I. INTRODUCTION

Magnetotransport properties of mesoscopic systems are sensitive to interferences between electronic waves and exhibit characteristic signatures of phase coherence. Among them are universal conductance fluctuations (UCF) leading to reproducible sample specific magnetoresistance patterns, which in a ring geometry are modulated by the flux periodic Aharonov-Bohm (AB) oscillations. Besides these effects on the linear conductance, it has been shown that mesoscopic systems exhibit rectifying properties related to the absence of spatial inversion symmetry of the disorder or confining potential. This gives rise to a quadratic term in the I-V relation:

\[ I = G_1 V + G_2 V^2 \]  

This nonlinearity was predicted theoretically and observed experimentally more than 15 years ago. It was understood as a direct consequence of the sensitivity of conductance fluctuations to the Fermi energy with a characteristic scale given by the Thouless energy

\[ E_T = \frac{h}{\tau_D} \]  

where \( \tau_D = L^2/D \) is the diffusion time across the sample of size \( L \). More recently, it has been pointed out that nonlinear transport coefficients do not obey Casimir-Onsager symmetry rules and may include, in a two-wire measurement, a component antisymmetric in magnetic field with a linear dependence at low field. On the experimental side, the existence of a term linear both in field and current was found in macroscopic helical structures and attributed to magnetic self-inductance effects. It was subsequently observed in carbon nanotubes and suggested to be related to their helical structure. More generally, this field asymmetry of nonlinear transport has been theoretically shown to be related to electron-electron interactions both in chaotic and diffusive systems. At the single impurity level, this effect can be simply viewed as the modification of the electron density \( dn(\vec{r}) \) around a scatterer in the presence of a current through the sample. 

Due to Coulomb interactions, this results in a modification of the potential around the impurity by a component linear in current. In a phase-coherent sample, this bias induced change of disorder potential \( dU_{dis} \) modifies the conductance fluctuations, giving rise to the nonlinear conductance \( G_2 \) defined as

\[ G(V) = G_1 + G_2 V + \cdots \]  

Just like the chemical potential measured in a multiprobe transport measurement, \( dU_{dis} \) exhibits field-dependent fluctuations, which have both symmetric and antisymmetric parts, including a component linear in \( B \) at low field. As a result, \( G_2 \) has a symmetric component in \( B \) and an antisymmetric one, respectively, equal to

\[ G_{2AS} = \frac{1}{2} [G_2(B) \pm G_2(-B)] / 2 \]

which both vary on the flux scale \( \Phi_c \) characteristic of conductance fluctuations. The antisymmetric component only exists in the presence of electron-electron interactions. The typical amplitudes of these components have been calculated in a two-dimensional (2D) system and yield the following for weak interactions:

\[ \delta G_2 \sim \frac{1}{2} \left( \frac{e}{E_c} \right), \quad \delta G_{2AS} \sim \frac{\gamma_{int}}{g} f \left( \frac{\Phi}{\Phi_c} \right) \delta G_2, \]  

where the function \( f(x) \) is equal to \( x \) for \( 0 < x \leq 1 \) and to \( 1 \) for \( x > 1 \). \( g = \langle G_1 \rangle \) and \( \langle G_1 \rangle \approx 1 \) are the average conductance and the typical amplitude of \( G_1 \) fluctuations in units of \( e^2/h \). In the weak interaction regime investigated in Ref. 11, the interaction constant is determined by the ratio of typical charging energy of the sample, \( \sim e^2/C \), and mean level spacing \( \Delta \) as

\[ \gamma_{int} = 1/(1 + C/2\Delta) \]  

[13] The limit \( \gamma_{int} \ll 1 \) of Eq. (1) corresponds to \( dU/dn \approx e^2/C \). The \( 1/g \) factor in Eq. (1) indicates that the field asymmetry should be easily detectable in low conductance samples, where \( g \) does not exceed 10, but is not observable in metallic mesoscopic systems.
guish the effects due to the tube helicity from mesoscopic ones. Semiconducting samples are well suited for such investigations, since they combine rather low conductance and large sensitivity to small fluxes. We have measured the nonlinear conductance of two terminal GaAs/GaAlAs rings with only a few conducting channels. We find that $G_2$, like $G_1$, exhibits both AB oscillations and UCF conductance fluctuations. We show evidence of the existence of an asymmetry in magnetic field in $G_2$ from which we deduce the amplitude of the electron interactions.

II. EXPERIMENTAL RESULTS

The square rings investigated in this experiment were obtained by shallow etching through an aluminum mask of a 2D electron gas (2DEG) of density $n_e=3.8 \times 10^{15}$ m$^{-2}$ at the interface of a GaAs/GaAlAs heterojunction with Si donors. Contrary to previous experiments, there is no electrostatic gate on the samples. Due to depletion effects, the real width of the rings is smaller than the etched one and is determined from magnetotransport data. We present data on two rings of circumference $L=4.8$ $\mu$m and respective widths $W=0.3$ and $0.45 \pm 0.05$ $\mu$m. The elastic mean free path $l_e$ extracted from the conductance at 4 K varies between 1 and 2 $\mu$m, which is less than the value of the initial 2DEG and comparable with the side of the square ring. In situ modifications of the samples were obtained by short illuminations with an electroluminescent diode resulting in an increase of width and conductance of the rings. It was also possible to change the disorder configuration by applying current pulses in the 10–50 $\mu$A range, which decrease the average conductance of the samples. Therefore, with only two samples, we could investigate a conductance ranging from $g=1$ to 20. The measurements were conducted via filtered lines in a dilution refrigerator between 25 mK and 1.2 K. The samples were biased with ac current of frequency $\omega/2\pi=30$ Hz in the few nanoampere range, and voltage was measured with a low noise amplifier followed by lock-in amplifiers detecting the first- and second-harmonic responses $V_1 \cos \omega t$ and $V_2 \cos 2\omega t$. The amplitude of the ac modulation was chosen to maximize the second-harmonic signal in the regime where it is still quadratic with the current modulation amplitude. In this region, the second-order conductance $G_2$ is simply related to $V_2$ and $V_1$ by $G_2=-2V_2/\bar{V}_1^2$. As shown in Fig. 1, both $G_1(B)$ and $G_2(B)$ exhibit $h/e$ periodic AB oscillations modulated by UCF fluctuations.

More remarkable, whereas $G_1(B)$ is a symmetric function of magnetic field as expected in a two-wire configuration, $G_2(B)$ exhibits a component antisymmetric in field $G_2^{\text{AS}}$. To compare the field asymmetry of $G_2$ on the various samples (Fig. 2), we extract the amplitude of UCF and AB oscillations from the Fourier transform of $G_2^{\text{AS}}(B)$ and $G_1(B)$. The integrated UCF and AB peaks are noted as $\delta G_2^{\text{AS}}$ and $\delta G_1^{\text{AS}}$, respectively. These averages performed on a flux range much larger than $\Phi_0$ do not depend on this range. We find that all these quantities are aligned on logarithmic plots as a function of $g$. We first note the relatively large amplitudes of the AB oscillations $\delta G_1^{\text{AS}}$, which are of the order of

![Figure 1](image1.png)

**FIG. 1.** (Color online) Field dependence of $G_1^{\text{AS}}$ and $G_2^{\text{AS}} (G_1^{\text{AS}}$ shifted). These data were obtained on ring 1 in its initial state for a current of 10 nA. Inset: micrograph of ring 1.

![Figure 2](image2.png)

**FIG. 2.** (Color online) (a) Amplitudes of the UCF and AB components of $G_1$ in units of $e^2/h$, and $G_2$ in $(\Omega$ V$)^{-1}$ as a function of the conductance of the rings. The open and closed symbols correspond to UCF and AB integrated peaks (straight lines are guides to the eyes). (b) Fourier transforms of $G_2$ and $G_1$ renormalized so that the amplitude of the AB peaks are identical. (c) Conductance dependence of the ratio $\delta G_2^{\text{AS}}/\delta G_1^{\text{AS}}$ for UCF (open stars) in comparison with Eq. (2) (diamonds), where $\gamma_{\text{int}}=0.85$ for ring 1 and 0.94 for ring 2.
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Fig. 3. (Color online) (a) Temperature dependence of the amplitude of the AB oscillations in $G_1$, $G_2^S$, and $G_2^{AS}$ (ring 2). Continuous lines are the fits corresponding to the regions (1) $k_b T \ll E_c$ and (2) $k_b T \gg E_c$ but $L \ll L_\phi$, (3) $L \gg L_\phi$. (b) Examples of temperature dependence of $G_2$ in ring 2 for different values of the magnetic field between $-1000$ and $1000$ G.

Fig. 4. (Color online) (a) Low-field dependence of $G_2$ on ring 1 for different temperatures. (b) Field dependence of $G_2^{AS}$ after low-pass (light line) and high-pass (bold line) filtering on ring 1 and ring 2, respectively, lower and upper curves (shifted for clarity). Note the extinction of AB oscillations in the vicinity of $B=0$.

The temperature dependence of $\delta G_1$ and $\delta G_2^{AS}$ are shown on Fig. 3(a). As already observed, $\delta G_1$ is only weakly $T$ dependent below the Thouless energy such as $1-AT^2$, with $A=2(k_b/E_c)^2$, and decays at higher temperature as $T^{-1/2}$. Deviations above $0.5$ K are consistent with $\exp(-L/L_\phi(T))$, with a $T$ dependence of the phase-coherence length as $T^{-1/2}$, but this last fit on a small range of temperature is not unique and just indicative. $\delta G_2^{S,AS}$ have similar $T$ dependences, which are nearly identical to $(\delta G_1(T))^2$ with a $T^{-1}$ decay in the limit $k_b T \gg E_c$ and $L \ll L_\phi$ in agreement with theoretical predictions. The temperature dependence of $G_2$ at fixed magnetic fields is depicted in Fig. 3(b). In the same way as observed on $G_1$, $G_2(T)$ exhibits a nonmonotonous variation with temperature on the scale $E_c$, which randomly fluctuates with magnetic field. In some cases, we also observed [see Fig. 4(a)] that the phase of the AB oscillations in $G_2$ depends on temperature. Surprisingly, it remains pinned either to 0 or $\pi$ at zero field with the appearance of a second-harmonic contribution in the region of temperature where the sign change occurs. At larger fields, the phase takes any value between 0 and $\pi$. Note that this last effect is observed both in $G_1$ and $G_2$ as a result of phase modulation of the AB oscillations by the UCF, but these phase modulations are symmetric in field on $G_1$ and not on $G_2$. In short, we find that AB oscillations on $G_2$ are symmetric in zero field and the asymmetry only appears at higher fields. This is also clearly seen in the AB oscillations on $G_2^{AS}$. We observe in all samples that their amplitudes vanish linearly at zero field [Fig. 4(b)].
III. MESOSCOPIC RECTIFICATION FROM SEMIClassICS

We now propose a simple explanation of the extinction of AB oscillations at low fields in $G_2^S$ linked to the larger AB component observed in $G_2$ compared to $G_1$. In the semiclassical approximation ($k_f l_s \gg 1$), it is possible to express the conductance of a phase-coherent ring in terms of interference between scattering amplitudes of electronic waves. It takes the following form at zero temperature:

$$G_Q = \sum_{i,j} A_i A_j \cos(\phi_{ij}) \cos \left( \frac{2\pi BS_{ij}}{\Phi_0} \right).$$

(3)

In the presence of a current through the sample associated with a potential drop $V$, the local electronic density and, consequently, the scattering potential are modified with a term $d U_{dis} = -V_{dis} r dt$, which induces phase shifts, $d\phi_{ij} = \int_{-r}^{r} dU_{dis}(r(t)) dt$. The quantity $G_2^S = dG(V)/dV|_{V=0}$ is then directly related to these phase shifts through

$$G_2^S = \sum_{i,j} A_i A_j \left( \frac{d\phi_{ij}}{dV} \right) \sin(\phi_{ij}) \cos \left( \frac{2\pi BS_{ij}}{\Phi_0} \right).$$

(4)

Since $d\phi_{ij}$ increases with the length of the interfering trajectories $i$ and $j$, long trajectories enclosing the ring (AB oscillations) contribute more than the trajectories within the same branch of the ring (UCF). This explains the larger relative amplitude of the AB oscillations and conductance fluctuations. This asymmetry is characterized by $\delta G_2^S / \delta G_2$ and analyzed within theoretical predictions expressing this ratio with only two parameters, namely, the dimensionless conductance of the rings and the interaction constant whose value can be determined as $\gamma_{int} = 0.90 \pm 0.05$. We have also found that the relative amplitude of the AB oscillations compared to the UCF is much larger in $G_2$ than in $G_1$, with the existence of a linear low-field modulation in the AB oscillations in the antisymmetric component of $G_2$. These effects can be understood within a simple semiclassical description of quantum interference. Recently, it has come to our attention that related experimental work on gated quantum dots and small Aharonov-Bohm rings have appeared.

IV. CONCLUSION

In conclusion, we have shown evidence of a field asymmetry on the second-order response of GaAs/GaAlAs rings of mesoscopic origin, which contains both AB oscillations and conductance fluctuations. This asymmetry is characterized by $\delta G_2^S / \delta G_2$ and was analyzed within theoretical predictions expressing this ratio with only two parameters, namely, the dimensionless conductance of the rings and the interaction constant whose value can be determined as $\gamma_{int} = 0.90 \pm 0.05$. We have also found that the relative amplitude of the AB oscillations compared to the UCF is much larger in $G_2$ than in $G_1$, with the existence of a linear low-field modulation in the AB oscillations in the antisymmetric component of $G_2$. These effects can be understood within a simple semiclassical description of quantum interference. Recently, it has come to our attention that related experimental work on gated quantum dots and small Aharonov-Bohm rings have appeared.

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