Entanglement swapping, light cones and elements of reality

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19 July 2006

Abstract

Recently, a number of two-participant all-versus-nothing Bell experiments have been proposed. Here, we give local realistic explanations for these experiments. More precisely, we examine the scenario where a participant swaps his entanglement with two other participants and then is removed from the experiment; we also examine the scenario where two particles are in the same light cone, i.e. belong to a single participant. Our conclusion is that, in both cases, the proposed experiments are not convincing proofs against local realism.

PACS numbers: 03.67.-a, 03.67.Mn
Keywords: Non-locality; Bell inequalities; Pseudo-telepathy; Foundations of quantum mechanics.

1 Introduction

Henry R. Stapp [1] once described the work of John S. Bell [2] as “the most profound discovery of science”. Indeed, the work of Bell showed that our intuition that the world should be local realistic is incorrect, thus changing our perception of the physical world, perhaps to the same extent as Isaac Newton’s work on classical dynamics and Albert Einstein’s work on relativity. Albert Einstein, Boris Podolsky and Nathan Rosen (EPR), defenders of the local realistic viewpoint, argued that quantum mechanics is not a complete theory for it
does not contain every element of physical reality in its formalism \cite{3}. Bell showed that these exact same elements of reality, weaved into a local model of Nature, lead to a theory which contradicts the predictions of quantum mechanics. The experimental verification of Bell’s predictions \cite{4,5} gives us strong evidence that Nature indeed does not have a local realistic description.

More recently, a new kind of refutation of the local realistic viewpoint has arisen \cite{6,7,8,9}. These local-hidden-variable no-go theorems are also called “Bell theorems without inequalities“. Like standard Bell theorems, these experiments must be repeated for many runs in order to rule out a local realistic viewpoint (if we observe a single successful run, we cannot conclude anything except maybe that quantum mechanics is right or that a local-hidden-variable (LHV) model was lucky!) but usually less runs are required in order to reach the same confidence level as for standard Bell theorems. Another advantage is that the proof that no LHV model can reproduce the quantum correlations is usually much more elegant and simple. Instead of only showing that no LHV model can reproduce the correlations predicted by quantum mechanics (as is the case for standard Bell theorems), Bell theorems without inequalities show that an LHV model which is to attempt a simulation of quantum mechanics will run into a contradiction with itself \cite{10}. Most of these Bell theorems can be recast into the framework of pseudo-telepathy \cite{11,12}. In the pseudo-telepathy paradigm, proofs of non-locality are presented in the form of games. These games consist of questions given to space-like separated players who must give answers satisfying a certain relation with the questions. We say that a game which cannot be won with certainty by classical players (who share common classical information), whereas it can be won with certainty by quantum players (who share entanglement), is a pseudo-telepathy game. In other words, any LHV model that is to attempt to simulate the quantum correlations will, once in a while, output something that is forbidden according to quantum mechanics. There exists a Bell theorem without inequalities that cannot be transformed into pseudo-telepathy: Lucien Hardy’s theorem \cite{9}. Hardy’s argument uses a pair of non-maximally entangled qubits, and such a state cannot produce correlations that yield a pseudo-telepathy game \cite{13}.

In the last few months, new scenarios that can be cast into the framework of two-player pseudo-telepathy games have been proposed \cite{14,15,16,17}. The authors claim that they present new proofs against local realism. Although the equations that they present are mathematically correct, it is not possible to interpret them in such a way as to rule out all LHV models for the proposed experiments. Our work aims to clarify this situation. The present paper is divided such that we first discuss candidates for pseudo-telepathy games that use entanglement swapping in Section II. In Section III, we analyse the treatment of LHVs which are time-like separated. Before concluding, we finish with a discussion on elements of reality in Bell experiments in Section IV.
2 Entanglement swapping

Daniel M. Greenberger, Michael A. Horne and Anton Zeilinger [14, 15], recently proposed schemes based on entanglement swapping that fit in the framework of pseudo-telepathy. Here, we present a simple proof that shows that there is an LHV model for any two-participant protocol based on entanglement swapping. Thus, without going into the details of the scheme, we show that the experiment of Greenberger, Horne and Zeilinger cannot rule out local realism. Afterwards, we show that even if we consider the three-participant version of the Greenberger, Horne and Zeilinger protocol, it still admits an LHV model.

Recall that the following are the four Bell states:

\begin{align}
|\psi^-\rangle &= \frac{1}{\sqrt{2}} |01\rangle - \frac{1}{\sqrt{2}} |10\rangle, \\
|\psi^+\rangle &= \frac{1}{\sqrt{2}} |01\rangle + \frac{1}{\sqrt{2}} |10\rangle, \\
|\phi^-\rangle &= \frac{1}{\sqrt{2}} |00\rangle - \frac{1}{\sqrt{2}} |11\rangle, \\
|\phi^+\rangle &= \frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{2}} |11\rangle.
\end{align}

In the entanglement swapping scheme, Bob shares a copy of the state \(|\psi^-\rangle\) with both Alice and Charlie, while Alice and Charlie are not entangled. In order to swap his entanglement, Bob then measures his two qubits in the Bell basis. Before the measurement, the state of the global system is, up to local unitaries,

\begin{equation}
\frac{1}{2} |\psi^-\rangle_B |\psi^-\rangle_{AC} + \frac{1}{2} |\psi^+\rangle_B |\psi^+\rangle_{AC} + \frac{1}{2} |\phi^-\rangle_B |\phi^-\rangle_{AC} + \frac{1}{2} |\phi^+\rangle_B |\phi^+\rangle_{AC},
\end{equation}

where the first two qubits belong to Bob, the third to Alice and the last to Charlie. After Bob’s measurement, Alice and Charlie are therefore left in a Bell state. The fact that the entanglement between Alice and Charlie comes from particles that never interacted means that they do not share LHV’s (but note that any experimental setup that uses this hypothesis would have to be extremely well orchestrated in order to ensure that Alice and Charlie were never in a situation where they could communicate). The argument presented in [14, 15] then goes on to analyse the correlations of a Bell state as to whether they can be simulated by an LHV model where Alice’s and Charlie’s particles do not share any variables. The assumption made here is that whatever happens in Bob’s lab is of no consequence. The reason given is that the LHV’s of the particles belonging to Alice and Charlie cannot depend on each other and cannot depend on what happened in Bob’s lab. In the given interpretation of the experimental scheme, Bob’s lab can be thrown into a black hole for all it matters.

Is this argument valid? The answer is no. If Bob’s knowledge of the outcome of the Bell measurement is lost, Alice and Charlie are left with a mixture of all the Bell states, each with equal probability. Obviously, this state is the totally mixed state and it is not entangled. Therefore, Alice’s and Charlie’s answers will not be correlated in any fashion. A simple
LHV model can then simulate measurements on Alice’s and Charlie’s particle: output at random! (while taking into account that for a general POVM on a totally mixed state, not every POVM element will be produced with equal probability, and adjusting the marginal probabilities accordingly). Hence, without even considering the specific measurements that are performed in the experiment, we conclude that any scheme with two participants that is based on entanglement swapping admits an LHV model.

What if, instead of sending Bob into a black hole, we take into consideration his outcome? If we know Bob’s measurement outcome, then we know the actual Bell state that is shared between Alice and Charlie. We will now rewrite the experiment of \cite{14} in the language of quantum information, and consider the case where Bob’s measurement results are taken into consideration. We show that this experiment also admits an LHV model that simulates the correlations, and that this is due to the fact that Bob shares LHVs with Alice and with Charlie.

Here is the scheme that we consider: Bob does a Bell state measurement on the state described in Equation (5). His outcome, say $b$, is therefore one of the four Bell states. Alice and Charlie are now left in a Bell state that depends on Bob’s measurement outcome. They are then both asked to perform the same measurement: either in the standard basis (standard von Neumann measurement, or $\sigma_z$) or in the Hadamard basis (sending $|0\rangle \rightarrow (|0\rangle + |1\rangle)/\sqrt{2}$ and $|1\rangle \rightarrow (|0\rangle - |1\rangle)/\sqrt{2}$, followed by a von Neumann measurement, or $\sigma_x$). Let $a \in \{-1, 1\}$ be Alice’s outcome and $c \in \{-1, 1\}$ be Charlie’s outcome. Alice and Charlie each output a single bit ($-1$ or $1$), but to ease the notation we will denote the outcomes $a_\pm$ and $c_\pm$ if the measurements were performed in the standard basis and $a_x$ and $c_x$ if the measurements were performed in the Hadamard basis. Depending on the state that they share after Bob’s measurement, their results will either be correlated ($a \cdot c = 1$) or anti-correlated ($a \cdot c = -1$).

Table 1 gives the measurements outcomes that are predicted by quantum mechanics. Note also, that according to these predictions, the local outcomes of Alice, Bob, and Charlie are uniformly distributed. Recall that we are in a scenario where Alice and Charlie do not share hidden variables. At first sight it seems reasonable to think that the correlations of Table 1 cannot be fulfilled. However, Alice and Charlie are allowed to share variables with Bob. We will now show how they can exploit this to reproduce the predictions of quantum mechanics using only LHVs.

| $b$   | $a_\pm \cdot c_\pm$ | $a_x \cdot c_x$ |
|-------|---------------------|-----------------|
| $|\phi^+\rangle$ | 1                   | 1               |
| $|\phi^-\rangle$ | 1                   | -1              |
| $|\psi^+\rangle$ | -1                  | 1               |
| $|\psi^-\rangle$ | -1                  | -1              |

Table 1: Measurement outcomes
Alice, Bob and Charlie share four LHVs that each take the value $-1$ or $1$ independently and with equal probability. We denote these values by $\lambda_{a+}, \lambda_{a-}, \lambda_{c+}$ and $\lambda_{c-}$. When challenged to output the result of a measurement, Alice answers $\lambda_{a+}$ if the measurement is in the standard basis and $\lambda_{a-}$ otherwise. Charlie does the same, answering $\lambda_{c+}$ and $\lambda_{c-}$ depending on his measurement. In order to give an answer that is consistent with Table 1, all that Bob needs to do is compute the values $\lambda_{a+} \cdot \lambda_{c+}$ and $\lambda_{a-} \cdot \lambda_{c-}$. He then outputs the Bell state that he finds in the corresponding row of Table 2. It is easy to see that this strategy that uses four bits of shared randomness satisfies all the conditions of Table 1 and that in addition, the local statistics correspond to those predicted by quantum mechanics. This technique works regardless of the order in which the participants are required to answer.

| $\lambda_{a+} \cdot \lambda_{c+}$ | $\lambda_{a-} \cdot \lambda_{c-}$ | $b$   |
|---------------------------------|---------------------------------|-------|
| 1                               | 1                               | $|\phi^+\rangle$ |
| 1                               | $-1$                            | $|\phi^-\rangle$ |
| $-1$                            | 1                               | $|\psi^+\rangle$ |
| $-1$                            | $-1$                            | $|\psi^-\rangle$ |

Table 2: LHV simulation

The technique that we have used is reminiscent to postselection: according to the answers that Alice and Charlie are to give, Bob selects an appropriate measurement outcome. This is similar to [20], which, surprisingly, rules out the results presented many years later in [15]. We can formulate a similar argument against the Bell theorem presented by Adán Cabello in [21], as well as the one presented by Zeng-Bing Chen, Yu-Ao Chen and Jian-Wei Pan [19]. In all these cases, postselection is used in order to generate quantum correlations that cannot be produced by any LHV model. These experiments omit to consider the possibility that, as we have shown above, an LHV model can use postselection to its advantage.

3 Inside the light cone

What if we consider particles that were created in space-like separated regions of space-time that are later brought together? Could experiments performed on such particles and analysed with the hypothesis that these particles cannot share any LHVs be convincing? In [16, 17], it is argued that different physical quantities of a particle are elements of reality in the EPR sense. Then, the values of these observables are analysed as being independent since they are elements of reality. One might be tempted to think that these assumptions are reasonable, however they are not. While creating the particles in space-like separated regions will ensure that they do not share any LHVs at that point, we cannot assume that this property is conserved for the entire evolution of the system. In an LHV model, we do
require that what happens to a particle outside the light cone of another cannot have any influence on the latter, but we can allow the particles to constantly broadcast information that is secret to us (hidden travelling information) which travels at the speed of light in all directions. Therefore, once we bring a particle in the forward light cone of the other, its LHV's can be influenced by those of the other particle. This type of model is consistent with the local realistic viewpoint and invalidates the assumption that the LHV's will stay independent. This argument applies *mutatis mutandis* to the assumption that different observables, which are elements of reality, do not share LHV's.

We now give a brief summary of the scheme proposed in [16], which uses the four-qubit state:

\[
|\psi\rangle = \frac{1}{2} \left( |0\rangle_1 |0\rangle_2 |0\rangle_3 |0\rangle_4 + |0\rangle_1 |1\rangle_2 |0\rangle_3 |1\rangle_4 + |1\rangle_1 |0\rangle_2 |1\rangle_3 |0\rangle_4 - |1\rangle_1 |1\rangle_2 |1\rangle_3 |1\rangle_4 \right).
\]

Qubits 1 and 2 belong to Alice and qubits 3 and 4 to Bob. Now consider the following three measurements \(X_j, Y_j\) and \(Z_j\), performed individually on qubits \(j\) (\(j = 1 \ldots 4\)):

\[
X_j = |0\rangle_j \langle 1| + |1\rangle_j \langle 0|
\]
\[
Y_j = i( |1\rangle_j \langle 0| - |0\rangle_j \langle 1|)
\]
\[
Z_j = |0\rangle_j \langle 0| - |1\rangle_j \langle 1|.
\]

Each of these measurements has two possible outcomes, +1 and −1. Let the outcome of measurement \(X_j\) be written \(x_j \in \{+1, -1\}\), and similarly for \(Y_j\) and \(Z_j\). Quantum mechanics tells us that when appropriate measurements are made on state \(|\psi\rangle\), the following four equalities always hold:

\[
x_1 = x_3 z_4,
\]
\[
y_1 = -y_3 z_4,
\]
\[
x_1 x_2 = y_3 y_4, \quad \text{and}
\]
\[
y_1 x_2 = x_3 y_4.
\]

In the scheme proposed in [16], Alice is asked one of two possible questions:

1a. What are \(x_1\) and \(x_2\)?
2a. What are \(y_1\) and \(x_2\)?

Bob is independently asked one of four possible questions:

1b. What are \(x_3\) and \(y_4\)?
2b. What are \(x_3\) and \(z_4\)?
3b. What are \(y_3\) and \(y_4\)?
4b. What are \(y_3\) and \(z_4\)?
The challenge that Alice and Bob face is to provide answers to these questions such that Equations (8)–(11) are satisfied. Although it is shown in [16] that there is an element of reality corresponding to each measurement result, it is also possible that particles inside the same light cone can exchange unlimited information. Therefore, measurements on separate particles can be seen, for LHV model purposes, as one measurement on a global system.

We now give an explicit LHV model that perfectly mimics the predictions of quantum mechanics for the above scenario. Alice and Bob share two random variables, $\lambda_1$ and $\lambda_2$. Regardless of the question she is asked, Alice always answers "$\lambda_1$" and "$\lambda_2$". Bob’s strategy is to first flip a fair coin. The outcome ($-1$ or $1$) of this coin flip is Bob’s first answer, call it $b_1$. Bob then computes his second answer, $b_2$ according to Table 3 by using the information that he has: the question that he was asked, $\lambda_1$, $\lambda_2$ and $b_1$. It is interesting to point out that our LHV model not only satisfies the rules of Equations (8)–(11), but also reproduces the predictions of quantum mechanics perfectly: it is easy to see that in the LHV model, the local outcomes of Alice and Bob are uniformly distributed, and that this corresponds exactly to the predictions of quantum mechanics!

A similar argument can be used to show an LHV model to simulate the experiment in [17]. At this point, it is important to stress that this LHV model is not a contextual model in the usual sense of the term. Non-contextuality has to do with the choice of output in a given POVM [22], while here the “context” is which POVM is done on what particle. This model is not contextual but uses hidden traveling information between the particles and is of course consistent with a local realistic viewpoint.

The argument presented by Cabello does rule out a certain class of LHV models, those that do not use hidden traveling information, also called the EPRLER model by Cabello. However, it does not rule out every LHV model. There is of course a simple solution to make the equations given in [16, 17] physically meaningful. We keep the elements of reality in space-like separated regions of space and give them to new players. We can thus convert the game presented in [16, 17] into convincing experimental proposals [18].

| question | $b_2$ |
|----------|-------|
| 1b       | $\lambda_1 \cdot \lambda_2 \cdot b_1$ |
| 2b       | $\lambda_1 \cdot b_1$ |
| 3b       | $\lambda_1 \cdot \lambda_2 \cdot b_1$ |
| 4b       | $-\lambda_1 \cdot b_1$ |

Table 3: Bob’s strategy in the LHV model for Cabello’s game
4 Discussion and conclusion

Since Bell’s 1964 discovery, new Bell experiments have continuously been proposed. The goal of such experiments is to demonstrate experimentally the nonlocality of the world in which we live. In order to circumvent imperfections in the laboratory setting, new experiments are proposed to close experimental loopholes, one of the most notorious being the detection loophole \[23\]. But as we have demonstrated, not all Bell experiments are created equally, and a careful analysis is required in order to verify the validity of the proposed experiments. The papers that we have analysed here have something in common: they start by arguing for the existence of elements of reality and then base their analysis of the experiment on these elements of reality. However, the existence or independence of these elements of reality is not tested in the final experimental setup. We believe that this is what sets these experiments apart from others and that allows an LHV model that explains the experiment.

One must also be careful using arguments that concern elements of reality. Einstein, Podolsky and Rosen gave a criterion to recognize elements of reality \[3\]:

> If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

It cannot be stressed enough that this criterion is “regarded not as a necessary, but merely as a sufficient, condition of reality” \[3\]. Said differently, not every element of reality can necessarily be measured without disturbing the system. Otherwise, EPR would have claimed, after showing that momentum and position can have simultaneous reality, that the Heisenberg uncertainty relation can be violated \[24\]!

In order to propose meaningful experiments, it is useful to use a higher level of abstraction to analyse the scenario: by placing the proposed experiments in the framework of pseudotelepathy, we have been able to show that an LHV model can explain the results of the experiments. In fact, we believe that there is much to gain by studying nonlocality in an adversarial context: when analysing nonlocality proofs, one should be just as paranoid about Nature cheating our senses as are cryptographers about the security of a protocol against attacks from a malicious adversary.

Acknowledgements

We would like to thank Adán Cabello, Hilary Carteret, Nicolas Gisin, Alain Tapp and Jonathan Walgate for very stimulating discussions and Sébastien Gambs for helpful comments on the manuscript. This work is supported in part by Canada’s NSERC.
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