Dilepton production with the SMASH model

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Abstract. In this work the SMASH model is presented ("Simulating Many Accelerated Strongly-Interacting Hadrons"), a next-generation hadronic transport approach, which is designed to describe the non-equilibrium evolution of hadronic matter in heavy-ion collisions. We discuss first dilepton spectra obtained with SMASH in the few-GeV energy range of GSI/FAIR, where the dynamics of hadronic matter is dominated by the production and decay of various resonance states. In particular we show how electromagnetic transition form factors can emerge in a transport picture under the hypothesis of vector-meson dominance.

1. Introduction

Lepton pairs are a useful probe for the regions of high density and temperature that are produced in a heavy-ion collision. They have been measured experimentally by detectors like NA60 \cite{1} and HADES \cite{2}. In this work, the production of dileptons is investigated within a hadronic transport approach, SMASH, which essentially solves the Boltzmann equation for a hadron resonance gas. Here we focus on a few aspects of the model that are important for dilepton production. A full description of SMASH will be provided in an upcoming publication \cite{3}.

At low energies (of a few GeV) the particle interactions in SMASH rely on cross sections that are dominated by the excitation and decay of hadronic resonance states. A typical dilepton spectrum contains contributions from direct and Dalitz decays of resonances such as:

\begin{itemize}
  \item $\rho, \omega, \phi \rightarrow e^+e^-$
  \item $\pi^0, \eta, \eta' \rightarrow e^+e^-\gamma$
  \item $\omega, \phi \rightarrow e^+e^-\pi$
  \item $N^*, \Delta \rightarrow e^+e^-N$
\end{itemize}

Here we mainly focus on the contributions from the $\omega$ meson. The major production channels of the $\omega$ in pp and AA collisions at low energies are assumed to be processes like $NN \rightarrow NN^* \rightarrow NN\omega$ or $NN \rightarrow \Delta N^* \rightarrow NN\pi\omega$, where the $N^* \rightarrow \omega N$ couplings for the $N^*$ states are taken from the PDG database \cite{4}. The dominant $\omega$ decay channel is $\omega \rightarrow 3\pi$, which in SMASH is treated via the two-step decay chain $\omega \rightarrow \pi\rho \rightarrow 3\pi$, the so-called GSW process (Gell-Mann, Sharp, Wagner) \cite{5}. This is done in order to be able to fulfill detailed balance with $1 \leftrightarrow 2$ and $2 \leftrightarrow 2$ processes only (avoiding the technically more challenging $1 \leftrightarrow 3$ processes). This assumption for the $\omega$ meson is also used in other related works \cite{6} \cite{7}. Further it should be noted that SMASH uses the decay-width parametrizations of Manley et al. \cite{8}, which are also employed in the GiBUU transport model \cite{9}. The above assumptions lead to the fact that in
SMASH the ω meson is exclusively produced via the decay of \( N^* \) resonances in pp collisions, while in AA collisions also the production via \( \pi \rho \) and \( \pi \pi \) collisions is possible.

In the following we first investigate the ω Dalitz decay into \( e^+e^-\pi^0 \) (which proceeds as a two-step chain via an intermediate \( \rho \)) and then the Dalitz decay \( N^* \rightarrow e^+e^-N \), which can proceed via an intermediate \( \rho \) or ω meson and is also treated as a two-step process.

2. ω Dalitz decay

We investigate the electromagnetic Dalitz decay of the ω meson, \( \omega \rightarrow \pi^0e^+e^- \), under the assumption that this decay proceeds in two steps as \( \omega \rightarrow \pi^0\rho \rightarrow \pi^0e^+e^- \). As mentioned above, this assumption is motivated mainly from the fact that the SMASH model currently emulates the \( \omega \rightarrow 3\pi \) decay by \( \omega \rightarrow \pi\rho \), in order to fulfill detailed balance without the need of introducing three-body collisions. Furthermore, this assumption is in fact equivalent to the two-step VMD treatment that was employed in earlier works for the electromagnetic decays of \( N^* \) and \( \Delta^* \) baryons [10] and later extended also to the \( \Delta(1232) \) [11].

![Diagram](image_url)

**Figure 1.** \( \omega \rightarrow \pi^0e^+e^- \) decay spectrum (left) and form factor (right) for a branching ratio of 89% for \( \omega \rightarrow \pi\rho \) and different angular momenta. The form factor is compared to data from the NA60 experiment [1].

In Fig. 1 (left) we show the dilepton mass spectrum originating from the process \( \omega \rightarrow \pi\rho \rightarrow \pi e^+e^- \), using three different values for the angular momentum \( L \) in the \( \omega \rightarrow \pi\rho \) decay and a branching ratio of \( \text{BR}(\omega \rightarrow \pi\rho) = 89\% \), such as to fully saturate the 3\( \pi \) decay. The values of \( L = 0, 1, 2 \) are the only ones allowed by angular momentum conservation, but in fact only \( L = 1 \) is allowed by parity conservation, which demands that \( L \) is odd in order to fulfill \( P_{\omega} = P_{\pi}P_{\rho}(-1)^L \), where all three mesons have negative intrinsic parity. We actually show all three cases in order to illustrate the effect of the angular momentum. The plot also shows the QED case, which corresponds to a point-like vertex in the ω Dalitz decay [12]. All curves assume that the decaying ω meson is on the pole mass of \( m_\omega = 782 \) MeV. The vertical dotted line indicates the maximum dilepton mass of \( m_\omega - m_\pi \approx 644 \) MeV.

The right panel of Fig. 1 shows the form factor of the ω Dalitz decay (which is obtained simply by dividing the “two-step VMD” curves by the QED baseline) compared to the data taken by NA60 [1] and a fit of the data using the pole approximation

\[
|F|^2 = \left(1 - m^2/\Lambda^2\right)^{-2},
\]

with \( \Lambda = 668 \) MeV. We note that the form factor is by definition normalized to one at the real-photon point (\( m = 0 \)). However, this is not given a priori for our three cases. Instead their
normalization is determined by the branching ratio, which we fix to 89% for now (i.e. the upper limit given by the $3\pi$ decay). While the $L = 2$ case is apparently in strong conflict with the data, $L = 0$ seems to be the best match for the data from a phenomenological point of view. However, it is physically forbidden, since parity conservation only allows $L = 1$. The curve for $L = 1$ touches the data only at the highest masses, while it overshoots the data over most of the spectrum and also overestimates the photon-point value. This is obviously due to the large branching ratio employed for $\omega \to \pi\rho$. From comparing the $L = 1$ form factor to the NA60 data, one has to conclude that the assumption that the whole $\omega \to 3\pi$ decay proceeds via an intermediate $\pi\rho$ state was too strong. Therefore, we reduce the $\omega \to \pi\rho$ branching, in order to avoid overshooting the form factor data. In particular we scale it down from 89% to 57% to obtain the right photon-point normalization. As shown in Fig. 2, this results in a reasonable description of the experimental data. A significant deviation is visible only for the two highest data points.

\begin{figure}[ht]
\centering
\includegraphics[width=0.5\linewidth]{omega_to_piphiphi}
\caption{$\omega \to \pi^0 e^+e^-$ transition form factor with reduced $\omega \to \pi\rho$ (p-wave) coupling.}
\end{figure}

For comparison we also show a 'simple' VMD curve, as presented in the NA60 paper [1], which only includes a plain $\rho$ propagator in pole approximation. Our treatment of $\omega \to \pi\rho \to \pi e^+e^-$ is of course also a VMD-like model, but in contrast to the 'simple' VMD it includes a hadronic form factor in the $\omega$-$\pi\rho$ vertex.

In our model the electromagnetic transition form factor of the $\omega$ meson is essentially determined by the mass distribution of the $\rho$ meson in the $\omega \to \pi\rho$ decay, which is given by the following formula (leaving out constant factors):

$$\frac{d\Gamma_{\omega \to \pi\rho}}{dm_\rho} \propto A_\rho(m_\rho) \cdot p_F(m_\omega, m_\rho, m_\pi) \cdot B^2_L(p_F R) \quad (2)$$

Following from eq. (172)-(173) in [9], this equation illustrates that there are three factors that influence the mass distribution of the $\rho$ meson in the decay of the $\omega$:

(i) the spectral function $A_\rho$ of the $\rho$ meson, which we assume to be a Breit-Wigner function with mass-dependent width [13],

(ii) the phase-space factor $p_F$, i.e. the center-of-mass momentum in the $\pi\rho$ final state and

(iii) the Blatt-Weisskopf factor $B_L$, which depends on the angular momentum $L$ and includes a cutoff parameter $R$. For $L = 1$, $B^2_1(x) = x^2/(1 + x^2)$ is obtained.
The Blatt-Weisskopf function contains both a phase-space factor as well as a hadronic form factor. The latter is characterized by the cutoff parameter $R$, which is taken to be $R = 1$ fm.

Our model suggests that the $\omega$ meson decays into $\pi \rho$ with a branching ratio of around 57%, which makes up more than half of the $3\pi$ final state and at the same time provides a rather good explanation of the electromagnetic transition form factor of the $\omega$ meson. We also show in Fig. 2 the result of the effective-Lagrangian approach of Terschlüsen et al. [14], which in fact is rather close to our result. Further we note that preliminary calculations seem to indicate that our approach can also be applied to the $\phi \rightarrow e^+ e^- \pi^0$ decay and yields results compatible with recent data from the KLOE-2 collaboration [15].

3. Baryonic Dalitz decays

Next we discuss the baryonic Dalitz decays, using as an example the process $pp \rightarrow e^+ e^- X$ at $E_{\text{kin}} = 3.5$ GeV, as measured by the HADES collaboration [2]. Our simulations do not yet include a detector-acceptance filtering, therefore we can not directly compare to data but only want to discuss qualitative effects here.

![Figure 3](image)

**Figure 3.** Dilepton mass spectra for p+p at 3.5 GeV. Left: Total inclusive spectrum. Right: $\omega \rightarrow e^+ e^-$ only, with contributions from different baryon resonances.

Figure 3 (left) shows the inclusive dilepton mass spectrum simulated with the SMASH model. We note that the $\pi^0$, $\eta$ and $\Delta$ Dalitz decays follow the standard treatment employed in many other transport approaches (see e.g. [13]), the first two with a parametrized form factor, the latter in QED approximation. The $\rho \rightarrow e^+ e^-$ contribution consists mainly of $N^* \rightarrow \rho N \rightarrow e^+ e^- N$ and $\Delta^* \rightarrow \rho N \rightarrow e^+ e^- N$ processes, since almost all $\rho$ mesons in pp collisions are produced via baryonic resonance decays in SMASH. This is why the $\rho$ contribution has a Dalitz-like shape with contributions below the $2\pi$ threshold, which has already been seen in earlier GiBUU simulations [10]. In addition to the baryonic Dalitz decays, the $\rho$ contains an additional feed-down from the $\omega \rightarrow \pi \rho$ decay, as discussed in the previous section. The most interesting feature of the SMASH dilepton spectrum is actually the $\omega \rightarrow e^+ e^-$ contribution. In addition to the peak at the $\omega$ pole mass of 782 MeV, it also contains a strong Dalitz-like tail in the low-mass region. This is due to the fact that the $\omega$ in SMASH is produced mainly via the excitation and decay of baryonic resonances in pp collisions, just like the $\rho$ meson. Due to isospin arguments, the production of the $\omega$ proceeds only via $N^*$ resonances, while the $\rho$ can also be produced from $\Delta^*$ decays.

The right panel of Fig. 3 shows the $\omega$ contribution to the dilepton spectrum, with all the contributions from different $N^*$ states. The branching ratios for $N^* \rightarrow \omega N$ are taken from the
current PDG database by default \cite{4}. We also show how the spectrum changes when using the branching ratios from a recent analysis of the Bonn-Gatchina group \cite{16}. This analysis yields strong $\omega$ couplings for sub-threshold resonances like the $N^*(1700)$ and $N^*(1720)$ that do not have an $\omega N$ mode in the current PDG, which further enhances the low-mass Dalitz tail (while the yield at the $\omega$ pole mass and above is basically unchanged). Due to the missing acceptance filtering, we can not yet comment whether these effects are compatible with experimental data. The general fact that effects from baryonic Dalitz decays exist not only for the $\rho$, but also for the $\omega$ meson is not extremely surprising. However, the sheer size of the effect is rather large in our preliminary simulations, and certainly requires further investigation.

4. Conclusions
We have shown that the first simulations of dilepton spectra with the SMASH model yield some interesting effects concerning the $\omega$ meson. Firstly, our model for the $\omega \to \pi \rho$ p-wave decay shows a rather good agreement with NA60 data for the electromagnetic $\omega$ transition form factor and suggests a branching ratio of roughly 57% for the GSW process $\omega \to \pi \rho$. Secondly, we have investigated effects of the baryonic coupling of the $\omega$ meson, which can have an impact on dilepton spectra from pp as well as AA collisions. Our preliminary results indicate that such effects could be surprisingly large, but need further investigation before final conclusions can be drawn.

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