Spontaneous Symmetry Breaking in Gauge Theories: a Historical Survey

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Abstract

The personal and scientific history of the discovery of spontaneous symmetry breaking in gauge theories is outlined and its scientific content is reviewed.

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1 Talks presented at the Award Ceremony of the 1997 High Energy and Particle Physics Prize of the European Physical Society (Jerusalem, 24 August 1997)
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Our discovery of spontaneous symmetry breaking in gauge theory is intimately linked to the history of our collaboration. Evoking this period of our life, I shall survey the scientific history of spontaneous symmetry breaking. A more detailed historical review can be found in the talk presented by Veltman at the International Symposium on Electron and Photon Interactions at High Energies in 1973 at Bonn.

I came to Cornell University in 1959 as a Research Associate to Robert Brout who was Professor there. My background was mainly in solid state physics and many body problems, and Robert Brout was already well known for his work in these fields. Our first contact was unexpectedly warm. It started right when he came to take me at the airport with his near century old Buick and took me to a drink which lasted up to the middle of the night. When we left, we knew that we would become friends.

At Cornell, we realized that, in our approach to physics, we were different. Robert had an amazing easiness in translating abstract concepts into tangible intuitive images, whereas I, with my latin oriented education, had on the contrary always a tendency to express images in terms of formal structures. But this difference turned into a fruitful complementarity because we quickly learned to understand the functioning of each others mind. Still today, talking physics, each of us gets somehow frustrated not to be able to terminate a sentence, as the other does it for him, and is apparently very happy to do so.

Playing together on many-body problems, we got involved in the study of phase transitions and particularly in ferromagnetism. We understood the importance of the spin wave excitations for the description of the ferromagnetic phase. The order parameter, the magnetization, is the manifestation of spontaneously broken rotation invariance and the spin waves are collective modes whose energy goes to zero when the wavelength goes to infinity. These are the massless Nambu-Goldstone bosons [1, 2, 3] associated to the spontaneously broken symmetry. Their dynamics essentially determine the magnetization curve. When the range of the forces between spins is extended to all spins instead of, as is usually considered, limited to near neighbors, the spin waves become effectively massive and the magnetization curve encodes the emergence of such a gap in the spectrum. This was the first time we realized that the Nambu-Goldstone bosons, which signal in general the spontaneous breakdown of a global symmetry, cannot survive in presence of long range forces.

Our study of phase transition led us also to the analysis of superconductivity. We were extremely impressed by Nambu’s formulation of the BCS theory [4]. This paper and the related papers of Nambu and Jona-Lasinio [4, 2] on spontaneously broken chiral invariance brought to light, in full field theoretic terms, the emergence of the massless Nambu-Goldstone boson. They also pointed out the existence of a massive scalar bound state in...
the channel orthogonal to the massless mode. The significance of this massive scalar is more transparent in the context of the Goldstone scalar field model when the symmetry breaking is driven by the scalar field potential itself: it describes the response to the order parameter. In the particular case of the abovementioned ferromagnetic transition, this response is the longitudinal susceptibility. These beautiful papers were certainly an element which later drove us into field theory.

In fall 1961, I was scheduled to return to Belgium. By that time our collaboration and our friendship had become deeply rooted. I received an offer of a professorship at Cornell but I was missing Europe very much. I decided not to accept it and to return to Belgium. Robert and his wife Martine had a similar attraction for the Old Continent; Robert got a Guggenheim fellowship and they joined me in Belgium. After a few months, the social life there and our personal relations decided Robert to resign from his professorship at Cornell University and to settle permanently at Brussels University.

We then resumed in Belgium our analysis of broken symmetry. We knew from our study of ferromagnetism that long range forces give mass to the spin waves and we were aware, from Anderson’s analysis of superconductivity, of the fact that the massless mode of neutral superconductors, which is also a Nambu-Goldstone mode, disappears in charged superconductors in favor of the usual massive plasma oscillations resulting from the long range coulomb interactions in metals. Comforted by these facts, we decided to confront, in relativistic field theory, the long range forces of Yang-Mills gauge fields with the Nambu-Goldstone bosons of a broken symmetry.

The latter arose from the breaking of a global symmetry and Yang-Mills theory extends the symmetry to a local one. Although the problem in this case is more subtle because of gauge invariance, the emergence of the Nambu-Goldstone massless boson is very similar. We indeed found that there were well defined gauges in which the broken symmetry induces such modes. But, as we expected, the long range forces of the Yang Mills fields were conflicting with those of the massless Nambu Goldstone fields. The conflict is resolved by the generation of a mass reducing long range forces to short range ones. In addition, gauge invariance requires the Nambu-Goldstone mode to combine with the Yang Mills excitations. In this way, the gauge fields acquire a gauge invariant mass!

This work was finalized in 1964. We obtained the mass formula for gauge fields, abelian and non abelian, where symmetry breaking arises from non vanishing expectation values of scalar fields. These play the role of “order parameters”. We also, guided by Nambu’s work on superconductivity, obtained through Ward identities a mass formula for a dynamical symmetry breaking by fermion condensate. Thus the scalar fields could be either fundamental or constitute a phenomenological description of a condensate. Only future experimental and theoretical development can tell. Their massive excitations is a property of global symmetry breaking and are to be identified with
the Goldstone (or with the Nambu) massive scalar bosons describing the response to the “order parameters”; they are not altered by the introduction of local symmetry. On the contrary, the Nambu-Goldstone bosons which arise in group space in directions orthogonal to the massive scalars, and also exist whether the scalar fields are fundamental or composite, are “eaten up” by the gauge fields which lay in these directions. These acquire mass. This fate of the Nambu-Goldstone bosons is the characteristic feature of the symmetry breaking mechanism in a local gauge symmetry.

I shall not dwell on subsequent developments but briefly recall the most relevant steps. First there is the work of Higgs who obtained essentially the same results in a somewhat different way [8]. He showed in simple field theoretic terms that the Nambu-Goldstone boson was unobservable as such but provided the required longitudinal polarization for the gauge fields to get mass [9]. This fact is deeply related to the unitarity of the scheme and is less explicit in our approach. On the other hand, our formulation of the problem, using covariant gauges and Ward identities, puts into evidence that power counting for Feynman graphs was consistent with renormalizability. This is why we were led in 1966 to suggest that the theory of vector mesons with mass generated by the symmetry breaking mechanism was renormalizable [10]. But the full proof of renormalizability was much more involved and in fact required a detailed analysis of the consistency of the power counting in covariant gauges and of the unitarity of the theory. This was worked out by Veltman and ’t Hooft and the proof was essentially completed in 1971 [11]. It rendered the electroweak theory [12, 13], hitherto the most impressive application of the mechanism discovered by Higgs and ourselves, a truly consistent and predictive scheme whose experimental verification confirmed the validity of the symmetry breaking mechanism.

This ends the story of the genesis of the unification scheme which relates short and long range forces in gauge field theory, indicating a path to the search for more general laws of nature. For us, it was only the beginning of our lasting collaboration and of our lasting friendship.
The acquisition of mass by gauge vector mesons results from the mutual coupling of two fields each of which, in other circumstances, had vanishing mass. These are, on one hand, the zero mass excitations which result from the spontaneous breakdown of symmetry (SBS) and, on the other, the zero mass vector field which is necessitated when this same symmetry is promoted from global to local, in which case it is called a gauge symmetry. In relativistic theory these are called respectively the Nambu-Goldstone (NG) and Yang-Mills (YM) fields, although they have had a long prior history, the first in statistical mechanics and solid state physics and the second, of course, dating from Maxwell. I shall first explain their masslessness in conceptual terms and then indicate how their coupling gives rise to the vector mass.

We begin with the NG field which arises in consequence of SBS. But firstly what is SBS? An amusing image has been given by Abdus Salam. Consider an ensemble of dinner companions seated round the table set with plates between which is placed a spoon. When the first guest chooses a spoon, to his right or to his left, all others must follow suit. This is SBS.

The above example is a case of discrete SBS. A spoon on the right (left) of a plate is represented by a spin which is on a lattice site that is polarized up (down). Interaction favors that neighboring spins be parallel. So the ground state is all spins up (or down). An angel who fixes the polarization of some spin then determines the polarization of all. Clearly it costs a finite amount of energy to turn one spin against the other so that this model presents an excitation spectrum which has a gap.

Contrast this to SBS for the case of continuous symmetry. For simplicity take the group $U(1)$. Then the spin has two components $(S_x, S_y)$ such that $S_x^2 + S_y^2 = 1$. Once more the ferromagnetic interaction favors neighbors to be parallel. Imagine a state where all sit at an angle $\theta$ with respect to the $x$-axis in group space. Clearly it costs no energy to rotate them all through an angle $\Delta \theta$, since they all remain parallel. The ground state is thus degenerate with respect to $\theta$. Now divide the system in two and rotate the two halves against each other. Only the spins that “rub” against each other require an energy to make such a configuration. Call this one node’s worth of energy. If one divides in thirds, it will cost two nodes’ worths of energy, and so on. Since translational symmetry of the lattice requires that the excitations be classed by wave number, $k$, we see that the energy grows with $k$, and furthermore $\omega(k = 0) = 0$. So $k = 0$ is the terminal point of a spectrum which starts at zero frequency. This is the NG excitation.

It is noteworthy that this process of the monotonic increase of energy with the number of nodes stops once $k$ reaches the inverse range of the force between spins, for then it cost no more energy to make more nodes. Thus as the range tends to infinity, the
spectrum develops a gap, that is a mass, i.e. \( \lim_{k \to 0} \omega(k) \) is finite. This is a precursor of what happens in general when long range forces are present.

I now explain why it is natural that gauge fields are also massless. Take once more the simple case of spins. Global symmetry is the invariance of the energy when all spins are rotated “en masse”. Local symmetry is realized when different portions can be rotated differently (in group space) at no cost of energy. This is possible only if there is a “messenger” which transmits from portion to portion the information that such local rotations indeed do not cost any energy and have no physical effect. In technical terms, this messenger is called a connection or a gauge field. It transforms under local rotations exactly in a way to compensate for the energy that would otherwise follow from relative rotations of neighboring spins. It is this beautiful idea which governs all the presently known interactions of nature.

From the above one understands that it is natural that the gauge field has zero mass. Indeed under global transformations a gauge field is not required to ensure invariance. So it should not manifest itself. But a global transformation corresponds to one whose wave vector \( k_\mu \) vanishes: it should cost no energy \( (k_0 = 0) \) to make a gauge field excitation which is everywhere the same \( (\vec{k} = 0) \). In relativity the condition \( k_\mu = 0 \) becomes the invariant statement \( k_\mu k^\mu = 0 \) (or \( k_0^2 - \vec{k}^2 = 0 \)) which is the statement of masslessness.

It is to be expected that a dramatic situation arises when these two kinds of zero mass excitations are put together in the context of a local symmetry. What happens is that they combine into one massy vector field. The gauge field, of its elf, due to relativistic constraints has two degrees of freedom. These are encoded in the polarization transverse to the direction of propagation. Massive vector fields have a longitudinal polarization as well and this is induced by the coupling to the NG field. To see how this mechanism works it is convenient to express things in terms of Feynman graphs.

In the case of no SBS, gauge fields propagate in vacuum by taking into account the dielectric constant of the vacuum. This is represented by loop insertion in the gauge field propagator. For matter represented by a scalar field, a single loop insertion is drawn as follows

These loops insertions in the YM propagator, represented by wavy lines, conserve the transverse character of the gauge field and keep it massless. Their effect is to change the value of the coupling constant of the gauge field to matter.
In the case of SBS, the finite expectation value of the scalar field causes additional graphs to arise which are found by cutting the loop, generating the so-called tadpole graphs. One must then addendum to the above the graphs

\[\text{\includegraphics[width=0.5\textwidth]{tadpole_graphs.png}}\]

In these graphs the wavy lines still represent the gauge field and the solid line tadpoles are the expectation values of the scalar fields which play the role of order parameters. The dashed line is the propagator of a NG excitation. These arise in directions orthogonal in group space to the order parameters. The latter graphs show how the NG field gets absorbed into the gauge field, the net effect being to give to the latter a mass proportional to the order parameter and to increase the number of degrees of freedom of the gauge field from two to three. Although this “order parameter” is here gauge dependent, there are Ward identities ensuring the gauge invariance of the mass arising from these graphs. These two elements are of utmost importance since the gauge invariance ensures that the divergences of the graphs remain under control, indicating that the theory could be renormalizable, and the new longitudinal degree of freedom renders the perturbation series unitary. It is the combination of these two elements that Veltman and ’t Hooft used in their masterful works to prove that the theory is indeed renormalizable, thereby really setting the standard model on a sound basis.

As just mentioned, the appearance of the massless NG bosons is guaranteed by the Ward identities, and as such does not rely on perturbation theory. They therefore also appear if SBS is realized dynamically through a fermion condensate, as in the BCS theory of superconductivity or in the Nambu Jona-Lasinio theory of broken global chiral symmetry. In presence of a local symmetry, they would then still generate a mass for the gauge vector mesons. In that case, the scalar fields would be phenomenological rather than fundamental objects but the mechanism would remain essentially the same. Whether fundamental or not, the scalar fields describing the order parameters, have massive quanta. These massive scalars are not a specific feature of the mechanism: they arise already in global SBS, and even in discrete ones such as our original discrete spin system. The free energy of such a system presents as a function of the magnetization, below the Curie point, the double dip shape typical of the Landau-Ginsburg potential \(V\) represented in the figure below. This potential is the same as the one driving global SBS in the Goldstone scalar field model. The distance of the dip to the origin and the curvature at this point are respectively the expectation value and the mass squared of the Goldstone massive scalar boson. The latter, or more precisely its inverse mass squared, measures the longitudinal susceptibility. This is the response of a field parallel to the order parameter and appears in any second order phase transition.
I conclude by restating the main results with some emphasis on their phenomenological implications.

Massive gauge vector mesons are an inevitable consequence of SBS, independently of the dynamical mechanism which causes the breaking, scalar fields, bound state condensate... Observationally in the electroweak sector they occur as narrow resonances because their coupling to the continuum is small. This latter is an observational fact: the electric charge is small and thus governs the scale of all gauge field couplings.

Massive scalar occur in channels orthogonal to the NG channels. They appear in any global SBS and are thus not a specific feature of mass generation for gauge fields; their physics is accordingly much more sensitive to dynamical assumptions. They too would appear as resonances whether or not they be manifestations of an elementary scalar field or as composites due to a more elaborate mechanism. Whether or not these resonances are swamped into the continuum depends on the parameters of the theory and we do not control these in the same way as we control coupling to the gauge vectors. Thus observation of both mass and width in these channels will deliver to us precious indications of the mechanism at work.

In all cases SBS in gauge theories is characterized by NG bosons which are “eaten up” by the YM fields, giving them longitudinal polarization and mass. It is this phenomenon which allows for consistent renormalizable theories of massive gauge vector mesons.

References

1Within the past few years, an interesting development has occurred principally due to the mathematician A. Connes who has applied techniques of non commutative geometry to construct the standard model. In this, the key point is that the scalar field plays the role of a gauge connection in the “motion” of a fermion (whose mass is generated by spontaneously broken chiral symmetry) during its zitterbewegung.
[1] Y. Nambu, Phys.Rev.Lett. 4 (1960) 380.

[2] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345; Phys. Rev. 124 1961 246.

[3] J. Goldstone, Il Nuovo Cimento 19 (1961) 154.

[4] Y. Nambu, Phys. Rev. 117 (1960) 648.

[5] P.W. Anderson, Phys. Rev. 112 (1958) 1900.

[6] C.N. Yang and R.L. Mills, Phys. Rev. 96 (1954) 191.

[7] F. Englert and R. Brout, Phys. Rev. Lett. 13 (1964) 321.

[8] P.W. Higgs, Phys. Rev. Lett. 13 (1964) 508.

[9] P.W. Higgs, Phys. Rev. 145 (1966) 1156.

[10] F. Englert, R. Brout and M. Thiry, Il Nuovo Cimento 43A (1966) 244; see also the Proceedings of the 1997 Solvay Conference, *Fundamental Problems in Elementary Particle Physics*, Interscience Publishers J. Wiley ans Sons, p 18.

[11] G. 't Hooft, Nucl. Phys. B35 (1971) 167.

[12] S.L. Glashow, Nucl. Phys. 22 (1961) 579.

[13] S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264.