Insulating Behavior of a Trapped Ideal Fermi Gas

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We investigate theoretically and experimentally the center-of-mass motion of an ideal Fermi gas in a combined periodic and harmonic potential. We find a crossover from a conducting to an insulating regime as the Fermi energy moves from the first Bloch band into the bandgap of the lattice. The conducting regime is characterized by an oscillation of the cloud about the potential minimum, while in the insulating case the center of mass remains on one side of the potential.

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Recent experiments have demonstrated the possibility of reaching the superfluid regime with trapped fermionic atoms \[ \text{[1,2]} \]. In this context, the combination of trapped cold fermions and optical lattices \[ \text{[3]} \], is very promising. On the one hand it represents the natural analogy with superconductivity of electrons in crystal lattices, while on the other it might be an additional tool to manipulate the interaction properties of the system \[ \text{[4]} \], and to investigate the superfluid phase. Atomic Fermi gases in lattices are interesting also in view of observing novel quantum phases that have been predicted for interacting Fermi \[ \text{[5]} \] and Fermi-Bose systems \[ \text{[6]} \].

In this work we investigate a Fermi gas of non interacting atoms trapped in a harmonic field and subjected to a one-dimensional optical lattice. We study both theoretically and experimentally the center-of-mass dynamics of the system after a sudden displacement of its equilibrium position along the lattice. We find that the statistical distribution makes the dynamical properties of such system highly non trivial. The most dramatic new feature is that the system can exhibit a conducting or an insulating behavior, depending on whether the Fermi energy lies in the first Bloch band of the lattice or in the gap region. In the first case the cloud oscillates symmetrically in the harmonic potential, while in the second case it remains trapped on one side of the potential. By tuning the width of the lattice band we observe the full crossover between the two different regimes.

Semiclassical Model. Let us first introduce our theoretical model to describe the system. Neglecting the interatomic collisions, the many-body Hamiltonian can be simply written as a sum of single particle Hamiltonians:

\[
H_0 = \left( \frac{p_x^2}{2m} + \frac{1}{2} m \omega_x^2 z^2 + s E_R \sin^2(2\pi/\lambda z) \right) + \left( \frac{p_y^2}{2m} + \frac{1}{2} m \omega_y^2 (x^2 + y^2) \right),
\]

where \( s \) is a tunable dimensionless parameter, \( E_R = \frac{\hbar^2}{2m} \left( \frac{\lambda}{2\pi} \right)^2 \) is the recoil energy and \( \lambda \) is the wavelength of the laser creating the lattice. Since the harmonic oscillator length is typically much larger than the lattice spacing \( d = \lambda/2 \) we can study the system in the semiclassical approximation.

We first concentrate on the \( z \)-component of the Hamiltonian \[ \text{[10]} \]. In the limit of a vanishing harmonic field \((\omega \rightarrow 0)\), the eigenfunctions of the stationary Schrödinger equation are Bloch waves. The eigenergies \( \epsilon_n(p_z) \) have the typical Bloch band structure as a function of the quasimomentum \( p_z \), defined in the first Brillouin zone \((-\pi\hbar/d \leq p_z \leq +\pi\hbar/d)\), and of band index \( n \) (in the following we consider only the lowest energy Bloch band \( n = 1 \)). In a semiclassical approach, we can incorporate the effects of the periodic potential modifying the kinetic part of the Hamiltonian: \( \frac{p_z^2}{2m} \rightarrow \epsilon(p_z) \), while the harmonic confinement generates a driving field. Therefore, Eq. \[ \text{(1)} \] can be approximated as:

\[
H_0 = \epsilon(p_z) + \frac{1}{2} m \omega_x^2 z^2 + \frac{p_x^2}{2m} + \frac{1}{2} m \omega_y^2 (x^2 + y^2). \tag{2}
\]

The effects of a sudden small displacement \( z_0 \) of the center of the harmonic trap can be included adding the perturbative term \( H_{\text{pert}}(z,t) = m \omega_z^2 \Theta(t)z_0 z \), where \( \Theta(t) \) is the unit step function. A first insight in the behavior of the system is provided by the study of the linear dynamics in a 1-D configuration.

1-D Analysis. The physics becomes particularly clear in the semiclassical phase space, as shown in Fig. \[ \text{1} \] The lines are isoenergetic single particle orbits; they belong to two different classes, separated by the dashed orbit, which has an energy \( 2\delta \), corresponding to the width of the first Bloch band. The orbits are close when their energy is in the band, i.e. the particles oscillate around the trap minimum. The orbits are instead open when their energy is in the gap, because the quasimomentum...
evolution (figs. C and D). The ordinate and abscissa are in
the trap (figs. A and B, respectively), and their dynamical
shows a strong dispersion with energy.

In general the oscillation frequency of both kinds of orbits
cillate on the sides of the harmonic potential, performing
is reversed at the band edges. Particles in these orbits os-
tial. Notice that in this 1-D configuration, the damping
hand, the yellow orbits remain open, trapping the center
are dominated by the particles moving around the cen-
the relaxation and the frequency of the oscillation mode
time scale with respect to the yellow ones. Therefore,
out of equilibrium and give rise to a collective dipole mo-
in equilibrium in the new configuration of the trapping
Fig. 1B). The blue region contains particles that are still

corresponds to a shift of the center of the harmonic potential
fill the phase space region with energy below
\[ \tilde{z}(t) = -1 + \frac{2m\omega_z^2}{\pi} \int_0^{+\infty} d\omega \left( \begin{array}{c} 1 - \cos(\omega t) \\ \omega \end{array} \right) \Im(\chi(\omega)) \] (4)
where \( \Im(\chi(\omega)) \) is the imaginary part of the dipole re-
sponse function \( \chi(\omega) \).
In order to calculate it we solve
Eq. 3 in the linear regime by the general method illus-
trated in [8], by using the energy of motion in z-direction
\( E_z = \varepsilon(p_z) + \frac{1}{2}m\omega_z^2 z^2 \) as an independent variable instead of \( p_z \).
The imaginary part of the linear response function then becomes
\[
\Im(\chi(\omega)) = \frac{1}{(2\omega_{\perp})^2 \pi} \sum_{n=1}^{+\infty} \int_0^{+\infty} dE_\perp \delta(\omega - n\omega_0(E_\perp)) \psi (E_\perp) \left[ \int_0^{+\infty} dE_\perp \frac{\partial f_0(E_\perp + E_\perp, T)}{\partial E_\perp} \right] Q^2(n, E_\perp)
\] (5)
where
\[
Q(n, E_\perp) = -\frac{1}{n\pi} \int_{z_1(E_\perp)}^{z_2(E_\perp)} dz \sin \left( n\pi \frac{\omega_0(E_\perp)}{\omega_0(E_\perp, z)} \right)
\] (6)
and
\[
\omega_0(E_\perp, z) = \pi \left( \int_{z_1(E_\perp)}^{z_2(E_\perp)} \frac{d\zeta}{(\partial c(P_z)/\partial P_z)_{P_z = P_z(\zeta, E_\perp)}} \right)^{-1}.
\] (7)
In the above equations \( \omega_0(E_\perp) = \omega_0(E_z, z = z_2(E_z)) \) is the single particle frequency, \( z_1(E_z) \) and \( z_2(E_z) \) are the inversion points of the motion (i.e. the maximum and the minimum positions reached by the particle of energy \( E_z \)). Notice that, due to the 3-D nature of the system, all val-
des \( E_z \leq E_F \) contribute to the response function \( \Im(\chi) \) even at zero temperature, where the integral inside the square brackets becomes \( (E_z - E_F)\Theta(E_F - E_z) \). As a con-
sequence a damping due to dephasing is in general present
also in the linear limit, which is different from the 1-D case. As shown by the Eqs. 37, in order to calculate
the dipole motion we need to know explicitly the Bloch
energy \( \varepsilon(p_z) \) as a function of the quasimomentum. In

tight binding approximation (large lattice height, corre-
ponding to small tunnelling between the potential bar-
rriers) the first band is \( \varepsilon(p_z) = 2\delta\sin^2(p_z \lambda/4\hbar) \) where
\( \delta \equiv \delta(s) \) is a decreasing function of the lattice height.
With this choice for the dispersion the frequency
\( \omega_0(E_z) \) can be expressed in terms of elliptic integrals [10] and we

### FIG. 1: Phase trajectories for a trapped 1-D Fermi gas in a
lattice at \( T = 0 \), just before and after the displacement of
the trap (figs. A and B, respectively), and their dynamical
shows a strong dispersion with energy.

- **A** and **B** show the phase space of the particles in the trap,
- **C** and **D** illustrate the evolution of the system after the displacement.

### 3-D Analysis

The semiclassical phase-space distribution \( f(\vec{r}, \vec{p}, t) \) is governed by the Liouville equation
\[
\frac{\partial f}{\partial t} + \frac{\partial H}{\partial \vec{p}} \frac{\partial f}{\partial \vec{r}} - \frac{\partial H}{\partial \vec{r}} \frac{\partial f}{\partial \vec{p}} = 0
\] (3)
where \( H = H_0 + H_{\text{pert}} \). Such an equation correctly de-
scribes the quantum dynamics up to orders of \( O(\hbar^2) \).
can calculate analytically the quantities of interest. In regimes out of tight binding we use a parametrization of the energy band [11].

**Experiment.** We realize experimentally the system by using a Fermi gas of $^{40}$K atoms. The sample is prepared in the $F = 9/2$, $m_F = 9/2$ state and sympathetically cooled to quantum degeneracy in a magnetic trap with frequencies $\omega_z = 2\pi \times 24$ s$^{-1}$ and $\omega_\perp = 2\pi \times 275$ s$^{-1}$. Subsequently a one-dimensional optical lattice is adiabatically superimposed along the weak axis of the trap (z direction) [12]. The lattice is created by a laser beam in standing-wave configuration, with wavelength $\lambda = 863$ nm. The lattice height can be adjusted in the range $U = 0.1 - 8E_R$, where $E_R/k_B = 317$ nK. The typical Fermi gas is composed by 25,000 atoms at a temperature that can be varied between $0.2 T_F$ and $T_F$, where the Fermi temperature is $T_F \approx 300$ nK. The system is brought out of equilibrium by displacing the magnetic trap minimum along the lattice. The typical displacement is $z_0=15$ $\mu$m, much smaller than the $1/e^2$ radius of the cloud (110 $\mu$m) [13]. After a variable evolution time in the trap the atoms are released from the combined potential. We detect the position of the center of mass of the cloud by absorption imaging after a ballistic expansion of 8 ms.

**Results.** In Fig. 2 we show both the theoretical prediction (solid line) and the experimental observation (solid circles) of the dipole oscillation of a Fermi gas at $T = 0.3 T_F$ in presence of a lattice with $s=3$. For comparison, we show also the dipole motion in the absence of the periodic potential (dashed line and open circles) which consists in an undamped harmonic oscillation at the trap frequency $\omega_z$. We observe a dramatic change of the dipole oscillation in presence of the lattice, which is well described by the 3-D model. The appearance of an offset in the oscillations is due to the significant fraction of particles moving along open orbits; for the given parameters the Fermi energy is indeed larger than the bandwidth $2\delta \approx 0.4 E_F$. This fraction behaves macroscopically as an insulator, because its center of mass does not move under the harmonic force but stays trapped on one side. The fraction of the gas occupying closed orbits can instead oscillate in the harmonic potential, and has therefore a conducting nature. A damping appears as expected because of the dephasing between different orbits. Also, the oscillation frequency is reduced because of the larger effective mass of the atoms in the lattice.

To understand how such phenomenology depends on the width of the first energy band, we have performed a series of measurements by keeping the atom number and temperature of the Fermi gas constant, and varying the lattice height. In this way it was possible to move the Fermi energy within the energy gap between the first and second band of the lattice. In Figs. 3 we plot the measured dependencies of the offset, damping rate and oscillation frequency, that are in good agreement with the theoretical calculations. Fig. 4 shows the crossover from a conducting behavior in low lattices to an almost completely insulating behavior in higher lattices. Here we have plotted the relative oscillation offset defined as $\delta z_0/z_0$, where $z_0$ is the center of oscillation of the system in the lattice, as a function of the lattice height. Note how the relative offset, which represents the insulating fraction of the Fermi gas, stays small as long as $2\delta < E_F$, and then raises quite rapidly towards unity. Since in the present experiment $E_F \approx E_R$, an insulating fraction appears already with low lattices; the theory however shows that the threshold for the insulation moves to higher lattices in case of smaller Fermi energies (dashed line in Fig. 3). The disagreement between experiment and theory at low lattice heights, $s < 3$, can arise from the population of higher bands due to the finite temperature and/or Landau-Zener tunnelling, which
are not included in the theoretical model. Increasing $s$, the energy gap between bands increases, and the single band calculations become more realistic.

In Fig. 4 we show the observed features of the conducting fraction of the gas. The damping rate of the oscillation shown in Fig. 4, also increases with the lattice height, because of an increased dispersion of the oscillation frequencies of atoms in closed orbits. As shown in Fig. 4, the oscillation frequency of this component of the gas is close to that expected for a particle in the bottom of the band $\omega_0(E_z = 0)$. Actually, the theoretical analysis shows that this value is strictly reached only asymptotically during the oscillation, because initially closed orbits with different frequencies contribute to the dipole motion.

As a consequence of the Pauli principle, which keeps the energy distribution broad, the Fermi gas exhibits an insulating behavior even at $T = 0$. The observed phenomena only weakly depend on the gas temperature, at least in the region 0.2-1 $T_F$ that we have explored so far in both experiment and theory, and in general we observe an increase of both offset and damping for increasing temperatures, as expected because of the broader energy distribution. An analogous insulating phenomenon can be observed in uncondensed Bose gases at temperatures larger than the bandwidth. Indeed also in this case a significative part of the single particle orbits will have energy in the gap region giving rise to insulation. However we have observed [15] that in the case of bosons, or even in the case of fermions admixed with bosons, interatomic collisions quench the open orbits and eventually bring the system into the equilibrium position.

**Conclusion.** In this paper we have investigated the dynamics of a non-interacting Fermi gas in presence of a combined periodic and harmonic potential, comparing the experimental observation with a semiclassical theory. We have shown that the system crosses from a conducting to an insulating behavior by tuning the Fermi energy to the equilibrium position.

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![Figure 4](image)

**FIG. 4: A)** Comparison between theory (line) and experiments (circles) for the damping rate of the dipole oscillations of the Fermi gas as a function of the lattice height. **B** Oscillation frequency of the Fermi gas as a function of the lattice height. The line is the expectation for a particle oscillating at the band bottom.

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