Chiral transition in dense, magnetized matter

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Abstract.
In the presence of a chemical potential, the effect of a magnetic field on chiral symmetry breaking goes beyond the well-known magnetic catalysis. Due to a subtle interplay with the chemical potential, the magnetic field may work not only in favor but also against the chirally broken phase. At sufficiently large coupling, the magnetic field favors the broken phase only for field strengths beyond any conceivable value in nature. Therefore, in the interior of magnetars, a possible transition from chirally broken hadronic matter to chirally symmetric quark matter might occur at smaller densities than previously thought.

Keywords: Chiral symmetry breaking, inverse magnetic catalysis, Sakai-Sugimoto model

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INTRODUCTION

Spontaneous breaking of (approximate) chiral symmetry is a fundamental, non-perturbative aspect of quantum chromodynamics (QCD). Within lattice gauge theory, the chiral condensate can be computed, and it has been shown that QCD exhibits a smooth transition (“crossover”) from the chirally broken to the chirally restored phase [1]. This crossover occurs at \( T \approx 150\text{MeV} \). Little is known from first principles about the transition at nonzero chemical potentials \( \mu \) and/or magnetic fields \( B \), the exception being asymptotically large values of \( \mu \), where chiral symmetry is broken by the formation of a diquark condensate in the color-flavor locked (CFL) phase [2, 3]. At \( \mu = 0 \), lattice studies of the chiral phase transition in a magnetic field have just begun recently, and first results indicate the decrease of the crossover temperature with increasing \( B \) [4, 5]. From phenomenological models, for instance the Nambu-Jona-Lasinio (NJL) model, one gets indications that can be summarized as follows,

- \( \mu = 0, B \neq 0 \). “Magnetic catalysis” [6, 7]: the magnetic field enhances the chiral condensate, the critical temperature is expected to increase with \( B \). (Note that this expectation is not borne out by the recent lattice results. For possible explanations of this discrepancy see Refs. [8, 9].)
- \( B = 0, \mu \neq 0 \). Chiral symmetry is expected to be restored at sufficiently large \( \mu \), most likely in a first-order phase transition. (If the hadronic phase is directly superseded by CFL, a crossover is conceivable at small temperatures [10, 11].)

How about the situation where both \( \mu \) and \( B \) are nonvanishing? Naively extrapolating the above two indications suggests a critical chemical potential that increases with
the magnetic field. Indeed, this is found for weak coupling in the NJL model. In the following, we discuss that at strong coupling this expectation is incorrect because of a nontrivial interplay between the magnetic field and the chemical potential. The resulting effect is called “inverse magnetic catalysis” [12].

MAGNETIC CATALYSIS

Magnetic catalysis can be explained with the help of an NJL model for one flavor of massless fermions,

\[
\mathcal{L} = \bar{\psi} (i\gamma^\mu \partial_\mu + \mu \gamma_0) \psi + G \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} \gamma_5 \psi)^2 \right],
\]

where \( G \) is a coupling constant with mass dimensions -2. Within the mean-field approximation, one can show that a dynamical fermion mass \( M \equiv -\frac{2}{G} \langle \bar{\psi} \psi \rangle \) is generated only for sufficiently large coupling strengths \( g > 1 \), where \( g \equiv G \Lambda^2/(2\pi) \) with an ultraviolet momentum cut-off \( \Lambda \). (The precise numerical value of the critical dimensionless coupling, here \( g = 1 \), depends on the regularization scheme; the results we are referring to here [13] are done within the proper time regularization where \( 1/\Lambda^2 \) is the lower bound of the proper time integral.)

If the same analysis is done in the presence of a background magnetic field \( B \), the conclusion is changed qualitatively. Now, a dynamical mass is generated for all (i.e., arbitrarily small) coupling strengths. The weak-coupling result (\( g \ll 1 \)) is

\[
M \simeq \sqrt{\frac{B}{\pi}} e^{-\frac{\pi^2}{2BG}}.
\]

(For simplicity, we have set the electric charge of the fermions to 1.) The physics behind this effect can be nicely understood in analogy to BCS theory: for an arbitrarily weak attractive interaction and sufficiently small temperatures, any Fermi surface is unstable with respect to the formation of a Cooper pair condensate. More technically speaking, an infrared divergence is induced by the Fermi surface because it renders the system effectively 1+1 dimensional. Here, the magnetic field plays the role of the Fermi surface in some sense because, due to the physics of the lowest Landau level (LLL), the system also becomes effectively 1+1 dimensional. As a consequence, an instability analogous to the Cooper instability arises, and a fermion–anti-fermion condensate is formed. The exponential form of the dynamical mass (2) is thus analogous to the weak-coupling gap in a BCS superconductor.

Magnetic catalysis manifests itself also in the critical temperature \( T_c \) for chiral symmetry breaking: within the NJL model, one finds that \( T_c \) increases monotonically with increasing magnetic field. The result for large \( B \) suggests that \( T_c \) can become arbitrarily large (although the NJL model does not allow for reliable predictions for magnetic fields beyond the cutoff scale).

Before we come to nonzero chemical potentials, let us, in addition to the NJL model, also introduce the Sakai-Sugimoto model [14], which is a specific example of the gauge/gravity duality. The Sakai-Sugimoto model is based on type-IIA string theory, and
is currently the holographic model whose dual comes closest to QCD. This is achieved by breaking supersymmetry through the compactification of an extra dimension on a circle, which gives mass to unwanted adjoint scalars and fermions of the order of the Kaluza-Klein mass $M_{\text{KK}}$ (inversely proportional to the radius of the compactification circle). Moreover, it realizes spontaneous symmetry breaking of the full chiral group $SU(N_f)_R \times SU(N_f)_L$ in a geometrical way with the help of $N_f$ D8 and $\overline{D8}$ branes, embedded in the background of $N_c$ D4 branes. In its original version, chiral symmetry breaking is rigidly coupled to the deconfinement phase transition, i.e., chiral symmetry is restored if and only if the deconfined phase is preferred. The chirally restored, deconfined phase occurs above a certain critical temperature which is independent of the values of $\mu$ and $B$. The reason for this trivial phase structure is the large-$N_c$ limit, to which the usually employed probe brane approximation is necessarily restricted. However, besides $M_{\text{KK}}$ there is a second parameter $L$ in the model (the asymptotic separation of the flavor D8 and $\overline{D8}$ branes in the compactified direction) which can be varied to obtain a less rigid behavior. The reason is that for small $L$ the gluon dynamics become less important. Effectively, confinement is switched off, and thus for $L \ll M_{\text{KK}}^{-1}$ the model can be expected to be dual to an NJL-like field theory. Although its physical content is less transparent than the one of the NJL model, it is a very useful tool since it yields a top-down, microscopic description of strongly coupled physics.

Within the Sakai-Sugimoto model, the usual magnetic catalysis is confirmed: the critical temperature as well as the constituent quark mass increase monotonically with $B$ [15]. In contrast to the NJL model, $T_c$ saturates at a finite value for asymptotically large $B$, i.e., in some sense magnetic catalysis becomes weaker for large $B$.

**Inverse Magnetic Catalysis**

To discuss the chiral phase transition in the presence of nonzero $B$ and $\mu$, we consider the free energy difference $\Delta \Omega$ between the chirally restored and chirally broken phases, such that, by convention, the ground state is chirally restored for $\Delta \Omega > 0$ and chirally broken for $\Delta \Omega < 0$. First, we are interested in the weak-coupling limit. This is easily done in the NJL model. In the Sakai-Sugimoto model, we are restricted to the strongly coupled limit. Interestingly, however, we recover the weak-coupling NJL result when we take the limit of large magnetic fields in the holographic calculation. Hence we can write for both models,

$$\Delta \Omega = \frac{B}{4\pi^2}[\mu^2 - \alpha M(B)^2],$$

where $\alpha = \frac{1}{2}$ (weak-coupling NJL) and $\alpha = \sqrt{\pi \Gamma\left(\frac{3}{2}\right) / \left[3\Gamma\left(\frac{1}{10}\right)\right]} \simeq 0.09$ (large-$B$ Sakai-Sugimoto). This expression for $\Delta \Omega$ is in complete accordance with magnetic catalysis: if we start from the chirally broken phase, i.e., $\Delta \Omega < 0$, we can never restore chiral

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2 For a better comparison, here we have omitted the effect of the topologically induced baryon density (“chiral spiral”). This effect is included in the Sakai-Sugimoto phase diagrams of Fig. 2 and does, within this particular model, not change the chiral phase transition qualitatively.
symmetry by increasing $B$ at fixed $\mu$ because the constituent quark mass $M(B)$ increases with $B$.

For supercritical couplings $g > 1$ in the NJL model the situation is different. In this regime, the small-$B$ limit of the NJL model is very similar to the small-$B$ limit of the Sakai-Sugimoto model (where the coupling is always large). For simplicity, we only consider the LLL. This is also possible in the Sakai-Sugimoto model, where a structure reminiscent of the LLL has been observed [12, 16]. The LLL approximation is of course inconsistent with the limit of very small magnetic fields. Nevertheless, we find that there is an intermediate regime where this approximation reproduces the full numerical result, and it captures the essential interplay between $\mu$ and $B$. We find

$$\Delta \Omega = -(a_0 + a_1 B^2) + \frac{B \mu^2}{4\pi^2},$$

with the positive constants $a_0 \propto M_0^2 \Lambda^2$ (NJL), $a_0 \propto M_0^{7/2} M_{KK}^{1/2}$ (Sakai-Sugimoto), where $M_0$ is the constituent quark mass in the absence of a magnetic field. Also $a_1$ is positive, for the explicit expressions see Ref. [13]. Now, the chiral phase transition does not behave as naively expected from magnetic catalysis. Even though the chiral condensate still increases with $B$, $\Delta \Omega$ can switch its sign upon increasing $B$. The reason is that the condensation energy only increases quadratically with $B$, while the positive term in the free energy difference is linear in $B$. As a consequence, a magnetic field can now restore chiral symmetry at a given chemical potential. This effect is called inverse magnetic catalysis and is confirmed by the full numerical results which are shown in Figs. 1 (NJL) and 2 (Sakai-Sugimoto), including all Landau levels. In the latter figure, we also show the chiral phase transition in the presence of homogeneous baryonic matter [17].

Inverse magnetic catalysis is thus a manifestation of a somewhat ambivalent role of the magnetic field. It enhances the condensation energy, but also leads to a larger free energy cost which, in the presence of a chemical potential, has to be paid for fermion–anti-fermion pairing. It is instructive to compare this situation with Cooper pairing in the presence of a mismatch of the Fermi momenta of the constituent fermions. In this case, one can imagine the condensation process by first filling the two fermion species up to a common Fermi momentum, and then forming the pairs [18]. The first step costs free energy depending on the mismatch, the second yields a gain, depending on the condensation energy (usually given by the coupling strength). Our situation is analogous, as Eqs. (3) and (4) illustrate: in the first step we need to overcome the mismatch of
fermions and antifermions, in the second we form the chiral condensate. Now, crucially, the change in free energy of both steps depend on $B$. Unless this dependence is the same, as in Eq. (3), inverse magnetic catalysis becomes possible.

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