Model-Rich Approaches to Eliciting Possibly Weak or Incomplete Preferences: Evidence from a Multi-Valued Choice Experiment

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Abstract

This paper contributes to the elicitation of a decision maker’s strict preferences and their possible indifference or incomparability/indecisiveness. Every subject in both treatments of an incentivized lab experiment could choose multiple alternatives from each of the 50 distinct menus of popular gift-card pairs that they saw. Subjects in the non-forced-choice treatment could, in addition, avoid/delay making an active choice at those menus. Applying a non-parametric optimization method on data collected from 273 subjects, we find that nearly 60% of them are well-approximated by an indifference-permitting model of complete- or incomplete-preference maximization. Most recovered preferences are unique, have a non-trivial indifference part and, where relevant, a distinct indecisiveness part. The two kinds of distinctions between indifference and indecisiveness uncovered by this method are theory-guided and documented empirically for the first time. These findings suggest that accounting for possible indifferences and/or incomparabilities in the data-collection process and analysis can be useful in eliciting transitive weak preferences. Two aspects of the experimental design, finally, allow for interpreting an additional 10% of subjects as revealing a systematic preference for randomization or satisficing.

Keywords: multi-valued choice; (non-)forced choice; indifference; indecisiveness; experiment; optimization.

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1 Introduction

The state in which a decision maker is indifferent between two or more choice alternatives plays a prominent role in many domains of economic analysis. Despite its significance, however, it is not very well-understood at present how one can elicit and analyse an individual’s observable choice behaviour in ways that allow one to extract their potentially weak preferences and distinguish between their strict-preference and indifference parts. Even more challenging, if one also accepts that the individual in question may be indecisive, with preferences that are transitive but incomplete, how could such observable data be used to separate indifference from incomparability/indecisiveness?

These questions are important from both a theoretical and a policy perspective. Since many economic models assume that agents have preferences with non-trivial indifference parts, elicitation of the agents’ weak preference relations would allow testing these models’ descriptive relevance more accurately than if indifferences were assumed away. Moreover, knowing for example how many decision makers within a community consider a tree-planting program to be equally good to the development of a playground, and how many have a strict preference instead, would generally allow the local policy maker to arrive at a better decision than if all preferences were mistakenly interpreted to be strict. Similarly, understanding when agents are indifferent and when they are indecisive can enable the policy maker to decide whether to immediately make an active choice that will impact everyone in the group or delay such a choice and aim instead to inform the group’s members towards resolving their indecisiveness, e.g. via targeted information campaigns.

This paper’s first contribution is to propose and apply an incentivized lab experimental design which elicits observable behavioural data that could be used to recover an individual’s weak preferences and distinguish between their strict part, indifference part and, where relevant, incomparability/indecisiveness part. The main feature of the design is that it allows subjects to make multiple choices from every menu of alternatives they see by introducing incentives that help them make such possibly multi-valued choices in a preference-guided way. Subjects in the Non-Forced-Choice treatment can additionally avoid/delay making a choice from any menu at a small cost, whereas in the Forced-Choice treatment they are always required to choose at least one option.

We implemented this experimental design in an environment of riskless choice from 50 distinct menus that were derived from 6 pairs of popular £10 gift cards. We analysed the data that were collected from a total of 273 subjects by applying an optimization-based computational method that allows for a model-rich approach towards recovering an individual’s potentially weak and/or incomplete preferences. This method builds on and extends in a
model-based way the classic Houtman and Maks (1985) technique that is routinely used in empirical revealed preference tests of the rational choice/utility maximization hypothesis.

More specifically, the original Houtman-Maks technique computes the maximal subset of a subject’s dataset that is consistent with rational choice, essentially allowing for an intuitive quantification of the subject’s behavioural proximity with that model. The extension that we use applies this approximation principle simultaneously to the model of rational choice and two additional models of incomplete-preference maximization: (i) undominated choice, whereby an individual chooses the feasible alternative(s) that are not worse than anything else; (ii) dominant choice, whereby they choose the most preferred feasible alternative(s) if and only if those exist and defer otherwise. Consistent with the questions that we raised earlier, all three models predict multi-valued choices under some of their admissible preference orderings, while the latter two also afford or predict two theoretical separations between revealed indifference and incomparability/indecisiveness. By finding which model is closest to explaining each subject’s behaviour, this method effectively allows for recovering –possibly approximately, and with the standard as if qualifications in place– both the individual’s deterministic decision rule and their preferences conditional on this decision rule.

Our main findings are as follows:

1. Despite the large number of decisions, 56% of all subjects were perfectly or approximately explained by these models, with 63% of these best-matched by rational choice and the rest by some model of incomplete-preference maximization.

2. The fit was slightly better in the Non-Forced-Choice treatment but insignificantly so; yet subjects in that treatment made significantly more consistent active choices than their Forced-Choice counterparts. This shows that the negative forced-choice effect on consistency that was recently documented in Costa-Gomes et al. (2021) extends to possibly multi-valued choices from twice as many menus of riskless alternatives.

3. For 71% of the above subjects the model-optimal recovered preference relation features a non-trivial indifference part. This highlights the potential usefulness of indifference elicitation for revealed preference and welfare analyses.

4. For 36 of the 57 subjects who were best-matched by the models of undominated or dominant choice with incomplete preferences the recovered preference relations document empirically –for the first time– the distinct behavioural separations between revealed indifference and incomparability/indecisiveness that were proposed under these models by Eliaz and Ok (2006) and Gerasimou (2018a), respectively.

5. Two aspects of the experimental design allow for inferring that an additional 10% of
all subjects could be thought of as exhibiting systematic *satisficing* behaviour (Simon, 1956, 1955; Caplin et al., 2011; Reutskaja et al., 2011) or a *preference for randomization* (Agranov and Ortoleva, 2017; Dwenger et al., 2018).

Clearly, while one of the goals in this paper is to suggest a way that preference-guided multi-valued (non-)forced choices could be elicited in the lab, survey methods can obviously be used instead to elicit such data whenever incentivized decision making in a control environment is not practical or of primary concern. Furthermore, both non-forced and multi-valued choice data could in principle be collected in real-world environments too. For example, an online retailer may analyse data from a customer who repeatedly makes use of the retailer’s generous returns policy by buying multiple products with every order and then returning all but one for a full refund. In this case the retailer knows the menus seen by that customer, the set of products they bought in each case, and the one they finally kept. Similarly, the retailer may analyse data from another customer who often browses the products they see in various menus but ultimately decides to buy none.

A higher-stakes real-world example of observable non-forced choices comes from high-court judicial decision making and the fact that justices in such courts often have the luxury of choosing which cases to hear—and make a decision on—and which to ignore. Specifically, as Hitt (2019) notes for the case of the US Supreme Court (pp. 6-7), “the Court could protect the logical consistency and quality of its opinions by ignoring complex and multifaceted cases. [...] Essentially, [the Supreme Court Case Selections Act of 1988] gave the justices almost total freedom to opt not to decide most disputes. As such, whether consciously or not, the modern Supreme Court actively evolved away from decisiveness”. Hitt further argued that such potentially very impactful choices may have traceable and behaviourally intuitive underlying patterns, noting that “prioritizing consistency means that the Court will leave numerous important questions and conflicts unresolved” because “a desire to produce ‘good’ (consistent) law may induce the justices to prioritize consistency over decisiveness.”

The next section presents the experimental design and its implementation. Section 3 discusses the main aggregate-level findings on subjects’ behaviour across the two treatments. Section 4—the heart of the paper—proceeds with the individual-level analysis and the recovery of subjects’ decision rules and preferences. Section 5 elaborates on the two models of incomplete preferences with the identification of some revealed-preference axioms that can be used to perform pass/fail tests of these models in arbitrary choice domains. Online Appendices A–E contain proofs and additional information about the experimental design and data analysis.
2 Experimental Design

2.1 Structure

We outline the structure of the more general Non-Forced-Choice treatment design. At the beginning of the experiment subjects are allocated an initial monetary amount, $I > 0$. During the main part they are presented sequentially with a series of menus of choice alternatives. When a menu is shown subjects are asked to either choose one or more of the alternatives in that menu or to delay their choice by selecting “I’m not choosing now”. Subjects know that one menu will be picked at random for them at the end of the experiment. They also know that they will be rewarded with an element of their randomly selected menu, and that the decision they made at that menu during the main part of the experiment will be reminded to them before they are asked to make their final, payoff-relevant decision at that menu. Once subjects are past a menu during the main part of the experiment they never see it again unless that menu later turns out to be their randomly selected one.

Table 1: Incentives in the Non-Forced-Choice treatment are structured around the subjects’ 1st and 2nd decisions at their randomly selected menu (denoted here by $A$).

| 1st decision (main stage) | 2nd decision (payoff stage) | Choice reward | Cash reward | Possible interpretation |
|---------------------------|-----------------------------|---------------|-------------|-------------------------|
| $C^1(A) = A$              | none                        | random        | initial     | Revealed total indifference is costless & responded to literally |
|                           | possible                    | $a \in A$     | endowment $I$ |                         |
| $\emptyset \neq C^1(A) \neq A$ | $C^2(A) \in C^1(A)$ | $C^2(A)$ | $I$ | Stable revealed preference (possibly with ties) is costless |
| $\emptyset \neq C^1(A) \neq A$ | $C^2(A) \notin C^1(A)$ | $C^2(A)$ | $I_{rev} \in (0,I)$ | Unstable revealed preference is costly, and more so than revealed indecisiveness |
| $C^1(A) = \emptyset$     | $C^2(A) \in A$              | $C^2(A)$ | $I_{def} \in (I_{rev},I)$ |                         |

If a subject chooses one or more -but not all- options from their randomly selected menu during the main part of the experiment and wants to choose something from those previously chosen options if that menu is selected as their payoff-relevant one, then they receive that item as an in-kind reward and $I$ as a cash reward. If they choose something that is not among their previously chosen options at that menu, then they receive that item, together with $I_{rev} \in (0,I)$. If they had delayed choice at that menu originally, they are asked to choose an item now. In this case they receive this item as well as $I_{def} \in (I_{rev},I)$. Finally, if they had chosen everything at that menu originally, they receive $I$ and a randomly selected element of that menu. Denoting by $A$ a subject’s payoff-relevant randomly selected menu and by $C^1(A)$, $C^2(A)$ the choices they made at that menu during the main and final parts, respectively, Table 1 summarizes the incentive structure in this treatment. As will be discussed in more detail below, $C^1(\cdot)$ is a possibly empty- and multi-valued choice correspondence; hence, it
satisfies $\emptyset \subseteq C^1(A) \subseteq A$ for every menu $A$. By contrast, $C^2(\cdot)$ assigns a single chosen alternative to the relevant subject’s randomly selected menu.

We stress that both treatments allow subjects to also choose all feasible alternatives. Naturally, an analyst may wonder whether a subject who behaves in this way at some menu effectively reveals that they are indifferent between all alternatives that are contained in it. To make this interpretation possible the design rewards a subject with a randomly selected alternative from any menu where everything is chosen if and only if the decision at that menu later turns out to be the subject’s payoff-relevant one at the end of the experiment. By contrast, if a decision maker either chooses and rejects some alternative(s) or defers making an active choice, then rewarding them with something from the relevant menu at random would be undesirable because they might end up, respectively, with an option they clearly dislike or one whose relative preference standing they haven’t determined yet.

More specifically, under this design it is clear that, for a utility-maximizing subject, choosing everything in a menu is only compatible with total indifference at that menu because, by definition, they do not care which alternative they end up with, or whether this is decided by someone else or randomly (see also Danan, 2010 for a formal elaboration of this point). Conversely, if such a subject has one or more most preferred alternative(s) at the menu and these are strictly better than something else, then choosing everything is dis-incentivized by the design because doing so comes with the risk of potentially receiving an inferior alternative as a reward. The design instead incentivizes such a subject to select those and only those options that they prefer the most, because doing so enables them to choose one of these superior options at the end and also receive their full cash endowment. For such individuals, therefore, the design is in line, for example, with Kreps’ (2012, p.2) account of multi-valued choice: “The story is that the consumer chooses one element of $A$. Nonetheless, we think of $c(A)$ as a subset of $A$, not a member or element of $A$. This allows for the possibility that the consumer is happy with any one of the several elements of $A$, in which case $c(A)$ lists all those elements. When she makes a definite choice of a single element, say $x$, out of $A$—when she says in effect, ‘I want $x$ and nothing else’—we write $c(A) = \{x\}$, or the singleton set consisting of the single element $\{x\}$. But if she says, ‘I would be happy with either $x$ or $y$’, then $c(A) = \{x, y\}$.”

Suppose now, instead, that the subject is not a utility maximizer but has incomplete preferences and cannot compare any two of the feasible options at a menu. Then, depending on the individual’s subjective perception of the decision’s importance, they may either opt to choose everything and end up with something at random or opt to incur the relatively small cost $I - I_{def}$ to delay making an active choice at that menu themselves. The former kind of behaviour could be seen as analogous to recent findings suggesting that decision makers
prefer to randomize when they are faced with a difficult decision repeatedly (Agranov and Ortoleva, 2017; Dwenger et al., 2018). Importantly though, it could also be compatible with the model of undominated choice with incomplete preferences –discussed below– whenever every feasible option is incomparable to all others. By contrast, deferring at a small cost could be seen as a manifestation of indecisiveness-driven deferral (Tversky and Shafir, 1992; Costa-Gomes et al., 2021) and may potentially be compatible with the model of dominant choice with incomplete preferences, also discussed below.

With regard to the related literature, this structure extends the one in the single-valued non-forced-choice design in Costa-Gomes et al. (2021) by enabling the analyst to also elicit multi-valued choices from all menus in a preference-guided way. We refer the reader to that paper for a review of and comparison to other approaches toward eliciting incomplete preferences that assume availability of different kinds of choice datasets. We also refer the reader to earlier versions of that paper for details on a more restrictive method that uses survey data to augment single-valued choices with indifference information. In work that predates the present study, moreover, Bouacida (2021) obtains multi-valued choices in a forced-choice environment from menus of real-effort tasks. In his experimental design subjects are paid extra for every additional alternative they choose from a menu, and are given one of these alternatives at random as a reward. Balakrishnan et al. (2021) propose instead to use data from a decision maker’s repeated choices from the same collection of menus to define that individual’s generally multi-valued choice correspondence. They do so by means of a parametric threshold rule whereby an individual’s choosable options at a menu is the set of all feasible options whose choice frequency exceeds a cut-off probability. The latter is defined by scaling down the highest choice frequency in the menu by some menu-independent factor. Evidently, our approach toward eliciting choice correspondences in this paper is significantly different from –and complements– those above.

2.2 Implementation

Table 2 summarizes the key information about the implementation of this experimental design. There were a total of 6 choice alternatives. Each was worth £20 and comprised a pair of distinct £10 gift cards from popular UK and/or international brands: two supermarkets, two coffee shops, one bookshop, and a gift card that enabled dining at one of nine restaurants. All 6 cards in these 6 pairs could be redeemed in at least one venue in the local town centre (subjects were explicitly informed about this), with the restaurant gift card being redeemable in 3 such venues. Each of the 6 gift cards appeared in exactly 2 of the 6 pairs, once showing up as the first item in the pair and once as the second (more details are available in Appendix B).
There are several reasons why the experimental choice alternatives were selected to be gift-card pairs, and those pairs in particular. First, gift cards can be thought of as restricted forms of cash that can be used for consumption, informally traded or, indeed, gifted. As such, they are intrinsically valuable. Second, these particular gift cards were issued by some of the most popular leisure and grocery destinations for the local student population, and were all within a short walking distance from each other. Hence, all were expected to be desirable to everyone in this experiment’s subject pool. Finally, presenting the choice alternatives as gift card pairs rather than as separate gift cards is realistic (many online retailers invite consumers to choose their “gift card bundle”) and could potentially lead to some relatively hard decisions.

Out of the 63 non-empty menus that are derivable from this set of 6 alternatives, subjects were presented with the 15 binary, 20 ternary and 15 quaternary menus. Each of the 50 distinct menus was presented once and the order of menu presentation was randomized and differed between subjects. This choice domain is therefore rich, heterogeneous and symmetric in the sense that all alternatives are feasible in exactly the same number of menus (see also Appendix C for more details on this). Furthermore, the total number of 50 decisions coincides with the one used in experiments on budget-constrained choice from Arrow-Debreu securities such as Choi et al. (2007) and Halevy et al. (2018). Unlike these studies, the riskless choice objects in this experiment were presented naturalistically, and subjects did not operate under a budget constraint.

Table 2: Summary information on the two experimental treatments.

|                              | Non-Forced-Choice treatment | Forced-Choice treatment |
|------------------------------|------------------------------|------------------------|
| **Initial endowment**        | £2.40                        | £2.40                  |
| **Choice-reversal cost**     | £1.20                        | £1.20                  |
| **Choice-deferral cost**     | £0.30                        | N/A                    |
| **Original sample**          | 141                          | 141                    |
| **Subjects excluded because they always chose everything** | 1                            | 3                      |
| **Subjects excluded because they always deferred** | 5                            | N/A                    |
| **Subjects excluded - total**| 6                            | 3                      |
| **Subjects after all exclusions** | 135                          | 138                    |
| **Location**                 | St Andrews lab               | St Andrews lab         |
| **Dates when the experimental sessions were conducted** | 17–20 Sept 2019              | 29 Jan 2020 (62)       |
|                              |                              | 22 Sept 2021 (79)      |

As soon as subjects finished all tasks their randomly selected menu showed up on their screens, together with the reminder of the decision they had made at this menu. As an additional incentive for subjects to make deliberated and non-rushed decisions, they were told from the beginning that no participant would be able to receive their rewards and
leave the lab in the first 50 minutes of the session. The experimenter (this author) went
to each subject’s desk once they were finished (after this threshold was exceeded), asked
them about their final choice at this menu, and later gave them their cash and gift card
rewards accordingly. Subjects who had chosen everything at their randomly selected menu
were in turn invited to the experimenter’s desk and, after the appropriate numerical range
was specified on the random-number generating website https://random.org, and the way
in which the numbers in this range were mapped to the relevant gift card pairs was agreed,
a random number was generated to determine the pair they would be rewarded with. A
nine-question understanding quiz preceded the main part of the experiment in each of the
two treatments. Subjects could not proceed to the main part of the experiment until they
answered all questions correctly.

The experiment’s computer interface was programmed in Qualtrics and executed in full-
screen mode on a web browser that prevented subjects from exiting the interface without
the experimenter’s intervention. Subject recruitment was done with ORSEE (Greiner, 2015).
All menus appeared as unnumbered vertical choice lists. The “I’m not choosing now” option
in the Non-Forced-Choice treatment was always the last item. Because subjects often had to
scroll down to find and select that option, this positioning means that it was often physically
harder for them to avoid/delay choice. There were a total of 141 participants in each of the
two treatments. The 9 subjects who always deferred or always chose everything were
excluded from the analysis because their choice behaviour is completely uninformative. Every
subject received both their in-kind and cash rewards as specified above.

3 Aggregate-Level Analysis

3.1 Choice Sizes and Response Times

We start with a key new variable of interest, which we refer to as the subjects’ choice size
and which is defined simply as the number of gift-card pairs that were chosen at each menu.
This variable ranges between 0 and 4 in the Non-Forced-Choice treatment, and between
1 and 4 in the Forced-Choice treatment. However, because choice sizes 1 and 2 (as well
as 0 in the former treatment) are always feasible whereas 3 and 4 are not, we adjust their
relative frequencies accordingly. Specifically, Figure 1 presents the distributions of menu-size
adjusted choice sizes in the two treatments, which are derived once their absolute frequencies
are divided by the total number of menus where these choice sizes might be observed. For
example, the denominators here are $50 \times N$ and $15 \times N$ for choice sizes 1 and 4, respectively,
where $N$ is the total number of subjects in the relevant treatment.
This comparison shows that the modal adjusted choice size was 1 in both treatments, at a rate of just over 50%, and was followed by 2 and 3. The deferral rate in the Non-Forced-Choice treatment, defined as the relative frequency of a zero choice size, is 7.4%. This is slightly higher than the 6.9% choose-everything rate, which is obtained by weighted-averaging the relative frequencies where \( n \) out of \( n \) items were chosen, for \( n = 2, 3, 4 \) (Table 3). The latter rate is also lower than the corresponding one in the Forced-Choice treatment (8.5%), and the difference is statistically significant (\( p < 0.001 \); two-sided Fisher’s exact test). This difference is consistent with the intuition that, in the absence of the possibility to delay choice when faced with a potentially difficult decision, subjects are more likely to choose everything that is available in the menu, either as a result of following a more general decision rule or, in the context of our experimental design, possibly due to a preference for randomization. Finally, the rate at which subjects chose all 4 alternatives was very low in both treatments, at 3.2% and 2.5% in the Forced- and Non-Forced-Choice treatments, respectively.

Table 3 elaborates further on this theme by also showing how the relative frequencies of different choice sizes vary with the number of alternatives at different menus, and by also presenting the corresponding average response times. These conditional relative frequencies are uniformly higher in the Forced-Choice treatment for choice sizes 1 and 2 in binary and ternary menus. In addition, proportionally more subjects in that treatment chose 2, 3 or 4 gift-card pairs in quaternary menus too. As far as deferrals in the Non-Forced-Choice treatment are concerned, these were more likely in binary menus (\( \approx 12\% \)) than in ternary
Table 3: Choice sizes and the corresponding relative frequencies and average response times at menus of different sizes.

| Alternatives chosen in the Non-Forced-Choice treatment | (average response times, in seconds, in parenthesis) |
|--------------------------------------------------------|------------------------------------------------------|
| 0 | 1 | 2 | 3 | 4 |
| Binary menus | 11.95% (6.19) | 74.52% (6.01) | 13.53% (7.93) | – | – |
| Ternary menus | 6.85% (9.62) | 44.67% (7.25) | 43.33% (8.93) | 5.15% (8.29) | – |
| Quaternary menus | 3.70% (12.71) | 33.53% (8.98) | 33.28% (9.93) | 27.02% (11.24) | 2.47% (12.92) |

| Alternatives chosen in the Forced-Choice treatment | (average response times, in seconds, in parenthesis) |
|--------------------------------------------------|------------------------------------------------------|
| 0 | 1 | 2 | 3 | 4 |
| Binary menus | – | 83.77% (5.82) | 16.23% (7.56) | – | – |
| Ternary menus | – | 46.19% (7.31) | 47.25% (8.44) | 6.56% (8.88) | – |
| Quaternary menus | – | 28.50% (8.27) | 41.01% (10.06) | 27.29% (11.25) | 3.18% (13.13) |

(≈ 7%) or quaternary menus (≈ 4%).

Notably, the average response time in both treatments was always lowest (≈ 6 secs) when subjects chose a single pair of gift cards, while it increased monotonically (and statistically significantly) in similar ways as the size of choices and menus increased (Figure 2). These facts might be seen as intuitive evidence suggesting that the subjects’ decision was easier at menus where they chose a single gift-card pair, perhaps because that was their clearly preferred one. However, they are also consistent with an alternative interpretation whereby the time was shorter in those cases because subjects only had to move the mouse cursor on their computer to check a single choice box on their screen, thereby mechanically resulting in a shorter response time than when multiple items were selected from relatively larger menus. This argument is less relevant in the case of deferral decisions, however, where the average response time increases monotonically from (approximately) 6 to 10 to 13 secs when the menu includes 2, 3 and 4 alternatives, respectively. This suggests that the decision to defer was not generally based on some menu-irrelevant strategy (e.g. to reach the end of the experiment quickly), but was influenced instead by the relevant menu’s composition and, presumably, the subjects’ preferences at that menu. Finally, there is no significant difference in the distributions of response times in the subjects’ active choices across treatments ($p = 0.612$; two-sided Mann-Whitney $U$ test).

Next, we define a subject’s choice proportion at a menu as the number of chosen alternatives divided by the number of feasible alternatives at that menu. The distributions of average choice proportions across the two treatments are shown in Figure 3 (top panel) and are significantly different (higher in the Forced-Choice treatment) both when deferrals are included and when they are not ($p = 0.006$ and $p < 0.001$, respectively; two-sided
Figure 2: Response times are positively correlated with menu and choice size in both treatments.

(a) Forced-Choice treatment

Spearman R = 0.38, p < 0.001

(b) Non-Forced-Choice treatment

Spearman R = 0.34, p < 0.001

Note: The violin plots are thicker (thinner) in regions with more (fewer) observations, and also show the conditional average response times and 95% confidence intervals per menu/choice size.
Figure 3: Distributions of the average choice proportions and menus where subjects chose everything or deferred.

(a) Forced-Choice treatment
(b) Non-Forced-Choice treatment

Notes: Unless otherwise noted, all p-values below are from two-sided Mann-Whitney U tests. The distribution of average choice proportions (top panel) is significantly different between treatments, both when deferrals are included (p = 0.006) and when they are not (p < 0.001; figure not shown). The per subject distribution of menus where everything was chosen (middle panel) is not significantly different between treatments (p = 0.127), although the rate at which subjects exhibited such behaviour is higher in the Forced-Choice treatment (8.4% vs 6.9%; p < 0.001; two-sided Fisher’s exact test), and so is the proportion of subjects who did so at least once (71% vs 60%; p = 0.057; two-sided Fisher’s exact test). See also main text. In the Non-Forced-Choice treatment the distribution of the number of menus where everything was chosen is not significantly different from the corresponding distribution where subjects deferred (bottom right panel; p = 0.498).
Mann-Whitney $U$ tests). The mean/median choice proportions across all subjects in the Forced- and Non-Forced-Choice treatments were 0.54/0.5 and 0.49/0.5, respectively. That is, subjects in both treatments tended to choose around half of the feasible alternatives, on average.

Additionally, 69 subjects (51.1%) in the Non-Forced-Choice treatment deferred at least once, with deferrals per subject ranging between 1 and 42 (Figure 3-(b); bottom panel), and with a mean/median occurrence of 7.28/5 menus. Moreover, 81 subjects in this treatment (60%) chose all feasible gift-card pairs at least once, with such occurrences per subject ranging between 1 and 38 menus (Figure 3-(b); middle panel), and with a mean/median occurrence of 3.42/1 menus. By contrast, in the Forced-Choice treatment there were 98 subjects (71%) who chose everything at least once, with occurrences per subject ranging between 1 and 45 (Figure 3-(a); middle panel), and with a mean/median of 4.22/2 menus. The difference in the two proportions is borderline (in)significant at the 5% level ($p = 0.058$; two-sided Fisher’s exact test). These findings lend further support to the possibility that, unable to delay their choice when faced with a hard decision, subjects are more likely to choose everything.

### 3.2 Choice Consistency

We now proceed to the analysis of choice consistency (Table 4). Following Costa-Gomes et al. (2021), we first compare the proportions of subjects who made perfectly consistent active choices in the two treatments while also accounting here for possibly non-trivial indifferences. That is, we find the proportions of subjects whose single- and/or multi-valued choices –ignoring any deferral decisions– that are compatible with indifference-permitting utility maximization (see Section 4 for a formal statement of this model). These are approximately 4% and 13% for Forced- and Non-Forced-Choice subjects, respectively, and the difference is significant ($p = 0.007$; two-sided Fisher’s exact test). Second, we compute the original Houtman-Maks (1985) index for each subject’s active choices and compare the two treatments’ distributions. Recall that this index gives the smallest number of active choices that need to be removed from a subject’s dataset in order for the remaining ones to be compatible with the model of rational choice. Forced- and Non-Forced-Choice subjects had a mean/median Houtman-Maks index of 10.81/10.5 and 9.05/6, respectively, and the two distributions are significantly different ($p = 0.033$; two-sided Mann-Whitney $U$ test).

Finally, to account for the fact that deferring at a menu can never decrease a decision maker’s active-choice consistency we also carry out subject-specific simulations to find how likely it would be for a possibly deferring subject to attain their Houtman-Maks score given that they made their active choices only at those particular menus where they did so. More specifically, in line with Selten (1991), Beatty and Crawford (2011) and Costa-Gomes et al.
Table 4: Active-choice consistency in the two treatments.

| Treatment                  | Proportion of perfectly consistent subjects | Average (median) number of active choices away from utility maximization |
|----------------------------|---------------------------------------------|------------------------------------------------------------------------|
| Forced-Choice treatment    | 3.62% (5/138)                               | 10.81 (10.5)                                                          |
| Non-Forced-Choice treatment| 12.59% (17/135)                             | 9.05 (6.0)                                                            |

Note: The first and second p-values are from two-sided Fisher’s exact and Mann-Whitney U tests, respectively.

Figure 4: Choice consistency is negatively correlated with average response times in both treatments.

(a) Forced-Choice treatment

(b) Non-Forced-Choice treatment

Notes: $R$ is the Spearman correlation coefficient and $p$ is the p-value. Shaded areas indicate 95% confidence intervals. There is no significant difference in the average response times between treatments ($p = 0.550$; two-sided Mann-Whitney U test).
we compute in each treatment the Selten measure of predictive success of utility maximization on subjects’ active choices. We do so as follows: (i) for every experimental subject we create 10,000 datasets from as many artificial subjects who were restricted to make uniform-random active choices only at those menus where the experimental subject did so; (ii) we find the proportion of such subject-specific simulated datasets within this block that have a zero Houtman-Maks score when both indifference-permitting and indifference-excluding utility maximization are accounted for. Then, from the proportion, \( p_i \), of perfectly consistent human subjects in treatment \( i \) we subtract the average –over all such blocks– proportion of artificial subjects who are also perfectly consistent, \( a_i \).

The closer the difference \( m_i := p_i - a_i \) is to 1, the higher the proportion of subjects who behaved as if they were utility maximizers, and the more likely it is that this could not have happened randomly. Low but positive values on the other hand could arise either because (i) relatively many experimental subjects were consistent but this could probably be due to chance; or (ii) because relatively few subjects were consistent and consistency in this environment was unlikely to occur by chance. It turns out that the latter case applies to the data from both our treatments, where we have \( m_{FC} = p_{FC} - a_{FC} = 0.0362 - 0 = 0.0362 \), and \( m_{NFC} = p_{NFC} - a_{NFC} = 0.126 - 0 = 0.126 \). Therefore, the results from these three analyses show that the negative forced-choice effect on consistency that was recently documented for single-valued choices in Costa-Gomes et al. (2021) extends to possibly multi-valued choices from twice as many menus (50 vs 26), over more alternatives (6 vs 5), and when indifference-permitting utility maximization is also accounted for.

We also ask how a subject’s choice consistency, as captured by the above Houtman-Maks index, is related to their response times. Figure 4 (top panel) shows that there is a significant –and nearly identical, with an approximate coefficient \( R = 0.4 \)– positive correlation in each treatment between subjects’ active-choice Houtman-Maks indices and their average response times. That is, subjects’ choice consistency is negatively correlated with the time it takes for them to make their active choices. A possible explanation for this interesting and perhaps counter-intuitive finding is that subjects with a higher cognitive ability (an unmeasured variable in our study) are both more consistent and faster than subjects with a lower cognitive ability. A distinct possible explanation is that spending more time before deciding is more characteristic of people who are prone to second thoughts and therefore more likely to be involved in choice reversals when presented with a series of decision problems. Conversely, it is also possible that subjects who had neither a stable preference relation over the choice alternatives nor a clear decision rule ended up spending more time on average at each menu, but without this extra time ultimately alleviating the effects of their ambivalence.

Figure 5 further shows that there is also a significant negative correlation between sub-
Figure 5: Choice consistency is negatively correlated with the average choice size.

(a) Forced-Choice treatment

(b) Non-Forced-Choice treatment

Note: $R$ is the Spearman correlation coefficient and $p$ is the p-value. Shaded areas indicate 95% confidence intervals.

jects’ active-choice consistency and their average choice sizes in each of the two treatments, with this relationship being more pronounced for Non-Forced-Choice subjects ($R = 0.54$ vs $R = 0.34$ on Houtman-Maks index). In line with our preceding findings and discussion, an intuitive explanation for this difference is that, unlike Non-Forced-Choice subjects, their Forced-Choice counterparts could not defer at menus where they might perhaps have wished to do so, opting instead for more alternatives per menu. But while delaying choice when confronted with a difficult problem safeguards the consistency of one’s behaviour, choosing more (possibly all) alternatives could do the opposite because it opens up more possibilities for choice reversals/cycles to emerge.

Figure 6: In the Non-Forced-Choice treatment, subjects who deferred are significantly more consistent.
Some additional support to this explanation, finally, is obtained by also comparing the active-choice consistency of Non-Forced-Choice subjects who deferred at least once to the consistency of those who did not. More specifically, 14 of the 69 (20%) deferring subjects were perfectly consistent in this sense, while 3 of the 66 (4.5%) non-deferring ones were such (equivalently, 14 out of the 17 perfectly consistent in this treatment deferred at least once). The difference between these two proportions is significant ($p = 0.008$; two-sided Fisher’s exact test). Moreover, the two groups had an average Houtman-Maks score of 6.83 and 11.38, respectively, and the difference in the two distributions is significant as well ($p = 0.001$; two-sided Mann-Whitney $U$ test). In fact, as shown in Figure 6, deferring subjects are uniformly more consistent than non-deferring ones in the sense that the distribution of their Houtman-Maks scores first-order stochastically dominates the corresponding distribution of the latter subjects. That is, for every number less than or equal to $n$, and for every $n$, non-deferring subjects were more likely to be strictly more than $n$ active choices away from utility maximization. This within-treatment effect is in the spirit of the negative forced-choice effects that were first reported in Costa-Gomes et al. (2021) and also above, and clarifies the important mediating role of deferrals for consistency, even in cases such as this where the average/median number of deferrals is 7.3/5 out of 50 and where decision makers are allowed to also make multi-valued choices.

4 Individual-Level Analysis

4.1 Three Deterministic Models of Preference-Maximizing Choice

As was mentioned in the Introduction, in the main part of our individual-level analysis we consider three simple choice models of deterministic preference maximization that impose a rich structure on observable behaviour:

1. Rational choice/utility maximization.
2. Undominated choice with incomplete preferences.
3. Dominant choice with incomplete preferences.

We focus on these models for several reasons. First, they all feature stable preferences and predict both single-valued and multi-valued choices under different preference orderings, with multi-valuedness potentially interpretable as revealing indifferences and single-valuedness as revealing strict preferences. Second, they impose strong behavioural restrictions. In particular, all three models satisfy the fundamental Property $\alpha$ (Sen, 1971) or Contraction Consistency principle, which requires an alternative to be chosen at a menu whenever it is
feasible at that menu and also chosen at a larger menu. In addition, the first and third models predict active choices that are actually consistent with the Congruence/Strong Axiom of Revealed Preference principle, which rules out all forms of choice cycles. Third, these models are defined in terms of (and hence in principle allow the analyst to recover) a single preference relation, thereby making the welfare-relevant parts of the analysis unambiguous. Fourth, all three models are uniquely identifiable. That is, if a decision maker’s observable behaviour is perfectly compatible with one of these models and the available data are sufficiently rich (as is the case in our experiment), then there is a unique preference relation with which that model explains the individual’s behaviour. Fifth, they are sufficiently computationally tractable to allow for the behaviourally intuitive optimization-based goodness-of-fit test that we describe below. Finally, in light also of the preceding discussion about the structure and possible interpretation of the experimental design, and in light also of the empirical findings presented above, the three models predict the kinds of active-choice and deferring behaviour that one might expect to observe in our data.

To state the models formally we first define a decision maker’s choice dataset $D = \{(A_i, C(A_i))\}_{i=1}^{k}$ on a finite grand choice set $X$ to be a collection of pairs that comprise a non-empty menu $A_i \subseteq X$ and a -possibly empty- set of alternatives that were chosen at this menu when the decision maker was presented with it. Thus, $\emptyset \subseteq C(A_i) \subseteq A_i$ holds for all $i \leq k$. Dataset $D$ is explainable by rational choice/utility maximization if there exists a complete and transitive preference relation $\succsim$ on $X$ such that, for all $i \leq k$,

$$C(A_i) = \{x \in A_i : x \succsim y \text{ for all } y \in A_i\}. \quad (1)$$

If, instead, (1) is true for all $i \leq k$ with respect to a reflexive and transitive but incomplete preference relation, then $D$ is explainable by the model of (maximally) dominant choice with incomplete preferences. In that case we have

$$C(A) \neq \emptyset \iff \text{there is } x \in A \text{ such that } x \succsim y \text{ for all } y \in A,$$

$$C(A) = \emptyset \iff \text{for all } x \in A \text{ there is } y \in A \text{ such that } x \not\succsim y.$$

Finally, $D$ is explainable by the model of undominated choice with incomplete preferences if there is a reflexive, transitive and incomplete preference relation $\succsim$ whose asymmetric part is $\succ$, such that, for all $i \leq k$,

$$C(A_i) = \{x \in A_i : y \not\succ x \text{ for all } y \in A_i\}. \quad (2)$$

The model of rational choice/utility maximization was characterized by Richter (1966) in
a general environment that encompasses the one within which we are operating here. The
model of undominated choice with incomplete preferences was introduced by Schmeidler
(1969) in a general equilibrium setting and was analyzed choice-theoretically under a vari-
ety of decision environments and preference structures by, most notably, Schwartz (1976),
Bossert et al. (2005), Eliaz and Ok (2006), Bossert and Suzumura (2010) and Stoye (2015).
Dominant choice with incomplete preferences was studied theoretically in Gerasimou (2018a)
and empirically in Costa-Gomes et al. (2021), with the latter analysis limited to an envi-
ronment of single-valued non-forced choice experimental data which, in particular, do not
allow for testing the model of undominated choice or distinguishing between indifference and
indecisiveness in either of the two models of choice with incomplete preferences.

The two models of incomplete-preference maximization are logically distinct. Moreover, if
we replace the term “incomplete” with “possibly incomplete” in their respective statements,
then these models generalize rational choice in different ways. The first does so by relaxing
active-choice consistency while retaining the decisiveness (non-emptiness) assumption that
requires $C(A_i) \neq \emptyset$ for all $i \leq k$. The second model does so by relaxing the decisiveness
assumption while retaining active-choice consistency.

We test our possibly multi- and/or empty-valued experimental choice data for compati-
bility with each of these three models. We do so by using a model-based extension of the
classic Houtman and Maks (1985) method that is standard in empirical revealed prefer-
ence analysis. Specifically, we ask what is the smallest number of decisions that need to be
dropped/changed in a subject’s dataset in order for it to be compatible with that model
under some of its instances/admissible preference orderings. We refer to this number as
the model’s distance score. These scores were computed using a method that was originally
introduced in a working-paper version of Costa-Gomes et al. (2021) and is now greatly ex-
tended and freely available online in the open-source desktop application Prest (Gerasimou
and Tejšíčák, 2018).

More specifically, a brute-force algorithm was implemented for these computations. This
involved the production of all choice datasets that are generated by all possible instances of
every model, and comparing each such dataset against every subject’s own dataset in order
to find the model(s) and instance(s) that are closest to it in the minimum distance-score
sense. Therefore, this method detects perfect as well as approximate model fits, and in the
latter case it quantifies the approximation in an intuitive way that may be interpreted as the
number of “mistakes” made by an agent who might be portrayed as following a particular
decision rule. For other approximations that have been proposed and/or used recently for
distinct classes of models and choice environments we refer the reader to Choi et al. (2007);
Echenique et al. (2011); Choi et al. (2014); Apesteguia and Ballester (2015); Dean and Martin
We stress that despite the vast numbers of weak orders (4,683), partial orders (130,023) and, especially, incomplete preorders (209,527) that are defined on a set of 6 alternatives (OEIS, 2021; see Section 5 for formal definitions of these terms), the above tool makes such an exact computation possible very quickly—in less than 12 seconds—for all subjects in each treatment, for each of the three models. We also stress that, although this tool lies within the rapidly expanding realm of (broadly interpreted) Artificial-Intelligence technologies it builds on combinatorial-optimization methods to arrive at exact solutions to a clearly defined and behaviourally intuitive problem. In particular, it does not feature Deep-Learning decision algorithms which build on neural-network structures that may be unclear/uninterpretable or prone to over-fitting. Finally, although the brute-force algorithm is linear in the number of subjects but exponential in the number of alternatives, the model-rich distance-score method itself is scalable and can be extended to analyse datasets that are derived from much larger sets of alternatives by employing powerful open-source constraint solvers such as Gecode (Gecode Team, 2006) and Z3 (Moura and Bjørner, 2008, Microsoft Research).

Table 5: Classification of subjects who are perfectly or approximately explainable by a model.

| Non-Forced-Choice treatment (N = 135) | Dominant Choice with Incomplete Preferences | Undominated Choice with Incomplete Preferences | Rational Choice |
|--------------------------------------|---------------------------------------------|-----------------------------------------------|------------------|
| % of subjects with Score = 0          | 2.22%                                       | 5.19%                                         | 7.41%            |
| % of subjects with Score ≤ 10         | 24.44%                                      | 5.19%                                         | 28.15%           |
| Mean (median) best score              | 4.15 (3)                                    | 5.57 (5)                                      | 3.39 (3)         |
| Minimum score in simulations          | 27                                           | 26                                            | 18               |
| Mean (median) best-model preference orderings | 1.00 (1)                                    | 1.29 (1)                                      | 1.08 (1)         |

| Forced-Choice treatment (N = 138)    | Dominant Choice with Incomplete Preferences | Undominated Choice with Incomplete Preferences | Rational Choice |
|--------------------------------------|---------------------------------------------|-----------------------------------------------|------------------|
| % of subjects with Score = 0          | 3.62%                                       | 0%                                            | 3.62%            |
| % of subjects with Score ≤ 10         | 46.38%                                      | 8.70%                                         | 55.07%           |
| Mean (median) best score              | 4.06 (4)                                    | 7.25 (8)                                      | 4.56 (4)         |
| Minimum score in simulations          | 25                                           | 23                                            | 23               |
| Mean (median) best-model preference orderings | 1.17 (1)                                    | 1.00 (1)                                      | 1.15 (1)         |

Notes: Model-score ties were always broken in favour of Rational Choice/Utility Maximization (no other ties emerged). When Rational Choice/Utility Maximization was not a subject’s optimal model, the difference between this and the optimal model’s score was on average 4.2 and 6.5 in the Forced- and Non-Forced-Choice treatments, respectively.

Table 5 and Figure 7 present summaries of this goodness-of-fit analysis for both the Forced- and Non-Forced-Choice subjects. In line with standard practices in revealed preference analysis whereby one also wishes to understand the extent to which a certain behaviour
Figure 7: Perfect and approximate fits for the models of rational choice, undominated and dominant choice with incomplete preferences, and the corresponding fits from simulated subjects.

(a) Forced-Choice treatment

(b) Non-Forced-Choice treatment

Notes: (i) For experimental subjects the histograms show the relative frequencies of those who are best-explained by the respective models and have a distance score between 0 and 10; (ii) ties between Rational Choice and each of the other two models were broken in favour of the former: 92 such ties occurred between Rational and Undominated Choice (60 and 32 ties under Forced and Non-Forced Choice) and 39 between Rational and Dominant Choice (Non-Forced Choice); there were no ties between the two incomplete-preference models.
or perfect/approximate model fit could have been generated randomly (Bronars, 1987), we also performed our model analysis on 100,000 simulated datasets of artificial uniform-random behaving subjects under both a forced- and a non-forced choice configuration (details on the simulations' structure are available in Appendix D). The distance-score distributions of simulated subjects are juxtaposed in the relevant graphs of Figure 7 to those of the experimental subjects that are perfectly/approximately explained by one of the three models, while the minima of the simulated-subject distributions are also presented in Table 5.

For experimental subjects Rational Choice often tied with one but never both of the other two models; our classification broke ties in favour of that model whenever a subject’s distance score was produced by rational choice and another model (see the notes of Table 5 for more details). Under this tie-breaking assumption, and in line also with model-approximation methods used in the above-cited studies, our two exhibits show the number, proportion and relative-frequency distributions of distinct subjects that are on average within 20% (equivalently, within 10 decisions) away from being explainable perfectly by an instance of some model, separately for each of the two experimental treatments. We chose this conservative approximation range for two factors: (i) simulations suggest that a distance score of 10 for any of the three models is extremely unlikely to occur randomly in this decision environment (see Table 5 and 7); (ii) for subjects with a distance score up to 10 there is typically only one best-matching preference ordering that explains their behaviour under the respective subject-optimal model.

The total number of Non-Forced-Choice subjects with a perfect or approximate fit was 78 (58%); 33, 7 and 38 of them were categorized under rational choice, undominated and dominant choice with incomplete preferences, and 3, 7 (7%) were perfectly compatible with the first and third of these models, respectively. The total number of Forced-Choice subjects with a perfect or approximate fit was 76 (55%), with 64 and 12 of them classified under rational choice and undominated choice with incomplete preferences, respectively. In addition, 5 of these subjects (4%) conformed with rational choice perfectly. The fit was slightly better on average in the Non-Forced-Choice treatment (mean/median: 3.91/3) than in the Forced-Choice one (mean/median: 4.56/4), although the distributions were not significantly different ($p = 0.173$; two-sided Mann-Whitney $U$ test). Similarly, and consistent with the results from the previous section, the proportion of subjects who were best-matched by one of two models of consistent active choices is higher in the Non-Forced-Choice treatment (53% vs 46%), while the proportion of subjects who were best-matched by the model of of imperfectly consistent active choices is lower in that treatment (5% vs 9%). Moreover, when rational choice was not the best-matching model, its distance score was on average 6.5 and 4.2 units higher in these treatments, respectively. Finally, the highest distance score of 10
As far as preference recovery is concerned, for the vast majority of these subjects there was a unique preference ordering that generated their distance score under their best-matching model, with the mean/median number of such orderings being 1.15/1 and 1.06/1 in the Forced- and Non-Forced-Choice treatments, respectively. The directed graphs of the subject-optimal preference orderings that were recovered by this method are shown in Appendix E for all 154 subjects that were included in this analysis. Importantly, for 110 (71%) of them the recovered optimal preference relation(s) has a non-trivial indifference component. This highlights the potential usefulness of eliciting indifferences for revealed preference and welfare analysis.

In summary, despite the relatively large number of decisions made, the behaviour of nearly 60% of all 273 subjects in the experiment was either perfectly or approximately matched by some simple but richly structured deterministic model of complete or incomplete preference maximization. Rational choice accounts for the behaviour of 63% of all subjects in this group (35.5% of the total), while the two models of incomplete-preference maximization together account for the remaining 37% (21% of the total). Consistent with the results from Section 3.2, the model-based analysis suggests that proportionately more subjects in the Non-Forced Choice treatment are well-approximated by a model of consistent active choices, although the difference is not significant. Additionally, the optimal preference relation that was recovered conditional on a subject’s best-matching model typically features a non-trivial indifference relation. Finally, it is highly unlikely that those subjects’ behaviour and model fit have been generated by chance. These findings together point to the potential relevance and applicability of this model-rich method for choice-based preference estimation and recovery.

4.2 Distinguishing Between Indifference and Indecisiveness

For a decision maker with incomplete preferences who is also indifferent between some alternatives, a non-trivial question that emerges naturally is how one might use observable behavioural data in conjunction with some model in order to separate those pairs of alternatives between which the agent is indifferent from those where the agent is indecisive/unable to compare. Eliaz and Ok (2006) were the first to raise and provide an answer to this question. Taking the model of undominated choice as their primitive, the authors focused on and characterized the special case where the model’s rationalizing incomplete preference relation \( \succcurlyeq \) is “regular” in the sense that whenever \( x \succcurlyeq y \) and \( y \succcurlyeq x \) are both true, then there is \( z \in X \) such that either \( x \succcurlyeq z, z \succcurlyeq x \) and \( y \succ z \) or \( z \succ y \), or \( y \succcurlyeq z, z \succcurlyeq y \) and \( x \succ z \) or \( z \succ x \). Their proposed distinction can then be summarized as follows:
An agent whose incomplete preferences are captured by a regular preorder and who maximizes these preferences according to the undominated-choice model is revealed to be

indifferent between $x$ and $y$ only if $[x, y \in A, y \in C(A)] \Rightarrow x \in C(A)$;

indecisive between $x$ and $y$ only if $x \in C(A), y \in A \setminus C(A), y \in C(B), x \in B$

for distinct menus $A$ and $B$.

In words, the agent is indifferent only if the two options are either chosen or rejected together when both are feasible, while they are indecisive only if one is chosen over the other in some menu and the latter is chosen in the presence of the former in another menu. Importantly, the revealed indifference relation here is transitive, whereas the revealed indecisiveness one is not (see also Mandler, 2009). Also importantly, although this choice-reversal-based distinction between the two notions is intuitive, it is not robust. Indeed, any behaviour that is compatible with such an indifference-permitting instance of that model is observationally equivalent to the same instance of the model where the decision maker is simply indecisive between any two alternatives that are not ranked by strict preference (see also Theorem 1 in Bossert et al., 2005 and Theorem 3.3 in Bossert and Suzumura, 2010).

In Gerasimou (2018a) we recently noted that a distinct and robust behavioural separation between indifference and indecisiveness is afforded by the dominant choice model. This can be summarized as follows:

A decision maker whose incomplete preferences are captured by a preorder and who maximizes these preferences according to the dominant-choice model is revealed to be

indifferent between $x$ and $y$ if and only if $[x, y \in A, y \in C(A)] \Rightarrow x \in C(A)$;

indecisive between $x$ and $y$ if and only if $x, y \in A \Rightarrow x, y \notin C(A)$.

In words, the agent is indifferent iff both options are either chosen or rejected together when both are feasible, and they are indecisive between these options iff neither is ever chosen in the presence of the other. This distinction is obviously robust because reducing a relation’s indifferences to incomparabilities in this case generates different active-choice and deferring behaviour. Notably, moreover, it turns out that the generally intransitive revealed indecisiveness relation that is derived from this model coincides with the incomparability relation that is derived from the strict welfare preference relation that was studied in Bernheim and Rangel (2009), which these authors motivated as possibly applicable in model-free environments of forced choice (see also Section 5).

The multi- and/or empty-valued experimental choice data and the non-parametric optimization goodness-of-fit method that were presented in the previous (sub)sections jointly
allow us to test for the potential presence of both these theory-guided distinctions for the first time. Additionally, going beyond the boundaries of the above two theoretical studies which assumed that the analyst observes active choices/deferrals at all menus that can be derived from the grand choice set, this method allows us to carry out this test and recover the individual’s strict preferences and, where relevant, indifferences and incomparabilities from an *incomplete* collection of menus.

We find that a total of 6 subjects (5 in the Forced-Choice treatment) out of the 19 who were best-matched by the undominated-choice model revealed “regular” incomplete preferences that afford the Eliaz-Ok distinction between indifference and indecisiveness (range of distance scores: [5,10]). Figure 8 illustrates one such example, with the graph on the right depicting the revealed incomplete weak preference relation and the graph on the left the observationally equivalent revealed incomplete strict preference relation.

Figure 8: A subject’s incomplete preferences, optimally recovered with the undominated-choice model.

![Diagram](image1)

Notes: Approximate fit (distance score: 5). The left (right) graph depicts the strict-(weak-)preference interpretation of the subject’s choices. In the right graph the subject is revealed indifferent between alternatives A and F and revealed indecisive between e.g. A and D. For example, $C(A, F) = C(A, F, D) = \{A, F\}$ while $C(A, D) = \{A, D\}$ and $C(A, C, D, E) = \{D, E\}$. In the left graph they are revealed indecisive between A and F too.

We also find that 30 out of the total 38 Non-Forced-Choice subjects for whom the dominant-choice model provided the best fit (including the 5 subjects for whom this fit was perfect) revealed incomplete preferences with a non-degenerate indifference part (range of distance scores: [0,10]). Figure 9 depicts such an example.

Figure 9: A subject’s incomplete preferences, optimally recovered with the dominant-choice model.

![Diagram](image2)

Notes: Perfect fit. The subject is revealed indifferent between A and F and revealed indecisive e.g. between B and E. For example, $C(A, F) = \{A, F\} = C(A, F, D, C)$ and $C(B, E) = \emptyset = C(B, C, D, E)$.

Although indifference is generically non-existent for a large class of incomplete preference relations that are defined on a space of bundles of continuous commodities or uncertain acts
(Gerasimou, 2018b), this is not so when such preferences are over finitely many indivisible goods, as in the choice environment we consider here. Indeed, the preceding analysis suggests that out of all 57 subjects that the two models of incomplete-preference maximization explain optimally in our sample, 36 (63%) exhibited behaviour that allows for recovering a transitive strict preference relation together with a transitive indifference relation and a generally intransitive incomparability/indecisiveness relation.

4.3 Satisficing

Simon (1956, 1955) famously coined the term *satisficing* to describe resource-constrained agents who, instead of searching through all available alternatives until they find the best, only search until they find one that meets an acceptability threshold. Two recent forced-choice studies that tested the satisficing hypothesis in economics are Reutskaja et al. (2011) and Caplin et al. (2011). The former found weak evidence for such behaviour in choice from lists of food snacks under intense time pressure, whereas the latter found strong evidence in choice from lists of monetary amounts that were described verbally through a series of additions and subtractions, without (or with limited) time pressure. The fact that all 50 menus in both treatments of our experiment were presented vertically as unnumbered lists allows us to add to this literature by complementing the preceding model-based analysis with a test for satisficing in the present time-unconstrained decision environment of multi-valued choice over pairs of gift cards.

To this end, we first conduct a relatively narrow but potentially informative test of satisficing by focusing on the frequency with which subjects opted to choose only the first alternative that appeared in the menus they saw, and then asking whether this frequency can be viewed as being above and beyond what might be reasonably interpretable differently. Specifically, we find that 5 and 4 subjects (3.3%) in the Forced- and Non-Forced-Choice treatments, respectively, who were not classified as approximately explainable by one of the three deterministic models discussed earlier chose only the first option at frequencies that strictly exceeded the 97.5% cut-off values of 0.28 and 0.29 that are derived from the relevant simulations. These numbers rise to 27 and 20 (17%), respectively, if the non-randomness criterion is retained but the model-classification requirement is dropped.

In addition, we compute and study the average position of each subject’s chosen item(s) in the 50 menus’ list orderings because, intuitively, an unusually low average value of this metric could also be indicative of satisficing behaviour for the subject in question. We find that for an additional 7 and 5 (4.4%) subjects in the Forced- and Non-Forced-Choice treatments, respectively, who were not classified as approximately explainable by one of the three deterministic models that were discussed above, the average positions of their chosen
item(s) in the 50 menus’ list orderings were strictly below the 2.5% percentile cut-off value of 1.84 that is derived from simulated uniform-random forced choices. These two distinct analyses therefore suggest that a total of 21 subjects (7.7%) who were not model-classified in Table 5 might be thought of as exhibiting a systematic satisficing behaviour.

Figure 10: Forced-Choice subjects are more likely to select items that are higher up in the menu list.

(a) Forced-Choice treatment

(b) Non-Forced Choice treatment

Interestingly, a cross-treatment comparison of how the average positions of all chosen alternatives are distributed (see Figure 10) indicates that Forced-Choice subjects may be significantly more likely to choose items that appear higher up in the menu list ($p = 0.009$; two-sided Mann-Whitney $U$ test). This finding could be explained intuitively as follows: for individuals who are either insufficiently motivated to make 50 careful decisions over 2–4 pairs of gift cards in an experimental lab or find the task to be cognitively challenging, being unable to avoid/defer the decision at a small cost could make the use of a non-compensatory decision heuristic such as satisficing more likely.

4.4 Preference for Randomization

Agranov and Ortoleva (2017) and Dwenger et al. (2018), among others, have recently documented a preference for randomization in repeated choices from binary menus of money lotteries. Such a preference refers to subjects’ frequent tendency to change their choices from one occurrence of a menu to the next, and their willingness to even incur a small cost in order to have their choice determined randomly. Cerreia-Vioglio et al. (2019) is a recent theoretical contribution to the axiomatic structure of such behaviour. In addition to the indifference-permitting goodness-of-fit analysis of the three models that was presented earlier, the aspect of our experimental design whereby subjects are rewarded with a random alternative at a menu if they chose all alternatives at that menu allows us to test for the existence of a similar preference for randomization in our data, even without any choice repetitions.
As with our analysis of satisficing, to proceed with this investigation we regard a subject as potentially exhibiting such a preference if: (i) they are not approximately explainable by one of the three deterministic models of preference maximization; and (ii) the number of menus were they chose everything strictly exceeds the 97.5% simulations-based cut-off values of 14 and 11 menus in the Forced- and Non-Forced-Choice treatments, respectively. Both criteria are satisfied by 3 and 4 subjects (2.5%) in the Forced- and Non-Forced-Choice treatment, respectively, with these numbers rising to 6 and 9 (5.5%) when the first criterion is dropped. In addition, none of these 7 subjects belongs to the “satisficing” category that was defined above. This finding adds to the existing and growing literature on preference for randomization by showing that such a preference could potentially manifest itself in binary as well as non-binary menus of riskless alternatives, even when the choice-deferral option is feasible and acts as another obvious way for an individual to deal with a difficult decision, and even in non-repeated-choice environments.

5 Incomplete-Preference Maximization in General Domains

The main part of the analysis in the previous section reported on the subjects’ perfect or approximate conformity with the three deterministic models of complete and incomplete preference maximization. But even though the axiomatic structure of these models is well-understood when the domain is finite and observations are available from all possible menus (Schwartz, 1976; Eliaz and Ok, 2006; Gerasimou, 2018a), it is only in the case of rational choice that a model’s behavioural implications are known for arbitrary domains and menu collections (Richter, 1966). We now study this problem for the two models of choice with incomplete preferences (the proofs of both results below are in Appendix A).

Let \( X \) be a set of choice alternatives. A weak preference relation \( \succsim \) on \( X \) is a reflexive and transitive binary relation –a preorder– on that set. Such a relation is incomplete if there exist \( x, y \in X \) such that \( x \not\succsim y \) and \( y \not\succsim x \). The incomparability relation that is derived from a weak preference relation \( \succsim \) is irreflexive and symmetric but not transitive in general. If a weak preference relation is not incomplete, then it is a complete preorder or a weak order. A strict preference relation \( \succ \) on \( X \) is an asymmetric and transitive binary relation –a (strict) partial order– on that set. Such a relation is incomplete if there exist \( x, y \in X \) such that \( x \not\succ y \) and \( y \not\succ x \). The incomparability relation derived from a strict preference relation is reflexive and symmetric but not transitive in general. If a strict preference relation is not incomplete, then it is a linear order. Finally, a strict preference relation is acyclic if there are are no \( x_1, \ldots, x_m \in X \) such that \( x_m \succ x_1 \) and \( x_i \succ x_{i+1} \) for all \( i \leq m - 1 \).

Given now a choice dataset \( D = (A_i, C(A_i))_{i=1}^k \) of the kind that was introduced in Section
4, let us define the revealed preference relations $\succeq_R$, $\succ_R$, $\succeq^{\bar{R}}$, $\succ^{\bar{R}}$ and $\succ^{R^*}$ as follows:

- $x \succeq_R y$ if $x \in C(A^i)$ and $y \in A^i$ for some $i \leq k$;
- $x \succ_R y$ if $x \in C(A^i)$ and $y \in A^i \setminus C(A^i)$ for some $i \leq k$;
- $x \succeq^{\bar{R}} y$ if there exist $x_1, \ldots, x_n \in X$ such that $x_1 = x$, $x_n = y$ and $x_i \succeq_R x_{i+1}$ for all $i = 1, \ldots, n - 1$;
- $x \succ^{\bar{R}} y$ if $x \succ_R y$ and $y \nless_R x$;
- $x \succ^{R^*} y$ if there exist $x_1, x_2, \ldots, x_n \in X$ such that $x_1 = x$, $x_n = y$ and $x_i \succ^{R^*} x_{i+1}$ for all $i = 1, \ldots, n - 1$.

The first three are, respectively, the direct weak, direct strict and indirect weak preference relations. The fourth relation, $\succ^{R^*}$, differs from $\succ^R$ because the latter is defined only in terms of the first requirement that appears in the definition of $\succ^{R^*}$. That is, $x \succ^{R^*} y$ is true iff $x$ is chosen over $y$ in some menu and at the same time $y$ is never chosen at any menu where $x$ is present. The relation $\succ^{R^*}$ also differs from the “model-free” revealed preference relation that was suggested in Bernheim and Rangel (2009), which is defined only in terms of the second requirement that appears in the definition of $\succ^{R^*}$. As was shown by these authors, that relation is generally acyclic –though not necessarily transitive– only in a full-domain environment. Finally, $\succ^{\bar{R}^*}$ is the indirect strict revealed preference relation that is derived from $\succ^{R^*}$, and is defined in terms of strict such comparisons along any two consecutive elements in a given chain.

We begin with the model of undominated choice with (possibly) incomplete preferences.

**Definition 1**

A dataset $\mathcal{D}$ on $X$ is rationalizable by undominated choice with partially ordered preferences if there is an asymmetric and transitive relation $\succ$ on $X$ such that, for all $i \leq k$,

$$C(A^i) = \{x \in A^i : y \not\succ x \text{ for all } y \in A^i\}. \quad (3)$$

To analyse this model we introduce the following axioms.

**Behavioural Decisiveness**

For every $i \leq k$, $C(A^i) \neq \emptyset$.

**Generalized Congruence**

If $x \succ^{\bar{R}^*} y$, then $y \not\succeq^R x$.

**Upward Consistency**

If $x \in A^i$ and $y \not\succ^{R^*} x$ for all $y \in A^i$, then $x \in C(A^i)$.
The first axiom requires that the agent never avoid/delay their choice, and is typically assumed implicitly in the literature. The second axiom generalizes Richter’s (1966) Congruence (stated below), which in turn is a version of the Strong Axiom of Revealed Preference. It does so because the revealed preference relation in its antecedent places more constraints than the relation $\succeq^R$ in Congruence does, thereby requiring their common consequent to hold in fewer situations. Upward Consistency appears to be new and states that if some feasible alternative is not dominated at some menu according to the strong indirect strict revealed preference relation $\succ^R$, then it is chosen at that menu.

**Proposition 1**

*If a dataset $D$ on a finite set $X$ satisfies Behavioural Decisiveness, Generalized Congruence and Upward Consistency, then it is rationalizable by undominated choice. The converse is not true in general.*

The set of sufficient conditions for undominated-choice rationalizability in general finite domains that appears in Proposition 1 is to our knowledge new. We note, however, that Bossert et al. (2005, Theorem 4) and Bossert and Suzumura (2010, Theorem 4.13) also studied this model at a similar level of generality and characterized it in terms of a general acyclicity condition, which is nevertheless not as obviously interpretable as those stated above. We refer the reader to these works for more details on that condition.

We now turn to the model of dominant choice with (possibly) incomplete preferences.

**Definition 2**

*A dataset $D$ on $X$ is rationalizable by dominant choice with preordered preferences if there is a reflexive and transitive relation $\succeq$ on $X$ such that, for all $i \leq k$,

$$C(A^i) = \{ x \in A^i : x \succeq y \text{ for all } y \in A^i \}. \quad (4)$$

We state the following axioms in relation to this model.

**Congruence**

*If $x \succeq^R y$, then $y \not\succ^R x$.***

**Expansion**

*If $x \in A^i$ and $x \succeq^R y$ for all $y \in A^i$, then $x \in C(A^i)$.***

**Desirability**

*If $A^i = \{ x \}$, then $C(A^i) = \{ x \}$.***

The Congruence axiom is as in Richter (1966) except that non-empty-valuedness (i.e. the Behavioural Decisiveness axiom) is not implicitly assumed. The Desirability axiom was
introduced in Gerasimou (2018a). It is consistent with an interpretation whereby whenever
the decision maker decides to avoid/delay choice at some non-singleton menu, this is not
because that alternative is considered to be insufficiently good according to some desirability
threshold. Finally, when defined in terms of the relation $\succsim^R$, Expansion is another classic
axiom that was introduced in Sen (1971) as Property $\gamma$. It is stated here in a stronger form
that features the indirect weak revealed preference relation $\succsim^R$ instead.

**Proposition 2**

The following are equivalent for a dataset $D$ on a set $X$:

1. $D$ satisfies Congruence, Desirability and Expansion.
2. $D$ is rationalizable by dominant choice with preordered preferences.

Proposition 2 includes the following result as a special case.

**Corollary 1 (Richter, 1966)**

The following are equivalent for a dataset $D$ on a set $X$:

1. $D$ satisfies Behavioural Decisiveness and Congruence.
2. $D$ is rationalizable by dominant choice with completely preordered preferences.

Indeed, under Behavioural Decisiveness the Congruence axiom implies Expansion and, under
standard extension theorems, the indirect revealed preference relation $\succsim^R$ in the proof of
Proposition 2 can assume a complete extension without any loss of generality.

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Online Appendix

A Proofs

Proof of Proposition 1.

The relation $\succ^R$ defined above is asymmetric by construction. Also by construction, the relation $\succ^\hat{R}$ is transitive and extends $\succ^R$ in the sense that $\succ^R \subseteq \succ^\hat{R}$. Now suppose to the contrary that $x \succ^\hat{R} y$ and $y \succ^\hat{R} x$. Since $\succ^R$ is asymmetric, it follows from $x \succ^\hat{R} y$, $y \succ^\hat{R} x$ and the definition of $\succ^\hat{R}$ that there is $x_i \in X$ such that $x_i \neq x, y$ and either $x \succ^{\hat{R}} x_i \succ^R y$ or $x \succ^R x_i \succ^{\hat{R}} y$. Without loss of generality, suppose the former is true. From $y \succ^{\hat{R}} x \succ^{\hat{R}} x_i$ and the transitivity of this relation we obtain $y \succ^R x_i$. Since $x_i \succ^R y$ and $y \succ^{\hat{R}} x_i$ are both true, and since $x_i \succ^R y$ implies $x_i \succ^R y$, Generalized Congruence is violated. A contradiction is therefore obtained. This establishes that $\succ^{\hat{R}}$ is also asymmetric, hence a strict partial order. It also readily follows from Generalized Congruence that if $x \in C(A^i)$ for some $i \leq k$, then $y \not\succ^{\hat{R}} x$ for all $y \in A^i$. Conversely, if $x \in A^i$ and $y \not\succ^{\hat{R}} x$ for all $y \in A^i$, then Upward Consistency and finiteness of $A^i$ together imply $x \in C(A^i)$. Therefore, $D$ is rationalizable by undominated choice with the strict partial order $\succ^{\hat{R}}$ whenever these two axioms and Behavioural Decisiveness (which is an obvious necessary condition that is independent of these axioms) hold.

Proof of Proposition 2.

That $2 \Rightarrow 1$ is straightforward. For the converse claim, let $\succeq^{\hat{R}}$ be the indirect revealed preference relation as defined earlier, and define further

$$\succeq := \succeq^{\hat{R}} \cup \{(x, x) : x \in X\}.$$ (5)

The relation $\succeq$ is a preorder by construction. Since, by Desirability, $C(A^i) = \{x\}$ whenever $A^i = \{x\}$ for some $i \leq k$, it also follows that $\succeq \supseteq \succeq^{\hat{R}}$, i.e. $\succeq$ extends $\succeq^{\hat{R}}$.

We will verify that $\succeq$ rationalizes $D$, as claimed. First, $x \in C(A^i)$ for some $i \leq k$ implies $x \succeq y$ for all $y \in A^i$, by definition. Conversely, let $A^i$ be such that $x \in A^i$ for some $i \leq k$. Suppose first that $A^i = \{x\}$. By Desirability, $C(A^i) = \{x\}$. Suppose now that $A^i \supseteq \{x\}$ and let $x \succeq y$ for all $y \in A^i$. By Congruence, there is no $y \in A^i$ such that $y \in C(A^i)$ and $x \in A^i \setminus C(A^i)$. Suppose $C(A^i) = \emptyset$ instead. Since $x \succeq y$ for all $y \in A^i$, Expansion and (5) together imply $x \in C(A^i)$, a contradiction. Therefore, for all $i \leq k$, $C(A^i) = \{x \in A^i : x \succeq y \text{ for all } y \in A^i\}$. ■
B  Experiment Details

B.1  Instructions: Non-Forced-Choice Treatment (verbatim)

Welcome and thank you for your participation!

At the beginning of the experiment you will be automatically allocated £2.40.

During the main phase you will be presented with 50 menus of pairs of gift cards, one menu at a time.

(Note: all gift cards come from well-known brands and each one is worth £10.)

For every gift card there is at least one branch in St Andrews town centre (on Market St. or South St.) where that gift card can be used instead of cash for making purchases.

At each of these 50 menus, you will be asked to choose one or more of the available pairs of gift cards, or to choose “I’m not choosing now”.

Once you have left a menu during that phase you will not be able to go back to it and change your decision.

At the end of the experiment one of the 50 menus will be randomly selected for you. Each menu is equally likely to be selected.

You will win a pair of gift cards (worth a total of £20) from your randomly selected menu.

First, you will be reminded which pair(s) of gift cards you chose at that menu, or if you chose “I’m not choosing now”.

Following that, your pair of gift cards and cash rewards will be determined as follows:

1. If you had previously chosen one or more— but not all— pairs from your randomly selected menu, then you will be asked to choose a single pair at this time:
   • If you now choose a pair among those previously chosen, you will win that pair and also receive the £2.40 that you were initially allocated.
   • If you now choose a pair not among those previously chosen, you will win that pair and also receive £1.20 from the amount that you were initially allocated.

2. If you had previously chosen all pairs from your randomly selected menu, then you will not be able to choose a pair now and the experimenter will choose one for you at random. You will win that pair and also receive the £2.40 that you were initially allocated.

3. If you had previously chosen “I’m not choosing now” at your randomly selected menu, then you will be asked to choose a single pair now. You will win that pair and also receive £2.10 from the amount that you were initially allocated.

(Note: the decisions you made at all menus other than your randomly selected one will not affect your gift card and cash rewards.)

After at least fifty minutes have passed since the experiment’s start, the experimenter will start giving the gift card and cash rewards to participants who have finished.

No participant will be given their rewards before such time, regardless of how early they finish.
B.2 Instructions: Forced-Choice Treatment (verbatim)

Welcome and thank you for your participation!

At the beginning of the experiment you will be automatically allocated £2.40.

During the main phase you will be presented with 50 menus of pairs of gift cards, one menu at a time.

(Note: all gift cards come from well-known brands and each one is worth £10.)

For every gift card there is at least one branch in St Andrews town centre (on Market St. or South St.) where that gift card can be used instead of cash for making purchases.

At each of these 50 menus, you will be asked to choose one or more of the available pairs of gift cards.

Once you have left a menu during that phase you will not be able to go back to it and change your decision.

At the end of the experiment one of the 50 menus will be randomly selected for you. Each menu is equally likely to be selected.

You will win a pair of gift cards (worth a total of £20) from your randomly selected menu.

First, you will be reminded which pair(s) of gift cards you chose at that menu.

Following that, your pair of gift cards and cash rewards will be determined as follows:

1. If you had previously chosen one or more –but not all– pairs from your randomly selected menu, then you will be asked to choose a single pair at this time:
   - If you now choose a pair among those previously chosen, you will win that pair and also receive the £2.40 that you were initially allocated.
   - If you now choose a pair not among those previously chosen, you will win that pair and also receive £1.20 from the amount that you were initially allocated.

2. If you had previously chosen all pairs from your randomly selected menu, then you will not be able to choose a pair now and the experimenter will choose one for you at random. You will win that pair and also receive the £2.40 that you were initially allocated.

(Note: the decisions you made at all menus other than your randomly selected one will not affect your gift card and cash rewards.)

After at least fifty minutes have passed since the experiment’s start, the experimenter will start giving the gift card and cash rewards to participants who have finished.

No participant will be given their rewards before such time, regardless of how early they finish.
B.3 The six pairs of £10 gift cards used in the experiment

![Gift Cards](image1.png)

B.4 Example decision problem shown in the main part

Please choose one or more of the gift card pairs that are shown in this menu, or select "I'm not choosing now":

![Decision Problem](image2.png)

- [ ] I’m not choosing now
B.5 Example randomly selected menu shown at the end

Your randomly selected menu is shown below:

You chose

at that menu.

Please wait for the experimenter to come to your desk.

Your rewards will be determined as described in your instructions.
Motivation for Strongly Symmetric Menu Collections

Recall that subjects were presented with the 50 menus that comprised all those with two (15), three (20) and four (15) alternatives. This excludes all singleton menus and all those with 5 or 6 alternatives. Obtaining decision data from the full collection of menus may be desirable at some levels (e.g. because they provide additional information) but undesirable at others (e.g. because choice fatigue could negatively affect decision quality) or even impractical (for example, when the grand choice set comprises 6, 7 or 8 alternatives the full collection includes 63, 127 or 255 menus). In situations such as this the researcher may be inclined to limit the menus seen by subjects to some manageable number. How should this be done?

We argue that the collection of menus presented to subjects should satisfy strong symmetry in the sense that the distribution of menu sizes where an alternative is feasible is the same for every alternative. This requirement in turn implies the weak symmetry condition whereby all alternatives are feasible at some menu the same number of times. For example, a collection that comprises menus \( P = \{x, y, z\} \), \( Q = \{w, z\} \) and \( R = \{x, y, w\} \) satisfies weak but not strong symmetry because, although each alternative appears twice in the dataset, \( x \) and \( y \) appear only in ternary menus while \( w \) and \( z \) do so in one binary and one ternary menu instead. By contrast, a collection consisting of \( P = \{x, y, z\} \), \( R = \{x, y, w\} \), \( S = \{w, y, z\} \) and \( T = \{w, x, z\} \) satisfies strong symmetry, with each alternative appearing three times in as many menus of the same size.

The motivation for the strong symmetry requirement, which is indeed satisfied in our experimental dataset, is intuitive: if the menu-size distributions differ across alternatives, this means that at least one of them is feasible in at least one larger/smaller menu compared to at least one other alternative. This could then pave the way for a potential bias in favour of or against that option in the ensuing analysis. In the first menu collection above, for example, suppose \( x \) is (uniquely) chosen at \( P \), \( z \) at \( Q \) and \( w \) at \( R \). Clearly, this is a revealed-preference cycle. Moreover, removing any of the three observations from this dataset breaks that cycle. Which one should be removed? One might be tempted to keep the choice at \( Q \) as the potentially more accurate of the three because it is derived from a binary menu. But doing so and removing \( P \), for example, would amount to giving a possibly unfair (dis)advantage to alternative \( z \) (\( x \)). Indeed, \( z \) (\( x \)) would then appear first (last) in the inferred revealed preference ordering even though \( x \) (\( z \)) would have been second (third) if \( Q \) had been removed instead. Yet even though it could be that \( x \) would continue to be chosen over \( z \) also at the binary menu \( \{x, z\} \), the (symmetry-breaking) fact that this observation is unavailable works against that alternative.
D Choice Probabilities in Simulations

The probability of an alternative being chosen at a menu under multi-valued choice simulations is interpreted here as the probability that this alternative belongs to the chosen submenu of that menu. Assuming Forced Choice first, and considering an arbitrary menu $A$ with $k$ alternatives, every non-empty submenu $A' \subseteq A$ is equally likely to be chosen, and is therefore chosen with probability $\frac{1}{2^k - 1}$. Since each of the $k$ feasible alternatives belongs to exactly $\frac{2^k}{2}$ of these submenus, it follows that each of them is chosen in the above sense with probability $\frac{2^k}{2(2^k - 1)}$. Under Non-Forced Choice now, since some non-empty submenu of $A$ is chosen with probability $\frac{k}{k+1}$ because choice is deferred with probability $\frac{1}{k+1}$, the corresponding probability for each of the $k$ active-choice alternatives is adjusted accordingly. This results to the probabilities described in Table 6 and Figure 12.

Table 6: Choice/deferral probabilities at a menu with $k$ alternatives under multi-valued choice simulations.

|                      | Probability of each non-empty submenu being chosen | Probability of each active-choice alternative being chosen | Probability of deferral |
|----------------------|--------------------------------------------------|--------------------------------------------------------|-------------------------|
| Forced-Choice simulations | $\frac{1}{2^k - 1}$                             | $\frac{1}{2} \cdot \frac{2^k}{2}$                     | NA                      |
| Non-Forced-Choice simulations | $\frac{1}{2^k - 1 \cdot k + 1}$                  | $\frac{1}{2} \cdot \frac{2^k}{2} \cdot \frac{k}{k + 1}$ | $\frac{1}{k + 1}$       |

Figure 12: Choice/deferral probabilities under multi-valued choice simulations at various menu sizes.

(a) Every feasible alternative is chosen with $\approx 0.5$ probability as menu size increases, while deferral becomes less likely.

(b) The probability of choosing any non-empty submenu decreases as menu size increases.
Graphs of Model-Optimal Revealed Preference Orderings

Note: arrows point to dominated options/indifference classes

E.1 Rational Choice/Utility Maximization

ID = 1140, Score = 0, 1 optimal relation
ID = 2582, Score = 0, 1 optimal relation
ID = 3843, Score = 0, 1 optimal relation
ID = 7228, Score = 0, 1 optimal relation

ID = 7538, Score = 0, 1 optimal relation
ID = 9139, Score = 0, 1 optimal relation
ID = 9425, Score = 0, 1 optimal relation
ID = 9507, Score = 0, 1 optimal relation

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ID = 1411, Score = 1, 1 optimal relation

ID = 1967, Score = 1, 1 optimal relation

ID = 2895, Score = 1, 1 optimal relation

ID = 3826, Score = 1, 1 optimal relation

ID = 4430, Score = 1, 1 optimal relation

ID = 4540, Score = 1, 1 optimal relation

ID = 4542, Score = 1, 1 optimal relation

ID = 6861, Score = 1, 1 optimal relation

ID = 6890, Score = 1, 2 optimal relations

ID = 6969, Score = 1, 1 optimal relation

ID = 7081, Score = 1, 1 optimal relation

ID = 8360, Score = 1, 1 optimal relation
ID = 7996, Score = 4, 2 optimal relations
ID = 8468, Score = 4, 2 optimal relations
ID = 8892, Score = 4, 1 optimal relation
ID = 8989, Score = 4, 1 optimal relation

ID = 9039, Score = 4, 1 optimal relation
ID = 9292, Score = 4, 1 optimal relation
ID = 9292, Score = 4, 1 optimal relation
ID = 9292, Score = 4, 1 optimal relation

ID = 4261, Score = 5, 1 optimal relation
ID = 5820, Score = 5, 1 optimal relation
ID = 5646, Score = 5, 1 optimal relation
ID = 5878, Score = 5, 1 optimal relation
E.2 Undominated Choice with Incomplete Preferences

ID = 6293, Score = 10, 1 optimal relation

ID = 6751, Score = 10, 1 optimal relation

ID = 7571, Score = 10, 2 optimal relations

ID = 7735, Score = 10, 1 optimal relation

ID = 8049, Score = 10, 1 optimal relation

ID = 1889, Score = 2, 1 optimal relation

ID = 8721, Score = 3, 1 optimal relation

ID = 9001, Score = 3, 1 optimal relation
ID = 8945, Score = 4, 1 optimal relation

ID = 6521, Score = 5, 1 optimal relation

ID = 6140, Score = 5, 1 optimal strict partial order (left), 1 optimal incomplete preorder (right)

ID = 2432, Score = 7, 1 optimal strict partial order (left), 1 optimal incomplete preorder (right)

ID = 9222, Score = 7, 1 optimal relation

ID = 3574, Score = 8, 1 optimal relation

ID = 5049, Score = 8, 1 optimal strict partial order (left), 1 optimal incomplete preorder (right)

ID = 8075, Score = 8, 1 optimal strict partial order (left), 1 optimal incomplete preorder (right)
ID = 7423, Score = 8, 1 optimal relation

ID = 8719, Score = 8, 1 optimal strict partial order (left), 1 optimal incomplete preorder (right)

ID = 5049, Score = 8, 1 optimal strict partial order (left), 1 optimal incomplete preorder (right)

ID = 9819, Score = 8, 1 optimal relation

ID = 2038, Score = 9, 1 optimal relation

ID = 3645, Score = 10, 1 optimal strict partial order (left), 1 optimal incomplete preorder (right)

ID = 7332, Score = 10, 3 optimal strict partial orders

ID = 7365, Score = 10, 1 optimal relation
E.3 Dominant Choice with Incomplete Preferences

```
| ID   | Score | Optimal Relations |
|------|-------|-------------------|
| 1716 | 0     | A ~ B ~ C ~ E     |
| 3487 | 0     | A ~ B ~ C ~ E     |
| 3746 | 0     | A ~ B ~ C ~ E     |
| 4486 | 0     | A ~ B ~ C ~ E     |
| 5649 | 0     | A ~ B ~ C ~ E     |
| 5813 | 0     | A ~ B ~ C ~ E     |
| 5992 | 1     | A ~ B ~ C ~ E     |
| 6499 | 1     | A ~ B ~ C ~ E     |
| 8655 | 1     | A ~ B ~ C ~ E     |
| 9072 | 2     | A ~ B ~ C ~ E     |
| 9142 | 2     | A ~ B ~ C ~ E     |
| 1052 | 3     | A ~ B ~ C ~ E     |
| 1073 | 3     | A ~ B ~ C ~ E     |
| 1194 | 3     | A ~ B ~ C ~ E     |
| 1215 | 3     | A ~ B ~ C ~ E     |
| 1236 | 3     | A ~ B ~ C ~ E     |
| 1257 | 3     | A ~ B ~ C ~ E     |
| 1278 | 3     | A ~ B ~ C ~ E     |
| 1299 | 3     | A ~ B ~ C ~ E     |
| 1320 | 3     | A ~ B ~ C ~ E     |
| 1341 | 3     | A ~ B ~ C ~ E     |
| 1362 | 3     | A ~ B ~ C ~ E     |
| 1383 | 3     | A ~ B ~ C ~ E     |
| 1404 | 3     | A ~ B ~ C ~ E     |
| 1425 | 3     | A ~ B ~ C ~ E     |
| 1446 | 3     | A ~ B ~ C ~ E     |
| 1467 | 3     | A ~ B ~ C ~ E     |
| 1488 | 3     | A ~ B ~ C ~ E     |
| 1509 | 3     | A ~ B ~ C ~ E     |
| 1530 | 3     | A ~ B ~ C ~ E     |
| 1551 | 3     | A ~ B ~ C ~ E     |
| 1572 | 3     | A ~ B ~ C ~ E     |
| 1593 | 3     | A ~ B ~ C ~ E     |
| 1614 | 3     | A ~ B ~ C ~ E     |
| 1635 | 3     | A ~ B ~ C ~ E     |
| 1656 | 3     | A ~ B ~ C ~ E     |
| 1677 | 3     | A ~ B ~ C ~ E     |
| 1698 | 3     | A ~ B ~ C ~ E     |
| 1719 | 3     | A ~ B ~ C ~ E     |
| 1740 | 3     | A ~ B ~ C ~ E     |
| 1761 | 3     | A ~ B ~ C ~ E     |
| 1782 | 3     | A ~ B ~ C ~ E     |
| 1803 | 3     | A ~ B ~ C ~ E     |
| 1824 | 3     | A ~ B ~ C ~ E     |
| 1845 | 3     | A ~ B ~ C ~ E     |
| 1866 | 3     | A ~ B ~ C ~ E     |
| 1887 | 3     | A ~ B ~ C ~ E     |
| 1908 | 3     | A ~ B ~ C ~ E     |
| 1929 | 3     | A ~ B ~ C ~ E     |
| 1950 | 3     | A ~ B ~ C ~ E     |
| 1971 | 3     | A ~ B ~ C ~ E     |
| 1992 | 3     | A ~ B ~ C ~ E     |
| 2013 | 3     | A ~ B ~ C ~ E     |
| 2034 | 3     | A ~ B ~ C ~ E     |
| 2055 | 3     | A ~ B ~ C ~ E     |
| 2076 | 3     | A ~ B ~ C ~ E     |
| 2097 | 3     | A ~ B ~ C ~ E     |
| 2118 | 3     | A ~ B ~ C ~ E     |
| 2139 | 3     | A ~ B ~ C ~ E     |
| 2160 | 3     | A ~ B ~ C ~ E     |
| 2181 | 3     | A ~ B ~ C ~ E     |
| 2202 | 3     | A ~ B ~ C ~ E     |
| 2223 | 3     | A ~ B ~ C ~ E     |
| 2244 | 3     | A ~ B ~ C ~ E     |
| 2265 | 3     | A ~ B ~ C ~ E     |
| 2286 | 3     | A ~ B ~ C ~ E     |
| 2307 | 3     | A ~ B ~ C ~ E     |
| 2328 | 3     | A ~ B ~ C ~ E     |
| 2349 | 3     | A ~ B ~ C ~ E     |
| 2370 | 3     | A ~ B ~ C ~ E     |
| 2391 | 3     | A ~ B ~ C ~ E     |
| 2412 | 3     | A ~ B ~ C ~ E     |
| 2433 | 3     | A ~ B ~ C ~ E     |
| 2454 | 3     | A ~ B ~ C ~ E     |
| 2475 | 3     | A ~ B ~ C ~ E     |
| 2496 | 3     | A ~ B ~ C ~ E     |
| 2517 | 3     | A ~ B ~ C ~ E     |
| 2538 | 3     | A ~ B ~ C ~ E     |
| 2559 | 3     | A ~ B ~ C ~ E     |
```
ID = 7286, Score = 3, 1 optimal relation

ID = 6055, Score = 4, 1 optimal relation

ID = 8801, Score = 4, 2 optimal relations

ID = 1124, Score = 5, 1 optimal relation

ID = 3116, Score = 5, 1 optimal relation

ID = 4155, Score = 5, 1 optimal relation

ID = 5860, Score = 5, 1 optimal relation

ID = 9650, Score = 5, 1 optimal relation

ID = 5440, Score = 6, 1 optimal relation

ID = 7869, Score = 6, 1 optimal relation

ID = 8133, Score = 6, 2 optimal relations

ID = 8781, Score = 6, 2 optimal relations
ID = 6324, Score = 7, 1 optimal relation

ID = 7071, Score = 7, 1 optimal relation

ID = 7577, Score = 7, 1 optimal relation

ID = 9305, Score = 7, 1 optimal relation

ID = 5580, Score = 10, 1 optimal relation