Direct calculation of the triple-pomeron coupling for diffractive DIS and real photoproduction

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Abstract

We present a unified direct calculation of the triple-pomeron coupling $A_{3\text{IP}}(Q^2)$ for diffractive real photoproduction ($Q^2 = 0$) and deep inelastic scattering at large $Q^2$ in the framework of the dipole approach to the generalized BFKL pomeron. The small phenomenological value of $A_{3\text{IP}}(0) \approx 0.16 \text{ GeV}^2$, which was a mystery, is related to the small correlation radius $R_c \approx 0.3 \text{ fm}$ for the perturbative gluons. We confirm the early expectations of weak $Q^2$ dependence of the dimensionfull coupling $A_{3\text{IP}}(Q^2)$ and predict that it rises by the factor $\sim 1.6$ from real photoproduction to deep inelastic scattering.

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1 Introduction

Salient feature of diffraction dissociation \( a + p \rightarrow X + p' \) of \((a = h)\) hadrons and \((a = \gamma)\) real photons \((Q^2 = 0)\) is the so called triple-pomeron regime

\[
\frac{M^2}{\sigma_{\text{tot}}(ap)} \cdot \frac{d\sigma_D(a \rightarrow X)}{dtdM^2} \bigg|_{t=0} \approx A_{3\text{IP}}. \tag{1}
\]

Here \( t \) is the \((p, p')\) momentum transfer squared, and the mass \( M \) of the excited state satisfies \( m_p^2 \ll M^2 \ll W^2 \), where \( W \) is the total c.m.s. energy. Eq. (1) with the approximately energy-and projectile-independent triple-pomeron coupling \( A_{3\text{IP}} \), holds at moderately high energies, such that the photoabsorption and hadronic cross sections are approximately constant ([1], for a review see [2]). Notice that \( A_{3\text{IP}} \) is a dimensional coupling: \([A_{3\text{IP}}] = [\text{GeV}]^{-2}\). The Fermilab data on the diffractive real photoproduction give \([A_{3\text{IP}}(Q^2 = 0) \approx 0.16 \text{ GeV}^{-2}] \) [3]. In the Regge theory language, the inclusive cross section of diffraction dissociation measures the projectile-pomeron cross section [1]

\[
\sigma_{\text{tot}}(a\text{IP}; M^2 \gg m_p^2) = \frac{16\pi M^2}{\sigma_{\text{tot}}(pp)} \cdot \frac{d\sigma_D(a \rightarrow X)}{dtdM^2} \bigg|_{t=0} \approx 16\pi A_{3\text{IP}} \frac{\sigma_{\text{tot}}(ap)}{\sigma_{\text{tot}}(pp)}. \tag{2}
\]

Why \( A_{3\text{IP}} \) is numerically small, and why the hadron-pomeron cross section \( \sigma_{\text{tot}}(a\text{IP}) \) is more than one order in magnitude smaller than the hadron-nucleon cross section, is one of outstanding mysteries of the pomeron.

The triple-pomeron regime will soon be explored in details in an entirely new domain of diffractive deep inelastic scattering (DIS) at HERA. Here the underlying process is a diffraction dissociation of the virtual photon,

\[
\gamma^* + p \rightarrow X + p', \tag{3}
\]

at \( x = Q^2/(Q^2 + W^2) \ll 1 \), where \( Q^2 \) is the virtuality of the photon. The variable \( x_{\text{IP}} = (M^2 + Q^2)/(W^2 + Q^2) \ll 1 \) can be interpreted as the fraction of proton’s momentum taken away by the pomeron, whereas \( \beta = Q^2/(Q^2 + M^2) \) is the Bjorken variable for DIS on the pomeron. Notice that

\[
x_{\text{IP}} \beta = x. \tag{4}
\]

\(^1\)Apart from \( \approx 5\% \) statistical error and 16% normalization uncertainty [3], this number contains \( \lesssim 10\% \) uncertainty from our extrapolation from \(|t| = 0.05 \text{ GeV}^2\) to \( t = 0 \) using the slope of the \( t\)-dependence as measured in [3].
The final-state proton $p'$ carries the fraction $(1 - x_{IP})$ of the beam proton’s momentum and is separated from the hadronic debris $X$ of the photon by a large (pseudo)rapidity gap $\Delta\eta \approx \log \frac{1}{x_{IP}} \gg 1$. In real photoproduction and hadronic interactions, the pomeron exchange was shown to dominate at $x_{IP} \sim x_{IP}^0 = (0.05-0.1)$ and/or $\Delta\eta \gtrsim \Delta\eta_c = (2.5-3)$ [1,2]. In hadronic interactions and/or real photoproduction, the triple-pomeron regime corresponds to high c.m.s. energy of the $aIP$ interaction, $M^2 \gg m_p^2$, in DIS it requires $\beta \ll 1$ and/or $M^2 > Q^2$. For the evaluation of $A_{3IP}(Q^2)$ for DIS, one must consider the moderately large rapidity gap such that $x_{IP} \sim x_{IP}^0 \lesssim x_{IP}^{c}$, and the moderately small $x$, such that the proton structure function $F_2^p(x, Q^2)$ is still approximately flat vs. $\frac{1}{x}$. For reference point, we will consider diffractive DIS at $x_{IP} = x_{IP}^0 = 0.03$. Then, one can define (for more precise definition of the related kinematical domain see below)

$$\frac{M^2 + Q^2}{\sigma_{tot}(\gamma^* p)} \cdot \frac{d\sigma_D(\gamma^* \to X)}{dt dM^2} \bigg|_{t=0} \approx A_{3IP}(Q^2).$$

(5)

In diffractive DIS, the triple-pomeron coupling $A_{3IP}(Q^2)$ controls [4] the normalization of the gluon and sea content of the pomeron structure function.

The subject of this paper is a direct calculation of $A_{3IP}(Q^2)$ starting from the microscopic QCD description [4-6] of diffraction dissociation in the framework of the (generalized) dipole BFKL pomeron [5,6,8,9]. In section 2 we briefly review how reaction (3) is described in terms of the diffraction excitation of multiparton Fock states of the photon, which interact with the target proton by the dipole BFKL pomeron exchange. In section 3 we show that in DIS, the virtual photon acts as an effective two-gluon (color octet-octet dipole) state with the size of the order of the correlation radius $R_c$ for the perturbative gluons. We demonstrate how the small $R_c \sim 0.3$ fm, as suggested by lattice QCD studies (for the recent review see [10]), gives a natural small scale for $A_{3IP}(Q^2)$. The case of real photoproduction is studied in section 4. Here the underlying mechanism of diffraction dissociation into large masses is an excitation of $gg$ ($q\bar{q}$) ‘clusters’ (”constituent” quarks) of size $R_c$, and we find $A_{3IP}(0)$ which agrees well with the experimental determination. This is the first direct calculation of $A_{3IP}$ and the first instance, when DIS and real photoproduction processes are shown to share the dimensionfull coupling, $[A_{3IP}] = [\text{GeV}]^{-2}$, which does not scale with $1/Q^2$, confirming earlier conjectures [4-7]. Furthermore, in the scenario [11,12] for the dipole cross section, we predict a slight, by the
factor $\sim 1.6$, rise of $A_{3\text{IP}}(Q^2)$ from real photoproduction to DIS. In section 5 we summarize our main results.

2 Dipole pomeron description of the diffraction excitation of photons

We rely upon the microscopic QCD approach to diffractive DIS developed in [4-7]. Diffraction excitation of the lowest $q\bar{q}$ Fock state of the photon has the cross section (hereafter we focus on the dominant diffraction dissociation of transverse photons)

$$
\frac{d\sigma_D(\gamma^* \to X)}{dt} \bigg|_{t=0} = \int dM^2 \frac{d\sigma_D(\gamma^* \to X)}{dt dM^2} \bigg|_{t=0} = \frac{1}{16\pi} \int_0^1 dz \int d^2\vec{r} |\Psi_{\gamma^*}(Q^2, z, r)|^2 \sigma^2(x, r). \tag{6}
$$

Here $\vec{r}$ is the transverse separation of the quark and antiquark in the photon, $z$ and $(1-z)$ are partitions of photon’s lightcone momentum between the quark and antiquark, $\sigma(x, r)$ is the dipole cross section for interaction of the $q\bar{q}$ dipole with the proton target (hereafter we use $\sigma(x, r)$ of Refs. [11,12]), and the dipole distribution in the transverse polarized photon $|\Psi_{\gamma^*}(Q^2, z, r)|^2$ derived in [7], equals

$$
|\Psi_{\gamma^*}(Q^2, z, r)|^2 = \frac{6\alpha_{em}}{(2\pi)^2} \sum_i \epsilon_i^2 \left\{ z^2 + (1-z)^2 \right\} \varepsilon^2 K_1(\varepsilon r)^2 + m_q^2 K_0(\varepsilon r)^2 \right\}, \tag{7}
$$

where $\alpha_{em}$ is the fine structure constant, $e_i$ is the quark charge in units of the electron charge, $m_q$ is the quark mass,

$$
\varepsilon^2 = z(1-z)Q^2 + m_q^2 \tag{8}
$$

and $K_\nu(x)$ is the modified Bessel function. The mass spectrum for the $q\bar{q}$ excitation was calculated in [4] and steeply decreases with $M^2$:

$$
\frac{d\sigma_D}{dM^2 dt} \bigg|_{t=0} \sim \frac{M^2}{(Q^2 + M^2)^3}. \tag{9}
$$

The $\propto 1/M^2$ component of the mass spectrum comes from the diffraction excitation of the $q\bar{q}g$ Fock state of the photon containing the soft gluon which carries the fraction $z_g \ll 1$ of photon’s lightcone momentum and gives rise to a large excited mass $M^2 \propto Q^2/z_g$. Let
\( \vec{r}, \vec{\rho}_1, \vec{\rho}_2 \) be the \( q \bar{q}, g-q \) and \( g \bar{q} \) separations in the impact parameter (transverse size) plane, \( \vec{\rho}_2 = \vec{\rho}_1 - \vec{r} \). Then, in the triple-pomeron regime of \( x_{IP}, \beta \ll 1 \),

\[
(Q^2 + M^2) \left. \frac{d\sigma_D}{dt dM^2} \right|_{t=0} = \int d\bar{r} d\bar{\rho}^2 \bar{\rho}_1 \left\{ z_g |\Phi(\vec{r}, \vec{\rho}_1, \vec{\rho}_2, z, z_g)|^2 \right\}_{z_g=0} \cdot \frac{\sigma_3^2(x_{IP}, r, \rho_1, \rho_2) - \sigma^2(x_{IP}, r)}{16\pi}
\]

(10)
in which the square of the 3-parton wave function \( |\Phi|^2 \) equals \([5,6]\)

\[
|\Phi(\vec{r}, \vec{\rho}_1, \vec{\rho}_2, z, z_g)|^2 = \frac{1}{z_g 3\pi^3} |\Psi_{\gamma^*}(Q^2, z, r)|^2 \mu_G^2 \left| g_S(R_1) K_1(\mu_G \rho_1) \frac{\vec{\rho}_1}{\rho_1} - g_S(R_2) K_1(\mu_G \rho_2) \frac{\vec{\rho}_2}{\rho_2} \right|^2 .
\]

(11)

Here \( g_S(r) \) is the running color charge, \( \alpha_S(r) = g_S(r)^2/4\pi \), the arguments of color charges are \( R_i = \min\{r, \rho_i\} \). In the wave function (11), the \( \mu_G K_1(\mu_G r) \vec{\rho}/\rho \) emerges as \( \vec{\nabla}_\rho K_0(\mu_G \rho) \), where \( K_0(\mu_G \rho) \) is precisely the two-dimensional Coloumb-Yukawa screened potential. This makes self-explanatory the interpretation \([5,6,8,9]\) of \( R_c = 1/\mu_G \) as the correlation (propagation) radius for perturbative gluons. The 3-body interaction cross section equals \([5,6]\)

\[
\sigma_3(r, \rho_1, \rho_2) = \frac{9}{8} [\sigma(\rho_1) + \sigma(\rho_2)] - \frac{1}{8} \sigma(r) .
\]

(12)

Hereafter we focus on \( x_{IP} = x_{IP}^0 \) and, for sake of brevity, \( \sigma(r) \) stands for \( \sigma(x_{IP}^0, r) \). Notice that the subtraction of \( \sigma^2(r) \) in the integrand of (11) corresponds to the renormalization of the wave function of the \( q\bar{q} \) state for the radiation of perturbative gluons \([5,6]\). For a detailed discussion of the consistency of the above formalism with color gauge invariance constraints see \([5,6]\).

### 3 \( A_{3IP}(Q^2) \) in DIS: photon as an effective gluon-gluon dipole

Because of \( K_\nu(z) \sim \exp(-z) \) at large \( z \), and by virtue of (8), the dipole distribution (7) gives the typical size of the \( q\bar{q} \) dipole

\[
r^2 \lesssim R_{q\bar{q}}^2 = \frac{1}{\varepsilon^2} \propto \frac{1}{Q^2} .
\]

(13)
Consequently, at $Q^2 \gg 1/R_c^2$, to the standard leading Log$Q^2$ approximation, the dominant contribution to the diffraction cross section (10) comes from

$$r^2 \ll \rho_1^2 \approx \rho_2^2 \sim R_c^2.$$  

(14)

In the region (14), we have

$$\sigma_3(r, \rho_1, \rho_2) \approx \frac{9}{4} \sigma(\rho) \gg \sigma(r),$$

(15)

where $\rho = \frac{1}{2}(\rho_1 + \rho_2)$, and the virtual photon interacts as an effective gluon-gluon dipole of size $\rho \sim R_c$, with the $q\bar{q}$ pair acting as an octet color charge. Also, in this region we have

$$\mu_G^2 |K_1(\mu_G \rho_1)\tilde{\rho}_1 - K_1(\mu_G \rho_2)\tilde{\rho}_2|^2 \approx \frac{r^2}{\rho^4} \mathcal{F}(\mu_G \rho),$$

(16)

so that the 3-parton wave function factorizes. The form factor $\mathcal{F}(z) = z^2[K^2_1(z) + zK_1(z)K_0(z) + \frac{1}{2}z^2K_0^2(z)]$ satisfies $\mathcal{F}(0) = 1$ and $\mathcal{F}(z) \propto \exp(-2z)$ at $z > 1$.

The resulting diffraction cross section (10) takes on the factorized form

$$(Q^2 + M^2) \frac{d^2\sigma_D}{dtdM^2}\bigg|_{t=0} = \int d\rho d^2\tilde{r} |\Psi_{\gamma^*}(Q^2, z, r)|^2 \cdot \frac{16\pi^2}{27} \cdot \alpha_S(r)r^2 \times \frac{1}{2\pi^4} \cdot \left(\frac{9}{8}\right)^3 \cdot \int d\rho^2 \left[\frac{\sigma(\rho)}{\rho^2}\right]^2 \mathcal{F}(\mu_G \rho),$$

(17)

Making use of the fact that, at moderately small $x_{\gamma^*}$ and $r^2 \lesssim R_c^2$, for the proton target and 3 active flavors, the exchange by two perturbative gluons gives [5,6]

$$\sigma(x_{\gamma^*}, r) \approx \frac{16\pi^2}{27} r^2 \alpha_S(r) \log \left[\frac{1}{\alpha_S(r)}\right] \approx \frac{16\pi^2}{27} r^2 \alpha_S(r),$$

(18)

modulo to the logarithmic factor $\sim \log[1/\alpha_S(Q^2)] \sim 1$, we have

$$\int d\rho d^2\tilde{r} |\Psi_{\gamma^*}(Q^2, z, r)|^2 \cdot \frac{16\pi^2}{27} \cdot \alpha_S(r)r^2 \approx \sigma_{\text{tot}}(\gamma^* p) = \int d\rho d^2\tilde{r} |\Psi_{\gamma^*}(Q^2, z, r)|^2 \sigma(r).$$

(19)

Consequently, (17) becomes equivalent to (3) [we will be back to a detailed calculation of $A_{3\gamma^*}(Q^2)$ in section 5] with

$$A_{3\gamma^*}(Q^2) \sim A_{3\gamma^*} = \frac{1}{2\pi^4} \cdot \left(\frac{9}{8}\right)^3 \cdot \int d\rho^2 \left[\frac{\sigma(\rho)}{\rho^2}\right]^2 F(\mu_G \rho).$$

(20)
As the factorization (16) holds simultaneously for all the \( q\bar{q}g \) states, we predict independence of \( A_{3\Pi}(Q^2) \) on the flavour "\( i \". \( A_{3\Pi}^* \) is dominated by \( \rho \sim R_c \). Making use of (18), we obtain the order of magnitude estimate of \( A_{3\Pi}^* \):

\[
A_{3\Pi}^* \sim \frac{1}{16}R_c^2 \sim 0.1 \text{ GeV}^{-2}.
\]  

(21)

Here the scale for \( A_{3\Pi}^* \) is set by the size \( \rho \sim R_c \) of the \( q\bar{q}g \) Fock state of the photon. Following [8,9], here we have taken \( \mu_G = 0.75 \text{ GeV} \) as suggested by lattice QCD studies [10].

Eq. (18) describes the contribution to the dipole cross section \( \sigma(x_{\Pi}, r) \) from the exchange by perturbative gluons. This component \( \sigma^{(pt)}(x, r) \) is a solution of the generalized BFKL equation [5,6,8,9] and rapidly rises towards large \( \frac{1}{x} \), dominating the observed growth and giving a good quantitative description of the proton structure function at HERA [11]. The phenomenological description of \( \sigma(x, r) \) at large dipole sizes, \( r \gtrsim R_c \), requires introduction of the nonperturbative component \( \sigma^{(npt)}(r) \) of the dipole cross section [11-14], which is expected to have a weak energy dependence (the scenario [11,12] introduces an energy-independent \( \sigma^{(npt)}(r) \)) and must be inferred from experimental data. Here we wish to recall that real photoproduction of the \( J/\Psi \) and exclusive leptoproduction of the \( \rho^0 \) at \( Q^2 \sim 10 \text{ GeV}^2 \), probe the (predominantly nonperturbative) dipole cross section at \( r \sim 0.5 \text{ fm} \lesssim 2R_c \) [12-14]. Real, and weakly virtual \( Q^2 \sim 10 \text{ GeV}^2 \), photoproduction of the open charm probes the (predominantly perturbative) dipole cross section at \( r \sim \frac{1}{m_c} \sim \frac{1}{2}R_c \) [11,12]. The proton structure function \( F_2^p(x, Q^2) \) probes the dipole cross section in a broad range of radii from \( r \sim 1 \text{ fm} \) down to \( r \sim 0.02 \text{ fm} \). Successful quantitative description of the corresponding experimental data in [11-14] implies that we have a reasonably good, to a conservative accuracy \( \lesssim (15-20)\% \), understanding of the dipole cross section at \( r \sim R_c \) of the interest for evaluation of \( A_{3\Pi}^* \). Quantitatively, at \( r \sim R_c \) the dipole cross section \( \sigma(x_{\Pi}^0, r) \) receives approximately \( 1:2 \) contributions from the exchange by perturbative gluons (18), and from the nonperturbative component \( \sigma^{(npt)}(r) \). The numerical calculation with the dipole cross section of Ref. [12] gives

\[
A_{3\Pi}^* = 0.56 \text{ GeV}^{-2},
\]

(22)

which is of the same order in magnitude as \( A_{3\Pi}(Q^2 = 0) \approx 0.16 \text{ GeV}^2 \) from the real photoproduction analysis [3].
4 Real photoproduction: diffraction excitation of ”constituent” quarks

At a first sight, the mechanism of diffraction dissociation of real photons, $Q^2 = 0$, is quite different from the above in DIS. For diffraction dissociation of real photons, in the dipole distribution (7) the typical size is large, of the hadronic scale,

$$r^2 \sim R^2_{qg} \approx \frac{1}{m_q^2} \gg R^2_c$$

(23)

Here, following [12,15], for light flavours we use $m_q = 0.15 \text{ GeV}$. Such a choice of $m_q$ in the wave function (7) gives, with the same dipole cross section $\sigma(x, r)$, a good quantitative description of the real photoabsorption cross section [12,4], of exclusive leptoproduction of vector mesons at moderate $Q^2$ [14], of nuclear shadowing in DIS on nuclei [15] and of color transparency effects in exclusive production of vector mesons on nuclei [13,14]. Because of (23), the 3-parton distribution $|\Phi|^2$ will be dominated by configurations with $\rho_1^2 \ll R^2_c \ll \rho_2^2 \sim r^2$ and $\rho_2^2 \ll R^2_c \ll \rho_1^2 \sim r^2$. The dipole distribution in the $q\bar{q}g$ state takes on the factorized form first considered in [4]:

$$|\Phi(\vec{r}, \vec{p}_1, \vec{p}_2, z, z_g)|^2 = \frac{1}{z_g} \frac{4}{3\pi^2} |\Psi_{\gamma^*}(Q^2, z, r)|^2 \mu_G^2 \left[ \alpha_S(\rho_1) K^2(\mu_G \rho_1) + \alpha_S(\rho_2) K^2(\mu_G \rho_2) \right].$$

(24)

We recover a sort of the constituent quark model, in which the gluon clusters with the (anti)quark into the $qg$ and/or $\bar{q}g$ cluster of size $\rho \sim R_c$, with the square of the $qg$ wave function of the ”constituent” quark $\propto \frac{1}{z_g} \alpha_S(\rho) K^2(\mu_G \rho)$ . Diffraction dissociation of the photon $\gamma^* \rightarrow q + \bar{q} + g$ proceeds via diffraction excitation of the ”constituent” (anti)quark $q(\bar{q}) \rightarrow q(\bar{q}) + g$ of the parent $q\bar{q}$ state of the photon. Eq. (12) leads to the estimate

$$\sigma_3(r, \rho_1, \rho_2) \approx \sigma(r) + \frac{9}{8} \sigma(\rho)$$

and

$$\sigma_3^2(r, \rho_1, \rho_2) - \sigma^2(r) \approx \frac{9}{4} \sigma(r) \sigma(\rho),$$

(25)

where $\rho = \min\{\rho_i\}$. Then, the diffraction cross section (10) takes the factorized form

$$M^2 \frac{d^2\sigma_D}{dt \, dM^2} \bigg|_{t=0} \approx \int dz \, d^2\vec{r} \, |\Psi_{\gamma^*}(Q^2 = 0, z, r)|^2 \sigma(r)$$

$$\times \frac{3}{8\pi^2} \int d\rho^2 \left[ \frac{\sigma(\rho)}{\rho^2} \right] f^2(\mu_G \rho) = \sigma_{tot}(\gamma p) A_{3\text{IP}}(0),$$

(26)
where
\[
\sigma_{\text{tot}}(\gamma p) = \int dz \, d^2 \vec{r} \, |\Psi_{\gamma^*}(Q^2, z, r)|^2 \sigma(r) \tag{27}
\]
and
\[
A_{3\text{IP}}(0) \approx \frac{3}{8\pi^2} \cdot \int d\rho^2 \alpha_S(\rho) \left[ \frac{\sigma(\rho)}{\rho^2} \right] f^2(\mu G\rho) \sim \frac{1}{18} R_c^2, \tag{28}
\]
where \( f(z) = z K_1(z) \). A comparison of estimates (21) and (28) shows that, modulo to the numerical, and logarithmic, factors \( \sim 1 \), we obtained \( A_{3\text{IP}}(Q^2) \approx A_{3\text{IP}}(0) \), which was conjectured some time ago in [4-7,16]. As a matter of fact, the exact large-\( r \) behaviour of \( \sigma(r) \) is not a main point here, we only should assume that (27) reproduces the observed total phototabsorption cross section. With the dipole cross section of Ref. [12] we find \( \sigma_{\text{tot}}(\gamma p) = 108 \mu \text{b} \), in good agreement with the Fermilab data [17]. A direct calculation from (10)-(12) gives
\[
A_{3\text{IP}}(0) = 0.23 \text{GeV}^{-2} \tag{29}
\]
in agreement with the real photoproduction determination \( A_{3\text{IP}} \approx 0.16 \text{GeV}^{-2} \) [3].

5 \( Q^2 \) dependence of \( A_{3\text{IP}}(Q^2) \). Discussion of results.

To have more insight into the \( Q^2 \) dependence of the triple-pomeron coupling, here we present the results of a direct evaluation of \( A_{3\text{IP}}(Q^2) \) from equations (5) and (10). We consider \( x_{3\text{IP}} = 0.03 \) and in the case of DIS, we take \( x = 0.004 \). In this range of \( x \) and moderate \( Q^2 \lesssim 10 \text{GeV}^2 \), the proton structure function is approximately flat vs. \( \frac{1}{x} \). The equivalent c.m.s. energy in the real photoproduction can be estimated as \( W^2 \sim m_p^2/x \sim 250 \text{GeV}^2 \), which corresponds to the energy range of the Fermilab experiment [3]. We calculate the corresponding real and virtual photoabsorption cross section from Eqs. (19,27) (for a detailed comparison with experiment see [11,12,14]).

Our results for the \( Q^2 \) dependence of the triple-pomeron coupling \( A_{3\text{IP}}(Q^2) \) are presented in Fig.1. The main feature of \( A_{3\text{IP}}(Q^2) \) is its weak \( Q^2 \) dependence, which was anticipated in [4-7,16]. Still, we predict a slight, by a factor \( \sim 1.6 \), growth of \( A_{3\text{IP}}(Q^2) \) from \( A_{3\text{IP}}(Q^2 = 0) = 0.23 \text{GeV}^2 \) to the DIS value \( A_{3\text{IP}}(Q^2 \approx 10 \text{GeV}^2) = 0.36 \text{GeV}^{-2} \). This rise of \( A_{3\text{IP}}(Q^2) \) is predicted to take place predominantly at \( Q^2 \lesssim Q^* \approx (2-3) \text{GeV}^2 \). The scale
for $Q^*^2$ corresponds to the transition from the regime of diffraction dissociation of the "constituent" quark of section 4 to the regime of diffraction dissociation of the octet-octet state of the photon of section 3. This transition takes place when the typical size of the $q\bar{q}$ pair $R_{q\bar{q}}$ becomes of the order of the size $\sim R_c$ of the "constituent" quark:

$$R_{q\bar{q}} = \frac{1}{\varepsilon} \approx \frac{2}{\sqrt{Q^2}} \sim R_c,$$

which leads to the estimate

$$Q^*^2 \sim \frac{4}{R_c^2} = 3 \text{GeV}^2.$$

The predicted $Q^2$ dependence of $A_{3\text{IP}}(Q^2)$ can be tested at HERA. The onset of the GLDAP evolution of the pomeron structure function requires $R_{q\bar{q}}^2 \ll R_c^2$, i.e., $Q^2 \gtrsim Q_{\text{IP}}^2 \gg Q^*^2$.

Following the analysis [11], a reasonable choice for the corresponding factorization scale is $Q_{\text{IP}}^2 = 10 \text{GeV}^2$. The value of $A_{3\text{IP}}(Q_{\text{IP}}^2)$ determines the normalization of the input sea structure function of the pomeron (more detailed analysis of the partonic structure of the pomeron and of its evolution properties is presented elsewhere [18]). Notice that for diffraction excitation of "constituent" quarks in real photoproduction, the calculated $A_{3\text{IP}}(0)$ is a linear functional of $\sigma(R_c)$, whereas for diffraction excitation of the octet-octet state of the photon in DIS, $A_{3\text{IP}}(Q_{\text{IP}}^2)$ is a quadratic functional of $\sigma(R_c)$. Hence the conservative $\sim$(15-20)% uncertainty in our present knowledge of $\sigma(R_c)$ implies the conservative theoretical uncertainty of $\sim$(15-20)% and $\sim$(30-40)% in the predicted value of $A_{3\text{IP}}(0)$ and $A_{3\text{IP}}(Q_{\text{IP}}^2)$, respectively.

The indirect experimental evidence for weak $Q^2$ dependence of $A_{3\text{IP}}(Q^2)$ comes from nuclear shadowing in DIS on nuclei. Diffractive excitation of large masses contributes significantly to nuclear shadowing at $x \ll 10^{-2}$, and in [15] it was shown that calculations using the above photoproduction value of $A_{3\text{IP}}(0)$ are in good agreement with the experiment. Crude evaluations [4,19] of the total rate of diffractive DIS, using the photoproduction value of $A_{3\text{IP}}(0)$, are also consistent with the HERA data [20] (a detailed treatment of diffractive DIS in the dipole approach to the BFKL pomeron is presented elsewhere [18]).

The so-called absorption corrections, not considered here, will slightly reduce $A_{3\text{IP}}(Q^2)$ (for the relevant formalism see [5,6]). The absorption correction is typically of the order of $\sigma_3/8\pi B$, where $B$ is the diffraction slope. In real photoproduction, $\sigma_3 \sim \sigma(r \sim 1/m_q)$ and the absorption correction to $A_{3\text{IP}}(0)$ will be larger than to $A_{3\text{IP}}(Q_{\text{IP}}^2)$, where $\sigma_3 \sim \frac{9}{4} \sigma(r \sim R_c)$.
is much smaller. Therefore, the increase of $A_{3\mathbf{P}}(Q^2)$ by the factor $\sim 1.6$ cf. $A_{3\mathbf{P}}(0)$ will be retained and, as a matter of fact somewhat enhanced, by the absorption corrections.

To summarize, we presented the first direct evaluation of the triple-pomeron coupling $A_{3\mathbf{P}}$. With $R_c \approx 0.3\text{ fm}$, we find good agreement with the experimental determination of $A_{3\mathbf{P}}(0)$. We related the small numerical value of $A_{3\mathbf{P}}$, which was a mystery, to the small correlation radius $R_c$ for the perturbative gluons. We predict a slight rise of $A_{3\mathbf{P}}(Q^2)$ with $Q^2$, by a factor $\sim 1.6$, from real photoproduction to DIS at $Q^2 \sim 10\text{ GeV}^2$.

In a somewhat related approach to the BFKL pomeron, the triple-pomeron regime was considered also in [21]. These authors consider the scaling BFKL regime of $R_c = \infty$ and fixed $\alpha_S = \text{const}$ [22], and their results are not applicable to the forward diffraction dissociation ($t = 0$) considered here.

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Figure captions

Fig.1 - Our prediction for the $Q^2$ dependence of $A_3^{\text{NP}}(Q^2)$. 
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