NEUTRON STAR STRUCTURE

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Abstract.
A review of properties of matter in the interior of neutron stars is given. Particular attention is paid to recent many-body theory calculations of the properties of dense matter. Among topics discussed are the strong increase of tensor correlations at relatively low densities, the “relativistic boost term” in the interaction, and the sensitivity of properties of neutron star models to three-body forces.

1. Introduction

Before turning to theory, it is salutary to consider what information about neutron stars may be learned from observations. One basic quantity is the stellar mass, and an important recent development is the tightening of the mass estimates for neutron stars in binary systems by Thorsett and Chakrabarty for radio pulsars [1], and for Chakrabarty and Thorsett for accreting neutron stars [2]. In addition, as we shall hear from van der Klis later in this symposium [3], (see also Ref. [4]), an exciting possibility of obtaining limits on masses of neutron stars from measurements of the frequencies of kilohertz quasiperiodic oscillations has recently arisen. Thus information about masses of neutron stars in binaries is becoming increasingly precise, and provides important constraints on models of neutron stars.

While a knowledge of the equation of state is the only input required for determining the mass-radius relationship for neutron stars, and hence the range of physically allowable neutron star masses, there are a number of observational phenomena of neutron stars that depend on other properties of matter in stars, apart from the equation of state. Among these are the surface emission from neutron stars, which will be discussed later in this symposium by Caraveo [5] and Shearer [6], and glitches.

In this talk we shall discuss a number of aspects of neutron star behaviour, and will describe some recent calculations of the equation of state of neutron star matter and the neutron star models that these equations of state predict. To set the scene, we give in Fig. 1 an overview of the structure of a neutron star. At the lowest densities matter is similar to terrestrial matter. Electrons become increasingly free as
the density increases, and at a density of about $10^6$ g/cm$^3$ they become relativistic. The increasing electron Fermi energy makes it energetically favorable to reduce the number of electrons, and convert a corresponding number of protons into neutrons. Consequently with increasing depth in the star matter becomes more neutron rich, and at a density of about $4 \times 10^{11}$ g/cm$^3$ matter becomes so neutron rich that the most energetic neutron states in nuclei become unbound, and begin to occupy continuum states. This is neutron drip. As the density increases further, the density of neutrons between nuclei also increases, and eventually when the density of neutrons outside nuclei is roughly the same as that of the matter in nuclei, nuclei merge to form a uniform fluid consisting mainly of neutrons, with a few percent of protons, together with electrons to ensure the electrical neutrality of the matter. At densities just below that at which nuclei merge, one expects nuclei to be aspherical, having the form of extended rods and plates [7].

Above nuclear density the physics of matter is more uncertain, but the equation of state and composition of matter at these densities are central ingredients in constructing neutron star models and predicting their evolution.
At the lowest densities, properties of matter may be predicted directly on the basis of measured nuclear data, since up to just below neutron drip density the nuclei predicted to occur are ones accessible to laboratory study. For the higher density region up to just above nuclear density, the basic constituents of matter are neutrons and protons, together with the electrons. The interaction of two nucleons is rather well understood at the energies of relevance, and the system is sufficiently dilute that the methods of many-body theory may be used with confidence to predict the properties of matter. Above nuclear density the situation is less clear. Basic questions are what the fundamental constituents are, what their interactions are, and how the energy of dense matter can be calculated. These questions are not independent, since whether or not a particular constituent is present depends on what its energy is.

There are basically two sorts of approach to calculating the properties of dense matter. The first is to take an interaction between constituents which is as realistic as possible, and fits known scattering data, and to then use the techniques of many-body theory to calculate correlations in the matter, from which one can then evaluate the energy and the equation of state. The second is to adopt a schematic model, often of the mean field type, where the coupling strengths for the various interactions are treated as parameters to be fitted to a limited set of observable quantities. Both approaches have their advantages and disadvantages. As far as the many-body approach is concerned, the nucleon-nucleon interaction is well understood, and the methods of many-body theory are well developed. However, the approach is generally based on non-relativistic theory. Some of the schematic models are better able to incorporate the effects of relativity, and can be easily generalized to allow for the presence of many constituents. The disadvantage of the approach is that the complicated correlations in dense matter, which are very density dependent, are represented by a number of parameters giving the strengths of the various mean fields. Both approaches, however, are hampered by the paucity of relevant experimental data. While pairwise interactions of nucleons are well understood at low energy, three- and higher-body interactions of nucleons are not well characterized, and even information about two-body interactions involving constituents other than nucleons is very limited.

2. Basic considerations

In this talk we shall describe some recent calculations of the properties of dense matter from many-body theory. The wave function, $\Psi$, for the many-nucleon system, whether it be pure neutron matter or isospin-symmetric nuclear matter with equal numbers of neutrons and protons, is assumed to be of the Jastrow form,

$$\Psi = \prod_{i<j} f_{ij} \Psi_0,$$

where the indices $i$ and $j$ label particles, $\Psi_0$ is a Slater determinant for a gas of free nucleons, and $f_{ij}$ is a correlation factor, which depends on the distance between two particles, and other variables such as the spin and isospin of the interacting pair of nucleons. The procedure is to evaluate the energy of the system with the wave function Eq. (1).

Information about two-body nucleon nucleon interactions is derived from the Nijmegen data set for nucleon-nucleon scattering at energies below 300 MeV in the center of mass frame. This was fitted by Wiringa, Stoks and Schiavilla to a non-relativistic interaction, denoted by Argonne v18 (or simply A18), which allows for
isospin dependent effects, such as those coming from the mass difference between neutral and charged pions. The “18” denotes the number of operators in the effective two-body interaction.

It has long been known that the two-body interaction alone is insufficient to account for the binding of nuclear matter and of light nuclei. This discrepancy has been attributed to the presence of three-body forces, in which three nucleons interact simultaneously. This takes into account non-nucleonic degrees of freedom, which are not present in the wave function \( \psi \). One contribution to this interaction arises from two nucleons scattering, with one of the nucleons being excited to a \( \Delta \) resonance state, which subsequently de-excites by scattering from a third nucleon. Experimental information about the three-body force is sparse, and in the calculations it is represented by a simple expression which has the theoretically predicted behaviour at large distances but with a strength to be obtained by fitting to experiment, and a parametrized form for the short-range part. Parameters in the interaction are adjusted to account for the binding energy and density of light nuclei and nuclear matter.

A novel feature of the present set of calculations is the inclusion of the so-called “relativistic boost correction”. As we mentioned above, the methods of many-body theory are based on non-relativistic concepts, but it is possible to include the leading effects of relativity by appropriate modification of the effective interaction. Such a formalism is familiar in the context of electrodynamics, where it leads to the so-called Breit interaction between electrons. Within the context of the nuclear many-body problem its importance has been appreciated for more than a quarter of a century \( [10] \), but it has generally not been included in previous calculations \( [11] \).

The effective Hamiltonian thus has the form

\[
H_{NR} = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} \left( v_{ij} + \delta v^b_{ij} \right) + \sum_{i<j<k} v_{ijk},
\]

where \( v_{ijk} \) is the effective three-body interaction, and \( \delta v^b_{ij} \) is the relativistic boost correction. The latter has the form

\[
\delta v^b(P) = -\frac{P^2}{8m^2} v + \frac{1}{8m^2} \left[ P \cdot r \nabla + (\text{spin-dependent terms}) \right].
\]

In this expression \( P \) is the relative momentum operator for the pair of particles and the \( \nabla \) operates on the relative coordinate \( r \) for the two particles, and the two-body interaction is written simply as \( v \).

The most complex part of the calculations is the evaluation of the energy for the trial wave function \( \psi \). We shall not describe the technicalities here, but will go directly to a discussion of some of the salient features of the results.

First of all we consider results for light nuclei \( ^3\text{H} \) and the alpha particle, \( ^4\text{He} \). The boost correction to their energies amounts to almost 40% of the difference between the experimental energies and the theoretical ones calculated using the two-body interactions alone. Consequently, if the boost correction is included, the strength of the three-body interaction required to produce agreement between theory and experiment is about 40% less than that required if the boost correction is neglected.

3. Strong tensor correlations

For uniform matter, a striking discovery is a strong increase in tensor correlations at relatively low densities. For pure neutron matter this occurs at a density of \( \sim 0.2 \)
fm$^{-3}$, just above nuclear matter density ($\sim 0.16$ fm$^{-3}$), while for symmetric nuclear matter it occurs at a density of $\sim 0.32$ fm$^{-3}$. For pure neutron matter these tensor correlations reflect a tendency for the spin components perpendicular to the vector joining two neutrons to be aligned, while the components along the vector joining the spins tend to be anti-aligned. Such correlations arise from the tensor force, whose dependence on angle is given by $3\hat{r}_{12}\cdot\sigma_1\cdot\hat{r}_{12}\cdot\sigma_2 - \sigma_1\cdot\sigma_2$. If such correlations were of long range, neutron matter would have a neutron spin density wave, or in other words it would be a layered antiferromagnet. Such a state, if described in terms of pionic degrees of freedom, would be a Bose condensation of neutral pions.

This result is noteworthy because for the past two decades, the accepted opinion has been that pion condensation was ruled out by strong repulsive central correlations which suppressed the tensor ones. However, in view of the recent results the possibility of pion condensation should be re-examined. The presence of the stronger pionic correlations results in the energy of dense matter rising less rapidly with increasing density than one would anticipate on the basis of calculations in which the tensor correlations are assumed to be insensitive to density.

Next we consider how predictions for the energy of dense matter are influenced by the inclusion of the boost correction. This is exhibited in Fig.(2). If the boost 

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**Figure 2.** Energy per baryon of pure nuclear matter as a function of density for various nuclear models: A18 is the Argonne v18 two-body interaction; $\delta vb$ is the relativistic boost correction to the energy; UIX refers to the 3-body interaction with parameters determined by fits to light nuclei and nuclear matter in the absence of the boost correction, while UIX$^*$ is the corresponding interaction fitted with the boost correction included.
correction is included, with a three-body interaction adjusted to give a fit to the properties of light nuclei and nuclear matter, the energy rises less rapidly than if one neglects the boost correction and uses a correspondingly stronger three-body interaction so that the properties of observed nuclei are reproduced. The reason for this effect is that the contribution to the energy of matter coming from the boost correction increases less rapidly with density than does that coming from three-body interactions. At a density of \( \sim 0.8 \text{ fm}^{-3} \) the contribution from three-body interactions is reduced by about a factor of two if the boost correction is included. In view of the uncertainty of the form of the three-body interaction, this reduced sensitivity of results to the three-body interaction is encouraging.

It is interesting to compare the results of calculations based on variational methods with ones based on Brueckner theory [12]. Within the Brueckner approach it is difficult to evaluate higher order correlations, and most calculations are performed at the two-body level. Such comparisons have been made neglecting the boost correction and three-body terms in the interaction, and for neutron matter one finds remarkably good agreement between the two approaches. This is reassuring, because it indicates that the effects of high-order correlations is not overwhelming, probably due in part to the fact that for pure neutron matter, the Pauli principle prohibits three particles from being at the same point in space. For symmetric nuclear matter short-range correlations are not suppressed by the Pauli principle, and the variational and Brueckner results differ increasingly for densities above a few times nuclear density [13]. A further point to notice that there are a number of different two-nucleon interactions based on the Nijmegen data set, but where comparisons have been made, results do not depend on which particular form of the interaction one uses, so long as it fits the Nijmegen data.

4. Other constituents

The next topic we address is the possibility of other constituents being present in matter. We have already mentioned pion condensates, and among the others are a kaon condensate, hyperons such as \( \Sigma^- \) and \( \Lambda \), and quark matter, in which hadrons are crushed and the basic degrees of freedom of the system become more or less free quarks. The appearance of new constituents in neutron stars is of importance for two reasons. First, if it becomes energetically favourable to exploit a new degree of freedom, the energy will be reduced compared with the energy of the system in which that degree of freedom is suppressed. In other words, the energy will rise less rapidly with density, a fact which implies a general softening of the equation of state, and a reduction in the maximum mass of a neutron star. Second, essentially all possible new degrees of freedom of the system that have been considered to date allow for neutrino emission to occur via the direct Urca process. This is much faster than the modified Urca process, which is the standard process for matter consisting of neutrons and a small fraction of electrons and protons. This is expected to have far-reaching consequences for the thermal evolution of neutron stars. The basic condition for a constituent of charge \( Ze \) and baryon number \( A \) to be present in equilibrium is that its chemical potential is given by

\[
\mu(A, Z) = -Z\mu_e + A\mu_n, \tag{4}
\]

where \( \mu_e \) is the electron chemical potential and \( \mu_n \) the neutron one. In Fig.(3) we show plots of \( \mu_n \) and \( \mu_e \) as functions of density for matter in beta equilibrium. The bars
denote the lowest density at which particular constituents appear if they are treated as non-interacting. Observe how the inclusion of the three-body interaction increases the neutron chemical potential, and thereby lowers the threshold densities for other baryonic constituents. The fact that interactions of the possible new constituents have been neglected here means that the threshold estimates given are unrealistic. One further observation is that the model with the boost correction and three-body interactions included predicts proton concentrations high enough for the direct Urca process for nucleons to occur at densities in excess of $\sim 0.75 \text{fm}^{-3}$. However, such densities are expected to occur only in the cores of stars with masses greater than about $2M_\odot$.

5. Equation of state and stellar models

The pressure may be calculated from the energy density $E(\rho)$, where $\rho$ is the number density of baryons, and the properties of neutron stars may then be obtained from the conditions for hydrostatic equilibrium in general relativity, the Oppenheimer-Volkoff equations. The mass-radius relationship that results is shown in Fig.(4), indicating a maximum mass of about $2.2M_\odot$.

One unfortunate feature of the equation of state obtained in these calculations is that matter is superluminal, that is the sound speed, given by $c_s = (\partial P/\partial \rho)^{1/2}$ exceeds...
Figure 4. Neutron star mass versus radius for the nuclear models discussed in this paper compared with the results of the older FPS interaction based on the results of Ref. (10). UIX denotes the three-body interaction fitted to nuclear properties if the relativistic boost correction is neglected, and UIX* is the corresponding interaction if the boost correction is included.

the speed of light. However, this occurs only at densities greater than about 1 fm$^{-3}$, which would be realized only in the centres of stars close to the upper mass limit. This pathology reflects the fact that the theory is not completely consistent with special relativity. Two sorts of approach, short of developing a complete theory consistent with special relativity, can be adopted to remedy this defect. One is to assume that the equation of state is known reliably up to some density, and then assume that at higher densities the sound velocity is equal to $c$, the maximum value consistent with relativity. Such a procedure, first used by Rhoades and Ruffini [15] and more recently by Kalogera and Baym [16], leads to an equation of state which is stiffer than is realistic, and thus gives larger stellar masses for a given central density. Another approach is to argue that physically it is a better approximation to use at high densities a model of matter that is based on degrees of freedom other than the nucleon ones that are appropriate at low densities. The particular example we consider is quark matter. The question to be addressed is then how to interpolate between the high and low density limits. This is a complicated problem since in the region where the crossover from one state to another occurs neither the low- nor high-density equations of state are expected to be good approximations. Putting this problem aside, the simplest way to make a thermodynamically consistent description of the crossover is to adopt the standard method of making a Maxwell construction, which leads generally to a first-
order phase transition at which the density of matter jumps discontinuously in the star. Glendenning pointed out that another solution might be possible because there could be an intermediate phase consisting of coexisting quark and nucleon phases in a uniform background of electrons [17]. The regions of quark matter and nucleons should have length scales small compared with the electron screening length, and consequently the quark and nuclear matter regions need not be electrically neutral locally, even though they would be neutral on length scales large compared with the electron screening length. The lack of local charge neutrality implies that the pressure need not be constant in the intermediate region, as it is if matter is locally electrically neutral. Which of the two ways of matching the two equations of state is closer to the truth depends on the surface tension between the two phases: if the surface tension is too high the length scale of the quark and nucleon regions will exceed the electron screening length and the assumption of a uniform background of electrons will fail. In this case the Maxwell construction for two phases of neutral matter would give the correct answer, while for a lower surface tension, the mixed phase would be a possibility. Assuming the mixed phase and neglecting surface energies and the Coulomb energies associated with the charge inhomogeneities clearly underestimates the energy of the system, and thus this procedure is expected to lead to the “softest” equation of state in the intermediate region. However, it must be borne in mind that these methods of describing matter at intermediate densities are just devices behind which we try to hide our basic ignorance about what is really going on there, and any phase transitions that are found in model calculations may be purely due to the inadequacy of our fundamental understanding.

Let us now consider the consequences of interpolating between the nucleon equation of state described earlier and one for a free gas of quarks at high densities, assuming that the equation of state in the intermediate region is calculated on the assumption of the coexistence of regions of nucleon and quark matter with different charge densities. First of all, matter is never superluminal for the values of the bag constant considered, $B = 200 \text{ MeV fm}^{-3}$ and $B = 122 \text{ MeV fm}^{-3}$. The maximum mass of a neutron star is reduced, to $2 \text{ M}_\odot$ for a bag constant $B = 200 \text{ MeV fm}^{-3}$ and to $1.9 \text{ M}_\odot$ for $B = 122 \text{ MeV fm}^{-3}$.

Another important question is how sensitive results are to the assumed form of the three-body interaction. As we have stressed earlier, the information available empirically is scanty. In our earlier discussion we remarked that the three-body force was made up of two parts, a long-range one, which we denote by $v^l_{ijk}$, and a short range one $v^s_{ijk}$. It turns out that the contributions to nuclear binding energies of the long and short range parts are in the same ratio for $^3\text{H}$ and $^4\text{He}$, the two light nuclei used to fit parameters, the long-range contribution being $-2.2$ times the short-range one. Thus data on these nuclei gives no information about the relative strengths of the long and short range parts of the interaction since any three-body interaction of the form

$$v_{ijk}(x) = (1 + 0.45x)v^s_{ijk} + (1 + x)v^l_{ijk}$$

will give an equally good fit to the properties of these two nuclei for an arbitrary value of the parameter $x$, provided the changes can be calculated to linear order in $x$. However, the properties of symmetric nuclear matter do depend on $x$, especially at densities higher than those of the light nuclei. The values appropriate for the parameter $x$ are such that the modifications that would be needed to make symmetric nuclear matter saturate correctly ($E = -16 \text{ MeV at } \rho = 0.16 \text{ fm}^{-3}$) are negative (i.e.
the energies obtained do represent an upper bound, as is necessary for a variational calculation), and that they are no larger than 5 MeV, a generous estimate of the errors incurred in the variational many-body calculations. This limits $x$ to the range 0 to 0.3, small enough that the first-order treatment of the $x$ contribution is reasonably accurate. The neutron-star analysis for the cases $x = 0$ (effectively the same as the $A_{18} + \delta v_b + U I X^*$ of Ref. except for the adjustment for saturation), and $x = 0.15$ and 0.30, are shown in Fig. 5. The maximum mass depends on the value of $x$, and now

Figure 5. Mass of neutron star plotted against radius for force models with different three-body parts corresponding to the values 0, 0.15 and 0.30 of the variable $x$ defined in the text. The dotted and dashed lines indicate the effect of matching the low-density equation of state onto a high-density one for free quarks for two values of the bag constant.

extends from $2.20 \, M_\odot$ to $2.35 \, M_\odot$. This result is merely an estimate of the tolerance permitted by the reliance on fitting the three-body interaction to the ground-state energies of only $^3\text{H}$ and $^4\text{He}$. A new three-nucleon interaction that will provide much improved fits to properties of a wider selection of light nuclei is under development [18].

6. Concluding remarks

There are many areas in the physics of neutron stars that we have not addressed in this paper. One of these is the effect that possible phase transitions in the core could have on observable properties of neutron stars. This is reviewed in a recent article
by Heiselberg and Hjorth-Jensen [19], who also investigate the effects of different interpolations between assumed low- and high-density equations of state. We have also not considered the question of what of importance to neutron star physics can be learned from laboratory experiments with heavy-ion beams. This promises to be a fruitful approach, and the development of understanding of the physics of heavy-ion collisions that is a necessary prerequisite for constructing models of dense matter is well underway [20]. In addition we have not treated the advances that have occurred in the calculation of neutrino emission rates and transport properties.

While many mysteries remain, the advances on a number of different fronts (observations of neutron stars, nuclear experiment, and theory) are leading to an increasing understanding of the properties of neutron stars.

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