The (de)-confinement transition in tachyonic matter at finite temperature

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In this paper we consider the effect of temperature on the net potential of the electric field confinement. We study the behavior of the electric field coupled with dielectric function in the presence of temperature ($T$). The aim, is to obtain an electric field confinement at low temperatures $T < T_c$ ($T_c$ is the critical temperature), similar to what we observe with the field of gluons that bind quarks in hadrons at low energies, and also to determine its associated string tension as a function of temperature. We also study the screening phenomenon at high temperatures, $T \geq T_c$.

To achieve this, we use the phenomenon of color dielectric function coupled with the dynamics of the gauge field in a tachyon matter. This method is efficient for obtaining the net potential associated with the confinement of quarks and gluons as a function of temperature. We show that the tachyon matter behaves the same way the associated dielectric function, modifies the Maxwell equations and influences the fields in a manner which results in confinement regime at $T < T_c$ and Coulombian-like regime at $T = T_c$ in three spatial dimensions.

I. INTRODUCTION

Quantum chromodynamics (QCD) is the theory that describes the strong interaction mediated by gluons which confine quarks in hadrons. The success of this theory depends on asymptotic freedom \cite{1–3}. QCD forms the basics of all nuclear physics and enables us to understand and describe many features of matter as we know it today \cite{4, 5}. In this paper, we will show that the phenomenon of confinement is achievable in an electric field immersed in a color dielectric medium ($G$) in the presence of temperature $T$. We shall also show that the net potential of the resulting confinement at $T = 0$ appears the same way as the Cornell’s potential for confinement of heavy quarks and gluons in hadron at low energies. The presence of the tachyon matter serves as the medium that enable us to achieve both the anti-screening (the confinement regime) at low temperatures ($T < T_c$) and screening behavior (the Coulomb potential behavior) at high temperatures ($T \geq T_c$). The purpose of the color dielectric function $G(\phi)$ coupled with the dynamics of the gauge field is to create the color dielectric field which brings about confinement due to the strong interaction it creates \cite{4–6}.

There exist some similarity between QCD and QED (Quantum Electrodynamics) in terms of the successes of both theories, but they depart from each other by their strength, medium and dynamics of interactions. QED explains the interaction between charged particles while QCD explains the strong interaction between atomic particles. QED creates a screening effect which decreases its effective electric charge as distance between electron-anti-electron decreases increases. The opposite effect is observed in QCD where an anti-screening is created, the effective color charge increases with increasing distance between quark and antiquark. This similarity and the differences makes it interesting to advance a study of one in terms of the other. In this work we shall investigate QCD in terms of QED.

The well-known potential for determining the potential of heavy quarks at confined state is the Cornell potential given by, $v_c = -\frac{a}{r} + br$, where $a$ and $b$ are positive constants. This potential comprises linearly increasing part (infrared interaction) and Coulombian part (ultraviolet interaction) \cite{2, 7}.

Many works have been done on determining the potential of quark confinement as a function of temperature, usually referred to as thermal QCD, by applying several approaches to both Wilson and Polyakov loops \cite{8–11}. Most of the challenges posed by these models stems from the proper behavior of the QCD string tension at all temperatures as compared with lattice simulation results. The expected behavior of the string as suggested by many simulation results are a sharp decrease with temperature at $T < T_c$, vanishes at $T = T_c$, and slowly decreases at $T > T_c$ \cite{1, 11–13}.

The main purpose of this paper is to define the net static potential of the quarks and gluon confinement in three spatial dimensions as a function of temperature. We shall also obtain the QCD string tension associated with it as a
function of temperature and study its behavior. We shall use an Abelian approach throughout our computations as it is applied in QED, but our dielectric function \( G(r) \) is defined such that it can produce both the expected confinement and deconfinement in the chosen tachyonic matter \([14, 15]\). It has already been established that the Abelian part of the non-Abelian QCD string tension represents 92% and comprises the confinement part of the net static potential. Hence we can make an approximation of the non-Abelian field using an Abelian approach \([16, 17]\). These facts also permit us to employ a phenomenological effective field theory for QCD in studying the confinement of quarks and gluons inside the hadron \([18–21]\). The color dielectric function \( G(\phi) \) creates the strong interaction between the elementary particles (quarks and gluons) that leads to confinement while the self-interacting scalar field \( \phi(r) \) describes the dynamics of the dielectric function in the tachyonic matter. Our color dielectric function \( G \) is associated with tachyon condensation \([6]\). With these known facts, we shall use a Lagrangian that would collectively describe both the dynamics of the gauge and the scalar field associated with the tachyon dynamics and its behavior with temperature \([21]\). The motivation for using this approach is that, we would be able to obtain a phenomenological field theory for QCD and also be able to identify the dielectric function automatically with the tachyon potential.

The paper is organized as follows. In Sec. II we review the theory of electromagnetism in a dynamical dielectric medium. In Sec. III we introduce the tachyon Lagrangian coupled with temperature and its associated effective potential. In the latter cases we find analytically the net potential of confinement of quarks and gluons as a function of temperature. We explore the physics given in the net confinement potential. We also explore the physics in the QCD string tension as a function of temperature. In Sec. IV we present our final considerations.

### II. MAXWELL’S EQUATIONS MODIFIED BY DIELECTRIC FUNCTION

In this section we study the theory of electromagnetism embedded in a color dielectric medium to describe the phenomenon of electric confinement. We start by writing the Maxwell Lagrangian in vacuum without sources

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}
\]  

(1)

The equations of motion for the electromagnetic field are given by

\[
\partial_{\mu} F^{\mu\nu} = 0
\]  

(2)

It is important to note that even in the absence of sources the equations of motion for the electromagnetic field produce spherically symmetric static vacuum solutions (the Coulomb field due to a point charge) \([22]\).

For electromagnetic fields immersed in a dielectric medium characterized by dielectric function \( G(\phi) \), where \( \phi(r) \) is a scalar field that governs the dynamics of the medium, we have the following Lagrangian

\[
\mathcal{L} = -\frac{1}{4} G(\phi) F_{\mu\nu} F^{\mu\nu}
\]  

(3)

The equations of motion of Eq.(3) are now described by

\[
\partial_{\mu}[G(\phi) F^{\mu\nu}] = 0
\]  

(4)

where \( \mu = 1, 2, 3 \). For the component \( \nu = 0 \), we simply have

\[
\nabla_{r}[G(\phi) E_{r}] = 0
\]  

(5)

We shall be neglecting the magnetic field throughout the paper. This is because the electric field is sufficient for our analysis.

Starting from Eq.(5) we find the electric field \( \mathbf{E} \) coupled to dielectric function \( G(\phi) \). Now, working with spherical coordinates and assuming that \( E(r) \) and \( \phi(r) \) are only functions of \( r \) and as a consequence \( G(\phi) \) follows the same condition, we have the following form

\[
\nabla_{r}[G(\phi) E_{r}] = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 G(\phi) E_{r}) = 0
\]  

(6)

By using this equation we can obtain the vacuum solutions mentioned above. Thus, now we integrate the differential equation to obtain

\[
E_{r} = \frac{\lambda}{r^2 G(\phi)}
\]  

(7)
Now it is easy to interpret the constant of integral as \( \lambda = \frac{q}{4\pi\varepsilon_o} \), to write the Coulomb electric field modified by the dielectric function \( G(\phi) \)

\[
E = \frac{q}{4\pi\varepsilon_o r^2 G(\phi)},
\]

where \( E = |E| = E_r \). Therefore, we observe that the dielectric function coupled with the electric field \( E \) changes its magnitude as a function of a dynamical field \( \phi \).

Let us now consider the following effective Lagrangian

\[
\mathcal{L} = -\frac{1}{4} G(\phi) F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V_{eff}(\phi)
\]

whose effective potential for the scalar field at finite temperature \( T \) has been found and is given by [23, 24]

\[
V_{eff}(\phi) = V(\phi) + \frac{T^2}{24} V_{\phi\phi}(\phi)
\]

where \( V_{\phi\phi} \) stands for second derivative of \( V(\phi) \). The behavior of the dielectric function \( G(\phi) \) will be obtained from the solutions of the equations of motion [25] of the above Lagrangian. The equations of motion for the electromagnetic field \( A_\mu \) and the scalar field \( \phi \), are given directly as follows

\[
\partial_\mu [G(\phi) F^{\mu\nu}] = 0
\]

\[
\partial_\mu \partial^\nu \phi + \frac{1}{4} \partial G(\phi) \partial_\phi F_{\mu\nu} F^{\mu\nu} + \frac{T^2}{24} \partial V_{\phi\phi}(\phi) + \partial V(\phi) \partial_\phi = 0
\]

Hence, the equation of motion for the dielectric medium and electric field in spatial coordinates are

\[
\frac{1}{r^2} \frac{d}{dr} \left( r^2 G(\phi) E \right) = 0
\]

\[
\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\phi}{dr} \right) = \frac{T^2}{24} \partial V_{\phi\phi}(\phi) - \frac{1}{2} \frac{\partial G(\phi)}{\partial \phi} E^2 + \frac{\partial V(\phi)}{\partial \phi} = 0
\]

Based on the previous discussion it is easy to show that the solution of Eq. (13) for the electric field is that given in equation (8).

To find confinement regime the dielectric function in equation (8) must have the following asymptotic behavior:

\[
G(\phi(r)) = 0 \quad \text{as} \quad r \to r_*
\]

\[
G(\phi(r)) = 1 \quad \text{as} \quad r \to 0
\]

where \( r_* \) stands for the scale where the confinement starts to become effective. Particularly, \( r_* = \infty \) for \( G(\phi(\infty)) \sim \frac{1}{r} \) and from (8) we find \( E \equiv \text{constant} \). This uniform electric field behavior agrees with confinement everywhere.

III. TACHYON CONDENSATION AND ELECTRIC CONFINEMENT

In this section we discuss the relationship between the phenomena of tachyon condensation and the confinement of the electric field. Tachyons are particles that are faster than light, they have negative mass squared and are associated with instabilities. They are theoretical particles whose existence is presumed the same way as magnetic monopoles. Tachyons have never been observed isolated in nature, although, specially from superstring point of view, they may also be interacting with other fields or self-interacting at higher orders to form the tachyon condensation [6, 26].
A. Tachyon Lagrangian with electromagnetic field and temperature

From equation (14) we have in principle, the potential $V(\phi)$ and the dielectric function $G(\phi)$. However, we can restrict these choices considering $G(\phi(r)) = V(\phi(r))$. We shall demonstrate bellow, that this choice is legitimate when we are dealing with a Lagrangian that describes the dynamics of tachyons represented by $\phi(r)$.

Firstly let us consider the Lagrangian at Eq. (9) without the temperature correction term as seen in [5]

$$\mathcal{L} = -\frac{1}{4} G(\phi) F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)$$

(17)

The equation of motion of this Lagrangian is given by

$$\partial_\mu \partial^\mu \phi + \frac{1}{4} \frac{\partial G(\phi)}{\partial \phi} F_{\mu\nu} F^{\mu\nu} + \frac{\partial V(\phi)}{\partial \phi} = 0$$

(18)

Now, let us consider the fields depending only on the spatial coordinate $x$, that is,

$$\phi = \phi(x), \quad A_\mu = A_\mu(x)$$

(19)

The associated equations of motion given in one dimension are

$$\frac{d}{dx}[G(\phi) E] = 0$$

(20)

$$-\frac{d^2 \phi}{dx^2} - \frac{1}{2} \frac{\partial G}{\partial \phi} E^2 + \frac{\partial V}{\partial \phi} = 0,$$

(21)

where we used the fact that $F^{01} = E$. Integrating Eq.(20) we have

$$G(\phi) E = q \quad \Rightarrow \quad E = \frac{q}{G(\phi)}$$

(22)

Substituting equation (22) into equation (21), we find

$$-\frac{d^2 \phi}{dx^2} - \frac{1}{2} \frac{\partial G}{\partial \phi} \frac{q^2}{G(\phi)^2} + \frac{\partial V}{\partial \phi} = 0$$

(23)

We now consider the tachyon Lagrangian well known from string theory with dynamics of tachyonic field, $\tilde{T}(x)$, coupled with the electric field $E(x)$. Thus, for slow varying fields, as usual in tachyon matter, we can power expand it as follows [6, 26]

$$e^{-1} \mathcal{L} = -V(\tilde{T}) \sqrt{1 - \tilde{T}^2} + F_{01} F^{01}$$

(24)

$$= -V(\tilde{T}) [1 - \frac{1}{2} (\tilde{T}^2 + F_{01} F^{01}) + ...]$$

$$= -V(\tilde{T}) + \frac{1}{2} V(\tilde{T}) \tilde{T}^2 - \frac{1}{2} V(\tilde{T}) F_{01} F^{01} + ...$$

$$= -V(\phi) + \frac{1}{2} \phi^2 - \frac{1}{2} V(\phi) F_{01} F^{01} + ...$$

(25)

where $e = \sqrt{|g|}$ in a general spacetime. This derivation is also valid in 3 + 1 dimensions for $\phi$ depending only on the radial coordinate $r$ which can be identified with coordinate $x$. In equation (25) we used the fact that

$$V(\tilde{T}(\phi)) = \left( \frac{\partial \phi}{\partial \tilde{T}} \right)^2 \Rightarrow \frac{1}{2} V(\tilde{T})(\tilde{T}^2)$$

$$= \frac{1}{2} \left( \frac{\partial \phi}{\partial \tilde{T}} \frac{\partial \tilde{T}}{\partial x} \right)^2 = \frac{1}{2} \phi^2,$$

(26)

where $\phi = f(\tilde{T})$, or $\tilde{T} = f^{-1}(\phi)$. Now comparing equation (25) with equation (17) we find the equality $G = V$. This result is also true for equation (9) up to the thermal correction term, though from the perspective of string theory where the thermal correction affects the tachyon potential $V(\tilde{T})$ — see e.g. [27] and references therein — of the original tachyon Lagrangian (24), one would also expect thermal corrections for the potential $V(\phi)$ in the effective theory (25), such that we had $G_{\text{eff}} = V_{\text{eff}}$. However, we restrict ourselves to the context of effective quantum field theory, where the one loop thermal corrections from the scalar sector affects only $V(\phi)$ as given in the Lagrangian (9).
1. Confinement potential for the electric field in three dimensions as a function of temperature

For the tachyon Lagrangian in equation (25) expanded polynomially, it is increasingly clear that the dielectric function $G(\phi)$ is equal to the potential $V(\phi)$. Thus, our dielectric function is naturally identified in the context of tachyon theory. Now we choose the appropriate classical tachyon potential that gives us the appropriate behavior for confinement and deconfinement in the presence of temperature. We choose a tachyon potential in the form

$$V(\phi) = \frac{1}{2}(\alpha^2\phi^2 - 1)^2$$

(27)

In three dimensions, (in the absence of magnetic field) in radial symmetry, equation (12) can be rewritten as

$$-\left[\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\phi}{dr} \right) \right] + \frac{T^2}{24} \frac{\partial V_{\phi\phi}}{\partial \phi} - \frac{1}{2} \frac{\partial G(\phi)}{\partial \phi} E^2 + \frac{\partial V(\phi)}{\partial \phi} = 0$$

(28)

Recalling that the solution for the electric field is given by

$$E(r) = \frac{q}{4\pi \varepsilon_0 G(\phi) r^2}$$

(29)

Substituting this solution into equation (28) one finds

$$-\left[\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\phi}{dr} \right) \right] + \frac{T^2}{24} \frac{\partial V_{\phi\phi}}{\partial \phi} - \frac{\lambda}{2} \frac{\partial V(\phi)}{\partial \phi} \frac{1}{V(\phi)} + \frac{1}{r^4} + \frac{\partial V(\phi)}{\partial \phi} = 0$$

(30)

Now considering the fact that $G(\phi) = V(\phi)$ and

$$\lambda = \frac{q}{4\pi \varepsilon_0}$$

(31)

we have

$$-\left[\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\phi}{dr} \right) \right] + \frac{T^2}{24} \frac{\partial V_{\phi\phi}}{\partial \phi} - \frac{\lambda^2}{2} \frac{\partial V(\phi)}{\partial \phi} \frac{1}{V(\phi)} + \frac{1}{r^4} + \frac{\partial V(\phi)}{\partial \phi} = 0$$

(32)

which implies

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\phi}{dr} \right) = \frac{\partial}{\partial \phi} \left[ \frac{T^2}{24} V_{\phi\phi} + \frac{\lambda^2}{2} \frac{1}{V(\phi)} + \frac{1}{r^4} + V(\phi) \right]$$

(33)

Now substituting the potential (27) into equation (33) gives

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\phi}{dr} \right) = \frac{\partial}{\partial \phi} \left[ \frac{T^2}{24} 2(-\alpha^2 + 3\alpha^4\phi^2) \right] + \frac{\lambda^2}{2} \frac{\partial}{\partial \phi} \left[ \frac{1}{r^4} \right] + \frac{\partial}{\partial \phi} \left[ \frac{1}{2} (\alpha^2\phi^2 - 1)^2 \right]$$

(34)

Assuming $\alpha\phi$ sufficiently small we can write equation (34) as follows

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\phi}{dr} \right) = \frac{T^2}{2} \alpha^4 \phi + \lambda^2 (4\alpha^2 \phi + 12\alpha^4\phi^3) \frac{1}{r^4} + 2(\alpha^4\phi^3 - \alpha^2 \phi)$$

(35)

where we have made the following power series expansion for small $\alpha\phi$:

$$[(\alpha^2\phi^2 - 1)^{-2}] = 1 + 2\alpha^2\phi^2 + 3\alpha^4\phi^4 + \mathcal{O}(\phi^6).$$

(36)
Now defining $T_c^2 = \frac{4}{\alpha^2}$ and taking only linear terms in $\phi$, equation (35) can be reduced to the form

$$\phi'' + \frac{2}{r} \phi' = 2\alpha^2 \left[ \frac{T^2}{T_c^2} - 1 \right] \phi + \frac{4\lambda^2}{r^4} \alpha^2 \phi$$  \hspace{1cm} (37)

Defining $A = \alpha^2 \left[ \frac{T^2}{T_c^2} - 1 \right]$ and $B = 4\lambda^2 \alpha^2$, equation (37) can be rewritten as

$$\phi''(r) + \frac{2}{r} \phi'(r) - \left( 2A + \frac{B}{r^4} \right) \phi(r) = 0$$  \hspace{1cm} (38)

Assuming that $(B/2A)^{1/4} \ll r$, where $A$ and $B$ are constants, equation (38) reduces to

$$\phi''(r) + \frac{2}{r} \phi'(r) - 2A \phi(r) = 0$$  \hspace{1cm} (39)

This equation has a solution given by

$$\phi(r) = \frac{\sin(\sqrt{2} |A| r)}{\alpha \sqrt{|A| r}}$$  \hspace{1cm} (40)

where we have considered the boundary condition $\phi(r \to 0) \to \text{finite}$ and $|A| = -A = \alpha^2 \left[ 1 - \frac{T^2}{T_c^2} \right]$ for $T \leq T_c$. Let us now set the limit $r \ll 1/\sqrt{2} |A|$. The discussion for large scales $r \gg 1/\sqrt{2} |A|$ and $T > T_c$ will be addressed shortly. For now we consider the regime $(B/2A)^{1/4} \ll r \ll 1/\sqrt{2} |A|$ such that we can approximate equation (40) as

$$\phi(r) = \frac{\sqrt{2}}{\alpha} \left[ 1 - \frac{|A| r^2}{3} \right]$$  \hspace{1cm} (41)

Substituting this into equation (27) (the tachyon potential) gives

$$V(r) = G(r) = \frac{1}{2} \left[ 2 \left( 1 - \frac{|A| r^2}{3} \right)^2 - 1 \right]^2$$

$$= \frac{1}{2} - \frac{4 |A| r^2}{3}$$  \hspace{1cm} (42)

Substituting this results into the electric field equation modified by dielectric function $G(r)$

$$E = \frac{\lambda}{r^2 G(r)} = \frac{\lambda}{r^2 \left( \frac{1}{2} - \frac{4 |A| r^2}{3} \right)}$$  \hspace{1cm} (43)

Using the well known relation for determining electric field potential, $V(r) = \int E \, dr$, to determine the confinement potential $V_c(r)$, we get the result

$$V_c(r, T) = -\frac{2\lambda}{r} + \frac{16\lambda |A| r}{3} + c$$  \hspace{1cm} (44)

Now, we can compare our results from equation (37) with the results of [7, 28], for confinement of quarks and gluons with $N_c$ colors

$$\frac{d^2 (\phi(r))}{dr^2} + \frac{2}{r} \frac{d \phi(r)}{dr} = -\frac{g^2}{64\pi^2} \frac{1}{f_\phi} \left( 1 - \frac{1}{N_c} \right) \exp \left( -\frac{\phi(r)}{f_\phi} \right) \frac{1}{r^4}$$  \hspace{1cm} (45)
We replace the exponential in equation (45) as \( \exp(-\phi(r)/f_\phi) \rightarrow 2(\alpha^4\phi^3 - \alpha^2\phi) \), in such a way that conforms with the derivative of our starting potential used to obtain equation (37).

Now using the fact that \( \alpha \phi \) is sufficiently small, and taking only linear terms in \( \phi(r) \), equation (45) becomes

\[
\frac{d^2\phi(r)}{dr^2} + \frac{2}{r} \frac{d\phi(r)}{dr} = \frac{g^2}{32\pi^2 f_\phi} \left( 1 - \frac{1}{N_c} \right) \phi(r)\alpha^2 \frac{1}{r^4}
\]

(46)

Now we can identify our electric charge \( q \) in terms of the gluon charge \( g \) by comparing equation (37) and equation (46) as follows

\[
4\lambda^2 = \frac{g^2}{32\pi^2 f_\phi} \left( 1 - \frac{1}{N_c} \right)
\]

(47)

Therefore identifying \( \alpha = \frac{1}{f_\phi} \) we find

\[
\lambda = \frac{g}{4\pi} \left( 1 - \frac{1}{N_c} \right) \frac{1}{2},
\]

(48)

where we have redefined \( g \rightarrow g/2\sqrt{2} \). Using equation (31) one can readily find the following relationship between the charges

\[
q = \varepsilon_0 g \sqrt{\left( 1 - \frac{1}{N_c} \right)}
\]

(49)

Substituting the results obtained above into equation (44), we have

\[
V_c(r, T) = \frac{g}{4\pi} \sqrt{\left( 1 - \frac{1}{N_c} \right)} \left[ -\frac{2}{r} + \frac{16\alpha^2}{3} \left( 1 - \frac{T^2}{T_c^2} \right) r \right]
\]

\[
+ c
\]

(50)

This represents the net static potential observed for the confinement of quarks and gluons in the tachyon matter. It shows the combined effect of Coulombian-like (deconfinement) potential and a linearly increasing (confinement) potential. As we shall see, the Coulombian-like potential dominates at some temperature \( T = T_c \) (high energy) eliminating the effect of confinement. At \( T = 0 \) (low energy) we observe confinement regime. Also, from the above equation as \( r \) sufficiently increases (low energy) the Coulombian-like part of the potential approaches zero, the linear part dominates and confinement is observed.

Writing equation (50) in a more compact form, we have

\[
V_c(r, T) = -\frac{k}{r} + \sigma(T)r + c,
\]

(51)

where \( c \) is the integration constant, \( k \) is a positive constant given by \( k = 2\frac{g}{4\pi} \sqrt{\left( 1 - \frac{1}{N_c} \right)} \) and \( \sigma \) is the QCD string tension which in this case depends explicitly on the temperature.

The QCD string tension can be expressed as

\[
\sigma(T) = \frac{g\alpha^2}{4\pi} \frac{16}{3} \sqrt{\left( 1 - \frac{1}{N_c} \right)} \left[ 1 - \frac{T^2}{T_c^2} \right]
\]

(52)

At \( T = 0 \), \( \sigma \) does not longer depend on temperature and equation (51) looks like the Cornell potential for confinement of heavy quarks and gluons in hadrons. At this temperature, the gluons are automatically in a confined state. At \( T = T_c \), \( \sigma(T = T_c) \) vanishes and equation (51) reduces to \( V_c(r) = -\frac{k}{r} + c \).

Plotting the results from equations (51) and (52) in Figs. 1 and 2 we assumed that \( \alpha = 1 \), \( \lambda = 1 \), with this, we get, \( g/4\pi = 1 \), \( N_c \gg 1 \) and \( c = 0 \).
The net potential for the linear confinement and Coulombian-like regimes is depicted in Fig. 1. At non zero temperatures $T < T_c$, the potential rises linearly as expected, but the slope decreases with steady increase in temperature from $T = 0$ to $T = T_c$, where the slope approaches zero. This represents an increase in the free energy and a decrease in the interactions between the quarks and gluons as temperature increases. At $T = T_c$ the potential approaches a constant given by $V(r, T) \equiv \frac{1}{2}$. This shows a rise in the energy and a reduction in the interactions of the gluons, making it free (asymptotically) in the hadrons. Fig. 2 shows the sharp decrease in $\frac{T}{T_c}$, vanishing at $T = T_c$.

Substituting the confinement solution from equation (41) into the tachyon potential, we obtain from (42)

$$G(r, T) = \frac{1}{2} - \frac{4\alpha^2}{3} \left[ 1 - \frac{T^2}{T_c^2} \right] r^2$$

This result is plotted in Fig. 3. As we have earlier shown that $V(r) = G(r)$ we can as well say that $V(r, T) = G(r, T)$.

In this sense, we can clearly see from figure (1) and (3) that the confinement regime/tachyon condensation (at the places $r_s$ where $V(r_s) = G(r_s) = 0$) within $0 \leq T \leq 0.9T_c$ coincides at $0.6 \lesssim r \lesssim 2$. Also, the phase at $T = T_c$ and $V(r, T) = G(r, T) = 1/2$ does not correspond to the tachyon condensation. Thus, by comparing figures (1) and (3) we can identify that the electric confinement is associated with tachyon condensation [5].

**Quark gluon plasma (QGP).** At large temperature the QCD matter is in a quark-gluon plasma state, and undergoes a confinement phase transition as the temperature cools down below a critical temperature. Before ending this section some further comments on the screening at $r, T_c$ are in order. One can check that the solution $\phi = e^{-mr}/\alpha$ satisfies Eq. (38) for large distances, i.e., $r \gg m^{-1}$ and $r \gg (B/2A)^{1/4}$. This imposes an interesting condition $m^2 = A$, which means that $m = \alpha T$ for large temperatures, i.e., $T > T_c$. Now plugging this solution into (43), integrating out in $r$ and expanding the result for large distance, the leading term gives us precisely the well known color averaged potential [34]

$$V_{av}(r, T) \approx -2\lambda e^{-4mr}$$

$$\approx \frac{e^{-2m_c(T)r}}{Tr^2}$$

where in the last step we have identified the electric screening mass $m_e \sim gT$ — Debye-type screening. Since from Eq. (48) we find $\lambda \sim g$ here we can naturally identify $g = 2\alpha$, to get the Eq. (54) in the standard form. Above the critical temperature the confinement potential between a quark and an antiquark should be replaced by the color averaged potential. However, for numerical reasons one works with the color singlet potential which decreases slower with $1/r$ instead of $1/r^2$ given by the color averaged potential. The computation of the former potential is out of the scope of this paper.

![Figure 1: The potential $V_c(r, T)$ against $r$ for $T = T_c, T_1 = 0.9T_c, T_2 = 0.6T_c, T_3 = 0.3T_c$ and $T = 0$.](image)

**IV. CONCLUSIONS**

In our investigations we found the net static potential for linear confinement and the Coulomb-like regimes of quarks gluons within a specific range of interquark distance as a function of temperature. We found the results
for QCD in terms of QED by employing the Abelian QED theory to approximate the non-Abelian QCD theory by applying phenomenological effective field theory. The color dielectric function coupled with the gauge field produced the required strong interaction between the gluons that results in confinement at $0 \leq T \leq 0.9T_c$ and deconfinement at $T \geq T_c$ in the tachyonic matter. The confinement of the quarks and gluons at $0 \leq T \leq 0.9T_c$ coincides with tachyon condensation within the same temperature range as it is shown in Fig. 1 and 3. The de-confining phase starting at $T = T_c$ does not correspond to the tachyon condensation as it is seen in Figs. 1 and 3. Hence, the tachyon condensation is associated with the electric confinement and thus, it represents a monopole condensation of the electric field confinement as predicted in a well-known dual scenario [30]. Also the tachyon plays the role of Higgs field and the Higgs mechanism is expected to proceed via tachyon condensation, this justifies our choice of the potential in this work. The tachyon is expected to condense to a value in a limit of the string scale [31]. Moreover, at the confining phase ($T < T_c$) there is a spontaneous chiral-symmetry breaking whilst at the de-confining phase ($T = T_c$) there is a restoration of the chiral-symmetry, because QCD-monopole condensation is essential for spontaneous chiral-symmetry breaking [29, 32, 33].

The QCD string tension was also found as a function of temperature. It decreases rapidly with temperature and breaks (vanishes) at $T = T_c$. Furthermore a screening phase was shown to appear at large temperatures and large distance for the color averaged potential, which can be further considered to address issues of quark gluon plasma (QGP). Finally, we intend to advance further studies in this subject by studying confinement of fermionic tachyons using similar approach.
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