Determination of Economic Lot Size between Suppliers and Manufacturers for Imperfect Production System with Probabilistic Demand

S Yuniar¹, R Wangsaputra¹,², A T Sinaga¹

¹Industrial Engineering and Management, Institut Teknologi Bandung, Ganeca Street 10, Bandung, West Java, Indonesia, 40132
²Manufacturing Systems Research Group, Institut Teknologi Bandung, Ganeca Street 10, Bandung, West Java, Indonesia, 40132

suciyuniar7@gmail.com; rwangsa@ti.itb.ac.id; astridtheresia27@gmail.com

Abstract. This study aims to develop a combined economical lot size model between supplier and manufacturer for imperfect production processes with probabilistic demand patterns and constant lead times. The supplier side produces the product within a certain time interval then sent to the manufacturer with a certain amount of lot size. Imperfect supplier production systems are characterized by the probability of defective product (γ). The model decision variables are the lot size of the manufacturer's ordering, supplier lot size, and the reorder point of the manufacturer. The optimal decision variables are obtained by minimizing the total expected cost of the combined costs between the suppliers and the manufacturers borne by both parties. The model is built compared to the transactional partnership model, in which the supplier does not participate in the efficiency of its inventory system. A numerical example is given as an illustration of the JELS model and the transactional partnership model. Sensitivity analysis of the model is done by changing the parameters aimed at analyzing the behavior of the developed model.

1. Introduction

In traditional management systems, inventory problems are viewed only from one aspect of the supplier or the manufacturer. Today, companies often manage inventory by involving outsiders directly related to the inventory system by determining the lot size of orders together. This process of integration of lot size determination can provide benefits to both parties known as Joint Economic Lot Sizing (JELS).

Some research has been done related to the development of this research. Goyal discusses the generic model of JELS on the problem of sub-division of product delivery from vendor to buyer[11]. The inventory of the entire system product that involves inventory on the vendor and inventory on the buyer will be more minimal when compared to the delayed equal-size shipment. Jonrinaldi and Suprayogi developed a combined economical lot size model between suppliers and buyers for the type of product that was deteriorated while in stock[16]. Jonrinaldi and Suprayogi conducted a model comparison considering the deterioration factor and a model that did not consider the deterioration factor[16]. Ben-Daya and Hariga developed a combined lot size model for one supplier and one buyer with deterministic demand with stochastic lead time[5]. Ouyang[19] developed research conducted by Ben-Daya and Hariga[5] where lead time can be controlled with normal distribution. The buyer orders a
product of size $Q$ to the supplier\textsuperscript{[19]}. The supplier produces a product of $nQ$ size with a one-time setup and sends the product to buyers of $Q$ size. Saraswati examines the effect of fluctuating demand on the determination of the size of the production lot and the delivery schedule on an integrated inventory system with the total inventory cost involves the manufacturing and buyer inventory system simultaneously\textsuperscript{[21]}. The search for a lot size production solution with fluctuating demand uses a forward dynamic programming approach. The integration model proposed considers two conditions, namely the condition of unlimited production capacity and limited production capacity. Jauhari developed a combined model of suppliers and buyers with probabilistic demand with equal shipping size. Every lot of orders will be shipped in multiple lot shipping and the supplier will produce the goods in batch production size which is an integer multiple of lot shipping\textsuperscript{[15]}. Ekawati developed a combined lot size model for one supplier and one buyer with demand and lead time is probabilistic\textsuperscript{[10]}. Buyers order goods of size $Q$ and suppliers will produce goods in size $nQ$, but suppliers deliver goods to buyers in size $Q$. This research is essentially similar to the research conducted by Ouyang\textsuperscript{[19]}, the difference is Ekawati\textsuperscript{[10]} develops in demand patterns and lead time buyers where deficiencies are permitted in research. In addition, Ekawati\textsuperscript{[10]} comparing proposed models to 2 (two) partnership systems namely transactional partnerships and cooperative partnerships conducted by Cohen and Roussel\textsuperscript{[8]}.

In previous studies, the JELS model was built on the assumption that demand patterns are constant and are known. Few JELS studies are considering probabilistic requests. A probabilistic demand pattern will affect the form of inventory system management. Given the demand variance, companies must provide safety stocks outside the operating stock to anticipate fluctuations in emerging demand. The demand fluctuations depend heavily on lead time. The demand during lead time is influenced by two things: the demand pattern and the length of lead time. In addition, previous studies have assumed that all goods orders are always accepted into inventory, where the product conditions during the production process are considered perfect, meaning that there are no defective or defective products. Hsu and Hsu develop a combined lot size model with a deterministic demand pattern with constant lead times. Hsu and Hsu assume that the production process is not perfect, where the proportion of defective products on the lot of production is assumed to be uniformly distributed\textsuperscript{[13]}. This study develops a combined lot size modeling for one supplier and one manufacturer for an imperfect production system with probabilistic demand patterns and constant lead times. This research uses 2 (two) reference paper that is research conducted by Ekawati\textsuperscript{[10]} as the basis for JELS model development on the research that will be conducted and research conducted by Hsu and Hsu\textsuperscript{[13]} which examines the determination of production lot size for the imperfect production system where the defect product produced is influenced by the limitations of the equipment and the operator.

2. Mathematical Model

The assumptions used in this research are:

- The rate of demand is probabilistic
- The supplier has a limited production rate and the production rate of the supplier is greater than the demand rate of the manufacturer (P>D)
- Suppliers deliver goods to manufacturers in $Q$ size with unlimited delivery capacity
- The process is free from failure
- Shortage is permitted in the study
- Raw materials arrived at the supplier company in accordance with the agreed time, so that the fulfilment of production needs did not experience obstacles due to lack of raw materials
- Defective products cannot be reworked

Variables and parameters notation used in this research listed in Table 1.

| Notation | Definition | Unit of Measurement |
|----------|------------|---------------------|
| $D$      | Demand rate| Unit/ year          |
| Symbol | Description                                      | Unit/Unitless                  |
|--------|-------------------------------------------------|--------------------------------|
| \( P \) | Production rate                                  | Unit/ year                     |
| \( Q \) | Ordering lot size                                |                                |
| \( nQ \) | Production lot size                              |                                |
| \( N \) | Order frequency (in integer)                     |                                |
| \( r \) | Reorder point                                    |                                |
| \( L \) | Lead time                                       |                                |
| \( \gamma \) | Probability of defective product                 | Unitless                       |
| \( f(\gamma) \) | Probabilistic density function                   |                                |
| \( \sigma \) | Standard deviation of demand                     | Unit/ year                     |
| \( k_b \) | Manufacturers’ shortage cost                     | Rupiah/unit                    |
| \( c_b \) | Manufacturers order cost                         | Rupiah/order                   |
| \( h_i \) | Supplier inventory cost                          | Rupiah/unit/ tahun             |
| \( h_o \) | Manufacture inventory cost                       | Rupiah/unit/ tahun             |
| \( S \) | Setup cost                                       | Rupiah/setup                   |
| \( s_i \) | Supplier setup cost                              | Rupiah/setup                   |
| \( P_i \) | Inspection cost                                  | Rupiah/unit                    |
| \( F_v \) | Supplier shipping cost                           | Rupiah/shipping                |
| \( E(\text{O}_{ka}) \) | Expectation of manufacturing shortage cost       | Rupiah/unit/ tahun             |
| \( E(\text{O}_{oa}) \) | Expectation on manufacturer’s order cost         | Rupiah/order/ tahun            |
| \( E(\text{O}_{ia}) \) | Expectation of supplier’s inventory cost         | Rupiah/unit/ tahun             |
| \( E(\text{O}_{ia}) \) | Expectation of manufacturer’s inventory cost     | Rupiah/unit/ tahun             |
| \( E(\text{O}_{ia}) \) | Expectation of supplier setup cost               | Rupiah/setup/ tahun            |
| \( E(\text{O}_{na}) \) | Expectation on inspection cost                   | Rupiah/unit/ tahun             |
| \( E(\text{O}_{na}) \) | Expectation of supplier shipping cost            | Rupiah/shipping/ tahun         |
| \( ETC_v \) | Total supplier cost                              | Rupiah/ tahun                  |
| \( ETC_o \) | Total manufacturer cost                          | Rupiah/ tahun                  |
| \( ETC_{Gab} \) | Total combined cost                              | Rupiah/ tahun                  |

### 2.1. Manufacturer

When the inventory reaches the reorder point \( r \), the manufacturer will place an order for the supplier in size \( Q \). The ordered goods will arrive based on lead time \( L \) and added to the inventory system. \( SS \) is the safety stock level supplied by the manufacturer to overcome the demand fluctuation during the \( L \) time. The goods of \( Q \) will decrease at the rate of \( D \) and will run out within a cycle of ordering \( Q / D \) where \( D \) is the monthly demand rate coming to the manufacturer. Thus in one cycle there is \( D / Q \) order cycle.

![Manufacturer inventory cycle in JELS](image)

Figure 1. Manufacturer inventory cycle in JELS

### 2.2 Supplier

To meet the demand of the manufacturer, the supplier produces the product within a certain time interval and then sent to the manufacturer with a certain amount of lot size. The imperfect supplier production process is characterized by the number of defective products with defective product.

\[
\text{Expectation of manufacturer inventory cost} = h_b \times \left( \frac{Q}{2} + r - DL \right) \quad (1)
\]

\[
\text{Expectation of manufacturer order cost} = \frac{D}{Q} \quad (2)
\]

\[
\text{Expectation of manufacturer shortage cost} = \frac{D}{Q} k_b B(r) \quad (3)
\]

\[
\text{Expectation of total manufacturer cost} = c_b \frac{D}{Q} + h_b \left( \frac{Q}{2} + r - DL \right) + \frac{D}{Q} k_b B(r) \quad (4)
\]
probability (γ) uniformly distributed. The defective product cannot be fixed so it becomes scrap. However, in this study, manufacturers receive products from suppliers in a state free of defects.

Based on Figure 2, the supplier accepts the order from the manufacturer according to the manufacturer's order size Q. The order size Q will be produced in one setup with the production rate P so that the goods will be finished in time Q/P. With the defective product (γ) detected during inspection, the production rate becomes P′ = P (1 - γ), so the goods will be finished in time Q/P(1-γ).

The assumption of a constant and known lead time, the supplier sends a product of size Q. If there is a process of repeating the ordering cycle by the manufacturer, then the production process will recur every Q/D. To ensure that the supplier has sufficient production capacity to meet manufacturers' demand, it is assumed that the maximum value γ is less than (1 - D/P). The probability of a defective product is uniformly distributed where the probability of the defective product being produced is likely to arise with the same value.

\[
f(x, a, b) = \begin{cases} 
\frac{1}{b - a}, & a \leq x \leq b \\
0, & \text{other } x 
\end{cases}
\]

Expectation of x value is:

\[
E(x) = \int_{a}^{b} \frac{1}{b - a} \, dx
\]

Expectation of supplier inventory cost

\[
= h_v \left( \frac{Q}{2} (n - 1) \left( 1 - \frac{D}{P(1-\gamma)} \right) + \frac{D}{P(1-\gamma)} \right) + \left[ \frac{nQ\gamma}{P(1-\gamma)} \right]
\]

Expectation of supplier inspection cost

\[
= P_v nQ
\]

Expectation of supplier setup cost

\[
= s_v \frac{nQ}{D}
\]

Expectation of supplier transportation cost

\[
= F_v \frac{nQ}{D}
\]

Expectation of supplier total cost

\[
= h_v \left( \frac{Q}{2} (n - 1) \left( 1 - \frac{D}{P(1-\gamma)} \right) + \frac{D}{P(1-\gamma)} \right) + \left[ \frac{nQ\gamma}{P(1-\gamma)} \right] \left( \frac{Q}{2} (n - 1) \left( 1 - \frac{D}{P(1-\gamma)} \right) + \frac{D}{P(1-\gamma)} \right) + \left[ \frac{nQ\gamma}{P(1-\gamma)} \right]
\]

3. Model Analysis

The analysis is performed to determine the value of Q*, r*, n* for combined lot size between the manufacturer and the manufacturer by adjusting the total expected combined cost of (ETC_Gab). The total expected combined cost is derived from the summation of the total expected cost of the manufacturer with the expected total cost of the supplier.

\[
ETC_{Gab} = c_b \frac{D}{Q} + h_b \left( \frac{Q}{2} + r - D_t \right) + \frac{D}{Q} k_b B(r) + P_v nQ + s_v \frac{D}{nQ} + F_v \frac{D}{Q}
\]

+ \left[ \frac{nQ\gamma}{P(1-\gamma)} \right] \left( \frac{Q}{2} (n - 1) \left( 1 - \frac{D}{P(1-\gamma)} \right) + \frac{D}{P(1-\gamma)} \right) + \left[ \frac{nQ\gamma}{P(1-\gamma)} \right]
\]
To determine the value of $Q^*$, it must meet the following conditions:

$$\frac{\partial ETC_{Gab}(Q,r,n)}{\partial Q} = 0$$  \hspace{1cm} (13)

To determine the value of $r^*$, it must meet the following conditions:

$$\frac{\partial ETC_{Gab}(Q,r,n)}{\partial r} = 0$$  \hspace{1cm} (14)

And to meet the minimum criteria, must meet the following conditions:

$$\frac{\partial ETC_{Gab}^2(Q,r,n)}{\partial Q^2} > 0$$  \hspace{1cm} (15)
$$\frac{\partial ETC_{Gab}^2(Q,r,n)}{\partial r^2} > 0$$  \hspace{1cm} (16)

Iteration solution research model is as follows:

1) Apply $n = 1, i = 1$, dan $ETC_{Gab}(Q_{n-1}, r_{n-1}, 0) = \infty$
2) Calculate the value of $Q_{n-1}$ by using equation (17) for $r_{n-1} = \infty$

$$Q_{n-1} = \frac{2D \left( c_b + F_v + \frac{s_v}{n} \right)}{h_b + h_v \left[ (n - 1) \left( 1 - \frac{D}{P(1 - \gamma)} \right) + \frac{D}{P(1 - \gamma)} + \left( \frac{ny}{P(1 - \gamma)} \right) \right] + 2P_l n}$$  \hspace{1cm} (17)

3) Calculate $r_{n-1}$ that completed

$$\alpha_{n,i} = \frac{h_b Q_{n-1}}{k_b D}$$

So that,

$$r_{n,i} = Z_{a_{n,i}} \sigma \sqrt{L} + DL$$  \hspace{1cm} (18)
$$ss_{n,i} = r_{n,i} - DL$$  \hspace{1cm} (19)

4) Calculate $Q_{n,i}$ by using equation (20).

$$Q_{n,i} = \frac{2D \left( c_b + F_v + \frac{s_v}{n} + k_b B(r_{n,i}) \right)}{h_b + h_v \left[ (n - 1) \left( 1 - \frac{D}{P(1 - \gamma)} \right) + \frac{D}{P(1 - \gamma)} + \left( \frac{ny}{P(1 - \gamma)} \right) \right] + 2P_l n}$$  \hspace{1cm} (20)

5) Calculate $ETC_{Gab}(Q_{n,i}^*, r_{n,i}^*, n)$.
6) Add $i = i + 1$, then calculate the value of $\alpha_{n,(i+1)}$ by using equation (21). Then calculate the value of $r_{n,(i+1)}$ and $ss_{n,(i+1)}$ by using equation (22) and (23).

$$\alpha_{n,(i+1)} = \frac{h_b Q_{n,(i+1)}^*}{k_b D}$$  \hspace{1cm} (21)

So that,

$$r_{n,(i+1)} = Z_{a_{n,(i+1)}} \sigma \sqrt{L} + DL$$  \hspace{1cm} (22)
$$ss_{n,(i+1)} = r_{n,(i+1)}^* - DL$$  \hspace{1cm} (23)

7) Calculate $ETC_{Gab}(Q_{n,(i+1)}^*, r_{n,(i+1)}^*, n)$. Ifs:

- $ETC_{Gab}(Q_{n,(i+1)}^*, r_{n,(i+1)}^*, n) > ETC_{Gab}(Q_{n,i}^*, r_{n,i}^*, n)$, then the optimal solution is $n^* = n, Q^* = Q_{n,i}$, dan $r^* = r_{n,i}$.

- $ETC_{Gab}(Q_{n,(i+1)}^*, r_{n,(i+1)}^*, n) < ETC_{Gab}(Q_{n,i}^*, r_{n,i}^*, n)$, then apply $n^* = n + 1$ and back to step (3) by replacing the value of $Q_{n-1}$ with $Q_{n,(i+1)}$ which has been calculated before.

Do each step to get the minimum $ETC_{Gab}$ value.

\vspace{1cm}

5
4. Numerical Example

**Table 2. Numerical Example**

| Notation | Value   | Measure               |
|----------|---------|-----------------------|
| D        | 500,000 | Unit/year             |
| P        | 900,000 | Unit/year             |
| s_i      | 100,000 | Rupiah/setup          |
| h_i      | 1500    | Rupiah/unit/year      |
| P_i      | 1000    | Rupiah/unit           |
| F_i      | 100,000 | Rupiah/shipping       |
| L        | 0.02    | Year                  |
| c_b      | 15,000  | Rupiah/order          |
| h_b      | 2000    | Rupiah/unit/year      |
| $\sigma$| 50,000  | Unit/year             |
| k_b      | 5,500   | Rupiah/unit           |

The search results for Q *, r *, and n * values for model solutions using joint economic lot sizing (JELS) can be seen in Table 3.

**Table 3. Result of Calculation of JELS Model Inventory Policy**

| Manufacturing Lot size (Q) | 19,229.7 Unit |
|----------------------------|---------------|
| Manufacturing reorder point level (r) | 26,820.6 Unit |
| N factor                    | 2             |
| Supplier’s lot size production | 38,459.4 Unit |
| Total cost (ETC(Q, r, n))   | Rp.106,822,810.9/year |

In comparison, this numerical example also presents an optimal solution for suppliers and manufacturers in the absence of labour in inventory management, a pattern of partnership called a transactional pattern. Mathematical model without cooperation in inventory system management can be seen in appendix. Full results can be seen on Table 4.

**Table 4. Calculation Result of Inventory Policy without Cooperation**

| Manufacturing Lot size (Q) | 23,547,127 Unit |
|----------------------------|-----------------|
| Manufacturing reorder point level (r) | 25,716,4488 Unit |
| Supplier’s lot size production | 23,547,127 Unit |
| Total cost (ETC(Q, r, n))   | Rp. 117,381,977.1/year |

Based on the above calculation, it can be seen that the inventory policy with JELS model is smaller than the model without any cooperation. This is because in the decision-making obtained from the results of mutual decisions from both parties.

5. Sensitivity Analysis

Sensitivity analysis is done by changing parameters in joint economic lot sizing model developed. This parameter change aims to analyze the behavior of the developed model, both in terms of decision variables and performance generated by the model. Changes in parameters made are changes in the manufacturer's message-cost parameters, manufacturer's shipping cost, manufacturing shortage cost, supplier checking fee, supplier's store cost, supplier setup cost, supplier transportation cost, demand rate, probability of supplier defective product (\(\gamma\)), and length of lead time. The magnitude of the parameter value change is 10%, 25%, and 50% of the parameter value in the numerical example.
Based on Figure 3, it can be seen that the parameters that are sensitive to the optimal lot lot size ($Q^*$) are the demand rate ($D$), supplier's store cost ($h_s$), and the manufacturing shelf cost ($h_b$). According to Figure 3, the greater the change in the cost parameters of the supplier's store cost ($h_s$) and the manufacturing shelf cost ($h_b$) the smaller the lot size of order ($Q$). Unlike the change of demand rate parameter ($D$) to order lot size ($Q$), the greater the demand rate parameter ($D$) parameter, the larger the order lot size ($Q$).

Based on Figure 4, it can be seen that the parameters that are sensitive to the optimal reorder point ($r^*$) are the delivery lead time ($L$), the demand ($D$) and the standard deviation ($\sigma$). The greater the parameter of the delivery lead time ($L$), the demand rate ($D$) and the standard deviation ($\sigma$), the greater the reorder point ($r$).

6. Conclusion
This study develops a combined economical lot size model between supplier and manufacturer for an imperfect production system with probabilistic demand patterns. The supplier side produces the product within a certain time interval then sent to the manufacturer with a certain amount of lot size. Incomplete supplier production systems are characterized by the probability of defective product ($\gamma$). The model decision variables are the lot size of the manufacturer's ordering, supplier lot size, and the reorder point of the manufacturer. The optimal decision variable is obtained by minimizing the total expected cost of the combined costs between the supplier and the manufacturer that is borne by both parties. The model is built compared to the transactional partnership model, in which the supplier does not participate in the efficiency of its inventory system. The comparison results show that JELS model gives more minimum cost compared with transactional model.
To see the effect of parameters that exist in the model to optimal lot size, then the sensitivity analysis is done. The parameters that are sensitive to optimal order lot size ($Q^*$) are demand rate ($D$), supplier's store cost ($h_v$), and manufacturer's shelf cost ($h_s$). The parameters that are sensitive to the optimal reorder point ($r^*$) are the delivery lead time parameter $L$, the demand rate ($D$) and the standard deviation ($\sigma$).

This research is conducted using a supply chain system consisting of one supplier and one manufacturer so that it can be developed by creating a model involving multiple suppliers and some manufacturers. In addition, in the study developed models with production systems that are not perfect on the supplier resulting in defective products. However, it is assumed that the defective product cannot be reworked and becomes scrap. So that can be developed for further research with the assumption that the defective product can be done rework.

**Acknowledgement**

We would like to express our gratitude to the ITB internal research grant and Manufacturing Systems Research Group-ITB for the financial support to the research.

**Appendix**

The mathematical model of economical lot size in the pattern of transactional partnership is as follows:

1. Manufacturer

   The manufacturer will place the order of goods to the supplier in size $Q$ when the inventory reaches the reorder point level. The ordered item will arrive in lead time ($L$) and added to the inventory system at once. $SS$ is the level of safety stock supplied by the manufacturer to overcome the demand fluctuations over time $L$. The goods of $Q$ will decrease with the demand rate per year ($D$) and will be exhausted within a cycle of ordering of $Q/D$. Thus within one year there is $D/Q$ order cycle. Manufacturing inventory systems in transactional partnerships are similar to inventory systems on JELS models.

2. Supplier

   In the transactional partnership pattern, the supplier receives an order from the manufacturer then manufactures it according to the manufacturer's ordering size $Q$. The order size $Q$ is produced in one setup at the production rate $P$ so that the product will be completed in $Q/P$ time. Then the supplier sends the finished product to the manufacturer in $Q$ size and if there is a repeat of the ordering cycle by the manufacturer, then the production process will repeat every $Q/D$. Supplier supply systems on transactional partnerships can be seen in Figure 5.

   ![Figure 5 Supplier Inventory System In Transactional Partnership](image)

   With the defective product ($\gamma$) detected during inspection, the production rate becomes $P' = P(1-\gamma)$, so that the goods will be finished in $Q/P'(1-\gamma))$. To ensure that the supplier has sufficient production capacity to meet manufacturers' demand, it is assumed that the maximum value $\gamma$ is less than $(1 - D/P)$.

\begin{itemize}
  \item Expectation of supplier inventory cost = $h_v \left( \left[ \frac{DQ}{2P(1-\gamma)} \right] + \left[ \frac{Q\gamma}{P(1-\gamma)} \right] \right)$
  \item Expectation of supplier setup cost = $s_v \frac{D}{Q}$
  \item Expectation of supplier transportation cost = $F_v \frac{D}{Q}$
  \item Expectation of supplier inspection cost = $P_iQ$
\end{itemize}
• Expectation of supplier total cost

\[ h_v \left( \frac{DQ}{2P(1-\gamma)} + \frac{Q\gamma}{P(1-\gamma)} \right) + s_v \frac{D}{Q} + F_v \frac{D}{Q} + P_iQ \]  

(28)

References

[1] Bahagia, S.N., (2006), Sistem Inventori, Penerbit ITB, Bandung.
[2] Banarjee, A., (1986), A joint economic-lot size for purchaser and vendor, Decision Science 185:292-311.
[3] Ben-Daya M., Darwish M., Ertogral K., (2008), The joint economic lot sizing problem: review and extensions, Eur J Oper Res 185 (2):726–742.
[4] Ben-Daya, M., dan Hariga, M., (2000), Economic lot scheduling problem with imperfect production processes. Journal of the Operational Research Society, 51, 875–881.
[5] Ben-Daya, M., and Hariga, M., (2004), Integrated Single Vendor Single Buyer Model with Stochastic Demand and Variable Lead Time, International Journal of Production Economic, Vol 92, pp. 75-80.
[6] Buchan, Joseph, dan Koenigsberg, E, Scientific Inventory Management, Prentice Hall, Englewood Cliffs, NJ, 1963.
[7] Buffa, Elwood S. dan Miller, J.F., Production-Inventory System:Planning and Control, 3nd, Richard D. Irwin Inc. 1979.
[8] Cohen, S., dan Roussel J., (2005), Strategic Supply Chain Management: The Five Disciplines for Top Performance, McGraw-Hill.
[9] Daellenbach, H.G., (1995), System and Decision Making, John Wiley & Sons, Inggris.
[10] Ekawati, I., Iskandar, B.P., Suprayogi, Cakravastia, A., (2010), Model penentuan ukuran lot gabungan ekonomis pemasok dan pemanufaktur dengan pola permintaan dan lead time probabilistik, Tesis S-2, Teknik dan Manajemen Industri, ITB.
[11] Goyal, SK., (1988), A joint economic-lot-size model for purchaser and vendor: a comment, Decis Sci 19(1): 236–241.
[12] Hadley, G., Whitin, T. M., (1963), Analysis of Inventory System, Prentice-Hall Inc, Englewood Cliffs, New Jersey.
[13] Hsu, J.T., dan Hsu, L.F., (2013), An integrated vendor-buyer cooperative inventory model for items with imperfect quality and shortage backordering. Advances in Decision Sciences, Article ID 679083, 19.
[14] Hsu, J.T., dan Hsu, L.F., (2015), Economic production quantity (EPQ) models under an imperfect production process with shortages backordered, International Journal of Systems Science, Department of Management, Zicklin School of Business, Baruch College, The City University of New York, New York, NY, USA.
[15] Jauhari, W.A., Pujawan, I.N., Wiratno, S.E., (2009), Model joint economic lot sizing kasus pemasok-pembeli dengan permintaan probabilistik, Jurnal Teknik Industri, Vol 11 no 1:1-14.
[16] Jonrinaldi dan Suprayogi, (2003), Model penentuan ukuran lot ekonomis gabungan antara pemasok dan pembeli untuk produk yang mengalami deteriorasi, Seminar Nasional Perencanaan Sistem Industri 2013 (SNPSI 2003).
[17] McLeavy, D.W., Narasimhan S.L., Production Planning and Inventory Control, Allyn and Bacon, Boston, MA, 1985.
[18] Monden, Yasuhiro, Toyota Production System: Practica Approach to Production Management, Institute of Industrial Engineers, Atlanta, GA, 1983.
[19] Ouyang, L.Y., Wu, K. S., dan Ho, C. H., 2004, Integrated Vendor-Buyer Cooperative Models with Stochastic Demand in Controllable Lead Time, International Journal of Production Economics, Vol. 92, pp. 255-266.
[20] Prasetyo, H., (2004), Model ukuran lot untuk proses produksi yang mengalami penurunan kinerja dengan pola permintaan berfluktuasi, Tesis S-2, Teknik dan Manajemen Industri, ITB.
[21] Saraswati, D., Halim, A.H., Iskandar, B.P., Cakravastia, A., (2009), Model kemitraan kooperasi antara pemasok dan pemanufaktur dalam penentuan ukuran lot gabungan. Proceedings of Joint Seminar Japan-Indonesia Seminar on Technology Transfer (JITT) & National Seminar on Industrial System Planning 2008 (SNPSI).

[22] Salameh, M.K., dan Jaber, M.Y., (2000), Economic production quantity model for items with imperfect quality, International Journal of Production Economics, 64(1), 59–64.

[23] Shimci-Levi, D., Kamisnksy, P., dan Simchi-Levi, E., (2003), Designing and Managing The Supply Chain: Concept, Strategy, and Case Studies, Second Edition, McGraw-Hill, New Jersey.

[24] Tersine, R.J., (1994), Principle of Inventory and Material Management, Prentice Hall Int. Ed.4

[25] Walpole, R.E., Myers, R.H., Myers, S.L., Ye, K., (2002), Probability and Statistics for Engineers and Scientists, Seventh Edition, Prentice-Hall, Inc., New Jersey.