BDS Stochastic Modeling and Impact Analysis

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Abstract. Carrier phase measurements are always used in high precision situations. Though in short baseline case, double difference technic can eliminate most of the systematic errors, some errors still remain. The stochastic model of carrier phase describes its stochastic characters. By modeling stochastic model, the solution accuracy and stability can be improved. To investigate the impact of stochastic models, the elevation model and SNR model are established respectively for GEO, IGSO and MEO satellites. Based on these stochastic models, their impact on azimuth solution is analyzed. In this paper, the basic function model is first introduced. Then the single difference (SD) carrier phase model and the estimation method of SD residuals are proposed. Finally, the stochastic models are established based on experiment data and the impact of those models are analyzed.

1. Introduction
BeiDou navigation system (BDS) is a global navigation system owned by China. At present, the construction of BDS-2 has completed. The BDS-3 is now under construction. Positioning and orientating using satellite navigation system has many advantages comparing to inertial navigation system.

The double differencing function model are often used in BDS orientating. In short baseline case, double difference technic can eliminate most of the systematic errors which make the solution unbiased [1-2]. However, besides systematic errors, the stochastic characters of observations also have influences on the solution optimality. The stochastic characters are related to lots of factors, such as hardware design, measurement snr ration and satellite elevation [3-6]. If the measurement quality representing factors can be chosen properly, the appropriate stochastic models can be established. Then the solution accuracy and stability can be improved [7].

In order to improve the orientating accuracy, the elevation model and SNR model are established respectively for GEO, IGSO and MEO satellite. In this paper, the basic function model is first introduced. Then the single difference (SD) carrier phase model and the estimation method of SD residuals are proposed. Finally, the stochastic models are established based on experiment data and the impact of those models are analyzed.

2. Basic model
The model of carrier phase observations is given as.
\[
\phi_i^s = \frac{1}{\lambda_i} \left( \rho + \delta_{\text{trop}} + \delta_{\rho} \right) + N_i + I_i + \epsilon_i
\]  

(1)

Where \( \phi_i^s \) is the carrier phase of satellite \( s \) on \( i \)th frequency, \( \lambda_i \) is wavelength, \( \rho \) is the geometry distance between receiver and satellite, \( \delta_{\text{trop}} \) is troposphere delay, \( \delta_{\rho} \) is other frequency independent errors, \( N_i \) is ambiguity, \( I_i \) is ionosphere delay, \( \epsilon_i \) is residual error.

In order to mitigate the errors, double difference technic is often used. In short baseline case, after double differencing, the atmosphere delay, receiver clock error and satellite clock error can be eliminated. The carrier phase double difference model is given by

\[
\lambda_i \Delta \nabla \phi_i^s = \Delta \nabla \rho + \Delta \nabla \delta_{\text{trop}} + \Delta \nabla \delta_{\rho} + \lambda_i \Delta \nabla N_i + \Delta \nabla I_i + \Delta \nabla \epsilon_i
\]

(2)

Where \( \Delta \nabla \) represents double differencing operator, \( \Delta \nabla \delta_{\text{trop}} \), \( \Delta \nabla \delta_{\rho} \) and \( \Delta \nabla I_i \) can be ignored.

When multiple satellites are observed and linearize \( \Delta \nabla \rho \), Eq.2 can be rewritten in a general form

\[
L_i = A_i X + B N_i + \epsilon_i
\]

(3)

Where \( X \) is baseline vector, \( L_i \) is double differenced measurements, \( A_i \) and \( B \) are coefficient matrix, \( \epsilon_i \) is residual error vector. Specifically,

\[
A_i = \begin{bmatrix}
x_{s1} - x_{rs} & y_{s1} - y_{rs} & z_{s1} - z_{rs} \\
x_{s2} - x_{rs} & y_{s2} - y_{rs} & z_{s2} - z_{rs} \\
\vdots & \vdots & \vdots \\
x_{sr} - x_{rs} & y_{sr} - y_{rs} & z_{sr} - z_{rs}
\end{bmatrix}
\]

(4)

\[
L_i = \begin{bmatrix}
\Delta \nabla \phi_{s1} \\
\Delta \nabla \phi_{s2} \\
\vdots \\
\Delta \nabla \phi_{sr}
\end{bmatrix}
\]

Where \( s_r \) is the reference satellite and \( (x_{sr}, y_{sr}, z_{sr}) \) is its coordinates.

Based on least square method, the solution of Eq. 3 can be obtained

\[
X = (A_i^T Q_i^{-1} A_i)^{-1} A_i^T Q_i^{-1} L_i
\]

(5)

Where \( Q_i \) is the covariance matrix of measurements.

3. Single difference residual estimation

Single differenced carrier phase always used to modeling stochastic model. Its equation can be written as

\[
\lambda_i \Delta \phi_i^s = \Delta \rho_i^s + \Delta \delta_i + \lambda_i \Delta N_i^s + \Delta \epsilon_i^s
\]

(6)
Where $\Delta$ represents single difference (SD) operator, $\Delta \delta_i$ is hardware delay. Reparameterizing Eq. 6 we can obtain

$$\Delta L_i^s = \Delta \delta_i + \lambda_i \Delta N_i^s + \Delta \epsilon_i^s$$  \hspace{1cm} (7)

Where $\Delta L = \lambda_i \Delta \phi_i^s - \Delta \rho_i^s + \lambda_i \Delta V N_i^s$ and after the double differenced ambiguities are resolved, there is no unknown parameters in $\Delta L$, $\Delta V N_i^{sr} = \Delta N_i^s - \Delta N_i^r$ is double differenced ambiguities, $\Delta N_i^r$ is the single differenced ambiguities of reference satellite. It is obvious that in Eq.7, $\Delta \delta_i + \lambda_i \Delta N_i^r$ will be a constant if there is no cycle slip. Therefore the residual estimation of satellite $k$ is

$$\Delta \hat{\epsilon}_i^k = \Delta L_i^s - \frac{\sum_{i=1}^{m} \Delta L_i^s}{m}$$  \hspace{1cm} (8)

Where $m$ is the number of satellite observed. Similarly, the residuals of rest satellites can also be calculated. The relationship between SD residual estimation accuracy and SD residual is

$$D(\Delta \hat{\epsilon}_i^k) = D(\Delta L_i^s) - \frac{\sum_{i=1}^{m} D(\Delta L_i^s)}{m^2}$$  \hspace{1cm} (9)

For $D(\Delta L_i^s) = D(\Delta \epsilon_i^s)$, then Eq. 9 can be further written as

$$D(\Delta \hat{\epsilon}_i^k) = \frac{(m-1)^2 + \sum_{s=1, s\neq k}^{m} D(\Delta L_i^s)}{m}$$

$$\approx \frac{(m-1)}{m}$$  \hspace{1cm} (10)

In other words, $D(\Delta \epsilon_i^k) = r \cdot D(\Delta \hat{\epsilon}_i^k)$, $r = (m-1)/m$. Assuming the measure accuracy of the two receivers are same. The the relationship between carrier phase accuracy and SD residual accuracy is $D(\lambda_i \Delta \phi_i^s) = D(\Delta \epsilon_i^s)/2$. Therefore carrier phase accuracy can be obtained by

$$D(\Delta \epsilon_i^k) = \frac{D(\Delta \hat{\epsilon}_i^k)}{2r}$$  \hspace{1cm} (11)

In short period, the standard deviation of $\Delta \hat{\epsilon}_i^k$ can be assumed as a constant. So its deviation can be obtained from multi-epoch data

$$D(\Delta \hat{\epsilon}_i^k) = \frac{\sum_{i=1}^{n} (\Delta \hat{\epsilon}_i^k)^2}{n}$$  \hspace{1cm} (12)
Where \( n \) is the number of satellites. Substituting Eq. 12 to Eq. 11, the final equation to calculate carrier phase accuracy is obtained.

\[
D(\hat{e}_i) = \frac{1}{2nr} \sum_{i=1}^{n} (\Delta \hat{e}_{i,j})^2
\]  

(13)

4. Stochastic modeling

Based on satellite elevation or SNR, two category of stochastic models can be established: Elevation Model and SNR Model. Elevation model this the function of elevation. There are may form of this model but in this paper the exponent model is used. Its equation is given by.

\[
\sigma = a + b \cdot e^{c - elev}
\]

(14)

Where \( \sigma \) is the standard derivation of measurements, \( a \), \( b \) and \( c \) are unknown parameters depend on receiver type and observation type, \( elev \) is the elevation angle of satellite.

The SNR model used in this paper is \( \sigma^-\epsilon \) model, it equation is written as

\[
\sigma^2 = a + b \cdot 10^{-\frac{C/N_0}{10}}
\]

(15)

Where \( \sigma^2 \) is the variance of measurements, \( a \) and \( b \) are unknown parameters depend on receiver type and observation type. The unknown parameters in Eq.14 and Eq.15 need to be calculated before used to obtain observation variance. Having obtained the observation variance, the covariance of double differenced carrier phase \( Q_i \) can be gained. When using different stochastic models, the \( Q_i \) will be different which will impact the solution accuracy.

5. Numerical experiment

The experiment equipment is mounted on the top of a building and the distance between the antennas is about 10m. The experiment lasts about 24 hour. Using the method analyzed in section 2, the residuals of all satellites are estimated. The relationship between carrier phase variance and elevation, SNR are then obtained.

5.1. Carrier phase variance with satellite elevation and SNR

Fig.1 illustrates carrier phase standard derivation and satellite elevation. It is obvious that for the GEO satellites there are no much variations in its accuracy. So it is reasonable to regard there variance as a constant. But for IGSO and MEO satellite there is an obvious proportional relationship between carrier phase accuracy and satellite elevation. In Fig1. (b), the data of IGSO satellite C6 and C9 are given. Their accuracy are same. Fig1.(c) gives the data of IGSO C6 and MEO C14 which indicates that the carrier phase accuracy difference of IGSO and MEO satellite can not be ignored. In general, when modeling BDS stochastic models, we should establish three kind of models respectively corresponding to GEO, IGSO and MEO.
Figure 1. GEO, IGSO and MEO phase accuracy with satellite elevation.
Figure 2 illustrates the carrier phase variance and SNR values. For the variance of GEO satellite can be regard as constants, they are not discussed here. From Fig. 2, it is easy to find out the proportional relationship between observation variance and SNR. Same to elevation case, the accuracy of IGSO satellites are same, but the difference between IGSO and MEO satellites can not be ignored. Therefore, same to elevation case, we also need to establish different kind of stochastic models for IGSO and MEO satellites.

(a) IGSO Satellites C6 and C9

(b) IGSO Satellite C6 和 MEO Satellite C14

Figure 2. Phase accuracy with C/N0

5.2. Stochastic modeling
Based on the observation data, the unknown parameters in Eq.14 and Eq.15 are fitted by least square method. The fitting result of elevation model is illustrated in Fig. 3. The fitting result of SNR model is illustrated in Fig. 4. The values of those parameters are showed in Table 1, 2 and 3.
Figure 3. Elevation stochastic model fitting result
Figure 4. SNR stochastic model fitting result

Table 1. Carrier phase accuracy of GEO satellite

| No. | STD(mm) | Elevation(°) |
|-----|---------|--------------|
| C1  | 0.8128  | 37           |
| C2  | 0.8057  | 43           |
| C3  | 0.6184  | 48           |
| C4  | 1.0610  | 23           |
| C5  | 0.8983  | 24           |

Table 2. Stochastic model parameters of IGSO satellite

| Model | a      | b       | c       |
|-------|--------|---------|---------|
| Elevation | 0.1528 | 2.036   | 0.0299  |
| SNR   | -0.1016| 39484   |         |
Table 3. Stochastic model parameters of MEO satellite

| Model | a     | b     | c     |
|-------|-------|-------|-------|
| Elevation | 0.3642 | 2.214 | 0.05935 |
| SNR    | -0.01767 | 35999 |

5.3. The impact of different stochastic models on azimuth solution

The elevation model and SNR model are respectively used to solve azimuth. The results are shown in Fig.5 where the blue curve is data from equal weight model. The result comparison between elevation model, SNR model and equal weight model are in Table 4. From this table, we can find out that the elevation model and SNR model can both improve azimuth accuracy about 7%, and reduce azimuth range about 11%. Therefore, the impact of those two model are same. This is because satellite elevation and SNR values have an proportional relationship as shown in Fig.6. So there impact will be similar.

![Figure 5. Solved azimuth using three stochastic models](image-url)
Figure 6. The relationship between C/N0 and satellite elevation

Table 4. Comparison of azimuth accuracy calculated using three stochastic models

| Model      | STD (") | Range (") |
|------------|----------|-----------|
| Elevation  | 18.8928  | 132.2640  |
| SNR        | 18.8964  | 131.7960  |
| Equal weight | 20.2788  | 148.0320  |

| Model | STD Improve | Range Reduce |
|-------|--------------|--------------|
| Elevation | 6.83%        | 10.65%       |
| SNR    | 6.82%        | 10.97%       |

6. Conclusion

In this paper, we established the stochastic models respectively for GEO, IGSO and MEO satellites. Besides, the impact of those stochastic models on azimuth solution are investigated. The results showed that the elevation model and SNR model can both improve azimuth accuracy about 7%, and reduce azimuth range about 11%. So in high precision situations, it is needed to establish appropriate stochastic models in order to improve solution precision and stability.

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