Quark-hadron continuity under rotation: vortex continuity or boojum?

Chandrasekhar Chatterjee,††† Muneto Nitta,‡‡‡ and Shigehiro Yasui†††

1Department of Physics & Research and Education Center for Natural Sciences, Keio University, Hiyoshi 4-1-1, Yokohama, Kanagawa 223-8521, Japan

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Quark-hadron continuity was proposed as crossover between hadronic matter and quark matter without a phase transition, based on matching of symmetry and excitations in both phases. In the limit of light strange quark mass, it connects hyperon matter and color-flavor locked (CFL) phase exhibiting color superconductivity. Recently, this conjecture was proposed to be generalized in the presence of superfluid vortices penetrating both phases [1], in which they suggested that one hadronic superfluid vortex in hyperon matter could be connected to one non-Abelian vortex (color magnetic flux tube) in the CFL phase. Here, we argue that their proposal is consistent only at large distances: Instead, we show that three hadronic superfluid vortices must join together to three non-Abelian vortices with different colors with the total color magnetic fluxes canceled out, where the junction was called a colorful boojum. We rigorously prove this both in a macroscopic theory based on the Ginzburg-Landau description, in which symmetry and excitations match including vortex cores, and a microscopic theory based on the Bogoliubov de-Gennes equation, in which the Aharanov-Bohm phase of quarks around vortices match.

INTRODUCTION

The presence or absence of phase transitions is the most important issue to understand phases of matter. In last few decades, a lot of efforts was paid to understand the phase structure of matter at high density and/or temperature [2]. In particular, the region of high density and low temperature is relevant for cores of compact stars such as neutron stars, in which nuclear matter and quark matter are present. Superfluidity of nucleon-nucleon pairing is expected in nuclear matter, and there appears, in high density region, nuclear matter of hyperons, nuclei containing strange quarks, is expected [3] (see Ref. [4] as a recent review). In higher density, quark matter is expected; in asymptotically high density much higher than strange quark mass, the color-flavor locked (CFL) phase is realized, in which three (up, down and strange) quarks participate in a diquark paring, exhibiting color superconductivity as well as superfluidity [5, 6], see Refs. [7, 8] as a review. In addition to superfluid vortices [9, 10], there are non-Abelian vortices or color magnetic flux tubes [11-17], see Ref. [18] as a review. The former is dynamically split into three of the latter with the total magnetic fluxes canceled out [12-17].

The quark-hadron continuity conjecture was proposed as crossover between hadronic matter and quark matter, based on the matching of elementary excitations and existing global symmetries in both the matter, in particular hyperon matter and CFL phase [19, 20], as summarized in Table I. The continuity was further studied in interior of neutron stars [21-23]. Since neutron stars are rapidly rotating, there appear superfluid vortices both in nuclear matter and quark matter, thereby it is natural to extend the quark-hadron continuity in the presence of vortices penetrating both the matter [1]. They defined a continuity of vortices by matching of the Onsager-Feynman circulation of vortices by matching of the Onsager-Feynman

| unbroken symmetry | hadronic phase | CFL phase |
|-------------------|--------------|-----------|
| SU(3)_F           | SU(3)_{C+\bar{F}} |
| # of NG bosons    | 8            | 8         |
| # of massive vector mesons | 8     | 8         |
| # of quasi fermions| 8            | 8+1       |
| vortices          | vortex configurations | 3 ΛA N |
|                   | circulation of 3 vortices | 2π \frac{2q_i}{2\mu_i} N π |
|                   | AB phase of quarks | u_r, d_u, s_b |

TABLE I. The properties in the CFL phase and in the hadron phase. The AB phases of quarks are acquired for 2π rotations around a bundle of N = 3 vortices.

In this Letter, with pointing out that their conclusion is consistent only in large distance behavior of vortices but is not compatible with symmetry structures of vortex cores, we reach a conclusion that arguably only possibility that is left is to form a connection of three hadronic vortices in the hyperon matter of ΛΛ condensation with three non-Abelian vortices in the CFL phase with total color magnetic fluxes cancelled out, forming a colorful boojum [24], analogous to a boojum in helium 3 superfluid [25, 26]. We prove this both in a macroscopic theory based on the Ginzburg-Landau (GL) description, in which symmetry and excitations match including vortex cores, and a microscopic theory based on the Bogoliubov de-Gennes (BdG) equation, in which the Aharanov-Bohm (AB) phase of quarks around vortices match.
The concept of continuity is defined by continuing of symmetries and elementary excitations at ground state while going through crossover. Now we would like to discuss the concept of continuity in the presence of general background. For example, the vortices which are present in two different phases should be joined together so that all physical quantities remain smoothly connected and symmetry structure remains the same through the crossover.

On the other hand, in the presence of solitonic objects may sometime break existing unbroken global symmetries present at the ground state. Since the condensate eventually reaches its ground state expectation value (modulus gauge transformations) at large distances, the large distance symmetry structure in general remains the same as the ground state. However, scenario may change inside solitonic objects and the existing bulk symmetry may be broken spontaneously inside. In this case there appear extra Nambu-Goldstone (NG) zero modes inside the solitons, which should be carefully handled during crossover. In other words, to maintain the continuity of solitonic objects along with elementary excitations, one should check the symmetry structure everywhere.

Let us focus our interest on crossover of the hadronic phase to the CFL phase. At high densities, one may expect the appearance of strange quarks as hyperon states in the hadronic side. In general, the first hyperon expected to appear is Λ, which is the lightest one with an attractive potential in nuclear matter. Here we consider only flavor symmetric ΛΛ pairing in 1S0 channel for our purpose to be fulfilled[1]. In this case we may consider the existence of superfluid vortices since ΛΛ would break U(1) baryon number symmetry and we may express the vortex ansatz as

\[ \Delta_{ΛΛ}(r, θ) = |\Delta_{ΛΛ}(r)| e^{iθ}, \]

with the distance r from the center of the vortex and the angle θ around the vortex axis. The exact nature of the profile function can be derived from the GL theory of the system and we are not going to discuss this here. Since the condensate \( \Delta_{ΛΛ} \) is flavor symmetric in this phase, the SU(3)F flavor symmetry would be intact everywhere including the vortex cores. The Onsager-Feynman circulation which is defined as \( C = \oint \vec{v} \cdot d\bar{l} = 2\pi n \), where \( n \) and \( μ \) are the winding number and chemical potential of the condensate, can be computed for a single ΛΛ vortex to be \( C_{ΛΛ} = \frac{2π}{2μ} \), where \( μ_B \) is the chemical potential for a single baryon. Here \( \vec{v} \) is the superfluid velocity at large distance from the core of the vortex.

In the CFL phase, the order parameter is a matrix \( \Delta^i_a = \Delta_{λa}^i = -\Delta_{ba}^i \) with a color index a = 1, 2, 3 (r, g, b) and a flavor index i = 1, 2, 3 (u, d, s), where \( \Delta_{ba}^i \sim \epsilon_{aμbνc}q_{μBC}\phi^C_{νC} \), \( \Delta^i_a \sim \epsilon_{aμbνc}q_{μBC}C_{νC} \). The Ginzburg-Landau formulation of the CFL phase has been derived in Refs. [28][30]. Symmetries in the CFL phase are summarized in Appendix. The order parameter for an Abelian superfluid vortex can be written as [9][10]

\[ Δ_{Λ}(r, θ) = Δ_{cfl}(r) e^{iθ} 1_{3x3}, \]

where \( φ(0) \) is a profile function vanishing at the center of the vortex, \( φ(0) = 0 \), and eventually reaching the ground state value \( φ(r → ∞) → 1 \) at large distances. \( Δ_{cfl} \) is the absolute value of the gap (condensate) at the bulk in the CFL phase. The Onsager-Feynman circulation of Abelian vortices in the CFL phase is found to be \( C_{Λ} = \frac{2π}{μ_B} \), since the chemical potential of di-quark is \( μ_{CFL} = \frac{2π}{3μ_B} \). So a single ΛΛ vortex cannot connect continuously to a single U(1) vortex in the CFL phase. Instead, we may conclude that three ΛΛ vortices would join to form one U(1) CFL vortex.

Now let us discuss non-Abelian vortices or color magnetic flux tubes. In this case, the simplest vortex ansatz can be expressed as [11][14]

\[ Δ_{ur}(r, θ) = Δ_{cfl}(r) \text{diag}(f(r)e^{iθ}, g(r), g(r)), \]

\[ A^ur(r) = -\frac{1}{3g_s} \frac{ε_{ij}x^j}{r^2} (1 - h(r)) \text{diag}(2, -1, -1), \]

with the gauge coupling constant \( g_s \). The profile functions \( f(r) \), \( g(r) \) and \( h(r) \) can be computed numerically with boundary conditions, \( f(0) = 0 \), \( ∂_r g(r)|_{r=0} = 0 \), \( h(0) = 1 \), \( f(∞) = g(∞) = 1 \), \( h(∞) = 0 \) [14]. We call this an up-red (ur) vortex since the ur component has a vortex winding. We also define two other vortices by changing the position of the vortex winding \( e^{iθ} \) from \( Δ_{11} \) to \( Δ_{22} \) and \( Δ_{33} \), which can be called as down-green (dg) and strange-blue (sb) vortices, respectively. At large distances, the order parameter of these three vortices behave as \( Δ ∼ Δ_{cfl} e^{iθ} \frac{2π}{3π} A^i d\bar{l} 1_{3x3} \), where \( A_i \) is the large distance configuration of the gauge field.

**FIG. 1.** A schematic diagram of connection of three ΛΛ vortices in hadronic phase with three different CFL vortices via single U(1) CFL vortex.
corresponding to the color flux present inside the vortex core. So it is easy to check that at large distances the $SU(3)_{C+F}$ symmetry remains unbroken. In this case one may derive superfluid velocity at large distances by replacing ordinary derivative to covariant derivative in the expression of the current. The Onsager-Feynman circulation of non-Abelian vortices in the CFL phase is found to be $C_{\text{NA}} = \frac{2\pi}{\mu_B}$, which coincides with the circulation of a single $\Lambda\Lambda$ vortex. Therefore one would expect that a single $\Lambda\Lambda$ vortex would be smoothly connected to a single non-Abelian vortex during the crossover [1]. Below we show that this is consistent only at large distances but contradict at short distance at the vortex core.

First let us consider the symmetry structures in the presence of non-Abelian vortices. According to hadron-quark continuity the unbroken $SU(3)_{C+F}$ symmetry can be smoothly connected to the unbroken flavor symmetry in the hadron phase. So it seems that as if there would not be any problem also for continuation of a non-Abelian vortex to a single $\Lambda\Lambda$ vortex. However, the missing point is that the $SU(3)_{C+F}$ symmetry present at the bulk is spontaneously broken at the core of a non-Abelian vortex to $SU(2) \times U(1)$. This generates $\mathbb{CP}^2 \simeq SU(3)/[SU(2) \times U(1)]$ NG modes inside the vortex core [12, 15]. The low-energy effective theory of the $\mathbb{CP}^2$ NG modes was obtained along the vortex line [15, 16, 31]. This helps us to distinguish two different vortices by flavor quantum numbers. The three kinds of vortices $ur$, $dg$ and $sb$, where the color part is chosen in a particular gauge for our own convenience, lie in three points of the $\mathbb{CP}^2$ moduli space, and they are continuously connected by the flavor symmetry. This can be understood directly from the structure of the order parameters at the center of vortices. We may write the order parameters at the center of vortices for these three cases as

$$\Delta_{ur}(0) = c \ \text{diag}(0, 1, 1), \ \Delta_{dg}(0) = c \ \text{diag}(1, 0, 1), \ \Delta_{sb}(0) = c \ \text{diag}(1, 1, 0),$$

with a constant $c$ which can be fixed numerically. The flavor symmetry $SU(3)_{C+F}$ is spontaneously broken by these matrices to three different unbroken $SU(2) \times U(1)$ subgroups of the $SU(3)_{C+F}$. Since the $SU(3)$ flavor is unbroken in the hadronic vortex, following symmetry principle of continuity we can say that a single $\Lambda\Lambda$ vortex cannot smoothly transform into any single non-Abelian vortex.

We need to have a construction where the $SU(3)$ flavor symmetry is recovered in a vortex core while connecting to the hadronic phase. In other words, we have to terminate the $\mathbb{CP}^2$ NG modes. This is possible only when three different non-Abelian vortices join to one $U(1)$ CFL vortex in whose core the $SU(3)$ flavor symmetry is not broken\(^2\). The $\mathbb{CP}^2$ NG modes of the three different kind of non-Abelian vortices describe fluctuations from the three different points of the $\mathbb{CP}^2$ moduli space. When we join all of them, these NG modes can smoothly move from one patch to another patch at the junction. As we already discussed, one $U(1)$ CFL vortex can be connected to three $\Lambda\Lambda$ vortices during the hadron-CFL crossover, and then we reach Fig. 1. The junction point was called a colorful boojum [24], analogous to those in helium 3 superfluids [25, 26].

One important notice is that we did not require cancellation of color magnetic fluxes at the junction point. Instead, we only required the termination of the $\mathbb{CP}^2$ NG modes. The color of a non-Abelian vortex is gauge dependent as emphasized in Ref. [1], but the termination of the $\mathbb{CP}^2$ NG modes implies the cancellation of the color magnetic fluxes in our gauge choice.

### VORTEX CONTINUITY IN MICROSCOPIC THEORY

We now prove the same result from a microscopic point of view, by requiring a continuity of quark wave functions in the presence of vortices penetrating the CFL and hadronic phases. More precisely, we achieve a continuity of AB phases of quarks encircling around vortices.

In the hadronic phase, the BdG equation in the $\Lambda\Lambda$ vortex, $\Delta_{\Lambda\Lambda}$ in Eq. (1), can be written as

$$\begin{pmatrix} -\frac{\sqrt{s}}{2m_B} - \mu_B & e^{i\theta} |\Delta_{\Lambda\Lambda}| \\ e^{-i\theta} |\Delta_{\Lambda\Lambda}| & \frac{\sqrt{s}}{2m_B} + \mu_B \end{pmatrix} \begin{pmatrix} u_B \\ v_B \end{pmatrix} = \mathcal{E} \begin{pmatrix} u_B \\ v_B \end{pmatrix},$$

for the $\Lambda$ baryon ($u_B$ particle component, $v_B$ hole component) in the Nambu-Gor’kov formalism. Here, $m_B$ is the $\Lambda$ baryon mass, $|\Delta|$ is a profile function of the vortex. By noting that the BdG equation has the symmetry $\theta \to \theta + \alpha, \ (u, v) \to (e^{i\alpha/2}u, e^{-i\alpha/2}v)$, we find that the rotation of a $\Lambda$ baryon around the vortex with an angle $\alpha$ induces the above phase transformations on particle and hole components of the $\Lambda$ baryon. A complete encircling yields an AB phase $1/2 \times 2\pi = \pi$. We call this $+1/2$ factor as a charge or simply a AB phase of the $\Lambda$ baryon around the vortex. Let us understand this in the quark level. Since the quarks are confined inside the $\Lambda$ baryon in a symmetric way, each quark quasi-particles $q_{\alpha i}$ in $\Delta_{\Lambda\Lambda}$ should have AB phases $Q_{\Lambda\Lambda}(q)$ given by

$$Q_{\Lambda\Lambda} = \frac{1}{6} \begin{pmatrix} +1 & +1 & +1 \\ +1 & +1 & +1 \\ +1 & +1 & +1 \end{pmatrix} \text{ for } (q)_{\alpha i} = \begin{pmatrix} u_r & d_r & s_r \\ u_g & d_g & s_g \\ u_b & d_b & s_b \end{pmatrix}.$$  

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\(^2\) The reason in the support of the above proposal is related to the fact that a $U(1)$ CFL vortex is energetically unstable to break into three non-Abelian vortices [12, 17].
This is because the phase should be independent of flavor and color. Here, we indicated the AB phases of particle components, while the hole components have simply opposite sign. The question is then how the phases of quarks can be connected smoothly from the hadronic phase to the CFL phase.

Before investigating the AB phases of quarks in the CFL phase, we remind us that, as a simpler case of a single-component Dirac fermion (ψ) in the BdG equation, it has an AB phase +1/2 because of the phase $\alpha/2$ as in $\psi \rightarrow e^{i\alpha/2}\psi$ for the rotation $\theta \rightarrow \theta + \alpha$ as shown in Appendix.

The BdG equation $\hat{H}\Psi = E\Psi$ in the presence of a sb non-Abelian vortex is given by [32][33] (see also Ref. [34])

$$
\left( \begin{array}{cccccc}
\hat{H}_0 & \hat{\Delta}_1 & \hat{\Delta}_0 & 0 & 0 & 0 \\
\hat{\Delta}_1 & \hat{H}_0 & 0 & 0 & 0 & 0 \\
\hat{\Delta}_0 & 0 & \hat{H}_0 & 0 & 0 & 0 \\
0 & 0 & 0 & \hat{H}_0 & 0 & 0 \\
0 & 0 & 0 & 0 & \hat{H}_0 & 0 \\
0 & 0 & 0 & 0 & 0 & \hat{H}_0
\end{array} \right) \left( \begin{array}{c}
u_r \\
d_g \\
s_b \\
d_r \\
u_g \\
s_r \\
u_b \\
d_g \\
s_b
\end{array} \right) = \mathcal{E} \left( \begin{array}{c}
u_r \\
d_g \\
s_b \\
d_r \\
u_g \\
s_r \\
u_b \\
d_g \\
s_b
\end{array} \right),
$$

where we have used the notation e.g., $u_r$ in the Nambu-Gor'kov representation. We define $\hat{H}_0 = \text{diag}(-i\gamma_0 \vec{\nabla} - \mu_q, -i\gamma_0 \vec{\nabla} + \mu_q)$ and

$$
\hat{\Delta}_i = \begin{pmatrix}
0 & \Delta r_{\gamma_0 \gamma_5} \\
-\Delta r_{\gamma_0 \gamma_5} & 0
\end{pmatrix} (i = 0, 1),
$$

where $\Delta r(r, \theta) = |\Delta r(r)| e^{i\theta}$ corresponds to the vortex configuration with winding number one, and $\Delta_0(r)$ does not have a winding number. We also obtain the BdG equations for the $ur$ and $dg$ vortices, where the quarks couple to the gap profile functions differently. The AB phases $Q_{ur}(q)$, $Q_{dg}(q)$, $Q_{sb}(q)$ of quarks around non-Abelian $ur$, $dg$, $sb$ vortices are obtained respectively as

$$
Q_{ur} = \frac{1}{2} \begin{pmatrix}
-1 & 0 & 0 \\
0 & +1 & +1 \\
0 & +1 & +1
\end{pmatrix},
Q_{dg} = \frac{1}{2} \begin{pmatrix}
+1 & 0 & +1 \\
0 & -1 & 0 \\
+1 & 0 & +1
\end{pmatrix},
$$

$$
Q_{sb} = \frac{1}{2} \begin{pmatrix}
+1 & +1 & 0 \\
+1 & +1 & 0 \\
0 & 0 & -1
\end{pmatrix}.
$$

Now we consider the connection of the vortices in the hadronic and CFL phases. The AB phase $\alpha/6$ of quarks around a $\Lambda\Lambda$ vortex in Eq. (6) is apparently different from those of quarks in any single non-Abelian vortex (either of $ur$, $dg$, $sb$) in the CFL phase:

$$
Q_{\Lambda\Lambda} \neq Q_{ur}, Q_{dg}, Q_{sb}.
$$

Therefore, one non-Abelian vortex cannot be connected to one $\Lambda\Lambda$ vortex without discarding the continuity at the quark level, although such a connection could be consistent only at large distance scale in the GL equation [1].

To achieve a smooth connection with the obtained charges in Eq. (9), we consider the case that the quark turns around a bundle of non-Abelian $sb$, $ur$, $dg$ vortices simultaneously as shown in Fig. 2. We notice that all the quarks acquire the AB phase $+\alpha/2$ irrespective to the flavor and color components. For example, the $u_r$ quark acquires a phase $-\alpha/2$ for the path around the $ur$ vortex, $+\alpha/2$ for the $dg$ and $sb$ vortices, and hence it acquires $+\alpha/2$ in total. The same phase is obtained for the other quarks. The AB phases in the path turning around the $ur$, $dg$, $sb$ vortices simultaneously are equal to the sum of the AB phases in the paths turning around each of them: $Q_{ur+dg+sb} = Q_{ur} + Q_{dg} + Q_{sb}$. As a result, the quarks with any flavor and color acquire a common charge, which turns out to be exactly equal to the AB

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3 The BdG equation was used to find a Majorana fermion zero mode on a non-Abelian vortex in Refs. [32][33], and it was applied to a non-Abelian statistics of exchanging multiple non-Abelian vortices [34][35]. The coupling of the Majorana fermion zero modes and the CP$^2$ NG modes was obtained in Ref. [35].
phase in the presence of three ΛΛ vortices:

\[ Q_{ur+dg+sb} = Q_{ur} + Q_{dg} + Q_{sb} = 3Q_{\Lambda \Lambda}. \]  

(11)

Therefore, the continuity of the AB phases of quarks is allowed only when the bundle of \( ur, dg, sb \) vortices is connected to the bundle of three ΛΛ vortices.

We prove that the \( ur, dg, sb \) vortices and the three ΛΛ vortices meet at one point in transverse directions. First of all, we notice that the quarks can take an arbitrary path. One may think of a path that does not necessarily encircle all the \( ur, dg, sb \) vortices when those vortices are separated in space. However, such a path precludes a continuity of the quark wave functions between the CFL and hadronic phases. Therefore, only the paths turning simultaneously around the \( ur, dg, sb \) vortices should be allowed to exist: the \( ur, dg, sb \) vortices meet at one point. There, they are connected to the \( U(1) \) vortex, as shown in Fig. 1.

In summary, the continuity of the quark wave function induces that the bundle of the \( ur, dg, sb \) vortices and the three ΛΛ vortices are connected via the \( U(1) \) vortex.

**SUMMARY AND DISCUSSION**

In this Letter we have discussed a continuity of vortices during the crossover between the hadronic and CFL phases. By using macroscopic (GL) and microscopic (BdG) descriptions, we have proved that three ΛΛ vortices in the hadronic phase must join together and transform to three different non-Abelian (BdG) descriptions, we have proved that three ΛΛ vortices are connected via the ΛΛ vortex.

We have ignored (strange) quark masses and electromagnetic interaction, whose effects on a non-Abelian vortex were investigated in Refs. [16] and [40–42], respectively. We should take into account them for more realistic situations. One of questions is whether fermion zero modes in a vortex core in the CFL phase have a continuity to the hadron phase. It is also interesting what a role a confined monopole in the CFL phase plays for quark-hadron duality. It is also interesting to study how vortex lattices are connected during continuity. Finally, it will be important to study impacts of the presence of vortex junctions on dynamics of neutron stars.

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The symmetries of CFL Phase

We summarize symmetry of the CFL phase. The color-flavor-locked phase can be expected when density becomes asymptotically high. The order parameters in CFL phase are defined by the di-quark condensates (close to the critical temperature $T_c$) as $\Delta_{ij} \sim \epsilon_{abc} c^{ij}_a q_b c^{jk}_c$, $\Delta_{i} \sim \epsilon_{abc} c^{ij}_b q_a c^{jk}_c$, where $q_{ij}/q_{ij}$ are left/right handed quarks carrying fundamental color indices $i$, $j$, $k$. The chiral symmetry is spontaneously broken at ground state $\Delta = -\Delta$. The order pa-
Parameter $\Delta$ transforms as $\Delta' = e^{i\theta_B} U_c \Delta U_f^{-1}$, $e^{i\theta_B} \in U(1)_B$, $U_c \in SU(3)_c$, $U_f \in SU(3)_f$. After subtraction of the redundant discrete symmetries the actual symmetry group is given by $G = \frac{SU(3)_c \times SU(3)_f \times U(1)_B}{Z_3 \times Z_3}$. At ground state the full symmetry group $G$ is spontaneously broken down to $H \simeq \frac{SU(3)_c \times SU(3)_f}{Z_3 \times Z_3}$ and the order parameter is defined as $\langle \Delta \rangle = \Delta_{cfl} 1_3$ where $\Delta_{cfl}$ depends on the GL parameters $[28-30]$. The existence of stable vortices can be confirmed by a non-trivial first homotopy group of the order parameter space $\pi_1(G/H) \simeq \mathbb{Z}$.

**BdG equation for a single component Dirac fermion**

We consider a single component (massless) Dirac fermion in the presence of a vortex with a winding number 1. The explicit form of the BdG equation is expressed as

$$
\begin{pmatrix}
-i\gamma_0 \vec{\gamma} \cdot \vec{\nabla} - \mu & e^{i\theta} |\Delta| \gamma_0 \gamma_5 \\
-e^{-i\theta} |\Delta| \gamma_0 \gamma_5 & -i\gamma_0 \vec{\gamma} \cdot \vec{\nabla} + \mu
\end{pmatrix}
\begin{pmatrix}
u \\
u
\end{pmatrix} = \mathcal{E}
\begin{pmatrix}
u \\
u
\end{pmatrix},
$$

(12)

with $u$ particle component and $v$ hole component in the Nambu-Gor’kov representation. Here, $\mu$ is the chemical potential. The rotation of the quark around the vortex changes $\theta$ to $\theta + \alpha$. This is compensated by the phase rotations for $u$ and $v$, to maintain the above equation, by changing $(u, v)$ to $(e^{i\alpha/2} u, e^{-i\alpha/2} v)$. Therefore, the particle (hole) in the presence of the vortex has an AB phase $+(-)1/2$.  
