Network Sensitivity to Geographical Configuration

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Abstract. Gravitational wave astronomy will require the coordinated analysis of data from the global network of gravitational wave observatories. Questions of how to optimally configure the global network arise in this context. We have elsewhere proposed a formalism which is employed here to compare different configurations of the network, using both the coincident network analysis method and the coherent network analysis method. We have constructed a network model to compute a figure-of-merit based on the detection rate for a population of standard-candle binary inspirals. We find that this measure of network quality is very sensitive to the geographic location of component detectors under a coincident network analysis, but comparatively insensitive under a coherent network analysis.

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1. Introduction

Considerations of network analysis have helped shape the growth of the global network of gravitational wave detectors. The sites of the Laser Interferometric Gravitational-wave Observatory (LIGO) detectors were chosen to facilitate a coincident network analysis [1]; far enough apart to produce a detectable direction-dependent relative time delay, but close enough together to have similar antenna patterns. Likewise, the proposed site for the Australian International Gravitational Observatory (AIGO) was chosen to be near-antipodal to the LIGO detectors and thus to share their antenna patterns [2]. Instruments such as VIRGO [3] and the proposed Laser Cryogenic Gravitational Telescope (LCGT) [4] have not been sited with regard to such considerations.

The coincident network analysis technique [5, 6], in its simplest form, allows independent searches to be performed by each detector in the network; a signal is only detected by the network when the signal is detected by each member detector. A more recently proposed technique is coherent network analysis [7, 8], whereby the output of all detectors is collected and then a single search is performed on the combined data. The coherent network analysis has a theoretical advantage over the coincident network analysis, but the practical merits of each are still under debate.

The increasing viability of the new coherent network analysis technique [7, 8] encourages us to reconsider existing results about the global network; in particular, the influence of instrument siting on the quality of the network as a whole [6].

We have introduced a formalism [9] to answer some of the questions arising about how best to configure a network of gravitational wave detectors; in particular, how to optimally site a new detector augmenting an existing network, so as to maximize the detection rate for a standard-candle binary inspiral, and whether the effect is significant enough to warrant consideration in planning the growth of the global network.

2. Network Model

Consider the parameter space representing a network of \( n \) interferometric gravitational wave detectors. The physical location and orientation of an interferometer can be defined by the latitude \( \theta \) and longitude \( \phi \) of the beam-splitter, and the orientation angle \( \psi \) of the x-arm with respect to north, under the assumption of a horizontal detector on a perfectly spherical Earth. If the detectors are further assumed to be identical up to location and orientation (that is, with identical arm lengths and noise curves), then a network may be described by the \( 3n \) parameters \( [(\theta_1, \phi_1, \psi_1), \ldots, (\theta_n, \phi_n, \psi_n)] \), forming a \( 3n \) dimensional parameter space \( \Theta \).

We have devised a \textit{figure of merit}, \( f : \Theta \rightarrow \mathbb{R} \), as a measure of the quality of the network. We use a quantity proportional to the detection rate for a population of randomly oriented standard-candle binary inspirals uniformly distributed in a flat space. For a network with identical detectors, uncorrelated noise and a fixed source class, the structure of the noise curve and strain waveform contribute only a constant
Figure 1. Relative merit of an additional site to augment the LIGO Livingston Observatory in a coincident analysis (lighter is better, contours every 2.5%). The minimum detection rate is 41% of the maximum.

multiplicative factor to the detection rate, so that we may neglect them and consider only the relative orientation and distance between the network and the source, and the analysis algorithm used. We follow Finn’s maximum likelihood test formulations of the coincident and coherent algorithms [7], and additionally assume, for computational amenability, that the threshold $\lambda$ is high with respect to the noise power (see [9] for details).

The $3n$-dimensional parameter space is too large to be computationally amenable, and so we consider only proper subsets of particular interest. For example, the best site (under our figure of merit) to augment an existing network of $n$ detectors can be found by fixing the first $n$ detectors of a $n + 1$ detector network, and varying only $(\theta_{n+1}, \phi_{n+1}, \psi_{n+1})$. Furthermore, noting that the figure of merit depends only weakly on the orientation $\psi_{n+1}$, we can fix it to an arbitrary value, and vary only the latitude and longitude. The figure of merit over this two-dimensional section of parameter space then corresponds to a map of the relative merit of different sites on the Earth for augmenting an existing network.

3. Results

Consider first a single interferometer, at the site [10] of LIGO Livingston Observatory (LLO). For a coincident network analysis, the merit of an additional site to augment

‡ See Figures 1, 2, 3, 4 at the upper and lower edges, corresponding to the north and south geographic poles, the detectors rotate in place as the longitude varies; despite this the figure of merit remains constant to a good approximation.
Figure 2. Relative merit of an additional site to augment the LIGO Livingston Observatory in a coherent analysis (lighter is better, contours every 2.5%). The minimum detection rate is 89% of the maximum.

LLO is given in Figure 1. It demonstrates, as expected, that sites near or near-antipodal to LLO are best to augment it. This is the rationale behind the siting of the LIGO detectors, and the proposed AIGO detector. The worst configurations produce a substantially reduced detection rate; approximately 40% that of the optimal configuration. Unfortunately, the locations of VIRGO and the proposed LCGT fall into this category.

It is interesting to note that for this simple case the map bears some resemblance to the “peanut” antenna pattern of the fixed single detector; the weak directionality of the varying detector, and the superiority of a co-aligned network [11] are responsible for this effect [9]. This resemblance breaks down for more complicated networks.

Considering the same configuration of a fixed LLO detector and a varying detector with a coherent network analysis in Figure 2 the qualitative structure of the map is similar, but quantitatively it is quite different. For a coherent analysis, the worst configurations produce a detection rate that is still 90% of optimal; site merit does not vary substantially with location.

We now move on to consider an approximation to the existing global network of the larger interferometric gravitational wave detectors. We model the LIGO-VIRGO network as three identical interferometers at the sites of LIGO Hanford Observatory (LHO), LIGO Livingston Observatory and VIRGO [10]. Note that this model neglects the 2 kilometre LHO instrument, and the differences between the LIGO and VIRGO instruments. Similarly, we augment this three-detector network with a fourth (identical) detector at different locations and compare the relative detection rates of the resulting network.
Figure 3. Relative merit of an additional site to augment a network consisting of the
LIGO Hanford (4km) Observatory, the LIGO Livingston Observatory and a 4km LIGO
I instrument at the VIRGO site, in a coincident analysis (lighter is better, contours
every 2.5%). The minimum detection rate is 69% of the maximum.

Using a coincident network analysis in Figure 3, we see that the merit of the
network varies moderately with location, with multiple minima of about 70% of the
best achievable detection rates.

Under a coherent network analysis in Figure 4, we once again see a qualitative
similarity to Figure 3 in the locations of maxima and minima, but quantitatively much
less variation than in the coincident case, with only 6% separating the best and worst
sites. As expected, this indicates that AIGO is an optimal site to augment the existing
global network; however, the weak dependence of event rate on geographical location
means that its advantage over other sites is slight.

4. Conclusion

Under our model, it is clear that the (binary inspiral) detection rate for a global
network is insensitive to the geographical configuration of its component detectors when
a coherent analysis is used, in contrast to when a simple coincident analysis is used.
Whilst the LIGO detectors and proposed AIGO detector are well sited to complement
one another under a coincident analysis, the sites of the VIRGO detector and proposed
LCGT detector are far from optimal; our results demonstrate that under a coherent
analysis the cost of this sub-optimal siting is substantially reduced, on at least one figure
of merit. In this sense, the global network is closer to optimal for a coherent analysis than
for a coincident analysis. Our results also indicate that since, under a coherent analysis,
detection rate is insensitive to detector siting, the location of an augmenting detector
Figure 4. Relative merit of an additional site to augment a network consisting of the LIGO Hanford (4km) Observatory, the LIGO Livingston Observatory and a 4km LIGO I instrument at the VIRGO site, in a coherent analysis (lighter is better, contours every 2.5%). The minimum detection rate is 94% of the maximum.

could be optimized for other network properties (for example, directional resolution) without compromising the event rate.

It is important to note that the model does not compare the absolute detection rates for the two analysis techniques; we cannot say that one method would produce a higher detection rate than the other for a given false alarm rate. We have considered only one class of source and one figure of merit, but our formalism permits us to refine and extend the network model and the range of questions with which it is used to answer.

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