NEARNESS THROUGH AN EXTRA DIMENSION

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It is shown that if our visible universe is a thin trapped shell in a five-dimensional universe, all matter in it may be connected almost instantaneously through the fifth dimension. What appears to be action at a distance is then understood as undetectable ultrafast communication.

1. The universe as a shell

Several authors in the physics literature speculated about the possibility that our universe is a thin shell in a large dimensional hyper-universe.

Wesson [1], tried to explain the very existence of mass and charge of elementary particles as originating from the properties of a universe with extra dimensions. He found that mass could be related to the fifth coordinate and charge to the momentum along it.

Visser [2] before him showed that the trapping of our universe can be implemented mathematically by using a large cosmological constant. Squires [3] later implemented the trapping by using a scalar field that becomes effectively confined to one dimension less than the original space, again, by using a cosmological constant. The trapped universe is essentially flat.

More formal arguments for multidimensional spaces were given by Beciu [4].

These proposals are an alternative to the usual claim that extra dimensions are curled up to an unobservable size, an assertion that is not devoid of problems.[5]

In the alternative approach of refs.[1-4], our universe becomes a thin shell in a larger hyper-universe.

The trapping has to be gravitationally repulsive in nature. A cosmological constant does indeed produce the effect, however this is not the only possibility. If the
inner-outer space to the shell is filled with negative energy, then it can trap such a shell effectively. In the present work we will show that, regardless of the mechanism of trapping, its very existence implies that all matter in the universe may be connected through the fifth dimension by means of electromagnetic (or other) signals in an undetectable amount of time.

All matter and energy in the universe is then tied up together in a manner that usual locality and causality on the shell would prohibit. This may in turn explain not only phenomena related to quantum mechanical behavior, but also macroscopic action at a distance.

A grotesque way to view the universe would be comparing the matter in it to puppets that are connected by cables to a puppeteer that holds all the strings together. A perturbation in the hands of the puppeteer causes a turmoil in all the other strings in unison.

In the next section a simple model will investigate these assertions and briefly comment about its implications.

The model addresses what appears to be action at a distance in terms of ultrafast communication. Action at a distance is, in our opinion, a mirage, the real situation is an incessant bombardment by ultrafast radiation that fills the universe’s shell.

2. Nearness through the fifth dimension

Consider a five-dimensional metric space with line element

\[ ds^2 = F(R, t) \, dt^2 - G(R, t) \, dR^2 - c(R, t) \, d\Omega_3^2 \]  \hspace{1cm} (1)

where \( F \) and \( G \) are for simplicity taken as depending only on the fifth coordinate and the cosmic time \( t \) and, \( d\Omega_3 \) is the volume element of the three-dimensional subspace on the surface.

The above ansatz corresponds to a hyperspherically symmetric solution of five dimensional Einstein gravity, provided Einstein’s gravity still works in the hyper-universe, as one might hope. We still regard space-time inside the shell as a Riemannian manifold.

Define the center of the hypersurface by \( R_0 \), such that \( F(R_0, t) = 1, \ c(R_0, t) = S(t) \), with \( S \) the scale parameter of our expanding universe. This coordinate choice defines what is understood by the cosmic time on the shell, at \( x = 0 \).

This is similar to the De-Sitter picture for which the intrinsic radius of the universe is merely the radius of curvature of the hypersurface embedded in the larger flat universe. The differences arise only from the fact that we do not constrain the shell to be of zero thickness as for the De-Sitter case, or the cases investigated in the past.[1,4] The shell has a finite -albeit minuscule- width.
We now simplify the problem a bit more by taking all the metric functions to be time independent, namely we choose $t = t_0$, the present epoch. We do so because the scale function of conventional Friedmann-Robertson-Walker expansion is the inverse Hubble constant of the order of the radius of the universe, extremely large as compared to the supposed thickness of the shell. Time evolution of the shell is not important for the time scales that will be involved in the present investigation.

We have chosen the signature of the extra dimension as space-like inspired in previous results.[1,2] However, as we will show later, the signature may change at the inner-outer boundaries of the shell.

Regardless of the trapping mechanism (cosmological constant, negative energy) the functions $F$ and $G$ may be expanded around the equilibrium (or quasiequilibrium, due to the time dependence) radius $R_0$. At fixed time the expansion has to be a constant and a quadratic term in the distance $x = R - R_0$. The linear term has to vanish due to the equilibrium condition, otherwise, all the mass in the universe would suffer a constant force that will eventually drag it to a new equilibrium position and we can always call this new position $R_0$.

In any event, we will assume, that the extension of the shell into the fifth dimension is so tiny as compared to any reasonable scale length -as it should be if its effect remains undetected in the dynamics of bodies-, that our approximation will be valid. As mentioned above, one way to actually obtain the expansion is to consider a positive mass shell in a cosmological constant filled universe as in ref.[4].

Moreover, as in any outer (to the inner-outer universe) Schwarzschild-like solution, we will take $F = G^{-1}$.[4]

Hence we have

$$F(R, t)|_{R_0} = A + B x^2$$

where $A$ and $B$ are constants.

In order to fulfill the constraint of a flat space-time on the shell at our present time, we redefine our coordinates to have $A = 1$[6].

We will further make the assumption that the trapping is due to the repulsion from the inner and outer regions, hence $B$ has to be positive, for the particles to be repelled into the shell. (Recall the relation between gravitational potential and metric in the Newtonian approximation).

Collapse is prevented by the pressure of the mass in the shell, although space where mass is sparse may be penetrated by the matter of the inner and outer regions, producing important effects. One such effect would be similar to that provided by the hypothetical dark matter.

Instead of having more matter inside the galaxies that is unseen, there could be unseen negative mass outside them. Also, if the shell is cracked suddenly and
mass from the inner and outer regions coalesces with the mass in the shell they
could annihilate producing enormous bursts of energy, an alternative to black hole
generated active galactic nuclei.

Hence

\[ F = G^{-1} = 1 + k x^2 \]  

with \( k > 0 \).

In order to find out the functional dependence of \( c(x = R - R_0) \), we insert the
metric of eq.(1), with the ansatz of eq.(3) for the metric functions \( F \), and \( G \) in the
five dimensional Einstein tensor.

It is found that the function \( c(x) \approx e^{-\beta x^2/2} \) yields, for small \( x \)

\[ G_{00} = -\frac{3}{2} \beta \]  

All the other components of the Einstein tensor vanish provided that \( \beta = k \).
This is not the most general solution, but, it will suffice for the present purposes.

Inserting eq.(4) in the Einstein equations with a cosmological term and an ideal
fluid inside the shell, we find that the energy momentum tensor in the comoving
frame of the shell can be chosen to have vanishing energy density and a pressure \( p \approx \)
\( \frac{3}{16\pi G} \) provided by the cosmological constant, whith \( G \) the gravitational constant.
This solution is only one amongst many other choices. Alternatively we could have
chosen a solution with no cosmological constant, no pressure, and an enormous
energy density in the shell, or any solution in between. Due to the fact that our
universe seems to be essentially vacuous, we favor the first solution.

If we choose the shell thickness to be microscopical in size, the value of the
cosmological constant will be enormous \( \lambda = 3/4 \ k \).

The cosmological constant prevents the collapse of the shell against the gravita-
tional squeeze produced by the harmonic potential. The effect of the constant is to
maintain the shell in equilibrium against the repulsion of the inner-outer regions.
The need for a large cosmological constant is consistent with previous works[2],

The constant \( k \) should be large enough, for the thickness of the shell to be
smaller than the radii of nuclei, smaller than the deep inelastic scattering scale of
the experiments carried until now, otherwise, its imprint should have become visible
by now. *

From eqs.(1,3), it appears that, space-time will change signature at the bound-
daries of the shell. The inner-outer universe is then shielded from the shell by a
horizon, at least classically. Quantum effects, like Hawking radiation might still be
able to cross the boundary, as for the case of a black hole.

*It would be intriguing to find out about the influence of the thickness of the shell on electromag-
netic processes in a Kaluza-Klein type of approach, without curled up extra dimensions
Let us now consider signal propagation along a path that goes in the $R$ direction, then across it and back.

Our measure of time on the shell is the cosmic time $t$, and we refer every process to it. Suppose a signal whose speed is the velocity of light $c = 1$ in our units, starts traveling from $x = 0$, the center of the shell to some fixed $x$ inside it, then travels a distance $L$ perpendicular to $R$ at fixed $x$ and returns from $x$ to $x = 0$ back to a point at a coordinate distance $L$ from the initial point. For such a scenario to occur, radiation has to propagate in a direction that is not completely perpendicular to the hypersurface, scatter inside it, or be reflected at some hypothetical edge. The tangential component of the velocity, can provide the motion along the surface. Reflection and scattering can modify the frequency of the radiation, but, it seems unlikely the time lapse of the trip will be affected by these processes. It is then sufficient to consider the path chosen.

In principle, zero mass radiation can reach any value of $x$, even large values as compared to $\sqrt{1/k}$. The farther out radiation travels, the larger the effect.

Using three dimensional spherical coordinates, $L$ along the radial distance in three-dimensional space, and for fixed angles, we find

$$ds^2 = (1 + kx^2)dt^2 - \frac{1}{(1 + k x^2)} dR^2 - e^{-k x^2/2} \, dl^2$$

(5)

The definition of $dl$ is such that at $x = 0$ space-time is essentially Minkowski, as observed.

Further, for radiation we still have $ds^2 = 0$.[6]

Our choice of $k$ will be of the order of $R_{GUT}^2$, where $R_{GUT} \approx 10^{-31}m$ stands for the grand unified theories scale. We do this because we want to encompass democratically all the known interactions, and, in order to avoid quantum gravity effects that will enter at much smaller scales of the order of $R_{Planck} \approx 10^{-35}m$.

However, any microscopic scale will be a viable choice. We take this value for the sake of exemplification.

Hence

$$t = 2 \int_0^x \frac{dx}{1 + k x^2} + \int_0^L dl \frac{e^{-k x^2/4}}{\sqrt{1 + kx^2}} \approx \frac{L e^{-k x^2/4}}{\sqrt{1 + kx^2}}$$

(6)

Where we have used $x = R - R_0$ and neglected the first integral because it is of the order of $t \approx 10^{-39}sec$.

If the signal climbs up the harmonic potential and back far enough in $x$, the coordinate time becomes negligible.

With $x = 15 \, R_{GUT} \approx 1.5 \, 10^{-30}m$, several times the radius of curvature of the shell, and $L = 100 Mpc$, the time taken by radiation to traverse this cosmic distance is $t = 2.5 \, 10^{-10}sec$. A ridiculously small time as compared to the 326 Million years
needed for the light to traverse this distance along the direction perpendicular to \( R \) on the surface. Recall that on the surface, \( x = 0 \).

Due to the crudeness of the approximations used in order to derive the above result, we should not attach too much rigor to the actual numbers. The effect is, nevertheless, evident.

This amazing mechanism might be at work for all the processes we recognize as action at a distance. Gravity waves are not an exception. However, we are still far from explaining in mathematical detail how will this actually generate static instantaneous-like interactions between far away bodies, that could not be communicated causally through the shell. It is, nevertheless a line of thought that has not been explored before and deserves further attention.

The scheme can fail if, either radiation is not able to climb up the potential very far uphill, because the harmonic is an extremely crude approximation and other terms may come into play creating perhaps a horizon, or, if it is absorbed in the inner and outer spaces. Both options seem unlikely due to the repulsive nature of the exterior-interior spaces.

The signal becomes red shifted and blue shifted enormously, but will arrive at its destination with the same frequency as emitted.

All the radiation that is emitted in the \( R \) direction is then detected in no-time by all the other particles in the universe at random. This radiation will oscillate back and forth in the \( R \) direction and most certainly fill the shell completely. This wandering radiation is a poor man’s model of what one would call an action at a distance. The fact that we do not violate causality and locality is because both are distorted enormously by the potential of the shell. This in turn might have some bearing to the nonlocality witnessed in quantum mechanics. Instead of having alternative hidden-variable theories we could think about hidden-dimension theories.

In summary, if our universe is a thin shell immersed in a higher dimensional hyperuniverse, and if the shell is trapped by some repulsive force or analogous mechanism, then what seems to be action at a distance, such as the action of static potentials could be due to action through the fifth dimension.

References

1. P. S. Wesson and H. Liu, Int. Jour. of Theor. Phys. 36, 1865 (1997) and references therein.

2. M. Visser Phys. Lett. B159, 22 (1985).

3. E. J. Squires, Phys. Lett. B167, 286 (1985).

4. M. I. Beciu, Europhys. Lett. 12, 229 (1990).
5. M. J. Duff, *Supersymmetry, Supergravity and related topics*, World Scientific, Singapore, 1984.

6. S. Weinberg, *Gravitation and Cosmology* J. Wiley and Sons, 1972.