On a theorem of impulse and energy to determine parameters of the electric jet engine KA

Nikolay Sidnyaev
Moscow State Technical University Bauman, Moscow, ul. Baumanskaya 2-ya, 5, Russia
E-mail: sidnyaev@yandex.ru

Abstract. The report presents the modern physical principles of acceleration of charged particles and plasma in electric and magnetic fields, which are the basis for the classification of electric rocket engines. It is shown that for reactive means of correction of orbit parameters the lifetime depends to a greater extent on the reserves of the working fluid and the corresponding thrust and energy for the propulsion system. It is postulated that the special value of the determination of pulses and energy for electric propulsion (ERD) is that their application makes it possible to obtain an idea of these phenomena on the boundary surface of a certain area, without knowledge of the individual phenomena occurring within the ERD. That is, often in cases where the differential equations of plasma flow can not be composed or at least can not be integrated, the theory of pulses or energy makes it possible to know the nature of the movement in its General features. It is convincingly proved that the theorems of impulses and energy are a convenient means for checking the correctness of the results, including experimental ones.

1. Introduction
A new, rapidly developing branch of electric propulsion is the space engine industry, which combines the design, development and manufacture of space propulsion systems. Electric propulsion systems (EDU) are commonly referred to as propulsion systems of spacecraft (SC) designed to perform ballistic maneuver in space flight [1,2]. During spacecraft flights on orbits of satellites of celestial bodies or on interplanetary trajectories creation of necessary control forces and moments for the purpose of preservation of parameters of orbits (trajectories) and the set position of the satellite in space. A characteristic element of the EDU is the engine, in which the input energy is converted into kinetic energy of the expiring working substance, and the reaction force arising at the expiration is directly the driving force (thrust force) [3]. The possible values of the effective flow rate in chemical engines are limited both by the relatively low energy of chemical bonds and by the high molecular weight of combustion products. The achievable effective flow rates in chemical engines do not exceed 4.5 - 5.0 km/s. To date, electric rocket engines have been used in orientation and correction systems of various spacecraft [4-6]. Both in Russia and abroad have repeatedly conducted flight and space tests of different classes of ERD, on a number of space objects ERD performed operational functions. It provides for the use of EDR in advanced propulsion systems. For a number of practically important tasks ERD essentially have no competitors [7-10]. Their main advantages, confirmed in the process of research and development, are as follows: high specific impulse, large resource, the possibility of a large number of inclusions up to 100 - 120 and above, extremely small single impulse up to 10 DIN-s, etc. The level of ERD characteristics achieved so far is not the maximum possible, work
on further more in-depth study of accelerators and engines is continuing [8]. At the new stage of research, we can expect further success: the expansion of the range of stable operation of accelerators with high efficiency and low vibration, the emergence of new types of accelerators with fundamentally important advantages [11,12].

2. Problem statement. Method of research

The test program of a pilot electric rocket engine usually includes an experimental determination of the grid of its orbital characteristics over the entire range of flight conditions [7]. The results of such an experiment are necessary for the analysis of satellite flight data, since the calculated orbital characteristics of the engine used in the early stages of satellite development do not always fully reflect the actual impact of flight conditions. Such, often very significant, flight factors as the change in density in all elements of the flow part, the violation of geometric similarity due to thermal and elastic deformations of parts, the change in the actual characteristics of the control system and the limitation of modes and a number of others, are not fully taken into account in the calculations, even if the mathematical model used is identified by the results of the earth bench tests of the engine [2]. The article deals with the problem of choosing the design characteristics of the spacecraft (SC), which has in its composition an electric propulsion system for correcting the orbit parameters. Mathematical models are considered to describe the relationship of environmental factors, design parameters of on-Board systems (including erdu) and design characteristics of the low-orbit SPACECRAFT as a whole. In modern electric propulsion engines (EDU), the jet principle of creating a driving force is used. A characteristic element of the EDU is the engine, in which the input energy is converted into kinetic energy of the expiring working substance, and the reaction force arising at the expiration is directly the driving force (thrust force). Propulsion systems with chemical and gas jet engines are widely used in modern SPACECRAFT. In chemical engines, the fuel stored on Board is both a source of energy and a source of working substance, and there are no special systems for converting and dissipating energy. The working process of chemical engines consists of two main stages: first, in the combustion chamber, the chemical energy of the fuel is converted into thermal energy of gaseous combustion products, and then in the nozzle, the thermal energy passes into the kinetic energy of the jet of gases. The possible values of the effective flow rate in chemical engines are limited both by the relatively low energy of chemical bonds and by the high molecular weight of combustion products. The carried out development and flight and space tests revealed a number of advantages of ERD in comparison with engines of other classes used for solving similar problems (see Fig.1), for example, in comparison with gas reactive systems and micro-LRE. For a number of practically important tasks ERD essentially have no competitors [1].

![Figure 1. The principle of creating thrust ERD](image-url)
3. The impetus for the steady motion of ERD

The special value of the definition of impulses and energy for ERD is that their application makes it possible to get an idea about these phenomena only their knowledge of the state on the boundary surface of a certain area, without knowing separately the phenomena occurring within the ERD, without understanding the "mechanism" of the phenomenon. That is, often in cases where the differential equations of plasma flow can not be composed or at least can not be integrated, the theory of pulses or energy makes it possible to know the nature of the movement in its General features, without going into the details of the phenomenon. On the other hand, the momentum and energy theorems are a convenient means to verify the correctness of the results, including experimental ones. The theorem of impulses is of practical importance only for the steady-state phenomena of motion or for the "average" steady-state movements, i.e. for the such vortex and seemingly irregular movements, which allow us to notice a steady main motion of the plasma. While the momentum theorem can be applied to phenomena in which energy loss occurs due to the flow of the anode grid - for the energy theorem, this is impossible, since here the thermal energy formed as a result of the influence of the magnetic field would remain as unknown, so that the applied theorem would no longer give the opportunity to draw conclusions about But with unsteady movements, the energy theorem in some cases makes it possible to draw conclusions about the nature of the movement; its application to steady movements always leads to trivial results. Let us first give the first conclusion, we proceed, therefore, from the theorem of General mechanics on the amount of motion of the system: referred to the unit of time of pulses or quantities of motion of a \((\sum m\omega)\) limited system of material points is the sum of forces acting on this system from the outside, i.e.

\[
\frac{d}{dt} (\sum m\omega) = \sum P,
\]

where \(\sum P\) means the sum of all external forces, i.e. such forces with which the masses of the system are not belonging to the system (internal forces, i.e. those forces with which the masses belonging to the ERD act on each other, are mutually destroyed according to the principle of equality of action and counteraction. In the transition from the system of material points to the plasma, considered as a continuum, the sum \(\sum m\omega\) passes into the integral \(\int \omega dm = J\), and, therefore, in this case we will have

\[
\frac{dJ}{dt} = \frac{d}{dt} \int \omega dm = \sum P.
\]

Consider a certain amount of plasma limited by the plasma surface. The time change \(t_1\) in the amount of motion of this limited plasma occurs because, on the one hand, the velocities within the region under consideration can change and, on the other hand, the limiting plasma surface moves. \(F\) If the movement is steady, hence each particle is replaced within the element of time in a fixed location particle plasma with the same velocity, the time variation of momentum of the mass of the plasma bounded by a surface \(F\), will be only in the change of momentum caused by the displacement of the plasma surface.

Take a stationary surface in space. Let it coincide with the plasma surface at any time \(t_1\). Then when you move the plasma surface \(F\) of the shift pulses through fixed in the space of the surface \(F\). Let it \(AA\) mean a part of a stationary surface \(BB\) in space, and – a part of a plasma surface at a time \(t_1 + dt\). If the external normals of closed surfaces are considered positive, then the volume passing into the element of time \(dt\) through the elementary platform \(dF\) of a fixed surface \(AA\) in space is equal \(dF \circ \omega dt\), and, consequently, the amount of motion transferred per unit time through the elementary platform \(dF\) is equal \(\rho dF \circ \omega \omega\). Thus, the change in the unit time of the total pulse caused by the movement of the liquid bounding surface is equal to the
resulting pulses passing per unit time through the fixed in space bounding surface, ie.

\[
\frac{dJ}{dt} = \iint \rho dF \circ \omega, \\
\frac{dJ_x}{dt} = \iint \rho dF |\omega| \cos(n, \omega)u, \\
\frac{dJ_y}{dt} = \iint \rho dF |\omega| \cos(n, \omega)v, \\
\frac{dJ_z}{dt} = \iint \rho dF |\omega| \cos(n, \omega)w,
\]

where it \( n \) means outward positive normal. If, in particular, \( dF \) perpendicular to the direction \( x \), the displaced mass \( \rho dF u \) will be, and if \( dF \) perpendicular to the direction, \( y \) the displaced mass will be \( \rho dF v \), and

\[
\frac{dJ_x}{dt} = \iint \rho dF uv.
\]

A closed stationary surface in space will be used as a reference surface. Thus, a theorem was obtained showing that with the steady-state motion phenomenon, the second change in the total impulse of the plasma quantity under consideration is equal to the flow of impulses through the control surface.

The right side of equality, i.e. the sum of all external forces is formed under the assumption of an ideal plasma - from:

1. pressure forces along the control surface \( \iint \rho dF \) (vector integration; the component \( x \) is equal to: \( -\iint \rho dF \cos(n, x) \), etc.). The negative sign is set because the pressure refers to the inward force acting on the reference surface, i.e. in the direction opposite to the outer normal, considered positive.

2. mass forces, in particular gravity \( \iiint \rho gdV \).

3. extraneous forces, i.e. forces acting from the outside on the catchers and the anode grid, which are in the plasma: \( \sum P_f \). The fact is that in a limited part of the plasma there may be bodies, for example, bodies that, with their fixing devices, protrude from a limited part electractor engine, and these bodies act outside the electractor engine, through the devices for holding them, forces act. The surface pressure integral \( \rho \) can also be extended to the surface of foreign bodies, and in this case, especially foreign bodies should not be taken into account. However, it is often more convenient to deal directly with outsiders. So, for the pulse method we get the following expression:

\[
\iint \rho dF \circ \omega = \iiint \rho gdV - \iint \rho dF + \sum P_f, \iint \rho dF \circ \omega + \iint \rho dF = \sum P_f.
\]

If we proceed from the theorem of areas of the mechanics of a system of material points, according to which the change in angular momentum relative to some point or axis relative to a unit of time is equal to the resulting moment of forces, i.e. the corresponding momentum theorem is obtained:

\[
\frac{d}{dt} \left( \sum mr \times \omega \right) = \sum r \times P, \iint \rho dF \circ \omega r \times \omega = \iiint \rho r \times gdV - \iiint pr \times dF + \sum r \times P_f.
\]

4. Findings

The design of several types of engines is based on the interaction between the electric and magnetic fields, which allows you to create a thrust at a high level. In the West, the main research went in the direction of the magnetnoplan-dynamic (MTD) engine. In the former USSR engines based on the Hall effect were developed. The technology of their creation was handed over to the United States - in the framework of a joint program. Relatively cheap installations were used in many satellites. An external radial magnetic field is necessary for the operation of such an engine. Inside it there is a constant axial electrical discharge in xenon, which is used as
fuel and is under low pressure. As a result of the interaction between axial and radial fields, a Hall current arises, perpendicular to the electric field and the azimuthal field inside the engine. Hall current interacts with the magnetic field responsible for the axial acceleration of the fuel. The performance depends on the size of the engine. The stated results indicate that the use of mathematical planning methods for an experiment really allows to increase the information content of one of the important types of testing of electric rocket engines on high-altitude stands and should be introduced into practice.

5. Conclusion
The analysis shows that, depending on the conditions, the electroract engine can operate in different modes, each of which has its own specific features. Acceleration of plasma in the electroract engine in typical conditions occurs with the simultaneous action of various mechanisms. Such a nature of the processes in a turbofan engine, on the one hand, complicates the regulation of its characteristics, and also complicates its calculation. To date, it has been possible to develop satisfactory calculation methods only for individual special cases. On the other hand, the possibility of using various engine operating modes undoubtedly expands the practical possibilities of its use. For example, the contribution of the gas-dynamic and dissipative acceleration mechanisms makes it possible to obtain flow rates that exceed the value determined by the applied potential difference. Conducting a comparison of end and linear Hall engines based on materials, it is easy to notice between them a lot in common. In both cases, the processes of anomalous conductivity and the development of oscillations are very similar.

6. References
[1] Gusev Yu G and Pilnikov A V 2012 The role and place of electric rocket engines in the Russian space program *Electronic journal "Trudy MAI"* 60
[2] Makridenko L A, Volkov S N, Khodnenko V P and Golden S A 2010 Conceptual issues of the creation and use of small spacecraft *Electromechanical Issues* 114 15-26.
[3] Grishin S D, Leskov L V and Kozlov N P 1975 *Electric rocket engines* (Moscow: Mashinostroenie) p 272
[4] Yarygin V I, Ruzhnikov V A and Sinyavsky V V 2012 *Space nuclear power plants: past, present, future. Part I. Space nuclear power plants of the first generation* (Obninsk: Ed. IATE NRNU MEPhI) p 50
[5] Grodzovsky G L, Ivanov Yu N and Tokarev V V 1975 *Space Flight Mechanics: Optimization Problems* (Moscow: Science) p 702
[6] Garrigues L 2011 Study of a Hall effect thruster working with ambient atmospheric gas as propellant for low earth orbit missions *Presented at the 32nd Int. Electric Propulsion Conf. (Wiesbaden, Germany)* IEPC-2011-142
[7] Diamant K D 2010 A 2-Stage Cylindrical Hall Thruster for Air Breathing Electrical Propulsion 46th AIAA/ASME/SAE/ASEE Joint Propulsion Conf. and Exhibit (United States) AIAA Paper 2010-6522
[8] Hruby V, Pote B, Brogan T, Holman K, Szabo J and Rostler P 2003 *Air Breathing Electrically Powered Hall Effect Thruster Patent* WO 03/098041 A2
[9] Malyshev G V 2006 The use of electric propulsion engines for the removal, correction of the orbit and the maintenance of satellite system groups *Flying* 34-40
[10] Sidnyaev N I, Omurfiev V V and Govor S A 2017 Determination of the height characteristics of electric rocket engines of a spacecraft using experiment planning methods *Control problems* 1 75-85
[11] Sidnyaev N I, Omurfiev V V and Gecha V Ya 2017 Parameters of the critical orbit in the conditions of the existence of the spacecraft *Abstracts of the International Scientific Conf. "Fundamental and applied problems of mechanics"* (Moscow) (Moscow: MGTU im. N.E. Bauman) 163-164