Repeating Fast Radio Bursts from Pulsar–Asteroid Belt Collisions: Frequency Drifting and Polarization

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Abstract

Fast radio bursts (FRBs) are a new kind of extragalactic radio transients. Some of them show repeating behaviors. Recent observations indicate that a few repeating FRBs (e.g., FRB 121102) present time–frequency downward drifting patterns and nearly 100% linear polarization. Following the model of Dai et al. who proposed that repeating FRBs may originate from a slowly rotating, old-aged pulsar colliding with an asteroid belt around a stellar-mass object, we focus on the prediction of time–frequency drifting and polarization. In this scenario, the frequency drifting is mainly caused by the geometric structure of a pulsar magnetosphere, and the drifting rate–frequency index is found to be 25/17. On the other hand, by considering the typical differential mass distribution of incident asteroids, we find that an asteroid with mass $m \geq 10^{17}$ g colliding with the pulsar would contribute abundant gravitational energy, which powers an FRB. A broad frequency band of the FRBs would be expected, due to the mass difference of the incident asteroids. In addition, we simulate the linear polarization distribution for the repeating FRBs, and constrain the linear polarization with $\geq 30\%$ for the FRBs with flux of an order of magnitude lower than the maximum flux.

Unified Astronomy Thesaurus concepts: Radio bursts (1339); Asteroids (72); Minor planets (1065); Neutron stars (1108); Radio transient sources (2008)

1. introduction

Fast radio bursts (FRBs) are extragalactic radio transients with millisecond durations and large observed dispersion measures (DMs; Lorimer et al. 2007; Keane et al. 2012; Thornton et al. 2013; Champion et al. 2016; Shannon et al. 2018; Petroff et al. 2019; Platts et al. 2019), but their physical origin remains a mystery. So far, there are more than 100 FRBs that have been observed, and a small fraction of them show repeating behaviors (Spitler et al. 2016; CHIME/FRB Collaboration et al. 2019a, 2019b; Kumar et al. 2019; Fonseca et al. 2020). The first reported repeating source, FRB 121102, was found to be localized in a dwarf galaxy with an extremely large observed rotation measure (Chatterjee et al. 2017; Michilli et al. 2018), and exhibits an intriguing downward structure that was then called the subpulse time–frequency drifting pattern (Hessels et al. 2019; Josephy et al. 2019; Caleb et al. 2020).

The subpulse drifting patterns were found in most of the repeating sources rather than any nonrepeating FRBs, suggesting that they are very likely to be a common property for repeaters. A lot of efforts have been made to explain the drifting pattern. Propagation effects seem to be possible, for instance, plasma lensing (Cordes et al. 2017), which is expected to show both upward and downward drifts. On the other hand, if the coherent radiation of an FRB is generated in the pulsar magnetosphere6 (Cordes & Wasserman 2016; Kumar et al. 2017; Ghisellini & Locatelli 2018; Katz 2018; Yang & Zhang 2018; Lu & Kumar 2019; Wang et al. 2019, 2020; Lu et al. 2020; Yang et al. 2020). Moreover, the recent work by Beniamini & Kumar (2020) shows that the light-curve variability timescales and the spectro-temporal correlations in the data would provide strong evidence in favor of the “close-in” models in which an FRB is proposed to occur near a magnetar. Wang et al. (2020) found that there is a small probability to generate the upward drifting but in most cases the downward drifting appears. Therefore, we mainly account for the downward drifting in this work.

Polarized observation is useful for identifying the origin of FRBs. The linear polarization component is almost 100% for FRB 121102 (Gajjar et al. 2018; Michilli et al. 2018) and is also quite high for other repeating sources (CHIME/FRB Collaboration et al. 2019b; Day et al. 2020; Fonseca et al. 2020). For curvature radiation, electromagnetic waves are polarized in the plane of the magnetic field lines. The flux reaches its maximum when the angle between the electron trajectory plane and line of sight (LOS) is limited within 1/$\gamma$, and the polarization degree is almost 100%. Besides, strong linear polarization can be generated by a magnetized plasma in the magnetospheres of pulsars (e.g., Melrose 1979).

The periodic activity could provide a new channel for studying the nature of FRBs. A periodicity of $\sim 16$ days (CHIME/FRB Collaboration et al. 2020a) and a possible periodicity of $\sim 159$ days (Rajwade et al. 2020) were recently detected to arise from FRB 180916.J0158+65 and FRB 121102, respectively. The periodicity was proposed to possibly result from precession of a magnetar (Levin et al. 2020; Zanazzi & Lai 2020), orbital precession of a binary (Yang & Zou 2020), spin of an ultra-long-period magnetar

6 The other kinds of models in which an FRB is generated by energy dissipation via an outflow interacting with an ambient medium, i.e., “far-away” models, have been proposed in the literature (e.g., Lyubarsky 2014; Beloborodov 2017, 2020; Ghisellini 2017; Waxman 2017; Gruzinov & Waxman 2019; Metzger et al. 2019; Margalit et al. 2020b).
Figure 1. Schematic geometry of collision between an elongated, compressed asteroid and a pulsar, producing subburst downward frequency drifting of repeating FRBs. As the asteroid gradually falls with the immediate emergence of numerous net charges, a stray electric field outside of the asteroid has a component parallel to the magnetic field, which causes abundant bunches to leave from the surface of the asteroid and move along the magnetic field lines. A higher-frequency signal is generated closer to the pulsar due to a smaller curvature radius. The dashed lines display the line of sight (LOS) and $\theta_D$ is the angle between the spark and the magnetic field lines transversely, an induced electric field can be produced, where $\Omega$ is the angular velocity of the subpulses of FRBs.

2. Time–Frequency Drifting

In order to let an asteroid fall freely over the stellar surface, a pulsar with a slow rotation and an old age is required. In this scenario, a low cooling luminosity and an extremely low spin-down power would cause evaporation and ionization to become ineffective for the asteroid falling into the magnetosphere of the pulsar (Cordes & Shannon 2008).

Figure 1 shows a higher frequency radio signal from a bunch observed earlier in a lower height, and vice versa. Coherent curvature emission from many bunches of different magnetic field lines can be observed. The curvature radius $\rho_0$ and the distance along the magnetic field lines ($\Delta l$) of the subpulses of high frequency and low frequency are different. The emission timescale of bunches is of order nanoseconds because of the frequency of $\sim$GHz, which is extremely shorter than millisecond-duration radio bursts, so a large number of bunches is required to sweep across the LOS (Yang & Zhang 2018).

2.1. Heating and Evaporation Mechanism of Asteroids

During each pulsar–asteroid collision, the heating and evaporation mechanism of an asteroid can determine whether the asteroid can be injected into the magnetosphere of an old-aged pulsar. Evaporation occurs inside the light cylinder and the evaporation temperature of an asteroid changes from 1500 to 2600 K (Cheng 1985). On the other hand, the asteroid temperature is determined by an equilibrium between the cooling rate and the heating rate. For an asteroid in thermal balance, due to surface radiation of the neutron star, the asteroid temperature $T_a$ is given by $T_a \sim T_{NS} \left( R_{NS} / 2 R \right)^{1/2} \sim 707 T_{NS} 5 R_{0}^{1/2}$ K (Cordes & Shannon 2008), where $T_{NS}$ and $R_{NS}$ are the surface temperature and the radius of the neutron star, respectively. The cooling mechanism is dominated by the stellar surface blackbody radiation, and the surface temperature of the old-aged neutron star reaches $\sim 10^3$ K (Shapiro & Teukolsky 1983). Their evaporation rates are too low to provide the full current density for $R \sim 10^{10}$ cm. Therefore, the asteroid cannot be evaporated outside of the light cylinder. As the asteroid reaches the surface of the neutron star, numerous net charges near the surface of the asteroid can be contributed instantly by the separation of electrons and ions in the interior of the asteroid due to reverse Lorentz forces. It is dominated by runaway ohmic heating because an induced electric field can provide sufficient current through the asteroid. As the asteroid crosses the magnetic field lines transversely, an induced electric field, $E_I = - (\Omega \times R) \times B / c$, can be produced, where $\Omega$ is the pulsar’s angular velocity. When the condition of the induced azimuthal magnetic field at the asteroidal surface $2 \pi r J / c \lesssim B$ can be satisfied, where $J$ is the current density, the ohmic heating power is given by $\dot{P} \approx \rho^3 / c$ (Cordes & Shannon 2008). Thus, the maximum current density can be estimated as $J_{\text{max}} \approx c B / 2 \pi r$. The maximum heating power can be estimated as

$$\dot{P}_{\text{max}} \approx \frac{r B_p R_{\text{NS}}^3}{P^{1/2}} \left( \frac{1}{2 \pi R} \right)^5 \approx 10^{33} \text{ erg s}^{-1} r_p^2 B_{p,14}^2 R_{\text{NS},6}^6 P_0^{-1} R_7^{-5},$$

and thus the asteroidal temperature by runaway ohmic heating is

$$T_a \approx \alpha \left( \frac{\dot{P}_{\text{max}}}{4 \pi r \sigma_{\text{SB}} r^2} \right)^{1/4} \approx 2600 \sigma_{\text{SB}} B_{p,14}^{1/2} R_{\text{NS},6}^{3/2} P_0^{-1/4} R_7^{-5/4} \text{ K},$$

where $\sigma_{\text{SB}}$ is the Stefan–Boltzmann constant, $c$ is the speed of light, $r \sim 10^3$ cm denotes the radius of the stretched asteroid, and $B_p = 10^{14}$ G is the surface magnetic field of the pulsar. If the thermal conversion efficiency $\sigma \approx 2 \times 10^{-4}$, then the pulsar’s rotation period can be limited to be $P \gtrsim 1$ s when the asteroidal surface reaches a maximum temperature 2600 K with the location of the asteroid $\sim 10^3$ cm.
2.2. The Downward Drift Pattern

The free-falling asteroid passing through the magnetosphere of the pulsar can generate an induced electric field $E_2$, which is shown as a force-free field inside the asteroid (Dai et al. 2016). This electric field outside of the asteroid has a component parallel to the magnetic field, making electrons dragged out of the surface of the asteroid and then accelerating these electrons to relativistic energies. These dragged electrons form bunches of charged particles, which give rise to coherent curvature radiation in the inner magnetosphere of the neutron star.

Generally, the phenomenon of time–frequency drifting can be interpreted by the geometrical effect within the framework of curvature radiation (Wang et al. 2019, 2020), since the typical frequency $\nu$ strongly depends on the curvature radius $\rho_c$, i.e., $\nu = \frac{3c\gamma^3}{4\pi\rho_c}$, where $\gamma$ is the Lorentz factor. As the asteroid elongated, transversely compressed asteroid falls down to the radius $R$ from the pulsar and the angle $\theta_0$ between the asteroidal falling direction with the magnetic axis. According to the magnetic dipole geometry, bunches leave from the surface of the asteroid and then they pass through the distance along the magnetic field lines $\Delta l = \nu c \Delta t$, where $\Delta l$ is given by the Appendix (bunches’ motion nearly at the speed of light). When the bunches move to $(R_1, \theta_1)$, their emission can be observed by us (see Figure 1). In this case, the bunches at a more-curved part of the magnetic field lines are always seen earlier, thus emitting higher-frequency waves. The difference of the distance along the different magnetic lines is given in the Appendix, and thus the delay of the high frequency bunches relative to the low frequency bunches is given by

$$dt = \frac{dR}{c \sin^2 \theta_0} [Z(\theta_0, \theta_1) - \cos(\beta - \theta_1) \sin^2 \theta_0], \quad (3)$$

where

$$Z(\theta_0, \theta_1) = \int_{\theta_0}^{\theta_1} \sqrt{1 + 3 \cos^2 \theta \sin \theta} \, d\theta, \quad (4)$$

and $\beta$ denotes the angle between the LOS and the magnetic axis. As shown in Equation (3), the time delay mainly depends on the separation in the radial direction. The relation between $\theta_1$ and $\beta$ can be written as (Lyutikov 2020)

$$\cos 2\theta_1 = \frac{1}{6} \left[ \sqrt{2} \cos \beta \sqrt{\cos(2\beta) + 17} + \cos(2\beta) - 1 \right]. \quad (5)$$

The time delay as a function of $\theta_0$ is shown in Figure 2. As shown in Figure 2, we set $\theta_1 = \pi/3, \pi/2, 2\pi/3$, respectively, the difference between the dashed line and the solid line corresponds to $\theta_1 - \theta_0$ for the observed time delay. The asteroidal length can be given by $l = l_0(R/R_0)^{-1/2}$ for $R \leq R_1$ (Colgate & Petschek 1981), where $R_0$ and $R_1$ denote the asteroids distorted tidally by the magnetar at breakup radius and the radius at which compression begins, respectively. The length of the asteroid can be estimated as $l \approx 9.6 \times 10^6 \text{ cm} \rho_0^{2/9} \rho_0^{5/18} s^{-1/6} (M/1.4 M_\odot)^{1/9} R_7^{-1/2}$, where $\rho_0$ and $s$ are the iron–nickel original’s mass density and tensile strength, respectively. If the distance between the bunches of high and low frequency from the surface of the asteroid is too close, no matter what angle of the asteroid falls, there is no delay effect in the direction of the LOS. There is always a low limit of the incident angle for the time delay of millisecond order between high and low frequencies to be observed in a given LOS. However, if $\theta_0$ is larger than $\theta_1$ (see Figure 1), then we are unable to observe the frequency drift.

For a magnetic dipole configuration, from a geometric point of view, during the asteroid’s fall, the bunches leave the asteroidal surface and move along the magnetic field line to sweep across the LOS, which can naturally produce subpulses with downward time–frequency drifting. Assuming that the Lorentz factor is a constant, we have

$$\frac{d\nu}{dt} = -\frac{4\pi c^2 C(\theta)}{9\gamma^3} \nu^2, \quad (6)$$

where $C(\theta)$ is given by

$$C(\theta) = \frac{\sin \theta_1 \sin^2 \theta_0 (1 + 3 \cos^2 \theta_0)^{3/2}}{(1 + \cos^2 \theta_1)[Z(\theta_0, \theta_1) - \cos(\beta - \theta_1) \sin^2 \theta_1]}. \quad (7)$$

This result deviates from the observation of FRB 121102 (Josephy et al. 2019), which shows a linear tendency. In the following discussion, we consider that the Lorentz factor evolves with $R$.

The coherent radiation leads to rapid cooling of emitting particles and therefore requires continuous acceleration of the bunched particles. Generally, the cooling timescale of curvature emission in the observer’s rest frame is given by $t_{\text{cool}} = \frac{27e^2\gamma^3}{16\pi^2 \nu^2 N_e}$, where $N_e$ is the number of electrons in the bunch (Kumar et al. 2017). Then the parallel electric field can be written as

$$E_1 = \frac{16\pi^2 e N_e [9 \sin \theta_1 (1 + \cos^2 \theta_0)]^{2/3} \nu^{4/3}}{27c^{2/3} (4\pi (1 + 3 \cos^2 \theta_1)^{3/2} \nu^{2/3} R_7^{2/3})}. \quad (8)$$

Owing to the fact that the induced electric field $E_1$ is perpendicular to the magnetic field line, it cannot accelerate electrons. While the asteroid travels through the magnetic field lines longitudinally, causing the induced electric field $E_2$ to accelerate electrons (Dai et al. 2016), we have

$$E_2 = -v_{\text{eff}} \times B/c = 2 \times 10^{13} \text{volt cm}^{-1} e_2 \times \left( \frac{M}{1.4 M_\odot} \right)^{1/2} \mu_3^2 R_7^{-7/2}, \quad (9)$$

![Figure 2](image_url)
where \( \mu = B_0 R_{eq}^3 \) denotes the magnetic dipole moment of the pulsar, \( v_{\text{eff}} \) is the asteroidal freewheel velocity, which can be written as \( v_{\text{eff}} = (2GM/R)^{1/2} \), and \( e_2 \) is the unit vector of \( E_2 \). The cooling electric field is equal to the induced electric field to prevent rapid cooling of electrons. From Equations (3), (8), and (9), the drifting rate is thus given by

\[
\frac{d\nu}{dt} = -1.9 \left( \frac{eN}{\mu} \right) \frac{v^{9/17} \pi^{8/17} c^{5/17}}{(GM)^{3/17}} D(\theta) \nu^{25/17},
\]

where \( D(\theta) \) is given by

\[
D(\theta) = \frac{\sin^2 \theta_0 [\sin \theta_1 (1 + \cos^2 \theta_1)]^{1/17}}{1 + 3 \cos^2 \theta_1)^{6/17} [Z(\theta_0, \theta_1) - \cos(\beta - \theta_1) \sin^2 \theta_0].}
\]

The frequency drifting rates are negative, i.e., downward drifting, matching the observations of most repeating FRBs (CHIME/FRB Collaboration et al. 2019a, 2019b; Hessels et al. 2019; Josephy et al. 2019; Caleb et al. 2020; Fonseca et al. 2020).

In Figure 3, we present the drifting rate as a function of the central frequency, and the fitted parameter \( N_e \sim 10^{27} \) matches the electron number of Dai et al. (2016), which indicates that our model is self-consistent. As shown in Figure 3, the drifting rate implied by Equation (10) is more consistent with the observations than Equation (6). The drifting rate at a higher frequency is larger than that at a lower frequency. Subpulses at the same central frequency may have fluctuated the drifting rate. This fluctuation may be related to the fluctuation of the emitting electron Lorentz factor and the angle of the asteroidal falling due to a random-falling asteroid.

### 3. Polarization

Polarization is an important clue to the radio radiation mechanism and related to radiation geometry. At present, polarization of dozens of FRBs has been observed.\(^7\) There are completely unpolarized FRBs that have been observed (e.g., FRB 150418; Keane et al. 2016). FRB 140514 displays only circular polarization (Petroff et al. 2015), a few show only linear polarization (e.g., FRB 121102 and FRB 180916.0158 +65; Michilli et al. 2018; CHIME/FRB Collaboration et al. 2019b), and several FRBs show both (e.g., FRB 110523; Masui et al. 2015). FRB 121102, FRB 180916.0158 +65, FRB 190604, and FRB 190303 (Fonseca et al. 2020) are repeating sources. The linear polarization is very high, nearly 100% linearly polarized for the first three FRBs. In particular, the linear polarization of FRB 190303 was constrained to be \( \geq 20\% \) (Fonseca et al. 2020), which could suggest the emission mechanism of the FRBs.

For curvature radiation, the electron trajectory is almost along a magnetic field line. We define the radiation angular frequency as \( \omega = \omega_0(1 + \gamma^2)^{1/2} \), the angle between the LOS and the charge trajectory plane as \( \chi \), and the Lorentz factor of relativistic electrons as \( \gamma \). In the direction of the electron’s motion, the radiation would be beamed in a narrow angle \( 1/\gamma \). The energy per unit frequency interval per unit solid angle is given by (e.g., Jackson 1975)

\[
\frac{dI}{d\Omega d\omega} = \frac{e^2}{3\pi^2 c} \left( \frac{\omega_0}{c} \right)^2 \left( \frac{1}{\gamma^2} + \chi^2 \right)^2 \times \left[ K_{2/3}(\xi) + \frac{\chi^2}{(1/\gamma^2) + \chi^2} K_{1/3}(\xi) \right],
\]

where \( K_{m}(\xi) \) denotes the modified Bessel function, \( \xi = (1/\gamma^2 + \chi^2)^{1/2}/\omega_0/3c \), the first term corresponds to the parallel component in the trajectory plane and the second term corresponds to the polarized component that is perpendicular to the LOS. The degree of linear polarization is given by Jackson (1975)

\[
\Pi = \frac{\chi^2 K_{2/3}(\xi) - \left( \frac{1}{\gamma^2} + \chi^2 \right) K_{1/3}(\xi)}{\chi^2 K_{2/3}(\xi) + \left( \frac{1}{\gamma^2} + \chi^2 \right) K_{1/3}(\xi)}.
\]

In the following calculation, we adopt the parameters: \( \gamma = 200 \) and \( \chi \) changes from 0 to 1/\( \gamma \). Therefore, we can obtain the relation between curvature radiation of electrons and the degree of the linear polarization. In Figure 4, we can see that the energy radiated per unit frequency interval per unit solid angle increases with the linear polarization for \( \omega \gg \omega_0 \). It is noted that \( dI/d\Omega d\omega \) increases first and then decreases slowly with \( \Pi \) at the high linear polarization for \( \omega < \omega_0 \), since the flux would reach the maximum when \( \chi \) deviates the LOS slightly. If the observed flux varies within an order of magnitude for different subbursts, the linear polarization would be \( \geq 30\% \) for the characteristic frequency, which is nearly consistent with the observed linear polarization constraint of the FRB 190303 (Fonseca et al. 2020). If \( \omega \gg \omega_0 \), the low limit of the linear polarization may be raised until nearly 100\%, while \( \omega \ll \omega_0 \), the linear polarization could be extended to the lower values.

As the asteroid falls, the kinetic energy density is equal to the magnetic energy density of the pulsar at the Alfvén radius, i.e.,

\[
\frac{\rho v^2}{2} \approx \frac{\mu^2}{8\pi R^6},
\]

where \( \rho \equiv m/\pi R^2l \) is the mass density of the asteroid. The emission position could be related to the compressed asteroid

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\(^7\) The FRB Catalog http://www.frbcat.org.
The gravitational energy release rate via \( \dot{E}_G \approx G M m / R \Delta t \), where \( \Delta t = 1 \text{ ms}, \ R \sim 10^7 \text{ cm} \). While the observed minimum luminosity of the extragalactic FRBs \( \sim 10^{50} \text{ erg s}^{-1} \) (see footnote 7), the low limit of the incident asteroid mass and radius can be constrained to \( m_{\min} \approx 5.4 \times 10^{16} \text{ g} \) and \( R_{0,\min} \approx 1.8 \times 10^5 \text{ cm} \), respectively. The observable FRBs could arise from \( m > m_{\min} \) and \( r_0 > r_{0,\min} \). Since \( m_{\min} \approx m_{\min} \), we can consider the asteroidal mass distribution for \( m > m_{\min} \). From the top panel of Figure 5, which indicates that a heavier asteroid would cause a stronger gravitational energy release rate.

We next consider the solid angle random distribution of the incident asteroid angle \( \sin \theta \) (from 0 to 1) and the emission angle \( \sin \chi \) (from 0 to \( \sin (10/\gamma) \)), and the asteroidal radius distribution for \( r_0 > r_{\min} \). We can get the distribution of the characteristic frequency. As shown in the middle panel of Figure 5, it is suggested that the curvature radius can be affected by a different asteroidal radius would lead to a different characteristic frequency. A larger incident radius could lead to a lower Alfvén radius, causing a lower curvature radius and a higher observed frequency. Owing to the mass difference of the incident asteroids, FRBs with a broad frequency band would be observed. This is consistent with the detected frequency range from 400 MHz to 6.5 GHz for FRB 121102 (Chatterjee et al. 2017; Michilli et al. 2018; Hessels et al. 2019; Josephy et al. 2019). It can be seen from the simulation results (see the bottom panel of Figure 5) that a smaller emission angle \( \chi \) would cause a higher linear polarization distribution due to the beaming effect. It is interesting that the polarization distribution is peaked at \( \Pi \approx 0 \) and \( \Pi \approx 1 \). The reason is as follows: for a single point source, the polarization decreases with \( \chi \), which means that the polarization is maximum at \( \chi = 0 \). However, by considering that the LOS is random, in most cases, one has \( \chi > 1/\gamma \), which makes the polarization become low (the corresponding flux is also much less than the maximum one). Therefore, the polarization distribution is peaked at \( \Pi \approx 0 \) and \( \Pi \approx 1 \) due to the two reasons mentioned above.

Figure 6 shows the linear polarization as a function of angular frequency. It can be seen that the linear polarization decreases with the angular frequency for a fixed \( \chi \). Owing to the mass difference of incident asteroids, FRBs have a broad frequency band so that we can get the corresponding broad linear polarization.

**4. Discussion**

Owing to the geometrical effect, bunches producing high-frequency emission from small curvature radii first arrive at an observer, and then bunches producing low-frequency emission are followed. Meanwhile, the NS rotates a small phase so that the magnetic configuration changes slightly. One can then calculate the time delay either via radial direction or rotational direction.

Moreover, some subbursts have extremely short time intervals due to a short distance between high and low bunches from the asteroidal surface, i.e., one may not distinguish between the subbursts when extracting signals. The drifting rate’s linear fitting index \( \sim 1 \) (Josephy et al. 2019) has a lot of uncertainties due to the drifting rate only observed in the 400–800 MHz and 6.5 GHz, respectively, mainly dominating the linear trend. On the other hand, the linear fitting curve does not cross the original point of the coordinate axis. In addition, the drifting rate detected at the central frequency of 1.4 GHz
deviates from a Gaussian distribution. Since the drifting rate itself fluctuates, an observation of a wider frequency range \((\nu \gtrsim 8 \text{ GHz} \text{ or } \lesssim 400 \text{ MHz})\) would provide significant information for understanding the relationship between frequency drifting rate and frequency, which depends on the geometric distribution of \(E_{\parallel}\).

Some bursts such as FRB 200428 and FRB 180916.J0158+65 exhibit an upward drifting pattern (Bochenek et al. 2020; CHIME/FRB Collaboration et al. 2020a, 2020b). A long-range \(E_{\parallel}\) and the time difference of bunch generation would lead to a time–frequency upward drifting pattern. If there is upward frequency drifting, from Figure 1, it is suggested that the bunches would move a longer distance than those typically observed \(<\text{ms}-\text{order FRBs}\) along the magnetic field lines. The long-range of an accelerating electric field may be triggered by the sudden distortion of the magnetic field lines around the asteroid or other globally accelerating electric fields such as \(E_{\parallel}\) in the slot gap (Muslimov & Harding 2003, 2004). Furthermore, similarly to Wang et al. (2020), by considering bunches produced at different times, the radiation from a more curved part is seen later, which also provides a small possibility for upward frequency drifting.

As shown in the bottom panel of Figure 5, we simulate the degree of linear polarization distribution for \(r_0 > r_{0,\text{min}}\) and \(\chi\) ranges from 0 to \(\sin(10/\gamma)\). We find that the curvature radius would have a slight difference due to the radius difference of an incident asteroid, which is not sensitive to the linear polarization distribution. From Figure 4, if the angular frequency is larger or smaller than the characteristic frequency, then the linear polarization could increase or decrease for FRBs with flux an order of magnitude lower than the maximum flux.

The polarization angle swings, due to different magnetic field lines of the pulsar sweeping across the LOS, depend on the rotational period of the pulsar and the angle between the rotation axis and magnetic axis. In our model, the constraints on the slowly spinning pulsar and the small angle between the spin axis and magnetic axis can explain small variations in the polarization position angles for those repeating sources.

In our model, the observed spectra during the downwards drift can be understood as coherent curvature radiation by bunches (Yang & Zhang 2018; Yang et al. 2020). The observed light-curve variability timescale may be related to the difference between \(\theta_0\) and \(\theta_1\). If the difference between \(\theta_0\) and \(\theta_1\) is very small, then we can find the light-curve variability timescale is smaller than the ms timescale, which provides strong evidence in favor of the magnetospheric model.

5. Summary

In this paper, we have shown that the time–frequency downward drifting patterns are caused by the geometrical effect, i.e., the motion of emitting bunches from asteroids along the magnetic field lines at different heights. In our model, the
production of FRBs requires that the pulsar is slowly spinning and old-aged (i.e., the low spin-down power and cooling luminosity) so that the asteroids can enter its magnetosphere smoothly. Based on the unipolar inductor accelerating electric field balanced with the cooling electric field to prevent the electrons cooled quickly, we derived the frequency drifting rate index is $25/17$ and the total radiating electron number $N_e \approx 10^{27}$. The frequency drifting does not occur in the case that the asteroids enter the magnetosphere of a pulsar along the magnetic axis (i.e., $\theta_0 = 0$ or $\pi$). We found that the asteroidal incident angle has a low limit for each given $\theta_1$ and the distance of high and low bunches from the asteroidal surface should be approximately the length of the asteroid since the time delay of ms-order between high and low frequencies is observed along the LOS. Furthermore, the variation of the drifting rate is probably a result of both the fluctuation of the electron number and the asteroidal random falling angle.

In addition, we considered the asteroidal mass distribution and found that a heavier asteroid colliding with the pulsar could release a more abundant gravitational energy. FRBs with a broad frequency band would be expected due to the mass difference of the incident asteroids. We also simulated the linear polarization distribution for the repeating FRBs and found that the linear polarization can be constrained with $>30\%$ for the FRBs with flux an order of magnitude lower than the maximum flux.

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Appendix

Frequency Drifting Rate in the Magnetosphere

For a dipole magnetic field configuration, the polar coordinates $(R, \theta, \phi)$ and the radial direction position can be described as

$$R = R_{\text{max}} \sin^2 \theta,$$

(A1)

where $R_{\text{max}}$ is the maximum distance of magnetic field lines across the equator. According to Equation (A1), $R_{\text{max}} = R/\sin^2 \theta$. Assuming the location of lower bunches and higher bunches at $R_1$ and $R_2$ to leave the surface of the asteroid. The distance along the magnetic lines can be written as, respectively (Yang & Zhang 2018),

$$\Delta l_1 = \frac{R_1}{\sin^2 \theta_0} \int_{\theta_0}^{\theta_1} \sqrt{1 + 3 \cos^2 \theta} \sin \theta \, d\theta,$$

(A2)

$$\Delta l_2 = \frac{R_2}{\sin^2 \theta_0} \int_{\theta_0}^{\theta_1} \sqrt{1 + 3 \cos^2 \theta} \sin \theta \, d\theta,$$

(A3)

and the difference of the distance can be written as

$$d\ell = \frac{dR}{\sin^2 \theta_0} \int_{\theta_0}^{\theta_1} \sqrt{1 + 3 \cos^2 \theta} \sin \theta \, d\theta. \quad \text{(A4)}$$

The difference of the distance, $d\ell$, mainly depends on the separation in the radial direction $dR$. Relativistic electrons along the magnetic field lines can cause coherent curvature radiation for the Lorentz factor $\gamma$ and the curvature radius $\rho_c$, whose characteristic frequency $\nu$ of curvature radiation is

$$\nu = \frac{3c\gamma^3}{4\rho_c}, \quad \text{(A5)}$$

where the curvature radius $\rho_c$ at $(R, \theta)$ is given by

$$\rho_c = \frac{R(1 + 3 \cos^2 \theta)^{3/2}}{3 \sin \theta(1 + \cos^2 \theta)}. \quad \text{(A6)}$$

The subscripts $x$ and $y$ indicate the location of lower bunches and higher bunches sweeping across the LOS, respectively. We have

$$\frac{R_x}{R_0} = \frac{R_1 \sin^2 \theta_1}{\sin^2 \theta_0}, \quad \text{(A7)}$$

the curvature radius and the frequency at lower bunches are given by

$$\rho_c = \frac{R_1 \sin^2 \theta_1(1 + 3 \cos^2 \theta_1)^{3/2}}{3 \sin^2 \theta_0(1 + \cos^2 \theta_1)}, \quad \text{(A8)}$$

and

$$\nu_x = \frac{9c\gamma^3 \sin^2 \theta_0(1 + \cos^2 \theta_1)}{4\pi R_1 \sin \theta_1(1 + 3 \cos^2 \theta_1)^{3/2}}. \quad \text{(A9)}$$

Similarly, the frequency at the higher bunches can be written in the same form,

$$\nu_y = \frac{9c\gamma^3 \sin^2 \theta_0(1 + \cos^2 \theta_1)}{4\pi R_2 \sin \theta_1(1 + 3 \cos^2 \theta_1)^{3/2}}. \quad \text{(A10)}$$

The difference of the frequency can be given by

$$d\nu = -\frac{9c\gamma^3 \sin^2 \theta_0(1 + \cos^2 \theta_1)}{4\pi R_1 \sin \theta_1(1 + 3 \cos^2 \theta_1)^{3/2} R^2} \frac{dR}{\gamma^3(1 + \cos^2 \theta_1)Z(\theta_0, \theta_1)} \nu_z, \quad \text{(A11)}$$

where we take the approximation $R \sim R_1 \sim R_2$. Thus, the frequency drifting rate is given by

$$\frac{d\nu}{dt} = -\frac{9c\gamma^3 \sin^2 \theta_0(1 + \cos^2 \theta_1)}{4\pi R_1 \sin \theta_1(1 + 3 \cos^2 \theta_1)^{3/2}} \nu_z. \quad \text{(A12)}$$

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