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A rapidly evolving high-amplitude $\delta$ Scuti star crossing the Hertzsprung Gap

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People cannot witness the stellar evolution process of a single star obviously in most cases because of its extremely secular time-scale, except for some special time nodes in it (such as the supernova explosion [1]). But in some specific evolutionary phases, we have the chances to witness such process gradually on human times-scales. When a star evolved leaving from the main sequence, the hydrogen nuclei fusion in its core is gradually transferring into the shell. In the Hertzsprung–Russell diagram, its evolutionary phase falls into the Hertzsprung gap, which is one of the most rapidly evolving phases in the life of a star [2]. Here we report a discovery of a rapidly evolving high-amplitude δ Scuti star KIC6382916 (J19480292+4146558) which is crossing the Hertzsprung gap. According to the analysis of the archival data, we find three independent pulsation modes of it, whose amplitudes and frequencies are variating distinctly in 4 years. The period variation rates of the three pulsation modes are one or two orders larger than the best seismic model constructed by the standard evolution theory, which indicates the current theory cannot precisely describe the evolution process in this rapidly evolving phase and needs further upgrades. Moreover, the frequency and amplitude interactions between the three independent pulsation modes and their harmonics/combinations open a new window to the future asteroseismology.

δ Scuti stars are a class of short-period pulsating variable stars with period between 0.02 and 0.25 day and the spectral classes A-F, which locate on the main sequence or post main sequence evolutionary stage at the bottom of the classical Cepheid instability strip and excited by the κ mechanism [3]. High-amplitude δ Scuti stars (hereafter HADS) are a subclass of δ Scuti stars, who always have larger amplitudes (ΔV ≥ 0.3 m) and slower rotations [3]. Most of the HADS show single or double radial pulsation modes, and some of them have three radial pulsation modes or even some non-radial pulsation modes [4–7]. Theoretically, the period variation rates ((1/P)(dP/dt)) of HADS caused by the evolution should have a value from 10^{-10} yr^{-1} on the main sequence to 10^{-7} yr^{-1} on the post-main sequence phase [8]. According to the seismic models, the observed period variation rates of some HADS (such as XX Cyg, AE UMa, and VX Hya [5, 6, 9]) can be interpreted by
the evolutionary effect self-consistently, and each of these stars should be located in the Hertzsprung gap in the H-R diagram with a helium core and a hydrogen-burning shell. In these researches, the period variation rates are obtained from the ground based time-series photometric data accumulated in several decades, but only that of the fundamental pulsation modes have a sufficient precision to be confirmed and tested because of the quality of the data. The high precision photometric data from *Kepler space telescope* lasted for about 4 years could provide us excellent opportunity to determine the period and amplitude variation rates not only for the fundamental mode, but also for the other pulsation modes. These observed quantities would tell us more secrets about the stellar evolution in this special rapidly evolving phase.

KIC6382916 (J19480292+4146558, also known as GSC 03144-595) is a HADS to be monitored extensively by the *Kepler space telescope* for its pulsation properties, which shows three independent frequencies (pulsation modes) in its light curves [10]. These three pulsation modes have been confirmed as the fundamental, the first overtone and the second overtone radial p-modes [11], and $T_{\text{eff}} = 6950 \pm 100$ K, $\log g = 3.7 \pm 0.1$ and $v \sin i = 50 \pm 10$ km s$^{-1}$ have been estimated by the high-dispersion spectra [10]. Using the long-cadence (LC) photometric data of KIC6382916 lasted from BJD 2454953 to 2456424 (Quarter 0-17), we extract the first 23 frequencies with largest amplitudes (which have a cut-off of 1.3 mmag, see Photometric Data Reduction in Methods and Extended Data Table I for more details) to study the variations and interactions of them. Theses 23 frequencies are composed as that of the fundamental ($f_0$), the first overtone ($f_1$), and the second overtone ($f_2$) modes, together with their harmonics/combinations. The IDs of these frequencies are also identified as the labels of the corresponding pulsation modes in this work.

In order to extract the variation information of the amplitudes and frequencies over time, we use the Short-Time Fourier Transformation to deal with the LC photometric data. The pre-whiting process in a time window of 150 days is performed when the window is moving from the start to the end time of the LC data, with a step of 30 days. In each step, the frequencies are fixed as that have been extracted in the complete LC data (see Extended Data Table I for more details) while the amplitudes and phases are obtained by the non-linear least square fitting. Then, we get the amplitude and phase variations of the 23 frequencies. Using the Fourier-Phase Diagram method [6,12,13], the variations of the frequencies can be derived from the variations of the phases.
The amplitude and phase variations of $f_0$, $f_1$, and $f_2$ are presented in Figure 1 and each one of them is performed by a quadratic fitting. In Figure 1 we find the amplitudes of $f_0$ ($A_{f_0}$) is relatively stable, that of $f_1$ ($A_{f_1}$) is slightly decreasing while $A_{f_2}$ is distinctly increasing by about 24% in 4 years. On the other hand, the phases (subtracted by its average) of $f_0$, $f_1$, and $f_2$ variate with a same trend but different levels, which is corresponding to the increasing periods of them with different period variation rates: $(1/P_0)(dP_0/dt) = (3.0 \pm 0.6) \times 10^{-7}\text{yr}^{-1}$, $(1/P_1)(dP_1/dt) = (3.0 \pm 0.5) \times 10^{-7}\text{yr}^{-1}$, and $(1/P_2)(dP_2/dt) = (2.3 \pm 0.2) \times 10^{-6}\text{yr}^{-1}$ (see Methods for more details).

Noting the decrement of $A_{f_1}$ and the increment of $A_{f_2}$, it is quit interesting to study the amplitude interactions between the 23 pulsation modes, which also indicate the energy transformation between them. Here, we introduce the Interaction Diagram (see Figure 2) to show the amplitude interactions between the pulsation modes. In Figure 2, the color of a small square represents the correlation coefficient between the amplitudes of the labeled pulsation modes whose column and row intersect at the square. The dendrograms on the upper and left represent the Agglomerative Hierarchical Clustering (AHC) process in the amplitude interaction space which is spanned by the vectors (corresponding to each of the pulsation modes) consisting of the correlation coefficients between a specific pulsation mode to all of them. In general, the AHC process reorders the pulsation modes in order to cluster those who have similar behaviors in the interaction space together.

At first glance of Figure 2, the dominating interactions occur between the pulsation modes including $f_1$ and $f_2$. In detail, all the pulsation modes including $f_2$ are clustered together and have increasing amplitudes, while the pulsation modes of $f_1$, $2f_1$, $2f_0 + f_1$, $f_0 + 2f_1$, and $f_0 + 3f_1$ are anti-correlated with the above $f_2$ related modes and have decreasing amplitudes. What’s more interesting is that the $3f_0$, $3f_1$, $-2f_0 + 2f_1$, and $-f_0 + 2f_1$ modes, which are not $f_2$ related, but have increasing amplitudes like that of the $f_2$ related modes. All these relations and structures shown in the Interaction Diagram of amplitudes indicate that there exists interactions or energy transformations between the independent pulsation modes and their harmonics/combinations, and this opens a new window for exploring the interior of the stars in future researches.

1 The variations of all the 23 pulsation modes are listed in Extended Data Figure 2, 3, and 4.
2 Here we use the Spearman’s rank correlation coefficient, which is a measure of how well the relationship between two variables can be described by a monotonic function.
3 Comparing with the complicated interactions shown in Figure 2, the Interactions Diagram of the phases (see Extended Data Figure 5) shows that almost all the pulsation modes have the same variation trend.
Figure 1. Variation of the amplitudes and phases (subtracted by their averages) of $f_0$, $f_1$, and $f_2$. In each of the subfigures, the data points are fitted by a quadratic fitting, the best-fit result is presented by a solid black line, and the corresponding residuals are plotted in the lower panel. The $2\sigma$ (deep red) and $3\sigma$ (light red) bounds of the fitting are also shown in the subfigures. In the lower panel of each subfigures, the $\sigma_{eff}$ is defined as $(Q_{obs} - Q_{cal})/\sigma$, where $Q_{obs}$ and $Q_{cal}$ are the values which come from the observation and model calculation, respectively; $\sigma$ is the uncertainty of the observed points.

except the $-2f_0 + 2f_1$, $f_0 + f_1 - f_2$, and $3f_0$ modes, which leads to the decreasing periods of these three modes.
The single star evolutionary models are constructed by using different initial masses with three groups of [Fe/H] and two different values of equatorial rotation velocities ($v_{eq}$), from the pre-main sequence to the red giant branch. At each of the steps on the evolutionary tracks, the pulsation frequencies are calculated based on the corresponding stellar structure. Figure 3 shows the best-fit seismic models of the observed independent frequencies ($f_0$, $f_1$, and $f_2$) together with the corresponding evolutionary tracks for specific combinations of ([Fe/H], $v_{eq}$). The detailed information of the best-fit seismic models is collected in Table I.
The results of the seismic models re-confirm the conclusion that $f_0$, $f_1$, and $f_2$ are the fundamental, first overtone, and second overtone radial p-modes, respectively [11]. Moreover, it shows that this star locates in the later evolutionary phase of the Hertzsprung gap, which is a more rapidly evolving phase compared with those stars in the earlier evolutionary phase of the Hertzsprung gap (such as AE UMa [5] and VX Hya [6]). The most incredible thing is that the observed period variation rates of $f_0$ and $f_1$ are an order of magnitude larger than the theoretical predicted ones, and two orders of magnitude larger for the case of $f_2$. If we ascribe these observed period variation rates to the stellar evolution (which seems to be the most reasonable case), it indicates that the star evolving more rapidly than theoretical prediction and the standard stellar evolution theory cannot precisely describe a single star’s evolution process in this rapidly evolving phase.

Figure 3. The best-fit seismic models for KIC6382916 along with the corresponding evolutionary tracks from the main-sequence to the red giant branch. The solid lines and the dash lines represent the evolutionary tracks of $v_{\text{eq}} = 40 \text{ km s}^{-1}$ and $v_{\text{eq}} = 120 \text{ km s}^{-1}$, respectively. The different colors represent different value of [Fe/H]. The colored pluses and crosses represent the best-fit seismic models of different value of $v_{\text{eq}}$.

For KIC6382916, following the current variation rates, the amplitude of $f_2$ will exceed that of $f_0$ at about BJD 2460331 (Jan, 2024), and exceed that of $f_1$ at about BJD 2460343.
Table I. The frequencies and period variation rates of the three independent pulsation modes from the best-fit seismic models and observations (which are listed in the last column).

| $f_0$ (c days$^{-1}$) | $f_1$ (c days$^{-1}$) | $f_2$ (c days$^{-1}$) | $(1/P_0)(dP_0/dt)$ (yr$^{-1}$) | $(1/P_1)(dP_1/dt)$ (yr$^{-1}$) | $(1/P_2)(dP_2/dt)$ (yr$^{-1}$) | Mass $[\text{Fe/H}]$ | Rotation M$_\odot$ dex km s$^{-1}$ |
|----------------------|----------------------|----------------------|-------------------------------|-------------------------------|-------------------------------|--------------------|-----------------------------|
| 4.9099157            | 6.4314933            | 8.0886817            | 3.97 $\times$ 10$^{-8}$        | 3.64 $\times$ 10$^{-8}$        | 3.12 $\times$ 10$^{-8}$        | 1.81               | 0.266 40                    |
| 4.9084637            | 6.4334156            | 8.1142769            | 3.59 $\times$ 10$^{-8}$        | 3.09 $\times$ 10$^{-8}$        | 2.38 $\times$ 10$^{-8}$        | 1.79               | 0.266 120                   |
| 4.9106769            | 6.4315033            | 8.0896883            | 3.85 $\times$ 10$^{-8}$        | 3.56 $\times$ 10$^{-8}$        | 3.10 $\times$ 10$^{-8}$        | 1.83               | 0.111 40                    |
| 4.9097575            | 6.4319370            | 8.1102328            | 3.51 $\times$ 10$^{-8}$        | 3.06 $\times$ 10$^{-8}$        | 2.40 $\times$ 10$^{-8}$        | 1.81               | 0.111 120                   |
| 4.9092663            | 6.4320905            | 8.1067212            | 3.41 $\times$ 10$^{-8}$        | 3.14 $\times$ 10$^{-8}$        | 2.73 $\times$ 10$^{-8}$        | 1.87               | 0.040 40                    |
| 4.9087663            | 6.4328544            | 8.1284759            | 3.10 $\times$ 10$^{-8}$        | 2.68 $\times$ 10$^{-8}$        | 2.09 $\times$ 10$^{-8}$        | 1.85               | 0.040 120                   |
| 4.909845(5)          | 6.431886(9)          | 8.03541(4)           | $(3.0 \pm 0.6) \times 10^{-7}$ | $(3.0 \pm 0.5) \times 10^{-7}$ | $(2.3 \pm 0.2) \times 10^{-6}$ | -                 | -                           |

(Feb, 2024). This prediction can be tested in the near future by the following photometric observations, which also could provide us an opportunity to witness the stellar evolution process of a single star gradually in this special evolutionary phase. Moreover, the current continuous photometric data from Kepler can sufficiently support us to carry out the researches on the interactions between the pulsation modes via the Interaction Diagram, which could not only be used as a new tool to classify the different pulsation modes of a pulsating star or classify the different kinds of pulsating stars, but also opens a new window for the future asteroseismology.
**METHODS**

**Photometric Data Reduction**

The long-cadence (LC) photometric data of KIC6382916 from the *Kepler space telescope* were used in this work, which covers from BJD 2454953 to 2456424 (Quarter 0-17) (publicly available PDC data [15, 16]). We downloaded the light curves (in the format of reduced BJD and magnitudes) of KIC6382916 from Mikulski Archive for Space Telescope (MAST) which were then normalized to be zero in the mean for each quarter. An overview of all the normalized LC data in time domain and frequency domain are shown in Extended Data Figure 1.

All the above normalized LC data were pre-whiten to extract the frequencies, amplitudes, and phases of the pulsation modes. This process was cut-off until the amplitude smaller than 1.3 mmg and we totally got 23 pulsation modes (see in Extended Data Table II), which is much higher than the typical noise level in *Kepler* data of this star (∼ 0.03 mmag). This choice ensured that the uncertainties of the amplitudes and phases were typically smaller than the variations of some interested pulsation modes in 4 years, which was determined by the subsequent Short-Time Fourier Transformation results. In each step in the pre-whiting process, the Lomb-Scargle algorithm [17] was used to help to find the initial value of frequency with largest amplitude, and then a non-linear least square fitting was performed to get the final values of the frequency, amplitude, and phase. The frequencies within $1.0 \leq f \leq 24.4 \text{ d}^{-1}$ were considered in this work, which was determined by the LC Nyquist frequency [13].

The Short-Time Fourier Transformation [13, 18-20] was then performed to the normalized LC data to get the variation of the amplitudes and phases. In this process, a time window of 150 days was moving from the start to the end time of the LC data, with a step of 30 days. In each step, the pre-whiting process was performed to extract the amplitudes and phases of the specific 23 pulsation modes, while the frequencies were fixed as the values obtained in the complete LC data in Extended Data Table II.

At last, the amplitude and phase (subtracted by its average value) for each of the 23 pulsation modes in each of the moving window were collected (with the times which were

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4 http://archive.stsci.edu/kepler
defined as the midpoints of the window). After removing the periodic signals between 350 to 400 days which is related to the 372.5 day Earth-trailing orbit of the Kepler Space Telescope, the variations of the amplitudes and phases of the 23 pulsation modes were presented in Extended Data Figure 2, 3 and 4.

In this work, the pre-whiting process was performed by the Fourier decomposition which can be presented by the formula

\[ m = m_0 + \sum A_i \sin [2\pi(f_i t + \phi_i)] , \]

where \( m_0 \) is the shifted value, \( A_i \) is the amplitude, \( f_i \) is the frequency and \( \phi_i \) is the corresponding phase.

The uncertainties all through the work were estimated by the following expressions [21, 22]:

\[ \sigma_A = D \sqrt{\frac{2}{N}} \sigma_N, \sigma_f = \frac{D \sqrt{6} \sigma_N}{\pi \sqrt{NAT}}, \sigma_\phi = \frac{D \sigma_N}{\pi \sqrt{2NA}}. \]

In these expressions, \( \sigma_A, \sigma_f, \) and \( \sigma_\phi \) are the uncertainties of the amplitude, frequency, and phase, respectively; \( \sigma_N \) can be approximated by the standard deviation of the final residual light curve; \( A \) is the amplitude; \( N \) and \( T \) are the total number of data points and the total time base-line employing in the pre-whiting process, respectively; \( D \) can be estimated as the square-root of the average number of consecutive data points of the same sign in the final residual light curves. In this work, the residual light curve were conservatively considered as the light curves in which the specific 23 pulsating signals had been removed.

**Theoretical Model Calculation**

In order to determine the stellar mass and evolutionary stage based on the single star evolutionary models, the open source 1D stellar evolution code Modules for Experiments in Stellar Astrophysics (MESA [23][27]) was used to construct the structural and evolutionary models. At each step along with the evolutionary tracks, the pulsation frequencies of the specific structure were calculated by the stellar oscillation code GYRE [28] (see, e.g., Refs. [5, 6, 29]).

The initial parameters used to construct pre-main sequence evolutionary models of KIC6382916 were configured as follows. Different metallicity [Fe/H] with values of 0.266, 0.111, and 0.040 dex were considered as the initial metallicity of the evolutionary models.
(see Extended Data Table I for more details). The following formulas were used to calculate the initial heavy element abundance $Z$ and initial hydrogen abundance $X$:

$$\frac{[\text{Fe/H}]}{\text{H}} = \log \frac{Z}{X} - \log \frac{Z_\odot}{X_\odot},$$  

(3)

$$Y = 0.24 + 3Z,$$  

(4)

$$X + Y + Z = 1,$$  

(5)

where $X_\odot = 0.7381$ and $Z_\odot = 0.0134$ [30]. Equation (4) was provided by Ref. [31]. Based on the given values of $[\text{Fe/H}]$, we got ($X = 0.672$, $Z = 0.022$), ($X = 0.695$, $Z = 0.016$), and ($X = 0.704$, $Z = 0.014$) as the initial inputs of the evolutionary models. The initial mass of the models was set in the interval from 1.5 $M_\odot$ to 2.5 $M_\odot$ with a step of 0.01 $M_\odot$, covering the typical mass range of HADS. The rotation of the star had also been considered in the model calculation. Because Ref. [10] provided us the projected rotational velocity $v \sin i = 50 \pm 10 \text{ km s}^{-1}$ from high-resolution spectroscopic observation, the equatorial rotation velocities $v_{eq} = 40 \text{ km s}^{-1}$ and $v_{eq} = 120 \text{ km s}^{-1}$ were set to be the inputs in the model calculation, which covered a reasonable range of $v_{eq}$. The mixing-length parameter was set to be a value of $\alpha_{MLT} = 1.89$ [9]. All the evolutionary tracks were calculated from the pre-main sequence to red giant branch.

Based on the pulsation frequencies calculated in every step along with the evolutionary tracks, we got the best-fit seismic models (which have the smallest $\chi^2$ with respect to the observed values of $f_0$, $f_1$, and $f_2$) with the different combinations of $([\text{Fe/H}], v_{eq})$ (see in Figure 3 and Table I).

The possibility that $f_0$, $f_1$, and $f_2$ could be the non-radial pulsation modes was also tested in our calculation. It can be excluded based on two reasons: (i) we explored the frequency spectrum carefully and did not find any hints of rotation splits of the identified frequencies; (ii) according to the model calculation, the non-radial pulsation modes represented negative period variation rates in post-main sequence phase, which was opposite to the observed values.
Fitting on the Three Independent Pulsation Modes

The amplitudes and phases of the three independent pulsation modes ($f_0$, $f_1$, and $f_2$) were fitted by the quadratic polynomial $a + b \cdot t + c \cdot t^2$. The Markov Chain Monte Carlo (MCMC) algorithm [32] was used to determine the coefficients and their uncertainties in above expression, and the fitted coefficients are listed in Extended Data Table III.

Based on the Fourier Phase Diagram method [6], the period variation rates can be directly derived via the fitted coefficients of the quadratic term of the phases.

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Author contributions J.S.N. contributed to the design of the work, performed the photometric data analysis, and wrote the manuscript. H.F.X. contributed to the theoretical model calculation.

Competing Interests The authors declare no competing interests.

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Extended Data Figure 1. Overview of the normalized LC data in time domain and frequency domain of KIC6382916.
Extended Data Figure 2. Variation of the amplitudes and phases of the 23 pulsation modes, Part I.
Extended Data Figure 3. Variation of the amplitudes and phases of the 23 pulsation modes, Part II.
Extended Data Figure 4. Variation of the amplitudes and phases of the 23 pulsation modes, Part III.
Extended Data Figure 5. Interaction diagram of phases of the 23 pulsation modes.
Extended Data Table I. The observed stellar parameters of KIC6382916.

| Parameter      | Value               | Reference      |
|----------------|---------------------|----------------|
| [Fe/H] (dex)   | 0.266 ± 0.021       | Ref. [33]      |
|                | 0.111 ± 0.021       | Ref. [34]      |
|                | 0.040^{+0.25}_{-0.35} | Ref. [35]      |
| $T_{\text{eff}}$ (K) | 7075 ± 281         | Ref. [36]      |
|                | 6950 ± 100          | Ref. [10]      |
|                | 6923.39 ± 24.99     | Ref. [33]      |
|                | 6737.92 ± 15.29     | Ref. [34]      |
|                | 6548^{+148}_{-181} | Ref. [35]      |
|                | 6557 ± 229          | Ref. [37]      |
| $v \sin i$ (km s$^{-1}$) | 50 ± 10            | Ref. [10]      |
Extended Data Table II. Frequency solution of the first 23 frequencies with largest amplitudes.

| ID       | Frequency (c days$^{-1}$) | Amplitude (mmag) | Phase (rad/2π) |
|----------|---------------------------|------------------|----------------|
| $f_0$    | 4.9098455 ± 0.0000002     | 79.88 ± 0.05     | -0.0218 ± 0.0001 |
| $f_1$    | 6.4318869 ± 0.0000003     | 77.84 ± 0.05     | -0.2356 ± 0.0001 |
| $f_0 + f_1$ | 11.3417324 ± 0.0000007   | 28.38 ± 0.05     | 0.2002 ± 0.0003 |
| $-f_0 + f_1$ | 1.522043 ± 0.000001    | 16.53 ± 0.05     | 0.0338 ± 0.0005  |
| $2f_0$   | 9.819690 ± 0.000001      | 16.11 ± 0.05     | -0.0881 ± 0.0005 |
| $f_2$    | 8.035414 ± 0.000002      | 12.54 ± 0.05     | -0.0011 ± 0.0007 |
| $2f_1$   | 12.863775 ± 0.000002     | 11.71 ± 0.05     | -0.3292 ± 0.0007 |
| $2f_0 + f_1$ | 16.251578 ± 0.000002   | 8.51 ± 0.05      | 0.376 ± 0.0001   |
| $-f_0 + 2f_1$ | 7.953928 ± 0.000003  | 6.94 ± 0.05      | -0.348 ± 0.0001  |
| $2f_0 - f_1$ | 3.387805 ± 0.000004    | 5.26 ± 0.05      | 0.251 ± 0.0002   |
| $f_0 + 2f_1$ | 17.773618 ± 0.000005   | 4.23 ± 0.05      | 0.291 ± 0.0002   |
| $f_0 + f_2$ | 12.945257 ± 0.000006   | 3.37 ± 0.05      | -0.034 ± 0.0003  |
| $2f_0 + 2f_1$ | 22.683465 ± 0.000006  | 3.24 ± 0.05      | 0.025 ± 0.0003   |
| $f_1 + f_2$ | 14.467308 ± 0.000006   | 3.19 ± 0.05      | -0.041 ± 0.0003  |
| $-f_1 + f_2$ | 1.603525 ± 0.000007    | 2.97 ± 0.05      | -0.060 ± 0.0003  |
| $-f_0 + f_2$ | 3.125577 ± 0.000008    | 2.58 ± 0.05      | -0.371 ± 0.0003  |
| $f_0 + f_1 - f_2$ | 3.306303 ± 0.000009 | 2.18 ± 0.05      | -0.108 ± 0.0004  |
| $3f_0$   | 14.72954 ± 0.000001     | 2.10 ± 0.05      | -0.153 ± 0.0004  |
| $-f_0 + f_1 + f_2$ | 9.55745 ± 0.000001 | 1.99 ± 0.05      | -0.191 ± 0.0004  |
| $3f_1$   | 19.29566 ± 0.000001     | 1.93 ± 0.05      | -0.212 ± 0.0004  |
| $f_0 + 3f_1$ | 24.20551 ± 0.000001   | 1.87 ± 0.05      | 0.286 ± 0.0005   |
| $3f_0 + f_1$ | 21.16143 ± 0.000001   | 1.37 ± 0.05      | -0.227 ± 0.0006  |
| $-2f_0 + 2f_1$ | 3.04408 ± 0.000001  | 1.38 ± 0.05      | 0.041 ± 0.0006   |
Extended Data Table III. Quadratic fitting results of the amplitudes and phases of $f_0$, $f_1$, and $f_2$.

| Variable | $a$ | $b$ | $c$ | $\chi^2$/d.o.f. |
|----------|-----|-----|-----|------------------|
| $A_{f_0}$ | 79.7(±0.1) | 7(±3) $\times 10^{-4}$ | $-4(\pm 2) \times 10^{-7}$ | 43.02/41 |
| $A_{f_1}$ | 78.1(±0.1) | $-6(\pm 30) \times 10^{-5}$ | $-2.4(\pm 1.8) \times 10^{-7}$ | 44.96/41 |
| $A_{f_2}$ | 12.0(±0.1) | $-1.8(\pm 0.3) \times 10^{-3}$ | $2.3(\pm 0.2) \times 10^{-6}$ | 143.34/41 |
| $\phi_{f_0} - <\phi_{f_0}>$ | $-1.1(\pm 0.2) \times 10^{-3}$ | $3.4(\pm 0.6) \times 10^{-6}$ | $-2.0(\pm 0.4) \times 10^{-9}$ | 34.34/41 |
| $\phi_{f_1} - <\phi_{f_1}>$ | $-1.6(\pm 0.2) \times 10^{-3}$ | $-4.6(\pm 0.6) \times 10^{-6}$ | $-2.6(\pm 0.4) \times 10^{-9}$ | 15.39/41 |
| $\phi_{f_2} - <\phi_{f_2}>$ | $-1.3(\pm 0.1) \times 10^{-2}$ | $4.1(\pm 0.4) \times 10^{-5}$ | $-2.5(\pm 0.2) \times 10^{-8}$ | 34.34/41 |
Figures

Figure 1

Variation of the amplitudes and phases (see the Manuscript file for full figure legend)
Figure 2

Interaction diagram of amplitudes of the 23 pulsation modes.
Figure 3

The best-fit seismic models for KIC6382916 along with the corresponding 122 evolutionary tracks from the main-sequence to the red giant branch. The solid lines and the dash lines represent the evolutionary tracks of $v_{eq} = 40 \text{ km s}^{-1}$ and $v_{eq} = 123 \text{ 120 km s}^{-1}$, respectively. The different colors represent different value of $[\text{Fe/H}]$. The 124 125 colored pluses and crosses represent the best-fit seismic models of different value of 126 $v_{eq}$. 