Research Article

Excitation Kinetics of Impurity Doped Quantum Dot Triggered by Gaussian White Noise

Suvajit Pal,1 Sudarson Sekhar Sinha,2 Jayanta Ganguly,3 and Manas Ghosh4

1 Department of Chemistry, Hetampur Raj High School, Hetampur, Birbhum, West Bengal 731124, India
2 Department of Physical Chemistry, Indian Association for the Cultivation of Science, Jadavpur, Kolkata, West Bengal 700032, India
3 Department of Chemistry, Brahmankhand Basapara High School, Basapara, Birbhum, West Bengal 731215, India
4 Department of Chemistry, Physical Chemistry Section, Visva Bharati University, Santiniketan, Birbhum, West Bengal 731 235, India

Correspondence should be addressed to Manas Ghosh; pcmg77@rediffmail.com

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We investigate the excitation kinetics of a repulsive impurity doped quantum dot initiated by the application of Gaussian white noise. In view of a comprehensive research we have considered both additive and multiplicative noise (in Stratonovich sense). The noise strength and the dopant location have been found to fabricate the said kinetics in a delicate way. Moreover, the influences of additive and multiplicative nature of the noise on the excitation kinetics have been observed to be prominently different. The investigation reveals emergence of maximization and saturation in the excitation kinetics as a result of complex interplay between various parameters that affect the kinetics. The present investigation is believed to provide some useful perceptions of the functioning of mesoscopic systems where noise plays some profound role.

1. Introduction

Throughout last couple of decades we came across a tremendous surge in theoretical and experimental researches on impurity states of low-dimensional heterostructures [1]. Among these heterostructures, quantum dots (QDs) have been found to take some leading role undoubtedly. The importance of QD originates because of its emergence as the destination in the journey towards miniaturization of semiconductor devices. Furthermore, the properties of doped QDs have made them potential candidates for scientific study and technological applications. With QD, the subtle interplay between new confinement sources and impurity potentials has opened up new areas of research in this field [2]. Under the confinement, the dopant location can tailor the electronic and optical properties of the system [3]. This resulted in a rich literature comprising of theoretical studies on impurity states [4–7] in general, and also on their optoelectronic properties, in particular, for a wide range of semiconductor devices [3, 8–15]. The widespread research trend elegantly explores new physics and indicates profound technological impact simultaneously. As a natural extension of the trend, dynamical aspects of doped QD have also been extensively studied. Out of them researches on excitation of strongly confined electron merit importance because of the crucial role played by the said phenomenon in optoelectronic devices and as lasers. From the perspective of technological applications such excitation further involves optical encoding, multiplexing, and photovoltaic and light emitting devices. The phenomenon also plays some important role in the population transfer among the exciton states in QD [16]. Of late, we have performed some inspections which deal with excitation in doped QDs that have been triggered by some external time-varying field [17, 18].

Noise in mesoscopic systems influences the functioning of the device to a great extent. This is because of the pivotal role played by the noise as the size of the electronic system reaches the nanometer scale [19]. In practice, noise appears as the main obstacle in the development of semiconductor heterostructure devices for diverse applications and more often than not it also limits the device performance [20]. Hastas et al. have also showed the importance of noise in
self-assembled InAs QDs embedded in GaAs [21]. The noise can result externally, or it may be intrinsic. Intrinsic noises generally result owing to changes in impurity configurations [22]. However, low-frequency noise is one of the prime requirements in QD heterostructures looking at its potential use in a wide range of optoelectronic devices. These applications include industrial manufacturing, medicine, remote sensing, space communications, and military uses [23]. Such noise measurements are also becoming useful as a nondestructive method for the determination of structural disorders entered during production or operation of the devices [24, 25]. Since QD suffers from a lot of stress during its growth, lattice defects are generated and they diffuse inside the QD structure. The said low-frequency noise also provides a diagnostic tool for defect related properties of materials and structures [26]. Asriyan et al. have extensively studied the off-center impurities invoking heterostructures. Particularly, Karabulut, Baskoutas, and their coworkers studied the off-center impurities invoking an accurate numerical method (potential morphing method (PMM)) [13, 15]. We therefore, looking at a comprehensive description of our investigation, have determined the time-average excitation rate \((R_{av})\) as a function of dopant location \((r_0)\) and noise strength \((\zeta)\). The various profiles unveil the subtle interplay between the dopant location and the noise strength that sensitively modulates the excitation kinetics.

**2. Method**

The model considers an electron subject to a harmonic confinement potential \(V(x, y)\) and a perpendicular magnetic field \(B\). The confinement potential assumes the form \(V(x, y) = (1/2)m^*\omega_c^2(x^2 + y^2)\), where \(\omega_c\) is the harmonic confinement frequency, \(\omega_c = eB/m^*c\) being the cyclotron frequency (a measure of magnetic confinement offered by \(B\)). In the present work a magnetic field of miliTesla (mT) order has been employed. \(m^*\) is the effective electronic mass within the lattice of the material to be used. We have taken \(m^* = 0.5m_0\) and set \(\hbar = e = m_0 = \omega_c = 1\). This value of \(m^*\) closely resembles Ge quantum dots \((m^* = 0.55\) a.u.). We have used Landau gauge \([A = (By, 0, 0)]\) where \(A\) stands for the vector potential. The Hamiltonian in our problem reads

\[
H_0 = -\frac{\hbar^2}{2m^*} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + \frac{1}{2} m^* \omega_c^2 x^2 + \frac{1}{2} m^* (\omega_x^2 + \omega_y^2) y^2 - i\hbar\omega_c y \frac{\partial}{\partial x},
\]

Define \(\Omega^2 = \omega_x^2 + \omega_y^2\) as the effective frequency in the \(y\)-direction. The model Hamiltonian [cf. (1)] sensibly represents a 2-d quantum dot with a single carrier electron [27, 28]. The form of the confinement potential conforms to kind of lateral electrostatic confinement (parabolic) of the electrons in the \(x - y\) plane [4, 11, 29].

In the present problem we have considered that the QD is doped with a repulsive Gaussian impurity [30, 31]. The noise consists of random term \((\xi(t))\) quite often assumed to be following a Gaussian distribution and characterized by the following equations:

\[
\langle \xi(t) \rangle = 0,
\]

the zero mean value condition, and

\[
\langle \xi(t) \xi(t') \rangle = 2\zeta\delta(t - t'),
\]

the two-time correlation condition with a negligible correlation time, \(\zeta\) being the noise strength. The highly fluctuating term \(\xi(t)\) is called white noise because of a flat spectrum in frequency space, like that of white light. The noise strength \(\zeta\) becomes simply a measure of intensity of fluctuation. We have invoked Box-Muller algorithm to generate \(\xi(t)\). An additive noise term is a random term that does not depend on coordinates of system so that \(V(t) = \xi(t)\), whereas a multiplicative noise gets coupled with the system coordinates making \(V(t) = \xi(t)(x + y)\) in Stratonovich sense [35].

The time-dependent Schrödinger equation (TDSE) containing the evolving wave function has now been solved numerically by 6th order Runge-Kutta-Fehlberg method. In the numerical solution appropriate initial conditions have been used and numerical stability has been checked. We define the quantity \(R_k(t) = |\psi_k(t)|^2\) to indicate the population of \(k\)th state of \(H_0\) at time \(t\) where \(\psi_k(t)\) is the time-dependent superposition coefficient for the \(k\)th eigenstate.
of the unperturbed Hamiltonian. We observe a continuous variation in the ground state population \([P_0(t)]\) during the time evolution. Naturally the quantity \(Q(t) = 1 - P_0(t)\) serves as a measure of excitation. In consequence, the quantity \(R_{\text{ex}}(t) = dQ/dt\) serves as the time-dependent rate of excitation from which the time-average rate of excitation \(\langle R_{\text{ex}} \rangle = (1/T) \int_0^T R_{\text{ex}}(t)dt\) can be calculated, where \(T\) is the total time of dynamic evolution \([17, 18]\).

### 3. Results and Discussion

#### 3.1. Role of Additive Noise

Figure 1 displays the excitation profile of the doped dot as a function of time \((t)\) when an additive noise of fixed strength \(\zeta = 1.0 \times 10^{-6}\) a.u. has been applied for three different dopant locations \((r_0)\). The plots reveal lot of undulations at all dopant locations indicating occurrence of considerable amount of excitation and deexcitation in tandem. However, extent of oscillation becomes more vigorous at off-center locations \((r_0 = 28.28\) a.u. and \(70.71\) a.u.) in comparison to the on-center dopant \((r_0 = 0.0\) a.u.). The variation in the extent of oscillation at on- and off-center locations can be attributed to varied influence of dot confinement center at these locations. The reduced strength of dot confinement at off-center locations makes the noise term more prominently active resulting in an enhancement in the oscillations of excitation profile. At on-center dopant location, because of the strong confinement offered by dot, the noise effect remains submissive to some extent which restricts the oscillation a bit.

In order to analyze the exclusive role played by the dopant location on excitation kinetics, we have plotted \(\langle R_{\text{ex}} \rangle\) as a function of dopant location for a fixed value of noise strength \((\zeta = 1.0 \times 10^{-6}\) a.u.) (Figure 2). The plot reveals a distinct maximization in the rate at \(r_0 \sim 14.0\) a.u. It needs to be realized that the role of dopant coordinate on excitation rate is not at all straightforward, rather it is quite delicate. An increase in \(r_0\) takes the dopant away from the confinement of dot potential favoring the excitation. However, the said shift also reduces dot-impurity repulsive interaction thereby suppressing excitation. Thus, a progressive shift of dopant to more and more off-center positions invites two opposite factors come into play that can affect excitation rate. It is the changeover of relative dominance of one of the factor over the other that causes maximization in the kinetics at some typical dopant location. At far off-center locations \((r_0 \geq 60.0\) a.u.) we envisage kind of saturation in the kinetics owing to sort of balance between the opposing factors.

Surprisingly enough, in the present case we do not envisage any influence of noise strength \((\zeta)\) on the excitation kinetics. The reluctance of noise strength to influence the excitation is interesting and presumably has origin in its very “additive” nature. It seems that, remaining uncoupled to the system coordinates, the additive noise simply adds on to the Hamiltonian and thus perturbs all the eigenstates to equal extents. Thus, on the whole, the relative separations between all the eigenstates remain unaltered making the excitation kinetics a mute spectator of the noise strength. This observation further necessitates investigation of the role of multiplicative noise in the present context.

#### 3.2. Role of Multiplicative Noise

Figure 3 exhibits the excitation profile induced by a multiplicative white noise for the same fixed value of \(\zeta\) at three different dopant locations. Unlike to the case of additive noise, here the extent of oscillation is found to be more or less equivalent at these dopant locations. Thus, insofar as excitation and deexcitation are concerned, coupling the noise term to the system coordinate brings about more subtle interplay between the dot confinement and the noise strength.
In the concluding section of our discussion we would like to demonstrate the variation of $\langle R_{ex} \rangle$ with $\zeta$ at on-center noise for different noise strengths (Figure 4). However, no such maximization is observed using multiplicative noise of medium ($\zeta = 1.0 \times 10^{-4}$ a.u.) and high ($\zeta = 1.0 \times 10^{-2}$ a.u.) strength. It is also evident from the figure that for all noise strengths the excitation rate attains steady value at far off-center dopant locations. A multiplicative noise of extremely low strength perturbs the system feebly and hardly leaves its signature on excitation kinetics. Thus, the excitation kinetics effectively depends on the dopant location and exhibits similar pattern as that of additive noise. As the noise strength assumes some intermediate value ($\zeta = 1.0 \times 10^{-5}$ a.u.), the rate profile after some small initial rise exhibits a steady behavior within the range $15.7 \text{ a.u.} \leq r_0 \leq 55.5 \text{ a.u.}$ Beyond $r_0 \geq 55.5 \text{ a.u.}$ the excitation rate jumps abruptly to a high value and permanently settles to steady value beyond $r_0 \geq 63.1 \text{ a.u.}$ Such rate profile reveals the nuances of combined influence of several factors that affect excitation. The initial small rise in $\langle R_{ex} \rangle$ at near off-center locations ($r_0 \leq 15.7 \text{ a.u.}$) indicates mild win of noise and dot-impurity repulsive terms over that of strong confinement potential. After that there is a sustained balance between the diverse factors that affect excitation over a quite extended range of dopant location ($15.7 \text{ a.u.} \leq r_0 \leq 55.5 \text{ a.u.}$). At $r_0 = 55.5 \text{ a.u.}$, the confinement becomes quite weak and factors that promote excitation (noise + repulsive interaction) suddenly become prominent leading to an abrupt rise in excitation rate. Beyond $r_0 \geq 63.1 \text{ a.u.}$, because of large dot-impurity separation, their mutual repulsive interaction gets considerably diminished and we land up at a permanent balanced condition of those factors revealed through a saturation in excitation kinetics. With a high value of noise strength ($\zeta = 1.0 \times 10^{-4}$ a.u.) the rate profile manifests kind of resemblance with that of medium noise strength counterpart. However, the variation in the rate with dopant location appears much less sharp. The initial rise of $\langle R_{ex} \rangle$ occurs over an extended region ($0.0 \text{ a.u.} \leq r_0 \leq 35.0 \text{ a.u.}$) in an indolent manner; the former extended zone of constant excitation rate becomes nearly absent and the rate begins to rise as $r_0 \geq 35.0 \text{ a.u.}$ From the figure it is also quite discernible that the rise in excitation rate within $35.0 \text{ a.u.} \leq r_0 \leq 56.6 \text{ a.u.}$ becomes much less abrupt in comparison with the medium noise strength counterpart. The regime of persistent saturation in excitation kinetics begins from $r_0 = 56.6 \text{ a.u.}$ Thus, an increase in noise strength from medium to high value does not much alter the intrinsic nature of interplay between itself, the dot confinement potential, and the dot-dopant repulsive interaction (the last two factors undergo change with a variation in dopant location) toward fabricating the excitation kinetics. The said increase seems to disseminate the complex interplay over a spatially more spread out domain making the overall variation of the excitation rate rather sluggish with change in dopant coordinate.

In the concluding section of our discussion we would like to demonstrate the variation of $\langle R_{ex} \rangle$ with $\zeta$ at on-center

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**Figure 4**: Plot of $\langle R_{ex} \rangle$ versus $r_0$ for Gaussian multiplicative white noise for different noise strengths ($\zeta$); (i) $\zeta = 1.0 \times 10^{-2}$ a.u., (ii) $\zeta = 1.0 \times 10^{-3}$ a.u., and (iii) $\zeta = 1.0 \times 10^{-4}$ a.u.
indifferent toward the excitation rate. The interplay between dopant location and noise strength becomes more subtle in case of multiplicative noise. Now excitation rate has been found to display distinct characteristics as dopant location and noise strength are varied over a wide range. Whereas the appearance of maxima in the excitation kinetics indicate preponderance of the parameters that favor excitation over the opposite ones, the saturation in the kinetics can be attributed to kind of compromise between these rival factors. The results are thus quite interesting and expected to convey important insights in related field of research.

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