The neutralino projector formalism for complex SUSY parameters.

G.J. Gounaris, C. Le Mouël

Department of Theoretical Physics, Aristotle University of Thessaloniki,
Gr-541 24, Thessaloniki, Greece.

Abstract

We present a new formalism describing the neutralino physics in the context of the minimal supersymmetric model (MSSM), where CP violation induced by complex $M_1$ and $\mu$ parameters is allowed. The formalism is based on the construction of neutralino projectors, and can be directly generalized to non-minimal SUSY models involving any number of neutralinos. It extends a previous work applied to the real SUSY parameter case. In MSSM, the method allows to describe all physical observables related to a specific neutralino, in terms of its CP eigenphase and three complex numbers called its "reduced projector elements". As an example, $\sigma(e^-e^+ \rightarrow \tilde{\chi}_1^0\tilde{\chi}_j^0)$ is presented in this language.

As the experimental knowledge on the neutralino-chargino sectors is being accumulated, the problem of extracting the various SUSY parameters will arise. Motivated by this, we consider various scenarios concerning the quantities that could be first measured. Analytical disentangled expressions determining the related SUSY parameters from them, are then derived, which emphasize the efficiency of the formalism.

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1 Introduction

The neutralinos are probably among the lightest supersymmetric particles [1, 2]. In fact, if R-parity is conserved, the lightest neutralino is one of the most promising candidates for the Dark Matter of the Universe [3, 4].

On the other hand, the description of neutralinos is quite complicated because, even in the minimal supersymmetric model (MSSM), it involves the diagonalization of a $4 \times 4$ matrix, which determines the neutralino masses and mixings, and thereby the various couplings [1, 2]. It is also quite possible that the true theory actually contains more than four neutralinos [5], which will further complicate the situation. Such thoughts motivate the idea to look for a formalism which will simplify the neutralino problem and, if possible, be easy to generalize to any neutralino number.

In case the Higgs mixing parameter $\mu$ and the soft breaking ones $M_1$ and $M_2$ are all real, the neutralino mass-matrix is real and symmetric, implying that the neutralino and chargino sectors are both CP conserving. The diagonalization of the real neutralino mass matrix in MSSM has been analytically studied since a long time [6, 7, 8].

Nevertheless in [9], inspired by Jarlskog’s treatment of the CKM matrix [10], we have added to the previous descriptions a new one, which describes each physical neutralino in terms of its projector matrix and its CP eigensign $\eta_j = \pm 1$. Physically, the projector matrix of a mass-eigenstate neutralino is identified with its density matrix in the space of the neutral gaugino and higgsino fields. If the neutralinos were Dirac particles, physical observables would only depend on these projectors; so that an explicit dependence on the CP eigensigns, signals contributions generated by the Majorana nature of the neutralinos [9]. Analytic expressions for these CP eigensigns and projector matrices, as well as the physical masses, in terms of the real SUSY parameters $M_1$, $\mu$ and $M_2$ were given in [9].

If $M_1$ and $\mu$ are complex, with non-trivial phases $\Phi_1$ and $\Phi_\mu$ respectively, then CP is violated in the neutralino and chargino sectors [9]. Diagonalizing the neutralino mass matrix is then more complicated; but nevertheless, explicit analytic solutions have already been constructed in [12].

The first purpose of the present work is to extend the neutralino projector formalism of [9], to the case of complex SUSY parameters. The formalism is applied to the MSSM case containing four neutralinos with complex (or real) couplings; but it can be straightforwardly extended to any neutralino number.

For the description of each physical neutralino we need, in addition to its projector matrix, one pseudo-projector matrix and one CP eigenphase. As in the real parameter case, if the neutralinos were Dirac particles, only the projectors would be needed. It is their Majorana nature that necessitates the introduction of the pseudo-projectors and CP-eigenphases. This is done in Section 2; while in Appendix A we present (for completeness) the formulae for calculating the neutralino physical masses in terms of the MSSM parameters at the tree level.

\footnote{\textit{M}_2 \textit{is chosen by convention to be real and positive. Recent analyses of the electric dipole moments suggest that $\Phi_\mu$ is probably very close to zero or $\pi$, provided the sfermions of the first two generations are not too heavy [11].}}
In Section 3 we introduce the notion of the "reduced projector elements" (RPE), which in MSSM constitute 12 complex numbers; three for each physical neutralino. The 12 RPE are not independent though; since the three RPE referring to just one physical neutralino, are sufficient to determine all the other ones. All projector, RPE and pseudo-projector matrix elements, as well as the CP-eigenphases, are expressed in terms of the neutralino mass matrix, in a way which can be immediately generalized to any neutralino number. Explicit expressions for the RPE in terms of the MSSM parameters at tree level, are also given. In the same Section 3, a new form of the necessary and sufficient condition for CP conservation in the neutralino-chargino sector, is also presented.

The RPE and the neutralino CP eigenphases offer an elegant way to describe the neutralino physical observables. As an example, the cross section for $e^{-}e^{+} \rightarrow \tilde{\chi}^{0}_{i}\tilde{\chi}^{0}_{j}$ with longitudinally polarized beams, appears in Section 4.

For the physical applications anticipated immediately after the first charginos and/or neutralinos will be discovered, it is actually not sufficient to simply express their masses and mixings in terms of the SUSY parameters at the electroweak scale. Inverse relations should also be provided, expressing the SUSY parameters in terms of the partial ino information which might become available. These relations depend of course on scenarios about such discoveries; and since the input information will be rather lacking, they will inevitably involve ambiguities, whose lifting calls for appropriate ideas.

Work in this spirit, has already appeared in [13, 14]. Concerning particularly the neutralino sector, very detail work has been presented in [15] for the CP-conserving case, and in [16, 12] for the CP-violating one. One kind of scenarios considered in these references, was based on the idea that, when the neutralinos will start being studied, the chargino parameters $M_{2}$, $|\mu|$, $\Phi_{\mu}$, and $\tan\beta$ will be already known from $\tilde{\chi}^{\pm}_{j}$ production experiments and studies of $e^{-}e^{+} \rightarrow \tilde{\chi}^{-}_{i}\tilde{\chi}^{+}_{j}$ [17]. Under these conditions, the only (generally complex) parameter to be determined from the neutralino studies is $M_{1}$. Explicit formulae for determining $M_{1}$, and suggestions on the neutralino information needed to lift the inherent ambiguities, have appeared in [15, 16, 12].

Another kind of scenarios was based on the idea that the chargino sector will not be fully known by the time the first neutralinos will start being studied. As an example, it was assumed that only the lightest chargino and its mixing angles are determined from Linear Collider (LC), together with the masses of one or two of the lightest neutralinos. Then, relations for determining the generally complex $M_{1}$ and $\mu$ parameters were suggested, in which information from $e^{-}e^{+} \rightarrow \tilde{\chi}^{0}_{1}\tilde{\chi}^{0}_{2}$ was used [12].

The second aim of this paper is to present the formalism for extracting the SUSY parameters, in the context of scenarios concerning the neutralino and chargino measurements. We essentially study the same scenarios as in [13, 14, 12]. This is done in Section 5. In all cases, disentangled expressions are given, determining the relevant SUSY parameters from the input data, and many numerical examples are presented. They are based on a recently proposed set of benchmark SUGRA models [4]; to which variations are made, in order for $M_{1}$ and $\mu$ to become complex. Scenarios involving real SUSY parameters, are also considered, based on the SUGRA benchmarks of [4]. It is checked that
the relations constructed in the various scenarios, really determine the SUSY parameters they are supposed to. Our treatment is theoretical though, simply aiming at supplying the relevant formulae. No error in the input data of the various scenarios, is taken into consideration.

The whole analysis is carried at tree level, but in principle it could be extended to any order of perturbation theory, essentially by simply using the appropriate renormalized neutralino mass matrix. In such a case of course, parameters from other SUSY sectors will enter, leading to the necessity of a more global analysis [18].

In Section 6 we give our conclusions. Finally, in Appendix B we compare to the approach of [12] for MSSM, and identify the relations between the elements of the two formalisms.

2 The neutralino projectors, pseudo-projectors and CP eigenphases.

A description of neutralinos in terms of their projectors has already been proposed in [9], where the restriction to real $M_1$, $M_2$, $\mu$ parameters was made, corresponding to CP conserving chargino and neutralino sectors.

In the present work, we extend this formalism to the most general CP-violating case in which $M_1$ and $\mu$ are taken complex, i.e.

$$M_1 \equiv e^{i\Phi_1} \bar{M}_1, \quad \mu \equiv e^{i\Phi_\mu} \bar{\mu},$$

where $\bar{M}_1 \equiv |M_1|$, $\bar{\mu} \equiv |\mu|$, while the phase angles are defined such that $-\pi < \Phi_1 \leq \pi$ and $-\pi < \Phi_\mu \leq \pi$. Without loss of generality, the soft SUSY breaking gaugino mass $M_2$, is always taken real and positive.

The neutralino mass term in the SUSY Lagrangian is written as [1]

$$\mathcal{L}_m = \frac{-1}{2} \Psi_0^\dagger C Y \Psi_0^L + \text{h.c.},$$

where the minimal complex symmetric neutralino mass-matrix is

$$Y = \begin{pmatrix} \bar{M}_1 e^{i\Phi_1} & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\bar{\mu} e^{i\Phi_\mu} \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -\bar{\mu} e^{i\Phi_\mu} & 0 \end{pmatrix},$$

$\mathcal{C} = i\gamma^2\gamma^0$ is the usual Dirac charge conjugation matrix, and $\Psi_0^L$ is the column vector describing the Left neutralino fields in the "weak basis" of the gauginos and Higgsinos as

$$\Psi_0^L = \begin{pmatrix} \bar{B}_L \\ \bar{W}_L^{(3)} \\ \bar{H}_0^L \\ \bar{H}_{2L}^0 \end{pmatrix}.$$
The corresponding Left mass eigenstate fields $\tilde{\chi}_j^0$ are related to them through the unitary transformation $U_N$ as

$$\Psi^0_{\alpha L} = \sum_{j=1}^{4} U_{N\alpha j} \tilde{\chi}_j^0 ,$$

(5)

with ($\alpha, j = 1 - 4$) being respectively the indices for the weak and mass eigenstates. The matrix $Y$ is diagonalized as

$$U_N^\dagger Y U_N = \begin{pmatrix}
  m_{\tilde{\chi}_1^0} & 0 & 0 & 0 \\
  0 & m_{\tilde{\chi}_2^0} & 0 & 0 \\
  0 & 0 & m_{\tilde{\chi}_3^0} & 0 \\
  0 & 0 & 0 & m_{\tilde{\chi}_4^0}
\end{pmatrix} ,$$

(6)

where $m_{\tilde{\chi}_j^0} (j = 1 - 4)$ describe the physical neutralino masses ordered as

$$0 \leq m_{\tilde{\chi}_1^0} \leq m_{\tilde{\chi}_2^0} \leq m_{\tilde{\chi}_3^0} \leq m_{\tilde{\chi}_4^0} .$$

(7)

Using (6) we get

$$U_N^\dagger Y U_N = \sum_{j=1}^{4} m_{\tilde{\chi}_j^0} E_j ,$$

(8)

$$U_N^\dagger Y U_N = \sum_{j=1}^{4} \sum_{k=1}^{4} m_{\tilde{\chi}_j^0} m_{\tilde{\chi}_k^0} E_j ,$$

(9)

where $E_j$ are the basic $4 \times 4$ matrices defined by $(E_j)_{ik} = \delta_{ji} \delta_{jk}$.

The neutralino projector matrices, which acting on the weak-basis fields, project out the neutralino mass eigenstate $\tilde{\chi}_j^0$, are defined by

$$P_j = P_j^\dagger = U_N E_j U_N^\dagger ,$$

(10)

so that

$$P_{j\alpha\beta} = U_{N\alpha j} U_{N\beta j}^* .$$

(11)

They satisfy the standard projector relations

$$P_i P_j = P_j \delta_{ij} , \quad Tr P_j = 1 , \quad \sum_{j=1}^{4} P_j = 1 ,$$

(12)

$$P_{j\alpha\alpha} P_{j\beta\beta} = |P_{j\alpha\beta}|^2 ,$$

(13)

where summation over the repeated indices is only done if it is explicitly indicated. Here $(i, j) = (1 - 4)$ describe the neutralino mass-eigenstate indices, while $(\alpha, \beta) = (1 - 4)$ are
the weak basis ones. Eq. (13) which follows from (11), may also be obtained by viewing the projector as the density matrix of a pure state (9). From (9, 10) we also get

$$Y^\dagger Y = \sum_{j=1}^{4} m_{\chi_j}^2 P_j$$ \hspace{1cm} (14)$$

The analytic expressions for the physical neutralino masses are obtained by solving the characteristic equation for $Y^\dagger Y$, which at tree level is given by \(\text{(A.8)}\) in Appendix A. They are ordered according to (7). Following Jarlskog (10), the neutralino projectors are then given by (3)

$$P_1 = \frac{(m_{\chi_4}^2 - Y^\dagger Y)(m_{\chi_3}^2 - Y^\dagger Y)(m_{\chi_2}^2 - Y^\dagger Y)}{(m_{\chi_4}^2 - m_{\chi_3}^2)(m_{\chi_3}^2 - m_{\chi_2}^2)(m_{\chi_2}^2 - m_{\chi_1}^2)}$$

$$P_2 = \frac{(m_{\chi_4}^2 - Y^\dagger Y)(m_{\chi_3}^2 - Y^\dagger Y)(Y^\dagger Y - m_{\chi_2}^2)}{(m_{\chi_4}^2 - m_{\chi_3}^2)(m_{\chi_3}^2 - m_{\chi_2}^2)(m_{\chi_2}^2 - m_{\chi_1}^2)}$$

$$P_3 = \frac{(m_{\chi_4}^2 - Y^\dagger Y)(Y^\dagger Y - m_{\chi_2}^2)(Y^\dagger Y - m_{\chi_1}^2)}{(m_{\chi_4}^2 - m_{\chi_3}^2)(m_{\chi_3}^2 - m_{\chi_2}^2)(m_{\chi_2}^2 - m_{\chi_1}^2)}$$

$$P_4 = \frac{(Y^\dagger Y - m_{\chi_2}^2)(Y^\dagger Y - m_{\chi_1}^2)}{(m_{\chi_4}^2 - m_{\chi_3}^2)(m_{\chi_3}^2 - m_{\chi_2}^2)(m_{\chi_2}^2 - m_{\chi_1}^2)}$$ \hspace{1cm} (15)$$

where (at tree level again) $Y^\dagger Y$ is taken from (A.3). In (13) the assumption is made, that all neutralino masses are different from each other\(^2\), and that there are only four neutralinos. The extension of this formalism to cases involving any number of neutralinos and any form of the $Y$-matrix\(^3\) is obvious (10).

As already mentioned, if the neutralinos were Dirac particles, the projectors in (13), would had been sufficient to describe any physical observable. Because of the Majorana nature of the neutralinos though, additional parameters are needed, denoted as their CP eigenphases and pseudo-projector matrices.

The CP-eigenphases, are analogous to the CP eigensigns of the real SUSY-parameter case treated in (3), and are defined as (compare (2))

$$\eta_j \equiv \frac{(U_N^*)_{1j}}{(U_N)_{1j}}$$ \hspace{1cm} (16)$$

where $|\eta_j| = 1$ is obvious. The phases in (16), chosen as\(^4\)

$$-\pi < \text{Arg}(\eta_j) \leq \pi ,$$

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\(^2\)In case of mass degeneracy, the instructions in (10) should be followed.

\(^3\)For more than four neutralinos, their physical masses may be obtained by solving \textit{e.g.} numerically, the characteristic equation for $Y^\dagger Y$.

\(^4\) The CP conserving case is obtained when Arg($\eta_j$) is either 0 or $+\pi$, (3).
are determined by those of the first row of $U_N$. Using $\eta_j$, a second unitary matrix $U^0$ is defined through\footnote{Eqs. (18, 21) look formally the same as Eqs. (7, 6) of \cite{[9]}, in which the SUSY parameters were assumed real. In the present formalism, the quantity $\tilde{\eta}_j$ of \cite{[9]} should be identified as $\tilde{\eta}_j = \sqrt{\eta_j^*}$.} \begin{equation}
abla_{(U_N)_{\alpha j}} \equiv \sqrt{\eta_j^* U^0_{\alpha j}}, \tag{17}
abla
abla \end{equation}
so that (5) may be rewritten as \begin{equation}
abla_{\alpha L} = \sum_{j=1}^{4} U_{N\alpha j} \bar{\chi}_j^0 = \sum_{j=1}^{4} \sqrt{\eta_j^* U^0_{\alpha j} \bar{\chi}_j^0}, \tag{18}
abla \end{equation}
Using (17, 11), the $P_j$ matrix elements may be expressed as \begin{equation}
P_{j\alpha\beta} = U_{N\alpha j}^* U_{N\beta j} = U^0_{\alpha j} U^0_{\beta j}^*, \tag{19}
abla \end{equation}
which clearly indicates that the projectors do not know about the CP-eigenphases.

The use of (8, 17) also give $U^0^\dagger Y U^0 = \left( \begin{array}{cccc}
\tilde{m}_{\tilde{\chi}_1^0} & 0 & 0 & 0 \\
0 & \tilde{m}_{\tilde{\chi}_2^0} & 0 & 0 \\
0 & 0 & \tilde{m}_{\tilde{\chi}_3^0} & 0 \\
0 & 0 & 0 & \tilde{m}_{\tilde{\chi}_4^0} \end{array} \right)$, \tag{20}
where the "complex" neutralino masses $\tilde{m}_{\tilde{\chi}_j^0}$ ($j = 1 - 4$) are related to the physical (positive) ones $m_{\tilde{\chi}_j^0}$ by \begin{equation}
\tilde{m}_{\tilde{\chi}_j^0} \equiv \eta_j m_{\tilde{\chi}_j^0}, \tag{21}
abla \end{equation}
and constitute a simple generalization of the "signed" masses used in the real SUSY parameter case \cite{[9]}. In addition, (20, 8) imply \begin{equation}
U^0^\dagger Y U^0 = \sum_{j=1}^{4} \tilde{m}_{\tilde{\chi}_j^0} E_j, \tag{22}
abla \end{equation}
which is analogous to (8), but involving the complex masses instead\footnote{Eqs. (18, 21) look formally the same as Eqs. (7, 6) of \cite{[9]}, in which the SUSY parameters were assumed real. In the present formalism, the quantity $\tilde{\eta}_j$ of \cite{[9]} should be identified as $\tilde{\eta}_j = \sqrt{\eta_j^*}$.}

We next turn to defining the pseudo-projector for the physical neutralino $\tilde{\chi}_j^0$ as \begin{equation}
\bar{P}_j = \bar{P}_j^\dagger = U_N^* E_j U_N^\dagger = \eta_j U^0^* E_j U^0^\dagger, \tag{23}
\end{equation}
with its matrix elements being \begin{equation}
\bar{P}_{j\alpha\beta} = U_{N\alpha j}^* U_{N\beta j}^* = \eta_j U_{\alpha j}^0 U_{\beta j}^0*, \tag{24}
\end{equation}
and satisfying \begin{equation}
\bar{P}_j^* \bar{P}_k = \delta_{jk} \bar{P}_j, \tag{25}
\end{equation}
\begin{equation}
\bar{P}_j \bar{P}_k = \bar{P}_k^\dagger \bar{P}_j = \delta_{jk} \bar{P}_j, \tag{26}
\end{equation}
\[ Y = \sum_{j=1}^{4} m_{\tilde{\chi}^0_j} \tilde{P}_j. \]  

As indicated by (12, 25, 26) and (14, 27), the pseudo-projector \( \tilde{P}_j \), shares the projector \( P_j \) property to isolate the \( \tilde{\chi}^0_j \) component, when acting on any neutralino state. This is the reason we call it pseudo-projector.

According to (24), the pseudo-projector matrix elements (contrary to the projector ones), do depend on the CP-eigenphases. In fact, (19, 24) indicate that the matrix elements of \( P_j \) and \( \tilde{P}_j \), just differ by phases, which in the CP-conserving limit (corresponding to real \( U^0 \)), are simply reduced to an \( \eta_j = \pm 1 \) overall factor.

### 3 The reduced projector elements.

The neutralino description in terms of projectors, pseudo-projectors and CP-eigenphases is, of course complete, but uneconomical. Moreover, we have not yet given the formulae from which the CP-eigenphases and the pseudo-projector matrix elements may be calculated. To cure this, we introduce below the notion of the reduced projector elements.

We first count the number of independent real parameters needed for a complete description. This is determined by the number of parameters entering the neutralino mass matrix, which at tree level has the form appearing in (3). If we temporarily ignore its specific form, and consider it as a general complex symmetric \( 4 \times 4 \) matrix, there would be 20 independent parameters. As such, we could take the four physical neutralino masses, the four CP eigenphases \( \eta_j \), and the 12 complex (but constrained\(^6\)) parameters defined as

\[ p_{j\alpha} \equiv \frac{P_{j1\alpha}}{P_{j11}}, \quad (28) \]

with \((j = 1 - 4)\) and \((\alpha = 2, 3, 4)\). In writing (28), we assume \( P_{j11} \neq 0 \), which means that there should always be some non-vanishing Bino contribution to the physical neutralino states \([3]\). We also define \( p_{j1} = 1 \), so that (28) may be extended to \( \alpha = 1 \).

The quantities \( p_{j\alpha} \), are directly calculated from the Jarlskog relation (15), and play a central role in the analysis below. We call them ”reduced projector elements”. In terms of them, the elements of \( P_j, \tilde{P}_j \) and \( U_N \) are expressed by the Ansatz

\[ P_{j\alpha\beta} = P_{j11} p_{j\alpha}^* p_{j\beta}, \quad (29) \]
\[ \tilde{P}_{j\alpha\beta} = P_{j11} \eta_j p_{j\alpha} p_{j\beta}, \quad (30) \]
\[ U_{N\alpha j} = \sqrt{\eta_j^* P_{j11} p_{j\alpha}^*}, \quad (31) \]

where

\[ P_{j11} = \frac{1}{1 + |p_{j2}|^2 + |p_{j3}|^2 + |p_{j4}|^2}. \quad (32) \]

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\(^6\)For the counting to be correct, \( p_{j\alpha} \) have to satisfy 12 real constraints discussed below.
We note that the CP-eigenphases entering \( \langle 34, 31 \rangle \), are the only quantities which are still needed in order to fully determine the neutralino properties, in terms of the initial SUSY parameters. They will be determined below using \( \langle 27 \rangle \).

The expressions \( \langle 29 - 32 \rangle \), together with \( \langle 20 - 16 \rangle \), guarantee that all projector and pseudo-projector properties \( \langle 11 \rangle \), and the unitarity of \( U_N \), are automatically satisfied, for any complex symmetric neutralino mass matrix. It is very important to remark that all these expressions, can be directly generalized to any neutralino number, and any complex symmetric mass matrix.

Returning to the counting of the independent variables for a general \( 4 \times 4 \) matrix \( Y \), we remark that the unitarity of \( U_N \) together with \( \langle 31 \rangle \) produce 12 real constraints on the 12 complex parameters \( \eta_j \), indeed provide 20 independent real parameters, matching those contained in a most general complex symmetric, \( 4 \times 4 \), neutralino mass matrix, as mentioned above.

At the tree level, in which \( \langle 3 \rangle \) is used, the number of independent parameters is reduced from 20 to the six basic SUSY parameters \( \tilde{M}_1 e^{i \Phi_1}, M_2, \bar{\mu} e^{i \Phi_2} \) and \( \tan \beta \). This allows to express all \( p_{j \alpha} \) in terms of them. Indeed using \( \langle 15, 28 \rangle \) we find \( \langle 1 \rangle \) and \( \alpha = 1 - 4 \)

\[
\begin{align*}
p_{j \alpha} &= \frac{q_{j \alpha}}{q_{j1}},
\end{align*}
\]

with

\[
\begin{align*}
q_{j1} &= -m_Z^2 - m_N^2 (m_{\tilde{N}^0}^2 - \bar{\mu}^2 s^2 \beta) (m_{\tilde{\chi}_j^0}^2 - \tilde{M}_1^2 c^4 - M_2^2 s^4 - 2\tilde{M}_1 M_2 s_W^2 c_W c_1) \\
&\quad + (m_{\tilde{\chi}_j^0}^2 - \bar{\mu}^2) [-2\tilde{M}_1 \bar{\mu} s_{2\beta} (\tilde{M}_1 M_2 c_W^2 c_\mu - (m_{\tilde{\chi}_j^0}^2 - M_2^2) s_W^2 c_\mu) \\
&\quad + m_{\tilde{\chi}_j^0}^2 ((m_{\tilde{\chi}_j^0}^2 - M_2^2) s_W^2 - 2\tilde{M}_1^2 s_W^2)] - \tilde{M}_1^2 (m_{\tilde{\chi}_j^0}^2 - M_2^2) (m_{\tilde{\chi}_j^0}^2 - \bar{\mu}^2)^2, \\
q_{j2} &= m_{\tilde{\chi}_j^0}^2 m_Z^2 s_W c_W [m_{\tilde{N}^0}^2 (m_{\tilde{\chi}_j^0}^2 - \bar{\mu}^2 s^2 \beta) \\
&\quad - (m_{\tilde{\chi}_j^0}^2 - \bar{\mu}^2) (m_{\tilde{\chi}_j^0}^2 + \bar{\mu} s_{2\beta} (\tilde{M}_1 e^{-i(\Phi_1 + \Phi_2)} + M_2 e^{i \Phi_2}))], \\
q_{j3} &= m_{\tilde{\chi}_j^0}^2 m_Z s_W [m_{\tilde{N}^0}^2 \bar{\mu} \bar{c}_\beta c_{2\beta} (\tilde{M}_1 M_2 c_W e^{i(\Phi_\mu - \Phi_1)} - (m_{\tilde{\chi}_j^0}^2 - M_2^2) s_W^2 e^{i \Phi_2}) \\
&\quad + m_{\tilde{\chi}_j^0}^2 c_\beta (m_{\tilde{\chi}_j^0}^2 - 2 \bar{\mu}^2 s^2 \beta) (M_2 - \tilde{M}_1 e^{-i \Phi_1}) \\
&\quad + (m_{\tilde{\chi}_j^0}^2 - \bar{\mu}^2) (m_{\tilde{\chi}_j^0}^2 - M_2^2) (\bar{\mu} s_{2\beta} e^{i \Phi_2})], \\
q_{j4} &= m_{\tilde{\chi}_j^0}^2 m_Z s_W [m_{\tilde{N}^0}^2 \bar{\mu} c_\beta c_{2\beta} (\tilde{M}_1 M_2 c_W e^{i(\Phi_\mu - \Phi_1)} - (m_{\tilde{\chi}_j^0}^2 - M_2^2) s_W^2 e^{i \Phi_2}) \\
&\quad + m_{\tilde{\chi}_j^0}^2 s_\beta (m_{\tilde{\chi}_j^0}^2 - 2 \bar{\mu}^2 c^2 \beta) (M_2 - \tilde{M}_1 e^{-i \Phi_1}) \\
&\quad + (m_{\tilde{\chi}_j^0}^2 - \bar{\mu}^2) (m_{\tilde{\chi}_j^0}^2 - M_2^2) (\bar{\mu} s_{2\beta} e^{i \Phi_2})],
\end{align*}
\]

where \( c_{2\beta} = \cos 2\beta \) and \( c_1 = \cos \Phi_1, c_\mu = \cos \Phi_\mu, c_{1\mu} = \cos(\Phi_1 + \Phi_\mu) \). We note that \( q_{j3} \leftrightarrow -q_{j4} \), when exchanging \( s_\beta \leftrightarrow c_\beta \). The physical neutralino masses appearing in \( \langle 34 \rangle \), are expressed in terms of the basic SUSY parameters using the formalism in Appendix A.
Using also (A.8), we thus get four equivalent relations expressing the CP-eigenphases \( \chi_j \) with the "complex" neutralino mass of \( \tilde{\chi}_j \),

\[
\sum_{\alpha=1}^4 Y_{\alpha j} \frac{p^*_j}{p_{j\beta}} = M_1 e^{i\Phi_1} - m_Z s_W (c_\beta p^*_j - s_\beta p^*_4),
\]

\[
= M_2 p^*_j + m_Z c_W (c_\beta p^*_j - s_\beta p^*_4),
\]

\[
= - \frac{\mu}{p_{j3}} e^{i\Phi_3} p^*_j + m_Z c_\beta (s_W - c_W p^*_2),
\]

\[
= - \frac{\mu}{p_{j4}} e^{i\Phi_4} p^*_j - m_Z s_\beta (s_W - c_W p^*_2).
\]

Using also (A.8), we thus get four equivalent relations expressing the CP-eigenphases \( \eta_j \) (\( j = 1 - 4 \)), in terms of the reduced projector elements and various SUSY parameters.

Inverting them, determines the fundamental SUSY parameters in terms of \( p_{j2}, p_{j3}, p_{j4} \), (for any fixed \( j \)), as

\[
M_1 e^{i\Phi_1} = \tilde{m}_j + m_Z s_W (c_\beta p^*_j - s_\beta p^*_4),
\]

\[
M_2 = \frac{p_{j3}}{p_{j2}} \left[ \frac{\tilde{m}_j - m_Z c_W (c_\beta p^*_j - s_\beta p^*_4)}{p_{j2}} \right],
\]

\[
\tilde{\mu} e^{i\Phi_\mu} = m_Z \frac{(c_\beta p_{j3} + s_\beta p_{j3})(s_W - c_W p^*_2)}{|p_{j3}|^2 - |p_{j4}|^2},
\]

with the "complex" neutralino mass of \( \tilde{\chi}_j \) given by (compare (21))

\[
\tilde{m}_j = \eta_j m_{\tilde{\chi}_j} = -m_Z \frac{(c_\beta p_{j3} + s_\beta p_{j3})(s_W - c_W p^*_2)}{|p_{j3}|^2 - |p_{j4}|^2},
\]

which may also be viewed as expressing \( \eta_j \) in terms of the reduced projector elements, the physical neutralino mass, and \( \tan \beta \).

Relations (40-43), together with their inverse (36-39), are very important. They will be heavily used below in exploring strategies for determining the SUSY parameters, under various conditions concerning the experimental knowledge of the neutralino and chargino masses.

In the true CP violating case where \( \Phi_1, \Phi_\mu \) are non-trivial, the vanishing of the imaginary part of (41) also gives

\[
\tan \beta = -\frac{t_W \text{Im}[p_{j3}(p^*_2)] + (|p_{j3}|^2 - |p_{j2}|^2 - |p_{j4}|^2) \text{Im}[p_{j3}p^*_2]}{t_W \text{Im}[p_{j4}(p^*_2)] + (|p_{j4}|^2 - |p_{j2}|^2 - |p_{j3}|^2) \text{Im}[p_{j4}p^*_2]},
\]

(44)
where $t_W = s_W/c_W$, and $\text{Im}[p]$ stands for the imaginary part of the complex numbers $p_{j\alpha}$. No such relation is obtained in the CP conserving case, in which $M_1$ and $\mu$ are both real.

Thus, in the CP-violating case, the 3 complex numbers $p_{j1}, p_{j2}, p_{j3}$ (for any fixed $j$) fully determine through \(40-44\), the 6 parameters ($M_2$, $M_1 e^{i\Phi_1}$, $\bar{\mu} e^{i\Phi_\mu}$, $\tan \beta$) entering the neutralino mass matrix $Y$ in \(3\).

The final topic in this Section concerns the condition for CP conservation in the neutralino-chargino sector; i.e. the condition for $M_1$ and $\mu$ to be real. Such a condition has already been presented in \(12\). Here we derive a new form for it. From \(33-34\), we deduce that CP conservation can only arise if the three $p_{j1}, p_{j2}, p_{j3}$ (for any fixed $j$), are all real. If this happens, then according to \(10-13\), $M_1$ and $\mu$ are also real; which subsequently implies that all reduced projector elements are in fact real; and all CP eigenphases satisfy $^7 \eta_i = \pm 1$, so that they may be called CP eigensigns. Thus, the necessary and sufficient condition for CP conservation in the neutralino-chargino sector is expressed by the equivalence

$$\text{CP conservation } \iff \text{Im}(p_{j\alpha}) = 0,$$  \hspace{2cm} (45)

for any fixed $j$, and all $\alpha$.

We note that the explicit formulae \(34, 36-44\) have been derived at the tree level, where the mass matrix is given by \(3\). It should be possible though to extend the formalism to any order, by simply using in \(15, 28, 35\), the renormalized neutralino mass matrix. We should then find that the CP-conservation condition \(45\) remains true, even if higher order effects are included.

It is amusing to compare the criterion \(45\), to the one of \(12\), where the quantities

$$D_{\alpha\beta} \equiv \sum_{j=1}^{4} U_{N_{\alpha}j} U_{N_{\beta}j}^* = \sum_{j=1}^{4} P_{j\alpha\beta} = \delta_{\alpha\beta},$$  \hspace{2cm} (46)

$$M_{ij} \equiv \sum_{\alpha=1}^{4} U_{N_{\alpha}i} U_{N_{\alpha}j}^* = \sqrt{\eta_i \eta_j} \sum_{\alpha=1}^{4} \frac{P_{i1\alpha} P_{j\alpha1}}{\sqrt{P_{i11} P_{j11}}} = \sqrt{\eta_i \eta_j} \delta_{ij},$$  \hspace{2cm} (47)

are constructed. According to \(12\), CP conservation is equivalent to requiring that all terms in each of the indicated summations in \(46, 47\), are either purely real or purely imaginary. Remembering \(29-32\), we see that this is indeed equivalent to requiring the reality of $p_{j\alpha}$. If $p_{j\alpha}$ are real, then all terms in \(46\) will also be real, while those in \(47\) will be either real or imaginary, depending on whether $\eta_i \eta_j = +1$ or $-1$.

In the CP conserving case, the solution of the real equations \(30-39\) gives

$$p_{j2} = -\frac{\tilde{m}_{\chi_0^0} - M_1}{t_W (\tilde{m}_{\chi_0^0} - M_2)}.$$

\(^7\)In the CP conserving case $\eta_i$ coincide with the corresponding quantities defined in \(9\).

\(^8\)See also Appendix B.

11
ters, are \[9\] 

The neutralino propagators to lowest order in the weak basis, or complex SUSY parameters, are 

\[
p_{j3} = -\frac{\mu (\tilde{m}_{\tilde{\chi}_j^0} - M_1)(\tilde{m}_{\tilde{\chi}_j^0} - M_2) + m_Z^2 s_\beta c_\beta (\tilde{m}_{\tilde{\chi}_j^0} - M_1 c_W^2 - M_2 s_W^2)}{m_Z s_W (\tilde{m}_{\tilde{\chi}_j^0} - M_2)(\tilde{m}_{\tilde{\chi}_j^0} s_\beta + \mu c_\beta)} ,
\]

\[
p_{j4} = \frac{\tilde{m}_{\tilde{\chi}_j^0}(\tilde{m}_{\tilde{\chi}_j^0} - M_1)(\tilde{m}_{\tilde{\chi}_j^0} - M_2) - m_Z^2 c_\beta^2 (\tilde{m}_{\tilde{\chi}_j^0} - M_1^2 - M_2^2) s_W^2}{m_Z s_W (\tilde{m}_{\tilde{\chi}_j^0} - M_2)(\tilde{m}_{\tilde{\chi}_j^0} s_\beta + \mu c_\beta)} ,
\] (48)

which determine \(p_{j2}, p_{j3}\) and \(p_{j4}\), in terms of \(M_1, M_2, \mu, \) and \(\tilde{m}_{\tilde{\chi}_j^0}\). In the CP conserving limit, we always take \(\mu\) and \(M_1\) to be real of any sign, while \(M_2 > 0\).

Eqs. (48) are analogous to (33, 34), of the more general CP violating case. Contrary to them though, (48) involve the "signed" neutralino masses \(\tilde{m}_{\tilde{\chi}_j^0}\), rather than the physical ones; compare (21). Partly because of this, the symmetry relation between \(p_{j3}\) and \(p_{j4}\) observed in (34), is lost in (48).

We also note that since (44) is not obtained in the CP conserving case, \(\tan \beta\) must be known from some other means, in order to use the three real \((p_{j1}, p_{j2}, p_{j3})\) for determining \(M_1, M_2\) and \(\mu\) from (10-13).

Before concluding this section, we recapitulate the procedure for diagonalizing the neutralino mass matrix, in the most general complex parameter case. For this, one first constructs the physical neutralino masses in terms of the SUSY parameters, using the formalism in Appendix A. Then, (13, 28) or (33, 34), allow to construct the projector and reduced projector elements, from which the pseudo-projectors and the CP-eigenphases may be obtained through (50) and any of (36-39, 43). If needed, \(U_N\) may then be obtained from (44).

The results in the CP conserving limit, may be obtained by simply putting \((\Phi_1, \Phi_\mu) = 0\) or \(\pi\), in the aforementioned formulae. Alternatively, they can also be derived using (48) and the "signed" neutralino masses calculated in the Appendix of [2].

4 The \(e^- e^+ \to \tilde{\chi}_i^0 \tilde{\chi}_j^0\) cross section.

The neutralino propagators to lowest order in the weak basis, or complex SUSY parameters, are [9]

\[
\langle 0 | T \Psi_{aL}^0(x) \Psi_{\beta L}^{0\dagger}(y) | 0 \rangle = -\sum_{j=1}^4 m_{\tilde{\chi}_j^0}^2 P_{j\alpha\beta}^* \Delta_F(x - y; m_{\tilde{\chi}_j^0}) C \frac{(1 - \gamma_5)}{2} ,
\] (49)

\[
\langle 0 | T \Psi_{aL}^{0\dagger}(x) \Psi_{\beta L}^0(y) | 0 \rangle = \sum_{j=1}^4 m_{\tilde{\chi}_j^0} P_{j\alpha\beta} \Delta_F(x - y; m_{\tilde{\chi}_j^0}) C \frac{(1 - \gamma_5)}{2} ,
\] (50)

\[
\langle 0 | T \Psi_{aL}^0(x) \Psi_{\beta L}^0(y) | 0 \rangle = \sum_{j=1}^4 P_{j\alpha\beta} S_F^{(1)}(x - y; m_{\tilde{\chi}_j^0}) \gamma_0 \frac{(1 - \gamma_5)}{2} ,
\] (51)

\footnote{As in [1], the "complex masses" of (21) are simply called "signed masses" in the CP conserving case.}
and

\[ \Delta_F(x - y; m) = i \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} \frac{1}{k^2 - m^2 + i\epsilon}, \]

\[ S_F^{(1)}(x - y; m) = i \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} \frac{k}{k^2 - m^2 + i\epsilon}. \] (52)

If the neutralinos were Dirac particles, only the propagator (51) would have been allowed, implying that the projector matrix elements would have been sufficient to express all physical observables. The Majorana nature of the neutralinos though, introduces the additional propagators (49, 50), and thereby necessitates the introduction of the pseudo-projectors also.

Using subsequently (29, 30, 32), we conclude that an elegant way to express the physical observables related to a neutralino \( \tilde{\chi}^0 \), is in terms of its three reduced projector elements \( p_{j\alpha} (\alpha = 2, 3, 4) \), its CP eigenphase \( \eta_j \) and its physical mass.

As an example, we consider the differential cross section for \( e^-e^+ \rightarrow \tilde{\chi}^0 \tilde{\chi}^0 \), using longitudinally polarized \( e^- \) and \( e^+ \) beams, with the polarizations denoted by \( \lambda_1 \) and \( \lambda_2 \) respectively\(^{10}\). The contributions to this process at tree level arise from \( s \)-channel \( Z \)-exchange, and \( t \)- and \( u \)-channel \( \tilde{e}_L \) and \( \tilde{e}_R \) exchanges. The differential cross section may then be written as \(^3\)

\[ \frac{d\sigma(e^-e^+ \rightarrow \tilde{\chi}^0\tilde{\chi}^0)}{dt} = \frac{\alpha^2 \pi}{s^2(1 + \delta_{ij})} P_{i11} P_{j11} \left[ \Sigma_Z(\lambda_1, \lambda_2) + \Sigma_{\tilde{e}_L}(\lambda_1, \lambda_2) + \Sigma_{\tilde{e}_R}(\lambda_1, \lambda_2) \right. \]

\[ + \left. \Sigma_{Z\tilde{e}_L}(\lambda_1, \lambda_2) + \Sigma_{Z\tilde{e}_R}(\lambda_1, \lambda_2) \right] , \] (53)

with the r.h.s. terms representing respectively the \( Z \)-, \( \tilde{e}_L \)- and \( \tilde{e}_R \)-square contributions, and the \( Z\tilde{e}_L \)- and \( Z\tilde{e}_R \)-interferences\(^{11}\). These are written as

\[ \Sigma_Z(\lambda_1, \lambda_2) = \frac{1}{8s_W^4c_W^4(s - m_{Z}^2)} [(g_{ve}^2 + g_{ae}^2)(1 - \lambda_1 \lambda_2) - 2g_{ve}g_{ae}(\lambda_1 - \lambda_2)] \]

\[ \cdot \left[ |p^*_i p_{j3} - p^*_j p_{i4}|^2 [(t - m_{\tilde{\chi}^0_i}^2)(t - m_{\tilde{\chi}^0_j}^2) + (u - m_{\tilde{\chi}^0_j}^2)(u - m_{\tilde{\chi}^0_i}^2)] \right. \]

\[ - 2sm_{\tilde{\chi}^0_i}m_{\tilde{\chi}^0_j} Re(\eta_i \eta_j [p^*_i p_{j3} - p^*_j p_{i4}]^2) \left. \right] , \] (54)

\[ \Sigma_{\tilde{e}_L}(\lambda_1, \lambda_2) = \frac{1}{4s_W^4c_W^4} \frac{(1 - \lambda_1)(1 + \lambda_2)}{4} \]

\[ \cdot \left[ |(c_W p_{i3} + s_W \cdot c_W p_{j2} + s_W) [t - m_{\tilde{\chi}^0_i}^2)(t - m_{\tilde{\chi}^0_j}^2) + (u - m_{\tilde{\chi}^0_j}^2)(u - m_{\tilde{\chi}^0_i}^2)] \right. \]

\[ \left. \cdot \frac{(t - m_{\tilde{\chi}^0_i}^2)^2}{(t - m_{\tilde{\chi}^0_i}^2)^2 + (u - m_{\tilde{\chi}^0_i}^2)^2} \right] \]

\(^{10}\)Transverse \( e^\pm \) polarizations are irrelevant, if the azimuthal distribution of the neutralino production plane is integrated over.

\(^{11}\)To the extent that we neglect the electron mass, there is never any \( \tilde{e}_L\tilde{e}_R \)-interference; neither any Higgsino-e\( \bar{e} \) coupling.
powers of $\Delta$ 

Close to threshold, the neutralino-pair production cross section can be expanded in powers of $\Delta s \equiv s - (m_{\chi_i^0}^2 + m_{\chi_j^0}^2)/2$, as 

\[
\sigma(e^- e^+ \rightarrow \chi_i^0 \chi_j^0) = \frac{\alpha^2 \pi}{s_W^4 c_W^4 (1 + \delta_{ij}) (m_{\chi_i^0}^2 + m_{\chi_j^0}^2)^2} \left[ P_{i11} P_{j11} \sqrt{\Delta s} \right]
\]

\[
\left[ \left( g_{ve} + g_{ae} \right) (1 - \lambda_1) (1 - \lambda_2) \right] \frac{(m_{\chi_i^0}^2 m_{\chi_j^0}^2)^{3/2}}{s_W^4 c_W^4 (1 + \delta_{ij}) (m_{\chi_i^0}^2 + m_{\chi_j^0}^2)^2} P_{i11} P_{j11} \sqrt{\Delta s}
\]

\[
+ \frac{(1 - \lambda_1) (1 + \lambda_2)}{4} \frac{2 \mathcal{J}_{ij}^2}{(m_{\chi_i^0}^2 m_{\chi_j^0}^2 + m_{\chi_i^0}^2 m_{\chi_j^0}^2)^2} + \frac{(1 + \lambda_1) (1 - \lambda_2)}{4} \frac{32 s_W^4 \mathcal{T}_{ij}^2}{(m_{\chi_i^0}^2 m_{\chi_j^0}^2 + m_{\chi_i^0}^2 m_{\chi_j^0}^2)^2}
\]

\[
+ \frac{(g_{ve} + g_{ae}) (1 - \lambda_1) (1 + \lambda_2)}{4} \frac{4 \mathcal{J}_{ij} \mathcal{K}_{ij}}{(m_{\chi_i^0}^2 m_{\chi_j^0}^2 + m_{\chi_i^0}^2 m_{\chi_j^0}^2)^2}
\]

\[
- \frac{(g_{ve} - g_{ae}) (1 - \lambda_1) (1 - \lambda_2)}{4} \frac{16 s_W^2 \mathcal{T}_{ij} \mathcal{K}_{ij}}{(m_{\chi_i^0}^2 m_{\chi_j^0}^2 + m_{\chi_i^0}^2 m_{\chi_j^0}^2)^2}
\]

\[
+ O(\Delta s)^{3/2}
\]

where $g_{ve} = -0.5 + 2 s_W^2$ and $g_{ae} = -0.5$ are the vector and axial $Zee$-couplings.
where

\[ I_{ij} = \text{Im}[\sqrt{\eta_i^* \eta_j}], \]
\[ J_{ij} = \text{Im}[\sqrt{\eta_i^* \eta_j} (c_W p_{i2}^2 + s_W)(c_W p_{j2} + s_W)], \]
\[ K_{ij} = \text{Im}[\sqrt{\eta_i^* \eta_j} (p_{i3}^* p_{j3} - p_{i4}^* p_{j4})]. \] (60)

Therefore, the leading behaviour \( \sim \Delta s^{1/2} \), can only be realized if the produced neutralinos are not identical, and such that at least one of \( (I, J, K,) \) of (60), is non vanishing. The next to leading term varies like \( \Delta s^{3/2} \).

In the special case that CP is conserved (i.e. \( p_{j\alpha} \) real), the above conditions allow the appearance of the leading \( \Delta s^{1/2} \) term, only if \( \eta_i = -\eta_j = \pm 1 \). \[ \text{[12]} \]

But, if instead CP is violated and the reduced projector elements are complex, then (60) could allow the appearance of the leading threshold term \( \Delta s^{1/2} \), even if \( \eta_i = \eta_j \), provided of course that the two neutralinos continue to be different.

Finally, we note that the vanishing of the leading \( \Delta s^{1/2} \) term in the production of two identical neutralinos, may be simply viewed as a consequence of the fact that the \( (e^-, e^+) \) helicities have to be opposite to each other. The reason is that the total angular momentum of the two neutralinos would then be at least one, which forces the S-wave neutralino wave function\[ ^{12} \] to be antisymmetric, implying a threshold behaviour like \( \Delta s^{3/2} \).

5 Determining SUSY parameters.

In this section we address the problem of determining the MSSM SUSY parameters, under the various conditions that will inevitably arise when some (presumably the lightest) charginos and neutralinos will start being discovered. In all cases, we assume that \( \tan \beta \) and some knowledge of the chargino parameters \( (M_2, \bar{\mu}, \Phi_\mu) \), will have been established from chargino and other measurements, before the neutralinos start being studied. Then, neutralino measurements may supply information on their physical masses, reduced projector elements and CP eigenphases.

Our general procedure starts from (37-39) which are solved for \( p_{j2}, p_{j3}, p_{j4} \) \( (j = 1 - 4) \), without introducing any explicit dependence on the Bino parameters \( M_1, \Phi_1 \). This gives

\[ p_{j2} = t_W + (m_{\tilde{\chi}_j^0}^2 - \bar{\mu}^2) Z_j^*, \] (61)
\[ p_{j3} = m_W (c_\beta \tilde{m}_{\tilde{\chi}_j^0}^* Z_j + s_\beta \bar{\mu} e^{i\Phi_\mu} Z_j^*), \] (62)
\[ p_{j4} = -m_W (s_\beta \tilde{m}_{\tilde{\chi}_j^0}^* Z_j + c_\beta \bar{\mu} e^{i\Phi_\mu} Z_j^*), \] (63)

where the auxiliary complex numbers \( Z_j \) \( (j = 1 - 4) \) are defined as

\[ Z_j = Z_{j1} - \tilde{m}_{\tilde{\chi}_j^0} Z_{j2}, \] (64)

\[ ^{12} \text{For S-wave, the total neutralino spin should be one. Neutralino states with higher orbital angular momentum cannot produce a } \Delta s^{1/2} \text{ term.} \]
with

\[ Z_{j1} = t_W D_j^{-1}(M_2^2 - m_{\chi_j^0}^2)(m_{\chi_j^0}^2 - \mu^2) + m_W^2 (m_{\chi_j^0}^2 + M_2 \mu e^{i\phi_s} s_{2\beta}) \]  \hspace{1cm} (65)

\[ Z_{j2} = m_Z^2 s_W c_W D_j^{-1}(M_2 + \mu e^{i\phi_s} s_{2\beta}) \]  \hspace{1cm} (66)

\[ D_j = (m_{\chi_j^0}^2 - \mu^2)(m_{\chi_j^0}^2 - M_2^2) + m_W^2 [2(\mu^2 - m_{\chi_j^0}^2)(m_{\chi_j^0}^2 + M_2 \mu s_{2\beta} c_{\mu}) + m_W^2 (m_{\chi_j^0}^2 - \mu^2 s_{2\beta}^2)] \]  \hspace{1cm} (67)

Note that \( Z_{j1}, Z_{j2}, D_j \) do not depend on the CP-eigenphase \( \eta_j \); but only on the physical neutralino mass \( m_{\tilde{\chi}^0_j} = |\tilde{m}_{\chi_j^0}| \).

Substituting (64) in (63) we then obtain

\[ M_1 = M_1 e^{i\phi_1} = A_j \eta_j + B_j \]  \hspace{1cm} (68)

where

\[ A_j = m_{\chi_j^0} D_j^{-1}[(m_{\chi_j^0}^2 - M_2^2)(m_{\chi_j^0}^2 - \mu^2)^2 + m_Z^2 m_W^2 (m_{\chi_j^0}^2 - \mu^2 s_{2\beta}^2)] \]

\[ B_j = -D_j^{-1} m_Z^2 s_W^2 [(m_{\chi_j^0}^2 - M_2^2)(m_{\chi_j^0}^2 - \mu^2) \mu e^{-i\phi_s} s_{2\beta} + m_W^2 M_2 (m_{\chi_j^0}^2 - \mu^2 s_{2\beta}^2)] \]  \hspace{1cm} (69)

and \( D_j \) is given in (67).

The set of equations (63-69) determines \( M_1 \) including its phase, under various conditions concerning the knowledge of the neutralino and chargino sectors.

If CP is conserved; then \( p_{j\alpha}, \eta_j \) and the SUSY parameters are all real, so that (61-63, 68) become

\[ p_{j2} = \frac{m_Z m_W (\tilde{m}_{\chi_j^0} + s_{2\beta} \mu) s_W}{(M_2 - \tilde{m}_{\chi_j^0})(\tilde{m}_{\chi_j^0}^2 - \mu^2) + m_W^2 (\tilde{m}_{\chi_j^0} + s_{2\beta} \mu)} \]  \hspace{1cm} (70)

\[ p_{j3} = -\frac{m_Z (M_2 - \tilde{m}_{\chi_j^0})(c_{\beta}\tilde{m}_{\chi_j^0} + s_{\beta} \mu) s_W}{(M_2 - \tilde{m}_{\chi_j^0})(\tilde{m}_{\chi_j^0}^2 - \mu^2) + m_W^2 (\tilde{m}_{\chi_j^0} + s_{2\beta} \mu)} \]  \hspace{1cm} (71)

\[ p_{j4} = \frac{m_Z (M_2 - \tilde{m}_{\chi_j^0})(s_{\beta}\tilde{m}_{\chi_j^0} + c_{\beta} \mu) s_W}{(M_2 - \tilde{m}_{\chi_j^0})(\tilde{m}_{\chi_j^0}^2 - \mu^2) + m_W^2 (\tilde{m}_{\chi_j^0} + s_{2\beta} \mu)} \]  \hspace{1cm} (72)

\[ M_1 = \tilde{m}_{\chi_j^0} + m_Z s_W (p_{j3} c_{\beta} - p_{j4} s_{\beta}) \]

\[ = \frac{\tilde{m}_{\chi_j^0} (\tilde{m}_{\chi_j^0} - M_2)(\tilde{m}_{\chi_j^0}^2 - \mu^2) - m_W^2 (\tilde{m}_{\chi_j^0} + s_{2\beta} \mu)(\tilde{m}_{\chi_j^0} - M_2 s_W^2)}{(\tilde{m}_{\chi_j^0}^2 - \mu^2)(\tilde{m}_{\chi_j^0} - M_2) - m_W^2 (\tilde{m}_{\chi_j^0} + s_{2\beta} \mu)} \]  \hspace{1cm} (73)

where the "signed" neutralino masses defined in (73) (of course for real \( \eta_i \) ) appear.

We next consider various scenarios in the CP conserving and CP violating cases for the neutralino-chargino sectors.
5.1 CP-conserving scenarios

- The scenario S1.
  This is a rather extreme scenario considered in [15], where we assume that the real MSSM parameters \((M_2, \tan \beta, \mu)\) are known from e.g. the chargino sector. In addition to them, the signed mass \(\tilde{m}_{\tilde{\chi}_i^0} = \eta_i m_{\tilde{\chi}_i^0}\) of one neutralino \(\tilde{\chi}_i^0\) is also assumed known. Knowledge of the value of the index \(i\) determining the ordering of \(\tilde{\chi}_i^0\), is not assumed. Then, (73) allows us to reconstruct \(M_1\), including its sign.

  It may turn out though, that only the physical mass of \(\tilde{\chi}_i^0\) is known, and not its CP eigensign \(\eta_i\). As an example, we consider the model SP1b of Table 1, which is essentially identical to the Snowmass benchmark SPS1b [4]. In this model, one neutralino is almost a pure Bino of mass \(|M_1|\), another one is approximately a Wino of mass \(M_2\), and the last two are almost degenerate Higgsinos of mass \(\sim |\mu|\). This property seems rather stable under model variations where \((M_2, \tan \beta, \mu)\) are kept fixed, while the eigensign \(\eta_i\) is changed. To see this, we present in Fig.1 the values of \((|M_1| - m_{\tilde{\chi}_i^0})/2\) predicted by (73) as a function of \(m_{\tilde{\chi}_i^0}\), for the two choices \(\eta_i = \pm 1\). It seems from this figure that if \((M_2, \mu)\) are known, and the mass of the considered neutralino is not very close to either \(M_2\) or \(|\mu|\); then it should be almost identical to \(|M_1|\), for any \(\eta_i\) sign.

  For the same data, we show in Fig.2, the diagonal elements of \(P_i\) as a function of \(m_{\tilde{\chi}_i^0}\), for the two possible values \(\eta_i = \pm 1\). This figure confirms that if \(m_{\tilde{\chi}_i^0}\) is close to \(|M_1|\), then \(\tilde{\chi}_i^0\) is approximately a pure Bino state.

  The lack of knowledge of \(\eta_i\) does not seem important, at least in the above example. Nevertheless, the ambiguity induced by it may be lifted, provided some additional information is used:

  One possibility could be that a second neutralino with mass \(m_{\tilde{\chi}_j^0}\) \((j \neq i)\) is also known. The ambiguity is then lifted by comparing the values of \(M_1\) obtained when trying all possible choices for \(\eta_i = \pm 1\) and \(\eta_j = \pm 1\). The correct \((\eta_i, \eta_j)\) choice, is the one which leads to the same \(M_1\) prediction.

  Another possibility arises from the fact that typically the \(M_1\) values, obtained for \(\eta_i = \pm 1\), are of opposite signs. Therefore, if a particular sign for \(M_1\) is favored (as e.g. in mSUGRA models with gaugino unification for which \(\text{Sign}[M_1] = \text{Sign}[M_2]\) is expected), the ambiguity is again lifted.

  A third possibility of determining \(\eta_j\), is from \(\sigma(e^- e^+ \rightarrow \tilde{\chi}_j^0 \tilde{\chi}_i^0)\) for \(j \neq i\), or from chargino decays to neutralinos, if such measurements exist.

- The scenario S2.
  In this scenario, it is assumed that the first quantities to be measured will be the physical masses of one chargino and two neutralinos [14]. As such, we take the mass of the lightest chargino \(\tilde{\chi}_1^\pm\), and the two lightest neutralino masses. If the signed neutralino masses \(\tilde{m}_{\tilde{\chi}_1^0}\) and \(\tilde{m}_{\tilde{\chi}_2^0}\) (compare (21)), were also known\(^{13}\), then (73) would give the \(M_2\)-quadratic equation

\[
a^{(0)}_M + a^{(1)}_M M_2 + a^{(2)}_M M_2^2 = 0 ,
\]

\(^{13}\)We come back to the determination of the signs \(\eta_1\) and \(\eta_2\) at the end of this subsection.
whose coefficients

\[ a^{(0)}_M = \tilde{m}_{\chi_1^0} \tilde{m}_{\chi_2^0} (\tilde{m}_{\chi_1^0} - \tilde{m}_{\chi_2^0}) (\tilde{m}_{\chi_1^0}^2 - \mu^2) (\tilde{m}_{\chi_2^0}^2 - \mu^2) + m_Z^2 [\tilde{m}_{\chi_1^0} \tilde{m}_{\chi_2^0} (\tilde{m}_{\chi_1^0} - \tilde{m}_{\chi_2^0}) (\tilde{m}_{\chi_1^0}^2 - \mu^2) + (1 + \alpha_W^2) \mu^2 + m_{Z^2}^2 \tilde{m}_{\chi_2^0}] + \mu s_{2\beta} (\tilde{m}_{\chi_1^0} - \tilde{m}_{\chi_2^0}) (\tilde{m}_{\chi_1^0}^2 - \mu^2) (\tilde{m}_{\chi_2^0}^2 - \mu^2) - c_W^2 (\tilde{m}_{\chi_1^0}^2 + \tilde{m}_{\chi_2^0}^2 - \mu^2 - m_Z^2)] + \mu m_Z^2 s_{2\beta} \alpha_W \]

\[ a^{(1)}_M = (\tilde{m}_{\chi_1^0}^2 - \tilde{m}_{\chi_2^0}^2) (\tilde{m}_{\chi_1^0}^2 - \mu^2) (\tilde{m}_{\chi_2^0}^2 - \mu^2) + m_Z^2 [(\tilde{m}_{\chi_1^0}^2 - \tilde{m}_{\chi_2^0}^2) (\alpha_W^2 - s_W^2) \tilde{m}_{\chi_1^0} \tilde{m}_{\chi_2^0} - \mu^2) + \mu s_{2\beta} (\tilde{m}_{\chi_1^0} - \tilde{m}_{\chi_2^0}) (\tilde{m}_{\chi_1^0}^2 - \mu^2) + m_Z^2 s_W (\tilde{m}_{\chi_1^0}^2 - \mu^2 - \tilde{m}_{\chi_2^0}^2 - \mu^2) + \mu s_{2\beta} (\tilde{m}_{\chi_1^0} - \tilde{m}_{\chi_2^0}) (\tilde{m}_{\chi_1^0}^2 - \mu^2)] , \]

\[ a^{(2)}_M = (\tilde{m}_{\chi_1^0}^2 - \tilde{m}_{\chi_2^0}^2) (\tilde{m}_{\chi_1^0}^2 - \mu^2) (\tilde{m}_{\chi_2^0}^2 - \mu^2) + m_Z^2 s_W (\tilde{m}_{\chi_1^0}^2 - \mu^2 - \tilde{m}_{\chi_2^0}^2 - \mu^2) + \mu s_{2\beta} (\tilde{m}_{\chi_1^0}^2 - \tilde{m}_{\chi_2^0}^2) , \]

depend on \( \mu \) in a complicated way.

A second independent quadratic \( M_2 \)-equation, with coefficients depending on \( \tan \beta, \mu \) and \( \text{e.g.} \) the lightest chargino mass\(^{[4]} \) \( m_{\chi_1^+} \), may be derived by noting that the chargino mass matrix in the CP conserving case implies

\[ m_{\chi_1^+}^2 + m_{\chi_2^+}^2 = M_2^2 + \mu^2 + 2 m_W^2 , \]

\[ m_{\chi_1^0}^2 + m_{\chi_2^0}^2 = (M_2 \mu - m_W^2 s_{2\beta})^2 . \]

Eliminating the heavier chargino mass then leads to

\[ M_2^2 (m_{\chi_1^+}^2 - \mu^2) + 2 \mu m_W^2 s_{2\beta} M_2 - m_{\chi_1^+}^4 (\mu^2 + 2 m_W^2) - m_{\chi_1^+}^2 (M_2 \mu - m_W^2 s_{2\beta}) = 0 . \]

Combining (74, 75, 76), we find

\[ M_2 = \frac{a^{(0)}_M (m_{\chi_1^+}^2 - \mu^2) + a^{(2)}_M (m_{\chi_1^+}^4 - m_{\chi_1^+}^2 (\mu^2 + 2 m_W^2) + m_W^2 s_{2\beta})}{a^{(1)}_M (m_{\chi_1^+}^2 - \mu^2) - 2 a^{(2)}_M \mu m_W s_{2\beta}} , \]

and the compatibility relation

\[ [a^{(2)}_M (m_{\chi_1^+}^4 - m_{\chi_1^+}^2 (\mu^2 + 2 m_W^2) + m_W^2 s_{2\beta}) + a^{(0)}_M (m_{\chi_1^+}^2 - \mu^2)]^2 - [a^{(1)}_M (m_{\chi_1^+}^2 - \mu^2) - 2 a^{(2)}_M \mu m_W s_{2\beta}]^2 = 0 . \]

Our next task is to solve (78) for \( \mu \), in terms of \( \tan \beta \), the lightest physical chargino mass \( m_{\chi_1^+} \), and the signed neutralino masses \( \tilde{m}_{\chi_1^0} \) and \( \tilde{m}_{\chi_2^0} \). In principle there could be many solutions, since (79) is of order 12 in \( \mu \). Their number is reduced though by restricting to real \( \mu \), such that (78) leads to positive \( M_2 \). Further restrictions arise by calculating also the masses of the other charginos and neutralinos, and demanding that those we have started with, are indeed the lightest ones.

\(^{[4]}\text{In our convention, the physical (positive) chargino masses are ordered so that } m_{\chi_1^+} < m_{\chi_2^+}.\)
As an example we consider again the benchmark mSUGRA models $SP1b$ of [4] and $D$ of [3], whose relevant electroweak scale parameters, calculated using the SUSPECT code [16], are presented in Table 1. We intend to look at how these models are reconstructed (always of course at the electroweak scale), using in the context of scenario S2, the formalism developed above. Solving (79) for $\mu$ in the $SP1b$ ($D$) case, we find 8 (10) real solutions, out of which only 6 (7) are consistent with of our starting convention $M_2 > 0$. Finally, when imposing the additional requirement that the solutions respect the starting mass hierarchies; then only two remain for each of the $SP1b$ and $D$ cases. Thus, apart from the models $SP1b$ and $D$ which are of course reconstructed, two new solutions are found denoted as $SP1b'$ and $D'$ in Table 1. By construction therefore, the pairs of models ($SP1b$, $SP1b'$) and ($D$, $D'$), have the same mass for $\tilde{\chi}_1^\pm$, and the same signed masses for $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, as well as the same value for $\tan \beta$. But their derived $M_1$, $M_2$, $\mu$ parameters and the masses for $\tilde{\chi}_2^\pm$, $\tilde{\chi}_3^0$, $\tilde{\chi}_4^0$, are generally different.

Table 1: Electroweak scale parameters for some CP conserving models; (Masses in GeV.)

| Model | $\tan \beta$ | $\mu$ | $M_1$ | $M_2$ | $m_{\tilde{\chi}_1^\pm}$ | $m_{\tilde{\chi}_2^\pm}$ | $m_{\tilde{\chi}_1^0}$ | $m_{\tilde{\chi}_2^0}$ | $m_{\tilde{\chi}_3^0}$ | $m_{\tilde{\chi}_4^0}$ |
|-------|---------------|-------|-------|-------|--------------------------|--------------------------|-----------------|-----------------|-----------------|-----------------|
| $SP1b$ | 30            | 496   | 166   | 310   | 297                      | 516                      | 164             | 297             | -501            | 516             |
| $SP1b'$ | 30           | -398  | 166   | 320   | 297                      | 431                      | 164             | 297             | -405            | 430             |
| $SP1b''$ | 30          | 298   | -168  | 1156  | 297                      | 1162                     | -164            | 297             | -307            | 1162            |
| $SP1b'''$ | 30        | -297  | -170  | 1627  | 297                      | 1631                     | -164            | 297             | -306            | 1631            |
| $D$    | 10            | -659  | 225   | 418   | 411                      | 673                      | 224             | 411             | -664            | 671             |
| $D'$   | 10            | -430  | 226   | 519   | 411                      | 546                      | 224             | 411             | -436            | 546             |
| $D''$  | 10            | -765  | -226  | 415   | 411                      | 776                      | -224            | 411             | -771            | 773             |
| $D'''$ | 10            | 639   | -225  | 425   | 411                      | 658                      | -224            | 411             | -644            | 657             |
| $D^{IV}$ | 10        | 416   | -226  | 973   | 411                      | 982                      | -224            | 411             | -422            | 982             |
| $D^V$  | 10            | -411  | -229  | 2673  | 411                      | 2676                     | -224            | 411             | -418            | 2676            |

We next turn to the CP eigensigns $\eta_1 = \pm 1$ and $\eta_2 = \pm 1$, which are needed in order to determine the signed masses from the physical ones. The lack of knowledge of these signs creates additional ambiguities whose characteristics should be studied on a-case-by-case basis. The aforementioned models $SP1b$, $SP1b'$, $D$ and $D'$ in Table 1, correspond to the choice $\eta_1 = \eta_2 = +1$.

To study the consequences of the eigensign choice, we consider in Table 1 variations of the $SP1b$ and $D$ models in which the same physical masses for the lightest chargino and the two lightest neutralino states are used, but we have set instead $\eta_1 = -\eta_2 = -1$. We then find two new solutions for ($M_1$, $M_2$, $\mu$) and the heavier chargino and neutralino masses, denoted as $SP1b''$ and $SP1b'''$ and displayed in Table 1. The same procedure for the $D$-case, produced four new solutions, denoted as $D''$, $D'''$, $D^{IV}$, $D^V$ and shown in Table 1. We note that in the models $SP1b''$, $SP1b'''$, $D^{IV}$ and $D^V$, the second and

\[15\] The interest in this model is that it features a negative $\mu$.

\[16\] Of course the conclusions below do not depend on the specific use of this code.
third neutralinos are almost degenerate and considerably smaller than the heaviest one. Another feature is that the two chargino masses tend to be very different.

In the same spirit, we have also tried variations to the $SP1b$ and $D$ models with $\eta_1 = -\eta_2 = 1$, but no consistent solution was found. It seems therefore that for an initial choice of the physical masses, not all $\eta_j$-signs are generally possible.

An efficient way of constraining $\eta_1$, $\eta_2$, and the other model parameters, is by looking at the $\sigma(e^- e^+ \rightarrow \tilde{\chi}^0_1 \tilde{\chi}^0_2)$ cross section, preferably using polarized beams. In Figs. 3a and 3b we present these cross sections for the CP-conserving models ($SP1b$, $SP1b^\prime$, $SP1b''$) and ($D$, $D''$, $D'''$) respectively. The necessary input data are given in Table 1, while the reduced projector elements are calculated either from ([70]-[72]) or ([18]). For the $\tilde{e}_L$, $\tilde{e}_R$ masses we used $m_{\tilde{e}_L} = 338$ GeV, $m_{\tilde{e}_R} = 251.3$ GeV for the $SP1b$-type models [14], and $m_{\tilde{e}_L} = 385.8$ GeV, $m_{\tilde{e}_R} = 240.2$ GeV for the $D$-type ones [8].

As seen in Figs. 3a, b, there is a striking difference between the cross sections for the models ($SP1b$, $SP1b^\prime$, $D$) which have $\eta_1 \eta_2 = 1$, and the models ($SP1b''$, $D''$, $D'''$) which have $\eta_1 \eta_2 = -1$. Close to the $\tilde{\chi}^0_1 \tilde{\chi}^0_2$ production threshold, such a difference may be understood on the basis of ([9], [10]), which suggest that the bigger cross section arises for models with $\eta_1 \eta_2 = -1$. At higher energies, the relative magnitude of the various polarization cross sections is model dependent. The threshold behaviour is preserved as the energy increases, in Fig.3b, but not in Fig.3a.

Since the neutralinos are rather heavy in the benchmark models [14], [3] we consider here, we contemplate a Linear Collider at 800 GeV c.m. energy, with an integrated luminosity of about 500 $fb^{-1}$ [21]. It should then be possible to discriminate models with opposite values of $\eta_1 \eta_2$; compare $SP1b$ or $SP1b'$ with $SP1b''$ in Fig.3a; or $D$ with $D''$ or $D'''$ in Fig.3b [4].

Even the discrimination between models with the same value of $\eta_1 \eta_2$ may be possible, at least if some knowledge of the $\tilde{e}_L$, $\tilde{e}_R$ masses is available [18]. This is inferred from the fact that for a 500$f b^{-1}$ luminosity, the difference between the unpolarized cross sections at 800 GeV for the models $SP1b$ and $SP1b'$ is at the level of 6.7 statistical Standard Deviations (SD); and it increases to 12 SD when longitudinally polarized beams are used with $\lambda_1 = -0.85$, and $\lambda_2 = +0.6$; see Fig.3a. The corresponding difference between the unpolarized cross sections for the $D''$ and $D'''$ models, (which both have $\eta_1 \eta_2 = -1$), is at the level of 5.7 statistical SD, and it increases to about 11 SD for longitudinal polarizations like those used in Fig.3b. Of course such signals will be reduced considerably, when the additional uncertainties due to the slepton masses and the experimental systematics are taken into account.

The discrimination between the models $SP1b''$, $SP1b'''$, or $D^{IV}$, and $D^V$, which could lift the remaining reconstruction ambiguities, may be possible if an accurate determination of the third neutralino mass is available. This may be obtained by measurements of the $\sigma(e^- e^+ \rightarrow \tilde{\chi}^0_1 \tilde{\chi}^0_3)$ cross section using polarized beams.

Finally, we note from Fig.3a, that not only the overall magnitude, but also the relative magnitudes of the various polarized cross sections may change, as $\eta_1 \eta_2$ changes. Therefore,
the measurement of the physical masses of the lightest chargino and the two lightest neutralinos, combined with some knowledge of the \( \sigma(e^-e^+ \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0) \) cross section (and possibly also the measurements of \( m_{\tilde{\chi}_3^0} \) and \( \sigma(e^-e^+ \rightarrow \tilde{\chi}_1^0\tilde{\chi}_3^0) \)), may be able to provide more or less unique results for \( M_2, M_1, \mu \). At least this is what happens in the CP conserving case considered above.

A nice feature of the present approach is that \( (78) \) and \( (79) \), offer an analytical way of disentangling the \( \mu \) and \( M_2 \) parameters.

5.2 The CP violating case

We now address the CP-violating case in which there exist six parameters affecting the ino sector; namely the set \((M_2, \tan \beta, \bar{\mu}, \Phi_\mu)\) affecting both charginos and neutralinos, and the set \((\bar{M}_1, \Phi_1)\) which influences only the neutralinos.

As shown in \([12, 13]\), if the charginos are not too heavy, a detailed measurement of \( \sigma(e^-e^+ \rightarrow \tilde{\chi}_i^+\tilde{\chi}_j^-) \) at a Linear Collider with polarized beams, should be able to determine the two chargino masses \( m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm} \), as well as the mixing angles \( \phi_L, \phi_R \) entering the diagonalization of the chargino mass matrix. If this is achieved, then \((M_2, \tan \beta, \bar{\mu}, \Phi_\mu)\) are fully determined \([10, 7]\). The measurement of neutralino production cross sections could then be used to measure the genuine \( U(1) \) parameters \( |\bar{M}_1| \) and \( \Phi_1 \).

Of course it may turn out that such a full separation of the information sources for the two aforementioned sets, is not possible. It may for example happen that \( m_{\tilde{\chi}_2^+} \) is too heavy to be produced on its mass shell in a Linear Collider. We therefore explore various scenarios concerning the chargino and neutralino measurements:

- **The scenario CPv1:**
  This is analogous to the S1 scenario of the CP conserving case \([15]\). We assume in it that the parameters \((M_2, \tan \beta, \bar{\mu}, \Phi_\mu)\) will be measured first from e.g. the chargino sector. Concerning the neutralinos, the assumption is that initially we will only know the physical mass \( m_{\tilde{\chi}_j^0} \) of one (not necessarily the lightest) neutralino, and may also have a limited knowledge of its associated CP-eigenphase \( \eta_j \). The problem will then be how to determine \( |\bar{M}_1| \) and at least constrain the phase angle \( \Phi_1 \) from \((28)\).

  A feeling of the possibilities concerning these phases may be obtained by looking at CP-violating variations of e.g. the model \( SP1b \), in which the values of \( \Phi_1, \Phi_\mu \) are varied as in \([39]\) Table 2, while \(|\bar{M}_1|, M_2, |\mu| \) and \( \tan \beta \) are kept fixed. The models thus obtained are labeled as \( SP1c, SP1d, SP1e, SP1g \). In the same Table 2 we also give the implied physical masses and \( \eta_j \) phases for the two lightest neutralinos calculated from the formulae in Appendix A, and \( (33, 36) \). There are in fact two models labeled as \( SP1c \) (and another two as \( SP1d \)), corresponding to the upper and lower signs for \( \Phi_1, \text{Arg}\eta_1 \) and \( \text{Arg}\eta_2 \). In all models listed in Table 2, the dimensional quantities are essentially the same, but the phases change considerably.

\(^{19}\)The same model \( SP1b \) appears both in Table 1 and in Table 2; but in the latter Table more significant digits are kept in order to indicate the small mass changes caused by the changes in the phase angles.
Table 2: Parameters for some CP-violating variations of $SP_1b$ of Table 1. (Phase angles in rad and masses in $GeV$, always at the electroweak scale.)

| Model | $|M_1|$ | $\Phi_1$ | $\Phi_\mu$ | $m_{\tilde{\chi}_1^0}$ | $m_{\tilde{\chi}_2^0}$ | $\text{Arg}[\eta_1]$ | $\text{Arg}[\eta_2]$ |
|-------|--------|----------|-----------|-----------------|-----------------|--------------------|--------------------|
| $SP_1b$ | 165.7  | 0        | 0         | 163.9          | 296.7           | 0                  | 0                  |
| $SP_1c$ | 165.7  | $\pm \frac{\pi}{3}$ | 0         | 164.1          | 296.6           | $\pm 1.049$       | $\pm 0.718$       |
| $SP_1d$ | 165.7  | $\pm \frac{\pi}{6}$ | $\pi$    | 164.4          | 299             | $\pm 1.046$       | $\pm 0.712$       |
| $SP_1e$ | 165.7  | $\frac{\pi}{2}$ | $\frac{\pi}{6}$ | 164.3          | 296.9           | 1.049              | 0.662              |
| $SP_1f$ | 166    | -1.33    | $\frac{\pi}{12}$ | 164.3          | 296.9           | -1.33              | -0.947             |
| $SP_1g$ | 165.7  | $\frac{\pi}{4}$ | $\frac{\pi}{4}$ | 163.9          | 296.9           | 1.0478             | 0.768              |
| $SP_1h$ | 165.4  | -0.761   | $\frac{\pi}{4}$ | 163.9          | 296.9           | -0.763             | -0.483             |

In the context of the CPv1 scenario, we concentrate on the models $SP_1b$, $SP_1c$, $SP_1d$, $SP_1e$, $SP_1g$ of Table 2. We now invert the logic within each of these models, and assume that only ($M_2$, $\tan \beta$, $\bar{\mu}$, $\Phi_\mu$) and the physical mass of one neutralino $\tilde{\chi}_j^0$, are known. Under these conditions, we explore the extent to which $|M_1|$ and $\Phi_1$ may be reconstructed.

If the known neutralino is the lightest one $\tilde{\chi}_1^0$, we present in Fig.4a the reconstructed value of $|M_1|$ calculated from (18), as a function of the phase angle $\text{Arg}(\eta_1)$. As seen from this figure, the reconstructed value of $|M_1|$ varies extremely little around the expected value of 165.7 GeV, as $\text{Arg}(\eta_1)$ takes all possible values. Under the same conditions, Fig.4b indicates that the reconstructed value of $\Phi_1$ closely follows that of $\eta_1$.

We conclude therefore that, at least in these models, $|M_1|$ can be accurately reconstructed from the physical mass of the lightest neutralino, even if we know nothing about its CP-eigenphase $\eta_1$.

In principle, the same method could have been also applied in the case where the known neutralino mass was $m_{\tilde{\chi}_2^0}$. But, as it is clearly seen from Figs.4a,b, the method is now not expected to be accurate.

We next turn to the $\Phi_1$ determination. From Fig.4b, we expect $\Phi_1 \simeq \eta_1$. For measuring therefore $\Phi_1$, we need to know the CP eigenphase $\eta_1$ of the lightest neutralino. Information on $\eta_1$ could be obtained, e.g. by looking at neutralino production in an $e^-e^+$ Linear Collider. In Figs.5(a) and (b) we present the cross sections for $\tilde{\chi}_1^0\tilde{\chi}_2^0$ and $\tilde{\chi}_2^0\tilde{\chi}_2^0$ production respectively. The results correspond to the models $SP_1b$ and $SP_1c$, the parameters of which appear in Table 2. As expected from (59, 60), the dominant $\Delta s^{1/2}$ term in the threshold region for the $\tilde{\chi}_1^0\tilde{\chi}_2^0$ production cross section, should be vanishing in the CP conserving model $SP_1b$ where $\eta_1 = \eta_2$, but not in the $SP_1c$ case in which $\eta_1 \neq \eta_2$ and complex.

For $\tilde{\chi}_2^0\tilde{\chi}_2^0$ production though, the first non vanishing threshold term is $\Delta s^{3/2}$. From Fig.5a,b and an expected luminosity of 500 $fb^{-1}$, we see that the $e^-e^+ \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ cross...
section measurement can indeed discriminate between the models \( SPb \) and \( SPc \), which differ essentially only for \( \Phi_1 \) and the neutralino eigenphases.

In the above numerical example we have thus found that if the measured neutralino is the lightest one; then, \(|M_1|\) and \( \Phi_1 \) may be accurately reconstructed. In that example, we have also found that \(|M_1| \approx m_{\tilde{\chi}_1^0} \) and \( \Phi_1 \approx \eta_1 \).

• The scenario \( CPv2 \):
  In addition to the chargino parameters \( M_2, \tan \beta, \tilde{\mu}, \Phi_\mu \), we assume here that we also know the physical masses of two neutralinos; say \( e.g. \) the masses of the two lightest neutralinos \( m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0} \). The CP eigenphases \( \eta_1, \eta_2 \) are taken as unknown [10].

  Then \( \tilde{M}_1 \equiv |M_1|, \Phi_1 \) and the CP-eigenphases \( \eta_1, \eta_2 \) can be extracted analytically, up to a two-fold discrete ambiguity. Indeed applying (88) to the two lightest neutralinos we obtain

\[
A_1 \eta_1 - A_2 \eta_2 = B_2 - B_1 ,
\]

where \( A_1, A_2, B_1, B_2 \) are defined in (89). Imposing then the condition \(|\eta_{1,2}| = 1\) in (80) and defining \( \Delta B \equiv B_2 - B_1 \), we obtain the constraint

\[
\Delta = 4A_1^2 A_2^2 - (A_1^2 + A_2^2 - |\Delta B|^2)^2 \geq 0 ,
\]

(81)

together with

\[
\eta_{1,2} = \frac{A_1^2 - A_2^2 \pm |\Delta B|^2 + i\epsilon \sqrt{\Delta}}{2 \Delta B^* A_{1,2}} ,
\]

(82)

and

\[
\tilde{M}_1 e^{i\Phi_1} = \frac{A_1^2 - A_2^2 + (B_1 + B_2) \Delta B^* + i\epsilon \sqrt{\Delta}}{2 \Delta B^*} ,
\]

(83)

where \( \epsilon = \pm 1 \) expresses the aforementioned two-fold ambiguity of the solution. In the context of this scenario, (82) fixes the CP eigenphases of the two lightest neutralinos, and (83) determines \( (\tilde{M}_1, \Phi_1) \), up to a two-fold ambiguity.

As an example we consider the models \( SP1e \) and \( SP1g \) of Table 2, considering as input \( (M_2, \tan \beta, \tilde{\mu}, \Phi_\mu) \) together with the physical masses of the two lightest neutralinos. As expected, (83, 82) reproduce the \( \tilde{M}_1, \Phi_1, \eta_1 \) and \( \eta_2 \) results quoted in Table 2 for \( SP1e \) and \( SP1g \) respectively; and in addition, due to the above ambiguity, they also respectively allow the models \( SP1f \) and \( SP1h \) shown in the same Table.

Exactly the same statements could be made when applying the above procedure to the upper signs version of the model \( SP1c \) (or \( SP1d \)) in Table 2; and obtaining, due to the aforementioned ambiguity, also the lower signs version of the same model.

As seen from Table 2, the models in each of the pairs \( (SP1e, SP1f) \) and \( (SP1g, SP1h) \) have the same values for \( M_2, \tan \beta, |\mu|, \Phi_\mu \) and the physical masses of both charginos and the two lightest neutralinos. They have also almost the same values for \(|M_1|\). Their

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20 According to (61-67), the upper and lower sign models appearing in Table 2 as \( SP1c \) (or \( SP1d \)) are characterized by complex conjugate projectors and \( (\eta_1, \eta_2) \) phases. They therefore give identical cross sections from (53).

21 It is equivalent to Eq.(3.15) of [16]. We thank Gilbert Moultaka for pointing this out to us.
The main difference is on the phase angles $\Phi_1$, Arg($\eta_1$), Arg($\eta_2$), which are found to be of opposite sign, in each of the above pairs. Apart from this, their only other difference is on the higher neutralino masses and CP eigenphases.

An efficient way to discriminate between models like e.g. $SP1e$ and $SP1f$, is by looking at $\sigma(e^-e^+ \to \tilde{\chi}_1^0\tilde{\chi}_1^0)$. These cross sections are presented in Fig. 8c for polarized beams. According to (53-58) and the reduced projectors of (61-63), the difference between the cross sections for these models solely comes from the differences between $\eta_1$ and $\eta_2$ given in Table 2. Thus, for polarized beams with longitudinal polarizations $\lambda_1 = -0.85$, $\lambda_2 = 0.6$, the cross section difference between the two models may reach the level of 20 fb; which means a difference of about ten thousands of events. The discrimination between such models should be amply possible at a Linear Collider.

• The scenario CPv3:
This scenario is a weak version of CPv2. We assume in it, that the first quantities to be measured will be the physical masses of the two lightest neutralinos, the mass of the lightest chargino $\tilde{\chi}_1^+$, and in addition to it, the chargino mixing angle $\phi_L$ and $\tan \beta$ [12, 16].

The motivation for this scenario is based on the observation that a study of the $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$ cross section for polarized beams, should be sufficient to extract $\tan \beta$ and the mixing angle $\phi_L$, even if it would be impossible to experimentally determine the mass of the heavier chargino [12, 16]. Thus, $m_{\tilde{\chi}_2^\pm}$ is not assumed known.

The relevant formulae in this scenario are still (81, 82, 83) reconstructing the neutralino eigenphases and complex $M_1$, in terms of $\tan \beta$ and $(M_2, \mu)$. But now $(M_2, \mu)$ should also be constructed from the lightest chargino mass and $\phi_L$. To this end we note that the chargino mass matrix gives [17]

\[
|\mu| = \sqrt{\frac{m_{\tilde{\chi}_2^+}^2[1 + \cos(2\phi_L)] + m_{\tilde{\chi}_1^+}^2[1 - \cos(2\phi_L)] - 4m_W^2c_{\beta}^2}{2}},
\]

\[
M_2 = \sqrt{\frac{m_{\tilde{\chi}_2^+}^2[1 - \cos(2\phi_L)] + m_{\tilde{\chi}_1^+}^2[1 + \cos(2\phi_L)] - 4m_W^2s_{\beta}^2}{2}},
\]

and

\[
\cos \Phi_\mu = 1 - \frac{m_{\tilde{\chi}_2^+}^2m_{\tilde{\chi}_2^\pm}^2 - (M_2|\mu| - m_W^2s_{2\beta})^2}{2m_W^4M_2|\mu|s_{2\beta}},
\]

\[
\sin \Phi_\mu = \epsilon' \sqrt{1 - \cos^2 \Phi_\mu},
\]

involving an additional two-fold ambiguity on the sign of $\Phi_\mu$, which is described by $\epsilon' = \pm 1$, [16].

To reconstruct $(|\mu|$, $M_2$, $\Phi_\mu)$ from (84-85), we first need to constrain or determine $m_{\tilde{\chi}_2^\pm}$. A very strong such constraint may be obtained by imposing on (85) the requirements $|\cos \Phi_\mu| \leq 1$, $|\sin \Phi_\mu| \leq 1$, [16]. To illustrate this we consider again the model $SP1e$ of Table 2, for which the chargino masses are $m_{\tilde{\chi}_1^+} = 296.9$ GeV, $m_{\tilde{\chi}_2^\pm} = 516.17$ GeV and $\cos(2\phi_L) = -0.77$. 

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If now, within the philosophy of the CPv3 scenario, we invert the logic and start from the \(SP1e\) values for \(m_{\tilde{\chi}^\pm_1}\), \(\tan \beta\), \(\cos(2\phi_L)\) and \(m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^0_2};\) while treating \(m_{\tilde{\chi}^\pm_2}\), as well as all other \(SP1e\) parameters, as unknown; then consistency of (83) implies the rather strong constraint
\[
507.2 \text{ GeV} \lesssim m_{\tilde{\chi}^\pm_2} \lesssim 517.7 \text{ GeV}.
\]
This constraint is generated from the chargino sector alone, and it is independent of the \(m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^0_2}\) values. It is further strengthened if we also demand that \(m_{\tilde{\chi}^\pm_2}\) should be such, that the induced values for \(|\mu|\) and \(M_2\) from (84), satisfy the constraint (81). Indeed, for the same input parameters as above, and taking also into account the information on the neutralino masses, we then find the very tiny interval
\[
515.8 \text{ GeV} \lesssim m_{\tilde{\chi}^\pm_2} \lesssim 516.9 \text{ GeV},
\]
independently of the \(\epsilon, \epsilon'\) ambiguities. The corresponding intervals from (84) are
\[
495.6 \text{ GeV} \lesssim |\mu| \lesssim 496.5 \text{ GeV}, \quad 309.5 \text{ GeV} \lesssim M_2 \lesssim 309.7 \text{ GeV}.
\]
\(\Phi_\mu\) is then restricted by (83) to the \(\epsilon'\)-dependent interval
\[
0.63 \text{ rad} \lesssim \epsilon' \Phi_\mu \lesssim 0.87 \text{ rad},
\]
while (83) gives
\[
165.69 \text{ GeV} \lesssim \bar{M}_1 \lesssim 165.72 \text{ GeV}, \quad 1 \text{ rad} \lesssim \epsilon' \Phi_1 \lesssim 1.1 \text{ rad},
\]
for \(\epsilon\epsilon' = -1\), and
\[
165.99 \text{ GeV} \lesssim \bar{M}_1 \lesssim 166.72 \text{ GeV}, \quad -1.34 \text{ rad} \lesssim \epsilon' \Phi_1 \lesssim -1.31 \text{ rad},
\]
for \(\epsilon\epsilon' = 1\).

We note that the appearances of \(\epsilon\) and \(\epsilon'\) above and in (82, 83), indicate a four fold ambiguity. If \(\epsilon' = 1\) is selected, then (87) and (88) correspond respectively to the models \(SP1e\) and \(SP1f\), in Table 2. On the other hand, the solution \(\epsilon' = -1\) combined with \(\epsilon = 1\) or \(\epsilon = -1\), allows two additional model possibilities, in which the reduced projector elements and the CP eigenphases for the two lightest neutralinos will be complex conjugate to those of the models \(SP1e\) and \(SP1f\) respectively; compare (82, 83). For clarity, we call these later models \(SP1e'\) and \(SP1f'\), respectively.\(^{22}\)

The above constraints are derived assuming that we only know the physical masses of the two lightest neutralinos, the mass of the lightest chargino \(\tilde{\chi}^+_1\), \(\tan \beta\) and the mixing angle \(\phi_L\). It turns out that we can go beyond this, and fully determine the mass of \(\tilde{\chi}^\pm_2\), even if (as in \(SP1e\)), it is too heavy to be produced on-shell in a 500 GeV LC. Such

\(^{22}\)They should not be confused with \(SP1g\) and \(SP1h\) of Table 2, in which the masses of the two lightest neutralino are different.
a determination may e.g. be done if we also measure $\sigma(e^-e^+ \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0)$, preferably for polarized beams.

Because of the aforementioned properties of the RPE and $\eta_{1,2}$, the models in each of the pairs $(SP1e, SP1e')$ or $(SP1f, SP1f')$ give identical results for $\sigma(e^-e^+ \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0)$. Thus, measurement of this cross section may only be used to discriminate between the pairs $(SP1e, SP1e')$ and $(SP1f, SP1f')$, corresponding to $\epsilon' = -1$ and $\epsilon' = 1$, respectively.

Thus, using (81, 83, 82), and the reduced projector elements $p_{1\alpha}, p_{2\beta}$ evaluated from (84, 85), the cross section $\sigma(e^-e^+ \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0)$ is calculated from (53). For a given energy $\sqrt{s}$ and polarizations ($\lambda_1, \lambda_2$), this cross section only depends on the chargino mass $m_{\tilde{\chi}_2^\pm}$, allowed to vary in the interval specified by $|c_\mu|, |s_\mu| \leq 1$ and $\Delta \geq 0$. The mass $m_{\tilde{\chi}_2^\pm}$ is obtained by comparison with the experimental cross section.

As an illustration of how well we can reproduce the specific ”experimental” value of $m_{\tilde{\chi}_2^\pm} = 516.17$ GeV for the above SP1e-generated parameters, we call ”experimental”, the cross section $\sigma(e^-e^+ \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0)_{\exp} \simeq 54.5$ fb for $m_{\tilde{\chi}_2^\pm} = 516.17$ GeV, $\epsilon' = -1$, $\sqrt{s} = 500$ GeV and $e^-e^+$-polarizations ($\lambda_1, \lambda_2$) = $(-0.8, 0.6)$. The ”theoretical” cross section is defined for varying $m_{\tilde{\chi}_2^\pm}$ in the range of (86), considering the two possibilities $\epsilon' = -1$ and $\epsilon' = 1$ corresponding to $(SP1e, SP1e')$ and $(SP1f, SP1f')$ respectively.

In Fig.[7], we display the ratio of these cross sections

$$\mathcal{R}_{-+} = \frac{\sigma(e^-e^+ \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0)_{\exp}}{\sigma(e^-e^+ \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0)_{\text{th}}}$$

as a function of $m_{\tilde{\chi}_2^\pm}$.

For a Linear Collider with an integrated luminosity of about $\mathcal{L} = 500$ fb$^{-1}$; the 1 SD error to $\mathcal{R}_{-+}$ is

$$\delta\mathcal{R}_{-+} \simeq \sqrt{\frac{\mathcal{R}_{-+}}{\sigma(e^-e^+ \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0)_{\exp}} \mathcal{L}}$$

Using this, we also present in Fig.[7] the ±1 SD results $\mathcal{R}_{-+} \pm \delta\mathcal{R}_{-+}$, for the two distinct cases $\epsilon' = -1$ and $\epsilon' = +1$. As seen in Fig.[7], the possibility $\epsilon' = +1$ (corresponding to the pair $SP1f$, $SP1f'$) is ruled out by many standard deviations; while for $\epsilon' = -1$ we find $m_{\tilde{\chi}_2^\pm} = 516.20 \pm 0.04$, in consistency with models SP1e and SP1e'.

After $m_{\tilde{\chi}_2^\pm}$ is thus fixed; then the choice $\epsilon' = 1$, together with (84, 85, 81, 82, 83) determines $|\mu|, \Phi_\mu, M_2, |M_1|, \Phi_1$, and the CP eigenphases of the two neutralinos we have started with $\eta_1, \eta_2$, to the values expected for the SP1e model. Of course the $\epsilon'$ ambiguity is not lifted, and an alternative set of results is allowed corresponding to SP1e'.

The same procedure could be applied also for a c.m. energy of $\sqrt{s} = 800$ GeV. For the same polarizations ($\lambda_1, \lambda_2$) = $(-0.8, 0.6)$, we again find that $\epsilon' = +1$ is strongly ruled out; while the reconstruction of the chargino mass $m_{\tilde{\chi}_2^\pm}$ for $\epsilon' = -1$ gives $m_{\tilde{\chi}_2^\pm} = 516.2 \pm 0.1$. It is interesting to note that the reconstruction error is larger in this case.

A similar procedure could also be applied to the pair of models labeled SP1c (or SP1d). In this particular case, where the chargino sector is CP conserving, we should

\[23\] As already stated, this cross section is identical for both SP1e and SP1e' models.

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only have the $\epsilon$ ambiguity; not the $\epsilon'$ one, [12]. But the measurement of $\sigma(e^-e^+ \rightarrow \tilde{\chi}_i^0\tilde{\chi}_j^0)$ will not be able to lift it. So finally again one ambiguity remains which is expressed by the upper and lower signs in each of these pairs of models.

6 Conclusions

In this paper we have first addressed the problem of constructing analytical expressions for the diagonalization of the most general complex symmetric neutralino mass matrix. The motivating idea was to extend to complex SUSY parameters, the approach of [9], which was hitherto applied only to the real SUSY parameter case. A very nice feature of this approach is that it can be straightforwardly extended to theories containing an arbitrary number of neutralinos.

The diagonalization starts from the observation that neutralino physical observables can only depend on their projectors (or density matrices); and what we have here defined as the neutralino pseudo-projectors, enforced by their Majorana nature.

In the next step, we introduce for each physical neutralino $\tilde{\chi}_j^0$, its CP-eigenphase $\eta_j$, and its ”reduced projector elements” $p_{j\alpha}$, which are complex numbers. The projectors and pseudo-projectors, as well as the diagonalization matrix, can be expressed in terms of them; and all may subsequently be calculated using Jarlskog’s formula (15) together with (29-32, 35). Analytic expressions for calculating the physical masses are given in Appendix A.

The end result is that all physical observables related to any specific neutralino, can be fully described in terms of its CP eigenphase and its reduced projector elements. The ensuing formulae constitute one of the basic contents of this paper. Their importance is further emphasized by noting that they can be directly generalized to any non-minimal SUSY theory, involving any number of neutralinos with real or complex couplings.

As an example, we have presented the tree-level expression for $d\sigma(e^-e^+ \rightarrow \tilde{\chi}_i^0\tilde{\chi}_j^0)/dt$ in the MSSM case; in which there are three reduced projector elements, and of course one CP eigenphase, for each neutralino. Arbitrary beam polarizations are used.

The above formulae are quite simple and easy to insert in a numerical code. Thus, they should be useful in any calculation involving either on-shell or virtual neutralinos. Since all masses and mixings are determined together at the same scale and level of approximation; it is guaranteed that no spurious enhancements or lack of divergence cancellations in neutralino loops, may creep in.

The next very important set of formulae of the present paper appears in (40-43) and expresses the various MSSM parameters in terms of the three reduced projector elements $p_{j\alpha}$ ($\alpha = 2 - 4$) for $\tilde{\chi}_j^0$, its mass $m_{\tilde{\chi}_j^0}$, and its CP eigenphase $\eta_j$. These relations are subsequently used in various applications.

The first concerns the CP conservation in the chargino-neutralino sector and may be stated as: CP conservation is equivalent to the reality of the reduced projector elements referring to any given neutralino state. But if this happens, then it is immediately
concluded that all reduced projector elements for all neutralinos, and all CP eigenphases, are in fact real.

As a second set of applications of (40-43) and their inverse (35-39), we have studied various scenarios of reconstructing the SUSY parameters; under various conditions concerning the acquirement of experimental knowledge in the chargino and neutralino sectors. To study the efficiency of our formalism, we have in fact considered the same scenarios as in [15, 16, 12]. In all cases, analytical expressions are given which disentangle the unknown SUSY parameters. There is never need to do this numerically [12, 16]. Ways to handle the various construction ambiguities are also discussed.

More explicitly, in the real SUSY parameter CP conserving case, we considered the scenario S1, in which $M_2$, $\mu$, $\tan \beta$, the lightest physical neutralino mass and (partly at least) its CP eigensign are known; and S2, in which measurements of only the masses of the lightest chargino and the two lightest neutralinos, are assumed.

For the complex parameters case, we have also considered scenarios in which chargino measurements are assumed to have already determined $M_2$, $|\mu|$, $\Phi_\mu$ and $\tan \beta$; while from the neutralino sector only the physical mass and CP-eigenphase of the lightest neutralino (CPv1), or just the physical masses of the two lightest neutralinos (CPv2), are assumed known.

As already said, analytical formulae have in all cases been derived which disentangle the unknown SUSY parameters.

In addition to them, we have also considered the complex parameter CPv3 scenario, in which only $\tan \beta$, the lightest chargino mass, its mixing angle $\phi_L$, and the physical masses of the two lightest neutralinos, are assumed known [16, 12]. Then, explicit expressions are constructed which may be used to determine $m_{\tilde{\chi}_2^\pm}$ from $\sigma(e^-e^+ \to \tilde{\chi}_1^0\tilde{\chi}_2^0)$ measurements; even if the energy is not sufficient to produce $\tilde{\chi}_2^\pm$. Once, $m_{\tilde{\chi}_2^\pm}$ is determined; expressions for finding $|\mu|$, $\Phi_\mu$, $M_2$, $|M_1|$, $\Phi_1$, as well as the CP eigenphases of the two lightest neutralinos, are also given.

In all previous scenarios, some knowledge of the chargino sector was assumed, and only the total neutralino production cross section at an LC was used. Since in many models $m_{\tilde{\chi}_2^0} \sim m_{\tilde{\chi}_1^\pm}$, it may turn out that the charginos will be too heavy to be produced, and that the neutralino sector will be the first to be studied. In such a case, the differential cross section $d\sigma(e^-e^+ \to \tilde{\chi}_l^0\tilde{\chi}_j^0)/dt$, together with the physical masses of the two lightest neutralinos, will be the only means we might have in order to reconstruct the neutralino parameters. We plan to explore such scenarios in the near future.

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Appendix A: The neutralino masses and projectors in the presence of the CP violating phases:

Here we present the formulae for the analytic determination of the neutralino masses. Analogous results have already appeared in [22]. Nevertheless we give them here also, following the same formalism as in the real SUSY parameter case treated in [3].

Using (3) we find

\[
Y^\dagger Y = \begin{pmatrix}
\bar{M}_1^2 + m_Z^2 s_W^2 & -m_Z^2 s_W c_W & y_{13} & y_{14} \\
-m_Z^2 s_W c_W & M_2^2 + m_Z^2 c_W^2 & y_{23} & y_{24} \\
y_{31} & y_{32} & \mu^2 + m_Z^2 c_\beta^2 - m_Z^2 s_\beta c_\beta \\
y_{41} & y_{42} & -m_Z^2 s_\beta c_\beta & \bar{\mu}^2 + m_Z^2 s_\beta^2
\end{pmatrix}, \tag{A.1}
\]

where

\[
y_{13} = y_{31} = -m_Z s_W (\bar{M}_1 c_\beta e^{-i\Phi_1} + \bar{\mu} s_\beta e^{i\Phi_\mu}) \tag{A.2},
\]
\[
y_{14} = y_{41} = m_Z s_W (\bar{M}_1 s_\beta e^{-i\Phi_1} + \bar{\mu} c_\beta e^{i\Phi_\mu}) \tag{A.3},
\]
\[
y_{23} = y_{32} = m_Z c_W (\bar{M}_2 c_\beta + \bar{\mu} s_\beta e^{i\Phi_\mu}) \tag{A.4},
\]
\[
y_{24} = y_{42} = -m_Z c_W (\bar{M}_2 s_\beta + \bar{\mu} c_\beta e^{i\Phi_\mu}) \tag{A.5}.
\]

The characteristic equation associated to \( Y^\dagger Y \) determine the physical neutralino masses \( m_{\tilde{\chi}_j^0} \):

\[
m_{\tilde{\chi}_j^0}^8 - Am_{\tilde{\chi}_j^0}^6 + Bm_{\tilde{\chi}_j^0}^4 - Cm_{\tilde{\chi}_j^0}^2 + D = 0 , \tag{A.6}
\]

with [22]

\[
A = \bar{M}_1^2 + M_2^2 + 2(\bar{\mu}^2 + m_Z^2) ,
\]
\[
B = -2\bar{\mu} m_Z^2 s_{2\beta} (\bar{M}_1 s_W^2 c_\mu + M_2 c_W^2 c_\mu) + (\bar{M}_1 M_2)^2 \\
+ 2\bar{\mu}^2 (\bar{M}_1^2 + M_2^2) + 2m_Z^2 (\bar{M}_1^2 c_W^2 + M_2^2 s_W^2) + (\bar{\mu}^2 + m_Z^2)^2 ,
\]
\[
C = \bar{\mu}^2 m_Z^2 s_{2\beta} - 2\bar{M}_1 m_Z^2 (\bar{M}_2^2 + \bar{\mu}^2) s_W^2 s_{2\beta} c_\mu - 2M_2 \bar{\mu} (\bar{M}_1^2 + \bar{\mu}^2) m_Z^2 c_W^2 s_{2\beta} c_\mu \\
+ \bar{M}_1^2 (2M_2^2 \bar{\mu}^2 + (\bar{\mu}^2 + m_Z^2 c_W^2)^2) + M_2^2 (\bar{\mu}^2 + m_Z^2 s_W^2)^2 + 2\bar{M}_1 M_2 m_Z^4 s_W^2 c_W^2 c_1 ,
\]
\[
D = (\bar{M}_1 M_2 \bar{\mu}^2)^2 + m_Z^4 \bar{\mu}^2 s_{2\beta} (\bar{M}_1^4 c_W^4 + M_2^4 c_W^4 + \bar{M}_1 M_2^2 s_W^2 c_W^2 c_1) \\
- 2(\bar{M}_1 M_2 \bar{\mu}^2) m_Z^2 s_{2\beta} (\bar{M}_1^2 c_W^2 c_\mu + M_2^2 s_W^2 c_\mu) , \tag{A.7}
\]

where \( s_{2\beta} = \sin(2\beta) \), \( c_1 = \cos \Phi_1 \), \( c_\mu = \cos \Phi_\mu \) and \( c_\mu = \cos(\Phi_1 + \Phi_\mu) \).

The four real and positive solutions of (A.6) determining \( m_{\tilde{\chi}_j^0}^2 \) are [3].

\[
m_{\tilde{\chi}_j^0}^2 = \frac{1}{2} \left\{ \frac{A}{2} - 2E \pm \sqrt{\left( \frac{A}{2} - 2E \right)^2 - 4 \left( \frac{B}{6} + 2\theta_N + F \right)} \right\} ,
\]
\[
\quad = \frac{1}{2} \left\{ \frac{A}{2} + 2E \pm \sqrt{\left( \frac{A}{2} + 2E \right)^2 - 4 \left( \frac{B}{6} + 2\theta_N - F \right)} \right\} , \tag{A.8}
\]

where

\[
E = \frac{1}{4} \left( A^2 - \frac{8B}{3} + 16\theta_N \right)^{1/2} , \quad F = \frac{1}{4E} \left( C - \frac{AB}{6} - 2A\theta_N \right) . \tag{A.9}
\]
and $\theta_N$ is a real solution of the auxiliary cubic equation

$$\theta_N^3 + a\theta_N + b = 0,$$

\[a \equiv -\frac{1}{4}\left(-\frac{AC}{4} + \frac{B^2}{12} + D\right),\]

\[b \equiv \frac{1}{4}\left(-\frac{A^2D}{16} + \frac{ABC}{48} - \frac{B^3}{216} + \frac{BD}{6} - \frac{C^2}{16}\right).\] (A.10)

Depending on the signs of $a$ and

$$\Delta \equiv \frac{b^2}{4} + \frac{a^3}{27},$$ (A.11)

the expression for $\theta_N$ is constructed using (14-16) of [9].
Appendix B: The projectors and the CP-eigenphases in terms of the rotation angles and the CP-phases:

The purpose of this Appendix is to clarify the connection between the present formalism and the one of [12]. To this aim, the $4 \times 4$ unitary matrix $U_N$ of [5] is factorized as [12]

$$U_N^\dagger = MD \ ,$$  \hspace{1cm} (B.1)

where

$$M = \text{Diag}\{e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}, e^{i\alpha_4}\} \ ,$$  \hspace{1cm} (B.2)

is used to define the four Majorana CP phases satisfying $0 \leq \alpha_i < 2\pi$ [12]; while the D matrix depends on 6 rotation angles $\theta_{ij} \in [0, \pi/2]$, ($i, j = 1 - 4$) and 6 CP-phases $\delta_{ij} \in [0, 2\pi)$, ($i, j = 1 - 4$), as [12]

$$D = R_{34}R_{24}R_{14}R_{23}R_{13}R_{12} \ ,$$  \hspace{1cm} (B.3)

with $R_{ij}$ being the rotation matrix in the plane $(ij)$. Thus, e.g.

$$R_{12} = \begin{pmatrix} c_{12} & s_{12}e^{-i\delta_{12}} & 0 & 0 \\ -s_{12}e^{i\delta_{12}} & c_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \ ,$$

where

$$c_{ij} \equiv \cos \theta_{ij} \ , \quad s_{ij} \equiv \sin \theta_{ij} \ , \quad t_{ij} \equiv \tan \theta_{ij} \ .$$  \hspace{1cm} (B.4)

In terms of these parameters, the reduced projector elements $p_{j2}, p_{j3}, p_{j4}$ for the four physical neutralinos, are expressed as

$$p_{12} = \kappa_2 t_{12} \ , \quad p_{13} = \kappa_3 \frac{t_{13}}{c_{12}} \ , \quad p_{14} = \kappa_4 \frac{t_{14}}{c_{12}c_{13}} \ ,$$  \hspace{1cm} (B.5)

$$p_{22} = \kappa_2 [s_{12}c_{24}s_{13}s_{23}\xi_1 - c_{23}c_{24}c_{12}\xi_2 + s_{12}c_{13}s_{14}s_{24}]p_{24}^{-1} \ ,$$  \hspace{1cm} (B.6)

$$p_{23} = \kappa_3 [s_{13}s_{14}s_{24} - c_{13}c_{24}s_{23}\xi_2]p_{24}^{-1} \ ,$$  \hspace{1cm} (B.7)

$$p_{24} = -\kappa_4 c_{14}s_{24}p_{24}^{-1} \ ,$$  \hspace{1cm} (B.8)

$$p_{32} = \kappa_2 [s_{12}s_{13}s_{23}s_{24}s_{34}\xi_1 - c_{23}c_{24}s_{23}\xi_2 - s_{12}c_{23}s_{34}s_{13}\xi_3 - c_{34}c_{12}s_{23}\xi_1^*\xi_2\xi_3 - s_{12}c_{13}c_{24}s_{14}s_{34}]p_{34}^{-1} \ ,$$  \hspace{1cm} (B.9)

$$p_{33} = \kappa_3 [-c_{13}s_{23}s_{24}s_{34}\xi_1 + c_{13}c_{23}s_{34}\xi_2 - c_{24}s_{13}s_{14}s_{34}]p_{34}^{-1} \ ,$$  \hspace{1cm} (B.10)

$$p_{34} = \kappa_4 c_{14}c_{24}s_{34}p_{34}^{-1} \ ,$$  \hspace{1cm} (B.11)

$$p_{42} = \kappa_2 [s_{12}s_{13}s_{23}s_{24}s_{34}\xi_1 - c_{23}c_{24}s_{23}\xi_2 + s_{12}c_{23}s_{34}s_{13}\xi_3 + s_{34}c_{12}s_{23}\xi_1^*\xi_2\xi_3 - s_{12}c_{13}c_{24}s_{14}s_{34}]p_{44}^{-1} \ ,$$  \hspace{1cm} (B.12)

$$p_{43} = \kappa_3 [-c_{13}s_{23}s_{24}s_{34}\xi_1 - c_{13}c_{23}s_{34}\xi_2 - c_{24}s_{13}s_{14}s_{34}]p_{44}^{-1} \ ,$$  \hspace{1cm} (B.13)

\hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm}

\hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm}

\textsuperscript{24}Note that our convention for $s_{ij}$ slightly differs from the one used in [12].

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with
\[ \kappa_j = e^{-i(\delta_{1j} + \alpha_1 - \alpha_j)} \]  \hspace{1cm} (B.14)

\[ p_2 = c_{12} c_{24} s_{13} s_{23} \xi_1 + c_{23} c_{24} s_{12} \xi_2 + c_{12} c_{13} s_{14} s_{24} \]  \hspace{1cm} (B.15)

and
\[ \xi_1 \equiv e^{i(\delta_{13} - \delta_{23} + \delta_{24} - \delta_{14})}, \quad \xi_2 \equiv e^{i(\delta_{12} + \delta_{24} - \delta_{14})}, \quad \xi_3 \equiv e^{i(\delta_{13} + \delta_{34} - \delta_{14})}; \]  \hspace{1cm} (B.16)

while the CP-eigenphases of the four physical neutralinos defined at (16) are
\[ \eta_1 = e^{2i\alpha_1}, \]
\[ \eta_2 = e^{2i(\alpha_1 + \delta_{14} - \delta_{24} + \text{Arg}[p_{14}] - \text{Arg}[p_{24}] - \text{Arg}[p_{34}])}, \]
\[ \eta_3 = e^{2i(\alpha_1 + \delta_{14} - \delta_{24} + \text{Arg}[p_{14}] - \text{Arg}[p_{24}] - \text{Arg}[p_{34}])}, \]
\[ \eta_4 = e^{2i(\alpha_1 + \delta_{14} - \text{Arg}[p_{14}] - \text{Arg}[p_{24}] - \text{Arg}[p_{34}])}. \]  \hspace{1cm} (B.17)

Eqs. (B.5)-(B.17) express the parameters of the present formalism in terms of those of 12. Conversely, the rotation angles \( \theta_{ij} \) and the \( \alpha_i \) phases of 12 (compare 12.3, 12.2), may also be written in terms of the projector elements and the neutralino CP eigenphases as
\[ s_{12} = \sqrt{\frac{P_{12}}{1 - P_{133} - P_{144}}}, \quad s_{13} = \sqrt{\frac{P_{133}}{1 - P_{144}}}, \quad s_{14} = \sqrt{P_{144}}, \]
\[ s_{24} = \sqrt{\frac{P_{244}}{1 - P_{144}}}, \quad s_{34} = \sqrt{\frac{P_{344}}{1 - P_{144} - P_{244}}}, \]
\[ s_{23} = \sqrt{(1 - P_{144})^2 P_{233} + P_{133} P_{144} P_{244} + 2(1 - P_{144}) \text{Re}[P_{134} P_{234}]} \]
\[ (1 - P_{133} - P_{144})(1 - P_{144} - P_{244}) \]  \hspace{1cm} (B.18)

\[ \alpha_i = \text{Arg}[\eta_i P_{144} P_{14}] + \tilde{\alpha}_i, \quad \delta_{ij} = \text{Arg}[\eta_i \eta_j P_{144}^* P_{14}] + \tilde{\delta}_{ij}, \]  \hspace{1cm} (B.19)

with
\[ \tilde{\alpha}_{1,4} \equiv 0, \quad \tilde{\alpha}_{2,3} \equiv \text{Arg}[\xi_{2,3}], \]
\[ \tilde{\delta}_{11} \equiv \tilde{\alpha}_i, \quad \tilde{\delta}_{23} = \text{Arg}[\xi_{3}^*], \quad \tilde{\delta}_{24} = \tilde{\delta}_{34} = 0, \]  \hspace{1cm} (B.20)

where the auxiliary complex numbers \( \xi_i \) read:
\[ \xi_1 = \frac{c_{13} c_{24} s_{23}}{s_{12} s_{13} \sqrt{P_{233} e^{i \text{Arg}[P_{134} P_{234}]} + c_{13} \sqrt{P_{222} e^{i \text{Arg}[P_{124} P_{224}]} + s_{12} s_{14} s_{24}}}, \]
\[ \xi_2 = \frac{c_{12} c_{13} c_{23} c_{24}}{s_{24} s_{34} \sqrt{P_{233} e^{i \text{Arg}[P_{134} P_{234}]} + c_{24} \sqrt{P_{333} e^{i \text{Arg}[P_{134} P_{334}]} + s_{13} s_{14} s_{34}}}}. \]  \hspace{1cm} (B.21)
Notice that the projectors elements and the neutralino CP eigenphases appearing above, may be expressed in terms of the reduced projector elements through (29, 32) and (33).

Concerning the condition for CP conservation, we note that the result of [12] may be expressed as

\[
CP - \text{conservation} \iff \left\{ \alpha_i = 0 \text{ or } \frac{\pi}{2} , \quad \delta_{ij} = 0 \text{ or } \pi \right\}, \tag{B.22}
\]

which can be immediately verified starting from (45) and using (B.5-B.16, B.18-B.21).

It may also be remarked that since CP conservation according to (45) is equivalent to the reality of the reduced projector elements \( p_{12}, p_{13}, p_{14}, \) (which also implies the reality of \( \eta_j \)), the validity of

\[
CP - \text{conservation} \iff \delta_{1i} = \alpha_i - \alpha_1 \mod \pi \tag{B.23}
\]

for \((i = 2, 3, 4)\), should be sufficient to imply the complete (B.22). The amusing feature of condition (B.23), is that it is equivalent to (B.22), in spite of the fact that it does not directly specify the values of any of the \( \alpha_j \) or \( \delta_{ij} \).

\[\text{Compare in particular (B.5) and (B.14).}\]
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Figure 1: Reconstruction of $M_1$ in the S1 CP-conserving scenario in which $M_2$, $\mu$, $\tan \beta$ are taken as in model SP1b of Table 1. $|M_1|$ is determined from (73), for varying $m_{\tilde{\chi}_i^0}$. Solid (broken) lines correspond to $\eta_i = 1$ ($-1$) respectively. For lower $m_{\tilde{\chi}_i^0}$-values than those indicated, the results remain the same as for $m_{\tilde{\chi}_i^0} \sim 200 \text{ GeV}$.

Figure 2: Diagonal projector elements $P_{i11}$, $P_{i22}$ (a) and $P_{i33}$, $P_{i44}$ (b), as a function of the physical mass $m_{\tilde{\chi}_i^0}$, for the same parameters as in Fig.1. The values of $P_{i22}[-1]$ are so small that they cannot be seen. For lower $m_{\tilde{\chi}_i^0}$-values than those indicated, the results remain the same as for $m_{\tilde{\chi}_i^0} \sim 200 \text{ GeV}$. 

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Figure 3: Total $\sigma(e^--e^+ \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0)$ cross sections for the CP conserving models $SP1b$, $SP1b'$, and $SP1b''$ (a), and $D$, $D''$, $D'''$ (b); see Table 1. The beams are taken either unpolarized, or with longitudinal polarizations of $\lambda_1 = \pm 0.85$ for $e^-$ and $\lambda_2 = \pm 0.6$ for $e^+$, indicated by Sign($\lambda_j$). The not plotted cross sections for the polarizations (+0) and (+-) are considerably smaller. For clarity, we also note that the marks on the (-0) lines, are actually pluses (+).
Figure 4: Reconstruction of $|M_1|$ and $\Phi_1$ in the CPv1 scenario, in which $M_2$, $\tan\beta$, $|\mu|$ and $\Phi_\mu$ and the physical mass of the lightest neutralino $\tilde{\chi}_0^1$ are assumed as in models $SP1b$, $SP1c$, $SP1d$, $SP1e$, $SP1g$; see Table 2. The neutralino CP-eigenphase $\eta_1$ is allowed to vary. In (b) the predictions for $SP1b$ and $SP1c$ overlap.

Figure 5: Same as in Fig. 1, but considering instead the $\tilde{\chi}_0^2$ physical mass of the neutralino. The neutralino CP-eigenphase $\eta_2$ is allowed to vary.
Figure 6: Comparison of the CP-conserving SP1b ($\eta_1 = \eta_2$), and the CP-violating model SP1c ($\eta_1 \neq \eta_2$ and complex) for $\sigma(e^--e^+ \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0)$ (a), and $\sigma(e^+e^- \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0)$ (b) respectively. In (c), the CP violating models SP1e and SP1f are compared for $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0)$. Polarizations as in the caption of Fig. 3a. For clarity, we also note that the marks on the (-0) lines, are actually pluses (+).
Figure 7: Reconstruction of \( m_{\tilde{\chi}_2^\pm} \) in the CPv3 scenario, for models \((SP1e, SP1e')\). Only the ambiguity between \( \epsilon \epsilon' = 1, (SP1f, SP1f') \) and \( \epsilon \epsilon' = -1, (SP1e, SP1e') \) may be lifted. As input we use \( m_{\tilde{\chi}_1^\pm} = 296.9 \text{ GeV}, \cos(2\phi_L) = -0.77 \), and the two lightest neutralino masses and \( \tan \beta \) given in Table 2. The \( \tilde{\chi}_2^\pm \) “experimental” value to be reproduced is \( m_{\tilde{\chi}_2^\pm} = 516.17 \text{ GeV} \). The LC energy is 500 GeV and \( \mathcal{L} = 500 \text{ fb}^{-1} \). The middle dash (dash-circles) lines indicate the central \( \mathcal{R}_{-4} \)-value, while the upper and lower lines describe the 1 SD changes.