Elastic Inflation

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Abstract

Inflation of a universe filled by an elastic continuous medium is considered. Elastic inflation, while capable of describing current observations, is qualitatively different from standard inflationary models. The scalar and tensor modes keep evolving after crossing the horizon. Due to this superhorizon evolution, the amplitude of the tensor mode (inflationary gravitational wave) is not simply proportional to the energy density of the universe at the time of horizon crossing. The spectral index of tensor modes can be positive.
I. INTRODUCTION

The cosmological theory, and cosmological perturbation theory in particular [1, 2], have been confirmed observationally only when applied to an ideal fluid. Yet inflation and inflationary perturbations are calculated assuming that the universe was dominated by a scalar field [2, 3, 4], or a higher-derivative gravity [5], a scalar field with non-minimal kinetic term [6], a higher-derivative scalar field [7], a self-coupled vector field [8], or by a self-coupled three form [9].

Since physics at inflationary energies is unknown, it makes sense to consider a fluid-like description of inflation, which may ultimately turn out to be closer to reality than any of the above-listed field models. An ideal fluid cannot drive inflation, but an ideal elastic medium can. Elastic medium was already considered as a model of dark energy [10].

We describe the relativistic elastic medium in §2, and the elastic inflation in §3, where we also introduce the speeds of longitudinal and transverse sound waves. We calculate the inflationary tensor spectrum in §4, scalar spectrum in §5, and discuss the results in §6.

II. ELASTIC MEDIUM

Continuous elastic medium is a collection of time-like world lines filling the space-time ("congruence", see [10] and references therein). The action of elastic medium is given by a time integral of potential energy along each of the world lines of the congruence. The potential is an arbitrary function of spatial distances between neighboring world lines.

It is convenient, at least for our purposes, to choose spatial coordinates coincident with the markers of the world lines, so that the world lines of the congruence become the lines of constant $x^i$, $i = 1, 2, 3$. Then the action of the elastic medium can be written in the following form

$$S = - \int d^4 x \sqrt{-g} V.$$  \hspace{1cm} (1)

Here $V$ is an arbitrary function of the invariants of the spatial metric

$$\gamma_{ij} = - g_{ij} + \frac{g_{0i}g_{0j}}{g_{00}}.$$  \hspace{1cm} (2)
As the three invariants, one may take
\[ t_1 = \gamma_{ii}, \quad t_2 = \gamma_{ij}\gamma_{ji}, \quad \gamma = \det(\gamma_{ij}). \] (3)

We will not go beyond the first-order perturbation theory (will not calculate the non-Gaussianities for elastic inflation). Then, given that deviations from isotropy are small and \( \gamma_{ij} \) is approximately diagonal, to sufficient accuracy, we may express the third invariant as a function of the other two. We choose the traces \( t_1 \) and \( t_2 \) as the variables of elastic energy, \( V = V(t_1, t_2) \). All the formulas written below are valid only up to first order in metric perturbation.

The energy-momentum tensor of elastic medium is obtained by varying the action with respect to \( g_{\mu\nu} \). (The non-covariant form of the following expressions is due to the special choice of coordinates. All our final results will be expressed in a coordinate-free language.)

\[ T^{00} = V g^{00}, \] (4)
\[ T^{0i} = V g^{0i} + 2V_1 \frac{g^{0i}g_{0k}g_{ki}}{g_{00}} - 4V_2 \frac{g^{0k}g_{ki}}{g_{00}}, \] (5)
\[ T^{ik} = V g^{ik} - 2V_1 \delta_{ik} + 4V_2 g_{ik}. \] (6)

Here and in the following, \( V_1 \equiv \partial t_1 V, \ V_2 \equiv \partial t_2 V \). With \( 8\pi G = 1 \), the Einstein equations are
\[ G^\mu_\nu = T^\mu_\nu. \] (7)

When the energy-momentum tensor is known, one calculates the unperturbed inflation and inflationary perturbations along standard lines [2].

### III. ELASTIC INFLATION

The background metric is flat FRW
\[ ds^2 = a^2(\eta) \left(d\eta^2 - \delta_{ij}dx^idx^j\right), \] (8)
and the corresponding Friedmann equations are
\[ 3a^{-2}\mathcal{H}^2 = V \equiv \epsilon, \] (9)
\[ a^{-2}(2\dot{\mathcal{H}} + \mathcal{H}^2) = V + 2a^2V_1 + 4a^4V_2 \quad \equiv -p, \]  

(10)

here and in what follows \( \mathcal{H} \equiv a'/a \) and the prime denotes \( \partial_y \). We have also defined the unperturbed energy density and pressure.

Inflationary stage requires \( p \approx -\epsilon \). On the other hand, we will show that

\[ \frac{dp}{d\epsilon} = c_s^2 - \frac{4}{3}c_v^2, \]

(11)

where \( c_s \) is the speed of scalar (longitudinal) sound and \( c_v \) is the speed of vector (transverse) sound. Thus, inflation requires

\[ c_s^2 - \frac{4}{3}c_v^2 \approx -1, \quad 0 < c_s^2 < 1, \quad 0 < c_v^2 < 1. \]

(12)

These requirements constrain inflationary potentials \( V(t_1, t_2) \).

In the above equations, \( c_s \) and \( c_v \) are the short-wavelength limits of the propagation speeds of scalar and vector perturbations. These can be calculated neglecting gravity (assuming a pure gauge metric and neglecting the expansion of the universe). As shown in Appendix A,

\[ c_s^2 = -\frac{3V_1 + 10a^2V_2 + 2a^2V_{11} + 8a^4V_{12} + 8a^6V_{22}}{V_1 + 2a^2V_2}, \]

(13)

\[ c_v^2 = -\frac{V_1 + 4a^2V_2}{V_1 + 2a^2V_2}, \]

(14)

where \( V_{11}, \ldots \) are second derivatives. Equation (11) follows.

For \( c_v = 0 \), one can show that \( V = V(\gamma) \). Then the energy depends only on the change of volume, and the elastic model reduces to an ideal fluid. We will assume that elastic inflation ends when \( c_v \) vanishes. If at this moment also \( c_s = 0 \), then one needs to add reheating. But reheating is not necessary for elastic inflation, one may assume that at the end of inflation \( c_s^2 = 1/3 \) – inflation ends when an ultrarelativistic fluid loses elasticity.

A simple elastic inflation model, which we use for numerical examples, assumes constant sound speeds \( c_{si}, c_{vi} \) during the inflationary epoch, satisfying \( c_{si}^2 - (4/3)c_{vi}^2 = -1 \). Inflation ends at the energy density \( \epsilon_r \). After inflation, for \( \epsilon < \epsilon_r \), we also assume constant sound speeds, \( c_{sr} \), and \( c_{vr} = 0 \). Equation (11) gives the pressure

\[ p = c_{sr}^2\epsilon, \quad \epsilon < \epsilon_r, \quad p = (1 + c_{sr}^2)\epsilon_r - \epsilon, \quad \epsilon > \epsilon_r. \]

(15)
Friedmann equations give energy

\[ \epsilon = \epsilon_r + 3(1 + c_{sr}^2)\epsilon_r N, \]  

(16)
as a function of the number of e-foldings until the end of inflation, \( N = -\ln a + \text{const.} \). We will see that this artificial “constant speeds” model satisfies all current observational constraints on inflation.

IV. TENSOR MODES

We start with tensor modes, because the calculation of the spectrum is much simpler than for scalars. The perturbed metric is

\[ ds^2 = a^2 \left( d\eta^2 - (\delta_{ij} + h_{ij})dx^i dx^j \right), \]  

(17)

where \( h_{ij} \) is a symmetric pure tensor, with \( h_{ii} = 0 \) and \( \partial_j h_{ij} = 0 \). Equations (4-6) give the energy-momentum tensor perturbation

\[ \delta T^i_j = 2a^2(V_1 + 4a^2V_2)h_{ij}. \]  

(18)
The perturbation of the Einstein tensor is

\[ \delta G^i_j = -\frac{1}{2}a^{-2} \left( h''_{ij} + 2H h'_i j + k^2 h_{ij} \right), \]  

(19)

where \( k \) is the comoving wavenumber. Dropping the indices, we get the following tensor mode equation

\[ h'' + 2H h' + k^2 h + 2c_v^2 a^2(\epsilon + p) h = 0. \]  

(20)

It is convenient to write the mode equation using cosmic time, \( dt = ad\eta \). Denoting the time derivative by the dot and introducing the Hubble constant \( H \equiv \dot{a}/a \), we have

\[ \ddot{h} + 3H \dot{h} + \frac{k^2}{a^2} h + 2c_v^2(\epsilon + p) h = 0. \]  

(21)

For zero elasticity, \( c_v = 0 \), the above equation reduces to the standard one [2].

Denote by \( F_T(k) \) the elastic transfer function for tensor modes defined as the large time limit of the ratio of the amplitudes of the mode described by equation (21) and the mode
described by (21) with the same $a(t)$ but with $c_v = 0$. The late-time ratio of amplitudes is calculated for the modes with equal early-time $(H a \ll k)$ amplitudes.

The transfer function can be calculated analytically assuming slow-roll inflation. The effect of elasticity remains small before horizon crossing and leads to a slow roll of $h$ after horizon crossing, according to

$$3H \dot{h} + 2c_v^2(\epsilon + p)h = 0. \quad (22)$$

The last equation gives $h \propto F_T(k)$ with

$$F_T(k) \approx \exp \left( -\frac{2}{3} \int_{\epsilon_r}^{\epsilon_k} \frac{c_v^2 d\epsilon}{\epsilon} \right) \quad (23)$$

Here $\epsilon_r$ is the energy density at the end of inflation, $\epsilon_k$ is the energy density at the time when the $k$ mode crosses horizon, that is when $H a = k$.

The slow-roll condition $c_v^2(\epsilon + p) \ll \epsilon$ will be violated near the end of inflation, leading to a multiplicative $k$-independent error of order few in expression (23). The correct expression for the transfer function is

$$F_T(k) = F_T \exp \left( -\frac{2}{3} \int_{\epsilon_r}^{\epsilon_k} \frac{c_v^2 d\epsilon}{\epsilon} \right) \quad (24)$$

where $F_T$ is a constant factor of order few.

The tensor perturbation spectrum is (here and for the scalar spectrum we use the definitions used in [11])

$$P^h(k) = \frac{128}{3} F_T^2 \frac{\epsilon_k}{M_p} \exp \left( -\frac{4}{3} \int_{\epsilon_r}^{\epsilon_k} \frac{c_v^2 d\epsilon}{\epsilon} \right) \quad (25)$$

The tensor spectral index (the tilt) is

$$n_T = \frac{d \ln P^h(k)}{d \ln k} = \left(4c_v^2 - 3\right) \left(1 + \frac{p}{\epsilon}\right) \quad (26)$$

For zero elasticity, $c_v = 0$, this reduces to the standard expression [11]. However, elastic inflation requires $c_s^2 - \frac{4}{3} c_v^2 \approx -1$, giving

$$n_T = 3c_s^2 \left(1 + \frac{p}{\epsilon}\right). \quad (27)$$

For elastic inflation the tensor index can be positive.
For the constant speeds model of elastic inflation described at the end of §3, we get the tensor power

\[ P^h(k) = \frac{128}{3} F_T^2 \frac{\epsilon_k}{M_P} \left( \frac{\epsilon_k}{\epsilon_f} \right)^{\frac{4}{3} \frac{v}{vi}} \]  

(28)

and the tilt

\[ n_T = \frac{c_s^2}{N_k} \]  

(29)

V. SCALAR MODES

Generic scalar perturbations of the metric are, [2],

\[ ds^2 = a^2 \left( (1 + 2\phi)d\eta^2 - 2\partial_i\partial^i \eta dx^j - \left( \delta_{ij} - 2\psi\delta_{ij} + 2\partial_i\partial_j E \right) dx^i dx^j \right). \]  

(30)

For this metric one finds the perturbed energy-momentum tensor from (4-6) and the perturbed Einstein tensor from [2]. We do this in Appendix B, here we just give a summary of results. Define a new variable \( \zeta \) by

\[- \left( 1 + 4 \frac{c_v^2 H^2}{k^2} \right) \zeta = \Psi + \frac{H}{H^2 - H'} \left( \Psi' + \left( 1 - \frac{4}{3} c_s^2 \right) H \Psi \right), \]  

(31)

where \( \Psi \equiv \psi - H(B - E') \) is the gauge-invariant potential. After inflation, \( c_v = 0 \), \( \zeta \) becomes the standard curvature perturbation (on superhorizon scales, for \( c_v = 0 \), \( \zeta \) is the perturbation of the scale factor on uniform energy hypersurfaces). Our \( \zeta \) also coincides with the usual \( \zeta \) at very early time when \( k \gg H \), allowing us to use standard results when calculating quantum fluctuations.

The \( \zeta \)-perturbation satisfies

\[ \ddot{\zeta} + \frac{z}{z} \dot{\zeta} + \frac{c_s^2 k^2}{a^2} \zeta + w\zeta = 0. \]  

(32)

Here

\[ z \equiv \frac{a^3(\epsilon + p)}{c_s^2 H^2} \]  

(33)

coincides with the corresponding function for an ideal fluid case [11]. The other function

\[ w \equiv 4c_v^2 H \left( \frac{\dot{\epsilon}_v}{c_v} - 2\frac{\dot{\epsilon}_a}{c_a} - \frac{1}{2} \frac{\dot{\epsilon}}{\epsilon} \right) \]  

(34)
turns to zero in the ideal fluid limit.

The scalar perturbation spectrum is calculated from (32) similarly to the tensor mode case. Using (11), we find

$$P_\zeta(k) = \frac{16}{9} F_S^2 \frac{\epsilon_k}{M_P} \frac{1}{c_s(1 + p_k/\epsilon_k)} \exp\left\{ -\frac{4}{3} \int_{\epsilon_r}^{\epsilon_k} \frac{c_s^2 d\epsilon}{\epsilon} \left( 1 + \frac{4}{3} \frac{d\ln c_s}{d\ln \epsilon} - \frac{4}{3} \frac{d\ln c_v}{d\ln \epsilon} \right) \right\}. \quad (35)$$

The scalar tilt is

$$n_S = -3 \left( 1 + \frac{p}{\epsilon} \right) \left( 2 - \frac{d\ln c_s}{d\ln \epsilon} - \frac{d\ln (p + \epsilon)}{d\ln \epsilon} - \frac{4}{3} c_v^2 \left( 1 + \frac{4}{3} \frac{d\ln c_s}{d\ln \epsilon} - \frac{4}{3} \frac{d\ln c_v}{d\ln \epsilon} \right) \right). \quad (36)$$

For the constant speeds inflation described at the end of §3, the scalar power is

$$P_\zeta(k) = \frac{16}{9} F_S^2 \frac{\epsilon_k}{M_P} \frac{3N_k}{c_{si}} \left( \frac{\epsilon_k}{\epsilon_r} \right)^{-\frac{4}{3} c_{si}} \quad (37)$$

and the tilt is

$$n_S = -\frac{1 - c_{si}^2}{N_k}. \quad (38)$$

VI. DISCUSSION

Elastic Inflation is capable of describing available observations. Currently, there exist three inflationary observables [12, 13]: the tensor to scalar ratio, the scalar tilt, and non-Gaussianity (all upper bounds). The scalar power, which has been measured to a great accuracy, simply gives the energy scale of inflation, and does not constrain phenomenological models like ours.

We have not calculated the non-Gaussianity, but one expects it to be either $\sim 1/N$ ([14]) or $\sim 1$ ([15]), easily passing the observational constraints. The other two constraints are satisfied even by the artificial constant speeds model of elastic inflation described at the end of §3. This model gives a negative scalar tilt $- (1 - c_{si}^2)/N$, which for $N \sim 60$ is consistent with observations for any $0 < c_{si} < 1$. The tensor to scalar ratio $\sim 8c_{si}/N$ is also consistent with current observations for arbitrary $0 < c_{si} < 1$.

Summarizing:

(i) Elastic Inflation is described by equations (9, 10, 11, 12).

(ii) The tensor and scalar spectra generated by elastic inflation are given by equations (25, 26, 35, 36), where the constant factors of order unity $F_{T,S}$ should be calculated by numerical integration of the mode equations (21, 32).
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APPENDIX A: SOUND SPEEDS

We consider perturbations on Minkowski background $ds^2 = dt^2 - \delta_{ij}dx^idx^j$. For short-wavelength perturbations we neglect gravity, meaning that the metric perturbation is a pure gauge, $x^i \rightarrow x^i - \xi^i$, giving the perturbed metric

$$g_{00} = 1, \quad g_{0i} = -\dot{\xi}_i, \quad g_{ij} = -\delta_{ij} - \partial_i \xi_j - \partial_j \xi_i. \quad (A1)$$

The perturbed traces and 4-Jacobian are

$$t_1 = 3 + 2\partial_k \xi_k, \quad t_2 = 3 + 4\partial_k \xi_k, \quad \sqrt{-g} = 1 + \partial_k \xi_k. \quad (A2)$$

The perturbed energy-momentum tensor is calculated from (4-6):

$$T_{0}^{0} = V + \delta V, \quad T_{i}^{0} = (2V_{1} + 4V_{2})\dot{\xi}_i, \quad T_{0}^{0} = -2(V + 2V_{1} + 4V_{2})\dot{\xi}_i, \quad (A3)$$

$$T_{k}^{i} = (\delta V + 2\delta V_{1} + 4\delta V_{2})\delta_{ik} + (2V_{1} + 8V_{2})(\partial_i \xi_k - \partial_k \xi_i), \quad (A4)$$

where $V_1 \equiv \partial_{i} V_{...}$ and the perturbations of potential energy and its derivatives are given by

$$\delta V = (2V_{1} + 4V_{2})\partial_k \xi_k, \quad \delta V_{1} = (2V_{11} + 4V_{12})\partial_k \xi_k, \quad \delta V_{2} = (2V_{21} + 4V_{22})\partial_k \xi_k. \quad (A5)$$

The equation of motion

$$0 = \nabla_{\nu}T_{\nu}^{\mu} = \frac{1}{\sqrt{-g}}\partial_{\mu}(\sqrt{-g}T_{\nu}^{\mu}) - \frac{1}{2}(\partial_{\nu}g_{\alpha\beta})T^{\alpha\beta} \quad (A6)$$

gives

$$(2V_{1} + 4V_{2})\ddot{\xi}_k + \partial_k(\delta V + 2\delta V_{1} + 4\delta V_{2}) + (2V_{1} + 8V_{2})(\partial^2_k \xi_k + \partial_k \partial_n \xi_n). \quad (A7)$$

For vector modes ($\partial_n \xi_n = 0$) we get the propagation speed (14), with $a=1$. For the scalar mode ($\xi_n \equiv \partial_n \chi$), we get the speed (13).
APPENDIX B: SCALAR PERTURBATIONS

The perturbed metric is

\[ ds^2 = a^2 \left( (1 + 2\phi) \text{d}t^2 - 2\partial_i B \text{d}t \text{d}x^i - (\delta_{ij} - 2\psi \delta_{ij} + 2\partial_i \partial_j E) \text{d}x^i \text{d}x^j \right). \]  \tag{B1} 

Perturbation of the energy-momentum tensor is found from (4-6)

\[ \delta T^0_0 = \delta V, \quad \delta T^i_0 = (2a^2 V_1 + 4a^4 V_2) \partial_i B, \]  \tag{B2} 

\[ \delta T^i_k = (\delta V + 2a^2 \delta V_1 + 4a^4 \delta V_2) \delta_{ik} + (4a^2 V_1 + 16a^4 V_2)(\partial_i \partial_k E - \psi \delta_{ik}). \]  \tag{B3} 

Here the perturbations of potential energy and its derivatives are

\[ \delta V = V_1 \delta t_1 + V_2 \delta t_2, \quad \delta V_1 = V_{11} \delta t_1 + V_{12} \delta t_2, \quad \delta V_2 = V_{21} \delta t_1 + V_{22} \delta t_2, \]  \tag{B4} 

and the perturbations of the traces are

\[ \delta t_1 = -2a^2(3\psi + k^2 E), \quad \delta t_2 = -4a^4(3\psi + k^2 E), \]  \tag{B5} 

where \( k \) is the comoving wavenumber.

With Einstein tensor written in [2], the Einstein equations are

\[ -3\mathcal{H}^2 \phi - 3\mathcal{H} \psi' - k^2 \psi + k^2 \mathcal{H}(B - E') = \frac{1}{2}a^2 \delta V, \]  \tag{B6} 

\[ \mathcal{H} \phi + \psi' = a^4(V_1 + 2a^2 V_2) B, \]  \tag{B7} 

\[ (2\mathcal{H}' + \mathcal{H}^2) \phi + \mathcal{H} \phi' + \psi'' + 2\mathcal{H} \psi' - \frac{k^2}{2} D = \]  \[ -\frac{1}{2}a^2(\delta V + 2a^2 \delta V_1 + 4a^4 \delta V_2) + 2a^4(V_1 + 4a^2 V_2) \psi, \]  \tag{B8} 

\[ D = 4a^4(V_1 + 4a^2 V_2) E, \]  \tag{B9} 

\[ D \equiv \phi - \psi + (B - E')' + 2\mathcal{H}(B - E'). \]  \tag{B10} 

This system is redundant. Due to time reparametrization invariance one equation can be dropped. Equivalently, one gauge condition may be imposed. We set

\[ B = 0. \]  \tag{B11}
Then (B7) gives

\[ \mathcal{H} \phi + \psi' = 0. \]  

(B12)

Introducing a gauge invariant potential (see [2] for discussion)

\[ \Psi = \psi + \mathcal{H} E', \]  

(B13)

we get a system of two first-order ODEs

\[ \Psi' + \mathcal{H} \Psi = a^4(V_1 + 2a^2V_2)E' - 4a^4(V_1 + 4a^2V_2)\mathcal{H}E \]  

(B14)

\[ k^2 \Psi = a^4(V_1 + 2a^2V_2)(3\Psi - 3\mathcal{H}E' + k^2 E). \]  

(B15)

Now we want to get an equation for the curvature perturbation \( \zeta \). More precisely, we would like to define a variable which becomes curvature perturbation in the ideal fluid limit \( c_v = 0 \) (because this is what we want to calculate), and also coincides with the standard \( \zeta \) variable at early times (this will allow to use standard results to normalize the mode). This can be done in the following (rather tedious) way.

One notes that the system (B14, B15) can be written as

\[ \left(2a\Psi + a^3(\epsilon + p)E\right)' = -3c_s^2 a^3(\epsilon + p)E, \]  

(B16)

\[ k^2 \left(2a\Psi + a^3(\epsilon + p)E\right) = -3a^3(\epsilon + p)(\Psi - \mathcal{H}E'). \]  

(B17)

With the help of [11], one defines a possible variable \( \zeta \):

\[ \zeta = \frac{k^2}{3} \left( E + \frac{2}{a^2(\epsilon + p)} \Psi \right). \]  

(B18)

The definitions and equations given in the main text follow.

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