One-loop Gauge Couplings in Orbifold Field Theories

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(Dated: December 17, 2021)

We discuss the gauge coupling renormalization in orbifold field theories in which the 4-dimensional graviton and/or matter fields are quasi-localized in extra dimension to generate hierarchically different mass scales and/or Yukawa couplings. In such theories, there can be large calculable Kaluza-Klein threshold corrections to low energy gauge couplings, enhanced by the logarithms of small warp factor and/or of small Yukawa couplings. We present the results on those Kaluza-Klein threshold corrections in generic 5-dimensional theory on $S^1/Z_2 \times Z_2$ containing arbitrary 5-dimensional gauge, spinor and scalar fields.

I. INTRODUCTION

It has been noticed that theories with extra dimension can provide an elegant mechanism to generate various hierarchical structures in 4-dimensional (4D) physics, e.g. the weak to Planck scale hierarchy $M_W/M_{Pl} \approx 10^{-16}$ [1, 2] and the hierarchically different Yukawa couplings ranging from $y_e \approx 10^{-6}$ to $y_t \approx 1$ [3]. A particularly interesting mechanism to generate the scale hierarchy $M_W/M_{Pl} \approx 10^{-16}$ is the quasi-localization of 4D graviton in extra-dimension [2]. In 5D theory on $S^1/Z_2$ which is parameterized by $y = [0, \pi]$, if the 5D cosmological constant is negative and the brane cosmological constants are appropriately tuned, the resulting geometry is a slice of AdS$_5$, yielding a 4D graviton quasi-localized at $y = 0$ with wavefunction $e^{-kRy}$ where $k$ is the AdS curvature and $R$ is the radius of $S^1$. If the Higgs boson for electroweak symmetry breaking is assumed to be confined at the orbifold fixed point $y = \pi$, its wavefunction-overlap with quasi-localized graviton is suppressed by the warp factor $e^{-kR\pi}$, leading to an exponentially small scale ratio $M_W/M_{Pl} \approx e^{-\pi k R}$. One can obtain also the hierarchically different Yukawa couplings by quasi-localizing fermions in extra dimension [3]. Again for the Higgs boson confined at $y = \pi$, fermions quasi-localized at $y = \pi$ have Yukawa couplings of order unity. On the other hand, fermions quasi-localized at the other fixed point $y = 0$ have an exponentially small wavefunction-overlap with the Higgs boson, and thus small Yukawa couplings.

Grand unification of the strong and electroweak forces is a highly persuasive idea for physics at high energy scales. However conventional 4D grand unified theories (GUTs) have suffered from well-known problems such as the doublet-triplet splitting problem and the issue of too rapid proton decay. GUTs in higher dimensional spacetime can avoid these problems through the mechanism of symmetry breaking by boundary conditions [4]. It is straightforward to implement the idea of quasi-localization in orbifold GUTs to generate the scale and/or Yukawa hierarchies [5].

In any GUT, heavy particle threshold effects at GUT-symmetry breaking scale should be taken into account for a precision analysis of low energy gauge couplings. In conventional 4D GUT, those heavy particle threshold corrections are not so important since they are subleading compared to the leading large log effects from light fields. However orbifold GUT contains (infinitely) many Kaluza-Klein (KK) modes, thus can have sizable GUT-scale threshold corrections [6]. There are two different type of KK threshold corrections in orbifold GUT: one which is power-law divergent [7] and the other which is either log-divergent or finite [8, 9]. The power-law divergent parts are sensitive to the unknown UV completion, and thus not calculable within orbifold field theory [8, 10, 11]. Still the GUT-symmetry in bulk spacetime guarantees that the power-law divergent parts are universal for all gauge couplings, so do not affect the gauge coupling differences which are of phenomenological interests. The log-divergent or finite parts are calculable within orbifold GUT, and not universal in general since the GUT-symmetry is broken by boundary conditions. As we will see, these calculable threshold corrections are enhanced by the large logarithm of an exponentially small warp factor and/or of an exponentially small Yukawa coupling when the scale and/or Yukawa hierarchies are generated by quasi-localization [11–15], and thus should be taken into account in the analysis of low energy gauge couplings.

The aim of this talk is to discuss and summarize the results on KK threshold corrections in generic 5D orbifold field theories in which the scale and/or Yukawa hierarchies are generated by quasi-localization [11, 14, 15]. In Section II,
we discuss briefly how the quasi-localization of graviton and matter fermions lead to the scale and Yukawa hierarchies. In Section III, we present the results on KK threshold corrections in generic 5D gauge theory on a slice of AdS5. In Section IV, we discuss an alternative way to compute KK threshold corrections in supersymmetric theories, which is based on 4D effective supergravity (SUGRA). In Section V, we briefly summarize the results for 5D theories on flat extra dimension in which matter fermions are quasi-localized to generate hierarchical Yukawa couplings.

II. SCALE AND YUKAWA HIERARCHIES FROM QUASI-LOCALIZATION

Since the mass scales in 4D physics are measured by 4D graviton, a dynamical quasi-localization of 4D graviton in extra dimension can generate hierarchically different mass scales in 4D physics. To see this, let us consider the Randall-Sundrum model [2] on S1/Z2 with 5D action:

\[
S = M_5^3 \int d^5x \sqrt{-G} \left[ -\frac{1}{2} R + 6k^2 - \frac{1}{\sqrt{G_{55}}} \left( \delta(y)6k - \delta(y - \pi) (6k - G^{\mu\nu} D_\mu H D_\nu H^* - M_H^2 H H^*) \right) \right],
\]

where \( M_5 \) and \( R \) are the 5D Planck scale and Ricci scalar for the 5D metric \( G_{55} \), \( H \) is the Higgs field for electroweak symmetry breaking which is confined at \( y = \pi \), and the 5D mass parameters \( k (> 0) \) and \( M_H \) are assumed to be comparable to \( M_5 \). The equations of motion from this action determine the spacetime geometry to be a slice of AdS5:

\[
ds^2 = g_{MN} dx^M dx^N = e^{-2kRy} g_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2,
\]

where \( g_{\mu\nu} \) corresponds to the massless 4D graviton, \( R \) is the orbifold radius, and \( y = [0, \pi] \) is the coordinate of \( S^1/\mathbb{Z}_2 \). This solution shows that the 4D graviton is quasi-localized at \( y = 0 \) due to the negative 5D cosmological constant.

The Higgs boson mass \( M_H \) measured by \( G_{MN} \) is generically of the order of \( M_5 \). However the observed electroweak scale is measured by the 4D graviton \( g_{\mu\nu} \). Since the 4D graviton wavefunction at \( y = \pi \) is exponentially small, the Higgs boson mass \( m_H \) measured by \( g_{\mu\nu} \) is red-shifted by the warp factor as

\[
m_H = e^{-\pi kR} M_H.
\]

This can be easily seen by considering the 4D effective action of \( g_{\mu\nu} \) and \( H \):

\[
S_{\text{eff}} = \int d^4x \sqrt{-g} \left[ -\frac{M_5^3}{2k} \left( 1 - e^{-\pi kR} \right) \mathcal{R}(g) - g_{\mu\nu} D_\mu D_\nu H^* - e^{-2\pi kR} M_H^2 H H^* \right],
\]

yielding the weak to Planck scale ratio measured by the 4D graviton:

\[
\frac{m_H}{M_{Pl}} = \frac{e^{-\pi kR} M_H}{\sqrt{M_5^3 \left( 1 - e^{-2\pi kR} \right) / k}} \approx e^{-\pi kR} \approx 10^{-16}
\]

for \( M_5 \approx k \approx M_H \) and \( kR \approx 12 \). In fact, the above red-shift applies to all mass scales at \( y = \pi \). For any dimensionful coupling \( \lambda_5 \) at \( y = \pi \) defined in the metric frame of \( G_{MN} \), the corresponding dimensionful coupling \( \lambda_4 \) measured by \( g_{\mu\nu} \) is red-shifted as

\[
\lambda_4 = (e^{-\pi kR})^{D_\lambda} \lambda_5,
\]

where \( D_\lambda \) is the mass-dimension of \( \lambda_5 \).

Hierarchical Yukawa couplings can be generated similarly by quasi-localizing matter fermions [3]. To see this, let us consider a 5D theory on \( S^1/\mathbb{Z}_2 \) containing 5D fermions and also a 4D Higgs field confined at \( y = \pi \):

\[
S = -\int d^5x \sqrt{-G} \left[ i\bar{\Psi}_I (\gamma^M D_M + M_I \epsilon(y)) \Psi_I + \frac{\delta(y - \pi)}{\sqrt{G_{55}}} \left( D_\mu H D^\mu H^* + \frac{\Lambda I J}{\Lambda} H \bar{\psi}_I \psi_J \right) \right].
\]

where \( \epsilon(y) = y/|y| \), \( \Lambda \) denotes the cutoff scale of 5D orbifold field theory, and \( \Lambda I J \) are dimensionless brane Yukawa couplings. Here we assume that the spacetime geometry is flat, so

\[
ds^2 = G_{MN} dx^M dx^N = g_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2.
\]

The 5D Dirac fermion \( \Psi_I \) has the boundary condition

\[
\Psi_I(-y) = z_I \gamma_5 \Psi_I(y), \quad \Psi_I(-y') = z_I \gamma_5 \Psi_I(y'),
\]
where \( y' = y - \pi, \, z_I = \pm 1, \) and \( \psi_I = \frac{1}{2}(1 + \gamma_5)\Psi_I \) \((z_I = 1)\) or \( \frac{1}{2}(1 + \gamma_5)\Psi_I^\dagger \) \((z_I = -1)\). For any value of the \( Z_2 \)-odd mass \( M_I \), the 5D fermion \( \Psi_I \) has a 4D chiral zero mode

\[
\psi_{0I} = \exp(-z_I M_I y),
\]

which is quasi-localized at \( y = 0 \) or \( \pi \), depending on the sign of \( M_I \). It is then straightforward to find that the 4D Yukawa couplings of canonically normalized fermion zero modes are given by

\[
y_{IJ} = \sqrt{Z(z_I M_I)Z(z_J M_J)} \lambda_{IJ}
\]

where

\[
Z(M) = \frac{M}{\Lambda} \frac{1}{e^{2\pi y R} - 1}.
\]

Obviously, the 4D Yukawa couplings \( y_{IJ} \) can have very different values, depending upon the values of \( M_I \), even when the 5D parameters \( \lambda_{IJ} \) have similar values. For instance, for \( z_I M_I \) and \( z_J M_J \lesssim -1/R \), we have

\[
y_{IJ} \approx \sqrt{\frac{M_I M_J}{\Lambda}} \lambda_{IJ},
\]

while for \( z_I M_I \) and \( z_J M_J \gtrsim 1/R \),

\[
y_{IJ} \approx \sqrt{\frac{M_I M_J}{\Lambda}} e^{-(z_I M_I + z_J M_J)\pi R} \lambda_{IJ}.
\]

The physical interpretation of this result is simple. If \( z_{I,J} M_{I,J} \gtrsim 1/R \), the corresponding zero modes are quasi-localized at \( y = 0 \), so the Yukawa couplings are exponentially suppressed as they originate from \( y = \pi \). On the other hand, for \( z_{I,J} M_{I,J} \lesssim -1/R \), the zero modes are localized at \( y = \pi \), so there is no suppression of Yukawa couplings.

### III. ONE LOOP GAUGE COUPLINGS IN AdS5

The model we study in this section is a 5D gauge theory defined on a slice of AdS5 with the spacetime metric (2) [14]. The lagrangian is given by

\[
S = -\int d^4x dy \sqrt{-G} \left[ \frac{1}{4g^2_5} F^{aMN} F_{aMN} + D_M \phi D^M \phi + \bar{\phi} \phi + 2i \bar{\phi} \gamma^y M \phi \right],
\]

where \( D_M \) is the covariant derivative containing the gauge connections as well as the spin connection of AdS5. Here we include the 4D gauge kinetic terms and scalar mass-squares confined on the orbifold fixed points as well as the conventional 5D kinetic and mass terms, so

\[
\frac{1}{g^2_5} = \frac{1}{g^2_5} + \frac{\delta(y)}{R} \frac{1}{g^2_5} + \frac{\delta(y - \pi)}{R} \frac{1}{g^2_5},
\]

\[
\bar{m}^2 = A^2 k^2 + \frac{2k}{R} [B_0 \delta(y) - B_\pi \delta(y - \pi)], \quad M = Ck.
\]

The 5D fields in the model can have arbitrary \( Z_2 \times Z'_2 \) orbifold boundary condition,

\[
\phi(-y) = z_\phi \phi(y), \quad \phi(-y') = z'_\phi \phi(y'),
\]

\[
\Psi(-y) = z_\Psi \gamma_5 \Psi(y), \quad \Psi(-y') = z'_\Psi \gamma_5 \Psi(y'),
\]

\[
A^a_\mu(-y) = z_a A^a_\mu(y), \quad A^a_\mu(-y') = z'_a A^a_\mu(y'),
\]

where \( z_\phi, z'_\phi = \pm 1 \) for \( \Phi = \{ \phi, \Psi, A^a_\mu \} \) and \( y' = y - \pi \).

The one-loop gauge couplings at low momentum scale \( p \) are given by

\[
\frac{1}{g^2_5(p)} = \left( \frac{1}{g^2_5} \right)_\text{tree} + \left( \frac{1}{g^2_5} \right)_\text{loop},
\]
where
\[
\left( \frac{1}{g_a^2} \right)_\text{tree} = \frac{\pi R}{g_{5a}^2} + \frac{1}{g_{0a}} + \frac{1}{g_{2a}^2}
\]
denote the tree-level couplings and the one-loop corrections can be written as
\[
\left( \frac{1}{g_a^2} \right)_\text{loop} = \frac{\gamma_a}{24\pi^3} \Lambda \pi R + \frac{1}{8\pi^2} \left[ \tilde{b}_a \ln \Lambda + \Delta_0(A,B_0,B_\pi,C,k,R) - b_a \ln p \right],
\]
\[
eq \frac{\gamma_a}{24\pi^3} \Lambda \pi R + \frac{1}{8\pi^2} \left[ \Delta_a(\ln \Lambda, A, B_0, B_\pi, C, k, R) + b_a \ln \left( \frac{\Lambda}{p} \right) \right],
\]
where the cutoff scale \( \Lambda \) is assumed to be large enough compared to other mass parameters of the theory, so the parts suppressed by \( 1/\Lambda \) are ignored. The linearly divergent one-loop corrections are UV-sensitive, i.e. regularization scheme-dependent, thus can not be computed within our orbifold field theory to a level better than just constraining them by 5D gauge symmetry. On the other hand, the logarithmically divergent or finite corrections are UV-insensitive, thus can be computed unambiguously within orbifold field theory. Note that the coefficients of \( \ln p, i.e. b_a, \) correspond to the standard one-loop beta function coefficients which are determined by the massless spectrum.

One may rewrite the one-loop corrections as
\[
\left( \frac{1}{g_a^2} \right)_\text{loop} = \Delta'_0 + \frac{b_a}{8\pi^2} \ln \left( \frac{M_{KK}}{p} \right),
\]
where \( M_{KK} \) denotes the mass of the lightest KK state which is still bigger than \( p \). Then
\[
\Delta'_0 = \frac{\gamma_a}{24\pi^3} \Lambda \pi R + \Delta_a + \tilde{b}_a \ln \Lambda - b_a \ln M_{KK} = \frac{\gamma_a}{24\pi^3} \Lambda \pi R + \Delta_a + b_a \ln(\Lambda/M_{KK})
\]
could be interpreted as the full threshold corrections due to the KK modes at scales between \( \Lambda \) and \( M_{KK} \). However \( \Delta'_0 \) contain the non-calculable power-law divergences. Also, since \( M_{KK} \) is a non-trivial function of \( R \) and the fundamental mass parameters of the model, \( \Delta'_0 \) do not represent directly the dependence of \( g_a^2(p) \) on \( R \) and the fundamental parameters of the model. It is thus more convenient to parameterize \( g_a^2(p) \) as
\[
\frac{1}{g_a^2(p)} = \left( \frac{1}{g_a^2} \right)_{\text{bare}} + \frac{1}{8\pi^2} \left[ \Delta_a(\ln \Lambda, A, B_0, B_\pi, C, k, R) + b_a \ln \left( \frac{\Lambda}{p} \right) \right],
\]
where all uncalculable parts are encoded in
\[
\left( \frac{1}{g_a^2} \right)_{\text{bare}} = \frac{\pi R}{g_{5a}^2} + \frac{1}{g_{0a}^2} + \frac{\gamma_a}{24\pi^3} \Lambda \pi R
\]
and all calculable dependences of \( g_a^2(p) \) on \( R \) and the fundamental mass parameters are encoded in \( \Delta_a \). In the following, we will simply call \( \Delta_a \) the KK threshold corrections.

In orbifold GUT, the unified higher dimensional gauge symmetry \( G_{GUT} \) assures that both \( g_{5a}^2 \) and the power-law divergent corrections \( \gamma_a \Lambda \) are universal. On the other hand, since \( G_{GUT} \) is generically broken at the fixed points by boundary condition, the bare fixed point gauge couplings, \( g_{0a}^2 \) and \( g_{2a}^2 \), are neither universal nor calculable. However in models with an orbifold radius significantly larger than the cutoff length scale, i.e. \( R\Lambda \gg 1 \), which would be required for 5D orbifold field theory to be a useful theoretical framework, we have \( g_{5a}^2 \approx \pi R g_{a}^2 \gg 1/\Lambda \), implying that the theory is strongly coupled at the cutoff scale \( \Lambda \). A simple naive dimensional analysis suggests that the most plausible parameter region is given by [16]
\[
\frac{1}{g_{5a}^2} \approx \frac{1}{\pi R} = O\left( \frac{\Lambda}{24\pi^3} \right), \quad \frac{1}{g_{0a}^2} \approx \frac{1}{g_{2a}^2} = O\left( \frac{1}{8\pi^2} \right).
\]
In this strong coupling limit, the uncalculable fixed point gauge couplings can be safely ignored, yielding
\[
\left( \frac{1}{g_a^2} \right)_{\text{bare}} = \frac{1}{g_{GUT}^2} + O\left( \frac{1}{8\pi^2} \right),
\]
and then the differences between low energy gauge couplings are dominated by the renormalization group (RG) running due to zero modes and also the calculable KK threshold corrections \( \Delta_a \).
The computation of KK threshold corrections involves the summation over all massive KK modes. However the involved KK summation can be replaced by a contour integration with a pole function $P(q)$ which has (simple) poles at $q = m_n$ where $\{m_n\}$ denote the KK mass eigenvalues [17]. The pole function we will use here is given by

$$P(q) = \frac{N'(q)}{2N(q)} ,$$

where $N(q)$ has zeroes at $q = m_n$. Then the summation over KK modes can be replaced by a counter integral

$$\sum_{m_n} \int d^d p f(p, m_n) = \int \frac{dq}{2\pi i} \int d^d p \frac{N'(q)}{2N(q)} f(p, q) ,$$

which allows us to compute $\Delta_a$ without having the detailed knowledge of $\{m_n\}$. The actual computation of $\Delta_a$ using this prescription is somewhat tedious, but still straightforward. By computing the one-loop effective action of gauge field zero modes in this approach, we find [14]

$$\Delta_a = -\frac{1}{6} T_a (\phi_{++}^{(0)}) [\ln R_{++} + \ln(\Lambda/k) + \pi kR]$$

$$- \frac{1}{6} T_a (\phi_{++}) [\ln Q_{++} - \ln(\Lambda/k)]$$

$$- \frac{1}{6} T_a (\phi_{+-}) \ln Q_{+-} - \frac{1}{6} T_a (\phi_{-+}) \ln Q_{-+}$$

$$- \frac{1}{6} T_a (\phi_{--}) [\ln Q_{--} + \ln(\Lambda/k)]$$

$$- \frac{2}{3} T_a (\Psi_{++}) \left[ \ln(\Lambda/k) + \frac{1}{2} \pi kR + \ln \left( \frac{e^{(C_{++} - \frac{k}{2})\pi kR} - e^{-(C_{++} - \frac{k}{2})\pi kR}}{2 (C_{++} - \frac{k}{2})} \right) \right]$$

$$+ \frac{2}{3} T_a (\Psi_{+-}) (C_{+-} \pi kR - \frac{2}{3} T_a (\Psi_{-+}) C_{-+} \pi kR)$$

$$- \frac{2}{3} T_a (\Psi_{--}) \left[ \ln(\Lambda/k) + \frac{1}{2} \pi kR + \ln \left( \frac{e^{(C_{--} + \frac{k}{2})\pi kR} - e^{-(C_{--} + \frac{k}{2})\pi kR}}{2 (C_{--} + \frac{k}{2})} \right) \right]$$

$$+ \frac{1}{12} T_a (A_{++}^M) \left[ 21 \ln(\Lambda \pi R) + 22 \pi kR \right]$$

$$- \frac{11}{6} T_a (A_{+-}^M) \pi kR + \frac{11}{6} T_a (A_{--}^M) \pi kR$$

$$+ \frac{1}{12} T_a (A_{++}^M) \left[ 21 \ln(\Lambda \pi R) - \pi kR + 21 \ln \left( \frac{e^{\pi kR} - e^{-\pi kR}}{2\pi kR} \right) \right] ,$$

where the subscripts $\pm$ represent the $Z_2 \times Z'_2$ boundary conditions, $T_a (\Phi) = \text{Tr}(T_a^2(\Phi))$ is the Dynkin index of the gauge group representation $\Phi$, and $C_{zz',k}$ is the kink mass of $\Psi_{zz'}$. Here $\phi_{++}^{(0)}$ denotes a 5D complex scalar field having a zero mode, i.e. a scalar field whose bulk and brane mass parameters (see Eq.10) satisfy

$$B_0(\phi_{++}^{(0)}) = B_+(\phi_{++}^{(0)}) \equiv B_{++} ,$$

$$\sqrt{4 + A^2(\phi_{++}^{(0)})} = |2 - B_{++}| ,$$

while $\phi_{zz'}$ ($z, z' = \pm 1$) stand for complex scalar fields without zero mode. The functions that appear in $\Delta_a$ for 5D scalar fields are given by

$$Q_{++} = \frac{1}{2\alpha_{++}} \left[ (\alpha_{++} + B_{0++} - 2)(\alpha_{++} - B_{\pm++} + 2)e^{\alpha_{++}\pi kR}$$

$$- (\alpha_{++} + B_{\pm++} - 2)(\alpha_{++} - B_{0++} + 2)e^{-\alpha_{++}\pi kR} \right] ,$$

$$Q_{+-} = \frac{1}{2\alpha_{+-}} \left[ (\alpha_{+-} + B_{0+-} - 2)e^{\alpha_{+-}\pi kR} + (\alpha_{+-} - B_{0+-} + 2)e^{-\alpha_{+-}\pi kR} \right] ,$$

$$Q_{--} = \frac{1}{2\alpha_{--}} \left[ (\alpha_{--} + B_{\pm--} + 2)e^{\alpha_{--}\pi kR} + (\alpha_{--} + B_{\pm--} - 2)e^{-\alpha_{--}\pi kR} \right] ,$$

$$\text{for } z = z' = \pm 1;$$

$$Q_{zz'} = \frac{1}{2\alpha_{zz'}} \left[ (\alpha_{zz'} + B_{0zz'} - 2)(\alpha_{zz'} - B_{\pmzz'} + 2)e^{\alpha_{zz'}\pi kR}$$

$$- (\alpha_{zz'} + B_{\pmzz'} - 2)(\alpha_{zz'} - B_{0zz'} + 2)e^{-\alpha_{zz'}\pi kR} \right] ,$$

$$\text{for } z = z' = \pm 1;$$
\[ Q_{--} = \frac{1}{2a_{--}} [e^{\alpha_{--} \pi kR} - e^{-\alpha_{--} \pi kR}], \]
\[ R_{++} = \frac{1}{2(1 - B_{++})} [e^{(1-B_{++})\pi kR} - e^{-(1-B_{++})\pi kR}], \]

where
\[ \alpha_{zz'} = \sqrt{4 + A_{zz'}^2} \]

and the parameters \( A_{zz'}, B_{0zz'}, B_{\pi zz'}, C_{zz'} (z, z' = \pm 1) \) are defined through the scalar and fermion masses (see Eq.(10)): \[
\begin{align*}
\Delta_{zz'} &= \frac{2k}{R} \left| B_{0zz'} \delta(y) - B_{\pi zz'} \delta(y - \pi) \right|, \\
M_{zz'} &= C_{zz'} \kappa.
\end{align*}
\]

The one-loop beta function coefficients \( b_a \) in (12) are given by
\[
b_a = -\frac{11}{3} T_a(A_+^M) + \frac{1}{6} T_a(A_-^M) + \frac{1}{3} T_a(\phi_+^0) + \frac{2}{3} T_a(\Psi_+) + \frac{2}{3} T_a(\Psi_-),
\]

which can be easily understood by noting that \( A_{++}^M \) gives a massless 4D vector, \( A_{--}^M \) gives a massless real 4D scalar, and \( \Psi_{zz} \) with \( z = z' = \pm 1 \) gives a massless 4D chiral fermion.

In the above, we considered only the KK threshold corrections which are \( \text{parametrically enhanced by the large logarithms of scale ratios} \), while ignoring the scheme-dependent constant parts of order unity. There appear a variety of \( \omega \)'s from \( \ln(e^{-\pi kR}) \) (\( \omega = 1, \alpha_{zz'}, C_{zz'} \)) to \( \ln(\Lambda/k) \) and \( \ln(\omega' \pi kR) \) (\( \omega' = 1, \alpha_{zz'}, B_{0zz'}, B_{\pi zz'}, C_{zz'} \)). It is obvious that \( \Delta_a = \mathcal{O}(\pi MR) \) in general, where \( e^{-\pi MR} \) corresponds to either the warp factor or the small Yukawa couplings of quasi-localized fermions. Thus, in orbifold field theories in which the 4D graviton and/or matter fermions are quasi-localized to generate hierarchically different scales and/or Yukawa couplings, we have
\[
\Delta_a = \mathcal{O}(\ln e^{-\pi kR}) \quad \text{and/or} \quad \mathcal{O}(\ln y).
\]

The expression of \( \Delta_a \) in (16) is based on the assumption that there exists a large mass gap between the lightest KK mass \( (M_{KK}) \) and the zero mode masses. Note that all KK states are treated as superheavy, while all zero modes are considered to be massless. The low energy gauge couplings at \( p \) below \( M_{KK} \) (but above the zero mode masses) are determined as (12) at one-loop approximation. In many 5D orbifold field theories, we have
\[
\frac{M_{KK}}{M_W} \gg \frac{\Lambda}{M_{KK}}
\]

by many orders of magnitude, where \( M_W \) is the weak scale. Then the dominant part of higher order corrections (beyond one-loop) to low energy couplings at \( M_W \) come from the energy scales below \( M_{KK} \), which can be systematically computed within 4D effective theory. To include those higher order corrections, one can start with the matching condition at \( M_{KK} \):
\[
\frac{1}{g_5^2(M_{KK})} = \left( \frac{1}{g_5^2} \right)_{\text{bare}} + \frac{1}{8\pi^2} \left[ \Delta_a + b_a \ln \left( \frac{\Lambda}{M_{KK}} \right) \right],
\]

where \( \Delta_a \) are given by (16), and then subsequently perform two-loop RG analysis over the scales between \( M_{KK} \) and \( M_W \).

In some case, there can be another large mass gap between the lightest KK mass \( M_{KK} \) and the next lightest KK mass \( M'_{KK} \). In fact, such additional mass gap is quite common in orbifold field theories in which the scale and/or Yukawa hierarchies are generated by the quasi-localization of 4D graviton and/or fermions. In such case with
\[
\frac{M_{KK}}{M_W} \gg \frac{M'_{KK}}{M_{KK}} \gg \frac{\Lambda}{M'_{KK}},
\]

the next important higher order corrections would come from energy scales between \( M_{KK} \) and \( M'_{KK} \). Those higher order corrections can be included by performing the two-loop RG analysis starting from \( M'_{KK} \). The corresponding matching condition at \( M'_{KK} \) is given by
\[
\frac{1}{g_5^2(M'_{KK})} = \left( \frac{1}{g_5^2} \right)_{\text{bare}} + \frac{1}{8\pi^2} \left[ \Delta'_a + b'_a \ln \left( \frac{\Lambda}{M'_{KK}} \right) \right].
\]
\[ \beta_a = b_a + \delta b_a, \]
\[ \Delta_a' = \Delta_a - \delta b_a \ln \left( \frac{\Lambda}{M_{KK}} \right) \]

for \( \Delta_a \) given by (16). Here \( b_a \) denote the one-loop beta function coefficients due to zero modes, while \( \delta b_a \) denote the coefficient due to the lightest KK states.

**IV. 4D SUPERGRAVITY CALCULATION**

In supersymmetric 5D theories, one-loop low energy gauge couplings can be computed using the gauged \( U(1)_R \) symmetry and chiral anomaly of 5D SUGRA on orbifold and also the known properties of gauge couplings in 4D effective SUGRA [11]. In this section, we discuss this alternative way to compute the KK threshold corrections in supersymmetric theories, and show that the SUGRA results agree with the results of the previous section. To proceed, let us briefly discuss supersymmetric 5D theory on \( \text{AdS}_5 \). The theory contains two types of 5D supermultiplets other than the SUGRA multiplet. One is the hypermultiplet \( \mathcal{H} \) containing two 5D complex scalar fields \( h^i \) \((i = 1, 2)\) and a Dirac fermion \( \Psi \), and the other is the vector multiplet \( \mathcal{V} \) containing a 5D vector \( A_M \), real scalar \( \Sigma \) and a symplectic Majorana fermion \( \lambda^i \). In supersymmetric model, all 5D scalar fields and their superpartner fermions have
\[ B_0 = B_\pi = B, \quad \sqrt{4 + A} = |2 - B|, \quad C = \pm (3 - 2B)/2. \]

See Eqs. (10) for the definitions of \( A, B_0, \pi \) and \( C \). Also the \( U(1)_R \) symmetry is gauged with the graviphoton \( \Omega_M \) in the following way [11]:
\[ D_M h^i = \partial_M h^i - \frac{3}{2} (\sigma_3)^j \partial_j h^i + \ldots \]
\[ D_M \Psi = \partial_M \Psi + iC\epsilon(y)\Omega_M \Psi + \ldots \]
\[ D_M \lambda^i = \partial_M \lambda^i - \frac{3}{2} (\sigma_3)^j \epsilon(y)\Omega_M \lambda^j + \ldots, \]

where \( \Psi \) has a kink mass \( C\epsilon(y) \) and the ellipses stand for the couplings with other gauge fields. Taking into account the \( Z_2 \times Z_2' \) parity, the supermultiplet structure is given by
\[ \mathcal{H}_{zz'}(C) = \left( h^i_{zz'}(B = \frac{3}{2} - C), h^i_{zz}(B = \frac{3}{2} + C), \Psi_{zz'}(C) \right), \]
\[ \mathcal{V}_{zz'} = \left( A^M_{zz} = (A^\mu_{zz'}, A^\sigma_{zz'}(B = 2)), \lambda^i = \lambda^i_{zz'}(C = \frac{1}{2}), \Sigma_{zz'}(B = 2) \right), \]

where \( z, z' = \pm 1, \quad \bar{z} = -z, \quad \bar{z}' = -z' \), \( B \) is the fixed point mass parameter and \( C \) is the kink mass parameter.

Let us assume that our 5D theory is compactified in a manner preserving \( D = 4, N = 1 \) supersymmetry. Then the low energy physics can be described by a 4D effective SUGRA action which can be written as
\[ S_{4D} = \int d^4x \left[ \int d^4\theta \left\{ -3 \exp \left( -\frac{K}{3} \right) \right\} + \left( \int d^2\theta \frac{1}{4} f_a W^a W^a + h.c. \right) \right], \]

where \( W^a \) is the chiral spinor superfield for the 4D gauge multiplet and the 4D SUGRA multiplet is replaced by their vacuum expectation values. The Kähler potential \( K \) can be expanded in powers of generic gauge-charged chiral superfield \( Q \):
\[ K = K_0(T, T^*) + Z_Q(T, T^*)Q^* e^{-\mathcal{V}} Q + \ldots, \]

where \( T \) denotes the radion superfield whose scalar component is given by
\[ T = R + i\Omega_5, \]

where \( \Omega_5 \) is the fifth-component of the 5D graviphoton, and the gauge kinetic function \( f_a \) is a holomorphic function of \( T \). Then the one-loop gauge couplings in effective 4D SUGRA can be determined by \( f_a \) containing the one-loop
threshold corrections from massive KK modes and also the tree-level Kähler potential $K$ [18]:

$$\frac{1}{g_5^2(p)} = \text{Re}(f_a) + \frac{b_a}{16\pi^2} \ln \left( \frac{M_{Pl}^2}{e^{-K_0/3}p^2} \right)$$

$$- \sum_Q T_a(Q) \frac{T_a(Q)}{8\pi^2} \ln \left( e^{-K_0/3}Z_Q \right) + \frac{T_a(\text{Adj})}{8\pi^2} \ln (\text{Re}(f_a)),$$

(29)

where $b_a = \sum T_a(Q) - 3T_a(\text{Adj})$ are the one-loop beta function coefficients and $M_{Pl}$ is the Planck scale of $g_{\mu\nu}$ which defines the momentum scale $p^2 = -g_{\mu\nu}\partial_\mu\partial_\nu$.

Let us consider the 4D effective SUGRA of a 5D theory which contains generic 5D hypermultiplets and vector multiplets, $\mathcal{H}_{zz'}$ and $\mathcal{V}_{zz'}$, with arbitrary boundary conditions. The vector multiplet $\mathcal{V}_{zz'}$ gives a massless 4D gauge multiplet containing $A^{zz'}_\mu$ whose low energy couplings are of interest for us, while $\mathcal{V}_{zz'}$ gives a massless 4D chiral multiplet containing $\Sigma_{zz} + iA^{zz'}_\mu$. $\mathcal{H}_{zz'}$ and $\mathcal{V}_{zz'}$ also give massless 4D chiral multiplets containing $h_1$ and $h_2$, respectively, whose tree level Kähler metrics are required to compute the one-loop gauge couplings (29). Other multiplets, i.e. $\mathcal{V}_{zz'}$, $\mathcal{V}_{zz'}$, $\mathcal{H}_{zz'}$ and $\mathcal{V}_{zz'}$ do not give any massless 4D mode.

Let $Z_Q (Q = H_{zz'}, H_{zz'}, V_{zz'})$ denote the Kähler metric of the 4D massless chiral superfields coming from the 5D multiplets $H_{zz'}, H_{zz'}$ and $V_{zz'}$. It is then straightforward to compute the tree level $Z_Q$ and also $f_a$ containing the $1$-loop threshold corrections from massive KK modes:

$$M_{Pl} = e^{-K_0/3} \Lambda^2 = \frac{M_5^2}{k} \left( 1 - e^{-k\pi(T+T^*)} \right),$$

$$e^{-K_0/3} Z_{H_{zz'}} = \frac{\Lambda}{(1 - C_{zz'})k} \left( e^{(1-C_{zz'})\pi k(T+T^*)} - 1 \right),$$

$$e^{-K_0/3} Z_{H_{zz'}} = \frac{\Lambda}{(1 + C_{zz'})k} \left( e^{(1+C_{zz'})\pi k(T+T^*)} - 1 \right),$$

$$e^{-K_0/3} Z_{V_{zz'}} = \frac{1}{\Lambda e^{k(T+T^*)} - 1},$$

$$f_a = \frac{\pi T}{g_5 a} + \frac{Z'}{8\pi^2} \left( \frac{3}{2} \sum_{H_{zz'}} T_a(\mathcal{V}_{zz'}) - \sum_{H_{zz'}} C_{zz'} T_a(\mathcal{H}_{zz'}) \right) k\pi T,$$

(30)

where $\Lambda$ and $M_5$ are the 5D cutoff scale and the 5D Planck scale, respectively, and $C_{zz'}$ is the kink mass of $H_{zz'}$. The scale ratio $\Lambda/M_5$ is not sensitive to the values of relevant physical variables like $\sigma_2$, $k$ and $R$, so can be set to be a constant of order unity, $\Lambda/M_5 \approx 1$. The KK threshold correction to $f_a$ can be entirely determined by the chiral anomaly w.r.t the following $\Omega_5$-dependent phase transformation:

$$\chi^a_i \rightarrow \left( e^{3iky_5\partial_5\sigma_2/2} \right)^i_j \chi^a_j, \quad \Psi \rightarrow e^{-i\sigma_2/2} \Psi.$$

(31)

Using the above results, we find [11]

$$(\Delta_a)_{\text{SUSY}} = -T_a(H_{zz'}) \left[ \ln \left( \frac{\Lambda}{k} \right) + C_{zz'} \pi kR + \ln \left( \frac{e^{(1-2C_{zz'})\pi kR} - 1}{1 - 2C_{zz'}} \right) \right]$$

$$+ C_{zz'} T_a(H_{zz'}) \pi kR - C_{zz'} T_a(H_{zz'}) \pi kR$$

$$- T_a(H_{zz'}) \left[ \ln \left( \frac{\Lambda}{k} \right) - C_{zz'} \pi kR + \ln \left( \frac{e^{(1+2C_{zz'})\pi kR} - 1}{1 + 2C_{zz'}} \right) \right]$$

$$+ T_a(V_{zz'}) \left[ \ln(\Lambda\pi R) + \frac{3}{2} \pi kR \right]$$

$$- \frac{3}{2} T_a(V_{zz'}) \pi kR + \frac{3}{2} T_a(V_{zz'}) \pi kR$$

$$+ T_a(V_{zz'}) \left[ \ln \left( \frac{\Lambda}{k} + \frac{1}{2} k\pi R + \ln \left( \frac{1 - e^{-2\pi kR}}{2} \right) \right) \right]$$

(32)

and also the 4D beta function coefficients

$$(b_a)_{\text{SUSY}} = -3T_a(V_{zz'}) + T_a(V_{zz'}) + T_a(H_{zz'}) + T_a(H_{zz'}).$$
The above result obtained by 4D SUGRA analysis perfectly agrees with the results of the previous section in supersymmetric limit. This provides a nontrivial check for the results of the previous section and assures that our results are regularization scheme-independent.

V. RESULTS FOR FLAT EXTRA DIMENSION

In fact, most of the 5D theories using the quasi-localization of fermions to generate hierarchical Yukawa couplings have been constructed on a flat orbifold, not on a slice of AdS. We thus summarize separately the KK threshold corrections in theories on flat orbifold, which can be obtained from the results for AdS orbifold, not on a slice of AdS

\[ \Delta_\alpha = \frac{1}{6} T_a (\phi^{(0)}_{++}) \ln \left( \frac{\Lambda (e^{m_{++} \pi R} - e^{-m_{++} \pi R})}{2m_{++}} \right) - \frac{1}{6} T_a (\phi_{++}) \ln \left( \frac{(m_{++} + \mu_{++})(m_{++} - \mu'_{++})e^{m_{++} \pi R} - (m_{++} - \mu_{++})(m_{++} + \mu'_{++})e^{-m_{++} \pi R}}{2m_{++} \Lambda} \right) - \frac{1}{6} T_a (\phi_{+-}) \ln \left( \frac{(m_{+-} + \mu_{+-})e^{m_{+-} \pi R} + (m_{+-} - \mu_{+-})e^{-m_{+-} \pi R}}{2m_{+-}} \right) - \frac{1}{6} T_a (\phi_{-+}) \ln \left( \frac{(m_{-+} - \mu'_{-+})e^{m_{-+} \pi R} + (m_{-+} + \mu'_{-+})e^{-m_{-+} \pi R}}{2m_{-+}} \right) - \frac{2}{3} T_a (\Psi_{++}) \ln \left( \frac{\Lambda (e^{m_{++} \pi R} - e^{-M_{++} \pi R})}{2M_{++}} \right) - \frac{2}{3} T_a (\Psi_{+-}) \ln \left( e^{-M_{+-} \pi R} \right) - \frac{2}{3} T_a (\Psi_{-+}) \ln \left( e^{M_{-+} \pi R} \right) - \frac{2}{3} T_a (\Psi_{-}) \ln \left( \frac{\Lambda (e^{M_{-} \pi R} - e^{-M_{-} \pi R})}{2M_{-}} \right) + \frac{21}{12} \left[ T_a (A^M_{++}) + T_a (A^M_{-+}) \right] \ln (\Lambda \pi R) \]

where \( m_{zz'}, \mu_{zz'} \) and \( \mu'_{zz'} \) denote the bulk and brane masses of \( \phi_{zz'} \),

\[ \hat{m}^2 = m^2 + \frac{2}{R} [ \mu \delta(y) - \mu' \delta(y - \pi) ] \]

and \( M_{zz'} \) is the kink mass of \( \Psi_{zz'} \). Again, \( \phi^{(0)}_{++} \) is a 5D scalar field having a zero mode, i.e. a scalar field with \( \mu = \mu' = m \), and \( \phi^{++} \) stands for scalar fields without zero mode. Similarly, in supersymmetric case, we have

\[ (\Delta_\alpha)_{\text{SUSY}} = -T_a (H_{++}) \ln \left( \frac{\Lambda (e^{M_{++} \pi R} - e^{-M_{++} \pi R})}{2M_{++}} \right) - T_a (H_{+-}) \ln \left( e^{-M_{+-} \pi R} \right) - T_a (H_{-+}) \ln \left( e^{M_{-+} \pi R} \right) - T_a (H_{-}) \ln \left( \frac{\Lambda (e^{M_{-} \pi R} - e^{-M_{-} \pi R})}{2M_{-}} \right) + T_a (V_{++}) \ln (\Lambda \pi R) + T_a (V_{-+}) \ln (\Lambda \pi R) \]

where \( M_{zz'} \) is the kink mass of the hypermultiplet \( H_{zz'} \).

Acknowledgements
This work is supported by KRF PBRG 2002-070-C00022 and KOSEF through Center for High Energy Physics of Kyungpook National University.

[1] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B429, 263 (1998).
[2] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999); Phys. Rev. Lett. 83, 4690 (1999).
[3] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D 61, 033005 (2000); E. A. Mirabelli and M. Schmaltz, Phys. Rev. D 61, 113011 (2000); D. E. Kaplan and T. M. Tait, JHEP 0111, 051 (2001); M. Kakizaki and M. Yamaguchi, arXiv:hep-ph/0110266; N. Haba and N. Maru, Phys. Rev. D 66, 055005 (2002); Y. Grossman and G. Perez, Phys. Rev. D 67, 015011 (2003); K. Choi, D. Y. Kim, I. W. Kim and T. Kobayashi, hep-ph/0305024.
[4] Y. Kawamura, Prog. Theor. Phys. 105, 691 (2001); Prog. Theor. Phys. 105, 999 (2001); A. Hebecker and J. March-Russell, Nucl. Phys. B 613, 3 (2001); Nucl. Phys. B 625, 128 (2002); G. Altarelli and F. Feruglio, Phys. Lett. B 511, 257 (2001); L. J. Hall and Y. Nomura, Phys. Rev. D 64, 055003 (2001); Phys. Rev. D 66, 075004 (2002); H. D. Kim, J. E. Kim and H. M. Lee, JHEP 0206, 048 (2002); H. D. Kim and S. Raby, JHEP 0301, 056 (2003); JHEP 0307, 014 (2003).
[5] A. Hebecker and J. March-Russell, Phys. Lett. B 541, 338 (2002); A. Hebecker, J. March-Russell and T. Yanagida, Phys. Lett. B552, 229 (2003); R. Kitano and T. j. Li, Phys. Rev. D 67, 116004 (2003).
[6] K. Choi, Phys. Rev. D 37, 1564 (1988); V. S. Kaplunovsky, Nucl. Phys. B 307, 145 (1988) [Erratum-ibid. B 382, 436 (1992)].
[7] K. R. Dienes, E. Dudas and T. Gherghetta, Phys. Lett. B436, 55 (1998); Nucl. Phys. B537 47 (19990.
[8] R. Contino, L. Pilo, R. Rattazzi and E. Trincherini, Nucl. Phys. B 622, 227 (2002).
[9] L. J. Hall and Y. Nomura, Phys. Rev. D65, 125012 (2002).
[10] A. Hebecker and A. Westphal, Annals Phys. 305, 119 (2003); J.F. Oliver, J. Papavassiliou and A. Santamaria, Phys. Rev. D67, 125004 (2003);
[11] K. Choi, H. D. Kim and I. W. Kim, JHEP 0211, 033 (2002); JHEP 0303, 034 (2003);
[12] A. Pomarol, Phys. Rev. Lett. 85, 4004 (2000); L. Randall and M. D. Schwartz, JHEP 0111, 003 (2001); Phys. Rev. Lett. 88, 081801 (2002).
[13] W. D. Goldberger and Ira Z. Rothstein, Phys. Rev. Lett. 89, 131601 (2002); hep-ph/0303158; hep-th/0208060; K. Agashe, A. Delgado and R. Sundrum, Nucl. Phys. B643, 172 (2002); Annals Phys. 304, 145 (2003); R. Contino, P. Creminelli and E. Trincherini, JHEP 0210, 029 (2002).
[14] K. Choi and I. W. Kim, Phys. Rev. D 67, 045005 (2003).
[15] K. Choi, I. W. Kim and W. Y. Song, hep-ph/0307365.
[16] Z. Chacko, M. A. Luty and E. Ponton, JHEP 0007, 036 (2000); Y. Nomura, Phys. Rev. D 65, 085036 (2002).
[17] S. Groot Nibbelink, Nucl. Phys. B 619, 373 (2001); R. Contino and A. Gambassi, J. Math. Phys. 44, 570 (2003).
[18] V. Kaplunovsky and J. Louis, Nucl. Phys. B 422, 57 (1994).