Relational Quantum Measurements, Information, and State Collapse

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Abstract

The quantum measurement problem considered for measuring system (MS) consist of measured state S (particle), detector D and information processing device (observer) O. It’s shown that O states selfreference structure results in principal nonobservability of MS interference terms which discriminate pure and mixed S states. Such observables restriction permit to construct for MS states subjective representation (SR) which describes probabilistic evolution for measurement events observed by O and his subjective information about S values. SR is dual and nonequivalent to MS Hilbert space $H$ for external observer $O'$. Due to it SR evolution is compatible with Schrodinger linear MS evolution observed by $O'$. It’s argued that SR evolution corresponds to S state collapse for individual events observed by $O$.

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1 Introduction

Despite that Quantum Mechanics (QM) describe perfectly most of experimental effects in microscopic domain there still some difficult questions and ‘dark spots’ connected with its Measurement Theory and more generally with its proper interpretation. Of them the problem of the state vector collapse is most remarkable and straightforward and it’s still open despite the multitude of the proposed models and theories (for the review see [1]). Eventually the measurements and collapse studies can help us to select the true QM interpretation out of many proposed. This paper analyses some microscopic dynamical models of quantum measurements which attempt to describe the evolution of the measuring system (MS) from the first QM principles. In our approach MS includes the measured state (particle) S, detector D amplifying S signal, environment E and observer O which processes and stores the information. Under observer we mean information gaining and utilizing system (IGUS) of arbitrary structure [2]. It can be human brain or some automatic device processing the information, but in both cases it’s the system with many internal degrees of freedom (DF) memorizing the large amount of information. In general the computer information processing or perception by human brain is the physical objects evolution which on microscopic level supposedly obeys to QM laws [3].

Copenhagen QM interpretation divide our physical world into microscopic objects which obeys to QM laws and macroscopic objects, also observers which are strictly classical. This artificial partition was much criticized, first of all because it’s not clear where to put this quantum/classical border. Moreover there are strong experimental evidences that at the dynamical level no such border exists and QM successfully describes large, complicated systems including biological ones.

The possible role of observer in quantum measurements was discussed for long time [4], but now it attracts the significant attention again due to the the progress of quantum information studies [3]. The different aspects of observer inclusion in QM formalism was discussed in Rovelli paper which includes extensive review of previous activity [3]. Such approach called Relational QM isn’t selfconsistent formalism at the current stage. Rather it’s phenomenological theory with many ad hoc assumptions especially concerning the measurement problem and we’ll investigate some of its difficulties. As its particular version can be regarded Kochen Witnessing QM interpretation [7].

Relational QM concedes (Hypothesis 1 of Rovelli paper) that QM description is applicable both for microscopic states and macroscopic objects including observer O which Dirac state vector |O⟩ can be defined relative to some other observer O′, which is also another quantum object. Of course this assumption it’s not well founded and real O state can be much more complicated, but it’s reasonable to start from that simple case. Consequently the evolution of any complex system C described by Schrodinger equation and for C the superposition principle holds true at any time. MS measurement description by O formally must include evolution of O own internal DFs which participate in the interaction with S [3].
The role of observer in the measurement and its selfmeasurement analyzed in some works as the implication of more general algebraic problem of selfreference \[1\]. Following this approach Breuer derived the general selfmeasurement restrictions for classical and quantum measurements \[8\]. This formalism don’t resolve the measurement problem, but its comparison with Relational QM will be shown to result in some important conclusions. From this analysis we propose modification of standard QM Hilbert space formalism which permit \(O\) to observe state collapse without contradiction with MS linear evolution. Its main feature is the extension of QM states manifold permitting to account observer selfmeasurement effects, which were qualitatively formulated by Rovelli \[1\].

In chap 2. we describe our measurement model and propose the particular variant of QM formalism modification. In chap 3. we’ll discuss gedankenexperiments which help to interpret our formalism and discuss its implications. In chap. 4 we’ll discuss interpretation of our results and their physical and philosophical implications.

Of course if some correction to quantum dynamics like in GRW model exist \[12\] then the state collapse can occur in macroscopic detector. But until such effects would be found and the standard QM Hamiltonian regarded well established, we must inspect in detail observer properties exploring Measurement problem.

Here it’s necessary to make some technical comments on our model premises and review some terminology. For our models we’ll suppose that MS always can be described completely (including Environment if necessary) by some state vector \(|MS\rangle\) relative to \(O'\) or by density matrix if it’s in the mixed state. MS can be closed system, like atom in the box or open pure system surrounded by electromagnetic vacuum or \(E\) of other kind. We don’t assume in our work any special properties of \(O\) internal states beyond standard QM.

We’ll use Rovelli approach to quantum information as the correlation between \(S\) and \(O\) states. It differs from the standard definition \[10\], but is more useful for pure states measurements. It defines that \(O\) have the information about \(S\), if \(O\) (internal) state correlates with \(S\) state \[6\]. Note that such correlation, if to exclude the noise and errors means \(S\) and \(O\) states causal connection realized via their interaction. In particular we’ll use projective or selected information \(I_Q\) related to arbitrary \(S\) observable \(Q\). \(I_Q\) measure can be some function of \(Q\) dispersion of the kind \(I_Q \approx \sigma_Q^{-1}\). More correct seems to use \(I_Q = -\text{ln}\sigma_Q^2\) with proper normalization, but our conclusions only slightly depend on its exact form \[11\]. \(\sigma_Q\) derived from \(O\) state after the measurement of some \(Q'\) on \(S\), which in general can differ from \(Q\) but gives some \(Q\) value estimate.

In this paper the brain-computer analogy used without discussing its reliability and philosophical implications \[4\]. We’ll ignore here quantum computer options having in mind only the standard solid-state dissipative computers. We must stress that throughout our paper the observer consciousness never referred directly. Rather in our model observer can be regarded as active reference frame (RF) which interacts with studied object. The terms ‘perceptions’, ‘impressions’ used by us in a Wigner sense \[4\] of observer subjective description of experimental results and so can be
defined in strictly physical and information theory terms.

2 Selfmeasurement and Weak Collapse

We’ll consider the measurement description for simple model for detector D and observer O each only with one DF each corresponding to Von Neuman scheme. The example of dynamical model with many DFs gives Coleman-Hepp model described in appendix [21]. Account of many DFs doesn’t change principally the results obtained below [9], but in addition it resolves the problem of ‘preferred basis’ arising for one DF detector model [11]. Let’s consider in this ansatz O’ description of the measurement by O of binary observable \( \hat{Q} \) on S state:

\[
\psi_s = a_1|s_1\rangle + a_2|s_2\rangle
\]

, where \(|s_{1,2}\rangle\) are Q eigenstates with eigenvalues \(q_{1,2}\). Initial D, O states are \(|D_0\rangle, |O_0\rangle\) relative to O’ RF. We assume that S-D-O measuring interaction starts at \(t = t_0\) and finished at some finite \(t_1\). It follows from the linearity of Schrödinger equation that for suitable interaction Hamiltonian \(\hat{H}_I\) at \(t > t_1\) the state of MS system relative to O’ observer is

\[
\Psi_{MS} = a_1|s_1^{f}\rangle|D_1\rangle|O_1\rangle + a_2|s_2^{f}\rangle|D_2\rangle|O_2\rangle
\]  (1)

Here \(|D_1\rangle, |O_1\rangle\) are D, O state vectors obtained after the measurement of particular Q eigenstate \(|s\rangle = |s_1\rangle\) and are eigenstates of \(Q_D, Q_O\) observables and correspondingly for \(s_2, O_2\) (below state vectors with \(n > 2\) components used with the same notations). All this states including O belongs to Hilbert space \(H'\) defined in O’ RF and Hilbert space \(H\) in O RF supposedly can be obtained performing unitary \(H'\) transformation \(\hat{U}'\) to O c.m.s.. \(\hat{U}'\) can be neglected if only internal or RF independent discrete states regarded permitting to take \(H = H'\). For realistic IGUS \(|O_{1,2}\rangle\) can correspond to some excitations of O internal collective DF like phonons, etc., which memorize this Q information, but we don’t consider its particular physical mechanism here.

For the simplicity in the following we’ll omit detector D in MS chain assuming that S directly interacts with O. It’s reasonable for our simple model, because if to neglect decoherence the only D effect is the amplification of S signal to make it conceivable for O. We’ll assume also that after interaction S leaves O volume, which can be regarded as ‘self-decoherence’, because final MS state quantum phase becomes unavailable for O. More realistic dynamic decoherence mechanisms will be discussed in final chapter [9]. In this case MS initial state is:

\[
\Psi_{MS}^0 = (a_1|s_1\rangle + a_2|s_2\rangle)|O_o\rangle
\]  (2)

and final state at \(t > t_1\):

\[
\Psi_{MS} = a_1|s_1^{f}\rangle|O_1\rangle + a_2|s_2^{f}\rangle|O_2\rangle
\]  (3)
to which corresponds the density matrix $\rho_{MS}$. In most cases one can take for the simplicity $|s_i^{1}\rangle = |s_i\rangle$ without influencing main results. We’ll suppose that for $t > t_1$ measurement definitely finished which simplifies all the calculations, but in fact that’s fulfilled exactly only for the restricted class of models like Coleman-Hepp. Thus QM predicts at time $t > t_1$ for external observer $O'$ MS is in the pure state $\Psi_{MS}$ of (3) which is superposition of two states. Yet we know from the experiment that $O$ observes some definite random $Q_O$ value and acquires the state $|O_1\rangle$ or $|O_2\rangle$, from which he concludes that $S$ state is $|s_1\rangle$ or $|s_2\rangle$. If detector D included into MS chain then this $O$ memory states results from observing detector pointer position $D_1$ or $D_2$, but if D omitted like in our case they result from direct $S-O$ interaction.

The standard QM conclusion is that MS final state coincides with the statistical ensemble of such individual final states for $O$ described by density matrix of mixed state $\rho_m$ which presumably means the state collapse:

$$\rho_m = |a_1|^2 |s_1\rangle\langle s_1| |O_1\rangle\langle O_1| + |a_2|^2 |s_2\rangle\langle s_2| |O_2\rangle\langle O_2|$$

(4)

It would be natural to expect that MS final states described by $O$ and $O'$ are connected by some unitary transformation, but it’s well known that such transformation between (4) and (3) don’t exist [1]. It’s quite difficult to doubt both in correctness of $O'$ description of MS evolution by Schrodinger equation and in collapse experimental observations. This contradiction constitutes famous Wigner dilemma for $O,O'$ [5]. Really if for $O$ MS state is (3) it formally describes the superposition of alternative $O$ impressions on measurement results and its meaning is difficult to interpret. We mentioned already that Relational QM suppose phenomenologically that $O'$ description is correct. But in standard QM formalism it’s incompatible with assignment of definite values $q_i$ to $Q$ in $O$ RF in particular event [14]. We’ll propose here alternative formalism in which MS state linear evolution description by $O'$ is correct, but the description (4) for $O$ is incomplete.

Because we include observer in our model it’s necessary to formulate some minimal assumptions about observer internal states which stipulate his reaction on the input quantum signal. We’ll suppose that for any $Q$ eigenstate $|s_i\rangle$ with probability close to 1 after $S$ measurement finished at $t > t_1$ and $O$ state becomes $|O_i\rangle$ observer $O$ have the impression that the measurement event occurred and the value of outcome is $q_i$. It means that at least in this case and also for mixed states the signal memorization described by Schrodinger equation and so it is reversible process which for $O$ perceived as individual event with definite outcome. Of course this is only subjective not universally objective event, but probably other kinds of events are impossible. This events or impressions are connected with the excitations of $O$ internal DFs. The simple $O$ toy-model of information memorization is hydrogen-like atom for which $O_0$ is ground state and $O_i$ are different metastable levels excited by $s_i$. If $S$ state is the superposition (1) then we’ll suppose that its measurement also result in appearance of some new $O$ impression without specifying it at the moment. This considerations have little importance for the following calculations, rather they explain our philosophy of impressions-states relation.
Here we assumed that at $t > t_1$ measurement finished with probability 1, but for realistic measurement Hamiltonians it’s only approximately true, because transition amplitudes have long tails. This assumption exactly fulfilled only for some simple models like Coleman-Hepp, but we’ll apply it to simplify our analysis. The more subtle question of exact time at which $O$ percepts its own final $O_j$ isn’t important at this stage.

To discuss $O$ selfdescription let’s consider Breuer selfmeasurement theorem which is valid for the large class both of classical and quantum measurements [3]. Any measurement of studied system MS is the mapping of MS states set $N_S$ on observer states set $N_O$. For the situations when observer $O$ is the part of the studied system MS - measurement from inside, $N_O$ is $N_S$ subset and $O$ state in this case is MS state projection on $N_O$ called MS restricted $O$ state $R_O$. From $N_S$ mapping properties some principal restrictions for $O$ states were obtained. The theorem claims that if for two arbitrary MS states their restricted $O$ states coincide then for $O$ this MS states are indistinguishable. The origin of this effect is easy to understand: $O$ has less number of DFs then MS and so can’t describe completely MS state. For quantum measurements Breuer supposed that $O$ restricted state can be the partial trace of complete MS state (3):

$$R_O = Tr_s \rho_{MS} = \sum |a_i|^2 |O_i\rangle \langle O_i|$$  \hspace{1cm} (5)

Note that for MS mixed state $\rho_m$ of (4) the restricted state is the same $R^m_O = R_O$. This equality doesn’t mean collapse of MS state $\Psi_{MS}$, because it holds for statistical quantum ensembles, but collapse in fact must be tested for individual events as explained below. Such restricted $R_O$ form suppose that $O$ can percept only his internal excitations independently of quantum correlations with $S$ state. This assumption can be wrong for quantum systems due to well known quantum entanglement. and in fact this effects study shows that from equality of restricted states doesn’t follows the transition of pure system state to mixed one. MS interference term observable:

$$B = |O_1\rangle \langle O_2||s_1\rangle \langle s_2| + j.c.$$

being measured by $O'$ gives $\bar{B} = 0$ for mixed MS state (4) and in general $\bar{B} \neq 0$ for pure MS state (3). It evidence that even for statistical ensemble the observed by $O'$ effects can differentiate pure and mixed MS states. Note that $B$ value principally can’t be measured by $O$ directly, because $O$ performs $Q$ measurement and $[Q,B] \neq 0$.

Considering individual events Breuer notices that for mixed incoming state $S$ their MS state is:

$$\rho^m_l = |a_l|^2 |O_l\rangle \langle O_l||s_l\rangle \langle s_l|$$

with arbitrary $l$ which in accordance with standard QM objectively exists, but can be unknown for $O$. Its restricted state $R^m_l = |a_l|^2 |O_l\rangle \langle O_l|$ and so differs from $R_O$. Due to it main condition of Breuer Theorem violated and so it don’t applicable for this problem. From that Breuer concludes that the restricted states ansatz doesn’t
prove the collapse appearance for individual events even with inclusion of observer
in standard QM formalism with Hilbert space states set. The analogous conclusions
follows from the critical analysis of Witnessing interpretation [14].
Breuer analyses is quite informative and useful for our attempt to modify QM
formalism, because it prompts the particular extension of MS states set $N_S$ which
can describe MS state collapse noncontroversially. We’ll demand that this extension
suits to Relational QM hypothesis that $O$ and $O'$ can make different conclusions
about MS measurement - MS final states relative to $O$ and $O'$ are nonequivalent.
Due to it results of $B$ measurement by $O'$ can be unimportant for $O$ description
of measurement and so at least statistically it can be the same for pure and mixed
MS state. For individual events to agree with Relational QM our formalism is
demanded to describe the weak (subjective) collapse having different conditions
then standard one. It means the following: For MS final state its description
(perception) by observer $O$ presents the probabilistic events realization for $O$ with
partial probabilities $|a_i|^2$. It means that any restricted statistical state $R_O$ has unique
physical realization for $O$ which coincide for pure and mixed S state with the same
probabilities $|a_i|^2$. This is subjective collapse observed only by $O$ and in the same
time MS state for $O'$ stays pure and evolves according to Schrodinger equation.

So our aim is to find minimal modification of QM states set which can incorporate
simultaneously both MS linear state evolution for $O'$ and random events observed
by $O$. $B$ nonobservability for $O$ hints that states manifold must be modified for
MS states description, because in standard Hilbert space all hermitian operators
are observables. It’s worth to remind that Hilbert space is in fact empirical con-
struction which choice advocated by fitting most of QM data, and so QM states set
modification doesn’t seems unthinkable in principle. Such modifications attempts
were published already and most famous is Namiki-Pascazio many Hilbert spaces
formalism. In standard QM formalism all its states manifold representations are
unitarily equivalent, but observers interactions and evolution aren’t considered in
it. It will be shown that new formalism to some extent is analog of the nonequivlent
representations of commutation relations. Analogous superselection systems are well
studied for quantum nonperturbative Field theory (QFT) with infinite DF number
[15]. QFT methods were applied to measurement problem, but it’s not clear its
applicability for finite even macroscopic systems [16, 17].

To illustrate the formalism features let’s regard how $O$ measures some stochastic
parameter $q$ with distribution $P(q)$ in Classical Physics. For some $q$ distribution $P(q)$
(or $P_l$ array for discrete $q$ ) when $O$ measures $q$ he acquires instantly information
about $q$ value and initial $O_0$ state changes to some $O_j$ correlated with measured
$q_j$. Formally at this moment $P(q)$ collapses to delta-function, but in classical case
this effect reflects only $O$ information change. Suppose that $q$ acquires discrete
random values with 1- dimensional probabilities matrix $P_l$ $(l = 1, n)$ and after its
measurement $O$ acquires state $O_i$ with probability $P_i'$ which in ideal case coincides
with $P_l$ and define $O$ ensemble statistics. To present recorded result $O_i$ in given event
random 1-dimensional matrix $V^O = (0, ..., O_j, ..., 0)$ is used with only one $v_{ij}^O \neq 0$ in
each event. We write formally this event-state manifold as \( N_d = P \otimes V^O \) assuming to apply in quantum case such dual form which unite probability distribution \( P \) and \( O \) information in event \( V^O \).

It’s worth to remind that experimentalist never observes state vector directly, but his data consists of individual random events like detector pointer counts and the initial state vector restored from observed random events statistics. This fact prompts us to explore QM dual representations, in which state vector and random events can coexist simultaneously. The phenomenological dual ansatz for statistical ensemble states was proposed in \([9]\). We describe first this new representation for our MS system evolution observed by \( O \) and \( O' \) which Hilbert spaces \( H, H' \) will be our starting point. We use QM density matrices manifold \( L_q = (\rho \geq 0, Tr\rho = 1) \) constructed of \( H \) state vectors. They evolve according to standard Schrodinger-Liouville equation with MS Hamiltonian \( \hat{H}_c \):

\[
\dot{\rho} = [\rho, \hat{H}_c] \quad (7)
\]

For initial MS state vector \( \Psi_{MS}^0 \) MS measurement result in \( \rho_{MS} \) of \([3]\). Inside \( L_q \) we extract \( O \) restricted states submanifold of \([3]\) \( R_O = Tr s \rho \) and calculate in \( O \) basis the weights matrix \( P_j(t) = Tr_O(\hat{P}_j^O R_O) \) where \( \hat{P}_j^O \) is \( O_j \) projection operator. For \([3]\) it gives \( P_j(t) = |a_j(t)|^2 \). As we stressed already discussing Breuer Theorem the restricted state \( R_O \) of \([3]\) by itself has no probabilistic meaning and \( Q \) values are uncertain and nonobjective in \( O' \) RF \([8]\). But in \( O \) RF in our dual framework we’ll suppose it becomes probabilistic distribution \( P_j \) which generates random \( O \) states \( V^O = |O_j\rangle\langle O_j| \) for given event. It describes the restricted subjective state \( |O_j\rangle\langle O_j| \) observed by \( O \) in given event and corresponding to random \( q_j, q_{Oj} \) values. But in distinction from standard QM in our formalism the complete state \( \rho \) don’t disappear after the measurement. On the opposite it evolves all the time according to Schrodinger-Liouville equation \([7]\) and doesn’t suffers the collapse stochastic jumps. Thus our dual state or event-state, which due to \( j \) randomness differs for each event in \( O \) RF is doublet \( \Phi = |\rho, |O_j\rangle\langle O_j|\rangle \). Thus it includes dynamical (objective) component \( \rho \) which evolves linearly according to \([4]\), but isn’t observed directly by \( O \) and \( O \) subjective component \( V^O \) which describe \( O \) impression about event and which probabilistic evolution controlled by \( \rho \). In this case \( O_j \) observes with probability 1 S state component with which it’s entangled i.e. \( s_j \) and MS subjective state component can be defined:

\[
V^{MS} = |O_j\rangle\langle O_j| |s_j\rangle\langle s_j|
\]  

(8)

, but for our problem it’s equivalent to \( V^O \).

Complete manifold in \( O \) RF for this event-states is \( N_T = L_q \otimes L_V \) i.e tensor product of dynamical and subjective components. \( L_V \) is the linear space of diagonal positive matrices with \( tr V^O = 1 \). If we restrict our consideration only to pure states then \( N_T \) is equivalent to \( \hat{H} \otimes L_V \) and state vector can be used as event-state dynamical component. Note that the physical meaning of Hilbert space \( H \) in our
formalism essentially differs from standard QM, because the operator $B$ of (8) isn’t observable for $O$. Before measurement starts event state is $\Phi_0 = | \Psi_{MS}^0, V_0^O \rangle$ where $V_0^O = |O_0\rangle\langle O_0|$ describe $O$ definite initial information.

In our formalism $O'$ has its own subjective linear space $L'_{V'}$ and in his RF the event-states manifold is $N'_T = H' \otimes L'_{V'}$ for pure states. From the above description it seems that subspace $L_V$ must be unobservable for $O'$ and vice versa, because $O'$ interacts directly only with $\rho$ component of event-state. But this is true if $O', O$ don’t interact and $V^O, V'^O$ events can be correlated if $O'$ measures $O$ state expressed by $\rho$ and below we’ll discuss such effects.

If $S$ don’t interact with $O$ then $V_0^O$ is time invariant and one obtains standard QM evolution for event-state dynamical component $\rho$. If one interested only to calculate $\bar{Q}$ after $S$ measurement by $O$ or any other expectation values ignoring event structure it’s possible to drop $V^O$ component and to make standard QM calculations for $\rho$.

Proposed doublet states ansatz can be regarded as the upgrade of standard reduction postulate, which describes for standard QM how state vector correlates with the changes of observer information in the measurement. The main difference is that in place of abrupt and irreversible state vector $\Psi_t$ in standard QM, in our formalism in $O'$ RF the dynamical component $\Psi_{MS}$ of MS event-state evolves linearly and reversibly in accordance with (7). It’s only subjective component $V^O$ which changes abruptly and probabilistically describing the change of $O$ subjective information about $S$. We must stress that subjective $V^O$ component is physical object which is new degree of freedom connected with $O$ internal state, which lays outside of MS Hilbert space $H$. We noticed already that in standard QM formalism in MS state (8) Q values aren’t objectively existing for $O'$ and $O$ which is serious argument against Witnessing interpretation [14]. That’s the same for $O'$ in our formalism, but in the same time $Q$ and $Q_O$ can have objective values $q_j, q_{Oj}$ in $O$ RF.

Equation (7) is in fact master equation for probabilities $P_j(t)$ which induce $V^O$ probabilistic distribution which becomes new random DF of final $O$ states $O_j$. Due to independence of MS dynamical state component of internal parameter $j$ of $V^O$ this $O$-$S$ evolution is reversible. Due to it no experiment performed by $O'$ on MS wouldn’t contradict to standard QM. If $O$ perform selfmeasurement experiment on MS the situation is more subtle and will be discussed in the next chapter. Note that in this formalism parameter $j$ don’t existed before $S$-$O$ interaction starts.

Now we regard in more detail relation between dynamical and subjective components of event-state. We proposed already that parameter $j$ of $V^O$ defined at random with probabilities $P_j$ in $S$ measurement. In general to calculate $\Phi$ evolution for arbitrary complex system MS Schrodinger-Lioville equation for MS Hamiltonian can be used and $P_j(t)$ found. Then from $P_j(t)$ at any time we find random $V^O$. So if we have several $S$-$O$ rescatterings each time after it we get new $V^O$ state component which effect in details will be discussed below. To exclude spontaneous $V^O$ jumps without effective interactions with external world we introduce additional $O$ identity condition [22] : if $S$ and $O$ don’t interact then the same $V^O$ conserved.
We supplement it by more general condition: if different $\Psi_{MS}$ $O_j$ branches don’t intersects i.e. $\langle O_i | \hat{H}_c | O_j \rangle = 0, i \neq j$ then $V^O$ conserved. It means that $O$ observes constantly only $s_j$ branch of $S$ state, despite that after measurement $s_j$ state can evolve. This conditions don’t influence MS dynamics, but only subjective information $V^O$. Here $L_V$ corresponds to the simplest measurement and in general it will can have more complicated form, which can be obtained demanding them to correspond to the probabilities definition of standard QM [1]. For example 2-dimensional values correlation measurement by $O$ has the distribution:

$$ P_{ij} = Tr(\rho \hat{P}^O_{1i} \hat{P}^O_{2j}) \quad (9) $$

where $\hat{P}^O_{ij}$ are projectors on corresponding $O$ substates. Corresponding subspace $L_V = V^{O1} \otimes V^{O2}$.

If one regards the statistical results for quantum ensembles then statistics in $L_V$ subspace corresponds to $|a_j(t)|^2$ the probabilities of particular $O$ observation. Note that their meaning differs from $O^'$ representation where they can’t be regarded as probabilities and can be regarded only like some weights. Note that this restricted or partial states $R_O$ gives naturally the values of outcome probabilities $|a_i|^2$ which is quite difficult to obtain in other theories explaining collapse like Many Worlds Interpretations [1].

## 3 Collapse and Quantum Memory Eraser

To discuss measurement dynamics in our formalism for more subtle situations let’s consider several gedankenexperiments for different selfmeasurement effects:

1) ‘Undoing’ the measurement. Such experiment was discussed by Vaidman [19] and Detsch [20] for many worlds interpretation but we’ll regard its slightly different version. Consider $S$ state (2) measurement by $O$ resulting in the final state (3). This $S$ measurement can be undone or reversed with the help of auxiliary devices - mirrors, etc., which reflects $S$ back in $O$ direction and make them reinteract. It means that final state $\Psi_{MS}$ obtained at time $t_1$ at the later time $t_2$ transformed backward to MS initial state $\Psi_{MS}^0$. In any realistic layout to restore state (2) is practically impossible but to get the arbitrary $S-O$ factorized state is more simple problem and that’s enough for our considerations. Despite that under realistic conditions the decoherence processes make this reversing immensely difficult it doesn’t contradict to any physical laws.

If we consider this experiment in standard QM we come to some strange conclusions. When memorization finished at $t_1$ in each event MS collapsed to some arbitrary state $|s_i\rangle |O_i\rangle$. Then at $t_2$ $O$ undergoes the external reversing influence, in particular it can be the second collision with $S$ during reversing experiment and its state changes again and such rescattering leads to a new state correlated with $|s_i\rangle$:

$$ |s_i\rangle |O_i\rangle \rightarrow |s'_i\rangle |O_o\rangle $$
It means that $O$ memory erased and he forgets measured $Q$ value $q_i$, but if he measure $S$ state again he would restore the same $q_i$ value. Its statistical state is

$$
\rho'_m = |O_0⟩⟨O_0| \sum |a_i|^2 |s'_i⟩⟨s'_i|
$$

But this $S$ final state differs from MS state (2) predicted from MS linear evolution observed by $O'$ and in principle this difference can be tested on $S$ state without $O$ measurement. In our doublet formalism it’s necessary also to describe subjective event-state component $V^O$ which after measurement becomes some random $V^O_j$. But after reversing independently of $j$ it returns to initial value $V^O_0$, according to evolution ansatz described in previous chapter. If such description of this experiment is correct, as we can believe because its results coincides with Schrodinger evolution in $O'$ it follows that after $q_i$ value erased from $O$ memory it lost unrestorably also for any other possible observer. If after that $O$ would measure $Q$ again obtained new value $q_j$ will have no correlation with $q_i$.

Of course one should remember that existing for finite time intermediate $O$ states are in fact virtual states and differ from really stable states used here, but for macroscopic time intervals this difference becomes very small and probably can be neglected.

The analogy of 'undoing' with quantum eraser experiment is straightforward: there the photons polarization carry the information which can be erased and so change the system state [18]. The analogous experiment with information memorization by some massive objects like molecules will be important test of collapse models.

Note that observer $O'$ can perform on $O$ and $S$ also the direct measurement of interference terms for (3) without reversing MS state. Such experiment regarded for Coleman-Hepp model in [6] doesn’t introduces any new features in comparison with 'Undoing' and so we don’t discuss it here.

2) After $O$ measures $Q$ value of $S$ at $t_1$ which results in MS state (3) for $O'$, this $S$ observable is measured again by observer $O'$ at $t_2 > t_1$. The interaction of $O'$ with MS results in entangled state of $S, O$, and $O'$ and so both observers acquire some information about $S$ state. $O'$, MS state relative to next observer $O^2$ is:

$$
Ψ_{MS} = |a_1|s_1⟩|O_1⟩|O'_1⟩ + |a_2|s_2⟩|O_2⟩|O'_2⟩
$$

(10)

Note that now there is interference term operator $B'$ on $O' + MS$ which is unobservable for $O'$. The experiments of such kind were discussed frequently, due to its relation to EPR-Bohm correlations, but here we regard in detail its timing sequence. In our formalism at $t_1 < t < t_2$ observer $O$ already have the information that $Q$ value is some $q_i$, reflected by $V^O = |O_i⟩⟨O_i|$. In the same time $Q$ value stays uncertain and objectively nonexisting for $O'$, because relative to him MS state is (3) , which isn’t $Q$ eigenvalue [4]. When measurement by $O'$ finished the obtained $Q$ value coincides with $q_i$, but it don’t contradicts to the previous assumption that for $O'$ before $t_2$ it was principally uncertain. The reason is that in between $O'$ interacts with $S$ and
this interaction transfer S information to O' and makes Q value definite for him. To check that Q value coincides for O' and O, O' can perform measurement both Q and \( Q_O \) which is described by (9). Together this experiments supposedly demonstrate the subjective character of collapse, which happens only after interaction of S with particular observer occurs. It differs from standard QM picture of objective collapse which occurs for all observers simultaneously independently of their participation in the measurement. This results contradict to first intuitive impression that if Q had some definite value relative to O then its objectively exists also for O' and any other observer. But it’s erroneous conclusion because at that time MS state relative to O' is pure state \( \Psi_{MS} \) of (3) which isn’t Q eigenstate. To demonstrate it experimentally O' can measure \( \hat{B} \) on MS which don’t commute with Q and for which \( \Psi_{MS} \) is eigenstate. Alternatively O' can perform 'undoing' on MS and Q value known to O will be erased unerestorably, which is impossible if Q value objectively existed for O'.

Let’s discuss why O, O’ MS descriptions can be compatible without contradictions. After MS selfmesurement finished in a state (3) it describes O' information about S, O states and in particular Q uncertainty. Even if O have definite information about Q value, as our doublet formalism assume, until any signal will be send by O to O' his information don’t change and described by \( \Psi_{MS} \) of (3). S measurement by O' discussed above corresponds to such signal and after it O' acquire new information expressed by \( V'^{O} \) value.

Relativistic analysis of EPR-Bohm pairs measurement also indicates subjective character of state vector and its collapse [24]. It’s shown that state vector can be defined only on space-like hypersurfaces which are noncovariant for different observers. This results supports nonequivalence of different observers assumed in Relational QM and our formalism, for which EPR-Bohm correlations seems to deserve detailed study.

In case of general S-O interaction consisting of several effective rescatterings alike in ‘undoing’, one should calculate \( P_i(t) \) each time and define \( V'^{O} \) anew. Note that we must be really interested only in final O state, because in this model O has no memory about intermediate states and no dynamical dependence on them.

In doublet formalism O percepts only \( O_j \) component of complete state vector \( \Psi_{MS} \) and it’s interesting to speculate why other its components aren’t observed. Our tempting explanation is prompted by Breuer theorem, but in doublet formalism we should reconsider it for probabilistic situation. It was shown that O selfmesurement is always noncomplete and it’s possible to assume that in given event O can percept only part \( O_j \) of his effective physical state. Note that in the same time O' can perform \( B \) measurement which demonstrates the existence of other O components.

Our doublet formalism can be interpreted if to concede that S initial state vector describes Q fundamental uncertainty for O i.e. limited amount of selected information \( I_Q \). When S-O start to interact corresponding O internal DFs excited and its internal state correlated with S. It’s tempting to assume that for O internal states any uncertainty is excluded - i.e. O knows his own state due to continuous interactions inside O and initially uncertain Q percepted as random but certain value \( Q_O \).
$O$ is the 'last ring' of measurement chain and it can have such singularity, because it related to $O$ uncertainty relative to himself and don’t contradicts to QM dynamics. It reminds Von Neuman and London - Bauer collapse theories, but our formalism agrees with applicability of Schrodinger equation to MS for external observer $O'$.

4 Discussion

In this paper the simple measurement model studied which accounts observer (IGUS) information processing and memorization. Real IGUSes are very complicated systems with many DFs, but the main quantum effects like superpositions or decoherence are the same for large and small systems and can be studied with the simple models.

Summing up our results we notice that by itself observer inclusion into QM measurement chain doesn’t lead to collapse explanation. If in addition QM states manifold changed to dual structure it results in consistent collapse description, which permit to change collapse postulate modifying it into more consistent and sensible form. Its most important feature is the absence of any special collapse dynamics for external observer $O'$ for which MS evolves according to Schrodinger equation.

So it seems that to avoid completely even weak or subjective collapse it’s necessary to reconsider the quantum theory foundations, not only its measurement part. For this purpose we develop in forcoming paper Information Causality Interpretation (ICI) which explain the appearance of doublet event-state formalism.

Our doublet formalism demonstrates that probabilistic evolution is generic and unavoidable for QM and without it QM can’t acquire any sensible observational realization. Wave-particle dualism was always regarded as characteristic QM feature, but in our formalism it has straightforward description.

Rovelli assumed that in QM all physical variables can have definite values only relative to some observer or RF. Correspondingly in Relational QM state collapse is subjective effect occurring in the interaction of $O$ with measured state or with the signal send by other observer. It seems that doublet formalism can describe this main effects predicted phenomenologically by Relational QM.

It’s widely accepted now that decoherence effects are very important in measurement dynamics [13, 2]. But the frequent claim that collapse phenomena can be completely explained in its framework was shown to be incorrect at least for simple models [22]. But in our model of subjective or weak collapse some kind of decoherence is also present in the form of self-decoherence when $S$ departs from $O$ after interaction and additional $O$-environment decohering interaction only will amplify this effects. Our approach to collapse is close to the decoherence attitude, where also any additional collapse dynamics don’t exist. The main difference is that we suppose that collapse isn’t objective phenomena and has relational or subjective character and observed only by observer inside decohering system.

The situation with the measurement problem for two quantum observers has
much in common with Quantum reference frames introduced by Aharonov [25]. Note that our formalism is principally different from hidden parameters Theories where this stochastic parameters influence Quantum state dynamics. In our model $V^O$ internal parameter $j$ is on the opposite is controlled by evolution equation for quantum state.

Our formalism deserves detailed comparison with formalisms of different MWI variants, due to their analogy - both are theories without dynamical collapse [1]. In Everett+brain QM interpretations eq. (3) describes so called observer $O$ splitting identified with state collapse [22]. In this theory it’s assumed that each $O$ branch describes the different reality and the state collapse is phenomenological property of human consciousness. Obviously this approach has some common points with our models which deserve further analysis. In general all our experimental conclusions are based on human subjective perception. Assuming the computer-brain perception analogy in fact means that human signal perception also defined by $\bar{Q}_O$ values. Despite that this analogy looks quite reasonable we can’t give any proof of it. In our model in fact the state collapse have subjective character and occurs initially only for single observer $O$, but as was shown by Rovelli it doesn’t results in any contradictions [6]. If it’s sensible to discuss any world partition prompted by QM results it seems to be the division between subject - observer $O$ which collect information about surrounding world and this world objects which can include other observer $O'$.

**Appendix**

The simple model of observer with many DFs is modified Coleman-Hepp (CH) model which used often for QM paradoxes discussion [21]. CH model considers fermion $S^0$ spin z-projection measurement via interaction with $N$ spin-half atoms $A_i$ linear chain - 1-dimensional crystal detector $D$. $A_i$ atoms are regularly localized at the distance $r_0$ by the effective potential $U_i(x_i)$. $S^0$ initial state $\psi_0^0 = \varphi(x, t_0)(a_1 |u_0\rangle + a_2 |d_0\rangle)$ where $u, d$ are up,down spin states and $\varphi(x, t_0)$ is localized $S^0$ wave packet spreading along D spin chain. For the comparison the measurement of corresponding mixed state $\rho^0_m$ with weights $|a_{1,2}|^2$ will be regarded. $S^0 - D$ interaction Hamiltonian is:

$$H_I = (1 - \sigma_z^0) \sum_{i=1}^{N} V(x - x_i) \sigma_z^i$$

(11)

where $V$ is $S^0 - A_i$ interaction potential. For suitable model parameters and for D initial polarized state $\psi_D^0 = \prod |u_i\rangle$ one obtains that if $S^0$ initial spin state is $|u_0\rangle$ this D state conserved after $S^0$ passed over the chain, but for initial state $|d_0\rangle$ D state transformed into $\psi_D^0 = \prod |d_i\rangle$. Thus for finite $N$ at $t > t_1$ for $S^0 - D$ final state

$$\psi_f(t) = \psi_1(t) + \psi_2(t) = \varphi(x, t)(a_1 |u_0\rangle \psi_D^0 + a_2 (-i)^N |d_0\rangle \psi_D^0)$$

(12)

we get macroscopically different values of D pointer which described by the polarization operator : $\mu_z = \frac{1}{N} \sum \sigma_z^i$ acting in $h_D$ subspace. It gives estimate $\bar{\mu}_z = \bar{\sigma}_z^0$
and $\Delta Q = 0$, so this is strict exact measurement. Despite, it doesn’t mean $S^0$ state collapse because $S^0 - D$ interference terms (IT) operator:

$$B = \sigma^0_z B_I = \sigma^0_z \prod_{i=1}^{N} \sigma^y_i$$

(13)

describing spin-flips of all $A_i$ and $S^0$ spins. In principle $B$ also can be measured by observer $O$ and discriminate $S^0 - D$ mixed and pure states. Its expectation value $\langle B \rangle = .5(a_1^* a_2 + a_1 a_2^*)$ for $S^0 - D$ final state $\psi_f$ differs from $\langle B \rangle = 0$ for $S^0$ mixed state $\rho^0_m [27]$. For the convenience we exclude from consideration $a_1, a_2$ values such that $\langle B \rangle = 0$, which doesn’t influence on our final results. Note that $S^0$ IT can be measured separately, but only before $S^0 - D$ interaction starts, after it only their joint IT operator have sense. $\mu_z, B$ don’t commute and can’t be measured simultaneously:

$$[\mu_z, B] = \frac{i}{N} \sum_{i=1}^{N} \sigma^z_i \prod_{i \neq j}^{N} \sigma^y_j$$

(14)

It’s easy to propose how to measure collective (additive) operator $\mu_z$, but also $B$ values can be destructively measured decomposing $D$ into atoms and sending $A_i$ one by one and also $S^0$ into Stern-Gerlach magnet. Then measuring $A_i$ amount in each channel and their correlations by some other detector $D'$ one obtains information on $B$ value from it. Standard QM don’t regard any special features of destructive measurements assuming that any hermitian operator is observable and can be measured by one way or another.

References

[1] P.Busch, P.Lahti, P.Mittelstaedt, 'Quantum Theory of Measurements’ (Springer-Verlag, Berlin, 1996)

[2] D.Guilini et al., 'Decoherence and Appearance of Classical World’, (Springer-Verlag,Berlin,1996)

[3] D.Z.Albert, Phyl. of Science 54, 577 (1986)

[4] R.Penrose, ‘Shadows of Mind’ (Oxford, 1994)

[5] E.Wigner, 'Scientist speculates’ , (Heinemann, London, 1962)

[6] C. Rovelli, Int. Journ. Theor. Phys. 35, 1637 (1995); quant-ph 9609002 (1996),

[7] S.Kochen 'Symposium on Foundations of Modern Physics’ , (World scientific, Singapour, 1985)

[8] T.Breuer, Phyl. of Science 62, 197 (1995), Synthese 107, 1 (1996)
[9] S. Mayburov, quant-ph 9911103, J. Mod. Opt (2000) to appear

[10] B. Schumaher, Phys. Rev. A51, 2738 (1995)

[11] A. Elby, J. Bub Phys. Rev. A49, 4213, (1994)

[12] G.C. Girardi, A. Rimini, T. Weber Phys. Rev. D34, 470 (1986)

[13] W. Zurek, Phys Rev, D26, 1862 (1982)

[14] P. Lahti Int. J. Theor. Phys. 29, 339 (1990)

[15] H. Umezawa, H. Matsumoto, M. Tachiki, 'Thermofield Dynamics and Condensed States' (North-Holland, Amsterdam, 1982)

[16] R. Fukuda, Phys. Rev. A, 35, 8 (1987)

[17] S. Mayburov, Int. Journ. Theor. Phys. 37, 401 (1998)

[18] M. Scully, K. Druhl Phys. Rev. A25, 2208 (1982)

[19] L. Vaidman, quant-ph 9609006

[20] D. Deutsch, Int J. Theor. Phys. 24, 1 (1985)

[21] K. Hepp, Helv. Phys. Acta 45, 237 (1972)

[22] W. D’Espagnat, Found Phys. 20, 1157, (1990)

[23] S. Snauder Found., Phys. 23, 1553 (1993)

[24] Y. Aharonov, D.Z. Albert Phys. Rev. D24, 359 (1981)

[25] Y. Aharonov, T. Kaufherr Phys. Rev. D30, 368 (1984)

[26] A. Whitaker, J. Phys., A18, 253 (1985)

[27] J. S. Bell, Helv. Phys. Acta 48, 93 (1975)