Approximation of amplitude-frequency characteristics using equidistants

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Abstract. Building an equidistant curve to amplitude-frequency characteristic of oscillatory system with one degree of freedom, we introduce the limited damping. The analogous studies were carried out for the systems with the large number of degrees of freedom and analogous conclusions were obtained. It is possible to conclude that the equidistant curve, as a volumetric curve, reflects the results, obtained in the experiment and observed on the screens of different instruments with the study of technical systems.

1. Introduction

Modern technical systems are quite complex objects for theoretical and experimental study and, as a rule, are multicomponent. Therefore, they can be considered from different points of view. One of the main components, of course, is the mechanical part, the basis of the technical system. It is in this aspect that the dynamics and control of technical systems are most often represented, more precisely, at the level of dynamics and control of mechanical systems.

The dynamic properties of these systems are sufficiently described by frequency characteristics. On their basis, it is possible to construct various controls, to estimate the possibilities of introducing structural changes, to evaluate limiting properties, and to ensure dynamic synthesis of the parameters. The characteristics of these systems also have a number of features associated with the existence of discontinuities, asymptotic approximations. At the same time, the determination of properties of systems is associated with the definition of reactions to single and periodic effects, which is related to the calculation of some integral estimates, for example, the areas of the figures.

The study of a fairly complex technical system, as a rule, is accompanied by an experiment, the results of which are important for correcting the initial positions in the formation of a mathematical model, as well as for determining the reliability of information obtained during the analysis. Experimental studies suggest confirmation of both the qualitative and quantitative characteristics of the object and are based on the widespread use of various types of devices. Note that any devices that record changes in the observed parameters are selective; they are characterized by dead zones, desensitization and a certain “erosion” of the results, which is well known in measurement theory. In connection with this, the statements have repeatedly appeared aimed at understanding and taking into account that the parameters of real objects with respect to data from an analytical study are in a certain
“erosion” zone, often called a “tube” [1, 2]. Such considerations could not but lead to proposals for the introduction of an equidistant as some kind of dependence, obtained analytically, but containing well-defined information, taking into account the real nature of the object. In turn, the construction of equidistants for the initial dependencies can be considered as one of the methods of approximation, simplification of models, thereby reducing the amount of subsequent experimental studies, often requiring high material costs.

The methodological value of introducing that kind of a specific approximation technique also lies in the fact that the discontinuous characteristics typical of analytical models are smoothed out. This kind of real processes one often has to face in practice.

2. Materials and methods

Experimental methods are widely used: experimental theory, measurement theory, error theory. These methods create methodological and scientific-methodological foundations for the experimental study of properties using modern instrumentation equipment and automated systems. Processing the results of the experiment is associated with the analysis of measurement errors, inaccuracies, taking into account the erosion of the graphs, curves, and phase patterns demonstrated by the devices, which creates prerequisites for understanding the need to introduce tools appropriate to the process. Analytically, the construction of equidistants can be attributed to the integro-differential transformations of the original curves. The equidistant has a certain property of smoothing the original curves, changes their properties, and, in turn, can be seen as the implementation of some approximation procedure over the initial analytical dependence, thus smoothing the transition to the results that can be obtained on an instrument basis in the experiment. The important parameter in these transformations is the height of equidistancing $p$, which determines the accuracy of the approximation, and further, the degree of closeness of the results to the experimental data.

The concept of equidistant was introduced by N. I. Lobachevsky. There are three main types of movements in his planimetry. For one of them, the notion of equidistants is introduced [3-5].

The lines that are invariant (that is, retain their position) with respect to all the shifts in a single straight line $u$ are not straight lines in Lobachevsky planimetry, as in the case of Euclidean, but are special curves called equidistant curves. N.I. Lobachevsky defined this curve as follows: if we have a sheaf of parallel straight lines and there is a straight line perpendicular to all straight lines of the sheaf, then the curve that lies at a distance from this straight line and intersects each straight line of the sheaf at a right angle is called equidistant. Or: “the equidistant is the locus of points, on one side of the straight line $u$ at one distance from it” [6]. In this case, the straight line $u$ is called the base of the equidistant, and the distance value $p$ is called the height. It is obvious that each straight line can be considered as an equidistant with height $p=0$. Note that the equidistant is a curved line, since the following theorem holds true: "Each straight line has no more than two common points with equidistant." Its proofs are given in the articles [6, 7]. It also holds true that “the height of equidistants is its normal” [3-6]. It is important that the equidistant is symmetric with respect to any of its normals and all its normals are perpendicular to one straight line and, therefore, diverge.

3. Results and discussion

Dynamic characteristics of technical systems, capabilities of management, targeted changes in parameters and the range of possible properties are most often considered using models in the form of oscillatory systems, which implies appropriate attention to research methods and approaches to interpreting results.

Oscillatory systems, by themselves, represent sufficiently informative objects for research, if we bear in mind the parallel study of the properties of oscillatory models, introducing the concept of dynamic constraints, the form and structure of which may differ from classical concepts.

It is known that the amplitude-frequency characteristics of oscillatory systems have features: one - in the case of damped oscillations of a system with one degree of freedom, and two - in the case of undamped oscillations (Figure 1).
It is impossible to calculate the exact area of the figures bounded by these graphs and coordinate axes, since the figures are open and their area is infinite. However in applied problems it is sometimes necessary to know this area. For example, in the tasks of automatic regulation it is used to assess the system regulation quality. The construction of equidistant allows calculating this value with a predetermined accuracy depending on the parameter of equidistant (Figure 2).

![Figure 1. AFRCs of the system with one degree of freedom.](image1)

However thereby we change the appearance of the amplitude-frequency characteristic. The curve becomes similar to the amplitude-frequency characteristic of a system with friction. That is, we can assume that by introducing an equidistant we thereby introduced some damping.

The direct and inverse dependencies of the parameter of equidistant \( p \) and the damping values \( D(p) \) are:

\[
p = \frac{\sqrt{(1-\eta^2)^2 + 4D^2\eta^2 - C(1-\eta^2)}}{\sqrt{1-\eta^2}\sin\phi\sqrt{(1-\eta^2)^2 + 4D^2\eta^2}}
\]

\[
D = \frac{\sqrt{1-\eta^2}}{2\eta(1-p)(1-\eta^2)\sin\phi}\sqrt{C^2 - (1-p)(1-\eta^2)\sin^2\phi}
\]
The studies have shown that the function $p(D)$ is bounded above and below. That is, with an unlimited increase of $D$, the value of $p$ can not exceed a certain specific value equal to

$$\frac{1}{|1-\eta|^{2} \sin \phi}.$$ 

Therefore, there is no limit to the function $D(p)$ with $p \to \infty$, since $p$ can not grow infinitely. Note that with increasing parameter, the equidistant assumes singularities, some of which are described in [8-16]. From all this we can conclude that by building an equidistant to the amplitude-frequency characteristic of a system without friction, we introduce damping, which can not be unlimitedly high.

For the amplitude-frequency characteristic of a system with friction, introducing an equidistant (Fig. 3), we increase the amount of damping of the system. Here the value of $p$ is limited by the properties of the equidistant itself, since starting from a certain value of $p$, the equidistant will acquire singularities and it will become ambiguous at point $\eta_p$. Moreover, with the growth of $p$ these features will increase.

![Equidistant curve with amplitude-frequency characteristics of the system with $D \neq 0$.](image)

**Figure 3.** Equidistant curve with amplitude-frequency characteristics of the system with $D \neq 0$.

### 4. Conclusion

Thus, building an equidistant curve to amplitude-frequency characteristic of oscillatory system with one degree of freedom, we introduce the limited damping.

Analogous studies were carried out and for the systems with the large number of degrees of freedom and analogous conclusions were obtained.

Thus, it is possible to conclude that the equidistant curve, as a volumetric curve, reflects the results, obtained in the experiment and observed on the screens of different instruments with the study of technical systems.

### References

[1] Efimov N V, Rozendorn E R 1974 *Linear algebra and multidimensional geometry* (Moscow: Nauka Publ.) p 545

[2] Ilinsky V S 1982 *Protection of radio and electronic devices and precision equipment from dynamic effects* (Moscow: Radiosvyaz' Publ.) p 295

[3] Lobachevsky N I 1945 *Geometric studies on the theory of parallel lines* (Moscow, Leningrad: Publishing House of the Academy of Sciences of the USSR) p 180

[4] Lobachevsky N I 1956 *Selected works on geometry* (Moscow: Publishing House of the Academy of Sciences of the USSR)

[5] Lobachevsky N I 1956 *Three essays on geometry* (Moscow: Gostekhizdat Publ.)

[6] Efimov N V 1971 *Higher Geometry* (Moscow: Nauka Publ.) p 578
[7] Trainin Ya L 1965 *Foundations of geometry* (Moscow: State educ. and paed. publishing house) p 325

[8] Gozbenko V E, Kargapoltsev S K, Kornilov D N, Minaev N V, Karlina A I 2016 *International Journal of Applied Engineering Research* 11(20) 10367-10373

[9] Balanovskii A E, Van Huy V 2018 *Journal of Friction and Wear* 39(4) 311-318

[10] Balanovskii A E 2018 *High Temperature* 56(4) 486-495

[11] Balanovskii A E 2018 *High Temperature* 56(1) 3-13

[12] Balanovskii A E 2018 *High Temperature* 56(3) 329-337

[13] Balanovsky A E, Grechneva M V, Van Huy V, Ponomarev B B 2018 *IOP Conference Series: Materials Science and Engineering* 042010

[14] Balanovskii A E, Grechneva M V, Van Huy V, Zhuravlev D A 2017 *IOP Conference Series: Earth and Environmental Science* 092003

[15] Balanovskii A E, Grechneva M V, Huy V V, Zhuravlev D A 2017 IOP Conference Series: Earth and Environmental Science 092004

[16] Gozbenko V E, Khomenko A P, Kargapoltsev S K, Minaev N V, Karlina A I 2017 *International Journal of Applied Engineering Research*. 12 22 12369-12372