Isolated photons
in perturbative QCD

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Abstract

I present a definition of the cross section for the production of an isolated photon plus $n$ jets which only depends upon direct photon production, and it is independent of the parton-to-photon fragmentation contribution. This prescription, based on a modified cone approach which implements the isolation condition in a smooth way, treats in the same way quarks and gluons and can be directly applied to experimental data in hadron-hadron, photon-hadron and $e^+e^-$ collisions. The case of several, isolated photons in the final state can also be dealt with in the very same way.

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1. Introduction

Photons are produced in scattering phenomena by two different mechanisms. In the direct process, the photon enters the partonic hard collision, characterized by a large energy scale. In the fragmentation process, a QCD parton (quark or gluon) fragments non-perturbatively into a photon, at a scale of the order of the typical hadronic mass. The former process is computable in perturbative QCD, while the latter is not; all the unknowns of the fragmentation mechanism are collected into two functions (the quark-to-photon and gluon-to-photon fragmentation functions) which, although universal, must be determined by comparison with the data. Direct photons are usually well isolated from the final state hadrons, while photons produced via fragmentation usually lie inside hadronic jets. From the experimental point of view, it is not difficult to select a data sample in which the direct mechanism is dominant over the fragmentation mechanism, by rejecting all those events where the photon is not isolated from hadronic tracks. A clean sample of well-isolated photons is extremely useful for a variety of topics, like a detailed understanding of the underlying parton picture, to constrain the gluon density of the proton in an intermediate $x$ range, and to obtain an efficient background rejection in Higgs searches at future colliders.

In perturbative QCD, the problem is more involved. It is not possible to separate sharply the photon from the partons; in fact, this would constrain the phase space of soft gluons, thus spoiling the cancellation of infrared divergences which is crucial in order to get a sensible cross section. Two methods have been devised to tackle this problem. In the cone approach [1], a cone is drawn around the photon axis; if only a small hadronic energy (compared to the photon energy) is found inside the cone, the partons accompanying the photon are clustered with a given jet-finding algorithm. In the democratic approach [2] the photon is treated as a parton as far as the jet-finding algorithm is concerned. At the end of the clustering procedure, the configuration corresponds to an isolated photon event only if the ratio of the hadronic energy found inside the jet containing the photon over the total energy of the jet itself is smaller than a fixed amount, usually less than 10%. The democratic approach is more suited than the cone approach to extract the non-perturbative parton-to-photon fragmentation functions from the data. This point of view was adopted in ref. [3].

In this paper, I will deal with the problem of defining an isolated-photon cross section which minimizes the contribution of the fragmentation mechanism. In particular, I will show that it is possible to modify the cone approach in order to get a cross
section which only depends upon the direct process. I argue that this prescription is infrared safe at any order in perturbative QCD. The paper is organized as follows: in section 2 I sketch the main ideas in a simple way. In section 3 I give a precise definition of the isolation conditions. Finally, section 4 contains my conclusions.

2. A simple formulation of the problem

In the cone approach, the first naive procedure is to draw a cone around the photon axis, and to impose that no quark or gluon is found inside the cone. With this definition, the configurations where a parton is collinear to the photon are rejected, and therefore the contribution of the fragmentation process is exactly zero. Unfortunately, this prescription is not infrared safe: soft gluons can not be emitted inside the cone, thus spoiling the cancellation of infrared singularities. One may relax the definition, by allowing a small amount of hadronic energy inside the cone. This restores the correct infrared behaviour, but at the same time introduces a dependence upon the fragmentation functions, since collinear configurations are not forbidden any longer. Another possibility, which only works at next-to-leading order, is to allow soft gluons inside the cone, but to exclude the quarks. This prescription, however, is non-physical and its predictions can not be straightforwardly compared with experimental results.

Therefore, to define an infrared-safe cross section, there should be no region of forbidden radiation in the phase space, while to eliminate the dependence upon the fragmentation functions such a region must exist; these two requirements are seemingly incompatible. I will argue in the following that actually this is not the case. In fact, the fragmentation mechanism in QCD is a purely collinear phenomenon; therefore, to eliminate its contribution to the cross section, it is sufficient to veto the collinear configurations only. In practice, this can be achieved in the following way (I restrict for the moment to the case of $e^+e^-$ collisions). A cone of (fixed) half-angle $\delta_0$ is drawn around the photon axis. Then, for all $\delta \leq \delta_0$, the total amount of hadronic energy $E_{\text{tot}}(\delta)$ found inside the cone of half-angle $\delta$ drawn around the photon axis is required to fulfill the following condition

$$E_{\text{tot}}(\delta) \leq K \delta^2,$$  \hspace{1cm} (2.1)

where $K$ is some energy scale (the form $K \delta^2$ is chosen for illustrative purposes; it will be generalized in the following). According to eq. (2.1), a soft gluon can be arbitrarily
close to the photon. On the other hand, eq. (2.1) implies that the energy of a parton emitted exactly collinear to the photon must vanish. Therefore, the contribution of the fragmentation process is restricted to the zero-measure set \( z = 1 \).

In the following section, I will give a precise definition of the isolation condition, refining eq. (2.1). Here, I stress that eq. (2.1) does not spoil the cancellation of soft gluon effects, and achieves the isolation of the photon in a smooth way, which can be easily implemented at the experimental level.

3. Isolated-photon plus jets cross section

I start from the class of scattering events whose final state contains a set of hadrons, labelled by the index \( i \), with four-momenta \( k_i \), and a hard photon with four-momentum \( k_\gamma \). I assume to be in a kinematic regime where the masses of the hadrons are small compared to their (transverse) energies. Also, in a real experimental situation, we may think of the \( k_i \) as the four momenta deposited in the \( i^{th} \) calorimetric cell, instead of the four momentum of the \( i^{th} \) hadron. Fix the parameter \( \delta_0 \), which defines the so-called isolation cone, and apply to each event the following procedure (isolation cuts).

1. For each \( i \), evaluate the angular distance \( R_{i\gamma} \) between \( i \) and the photon. The angular distance is defined, in the case of \( e^+e^- \) collisions, to be
   \[
   R_{i\gamma} = \delta_{i\gamma},
   \]
   where \( \delta_{i\gamma} \) is the angle between the three-momenta of \( i \) and \( \gamma \). In the case of hadronic collisions I define instead
   \[
   R_{i\gamma} = \sqrt{(\eta_i - \eta_\gamma)^2 + (\varphi_i - \varphi_\gamma)^2},
   \]
   where \( \eta \) and \( \varphi \) are the pseudorapidity and azimuthal angle respectively.

2. Reject the event unless the following condition is fulfilled
   \[
   \sum_i E_i \theta(\delta - R_{i\gamma}) \leq \mathcal{X}(\delta) \quad \text{for all} \quad \delta \leq \delta_0,
   \]
   where \( E_i \) is the energy of hadron \( i \) and, due to \( \theta(\delta - R_{i\gamma}) \), the sum gets contribution only from those hadrons whose angular distance from the photon is
smaller than or equal to $\delta$. The function $\mathcal{X}$, which plays the rôle of $K\delta^2$ in eq. (2.1), is fixed and will be given in the following. The function $\mathcal{X}$ must vanish when its argument tends to zero, $\mathcal{X}(\delta) \to 0$ for $\delta \to 0$. At hadron colliders, the transverse energy $E_{\gamma T}$ must be used instead of $E_i$.

3. Apply a jet-finding algorithm to the hadrons of the event (therefore, the photon is excluded). This will result in a set of $m + m'$ bunches of well-collimated hadrons, which I denote as candidate jets. $m$ ($m'$) is the number of candidate jets which lie outside (inside) the isolation cone, in the sense of the angular distance defined by eqs. (3.1) or (3.2).

4. Apply any other additional cuts to the photon and to the $m$ candidate jets which lie outside the cone (for example, the cut over the minimum observable (transverse) energy of the jets must be applied here).

An event which is not rejected when the isolation cuts are applied is by definition an *isolated-photon plus $m$-jet* event. The key point in the above procedure is step 2: hadrons are allowed inside the isolation cone, provided that eq. (3.3) is fulfilled. This in turn implies the possibility for a candidate jet to be inside the isolation cone. It would not make much sense to define a cross section exclusive in the variables of such a jet, which can not be too hard. For this reason, in the physical observable that I define here, the jets which accompany the photon are the candidate jets outside the isolation cone which also pass the cuts of step 4. The resulting cross section is therefore totally exclusive in the variables of these jets and of the photon, and inclusive in the variables of the hadrons found inside the isolation cone. Notice that this is not equivalent to applying a jet-finding algorithm only to the hadrons lying outside the isolation cone; in fact, such a procedure is not infrared-safe.

I define

$$
\mathcal{X}(\delta) = E_{\gamma}\epsilon_{\gamma} \left( \frac{1 - \cos \delta}{1 - \cos \delta_0} \right)^n, \quad (3.4)
$$

where $E_{\gamma}$ is the photon energy (in the case of hadron collisions, $E_{\gamma}$ must be replaced by the transverse energy of the photon, $E_{\gamma T}$). I will use

$$
\epsilon_{\gamma} = 1, \quad n = 1. \quad (3.5)
$$

The reason for this choice will be discussed in the following. Here, I stress that this choice is arbitrary to a large extent. The main feature of the function $\mathcal{X}$ is that

$$
\lim_{\delta \to 0} \mathcal{X}(\delta) = 0. \quad (3.6)
$$
In QCD, any jet cross section is easily written in terms of measurement functions [4]. Given a \(N\)-parton configuration \(\{k_i\}_{i=1}^N\), the application of a jet-finding algorithm results in a set of \(M\) jets with momenta \(\{q_a\}_{a=1}^M\). This can be formally expressed by the measurement function

\[
S_N \left( \{q_a\}_{a=1}^M; \{k_i\}_{i=1}^N \right), \tag{3.7}
\]

which embeds the definition of the jet four-momenta in terms of the parton four-momenta. It has been shown [4,5,6] that, at next-to-leading order and for an arbitrary type of collisions, the infrared-safeness requirement on the cross section can be formulated in terms of conditions relating the measurement functions \(S_N\) for different \(N\) (I refer the reader to the original publications for details). These conditions can be extended without any difficulties to higher perturbative orders. Here, I stress that the measurement function in eq. (3.7) implements an infrared-safe jet cross section definition, which I will apply to the partons accompanying the photon in a candidate isolated-photon event. By labeling the partons in such a way that

\[
R_{i\gamma} \geq R_{j\gamma} \quad \text{if} \quad i > j, \tag{3.8}
\]

I define

\[
S_{\gamma,N} \left( k_{\gamma}, \{q_a\}_{a=1}^M; \{k_i\}_{i=1}^N \right) = S_N \left( \{q_a\}_{a=1}^M; \{k_i\}_{i=1}^N \right) \times \prod_{i=1}^N I_i, \tag{3.9}
\]

\[
I_i = \theta \left( \mathcal{X} \left( \min(R_{i\gamma}, \delta_0) \right) - \sum_{j=1}^i E_j \theta(\delta_0 - R_{j\gamma}) \right). \tag{3.10}
\]

It is easy to understand that eq. (3.9) is equivalent to the isolation cuts described above. In particular, the quantity \(\prod_{i=1}^N I_i\) is equivalent to step 2. Therefore, \(S_{\gamma,N}\) is the measurement function relevant for the isolated-photon plus jets cross section: it vanishes when applied to those parton configurations where the photon is non-isolated.

I now turn to the discussion of the main features of the definition of isolated photon given in this paper. Clearly, it is sufficient here to investigate the behaviour of the cross section when one or more partons are inside the isolation cone. First of all, I observe that for a soft parton \((E \to 0)\) the isolation cone does not exist at all (see eq. (3.10)). This ensures that the cancellation of soft gluon effects will take place as in ordinary infrared-safe jet cross sections. Secondly, eqs. (3.3) and (3.4)
imply that a parton is softer the closer to the photon axis: a parton exactly collinear to the photon is necessarily soft. Thus, when a quark gets collinear to the photon, the damping associated with the quark vanishing energy suppresses the collinear divergence. Therefore, there is no need for a final-state collinear counterterm.

To be more quantitative, I start by considering the case of a quark inside the isolation cone. I restrict for the moment to the case of $e^+e^-$ collisions. The leading behaviour of the partonic amplitude squared is $1/(1 - y)$, where $y$ is the cosine of the angle between the quark and the photon. The contribution to the isolated-photon cross section from the region inside the isolation cone is therefore

$$\sigma_{\text{cone}} \sim \int_{\cos \delta_0}^1 dy \int_0^E dE \frac{\theta(\mathcal{X}(\delta(y)) - E)}{1 - y},$$

where the factor $E$ originates from the phase space, and the condition of eq. (3.3) has been enforced with a theta function. The upper integration bound in $E$ is irrelevant in what follows, and will be neglected. Using eq. (3.4) we get

$$\sigma_{\text{cone}} \sim E_{\gamma\gamma}^2 \int_{\cos \delta_0}^1 dy \frac{1}{1 - y} \left( \frac{1 - y}{1 - \cos \delta_0} \right)^{2n} = \frac{E_{\gamma\gamma}^2 \epsilon_{\gamma}^2}{4n},$$

provided that $n \geq 1/2$. As previously anticipated, the damping associated with the energy of the quark which gets soft cancels the effects of the collinear divergence, for reasonable choices of $\mathcal{X}$. The fact that there is no need for a final-state collinear counterterm is consistent with the fact that the contribution from the fragmentation function is also vanishing; in QCD, fragmentation is a rigorously collinear phenomenon, and therefore eq. (3.3) would imply $z = 1$. Thus, the contribution of the fragmentation function is restricted to a zero-measure set in the phase space.

I now turn to the case of a gluon inside the isolation cone. The leading behaviour of the partonic amplitude squared is $1/E^2$. Using the subtraction method the contribution to the finite part of the cross section (again, from the region inside the isolation cone) will read

$$\sigma_{\text{cone}} \sim \int_{\cos \delta_0}^1 dy \int_0^E dE \frac{\theta(\mathcal{X}(\delta(y)) - E) - 1}{E}.$$

\footnote{Notice that, by integrating $1/(1 - y)$ outside the isolation cone, one gets a term proportional to $\log(1 - \cos \delta_0)$, as expected in isolated-photon production.}
\footnote{With the subtraction method, the divergent part of the cross section is evaluated strictly in the soft limit, $E = 0$. As previously stressed, the isolation cone does not constrain at all the phase space of soft partons, and therefore this divergent part will be cancelled by the corresponding virtual contribution as customary in perturbative QCD.}
Using eq. (3.4) we get

\[ \sigma_{\text{cone}} \sim \int_{\cos \delta_0}^{1} dy \log \left( E_\gamma \epsilon_\gamma \left( \frac{1 - y}{1 - \cos \delta_0} \right)^n \right) = (1 - \cos \delta_0)(\log(E_\gamma \epsilon_\gamma) - n). \tag{3.14} \]

The case of hadronic collisions is only slightly more complicated. The \( \theta \) function in eqs. (3.11) and (3.13) is now

\[ \theta(\mathcal{X}(R(y)) - E_\gamma). \tag{3.15} \]

This function clearly constrains the energy of the parton, since \( E = E_\gamma \cosh \eta(y) \), where I have explicitly indicated that the pseudorapidity of the parton depends upon \( y \). In order not to spoil the conclusions of eqs. (3.12) and (3.14), one must have \( E \to 0 \) when \( y \to 1 \), or, which is equivalent, \( \cosh \eta(y) \) must tend to a finite constant when \( y \to 1 \) (notice that \( 1 - \cos R(y) \) tends to zero at the same rate of \( 1 - y \) for \( y \to 1 \)).

This is indeed the case, since for the very definition of \( y \) one gets \( \cosh \eta(y) \to \cosh \eta_\gamma \).

In isolated-photon production, the photon is observed in the central region of the detector, and \( \cosh \eta_\gamma \) is of order one.

By increasing the number of partons inside the isolation cone the situation rapidly becomes rather involved, preventing us to perform analytical calculations. However, from the definition of the isolation cuts, it should be clear that, given a configuration which fulfills the isolation criteria, it is always possible that a parton emits a soft gluon, or splits into two collinear partons, without changing abruptly the physical observables. On the other hand, if a configuration does not fulfill the isolation criteria, if a parton emits a soft gluon or splits into two collinear partons, we get a configuration which still does not fulfill the isolation cuts. Therefore, any QCD infrared configuration (soft gluons, quarks and gluons collinear to each other) can be locally (that is, prior to any phase-space integration) subtracted, and the corresponding singularities are cancelled by the virtual contributions. The only singularities which are left unsubtracted are the QED ones (quarks collinear to the photon). However, in this case the singularity is damped by the mechanism described in eq. (3.12).

The fact that soft gluon emission and collinear splitting do not modify the physical observables is formally equivalent to the infrared safeness of the cross section to all perturbative orders. However, it has to be stressed that the isolation cuts have an impact on the local subtraction of singularities. This can be clearly seen from eqs. (3.13) and (3.14), where the integration over the gluon energy \( E \), constrained by the isolation cuts, results in a singular (but integrable) function of \( y \) (without any
isolation condition, the integral over $E$ would simply give zero). It seems reasonable to assume that, at higher orders, the isolation cuts always define a function which, although singular in some regions of the phase space, has a finite integral: however, no formal proof will be given here. This fact has been explicitly verified in the case where a quark and a gluon are inside the isolation cone, which is relevant for the next-to-leading order cross section of isolated-photon production in $e^+e^-$ collisions.

We can understand the impact of the isolation condition on the radiative corrections by looking at eq. (3.14). The definitions of isolated photon in the cone approach which are usually adopted in the literature can be recovered by setting $n = 0$ and $\epsilon_\gamma = \epsilon_c$. $\epsilon_c$ is the maximum amount of hadronic energy (normalized to the photon energy) allowed inside the cone, and to minimize the contribution of the fragmentation mechanism it must be a small number. Therefore, we see from eq. (3.14) that, comparing to the case where the isolation condition is extremely loose ($n = 0, \epsilon_\gamma = 1$), there is a sizeable negative correction $\log \epsilon_c$. On the other hand, with the definition given in this paper, $\epsilon_\gamma$ does not need to be small. The isolation condition is obtained in a smooth way, controlled by the function $\mathcal{X}$. From eq. (3.14), the smaller is $n$, the more moderate will be the (negative) corrections. This is the reason for the choice of $\epsilon_\gamma$ and $n$ made in eq. (3.3). Finally, notice that in standard approaches $\delta_0$ is usually taken to be of the order of $20^\circ$. With the current definition, $\delta_0$ can be chosen to be larger. This implies that the logarithms $\log(1 - \cos \delta_0)$, which are present in the isolated-photon cross section, do not get large.

A final remark is still in order. When $\delta \simeq 0$, the value of $\mathcal{X}(\delta)$ is below the energy threshold $E_{th}$ of hadronic calorimeters, and therefore the condition of eq. (3.3) is not experimentally meaningful any longer. The data can therefore get a contribution from the fragmentation process, with $z > z_{th} = 1 - E_{th}/E_\gamma$. However, from the experimental point of view there is nothing special in the region $\delta \simeq 0$ (obviously, there are no singularities in particle detectors); since $E_{th}$ is typically of the order of few hundred MeV, and the photon is required to be hard, $z_{th} \simeq 1$, and the contribution of the fragmentation mechanism will be small. In other words, one could relax eq. (3.3) by allowing quasi-soft partons to be collinear with the photon; this would imply the presence of a collinear singularity in the direct cross section, which would be cancelled by a proper fragmentation cross section. After the cancellation, one would be left with a very small finite contribution. By strictly imposing eq. (3.3), this small contribution is zero from the beginning, and the fragmentation cross section is simply not there.
4. Conclusions

I presented a definition of isolated-photon plus jets cross section which is based upon an isolation cone and a jet-finding algorithm which excludes the photon. It has been argued that this prescription, which treats identically quarks and gluons, is infrared safe to all orders. Hadrons are allowed inside the isolation cone, at the border of which they can be as energetic as the photon itself, but are required to be softer the closer they are to the photon axis, eventually becoming soft in the exact collinear limit. This fact implies that the contribution of the photons coming from the fragmentation of quarks and gluons can be neglected in QCD. Therefore, for hadron-hadron and photon-hadron collisions, the only non-calculable parts which enter the theoretical prediction are the parton densities of the incoming hadrons. In the case of $e^+e^-$ collisions, the isolated photon plus jets cross section is fully calculable in perturbation theory. The prescription given here is also applicable without any modification to the case of several, isolated photons in the final state.

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