Nonlinear dynamic analysis of dragonfly-inspired piezoelectric unimorph actuated flapping and twisting wing

Sujoy Mukherjee and Ranjan Ganguli

Department of Aerospace Engineering, Indian Institute of Science, Bangalore – 560012, Karnataka, India

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The nonlinear equations for coupled elastic flapping–twisting motion of a dragonfly-inspired smart flapping wing are used for a flapping wing actuated from the root by a PZT unimorph in the piezofan configuration. Excitation by the piezoelectric harmonic force generates only the flap bending motion, which, in turn, induces the elastic twist motion due to interaction between flexural and torsional vibration modes. An unsteady aerodynamic model is used to obtain the aerodynamic forces. Numerical simulations are performed using a wing whose size is the same as the dragonfly Sympetrum frequens wing. It is found that the value of average lift reaches its maximum when the smart flapping wing is excited at a frequency closer to the natural frequency in torsion. Moreover, consideration of the elastic twisting of the flapping wing leads to an increase in the lift force. It is also found that the flapping wing generates sufficient lift to support its own weight and carry a small payload. Therefore, the piezoelectrically actuated smart flapping wing based on the geometry of a Sympetrum frequens wing and undergoing flapping–twisting motions can be considered as a potential candidate for use in micro air vehicle applications.

Keywords: piezofan; nonlinear vibration; coupled flapping–twisting; dragonfly; smart material; micro air vehicle

1. Introduction

Flapping flight has fascinated humans for many centuries and research interest in flapping flight was recently renewed because of the need to develop a class of very small flight vehicles called micro air vehicles (MAVs), which have maximum dimensions of 15 cm and a gross weight of 100 g. MAVs are envisaged for sensing and information gathering in areas such as environmental monitoring and national security. Additionally, they have a large number of potential military and commercial applications, such as outdoor NBC (nuclear–biological–chemical) reconnaissance, crowd control, snap inspection of pollution, road accident documentation, urban traffic management, search for survivors, pipeline inspection, high risk indoor inspection, etc. [1]. In order to perform these tasks, MAVs should be able to fly slowly, maneuver with precision and navigate through complex environments. Nature provides flapping flyers such as birds and insects, which routinely exhibit such performance [2]. These natural flapping flyers present a very successful design for intelligent MAVs with much better performance than conventional wings and rotors [3].
Therefore, birds and insects provide biomimetic inspiration for the development of MAVs. For a flapping-wing MAV, the wings are not only responsible for lift, but also for propulsion and maneuvers. Therefore, MAV flapping wing design represents one of the major challenges to efficient flight in the low Reynolds-number regime [4].

Researchers have developed flapping wing mechanisms which mimic the wing movements of birds [5–8]. Several flapping wing mechanisms which mimic an insect’s wingbeat kinematics have been developed. These mechanisms have greatly helped in increasing the fundamental understanding of flapping flight [9–14]. Unfortunately, these dynamically scaled flapping wing mechanisms are bulky and flap at very low frequency. Therefore, they may not be suitable for use in small or micro-scale flying vehicles. Moreover, current flapping wing mechanisms rely on pneumatic and motor-driven flapping actuators which lead to high weight and system complexity [15]. Additionally, it may not be easy to mimic the complex wingbeat kinematics of the natural flyers with these conventional actuators. Another plausible alternative for developing flapping wings is to use actuators made of smart materials.

Smart materials, especially piezoelectric materials, are widely used in smart structures such as sensors and actuators for various engineering applications [16,17]. Piezoelectrics are attractive for flapping wing mechanisms due to their high bandwidth, high output force, compact size and high power density. Several flapping wing mechanisms have been developed using piezoelectric actuators. Cox et al. [18] developed lead zirconate titanate (PZT)-actuated three flexure-based mechanisms for the electromechanical emulation of mesoscale flapping flight. Sitti [19] developed a flapping wing mechanism based on a compliant four-bar structure. He used an integrated piezoelectric unimorph actuator at the input link for actuation. Syafuddin et al. [15] introduced a flapping wing mechanism actuated by a unimorph piezoceramic actuator called LIPCA (lightweight piezo-composite actuator). In the flapping system, the limited actuation displacement produced by bending motion of the simply supported actuator was transformed into a large flapping angle by using a four-bar linkage system. A passive wing rotation mechanism was also implemented in the flapping system. Nguyen et al. [20] introduced an insect-mimicking flapping-wing system actuated by unimorph piezoceramic actuator LIPCA, where the rotation, corrugation and clapping of insect wings were mimicked. Kim et al. [21] developed a smart flapping wing with a macro-fiber composite (MFC) as a surface actuator to generate the camber motion of a local wing section. The flapping motion was generated using an electric DC motor and a transmission system with a reduction gear system that converts the rotary motion of the motor into the flapping motion. The twisting motion was induced passively by the flexibility of the wing during up and down strokes. In another study, Kim et al. [22] developed a biomimetic flexible flapping wing of a real ornithopter scale by using MFC actuators. They performed aerodynamic tests in a low-speed wind tunnel to investigate the aerodynamic characteristics, particularly the camber effect, the chord-wise flexibility effect and the unsteady effect. Recently, Kummari et al. [23] developed a piezoelectric actuated two-bar, two-flexure motion amplification mechanism for flapping wing micro-aerial vehicle application. They used $\frac{f_r^* A}{A}$ as an optimization criterion for obtaining the best piezoelectric actuation mechanism with the best energy transmission coefficient where $f_r$ is the fundamental resonant frequency of the system and $A$ is the vibration amplitude at the wing tip, or the free tip deflection at quasi-static operation. The above studies show that a mechanism is needed to amplify the displacement obtained from the piezoelectric effect. This is because the piezoelectric effect is intrinsically small and leads to a small displacement when expected directly from the bending piezoelectric unimorph/bimorph [15,24].
A piezoelectric fan (piezofan) is a simple motion amplification mechanism which was first proposed by Toda and Osaka [25]. The piezofan can produce large deflection, especially at resonance. Piezofans are very popular as cooling fans for portable electronic devices such as automobile multimedia boxes, laptop computers, cellular phone, etc. [26,27]. Several researchers have used the piezofan as the actuator for flapping wing mechanisms. Chung et al. [28] placed two coupled piezofans in parallel and applied sinusoidal voltages with a different phase delay between them to control the flapping and twisting motions of the wing. Their experimental investigation showed that the bending amplitude of the wing reduced with increasing phase delay and the twisting movement increased with an increasing phase delay. In another study, Chung et al. [24] proposed a form of design optimization using the Strouhal number and a limited number of material parameters to select the best piezofans for flapping wing MAV applications. They used a linear analytical model to analyze the performance of the piezofan structure at dynamic operation. However, the wings of birds and insects move through a large angle which can be obtained using piezofan through large deflection. Recently, Mukherjee and Ganguli [29] performed a nonlinear dynamic analysis of a flapping wing actuated from the root by a PZT unimorph in the piezofan configuration. They analyzed the large flapping angle of the wing obtained through large deformation. It can be also noted that natural fliers flap their wings in a vertical plane with a change in the pitch of the wings during a flapping cycle. Therefore, a nonlinear dynamic model capturing coupled flapping–pitching motion is needed to perform dynamic analysis. However, Mukherjee and Ganguli [29] did not consider twisting motion of the wing. Mahmoodi and Jalili [30] investigated the nonlinear coupled flexural–torsional vibrations of a microcantilever beam actuated by a piezoelectric patch attached on surface of the beam. The nonlinear terms appeared due to the coupling between the flexural and torsional vibrations as a result of nonlinearity in geometry and the presence of piezoelectric layer.

In order to consider the piezofan for flapping wing applications, it is necessary to analyze its aerodynamic performance. Hence, an aerodynamic model is needed for obtaining the aerodynamic forces. DeLaurier [31] proposed an unsteady aerodynamic model based on modified strip theory. This aerodynamic model makes it possible to estimate the aerodynamic performance of harmonically flapping wings during preliminary design and development [32]. Various aerodynamic effects can be considered in this model, such as camber effect, partial leading edge suction effect, viscous effect, unsteady wake effect and dynamic stall model of pitching motion. Therefore, the DeLaurier [31] model is useful for estimating the lift generated by a flapping wing.

In this paper, nonlinear equations of motion of the coupled flapping–twisting wing are obtained using Hamilton’s principle. A PZT unimorph, in piezofan configuration, is used to actuate the flapping wing from the root. The piezoelectric layer generates flapping motion, which in turn induces twisting motion due to interaction between the flapping and twisting motion. Numerical simulations are performed to analyze the dynamic characteristics of a wing, which has the same size as the wing of the Sympetrum frequens dragonfly. Finally, an unsteady aerodynamic model based on modified strip theory is used to obtain the aerodynamic forces.

2. Structural modeling

Figure 1 shows a schematic diagram of the flapping wing where a piezoelectric unimorph is attached to the uniform flexible wing. Here, the piezoelectric unimorph actuates the wing by supplying a voltage, $P_e(t)$. A schematic diagram of the wing geometry used for the structural modeling is shown in Figure 2. Figure 3 shows the inertial ($x, y, z$) and principal ($\xi$, $\bar{\theta}$, $\zeta$) coordinate systems of the beam cross-section for an arbitrary position $s$. 
The relationship between the principal axes and the inertial axes is described by two Euler angle rotations. The rotation angle $\phi(s,t)$ takes $x$ to $\xi$ and the rotation angle $\psi(s,t)$ takes $z$ to $\zeta$. Therefore, angle $\phi(s,t)$ and $\psi(s,t)$ represent the measure of torsional vibrations and flexural angle between $x$ and $\xi$, respectively. Using Figure 3, angle $\psi(s,t)$ for an element of length $ds$ can be expressed as

$$\psi = \tan^{-1} \frac{v'}{1 + u'},$$

where the prime denotes derivative with respect to the arc length ($s$). The longitudinal and transverse displacement are described by $u(s,t)$ and $v(s,t)$, respectively.

The Green’s strain ($\varepsilon_0$) associated with the material located at the neutral axis is given by [33]

$$\varepsilon_0 = \sqrt{(1 + u'^2) + v'^2} - 1,$$
which is utilized to relate longitudinal and bending vibrations through inextensibility condition which demands no relative elongation of neutral axis. Using Taylor series expansion, Equation (2) reduces to

\[ u' = -\frac{1}{2} v'^2. \] (3)

The piezoelectric layer is not attached on the entire length of the wing. Therefore, the neutral surface, \( y_n \), changes for each section and can be expressed as

\[
y_n = \begin{cases} 
0, & s < l_1 \text{ or } s > l_2 \\
\frac{E_p h_p (h_p + h_b)}{2 (E_p h_p + E_b h_b)}, & l_1 < s < l_2 
\end{cases}
\] (4)

Since the electrical and mechanical phenomena are coupled for piezoelectric materials, the stress–strain relations for the piezoelectric materials can be written

\[
\varepsilon = S^e \sigma + d^T D,
\] (5)

\[
Q = d \sigma + \mu^T D,
\] (6)

where \( S^e_{6 \times 6} \) is the compliance matrix, \( \sigma_{6 \times 1} \) is the stress vector, \( d_{3 \times 6} \) is the matrix of piezoelectric constants, \( D_{3 \times 1} \) is the electrical displacement vector, \( Q_{3 \times 1} \) is the electrical field vector and \( \mu^T_{3 \times 3} \) is the matrix of the dielectric permittivity. In the present study, the electrical displacement is one-dimensional and defined as

\[
D_1 = D_3 = 0, \quad D_2 (t),
\] (7)

\[
\sigma^p_{11} = E_p \varepsilon^p_{11} - E_p d_{31} \frac{P_e (t)}{h_p},
\] (8)

where \( P_e (t) \) is the applied voltage to the piezoelectric material.

The equations of motions are derived using an energy method [30]. The total kinetic energy (\( T \)) of the system can be expressed as

\[
T = \frac{1}{2} \int_0^l \left\{ m (s) \left( \dot{u}^2 + \dot{v}^2 \right) + J_\xi \dot{\phi}^2 + J_\theta \phi^2 \dot{v}^2 + J_\zeta \left( \dot{v}^2 - 2 \dot{v}^2 u' - 2 \dot{v} \dot{u} \dot{v}' - 2 \dot{v}^2 v'^2 \right) \right\} ds,
\] (9)

where

\[
m (s) = w_b \left( \rho_b h_b + (H_{l_1} - H_{l_2}) \rho_p h_p \right),
\]

\[
J_\xi (s) = \frac{1}{12} m (s) \left[ w^2_b + \left( h_p + (H_{l_1} - H_{l_2}) h_p \right)^2 \right],
\]

\[
J_\theta (s) = \frac{1}{12} m (s) w^2_b,
\]

\[
J_\zeta (s) = \frac{1}{12} m (s) \left[ h_b + (H_{l_1} - H_{l_2}) h_p \right]^2.
\] (10)
and

\[ H_{l_1} = H (s - l_1), \quad H_{l_2} = H (s - l_2). \]  

(11)

Similarly, the total potential energy \( (V) \) of the system can be written as

\[
V = \frac{1}{2} \int_{l_1}^{l_2} \int_A \left( \sigma_{11}^b \varepsilon_{11}^b + \sigma_{12}^b \gamma_{12}^b + \sigma_{13}^b \gamma_{13}^b \right) dAds + \frac{1}{2} \int_{l_1}^{l_2} \int_A \left( \sigma_{11}^b \varepsilon_{11}^b + \sigma_{12}^b \gamma_{12}^b + \sigma_{13}^b \gamma_{13}^b \right) dAds
\]

\[+ \frac{1}{2} \int_{l_1}^{l_2} \int_A \left( \sigma_{11}^b \varepsilon_{11}^b \right) dAds + \frac{1}{2} \int_{l_1}^{l_2} \int_A \left( \sigma_{11}^b \varepsilon_{11}^p + \sigma_{12}^b \gamma_{12}^b + \sigma_{13}^b \gamma_{13}^b \right) dAds
\]

\[+ \frac{1}{2} \int_0^{l_1} EA(s) \left( a'' + a' v'^2 + \frac{1}{4} a^2 \right) ds \]  

(12)

where

\[ EA(s) = (H_0 - H_{l_1})E_bw_bh_b + (H_{l_1} - H_{l_2})E_pw_ph_p. \]  

(13)

The following fundamental assumptions are made for the analysis:

(1) The wing is initially straight and it is clamped at one end and free at the other end.

(2) The Euler–Bernoulli beam theories are followed, where shear deformation and rotary inertia terms are negligible.

(3) The wing is inextensible.

(4) The bonding layer is assumed to be perfect.

Considering all the assumptions and applying the extended Hamilton’s principle, governing equations of motion can be obtained as

\[ J_\xi \ddot{\phi} - (C_\xi \phi)' + (C_\theta - C_\xi) \phi v'' + \frac{1}{2} C_c v' \phi P_e (t) = 0, \]  

(14)

\[ m \ddot{v} + (C_\xi v'')' + \left[ v' \int_i^s m \int_0^s (\ddot{v} v' + \ddot{v} v'^2) ds ds \right] + \left[ \left( C_\xi v' v'' \right)' \right] + \left[ \left( C_\theta - C_\xi \right) v'' \phi^2 \right]' \]

\[\frac{1}{2} \left[ \left(C_c \left( \frac{v'^2}{2} + \frac{\dot{\phi}^2}{2} \right) \right)'' - \left[ v' \left( C_c v' \right)' \right]' \right] P_e (t) = \frac{1}{2} C'' \phi P_e (t) \]  

(15)

with boundary conditions

\[ \phi = 0 \text{ at } s = 0, \quad \phi = 0 \text{ at } s = l, \]  

(16)

\[ v = v' = 0 \text{ at } s = 0, \quad v'' = v''' = 0 \text{ at } s = l, \]  

(17)

where
\[ C_\xi (s) = (H_0 - H_{l_1}) G_b I^b_\xi + (H_{l_1} - H_{l_2}) G_b (I^b_\xi + w_b h_b y_n^2) + (H_{l_1} - H_{l_2}) G_b I^b_\xi \]
\[ \quad + (H_{l_2} - H_l) G_b I^b_\xi \]  
\[ (18) \]

\[ C_\theta (s) = (H_0 - H_{l_1}) E_b I^b_\theta + (H_{l_1} - H_{l_2}) E_b I^b_\theta + (H_{l_2} - H_l) E_b I^b_\theta \]  
\[ (19) \]

\[ C_\zeta (s) = (H_0 - H_{l_1}) E_b I^b_\zeta + (H_{l_1} - H_{l_2}) E_b (I^b_\zeta + w_b h_b y_n^2) + (H_{l_2} - H_l) E_b I^b_\zeta \]
\[ \quad + (H_{l_2} - H_l) E_b I^b_\zeta \]  
\[ (20) \]

\[ C_c (s) = (H_{l_1} - H_{l_2}) \frac{w_p}{2} E_p d_{31} (h_p + h - 2y_n) \]  
\[ (21) \]

and

\[ I^b_\xi = w_b h_b^3 \kappa^b_\xi, \quad \kappa^b_\xi = \frac{1}{3} \left[ 1 - \frac{192 h_b}{\pi^5 w_b} \sum_{n=1,3,\ldots}^{\infty} \frac{1}{n^5} \tanh \left( n\pi w_b \right) \right], \]

\[ I^b_\theta = \frac{w_b h_b^3}{12}, \]

\[ I^b_\zeta = \frac{w_b h_b^3}{12}, \]

\[ I^p_\xi = w_p h_p^3 \kappa^p_\xi, \quad \kappa^p_\xi = \frac{1}{3} \left[ 1 - \frac{192 h_p}{\pi^5 w_p} \sum_{n=1,3,\ldots}^{\infty} \frac{1}{n^5} \tanh \left( n\pi w_p \right) \right], \]

\[ I^p_\theta = \frac{w_p h_p^3}{12}, \]

\[ I^p_\zeta = w_p \left[ h_p y_n^2 - (h_p^2 + h_b h_p) y_n + \frac{1}{3} \left( h_p^3 + \frac{3}{2} h_b h_p^2 + \frac{3}{4} h_b^2 h_p \right) \right]. \]
\[ (22) \]

Cubic nonlinear terms of inertia and stiffness appear in the governing equations of motion due to the geometry, which can be attributed to large-amplitude coupled flapping–twisting vibrations of the wing. Moreover, coupling of electrical and mechanical fields introduces quadratic nonlinearities due to piezoelectric effect. The Galerkin approximation is used to produce the ordinary differential equations governing the time functions of equations of motion. Therefore, equations of motion are separated into position and time components as

\[ \phi (s, t) = \sum_{m=1}^{\infty} \phi_m (s, t) = \sum_{m=1}^{\infty} \alpha_m (s) q_m (t), \]
\[ (23) \]

\[ v (s, t) = \sum_{n=1}^{\infty} v_n (s, t) = \sum_{n=1}^{\infty} \beta_n (s) r_n (t), \]
\[ (24) \]
where $\alpha_m$ and $\beta_n$ are the comparison functions which satisfy only geometric boundary conditions of the wing and not necessarily the equations of motion, i.e. Equations (14) to (17), for flap and twist. Both $q_m$ and $r_n$ represent the generalized time-dependent coordinates. The linear mode shapes for the flap and twist are considered to be the following comparison functions because the boundary conditions of the wing are clamped-free. Therefore, $\alpha_m$ and $\beta_n$ can be expressed as

$$\alpha_m (s) = A_m \sin (\gamma_m s), \quad (25)$$

$$\beta_n (s) = B_n \left\{ \cosh (\lambda_n s) - \cos (\lambda_n s) + [\sin (\lambda_n s) - \sinh (\lambda_n s)] \right\} \frac{\cosh (\lambda_n l) + \cos (\lambda_n l)}{\sin (\lambda_n l) + \sinh (\lambda_n l)}, \quad (26)$$

where

$$\gamma_m = \left( 2m - 1 \right) \frac{\pi}{2l}, \quad (27)$$

and $\lambda_n$ are the roots of frequency equation,

$$1 + \cos (\lambda_n l) \cosh (\lambda_n l) = 0. \quad (28)$$

Substituting Equations (25) and (26) into Equations (14) to (17) and taking the inner product of the resulting equations with $\alpha_m(s)$ and $\beta_n(s)$, respectively, yields

$$k_{1mn} \ddot{q}_m + k_{2mn} q_m + k_{3mn} q_m r^2_n + k_{4mn} q_m r_n P_e (t) = 0, \quad (29)$$

$$k_{5mn} \ddot{r}_n + k_{6mn} r_n + k_{7mn} r_n^2 P_e(t) + k_{8mn} r_n^3 + k_{9mn} \left( r_n^2 \dot{r}_n + r_n r_n^2 \right)$$

$$+ k_{10mn} r_n q_m^2 + k_{11mn} q_m^2 P_e(t) = k_{12mn} P_e(t) \quad (30)$$

where

$$k_{1mn} = \int_0^l J_{\xi} (s) \alpha_m^2 (s) \, ds, \quad (31)$$

$$k_{2mn} = \int_0^l \left( -C_{\xi} (s) \alpha_m' (s) \right)' \alpha_m (s) \, ds, \quad (32)$$

$$k_{3mn} = \int_0^l \left( C_{\xi} (s) - C_{\xi} (s) \right) \alpha_m^2 (s) \beta_n'' (s) \, ds, \quad (33)$$

$$k_{4mn} = \frac{1}{2} \int_0^l C_{c} (s) \alpha_m^2 (s) \beta_n'' (s) \, ds, \quad (34)$$

$$k_{5mn} = \int_0^l m (s) \beta_n^2 (s) \, ds, \quad (35)$$

$$k_{6mn} = \int_0^l \beta_n (s) \left( C_{\xi} (s) \beta_n'' (s) \right)'' \, ds, \quad (36)$$
Finally, the nonlinear coupled Equations (29) and (30) are solved numerically. Subsequently, the twist angle and the deflection at the tip of the wing are obtained using Equations (23) and (24), respectively.

3. Aerodynamic modeling

The aerodynamic model is based on the modified strip theory as proposed by DeLaurier [31], in which the aerodynamic forces of the flapping wing are obtained by integrating the sectional aerodynamic forces calculated in each section. The aerodynamic model is a well validated model and several researchers have used it in order to estimate the aerodynamic performance of harmonically flapping wings in the phase of preliminary design and development [32,34]. In this unsteady aerodynamic model, the kinematics for a section of the wing is represented by a plunging velocity $\dot{h}$ and a pitch angle of the chord $\theta$ relative to the free stream velocity, as shown in Figure 4.

The local parameters determining the forces includes the section’s geometry, relative angle of attack at the $\frac{3}{4}$-chord location, pitch rates and the dynamic pressure at the $\frac{1}{4}$-chord location. The aerodynamic forces acting on each section of the wing are divided into the normal force $dN$, and the chordwise force, $dF_X$. The components of the normal force are: (i) $dN_c$, a circulatory force normal to the chord at the $\frac{1}{4}$-chord location and (ii) $dN_a$, an...
apparent-mass force normal to the chord at the $\frac{1}{2}$-chord location. The expressions for the section’s total attached flow normal force is

$$dN = \frac{\rho_{\text{air}}U}{2} C_n c dy + \frac{\rho_{\text{air}} \pi c^2}{4} \dot{\gamma} dy.$$  \hspace{1cm} (43)$$

The components of the chordwise force are: (i) $dT_s$, a chordwise leading edge suction force, (ii) $dD_{\text{camber}}$, a chordwise drag due to camber, and (iii) $dD_f$, a chordwise drag due to skin friction. The expressions for the total chordwise force is

$$dF_X = \eta_s 2\pi \left( \alpha' + \bar{\theta}_a - \frac{c\dot{\theta}}{4U} \right)^2 \frac{\rho_{\text{air}}U}{2} c dy - 2\pi \alpha_0 (\alpha' + \bar{\theta}_a + \bar{\theta}_w) \frac{\rho_{\text{air}}U}{2} c dy$$

$$+ (C_d)_f \frac{\rho_{\text{air}} V_s^2 \alpha}{2} c dy.$$  \hspace{1cm} (44)$$

The whole wing’s instantaneous lift and thrust can be obtained by integrating the segment’s instantaneous lift ($dL$) and thrust ($dT$) along the span:

$$L(t) = 2 \int_{0}^{\frac{\pi}{2}} \cos \gamma dL,$$  \hspace{1cm} (45)$$

$$T(t) = 2 \int_{0}^{\frac{\pi}{2}} dT,$$  \hspace{1cm} (46)$$

where $\gamma(t)$ is the section’s dihedral angle at that instant in the flapping cycle.

The wing’s average lift and thrust are obtained by integrating $L(t)$ and $T(t)$ over the cycle. Integrating with respect to cycle angle, $\phi$, instead of time, $t$, where

$$\phi = \omega t$$  \hspace{1cm} (47)$$

so that the average lift and thrust are expressed as

$$\bar{L} = \frac{1}{2\pi} \int_{0}^{2\pi} L(\phi) d\phi,$$  \hspace{1cm} (48)$$

$$\bar{T} = \frac{1}{2\pi} \int_{0}^{2\pi} T(\phi) d\phi.$$  \hspace{1cm} (49)$$

The complete details of the aerodynamic model are given by DeLaurier [31].

4. Results and discussion

4.1. Model validation

A numerical analysis was carried out for the structure that was used by Mahmoodi and Jalili [30], as shown in Figure 5, to validate the implementation of the dynamic model. Table 1 shows the physical properties used for this analysis. It can be noted from Figure 5 that the
piezoelectric layer only excites the flexural mode of vibration. However, piezoelectric harmonic force induces the torsional vibration mode because there is an interaction between the flexural and torsional vibration, as shown in Equations (29) and (30). This results in coupled flapping–twisting motion of the wing. The numerical simulation for frequency response of coupled flexural–torsional vibrations for the system is shown in Figure 6. It can be seen from Figure 6 that the present result matches well with the result presented in Mahmoodi and Jalili [30]. Therefore, the model implementation is validated and subsequently used for dynamic analysis of dragonfly-inspired smart flapping wing.

4.2. Dragonfly-inspired wing

In this study, the size of the flapping wing is based on the geometry of dragonfly Sympetrum frequens wing [35]. The dragonfly is a very agile flyer and its flight satisfies the entire flight envelope of an MAV. Therefore, flight characteristics of the dragonfly provide inspiration for MAV design and development. We consider that a Mylar flapping wing is actuated from the root by a PZT unimorph in the piezofan configuration, as shown in Figure 1. A harmonic input excitation voltage is used to generate flapping motion. As mentioned above, the flapping motion induces torsion motion of the wing due to coupling between the flapping and elastic twist deflections as seen in Equations (14) and (15). Material and geometric properties pertaining to the smart flapping wing are given in Table 2. These properties are used to perform dynamic analysis of the dragonfly-inspired flapping wing.

Tip deflection of the flapping wing is shown in Figure 7 when actuated at 120 V. The flapping wing is excited at its first natural frequency, which is found to be 24 Hz. It can be noted from Figure 7 that a tip deflection of 52.35 mm is obtained at 120 V. Figure 8
Figure 6. Frequency response of coupled flexural–torsional vibrations.

Table 2. Material and geometric properties of the flapping wing.

| Parameter                  | Wing PZT-5H Layer |
|----------------------------|-------------------|
| Parameter | Value [26] | Parameter | Value [24] |
| Young’s modulus | 4.6 GPa | Young’s modulus | 62 GPa |
| Wing length | 67 mm | Piezoelectric length | 10 mm |
| Width | 6.6 mm | Width | 6.6 mm |
| Poisson’s ratio | 0.44 | Thickness | 200 µm |
| Density | 1240 kKg/m³ | Piezoelectric constant | $-320 \times 10^{-12}$ m/V |
| Thickness | 200 µm | Density | 7800 kKg/m³ |
| Mass | 109.67 mg | Mass | 102.96 mg |

shows the definition of the flapping angle and twist angle. Figure 9 shows the flapping angle variations at 120 V and it is an important measure of the actuation performance. It can be seen from Figure 9 that a flapping angle of $38^\circ$ is obtained at 120 V, 24 Hz. This relatively large deflection clearly justifies the need for a nonlinear analysis. Moreover, the ability to produce large motion using a piezofan at resonance has been reported by several researchers [36,37]. Yao and Uchino [37] experimentally obtained a deflection angle of the piezofan as high as $47^\circ$ at resonance.

As mentioned earlier, twisting motion of the wing is induced by the flapping motion because there is an interaction between flexural and torsional vibrations modes [18,30]. Cox et al. [18] experimentally obtained torsional motion from the single-degree-of-freedom wing root excitation due to dynamical tuning. The twist angle variation observed in the present study is shown in Figure 10 at 120 V at an excitation frequency of 37 Hz, which is the natural frequency in twisting of the smart flapping wing. The twist angle is obtained from the wing deflection at the tip following the procedure as explained schematically in Figure 8(b). It can be seen from Figure 10 that a twist angle of $12^\circ$ is obtained at 120 V, 37 Hz.
Figure 7. Tip deflection of the flapping wing.

Figure 8. Schematic diagram of calculating (a) flapping angle and (b) twisting angle.

Figure 9. Flapping angle variation of the wing.
Next, the smart flapping wing was numerically actuated at different excitation frequencies, which include both the natural frequencies in flapping (24 Hz) and twisting (37 Hz).

4.3. Lift and thrust

Average lift was calculated at each excitation frequency using the unsteady aerodynamic model discussed in Section 3. Figure 11 shows the average lift produced by the smart flapping wing at different excitation frequencies. Figure 11 shows that the value of average lift reaches to its maximum at the excitation frequency of 32 Hz which is closer to the natural frequency in twisting motion. Moreover, a phase difference of $60^\circ$ is obtained between flapping and twisting motion at this excitation frequency. Subsequently, average thrust force at different excitation frequencies are obtained, as shown in Figure 12, which shows that thrust force increases with increase of excitation frequency. It is important to mention here that the calculation of aerodynamic forces is done based on the kinematics pertaining to the wing section at 75% of the wing span.

Selection of the pitch angle of the flapping axis ($\bar{\theta}_a$), as shown in Figure 4, is important for the performance of the flapping wing. Here, $\bar{\theta}_a$ is a fixed angle at which the flapping wing operates. The average lift pertaining to a single wing at different pitch angles of the flapping axis is shown in Figure 13. Similarly, Figure 14 shows the average thrust at different pitch angles of the flapping axis. Both the average lift and thrust are obtained at a flight speed of 3 m/s, which is a typical flight speed for dragonflies. It can be seen from Figure 13 that the lift is maximum at the pitch angle of $7.6^\circ$. However, at this pitch angle, thrust has a negative value, as can be seen from Figure 14. Since average thrust force must be positive to satisfy the condition for cruise flight, therefore the pitch angle value is selected as $6.5^\circ$. After selecting the flapping frequency and the pitch angle of the flapping axis for this particular wing configuration, average lift and thrust are obtained for each smart flapping wing by varying the flight speeds.
Figure 11. Average lift force produced by the smart flapping wing at different excitation frequencies.

Figure 12. Average thrust force produced by the smart flapping wing at different excitation frequencies.

Figure 15 shows the average lift produced by smart flapping wings at different flight speeds. It can be seen from Figure 15 that the smart flapping wing produces maximum lift force of 1.78 g. Moreover, maximum lift force occurs at a flight speed of 2.4 m/s. Next, average thrust force at different flight speeds are shown in Figure 16. The average thrust force is found to be 0.016 N at the flight speed of 2.4 m/s.

In order to study the effect of the elastic twist on the performance of the smart flapping wing, the value of the average lift was obtained without considering the twisting angle.
Figure 13. Average lift force produced by the smart flapping wing at different pitch angles of flapping axis.

Figure 14. Average thrust force produced by the smart flapping wing at different pitch angles of flapping axis.

[29,38]. It is interesting to note that the value of the average lift force obtained in this case is found to be 1.61 g at 120 V. A comparison of the average lift value obtained with and without considering the elastic twist is shown in Figure 17. Figure 17 shows that consideration of the elastic twist yields an increase of average lift value. Similar experimental observations have also been made and reported to the literature by several researchers [39–41]. Yoon et al. [39] clearly showed in their experiment that consideration of the twist angle increases the vertical force values due to the change of the effective wing
area. Therefore, it is necessary to consider the elastic twist for more accurate performance estimation of the smart flapping wing.

Finally, Figure 18 shows the net lift force, obtained by subtracting the total wing weight from the total lift force, when two smart flapping wings are used. It can be seen from Figure 18 that flapping wings can carry a payload of 3.15 g at a flight speed of 2.4 m/s. This net lift may be used to carry several sub-systems, such as power supply unit, control unit, sensory systems, etc, required for autonomous flight.

Before concluding, it is important to mention here that the present numerical investigation is a feasibility study to design a dragonfly-inspired smart flapping wing using piezofan.
The experimental demonstration of the present flapping wing configuration is taken up as a future study.

5. Conclusions

The nonlinear equations of coupled flapping–twisting motion of a dragonfly-inspired smart flapping wing are derived using Hamilton’s principle. The flapping wing is actuated from the root by a PZT unimorph in the piezofan configuration. Dynamic characteristics of the wing, having the same size as dragonfly *Sympetrum frequens* wing, are analyzed using
numerical simulations. An unsteady aerodynamic model is used to obtain the aerodynamic forces. The value of average lift reaches to its maximum at the excitation frequency of 32 Hz which is closer to the natural frequency in twisting motion. The smart flapping wing can generate a lift and thrust force of 1.78 g and 0.016 N, respectively, at a flight speed of 2.4 m/s and 120 V. Additionally, the value of lift force obtained in this case is 10.57% higher than the lift force obtained without considering the elastic twisting. Finally, it is found that the net lift force produced by the flapping wing is 3.15 g which may be used to carry several sub-systems, such as power supply unit, control unit, sensory systems etc., required for autonomous flight. Therefore, the piezoelectrically actuated smart wing based on the geometry of *Sympetrum frequens* wing and undergoing flapping–twisting motion may be considered as a potential candidate for use in MAV applications.

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