Evaluating The Two Loop Diagram Responsible For Neutrino Mass In Babu’s Model

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Babu studied the neutrino spectrum obtained when one adds a charged singlet and a doubly charged singlet to the standard model particle spectrum. It was found that the neutrinos acquire a mass matrix at the two-loop level which contains one massless eigenstate. The mass matrix of Babu’s model depends on an integral over the undetermined loop momenta. We present the exact calculation of this integral.

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I. INTRODUCTION

Zee observed that the addition of a charged singlet and additional Higgs doublets to the Standard Model particle spectrum resulted in radiatively generated neutrino mass at the one-loop level [1,2]. It was consequently noted that if only one doublet couples to the leptons, the mass matrix takes a simple form and produces one mass eigenstate much lighter than the other two [3,4]. Babu studied the neutrino spectrum obtained when one retains Zee’s charged singlet and adds a doubly charged singlet [5]. It was found that the neutrinos develop a mass matrix at the two-loop level, which contains one massless eigenstate, to lowest order. The mass matrix depends on an integral over the undetermined loop momenta. We present the exact calculation of this integral. The analytic evaluation of related integrals, occurring in Sec. IV for the relevant hierarchies of mass parameters involved.

where $f_{ab} = -f_{ba}$ and $h_{ab} = h_{ba}$. Gauge invariance precludes the singlet Higgs fields from coupling to the quarks. The Higgs potential contains the terms:

$$V(\phi, h^+, k^{++}) = \mu (h^- h^- k^{++} + h^+ h^+ k^-) + \ldots$$

which violate lepton number by two units and give rise to neutrino Majorana mass contributions at the two-loop level (see Figure 1). The neutrino masses are calculable and to lowest order the mass matrix takes the form:

$$M_{ab} = 8 \mu f_{ac} h_{cd} m_c m_d I_{cd}(f^\dagger)_{db},$$

where $h_{ab} = \eta h_{ab}$ with $\eta = 1$ for $a = b$ and $\eta = 2$ for $a \neq b$. $m_{c,d}$ are the charged lepton masses and:

$$I_{cd} = \int \frac{d^4 q}{(2\pi)^4} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 - m_h^2)} \frac{1}{(q^2 - m_{h'}^2)} \frac{1}{(q^2 - m_{h''}^2)} \frac{1}{(q^2 - m_{h''}^2)} \frac{1}{(q^2 - m_{h''}^2)}$$

× \frac{1}{(q^2 - m_{h'}^2)} \frac{1}{(q^2 - m_{h''}^2)} \frac{1}{(q^2 - m_{h''}^2)} \frac{1}{(q^2 - m_{h''}^2)} \frac{1}{(q^2 - m_{h''}^2)} \frac{1}{(q^2 - m_{h''}^2)}.$$

If one defines:

$$K_{cd} = 8 \mu \tilde{h}_{cd} m_c m_d I_{cd},$$

where no summation is implied by the repeated indices, the mass matrix may be written as $M_{ab} = (f K f^\dagger)_{ab}$. Thus $\det M = |\det f|^2 \det K = 0$ for an odd number of generations (due to the anti-symmetry of $f$) and to lowest order the

![Fig. 1: Two-loop diagram responsible for neutrino mass in the Babu model.](image)
neutrino spectrum contains one massless state. In what follows, the exact calculation of the integral \( I_{cd} \) is performed and the asymptotic behaviour for the cases \( m_k \gg m_h \gg m_c, m_d \) and \( m_k \gg m_k \gg m_c, m_d \) is presented. Note that the hierarchy \( m_h, m_k \gg m_c, m_d \) is a phenomenological constraint, whilst the relative size of \( m_k \) and \( m_h \) is not predetermined.

### III. Evaluation of \( I_{cd} \)

After performing a Wick rotation and letting \( q \rightarrow -q \), the integral \( I_{cd} \) may be written as:

\[
I_{cd} = \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{1}{(p^2 + m_1^2)} \frac{1}{(p^2 + m_1^2)} \frac{1}{(q^2 + m_k^2)} \frac{1}{(q^2 + m_k^2)} \times 
\]

\[
\frac{1}{(p^2 + M_{1i}^2)} \frac{1}{((p + q)^2 + M_{jk}^2)},
\]

where the momenta \( p \) and \( q \) are now Euclidean four-vectors. We adopt the notation of [7] and define:

\[
(M_{11}, M_{12}...M_{1n1}, M_{21}, M_{22}...M_{2n2}|M_{31}, M_{32}...M_{3n3})
\]

\[
= \int d^n p \int d^n q \prod_{i=1}^{n1} \prod_{j=1}^{n2} \prod_{k=1}^{n3} \frac{1}{(p^2 + M_{1i}^2)} \times 
\]

\[
\frac{1}{(q^2 + M_{2j}^2)} \frac{1}{((p + q)^2 + M_{jk}^2)},
\]

so that \( I_{cd} = \frac{1}{(2\pi)^8} (m_h, m_c|m_h, m_d|m_k) \) when \( n = 4 \), where \( n \) is the space-time dimensionality. Note that all momenta in [6] are Euclidean whilst the definition used in [7] contains Minkowski vectors. It is possible to express \( I_{cd} \) as a linear combination of integrals with less than five propagators. One may use partial fractions to obtain relations like:

\[
(m, m_0|m_1|m_2) = \frac{1}{m^2 - m_0^2} \{ (m_0|m_1|m_2) - (m|m_1|m_2) \},
\]

which may be applied to \( I_{cd} \) to give:

\[
I_{cd} = \frac{1}{(2\pi)^8} \frac{1}{(m_h^2 - m_c^2)} \frac{1}{(m_h^2 - m_d^2)} \frac{1}{(m_h^2 - m_k^2)} \frac{1}{(m_h^2 - m_k^2)} \times 
\]

\[
\{ (m_c|m_d|m_k) - (m_h|m_d|m_k) - (m_0|m_h|m_k) + (m_h|m_h|m_k) \}.
\]

At this point one may proceed to evaluate the integral \( (m_0|m_1|m_2) \) to determine \( I_{cd} \) but it is preferable to express \( I_{cd} \) as a combination of integrals of the form \( (2m_0|m_1|m_2) \) where \( (2m_0|m_1|m_2) \) is shorthand for \( (m_0, m_0|m_1|m_2) \). If one evaluates integrals of the form \( (m_0|m_1|m_2) \) by introducing Feynman parameters, some of the ultraviolet divergences are transferred from the radial integrals in momentum space to the Feynman parameter integrals. It is easier to handle the divergences when evaluating \( (2m_0|m_1|m_2) \), as they may be completely contained in the momentum space integrals. Integrals of the type \( (m_0|m_1|m_2) \) have been considered in [8, 9] and single integral representations have been obtained [10, 11, 12].

To express \( (m_0|m_1|m_2) \) in terms of integrals of the form \( (2m_0|m_1|m_2) \) one may use the partial \( p \) operation of ’t Hooft [13]. Essentially one inserts the identity expression:

\[
1 = \frac{1}{2n} \sum_{i=1}^{n} \left\{ \frac{\partial p_i}{\partial p_i} + \frac{\partial q_i}{\partial q_i} \right\}
\]

into the integrand of \( (m_0|m_1|m_2) \), performs integration by parts and rearranges the resulting expressions to obtain the relationship:

\[
(m_0|m_1|m_2) = \frac{1}{3 - n} \times \{ m_0^2 (2m_0|m_1|m_2) 
\]

\[
+ m_1^2 (2m_1|m_0|m_2) + m_2^2 (2m_2|m_0|m_1) \}.
\]

We note that equation (8) is defined for integral \( n \) but the resulting relationship may be analytically continued to non-integral \( n \). Using equation (9) on \( I_{cd} \) gives:

\[
I_{cd} = \frac{1}{(2\pi)^8} \frac{1}{(m_h^2 - m_c^2)} \frac{1}{(m_h^2 - m_d^2)} \frac{1}{(m_h^2 - m_k^2)} \times 
\]

\[
\{ m_0^2 \left[ (2m_c|m_h|m_k) - (2m_c|m_d|m_k) + (2m_d|m_h|m_k) - (2m_d|m_c|m_k) \right] 
\]

\[
+ m_h^2 \left[ (2m_k|m_h|m_d) + (2m_k|m_c|m_h) - (2m_k|m_c|m_d) - (2m_k|m_h|m_k) \right] 
\]

\[
+ m_k^2 \left[ (2m_h|m_k|m_d) + (2m_h|m_k|m_c) - 2(m_h|m_k|m_h) \right] \}.
\]

Thus evaluation of the generic integral \( (2m|m_1|m_2) \) allows one to determine \( I_{cd} \). We obtain:

\[
(2m|m_1|m_2) = \frac{\pi^4 (\pi m^2)^{n-4} \Gamma(2 - \frac{1}{n})}{\Gamma(3 - \frac{1}{n})} \int_0^1 dx \int_0^1 dy (x(1-x))^{n/2-2} (y(1-y))^{2-n/2} \times 
\]

\[
\left[ \Gamma(5-n) \frac{\mu^2}{(y + \mu^2(1-y))^{5-n}} + \frac{1}{2} \frac{n \Gamma(4-n)}{(y + \mu^2(1-y))^{4-n}} \right],
\]
where
\[ \mu^2 = \frac{ax + b(1 - x)}{x(1 - x)}, \quad a = \frac{m_1^2}{m^2}, \quad b = \frac{m_2^2}{m^2}. \]

This result is in agreement with the result for \( G(m, m_1, m_2; 0) \) in \([14]\) and differs by an overall minus sign to that obtained in \([7]\) due to our different definition of \((2m|m_1|m_2)\) in terms of Euclidean momenta. Letting \( n = 4 + \varepsilon \) and expanding for small \( \varepsilon \) gives:
\[
(2m|m_1|m_2) = -\pi^2 \left[ \frac{2}{\varepsilon^2} + \frac{1}{\varepsilon}(1 - 2\gamma_E - 2 \log(\pi m^2)) \right] - \pi^4 \left[ -\frac{1}{2} - \frac{1}{12}\pi^2 - \gamma_E^2 + (1 - 2\gamma_E) \log(\pi m^2) - \log^2(\pi m^2) - f(a, b) \right] + O(\varepsilon) \tag{11}
\]

where the function \( f(a, b) \) is given by:
\[
f(a, b) = \frac{1}{2} \log a \log b - \frac{1}{2} \left( \frac{a + b - 1}{\sqrt{-a}} \right) \left\{ \text{Li}_2 \left( \frac{-x_2}{y_1} \right) + \text{Li}_2 \left( \frac{-y_2}{x_1} \right) - \text{Li}_2 \left( \frac{-x_1}{y_2} \right) - \text{Li}_2 \left( \frac{-y_1}{x_2} \right) \right. \nonumber \\
\left. + \text{Li}_2 \left( \frac{b - a}{x_2} \right) + \text{Li}_2 \left( \frac{a - b}{y_2} \right) - \text{Li}_2 \left( \frac{b - a}{x_1} \right) - \text{Li}_2 \left( \frac{a - b}{y_1} \right) \right\}. \tag{12}
\]

We have introduced:
\[
x_1 = \frac{1}{2}(1 + b - a + \sqrt{a - b}), \quad x_2 = \frac{1}{2}(1 + b - a - \sqrt{a - b}), \quad y_1 = \frac{1}{2}(1 + a - b + \sqrt{a - b}), \quad y_2 = \frac{1}{2}(1 + a - b - \sqrt{a - b}),
\]
and:
\[ \sqrt{a - b} = (1 - 2(a + b) + (a - b)^2)^{1/2}, \]
where under \( a \leftrightarrow b \) we have \( x_i \leftrightarrow y_i \) for \( i = 1, 2 \). The expression \( \text{12} \) can be shown to be equivalent to:
\[
f(a, b) = -\frac{1}{2} \log a \log b - \left( \frac{a + b - 1}{\sqrt{a - b}} - \frac{1}{2} \right) \times \nonumber \\
\left\{ \text{Li}_2 \left( \frac{-x_2}{y_1} \right) + \text{Li}_2 \left( \frac{-y_2}{x_1} \right) + \frac{1}{4} \log^2 \frac{x_2}{y_1} + \frac{1}{4} \log^2 \frac{y_2}{x_1} \right. \nonumber \\
+ \frac{1}{4} \log^2 \frac{x_1}{y_2} - \frac{1}{4} \log^2 \frac{x_2}{y_1} + \text{Li}_2(1) \right\}. \tag{13}
\]

which is the symmetrized version obtained in \([7]\). For numerical evaluation via mathematica etc, it may be more convenient to use the form \( \text{12} \). If one uses \( \text{12} \) for arbitrary masses it is possible for the logarithms to go negative, thus producing a non-zero imaginary component for \( f(a, b) \). By judicious logarithmic branch choice one can ensure that the imaginary components of \( f(a, b) \) cancel, whilst the real part of \( f(a, b) \) is independent of the branch choices. This real part is always in agreement with the form \( \text{12} \) of \( f(a, b) \), but in the latter the imaginary parts cancel when using the principal branch for all dilogarithms. The result for \( I_{cd} \) will ultimately be a combination of functions of the form \( f(a, b) \). As \( I_{cd} \) is evaluated in the neutrino rest frame, it is not possible to cut the two-loop diagram it represents and obtain an energy- and momentum-conserving sub-graph (ie the neutrino at rest doesn’t have enough energy for the internal particles to be real and on the mass shell). Thus the amplitude for the graph must be real \([15]\), forcing the function \( f(a, b) \) to be real. Equation \( \text{11} \) together with either \([13]\) or \([12]\) gives the final result for \((2m|m_1|m_2)\).

\( I_{cd} \) is now obtained by substituting the result for \((2m|m_1|m_2)\) into the expression \([10]\). Inspection of \([10]\) shows that we may use \((2m|m_1|m_2) \rightarrow \pi^4 f(a, b)\) when evaluating \( I_{cd} \), as the constants and logarithms with massive arguments occurring in \((2m|m_1|m_2)\) cancel amongst the terms with a given mass coefficient.

### IV. DOMINANT BEHAVIOUR OF \( I_{cd} \)

We now find the asymptotic behaviour for the cases \( m_k \gg m_h \) and \( m_h \gg m_k \). The dominant terms are found by using the expansions for \((2m|m_1|m_2)\) presented in \([7]\). Utilising the symmetry properties of \((2m|m_1|m_2)\) and the structure of the expression \([10]\) for \( I_{cd} \) allows one to obtain the leading terms relatively easily.
In the expression for $I_{cd}$, equation (10), we may safely neglect the terms with lepton mass coefficients. The terms with coefficient $m_k^2$ have the form $(2m_k|m_x|m_h)$ and the expansion for $(2m_k|M_1|M_2)$, where $m \gg M_1, M_2$, given in [7], may be used. All constants and logarithms of $m^2$ may be ignored as they cancel among the various terms. The expansion is given in terms of $a = (M_1/m)^2$ and $b = (M_2/m)^2$. By noting the form of (10) and that $(2m_1|M_1|M_2) = (2m_2|M_2|M_1)$, it is seen that in the expansion of a term $(2m_k|m_x|m_h)$, any term depending only on $a$ or only on $b$ will occur in the expansion of another term with a relative minus sign. So one need only retain terms containing both $a$ and $b$. Any terms which contain a lepton mass will be suppressed, meaning the term $(2m_k|m_x|m_h)$ dominates. Defining $h = (m_k/m_x)^2$ and using $(2m_1|M_1|M_2)$ in [7], keeping only the terms containing both $a$ and $b$, gives:

$$
(2m_k|m_x|m_h) = -\pi^4 \left\{ h^2 + 6h^3 - 2 \log(h) \left\{ h^2 + 5h^3 \right\} - \log^2(h) + \frac{\pi^2}{3} \left\{ h^2 + 4h^3 \right\} + \ldots \right\}. \quad (14)
$$

All the terms with a coefficient $m_h^2$ in $I_{cd}$ have the form $(2m_h|m_x|m_h)$ and will be expanded in terms of $h$ and $(m_x/m_h)^2$. The form of (10) shows that any term in an expansion depending only on $h$ (not including the cross terms when $m_x = m_h$) will occur in another expansion with a relative minus sign. If $m_x$ is a lepton mass, the leading contributions from $(2m_h|m_x|m_h)$ will contain factors $(m_x/m_h)^2$ and consequently be suppressed. Thus leading order:

$$
m_h^2 \left\{ (2m_h|m_d|m_h) + (2m_h|m_e|m_h) - (2m_h|m_v|m_h) \right\} \approx -2m_h^2 (2m_h|m_h|m_h). \quad (15)
$$

Using the expansion of $(2M_1|M_2|m)$ for $m \gg M_1, M_2$ from [7] and noting the above gives:

$$
(2m_h|m_h|m_h) = -\pi^4 \left\{ -h - \frac{21}{4}h^2 + \frac{\pi^2}{3} \left\{ h + 2h^2 \right\} + \log^2(h) \left\{ h + \frac{13}{2}h^2 \right\} + \ldots \right\} \quad (16)
$$

as the leading terms which contribute to $I_{cd}$. Combining the expressions for $(2m_h|m_h|m_h)$ and $(2m_h|m_v|m_h)$ and noting the appropriate coefficients from equation (10) gives:

$$
I_{cd} \approx \frac{1}{(4\pi)^2} \frac{1}{m_k^2} \left\{ \log^2 \frac{m_h^2}{m_k^2} + \frac{\pi^2}{3} \left\{ \frac{1}{m_k^2} \right\} + O \left( \frac{1}{m_k^2} \right) \right\}, \quad (17)
$$

where the lepton masses have been neglected relative to the scalar masses in the factors $(m_h^2 - m_e^2)^{-1}(m_h^2 - m_o^2)^{-1}$. Babu’s approximate form reproduces the $\log^2$ term of (17) in the $h \to 0$ limit. It is necessary to retain both $(2m_h|m_x|m_h)$ and $(2m_h|m_x|m_h)$ to obtain the leading terms of (17). The leading terms of $I_{cd}$ may be obtained by setting $m_e = m_d = 0$, but one may not take $m_h = 0$ when $m_k \gg m_h$ as $I_{cd}$ develops an infra-red singularity. This singularity is seen to manifest itself in the leading terms of equation (17) as a logarithmic singularity in the limit $m_h \to 0$.

The terms with lepton mass coefficients may again be neglected to obtain an expansion in terms of $k = (m_k^2/m_h^2)$. The presence of the factors $(m_h^2 - m_e^2)^{-1}(m_h^2 - m_o^2)^{-1} \approx m_h^{-4}$ in (10) means that the terms with coefficient $m_h^2$ will be suppressed relative to those with coefficient $m_k^2$. The expansion of $(2m_k|M_1|M_2)$ for $m > M$ in [7] may be used for $(2m_h|m_x|m_h)$, where only terms originating in the $f(a, b)$ portion, as defined in (11) need be retained. The result is:

$$
(2m_h|m_h|m_k) = -\pi^4 \left\{ k + \frac{5}{36}k^2 + \log(k) \left\{ \frac{1}{2} - \frac{1}{12}k^2 \right\} + \ldots \right\}. \quad (18)
$$

For $m_h \gg m_k$, contributions from the terms $(2m_h|m_x|m_k)$, where $m_x$ is a lepton mass, must be included. The leading terms are those which do not contain factors $(m_x^2/m_h^2)$. The expansion for $(2m_1|M_2|^2)$ with $m > M_1, M_2$ in [7] is again employed. Retaining only those contributions which stem from $(a, b)$ gives:

$$
(2m_h|m_x|m_k) = -\pi^4 \left\{ -\frac{\pi^2}{6} + k + \frac{1}{4}k^2 + \log(k) \left\{ -\frac{1}{2}k^2 \right\} + \ldots \right\}, \quad (19)
$$

This expression holding for $m_x = m_e$ and $m_x = m_d$. Using (18) and (19) and neglecting terms with coefficients $m_x^2, m_y^2$, and $m_z^2$ in (10), gives the result:

$$
I_{cd} \approx \frac{1}{2\pi^4} \frac{1}{m_h^2} \left\{ \frac{\pi^2}{6} + \frac{1}{2m_h^2} \log \frac{m_k^2}{m_h^2} \right\} + O \left( \frac{1}{m_h} \right). \quad (20)
$$

Thus for large $m_h^2$ the leading term is:

$$
I_{cd} \approx \frac{1}{(4\pi)^2} \frac{2}{m_h^2} \frac{\pi^2}{6}. \quad (21)
$$

The $m_h^{-2}$ factor is expected on dimensional grounds and in this limit the coefficient of $m_h^{-2}$ contains no logarithmic singularities. We see that the dominant terms for $I_{cd}$, when $m_h \gg m_k$, are found by considering the terms with coefficient $m_k^2$ in (10). At higher order in $(m_k/m_h)^2$ contributions from the terms with coefficient $m_k^2$ will also become important. Note that (21) is independent of $m_k$ and the lepton masses and that its form may be understood as follows. Define:

$$
I_2(k^2) = \frac{-k^2}{\pi^4} \int d^4p \int d^4q \frac{1}{(p^2 - m_e^2)(p - k)^2} \frac{1}{(q^2 - m_h^2)(q - q)^2} \frac{1}{(p - q)^2}. \quad (22)
$$

This corresponds, up to multiplicative factors, to $I_{cd}$ with $m_e = m_d = m_k = 0$ and $m_h \rightarrow m$ when the external momenta $k$ is taken to be zero:

$$
I_{cd}|_{m_e=m_d=m_k=0} = \lim_{k^2 \rightarrow 0} \frac{1}{(2\pi)^8} \left( \frac{-\pi^4}{k^2} \right) I_2(k^2) \bigg|_{m \rightarrow m_k}. \quad (23)
$$
The integral $I_2(k^2)$ has been evaluated [9], leading to the result:

$$I_{cd}|_{m_c=m_d=m_k=0} = \frac{1}{(4\pi)^4} \frac{2}{m_h} \zeta(2).$$  \hspace{1cm} (23)

Noting that $\zeta(2) = \frac{\pi^2}{6}$ and comparing (23) with (21) it is seen that the leading term of $I_{cd}$, for $m_h \gg m_k$, is simply the result one obtains for $I_{cd}$ when the lepton masses and $m_k$ are taken to be zero (observe that for $m_h \gg m_k$ we may safely set $m_c, m_d$ and $m_k$ to zero to obtain the leading term as $I_{cd}$ remains infra-red finite in this limit).

After completing this work we became aware that the explicit asymptotic terms in equations (17) and (21) were given in [16].

V. CONCLUSION

The analytic result for the integral:

$$I_{cd} = \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{1}{(p^2 - m_h^2)(q^2 - m_k^2)} \frac{1}{(p^2 - m_c^2)} \frac{1}{(q^2 - m_d^2)} \frac{1}{(p-q)^2 - m_k^2},$$  \hspace{1cm} (24)

which appears in the lowest order neutrino mass matrix in Babu’s model, has been obtained. The result comprises of equation (10) together with (11) and either (13) or (12) for the terms $(2m|m_1|m_2)$. We note that for $m_k \gg m_h$ the leading term is:

$$I_{cd} \approx \frac{1}{(4\pi)^4} \frac{1}{m_h^2} \log^2 \left( \frac{m_h^2}{m_k^2} \right).$$

The $m_k^{-2}$ term in the asymptotic expansion was extracted and is in agreement with the result of [16]. The leading term when $m_h \gg m_k$ is independent of the lepton masses but requires a non-zero value of $m_h$ to avoid an infra-red singularity. For the reversed hierarchy, $m_h \gg m_k$, the leading term is:

$$I_{cd} \approx \frac{1}{(4\pi)^4} \frac{2}{m_h^2} \frac{\pi^2}{6},$$

which is the result one obtains for $I_{cd}$ upon setting the lepton masses and $m_k$ to zero and is in agreement with that obtained in [16].

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