Platicon Stability in Hot Cavities

Valery E. Lobanov1,*, Nikita M. Kondratiev2, Artem E. Shitikov1, Olga V. Borovkova3, Steevy J. Cordette2, and Igor A. Bilenko1,3

1Russian Quantum Center, 143026, Skolkovo, Russia
2Directed Energy Research Centre, Technology Innovation Institute, Abu Dhabi, United Arab Emirates
3Faculty of Physics, Lomonosov Moscow State University, 119991, Moscow, Russia
*Corresponding author: v.lobanov@rqc.ru

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The stability of platicons in hot cavities with normal group velocity at the interplay of Kerr and thermal nonlinearities was addressed numerically. The stability analysis was performed for different ranges of pump amplitude, thermal nonlinearity coefficient and thermal relaxation time. It was revealed that for the positive thermal effect, the high-energy wide platicons are stable, while the negative thermal coefficient provides the stability of narrow platicons.

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Over the past decades, the high-Q optical microresonators have proven to be an ideal platform for implementing, investigation, and application of various nonlinear optical phenomena [1–3]. Numerous studies have shown that the dynamics of nonlinear processes in optical microresonators pumped by continuous wave lasers are greatly affected by the thermal effects, such as thermo-optic and thermal expansion effects [4–6]. The influence of these effects is almost inevitable in microresonator-based photonic crystal resonator [27] for dispersion control or pump modulation systems [28, 29]. Moreover, the possibility of the turn-key regime of platicon generation enabled by the negative thermal effects was demonstrated. However, the applicability range of this approach is limited by the stability domain of platicons defined by the interplay of Kerr and thermal nonlinearities.

In this Letter, we perform a numerical analysis of platicon stability in the presence of both Kerr and thermal nonlinearities for a wide range of parameters. It is fascinating for many practical applications since the thermal effects are unavoidable at the pump powers required for Kerr combs excitation. We found that high-energy wide platicons (or narrow dark solitons) are stable for the positive thermal effect, while the negative effect stabilizes the narrow platicons.

We analyzed numerically the system of two equations: the Lugato-Lefever equation [23] for the slowly varying envelope of the intracavity field $\Psi$ and the rate equation for the normalized thermally induced detuning $\Theta$ [6–9]:

$$\begin{align*}
\frac{\partial \Psi}{\partial t} &= \frac{i}{2} \frac{\partial^2 \Psi}{\partial \varphi^2} - \left[1 + i(\alpha - \Theta)\right] \Psi + i|\Psi|^2 \Psi + f, \\
\frac{\partial \Theta}{\partial t} &= \frac{2}{\kappa T} \left( \frac{\partial^2 U}{\partial \varphi^2} - \Theta \right).
\end{align*}$$

(1)

Here $\tau = kt/2$ denotes the normalized time, $\kappa = \omega_0/Q$ is the cavity total decay rate (Q is the loaded quality factor), $\omega_0$ is the pumped mode resonant frequency, $\varphi \in [-\pi, \pi]$ is an azimuthal angle in the coordinate frame rotating with the rate equal to the microresonator free spectral range (FSR) $D_1$, $D_2 = 2D_2/\kappa$ is the normalized GVD coefficient, positive for the anomalous GVD and negative for the normal GVD [the microresonator eigenfrequencies are assumed to be $\omega_p = \omega_0 + D_1 \mu + \frac{D_2}{\kappa} \mu^2$, where $\mu$ is the mode number, calculated from the pumped model, $\alpha = 2(\omega_0 - \omega_p)/\kappa$ is the normalized detuning from the pump frequency $\omega_p$ from the pumped resonance. The normalized pump amplitude for matched beam area and coupler refraction is $f = \frac{\sqrt{\omega_0 c_0 n_p n_{0c} V_p}}{k r^2 V_{eff}}$ [9], where $c$ is the speed of light, $n_2$ is the...
microresonator nonlinear index, \( P_{in} \) is the input pump power, \( n \) is the refractive index of the microresonator mode, \( V_{eff} \) is the effective mode volume, \( \eta \) is the coupling efficiency \( [\eta = 1/2 \text{ for critical coupling}, \eta \to 1 \text{ for overloaded},] \), \( U = \int |\Psi|^2 d\varphi \).

The thermal relaxation time is not as important as its relation to the photon lifetime \( \tau_n \) since \( 2/\tau_n = 2t_{ph}/t_\Gamma \). Parameters \( t_\Gamma \) and \( n_{2\Gamma} \) both depend on material properties and the geometry of the resonator, as a result their values can be significantly different for thermal refraction and thermal expansion [4, 7]. However, sometimes it is possible to neglect one of the effects, for example, when special composite structures are used for the compensation of thermal expansion [30, 31]. Thus, as a first step we consider a single composite effect.

First, we searched for stationary solutions of Eq. (1) (when \( \frac{\partial \Psi}{\partial t} = 0, \frac{\partial^2 \Psi}{\partial t^2} = 0 \) ) in the form of dark solitons or platicons by means of the relaxation method using the following equation:

\[
\frac{d^2}{dt^2} \Psi - \left[ 1 + i \left( \alpha - \frac{n_{2\Gamma}}{n_2} \frac{U}{2\pi} \right) \right] \Psi + i |\Psi|^2 \Psi + f = 0.
\]

Then the stability of obtained dark soliton solutions was tested by imposing random perturbations (additive and/or multiplicative) in the solutions and simulating their subsequent evolution up to \( \tau \approx 10^4 \) using Eq. (1). Solitons that kept their shape during propagation were considered as the stable ones. Such analysis was performed for wide ranges of pump amplitude \( f = 1.5...5 \), normalized thermal relaxation time \( \tau_n/\tau_\Gamma = 0.001...10 \), and for wide range of thermal nonlinearity coefficient both positive and negative ones \( -20 \leq \frac{n_{2\Gamma}}{n_2} \leq 20 \). We set \( d_2 = -0.02 \) and checked that results were qualitatively the same for other values of the GVD coefficient.

In thermally-compensated microresonators several discrete energy levels corresponding to stable dark solitons or platicons with different widths may exist in the same spectral range organized in a bifurcation structure known as collapsed snaking [22, 32]. The dependence of platicon energy \( U \) on the pump frequency detuning \( \alpha \) is shown in the left panel of Fig. 1 for \( f = 3, \frac{n_{2\Gamma}}{n_2} = 0 \). Stable solutions are indicated by red lines. Upper energy levels correspond to wider platicons. The stability domains become narrower with the decrease of the platicon width.

Let’s start with positive values of the thermal nonlinearity coefficient. In this case the thermal resonance shift had the same direction as nonlinear shift, we observed the shift, stretching and transformation of energy levels of platicons [compare left and right panels of Fig. 1]. Thermal nonlinearity partially lifts the degeneracy of platicon energy levels. However, it should be noted that at rather large values of thermal nonlinearity coefficient, the transition between different platicon levels by pump frequency scanning is impossible: at forward scan with linear-in-time increase of detuning, the platicon transforms into low-intensity homogeneous state, while at backward scan the platicon comes into high-intensity homogeneous state.

Platicon stability was examined for different combinations of the pump amplitude, thermal nonlinearity coefficient and thermal relaxation time. It was revealed that for positive thermal nonlinearity only high-energy wide platicons from several upper energy levels can remain stable. Stability range becomes wider and shifts to the larger detuning values with the growth of the nonlinearity coefficient [see right panel in Fig. 1].

At the small values of the thermal nonlinearity coefficient the stability of platicon does not depend on the thermal relaxation time value. However, if the normalized thermal nonlinearity coefficient exceeds some threshold value, there appears a range of thermal relaxation time values providing platicon instability even for upper platicon levels. The dependence of the instability region on the pump amplitude \( f \) and thermal nonlinearity coefficient \( \frac{n_{2\Gamma}}{n_2} \) is shown in Fig. 2. It should be noted that the form of the collapsed snaking diagram does not depend on the thermal relaxation time as the Eq. (2) does not depend on it.

![Fig. 1. Left: The platicon energy levels as the dependence of platicon energy \( U \) on the pump frequency detuning \( \alpha \) in the absence of thermal effects \( \frac{n_{2\Gamma}}{n_2} = 0 \). Red lines correspond to stable platicons. Right: The platicon energy levels in presence of the positive thermal effects. Red lines correspond to stable platicons at \( 2/\tau_n \approx 0.1 \). In all cases \( d_2 = -0.02 \).](image1)

![Fig. 2. The platicon stability domains on the thermal nonlinearity coefficient (left panel) and on the pump amplitudes (right panel). For the parameters indicated in the panels the platicons are unstable between solid lines of the corresponding color. In all cases \( d_2 = -0.02 \).](image2)
with the growth of the thermal nonlinearity strength [compare panels in Fig. 5]. If photon lifetime to thermal relaxation time value or on the ratio of the thermal relaxation time and photon lifetime at $\frac{2}{\kappa t} > 0.01$ (red line) and $\frac{2}{\kappa t} = 0.001$ one virtually shifted along vertical axes by 5, 10 and 15, correspondingly for clarity.

Fig. 4. The platicon energy levels in the presence of negative thermal effects. Red lines correspond to stable platicons at $2/\kappa t = 0.1$. In all cases $f = 3$, $d_2 = -0.02$.

With the growth of the thermal nonlinearity strength, high-energy wide platicons become unstable and most of low-power narrow platicons remain stable. The dependence of the platicon energy $U$ on pump frequency detuning $\alpha$ is shown in Fig. 4 for $f = 3$ at $\frac{2}{\kappa t} = -1.0$ (left panel) and at $\frac{2}{\kappa t} = -3.0$ (right panel). Stable solutions are indicated by red lines. The structure of the spectral stability domains may be rather complicated and strongly depends on the normalized thermal relaxation time value or on the ratio of the thermal relaxation time and photon lifetime at $2/\kappa t$. It is shown in the Fig. 5 where each panel corresponds to the particular value of thermal nonlinearity coefficient $T$.

One may notice that the difference between structures of the stability domains shown by red lines for different values of the thermal relaxation time becomes more and more pronounced with the growth of the thermal nonlinearity strength [compare panels in Fig. 5]. If photon lifetime to thermal relaxation time ratio $2/\kappa t$ exceeds the critical value, then there is comparatively wide stability domain close to the right boundary of the existence domain. This domain becomes narrower with decrease of $2/\kappa t$. If $2/\kappa t$ is less than the critical value for this domain, then the platicon experiences a decay upon propagation. This is shown in Fig. 6 for the parameters from top right panel in Fig. 5. Outside this widest stability domain there may be several narrow stability domains, but they can exist if $2/\kappa t$ is less than some critical value [see Figs. 5 and 7]. Different scenarios of the evolution of the platicon from this domain are shown in Fig. 7 for the parameters from top right panel in Fig. 5. Note, that with the further growth of the nonlinearity strength the upper threshold for $2/\kappa t$ also appears for this wide right stability domain. Upper threshold value decreases with the growth of $|\frac{n_{17}}{n_2}|$ and increases with decrease of pump frequency detuning. For example, for $f = 3$ and $\frac{n_{17}}{n_2} = -3.0$ it varies from $2/\kappa t = 0.68$ at right boundary of the platicon existence domain ($\alpha = -8.29$) to $2/\kappa t = 2.19$ at the left boundary of the widest stability domain ($\alpha = -25$).

We have verified that the thermally induced generation of stable platicons [24] upon forward scan (with linear-in-time increase of pump frequency detuning) occurs within the right stability domain. Moreover, it was revealed that the platicon generation is also possible upon backward scan ($\alpha(t) = \alpha(0) - \nu t$) that is shown in the Fig. 8. In that case the platicon tuning
range is limited by the width of the right stability domain. One may see in the Fig. 8 that at $f = 3$, $d_2 = -0.02$, $2/\kappa T = 0.1$, $v = 0.00025$ generation of stable platicons takes place for $\frac{\kappa}{\nu} = -1.0$ (left panel), while for $\frac{\kappa}{\nu} = -5.0$ platicons become unstable at some detuning value (right panel; corresponding stability domains are shown in the bottom left panel of Fig. 5). This also impose limitations on the maximum power of the stable platicon.

It should be noted that the spectral width of the stability range at the large negative thermal nonlinearity coefficient is significantly smaller than for the same positive thermal effect value. For example, at $\frac{\kappa}{\nu} = -20.0$, $f = 3$ the normalized width of the stability domain for $2/\kappa T = 0.1$ is $\delta_{\text{stab}} \approx 10$, while for $\frac{\kappa}{\nu} = 20.0$, $\delta_{\text{stab}} \approx 35$.

To sum up, we have analyzed the stability of platicons in hot cavities for different combinations of pump amplitude, thermal nonlinearity coefficient and thermal relaxation time. It was revealed that if the thermal effects are positive, the high-energy wide platicons are stable; if thermal effects are negative, low-energy narrow platicons are mostly stable. The obtained results provide a deep insight in complex nonlinear dynamics in microresonator-based photonic platforms at normal GVD and can help to estimate the applicability range of the platicon generation method based on thermal effects.

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