Fractional Quantum Hall Effect and Wigner Crystal of Two-Flux Composite Fermions

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(Dated: today)

In two-dimensional electron systems confined to GaAs quantum wells, as a function of either tilting the sample in magnetic field or increasing density, we observe multiple transitions of the fractional quantum Hall states (FQHSs) near filling factors $\nu = 3/4$ and $5/4$. The data reveal that these are spin-polarization transitions of interacting two-flux composite Fermions, which form their own FQHSs at these fillings. The fact that the reentrant integer quantum Hall effect near $\nu = 4/5$ always develops following the transition to full spin polarization of the $\nu = 4/5$ FQHS strongly links the reentrant phase to a pinned ferromagnetic Wigner crystal of two-flux composite Fermions.

Fractional quantum Hall states (FQHSs) are among the most fundamental hallmarks of ultra-clean interacting two-dimensional electron systems (2DESs) at a large perpendicular magnetic field ($B_z$) $[1]$. These incompressible quantum liquid phases, signaled by the vanishing of the longitudinal resistance ($R_{xy}$) and the quantization of the Hall resistance ($R_{xy}$), can be explained by mapping the interacting electrons to a system of essentially non-interacting, 2$p$-flux composite Fermions ($2p$CFs), each formed by attaching $2p$ magnetic flux quanta to an electron ($p$ is an integer). The $2p$CFs have discrete energy levels, the so-called A-levels, and the FQHSs of electrons seen around Landau level (LL) filling factor $\nu = 1/2$ (1/4) would correspond to the integer quantum Hall states of $2p$CFs ($4p$CFs) at integral $\nu^{CF}$ $[2]$. In state-of-the-art, high-mobility 2DESs, FQHSs also develop around $\nu = 3/4$, and are usually understood as the particle-hole counterparts of the FQHSs near $\nu = 1/4$ through the relation $\nu \leftrightarrow (1 - \nu)$ $[3, 4]$. Alternatively, these states might also be the FQHSs of interacting $2p$CFs at $\nu^{CF} = \nu/(1 - 2\nu)$. For example, the $\nu = 4/5$ state is the $\nu^{CF} = -4/3$ FQHS of $2p$CFs, and has the same origin as the unconventional FQHS seen at $\nu = 4/11$ ($\nu^{CF} = 4/3$) $[5, 6]$.

Another hallmark of clean 2DESs is an insulating phase that terminates the series of FQHSs at low fillings, near $\nu = 1/3$, $[7, 8]$. This insulating phase is generally believed to be an electron Wigner crystal, pinned by the small but ubiquitous disorder potential $[9]$. Recently, an insulating phase was observed near $\nu = 4/5$ in clean 2DESs $[10, 11]$. This phase, which is signaled by a reentrant integer quantum Hall state (RIQHS) near $\nu = 1$, was interpreted as the particle-hole symmetric state of the Wigner crystal seen at very small $\nu$ $[10, 11]$. In this picture, the holes, unoccupied states in the lowest LL, have filling factor $\nu^h \sim 1/5 (= 1 - 4/5)$ and form a liquid phase when the short-range interaction is strong; see the left panel of Fig. 1(a). They turn into a solid phase when the thickness of the 2DES increases and the long-range interaction dominates (right panel of Fig. 1(a)). This interpretation is plausible, since the RIQHS only appears when the well-width ($W$) is more than five times larger than the magnetic length. However, it does not predict or allow for any transitions of the $\nu = 4/5$ FQHS, which is always seen just before the RIQHS develops $[10]$.

Here we report our extensive study of the FQHSs near $\nu = 3/4$ (at 4/5 and 5/7) and their particle-hole counterparts near $\nu = 5/4$ (at 6/5 and 9/7) $[8]$. Via either increasing the 2DES density or the tilt angle between the magnetic field and the sample normal, we increase the ratio of the Zeeman energy ($E_Z = |g|\mu B$ where $B$ is the total magnetic field) to Coulomb en-
ergy \( V_C = \frac{e^2}{4\pi\epsilon l_B} \) where \( l_B = \sqrt{\hbar/cB} \) is the magnetic length), and demonstrate that these FQHSs undergo multiple transitions as they become spin polarized. The number of observed transitions, one for the FQHSs at \( \nu = 4/5 \) and 6/5, and two for the states at \( \nu = 5/7 \) and 9/7, is consistent with what is expected for polarizing the spins of interacting 2CFs. Note that these interacting 2CFs form FQHSs at fractional CF fillings \( \nu^{CF} = -4/3 \) (\( \nu = 4/5 \) and 6/5) and -5/3 (\( \nu = 5/7 \) and 9/7). Even more revealing is the observation that, whenever the RIQHS near \( \nu = 4/5 \) develops, it is preceded by a transition of the FQHS at \( \nu = 4/5 \) to a fully spin-polarized 2CF state. This provides evidence that the RIQHS is the manifestation of a ferromagnetic 2CF Wigner crystal (see Fig. 1(b)).

We studied 2DESs confined to wide GaAs quantum wells (QWs) bounded on each side by undoped Al\(_{0.25}\)Ga\(_{0.75}\)As spacer layers and Si \( \delta \)-doped layers, grown by molecular beam epitaxy. We report here data for two samples, with QW widths \( W = 65 \) and 60 nm, and as-grown densities of \( n \approx 1.4 \) and 0.4, in units of \( 10^{11} \) cm\(^{-2} \) which we use throughout this report. The samples have a van der Pauw geometry with InSn contacts at their corners, and each is fitted with an evaporated Ti/Au front-gate and an In back-gate. We carefully control \( n \) while keeping the charge distribution symmetric. The measurements were carried out in dilution refrigerators with a base temperature of \( T \approx 25 \) mK and superconducting magnets. We used low-frequency (\( \lesssim 40 \) Hz) lock-in techniques to measure the transport coefficients.

Figure 2(a) shows \( R_{xx} \) and \( R_{yy} \) magnetoresistance traces near \( \nu = 3/4 \) measured in a symmetric 65-nm-wide QW, at densities ranging from 1.00 to 1.54. The deep \( R_{xx} \) minimum seen at \( \nu = 4/5 \) in the lowest density \( (n = 1.00) \) trace disappears at \( n = 1.13 \) and reappears at higher densities. With increasing \( n \), an \( R_{xx} \) minimum also develops to the left of \( \nu = 4/5 \), and merges with the \( \nu = 1 \) \( R_{xx} = 0 \) plateau at the highest density \( n = 1.54 \) (see down arrows in Fig. 2(a)). Meanwhile, two minima appear in \( R_{xy} \) on the sides of \( \nu = 4/5 \) when the \( \nu = 4/5 \) FQHS reappears (up-arrows in Fig. 2(a)). These two \( R_{xy} \) minima become deeper at higher densities and, at \( n \approx 1.54 \), the \( R_{xy} \) minimum on the left side of \( \nu = 4/5 \) merges into the \( \nu = 1 \) \( R_{xy} = h/e^2 \) plateau [12].

The \( R_{xx} \) and \( R_{yy} \) data of Fig. 2(a) provide evidence for the development of a RIQHS between \( \nu = 4/5 \) and 1, as reported recently and attributed to the formation of a pinned Wigner crystal state [10, 11]. Note that at the onset of this development, the \( \nu = 4/5 \) FQHS shows a transition manifested by a weakening and strengthening of its \( R_{xx} \) minimum. The central questions we address here are: What is the source of this transition, and what does that imply for the origin of the \( \nu = 4/5 \) FQHS and the nearby RIQHS?

As Fig. 2(b) illustrates, the \( R_{xx} \) and \( R_{xy} \) traces measured in the same QW at a fixed density \( n = 1.00 \) and different tilting angles \( \theta \) reveal an evolution very similar to the one seen in Fig. 2(a) (\( \theta \) denotes the angle between the magnetic field and normal to the 2D plane). At \( \theta = 0^\circ \), a strong FQHS is seen at \( \nu = 4/5 \). The \( R_{xx} \) minimum at \( \nu = 4/5 \) disappears at \( \theta \approx 30^\circ \) and reappears at higher \( \theta \), signaling the destruction and resurrection of the FQHS. Two minima in \( R_{xy} \) on the sides of \( \nu = 4/5 \), marked by the up-arrows, develop at \( \theta > 30^\circ \). As \( \theta \) is further increased, the \( R_{xy} \) minimum to the left

![FIG. 2. Longitudinal \( R_{xx} \) and Hall \( R_{xy} \) magnetoresistance traces for 2D electrons confined to a 65-nm-wide GaAs QW near \( \nu = 3/4 \) as a function of (a) increasing charge density, and (b) tilting the sample in the magnetic field. The density \( n \) (in units of \( 10^{11} \) cm\(^{-2} \)) or tilting angle \( \theta \) for each trace is indicated, and traces are shifted vertically for clarity. (c) Energy gap of the \( \nu = 4/5 \) FQHS as a function of \( n \). (d) Arrhenius plot of \( R_{xx} \) vs. \( 1/T \) at \( n = 0.93 \) and \( 1.41 \times 10^{11} \) cm\(^{-2} \).](image-url)
FIG. 3. (a) $R_{xx}$ measured in a 60-nm-wide QW near $\nu = 3/4$, at a fixed density $n = 0.44 \times 10^{11}$ cm$^{-2}$ and different tilting angles $\theta$. (b) $R_{xx}$ for a 65-nm-wide QW near $\nu = 5/4$, at $\theta = 0^\circ$ and different densities $n = 0.86$ to 1.94 $\times 10^{11}$ cm$^{-2}$. (c, d) Schematic plots showing multiple configurations of the $\nu = 4/5$ and 6/5, and 5/7 and 9/7 FQHSs with different spin-polarizations.

FIG. 4. Summary of the spin-polarization energy in units of the Coulomb energy, $E_Z/V_C$, at different filling factors ($\nu$). Dotted lines are guides to the eye. All data points were measured in symmetric QWs. The transitions at $\nu = 2/3, 3/5$ and 4/7 were measured in perpendicular magnetic field by changing $n$, and the rest at a fixed density $n = 0.44$ by changing $\theta$. For each filling, only the (last) transition into a fully spin-polarized configuration is shown.

Observing a spin-polarization transition for the $\nu = 4/5$ FQHS, however, is surprising as this state is usually interpreted as the particle-hole counterpart of the $\nu = 1/5$ FQHS, which is formed by non-interacting four-flux $^4$CFs. Such a state should be always fully spin-polarized and no spin-polarization transition is expected [4].

On the other hand, if the $\nu = 4/5$ FQHS is interpreted as the FQHS of interacting two-flux CFs ($^2$CFs), then it corresponds to the $\nu^{CF} = -4/3$ FQHS of $^2$CFs, and has two possible spin configurations as shown in Fig. 3(c). The system has one fully-occupied, spin-up, $\Lambda$-level and
one 1/3-occupied Λ-level. Depending on whether \( E \)
smaller or larger than the Λ-level separation of the \( ^2 \)CFs,
the 1/3-filled Λ-level may be either spin-down or spin-up
(see Fig. 3(c)) [17].

To further test the validity of the above interpretation,
we measured \( R_{xx} \) in a 60-nm-wide QW at a very low density
\( n = 0.44 \) and different \( \theta \) in Fig. 3(a). The \( \nu = 4/5 \)
FQHS exhibits a clear transition at \( \theta = 60^\circ \), manifested
by a weakening of the \( R_{xx} \) minimum. Note that the
transition of the \( \nu = 4/5 \) FQHS appears in Figs. 2(a),
2(b) and 3(a) when the ratio of the Zeeman to Coulomb
energies \( (E_Z/V_C) \) is about 0.0145, 0.0157 and 0.0177,
respectively. The electron layer-thicknesses at these three
transitions, parameterized by the standard deviation \( (\lambda) \)
of the charge distribution in units of \( \lambda/\lambda_B \), are 1.66, 1.52 and
0.75, respectively. The softening of the Coulomb interaction
due to the finite-layer-thickness effect is less and the spin-polarization energy should be higher for smaller
\( \lambda/\lambda_B \) (see [16] for the dependence of spin-polarization energy
on the finite-layer-thickness). Therefore, these values
are consistent with each other.

In Fig. 3(a), we also observe two transitions for the \( \nu = 5/7 \)
FQHS at \( \theta = 37.5^\circ \) and \( 50^\circ \), suggesting three
different phases. This observation is consistent with the
\( \nu = 5/7 \) FQHS being formed by interacting \( ^2 \)CFs. In such
a picture, the \( \nu = 5/7 \) FQHS, which is the \( \nu^{\text{CF}} = -5/3 \)
FQHS of the \( ^2 \)CFs, has three different possible spin
configurations, as shown in Fig. 3(d). Similar to Fig. 3(c),
the lowest spin-up Λ-level is always fully occupied. The
second Λ-level is 2/3-occupied spin-up (spin-down), if \( E_Z \)
is larger (smaller) than the Λ-level separation. If \( E_Z \)
equals the Λ-level separation, the \( ^2 \)CFs form a novel spin
singlet state when the spin-up and spin-down Λ-levels are
both 1/3-occupied; see the middle panel of Fig. 3(d).

Data near \( \nu = 5/4 \) measured in the 65-nm-wide QW
at different densities, shown in Fig. 3(b), further
confirm our picture. The \( \nu = 6/5 \) and 9/7 FQHSs ex-
hibit transitions similar to their particle-hole conjugate
states at \( \nu = 4/5 \) and 5/7, respectively. The \( \nu = 6/5 \)
FQHS shows a transition at \( n = 1.78 \). At this transition,
\( E_Z/V_C \simeq 0.0149 \) and \( \lambda/\lambda_B \simeq 1.86 \), very similar to
the corresponding values (0.0145 and 1.66) at \( \nu = 4/5 \)
in Fig. 2(a), suggesting that the particle-hole symmetry \( \nu \leftrightarrow (2 - \nu) \) is conserved in this case [18].
Furthermore, the \( \nu = 9/7 \) FQHS becomes weak twice, at \( n = 1.17 \)
and 1.55, also consistent with the \( \nu = 5/7 \) FQHS transitions.

It is instructive to compare the transitions we observe
at fractional \( \nu^{\text{CF}} \) with the spin-polarization transitions
of other FQHSs at integer \( \nu^{\text{CF}} \). In Fig. 4, we sum-
mearize the critical \( E_Z/V_C \) above which the FQHSs between
\( \nu = 1/2 \) and 3/2 become fully spin-polarized. The meas-
urements were all made on the 60-nm-wide QW. The
\( x \)-axis is \( 1/\nu^{\text{CF}} \), and we mark the electron LL filling
factor \( \nu \) in the top axis. The dotted lines, drawn as guides
to the eye, represent the phase boundary between fully
spin-polarized (above) and partially spin-polarized (be-
low) \( ^2 \)CFs. Note that the system is always fully spin-
polarized at \( \nu^{\text{CF}} = -1 \) \( (\nu = 1) \) [19, 20]. The critical
\( E_Z/V_C \) of FQHSs with integral \( \nu^{\text{CF}} \) increases with
\( \nu^{\text{CF}} \) and reaches maxima at \( \nu^{\text{CF}} = -\infty \) \( (\nu = 1/2 \) and
3/2). Secondary maxima in the boundaries appear at
\( \nu^{\text{CF}} = -3/2 \) \( (\nu = 3/4 \) and 5/4), and seem to have
approximately the same height as at \( \nu = 1/2 \) and 3/2.

While Fig. 3 data strongly suggest that we are ob-
serving spin transitions of various FQHSs, there is also
some theoretical justification. It has been proposed that
the enigmatic FQHSs observed at \( \nu = 4/11 \) and 5/13
in the highest quality samples can be interpreted as the
FQHSs of interacting \( ^2 \)CFs at \( \nu^{\text{CF}} = +4/3 \) and \(+5/3 \)
[5, 6]. A recent theoretical study predicts a transition of the
\( \nu = 4/11 \) FQHS to full spin polarization when
\( E_Z/V_C \) is about 0.025 [21]. Our observed transition of the
\( \nu = 4/5 \) \( (\nu^{\text{CF}} = -4/3) \) FQHS appears at \( E_Z/V_C \simeq 0.015 \)
to 0.024, in different QWs with well width ranging from
65 to 31 nm and corresponding \( \lambda/\lambda_B \simeq 1.7 \) to 1.1 [10],
consistent with this theoretically predicted value.

Another useful parameter in characterizing the origin of
the \( \nu = 4/5 \) FQHS and its transition are the energy
gaps on the two sides of the transition. We show in Fig.
2(c) the measured excitation gaps for this state at differ-
et densities in the 65-nm-wide QW. The \( \nu = 4/5 \) FQHS
transition for this sample occurs at density \( n \simeq 1.1 \),
see Fig. 2(a). Before and after the transition, e.g. at
\( n = 0.86 \) and 1.64, the \( \nu = 4/5 \) FQHS has very similar
energy gaps \( (\sim 0.35 \text{ K}) \) although the densities are differ-
ent by nearly a factor of two. Since the FQHS energy
gaps at a given filling are ordinarily expected to scale with
\( V_C \sim \sqrt{n} \), this observation suggests that the exci-
tation gap at \( \nu = 4/5 \) is reduced when the FQHS becomes
fully spin polarized.

Finally we revisit the RIQHSs we observe near \( \nu = 4/5 \)
(see, e.g., Figs. 1(a) and 1(b)). These RIQHSs were
interpreted as pinned Wigner crystal states [10], and
the recent microwave resonance experiments confirm this
interpretation [11]. Moreover, the data in Ref. [10]
as well as the data we have presented here all indicate that,
whenever a transition to a RIQHS occurs, it is initiated
by a transition of the \( \nu = 4/5 \) FQHS. As we have shown
here, the transition we see for the \( \nu = 4/5 \) FQHS is a
transition to a fully spin polarized state of interacting
\( ^2 \)CFs. Combining these observations leads to a tantaliz-
ing conclusion: The fact that the RIQHS near \( \nu = 4/5 \) always
develops following a spin polarization transition of \( ^2 \)CFs strongly links the RIQHS to a pinned, ferromag-
netic Wigner crystal of \( ^2 \)CFs, as schematically illustrated
in Fig. 1(b).

We acknowledge support through the NSF (DMR-
1305691) for measurements, and the Gordon and Betty
Moore Foundation (Grant GBMF2719), Keck Foundation,
the NSF MRSEC (DMR-0819860), and the DOE
BES (DE-FG02-00-ER45841) for sample fabrication. A
portion of this work was performed at the National High
Magnetic Field Laboratory, which is supported by the NSF (Cooperative Agreement No. DMR-1157490), State of Florida, and DOE. We thank S. Hannahs, G. E. Jones, T. P. Murphy, E. Palm, and J. H. Park for technical assistance, and J. K. Jain for illuminating discussions.

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