New Agegraphic Dark Energy in $f(R)$ Gravity

M.R. Setare *
Department of Science, Payame Noor University. Bijar, Iran

Abstract

In this paper we study cosmological application of new agegraphic dark energy density in the $f(R)$ gravity framework. We employ the new agegraphic model of dark energy to obtain the equation of state for the new agegraphic energy density in spatially flat universe. Our calculation show, taking $n < 0$, it is possible to have $w_\Lambda$ crossing $-1$. This implies that one can generate phantom-like equation of state from a new agegraphic dark energy model in flat universe in the modified gravity cosmology framework. Also we develop a reconstruction scheme for the modified gravity with $f(R)$ action.

*E-mail: rezakord@ipm.ir
1 Introduction

Many cosmological observations, such as SNe Ia [1], WMAP [2], SDSS [3], Chandra X-ray observatory [4] etc., discover that our universe is undergoing an accelerated expansion. They also suggest that our universe is spatially flat, and consists of about 70 % dark energy (DE) with negative pressure, 30 % dust matter (cold dark matter plus baryons), and negligible radiation. In order to explain why the cosmic acceleration happens, many theories have been proposed. It is the most accepted idea that a mysterious dominant component, dark energy, with negative pressure, leads to this cosmic acceleration, though its nature and cosmological origin still remain enigmatic at present. An alternative proposal for dark energy is the dynamical dark energy scenario. The cosmological constant puzzles may be better interpreted by assuming that the vacuum energy is canceled to exactly zero by some unknown mechanism and introducing a dark energy component with a dynamically variable equation of state. The dynamical dark energy proposal is often realized by some scalar field mechanism which suggests that the energy form with negative pressure is provided by a scalar field evolving down a proper potential.

In recent years, many string theorists have devoted to understand and shed light on the cosmological constant or dark energy within the string framework. The famous Kachru-Kallosh-Linde-Trivedi (KKLT) model [5] is a typical example, which tries to construct metastable de Sitter vacua in the light of type IIB string theory. Furthermore, string landscape idea [6] has been proposed for shedding light on the cosmological constant problem based upon the anthropic principle and multiverse speculation. Although we are lacking a quantum gravity theory today, we still can make some attempts to probe the nature of dark energy according to some principles of quantum gravity. An interesting attempt in this direction is the so-called “holographic dark energy” proposal [7, 8, 9, 10]. Such a paradigm has been constructed in the light of holographic principle of quantum gravity [11], and thus it presents some interesting features of an underlying theory of dark energy. More recently a new dark energy model, dubbed agegraphic dark energy has been proposed [12] (see also [13]), which takes into account the Heisenberg uncertainty relation of quantum mechanics together with the gravitational effect in general relativity.

Because the holographic energy density belongs to a dynamical cosmological constant, we need a dynamical frame to accommodate it instead of general relativity. Einstein’s theory of gravity may not describe gravity at very high energy. The simplest alternative to general relativity is Brans-Dicke scalar-tensor theory [14]. Modified gravity provides the natural gravitational alternative for dark energy [15]. Moreover, modified gravity present natural unification of the early-time inflation and late-time acceleration thanks to different role of gravitational terms relevant at small and at large curvature. Also modified gravity may naturally describe the transition from non-phantom phase to phantom one without necessity to introduce the exotic matter. But among the most popular modified gravities which may successfully describe the cosmic speed-up is $F(R)$ gravity. Very simple versions of such theory like $1/R$ [16] and $1/R + R^2$ [17] may lead to the effective quintessence/phantom late-time universe (to see solar system constraints on modified dark energy models refer to [20]). Another theory proposed as gravitational dark energy is scalar-Gauss-Bonnet gravity [18] which is closely related with low-energy string effective action.

In present paper, using the new agegraphic model of dark energy in spatially flat universe, we obtain equation of state for agegraphic dark energy density in framework of modified
gravity. We show the phantomic description of the new agegraphic dark energy in flat universe with \( n < 0 \). Also we develop a reconstruction scheme for the modified gravity with \( f(R) \) action, the known new agegraphic energy density is used for this reconstruction.

## 2 Modified gravity and new agegraphic dark energy

The action of modified gravity is given by

\[
S = \int \sqrt{-g} d^4x [f(R) + L_m].
\]

where \( L_m \) is the matter Lagrangian density. The equivalent form of above action is [15]

\[
S = \int d^4x \sqrt{-g} [P(\phi)R + Q(\phi) + L_m].
\]

where \( P \) and \( Q \) are proper functions of the scalar field \( \phi \). By the variation of the action (2) with respect to the \( \phi \), we obtain

\[
P'(\phi)R + Q'(\phi) = 0
\]

which may be solved with respect to \( \phi \):

\[
\phi = \phi(R)
\]

By the variation of the action (2) with respect to the metric \( g_{\mu\nu} \), one can obtain

\[
-\frac{1}{2} g_{\mu\nu} [P(\phi)R + Q(\phi)] - R_{\mu\nu} P(\phi) + \nabla_\mu \nabla_\nu P(\phi) - g_{\mu\nu} \nabla^2 P(\phi) + \frac{1}{2} T_{\mu\nu} = 0
\]

where \( T_{\mu\nu} \) is the energy-momentum tensor. The equations corresponding to standard spatially-flat FRW universe are

\[
\rho = 6H^2 P(\phi) + Q(\phi) + 6H \frac{dP(\phi)}{dt}
\]

\[
p = -(4\dot{H} + 6H^2) P(\phi) - Q(\phi) - 2\frac{d^2P(\phi)}{dt^2} - 4H \frac{dP(\phi)}{dt}
\]

where, \( p \) and \( \rho \) are the pressure and energy density due to the scalar field in the modified gravity framework. By combining (6) and (7) and deleting \( Q(\phi) \), we find

\[
p + \rho = -2\frac{d^2P(\phi)}{dt^2} + 2H \frac{dP(\phi)}{dt} - 4\dot{H} P(\phi)
\]

Now we suggest a correspondence between the new agegraphic dark energy scenario and the above modified dark energy model. According to the new agegraphic dark energy we have following relation for energy density [19]

\[
\rho_\Lambda = 3n^2 M_p^2 \eta^{-2}
\]

where the numerical factor \( 3n^2 \) is introduced to parameterize some uncertainties, such as the species of quantum fields in the universe, \( \eta \) is conformal time, and given by

\[
\eta = \int \frac{dt}{a} = \int \frac{da}{a^2 H}.
\]
The critical energy density, $\rho_{cr}$, is given by following relation

$$\rho_{cr} = 3H^2 \quad (11)$$

Using definitions $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}$ and $\rho_{cr} = 3M_p^2H^2$, we get

$$H\eta = \frac{n}{\sqrt{\Omega_\Lambda}} \quad (12)$$

we obtain the equation of state for the agegraphic energy density. Let us consider the dark energy dominated universe. In this case the dark energy evolves according to its conservation law

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + P_\Lambda) = 0 \quad (13)$$

By considering the definition of agegraphic energy density $\rho_\Lambda$, one can find:

$$\dot{\rho}_\Lambda = \frac{-2}{a\eta}\rho_\Lambda \quad (14)$$

Substitute this relation into Eq.(13) we obtain

$$w_\Lambda = \frac{2\sqrt{\Omega_\Lambda}}{3an} - 1, \quad (15)$$

then we can see that $w_\Lambda$ can cross the phantom divide if $n < 0$.

As one can redefine the scalar field $\phi$ properly, we may choose

$$\phi = t. \quad (16)$$

Now using Eqs.(9), (15), one can rewrite Eq.(8) as

$$2\frac{d^2P(t)}{dt^2} - 2H\frac{dP(t)}{dt} + 4\dot{H}P(t) - \frac{2\Omega_\Lambda^{3/2}H^2}{an} = 0 \quad (17)$$

In principle, by solving Eq.(17) we find the form of $P(\phi)$. Using Eqs. (6), (9), we also find the form of $Q(\phi)$ as

$$Q(\phi) = 3\Omega_\Lambda H^2 - 6H^2P(\phi) - 6H\frac{dP(\phi)}{dt} \quad (18)$$

### 3 Modified gravity and its reconstruction from the new agegraphic dark energy

In this section we consider another approach [21] to realistic cosmology in new agegraphic modified gravity. We start with general $f(R)$-gravity action (1) but without the matter term. For the spatially flat FRW universe we have

$$\rho = f(R) - 6(H + H^2 - H\frac{d}{dt})f'(R) \quad (19)$$

$$p = f(R) - 2(-\dot{H} + 3H^2 + \frac{d^2}{dt^2} + 2H\frac{d}{dt})f'(R) \quad (20)$$
where
\[ R = 6\dot{H} + 12H^2 \] (21)
Again we use the new agegraphic dark energy density and substitute Eq.(9) into Eq.(19)
\[ 3\Omega_\Lambda H^2 = f(R) - 6(\dot{H} + H^2 - H\frac{d}{dt})f'(R) \] (22)
thus
\[ f(R) = 3\Omega_\Lambda H^2 + 6(\dot{H} + H^2 - H\frac{d}{dt})f'(R) \] (23)
Using Eqs.(9), (15), and substituting \( f(R) \) into Eq.(20) one can obtain
\[ \frac{d^2}{dt^2}f'(R) - H\frac{d}{dt}f'(R) + 2\dot{H}f'(R) + f(R) + \frac{\Omega_\Lambda^3/2H^2}{a} = 0 \] (24)
We shall consider the following simple solution
\[ a = a_0(t_s - t)^{h_0}, \] (25)
where \( a_0, h_0 \) and \( t_s \) are constant. Substituting Eq.(25) into Eq.(21), give us following relation for scalar curvature
\[ R = \frac{12h_0^2 - 6h_0}{(t_s - t)^2} \] (26)
Using Eqs.(10, 25) we can write
\[ \eta = \int_t^{t_s} \frac{dt}{a_0(t_s - t)^{h_0}} = \frac{1}{a_0(1 - h_0)(t_s - t)^{h_0 - 1}} \] (27)
Now using definition \( \rho_\Lambda \) and above relation we obtain the time behaviour of agegraphic dark energy as
\[ \rho_\Lambda = \frac{3n^2a_0^2(1 - h_0)^2}{(t_s - t)^{2 - 2h_0}} \] (28)
Substituting the above \( \rho_\Lambda \) into Eq.(19), and using Eqs.(25,26) one can obtain
\[ \frac{72h_0^2(1 - 2h_0)}{(t_s - t)^4}f''(R) - \frac{6h_0(h_0 - 1)}{(t_s - t)^2}f'(R) + f(R) = \frac{3n^2a_0^2(1 - h_0)^2}{(t_s - t)^{2 - 2h_0}} \] (29)
Again we use Eq.(26) and rewrite the above differential equation as following
\[ f''(R) + \frac{a}{R}f'(R) + \frac{b}{R^2}f(R) = \frac{d}{R^{1+h_0}} \] (30)
where
\[ a = \frac{h_0 - 1}{2}, \quad b = \frac{1 - 2h_0}{2}, \quad d = \frac{-a_0^2n^2(1 - h_0)^2}{4(6h_0(2h_0 - 1))^{h_0}}. \] (31)
The solution of differential equation (30) is given by
\[ f(R) = C_1R^{\frac{1}{2\left(\frac{3-h_0}{2} - \sqrt{\frac{(h_0-3)^2}{4}+4h_{0^2}}\right)}} + C_2R^{\frac{1}{2\left(\frac{3-h_0}{2} + \sqrt{\frac{(h_0-3)^2}{4}+4h_{0^2}}\right)}} + \frac{n^2a_0^2(1 - h_0)^2(6h_0(2h_0 - 1))^{h_0}}{2h_0(2 - h_0)R^{h_0 - 1}}. \] (32)
where $C_1, C_2$ are constant. Therefore, a consistent modified gravity with new agegraphic dark energy in flat space has the above form. In order that the accelerating expansion in the present universe could be generated, let us consider that $f(R)$ could be a small constant at present universe, that is,

$$f(R_0) = -2R_0, \quad f'(R_0) \approx 0, \quad \text{(33)}$$

where $R_0 \sim (10^{-33}eV)^2$ is current curvature [20]. By impose the conditions (33) on the solution (32) we can obtain the constants $C_1$ and $C_2$ as following

$$C_1 = -\frac{(1 - h_0)^2a_0^2n^2(6h_0(2h_0 - 1))^{h_0}(v + h_0 - 1)}{(v - uR_0)R_0^{h_0 + u - 1}} - \frac{2v}{(v - uR_0)R_0^{u - 1}}, \quad \text{(34)}$$

$$C_2 = \frac{(1 - h_0)^2a_0^2n^2(6h_0(2h_0 - 1))^{h_0}(uR_0 + h_0 - 1)}{(v - uR_0)R_0^{h_0 + v - 1}} + \frac{2u}{(v - uR_0)R_0^{v - 2}}, \quad \text{(35)}$$

where

$$u = \frac{1}{2} \left( \frac{3 - h_0}{2} - \sqrt{\left(\frac{h_0 - 3}{4}\right)^2 + 4h_0 - 2} \right), \quad v = \frac{1}{2} \left( \frac{3 - h_0}{2} + \sqrt{\left(\frac{h_0 - 3}{4}\right)^2 + 4h_0 - 2} \right) \quad \text{(36)}$$

4 Conclusions

In order to solve cosmological problems and because the lack of our knowledge, for instance to determine what could be the best candidate for DE to explain the accelerated expansion of universe, the cosmologists try to approach to best results as precise as they can by considering all the possibilities they have. Within the different candidates to play the role of the dark energy, the new agegraphic dark energy model, has emerged as a possible model with EoS across $-1$. In the present paper we have studied cosmological application of new agegraphic dark energy density in the $f(R)$ modified gravity framework. By considering the agegraphic energy density as a dynamical cosmological constant, we have obtained the equation of state for the agegraphic energy density in the $f(R)$ gravity framework. We have shown if $n < 0$, the new agegraphic dark energy model also will behave like a phantom model of dark energy the amazing feature of which is that the equation of state of dark energy component $w_\Lambda$ crosses $-1$. Also we have developed a reconstruction scheme for modified gravity with $f(R)$ action. We have considered the energy density in Eq.(19) in new agegraphic form, then by assumption a simple solution as Eq.(25) we could obtain a differential equation for $f(R)$, the solution of this differential equation give us a modified gravity action which is consistent with new agegraphic dark energy scenario.

References

[1] A. G. Riess et al. [Supernova Search Team Collaboration], Astrophys. J. 607, 665 (2004) [astro-ph/0402512]; R. A. Knop et al., [Supernova Cosmology Project Collaboration], Astrophys. J. 598, 102 (2003) [astro-ph/0309368]; A. G. Riess et al. [Supernova Search Team Collaboration], Astron. J. 116, 1009 (1998) [astro-ph/9805201]; S. Perlmutter et al. [Supernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999) [astro-ph/9812133].
[2] C. L. Bennett et al., Astrophys. J. Suppl. 148, 1 (2003) [astro-ph/0302207]; D. N. Spergel et al., Astrophys. J. Suppl. 148, 175 (2003) [astro-ph/0302209].

[3] M. Tegmark et al. [SDSS Collaboration], Phys. Rev. D 69, 103501 (2004) [astro-ph/0310723]; M. Tegmark et al. [SDSS Collaboration], Astrophys. J. 606, 702 (2004) [astro-ph/0310725]; U. Seljak et al., Phys. Rev. D 71, 103515 (2005) [astro-ph/0407372]; J. K. Adelman-McCarthy et al. [SDSS Collaboration], astro-ph/0507711; K. Abazajian et al. [SDSS Collaboration], astro-ph/0410239; astro-ph/0403325; astro-ph/0305492;

[4] S. W. Allen, R. W. Schmidt, H. Ebeling, A. C. Fabian and L. van Speybroeck, Mon. Not. Roy. Astron. Soc. 353, 457 (2004) [astro-ph/0403325]; J. K. Adelman-McCarthy et al. [SDSS Collaboration], astro-ph/0507711; K. Abazajian et al. [SDSS Collaboration], astro-ph/0410239; astro-ph/0403325; astro-ph/0305492;

[5] M. Tegmark et al. [SDSS Collaboration], Phys. Rev. D 69, 103501 (2004) [astro-ph/0310723]; M. Tegmark et al. [SDSS Collaboration], Astrophys. J. 606, 702 (2004) [astro-ph/0310725]; U. Seljak et al., Phys. Rev. D 71, 103515 (2005) [astro-ph/0407372]; J. K. Adelman-McCarthy et al. [SDSS Collaboration], astro-ph/0507711; K. Abazajian et al. [SDSS Collaboration], astro-ph/0410239; astro-ph/0403325; astro-ph/0305492;

[6] S. W. Allen, R. W. Schmidt, H. Ebeling, A. C. Fabian and L. van Speybroeck, Mon. Not. Roy. Astron. Soc. 353, 457 (2004) [astro-ph/0403325]; J. K. Adelman-McCarthy et al. [SDSS Collaboration], astro-ph/0507711; K. Abazajian et al. [SDSS Collaboration], astro-ph/0410239; astro-ph/0403325; astro-ph/0305492;

[7] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Phys. Rev. Lett. 82, 4971 (1999); P. Horava and D. Minic, Phys. Rev. Lett. 85, 1610 (2000); S. D. Thomas, Phys. Rev. Lett. 89, 081301 (2002).

[8] S. D. H. Hsu, Phys. Lett. B 594, 13 (2004).

[9] M. Li, Phys. Lett. B 603, 1 (2004); D. Pavon and W. Zimdahl, Phys. Lett. B 628, 206 (2005).

[10] K. Enqvist and M. S. Sloth, Phys. Rev. Lett. 93, 221302 (2004); K. Ke and M. Li, Phys. Lett. B 606, 173 (2005); Q. G. Huang and M. Li, JCAP 0503, 001 (2005); E. Elizalde, S. Nojiri, S. D. Odintsov and P. Wang, Phys. Rev. D 71, 103504 (2005); B. Wang, Y. Gong and E. Abdalla, Phys. Lett. B 624, 141 (2005); H. Kim, H. W. Lee and Y. S. Myung, Phys. Lett. B 632, 605 (2006); B. Hu and Y. Ling, Phys. Rev. D 73, 123510 (2006); H. Li, Z. K. Guo and Y. Z. Zhang, Int. J. Mod. Phys. D 15, 869 (2006); M. R. Setare, Phys. Lett. B 642, 421 (2006); M. R. Setare, Phys. Lett. B 644, 99 (2007); M. R. Setare, J. Zhang and X. Zhang, JCAP 0703, 007 (2007); M. R. Setare, Phys. Lett. B 648, 329 (2007); M. R. Setare, Phys. Lett. B 654, 1 (2007); W. Zhao, Phys. Lett. B 655, 97, (2007); M. Li, C. Lin and Y. Wang, JCAP 0703, 007 (2007); M. R. Setare, Phys. Lett. B 642, 421 (2006); M. R. Setare, Phys. Lett. B 644, 99 (2007); M. R. Setare, Phys. Lett. B 648, 329 (2007); M. R. Setare, Phys. Lett. B 654, 1 (2007); W. Zhao, Phys. Lett. B 655, 97, (2007); M. Li, C. Lin and Y. Wang, JCAP 0805, 023 (2008).

[11] G. ’t Hooft, [arXiv:gr-qc/9310026]; L. Susskind, J. Math. Phys. 36, 6377 (1995).

[12] R. G. Cai, Phys. Lett. B 657, 228, (2007).

[13] I. P. Neupane, Phys. Lett. B 673, 111, (2009); K. Y. Kim, H. W. Lee, Y. S. Myung, Phys. Lett. B 660, 118, (2008); J. Zhang, X. Zhang, H. Liu, Eur. Phys. J. C54, 303, (2008).

[14] C. Brans and C. H. Dicke, Phys. Rev. 124, 925 (1961).
[15] S. Nojiri, S. D. Odintsov, H. Stefancic, Phys. Rev. D 74, 086009, (2006); S. Nojiri, S. D. Odintsov, J. Phys. A 40, 6725, (2007); G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov and S. Zerbini, Phys. Rev. D75 , 086002, (2007); S. Nojiri and , S. D. Odintsov, J. Phys. Conf. Ser.66, 012005, (2007).

[16] S. Capozziello, Int. J. Mod. Phys. D 11, 483 (2002); S. Capozziello, S. Carloni and A. Troisi, arXiv:astro-ph/0303041; S. M. Carroll, V. Duvvuri, M. Trodden and S. Turner, Phys. Rev. D 70 (2004) 043528.

[17] S. Nojiri and S. D. Odintsov, Phys. Rev. D 68, 123512 (2003).

[18] S. Nojiri, S. D. Odintsov and M. Sasaki, Phys. Rev. D 71, 123509 (2005); S. Nojiri, S. D. Odintsov and M. Sami, Phys. Rev. D 74, 046004, (2006); B. M. N. Carter, I. P. Neupane, Phys. Lett. B638, 94, (2006); B. M. N. Carter, and I. P. Neupane, JCAP 0606, 004, (2006); J. W. Moffat, and V. T. Toth. arXiv:0710.0364 [astro-ph].

[19] H. Wei, R. G. Cai, Phys. Lett. B 660, 113, (2008); K. Y. Kim, H. W. Lee, Y. S. Myung, M. I. Park, Mod. Phys. Lett. A23, 3049, (2008); J. P Wu, D. Z. Ma, Y. Ling, Phys. Lett. B663, 152, (2008); J. Cui, L. Zhang, J. Zhang, X. Zhang, arXiv:0902.0716 [astro-ph].

[20] S. Nojiri, S. D. Odintsov, 0707.1941v2 [hep-th];S. Nojiri, S. D. Odintsov, 0710.1738v2 [hep-th]; G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov, L. Sebastiani, S. Zerbini, 0712.4017v1 [hep-th].

[21] S. Capozziello, S. Nojiri, S. D. Odintsov and A. Troisi, Phys. Lett. B 639, 135, (2006) ; S. Nojiri and S. D. Odintsov, Phys. Rev. D74, 086005, (2006).