A Precision Age Determination Technique for Globular Clusters

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ABSTRACT
Globular cluster age estimates based on the absolute magnitude of the main sequence turn-off ($M_v$(TO)) are generally considered to be the most reliable from a theoretical viewpoint. However, the difficulty in determining $M_v$(TO) in observed colour-magnitude diagrams leads to a large error in the derived age. In this paper, we advocate the use of the absolute magnitude of the point which is brighter than the turn-off and 0.05 mag redder ($M_v$(BTO)) as a precision age indicator. It is easy to measure this point on observed colour-magnitude diagrams, leading to small observational error bars. Furthermore, an extensive Monte Carlo calculation indicates that the theoretical uncertainty in $M_v$(BTO) is similar to $M_v$(TO). As a result, ages derived using $M_v$(BTO) are at least a factor of 2 more precise than those derived using $M_v$(TO). This technique is applied to the globular cluster M68 and an age of $12.8 \pm 0.3$ Gyr is derived, indicating that M68 is a ‘young’ globular cluster. A homogeneous set of globular cluster age estimates with this precision would provide unprecedented insight into the formation of the Galactic halo.

Key words: globular clusters: general — methods: data analysis — stars: evolution — stars: interiors — stars: Population II

1 INTRODUCTION
There are a number of different techniques which may be used to determine the age of a globular cluster (GC). All of these methods rely on comparing some aspect of theoretical stellar evolution models to the observations. Thus, in order to evaluate the reliability of the various age indicators, one must be aware of the uncertainties in theoretical stellar evolution models. The correct treatment of convection in stellar models is an area of active research (e.g. Kim et al. 1995, 1996; Demarque, Guenther & Kim 1996a,b) and remains the largest possible source of error in stellar models. For this reason, properties of the stellar models which depend on the treatment of convection are the most uncertain. The main sequence and red giant branch stars in GCs have surface convection zones, and so the predicted radii (and hence, colours) are subject to large theoretical uncertainties. The helium burning stars (horizontal branch, and asymptotic giant branches) are convective in the energy generation regions, and so even the predicted lifetimes and luminosities of stars in this phase of evolution are somewhat uncertain. An additional consideration when considering the reliability of stellar models is that observed CNO abundances in stars on the red giant branch indicate that some form of deep mixing occurs in these stars, which is not present in the models (e.g. Langer et al. 1983; Kraft 1994; Chaboyer...
This indicates that the red giant branch models are in need of revision. In contrast, low mass main sequence models are in excellent agreement with the observations. Indeed, inversions of solar models which use the observed p-modes indicate that the run of density and sound speed in solar models agree with the Sun to within 1% (Basu et al. 1996).

The relative reliability of the age-luminosity relationship for low mass stars is well known, and it is for this reason that the absolute magnitude of main sequence turn-off ($M_v(\text{TO})$) results in GC ages with the smallest theoretical error (e.g. Renzini 1991). Operationally, $M_v(\text{TO})$ is defined to be the magnitude of the bluest point on the main sequence. (Since this definition involves the use of colour $M_v(\text{TO})$ is not strictly independent of the uncertainties in stellar radii.) Unfortunately, the turn-off region has nearly the same colour over a large range in magnitude. This leads to difficulties measuring $M_v(\text{TO})$ observationally, due to the scatter in the observed points around the turn-off. Observers typically quote errors of order 0.10 mag in $M_v(\text{TO})$, which leads to an error in the derived age around $\pm 1.5 \text{ Gyr}$ (e.g. Sarajedini & King 1989; Chaboyer, Demarque & Sarajedini 1996, hereafter CDS). This large error in the derived age of any individual GC is a great obstacle in furthering our understanding of galaxy formation. This problem has led Sarajedini & Demarque (1990) and VandenBerg, Bolte & Stetson (1990) to advocate the use of the difference in colour between the main sequence turn-off and the base of the giant branch ($\Delta(B-V)$) as an age indicator. This method has the advantage that the colour of the turn-off and the base of the giant branch can be accurately determined in observed colour-magnitude diagrams (CMDs), and is independent of the distance modulus. As a result, this method can lead in principle to very precise age estimates (of order $\pm 0.5 \text{ Gyr}$). However, the theoretical colours are subject to large uncertainties and $\Delta(B-V)$ only yields reliable relative age differences between clusters of a similar metallicity (see, however, the case of M68 and M92 (§3) for which $\Delta(B-V)$ fails). In this paper, we advocate the use of a point which is brighter than the turn-off, and 0.05 mag redder in B-V (hereafter referred to as $M_v(\text{BTO})$). This point is easy to measure in observational data and has a small theoretical uncertainty. As it still requires knowledge of the distance modulus, $M_v(\text{BTO})$ complements the $\Delta(B-V)$ technique in providing precision age estimates for GCs. In §4 a Monte Carlo set of isochrones is described and analyzed in order to estimate the theoretical error associated with $M_v(\text{BTO})$ and $M_v(\text{TO})$. The well studied GC M68 is used to illustrate the relative precision of ages derived using $M_v(\text{BTO})$ and $M_v(\text{TO})$ in §5. Finally, §6 summarizes the results of this work and suggests that observers should quote $M_v(\text{BTO})$ in their papers in addition to $M_v(\text{TO})$. Simple formulae are provided to determine GC ages, given $M_v(\text{BTO})$ and [Fe/H].

## 2 THEORETICAL ANALYSIS

The basic problem in measuring $M_v(\text{TO})$ is that the turn-off region is nearly vertical in the HR diagram. Thus, the colour of the main sequence turn-off is well defined, but its magnitude is not. As stars evolve off the main sequence they quickly expand, and so points somewhat brighter than the turn-off are more horizontal in the HR diagram. Thus, it is easy to measure the magnitude of $M_v(\text{TO})$, and ages derived using $M_v(\text{BTO})$ will have small observational error bars. The main reason for using $M_v(\text{TO})$ as an age indicator is that it is widely perceived to be the most robust of the theoretical age estimators. Thus, ages derived using $M_v(\text{TO})$ have small theoretical ‘error bars’. The key question then is whether the theoretical uncertainty in $M_v(\text{BTO})$ is similar to that in $M_v(\text{TO})$. If that is the case, then there would be no reason to use $M_v(\text{TO})$ in GC age estimates.

The theoretical error in $M_v(\text{BTO})$ may be estimated by constructing a series of isochrones under a variety of assumptions. In a study designed to provide an estimate of the error associated with GC ages Chaboyer, Demarque, Kernan & Krauss (1996, hereafter CDKK) calculated 1000 independent sets of isochrones. These isochrones were constructed via a Monte Carlo analysis, whereby the various input elements needed to compute a stellar model and isochrone (such as opacity, mixing length, etc.) were picked at random from distributions based on a careful analysis of the recent literature. Table 1 provides an outline of the various input parameters and their distribution. Further details are provided in CDKK. This Monte Carlo study was designed to yield a set of isochrones which span the range of relevant uncertainties in modern stellar evolution calculations.

In the original work, each set of isochrones consisted of 45 isochrones in the age range 8 – 22 Gyr, with metallicities [Fe/H] = −2.5, −2.0 and −1.5. We have supplemented this with additional calculations with [Fe/H] = −1.0 and −0.5 to span the majority of the GC metallicity range. The three lowest metallicity isochrones in the original set assumed that the helium abundance was equal to its primordial value ($Y_p$). The higher metallicity isochrones in the new calculations allowed for some helium enrichment, assumed to be $Y = Y_p + 1.8 \Delta Z$, where $Z$ is the mass fraction of heavy elements. In addition, the [Fe/H] = −0.5 isochrone assumed that the oxygen enhancement was one-half (in dex) of that in the more metal-poor isochrones. In total, about $5 \times 10^4$ stellar evolutionary runs were performed, involving the calculations of nearly 8 million stellar models.

The Monte Carlo set of 14 Gyr, [Fe/H] = −2.0 isochrones is shown in Figure 1, with the turn-off and brighter points highlighted. This figure demonstrates the very wide range in colour which is possible in theoretical isochrones, given the uncertainties in present stellar models and isochrone construction. As expected, the range $M_v(\text{TO})$ is rather small. The 1-σ (68% confidence limits) range in $M_v(\text{TO})$ is $\pm 0.0625 \text{ mag}$. The 1-σ range in $M_v(\text{BTO})$ is nearly identical, $\pm 0.0625 \text{ mag}$. This analysis has been repeated for the other metallicities, and in all cases the spread in $M_v(\text{BTO})$ was found to be quite similar to the spread in $M_v(\text{TO})$. The mean 1-σ range was $\pm 0.066 \text{ mag}$ in $M_v(\text{TO})$, and $\pm 0.068 \text{ mag}$ in $M_v(\text{BTO})$. This analysis was repeated on a subset of 400 Monte Carlo isochrones with ages of 10 and 18 Gyr and similar results were obtained. This strongly

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suggests that the theoretical error associated with ages derived from $M_v$(BTO) will be similar to those derived from $M_v$(TO).

This issue may be addressed directly by comparing the spread in derived ages for a given value of $M_v$(TO) and $M_v$(BTO). Using the standard set of isochrones described in the next paragraph, values of $M_v$(TO) and $M_v$(BTO) were chosen which yielded an age of 15 Gyr. These fixed values of $M_v$(TO) and $M_v$(BTO) (along with the corresponding metallicity) were used as input parameters for a program which determined the corresponding ages in each of our 1000 independent sets of isochrones. The resulting set of 1000 $M_v$(TO) and $M_v$(BTO) ages were analyzed. The dispersion in age was only slightly larger for ages derived using $M_v$(BTO) as compared to $M_v$(TO). For example, the 1-$\sigma$ dispersion at [Fe/H] $= -2.0$ was $\pm 0.9$ Gyr for the $M_v$(TO) ages versus $\pm 1.0$ Gyr for the ages derived using $M_v$(BTO). This indicates that the theoretical uncertainty in ages derived using $M_v$(BTO) is similar to the theoretical uncertainty in ages derived using $M_v$(TO).

Given that $M_v$(BTO) has a similar theoretical uncertainty to $M_v$(TO) the next important issue to address is the sensitivity of $M_v$(BTO) to age changes. In order to evaluate this issue, a single set of isochrones was constructed, with our best estimate for the input physics and composition. For the primordial helium abundance, a value of $Y = 0.235$ was chosen. The effect of the enhancement of the $\alpha$-capture elements (O, Mg, Si, S, and Ca) was taken into account by modifying the relationship between Z and [Fe/H], as prescribed by Salaris, Chieffi & Straniero (1993). Over the range 2.5 $\leq$ [Fe/H] $\leq -1.0$ a value of $[\alpha/Fe] = +0.55$ was employed (Nissen et al. 1994), while at [Fe/H] $= -0.5$, $[\alpha/Fe] = +0.275$ was assumed. In this set of isochrones, the points $M_v$(TO) and $M_v$(BTO) were determined and fit to a simple quadratic of the form

$$t_0 = a_0 + a_1 M_V + a_2 M_V^2,$$

where $t_0$ is the age in units of Gyr. The coefficients of the fit at each metallicity are given in Table 2. These coefficients are valid for derived ages in the range 8 $-$ 22 Gyr.

Figure 2 plots a subset of the standard set of isochrones for ages of 10, 14, 18 Gyr, and [Fe/H] $= -2.0$ (the same metallicity as in Fig. 2). This figure graphically illustrates that $M_v$(TO) and $M_v$(BTO) have similar sensitivity to age changes. The sensitivity of $M_v$(TO) and $M_v$(BTO) as age indicators may be evaluated analytically by taking the deriva-

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Table 1. Monte Carlo Input Parameters

| Parameter                          | Distribution            | Comment                               |
|------------------------------------|-------------------------|---------------------------------------|
| mixing length                      | $1.85 \pm 0.25$ (stat.) | fits GC observations                   |
| helium diffusion coefficients      | $0.3 - 1.2$ (syst.)     | possible systematic error dominate     |
| nuclear reaction rates             | see CDKK                |                                       |
| OPAL high temperature opacities    | $1 \pm 0.01$ (stat.)    | comparison of OPAL & LAOL opacities   |
| surface boundary condition         | grey or Krishna-Swamy (1966) |                                       |
| colour table                       | Green et al. (1987) or Kurucz (1992) |                                       |
| primordial $^4$He abundance        | $0.22 - 0.25$ (syst.)   | possible systematic error dominate     |
| oxygen abundance, [O/Fe]           | $+0.55 \pm 0.05$ (stat.)|                                       |
|                                   | $\pm 0.20$ (syst.)      |                                       |

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Figure 1. The turn-off region in the Monte Carlo set of 14 Gyr, [Fe/H] $= -2.0$ isochrones. The turn-off ($M_V \sim 4.2$) and $M_v$(BTO) ($M_V \sim 3.6$) points have been highlighted.

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$^5$ High temperature opacities from Iglesias & Rogers (1991); low temperature opacities from Kurucz (1991); nuclear reaction rates from Bahcall & Pinsonneault (1992) and Bahcall (1988); an equation of state which includes the effects of Coulomb interactions (Guenther et al. 1992); Chaboyer & Kim (1995); helium diffusion coefficients from Michaud & Proffitt (1993) multiplied by 0.75; a grey model atmosphere was used for the surface boundary conditions, a near solar mixing length of $\alpha = 1.85$ and the colour transformation of Green, Demarque & King (1987) was used to transform the isochrones to the observational plane.
A sample of the [Fe/H] = −2.0 standard set of isochrones, with ages of 10, 14 and 18 Gyr. The turn-off and main sequence fitting to local sub-dwarfs with well measured parallaxes, and (2) using the observed magnitude of the horizontal branch (HB) combined with a relationship for the absolute magnitude of the HB (derived using RR Lyrae stars). Unfortunately, there is only one sub-dwarf with a well measured parallax (error in the absolute magnitude less than 0.05 mag), Groenewegen et al. 1990, so the application of main sequence fitting to GCs is still rather uncertain. Until improved sub-dwarf parallaxes become available, the use of the HB to set the distance scale to globular clusters will remain popular. The HB has the advantage that the difference in magnitude between the main-sequence turn-off and the horizontal branch (∆V(TO – HB)) is independent of reddening. Thus, ∆V(TO – HB) is a widely used age determination technique, which uses Mv(TO) as its age diagnostic (e.g. CDS). Although there are significant uncertainties in the absolute magnitude of the RR Lyrae stars (used to determine ∆V(TO – HB) in the theoretical calibration), CDS have shown that many statements can be made regarding the relative GC ages which are independent of the RR Lyrae calibration. This is very important, as the error in the absolute ages is dominated by the error in the distance modulus (CDKK). To exploit the merits of Mv(BTO) as an age indicator on observations, we suggest the use of a modified ∆V approach, using the difference in magnitude between Mv(BTO) and the HB, ∆V(BTO – HB) in order to study relative GC ages, and the formation of the Galactic halo.

Table 2. Mv(TO) and Mv(BTO) Fit Coefficients (see eq. 1)

| [Fe/H] | Mv(TO)   | Mv(BTO)   |
|-------|----------|----------|
|       | a₀      | a₁      | a₂      | a₀      | a₁      | a₂      |
| −2.5  | 2.261   | 0.1641   | −0.003048| 1.886   | 0.1452   | −0.002653|
| −2.0  | 2.513   | 0.1506   | −0.002696| 1.996   | 0.1521   | −0.002866|
| −1.5  | 2.596   | 0.1626   | −0.003157| 2.190   | 0.1546   | −0.002913|
| −1.0  | 2.981   | 0.1343   | −0.002353| 2.449   | 0.1563   | −0.002654|
| −0.5  | 3.963   | 0.1288   | −0.002046| 2.563   | 0.1529   | −0.002645|

Figure 2. A sample of the [Fe/H] = −2.0 standard set of isochrones, with ages of 10, 14 and 18 Gyr. The turn-off and main sequence fitting to local sub-dwarfs with well measured parallaxes, and (2) using the observed magnitude of the horizontal branch (HB) combined with a relationship for the absolute magnitude of the HB (derived using RR Lyrae stars). Unfortunately, there is only one sub-dwarf with a well measured parallax (error in the absolute magnitude less than 0.05 mag), Groenewegen et al. 1990, so the application of main sequence fitting to GCs is still rather uncertain. Until improved sub-dwarf parallaxes become available, the use of the HB to set the distance scale to globular clusters will remain popular. The HB has the advantage that the difference in magnitude between the main-sequence turn-off and the horizontal branch (∆V(TO – HB)) is independent of reddening. Thus, ∆V(TO – HB) is a widely used age determination technique, which uses Mv(TO) as its age diagnostic (e.g. CDS). Although there are significant uncertainties in the absolute magnitude of the RR Lyrae stars (used to determine ∆V(TO – HB) in the theoretical calibration), CDS have shown that many statements can be made regarding the relative GC ages which are independent of the RR Lyrae calibration. This is very important, as the error in the absolute ages is dominated by the error in the distance modulus (CDKK). To exploit the merits of Mv(BTO) as an age indicator on observations, we suggest the use of a modified ∆V approach, using the difference in magnitude between Mv(BTO) and the HB, ∆V(BTO – HB) in order to study relative GC ages, and the formation of the Galactic halo.

The well studied GC M68 (Walker 1994) provides an ideal database to test this new age determination technique. This is a relatively metal-poor cluster with [Fe/H] = −2.0 ± 0.11 Zinn & West 1984. Its HB morphology is predominantly blue but significantly redder than other clusters with comparable metallicity. This led Zinn (1993) to classify M68 as a relatively young halo cluster (see also Da Costa & Armandroff 1995). From Table 5 of Walker (1994), the mean V magnitude of the RR Lyraes is < V(RR) > = 15.635 ± 0.006, where the uncertainty is the standard error of the mean. There is also an error in the photometric zero-point, but that is not included here because < V(RR) > will be combined with V(TO) making the error in the absolute photometric scale irrelevant. For the theoretical HB magnitude, the preferred relationship of CDS and CDKK, Mv(RR) = 0.20[Fe/H] + 0.98 was arrived at after a review of the current literature. Recent HST data (Ajhar et al. 1996).
indicates that the slope is shallow, suggesting that $M_v(RR) = 0.15 [\text{Fe/H}] + 0.885$ may be a somewhat better choice. However, as emphasized by CDS, most statements regarding relative GC ages are independent of the particular choice of $M_v(RR)$. Walker (1994) measured a turn-off magnitude of $V(TO) = 19.05 \pm 0.05$, and hence $\Delta V(TO - HB) = 3.415 \pm 0.05$. Due to the difficulty in determining the turn-off point, Walker (1994) elected to increase his error in $\Delta V(TO - HB)$ to $\pm 0.10$ mag. Using our standard set of isochrones and $M_v(RR) = 0.20 [\text{Fe/H}] + 0.98$ the $\Delta V(TO - HB)$ age of M68 is $12.7 \pm 1.3 \text{Gyr}$ using the larger error, and $12.7 \pm 0.7 \text{Gyr}$ with the smaller error bar in $\Delta V(TO - HB)$. If the shallower slope for $M_v(RR)$ is chosen ($M_v(RR) = 0.15 [\text{Fe/H}] + 0.885$), an age of $12.8 \pm 1.3$ is derived. This age is 18% younger than that derived by CDS, using the same $M_v(RR)$ relation. The difference in age is due to the different input physics used to construct the isochrones. CDS ignored the effects of diffusion (7% increase in age), the Coulomb correction to the equation of state (7%) and assumed a somewhat smaller α-element enhancement ($[\alpha/\text{Fe}] = -0.40$).%

In order to derive the age using $\Delta V(\text{BTO} - \text{HB})$, an objective technique was used to measure $V(\text{BTO})$. A fifth order polynomial ($V = f(B-V)$) was fitted to the stars in the region of the turnoff using a 2-$\sigma$ rejection scheme. This polynomial yields ($B-V)_{TO}$ and $V(\text{BTO})$. The error in $V(\text{BTO})$ is determined by constructing a histogram of the $V$ deviations from the fit using stars within $\pm 0.03$ mag in ($B-V$) of the $V(\text{BTO})$ point. A gaussian is then fitted to the histogram. The standard deviation of this gaussian divided by the square root of the number of data points used is then the 1-$\sigma$ error in $V(\text{BTO})$. Application of this technique to the M68 data (Walker 1994) results in $V(\text{BTO}) = 18.519 \pm 0.006$. Note that the error in $V(\text{BTO})$ is an order of magnitude smaller than the error in $V(TO)$.

Combining the above value of $V(\text{BTO})$ with the mean RR Lyrae magnitude results in $\Delta V(\text{BTO} - \text{HB}) = 2.884 \pm 0.008$ which implies an age of $12.8 \pm 0.3 \text{Gyr}$. Due to the extremely high precision in the determination of $\Delta V(\text{BTO} - \text{HB})$ the error in the age is dominated by the error in metallicity (±0.11 dex), and not by the error in measuring $\Delta V(\text{BTO} - \text{HB})$. If the error in $\Delta V(\text{BTO} - \text{HB})$ is increased to $\pm 0.025 \text{mag}$, then the error in the age increases slightly to $\pm 0.4 \text{Gyr}$. An error of order $\pm 0.025$ in $\Delta V(\text{BTO} - \text{HB})$ allows for a greater error in the determination of the HB level, and is perhaps more typical of most data in the literature. This example clearly demonstrates that ages derived using $\Delta V(\text{BTO} - \text{HB})$ are at least a factor of two more precise than those derived using $\Delta V(TO - HB)$.

CDKK constructed a sample of 17 metal-poor GCs (mean [Fe/H] = −1.9), which were believed to be old based on $(B-V)$ or the horizontal branch morphology. The mean age of these 17 GC using our standard set of isochrones is $15.2 \pm 0.4 \text{Gyr}$. Note that the error in the $\Delta V(\text{BTO} - \text{HB})$ age of M68 is smaller than the error in the mean age of 17 GCs determined using $\Delta V(TO - HB)$. The difference in age between M68 and these old clusters is $2.4 \pm 0.5 \text{Gyr}$, showing that M68 is indeed a young GC for its metallicity. This statement could not be made using the $\Delta V(TO - HB)$ data and demonstrates the usefulness of using $\Delta V(\text{BTO} - \text{HB})$ to probe relative GC ages. As high precision $\Delta V(\text{BTO} - \text{HB})$ age estimates become available for other metal-poor GCs, it will become possible to test the assumption of CDKK that the 17 GCs in their sample had the same age.

In their $\Delta(B-V)$ study, VandenBerg et al. (1990) determined that the difference in colour between the turn-off and the base of the red giant branch was the same for M68 and M92. This implied that M68 had the same age as M92 and other metal-poor GCs. This is somewhat surprising, in light of the young age for M68 determined via $\Delta V(\text{BTO} - HB)$. To explore this question further, we have performed a detailed comparison of the CMDs for M92 and M68 using the deep M92 photometry of Stetson & Harris (1988), the M92 RR Lyrae photometry of Carney et al. (1991), and the M68 photometry from Walker (1994). This comparison reveals that, while VandenBerg et al. (1990) showed that M68 and M92 have nearly identical $\Delta(B-V)$ values, the $\Delta V(\text{BTO} - HB)$ and $\Delta V(TO - HB)$ values of M68 differ by approximately 0.1 magnitude as compared with those of M92. Thus, whereas $\Delta(B-V)$ appears to indicate that these two clusters have identical ages (to within $\sim 0.3 \text{Gyr}$), $\Delta V$ shows that M68 is $\sim 2 \text{Gyr}$ younger than M92. Salaris et al. (1993) and Zinn (1993) have shown that, in general, ages derived via $\Delta(B-V)$ and $\Delta V(\text{TO} - HB)$ are in good agreement. However, Salaris et al. (1993) also point out that M68 is one of a small number of clusters for which this is not the case, in accordance with our result. This implies that age is not the only variable which differs between these two clusters. It is not clear what this other variable could be. It could simply be different compositions (such as $[\alpha/\text{Fe}]$), or it could be something more exotic (such as rotation, or convection) which differs between stars in these two clusters. This is an interesting issue which we are currently investigating. Our ability to distinguish such small differences between the $\Delta V$ values of M68 and M92 is a testament to the improvements made in the reduction and calibration of globular cluster photometry and the use of the $\Delta V(\text{BTO} - HB)$ age diagnostic.

### 4 SUMMARY

An extensive Monte Carlo analysis indicates that the theoretical uncertainty in $M_v(\text{BTO})$ is similar to $M_v(\text{TO})$. The sensitivity of $M_v(\text{TO})$ to age changes is similar to $M_v(\text{TO})$. The objective fitting technique described in indicates that the error in measuring $V(\text{BTO})$ in observational data is $\sim 0.006 \text{mag}$, at least an order of magnitude smaller than the error typically quoted in $V(\text{TO})$. Hence, $M_v(\text{TO})$ is a superior age indicator to $M_v(\text{TO})$. We suggest that observers should measure $V(\text{BTO})$ as outlined in , and provide this value as a routine part of the analysis of GC CMDs. A calibration of age as a function of $M_v(\text{TO})$ (for $B-V$ data) is presented in eq. 1 and Table A. A similar calibration for $V-I$ data is presented in appendix D.

The use of $M_v(\text{BTO})$ as an age indicator requires a
knowledge of the distance modulus. We suggest the use of the absolute magnitude of the horizontal branch so that ages can be derived using the difference in magnitude between the horizontal branch, and $V(BTO)$, $\Delta V(BTO - HB)$. This leads to an age for M68 ([Fe/H] = $-$2.1) of 12.8 ± 0.3 Gyr, assuming $M_V(RR) = 0.20[Fe/H] + 0.98$. The error in the derived age is dominated by the error in metallicity. This is an internal error, useful in comparing relative ages, and does not include the error due to the uncertainty in the $M_V(RR)$ calibration. However, as shown by CDS, many statements concerning relative ages are true independently of the choice of $M_V(RR)$. For example, M68 is significantly younger ($\sim$ 2.5 Gyr) than the mean age of 17 other low metallicity GC. This demonstrates the unique advantage of using $M_V(BTO)$ to probe relative GC ages, which should lead to new insights into the formation of the Milky Way.

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APPENDIX A: APPLICATION TO $(V - I)$ DATA

The discussion of this paper (and calibration presented in Table 3) has centered on the use of $(B - V)$ data. However the use of $M_V(BTO)$ to measure ages can be easily extended to other colours. Increasingly, observers have obtained GC CMDs using $V$, $(V - I)$. The point $M_V(BTO)$ may be defined as the magnitude of the point which is brighter than the turn-off, and 0.05 mag redder in $(V - I)$. This will not correspond exactly to the same point defined in $(B - V)$ data, but will be in a similar region of the HR diagram. In order to facilitate the use of $M_V(BTO)$ with $(V - I)$ data, Table 3 presents the fitting coefficients which can be combined with eq. 1 to obtain the age of a GC, given $M_V(BTO)$ in $V$, $(V - I)$ data.

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