A New Test for Dark Energy Models

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One of the greatest challenges in cosmology today is to determine the nature of dark energy (DE), the source of the observed present acceleration of the Universe. Besides the vacuum energy, various DE models have been suggested. The tests that have been proposed to differentiate among these models are based on observations of galaxies at high redshift (z > 0), to be obtained in the future. We suggest here a new test that is valid at z ≃ 0. It is based on existing observational data, numerical simulations, and three well known analytic models that evaluate the bias parameter b, the ratio of galaxy to dark matter (DM) fluctuations. These analytic models are based on the physical processes involved in the formation of stars and in the formation and merging of galaxies. The value of b(z) obtained in each model is a function of the DM growth factor D(z), which, in turn, is a function of the DE. We show that the equations for b in all three analytic models can be reduced to the form of a known constant plus the term E[D(z = 0)/D(z)]^α, where α = 1 or 2 and E is a free parameter. Using the value of b obtained by the 2dFGRS consortium for the ΛCDM model, to normalize E, we find that all three analytic models predict b(0)^2 = 1 ± 0.1 for all DE models. Since we use the result that b(0)^2 ≃ 1 from the 2dFGRS consortium for the ΛCDM model, the L* galaxy used in our text is the same as that obtained by the consortium for a broad range of galaxy types, using the Schechter function fit to the overall luminosity function of their ~ 220,000 galaxies. Numerical simulations that evaluated b^2(0) for the ΛCDM and CDM (Λ = 0) models also obtained b^2(0) = 1 ± 0.1. Since this value of b^2(0) is indicated by numerical simulations as well as by all three popular analytic models, which are normalized by the 2dFGRS consortium result for the ΛCDM model, we suggest the condition that b^2(0) = 1 ± 0.1 at z = 0 as a new test for the viability of DE models. Thus, for a given observed galaxy fluctuation spectrum such as that of the 2dFGRS consortium, if the DM fluctuations are greater or less than the galaxy fluctuations by more than 10%, the DE model can be discarded. As examples of this test, we show that three popular DE models do not satisfy this test: the vacuum metamorphosis model deviates from b^2(0) = 1.0 at z = 0 by 20%, the brane-world model by 26% and the supergravity (SUGRA) model by 38%.

Keywords: Observational cosmology; Dark energy; Spatial distribution of galaxies.
1. Introduction

The nature of dark energy (DE), the existence of which was first indicated to explain the recent SNIa observation of the acceleration of the Universe, is one of the major problems in cosmology.\(^1\) Theories in which gravity is modified as well as those that include parametrizations of the DE equation of state (EOS), \(w(z) = p/\rho\), where \(p(\rho)\) is the pressure (energy density) of the DE, have been suggested to explain it.\(^2\) Based on observations, various constraints have been put on the EOS for a variety of models (see e.g., Refs.\(^7\)–\(^13\)). In order to investigate DE models, we used the bias parameter \(b^2\), the ratio of galaxy clustering \(\xi_{gg}\) to dark matter (DM) clustering \(\xi_{dm}\).

Observed galaxy clustering \(\xi_{gg}\) [or its power spectrum \(P(k)\)] does not directly provide information about the DE. The DE affects the DM clustering \(\xi_{dm}\) which, in turn, affect \(\xi_{gg}\). In order to obtain information about DE, we need to know the ratio \(b^2 = \xi_{gg}/\xi_{dm}\). In the future, observations will be used to determine \(\xi_{dm}\). Thus, along with observations of \(\xi_{gg}\), we will be able to obtain \(b^2\). One very important observational project to determine \(\xi_{dm}\) is the DES (the Dark Energy Survey) (http://www.darkenergysurvey.org). DES will study the growth of DM fluctuations as a function of redshift with “weak” gravitational lensing, produced by the DM fluctuations. The results of DES will take some time, however, to become available.

In the present paper, we use observations to normalize existing analytic models to predict \(b^2(0) \simeq 1\) at \(z = 0\), which we propose as a new test for DE models. Recently, Grande et al. used it to limit the validity of two DE models.\(^{14}\)

Somerville et al. define the general bias parameter as \(b(\delta)\delta \equiv < \delta_g|\delta >= \int d\delta_g P(\delta_g|\delta) \delta_g\) for a DM fluctuation \(\delta\). The bias parameter \(b(\delta)\) is the average of the probability \(P(\delta_g|\delta)\) that there is a galaxy fluctuation \(\delta_g\) within the matter fluctuation \(\delta\). This relation fully characterizes the mean non-linear biasing and reduces to the linear biasing relation, \(b(\lambda) = \delta_g(\lambda)\), if \(b\) is independent of \(\delta\), where \(\delta_g(\lambda)\) is the galaxy fluctuation in a sphere of radius \(\lambda\). In order to track the formation of galaxies and quasars in their simulations to evaluate \(b\), Somerville et al. used a semi-analytic model to follow gas, star, and supermassive black hole processes within the merger trees of DM halos and substructures. This semi-analytic model is described in Refs.\(^{16}\)–\(^{18}\). The modelling assumptions and parameters were adjusted in order to fit the observed properties of low redshift galaxies, primarily their joint luminosity-color distribution and their distributions of morphology, gas content, and central black hole mass.

The observed galaxy power spectrum from the final 2dFGRS catalogue can be found in Ref.\(^{19}\) (Cole et al.). According to Cole et al., the large-scale linear bias factor for \(L_\star\) galaxies, \(k \leq 0.1 h \text{Mpc}^{-1}\) is \(b = 1.03\) for the \(\Lambda\)CDM model. This is consistent with their previous result for brighter \(L_S\) galaxies, \(b(L_S, z = 0) = 1.10 \pm 0.08\), obtained from APM-selected massive galaxies \((L_S = 1.9 L_\star)\).\(^{20}\)

Bias depends on the luminosities of the galaxies studied and is known as “luminosity segregation”. In order to normalize the bias to \(L_\star\) galaxies, the 2dFGRS
consortium used the expression \( b(L) = 0.85 + 0.15(L/L_\star) \) to take into account luminosity segregation. Since \( L_\star \) galaxies are almost unbiased, the 2dFGRS consortium result indicates that \( b^2(z \approx 0) \approx 1.0 \) to a 10% accuracy for the \( \Lambda \)CDM model. We use this important result in our new test for DE models.

The comparison between the DM fluctuations at the CMB epoch and those at the present, involves two steps:

(i) the calculation of the DM perturbation amplitude at the CMB last scattering epoch \( \xi_{dmls} \) from the observed \( C_l \) of the CMB and the cosmological parameters; and

(ii) the calculation of the DM perturbation amplitude at the present epoch \( \xi_{dm0} \) from the growth rate and \( \xi_{dmls} \).

We assume negligible DE at the recombination era in the present paper, making the perturbation amplitude at recombination independent of DE models. This is a reasonable assumption since the flatness of the Universe, which we assume for all our DE models, primarily determines the sound horizon for the CMB data and the mapping between the matter power spectrum \( k \) and the CMB \( l' \)s [step (i)], while the DE primarily determines the growth rate [step (ii)].

The perturbation amplitude at the recombination era depends primarily on the cosmological parameters \( \Omega_M, \Omega_B, \) and \( H_0 \) in a flat Universe. The parameter \( \Omega_M \) is generally attributed to neutralinos produced in the supersymmetric dominated primordial Universe, \( \Omega_B \) is produced in the baryogenesis primordial Universe, and the present Hubble parameter \( H_0 \) in a flat Universe is due to \( \Omega_M^\Lambda \) and \( \Omega_B^0 \).

These parameters can be obtained from observations, using methods that are not in any way at all connected with the CMB and DE models. The \( \Omega_M \) can be obtained from high precision observations of galaxy clusters, using the relation \( \Omega_M = (M/L)J/\rho_c \), where \( M(L) \) is the mass (luminosity) of a given cluster, \( J \) is the luminosity density in the field around the cluster, and \( \rho_c \) is the critical density. Other methods using clusters and galaxies that depend on the linear theory growth factor (and thus, on the DE model), are also used to determine \( \Omega_M \). The parameter \( \Omega_M \) can be obtained from the observed number of clusters as a function of redshift, compared with the number predicted by the Press-Schechter relation or the Sheth-Tormen model, which depends on the linear theory growth factor and the DE model. Yet another method of obtaining \( \Omega_M \), which depends on the linear theory growth factor, uses \( \sigma_8 \), the present average amplitude of the dark matter fluctuations in a sphere of radius \( 8h^{-1}\text{Mpc} \). The parameter \( \Omega_B \) can be determined from high precision deuterium abundance observations of quasar absorption lines, and the present parameter \( H_0 \), from data of Cepheids in nearby galaxies.

Analytic models that evaluate the linear bias evolution \( b(z) \) as a function of redshift, depend on the EOS of the DE. These analytic models predict \( b^2(z \sim 0) \approx 1.0 \) for the flat DE \( \Lambda \)CDM model. The reason for this was noted by Tegmark and Peebles: “...even if galaxies initially were uncorrelated with the mass, they would gradually become correlated as gravity draws them toward overdense regions.
and one might expect this process to drive $b$ toward unity”. Numerical simulations, such as those of Somerville et al., also obtain this value for $b^2(0)$ for very different DE models, for example, the CDM ($\Lambda = 0$) and $\Lambda$CDM models.

From the analysis of the 2dFGRS data by Cole et al., $b^2_{\Lambda \text{CDM}}(0) \simeq 1.06$. Normalizing the analytic bias models, discussed in Section 2, to $0.9 \leq b^2(0) \leq 1.1$, we find that $0.9 \leq b^2(0) \leq 1.1$ for viable DE models, which we propose as a new test for DE models. Since we use the result that $b^2(0) \simeq 1$ from the 2dFGRS consortium for the $\Lambda$CDM model, the $L_*$ galaxy used in our text is the same as that obtained by the consortium for a broad range of galaxy types, using the Schechter function fit to the overall luminosity function of their $\sim 220,000$ galaxies.

We apply our DE model test to several popular DE models. We relate the $b^2$ of the DE model at $z \sim 0$ to a factor, $F = |b^2 - b^2_{\Lambda \text{CDM}}|/b^2_{\Lambda \text{CDM}}$, where $b^2_{\Lambda \text{CDM}}$ is the $b^2$ of the flat $\Lambda$CDM model at $z \sim 0$. A 10% deviation of $b^2$ from $b^2_{\Lambda \text{CDM}}$ normalized to unity implies a 10% deviation of $F$ from zero, $F = 0.1$. Viable DE models then need to have $F \leq 0.1$ (i.e., $b^2 = 1.0$ to a 10% accuracy at $z = 0$).

Three well-known analytic models of linear bias evolution are discussed in Section 2. We discuss the effect of DE on the linear growth of $(\delta \rho/\rho)$ in Section 3. DE models can be described by an EOS, $P/\rho = w(a) = w_0 + w_a(1 - a)$, where $P(\rho)$ is the pressure (energy density) at the cosmic scale factor $a$. It is shown in this section, that the permissible values of $F \leq 0.1$ limits the parameters $w_0$ and $w_a$. We then go on to discuss three popular DE models: the five-dimensional brane-world model (BWM), the vacuum metamorphosis model (VMM), and the supergravity (SUGRA) model. Conclusions and discussion are presented in Section 4.

2. Analytic bias models as a function of DE

As noted above, the 2dFGRS final results as well as the numerical calculations of Cole et al. indicate a bias parameter $b(z \equiv 0) \simeq 1.03$ for the $\Lambda$CDM model. From these results, we have $b^2_{\Lambda \text{CDM}} \simeq 1.0$ to better than a 10% accuracy. We use this result to normalize well-known analytic models for $b^2(0)$ in order to obtain $b^2(0)$ for viable DE models.

2.1. Galaxy Conserving Model (GCM)

In this model, galaxies behave as test particles, with their intrinsic properties conserved (see e.g., Refs. 23, 27, 28). An $L_*$ galaxy is a present massive galaxy. It probably started to form as a small galaxy at a high redshift, gradually building up by the process of merging. The assumption of the creation of an $L_*$ galaxy at a high redshift, with no further creation or merging, is an approximation of the evolution of $L_*$ galaxies. The linear bias parameter relating the density distribution to its mass density for a galaxy population formed at a given cosmic epoch $z_*$ is given by

$$b_{\text{GCM}}(z) = 1 + [b(z_*) - 1] \frac{D(z_*)}{D(z)} = 1 + (b_0 - 1) \frac{D(z = 0)}{D(z)},$$

(1)
where $b_0$ is the bias parameter at the present epoch and $D(z) = \delta \rho / \rho$ is the linear growth of the density fluctuations (details of $D(z)$ are presented in the Section 3).

### 2.2. Galaxy Merging Model (GMM)

The evolution of galaxy clustering is associated with host dark matter halos in this model. We use the analytical expression obtained by Ref. [29] for the halo-halo correlation,

$$\xi_{hh}(r, M) = b^2(M) \xi_{mm}(r),$$

where the bias parameter, according to the Press-Schechter formalism, [30] can be written as

$$b_{GMM}(M, z) = (1 - \frac{1}{\delta_c}) + \left(\frac{\nu^2}{\delta_c}\right)\frac{D^2(z = 0)}{D^2(z)},$$

where $\nu$ is the ratio of $\delta_c$ to the average value of the DM fluctuation $\sigma(M)$, for the mass $M$, [Eq.(5)]. From the dynamics of the spherical collapse in an expanding background, the factor $\delta_c$ was derived, and shown to be $\delta_c \simeq 1.69$, independent of the DE model by Ref. [31]. The factor

$$\nu^2/\delta_c = \delta_c/\sigma^2(M)$$

is independent of $z$ and $\sigma^2(M)$ is defined as

$$\sigma^2(M) = \frac{\sigma^2(M, z)}{D^2(z)/D^2(z = 0)},$$

where

$$\sigma^2(M, z) = \frac{D^2(z)/D^2(z = 0)}{2\pi^2} \int_0^\infty k^2 P(k)W^2(k, M)dk$$

is the rms of the linear density fluctuation of top-hat spheres containing an average mass $M$. The function $P(k)$ is the linear power spectrum at redshift zero and $W(k, M)$ is the Fourier space representation of the real space top-hat enclosing the average mass $M$.

### 2.3. Star Forming Model (SFM)

DM halos with masses greater than a given mass $M$ can be identified with galaxies with luminosities greater than a corresponding luminosity $L$ at a redshift $z$ in the star-forming model (see e.g., Ref. [32]). In this model, the bias evolution is

$$b_{SFM}(M, z) = 1 + \left(\frac{\nu^2}{\delta_c}\right)\frac{D^2(z = 0)}{D^2(z)}.$$  

The equations for $b(z)$ in the above three models (i.e., Eqs. (1), (3) and (7)) can all be written in the form

$$b(z) = A + E \left[ \frac{D(z = 0)}{D(z)} \right]^\alpha,$$
where $A$ is a known constant, $\alpha = 1$ or 2, and $E$ is a free parameter. For the $\Lambda$CDM model obtained by the 2dFGRS consortium, $b^2 = 1.0 \pm 0.1$ at a mean redshift $z = 0.17$. [18] The curves for the star forming and galaxy conserving models in Fig. 1 were normalized to $b^2(0) = 1.1$, the approximate maximum, value of the 2dFGRS consortium. From Eqs. (1), (7) and Fig. 1 we note that the galaxy conserving model has $b^2$ equal to a constant plus a term $\propto z$ and the star forming model a constant plus a term $\propto z^2$. Both models predict a change of $b^2$ from $z = 0.17$ to $z = 0$ by a very small factor $\sim 0.1\%$. The 2dFGRS result $b^2 = 1.0 \pm 0.1$ can then be used at $z = 0$:

$$b^2(0) = 1.0 \pm 0.1.$$  

(9)

We use Eq. (9) to normalize $E$ in Eq. (8) for all DE models.

This article is based on the requirement that $b^2(z = 0) \cong 1$ to an accuracy of 10% for all viable DE models, which is consistent with the results of the above analytic models. It is also consistent with the numerical calculations of Somerville et al., who showed that $b^2_0 \cong 1.0$ for both the CDM and $\Lambda$CDM models, $(\Omega_M = 1.0, \Omega_\Lambda = 0)$ and $(\Omega_M = 0.3, \Omega_\Lambda = 0.7)$, respectively.

We can compare the predictions of the above analytic models against observations of galaxy formation as a function of redshift. In Fig. 1, we show the galaxy conserving model (short-dashed curve) and the star forming model (continuous curve) for $\Lambda$CDM, normalized to $b^2 = 1.1$ at $z = 0$. The curves are in agreement.

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**Fig. 1.** Analytic models of linear bias evolution, $b^2(z)$ for the flat $\Lambda$CDM model $(\Omega_M = 0.3, \Omega_\Lambda = 0.7)$, normalized at $b^2(0) = 1$ with a deviation $F = +10\%$, compared with the $b^2_0$ for the volume-limited sample data with $M_B^0 = -20 + 5 \log h$ from the first-epoch VIMOS-VLT Deep Survey (VVDS). The solid and the short-dashed lines correspond to the star-forming and galaxy conserving models of $b^2(0)$, respectively. The long-dashed line is $b^2(z) \equiv 1$ for comparison. The bias at $z = 0.17$ (the effective depth of the 2dFGRS survey) was inferred from the 2dFGRS galaxies, which had the same median luminosity of the volume-limited VVDS sample, $L/L^* \sim 2$.  

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with the $\Lambda$CDM $b^2(z)$ from the first-epoch VIMOS-VLT Deep Survey (VVDS) in Ref.33 (Marinoni et al.). (We do not plot the merging model since, as was shown by Marinoni et al., it does not describe the redshift evolution of the bias of their galaxies well.)

In Fig. 1, the values of $E$ and $A$ in Eq. (8) are $E = 0.1$ and $A = 1.0$ for the galaxy conserving and star forming models. Using the value of $E$ for the star-forming model in Eqs. (4)-(7), we find that the dark matter fluctuations for the massive $L_\ast$ galaxies, $\sigma(M) = 4.1$, its present linear extrapolated value. The fact that $\sigma(M) = 4.1$ is greater than $\delta_c = 1.69$, implies that the halo of the $L_\ast$ galaxy has already collapsed, which is reasonable.

### 3. DE and the growth of density fluctuations

The nature of DE is still unknown and there are many alternative models which try to explain it. In addition to the popular cosmological model of a constant vacuum energy, described by a cosmological constant, there are models that modify gravity as well as DE those that parametrize the DE EOS, $w(a) = w_0 + w_a(1 - a)$, setting values for $w_0$ and $w_a$. For these models, the Friedmann equation can be written in a general form in terms of an effective EOS.37 Modelling the DE as an ideal fluid in a flat Universe, we can write the Friedmann equation as

$$\frac{H^2(z)}{H_0^2} = \Omega_M^0 (1 + z)^3 + (1 - \Omega_M^0) e^{\int^z_0 \frac{d\ln(1+z')}{(1+z')^{1+w(z')}}} ,$$

or

$$\frac{H^2(z)}{H_0^2} = \Omega_M^0 (1 + z)^3 + \frac{\delta H^2}{H_0^2} ,$$

where $H_0$ is the present value for the Hubble parameter, $\Omega_M^0$ is the present normalized matter density, and $\delta H^2/H_0^2$ depends upon the DE model. The EOS $w(z)$ for the DE can be written as

$$w(z) \equiv -1 + \frac{1}{3} \frac{d \ln \delta H^2/H_0^2}{d \ln (1+z)} .$$

Linear growth of a density fluctuation, $D = \delta \rho/\rho$, depends on the EOS. We define the growth factor, $G \equiv D/a$, where the cosmological scale factor is $a \equiv 1/(1+z)$ and $G$ is normalized to unity at $z \sim 1100$, the recombination epoch.37 In terms of $G$, we have

$$G''(a) + \left[ \frac{7}{2} - \frac{3}{2} \frac{w(a)}{1 + X(a)} \right] \frac{G'(a)}{a} + \frac{3}{2} \frac{1 - w(a)}{1 + X(a)} \frac{G(a)}{a^2} = 0 ,$$

where $X(a)$ is defined as

$$X(a) = \frac{\Omega_M^0 a^{-3}}{\delta H^2/H_0^2} .$$

For large $X$ we recover the matter dominated behavior $D \sim a$. 
It is generally required that the DE was very small compared to the cold DM for redshifts \( z \gtrsim 10 \) in order for the latter to create the structure in the Universe at redshifts \( z \lesssim 10 \). In this paper, we analyze only dark energy models in which the DE density at the recombination redshift or higher \( (z \gtrsim 1100) \) was negligible, as in the ΛCDM model.

One of the DE models studied was the Vacuum Metamorphosis Model (VMM), which has zero DE at recombination. A second model was the Brane World Model (BWM), in which the ratio of the DE density to the DM density is proportional to \( (1 + z)^{-3} \) and negligible at high redshifts. A third model was the supergravity SUGRA model, which becomes a CDM model at high redshifts with negligible DE at recombination.

Much effort has been made to obtain the density power spectrum normalization at the recombination era. For example, Spergel et al. used the three year WMAP data with a ΛCDM model to obtain the density power spectrum normalization \( A_S \) at recombination.\(^{34}\)

Our DE model test is independent of \( A_S \) since it is primarily dependent on the evolution of the DM and radiation densities at \( z = 1100 \), the recombination era, or higher, when DE was small or negligible. The models that we examine here have negligible DE for \( z \gtrsim 1100 \) and, thus, a negligible effect on \( A_S \). It is to be noted that the ΛCDM model, used by Spergel et al., is one such model, in which DE density was one billionth that of the total energy density at the recombination era.

For our DE model test, it is not necessary to obtain the exact value of the normalization factor. At a given epoch, the only relevant difference between the different models studied is the value of the DE density, all other factors being equal. Since at the recombination epoch, all of the models we examined had a negligible DE density, they are essentially identical and their effect on the normalization factor is identically negligible. Thus, an uncertainty in the normalization factor (which is not known to better than \( \sim 20\% \)) does not enter into the uncertainty of \( \sim 10\% \) in the factor \( F \) [Eqs. (15) and (16)], used in our DE model test, since all models are normalized at the recombination era.

It is important to emphasize the difference between the objective of Spergel et al. and ours. The objective of Spergel et al. was to obtain a value for \( A_S \) as accurate as possible as well as other parameters, such as the normalized DM density \( \Omega_M \), using one DE model, the ΛCDM model. Our objective here, is to discard DE models which have negligible DE densities at the recombination era, but that dominate the total energy density of the Universe at present.

Galaxies began to form at a redshift \( z \sim 20 \), growing until the present time \( (z \sim 0) \). The formation and growth of the galaxies, described by their fluctuations \( \xi_{gg} \), are greater for larger DM fluctuations \( \xi_{dm} \) in the redshift interval \( z \sim 20 \) to \( z \sim 0 \). Since \( \xi_{dm} \) monotonically increases with a decrease in redshift and DE does not effect it at very high redshifts \( (z \gg 20) \), \( \xi_{dm}(0) \) is sensitive to the DE model and the growth factor. Our test, \( b^2(0) \simeq 1 \), does not tell us whether the growth factor or \( \xi_{dm}(0) \) needs to be big or small. What \( b^2(0) \simeq 1 \) does say is that whatever
\( \xi_{gg}(0) \) is, \( \xi_{dm}(0) \) must accompany it such that \( b^2(0) \equiv \xi_{gg}(0)/\xi_{dm}(0) \simeq 1 \).

We define the deviation from the standard \( \Lambda \)CDM model by

\[
F = \left| \frac{D^2 - D^2_{\Lambda}}{D^2} \right| \bigg|_{z=0},
\]

where \( D^2_{\Lambda} \) are the density fluctuations in the standard flat \( \Lambda \)CDM model and \( D^2 \) are the density fluctuations of an arbitrary DE model. The functions \( D/a \) and \( D_{\Lambda}/a \) are normalized to unity at the recombination era, \( z \simeq 1100 \), where \( a \equiv 1/(1 + z) \) is the cosmic scale factor. Therefore \( F \) in Eq. (15) is concerned only with the relative growth of the density fluctuations between a given DE model and the flat \( \Lambda \)CDM model, from the recombination epoch to the present era. It thus has the important characteristic of being independent of the power spectrum normalization at recombination.

The linear bias parameter is defined by \( b^2 = P_{gg}/P_{mm} \), where \( P_{gg} \) is a galaxy distribution power spectrum (e.g., that of 2DFGRS or SDSS) and \( P_{mm}(z) \propto (\delta \rho/\rho)^2 = D^2(z) \) is the DM power spectrum. The difference in the DE models is in the evolution of \( \delta \rho/\rho \), which occurs after the recombination era at redshifts \( z \lesssim 10 \). For this reason, we use the factor \( F \), which examines the effect of DE models on the evolution of \( \delta \rho/\rho \) after the recombination era. The DE models that we investigate do not effect the evolution of \( \delta \rho/\rho \) before the recombination era and, therefore, \( (\delta \rho/\rho)_{\text{rec}} \) is the same for all the DE models studied. Thus, Eq. (15) becomes

\[
F = \left| \frac{b^2 - b^2_{\Lambda}}{b^2_{\Lambda}} \right| \bigg|_{z=0}.
\]

A maximum 10\% deviation of \( b^2 \) from \( b^2_{\Lambda} \) with \( b^2_{\Lambda} \simeq 1.0 \) at \( z = 0 \) implies that the maximum \( F \) is \( F_{\text{max}} = 0.1 \).

| \( w_a \) | \( w_0 \) |
|---|---|
| 1.53 | -1.72 \( \pm \) 0.02 |
| 1.63 | -1.77 \( \pm \) 0.02 |
| 1.73 | -1.82 \( \pm \) 0.02 |
| 1.83 | -1.86 \( \pm \) 0.02 |
| 1.93 | -1.82 - 0.01 |
| 2.03 | -1.89 - 0.01 |
| 2.9 | -1.86 - 0.02 |
3.1. DE models described by a parametrized EOS

We first discuss a parametrization for the EOS, \( w(a) = w_0 + w_a (1 - a) \), which has been widely used for DE models since it is well-behaved at high redshifts, unlike \( w(z) = w_0 + w_1 z \), for example, which diverges at high \( z \). This parametrization was introduced by Ref. [5]. The best fit parameters, \( w_0 \) and \( w_a \), which are consistent with the Gold SNIa dataset, were found to be in the intervals \(-1.91 \leq w_0 \leq -1.25 \) and \( 1.53 \leq w_a \leq 5.05 \). Assuming that \( F = 0.10 \pm 0.02 \) and \( \Omega_M^0 = 0.28 \pm 0.02 \), we obtain the best fit values, \(-1.91 \leq w_0 \leq -1.72 \) and \( 1.53 \leq w_a \leq 2.9 \), shown in Table 1.

\[
\begin{array}{cccccc}
\Omega_M^0 & F_{\text{BWM}} & H_0 r_c & F_{\text{VMM}} & m^2 & z_j \\
0.26 & 0.27 & 1.4 & 0.29 & 11 & 1.4 \\
0.28 & 0.27 & 1.4 & 0.24 & 11 & 1.4 \\
0.30 & 0.26 & 1.4 & 0.20 & 11 & 1.3 \\
0.32 & 0.25 & 1.5 & 0.16 & 11 & 1.2 \\
0.34 & 0.25 & 1.5 & 0.13 & 11 & 1.2 \\
0.36 & 0.24 & 1.6 & 0.095 & 10 & 1.1 \\
0.72 & 0.11 & 3.6 & 0.24 & 8 & 0.5 \\
\end{array}
\]

3.2. Brane-world and vacuum metamorphosis models

In the BWM\,[24] gravity is modified by adding a five-dimensional Einstein-Hilbert action, that dominates at distances that are larger than the crossover length \( r_c \), which defines an effective energy density, \( \Omega_{bw} = (1 - \Omega_M^0)^2/4 = 1/(4H_0^2 r_c^2) \), for a flat Universe. The factor \( \delta H^2/H_0^2 \) in Eq. (11) then becomes

\[
\delta H^2/H_0^2 = 2\Omega_{bw} + 2\sqrt{\Omega_{bw}}\sqrt{\Omega_M^0 (1 + z)^3} + \Omega_{bw}. \tag{17}
\]

In the VMM\,[25] the vacuum contributions are due to a quantized massive scalar field, which is coupled to gravity. For \( z < z_j \), \( \delta H^2/H_0^2 \) in Eq. (11) is

\[
\delta H^2/H_0^2 = (1 - m^2/12)(1 + z)^3 + m^2/12 - \Omega_M^0 (1 + z)^3, \tag{18}
\]

where \( z_j = [m^2/(3\Omega_M^0)]^{1/3} - 1 \) and \( m^2 = 3\Omega_M^0 [(4/m^2) - (1/3)]^{-3/4} \). Both the BWM and the VMM can be described by the EOS, \( w(a) = w_0 + w_a (1 - a) \), with \((w_0, w_a) = (-0.78, 0.32)\) and \((w_0, w_a) = (-1, -3)\), respectively\,[36].

It can be seen from Table 2 that an agreement within 10\% between the VMM and the \( \Lambda \)CDM model is possible only for a matter density \( \Omega_M^0 \approx 0.36 \). For the BWM, an agreement with the \( \Lambda \)CDM model within 10\% is possible only if the matter density \( \Omega_M^0 \approx 0.72 \). However, both of these values for \( \Omega_M^0 \) are greater than the observed value.
3.3. Supergravity model

The SUGRA model is very attractive for explaining the acceleration of the Universe.\footnote{26} It can be described by the EOS of § 3.1, with \( w_0 = -0.82 \) and \( w_a = 0.58 \).\footnote{37} This equation of state is in agreement with observations for the low redshift SNIa dataset and galaxy distribution data.\footnote{[1][38]} However, \( F \) for this model makes it inviable.

Figure 2 shows the growth of the density fluctuations \( G = \left[ \left( \frac{\delta \rho}{\rho} \right) / a \right] \) as a function of \( a \) for the BWM, the VMM, and the \( \Lambda CDM \) model. It shows that the growth of \( \delta \rho/\rho \) is smaller for the BWM and SUGRA model than for the \( \Lambda CDM \) model. The deviation \( F \) for the SUGRA model is \( F_{SUGRA} \approx 0.38 + 0.04 \) for \( \Omega_M^0 = 0.30 - 0.04 \), which gives a bias parameter, normalized to the \( \delta_{\Lambda CDM}^2(z = 0) = 1 \), appreciably greater than unity: \( b_{SUGRA}^2(z = 0) = 1.38 \).

4. Conclusions and Discussion

In this paper we suggested a new test for the viability of DE models, based on the value of the bias parameter \( b \), the ratio of galaxy to DM fluctuations, at \( z = 0 \). If it were the case that we knew nothing about galaxy formation, \( b \) could, in principle, have been anything at all, i.e., very much less or very much greater than unity. However, our present knowledge of galaxy formation from analytic models and numerical simulations, indicates that \( b^2 \) is close to unity at \( z \approx 0 \). This information can be used to discard DE models that do not predict \( b \) close to unity at \( z = 0 \).

We studied three popular analytic models for \( b(z) \). We showed that the equations for \( b \) in all three analytic models can be reduced to the form of a known constant plus the term \( E[D(z = 0)/D(z)]^\alpha \), where \( \alpha = 1 \) or 2 and \( E \) is a free parameter.

![Fig. 2. The growth of density fluctuations, \( G = \left[ \left( \frac{\delta \rho}{\rho} \right) / a \right] \), for the VMM, \( \Lambda CDM \), BWM, and the SUGRA models. The dashed lines show the deviation of the matter density, \( \Omega_M^0 = 0.28 \pm 0.02 \).](image-url)
Using the value for \( b \) obtained by the 2dFGRS consortium\cite{11} for all three models predict \( b^2(0) = 1.0 \pm 0.1 \) for all DE models. This value of \( b \) is also in agreement with numerical simulations that evaluated \( b^2(0) \) for the \( \Lambda \)CDM and CDM (\( \Lambda = 0 \)) models.\cite{10} Since this value of \( b^2(0) \) is indicated by numerical simulations as well as by all three popular analytic models, which are normalized by the 2dFGRS consortium result for the \( \Lambda \)CDM model, we suggest the condition that \( b^2(0) = 1 \pm 0.1 \) at \( z = 0 \) as a new test for the viability of DE models.

Obtaining \( b(0) \) from galaxy observations involves a complex process of combining data from all types of galaxies (see e.g., Ref.\cite{39}). These complexities reflect the galaxy formation process. As in the standard analysis for the evaluation of the bias parameter \( b \) (see e.g., Cole et al. Ref.\cite{19}), we assume that \( b \) is independent of scale for \( k \leq 0.1 \)h Mpc\(^{-1} \) and that all galaxies are normalized to a standard massive bright galaxy with luminosity \( L_\star \). Cole et al. made the following normalization of the bias parameter to a galaxy of luminosity \( L_\star \) for the galaxies in the 2dF Galaxy Redshift Survey:

- Luminosity Normalization: \( b(L) = 0.85 + 0.15(L/L_\star) \);
- Red Galaxy Subset Normalization: \( b(\text{red}) = 1.3[0.85 + 0.15(L/L_\star)] \); and
- Blue Galaxy Subset Normalization: \( b(\text{blue}) = 0.9[0.85 + 0.15(L/L_\star)] \).

Obtaining \( b(0) \) from numerical simulations of galaxy formation is also not simple. It is difficult to build models for galaxy populations of dark halos that can robustly relate the amplitude of large-scale galaxy clustering at better than the 10% level.\cite{40} This, in part, is the reason that we have a 10% limit on the accuracy of our test, which examines whether \( b = 1 \) at \( z = 0 \) for a viable DE model.

The numerical simulations of the Virgo Consortium are consistent with our test for DE models. Very high-resolution simulations have been made by the Virgo Consortium for the \( \Lambda \)CDM model.\cite{11} They found that for galaxies with \( M_B < -17 \) at \( z = 0 \) on the largest scales, the galaxy power spectrum has the same shape as that of the DM, but with a slightly lower amplitude, corresponding to a bias \( b \simeq 1.0 \). Samples of brighter galaxies have a bias close to \( b \simeq 1.0 \).

In this article, we related \( b(0) \) to the \( \Lambda \)CDM value of \( b \), \( b_{\Lambda}(0) \), using a function \( F \). The bias parameter \( b \) is related to the factor \( F \) by \( F = |(b^2/b_{\Lambda}^2) - 1| \) at \( z = 0 \) (Eq. 16). Thus, a maximum deviation of 10% of \( b^2(z = 0) \) from \( b_{\Lambda}^2(z = 0) \) with \( b_{\Lambda}^2(z = 0) \simeq 1.0 \) implies a \( \sim 10\% \) deviation of \( F \) from zero or \( F_{\max} = 0.1 \). We investigated DE models that make a negligible contribution to the total energy density before the recombination era and, thus, a negligible contribution to the density power spectrum normalization factor \( A_S \) at the recombination era. The function \( F \) has the important characteristic of being independent of \( A_S \). We calculated the value of \( F \) numerically for several well-known DE models from the growth equation for \( \delta \rho/\rho \). The constraints from the Gold SNIa data\cite{35} and the condition that \( F = 0.1 \) restrict the values of the parameters of the linear EOS, \( w(a) = w_0 + w_a(1 - a) \), for dark energy. It was found that the best fit values of \( w_0 \) and \( w_a \) are \(-1.86 < w_0 < -1.72, \)
with $1.53 < w_a < 2.9$. For $z \sim 0.5 - 1$, where $w(a)$ is sensitive to the supernova data, and $w_0 \sim -1$.

The BWM and VMM were then studied using the factor $F$. We showed that these DE models do not satisfy our DE model test, with $F = 0.1$ and $\Omega_M^0 = 0.28 \pm 0.02$. The BWM has $F = 0.26$ and the VMM, $F = 0.20$. Thus BWM and VMM are not viable DE models.

Finally, we analyzed the SUGRA model for the above parametrized EOS with $w_0 = -0.82$ and $w_a = 0.58$. $F$ was found to be very large: $F_{\text{SUGRA}} \approx 0.38^{+0.04}_{-0.02}$ for $\Omega_M^0 = 0.30^{+0.04}_{-0.02}$ giving $b_{\text{SUGRA}}(z \approx 0) = 1.38$, which is appreciably greater than unity. Thus the SUGRA model is also not a viable DE model.

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