RELATIVISTIC MODELS OF KHZ QPOS

W. KLUŹNIAK

Copernicus Astronomical Center
ul. Bartycka 18, 00-716 Warszawa, Poland
wlodek@camk.edu.pl

Abstract. After reviewing the general-relativistic “gap” model of accretion, I discuss its relation to the high frequency quasi-periodic oscillations observed in low-mass X-ray binaries. The “300” Hz frequency seen in some X-ray bursts may be a relativistic signature of keplerian rotation of the neutron star.

It is easy to see how much the field has advanced in the past decade by comparing the topics under discussion here in Elounda, with those discussed at that previous conference of this series which also took place in Crete, in Agia Pelagia, at the beginning of the decade.

Back then, magnetic fields were all the rage. Gamma-ray bursts supposedly showed in their spectra cyclotron absorption lines suggesting (to many) that the sources are Galactic neutron stars, a view completely ruled out in the decade of Compton GRO, Beppo SAX and the observations of afterglows (described carefully in Dr. Fishman’s talk here in Elounda). Another view much discussed at Agia Pelagia was that low-mass X-ray binaries contain direct counterparts of millisecond pulsars, i.e., $10^9$ to $10^{10}$ Gauss neutron stars, rotating at periods of a few milliseconds, and therefore accreting through a disk in which the orbital frequency (supposedly) differed from the stellar rotational frequency by about 50 Hz. Today, after the discovery of kHz QPOs (described in these proceedings by M. van der Klis), there is hardly any doubt that the characteristic orbital frequency in the inner disk is at least 1 kHz, very different from the value of about 300 Hz promoted at Agia Pelagia.

So let us forget about magnetic fields in LMXBs and ask what would be expected then. The answer depends on the equation of state (e.o.s.) of matter at supranuclear densities and on the mass of the neutron star, as well as its angular momentum. Here I will only discuss the general relativistic
“gap” regime of accretion, in which the accretion disk does not extend to the stellar surface—there are good reasons for that.

1. The relativistic gap regime

LMXBs are old accreting systems, so a priori one would expect that the central neutron stars have each gained a few tenths of a solar mass since their early days. Now, as pointed out some time ago (Kluźniak and Wagoner 1985), for all e.o.s., at sufficiently high stellar mass (which need not be very large), a slowly rotating neutron star is within the innermost stable circular orbit (ISCO) allowed by general relativity (GR), a.k.a. the marginally stable orbit. For rapidly rotating neutron stars this is not always so, but according to the tables of Cook et al. (1994), for most e.o.s. the maximally rotating models are also within the ISCO. For strange (quark) stars this is also true (Stergioulas et al. 1999). In short, it seems reasonable to assume that in LMXBs, the compact object is inside the ISCO, so let us do so.

The three-dimensional flow in accretion disks is still poorly understood (it may resemble the flow of waves crashing on the beach, particularly the rip tide dreaded by ocean swimmers—see the figure from Kita’s 1995 thesis reproduced in Kluźniak 1998b). But in any case, in the relativistic gap regime, the disk should be terminated by GR effects, as in the black hole disks, whose essential properties were discussed in numerous papers, e.g., of the Warsaw school some two decades ago (by Paczyński, Abramowicz, Sikora, Muchotrzeb and others, in various combinations). Without further ado, let us accept the view that the maximum observable frequency in LMXB disks is close to the ISCO frequency and that this frequency may modulate the X-ray flux (Kluźniak et al. 1990). Then it will be easy to believe that the saturation (at 1.07 kHz) of QPO frequency in 4U 1820-30 is a signature of the ISCO (Zhang 1998), and that the e.o.s. is severely constrained by the observed maximum frequency value (Kluźniak 1998a).

What happens to the matter which leaves the disk through its inner edge (assumed to be close to the ISCO radius)? It goes into free-fall and approaches the surface at a rather shallow angle. Under these conditions, a sheared atmosphere heated by the incoming fluid is set up in the equatorial regions (or even the tropics, as in Dr. Sunyaev’s talk), whose vertical structure has been found in a 1+1–d calculation with full radiative transfer (Kluźniak and Wilson 1991): the atmosphere is hot and gives off radiation with a power law spectrum extending to about 200 keV. This would agree with reports of hard radiation from several X-ray bursters. Of course this spectrum may be downgraded as the radiation interacts with the (relatively) cool disk and the accretion stream, this interaction has not yet
been computed, but it is clear that on such a picture one would expect the down-scattered softer photons to lag in time the harder photons (as has been reported in SAX J 1808.4–3658).

My prejudiced view is that the balance of observations is in favour of the gap regime. It really seems that several separate facts (especially these: the presence of kHz QPOs and their frequency values, hard spectra, soft lags) suggest that the disk is terminated outside the stellar surface by effects of general relativity.

2. Model independent conclusions about QPOs?

The power spectrum of both neutron star and candidate black hole systems has been studied over the whole range of observed frequencies, and the phenomenology of QPOs in both types of systems was found to be remarkably similar (Psaltis et al. 1999). The neutron star systems show two kHz QPOs, the one with lower frequency has a clear counterpart in black hole sources (for example, in both types of systems it has identical correlations with lower frequency features in the spectrum), only the highest frequency QPO has a different phenomenology (Psaltis et al. 1999).

If the black hole candidates are indeed black holes, all black hole QPOs and their counterparts in neutron star systems must be accretion disk phenomena, reflecting fundamental properties of flow in the gravitational field of the compact object. A logical conclusion would be that in the neutron star systems it is the lower frequency kHz QPO which could be connected with orbital motion, with its characteristic cut-off frequency in the ISCO. This would make constraints on the e.o.s. even more stringent than the ones inferred from the higher frequency QPO, and discussed in the previous section (but in either case, these constraints would be relaxed if the QPO frequency were lower than the orbital frequency). The same model should then describe the power spectra of neutron stars in LMXBs as of accreting black holes, including those in AGNs (where the frequency would be scaled down in inverse proportion to the black hole mass).

On this picture, it would be the highest frequency kHz QPO alone, which would need a special explanation for neutron star systems. A special topic of attention must be the similar value of the difference in the two “kHz” QPO frequencies, to the ∼ 300 Hz frequency of the coherent peak in the power spectrum seen in X-ray bursts (Strohmeyer et al. 1997).

3. Keplerian rotation?

While accreting mass in LMXBs, the neutron stars are also accreting angular momentum, a lot of it. Exact models (Cook et al. 1994) show that maximally rotating neutron stars have angular momentum $J \approx 0.6GM^2/c$,
this amount of momentum can be accreted already with $\sim 0.2M_\odot$ in mass (Kluźniak and Wagoner 1985). Several instabilities are known which can limit the spin rate of a neutron star (mostly through emission of gravitational radiation), but it is not known whether they actually operate in practice. The most recently discovered $r$-mode instability could, in principle, limit neutron star periods to values even as long as a few milliseconds in LMXBs (Andersson et al. 2000).

On the observational side, there is no compelling evidence of the periodicity of persistent accretors in LMXBs (with the exception of the one or two strongly magnetized X-ray pulsars which have a low mass companion, but have nothing to do with the atoll, banana, and other LMXBs sources so colourfully described by Michiel van der Klis). This is why the 2.5 ms coherent period discovered in the transient SAX J 1808.4–3658 gave rise to much excitement. Of course, the famous “300 Hz periodicity” discovered in several X-ray bursts has been interpreted as the stellar rotational frequency (Strohmyer et al. 1997), but there seems to be no good model for its observed properties (the “hot spot model” has been criticized here in Elounda, on different grounds, by Fred Lamb and Rashid Sunyaev). The argument for rotation seems to be: what else could it be? Clearly there is a clock in the system with a very good memory of frequency, and yet one which wanders on short time scales.

The coherent peak in the power spectrum of bursters appears during the X-ray bursts, and persists for several seconds, during which the frequency increases usually, and yet from burst to burst the frequency is amazingly stable. Usually these are hallmarks of an (anharmonic) oscillator. Can we find one in the system?

Marek Abramowicz and I think that we have found an anharmonic oscillator on the road to Knossos, at least in our minds. In the equatorial plane outside of (even a spinning) gravitating body the description of test-particle motion can be reduced to one dimensional motion in an effective potential, $V$. The characteristic shape of the effective potential in GR is shown in the figure. Stable motion in circular orbits, as in the Newtonian case, is possible in the minimum of the potential—for the metric and angular momentum chosen in the figure, this orbit would have radius $r_o = 8GM/c^2$; for a different angular momentum of the test particle in the same metric the potential would have a minimum at a different radius, but always outside the ISCO, which has a radius $r_{ms}$ uniquely fixed by the metric. A characteristic feature of GR metrics is the existence of a maximum of the effective potential (in the figure at $u = 5GM/c^2$), at which unstable circular motion is possible. In all cases, $r_u \leq r_{ms} \leq r_o$, with the equality occurring at the minimum value of angular momentum possible in circular motion of a test particle in the (fixed) metric.
The maximum ("Keplerian") rotation rate of a star with radius $R$ inside the ISCO (i.e., $R < r_{\text{ms}}$) occurs, when $R = r_u$. Imagine, then, that the effective potential in the figure is that of test particles in the external metric of a maximally rotating neutron star of radius $R = 5M$, or just a little bit less, for the same value of specific angular momentum as that of matter on the equator. Now imagine that an explosion (an X-ray burst) lifts some matter off the surface. The radial motion of the matter is an oscillation in the potential well. If the energy of the matter were constant, it would travel to a turning point (at about $r = 12M$) and return to the maximum of the potential. However, if a little energy is removed, the matter oscillates back and forth between the right turning point, and the left one, just below the maximum of $V$. This is the anharmonic oscillator suggested as the
origin of the “300 Hz burst oscillation,” with the frequency increasing as the amplitude of motion (range in \( r \)) decreases. At last, the matter settles in circular orbit at the bottom of the potential well.

3.1. PREDICTION

Clearly, the model presented here requires maximal rotation of the neutron star, which is expected to be higher than 600 Hz (because two radio pulsars with 1.6 ms periods have already been observed).

We (Marek and I), would then predict that no X-ray burst oscillation will be seen in X-ray bursts of sources with a clearly detected (phase connected solution) rotational period of more than 1.6 ms. For instance, in the transient SAX J 1808.4–3658, where a period of \( P = 2.5 \) ms has been measured, no such burst oscillation should be discovered.

4. Summary

It seems that the relativistic gap regime—the expected basic mode of accretion onto neutron stars with very weak magnetic fields—fits most observations of LMXBs, including the essential phenomenology of QPOs. The regime allows for neutron star rotation rates higher than orbital frequencies in the disk. For maximally rotating neutron stars, an oscillation in the relativistic “potential well” is possible, with properties similar to those of the “300 Hz” oscillation observed in some X-ray bursts.

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