A Fault-Tolerant Superconducting Associative Memory

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The demand for high-density data storage with ultrafast accessibility motivates the search for new memory implementations. Ideally such storage devices should be robust to input error and to unreliability of individual elements; furthermore information should be addressed by its content rather than by its location. Here we present a concept for an associative memory whose key component is a superconducting array with natural multiconnectivity. Its intrinsic redundancy is crucial for the content-addressability of the resulting storage device and also leads to parallel image retrieval. Because patterns are stored nonlocally both physically and logically in the proposed device, information access and retrieval are fault-tolerant. This superconducting memory should exhibit picosecond single-bit acquisition times with negligible energy dissipation during switching and multiple non-destructive read-outs of the stored data.

The key component of our proposed associative memory is a superconducting array with multiple interconnections (Figure 1), where each bit is represented by a wire and thus is physically delocalized. More specifically this network consists of a stack of two perpendicular sets of $N$ parallel wires separated by a thin oxide layer.\textsuperscript{1,2} At low temperatures a superconductor-insulator-superconductor layered structure, known as a Josephson junction,\textsuperscript{3,4} exists at each node of this array; logically each pattern in our proposed memory is stored nonlocally in these $N^2$ interconnections. We note that in this network each horizontal/vertical wire is coupled to each vertical/horizontal one by a Josephson junction so that in the thermodynamic
limit \( N \to \infty \) for fixed area) the number of neighbors diverges.

The important energy-scales of this long-range array are those associated with the superconducting wires and with the Josephson junctions. Each superconducting wire is characterized by a macroscopic phase which is constant in equilibrium; here we assume that phase slips in each wire are energetically unfavorable. Application of a magnetic field results in the rotation of this phase, where the rotation rate is determined by the amplitude of the applied field. The interaction energy of a Josephson junction is minimized when the phase difference across its insulating layer is zero. In the absence of a field this condition is satisfied at each junction of the array. However application of a field transverse to the network results in an overconstrained system since the \( 2N \) phases and the \( N^2 \) Josephson junctions have competing energetic requirements. The identification of the ground-state in such a system is a hard combinatorial optimization problem as the number of metastable states scales exponentially with the number of wires, \( N \).

Because of its high-connectivity, the long-range Josephson array is accessible to analytic treatment; furthermore it can be fabricated and studied in the laboratory. A detailed theoretical characterization of this network has been performed. At low temperatures the system has an extensive number of states, \( N_{\text{states}} \sim e^{cN} \) where \( c \sim O(1) \), separated by free energy barriers that scale with the number of wires, \( N \). Its specific low-temperature configuration is determined by sample history, a feature also observed in glassy materials. Experiments have confirmed predicted static properties of this multi-connected array, though detailed dynamical investigations in the laboratory remain to be performed.

The proposed superconducting network (Figure 1) has long-range temporal correlations (memory) and an extensive number of metastable states, and thus it is natural to explore its possible use for information storage. Indeed high-connectivity and nonlinear elements (e.g. Josephson junctions) are key features required for the construction of associative memories. Here one would like to store \( p \) patterns in such a way that if the memory is exposed to another one \( (\xi_i) \) with a significant \( \geq \frac{1}{\sqrt{N}} \) overlap with a stored image \( (\tilde{\xi}_i) \), \( q = \frac{1}{N} \sum_i \xi_i \tilde{\xi}_i \), then it produces \( \tilde{\xi} \). A simple model for such a memory is based on an array of McCulloch-Pitts
neurons (Figure 2). The patterns are stored in the couplings, $J_{ij}$. Each nonlinear element has multiple inputs, and the output is a nonlinear function of the weighted sum of the inputs. The McCulloch-Pitts network, with inputs $n_i \in \{0, 1\}$, can be reformulated as a spin model where $\xi_i = 2n_i - 1$; then the output is

$$\xi_i = \text{sgn}\left( \sum_j J_{ij} \xi_j \right)$$

(1)

where

$$\text{sgn}(x) = \begin{cases} +1 & x \geq 0 \\ -1 & x \leq 0 \end{cases}$$

(2)

and the couplings $J_{ij}$ can have arbitrary sign. Clearly the output is robust to errors in the input due to the multiple connections present.

In order to ensure that the McCulloch-Pitts array is content-addressable, the couplings must be chosen so that the stored images correspond to stable configurations of the network. Hopfield has proposed an algorithm\textsuperscript{12} where the desired patterns are local minima of an energy function, e.g. $H = -\frac{1}{2} \sum_{ij} J_{ij} S_i S_j$ where $S_i \in \{-1, +1\}$. The couplings are chosen so that the energy is minimized for maximal overlap of $S_i$, the array configuration, and the desired output, $\xi_i$. For one pattern, this condition is satisfied for $J_{ij} = \frac{1}{N} \xi_i \xi_j$ where $N$ is the number of elements in the array; then $H = -\frac{1}{2N} (\sum_i S_i \xi_i)^2$. We note that with this choice of weights $J_{ij}$ the output is

$$\xi_i = \text{sgn}\left( \sum_{j=1}^{N} J_{ij} \xi_j \right)$$

(3)

for all $i$ since $\xi_j^2 = 1$ where $\xi_j$ and $\xi_i$ are the desired inputs and outputs respectively. We note that, according to (3) with the discussed choice of $J_{ij}$, a real input $S_j \approx \xi_j$ yields the desired output if it has errors in less than half its bits. In the Hopfield model, the couplings associated with several stored images are simple superpositions of the one-pattern case such that

$$J_{ij} = \frac{1}{N} \sum_{\mu=1}^{p} \xi_{i}^{\mu} \xi_{j}^{\mu}$$

(4)
where \( \mu \) labels each pattern. The total storage capacity of the network, \( p_{\text{max}} \), is dependent on the acceptable error rate; in general \( p_{\text{max}} = \alpha N \) where \( \alpha \leq 0.138 \) if the probability of an erroneous bit in each pattern is \( P_{\text{error}} < 0.01 \). Here we discuss the Hopfield algorithm because of its simplicity, but we note that other more efficient algorithms can also be implemented in this network.

The long-range Josephson array (Fig. 1) can be adapted to become a superconducting analogue of a McCulloch-Pitts network. It is described by the Hamiltonian

\[
\mathcal{H} = \text{Re} \sum_{j\bar{j}} S_j^* J_{j\bar{j}} S_{\bar{j}}
\]

with \( 1 \leq (j, \bar{j}) \leq N \) where \( j \) and \( \bar{j} \) are the indices of the horizontal and vertical wires respectively; \( S_j \) are effective complex spins with unit amplitude, \( S_j = e^{i\phi_j} \) where \( \phi_j \) is the phase of the \( j \)-th superconducting wire. The couplings are site-dependent and are related to the enclosed flux, \( \Phi_{j\bar{j}} \), in a given area whose edges are defined by wires \( j \) and \( \bar{j} \) such that

\[
J_{j\bar{j}} = \frac{J}{\sqrt{N}} \exp \frac{2\pi i \Phi_{j\bar{j}}}{\Phi_0}. (6)
\]

where \( \Phi_0 \) is the flux quantum. For a uniform magnetic field \( H \), \( \Phi_{j\bar{j}} = H(\bar{j}l^2) \) where \( l \) is the interwire spacing. We emphasize that the sign of \( J_{j\bar{j}} \) in (6) can be both positive and negative depending on the value of \( \frac{\Phi_{j\bar{j}}}{\Phi_0} \). In complete analogy with our previous discussion of the McCulloch-Pitts network, patterns are stored in this superconducting associative memory by appropriate choice of the weights, \( J_{j\bar{j}} \). Because the \( J_{j\bar{j}} \) are functions of the enclosed fluxes, \( \Phi_{j\bar{j}} \), they can be set to their desired values by appropriately tuning the local applied field \( H_{j\bar{j}} \).

In practice this writing procedure could be accomplished by an array of superconducting quantum interference devices (SQUIDs) superimposed on the multi-connected Josephson network.

The stored patterns are encoded in the Josephson couplings of the long-range array and correspond to stable configurations of the \( 2N \) superconducting phases. A “fingerprint” of each image can then be determined using voltage pulses and the Josephson phase-voltage relation \( \Delta \phi = \frac{2\pi}{\Phi_0} \int V dt \) where \( \Delta \phi \) is the phase difference across the relevant junction.
More specifically a set of voltage pulses can be applied to a small subset (> \( \sqrt{N} \)) of the horizontal wires, a “key”, thereby altering the phase differences at the associated nodes. The phases of the vertical wires must readjust in order for the system to settle into a stable configuration, a process which results in the absence/presence of a voltage pulse. The set of key input and \( N \) output voltage pulses therefore constitutes a signature of each stored image. Single-flux quantum (SFQ) voltage pulses, where \( \int V \, dt = \Phi_0 \), may be used for direct analogy with the McCulloch-Pitts array where inputs \( n_j \in \{0, 1\} \) now refer to the absence/presence of a SFQ pulse. Again it is convenient to describe the network in terms of the variables \( \xi_i = 2n_i - 1 \). Following the Hopfield algorithm, the local coupling associated with one pattern is \( J_{jj} = \frac{1}{N} \xi_j \xi_j \) in accordance with (3) so the desired output \( \xi_j = J_{jj} S_j \) is robust for \( S_j \approx \xi_j \). The weights coding many stored patterns are superpositions, (4), of the one-pattern cases. From a practical standpoint, these couplings are “written” by local (uniform) fields applied to individual plaquettes of area \( l^2 \); for a cell with its lower left-hand corner defined by the Cartesian coordinates \( (j, \bar{j}) \), the plaquette flux, \( \Phi_{\text{plaquette}}^{jj} \), is related to the weights by the expression

\[
\Phi_{\text{plaquette}}^{jj} = \frac{\Phi_0}{2} \Theta \left\{ -J_{jj} J_{j+1j} J_{j+1j+1} J_{j+1j+1} \right\}
\]

(7)

where \( \Theta(x) = 1 \) if \( x \geq 0 \) and \( \Theta(x) = 0 \) otherwise. Again we note that other algorithms can be used to determine the couplings; here we use the Hopfield model as an illustrative example.

In the proposed superconducting memory, each stored image is coded by a distinct set of superconducting phases associated with weak supercurrents and negligible induced fields. In conventional superconducting memories/logic, digital information is stored locally in trapped magnetic fluxes that are switched between SFQ states, \( \Phi \in \{0, \Phi_0\} \), by Josephson junctions; therefore the associated supercurrents should be large and can lead to unwanted crosstalk between adjacent elements. However in order to maintain their advantage in speed compared to other memory technologies, Josephson junction devices must use SFQ for both information storage and retrieval. This condition is satisfied by the design
of our proposed memory where the Josephson junctions switch fluxoids while the applied magnetic fluxes remain fixed; it is not the local fields but the supercurrents that store the information.

The fault-tolerance of the long-range Josephson network discussed here is due to the non-local nature of its data storage both at the physical and the logical levels. In conventional planar superconducting arrays there are $O(N^2)$ individual short superconducting wires and the fluxoids are spatially confined to areas $A \sim l^2$ where $l$ is the internode spacing. By contrast, in the multi-connected network the phases reside on the $2N$ wires of length $Nl$; thus the fluxoids here are extended to the entire system. Data is coded nonlocally in configurations of these superconducting phases, similar to the situation in an optical holographic storage device. There the stored patterns are independent of the input and an analogous superconducting holographic memory can be constructed. For example, let us consider the input wavefunction

$$\Psi^p_j = \exp 2\pi i \phi^p_j = \exp \frac{2\pi i j p}{N}$$

where $j$ and $p$ are indices labelling the horizontal wires and the stored patterns respectively. Then the input voltage pulses would be $\int V dt = \left[\frac{ip}{N}\right] \Phi_0$ where $\left[\quad\right]$ refers to the fractional part. Using the Hopfield algorithm, we have $J_{jj} = \frac{1}{N} \sum_p \xi^p_j \Psi^p_j$ which yields the desired output $\xi^p_j = \sum_j J^*_{jj} \Psi^p_j$. We note that any orthogonal basis for the inputs will work; therefore this long-range Josephson array can be used as a key component of both an associative and a holographic memory.

Practically, the proposed superconductor memory cell consists of the superimposed SQUID and long-range networks for writing and reading respectively, and a phase reset circuit. Each data retrieval event in the READ array must be followed by a reset operation since the output signals correspond to phase differences with respect to a reference state. This reset circuit can be constructed from a series of double-junction SQUID loops connected to each horizontal wire of the multi-connected array with a variable coupling to ground; if finite this coupling locks the relevant phase into a reference state, whereas if zero (e.g. in the
presence of a control line current) the next data retrieval process can be performed. This memory cell can then be embedded in an environment with known input/output SFQ circuitry that includes DC/SFQ voltage pulse converters and SFQ transmission/amplification lines. The network parameters, the charging ($E_c$) and the coupling ($E_J$) energy scales, should be chosen to optimize performance, particularly to maximize access rate, $\omega \sim \min(\Delta, \sqrt{E_c E_J})$ where $\Delta$ refers to the superconducting gap. An additional constraint on $E_C$ ($E_C \leq 0.01 E_J$) results from the condition that phase fluctuations remain weak and do not result in errors. Given these constraints, the minimal dissipation per bit ($\sim E_J$) is roughly $10\Delta$ which is $\sim 10^{-22}$ joule for aluminum in contrast to its value of $\sim 10^{-15}$ joule for conventional semiconducting electronics.

In summary, we have proposed an associative memory device that is a superconducting analogue of a McCulloch-Pitts network. It is content-addressable with a single-bit data accessible time, $\tau_A$, that is determined by the superconducting gap and the charging and coupling energy-scales in the network. Moreover because this memory is intrinsically parallel due to its crossbar design, an image of $N$ bits can be retrieved in a time (per bit) $\tau_{DT} \sim \frac{\tau_A}{N}$; by contrast $\tau_{DT} = \tau_A$ in a conventional local memory. For example, an array of $N = 1000$ wires with $l = 0.5\mu$ has a capacity of $C = 0.1N^2 = 10^5$ bits; a set of such arrays on a typical $1cm^2$ chip would then have a capacity of 1 Gigabyte with an image access time (per bit) of $\tau_{DT} = 10^{-15}$ seconds. The fault-tolerance of this superconducting memory enhances its appeal as a candidate for ultrafast high-density information storage without conventional problems of power dissipation and subsequent heat removal. As a point of comparison, we remark that this proposed device is faster but has lower absolute capacity than the best optical holographic memories; this is because the latter are intrinsically three-dimensional. We end by noting that we can tune the long-range array such that its stored images are maximally distant from each other in phase space. In this case the matrix elements associated with external noise will be negligible, and these patterns will have long decoherence times. Such orthogonal configurations could be promising as basis states for quantum computation.
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Figure 1. A schematic of the long-range superconducting array discussed in the text; the horizontal and vertical wires are coupled by Josephson junctions at each node and the vertical arrows refer to the local fields applied to individual plaquettes that determine the weights.
\[ \xi_i = \text{sgn} \left( \sum_j J_{ij} \xi_j \right) \]

**Figure 2.** Schematic diagrams of (a) a McCulloch-Pitts neuron (b) a McCulloch-Pitts network where \( \xi_i = \pm 1 \) and the \( J_{ij} \) can have arbitrary sign. The high-connectivity and the nonlinear elements in the McCulloch-Pitts array are crucial for its content-addressability and fault-tolerance.