Parametric study of the stress state of beams with round holes in pure bending

S Saiyan and A Paushkin
Moscow State University of Civil Engineering, National Research University, 26 Yaroslavskoe shosse, Moscow, 129337, Russia
E-mail address: berformert@gmail.com

Abstract. A parametric study of the stress state of a rectangular cross-section beam with round holes in pure bending is performed by the finite element method implemented in the ANSYS Mechanical APDL software package. Correction coefficients were obtained for normal stresses determined by strength of materials methods. The coefficients of stress concentration during bending were determined depending on the size of the holes, as well as the distance between them. The research results obtained can be used in practical problems of designing perforated beams.

1. Introduction
Perforated beams are often used as load-bearing structures that work on bending. Such beams work well under static loads, while it is possible to save material and lighten the weight of building structures [1]. Existing methods for calculating perforated beams [2-7] are not able to consider the stress distribution across the section of perforated beams and do not answer the question of how exactly the stresses are distributed taking into account the holes. In this work, the objective was to determine the correction factors of stress concentration for results obtained from the engineering methods of strength of materials. For reliable determination of the stress state of perforated beams with circular cutouts subjected to pure bending, depending on the size of the perforation and the perforation step, was conducted the parametric computer simulation of beams using the software ANSYS Mechanical APDL.

2. Research
To conduct the numerical study, we considered a rigidly pinched rectangular cross-section beam with a perforation in the form of round notches, the radius of which changes parametrically to the height of the beam. The cross-section of a detail of the perforation in the wall beams for parametric studies of the stress state shown in figure 1. For the parametric study varied the radius of the holes \( R \), and the distance between them (pitch) \( L \). The value of the beam height was taken \( H = 0.9 \) m, a thickness of the beam \( T = 0.1 \) m. The beam material is steel with an elastic modulus \( E = 2.1 \times 10^5 \) MPa and a Poisson's ratio \( \nu = 0.3 \). The beam's own weight was not taken into account. The value of the bending moment \( M = 900 \) kN·m. The material model is selected as linear-elastic.
As an analytical solution, we will use a solution using strength of materials formulas that gives acceptable accuracy results for solid beams when bending, but does not take into account the perforation effects of stress concentration near the holes. The finite element calculation was performed in a two-dimensional setting for a generalized plane stress state. The size of the end element varies depending on the perforation parameters and is determined based on the number of end elements with local thickening in the hole areas. Due to the constancy of the bending moment along the beam, normal stresses along the perforated beam (away from kinematic and static boundary conditions) [8-13] will only change due to the influence of holes, which allows us to study the influence of perforation on the distribution of normal stresses in local areas near the holes.

Based on the considered scheme in figure 1, we present the Isofields of normal cross-section stresses at $R = H/6$ and $L = 5R$, determined by the finite element method in the ANSYS Mechanical APDL software package (figure 2).

As you can see, if there is a perforation, there is a stress concentration that occurs at the border of the cutout. Moreover, the maximum stress values occur directly in the section at the center of the hole. This effect is characterized by a non-linear distribution of normal stresses across the cross-section (figure 2), while the linear law is preserved by methods of strength of materials (figure 1).
Having analyzed on the basis of parametric studies the changes in normal stresses at different radii of holes $R$, we present graphs of changes in normal stresses at the level of the border of the holes (figure 3) and at the level of the extreme fibers (figure 4), depending on the radius of the holes $R$ at a step between them $L = 5R$. These graphs show the stress values determined numerically by the finite element method in the ANSYS Mechanical APDL software package, as well as analytically by the strength of materials formulas.

**Figure 3.** Graph of changes in normal stresses at the level of the hole boundary depending on their radius $R$ at $L = 5R$ for the beam shown in figure 1.

**Figure 4.** Graph of changes in normal stresses at the level of the extreme fibers depending on the radius of the holes $R$ at $L = 5R$ for the beam shown in figure 1.
Let’s consider some features of changing the stress state when changing the parameters of the perforation:

1. The normal stresses $\sigma_x$ defined in the cross section along the center of the holes in the finite element solution are distributed according to a nonlinear law, while in the solution by methods of strength of materials, the law of variation of normal stresses is linear.

2. At the level of the hole boundary, the stress concentration coefficient at the step $L = 5R$ for different hole sizes practically does not change and corresponds to $K \approx 2$ (figure 3).

3. As in the case with the solution by methods of strength of materials, in the numerical solution the value of the normal stress at the level of the border of the holes tends to infinity $\sigma_x \to \infty$ with increasing diameter (figure 3).

4. Normal stresses $\sigma_x$ at the level of extreme fibers almost completely coincide when solving by methods of strength of materials and in a finite element solution up to the value of the ratio of the hole radius to the height of the section equal to 1/6, after which large discrepancies are observed (figure 4).

5. At the level of the extreme fibers when the ratio of the radius of the holes to the height of the section more than 1/6, have seen a significant increase of the normal stresses in the solution by methods of strength of materials ($\sigma_x \to \infty$), while in the finite-element solution is observed a slight decrease of the stress level (figure 4).

6. When the ratio of the radius of the holes to the height of the section is approximately 1/4, the effect of equal stresses is observed in the finite element solution, at which the values of the normal stresses $\sigma_x$ at the level of the boundary of the holes and at the level of the extreme fibers are equal to each other.

7. When the ratio of the hole radius to the cross-section height is approximately 5/18 or higher, the values of normal stresses $\sigma_x$ in the finite element solution at the level of the hole boundary become higher than at the level of the extreme fibers in the solution by strength of materials methods, which indicates that such perforated structures should be designed from the strength condition for normal stresses with a radius-to-height ratio greater than 5/18, taking into account the influence of the stress concentration coefficient for stresses.

![Graph](image.png)

**Figure 5.** Graph of the dependence of the concentration coefficient $K$ of normal stresses $\sigma_x$ at the level of the hole boundary on the ratio of the radius $R$ to the height of the beam $H$ at a different step $L$. 
By examining these graphs, the values of normal stresses $\sigma_x$ determined by strength of materials methods can be correlated through correction values, which can be called stress concentration coefficients $K$.

After analyzing some types of perforation sizes for different perforation steps, we have plotted the dependence of stress concentration coefficients $K$ for normal stresses at the level of the hole border (figure 5) and at the level of the extreme fibers (figure 6) depending on the radius of the holes $R$ and the step between them $L$.

The ratio of the hole radius to the cross-section height $1/12$, $1/6$, $1/4$, $1/3$ at steps $L = 0.5R$, $L = R$, $L = 1.5R$, $L = 2R$, $L = 3R$ was considered as the standard sizes.

![Graph of stress concentration coefficients](image)

**Figure 6.** Graph of the dependence of the concentration coefficient $K$ of normal stresses $\sigma_x$ at the level of the extreme fibers on the ratio of the radius $R$ to the height of the beam $H$ at a different step $L$ between the holes.

Let's do some analysis of these graphs:

1. Normal stresses $\sigma_x$ at the level of the hole boundary are most sensitive to changes in the perforation parameters. For almost all values of the step $L$, the stress concentration coefficient $K$ increases with increasing hole sizes. Almost independent of the size of the holes, the concentration coefficient $K \approx 2$ occurs at $L = 5R$ (figure 3).

2. The influence of stress concentration at the level of the extreme fibers practically does not depend on the parameters of perforation, but when the size of the holes increases, the normal stress at the level of the extreme fibers decreases (figure 6).

3. The greater the step $L$ between the holes, the more flat the graph of stress concentration $K$ becomes at the level of the border of the holes, which indicates that the value of stress concentration ceases to depend on their size (figure 5).

4. Verification studies of intermediate values of perforation parameters have shown that the use of linear interpolation based on the data of graphs of stress concentration coefficients shows fairly reliable results in comparison with the finite element solution.

**3. Conclusions**

1. The presence of a perforation changes the character of normal stress $\sigma_x$ diagrams to a non-linear one with a stress concentration near the boundaries of the holes.

2. The normal stress concentration factor $K$ depends on the ratio of the hole radius to the cross-section height $R/H$, as well as on the pitch $L$. 
3. Based on the finite element solution, when the ratio of the radius of the holes \( R \) to the height of the section becomes equal 1/4, an effect is observed in which the values of normal stresses \( \sigma_x \) at the level of the border of the holes and at the level of the extreme fibers are equal to each other.

4. The use of correction coefficients of stress concentration \( K \) in determining stresses by methods of strength of materials gives acceptable results in terms of accuracy when performing engineering calculations.

5. Further research can be devoted to the study of the behavior of shear stresses during transverse bending in beams with holes.

References

[1] Saiyan S and Paushkin A 2019 Fatigue cracks in castellated I-beams under cyclic loads *J. of Physics: Conference Series* **1425** 012163

[2] Drobachev V M and Litvinov E V 2003 Analytical determination of the stress-strain state of the wall-bridge of a perforated beam *University news. Construction* **5** pp 128 –33

[3] Filatov V V 2013 Calculation of through beams based on the theory of composite rods by A. R. Rzhanitsyn *MGSU Bulletin* **9** pp 23 – 31

[4] Bondarenko V M, Zaitsev P I and Lyubimov A A 1963 Calculation of steel beams from split rolled I-beams with holes in the wall *Collection of Scientific Papers Kharkov Civil Engineering Institute* (Kharkov: Kharkov Civil Engineering Institute) **16** pp 19 – 25

[5] Besedin M T 1962 Beams from developed rolled I-beams with holes in the wall *Scientific works of the Kharkov Civil Engineering Institute* (Kharkov: Kharkov Civil Engineering Institute) **19** pp 15 – 19

[6] Hahnemann G A and Kikot A A 2017 Analysis of metal beams with a perforated wall *Polzunovsky Almanac* **4** (2) pp 49 – 52

[7] Al Khetari A A 2018 Features of work and calculation of beams with a perforated wall *Symbol of science* **6** pp 11 – 4

[8] Saiyan S G and Paushkin A G 2018 Investigation of shear stresses in a straight rod near the load application zone *BST: Byulleten’ Stroitel’noy Tehniki* **1** pp 56 – 9

[9] Saiyan S and Paushkin A 2017 Numerical study of the applicability of the Saint-Venant principle *MATEC Web of Conferences* **117** 00134

[10] Saiyan S and Paushkin A 2018 Investigation of shear stress attenuation in a straight rod near the load application point *IOP Conference Series: Materials Science and Engineering* **365** 042026

[11] Saiyan S and Paushkin A 2018 Investigation of perturbations of normal stresses in an I-beam rod in the load application zone *MATEC Web of Conferences* **251** 04025

[12] Saiyan S and Paushkin A 2020 Saint-Venant principle for kinematic boundary conditions *Journal of Physics: Conference Series* **1425** 012198

[13] Saiyan S and Paushkin A 2019 Numerical study of the shear stress distribution in an i-beam in the load application zone *Material Science Forum* **974** pp 659 – 64