Study of light-cone distribution amplitudes for $p$-wave heavy mesons

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Abstract

In this paper, a study of light-cone distribution amplitudes for $p$-wave heavy mesons is presented in both general and heavy quark frameworks. Within the light-front approach, the leading twist light-cone distribution amplitudes, $\phi_M(u)$ and their relevant decay constants of heavy scalar, axial-vector and tensor mesons, $f_M$, are formulated. The relations of some decay constants can be simplified when the heavy quark limit is taken into account. After fixing the parameters which appear in a Gaussian wave function, the corresponding decay constants are calculated and compared with those of other theoretical approaches. The curves and the first six $\xi$ moments of $\phi_M(u)$ are plotted and estimated. These results all endorse the requirements of heavy quark symmetry.

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I. INTRODUCTION

Light-cone distribution amplitudes (LCDAs) of hadrons are key ingredients in the description of various exclusive processes of quantum chromodynamics (QCD), and their role can be analogous to those of parton distributions in inclusive processes. In terms of Bethe-Salpeter wave functions $\varphi(u_i, k_{i\perp})$, LCDAs $\phi(u_i)$ are defined by retaining the momentum fractions $u_i$ and integrating out the transverse momenta $k_{i\perp}$. They provide essential information on the nonperturbative structure of the hadron for QCD treatment of exclusive reactions. Specifically, the leading twist LCDAs describe the probability amplitudes to find the hadron in a Fock state with the minimum number of constituents. In the literature, there have been many nonperturbative approaches to estimate LCDAs, such as the QCD sum rules [2–6], lattice calculation [7, 8], chiral quark model from the instanton vacuum [9, 10], Nambu-Jona-Lasinio model [11, 12], and the light-front quark model [13–15]. These studies have dealt with LCDAs of pseudoscalar [3, 8–14], vector [4, 7, 13, 14], axial-vector [5, 6, 15], and tensor mesons.

The fact that $B$-physics exclusive processes are under investigation in BABAR, Belle, and LHC experiments also urges the detailed study of hadronic LCDAs. Recently, many $p$-wave heavy mesons were observed and confirmed. They include $D_0^*, D_1, D_s^*, D_s^+, D_s^{*+}, B_s^0, B_s^{*0}, \bar{B}_s^{*0}$ [21–23]. The present paper is devoted to the study of leading twist LCDAs of $p$-wave heavy mesons which include the scalar, axial-vector, and tensor mesons. We hope a thorough understanding of their properties, such as LCDAs which are universal nonperturbative objects, will be of great benefit when analyzing the hard exclusive processes with heavy meson production.

In the past decade, the most significant progress made in the QCD description of hadronic physics was, perhaps, in the avenue of heavy quark dynamics. The analysis of heavy hadron structures has been tremendously simplified by the heavy quark symmetry (HQS) proposed by Isgur and Wise [24, 25], and the heavy quark effective theory (HQET) developed from QCD in terms of $1/m_Q$ expansion [26–28]. HQET has provided a systematic framework for studying symmetry breaking $1/m_Q$ corrections (for a review, see Ref. [29]). Moreover, in terms of heavy quark expansion, HQET offered a new framework for the systematic study of the inclusive decays of heavy mesons [30–33]. However, the general properties of heavy hadrons, namely, their decay constants, transition form factors, structure functions, etc, are
still incalculable within QCD, even in the infinite quark-mass limit with the utilization of HQS and HQET. Hence, although HQS and HQET have simplified heavy quark dynamics, a complete first-principles QCD description of heavy hadrons is still lacking due to the unknown nonperturbative QCD dynamics.

In this study, the \( p \)-wave heavy meson is explored in a light-front quark model (LFQM) with both general and heavy quark frameworks. LFQM is a promising analytic method for solving the nonperturbative problems of hadron physics \[34\], as well as offering much insights into the internal structures of bound states. The basic ingredient in LFQM is the relativistic hadron wave function which generalizes distribution amplitudes by including transverse momentum distributions; it contains all the information of a hadron from its constituents. The hadronic quantities are represented by the overlap of wave functions and can be derived in principle. The light-front wave function is manifestly a Lorentz-invariant, expressed in terms of internal momentum fraction variables which are independent of the total hadron momentum. Moreover, the fully relativistic treatment of quark spins and center-of-mass motion can be carried out using the so-called Melosh rotation \[35\]. This treatment has been successfully applied to calculate phenomenologically many important meson decay constants and hadronic form factors \[36–40\]. Therefore, the main purpose of this study is the calculation of the leading twist LCDAs of \( p \)-wave heavy mesons within LFQM.

The remainder of this paper is organized as follows. In Sec. II, the leading twist LCDAs of \( p \)-wave meson states are shown in cases of vector and tensor currents with general and heavy quark frames. In Sec. III, the formulism of LFQM is reviewed briefly; then, the leading twist LCDAs are extracted within the LFQM. In Sec. IV, numerical results of the decay constants and LCDAs are recorded. The \( \xi \) moments of these LCDAs are also calculated and presented. Finally, conclusions are given in Sec. V.

II. LEADING TWIST LCDAS OF \( p \)-WAVE MESONS

A. General Framework

Amplitudes of hard processes involving \( p \)-wave mesons can be described by the matrix elements of gauge-invariant nonlocal operators, which are sandwiched between the vacuum
and the meson states:

$$\langle 0|\bar{q}(x)\Gamma[x,-x]q(-x)|H(P,\epsilon)\rangle,$$  (2.1)

where $P$ is the meson momentum, $\epsilon$ is the polarization vector or tensor ($\epsilon$ does not exist in the case of the scalar meson), $\Gamma$ is a generic notation for the Dirac matrix structure, and the path-ordered gauge factor is

$$[x,y] = P \exp \left[ ig_s \int_0^1 dt (x - y)\mu A_\mu(t x + (1 - t)y) \right].$$  (2.2)

This factor is equal to unity in the light-cone gauge which is equivalent to the fixed-point gauge, $(x - y)\mu A_\mu(x - y) = 0$, as the quark-antiquark pair is at the lightlike separation [41]. For simplicity, the gauge factor will not be shown below.

The asymptotic expansion of exclusive amplitudes, in powers with large momentum transfer, is governed by the expanding amplitude Eq. (2.1), shown in powers of deviation from the light-cone $x^2 = 0$. There are two lightlike vectors, $p$ and $z$, which can be introduced by

$$p^2 = 0, \quad z^2 = 0,$$  (2.3)

such that $p \to P$ in the limit $M_H^2 \to 0$ and $z \to x$ for $x^2 = 0$. From this, it follows that [4]

$$z^\mu = x^\mu - P^\mu \frac{1}{M_H^2} \left[ P x - \sqrt{(P x)^2 - x^2 M_H^2} \right] = x^\mu - P^\mu \frac{x^2}{2Pz} + O(x^4),$$

$$p^\mu = P^\mu - z^\mu \frac{M_H^2}{2Pz},$$  (2.4)

where $P x \equiv P \cdot x$ and $P z = pz = \sqrt{(P x)^2 - x^2 M_H^2}$. In addition, if it is assumed that the meson moves in a positive $\hat{e}_3$ direction, then $p^+$ and $z^-$ are the only nonzero components of $p$ and $z$, respectively, in an infinite momentum frame. For the axial-vector meson, the polarization vector $\epsilon^\mu$ is decomposed into longitudinal and transverse projections as

$$\epsilon^\mu = \frac{\epsilon z}{Pz} \left( p^\mu - z^\mu \frac{M_H^2}{2Pz} \right), \quad \epsilon^\mu_\perp = \epsilon^\mu - \epsilon^\mu_\parallel,$$  (2.5)

respectively. For the tensor meson, the polarization tensor is

$$\epsilon^{\mu\nu}(m) = \langle 11; m' m''|11; 2m\epsilon^\mu(m')\epsilon^\nu(m'')\rangle,$$  (2.6)
(m is the magnetic quantum number) or

\[ \epsilon_{\pm 2}^\mu = \epsilon_{\pm 1}^\mu, \]  
\[ \epsilon_{\pm 1}^\mu = \sqrt{\frac{1}{2}} [\epsilon_{\pm 1}^\mu \epsilon_0^\nu + \epsilon_0^\mu \epsilon_{\pm 1}^\nu], \]  
\[ \epsilon_0^\mu = \sqrt{\frac{1}{6}} [\epsilon_{+1}^\mu \epsilon_{-1}^\nu + \epsilon_{-1}^\mu \epsilon_{+1}^\nu] + \sqrt{\frac{2}{3}} \epsilon_0^\mu \epsilon_0^\nu, \]  

and \(\epsilon_{\mu}(\equiv \epsilon^{\mu\nu}z_\nu)\) can also be decomposed into longitudinal and transverse projections as

\[ \epsilon_{\parallel}^\mu = \frac{\epsilon_{\parallel\parallel}^\mu}{p_z} \left(p^\mu - z^\mu \frac{M_H^2}{2p_z^3}\right), \]
\[ \epsilon_{\perp}^\mu = \epsilon_{\mu}^\parallel - \epsilon_{\parallel\parallel}^\mu. \]  

LCDAs are defined in terms of the matrix element of a nonlocal operator in Eq. (2.4). For scalar \((S)\), axial vector \((A)\), and tensor \((T)\) mesons, the leading twist LCDAs can be defined as

\[ \langle 0 | \bar{q}(z) \gamma^\mu q(-z) | S(P) \rangle = f_S \int_0^1 du \ e^{i p \cdot z} \left[p^\mu \phi_S(u) + z^\mu \frac{M_S^2}{2p_z^3} g_S(u)\right], \]  
\[ \langle 0 | \bar{q}(z) \gamma^\mu \gamma_5 q(-z) | A(P, \epsilon_\lambda=0) \rangle = i f_A M_A \int_0^1 du \ e^{i p \cdot z} \left\{ \epsilon_{\parallel\parallel}^\mu \epsilon_{\parallel\parallel}^\nu \phi_A(u) + \epsilon_{\parallel\perp}^\mu g_A(u) \right\}, \]  
\[ \langle 0 | \bar{q}(z) \sigma^{\mu\nu} \gamma_5 q(-z) | A(P, \epsilon_{\lambda\pm 1}) \rangle = f_A^\perp \int_0^1 du \ e^{i p \cdot z} \left\{ \left( \epsilon_{\parallel\parallel}^\mu \epsilon_{\parallel\parallel}^\nu - \epsilon_{\parallel\perp}^\mu \epsilon_{\parallel\perp}^\nu \right) \phi_{A\perp}(u) \right\}, \]
\[ \langle 0 | \bar{q}(z) \gamma^\mu q(-z) | T(P, \epsilon_\lambda=0) \rangle = f_T M_T^2 \int_0^1 du \ e^{i p \cdot z} \left\{ \epsilon_{\parallel\parallel}^\mu \epsilon_{\parallel\parallel}^\nu \phi_{T\parallel}(u) + \epsilon_{\parallel\perp}^\mu g_{T\perp}(u) \right\}, \]
\[ \langle 0 | \bar{q}(z) \sigma^{\mu\nu} q(-z) | T(P, \epsilon_{\lambda\pm 1}) \rangle = i f_T^\perp M_T \int_0^1 du \ e^{i p \cdot z} \left\{ \left( \epsilon_{\parallel\parallel}^\mu \epsilon_{\parallel\parallel}^\nu - \epsilon_{\parallel\perp}^\mu \epsilon_{\parallel\perp}^\nu \right) \phi_{T\perp}(u) \right\}, \]

where \(u\) is the momentum fraction and \(\xi \equiv (1-u) - u = 1 - 2u\). Here \(\phi_S, \phi_{A,T\parallel}\) and \(\phi_{A,T\perp}\) are the leading twist-2 LCDAs, and the others contain contributions from higher-twist operators.
The leading twist LCDAs are normalized as
\[
\int_0^1 du \phi_{S,A}(u) = 1, \quad (2.16)
\]
\[
\int_0^1 du \xi \phi_T(u) = 1 \quad (2.17)
\]
and can be parametrized as the so-called \(\xi\) moments,
\[
\langle \xi^n \rangle = \int_{-1}^1 d\xi \, \xi^n \phi(\xi). \quad (2.18)
\]

To disentangle the twist-2 LCDAs from higher-twist operators in Eqs. (2.11) \(\sim\) (2.15), a twist-2 contribution of the relevant nonlocal operator \(\bar{q}(z) \Gamma q(-z)\) must be derived. In the case of \(\Gamma = \gamma^\mu (\gamma_5)\), the leading twist-2 contribution contains contributions from the operators which are fully symmetric in Lorentz indices \([42, 43]\):
\[
[\bar{q}(-z) \gamma^\mu (\gamma_5) q(z)]_2 = \sum_{n=0}^{\infty} \frac{1}{n!} \bar{q}(0) \left\{ \frac{(z \cdot \vec{D})^n}{n + 1} \gamma^\mu + \frac{n(z \cdot \vec{D})^{n-1}}{n + 1} \vec{D}^\mu \right\} (\gamma_5) q(0), \quad (2.19)
\]
where \(\vec{D} = \vec{D} - \vec{D}\) and \(\vec{D} = \vec{D} - igB^a(\lambda^a/2)\). The sum can be represented in terms of a nonlocal operator,
\[
[\bar{q}(-z) \gamma^\mu (\gamma_5) q(z)]_2 = \int_0^1 dt \frac{\partial}{\partial z_\mu} \bar{q}(-tz) \not{D}(\gamma_5) q(tz). \quad (2.20)
\]
Taking the matrix element between the vacuum and the \(p\)-wave meson state, we obtain
\[
\langle 0 | [\bar{q}(-z) \gamma^\mu q(z)]_2 | S(P) \rangle = f_S \int_0^1 du \phi_S(u) \left\{ p^\mu e^{i\xi p z} + (P^\mu - p^\mu) \int_0^1 dt e^{i\xi tp z} \right\}, \quad (2.21)
\]
\[
\langle 0 | [\bar{q}(-z) \gamma^\mu \gamma_5 q(z)]_2 | A(P, \epsilon_{\lambda=0}) \rangle = if_{AM} M_A \int_0^1 du \phi_{A\parallel}(u) \left\{ p^\mu \frac{\epsilon z}{p z} e^{i\xi p z} + \left( \epsilon^\mu - \frac{\epsilon^\mu \epsilon z}{p z} \right) \int_0^1 dt e^{i\xi tp z} \right\}, \quad (2.22)
\]
\[
\langle 0 | [\bar{q}(-z) \gamma^\mu q(z)]_2 | T(P, \epsilon_{\lambda=0}) \rangle = f_T M_T^2 \int_0^1 du \phi_{T\parallel}(u) \left\{ p^\mu \frac{\epsilon^{\mu\ast}}{(p z)^2} e^{i\xi p z} + 2 \left( \frac{\epsilon^{\mu\ast}}{p z} - \frac{\epsilon^\mu \epsilon^{\mu\ast}}{(p z)^2} \right) \int_0^1 dt e^{i\xi tp z} \right\}, \quad (2.23)
\]
For the derivations in Eqs. (2.21) \(\sim\) (2.23), we refer to Ref. [42], which dealt with the vector meson state. We can use Eq. (2.19), and then expand the right-hand sides of Eqs. (2.21) \(\sim\) (2.23), as follows:
\[
\sum_{n=0}^{\infty} \frac{1}{n!} \langle 0 | \bar{q}(0) \left\{ \frac{(z \cdot \vec{D})^n}{n + 1} \gamma^\mu + \frac{n(z \cdot \vec{D})^{n-1}}{n + 1} \vec{D}^\mu \right\} q(0) | S(P) \rangle
\]
we obtain:

Taking the matrix element between the vacuum and the axial-vector and tensor meson state, the sum can also be represented in terms of nonlocal operators:

\[ \langle 0 | \bar{q}(0) \gamma^\mu q(0) | S(P) \rangle = f_s P^\mu \int_0^1 du \phi_S(u), \]

Note that the tensor meson cannot be produced by the \( V - A \) current. We then pick \( n = 1 \) in Eq. (2.26) and obtain

\[ \frac{1}{2} \langle 0 | \bar{q}(0) (\gamma^\mu z \cdot \bar{D} + z \bar{D}^\mu) q(0) | T(P, \epsilon_{\lambda=0}) \rangle = f_T M_T^3 \epsilon^\mu \int_0^1 du \xi \phi_T\parallel(u). \]

From the normalization Eq. (2.16), we have \( \langle 0 | \bar{q} \gamma^\mu \gamma_5 q | 3A_1(P, \epsilon) \rangle = i f_{3A_1} M_{3A_1} \epsilon^\mu \) which is consistent with the results of Ref. [44].

Next, we consider the case of \( \Gamma = \sigma_{\mu\nu}(\gamma_5) \), where the leading twist-2 contribution contains contributions from the operators:

\[ [\bar{q}(-z) \sigma^{\mu\nu}(\gamma_5) q(z)]_2 = \sum_{n=0}^{\infty} \frac{1}{n!} \bar{q}(0) \left\{ (z \cdot \bar{D})^n \sigma^{\mu\nu} + \frac{n(z \cdot \bar{D})^{n-1}}{2n+1} \bar{D}^\mu \sigma^{\nu\bullet} + \frac{n(z \cdot \bar{D})^{n-1}}{2n+1} \bar{D}^\nu \sigma^{\mu\bullet} \right\} (\gamma_5) q(0). \]

The sum can also be represented in terms of nonlocal operators:

\[ [\bar{q}(-z) \sigma^{\mu\nu}(\gamma_5) q(z)]_2 = \int_0^1 dt \left[ \frac{\partial}{\partial t} \bar{q}(-t^2 z) \sigma^{\nu\bullet}(\gamma_5) q(t^2 z) + z_\alpha \frac{\partial}{\partial z_\nu} \bar{q}(-t^2 z) \sigma^{\mu\alpha}(\gamma_5) q(t^2 z) \right]. \]

Taking the matrix element between the vacuum and the axial-vector and tensor meson state, we obtain:

\[ \langle 0 | [\bar{q}(-z) \sigma^{\mu\nu} \gamma_5 q(z)]_2 | A(P, \epsilon_{\lambda=\pm 1}) \rangle \]
\[ f_A^\perp \int_0^1 du \left\{ \phi_{A\perp}(u) \left[ S^{\mu\nu} e^{i\xi Pz} + \left( (\epsilon^\mu P^\nu - \epsilon^\nu P^\mu) - S^{\mu\nu} \right) \int_0^1 dt e^{i\xi t^2pz} \right] \\
+ \left( h_{A\parallel}(u) - \phi_{A\perp}(u) \right) \left[ T^{\mu\nu} e^{i\xi Pz} + \left( U^{\mu\nu} - T^{\mu\nu} \right) \int_0^1 dt e^{i\xi t^2pz} \right] \right\}, \quad (2.32) \]

\[ \langle 0 | [\bar{q}(z)\sigma^{\mu\nu}\epsilon_{\perp\mu}\gamma_5 q(z)]_2 | A(P, \epsilon_{\lambda=\pm1}) \rangle = f_A^T \int_0^1 du \phi_{T\perp}(u) \frac{1}{2} (\epsilon \cdot \epsilon_{\perp} Pz) e^{i\xi Pz} + \int_0^1 dt e^{i\xi t^2pz} \]

\[ \langle 0 | [\bar{q}(z)\sigma^{\mu\nu}\epsilon_{\perp\mu}\gamma_5 q(z)]_2 | T(P, \epsilon_{\lambda=\pm1}) \rangle = i f_T^T M_T \int_0^1 du \phi_{T\perp}(u) \frac{1}{2} \epsilon^{\mu\nu} \epsilon_{\perp\mu} e^{i\xi Pz} + \int_0^1 dt e^{i\xi t^2pz} \]

Then, we use Eq. (2.30) and expand the right-hand sides of Eqs. (2.35) and (2.36) as

\[ \sum_{n=0}^{\infty} \frac{1}{n!} \langle 0 | \bar{q}(0) \frac{(n+1)(z \cdot \vec{D})^n}{2n+1} \sigma^{\mu\nu} \epsilon_{\perp\mu} \gamma_5 q(0) | A(P, \epsilon_{\lambda=\pm1}) \rangle = f_A^T \sum_{n=0}^{\infty} \frac{i^n}{n!} \int_0^1 du \phi_{A\perp}(u) \frac{1}{2} (\epsilon \cdot \epsilon_{\perp} Pz)(\xi Pz)^n \left[ 1 + \int_0^1 dt e^{i\xi t^2pz} \right], \quad (2.37) \]

\[ \sum_{n=0}^{\infty} \frac{1}{n!} \langle 0 | \bar{q}(0) \frac{(n+1)(z \cdot \vec{D})^n}{2n+1} \sigma^{\mu\nu} \epsilon_{\perp\mu} q(0) | T(P, \epsilon_{\lambda=\pm1}) \rangle = i f_T^T M_T \sum_{n=0}^{\infty} \frac{i^n}{n!} \int_0^1 du \phi_{T\perp}(u) \frac{1}{2} \epsilon^{\mu\nu} \epsilon_{\perp\mu} (\xi Pz)^n \left[ 1 + \int_0^1 dt e^{i\xi t^2pz} \right], \quad (2.38) \]
Picking \( n = 0 \) and \( n = 1 \) in Eqs. (2.37) and (2.38), respectively, we obtain

\[
\langle 0 | \bar{q}(0) \sigma^{\mu \nu} \epsilon_{\mu \nu} q(0) | A(P, \epsilon_{\lambda = \pm 1}) \rangle = f_\perp \int_0^1 du \phi_A(u) (\epsilon \cdot \epsilon P z),
\]

\[ (2.39) \]

\[
\langle 0 | \bar{q}(0) (z \cdot \vec{D}) \sigma^{\mu \nu} \epsilon_{\mu \nu} q(0) | T(P, \epsilon_{\lambda = \pm 1}) \rangle = f_\perp M_T \int_0^1 du \xi \phi_T(u) (\epsilon \cdot \epsilon P z).
\]

\[ (2.40) \]

B. Heavy Quark Framework

In general, the theoretical description of meson properties relies on the bound state models with a relativistic normalization:

\[
\langle M(P') | M(P, \epsilon) \rangle = 2 P^0 (2\pi)^3 \delta^3(P' - P) \delta_{\epsilon' \epsilon}.
\]

\[ (2.41) \]

At low energies, however, these models have little connection to the fundamental theory of QCD. The reliable predictions are often made based on symmetries. A well-known example is HQS \[29\], which arises since the Compton wavelength, \( 1/m_Q \), of a heavy quark bound inside a hadron is much smaller than a typical hadronic distance (about 1 fm), and \( m_Q \) is unimportant for the low-energy properties of the state. For a heavy-light meson system, it is more natural to use velocity \( v^\mu \) instead of momentum variables. Then, it is appropriate to work with a mass-independent normalization of a heavy-light meson state:

\[
\langle \tilde{M}(v', \epsilon') | \tilde{M}(v, \epsilon) \rangle = 2v^0 (2\pi)^3 \delta^3(\tilde{\Lambda} v' - \tilde{\Lambda} v) \delta_{\epsilon' \epsilon},
\]

\[ (2.42) \]

where \( \tilde{\Lambda} = M - m_Q \) is the so-called residual center mass of a heavy-light meson. The relation between these two bound states is

\[
|M(P, \epsilon)\rangle = \sqrt{M} |\tilde{M}(v, \epsilon)\rangle.
\]

\[ (2.43) \]

In addition, the heavy quark field can be expanded as \[29\]

\[
Q(x) = e^{-im_Q v \cdot x} \left[ 1 + \frac{1}{iv \cdot D + 2m_Q - i\epsilon} i \vec{D}_\perp \right] h_v(x),
\]

\[ (2.44) \]

where \( h_v^*(x) \) is a field describing a heavy antiquark with velocity \( v \). Then, the current \( \bar{q} \Gamma Q \) can be represented as

\[
\bar{q} \Gamma Q = \bar{q} \Gamma \left( 1 + i \frac{\vec{D}_\perp}{2m_Q} + \cdots \right) h_v.
\]

\[ (2.45) \]
are obtained as heavy meson $M$, namely, \( \omega = uM \), and $\omega$ was first introduced in Ref. [46] as the product of the longitudinal momentum fraction $u$ of the light (anti)quark and the mass of heavy meson $M$, namely, $\omega = uM$. Following a similar process, the leading twist LCDAs are obtained as

\[
\langle 0 | \bar{q}(0) \gamma^\mu h_v(0) | \bar{S}(v) \rangle = F_S v^\mu \int_0^\infty d\omega \Phi_S(\omega),
\]

\[
\langle 0 | \bar{q}(0) \gamma^\mu \gamma_5 h_v(0) | \bar{A}(v, \hat{c}_{\lambda=0}) \rangle = i F_A \hat{e}^\mu \int_0^\infty d\omega \Phi_{A\parallel}(\omega),
\]

\[
\frac{1}{2} \langle 0 | \bar{q}(0) (\gamma^\mu z \cdot \vec{D} + \vec{z} \cdot \vec{D}^\mu) h_v(0) | \bar{T}(v, \hat{c}_{\lambda=0}) \rangle = F_T \hat{e}^{\mu\bullet} \int_0^\infty d\omega \Phi_{T\parallel}(\omega),
\]

\[
\langle 0 | \bar{q}(0) \sigma^{\mu\nu} \gamma_5 h_v(0) | \bar{A}(v, \hat{c}_{\lambda=\pm 1}) \rangle = F_A \hat{e}_{\perp} \int_0^\infty d\omega \Phi_{A\perp}(\omega),
\]

\[
\langle 0 | \bar{q}(0) (z \cdot \vec{D}) \sigma^{\mu\nu} \gamma_5 h_v(0) | \bar{T}(v, \hat{c}_{\lambda=\pm 1}) \rangle = F_T \hat{e}_{\perp} \int_0^\infty d\omega \Phi_{T\perp}(\omega).
\]
III. GENERAL FORMULISM IN LFQM

A. General Framework

A meson bound state, consisting of a quark $q_1$ and an antiquark $\bar{q}_2$ with total momentum $P$ and spin $J$, can be written as (see, for example, Ref. [37])

$$|M(P, L, J)\rangle = \int \{d^3k_1\}\{d^3k_2\} \frac{2(2\pi)^3\delta^3(\vec{P} - \vec{k}_1 - \vec{k}_2)}{2(2\pi)^3} \times \sum_{\lambda_1, \lambda_2} \Psi^{J_L L_S}_{\lambda_1}((\vec{k}_1, \vec{k}_2, \lambda_1, \lambda_2) | q_1(k_1, \lambda_1)\bar{q}_2(k_2, \lambda_2)), \tag{3.1}$$

where $k_1$ and $k_2$ are the on-mass-shell light-front momenta,

$$\vec{k} = (k^+, k^\perp), \quad k^\perp = (k^1, k^2), \quad k^- = \frac{m_q^2 + k_1^2}{k^+}, \tag{3.2}$$

and

$$\{d^3k\} = \frac{dk^+dk^\perp}{2(2\pi)^3},$$

$$|q(k_1, \lambda_1)\bar{q}(k_2, \lambda_2)) = b_{\lambda_1}^d(k_1)\tilde{d}_{\lambda_2}^i(k_2)|0\rangle,$$

$$\{b_{\lambda'}(k'), b_{\lambda}^i(k)\} = \{d_{\lambda'}(k'), d_{\lambda}^i(k)\} = 2(2\pi)^3 \delta^3(\vec{k}' - \vec{k}) \delta_{\lambda'\lambda}. \tag{3.3}$$

In terms of the light-front relative momentum variables $(u, \kappa_\perp)$ are defined by

$$k_1^+ = (1 - u)P^+, \quad k_2^+ = uP^+,$$

$$k_1^\perp = (1 - u)P_\perp + \kappa_\perp, \quad k_2^\perp = uP_\perp - \kappa_\perp. \tag{3.4}$$

The momentum-space wave function $\Psi^{J_L L_S}_{\lambda_1, \lambda_2}$ for a $2S+1L_J$ meson can be expressed as

$$\Psi^{J_L L_S}_{\lambda_1, \lambda_2}(\vec{k}_1, \vec{k}_2, \lambda_1, \lambda_2) = \frac{1}{\sqrt{N_c}} \langle LS; L_zS_z|LS; J J_z\rangle R^{SS}_{\lambda_1\lambda_2}(u, \kappa_\perp) \varphi_{LL_z}(u, \kappa_\perp), \tag{3.5}$$

where $\varphi_{LL_z}(u, \kappa_\perp)$ describes the momentum distribution of the constituent quarks in the bound state with the orbital angular momentum $L$, $\langle LS; L_zS_z|LS; J J_z\rangle$ as the corresponding Clebsch-Gordan coefficient and $R^{SS}_{\lambda_1\lambda_2}$ constructing a state of definite spin $(S, S_z)$ out of light-front helicity $(\lambda_1, \lambda_2)$ eigenstates. Explicitly,

$$R^{SS}_{\lambda_1\lambda_2}(u, \kappa_\perp) = \sum_{s_1, s_2} \langle \lambda_1|\mathcal{R}^+_M(1 - u, \kappa_\perp, m_1)|s_1\rangle \langle \lambda_2|\mathcal{R}^-_M(u, -\kappa_\perp, m_2)|s_2\rangle \left(\frac{1}{2}; s_1s_2|\frac{1}{2}; SS\right), \tag{3.6}$$

where $|s_i\rangle$ are the usual Pauli spinors, and $\mathcal{R}_M$ is the Melosh transformation operator $^{30}$. 

$$\langle s|\mathcal{R}_M(u, \kappa_\perp, m_i)|\lambda\rangle = \frac{m_i + u_iM_0 + i\vec{s}_\lambda \cdot \vec{n}_\perp \times \vec{n}}{\sqrt{(m_i + u_iM_0)^2 + \kappa_\perp^2}}, \tag{3.7}$$
with $u_1 = 1 - u$, $u_2 = u$, and $\vec{n} = (0, 0, 1)$ is a unit vector in the $\hat{z}$-direction. In addition,

$$M_0^2 = (e_1 + e_2)^2 = \frac{m_1^2 + \kappa_{\perp}^2}{u_1} + \frac{m_2^2 + \kappa_{\perp}^2}{u_2},$$

$$e_i = \sqrt{m_i^2 + \kappa_{\perp}^2 + \kappa_z^2}, \quad \frac{e_1 - \kappa_z}{e_1 + e_2} = 1 - u, \quad \frac{e_2 + \kappa_z}{e_1 + e_2} = u,$$

where $\kappa_z$ is the relative momentum in the $\hat{z}$ direction and can be written as

$$\kappa_z = \frac{uM_0}{2} - \frac{m_1^2 + \kappa_{\perp}^2}{2uM_0}. \quad (3.8)$$

$M_0$ is the invariant mass of $q\bar{q}$ and is generally different from the mass $M$ of a meson which satisfies $M^2 = P^2$. This is due to the fact that the meson, quark and antiquark cannot be simultaneously on-shell. We normalize the meson state as

$$\langle M(P', J', J_z') | M(P, J, J_z) \rangle = 2(2\pi)^3 P^+ \delta^3(\tilde{P}' - \tilde{P})\delta_{J'J}\delta_{J_z'J_z}, \quad (3.9)$$

in order that

$$\int \frac{du d^2\kappa_{\perp}}{2(2\pi)^3} \varphi^*_{L'L_z}(u, \kappa_{\perp})\varphi_{L_z}(u, \kappa_{\perp}) = \delta_{L_L, L_L}\delta_{L_z, L_z}. \quad (3.10)$$

Explicitly, we have

$$\varphi_{1L_z} = \kappa_L \varphi_p, \quad (3.11)$$

where $\kappa_{L_z = \pm 1} = \mp (\kappa_{\perp} \pm i\kappa_{\perp y})/\sqrt{2}$, $\kappa_{L_z = 0} = \kappa_z$ are proportional to the spherical harmonics $Y_{1L_z}$ in momentum space, and $\varphi_p$ is the distribution amplitude of the $p$-wave meson. In general, for any function $F(|\vec{\kappa}|)$, $\varphi_p(u, \kappa_{\perp})$ has the form of

$$\varphi_p(u, \kappa_{\perp}) = N \sqrt{\frac{d\kappa_z}{du}} F(|\vec{\kappa}|), \quad (3.12)$$

where

$$\frac{d\kappa_z}{du} = \frac{e_1 e_2}{u(1 - u)M_0} \quad (3.13)$$

is the Jacobian of transformation from $(u, \kappa_{\perp})$ to $\vec{\kappa}$ and the normalization factor $N$ is determined from Eq. (3.10).

In the case of a $p$-wave meson state, it is more convenient to use the covariant form of $R^{SSZ}_{\lambda_1\lambda_2} \var{36, 39, 47}$:

$$\langle 1S; L_z S_z | 1S; JJ_z \rangle k_{L_z} R^{SSZ}_{\lambda_1\lambda_2}(u, \kappa_{\perp}) = \frac{\sqrt{k_1^+ k_2^+}}{\sqrt{2} M_0(M_0 + m_1 + m_2)} \times \tilde{u}(k_1, \lambda_1)(\vec{P} + M_0)\Gamma_{2s+1P_j} v(k_2, \lambda_2), \quad (3.14)$$
where

\[ \tilde{M}_0 \equiv \sqrt{M_0^2 - (m_1 - m_2)^2}, \quad \tilde{P} \equiv k_1 + k_2, \]

\[ \bar{u}(k, \lambda) u(k, \lambda') = \frac{2m}{k^+} \delta_{\lambda, \lambda'}, \quad \sum_{\lambda} u(k, \lambda) \bar{u}(k, \lambda) = \frac{k + m}{k^+}, \]

\[ \bar{v}(k, \lambda) v(k, \lambda') = -\frac{2m}{k^+} \delta_{\lambda, \lambda'}, \quad \sum_{\lambda} v(k, \lambda) \bar{v}(k, \lambda) = \frac{k - m}{k^+}. \] (3.15)

For the scalar, axial-vector, and tensor mesons, we have

\[ \Gamma_{P_0} = \frac{1}{\sqrt{3}} \left( K - \frac{K \cdot \tilde{P}}{M_0} \right), \]

\[ \Gamma_{P_1} = \epsilon \cdot \tilde{K} \gamma_5, \]

\[ \Gamma_{P_1} = \frac{1}{\sqrt{2}} \left( (K - \frac{K \cdot \tilde{P}}{M_0}) \gamma^5 - \epsilon \cdot \tilde{K} \right) \gamma_5, \]

\[ \Gamma_{P_2} = \epsilon_{\mu\nu} \gamma^\mu (-K^\nu), \] (3.16)

where \( K \equiv (k_2 - k_1)/2 \) and

\[ \epsilon_{\lambda=\pm 1}^\mu = \left[ \frac{2}{P_+} \vec{\epsilon}_\perp (\pm 1) \cdot \vec{P}_\perp, 0, \vec{\epsilon}_\perp (\pm 1) \right], \]

\[ \vec{\epsilon}_\perp (\pm 1) = \mp (1, \pm i)/\sqrt{2}, \]

\[ \epsilon_{\lambda=0}^\mu = \frac{1}{M_0} \left( -M_0^2 + P_+^2 \right), P_+, P_\perp. \] (3.17)

Note that the polarization tensor of a tensor meson satisfies the relations \( \epsilon_{\mu\nu} = \epsilon_{\nu\mu} \) and \( \epsilon_{\mu\nu} \bar{P}^\mu = \epsilon_\mu = 0 \). Equations (3.14) and (3.16) can be further reduced by the applications of equations of motion on spinors:

\[ \langle 1S; L_2S_2|1S; JJ_z \rangle k_{L_2} R_{\lambda_1\lambda_2}^{SS}(u, \kappa) = \frac{\sqrt{k_1^+ k_2^+}}{\sqrt{2} M_0} \bar{u}(k_1, \lambda_1) \Gamma_{P_0}^{\lambda_2+1} v(k_2, \lambda_2), \] (3.18)

where

\[ \Gamma_{P_0}^{\lambda_2+1} = -\frac{\tilde{M}_0^2}{2\sqrt{3} M_0}, \]

\[ \Gamma_{P_1}^{\lambda_2+1} = \epsilon \cdot \tilde{K} \gamma_5, \]

\[ \Gamma_{P_1}^{\lambda_2+1} = \frac{-1}{2\sqrt{2} M_0} \left( \bar{g} M_0^2 - 2\epsilon \cdot K (m_1 - m_2) \right) \gamma_5, \]

\[ \Gamma_{P_2}^{\lambda_2+1} = \epsilon_{\mu\nu} \left( \gamma^\mu + \frac{2K^\mu}{M_0 + m_1 + m_2} \right) (-K^\nu). \] (3.19)

Next, the matrix elements of Eqs. (2.27), (2.28), (2.29), (2.39), and (2.40) will be calculated within the LFQM, and the relevant leading twist LCDAs are extracted. For the scalar
meson state, we substitute Eqs. (3.1), (3.5), and (3.18) into Eq. (2.27) to obtain

\[
\langle q_2 \gamma^\mu q_1 | S(P) \rangle = N_c \int \{ d^3 k_1 \} \sum_{\lambda_1, \lambda_2} \Psi_{LS}^{T_I}(k_1, k_2, \lambda_1, \lambda_2) \langle 0 | q_2 \gamma^\mu q_1 | q_1 q_2 \rangle \\
= -\sqrt{N_c} \int \{ d^3 k_1 \} \frac{\sqrt{k_1^+ k_2^+}}{\sqrt{2} M_0} \varphi_p \left( \gamma^\mu \left( \frac{k_1 + m_1}{k_1^+} \right) \right) \frac{\tilde{M}_0}{2 \sqrt{3} M_0} \left( -\frac{k_2 + m_2}{k_2^+} \right) \\
= f_S P^\mu \int du \varphi(u).
\]

For the “good” component, \( \mu = + \), the leading twist LCDA \( \phi_S \) can be extracted as

\[
\phi_S(u) = \frac{\sqrt{2N_c}}{f_S} \int \frac{d^2 \kappa_\perp}{2(2\pi)^3} \frac{[um_1 - (1 - u)m_2]\tilde{M}_0}{\sqrt{3u(1 - u)M_0} \varphi_p(u, \kappa_\perp)}.
\]

A similar process can be used for the axial-vector and tensor mesons which correspond to Eqs. (2.28), (2.39), and (2.29), (2.40), respectively, and the leading twist LCDAs are extracted as

\[
\phi_{3A_1}^u(u) = -\frac{\sqrt{3}}{f_{3A_1}} \int \frac{d^2 \kappa_\perp}{2(2\pi)^3} \varphi_p(u, \kappa_\perp) \left[ \frac{\tilde{M}_0^2}{2M_0} (\tilde{M}_0^2 - 4m_1 m_2) + \tilde{M}_0^2 \kappa_z (1 - 2u) \right] - 2\kappa_z(m_1 - m_2)(um_1 + (1 - u)m_2),
\]

\[
\phi_{1A_1}^u(u) = \frac{2\sqrt{6}}{f_{1A_1}} \int \frac{d^2 \kappa_\perp}{2(2\pi)^3} \frac{um_1 + (1 - u)m_2}{\sqrt{u(1 - u)M_0}} \varphi_p(u, \kappa_\perp) \kappa_z,
\]

\[
\phi_{3A_1}^u(u) = -\frac{\sqrt{3}}{f_{3A_1}} \int \frac{d^2 \kappa_\perp}{2(2\pi)^3} \frac{\varphi_p(u, \kappa_\perp)}{\sqrt{u(1 - u)M_0}} \left[ \tilde{M}_0(u m_1 - (1 - u) m_2) + \frac{\kappa_z^2}{M_0} (m_1 - m_2) \right],
\]

\[
\phi_{1A_1}^u(u) = \frac{\sqrt{6}}{f_{1A_1}} \int \frac{d^2 \kappa_\perp}{2(2\pi)^3} \frac{\kappa_z^2}{\sqrt{u(1 - u)M_0}} \varphi_p(u, \kappa_\perp),
\]

\[
\phi_{T}^u(u) = \frac{\sqrt{6}}{f_T} \int \frac{d^2 \kappa_\perp}{2(2\pi)^3} \varphi_p(u, \kappa_\perp) \left[ \kappa_z \left( um_1 - (1 - u) m_2 \right) - \frac{\kappa_z^2}{2M_0} (m_1 - m_2) \\
+ (2\kappa_z^2 - \kappa_\perp^2) \left( (1 - 2u) + 2 \frac{um_1 - (1 - u) m_2}{M_0 + m_1 + m_2} \right) \right],
\]

\[
\phi_{T}^u(u) = \frac{2\sqrt{6}}{f_T} \int \frac{d^2 \kappa_\perp}{2(2\pi)^3} \frac{\varphi_p(u, \kappa_\perp)}{\sqrt{u(1 - u)M_0}} \left[ \kappa_z (um_1 + (1 - u)m_2) - \frac{\kappa_z^2}{2M_0} (m_1 - m_2) \\
+ \frac{2\kappa_\perp^2}{M_0 + m_1 + m_2} \kappa_z \right].
\]
B. **Heavy Quark Framework**

If one takes \( m_1 = m_Q \to \infty \), that is, the heavy quark limit in the heavy-light meson, then two inequalities, \( m_Q \simeq M_0 \gg m_2, \kappa_\perp \) and \( u \to 0 \), are obtained. The exact form of \( \Phi_M \) can be derived by the redefinition of the meson bound state. Let us consider the bound states of heavy mesons in the heavy quark limit:

\[
|\tilde{M}(v; L, J)\rangle = \int \{d^3q\}\{d^3k_2\} 2(2\pi)^3\delta^3(\tilde{X} - \tilde{q} - \tilde{k}_2)
\times \sum_{\lambda_1,\lambda_2} \tilde{\Psi}_{LS}(\omega, \kappa_\perp, \lambda_1, \lambda_2) b^\dagger_v(q, \lambda_1) d^\dagger(k_2, \lambda_2)|0\rangle,
\]  

(3.28)

where \( q = k_1 - m_Q v \) is the residual momentum of the heavy quark. Operators \( b^\dagger_v(q, \lambda_1) \) create a heavy quark with

\[
\{b_v(q, \lambda_1), b^\dagger_{v'}(q', \lambda'_1)\} = 2(2\pi)^3\delta_{vv'}\delta^3(\tilde{q} - \tilde{q}')\delta_{\lambda_1\lambda'_1}.
\]  

(3.29)

The variable \( \omega \) and the relative transverse and longitudinal momenta, \( \kappa_\perp \) and \( \kappa_z \), are obtained by

\[
\omega = e_2 + \kappa_z, \quad \kappa_\perp = k_2 - \omega v_\perp, \quad \kappa_z = \frac{\omega}{2} - \frac{m_2^2 + \kappa_\perp^2}{2\omega}.
\]  

(3.30)

The momentum-space wave function \( \tilde{\Psi}_{LS}^{Jz} \) can be expressed as

\[
\tilde{\Psi}_{LS}^{Jz}(\omega, \kappa_\perp, \lambda_1, \lambda_2) = \frac{1}{\sqrt{N_c}} \langle LS; L_z S_z | LS, J J_z \rangle \tilde{R}_{\lambda_1 \lambda_2}^{SS_z}(\omega, \kappa_\perp) \tilde{\phi}_{LL_z}(\omega, \kappa_\perp),
\]  

(3.31)

where

\[
\langle LS; L_z S_z | LS, J J_z \rangle_{\kappa_L \kappa_z} \tilde{R}_{\lambda_1 \lambda_2}^{SS_z}(\omega, \kappa_\perp, \lambda_1, \lambda_2) = \frac{k_2^z}{\sqrt{2}\sqrt{(\omega + m_2)^2 + \kappa_\perp^2}} \bar{u}(v, \lambda_1) \hat{\Gamma}v(k_2, \lambda_2)
\]  

(3.32)

with

\[
\hat{\Gamma}_{3P_0} = -\frac{1}{\sqrt{3}}(v \cdot k_2 + m_2),
\]

\[
\hat{\Gamma}_{1P_1} = \hat{\epsilon} \cdot k_2 \gamma_5,
\]

\[
\hat{\Gamma}_{3P_1} = -\frac{1}{\sqrt{2}}\left[(v \cdot k_2 + m_2) \hat{\phi} - \hat{\epsilon} \cdot k_2 \right] \gamma_5,
\]

\[
\hat{\Gamma}_{3P_2} = -\hat{\epsilon}_\mu \gamma_\mu k_2^0,
\]  

(3.33)

and

\[
\hat{\epsilon}_{\lambda = \pm 1}^\mu = \left[ \frac{2}{v^+} \hat{\epsilon}_\perp^\mp(\pm 1) \cdot \bar{v}_\perp, 0, \hat{\epsilon}_\perp(\pm 1) \right],
\]

\[
\hat{\epsilon}_{\lambda = 0}^\mu = \left( -\frac{1 + v_\perp^2}{v^+}, v^+, v_\perp \right).
\]  

(3.34)
\[ u(v, \lambda_1) \] is the spinor for the heavy quark, 
\[
\sum_{\lambda} u(v, \lambda) \overline{u}(v, \lambda) = \frac{1}{v^+}, \tag{3.35}
\]

The normalization of the heavy meson bound states can then be given by
\[
\langle \hat{M}(v', J', J_z') | \hat{M}(v, J, J_z) \rangle = 2(2\pi)^3 v^+ \delta^3(\hat{\Lambda} v' - \hat{\Lambda} v) \delta_{J J'} \delta_{J_z, J'_z}, \tag{3.36}
\]
which not only leads to Eq. (2.43), but also to the space part \( \hat{\varphi}_{LL}^z(\omega, \kappa^2_\perp) \) (called the light-front wave function) in Eq. (3.28) which has the following wave-function normalization condition:
\[
\int d\omega d^2\kappa_\perp \frac{|\hat{\varphi}_{LL}^z(\omega, \kappa^2_\perp)|^2}{2(2\pi)^3} = 1, \tag{3.37}
\]
where \( \hat{\varphi}_{LL}^z = \kappa_{Lz} \hat{\varphi}_p \). In principle, the heavy quark dynamics are completely described by HQET, which is given by the \( 1/m_Q \) expansion of the heavy quark QCD Lagrangian:
\[
\mathcal{L} = \overline{Q}(i \not{D} - m_Q)Q = \sum_{n=0}^{\infty} \left( \frac{1}{2m_Q} \right)^n \mathcal{L}_n. \tag{3.38}
\]
Therefore, \( |\hat{M}(v, J)| \) and \( \hat{\varphi}_p(\omega, \kappa^2_\perp) \) are determined by the leading Lagrangian \( \mathcal{L}_0 = \bar{h}_v i\not{v} \cdot D h_v \). From the normalization conditions of Eqs. (3.10) and (3.37), we obtain the relation between wave functions \( \varphi_p(u, \kappa^2_\perp) \) and \( \hat{\varphi}_p(\omega, \kappa^2_\perp) \):
\[
\varphi_p(u, \kappa^2_\perp) = \sqrt{M} \hat{\varphi}_p(\omega, \kappa^2_\perp). \tag{3.39}
\]
In addition, in the heavy quark limit \( (m_1 \to \infty) \), the heavy quark spin \( s_Q \) decouples from the other degrees of freedom so that \( s_Q \) and the total angular momentum of the light antiquark \( j \) are separately good quantum numbers. Hence, it is more convenient to use the \( L_j = \frac{2}{3}P_2^{3/2}, \quad P_1^{3/2}, \quad P_1^{1/2} \) and \( P_0^{1/2} \) basis. It is obvious that the first and the last of these states are \( ^3P_2 \) and \( ^3P_0 \), respectively, while
\[
|P_1^{1/2}\rangle = \frac{1}{\sqrt{3}} |^1P_1\rangle - \sqrt{\frac{2}{3}} |^3P_1\rangle, \\
|P_1^{3/2}\rangle = \sqrt{\frac{2}{3}} |^1P_1\rangle + \frac{1}{\sqrt{3}} |^3P_1\rangle. \tag{3.40}
\]
In terms of \( P_1^{1/2} \) and \( P_1^{3/2} \) states, the relevant vertex functions read
\[
\hat{\Gamma}_{P_1^{1/2}} = \frac{1}{\sqrt{3}} (v \cdot k_2 + m_2) \not{v} \gamma_5, \\
\hat{\Gamma}_{P_1^{3/2}} = -\frac{1}{\sqrt{6}} [(v \cdot k_2 + m_2) \not{v} - 3\not{\epsilon} \cdot k_2] \gamma_5. \tag{3.41}
\]
Next, the matrix elements of Eqs. (2.51), (2.52), (2.53), (2.54) and (2.55) can be calculated, and the relevant leading twist LCDAs are extracted as

\[ F_{S} \Phi_{S}(\omega) = F_{A_{1}^{1/2}} \Phi_{A_{1}^{1/2}}(\omega) = F_{A_{1}^{3/2}} \Phi_{A_{1}^{3/2}}(\omega) \]

\[ = \sqrt{2} \int \frac{d^{2}\kappa_{\perp}}{2(2\pi)^{3}} \frac{\omega - m_{2}}{\omega} \sqrt{(\omega + m_{2})^{2} + \kappa_{\perp}^{2}} \hat{\phi}_{p}(\omega, \kappa_{\perp}^{2}), \]

\[ F_{A_{1}^{3/2}} \Phi_{A_{1}^{3/2}}(\omega) = \int \frac{d^{2}\kappa_{\perp}}{2(2\pi)^{3}} \frac{\hat{\phi}_{p}(\omega, \kappa_{\perp}^{2})}{(\omega + m_{2})^{2} + \kappa_{\perp}^{2}} \left[ \omega^{2} - m_{2}^{2} - 3\kappa_{1}^{2} + 2\kappa_{2}^{2} + 2m_{2}\kappa_{z} \right], \quad (3.43) \]

where we have denoted the \( P_{1}^{1/2} \) and \( P_{1}^{3/2} \) states by \( A_{1}^{1/2} \) and \( A_{1}^{3/2} \), respectively. In the literature, we read that [49, 50] HQS requires

\[ f_{A_{1}^{1/2}} = f_{S}, \quad f_{A_{1}^{3/2}} = 0. \quad (3.47) \]

These relations can be understood from the fact that \( (P_{0}^{1/2}, P_{1}^{1/2}) \) and \( (P_{0}^{3/2}, P_{2}^{3/2}) \) form two doublets in the heavy quark limit and that the tensor meson cannot be induced from the \( V - A \) current. From Eq. (3.42), it is easy to find that

\[ F_{S} = F_{A_{1}^{1/2}} = F_{A_{1}^{3/2}}^{\perp}, \quad \Phi_{S}(\omega) = \Phi_{A_{1}^{1/2}}(\omega) = \Phi_{A_{1}^{3/2}}^{\perp}(\omega). \quad (3.48) \]

Combining the former of Eq. (3.48) and the scaling \( F_{M} = \sqrt{M} f_{M} \), we obtain: \( f_{S} = f_{A_{1}^{1/2}} = f_{A_{1}^{3/2}}^{\perp} \), which are consistent with the former of Eq. (3.47). On the other hand, if wave function \( \hat{\phi}_{p} \) has a similar form to Eq. (3.12),

\[ \hat{\phi}_{p}(\omega, \kappa_{\parallel}) = N' \sqrt{\frac{d\kappa_{z}}{d\omega}} F(|\vec{k}|), \quad (3.49) \]

where \( N' \) is the normalization constant and \( F(|\vec{k}|) \) is a function of \( |\vec{k}| \), then, Eqs. (3.43) and (3.44) can be shown as \( F_{A_{1}^{3/2}} = F_{A_{1}^{1/2}}^{\perp} = 0 \) or \( f_{A_{1}^{3/2}} = f_{A_{1}^{1/2}}^{\perp} = 0 \), which are also consistent with the latter of Eq. (3.47). The derivations are shown in Appendix A. In addition, the tensor meson can be created through the \( V - A \) currents with covariant derivatives [see Eqs. (2.29) and (2.40)]. Thus, we can study its decay constant \( F_{T} \) and \( F_{T}^{\perp} \) here. From Appendix A, we find that Eqs. (3.45) and (3.46) lead to \( F_{T} = F_{T}^{\perp} = 0 \) and then \( f_{T} = f_{T}^{\perp} = f_{A_{1}^{3/2}} = f_{A_{1}^{1/2}}^{\perp} = 0 \), which is consistent with the fact that \( (P_{1}^{3/2}, P_{2}^{3/2}) \) forms a doublet in the heavy quark limit.
IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, the decay constants and LCDAs for the $p$-wave states of $D$, $D_s$, $B$ and $B_s$ systems are studied with the wave function $\varphi_p(u, \kappa_\perp)$. In principle, $\varphi_p$ is obtained by solving the light-front QCD bound-state equation $H_{LF}|M\rangle = M|M\rangle$, which is the familiar Schrödinger equation in ordinary quantum mechanics, and $H_{LF}$ is the light-front Hamiltonian. However, except in some simple cases, achieving the full solution has remained a challenge. There are several popular phenomenological light-front momentum distribution amplitudes which have been employed to describe various hadronic structures in the literature. A widely used one is the Gaussian type. If

$$F^g(|\vec{\kappa}|) = \exp\left(-\frac{||\vec{\kappa}||^2}{2\beta^2}\right),$$

then the corresponding wave functions are

$$\varphi^g_p(u, \kappa_\perp) = 4\sqrt{\frac{2}{\beta^2}} \left(\frac{\pi}{\beta^2}\right)^{3/4} \sqrt{\frac{e_1 e_2}{u(1-u)M_0}} \exp\left[-\frac{\kappa_\perp^2 + (\frac{uM_0}{2} - \frac{m^2 + \kappa_\perp^2}{2uM_0})^2}{2\beta^2}\right].$$

Prior to numerical calculations, the parameters $m_1$, $m_2$, and $\beta$, which appeared in the wave function, have to first be determined. Here, we use the parameters obtained in the ISGW2 model [51], an update of the ISGW quark model [52] for semileptonic meson decays. They are listed in Table I. Next, we use the parameters in Table I to evaluate the relevant decay constants, Eqs. (3.21) $\sim$ (3.27), for the $p$-wave heavy mesons. Note that the decay constant of the scalar and axial vector mesons are calculated by normalization of Eq. (2.16), and that of the tensor mesons are calculated by normalization of Eq. (2.17). The results, which compare with other theoretical evaluations, are listed in Table II. We find two things: the first is $f^{(1)}_{A_1^{1/2}} \gg f^{(1)}_{A_3^{1/2}}$, and the second is $f_T$ and $f_T^\perp$ for the charm sector are larger than those for the bottom sector. Both are qualitatively consistent with the results in Section 3B, which were derived from the HQS. In addition, in this work, the ratio of the decay constants $f_{A_1^{1/2}}$...
TABLE II: Mesonic decay constants (in units of MeV) obtained in this work and other theoretical evaluations.

| $2S+1L_J(L^J_J)$ | $^3P_0$ | $^3P_1$ | $^1P_1$ | $^3P_2$ | $P^{1/2}_1$ | $P^{3/2}_1$ |
|-------------------|--------|--------|--------|--------|-----------|-----------|
| $f_{cu}$          | 78     | -113   | 45     | -50    | 118       | -29       |
|                   | $f_{cu}$ [39] | 86     | -127   | 45     | 130       | -36       |
|                   | $f_{cu}$ [50] | 139 ± 30 |        |        | 251 ± 37  | 77 ± 18   |
| $f_{cu}$ [53]     | 133    | -211   | 72     |        |           |           |
| $f_{cu}^\perp$    |        | -127   | 80     | -48    | 150       | -8        |
| $f_{cs}$          | 66     | -123   | 38     | -65    | 123       | -40       |
|                   | $f_{cs}$ [39] | 71     | -121   | 38     | 122       | -38       |
|                   | $f_{cs}$ [50] | 110 ± 18 |        |        | 233 ± 31  | 87 ± 19   |
| $f_{cs}$ [53]     | 112    | -219   | 62     |        |           |           |
| $f_{cs}^\perp$    | -107   | 87     | -62    | 138    | 9         |
| $f_{bu}$          | 73     | -73    | 42     | -17    | 84        | -8        |
|                   | $f_{bu}$ [39] | 112    | -123   | 68     | 140       | -15       |
|                   | $f_{bu}$ [50] | 162 ± 24 |        |        | 206 ± 29  | 32 ± 10   |
| $f_{bu}$ [53]     | 145    | -150   | 76     |        |           |           |
| $f_{bu}^\perp$    | -119   | 52     | -17    | 127    | -26       |
| $f_{bs}$          | 75     | -82    | 43     | -25    | 92        | -12       |
|                   | $f_{bs}$ [50] | 146 ± 19 |        |        | 196 ± 26  | 36 ± 10   |
|                   | $f_{bs}$ [53] | 140    | -157   | 76     |           |
| $f_{bs}^\perp$    |        | -122   | 58     | -25    | 133       | -23       |

and $f_{S}$ in the $D_s$ system is

$$f_{A^{1/2}}/f_{S} = 1.9.$$ (4.3)

This is very close to the predictions of Refs. [50] and [54]: $f_{A^{1/2}}/f_{S} = 2.12$ and $2.26 ± 0.41$, which are based on the mock-meson approach and the factorization hypothesis, respectively.

For these heavy mesons, it is convenient to study the leading twist LCDAs using the $L^J_J$ basis. Thus, the curves of $\phi(\xi)$ for the $p$-wave states in $D$, $D_s$, $B$, and $B_s$ systems are evaluated and shown in Figs. 1 $\sim$ 7. From these figures, we find the curves of
FIG. 1: Leading twist-2 LCDAs $\phi_S(\xi)$ of the heavy meson. The solid and dotted lines, and short and long dashes correspond to the $D$, $D_s$, $B$, and $B_s$ systems, respectively.

$\phi_S(\xi)$

FIG. 2: Leading twist-2 LCDAs $\phi_{A_1^{1/2}}(\xi)$ of the heavy meson. The solid and dotted lines, and short and long dashes correspond to the $D$, $D_s$, $B$, and $B_s$ systems, respectively.

$\phi_{A_1^{1/2}}(\xi)$

$\phi_{A_1^{1/2}}(\xi)$ and $\phi_{A_1^{1/2}}(\xi)$ are very similar to those of $\phi_S(\xi)$, but are quite different from those of $\phi_{A_3^{1/2}}(\xi)$ and $\phi_{A_3^{1/2}}(\xi)$. The scales of Figs. 3, 5, 6, and 7 are much larger than those of the others because the values of $f_{A_1^{1/2}}^{(1)}$ and $f_{A_1^{1/2}}^{(1)}$ are relatively small. Finally, we parametrize the LCDAs in terms of the first six $\xi$ moments with Eq. (2.18). The results are shown in Tables III, IV, V, and VI. We find that the similarities between the longitudinal and transverse projections of the tensor meson are displayed not only in ratio $f_T/f_T^\perp \simeq 1$, but also in the approximations $\langle \xi^i \rangle_T \simeq \langle \xi^i \rangle_{T\perp}$ for all heavy meson systems.
FIG. 3: Leading twist-2 LCDAs $\phi_{\frac{3}{2}\parallel}(\xi)$ of the heavy meson. The solid and dotted lines, and short and long dashes correspond to the $D$, $D_s$, $B$, and $B_s$ systems, respectively.

FIG. 4: Leading twist-2 LCDAs $\phi_{\frac{1}{2}\perp}(\xi)$ of the heavy meson. The solid and dotted lines, and short and long dashes correspond to the $D$, $D_s$, $B$, and $B_s$ systems, respectively.

V. CONCLUSIONS

This study discussed the leading twist LCDAs of $p$-wave heavy mesons within the light-front approach. These LCDAs have been displayed in terms of light-front variables $(u, \omega, \kappa_\perp)$ and the relevant decay constants in both general and heavy quark frameworks. In the heavy quark framework, we analytically found that the decay constants and LCDAs had the following relations: $f_S = f_{A_1^{1/2}} = f_{A_1^{1/2}}^{\perp} = \Phi_S(\omega) = \Phi_{A_1^{1/2}\parallel}(\omega) = \Phi_{A_1^{1/2}\perp}(\omega)$ and $f_T = f_T^{\perp} = f_{A_1^{3/2}} = f_{A_1^{3/2}}^{\perp} = 0$, which are consistent with the requirements of HQS-$(P_0^{1/2}, P_1^{1/2})$ and $(P_1^{3/2}, P_2^{3/2})$ form two doublets. It was worth noting that we could study $f_T^{(1)}$ because
FIG. 5: Leading twist-2 LCDAs $\phi_{A_1^{3/2,\perp}}(\xi)$ of the heavy meson. The solid and dotted lines, and short and long dashes correspond to the $D$, $D_s$, $B$, and $B_s$ systems, respectively.

FIG. 6: Leading twist-2 LCDAs $\phi_{T\parallel}(\xi)$ of the heavy meson. The solid and dotted lines, and short and long dashes correspond to the $D$, $D_s$, $B$, and $B_s$ systems, respectively.

the tensor meson was created through the $V - A$ currents with covariant derivatives. In the general framework, we quoted the parameters $m_i$ and $\beta$, which appear in the Gaussian-type wave functions, from the ISGW2 model, and numerically found that: i) an inequality $f_{A_1^{1/2}}^{(\perp)} \gg f_{A_1^{3/2}}^{(\perp)}$ is existent for all systems; ii) the decay constants $f_T$ and $f_T^\perp$ for the charm sector are larger than those for the bottom sector; iii) the ratio $f_{A_1^{1/2}}^{(\perp)} / f_S = 1.9$ for the $D_s$ system is close to the predictions in Ref. [50] and [54]; iv) the curves of $\phi_{A_1^{1/2,\perp}}(\xi)$ were very similar to those of $\phi_S(\xi)$, but were quite different from those of $\phi_{A_1^{3/2,\perp}}(\xi)$ and $\phi_{T\parallel,\perp}(\xi)$; and v) the ratio $f_T / f_T^\perp \simeq 1$ and approximations $\langle \xi^{(i)} T\parallel \rangle \simeq \langle \xi^{(i)} T\perp \rangle$ were satisfied for all heavy
FIG. 7: Leading twist-2 LCDAs $\phi_{T\perp}(\xi)$ of the heavy meson. The solid and dotted lines, and short and long dashes correspond to the $D$, $D_s$, $B$, and $B_s$ systems, respectively.

TABLE III: The first four $\xi$ moments of $\phi_M(\xi)$ for the $p$-wave states of $D$ system.

| $M$ | $S$ | $P_{1/2}\parallel$ | $P_{1/2}\perp$ | $P_{3/2}\parallel$ | $P_{3/2}\perp$ | $T\parallel$ | $T\perp$ |
|-----|-----|-------------------|---------------|------------------|---------------|-------------|-------------|
| $\langle \xi \rangle$ | 0.14 | 0.25 | 0.30 | 1.7 | $-1.7$ | 1 | 1 |
| $\langle \xi^2 \rangle$ | 0.0029 | 0.091 | 0.13 | 1.3 | $-1.5$ | 0.83 | 0.83 |
| $\langle \xi^3 \rangle$ | $-0.050$ | 0.023 | 0.055 | 1.0 | $-1.3$ | 0.68 | 0.68 |
| $\langle \xi^4 \rangle$ | $-0.061$ | $-0.0031$ | 0.023 | 0.79 | $-1.1$ | 0.54 | 0.54 |
| $\langle \xi^5 \rangle$ | $-0.062$ | $-0.015$ | 0.0061 | 0.62 | $-0.87$ | 0.43 | 0.44 |
| $\langle \xi^6 \rangle$ | $-0.057$ | $-0.019$ | $-0.0021$ | 0.50 | $-0.72$ | 0.35 | 0.35 |

TABLE IV: The first four $\xi$ moments of $\phi_M(\xi)$ for the $p$-wave states of $D_s$ system.

| $M$ | $S$ | $P_{1/2}\parallel$ | $P_{1/2}\perp$ | $P_{3/2}\parallel$ | $P_{3/2}\perp$ | $T\parallel$ | $T\perp$ |
|-----|-----|-------------------|---------------|------------------|---------------|-------------|-------------|
| $\langle \xi \rangle$ | $-0.13$ | 0.093 | 0.13 | 1.5 | 2.9 | 1 | 1 |
| $\langle \xi^2 \rangle$ | $-0.14$ | 0.011 | 0.033 | 0.96 | 1.9 | 0.66 | 0.66 |
| $\langle \xi^3 \rangle$ | $-0.14$ | $-0.029$ | $-0.013$ | 0.69 | 1.4 | 0.50 | 0.50 |
| $\langle \xi^4 \rangle$ | $-0.11$ | $-0.032$ | $-0.020$ | 0.48 | 1.0 | 0.35 | 0.36 |
| $\langle \xi^5 \rangle$ | $-0.092$ | $-0.030$ | $-0.022$ | 0.35 | 0.74 | 0.26 | 0.26 |
| $\langle \xi^6 \rangle$ | $-0.072$ | $-0.026$ | $-0.020$ | 0.26 | 0.55 | 0.20 | 0.20 |

meson systems. It is easily realized that the results of i, ii, and iv qualitatively supported
the requirements of HQS. Due to the lack of relevant experimental data, the consistencies between our estimations and the predictions of HQS are important.

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Appendix A: Some useful identities

We consider an integration as

\[
\int \frac{d\omega d^2k_\perp}{2(2\pi)^3} \frac{\tilde{\phi}_p(\omega, \kappa_\perp^2)}{\sqrt{(\omega + m_2)^2 + \kappa_\perp^2}} g(m_2, \kappa_z, \kappa_\perp^2). 
\] (A1)
Substituting Eq. (3.49) with Eq. (A1), we obtain

$$N' \int \frac{d\omega d^2 \kappa_\perp}{2(2\pi)^3} \sqrt{\frac{d \kappa_z}{d \omega}} \frac{F(|\vec{\kappa}|)}{\sqrt{\omega + m_2^2 + \kappa_\perp^2}} g(m_2, \kappa_z, \kappa_\perp^2).$$ (A2)

Taking the heavy quark limit for Eq. (3.13), we obtain $\sqrt{\frac{d \omega \kappa_z}{d \omega}} = \sqrt{\frac{e_2}{\omega}}$, and Eq. (A2) can be rewritten as

$$N' \int \frac{d^3 \vec{\kappa}}{2(2\pi)^3} \frac{\sqrt{\omega + m_2^2 + \kappa_\perp^2}}{e_2 \sqrt{\omega + m_2^2 + \kappa_\perp^2}} g(m_2, \kappa_z, \kappa_\perp^2).$$ (A3)

From Eq. (3.30), the variables $\omega$, $e_2$, and $\kappa_z$ have the following relations:

$$\omega = e_2 + \kappa_z, \quad \frac{m_2^2 + \kappa_\perp^2}{\omega} = e_2 - \kappa_z.$$ (A4)

Thus, Eq. (A3) can be rewritten as

$$N' \int \frac{d^3 \vec{\kappa}}{2(2\pi)^3} \frac{F(|\vec{\kappa}|)}{\sqrt{\frac{e_2}{2}(e_2 + m_2)}} g(m_2, \kappa_z, \kappa_\perp^2).$$ (A5)

Besides the function $g$, the only variable in Eq. (A5) is $|\vec{\kappa}|$ because $e_2 = \sqrt{m_2^2 + |\vec{\kappa}|^2}$. Therefore, if we designate $g$ to some specific function in Eq. (A1), the integration can be made aware by symmetry. For example, if $g = \kappa_z$,

$$\int \frac{d\omega d^2 \kappa_\perp}{2(2\pi)^3} \frac{\hat{\varphi}_p(\omega, \kappa_\perp^2)}{\sqrt{\omega + m_2^2 + \kappa_\perp^2}} \kappa_z = 0,$$ (A6)

because $\kappa_z$ is an odd function. The second case is $g = \kappa_\perp^2 - \kappa_\perp^2/2$,

$$\int \frac{d\omega d^2 \kappa_\perp}{2(2\pi)^3} \frac{\hat{\varphi}_p(\omega, \kappa_\perp^2)}{\sqrt{\omega + m_2^2 + \kappa_\perp^2}} \left(\kappa_\perp^2 - \frac{\kappa_\perp^2}{2}\right) = 0,$$ (A7)

because the contributions of $\kappa_\perp^2$, $\kappa_\perp^2$, and $\kappa_\perp^2$ are equal. The third case is $g = 2\kappa_z(e_2 + \kappa_z) = \omega^2 - m_2^2 - \kappa_\perp^2$,

$$\int \frac{d\omega d^2 \kappa_\perp}{2(2\pi)^3} \frac{\hat{\varphi}_p(\omega, \kappa_\perp^2)}{\sqrt{\omega + m_2^2 + \kappa_\perp^2}} \left(\omega^2 - m_2^2 - \kappa_\perp^2\right) = \int \frac{d\omega d^2 \kappa_\perp}{2(2\pi)^3} \frac{\hat{\varphi}_p(\omega, \kappa_\perp^2)}{\sqrt{\omega + m_2^2 + \kappa_\perp^2}} \kappa_\perp^2,$$ (A8)

where Eqs. (A6) and (A7) are applied. We can employ Eqs. (A6), (A7) and (A8) to prove that the integrations of $\omega$ for Eqs. (3.43), (3.44), (3.45), and (3.46) are all equal to zero.

[1] G. P. Lepage and S. J. Brodsky, Phys. Rev. D 22, 2157 (1980).
[2] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. 112, 173 (1984).

[3] A. P. Bakulev, S. V. Mikhailov, and N. G. Stefanis, Phys. Rev. D 73, 056002 (2006).

[4] P. Ball, V. M. Braun, Y. Koike, and K. Tanaka, Nucl. Phys. B 529, 323, (1998).

[5] K. C. Yang, Nucl. Phys. B 776, 187 (2007).

[6] V. V. Braguta, A. K. Likhoded, and A. V. Luchinsky, Phys. Rev. D 79, 074004 (2009).

[7] A. Ali Khan et al. (CP PACS Collaboration), Phys. Rev. D 65, 054505 (2002).

[8] V. M. Braun et al. (QCDSF/UKQCD Collaboration), Phys. Rev. D 74, 074501 (2006).

[9] V. Y. Petrov, M. V. Polyakov, R. Ruskov, C. Weiss, and K. Goeke, Phys. Rev. D 59, 114018 (1999).

[10] S. I. Nam, H. C. Kim, A. Hosaka, and M. M. Musakhanov, Phys. Rev. D 74, 014019 (2006).

[11] E. R. Arriola and W. Broniowski, Phys. Rev. D 66, 094016 (2002).

[12] M. Praszalowicz and A. Rostworowski, Phys. Rev. D 64, 074003 (2001).

[13] H. M. Choi and C. R. Ji, Phys. Rev. D 75, 034019 (2007).

[14] C. W. Hwang, Eur. Phys. J. C 62, 499 (2009).

[15] C. R. Ji, P. L. Chung, and S. R. Cotanch, Phys. Rev. D 45, 4214 (1992).

[16] B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 79, 112004 (2009).

[17] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. 103, 051803 (2009).

[18] S. Chekanov et al. (ZEUS Collaboration), Eur. Phys. J. C 60, 25 (2009).

[19] V. M. Abazov et al. (D0 Collaboration), Phys. Rev. Lett. 102, 051801 (2009).

[20] R. Aaij et al. (LHCb Collaboration), Phys. Lett. B 698, 14 (2011).

[21] T. Aaltonen et al. (CDF Collaboration), Phys. Rev. Lett. 102, 102003 (2009).

[22] T. Aaltonen et al. (CDF Collaboration), Phys. Rev. Lett. 100, 082001 (2008).

[23] V. M. Abazov et al. (D0 Collaboration), Phys. Rev. Lett. 100, 082002 (2009).

[24] N. Isgur and M. B. Wise, Phys. Lett. B 232, 113 (1989).

[25] N. Isgur and M. B. Wise, Phys. Lett. B 237, 527 (1990).

[26] H. Georgi, Phys. Lett. B 240, 447 (1990).

[27] E. Eichten and B. Hill, Phys. Lett. B 234, 511 (1990).

[28] E. Eichten and B. Hill, Phys. Lett. B 243, 427 (1990).

[29] M. Neubert, Phys. Rep. 245, 259 (1994).

[30] J. Chay, H. Georgi, and B. Grinstein, Phys. Lett. B 247, 399 (1990).

[31] I. I. Bigi, M. Shifman, N. G. Uraltsev, and A. Vainshtein, Phys. Rev. Lett. 71, 496 (1993).
[32] A. V. Manohar and M. B. Wise, Phys. Rev. D 49, 1310 (1994).
[33] T. Mannel, Nucl. Phys. B 413, 396 (1994).
[34] S. J. Brodsky, H. C. Pauli and S. S. Pinsky, Phys. Rep. 301, 299 (1998).
[35] M. V. Terent’ev, Sov. J. Phys. 24, 106 (1976); V. B. Berestetsky and M. V. Terent’ev, Sov.
    J. Phys. 24, 547 (1976); 25, 347 (1977).
[36] W. Jaus, Phys. Rev. D 41, 3394 (1990); Phys. Rev. D 44, 2851 (1991).
[37] H. Y. Cheng, C. Y. Cheung and C. W. Hwang, Phys. Rev. D 55, 1559 (1997).
[38] W. Jaus, Phys. Rev. D 60, 054026 (1999).
[39] H. Y. Cheng, C. K. Chua and C. W. Hwang, Phys. Rev. D 69, 074025 (2004).
[40] C. W. Hwang, Phys. Rev. D 64, 034011 (2001).
[41] K. C. Yang, J. High Energy Phys. 10 (2005) 108.
[42] P. Ball and V. M. Braun, Phys. Rev. D 54, 2182 (1996).
[43] I. I. Balitsky and V. M. Braun, Nucl. Phys. B 311, 541 (1989).
[44] H. Y. Cheng and K. C. Yang, Phys. Rev. D 76, 114020 (2007).
[45] C. W. Hwang, J. High Energy Phys. 10 (2009) 074.
[46] C. Y. Cheung, W. M. Zhang, and G. L. Lin, Phys. Rev. D 52, 2915 (1995).
[47] C. Y. Cheung, C. W. Hwang, and W. M. Zhang, Z. Phys. C 75, 657 (1997).
[48] N. Isgur and M. B. Wise, Phys. Rev. D 43, 819 (1991).
[49] A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, Phys. Lett. B 387, 582 (1996).
[50] S. Veseli and I. Dunietz, Phys. Rev. D 54, 6803 (1996).
[51] D. Scora and N. Isgur, Phys. Rev. D 52, 2783 (1995).
[52] N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D 39, 799 (1989).
[53] G. L. Wang, Phys. Lett. B 650, 15 (2007).
[54] D. S. Hwang and D. W. Kim, Phys. Lett. B 606, 116 (2005).