Two-dimensional Lifshitz-like AdS black holes in $F(R)$ gravity

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Two-dimensional (2D) Lifshitz-like black holes in special $F(R)$ gravity cases are extracted. We indicate an essential singularity at $r = 0$, covered with an event horizon. Then conserved and thermodynamic quantities such as temperature, mass, entropy, and the heat capacity of 2D Lifshitz-like black holes in $F(R)$ gravity are evaluated. Our analysis shows that 2D Lifshitz-like black hole solutions can be physical solutions, provided that the cosmological constant is negative ($\Lambda < 0$). Indeed, there is a phase transition between stable and unstable cases by increasing the radius of AdS black holes. In other words, the 2D Lifshitz-like AdS black holes with large radii are physical and enjoy thermal stability. The obtained 2D Lifshitz-like AdS-black holes in $F(R)$ gravity turn to the well-known 2D Schwarzschild AdS-black holes when the Lifshitz-like parameter is zero ($s = 0$). Also, correspondence between these black hole solutions and the 2D rotating black hole solutions is found by adjusting the Lifshitz-like parameter.

I. INTRODUCTION

One of the exciting observations in cosmology is related to the fact that our Universe has an accelerated expansion [1, 2]. Modified theories of gravity are the simplest way to address this accelerating behavior. In this regard, $F(R) = R + f(R)$ theory of gravity is the straightforward modification of General Relativity (GR). The gravitational action in this theory is a general function of $R$ [3–5]. $F(R)$ gravity includes some exciting features such as i) $F(R)$ models can be fixed according to the cosmological and astrophysical observations ranging from local to cosmological scale (for more details, see Refs. [6–12]). ii) it may explain the structure formation of the Universe without considering dark energy or dark matter. iii) this theory of gravity is coincident with Newtonian and post-Newtonian approximations [13, 14]. iv) $F(R)$ gravity may give the unified description of the early-time inflation and late-time acceleration. v) the whole sequence of the Universe’s evolution epochs: inflation, radiation/matter dominance, and dark energy may be extracted in this theory of gravity. According to the mentioned features of $F(R)$ gravity, some people have studied different solutions of this gravity in Refs. [15–21].

On the other hand, a new quantum theory of gravity based on Lifshitz’s idea in the quantum system was proposed by Horava [22, 23], which takes into account different spacetime footing. This theory is known as Horava-Lifshitz gravity, which modifies GR in small-scale or ultraviolet (UV) regimes. Notably, this theory reduces to GR in the infrared (IR) limit. The Lorentz symmetry is broken based on this assumption. Consequently, the theory is renormalizable at a quantum level. Indeed, Horava-Lifshitz gravity introduces a new metric that includes an anisotropic scale invariant between time and space as

$$t \rightarrow b^z t \quad \& \quad x \rightarrow bx,$$

where $z$ is a dynamical critical exponent that makes the theory renormalizable for $z \geq d$ in $(d + 1)$–dimensional spacetime. However, Horava-Lifshitz’s theory of gravity does not explain the dark energy epoch in its original form as it also occurs in GR. To have a unified theory of gravity in UV and IR regimes, $F(R)$ Horava-Lifshitz gravity has been introduced. $F(R)$-Horava-Lifshitz’s theory of gravity with different points of view has been evaluated in Refs. [24–29].

Two-dimensional (2D) quantum theories of gravity were introduced to understand the principles and puzzles of quantum gravity [30–34]. Such ideas of gravity are much simpler than four-dimensional cases. Also, these theories share some exciting features with four-dimensional gravity. Further, the 2D theories of gravity could be identified with non-critical string theories, models with spacetime super-symmetry have also been proposed in Refs. [35–36]. In this regard, 2D gravity with different motivations has been introduced, and their properties are investigated in Refs. [37–47].

The main motivation for considering $F(R)$-Horava-Lifshitz’s theory of gravity is related to these facts that $F(R)$ gravity modifies GR in the infrared (IR) limit, and also Horava-Lifshitz gravity modifies GR in small-scale or ultraviolet...
(UV) regime. On the other hand, to understand quantum gravity, 2D theories of gravity can be helpful. So the investigation of 2D black hole solutions in the context of such a unified theory of gravity can give us some essential information. Therefore we consider 2D Lifshitz-like spacetime and extract black hole solutions in $F(R)$ gravity. Notably, the black hole solutions in four [48], and three [49] dimensional Lifshitz-like spacetime have been studied in $F(R)$ gravity. The structure of this paper is as follows: at first, we introduce the fundamental equation of $F(R)$ gravity. Then we obtain 2D Lifshitz-like black holes in the context of such a unified theory of gravity can give us some essential information. Therefore we consider 2D Lifshitz-like spacetime and extract black hole solutions in $F(R)$ gravity. The final section is devoted to concluding remarks.

II. BASIC EQUATIONS

The action of $F(R)$ gravity in 2D spacetime is given by [50]

$$I = \frac{1}{2\kappa^2} \int d^2x \sqrt{-g} F(R),$$

(2)

where $\kappa^2 = 8\pi G$, and $G$ is the Newtonian gravitational constant. Also, $g = \text{det}(g_{\mu\nu})$ is the determinant of metric tensor $g_{\mu\nu}$. Here, we set the Newtonian gravitational and the speed of light equal to 1 ($G = c = 1$). The above action explains a theory of 2D gravity where $F(R) = R + f(R)$ and $f(R)$ is an arbitrary function of the Ricci scalar $R$. It is worthwhile to mention that for $F(R) = R$, the action (2) recovers the Hilbert-Einstein action.

The field equations are obtained by variation with respect to metric tensor $g_{\mu\nu}$, in the following form

$$R_{\mu\nu} F_R - \nabla_{\mu} \nabla_{\nu} F_R + \left( \Box F_R - \frac{1}{2} F(R) \right) g_{\mu\nu} = 0,$$

(3)

where $F_R \equiv dF(R)/dR$, and $R_{\mu\nu}$ is the Ricci tensor. Also, $\Box = \nabla^\nu \nabla_{\nu}$.

III. 2D LIFSHITZ-LIKE BLACK HOLES

We consider the following 2D Lifshitz spacetime ansatz

$$ds^2 = -\left( \frac{r^2}{l^2} \right)^{z} h(r) dt^2 + \frac{l^2 dr^2}{r^2 h(r)},$$

(4)

where $z$ is a real number called the Lifshitz parameter, and $l$ is an arbitrary positive length scale. To extract exact solutions, we apply the following transformations,

$$\left( \frac{r^2}{l^2} \right) h(r) \rightarrow g(r), \quad & l \rightarrow r_0, \quad & 2z - 2 \rightarrow s,$$

in which the metric (4) becomes

$$ds^2 = -\left( \frac{r}{r_0} \right)^{s} g(r) dt^2 + \frac{dr^2}{g(r)},$$

(5)

where $s$ and $r_0$ are Lifshitz-like parameter and an arbitrary positive length scale, respectively. In order to obtain exact 2D Lifshitz-like black hole solutions in $F(R)$ gravity, we consider the metric (5), and the field equation (3). In addition, to have a theory renormalizable in two-dimensional spacetime, we must respect to $s \geq 0$ (or $z \geq 1$).

It is worthwhile to mention that in this paper, we intend to extract 2D black hole solutions in Lifshitz-like spacetime for the special class of $F(R)$ gravity which their Ricci scalars are constant, i.e., $R = R_0 = \text{constant}$. Indeed, we use the mentioned method in Ref. [51], which is applicable for $F(R)$ models with two constraints, simultaneously, $F(R_0) = 0$ and $F_R = 0$. This class of $F(R)$ models does not lead to the usual GR since the vacuum field equation of GR identically satisfies $F_R = 1$ with vanishing Ricci scalar. Some of the important features of this model of $F(R)$ gravity are
1) The early-time inflation or late-time accelerated expansion fall in this class of theories. In other words, this theory of gravity can unveil some cosmological scenarios that cannot be described in the frame of standard general relativity [51].

2) This class of \( F(R) \) theories of gravity may explain the anomalous rotation curve typical of spiral galaxies. These models can also mimic the influence of the dark matter halo that is supposed to surround spiral galaxies. In addition, in this case there is a potential for fixing the parameters of the theory by analyzing the rotation curves of a large number of galaxy [51].

3) There are a few solutions in this theory that lead to traversable wormholes. The conceptual advantage of these solutions is the fact that there is no need for unnatural matter sources that violate general relativity energy conditions [51].

4) The charged (A)dS black hole solutions were extracted without considering the matter field and the cosmological constant. Indeed, the electrical charge and the cosmological constant create as a result of the geometry of spacetime in this theory [17].

5) We can find some new solutions to the equations of motion of this class of \( F(R) \) theories of gravity without solving them explicitly. In the case of black holes, the metric is often different from the standard Schwarzschild one.

Using the metric (5), we can obtain the metric function for \( R = R_0 \), from

\[
R_0 = g'' + \frac{3sg'}{2r} + \frac{s(s - 2)g'}{2r^2},
\]

where \( g = g(r) \), \( g' = \frac{dg}{dr} \), and \( g'' = \frac{d^2g}{dr^2} \). Considering the Eq. (11) and after some calculations, we can find an exact solution in the following form

\[
g(r) = -\frac{m}{r^s-1} + \frac{c}{r^s} - \Lambda r^2,
\]

where \( m \) and \( c \) are two integration constants. Also, \( \Lambda \) is a constant which depends on \( R_0 \), as \( \Lambda = \frac{2R_0}{(s+2)r} \). In addition, the Kretschmann scalar can be written in the following form

\[
\mathcal{K} = R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} = \frac{(s + 2)^2 \Lambda^2}{4}.
\]

In order to find a singularity of the obtained solution (7), we calculate all of the components of Ricci and Riemannian tensors. The Ricci tensors are

\[
R_{tt} = R_{rr} = \frac{\mathcal{K}}{\Lambda}, \quad \& \quad R_{tt} = \frac{R_{tt}}{g^2(r) \left( \frac{r}{r_0} \right)^{2s}}, \quad \& \quad R_{rr} = g^2(r) R_{rr},
\]

where \( R_{tt} = -\frac{\mathcal{K}g(r)}{\Lambda} \left( \frac{r}{r_0} \right)^s \) and \( R_{rr} = \frac{\mathcal{K}}{Ag(r)} \). It is evident that \( R_{rr} \) diverges at \( r = 0 \) (i.e., \( \lim_{r \to 0} R_{rr} \to \infty \)), by considering \( s = 0 \) and \( s > 0 \). Besides for the Riemannian tensor, we have

\[
\begin{align*}
\lim_{r \to 0} R_{trr}^t & = \frac{\mathcal{K}}{Ag(r)} \to \infty \quad s = 0 \\
\lim_{r \to 0} R_{trr}^t & = -\frac{\mathcal{K}}{A} \left( \frac{r}{r_0} \right)^{-s} \to \infty \quad s > 0,
\end{align*}
\]

where confirms that there is at least a singularity at \( r = 0 \) for different values of \( s \). In addition, according to the Eq. (7), the asymptotical behavior of the obtained solution is Anti-de Sitter (AdS) or de Sitter (dS) spacetime for \( \Lambda < 0 \) or \( \Lambda > 0 \), respectively.

In order to find roots of the solution (7), we solve \( g(r) = 0 \), which becomes

\[
r_{\pm} = \left( \frac{2\Lambda}{-m \mp \sqrt{m^2 + 4\Lambda c}} \right)^{-2/(2+s)},
\]

which \( r_- \) and \( r_+ \) are the inner and outer (event) horizons, respectively. To have real and positive event horizon \( (r_+) \), we should respect to \( m^2 > -4\Lambda c \), and \( \Lambda < 0 \). As a result, the 2D Lifshitz-like black holes have real and positive event horizons when \( \Lambda < 0 \). In other words, according to our finding, the 2D Lifshitz-like AdS-black holes can be valid in
$F(R)$ gravity. As a result, the solution (7) includes a singularity at $r = 0$, which is covered by an event horizon. In other words, the solution (7) is related to the 2D AdS-black hole in Lifshitz-like spacetime.

To more investigate the obtained 2D Lifshitz-like AdS-black holes in $F(R)$ gravity, we determine the admissible space of the parameters of the metric function $g$. For this purpose, we can do it by studying the extremal black hole criteria

$$g(r_+) = 0 = g'(r_+).$$

The equation (12) indicates that the metric function has one degenerate horizon at $r_+$, which corresponds to the coincidence of the inner and outer horizons. Solving these two equations simultaneously, leads to

$$m = \frac{2c}{r_+^{\frac{s}{s+1}}}, \quad \& \quad \Lambda = \frac{-c}{r_+^{\frac{s}{s+1}}}.$$

Using the equation (13), we can plot the admissible parameter space of metric function in the $\Lambda - m$ plan. The resultant curve is depicted in the left panel of Fig. 1 by the black line and the blue area. In the left panel of Fig. 1, the black line and the blue area denote the extremal limit and black holes with two horizons (inner and event horizons), respectively. Notably, no black hole exists out of the black line and the blue area. Our results show that there are only black holes with the negative values of the cosmological constant ($\Lambda < 0$). In the same way, by using the equation (12), one can determine the admissible parameter space of metric function in the $\Lambda - c$ plan (see the right panel in Fig. 1). We use the following equations for the $\Lambda - c$ plan

$$c = \frac{m}{2} r_+^{\frac{s}{s+1}}, \quad \& \quad \Lambda = \frac{-m}{2r_+^{\frac{s}{s+1}}}.$$

It is clear that the 2D Lifshitz-like black holes only exist for the negative value of the cosmological constant ($\Lambda < 0$).

![FIG. 1: The admissible parameter space of metric function ($g(r)$).](image)

To more investigate the behavior of the obtained solution (7), we plot the metric function versus $r$ in Fig. 2. Similar to Reissner-Nordstrom black holes, by adjusting the values of free parameters, the metric function (7) may have two roots (black hole with an inner horizon and an outer (event) horizon), an extreme root (extremal black hole) or no root (naked singularity: no black hole). Indeed, for $\Lambda < 0$ and $c > 0$, the black hole may have two roots (see the up panels in Fig. 2). For $\Lambda < 0$ and $c < 0$, there is one root (see the down left panel in Fig. 2). On the other hand, for $\Lambda > 0$ with $c > 0$ (or $c < 0$), there is no event horizon. Also, by considering $r_+ = r_-$ in Eq. (14), the 2D Lifshitz-like extreme black hole is obtained as

$$m = \pm 2\sqrt{-\Lambda c},$$

where the above relation reveals that for the 2D Lifshitz-like extreme black holes in AdS spacetime, $c$ has to be positive.

In short, our analysis determines that black holes in the 2D Lifshitz-like spacetime are AdS, and also the extreme case exists when $c > 0$. 
IV. CONSERVED AND THERMODYNAMIC QUANTITIES AND THERMAL STABILITY

In this section, we study the thermodynamic properties of the 2D Lifshitz-like AdS-black holes. To calculate thermodynamic quantities, we start with the Hawking temperature. The Hawking temperature of the black hole on the event horizon \((r_+)\) may be obtained through the use of the definition of surface gravity,

\[
T = \frac{1}{2\pi} \sqrt{-\frac{1}{2} (\nabla_{\mu} \chi_{\nu}) (\nabla^{\mu} \chi^{\nu})},
\]

where \(\chi\) is the Killing vector. It is notable that the mentioned spacetime contains a temporal Killing vector \((\chi = \partial/\partial t)\). Now we can obtain the Hawking temperature of the 2D Lifshitz-like AdS-black hole in the following form

\[
T = -\frac{(s + 2)}{8\pi} \left( r_+ + \frac{c}{r_+^{s+1}} \right) \left( \frac{r_+}{r_0} \right)^{s/2}.
\]

After some calculation, we can find the root of temperature \((17)\), as

\[
r_{\text{root}(T)} = \left( -\frac{\Lambda}{c} \right)^{-1/(s+2)}.
\]
According to the equation (18), in order to have a real and positive root, we have to the following conditions $\Lambda < 0$, and $c > 0$. We plot the temperature (17) versus $r_+$ in Fig. 3.

FIG. 3: $T$ versus $r_+$ for different values of $c$ (up left panel), $\Lambda$ (up right panel), $s$ (down left panel) and $r_0$ (down right panel).

The equation (18) and Fig. 3 show that the temperature root ($r_{root(T)}$) of the 2D Lifshitz-like AdS-black holes becomes large by increasing (decreasing) $c$ and $\Lambda$ ($s$), see the up panels and down left panel in Fig. 3. But for different value of $r_0$, the root of temperature does not change (see the down right panel in Fig. 3).

To obtain the finite mass of 2D Lifshitz-like AdS-black holes, we use the Ashtekar-Magnon-Das (AMD) formula for a far away observer [53, 54]. Considering the AMD approach, the total mass can be written as [53–56]

$$M = m_0 = m r_0^{1-\frac{s}{2}},$$

(19)

where $m$ is related to the geometrical mass ($m = m_0 r_0^{\frac{s}{2}-1}$) of the metric function (7). Using the above relation and the metric function (7) on the event horizon ($g(r_+) = 0$), one can obtain $M$ as a function of $r_+$ as

$$M = \left(\frac{c}{r_+^s} - \Lambda r_+^{s-1}\right) \left(\frac{r_+}{r_0}\right)^{\frac{s}{2} - 1}.$$

(20)

Considering the obtained mass (20), we have plotted the mass of the 2D Lifshitz-like AdS-black holes versus $r_+$, in Fig. 4.
FIG. 4: $M$ versus $r_+$ for different values of $c$ (left panel), $\Lambda$ (middle panel) and $s$ (right panel).

The results in Fig. 4 show that the mass of 2D Lifshitz-like AdS-black hole is positive. Also, there is a minimum mass for this black hole, where changes for different values of $c$ and $\Lambda$ (see the left and middle panels in Fig. 4). But this minimum does not change for different values of $s$ (see the right panel in Fig. 4).

To extract the entropy of the 2D Lifshitz-like AdS-black holes, one can respect the validity of thermodynamics first law. Therefore we can obtain the entropy as

$$\delta S = \frac{1}{T} \delta M,$$

(21)

by inserting Eqs. (17) and (20), in the above equation, the entropy is obtained as

$$S = \int \frac{1}{T} \delta M = \frac{4\pi r_0}{s + 2} \ln \left( \frac{r_+}{r_0} \right)^{s+2} = 4\pi r_0 \ln \left( \frac{r_+}{r_0} \right),$$

(22)

where is independent of the Lifshitz-like parameter $(s)$. It is notable that the entropy in $F(R)$ gravity is extracted by using generalized area law as $S = \frac{A}{4} F_R = \frac{A}{4} (1 + f_R)$ in Ref. [57], where $A$ is the event horizon area of the black holes. But here, we have used two constraints $F(R) = 0$ and $F_R = 0$, and therefore, the generalized area law leads to zero entropy. So, to obtain the entropy, we considered the thermodynamics first law.

The heat capacity is one of the thermodynamical quantities carrying important information regarding the black holes thermal structure in the canonical ensemble. Notably, the discontinuities of this quantity mark the possible thermal phase transitions that the system can undergo. Also, the heat capacity’s (negativity) positivity determines whether the system is thermally (in)stable. In addition, the roots of this quantity may give information about the possible changes between stable/unstable states or bound points [58]. Due to these critical points, we evaluate the 2D Lifshitz-like AdS-black holes’ heat capacity to study the thermal structure of such black holes. Indeed, we will indicate that by using the heat capacity alongside the temperature, we can draw a picture regarding stability/instability of the 2D Lifshitz-like AdS-black holes in $F(R)$ gravity.

Using the Eqs. (17) and (22), one can obtain the heat capacity in the following form

$$C = T \left( \frac{\partial S}{\partial T} \right) = T \left( \frac{\partial S}{\partial r_+} \right) = \frac{8\pi (Ar_+^{s+2} + c)}{(Ar_+^{s+2} - c)(s + 2)},$$

(23)

which has two interesting points, namely the root ($r_{\text{root}}$) and divergence ($r_{\text{div}}$) points, where are

$$r_{\text{root}} = \left( \frac{-\Lambda}{c} \right)^{-1/(s+2)}, \quad \& \quad r_{\text{div}} = \left( \frac{\Lambda}{c} \right)^{-1/(s+2)}.$$
holes with small radii are non-physical and unstable. Nevertheless, for \( r_+ > r_{\text{root}} \), the temperature and the heat capacity of these black holes are positive. In other words, the 2D Lifshitz-like AdS-black holes with large radii (i.e., \( r_+ > r_{\text{root}} \)) are physical and enjoy thermal stability. In addition, the physical and thermal stability areas of these black holes decrease by increasing \( c \) (up left panel in Fig. 5) and \( \Lambda \) (up right panel in Fig. 5), and also by increasing \( s \), this area increases (down left panel in Fig. 5). On the other hand, different values of \( r_0 \), do not affect this area (down right panel in Fig. 5).

![Graphs showing C and T versus r_+ for different values of c, s, A, and r_0.](image)

FIG. 5: \( C \) (thick lines) and \( T \) (thin lines) versus \( r_+ \) for different values of \( c \) (up left panel), \( \Lambda \) (up right panel), \( s \) (down left panel) and \( r_0 \) (down right panel).

As a result, the obtained 2D Lifshitz-like AdS-black holes in \( F(R) \) gravity with large radii are physical and enjoy thermal stability.

In the following, different values of Lifshitz-like parameters are evaluated for the 2D Lifshitz-like AdS-black holes in \( F(R) \) gravity. Indeed, we indicate that the obtained 2D Lifshitz-like AdS-black hole in \( F(R) \) gravity turns to the well-known 2D Schwarzschild AdS-black hole when the Lifshitz-like parameter is zero, \( s = 0 \). Besides, there is a correspondence between this black hole solution and the 2D rotating black hole solution when the Lifshitz-like parameter is two, \( s = 2 \).
A. Case $s = 0$: 2D Schwarzschild black holes

The 2D Schwarzschild AdS-black holes are obtained by adjusting $s = 0$ in the metric function (7). In other words, by replacing $s = 0$ in Eq. (7), we have

$$g_{s=0} (r) = -mr + c - \Lambda r^2,$$

(25)

where the solution (25) is known as 2D black holes of the Schwarzschild-AdS spacetime when $c = 1$ [50]. So, we have

$$g_{s=0} (r) = -mr + 1 - \Lambda r^2.$$  

(26)

Considering the obtained solution (Eq. (26)), we look for the essential singularity(ies). The Ricci and the Kretschmann scalars and also $tt$-component of the Ricci tensor are given

$$R_0 = 2\Lambda, \quad & \quad K = \Lambda^2, \quad & \quad R_{tt} = -\frac{\Lambda}{mr - 1 + \Lambda r^2},$$

(27)

which $\lim_{r \to 0} R_{tt} \to \infty$. So, there is a curvature singularity at $r = 0$.

Two roots of the solution (26), are given by $g(r) = 0$, as

$$r_{\pm} = \frac{-m \mp \sqrt{m^2 + 4\Lambda}}{2\Lambda}.$$  

(28)

Another interesting result is the existing two roots for the 2D Schwarzschild AdS-black holes. As we know, there is one root for Schwarzschild black holes in four-dimensional spacetime, whereas the 2D Schwarzschild-like AdS black holes have two roots (similar to Reissner-Nordström black holes). In order to study the 2D Schwarzschild-like AdS-black holes in more details, we plot $g_{s=0} (r)$ versus $r$ in Fig. 6.

![Figure 6: $g_{s=0} (r)$ versus $r$ for $\Lambda = -1$, and different values of $m$.](image)

Figure. 6 shows that by increasing the parameter $m$, the black holes have two horizons. In other words, massive 2D Schwarzschild-like AdS-black holes may have two horizons.

We calculate the Hawking temperature, mass, entropy, and the heat capacity of the 2D Schwarzschild-like AdS-black holes in the following forms

$$T_{s=0} = \frac{1 + \Lambda r_+^2}{4\pi r_+}, \quad & \quad M_{s=0} = \left(1 - \Lambda r_+^2\right) \left(\frac{r_0}{r_+}\right),$$

$$S_{s=0} = 4\pi r_0 \ln \left(\frac{r_+}{r_0}\right), \quad & \quad C_{s=0} = \frac{4\pi \left(\Lambda r_+^2 + 1\right) r_0}{\Lambda r_+^2 - 1}.$$  

(29)
To study the behaviors of temperature, mass, and heat capacity, we plot Fig. 7.

Our findings in Fig. 7 indicate that the 2D Schwarzschild-like AdS-black holes with large radii are physical and enjoy thermal stability. In other words, for \( r_+ > r_{\text{root}} \), the temperature and the heat capacity are positive. But for \( r_+ < r_{\text{root}} \), the temperature and the heat capacity of these black holes are negative.

**B. Case \( s = 2 \): 2D rotating-like black hole**

By replacing \( s = 2 \), in the presented solution (7), we have

\[
 g_{s=2} (r) = -m - \Lambda r^2 + \frac{c}{r^2}. \tag{30}
\]

It is worthwhile to mention that there is an interesting correspondence between the black hole solution (30) and the 2D rotating black hole (which is obtained from an appropriate, effective action for looking at \( S^- \)wave scattering off a spinning BTZ black hole [59]) when \( c = \frac{J^2}{4} \). So, by considering \( c = \frac{J^2}{4} \), the solution (30) turns to

\[
 g_{s=2} (r) = -m - \Lambda r^2 + \frac{J^2}{4r^2}. \tag{31}
\]

The Ricci and the Kretschmann scalars are given

\[
 R_0 = 8\Lambda, \quad \& \quad K = 4\Lambda^2, \tag{32}
\]

and also \( tt \)-component of the Ricci tensor is \( R_{tt} = \frac{16\Lambda r^2}{4m r^2 + J^2 - 4\Lambda r^4} \), in which indicates that there is a curvature singularity at \( r = 0 \), because the Ricci tensor diverges at \( r = 0 \) (i.e., \( \lim_{r \to 0} R_{tt} \to \infty \)).

Fig. 8 shows that by increasing (decreasing) the parameter \( m \) (\( J \)), the black holes may have two horizons. In other words, massive 2D rotating-like AdS-black holes (or 2D slowly rotating-like AdS-black holes) may have two horizons.

The Hawking temperature, mass, entropy, and also the heat capacity of the 2D rotating-like black holes are obtained as

\[
 T_{s=2} = -\frac{(J^2 + 4\Lambda r_+^4) \left( \frac{r_+}{r_0} \right)}{8\pi r_+^3}, \quad \& \quad M_{s=2} = \frac{J^2 - 4\Lambda r_+^4}{4r_+^2}, \tag{33}
\]

\[
 S_{s=2} = 4\pi r_0 \ln \left( \frac{r_+}{r_0} \right), \quad \& \quad C_{s=2} = \frac{2\pi r_0 \left( J^2 + 4\Lambda r_+^4 \right)}{J^2 - 4\Lambda r_+^4}.
\]
The behavior of temperature, mass, and entropy in Fig. 9 indicate that 2D rotating-like AdS-black holes can be physical and enjoy thermal stability in the range $r_+ > r_{\text{root}}$. Another interesting result is related to the rotating parameter effect ($J$). According to the obtained results in Fig. 9, the 2D rapidly rotating-like black holes are physical and enjoy thermal stability in the large radius compared with the slow case.

**V. CONCLUSIONS**

By considering the 2D Lifshitz-like spacetime, the black hole solutions were extracted in $F(R)$ gravity. Our analysis indicated that the 2D Lifshitz-like AdS black holes with large radii could exist in this theory of gravity.

Next, the thermodynamic quantities such as temperature, mass, and entropy were obtained. The temperature and mass of the 2D Lifshitz-like AdS black holes with large radii were positive simultaneously. In other words, the 2D Lifshitz-like large AdS black holes were physical. The heat capacity was evaluated for the 2D Lifshitz-like black holes in $F(R)$ gravity. The results indicated two critical points: i) the existence of one root for the heat capacity. In this case, the small black holes were non-physical and unstable. But the 2D Lifshitz-like black holes with large radii were...
physical and enjoyed thermal stability. ii) although there was a divergent point for the heat capacity. But it could not be real since the cosmological constant has to be negative. Our results indicated a phase transition for the 2D Lifshitz-like black holes between unstable non-physical and stable physical cases. Briefly, according to the behavior of temperature and heat capacity, the large AdS black holes in the 2D Lifshitz-like spacetime were physical and enjoyed thermal stability.

In addition, the 2D Lifshitz-like black holes in $F(R)$ gravity might be recovered two other interesting black holes by adjusting the Lifshitz-like parameter. In other words, there was a correspondence between the 2D Lifshitz-like black holes in $F(R)$ gravity with Schwarzschild and or rotating-like AdS black holes. These black hole solutions are:

I) the 2D Schwarzschild AdS black holes: the 2D Lifshitz-like AdS black holes were reduced to the 2D Schwarzschild black holes when the Lifshitz-like parameter was zero ($s = 0$). For these black holes, the thermodynamic quantities and heat capacity are obtained. The results revealed that massive 2D Schwarzschild black holes might encounter two horizons.

II) the 2D rotating-like AdS black holes: these solutions are extracted by adjusting $s = 2$. There were two horizons (inner and outer horizons) for massive or slowly rotating-like AdS black holes in 2D Lifshitz-like spacetime.

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Data Availability

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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