Improved bound on isotropic Lorentz violation in the photon sector from extensive air showers

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Abstract

Cosmic rays have extremely high particle energies (up to $10^{20}$ eV) and can be used to search for violations of Lorentz invariance. We consider isotropic nonbirefringent Lorentz violation in the photon sector for the case of a photon velocity larger than the maximum attainable velocity of the standard fermions. Up to now, Earth-based bounds on this type of Lorentz violation have been determined from observations of TeV gamma rays. Here, we elaborate on a novel approach to test Lorentz invariance with greatly improved sensitivity. This approach is based on investigating extensive air showers which are induced by cosmic-ray particles in the Earth’s atmosphere. We study the impact of two Lorentz-violating decay processes on the longitudinal development of air showers, notably the atmospheric depth of the shower maximum $X_{\text{max}}$. Specifically, the two Lorentz-violating decay processes considered are photon decay into an electron-positron pair and modified neutral-pion decay into two photons. We use Monte Carlo simulations performed with the CONEX code which was extended to include these two Lorentz-violating decay processes at a magnitude allowed by the best previous Earth-based bound. Compared to standard physics, these Lorentz-violating decay processes reduce the average $X_{\text{max}}$ for showers with primary energies above $10^{18}$ eV by an amount that is significantly larger than the average resolution of current air shower experiments. Comparing the simulations of the average $X_{\text{max}}$ to observations, new Earth-based bounds on this type of Lorentz violation are obtained, which are better than the previous bounds by more than three orders of magnitude. Prospects of further studies are also discussed.
I. INTRODUCTION

Ever since its inception, the standard model (SM) of elementary particle physics has been extremely successful with its predictions tested to high precision. However, it is well known that the SM is not complete, as it does not describe gravity or dark matter, for example. Current approaches to establish a comprehensive and fundamental theory allow for deviations from exact Lorentz symmetry. The determination of some of the best current bounds on Lorentz violation in the various sectors of the SM has taken advantage of the high energies of cosmic rays and gamma rays (see, e.g., Refs. [1–3] for three research papers and Refs. [4, 5] for two reviews).

We study the impact of Lorentz violation (LV) on extensive air showers initiated by cosmic rays in the Earth’s atmosphere with a focus on ultrahigh energies above \(1 \text{ EeV} = 10^{18} \text{ eV}\). In particular, we consider isotropic nonbirefringent LV in the photon sector, specializing to the case of a photon velocity larger than the maximum attainable velocity of the standard fermions. This approach was explored in Ref. [6], where an analytical Ansatz was used which modifies the well-known Heitler model for electromagnetic cascades by including Lorentz-violating photon decays. A significant impact on the longitudinal shower development of electromagnetic cascades was found.

Here, we build upon that previous work and extend it in several essential ways. We employ a full Monte Carlo (MC) procedure, which allows us to study in detail the impact of LV not only on purely electromagnetic cascades but also on air showers initiated by hadrons. In addition to the Lorentz-violating decays of secondary photons with energies above the threshold, also a modification of the decays of neutral pions has to be accounted for in the case of hadron primaries. With these extensions, we are then able to compare the theoretical expectations to shower observations.

The theory background on LV in the context of our study is briefly summarized in Sec. II. The results of our MC study are presented in Sec. III, in particular the changes in the average atmospheric depth of the shower maximum \(\langle X_{\text{max}} \rangle\) due to LV for purely electromagnetic cascades in Sec. IIIA (in order to compare with Ref. [6]) and for hadron-induced air showers in Sec. IIIB. Comparing the latter simulations to \(\langle X_{\text{max}} \rangle\) observations, we present an improved bound on this type of LV (Sec. IIIC). While the focus of the present work is on \(\langle X_{\text{max}} \rangle\), the impact of the LV modifications on other shower observables (shower fluctuations and muon content) is briefly discussed in Sec. IID. Section IV contains a summary and an outlook on future prospects. Appendix A gives some details of the Lorentz-violating photon decays possibly occurring in the extensive air showers.
II. THEORY

In a relatively simple extension of standard quantum electrodynamics (QED), a single term which breaks Lorentz invariance but preserves CPT and gauge invariance is added to the Lagrange density \([7, 8]\),

\[
\mathcal{L}(x) = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + \bar{\psi}(x) \left( \gamma^{\mu} \left[ i\partial_\mu - eA_\mu(x) \right] - m \right) \psi(x) \\
-\frac{1}{4} (k_F)_{\mu\nu\rho\sigma} F^{\mu\nu}(x) F^{\rho\sigma}(x),
\]

where the first two terms on the right-hand side correspond to standard QED and the last term gives CPT-invariant Lorentz violation in the photon sector [the CPT transformation corresponds to the combined operation of charge conjugation (C), parity reflection (P), and time reversal (T)]. The Maxwell field strength tensor is defined as usual, \(F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu\).

Throughout this article, we use the Minkowski metric \(\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)\) and its inverse \(\eta^{\mu\nu}\) to lower and raise spacetime indices. In addition, we employ natural units with \(\hbar = c = 1\).

The fixed constant “tensor” \((k_F)_{\mu\nu\rho\sigma}\) in (1) has 20 independent components, ten of which produce birefringence and eight of which lead to direction-dependent modifications of the photon propagation. The remaining two components correspond to an isotropic modification of the photon propagation and an unobservable double trace that changes the normalization of the photon field. Isotropic nonbirefringent LV in the photon sector is then controlled by a single dimensionless parameter \(\kappa\), which enters the fixed tensor \(k_F\) from (1) in the following way:

\[
(k_F)_{\mu\nu\rho\sigma} = \frac{1}{2} \left( \eta_{\mu\rho} \tilde{\kappa}_{\nu\sigma} - \eta_{\mu\sigma} \tilde{\kappa}_{\nu\rho} + \eta_{\nu\sigma} \tilde{\kappa}_{\mu\rho} - \eta_{\nu\rho} \tilde{\kappa}_{\mu\sigma} \right),
\]

\[
\tilde{\kappa}_{\mu\nu} = \frac{\kappa}{2} \left[ \text{diag}(3, 1, 1, 1) \right]_{\mu\nu},
\]

where Ansatz (2a) gives nonbirefringence and Ansatz (2b) isotropy. From microcausality and unitarity, there is the following restriction on the “deformation parameter” \(\kappa\) of the photon theory [9]: \(\kappa\) can only take values in the half-open interval \((-1, 1]\). Note that the parameter \(\kappa\) of the Ansatz (2) is denoted by \(\tilde{\kappa}_{\text{tre}}\) in, e.g., Refs. [3, 5, 8, 9].

The photon propagation is determined by the field equations obtained from (1) and (2). The phase velocity of the photon is found to be given by

\[
v_{\text{ph}} = \frac{\omega}{|\mathbf{k}|} = \sqrt{\frac{1 - \kappa}{1 + \kappa}} c,
\]

where the velocity \(c\) corresponds to the maximum attainable velocity of the massive Dirac fermion of \([1]\). The photon velocity (3) is smaller (larger) than \(c\) for positive (negative) values of \(\kappa\). Indeed, the “operational definition” of the LV parameter \(\kappa\) is the relative difference
of the squared maximum attainable fermion velocity \( (c, \text{ now written as } v_{\text{fermion, max}}) \) and the squared photon velocity \( (v_{\text{photon}}, \text{ now written as } v_{\text{photon}}) \),

\[
\kappa = \frac{(v_{\text{fermion, max}})^2 - (v_{\text{photon}})^2}{(v_{\text{fermion, max}})^2 + (v_{\text{photon}})^2} \sim 1 - \frac{v_{\text{photon}}}{v_{\text{fermion, max}}}.
\]  (4)

As mentioned above, we set \( v_{\text{fermion, max}} \equiv c = 1 \) by using natural units.

For nonzero values of \( \kappa \), certain decay processes that are forbidden in the conventional Lorentz-invariant theory become allowed. Theoretically, these nonstandard decays are rather subtle; see Ref. \([10]\) for a general discussion and Refs. \([3, 11, 12]\) for detailed calculations of the two decay processes considered in this article.

In fact, we now focus on theory \((1) - (2)\) for the case of negative \( \kappa \), that is, having a “fast” photon, compared to the maximum attainable velocity of the standard Dirac fermion. For sufficiently high energy, this photon can then decay into an electron-positron pair,

\[
\tilde{\gamma} \rightarrow e^{-} + e^{+},
\]  (5)

where \( \tilde{\gamma} \) denotes the nonstandard photon as described by the Lagrange density \((1)\) with \textit{Ansatz} \((2)\). Specifically, the energy threshold for photon decay is given by \([3]\)

\[
E_{\text{thresh}}^\gamma(\kappa) = 2m_e \sqrt{\frac{1 - \kappa}{-2\kappa}} \sim \frac{2m_e}{\sqrt{-2\kappa}},
\]  (6)

in terms of the electron rest mass \( m_e \approx 0.511 \text{ MeV} \). At tree level, the exact decay rate of photon decay (PhD) as a function of the photon energy \( E_{\gamma} \geq E_{\gamma}^{\text{thresh}} \) has been calculated \([3, 11]\),

\[
\Gamma_{\text{PhD}}(E_{\gamma}) = \frac{\alpha}{3} \frac{-\kappa}{1 - \kappa^2} \sqrt{(E_{\gamma})^2 - (E_{\gamma}^{\text{thresh}})^2} \left( 2 + (E_{\gamma}^{\text{thresh}}/E_{\gamma})^2 \right),
\]  (7)

with the fine-structure constant \( \alpha \equiv e^2/(4\pi) \approx 1/137 \) and \( E_{\gamma}^{\text{thresh}} \) from \((6)\).

The photon decay length drops to scales of a centimeter right above threshold (cf. Fig. 7 of Ref. \([11]\)) and the decay process resembles a quasi-instantaneous conversion of photons into electron-positron pairs. Therefore, ground-based Cherenkov-telescope observations of gamma rays with energies of order \( 10^-100 \text{ TeV} \) can be used to impose bounds on negative \( \kappa \), with the most stringent Earth-based bound up to now \([3]\)

\[
\kappa > -9 \times 10^{-16} \quad (98\% \text{ C.L.}).
\]  (8)

It may be of interest to mention that there is also a qualitative astrophysics bound \([13]\) at the level of \( -10^{-19} \), but this bound is less reliable than \((8)\) for reasons explained in Ref. \([6]\).

In order to improve upon bound \((8)\), photons of higher energy than currently observed would be needed. So far, despite extensive searches for astrophysical (primary) photons with PeV or EeV energies, no unambiguous photon detection could be reported at these energies (see, e.g., Ref. \([14]\)). However, photons with energies far above 100 TeV are expected to


be produced as secondary particles when an ultrahigh-energy hadron enters the Earth’s atmosphere and initiates an air shower. In the first interaction of the primary hadron with an atmospheric nucleus, mostly charged and neutral pions are produced. The charged pions further interact with particles from the atmosphere, producing more pions, while the neutral pions, in standard physics, rapidly decay into pairs of photons, which in turn trigger electromagnetic sub-showers. Especially in the start-up phase of the air shower, where the energies of the secondary particles are still very high, a modification of the particle dynamics due to LV can drastically change the overall development of the air shower, as shown in Ref. [6] for electromagnetic cascades with the immediate decay of above-threshold photons.

Since we intend to study air showers induced by hadrons, we have to take into account possible related modifications of other processes due to LV, which may also have an influence on the development of the air shower. The relevant process here is the decay of the neutral pion into two nonstandard photons [6, 12],

\[ \pi^0 \rightarrow \tilde{\gamma} + \tilde{\gamma}. \]  

In the context of the theory considered in this work, the decay time \( \tau \) of the neutral pion is modified by a factor depending on the energy \( E_{\pi^0} \) of the pion and the negative LV parameter \( \kappa \):

\[ \tau(E_{\pi^0}, \kappa) = \frac{\tau_{\text{SM}}}{g(E_{\pi^0}, \kappa)}, \]  

with \( \tau_{\text{SM}} \) denoting the neutral-pion decay time in the conventional Lorentz-invariant theory (standard model) and \( g(E_{\pi^0}, \kappa) \) given by [12]

\[ g(E_{\pi^0}, \kappa) = \begin{cases} \sqrt{\frac{1 - \kappa^2}{(1 - \kappa)^3}} \left[ 1 - \left( \frac{E_{\pi^0}}{E_{\text{cut} \pi^0}} \right)^2 - \left( \frac{m_{\pi^0}}{E_{\text{thresh} \gamma}} \right)^2 \right]^2, & \text{for } E_{\pi^0} < E_{\pi^0}^{\text{cut}}, \\ 0, & \text{otherwise}. \end{cases} \]  

The pion cutoff energy \( E_{\pi^0}^{\text{cut}} \) in (11) is

\[ E_{\pi^0}^{\text{cut}} = m_{\pi^0} \sqrt{\frac{1 - \kappa^2}{-2\kappa}} \sim \frac{m_{\pi^0}}{\sqrt{-2\kappa}} \sim \frac{m_{\pi^0}}{2m_e} E_{\gamma}^{\text{thresh}} \approx 132 E_{\gamma}^{\text{thresh}}, \]  

where the asymptotic photon-threshold expression [6] was used and the numerical values of the neutral-pion and electron rest masses, \( m_{\pi^0} \approx 135 \text{ MeV} \) and \( m_e \approx 0.511 \text{ MeV} \). Thus, neutral pions start to be affected significantly at energies about two orders of magnitude above the photon-decay threshold \( E_{\gamma}^{\text{thresh}} \).

III. ANALYSIS

In the following, we discuss the results of our LV study using a full MC approach. Since we are mainly interested in changes of the longitudinal development of air showers, we use
the MC code CONEX v2r5p40 [15] [16], which we extend to include the decay of photons as well as the modified decay of neutral pions. Photon decays into electron-positron pairs are implemented as the immediate decay of photons above the threshold given by (6), with the energy distributions of the secondary particles modeled according to Ref. [11]. An example of the energy distribution is given in Fig. 8 of App. A. For the modified decay time of neutral pions, the results (10)–(12) have been incorporated into the CONEX code.

To describe hadronic interactions in the MC simulation of the air showers, we use as default the EPOS LHC model [17] (see below for discussion of other models). For all other settings, we use the defaults provided by the CONEX code.

A. Primary photons

In order to compare the results of the full MC simulation to those of the analytical Heitler-type model of Ref. [6], we first consider the case of primary photons. In Fig. 1, the average atmospheric depth of the shower maximum \( \langle X_{\text{max}} \rangle \) is shown as a function of the primary energy for different values of \( \kappa \), including \( \kappa = -9 \times 10^{-16} \) corresponding to the maximum LV allowed so far by the previous bound [8]. Overall, the MC results confirm the significant reduction of \( \langle X_{\text{max}} \rangle \) and the change of elongation rate (increase of \( \langle X_{\text{max}} \rangle \) with energy) as expected from Ref. [6]. For instance, \( \langle X_{\text{max}} \rangle \) at \( 10^{19} \) eV is reduced by some 320 g cm\(^{-2}\). The full MC simulation shows a larger decrease compared to the analytical Heitler-type model. This can be understood as follows: the previous analytical approach neglected the fact that showers initiated by electrons/positrons are shorter than those initiated by photons of the same energy. A further difference comes from using realistic energy distributions, compared to the equal-energy-sharing assumption employed in the analytical estimate of Ref. [6].

The substantial reduction in \( \langle X_{\text{max}} \rangle \) of approximately 55 g cm\(^{-2}\) just above the threshold energy for photon decay into an electron-positron pair is due to two effects: first, having an electron/positron as primary instead of a photon, which leads to a shower shortened by approximately 30 g cm\(^{-2}\), and, second, having two lower-energy primaries instead of one higher-energy primary, which again leads to a shorter shower. Since a symmetric share of energies is favored just above threshold (see Fig. 8 of App. A), the latter effect can be estimated as approximately \((85 \times \log_{10} 2)\) g cm\(^{-2}\) \(\approx 25\) g cm\(^{-2}\) (using an elongation rate of approximately 85 g cm\(^{-2}\) per decade for electromagnetic showers).

For energies just above threshold, there is a somewhat larger elongation rate than at higher energies. The reason is that, up to about twice the threshold energy, the primary energy typically allows for just a single case of photon decay. Only at larger energies, can photon decay happen more than once, leading to a reduction of the elongation rate.

We can also try to obtain a better understanding of the magnitude of the \( \langle X_{\text{max}} \rangle \) reduction at primary energies far above the photon-decay threshold. In the cascade, a large number of
A simplified description is that each conversion of a secondary photon into an electron-positron pair will reduce the corresponding sub-shower in a similar manner as observed for the primary conversion. A difference is due to the energy distribution of the electron-positron pair which favors a more asymmetric share if the photon energy surpasses the threshold energy by a factor of 2 or more. Correspondingly, the effect of a shorter shower due to the production of lower-energy particles is reduced to a value of approximately $15 \text{ g cm}^{-2}$ (instead of approximately $25 \text{ g cm}^{-2}$ just above threshold) and leads to a net reduction of approximately $(30+15) \text{ g cm}^{-2} = 45 \text{ g cm}^{-2}$ for a sub-shower. The relative contribution of this sub-shower to the total shower may be approximated by the fractional energy $f_i$ carried by the secondary photon relative to the primary energy. The total reduction of $\langle X_{\text{max}} \rangle$ of the whole shower is then expected to contain the sum over all sub-showers from photon decay,

$$\Delta \langle X_{\text{max}} \rangle^{\text{expected}} \approx 55 \text{ g cm}^{-2} + 45 \text{ g cm}^{-2} \left( \sum f_i - 1 \right),$$

where the first term on the right-hand side comes from the primary conversion and the second term accounts for secondary conversions. An explicit study of this argument has been performed for a primary photon energy of $10^{18} \text{ eV}$. An example of a typical $f_i$ distribution in a shower is shown in Fig. 9 of App. A. For this case, $\sum f_i \approx 5.5$ gives $\Delta \langle X_{\text{max}} \rangle^{\text{expected}} \approx 258 \text{ g cm}^{-2}$, in reasonable agreement with the result of the MC simulation in Fig. 1.

**B. Primary hadrons**

We now consider the case of secondary photons produced in air showers initiated by primary hadrons. This has not been studied before and requires an account of the modification of the neutral-pion decay as well (see Sec. II). Compared to the case of primary photons, where already the initial particle is modified by LV, a smaller impact on $\langle X_{\text{max}} \rangle$ is expected here, as hadronic interactions (dominating the start of the cascade) are unaffected by LV in the photon sector.

The results are displayed in Fig. 2 for primary protons and iron nuclei. Also hadron-induced air showers are seen to be significantly affected in terms of $\langle X_{\text{max}} \rangle$, which is approximately $100 \text{ g cm}^{-2}$ smaller compared to the unmodified case, for protons at $10^{19} \text{ eV}$ and a $\kappa$ value saturating bound (8). The impact is large with respect to the experimental resolution. For instance, the $X_{\text{max}}$ resolution of the Pierre Auger Observatory is better than $26 \text{ g cm}^{-2}$ at $10^{17.8} \text{ eV}$, improving to about $15 \text{ g cm}^{-2}$ above $10^{19.3} \text{ eV}$ [18]. Figure 2 also shows that the elongation rate is modified as well. The reduction amounts to about $25 \text{ g cm}^{-2}$ per decade, relatively independent of the $\kappa$ value and the primary type considered.

The modification of $\langle X_{\text{max}} \rangle$ occurs if the energy per nucleon of the primary particle exceeds the LV threshold energy $E_{\gamma}^{\text{thresh}}$ by a factor of approximately 10, because secondary
photons above threshold then start to be produced. Neutral pions become stable at sufficiently high energies and the change occurs at energies about two orders of magnitude above the photon-decay threshold energy, according to the estimate from (12). The onset of neutral pions becoming stable is noticed only as a minor effect on \( \langle X_{\text{max}} \rangle \), i.e., a slight upturn of the modified curve. Changes in the values of \( \kappa \) and in the type of primary (e.g., primary iron nuclei instead of protons) give shifts of the curves in the ways naively expected.

Using different primary nuclei (proton, helium, oxygen, and iron), we have determined the values of \( \langle X_{\text{max}} \rangle \) as a function of \( \kappa \) at a fixed primary energy of \( 10^{19} \text{ eV} \). As shown in Fig. 3, \( \langle X_{\text{max}} \rangle \) scales linearly with \( \log_{10}(\kappa) \) if the photon-decay threshold energy is well below the energy per nucleon of the primary particle. This can be used to obtain a parametrization of \( \langle X_{\text{max}} \rangle \) as a function of the negative LV parameter \( \kappa \), the primary energy \( E \), and the primary mass \( A \). For EPOS LHC, this parametrization is given by

\[
\langle X_{\text{max}} \rangle (\kappa, E, A) \Big|^{(\text{above-threshold})} = p_0 + p_1 \log_{10}(\kappa) + p_2 \left( \log_{10}(E \text{[eV]}) - 19 \right) + p_3 \ln(A),
\]

where \( p_0 = (550 \pm 3) \text{ g cm}^{-2} \) and \( p_1 = (-10.7 \pm 0.1) \text{ g cm}^{-2} \) are determined by a fit to the proton distribution shown in Fig. 3, \( p_2 = (34.3 \pm 0.3) \text{ g cm}^{-2} \) is the \( \kappa \)- and \( A \)-independent elongation rate taken from Fig. 2, and \( p_3 = (-15.8 \pm 0.1) \text{ g cm}^{-2} \) is obtained from a fit to the distributions for the different primary particles shown in Fig. 3. It should be noted that for the determination of the improved bound on \( \kappa \) (to be discussed in Sec. III C), only the parametrization of \( \langle X_{\text{max}} \rangle \) for the proton case is needed. For completeness, we have included the generalization of the parametrization to heavier nuclei.

We have also checked the dependence of the simulations on the choice of the hadronic-interaction model. The uncertainty of \( \langle X_{\text{max}} \rangle \) due to modeling hadronic interactions is about \( \pm 20 \text{ g cm}^{-2} \) around the predictions of EPOS LHC [19]. Likewise, the predictions from the alternative models QGSJET-II-04 [20] and SIBYLL 2.3c [21–23] may be regarded as resembling the lower (QGSJET-II-04) and upper (SIBYLL 2.3c) range of possible \( \langle X_{\text{max}} \rangle \) predictions [19]. The results for proton primaries using QGSJET-II-04 and SIBYLL 2.3c are displayed in Fig. 4. As expected, the LV-modified curves reflect the differences between the unmodified curves for the different hadronic-interaction models. Differences between the models of the modified \( \langle X_{\text{max}} \rangle \) values are smaller compared to the unmodified \( \langle X_{\text{max}} \rangle \) values. At \( 10^{19} \text{ eV} \), for instance, the predictions cover a range of approximately \( 22 \text{ g cm}^{-2} \) for the modified case \( (\kappa = -1 \times 10^{-21}) \) and approximately \( 28 \text{ g cm}^{-2} \) for the unmodified case \( (\kappa = 0) \). Compared to EPOS LHC, the differences in \( \langle X_{\text{max}} \rangle \) are about \( 8 - 13 \text{ g cm}^{-2} \), where QGSJET-II-04 gives smaller \( \langle X_{\text{max}} \rangle \) values and SIBYLL 2.3 larger \( \langle X_{\text{max}} \rangle \) values. These differences are small compared to the overall reduction of \( \langle X_{\text{max}} \rangle \) by the LV modification allowed by previous bound [8].
FIG. 1. Average atmospheric depth of the shower maximum $\langle X_{\text{max}} \rangle$ as a function of the primary energy of a primary photon, taken from MC simulations performed with the CONEX code which was modified to include Lorentz violation controlled by a negative parameter $\kappa$. The dashed lines indicate the $\langle X_{\text{max}} \rangle$ values expected from the analytical Heitler-type model of Ref. [6].

FIG. 2. Simulated values of $\langle X_{\text{max}} \rangle$ as a function of the primary energy for primary protons and iron nuclei, where different values of the Lorentz-violating parameter $\kappa$ are considered.
FIG. 3. Simulated values of $\langle X_{\text{max}} \rangle$ as a function of $-\kappa$ for different primary nuclei (proton, helium, oxygen, and iron) at a fixed primary energy of $10^{19}$ eV. The dotted lines correspond to the parametrization (14) and the hatched area on the right indicates the range of $-\kappa$ values that is excluded by the previous bound (8).

FIG. 4. Simulated values of $\langle X_{\text{max}} \rangle$ as a function of the primary energy for primary protons, where different hadronic-interaction models are used.
C. Comparison to \(\langle X_{\text{max}}\rangle\) measurements

A comparison of our \(\langle X_{\text{max}}\rangle\) simulations with \(\langle X_{\text{max}}\rangle\) measurements is given in Fig. 5. Only measurements from the Pierre Auger Observatory [18] are shown here, since the \(\langle X_{\text{max}}\rangle\) measurements from the Telescope Array (TA) are not corrected for detector effects. In any case, the TA measurements have been found to be consistent with the Auger measurements [24]. For the maximum LV allowed by previous bound (8) from Ref. [3], it can be seen that \(\langle X_{\text{max}}\rangle\) predictions are significantly below the observations, regardless of assumptions on the primary mass and the interaction model. Thus, stricter constraints than before can be placed on negative \(\kappa\) by the method presented in this article.

Allowing, most conservatively, for the case of a pure proton composition, one can see that also the \(\kappa = -10^{-19}\) case in Fig. 5 appears to be on the lower side of the observed \(\langle X_{\text{max}}\rangle\) values above \(10^{18}\) eV. While we concentrate here on obtaining a bound, it is interesting to note that the elongation rate of a constant proton composition in the modified case for \(\kappa \sim -10^{-20}\) turns out to agree reasonably well with the observations at or above \(2 \times 10^{18}\) eV. Focusing on the energy bin around \(2.8 \times 10^{18}\) eV and taking the uncertainties on the measured \(\langle X_{\text{max}}\rangle\) values into account, we obtain the following bound:

\[
\kappa > -3 \times 10^{-19} \quad (98\% \text{ C.L.}) .
\]  

(15)

For completeness, the corresponding bound at 99.9\% C.L. is \(\kappa > -1.2 \times 10^{-18}\).

The bound (15) is based on EPOS-LHC simulations and the parametrization (14), where a systematic uncertainty of \(20 \text{ g cm}^{-2}\) has been assumed to account for the uncertainties related to the description of hadronic interactions. We note that the value of \(20 \text{ g cm}^{-2}\) is conservative, as it is taken from the uncertainty of the unmodified simulations, while the uncertainty for the modified case is likely to be reduced (see Sec. III B). Without additional systematic model uncertainties, the 98\% C.L. bounds derived from SIBYLL 2.3c, EPOS LHC, and QGSJET-II-04 are \(-2 \times 10^{-19}\), \(-0.2 \times 10^{-19}\), and \(-0.02 \times 10^{-19}\), respectively. These three bounds are even tighter than the bound quoted in (15).

The bound on negative \(\kappa\) as given by (15) improves the previous bound (8) by a factor of approximately 3000. As a comparison to the previous approach of Ref. [3], the observation of a primary photon with an energy of about \(2 \text{ PeV} = 2 \times 10^{15}\) eV would be needed to get a similar bound on negative \(\kappa\).

D. Other shower observables

While the focus of the present work has been on \(\langle X_{\text{max}}\rangle\), additional shower observables can be studied with the simulation setup.

The impact of the LV modifications on the shower-to-shower fluctuations \(\sigma(X_{\text{max}})\) is
shown in Fig. 6. Interestingly, the LV modifications leave \( \sigma(X_{\text{max}}) \) essentially unchanged, different from the behavior of \( \langle X_{\text{max}} \rangle \). This is understandable, as the shower fluctuations are dominated by fluctuations in the first interactions of the highest-energy hadronic primaries, which remain unmodified here (the LV in the photon sector is expected to lead to negligible LV loop corrections of the hadronic interactions).

Further, we checked for a possible effect on the muon content of the shower. Since neutral pions above \( E_{\pi^0}^{\text{cut}} \) re-interact, a corresponding increase of the number of muons could result (see also Refs. [25, 26] for considerations of shower muons and LV). As can be seen in Fig. 7, the number of muons is indeed affected. For primary protons with \( \kappa = -9 \times 10^{-16} \), the increase of the muon number starts for primary energies of about a factor 3 above the pion cutoff energy (\( E_{\pi^0}^{\text{cut}} \approx 3.2 \times 10^{15} \text{eV} \) in this case). The muon excess grows with energy, reaching a factor 1.5 in the EeV range. A similar effect is observed for primary iron nuclei, with the primary energy scaled according to the number of nucleons. Remarkably, the muon number at the highest energies turns out to be quite similar for proton and iron primaries in the modified case, in contrast to the unmodified case. As \( \kappa = -9 \times 10^{-16} \) is excluded by our new bound (15), Fig. 7 also shows the results for proton primaries at the value \( \kappa = -1 \times 10^{-19} \). The increase of muon number then starts at higher energies.
FIG. 6. Shower fluctuations $\sigma(X_{\text{max}})$ as a function of the primary energy for primary protons and iron nuclei.

FIG. 7. Average number of ground muons, normalized to the case of unmodified proton primaries, as a function of the primary energy for primary protons and iron nuclei.
IV. SUMMARY AND OUTLOOK

In the present article, we have considered isotropic Lorentz violation in a simplified theory of photons and electrically charged Dirac fermions. This Lorentz violation is described by a single dimensionless parameter $\kappa$, whose physical meaning is clarified by (4). Our focus has been on the case of negative $\kappa$ with a "fast" photon.

By implementing Lorentz-violating effects (photon decay and the suppression of neutral-pion decay) in a full MC shower simulation, we have studied the impact of LV on air showers initiated by ultrahigh-energy cosmic rays. This method exploits the expected production of secondary photons with energies far above 100 TeV and the accelerated shower development due to photon decay. The average depth of the shower maximum $\langle X_{\text{max}} \rangle$ can be reduced by some 100 g cm$^{-2}$ at $10^{19}$ eV and the difference increases with energy due to a reduced elongation rate. This reduction value of 100 g cm$^{-2}$ at $10^{19}$ eV is well above the typical $X_{\text{max}}$ resolution of 15--20 g cm$^{-2}$ in current air shower experiments. The shower fluctuations $\sigma(X_{\text{max}})$ are not affected by the LV modification. The number of muons at ground level has been found to increase significantly above the LV cutoff energy of neutral-pion decay.

With a value of $\kappa = -9 \times 10^{-16}$ as allowed so far by the previous bound \[3\], the predictions of $\langle X_{\text{max}} \rangle$ are at odds with the measurements, irrespective of the primary mass and the interaction model: much deeper showers are observed than the showers expected theoretically. From $\langle X_{\text{max}} \rangle$ alone, an improved bound of $\kappa = -3 \times 10^{-19}$ (at 98 % C.L.) has been obtained in Sec. III C. For this new bound (15), the primary composition has, most conservatively, been assumed to consist of protons only. Heavier primaries have smaller $\langle X_{\text{max}} \rangle$ values, which would lead to even stronger bounds.

Remark that the magnitude of the improved negative-$\kappa$ bound (15) is only a factor 5 larger than the positive-$\kappa$ bound (15a) from Ref. \[3\], $\kappa < 6 \times 10^{-20}$ (98 % C.L.). Future improved negative-$\kappa$ bounds using only $\langle X_{\text{max}} \rangle$ can come from further data, also at higher energies, for instance by the present upgrade of the Pierre Auger Observatory \[27, 28\]. Moreover, reducing present uncertainties helps: in case of a model uncertainty of 15 g cm$^{-2}$, the negative-$\kappa$ bound (15) improves by a factor of approximately 2. A similar effect is obtained by reducing the experimental uncertainty to half of its present value.

Significantly improved negative-$\kappa$ bounds appear possible if other observables beyond $\langle X_{\text{max}} \rangle$ are incorporated. In fact, a pure proton composition above $3 \times 10^{18}$ eV, as conservatively assumed here, is already excluded by other air shower measurements. Specifically, observations of $\sigma(X_{\text{max}})$ \[18\] and observations of the correlation between $X_{\text{max}}$ and the ground signal \[29\] show a mixed composition with a significant fraction of heavier nuclei. A mixed composition will provide further improvements of the negative-$\kappa$ bound (15). As illustration, let us assume an average primary mass $\langle A \rangle \approx 4$. Then, the resulting predicted $\langle X_{\text{max}} \rangle$ would coincide with that of primary helium. For a given observed value of $\langle X_{\text{max}} \rangle$
and according to Fig. 3, this would imply a further factor of approximately 100 improvement on the negative-$\kappa$ bound (15). We leave this analysis to the future.

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Appendix A: Lorentz-violating photon decays

In this appendix, we present some further details on the Lorentz-violating photon decay process, based on the theoretical results from Refs. [3, 11]. Figure 8 gives the differential decay rate for photon decay into an electron-positron pair and Fig. 9 shows the energy fraction of decaying photons.

![Graph showing differential decay rate](image)

**FIG. 8.** Differential decay rate $d\Gamma/da$ for photon decay into an electron-positron pair as a function of the fraction $a$ of the energy of the resulting electron with respect to the initial energy $E_\gamma$ of the decaying photon. Shown are the decay rates for different initial energies in units of the threshold energy $E_\gamma^{\text{thresh}}$ from (6) for the case $\kappa = -9 \times 10^{-16}$.

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FIG. 9. Distribution of fractions $f_i$ of the energy of decaying photons relative to the primary energy for a single photon-induced air shower with a primary energy of $10^{18}$ eV. Each decaying photon in the shower contributes to the distribution. The decay of the primary photon is seen as $f_1 = 1$. The minimum value of $f_i$ is determined by the Lorentz-violating energy threshold [6]. The simulation has been performed with the modified CONEX code and Lorentz-violating parameter $\kappa = -9 \times 10^{-16}$.

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