Dependence of mis-alignment sensitivity of ring laser gyro cavity on cavity parameters

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Abstract. The ring laser gyroscope (RLG), as a rotation sensor, has been widely used for navigation and guidance on vehicles and missiles. The environment of strong random-vibration and large acceleration may deteriorate the performance of the RLG due to the vibration-induced tilting of the mirrors. In this paper the RLG performance is theoretically analyzed and the parameters such as the beam diameter at the aperture, cavity mirror alignment sensitivities and power loss due to the mirror tilting are calculated. It is concluded that by carefully choosing the parameters, the significant loss in laser power can be avoided.

1. Introduction

Ring laser gyroscope (RLG) is an angular rate sensor based on Sagnac effect. The typical RLG is fabricated by boring or drilling connected optical cavities through a body block, which is made of ultra-low expansion materials. Flat or curved mirrors are attached to the corner of the cavities through the optical bonding process which is applied on the corresponding surface of the block to reflect the light and form a triangular or square shaped loop-path. In an ideal laser gyro, the mirrors can reflect the light to the perfect direction as designed. However, practically the angle may have a mismatch problem [1,2]. This mismatch may be caused by the tolerance of the manufacture or fabrication, or the distortion of the block when the RLG is placed in a circumstance of large acceleration.

The mismatch may deteriorate the performance of the RLG. In this paper, the power loss of a triangular RLG due to the mirrors mismatch is studied theoretically using the ABCD matrix [3]. For a given triangular RLG, the optimized parameters of the mirrors are obtained to reduce the influence of the mirror mismatch.

2. The triangular RLG
The RLG with a triangular optical path are presented in Fig. 1 schematically. The optical resonator is composed of 3 mirrors, \( M_1, M_2, \) and \( M_3 \), and the corresponding focus and curvature radii are defined as \( f_1, f_2, f_3 \) and \( R_1, R_2, R_3 \), respectively. The lateral of the triangle is \( \alpha \) so the perimeter is \( 3\alpha \). Since the difference of the laser beam radii among the different positions is typically less than 5%, the following approximation can be made in the calculation: the beam energy distribution is Gaussian in the cross section of the beam, and the beam is propagating as plane wave. It is assumed that a diaphragm is placed on the surface of \( M_3 \), and each mirror has an angular mismatch \( \theta \). The beam which starts from \( M_3 \) with the coordinate vector \( R_0 \), travels one loop of the optical path via the mirrors \( M_i \) (\( i=1,2,3 \)) with the coordinate vector \( R_i \), and reaches \( M_3 \) with the coordinate vector \( R_f \). The equation \( R_f = R_0 \) must be satisfied to meet the requirement of the laser oscillation. The following equations can be deduced to describe this process.

\[
\begin{align*}
R_1 &= M_1 R_0 \\
R_i &= M_i (R_{i-1} + \Theta), \quad \text{where} \quad i = 2, 3 \\
R_i &= R_3 + \Theta = R_0 \\
\Theta &= \begin{bmatrix} 0 \\ 2\theta \end{bmatrix}, \quad R_j = \begin{bmatrix} r_j \\ r_j' \end{bmatrix}, \quad \text{where} \quad j = 0, 1, 2, 3, f \\
M_k &= T_{f,k} T_{a,k}, \quad T_{f,k} = \begin{bmatrix} 1 & 0 \\ -f_k^{-1} & 1 \end{bmatrix}, \quad \text{where} \quad k = 1, 2, 3 \\
T_o &= \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}
\end{align*}
\]

(1)

Where \( r_j \) and \( r'_j \) denotes the position and angular part of the position vector \( R_j \). The gain medium is treated as vacuum approximately in Eq. (1). The ABCD matrix which describes the light travels through a distance \( \alpha \) in vacuum is denoted as \( T_{a} \). \( T_{f,k} \) denotes the ABCD matrix, describing the light being reflected by the mirror \( M_k \). For meridian ray and sagittal ray, the relation between the focus and the radius of the mirror can be written as:

\[
\begin{align*}
\phi_{\text{meridian}} &= \frac{R \cos \phi}{2} \\
\phi_{\text{sagittal}} &= \frac{R \sec \phi}{2}
\end{align*}
\]

(2)

Where, \( \phi \) is the angle of incidence, and \( \phi = 30^\circ \) for the equilateral triangular optical path. It can be deduced from Eq. (1) that the original coordinate vector satisfies the linear equation:

\[
M R_o = U
\]

(3)
Where

\[
M = M_1 M_2 M_3 - I
\]
\[
U = -(M_1 M_2 + M_3 + I) \Theta
\]  
(4)

Where \(I\) is the \(2 \times 2\) identity matrix. By solving Eq. (3), the shift of the laser beam due to the mismatch of the mirrors can be obtained:

\[
r_o = \frac{2\theta(a - 3f_1)(a - 3f_2)f_3}{a^2 - 2a(f_1 + f_2 + f_3) + 3(f_1 f_2 + f_2 f_3 + f_3 f_1)}
\]  
(5)

\[
r_o' = \frac{2\theta(L - 3f_2)(f_1 - f_3)}{a^2 - 2a(f_1 + f_2 + f_3) + 3(f_1 f_2 + f_2 f_3 + f_3 f_1)}
\]  
(6)

By ignoring the influence of the mismatch, the transform matrix which describes the beam traveling around the optical path can be written as:

\[
T_3 = M_1 M_2 M_3 = \begin{bmatrix} A & B \\ C & D \end{bmatrix}
\]  
(7)

The following condition should be satisfied to meet the stable resonator requirement:

\[-1 \leq \frac{1}{2}(A + D) < 1\]  
(8)

It can be obtained from Eq. (7) and (8):

\[-1 < 2 - 3a\left(\frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}\right) + 2a^2\left(\frac{1}{f_1 f_2} + \frac{1}{f_2 f_3} + \frac{1}{f_3 f_1}\right) - \frac{a^3}{f_1 f_2 f_3} < 1\]  
(9)

Since the variety of the beam width is very small at different positions of the optical path, for the readout system of the RLG, the position shift part \(r_o\) is a significant factor of the power loss due to the mirror mismatch. For given resonator, the influence of the mirror mismatch can be calculated using Eq.(5) and (9). The following discussions are based on a typical triangular resonator of a RLG which is composed of 2 concave mirrors and 1 flat mirror, or 2 flat mirrors and 1 concave mirror. The detail of the beam shift due to the mirror mismatch for a triangular resonator is discussed as follows:

1) The resonator is composed of 2 concave mirrors and 1 flat mirror.

If the diaphragm is placed on the surface of one concave mirror, \(f_2 = f_3 = f_1 \rightarrow \infty\), then

\[
r_o^{(1a)} = 3f \theta
\]  
(10)

If the diaphragm is placed on the surface of the flat mirror, \(f_1 = f_2 = f_3 \rightarrow \infty\), we have

\[
r_o^{(1b)} = (3f - a) \theta
\]  
(11)

Since \(r_o^{(1a)} > r_o^{(1b)}\), the result of placing 3 diaphragms on 3 mirrors is equivalent to placing only one mirror on the curved mirror. When estimating the influence of the mirror mismatch, the larger beam shift is used. Thus, \(r_o^{(1a)}\) is introduced, and the discussion is generally based on sagittal ray. However, when building the practical RLG, the diaphragm is placed on the flat mirror to reduce the influence of mirror mismatch.

The stable condition requirement should be met for both meridian ray and sagittal ray:

\[
\left\{ \begin{array}{l}
R > \frac{4\sqrt{3}}{3} a \\
or \frac{\sqrt{3}}{2} l > R > \frac{4}{9} \sqrt{3} l
\end{array} \right.
\]  
(12)

It is difficult to restrict the radius in the second region to keep the resonator stable [4], so the following region is chosen as the stable region.
\[ R > \frac{4\sqrt{3}}{3} a \]  \hspace{1cm} (13)

(2) The resonator is composed of 2 flat mirrors and 1 concave mirror.

If the diaphragm is placed on the surface of one flat mirrors, \( f_1 = f_2 = f_3 \to \infty \), then
\[ r_o^{(2a)} = (6f - 2a)\theta \]  \hspace{1cm} (14)

If the diaphragm is placed on the surface of the concave mirror, \( f_1 = f_2 = f_3 \to \infty \), then
\[ r_o^{(2b)} = 6f\theta \]  \hspace{1cm} (15)

The stable condition is
\[ R > \sqrt{3}a \]  \hspace{1cm} (16)

According to the aforementioned approximation, the beam has a Gaussian energy distribution on the cross section and the variety of the beam at different location along the propagating direction is neglected. The two concave mirrors have the same radius R. The beam width of the fundamental mode laser is determined if the radius of the concave mirrors is fixed.

\[ w^2 = \frac{2\lambda B}{\pi \sqrt{4 - (A + D)^2}} \]  \hspace{1cm} (17)

Where, A, B and D are the ABCD matrix element in Eq. (7).

3. Power loss due to the mirror mismatch

To get the high fundamental mode volume, the diagram of the diaphragm is chosen to be \( D_a = 3.6w \). If there is no mirror angle mismatch, the energy power which can be received should be written as
\[
P_0 = \int_{\sigma_1 + \sigma_2} I(r)2\pi r \, dr = \frac{\pi w^2}{2} [1 - \exp(-\frac{2D_a^2}{w^2})] \]  \hspace{1cm} (18)

If each mirror has a angle mismatch \( \theta \), and the beam has a shift \( d \), the energy power can be written as
\[
P_0 = \int_{\sigma_i} I(r)2\pi r \, dr \]  \hspace{1cm} (19)

Where \( \sigma_i \) and \( \sigma_j \) are the regions which are displayed in Fig. 2, and show the cross section of the shifted beam.

\[ \text{Figure 2. The shift of the beam.} \]

Suppose the optical path length is 320mm, which is similar to the Honeywell GG1342 [5]. The power loss due to the mirror mismatch of the two triangular resonators is calculated. And the result is demonstrated in Fig. 3. The power is reduced as the tilting angle increases. A critical angle is defined as the tilting angle which is required to reduce the output power by 20% (the horizontal line). A vertical line of 50 micro radians defines the maximum expected mirror tilt due to the flexure of the
RLG cavity block in a 40g acceleration extreme environment [6]. If the critical angle is less than 50 micro radians, there may be a problem for the RLG in the extreme environment. It can be seen that for resonator (a) the radius should be smaller than 5 meters so that the RLG performance won’t be deteriorated significantly in the extreme environment. However, for the resonator (b), this radius should be smaller than 1 meter. It is revealed that the influence of the mirror tilting for resonator (a) is much smaller than that for resonator (b).

![Power loss due to the mirror mismatch of the triangular resonator](image)

**Figure 3.** Power loss due to the mirror mismatch of the triangular resonator (a) composed of 1 flat mirror and 2 concave mirrors, (b) composed of 2 flat mirrors and 1 concave mirror.

4. Conclusion

In this paper, the mirror-tilting induced power loss of the triangular RLG is calculated. It is revealed that the large radius may be helpful to increase the laser mode volume and increase the efficiency of mode selection, while it may depress the performance of the RLG in extreme environment with very large acceleration. Besides, it can be seen that the triangular resonator which is compose of 2 concave mirrors and 1 flat mirror has a better capability to survive in the extreme environment than the triangular resonator composed of 2 flat mirrors and 1 concave mirror.

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