Searching for Low-mass Population III Stars Disguised as White Dwarfs

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Abstract

It is uncertain whether or not low-mass Population III stars ever existed. While limits on the number density of Population III stars with \(M_\star \approx 0.8 \, M_\odot\) have been derived, using Sloan Digital Sky Survey (SDSS) data, little is known about the occurrence of Population III stars at lower masses. In the absence of reliable parallaxes, the spectra of metal-poor main-sequence (MPMS) stars with \(M_\star \lesssim 0.8 \, M_\odot\) can easily be confused with those of cool white dwarfs. To resolve this ambiguity, in this paper we present a classifier that differentiates between MPMS stars and white dwarfs, based on photometry and/or spectroscopy without the use of parallax information. We build and train our classifier using state-of-the-art theoretical spectra, and evaluate it on existing SDSS-based classifications for objects with reliable Gaia DR2 parallaxes. We then apply our classifier to a large catalog of objects with SDSS photometry and spectroscopy to search for MPMS candidates. We discover several previously unknown, extremely metal-poor (EMP) candidate stars, and recover numerous confirmed EMP stars already noted in the literature. We conclude that archival SDSS spectroscopy has already been exhaustively searched for EMP stars. We predict that the lowest-mass stars of primordial composition will have redder optical-to-infrared colors than cool white dwarfs at constant effective temperature, due to surface gravity-dependent collision-induced absorption from molecular hydrogen. We suggest that the application of our classifier to data produced by next-generation spectroscopic surveys will set stronger constraints on the number density of low-mass Population III stars in the Milky Way.

1. Introduction

The first generation of stars formed in the universe was made of only the stable products of Big Bang nucleosynthesis: hydrogen, helium, and a tiny amount of lithium. These Population III stars are predicted to start forming around 100 Myr after the Big Bang (e.g., Bromm 2013; Glover 2013; Greif 2015). The earliest calculations of Population III star formation suggested that inefficient cooling would require large Jeans masses and therefore that Population III stars would form with a characteristic stellar mass \(M_\star \sim 100 \, M_\odot\) (e.g., Silk 1983; Tegmark et al. 1997; Bromm et al. 1999, 2002; Abel et al. 2000, 2002). However, more recent simulations have shown that fragmentation in the accretion disks around massive Population III protostars could potentially form pristine stars at much lower masses (e.g., Stacy et al. 2010, 2012, 2016; Clark et al. 2011a, 2011b; Greif et al. 2011, 2012; Dopcke et al. 2013; Stacy & Bromm 2013, 2014; Riaz et al. 2018; Wollenberg et al. 2020). While it is theoretically uncertain whether these fragments survive or merge with the more massive protostar growing at the center of their parent accretion disk (e.g., Hirano & Bromm 2017), there is at least circumstantial observational evidence to suggest that they might survive (Schlaufman et al. 2018). If these fragments do avoid merging, then low-mass Population III stars might persist to the present day in the local universe.

While no Population III star has been directly observed to date, according to the latest Stellar Abundances for Galactic Archaeology (SAGA) database observational searches have instead found more than 500 extremely metal-poor (EMP) stars with metallicity \([\mathrm{Fe/H}] \lesssim -3\) (Suda et al. 2008, 2011, 2017; Yamada et al. 2013). The chemical abundances of these extreme Population II stars can be used to infer the properties of Population III stars (e.g., Hartwig et al. 2015; Placco et al. 2016; Fraser et al. 2017; Ishigaki et al. 2018; Hansen et al. 2020), as well as the early chemical evolution of the Milky Way (e.g., Beers & Christlieb 2005; Frebel & Norris 2015; Kobayashi et al. 2020).

Because stars with bright apparent magnitudes are more easily studied than stars with faint apparent magnitudes, aside from early examples found in studies of stars with high proper motion (e.g., Ryan & Norris 1991a, 1991b; Ryan et al. 1991), the vast majority of metal-poor stars studied in detail to date are at least as luminous as the main-sequence turnoff (Cohen et al. 2013; Yong et al. 2013; Roederer et al. 2014). Of the 8419 unique stars in the SAGA database with inferences of effective temperature \(T_{\text{eff}}\) and surface gravity \(\log g\), only 199 or 2.3% have \(T_{\text{eff}} \lesssim 6000\, \text{K}\) and \(\log g \gtrsim 4.6\), indicative of metal-poor main-sequence (MPMS) stars. As a result, only a small fraction of the Milky Way’s metal-poor stellar population has been characterized to date. Indeed, assuming the evolutionary tracks from PARSEC version 1.2S (Bressan et al. 2012; Chen et al. 2014, 2015) and an initial mass function according to Kroupa (2001, 2002) for a 10 Gyr stellar population, the luminosity function for stars with \([\text{Fe}/\text{H}] \approx -2.2\) indicates that for every metal-poor turnoff star with \(T_{\text{eff}} \gtrsim 6000\, \text{K}\) and \(3.8 \lesssim \log g \lesssim 4.6\) there are about 20 MPMS stars with \(T_{\text{eff}} \lesssim 6000\, \text{K}\) and \(\log g \gtrsim 4.6\) in the same volume.

One significant obstacle to the identification of MPMS stars is the optical spectra of metal-poor (and metal-free) main-sequence stars are virtually indistinguishable from the optical spectra of more numerous cool hydrogen-atmosphere white dwarfs (WDs). While a significant fraction of white dwarf photospheres are externally seeded with metals by ongoing accretion from circumstellar disks (e.g., Gänsicke et al. 2012; Koester et al. 2014), the high surface gravities of white dwarfs
cause metals to rapidly sink out of their photospheres via gravitational settling (Schatzman 1948). The spectra of most white dwarf photospheres are therefore very similar to those of metal-free stars, featuring strong Balmer lines and possibly other absorption features due to molecular hydrogen. While in the absence of precise parallaxes it is sometimes possible to separate MPMS stars from white dwarfs using reduced proper motion as an analog for absolute magnitude (e.g., Luyten 1922), reduced proper motions are ambiguous for individual stars and can be confounded if the samples of interest belong to distinct kinematic populations.

The traditional method to distinguish MPMS stars from white dwarfs has been to visually identify MPMS stars on the basis of their narrower Balmer absorption lines at a given photometric color (Kepler et al. 2019). This process is prone to human error and very difficult for large data sets. These problems have historically been aggravated by imperfect models for the atmospheres of cool white dwarfs (Kepler et al. 2019). Recent advances have improved the fidelity of atmosphere models for white dwarfs with \( T_{\text{eff}} \leq 5000 \) K (Blouin et al. 2018a, 2018b), facilitating a theoretical comparison between the spectra of MPMS stars and white dwarfs at the lowest temperatures, where spectroscopic data are currently lacking.

Tens of thousands of spectra with weak or non-existent metal lines produced by either cool white dwarfs or metal-poor/metal-free main-sequence stars will be collected by ongoing and next-generation spectroscopic facilities/surveys such as the Large Sky Area Multi-Object Fibre Spectroscopic Telescope (LAMOST; Cui et al. 2012), the Dark Energy Spectroscopic Instrument (DESI; DESI Collaboration et al. 2016), the fifth phase of the Sloan Digital Sky Survey (SDSS-V; Kollmeier et al. 2017), and the WHT Enhanced Area Velocity Explorer (WEAVE; Dalton et al. 2012). The volume of data produced by these ongoing and future surveys will make it impossible to rely on the traditional human classification of MPMS stars and white dwarfs. This implies that there is a need for an automated way to differentiate between MPMS stars and white dwarfs.

In this paper, we develop an automated framework to identify MPMS stars in large spectroscopic surveys with a focus on the problem of differentiating them from spectroscopically similar white dwarfs. We describe in Section 2 state-of-the-art theoretical spectra for both MPMS stars and white dwarfs. We then introduce the SDSS photometric and spectroscopic data for MPMS stars and white dwarfs securely classified using parallaxes from Gaia Data Release 2 (DR2) that we use to validate and test our methods. We outline our classification algorithm in Section 3, including our extraction of features from photometric and spectroscopic observables and the validation of our classifier. In Section 4 we perform a search for candidate extremely metal-poor stars in a sample of possible MPMS stars with SDSS photometry and spectroscopy. We review our results and their implications in Section 5. We conclude by summarizing our findings in Section 6.

2. Data

Our goal is to differentiate MPMS stars from white dwarfs on the basis of photometry and spectroscopy alone. While there is enough observational data for MPMS stars and white dwarfs with \( T_{\text{eff}} \gtrsim 6000 \) K, the relative faintness of MPMS stars and white dwarfs with \( T_{\text{eff}} \lesssim 6000 \) K has resulted in few examples in the SDSS spectroscopic archive. Even though these cool MPMS stars and white dwarfs were largely out of reach of the Sloan Foundation 2.5 m Telescope, spectroscopic surveys using 4 m class telescopes such as DESI will target many cool MPMS stars and white dwarfs. It is therefore necessary for us to use both theoretical spectra for cool MPMS stars and white dwarfs and empirical spectra for warmer MPMS stars and white dwarfs to train and validate our classifier. We describe those data in the following two subsections.

2.1. Theoretical Spectra

We first construct grids of theoretical spectra for white dwarfs and MPMS stars using state-of-the-art models. We consider temperatures in the range 4000 K \( \leq T_{\text{eff}} \leq 8000 \) K in 40 K steps. For each temperature, we generate white dwarf spectra with \( 7 \leq \log g \leq 9 \) and MPMS star spectra with \( 3.5 \leq \log g \leq 5.5 \) in steps of 0.5 dex. This procedure results in grids of 505 theoretical spectra that we use to train our classifier. For the white dwarfs, our grid of theoretical spectra is described in Blouin et al. (2018a, 2018b). These spectra were computed assuming pure-hydrogen atmospheres, no magnetic fields, and metal-free photospheres. Blouin et al. (2018a, 2018b, 2019) describe the improved equation of state and radiative opacities featured in this new generation of models that are especially important for cool white dwarfs with \( T_{\text{eff}} \gtrsim 5000 \) K.

For MPMS stars, we use theoretical spectra computed with the PHOENIX code (Husser et al. 2013). Those models cover a broad range of temperatures and surface gravities and include metallicities \(-4.0 \leq [\text{Fe}/\text{H}] \leq +1.0\) and \( \alpha \) abundances \(-0.2 \leq [\alpha/\text{Fe}] \leq +1.2\). In our theoretical grid of MPMS star spectra, we use the \([\text{Fe}/\text{H}] = -4.0\) spectra with solar \( \alpha \) abundances. The spectroscopic classifier we describe in Section 3 relies on Balmer lines, so the assumed metallicity has only a negligible impact on our analyses. For both the MPMS and WD grids of theoretical spectra spanning \( T_{\text{eff}} \) and \( \log g \), we tri-linearly interpolate the logarithm of the fluxes with respect to effective temperature, surface gravity, and the logarithm of wavelength. We convolve all theoretical spectra to match an instrumental resolution of 1.5 A, comparable to the resolution of large-scale spectroscopic surveys such as SDSS, LAMOST, and DESI. We illustrate some sample theoretical spectra in Figure 1.

2.2. SDSS Data

Our empirical data for warmer MPMS stars and white dwarfs come mostly from SDSS DR16 (Ahumada et al. 2020). They were collected during the first four phases of the SDSS (York et al. 2000; Eisenstein et al. 2011; Blanton et al. 2017), including its Sloan Extension for Galactic Understanding and Exploration, Baryon Oscillation Spectroscopic Survey (BOSS), and extended BOSS programs (Yanny et al. 2009; Dawson et al. 2013, 2016). The data were collected using the Sloan Foundation 2.5 m Telescope and its imager and optical spectrographs (Gunn et al. 1998, 2006; Doi et al. 2010; Smee et al. 2013) then placed on the SDSS photometric system using the methods described in Fukugita et al. (1996), Smith et al. (2002), and Padmanabhan et al. (2008). We also make use of Gaia DR2 astrometry (Gaia Collaboration et al. 2016, 2018; Salgado et al. 2017; Arenou et al. 2018; Lindegren et al. 2018; Luri et al. 2018; Marrese et al. 2019). We deredden the SDSS
The optical spectrum of a very low-mass Population III star would be with scaled solar abundances. We offset the top panel.

Assuming $\log g = 8$ and $\log g = 4.5$ for the white dwarf and MPMS star spectra, respectively. For the MPMS star spectra, we assume $[\text{Fe/H}] = -4.0$ with scaled solar abundances. We offset the fluxes vertically for visual clarity. The optical spectrum of a very low-mass Population III star would be spectroscopically indistinguishable from the $T_{\text{eff}} = 4000$ K white dwarf in the top panel.

The ugriz magnitudes using the mw dust utility (Bovy et al. 2016) and the combined dust maps of Drimmel et al. (2003), Marshall et al. (2006), and Green et al. (2019).

We focus on the MPMS star and white dwarf classifications provided by Kepler et al. (2019). Those authors examined 500,000 SDSS spectra plausibly produced by white dwarfs. They provide classifications for 37,053 spectra based on 11 criteria, of which 15,716 were classified as DA white dwarfs and 15,855 were classified as subdwarf A or sdA stars (i.e., likely MPMS stars). Among the white dwarfs, over 78% were classified as DA white dwarfs with hydrogen-rich atmospheres on the basis of broad Balmer absorption lines with no other strong spectral features. Kepler et al. (2019) visually differentiated these DA white dwarfs from subdwarf A stars based on the fact that DA white dwarfs have broader Balmer lines at a given photometric color due to their high surface gravities and increased pressure broadening. Together these two classes comprise more than 85% of the classifications provided in Kepler et al. (2019).

We use in our analysis only those white dwarfs or MPMS stars classified by Kepler et al. (2019) as members of their “DA,” “sdA,” or “sdA/F” classes. The class “DA” designates hydrogen-atmosphere white dwarfs, the class “sdA” refers to subdwarf A stars, and the class “sdA/F” indicates ambiguous subdwarf A or F stars. This limits our sample to 14,522 spectra with prominent Balmer lines and no strong metal features. It also removes other white dwarf spectral types such as helium-rich DB white dwarfs. While most of the stars classified as “sdA” or “sdA/F” are likely MPMS stars (e.g., Brown et al. 2017; Pelisoli et al. 2018a, 2018b), a very small fraction could also be extremely low-mass (ELM) white dwarfs (e.g., Kosakowski et al. 2020).

As we argued above, visually separating MPMS stars from white dwarfs is prone to error, especially at low temperatures where MPMS star and white dwarf spectra are visually similar. For that reason, Kepler et al. (2019) appealed to Gaia DR2 parallaxes to distinguish MPMS stars and DA white dwarfs. At a given color, white dwarfs are far less luminous than MPMS stars due to their smaller radii. A high-quality parallax measurement therefore provides accurate classifications via absolute magnitudes. However, only about 15% of the 14,522 stars in this sample have reliable parallaxes (i.e., $\pi / \sigma_\pi > 10$). As a result, the classification of MPMS stars and white dwarfs cannot be confirmed for most of this sample. Consequently, when validating our methods we only use those stars for which a definite classification can be made based on a high-quality Gaia DR2 parallax. We require (1) $\text{parallax}_{\text{over\_error}} > 10$, (2) $\text{visibility\_periods\_used} > 8$, and (3) $\text{astrometric\_sigma}\_5d_{\text{max}} < 1$. This results in an empirical validation sample of 1807 stars with reliable MPMS star/white dwarf classifications. Of these, 65% are classified as DA white dwarfs and the remaining 35% are classified as MPMS stars. We illustrate this empirical validation sample in the Sloan color–magnitude diagram depicted in Figure 2.

3. Methods

Even though visual classifications have been sufficient to separate MPMS stars from white dwarfs in the past, the volume of data expected to be produced by ongoing and next-generation spectroscopic surveys will make the traditional approach impractical. We therefore use the grids of theoretical
spectra described in Section 2.1 to build an automated process to differentiate MPMS stars from white dwarfs using photometric and spectroscopic observables alone without appealing to accurate parallax measurements. We then validate our classifier using the empirical labels in Section 2.2.

3.1. Balmer Lines

Both relatively warm MPMS stars and white dwarfs exhibit strong Balmer lines in their spectra. On this basis, in the absence of high-quality parallaxes MPMS stars have usually been differentiated from white dwarfs based on Balmer lines for temperatures $T_{\text{eff}} \gtrsim 6000$ K. As we described above, MPMS stars have narrower Balmer lines than white dwarfs at a given temperature (Kepler et al. 2019). We now quantify this difference across several Balmer lines at once and use it to produce a rigorous selection function.

For each spectrum in the theoretical grids described in Section 2.1 we fit each of the Balmer absorption lines H$\alpha$, H$\beta$, H$\gamma$, and H$\delta$ with a Voigt profile.$^1$ From each profile we derive two summary statistics: the FWHM in angstroms and the minimum of the profile in continuum-normalized flux units that we define as the line amplitude. For each spectrum we therefore derive eight line statistics in total, which together quantify the phase space of the Balmer lines. This results in a vector of eight Balmer features per spectrum plus a “label” identifying it as the spectrum of an MPMS star or white dwarf. Our classifier then makes use of all eight features to differentiate MPMS stars from white dwarfs. We focus on H$\alpha$, H$\beta$, H$\gamma$, and H$\delta$ in this study because Balmer lines bluer than H$\delta$ occur in regions of the SDSS spectra of cool stars with lower signal-to-noise ratio (S/N).

We illustrate in Figure 3 two 2D “slices” through the Balmer line phase space that demonstrate why Balmer line features are useful for classification. Because of the faintness of cooler white dwarfs, most white dwarfs targeted by the SDSS have $T_{\text{eff}} \lesssim 6000$ K. As a consequence of the low space density of massive stars in fields at high Galactic latitude, most MPMS stars targeted by the SDSS have temperatures $T_{\text{eff}} \lesssim 8000$ K. In the overlap region between these two temperature ranges, the theoretical spectra confirm the expectation that MPMS stars have much narrower (i.e., smaller FWHM) H$\alpha$ profiles than white dwarfs.

Figure 3 is representative of the relationship between Balmer line features as a function of $T_{\text{eff}}$ and $\log g$. Above $T_{\text{eff}} \approx 6000$ K, MPMS star and white dwarf spectra are visually differentiable on many 2D slices through Balmer line phase space. Below $T_{\text{eff}} \approx 6000$ K, the higher-order Balmer lines in white dwarf spectra start to rapidly lose their intensities. In the range $5000 \lesssim T_{\text{eff}} \lesssim 6000$ K, combining statistics from several Balmer lines can still separate MPMS stars from white dwarfs. Below $T_{\text{eff}} \approx 5000$ K, the Balmer lines for both metal-free low-mass stars and white dwarfs begin to disappear altogether, resulting in pure-continuum spectra (Blouin et al. 2019). In other words, below $T_{\text{eff}} \approx 5000$ K there is no way to use optical spectra to distinguish low-mass Population III stars from white dwarfs. We discuss this special case of very low-mass Population III stars further in Section 4.

$^1$ A Voigt profile is a convolution of Gaussian and Lorentzian profiles that well approximates the pressure-broadened wings of the Balmer lines (e.g., Tremblay & Bergeron 2009).

Figure 3. Two “slices” of the Balmer line phase space for MPMS stars (solid lines colored by effective temperature) and white dwarfs (dashed lines). The thickness of each line is proportional to surface gravity, $\log g = 4, 4.5, 5$ for MPMS star models and $\log g = 7.5, 8, 8.5$ for white dwarf models.

3.2. Synthetic Photometry

In addition to the Balmer features we extract from each theoretical spectrum, we also calculate synthetic photometry in several photometric systems using the pyphot utility (Fouesneau 2020). We calculate synthetic absolute magnitudes in the SDSS ugriz, Pan-STARRS grizy (Chambers et al. 2016), SkyMapper uvgriz (Bessell et al. 2011), DECam uvgrizY (Flaugher et al. 2015), and Vera Rubin Observatory ugrizY (Ivezic et al. 2019) systems. While the software accompanying this paper can be used with any of these photometric systems, we focus on SDSS photometry colors in this paper since we validate our method on SDSS data.

We plot in Figure 4 a comparison between MPMS stars and white dwarfs in Sloan color–color space. Sloan $u - g$ color is strongly affected by the Balmer jump, a sensitive probe of surface gravity. A plot of $u - g$ color versus $g - r$ color can therefore be used to cleanly differentiate relatively warm MPMS stars and white dwarfs. At lower temperatures the difference becomes less pronounced, so this color–color diagram can only differentiate MPMS stars and white dwarfs when $T_{\text{eff}} \gtrsim 6000$ K. While Figure 4 depicts one well known photometric difference between MPMS stars and white dwarfs, we consider the complete color–color spaces defined by each of the SDSS, Pan-STARRS, SkyMapper, DECam, and Rubin
Figure 4. Synthetic color–color diagram using SDSS bands. The blue line corresponds to white dwarfs with log $g = 8$ while the shaded region indicates the locus of white dwarfs with surface gravities in the range 7 < log $g$ < 9. We also plot MIST isochrones for MPMS stars with [Fe/H] = −1.5 and age 10 Gyr (orange) and [Fe/H] = −4.0 and age 13.5 Gyr (red). We overlay as gray points all stars classified as “DA,” “sDA,” or “sDA/P” objects by Kepler et al. (2019).

Observatory photometric systems, computing all unique pairwise differences of the magnitudes in each photometric band. While the approach that we describe in the following subsection is related to traditional color–color selections (e.g., Lokhorst et al. 2016; Pelisoli et al. 2018b), our approach uses the entire N-dimensional color space distribution at once rather than relying on individual 2D slices through that space.

3.3. Logistic Regression Classifier

To map the spectroscopic and photometric features described above to an MPMS star or white dwarf classification, we use a logistic regression classifier. A logistic regression is a model that assumes a linear relationship between the input features and the log-odds of a binary (i.e., Bernoulli) random variable taking on the value 1 (or “true”). Given input features $x = [x_0, x_1, x_2, \ldots]$, fitting a logistic regression involves solving for the coefficients $\beta = [\beta_0, \beta_1, \beta_2, \ldots]$ such that

$$\beta x = \log \left( \frac{p}{1 - p} \right),$$

where $p$ is the probability that the Bernoulli random variable is 1 (or “true”). A logistic regression model is typically fit to data with known labels (i.e., where $p$ is known to be either zero or one), and the coefficients $\beta$ are subsequently used to estimate $\hat{p}$ for new input data. There is no closed-form solution to determine $\beta$, so an iterative gradient descent algorithm is often used to find the optimal coefficients.

We define the underlying Bernoulli variable to have value 0 if an object is a confirmed white dwarf and 1 if an object is a confirmed metal-poor main-sequence star. The associated probability in the model can therefore be defined as $p = P_{\text{MPMS}}$, the probability that a given object is an MPMS star as opposed to a white dwarf. We demonstrate three possible input configurations: the Balmer line summary statistics (eight features), ugriz photometric colors (ten features), and griz photometric colors (six features). We also consider a combined classifier that uses ugriz colors and Balmer features simultaneously.

We use a logistic regression model due to its simplicity and ease of interpretation. The classification probability $P_{\text{MPMS}}$ returned by a logistic regression is well calibrated by default (Yu et al. 2011), providing confidence in the classification. We found that using a more complex classification algorithm such as a random forest or support vector machine increased the complexity of the method without much increase in accuracy. In addition, these other algorithms require an external “calibration function” to transform the returned classification probabilities to statistically meaningful values (Niculescu-Mizil & Caruana 2005). This step is unnecessary for the logistic regression.

When training our logistic regression models, we reserve 5% of the stars in our grid of theoretical models as a synthetic validation set. We evaluate our logistic regression model on these unseen synthetic validation data and confirm that the model correctly predicts their classifications virtually 100% of the time. This validation step ensures that the logistic regression is working as expected on noiseless synthetic data. We will further validate our logistic regression model with SDSS photometry and spectroscopy for objects with secure parallax-based classifications in Section 3.4. Our classifier and synthetic training data for the SDSS, Pan-STARRS, SkyMapper, DECam, and Rubin Observatory photometric systems are publicly available.\(^2\)

3.4. Model Validation with Objects That Have Secure Empirical Classifications

We now further validate our classifier with the empirical validation sample of 1807 objects described in Section 2.2. These objects were classified by Kepler et al. (2019) based on spectroscopic features with Gaia DR2-confirmed classifications as either MPMS stars or white dwarfs. We confirm that the classifications of Kepler et al. (2019) for these objects are accurate based on our inspection of their locations in a color–magnitude diagram. We use the available SDSS photometry and spectroscopy for this empirical validation sample to compute Balmer features and ugriz colors. We then run the logistic regression classifier described in Section 3.3 on these features to predict—based solely on SDSS photometry and spectroscopy—the probability that each object is an MPMS star.

Receiver operating characteristic (ROC) curves are one way to graphically evaluate classifiers. ROC curves describe the relationship between the true-positive and false-positive rates of a classifier as a function of its acceptance threshold (i.e., the output probability above which a “positive” classification is made). ROC curves provide a high-level summary of the sensitivity (i.e., probability of detection) and specificity (i.e., probability of false alarm). Precision-recall (PR) curves provide an alternative graphical classifier diagnostic. They are particularly useful for our application, because precision (i.e., the probability that an MPMS star is accurately classified by our

\(^2\) https://github.com/vedanthchandra/mpms
classifiers trained on theoretical models and validated with classifications confirmed by high-quality Gaia DR2 parallaxes. The classifier that uses both spectroscopic and photometric features achieves the best performance, followed closely by the classifier that uses only spectroscopic features. Classifiers that use only photometric features achieve acceptable performance but lag behind classifiers that utilize spectroscopic features.

The ugriz colors. As expected, the Balmer spectroscopic features provide the most discriminating power. The inclusion of $u - g$ color greatly improves the quality of classifications compared to $griz$ colors alone, because $u - g$ color is sensitive to log $g$ via the Balmer break.

In terms of precision, the classifier using both photometric and spectroscopic features far outperforms the spectroscopy-only classifier. This implies that a degeneracy in spectroscopic features in MPMS star and white dwarf spectra can be overcome by including photometric features. The natural interpretation is that the photometric colors provide a strong constraint on temperature, breaking the $T_{\text{eff}}$-log $g$ degeneracy of the absorption line features. We validate the naive expectation that classifiers using more than one Balmer line are superior to any classifier based on a single line. We find that classification performance is most enhanced when lower-order Balmer lines such as H$\alpha$ are combined with higher-order lines such as H$\beta$ and confirm that the best performance is achieved when all four Balmer lines are included. We conclude that our combined photometric and spectroscopic classifier has the accuracy and precision necessary to separate MPMS stars from white dwarfs using SDSS data or equivalent LAMOST or DESI data.

While our photometry-only classifier can be used to separate MPMS stars from white dwarfs, it is susceptible to many of the issues with traditional color–color selections. For example, the colors of white dwarfs with heavily metal-polluted photospheres will change due to line blanketing and changes in continuum opacity. While metal-polluted white dwarfs could be misclassified by our photometry-only classifier, they are easy to remove from a joint photometric/spectroscopic search for metal-poor stars due to their prominent metal absorption lines. Our classifier is therefore most powerful when spectroscopic data are available.

### 3.5. Approximate Determination of Metallicity

After differentiating MPMS stars from white dwarfs, another important task is to select the most metal-poor candidates among the predicted MPMS stars. In order to select the most metal-poor MPMS star candidates, we implement a simple method to approximately quantify stellar metallicity from MPMS star spectra by fitting them with theoretical spectra.

We first use SDSS photometry and any available Gaia parallax information to generate an initial set of spectroscopic stellar parameters $T_{\text{eff}}$, log $g$, and [Fe/H]. We define a $\chi^2$ statistic by comparing the observed SDSS $ugriz$ photometry dereddened as described in Section 2.2 to MESA Isochrones & Stellar Tracks (MIST) $ugriz$ photometry (Paxton et al. 2011, 2013, 2015, 2018; Choi et al. 2016; Dotter 2016). To account for the low-precision parallaxes available in most cases, we use an exponentially decreasing space density prior on parallax (Bailer-Jones 2015). We sample with emcee (Foreman-Mackey et al. 2013, 2019) the posterior distribution of the fundamental stellar parameters (e.g., mass, age, and metallicity) conditioned on the data, taking $-0.5$ $\chi^2$ as the log-likelihood of the data. We then identify the maximum-likelihood sample and interpolate the MIST isochrone grid to define an initial set of spectroscopic stellar parameters $T_{\text{eff}}$, log $g$, and [Fe/H].

We next fit PHOENIX theoretical spectra from Husser et al. (2013) spanning the range $3000 \text{ K} \leq T_{\text{eff}} \leq 8000 \text{ K}$, $3.5 \leq \log g \leq 6.5$, and $-4 \leq [\text{Fe/H}] \leq 0$ to the observed
SDSS MPMS star spectra using a two-step fitting procedure. We initialize the stellar parameters at the photometrically inferred values described above. We first fit the sky emission line at 4047 Å in each SDSS spectrum with a Gaussian to estimate the instrumental line-spread function. We then convolve the grid of theoretical spectra with this kernel to bring them to the instrumental resolution. We next fit a straight line to a region around Hα in each SDSS spectrum and then divide the flux by that line to approximately normalize the spectrum in the vicinity of the line. We then fit a Gaussian to Hα and use its known wavelength to correct each SDSS spectrum to the rest frame. To spectroscopically infer \( T_{\text{eff}} \) and \( \log g \), we assume a fixed \( [\text{Fe}/\text{H}] = -2 \) and fit the grid of theoretical spectra to the continuum-normalized Balmer lines. To spectroscopically infer \( [\text{Fe}/\text{H}] \), we fix the \( T_{\text{eff}} \) and \( \log g \) derived in the previous step and fit the continuum-normalized Ca II K absorption line to infer \([\text{Ca}/\text{H}] \) and thereby \([\text{Fe}/\text{H}] \) assuming \([\text{Ca}/\text{Fe}] = 0 \). For each absorption line, we fit the theoretical spectra using a nonlinear least-squares algorithm \texttt{lmfit} (Newville et al. 2014) that minimizes the squared residual between the observed and theoretical spectra. We alternate the Balmer-based \( T_{\text{eff}} \) and \( \log g \) and the Ca II K-based \([\text{Fe}/\text{H}] \) inferences until the spectroscopic stellar parameters are self-consistent. In practice this only requires one or two iterations since the Balmer-based inference is quite independent of metallicity.

For the metal-poor main-sequence stars with published spectroscopic stellar parameters from data with higher resolution or higher S/N, we find that the spectroscopically derived parameters are better than those based on photometry and astrometry. This is likely due to the low-quality parallaxes available in most cases. We note that spectroscopic \([\text{Fe}/\text{H}] \) inferences based on the Ca II K absorption line can be overestimated if a star is carbon-enhanced, as in that case the Ca II K line can be blended with nearby carbon features in a moderate-resolution spectrum (e.g., Frebel et al. 2015). Since our primary goal is to find the most metal-poor candidates (including those depleted in carbon), we do not correct for this effect explicitly. We therefore suggest that our spectroscopically inferred metallicities should be treated as indicative estimates only.

4. Results

With the accuracy and precision of our classifier now established, we can now classify the entire sample of 14,522 MPMS star/white dwarf candidates from Kepler et al. (2019) with SDSS photometry and spectroscopy. We fit the Balmer lines of these spectra to derive the line summaries described in Section 3.1. We then apply our combined logistic regression classifier to the Balmer features and \texttt{ugriz} colors of these stars. The result of this process is a prediction of the probability \( P_{\text{MPMS}} \) that a spectrum was produced by an MPMS star as opposed to a white dwarf. Reassuringly, the distribution of probabilities is bimodal: most stars have probabilities close to zero or one. This distribution provides even more confidence that our classifier is both precise and accurate. We decide on a decision threshold of 0.5 and classify all objects with \( P_{\text{MPMS}} > 0.5 \) as MPMS star candidates.

We find that approximately 95% of our logistic regression-based classifications agree with the visual inspection-based classifications reported in Kepler et al. (2019). The 5% of cases where our classifier disagrees with Kepler et al. (2019) are due to objects with extremely low-S/N spectra. Spectroscopic surveys such as DESI will produce hundreds of thousands of spectra plausibly produced by MPMS stars or white dwarfs, and only our automated classification approach will be able to scale to that data volume.

We emulate future searches for candidate EMP stars in upcoming surveys such as SDSS-V and DESI by mining the stars in our SDSS data set for EMP candidates. Numerous studies have already searched these SDSS data for metal-poor stars (e.g., Allende Prieto et al. 2006, 2008, 2014, 2015; Caffau et al. 2011a, 2011b; Bonifacio et al. 2012, 2015; Aoki et al. 2013; Frebel et al. 2015; Placco et al. 2015; Aguado et al. 2016, 2017a, 2017b, 2018a, 2018b; Yoon et al. 2016; Carbon et al. 2017), so our expectation is that many of the most promising candidates have already been discovered. We select all stars from the 12,000 star subsample of the study of Kepler et al. (2019) referenced above with \( P_{\text{MPMS}} > 0.5 \) according to our combined classifier as possible MPMS candidates. This leaves us with 10,500 MPMS star spectra for which we compute approximate metallicities as described in Section 3.5.

We selected stars with the lowest estimated metallicities and visually inspected their spectra, examining the Ca II K (3934.8 Å), Na I (5895.6 Å), and Mg I (5176.7 Å) lines in particular. We report our most promising candidates in Table 1, along with our spectroscopically inferred estimates of \( T_{\text{eff}} \), \( \log g \), and \([\text{Fe}/\text{H}] \).

In addition, we use SIMBAD to search for literature references to our candidate MPMS stars (Wenger et al. 2000). While several of our MPMS star candidates have no prior reference in the literature, we find that our selection turns up dozens of previously known metal-poor stars with confirmed metallicities \([\text{Fe}/\text{H}] < -3.0 \). A selection of these stars is included in Table 1. We therefore conclude that SDSS data have already been exhaustively searched for extremely metal-poor stars. Nevertheless, this serves as a comprehensive validation of our methodology to search for metal-poor stars in future spectroscopic surveys.

While the existence of low-mass Population III stars is still debated, we assert that it will be possible to uniquely classify a featureless optical spectrum plausibly produced by either a low-mass Population III star or a cool white dwarf based on the optical-to-infrared colors of the object. The lowest-mass object of primordial composition that can burn hydrogen should have \( M_\odot \approx 0.1 \). Stellar models therefore predict a lower limit on the effective temperature of Population III stars of \( T_{\text{eff}} \approx 3600 \) K (Burrows et al. 1993; Saumon et al. 1994). At these temperatures, the optical spectra of the coolest primordial stars will be indistinguishable from the optical spectra of white dwarfs because both will have pure-continuum spectra with no Balmer or metal lines.

However, collision-induced absorption (CIA) from molecular hydrogen will produce surface gravity-dependent continuum features in the red optical and infrared (Saumon et al. 1994; Blouin et al. 2017, 2019). At a given temperature (or \( g - r \) color), a white dwarf will have more CIA than a low-mass Population III star due to its higher surface gravity. White dwarfs will consequently be fainter than low-mass Population III stars in the red optical \( i \) and infrared bands such as \( JHK \). Therefore, at constant temperature photometric colors comparing optical to red optical or infrared bands (e.g., \( r - z, V - J \), etc.) will be bluer for white dwarfs than for low-mass Population III stars. This effect is expected to be larger than...
the typical photometric uncertainties in ground-based surveys such as SDSS, Pan-STARRS, SkyMapper, or DES (Saumon et al. 1994; Blouin et al. 2017).

While future work on CIA opacities will enable a more quantitative comparison, current uncertainties in the theoretical modeling of CIA make it challenging to precisely quantify the expected difference between low-mass Population III stars and white dwarfs, and thereby build a predictive model like the one we have presented for warmer stars. Nevertheless, we predict that the photometric signature of CIA should be detectable via infrared photometry and consequently able to differentiate between low-mass Population III stars and white dwarfs even without parallax information.

5. Discussion

We have developed a classification framework that can be used to differentiate low-mass Population III stars from white dwarfs in future spectroscopic surveys. Since our techniques primarily rely on Balmer lines and broadband photometry, our methods naturally extend to the identification of metal-poor stars including the sought-after EMP and ultra-metal-poor (UMP) stars (i.e., stars with $\mathrm{[Fe/H]} \lesssim -4$). Another natural application of our technique is the identification and removal of the subdwarf stars that frequently contaminate catalogs of white dwarf candidates (Kepler et al. 2019).

The typical white dwarf has $M_\ast \approx 0.6 M_\odot$, $R_\ast \approx R_\odot$, and consequently $\log g \sim 8$. Its surface gravity is therefore orders of magnitude larger than that of a typical metal-poor main-sequence star with $\log g \lesssim 5$. As a result, our classifier gains most of its discriminatory power from an object’s surface gravity. While MPMS stars are well separated from typical white dwarfs in $\log g$, they can overlap in $\log g$ with EMP white dwarfs produced by mass transfer in multiple systems (e.g., Brown et al. 2010; Pelisoli et al. 2017, 2019; Pelisoli & Vos 2019). Because of their lower masses and larger radii than typical white dwarfs, ELM white dwarfs can have lower $\log g$ and much narrower Balmer lines than typical white dwarfs. Their narrow Balmer lines could cause our classifier to misidentify them as MPMS stars. While the identification of these relatively rare ELM white dwarfs is a worthwhile goal in its own right, it is not the main objective of our current analysis. We argue that in future searches for MPMS or low-mass Population III stars, ELM white dwarfs can be filtered out by directly fitting theoretical spectra to Balmer lines to infer $\log g$. Indeed, even the least-massive ELM white dwarfs should have $\log g \gtrsim 5$ while MPMS stars with $M_\ast \gtrsim 0.1 M_\odot$ will have $\log g \lesssim 5$ (Brown et al. 2010; Choi et al. 2016; Dotter 2016).

It remains an open question whether or not low-mass Population III stars exist in the local universe. Beyond awaiting serendipitous direct detections, Galactic/stellar archeology provides a promising technique to probe the hypothesis of surviving Population III stars and to constrain the primordial initial mass function. Hartwig et al. (2015) proposed that a total sample size of order $10^3$ halo stars is required to rule out Population III survivors at the 95% confidence level. However, this is likely an overestimate since it only considers blind surveys. In practice, searching for EMP and UMP stars is far more efficient. Magg et al. (2019) suggested instead to use the occurrence of EMP and UMP stars to constrain the existence of Population III survivors. The largest uncertainty in this method by far is the total number of EMP and UMP stars in the Milky Way’s stellar halo. As a result, a comprehensive search for such stars is an important need.

6. Conclusion

While it is trivial to separate metal-poor main-sequence stars and white dwarfs when high-quality parallaxes are available, even post-Gaia many objects with DESI or Sloan spectroscopy will lack reliable parallaxes. For that reason, we developed a classifier capable of separating metal-poor main-sequence stars from cool white dwarfs using a range of photometric and spectroscopic features. We trained and validated our classifier

### Table 1

Extremely Metal-poor Candidates Identified in This Study

| Gaia DR2 Source ID | R.A. (deg) | Decl. (deg) | Sloan Plate-MID-Fiber | $g$ (mag) | $T_{\text{eff}}$ (K) | $\log g$ | $\mathrm{[Fe/H]}$ | References |
|-------------------|-----------|-------------|----------------------|-----------|---------------------|---------|-----------------|------------|
| 3724067121891339648 | 211.826 | 10.505 | 1703-53799-0013 | 16.9 | 6441 | 4.0 | −4.0 | Aoki et al. (2013) |
| 874891322185786752 | 116.953 | 26.762 | 2055-53792-0121 | 17.3 | 6450 | 3.7 | −4.0 | Aoki et al. (2013) |
| 446521507285008640 | 245.076 | 16.141 | 2188-54595-0026 | 18.5 | 6550 | 4.1 | −3.9 | Aoki et al. (2013) |
| 1448863970892447744 | 203.103 | 27.516 | 2245-54208-0450 | 16.8 | 6435 | 4.0 | −3.8 | Aoki et al. (2013) |
| 395882559788943232 | 189.109 | 23.019 | 3374-54948-0220 | 18.2 | 6453 | 3.7 | −3.8 | Aoki et al. (2013) |
| 390521400829451841 | 184.325 | 8.870 | 5396-55947-0370 | 19.4 | 6560 | 4.5 | −3.8 | Aoki et al. (2013) |
| 1493736818927192576 | 220.706 | 44.954 | 6046-56996-0550 | 18.8 | 6359 | 4.0 | −3.8 | Aoki et al. (2013) |
| 3092216989574463616 | 126.339 | 4.059 | 1185-52642-0515 | 17.1 | 6441 | 4.0 | −3.9 | Aoki et al. (2013) |
| 290930261314166528 | 25.151 | 23.749 | 2044-53327-0002 | 15.8 | 6354 | 4.8 | −3.5 | Arentsen et al. (2019) |
| 127688247957416288 | 230.509 | 30.924 | 1651-54402-0330 | 16.6 | 6335 | 4.7 | −3.4 | Aoki et al. (2013) |
| 41908373987569490112 | 301.306 | −10.751 | 2303-54629-0377 | 17.0 | 6536 | 3.8 | −3.6 | Francois et al. (2018) |
| 1184737183522291712 | 221.669 | 12.822 | 1712-53531-0636 | 16.2 | 6445 | 4.0 | −2.9 | Aoki et al. (2013) |
| 3740963179636229768 | 207.345 | 14.127 | 1777-53587-0479 | 16.6 | 6340 | 3.8 | −3.5 | Bonifacio et al. (2018) |
| 3976087278282022272 | 175.848 | 20.349 | 2506-54179-0576 | 16.9 | 6426 | 4.0 | −2.9 | Yoon et al. (2016) |
| 389062736936883296 | 157.313 | 17.491 | 2853-54440-0113 | 16.9 | 6047 | 4.7 | −4.0 | Caffau et al. (2011a) |
| 16099144733432192 | 213.030 | 56.159 | 2447-54498-0274 | 16.0 | 6488 | 3.8 | −4.0 | Aoki et al. (2013) |
| 1195572485297534080 | 233.443 | 15.950 | 2782-54592-0411 | 16.9 | 6562 | 4.0 | −4.0 | Caffau et al. (2013) |
| 93127322991970560 | 123.976 | 47.496 | 3693-55208-0408 | 17.1 | 6523 | 4.6 | −4.0 | Aguado et al. (2018b) |
| 254851849254506896 | 5.808 | 3.133 | 4299-55827-0002 | 17.9 | 6404 | 4.0 | −4.0 | Aguado et al. (2018a) |

Note. We indicate SIMBAD references for stars with confirmed $\mathrm{[Fe/H]} < -3.0$ inferences. $T_{\text{eff}}$, $\log g$, and $\mathrm{[Fe/H]}$ are estimated from Sloan spectra with the spectroscopic fitting routine described in Section 3.5.
using theoretical spectra and synthetic photometry. We also validated the classifier using objects securely classified as metal-poor main-sequence stars or white dwarfs based on Sloan spectroscopy and high-quality Gaia DR2 parallaxes. We then applied our classifier to a sample of candidate metal-poor main-sequence stars and white dwarfs with visual classifications and confirmed that our automated approach reproduces the human classifications. We make our classifier and its underlying source code publicly available (see footnote 2) for reproducibility and application to future survey data. For the stars classified as metal-poor main-sequence stars, we executed a search for extremely metal-poor stars. We uncovered some previously unidentified candidates and flagged dozens of already confirmed extremely metal-poor stars. We predict that the application of our classifier to future DESI and SDSS-V spectroscopy will discover thousands of metal-poor main-sequence stars. We further predict that even in the absence of high-quality parallaxes, any candidate low-mass Population III stars identified by their featureless optical spectra can be separated from cool white dwarfs due to their redder optical-to-infrared colors. These red colors are produced by relatively weak collision-induced absorption from molecular hydrogen in the relatively low-surface gravity photospheres of low-mass Population III stars. The methods presented in this work will facilitate the rapid and reliable identification of MPMS stars, providing improved constraints on the uncertain existence of surviving pristine stars from the primordial universe.

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Facilities: Gaia, Sloan.

Software: numpy (Harris et al. 2020), scipy (Virtanen et al. 2020), matplotlib (Hunter 2007), astropy (Robitaille et al. 2013), scikit-learn (Pedregosa et al. 2011).
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