Evidence that turbulent superstructures are concatenations of geometrically self-similar coherent motions

Rahul Deshpande\textsuperscript{1}\textsuperscript{†}, Charitha M. de Silva\textsuperscript{2}, and Ivan Marusic\textsuperscript{1}

\textsuperscript{1}Department of Mechanical Engineering, University of Melbourne, Parkville, VIC 3010, Australia
\textsuperscript{2}School of Mechanical and Manufacturing Engineering, University of New South Wales, Sydney, NSW 2052, Australia

(Received xx; revised xx; accepted xx)

We present evidence suggesting that the superstructures in turbulent boundary layers are concatenations of smaller, geometrically self-similar coherent motions. The evidence comes from identifying and analyzing instantaneous superstructures from large-scale particle image velocimetry datasets, capable of capturing streamwise elongated motions extending up to 12 times the boundary layer thickness. Given the challenge in identifying the constituent motions of the superstructures based on streamwise velocity signatures, an approach is adopted that investigates the wall-normal velocity fluctuations within these very long motions, which reveals the constituent motions unambiguously. The conditional streamwise energy spectra of the wall-normal fluctuations, corresponding exclusively to the superstructure region, are found to exhibit the well-known distance-from-the-wall scaling in the intermediate scale range. This indicates geometrically self-similar motions co-exist within the superstructures. The same spectra also exhibit a unique energy enhancement at the large scales, suggesting the spatial organization of self-similar motions (i.e. concatenation) over extended streamwise distances, which plausibly leads to the appearance of superstructures. This interpretation is also confirmed by analyzing synthetically generated flow fields comprising self-similar structures. The association of the superstructures with the self-similar motions is further reinforced by computing conditionally averaged two-point correlations, representing the vertical coherence of the constituent motions, which are found to match with the mean correlations. The mean vertical coherence of the Reynolds-shear-stress carrying motions, which is investigated for the log-region across three decades of Reynolds numbers, is found to exhibit universal distance-from-the-wall scaling. The findings support the prospect for modelling these dynamically significant motions via data-driven coherent structure-based models.

**Key words:** turbulent boundary layers, turbulence modelling, boundary layer control.

† Email address for correspondence: raadeshpande@gmail.com
1. Introduction and motivation

Over the past two decades, the study of high Reynolds number \((Re_\tau \gtrsim \mathcal{O}(10^4))\) wall-bounded flows has become synonymous with very-large-scale motions (VLSMs), also known as ‘superstructures’, which play a predominant role in the dynamics and spatial organization of wall turbulence. Here, \(Re_\tau = \delta U_\tau / \nu\), where \(\delta\) is the boundary layer thickness, \(\nu\) is the kinematic viscosity and \(U_\tau\) is the skin-friction velocity, with the latter two used to normalize the statistics in viscous units (indicated by superscript ‘+’). The superstructures can extend beyond \(20\delta\) in the streamwise direction (Kim & Adrian 1999; Hutchins & Marusic 2007) and also exhibit ‘meandering’ when viewed on a wall-parallel plane (Eich et al. 2020), particularly in the logarithmic region of the flow. Such a large spatial footprint permits these motions to carry significant proportions of the total turbulent kinetic energy and the Reynolds shear stresses of the flow (Liu et al. 2001; Ganapathisubramani et al. 2003; Guala et al. 2006; Balakumar & Adrian 2007). Given that the shear stress is responsible for the wall-normal momentum transfer, this suggests that the VLSMs/superstructures also contribute significantly to the high \(Re_\tau\) turbulent skin-friction drag (Deck et al. 2014). Hence, an improved understanding of the origin of these VLSMs/superstructures, which this study aims to undertake, stands to advance our knowledge in both a fundamental and an applied perspective.

Hutchins & Marusic (2007) used the terminology ‘superstructures’ when referring to the spectrogram of the streamwise velocity fluctuations \((u)\) from a high \(Re_\tau\) boundary layer, as shown in figure 1. The spectrogram presents the premultiplied \(u\)-energy spectra as a function of the viscous-scaled streamwise wavelengths \((\lambda_x^+ = \lambda_x U_\tau / \nu)\) and wall-normal distance \((z_+ = z U_\tau / \nu)\), with \(\lambda_x = 2\pi / k_x\), where \(k_x\) is the streamwise wavenumber. The high \(Re_\tau\) \(u\)-spectrogram is seen to have two prominent peaks. One is located in the inner-region synonymous with the well-documented near-wall cycle (Kline et al. 1967), consisting of high and low-speed viscous-scaled streaks \((\lambda_x^+ \approx 1000)\), which are responsible for intense local production of turbulent kinetic energy. The second peak is in the outer region of the flow (typically in the logarithmic/inertial region), and corresponds to the superstructures, which have a spectral signature at very long wavelengths \((\lambda_x \sim 6\delta)\) and also extend down to the wall (Hutchins & Marusic 2007). Between these two peaks, a nominal plateau is seen in the spectrogram which corresponds to the distance-from-the-wall \((z)\)-scaled eddies coexisting in the log-region; these eddies make up the increased range of scales with increasing \(Re_\tau\). In the literature, these intermediate scaled eddies have been described by various structures or motions, including the large-scale motions (LSMs; Kim & Adrian (1999), Adrian et al. (2000)), uniform momentum zones (UMZs; Meinhart & Adrian (1995), de Silva et al. (2016)), attached eddies (Baars et al. 2017; Marusic & Monty 2019; Hu et al. 2020; Deshpande et al. 2021a) and so forth. In the remainder of this section, for simplicity, we will refer to these motions as LSMs. It should also be noted that the terminology ‘VLSMs’ and ‘superstructures’ have been conventionally associated with the very-large-scale motions in the internal (Kim & Adrian 1999) and external wall-bounded flows (Hutchins & Marusic 2007), respectively. Considering this study focuses solely on zero-pressure gradient turbulent boundary layers, we henceforth refer to either of these structures simply as superstructures.

To date, several studies have investigated the probable mechanisms responsible
for the formation of the superstructures, with two theories hypothesized most often: (i) the formation of superstructures via concatenation of the LSMs (Kim & Adrian 1999; Adrian et al. 2000; Lee & Sung 2011; Dennis & Nickels 2011b), or (ii) the emergence of the superstructures due to a linear instability mechanism (Del Alamo & Jimenez 2006; McKeon & Sharma 2010; Hwang & Cossu 2010). The present study does not focus on comparing and contrasting the likelihood of one mechanism over the other. But rather, it builds upon recent compelling evidence in support of the concatenation mechanism (Wu et al. 2012; Baltzer et al. 2013; Lee et al. 2014, 2019), to investigate the characteristics of the constituent motions forming the superstructures. The formation of superstructures via streamwise concatenation of the relatively smaller motions has been confirmed by several studies conducted across all canonical wall-bounded flows (turbulent boundary layers, channels, pipes), through: (i) investigation of the time evolution of instantaneous flow fields (Lee & Sung 2011; Dennis & Nickels 2011b; Wu et al. 2012; Lee et al. 2019), (ii) statistical analysis of the superstructure formation frequency/population density (Lee et al. 2014) and (iii) spatial correlations of the low-pass filtered velocity fields (Baltzer et al. 2013; Lee et al. 2019). In comparison, few studies have presented similar statistical arguments in favour of the linear instability mechanism. For instance, Bailey et al. (2008) supported the linear instability argument by noting different spanwise widths of the superstructures and LSMs in the inertial region of a turbulent pipe flow. Their estimates, however, were limited to two-point velocity correlations.
reconstructed in a particular wall-parallel plane, which cannot be uniquely associated with the LSMs responsible for the superstructure formation (Deshpande et al. 2020). Considering that superstructures extend down from the log-region to the wall, Deshpande et al. (2021b) reconstructed two-point velocity correlations across two wall-parallel planes located in the near-wall and the log-region. These statistics, which are purely representative of the large ‘wall-coherent’ motions, revealed similar spanwise extents of the coexisting superstructures and LSMs for all canonical wall flows, thereby favouring the concatenation argument.

Despite the substantial support for the concatenation argument, several unanswered questions are still associated with this mechanism. For instance, there is no universal agreement on what facilitates the streamwise concatenation of LSMs to form superstructures. While few studies have associated this with the spanwise alternate positioning of low and high momentum LSMs (Lee et al. 2014), others have conjectured the role played by secondary roll cells (Baltzer et al. 2013; Lee et al. 2019) in favourably organizing the relatively smaller motions. Progress in this regard has been hindered by the lack of understanding of the constituent motions forming the superstructures; for instance, are superstructures purely composed of the inertial $\delta$-scaled motions corresponding to the extreme right end of region II in figure 1? Or do they also comprise of the geometrically self-similar, i.e. $z$-scaled hierarchy of eddies encompassing the entire region II?

In the past, clarifying such information on the constituent motions has not been possible due to the low to moderate $Re_\tau (\lesssim 2000)$ of the experiments/simulations analyzing the concatenation argument, which severely constricts the extent of region (II) in figure 1. This prevents an unambiguous delineation between the $\delta$-scaled and $z$-scaled inertial motions coexisting in region II. However, increased access to high $Re_\tau$ data over the past decade has substantially increased our knowledge of these inertial eddies coexisting in the log and outer regions (Marusic et al. 2010, 2015; Baidya et al. 2017). This has also led to growing acceptance of the existence of the geometrically self-similar attached eddy hierarchy in the inertial region (de Silva et al. 2016; Baars et al. 2017; Hwang & Sung 2018; Hu et al. 2020; Deshpande et al. 2020, 2021a), which can be modelled conceptually (Marusic & Monty 2019). These advancements make it compelling to investigate whether these self-similar inertial motions are associated with the formation of superstructures, a conjecture that has previously shown promising results when implemented in coherent structure-based models (Chandran et al. 2020; Deshpande et al. 2021b). If this conjecture is proven true, then the preferred streamwise alignment of this energy-containing hierarchy of motions (to form superstructures) would have implications on Townsend’s attached eddy hypothesis, which otherwise assumes a random distribution of attached eddies in the flow field (Townsend 1976; Marusic & Monty 2019). The investigation can also help answer the long-standing contradiction (Guala et al. 2006; Balakumar & Adrian 2007; Wu et al. 2012) between: (i) the attached eddy hypothesis, which classifies turbulent superstructures to be ‘inactive’ (Deshpande et al. 2021a), and (ii) instantaneous flow field observations, per which these streamwise elongated motions carry significant Reynolds shear stresses (and hence behave as ‘active’ motions).

To this end, the present study investigates the geometric scalings exhibited by the constituent motions of the superstructures. Experimental data is employed from a moderate to high $Re_\tau$ turbulent boundary layer ($2500 \lesssim Re_\tau \lesssim 7500$), which is an order of magnitude higher than the simulation studies reported previously, to ensure coexistence of a broad range of inertial scales (region II). The dataset comprises of sufficiently resolved large-scale velocity fluctuations acquired in a physically thick
boundary layer via unique, large field-of-view (LFOV) particle image velocimetry (PIV), capturing instantaneous flow fields with an extent of $12\delta$ in the streamwise direction ($x$). In contrast to most studies to date, which have investigated the superstructures by analyzing the large-scale $u$-fluctuations, here we adopt a unique strategy to investigate the wall-normal ($w$) velocity fluctuations within the superstructure region. This is because deciphering smaller constituent $u$-motions from within a larger $u$-motion can be inconclusive, as can be noted from a sample DNS flow field shown in figures 2(a,b). On the other hand, the $w$-fluctuations can bring out the individual constituent motions more distinctly, which is evident from figure 2(c) and will be analyzed here by computing conditional statistics. The clustering/packing of the $u$- and $w$-carrying eddies leading to the appearance of the superstructure in a synthetic field, as shown using an idealized distribution of prograde vortices in figure 2(d,e), suggests the possibility of strong $u$- as well as $w$-correlations extending across large streamwise separations. Such a flow organization, which adds further credibility to the streamwise concatenation hypothesis, will be investigated here via conditional statistics. It is important to note that in the present study, any reference to concatenation henceforth refers to the spatial organization of constituent motions over extended streamwise distances, such as in figures 2(d,e). Given the experimental limitations, the study cannot conclude on the mechanism behind how this spatial organization comes into existence. Also, the terminology ‘attached eddies’ is used here to refer to any eddies/motions scaling with their distance from the wall, and hence is not limited to the eddies physically extending to the wall.
Table 1: Table summarizing the details of the datasets comprising synchronized measurements of $u$- and $w$-fluctuations at various wall-normal locations. $Re_\tau$ for the various PIV datasets is based on $\delta$ estimated at the centre of the flow field (figure 3(a)). Terminology has been defined in §2. $\Delta x^+$ and $\Delta z^+$ indicate viscous-scaled spatial resolution along $x$ and $z$ directions, respectively.

| Measurement | Facility     | $Re_\tau$ | $\nu/U_\tau$ (in $\mu$m) | $\Delta x^+$ | $\Delta z^+$ | FOV $(x \times z)$          | Reference                  |
|-------------|--------------|-----------|----------------------------|--------------|--------------|----------------------------|----------------------------|
| LFOV PIV    | HRNBLWT     | 2500      | 42                         | 26           | 26           | $12\delta \times 1.2\delta$ | de Silva et al. (2015, 2020) |
| LFOV PIV    | HRNBLWT     | 5000      | 22                         | 52           | 52           | $12\delta \times 1.2\delta$ | de Silva et al. (2015, 2020) |
| LFOV PIV    | HRNBLWT     | 7500      | 15                         | 75           | 75           | $12\delta \times 1.2\delta$ | de Silva et al. (2015, 2020) |
| PIV         | HRNBLWT     | 14500     | 24                         | 37           | 37           | $2\delta \times 0.4\delta$  | de Silva et al. (2014)      |
| Sonics SLTEST |            | $O(10^6)$ | 78                         | 1000         | 90           | –                          | Hutchins et al. (2012)      |

2. Experimental datasets and methodology

2.1. Description of the experimental datasets

Five multipoint datasets are used from previous high $Re_\tau$ experiments (table 1). Four of these are acquired via two-dimensional (2-D) two-component PIV in the Melbourne wind tunnel (HRNBLWT; Marusic et al. (2015)) and span the $Re_\tau$ range ~2500–14500. The test section of this wind tunnel has a cross-section of 0.92 m × 1.89 m, and has a large streamwise development length of ~27 m, with maximum possible free-stream speeds ($U_\infty$) of up to 45 ms$^{-1}$. Such a large-scale facility permits the generation of a sufficiently high $Re_\tau$ canonical boundary layer flow facilitated by substantial increment in its boundary layer thickness, along its long streamwise fetch. This capability is leveraged in the four PIV datasets employed in the present study, which will be described next.

Three of the PIV datasets comprise snapshots of very large streamwise wall-normal flowfields of a turbulent boundary layer ($x \times z \sim 12\delta \times 1.2\delta$), and are thus henceforth referred to as the large field-of-view (LFOV) PIV datasets (de Silva et al. 2015, 2020). The analysis presented in this paper is predominantly conducted on these datasets. The LFOV is made possible by stitching the imaged flow fields from eight high-resolution 14 bit PCO 4000 PIV cameras, having a sensor resolution of 4008 × 2672 pixels each. Figure 3(a) shows a schematic of the experimental setup for the LFOV PIV, where the region shaded in orange indicates the individual FOVs combined from the eight cameras. These measurements were conducted at the upstream end of the test section, with the LFOV starting at $x \approx 4.5$ m from the start of the test section. The experiments were conducted at three free-stream speeds ($U_\infty \approx 10$, 20 and 30 ms$^{-1}$), which led to a corresponding variation in $Re_\tau$ of 2500, 5000 and 7500, respectively. Here, $U_\tau$ and $\delta$ used to estimate the flow $Re_\tau$, were computed at the middle of the LFOV, using the method outlined in Chauhan et al. (2009). The boundary layer thickness is nominally $\delta \approx 0.11$ m for all three $Re_\tau$ cases.

Considering the focus of the experiment was on a LFOV, a homogeneous seeding density was ensured across the entire test section of the tunnel for these measurements, and the particles were illuminated by a Big Sky Nd-YAG double pulse laser (~1 mm thickness), delivering 120 mJ/pulse. The last optical mirror to direct this laser sheet was tactically placed within the test section (figure 3(a)), for ensuring adequate laser illumination levels across the LFOV. This optic arrangement, however, was sufficiently downstream of the PIV flow field and introduced no adverse effects
Figure 3: Schematic of the experimental setup used to conduct LFOV PIV experiments in the streamwise wall-normal plane \((x,z)\) in the HRNBLWT. Green shading indicates flow illuminated by the laser while the orange shading indicates the flow field cumulatively captured by the PIV cameras (shown in the background). Dash-dotted black line represents the streamwise evolution of the boundary layer thickness, with \(\delta\) defined at the centre of the full flow field. (b,c) Instantaneous (b) \(u^+\) and (c) \(w^+\)-fluctuations from the LFOV PIV dataset at \(Re_\tau \approx 2500\). The dashed green box in (b,c) identifies a low-momentum turbulent superstructure \((-u_{ss})\) of length \(L_x/\delta\) based on the turbulent superstructure algorithm described in §2.2. (d,e) shows an expanded view of the \(u\)- and \(w\)-fluctuations within \(-u_{ss}\), as identified in (b,c), respectively. Alternatively, the dashed brown box in (b,c) represents flow field of the same length×height as the dashed green box, but not associated with a turbulent superstructure.

(such as blockage, etc) on the measurement (de Silva et al. 2015). Figure 3(b,c) gives an example of the viscous-scaled \(u\)- and \(w\)-fluctuations estimated from the LFOV PIV experiment at \(Re_\tau \approx 2500\), which successfully captures a turbulent superstructure, as highlighted by a dashed green box in the \(u\)-field. Analysis on such a dataset not only avoids uncertainties due to Taylor’s hypothesis approximation (Dennis & Nickels 2008; Wu et al. 2012), but also permits identification of these superstructures directly from an instantaneous flow field. The latter represents another unique feature of the present study, and overcomes the limitations experienced by past studies, which were restricted to isolating superstructure characteristics based on Fourier-filtering or POD-based decomposition of ensemble/time-averaged statistics. The accuracy of these LFOV PIV datasets have been firmly established in Appendix 1 (§6), wherein figure 14 compares the premultiplied 1-D spectra obtained from the present data, with those acquired via multiwire anemometry published previously (Morrill-Winter et al. 2015; Baidya et al. 2017). The excellent match between the PIV and hotwire estimates, in the region of interest for the present study \((2.6 \sqrt{Re_\tau} \lesssim z^+ \lesssim 0.5Re_\tau)\), establishes confidence in the LSMs and superstructures captured by these PIV datasets. The agreement also confirms the insignificant evolution of the boundary
layer thickness across the streamwise extent of the PIV field, thereby not requiring any special considerations for extracting data at a constant $z^+$ along the $x$-direction.

The fourth and final PIV dataset comprises of relatively smaller flow fields in the $x-z$ plane (in terms of $\delta$-scaling), and is hence referred to as the PIV dataset. This was acquired at $U_\infty \approx 20 \text{ ms}^{-1}$, close to the downstream end of the test section ($x \approx 21 \text{ m}$ from the trip), where $\delta \approx 0.3 \text{ m}$, yielding a high $Re_\tau \approx 14500$. The full velocity field captured in this experiment was also made possible by using the same eight PCO 4000 cameras, arranged in two vertical rows of four cameras each, to capture the significantly thicker boundary layer (refer figures 1-2 of de Silva et al. (2014)). This limits the streamwise extent of the flow field to $x \sim 2\delta$ in this case, and is hence not used for identifying the turbulent superstructures in instantaneous fields, but rather used to compute the two-point correlations of $u$- and $w$-fluctuations along the $z$-direction (limited to the inner-region). It is owing to this reason that only a part of the full flow field ($x \times z \sim 2\delta \times 0.4\delta$), from this dataset, has been considered in the present study. The image pairs from all four PIV datasets were processed via an in-house PIV package developed by the Melbourne group (de Silva et al. 2014), with the final window sizes ($\Delta x^+, \Delta z^+$) used for processing given in table 1. Interested readers may refer to the cited references for further details about the experimental setup and methodology adopted for acquiring these datasets.

The fifth dataset, which is at the highest $Re_\tau \sim O(10^6)$, was acquired at the Surface Layer Turbulence and Environmental Science Test (SLTEST) facility in the salt flats of western Utah. The data is acquired from a spanwise and wall-normal array of 18 sonic anemometers (Campbell Scientific CSAT3) arranged in an ‘L’-shaped configuration (refer to figure 1 of Hutchins et al. (2012)). While the full dataset comprises of continuous measurements of all three velocity components as well as the temperature at the SLTEST site over a duration of nine days, here we limit our attention solely to one hour of data associated with near-neutral (i.e. near canonical) atmospheric boundary layer conditions (Hutchins et al. 2012). These conditions were confirmed based on estimation of the Monin–Obukhov similarity parameter, determined on averaging across the 10 sonic anemometers placed along the spanwise array, at a fixed distance from the wall ($z \approx 2.14 \text{ m}$). For the present analysis, we are solely interested in the $u$- and $w$-fluctuations measured synchronously by the 9 sonic anemometers on the wall-normal array, which were placed between $1.42 \text{ m} \leq z \leq 25.69 \text{ m}$ with logarithmic spacing. Mean streamwise velocity measurements reported by Hutchins et al. (2012) confirm that all these $z$-locations fall within the log-region of the atmospheric boundary layer. This data is also used here to compute the two-point correlations of $u$- and $w$-fluctuations along the $z$-direction, for comparison with those obtained from the four PIV datasets acquired in the laboratory.

2.2. Methodology employed to identify and extract turbulent superstructures

In the present study, we are interested in computing conditional statistics of the velocity fluctuations associated with the superstructures, identified from the individual flow fields in the LFOV PIV dataset. Identification of such structures inevitably requires establishing logical thresholds to the geometric and kinematic properties of the fluctuating $u$-field (Hwang & Sung 2018; de Silva et al. 2020). Here, we draw inspiration from previous findings in the literature to establish the following thresholds to identify a superstructure (both $-u$ and $+u$) in a $u$-flow field:

(i) streamwise extent of the $u$-structures, $L_x$ should be greater than $3\delta$ (Adrian et al. 2000; Lee & Sung 2011; Hwang & Sung 2018; Deshpande et al. 2021b).
Figure 4: Probability distribution function (pdf) of the large and intense (a) low- and (b) high streamwise momentum motions detected by the superstructure extraction algorithm at various $Re_{τ}$. Background shading indicates the bin sizes used to estimate the pdf, for which the total number of detected superstructures (i.e. addition of $+u_{ss}$ and $-u_{ss}$) was used for normalization.

(ii) they should exist across the log-region and extend up to $z^+ ≲ 0.5δ^+$ (Baars et al. 2017; Hwang & Sung 2018; Deshpande et al. 2021b).

(iii) Since superstructures carry significant streamwise turbulent kinetic energy (Liu et al. 2001; Ganapathisubramani et al. 2003), the identified motions should comprise intense $u$-fluctuations across the 2-D domain, i.e. $|u(x,z)| > \sqrt{u^2(z)}$.

On detection of a structure which satisfies the above three thresholds, our superstructure extraction algorithm defines a rectangular box surrounding this structure, to extract the 2-D flow field. Figure 3(d) gives an example of a -$u$ superstructure identified and extracted by the algorithm ($u|_{SS}$), from the full flow field depicted in figure 3(b) (highlighted by the dashed green box). A dashed brown box is also highlighted in the same flow fields (figure 3(b,c)), which is of the same length×height as the dashed green box but isn’t associated with a superstructure ($u|_{noSS}$). This practice of extracting $u|_{noSS}$, from the same PIV field from which $u|_{SS}$ is extracted, is conducted across all three LFOV datasets to form a set of $u|_{noSS}$ and $u|_{SS}$ of equal ensembles. Conditional statistics are computed and compared from both $u|_{SS}$ and $u|_{noSS}$, with the latter considered to confirm that the trends depicted by the former are not an artefact of aliasing or insufficient ensembling.

The superstructure extraction algorithm described above identified several superstructures of both $+u$ and -$u$ signatures from the three LFOV PIV datasets, a summary of which has been presented in the probability distribution function (pdf) plot in figure 4. It is interesting to note that the probability of the identified superstructures remains nominally unchanged irrespective of the change in $Re_{τ}$ or momentum ($-u$ or $+u$) of the structures. Further, the plots indicate that the population density decreases near exponentially as the criteria (i) to identify a superstructure (i.e. minimum length, $L_x$) is increased. This suggests availability of fewer ensembles to compute conditional statistics associated with very long superstructures ($\gtrsim 4.5δ$). The effect of increasing the minimum streamwise extent of a $u$-structure to qualify as a superstructure, on the conditionally averaged statistics, has been documented in figure 15 in Appendix 2 ($\S$6). Given that an increase in $L_x$ does not change the scaling behaviour, but significantly reduces the convergence of the statistics, reinforces the choice of $L_x \gtrsim 3δ$ in condition (i) above.
Figure 5: Iso-contours of the premultiplied streamwise 1-D (a) energy spectra of $w$-fluctuations and (b) co-spectra of the Reynolds shear stress plotted against $z^+$ and $\lambda_x^+$, computed from the LFOV PIV dataset at various $Re_\tau$. Dash-dotted golden and magenta lines represent the relationships $\lambda_x \approx 2z$ and $\lambda_x \approx 15z$, respectively following Baidya et al. (2017). (c) Schematic of representative $w$ and $uw$-carrying eddies centred at various distances from the wall ($z_r$) in the log region, with light to dark shading used to suggest an increase in $z_r$. $R_{ww}(z/z_r)$ and $R_{uw}(z/z_r)$ respectively represent the vertical coherence of the $w$- and $uw$-carrying eddies centred at $z_r$.

3. Mean statistics

Before investigating the conditionally averaged statistics associated with the superstructures, it is worth revisiting the scaling behaviour of the mean statistics, against which the former would be compared. Here, the mean statistics have been obtained by averaging across all 3000 flow fields, and considering the entire $12\delta$ long flow fields in case of the LFOV PIV datasets. In the present study, since we are primarily interested in the $w$-velocity behaviour associated with superstructures, we investigate the mean spatial coherence of the $w$-carrying eddies in the log-region of a high $Re_\tau$ boundary layer. We look at the spatial coherence in both the streamwise (figure 5) as well as wall-normal direction (figure 6), for both the $w$-fluctuations and the Reynolds shear stress ($uw$). Previous investigations on the vertical coherence have been rare compared to the streamwise coherence, particularly for the log-region of a high $Re_\tau$ boundary layer, owing to the lack of large-scale PIV experiments of the kind utilized here. This makes the present investigation (figure 6) unique by itself.

Figures 5(a,b) depict the iso-contours of the premultiplied spectrogram of the $w$-velocity and the Reynolds shear stress respectively, computed from the three large FOV PIV datasets. These are plotted as a function of $\lambda_x^+$ and $z^+$. The iso-contours for the $w$-velocity spectrograms can be seen centred around the linear ($z$-)scaling indicated by $\lambda_x = 2z$ for all $Re_\tau$, which is consistent with previous observations in the literature (Baidya et al. 2017). Similarly, the iso-contours for the Reynolds shear stress spectrograms also follow a linear scaling ($\lambda_x = 15z$) for all $Re_\tau$, again consistent with the literature (Baidya et al. 2017). This analysis not only validates the spectra estimated from the LFOV PIV, but also assists with the construction of a simplified 2-D conceptual picture of the $w$- and $uw$-carrying eddies in the log-region of a high $Re_\tau$ boundary layer (figure 5(c)). Here, based on the $z$-scaling exhibited by the data, the lengths ($\lambda_x$) of the $w$- and $uw$-carrying eddies have been defined as $2z_r$ and $15z_r$, respectively, where $z_r$ represents the distance of the eddy centre from the wall. This scaling confirms the association of these $w$- and $uw$-carrying eddies with Townsend’s attached eddy hierarchy, according to which attached eddies scale with $z_r$ (Townsend 1976; Baidya et al. 2017; Deshpande et al. 2021a).
While both these linear scalings, which represent the streamwise coherence of the $w$- and $uw$-carrying eddies, are well accepted in the literature, not much is known about the vertical/wall-normal coherence of the same eddying motions (indicated by $R(z/z_r)$ in Figure 5(c)). Here, we quantify the vertical coherence of these dynamically significant eddies by computing the two-point correlation coefficients defined as:

$$R_{ww}\left(\frac{z}{z_r}\right) = \frac{w(z)w(z_r)}{w^2(z_r)}$$ and $$R_{uw}\left(\frac{z}{z_r}\right) = \frac{w(z)u(z_r)}{uw(z_r)},$$

with $z_r$ acting as the reference wall-normal location fixed within the log-region. Similar statistics have been used previously by Hunt et al. (1987, 1988) to investigate the vertical coherence of the momentum carrying eddies in the very high $Re_\tau$ atmospheric boundary layers and low $Re_\tau$ channel simulations. Here, we compute them for the four high $Re_\tau$ boundary layer datasets considered and plot them in Figure 6, for various $z_r$ limited to the log-region. It can be clearly observed that both $R_{ww}$ and $R_{uw}$ follow universal scalings (represented graphically by golden and teal lines, respectively), owing to the collapse of the two-point correlations. The analytical expressions associated with these golden and indigo lines were found to be:

$$R_{ww}^a\left(\frac{z}{z_r}\right) = 1.007\left(\frac{z}{z_r}\right)^3 - 0.56\left(\frac{z}{z_r}\right)^2 + 0.58\left(\frac{z}{z_r}\right) - 0.027,$$ and

$$R_{uw}^a\left(\frac{z}{z_r}\right) = -0.65\left(\frac{z}{z_r}\right)^3 + 0.65\left(\frac{z}{z_r}\right)^2 + 1.03\left(\frac{z}{z_r}\right) - 0.03.$$
Although the collapse in $R_{uu}$ starts to breakdown to a certain extent at the lowest $Re_{r}$ ($\sim 5000$), that is certainly not the case for high $Re_{r}$ datasets. The fact that the same scaling is exhibited by the atmospheric boundary layer data, which has a log-region significantly thicker than that generated in the lab, confirms the universality of $R_{uu}$ and $R_{ww}$. These universal analytical forms (in (3.2)) represent geometric self-similarity in the vertical coherence of the $w$- and $uw$-carrying inertial eddies, reaffirming their association with Townsend’s attached eddies. They can be used directly in data-driven coherent structure-based modelling efforts. It is worth noting that the self-similarity observed in figure 6 vanishes for both $w$- and $uw$-carrying eddies centred far outside the log-region of the boundary layer (i.e. $z_r > 0.26$; not shown here), which may be due to the growing influence of the turbulent/non-turbulent interface in the outer-region (de Silva et al. 2014). Investigations for $z_r$ below the log-region, however, were not possible owing to insufficient data points captured by the LFOV PIV.

4. Conditionally averaged statistics associated with superstructures

With the scaling behaviour of the mean statistics established in §3, we progress next towards analyzing the conditionally averaged statistics (spectra and correlations) associated with the superstructures. Figure 7 plots the conditionally averaged, pre-multiplied $u$-spectra computed from the extracted flow fields associated with the superstructures (figure 3(d)), from the three LFOV PIV datasets. The spectra are plotted for $z^+ \approx 2.6 \sqrt{Re_{r}}$ and $0.5Re_{r}$, and estimated individually from the extracted flow fields associated with low-momentum ($k_x\phi_{uu}^+|_{-u_{ss}}$; in blue) and high-momentum superstructures ($k_x\phi_{uu}^+|_{+u_{ss}}$; in red). Also plotted is the conditionally averaged spectra considering both $-u_{ss}$ and $+u_{ss}$ ($k_x\phi_{uu}^+|_{-u_{ss},+u_{ss}}$; in green), which is compared against the mean $u$-spectra also plotted in figures 14(a-c). A noteworthy observation from the conditionally averaged spectra ($k_x\phi_{uu}^+|_{-u_{ss},+u_{ss}}$) is the enhanced large-scale energy ($\lambda_{z}^{2} \approx 10^{4}$) seen for all three $Re_{r}$ cases. These enhanced energy levels are due to the significant streamwise turbulent kinetic energy associated with the superstructures, which is captured in the extracted flow fields and averaged across fewer ensembles, than those used for obtaining the mean spectra. To confirm that these trends are not an artefact of aliasing or ensembling, figure 8 compares the conditionally averaged spectra associated with superstructures (green boxes in figure 3(c)) with that not associated with the superstructures (brown boxes in figure 3(c)). Given that both the conditional spectra are estimated from the same number of extracted flow fields, of the same length x height, the enhanced energy in the largest scales for $k_x\phi_{uu}^+|_{-u_{ss},+u_{ss}}$ (compared to $k_x\phi_{uu}^+|_{noSS}$) can be unambiguously associated with the turbulent superstructures. These trends give us confidence regarding the efficacy of the superstructure extraction algorithm. Also, they indicate that the scalings observed from the conditionally averaged $w$, $w$-statistics can be uniquely associated with the constituent motions of the superstructures.

Another interesting observation from the conditional spectra for low- and high-momentum motions, $k_x\phi_{uu}^+|_{-u_{ss}}$ and $k_x\phi_{uu}^+|_{+u_{ss}}$, is their starkly different behaviour in the lower portion of the log-region (figures 7(a-c)) and outside of it (figures 7(d-e)). While $k_x\phi_{uu}^+|_{+u_{ss}} > k_x\phi_{uu}^+|_{-u_{ss}}$ for $z^+ \approx 2.6 \sqrt{Re_{r}}$, it is vice versa for $z^+ \approx 0.5Re_{r}$, which is in accordance with previous observations made by Hutchins & Marusic (2007) and Mathis et al. (2009). These studies noted that the turbulent structures associated with $+u_{ss}$ are more energetic than those associated with $-u_{ss}$, in the region below the geometric mean of the log-region (nominally, $z^+ < 3.9 \sqrt{Re_{r}}$).
Figure 7: (a-f) Premultiplied 1-D spectra of the $u$-fluctuations plotted versus $\lambda_x^+$ at (a-c) $z^+ \approx 2.6 \sqrt{Re_\tau}$ and (d-f) $z^+ \approx 0.5 Re_\tau$ for LFOV PIV data at $Re_\tau \approx (a,d) 2500$, (b,e) 5000 and (c,f) 7500. Dashed black lines correspond to the mean spectra obtained by ensembling across 3000 PIV images of the full flow field. While, the solid blue and red lines represent conditional spectra computed from the $u$-flow field extracted based on identification of $-u_{ss}$ and $+u_{ss}$, respectively. The spectra in green is computed by ensembling across both $-u_{ss}$ and $+u_{ss}$.

Figure 8: Premultiplied 1-D spectra of the $u$-fluctuations at $z^+ \approx 2.6 \sqrt{Re_\tau}$ for $Re_\tau \approx (a) 2500$, (b) 5000 and (c) 7500. The mean spectra estimated from the full flow field (in black lines) is ensembled across all 3000 fields. While, the conditional spectra corresponds to extracted flow fields ($\sim 300$) of the same length x height associated (in green) and not associated (in brown) with the superstructures.

This behaviour, however, reversed for $z^+ > 3.9 \sqrt{Re_\tau}$, which is consistent with our observations from figure 7, reaffirming confidence in the extracted flow fields.

With the efficacy of the superstructure extraction algorithm now established, we shift our focus to the statistical quantity of primary interest: conditionally averaged, premultiplied $w$-spectra associated with the superstructures. Figures 9(a-f) plot $k_x \phi_{uw}^+ | -u_{ss} , k_x \phi_{uw}^+ | +u_{ss}$ and $k_x \phi_{uw}^+ | -u_{ss} , +u_{ss}$ for $z^+ \approx 2.6 \sqrt{Re_\tau}$ and $0.5 Re_\tau$ computed from all three LFOV PIV datasets. Here again, $k_x \phi_{uw}^+ | +u_{ss} > k_x \phi_{uw}^+ | -u_{ss}$ for $z^+ \approx 2.6 \sqrt{Re_\tau}$, and vice versa for $z^+ \approx 0.5 Re_\tau$, which is consistent with the behaviour noted for the $u$-spectra in figure 7. Interestingly $k_x \phi_{uw}^+ | -u_{ss} , +u_{ss}$, which represents $w$-energy associated with both $-u_{ss}$ and $+u_{ss}$, can be seen overlapping with the mean $w$-spectra, for both $z^+$ and all three $Re_\tau$ considered in figure 9 (except at the largest $\lambda_x^+$). This suggests that the $w$-carrying eddies within the superstructures conform to the inertia-dominated z-scaled eddies predominant in the log-region (figure 5). To test the $z$-scaling characteristics of $k_x \phi_{uw}^+ | -u_{ss} , +u_{ss}$, we plot it for various $z^+$ corresponding to the log-region in figure 9(g-i). Remarkably, $k_x \phi_{uw}^+ | -u_{ss} , +u_{ss}$ exhibits $z$-scaling behaviour similar to the mean spectra for $\lambda_x^+$.
Figure 9: (a-f) Premultiplied 1-D spectra of the $w$-fluctuations plotted versus $\lambda_x^+$ at (a-c) $z^+ \approx 2.6 \sqrt{Re_\tau}$ and (d-f) $z^+ \approx 0.5 Re_\tau$ for large FOV PIV data at $Re_\tau \approx (a,d,g) 2500$, $(b,e,h) 5000$ and $(c,f,i) 7500$. Dashed black lines correspond to the mean spectra obtained by ensembling across 3000 PIV images of the full flow field. While, the solid blue and red lines represent conditional spectra computed from the extracted $w$-flow fields associated with $-u_{ss}$ and $+u_{ss}$, respectively. The spectra in green corresponds to all extracted $w$-flow fields, associated with both $-u_{ss}$ and $+u_{ss}$. (g-i) Premultiplied 1-D spectra of $w$-velocity plotted vs $\lambda_x/z$ at various $z^+$ within the log region ($2.6 \sqrt{Re_\tau} \leq z^+ \leq 0.15 Re_\tau$). Colour coding is the same as that defined for plots (a-f). Line plots in brown correspond to conditional spectra $k_x \phi_{ww}^+|_{-u_{ss},+u_{ss}}$ computed from extracted flow fields of the same length x height as the $k_x \phi_{ww}^+|_{-u_{ss},+u_{ss}}$, but not associated with the superstructures. Dashed golden lines represents the linear scaling, $\lambda_x = 2z$. Shaded yellow background indicates enhanced energy at large $\lambda_x$ for $k_x \phi_{ww}^+|_{-u_{ss},+u_{ss}}$. 

$\geq 2z$, across all three $Re_\tau$, suggesting that the geometrically self-similar attached eddies are the constituent motions forming the superstructures. Further, the peak of $k_x \phi_{ww}^+|_{-u_{ss},+u_{ss}}$ also scales with $\lambda_x = 2z$, which is consistent with the mean spectra. A unique observation associated with $k_x \phi_{ww}^+|_{-u_{ss},+u_{ss}}$, which aligns with the streamwise concatenation argument, is the slightly enhanced energy at the large $\lambda_x$ compared to the mean spectra (indicated with shaded background in figures 9(g-i)). This can be ascertained to the unique spatial organization of the $w$-carrying eddies within the superstructures. It aligns well with our previous discussion based on figures 2(c,e), wherein the clustering/packing of the individual $w$-carrying eddies within the superstructures was expected to yield enhanced spatial correlations of the $w$-velocity at large streamwise extents ($\geq 3\delta$). The fact that this observation represents a physical phenomena associated with the superstructures (and not an artefact of aliasing or ensembling) can be reaffirmed by considering $k_x \phi_{ww}^+|_{noSS}$ plotted for the same $z^+$-range in figure 9(g-i). $k_x \phi_{ww}^+|_{noSS}$, which is estimated based
Figure 10: (a-f) Conditionally averaged correlations between \( w \)-fluctuations at \( z \) and \( z_r \), normalized by \( \overline{w^2(z_r)} \) for various \( z_r \). The correlations have been computed from the extracted \( w \)-flow fields associated with both \(-u_{ss}\) and \(+u_{ss}\). Dashed black line corresponds to the linear relationship, \( z/z_r \) while dashed dotted golden line corresponds to \( R^a_{ww} \) defined in (3.2).

5. Support from synthetically generated fields

The scaling arguments in §3 and §4 make a compelling case in favour of geometrically self-similar attached eddies concatenating along the streamwise direction within the superstructures. To test if this observation could potentially lead to the formation on the same number of ensembles and length\times height of the extracted flow fields as \( k_x \phi^+_{ww} | -u_{ss}, +u_{ss} \), is not associated with the superstructures and hence, does not have enhanced energy levels at large \( \lambda_x^+ \). Thus, \( k_x \phi^+_{ww} | -u_{ss}, +u_{ss} \) strongly suggests that the superstructures are associated with the streamwise concatenation of geometrically self-similar attached eddies.

While \( k_x \phi^+_{ww} | -u_{ss}, +u_{ss} \) brings out the geometric characteristics and organization of the constituent motions along the streamwise direction, the same can be understood for the wall-normal direction by computing the two-point correlations \( (R_{ww}; (3.1)) \) for the extracted flow fields. Figure 10 plots \( R_{ww} | -u_{ss}, +u_{ss} \), i.e. the two-point correlations computed from the \( w \)-fluctuations associated with both \(-u_{ss}\) and \(+u_{ss}\), for \( z_r \) limited to the log-region. These are estimated for all three LFOV PIV datasets and compared with the universal form (in (3.2)) exhibited by the mean statistics (in golden line). Remarkably, \( R_{ww} | -u_{ss}, +u_{ss} \) can be found to collapse for varying \( z_r \) for all three \( Re_\tau \), in a manner similar to that observed for the mean statistics. Consistent with our conclusions from figure 9, the investigation of the vertical coherence of the \( w \)-carrying eddies (associated with the superstructures) also indicates that the geometrically self-similar eddies coexist within the superstructures.

Whether it is actually these intermediate-scaled eddies which concatenate to form the superstructures (Adrian et al. 2000) through some unknown mechanism, or if it is the secondary roll modes that are responsible for aligning them along the streamwise direction (Lee et al. 2019), remains an interesting open question, one that is beyond the scope of the present study. Further, while the present study lacks the analysis to investigate the spanwise coherence of the extracted motions, consideration of the present findings in light of the recent knowledge on the log region (Hwang & Sung 2018; Chandran et al. 2020; Deshpande et al. 2020, 2021a,b) suggests that they would likely exhibit self-similar characteristics along the span as well.
of superstructures, here we make use of synthetically generated flow fields. The idea is to carefully understand the influence of the geometry and organization of the coherent motions, on the scalings/trends exhibited by the statistics (spectra and correlations). Three different types of synthetic fields are chosen for this purpose, each comprising of a ‘hairpin’ or simple arch-shaped (Λ) eddy as the representative coherent structure/’hierarchy’ to model the inertial region of a boundary layer (Head & Bandyopadhyay 1981; Adrian et al. 2000; Wu & Moin 2009; Dennis & Nickels 2011a). The three fields essentially represent characteristically different distributions of variable sized Λ-eddies in the flow field, in a way that would test the hypothesis brought out by the preceding empirical analysis. For this, the simplest version of the Λ-eddy is considered, which is made up of two vortex rods arranged in a Λ-shape, with each rod comprising of a Gaussian distribution of vorticity about its core. Based on the previous experimental evidence (Head & Bandyopadhyay 1981; Deshpande et al. 2019), each of these Λ-eddies are forced to be inclined forwards, at 45° with respect to the mean flow direction. The three component velocity fields associated with each eddy is obtained by performing Biot-Savart calculations. Each Λ-eddy also has a corresponding image eddy in the plane of the wall, which is implemented to enforce impermeability conditions at the wall (w = 0 at z = 0). Figure 11 schematically depicts the three synthetic fields considered here, with the green rods representing the vorticity carrying Λ-eddies and the red and blue iso-contours representing the induced +w and −w, respectively.

In case of the synthetic field 1, the inertial region of the boundary layer is simply represented by superposition of six hierarchies of the Λ-eddies, of varying sizes (H_i; with i being the hierarchy number) and population densities, randomly distributed in the flow field. Figure 11(a) shows a schematic of four Λ-eddy hierarchies (in green), organized randomly in the flow domain, for representative purposes. For convenience in setting up this synthetic field, the height (H) of the relatively taller hierarchy is defined to be twice the size of the previous hierarchy (i.e. H_{i+1} ∼ 2H_i), while the population density varies inversely proportional to the size of the hierarchy (P(H_i) ∼ 1/H_i). The height of the largest hierarchy is, by definition, the boundary layer thickness δ. Similar conventions have been followed in previous studies (de Silva et al. 2016; Eich et al. 2020; Chandran et al. 2020; Deshpande et al. 2021b) to set up attached eddy model simulations, which ensure a geometrically self-similar variation in size of the Λ-eddy hierarchies introduced in the present synthetic flow fields. A noteworthy difference between the present and past simulations, however, is the consideration of an individual Λ-eddy as the representative eddy for the present synthetic fields, as opposed to that of a Λ-eddy packet in the previous simulations (which is the recommended pathway). It is owing to this reason that the present fields can’t be expected to replicate the same statistical trends/features as those noted in the previous simulations. The choice for the present study, however, is justified considering the aim of investigating whether the spatial organization of smaller self-similar motions can lead to streamwise elongated motions.

Synthetic field 2 comprises essentially the same Λ-eddy hierarchies (with the same population density) as in synthetic field 1, with the only difference being that the hierarchies are forced to be streamwise aligned in case of the former, as per the concatenation hypothesis (Adrian et al. 2000). The synthetic field 2, hence, is a conceptual representation of the experimental observations from the LFOV PIV fields, that suggests superstructures may be concatenations of geometrically self-similar motions. Figure 11(b) schematically represents the unique spatial distribution of the various hierarchies of Λ-eddies in the case of synthetic field 2, with hierarchies
Figure 11: Schematic describing the various 3-D synthetic flow fields generated by hierarchies $(H_i, i = 1, 2, ...)$ of Λ-shaped vortex rods (in green) distributed in the flow domain. Blue and red regions represent the 3-D iso-contours of $-w$ and $+w$ induced due to the vorticity in the individual hairpins. Both, (a) synthetic field 1 and (b) synthetic field 2 comprise of geometrically self-similar Λ-eddies, with the only difference being these eddies are distributed randomly in space in field 1, while the various hierarchies are forced to align along $x$ in field 2 to form a streamwise elongated motion (as seen in figure 2(c,e)). (c) Synthetic field 3 is essentially a combination of synthetic field 1 in (a) and multiple geometrically non-self-similar Λ-eddies aligned along $x$ to also form an elongated motion. $P(H_i)$ represents the population density of the hierarchy, $H_i$.

$H_4$, $H_3$ and $H_2$ only organized in certain streamwise alignments to maintain the same population density as in synthetic field 1. Consequently, while synthetic field 2 would be expected to reveal recurrent streamwise elongated motions (superstructures) in the instantaneous flow field, synthetic field 1 won’t. In contrast to these two fields, synthetic field 3 is a conceptual representation of a hypothetical scenario wherein a superstructure has no association with the geometrically self-similar (i.e. $z$-scaled) inertia-dominated motions. For this purpose, synthetic field 3 considers multiple streamwise aligned non-self-similar Λ-eddies which are added to the same flow field as the synthetic field 1. Thus, comparing and contrasting the scalings/trends exhibited by the velocity statistics, from these three synthetic fields, could shed light on the possibility of whether the streamwise alignment of self-similar motions could lead to the formation of superstructures.

Figure 12 depicts the premultiplied 1-D spectra of the $w$-fluctuations (figures 12(a-c)) and the two-point correlations ($R_{ww}$; figure 12(d-f)) from the three synthetic fields considered in the present study. Both $k_x\phi_{ww}$ and $R_{ww}$ from synthetic field 1 exhibit $z$-scalings consistent with that expected from a flow field made up of purely self-similar attached eddies. The trend of the statistics are qualitatively similar to that observed for the real flow in figures 5(a) and 6(a-d); there is, however, a quantitative mismatch in the linear scaling for $k_x\phi_{ww}$ and universal form for $R_{ww}$. This is a function of the exact shape and vorticity defined for the representative Λ-eddy, which hasn’t been explored in the present study, to maintain simplicity. Considering synthetic field 2, the same $z$-scaling is also clearly noted for both $k_x\phi_{ww}$ and $R_{ww}$. One can also
Figure 12: (a-c) Premultiplied 1-D spectra of $w$-fluctuations plotted vs $\lambda_x/z$ at various wall-normal locations of the synthetic flow fields. (d-f) Cross-correlation of $w$-fluctuations measured at $z$ and $z_r$, normalized by $\overline{\sigma^2}(z_r)$ at various $z_r$. In (a-b), the dashed golden line represents the linear scaling, $\lambda_x \sim z$. In (d-f), the dashed green line corresponds to the linear relationship, $z/z_r$ while dash-dotted golden line corresponds to $R_{ww}$ defined in (3.2).

note an interesting trend of enhanced energy at large $\lambda_x$ (highlighted with shaded background), which can be associated with the imposed streamwise alignment of the $\Lambda$-eddy hierarchies. This is consistent with our experimental observations from the conditionally averaged $w$-spectra $(k_x \phi_{ww}^\pm - u_{ss}^\pm + u_{ss})$, which also exhibits enhanced energy at the large scales (figure 9). The good correspondence of the empirical observations with synthetic field 2 confirms the large-scale spatial organization of the geometrically self-similar eddies within the superstructures. Interestingly, when the spectra and correlations are investigated for synthetic field 3, neither of them exhibit $z$-scalings. Both $k_x \phi_{ww}$ and $R_{ww}$ clearly depict a trend dependent on $z_r$, which is not noted in synthetic fields 1 and 2, nor in the experimental statistics. This deviation from the $z$-scaling can be associated to the consideration of streamwise aligned non-self-similar eddies in synthetic field 3, further reaffirming the relationship between the attached eddies and superstructures.

6. Conclusions and outlook
The present study analyzes large-scale PIV datasets, acquired in moderate to high $Re_{\tau}$ turbulent boundary layers, to investigate the constituent motions of the turbulent superstructures. These unique datasets, which are acquired in a physically thick boundary layer, accurately capture the inertia-dominated $w$- and $w$-fluctuations across a large streamwise wall-normal plane, extending up to $12\delta$ in the $x$-direction. This facilitates a comprehensive investigation of the horizontal (via 1-D spectra) as well as vertical coherence (via two-point correlations) of the Reynolds shear stress
superstructure momentum carrying z-scaled eddies

Figure 13: Conceptual representation of the main conclusion of this study: z-scaled eddies co-exist within the superstructures in a spatially organized manner across extended streamwise separations.

carrying eddies coexisting in the log-region, which are responsible for the momentum transfer in a high $Re_\tau$ boundary layer (Marusic et al. 2010; Deshpande et al. 2021a). The statistics bring out the geometric self-similarity of these energetically significant eddies, which complements the well-established knowledge on the self-similarity exhibited by the wall-parallel velocity components in a canonical flow (Baars et al. 2017; Hwang & Sung 2018; Deshpande et al. 2020). We note that this motivates undertaking similar investigations of the momentum and heat flux in thermally stratified wall-bounded flows (Krug et al. 2019; Li et al. 2022), which can likely assist with coherent structure-based modelling of these practically relevant flows.

The empirically derived scaling behaviour observed from these mean statistics (spectra and correlations) provide a benchmark for comparing and contrasting with the conditionally averaged statistics, associated with the turbulent superstructures. The availability of such large-scale PIV flow fields permits identification of the superstructures directly from instantaneous flow fields, from which the conditional statistics are computed. Considering the ambiguity involved while interpreting the smaller constituent motions from a $u$-flow field, the present study adopts the approach of investigating the $w$-fluctuations within the superstructure region, to understand its constituent motions. Notably, the conditional streamwise $w$-spectra exhibits the classical $z$-scaling ($\lambda_x = 2z$) in the intermediate scale range, clearly suggesting that geometrically self-similar eddies co-exist within the superstructure region (represented schematically in figure 13). The same spectra also exhibits a energy enhancement at the large scales, indicating a spatial organization of the $z$-scaled $w$-carrying eddies over extended streamwise separations, plausibly leading to the appearance of the superstructures. Conditional two-point $w$-correlations along the vertical direction also exhibit self-similar scaling, similar to that noted for the mean flow, further reinforcing that the attached eddies coexist within the superstructure region. This explains the longstanding contradiction noted in the literature (Guala et al. 2006; Balakumar & Adrian 2007; Wu et al. 2012) about the instantaneous streamwise elongated motions (superstructures) carrying a significant proportion of Reynolds shear stresses (i.e. ‘active’ motions), despite Townsend describing these as ‘inactive’ in a statistically average sense (Townsend 1976; Deshpande et al. 2021a). The present observations encourage further investigations on explaining the mechanism behind this relationship between the geometrically self-similar motions and superstructures, i.e. does the streamwise concatenation (Adrian et al. 2000) of the $z$-scaled eddies generate the superstructures? Or are the secondary roll modes (Lee et al. 2019) responsible for aligning these eddies in the streamwise direction?

The plausibility of self-similar motions aligning along the streamwise direction to form superstructures was also tested here using synthetically generated flow fields. On consideration of a flow field comprising of $z$-scaled eddies forcibly aligned along the streamwise direction, the corresponding 1-D $w$-spectra was found to exhibit similar trends to those observed empirically. Significantly different trends, on the other hand, were exhibited by the synthetic field wherein the superstructure was represented by
a set of streamwise aligned non-self-similar eddies. These results, hence, support and endorse the prospect of using low-order conceptual models (such as the attached eddy model) to model the dominant skin-friction contributing motions in high $Re\tau$ boundary layers (Deck et al. 2014). They can also be used to further improve the attached eddy model (Marusic & Monty 2019), by extending the data-driven approach proposed recently in Deshpande et al. (2021b). This can be achieved by considering definition of the geometry and organization of the representative $\Lambda$-eddies, based on the empirically obtained $Re\tau$-invariant scalings in figures 5 and 6. Further, the comparison of the synthetic fields in §5 highlights the importance of maintaining the population density and geometric self-similarity of the individual eddies forming the large-scale motions (including superstructures), which should be considered in future versions of the model. The fact that some of the attached eddies would need to be forced to align along the streamwise direction, to form the elongated motions such as superstructures, also questions the aspect of ‘randomly distributed eddies’ in Townsend’s original attached eddy hypothesis (Townsend 1976).

**Acknowledgements**

The authors wish to acknowledge the Australian Research Council for financial support and are grateful to Prof. N. Hutchins for insightful discussions regarding this work.

**Declaration of Interests**

The authors report no conflict of interest.

**Appendix 1: Comparisons of energy spectra obtained from large FOV PIV with hotwire data**

Figure 14 plots the premultiplied streamwise 1-D spectra of the streamwise and wall-normal velocity components, as well as the Reynolds shear stress obtained from the LFOV PIV dataset. The spectra are plotted for $z^+ \approx 2.6 \sqrt{Re\tau}$ (nominal start of the log-region) and the middle of the boundary layer ($\approx 0.5 Re\tau$), and compared against published spectra estimated from multiwire experiments (Morrill-Winter et al. 2015; Baidya et al. 2017) conducted in the same facility and at similar $Re\tau$. It should be noted here that the hotwire spectra are plotted based on assumption of Taylor’s hypothesis, with the mean velocity at $z^+$ considered as the mean convection velocity of the turbulent scales. One can clearly observe that the PIV and hotwire spectra match reasonably well, especially in the large $\lambda_x/z$ ($\gtrsim 1$) range, corresponding to the LSMs and superstructures in the boundary layer. Some discrepancy at smaller $\lambda_x$ is expected for high $Re\tau$ PIV ($\sim 5000, 7500$), due to relatively poor spatial resolution of PIV compared to the hotwire sensor resolution (table 1).

**Appendix 2: Effect of threshold established to identify superstructures**

Figures 15(a-c) present the conditionally averaged, premultiplied $w$-spectra and figures 15(d-f) present the conditionally averaged two-point correlations of the $w$-fluctuations. Both are computed from the flow fields extracted based on varying thresholds ($L_x$) on the streamwise extents of the identified superstructures. The
statistics are computed for the $Re_\tau \approx 7500$ LFOV PIV dataset at $z^+$ corresponding to the log-region. It is evident from the comparison that the $z$-scalings, exhibited by both the statistics, remain unchanged despite the change in $L_x$. The most prominent effect of increasing the $L_x$ is the reduced number of ensembles (of the extracted flow fields), leading to poorly converged conditionally averaged statistics.

REFERENCES

Adrian, R. J., Meinhart, C. D. & Tomkins, C. D. 2000 Vortex organization in the outer region of the turbulent boundary layer. Journal of Fluid Mechanics 422, 1–54.

Baars, W. J., Hutchins, N. & Marusic, I. 2017 Self-similarity of wall-attached turbulence in boundary layers. Journal of Fluid Mechanics 823, R2.

Baidya, R., Philip, J., Hutchins, N., Monty, J. P. & Marusic, I. 2017 Distance-from-the-wall scaling of turbulent motions in wall-bounded flows. Physics of Fluids 29 (2), 020712.

Bailey, S.C.C., Hultmark, M., Smits, A.J. & Schultz, M.P. 2008 Azimuthal structure of turbulence in high Reynolds number pipe flow. Journal of Fluid Mechanics 615, 121–138.

Balakumar, B.J. & Adrian, R.J. 2007 Large-and very-large-scale motions in channel and boundary-layer flows. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 365 (1852), 665–681.

Baltzer, J.R., Adrian, R.J. & Wu, X. 2013 Structural organization of large and very large scales in turbulent pipe flow simulation. Journal of Fluid Mechanics 720, 236–279.

Chandran, D., Monty, J. P. & Marusic, I. 2020 Spectral-scaling-based extension to the attached eddy model of wall turbulence. Physical Review Fluids 5 (10), 104606.

Chauhan, K. A., Monkewitz, P. A. & Nagib, H. M. 2009 Criteria for assessing experiments in zero pressure gradient boundary layers. Fluid Dynamics Research 41 (2), 021404.
Figure 15: (a–c) Conditionally averaged premultiplied 1-D spectra of $w$-fluctuations plotted vs $\lambda_x/z$ at various $z^+$ within the log-region. (d–f) Correlation between $w$-fluctuations at $z$ and $z_r$, normalized by $w^2(z_r)$ for various $z_r$. Here, both the spectra and the cross correlations are computed from the extracted $w$-flow fields associated with $-u_{ss}$ and $+u_{ss}$ detected in the LFOV PIV data at $Re_\tau \approx 7500$, for varying thresholds of the streamwise lengths associated with the superstructures: (a) $L_x \geq 3\delta$, (b) $L_x \geq 3.8\delta$ and (c) $L_x \geq 4.5\delta$. In (a–c), the dashed golden line represents the linear scaling, $\lambda_x = 2z$. In (d–f), the dashed black line corresponds to the linear relationship, $z/z_r$ while the dash-dotted golden line corresponds to $R^a_{ww}$ defined in (3.2).

DECK, S., RENARD, N., LARAUFIE, R. & WEISS, P. 2014 Large-scale contribution to mean wall shear stress in high-Reynolds-number flat-plate boundary layers up to 13650. *Journal of Fluid Mechanics* 743, 202–248.

DEL ALAMO, J. C. & JIMENEZ, J. 2006 Linear energy amplification in turbulent channels. *Journal of Fluid Mechanics* 559, 205–213.

DENNIS, D.J.C. & NICKELS, T.B. 2011a Experimental measurement of large-scale three-dimensional structures in a turbulent boundary layer. Part 1. Vortex packets. *Journal of Fluid Mechanics* 673, 180–217.

DENNIS, D.J.C. & NICKELS, T.B. 2011b Experimental measurement of large-scale three-dimensional structures in a turbulent boundary layer. Part 2. Long structures. *Journal of Fluid Mechanics* 673, 218–244.

DENNIS, D.J.C. & NICKELS, T. B. 2008 On the limitations of Taylor’s hypothesis in constructing long structures in a turbulent boundary layer. *Journal of Fluid Mechanics* 614, 197–206.

DESHPANDE, R., CHANDRAN, D., MONTY, J.P. & MARUSIC, I. 2020 Two-dimensional cross-spectrum of the streamwise velocity in turbulent boundary layers. *Journal of Fluid Mechanics* 890, R2.

DESHPANDE, R., MONTY, J.P. & MARUSIC, I. 2019 Streamwise inclination angle of large wall-attached structures in turbulent boundary layers. *Journal of Fluid Mechanics* 877, R4.

DESHPANDE, R., MONTY, J. P. & MARUSIC, I. 2021a Active and inactive components of the streamwise velocity in wall-bounded turbulence. *Journal of Fluid Mechanics* 914, A5.

DESHPANDE, R., DE SILVA, C. M., LEE, M., MONTY, J. P. & MARUSIC, I. 2021b Data-driven enhancement of coherent structure-based models for predicting instantaneous wall turbulence. *International Journal of Heat and Fluid Flow* 92, 108879.

EICH, F., DE SILVA, C. M., MARUSIC, I. & KÄHLER, C. J. 2020 Towards an improved spatial
representation of a boundary layer from the attached eddy model. *Physical Review Fluids* **5** (3), 034601.

Ganapathisubramani, B., Longmire, E. & Marusic, I. 2003 Characteristics of vortex packets in turbulent boundary layers. *Journal of Fluid Mechanics* **478**, 35–46.

Guala, M., Hommem, S. E. & Adrian, R. J. 2006 Large-scale and very-large-scale motions in turbulent pipe flow. *Journal of Fluid Mechanics* **554**, 521–542.

Head, M. R. & Bandypadhyay, P. 1981 New aspects of turbulent boundary-layer structure. *Journal of Fluid Mechanics* **107**, 297–338.

Hu, R., Yang, X. I. A. & Zheng, X. 2020 Wall-attached and wall-detached eddies in wall-bounded turbulent flows. *Journal of Fluid Mechanics* **885**, A30.

Hunt, J.C.R., Moin, P., Moser, R.D. & Spalart, P.R. 1987 Self similarity of two point correlations in wall bounded turbulent flows. In *CTR Annu. Res. Briefs*, pp. 25–36.

Hunt, J. C. R., Kaimal, J. C. & Gaynor, J. E. 1988 Eddy structure in the convective boundary layer-New measurements and new concepts. *Quarterly Journal of the Royal Meteorological Society* **114** (482), 827–858.

Hutchins, N., Chauhan, K., Marusic, I., Monty, J. & Klewicki, J. 2012 Towards reconciling the large-scale structure of turbulent boundary layers in the atmosphere and laboratory. *Boundary-layer Meteorology* **145** (2), 273–306.

Hutchins, N. & Marusic, I. 2007 Evidence of very long meandering features in the logarithmic region of turbulent boundary layers. *Journal of Fluid Mechanics* **579**, 1–28.

Hwang, J. & Sung, H. J. 2018 Wall-attached structures of velocity fluctuations in a turbulent boundary layer. *Journal of Fluid Mechanics* **856**, 958–983.

Hwang, Y. & Cossu, C. 2010 Self-sustained process at large scales in turbulent channel flow. *Physical Review Letters* **105** (4), 044505.

Kim, K.C. & Adrian, R.J. 1999 Very large-scale motion in the outer layer. *Physics of Fluids* **11** (2), 417–422.

Kline, S. J., Reynolds, W. C., Schraub, F.A. & Runstadler, P.W. 1967 The structure of turbulent boundary layers. *Journal of Fluid Mechanics* **30** (4), 741–773.

Krug, D., Baars, W. J., Hutchins, N. & Marusic, I. 2019 Vertical coherence of turbulence in the atmospheric surface layer: connecting the hypotheses of Townsend and Davenport. *Boundary-Layer Meteorology* **172** (2), 199–214.

Lee, J., Lee, J.H., Choi, J. & Sung, H.J. 2014 Spatial organization of large-and very-large-scale motions in a turbulent channel flow. *Journal of Fluid Mechanics* **749**, 818–840.

Lee, J.H. & Sung, H.J. 2011 Very-large-scale motions in a turbulent boundary layer. *Journal of Fluid Mechanics* **673**, 80–120.

Lee, J. H., Sung, H. J. & Adrian, R. J. 2019 Space–time formation of very-large-scale motions in turbulent pipe flow. *Journal of Fluid Mechanics* **881**, 1010–1047.

Li, X., Hutchins, N., Zheng, X., Marusic, I. & Baars, W. J. 2022 Scale-dependent inclination angle of turbulent structures in stratified atmospheric surface layers. *Journal of Fluid Mechanics* **942**, A38.

Liu, Z., Adrian, R. J. & Hanratty, T. J. 2001 Large-scale modes of turbulent channel flow: transport and structure. *Journal of Fluid Mechanics* **448**, 53–80.

Marusic, I., Chauhan, K.A., Kulandaivelu, V. & Hutchins, N. 2015 Evolution of zero-pressure-gradient boundary layers from different tripping conditions. *Journal of Fluid Mechanics* **783**, 379–411.

Marusic, I., Mathis, R. & Hutchins, N. 2010 High Reynolds number effects in wall turbulence. *International Journal of Heat and Fluid Flow* **31** (3), 418–428.

Marusic, I. & Monty, J. P. 2019 Attached eddy model of wall turbulence. *Annual Review of Fluid Mechanics* **51**, 49–74.

Mathis, R., Hutchins, N. & Marusic, I. 2009 Large-scale amplitude modulation of the small-scale structures in turbulent boundary layers. *Journal of Fluid Mechanics* **628**, 311–337.

McKeon, B. J. & Sharma, A. S. 2010 A critical-layer framework for turbulent pipe flow. *Journal of Fluid Mechanics* **658**, 336–382.

Meinhart, C. D. & Adrian, R. J. 1995 On the existence of uniform momentum zones in a turbulent boundary layer. *Physics of Fluids* **7** (4), 694–696.

Morrill-Winter, C., Klewicki, J., Baidya, R. & Marusic, I. 2015 Temporally optimized
spanwise vorticity sensor measurements in turbulent boundary layers. *Experiments in Fluids* **56** (12), 216.

Sillero, J. A., Jiménez, J. & Moser, R. D. 2013 One-point statistics for turbulent wall-bounded flows at Reynolds numbers up to $\delta^+ \approx 2000$. *Physics of Fluids* **25** (10), 105102.

De Silva, C. M., Chandran, D., Baidya, R., Hutchins, N. & Marusic, I. 2020 Periodicity of large-scale coherence in turbulent boundary layers. *International Journal of Heat and Fluid Flow* **83**, 108575.

De Silva, C. M., Gnanamanickam, E. P., Atkinson, C., Buchmann, N. A., Hutchins, N., Soria, J. & Marusic, I. 2014 High spatial range velocity measurements in a high Reynolds number turbulent boundary layer. *Physics of Fluids* **26** (2), 025117.

De Silva, C. M., Hutchins, N. & Marusic, I. 2016 Uniform momentum zones in turbulent boundary layers. *Journal of Fluid Mechanics* **786**, 309–331.

De Silva, C. M., Squire, D. T., Hutchins, N. & Marusic, I. 2015 Towards capturing large scale coherent structures in boundary layers using particle image velocimetry. In *Proceedings of the 7th Australian Conference on Laser Diagnostics in Fluid Mechanics and Combustion, Melbourne, Australia*.

Townsend, A.A. 1976 *The structure of turbulent shear flow*, 2nd edn. Cambridge University Press.

Wu, X., Baltzer, J.R. & Adrian, R.J. 2012 Direct numerical simulation of a 30R long turbulent pipe flow at $R+ = 685$: large- and very large-scale motions. *Journal of Fluid Mechanics* **698**, 235–281.

Wu, X. & Moin, P. 2009 Direct numerical simulation of turbulence in a nominally zero-pressure-gradient flat-plate boundary layer. *Journal of Fluid Mechanics* **630**, 5–41.