The Effect of Gross Domestic Product and Population Growth on CO₂ Emissions in Indonesia: An Application of the Ant Colony Optimisation Algorithm and Cobb-Douglas Model

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ABSTRACT

Gross domestic product (GDP) is one indicator for measuring a country’s economic growth. However, the increase in GDP and population growth are affecting CO₂ emissions. This study analyses the effects of GDP and population density on CO₂ emissions in Indonesia. To this end, it used the Cobb-Douglas model, and parameter estimation using Ant Colony Optimisation algorithm. The analysis of the results reveals that GDP and population density influence CO₂ emissions in Indonesia significantly, and significantly follows the Cobb-Douglas model with increasing return to scale characteristics. Thus, an increase in GDP and population density will lead to increased CO₂ emissions in Indonesia.

Keywords: Economic Growth, Gross Domestic Product, Population Growth, CO₂ Emission, Ant Colony Optimisation Algorithm, Cobb-Douglas Model

JEL Classifications: C61, O47, O150, Q530

1. INTRODUCTION

Gross domestic product (GDP) is the total production value in the form of goods and services produced by production units within the boundaries of a country (domestic) for 1 year. GDP shows a country’s flow of income and expenditure in the economy over a period of 1 year (Kasperowicz, 2015). Indonesia is the fourth most populous country in the world. Based on data published by the Indonesian Central Bureau of Statistics 2017 entitled “Statistik Indonesia 2017” (Statistical Yearbook of Indonesia 2017), the population in Indonesia was 258,704,900 in 2016. This figure is 8.5% higher or 20,186,200 more people compared to 2015 which amounted to 238,518,800 inhabitants. This is cause for worry in an environmental sense, as a country’s economic growth is directly proportional to the decline in environmental function and quality, including the increased emissions of carbon dioxide (CO₂).

CO₂ emissions are substances, energy or other components resulting from an activity in the form of CO₂ gas. Studies often sample CO₂ emissions to illustrate the level of pollution (Ru et al., 2012). For example, to increase economic growth means that people must perform economic activities and consume energy which causes air pollution. The higher a country’s value of GDP and population density, the higher a citizen’s purchase power. Similarly, the higher the activity and energy consumption, the higher the CO₂ emissions produced. According to Alam (2014), global warming due to climate change is a critical global problem wherein CO₂ is considered a significant contributor to the problem.
Therefore, it is essential to analyse the effect of GDP and the population density on CO₂ emissions in Indonesia.

Many researchers have conducted studies on the relationship between GDP and CO₂ emissions. Among others, Alam (2014) researched changes in economic structure and CO₂ emissions trends with GDP per capita of Bangladesh. His study is based on the Environmental Kuznets Curve hypothesis, using World Bank data throughout 1972-2010. The results show a faster structural shift from the agricultural sector to the non-agricultural sector, and the emergence of the service sector as a dominant part of the economy, resulting in a rapidly increasing trend of CO₂ emissions.

Bozkurt and Akan (2014) researched economic growth, CO₂ emissions, and energy consumption relations in Turkey using a cointegration test based on GDP data, CO₂ emissions, and energy consumption for the 1960-2010 period. The results showed that CO₂ emissions negatively affect GDP growth, while energy consumption has a positive effect on CO₂ emissions. Similar research has also been conducted by Farhani and Rejeb (2012), Sharif et al. (2014), Kemal and Hizarci (2017), Magazino (2016), Salahuddin et al. (2018), Al mamun et al. (2014) and Tiwari (2011) among others.

To analyse a problem requires a model and a hypothesis that something will happen in uncertain conditions called parameters. In research on the influence of input variables on an output variable, one parameter could be the production function of the Cobb-Douglas model. According to Reynes (2017), to date, the Cobb-Douglas model production function is the most commonly used analysis of growth and productivity. Estimation of aggregate production function parameters is critical in growth analysis, technological change, productivity, and labour. Samsami (2013) conducted a study on the application of Ant Colony Optimisation (ACO) to predict CO₂ emissions in Iran based on socio-economic indicators. Forms of linear and non-linear equations were developed to predict CO₂ emissions using ACO. The results provide useful insights into energy systems and technology. Similarly, Zhao et al. (2017) applied the ACO algorithm to find the optimal path of a robot in an environment with random obstacles.

In equation (1), if the left and right segments are taken as a natural logarithm, the following linear equations are obtained:

\[
\log Q_t = \log \beta_0 + \beta_1 \log K_t + \beta_2 \log L_t + \varepsilon_t
\]

(4a)

If \( Y_t = \log Q_t \), \( a_0 = \log \beta_0 \) and \( X_{1t} = \log K_t \), and \( X_{2t} = \log L_t \), then the last equation can be expressed as:

\[
Y_t = a_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \varepsilon_t
\]

(4b)

### 2.2. Cobb-Douglas Model

The Cobb-Douglas model is used to estimate a production function (Douglas, 1928; Reynes 2017; Soukhovolovsky and Ivanova 2018). These equations involving two or more variables, a dependent variable, and independent variables. The production function of the Cobb-Douglas model with multiplicative error terms is given as the following equation:

\[
Q_t = \beta_0 K_t^{\beta_1} L_t^{\beta_2} e^\varepsilon_t
\]

(1)

Where in this study, \( Q_t \) is output as CO₂ emissions; \( K \) is the input as the GDP; \( L \) is the input as the population density; \( \beta_1 \), and \( \beta_2 \) are the Cobb-Douglas model parameters, and \( \varepsilon_t \) is the exponential of the residual. The elasticity of production \( E \) is the percentage change in output, divided by the percentage of input changes. Production elasticity is the ratio of the relative change of output produced to the relative changes in the number of inputs that effect. The output elasticity of the GDP \( E_k \) is measured using the following equation:

\[
E_k = \frac{\% \Delta Q}{\% \Delta K} = \beta_1
\]

(2)

The output elasticity of GDP can also be measured using coefficient parameters \( \beta_1 \) of the production function of the Cobb-Douglas model. The output elasticity of the population density \( E_z \) is measured using the equation:

\[
E_z = \frac{\% \Delta Q}{\% \Delta L} = \beta_2
\]

(3)

The output elasticity of the population can also be measured using coefficient parameters \( \beta_2 \) of the production function of the Cobb-Douglas model. The sum of the elasticity of production \( \sum \beta_i \) explains the size of a venture scale or called a return to scale. There are three characteristics of the return to scale as follows:

- If \( \sum \beta_i = 1 \), then the function shows a scale with a constant return (constant return to scale), meaning that an increase in proportional output will follow the increase in the input.
- If \( \sum \beta_i < 1 \), then the function shows the scale with decreasing return (decreasing return to scale), meaning the percentage increase output is smaller than the percentage of input addition.
- If \( \sum \beta_i > 1 \), then the function shows the scale with the increase (increasing return to scale), meaning the percentage of output addition is higher than the percentage of input addition.

In research on the influence of input variables on an output variable, one parameter could be the production function of the Cobb-Douglas model. According to Reynes (2017), to date, the Cobb-Douglas model production function is the most commonly used analysis of growth and productivity. Estimation of aggregate production function parameters is critical in growth analysis, technological change, productivity, and labour. Samsami (2013) conducted a study on the application of Ant Colony Optimisation (ACO) to predict CO₂ emissions in Iran based on socio-economic indicators. Forms of linear and non-linear equations were developed to predict CO₂ emissions using ACO. The results provide useful insights into energy systems and technology.
Thus, the estimator obtained from the regression equation (5) is:

$$\hat{Y}_i = \alpha_0 + \beta_1 X_{1i} + \beta_2 X_{2i}$$ (5)

Equation (5) is linear in the parameters $\alpha_0$, $\beta_1$ and $\beta_2$ as well as residuals $\varepsilon_i$. Thus, it is shaped as a linear regression model. Constant $\alpha_0$ is an intercept, and $\beta_1$, $\beta_2$ are a parameter of elasticity of production. For parameter estimation, an optimisation equation can be formed from equation (5) as follows:

Minimisation

$$\sum \varepsilon_i^2 = \sum (Y_i - \hat{Y}_i)^2 = \sum (Y_i - \alpha_0 - \beta_1 X_{1i} - \beta_2 X_{2i})^2$$ (6)

Equation (7) is used to estimate the value of $\alpha_0$, $\beta_1$ and $\beta_2$ which can minimise the sum of squares of residuals $\sum \varepsilon_i^2$ . The process of minimising the sum of square residual in this study was performed using the ACO algorithm.

### 2.3. ACO Algorithm

According to Samsami (2013), Deif and Gadallah (2017), and Bouarafa et al. (2018), ACO is derived from ant treatment, known as the ant system. Naturally, ant colonies can find the shortest route from the nest to the source of food and back again. As the ants walk, they leave information called pheromone where it passes and marks the route. Pheromones are used to communicate between ants while constructing routes. The path of ants from the nest to their food is illustrated in Figure 1 using the ACO algorithm.

According to Samsami (2013), Deif and Gadallah (2017), and Bouarafa et al. (2018), the ant algorithm functions as follows: (i) First, the ants move randomly. (ii) When ants find different paths, such as an intersection, they begin to determine the direction of the path at random. (iii) Some ants walk up, and others choose to walk down.

- When they have found their food, they return to the colony while marking it with pheromone traces.
- Since the path taken down the path is shorter, the lower ant arrives first, assuming the velocity of all the ants is the same.
- The pheromone left by the ants on the shorter path of the aroma is stronger, compared to the pheromone on the longer path.

**Figure 1:** (a-d) Ants’ path from the nest to their food

- The other ants are more interested in following the lower path, because of the stronger pheromone scent.

Also, according to Samsami (2013), Deif and Gadallah (2017), and Bouarafa et al. (2018), the ant algorithm requires several variables and steps to determine the shortest distance.

**Step 1:**

- The parameters required in the ant algorithm are as follows:
  - The intensity of ant traces between places $\tau_{ij}$ and changes $\Delta \tau_{ij}$
  - Intensity $\tau_{ij}$ must be initialised before starting the cycle. Change $\Delta \tau_{ij}$ initialised after one cycle. Change $\tau_{ij}$ used to specify $\tau_{ij}$ for the next cycle.
  - Ant cycle constant $Q$
  - Ant trace intensity control constant $\alpha$.
  - The traceability control constant is used in the probability of the place visited and served as the ant trace intensity controller. Value $\alpha$ is determined by the user.
  - Time visibility controller $\beta$.
  - Visibility between places $\eta_{ij}$.
  - Visibility between places $\eta_{ij}$ used in the probability of places visited. Value $\eta_{ij}$ is the result of $1/d_{ij}$ (the distance of the place).
  - Lots of Ants $m$.

Lots of Ants $m$ are many ants that cycle in the ant algorithm. The value $m$ is determined by the user.

- The ant trap evaporation constant $\rho$.
- The ant trap evaporation constant $\rho$ is used to determine $\tau_{ij}$ for the next cycle. Value $\rho$ is determined by the user.
- Maximum number of cycles $N_{C_{max}}$.
- The maximum number of cycles $N_{C_{max}}$ is the maximum number of cycles that will take place. The cycle will stop according to the value of $N_{C_{max}}$ which has been determined by the user.

**Step 2:**

Inputting the data of the starting point into the taboo list. The initialisation result of the first place of each ant in step 1 should be loaded as the first element of the taboo list. The result of this step is the data input of the taboo of each ant with the index of a certain place.

**Step 3:**

Arranging the route of each ant visit to every destination. Ant colonies that have been distributed to a number or destinations...
begin to travel from their place of origin to a destination. From the second place, each ant colony continues its journey, choosing one of the places not on the taboo list as the next destination. Ant colony trips continue continuously until all the places are visited one by one. If site specifies the index of the order of visits, the place of origin is expressed as $\text{tabu}(s)$, and other places are declared as $\{N-\text{tabu}(s)\}$. To determine the destination, the probability equations of places to visit are as follows:

$$P^s_i(s) = \left\{ \begin{array}{ll}
\frac{[\tau^s_i(s)]^\alpha [\eta^s_i]^\beta}{\sum_{k \in \{N-\text{tabu}(s)\}} [\tau^s_k(s)]^\alpha [\eta^s_k]^\beta}; & \text{if } j \in \{N-\text{tabu}(s)\}, \\
0; & \text{for other } j.
\end{array} \right.$$  

Where $i$ is the index of the place of origin, and $j$ as the destination index’s.

Step 4:

a. Calculation of the route length of each ant or $L_k$ of each ant is done after all ants complete one cycle. The calculation is based on $\text{tabu}(s)$ each with the following equation:

$$L_k = d_{\text{tabu}(s),\text{tabu}(1)}(s) + \sum_{r=1}^{n-1} d_{\text{tabu}(s),\text{tabu}(r+1)}(s).$$

Where $d_{ij}$ is the distance between place $i$ to place $j$ and calculated based on the equation:

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}.$$

b. Shortest distance search

After the $L_k$ is calculated for each ant, it will get the minimum price of the closed route length of each cycle, or $L_{\text{min NC}}$ and the minimum price of the overall closed line length is $L_{\text{min}}$.

c. Calculation of price changes in the intensity of ant footprints between places $\Delta \tau^s_g$

An ant colony leaves footprints on the path between the places it passes. The existence of evaporation and differences in the number of ants that passes causes the possibility of changes in the price of ant footprints intensity between places. The equation of the change is:

$$\Delta \tau^s_g = \sum_{k=1}^{m} \Delta \tau^s_{g k}$$

Where $m$ is the number of ants; $\tau^s_{g k}$ is the path length of each ant. $\Delta \tau^s_{g k}$ is the price change of the ant footprint intensity between places which is calculated for each ant using equation (12):

$$\Delta \tau^s_{g k} = \left\{ \begin{array}{ll}
\frac{Q}{L_k}; & \text{for } (i, j) \in \text{place of origin and destination in } \text{tabu}, \\
0; & \text{for other } (i, j)
\end{array} \right.$$  

Where $Q$ is the ant cycle constant; and $L_k$ is close length tour (let).

Step 5:

a. Calculation of the price of the ant footprint intensity between places for the next cycle. The intensity of the ant footprints between places on all tracks is likely to change as there is evaporation, as well as the difference in the number of ants that pass through. For the next cycle, the ants passing through the trajectory of the intensity price have changed. The equation calculates the price of the ant footprint intensity between places for the next cycle:

$$\tau^s_{g k} = (1 - \rho) \times \tau^s_{g k} + \Delta \tau^s_{g k}.$$  

b. Reset the price of the ant footprint intensity changes between places. For subsequent cycles, the change in the intensity value of ant traces between places needs to be reset to have a value equal to zero.

Step 6:

Dismiss the taboo list and repeat Step 2 if needed. The taboo list needs to be emptied to be filled again with a new place order in the next cycle. If the maximum number of cycles has not been reached, the algorithm is repeated from the taboo input step, with the price of the ant footprint intensity parameter between the updated places.

2.4. Model Significance Test

In this section, we discuss the significance test of the model, with the aim of finding the viability of the model resulting from the estimation process. The model significance test includes the partial parameter significance test, the test of the significance of the parameters, and the assumption of residual normality test. Also, this study measured the strength of the relationships between independent and dependent variables, along with the accuracy of forecasting (prediction).

1. Partial significance test: This partial significance test examines the significance of each coefficient parameter $\theta_\epsilon(i=1,2,3)$, where $\theta_\epsilon = \{\alpha_\epsilon, \beta_\epsilon\}$ of equation (2), in affecting the dependent variable. For the test parameter $\theta_\epsilon$, the hypothesis used is $H_{\theta_\epsilon0} : \theta_\epsilon = 0$ and $H_{\theta_\epsilon1} : \theta_\epsilon \neq 0$.

Testing is done using statistic $t$, where the equation is:

$$t_{\text{Statistic}} = \frac{\theta_\epsilon}{SE(\theta_\epsilon)}.$$  

Where $SE(\theta_\epsilon)$ is the standard error of the parameter $\theta_\epsilon$.

Reject the hypothesis $H_{\theta_\epsilon0}$ when $|t_{\text{Statistic}}| > t_{(n-2,\nu)}$, or $Pr(t_{\text{Statistic}}) < c$ where $t_{(n-2,\nu)}$ the critical value of the distribution $t$ at a level of significance $100(1-c)\%$, and $n$ the number of data (Sukono et al., 2016).

2. Simultaneously parameter test: This simultaneous significance test examines the significance of the coefficient parameters simultaneously $\theta_\epsilon(i=1,2,3)$, where $\theta_\epsilon = \{\alpha_\beta, \beta_\beta\}$ of equation (2), in affecting the dependent variable. The hypothesis used is $H_{\theta_\epsilon1} : \theta_\epsilon = \theta_\epsilon = 0$ and $H_{\theta_\epsilon0} : \theta_\epsilon \neq 0$. Testing has done using statistic $F$, where the equation is:

$$F_{\text{Statistic}} = \frac{MS_{\text{Reg}}}{s^2}.$$  

Where MSReg is the mean square due to regression, and $s^2$ mean square due to residual variation.

Reject the hypothesis $H_{\theta_\epsilon0}$ when $F_{\text{Statistic}} > F(1,n-2,1-c)$, or $Pr(F_{\text{Statistic}}) < c$, where $t_{(\frac{1-c}{2},\nu)}$ the critical value of the distribution $F$ at the level of significance $100(1-c)\%$, and $n$ the number of data (Sukono et al., 2016).

3. Residual normality test: According to Jäntschi and Bolboaca (2018), the cumulative distribution function (CDF) $F_\epsilon$ follows the empirical distribution function $F_\epsilon$. Given the ordered sample data $X_1 \leq X_2 \leq \ldots \leq X_n$, it is assumed that $H_{\epsilon}$: CDF
following distribution \( F_0(x) \), and \( H_1 \); CDF not following distribution \( F_0(x) \).

The Aderson-Darling (AD) test is a normality test performed using the equation:

\[
AD_{\text{Statistic}} = \sum_{t=1}^{n} 1 - \frac{2}{n} \left[ \log(F_0(z(t))) + \log(1 - F_0(z_{(n+1-t)})) \right] - n 
\]

Where \( F_0 \) is the normal distribution assumed by the parameter estimator \((\mu, \sigma^2)\); \( z(t) \) is the sample sequence value to \( t \); \( n \) is the sample size; \( \log \) is the natural logarithm (base \( e \)); and \( t = 1, 2, \ldots, n \).

The null hypothesis \( H_0 \) is rejected if the value of \( AD_{\text{Statistic}} > AD_{\text{Critical}} \) with \( AD_{\text{Critical}} = 0.752/(1 + 0.75/n + 2.25/n^2) \).

4. The coefficient of Determination: Coefficient of determination \( r^2 \) is used to determine the strength of the relationship between independent variables with the dependent variable in a regression model. The value of the coefficient of determination \( r^2 \) can be determined using the equation:

\[
r^2 = \frac{\sum_{t=1}^{n} (\hat{Y}_t - \bar{Y})^2}{\sum_{t=1}^{n} (Y_t - \bar{Y})^2}
\]

Where the value \( r^2 \) ranged \( 0 \leq r^2 \leq 1 \). If \( r^2 \) is close to zero, then it explains that the relationship between the independent variable and the dependent variable is weak. If \( r^2 \) is close to one, then it indicates that the relationship between independent variables and dependent variables is strong (Sukono et al., 2016).

5. Precision Forecast Size: According to Karmaker (2017) and Khair et al. (2017), accuracy is vital in forecasting and measures the suitability between existing data and forecasting data. Certain calculations are commonly used to determine total forecast errors, one of which is Mean Absolute Deviation (MAD). MAD statistics measure the accuracy of the prediction by averaging the absolute error (the absolute value of each error). MAD statistics are the size of the overall forecasting error for a model. The formula for calculating MAD is as follows:

\[
MAD = \frac{\sum_{t=1}^{n} |Y_t - \hat{Y}_t|}{n}
\]

Where \( Y_t \) is the actual data in the period \( t \), \( \hat{Y}_t \) the value of forecasting in the period \( t \), and \( n \) is the number of data points.

3. RESULTS AND DISCUSSION

This section discusses the result and provides detailed statistical data, estimates the model parameters, tests the significance of the model estimator, determines the Cobb-Douglas model estimator, and forecasts CO₂ emissions for the 2015-2017 period.

3.1. Descriptive Statistics Data

Descriptive statistics are structured to provide an overview of the quantitative data used in this study. Let us say \( Q_t \) is the CO₂ emissions; \( K_t \) is the GDP, and \( L_t \) is the population density. The descriptive statistical data of this research is presented in Table 1.

The graph of GDP data is illustrated in Figure 2, the population density in Figure 3, and the graph of CO₂ emission data in Figure 4.

3.2. Estimating the Model Parameters

In this section, the parameters of the linear regression model (5) are calculated using the ACO algorithm. In this estimate, the objective function is the minimisation of the sum of residual squares in equation (6). The model parameter estimation steps using the ACO algorithm is done with the help of Matlab R2015. The result of parameter estimation using the ACO algorithm gives the parameter estimator values of \( \hat{\alpha}_0 = -30.967 \), \( \hat{\beta}_0 = 0.19552 \) and \( \hat{\beta}_2 = 1.558 \). Substitute values are \( \alpha_0, \beta_1, \) and \( \beta_2 \) in equation (5) to obtain the multiple linear regression equation:

\[
Y_t = -30.967 + 0.19552X_{1t} + 1.558X_{2t} + \epsilon_t
\]

3.3. Testing the Model Estimator Significance

First, examine the partial effect of the parameters \( \alpha_0, \beta_1, \) and \( \beta_2 \). For parameter estimator of \( \alpha_0 \) assume \( H_0: \hat{\alpha}_0 = 0 \) and \( H_1: \hat{\alpha}_0 \neq 0 \). The result of the calculation using equation (11) gives \( t_{\text{Statistic}} = -10.80 \), while at a significant level \( \alpha = 0.05 \) with degrees of freedom \( df = 51-2=49 \) the critical value is \( t_{(49;0.05)} = -2.0105 \). So, it shows that \( t_{\text{Statistic}} > t_{(49;0.05)} \) thus the hypothesis, \( H_0 \) is rejected which means parameter \( \hat{\alpha}_0 = -30.967 \) is significant. Using the same method, testing is done for parameter estimator \( \hat{\beta}_1 = 0.19552 \) and \( \hat{\beta}_2 = 1.558 \). The test results conclude that parameter estimators \( \hat{\beta}_1 = 0.19552 \) and \( \hat{\beta}_2 = 1.558 \) are significant.

Second, we tested the simultaneous effect and significance value of parameter estimators \( \alpha_0, \beta_1, \) and \( \beta_2 \). The hypothesis used is \( H_0: \alpha_0 = \hat{\beta}_1 = \hat{\beta}_2 = 0, \) and \( H_1: \exists \alpha_0 \neq \beta_1 \neq \beta_2 \neq 0 \). The result of calculation using equation (12) gives the value \( F_{\text{Statistic}} = 1114.50 \), while at the level of significance \( \alpha = 0.05 \) the critical value is \( F_{(149;0.05)} = 4.02 \) which shows that \( F_{\text{Statistic}} > F(149;0.95) \). Thus hypothesis \( H_0 \) is rejected which means parameters \( \hat{\alpha}_0 = -30.967, \hat{\beta}_1 = 0.19552, \) and \( \hat{\beta}_2 = 1.558 \) significantly affects the dependent variable.

Third, test the assumption of residual normality \( \epsilon_t, H_0: \epsilon_t \) normally distributed with zero mean and certain variance, and \( H_1: \epsilon_t \) not normally distributed with mean zero and variance one. The calculation using equation (16) yields a value of \( AD_{\text{Statistic}} = 0.2222 \), while the critical value \( AD_{\text{Critical}} = 0.74047 \). It shows that \( AD_{\text{Statistic}} < AD_{\text{Critical}} \) thus the hypothesis \( H_0 \) accepted, which means \( \epsilon_t \) is normally distributed. The estimation result also generated that mean \( \hat{\mu} = 0.005426 \) with variance \( \hat{\sigma}^2 = 0.014568 \). Therefore, we conclude that \( \epsilon_t \sim N(0, 0.014568) \).

### Table 1: Descriptive statistics data

| Statistic | \( Q_t \) | \( K_t \) | \( L_t \) |
|-----------|-----------|-----------|-----------|
| Mean      | 1.02753743| 974.481835| 181.239.285|
| Median    | 0.89894000| 584.263600| 183.000.000|
| Maximum   | 2.55975023| 3,687.9540| 255.000.000|
| Minimum   | 0.23191548| 53.5161517| 105.907.403|
| SD        | 0.57736700| 1011.15700| 446.693.252|
This section also measured the strength of the relationship between independent and dependent variables. The strength of this relationship is measured by determining the deterministic coefficient value using equation (17). The calculation yields the value of the deterministic coefficient $r^2 = 98.0\%$, showing that the relationship between the independent and dependent variable is very strong.

### 3.4. Establishing a Cobb-Douglas Model Estimator

After testing the significance, it shows that the regression model given in equation (19) is well suited to model the effect of GDP and the population density on CO$_2$ emissions in Indonesia. In equation (6), the model estimator is used for forecasting and obtained the equation:

\[
\hat{Y} = -30.967 + 0.19552X_{1t} + 1.558X_{2t}, \text{ or}
\]

\[
\log \hat{Q}_t = \log e^{-30.967} + 0.19552 \log K_t + 1.558 \log L_t, \text{ or}
\]

\[
\hat{Q}_t = e^{-30.967} K_t^{0.19552} L_t^{1.558}
\]

Equation (20) is a production function of the Cobb-Douglas model estimator, which describes the effect of GDP $K_t$ and population density $L_t$ on CO$_2$ emissions in Indonesia $Q_t$.

### 3.5. CO$_2$ Emissions Forecast for 2015-2017

Using equation (20), CO$_2$ emissions in Indonesia generated in 2015-2017 are predicted by including the increased in each variable from 2015 to 2017. The results of these predictions are shown in Figure 5.

The forecast accuracy is measured using equation (15). The calculation obtained the value of MAD = 6.67\%. Thus, the error of the CO$_2$ emission model influenced by GDP and population density using MAD is relatively small, indicating that the model estimator is very good.

### 3.6. Discussion

Taking into account the graph of the GDP data in Figure 2, it appears that the value of the GDP in Indonesia has increased year on year. Although there is a decrease in 2000, there is a very sharp rise after the year 2000. The year 2010 recorded a slight decrease, but the trend rose immediately afterwards. It shows that the GDP in Indonesia is increasing steadily. Similarly, the population density data in Figure 3 is a straight line with a rising trend. It illustrates that the population density in Indonesia also increased year on year.
As a consequence of the increase in the value of GDP and population density, there is a sharp rise in CO₂ emissions in Indonesia. Even the year 2010 experienced an increase in CO₂ emissions, although there is a slight decline in the rate of increase. In general, CO₂ emissions in Indonesia from year to year have increased. Many studies CO₂ emissions from exhaust gases to illustrate the degree of pollution. High CO₂ emissions can be caused by high energy consumption and deforestation. The higher the income of a country, the higher the community’s ability to pay for energy consumption. Similarly, the higher the increase in population, the higher the rate of deforestation exploited for economic purposes. Figures 2-4 indicate increased CO₂ emissions in Indonesia accompanied by increases in GDP and population density.

The estimation of the effect of GDP and population density on CO₂ emissions is expressed as a production function of the Cobb-Douglas model of equation (20). Taking into account the Cobb-Douglas model estimator in equation (20), the elasticity of output as the GDP can be measured using the estimator coefficient parameter \( \hat{\beta}_1 = 0.19552 \) while the output elasticity as the population density can be measured using the estimator coefficient parameter \( \hat{\beta}_2 = 1.558 \). It indicates that population density is more elastic compared to GDP. That is, the increase in CO₂ emissions in Indonesia is influenced more by the population density compared to the GDP growth rates. Similarly, the sum of parameters is \( \hat{\beta}_1 + \hat{\beta}_2 = 1.75352 \) or \( \hat{\beta}_1 + \hat{\beta}_2 > 1 \), so that the production function of the Cobb-Douglas model of equation (20) has the characteristics of increasing return to scale. That is, an increase in GDP and population density will lead to an increase in CO₂ emissions with an elasticity of 1.75352. It should be a concern for the Indonesian people and recognise the roles played by economic growth and population density on CO₂ emissions and air pollution in Indonesia.

4. CONCLUSION

In this paper, we have analysed the effect of GDP and population density on CO₂ emissions in Indonesia. Based on the results, we conclude that GDP and population density significantly affect CO₂ emissions in Indonesia. The effect of GDP and population density on CO₂ emissions can be modelled as a production function of the Cobb-Douglas model. The estimator of the production function of the Cobb-Douglas model has the characteristics of increasing return to scale. It means that any increase in input in the form of GDP and population density will lead to a rise in output in the form of CO₂ emissions in Indonesia.

Forecasting (prediction) of CO₂ emissions using the Cobb-Douglas model estimators gives a MAD error rate of 6.67%. The error rate is small meaning that the estimated Cobb-Douglas model estimator is considered a suitable tool to measure the relationship between GDP and population density and CO₂ emissions.

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