Students’ understanding of system of linear equations through mathematical modeling

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Abstract. This study aims to develop a learning trajectory of students’ understanding of system of linear equations in three variables using mathematical modelling. This study was conducted in SMA Negeri 4 Palembang participated by 6 tenth grade students in pilot experiment and 30 tenth grade students in teaching experiment. The results show that the mathematical modelling based activities can help students build mathematical models as system of linear equations in three variables and solve the system using modelling steps.

1. Introduction
In the tenth grade math book of High School, the system of linear in three variables is an important material, as it is a prerequisite material for continuing to linear program material [1]. Within the third and fourth core competencies of the 2013 Curriculum, there is a basic competency that contains material of a system of linear in three variables associated with contextual or real-world problems. One of the basic competencies of skills in the revised edition of the mathematics class of the tenth edition of 2016 is to be able to solve contextual problems related to the system of linear in three variables [2]. The system of linear equation material invites students to be able to write variables, create algebraic symbols as representations and analyze mathematical situations, can build mathematical knowledge through real-world problems and problem solving, and can use mathematical language to express ideas [3].

However, based on research results said that students are not able to understand the problem so that students cannot create equations that represent what is known and students' understanding of the system of linear equations in three variables is limited to procedural knowledge [4]. Students also have difficulties in solving mathematical problems in linear equations system in three variables ie students still cannot use the information to plan the settlement so that difficulties in determining the steps and further calculation [5].

From the problems encountered required a solution that can bridge the students in understanding and solving problems related to the system of linear equations in three variables. In the METAL project to study the system of three-variable linear equations, its application is very closely related to everyday life such as in the market, business and economic balance including macro and microeconomics. The demand and supply model are a standard application of the system of linear equation in three-variables, where demand and supply equations are provided and market clearing equilibrium is found [6].
Mathematical modeling is one alternative that can teach students to be able to change the contextual problem into a mathematical form. Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena, the meaning of mathematical modeling is a process that uses mathematics to present/describe (represent), describe (analysis), make predictions or otherwise give insight into real-world phenomena [7]. Mathematical modeling can be defined as the use of mathematics to explain and define events in real life, to test ideas and to make estimates about real-life events [8-10].

Mathematical modeling has been applied in Indonesia, both at an elementary school (SD), junior high and high school (SMA) up to university. Learning with mathematical modeling can be used as one of the learning that can bridge abstract mathematical concepts with real world learning [11]. This is supported also by Bliss [7] that mathematical modeling should be taught in every level of mathematics education. Mathematical modeling is the topic most discussed and developed over the last few decades in mathematics education. The topic of models and modelling become important for science and mathematics education in recent years. The topic of "Modeling" topic is especially important for examinations such as PISA which is conducted at an international level and measures a student's success in mathematics [8]. From the above discussion, this study aims to aim to produce trajectory of students in learning system of linear equations of three variables by through mathematical modeling.

2. Methods
This study involves 30 students of state senior high school SMA N 4 Palembang. This study also involves a 10th grade classroom teacher of state senior high school SMA N 4 Palembang. This study uses design research method which is a cyclical process of the thought experiment and instruction experiment, according to [12], design research is a research method that aims to develop Local Instructional Theory (LIT) with the cooperation between researchers and teachers to improve the quality of learning. There are three stages in the research design: preliminary design, design experiment and retrospective analysis [13,14,15,16]. The cyclic process (repeated) is a thought experiment to an experiment then of learning in the form of a diagram with illustrations about the experimental idea [17] shown in the image below;

![Diagram of design research phase](image)

**Figure 1.** Design research phase

During conducting research, data collection in this research will be done through several activities that are as follows: 1) Observation, 2) Interview; conducted against teachers as partners in designing learning plans with researchers. Interviews with teachers aim to discuss the design of HLT that researchers have designed. Interviews were also conducted on 6 students who were not research subjects. It aims to estimate whether activities that have been designed are capable of being followed by research subjects. 3) Pretest, 4) Documentation, documentation data is collected through the scan of
student activity sheet, photo taking and video recording during classroom learning process, 5) Posttest; this data is the answers, strategies, and reasons students use to solve a given problem.

3. Result and Discussion

In the first stage of the study (preparing for the experiment), the researcher analysed the literature on the system of linear equations of three variables, and concepts that can be used as teaching materials in three-linear equations, mathematical modeling, linear equations of three variables in the curriculum, and research designs used as research methods. Furthermore, researchers designed the HLT (Hypothetical Learning Trajectory) students consisting of three components namely, learning objectives, mathematical ideas, and learning activities. In addition, researchers also designed learning tools that support early tests (pre-test), lesson plans, teacher guidance, student activity sheets, observation sheets and final tests (posttest).

After the first stage is completed, the researcher continues the second phase of experimental design for the first cycle (pilot experiment). This stage aims to determine students' early abilities and test the previously designed HLT. The result of this stage is the researcher did revision in terms of language, structure question on activity and construct on activity. Then the teacher advises to clarifying the statement or information that exist on the given problem so that it is easier to be understood by the students. Therefore, the researcher makes the sentence in the problem more clearly and easily understood by the students. In this case, for the step of making assumptions, the sentences in the statement are made clearer and easier to understand so that students are not confused in giving answers. Figure 2 shows the results of student work on cycle 1.

![Figure 2. Students’ answer pilot experiment stage](image)

Based on the students' work in Figure 2, the students have not understood the purpose of the problem. This is seen from the work of students when students try to find sales bonuses for the store. Students determine a bonus by using a multiple of ten million instead of through a multiple of one million. In the first answer the student we can see that the first answer is correct but because students do not understand the sentence that is on the problem then the students get the wrong answer.

The next stage in the design experiment is teaching experiment (second cycle). In the second cycle of this study, the student activity designed is the result of improvement from the first cycle. This stage was carried out in the research subject class that is X IPA 1 consisting of 30 students and researchers as an observer. At the beginning of the study, before the students begin the lesson will be given two pretest questions to determine the initial ability of students for 15 minutes. Based on the results of these tests the researchers concluded that students are able to follow the activities - activities
that have been designed researchers. Student activity consists of constructing and developing the concept of a linear equation system of three variables using mathematical modeling steps, and solving contextual problems related to linear equations of three variable variables using modeling steps. The modeling steps in this study are adaptation researchers from the modeling steps written by Bliss and Libertiny [7] that is identifying problems, making assumptions, making equations, solving equations (do math), recheck and report results. The following is the student's answer to the revised activity from the previous stage.

![Figure 3](image1.png)

**Figure 3.** Students’ answers in making assumptions the number of shoes

![Figure 4](image2.png)

**Figure 4.** Students' answers in making equations based on previous assumptions

Based on Figure 3 it can be seen from the student's answer in making the assumptions contained in the second step of modeling that students can make guesses of how many numbers each type of shoe might sell in each store. After making the assumption then the next step is the students make the equations that we can see in figure 4. To create equations that represent the assumptions they have made, students change each type of shoes into a variable form or symbolize the number of each type of shoe.
Then students construct assumptions that have been symbolized into the form of equations. From these two images, students can follow the modeling steps until they create an equation. After the students have succeeded in making assumptions and equations, the student continues the next step of completing the equation. At this stage, students reorganize the mathematical model that has been made in the form of equations that have been made in the previous step then students solve the equation using their own chosen strategy to determine the biggest bonus of the three stores. Here is a student's answer to solving the equation that has been made.

![Equation Solution](image)

**Figure 5.** Students answer in solving equations to get results

Figure 5 shows that students solve equations by substituting the price of each type of shoe to find bonuses to be gained from selling the shoes. However, when the final results for store A, students experience errors in determining the bonus earned. This means students do not understand the information contained in the problem so that teachers need to guide students in order to calculate the bonuses that will be obtained by the store A. In the picture 5 students determine the bonus by way of bonuses will be obtained if the sales turnover multiples of 10 million, the store should get a bonus of 100,000 if the turnover of the store exceeds 10 million and applies to every multiple of one million.

From this activity, it can be seen that the students have been able to create equations and construct the concept of linear equation system of three variables which can be seen in the 2nd and 3rd drawings. Then the students can use the existing information on the problem to make assumptions or guesses and change them into equation form. This is in line with the objective of HLT which is to compile the concept of the system of linear equation in three variables and solve the contextual problems related to the system of linear equation in three variables.

At the end of the lesson at the second meeting, the students are given posttest (end) to see the progress that students have achieved during the teaching experiment. Some students are mistaken in answering the problem number 2, this error occurs because students have not fully understood the information contained in the problem so that students are mistaken and confused and wrong in making a mathematical model (equation). Because of this mistake ultimately the results obtained by students were wrong. Here are some results of student answers on the final test.
In general, the results of the student's answer analysis on the final test conducted at the end of the meeting or at the end of the study showed progress when compared with the initial test conducted at the beginning of the research phase of the teaching experiment. This proves that the students are able to
solve the problem by using the concept of the system of linear equation in three-variables with modeling steps. In the first problem in Figure 6a students can search for each furniture price by using the concept of the system of linear equation three variables.

As for the second problem in Figure 6b shows that the student has understood how to make the equation through the information available on the problem even though the equation contained in the problem is more than two equations. The second question on posttest is a problem solving the problem but most students have successfully solved and looked for solutions to the problem. This is supported by Lee's research [17] involving high school students, demonstrating that modeling activities successfully help students improve problem-solving skills. From the results of the analysis of these two problems, students have been able to compile and create a mathematical model of the problem so that students can find a solution for each problem that cannot be separated from the concept of the system of linear equation in three variables that have been studied.

4. Conclusion
Based on the results of the discussions that have been described, it can be concluded that the activities in the student activity sheet that has been designed, can really help students in understanding the material system of linear equation in three variables. The results showed that the learning trajectory in this study can help students to help students understand and solve problems concerning systems of linear equations in three variables using mathematical modeling and mathematical modeling can be used as an alternative learning that can motivate students during learning, making students more courageous to express opinions during making assumptions and can train student problem-solving skills.

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