DEVELOPMENT OF A TURBULENT OUTFLOW DURING ELECTRON-POSITRON MAGNETIC RECONNECTION

M. SWISDAK, YI-HSIN LIU, AND J. F. DRAKE

IREAP, University of Maryland, College Park, MD 20740-3511; swisdak@umd.edu

Received 2007 November 14; accepted 2008 March 4

ABSTRACT

The mass symmetry between the two species in electron-positron (pair) plasmas has interesting consequences for collisionless magnetic reconnection, because the Hall term, which plays a crucial role in supporting fast reconnection in electron-proton plasmas, vanishes. We perform kinetic simulations of pair reconnection in systems of various sizes, show that it remains fast, and identify the reason why this occurs. For sufficiently large systems a Weibel-like temperature anisotropy instability develops in the outflow from the X-point that causes the current layer to broaden and form a Petschek-like open outflow. We discuss the parameter regimes in which pair reconnection should be fast and the implications for astrophysical pair plasmas.

Subject headings: magnetic fields — methods: numerical — plasmas

1. INTRODUCTION

The energy density of magnetic fields can be significant, and even dominant, in certain astrophysical systems, and so the question as to which processes act as sinks and convert that energy into other forms naturally arises. Observations (including in situ measurements) show that magnetic reconnection, in which topological changes in an embedded field lead to increases in a plasma’s kinetic and thermal energy, plays such a role in the terrestrial magnetosphere, the solar wind, and the solar corona (Mozer et al. 2002; Phan et al. 2006; Masuda et al. 1994). The natural extension to other astrophysical locations has led to the consideration of the role of reconnection in accretion disks (Eardley & Lightman 1975; Miller & Stone 1997), gamma-ray bursts (Drenkhahn & Spruit 2002), and pulsar winds (Coroniti 1990; Michel 1994; Zenitani & Hoshino 2001). Unlike the heliosphere, positrons, rather than protons, can comprise the positively charged species in the plasmas of these systems. The equality of the electron and positron masses introduces a symmetry that can have a potentially large effect on the development and role of reconnection.

Parker and Sweet (Parker 1957; Sweet 1958) proposed the first model of electron-proton reconnection, a magnetohydrodynamic (MHD) formulation that attempted to explain the fast timescales (minutes) observed. The GEM Challenge demonstrated that inclusion of the Hall term is a sufficient requirement for fast reconnection, the question of whether it is a necessary requirement remains open. Although the GEM Challenge demonstrated that inclusion of the Hall term is a sufficient requirement for fast reconnection, the question of whether it is a necessary requirement remains open. Pair reconnection provides an excellent test because one effect of the mass symmetry is the disappearance of the Hall term from the governing equations. If it is necessary for fast reconnection, then pair plasmas will reconnect slowly, on Sweet-Parker timescales, and magnetic reconnection will be an inefficient mechanism for releasing the magnetic energy in pair plasmas.

Despite this interesting limit the simulation community has only recently begun to consider pair reconnection. Bessho & Bhattacharjee (2005, 2007) observed fast pair reconnection in a small system and attributed it to a localized resistivity-like effect caused by the off-diagonal components of the pressure tensor. Bessho & Bhattacharjee (2005, 2007) observed fast pair reconnection in a small system and attributed it to a localized resistivity-like effect caused by the off-diagonal components of the pressure tensor. Daughton & Karimabadi (2007) did a careful examination of pair reconnection in large systems and arrived at similar conclusions for the early, impulsive phase of reconnection. Over longer periods they suggested that generation of large numbers of magnetic islands served to localize the current layer and support fast reconnection. Zenitani & Hoshino (2001), Jaroschek et al. (2004), and Fujimoto & Hoshino (2001) explored other aspects of pair reconnection but, although they observed fast reconnection, did not advance any causal explanations.

We also find that fast reconnection occurs in pair plasmas. However, we reach different conclusions from Daughton & Karimabadi (2007) on the role of magnetic islands. In the initial phase of electron-positron reconnection many magnetic islands typically form, especially if the ratio of the initial width of the current layer to the length of the system is very small. These islands have a transient influence on the development of reconnection until they convect downstream. Secondary islands also form later in time, and in greater numbers than in electron-proton reconnection. But, we find that these islands do not seem to control the overall length of the current layer and therefore the rate of reconnection. Based on our simulations we instead propose that the outflow jet in large pair plasma systems opens as a result of turbulence. We show that as the system size grows the current layer lengthens, just as Hall-less Sweet-Parker theory predicts. Interestingly, the reconnection rate remains constant during this change for small systems because other parameters, primarily the speed of the outflow jet,
also vary. This cannot continue for arbitrarily large boxes because of the Alfvenic limitation on the outflow speed. Instead, once the system is big enough (and the current layer long enough) an electromagnetic pressure-anisotropy instability similar to the Weibel mode begins to grow in the outflow jet. The outflow jet becomes strongly turbulent and the outflow layer broadens downstream as proposed by Petschek (1964). We present a simple analytic argument that suggests that as a result of this turbulence the rate of reconnection should take on a constant value, independent of system size.

In § 2 of this paper we describe the details of our computations. Section 3 outlines the role of the Hall term and how it disappears in pair plasmas. We discuss the simulations in § 4 and the instability and its consequences in § 5. Finally, § 6 explores the implications of our findings.

2. COMPUTATIONAL DETAILS

We use p3d, a massively parallel particle-in-cell (PIC) code (Zeiler et al. 2002), to perform our simulations. As in other PIC codes we divide the computational domain into cells, track the electromagnetic fields only on the gridpoints, and allow the particles to move freely. During a time step the Boris algorithm advances the Lorentz equation of motion for each particle: the electric field \( \mathbf{E} \) provides an acceleration for half a time step, the magnetic field \( \mathbf{B} \) rotates the velocity vector, and \( \mathbf{E} \) accelerates for the second half-time step. An explicit trapezoidal-leapfrog method employing second-order spatial derivatives in Faraday’s and Ampere’s laws advances the fields in time. We use uniform, square computational cells that force \( \nabla \cdot \mathbf{B} = 0 \). However, our discretization scheme leads to a discrepancy between \( \nabla \cdot \mathbf{E} \) and \( 4\pi \rho \). To address this problem we use a multigrid algorithm to solve Poisson’s equation for a correction term to \( \mathbf{E} \). Our particles do not directly interact, and, in particular, no electron-positron annihilation occurs.

To help elucidate the underlying physics we write the code equations in normalized units. Masses are normalized to the ion mass \( m_i \), magnetic fields to the asymptotic value of the reversed field, and the density to the value at the center of the current sheet. Other normalizations derive from these: velocities to the Alfven speed \( v_A \), lengths to the ion inertial length \( c/\omega_p \), times to the inverse ion cyclotron frequency \( \Omega_i^{-1} \), and temperatures to \( m_i v_A^2 \). In electron-positron plasmas \( m_e/m_i = 1 \).

In our coordinate system the inflow and outflow for an X-point lie parallel to \( \hat{\mathbf{y}} \) and \( \hat{\mathbf{z}} \), respectively. The reconnection electric field is parallel to \( \hat{\mathbf{z}} \). In the simulations presented here we assume out-of-plane derivatives vanish, i.e., \( \partial/\partial z = 0 \); this choice eliminates any structure in the \( \hat{\mathbf{z}} \)-direction. The reasonableness of simulating pair plasmas with this restriction remains unknown. For electron-proton plasmas, a comparison of 2D and 3D reconnection simulations suggests that the qualitative features remain unchanged (Hesse et al. 2001). However for many instabilities that can have large effects on the out-of-plane structure, e.g., the drift-kink, the onset threshold varies with the mass ratio, and so further simulations are needed to explore their importance in the \( m_e/m_i = 1 \) limit.

The initial equilibrium consists of two Harris current sheets (Harris 1962) superimposed on a ambient population of uniform density. The reconnecting magnetic field is given by \( B_x/B_0 = \tanh[(y - L_y/4)/w_0] - \tanh[(y - 3L_y/4)/w_0] - 1 \), where \( w_0 \) and \( L_y \) denote the half-width of the current sheets and the box size in the \( \hat{\mathbf{y}} \)-direction, respectively. This configuration allows us to use fully periodic boundary conditions. Both species have the same uniform initial temperature, \( T_e = T_i = 0.25 \). Pressure balance uniquely determines the density profile, aside from a uniform background population that can have arbitrary density (here \( n_0 = 0.2 \)); in this equilibrium \( n(y = L_y/4) = n(y = 3L_y/4) = 1.2 \).

At \( t = 0 \) we perturb the magnetic field—\( B_y/B_0 \) varies with system size but is always \( < 0.07 \) to seed X-points at \( (x, y) = (L_x/4, 3L_y/4) \) and \( (3L_x/4, L_y/4) \).

We set the speed of light (in normalized units) to 5. The spatial resolution is such that there are \( >4 \) gridpoints per inertial length and \( \approx 1 \) per Debye length. The Courant condition determines the particle time step; we substep the advancement of the electromagnetic fields. A typical cell contains \( \approx 200 \) particles and our largest simulations follow \( >10^6 \) particles. All of our simulations conserve energy to better than 1 part in 200.

3. THE HALL TERM

The current theory of electron-proton reconnection emphasizes the role of the Hall term in the generalized Ohm’s law. By combining the collisionless fluid momentum equations for electrons and a singly charged positive ion species of mass \( m_i \) we can write

\[
(1 + \mu) \mathbf{E} = -\frac{1 + \mu}{c} \mathbf{v} \times \mathbf{B} + \frac{1 - \mu}{\rho} \mathbf{J} \times \mathbf{B} - \frac{1}{\rho} \nabla \cdot (\mathbf{P}_e - \mu \mathbf{P}_i) + \frac{m_e}{\rho} \frac{\partial \mathbf{J}}{\partial t} + \nabla \cdot \left( \mathbf{J} \mathbf{v} + \mathbf{v} \mathbf{J} - \frac{1}{\rho} \frac{1 - \mu}{1 + \mu} \mathbf{J} \right),
\]

(1)

where \( \mu = m_e/m_i \), \( \mathbf{v} = (v_e \mathbf{v}_e + m_i \mathbf{v}_i)/(m_e + m_i) \) is the velocity in the center of mass frame, \( \mathbf{P} \) is the pressure tensor, and we assume quasi-neutrality, \( n_e = n_i = n \). For electron-proton plasmas \( \mu \approx 0 \).

By scaling equation (1) one can show that the first term on the right-hand side dominates at large lengthscales. The MHD model, and hence Sweet-Parker theory, discards the remaining terms. The second term on the right-hand side is the Hall term; it becomes important for lengthscales equal to or smaller than \( d_e \). With the addition of the Hall term the principal wave mode on these scales changes from the Alfven wave of MHD (\( \omega \sim k \)) to the dispersive (\( \omega \sim k^2 \)) whistler wave. The localization of the current layer along the outflow direction due to the action of dispersive waves has been proposed as the mechanism that facilitates fast reconnection in electron-proton plasmas (Birn et al. 2001; Rogers et al. 2001). The terms proportional to the diagonal portion of the pressure tensor can be important for systems with large guide fields (since they are the source of kinetic Alfven waves), while the off-diagonal terms and final inertial term matter only on electron scales, \( d_e = c/\omega_p \), and likely play only a small role outside the immediate vicinity of the X-point and magnetic separatrices.

In pair plasmas \( \mu = 1 \) and equation (1) simplifies to the form derived in Bessho & Bhattacharjee (2005):

\[
\mathbf{E} = -\frac{1}{c} \mathbf{v} \times \mathbf{B} - \frac{1}{2 \rho} \nabla \cdot (\mathbf{P}_e - \mathbf{P}_i) + \frac{m_e}{2 \rho} \frac{\partial \mathbf{J}}{\partial t} + \nabla \cdot \left( \mathbf{J} \mathbf{v} + \mathbf{v} \mathbf{J} - \frac{1}{\rho} \frac{1 - \mu}{1 + \mu} \mathbf{J} \right).
\]

(2)

The mass symmetry eliminates the Hall term, and with it the usual dispersive modes, including the whistler. If we make the reasonable assumption that \( \mathbf{v} \) vanishes at a steady state X-point, equation (2) becomes

\[
\mathbf{E} = -\frac{1}{2 \rho} \nabla \cdot (\mathbf{P}_e - \mathbf{P}_i).
\]

(3)
Hence, a nonzero reconnection electric field (Ez) requires an asymmetry between the two species, Pe ≠ Pp. In our quasi—two-dimensional system the balance must come from the off-diagonal components of the pressure tensors, which describe the transport of momentum away from the X-point.

4. SIMULATIONS

4.1. Overview

Figure 1 shows the steady state behavior of a typical simulation of pair reconnection in a large domain. Figure 1a displays magnetic field lines superimposed on the component of the electron velocity in the 3-direction; the positron velocity (not shown) is similar. The solid and dashed portions of the field lines indicate positive and negative signs of Bz, the reconnecting component of the field. The tension in the highly bent field lines to the left and right of the X-point (x = 0, y = 0) accelerates the plasma downstream until it reaches the local Alfvén velocity (≈1 in our normalization). A more gradual inflow with speed ≈0.1 (not shown) replenishes the plasma.

Figure 1b shows the out-of-plane component of the magnetic field Bz. The quadrupolar Bz pattern that is one of the signatures of Hall reconnection is not present. Instead Bz becomes large and highly structured farther downstream from the X-point where the current layer opens up. We discuss the turbulence, the instability responsible for it, and its linkage to the structure of the outflow jet later.

Although Ez has a structure similar to Bz in the turbulent outflow, along the thin central portion of the current layer it exhibits a narrow bipolar feature. Figure 1c shows a vertical cut of Ez averaged over the region −50 < x < 50. The central structure has a width of ≈2de and breaks the symmetry between the electrons and positrons. It grows from the ambient noise in all of our simulations, regardless of box size, but the sign (whether electrons or positrons accelerate toward the current layer) appears to arise spontaneously. In this case electrons are accelerated toward, and positrons repelled from, the center, which leads to an asymmetry in the respective particle distribution functions. The bipolar signal appears much earlier in the simulations than the temperature anisotropy instability we discuss later, and it appears in runs for which this other instability is absent. Instead, the counterstreaming inflowing plasma likely triggers a two-stream instability at the current layer in which the polarity of Ez is determined randomly.

To maintain this field the species’ number densities exhibit a slight asymmetry (i.e., quasi-neutrality is slightly violated). Figure 1d displays the distribution of v∥ for the electrons (green line) and positrons (blue line) within the box |x| < 50, |y| < 2 and shows the effects of this field. The v∥ distributions in Figure 1e are mirror-symmetric due to the effect of Ez (the reconnection electric field) on the oppositely charged electrons and positrons. The v∥ distributions are the same for both species because Ez is small. They are plotted in black.

In some of our preliminary simulations the long initial current layers produced a larger number of islands that played a transient role before being swept away. We find that varying the thickness of the initial layer affects the initial rate of island production, with a broader layer producing fewer islands. At the time of Figure 1 these initial islands have all disappeared downstream. However, we also find that secondary islands self-generated on the current layer remain a common occurrence in pair reconnection and are much more prevalent than in reconnection with me ≪ mi (Shay et al. 2007). Two such islands (at x ≈ 20 and x ≈ −80) appear in Figure 1. These remnant islands, however, are not responsible...
for the turbulent fluctuations in $B_z$ in Figure 1. The island at $x \approx 20$, occurs in a region with no turbulence and has no $B_z$ signature while the other, at $x \approx -80$, lies in the turbulent outflow where any independent $B_z$ signature (if it exists) is difficult to distinguish.

4.2. Scaling of the Reconnection Rate

Fast reconnection ultimately arises from the localization of the current layer (Biskamp & Schwarz 2001). This can be seen from a straightforward analysis of the fluid continuity equation

$$\frac{\partial n}{\partial t} + \nabla \cdot (nv) = 0.$$  \hspace{1cm} (4)

In a steady state the time derivative vanishes, leaving a term that the divergence theorem transforms into a surface integral. For one quadrant of a rectangular domain with length (parallel to $\hat{x}$) $2\Delta$ and width (parallel to $\hat{y}$) $2\delta$ centered on the X-point, the surface integrals imply that

$$v_{in} = v_{out} \left( \frac{\delta}{\Delta} \right) \left( \frac{n_{out}}{n_{in}} \right),$$ \hspace{1cm} (5)

where the subscripts “in” and “out” refer to inflowing and outflowing plasmas, respectively. Both the outflow speed, which approaches the local Alfvén speed, and the density ratio $n_{out}/n_{in}$, which does not radically change from its initial value, are $O(1)$. If, as seems a reasonable assumption, the inflow scale $\delta$ equals an intrinsic kinetic scale length such as $d_e$ and remains roughly constant as the system size changes, then $v_{in}$ scales inversely with the outflow dimension $\Delta$. In particular, if, as in the Sweet-Parker picture, $\Delta$ scales with the size of the system, $v_{in}$ should decrease for larger and larger simulation domains. In electron-proton reconnection the Hall term keeps $\Delta$ fixed, regardless of the system size, and maintains a fast reconnection rate $v_{in}/v_{out} \sim \delta/\Delta \sim O(0.1)$ (Shay et al. 1999, 2007).

If the Hall term is the only mechanism that can localize the current layer then pair reconnection should feature current sheets that scale with the system size. In order to test this behavior we began with a relatively small system and then doubled the size repeatedly. Table 1 contains the parameters of the four runs we discuss in this paper; Figure 1 comes from run $d$. Because of computational constraints we doubled $L_x$ but not $L_y$, between $c$ and $d$, and so the latter’s aspect ratio differs from the other three runs. Earlier work showed that if the system is large enough to reach a

| Run Label | Domain Size  | Grid Points  | $w_0$ |
|-----------|--------------|--------------|-------|
| a........... | 100 × 50     | 512 × 256    | 2     |
| b........... | 200 × 100    | 1024 × 512   | 2     |
| c........... | 400 × 200    | 2048 × 1024  | 4     |
| d........... | 800 × 200    | 4096 × 1024  | 4     |

Note.—$w_0$ is the initial half-width of the current sheet.

![Fig. 2.—Out-of-plane electron velocities for the simulations of Table 1, showing the evolution of the current layer with system size. Each panel shows about one-fourth of the total simulation domain.](image)
steady state the reconnection rate in a double tearing mode configuration is independent of the aspect ratio (Shay et al. 1999).

Figure 2 shows the steady state current layers for the four runs. In the smallest simulations (Figs. 2a and 2b) the current layers lengths approach the system size. But, when the system size doubles between Figs. 2b and 2c the current layer does not; it lengthens moderately, but the turbulence evident at both ends of the layer broadens it along the y direction and prevents the narrow portion of the layer from expanding further. A final lengthwise doubling of the system, Fig. 2d, barely increases the length of the current layer.

In Figure 3 we show the reconnection rates as a function of time. To measure this rate we first integrate the magnetic flux between the initial X-point and the large magnetic island formed by the reconnected field. (The large island is a consequence of our periodic boundaries and is not shown in the figures.) We check for the growth of other X-points, but find that the initially seeded one remains dominant throughout the runs, despite the formation of secondary magnetic islands. Figure 3 shows the (unsmoothed) temporal derivative of the reconnected flux. The vertical lines mark the times of the snapshots of $v_{\text{rec}}$ displayed in Figure 2. All of the runs exhibit essentially the same behavior: after an initial quiescent period the reconnection rate rises to an asymptotic value of $v_{\text{in}} \approx 0.1$, because the box is relatively small. In fact the outflow velocity does not even reach the Alfvén speed. Thus, exploring the scaling of reconnection in pair plasmas requires much larger simulation domains than in normal reconnection. In an electron-proton simulation of this size (measured in ion inertial lengths) the ion outflow velocity would have reached the full Alfvén speed. After doubling the box size (run b), the current layer length also increases by a factor of 2. However the larger box contains more room for the acceleration of the outflow and so $v_{\text{out}}$ also increases. The densities, both upstream and downstream, as well as the thickness of the current layer remain relatively constant.

So, despite the lack of a Hall term, pair reconnection remains fast when the system size increases. A more quantitative description of the reconnection can be found in Table 2, which lists the various quantities from equation (5) for each run. The $v_{\text{in,calc}}$ column contains the inflow speed measured in the simulation while $v_{\text{in,calc}}$ is the value calculated from the other data using equation (5).

For our smallest domain, run a, $2\Delta \approx 35$ approaches $L_x/2 = 50$. (The periodic boundaries in our simulations restrict the maximum current sheet length to $L_x/2$. In practice $\Delta$ never reaches this maximum because the transition from outflow to island has a finite scale.) Even though the current layer stretches to the system size in a Sweet-Parker–like manner, reconnection remains fast [$v_{\text{in}} \sim \mathcal{O}(0.1)$], because the box is relatively small. In fact the outflow velocity does not even reach the Alfvén speed. Thus, exploring the scaling of reconnection in pair plasmas requires much larger simulation domains than in normal reconnection. In an electron-proton simulation of this size (measured in ion inertial lengths) the ion outflow velocity would have reached the full Alfvén speed.

After doubling the box size (run b), the current layer length $\Delta$ also increases by a factor of 2. However the larger box contains more room for the acceleration of the outflow and so $v_{\text{out}}$ also increases. The densities, both upstream and downstream, as well as the thickness of the current layer remain relatively constant.

| Run Label | $n_{\text{in}}$ | $n_{\text{out}}$ | $2\delta$ | $2\Delta$ | $v_{\text{out}}$ | $v_{\text{in,meas}}$ | $v_{\text{in,calc}}$ |
|-----------|----------------|-----------------|----------|-----------|----------------|------------------|-----------------|
| a.......... | 0.16           | 0.27            | 4.0      | 35        | 0.5            | 0.13             | 0.10            |
| b.......... | 0.12           | 0.32            | 4.0      | 80        | 0.8            | 0.15             | 0.11            |
| c.......... | 0.13           | 0.33            | 4.5      | 120       | 1.3            | 0.16             | 0.12            |
| d.......... | 0.13           | 0.30            | 5.0      | 135       | 1.3            | 0.13             | 0.11            |

Notes.—The headings $2\delta$ and $2\Delta$ denote the smaller and larger spatial extents, respectively, of the current sheet. The headings $v_{\text{in,meas}}$ and $v_{\text{in,calc}}$ are the measured and calculated (from eq. [5]) velocities of plasma into the X-point.
between these two runs, and all of our other runs. The net effect of the variation of $\Delta$ and $v_{\text{out}}$ is to leave the reconnection rate unchanged. Since the outflow speed has an upper limit, the Alfvén speed, this progression cannot continue indefinitely.

After the next doubling (run d) the outflow velocity reaches its expected Alfvénic limit. Since the increase was less than a factor of 2 (1.3/0.8 $\approx$ 1.6) a Sweet-Parker–like doubling of $\Delta$ would be accompanied by a decrease in the reconnection rate. Instead of doubling, however, $\Delta$ only increases by a factor of 120/80 $\approx$ 1.5 and the reconnection rate stays approximately constant. Since the Hall term cannot halt this increase, some other process must be at work.

As a check we again doubled the length of the box (but not its width) for run d. Unlike in the previous doublings the outflow speed cannot increase further. Its maximum value is the Alfvén speed based on the downstream density and the upstream magnetic field; in these simulations max ($v_{\text{out}}$) $\approx$ 1.3. Absent any other localizing effect the current layer length should approximately double and the reconnection rate should halve. Instead the current layer length and the reconnection rate remain basically unchanged. In § 5 we argue that the length of the current sheet is limited by the development of an instability driven by a temperature anisotropy. The instability develops in the outflow from the X-point before eventually growing strong enough to open the layer.

5. ANISOTROPY INSTABILITY

In sufficiently large simulations the outflow from the X-point is subject to a temperature anisotropy instability. Figure 4a shows $B_z$ from run c at $t = 450$, soon after the instability develops. In Figure 4b we plot two of the diagonal components of the positron temperature tensor, $T_{xx}$ and $T_{yy}$, along the line $y = -1$. (The influence of the $E_z$ feature shown in Fig. 1 makes the anisotropy somewhat weaker exactly on the symmetry axis.) At the X-point ($x = 0$) the plasma temperature is isotropic and equal to the initial value of 0.25. As the plasma moves away from the X-point $T_{xx}$ increases sharply while $T_{yy}$ remains nearly unchanged, leading to an anisotropy that reaches a peak value of $\approx$4. At $|x| \approx 50$ the disturbance in $B_z$ becomes visible, indicating onset of an instability. Further downstream the $B_z$ pattern strengthens and the anisotropy simultaneously weakens. Movies of $B_z$ (not shown) demonstrate that the disturbances propagate with the mean flow of the plasma in the outflow jet.

In order to investigate the source of the anisotropy we show the 1D velocity distributions taken between the dashed lines in Figure 4b and $-2 < y < -1$ in the final three panels. Two populations intermingle in this region, the inflowing plasma with $v_x < 0$ and a jet with $v_z < 0$ moving outward from the X-point. As can be seen in Figure 4c the superposition of these populations results in a wide, hot distribution of $v_y$. Meanwhile, since the sample box is upstream of the bipolar $E_z$ signal, the $v_x$ distributions remain essentially unchanged from their initial Maxwellians. A mixture of the inflowing ambient plasma and plasma accelerated by the reconnection electric field form the $v_z$ distributions. The final result is a plasma with $T_x > T_y \approx T_z$.

Plasmas with hot bi-Maxwellian distribution functions are typically stable to electrostatic perturbations (the thermal spread stabilizes the electrostatic ion-ion streaming instability), but unstable to electromagnetic instabilities. One possibility is that the magnetic perturbations in Figures 1 and 4 result from the firehose instability. If that were the case the perturbed motion of the plasma in the $z$-direction with $k = \hat{k}$ would produce perturbations in $B_z$. We can rule out this instability, however, since along the symmetry axis ($y = 0$) motion of the plasma in $z$ would not produce a perturbation of $B_z$, since the equilibrium field $B_z$ is zero in this region. The simulations clearly have large perturbations of $B_z$ centered at $y = 0$. Instead the anisotropy is sufficiently large that
the magnetic field in the region around \( y = 0 \) is negligible and the instability is a Weibel-like mode that self-generates the magnetic field perturbation by separating the electrons and positrons moving in opposite direction to create a current in the \( T_x \)-direction. To demonstrate the viability of the instability we begin with a tri-Maxwellian plasma described by the distribution function

\[
 f = \prod_{\alpha=x,y,z} \frac{m}{2\pi k_B T_\alpha}^{1/2} \exp \left[ -\frac{m}{2k_B T_\alpha} v_\alpha^2 \right],
\]

with, in general, \( T_x \neq T_y \neq T_z \), and look for instabilities with \( \vec{B}_z \) and \( \vec{E}_y \) varying as \( \exp(ik_y y - i\omega t) \). Following the method of Krall & Trivelpiece (1986) the instability condition for a pair plasma is

\[
 \left( \frac{T_x}{T_y} - 1 \right) > \frac{k_y^2 c^2}{2\omega_p^2}.
\]

Setting the plasma dielectric function to zero, we obtain the dispersion relation relating the growth rate \( \gamma = -i\omega \) and the wavenumber:

\[
 k_y c^2 + \gamma^2 + 2\omega_p^2 \left( 1 - \frac{T_x}{T_y} \right) = 2\omega_p^2 \frac{T_x}{T_y} \zeta Z(\zeta),
\]

where the thermal speed \( v_{th} \) enters through

\[
 \zeta = \frac{\omega}{k_y \sqrt{2 T_y/m}} = \frac{\omega}{k_y v_{th}},
\]

and \( Z(\zeta) \) is the plasma dispersion function

\[
 Z(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp(-t^2) dt.
\]

In the limit \(|\zeta| \gg 1\) equation (8) has a solution for the growth rate of

\[
 \gamma \approx k_y v_{th,x} \frac{\omega_p}{\sqrt{k_y^2 c^2 + 2\omega_p^2}},
\]

where \( v_{th,x} = (2T_x/m)^{1/2} \). Thus, the growth rate peaks at \( \gamma \sim v_{th,x}/d_x \) for \( k_x d_x > \sqrt{2} \). Taking \(|\zeta| \gg 1\) is only possible if the anisotropy is strong, \( T_x/T_y \gg 1 \). Equation (11) can also be derived in the fluid limit by considering the stability of a cold plasma beam.

Strictly speaking these results apply only to the case of a homogeneous infinite plasma, while in the case under consideration here the instability develops within a thin layer. To further explore the behavior of this instability, particularly the behavior of its growth rate, we again turn to simulations. First, in order to test our code’s ability to reproduce the results of equation (8), we consider a homogeneous electron-positron system with a bi-Maxwellian distribution (for both species), no initial magnetic field, and a constant particle density, \( n = 1.2 \). We seed the system with the theoretical fastest growing wavenumber (with \( k_x = 0 \)) and monitor the growth of the instability. In Figure 5c we plot the growth rates calculated from the simulations (stars) compared with the values predicted by equation (8) (solid line). The two are in near perfect agreement.

In order to study the effect of the narrow current layer on the development of the instability we next turn to a more complicated...
system. We begin with a Harris sheet equilibrium similar to that discussed in § 2 except that one component of the temperature, $T_x$, increases along with the density in the center of the current layer. Note that $T_x$ does not affect force balance in the $y$-direction and therefore does not affect the initial equilibrium. For our usual box sizes this equilibrium is also unstable to the tearing mode, so we consider narrow domains (e.g., $L_x = 4$, $L_y = 50$) where the tearing mode is stabilized but the temperature anisotropy mode is not. In Figure 5d we show the growth rates (stars) for the instability in this geometry (again with $k_x = 0$) compared with the homogeneous growth rate (solid line). In general the growth rates are only marginally reduced from the homogeneous values. Thus, neither the finite geometry of the current layer nor the finite magnetic field in the layer significantly affect the growth of the instability, confirming that the Weibel instability can exist in a magnetically supported current layer.

In Figure 5a and 5b we show the nonlinear development of the instability in a 2D homogeneous system with no initial magnetic field and $T_x/T_y = 4$. The system is seeded with a magnetic perturbation corresponding to the fastest growing mode that dominates the structure at early time. At late time the dominant mode develops a finite value of $k_x$ ($k_x = 0.25$, $k_y = 1.0$), as also occurs in the reconnection simulations. The amplitude of the late-time perturbations of $B_x$ is about 0.5; compared with the turbulence seen in the reconnection simulations these perturbations are at somewhat longer wavelength and have modestly larger amplitudes. These differences may reflect constraints on the turbulence due to the finite width (in the $y$-direction) of the temperature anisotropy in the reconnection simulations.

For the full reconnection simulation a typical temperature anisotropy in the outflow is $\approx 3$ (see Fig. 4), which corresponds to a typical growth rate of $\gamma \approx 0.2$. (The density in the reconnection outflow is $\approx 4$ times smaller than in the instability simulations, which, since $\gamma \propto \omega_p$, implies a factor of 2 decrease in the growth rate.) In plasma traveling at the typical outflow velocity of $v_{\text{out}} \approx 1$ the lengthscale associated with one $e$-folding of the instability is $\ell \approx v_{\text{out}}/\gamma = 5$. A current layer of half-length $\Delta = 60$ can then fit $\Delta/\ell \approx 12$ growth periods within it, more than enough to amplify any initial fluctuations to nonlinear amplitudes.

A fundamental question is whether the dynamics of the Weibel instability scale in a manner such that the resultant length of the current layer can produce reconnection rates insensitive to the size of the computational domain. A complete model should incorporate the development of the temperature anisotropy in the outflow from the X-point by using the full dispersion relation of equation (8). We consider instead a cruder model in which we ignore the anisotropy threshold. That is, we take the plasma near the X-point to be cold so that the Weibel mode is unstable as soon as the anisotropy develops downstream. Strictly speaking this assumption does not apply near the X-point and will lead to an overstatement of the mode’s growth rate. But since the principal effects of the instability manifest themselves in the downstream region, the key result derived from this simpler approach — the independence of the reconnection rate on the system size — should remain valid in a more complete theory.

In its most primitive form, neglecting onset thresholds, the growth rate of the Weibel is given by $\gamma = \gamma_{\text{th}} s/\delta d_e$. In Figure 4 we show that the effective electron thermal speed in the $x$-direction is related to the relative drift between the basically stationary upstream plasma crossing into the outflow jet and the high-speed particles accelerated from the X-point. Hence, $v_{\text{th},e}$ is simply one-half the drift speed of those fast particles, which implies $\gamma = v_{e}/2d_e$. As discussed previously, the amplitude of the turbulent fluctuations is controlled by the convective amplification of the initial electromagnetic perturbations. The number of $e$-foldings between the X-point and an position $x$ downstream is therefore given by

$$\Gamma = \int_0^s \frac{dx}{v_e}. \quad (12)$$

But the growth rate $\gamma$ is itself proportional to $v_e$, so $\gamma$ drops out of the calculation and

$$\Gamma = x/2d_e. \quad (13)$$

If we assume that $\Gamma_0$ is the number of $e$-foldings required to amplify initial fluctuations to the values of order unity required to broaden the current layer, we obtain the length of the current layer $\Delta = 2\Gamma_0 d_e$. The resulting rate of reconnection can then be calculated from equation (5) (ignoring density changes and any variations in the current layer width) to be

$$v_{\text{in}} \approx v_A/\Gamma_0. \quad (14)$$

The rate of reconnection resulting from the broadening of the current layer by Weibel generated turbulence should therefore remain fast, independent of system size, and independent of kinetic scales, as we see in our simulations.

6. DISCUSSION

We have shown that reconnection in a pair plasma remains fast despite the absence of the Hall term. For small systems, up to $100d_e$ along the outflow direction, the length of the current layer that develops during reconnection scales with the size of the system, as expected based on the classical Sweet-Parker model with no dispersive waves. When the domain size exceeds around $200d_e$, a Weibel-like instability develops that forces the development of a turbulent outflow jet and limits the length of the narrow current layer, allowing reconnection to remain fast. Note that, despite being big enough to show the transition to system size independent reconnection, our simulation boxes remain small in astrophysical terms. A scale of $L = 10^4$ inertial lengths, modestly larger than our biggest boxes, corresponds to

$$L \sim 5 \times 10^3 \left(\frac{1 \, \text{cm}^{-3}}{n_e}\right)^{1/2} \text{km.} \quad (15)$$

The length $\Delta \approx 70d_e$ and width $\delta \approx 5d_e$ of the current layer are even smaller still. The wide separation of scale between the system size and the scale where the important reconnection physics occurs is a well-known phenomenon in magnetospheric physics where in situ satellite measurements have studied the inner scales near the X-point (Mozer et al. 2002).

Although fast pair reconnection has been noted in previous work, the underlying cause was attributed to different mechanisms. In particular Daughton & Karimabadi (2007) suggest that the generation and ejection of magnetic islands play an important role by continually disrupting the growth of a current layer that would otherwise scale with the system size. Magnetic islands form in our simulations, but appear to play little or no role in maintaining the length of the current layer. Figure 6 shows snapshots at intervals of $\Omega_e t = 25$ of the out-of-plane electron velocity for run d, beginning at $\Omega_e t = 600$. During this period the reconnection rate remains steady and fast. Figures 6a–6d the instability described in § 5 does not have enough room to grow and the current layer lengthens, in a process analogous to what occurs in our smaller simulation domains. In Figures 6e–6h the current layer grows
long enough for the Weibel-like instability to saturate and stop further lengthening of the narrow portion of the current layer. Although several islands form and travel downstream during this time, they do not appear to have any significant effect on the length of the current layer or on the system’s reconnection rate (see Fig. 3). However, several factors may play a role in island generation in numerical simulations including the initial width of the current sheet, the strength of the initial perturbation, the number of simulated particles, and the size of the grid. The relative importance of these various effects, and what they imply about island generation in real plasmas, remains unknown.

A surprise of the simulations of pair reconnection shown here and in earlier work is that, just as in the electron-proton system, the inflow velocity is $O(0.1c_A)$. This agreement is unexpected since different mechanisms are clearly at play in the two cases—while there is some turbulence in the outflow jets of electron-proton reconnection, the amplitude is far smaller than in the pair system. However, since the growth of the Weibel instability appears to be crucial for maintaining fast reconnection in our pair plasma simulations, any suppression of the growth rate would serve to slow the process. One possible mechanism for doing so is the addition of a constant guide field component, $B_z$. Such a field would tend to isotropize the temperatures in the current layer ($T_x \approx T_y$) simply by keeping the particles on Larmor orbits and either substantially decrease or stabilize the growth of the instability. On the other hand, we also cannot rule out the possibility that another instability would develop when a guide field is present. It seems clear that any elongated current layer will ultimately develop strong pressure anisotropies, and in the presence of a guide field the parallel pressure $p_{||}$ will likely become much larger than that perpendicular to the magnetic field $p_{\perp}$. For randomly oriented magnetic fields most reconnection is guide field reconnection, so the antiparallel geometry we consider here is in some sense a special case. But, if guide field reconnection is slow due to the suppression of the anisotropy instabilities, antiparallel reconnection, even if rare, may dominate in pair plasmas.

Even during antiparallel reconnection the Weibel instability will be suppressed for large enough asymptotic plasma temperatures. As can be seen in Figure 4 a large $T$ can minimize the effects of the extra heating provided by the beam of outgoing plasma in the current layer. The critical temperature can be roughly estimated as $T_{\text{crit}} \approx m_{e}v_{d}^2/2$, where $v_d \sim c_A$ is the drift velocity of the outflowing electrons. Thus, for asymptotic temperatures $T > T_{\text{crit}} \approx m_{e}c_A^2/2$, or $\beta > 1$, the instability should be stabilized. Further simulations in this regime are being performed.

After submitting this paper we became aware that Zenitani and Hesse also concluded that the Weibel instability plays a similar role during reconnection in very relativistic pair plasmas (Zenitani & Hesse 2008).

REFERENCES

Bessho, N., & Bhattacharjee, A. 2005, Phys. Rev. Lett., 95, 245001
———. 2007, Phys. Plasmas, 14, 056503
Birn, J., et al. 2001, J. Geophys. Res., 106, 3715
Biskamp, D., & Schwarz, E. 2001, Phys. Plasmas, 8, 4729
Coroniti, F. V. 1990, ApJ, 349, 538
Daughton, W., & Karimabadi, H. 2007, Phys. Plasmas, 14, 072303
Drenkhahn, G., & Spruit, H. C. 2002, A&A, 391, 1141
Eardley, D. M., & Lightman, A. P. 1975, ApJ, 200, 187
Fujimoto, K. 2006, Phys. Plasmas, 13, 072904
Harris, E. G. 1962, Nuovo Cimento, 23, 115

Fig. 6.— Out-of-plane electron velocity at $\delta t = 25$ intervals beginning at $t = 600$ for run d.
Hesse, M., Kuznetsova, M., & Birn, J. 2001, J. Geophys. Res., 106, 29831
Jaroschek, C. H., Treumann, R. A., Lesch, H., & Scholer, M. 2004, Phys. Plasmas, 11, 1151
Krall, N. A., & Trivelpiece, A. W. 1986, Principles of Plasma Physics (San Francisco: San Francisco Press)
Masuda, S., Kosugi, T., Hara, H., Tsuneta, S., & Ogawara, Y. 1994, Nature, 371, 495
Michel, F. C. 1994, ApJ, 431, 397
Miller, K. A., & Stone, J. M. 1997, ApJ, 489, 890
Mozer, F. S., Bale, S. D., & Phan, T. D. 2002, Phys. Rev. Lett., 89, 015002
Parker, E. N. 1957, J. Geophys. Res., 62, 509
Petschek, H. E. 1964, in Proc. AAS-NASA Symp. 50, The Physics of Solar Flares (Washington: NASA), 425
Phan, T. D., et al. 2006, Nature, 439, 175
Rogers, B. N., Denton, R. E., Drake, J. F., & Shay, M. A. 2001, Phys. Rev. Lett., 87, 195004
Shay, M. A., Drake, J. F., Rogers, B. N., & Denton, R. E. 1999, Geophys. Res. Lett., 26, 2163
Shay, M. A., Drake, J. F., & Swisdak, M. 2007, Phys. Rev. Lett., 99, 155002
Sweet, P. A. 1958, in Electromagnetic Phenomena in Cosmical Physics, ed. B. Lehnert (New York: Cambridge Univ. Press), 123
Zeiler, A., Biskamp, D., Drake, J. F., Rogers, B. N., Shay, M. A., & Scholer, M. 2002, J. Geophys. Res., 107, 1230
Zenitani, S., & Hesse, M. 2008, Phys. Plasmas, 15, 022101
Zenitani, S., & Hoshino, M. 2001, ApJ, 562, L63