A double-flux-quantum amplifier with a single flux-biasing line

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Abstract. A Double-Flux-Quantum (DFQ) amplifier can multiply its input voltage with quantum accuracy using DFQ generation at under-damped Josephson junctions. We conducted research on digital-to-analogue converters that were implemented using DFQ amplifiers for metrological applications. Conventional DFQ amplifiers were usually operated using two flux-biasing lines. In this study, we redesigned the circuit layout to realize operation using a single flux-biasing line. We designed a new layout of a 20-fold DFQ amplifier and fabricated it using a 2.5-kA/cm² Nb/AlOₓ/Nb process. The measurement was performed in liquid He. The Single-Flux-Quantum (SFQ) pulse train was fed using an over-biasing method, and the input-output characteristics are acquired using an oscilloscope. When the magnetic flux bias current $I_{fb1}$ was not supplied, the maximum input voltage $V_{inmax}$ was 68 μV, which was 4.2 times larger than the value of 17 μV that was observed in our previous study. $V_{inmax}$ was enhanced to 120 μV when we supplied an $I_{fb1}$ of 1.00 mA.

1. Introduction

Several studies are being conducted to investigate the development of next-generation AC voltage standards based on applying the AC Josephson effect using several Josephson circuits including single-flux-quantum (SFQ) digital-to-analogue (D/A) converters [1–4]. One example of this is a frequency-modulation type SFQ-D/A converter [2]. This method is proposed as a means to synthesize sinusoidal 10–100 mV voltage waves at 10–100 MHz frequencies [5, 6]. However, so far, the maximum achieved output of the AC waveform generated by SFQ-D/A converters were 2.5 mVpp [4]. In addition to the SFQ-D/A converters, the AC programmable Josephson voltage standards (ACPJVS) and pulse-driven JVS are also candidates for next-generation AC voltage standards [5, 6]. Although ACPJVS generates large voltages that can at times be as high as 10 V, it also suffers from transient errors. Pulse-driven JVS depicts technical difficulties in case of high-frequency packaging. However, even though the output voltages of SFQ-D/A converters are small, they can negate the limitations of other JVS systems [5, 6].

![Figure 1. Block diagram of a frequency-modulation SFQ-D/A converter.](image-url)
The main circuit components of a frequency-modulation SFQ-D/A converter include a DC-to-SFQ (DC/SFQ) converter, a variable pulse number multiplier (V-PNM), and a voltage multiplier (VM), as depicted in figure 1. The DC/SFQ converter varies the reference AC signal of frequency $f_{\text{ref}}$ into an SFQ pulse train that has a repetition frequency of $f_{\text{ref}}$. Further, the V-PNM multiplies each SFQ pulse by $\ell(t)$ according to the digital code. In other words, the SFQ repetition frequency is modulated. Meanwhile, the VM multiplies the output voltage of $\Phi_0 \cdot f_{\text{ref}} \cdot \ell(t) \cdot N$ times. The final output voltage is expressed using the following equation:

$$V = \Phi_0 \cdot f_{\text{ref}} \cdot \ell(t) \cdot N,$$  \hspace{1cm} (1)

where the flux quantum $\Phi_0 = h/2e$ is a physical constant that is obtained using the Planck constant $h$ and the elementary charge $e$, whereas $\ell(t)$ and $N$ are integers. The output voltage determined using equation (1) can depict the quantum accuracy, which is similar to that observed in case of conventional Josephson voltage standards.

Therefore, a VM is also required to achieve quantum accuracy in case of voltage multiplication. A Double-Flux-Quantum (DFQ) amplifier is one of the VM [7]. The DFQ amplifier is a superconducting voltage multiplication circuit that utilizes DFQ generation at an under-damped Josephson junction (JJ). Since voltage is multiplied by increasing the number of flux quanta, the voltage multiplication of quantum accuracy is attainable. Moreover, the DFQ amplifier depicts an additional advantage as compared to other approaches because it operates with constant bias currents even if the voltage multiplication factor is increased by stacking the fundamental circuit cells.

As the AC Josephson effect describes, the voltage across a Josephson junction is proportional to the repetition frequency of SFQ pulses. We have fixed our target operation frequency for the DFQ amplifier to 20.0 GHz of which the corresponding Josephson voltage is 41.4 μV [8–10]. These values are high enough to be used with our recent V-PNM research where the output frequency was 16.6 GHz [11]. In addition, for stable circuit operation, the DFQ amplifier should have bias margins (operation margins for the bias current $I_{sb}$) of ±20% or more at 20 GHz.

In all the tests that were conducted so far, a critical-current-controlled type ($I_c$-type) 100-fold DFQ amplifier demonstrated accurate operation for an $f_{\text{in}}$ value of 20.0 GHz or higher [4, 8]. As an alternative to the $I_c$-type, a phase-damping-controlled type ($\beta_c$-type) DFQ amplifier was also proposed as a solution [12] where $\beta_c$ is the McCumber parameter defined as $2 \pi I_c R^2 C / \Phi_0$[13]. Here, $I_c$, $R$, and $C$ are the junction critical current, the resistance in parallel to the junction, and the junction capacitance, respectively. Numerical simulations predicted that the $\beta_c$-type DFQ amplifier depicted wider operating margins for $f_{\text{in}}$ at 20.0 GHz. However, we did not confirm the correct voltage multiplication in experiments at $f_{\text{in}} \geq 20$ GHz.

In this study, we present our refinements of the circuit parameters and the physical layout of the $\beta_c$-type DFQ amplifier. Moreover, the redesigned $\beta_c$-type DFQ amplifier works using a single flux-biasing line instead of dual lines.

2. Two types of DFQ amplifiers

2.1. Operating principle of the DFQ amplifier

The DFQ amplifier comprises 2-junction loops (2JLs) that work as Josephson transmission lines, 3-junction loops (3JLs) that generate DFQs through under-damped JJs, and flux bias lines. Figures 2 and 3 depict the equivalent 3JLs circuits that were employed in $I_c$-type and $\beta_c$-type DFQ amplifiers, respectively. The hourglass symbols indicate the over-damped JJs (JJ.A and JJ.C in $I_c$-type, JJ.A in $\beta_c$-type), whereas those with crosses indicate under-damped JJs (JJ.B in $I_c$-type, JJ.B and JJ.C in $\beta_c$-type). By definition, an over-damped junction is where $\beta_c \leq 1$, whereas an under-damped junction is where $\beta_c >> 1$. An input SFQ pulse is fed through JJ.A to JJ.B, which causes the generation of DFQ. The DFQ generation at JJ.B allows the opposing SFQ to be induced in the 3JL. The opposite SFQ switches from
JJ.C and propagates to the subsequent 3JL (or a 2JL at the end of the DFQ amplifier). A malfunction of the DFQ amplifier indicates that the opposing SFQ flows backwar d through JJ.A instead of JJ.C. Therefore, it is important to determine the circuit parameters not only to make JJ.B generate the DFQ but also to make the opposite SFQs to switch to JJ.C instead of JJ.A. The difference between two DFQ amplifiers that is depicted in figures 2 and 3 is the control method that is used to control the propagation direction of opposing SFQs. Various parameters, including the $\beta_c$ of the Josephson junction, are illustrated in Table 1.

| Parameter        | $I_c$-type | $\beta_c$-type (past activity[12]) | $\beta_c$-type (this research) |
|------------------|------------|-----------------------------------|-------------------------------|
| $L_1$ (pH)       | 1.6        | 2.9                               | 2.9                           |
| $L_2$ (pH)       | 1.9        | 2.1                               | 0.68                          |
| $L_3$ (pH)       | 0.7        | 0.62                              | 0.62                          |
| $L_4$ (pH)       | -          | -                                 | 1.5                           |
| JJ.A $I_c$ (mA)  | 0.24       | 0.1                               | 0.1                           |
| JJ.A $R$ (Ω)     | 1.6        | 0.51                              | 0.58                          |
| JJ.A $\beta_c$   | 0.98       | 0.018                             | 0.018                         |
| JJ.B $I_c$ (mA)  | 0.22       | 0.21                              | 0.21                          |
| JJ.B $\beta_c$   | > 42000    | > 42000                           | > 42000                       |
| JJ.C $I_c$ (mA)  | 0.14       | 0.13                              | 0.13                          |
| JJ.C $R$ (Ω)     | 2.8        | 33                                | 35                            |
| JJ.C $\beta_c$   | 1          | 120                               | 135                           |

2.2. DFQ Amplifier of the critical-current-controlled type ($I_c$-type)

The first proposed DFQ amplifier belonged to the $I_c$-type that is depicted in figure 2. It has already been implemented in frequency-modulation SFQ-D/A converters [4]. The critical current of JJ.C was fixed to be smaller than that of JJ.A to ensure that the opposing SFQ switches to JJ.C. The two magnetic flux bias lines of $I_{b1}$ and $I_{b2}$ are essential for the $I_c$-type DFQ amplifier. All parameters of $I_c$-type DFQ

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**Table 1.** Parameters of the $I_c$-type and $\beta_c$-type DFQ amplifiers.

**Figure 2.** Equivalent circuit of a 3JL used in an $I_c$-type DFQ amplifier developed in the previous work.

**Figure 3.** Equivalent circuits of a 3JL used in a $\beta_c$-type DFQ amplifier designed in this work.
amplifier shown in Table 1 were used in the previous research. In this study, we did not redesign $I_c$-type DFQ amplifier because we wanted to focus on the designing of $\beta_c$-type DFQ amplifier.

2.3. DFQ Amplifier of the phase-damping-controlled type ($\beta_c$-type)

We searched for parameters that could expand the operational margins as compared to that of the initial $I_c$-type DFQ amplifier. We established that the operational margin became wider when the propagation direction control was performed based on the shunting resistor differences. As a result, the $\beta_c$-type DFQ amplifier was proposed as a means to exclude the flux-biasing lines [12]. In this study, we revised the equivalent circuit and device parameters, and the results are illustrated in figure 3. The shunting JJ.A and JJ.C resistors were tuned to make the opposing SFQ propagate through the JJ.C. Further, the 3JL parameters were determined using the SCOPE2 parameter optimization tool [14]. It should be noted that the JJ.C was tuned not to generate DFQ.

3. Layout of the $\beta_c$-type DFQ amplifier with a single magnetic-flux bias line

It should be reemphasized that we set the target values for the bias margins and maximum input frequency of the DFQ amplifier to be $\pm 20\%$ and 20.0 GHz, respectively and that the corresponding Josephson voltage is $41.4 \mu V$. In our previous study, the maximum input voltage was 17.0 $\mu V$ (8.2 GHz) for the $\beta_c$-type 20-fold DFQ amplifier with no magnetic-flux biasing [12].

Initially, we refined the equivalent circuit. We analyzed the cause of difference between the simulation and experimental results. We observed that, in the previous study, the inductance $L_4$ was not included in the equivalent circuit. Therefore, we re-optimized the circuit parameters, including $L_4$, using SCOPE2. The results are summarized in Table 1. In the layout design, InductEX [15] was used to perform inductance extraction of the superconducting electrodes.

In the previous study, we focused on zero flux bias operation. However, we could not demonstrate this for a 20-GHz operation. In this study, we focused on the improvement of the direct bias $I_{sb}$ operation margin; hence, one magnetic flux bias line was implemented again. The other flux bias line, which was less important for the operation, was still excluded.

Test circuits were fabricated using a 2.5-kA/cm$^2$ Nb technology process that was obtained from the National Institute of Advanced Industrial Science and Technology. Figure 4 depicts the micrographs of the $I_c$- and the $\beta_c$-type DFQ amplifiers.

![Figure 4](image_url)
4. Operation verification of the $\beta_c$-type DFQ amplifier with a single magnetic-flux bias line

4.1. Input–output voltage characteristics

A measurement circuit diagram of a 20-fold DFQ amplifier is depicted in figure 5. The over-biasing method was adopted to feed the input SFQ pulse train where a Josephson junction in the input terminal was over-biased to continuously vary its phase. While conducting the measurements, the bias currents $I_{sb}$, $I_{2J}$, and $I_{b1}$ were constantly supplied. The I–O voltage characteristics were acquired with an oscilloscope using 100-fold low-noise preamplifiers.

![Figure 5](image-url)  
**Figure 5.** Measurement set-up for the I–O characteristics of a 20-fold DFQ amplifier.

![Figure 6](image-url)  
**Figure 6.** Relation between the $V_{in}$ and $V_{out}$ that were obtained in experiments. The lower and upper horizontal axes represent the input voltage $V_{in}$ and the corresponding repetition frequency $f_{in}$, respectively. (a) Results for no-flux biasing; this study and the previous work. (b) Results for finite flux biasing; this study and the previous study for the $I_c$-type. $V_{inmax}$ of 85 $\mu$V in the previous study, where the largest value of the measured $V_{in}$ was 89 $\mu$V, was also determined for the relative errors less than 1%.
An example of the experimental results is depicted in figure 6 (a). The dotted line represents the ideal 20-fold voltage multiplication. In this study, we defined the correct voltage multiplication for the relative errors to be within ±1%. The red points are the resultant points using zero magnetic-flux biasing ($I_{b1} = 0$). The maximum input voltage ($V_{\text{inmax}}$) was determined to be 68 μV for which the bias conditions were $I_{b1} = 0.00 \text{ mA}$, $I_{ab} = 0.410 \text{ mA}$, and $I_{2J} = 1.52 \text{ mA}$. The corresponding repetition frequency of the SFQ pulse train, which was determined by the AC Josephson effect, was 32.7 GHz. This result was 4.2 times larger than our previous result (17.0 μV and 8.2 GHz, as depicted by the blue points in figure 6 (a)) [12].

In addition, the characteristics with finite magnetic-flux bias ($I_{b1} = 1.00 \text{ mA}$) are depicted in figure 6 (b) as blue points. $V_{\text{inmax}}$ was 120 μV of which the corresponding repetition frequency was 58.0 GHz under a biasing condition of $I_{b1} = 1.00 \text{ mA}$, $I_{ab} = 0.379 \text{ mA}$, and $I_{2J} = 1.52 \text{ mA}$. It was approximately 1.4 times larger than that of the $I_c$-type at 85 μV. These results are plotted as green points in figure 6 (b) [16].

4.2. Bias margins

The I–O characteristics were measured for various biasing conditions. We extracted the operable region for an input voltage exceeding 41.4 μV (20.0 GHz), which is plotted on the $I_{b1}$-$I_{ab}$ plane (as depicted in figure 7). The vertical axis is the direct bias $I_{ab}$, whereas the horizontal axis is the normalized magnetic-flux bias $I_{b1}M_1/\Phi_0$. The experimental results for the upper and lower limits are plotted with red circles (●), whereas the simulation results are plotted with black squares (■).

We initially look into the numerical results. Moderately wide margins, including extremes close to ±20.8% for $I_{ab}$, were identified near zero magnetic-flux biasing. It indicates that this DFQ amplifier could inherently work without magnetic-flux biasing. The operational region along the horizontal axis ranges from −0.4 to +0.55.

Further, we observe the experimental results. It is obvious that the experimental operation regions that are depicted in figure 7 are quite reduced in comparison with that of the numerical results. The bias margin is ±13.6% at the maximum. There are two possible reasons for these reductions. First, it is conceivable that the layout design and/or the inductance extraction were imperfect. Specifically, for the results along the horizontal axis, it should be noted that the mutual inductance $M_1$ was numerically, as opposed to experimentally, determined only by InductEX [15]. The 0.36-pH value was the result obtained using InductEX, whereas 0.47 pH was a 30% larger value than that was empirically obtained in our previous experiments using similar layouts. For both the $M_1$ values, the operational margins along the horizontal axis were reduced by more than 30%. Second, it could be a potential reason for the presence of parameter variations in chip fabrication.

From another point of view, this study demonstrated correct voltage multiplication for an input voltage of 41.4 μV (20.0 GHz) under a zero magnetic flux-biasing condition where the $I_{ab}$ margin was ±8.1%. This proves that the design of a $\beta_c$-type DFQ amplifier has improved.

![Figure 7. Bias margins for $V_{\text{in}} \geq 40.1 \text{ μV}$ plotted on the $I_{ab}$-$I_{b1}M_1/\Phi_0$ plane.](image-url)
5. Conclusion
In this study, the design and operational verification of the phase-damping-controlled-type DFQ amplifier were described. The number of magnetic-flux biasing lines that was used in our previous research was reduced from 2 to 1. It was confirmed that the maximum input voltage was 68 μV with no magnetic-flux bias, whereas the maximum input voltage with a finite magnetic-flux bias was 120 μV. This was a significant improvement over that observed in our previous result (85 μV for the Ic-type). The maximum Isb margin for an input voltage larger than 41 μV was ±13.6% in case of finite magnetic-flux biasing, whereas that for zero magnetic-flux biasing was ±8.1%.

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References
[1] Hamilton C A 1992 Josephson voltage standard based on single-flux-quantum voltage multiplier IEEE Trans. Appl. Supercond. 2 139–42
[2] Semenov V K 1993 Digital to analog conversion based on processing of the SFQ pulses IEEE Trans. Appl. Supercond. 2 2637-40
[3] Maezawa M, Hirayama F and Suzuki M 2005 Rapid single flux quantum digital-to-analog converter for ac voltage standard Physica C 426-31 1674-9
[4] Mizugaki Y, Takahashi Y, Shimada H and Maezawa M 2014 9-bit superconductive single flux quantum digital-to-analog converter Electronics Letters 50 1637–9
[5] Maruyama M 2008 Review on Josephson voltage standard AIST measurement standard report 8 263-278 (in Japanese)
[6] Benz S P, Hamilton C A 2004 Application of the Josephson Effect to Voltage Metrology Proceedings of the IEEE 92 1617-29
[7] Herr Q P 2005 Stacked double-flux-quantum output amplifier IEEE Trans. Appl. Supercond. 15 259–62
[8] Sato Y, Moriya M, Shimada H, Mizugaki Y and Maezawa M 2013 Design and operation of 1000-fold voltage multiplier based on double-flux-quantum amplifier Physics Procedia 45 221–4
[9] Watanabe T, Takahashi Y, Shimada H, Maezawa M and Mizugaki Y 2015 4-bit bipolar triangle voltage waveform generator using single-flux-quantum circuit Phys. Procedia 65 213–6
[10] Mizugaki Y, Urai Y and Shimada H 2017 Thermally-fluctuated single-flux-quantum pulse intervals reflected in input-output characteristics of a double-flux-quantum amplifier J. Phys. Conf. Ser. 871 012066
[11] Arai Y, Watanabe T, Sawada K, Shimada H and Mizugaki Y 2016 Improvement of SFQ pulse frequency in an SFQ-D/A converter (The 77th JSAP Autumn Meeting, Niigata) 15p-D61-12 Sep. 13-16 (in Japanese)
[12] Mizugaki Y and Watanabe T 2016 Design and operation of a double-flux-quantum amplifier excluding flux bias lines IEEE Trans. Appl. Supercond. 26 1301104
[13] McCumber D E 1968 Effect on dc Voltage-Current Characteristics of Superconductor Weak-Link Junctions J. phys. Appl. Phys. 39 3113-8
[14] Mori N, Akahori A, Sato T, Tekeuchi N, Fujimaki A and Hayakawa H 2001 A new optimization procedure for single flux quantum circuits Physica C 357-60 1557-660
[15] Fourie C J, Wetzstein O, Kunert J and Meyer H G 2013 SFQ circuits with ground plane hole-assisted inductive coupling designed with InductEx IEEE Trans. Appl. Supercond. 23 1300705

[16] Urai Y, Watanabe T, Takahashi Y and Mizugaki Y 2013 Mutually-coupled dc/SFQ converter tested using an on-chip pulse generator (Superconducting SFQ VLSI Workshop, Tsukuba) p 54 Nov. 21-22