Two-mode excited entangled coherent states and their entanglement properties

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We introduce two types of two-mode excited entangled coherent states (TMEECSs) $|\Psi_\pm(\alpha, m, n)\rangle$, study their entanglement characteristics, and investigate the influence of photon excitations on quantum entanglement. It is shown that for the state $|\Psi_+(\alpha, m, n)\rangle$ the two-mode photon excitations affect seriously entanglement character while the the state $|\Psi_-(\alpha, m, n)\rangle$ is always a maximally entangled state. We show how such states can be produced by using cavity QED and quantum measurements. It is found that the entanglement amount of the TMEECSs is larger than that of the single-mode excited entangled coherent states with the same photon excitation number.

I. INTRODUCTION

As it is well known that quantum entanglement has been viewed as an essential resource for quantum information processing, and creating and manipulating of entangled states are essential for quantum information applications. Among these applications are quantum computation [1], quantum teleportation, and quantum cryptography. In recent years, much attention has been paid to continuous variable quantum information processing [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20] in which continuous-variable-type entangled pure states of a light field [11] by using entangled two-mode squeezed vacuum states. Therefore, it is an interesting topic to create and apply continuous-variable-type entangled pure states. On the other hand, a coherent state is the simplest continuous-variable state. Based on coherent states, two types of continuous-variable states, called photon-added coherent states [21] and entangled coherent states, two types of continuous-variable states, have been introduced and shown to have wide applications in both quantum physics [22] and quantum information processing [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. In a previous paper [25], single-mode excited entangled coherent states (SMEECSs) are introduced. It has been shown that the SMEECSs form a type of cyclic representation of the Heisenberg-Weyl algebra and exhibit rich entanglement properties. The purpose of this paper is to propose the concept of two-mode excited entangled coherent states (TMEECSs), study their preparation and entanglement properties. This paper is organized as follows. In Sec. II, we present the definition of the TMEECSs and discuss their preparation. In Sec. III, we study entanglement character of the TMEECSs and compare them with the SMEECSs. We shall conclude this paper with discussions and remarks in the last section.

II. TWO-MODE EXCITED ENTANGLED COHERENT STATES AND THEIR PREPARATION

In this section we introduce the definition of the TMEECSs and present a possible scheme of producing them from the SMEECSs through atom-field interaction. Let us begin with the following two-mode entangled coherent states (ECSs)

$$|\Psi_\pm(\alpha, 0, 0)\rangle = A_\pm(\alpha, 0)|\alpha\rangle \pm |\alpha, -\alpha\rangle, (1)$$

where $|\alpha\rangle = 1^{|\alpha|}\sqrt{|\alpha|!}|\alpha\rangle$ with $|\alpha\rangle = D(\alpha)|0\rangle$ being the usual Glauber coherent state defined by the action of the displacement operator $D(\alpha) = \exp(\alpha \hat{a}^\dagger - \alpha^* \hat{a})$ upon the vacuum state $|0\rangle$. The normalization constants are given by

$$A_\pm^2(\alpha, 0) = 2 \left[ 1 \pm \exp(-4|\alpha|^2) \right]. (2)$$

For convenience, we denote the first and second modes in two-mode ECSs given by Eq. (1) by modes $a$ and $b$, respectively. Then for the ECSs we consider $m$-photon excitations in mode $a$ and $n$-photon excitations in mode $b$ with $m \neq 0$ and $n \neq 0$, respectively, and introduce the TMEECSs defined by

$$|\Psi_\pm(\alpha, m, n)\rangle = \mathcal{N}_\pm(\alpha, m, n)\hat{a}^m\hat{b}^n|\alpha, \alpha\rangle \pm |\alpha, -\alpha\rangle, (3)$$

where $\hat{a}^\dagger$ and $\hat{b}^\dagger$ are the creation operators of the modes $a$ and $b$, respectively. It is straightforward to calculate the normalization constants in Eq. (3) with the following expressions

$$\mathcal{N}_\pm^{-2}(\alpha, m, n) = 2mn! \left[ L_m(-|\alpha|^2)L_n(-|\alpha|^2) \pm e^{-4|\alpha|^2} L_m(|\alpha|^2)L_n(|\alpha|^2) \right], (4)$$

where $L_m(x)$ is a Laguerre polynomials of order $m$ defined by

$$L_m(x) = \sum_{n=0}^{m} \frac{(-1)^n n! x^n}{(n!)^2(m-n)!}. (5)$$
In the derivation of the Eq. (4) we have used the following expressions

\[ \langle \alpha | \hat{a}^m \hat{a}^\dagger^m | \alpha \rangle = m! L_m(-|\alpha|^2), \]
\[ \langle \alpha | \hat{a}^m \hat{a}^\dagger^m | - \alpha \rangle = m! e^{-2|\alpha|^2} L_m(|\alpha|^2), \quad (m \neq 0). \quad (6) \]

If we introduce the following normalized states with respect to mode \( a \) and mode \( b \)

\[ |a(\pm \alpha, m)\rangle = N(a, m) \hat{a}^\dagger^m | \pm \alpha \rangle, \]
\[ |b(\pm \alpha, m)\rangle = N(a, m) \hat{b}^\dagger^m | \pm \alpha \rangle, \quad (7) \]

We now present a scheme to produce them from the two-mode ECSs through atom-field interaction. Consider an interaction between a two-level atom with a cavity field. The atom makes a transition from the excited state \( |e\rangle \) to the ground state \( |g\rangle \) by emitting a photon. In the interaction picture, the Hamiltonian of resonant interaction is given by

\[ \hat{H} = \hat{\sigma}_+ \hat{b} + g^* \hat{\sigma}_- \hat{b}^\dagger, \quad (11) \]

where \( \hat{\sigma}_+ \) and \( \hat{\sigma}_- \) are Pauli operator corresponding to the two-level atom, \( \hat{b}^\dagger \) and \( \hat{b} \) are the creation and annihilation operator of the field mode \( b \), \( g \) is the coupling constant.

Suppose that the atom is initially in the excited state and the two field modes is initially in the SMECS with \( m \) photon excitations \( (m \neq 0) \) given by

\[ |\Psi_\pm(a, m)\rangle = N_\pm(a, m) \hat{a}^\dagger^m | \pm \alpha, -\alpha \rangle, \quad (12) \]

where the normalization constant is given by

\[ N^{-2}_\pm(a, m) = 2m! \left[ L_m(-|\alpha|^2) \pm e^{-2|\alpha|^2} L_m(|\alpha|^2) \right]^2, \quad (13) \]

where \( L_m(x) \) is \( m \)-th Laguerre polynomial defined in Eq. (5).

Suppose that only the mode \( b \) interacts with the atom. Then for the weak coupling case, the state of the atom-field system at time \( t \) can be approximated by

\[ |\psi(t)\rangle \approx |\Psi_\pm(a, m)\rangle \otimes |e\rangle - i \hat{H} t |\Psi_\pm(a, m)\rangle \otimes |e\rangle, \quad (14) \]

which is approximately valid for interaction times such that \( gt \ll 1 \). Making use of Eq. (11), one can reduce (14) to

\[ |\psi(t)\rangle \approx |\Psi_\pm(a, m)\rangle \otimes |e\rangle - i (g^* t) \hat{b}^\dagger |\Psi_\pm(a, m)\rangle \otimes |g\rangle, \quad (15) \]

which indicates that if the atom is detected to be in the ground state \( |g\rangle \), then after normalization the state of the two optical fields is reduced to the TMEECS \( |\Psi_\pm(a, m, n)\rangle \) given by Eq. (8). If we consider a succession of \( m \) atoms through the cavity and if we detect all the atoms in the ground state \( |g\rangle \), then the state of the two optical fields is reduced to the desired state, the TMEECS with \( (m + n) \)-photon excitations. Hence, we can, in principle, produce the TMEECS \( |\Psi_\pm(a, m, n)\rangle \).

\[ \mathcal{M}_\pm^2(a, m, n) = \frac{N^{-2}(a, m) L_m(|\alpha|^2) L_n(|\alpha|^2)}{2 \left[ L_m(-|\alpha|^2) L_n(-|\alpha|^2) \pm e^{-4|\alpha|^2} L_m(|\alpha|^2) L_n(|\alpha|^2) \right]^2}. \quad (10) \]

\[ C = \langle \Psi | \sigma_y \otimes \sigma_y | \Psi^* \rangle, \quad (16) \]

where \( |\Psi^*\rangle \) is the complex conjugate of \( |\Psi\rangle \). The concurrence equals one for a maximally entangled state.

For continuous-variables-type entangled states like (9), we consider a general bipartite entangled state

\[ |\psi\rangle = \mu |\eta\rangle \otimes |\gamma\rangle + \nu |\xi\rangle \otimes |\delta\rangle, \quad (17) \]

where \( |\eta\rangle \) and \( |\xi\rangle \) are normalized states of subsystem 1 and \( |\gamma\rangle \) and \( |\delta\rangle \) normalized states of subsystem 2 with complex \( \mu \) and \( \nu \). Through transforming continuous-variables-type components to discrete orthogonal basis and making use of a Schmidt decomposition (28), it is found the concurrence of the entangled state (17) to be
given by the following expression \[ C = \frac{2|\mu| |\nu| \sqrt{(1 - |p_1|^2)(1 - |p_2|^2)}}{|\mu|^2 + |\nu|^2 + 2\Re(\mu^* \nu p_1 p_2^*)}, \] where the two overlapping functions are defined by \[ p_1 = \langle \eta | \xi \rangle, \quad p_2 = \langle \delta | \gamma \rangle. \] From Eqs. (18) and (19) we find the corresponding concurrence to be \[ C_-(\alpha, 0) = 1, \quad C_+(\alpha, 0) = \frac{1 - e^{-4|\alpha|^2}}{1 + e^{-4|\alpha|^2}}, \] which implies that the degree of entanglement of the ECS \( |\Psi_-(\alpha, 0)\rangle \) is independent of the state parameter \( \alpha \), and it is a maximally entangled state while the amount of entanglement of the ECS \( |\Psi_+(\alpha, 0)\rangle \) is less than that of the ECS \( |\Psi_-(\alpha, 0)\rangle \). The concurrence \( C_+(\alpha, 0) \) increases with \( |\alpha|^2 \) and the state \( |\Psi_+(\alpha, 0)\rangle \) approaches the maximally entangled coherent state with \( C_+(\alpha, 0) \approx 1 \) for the strong filed case of the large \( |\alpha|^2 \).

When there exist photon excitations for both modes, i.e., \( m \neq 0 \) and \( n \neq 0 \), the TMEECSs are given by Eqs. (15) and (19). From Eqs. (18) and (19) we find the corresponding concurrence to be

\[ C_\pm(\alpha, m, n) = \frac{\sqrt{(1 - |p_1(\alpha, m)|^2)(1 - |p_2(\alpha, n)|^2)}}{1 \pm p_1(\alpha, m)p_2(\alpha, n)}, \] where the two overlapping functions are given by

\[ p_1(\alpha, m) = e^{-2|\alpha|^2} \frac{L_m(|\alpha|^2)}{L_m(-|\alpha|^2)}, \]
\[ p_2(\alpha, n) = e^{-2|\alpha|^2} \frac{L_n(|\alpha|^2)}{L_n(-|\alpha|^2)}. \]

From Eqs. (21) and (22) we can see that the TMEECSs exhibit the exchanging symmetry with respect to the exchange of two-mode photon excitations

\[ C_\pm(\alpha, m, n) = C_\pm(\alpha, n, m), \]

which implies that in the weak field regime of \( |\alpha|^2 \ll 1 \), we can get

\[ C_+(\alpha, 2m) = \sqrt{1 + 2m|\alpha|^2}, \]

which indicates that the photon excitation can enhance the entanglement amount for the SMEECs \( |\Psi_+(\alpha, 2m)\rangle \). In particular, when \( |\alpha|^2 \ll 1 \), we have \( C_+(\alpha, 2m) \ll 1 \). Therefore, in the weak field regime the TMEECS \( |\Psi_+(\alpha, m, m)\rangle \) exhibits similar entanglement character to that of the SMEECs \( |\Psi_+(\alpha, m)\rangle \).
Secondly, we consider the TMEECS $|\Psi_{-}(\alpha, m, m)\rangle$. In this case the corresponding concurrence is found to be

$$C_{-}(\alpha, m, m) = 1,$$

(30)

which implies that the TMEECS $|\Psi_{-}(\alpha, m, m)\rangle$ is always a maximally entangled state, and 2m photon excitations do not affect the entanglement amount of the state. This property is very different from that of the SMEECS $|\Psi_{-}(\alpha, 2m)\rangle$ defined in Eq. (26). The concurrence of the SMEECS $|\Psi_{-}(\alpha, 2m)\rangle$ is given by

$$C_{-}(\alpha, 2m) = \frac{1 - e^{-4|\alpha|^2}}{L_{2m}^2(|\alpha|^2) - e^{-4|\alpha|^2}L_{2m}^2(|\alpha|^2)} \left(1 - e^{-4|\alpha|^2}\right)^{1/2}.$$  

(31)

In the weak field regime of $|\alpha|^2 \ll 1$, we find that

$$C_{-}(\alpha, 2m) \approx \frac{1}{\sqrt{1 + 2m}}.$$

(32)

which indicates that the concurrence $C_{-}(\alpha, 2m)$ decreases with the increase of the photon excitation number $2m$. In particular, when $m \gg 1$ we have $C_{-}(\alpha, 2m) \ll 1$. Hence, the photon excitation suppresses the amount of entanglement for the the SMEECS $|\Psi_{-}(\alpha, 2m)\rangle$ in the weak field regime.

IV. CONCLUDING REMARKS

We have proposed two types of TMEECSs $|\Psi_{\pm}(\alpha, m, n)\rangle$, studied their entanglement characteristics, and investigated the influence of photon excitations on quantum entanglement. We have indicated it is possible to produce such states by using cavity QED and quantum measurements. It is found that the two TMEECSs $|\Psi_{\pm}(\alpha, m, n)\rangle$ exhibit quite different entanglement properties. In particular, for the state $|\Psi_{+}(\alpha, m, m)\rangle$ the two-mode photon excitations affect seriously entanglement character, and the entanglement amount decreases with the two-mode photon excitations in the weak field regime. However, the state $|\Psi_{-}(\alpha, m, m)\rangle$ is always a maximally entangled state, the two-mode photon excitations does not change the amount entanglement of the state. We have also made comparisons between the TMEECSs $|\Psi_{\pm}(\alpha, m, m)\rangle$ and the SMEECSs $|\Psi_{\pm}(\alpha, 2m)\rangle$. It has been shown that two-mode photon excitations have more advantages than single-mode photon excitations such as the entanglement amount of the TMEECSs $|\Psi_{\pm}(\alpha, m, m)\rangle$ is larger than that of the SMEECSs $|\Psi_{\pm}(\alpha, 2m)\rangle$ for the same photon excitation number $2m$. This approach of enhancing entanglement by using two-mode excitations of continuous-variable quantum states opens a new way to create new entanglement resources with continuous variables.

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