Gamma-ray bursts (GRBs) show evidence of different light curves, duration, afterglows, and host galaxies and explode within a wide redshift range. However, their spectral energy distributions (SEDs) appear to be very similar, showing a curved shape. Band et al. proposed a phenomenological description of the integrated spectral shape for the GRB prompt emission, the so-called Band function. In this Letter, we suggest an alternative scenario to explain the curved shape of GRB SEDs: the log-parabolic model. In comparison with the Band spectral shape our model is statistically favored because it fits the GRB spectra with one parameter less than the Band function and is motivated by a theoretical acceleration scenario. The new Fermi observations of GRBs will be crucial for disentangling these two models.

Key words: acceleration of particles – gamma-ray burst: general – radiation mechanisms: non-thermal

Online-only material: color figures

1. INTRODUCTION

Physical mechanisms behind the gamma-ray burst (GRB) prompt emission are still under debate. Band et al. (1993), investigating the BATSE GRBs sample, proposed a phenomenological description of the integrated spectral shape for the GRB prompt emission, the so-called Band function. The introduction of this function was strongly suggested by the observational evidence that the shape of the spectral energy distribution (SED) of the GRB prompt emission is convex and broadly peaked. It is remarkable that there has not been any physical explanation in terms of acceleration processes and non-thermal radiative losses that can lead to the Band spectral shape.

In this Letter, we propose to describe the curved shape of the GRB prompt emission using the log-parabolic function (Massaro et al. 2004), which has been successfully used to describe the SEDs of BL Lac objects over several decades. First, we consider the differences between this model and the Band function investigating the different γ -ray flux predictions in the Fermi Large Area Telescope (LAT) energy range. Second, we point out the physical interpretation of the log-parabolic shape in terms of Fermi acceleration mechanisms. Finally, we present a simple synchrotron emission model to explain the GRB prompt emission that appears to be the most reasonable scenario.

For our numerical results, we use cgs units unless stated otherwise and we assume a flat cosmology with \( H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1} \), \( \Omega_M = 0.27 \), and \( \Omega_\Lambda = 0.73 \) (Spergel et al. 2007).

2. THE SHAPE OF THE SPECTRAL ENERGY DISTRIBUTION

GRBs have a non-thermal spectrum that varies strongly from one burst to another. It is generally found that a simple power law does not fit well their spectra because of a steepening toward high energies. The Band function phenomenological model (Band et al. 1993) describes the prompt time-integrated GRB spectra, composed of two power laws joined smoothly at a break energy \( E_b \),

\[
F(E) = \begin{cases} 
    F_0 \left( \frac{E}{E_b} \right)^\alpha \exp \left( -\frac{E}{E_c} \right) & E \leq E_b \\
    F_1 \left( \frac{E}{E_b} \right)^\beta & E \geq E_b
    \end{cases}
\]

(1)

where \( F(E) \) is the number of photons per unit of area, energy, and time, while \( E_0 \) is a reference energy usually fixed to a value of 100 keV. Under the continuity requirement for the function \( F(E) \) and its first derivative, the break energy and normalization are given by

\[
E_b = (\alpha - \beta)E_c
\]

(2)

\[
F_1 = F_0 \left( \frac{E - \alpha E_c}{E_0} \right)^{\alpha - \beta} e^{(\beta - \alpha)}. \]

(3)

There are no particular theoretical scenarios that predict this spectral shape making it only a phenomenological model. However, it provides good fits to most of the observed spectra in terms of four parameters, namely, the two photon indices \( \alpha \) and \( \beta \), the exponential cutoff \( E_c \), and the normalization constant \( F_0 \), with all four parameters directly estimated during the fitting procedure. The peak energy \( E_p \) of the SED (i.e., \( S(E) = E^2 F(E) \)) is related to the spectral parameters by

\[
E_p = (\alpha + 2)E_c < E_c
\]

(4)

for which, typically, \(-2 < \alpha < -1\). We also note that for typical values \( \alpha \simeq -1.4 \) and \( \beta \simeq -2.4 \) (Band et al. 1993), \( E_b \approx E_c \).

We propose to describe and interpret the shape of the SED in the GRB prompt emission using a model defined by the following equation:

\[
F(E) = F_0 \left( \frac{E}{E_0} \right)^{-a-b \log(E/E_0)}, \]

(5)

where \( F_0 \) is the normalization, \( a \) is the spectral index at energy \( E_0 \), and \( b \) is the parameter which measures the spectral curvature.
This model is known as log-parabolic, a curve with a parabolic shape in a log–log plot (Massaro et al. 2004, 2006). We remark that this spectral distribution is the classical log-normal statistical distribution.

In particular, for this function, it is possible to define an energy-dependent photon index $\Gamma(E)$ given by the log-derivative of Equation (5),

$$\Gamma(E) = a + 2b \log(E/E_0)$$  \hspace{1cm} (6)

which describes the continuous change in the spectral slope.

The peak energy $E_p$ and the height of the SED $S(E)$ calculated at its peak frequency $S_p$ can be evaluated by the following relations:

$$E_p = E_0 10 \frac{a}{2b}$$  \hspace{1cm} (7)

$$S_p = S_0 10^{\frac{(2b-a)}{2b}}$$  \hspace{1cm} (8)

where $S_0$ is $S(E_0)$.

Consequently, the spectral shape can be expressed in terms of $b$, $E_p$, and $S_p$ using the relation

$$S(E) = E^2 F(E) = S_p 10^{-b \log^2(E/E_p)},$$  \hspace{1cm} (9)

where $S_p = E_p^2 F(E_p)$. In this form, the values of the parameters $b$, $E_p$, and $S_p$ are estimated independently in the fitting procedure, whereas those derived from Equations (7) and (8) are affected by intrinsic correlations (Tanihata et al. 2004; Tramacere et al. 2007).

As an example of the goodness of the fitting procedure, we show in Figure 1 the best fits of two GRB SEDs evaluated with the log-parabolic model, namely, GRB 910601 and GRB 920622, of the most bright and well-studied GRBs present in the literature, (data taken from Schaefer et al. 1994 and Tavani 1996). Their best-fit parameters are $E_p = 0.47 \pm 0.01$ MeV, $b = 0.74 \pm 0.03$, and $S_p = (9.99 \pm 0.12) \times 10^{-6}$ erg s$^{-1}$ cm$^{-2}$ for GRB 910601 while $E_p = 0.460 \pm 0.003$ MeV, $b = 0.95 \pm 0.02$, and $S_p = (1.098 \pm 0.005) \times 10^{-5}$ erg s$^{-1}$ cm$^{-2}$ for GRB 920622.

We note that GRB spectra appear to be narrower with respect to the BL Lac objects (Massaro et al. 2008), having the curvature parameters close to 1.
described in terms of this model as already shown in Figure 1. We found that extrapolating the log-parabolic spectrum only two GRBs, out of 20 in total, are expected to be marginally detected in the Fermi LAT band within 100 s, in good agreement with the detection rate of the first-year Fermi observations with respect to the high Fermi LAT detection rate estimated using the Band model extrapolation (Omodei 2009; Band et al. 2009).

In Figure 3, it is evident how the curved shape described by a log-parabolic function is in good agreement with the Fermi LAT first-year GRB detections. As in Figure 2, the Fermi LAT sensitivity has been evaluated from that reported in Atwood et al. (2009) rescaled for 100 s.

4. ACCELERATION MECHANISMS AND SYNCHROTRON RADIATION IN THE PROMPT EMISSION

The energy spectrum of accelerated particles by some statistical mechanism, such as those occurring in shock waves, is usually written as a power law. The origin of this interpretation resides in the first-order Fermi acceleration mechanism (Bell 1978; Blandford & Eichler 1987; Protheroe 2004), originally presented to explain the cosmic-ray spectrum. However, the observational evidence that the SED of BL Lacs objects has a curved shape has demanded a different interpretation. In particular, Landau et al. (1986) provided a useful description of the synchrotron component of BL Lac objects in terms of a log-parabolic model that has been recently applied to describe the synchrotron X-ray spectra of TeV BL Lacs (Massaro et al. 2008).

The theoretical interpretation of the log-parabolic model resides in the energy distribution of the emitting particles. The general solution of the kinetic equation, for the time-dependent distributions with respect to the energy, yields curved particle energy distributions (PEDs) in the form of log-normal function, where terms taking into account the stochastic and systematic acceleration by the Fermi mechanisms are considered (Kardashev 1962).

Assuming a simple $\delta$ function as an initial condition for the PED, the analytical solution of the particle kinetic equation yields the log-parabolic shape in the form

$$N(\gamma) = N_0 \left( \frac{\gamma}{\gamma_0} \right)^{-s-r \log(\gamma/\gamma_0)},$$

(10)

where the parameters $s$, $r$ and the normalization $N_0$ are directly linked to the physical parameters $\lambda_1$ and $\lambda_2$, while $\gamma_0$ is a reference energy. We note that the PED curvature $r$ is only directly linked to the diffusion coefficient; this means that the curved shape of the particle distribution depends on considering the second-order Fermi acceleration mechanisms (Kardashev 1962; Paggi et al. 2009). The case $r = 0$ corresponds to the simple power-law electron distribution as expected by a first-order acceleration mechanism. The mean quadratic energy of the PED, $(\langle \gamma^2 \rangle)$, corresponds to the second normalized momentum and can be expressed as

$$\langle \gamma^2 \rangle = \gamma_0 10^{(2-s-r)/2r} = \gamma_p$$

(11)

which corresponds to the peak of $\gamma^2 N(\gamma)$. By freezing the PED slope, $s$, to the value of 2, as expected in the first-order Fermi acceleration mechanism, it is possible to describe its shape in terms of the PED curvature $r$, $\gamma_p$ and the normalization $N_0$, and Equation (8) can be written as

$$N(\gamma) = N_0 \left( \frac{\gamma}{\gamma_p} \right)^{-2-r \log(\gamma/\gamma_p)}.$$  

(12)

Finally, we remark that it has been recently shown that including synchrotron and inverse Compton radiative losses as well as the “disappearance” of fast particles that escape from the acceleration region, either as a result of nuclear collisions or escape from the acceleration region, the numerical solution of the kinetic equation can be successfully described in terms of a log-parabolic shape (Tramacere et al. 2009; Paggi et al. 2009).

Under the assumption that the synchrotron radiation is emitted by a log-parabolic PED (Equation (10)), the resulting flux density and consequently the SED can be well approximated by a log-parabolic shape, expressed as Equation (5).

An alternative scenario is based on the assumption that the probability of accelerating particles depends on energy in a simple relation given by $P \propto \gamma^{-3}$, and in this case the resulting PED yields toward a log-parabolic shape (Massaro et al. 2004, 2006).

Applying the numerical code developed by Massaro (2007) and presented in Paggi et al. (2009), we calculated the synchrotron emission by a log-parabolic PED to describe the observed SED of GRB 910601. This is a clear example of how the log-parabolic scenario successfully describes the shape of the

Table 1

| Parameter | Symbol | Units | Value |
|-----------|--------|-------|-------|
| Redshift | $z$    |       | 1.0   |
| PED slope| $s$    |       | 2     |
| PED curvature | $r$ |          | 7.58  |
| PED energy peak | $\gamma_p$ | | $1.97 \times 10^4$ |
| PED minimum energy | $\gamma_{min}$ | | $10^3$ |
| PED maximum energy | $\gamma_{max}$ | | $5 \times 10^5$ |
| Electron density | $n_{el}$ | cm$^{-3}$ | $1.13 \times 10^4$ |
| Beaming factor | $\delta$ | | 30 |
| Magnetic field | $B$ | G | $10^4$ |
| Volume | $V$ | cm$^3$ | $10^{12}$ |

Figure 3. Log-parabolic models of the 20 brightest GRBs in the BATSE catalog detected during 1991 (Shaaf et al. 1994). It is clear how the extrapolation of the log-parabolic function led to conclude that only few GRBs are expected to have a detection in the Fermi LAT energy range in contrast to the expectations of the Band model.

(A color version of this figure is available in the online journal.)
GRB prompt emission. The parameters assumed for our calculations are given in Table 1, and in Figure 4 we show the GRB 910601 SED with the model adopted. We fixed the redshift of this GRB to 1 because it is unknown. The parameters in Table 1 are all consistent with plausible values of the GRB emitting region (e.g., Mészáros 2002; Zhang & Mészáros 2002). The synchrotron model, evaluated with a log-parabolic PED, is in agreement with the data, appearing to be a good description of the GRB SEDs of the prompt emission.

5. CONCLUSIONS

In this Letter, we propose to interpret the SEDs of the GRB prompt emission using the log-parabolic shape. In comparison with the Band function (Band et al. 1993), the log-parabolic shape is favored for two main reasons.

First, it is statistically better, because it requires only three parameters, namely, the curvature $b$, the peak energy $E_p$, and the height of the SED evaluated at the peak energy $S_p$ (see Equations (5) and (9)), while the usual Band model needs four spectral parameters, namely, $\alpha$, $\beta$, $E_c$, and $F_0$ or five if another high-energy exponential cutoff is introduced. Second, the proposed function has a strong physical motivation. This shape is directly related to the solution of the kinetic equation for the particles accelerated by Fermi mechanisms when the random acceleration is also taken into account with all the other terms (e.g., first-order Fermi mechanisms; Kardashev 1962).

In the recent Fermi LAT observations, only few GRBs have been detected in contrast to the predictions of the Band function. The high-energy curvature of the log-parabolic shape has a natural explanation for the Fermi observations without introduction of any new parameter, as the exponential cutoff in the Band function.

As shown in Massaro et al. (2006) or more recently in Tramacere et al. (2009) and Paggi et al. (2009), the synchrotron emission of a log-parabolic electron distribution yields a curved SED near its peak, well described in terms of the same spectral shape. In this Letter, we also show how the synchrotron scenario using a log-parabolic PED can describe the spectrum of the GRB prompt emission, successfully.

Finally, we remark that a different scenario, including other synchrotron or inverse Compton components and their spectral evolution with time, can make the spectral shape of the GRB prompt emission more complex in the Fermi LAT energy range as, for example, recently observed in the case of GRB 090902B (Abdo et al. 2009).

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