The establishment of the normal contact force model for a one-dimensional sphere chain subjected to impact load

Jun Liu¹² | Futian Zhao¹² | Zhimin Xiao¹² | Yue Wang¹² | Zheng Liu¹² | Haowen Zheng¹²

¹Institute of Engineering Safety and Disaster Prevention, Hohai University, Nanjing, China
²College of Civil and Transportation Engineering, Hohai University, Nanjing, China

Abstract

The normal contact force determines the behavior of a particle system. To investigate the normal contact force in a one-dimensional sphere chain subjected to impact load, by comparing the simulation results of the existing typical normal contact force models embedded in the discrete element program, an improved normal contact force model was proposed in this paper. The improved model consists of two parts: the Cundall model for loading and the Daniel model for unloading. Moreover, a systematic test was designed to verify the accuracy and applicability of the improved model. The results showed that the calculated contact force curves agree well with the experimental results. Furthermore, the improved model is implemented in the solution algorithm without need for complex numerical methods and parameters fitting, leading to more efficient simulations.

Key words
discrete element method, experiment and simulation, normal contact force, one-dimensional sphere chain

1 INTRODUCTION

The discrete element method (DEM)¹ has been widely used to investigate the behavior of particle flows,² fluidized beds,³ and cold powder compaction,⁴⁵ due to its excellent ability in the dynamic analysis of particle systems on both macroscopic and microscopic scales. Especially for granular systems in comminution⁶ and mining engineering,⁷ it is difficult to obtain enough detailed data from laboratory tests and traditional continuous theory, which can be supplemented and extended by DEM simulation.

Accuracy of calculation of contact force between particles is essential to describe motions of particles and mechanical response of granular systems. Due to the complexity of granular systems, which may involve spherical, nonspherical, smooth, rough, homogeneous, and inhomogeneous particles, numerous contact models have been proposed and implemented in the DEM computer-simulation algorithms in the past decades. Some contact physical properties, such as elastoplastic, visco-elastoplastic, adhesive, fracture and spoiling interactions, have been discussed. However, adhesive elastoplastic contacts between smooth, homogeneous, spherical particles have been important topics in several related fields.

While Hertz contact theory⁷ has been widely used in solving elastic contact problems, such a theory is not applicable for nonelastic contact, since it neglects effect of energy dissipation. Generally, use of a spring with a parallel dashpot to define contact-force model is common for solving the non-elastic contact problem. Based on the Hertz theory, researchers proposed several nonlinear spring-dashpot models, which use different displacement power factors in damping
terms. For instance, Tsuji et al.\textsuperscript{10} suggested a damping term proportional to the fourth root of the sphere displacement. The damping term was modified to be proportional to the 1.5 power of the displacement by Hu et al.\textsuperscript{11} and 1–1.5 power of the displacement by Hunt et al.\textsuperscript{12} Kruggel-Emden et al.\textsuperscript{13} proposed a nonlinear damping term that is exponential to the displacement of spheres, and the order was suggested to be 0.5–1. Zdancevicius et al.\textsuperscript{14} adjusted the exponent of displacement by fitting the impact velocity-dependent restitution coefficient.

Furthermore, other researchers proposed some improved elastic–plastic deformation calculation methods for soft spheres based on Hertz theory, which is only valid for rigid spheres. The Thornton model\textsuperscript{15} subdivided contact process into nonlinear Hertz elastic loading, linear plastic loading, and nonlinear elastic unloading. The stress at the contact surface was assumed to remain constant during plastic loading, and the influence of plastic deformation was considered during unloading. Based on the Thornton model, He model\textsuperscript{16} considered the stress evolution law after the plastic stage and proposed a strength coefficient to describe the strengthening or softening of materials. Meanwhile, Brake\textsuperscript{17} added an elastic–plastic loading stage between the elastic loading and the plastic loading in the Thornton model and considered the residual deformation. Zhang and Lesburg\textsuperscript{18} divided the radius of contact surface into elastic and plastic parts according to the continuum theory, and considered the elastic–plastic deformation of the sphere during loading and unloading stage combined with Hertz theory. Rathbone et al.\textsuperscript{19} developed a new contact model based on the finite element analysis of the normal force–displacement relationship between an ideal elastic–plastic sphere and a rigid plane, which divides the contact process into four stages: Hertz elastic loading, elastic–plastic loading, plastic loading, and unloading–reloading.

Deriving the contact force directly considering the viscoelastic property is also an effective way to improve DEM contact model. Hui et al.\textsuperscript{20} investigated the bonding and failure micromechanical process of viscoelastic materials based on the Johnson–Kendall–Roberts (JKR) theory\textsuperscript{21} of contact. By combining the viscoelastic contact for nonadhesive spheres\textsuperscript{22} and the restricted self-consistent model for adhesive axisymmetric bodies, Haiat et al.\textsuperscript{23} generated a restricted self-consistent model for the adhesive contact of linear viscoelastic spheres. Hunter and Graham et al.\textsuperscript{24,25} systematically analyzed the viscoelastic collision and contact problem between a rigid spherical indenter and a linear viscoelastic half-space. However, due to the complication and time-consuming calculations of the above models, such models are not ideally suited for implementation in DEM computational algorithms.

Cundall and Strack\textsuperscript{26} proposed a soft-sphere DEM model, which uses a spring to model the elastic contribution of the collision, while a damper is used to account for energy dissipation mechanism. The viscous damping influences solution convergence and stability of numerical algorithm used to solve motion equations in the Cundall model. Based on the Cundall model, Hunt and Crossley\textsuperscript{12} and Lee and Herrmann\textsuperscript{27} integrated the Hertz theory into a Cundall model and included nonlinear damping. Olsson and Jelagin\textsuperscript{28} proposed a general contact model for viscoelastic spheres considering material and size, and developed a computationally efficient procedure to solve the contact-force problem. Kuwabara and Kono\textsuperscript{29} studied the internal friction or viscosity of solids, and proposed a dissipative term for the normal contact force. Brilliantov et al.\textsuperscript{30} and Zheng et al.\textsuperscript{31} assumed that the displacement field of a viscoelastic sphere is equal to that of an elastic sphere or consistent with that of the static case, and formulated different expressions of the dissipative term. Brilliantov et al.\textsuperscript{32} developed a perturbation approach, which makes the derivation of dissipative force more rigorous without quasi-static approximation.

While many contributions have been made for developing and improving normal contact-force model, a physically realistic model for the sphere chain subjected to impact load has not been yet established. In this paper, five typical normal contact force models are implemented in a unified DEM algorithm used in the simulations of the collisions of sphere particles. An improved normal contact-force model was proposed by comparing simulation results of different existing models. A systematic test method is also proposed to investigate the normal force of the one-dimensional sphere chain under the impact load and verify feasibility and applicability of the improved model for one-dimensional sphere chain. In particular, three typical existing normal contact models and the improved model for a one-dimensional sphere chain subjected to impact load were verified by comparing test and numerical results.

2 | DEM ALGORITHM DESIGN

The DEM program was developed using a modular structure on the platform of VC++. The modular design allows dividing a complex program into several independent simple programs. The program used in this investigation is designed to include three main modules: pre-processor, main processor, and visualization post-processor. The pre-processing module reads the geometric characteristics, mechanical parameters, coordinates of the points of interest, and granular material’s parameters to generate a particle system based on the input data. The main processor module represents the solver. First, the contact force is calculated at every time step according to the applied force. Then, every particle state is iteratively updated at acceleration, velocity, and displacement levels. Finally, particle-contact information, including contact force at every time step, is saved to have the data used in the visualization. The visualization module uses the OpenGL graphics library, which has a visual output function. During the simulations, according to the contact information, the post-processor can be used for online motion visualization and plotting. The detailed algorithm is shown in Figure 1. Five typical normal contact force models (as listed in Table 1) are implemented in the DEM computer program for the purpose of comparative analysis.
The collision of two sphere particles under impact was simulated by the finite element method (FEM) and DEM using different normal contact force models. ANSYS software was used for FEM modeling, and 164 solid elements were selected to establish sphere particles (as shown in Figure 2). Since a spherical particle is prone to mesh distortion during mesh section, a 1/8 model is established and meshes are divided first, and then the whole particle is generated by coordinate axis mirroring. The contact type is automatic surface-to-surface contact and the particle system is constrained laterally, that is, only normal deformation is allowed. Additionally, the bottom of the particle system is modeled as a fixed boundary. A triangular pulse load with a 0.1 s loading duration and a 10 N peak value was applied at the central vertex of the particle system. The material of spherical particles is cement, and the diameters of the sphere are set as 30, 40, and 50 mm. Furthermore, the material properties were tested using a TAJW-2000 electrohydraulic servo testing machine and a rock density tester in the laboratory (Table 2).

Figure 3 shows the contact stress of sphere particles at different times under impact load. When \( t = 0 \) s, the two particles were in a critical state, and the contact stress was zero. When \( t = 0.025 \) s, they entered into the loading stage, and the stress of the whole sphere particle began to increase. When \( t = 0.05 \) s, the contact point stress reached the maximum. When \( t = 0.075 \) s, the two particles arrived at the unloading stage. The maximum stress occurs at the sphere center, while the side stress increases and the contact point stress decreases.
When $t = 0.1$ s, the stress at the contact point almost disappeared and approaches zero. The stress at the center of the sphere is the largest, and the stress wave is diffused throughout the sphere particle. When $t = 0.18$ s, the two particles are disconnected.

Figure 4 shows contact force–deformation curves of three particles with different sizes under the condition of the same material and impact load. The results were obtained using FEM and DEM approaches with different normal contact force models. Table 3 lists the predicted maximum contact force and residual deformation of different particle sizes using different DEM models and FEM approach.

By comparing the simulation curves of the Hertz and FEM models, it can be seen that they intersect at the loading stage in Figure 4A, and the intersection point is near the critical point of loading and unloading of the FEM curve in Figure 4B, and at the unloading stage in Figure 4C. This result suggests that the mechanical response becomes weaker with increasing particle size, which is consistent with the real situation. Since the Hertz theory is an elastic...
contact theory, it considers geometrical characteristics of the contact surface and nonlinearity of materials; at the same time, the contact force is regarded as a nonlinear function of contact depth. However, the energy loss during the collision process is not accounted for, and therefore, the elastic theory cannot accurately describe the actual collision process.

The Cundall model uses a Kelvin model with a parallel spring damper to simulate a linear viscoelastic contact process. The

| Density (kg/m³) | Young’s modulus (GPa) | Poisson’s ratio | Normal stiffness coefficient | Tangential stiffness coefficient | Damping coefficient | Compressive strength (MPa) | Yield strength (MPa) |
|----------------|-----------------------|----------------|-----------------------------|-------------------------------|-------------------|--------------------------|---------------------|
| 2500           | 30                    | 0.2            | $1.25 \times 10^8$          | $1.00 \times 10^5$             | 0.2               | 10                       | 7                   |

**Figure 3** Stress nephogram of sphere particles at different times: (A) $t = 0$ s; (B) $t = 0.025$ s; (C) $t = 0.05$ s; (D) $t = 0.075$ s; (E) $t = 0.1$ s; and (F) $t = 0.18$ s
contact force has two parts: elastic force and damping force, and the collision speed and energy loss are considered. The results show that the contact force–deformation curve of the Cundall model agrees well with that of FEM model in the loading stage; however, the plastic deformation is not considered during the unloading stage. Elastic stiffness and particle size highly influence the results, and therefore, the relevant parameters need to be adjusted to ensure solution accuracy.

The Thornton model can predict the residual deformation, but the maximum contact force is much lower than that of the FEM simulation. The Thornton model considers plastic deformation and divides the contact process into three parts: Hertz elastic loading, linear plastic loading, and nonlinear elastic unloading. Because of existing plastic deformation, the effective contact radius during unloading will be greater than the original value, so the original effective contact radius should be replaced by the effective contact radius during the unloading. For the Thornton model, the effective contact radius is a constant determined by the maximum deformation, maximum contact force, and material properties; the speed during the collision separation process is not considered.

Based on the Thornton model, and in order to consider the stress adjustment and release of material after entering the plastic stage, a reinforcement factor \( k \) is used to describe the strengthening or softening of the material in the He model. Actually, the value of the reinforcement coefficient greatly affects the predicted results. When the reinforcement coefficient is high enough, the force–deformation curve of He model is close to that of the Hertz model. Also, the lower the value of the reinforcement coefficient, the closer the prediction curve is to the Thornton model. The final value of the reinforcement coefficient \( k \) is set as \( 3 \times 10^8 \), considering material elastic–plastic hardening.\textsuperscript{16}

Daniel model can predict the maximum contact force and residual deformation well within a certain range, but there can be large deviation outside this range. Nevertheless, the predicted residual-deformation trend of Daniel model is still in good agreement with the FEM simulation. In Daniel model, there are four parts: Hertz elastic loading, elastic–plastic loading, unloading, and reloading. In the unloading stage, Daniel model is similar to Thornton model; however, Daniel model considers the collision velocity when calculating effective contact radius. Therefore, effective contact radius is a dynamically changing value in the particle separation phase, which is consistent with actual situation.

![FIGURE 4](image_url)

**FIGURE 4** Comparison of contact force–deformation curves with different methods for different particle sizes: (A) 3 cm particle diameter; (B) 4 cm particle diameter; and (C) 5 cm particle diameter

**TABLE 3** Comparison of the maximum contact force \( P_{\text{max}} \) and residual deformation \( \delta_p \)

| Models     | 3 cm particle diameter | 4 cm particle diameter | 5 cm particle diameter |
|------------|------------------------|------------------------|------------------------|
|            | \( P_{\text{max}} \) (N) | \( \delta_p \) (x10\(^{-6}\) m) | \( P_{\text{max}} \) (N) | \( \delta_p \) (x10\(^{-6}\) m) | \( P_{\text{max}} \) (N) | \( \delta_p \) (x10\(^{-6}\) m) |
| FEA        | 9.538                  | 2.994                  | 9.480                  | 1.919                  | 9.510                  | 1.25                  |
| Hertz      | 15.989 /               | /                      | 9.477                  | /                      | 5.890                  | /                      |
| Thornton   | 2.197                  | 3.484                  | 1.792                  | 2.056                  | 1.763                  | 1.443                  |
| S.M. He    | 2.350                  | 3.460                  | 1.879                  | 2.073                  | 1.676                  | 1.348                  |
| Daniel     | 11.150                 | 1.775                  | 9.527                  | 1.812                  | 5.489                  | 1.486                  |
| Cundall    | 9.786 /                | /                      | 9.128                  | /                      | 9.640                  | /                      |
For models including the equivalent radius \( R \), such as the Hertz model, the Daniel model, the Thornton model, and the S.M. He model, the effect of particle size on the force–displacement curves from different models is consistent. The expression of the equivalent radius is \( 1/R = 1/R_1 + 1/R_2 \). According to the expressions of contact models, the contact deformation decreases under the same load, while the equivalent radius increases with increasing radii \( R_1 \) and \( R_2 \). In general, the larger the particle radius, the stronger the deformation resistance of spherical particle. As shown in Table 3, the maximum contact force decreases with increasing particle diameters. For the Cundall model, the values of stiffness and the damping coefficient highly influence the calculated results as compared to particle size.

4 | IMPROVEMENT AND VALIDATION OF THE NORMAL CONTACT FORCE MODEL

The advantages and disadvantages of each normal contact model were analyzed by simulating sphere particle collision in Section 3. To summarize, the contact model based on the Hertz theory is inconsistent with the actual situation at the loading stage. Due to ignoring collision speed and energy dissipation, contact-force results are usually higher than the experimental results. Cundall model uses acceleration and damping to account for the collision speed and energy dissipation, making it more flexible and adaptable to different situations. Since the elastic stiffness and the damping coefficient are different for different materials, it is necessary to adjust the parameters to obtain accurate solutions. During the unloading stage, Cundall model ignores effects of yield stress and residual deformation; by contrast, Daniel model not only considers influence of plastic deformation but also account for the separation velocity of particles.

Based on the above analysis, an improved normal contact force model is proposed and can be expressed as

\[
P = \begin{cases} 
  k\delta + \beta\dot{\delta} & \delta \geq 0, \\
  \frac{4ER_1^{1/2}}{3} (\delta - \delta_p)^{3/2} & \delta < 0,
\end{cases}
\]  

(1)

where definitions of the variables can be found in Table 1.

The improved model consists of Cundall model for the loading process and Daniel model for the unloading process. The model not only retains the advantages of Cundall model during the loading stage but also compensates for its deficiency of elastic unloading through Daniel model. Moreover, the improved model does not require use of complex numerical methods and parameters fitting.

Figure 5 shows the force–deformation curves predicted using the improved and FEM models. The calculated results, including maximum contact force, maximum contact deformation, and residual deformation, are shown in Table 4. The results of the improved model are all in good agreement with the FEM results, as shown in Table 4. However, while the force–deformation curves of the two methods show good consistency in the unloading stage, there is an obvious difference in the loading stage. According to the definition of the improved model, it can be clearly seen that the theoretical force–displacement relationship is linear. In FEM model, on the other hand, the contact force depends on the area of contact interface, the relative position, and the contact state of two spheres, which are unknown in advance and change with time, and need to be determined during the solution process.

The penalty function method is used to solve the contact problem in this paper, and the expression of the penalty parameter for the solid element is \( a = \gamma kA R^2/V \), where \( \gamma \) is a scaling factor, \( K \) is the stiffness of the contact element, and \( A \) and \( V \) are the areas and volumes of elements at the contact interface. At each iteration step, the contact information are determined, and the expression of the penalty parameter determines the nonlinear growth of the contact force and convexity of force–displacement curve in the loading stage.

To summarize, the improved model shows clear advantages in computing the normal contact between spherical particles, especially for the prediction of the maximum contact force and residual deformation. Furthermore, the stiffness coefficients were constant under different impact loads in the improved model, which shows that the improved model has good adaptability.

5 | MODEL TEST AND DEM SIMULATION FOR A ONE-DIMENSIONAL SPHERE CHAIN

5.1 | Model test

As shown in Figures 6 and 7, the organic glass frame is designed as a cylinder with a diameter of 5.1 cm, and cement mortar balls with a diameter of 5 cm are successively placed into the cylinder to form a one-dimensional granular chain. From the bottom of the cylinder to the top, balls and piezoelectric sensors are numbered 1–9 and 1–8, respectively.
The cement mortar balls are made using an aluminum mould with two separate hollow hemispheres. The balls are polished to reduce the influence of friction between the balls and the inner wall of the cylinder.

In the cylinder wall, 4 mm holes are drilled to connect the piezoelectric sensors. The piezoelectric film sensors, whose effective area includes a central circular film with a diameter of 20 mm and a thickness of 0.25 mm, are placed between particles to measure the contact force. The contact point of two adjacent balls is at the center of the film, which can be regarded as the common tangent plane. The weight of the sensors is much lower than the weight of the ball, so its gravity effect can be ignored.

---

**TABLE 4** Results of DEM with an improved and FEM models

| Impact load (N) | Maximum contact force $P_{\text{max}}$ (N) | Maximum deformation $\delta_{\text{max}}$ (m) | Residual deformation $\delta_{\text{f}}$ (m) |
|----------------|---------------------------------------------|---------------------------------------------|---------------------------------------------|
|                | FEA | Improved model | FEA | Improved model | FEA | Improved model |
| 20             | 19.70 | 19.73          | 5.61 $\times 10^{-6}$ | 5.79 $\times 10^{-6}$ | 4.09 $\times 10^{-6}$ | 4.43 $\times 10^{-6}$ |
| 50             | 48.40 | 48.27          | 1.30 $\times 10^{-5}$ | 1.30 $\times 10^{-5}$ | 9.80 $\times 10^{-6}$ | 1.03 $\times 10^{-5}$ |
| 100            | 95.90 | 99.54          | 2.86 $\times 10^{-5}$ | 2.94 $\times 10^{-5}$ | 2.34 $\times 10^{-5}$ | 2.54 $\times 10^{-5}$ |

Abbreviations: DEM, discrete element method; FEM, finite element method.

---

**FIGURE 6** Sketch of the experimental setup: (A) framework and (B) instruments

**FIGURE 7** One-dimensional granular chain: (A) experimental design and (B) DEM model. DEM, discrete element method

**FIGURE 8** Measured waveform of sensors in the impact test
The impact loading is performed by striking the specimen with a hammer, and the waveforms of the applied load are recorded by a sensor embedded in the hammer. To ensure that the load direction is in the center of the granular chain, a steel plate is installed above the cylinder, and a reset system is set on top of the steel plate (see Figure 6A). The steel plate is made of a lightweight and high-strength composite metal material, and the reset system uses a small damping spring, so the effect of the steel plate on loading can be ignored.

The waveforms of the contact force are shown in Figure 8. The main part of the contact-force waveform is concentrated in the range of 0.01 s. The peak value of the contact force measured by the 8th sensor closest to the top of the cylinder is close to the peak value of

**FIGURE 9** Waveform of contact force of DEM simulation using different models: (A) Hertz model; (B) Cundall model; (C) Daniel model; and (D) improved model. DEM, discrete element method
the applied load. From the top of the cylinder to the bottom, the peak value of the contact force decreases first and then increases.

5.2 | DEM numerical simulation

The improved model and three existing normal contact force models, including the nonlinear elastic model (Hertz model), the linear viscoelastic model (Cundall model), and the nonlinear elastic–plastic model (Daniel model), were embedded in the program mentioned above. The waveforms of the contact force are shown in Figure 9.

5.3 | Results analysis and discussion

Analysis of the three groups of contact force curves considered in the preceding section indicates that, as the Hertz and Daniel models are based on the elastic theory, the waveforms of the contact force predicted by these two models show a long oscillation duration that lasts for the entire simulation. After reaching a peak of around 0.1 s, the amplitude gradually decreases with noticeable fluctuation. Due to consideration of the influence of the viscous effect and velocity change, Cundall and improved models showed a waveform pattern similar to the experiments, and the oscillation duration was basically within 0.05–0.1 s. The waveforms of particles Nos. 1, 3, 5, and 8 are extracted for further analysis (see Figure 10).

The curves of contact force calculated by Hertz and Daniel models have long vibration durations, which is inconsistent with the actual situation. The curves of Cundall and the improved models are close to experimentally-measured curve in terms of the waveform, oscillation duration, and peaks. However, the predicted through time and the value of the improved model are closer to the experimental results than that of the Cundall model, which does not consider the effect of plastic deformation.

The peaks of the contact force calculated using different methods are extracted and shown in Figure 11. The abscissa is
the distance from the applied load location to piezoelectric sensors. The changing trend of the contact-force peaks simulated predicted by different models is basically the same as that of the experiments. From the top to the bottom of the cylinder, the contact force first decreased and then increased slightly. The peak contact force at the top of the cylinder was the maximum. First, the contact force showed a decreasing trend following the force propagation in the chain, an indication of energy dissipation. Then, the contact force showed an increasing trend, which may be attributed to compression caused by bottom plate resistance and impact to particles by rebounding upper particles. Therefore, the speed, energy loss, and material properties should be considered in the calculation. For the improved model, the following parameters are considered: speed, elastic stiffness, and damping ratio to allow simulating different materials by changing the elastic stiffness and energy loss by damping.

The error of contact-force peaks in the model measured with respect to that of the experiment is calculated (see Table 5). The changing trend of the contact force in the improved model is closer to that of the experiment for the five peak curves. Daniel model is based on Hertz theory, and the calculated peaks of contact force are similar to the peaks in Hertz model. As the balls in the experiments are not perfectly elastic, the difference in the contact-force peaks between the results of Daniel-model and experimental results is larger. The improved model is based on Cundall model in the loading stage, and as a result, the error of contact-force peaks is reduced by considering speed and energy loss.

| Models            | Hertz model | Cundall model | Daniel model | Improved model |
|-------------------|-------------|---------------|--------------|---------------|
| Error of the first group | 24.0%       | 16.5%         | 16.4%        | 12.2%         |
| Error of the second group | 19.8%       | 11.3%         | 15.0%        | 10.2%         |
| Error of the third group | 15.4%       | 8.4%          | 10.6%        | 7.6%          |
| Average error     | 19.7%       | 12.1%         | 14.0%        | 10.0%         |

Finally, it is worth noting that the improved model was implemented in the DEM software without the need for using complex numerical methods and parameter fitting, which demonstrates the generality and ease of implementation of the improved model.

CONCLUSIONS
In this paper, five normal-contact force models have been investigated using a DEM software designed for the analysis of a spherical particle element normal contact forces. By comparing the advantages and disadvantages of each model, an improved normal contact-force model has been proposed with two parts: Cundall model for loading and Daniel model for unloading. The model retains the advantages of Cundall model during the loading stage and compensates for the deficiency of elastic unloading of Cundall model through the Daniel model.

A one-dimensional sphere chain impact test has been conducted to verify the feasibility and applicability of the improved model. The comparison between experimental data and DEM simulation results shows that the simulated waveform obtained by the improved model agrees with the measured waveform in the duration and the concentration degree of the main waveform. The average error of the waveform peak is only 10%, which is the smallest error among the models examined in this paper.

ACKNOWLEDGMENTS
This study was supported by the National Natural Science Foundation of China (Nos. 51874118, 51778211).

CONFLICT OF INTEREST
The authors declare that there are no conflict of interest.

AUTHOR CONTRIBUTIONS
Jun Liu contributed data. Futian Zhao analyzed the data and wrote the paper. Zhimin Xiao, Yue Wang, Haowen Zheng, and Zheng Liu contributed toward experimental testing.

DATA AVAILABILITY STATEMENT
The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

ORCID
Futian Zhao http://orcid.org/0000-0002-9973-122X

REFERENCES
1. Renzo AD, Maio F. Comparison of contact-force models for the simulation of collisions in DEM-based granular flow codes. Chem Eng Sci. 2004;59(3):525-541.
2. Doucet J, Bertrand F, Chaouki J. An extended radioactive particle tracking method for systems with irregular moving boundaries. Powder Technol. 2008;181(2):195-204.
3. Nakagawa M, Altobelli SA, Caprihan A, Fukushima E, Jeong EK. Non-invasive measurements of granular flows by magnetic resonance imaging. Exp Fluids. 1993;16(1):54-60.
4. Ding YL, Forster R, Seville J, et al. Segregation of granular flow in the transverse plane of a rolling mode rotating drum. Granular Matter—Proceedings of CCAST (World Laboratory) Workshop. 2003.
5. Yamane K, Nakagawa M, Altobelli SA, et al. Steady particulate flows in a horizontal rotating cylinder. Phys Fluids. 1998;10(6):1419-1427.
6. Khakhar DV, Mccarthy JJ, Ottino JM. Radial segregation of granular mixtures in rotating cylinders. *Phys Fluids*. 1997;9(12):3600-3614.

7. Aranson IS, Tsimring LS. Dynamics of axial separation in long rotating drums. *Phys Rev Lett*. 1999;82(23):4643-4646.

8. Khakhar DV, Orpe AV, Ottino JM. Continuum model of mixing and size segregation in a rotating cylinder: concentration-flow coupling and streak formation. *Powder Technol*. 2016;282(3):232-245.

9. Hertz H. ber Die Berührung Fester Elastischer Krper. *J für Reine Angew Math*. 1882;92:156-171.

10. Tsuji Y, Tanaka T, Ishida T. Lagrangian numerical simulation of plug flow of cohesionless particles in a horizontal pipe. *Powder Technol*. 1992;71(3):239-250.

11. Hu G, Hu Z, Jian B, Liu L, Wan H. On the determination of the damping coefficient of non-linear spring-dashpot system to model Hertz contact for simulation by discrete element method. *J Comput. 2011;6(5):984-988.*

12. Hunt KH, Crossley F. Coefficient of restitution interpreted as damping in vibroimpact. *J Appl Mech*. 1975;42(2):440-445.

13. Kruggel-Ehmsen H, Simsek E, Rickelt S, Wirtz S. Review and extension of normal force models for the Discrete Element Method. *Powder Technol*. 2007;171(3):157-173.

14. Ždanczukiewicz E, Kacianauskas R, Zabulionis D. Improvement of viscoelastic damping for the Hertz contact of particles due to impact velocity. *Procedia Eng*. 2017;172:1286-1290.

15. Thornton C. Coefficient of restitution for collinear collisions of elastic-perfectly plastic spheres. *Trans ASME J Appl Mech*. 1997;64(2):383-386.

16. He SM, Wu Y, Li XP. Theoretical model on elastic-plastic granule impact. *Eng Mech*. 2008;25(12):19-24 (in Chinese).

17. Brake MR. An analytical elastic-perfectly plastic contact model. *Int J Solids Struct*. 2012;49(22):3129-3141.

18. Zhang X, Lesburg LA. Normal force-displacement model for contacting spheres accounting for plastic deformation: force-driven formulation. *Trans ASME J Appl Mech*. 2000;67(2):363-371.

19. Rathbone D, Marigo M, Dini D, van Wachem B. An accurate force-displacement law for the modelling of elastic–plastic contacts in discrete element simulations. *Powder Technol*. 2015;282(3):2-9.

20. Hui CY, Baney JM, Kramer EJ. Contact mechanics and adhesion of viscoelastic spheres. *Langmuir*. 1998;14(22):2075-2080.

21. Johnson KL, Kendall K, Roberts AD. Surface energy and the contact of elastic solids. *Proc R Soc Lond Ser A Math Phys Eng Sci*. 1971;324:301.

22. Ting TCT. The contact stresses between a rigid indenter and a viscoelastic half-space. *J Appl Mech*. 1966;33(4):845-854.

23. Haiat G, Huy M, Barthel E. The adhesive contact of viscoelastic spheres. *J Mech Phys Solids*. 2003;51(1):69-99.

24. Hunter SC. The Hertz problem for a rigid spherical indenter and a viscoelastic half-space. *J Mech Phys Solids*. 1960;8(4):219-234.

25. Graham G. The contact problem in the linear theory of viscoelasticity. *Int J Eng Sci*. 1965;3(1):27-46.

26. Cundall PA, Strack O. A discrete numerical model for granular assemblies. *Geotechnique*. 2008;30(3):331-336.

27. Lee J, Herrmann HJ. Angle of repose and angle of marginal stability: molecular dynamics of granular particles. *J Phys A Gen Phys*. 1999;26(2):373-383.

28. Olsson E, Jelagin D. A contact model for the normal force between viscoelastic particles in discrete element simulations. *Powder Technol*. 2018;342:992-999.

29. Kuwabara G, Kono K. Restitution coefficient in a collision between two spheres. *Jpn J Appl Phys*. 1987;26(1, No. 8):1230-1233.

30. Brilliantov NV, Spahn F, Hertzsch JM, Pöschel T. A model for collisions in granular gases. *Phys Rev E*. 1996;53:5382-5393.

31. Zheng QJ, Zhu HP, Yu AB. Finite element analysis of the normal force between a viscoelastic sphere and rigid plane. *Powder Technol*. 2012;226:130-142.

32. Brilliantov NV, Pimenova AV, Goldobin DS. A dissipative force between colliding viscoelastic bodies: rigorous approach. *EPL (Europhys Lett)*. 2015;109(1):1946-1974.

**How to cite this article:** Liu J, Zhao F, Xiao Z, Wang Y, Liu Z, Zheng H. The establishment of the normal contact force model for a one-dimensional sphere chain subjected to impact load. *Int J Mech Syst Dyn*. 2022;2:131-142. doi:10.1002/msd2.12023