Computer simulation of the process of magnetic reconnection in the vector configuration

Eu.Echkina, I.Inovenkov
CMC, Moscow State University, Russia, 119991
E-mail: ejane@cs.msu.su, inov@cs.msu.su

Abstract. Computational simulation of processes or magnetic reconnection an play an important role of studying of laboratory and space plasma. 3D nonlinear magnetohydrodynamic allow us to more accurately describe such phenomena as the current layer, solar flashes, solar wind, magnetic islands in the tokamak. Here we demonstrated the results of computer simulation using the parallel algorithm of process of the magnetic reconnection in 3D magnetic configuration.

1. Introduction

Process of magnetic reconnection of power lines is connected with disturbance of their freezing into plasma accompanied by release of energy accumulated in different plasma structures and its conversion into energy of fast particles. Magnetic islands, locks, current layers resulting from magnetic reconnection are ”thin, unstable” structures.

Therefore reconnection processes are important for numerous applications. A very dense space grid and a very little time step as a result of Courant condition are required for the accurate 3D simulation of characteristic features of magnetic reconnection.

This leads to two major issues that could arise when using a single computer configuration. On the one hand the volume of data which has to be kept on a single computer is very large. On the other hand the single processor loading is very high also. Therefore it is challenging to solve the MHD system numerically on a single processor computer.

Thus using of multiprocessor platform helps to solve two major issues. Firstly the data can be distributed among several computers instead of one. Secondly computations can be distributed among several processors and reduce the computation loading of one processor.

In the work we presented three models of splitting of cubic calculation region with calculation of parallelization productivity

2. System of MHD equations and mathematical model

The MHD equations for plasma density $\rho$, velocity $\mathbf{u}$, temperature $T$ and magnetic vector-potential $\mathbf{A}$ [1] can be written in next form:
\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0,
\rho(\partial_t \mathbf{u} + (\mathbf{u} \nabla) \mathbf{v}) = -\frac{\beta}{2} \nabla p + [\nabla \times \nabla \times \mathbf{A}] \times [\nabla \times \mathbf{A}],
\partial_t \mathbf{A} = [\mathbf{u} \times [\nabla \times \mathbf{A}]] + \nu_m \Delta \mathbf{A},
(1)
\frac{\rho}{\gamma - 1}(\partial_t T + (\mathbf{u} \nabla) \cdot T) + p \nabla \cdot \mathbf{u} = -\nabla \cdot (k \nabla T) + 2\nu_m \frac{(\Delta \mathbf{A})^2}{\beta},
p = \rho T.

In our case we consider for simplicity that plasma transport coefficients: magnetic viscosity \(\nu_m\), electric conductivity \(\sigma\), thermal conductivity \(k\), parameter \(\beta\), adiabatic index \(\gamma\) are constant.

The system is completed by boundary and initial conditions that correspond to the current sheet formation. Current layer is formed under action of magnetohydrodynamic wave.

Magnetosonic wave is created by excitation of vector-potential at the boundary of computational region \(x = \pm 1\) and \(y = \pm 1\) of the form

\[A_z(x, y, z, t) = A_z(x, y, z, t = 0) + \frac{r_1}{2} \mathcal{G}(t - 1/r + 1),\]

where \(A_z(x, y, z, t)\) is the \(z\)-component of the vector-potential and \(r^2 = x^2 + y^2\). The function \(\mathcal{G}(\zeta)\) it can be written in next form

\[\mathcal{G}(\zeta) = \begin{cases} -E_1(\zeta - 1)^2/\zeta & \text{for } \zeta > 1 \\ 0 & \text{for } \zeta < 1 \end{cases},\]

and is chosen so as to impose the electric field gradually.

Boundary conditions for other values were set according to the solution of MHD equations system. In the region where plasma flown into the computational domain the density and the pressure are equal to constants \(\rho = 1, p = 1\), while the conditions of free outflow are imposed on the part of the boundary where the plasma leaves the computational domain.

3. Computational algorithm

In numerical modeling of MHD processes, the greatest difficulty is that there are physical processes at different time intervals. Consequently, we must count with a small step in time to see fast processes, and count long to analyze a long process. At the same time forming thin current configurations require a small spatial grid.

In this paper we used the explicit finite-difference scheme which is more convenient for the numerical parallel implementation [2, 3].

System of equations(1) numerically solve in cubic area:

\[-1 \leq x \leq 1, -1 \leq y \leq 1, -1 \leq z \leq 1.\]

We considered the homogenous mesh in space and time \(w_{ht} = w_h \times w_t\) where

\[w_h = \{x_i = ih, i = 0, ..., M; y_j = jh, j = 0, ..., M; z_k = kh, k = 0, ..., M\},\]

\[w_t = \{t_n = n\tau, n = 0, \ldots\}.\]

\(M\) - dimension of a grid.

The difference scheme is

\[\bar{\mathbf{u}}^{n+1} = \bar{\mathbf{u}}^n + \frac{1}{\rho^n} \tau \cdot \bar{\mathbf{F}}(\rho, \bar{\mathbf{u}}, \bar{\mathbf{A}}, P)^n,\]
\[\bar{\mathbf{A}}^{n+1} = \bar{\mathbf{A}}^n + \tau \cdot \bar{\mathbf{F}}_1(\bar{\mathbf{u}}, \bar{\mathbf{A}})^n,\]
\[T^{n+1} = T^n + \tau \cdot \bar{\mathbf{F}}_2(\rho, \bar{\mathbf{A}}, T)^n,\]
\[\rho^{n+1} = \rho^n - \tau \cdot \nabla \cdot (\rho \bar{\mathbf{u}})^n,\]

(6)
denote through $F$ the expression on the right side of the Euler equation, $F_1$ is the right side term in the magnetic induction equation, $F_2$ is the right side term in the energy equation.

The operator $\nabla \cdot (\rho \vec{U})^n$ is approximated in the following form

$$
\rho = (\rho_{ijk}^n), U_x = (U_{x_{ijk}}^n), U_y = (U_{y_{ijk}}^n), U_z = (U_{z_{ijk}}^n).
$$

$$
\nabla \cdot (\rho \vec{U})^n = L_x(\rho_{ijk}^n) + L_y(\rho_{ijk}^n) + L_z(\rho_{ijk}^n)
+ \rho_{ijk}^n \left( \left( \frac{(U_x^n)_{i+1,j,k} - (U_x^n)_{i,j,k}}{2h} \right), \left( \frac{(U_y^n)_{i,j+1,k} - (U_y^n)_{i,j,1,k}}{2h} \right), \left( \frac{(U_z^n)_{i,j,k+1} - (U_z^n)_{i,j,k-1}}{2h} \right) \right).
$$

$$
L_x(f_{ijk}^n) = \frac{(U_x^n)_{ijk} + |(U_x^n)_{ijk}|}{2h} (f_{ijk}^n - f_{i-1,j,k}^n) + \frac{(U_x^n)_{ijk} - |(U_x^n)_{ijk}|}{2h} (f_{i+1,j,k}^n - f_{i,j,k}^n),
$$

$$
L_y(f_{ijk}^n) = \frac{(U_y^n)_{ijk} + |(U_y^n)_{ijk}|}{2h} (f_{ijk}^n - f_{i-1,j,k}^n) + \frac{(U_y^n)_{ijk} - |(U_y^n)_{ijk}|}{2h} (f_{i+1,j,k}^n - f_{i,j,k}^n),
$$

Boundary and initial conditions are approximated accordingly.

4. Multiprocessor implementation. Dependence of productivity at parallelization in the different directions

Let’s assume for calculations we have $n$ of identical processors. Calculations are carried out on a cubic grid with $M$ points in one direction. We distribute all calculations evenly among the processors.

When performing computational experiments, we must solve two algorithm problems: perform optimal parallelization and optimal use of computer memory and time.

The our equations system defines 8 independent scalar functions: 3 components of velocity, vector-potential, temperature and density. These will be defined as primary functions.

Let’s define the secondary functions of our system of equations pressure, 3 components of exchange $n$ - number of processes In this case at parallelization in one direction we have number of exchanges $E_1 = 2nM^2$ for two direction $E_2 = 4\sqrt{n}M^2$ for three directions $E_3 = 6\sqrt{n}M^2$

When using a uniform grid, we see the performance gains in the third case (as expected).

5. Results of the computer simulation

In this chapter we will present the results of a numerical experiment. Corresponding constants in MHD equations were taken equal to magnetic viscosity $\nu_m = 0.006$, pressure to $\beta = 0.012$, thermal conductivity $k = 0.01$, dimensionless electric field $E_1 = 0.06 [4],[5]$.

First we describe the results obtained for the magnetic field given by

$$
\vec{B} = x\vec{e}_x + y\vec{e}_y - 2z\vec{e}_z.
$$

The figure Fig 1(a) shows the force lines of the magnetic field at the beginning of time.

In this magnetic configuration the separatrix is located along axis $z$. As a result of excitation of electric current from side borders of estimated area, the topology of a field changes, we see
twisting of power lines, the current layer is not formed and plasma kind of is thrown out of area through the upper and lower bound. (Fig. 1(b)).

In the second case the magnetic configuration has a form

$$\vec{B} = x\vec{e}_x - 2y\vec{e}_y + z\vec{e}_z. \quad (9)$$

the separatrix surface of the magnetic field lies in the $y = 0$ plane. It is shown Fig 2(a).

After the MHD cylindrical wave induced from the boundaries has reached the center of the computational region the magnetic configuration changes Fig 2(b). We see formation of current structure directed along axis $z$. By its geometric characteristics, this structure likes to the current sheet.

This magnetic configuration is the equivalent of a two-dimensional one.

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**Figure 4.** The magnetic configuration at the initial time (a), the magnetic configuration at the finish time (b)

**Figure 5.** The magnetic configuration at the initial time (a), the magnetic configuration at the finish time (b)

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