Quantum Chaos, Superconductivity, and Information Scrambling in Disordered Magic-Angle Twisted Bilayer Graphene

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We use stochastic expansion and exact diagonalization to study the magic-angle twisted bilayer graphene (TBG) on a disordered substrate. We show that the substrate-induced strong Coulomb disorder in TBG with the chemical potential in the flat band drives the system to a network of weakly coupled Sachdev-Ye-Kitaev (SYK) bundles, stabilizing an emergent quantum chaotic strange metal phase of TBG that exhibits the absence of quasiparticles. The Gaussian orthogonal ensemble dominates TBG’s long-time chaotic dynamics at strong disorder, whereas fast quantum scrambling appears in the short-time dynamics. In weak disorder, TBG exhibits exponentially decaying specific heat capacity and exponential decay in out-of-time-ordered correlators. The latter follows the Larkin-Ovchinnikov behavior of the correlator suggesting the superconducting transition upon weakening the disorder strength. We propose a finite-temperature phase diagram for Coulomb disordered TBG and discuss the experimental consequences of the emergent strange metal phase.

Introduction. The experimental discovery of superconductivity and correlated insulating states in the twisted bilayer graphene (TBG) samples[1, 2] boosted the interest of the community in studying the effect of the flatbands in the TBG originating from the "magic" twist angles [3, 4]. The presence of non-dispersive bands enhances the effect of electron correlations and makes TBG an ideal venue to study the emergent phenomena stabilizing unconventional phases of strongly correlated electrons [5–7]. The recent extensive study of the correlated many-body properties of the TBG uncovered a variety of interesting phenomena, including flatband ferromagnetism [8], SU(2) collective excitations [9], Kohn-Luttinger instability [10], near Planckian dissipation [11]. The effect of the angular disorder is especially interesting, which is shown to smear the flatband and enhance the transmission [12, 13].

At the same time, the effects of residual impurities and smooth Coulomb disorder in such correlated flatband systems caused by the substrate and the interplay with strong electron-electron interactions are less well-understood. Emergence of quantum chaotic Sachdev-Ye-Kitaev (SYK)-like phase, i.e., a maximum chaotic non-Fermi liquid with its holographic dual [14–17], is one of the outcomes if point-like impurities are present in the flatband systems such as in the quantum matter with kagome lattice structure [18]. The geometry of such line-graph lattice protects the flatbands [19]. However, the origin of the numerous flatbands in magic-angle TBG is different: the chiral symmetry of the Moiré superlattice enables the emergence of the non-localized flatbands that do not disperse [4]. Another intuitive understanding of the flatband is based on the collapse of the Van Hove singularity [20–22], which applies even if the chiral symmetry is not exact. The Van Hove singularities of two overlapping Dirac cones appear at energies $\sim \pm \hbar v_0 k_y$, where $v_0$ is the Fermi velocity of the single-layer graphene, $k_y = 2k_D \sin(\theta/2)$ and $k_D$ is the magnitude of the wave vector at the corner of the Brillouin-zone for a single layer. The Van Hove singularities will be perturbed and extended by the inter-layer hopping $t'$. When $\hbar v_0 k_y \sim 2t'$, namely $\theta \sim \frac{2\pi}{k_D v_F}$, the nearly flatbands will appear. The flatband states are localized within the Moiré cell. The direct result of this localization is that the overlaps between the flatband states become sparse. Interestingly, compared to the disordered kagome lattice, this effect leads to different thermodynamics of the disordered TBG.

The dynamics, unlike thermodynamics, is dominated by local structure overlaps of states and is described by the out-of-time-order correlators (OTOCs). For quantum scrambling dynamics, $OTOC \equiv \langle X(x, \tau) Y(0, 0) \rangle_{\beta}$ is the exponential growing part of the commutator $-\langle [X(x, \tau), Y(0, 0)] \rangle_{\beta}$, where $X(x, \tau)$ and $Y(0, 0)$ are operators in Heisenberg picture and $\langle \ldots \rangle_{\beta}$ is the regularized thermal average at inverse temperature $\beta$, which describes the rate of local information spreading over the entire system in the semi-classical limit. Namely, such commutator of a pair of canonical variables is proportional to the $\langle \partial X(x, \tau) \partial Y(0, 0) \rangle$. Inspired by the gravity duality of black holes, it is proposed that the Lyapunov exponent, $\lambda$, of the correlator, i.e., $OTOC \sim e^{\lambda (\tau/\beta)}$ (where $v_B$ is the butterfly velocity [23]), has a maximal bound $\lambda \leq \lambda_{\text{max}} \equiv 2\pi T[24, 25]$. Even though the coefficient, $2\pi$, in the upper bound, can be reduced in the absence of dilatons and when the dual AdS spacetime is close to the flat spacetime, the linear dependence on $T$ is still strong evidence for quantum scrambling [26]. Importantly, for Fermi liquid, the Lyapunov exponent $\lambda$ scales as $T^2$ at low temperatures. In general, $\lambda$ can scale as $T^\gamma$, where $1 \leq \gamma \leq 2$, as a result of the quantum critically and the many-body effects. The lower bound $\gamma = 1$ is an evidence for lacking the long-lived quasiparticles [27–29]. On the other hand, in the superconducting phase, A. Larkin and Yu. Ovchinnikov showed that the semiclassical limit of $OTOC$ decays exponentially in $T[30].$

In the present letter, we study the behavior of $OTOC$s in Coulomb disordered magic-angle twisted TBG to pin-
point the region where superconductivity is stabilized[31–35] and understand the nature of the phase outside the superconducting region. Our analysis suggests the emergence of SYK physics in TBG and establishes a connection with superconductivity. Our central finding is that in a broad region of temperatures and disorder strengths, the TBG system exhibits the emergent weakly and randomly coupled SYK granules, stabilizing a kind of the strange metal phase. Our approach proposes a novel scenario of superconductivity in TBG, based on the flatband and disorder-induced SYK granules. Moreover, our theory proposes a finite-temperature phase diagram depicted in Fig. 1a. This is especially important because superconductivity is experimentally observed not in most available magic-angle TBG samples but only in some of them. To this end, the suggested phase diagram can be a systematic tool for characterizing the behavior of the TBG samples based on the disorder strength and corresponding observable characteristics.

Our considered disorder type originates from the disordered Dirac substrate. The latter induces hills and valleys of random potential with electrons forming puddles of random shapes[36]. This is the origin of Coulomb disorder in the magic-angle TBG samples. Thus, we first discuss the model describing the disordered TBG system and briefly review the stochastic expansion to calculate the physical observables numerically. The thermodynamics of the disordered TBG is then compared to the SYK model. We explore the quantum chaotic dynamics at short and long times, and demonstrate quantum scrambling dynamics of the strongly disordered TBG with typical disorder strength, w (see the model description below for the exact definition). The Gaussian orthogonal ensemble dominates the late-time statistics of the disordered TBG. We computed the OTOC at short time scales and the extracted low-temperature Lyapunov exponent, λ. We show that it scales linearly with temperature, T, as \( \lambda = 2\pi\alpha T \) where \( \alpha \approx 0.56 \), indicating that the disordered TBG is a non-Fermi liquid exhibiting the absence of quasiparticles. Finally, the magic-angle TBG supports superconductivity at weak disorder strength, manifested by the exponentially decreasing OTOC in time, \( \tau \).

Experimentally observable characteristics of the strange metal phase are (i) absence of the logarithmic zero-bias anomaly in the tunneling density of states; (ii) linear in a temperature specific heat capacity whose slope is essentially different from that of the FL; (iii) thermoelectric transport typical for disordered metals without quasiparticles[16]. We remind the reader that the logarithmic zero bias tunneling anomaly [37, 38] in the quasiballistic regime can be observed in disordered Fermi liquid due to interference and cancellation of the quasiparticles’ Friedel oscillation at low energy. Such anomaly also exists in systems with smooth density variations[39].

A defining feature of the strange metal phase without quasiparticles in Coulomb disordered TBG is the absence of the zero-bias anomaly due to its non-Fermi liquid nature. Finally, we argue that the linear in temperature Lyapunov exponent and emergent quantum chaotic behavior defined by the weakly coupled SYK bundles explains the experimentally observed linear-in-temperature resistivity in TBG [11, 40]. A schematic phase diagram with the emergent phases is depicted in fig. 1a.

**Model.** As demonstrated in Ref. 41, the proximity effect of the substrate can be used to modify the chemical potential. Thus, a TBG with coulomb disordered Dirac substrate[36] can be described by the Hamiltonian with three terms

\[
H = H_{tt'} + H_\mu + H_I. 
\]  

(1)

Here \( H_{tt'} \) is the sum of the nearest-neighbor hopping Hamiltonians between layers and within each of the lay-
The latter is fitted by a cubic polynomial function at low temperatures. The screened interaction potential is taken to be

$$V_{\sigma,\chi}(r-r') = \begin{cases} U, & \sigma = \bar{\sigma}', \chi = \bar{\chi}', r = r' \\ V, & \chi = \bar{\chi}', 0 < |r - r'| \leq a \\ V, & \chi \neq \bar{\chi}', 0 < |r - r' + d| \leq a \\ 0, & \text{otherwise} \end{cases}$$

where $a$ is the distance of the nearest neighbors within a single layer, $d$ is the perpendicular vector between two layers, $U$ is the on-site interaction for electrons with the same spin, and $V$ is the “nearest-neighbor” interaction potential. The nearest neighbor interactions are restricted to the screening radius $a$ for interlayer and intralayer interactions.

Finally, $H_{\mu}$ represents the sum of local random on-site potentials

$$H_{\mu} = - \sum_{\mathbf{r},\sigma,\chi} \mu(\mathbf{r}) a_{\sigma,\chi}^\dagger(\mathbf{r}) a_{\sigma,\chi}(\mathbf{r}).$$

Here $\mu(\mathbf{r})$ is the mean-field random chemical potential which can be approximated by the Gaussian function

$$\mu(\mathbf{r}) = \sum_i w_i e^{-(\mathbf{r}-\mathbf{r}_i)^2/2\sigma^2},$$

where $w_i$ is uniformly distributed random variable on $[-w, w]$ interval with $w$ representing the energy scale responsible for the strength of the disorder potential. Here, without loss of generality, we assume that centers $\mathbf{r}_i$ are positioned at the cites of a square lattice. The parameter $\sigma$ controls the correlation length of the chemical potential, i.e., $\langle \mu(\mathbf{r})\mu(\mathbf{r}') \rangle \sim e^{-(\mathbf{r}-\mathbf{r}')^2/4\sigma^2}$. The correlation originated from the Coulomb disordered materials can be estimated $\sigma \approx (\frac{1}{2})^{1/6} \frac{1}{\epsilon \nu_F} \frac{e^{n_{\text{eff}}^2/2}}{r}$ [36, 43], where $\alpha = \frac{e^2}{4\pi\epsilon\epsilon_0 \nu_F}$ is the effective fine-structure constant, $\epsilon$ is the dielectric constant, $n$ is the density of the carriers and $\nu_F$ is the Fermi velocity of the Dirac system. Throughout this paper, we consider the regime when the correlation length of the disordered potential, $\sigma$, is much smaller than the Moiré unit cell of the magic-angle twisted TBG. For example, a particular realization of the random chemical potential is shown in fig. 1b. The radius of the TBG $R = \sqrt{3a}/\theta$, which corresponds to about 19000 unit cell for each single layer.

**Projection onto flatbands.** The optimized parameters that ensure the flatness of the bands of $H_I$ with minimal variance are: $\theta = 1.1^{\circ}$ which is the magic twist angle,
FIG. 4. Panel (a) shows the histogram of the level statistics parameter $r$. The solid lines with narrower widths correspond to the Gaussian ensemble with orthogonal, unitary, and symplectic matrices. Panel (b) shows the Lyapunov exponent extracted from the $OTOC$ and its low-temperature trend. The left inset shows the $OTOC$ with the intermediate disorder or the strange metal (SM) phase. The right inset shows the $OTOC$ with the weak disorder supporting the superconducting (SC) phase. Panel (c) shows the algebraic increase of $v_B$ at low temperatures. Panel (d) depicts the averaged local density of states at low energy, exhibiting the absence of a zero-bias anomaly.

$t' = 0.57t$, and $r_c = 0.6\sqrt{3}a$. For these parameters, the spectrum of TBG is shown in fig. 1c. However, in the presence of the disorder potential with the medium strength chemical potential $H_p$, that is the typical energy corresponding to $H_p$, is of the same order as that of the hopping $H_{tt'}$, the flatbands are disturbed but generally remain localized, as shown in fig. 2a. In the numerical simulation, the disorder is chosen to be $w = t$, and the number of the nearly flatbands is 58 for the intermediate disorder phase which is identified as the chaotic strange metal phase in the following, and $w = 0.001t$ for the weak disordered phase which is identified as superconducting phase. Here, we will adopt the approach of flatband projection developed and applied in Refs. 18 and 44, and project the Hamiltonian $H$ onto the nearly flatband subspace to obtain the low energy effective Hamiltonian. The result is a sum of the interacting Hamiltonian, $H_{\text{flatband}}$, of flatband states and a dispersive Hamiltonian, $H_{\text{dispersive}}$. The latter is the bilinear part of the effective Hamiltonian $H_{\text{eff}}$ due to the non-flatness of the nearly flatbands:

$$H_{\text{eff}} = H_{\text{flatband}} + H_{\text{dispersive}},$$  \hfill (7)

$$H_{\text{flatband}} = \sum_{i<j,k<l} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l,$$  \hfill (8)

where $c_i^\dagger$ and $c_i$ are creation and annihilation of the nearly flatband states. The coupling constants are given by

$$J_{ijkl} = \sum_{r,r',\sigma,\sigma'} V_{\sigma,\sigma'}(r-r') \times \left[ \phi_{i,\sigma,\chi}^*(r) \phi_{j,\sigma',\chi'}^*(r') - \phi_{j,\sigma,\chi}(r) \phi_{i,\sigma',\chi'}(r') \right] \times \left[ \phi_{k,\sigma',\chi'}(r') \phi_{l,\sigma,\chi}(r) - \phi_{l,\sigma',\chi}(r') \phi_{k,\sigma,\chi}(r) \right],$$  \hfill (9)

where $\phi_{i,\sigma,\chi}(r)$ is the wavefunction of the $i$-th nearly flatband states.

Because most of the flatband states are localized and non-extended, the majority of the couplings is $J_{ijkl}$ is very small. The numerical result shows that, after filtering out these near-zero couplings $J_{ijkl}$, the remaining couplings are nearly Gaussian with variance $J'^2/6N^3$ ($N$ is the number of flatband states). Figure 2b depicts how randomly interacting bundles with couplings, $J_{ijkl}$, that are above a certain threshold emerge. Here each dot represents a pair of flatband states. Two dots are connected if the corresponding coupling, $|J_{ijkl}|$, is larger than $0.1\sqrt{J'^2/6N^3}$. The flatband states are bundled in the dual graph, indicating the sparsely connected feature of the couplings $J_{ijkl}$. We notice that such a bundled structure resembles the spinful version of the SYK model of Ref. 31 with superconducting instabilities and generalizes it to the multi-dot situation. Although we do not explicitly show in fig. 2b how the SYK bundles are connected through weak couplings $|J_{ijkl}| < 0.1\sqrt{J'^2/6N^3}$, these connections do exist. The distribution of the weak couplings is shown in the inset to fig. 2b. The standard deviation of the distribution is $\sigma_{\text{weak}} = 0.02\sigma_{\text{strong}}$, which confirms that the chosen splitting scale well separates the two types of couplings. Generally, weak connections play an important role for such bundled systems with zero-energy modes to be conductors[45].

**Thermodynamics.** We use the stochastic expansion[46] to numerically tackle the disordered TBG whose system size is large enough to have several Moiré cells. In the stochastic expansion, the trace is replaced by the average of random vectors. Ref. 46 has shown that the average converges to the expectation value sandwiched in the true state, and the fluctuation is proportional to $1/\sqrt{NR}$, where $N$ is the dimension of flatband Hamiltonian eq. (7), and $R$ is half of the degrees of freedom of the random vectors, allowing access to physical quantities for disordered TBG of intermediate size.
We start with the low temperature entropy,
\[
S(\beta) = -\text{tr} \rho \ln \rho, \quad \text{where} \quad \rho = \frac{e^{-\beta H_{\text{eff}}}}{Z},
\]
and \(Z\) is the partition function. The result of the simulation is shown in fig. 3a.

Choosing the 10\(J\) as the energy scale, we observe that the entropy of the effective Hamiltonian eq. (7) of disordered TBG decreases to a small value at temperatures much higher than for the SYK model[47] with the same value of the parameter \(J\). Despite the energy scale, the entropies show similarity in the shape[18], which leads to a conjecture \(S(T) = S_{\text{SYK}}(\zeta T)\), where \(\zeta \approx 0.1\). This is shown in fig. 3a. This is an indication that, unlike the SYK model, the ground state of the disordered TBG is not as largely degenerate, which is a result of the bundling structure of the \(J_{ijkl}\) couplings. We also checked that the dispersive part in the Hamiltonian eq. (7) does not affect the described picture, and that part can be safely neglected.

At the superconducting phase, the specific heat decays exponentially as \(\sim e^{-T/\Delta}\) where \(\Delta\) is the superconducting gap. At the same time, the specific heat of the chaotic phase shows more complicated behavior where the low-temperature part can be fitted by the cubic function \(c_1 T + c_2 T^2 + c_3 T^3\). The linearity coefficient \(c_1 = 0.04\) is close to one-tenth of the analytical and numerical results for the SYK model[48, 49]. This also agrees with the scale conjecture about the entropies, \(C(T) = \frac{T}{N} \frac{\partial S(T)}{\partial T} = \frac{T}{N} \frac{\partial S_{\text{SYK}}(\zeta T)}{\partial (\zeta T)} = C_{\text{SYK}}(\zeta T)\).

Quantum chaos. As we observed, thermodynamically, the disordered TBG states close to the ground state are not largely degenerate at energy scale \(J\), which is a result of the sparsely connected topology of the couplings \(J_{ijkl}\). However, the dynamics is determined by the local structure of the bundles in the coupling connection. To understand the long time dynamical behavior of the disordered TBG, we compute the level statistics, namely the distribution of the level-spacing parameter
\[
r = \frac{E_{n+1} - E_n}{E_n - E_{n-1}},
\]
corresponding to energy levels \(E_n, n = 1, 2, \ldots\). Here the energy of the \(n\)‘th level can be determined by the min-max theorem[50]
\[
E_n = \min_{U} \max_{x} \langle x | H_{\text{eff}} | x \rangle: x \in U, |x| = 1, \dim(U) = n,
\]
which is a variational method minimizing the \(n\)-th eigenenergy with in subspace \(U\) whose dimension is \(n\).

As can be seen from the fig. 4a, the distribution of level spacings follows the level statistics of the Gaussian orthogonal ensemble. This is natural because the time-reversal symmetry is respected in our model. To further understand the short-time behavior of the model, we compute the OTOC of the flatband states,
\[
\text{OTOC}(\tau, r, \beta) = \text{tr} e^{-\beta H_{\text{flatband}}} c^\dagger_i(\tau) c_i(0) \times e^{-\beta H_{\text{flatband}}} c^\dagger_i(\tau) c_i(0),
\]
where
\[
c^\dagger_i(\tau) = e^{iH_{\text{flatband}} \tau} c^\dagger_i e^{-iH_{\text{flatband}} \tau}
\]
is the Heisenberg picture operator. It should be noted that the Boltzmann factor \(e^{-\beta H_{\text{eff}}}\) is distributed in the definition of the OTOC to regularize it[51]. The Lyapunov exponent is extracted as a function of temperature from the plot shown in fig. 4b from the exponential growing portion of the OTOC around the scrambling time \(\tau_s[24]\). Scaling of \(\lambda\) versus \(T\) is linear, and the slope approaches 3.52 as the temperature goes to zero, which is the \(\alpha \approx 0.56\) portion of the conjectured maximal bound. A similar effect was reported in a non-Fermi liquid spin-fermion model[28]. This indicates substantial quantum scrambling in the disordered TBG.

To study the butterfly velocity, one must turn to the local electronic operators. The OTOC is now not only time and temperature dependent, but it also depends on real-space coordinates of electrons:
\[
\text{OTOC}(\tau, r, \beta) = \text{tr} e^{-\beta H_{\text{flatband}}} a_{\chi}^\dagger(\tau, r) a_{\sigma \chi}(0) \times e^{-\beta H_{\text{flatband}}} a_{\chi}^\dagger(\tau, r) a_{\sigma \chi}(0).
\]
For a given \(\tau\), the butterfly velocity \(v_B\), can be extracted from the exponential behavior of OTOC. fig. 4c depicts the actual data and fitted \(v_B\) versus temperature showing \(v_B \sim T^\delta\), with \(\delta \approx 1.22\).

For Fermi liquid with dispersion relation \(\omega \sim k^z\) at finite temperature \(T\), the butterfly velocity defines the velocity of the operator/information spreading, so \(v_B = \frac{d\omega}{dk} \sim w^{1-1/2} \sim T^{1-1/2}\). However, this argument does not apply to flatband non-Fermi liquid, even perturbatively. As a result, \(\delta > 1\) becomes a signature for breaking Fermi liquid theory and the absence of quasiparticles. As a result of the absence of quasiparticles, the zero-bias anomaly is missing. The averaged local density of states, \(\rho\), is exponentially decaying at small energies as shown in fig. 4d.

At weak disorder, \(w = 0.001t\), we observed a negative Lyapunov exponent of the disordered TBG. This behavior signals a transition to superconducting state, as it is consistent with the superconducting Larkin-Ovchinnikov behavior of OTOC at \(w \to 0\).

Conclusion. We investigated the characteristics of the TBG on a disordered substrate or sandwiched in the Coulomb disordered materials. We used stochastic expansion combined with numerical diagonalization. The thermodynamic entropy of disordered TBG and the SYK model are compared to determine the energy scale and the bundling structure of the coupling \(J_{ijkl}\) in the disordered TBG. The quantum chaos of the disordered TBG
is demonstrated to originate from the emergence of the weakly coupled network of the SYK bundles and follows the Gaussian orthogonal ensemble statistics. The exponential growth of the OTOC probes quantum scrambling in the disordered TBG. The temperature scaling of the butterfly velocity, \( v_B \), of the system implies anomalous information spreading with \( v_B \sim T^{1.22} \). When \( T \to 0 \), the corresponding Lyapunov exponent has a linear dependence on the temperature, and the slope is \( \sim 0.56 \)th of the conjectured upper bound. Upon decreasing the strength of the disorder, the OTOC undergoes a transition to the exponentially decaying Larkin-Ovchinnikov behavior inherent to the superconducting phase. Simultaneously, the specific heat shows exponential decay, a typical characteristic of superconductors. These observations suggest that the superconductivity in the magic angle TBG stems from the increase of the couplings between spatially extended SYK bundles of the strange metal phase. The superconductivity in such a non-Fermi-liquid scenario can be stabilized upon the emergence of the attractive Hubbard interaction between flatband states. It would be interesting to see if the interplay between weak Coulomb disorder and strong random interactions may lead to the effective attraction that leads to pairing. Although we focus on the disordered TBG here, we believe such chaotic non-Fermi liquid behavior exists in other disordered Morié flatband systems, which may also stabilize superconducting phases.

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black hole, arXiv preprint arXiv:2203.07298 (2022).

[27] S. A. Hartnoll and A. P. Mackenzie, Planckian dissipation in metals, arXiv preprint arXiv:2107.07802 (2021).

[28] M. Tikhanovskaya, S. Sachdev, and A. A. Patel, Maximal quantum chaos of the critical Fermi surface, arXiv preprint arXiv:2202.01845 (2022).

[29] T. A. Sedrakyan and A. V. Chubukov, Fermionic propagators for two-dimensional systems with singular interactions, Phys. Rev. B 79, 115129 (2009).

[30] A. Larkin and Y. N. Ovchinnikov, Quasiclassical method in the theory of superconductivity, Sov Phys JETP 28, 1200 (1969).

[31] E. Lantagne-Hurtubise, V. Pathak, S. Sahoo, and M. Franz, Superconducting instabilities in a spinful Sachdev-Ye-Kitaev model, Phys. Rev. B 104, L020509 (2021).

[32] L. Classen and A. Chubukov, Superconductivity of incoherent electrons in the Yukawa Sachdev-Ye-Kitaev model, Phys. Rev. B 94, 035135 (2016).

[33] G. A. Inkof, K. Schalm, and J. Schmalian, Quantum critical eliashberg theory, the SYK superconductor and their holographic duals, arXiv preprint arXiv:2108.11392 (2021).

[34] B. Skinner, Coulomb disorder in three-dimensional dirac systems, Phys. Rev. B 90, 060202 (2014).

[35] A. M. Rudin, I. L. Aleiner, and L. I. Glazman, Tunneling zero-bias anomaly in the quasiballistic regime, Phys. Rev. B 55, 9322 (1997).

[36] E. Mariani, L. I. Glazman, A. Kamenev, and F. von Oppen, Zero-bias anomaly in the tunneling density of states of graphene, Phys. Rev. B 76, 165402 (2007).

[37] T. A. Sedrakyan, E. G. Mishchenko, and M. E. Raikh, Zero-bias tunneling anomaly in a clean 2d electron gas caused by smooth density variations, Phys. Rev. Lett. 99, 206405 (2007).

[38] Y. Cao, D. Chowdhury, D. Rodan-Legrain, O. Rubies-Bigorda, K. Watanabe, T. Taniguchi, T. Senthil, and P. Jarillo-Herrero, Strange metal in magic-angle graphene with near Planckian dissipation, Phys. Rev. Lett. 124, 076801 (2020).

[39] J. M. Pizarro, M. Rössner, R. Thomale, R. Valentí, and T. O. Wehling, Internal screening and dielectric engineering in magic-angle twisted bilayer graphene, Phys. Rev. B 100, 161102 (2019).

[40] R. Conti, H. Topchyran, R. Tateo, and A. Sedrakyan, Geometry of random potentials: Induction of two-dimensional gravity in quantum hall plateau transitions, Phys. Rev. B 103, L041302 (2021).

[41] B. I. Shklovskii, Simple model of Coulomb disorder and screening in graphene, Phys. Rev. B 76, 233411 (2007).

[42] A. Chen, R. Ilan, F. de Juan, D. I. Pikulin, and M. Franz, Quantum holography in a graphene flake with an irregular boundary, Phys. Rev. Lett. 121, 036403 (2018).

[43] E. Day-Roberts, R. M. Fernandez, and A. Kamenev, Nature of protected zero-energy states in penrose quasicrystals, Phys. Rev. B 102, 064210 (2020).

[44] A. Weiße, G. Wellein, A. Alvermann, and H. Fehske, The kernel polynomial method, Rev. Mod. Phys. 78, 275 (2006).

[45] W. Fu and S. Sachdev, Numerical study of fermion and boson models with infinite-range random interactions, Phys. Rev. B 94, 035135 (2016).

[46] J. Maldacena and D. Stanford, Remarks on the sachdev-ye-kitaev model, Phys. Rev. D 94, 106002 (2016).

[47] A. M. García-García and J. J. M. Verbaarschot, Spectral and thermodynamic properties of the Sachdev-Ye-Kitaev model, Phys. Rev. D 94, 126010 (2016).

[48] G. Teschl, Mathematical methods in quantum mechanics, Vol. 157 (American Mathematical Society Providence, RI, USA, 2014).

[49] E. Lantagne-Hurtubise, S. Plugge, O. Can, and M. Franz, Diagnosing quantum chaos in many-body systems using entanglement as a resource, Phys. Rev. Research 2, 013254 (2020).