Novel Modelling and Control Strategies for a Steam Boiler under Fast Load Dynamics*

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October 27, 2022

Abstract

This paper describes a new nonlinear dynamic model for a natural circulation boiler. The model is based on physical principles, i.e. mass, energy and momentum balances. A systematic approach is followed leading to new insights into the physics of drum water level and downcomer mass flow. The model captures fast dynamic responses that are necessary to describe the operation of a boiler under highly variable load conditions. New features of the model include (i) a multi-compartment model for the risers, (ii) a new model for drum water level, and (iii) a new dynamic model for the flow of water in the downcomers. Implications of the model for control system design are explored in detail. Finally, the suggested improvements are validated in a sugar mill boiler.

1 Introduction

The current paper describes the outcomes of a three year project carried out in collaboration with Wilmar Sugar at their Proserpine Mill in Queensland, Australia. Sugar mills burn sugar cane residue (bagasse) to produce steam in a boiler. The generated steam is used for many purposes, including to power the factory, to crystallise sugar, and to co-generate electricity.

Unlike boilers used in conventional gas or coal-fired power stations, boilers in sugar mills are subject to large and rapidly changing loads, e.g. when cane crushers are started or stopped. In addition, the fuel (bagasse) has a highly variable calorific value due to the different moisture content in the original cane. The result of these two factors is that (i) boilers in sugar mills must necessarily cope with large and rapid load changes, e.g. 50% load change over the span of 6 – 8 seconds, and (ii) the high moisture content in the bagasse can make it very difficult to burn, thus impacting furnace dynamics. As a consequence, boilers in sugar mills can experience severe operational difficulties, including frequent stoppages due to large drum water level excursions. To address the aforementioned problems, a novel boiler model was developed aimed specifically at capturing fast load dynamics. The model was then used to redesign the associated control system.

Dynamic models for Boilers have appeared in the literature for many years [2, 9, 10, 12, 13, 14]. Early work focused on obtaining empirical models capable of describing the internal dynamics with limited accuracy [5, 6]. Throughout the years, the focus has shifted to develop models based on physical principles, i.e. first-principle models [4, 8, 12]. Models for boilers have taken many different formats, e.g. linear/nonlinear [4, 17], high/low order [7], one/two fluid [1], lumped/distributed parameter [4, 16], and have also been tailored to different thermal power plant technologies [1].

*This research was supported partially by the Australian Government through the Australian Research Council’s Linkage Projects funding scheme (LP170100576)
In the seminal work of [4], a model using mass and energy balances is described. Several simplifications were used when developing this model, namely (i) a steady state equation for the downcomer mass flow, and (ii) an assumption that steam quality varies linearly as a function of height in the risers. The more recent work described in [15] derives a first principles model using mass, energy and momentum conservation equations. However, other simplifications are used when developing this model, including the fact that pressure and other internal variables are reconstructed by first order filters. It is shown in the current paper that the simplifications and assumptions used in [4,15] are not valid under rapidly changing load conditions.

Based on the above background, this paper presents a new nonlinear model for natural a circulation boiler based entirely on physical principles, including mass and energy balances and implications of constant volume of the different sections of the boiler. The goal is to develop a model that (i) captures the internal fast dynamics needed to account for large and rapid load variations and fuel variability, and (ii) is simple enough to support controller design. Another aspect of the model development is that all assumptions are clearly and explicitly stated. Thus they can be readily assessed for their validity in specific cases.

The key new features of the model described in this paper are:

• A multi-compartment model is developed for the risers. It will be shown that the spatially distributed nature of boiling water in the risers plays a central role in drum water level dynamics under fast and large load changes.

• A new model for drum water level is developed. The model gives rise to a key controller design insight, i.e. drum water level deviations are proportional to steam flow out of the boiler.

• A new model for downcomer water flow in a natural circulation boiler is described. It will be shown that a general momentum balance approach provides a link between water flow and pressure derivatives.

The work presented here embellishes and extends the work in [4]. The pressure model turns out to be equivalent to that presented in [4]. The novelty of the current paper lies in (i) the three key features described above, and (ii) the manner in which the equations are used to describe the model. It will be seen that, through simple algebraic manipulations, the final model format allows for easy understanding and facilitates the design and (re)-tuning of controllers.

The remainder of the paper is organised as follows: In Section 2 the equations for mass balance, energy balance and volume constraints on the drum are described. In Section 3 the equations for mass balance, energy balance and volume constraints on the risers are described. In Section 4 a model for boiler pressure is developed. In Section 5 a model for drum water level is developed. It is shown that further equations are necessary. In Section 6 the equations corresponding to spatial discretisation of the risers are developed. An additional assumption of homogeneous mixing in the risers is introduced. In Section 7 an equation for the mass flow of water in the downcomers is developed based on momentum balance. In Section 8 a model for a superheater based on constant volume, plus energy and mass balances is developed. In Section 9 key consequences of the new model are discussed and simulations are presented highlighting the key new features. In Section 10 the implications of the new model relative to the control of drum water level are explored. In Section 11 experimental results obtained from a boiler operating in the sugar industry are presented. This results confirm the benefits of the new model and the associated control strategies. In Section 12 conclusions are drawn. For ease of reference, a list of the variables used throughout the paper is presented in Table 1 and a schematic of the boiler is given in Fig. 1.
| Symbol | Description | See Eq. |
|--------|-------------|--------|
| $\alpha_i$ | Steam quality in riser section $i$ | (39) |
| $\delta$ | Drum water level deviations | (29) |
| $f_r$ | Mass flow of water converted into steam | (19) |
| $f_s$ | Steam mass flow from risers into drum | (41) and (34d) |
| $f_w$ | Water mass flow from risers into drum | (43) and (34c) |
| $f_w^*$ | Water mass flow from drum into risers | (52) |
| $h_s$ | Enthalpy of steam | Steam tables |
| $h_w$ | Enthalpy of water | Steam tables |
| $h_s^*$ | Enthalpy of feedwater | Assumed known |
| $M_{sD}$ | Mass of steam in the drum (above water line) | (1) |
| $M_{wD}$ | Mass of water in the drum | (2) |
| $M_{sR}$ | Mass of steam in the risers | (14) |
| $M_{wR}$ | Mass of water in the risers | (15) |
| $M_{sBW}$ | Mass of steam below the water line | (26) |
| $P$ | Drum Pressure | (25) |
| $Q^B$ | Heat flow used to turn water into steam | Control variable |
| $q_f$ | Mass flow of feedwater | Control variable |
| $q_s$ | Mass flow of steam exiting the drum | Assumed known |
| $\rho_s$ | Steam density | Steam tables |
| $\rho_w$ | Water density | Steam tables |
| $V^D$ | Total volume of the drum | Assumed known |
| $V_{sD}$ | Volume of steam in the drum | (9) |
| $V_{wD}$ | Volume of water in the drum | (9) |
| $V_{sBW}$ | Volume of steam below the water line | (28) |
| $V^R$ | Total volume of the risers | Assumed known |
| $V_{sR}$ | Volume of steam in the risers | (20) |
| $V_{wR}$ | Volume of water in the risers | (20) |

Table 1: List of Variables

## 2 Steam Drum

In this Section, mass balance, energy balance and volume constraint equations for the drum section of the boiler will be derived. Algebraic manipulations of the latter two will give rise to equations used in the final model. Two assumptions are needed for the subsequent derivations, namely:

**Assumption A.** Water in the boiler is at its saturation temperature $T_s$. □

**Assumption B.** The temperature of the metal, $T_{m}$, is the same as the saturation temperature, $T_s$. □

### 2.1 Mass Balance

The mass of steam and water in the drum satisfy conservation equations. In particular, the time rate of change of mass contained in an open system is equal to the difference between mass inflow and outflow
downcomers risers

\[ \dot{M}_D^s = f_s - q_s - f_{cd} \]  
\[ \dot{M}_D^w = q_f + f_w - f_{w}^* + f_{cd} \]

where \( M_D^s, M_D^w \) denote the mass of steam in the drum, and the mass of water in the drum, respectively, and where \( f_s, q_s, q_f, f_w, f_w^*, f_{cd} \) denote the mass flow of steam from the risers into the drum, the mass flow of steam out of the drum, the mass flow of feedwater into the drum, the mass flow of water from the top of the risers into the drum, the mass flow of water from the drum into the downcomers, and the mass flow due to steam condensation, respectively.

Remark 2.1. Note that \( M_D^s \) is defined as the total mass of steam in the drum, i.e., it includes both the mass of steam above and below water.

2.2 Energy Balance

The time rate of change of the energy contained in an open system is equal to the difference between energy inflows and outflows of the system. The energy contained in the drum is given by the energy contained in the masses of water and steam in the drum, plus the energy contained in the metal walls of the system. This leads to:

Figure 1: Cross section of boiler
the drum. The following energy balance equation results from these considerations:

\[
\frac{d}{dt}\left\{ M_s^D u_s + M_w^D u_w + M_m^D C_p T_m \right\} = (f_s - q_s) h_s + (f_w - f_w^*) h_w + q_f h_w^f
\]  \hspace{1cm} (3)

where \( u_s, u_w, M_m^D, C_p, T_m \) denote the internal energy of steam, internal energy of water, mass of metal in the drum, heat capacity of metal, and temperature of metal, respectively. Also \( h_s, h_w, h_w^f \) denote the enthalpy of steam, enthalpy of water, and enthalpy of feedwater, respectively.

By definition, internal energy is related to enthalpy and pressure by

\[
u \equiv h = h - \frac{P}{\rho}.
\]

Substituting (5) and (6) into (4), and using the mass balance equations (1) and (2), leads to:

\[
\frac{d}{dt}\left\{ M_s^D h_s + M_w^D h_w - \left( \frac{M_s^D}{\rho_s} + \frac{M_w^D}{\rho_w} \right) P + M_m^D C_p T_m \right\} = (f_s - q_s) h_s + (f_w - f_w^*) h_w + q_f h_w^f
\]

Noting that

\[
\frac{M_s^D}{\rho_s} + \frac{M_w^D}{\rho_w} = V_s^D + V_w^D = V^D
\]

then,

\[
\frac{d}{dt}\left\{ M_s^D h_s + M_w^D h_w - V^D P + M_m^D C_p T_m \right\} = (f_s - q_s) h_s + (f_w - f_w^*) h_w + q_f h_w^f
\]  \hspace{1cm} (4)

Considering Assumption [A] it follows that:

\[
h = \frac{dh}{dt} = \frac{\partial h}{\partial P} \cdot \frac{dP}{dt} = \frac{\partial h}{\partial P} \cdot \dot{P}
\]  \hspace{1cm} (5)

Considering Assumption [B] it follows that:

\[
\dot{T}_m \approx \frac{dT_s}{dt} = \frac{\partial T_s}{\partial P} \cdot \frac{dP}{dt} = \frac{\partial T_s}{\partial P} \cdot \dot{P}
\]  \hspace{1cm} (6)

Substituting (5) and (6) into (4), and using the mass balance equations (1) and (2), leads to:

\[
(f_s - q_s - f_cd) h_s + (q_f + f_w - f_w^* + f_cd) h_w + \left( \frac{M_s^D \partial h_s}{\partial P} + \frac{M_w^D \partial h_w}{\partial P} - V^D + M_m^D C_p \frac{\partial T_s}{\partial P} \right) \dot{P} = (f_s - q_s) h_s + (f_w - f_w^*) h_w + q_f h_w^f
\]

Cancelling the common terms on both sides, and introducing the following definition

\[
K_1 = \frac{M_s^D \partial h_s}{\partial P} + \frac{M_w^D \partial h_w}{\partial P} - V^D + M_m^D C_p \frac{\partial T_s}{\partial P}
\]  \hspace{1cm} (7)

leads to:

\[
(-f_cd) h_s + (q_f + f_cd) h_w + K_1 \dot{P} = q_f h_w^f
\]

Rearranging the above equation leads to:

\[
f_cd = (h_s - h_w)^{-1} \left( K_1 \dot{P} + q_f (h_w - h_w^f) \right)
\]  \hspace{1cm} (8)

This equation will form part of the final model described in Section 4.
2.3 Volume Constraint

The total volume of the steam drum is constant. This leads to the following equation:

\[ V^D = V_s^D + V_w^D \]  \hfill (9)

where \( V^D, V_s^D, V_w^D \) denote the volume of the drum, the volume of steam in the drum (both above and below the water line), and the volume of water in the drum, respectively.

Since the volume of the drum is constant, using (9) and taking its derivative leads to:

\[
0 = \frac{d}{dt} \left( V_s^D + V_w^D \right) = \frac{d}{dt} \left( \frac{M_s^D}{\rho_s} + \frac{M_w^D}{\rho_w} \right)
\]

where the fact that \( V = M/\rho \) has been used, where \( M, \rho \) denote mass and density, respectively. Expanding the derivative leads to:

\[
0 = \frac{\dot{M}_s^D}{\rho_s} - \frac{M_s^D}{\rho_s^2} \cdot \dot{\rho}_s + \frac{\dot{M}_w^D}{\rho_w} - \frac{M_w^D}{\rho_w^2} \cdot \dot{\rho}_w
\]  \hfill (10)

A consequence of Assumption [A] is that density is a function of pressure only. Thus,

\[
\dot{\rho} = \frac{d\rho}{dt} = \frac{\partial \rho}{\partial P} \cdot \frac{dP}{dt} = \frac{\partial \rho}{\partial P} \cdot \dot{P}
\]  \hfill (11)

where \( P \) denotes the pressure of the boiler. Using (11), and substituting the mass balance equations (2) and (1) into (10) leads to:

\[
0 = \frac{f_s - q_s - f_{cd}}{\rho_s} + \frac{q_f + f_w - f_{cd}^* + f_{cd}}{\rho_w} - \left( \frac{M_s^D}{\rho_s^2} \frac{\partial \rho_s}{\partial P} + \frac{M_w^D}{\rho_w^2} \frac{\partial \rho_w}{\partial P} \right) \cdot \dot{P}
\]

Introducing the following definition

\[
C_1 \equiv \frac{M_s^D \partial \rho_s}{\rho_s^2 \partial P} + \frac{M_w^D \partial \rho_w}{\rho_w^2 \partial P}
\]  \hfill (12)

and rearranging terms, leads to:

\[
\dot{P} = C_1^{-1} \left( \frac{-q_s - f_{cd}}{\rho_s} + \frac{q_f - f_{cd}^* + f_{cd}}{\rho_w} \right) \frac{f_s + f_w}{\rho_s}
\]  \hfill (13)

This equation will also form part of the final model described in Section 4.
3 Risers

In a similar fashion to Section 2, mass balance, energy balance and volume constraint equations for the risers are presented in this Section. Algebraic manipulations of the latter two will give rise to equations used in the final model.

3.1 Mass Balance

Mass balance in the risers leads to:

\[ \dot{M}^R_s = f_\ell - f_s \]  
\[ \dot{M}^R_w = f^*w - f_w - f_\ell \]

(14) (15)

where \( M^R_s, M^R_w \) denote the mass of steam in the risers and the mass of water in the risers, respectively, and where \( f_s, f_w, f^*_w, f_\ell \) denote the mass flow of steam from the risers into the drum, the mass flow of water from the top of the risers into the drum, the mass flow of water from the drum into the downcomers, and the mass flow of water converted into steam in the risers, respectively.

Remark 3.1. The above equations consider the risers as one section. That is sufficient for the moment. However, in Section 6, a multiple-compartment model for the risers will be introduced. This will be necessary to more accurately describe the drum water level dynamics.

3.2 Energy Balance

Energy balance in the risers leads to:

\[ \frac{d}{dt} \left\{ M^R_s u_s + M^R_w u_w + M^R_m C_p T_m \right\} = - f_s h_s + (f^*_w - f_w) h_w + Q^B \]

(16)

where \( u_s, u_w, M^R_m, C_p, T_m \) denote the internal energy of steam, internal energy of water, mass of metal in the risers, heat capacity of metal, and temperature of metal, respectively. Also \( h_s, h_w, Q^B \) denote the enthalpy of steam, enthalpy of water, and heat flow used for boiling water in the risers, respectively.

Substituting \( u = h - P/\rho \) into the left hand side of (16) leads to:

\[ \frac{d}{dt} \left[ M^R_s h_s + M^R_w h_w - \left( \frac{M^R_s}{\rho_s} + \frac{M^R_w}{\rho_w} \right) P + M^R_m C_p T_m \right] = - f_s h_s + (f^*_w - f_w) h_w + Q^B \]

Again, noting that

\[ \frac{M^R_s}{\rho_s} + \frac{M^R_w}{\rho_w} = V^R_s + V^R_w = V^R \]

Then,

\[ \frac{d}{dt} \left[ M^R_s h_s + M^R_w h_w - V^R P + M^R_m C_p T_m \right] = - f_s h_s + (f^*_w - f_w) h_w + Q^B \]

Expanding the LHS leads to

\[ M^R_s h_s + M^R_s h_s + M^R_w h_w + M^R_w h_w - V^R P + M^R_m C_p T_m = - f_s h_s + (f^*_w - f_w) h_w + Q^B \]

(17)
In view of Assumption A, equations (5) and (6) can be used here. In addition, substituting the mass balance equations, (14) and (15), into (17), leads to:

\[
(f_ℓ - f_s)h_s + (f_w^* - f_w - f_ℓ)h_w + \left( M_s^R \frac{∂h_s}{∂P} + M_w^R \frac{∂h_w}{∂P} - V^R + M_m^R C_p \frac{∂T_s}{∂P} \right) \dot{P}
\]

\[
= -f_s h_s + (f_w^* - f_w + Q^B)
\]

Cancelling the common terms on both sides, and introducing the following definition

\[ K_2 \triangleq M_s^R \frac{∂h_s}{∂P} + M_w^R \frac{∂h_w}{∂P} - V^R + M_m^R C_p \frac{∂T_s}{∂P}, \]

leads to:

\[
(f_ℓ h_s - f_ℓ h_w + K_2 \dot{P}) = Q^B
\]

Rearranging the above equation yields:

\[
f_ℓ = (h_s - h_w)^{-1} \left( -K_2 \dot{P} + Q^B \right)
\]

This equation will form part of the model for boiler pressure described in Section 4.

### 3.3 Volume Constraint

The volume of the risers is constant, leading to the following equation:

\[
V^R = V_s^R + V_w^R
\]

where \( V^R, V_s^R, V_w^R \) denote the volume of the risers, the volume of steam in the risers, and the volume of water in the risers, respectively.

In the same fashion as the volume constraint for the drum, taking the derivative of (20) leads to:

\[
0 = \frac{d}{dt} \left( V_s^R + V_w^R \right)
\]

\[
= \frac{d}{dt} \left\{ \frac{M_s^R}{ρ_s} + \frac{M_w^R}{ρ_w} \right\}
\]

\[
= \frac{M_s^R}{ρ_s} - \frac{M_s^R}{ρ_s^2} \cdot \dot{ρ}_s + \frac{M_w^R}{ρ_w} - \frac{M_w^R}{ρ_w^2} \cdot \dot{ρ}_w
\]

\[
= \frac{f_ℓ - f_s}{ρ_s} + \frac{f_w^* - f_w - f_ℓ}{ρ_w} - \left( \frac{M_s^R}{ρ_s^2} \frac{∂ρ_s}{∂P} + \frac{M_w^R}{ρ_w^2} \frac{∂ρ_w}{∂P} \right) \cdot \dot{P}
\]

Introducing the following definition

\[ C_2 \triangleq \frac{M_s^R}{ρ_s^2} \frac{∂ρ_s}{∂P} + \frac{M_w^R}{ρ_w^2} \frac{∂ρ_w}{∂P} \]

and rearranging terms, leads to:

\[
\frac{f_s}{ρ_s} + \frac{f_w}{ρ_w} = \frac{f_ℓ}{ρ_s} + \frac{f_w^* - f_ℓ}{ρ_w} - C_2 \cdot \dot{P}
\]

This equation will also form part of the final model described in Section 4.
4 A Model for Boiler Pressure

So far, the model for the boiler dynamics comprises: (i) the four conservation of mass equations, (1)–(2) and (14)–(15), (ii) the two equations derived from the constant volume of the drum and risers, (13) and (22), and (iii) the two equations derived from energy balance in the drum and in the risers, (8) and (19). Here an expression for boiler pressure, \( P \), will be derived based only on the latter four equations. Substituting (22) into (13) leads to:

\[
P = C_1^{-1} \left( \frac{-q_s - f_{cd}}{\rho_s} + \frac{q_t + f_{cd}}{\rho_w} + \frac{f_t}{\rho_s} + \frac{f_w - f_t}{\rho_w} - C_2 \cdot \dot{P} \right)
\]

Grouping common terms for \( \dot{P} \) yields:

\[
(C_1 + C_2) \dot{P} = \frac{-q_s - f_{cd}}{\rho_s} + \frac{q_t + f_{cd}}{\rho_w} + \left( \frac{1}{\rho_s} - \frac{1}{\rho_w} \right) f_t
\]

Substituting with (19), it follows that:

\[
(C_1 + C_2) \dot{P} = \frac{-q_s - f_{cd}}{\rho_s} + \frac{q_t + f_{cd}}{\rho_w} + \left( \frac{1}{\rho_s} - \frac{1}{\rho_w} \right) (h_s - h_w)^{-1} \left( -K_2 \dot{P} + Q^B \right)
\]

The following variable is then introduced to simplify the equations:

\[
C_3 \triangleq \left( \frac{1}{\rho_s} - \frac{1}{\rho_w} \right) (h_s - h_w)^{-1}
\]

Rearranging the above equation leads to:

\[
(C_1 + C_2 + C_3 K_2) \dot{P} = \frac{-q_s - f_{cd}}{\rho_s} + \frac{q_t + f_{cd}}{\rho_w} + C_3 Q^B - C_3 \left( K_1 \dot{P} + q_f(h_w - h_w^*) \right)
\]

Using (8), it follows that

\[
(C_1 + C_2 + C_3 K_2) \dot{P} = \frac{-q_s - f_{cd}}{\rho_s} + \frac{q_t + f_{cd}}{\rho_w} + C_3 Q^B - C_3 \left( K_1 \dot{P} + q_f(h_w - h_w^*) \right)
\]

Finally, solving for \( \dot{P} \) yields:

\[
\dot{P} = \left( C_1 + C_2 + C_3(K_1 + K_2) \right)^{-1} \left( \frac{-q_s - f_{cd}}{\rho_s} + \frac{q_t + f_{cd}}{\rho_w} + C_3 \left( Q^B - q_f(h_w - h_w^*) \right) \right)
\]

In summary, equation (25) is a differential equation for \( \dot{P} \), equations (19) and (8) are algebraic equations for \( f_t \) and \( f_{cd} \), respectively, and equations (1)–(2) and (14)–(15) are differential equations for the steam and water masses in the boiler. Together, they provide a complete model for boiler pressure.

Remark 4.1. Note that, to evaluate (25), it is not necessary to know the values of \( f_s \), \( f_w \) and \( f_w^* \). The variable \( C_3 \) is determined only by the boiler pressure, while \( K_1 + K_2 \) and \( C_1 + C_2 \) depend on pressure and the total mass of water and steam in the boiler, since \( M^D_w + M^R_w = q_f - f_t \) and \( M^D_s + M^R_s = f_t - q_s \). Pressure is a global property of the boiler, and therefore, it stands to reason that it should not depend on internal properties of the boiler.

Remark 4.2. On the other hand, unlike pressure, drum water level is not a global property of the boiler. It will be shown in the next section that, in order to obtain a model for drum water level, it is necessary to know other internal quantities such as \( f_s \), \( f_w \) and \( f_w^* \).
5 A Model for Drum Water Level

Consider a section of the boiler drum as shown in Fig. 1, where $L$ is the steady state (nominal) height of the water, and $\delta$ denotes variations around $L$. Note that, to determine the height of the water it is necessary to know, not only the mass of water in the drum, $M_{Dw}$, but also the mass of steam below the water line, $M_{BW}$. An expression for $M_{BW}$ can be obtained by again applying the conservation of mass principle, i.e.:

$$\dot{M}_{BW}(t) = f_s(t) - f_s(t - a)$$

(26)

where $f_s(t)$, $f_s(t - a)$ denote the mass flow of steam out of the risers, and a delayed version of $f_s(t)$, respectively. The time delay $a$ can be easily computed by noting that it is the time taken for a given mass to cover a certain distance, i.e.:

$$a = \frac{\text{distance}}{\text{speed}} = \frac{L + \delta}{f_s(t)/(\rho_s A^{R})}$$

(27)

where $A^{R}$ denotes the total area of the risers.

**Remark 5.1.** Equation (27) implies that the mass flow of steam below the water line follows a straight trajectory from the risers to the surface of the drum water. It may prove beneficial to consider a free parameter $k \in \mathbb{R}_0^+$, so that $f_s(t - k \cdot a)$ can account for other trajectories. Note that $k$ would be the only free parameter in the model. □

**Assumption C.** In steady state, the nominal drum water level, $L$, corresponds to the centre of the drum. □

An immediate consequence of Assumption C is:

$$V_{Dw} + V_{BW} = \frac{V^D}{2} + A^D \delta$$

(28)

where $A^D$ denotes the area at the centre line of the drum. Using the fact that $V = M/\rho$, then equation (28) can be rewritten as:

$$\frac{M_{Dw}}{\rho_w} + \frac{M_{BW}}{\rho_s} = \frac{V^D}{2} + A^D \delta$$

Finally, the following expression for the drum water level deviation, $\delta$, is obtained:

$$\delta = \left(A^D\right)^{-1} \left[\frac{M_{Dw}}{\rho_w} + \frac{M_{BW}}{\rho_s} - \frac{V^D}{2}\right]$$

(29)

**Remark 5.2.** Equations (29), (26) and (2) show that it is necessary to be able to independently describe $f_s$, $f_w$, $f^*_w$ in order to obtain drum water level. □

**Remark 5.3.** Note that (13) and (22) are linearly dependent equations in $f_s$ and $f_w$. Therefore, with these two equations alone it is not possible to obtain independent expressions for $f_s$ and $f_w$. This problem will be resolved in Section 6. □

**Remark 5.4.** An expression for $f^*_w$ will be derived in Section 7 using conservation of momentum in the downcomer-riser system. □
6 Spatial Discretisation and Homogeneous Mixing in the Risers

In this section, the model of the risers will be embellished to account for spatial distribution of the boiling process. An additional assumption will be introduced which allows separation of the expressions for $f_s$ and $f_w$ in the model.

6.1 Spatial discretisation

Consider a uniform subdivision of the volume of the risers into $n$ sections. For each section, there are three core equations, describing mass balance, constant volume and energy balance. This leads to:

\begin{align*}
\dot{M}_{Rs}^i &= f_s^{i-1} - f_s^i + f_\ell^i \\
\dot{M}_{Rw}^i &= f_w^{i-1} - f_w^i - f_\ell^i \\
V_{Ri} &= V_{Rw}^i + V_{Rs}^i
\end{align*}

\begin{align*}
\frac{d}{dt} \left\{ M_{Rs}^i \rho_s \frac{\partial \rho_s}{\partial P} + M_{Rw}^i \rho_w \frac{\partial \rho_w}{\partial P} + M_{Rm}^i C_p T_m \right\} &= (f_s^{i-1} - f_s^i) h_s + (f_w^{i-1} - f_w^i) h_w + Q_{B_i}^i
\end{align*}

where the superscript $R_i$ denotes the $i$-th section of the risers, $f_\ell^i$ denotes the mass flow of water converted into steam in section $i$, and $f_s^i$, $f_w^i$ denote the mass flow of steam and water leaving section $i$, respectively. Similarly, $f_s^{i-1}$, $f_w^{i-1}$ denote the mass flow of steam and water entering section $i$, respectively. $Q_{B_i}^i$ denotes the heat flow directly affecting section $i$, for $i = 1, \ldots, n$.

**Assumption D.** The heat flow used for boiling water, $Q_{B_i}^i$, is distributed uniformly across the $n$ sections of the risers, i.e. $Q_{B_i}^i = Q_{B_i}^\text{avg} / n, \forall i$. □

The following equations are immediate:

\begin{align*}
&f_w^0 = f_w^* \\
&f_s^0 = 0 \\
&f_w^n = f_w \\
&f_s^n = f_s \\
&\sum_{i=1}^{n} f_\ell^i = f_\ell \\
&\sum_{i=1}^{n} M_{Rw}^i = M_{Rw} \\
&\sum_{i=1}^{n} M_{Rs}^i = M_{Rs}
\end{align*}

To simplify the equations in the sequel, the following variables are defined:

\begin{align*}
C_2^i &\triangleq \frac{M_{Rs}^i}{\rho_s^2} \frac{\partial \rho_s}{\partial P} + \frac{M_{Rw}^i}{\rho_w^2} \frac{\partial \rho_w}{\partial P} \\
K_2^i &\triangleq \frac{M_{Rs}^i}{\rho_s^2} \frac{\partial h_s}{\partial P} + \frac{M_{Rw}^i}{\rho_w^2} \frac{\partial h_w}{\partial P} - V_{Ri} + M_{Rm}^i C_p \frac{\partial T}{\partial P}
\end{align*}
Then, using the same procedure as in Sections 3.3 and 3.2 for equations (32) and (33), it follows that:

\[
\frac{f_i^w}{\rho_w} + \frac{f_i^s}{\rho_s} = \frac{f_i^{w-1} - f_i^s}{\rho_w} + \frac{f_i^{s-1} + f_i^s}{\rho_s} - C_2^i \cdot \dot{P} \quad (37)
\]

\[
f_i^s = (h_w - h_s)^{-1} \left( -K_2 \dot{P} + Q_i^B \right) \quad (38)
\]

Equations (37) and (38), together with (30) and (31), give a complete account of the dynamics of the \( i \)-th section of the risers. However, \( f_i^s \) and \( f_i^w \) are still linearly dependent. In the next subsection, an additional assumption is introduced which allows \( f_i^s \) and \( f_i^w \) to be separately described.

### 6.2 Homogeneous mixing in a section of the risers

Consider a section of the risers. Then, over an infinitesimal period of time \( \Delta \), the mass of steam and water leaving the section are given by \( f_i^s \Delta \) and \( f_i^w \Delta \), respectively. The steam quality of each section is defined as:

\[
\alpha_i = \frac{M_{Ri}^s}{M_{Ri}^s + M_{Ri}^w} \quad (39)
\]

The following assumption is next introduced:

**Assumption E.** Perfect mixing of water and steam occurs in each section of the risers. □

An immediate consequence of the above assumption is that the mass of steam and water leaving a specific section over a period of time \( \Delta \) must have the same ratio \( \alpha_i \). Therefore,

\[
\alpha_i = \frac{M_{Ri}^s}{M_{Ri}^s + M_{Ri}^w} = \frac{f_i^s \Delta}{f_i^s \Delta + f_i^w \Delta} \quad (40)
\]

Solving for \( f_i^s \) leads to:

\[
f_i^s = \frac{M_{Ri}^s}{M_{Ri}^s + M_{Ri}^w} f_i^w \quad (41)
\]

Substituting (41) into (37) yields:

\[
\frac{f_i^w}{\rho_w} + \frac{1}{\rho_s} \frac{M_{Ri}^s}{M_{Ri}^w} f_i^s = \frac{f_i^{w-1} - f_i^s}{\rho_w} + \frac{f_i^s + f_i^{s-1}}{\rho_s} - C_2^i \cdot \dot{P} \quad (42)
\]

Solving for \( f_i^w \) leads to:

\[
f_i^w = \left( \frac{1}{\rho_w} + \frac{1}{\rho_s} \frac{M_{Ri}^s}{M_{Ri}^w} \right)^{-1} \left( \frac{f_i^{w-1} - f_i^s}{\rho_w} + \frac{f_i^s + f_i^{s-1}}{\rho_s} - C_2^i \cdot \dot{P} \right) \quad (43)
\]

In summary, equations (41) and (43) provide a separate account of the mass flow of steam and water leaving the \( i \)-th section of the risers. Together with equations (38), (30) and (31), this constitutes a complete description of the dynamics of a section of the risers. Using equations (34), the model for the sections of the risers can be interfaced with the pressure and drum water level models presented in Sections 4 and 5.

**Remark 6.1.** Note that the concept of homogeneity of the steam-water mix is directly related to the concept that no slip occurs between the steam mass flow and the water mass flow – see [4]. Indeed, the no slip condition implies the linear speed of both steam and water masses leaving each section of the risers are the same, and therefore, the mass flows must be locked together. □
7  Water Flow in the Downcomers (Momentum Balance)

In order to obtain an expression for $f_w^*$, conservation of momentum is applied along the downcomers and risers. The fixed control volume is defined as the total volume of the downcomer-riser configuration as shown in Fig. 1. The control surface is defined as the surface of the control volume. A general expression for momentum balance is given by (see [3, Section 2.5]):

$$\frac{\partial}{\partial t} \iiint_V \rho \mathbf{v} dV + \oiint_S (\rho \mathbf{v} \cdot dS) \mathbf{v} = -\oiint_S \mathbf{P} dS + \oiint_V \rho f dV + F_{visc} \quad (44)$$

where the term A denotes the time rate change of the linear momentum of the contents of the control volume, the term B denotes the nett flow of linear momentum out of the control surface by mass flow, the term C denotes the force exerted by pressure on the control surface, the term D denotes the body force acting on the control volume, and $F_{visc}$ denotes the viscous forces acting on the control surface.

**Assumption F.** The pressure dynamics are much slower than the momentum dynamics.  

A consequence of the above assumption is that density can be considered to be uniform across the volume of the downcomers/risers system. Therefore,

$$\frac{\partial}{\partial t} \iiint_V \rho \mathbf{v} dV = \frac{\partial}{\partial t} \left[ M_w^{DC} f_w^* + M_w^R f_w + M_s^R f_s \right] \quad (45)$$

$$\oiint_S (\rho \mathbf{v} \cdot dS) \mathbf{v} = k_w^* \left( \frac{f_w^*}{\rho_w A^{DC}} + \frac{f_w^2}{\rho_w A^R} + \frac{f_s^2}{\rho_s A^R} \right) \quad (46)$$

$$-\oiint_S \mathbf{P} dS = 0 \quad (47)$$

$$\oiint_V \rho f dV = (M_w^{DC} - M_w^R - M_s^R) g \quad (48)$$

$$F_{visc} = 0 \quad (49)$$

Note that, since the downcomers contain only water, then

$$\frac{M_w^{DC}}{\rho_w A^{DC}} = \frac{V_w^{DC}}{A^{DC}} = L^{DC} \quad (50)$$

where $L^{DC}$ is the length of the downcomers. Therefore, equation (44) can be written as:

$$\frac{\partial}{\partial t} \left[ L^{DC} f_w^* + M_w^R f_w + M_s^R f_s \right] = k_w^* \left( \frac{f_w^*}{\rho_w A^{DC}} + \frac{f_w^2}{\rho_w A^R} + \frac{f_s^2}{\rho_s A^R} \right) + \frac{f_w^2}{\rho_w A^R} - k_w^* \frac{f_w^*}{\rho_w A^{DC}} + \frac{f_s^2}{\rho_s A^R} - \frac{f_s^*}{\rho_s A^{DC}} = (M_w^{DC} - M_w^R - M_s^R) g \quad (51)$$

Solving for $f_w^*$ leads to:

$$L^{DC} \frac{d}{dt} \left[ f_w^* \right] = (M_w^{DC} - M_w^R - M_s^R) g - k_w^* \left( \frac{f_w^*}{\rho_w A^{DC}} - \frac{f_w^2}{\rho_w A^R} - \frac{f_s^2}{\rho_s A^R} \right) - \frac{\partial}{\partial t} \left[ M_w^R \frac{f_w}{\rho_w A^R} + M_s^R \frac{f_s}{\rho_s A^R} \right] \quad (52)$$

**Remark 7.1.** Note that (52) represents a significant departure from the equations used in [3].  

13
8 Superheater Model

A superheater is a heat exchanger used to convert saturated steam generated in a boiler into superheated steam by adding heat, thus drying the steam. Superheated steam is used to power turbines to generate electricity. For the current purpose, the main difference between saturated and superheated steam is that, when considering saturated steam, it sufficed to use one state variable, namely the pressure of the water/steam mixture. This made it possible to unequivocally describe density, enthalpy, temperature, and other state variables, for both liquid and vapour phases. However, to describe the state of superheated steam it is necessary to consider two independent state variables. In the sequel, pressure and enthalpy will be used for this purpose.

Let the superheater have volume $V_{SH}$ and a heat flow input $Q_{SH}$. Then mass balance, energy balance and constant volume equations for a superheater can immediately be obtained as shown below.

8.1 Mass balance

$$\dot{M}_{SH}^S = q_s - q_{s}^{SH}$$

(53)

where $M_{SH}^S$, $q_s$, $q_{s}^{SH}$ denote the mass of steam in the superheater, the steam mass flow out of the drum into the superheater, and the steam mass flow out of the superheater.

8.2 Energy balance

$$\frac{d}{dt}\{M_{SH}^{S\;H} u_{s}^{S\;H}\} = q_s h_s - q_{s}^{SH} h_{s}^{S\;H} + Q^{SH}$$

(54)

By definition $u = h - P/\rho$, therefore,

$$\frac{d}{dt}\left\{M_{s}^{S\;H} h_{s}^{S\;H} - \frac{M_{s}^{S\;H}}{\rho_{s}^{S\;H}} P^{S\;H}\right\} = q_s h_s - q_{s}^{SH} h_{s}^{S\;H} + Q^{SH}$$

Noting that $M_{s}^{S\;H}/\rho_{s}^{S\;H} = V^{S\;H}$, and expanding the derivative, leads to:

$$M_{s}^{S\;H} h_{s}^{S\;H} - M_{s}^{S\;H} h_{s}^{S\;H} - V^{S\;H} {\dot P}^{S\;H} = q_s h_s - q_{s}^{SH} h_{s}^{S\;H} + Q^{SH}$$

Finally, using equation (53) and cancelling the common terms yields:

$$M_{s}^{S\;H} h_{s}^{S\;H} - V^{S\;H} {\dot P}^{S\;H} = q_s (h_s - h_{s}^{S\;H}) + Q^{SH}$$

(55)

8.3 Volume Constraint

$$\frac{d}{dt}\{V^{S\;H}\} = 0$$

(56)

By definition we know that $V = M/\rho$, thus

$$\frac{d}{dt}\{V^{S\;H}\} = \frac{d}{dt}\left\{\frac{M_{s}^{S\;H}}{\rho_{s}^{S\;H}}\right\} = \frac{\dot{M}_{s}^{S\;H}}{\rho_{s}^{S\;H}} - \frac{M_{s}^{S\;H}}{(\rho_{s}^{S\;H})^{2}} {\dot \rho}_{s}^{S\;H}$$

(57)
However, as mentioned earlier, density of superheated steam is no longer a function of pressure only. Therefore, the time derivative of density must now be expanded as follows:

$$\dot{\rho}^{SH} = \frac{\partial \rho^{SH}}{\partial h^{SH}} \dot{h}^{SH} + \frac{\partial \rho^{SH}}{\partial P^{SH}} \dot{P}^{SH}$$  \hspace{1cm} (58)$$

Substituting (57) and (58) into (56), and noting that $M/\rho = V$ leads to:

$$\frac{M_{v}^{SH}}{\rho_{v}^{SH}} \dot{V}^{SH} - \frac{V^{SH}}{\rho_{v}^{SH}} \left( \frac{\partial \rho_{v}^{SH}}{\partial h_{v}^{SH}} \dot{h}^{SH} + \frac{\partial \rho_{v}^{SH}}{\partial P_{v}^{SH}} \dot{P}^{SH} \right) = 0$$

Using (53) and reordering terms yields:

$$V^{SH} \left( \frac{\partial \rho_{v}^{SH}}{\partial h_{v}^{SH}} \dot{h}^{SH} + \frac{\partial \rho_{v}^{SH}}{\partial P_{v}^{SH}} \dot{P}^{SH} \right) = q_{v} - q_{s}^{SH}$$  \hspace{1cm} (59)$$

### 8.4 A model for the superheater

Equations (53), (55) and (59) provide a complete model describing the dynamics of a superheater. To implement the model, equations (55) and (59) should be decoupled. From (59), it follows that:

$$V^{SH} \left( \frac{\partial \rho_{v}^{SH}}{\partial h_{v}^{SH}} \dot{h}^{SH} + \frac{\partial \rho_{v}^{SH}}{\partial P_{v}^{SH}} \dot{P}^{SH} \right) = q_{v} - q_{s}^{SH}$$

Substituting into (55) leads to:

$$M_{v}^{SH} \dot{h}^{SH} + \left( \frac{\partial \rho_{v}^{SH}}{\partial P_{v}^{SH}} \right)^{-1} V^{SH} \frac{\partial \rho_{v}^{SH}}{\partial h_{v}^{SH}} \dot{h}^{SH} = q_{v}(h_{s} - h_{v}^{SH}) + Q^{SH} + \left( \frac{\partial \rho_{v}^{SH}}{\partial P_{v}^{SH}} \right)^{-1} \left( q_{v} - q_{s}^{SH} \right)$$

Solving for $\dot{h}^{SH}$ yields:

$$\dot{h}^{SH} = \left( M_{v}^{SH} + \left( \frac{\partial \rho_{v}^{SH}}{\partial P_{v}^{SH}} \right)^{-1} V^{SH} \frac{\partial \rho_{v}^{SH}}{\partial h_{v}^{SH}} \right)^{-1} \left( q_{v}(h_{s} - h_{v}^{SH}) + Q^{SH} + \left( \frac{\partial \rho_{v}^{SH}}{\partial P_{v}^{SH}} \right)^{-1} \left( q_{v} - q_{s}^{SH} \right) \right)$$

Then, from (55) it follows that:

$$\dot{P}^{SH} = \left( V^{SH} \right)^{-1} \left( M_{v}^{SH} \dot{h}^{SH} - q_{v}(h_{s} - h_{v}^{SH}) - Q^{SH} \right)$$

In summary, the model for a superheater is given by the following equations:

$$\dot{M}_{v}^{SH} = q_{v} - q_{s}^{SH}$$  \hspace{1cm} (60)$$

$$\dot{h}_{v}^{SH} = \left( M_{v}^{SH} + \left( \frac{\partial \rho_{v}^{SH}}{\partial P_{v}^{SH}} \right)^{-1} V^{SH} \frac{\partial \rho_{v}^{SH}}{\partial h_{v}^{SH}} \right)^{-1} \left( q_{v}(h_{s} - h_{v}^{SH}) + Q^{SH} + \left( \frac{\partial \rho_{v}^{SH}}{\partial P_{v}^{SH}} \right)^{-1} \left( q_{v} - q_{s}^{SH} \right) \right)$$  \hspace{1cm} (61)$$

$$\dot{P}^{SH} = \left( V^{SH} \right)^{-1} \left( M_{v}^{SH} \dot{h}_{v}^{SH} - q_{v}(h_{s} - h_{v}^{SH}) - Q^{SH} \right)$$  \hspace{1cm} (62)$$
9 Key consequences of the new model

This section summarises and illustrates the main consequences of the model derived in this paper. In particular, three key points are made, namely: (i) drum water level is proportional to steam flow out of the boiler, (ii) spatial discretisation of the risers is necessary for fast transient dynamic modelling, and (iii) the relationship between downcomer mass flow and pressure derivatives leads to a model that can describe fast transients in the drum water level responses.

In the sequel, the simulations and data presented correspond to Boiler 1 at Proserpine Mill. Boiler 1 has a maximum continuous rating (MCR) of 17.5 [kg/s] at 1650 [kPa]. Boiler 1 does not have a superheater. The details of the physical parameters used in the simulation are as follows:

\[
\begin{align*}
A^R &= 1.5 [m^2] \quad (63a) \\
V^R &= 10.5 [m^3] \quad (63b) \\
A^D &= 9.46 [m^2] \quad (63c) \\
V^D &= 10.2 [m^3] \quad (63d) \\
L^{DC} &= 7 [m] \quad (63e) \\
A^{DC} &= 0.62 [m^2] \quad (63f) \\
V^{DC} &= 9.32 [m^3] \quad (63g) \\
M^D_m &= 7400 [kg] \quad (63h) \\
M^R_m &= 40700 [kg] \quad (63i) \\
P_0 &= 1.60 \cdot 10^6 [Pa] \quad (63j) \\
h_f^T &= 399900 [J/kg] \quad (63k) \\
C_p &= 470 [J/(kg \cdot K)] \quad (63l)
\end{align*}
\]

**Remark 9.1.** It is very important to note that the above parameters have all been obtained from physical properties of the boiler and its associated datasheets. No estimation of parameters has been performed. This avoids the problem of overfitting due to the presence of many degrees of freedom [11]. □

9.1 Drum Water Level proportional to Steam Flow

One advantage of having a physical model is that particular occurrences observed in real life can be substantiated by using the model. Fig [2] shows real data from Boiler 1 at Proserpine Mill for a 30 [min] period. It can be seen that positive changes in Steam Flow are correlated with positive changes in Drum Water Level and vice versa. It is hypothesised that this is a general fact that can be explained by the model. In the following, the model presented in this paper will be used to show that this hypothesis is, in fact, true. First it will be proven that the derivative of pressure is proportional to steam flow, then it will be proven that drum water level deviations are proportional to the derivative of pressure. Combining these two observations leads to the final conclusion that drum water level is indeed proportional to steam flow.

9.1.1 Derivative of Pressure is proportional to Steam Flow

Consider equation (25). It can be seen that equation (25) can be rewritten as:

\[
\dot{P} = -\lambda(P, M_a, M_w, V^T) \cdot q_s + \beta(P, M_a, M_w, V^T, q_f, Q^T) \quad (64)
\]

where \(\lambda(\cdot)\), \(\beta(\cdot)\) are nonlinear functions. Therefore \(\dot{P}\) is proportional to \(q_s\).
Remark 9.2. Consider the following quantities for Boiler 1 evaluated at the nominal operating point:

\[ \frac{1}{\rho_w} = 1.16 \times 10^{-3}, \]
\[ \frac{1}{\rho_s} = 1.12 \times 10^{-1}, \]
\[ C_3 = 6.16 \times 10^{-8}. \]

It can be seen that, in equation (25), the coefficient multiplying \( q_s \) is at least 100 times larger than the others. This implies that \( q_s \) is the main factor affecting pressure changes.

9.1.2 Drum Water Level deviations are proportional to Derivative of Pressure

Next, consider equation (29) for the drum water level deviations \( \delta \) and equation (26) for the mass of steam below the water line \( M_{BW} \). It can be seen that \( \delta \) is proportional to \( M_{BW} \). Using Laplace transforms and a Padé approximation for the time delay, equation (26) leads to:

\[ s \cdot M_{BW}(s) = F_s(s) - e^{-as} F_s(s) \]
\[ = \left( 1 - \frac{2 - as}{2 + as} \right) F_s(s) \]
\[ = \frac{2as}{2 + as} F_s(s) \]

where \( s \) is the Laplace Transform variable. Cancelling the \( s \) (derivative) on both sides of the above equation leads to:

\[ M_{BW}(s) = \frac{2as}{2 + as} F_s(s) \]

Using the inverse Laplace transform yields:

\[ M_{BW}(t) = 2 \cdot \int_0^t f_s(\tau) \cdot e^{-2(t-\tau)} d\tau \]

Because of the convolution with the exponential decay, the above equation can also be written as:

\[ M_{BW}(t) = 2f_s(t) + \lambda_2(f_s(t-\tau), \tau), \quad 0 \leq \tau < t \]
where $\lambda_2(\cdot)$ denotes the tail of the convolution integral. Hence, any change in $f_s(t)$ will appear over a short interval in $M_{BW}^f(t)$, i.e. they are proportional. Finally, consider equations (41) and (43) for the top section of the risers, i.e. $i = n$. Then it can be seen that $f_s(t)$ is proportional to $f_w(t)$, and that $f_w(t)$ is proportional to $\dot{\bar{P}}$.

In summary, Drum Water Level deviations are indeed (approximately) proportional to the Derivative of Pressure.

### 9.1.3 Drum Water Level deviations are proportional to Steam Flow

The two facts established in the previous subsections have a major consequence, namely Drum Water Level is proportional to Steam Flow. This provides a physical explanation to the experimental results shown earlier in Fig. 2.

### 9.2 Alpha is not a linear function of height in the risers under transient conditions

A common assumption in the literature is that the (mass) steam quality increases linearly with height in the risers at all times – see e.g. [4]. It will be shown below that, under transient conditions, such an assumption is not valid and in fact, leads to large errors.

Let $h = 7 [\text{m}]$ be the height of the risers, and let the risers be divided in 7 sections. Let $\alpha_k$, $k = 1, \ldots, 7$ denote the (mass) steam quality in each of the $k$ sections, where $\alpha_1$ corresponds to the section at the bottom of the risers and $\alpha_7$ to the section at the top.

Define the ratio $\bar{\alpha}_k = \alpha_k / (k \cdot \alpha_1)$ for $k = 1, \ldots, 7$. If the assumption that $\alpha_k$ is linear with height, i.e. $h/k$, were to be valid, then $\bar{\alpha}$ would be equal to 1, $\forall k$, and for all times.

The full model described in this paper was used to simulate the boiler response to the real steam flow profile shown in Fig. 4. Fig. 3 shows $\bar{\alpha}_k$ for $k = 1, \ldots, 7$, for the first 300 $[\text{s}]$. It can be seen that $\bar{\alpha}_k = 1$, $k = 1, \ldots, 7$ does not hold under transient conditions, although it does hold in steady state. Under transient conditions the discrepancy gets larger the further one moves up the risers. Indeed, at the top of the risers, there is an error of almost 50\% at $t = 210 [\text{s}]$ in the maximum steam quality predicted. Furthermore, the transient response persists for more than 20 $[\text{s}]$ after a load change.

![Figure 3: Mass Steam Quality Ratio](image_url)
This is an important conclusion because the steam quality at the top of the risers is the main driving factor in the amount of water and steam entering the drum, and thus, it has a major impact on drum water level. A significant transient response such as the ones shown in Fig. 3 cannot be ignored if the goal is to capture large and fast drum water level excursions.

9.3 Tracking Fast Changes

The complete model, using the parameters shown in (63), will be used to simulate the Drum Water level response to a steam flow dataset obtained from Boiler 1 at Proserpine Mill. Fig. 4 shows this specific steam flow profile. It can be seen that large steam flow variations occur in a matter of seconds. In particular, at \( t = 200 \) [s] there is a 50\% spike in demand which occurs over a period of 8 [s].

As a comparison, the boiler model presented in [4] has been implemented, fitted and tuned to match Boiler 1 at Proserpine Mill. The same initial conditions and controllers have been used in both simulations. Fig. 5 shows a comparison between the real Drum Water Level response from Boiler 1, and the response predicted by both models. It can be seen that the new model accurately tracks the negative peak at \( t = 180 \) [s], the positive peak at \( t = 208 \) [s], and it maintains a non-increasing offset to the real drum water level data. On the other hand the model taken from [4] only tracks the positive peak accurately. As a performance metric, the mean squared error (MSE) was computed for both models. The MSE for the new model is 876.4 whereas the MSE for the Åström and Bell model [4] is 2660.8, i.e. the new model provides an MSE reduction of 67\% in this particular case.

10 Implications for Boiler Control

This section explores the impact that the new model has on boiler control architecture and tuning. Two controllers are explored, namely the steam flow controller and the drum water level controller.

The ideas presented below are based on two key observations:
Drum Water Level deviations are proportional to Steam Flow. Hence, if large Steam Flow fluctuations from downstream can be prevented from reaching the boiler, the deviations in Drum Water Level can be greatly reduced, and

(ii) Feedwater mass flow cannot be used to correct fast Drum Water Level deviations. Considering the geometry of the drum, then the maximum available feedwater flow can change the water level at a rate of $2 \text{[mm/s]}$. The disturbances considered in this paper are of the order of $10 \text{[mm/s]}$. Thus, the feedwater controller is ineffective for fast corrections.

Indeed, as mentioned in Section 1, controlling Drum Water Level under highly variable load conditions has been the main concern at Proserpine Mill. The insight provided in this section has proven crucial when developing new steam flow and drum water level controllers.

### 10.1 Steam Flow Controller

The steam flow controller regulates the opening of the steam valve, based on an external setpoint and the measurement of the current steam flow through the valve. The valve is positioned between the drum and the steam receiver. The mass flow of steam from the boiler to the steam receiver is proportional to the pressure difference, $\Delta P$, between them and is also dependent on the opening of the steam flow valve.

The observation that drum water level deviations are proportional to steam flow implies that it is highly desirable to prevent large and rapid steam flow variations from reaching the boiler. Two scenarios are studied, namely:

1. When there is a sudden load increase, then pressure in the steam receiver will decrease. In turn, this means that $\Delta P$ will increase and thus the steam flow out of the boiler will also increase. An appropriate control response under these conditions is to quickly reduce the opening of the steam flow valve. Therefore the steam flow valve controller time constant must be of the same order as the time constant of the steam flow perturbations. The tradeoff associated with this is that there will be greater deviations in the steam receiver pressure.
2. When there is a sudden load decrease, then pressure in the steam receiver will increase. In turn, this means that $\Delta P$ will decrease (possibly to zero) and thus the steam flow out of the boiler will also decrease. Unlike the previous scenario, the opening of the control valve under this scenario is ineffective as an appropriate control response, since no matter how open the valve is, the flow of steam is limited by $\Delta P$. Hence another approach is needed. One option is to use a let-down valve to release steam either to the atmosphere or other independent machinery. Two considerations must be made, namely (i) the letdown valve should be located as close as possible either to the source of the steam flow perturbation or to the steam receiver, and (ii) the time constant of the letdown valve controller must be of the same order as the time constant of the steam flow perturbations.

In conjunction, the two scenarios mentioned above provide a viable strategy for reducing drum water level excursions due to steam flow variations. The efficacy of these considerations will be illustrated in Section 11 for a boiler in Proserpine Mill.

10.2 Drum Water Level Controller

The drum water level controller regulates the feedwater mass flow into the drum, based on a given setpoint (0 [mm]) and the measurement of the current water level. The largest drum water level perturbation in Fig. 5 has variations of 70 [mm] in 8 [s]. This means that the drum water level can change at a rate of at least 8.75 [mm/s]. Due to the geometry of the drum, if 0 [mm] is considered to be at the center of the drum, then an increase of 17 [kg/s] in feedwater (which corresponds to the maximum available flow, from fully closed to fully open) can only change the Drum Water Level by about 2 [mm/s]. In conclusion, fast drum water level disturbances cannot be compensated with feedwater flow. Specifically, the available control authority is deficient by a factor of at least 4 : 1.

Traditional drum water level control consists of a classical feedback controller driven by the error in water level, and a feedforward controller that uses the measurement of the steam flow out of the boiler to act ahead of a steam disturbance, i.e. if steam flow increases then drum water level will increase as well, so the feedforward will decrease the controller output to preempt the incoming high water level before slowly increasing the controller output to balance the new rate at which steam leaves the drum.

The above strategy fails to acknowledge that fast drum water level excursions can never be compensated with feedwater. Indeed, in practice, the total controller output is a mirror of the Drum Water Level measurement, which indicates an almost pure Proportional controller – see the top plot in Fig. 6. The problem with this controller behaviour is that it adds water when the drum water level is low, but because the reason for it being low is most likely a steam flow disturbance, it will very likely go in the other direction. When that happens, the drum water level will be high due to the steam flow variation, but it will be higher than what it should be because for the past minute the controller has been putting in more water than necessary. The same occurs in the other direction. In summary, the current feedback and feedforward controllers make the Drum Water Level excursions worse.

The idea behind the new feedforward is that it is impossible to correct the effects of large load swings on drum water level with any controller. Therefore, we focus on stopping the feedback controller from making matters worse, i.e., the new feedforward blinds the feedback controller to large load swings. The result is that there is no more mirror behaviour – see bottom plot in Fig. 6.

11 Quantifying Boiler Improvement

In order to illustrate the improvements in drum water level deviations in the day-to-day operation of Boiler 1 at Proserpine Mill six datasets were compared, each comprising a minimum period of 5 days. Three datasets correspond to boiler operation before the changes in were made (1-3), and three datasets correspond to after the fact (4-6). The dates and times in question are as shown in Table 2. In the sequel,
Figure 6: Drum Water Level performance with existing controllers (top) and new controllers (bottom)

Fig 7 shows a histogram of Drum Water Level deviations with mean values removed and where the data in each bin has been normalised with respect to the total number of datapoints. The standard deviation of each case is given in Table 3. Several conclusions follow, namely:

- The histograms presented in Figure 7 show that the new control laws lead to narrower distribution than those corresponding to the original control laws.
The above can be interpreted as tighter control over drum water excursions resulting from the new controller, i.e. drum water level is regulated closer to the setpoint.

Table 3 shows that the new controller has resulted, on average, in a 32.5\% reduction in the standard deviation of drum water level.

Another measure of improvement is to analyse the number of times the drum water level surpasses a specified safety threshold, e.g. ±250 [mm]. Table 4 summarises this information. It can be seen from

| Dataset | < −250 | > 250 |
|---------|--------|-------|
| DS1     | 27     | 12    |
| DS2     | 38     | 15    |
| DS3     | 13     | 9     |
| Total   | 78     | 36    |

| Dataset | < −250 | > 250 |
|---------|--------|-------|
| DS4     | 0      | 3     |
| DS5     | 14     | 2     |
| DS6     | 1      | 13    |
| Total   | 15     | 18    |

Table 4: Water level excursions outside threshold (DS1-3 existing controller. DS4-6 new controller)

Table 4 that datasets 4-6 (with the new controllers) have a reduction of 80\%, on average, in the excursions below −250 [mm] and a reduction of 50\%, on average, in the excursions above 250 [mm].

Figure 7: Normalised histogram for Datasets 1-3 (top) and Datasets 4-6 (bottom)

12 Conclusions

This paper has described a new model for a Boiler operating under highly variable loads. The model is based on first principles. Significant departures have been made from the assumptions previously used in the literature. New features of the model include (i) a multi-compartment model for the risers, (ii) a new model for drum water level, and (iii) a new dynamic model for the flow of water in the downcomers. A comparison between simulations made with the model and real data from a boiler has been presented which (i) confirm the validity of the new model, and (ii) highlight the advantages of the new model under rapid load changing conditions. Implications of the model for boiler control have also been described with special emphasis on reducing drum water level excursions under large and rapid steam flow changes. Experimental results from a boiler at Wilmar Sugar’s Proserpine Mill have confirmed the improvements in drum water level excursions achieved by the revised control law.
Acknowledgements

The authors gratefully acknowledge the extraordinary help and support from Wilmar Sugar, in particular from Danny Ferraris, Matt Linneweber, John Andrews and Damien Kelly.

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