SAR image segmentation based on the advanced level set

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Abstract. Image segmentation takes an important role in SAR image processing. In this paper, a SAR image segmentation method based on level set evolution combining edge feature and statistic information is proposed. In order to enhance the impact of edge on image segmentation, all edge values are homogenized according to the calculated ROA operator. Different from traditional method where the SAR distribution is often specified based on human experiences, the Edgeworth algorithm, an approximation method for statistical distribution model, gives any SAR image distribution a statistical expression. Considering the practicability of ROA operator and the adaptivity of Edgeworth series expansion at fitting statistical distribution, an energy function based on edge and region properties is defined. To implement image division, partial differential equation (PDE) of curve evolution is obtained by minimizing the function. The proposed approach uses more information from SAR images and is appropriate for any SAR images without the need for human-specified distribution pattern. Finally, the experimental results which are obtained from the SAR images of some typical regions such as rivers and buildings show the applicability of the proposed method.

1. Introduction
Synthetic Aperture Radar (SAR) image segmentation is one of the key techniques for automatic interpretation of SAR image. Recently, a novel approach for image division, called the level set evolution under partial differential equation (PDE), develops rapidly. This method is very promising due to its simplicity at expansion, processing variation of zero level set topological structure and the good result at division of complex and multi-objective images. However, the traditional segmentation method based on level set evolution cannot be directly applied to SAR images due to the multiplicative speckle noise of SAR images.

SAR data distribution is considered to follow some statistical properties, which can be described by probability distribution models [1][2][3], such as Gamma distribution, Weibull distribution and G0 distribution. Gamma distribution is often used to describe statistical distribution model of SAR images [1][4][5][6]. However, the statistical property of SAR images does not fit Gamma distribution absolutely when the image resolution increases. Therefore, classical level set segmentation based on gamma distribution is not suitable for some SAR images.

In this paper, a SAR image segmentation method based on level set evolution combining edge feature and statistic information is proposed, in which the edge feature is obtained by ratio of average (ROA) operator and the statistic information is obtained by Edgeworth expansion. The ROA operator, which has constant false alarm rate property [7], is a common algorithm to extract the edge information in SAR images. The Edgeworth expansion can adaptively give any SAR image distribution a statistical expression in various regions [8][9], which generally is different from the SAR
distribution specified by human experience. To implement SAR image division, PDE of curve evolution is obtained by minimization of the function according to variation principle and gradient descent flow theory. The solution of the PDE is achieved by the variational level set approach. Finally, two SAR images of typical regions including rivers and buildings are processed by the proposed method and level set segmentation based on Gamma distribution respectively. Compared with the classical method, the results of the proposed method show the better performance and applicability.

2. Energy function model

2.1. Statistical distribution model approximation of Edgeworth expansion

Due to the fact that different scenes, such as mountainous region, forest and river, correspond to different distribution models and regularity of distribution changes with various bands and resolutions of images [6], the statistical distribution of SAR images cannot be described by a specific statistical distribution. Therefore, it is necessary to develop an adaptive distribution model obtained from the actual data of SAR images.

According to the theory of Edgeworth series expansion, distribution function can be fit by standard normal distribution and polynomial [8][9]. Suppose that $X$ is a random variable, the mean value is $\mu$, the variance is $\delta$, and equation (1) approximates to standard normal distribution according to central-limit theorem. The distribution of variable $Y$ can be described by equation (2).

$$ Y = \left( \frac{1}{n} \sum X - \mu \right) \left( \frac{\sigma}{\sqrt{n}} \right)^{-1} $$

$$ f(y) = g(y) \left( 1 + \frac{1}{6} \rho_3 H_3(y) + \frac{1}{24} \rho_4 H_4(y) + \frac{1}{72} \rho_5 H_5(y) \right) $$

where $g(y)$ represents standard normal distribution, $H_k(y)$ is the Hermite polynomial of $k$ degree of freedom, and $\rho_k$ is the standard cumulant of $r$ order, $g(y), H_k(y)$ and $\rho_k$ are given by:

$$ H_3(y) = y^3 - 3y, \quad H_4(y) = y^4 - 6y^2 + 3, \quad H_5(y) = y^5 - 15y^3 + 45y - 15 $$

$$ \rho_3 = K_3(x) \left( n \delta^3 \right)^{-1}, \quad \rho_4 = K_4(x) \left( n \delta^4 \right)^{-1} - 3 $$

where $K_3(x), K_4(x)$ are the cumulant of 3 and 4 order of variable $X$ respectively.

2.2 Energy function based on ROA operator and Edgeworth expansion

In existing SAR image segmentation methods based on level set, energy function is established assuming that SAR images are Gamma distributed [1][4][5]. Since Gamma distribution is only suitable for low resolution SAR images in homogeneous regions, Edgeworth expansion is adopted to describe the statistical properties of SAR images.

In addition, statistical properties are just regional information of images [10]. In order to leverage more image information, ROA operator is introduced to get edge information in SAR images. We choose to use ROA operator because it has constant false alarm, is simple to compute, and is one of the most common algorithms for obtaining edge information.

Assume that segmentation of SAR image $I(x, y)$ consists two parts, foreground $\Omega_f$ and background $\Omega_b$. Image segmentation based on statistical model requires maximizing likelihood function. Hence, the log likelihood is given by:

$$ \ln(l) = \sum_{(x,y) \in \Omega_f} \left[ \ln \omega_f + \ln(P_f(I(x, y))) \right] + \sum_{(x,y) \in \Omega_b} \left[ \ln \omega_b + \ln(P_b(I(x, y))) \right] $$

where $P_f, P_b$ obtained by Edgeworth expansion are the statistical distributions of foreground and
background respectively, $\omega_f, \omega_b$ are the prior probability respectively, and $\omega_f + \omega_b = 1$.

In consideration of the practicability of ROA operator and the adaptivity of Edgeworth series expansion fitting statistical distribution, an energy function is defined as below:

$$E = \alpha \int R(x, y) ds + \beta \int (-\ln(l)) ds dy + \gamma \int ds$$

(6)

where $\alpha, \beta, \gamma$ are the weights, $R(x, y)$ is edge intensity that calculated by ROA operator, $\ln(l)$ is achieved by equation (5). The third term is to guarantee the smoothness of the curve.

To achieve a better computational stability, variational level set method is adopted. Besides, as the inner effect of numerical calculation method, re-initialization of level set function is necessary in iteration. Moreover, in order to improve the speed of iteration, the variational approach proposed by Li is introduced[11]. So the energy function is modified as below:

$$E = \int \left[ \alpha \delta_f(u) R(x, y) - \beta \left[ \int \omega_f P_f(x, y) H_x(u) + \int \omega_b P_b(x, y) (1 - H_x(u)) \right] + \gamma \delta_i(u) \nabla u \right] ds dy + \lambda \left[ \|u\|^2 - 1 \right]$$

(7)

where $H_x(u)$ and $\delta_i(u)$ are given by:

$$H_x(u) = \frac{1}{2} + \frac{1}{\pi} \arctan \left( \frac{u}{\varepsilon} \right), \quad \delta_i(u) = \left( \frac{1}{\pi} \right) \left( \frac{\varepsilon^2 + u^2}{\varepsilon^2 + 1} \right)$$

(8)

2.3 Solution of minimizing energy function

According to the variation principle and gradient descent flow, the minimization of equation (7) is:

$$\frac{\partial u}{\partial t} = \delta_i(u) \left[ \alpha \nabla \nabla \nabla \left[ R(x, y) \nabla u \right] + \beta \left[ \nabla \left( \omega_f P_f(x, y) \right) - \nabla \left( \omega_b P_b(x, y) \right) \right] + \gamma \nabla \nabla \nabla \left( \frac{\nabla u}{\nabla u} \right) \right] + \lambda \left[ \nabla \nabla \nabla \left( \frac{\nabla u}{\nabla u} \right) \right]$$

(9)

$R(x, y) \in [0, 1]$, the value tends to 0 indicating that the pixel is edge while the value tends to 1 indicating that the pixel is in the uniform area. In order to enhance impact of edge information on image segmentation, all edge values are homogenized. In equation (9), $\omega_f, \omega_b$ can be estimated by:

$$\omega_f = \frac{N_f}{N}, \quad \omega_b = \frac{N_b}{N}$$

(10)

where $\omega_f, \omega_b$ and $N$ are the pixel number of foreground, background and image respectively.

Since equation (9) cannot be directly solved, iteration is required. The iteration function is:

$$\frac{u_{i+1} - u_i}{\Delta t} = \frac{\partial u}{\partial t}$$

(11)

Re-estimation of statistical distributions of foreground and background is necessary in each iterative step. The third origin moments $r_f$ and $r_b$ of foreground and background SAR images need to be calculated in statistical distribution approach of Edgeworth expansion, which are expressed by:

$$r_f = \frac{\sum \omega_f P_f(x, y) H_x(u)}{\sum \omega_f P_f(x, y)}, \quad r_b = \frac{\sum \omega_b P_b(x, y) (1 - H_x(u))}{\sum \omega_b P_b(x, y)}$$

(12)

The algorithm process is as below:

Step1. Calculate the edge value by ROA operator and homogenize them.

Step2. Initialize the level set function $u$, set $u = -1$ inside and $u = 1$ outside.

Step3. Estimate the foreground and the background distribution by equation (2) respectively.

Step4. Update the level set function $u$ by equation (9).

Step5. If the $u$ is stable, go to the next step, otherwise go to step 3.

Setp6. Get the divide curve by setting $u = 0$ to implement SAR image segmentation.
3. Experimental results

Compared with classical level set segmentation based on gamma distribution in the experiments, the performance of the proposed method has been verified by two SAR images. The window size of ROA operator is $7 \times 7$ in the experiment, which helps to detect main edge. Commonly, set $\lambda = 0.04$ and $\varepsilon = 1$ in iteration [11]. Since inner energy function exists, we can set $\Delta t = 5$ to improve iteration speed. Parameters $\alpha$, $\beta$ and $\gamma$ are changed with different images.

Figure 1(a) and Figure 3(a) are both captured from Radarsat-2 images acquired in Dujiangyan, Sichuan province, the beam mode is Ultra-Fine, the resolution is $1.6 \times 2.8$ m. The size of Figure 1(a) is $100 \times 100$, it includes river, bridge and bank. In this experiment, the parameters are $\alpha = 0.9$, $\beta = 0.8$ and $\gamma = 1.4$, the runtime is 4.8s.

![Figure 1](image1)

Figure 1. (a)(b)(c) are initial segmentation, result based on Gamma distribution and result by proposed method respectively.

![Figure 2](image2)

Figure 2. (a)(b)(c) are the means plots, foreground histogram and background histogram of Figure 1(b) respectively; (d)(e)(f) are the means plots, foreground histogram and background histogram of Figure 1(c) respectively.

Figure 1(b) shows that the result of level set segmentation based on Gamma distribution has many overdivided regions, which proves that Gamma distribution is not suitable for inhomogeneous regions.
with increasing resolution. Figure 2(b)(c) indicate that foreground(river) generally can be described by Gamma distribution while background(bank) does not fit Gamma distribution absolutely. Figure 1(c) and Figure 2(d)(e)(f) demonstrate that the result of the proposed method is better than the classical method and both river and bank are described by fitting distribution completely.

The size of Figure 3(a) is 160×160. It includes buildings and other objects. In this experiment, the parameters are \( \alpha = 1.2, \beta = 0.9, \gamma = 1.6 \), and the runtime is 16.3s.

![Figure 3](image)

**Figure 3.** (a)Initial segmentation; (b)Result based on Gamma distribution; (c)Result by proposed method.

![Figure 4](image)

**Figure 4.** (a)(b)(c) are the means plots, foreground histogram and background histogram of Figure 3(b) respectively; (d)(e)(f) are the means plots, foreground histogram and background histogram of Figure 3(c) respectively.

Figure 3(b) and Figure 4(a) show that buildings can be divided from the image. However, in Figure 4(a)(b), because of the existence of noises, there is a lot of false segmentations and the foreground(buildings) does not fit Gamma distribution. Additionally, Figure 4(c) indicates that the
background(others) does not fit Gamma distribution between pixels 0 to 20 as well. The proposed method can correctly divide buildings and the false segmentation is much less than the classical method as shown in Figure 4(d). Besides, the distributions of foreground and background can almost fit as shown in Figure 4(e)(f).

The results demonstrate that the proposed method can divide the objective correctly from SAR images in most of time and confirm the effectiveness of segmentation, which offers a useful method for later processing.

4. Conclusion
In this paper, a method based on level set combining ROA operator and Edgeworth expansion is presented. Because of the applicability of ROA operator for extracting edge information and effectiveness of Edgeworth expansion for fitting statistical distribution, the proposed method is able to segment objective correctly, and has desirable performance for SAR images with different regions. However, since a unique level set function is introduced, the proposed method only segment images into two parts. If there are more than two objectives in one image, the result is not satisfying. In order to divide multi-objectives and improve the correctness of segmentation, the development of several level set functions need to be studied in future research.

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