Application of DWF to heavy-light mesons

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We consider application of domain wall fermions to quarks with relatively heavy masses, aiming at precision calculations of charmed meson properties. Preliminary results for a few basic quantities are presented.

1. INTRODUCTION

After the remarkable accomplishment of the $B$ factories, theoretical uncertainty remaining in the decay amplitudes of heavy-light mesons is an important obstacle to the stringent test of the Standard Model. Lattice QCD offers a promising method to reduce this uncertainty in a systematic way and numerous attempts have been made to achieve this purpose so far \cite{1,2}. While the accuracy we can attain in simulating $b$ quark is still limited, mainly due to difficulty in taking a continuum limit, present computational resources should make it possible to perform precision calculations of charmed hadron properties. It is expected that the charm factories will allow us to calibrate lattice calculations of the heavy-light system with high precision, and the experiences gained in $c$-physics will be advantageous to the $b$-physics simulations.

Domain wall fermions (DWF) \cite{3,4,5} are expected to have only a negligible $O(a)$ error as well as better control over chiral extrapolations. Nonperturbative renormalization, which is essential to high precision calculations, also seems to work remarkably well \cite{6}. Simulating light quarks using DWF benefits from these advantages and has been successfully used in the light hadron system \cite{7,8,9}. It is, then, natural to consider its application to the massive quarks. As a first step toward the precision calculations, we have started the simulation of $D$ mesons using DWF for both heavy and light quarks.

2. SIMULATION PARAMETERS

The first exploratory study of massive DWF \cite{10} observed that with an improved gauge action at weaker gauge coupling DWF do not show clear failure for $am_q \lesssim 0.4$. Based on this observation, we take the DBW2 gauge action \cite{11} with $\beta=1.22$ at which the charm quark is realized around $am_q \sim 0.4$. The simulation is carried out using quenched lattices with $24^3 \times 48$. We set the extension of the fifth dimension and the domain wall height to $L_s=10$ and $aM_5=1.65$, respectively. The light and heavy quark masses range in $0.008 \leq am_lq \leq 0.040$ and $0.1 \leq am_hq \leq 0.5$, respectively. To avoid the finite size effect in the time direction, all the meson two-point functions are calculated under both the periodic and anti-periodic boundary conditions and then averaged.

The magnitude of the explicit chiral symmetry breaking can be probed by the residual mass, $m_{\text{res}}$, defined by

$$am_{\text{res}} = \frac{\langle \sum_x J_5^a(\vec{x},t)\pi^a(0)\rangle}{\langle \sum_x J_5^5(\vec{x},t)\pi^5(0)\rangle},$$

where the explicit forms of the operators are found in Ref. \cite{7,9}, and we obtain $am_{\text{res}} = 9.72 \times 10^{-5}$, which roughly corresponds to $m_{\text{res}} \sim 0.3$ MeV.

3. PRELIMINARY RESULTS

To see if the large mass parameters cause any unexpected behavior, we first examine the effective mass plots. The plots shown in Figure \ref{fig} are
obtained from the \( \langle A_4 P \rangle \) correlation functions with wall source and point sink, where \( A_4 \) and \( P \) are heavy-light bilinears. The light quark mass is fixed to \( am_{lq} = 0.040 \) while the heavy quark mass varies from 0.1 (bottom) to 0.5 (top) by 0.1. The data with \( am_{hq} = 0.4 \) and 0.5 show a gradual increase at \( t \sim 30 \) and it looks getting more conspicuous as \( am_{hq} \) gets larger, otherwise no odd behavior is seen for all heavy quark masses. We repeated the same calculation for one configuration with more stringent stopping condition and found that in the heaviest case (\( am_{hq} = 0.5 \)) the correlation function with the smaller stopping conditions differs by only \(+0.05\%\) at \( t = 20 \) while the difference reaches to \(+10\%\) around \( t \sim 30 \). Therefore it seems that the increase found in Figure 1 is caused by the loose stopping condition. Since all the correlation functions for \( t \leq 20 \) show no odd behavior and are seen to have a reasonable plateau, we take this range to perform a conservative analysis. The errors given below are statistical only, unless otherwise specified.

Figure 2 shows the chiral extrapolation of the heavy-light pseudo-scalar (circles) and vector (triangles) meson masses with \( am_{hq} = 0.4 \). Both the linear and quadratic fits describe the data very well, and the difference due to the different fits does not exceed 0.5 % in the chiral limit for the both mesons. We choose the linear fit to obtain the heavy-light masses at \( am_{ud} = \text{am}_{res} \) and \( am_{s} = 0.032(2) \) for later use, where \( am_{s} \), corresponding to the strange quark mass, is determined using \( m_K/m_\rho \) as a representative among other possible inputs. To find \( am_{charm} \), the pseudo-scalar masses are interpolated to the physical value of \( m_D \) using a quadratic function of heavy quark mass, and we obtain \( am_{charm} = 0.380(4) \). The mass splitting of \( m_D^* - m_D \) is, then, obtained as

\[
m_D^* - m_D = 114(6) \text{ MeV.} \tag{2}
\]

While this value is about 15% larger than the experimental value (99MeV), quenching may account for much of the discrepancy.

The lattice calculation of the 1S hyperfine splitting has been a long standing problem because the lattice results are significantly smaller than the experimental values, independent of the heavy quark action adopted. In this work we obtain

\[
m_{D^*} - m_D = 69(6) \text{ MeV,} \tag{3}
m_{D_s^*} - m_{D_s} = 70(4) \text{ MeV.} \tag{4}
\]

Comparing to the experimental results of \( m_{D^*} - m_D = 142 \text{ MeV} \) and \( m_{D_s^*} - m_{D_s} = 144 \text{ MeV} \), we see that this problem persists when using DWF.
The leptonic decay constants are obtained in a standard way from two-point functions with wall source and wall sink. Such correlation functions are still rather noisy, and the resulting decay constants depend on the fit ranges by as much as 10%. In the following analysis, we thus adopt a set of fit ranges and take 10% uncertainty as a systematic error. Figure 3 shows the chiral behavior of the decay constant at $am_{hq} = 0.4$. Since the data with $am_{hq} = 0.008$ looks unnaturally high, we performed linear extrapolations with and without this data point. The difference between the two extrapolations are taken as a systematic error again. After interpolation to $am_{hq} = am_{charm}$, we obtain

$$f_D^{latt} = 200(9)(^{+20}_{-21}) \text{ MeV}, \quad (5)$$

$$f_{Ds}^{latt} = 216(7)(22) \text{ MeV}, \quad (6)$$

where the systematic errors are summed up in quadrature. At the moment we quote the lattice bare values because we do not know how significant the $O(a^n m_{charm}^n)$ error is.

The SU(3) flavor breaking ratio of the decay constant is one of the quantities for which lattice QCD can attain a high precision because most of uncertainties are expected to cancel in the ratio. We can anticipate that the on-going charm experiments make it possible to measure this ratio at a few % accuracy, and so this is a good quantity for which to compare the lattice result with the experimental one although the uncertainties in $m_s$, the chiral extrapolation and quenching effects must first be under control. Our current result is

$$f_{Ds}/f_D = 1.08(3)(^{+2}_{-0}). \quad (7)$$

4. SUMMARY

We have started a exploratory study of massive DWF aiming at precision lattice calculations of the $D$ meson system. The preliminary results show no evidence of the DWF mechanism failing at large masses, while it may be better to improve how to impose a stopping condition. However, for precision calculations, we need to understand scaling violations, especially how serious the $O(a^n m_{charm}^n)$ error is. We also need more statistics to have better control over chiral extrapolations.

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