Generation of large-scale winds in horizontally anisotropic convection

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We simulate three-dimensional, horizontally periodic Rayleigh-Bénard convection between free-slip horizontal plates, rotating about a horizontal axis. When both the temperature difference between the plates and the rotation rate are sufficiently large, a strong horizontal wind is generated that is perpendicular to both the rotation vector and the gravity vector. The wind is turbulent, large-scale, and vertically sheared. Horizontal anisotropy, engendered here by rotation, appears necessary for such wind generation. Most of the kinetic energy of the flow resides in the wind, and the vertical turbulent heat flux is much lower on average than when there is no wind.

When a horizontal layer of ordinary fluid is heated from below, it overturns if the imposed vertical temperature inhomogeneity is large enough. This phenomenon, called Rayleigh-Bénard convection (RBC) by Avsec \(^1\), has been studied in numerous laboratory and computational experiments \(^2\) \(^3\). Buoyancy-driven convection is a primary mechanism of heat transport in the atmospheres of planets and cool stars, the cores of massive stars, and the Earth’s oceans, mantle, and outer core. Apart from the inherent interest of the process, these natural occurrences of convection have been a major motivation for its study throughout the past century.

Large-scale horizontal winds have been seen in many physical and numerical RBC experiments, as in the early observations of Malkus \(^4\) and the more detailed study of Krishnamurti and Howard \(^5\). The strongest winds yet reported occur in simulations of horizontally periodic, two-dimensional (2D) RBC between free-slip top and bottom boundaries. These winds have a horizontal wavenumber of zero, and they are vertically sheared, meaning that their velocities change with height. For weak thermal forcing, the bifurcations by which ordinary 2D convective cells can lose stability to states with wind have been studied using the full governing equations \(^6\) \(^7\) and reduced models \(^8\) \(^9\). When the thermal forcing is strengthened, the wind in 2D simulations intensifies to dominate the underlying convection, leading to drastically reduced heat transport \(^10\) \(^11\). Comparably strong winds in 3D RBC have not been reported heretofore.

The simulations described below reveal that strong, wavenumber-zero winds can arise also in 3D RBC if the horizontal isotropy is broken by uniform rotation about a horizontal axis. The resulting flows, which we call windy convection, are more complicated than their 2D counterparts but can develop winds of comparable strength. Three-dimensional windy convection resembles flow in the equatorial regions of rotating, convecting planets \(^17\) \(^20\) and in the outer edge of tokamak plasmas in the high-confinement mode \(^21\).

Horizontal anisotropy seems to be necessary for strong wavenumber-zero winds to persist. The winds ceased in our simulations whenever rotation was slowed sufficiently, and winds in the initial conditions invariably died out in the non-rotating simulations of Parodi et al. \(^22\). Likewise, solutions with mean winds were found in truncated modal expansions of 3D RBC only when the imposed convective pattern was sufficiently anisotropic \(^20\). Under horizontally isotropic conditions, other sorts of large-scale circulation have been seen, but none has had much effect on the mean heat flux. Circulation with a horizontal scale comparable to the layer depth can spatially organize smaller-scale plumes but does not inhibit their heat transport \(^21\) \(^22\), and the wind seen by Krishnamurti and Howard \(^5\) was apparently not strong enough to prevent thermal plumes from traversing the layer.

We adopt the Boussinesq approximation \(^20\) \(^21\), in which the fluid has constant kinematic viscosity, \(\nu\), thermal diffusivity, \(\kappa\), and coefficient of thermal expansion, \(\alpha\). The natural units we use are the layer thickness, \(d\), the imposed vertical temperature difference between the bounding plates, \(\Delta\), and the thermal time, \(\tau_{\text{th}} = d^2/\kappa\). Other time scales include the viscous time, \(\tau_{\text{vis}} = d^2/\nu\), the dynamical time, \(\tau_{\text{dyn}} = \sqrt{d/(go\Delta)}\), where \(g\) is gravitational acceleration, an effective dissipative time, \(\tau_{\text{diss}} = \sqrt{\tau_{\text{th}}/\tau_{\text{vis}}}\), and the rotation period.

The ability of buoyancy forces to overcome viscous drag is measured by the Rayleigh number, \(R = (\tau_{\text{diss}}/\tau_{\text{dyn}})^2\), and the relative importance of the two dissipative mechanisms is measured by the Prandtl number, \(\sigma = \tau_{\text{th}}/\tau_{\text{vis}}\). Here we set \(\sigma = 0.71\) (air) throughout. The dimensionless vertical extent is \(0 \leq z \leq 1\), and we set horizontal periods of \(2\pi\) in the \(x\) and \(y\) directions. The system rotates about a suitably placed axis parallel to the \(y\)-axis with dimensionless angular velocity \(\Omega\), expressed in units of \(1/\tau_{\text{th}}\). Cyclonic motion, with net vorticity parallel to the rotation vector, will appear clockwise when observed from the direction of negative \(y\).

The dimensionless Boussinesq equations governing the velocity \(\mathbf{u} = (u, v, w)\), temperature \(T\), and pressure \(p\) are

\[ \nabla \cdot \mathbf{u} = 0 \]  
\[ \partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \sigma \nabla^2 \mathbf{u} + \sigma R T \mathbf{z} - 2\Omega \mathbf{y} \times \mathbf{u} \]  
\[ \partial_t T + \mathbf{u} \cdot \nabla T = \nabla^2 T. \]

At the top and bottom boundaries, the temperatures are...
fixed and uniform, and the velocity conditions are either free-slip or no-slip:

\[ T|_{z=0} = 1/2, \quad T|_{z=1} = -1/2 \]  (4)

free-slip: \[ \partial u/\partial z|_{z=0,1}, \partial v/\partial z|_{z=0,1}, w|_{z=0,1} = 0 \]  (5)

no-slip: \[ u|_{z=0,1} = 0. \]  (6)

Free-slip boundaries favor strong winds, letting them develop at \( R \) that are small enough to be computationally accessible. In 2D, winds arise readily even when only one boundary is free-slip \[28\].

The governing equations are integrated with a spectral method in the periodic directions with 2/3 dealiasing, second-order finite differences in the vertical, and a third-order fractional step method for time advancement \[22, 29\]. We used 256 modes in each horizontal direction and 192 vertical points spaced more closely near the boundaries.

Figure 1 shows temperature slices in the \((x, z)\) plane at two instants of a simulation with \( R = 10^7 \) and \( 2\Omega = 10^4 \), and free-slip boundaries. (Slices at other values of \( y \) look qualitatively similar since the rotation makes variation in \( y \) weak.) A video of this simulation is available at \[30\]. Beginning from statistically homogeneous, random initial conditions, a pair of turbulent convective cells develop, and the temperature field is dominated by a hot plume and a cold plume (panel a). This stage persists for many turnover times, during which the cyclonic cell slowly (and non-monotonically) widens while the anticyclonic cell narrows. Eventually, the narrowing cell shrinks out of existence as the hot and cold plumes collide, and the widening cell breaks through to form a cyclonic wind that then dominates the system (panel b). Once the wind forms, fluid flows quickly in the \(+x\)-direction along the top boundary and in the \(-x\)-direction along the bottom one. Various thermal plumes come into and out of existence, but they are sheared out horizontally and rarely reach the opposite boundaries. Anticyclonic wind is also possible, as described in the following section.

The onset of the wind significantly alters not only the appearance of the flow but also its integral quantities. The great strength of the wind is indicated by the fact that the specific kinetic energy of motion in the wind direction, \( \frac{1}{2}\langle u^2 \rangle \), is much larger than that in the other two directions, \( \frac{1}{2}\langle v^2 + w^2 \rangle \). (Angular brackets denote volume averages.) The decrease in vertical heat flux that accompanies the wind’s onset is captured by the Nusselt number, \( N(t) = 1 + \langle wT \rangle \), which is the factor by which convection amplifies heat transport.

Figure 2a shows the time variation of the specific kinetic energy in different velocity components. These energies are comparable in the transient stage, during which the mean flow slowly strengthens. After the plumes collide near \( t = 0.08 \) and the wind develops fully, convective motions normal to the wind are much suppressed. Still, these motions never cease completely, as shown by the small but finite values of \( \frac{1}{2}\langle v^2 + w^2 \rangle \) in Fig. 2a. The evolution of the wind and its structure once developed are shown in Fig. 2b, which gives vertical profiles of \( u \), averaged over the horizontal directions and 0.025 thermal times. Even during the transient stage, the wind becomes fairly strong as the cyclonic convective cell grows to dominate the anticyclonic one (see Fig. 1a). Ultimately, the vertical profile of \( u \) becomes almost linear, with transitory deviations created by plume outbursts.

Figure 2c shows the time evolution of the Nusselt number, which first displays a strong peak as convective plumes emerge explosively from the linear instability. Then \( N(t) \) changes little as the wind builds in strength. When the wind takes over as the cyclonic cell annihilates the anticyclonic one, convective heat transport rapidly becomes feeble, as reflected by \( N(t) \) dropping almost to
FIG. 2. (a) Time series of volume-averaged specific kinetic energy in the direction of the wind, \( \frac{1}{2} \langle u^2 \rangle \), and normal to the wind, \( \frac{1}{2} \langle v^2 + w^2 \rangle \); (b) vertical profiles of the wind velocity averaged horizontally and over 0.025 thermal times; (c) time series of the Nusselt number; (d) vertical profiles of the temperature averaged horizontally and over 0.025 thermal times. The dots in panels (a) and (d) mark the starts of the time windows over which the vertical profiles in panels (b) and (d) have been averaged.

unity. Heat transport recovers only partially thereafter since the system remains dominated by the horizontal wind. The time-averaged Nusselt number is 7.1 after the wind has fully formed (0.2 \( \leq t \leq 0.75 \)), as compared with 22.6 during the transient stage (0.05 \( \leq t \leq 0.08 \)). Figure 2d shows the evolution of mean vertical temperature profiles, which are roughly isothermal in the interior before the wind develops but have a strongly unstable stratification afterward.

In addition to being smaller when the wind dominates, \( N(t) \) varies more strongly; at irregular intervals, there are deep minima of convective transport during which the Nusselt number dips nearly to its conductive value of unity. These minima occur when the wind becomes especially strong and nearly shuts off the convection from which it draws its energy. Lacking an energy source, the wind decays until it is too weak to inhibit the small-scale convection, which then rebounds. Such behavior is an instance of on-off intermittency [31]. Very strong intermittency occurs at similar parameters in 2D RBC [15, 16], which may be thought of as an extreme limit of horizontal anisotropy in 3D RBC.

Starting from statistically isotropic initial conditions, most of our simulations developed cyclonic winds like the one in Fig. 1b—that is, winds with \( u > 0 \) near the top and \( u < 0 \) near the bottom. However, at the parameters of the simulation described above, it seems that a cyclonic attractor and an anticyclonic attractor coexist. Fig. 3a shows a time series of \( N(t) \) and mean wind profiles for a cyclonic simulation and for an anticyclonic simulation. The latter state was obtained by restarting the former at \( t = 0.25 \), after reflecting \( x \mapsto -x \). The winds are of comparable energy, but the anticyclonic wind bursts much more strongly, resembling bursting seen in 2D [15, 16, 28].

It is rotation that distinguishes between the two wind directions. Without rotation, winds in 2D RBC spontaneously break the reflectional symmetry \( x \mapsto -x \) to form a vertical shear with one sign or the other. Here, Coriolis force breaks symmetry not only between the \( x \)- and \( y \)-directions but also between cycloonic and anticyclonic winds. For instance, it acts in the \(-x\)-direction on rising fluid, as found in warm plumes, and acts in the \(+x\)-direction on falling fluid, as found in cold plumes. (The broken \( x \mapsto -x \) symmetry recalls the cyclone-anticyclone asymmetry seen in stably stratified geophysical turbulence [32].) Coriolis force cannot help sustain the wind against viscous dissipation, however, since it performs no work on the fluid. With or without rotation, the wind can be driven only by the transfer of energy, via the ve-
N(t)
N(t)
10
10
20
20
30
30
40
40
50
50
60
60

FIG. 3. (a) Time series of Nusselt numbers for $2\Omega = 10^4$. The anticyclonic simulation (A) is started at time $t = 0.25$ by applying the reflection $x \mapsto -x$ to the cyclonic simulation (C). The inset shows mean wind profiles, averaged horizontally and over $0.3 \leq t \leq 0.75$. (b) Time series of Nusselt numbers for free-slip boundary conditions with $2\Omega = 3 \cdot 10^3$, where anticyclonic wind (A) and non-windy convection (B) result from different initial conditions, and for no-slip boundary conditions with $2\Omega = 10^4$. The inset shows mean wind profiles, averaged horizontally and over $0.1 \leq t \leq 0.2$. In all cases $R = 10^7$.

Locivity nonlinearity, from the smaller-scale motions that are buoyantly driven.

Figure 3b shows $N(t)$ series and mean horizontal velocities for a simulation with no-slip boundaries and two free-slip simulations at the lower rotation rate of $2\Omega = 3 \cdot 10^3$. A slight wind develops in the no-slip simulation, but it is not nearly strong enough to destroy the convective cells and create the qualitatively different state that we have called windy convection. With $2\Omega = 3 \cdot 10^3$ and free-slip boundaries, one simulation develops no strong wind, and the other develops anticyclonic wind. As in 2D [16, 28], it seems there is bistability between windy and non-windy convection. At this lower rotation rate, however, the wind does not display on-off intermittency and suppresses heat transport only slightly. We have not been able to induce sustained cyclonic wind at this lower rotation rate. Many more simulations would be needed to characterize the parameter regimes in which cyclonic, anticyclonic, and non-windy states exist.

We have observed the formation of strong horizontal winds in 3D RBC when a forced symmetry breaking, here brought about by imposed rotation, is strong enough. Quantification of the minimum necessary rotation must await a systematic parameter study. When rotation is too weak for wind to be sustained, any wind that starts to form is quickly disrupted by motions normal to it. This seems to be inevitable because the wind does not suppress convective instability normal to it, whereas the wind does partially suppress motions that are not normal to it and from which it draws its energy. This effect has been demonstrated in the related system where Couette shear is imposed on RBC [33, 34]. The tendency of strong winds to quench the motions that generate them has analogs also in the large-scale magnetic fields of convective dynamo theory [35, 36] and the zonal flows of tokamak plasmas [37].

The strength of rotation, which controls the degree of anisotropy in our simulations, may be thought of as parametrizing a transition from fully 3D RBC to 2D rotating RBC. The winds that arise when anisotropy is strong enough derive their energy from smaller-scale motions that are driven by buoyancy, as in 2D RBC [16]. The situation is reminiscent of 2D turbulence models that are driven by prescribed body forces at intermediate scales [38–41]. Energy in such models, as in the present one, tends to cascade upscale into the largest possible modes. Unlike prescribed forces, however, convective forces are dynamically affected by the very same large-scale winds that they sustain.

Our findings support the conjecture that the upscale cascade of energy in anisotropic turbulent convection, which here drives sheared winds, drives differential rotation in the equatorial regions of planetary atmospheres and stellar convective zones. Such differential rotation may have interesting consequences. At the bottom of the solar convection zone, for example, the shearing tachocline [42] is suspected of generating the solar activity that is manifested by sunspots [43], and the intermittent behavior of the heat transport described here recalls the irregularity of such activity [44]. Other possible locations of windy convection are in the convective cores of massive stars that rotate rapidly. Large-scale shear arising there might influence nuclear reaction rates [45], with implications for stellar evolution. More immediately, the desire to extrapolate windy convection to astrophysically or geophysically interesting regimes calls for further exploration of parameter space.
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