An Improved Artificial Potential Field Escape Method with Weight Adjustment

Hongyun Wang*, Min Gao, Weiwei Gao, Yi Wang and Haijun Zhou

Army Engineering University of Shijiazhuang Campus Shi Jiazhuang, Hebei Province, China

*Corresponding author email: 258033232@qq.com

Abstract. Aiming at the problems of obstacle avoidance and bullet avoidance during the patrol swarm, this paper analyzed the defects of the classical artificial potential field, proposed an adjustable escape method, which establish the relationship between the adjustment coefficient and the distance. This method avoid too large or too small escape force that get the bullet into new local shock problem near the target. Then given the weight calculation and parameter selection method, restricted the escape motion by kinematics according to the constraints in the actual motion. This improved method can effecting solve the problem of avoidance in dynamic and complex environment.

Keywords: Artificial potential field method; Minimum value problem; Weight adjustment; Escape method.

1. Introduction

The problem of collision between swarm bullets and obstacle \ swarm bullets is a very important issue as the number of patrol swarm flights increases. Because the actual flight may exist buildings, peaks, trees, birds, enemy aircraft and other obstacles, these obstacles include both static and dynamic obstacles, will pose a threat to the flight safety of the distance between the aircraft will also change with the formation of obstacle avoidance maneuver, improper handling is very easy to collision. Therefore, drone formations should be able to make decisions based on different environmental conditions, while avoiding threatening obstacles and other drones. This paper analyzes the defects of the classical artificial potential field and improves the artificial potential field method.

Due to the principle defects of the artificial potential field method, the necessary improvement needs to be made, especially when the flying bullet and obstacles, the target is a three - point line, or multiple obstacles are scattered around the target, or in the bullet and the target obstacle is type L or U. The distance of the bullet gets closer to the target, the repulsion generated by the obstacle slowly tends to the attraction of the target, and gradually reaches the equilibrium state, when the equilibrium potential field point is the minimum of the potential field . When the bullet falls into a local minimum, the bullet stays at a point or shocks back and forth in a small range, unable to search the path normally, resulting in failure. Document [1][2] aims at the local minimum problem , introducing the sub target point to fly to the sub target point and then to the final target point ; Document [3] proposed an obstacle connection method based on fusion scenario behavior, adding repulsion function within the minimum state to pull the local minimum state and ensure the mobile robot can reach the target in the unknown environment. Document [4] proposed a heuristic sub - target point to select the best sub target point to jump out of the minimum; The literature [5] adopts the escape force method to eliminate the local shocks in the search process, and realizes the automatic search of the feasible channels, compensating for the shortage of
easy minimum value in the traditional artificial potential field method. Document[6] (a force based on escape force) proposes a strategy based on fuzzy control, the size of escape force is controlled by the preset fuzzy controller, the direction of escape force and gravity direction are vertical[7][8] proposes a substitution method to return the avoidance zone problem. Document [3-7] all use the escape force method to solve the shock problem at the minimum value, but if the increased escape force is too large, if the escape force is too small, it is easy to lead to the bullet into the new local shock. In order to make the bullet escape near the target point, a weight adjustment escape method is proposed, and no escape force is too large or too small into a new local shock.

2. Classic Artificial Potential Field Method and Problem Analysis

Artificial potential field method is a typical global path planning algorithm[9][10][11]. The basic idea is that the surrounding environment of a patrol bullet has a mixed force field of gravity and repulsion. The target produces gravity to the patrol bullet and the direction is directed to the target by the patrol bullet. Assumption: $X = (x, y, z)^T$ represents the current position of the patrol bullet. As the distance between the patrol bullet and the target changes, the gravitational potential field function is as follows:

$$U_{att} = -\frac{1}{2} k_a d_{goal}^2(X)$$

(1)

Among them, $d_{goal} = \|X - X_{goal}\|$ is the distance between the cruiser and the target, $k_a$ is gravity coefficient. So the attraction can be represented by the negative gradient of the gravitational potential field function:

$$F_{att} = -k_a (X - X_{goal})$$

(2)

By setting the velocity vector of the patrol bullet is proportional to the vector field force, when the patrol bullet is close to the target, the patrol bullet is driven to the target at a reduced speed. The repulsion potential field keeps the patrol bullets away from the obstacles. The stronger this repulsion is when the patrol bullet approaches the obstacle; the impact of the repulsion potential field decreases when the patrol bullet is far away from the obstacle. The repulsion potential field generated by the obstacle is:

$$U_{repi}(X) = \begin{cases} \frac{1}{2} k_{repi} \left( \frac{1}{d_{obsi}(X)} - \frac{1}{d_0} \right)^2 & d_{obsi}(X) \leq d_0 \\ 0 & \text{others} \end{cases}$$

(3)

$i$ indicates the number of obstacles near a cruiser in the environment, $d_{obsi}$ Represents the distance from an obstacle, $d_o$ Represents the range of influence of obstacle repulsion field.

$$F_{repi}(X) = \begin{cases} k_{repi} \left( \frac{1}{d_{obsi}(X)} - \frac{1}{d_0} \frac{1}{d_{obsi}(X)} \right) \hat{e}_i & d_{obsi}(X) \leq d_0 \\ 0 & \text{others} \end{cases}$$

(4)

$\hat{e}_i = \frac{\partial d_{obsi}(X)}{\partial X}$ represent the unit vector of the repulsion direction.

By analyzing the function of gravitational potential field and repulsive potential field, the defects of classical potential field method are mainly manifested in the following aspects: (1) From the above gravity function, when the cruiser bullet is far from the target, the gravity is relatively large, which helps to lead the cruiser bullet to the target. If there is an obstacle near the cruiser bullet, and the repulsion force of the obstacle can be ignored relative to the gravity, the cruiser bullet may collide with the obstacle in the path to the target; (2) When the distance between the patrol bullet and the obstacle is
small, the repulsion force will be very large, which will make it difficult for the patrol bullet to fly to
the target; (3) When \( F_{\text{att}} = F_{\text{req}} \), in the opposite direction, the cruiser will oscillate back and forth at a
certain point, with a local minimum; (4) This algorithm is not suitable for collision avoidance in
dynamic complex environment. Due to the limitation of space, this study mainly analyzes the local
minimum problem.

3. Analysis and Algorithm Improvement of Minimum Value Problem

When there are many obstacles scattered around the target and close to the target, as the distance
between the bullet and the target decreases, the gravity of the target decreases gradually, but the
repulsion force produced by the obstacle increases gradually \(^{12}\)[13]. One of the reasons for the
minimum value problem is that the construction of the repulsive potential field function only considers
the relative distance between the bullet and the obstacle.

Add the following influence factors: the relative distance \((X - X_g)^n\) between the bullet and the target
in the repulsive field function. Influence factors of relative velocity bullets \(v_{io}(t)\), the obstacles
relative acceleration \(a_{io}(t)\), the influence of the minimum obstacle avoidance safe distance \(d_{min}\). Then
the improved repulsion potential field functions are:

\[
U_{\text{rep}}(X, v, a) = \begin{cases}
\frac{1}{2} k_{\text{rep}1} \left( \frac{1}{d_{\text{obs}}(X) - d_{\text{min}}} - \frac{1}{d_0} \right)^2 (X - X_g)^n \\
+ k_{\text{rep}2} v_{io}(t) + k_{\text{rep}3} a_{io}(t) & d_{\text{obs}}(X) \leq d_0 \\
0 & d_{\text{obs}}(X) > d_0
\end{cases}
\]

\(k_{\text{rep}1}, k_{\text{rep}2}, k_{\text{rep}3}\) repulsion coefficient greater than 0.

\[
v_{io}(t) = v(t) - v_{\text{obs}}(t)
\]

\[
a_{io}(t) = a(t) - a_{\text{obs}}(t)
\]

\(v_{\text{obs}}(t)\) and \(a_{\text{obs}}(t)\) are the velocity and acceleration of the obstacle, respectively,

\[
F_{\text{rep}1}(X, v, a) = -\nabla U_{\text{rep1}}(X, v, a) = -\nabla_x U_{\text{rep1}}(X, v, a) - \nabla_v U_{\text{rep1}}(X, v, a) - \nabla_a U_{\text{rep1}}(X, v, a)
\]

\[
F_{\text{rep1}}(X) = F_{\text{rep1}}(X) + F_{\text{rep1}}(v) + F_{\text{rep1}}(a)
\]

\[
F_{\text{rep1}}(X) = -k_{\text{rep1}} \left( \frac{1}{d_{\text{obs}}(X) - d_{\text{min}}} - \frac{1}{d_0} \right)^2 (X - X_g)^n
\]

\[
F_{\text{rep2}}(X) = \frac{n}{2} k_{\text{rep2}} \left( \frac{1}{d_{\text{obs}}(X) - d_{\text{min}}} - \frac{1}{d_0} \right)^2 (X - X_g)^{n-1}
\]

\(\hat{e}_i\) Represents the unit vector of bullets pointing to obstacles during flight.

The effects of the influence factors \(n\) on improving the repulsion function are discussed below:

(1) When \(n = 0\), \(F_{\text{rep}2}(X) = 0\). At this time, the influence factor \((X - X_g)^0 = 1\) is equivalent to no
influence factor added in the original repulsive potential field function, so the new repulsion field
function and the traditional repulsion field function are the same, which does not apply for the case of obstacles scattered near the target.

(2) When \(0 < n < 1\),
As the bullet gradually approaches the target
\[
\lim_{X \to X_g} \|F_{\text{req}}\| = \lim_{X \to X_g} \left[ k_{\text{req}} \left( \frac{1}{d_{\text{obs}}(X) - d_{\text{min}}} - \frac{1}{d_0^2} \right) \frac{(X - X_g)^n}{d_0^2} \right] = 0
\]
The repulsion is not equal to zero, but greater than zero and points to the target, so in this case, the bullet can achieve the target.

(3) When \(n = 1\),
As the bullet gradually approaches the target,
\[
\lim_{X \to X_g} \|F_{\text{req}}\| = \lim_{X \to X_g} \left[ k_{\text{req}} \left( \frac{1}{d_{\text{obs}}(X) - d_{\text{min}}} - \frac{1}{d_0^2} \right) \right] = C
\]
It can be seen from the formula that as the bullet approaches the target, the repulsive force approaches a constant greater than zero and points to the target, so the bullet can also achieve the obstacle avoidance under the action of the resultant force.

(4) When \(n > 1\),
\[
\lim_{X \to X_g} \|F_{\text{req}}\| = \lim_{X \to X_g} \left[ k_{\text{req}} \left( \frac{1}{d_{\text{obs}}(X) - d_{\text{min}}} - \frac{1}{d_0^2} \right) \frac{(X - X_g)^n}{d_0^2} \right] = 0
\]
It can be seen from the formula that as the bullet approaches the target, the repulsive force approaches to zero, but the force direction of the bullet gradually tends to the target, so the bullet can also achieve the obstacle avoidance under the action of the resultant force. From the above analysis of (1)-(4), it can be seen that the improved repulsive potential function can safely avoid obstacles and reach the target as the bullet approaches the target, but in order to ensure the total potential energy is the smallest when the target point is reached, it is best to be zero, \(n > 1\) should be selected.

4. Proposed Method of Weight Adjustment Escape
When the bullet in flight is in a three-point line with the obstacle, the target, as shown in figure 1(a)(b), or when multiple obstacles are scattered around the target, figure1(c), or when the bullet and the target obstacle are L or U, as shown in figure 1(d)(e), it is possible that the bullet oscillates back and forth in a certain range and falls into the minimum problem \([14][15][16]\).

(a) Obstacle between patrol bullets and targets (b) Obstacles outside of the cruising bullets and targets

(c) Multiple obstacles (d) Type L obstacle (e) Type U obstacle

Figure 1. Force of patrol bullets under different obstacles.
\[ F_{at}(X, v, a) = F_{at}(X) + F_{at}(v) + F_{at}(a) \]
\[ F_{repi}(X, v, a) = F_{repi}(X) + F_{repi}(v) + F_{repi}(a) \]
\[ F_{\tilde{r}}(X, v, a) = \omega_1 F_{at}(X, v, a) + \omega_2 F_{repi}(X, v, a) \]

(9)

The basic idea: set the distance threshold between the bullet and the target is \( l \). When detected \( d_{\text{goal}} = ||X - X_{\text{goal}}|| \geq l \), make \( \omega_1 = 1, \omega_2 = 1 \); When \( d_{\text{goal}} = ||X - X_{\text{goal}}|| < l \), to prevent falling into the minimum, make \( \omega_2 = 1, \omega_1 \). As the distance between the missile and the target decreases, the total potential energy of the bullet reaches the target point is minimized. To speed up the escape, you can also make \( \omega_2 \) decrease and \( \omega_1 \) increase. But, as shown above, if there is an obstruction on the connection between the bullet, the object may collide even if increasing, so an escape angle problem, \( \alpha \) assuming that the escape angle is involved.

5. Analysis of Adjusting Weights and Other Parameters

5.1. Weight Determination Method

For convenience, the relationship between bullets, targets and obstacles is discussed in the plane, as shown in the following Figure 2:

![Figure 2](image-url)

Figure 2. Geometrical relationship on a straight line of patrol bullets, targets and obstacles.

The current coordinates of the bullet \((x, y)\), the coordinates of the target \((x_g, y_g)\), the coordinates of the obstacle \((x_o, y_o)\), then the distance between the bullet and the target \( l = \sqrt{(x_g - x)^2 + (y_g - y)^2} \)

Distance between bullets and obstacles
\[
d = \sqrt{(x_o - x)^2 + (y_o - y)^2} \]
\[
\alpha = \arcsin \frac{r}{d} \quad \beta = \arctan \frac{x_o - x}{y_o - y} \]

Distance between bullet and obstacle
\[
s_1 = \sqrt{d^2 - r^2} = \sqrt{(x_o - x)^2 + (y_o - y)^2 - r^2} \]

(10)

Coordinates of right tangent: \((x_1, y_1)\)

Similarly, the distance between the left cut point of the bullet and the obstacle is
\[
s_2 = \sqrt{d^2 - r^2} = \sqrt{(x_o - x)^2 + (y_o - y)^2 - r^2} \]

(11)

Coordinates of left tangent: \((x_2, y_2)\)
The coordinates of the left and right tangent points can be obtained by bringing the relevant parameters in.

Take the tangent point as an example, the distance from the tangent point to the target point is \( s' \),

\[ s'_1 = \sqrt{(x'_1 - x_1)^2 + (y'_1 - y_1)^2} \]  

If the bullet moves along the bullet-tangent-target, the range of the bullet to the target is \( l' = s_1 + s'_1 \).

Set \( \omega_1 = l'/l > 1 \) is the gravitational gain coefficient, and the direction of the force shifts to the left or to the right is

\[ \alpha = \arcsin \frac{r}{\sqrt{(x_o - x)^2 + (y_o - y)^2}} \]  

In order to speed up the escape speed, the rep weight is reduced appropriately \( \omega_2 = s_1 / d < 1 \),

\[ F_{\beta} (X, v, a) = \frac{l'}{l} F_{an} (X, v, a) \Delta \alpha + \frac{s_1}{d} F_{req} (X, v, a) \]  

The selection of tangent points for L type and U type obstacles is shown in the Figure 3. The conclusion of the above derivation is also applicable. The L type obstacle should choose the one side of the reasonable direction to bypass the obstacle, as shown in the diagram; the U type obstacle can bypass the obstacle from the left and right ends.

**Figure 3.** Geometric relationship between patrol bullets, targets and L (u) obstacles.

5.2. Choice of Parameters

The following situation analysis of the improved bullet by the resultant force parameter selection\[17\][18]. Ignoring the effects of the phase degree velocity and acceleration.
\[ F_{\text{rep}}(X) = k_{\text{req}} \left( \frac{1}{d_{\text{obi}}(X) - d_{\text{min}}} - \frac{1}{d_0^2} \right) (X - X_g)^n \]

\[ F_{\text{rep}}(X) = \frac{n}{2} k_{\text{req}} \left( \frac{1}{d_{\text{obi}}(X) - d_{\text{min}}} - \frac{1}{d_0^2} \right)^2 (X - X_g)^{n-1} \]  

(16)

In order to avoid local minima, the total potential field must be directed to the target, that is

\[ F_{\text{tot}}(X) = F_{\text{atr}}(X) + [F_{\text{req}}(X) + F_{\text{rep}}(X)] > 0 \]  

(17)

\[ k_X, \ k_{\text{req}}, \ m \text{ and } n \text{ all normal, if } d = (X - X_g), \ A = 1 / d_{\text{obi}}(X) - 1 / d_0, \]

\[ mk_X [X - X_g]^{m-1} - k_{\text{req}} \left[ \frac{1}{d_{\text{obi}}(X) - d_{\text{min}}} - \frac{1}{d_0^2} \right] (X - X_g)^n + \frac{n}{2} k_{\text{req}} \left[ \frac{1}{d_{\text{obi}}(X) - d_{\text{min}}} - \frac{1}{d_0^2} \right]^2 (X - X_g)^{n-1} > 0 \]  

(18)

After simplified

\[ mk_X d^{m-1} - k_{\text{req}} \frac{d^n}{d_{\text{obi}}^2(X)} A + \frac{n}{2} k_{\text{req}} A^2 d^{n-1} > 0 \]

\[ mk_X d^{m-1} > k_{\text{req}} \frac{d^n}{d_{\text{obi}}^2(X)} A - \frac{n}{2} k_{\text{req}} A^2 d^n \]

(19)

\[ \frac{k_X}{k_{\text{req}}} > \frac{1}{md^{m-1}} \left[ \frac{d^n}{d_{\text{obi}}^2(X)} A - \frac{n}{2} A^2 d^{n-1} \right] = \frac{d^{n-1}}{md^{m-1}} \left[ \frac{d}{d_{\text{obi}}^2(X)} A - \frac{n}{2} A^2 \right] \]

(20)

\[ f(A) = f(A) = \frac{d}{d_{\text{obi}}^2(X)} A - \frac{n}{2} A^2 \]

(21)

\[ f'(A) = \frac{d}{d_{\text{obi}}^2(X)} - nA \]

(22)

\[ A = \frac{d}{nd_{\text{obi}}^2(X)} \quad f_{\text{max}}(A) = \frac{d}{d_{\text{obi}}^2(X)} A - \frac{n}{2} A^2 = \frac{d^2}{2nd_{\text{obi}}^2(X)} \]

(23)

\[ \frac{k_X}{k_{\text{req}}} > \frac{d^{n-1}}{md^{m-1}} \left[ \frac{d}{d_0^2} A - \frac{n}{2} A^2 \right] = \frac{d^{n-1}}{md^{m-1}} \left[ \frac{d}{md^{m-1}d_0^2} = \frac{d_n^2}{2md^{m-1}d_0^2} \right] \]

(24)

In practical application, the parameters \( m \) and \( n \) should be determined first. For convenience, now assuming \( m = n = 2 \), the relationship between gravitational gain factor and repulsive gain factor should be discussed.

\[ \frac{k_X}{k_{\text{req}}} > \frac{1}{8} \left( \frac{d}{d_0} \right)^2 = \frac{1}{8} \left( \frac{L}{s_1} \right)^2 \]

(25)

According to the above formula, when the position of the missile, the target, the obstacles is given. The size of the gain factor can be determined.
6. Escape Motion Constraints
In order to make the bullet escape the minimum value near the target point, a weight adjustment escape method is proposed, and the escape angle \( \alpha \) is set. When the bullet follows the adjusting force and escapes to the \( OO_1 \), and then take along \( O_1O \) to the target, At this time angle \( \eta \), angle \( \eta \) should satisfy \(-\eta_{\text{max}} < \eta < \eta_{\text{max}}\) as shown in the following Figure2. The force in the vertical plane is shown in the diagram, and in the horizontal plane as shown in the diagram, it can be seen from the diagram that the turning force is the partial force of the lift in the horizontal direction and the steering angle is the horizontal force, Steering angle is \( \eta \), \( F_{\text{horizontal}} \). According to the relationship of motion and overload, the turning radius and the speed and the overload of the missile are as follows:

\[
R = \frac{v^2}{g(n - \cos \eta)} \quad n = \frac{F \sin \eta}{mg} \quad (26)
\]

But the bullet in flight, must make the overload less than the specified maximum, with constant turning speed \( R_{\text{min}} > \frac{v^2}{g(n_{\text{max}} - \cos \eta)} \). That is, when the bullet is flying, the turning radius can not be infinitely small, if the minimum turning radius is limited to \( R_{\text{min}} \), then the maximum turning angle of the adjacent three points is: \( \eta_{\text{max}} = \frac{\zeta}{R_{\text{min}}} \), \( \zeta \) refers to the step size of the bullet flight.

![Figure 4. Force of bullet in vertical plane and horizontal plane.](image)

![Figure 5. Maximum corner of adjacent bullet points.](image)

7. Conclusion
This paper focuses on the local minimum problem in classical artificial potential field due scattering multiple obstacles or other shapes around the target. The artificial potential field method is improved by increasing relative distance and relative velocity in potential field function. On the basis of improving potential field function, the problem of minimum value in different shape obstacles is further analyzed. A weight adjustment escape idea is proposed. Assuming that the repulsion and gravity are equal at a certain point, the distance between the bullet and the target is a set threshold. Increase gravity, reduce repulsion, so that the bullet quickly escape the minimum value. Due to the certain mathematical relationship of the weight adjustment value and distance in the present method, the problem of too large or too small escape force is avoided. Finally, the necessary kinematic constraints on the escape motion according on vehicle constraints in actual motion. This improved method can effectively solve the problem of local minimum value and obstacle avoidance in dynamic and complex environment because of the relative position influence between bullets, targets and obstacles.

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