Two falling-chain demonstrations based on Einstein’s equivalence principle

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Simple demonstrations based on the equivalence principle are given of how a folded chain and a horizontal flat chain fall down when one chain end is fixed to a rigid support.

We describe here how Einstein’s equivalence principle can be used to show how the falling ends of two flexible chains fall when one chain end in each chain is held fixed to a rigid support. The first of these falling chains is a folded chain that has its two ends initially close together, with an initial horizontal separation $\Delta x = 0$. Kucharski has given a one-dimensional continuum model of this falling chain. He finds from its Lagrangian equation of motion that its energy is conserved and is concentrated in the falling arm. The falling arm is thus forced to fall faster than $g$, the acceleration due to gravity. The falling time turns out to be 85% of that for free fall, making it relatively straightforward to confirm the theoretical description. A description of this falling chain based directly on energy conservation can be found in many places, including Ref. 4–6.

Calkin and March have obtained experimental confirmation of energy conservation of the falling folded chain by measuring its tension at the support as a function of the vertical falling distance. They show that their experimental result can be reproduced closely by the Kucharski one-dimensional continuum model, except near the end of fall when the last link in the chain turns over as a rigid body. The recent history of this falling chain problem has been reviewed by Wong and Yasui who point out that energy conservation arises from the fact that links are transferred to the stationary arm at the bend of the flexible chain by elastic collisions, not inelastic ones. The falling chain end falls faster than $g$ because that part of the chain immediately next to the falling end is below it and pulls it down further. Wong and Yasui are able to isolate the downward chain tension involved with the help of the Lagrangian formulation of mechanics.

Recently, Tomaszewski, Pieranski and Geminard have re-confirmed the faster than $g$ fall of the folded chain both by experiment and by numerical simulation. They have also extended their study to falling chains with nonzero initial separations $\Delta x$. For the limiting case of $\Delta x \approx L$ (the chain length), where the chain is initially stretched taut as an almost flat catenary, they find that the chain end falls (almost) freely on release, with an acceleration very close to $g$. Their video capture pictures show clearly that the falling end is part of a freely falling horizontal chain segment that shrinks in length as the chain falls.

A simple way to “see” that the falling end is falling freely is to note that the chain near the falling end is (almost) horizontal so that there is little mass next to and below the falling end to drag it down further, and no mass next to and above the falling end to pull it up. In other words, the falling end falls freely because it is part of a freely falling horizontal chain segment. The reason why the horizontal chain segment itself falls freely is because the chain is flexible and bends smoothly and without kink to the original horizontal shape towards the falling end. This bend starts from the supported end of the chain and travels outward towards the falling end as the chain falls. The chain tension begins at the supported end and follows the direction of the chain itself, becoming horizontal beyond the bend. It can thus only pull the horizontal chain segment towards the supported end without affecting the horizontal segment’s acceleration in the vertical direction. Finally, the smoothness of the chain at the bend is the result of Newton’s third law that states that action and reaction forces must be equal in strength and opposite in direction. This is possible only when the chain has no kink. If there were a kink, the tangent direction of the chain would change discontinuously across the kink to give a reaction force that is not opposite in direction to the action, in violation of Newton’s third law.

Historically, the nature of these falling chains have been clarified by using high-speed photography, sensor and electronic circuitry, and video capture. We shall demonstrate the motion of these falling chains using only a chain and a smooth horizontal surface provided by a desk or table, with the result interpreted by the equivalence principle of Einstein.

The principle of mass equivalence, proposed by Einstein in 1907, states that the gravitational and inertial masses of an object are always equal. This means that the acceleration acquired by a freely falling observer in a gravitational field is just the acceleration $g$ due to gravity. A freely falling observer then sees no gravity. The sudden realization of this connection between inertia and weight was for Einstein the “happiest thought” of his life.

The weightlessness of a freely falling observer is beautifully demonstrated by a toy made by Eric Rogers of Princeton University and given to Einstein as a puzzle on the occasion of Einstein’s 76th and last birthday in 1955. A cup with a hollow tube at its bottom, looking like a champagne glass with a long stem, is placed at the end of a broomstick. A long soft spring is attached to the inside base of the hollow tube. The soft spring is stretched at the free end by a long thread that connects it to a metal ball outside the cup so that the ball hangs over the lip of the cup, as shown schematically in Fig. 1. The problem is to find a foolproof way to move the hang-
FIG. 1: The cup, soft spring and hanging metal ball in the toy made by Eric Rogers for Einstein’s 76th birthday.

The old man solved the problem “at once”. Holding the toy high by the end of the broomstick, he allowed the toy to fall freely along the length of the long broomstick. The outside mass became weightless. The soft spring then retracted, pulling the ball into the cup, to the old man’s great delight.

This story is behind the title “An Old Man’s Toy” of Zee’s book on gravity.

Let us now return to our falling folded chain. If its falling end falls freely, it will remain at rest relative to a freely falling observer. We now propose to show what this freely falling observer actually sees by putting a light folded ball or beaded chain on a smooth horizontal table top made of glass, marble, steel, Formica or polished wood where the friction is likely to be relatively small. Hold one chain end by the hand above the table. This chain end represents the original chain end at the fixed support. It is next made to move relative to the freely falling observer by being pulled suddenly away from the fold of the chain in a horizontal direction along the length of the folded chain, as shown in Fig. 2(a). You the observer will see that the free end still on the table will not remain at rest on the table, as it should if it were falling freely in a falling folded chain. You will find instead that it will move in the direction opposite to the pulling direction, thereby showing that the free end of the falling folded chain is pulled by the chain next to and below it to fall faster than $g$. The pull and the motion to be observed are sketched in the figure. Marking the initial position of the chain end on the table with a small object placed next to it will help to define its subsequent motion. We recommend using about a meter length of a light ball chain like those used to operate overhead fans. (It is sometimes called a No. 6 ball chain in the United States). These light chains will not scratch your table if you do it only a few times.

The demonstration of a falling flat chain is just as easy. Arrange the ball chain on the table in a straight line. Lift the chain by the end representing the support, and pull it suddenly in a horizontal direction perpendicular to the length of the chain on the table, as shown in Fig. 2(b). You will see that the chain on the table is pulled along its length towards the bend of the chain, but it will not move in the direction of the pull or opposite to it, thereby showing that the falling end of the original chain falls freely. In both demonstrations, the equivalence principle allows the observer to effectively fall freely in space so that the relative motion of the falling chain end can be seen more clearly by the unaided eye.

These demonstrations are most effective when the pull at the “support” end is of middling strength, fast enough to make friction unimportant but slow enough to make the motion to be demonstrated clearly seen by the eye. Of course, our pull is unlikely to generate a constant acceleration or a value necessarily close to $g$. The demonstrations work even with variable accelerations of any convenient strength. This is indeed one of their charms.

The falling flat chain can also be demonstrated directly but perhaps not as persuasively in the following way: A demonstrator stretches the chain horizontally before letting one end fall down. The observer should stand some distance away from the demonstrator in order to follow the fall readily with the eye. It is possible to see a falling horizontal chain segment before it finally merges into the rotating arm at the support end. One can also see that the kinetic energy of the rotating arm is supplied by the energy carried in by the chain links transferred from the falling horizontal chain segment. However, the observer cannot tell for sure that the falling horizontal chain segment is falling freely.

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2 Experimental confirmation of Kucharski’s observation in early years has been mentioned in other papers, and we remember seeing in an old book on mechanics a photograph
of a falling folded chain falling faster than a small freely falling ball. However, we are not able to locate any specific reference on these observations.

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