Stress-deformed state of vertical cylindrical metal shell under temperature climate impact

A A Zhdanov1,∗ and V N Petrov1

1Odesa State Academy of Civil Engineering and Architecture, 4, Didrihsona str., Odesa 65029, Ukraine

∗E-mail: zhdanov@ogasa.org.ua

Abstract. There are continued the previous executed researches of the tense-deformed condition of a vertical metal thin cylindrical shell under temperature climatic influences in this work. So, in [1] the temperature climatic influences on cylindrical metal shells are investigated, in [2] the parameters of the elastic base simulating the wheat repulse are investigated, in [3] the tense-deformed condition of a shell on the elastic base with piecewise-constant is investigated, and in [4] with a piecewise-linear Winkler coefficient for axisymmetric cooling of the shell. The tense-deformed condition of a vertical cylindrical thin-wall shell lies on a piecewise-constant elastic base under non-axisymmetric temperature climatic influences has been studied in the present study on the basis of membrane theory of shells and a simple edge effect. The decision is received in the Fourier series, containing in the general case of heating-cooling the shell, ten members of the Fourier series.

1. Introduction
Storages of bulk materials in the form of vertical cylindrical metal thin-walled shells, during operation, are empty and filled with bulk material and experience the temperature climatic effects described in [1]. A change in the temperature of the outside air, the intensity of direct and scattered solar radiation induces a flat temperature field in the shell - the temperature of the shell is constant along the generatrix and variable in the circumferential direction, approximated by a Fourier series containing 5 terms of the cosine expansion series for the shell fully illuminated by the sun and 10 members of the series expansion in cosines and sines - for a shell half of which is covered by a screen at its full height. The temperature drop across the thickness of the shell can be neglected due to the small thickness of the shell (3 ... 10 mm) and a significant coefficient of thermal diffusivity of metals (steel, aluminum). has a direct effect on the stress-strain state (SSS) of the shell. In this case, it is necessary to take into account the variability in time of the position of the sun in the sky and the corresponding change in the position of the normal of the incidence of sunlight on the shell. The stress-strain state of the walls of bulk materials storages under temperature climatic influences is necessary to know when designing new bulk materials storages and assessing the strength of existing ones.

2. Analysis of recent research and publications
In [1], the temperature climatic effects on cylindrical metal shells were investigated, in [2, 3], the stress-strain state of the shell on an elastic foundation with a piecewise constant was investigated, and in [4] with a piecewise linear bed coefficient with axisymmetric cooling of the shell.

The small thickness of the shells, the absence of significant gradients of changes in the shell temperature led to the application in [2-4] of the momentless theory of shells, supplemented, to eliminate the residuals of the momentless solution, the theory of a simple edge effect. In this study, we use a similar proven approach.

3. Goals and objectives
The purpose of this work is to study the stress-strain state of an empty vertical cylindrical metal thin-walled shell of piecewise constant (in height) thickness, (Figure 1, a), caused by temperature climatic
influences - non-axisymmetric heating-cooling of the shell to the state of the same temperature in all of its points.

4. Methods and Results
Let us write down the basic equations of the momentless theory of thin cylindrical shells [5, 6]:

Geometric Equations:

\[
\begin{align*}
\varepsilon_1 &= \frac{1}{R} \frac{\partial u}{\partial \alpha} + \alpha_t, \\
\varepsilon_2 &= \frac{1}{R} \left( \frac{\partial v}{\partial \beta} + w \right) + \alpha_t,
\end{align*}
\]

where \(u, v, w\) - components of displacements of points of the middle surface of the shell (positive directions are shown in Figure 1, a);
\(\varepsilon_1, \varepsilon_2\) - deformation of the middle surface, respectively, in the axial and circumferential directions;
\(\alpha, \beta\) - dimensionless coordinates of the points of the middle surface: \(\alpha = x/R\); \(\beta\) - angular coordinate;
\(t\) - shell temperature is a given function of coordinates \(\alpha\) and \(\beta\);
\(R\) - radius of the middle surface of the shell;
\(h\) - shell thickness;
\(E, \mu, \alpha\) - Young's modulus, Poisson's ratio and coefficient of linear thermal expansion of the shell material.

Positive directions of efforts and displacements are shown in Figure 1.

Physical equations:

\[
\begin{align*}
N_1 &= \frac{Eh}{(1 - \mu^2)R} \left[ \frac{\partial u}{\partial \alpha} + \mu \left( \frac{\partial v}{\partial \beta} + w \right) - (1 + \mu) \cdot \alpha_t \cdot R \cdot t \right], \\
N_2 &= \frac{Eh}{(1 - \mu^2)R} \left[ \frac{\partial v}{\partial \beta} + w + \mu \frac{\partial u}{\partial \alpha} - (1 + \mu) \cdot \alpha_t \cdot R \cdot t \right], \\
S &= \frac{Eh}{2(1 + \mu)R} \left( \frac{\partial u}{\partial \beta} + \frac{\partial v}{\partial \alpha} \right),
\end{align*}
\]

Equilibrium equations in displacements have the form:

\[
\begin{align*}
\frac{\partial^2 u}{\partial \alpha^2} + \frac{1 - \mu}{2} \frac{\partial^2 u}{\partial \beta^2} + \frac{1 + \mu}{2} \frac{\partial^2 v}{\partial \alpha \partial \beta} + \mu \frac{\partial w}{\partial \alpha} &= (1 + \mu) \cdot \alpha_t \cdot R \cdot \frac{\partial t}{\partial \alpha}, \\
\frac{1 + \mu}{2} \frac{\partial^2 u}{\partial \alpha \partial \beta} + \frac{1 - \mu}{2} \frac{\partial^2 v}{\partial \alpha^2} + \frac{\partial^2 v}{\partial \beta^2} + \frac{\partial w}{\partial \beta} &= (1 + \mu) \cdot \alpha_t \cdot R \cdot \frac{\partial t}{\partial \beta}, \\
\mu \frac{\partial u}{\partial \alpha} + \frac{\partial v}{\partial \beta} + w &= (1 + \mu) \cdot \alpha_t \cdot R \cdot t.
\end{align*}
\]
Figure 1. Positive directions of effort and displacement: 
\(a\) - momentless stress-strain state; \(b\) - the state of the edge effect.

The external influence is the flat temperature field of the shell of the form:

\[
t(\beta) = t_0 + \sum_{n=1}^{\infty} (t_{1n} \cos n\beta + t_{2n} \sin n\beta).
\]  

(4)

The amplitudes of harmonics in (4) can be calculated according to the recommendations [1].

For stitching the belts and on the supporting edges of the shell, we use the theory of a simple edge effect, the main dependences of which are presented below [5]:

The resolving equation of the simple edge effect for the \(i\)th belt has the form [5,6]:

\[
\frac{d^4w_i^*}{d\alpha^4} + 4g_i^4w_i^* = 0.
\]  

(5)

where \(g_i^4 = 3(1 - \mu^2) \frac{R_i^2}{h_i^2}\), \(D_i = \frac{Eh_i^3}{12(1 - \mu^2)}\).

Geometric Equations:

\[
\varepsilon_2^* = \frac{w^*}{R}, \quad \gamma^* = \frac{1}{R} \frac{dw^*}{d\alpha}, \quad \varepsilon_1^* = 0.
\]  

(6)

Physical equations

\[
\begin{align*}
N_1^* &= 0, \quad N_2^* = \frac{Eh}{R} w^*, \quad M_1^* = -\frac{D}{R^2} \frac{d^2w^*}{d\alpha^2}, \\
M_2^* &= \mu \cdot M_1^*, \quad Q_i^* = \frac{D}{R^3} \frac{d^3w^*}{d\alpha^3}.
\end{align*}
\]  

(7)

The positive directions of forces, bending moments and displacements of the edge effect are shown in Figure 1, \(b\).

Taking into account the form of the temperature field (4), the equilibrium equations (3) for each belt of the shell take the form:
\[\frac{\partial^2 u}{\partial \alpha^2} + \frac{1 - \mu}{2} \frac{\partial^2 u}{\partial \beta^2} + \frac{1 + \mu}{2} \frac{\partial^2 v}{\partial \alpha^2} + \mu \frac{\partial w}{\partial \alpha} = 0,\]
\[\frac{1 + \mu}{2} \frac{\partial^2 u}{\partial \alpha \partial \beta} + \frac{1 - \mu}{2} \frac{\partial^2 v}{\partial \alpha^2} + \frac{\partial^2 v}{\partial \beta^2} + \frac{\partial w}{\partial \beta} = (1 + \mu) \cdot \alpha_t \cdot R \cdot \frac{\partial t}{\partial \beta},\]
\[\mu \frac{\partial u}{\partial \alpha} + \frac{\partial v}{\partial \beta} + w = (1 + \mu) \cdot \alpha_t \cdot R \cdot t.\]

In expressions (6-8), the index of the belt, for which the system of equations is written, is omitted.

4.1. Axisymmetric temperature action \((n = 0)\)

In this case, \(v = 0\), all derivatives with respect to \(\beta\) are equal to zero, and the system of equations (8) takes the form:

\[\frac{d^2 u}{d \alpha^2} + \mu \frac{dw}{d \alpha} = 0,\]
\[\mu \frac{du}{d \alpha} + w = (1 + \mu) \cdot \alpha_t \cdot R \cdot t.\]

Integrating the first equation (9) and substituting it into the second, we find:

\[\frac{du}{d \alpha} = -\mu w + C_1,\]
\[w = \frac{\alpha_t R}{1 - \mu} - \frac{\mu}{1 - \mu^2} C_1.\]

Substituting (11) into (10) and performing the integration, we obtain:

\[u_o(\alpha) = -\mu \frac{\alpha_t R}{1 - \mu} \cdot \alpha + \frac{C_1}{1 - \mu^2} \cdot \alpha + C_2.\]

Arbitrary constants \(C_1\) and \(C_2\) included in (11) and (12) must be determined from the tangential boundary conditions:

\[u_{o_i}(0) = 0, \quad N_{i,m}(l) = N_{i,m} - \theta u_{m}(l)\]

and tangential conditions for joining the belts:

\[u_i(l) = u_{i+1}(0), \quad N_{i,l}(l) = N_{i+1,l}(0),\]

where \(m\) - the number of belts of the shell of different thickness, the numbering of the belts starts from the fixed edge of the shell;
\(N_{i,m}\) - pre-tensioning of the suspension shell in the axial direction;
\(\theta\) - stiffness coefficient of elastic fastening of the shell at the edge \(\alpha_o = l_m\).

Taking into account [2], tangential conditions (13) and (14) explicitly look like this:

\[C_{2i} = 0,\]
Discrepancies of the momentless solution arise due to different thicknesses of adjacent belts. To satisfy the nontangential boundary conditions and the conditions for the continuity of radial displacements and angles of rotation at the joints of the chords, we will use the theory of a simple edge effect, with success used earlier [2,3,4]. In the corresponding formulas [4], it is necessary to set the bedding coefficient of the base \( k(\alpha) = 0 \).

### 4.2. Impact of non-axisymmetric components of the temperature field \((n \geq 1)\)

We seek the solution of the system of inhomogeneous equations (8) in the form of the sum of the general solution of the corresponding homogeneous and particular solution of the inhomogeneous system. Let’s get a solution for an arbitrary belt. We omit the belt index in the calculations.

We represent the sought solutions in terms of two scalar functions \( \Phi(\alpha, \beta) \) and \( \Phi_0(\alpha, \beta) \). In this case, the function \( \Phi(\alpha, \beta) \) determines the solution of the homogeneous system of equations and is introduced in accordance with [6] as follows:

\[
\begin{align*}
\frac{\partial^4 \Phi}{\partial \alpha^4} + \frac{\partial^4 \Phi}{\partial \beta^4} &= \frac{2 + \mu}{1 - \mu^2} \frac{\partial^4 \Phi}{\partial \alpha^2 \partial \beta^2}, \\
\frac{\partial^4 \Phi}{\partial \alpha^2 \partial \beta^2} &= \frac{2 + \mu}{1 - \mu^2} \frac{\partial^4 \Phi}{\partial \alpha^2 \partial \beta^2} + \nabla^2 \Phi = 0. 
\end{align*}
\]

(16)

A particular solution is determined by the function \( \Phi(\alpha, \beta) \), which, taking into account [7], we introduce so that through it the displacements \( u_0, v_0 \) and \( w_0 \) are expressed as follows:
\begin{align}
  u_i &= (1 - \mu) \frac{\partial^4 \Phi_i}{\partial \alpha^4}, \\
  v_i &= -(1 - \mu) \frac{\partial^3 \Phi_i}{\partial \alpha^3 \partial \beta}, \\
  w_i &= (1 - \mu) \frac{\partial^2 \Phi_i}{\partial \alpha^2 \partial \beta^2} + \nabla^2 \Phi_i,
\end{align}

(19)

Taking into account the form of the temperature field of the shell, the functions \( \Phi(\alpha, \beta) \) and \( \Phi_i(\alpha, \beta) \) can be represented as a product of two functions, one of which depends only on \( \alpha \), the other only on \( \beta \):

\begin{align}
  \Phi(\alpha, \beta) &= \sum_{n=1}^{10} \left[ \Phi_n^\alpha \cos n\beta + \Phi_n^\beta \sin n\beta \right], \\
  \Phi_i(\alpha, \beta) &= \sum_{n=1}^{10} \left[ \Phi_n^\alpha \cos n\beta + \Phi_n^\beta \sin n\beta \right].
\end{align}

(20)

(21)

Further calculations, without loss of generality of transformations, are carried out for the shell completely illuminated by the sun, i.e. containing 5 terms of the cosine expansion series. Substitution of (19) into (3) leads to the fact that all three equations coincide and the resolving equation for the function \( \Phi_i(\alpha, \beta) \) can be written in the form:

\[ \frac{\partial^4 \Phi_i}{\partial \alpha^4} = \frac{\alpha, R_t}{1 - \mu}. \]

(22)

Taking into account (20) and (21), the resolving equations (18) and (22) for the nth term of the series are transformed into ordinary differential equations of the fourth order:

\begin{align}
  d^4 \Phi_n^\alpha &= 0, \\
  d^4 \Phi_n^\beta &= \frac{\alpha, R_t}{1 - \mu}.
\end{align}

(23)

(24)

The solutions to these equations are written in the form:

\[ \Phi_n^\alpha(\alpha) = C_{1c}^n \alpha^3 + C_{2c}^n \alpha^2 + C_{3c}^n \alpha + C_{4c}^n, \]

(25)

\[ \Phi_n^\beta(\alpha) = \frac{\alpha, R_t, \alpha^4}{1 - \mu} \frac{1}{24}. \]

(26)

Taking into account (25) and (26) \( n \) - e terms of the series of displacements of the momentless solution look like this:

\begin{align*}
  u_n^\alpha(\alpha) &= \alpha, R_t, \alpha - \left( 3n^2 \alpha^2 + 6\mu \right) C_{1c}^n - 2n^2 \alpha C_{2c}^n - n^2 C_{3c}^n, \\
  v_n^\alpha(\alpha) &= \alpha, R_t, \frac{n\alpha^2}{2} - \left[ n^3 \alpha^3 - (2 + \mu) 6n \alpha \right] C_{1c}^n - \left[ n^3 \alpha^3 - (2 + \mu) 2n \right] C_{2c}^n - n^3 \alpha C_{3c}^n - n^3 C_{4c}^n, \\
  w_n^\alpha(\alpha) &= \alpha, R_t, \left( 1 - \frac{n^2 \alpha^2}{2} \right) + \left( n^4 \alpha^3 - 12n^2 \alpha \right) C_{1c}^n + \left( n^4 \alpha^2 - 4n^2 \right) C_{2c}^n + n^4 \alpha C_{3c}^n + n^4 C_{4c}^n.
\end{align*}

(27)
Changing the index "c" to "s" in (27), we obtain formulas for calculating the amplitudes of the harmonics of the corresponding displacements at \( \sin n\beta \). The first terms in formulas (27) are particular solutions.

Substituting (27) into physical equations (2) after appropriate transformations, we obtain expressions for tangential forces:

\[

c_{10c}(\alpha) = \sum_{n=1}^{10} n_{10c}(\alpha) \cos n\beta, \\
c_{20c}(\alpha) = \sum_{n=1}^{10} n_{20c}(\alpha) \cos n\beta, \\
n_{0c}(\alpha) = \sum_{n=1}^{10} n_{0c}(\alpha) \sin n\beta,
\]

where

\[

N_{10c}(\alpha) = \frac{Eh}{R} \sum_{n=1}^{10} \left( 3\alpha c_{1c}^{n} + c_{2c}^{n} \right), \\
N_{20c}(\alpha) = 0, \\
n_{0c}(\alpha) = \frac{Eh}{R} 6n c_{1c}^{n}.
\]

The formulas for the amplitudes of the harmonics of tangential forces (29) do not include particular solutions - they are identically equal to zero. It is characteristic that the circumferential force identically equal to zero, this is a consequence of the free movement of the shell in the radial direction.

Arbitrary constants in the expressions for the displacements (27) and effort (29), are determined from the tangential boundary conditions and docking conditions zones recorded for the amplitudes of the \( n \)-th term of a similar (15).

Supplementing the obtained solutions with the edge effect written for the \( n \)-th harmonic similarly to [2, 3], we obtain a complete solution of the problem posed.

An empty vertical metal shell far from the fixed edges freely deforms in the radial direction, which explains the vanishing of the circumferential force of the momentless state \( N_{2c} \) in the shell.

A feature of the temperature climatic effect on a vertical circular cylindrical shell is that the temperature field of the shell seems to follow the sun, enveloping the shell, while in the shell without a screen, the temperature field differs slightly from the one that is symmetric relative to the normal incidence of sunlight.

5. Conclusions

1. For a vertical empty cylindrical shell, with one-sided heating-cooling of the shell, a momentless solution in the Fourier series for the components of the stress-strain state (SSS) is obtained.
2. A simple edge effect in the Fourier series eliminates the residuals of the momentless solution.
3. Taking into account [1], five terms of the cosine expansion series are sufficient to describe the SSS components during cooling of a shell heated without a screen.
4. To describe the components of the stress-strain state of a shell during cooling of a shell heated in the presence of a screen covering half of the shell along its entire height [1], it is necessary to take into account ten terms of the expansion series in the circumferential direction in sines and cosines.
5. The circumferential force \( N_{2c}(\alpha) \) of the momentless state is equal to zero as a consequence of the absence in the empty shell of restrictions on the shell displacement in the radial direction.

References

[1] Zhdanov A A 2014 Vliyanie solnechnoj radiacii na temperaturnye polya vertikal'nyh cilindricheskij hranilishch sypuchih materialov Visnik ODABA 56 (Odesa:
[2] Zhdanov A A 2015 Cilindricheskaya metallicheskaya obolochka na uprugom osnovanii pri temperaturnom klimaticheskom vozdejstvii Visnik ODABA 57 (Odesa: Zovnishreklamservis) pp 152-160

[3] Zhdanov A A 2017 Napryazheno-deformirovannoe sostoyanie cilindricheskoj metallicheskoj obolochki na kusochno-postoyannom uprugom osnovanii pri temperaturnom klimaticheskom vozdejstvii Visnik ODABA 68 (Odesa: ODABA) pp 9-19

[4] Zhdanov A A 2016 Cilindricheskaya metallicheskaya obolochka na uprugom osnovanii pri osesimmetricnom ohlazhdenii Tezisi dokladov III Mezhdunarodnyy nauchno-prakticheskoy konferentsii Aktualnye problemy inzhenernyy mehaniki (Odesa: Vnishreklamservis) pp 73-78

[5] Goldenveyzer A L 1953 Teoriya uprugih tonkih obolochek (Moscow: Gostekteorizdat) 544

[6] Vlasov V Z 1949 Obshchaya teoriya obolochek i ee prilozheniya v tekhnike (Moscow: Gostekteorizdat) 784

[7] Fridman L I 1956 Temperaturnye napryazheniya v kozhuhe koltevoy kamery sgoraniy Vibratsionnaya prochnost i nadezhnost aviaitsionnykh dvigateley Trudy Kujbyshevskogo aviacionnogo instituta (Kuybishev: Izd-vo KuAI) 19 pp 299-306