The Boundaries of the GW Vir Stars in the Effective Temperature - Surface Gravity Domain

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Abstract. We derive the theoretical red edge of the pulsating GW Vir stars by using full evolutionary calculations that involve mass loss and diffusion. We put the emphasis on the fact that the specific mass loss law used in the evolutionary computations determines the red edge’s position. By combining this specific property with the observed location of the red edge in the effective temperature-surface gravity domain, we obtain interesting constraints on possible mass loss laws for PG 1159 stars.

1. Astrophysical Context

The GW Vir stars constitute a class of pulsating stars descending from post-Asymptotic Giant Branch (post-AGB) evolution. They exhibit low-amplitude multiperiodic luminosity variations caused by low degree, intermediate-order nonradial \( g \)-mode oscillations. Their observed periods are found in the range from 300 s to upward of 5000 s. Although there has been some debate in the past as to the exact cause of the pulsational instabilities, it is now fair to say that there can no longer be any doubt that the physical process responsible for the excitation of \( g \)-modes in these stars is a \( \kappa \)-mechanism associated with the partial ionization of the K-shell electrons in carbon and oxygen, two important constituents of the envelopes of these stars (see, e.g., Saio 1996, Gautschy 1997, Quirion, Fontaine, & Brassard 2004, and Gautschy, Althaus, & Saio 2005). The GW Vir pulsators are found in a very wide area of the \( \log g - T_{\text{eff}} \) diagram covering the ranges \( 5.5 \leq \log g \leq 7.5 \) and \( 80,000 \, \text{K} \leq T_{\text{eff}} \leq 170,000 \, \text{K} \). This area overlaps with two spectroscopic classes: the Wolf-Rayet Central Stars of Planetary Nebula ([WCE]) and the hottest white dwarfs, the PG 1159 stars. As discussed in the excellent review paper presented recently by Werner & Herwig (2006), quantitative spectroscopy and detailed evolutionary calculations imply very strongly a tight evolutionary connection following the sequence [WCE]→PG 1159. In this context, it has been suggested that pulsating stars from these two spectral types and sub-types should all be included under the unique vocable of GW Vir stars \cite{Quirion06}.

The one factor that differentiates the PG 1159 from the [WC] stars despite their similar atmospheric compositions is the presence of much stronger winds in the latter. The wind produces wide carbon emission lines, giving the [WC] type its characteristic spectral signature. With time and decreasing luminosity, the
magnitude of the outgoing wind diminishes, thus weakening the carbon emission lines in the [WC] stars and letting the PG 1159 features appear in the spectra of the [WC]-PG 1159 transition objects. A further decrease of the mass loss and wind completes this scenario whereby the wide carbon emission lines disappear, leading to a full characteristic PG 1159 spectrum.

The chemical composition of the envelope of GW Vir stars is critical for the determination of their stability. It has been shown in Quirion et al. (2006) that the temperature of the blue edge drawn for this class is directly dependent on the quantity of carbon and oxygen present in their envelope. In general, the blue edge moves from lower to higher temperatures as the quantity of carbon and oxygen increases. When considering the dispersion in atmospheric parameters and in chemical composition in GW Vir stars, we must understand that the blue edge is a "fuzzy" notion for this type of pulsating stars. The positions of the blue edges for GW Vir stars, calculated for a large spectrum of chemical composition, can be found in Quirion et al. (2006).

The evolutionary connection following the sequence PG 1159→DO, where the spectral type DO belongs to He-dominated atmosphere white dwarfs cooler than $\sim 80,000$ K, is also strongly supported by quantitative spectroscopy (Werner & Herwig 2006). The strong wind present in [WC] and PG 1159 stars tends to homogenize their envelope (see, e.g., Chayer et al. 1997; Unglaub & Bues 2001), but as this wind weakens with lowering luminosity, it progressively losess its homogenizing capacity and gravitational settling takes over and will cause carbon and oxygen to precipitate while helium will float at the surface of the star. As carbon and oxygen are responsible for the $\kappa$-mechanism in these stars, it is expected that gravitational settling will ultimately stop pulsational driving in GW Vir stars, drawing the red edge of this class. We discuss here representative calculations showing the effects of the interaction between diffusion and mass loss on the position of the red edge of GW Vir stars.

2. Results and Discussion

We used a improved version of the evolutionary code based on a finite element method introduced in Fontaine et al. (2001) to model the effects of diffusion and mass loss on the red edge's position. Diffusion is treated by resolving the following mass conservation equation

$$\frac{\partial n_i}{\partial t} = -\frac{1}{r^2} \frac{\partial}{\partial r} r^2 [n_i (w_i + W)],$$

where $n_i$, $t$, $r$, $w_i$, $W$ are respectively, the number abundance of element $i$ (= He, C, O in the case of GW Vir stars), the time, the radial coordinate, the velocity of element $i$, and the wind velocity. The microscopic velocity $w_i$ of each element is calculated from first principles (effects of gravity, composition gradient, temperature gradient, and electric field) as explained in Iben & MacDonald (1985).

On the other hand, we cannot compute the macroscopic wind speed $W$ from first principles, as there is no complete physical theory addressing this problem yet. The usual method to model a stellar wind is to scale the mass loss of a star on its luminosity. When a suitable mass loss law has been selected, we link it to
the wind speed $W$ via flux conservation

$$W = \frac{\dot{M}}{4\pi r^2 \rho},$$

where $\dot{M}$ is the mass loss rate and $\rho$ the density. In this equation, the mass loss is assumed to be small enough to have negligible effects on the total mass of the star.

The mass loss laws used in the present calculation are given in Table 1. WM1 is a fit to the mass loss rates measured in five PG 1159 type stars — three of which are also GW Vir stars — published in Koesterke & Werner (1998). WM2 is a fit to the five previous stars plus nine nuclei of planetary nebulae of similar luminosity found in Pauldrach et al. (2004). For WM3, we chose the theoretical model used for post-AGB calculations in Blöcker (1995). Finally, WM4 is an empirical law derived in such a way that the theoretical red edge falls directly on the empirical red edge observed for the GW Vir class.

### Table 1. The Four Wind Models

| Model | $\dot{M} [M_\odot \text{yr}^{-1}]$ |
|-------|---------------------------------|
| WM1   | $1.14 \times 10^{-11} L^{0.93}$ |
| WM2   | $1.82 \times 10^{-13} L^{1.36}$ |
| WM3   | $1.29 \times 10^{-15} L^{1.86}$ |
| WM4   | $1.00 \times 10^{-17} L^{2.38}$ |

In Figure 1, we chose four temperatures along our $0.6 M_\odot$ evolutionary track, under the effect of diffusion and WM3, to represent, in a chronological way, the evolution of the internal structure of our model and the formation of the red edge. We see that, as the star cools down, the carbon and oxygen abundance in its driving/damping layers get lower. This reduction leads to a smoothing of the opacity bump created by the partial ionization of C/O, making the $\kappa$-mechanism, responsible for the pulsations, less and less efficient. The red edge is drawn in between the 69,877 K and the 67,865 K models. It is interesting to note that the last unstable model, at 69,877 K, shows only a small amount of carbon and essentially no oxygen at its surface with $X(\text{He}) \simeq 0.97$ and $X(C) \simeq 0.03$. This composition ratio would prevent a GW Vir model with a strictly uniform envelope composition (from the surface to below the driving/damping region) to be unstable. We come back on this phenomenon in the following paragraphs. The coolest model, stable at 44,694 K, has been transformed into a DO white dwarf. Indeed, helium completely fills the envelope from the surface down to $\log q \sim -7.0$, below the driving region. The residual opacity bump at $\log q \sim -6.0$ only marks the transition from almost pure helium to mixed He/C/O in deeper layers.

The qualitative picture of gravitational settling with the WM3 model is representative of what is happening with the other wind models. With the more important mass loss rates of WM1 and WM2, however, the settling of C and O is comparatively slowed down and the red edge is pushed to lower temperatures.
Figure 1. Four models taken along a $0.6 M_\odot$ track showing the effects of diffusion on the chemical composition of the envelope of a star undergoing mass loss according to the WM3 law. The composition of the envelope at the start of the calculation was $X(\text{He}) = 0.40$, $X(\text{C}) = 0.40$ and $X(\text{O}) = 0.20$ in mass units. The solid curves associated with helium, carbon and oxygen (in red, green and blue respectively in the electronic version) depict the progressive rise of the helium content at the surface, prompted by gravitational settling of carbon and oxygen, as the mass loss decreases with decreasing luminosity. The two models in the top panel, at $T_{\text{eff}} = 88,342$ K and 69,877 K, are unstable, as shown by the positive value of the work integral $W$ (running from left to right) at the surface of the models. The driving/dampening region, defined by the zone between the two vertical dashed lines, indicates where the magnitude of the derivative of the work integral $dW/d\log q$ is greater than 1% of its maximal value (normalized at either +1 or −1). The maximum of the driving occurs, as is always the case for a classic $\kappa$-mechanism, near the maximum of the opacity bump. The bump is seen in the Rosseland opacity $\kappa$ around $\log q \sim -9.5$. For the two stable models in the lower panels, at $T_{\text{eff}} = 67,865$ K and 44,694 K, we see negative values of the work integral at the surface of the star along with the vanishing opacity bump. The dotted extensions of the $\kappa$ curves mark the extent of the atmosphere, defined here as those layers with an optical depth $\tau_{\text{Ross}} < 100$. 
By the same token, a hotter red edge is found for the weaker mass loss rate associated with the WM4 law. We mention that for models initially unstable above 100,000 K, the position of the red edge has only a marginal dependence on the initial chemical composition in the envelope of the model. Thus, the dominant factor affecting the position of the red edge is the mass loss law. A corollary to this affirmation is that the average magnitude of the mass loss in GW Vir stars (and more generally in PG 1159) could be estimated from the actual position of the red edge.

The effects of the mass loss over the position of the red edge is pictured in Figure 2. We have calculated the evolution of three different models having masses of 0.5, 0.55 and 0.6 $M_\odot$. These models were all allowed to evolve under the effects of the WM1, WM2, WM3, and WM4 wind models. We then used our nonadiabatic pulsation code, briefly presented in [Fontaine et al. 1994], to probe the stability of the models along each track. The red edges obtained in the figure are simply fits along the three tracks calculated with the different mass loss laws WM2, WM3 and WM4. The position of the WM1 red edge, around $T_{\text{eff}} = 30,000$ K, is not shown here as it is offscale. As is well known, no PG 1159 stars exists at this low temperature. This temperature is clearly too cool and that particular mass loss law must be abandoned. The same conclusion could be drawn for WM2, but we prefer to set conservatively this law as a maximum value for the magnitude of the mass loss in GW Vir stars, $\dot{M} < \text{WM2}$.

By construction, and as indicated above, we have devised the WM4 model in order to match fairly closely the empirical red edge as defined by the position of the coolest known GW Vir star, PG 0122+200 at 80,000 K. However, it should be noted that the predicted surface composition according to the WM4 model at that effective temperature is highly deficient in carbon and oxygen as compared to the real atmospheric chemical composition of PG 0122+200 ($X(\text{He}) = 0.43$, $X(\text{C}) = 0.39$, and $X(\text{O}) = 0.17$). Hence, it would appear that the WM4 model underestimates the true average mass loss in the GW Vir stars. On the other end, the empirical red edge could also be actually somewhat cooler than the effective temperature of PG 0122+200. In any case, we can use the WM4 model to set a minimum value for the mass loss in GW Vir stars, $\dot{M} > \text{WM4}$.

On the other hand, we see that the WM3 wind model depicted in Figure 1 still permits relatively high abundances of carbon and oxygen in its 88,342 K unstable model, situated close to the empirical red edge. Also, the red edge produced by this wind model is not far, in the log $g - T_{\text{eff}}$ diagram, from the coolest known GW Vir stars. This model with $\dot{M} = \text{WM3}$, is therefore more likely to be representative of the actual red edge of GW Vir stars.

This brief analysis could be extended by taking into account the position of the hottest DO white dwarfs. However, the cornerstone for the accurate determination of the position of the red edge, and perhaps also the precise determination of the actual mean mass loss rate in GW Vir stars, would be the discovery of a low carbon and low oxygen atmosphere PG 1159 or DO star, showing luminosity variations caused by nonradial gravity modes. Such an object would have to reside almost exactly at the red edge itself.
Figure 2. Positions of the known GW Vir stars in the $\log g - T_{\text{eff}}$ diagram. The four spectroscopic types and sub-types of GW Vir are shown along with a representative 0.604 $M_\odot$ track from F. Herwig and the calculated red edge for WM2, WM3 and WM4.

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