POSSIBLE EFFECTS OF QUARK AND GLUON CONDENSATES IN HEAVY QUARKONIUM SPECTRA

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\textbf{Abstract}

The Cornell potential with the best fitted parameters $\alpha_s$ and $\kappa$ are modified by adding terms derived from non-perturbative QCD, which are characterized by a series of non-vanishing vacuum condensates of quarks and gluons. In terms of this potential, we study the system of heavy quarkonia. The results show that the correction caused by the additional terms reduces the deviation between the data and the values calculated with the pure Cornell potential and improves the splittings of energy levels. The achievements indicate that the non-perturbative effects induced by vacuum condensates play an important role for the correction to $1/q^2$, which in general was phenomenologically put in by hand. This result would be helpful for understanding non-perturbative QCD along a parallel direction to the QCD sum rules.

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I. Introduction

The success of the potential model in explaining hadronic spectra and hadronic properties has been remarkable. Especially, for the $J/\Psi$ and $\Upsilon$ families, various potential forms [1], in which both the Coulomb and confinement potentials are employed, can give results which are reasonably consistent with the data at the charm and bottom energy scales within a certain error. In general, it is believed that confinement comes from the non-perturbative effect of QCD, but unfortunately, so far, that is an unsolved problem.

Along another line, years ago, Shifman et al. [2] proposed to study hadronic properties in terms of the QCD sum rules where a few non-vanishing vacuum condensates of quarks and gluons $m_q < \bar{q}q >, \frac{\alpha_s}{\pi} < G_{\mu\nu}^a G^{a\mu\nu} >$ etc. describe the nonperturbative effect. They started from short distance, where the quark-gluon dynamics is essentially perturbative, and extrapolated the dynamics to larger distance by introducing non-perturbative effects step by step [3]. The applications of the theory extrapolated in studying hadronic properties such as the spectrum, decay width and hadronic matrix elements etc., indicate that one can trust the validity of this approach.

Inspired by these successes, we have been trying to introduce the non-perturbative QCD effects, characterized by non-vanishing quark and gluon vacuum condensates, into the traditional potential model [4], so that a deeper understanding of the hadronic structure and the underlying mechanisms which determine how quarks are bound into hadrons can be obtained. In our derivation of correction terms, the vacuum condensates were employed to modify the free gluon propagator. As a consequence, the quark-quark potential and the spectrum of heavy quarkonium are affected. Studying these possible effects is the main purpose of this paper. Meanwhile, we have noticed that beside the modification of the gluon propagator, characterized by the vacuum condensates, the closed-loop correction can also contribute a comparable effect, because these two kinds of corrections are in the same order of $\alpha_s$. This has been shown by Gupta et al [5,6], Fulcher [7] and Pantaleone et al [8]. In this investigation, as a preliminary study, we temporarily treat the condensate effect alone and will collect the closed-loop correction in our later paper.

As to the condensate correction, there are two different results, ours [4] and Larsson’s [9]. The effect of the difference may show up in the spin splittings of heavy quarkonia. Thus our additional effort in this paper will be donated to search the possible effects of these two approaches. We hope this investigation may help us to understand the condensate correction deeply.

As mentioned above, the non-vanishing vacuum condensates can only be used to describe
an extrapolation from the short distance to the medium range [3], therefore, the longer distance effect at the energy scale $\leq \Lambda_{QCD}$ cannot be included in the scenario. In other words, this scenario is only valid within the intermediate range if only a finite number of vacuum condensates are kept, and for the bound states, the potential term responsible for the confinement should come from the larger distance $\geq 1/\Lambda_{QCD}$. Therefore, we are attempting to introduce a reasonable picture where the main contribution to the confinement is caused by the interaction at $\leq \Lambda_{QCD}$, where the physical picture is not clear yet. Since this part is not derivable at the present stage, we keep a phenomenological confinement form, generally the linear $\kappa r$ term which is the main part of the confinement potential and universal to all of the heavy flavors. Then we perturbatively introduce the corrections induced by the non-trivial vacuum condensates into our framework. It would make observable contributions to some hadronic properties of $J/\Psi$ and $\Upsilon$ families, for example the spin splitting between $1^3S_1$ and $1^1S_0$ could be one of the sensitive quantities for the correction. Moreover, we would claim that there is no double-counting between our derived correction and the contribution from the larger distance $1/\Lambda_{QCD}$.

It should be emphasized that the amplitude of the wave function at the origin, which is essential to the spin splitting, depends on the potential model adopted. For instance, Richardson potential [10] wave function at the origin is almost the half of Cornell’s. In this investigation, we start with the Cornell potential, which has the simplest form among existant potential models, as the zeroth order approximation and then add in the condensate corrections in a perturbative way. Therefore, our wave function at the origin is close to that of the Cornell potential, and the difference between ours and Cornell’s starts to show up only in the approximation higher than the zeroth order. In our later work, we will study these differences and effects caused by adopting different model wave function as the zeroth order wave function.

In the next section, the potential corrections derived with vacuum condensates are briefly reviewed and different formulae are analyzed. In section III, the numerical results are presented and compared with the data. Finally, our results are discussed and conclusions are drawn.

II. Our Model

As SVZ [3] suggested, there are non-zero vacuum expectation values of the quark ($\langle \bar{\psi} \gamma_5 \psi \rangle$) and gluon ($\alpha_s \langle GG \rangle$) fields, so the propagator of the gluon should include the effects of these condensates. In fact, in the propagator, all the terms associated with vacuum
condensates are proportional to $\alpha_s^n \ (n \geq 1)$. On the other hand, the contributions of higher order perturbative QCD corrections to the potential were discussed in Ref.[5]. In comparison with such corrections, the condensate terms do not suffer from the loop suppression. SVZ noticed that fact and then suggested that the condensates could be considered as a larger contribution. Therefore, at the order of $\alpha_s^2$, we only include the condensates, but not the perturbative correction.

There have been various ways to modify the potential. Except the higher order perturbative QCD correction, all of them attempt to include some non-perturbative effects to the potential, because it is sure that such effects must be taken into account. Richardson et al proposed a form [10]

$$\bar{U}(q^2) = -\frac{4}{3} \frac{12\pi}{(33 - 2N_f) q^2 \ln(1 + \frac{q^2}{\Lambda^2})}$$

where $q^2$ is the momentum of the gluon exchanged between quarks and $N_f$ represents the flavor number, while the linear confinement still remains unchanged as $\kappa r$. It gives an effective correction which indeed involves the non-perturbative effects. Besides, Fulcher [7] gave

$$V(r) = Ar - \frac{8\pi}{(33 - 2N_f) r} f(\Lambda r)$$

where $f(\Lambda r)$ has a very complicated integration form, which can be found in ref.[7]. Recently, Gupta et al. considered not only the higher order perturbative radiative corrections, but also a more complicated non-perturbative term [6]. In all these works, the corrections related to non-perturbative effects are phenomenologically put in according to the observation or hint from lattice gauge results. Some very good results which coincide with data within an error of a few MeV were reported [3]. We will discuss this problem in some detail in the last section.

On the other hand, in this work we are trying to understand such corrections in terms of some well-established theories which can handle the non-perturbative QCD in a more natural way.

Within the QCD scenario, where the non-vanishing vacuum condensates of quarks and gluons characterize the non-perturbative effects, the modified gluon propagator in momentum space can be written as [4]

$$G_{\mu\nu} = \frac{-i}{q^2} (g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2}) F(q^2)$$

(1)

where

$$F(q^2) = 1 + \frac{1}{3} g_s^2 \sum_{\beta=u,d,s} \frac{m_{\beta} < \psi_{\beta} \bar{\psi}_{\beta} >}{q^2(q^2 - m_{\beta}^2)} + \frac{9}{32} g_s^2 < G^2 > \frac{1}{q^4}$$

(2)
In this expression we only keep the lowest dimensional condensates $<\psi\bar{\psi}>$ and $<GG>$. We derived this expression in the standard way [4], in which the normal product operators such as $\psi\bar{\psi}(0)$ and $G^2(0)$ have non-vanishing matrix elements in the physical vacuum, i.e., $<\psi\bar{\psi}>$ and $<G^2>$ are left as parameters to describe non-perturbative effects, and their values have already been determined in literature [2].

In another way, by comparing the $(2n + 1)$-point Green’s function ($n$ is the number of external legs of $\psi$ or $A^\mu$) and the $n$-point Green’s function with the insertion of the operators $\psi\bar{\psi}(0)$ or $G^2(0)$ (these Green’s functions are with respect to the physical vacuum), Larsson achieved

$$D_{\mu\nu} = [1 - \sum_\beta \frac{g^2m_\beta <\psi_\beta\bar{\psi}_\beta>}{q^2(q^2 + m_\beta^2)} + \frac{5g^2 <G^2>}{288q^4}]^{-1} \left(\frac{-i}{q^2}\right) (\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})$$

where the propagator was derived in the Euclidean space. Expanding the first piece with respect to $\alpha_s$, one notices that not only the coefficients at the lowest order of $\alpha_s$ are different from those in Eq.(2), but also the sign of the coefficient associated with the gluon condensate is different from that in Eq.(2). These differences arise from different approaches used to deal with the non-perturbative effects and perhaps due to the the improper usage of the fixed point gauge [2] which violates translational invariance. [11]

In this work, we also determine phenomenologically whether Eq.(2) or Eq.(3) is more consistent with the data.

We write down a proper scattering amplitude between two quarks as

$$M = (-ig_s)^2 \bar{u}_q(p_1)\gamma_\mu \frac{\lambda^a}{2} u_q(p_1') D^{\mu\nu}(q^2) \bar{u}_{q'}(p_2)\gamma_\nu \frac{\lambda^a}{2} u_{q'}(p_2')$$

with

$$p_1 - p'_1 = p'_2 - p_2 = q.$$

and carry out the Breit-Fermi expansion with the spinors $u_q(p_i)$ being the solutions of free quarks. In deriving an effective potential, the spontaneous approximation, $q_0 = 0$, has been taken. It should be mentioned that this approximation is a traditional treatment in literatures although it is not quite valid in the potential derivation. Moreover, in ref. [12], the term $q_\mu q_\nu$ was kept as $q_i q_j$ ($q_0 = 0$). If the CVC theorem is respected, i.e. $q^\mu \bar{u}_q(p_1)\gamma_\mu u_q(p'_1) = 0$, the term $q_\mu q_\nu / q^2$ vanishes, and then the derived potential will have a small difference with that without CVC theorem, even though it is not very extravagantly apart. Then one can apply the three-dimensional Fourier transformation to convert the propagator in momentum space to coordinate space and derive an effective potential between quarks.
The potential derived in the way used in ref.[4], in which leading order non-perturbative QCD effects are considered, together with the phenomenological linear confinement can be written in the following form:

\[ V(r) = -\frac{4\alpha_s}{3r} + \kappa r + V_1^{corr}(r) + V_2^{corr}(r) + V_3^{corr}(r), \]

where \( V_1^{corr} \) is the correction from the non-trivial physical vacuum condensates, while \( V_2^{corr} \) and \( V_3^{corr} \) are the Breit-Fermi corrections to the Coulomb and linear confinement terms, respectively. Due to the lengthy expressions for these potentials, we do not present them here. The explicit forms of these potentials can be found in Appendix.

It is noted that:

(1). In the preceding literature, for instance ref.[14], a comparison with the experimental data shows that the potential \( \kappa r - \frac{4\alpha_s}{3r} \), with universal values of \( \kappa \) and \( \alpha_s \) is applicable to both \( J/\Psi \) and \( \Upsilon \) in solving the Schrödinger equation. Hence, this confining potential should be independent of the quark mass \( m_c \) or \( m_b \). In other words, it corresponds to the potential in the limit \( m_Q \rightarrow \infty \) (\( m_Q \gg \Lambda_{QCD} \)). It is universal and exists in all \( Q\bar{Q} \) systems. On the other hand, all correction terms associated with vacuum condensates are of the order of \( 1/m_Q^2 \), which can be directly read from the Feynman diagrams. Therefore, they appear as mass-dependent corrections to \( \kappa r - \frac{4\alpha_s}{3r} \). Consequently, there is no double-counting involved in \( \kappa r - \frac{4\alpha_s}{3r} \). Moreover, the Coulomb term \( -\frac{4\alpha_s}{3r} \) comes from one-gluon-exchange and represents a short-distance effect where perturbative QCD works perfectly well. Therefore the correction induced by the condensates which manifest the non-perturbative effects does not overlap with the pure Coulomb part either. The \( \kappa r \) term may be understood in the following picture. When \( m_Q \) is very large, one can define the total momentum \( k \) of the quark \( Q \) as \( P_Q = m_Q v + \delta \) where \( Q \) is almost on the mass shell, \( v \) is the four-velocity of \( Q \) and \( \delta \) is the so-called residual momentum of the order of \( \Lambda_{QCD} \). From Eq.(4), one finds that in the large \( m_Q \) limit the emitted gluons are soft. This leads to the conclusion that the confinement term, \( \kappa r \), plays a role at the energy scale \( \Lambda_{QCD} \). In this picture, \( \kappa r \) is independent of \( m_Q \).

(2). Since all the correction terms associated with the vacuum condensates are proportional to \( 1/m_Q^2 \), under this meaning, they are of the same order as the relativistic corrections in the Breit-Fermi expansion.

(3). Since the higher order terms are omitted, the derived potential is not appropriate in dealing with higher resonances.

(4). Our numerical results indicate that beyond the 2S state the calculated values would deviate from the data more and more, but for the 1S, 1P and 2S states, these values indeed make sense (see below).
III. Numerical Results

In this work, to elucidate the significance of the correction, we present a few very typical quantities which are calculated in the framework of QCD.

The Cornell potential $-4\alpha_s/3r + \kappa r$ with corresponding parameters $\alpha_s$ and $\kappa$ [14], which gave the best fit to the $J/\Psi$ and $\Upsilon$ family data, is adopted as a basic condition, and the values of vacuum condensates are taken from ref. [2]. Thus there is no free parameters at all in the derived expressions (2) and (3).

To our understanding, the term $\kappa r$ is universal to $J/\Psi$ and $\Upsilon$ and dominates the confinement part. This is consistent with the consideration in the literature which deals with non-perturbative corrections. Then one can consider the additional part from condensates as a $1/m_Q^2$ correction to the potential. It is noticed that, as discussed in most of the preceding literatures, one always assumed that $\kappa r$ was caused by a scalar exchange. As a consequence, it would not induce a spin splitting. On the other hand, the only term responsible for the spin splitting is the Coulomb term which contributes as a vector potential. It should be mentioned that that assumption was based on phenomenological requirements — i.e. for the Cornell potential, if $\kappa r$ was induced by scalar exchange, a better fit to the spin splitting data could be obtained. Now, the new correction induced by the condensates contributes to the spin splitting, so the whole picture changes. In particular, this modification demands that $\kappa r$ comes not only from scalar exchange, but also from vector exchange. Thus, we can write

$$\kappa r = \beta \kappa r + (1 - \beta) \kappa r \quad 0 \leq \beta \leq 1$$

(5)

where the factor $\beta$ characterizes the fraction of the confinement potential which comes from vector exchange, while $(1 - \beta)$ denotes that from scalar exchange. If $\kappa r$ is fully caused by the scalar exchange, then $\beta = 0$ and it is the same assumption as in the preceding literature. The explicit value of $\beta$ can be fixed by data fitting. Then the spin splitting $\Delta = (M_1^3 S_1 - M_2^3 S_2)$ for $c\bar{c}$ and $b\bar{b}$ systems can be calculated. The correction term which contributes to the spin splitting can be read as

$$- \left( \frac{g^2}{4\pi} \right) \frac{\lambda^a \lambda^a}{4} \left[ \frac{1}{6} \sum_\beta a_{\beta} \frac{1}{r} - \frac{1}{12} b r - \frac{1}{6} \sum_\beta a_{\beta} \frac{e^{-m_{\beta} r}}{r} \right] + \frac{\pi}{3} c \delta(\vec{r}) (\vec{\sigma}_1 \cdot \vec{\sigma}_2),$$

(6)

where

$$c = \frac{2}{m_1 m_2} + \sum_\beta \frac{A_\beta'}{m_1 m_2 m_\beta}, \quad a_\beta = \frac{A_\beta'}{m_1 m_2 m_\beta}, \quad b = \frac{B'}{m_1 m_2},$$
\[ A'_\beta = \begin{cases} \frac{g^2}{3} < \psi_\beta \bar{\psi}_\beta > & \text{ref.[4]} \\ \frac{g^2}{5} < \psi_\beta \bar{\psi}_\beta > & \text{ref.[9]} \end{cases} \]

\[ B' = \begin{cases} \frac{9}{32} g^2 < G^2 > & \text{ref.[4]} \\ -\frac{5}{288} g^2 < G^2 > & \text{ref.[9]} \end{cases} \]

and \( m_1 = m_2 \) are the masses of the heavy quarks in quarkonium.

It is noted that, the terms with \( a_\beta \) in refs.[4] and [9] have the same sign, but the numerical value of this term in ref.[9] is three times larger than that in ref.[4]. Since the contribution from this term has the same sign (though \( < \psi \bar{\psi} > \)) as that from the term with \( \delta(\bar{r}) \) \( ( < \psi \bar{\psi} > = 0) \), it enhances the spin splitting between \( 1^1S_0 \) \( ( < \bar{\sigma} \cdot \bar{\sigma} > = -3) \) and \( 1^3S_1 \) \( ( < \bar{\sigma} \cdot \bar{\sigma} > = 1) \).

However, for the terms with \( b \) in refs.[4] and [9], they have not only different numerical values, but also opposite signs. Therefore, the term with \( b \) given by ref.[9] tends to increase the spin splitting, while the corresponding term in ref.[4] reduces the spin splitting. Since \( \frac{5}{288} < G^2 > \frac{1}{r} \sim 0.016 \frac{\alpha_s}{\alpha_s} \) is a small number, the existence of \( < GG > \) does not give rise to a more significant influence to the spin splitting than that of \( < \psi_q \bar{\psi}_q > \). Anyway, a measurement of the spin splitting between \( 1^3S_0 \) and \( 1^3S_1 \) tells us that the non-perturbative effects characterized by the non-vanishing vacuum condensates indeed play an important role, so manifest themselves in phenomenology.

The spin splitting of \( 1^3P_J \) state is also influenced by the above mentioned correction as well as the parameter \( \beta \). The correction terms can be re-written in the following way:

\[ V_i^{corr}(r) = V_{i}^{cen}(r) + V_{i}^{s-o}(r) + V_{i}^{ten}(r) + \cdots \]

\[ V_1^{corr}(r) = V_{1}^{cen}(r) + V_{1}^{s-o}(r) + V_{1}^{ten}(r) + \cdots \]

\[ V_2^{corr}(r) = V_{2}^{cen}(r) + V_{2}^{s-o}(r) + V_{2}^{ten}(r) + \cdots \]

\[ V_3^{corr}(r) = V_{3}^{s-o}(r) + V_{3}^{ten}(r). \]

with

\[ V_3^{s-o} = \frac{1}{2m_Q^2} \left[ \frac{3V'}{r} - \frac{S'}{r} \right] \]

\[ V_3^{ten} = \frac{1}{12m_Q^2} \left[ \frac{V'}{r} - V'' \right], \]

where the superscripts \( cen, s-o \) and \( ten \) denotes the central, spin-orbit and tensor parts of the corresponding \( V_i^{corr}(r) \), respectively, and \( V \) and \( S \) represent the vector and scalar parts of the confinement \( \kappa r \) in Eq.(5), respectively. Then the spectrum of \( 1^3P_J \) states can be expressed as

\[ M(1^3P_2) = M + f - \frac{2}{5} g \]

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\begin{align*}
M(1^3p_1) &= \overline{M} - f + 2g \\
M(1^3p_0) &= \overline{M} - 2f - 4g,
\end{align*}
\tag{9}

where

\begin{align*}
f &= <V_1^{s-o}(r)> + <V_2^{s-o}(r)> + <V_3^{s-o}(r)> \\
g &= <V_1^{ten}(r)> + <V_2^{ten}(r)> + <V_3^{ten}(r)>,
\tag{10}
\end{align*}

and \( \overline{M} \) is the weighted average for the \( 1^3P_J \) states.

It is noticed that in Eq. (10) the \( V \) and \( S \) terms are of opposite sign and the newly achieved correction term is opposite in sign to the Coulomb term. Then one may easily be convinced that to keep a good fit to data, the appearance of non-perturbative correction terms requires \( \beta \neq 0 \). In fact, Gupta et al. [6] also noticed that with a reasonable contribution from non-perturbative effects, the linear confinement \( \kappa r \) must be a superposition of two parts as given in Eq.(5). As a result, they obtained an approximate \( \beta \)-value of 0.25 with their model potential. Alternatively, with our modified potential, we have \( \beta \approx 0.5 \sim 0.6 \) for a better fit.

The numerical calculation is performed in the following way. The Cornell potential which is universal to \( c \) and \( b \) families, i.e., independent of \( m_Q \), is considered as the dominant part of the potential, and the corresponding parameters \( \alpha_s \) and \( \kappa \) are determined before adding in the corrections. The values of \( \alpha_s \) and \( \kappa \) are 0.381 and 0.182\,GeV\(^2\), respectively. And then, the newly derived corrections due to the non-vanishing vacuum condensates as well as the Breit-Fermi corrections are treated as a perturbation adding onto the dominant part. The resultant values for the \( c\bar{c} \) and \( b\bar{b} \) systems are tabulated in Table Ia and Ib, respectively.
### Table Ia. $c\bar{c}$ system

| exp’t. | $\nu_{Cornell} = \frac{-4\alpha_s}{3\pi} + \kappa r$ | $\nu_{Cornell} + \nu_{2}^{corr}$ | $\nu_{Cornell} + \nu_{2}^{corr}$ | $\nu_{Cornell} + \nu_{2}^{corr}$ | $\nu_{Cornell} + \nu_{2}^{corr}$ |
|--------|-------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $1^+s_0$ | $2978.8 \pm 1.9$ | $3074.0$ | $2979.3$ | $3026.6$ | $3010.8$ | $2999.5$ |
| $2^+s_0$ | $3594.0 \pm 5.0$ | $3662.1$ | $3446.3$ | $3676.0$ | $3625.0$ | $3618.4$ |
| $1^+s_1$ | $3096.88 \pm 0.04$ | $3074.0$ | $3000.6$ | $3157.7$ | $3095.1$ | $3098.8$ |
| $2^+s_1$ | $3686.00 \pm 0.09$ | $3662.1$ | $3493.1$ | $3754.0$ | $3674.5$ | $3676.7$ |
| $1^+p_0$ | $3415.1 \pm 1.0$ | $3321.2$ | $3452.7$ | $3440.8$ | $3418.0$ |
| $1^+p_1$ | $3510.53 \pm 0.12$ | $3497.1$ | $3395.1$ | $3533.1$ | $3493.9$ | $3480.7$ |
| $1^+p_2$ | $3556.17 \pm 0.13$ | $3445.1$ | $3605.8$ | $3514.4$ | $3527.5$ |
| $E_{20}$ | $141.07$ | $0$ | $123.9$ | $133.1$ | $73.6$ | $109.4$ |
| $E_{21}$ | $45.64$ | $0$ | $50.0$ | $72.7$ | $25.1$ | $46.7$ |
| $\Delta_{ss}^{(1)}$ | $118.08$ | $0$ | $111.3$ | $131.1$ | $84.3$ | $99.3$ |
| $\Delta_{ss}^{(2)}$ | $92.0$ | $0$ | $46.8$ | $78.0$ | $49.5$ | $58.3$ |

### Table Ib. $b\bar{b}$ system

| exp’t. | $\nu_{Cornell} = \frac{-4\alpha_s}{3\pi} + \kappa r$ | $\nu_{Cornell} + \nu_{2}^{corr}$ | $\nu_{Cornell} + \nu_{2}^{corr}$ | $\nu_{Cornell} + \nu_{2}^{corr}$ | $\nu_{Cornell} + \nu_{2}^{corr}$ |
|--------|-------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $1^+s_1$ | $9460.37 \pm 0.21$ | $9427.0$ | $9466.8$ | $9503.8$ | $9452.8$ | $9453.7$ |
| $2^+s_1$ | $100023.30 \pm 0.31$ | $10007.0$ | $9987.7$ | $10073.0$ | $10017.0$ | $10017.0$ |
| $1^+p_0$ | $9859.8 \pm 1.3$ | $9843.6$ | $9907.6$ | $9865.8$ | $9861.4$ |
| $1^+p_1$ | $9891.9 \pm 0.7$ | $9897.5$ | $9950.3$ | $9902.5$ | $9900.8$ |
| $1^+p_2$ | $9913.2 \pm 0.6$ | $9920.5$ | $9985.9$ | $9929.1$ | $9931.7$ |
| $E_{20}$ | $53.4$ | $0$ | $76.9$ | $78.3$ | $63.3$ | $70.3$ |
| $E_{21}$ | $21.3$ | $0$ | $34.4$ | $35.6$ | $26.6$ | $30.9$ |

In this table, $E_{20} \equiv M_{1^+p_2} - M_{1^+p_0}$, $E_{21} \equiv M_{1^+p_2} - M_{1^+p_1}$, $\Delta_{ss}^{(1)} \equiv M_{1^+s_1} - M_{1^+s_0}$ and $\Delta_{ss}^{(2)} \equiv M_{2^+s_1} - M_{2^+s_0}$.

The experimental data are taken from "Partial Physics Booklet", July 1994, Partical Data Group.

The decay width $\Gamma(J/\Psi \rightarrow e^+e^-)$ is also a good test for various models. For $\Gamma(e^+e^-)$, the annihilation process is related to the zero-point wavefunction of $J/\Psi$. Considering the QCD correction, we have

$$\Gamma(e^+e^-) = \Gamma_0(e^+e^-)(1 - 16\alpha_s/3\pi)$$

and

$$\Gamma_0(e^+e^-) = \frac{16\pi e^2 \alpha^2}{M^2} |\phi(0)|^2,$$

where $\phi(0)$ is the zero-point value of the $J/\Psi$ wavefunction. In this scenario, the wave function $\phi(0)$ undergoes a modification due to the addition of the non-perturbative QCD terms. With
expression (2) one has $4.85\,KeV$, while with expression (3), $4.72\,KeV$ and the experimental data is $4.69\,KeV$. Since the zero-point wavefunction is not sensitive to the new correction, the modified $\Gamma(e^+e^-)$ does not deviate far from that calculated directly with the Cornell potential.

\textbf{IV. Discussion and Conclusion}

The potential model is successful in explaining the hadronic spectra and other properties of heavy quarkonia. First, people believed that quarks are confined inside hadrons, therefore the potential must include a confinement term. The simplest form is the linear confinement, $kr$. Besides, at short-distance, where perturbative QCD works well, the one-gluon-exchange provides a Coulomb-type potential. By including these two extreme sides, the Cornell potential, in the form of $V(r) = -\frac{4\alpha_s}{3r} + kr$ where $\alpha_s$ and $\kappa$ are treated as free parameters, indeed gives reasonable results for both $c\bar{c}$ and $b\bar{b}$ families. However, there must be some non-perturbative effects which are not included in the linear term of the Cornell potential and they definitely make substantial contributions to the evaluation of spectra and other properties.

In the general approaches of Richardson, Fulcher and others, the universal linear confinement $kr$ were usually kept unchanged, but the simple propagator of the gluon $\frac{-i}{q^2}(g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2})$ was modified by multiplying a $q$-dependent factor $F$ which is model dependent. Usually, this factor was phenomenologically introduced based on some physical arguments or hints from the lattice calculations or obtained from the higher order perturbative QCD corrections[6].

Along the other line [2], the QCD sum rules are also successful in explaining hadronic effects. It implies that there should be crossing between two lines. Thus the non-vanishing vacuum condensates which characterize the non-perturbative effects must be somewhat involved in the modification factor $F$ and may be the dominant piece or at least an important one.

In this work, by including the effects of quark and gluon condensates, we derive the modified gluon propagator. Namely, a $q$-dependent factor $F$ which is similar to that shown in the recent literature is obtained in the framework of QCD. There are no free parameters in the derived expressions.

It is important to notice that as pointed out by Shifman, this framework is an extrapolation from short distances where perturbative QCD is reliable. Therefore, one cannot expect that this factor $F$ can include as much as a purly phenomenological ansatz. But, it does shed light on the physical picture and enrich our understanding of the physical mechanism which binds quarks into hadrons.
For fitting experimental data, it requires that the linear confinement comes not only from the scalar exchange but also from the vector exchange. This is consistent with Gupta et al.’s model. However, the value of $\beta$ depends on the model. In ref. [3], it is about 0.25, but in our case, it must be a value of 0.5~0.6, otherwise the result is not meaningful. This indicates that the vector exchange gives a large contribution to the linear potential.

Our numerical results show that by considering the effects of the quark and gluon condensates, a more reasonable results, especially a better fit to the spin splitting $E_{20}, E_{10}$ and $\Delta_{ss}^{(1)}$ can be obtained. However, $\Delta_{ss}^{(2)}$ does not change in the right direction. The reason is that we only take the lower dimensional condensates $< q\bar{q} >$ and $< G^2 >$ into the consideration. For higher excited states, interaction range becomes larger, we then have to extrapolate the scenario to the larger distance by introducing higher dimensional condensates such as $< q\bar{q}G >$, $< GGG >$ and etc.

There is a discrepancy between refs.[4] and [9] on the sign of the coefficient of term $< GG >$. As we pointed above, maybe it is caused by using different methods or the fixed point gauge. Unfortunately, the contribution from this term is small compared to that from $< q\bar{q} >$, therefore the numerical values calculated by using the formulae of refs.[4] and [9] are not very far apart. We are going to pursue this problem based on both the first principle and phenomenology in our next work.

In our evaluation, the closed-loop corrections are omitted since we only try to analyze the nonperturbative effect in the present paper. However, the closed-loop corrections are comparable with those from the quark and gluon condensates. Therefore, a complete analysis where both the condensate and loop contributions would be considered will be the aim of our next work. It will be helpful to clarify that the result as well as the method in refs. [4] is better or those in ref. [9] is more appropriate.

It should be mentioned that in this work we take the Cornell potential as the zeroth order approximation. Due to the fact that the wave function at the origin depends on the model sensitively, it will be interesting to choose other models as the zeroth order approximation and compare the calculated results. This will also be done in our next work.

Finally, the nonperturbative effects are considered by using perturbative treatment. Although it is simple, sometimes it does not work well. Since our purpose is to illustrate the effects of nonperturbative QCD effects, it would be not a serious issue. It is also interesting to employ other ways such as the variational method adopted by Gupta et al to re-carry out the analysis. It deserves further study.

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Appendix

By using the method mentioned in the text and ref.[4], the quark-quark potential is re-derived. The explicit expression of the revised quark-quark potential can be written as:

\[
U(\vec{p}_1, \vec{p}_2, \vec{r}) = \frac{g^2}{4\pi} \left( \lambda^a_1 \cdot \lambda^a_2 \right) \left\{ A_1 \delta(\vec{r}) + A_2 \frac{1}{r} + A_3 r + A_4 r^3 + A_5 \frac{e^{-m_\beta r}}{r} \right. \\
+ \left[ A_6 \frac{1}{r^3} + A_7 \frac{1}{r} + A_8 + A_9 \frac{m_\beta^2 r^2 + 3m_\beta r + 3}{r^3} \frac{e^{-m_\beta r}}{r} \right] S_{12} \\
+ \left[ A_9 \frac{1}{r^3} + A_{10} \frac{1}{r} + A_{11} r + A_{11} \frac{m_\beta^2 r + 1}{r^3} \frac{e^{-m_\beta r}}{r} \right] \cdot \left[ \vec{f}_1 \cdot (\vec{r} \times \vec{p}_1) - \vec{f}_2 \cdot (\vec{r} \times \vec{p}_2) \right] \\
\left. + \ p-dependent \ terms \right\}
\]

with

\[
A_1 = -\pi \left[ \frac{1}{2m_1^2} + \frac{1}{2m_2^2} - \frac{1}{m_1 m_2} + \frac{2}{3m_1 m_2} + \frac{A'_\beta}{3m_\beta m_1 m_2} \right] (\sigma_1 \cdot \sigma_2) \\
A_2 = 1 - \frac{A'_\beta}{m_\beta} \left[ \frac{1}{8m_1^2} + \frac{1}{8m_2^2} - \frac{1}{4m_1 m_2} + \frac{1}{m_\beta} + \frac{1}{6m_1 m_2} (\sigma_1 \cdot \sigma_2) \right] \\
A_3 = -\left[ \frac{A'_\beta}{2m_\beta} - B' \left( \frac{1}{16m_1^2} + \frac{1}{16m_2^2} - \frac{1}{8m_1 m_2} + \frac{(\sigma_1 \cdot \sigma_2)}{12m_1 m_2} \right) \right] \\
A_4 = \frac{1}{24} B' \\
A_5 = \frac{A'_\beta}{m_\beta} \left[ \frac{1}{8m_1^2} + \frac{1}{8m_2^2} - \frac{1}{4m_1 m_2} + \frac{1}{m_\beta} + \frac{(\sigma_1 \cdot \sigma_2)}{6m_1 m_2} \right] \\
A_6 = -\left[ \frac{1}{4m_1 m_2} - \frac{A'_\beta}{4m_1 m_2 m_\beta^3} \right] \\
A_7 = -\frac{A'_\beta}{24m_1 m_2 m_\beta} \\
A_8 = -\frac{1}{96m_1 m_2} B' \\
A_8 = -\frac{A'_\beta}{12m_1 m_2 m_\beta^3}
\]
\[ A_9 = - \left( 1 - \frac{A'_\beta}{m_\beta} \right) \]
\[ A_{10} = \frac{A'_\beta}{2m_\beta} \]
\[ A_{11} = \frac{1}{8} B' \]
\[ A_{11\beta} = - \frac{A'_\beta}{m_\beta} \]

where the summation over \( \beta \) is implied and the expressions of \( A'_\beta \) and \( B' \) are given in the text.

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