We investigate decay modes of spin-1 heavy vector bosons ($V'$) from the viewpoint of perturbative unitarity in a model-independent manner. Perturbative unitarity requires some relations among couplings. The relations are called unitarity sum rules. We derive the unitarity sum rules from processes that contain two fermions and two gauge bosons. We find the relations between $V'$ couplings to the SM fermions ($f$) and $V'$ couplings to the SM gauge bosons ($V$). Using the coupling relations, we calculate partial decay widths for $V'$ decays into $VV$ and $ff$. We show that $\mathrm{Br}(W' \to WZ) \lesssim 2\%$ in the system that contains $V'$ and CP-even scalars as well as the SM particles. This result is independent of the number of the CP-even scalars. We also show that contributions of CP-odd scalars help to make $\mathrm{Br}(W' \to WZ)$ larger than $\mathrm{Br}(W' \to ff)$ as long as the CP-odd scalars couple to both the SM fermions and the SM gauge bosons. The existence of the CP-odd scalar couplings is a useful guideline to construct models that predict $\mathrm{Br}(W' \to WZ) \gtrsim 2\%$. Our analysis relies only on the perturbative unitarity of $f\bar{f} \to WW'$. Therefore our result can be applied to various models.

1. Introduction

The ATLAS and the CMS experiments at the Large Hadron Collider (LHC) are capable of discovering physics beyond the standard model (BSM) around the TeV scale. In 2012, the LHC successfully discovered a scalar particle \cite{Higgs2012} that is consistent with the Higgs boson predicted in the standard model (SM). The next target of the LHC is particles predicted in BSM models.

Spin-1 heavy vector particles ($V'$) are popular particles predicted in BSM models such as composite Higgs models \cite{composite1-composite8}. An efficient way to study $V'$ phenomenology at the LHC is to use effective Lagrangians. For example, the effective Lagrangian given in Refs. \cite{effective1-effective10} provides a simple framework for $V'$ phenomenology at the LHC, and the framework is used in the analysis of ATLAS and CMS \cite{ATLAS-CMS1-ATLAS-CMS16}. On the other hand, the effective Lagrangian violates perturbative
unitarity at higher energy scales in general. If the unitarity violation scale is as low as the TeV scale, we have to take into account a lot of higher-dimensional operators. The higher dimensional operators accompany unknown coefficients, which make it difficult to give a definite prediction.

We can avoid the perturbative unitarity violation if the Lagrangian is renormalizable. The effective Lagrangian given in Refs. [9, 10] includes a renormalizable model called HVT model A in special regions of parameter space. This model predicts that \( V' \) particles mainly decay into two SM fermions \((f)\) and the decay mode into two SM gauge bosons \((V)\) is much smaller: \( \text{Br}(V' \rightarrow VV) \ll \text{Br}(V' \rightarrow ff) \). However, we do not know the main decay mode of \( V' \) in advance of the discovery. \( V' \) could be discovered in the \( VV \) decay mode in the future. It is thus important to prepare for discovery in any decay mode, and thus we should prepare other Lagrangians that predict \( \text{Br}(V' \rightarrow VV) \geq \text{Br}(V' \rightarrow ff) \) without violating perturbative unitarity.

In this paper, we investigate decay modes of spin-1 heavy vector bosons from the viewpoint of perturbative unitarity in a model-independent manner. Our purpose is to figure out the conditions under which \( \text{Br}(V' \rightarrow VV) \geq \text{Br}(V' \rightarrow ff) \) without relying on specific models. To this purpose, we need to know the \( V' \) couplings to the gauge bosons and to the fermions. We can find coupling relations by imposing perturbative unitarity on scattering amplitudes that contain both the gauge bosons and the fermions. These relations are called unitarity sum rules. Using the unitarity sum rules, we can obtain the coupling relations to calculate \( \text{Br}(V' \rightarrow VV) \) and \( \text{Br}(V' \rightarrow ff) \). The unitarity sum rules depend on the matter contents of the system, and thus we can understand what kinds of matter contents and interactions can make \( \text{Br}(V' \rightarrow VV) \) larger than \( \text{Br}(V' \rightarrow ff) \) from the unitarity sum rules.

The rest of this paper is organized as follows: In Sec. 2, we derive the unitarity sum rules for the process where two fermions are in the initial state and two charged gauge bosons are in the final state. The amplitudes contain terms that are proportional to energy and energy squared in the high-energy limit. We require that these terms vanish, and obtain the unitarity sum rules. In Sec. 3, we analyze a system that contains \( V' \) and an arbitrary number of CP-even scalars as well as the SM particles. This system includes HVT model A. Using the sum rules, we investigate the ratio of the two branching ratios, \( \text{Br}(W' \rightarrow WZ) \) and \( \text{Br}(W' \rightarrow ff) \). We show that the system predicts \( \text{Br}(W' \rightarrow WZ) \lesssim 2\% \) without depending on the number of CP-even scalars. We need other particles to make \( \text{Br}(W' \rightarrow WZ) \) larger than \( \text{Br}(W' \rightarrow ff) \) without violating perturbative unitarity. In Sec. 4, we add CP-odd scalar bosons to the system, and show that the contributions of the CP-odd scalars significantly modify the relation between the two branching ratios. We summarize our discussion in Sec. 5.
2. Unitarity sum rules for \( f_1 f_2 \to V_3^- V_4^+ \)

In this section, we calculate the scattering amplitude of \( f_1 f_2 \to V_3^- V_4^+ \) denoted by \( iM_{jk} \), where \( j \) and \( k \) are twice the helicity of \( f_1 \) and \( f_2 \), respectively. We include all the SM fermions, the SM gauge bosons \((W^\pm, Z, \gamma)\), new heavy vector bosons \((W'^\pm, Z')\), CP-even scalars \((h's)\), and CP-odd scalars \((\Delta^0's)\) in our analysis. We assume CP conservation in the scalar sector.

We begin by listing all the relevant interaction terms. The quark couplings to the gauge bosons are defined by

\[
\mathcal{L}_{f f V} = - \sum_{V=W, W'} \bar{u}_i \gamma^\mu V^\mu_i + \left( \frac{1}{\sqrt{2}} g_{U, d, V^+} P_L \right) d_j - \sum_{V=W, W'} \bar{d}_j \gamma^\mu V^\mu_j - \left( \frac{1}{\sqrt{2}} (g_{U, d, V^+})^* P_L \right) u_i
\]

\[
- \sum_{V=Z, Z', \gamma} \bar{u}_i \gamma^\mu V^\mu_i (g_{U, u, V^0} P_L + g_{U, u, V^0} P_R) u_j
\]

\[
- \sum_{V=Z, Z', \gamma} \bar{d}_j \gamma^\mu V^\mu_j (g_{D, d, V^0} P_L + g_{D, d, V^0} P_R) d_j,
\]

where \( g_{U, u, V^0} = (g_{U, u, V^0})^* \), and \( g_{D, d, V^0} = (g_{D, d, V^0})^* \) to keep Hermiticity of the Lagrangian. The quark couplings to the neutral scalar bosons are

\[
\mathcal{L}_{f f S} = - \sum_h \bar{h} u_i (g_{U, u, h}) u_j - \sum_h \bar{h} d_j (g_{D, d, h}) d_j
\]

\[
+ i \sum_{\Delta^0} \bar{u}_j (g_{U, u, \Delta^0}) u_k + i \sum_{\Delta^0} \bar{d}_j (g_{D, d, \Delta^0}) d_k.
\]

The couplings in the lepton sector are defined similarly to Eqs. (2.1) and (2.2). The CP-even scalar couplings to the charged gauge bosons are

\[
\mathcal{L}_{VVh} = \sum_h g_{W W h} h W^\mu_- W^{-\mu} + \sum_h g_{W' W' h} h W'^{\mu+} W'^{-\mu} + \sum_h g_{W W' h} (h W^\mu_- W'^{\mu+} + h W'^{\mu+} W^{-\mu}),
\]

where all the couplings are real. The CP-odd scalar couplings to the gauge bosons are

\[
\mathcal{L}_{VV\Delta} = i \sum_{\Delta^0} g_{W W' \Delta^0} W^\mu_- W'^{\mu+} \Delta^0 - i \sum_{\Delta^0} g_{W W' \Delta^0} W'^{\mu+} W^{-\mu} \Delta^0.
\]

The structure of the triple gauge boson couplings are the same as in the SM case,

\[
\mathcal{L}_{gauge} = \sum_{V=Z, Z', W} \sum_{\Delta^0} \Delta^0 \left( \partial_{\mu} V_{\mu}^- - \partial_{\nu} V_{\nu}^- \right) V^{\mu+} V^{\mu-}
\]

\[
+ \left( \partial_{\mu} V_{\mu}^+ - \partial_{\nu} V_{\nu}^+ \right) V^{\mu+} V^{\mu-} + \left( \partial_{\mu} V_{\mu}^- - \partial_{\nu} V_{\nu}^- \right) V^{\mu+} V^{\mu+}.
\]
The Feynman rules for the triple gauge bosons are obtained by replacing the coupling in the SM appropriately such as $g_{WWZ}^{SM} \to g_{WWZ}^\prime$, $g_{WWW}^{SM} \to g_{WWW}^\prime$.

Using the above interaction terms, we calculate the amplitude of $f_1 f_2 \to V_3^- V_4^+$ in the high-energy limit. We find that

\[
M_{--} = \frac{s}{2m_V^2 m_{V_4}} A \sin \theta + \mathcal{O}(s^0),
\]

\[
M_{++} = \frac{s}{2m_V^2 m_{V_4}} B \sin \theta + \mathcal{O}(s^0),
\]

\[
M_{+-} = \frac{\sqrt{s}}{2m_V^2 m_{V_4}} \left( C^{(0)} + C^{(1)} \cos \theta + \mathcal{O}(s^0) \right),
\]

\[
M_{-+} = \frac{\sqrt{s}}{2m_V^2 m_{V_4}} \left( D^{(0)} + D^{(1)} \cos \theta + \mathcal{O}(s^0) \right),
\]

where $\theta$ is the angle between the two momenta of $f_1$ and $V_3^-$, $\sqrt{s}$ is the center-of-mass energy, and

\[
A = -\frac{1}{2} \sum_F g_{f_1 V_3}^L (g_{f_2 V_4}^L)^* + \frac{1}{2} \sum_F (g_{f_1 V_4}^L)^* g_{f_2 V_3}^L - \sum_V g_{V_3^- V_4^+} g_{f_2 f_1 V}^L,
\]

\[
B = -\sum_V g_{V_3^- V_4^+} g_{f_2 f_1 V}^R,
\]

\[
C^{(0)} = -\frac{1}{2} \sum_F g_{f_1 V_3}^L (g_{f_2 V_4}^L)^* m_{f_1} - \frac{1}{2} \sum_F (g_{f_1 V_4}^L)^* g_{f_2 V_3}^L m_{f_1}

- \sum_{V'} \frac{m_{V_3^+}^2 - m_{V_4^-}^2}{m_{V'}^2} g_{V_3^- V_4^+} (g_{f_2 f_1 V}^L m_{f_1} - g_{f_2 f_1 V}^R m_{f_2})

+ \sum_{h} g_{V_3 V_4} g_{f_2 f_1 h}

+ \sum_{\Delta^0} g_{f_2 f_1 \Delta^0} g_{V_3 V_4 \Delta^0},
\]

\[
D^{(0)} = +\frac{1}{2} \sum_F g_{f_1 V_3}^L (g_{f_2 V_4}^L)^* m_{f_2} + \frac{1}{2} \sum_F (g_{f_1 V_4}^L)^* g_{f_2 V_3}^L m_{f_2}

+ \sum_{V'} \frac{m_{V_3^+}^2 - m_{V_4^-}^2}{m_{V'}^2} g_{V_3^- V_4^+} (g_{f_2 f_1 V}^R m_{f_1} - g_{f_2 f_1 V}^L m_{f_2})

- \sum_{h} g_{V_3 V_4} g_{f_1 f_2 h}

+ \sum_{\Delta^0} g_{f_1 f_2 \Delta^0} g_{V_3 V_4 \Delta^0},
\]

\[
C^{(1)} = m_{f_1} A + m_{f_2} B,
\]

\[
D^{(1)} = -m_{f_2} A - m_{f_1} B.
\]

We obtain unitarity sum rules by imposing $A = B = C^{(0)} = D^{(0)} = 0$. This condition is automatically satisfied in renormalizable models. Effective $V'$ models with a sufficiently high cutoff
scale also satisfy this condition within a good approximation. If $A = B = 0$, then $C^{(1)}$ and $D^{(1)}$ are automatically equal to zero. The sum rules from $C^{(0)}$ and $D^{(0)}$ contain both the CP-even and CP-odd couplings. We can separate these couplings by taking linear combinations, $C^{(0)} \pm D^{(0)}$. We finally obtain the following four independent unitarity sum rules.

\[
\sum_V g_{V_4^+V_4^-}^V g_{f_2f_1V}^L = -\frac{1}{2} \sum_F g_{F_1V_3^+}^L (g_{F_2V_4^+}^L)^* + \frac{1}{2} \sum_F (g_{f_1F_2V_4^+}^L)^* g_{f_2F_3V_4^+}^L, \quad (2.16)
\]

\[
\sum_V g_{V_4^+V_4^-}^V g_{f_2f_1V}^R = 0, \quad (2.17)
\]

\[
\sum_{\Delta^0} g_{f_2f_1D^0}^V g_{V_3V_4\Delta^0}^V = +\frac{m_{f_1} - m_{f_2}}{4} \sum_F \left( g_{F_1V_3^+}^L (g_{F_2V_4^+}^L)^* + (g_{f_1F_2V_4^+}^L)^* g_{f_2F_3V_4^+}^L \right)
+ \frac{m_{f_1} + m_{f_2}}{2} \sum_V \frac{m_{V_4^+}^2 - m_{V_4^-}^2}{m_{V_4^+}^2} g_{V_4^+V_4^-}^V (g_{f_2f_1V}^L - g_{f_2f_1V}^R), \quad (2.18)
\]

\[
\sum_{h} g_{V_3V_4h}^V g_{f_2f_1h} = +\frac{m_{f_1} + m_{f_2}}{4} \sum_F \left( g_{F_1V_3^+}^L (g_{F_2V_4^+}^L)^* + (g_{f_1F_2V_4^+}^L)^* g_{f_2F_3V_4^+}^L \right)
+ \frac{m_{f_1} - m_{f_2}}{2} \sum_V \frac{m_{V_4^+}^2 - m_{V_4^-}^2}{m_{V_4^+}^2} g_{V_4^+V_4^-}^V (g_{f_2f_1V}^L + g_{f_2f_1V}^R). \quad (2.19)
\]

In the following sections, we apply these sum rules to two simple setups and discuss the relation between $\Gamma(W' \to WZ)$ and $\Gamma(W' \to ff)$.

### 3. SM + $V'$ + CP-even scalars

In this section, we apply the unitarity sum rules we found in the previous section to the following simple setup. We consider SU(2)$_0 \times$SU(2)$_1 \times$U(1)$_2$ electroweak gauge symmetry. Left-handed fermions are SU(2)$_1$ doublets. Right-handed fermions are singlet under both SU(2)$_0$ and SU(2)$_1$. Both the left- and right-handed fermions have appropriate U(1)$_2$ charge. This charge assignment implies that the charged gauge bosons do not couple to the right-handed currents. All scalars are CP even. We do not include CP odd scalars in the setup here. We do not specify the number of the CP-even scalars. For simplicity, we assume the minimal flavor violation (MFV) [17], namely all the flavor-changing structures are embedded in the CKM matrix. This setup contains HVT model A [9, 10]. Thanks to these assumptions, the couplings in this setup are simplified as follows:

\[
g_{\bar{u}_i d_j V^+}^L = V_{ij}^{ij}^L g_V, \quad g_{\bar{u}_i \ell_j V^+}^L = \delta_{ij} g_V, \quad (3.1)
\]

\[
g_{\bar{u}_i u_j V^0}^L = \delta_{ij} g_{\bar{u}_i u_j V^0}, \quad g_{\bar{d}_i d_j V^0}^L = \delta_{ij} g_{\bar{d}_i d_j V^0}, \quad g_{\bar{d}_i \ell_j V^0}^L = \delta_{ij} g_{\bar{d}_i \ell_j V^0}, \quad (3.2)
\]

\[
g_{\bar{u}_i u_j h} = \delta_{ij} g_{\bar{u}_i u_j h}, \quad g_{\bar{d}_i d_j h} = \delta_{ij} g_{\bar{d}_i d_j h}, \quad g_{\bar{d}_i \ell_j h} = \delta_{ij} g_{\bar{d}_i \ell_j h}. \quad (3.3)
\]
Using these couplings, we can simplify the \( u\bar{u} \to V_3^- V_4^+ \) unitarity sum rules as follows.

\[
\frac{1}{2}g_{V_3}g_{V_4} = \sum_{V=\gamma,Z,Z'} g_{V_3}^L V_{V_4}^V - g_{uuV}, \quad (3.4)
\]

\[
0 = \sum_{V=\gamma,Z,Z'} g_{V_3}^L V_{V_4}^V g_{uuV}, \quad (3.5)
\]

\[
g_{V_3}g_{V_4} = 2 \sum_h g_{V_3}^h g_{uuh}, \quad (3.6)
\]

\[
0 = \sum_{V=Z,Z'} \frac{m^2_{V_3} - m^2_{V_4}}{m_V} g_{V_3}^L V_{V_4}^V (g_{uuV}^L - g_{uuV}^R). \quad (3.7)
\]

These unitarity sum rules are sufficient to discuss the ratio of \( \Gamma(W' \to W Z) \) to \( \Gamma(W' \to u_d \bar{d}_j) \). By combining Eqs. (3.4), (3.5), and (3.7), and taking \( V_3 = W \) and \( V_4 = W' \), we find

\[
\frac{1}{2}g_W g_{W'} = g_{WW'} (g_{uuZ}^L - g_{uuZ}^R) + g_{WW'} (g_{uuZ'}^L - g_{uuZ'}^R), \quad (3.8)
\]

\[
g_{WW'} (g_{uuZ}^L - g_{uuZ}^R) = - \frac{m^2_{Z'}}{m^2_Z} g_{WW'} (g_{uuZ}^L - g_{uuZ}^R). \quad (3.9)
\]

Combining these unitarity sum rules, we can erase \( g_{WW',Z'} \) and obtain

\[
g_{WW'} = - \frac{m^2_{Z'}}{m^2_Z} \frac{g_W g_{W'}}{2(g_{uuZ}^L - g_{uuZ}^R)} \frac{1}{1 - \frac{m^2_{Z'}}{m^2_{Z'}}}. \quad (3.10)
\]

In general, \( g_W, g_{uuZ}^L, \) and \( g_{uuZ}^R \) are different from the SM prediction but should become the same as in the SM in the decoupling limit \( (m_{W',Z'} \to \infty) \). Thus, the relations among \( g_W, g_{uuZ}^L, g_{uuZ}^R, m_W, \) and \( m_Z \) are the same as in the SM at the leading order in the large-\( m_{W',Z'} \) limit. We find

\[
g_{WW',Z'} \simeq \frac{m_W m_Z}{m^2_{Z'}} g_{W'}. \quad (3.11)
\]

In a similar manner, we obtain the \( \ell \bar{\ell} \to W^- W'^+ \) perturbative unitarity sum rules and we find the following relation.

\[
g_{WW',Z'} \simeq \frac{m_W m_Z}{m^2_{Z'}} g_{W'}. \quad (3.12)
\]

By comparing this equation with Eq. (3.11), we find \( g_{W'}^L \simeq g_{W'} \). Thus, the relation given in Eq. (3.11) is flavor independent. The partial widths for the \( W' \) decays into \( WZ \) and \( \ell \bar{\ell} \) are given by

\[
\Gamma(W' \to W Z) \simeq \frac{1}{192\pi} \frac{m^5_{W'}}{m^4_W m^2_Z} g_{WW',Z'}, \quad (3.13)
\]

\[
\Gamma(W' \to u_d \bar{d}_j) \simeq \frac{1}{16\pi} |V^{-ij}_{CKM}|^2 m^2_{W'} g^2_{W'}, \quad (3.14)
\]

\[
\Gamma(W' \to \ell \nu) \simeq \frac{1}{48\pi} m^2_{W'} g^2_{W'}. \quad (3.15)
\]
where the terms of order $m^2_{W,Z,f}/m^2_W$ have been neglected. From these equations, we find

$$
\frac{\Gamma(W' \rightarrow WZ)}{\Gamma(W' \rightarrow f_i f_j)} \simeq \frac{1}{4c_{ij}} \frac{m^4_W}{m^4_{Z'}} \tag{3.16}
$$

where

$$
c_{ij} = \begin{cases} 
N_c |V_{CKM}^{ij}|^2 & \text{(for quarks)} \\
\delta_{ij} & \text{(for leptons)}
\end{cases}
$$

(3.17)

where $N_c = 3$. Here we assume $g_{W'} \neq 0$. The case where $g_{W'} = 0$ is discussed in the end of this section. The mass difference between $W'$ and $Z'$ depends on the amount of custodial SU(2) symmetry violation due to the $U(1)_2$ gauge coupling and Yukawa couplings. We estimate that

$$
|m^2_{Z'} - m^2_{W'}| \simeq (g_2^2 \text{ and/or } y^2)v^2 \ll m^2_{W,Z'},
$$

(3.18)

Therefore, Eq. (3.16) is simplified as follows.

$$
\frac{\Gamma(W' \rightarrow WZ)}{\Gamma(W' \rightarrow f_i f_j)} \simeq \frac{1}{4c_{ij}}. \tag{3.19}
$$

The important consequence of Eq. (3.19) is the suppression of $\text{Br}(W' \rightarrow WZ)$. Since there are three generations in both the lepton and quark sectors,

$$
\frac{\Gamma(W' \rightarrow WZ)}{\sum_f \Gamma(W' \rightarrow f_i f_j)} \simeq \frac{1}{4(N_c + 1) \times 3} = \frac{1}{48}.
$$

(3.20)

where we use $\sum_{i,j} |V_{CKM}^{ij}|^2 = 3$. This equation implies that $\text{Br}(W' \rightarrow WZ) \lesssim 2\%$. If we assume the equivalent theorem relation $\Gamma(W' \rightarrow WZ) \simeq \Gamma(W' \rightarrow Wh)$ and the $W'$ decay to $W$, and the heavy neutral scalar is highly suppressed, we find $\text{Br}(W' \rightarrow WZ) \simeq 2\%$ and $\text{Br}(W' \rightarrow e\nu) \simeq 8\%$ [$\sum_f \text{Br}(W' \rightarrow f f) \simeq 96\%$]. The assumption is justified in the case where $g_{WWZ} \simeq g_{WWh}^{\text{SM}}$ [18]. Therefore, the branching ratio of $W'$ to the gauge bosons is much smaller than its ratio to the fermions. This result has been derived from the $f \bar{f} \rightarrow WW'$ unitarity sum rules and does not need perturbative unitarity of other processes such as $WW \rightarrow WW$. In addition, the result does not depend on the number of CP even scalars. Therefore our result in this section can be applied to various models.

If $\text{Br}(W' \rightarrow WZ)$ is measured in the future and if it is larger than $2\%$, it is implied that the perturbative unitarity of $f \bar{f} \rightarrow WW'$ is violated or that other new particles in addition to $W', Z'$, and CP-even scalar bosons exist.

We briefly discuss the case where $g_{W'} = 0$. In that case, $g_{WW'Z} = 0$, as we can see from Eq. (3.10). Therefore $W'$ decouples from the SM sector if $g_{W'} = 0$. 

7
4. SM + $V'$ + CP-even scalars + CP-odd scalars

In this section, we extend the analysis in the previous section by introducing CP-odd scalars, and show that $\text{Br}(W' \to WZ)$ can become larger than $\text{Br}(W' \to f f)$. The CP-odd scalar couplings are given in Eqs. (2.2) and (2.4). As in the previous section, we assume MFV and CP conservation in the scalar sector. Under these assumptions, all the CP-odd scalar couplings are simplified as follows:

$$
\begin{align*}
  g_{\bar{u}u_i \Delta^0} &= + \frac{1}{2} g_{\bar{u}u_i \Delta^0} \delta^{ij}, \\
  g_{d_i d_j \Delta^0} &= - \frac{1}{2} g_{d_i d_j \Delta^0} \delta^{ij}, \\
  g_{\ell_i \ell_j \Delta^0} &= - \frac{1}{2} g_{\ell_i \ell_j \Delta^0} \delta^{ij}.
\end{align*}
$$

(4.1)

We focus on the amplitude for $\bar{u}u \to W^- W'^+$ and obtain the same sum rules given in Eqs. (3.4)–(3.6) again. The unitarity sum rule in Eq. (3.7) is modified by the contributions of the CP-odd scalars. Instead of Eq. (3.7), we obtain the following unitarity sum rule.

$$
\sum_{\Delta^0} \frac{g_{\bar{u}u \Delta^0} g_{V_3 V_4 \Delta^0}}{m_u} = \sum_{V=Z,Z'} 2 g_{V_3 V_4} \frac{m^2_{V_3} - m^2_{V_4}}{m^2_V} (g_{L \bar{u}u Z} - g_{R \bar{u}u Z}).
$$

(4.2)

The difference of this equation from Eq. (3.7) is that the left-hand side can be nonzero because of the contributions of the CP-odd scalars. This is the only difference of the sum rules in this section from the previous section. This difference can make $\Gamma(W' \to WZ)$ change drastically, as we will see in the following. Using Eqs. (3.4), (3.5), and (4.2), we find that

$$
g_{WW'Z} \simeq - \frac{m_W m_Z}{m^2_Z'} (g_{W'} + x_{\Delta}), \quad \text{where} \quad x_{\Delta} = \sum_{\Delta} \frac{g_{\bar{u}u \Delta} g_{WW' \Delta}}{g_W}.
$$

(4.3)

Here we have neglected the terms of order $m^2_{W,Z,f}/m^2_{W'}$, and we have estimated that $m_{W'} \simeq m_{Z'}$, as we did in Eq. (3.18).

We have two comments on Eq. (4.3): First, this equation is independent of the quark flavor, although $x_{\Delta}$ looks quark-flavor dependent. This is because $x_{\Delta} \simeq -g_{W'} - g_{WW'Z} m^2_{Z'}/m_W m_Z$ and the right-hand side of this equation is independent of quark flavor. Second, Eq. (4.3) implies that $g_{WW'Z} \neq 0$ even if $g_{W'} = 0$ as long as the CP-odd couplings exist. In the case where $g_{W'} = 0$ and $g_{WW'Z} \neq 0$, the $W'$ decay to $WZ$ can be the dominant decay mode. This is a big difference of the current setup from the setup discussed in Sec. 3.

We find similar unitarity sum rules from the amplitude for $\ell \bar{\ell} \to W^- W'^+$. The sum rule that corresponds to Eq. (4.3) is given by

$$
g_{WW'Z} \simeq - \frac{m_W m_Z}{m^2_Z'} (g_{W'} + x'_{\Delta}), \quad \text{where} \quad x'_{\Delta} = \sum_{\Delta} \frac{g_{\ell \ell \Delta} g_{WW' \Delta}}{m_{\ell \ell}}
$$

(4.4)
Unlike the setup discussed in Sec. 3, we cannot conclude that $g_{W'} \simeq g_{W'}$ in this setup.

Using Eqs. (4.3) and (4.4), we find

$$\frac{\Gamma(W' \rightarrow WZ)}{\Gamma(W' \rightarrow \nu, \bar{\nu})} \simeq \frac{(g_{W'} + x_\Delta)^2}{4N_c |V^\nu_{CKM}|^2 g_{W'}^2},$$

$$\frac{\Gamma(W' \rightarrow WZ)}{\Gamma(W' \rightarrow \ell \nu)} \simeq \frac{(g_{W'} + x_\Delta')^2}{4(g_{W'})^2},$$

(4.5)

(4.6)

$$\frac{\Gamma(W' \rightarrow WZ)}{\sum \Gamma(W' \rightarrow f_i f_j)} \simeq \left( \frac{3}{4N_c (1 + \frac{x_\Delta}{g_{W'}})^2} + 4 \frac{3}{(1 + \frac{x_\Delta'}{g_{W'}})^2} \right)^{-1} \left( \frac{3}{4N_c (1 + \frac{x_\Delta}{g_{W'}})^2} + 4 \frac{3}{(1 + \frac{x_\Delta'}{g_{W'}})^2} \right)^{-1},$$

(4.7)

where the terms of order $m_{W, Z, f}^2 / m_{W'}^2$ have been neglected. The factor 3 in Eq. (4.7) is the number of the generation. We use $\sum_{i,j} |V^\nu_{CKM}|^2 = 3$.

We find that $g_{W'WZ}$ and $\Gamma(W' \rightarrow WZ)$ depend on $g_{W'}$, $x_\Delta$, and $x_\Delta'$. This dependence is a different feature from Eqs. (3.19) and (3.20). If the CP-odd scalars are absent, then the ratio of the two partial decay widths is uniquely determined, as we have discussed in Sec. 3; see Eq. (3.20). On the other hand, in the system with the CP-odd scalars, the ratio of the two partial decay widths is controlled by $g_{W'}$, $x_\Delta$, and $x_\Delta'$, which are model-dependent parameters controlled by the CP-odd scalar couplings. Thanks to this feature, $\Gamma(W' \rightarrow WZ)$ can be comparable to or even larger than the other decay modes. For example, $\Gamma(W' \rightarrow WZ) \simeq \Gamma(W' \rightarrow \ell \nu)$ if $x_\Delta \simeq g_{W'}$ or $\simeq -3g_{W'}$. $\Gamma(W' \rightarrow WZ)$ is larger than $\Gamma(W' \rightarrow ff)$ in the large-$|x_\Delta / g_{W'}|$ regime. $\Gamma(W' \rightarrow WZ)$ also can become small and even vanish for $x_\Delta \simeq x_\Delta' \simeq -g_{W'}$. In any case, the contributions of the CP-odd scalars significantly change the ratio of $\Gamma(W' \rightarrow WZ)$ to $\Gamma(W' \rightarrow ff)$ from the prediction without the CP-odd scalars. In particular, it is an important feature that $W' \rightarrow ff$ is highly suppressed and $W' \rightarrow WZ$ can be the dominant decay mode in this setup with large $|x_\Delta / g_{W'}|$. This is a very different feature from the setup in Sec. 3.

We estimate the maximum value of $\text{Br}(W' \rightarrow \ell \nu)$ as follows:

$$\text{Br}(W' \rightarrow \ell \nu) = \frac{\Gamma(W' \rightarrow \ell \nu)}{\Gamma(W' \rightarrow WZ) + \sum f \Gamma(W' \rightarrow ff) + \sum X \Gamma(W' \rightarrow X)} \leq \frac{\Gamma(W' \rightarrow \ell \nu)}{\Gamma(W' \rightarrow WZ) + \sum f \Gamma(W' \rightarrow ff)} \simeq \frac{4}{(1 + \frac{x_\Delta}{g_{W'}})^2} \left( 1 + \frac{36}{(1 + \frac{x_\Delta}{g_{W'}})^2} + \frac{12}{(1 + \frac{x_\Delta'}{g_{W'}})^2} \right)^{-1} \equiv \text{Br}_{\text{max}}(W' \rightarrow \ell \nu),$$

(4.8)
Figure 1: $\text{Br}_{\text{max}}(W' \rightarrow e\nu)$ as a function of $x_\Delta/g_{W'}$. We can easily see the two extreme cases. One is at the $x_\Delta \to 0$ limit where we can see $\text{Br}_{\text{max}}(W' \rightarrow e\nu) \simeq 8\%$. This is consistent with the result in Sec. 3. The other case is at the $g_{W'} \to 0$ limit where $\text{Br}(W' \rightarrow f f) = 0$ and $W'$ to $W Z$ can be the main decay mode. This is again consistent with our discussion below Eq. (4.3).
The existence of the CP-odd scalar couplings is important to the increase of $\text{Br}(W' \to WZ)$, because $x_\Delta$ and $x_\Delta^f$ can be zero if the CP-odd scalars do not couple to the fermions or to the gauge bosons, as can be seen from Eqs. (4.3) and (4.4). Both $g_{f\bar{f}\Delta}$ and $g_{WW'\Delta^0}$ need to be nonzero in order to make $\text{Br}(W' \to WZ)$ larger than 2% by the effect of the CP-odd scalars. To obtain a nonzero $g_{f\bar{f}\Delta}$, the scalar fields in the Yukawa terms need to contain CP-odd scalars. For nonvanishing $g_{WW'\Delta^0}$, the CP-odd scalars have to be components of scalar fields that develop vacuum expectation values (VEVs), because $g_{WW'\Delta^0}$ originates from kinetic terms of the scalar fields. These two conditions are useful guidelines to construct models that predict $\text{Br}(W' \to WZ) \gtrsim 2\%$.

Let us discuss how to construct models that predict $\text{Br}(W' \to WZ) \gtrsim 2\%$ by extending HVT model A. It contains two scalar fields $H$ and $\Phi$, which are $(2, 1)_{1/2}$ and $(2, 2)_0$ under $\text{SU}(2)_0 \times \text{SU}(2)_1 \times \text{U}(1)_2$, respectively. Since all the CP-odd scalars are eaten by the gauge bosons, we need to add other scalar fields to increase $\Gamma(W' \to WZ)$. New scalar fields should not have the same gauge charge as $H$ and $\Phi$. If there is more than one scalar field with the same gauge charge, we can redefine the scalar fields by taking their linear combination of them and go to a basis where only one of the scalars develops a VEV. This is equivalent to adding scalars that do not develop VEVs, and thus $g_{WW'\Delta^0} = 0$. A simple choice for obtaining a nonzero $g_{WW'\Delta^0}$ is to add a scalar field that is $(1, 2)_{1/2}$. The model with this choice is discussed in Refs. [19, 20], and it certainly predicts a large $\text{Br}(W' \to WZ)$ with appropriate parameter choices. Another possible choice is to add scalar fields that are representations of SU(2)$_0$ and/or SU(2)$_1$ larger than 2, for example, $(3, 3)_0$. However, such scalar fields break the custodial symmetry by their VEVs in many cases, and thus constraints should be studied carefully.

In this section, we have shown that the CP-odd scalars can drastically change the ratio of $\Gamma(W' \to WZ)$ to $\Gamma(W' \to ff)$. This result is the major difference from the result shown in Sec. 3. The difference originates from Eq. (2.18). The difference is very simple from the viewpoint of the unitarity sum rules, although the phenomenological consequence drastically changes.

5. Conclusions

Spin-1 heavy vector bosons are popular particles predicted in models beyond the SM. They decay into various particles, such as two SM gauge bosons. In this paper, we have investigated the relation between two decay modes, $W' \to WZ$ and $W' \to ff$, from the viewpoint of perturbative unitarity. We have focused on the amplitudes of $f\bar{f} \to V^-V^+$, where $V = W, W'$, and required
that these processes do not have bad high-energy behavior at the tree level. This requirement relates the couplings in the amplitudes to each other. The coupling relations obtained from this requirement are called the unitarity sum rules, which we have shown in Sec. 2. Using the unitarity sum rules, we can investigate the ratio of $\Gamma(W' \to WZ)$ to $\Gamma(W' \to ff)$.

In Sec. 3, we have applied the unitarity sum rules to the system that contains spin-1 heavy vector bosons and CP-even scalars as well as all the SM particles. We have shown that $\Gamma(W' \to WZ) / \sum_f \Gamma(W' \to ff) \simeq 1/48$ where we sum over the contributions from three generations in both the quark and the lepton sectors. Using this result, we have shown that $\text{Br}(W' \to WZ) \lesssim 2\%$ in the system. This result has been derived by imposing perturbative unitarity only on $f \bar{f} \to W^-W'^+$. The same result is thus obtained even if perturbative unitarity is violated in other processes such as $WW \to WW$. The result is independent of the number of CP-even scalars. Moreover, the ratio of the two decay modes only depends on the color factor and $V_{CKM}$, and thus is independent of details of models. Hence we conclude that the result can be applied to various models. If $\text{Br}(W' \to WZ)$ is measured in the future and is larger than 2\%, then perturbative unitarity requires new particles in addition to $V'$ and CP-even scalars.

In Sec. 4, we have shown that CP-odd scalars help to increase $\text{Br}(W' \to WZ)$ if they couple to both the SM fermions and the SM gauge bosons. In contrast to the case without the CP-odd scalars, $\text{Br}(W' \to WZ)$ depends on the parameters that are determined by the CP-odd scalar couplings and can be much larger than 2\%. Depending on the couplings, the decay mode of $W'$ to the SM fermions is highly suppressed, and the decay mode of $W'$ to the gauge bosons can be dominant. This is a big difference of the models with CP-odd scalars from the models with only CP-even scalars. The measurement of the decay properties of $W'$ is thus important not only for understanding the property of $W'$ itself, but also for revealing the structure of the system containing $W'$. For example, we can estimate the strength of the CP-odd scalar couplings to the SM particles from the measurement of $\text{Br}(W' \to e\nu)$ before the discovery of the CP-odd scalars. The result is also useful for model building. For example, we can see that the CP-odd scalars must be components of scalar fields that develop vacuum expectation values in order to obtain large $\text{Br}(W' \to WZ)$, because the nonzero CP-odd couplings to $W$ and $W'$ are required for large $\text{Br}(W' \to WZ)$ and the couplings arise from the scalar kinetic terms.

The sum rule given in Eq. (2.18) plays a crucial role in the prediction of $\text{Br}(W' \to WZ)$. Only this sum rule contains the CP-odd scalar couplings among the sum rules in Eqs. (2.16)–(2.19). We have shown the importance of the CP-odd scalar couplings for $\text{Br}(W' \to WZ)$. $\text{Br}(W' \to WZ)$ can be larger than 2\% if the left-hand side of Eq. (2.18) does not vanish. The sum rule given in
Eq. (2.18) is also important in setups without the assumptions we made in this paper—namely, the minimal flavor violation and CP conservation in the scalar sector. The analysis without these assumptions is straightforward but tedious, because it increases the number of parameters. We leave this analysis for future work.

We have used the $f \bar{f} \rightarrow W^- W'^+$ unitarity sum rules in our analysis. Our result does not change even if perturbative unitarity is violated in other processes such as $WW \rightarrow WW$. Therefore, our result can be applied to various models.

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