The electromagnetic instabilities propagation in weak relativistic quantum plasmas

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Abstract

The electromagnetic instabilities excited by the temperature anisotropy have been always one of the interesting issues in real high-density physical systems, where the relativistic and quantum effects due to spin can be important. This paper discusses the case where plasma is not strongly coupled but is still in regimes where a classic plasma description is not fully adequate. The length scale of the plasma can be larger than the de-Broglie length so that the quantum effects relevance to the spin can be significant. In addition, the only relativistic effects are due to the electrons spin, for example the spin-orbit coupling effect (the weak (semi) relativistic effects). Obtained results imply that these effects can not be important in the ICF subjects while these can lead to significant results in the astrophysical subjects because of the strong magnetic fields. It is found that the weakly relativistic effects can cover the magnetic dipole force and the spin precession effects so that the growth rate of the instability can increase compared to the non relativistic spin polarized cases.

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Indeed, it is expected that the probability of electron capture in the background magnetic fields and the particle energy dissipation will be reduced so that there will be a high portion of free energy in the system.

**Key words:** Spin, Relativistic effects, Spin-Orbit coupling, Electromagnetic instability, Temperature anisotropy.
1 Introduction

Plasma includes a highly complex physical system where there are a wide variety of theoretical methods with a common general principle to introduce them. It can be expected that one method can be transferred between different plasma system, for example transfer method for treating nonlinearities in classical plasmas to quantum plasmas [1]. In this order, lowest order corrections have been applied in terms of nonlocal terms related to the tunneling aspects of the electrons. In other words, the electron spin (the possibility of large-scale magnetization of plasma) can be effective on the dynamics of classical systems where the spin waves can be excited by flux of the intensity neutrons in dens magnetized plasmas [2]. In addition, the relativistic effects may have significant effects, too. In this case, for example in laboratory plasmas such as laser generated plasmas and in nature in particular for planetary interiors and stars [3], combining the quantum relativistic effects need more complex dynamic methods for obtaining accurate descriptions of a host of phenomena. There have been more studies related to the quantum effects due to the particle dispersive and the Fermi pressure where the magnetization current and dipole force due to the electron spin are included [2,4-6]. Studies show that, the spin quantum hydrodynamic theories (for example fluid models like MHD or two-fluid models) are not relevant to conventional laboratory or astrophysical plasmas while the kinetic theory needs more exact studies [7]. One of the most accurate theories is based on the Kandanoff-Bay kinetic equations [8]. This theory (containing memory effects (non local terms) both in space and time) can be effective even on time scale shorter than the typical relaxation time of system but is not specially adopted to some of the applications for example, a high-intensity laser-plasma interaction and high-energy density physics. Therefore, here, we are interested in a method similar to Asenjo et al. [9] where plasmas are not strongly coupled but still in regimes where, a classical plasma description is not fully adequate.
Notice that, increasing effects of the magnetic fields can add more phenomena to plasma-wave problems where anisotropy problem is one of the most important. Therefore, here, we are particularly interested in velocity space instabilities excited by the temperature anisotropy in weak relativistic spin-1/2 quantum plasmas. In this order, in section 2, first the dispersion relation and as a result, the growth rate of the instabilities are derived in presence of the weak (semi-) relativistic effects. A comparison of results to our previous study will be made in section 3 for two real physical systems ie. ICF and astrophysical plasmas and finally, our main conclusions are summarized in section 4.

2 Basic theory

Introducing suitable evolution equation is based on the Asenjo et al. work [9] where a semi-relativistic transformation is introduced by applying the Foldy-Wouthuysen transformation for particle in external fields [9,10]:

\[
\hat{H} = mc^2 + q\phi + \frac{1}{2m}\left(\hat{p} - \frac{q}{c} A\right)^2 - \frac{q\hbar}{2mc}\sigma \cdot B
\]

\[
+ \frac{\hbar^2 q}{8m^2c^2}\nabla \cdot E - \frac{q\hbar}{4m^2c^2}\sigma \cdot \left[ E \times \left(\hat{p} - \frac{q}{c} A\right)\right]
\]

\[
- \frac{i\hbar^2 q}{8m^2c^2}\sigma \cdot \nabla \times E + \frac{1}{8m^2c^2}\left(\hat{p} - \frac{q}{c} A\right)^4
\]  

(1)

where \( m \) is the electron mass, \( q = -e \) is the electron charge, \( c \) is the speed of light and \( \hat{p}, \phi \) and \( A \) are the momentum operator, the scalar and vector potential, respectively. The quantity \( \hbar \) is the reduced Planck constant, \( \sigma \) denotes a vector containing the 2 \( \times \) 2 Pauli matrices and \( B \) and \( E \) are the magnetic and electric field, respectively. The first four terms constitute the Pauli Hamiltonian while the fifth and eighth terms are the Darwin and mass-velocity correction terms, respectively. The sixth and seventh terms together give Thomas precession and spin-orbit coupling due to higher order corrections, too.

It is well known, plasma is introduced in a quantum regime where the most suitable
kinetic model is the Wigner model \cite{11} (in the absence of interaction between particles) when the characteristic de-Broglie wave length is equal to the Fermi wave length. The quantum effects associated with the particle dispersive effects disappear at the spatial length-scale higher than the de-Broglie length while the only reminded quantum effects are relevant to the spin of particle. Here, according to this, the evolution equation can be found for above Hamiltonian using the Wigner transformation in phase space and the Q-transformation in spin space as \cite{9}:

\[
\frac{\partial f}{\partial t} + \left\{ \frac{\dot{p}}{m} + \frac{\mu}{2mc} E \times (s + \nabla s) \right\} \cdot \nabla_x f \\
+ q \left( E + \frac{1}{c} \left\{ \frac{\dot{p}}{m} + \frac{\mu}{2mc} E \times (s + \nabla s) \right\} \times B \right) \cdot \nabla_p f \\
+ \frac{2\mu}{\hbar} s \times \left( B - \frac{P \times E}{2mc} \right) \cdot \nabla_s f + \mu (s + \nabla_s) \cdot \partial^i_x \left( B - \frac{P \times E}{2mc} \right) \partial^i_p f \\
- \frac{\hbar^2 q}{8mc^2} \partial^i_x (\nabla \cdot E) \partial^i_p f = 0
\]  

(2)

here, the only terms up to first order in the velocity is kept and the gamma factor is put to unity where the relation between the rest frame spin, \(s\), and the spatial part of the spin four-vector, \(S\), implies on \(S = s + \left[ \gamma^2 / (\gamma + 1) \right] (v \cdot s) v/c^2\). The quantity \(\mu = \frac{\hbar q}{4mc}\) is the intrinsic magnetic moment with the spin factor \(g = 2.00232\). Notice that, the Hamiltonian (1) and the Dirac theory have been started from the exactly value \(g = 2\) while we will use the value \(g = 2.00232\) in next. This may suggest that the corrections (the QED corrected value \cite{12}) should be added to Hamiltonian. It can add new terms to the evolution equation so that, extra terms are smaller than those kept by a factor of the order \((g - 2)\). Since, the QED-corrections to the Dirac Hamiltonian will not be included in theory otherwise modifying the spin \(g\)-factor value. The distribution function, \(f\), is sum of the time-independent unperturbed and time-dependent perturbed distribution functions as \(f = f_0 + f_1\), where it is normalized to the number density. In the absence of the background electric field and in the presence of the background magnetic field, such as
\[ B = B_0 + B_1, \text{ Eq. (2) can be rewritten as:} \]

\[
\frac{\partial f_1}{\partial t} + \frac{p}{m} \cdot \nabla_x f_1 + \frac{q}{mc} p \times B_0 \cdot \nabla_p f_1 + \frac{2\mu}{\hbar} s \times B_0 \cdot \nabla_s f_1 = \\
-qE_1 \cdot \nabla_p f_0 - \frac{q}{mc} p \times B_1 \cdot \nabla_p f_0 - \mu \nabla_{xi} [B_1 \cdot (s + \nabla_s)] \nabla_{pi} f_0 \\
+ \frac{\mu}{2mc} \nabla_{xi} [(p \times E_1) \cdot (s + \nabla_s)] \nabla_{pi} f_0 - \frac{q}{mc} \frac{2\mu}{\hbar} [E_1 \times (s + \nabla_s)] \times B_0 \cdot \nabla_p f_0 \\
- \frac{2\mu}{\hbar} s \times B_1 \cdot \nabla_s f_0 + \frac{\mu}{\hbar mc} s \times (p \times E_1) \cdot \nabla_s f_0
\]

(3)

where the last term in Eq. (2) is ignored because of the smaller contribution compare to other terms. In fact, we can ignore the two and higher orders of the Plank constant because of the applied assumption (the scale length larger than the de-Broglie length).

Notice that, different quantum effects can be contained in the unperturbed distribution function, such as Fermi-Dirac statistics, Landau quantization, and spin-splitting of the energy states [6, 13]. The electrons behavior as a degenerate electron gas follows the known Fermi-Dirac statistics in plasmas with low temperature and high density. The Landau quantization (quantization of the perpendicular energy states) is important in the regime of strong magnetic fields or very low temperatures (when \( \frac{\hbar \omega_c}{K_B T} \rightarrow 1 \) with the electron cyclotron frequency, \( \omega_c \), and the Boltzmann constant, \( K_B \)). In the presence of spin quantum effects, the unperturbed distribution function must contain the properties of the spin space. In this situation, there are different spin states with different probability distributions for orienting the particles magnetic moment in the background magnetic field. For cases where the chemical potential, \( \mu_c \), is large and the difference between the nearby Landau levels is smaller than the thermal energy, the velocity (momentum) distribution approaches the classic Maxwellian distribution, while the remaining quantum effect is due to the probability distribution of the spin-up and spin-down population states. Thus, the distribution can be approximated by [14]:

\[
f_0 = \sum_{\nu=\pm 1} F_{0\nu} (p) (1 + \nu \cos \theta_s)
\]

(4)
where the function $F_{0\nu}$ is the normalized function of the momentum variables and the indexes $+$ and $-$ define the spin-up and the spin-down states, respectively. Now, let us assume that the background magnetic field is static, homogeneous, and points in the $\hat{z}$-direction, ie. $B_0 = B_0 \hat{z}$ and the unstable electromagnetic waves propagate in the direction of the $z$-axis, such that the wave vector will be defined as $\vec{k} = k_z \hat{z}$. Therefore $f_1$ can be expanded in eigenfunctions to the operator of the right-hand side as:

$$f_1 = \frac{1}{2\pi} \sum_{a,b=-\infty}^{\infty} g_{a,b}(p_\perp, p_z, \theta_s) e^{-i(a\phi_p + b\phi_s)} + c.c. \quad (5)$$

where c.c. stands for complex conjugate and a standard ansatz of quasi-monochromatic harmonic variation on the perturbed quantities is used, ie. $\mathbf{E}_1 = \tilde{\mathbf{E}}_1 e^{i(\vec{k} \cdot \vec{x} - \omega t)}$, $\mathbf{B}_1 = \tilde{\mathbf{B}}_1 e^{i(\vec{k} \cdot \vec{x} - \omega t)}$ and $f_1 = \tilde{f}_1 e^{i(\vec{k} \cdot \vec{x} - \omega t)}$. Linearizing Eq. (3) can produce an exact form of the function $g_{a,b}$ where it will result to:

$$\tilde{f}_1 = \sum_{b=\pm 1} \frac{\mu_i}{\pi} \left( \tilde{B}_{1y} - \frac{p_z \tilde{E}_{1x}}{2mc} - \frac{ib}{2mc} p_z \tilde{E}_{1y} \cos 2\theta_s - ib \tilde{B}_{1x} \right)$$

$$\times \frac{\partial f_0}{\partial \phi_s} (\omega - k_z p_z/m - \omega_c) e^{-ib\phi_s}$$

$$- \sum_{b=\pm 1} \frac{\mu_i}{4mc} \left( b p_z \tilde{E}_{1x} + ip_z \tilde{E}_{1y} - 2mcB_1y + i2mc \tilde{B}_{1y} \right) k_z \cos \theta_s$$

$$\times \frac{\partial^2 f_0}{\partial \phi_s \partial p_z} (\omega - k_z p_z/m - \omega_c) e^{-ib\phi_s}$$

$$\pm 2mcB_{1y} \sin \theta_s - 2imc \tilde{B}_{1y} \sin (\theta_s) \right) k_z \times \frac{\partial f_0}{\partial p_z} (\omega - k_z p_z/m - \omega_c) e^{-ib\phi_s}$$

$$- \sum_{a=\pm 1} \frac{\mu_i}{4mc} \left( \sin \theta_s \left( -i B_0 \tilde{E}_{1x} - \frac{2ic}{\mu} p_z \tilde{B}_{1y} - \frac{2ac}{\mu} p_z \tilde{B}_{1x} \right) \right)$$

$$- \frac{2mc^2}{\mu} \tilde{E}_{1y} + \frac{2mc^2}{\mu} \tilde{E}_{1x} \frac{\partial f_0}{\partial p_\perp} (\omega - k_z p_z/m - \omega_c) e^{-ia\phi_p}$$

$$- \sum_{a=\pm 1} \frac{\mu_i}{4mc} \left( p_\perp \tilde{E}_{1y} \cos \theta_s + \frac{2a}{\mu} p_\perp \tilde{B}_{1x} + \frac{2a}{\mu} p_\perp \tilde{B}_{1y} \right) k_z$$

$$\times \frac{\partial f_0}{\partial p_z} (\omega - k_z p_z/m - \omega_c) e^{-ic\phi_s}$$

$$+ \sum_{a=\pm 1} \frac{\mu_i}{4mc} p_\perp (1 - a) \tilde{E}_{1z} \frac{\partial f_0}{\partial \phi_s} (\omega - k_z p_z/m - \omega_c) e^{-ia\phi_p - i\phi_s}$$

$$+ \sum_{a=\pm 1} \frac{\mu_i}{4mc} p_\perp (1 + a) \tilde{E}_{1z} \frac{\partial f_0}{\partial \phi_s} (\omega - k_z p_z/m - \omega_c + \omega_c) e^{-ia\phi_p + i\phi_s}$$
- \sum_{a=\pm 1} \frac{\mu a}{8mc} B_0 \tilde{E}_{1z} \sin \theta_s (1 - a) \frac{\partial f_0}{\partial p_\perp} e^{-ia\phi} e^{-iqs} e^{-i\omega_c \tau} \\
+ \sum_{a=\pm 1} \frac{\mu a}{8mc} B_0 \tilde{E}_{1z} \sin \theta_s (1 - a) \frac{\partial f_0}{\partial p_\perp} e^{ia\phi} e^{iqs} e^{-i\omega_c \tau} \\
- \sum_{a=\pm 1} \frac{\mu a}{8mc} B_0 \tilde{E}_{1z} \sin \theta_s (1 - a) \frac{\partial f_0}{\partial p_\perp} e^{ia\phi} e^{-iqs} e^{-i\omega_c \tau} \\
- \sum_{a=\pm 1} \frac{\mu a}{8mc} B_0 \tilde{E}_{1z} \cos \theta_s (1 + a) \frac{\partial^2 f_0}{\partial \theta_s \partial p_\perp} e^{-ia\phi} e^{-iqs} e^{-i\omega_c \tau} \\
- \sum_{a=\pm 1} \frac{\mu a}{8mc} B_0 \tilde{E}_{1z} \cos \theta_s (1 - a) \frac{\partial^2 f_0}{\partial \theta_s \partial p_\perp} e^{-ia\phi} e^{iqs} e^{-i\omega_c \tau} \\
- \sum_{a=\pm 1} \frac{\mu a}{8mc} p_\perp \tilde{E}_{1z} k_z \cos \theta_s (1 + a) \frac{\partial^2 f_0}{\partial \theta_s \partial p_\perp} e^{-ia\phi} e^{-iqs} e^{-i\omega_c \tau} \\
+ \sum_{a=\pm 1} \frac{\mu a}{8mc} p_\perp \tilde{E}_{1z} k_z \cos \theta_s (1 + a) \frac{\partial^2 f_0}{\partial \theta_s \partial p_\perp} e^{ia\phi} e^{iqs} e^{-i\omega_c \tau} \\
+ \sum_{a=\pm 1} \frac{\mu a}{4mc} B_0 \tilde{E}_{1y} \sin \theta_s \frac{\partial^2 f_0}{\partial \theta_s \partial p_\perp} e^{-ia\phi} e^{-iqs} e^{-i\omega_c \tau} \\
+ \sum_{a=\pm 1} \frac{\mu a}{4mc} B_0 \tilde{E}_{1z} \sin \theta_s \frac{\partial^2 f_0}{\partial \theta_s \partial p_\perp} e^{ia\phi} e^{iqs} e^{-i\omega_c \tau} \\
+ \sum_{a=\pm 1} \frac{\mu a}{4mc} p_\perp \tilde{E}_{1z} k_z \sin \theta_s \frac{\partial^2 f_0}{\partial \theta_s \partial p_\perp} e^{-ia\phi} e^{-iqs} e^{-i\omega_c \tau} \\
- \mu \tilde{B}_{1z} k_z \sin \theta_s \frac{\partial^2 f_0}{\partial \theta_s \partial p_\perp} e^{-ia\phi} e^{-iqs} e^{-i\omega_c \tau} \\
+ (\mu \tilde{B}_{1z} k_z \cos \theta_s - iq \tilde{E}_{1z}) \frac{\partial f_0}{\partial p_\perp} e^{-ia\phi} e^{-iqs} e^{-i\omega_c \tau} \\
\text{(6)}

Here, \( \omega_c = -\frac{eB_0}{mc} \) and \( \omega_{cg} = \frac{2\mu B_0}{\hbar} \) are the electron cyclotron frequency and the spin precession frequency, respectively.

Derivation of the general dispersion relation, in the same way as in the classical cases, is based on the relation \( \text{det} D_{ij} = 0 \) with \( D_{ij} = \delta_{ij} \left( 1 - \frac{e^2 k^2}{\omega^2} \right) - k_i k_j \frac{c^2}{\omega^2} \delta_{ij} + i\frac{\sigma_{ij}}{\epsilon_0 \omega} \), where the quantities \( \delta_{ij}, \epsilon_0 \) and \( \sigma_{ij} \) are the Kronecker delta function, the permittivity constant and the conductivity tensor, respectively. The conductivity tensor is proportional to the current density and the electric field as \( J_i = \sum_j \sigma_{ij} E_j \). Here, the total current density
includes three different contributions so that:

\[ J = J_F + \nabla \times M + \frac{\partial P}{\partial t} \]  

where, the first term is introducing the free current density, the second and last ones are introducing the magnetization, \( M \), and the polarization, \( P \), contribution, respectively, due to the spin which are defined respectively as:

\[ J_F = q \int d\Omega \left( \frac{p}{m} + \frac{3\mu}{2mc} E \times s \right) f \]  

(8)

\[ M = 3\mu \int d\Omega s f \]  

(9)

and

\[ P = -3\mu \int d\Omega \frac{s \times p}{2mc} f \]  

(10)

Here, \( d\Omega = d^3vd^2s \) is the integration measure performed over the three velocity variable in the cylindrical space (which will be transformed to the momentum variations for simplicity) and the two spin degrees of freedom in the spherical space. Notice that, the current density is only included the electron contribution and the ions contributions are ignored because of the their larger mass and intrinsic magnetic moment lower than the electrons one. Now, let us restrict the results to the specific situation. we consider the polarization as \( \mathbf{E}_1 = E_1 \hat{y} \mathbf{y} \) so that \( \mathbf{B}_1 = B_{1x} \hat{x} \). This polarization can be justified only when \( \sigma_{yy} \gg \sigma_{xy}, \sigma_{zy} \), so the dispersion relation can be presented (by the Amperes law) as follows:

\[ \omega^2 - c^2 k_z^2 + \frac{i\omega}{\varepsilon_0} \sigma_{yy} = 0 \]  

(11)

where

\[ \sigma_{yy} = + \sum_{\nu=+,-} \nu \frac{q^2 \mu_i}{16m^2c^2} \int p_\perp B_y \frac{\partial F_{nu}}{\partial p_\perp} \frac{d^3p}{(\omega - k_z p_z/m - \omega_c)} \\
+ \sum_{\nu=+,-} \nu \frac{q\mu_i}{8m^2c} \int p_\perp^2 k_z^2 \frac{\partial F_{nu}}{\partial p_z} \frac{d^3p}{(\omega - k_z p_z/m - \omega_c)} \\
- \sum_{\nu=+,-} \frac{q^3 \mu_i}{4m^2} \int p_\perp^2 k_z \frac{\partial F_{nu}}{\partial p_z} \frac{d^3p}{(\omega - k_z p_z/m - \omega_c)} \]
The quantity $T_{sp}$ is defined as the spin temperature [14]. In addition to the free energy
of the velocity space, the plasma can be confronted with another free energy, which is supposed to be the difference between the high- and low-energy spin. In that case, the number of particles in the two spin states does not correspond to thermodynamic equilibrium. As we know, we have two instabilities, namely spin instability and velocity space instability. The first one comes from the spin temperature, $T_{sp}$, and the second one comes from the kinetic energy; the two together form the source of such instability. In fact, it is expected that spin instabilities exist together with velocity space instabilities when the deviation in the spin temperature from the kinetic anisotropic temperature can be the source of this instability. Notice that when the only degree of particle freedom is spin, and all particles have the lowest energy (down spin state), the entropy will be equal to zero. In this case, adding the energy and flip-up particle spin leads to increase in the entropy and positive temperature until reaching the maximum entropy, where one-half of particles have the down spin state. After that, increasing the particle number in the up spin state leads to a decrease in the entropy and temperature, so that when all particles have the up spin state, the entropy is equal to zero and the temperature is negative. In the presence of other degrees of freedom (here, kinetic degrees relevant to the velocity space), the condition is special. In this situation, when the spin variations are independent of the other freedom variables, the definition of spin temperature can be important. In this condition, in the presence of a strong external magnetic field, the coupling between the degrees of freedom of spin and velocity will be sufficiently weak, while the coupling between spin freedoms is strong; therefore the timescale of energy flow between the degrees of freedom is large and the spin temperature will be negative. Here it is assumed that the energy difference between the high- and low-energy spin states is small, but even this small value can be important for generating the free energy in the background magnetic field [14].

Now, replacing the above distribution in the Eq. (12), the dispersion relation can be
derived as:

\[
\begin{align*}
\omega^2 - c^2 k_z^2 &- \omega_p^2 (1 - \frac{T_z}{T_e}) + \frac{\omega h^2 k_z^2}{32 m_k B T_z} \omega_p^2 (2 + \zeta_2 Z(\zeta_2) + \zeta_1 Z(\zeta_1)) \\
&- \frac{\omega h}{8\sqrt{2\pi} m c^2} \frac{B_0 \omega^2}{K_B T_z} \frac{\sqrt{m}}{\sqrt{2m_k B T_z}} [Z(\zeta_4) + Z(\zeta_3)] + \frac{\omega h}{2} \frac{T_z}{T_e} [\zeta_4 Z(\zeta_4) + \zeta_3 Z(\zeta_3)] \\
&- \frac{\omega h}{16 c \omega_p} \frac{\sqrt{m}}{\sqrt{2m_k B T_z}} [\zeta_2 Z(\zeta_2) - \zeta_1 Z(\zeta_1)] + \frac{\omega h}{16 \sqrt{2m_k B T_z}} [\zeta_2 Z(\zeta_2) - \zeta_1 Z(\zeta_1)] \\
&+ \frac{3 \omega h}{160 m c^2} \frac{\omega_p^2 \zeta_2 \zeta_1}{\sqrt{2m_k B T_z}} [\zeta_2 Z(\zeta_2) - \zeta_1 Z(\zeta_1)] + \frac{\omega h}{16 \sqrt{2m_k B T_z}} \omega_p^2 \zeta_2 (\zeta_1) \\
&- \frac{\omega h}{16 \sqrt{2m_k B T_z}} \omega_p^2 \zeta_2 (\zeta_1) + \frac{\omega h}{16 \sqrt{2m_k B T_z}} \omega_p^2 \zeta_2 (\zeta_1) - \zeta_2 (\zeta_1) \\
&- \frac{\omega h}{128 m c^2} \frac{\omega_p^2}{\sqrt{2m_k B T_z}} [\zeta_4 z(\zeta_4) - z(\zeta_3)] + \frac{\omega h^2}{128 m c^2} \frac{\omega_p^2}{\sqrt{2m_k B T_z}} [\zeta_4 z(\zeta_4) + \zeta_3 z(\zeta_3)] \\
&+ \frac{\omega h}{32 m c^2} \frac{\omega_p^2}{\sqrt{2m_k B T_z}} (\sqrt{m} - \sqrt{m}) \frac{T_z}{T_e} - \frac{\omega h}{16 \sqrt{2m_k B T_z}} \omega_p^2 (\frac{T_z}{T_e} - 1) [\zeta_4 Z(\zeta_4) - \zeta_3 Z(\zeta_3)] \\
&- \frac{\omega h^2}{16 \sqrt{2m_k B T_z}} \omega_p^2 (\frac{T_z}{T_e} - 1) [\zeta_4 Z(\zeta_4) - \zeta_3 Z(\zeta_3)] = 0
\end{align*}
\]

(14)

The function \(Z(\zeta)\) is the plasma dispersion function [15] given by \(Z(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-x^2}}{x - \zeta} dx\) with the arguments \(\zeta_1 = \frac{\sqrt{m}}{\sqrt{2m_k B T_z}} (\omega + \omega_c)\), \(\zeta_2 = \frac{\sqrt{m}}{\sqrt{2m_k B T_z}} (\omega - \omega_c)\), \(\zeta_3 = \frac{\sqrt{m}}{\sqrt{2m_k B T_z}} (\omega + \omega_c)\), \(\zeta_4 = \frac{\sqrt{m}}{\sqrt{2m_k B T_z}} (\omega - \omega_c)\) and \(x = \frac{\sqrt{m}}{\sqrt{2m_k B T_z}}\). Here, let us follow the results to the non resonant electromagnetic instabilities. In this order, it is necessary, the arguments of the plasma dispersion function be comparable to or smaller than unity [16]. Therefore, we consider, two specific limiting conditions: the arguments of the plasma dispersion function higher than unity and the arguments smaller than unity. In the limit of arguments larger than the unity, the function \(Z(\zeta)\) can be approximated by \(Z(\zeta) = -\frac{1}{\zeta} - \frac{1}{\zeta} + \cdots\), where the dispersion relation will be a fully real relation that cannot produce a simplified form of the real frequency and the growth rate of the instability. In the opposite condition, the plasma dispersion function can be approximated as \(Z(\zeta) = -2\zeta + \cdots + i\sqrt{\pi}\), so that it is obtained (for the sub-luminal waves):

\[
c^2 k_z^2 + \omega_p^2 - i\sqrt{\pi} \frac{\omega h}{8 m c} \frac{\sqrt{m}}{\sqrt{2m_k B T_z}} (\frac{T_z}{T_e} - 1) - \omega_p^2 \frac{T_z}{T_e}
\]
The wave frequency, $\omega$, is sum of the real and imaginary parts which define the frequency and the growth rate of the instability as $\omega_r$ and $\omega_i$, respectively. According this, it is obtained:

$$\omega_r = 0$$

and

$$\omega_i = \frac{v_{th,z} k_z}{\sqrt{\pi}} \left[ 1 - \frac{(1 + c^2 k_z^2)}{T_\perp} + \frac{g^2 \hbar^2 k_z^2}{16 m \omega \omega_p} \left( 1 + 2 \frac{m \hbar k_z}{c} \omega \tanh \alpha \right) \right] \times \left[ 1 + \frac{\hbar \omega \omega_p}{4 \omega_p} \left( 1 + \frac{T_\perp}{T_\parallel} \right) + \omega \tanh \alpha \right] + \frac{T_\perp}{T_\parallel} \left[ \frac{g^2 \hbar^2}{4 \omega_p} \omega \omega_p \tanh \alpha + \frac{g^2 \hbar^2 k_z^2}{8 m K_B T_{sp}} \right]^{-1} \omega_k^2 \mu B_0 \tanh \alpha = 0 \quad (17)$$

where, the instability does not have any fluctuation in time and the results are similar to the non relativistic cases [14].

In the absence of the spin relativistic effects, $\frac{\hbar \omega \omega_p}{mc^2} \rightarrow 0$, the results goes to the previous results where the quantum effects due to the electrons spin were investigated in a fully non relativistic plasmas [14]. For the non resonant instabilities, it is necessary the instability growth rate be larger than the instability frequency so that the growth condition of the instability is satisfied as follows:

$$k_z < k_{cutoff} = \left[ 1 - \frac{T_\perp}{T_\parallel} + \frac{g^2 \hbar}{8 K_B T_{sp}} \omega \tanh \left( \frac{\mu_e B_0}{K_B T_{sp}} \right) \right] \left[ \frac{\omega_p^2}{\omega_e^2} \frac{T_\parallel}{T_\perp} - \frac{g^2 \hbar^2}{16 m_e K_B T_{sp}} \right]^{-\frac{1}{2}} \quad (18)$$

The astrophysical and laboratory plasmas in particular inertial confinement fusion (ICF) are always two of the interesting plasma systems for studying the electromagnetic instabilities due to the temperature anisotropy, where these can play important roles because

\section{Physical systems}

The astrophysical and laboratory plasmas in particular inertial confinement fusion (ICF) are always two of the interesting plasma systems for studying the electromagnetic instabilities due to the temperature anisotropy, where these can play important roles because
of generating strong magnetic fields [17-19]. It is well known, these instabilities can be excited because of all laser-plasma energy deposition processes and as a result of the propagation of a shock wave in the first and second ones, respectively. In other hand, it is well known, the quantum effects can start playing a role in the high-density regime where, such dense relativistic plasmas can be observed in particular in stars and planetary interiors for astrophysical subjects and the transmission of the corona of the fuel plasma to the core of fuel pellet in ICF plasmas [20, 21]. Therefore, in the following work, the obtained results in the last section are investigated for the ICF and astrophysical subjects respectively.

3.1 Inertial confinement fusion plasmas (ICF)

The obtained growth rate (Eq. (17)) includes different parameters every one of which can play a significant role in exact numerical investigations, for example the temperature anisotropy fraction \( \frac{T_\perp}{T_z} \), the number density relevant to the plasma frequency, the field intensity relevant to the electron cyclotron and the spin precession frequency. The results show that, otherwise it is expected that the new effects lead to new findings but there will not be significant difference in values of the normalized growth rate compared to the last results in the fully non relativistic spin polarized cases [14] and even to the classical cases [22] because of the weak magnetic field (see the Fig. (1)). Here, increasing the strength of the magnetic field (equivalent to compression the cyclotron movement) leads to decreasing the growth rate of the instability where this can not be high because of the small contribution of the spin effects. The variation of the normalized growth rate is shown in Fig. (2) for two different strengths of the magnetic field 8\( T \) and 25000\( T \) and the fixed \( T_z = 5000eV \), \( \frac{T_\perp}{T_z} = 5 \), \( n_0 = 10^{30}m^{-3} \) and \( \frac{\Delta \nu}{\nu} = 2 \). The contribution of the sentences dependent on the weak relativistic effects can be affected by the variations of the temperature anisotropy fraction \( \frac{T_\perp}{T_z} \), too. The illustration, the Fig. (3), shows...
that decreasing this (equivalent to decreasing the free energy) can lead to decreasing the maximum normalized growth rate about 61.6% for example by decreasing the temperature anisotropy fraction about two units. In other words, variations of the the electron number density can be important for investigating the variations of the normalized growth rate. Here, it is observed, that decreasing the electron number density about 10 times can decrease the normalized growth rate about 68.75% (Fig. (4). Notice that, all of these variations can be equal to the obtained results for the non relativistic spin polarized cases by ignorable values in similar situation.

### 3.2 Astrophysical plasmas

Variations of the normalized growth rate are investigated for the astrophysical plasmas, too. It was observed, that the weak relativistic effects due to the spin will increase the instability growth rate compared to that our previous non relativistic work [14] (see the Fig. (5)). In fact, it is expected, the relativistic effects due to the spin, lead to increasing the free energy in the system and decreasing the electrons dissipation energy. Increasing of the instability growth rate can be affected by strength of the magnetic field. The results imply that, increasing the strength of the magnetic field about 10 times leads to decreasing the maximum normalized growth rate about 1.26%. Fig. (6) is illustrating the curve variations of the normalized growth rate for the strength of the magnetic field equal to $10^7 T$ and $10^8 T$ in the fixed $\frac{T_\perp}{T_z} = 5$, $n_0 = 10^{32} m^{-3}$, $\frac{T_m}{T_z} = 2$ and $T_z = 20000 eV$. The temperature anisotropy fraction can be important, too, where it can play a role in the sentences relevant to the weak relativistic effects. It is expected, that the normalized growth rate decrease about 63.4% by decreasing the values of the temperature anisotropy fraction about 2 unit (Fig.(7)). Notice that, here, increasing the temperature anisotropy fraction and strength of the magnetic field can lead to excited relativistic effects and the quantization Landau effects which are ignored here. Finally, the results are investigated
for the variation of the electron number density too. Fig. (8) shows, that decreasing the electron number density about 10 times can lead to decreasing the maximum value of the normalized growth rate about 68.79%.

4 Conclusion

In this paper, the weak relativistic effects due to the electrons spin are investigated on the electromagnetic instabilities excited by the temperature anisotropy for two real physical situations ie. the inertial confinement fusion and the astrophysical plasma. Calculation model is based on the kinetic theory. This order, the Foldy-Wouthuysen transformation for introducing the particle Hamiltonian in external fields and the the Wigner transformation in the phase space and the Q-transformation in the spin space were applied for the evolution equation at the spatial scale higher than the thermal de-Broglie length. The intrinsic relativistic effects (such as the relativistic mass correction) have been ignored and only the weak relativistic effects due to the electrons spin are considered where the only presented quantum effects are relevant to the spin. The results imply that, similar to our previous results, the weak relativistic spin polarized effect can not be effective on the electromagnetic instabilities in the ICF plasmas because of the low strength of the magnetic field while this is different for the astrophysical subjects. In the astrophysical plasmas, the weak relativistic effect due to the electrons spin leads to increasing the instability growth rate where the condition governed on growth of the instability is unchanged (compared to the non relativistic cases). In fact, here, it is expected, that the weak relativistic effect due to the electron spin lead to increasing the free energy in plasma so that the particles will be able to transmit in the magnetic field more simply.
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Figure 1: The normalized growth rate of the instability, $\frac{\omega_i}{\omega_{pe}}$, as a function of the wave number, $\frac{\omega}{\omega_{pe}}$, for the non-relativistic spin, the classical and the weekly relativistic spin ICF plasmas in the fixed $T_\perp = 5000eV$, $n_0 = 10^{30}m^{-3}$, $\frac{T_\perp}{T_z} = 5$, $B_0 = 8T$ and $\frac{T_{\parallel}}{T_z} = 2$. 
Figure 2: a) The normalized growth rate of the instability, $\frac{\omega}{\omega_{pe}}$, as a function of the wave number, $\frac{c k_z}{\omega_{pe}}$, for different values of magnetic field in the fixed $T_z = 5000 \text{eV}$, $\frac{T_{sp}}{T_z} = 2$ and $n_0 = 10^{30} \text{m}^{-3}$. b) The normalized growth rate is illustrated only for the sentences including spin effects.
Figure 3: The normalized growth rate of the instability, $\frac{\omega}{\omega_{pe}}$, as a function of the wave number, $\frac{c k_z}{\omega_{pe}}$, for different values of the temperature anisotropy fraction $T_{\perp}/T_z$ in the fixed $\frac{T_{\perp}}{T_z} = 2$, $n_0 = 10^{30} m^{-3}$, $T_z = 5000 eV$ and the magnetic field $8 T$ for ICF subjects.
Figure 4: The normalized growth rate of the instability, $\frac{\omega_i}{\omega_{pe}}$, as a function of the wave number, $c k_z / \omega_{pe}$ for different values of the electron number density in the fixed $\frac{T_{\perp}}{T_z} = 2$, $\frac{T_{\parallel}}{T_z} = 5$, $T_z = 5000eV$ and the magnetic field equal to $8T$. 
Figure 5: a) The normalized growth rate of the instability, $\frac{\omega}{\omega_{pe}}$, as a function of the wave number, $ck_z / \omega_{pe}$, for the non relativistic spin, the classical and the weekly relativistic spin astrophysical plasmas in the fixed $T_z = 20000\text{eV}$, $n_0 = 10^{32}\text{m}^{-3}$, $T_\perp / T_z = 5$, $B_0 = 10^8\text{T}$ and $T_{\text{sp}} / T_z = 2$. b) The variations of the normalizes growth rate in the part (a) at the larger scale.
Figure 6: The normalized growth rate of the instability, $\frac{\omega_i}{\omega_{pe}}$, as a function of the wave number, $ck$, in the fixed $T_z = 20000$ eV, $n_0 = 10^{32} m^{-3}$, $T_\perp/T_z = 5$ and $T_\parallel/T_z = 2$ for different values of the magnetic field in the astrophysical subjects.
Figure 7: The normalized growth rate of the instability, $\frac{\omega_i}{\omega_{pe}}$ as a function of the wave number, $\frac{ck}{\omega_{pe}}$ for different values of the temperature anisotropy fraction in the fixed $T_z = 20000 \text{eV}$, $n_0 = 10^{32} \text{m}^{-3}$, $\frac{T_{sp}}{T_z} = 2$ and the magnetic field equal to $8T$. 


Figure 8: The normalized growth rate of the instability, $\frac{\omega_i}{\omega_{pe}}$, as a function of the wave number, $ck$, for different values of the electron density in the fixed $T_z = 20000eV$, $T_\perp = 5$, $T_{sp} = 2$ and the magnetic field, $10^8T$, for the astrophysical subjects.