On the Sudakov suppression and enhancement for electroweak reactions

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Accounting for the double-logarithmic (DL) contributions to amplitude of $Z \rightarrow f \bar{f}$ leads to the Sudakov suppression. In contrast, resumming DL contributions to amplitudes of $W^\pm \rightarrow f \bar{f}'$ results into an exponential with a positive exponent, i.e. into the enhancement. This effect is intrinsic property of the theories with non-Abelian gauge groups.

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INTRODUCTION

Among different double-logarithmic (DL) contributions, there are the contributions that depend on the infrared cut-off. Results of resuming them to all orders in the coupling(s) can be often written in an exponential form, with a negative exponent, so that it falls when the total energy grows. This is known as the Sudakov suppression. It means the suppression of cross-sections compared to cross sections of the processes where bremsstrahlung is allowed. Such a suppression in QED (see e.g review) and in QCD was studied in details to all orders in the couplings. Exponentiation of the first-loop electroweak (EW) contributions was obtained in Ref. and then was confirmed in Refs. and by the fixed orders calculations. The complete list of publications on this subject can been found in the recent Refs. [5]-[9].

However, resumming DL contributions of the same infrared origin sometimes leads to different results. In particular, it was found in Ref. that one-loop DL contribution to the $W^\pm \rightarrow f \bar{f}'$ has the opposite sign compared to the one for $\gamma \rightarrow f \bar{f}$ and $Z \rightarrow f \bar{f}$ -decays. It suggests that the total resummation of the DL corrections to the amplitude of the $W^\pm \rightarrow f \bar{f}'$ may be an exponential with a positive exponent.

In the present paper we consider amplitudes for the $W$ -decay. We show that the result of accounting for DL contributions to all orders in the EW couplings has an exponential form with the positive exponent. In other words, there is the enhancement instead of the suppression for such amplitudes. We consider the decays of the electroweak bosons into the left fermions $f$ and the right antifermions $f'$ though drop the subscripts $L$ and $R$ through the paper.

The paper is organised as follows: in Sect. II we briefly review the Sudakov suppression for amplitude of $Z \rightarrow f \bar{f}$, comparing the use the Feynman and the Coulomb gauges. In Sect. III we discuss the decay of $W$-bosons into fermions, again with using both the Feynman and the Coulomb gauges, and arrive at the enhancement. The origin of the enhancement is considered in in Sect. IV. Finally, Sect. V is for concluding remarks.

SUDAKOV SUPPRESSION

The Sudakov suppression is well-known, so we discuss it very briefly. Let us consider amplitude $A_Z$ of the decay $Z \rightarrow f \bar{f}$ where $f$ stands for any (charged) fermion from the left doublets of the Standard Model. As we assume all external particles to be on-shell, $A_Z$ is gauge-independent. Among the often used gauges for propagators $D_{\mu
u}(k)$ of the EW bosons, there are the Feynman gauge and the Coulomb (temporal) gauge.

In the first-loop approximation and when the Feynman gauge is used, the DL contribution to $A_Z$ comes from the graph in Fig. 1. The vertical waved line in Fig. 1 denotes the external EW boson. The horizontal waved line may correspond to any virtual electroweak boson though the DL contribution to $A_Z$ appears only when this boson is a photon. When the Coulomb (temporal) gauge is used, DL contributions can come from the self-energy graphs depicted in Fig. 2. Again, DL contributions come only when the virtual bosons in Fig. 2 are photons.

Using the Feynman gauge, it is easy to arrive at the well-known one-loop DL contribution $A_Z^{(1)}$ to $A_Z$:

$$A_Z^{(1)} / A_Z^{Born} = -Q_f^2 \left( \frac{\alpha}{4\pi} \ln^2 \left( -M^2 / \mu^2 \right) \right) = -Q_f^2 L \quad (1)$$

FIG. 1:
where $Q_f$ is the electric charge of the fermion $f$, $M$ is the mass of the $Z$-boson and $\mu$ is the infrared cut-off. With DL accuracy, one can neglect the difference between masses of $Z$ and $W$-bosons. The double logarithm in Eq. (1) is written in the simplest way for the particular case when $\mu \geq m$, with $m$ being the mass of the fermion $f$. It also easy to arrive at Eq. (1) when the Coulomb gauge is used. In this case, DL contributions come from the self-energy graphs depicted in Fig. 2. The DL contributions $\Sigma_f^{DL}$ from the fermion self-energy graphs (graphs (b) and (c) in Fig. 2) are $\Sigma_f^{DL} = -Q_f^2 L/2$. In order to get a DL contribution from graph (a) in Fig. 2, at least one of the virtual bosons in Fig. 2a has to be a photon. However, there is not such a vertex in the Lagrangian of the Standard Model. It leaves us with two contributions $\Sigma_f^{DL}$ and we are back to Eq. (1). Using the Coulomb gauge for the Sudakov logarithms was discussed in details in Refs. [13, 14]. The DL asymptotics of $A_Z$ accounting for contributions both $\sim \gamma_\mu$ and $\sim \sigma_{\mu \nu} = (1/2)(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$ can be obtained by direct graph-by-graph calculations like it was done in Ref. [14]. The complete expression for the DL asymptotics for $A_Z$ is

$$A_Z = [\gamma_\mu + \sigma_{\mu \nu} \frac{q_\nu}{m} \frac{\partial}{\partial \rho}] e^{-Q_f^2 (\alpha/4\pi) \ln^2 \rho}$$

(2)

where we have omitted the spinors of the fermions and the $Z$-boson polarization vector, $q$ is the sum of the fermion momenta ($q^2 = M^2$); we have chosen in Eq. (2) $\mu \approx m$ and denoted $\rho \equiv -M^2/m^2$. The minus sign in the exponential of Eq. (2) means the Sudakov suppression. It suppresses the non-radiative decays of $Z$-bosons.

**SUDAKOV ENHANCEMENT**

Now let us consider DL contributions to amplitude $A_W$ of the decay $W^\pm \to f f'$ where $f$ and $f'$ ($f \neq f'$) belong the same doublet of the Standard Model. When the Feynman gauge is used, DL contribution comes from the graph in Fig. 1 where the horizontal waved line denotes again the virtual photon. This contribution is

$$A_W^{(1)} / A_W^{(Born)} = -Q_f Q_{f'} L$$

(3)

where $Q_f (Q_{f'})$ is the electric charge of the fermion $f$ ($f'$). Invoking results of Ref. [14] leads immediately to the DL asymptotics for $A_W$:

$$A_W = [\gamma_\mu + \sigma_{\mu \nu} \frac{q_\nu}{m} \frac{\partial}{\partial \rho}] e^{-Q_f Q_{f'} (\alpha/4\pi) \ln^2 \rho}.$$ 

(4)

The term $\sim \alpha$ in the expansion of Eq. (4) into series in $\alpha$ was obtained in Ref. [14].

Let us notice that when the fermions $f$ and $f'$ in Eq. (3) are quarks, there have opposite signs. It means that the exponent in Eq. (3) is positive and therefore there is the DL enhancement in Eq. (3) instead of the DL suppress of Eq. (2). For reactions $W^+ \to u d$ and $W^- \to d \bar{u}$, the DL amplitude of the process is

$$A_W^{(quarks)} = A_W^{(0)} (quarks) e^{(2/3)\alpha L}.$$ 

(5)

Notation $A_W^{(0)} (quarks)$ in Eq. (5) stands for the expression in the squared brackets in Eq. (4). At the same time, Eq. (4) reads that there are no DL contributions to the lepton decays $W \to l \nu l, W \to l \nu l$. Although amplitudes $A_W$ are gauge-invariant, it is useful to demonstrate how the first-loop contribution $A_W^{(1)}$ of Eq. (3) can be easily obtained when the Coulomb gauge is used. In this case, DL contributions come from the self-energy graphs. Contributions of graphs (b) and (c) in Fig. 2 are $\Sigma_f^{DL} = -Q_f^2 L/2$ and $\Sigma_f^{DL} = -Q_f^2 L/2$ respectively. Contrary to the case of the $Z$-boson decay, graph (a) in Fig. 1 now yields a DL contribution

$$P_W^{DL} = Q_f^2 L/2$$

(6)

and therefore

$$A_W^{(1)} = A_W^{Born} [P_W^{DL} + \Sigma_f^{DL} + \Sigma_f^{DL}] = A_W^{Born} [Q_f^2 - Q_f^2 L/2].$$

(7)

The electric charge conservation states that $Q_f - Q_{f'} = Q_W$. It allows to rewrite Eq. (7) in the form of Eq. (3).
ORIGIN OF THE ENHANCEMENT

Both the Sudakov suppression of Eq. (3) and the enhancement of Eq. (11) have a very simple origin in terms of the algebra of the SU(2) -group factors. After accounting for the Coulomb gauge in expressions for inclusive cross-sections. The results we have shown that when the EW bosons are charged, accounting for the DL radiative corrections leads to exponentiation of the first-loop contribution, however with the positive exponent, i.e. instead of the Sudakov suppression there is the enhancement for such decays. On the other hand, accounting for the soft photon emission accompanying such decays leads to the falling exponentials. Therefore, these DL contributions cancel each other in expressions for inclusive cross-sections. The results we obtained can be useful for analyses of 1 → 2 processes in theories with spontaneously broken non-Abelian gauge groups which involve different mass scales for charged massive bosons.

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