A Galileon Design of Slow Expansion: II

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We parameterize the evolutions of the slow expansion, which emerges from a static state in infinite past, into different classes. We show that the scale invariant adiabatical perturbation may be generated during these evolutions, and the corresponding evolutions can be realized with generalized Galileon Lagrangians, in which there is not the ghost instability.

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I. INTRODUCTION

How to apprehend the beginning of the hot “big bang” model has been still a significant issue. The inflationary scenario [1, 2, 3] is the current paradigm of the early universe, which not only homogenizes the universe but provides the scale invariant primordial perturbation responsible for the observable universe [4]. However, the universe still requires a beginning, since in inflationary scenario any backward null or timelike geodesic has a finite affine length [5].

The idea of the emergent universe is interesting [6, 7], in this scenario the universe originates from a static state in the infinite past. In emergent universe scenario, it seems that there might be not a beginning, since the affine length of backward null or timelike geodesic is classical infinite. However, for the model, in which the initial static state is constructed by applying a positive curvature, the static universe will inevitably collapse quantum mechanically [8, 9].

In Refs. [6, 7], after the universe emerges from the static state, the inflation will be required to set the initial conditions of “big bang” model, i.e. the homogenizing and the scale invariant primordial perturbation.

However, when the universe emerges, or begins to deviate from static state, it might be slowly expanding. In Ref. [10], it has been for the first time observed that the slow expansion might adiabatically generate the scale invariant curvature perturbation, see [11] for that induced by the entropy perturbation. Thus it might be imaginable that in emergent universe scenario, the initial state of “big bang” evolution could be set during this slowly expanding period. During the slow expansion, \( \epsilon < 0 \) is required [10, 12]. Thus in Ref. [10], the phantom has been applied for a phenomenological studying.

Recently, the cosmological application of Galileon, [13], or its nontrivial generalization [14, 15, 16], has acquired increasing attentions, e.g. see [14, 17] for dark energy, [15, 18] for inflation, [19] for curvaton and [20] for bouncing universe, in which \( \epsilon < 0 \) can be implemented stably, there is not the ghost instability.

The models of the emergent universe scenario can be realized with Galileon, e.g. [21], and also [22]. However, in Refs. [21, 22], the adiabatic perturbation is not scale invariant, thus the obtaining of the scale invariant curvature perturbation has to appeal to the conversion of the perturbations of other light scalar fields, i.e. the mechanism similar to that in Refs. [24, 25]. However, we showed that actually the scale invariant curvature perturbation may be adiabatically generated [22]. Here, with Ref. [2], we will make the relevant scenario clearer.

Here, we will parameterize the evolutions of the slow expansion, which emerges from a static state in infinite past, into different classes, and clarify how the adiabatical perturbation generated could be scale invariant for these parameterizations. When the slow expansion phase ends, the universe reheats and the evolution of “big bang” model begins. We have showed in Ref. [22] that one of these parameterizations can be realized by applying a generalized Galileon. Here, we will show that the other parameterization can also be realized similarly, and the results may be consistent with the observations.

II. AS A GENERAL RESULT

A. The parameterization of “emergence” or slow expansion

That in infinite past the universe is a static state, in which \( l_{\text{init}} > l_P \) is constant, \( l_P = 1/M_P \) and \( l_{\text{init}} \) is the physical size of initial universe, requires

\[
H \to 0, \text{ when } t \to -\infty.
\]

Thus the corresponding evolutions can be parameterized as

\[
H \sim \frac{1}{\beta^b(t_* - t)^b + 1},
\]

or\[ e^{-\beta(t_* - t)}, \]

where \( t < t_* \), and \( b > 0 \) and \( \beta \sim 1/|t_*| > 0 \) are constant. Thus in infinite past the behavior of \( H \to 0 \) is power law or is exponential. However, of course, the behavior of \( H \) could be also double exponential

\[
H \sim e^{-\beta(t_* - t)},
\]

or higher exponential.

Here, we will be only limited to (2) and (3). Thus \( a = e^{H dt} \) is given by

\[
a \sim e^{1/|\beta^b(t_* - t)^b|}, \text{ (parameterization I)}
\]

or\[ e^{-\beta(t_* - t)}, \text{ (parameterization II)},
\]
where $a_{inf} = 1$ is set in the infinite past. When $t_f \simeq \mathcal{O}(1) t_*$, $a \simeq e$ for (6) and $a \simeq e^{1/t}$ for (7). Thus in the regime $-\infty < t < t_f$, the universe is slowly expanding. Thus we may define the “emergence” as a period of slow expansion, which emerges from a static state in infinite past.

In Ref. [6], it has been showed that in a cosmological scenario, if $H_{average} > 0$, this scenario will be incomplete in the infinite past, since any backward null geodesic will have a finite affine length, i.e. a beginning is required. Here, $H_{average}$ is an average over the affine parameter, and is defined as

$$H_{average} = \frac{\int_{t_f}^{t_f} H(\lambda) d\lambda}{\int_{t_f}^{t_f} d\lambda} < \frac{1}{\int_{t_f}^{t_f} d\lambda}. \quad (8)$$

where $\lambda$ is the affine parameter of the backward null geodesic, $d\lambda = adt/a(t_f)$, which gives

$$\int_{t_f}^{t_f} H(\lambda) d\lambda = \int_{\alpha(t_f)}^{\alpha(t_\inf)} da < 1. \quad (9)$$

Eq. (8) is universal, independent of models. Thus with (9), $H_{average} > 0$ implies that the affine length $\int d\lambda$ must be finite.

We can calculate the affine length of the backward null geodesic for the slow expansion, which is parameterized as (6) or (7), which is

$$\int_{t_f}^{t_f} d\lambda = \int_{t_f}^{t_f} \frac{a}{a(t_f)} dt,
\sim \frac{e^{x_1}}{x_1^{1/6}} \bigg|_{0}^{x_1 f} + \int_{0}^{x_1 f} \frac{1}{x_1^{2+1/6}} e^{x_1} dx_1, \quad (10)$$

or $$\sim \frac{e^{x_2}}{x_2^{1/6}} \bigg|_{0}^{x_2 f} + \int_{0}^{x_2 f} \frac{1}{x_2^{2+1/6}} e^{x_2} dx_2, \quad (11)$$

where

$$x_1 = \frac{1}{\beta^b(t_* - t)^b}, \quad x_2 = e^{-\beta(t_* - t)}. \quad (12)$$

We see that both affine lengths are diverged, which indicates that the emergent universe scenario, parameterized as (6) or (7), could be complete in the infinite past, and there is not the beginning. When $b \gg 1$, the result of (11) is the same with that of (10), thus the affine length of the backward null geodesic in parameterization (6) is same with that in parameterization (7), which implies that (7) is actually the limit case of (6).

Here, the definition of $\epsilon$ is $\frac{d\epsilon}{dt}(1/H)$. Thus in the slowly expanding phase $\epsilon < 0$, since $H$ is rapidly increasing. In infinite past,

$$|\epsilon| \sim \beta^b(t_* - t)^b, \text{ for parameterization I,} \quad (13)$$

or $e^{\beta(t_* - t)}$, for parameterization II \quad (14)

is diverged. This is actually a result of the condition (11), i.e. in infinite past the universe is a static state, in which $l_{init} > l_P$ is constant.

What if $\epsilon$ is not divergent. When $|\epsilon| > 0$ tends to be constant in infinite past, we have

$$H \sim \frac{1}{|\epsilon| (t_* - t)} \rightarrow 0, \text{ in infinite past,} \quad (15)$$

which seems satisfy (11). However,

$$a \sim e^{\int H dt} \sim \frac{1}{(t_* - t)^{1/|\epsilon|}} \rightarrow 0 \quad (16)$$

in the meantime, which is not consistent with the requirement of $l_{init} > l_P$. In principle, $l_{init}$ should be larger than $l_P$, thus $a \rightarrow 0$ in infinite past in certain sense implies that the corresponding evolution requires a beginning.

When $|\epsilon| \rightarrow 0$ in infinite past, we have $b < 0$ for the parameterization (6), thus

$$a \sim e^{-b|\epsilon| (t_* - t)^{1/|\epsilon|}} \rightarrow 0, \text{ in infinite past,} \quad (17)$$

which is also not consistent with the requirement of $l_{init} > l_P$, as has been argued.

The parameterization (6) has been implemented by applying Galileon Lagrangians. In Refs. [12, 22], it has been showed that the adiabatic perturbation is scale invariant requires $b = 4$, i.e. $a$ behaviors as

$$a \sim e^{1/t - t/4}. \quad (18)$$

In Ref. [21], $a \sim e^{1/(t_* - t)}$, thus the adiabatic perturbation is not scale invariant.

While for the parameterization (7), $e^{-\beta(t_* - t)} \sim 0$ in infinite past i.e. $\beta(t_* - t) \gg 1$, thus we have

$$a \sim e^{-\beta(t_* - t)} \sim 1 + e^{\beta(t_* - t)} \quad (19)$$

which is just that in original emergent universe scenario [4, 5], in which the initial static state is constructed by introducing a positive curvature. In Ref. [6], initially $\beta(t_* - t) \gg 1$, the universe is slowly expanding from the static state, after the slowly expanding phase ends, a period of inflation is required, and the scale invariant adiabatic perturbation is generated during inflation. Here, we will show that the scale invariant adiabatic perturbation may be actually generated during the slow expansion without the help of inflation and the corresponding evolutions can be realized with generalized Galileon Lagrangians.

### B. How the adiabatic perturbation is scale invariant?

We will clarify how the adiabatic perturbation generated during the slow expansion, parameterized as (6) or (7), could be scale invariant. The quadratic action of the curvature perturbation $R$ is [24, 14, 15],

$$S_R \sim \int d\eta d^3 x \frac{a^2 M_P^2 Q}{c_s^2} \left( \frac{R^2 - c_s^2 (\partial R)^2}{c_s^2} \right), \quad (20)$$
where $Q > 0$ and $c_s^2 > 0$ should be satisfied for the avoidance of the ghost and gradient instabilities.

$Q = M_p^2 \phi$ for $P(X, \phi)$ \cite{27}, but $Q$ is complicated for Galileon \cite{14, 15}. However, as has been confirmed in Ref.\cite{22} and will be confirmed again here, $Q \sim |\epsilon|$ for the slow expansion with $\epsilon < 0$.

The equation of $R$ is

$$\ddot{u}'' + \left( k^2 - \frac{z''}{\bar{z}} \right) \dot{u} = 0, \quad (21)$$

after defining $\ddot{u}_k \equiv \ddot{z}R_k$, where $'$ is the derivative for $y = \int c_s \ddot{\eta}$, $\bar{z} \equiv a \sqrt{2M_\phi^2 Q/c_s}$. When $k^2 \ll \bar{z}''/\bar{z}$, the solution of $R$ given by Eq.\cite{21} is

$$R \sim C \ \text{is constant mode} \quad (22)$$

or

$$D \int \frac{dy}{\bar{z}'} \ \text{is changed mode}, \quad (23)$$

where $D$ mode is increasing or decaying dependent of the evolution of $\bar{z}$.

The scale invariance of $R$ requires $\frac{\bar{z}''}{\bar{z}} \sim \frac{2}{(y_s - y)}$, which implies

$$\bar{z} \sim a \sqrt{\frac{Q}{c_s}} \sim \frac{1}{y_s - y} \ \text{for constant mode} \quad (24)$$

or

$$(y_s - y)^2 \ \text{for increasing mode} \quad (25)$$

has to be satisfied. In principle, $a$, $Q$ and $c_s$ can be changed, all together contribute the change of $\bar{z}$. When $a$ is rapidly changed while others are hardly changed, the inflationary background and the contraction dominated by the matter \cite{27, 28} are obtained, respectively.

When $Q$ is rapidly changed while $a$ is hardly changed, the scale invariant adiabatical perturbation can be induced similarly by either its constant mode \cite{30}, also criticism for it \cite{31}, or its increasing mode \cite{12}.

We have showed that for the parameterization \cite{6}, the scale invariance of adiabatical perturbation requires \cite{12, 22},

$$a \sim e^\frac{\beta}{2}(t_s - t), \quad (26)$$

i.e. $|\epsilon| \sim (t_s - t)^4$, in which $c_s^2$ is constant.

Here, we will detailed clarify how to obtain scale invariant adiabatical perturbation for the parameterization \cite{7}.

The scale invariance of the increasing mode of $R$ requires

$$Q \sim \frac{c_s}{a^6} \left( \int c_s \ddot{\eta} \right)^4, \quad (27)$$

where $\ddot{\eta} \sim \dot{a}$, since $a$ is almost constant. Thus

$$c_s^2 \sim e^\frac{\beta}{2}(t_s - t), \quad (28)$$

where $Q \sim |\epsilon|$ and \cite{14} are applied. This implies that $c_s^2$ has to be infinite large in infinite past, and then rapidly decreased.

In Ref.\cite{22}, it has been observed that the rapid running of $c_s^2$ helps to obtain the scale invariant adiabatical perturbation in noninflationary background. In Refs.\cite{33, 34}, this issue is estimated again. In Ref.\cite{35}, it has been argued that the adiabatical perturbation in the model of the slow expansion with rapidly decreasing $c_s^2$ is not scale invariant. However, in their settings, the adiabatical perturbation is dominated by its constant mode, while here the perturbation is dominated by its increasing mode.

When $k^2 \gg \bar{z}''/\bar{z}$, the perturbation mode of $R$ leaves its horizon, which is called the $R$ horizon

$$\frac{a}{H_{freeze}} \sim \frac{\bar{z}'}{\bar{z}''} \sim \frac{1}{y_s - y}. \quad (29)$$

While the Hubble horizon is $1/H$. Thus the evolutions of the $R$ horizon and the Hubble horizon are different. We have

$$\frac{a}{H_{freeze}} \sim \left( \frac{1}{H} \right)^{1/5}. \quad (30)$$

In Ref.\cite{22}, it has been showed how the scale invariant adiabatical perturbation is obtained for \cite{6}, the result also implies \cite{30} is satisfied. The evolutions of both the $R$ horizon and the Hubble horizon are same only when $a$ is rapidly changed and $|\epsilon|$ is unchanged, e.g.inflation, since $z''/z \sim \alpha''/a$. When $k^2 \gg \bar{z}''/\bar{z}$, i.e. the perturbation is deep inside the $R$ horizon, $\ddot{u}_k$ oscillates with a constant amplitude.
The quantization of \( \tilde{u}_k \) is well defined since \( Q \sim |c| > 0 \), which gives its initial value.

\[
\tilde{u}_k \sim \frac{1}{\sqrt{2k}} e^{-ikc} dt. \tag{31}
\]

Thus initially

\[
k^{3/2}R = k^{3/2} \left| \frac{\tilde{u}_k}{z} \right| \sim k \sqrt{\frac{c_s}{Q}} \sim k e^{-\frac{3}{2} \beta (t_0 - t)} \to 0. \tag{32}
\]

This insures that initially the background of static state is not spoiled by the perturbations. Thus in slow expansion scenario \([10],[22],[29]\), the primordial perturbation must be induced by the increasing mode, or it is hardly possible to consistently define the initial perturbation in the infinite past.

When \( k^2 \ll \frac{2}{z} \), \( \tilde{u}_k \) is

\[
\tilde{u}_k = \sqrt{\frac{\pi}{2}} e^{it} \sqrt{-k \int c_s dt H(t_1)} \sim k e^{-\frac{3}{2} \beta (t_0 - t)}/Q. \tag{33}
\]

Thus the amplitude of perturbation spectrum is \( P_{R}^{1/2} \approx \sqrt{k^2} \left| \frac{\tilde{u}_k}{z} \right| \). The perturbation is given by the increasing mode \([23]\), which implies that the spectrum of \( R \) should be calculated around \( t_f \). Thus

\[
P_{R}^{1/2} \sim \frac{1}{M_p} \sqrt{\frac{c_s}{Q}} \sim \frac{\beta}{M_p}. \tag{34}
\]

The universe will reheat around or after \( t_f \), which has been studied in \([22]\). Hereafter, the evolution of hot “big bang” model begins, the perturbation mode outside of horizon will preserve constant.

Thus during the slow expansion, which emerge from a static state in infinite past, the initial conditions of hot “big bang” evolution could be set.

III. A GALILEON DESIGN OF SLOW EXPANSION:II

In Ref.\([22]\), we have showed that the parameterization \([3] \) can be realized in the effective field theory setting, in which there is not the ghost instability and the perturbation spectrum is consistent with the observations. Here, we will show the parameterization \([7] \) can be also realized similarly.

A. The background

We consider a generalized Galileon as follows

\[
\mathcal{L} \sim -\frac{\varphi^5}{M^5} X + \frac{1}{M^{10}} X^{7/2} - \frac{1}{M^7} X^2 \square \varphi, \tag{35}
\]

where the sign before \( \frac{\varphi^5}{M^5} X \) is ghostlike, however, as will be showed that there are not the ghost and gradient instabilities, since \( Q > 0 \) and \( c_s^2 > 0 \). The evolution of background is determined by

\[
-\frac{\varphi^5}{M^5} \ddot{\varphi} + 3 \left( \frac{7}{M^{10}} X^{5/2} + \frac{8}{M^7} H \dot{\varphi} X \right) \dot{\varphi} + 3 \left( -\frac{\varphi^5}{M^5} + \frac{7}{2M^{10}} X^{5/2} \right) \ddot{\varphi} + \left( -\frac{5\varphi^4}{M^5} + \frac{6H^2}{M^7} + \frac{18H^2 \varphi^2}{M^7} \right) X = 0, \tag{36}
\]

\[
3H^2 M_p^2 = -\frac{\varphi^5}{M^5} X + \frac{6}{M^{10}} X^{7/2} + \frac{6}{M^7} X^3 H. \tag{37}
\]

We require \( \frac{\varphi^5}{M^5} X \approx \frac{6X^{7/2}}{M^7} \) in Eq.\([37]\), which gives

\[
\varphi = \varphi_* e^{-\frac{\sqrt{X}}{\sqrt{2} M} M (t_0 - t)}. \tag{38}
\]

Thus

\[
H \sim \frac{\varphi^5}{M^5 M_p^3}, \tag{39}
\]

is induced. Thus for \( \beta = \frac{5\sqrt{X}}{M} \), Eq.\([7]\) is obtained, which is just required evolution.

We have \( H \dot{\varphi} \ll \dot{\varphi} \) for \( \beta (t_0 - t) \gg 1 \), since

\[
H \sim \frac{\varphi^5}{M^5} \sim e^{-\frac{5\sqrt{X}}{M} M (t_0 - t)} \ll e^{-\frac{5\sqrt{X}}{M} M (t_0 - t)}. \tag{40}
\]

Thus Eq.\([36]\) is approximately

\[
\left( -\frac{\varphi^5}{M^5} + \frac{21}{M^{10}} X^{5/2} \right) \ddot{\varphi} - \frac{5\varphi^4}{M^8} X \simeq 0, \tag{41}
\]

which is consistent with the solution \([38]\).

The numerical solutions of Eqs.\([36]\) and \([37]\) are plotted in Fig.2, which is consistent with \([39]\) and will be applied to the numerical calculations for the power spectrum of the adiabatical perturbation plotted in Fig.3.
B. The curvature perturbation

\( R \) satisfies Eq. (21). We follow the definitions and calculations in Ref. [13], and will only list the calculating results in term of the Lagrangian (35) with the solutions (39) and (38) of the background and field. The calculating steps are the same with section III.B in Ref. [22], which will be neglected here.

We have, for \( \beta(t_s - t) \gg 1, \)
\[
Q \approx \frac{4M^3 M_P^{4/5}}{\sqrt{2} \bar{\varphi}^2} \phi^5(t_s - t) \sim e^{\beta(t_s - t)}.
\]

Thus \( Q \sim |\epsilon| > 0 \), which is just required here, satisfies Eq. (14). The \( c_s^2 \) is given by
\[
c_s^2 \approx \frac{16}{\dot{q}_s^2} \frac{M^2}{(14\sqrt{2} - 3)} \frac{\phi^2}{\bar{\varphi}^2} \sim e^{2\beta(t_s - t)}.
\]

Thus \( c_s^2 > 0 \) is also just required, and satisfies Eq. (28). Thus there are not the ghost and gradient instabilities, the effective field theory is healthy.

Thus the spectrum of \( R \) is scale invariant. The amplitude of spectrum is given by Eq. (41)
\[
P_R^{1/2} \sim \frac{M}{M_P}.
\]

Thus \( P_R^{1/2} \sim 10^{-5} \) requires \( M \sim 10^{-5} M_P \), which implies that the only adjusted parameter in this model is set by the observation. There is not other finetuning.

The universe will reheat after the “emergence” or the slowly expanding phase ends. The reheating mechanism has been discussed in Refs. [22, 36]. Hereafter, the evolution of hot “big bang” model begins, the perturbation mode outside of horizon will preserve constant.

We can reformulate Eq. (21) as
\[
u_k'' + \left( c_s^2 k^2 - \frac{z''}{z} \right) u_k = 0,
\]
where \( \nu \) denotes the differential with conformal time, and \( u_k \equiv \dot{\nu}k \) and \( z \equiv a\sqrt{2M_P Q/c_6} \). We numerically solve Eq. (15) with the numerical solutions of Eqs. (36) and (37), and plot the evolution of the amplitude of the adiabatical perturbation in Fig. 3.

We can see that the perturbation spectrum is scale invariant, which is consistent with analytical result. The amplitude of perturbation is increasing with the time, however, only is its amplitudes increasing but the shape of the spectrum is not altered [37, 38].

IV. DISCUSSION

The idea of the emergent universe, i.e. the universe might originate from a static state in the infinite past,
obviously significant to find a Lagrangian with a better theoretical motivation, which is interesting for investigating.

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