We formulate ($N = 1, d = 11$) supergravity in components in light-cone gauge ($LC_2$) to order $\kappa$. In this formulation, we use judicious gauge choices and the associated constraint relations to express the metric, three-form and gravitino entirely in terms of the physical degrees of freedom in the theory.
1 Introduction

Eleven-dimensional supergravity is an interesting theory for a number of reasons. Chief among these is that the theory is the higher dimensional progenitor for \((N = 8, d = 4)\) supergravity. There are indications that the \(N = 8\) theory is perturbatively finite making it a candidate for a finite quantum field theory of gravity - this makes understanding the parent theory important as well. In this paper, we formulate \(d = 11\) supergravity in components, to order \(\kappa\) in light-cone gauge. This is an interesting exercise in itself: given the three very different fields in the theory, judicious gauge choices can vastly simplify the structure of the Lagrangian. We will make a number of such choices to make manifest the physical degrees of freedom and highlight the close ties between the graviton, gravitino and three-form. \((N = 1, d = 11)\) supergravity is ultra-violet divergent. A model like M-theory, also in eleven dimensions, presumably tames these divergences and an understanding of how may arise from a study of the ultra-violet properties of \(d = 11\) supergravity. Another point of interest is the role of the little group in divergence analysis. Hughes \[1\] conjectured that divergence cancellations in field theories could be traced back to the space-time little group. Curtright \[2\] made this proposal more concrete by considering loop integrals arising from theories in higher dimensions

\[ \Pi_{mn}(q) \propto \left( I^{(2)} - I^{(0)} \right) \left( q_m q_n - q^2 g_{mn} \right) f(q^2) \]  

(1)

\(f(q^2)\) represents a generic one-loop integral, \(D\) the dimension of space-time and \((-)^s = +1\) for bosons and \(-1\) for fermions. \(I^{(0)}\) and \(I^{(2)}\) represent Dynkin indices corresponding to \(O(D - 2)\) thus emphasizing the central role played by the space-time little group in determining ultra-violet behavior. The structure of \(SO(9)\), for example, offers insights into the divergent nature of the \((N = 1, d = 11) \leftrightarrow (N = 8, d = 4)\) system \[3\]. A systematic derivation of \(\Pi\), in the context of Yang-Mills, was undertaken in \[4\]. Extending this analysis to supergravity requires an \(LC_2\) formulation in components\[4\] and this motivates the present paper. A light-cone formulation where the unphysical degrees of freedom are eliminated is referred to as \(LC_3\) as opposed to \(LC_4\) \[6\].

2 Eleven-dimensional supergravity

The bosonic field content of eleven-dimensional supergravity consists of the elfbein, \(e_\mu^a\) and a completely antisymmetric 3-form potential \(A_{\mu\nu\rho}\) with field strength \(F_{\mu\nu\rho\sigma} = \partial_{[\mu} A_{\nu\rho\sigma]}\). In terms of the \(SO(9)\) little group in eleven dimensions, these correspond to a total of 128 bosonic states. The fermionic content consists of a single Majorana field, \(\Psi_\mu\) which has 128 fermionic states. The \(N = 1\) supergravity action in eleven dimensions is \[7\]

\[ S = \int d^{11} x \ e \left\{ L_1 + L_2 + L_3 + L_4 \right\} , \]  

(2)

\[ ^1\text{As opposed to a superspace or manifestly supersymmetric formulation [5].} \]
where \( e \) is the elfbein determinant and the individual Lagrangians are [8]

\[
L_1 = -\frac{1}{2} e R(e, \omega),
\]

\[
L_2 = -\frac{1}{48} e F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma} + \frac{2\kappa}{(12)!)^2} e^{\mu_1 \cdots \mu_{12}} F_{\mu_1 \cdots \mu_4} F_{\mu_5 \cdots \mu_8} A_{\mu_9 \mu_{10} \mu_{11}},
\]

\[
L_3 = -\frac{i}{2} e \bar{\Psi}_\mu \gamma^{\mu\rho\sigma} D_\rho (\omega + \hat{\omega}) \Psi_\sigma,
\]

\[
L_4 = \frac{\kappa}{192} e (\bar{\Psi}_\mu \gamma^{\mu\rho\sigma\alpha\beta} \Psi_\beta + 12 \bar{\Psi}_\mu \gamma^{\rho\sigma} \Psi_\beta) [F_{\nu\rho\sigma\alpha} + \hat{F}_{\nu\rho\sigma\alpha}].
\]

We will work with both the metric and elfbein fields \( e_\mu^a \), since we need to introduce fermions. We use light-cone coordinates for both space-time and the locally flat indices. Whenever necessary, we will circle space-time light-cone indices to differentiate them from locally flat light-cone indices. \( \mu, \nu \ldots \) represent space-time indices, \( a = +, -, i \) where \( i = 1 \ldots 8 \) while \( a, b \ldots \) are the locally flat indices, \( a = +, -, i \) with \( i = 1 \ldots 8 \).

Working in the 1.5 order formalism [9], we formulate the \( d = 11 \) theory in light-cone gauge to order \( \kappa \). The spin-connection, determined by the variation \( \frac{\delta}{\delta \omega} S = 0 \) is

\[
\omega_{\nu ab}(e) = e_a^\rho (\partial_\nu e_{b \rho} - \partial_\rho e_{b \nu}) - e_b^\rho (\partial_\nu e_{a \rho} - \partial_\rho e_{a \nu}) + e_a^\rho e_b^\sigma (\partial_\nu e_{c \rho} - \partial_\rho e_{c \nu}) e_c^\nu
\]

\[
+ \frac{\kappa^2}{4} (\bar{\Psi}_\nu \gamma_a \Psi_b - \bar{\Psi}_\nu \gamma_b \Psi_a + \bar{\Psi}_a \gamma_\nu \Psi_b) - \frac{\kappa^2}{8} \bar{\Psi}_a \gamma_{\nu ab} \Psi_\beta.
\]

The curvature is defined as

\[
R_{\mu \nu a b} = \partial_\mu \omega_{\nu a b} - \partial_\nu \omega_{\mu a b} + \omega_{\mu a c} \omega_{\nu c b} - \omega_{\nu a c} \omega_{\mu c b}.
\]

Space-time \( \gamma \) matrices are written in terms of locally flat coordinates as

\[
\gamma^\mu = e^\mu_a \gamma^a,
\]

and these flat gamma matrices satisfy

\[
\{ \gamma^a, \gamma^b \} = -2 \eta^{ab}.
\]

\( \eta^{ab} \) is flat with signature \((-1, +1, \ldots, +1)\) and \( \gamma^{\mu_1 \cdots \mu_n} \) is the completely antisymmetric product of \( n \) \( \gamma \) matrices.

\[
D_\nu = \partial_\nu + \frac{1}{8} [\gamma^a, \gamma^b] \omega_{\nu ab},
\]

is the covariant derivative.
3 \textbf{$d=11$ supergravity in $LC_2$ to order $\kappa$}

Pure gravity has been previously formulated in light-cone gauge in many mildly differing forms [10–15] while gravitino interactions have been analyzed in light-front variables in [16]. The three fields in the $d=11$ theory are the metric, the three-form and the Rarita-Schwinger field. The metric has 44 components, the three-form 84 and the gravitino 128 degrees of freedom. With the spacetime metric $(-,+,\ldots,+)$ we define

$$x^\pm = \frac{1}{\sqrt{2}} (x^0 \pm x^{10}) , \quad \partial^\pm = \frac{1}{\sqrt{2}} (\partial_0 \pm \partial_{10}) .$$  \hspace{1cm} (8)

$x^+$ plays the role of time and $-i \partial_+ \phi$ the Hamiltonian. $\partial_- = -\partial^+$ is now a spatial derivative and its inverse, $\frac{1}{\partial_-}$, is defined using the prescription in [17]. We will now systematically formulate each of the four terms in (2) in light-cone gauge.

\subsection*{3.1 $L_1$: gravity}

This subsection is not as detailed as the following two because light-cone gravity has been treated in detail before. We therefore keep this subsection short highlighting only important results and refer the reader to [12, 13] for additional details. In this subsection alone, we circle the space-time light-cone indices to differentiate them from the locally flat light-cone indices. We parametrize the elfbeins as follows

$$ e_\oplus^+ = e^{\frac{1}{2} \phi} , \quad e_\ominus^- = e^{\frac{1}{2} \phi} .$$  \hspace{1cm} (9)

The symmetric object $g_{ij}$ (transverse metric) is parametrized as

$$ g_{\ominus \ominus} = e^\xi \beta_{ij} ,$$  \hspace{1cm} (10)

where $\xi$ is a real field and $\beta_{ij}$ is a real symmetric unimodular matrix that satisfies

$$ \beta_{ij} \beta_{jk} = \delta_{ik} .$$  \hspace{1cm} (11)

We choose light-cone gauge by setting

$$ e_\ominus^+ = 0 , \quad e_\ominus^k = 0 , \quad e_{\ominus}^+ = 0 ,$$

and $\phi = \frac{1}{2} \xi$. We also choose [12]

$$ \beta_{ij} = (e^{\kappa h})_{ij} ,$$  \hspace{1cm} (12)

where $h_{ij}$ is a symmetric trace-free matrix. We expand $\beta$ as

$$ \beta_{ij} = \delta_{ij} + \kappa h_{ij} + \mathcal{O}(\kappa^2) .$$  \hspace{1cm} (13)
To order $\kappa$, the relevant tensor contributions from (3) read

$$T_{\mu\nu}(A) = -\frac{1}{12}(F_{\mu\alpha\beta\gamma}F^{\alpha\beta\gamma} - \frac{1}{12}g_{\mu\nu}F^2), \quad (14)$$

and

$$T_{\mu}^{\sigma}(\Psi) = -\frac{i}{2}e^{\frac{1}{2}}e^{\sigma\rho\mu\nu}(\psi_{m}^\rho\psi_{n}^\mu - \frac{1}{2}g_{\mu\nu}(\psi_{m}^\rho\psi_{n}^\mu - \psi_{m}^\mu\psi_{n}^\rho)). \quad (15)$$

There are many cancellations because most contributions to order $\kappa$ occur through the transverse metric $g_{ij} = \delta_{ij} + \kappa h_{ij}$ and its inverse $g^{ij} = \delta_{ij} - \kappa h_{ij}$ which differ in sign. From the various components of $R_{\mu}^{a} - \frac{1}{2}e_{a}^{\mu}R = T_{\mu}^{a}(A,\Psi)$ we infer that

$$\xi \sim 0 + O(\kappa^2), \quad e \sim 1 + O(\kappa^2), \quad (16)$$

$$e_{(1)}^{r} = -\kappa \frac{\partial m}{\partial h_{jm}} + O(\kappa^2), \quad (17)$$

and

$$e_{\parallel} = -\kappa \frac{\partial h_{ij}}{\partial h_{lm}} + O(\kappa^2). \quad (18)$$

The gravity Lagrangian

$$L_1 \propto e e^{\rho c} R_{\rho\sigma c b}, \quad (19)$$

is now expressible entirely in terms of the physical variables and to order $\kappa$ reads

$$L_1 = + \frac{1}{4} h_{ij} \Box h_{ij} + \frac{\kappa}{4} h_{ij} \frac{\partial h_{ij}}{\partial \omega} (\partial_+ h_{mn} \partial_- h_{mn})$$

$$+ \frac{\kappa}{2} h_{mk} (\partial_+ h_{mk} \partial_- h_{kl} + \kappa h_{mk} (\partial_+ h_{im} \partial_- h_{kl})$$

$$+ \frac{\kappa}{2} h_{mk} (\partial_+ h_{mk} \partial_+ h_{il} + \kappa (\partial_+ h_{im} \partial_+ h_{kl})$$

$$+ \frac{\kappa}{2} h_{ij} (\partial_+ h_{kl} \partial_+ h_{lkm} - \frac{\kappa}{2} h_{mk} (\partial^- h_{il} \partial_+ h_{im})$$

$$- \frac{\kappa}{2} h_{ji} (\partial_+ h_{ij} \partial^- h_{il} + 18\kappa(\partial_+ h_{klm} \partial_- h_{ijkl}))$$

$$- 6\kappa(\partial_+ h_{ijkl} \partial^- h_{ijkl}) + \frac{\kappa}{12} F_{ijklm} F_{ijklm} h_{ij}$$

$$+ \frac{3}{\sqrt{2}} i\kappa \chi^{j+1} \chi^{m} \frac{\partial^{m} \partial^{n}}{\partial_-} \chi^{l} \chi^{i} h_{ij}$$

$$- \frac{3}{\sqrt{2}} i\kappa \chi^{j+1} \chi^{m} \frac{\partial^{m} \partial^{n}}{\partial_-} \chi^{l} \chi^{i} h_{ij}$$

$$+ \frac{1}{2\sqrt{2}} i\kappa \chi^{j+1} \chi^{m} \chi^{p} \chi^{l} \chi^{i} \chi^{j} h_{ij}$$

$$- \frac{3}{\sqrt{2}} i\kappa \chi^{j+1} \chi^{m} \frac{\partial^{m} \partial^{n}}{\partial_-} \chi^{l} \chi^{i} \chi^{j} h_{ij}$$

$$+ \frac{1}{2\sqrt{2}} i\kappa \chi^{j+1} \chi^{m} \chi^{p} \chi^{l} \chi^{i} \chi^{j} h_{ij}$$

$$- \frac{3}{\sqrt{2}} i\kappa \chi^{j+1} \chi^{m} \frac{\partial^{m} \partial^{n}}{\partial_-} \chi^{l} \chi^{i} \chi^{j} h_{ij}$$

$$+ \frac{3}{\sqrt{2}} i\kappa \chi^{j+1} \chi^{m} \frac{\partial^{m} \partial^{n}}{\partial_-} \chi^{l} \chi^{i} \chi^{j} h_{ij}.$$
and yields the following equations of motion
\[ kl \]
\[ We choose light-cone gauge by setting \]
\[ kl \]
\[ explicitly involve \( \partial^\perp \text{form and gravitino) derived in the following two subsections. Also, terms that \]
\[ important to note that (20) makes significant use of results (for the three-
\[ resolution - this procedure is detailed in [12]. \]
\[ 3.2 \text{ } L_2: \text{ three-form} \]
\[ The 3-form part of the supergravity Lagrangian is \]
\[ L_2 + L_4 = -\frac{1}{48} e F_{\mu\nu\rho} F^{\mu\nu\rho} + \frac{2\kappa}{(12)^3} e^{\mu_1\ldots\mu_{13}} F_{\mu_1\ldots\mu_4} F_{\mu_5\ldots\mu_8} A_{\mu_9\mu_{10}\mu_{11}} \]
\[ + \frac{\kappa}{96} e (\Psi_\mu \gamma^{\mu\nu\rho\sigma} \Psi_\rho + 12 \Psi^\gamma \gamma^{\rho\sigma} \Psi^\beta) F_{\nu\rho\sigma} \]
\[ and yields the following equations of motion \]
\[ \partial_\mu F^{\mu\nu\rho\sigma} = -\frac{\kappa}{576} e^{\mu_1\ldots\mu_8\nu\rho\sigma} F_{\mu_1\ldots\mu_4} F_{\mu_5\ldots\mu_8} \]
\[ \text{We choose light-cone gauge by setting} \]
\[ A_{-ij} = -A^{+ij} = 0 \quad A_{-+k} = A^{+k} = 0 \].
\[ \text{The} +kl \text{ component of the equations of motion determines} \]
\[ A^{-kl}(\kappa^0) = -\frac{\partial}{\partial\tau} A^{ikl} \]
\[ \text{and} \]
\[ 6 \partial^2 A^{-kl}(\kappa) = -\frac{\kappa}{72} \epsilon^{+i_1\ldots i_{12}k l} \partial_i A_{i_1 i_2 i_3 i_4} \partial_j A_{i_5 i_6 i_7 i_8} - \frac{1}{2\sqrt{2}} \kappa \partial_\rho (\gamma^\rho \gamma^k \gamma^l) \]
\[ - \frac{1}{8\sqrt{2}} \kappa \partial_\rho (\gamma^\rho \gamma^j k l m \gamma^m) + \frac{5}{16\sqrt{2}} \kappa \partial_\rho (\gamma^\rho \gamma^j k l m \gamma^m) \]
\[ - \frac{5}{16\sqrt{2}} \kappa \partial_\rho (\gamma^\rho \gamma^j k l m \gamma^m) + \frac{1}{2\sqrt{2}} \kappa \partial_\rho (\gamma^\rho \gamma^k \gamma^l) \]
\[ - \frac{\kappa}{576} \epsilon^{+i_1\ldots i_{12}k l} \partial_i A_{i_1 i_2 i_3 i_4} \partial_j A_{i_5 i_6 i_7 i_8} \].
Note that this result uses some additional information (about the gravitino) derived in the next subsection. The \(L_2\) 3-form Lagrangian is

\[
L_2 = -18A_{ijk} \square A_{ijk} + \frac{2\kappa}{12^2} \epsilon^{+ijk} A_{mnq} A_{prs} \left\{ -8 (\partial_+ A_{ijk}) (\partial_- A_{mnq}) A_{prs} \\
+ 24 \left( \frac{\partial_+ \partial_- A_{ijk}}{\partial_- A_{mnq}} A_{prs} - 24 (\partial_+ A_{qij}) (\partial_- A_{mnq}) A_{prs} \\
+ 24 (\partial_+ A_{ijk}) (\partial_m A_{npq}) \partial_\nu A_{qrs} \right) + \frac{\kappa}{2} (\partial_- A_{ijkl}) \partial_\nu \left[ A^{-kl}(\kappa) \right] \right\},
\]

where \(A^{-kl}(\kappa)\) is given by (25). It is important to point out that \(L_4\) is not included above and will be dealt with in subsection (3.4). As in gravity, the first term in (26) involves an explicit \(\partial_+\) and this is easily removed using a field redefinition analogous to (29) in [12]. \(A_{\mu\nu\rho}\) has 165 components. The first and second gauge choices in (23) eliminate \(9 \cdot 8 = 72\) components and 9 components respectively while (24) eliminates an additional \(9 \cdot 8 = 72\) components leaving us with 84 “physical” components for \(A_{ijk}\).

### 3.3 \(L_3\): gravitino

The gravitino-dependent terms in eleven-dimensional supergravity are (with spinor indices suppressed)

\[
L_3 + L_4 = -\frac{i}{2} e \bar{\Psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \Psi_\lambda + \frac{\kappa}{96} e (\bar{\Psi}_\mu \gamma^{\mu\nu\rho} \sigma_\beta \Psi_\beta + 12 \bar{\Psi}^\rho \Psi_\alpha F_{\nu\rho\sigma\alpha}).
\]

As mentioned earlier, the determinant \(e \sim 1 + O(\kappa^2)\) and

\[
\gamma^{\mu\nu\lambda} = \left[ \gamma^{\mu\nu\lambda} - \gamma^{\nu\rho\sigma} \gamma_\rho \gamma_\sigma + \gamma^{\rho\lambda} \gamma^{\nu\rho} \gamma_\lambda + \gamma^{\lambda\nu} \gamma^{\rho\lambda} \gamma_\rho + \gamma^{\nu\rho\lambda} \gamma^{\rho\lambda} \gamma_\nu - \gamma^{\rho\lambda} \gamma^{\nu\rho} \gamma_\lambda \right].
\]

We define

\[
\gamma^\pm = \frac{1}{2} (\gamma^0 \pm \gamma^{10}) , \quad \gamma^+ \gamma^- \Psi^\mu = 2\Psi^\mu(+) , \quad \gamma^- \gamma^+ \Psi^\mu = 2\Psi^\mu(-),
\]

and go to light-cone gauge by setting

\[
\Psi_- = -\Psi^+ = 0.
\]

We also make the additional gauge choice

\[
\gamma \cdot \Psi = \gamma^i \Psi^i - \gamma^+ \Psi^- = 0,
\]

implying that

\[
\gamma^i \Psi^i(\cdot) = 0.
\]

\(^2\text{Technically, the }\partial_+\text{ reappears in terms involving higher orders of }\kappa.\)
This allows us to define the “physical” gravitino field
\[ \chi^I = (\delta^I + \frac{1}{9} \gamma^I \gamma^J) \Psi^{(-)} . \] (33)

Equations of motion corresponding to (27) are
\[ i \gamma^{\mu
\nu\lambda} D_\nu \Psi = \frac{\kappa}{96} \gamma^{\mu
\nu\rho\sigma} \Psi \beta F_{\nu\rho\sigma} + \frac{\kappa}{8} \gamma^\mu \Psi^\alpha F_{\rho\sigma}^\mu \alpha , \] (34)
with \( D_\nu \) defined by (7). The \( \mu = + \) component implies that
\[ \partial^+ \Psi^{-(-)} = \partial^+ \Psi^{-(-)} + \frac{\kappa}{2} (\partial^j h_{jk}) \Psi^{-(-)} - \frac{\kappa}{2} \gamma^j \gamma^l (\partial^l h_{jk}) \Psi^{-(-)} \]
\[ + \frac{\kappa}{2} \gamma^j (\partial^+ h_{jk}) \gamma^m \partial^m \Psi^{-(-)} - \frac{5}{8} \kappa \gamma^j \kappa \mu \nu \gamma^\gamma \gamma^\mu \partial^\gamma \Psi^{(-)} \partial_- A^{[jk]} \]
\[ + \frac{\kappa}{16} \gamma^j \kappa^m \partial^m \Psi^{-(-)} \partial_- A^{-}[jk] \] (35)
while the lower and upper components of the \( \mu = i \) equation yield
\[ \partial^+ \Psi^{-(+)} = \frac{1}{2} \gamma^+ \gamma^m \partial^m \partial^k \Psi^{-(-)} + O(\kappa) . \] (36)
and
\[ \partial^+ \Psi^{(+)} = \frac{1}{2} \gamma^+ \gamma^l \partial^l \Psi^{(-)} \]
\[ + \frac{1}{4} \kappa \gamma^+ \gamma^l \{ (\partial^k h_{jk}) \Psi^{(-)} \} + \frac{1}{2} \kappa \gamma^+ \gamma^l \{ (\partial^l h_{ij}) \Psi^{(-)} \} \]
\[ - \frac{1}{2} \kappa \gamma^+ \gamma^l \{ (\partial^+ h_{ij}) \gamma^m \partial^m \Psi^{(-)} \} - \frac{5}{8} \kappa \gamma^+ \gamma^l \gamma^k \mu \nu \gamma^\gamma \gamma^\mu \partial^\gamma \Psi^{(-)} \partial_- A^{[jk]} \]
\[ + \frac{1}{2} \kappa \gamma^+ \gamma^l \{ \partial^k h_{jk} \} \gamma^m \partial^m \Psi^{(-)} - \frac{5}{8} \kappa \gamma^+ \gamma^l \gamma^k \mu \nu \gamma^\gamma \gamma^\mu \partial^\gamma \Psi^{(-)} \partial_- A^{[jk]} \]
\[ + \frac{1}{4} \kappa \gamma^+ \gamma^l \gamma^m \partial^- A_{[jk]} \] (37)
\[ - \frac{15}{8} \kappa \gamma^+ \gamma^j \kappa^m \mu \nu \gamma^\gamma \gamma^\mu \partial^\gamma \Psi^{(-)} \partial_- A^{[jk]} \]
\[ - \frac{1}{16} \kappa \gamma^+ \gamma^j \kappa^m \mu \nu \gamma^\gamma \gamma^\mu \partial^\gamma \Psi^{(-)} \partial_- A^{[jk]} \]
\[ - \frac{1}{4} \kappa \gamma^+ \gamma^j \kappa \mu \nu \gamma^\gamma \gamma^\mu \partial^\gamma \Psi^{(-)} \partial_- A^{[jk]} + O(\kappa^2) , \]
The next subsection will focus exclusively on \( L_4 \) which was also ignored earlier when deriving (26). At present, we simply substitute the above results, to order \( \kappa \), into the first line in (27) to obtain
with $\Psi^{i(+)}$ given by (37).

3.4 $L_4$: gravitino three-form coupling

We now turn to the final piece $L_4$ in (2). A straightforward substitution of all $LC_2$ results derived thus far yields

$$L_4 = \frac{i}{\sqrt{2}}\chi^{ij}_0 \left[ \gamma^i \partial^j \Psi^{i(+)} + \gamma^+ \partial_+ \chi^i + \frac{\kappa}{4} \gamma^+ \gamma^j \gamma^k (\partial^k h_{ij}) \frac{\partial^l}{\partial^-} \chi^l \right.$

$$+ \frac{\kappa}{2} \gamma^+ \gamma^j \gamma^k (\partial^l h_{ij}) \frac{\partial^p}{\partial^-} \chi^l + \frac{\kappa}{2} \gamma^+ (\partial_+ h_{ij}) \chi^j - \frac{\kappa}{2} \gamma^+ \gamma^j (\partial^k h_{ij}) \frac{\partial^l}{\partial^-} \chi^k$

$$- \frac{\kappa}{2} \gamma^j (\partial_+ h_{ij}) \chi^j \right] + \frac{3}{8} \gamma^j \frac{\partial^m}{\partial^-} \chi^m \partial_- A_{ijkl} - \frac{15}{16} \gamma^j \gamma^k \chi^m \partial_m A_{ijkl}$$. (38)

$$= - \frac{1}{8} \gamma^{ijklm} \frac{\partial^p}{\partial^-} \chi^m \partial_- A_{ijkl} + \frac{3}{16} \gamma^j \gamma^k \chi^m \partial_m A_{ijkl} \right),$$

$$\text{with } \Psi^{i(+)} \text{ given by (37).}$$

3.4 $L_4$: gravitino three-form coupling

We now turn to the final piece $L_4$ in $[2]$. A straightforward substitution of all $LC_2$ results derived thus far yields

$$\frac{1}{\sqrt{2}} L_4 = - \frac{45}{4} \frac{\partial^m}{\partial^-} \chi^{ij} \gamma^{ijkl} \chi^l \partial_q A_{ijkl} + \frac{45}{4} \chi^{ij} \gamma^{ijkl} \chi^l \partial_q A_{ijkl}$$

$$- \frac{3}{2} \frac{\partial^m}{\partial^-} \gamma^{ijkl} \chi^l \partial_+ A_{jkp} - \frac{15}{4} \frac{\partial^i}{\partial^-} \chi^{ij} \gamma^{ijkl} \chi^l \partial_q A_{ijkp}$$

$$+ \frac{3}{4} \frac{\partial^m}{\partial^-} \chi^{ij} \gamma^{ijkl} \chi^l \partial_- A_{jkp} + \frac{9}{2} \chi^{ij} \gamma^{ijkl} \chi^l \partial_\Omega A_{ijkp}$$

$$+ \frac{15}{4} \frac{\partial^m}{\partial^-} \chi^{ij} \gamma^{ijkl} \chi^l \partial_- A_{jkp} + \frac{3}{4} \frac{\partial^i}{\partial^-} \chi^{ij} \gamma^{ijkl} \chi^l \partial_\Omega A_{ijkp}$$

$$\text{ (39)}$$

$$- \frac{15}{4} \chi^{ij} \gamma^{ijkl} \chi^l \partial_- A_{jkp} + \frac{15}{4} \frac{\partial^m}{\partial^-} \chi^{ij} \gamma^{ijkl} \chi^l \partial_- A_{jkp}$$

$$- \frac{3}{2} \chi^{ij} \gamma^{ijkl} \chi^l \partial_\Omega A_{jkp} - \frac{1}{8} \frac{\partial^m}{\partial^-} \chi^{ij} \gamma^{ijkl} \chi^l \partial_\Omega A_{jkp}$$

$$- \frac{1}{8} \chi^{ij} \gamma^{ijkl} \chi^l \partial_\Omega A_{jkp}$$.
4 Conclusions

Eleven-dimensional space-time houses $N = 1$ supergravity, the largest supersymmetric local field theory with helicity two on reduction to four dimensions. In this paper we have formulated this theory to order $\kappa$, in light-cone gauge. We have made a number of gauge choices which helped differentiate between the unwanted degrees of freedom and the actual physically relevant variables. It is interesting to note how every component of $L$ depends on all three fields thanks to the maximal supersymmetry that closely links them. One very nice thing about the $LC_2$ procedure is the way the gauge conditions make the counting of degrees-of-freedom obvious. In particular, the 128 fermionic degrees of freedom are captured entirely by (33). In momentum space, these structures collapse considerably and Feynman rules are therefore the next step. A necessary next step is the Lagrangian to order $\kappa^2$ which will then make an explicit check of (1) for the $(N = 1, d = 11) \leftrightarrow (N = 8, d = 4)$ system possible.

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