17 MeV Atomki anomaly from short-distance structure of spacetime

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An anomaly observed recently in the $^8$Be nuclear transition by the Atomki collaboration hints at a weakly-coupled, light new gauge boson with a mass about 17 MeV. In this paper, we propose that this new gauge boson comes from a short-distance structure of the spacetime, rather than from an extension of the Standard Model through adding an extra U(1) gauge symmetry. The dominant contribution to the relevant matrix element of the $^8$Be nuclear transition is given by the axial couplings. For accounting for the $^8$Be anomaly and satisfying the current experimental constraints, the coupling constant of the new gauge boson should be about $O(10^{-4} - 10^{-5})$. Our theoretical model allows to understand the origin of the smallness of the coupling constant, which is still missing or being incompletely understood in the models at which the new gauge boson has the axial couplings.

I. INTRODUCTION

There have been the experimental evidences which indicate new physics beyond the Standard Model (SM), such as the tiny masses of the neutrinos and their mixing, dark matter, matter-antimatter asymmetry. The simplest extension of the SM is to add new Abelian gauge forces corresponding to the U(1) symmetry groups. So far no signal for new Abelian gauge forces has been detected at the high energy colliders such as the LHC as well as they have been tested indirectly through the high accurate measurements, since they should be heavy with the masses (much) larger than the electroweak scale. On the contrary, the new Abelian gauge forces can be light but they are coupled very weakly to the SM particles, which allows them to be hidden under the searches at the lower energy collider experiments such as BABAR and BELLE.

The popular approach to add an extra U(1) symmetry is through the extension of the SM gauge symmetry group. There, the additional U(1) symmetry could arise from grand unified theories (GUTs) [1-3], from the left-right symmetric models [7-9], from various simple extra U(1) gauge symmetries [10-34]. However, there has an alternative possibility that the additional U(1) symmetry arises from a more fundamental structure of the spacetime. In this sense, this additional
U(1) symmetry is actually emerged from the more fundamental structure of the spacetime, rather than being considered as a fundamental gauge symmetry. In this direction, starting fundamentally from a generally covariant theory which consists the fields propagating dynamically in a 5D fiber bundle spacetime $M_5$ and respecting for the SM gauge symmetry group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, Ref. [35] obtained a U(1) extension of the SM in the 4D effective spacetime.

The Atomki Collaboration has recently reported an excess of the electron-positron pairs, with a high statistical significance of 6.8σ, produced in the $^8\text{Be}^* (1+) \rightarrow ^8\text{Be} (0^+) + e^+ e^-$ transitions [36] (see also Refs. [37–41]). The $^8\text{Be}$ anomaly can be interpreted by a weakly-coupled, light new gauge boson produced on-shell in the decay of the excited state $^8\text{Be}^*$ and subsequently decaying into the electron-positron pairs. The best fit to the mass of the new gauge boson is $16.70 \pm 0.35 \text{(sta)} \pm 0.5 \text{(sys)}$ MeV. Various models have been proposed to explain the $^8\text{Be}$ anomaly in other U(1) gauge symmetries [42–55]. In these studies, the U(1) gauge symmetry corresponding to the new gauge boson is introduced through the extension of the SM gauge symmetry group.

The goal of the present paper is to investigate whether the weakly-coupled, light new gauge boson accounting for the $^8\text{Be}$ anomaly can come from the more fundamental structure of the spacetime. The new gauge boson in this scenario has both axial and vector couplings to the quarks. (Note that, it is not easy to construct a model with only vector couplings to the quarks, because the coupling constant to the quarks is relatively large to explain the $^8\text{Be}$ anomaly, which is thus difficult to evade various experimental bounds.) As indicated in Refs. [51, 52, 55], in such models, the coupling constant of the new gauge boson is about $O(10^{-4} - 10^{-5})$ which the origin of this smallness is still missing or being incompletely understood. In our scenario, the coupling constant of the new gauge boson is proportional to the ratio between the inverse radius of the extra space and the (reduced) 4D Planck scale. And, since it is very small if the inverse radius of the extra space is much smaller than the 4D Planck scale.

This paper is organized as follows. In Sec. I we introduce a model at which a U(1)$_X$ gauge symmetry, corresponding to a new neutral gauge boson, is emerged from the more fundamental structure of the spacetime. We determine the charges of the SM particles under this U(1)$_X$ group and the coupling terms. In Sec. III we show that our model can provide an interpretation of the $^8\text{Be}$ anomaly consistent with the current experimental bounds. The last section is devoted to our conclusions.
II. MODEL SETUP

In this section, we will review briefly the model proposed in Ref. [35], with including the modifications to have a consistent model to explain the $^8$Be anomaly. (For the more details of this model we refer the reader to Ref. [35].) It was proposed that the spacetime at more fundamental level is a 5D fiber bundle $M_5$ whose base manifold and fiber are the 4D Minkowski-flat manifold $M_4$ and the Lie group manifold $U(1)$, respectively. With this structure, the local coordinates for a point in the spacetime $M_5$ are given by, $(x^\mu, e^{i\theta})$, where $\{x^\mu\} \in M_4$ and $e^{i\theta} \in U(1)$ with $\theta$ to be dimensionless real parameter. The general coordinate transformation is given by

$$
\begin{align*}
x^\mu &\rightarrow x'^\mu = x^\mu, \\
e^{i\theta} &\rightarrow e^{i\theta'} = h(x)e^{i\theta'}, \text{ or } \theta \rightarrow \theta' = \theta + \alpha(x).
\end{align*}
$$

(1)

The theory is covariant with respect to this general coordinate transformation. A metric which defines the invariant distance between nearby points in the spacetime $M_5$ is given by

$$
ds^2 = G_H + G_V = \eta_{\mu\nu}dx^\mu dx^\nu - T^2(x, e^{i\theta})\left(\frac{d\theta + g_X X_\mu dx^\mu}{\Lambda^2}\right)^2,
$$

(2)

where the field $T(x, e^{i\theta})$ determines the geometric size of the fiber, $\Lambda$ is a new constant of the energy dimension, $X_\mu$ is the gauge field which arises naturally from the structure of the spacetime $M_5$ and transforms under the general coordinate transformation [1] as

$$
X_\mu \rightarrow X'_\mu = X_\mu - \frac{1}{g_X} \partial_\mu \alpha(x),
$$

(3)

and $g_X$ is the coupling constant characterizing the strength of the interaction mediated by $X_\mu$. It should be noted that $G_H$ is called the horizontal metric which defines the invariant distance between nearby points along the horizontal directions (pointing from one fiber to another). Whereas, $G_V$ is called the vertical metric which defines the invariant distance between nearby points along the vertical direction (the tangent direction to the fiber). Here, the vertical and horizontal directions are defined independently on the choice of the local coordinates. We consider theory at the vacuum $\langle T(x, e^{i\theta}) \rangle = T_0$, and thus the geometric size of the fiber is determined by the radius $R = T_0/\Lambda$.

The model which will study is a set of the fields propagating dynamically in the spacetime $M_5$ and respecting for the SM gauge symmetry group. The fermion content is given as

$$
L_a(x, e^{i\theta}) = \frac{1}{\sqrt{2\pi R}} \left(\nu_a L(x) e^{iX_a \theta}\right) e^{iX_a \theta} \equiv \frac{L_a(x)}{\sqrt{2\pi R}} e^{iX_a \theta} \sim \left(1, 2, -\frac{1}{2}\right),
$$
\[ E_{aR}(x, e^{i\theta}) = \frac{e_{aR}(x)}{\sqrt{2\pi R}} e^{iX_{aR}} \sim (1, 1, -1), \]
\[ N_{aR}(x, e^{i\theta}) \sim (1, 1, 0), \]
\[ Q_{a}(x, e^{i\theta}) = \frac{1}{\sqrt{2\pi R}} \begin{pmatrix} u_{aL}(x) \\ d_{aL}(x) \end{pmatrix} e^{iX_{aQ}} \equiv \frac{Q_{a}(x)}{\sqrt{2\pi R}} e^{iX_{aQ}} \sim \left( 3, 2, \frac{1}{6} \right), \]
\[ D_{aR}(x, e^{i\theta}) = \frac{d_{aR}(x)}{\sqrt{2\pi R}} e^{iX_{aD}} \sim \left( 3, 1, -\frac{1}{3} \right), \]
\[ U_{aR}(x, e^{i\theta}) = \frac{u_{aR}(x)}{\sqrt{2\pi R}} e^{iX_{aU}} \sim \left( 3, 1, \frac{2}{3} \right), \]

where the numbers given in parentheses are the quantum numbers corresponding to the gauge symmetries \{SU(3)_{C}, SU(2)_{L}, U(1)_{Y}\}, respectively, and \( a = 1, 2, 3 \) are the generation indices. As indicated in Ref. [35], because the fermion fields except the right-handed neutrinos \( N_{aR} \) transform non-trivially under the SM gauge symmetry group, the vertical kinetic term describing their propagation along the vertical direction in the spacetime \( M_{5} \) is not invariant and thus is forbidden. As a result, these fermion fields themselves have no the term determining the \( \theta \)-dependence or the dynamics along the vertical direction in the spacetime \( M_{5} \). And, thus their \( \theta \)-dependence must be determined by a certain special property which nothing but they are invariant under the active action of the Lie group \( U(1) \). In (4), the numbers \( (X_{La}, X_{Ea}, X_{Qa}, X_{Da}, X_{Ua}) \) are quantum numbers characterizing the active action of the Lie group \( U(1) \) on the corresponding fermion fields, and the fields \([L_{a}(x), e_{aR}(x), Q_{a}(x), d_{aR}(x), u_{aR}(x)]\) should be identified as the SM (or 4D effective) fermion fields.

It is easily to see from (4) that the transforming parameters of the SM gauge symmetry group are completely independent on the fiber coordinate \( \theta \) but only dependent on the \( x \)-coordinates. Since it leads to the simplest form for the gauge fields of the SM gauge symmetry group as
\[
G_{aM} = \left( \frac{G_{aM}(x)}{\sqrt{2\pi R}}, 0 \right),
\]
\[
W_{iM} = \left( \frac{W_{iM}(x)}{\sqrt{2\pi R}}, 0 \right),
\]
\[
B_{M} = \left( \frac{B_{M}(x)}{\sqrt{2\pi R}}, 0 \right).
\]

Bulk action for the gauge boson and fermion fields, up to the gauge fixing and ghost terms, is given by
\[
S_{\text{FG}}^{\text{bulk}} = \int dx^4 d\theta \sqrt{|\det G|} \left( \mathcal{L}_{\text{gauge}}^{\text{bulk}} + \mathcal{L}_{\text{fer}}^{\text{bulk}} \right),
\]
\[ \mathcal{L}_{\text{gauge}}^{\text{bulk}} = \frac{1}{4} G_{aMN} G_{a}^{MN} - \frac{1}{4} W_{iMN} W_{i}^{MN} - \frac{1}{4} B_{MN} B^{MN} + \frac{M_{s}^{3}}{2} \mathcal{R}, \]

\[ \mathcal{L}_{\text{fer}}^{\text{bulk}} = \sum_{F} \bar{f} i \gamma^{\mu} \hat{D}_{\mu} F + \frac{i}{\sqrt{2}} \bar{N}_{aR} i \gamma^{\mu} \hat{D}_{\mu} N_{aR} + \frac{1}{2\Lambda} \left( \partial^{\theta} \bar{N}_{aR} \partial_{\theta} N_{aR} - M_{N_{a}}^{2} \bar{N}_{aR} N_{aR} + \text{H.c.} \right), \] (6)

where \{G_{aMN}, W_{iMN}, B_{MN}\} are the field strength tensors of the gauge fields \{G_{aM}, W_{iM}, B_{M}\}, which have the non-zero components given by (up to a normalized factor)

\[ G_{a\mu} = \partial_{\mu} G_{a\nu} - \partial_{\nu} G_{a\mu} + g_{s} f_{abc} A_{b\mu} A_{c\nu}, \]

\[ W_{i\mu} = \partial_{\mu} W_{i\nu} - \partial_{\nu} W_{i\mu} + g_{\varepsilon \mu \nu} W_{jk} W_{\mu \nu}, \]

\[ B_{\mu \nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu}, \] (7)

\( \mathcal{R} \) is the scalar curvature of the spacetime \( M_{5}, M_{s} \) is the 5D Planck scale which is related to the 4D one \( M_{Pl} \) as, \( 2\pi R M_{s}^{3} = M_{Pl}^{2} \), \( M_{Na} \) are the vertical mass parameters of the right-handed neutrinos \( N_{aR} \) which are naturally in the order of the scale \( \Lambda \), and the covariant derivative \( \hat{D}_{\mu} \) reads

\[ \hat{D}_{\mu} = \partial_{\mu} - i g_{s} \frac{\chi^{a}}{2} G_{a\mu} - i g \sigma^{\mu \nu} W_{\mu \nu} - ig' Y_{\mu} B_{\mu}, \] (8)

with \( \partial_{\mu} \equiv \partial_{\mu} - g_{X} X_{\mu} \partial_{\theta} \) and \{\( g_{s}, g, g' \)\} to be coupling constants of the gauge symmetries \{SU(3)_{C}, SU(2)_{L}, U(1)_{Y}\}. Note that, the sum in \( \mathcal{L}_{\text{fer}}^{\text{bulk}} \) is taken over all fermion fields, except the right-handed neutrinos \( N_{aR} \), given in (4). From the bulk action (6), we can find the effective action in the 4D effective spacetime as

\[ S_{\text{eff}}^{\text{FG}} = \int dx^{4} \left( -\frac{1}{4} G_{a\mu \nu} G^{a\mu \nu} - \frac{1}{4} W_{i \mu \nu} W^{i \mu \nu} - \frac{1}{4} B_{\mu \nu} B^{\mu \nu} - \frac{1}{4} X_{\mu \nu} X^{\mu \nu} + \sum_{f} f i \gamma^{\mu} D_{\mu} f + \mathcal{L}_{N} \right), \]

\[ \mathcal{L}_{N} = \mathcal{L}_{\nu} + \mathcal{L}_{\bar{\psi}} + \mathcal{L}_{X} + \mathcal{L}_{\text{int}}, \]

\[ \mathcal{L}_{\nu} = \bar{\nu}_{aR} i \gamma^{\mu} \partial_{\mu} \nu_{aR} - \frac{M_{a0}}{2} \bar{\nu}_{aR} \nu_{aR} + \text{H.c.}, \]

\[ \mathcal{L}_{\bar{\psi}} = \sum_{n=1}^{\infty} \left( \bar{\psi}_{naR} i \gamma^{\mu} \partial_{\mu} \psi_{naR} - M_{an} \bar{\psi}_{naR} \psi_{naR} + \text{H.c.} \right), \]

\[ \mathcal{L}_{X} = \sum_{n=1}^{\infty} \left( \bar{X}_{naR} i \gamma^{\mu} \partial_{\mu} X_{naR} - M_{an} \bar{X}_{naR} X_{naR} + \text{H.c.} \right), \]

\[ \mathcal{L}_{\text{int}} = ig_{X} \sum_{n=1}^{\infty} n \left( \bar{X}_{naR} \gamma^{\mu} \psi_{naR} - \bar{\psi}_{naR} \gamma^{\mu} X_{naR} \right) X_{\mu}, \] (9)

where \( X_{\mu \nu} = \partial_{\nu} X_{\mu} - \partial_{\mu} X_{\nu} \) is the field strength tensor of the gauge field \( X_{\mu} \), the 4D effective fermion \( f(x) \) is related to the fundamental fermion \( F(x, e^{i\theta}) \) as

\[ F(x, e^{i\theta}) = \frac{f(x)}{\sqrt{2\pi R}} e^{iX_{\mu} \theta}, \] (10)
the covariant derivative $D_\mu$ reads

$$D_\mu = \partial_\mu - ig s \frac{\lambda^a}{2} G_{a\mu} - ig \gamma^i W_{i\mu} - ig' Y_f B_\mu - ig X_f X_\mu,$$

(11)

(with $Y_F$ and $X_F$ to be replaced by $Y_f$ and $X_f$, respectively, for a convenient reason), $\nu_{aR}(x)$ are identified as the usual right-handed neutrinos and $\{\psi_{naR}, \chi_{naR}\}$ are their Kaluza-Klein (KK) excitations whose masses are given by

$$M_{a0} = \frac{M_{N_a}^2}{\Lambda} \sim \Lambda,$$

$$M_{an} = \frac{1}{\Lambda} \left( M_{N_a}^2 + \frac{n^2}{R^2} \right) \sim \Lambda.$$  

(12)

The effective action [9] looks like an extension of the SM based on the gauge symmetry group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$. Of course, we know here that $U(1)_X$ is not the fundamental gauge symmetry but it is emerged from the short-distance structure of the spacetime. The emergent $U(1)_X$ charges of the SM fermions are defined in Table I. The coupling constant $g_X$ corresponding to the emergent $U(1)_X$ gauge group is determined in terms of the radius $R$ of the fiber and the 4D Planck scale $M_{Pl}$ as [35]

$$g_X = \frac{\sqrt{2}}{M_{Pl} R}.$$  

(13)

This relation suggests that the coupling constant $g_X$ is completely fixed by the difference between the mass of the zero mode and that of its first KK excitation. This is one of the essential predictions in this emergent $U(1)_X$ model. In particular, the relation (13) suggests that the coupling of the gauge field $X_\mu$ to other fields should be very small but technically natural, if the inverse radius of the fiber is much smaller than the Planck scale, $R^{-1} \ll M_{Pl}$.

The scalar sector is given by

$$H(x, e^{i\theta}) = \frac{1}{\sqrt{2\pi R}} \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} e^{iX_H \theta} \equiv \frac{\phi(x)}{\sqrt{2\pi R}} e^{iX_H \theta} \sim \left( 1, 2, \frac{1}{2} \right),$$

$$\Phi(x, e^{i\theta}) = \frac{\varphi(x)}{\sqrt{2\pi R}} e^{iX_{\Phi} \theta} \sim (1, 1, 0).$$

(14)
It is important to note that comparing to Ref. [35] we have modified the scalar doublet $H$ in such a way that $H$ is invariant under the active action of the Lie group U(1) and thus has the specific $\theta$-dependence given as in (14). As seen later, this modification leads to no the couplings between the new gauge boson and all neutrinos, which allows to avoid the stringent constraints from the $\nu_e - e^-$ scattering experiments such as TEXONO experiment. The bulk action for the scalar sector is given by

$$S[H, \Phi] = \int d^4 x d\theta \sqrt{|\det G|} \left[ \left( \partial_\mu - ig^q_2 W_\mu - ig'_2 B_\mu \right) H \right]^2 + \left| \bar{\partial}_\mu \Phi \right|^2 - V(H, \Phi),$$

where the scalar potential is given by

$$V(H, \Phi) = \mu^2 H^\dagger H + \bar{\lambda}_1 (H^\dagger H)^2 + \mu_2^2 \Phi^\dagger \Phi + \bar{\lambda}_2 (\Phi^\dagger \Phi)^2 + \bar{\lambda}_3 (H^\dagger H) (\Phi^\dagger \Phi).$$

For simplicity, we set $X_\Phi = 1$ in this work. (If $X_\Phi \neq 1$, the related constraints will be imposed on the coupling constant by scaling as $g_X/X_\Phi$.) The effective action $S_{\text{eff}}$ in the 4D effective spacetime reads

$$S_{\text{eff}} = \int d^4x \left[ |D_\mu \phi|^2 + |(\partial_\mu - ig_X X_\mu) \varphi|^2 - V(\phi, \varphi) \right],$$

$$V(\phi, \varphi) = \mu_2^2 \phi^\dagger \phi + \lambda_1 (\phi^\dagger \phi)^2 + \mu_2^2 \varphi^\dagger \varphi + \lambda_2 (\varphi^\dagger \varphi)^2 + \lambda_3 (\phi^\dagger \phi) (\varphi^\dagger \varphi).$$

where $D_\mu = \partial_\mu - ig^q_2 W_\mu - ig'_2 B_\mu - ig_X X_H X_\mu$, $\lambda_1 = \bar{\lambda}_1/2\pi R$, $\lambda_2 = \bar{\lambda}_2/2\pi R$, and $\lambda_3 = \bar{\lambda}_3/2\pi R$.

The gauge symmetry SU(2)$_L \otimes$ U(1)$_Y$ is spontaneously broken due to that the scalar doublet $\phi$ develops the VEV, whereas the emergent U(1)$_X$ gauge symmetry is spontaneously broken by the the VEV of the scalar $\phi$. These VEVs are given by

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \varphi \rangle = \frac{v'}{\sqrt{2}},$$

where

$$v^2 = 2\frac{2\lambda_2 \mu_1^2 - \lambda_3 \mu_2^2}{\lambda_3^2 - 4\lambda_1 \lambda_2}, \quad v'^2 = 2\frac{2\lambda_1 \mu_2^2 - \lambda_3 \mu_1^2}{\lambda_3^2 - 4\lambda_1 \lambda_2}. \quad \text{(19)}$$

We expand these scalar fields around the vacuum as

$$\phi = \begin{pmatrix} w^+(x) \\ v + h(x) + iz(x) \end{pmatrix}, \quad \varphi = \frac{v' + h'(x) + iz'(x)}{\sqrt{2}}. \quad \text{(20)}$$

Here, the $CP$-odd fields $w^+(x)$, $z(x)$ and $z'(x)$ are Nambu-Goldstone bosons which should be absorbed by the weak gauge bosons and the U(1)$_X$ gauge boson. The $CP$-even fields mixes
together at which their squared mass matrix is given by

\[ \mathcal{L}_{\text{mass}}(h, h') = \frac{1}{2} \begin{pmatrix} h & h' \end{pmatrix} \begin{pmatrix} 2\lambda_1 v^2 & \lambda_3 v' \\ \lambda_3 v' & 2\lambda_2 v'^2 \end{pmatrix} \begin{pmatrix} h \\ h' \end{pmatrix}. \quad (21) \]

The physical states are found as, \( h_1 = c_\alpha h - s_\alpha h' \) and \( h_2 = s_\alpha h + c_\alpha h' \), corresponding to the following masses

\[ m_{h_1,h_2}^2 = \lambda_1 v^2 + \lambda_2 v'^2 \mp \left[ \left( \lambda_1 v^2 - \lambda_2 v'^2 \right)^2 + \lambda_3^2 v^2 v'^2 \right]^{1/2}. \quad (22) \]

The mixing angle \( \alpha \) is defined as, \( \sin(2\alpha) = \frac{2\lambda_3 v'}{m_{h_2}^2 - m_{h_1}^2} \). It is constrained by the measurements of the Higgs production cross section and its decay branching ratio at the LHC as \( s_\alpha \lesssim 0.2 \) \[56, 57\] which leads to the following constraint

\[ \left| \frac{\lambda_3 v'}{\lambda_1 v^2 - \lambda_2 v'^2} \right| \lesssim 0.426. \quad (23) \]

The mixing mass matrix between the bosons \( W^3_\mu, B_\mu \) and \( X_\mu \) is given by

\[ M^2 = \frac{1}{4} \begin{pmatrix} g^2 v^2 & -gg' v^2 & -2gg_X X_H v^2 \\ -gg' v^2 & g^2 v'^2 & 2g' g_X X_H v^2 \\ -2gg_X X_H v^2 & 2g' g_X X_H v^2 & 4g^2 (X_H^2 v^2 + v'^2) \end{pmatrix}, \quad (24) \]

This mass matrix is diagonalized by two matrices \( V \) and \( U \) as

\[ \text{Diag} \left( M^2_Z, 0, M^2_{Z'} \right) = U^T V^T M^2 V U, \quad (25) \]

where

\[ M^2_{Z,Z'} = \frac{1}{8} \left[ \left( g^2 + g'^2 \right) v^2 + 4g^2_X \left( v'^2 + X_H^2 v^2 \right) \right] \pm \sqrt{\left[ \left( g^2 + g'^2 \right) v^2 + 4g^2_X \left( v'^2 + X_H^2 v^2 \right) \right]^2 - 16 \left( g^2 + g'^2 \right) g^4_X v^2 v'^2}, \quad (26) \]

corresponding to the physical states which are the SM neutral gauge boson \( Z \) and the new gauge boson \( Z' \), respectively, and the matrices \( V \) and \( U \) are given by

\[ V = \begin{pmatrix} c_W & s_W & 0 \\ -s_W & c_W & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U = \begin{pmatrix} c_\beta & 0 & s_\beta \\ 0 & 1 & 0 \\ -s_\beta & 0 & c_\beta \end{pmatrix}, \quad (27) \]

where the mixing angle \( \beta \) is determined by

\[ \tan(2\beta) = \frac{4g_X X_H \sqrt{g^2 + g'^2}}{(g^2 + g'^2) - 4g^2_X (X_H^2 + v'^2/v^2)}. \quad (28) \]
Because $g_X$ is very small to explain the $^8$Be anomaly, thus the mixing angle $\beta$ should be small which is approximately given by

$$\beta \simeq \frac{2g_X X_H}{\sqrt{g^2 + g'^2}}.$$  \hspace{1cm} (29)

The new gauge boson $Z'$ couples to the SM fermions through the neutral current as

$$\mathcal{L} \supset \sum_f \bar{f} \gamma^\mu \left( C_{f,V} + C_{f,A} \gamma^5 \right) f Z'_\mu,$$  \hspace{1cm} (30)

where the vector and axial couplings are given by

$$C_{f,V} = \frac{gs \beta}{2c_W} \left( T_{f_L}^3 - 2s_W^2 Q_f \right) + c_\beta g_X \frac{X_{f_R} + X_{f_L}}{2},$$

$$\simeq g_X \left[ X_H \left( T_{f_L}^3 - 2s_W^2 Q_f \right) + \frac{X_{f_R} + X_{f_L}}{2} \right],$$

$$C_{f,A} = -\frac{gs \beta}{2c_W} T_{f_L}^3 + c_\beta g_X \frac{X_{f_R} - X_{f_L}}{2},$$

$$\simeq -g_X \left( X_H T_{f_L}^3 + \frac{X_{f_R} - X_{f_L}}{2} \right),$$ \hspace{1cm} (31)

where $Q_f$ and $T_{f_L}^3$ refer to the electric charge and the weak isospin of the fermion $f$. For the neutrinos, we have $C_{\nu_a,V} = C_{\nu_a,A} = g_X \left( X_H + X_{L_a} \right)/2$. We expect that there are no the $Z'$ couplings to all neutrinos. This is satisfied if $X_H = -X_{L_a}$ and $X_{L_1} = X_{L_2} = X_{L_3}$.

With the vector and axial couplings given in (31) and the $U(1)_X$ charges of the SM fermions given in Table 1 of Ref. [35], one can find that $C_{e,A} = C_{\mu,A} = C_{\tau,A} = 0$ which allows to evade the very stringent constraints such as the atomic parity violation in Cesium or the $(g-2)_{e,\mu}$ constraints. Whereas, in analogy the axial couplings for the quarks vanish all. However, a model with primarily vector couplings to the quarks should have the relatively large $g_X$ coupling constant to explain the $^8$Be anomaly, which is difficult to evade various experimental bounds. Thus, we will modify the $U(1)_X$ charges of the quarks to obtain a consistent model to explain the $^8$Be anomaly. Because the $U(1)_X$ charges of the SM fermions given in Table 1 of Ref. [35] is the unique solution with the universality among the generations, modifying the $U(1)_X$ charges of the quarks suggests that there has no the universality of the $U(1)_X$ charge among the generations of the quarks. The absence of the nontrivial anomalies associated with $U(1)_X$ lead to the following equations for the $U(1)_X$ charges of the SM fermions

$$\sum_{a=1}^3 \left( 2X_{Q_a} - X_{u_{aR}} - X_{d_{aR}} \right) = 0,$$

$$\sum_{a=1}^3 X_{Q_a} + X_L = 0,$$
\[
\sum_{a=1}^{3} (X_{Q_a} - 8X_{u_a R} - 2X_{d_a R}) + \frac{9}{4} (X_{L} - 2X_{e}) = 0, \\
\sum_{a=1}^{3} \left( X_{Q_a}^2 - 2X_{u_a R}^2 + X_{d_a R}^2 \right) - 3 \left( X_{L}^2 - X_{e}^2 \right) = 0, \\
\sum_{a=1}^{3} \left( 2X_{Q_a}^3 - X_{u_a R}^3 - X_{d_a R}^3 \right) + 2 \left( X_{L}^3 - X_{e}^3 \right) = 0, \\
\sum_{a=1}^{3} (2X_{Q_a} - X_{u_a R} - X_{d_a R}) + (2X_{L} - X_{e}) = 0,
\]

(32)

where

\[
X_{L_1} = X_{L_2} = X_{L_3} \equiv X_L, \\
X_{e_1 R} = X_{e_2 R} = X_{e_3 R} \equiv X_e.
\]

(33)

It should be noted here that, the usual right-handed neutrinos \(\nu_{a R}\) do not contribute to the above six anomalies because they have no the charges under both the SM gauge symmetry group and the \(U(1)_X\) one. In addition, their KK counterparts \(\{\psi_{na R}, \chi_{na R}\}\) do not contribute to the first four anomalies because they have no the charges under the SM gauge symmetry group. Whereas, because \(\{\psi_{na R}, \chi_{na R}\}\) couple to the gauge boson \(X_{\mu}\) in an unusual way given in \([9]\), they should not appear in the last two anomalies. One solution of this system of equations is found as

\[
X_{Q_1} = \frac{X_{Q_2}}{4} = \frac{X_{Q_3}}{3} = \frac{y}{3}, \\
X_{u_{1 R}} = \frac{X_{u_{2 R}}}{4} = \frac{X_{u_{3 R}}}{7} = \frac{y}{3}, \\
X_{d_{1 R}} = -\frac{X_{d_{2 R}}}{2} = -\frac{X_{d_{3 R}}}{5} = \frac{y}{3}, \\
X_{L_1} = X_{L_2} = X_{L_3} = -y, \\
X_{e_{1 R}} = X_{e_{2 R}} = X_{e_{3 R}} = -2y.
\]

(34)

where \(y\) is a free parameter.

Note that, the mass eigenstates \(u'_{L,R} = (u, c, t)^T_{L,R}\) and \(d'_{L,R} = (d, s, b)^T_{L,R}\) are related to the weak states \(u_{L,R} = (u_1, u_2, u_3)^T_{L,R}\) and \(d_{L,R} = (d_1, d_2, d_3)^T_{L,R}\) as

\[
u'_{L} = U^\dagger_{u} u_{L}, \quad u'_{R} = V^\dagger_{u} u_{R} \quad d'_{L} = U^\dagger_{d} d_{L}, \quad d'_{R} = V^\dagger_{d} d_{R}.
\]

(35)

In the basis of the mass eigenstates, the couplings of the gauge boson \(Z'\) with the quarks are given by

\[
\mathcal{L} \supset (\bar{u}'_{L} \gamma^\mu \Gamma_{L,u} u'_{L} + \bar{u}'_{R} \gamma^\mu \Gamma_{R,u} u'_{R} + \bar{d}'_{L} \gamma^\mu \Gamma_{L,d} d'_{L} + \bar{d}'_{R} \gamma^\mu \Gamma_{R,d} d'_{R}) Z'^\mu,
\]

(36)
corresponding to the Atomki experiment \cite{36}. As reported by the Atomki Collaboration, a ratio of the branching ratios

\begin{align*}
\Gamma_{L,u} &= U_u^\dagger \text{Diag} \left( C_{u_1,V} - C_{u_1,A}, C_{u_2,V} - C_{u_2,A}, C_{u_3,V} - C_{u_3,A} \right) U_u, \\
\Gamma_{R,u} &= V_u^\dagger \text{Diag} \left( C_{u_1,V} + C_{u_1,A}, C_{u_2,V} + C_{u_2,A}, C_{u_3,V} + C_{u_3,A} \right) V_u, \\
\Gamma_{L,d} &= U_d^\dagger \text{Diag} \left( C_{d_1,V} - C_{d_1,A}, C_{d_2,V} - C_{d_2,A}, C_{d_3,V} - C_{d_3,A} \right) U_d, \\
\Gamma_{R,d} &= V_d^\dagger \text{Diag} \left( C_{d_1,V} + C_{d_1,A}, C_{d_2,V} + C_{d_2,A}, C_{d_3,V} + C_{d_3,A} \right) V_d,
\end{align*}

(37)

where \( C_{q_a,V}, C_{q_a,A} \) with \( q_a \) referring to the quark are given in \cite{31}. We assume \( U_u = V_u \) and \( U_d = I \) with \( I \) to be the identity matrix, meaning that the presence of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix \( V_{\text{CKM}} \) is due to the up-type quarks. In this sense, we have \( U_u = V_u = V_{\text{CKM}}^\dagger \). Note that, in the following computation, the CKM matrix \( V_{\text{CKM}} \) is given in the Wolfenstein parameterization as

\begin{equation}
V_{\text{CKM}} = \begin{pmatrix}
1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4),
\end{equation}

(38)

where \( \lambda \approx 0.22453, A \approx 0.836, \rho \approx 0.12515 \) and \( \eta \approx 0.36418 \) \cite{56}.

### III. \(^8\text{Be} \) Anomaly and Other Constraints

In the previous section, we have proposed a U(1)\(_X\) extension of the SM, which is emerged from the short-distance structure of the spacetime. There, the coupling constant \( g_X \) of the new gauge boson \( Z' \) to the SM fermions is determined by the ratio between the inverse radius \( R^{-1} \) of the fiber and the 4D Planck scale \( M_{\text{Pl}} \) as, \( g_X = \sqrt{2R^{-1}/M_{\text{Pl}}} \). This suggests that, if \( R^{-1} \) is much lower than \( M_{\text{Pl}} \), the coupling of \( Z' \) to the SM fermions is small and it is light but technically natural. Thus, in this section we are interested in explaining the new gauge boson \( Z' \) appearing as a resonance in the Atomki experiment \cite{36}. As reported by the Atomki Collaboration, a ratio of the branching ratios corresponding to the \(^8\text{Be} \) anomaly is given by

\begin{equation}
\frac{\text{BR} \left( ^8\text{Be}^* \rightarrow ^8\text{Be} + Z' \right)}{\text{BR} \left( ^8\text{Be}^* \rightarrow ^8\text{Be} + \gamma \right)} \times \text{BR} \left( Z' \rightarrow e^+e^- \right) = 5.8 \times 10^{-6},
\end{equation}

(39)

or in terms of the partial decay width as

\begin{equation}
\frac{\Gamma \left( ^8\text{Be}^* \rightarrow ^8\text{Be} + Z' \right)}{\Gamma \left( ^8\text{Be}^* \rightarrow ^8\text{Be} + \gamma \right)} \times \text{BR} \left( Z' \rightarrow e^+e^- \right) = 5.8 \times 10^{-6},
\end{equation}

(40)

whose statistical significance is about 6.8\( \sigma \) \cite{36}. Here, the partial decay width for \(^8\text{Be}^* \rightarrow ^8\text{Be} + \gamma \) is well-known as \( \Gamma \left( ^8\text{Be}^* \rightarrow ^8\text{Be} + \gamma \right) = 1.9 \times 10^{-6} \) MeV.
First, we need to calculate the partial decay width for $^{8}\text{Be}^* \rightarrow ^{8}\text{Be} + Z'$ to determine what the inverse radius $R^{-1}$ of the fiber (or the coupling constant $g_X$ of the gauge boson $Z'$) and the elements of the mixing matrix $\nu_d$ must be to explain the $^8\text{Be}$ anomaly. The gauge boson $Z'$ has both vector and axial couplings to the quarks. Ref. [51] showed that the contributions coming from the vector and axial couplings to the $^{8}\text{Be}^* \rightarrow ^{8}\text{Be} + Z'$ decay are proportional to $\frac{k^3}{M_{Z'}}$ and $\frac{k}{M_{Z'}}$, respectively, with $k$ to be the momentum of the gauge boson $Z'$. Because $\frac{k}{M_{Z'}} \ll 1$, the contribution of the axial coupling is dominant and since we can neglect the contribution of the vector coupling as well as its interference effect with the axial coupling part. We can use the result of the partial decay width for $^{8}\text{Be}^* \rightarrow ^{8}\text{Be} + Z'$ mediated by an axial-vector boson in Ref. [51] as

$$\Gamma_X = \frac{k}{18\pi} \left( 2 + \frac{E_k^2}{M_{Z'}^2} \right) \left| \frac{a_0 - a_1}{2} \langle 0|\sigma_n||S\rangle + \frac{a_0 + a_1}{2} \langle 0|\sigma_p||S\rangle \right|^2,$$

where $M_{Z'} = 16.7$ MeV, $k = \sqrt{E_k^2 - M_{Z'}^2}$, $E_k = E(8\text{Be}^*) - E(8\text{Be}) = 18.15$ MeV is the excitation energy of the isoscalar $^8\text{Be}^*$, the couplings of the neutron and proton are defined by

$$a_0 = \left( \Delta u^{(p)} + \Delta d^{(p)} \right) (C_{u,A} + C_{d,A}) + 2C_{s,A} \Delta s^{(p)},$$
$$a_1 = \left( \Delta u^{(p)} - \Delta d^{(p)} \right) (C_{u,A} - C_{d,A}),$$

with

$$\Delta u^{(p)} = 0.897(27), \quad \Delta d^{(p)} = -0.367(27), \quad \Delta s^{(p)} = -0.026(4),$$

and the matrix elements are given as, $\langle 0|\sigma_n||S\rangle = -0.132(33)$ and $\langle 0|\sigma_p||S\rangle = -0.047(29)$, with $|S\rangle$ denoted to the isoscalar $^8\text{Be}^*$. In this work, the couplings $C_{u,A}$, $C_{d,A}$ and $C_{s,A}$ are given by

$$C_{u,A} = \frac{(\Gamma_{R,u})_{11} - (\Gamma_{L,u})_{11}}{2} \approx -g_X \frac{y}{2},$$
$$C_{d,A} = \frac{(\Gamma_{R,d})_{11} - (\Gamma_{L,d})_{11}}{2} = g_X \frac{y}{2} \left[ 2|\nu_d|_{11}^2 + |\nu_d|_{21}^2 - 1 \right],$$
$$C_{s,A} = \frac{(\Gamma_{R,s})_{22} - (\Gamma_{L,s})_{22}}{2} = -g_X \frac{y}{2} \left[ 2 - |\nu_d|_{22}^2 - 2|\nu_d|_{12}^2 \right].$$

From the branching fraction of the $^8\text{Be}$ anomaly and the uncertainties in the nuclear matrix elements, we can find upper and lower bounds on the inverse radius $R^{-1}$ of the fiber as

$$\frac{1}{R} \gtrsim \frac{9.07 \times 10^{13}}{1.46|\nu_d|_{11}^2 - 0.03|\nu_d|_{22}^2 + 0.73|\nu_d|_{21}^2 - 0.06|\nu_d|_{12}^2 - 0.43 \text{ GeV}},$$
$$\frac{1}{R} \lesssim \frac{5.71 \times 10^{14}}{3.81|\nu_d|_{11}^2 - 0.12|\nu_d|_{22}^2 + 1.9|\nu_d|_{21}^2 - 0.24|\nu_d|_{12}^2 - 2.69 \text{ GeV}}.$$
In the Atomki pair spectrometer, the distance between the target (where the $^8$Be nucleus is excited) and the detectors is a few cm. The requirement that the gauge boson $Z'$ must decay promptly in the detectors imposes a constraint on the electron coupling as \[42, 46\]

\[
\frac{\sqrt{C_{e,V}^2 + C_{e,A}^2}}{e \sqrt{\text{BR}(X \rightarrow e^+e^-)}} \gtrsim 1.3 \times 10^{-5}.
\] \tag{46}

This leads to a lower bound for the inverse radius $R^{-1}$ of the fiber as

\[
\frac{1}{R} \gtrsim \frac{4.43 \times 10^{12}}{|y|} \text{ GeV},
\] \tag{47}

which is clearly weaker than the lower bound given in (45).

The constraint (45) indicates a basic requirement to explain the $^8$Be anomaly. In what follows, we will discuss the constraints from the various current experiments to obtain the region of the allowed parameter space for explaining this anomaly.

The most precise measurement of the parity-violating Møller scattering from the SLAC E158 experiment [58] imposes a constraint on the coupling of the gauge boson $Z'$ to the electron as [59]

\[
|C_{e,V}C_{e,A}| \lesssim 10^{-8},
\] \tag{48}

for $M_X \approx 17 \text{ MeV}$. The atomic parity violation in Cesium ($^{133}_{55}$Cs) whose value is measured by the experiment [60–62] and is predicted by the SM [63, 64] as

\[
Q_{W}^{\text{exp}}(^{133}_{55}\text{Cs}) = -73.16(29)_{\text{exp}}(20)_{\text{th}},
\]

\[
Q_{W}^{\text{th}}(^{133}_{55}\text{Cs}) = -73.16(3).
\] \tag{49}

These lead to the requirement for the new physics contribution to the nuclear weak charge of Cesium as, $|\Delta Q_{W}(^{133}_{55}\text{Cs})| \lesssim 0.52$, corresponding to the following bound on the couplings

\[
C_{e,A} [(2Z + N)C_{u,V} + (Z + 2N)C_{d,V}] \lesssim 6 \times 10^{-10},
\] \tag{50}

for $M_X \approx 17 \text{ MeV}$ where $Z = 55$ and $N = 78$. Clearly, the bounds (48) and (50) all are automatically satisfied because in our model we have $C_{e,A} = C_{\mu,A} = 0$.

Since the gauge boson $Z'$ has the coupling to the electron, it can be produced in the electron beam dump experiments [65, 66]. In these experiments, the gauge boson $Z'$ could be produced in the bremsstrahlung reaction and then it would escape the beam dump and subsequently decay into the $e^+e^-$ pair. A signal of the gauge boson $Z'$ has been not observed so far in such experiments, and thus for $M_X \approx 17 \text{ MeV}$ the SLAC E141 dump experiment puts a constraint as [52, 55, 59]

\[
C_{e,V}^2 + C_{e,A}^2 < 10^{-17},
\] \tag{51}
if $Z'$ has been not produced, or

$$
\frac{C^2_{e,V} + C^2_{e,A}}{\text{BR}(X \rightarrow e^+e^-)} \gtrsim 3.7 \times 10^{-9},
$$

(52)

if $Z'$ has been caught in the dump. Of course, the first constraint (51) is not consistent to the $^8\text{Be}$ anomaly and hence it is excluded. On the other hand, the coupling of the gauge boson $Z'$ to the electron is constrained by the last constraint which places a lower bound on the inverse radius $R^{-1}$ of the fiber as

$$
\frac{1}{R} \gtrsim \frac{6.81}{|y|} \times 10^{13} \text{ GeV.}
$$

(53)

The analogous searches, from Orsay [67] and the SLAC E137 experiment [68] lead to less stringent constraints on the coupling constant of the gauge boson $Z'$ or the inverse radius of the fiber. Whereas, the E774 experiment [69] is only sensitive for the mass of the gauge boson $Z'$ lighter than 10 MeV.

Recently, collaboration of the NA64 beam dump experiment [70] reported the first results in attempting to search for the hypothetical 17 MeV gauge boson. This showed that, for the explanation of the gauge boson $Z'$ as the $^8\text{Be}$ anomaly, its coupling to the electron must satisfy the following constraint

$$
\frac{C^2_{e,V} + C^2_{e,A}}{\text{BR}(Z' \rightarrow e^+e^-)} \gtrsim 1.6 \times 10^{-8},
$$

(54)

which leads to a lower bound on the inverse radius of the fiber as

$$
\frac{1}{R} \gtrsim \frac{1.42}{|y|} \times 10^{14} \text{ GeV.}
$$

(55)

It is clearly that this lower bound is stronger than one placed by the SLAC E141 dump experiment.

Also, the gauge boson $Z'$ could be emitted in the decays of the $\eta$ and $\eta'$ neutral mesons which are produced by the high energy proton beam in a neutrino target [71]. Using the constraints from search for the signature of the heavy neutrino decay $\nu_h \rightarrow \nu e^+e^-$, the CHARM experiment at CERN puts a bound as, $(C^2_{e,V} + C^2_{e,A})/\text{Br}(Z' \rightarrow e^+e^-) \gtrsim 3.7 \times 10^{-11}$ for $M_{Z'} \approx 17$ MeV [46], which is clearly much weaker than the bound of the NA64 experiment.

Furthermore, the coupling of the gauge boson $Z'$ to the electron is constrained by the electron-positron colliding experiment, e.g. KLOE2 [72]. The search for the process $e^+e^- \rightarrow \gamma(Z' \rightarrow e^+e^-)$ has set a constraint as, $(C^2_{e,V} + C^2_{e,A}) \text{Br}(Z' \rightarrow e^+e^-) \lesssim 3.7 \times 10^{-7}$ for $M_X \approx 17$ MeV [52 55 59], which leads to an upper bound on the inverse radius of the fiber as

$$
\frac{1}{R} \lesssim \frac{6.81}{|y|} \times 10^{14} \text{ GeV.}
$$

(56)
An analogous search, e.g. the BABAR experiment \cite{73}, is only sensitive to the mass of the gauge boson $Z'$ heavier than 20 MeV.

In addition, the precise measurement of the mass of the SM boson $Z$ which is $M_Z = 91.1876 \pm 0.0021$ GeV \cite{56} leads to

$$|\Delta M_Z| = \left( M_Z - \frac{\sqrt{g^2 + g'^2}}{2} v \right) \simeq \frac{g_X y^2 v}{\sqrt{g^2 + g'^2}} \lesssim 0.0021 \text{ GeV},$$

This corresponds to the following upper bound

$$\frac{1}{R} \lesssim \frac{4.33}{|y|} \times 10^{15} \text{ GeV}. \quad (58)$$

Because the gauge boson $Z'$ is coupled to the charged leptons, it should contribute to the anomalous magnetic moments of the electron and muon. In our model, the gauge boson $Z'$ has only the vector couplings to the electron and muon. Thus, the one-loop contributions for the anomalous magnetic moments of the electron and muon, mediated by $Z'$, are given by

$$\delta a_e = \frac{C_{\mu,V}^2}{4\pi^2} \int_0^1 \frac{dz}{M_{Z'}^2 m_e} \frac{z^2 (1 - z)}{(1 - z) + z^2} \simeq (g_X y)^2 \times 1.84 \times 10^{-5},$$

$$\delta a_\mu = \frac{C_{\mu,V}^2}{4\pi^2} \int_0^1 \frac{dz}{M_{Z'}^2 m_\mu} \frac{z^2 (1 - z)}{(1 - z) + z^2} \simeq (g_X y)^2 \times 2.04 \times 10^{-2}. \quad (59)$$

Now we use the discrepancy between the measured values of the anomalous magnetic moments of the electron and muon and the SM predictions to impose the constraints on the coupling of the gauge boson $Z'$ or the inverse radius $R^{-1}$ of the fiber. For the anomalous magnetic moments of the electron, the new contributions must satisfy $-26 \times 10^{-13} \lesssim \delta a_e \lesssim 8 \times 10^{-13}$ \cite{74} which leads to the following bound

$$\frac{1}{R} \lesssim \frac{3.6}{|y|} \times 10^{14} \text{ GeV}. \quad (60)$$

For the anomalous magnetic moments of the muon, the positive discrepancy between the measurement and the SM prediction is about $\delta a_\mu = a_\mu^{\exp} - a_\mu^{\text{th}} = 306 \pm 72 \times 10^{-11}$ which is at $4.3\sigma$ \cite{75, 76}. Since the new contribution coming from the gauge boson $Z'$ is required to satisfy $\delta a_\mu \lesssim 3.78 \times 10^{-9}$ in 90\% C.L. from which we obtain an upper bound as

$$\frac{1}{R} \lesssim \frac{7.4}{|y|} \times 10^{14} \text{ GeV}. \quad (61)$$

Note that, in the scenarios in which the new neutral gauge boson has both axial and vector couplings to the muon, the corresponding one-loop contribution is given about $0.009 C_{\mu,V}^2 - C_{\mu,A}^2$ for $M_{Z'} \approx 17$ MeV which is negative and thus disagrees \cite{51, 52}. On the other hand, the constraint
is imposed on the axial coupling whose contribution is required to be less than the 2σ uncertainty of the discrepancy between the measurement and the SM prediction.

We now analyze the constraints from the couplings of the gauge boson $Z'$ to the quarks. The axial $Z'$ couplings would contribute to the amplitude of the rare $\eta$ decay $\eta \to \mu^+\mu^-$. This imposes the constraint on the product $C_{\mu,A}(C_u + C_d - cC_s)$ where $C_f = \sqrt{C_{f,V}^2 + C_{f,A}^2}$ and $c$ is the parameter relating to the $\eta$-$\eta'$ mixing. In our model, this constraint is avoided because $C_{\mu,A} = 0$.

Also, the rare neutral pion decay $\pi^0 \to \gamma(Z' \to e^+e^-)$ from the NA48/2 experiment [77] imposes the constraint on the coupling of the gauge boson $Z'$ to the first generation of the quarks. The contribution coming from the axial couplings is suppressed by the masses of the light quarks. Since it is mainly imposed the bound on the vector couplings as [55]

$$|2C_u,V + C_d,V| < \frac{3.6 \times 10^{-4}}{\sqrt{\text{BR}(X \to e^+e^-)}}.$$  \hspace{1cm} (62)

for $M_X \approx 17$ MeV. This leads to the following upper bound on the inverse radius of the fiber as

$$\frac{1}{R} < \frac{5.44 \times |y|^{-1} \times 10^{14}}{0.12 + 0.88 [(|V_{d11}|^2 + |V_{d21}|^2/2)^{1/2}} \text{ GeV.}  \hspace{1cm} (63)$$

Furthermore, the measurements of the neutron-nucleus scattering set a bound on the combination of the $Z'$ couplings of the up and down quarks as $(2C_d + C_u)^2 \lesssim 13.6 \times \pi \times 10^{-11} \times (M_{Z'}/\text{MeV})^4$ [79]. Clearly, this bound is weaker than the bound obtaining from the rare neutral pion decay.

There are additionally the constraint on the $Z'$ couplings to the second generation of the quarks. The decay of $\phi \to \eta(Z' \to e^+e^-)$ sets a bound on the $Z'$ coupling of the strange quark as, $|C_s|\sqrt{\text{Br}(Z' \to e^+e^-)} \lesssim 1.0 \times 10^{-2}$ for $M_{Z'} \approx 17$ MeV [72]. In the case $C_s \approx C_d$, this constraint is much weaker than the constraint obtaining from the rare neutral pion decay.

The gauge boson $Z'$ has the flavor-nondiagonal couplings to quarks and thus it would lead to the quark-flavor violating processes at the tree-level. As a result, the $Z'$ couplings to the quarks (or the inverse radius of the fiber) and the mixing matrix $V_d$ should be constrained by the relevant experimental bounds. With $M_{Z'} \approx 17$ MeV, the gauge boson $Z'$ would contribute to the decay of the mesons, such as $K^0 \to \pi^0 e^+e^-$ or $B^0 \to K^{*0} e^+e^-$. However, in search for such decays in LHCb, the $e^+e^-$ invariant mass is above 20 MeV [80, 81]. And, since the constraints on the $Z'$ couplings to the quarks and the mixing matrix $V_d$ from these decays require the future upgrades of the LHCb experiment.

Now we study the contribution of the gauge boson $Z'$ to the $\Delta F = 2$ transition or the mixing of the neutral meson systems. The effective Lagrangian, that describes the mixing of the neutral
meson systems mediated by the gauge boson $Z'$, is given by

$$
\mathcal{L}_{\text{eff}}^{Z'}(q_i, q_j) = - \frac{1}{2} \left[ C_{ij}^{LL}(q^2) (\bar{q}_i \gamma \mu q_j L) (\bar{q}_i \gamma \mu q_j L) + C_{ij}^{RR}(q^2) (\bar{q}_i \gamma \mu q_j R) (\bar{q}_i \gamma \mu q_j R) \right] + H.c.,
$$

(64)

where

$$
C_{ij}^{LL}(q^2) = \frac{(\Gamma_{L,q})_{ij}^2}{q^2 - M_{Z'}^2},
$$

$$
C_{ij}^{RR}(q^2) = \frac{(\Gamma_{R,q})_{ij}^2}{q^2 - M_{Z'}^2},
$$

$$
C_{ij}^{LR}(q^2) = \frac{(\Gamma_{L,q})_{ij} (\Gamma_{R,q})_{ij}}{q^2 - M_{Z'}^2},
$$

(65)

and $q^2$ refers to the momentum transfer. It should be noted here that, for the down-type quarks in our model, $C_{ij}^{LL}(q^2) = 0$ with $i \neq j$. With this effective Langrangian, one can find the mass difference for the mixing of the $S - \bar{S}$ meson system, as

$$
\Delta M_S = -2 \text{Re} \langle \bar{S} | \mathcal{L}_{\text{eff}}^{Z'}(q_i, q_j) | S \rangle,
$$

$$
= \frac{2M_S f_S^2}{3} \text{Re} \left[ C_{ij}^{LL}(q^2) + C_{ij}^{RR}(q^2) - 2C_{ij}^{LR}(q^2) \right].
$$

(66)

Using the results in Ref. [82] for the $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixings, we can place the corresponding bounds as

$$
\left| C_{ds}^{RR}(M_K^2) \right| \lesssim \frac{8.8 \times 10^{-7}}{\text{TeV}^2},
$$

$$
\left| C_{uc}^{LL}(M_D^2) + C_{uc}^{RR}(M_D^2) - 2C_{uc}^{LR}(M_D^2) \right| \lesssim \frac{5.9 \times 10^{-7}}{\text{TeV}^2}.
$$

(67)

This leads to

$$
\frac{1}{R} \lesssim \frac{2.41 \times \left| y \right|^{-1} \times 10^{12}}{\left| (\mathcal{V}_d)_{12} - 2(\mathcal{V}_d^*)_{21} + 2s_W^2 \left[ (\mathcal{V}_d)_{12} + (\mathcal{V}_d^*)_{21} \right] \right|} \text{ GeV},
$$

(68)

$$
\frac{1}{R} \lesssim \frac{8.03 \times 10^{15}}{\left| y \right|} \text{ GeV}.
$$

(69)

Note that, we have taken approximately the diagonal elements of the mixing matrix $\mathcal{V}_d$ to be equal to one and we have neglected the quadratic term in the off-diagonal elements, because the off-diagonal elements of $\mathcal{V}_d$ are small due to the constraints. Experimental and SM values for the mass difference in the $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ meson systems are given by [83]

$$
\Delta M_{B_d}^{\text{exp}} = (0.5064 \pm 0.0019) \text{ ps}^{-1}, \quad \Delta M_{B_d}^{\text{SM}} = (0.547^{+0.035}_{-0.046}) \text{ ps}^{-1},
$$

$$
\Delta M_{B_s}^{\text{exp}} = (17.757 \pm 0.021) \text{ ps}^{-1}, \quad \Delta M_{B_s}^{\text{SM}} = (18.5^{+1.2}_{-1.5}) \text{ ps}^{-1}.
$$

(70)
From this, we can obtain the 2σ lower bounds on the $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixings as, $\Delta M^\text{SM}_{B_d} > 0.455$ ps$^{-1}$ and $\Delta M^\text{SM}_{B_s} > 15.5$ ps$^{-1}$, respectively. For the $B_d - \bar{B}_d$ mixing, the contribution of the gauge boson $Z'$ satisfies the 2σ bound as

$$\frac{1}{R} \lesssim \frac{1.43 \times |y|^{-1} \times 10^{13}}{\left(\text{Re} \left\{ (V_d)_{13} - 5(V_d^*)_31 + 2s_W^2 \left[ (V_d)_{13} + (V_d^*)_31 \right] \right\} \right)^{1/2}} \text{GeV.} \quad (71)$$

Whereas, for the $B_s - \bar{B}_s$ mixing, the 2σ bound is given by

$$\frac{1}{R} \lesssim \frac{7.8 \times |y|^{-1} \times 10^{13}}{\left(\text{Re} \left\{ 2(V_d)_{23} + 5(V_d^*)_32 - 2s_W^2 \left[ (V_d)_{23} + (V_d^*)_32 \right] \right\} \right)^{1/2}} \text{GeV.} \quad (72)$$

We can see that the bound coming from the $D^0 - \bar{D}^0$ mixing almost satisfies the requirement to explain the $^8\text{Be}$ anomaly. Whereas, for the remaining bounds, the bound coming from the $K^0 - \bar{K}^0$ mixing is the most stringent one if the amplitudes of the off-diagonal elements of $V_d$ are approximately equal together.

In summary, combining all the required bounds obtained above, we have the region of the allowed parameter space to explain the $^8\text{Be}$ anomaly. The region of the allowed parameter is essentially determined by the basic requirement (45), the NA64 bound (55), the $g - 2)_e$ bound (60), the NA48/2 bound (63), the bounds (68), (71), and (72) from the mixing of the neutral meson systems. In Fig. 1 we plot the region of the allowed parameter space in the plane of the inverse radius $R^{-1}$ of the fiber and the element $|(V_d)_{12}|$ of the mixing matrix $V_d$. For doing this (as well as for the following figures), we take $(V_d)_{11}, (V_d)_{22}, (V_d)_{33} \approx 1, (V_d)_{12} = -(V_d)_{21}, (V_d)_{13} = -(V_d)_{31},$ and $(V_d)_{23} = -(V_d)_{32}$. The allowed parameter space is given by the white region.

The allowed range of the gauge coupling constant $g_x$ is given by $5.5 \times 10^{-5} \lesssim g_x \lesssim 1.4 \times 10^{-4}$ and $5.5 \times 10^{-5} \lesssim g_x \lesssim 1.2 \times 10^{-4}$ corresponding to the left and right panels of Fig. 1 respectively.

From the expression for the squared masses of the SM neutral boson $Z$ and the new neutral one $Z'$, given in (26), we find the following relation for the VEV $v'$ of the exotic Higgs field $\varphi$ as

$$v' = \frac{2M_Z M_{Z'}}{g_x \sqrt{g^2 + g'^2 v}} \approx 2.88 \times 10^{16} \times \frac{\text{GeV}^2}{R^{-1}}. \quad (73)$$

In Fig. 2 we plot the region of the allowed parameter space in the plane of the VEV $v'$ and the element $|(V_d)_{12}|$ of the mixing matrix $V_d$. Here, the allowed parameter region is specified by the white region.

Furthermore, using the bound (23) and the relation (73), one can obtain the constraint on $|\lambda_3/\lambda_1|$ as

$$\frac{|\lambda_3|}{\lambda_1} \lesssim \left| 0.63g_x - \frac{3 \times 10^{-9}}{g_x} \right| \times 10^4,$$
The Atomki pair spectrometer experiment has recently reported an anomaly in the $^8$Be nuclear transition, which hints at a new weakly-coupled, light neutral gauge boson. Motivated by this, many theoretical models have been constructed to explain this anomaly, at which the new gauge boson comes from the extension of the SM gauge symmetry group through adding an extra U(1) group(s). In this work, based on our recent work [35], we proposed a realistic model where the new gauge boson arises from a short-distance structure of the spacetime. We depart fundamentally...
FIG. 3: The allowed parameter space (white region) for $|\lambda_3/\lambda_1|$ and the inverse radius $1/R$ of the fiber, at $y = 3/2$, $(V_d)_{13} = 0.006$ and $(V_d)_{23} = 0.055$. Left panel: $(V_d)_{12} = 0.002$. Right panel: $(V_d)_{12} = 0.003$. The blue lines correspond to the constraint (74).

from a generally covariant theory in the five-dimensional fiber bundle spacetime with the SM gauge symmetry. We obtained in the four-dimensional effective spacetime an extension of the SM with the gauge symmetry $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$. We identify the new gauge boson as the physical state which mixes between the SM neutral boson and the gauge boson corresponding to $U(1)_X$. Interestingly, the coupling constant of the new gauge boson is naturally defined by the ratio between the inverse radius of the fiber and the four-dimensional Planck scale. We determined the required inverse radius of the fiber to explain the $^8$Be anomaly and satisfy the constraints from various experiments, which is in order of $O(10^{-14})$ GeV.

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