Preliminary Study of 2D Fracture Upscaling of Geothermal Rock Using IFS Fractal Model

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Abstract. Fractured rock plays important role in reservoir production. In larger scale, fractures are more likely to be heterogeneous and considered to be fractal in its nature. One of the characteristics of fractal structure is the scale independence. An investigation of fractal properties on natural fractured rock is therefore needed for modelling larger fracture. We have investigated the possibilities of fractal upscaling method to produce a larger geothermal fracture model based on smaller fracture data. We generate Iterated Function System (IFS) fractal model using parameters e.g. scale factor, angle between branch, initial line direction, and branch thickness. All the model parameters are obtained from smaller fracture data. We generate higher iteration model to be compared with larger geothermal fracture. The similarity between the IFS fractal model and natural fracture is measured by 2D box counting fractal dimension (D). The fractal dimension of first to fourth generation fractal model is (1.86 ± 0.02). The fractal dimension of the reference geothermal site is (1.86 ± 0.04). Besides of D, we found significant similarity of fracture parameters there are intensity and density between fracture model and natural fracture. Based on these result, we conclude that fractal upscaling using IFS fractal model is potential to model larger scale of 2D fracture.

1. Introduction
Fracture rock plays important role in reservoir both oil and geothermal production. The measurement of permeable fractures is important in engineering fields, such as the extraction of geothermal energy through subsurface fractures [1]. Not only because of their actual oil storage capability, they also provide an interface with the matrix which is much larger than the borehole and the fracture volume may be negligible in comparison with the total reservoir volume [2]. Even though a very small primary permeability values are sufficient in the production of geothermal reservoir, most geothermal production comes from highly fractured reservoirs. But in larger scale, natural fracture is more complicated in structure.

Fractal is an object that has self-similarity in different scale. That means, parts of the object will look the same as the object itself when viewed as a whole [3]. One of the characteristics of fractal structure is its scale independence. Therefore an investigation fractal properties on natural fracture rock is needed for modeling larger fracture. Kagan [4] have investigated fractal dimension of fracture due to earthquake
and he found the fractal dimension decline to 1.8-1.9. Nolte [5] have investigated fractal properties of natural fractured in quartz monzonite and found the fractal dimension of its fractures ranged 1.96 to 2. Park et al. [6] have explored granite fracture in SE Korea, they use 2D box-counting to analyses and they got the fractal dimension is ranged 1.11 to 1.51. In larger scale, natural fracture is more complicated in structure and became heterogeneity, Upscaling is needed in this case.

Upscaling is transformation method from microscopic data to macroscopic data. To predict natural fracture in larger scale, we need to apply appropriate Upscaling method. Upscaling methods has been widely introduced, e.g. Effective Medium Theory (EMT), Numerical Method, and Fractal Model. Fractal model have become popular precisely than other, because fractal model is free of the assumption of statistical homogeneity and yet do not involve numerical method [7].

In this paper, first we investigated fractal properties of geothermal rock fractures. Then, we investigated the fractal dimension invariance of fractal fracture models generated by Iterated Function System (IFS).

2. Characterization of 2D Natural Fracture

In this paper, the sample was obtained from geothermal rock fracture in West Java of Indonesia (see Figure 1). The geothermal rock sample has spatial dimensions of 3 cm × 5 cm. We are interested in analyzing void fractures, thus in order to isolate the fractures, we must first convert the image of the sample into binary (black and white) image and then remove the granular type of pore space and the noises.

![Figure 1. Geothermal fractured rock sample](image)

By applying image processing to Figure 1b, we can characterize the fracture to see the distribution of the fracture and orientation of fracture. The result of characterization can be seen in Table 1.

| Fracture Length Distribution (σ) | Fracture Orientation |
|---------------------------------|----------------------|
| 0 < σ < 50                      | 1<sup>st</sup> Orientation |
| 50 < σ < 100                    | 2<sup>nd</sup> Orientation |
| 100 < σ < 150                   | 21.13°                 |
| σ ≥ 150                         | -31.49°                |

In the Table 1, we can see that the length of the fractures are between 0 to 50 pixels and 50 to 100 pixels have almost same the number of fractures. Besides the basic fracture characteristic listed in Table 1, there are also several other important physical properties of the fracture, i.e., 2D fracture intensity (I), 2D fracture density (ρ), and fractal dimension (D). Intensity and density of the fracture can be calculated by applying the following formula:

\[ I = \frac{1}{A} \sum_{i=1}^{N} l_i \]  \hspace{1cm} (1)

\[ \rho = \frac{A}{A_f} \]  \hspace{1cm} (2)
where $l$ is the length of the fracture, $A$ is the cross sectional area of the fracture, and $A_f$ is the total of cross sectional area of the fracture. Fractal dimension is the most important of fractal sample like the geothermal rock fracture sample. Fractal dimension was defined by Besicovitch and Ursell [8], they assumed that the sample has to divide by many squares. Fractal dimension can be calculated by applying following formula:

$$D = \frac{\log N(r)}{\log r}$$

with $N(r)$ is the number of grid squares that cover the fractal object or 0 pixel of grid squares and $r$ is the total of squares side length [9]. By applying above formulas to the Figure 1b, we get the properties, there are $1.3 \times 10^{-2}$ cm$^{-1}$ of intensity, 0.86 of density and 1.83 of fractal dimension.

We can divide fracture in figure 1 to capture fracture in four different size from smallest to bigger scale, level 1, level, 2 level 3 and level 4 respectively as shown in Figure 2.

Figure 2. (a) 2D natural fracture sample; (b) 1$^{st}$ Level; (c) 2$^{nd}$ Level; (d) 3$^{rd}$ Level; (e) 4$^{th}$ Level

In the Figure 2, the number of fractures on scale 1 to 4 respectively are 3, 7, 15, 31, that number of fractures follow the series formula $\left(2^{n+1} - 1\right)$, with $n$ is the scale fractures. Result of fracture intensity and density properties for each level of natural fracture can be seen in Figure 3 below.

![Density vs Level Scale](image)
Figure 3 above says that the density is almost constant for increase of fracture level scale with an average value \((8.61 \pm 0.02) \times 10^{-1}\), while the intensity of fracture increases exponentially with increase of fracture level scale with the graph equation

\[
I = 0.0004 e^{0.8681n} \\
R^2 = 0.9921
\]

3. Fractal Fracture Model Generated by IFS
Modeling using IFS fractal models has been carried out by many researchers e.g., Perfect et al [10] who generated mono-fractal and multi-fractal of Sierpinski carpets using IFS in their research. Mistakidis [11] have modeled the fractures with IFS to get an overview description of the mechanical properties of the material being modeled, whereas He [12] have described a few of inner boundaries and separating map and then they have used the filter algorithms of inner boundaries to determine the spatial boundaries of IFS attractor quickly and accurately. Cen [13] explored the use of 3D IFS as time graph model including coloring, lighting and fog effects. IFS is one way to generate fractals with number iteration of affine transformations including translation, rotation, and scaling. Affine transformation more easily expressed in matrix form:

\[
\begin{bmatrix}
\tilde{x} \\
\tilde{y}
\end{bmatrix} =
\begin{bmatrix}
    s_x \cos \theta & -s_x \sin \theta \\
    s_x \sin \theta & s_x \cos \theta
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} +
\begin{bmatrix}
t_x \\
t_y
\end{bmatrix}
\]

where \(s\) is scale factor, \(t\) is translation, and \(\theta\) is the rotation. In this paper the modeling parameters are position, scale factor \((s)\), the angle of the previous branch \((\theta)\) and thickness. Figure 4 shows an example of fracture model that has been built by IFS.
Figure 4. Fractal fracture model constructed by IFS with (0.818, 0.900) of position input, 1.5 scale factor, 55° of the angle, and 0.015 pixel of thickness. The fractal dimension is (1.8635±0.0241).

We have conducted experiment to find the correlation between fractal dimension and the variation of the model’s scale factor. The experiment yield the results as shown in Table 2. We can observe that the scale factor does not significantly affect the fractal dimension of the fracture which can be seen from the standard deviations.

| Iteration | Fractal Dimension (D) |
|-----------|------------------------|
|           | s = 1.0 | s = 1.1 | s = 1.2 | s = 1.3 | s = 1.4 | s = 1.5 | s = 1.6 | s = 1.7 |
| 2         | 1.8766  | 1.8662  | 1.8586  | 1.8499  | 1.8449  | 1.8988  | 1.894   | 1.8904  |
| 3         | 1.8476  | 1.8959  | 1.8875  | 1.8837  | 1.8938  | 1.8977  | 1.8416  | 1.8445  |
| 4         | 1.8404  | 1.8482  | 1.8544  | 1.8573  | 1.8634  | 1.8668  | 1.8707  | 1.8735  |
| 5         | 1.8471  | 1.8514  | 1.8576  | 1.8594  | 1.865   | 1.8623  | 1.8644  | 1.8661  |
| 7         | 1.8392  | 1.8317  | 1.8279  | 1.8257  | 1.8265  | 1.8216  | 1.8194  | 1.822   |
| std dev   | 0.026105| 0.031547| 0.032348| 0.030056| 0.030478| 0.031451| 0.034369| 0.03136  |
| average   | 1.8593  | 1.8681  | 1.8679  | 1.8648  | 1.8671  | 1.8750  | 1.8674  | 1.8677  |

From the Table 2, we can see that the fractal dimension is almost same for increase of iteration, it means that IFS fractal model has self-similarity properties. The size effect of intensity and density on fractal model can be seen in Figure 5.
Figure 5. The size effect on intensity and density fractal fracture model.

The plots show that there are similarities of the intensity and density between natural fracture and the fractal models. Thus we infer that the IFS fractal model can be used to represent the 2D natural fracture.

4. Modelling of Natural Rock Fracture with IFS Fractal Tree Model and Its Comparison

In the previous section we have deduced that IFS fractal model can represent the natural fracture, therefore, in this section we will discuss about the comparison between fractal model and natural fracture using the following modeling parameters (see Figure 6 for the produced models):
In order to compare the model with the natural fracture, the sample is divided into 4 images (see Figure 2). Each level 1, 2, 3 and 4 is associated with IFS model first, second, third and fourth iteration respectively with the numbers of fracture are follow \(2^i - 1\), with \(i\) is the number of iteration.

\[
\begin{array}{ccc}
\hline
\text{Parameters} & \text{Value} \\
\hline
\text{Initial Input} & (0.828, 0.915) \\
\text{Factor Scale} & 1.1 \\
\text{Previous Branch Angle} & 55^\circ \\
\text{Thickness} & 0.015 \text{ pixel} \\
\hline
\end{array}
\]

Table 3. Modeling Natural Rock Fracture Parameters

Figure 6. Fracture model based on natural fracture sample. (a) 1\(^{st}\) iteration; (b) 2\(^{nd}\) iteration; (c) 3\(^{rd}\) iteration; (d) 4\(^{th}\) iteration

Digital image analysis is then applied to both images (fracture model & the natural fracture) to obtain the value of fractal dimension, intensity, and density. The obtained value from both images are then compared in Table 4. The difference between each properties is not significant. Here we can conclude that the generated IFS model with parameters as listed in Table 3 is suitable to describe the physical properties on larger scale to the natural fracture.

Table 4. Comparison of physical properties between fracture model and natural fracture

| Iteration | Fracture Model | Natural Fracture |
|-----------|----------------|------------------|
| Dimension | Intensity (cm\(^{-1}\)) | Density | Level | Dimension | Intensity (cm\(^{-1}\)) | Density |
|-----------|-----------------|----------|-------|-----------|-----------------|----------|
| 2         | 1.8856          | 0.0029   | 0.878 | 2         | 1.8903          | 0.0027   | 0.86   |
| 3         | 1.8600          | 0.0068   | 0.880 | 3         | 1.8767          | 0.0054   | 0.86   |
| 4         | 1.8450          | 0.0154   | 0.883 | 4         | 1.8160          | 0.0129   | 0.864  |

From Table 4 above can be seen the between the fracture model and the natural fracture have the dominant same value for all fracture properties parameters. The fractal dimension for fracture model
and natural fracture respectively are (1.86 ± 0.02) and (1.86 ± 0.04), density of fracture model and natural fracture are almost constant with increase scale with value respectively are (8.80 ± 0.02)×10^{-1} and (8.61 ± 0.02)×10^{-1}, while the intensity of fracture model and natural fracture will increase exponentially for increase scale with the graph function respectively are $I_e = 9×10^{-4}e^{0.74i}$ cm$^{-1}$ and $I_e = 4×10^{-4}e^{0.874n}$ cm$^{-1}$, with $i$ is the iteration and $n$ is the level scale.

5. Results
Calculation of the physical properties of fracture on natural fracture and fracture fractal model that generated by IFS was done by means of digital image analysis. For the fractal models, the relationship between scale factor parameter and fractal dimension has investigated and yield the following conclusions: the change of scale factor number does not affect to the fractal dimension number, the natural sample has fractal dimension of (1.86 ± 0.04) which is in good agreement with the ones obtained from other researches [4][5][14][15], the intensity of fracture increases exponentially with increase level scale/iteration for both the natural fracture and the fractal model, while the density is almost constant with increasing level scale/iteration.

6. Conclusions
Based on the results, we conclude that the IFS model can represent the 2D fracture, it means that IFS fracture model parameters can represent 2D natural fracture on larger scale with upscaling application. Nevertheless, the fracture modeling with IFS still requires optimization of more parameters. For future work the model can be applied for different type of rock fracture, which can also explain the physical meaning of fractal dimension in fractured media.

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