Holographic Principle and AdS/CFT Correspondence

Victor O. Rivelles

Instituto de Física, Universidade de São Paulo
C.Postal 66318, 05315-970, S.Paulo, SP, Brazil
rivelles@fma.if.usp.br

Abstract

According to the holographic principle all information in the bulk of a space is coded at its border. We will check this statement in three situations involving the AdS/CFT correspondence. There is a well known equivalence between the Maxwell-Chern-Simons theory and the self-dual model in 3 dimensions when the parameters of both theories are related in a given way. We will show that when this relation holds the corresponding CFT’s at the border are the same. Then we will study scalar fields. There are two quantum theories for the scalar field in AdS space. The usual prescription of the AdS/CFT correspondence which takes Dirichlet boundary conditions at the border corresponds to one of the quantum theories. We will show that changing boundary conditions will allow us to get the other quantum theory. Finally we consider an Abelian gauge theory in AdS. We will show that the corresponding CFT is independent of the gauge choice and that the gauge dependence stays only in the contact terms at the border.

---

1 Talk presented at the International Workshop “Supersymmetries and Quantum Symmetries”, held at JINR, Dubna, July 26-31, 1999
1 INTRODUCTION

The holographic principle states that a quantum theory with gravity must be describable by a boundary theory [1]. Of course this raises the question on how the detailed information in the bulk can be completely stored at the border and this surely deserves a deep investigation. A possible way to investigate this connection is through the AdS/CFT correspondence [2]. It says that the large $N$ limit of a certain conformal field theory (CFT) in a $d$-dimensional Minkowski space can be described by string/M-theory on $AdS_{d+1} \times K$ where $K$ is a suitable compact space. The precise relation between both theories is given by [3]

$$Z_{AdS}[\phi_0] = \int_{\phi_0} D\phi \exp(-I[\phi]) \equiv Z_{CFT}[\phi_0] = \langle \exp \left( \int_{\partial \Omega} d^d x \mathcal{O}\phi_0 \right) \rangle, \quad (1)$$

where $\phi_0$ is the value taken by $\phi$ at the border. On the right handed side $\phi_0$ is the external current coupling to the operator $\mathcal{O}$ in the boundary CFT. Hence the partition function in $AdS_{d+1}$ allows us to obtain the correlation functions of the boundary CFT. In this sense the AdS/CFT correspondence is a realization of the holographic principle.

The AdS/CFT correspondence in the form Eq.(1) has been studied in several situations [4]. We will analyze it the three different situations. In the first case we will concentrate in $d = 3$. As it is well known there is a equivalence between the Maxwell-Chern-Simons theory [6] and the self-dual model [7] in Minkowski space. Both theories are equivalent when the parameters have a precise relation. We have been able to show that this equivalence also holds in AdS space [8]. We can then use the AdS/CFT correspondence to compute the two point functions at the border. When the parameters are chosen so that the equivalence holds we can show that the corresponding CFT’s at the border are the same [8]. This shows that a relationship between two quantum field theories in AdS is directly reflected in the corresponding CFT’s at the border. This is discussed in Section 2.

The second case we study regards a scalar field. There are two distinct quantum field theories for the scalar field in AdS. They depend on which energy-momentum tensor is chosen [9]. Using the AdS/CFT correspondence in the form Eq.(1), which uses Dirichlet boundary conditions, one of the two quantum theories for the scalar field is reproduced. The other one can be obtained by considering a different boundary condition [10]. Therefore, as expected, both quantum theories can be obtained using the AdS/CFT correspondence. This will be presented in Section 3.
Finally we will consider the gauge dependence in the AdS/CFT context. We take an Abelian gauge theory in arbitrary gauge in an AdS background. We will show that the conformal correlators do not depend on the gauge parameter. However the holographic principle asserts that the information about the gauge degrees of freedom must survive at the border. We indeed find a gauge dependence only in the contact terms which usually are thrown away. This will be the subject of Section 4.

2 CHERN-SIMONS THEORIES

Since we are going to consider the Euclidean version of $AdS_3$ we start with the Euclidean signature action for the Proca-Chern-Simons theory which is given by

\[ I_{PCS} = \int d^3x \sqrt{g} \left( \frac{1}{8} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4} m^2 A_\mu A^\mu + \frac{1}{\sqrt{g}} \frac{i\mu}{8} \epsilon^{\mu\nu\alpha} F_{\mu\nu} A_\alpha + \text{c.c.} \right), \]

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\epsilon^{\mu\nu\alpha}$ is the Levi–Civita tensor density with $\epsilon^{012} = 1$. The field equations which follow from Eq.(2) are

\[ \nabla_\mu F^{\mu\nu} - m^2 A_\nu - i\mu \frac{1}{\sqrt{g}} \epsilon^{\nu\alpha\beta} \partial_\alpha A_\beta = 0, \]

and they can be manipulated to give

\[ \left( \nabla^2 - m_+^2 - \frac{R}{3} \right) \left( \nabla^2 - m_-^2 - \frac{R}{3} \right) A^\mu = 0, \]

where

\[ m_{\pm}^2(m, \mu) = \left[ \left( m^2 + \frac{\mu^2}{4} \right) + \frac{\mu}{2} \right]^2, \]

and $R = -6$ is the AdS radius. We notice that the solutions of Eq.(4) must satisfy

\[ \left( \nabla^2 - m_+^2 - \frac{R}{3} \right) A^\mu = 0, \]

or

\[ \left( \nabla^2 - m_-^2 - \frac{R}{3} \right) A^\mu = 0. \]

Therefore the general solution of Eq.(4) is a superposition of solutions of the Proca theory with masses $m_+$ and $m_-$. In the flat space limit we recover the
The fact that the Proca-Chern-Simons theory describes two massive excitations.
The Proca theory in AdS space has been analyzed in detail in [12].

We can now calculate the bulk to bulk propagator. The details of the calculation can be found in [8]. Then we can evaluate the classical action near the boundary surface using the action Eq.(2). After an integration by parts and using the equations of motion we find that there is only a contribution from the boundary

\[ I_{PCS} = \frac{1}{4} \int d^3 x \, \partial_\mu (\sqrt{g} F^{\mu\nu} A_\nu) + c.c., \]  

which evaluated on the near boundary surface gives

\[ I_{PCS} = -\frac{1}{4} \int d^2 x \, \epsilon^{-2} \tilde{A}_{\epsilon,i} \left( -\tilde{A}_{\epsilon,i} + \epsilon \tilde{F}_{\epsilon,0i} \right) + c.c.. \]

Here \( \tilde{A}_{\epsilon,i} \) is the value of the field at the near boundary surface. When inserting the propagator to evaluate \( I_{PCS} \) we find that the result is divergent in the limit \( \epsilon \to 0 \) so that a regularization has to be introduced. In order to have a finite action we take the limit

\[ \lim_{\epsilon \to 0} \epsilon^{m-(m,|\mu|)-1} \tilde{A}_{\epsilon,i}(\vec{x}) = \tilde{A}_{0,i}(\vec{x}). \]

Then we use the AdS/CFT correspondence in the form

\[ \exp \left( -I_{AdS} \right) \equiv \left\langle \exp \left( \int d^2 x \, J_i A_{0,i}(\vec{x}) \right) \right\rangle, \]

and we find the two point function

\[ \left\langle J_i^{PCS}(\vec{x}) \, J_j^{PCS}(\vec{y}) \right\rangle = \tilde{c}_{PCS} \tilde{\Delta}_{PCS} \left( \delta_{ij} - 2 \frac{(x-y)_i(x-y)_j}{|\vec{x}-\vec{y}|^2} \right) |\vec{x}-\vec{y}|^{-2\tilde{\Delta}_{PCS}}, \]

where \( \tilde{\Delta}_{PCS} = \tilde{\Delta}_-(m, |\mu|) \), and \( \tilde{c}_{PCS} = \tilde{c}_-(m, |\mu|) \), so that \( J_i^{PCS} \) has conformal dimension \( \tilde{\Delta}_{PCS} \). It is important to note that the identification Eq.(10) agrees with the requirement that the isometries of \( AdS_3 \) correspond to the conformal isometries in \( CFT_2 \).

In order to get the boundary CFT associated to the Maxwell-Chern-Simons theory we take \( m = 0 \) in Eq.(12), which gives

\[ \left\langle J_i^{MCS}(\vec{x}) \, J_j^{MCS}(\vec{y}) \right\rangle = \tilde{c}_{MCS} \tilde{\Delta}_{MCS} \left( \delta_{ij} - 2 \frac{(x-y)_i(x-y)_j}{|\vec{x}-\vec{y}|^2} \right) |\vec{x}-\vec{y}|^{-2\tilde{\Delta}_{MCS}}, \]
where $\tilde{\Delta}_{MCS} = |\mu| + 1$, and $\tilde{c}_{MCS} = \frac{|\mu|}{\pi}$. Therefore $J^M_{MCS}$ has conformal dimension $\tilde{\Delta}_{MCS}$. As it is well known the Maxwell-Chern-Simons theory describes a particle with mass $\mu$ [6] and this fact is reflected in the conformal dimension $\tilde{\Delta}_{MCS}$. Furthermore, our result is consistent with the holographic principle since the mass $m_-(0, |\mu|) = 0$ is not physical in the bulk [6] and does not contribute to the border two-point function.

For the self-dual model we start with the Euclidean signature action

$$I^0_{SD} = \int d^3x \sqrt{g} \left( \frac{1}{\sqrt{g}} \epsilon^{\mu\alpha} F_{\mu\nu} A_{\alpha} + \frac{1}{4} M^2 A_\mu A^\mu + \text{c.c.} \right). \quad (14)$$

In order to have a stationary action we must supplement the action Eq.(14) with a surface term which cancels its variation [13]. The variational principle generates a boundary term

$$-\kappa \int d^2x \epsilon^{0ij} \left[ A^R_i(\vec{x}) \delta A^I_j(\vec{x}) + A^I_i(\vec{x}) \delta A^R_j(\vec{x}) \right], \quad (15)$$

which is written in terms of the real and imaginary parts of the vector potential. Since the field equations derived from Eq.(14) are first order differential equations we can not choose boundary conditions which fix simultaneously the real and imaginary parts of the $A_i$'s. Then we choose boundary conditions on the $A^R_i$'s leaving a non-vanishing term proportional to the $\delta A^I_i$'s in the boundary term Eq.(15). So we add to the action Eq.(14) a surface term of the form

$$I^\text{surface}_{SD} = \frac{\kappa}{2} \int d^2x \epsilon^{0ij} A^R_i(\vec{x}) A^I_j(\vec{x}), \quad (16)$$

and the action

$$I_{SD} = I^0_{SD} + I^\text{surface}_{SD}, \quad (17)$$

is now stationary.

The field equations which follow from the action Eq.(17) are

$$i\kappa \frac{1}{\sqrt{g}} \epsilon^{\nu\alpha\beta} \partial_\alpha A_\beta + M^2 A^\nu = 0, \quad (18)$$

and it implies again

$$\nabla_\mu A^\mu = 0. \quad (19)$$

As in the case of the Proca-Chern-Simons theory we can eliminate the Levi–Civita tensor density by increasing the order of the equations of motion. We then get

$$\left( \nabla^2 - \frac{M^4}{\kappa^2} - \frac{R}{3} \right) A^\mu = 0. \quad (20)$$
and the results of the Proca theory can again be used.

The bulk to bulk propagator can be calculated [8] and a regularization has to be introduced
\[
\lim_{\epsilon \to 0} \epsilon^{\frac{M^2}{|\kappa|} - 1} \tilde{A}_{\epsilon, i}(\vec{x}) = A_{0, i}(\vec{x}).
\]

Using the AdS/CFT correspondence Eq.(17) we find the two-point function of the conformal field \(J^SD_i\) coupled to the field \(\tilde{A}_i\) on the boundary
\[
\langle J^SD_i(\vec{x}) J^SD_j(\vec{y}) \rangle = 2\tilde{c}_SD \tilde{\Delta}_{SD} \left( \delta_{ij} - 2 \frac{(x - y)_i(x - y)_j}{|\vec{x} - \vec{y}|^2} \right) |\vec{x} - \vec{y}|^{-2\tilde{\Delta}_{SD}},
\]
where \(\tilde{\Delta}_{SD} = \frac{M^2}{|\kappa|} + 1\), and \(\tilde{c}_SD = \frac{|\kappa|}{\pi}\). We then find that the field \(J^SD_i\) has conformal dimension \(\tilde{\Delta}_{SD}\). Therefore the conformal dimensions of the conformal fields corresponding to the Maxwell-Chern-Simons theory and the Self-Dual model are the same for \(\frac{M^2}{|\kappa|} = |\mu|\) in agreement with the equivalence between those models [5]. The fact that we obtain the same conformal dimension for the corresponding CFT’s in the border is in support of the holographic principle. Not only the conformal dimensions are the same but the coefficients \(\tilde{c}\) of the two-point functions can be made the same by an appropriate normalization of the Self-Dual action. Since we started with two independent parameters in Eq.(14) we can now choose \(M = |\kappa|\) so that the model describes a particle with mass \(M\). Now our results have an universal form in which the conformal dimension and the two-point function coefficient can be written as \(\tilde{\Delta} = m + 1\) and \(\tilde{c} = m/\pi\) respectively, where \(m\) is the mass of the bulk theory.

3 SCALAR FIELD THEORY

Scalar fields in AdS spaces have been intensively studied. If the scalar field has mass-squared in the range \(-d^2/4 < m^2 < -d^2/4 + 1\) then there are two possible quantum field theories in the bulk depending on the choice of the energy-momentum tensor [14]. The AdS/CFT correspondence with Dirichlet boundary condition can easily account for one of the theories. The other one appears in a very subtle way by identifying a conjugate field through a Legendre transform as the source of the boundary conformal operator [15]. The existence of two conjugated boundary operators has been first pointed out in [16].

Since a field theory is determined not only by its Lagrangian but also by its boundary terms we expect that the AdS/CFT correspondence must be sensitive to these boundary terms. We will consider Dirichlet and Neumann
boundary conditions, and a combination of both of them which we will call mixed boundary condition. Each type of boundary condition requires a different boundary term. We will show that the mixed boundary conditions are parametrized by a real number so that there is a one-parameter family of boundary terms consistent with the variational principle. We will also show that different types of boundary condition give rise to different conformal field theories at the border.

The two solutions found in [14] correspond to two different choices of energy-momentum tensor. For the Dirichlet boundary condition it is well known that the scalar field behaves as \( x^{d/2 - \sqrt{d^2/4 + m^2}} \) near the border at \( x_0 = 0 \). There is no upper restriction on the mass in this case. It corresponds to one of the solutions found in [14] and gives rise to a boundary conformal operator with conformal dimension \( d/2 + \sqrt{d^2/4 + m^2} \). We will show that for a particular choice of mixed boundary condition and when the mass squared is in the range \(-d^2/4 < m^2 < -d^2/4 + 1\) the scalar field behaves as \( x^{d/2 + \sqrt{d^2/4 + m^2}} \) near the border. It corresponds precisely to the second solution of [14] and gives rise to a boundary conformal operator with conformal dimension \( d/2 - \sqrt{d^2/4 + m^2} \). Note that the upper limit for the mass squared \(-d^2/4 + 1\) is consistent with the unitarity bound \((d - 2)/2\). Another important point that we will show is the existence of boundary conditions which give rise to boundary conformal operators for which the unitarity bound \((d - 2)/2\) is reached. They correspond to a massless scalar field with Neumann boundary condition or to a massive scalar field with \( m^2 > -d^2/4 + 1 \) with a particular choice of the mixed boundary condition (the same choice which gives the boundary operator with conformal dimension \( d/2 - \sqrt{d^2/4 + m^2} \)). In this way, using different boundary conditions, we obtain all scalar conformal field theories allowed by the unitarity bound.

The action for the massive scalar field theory is given by
\[
I_0 = \frac{1}{2} \int d^{d+1}x \sqrt{g} \left( g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi + m^2 \phi^2 \right),
\] (23)
and the corresponding equation of motion is
\[
\left( \nabla^2 - m^2 \right) \phi = 0.
\] (24)

In order to have a stationary action we must supplement the action \( I_0 \) with a boundary term \( I_S \) which cancels its variation. The appropriate action is then \( I = I_0 + I_S \).
In order to capture the effect of the Minkowski boundary of the $AdS_{d+1}$, situated at $x_0 = 0$, we first consider a boundary value problem on the boundary surface $x_0 = \epsilon > 0$ and then take the limit $\epsilon \to 0$ at the very end. Then the variational principle applied to the action $I$ gives

$$\delta I = \int d^d x \, \epsilon^{-d+1} \partial_0 \phi_\epsilon \, \delta \phi_\epsilon + \delta I_S = 0,$$

where $\phi_\epsilon$ and $\partial_0 \phi_\epsilon$ are the value of the field and its derivative at $x_0 = \epsilon$ respectively. This equation will be used below to find out the appropriate boundary term $I_S$ for each type of boundary condition.

For Dirichlet boundary condition the variation of the field at the border vanishes so that the first term in Eq.(25) also vanishes and the usual action $I_0$ is already stationary. Making use of the field equation the action $I$ takes the form

$$I_D = \frac{1}{2} \int d^{d+1} x \, \partial_\mu (\sqrt{g} \, \phi \, \partial^\mu \phi) = -\frac{1}{2} \int d^d x \, \epsilon^{-d+1} \phi_\epsilon \, \partial_0 \phi_\epsilon .$$

(26)

It is to be understood that $\partial_0 \phi_\epsilon$ in Eq.(26) is evaluated in terms of the Dirichlet data $\phi_\epsilon$.

To consider Neumann boundary conditions we first take a unitary vector which is inward normal to the boundary $n^\mu(x_0) = (x_0, \mathbf{0})$. The Neumann boundary condition then fixes the value of $n^\mu(\epsilon)\partial_\mu \phi_\epsilon \equiv \partial_n \phi_\epsilon$. The boundary term to be added to the action reads

$$I_S = -\int d^{d+1} x \, \partial_\mu (\sqrt{g} \, g^{\mu\nu} \phi \, \partial_\nu \phi) = \int d^d x \, \epsilon^{-d+1} \phi_\epsilon \, \partial_0 \phi_\epsilon ,$$

(27)

so that we find the following expression for the action at the boundary

$$I_N = \frac{1}{2} \int d^d x \, \epsilon^{-d} \phi_\epsilon \, \partial_n \phi_\epsilon .$$

(28)

Here $\phi_\epsilon$ is to be expressed in terms of the Neumann value $\partial_n \phi_\epsilon$. Notice that the on-shell value of the action with Neumann boundary condition Eq.(28) differs by a sign from the corresponding action with Dirichlet boundary condition Eq.(26).

We now consider a boundary condition which fixes the value of a linear combination of the field and its normal derivative at the border

$$\phi(x) + \alpha n^\mu \partial_\mu \phi(x) \equiv \psi^\alpha(x) .$$

(29)

We will call it mixed boundary condition. Here $\alpha$ is an arbitrary real but non-zero coefficient. In this case the surface term to be added to the action is

$$I_S^\alpha = \frac{\alpha}{2} \int d^{d+1} x \, \partial_\mu (\sqrt{g} \, g^{\mu\nu} \partial_\nu \phi \, n^\rho \partial_\rho \phi) = -\frac{\alpha}{2} \int d^d x \, \epsilon^{-d+2} \partial_0 \phi_\epsilon \, \partial_0 \phi_\epsilon .$$

(30)
and we find the following expression for the action at the boundary

\[ I_M^\alpha = -\frac{1}{2} \int d^d x \, \epsilon^{-d+1} \psi_\epsilon^\alpha \partial_\epsilon \phi_\epsilon. \]  

(31)

Clearly \( \partial_\epsilon \phi_\epsilon \) in the above expression must be written in terms of the boundary data \( \psi_\epsilon^\alpha \). We then have a one-parameter family of surface terms since the variational principle does not impose any condition on \( \alpha \). In this way the value of the on-shell action Eq.\( (31) \) also depends on \( \alpha \).

We can now calculate the bulk to bulk propagator for each boundary condition. The details are given in [10]. We then find the following two-point functions. For Dirichlet boundary conditions:

\[ \langle O_D^{\nu \neq 0}(\vec{x}) O_D^{\nu \neq 0}(\vec{y}) \rangle = \frac{2\nu}{\pi^\frac{d}{2}} \frac{\Gamma\left(\frac{d}{2} + \nu\right)}{\Gamma(\nu)} \frac{1}{|\vec{x} - \vec{y}|^{2(\frac{d}{2} + \nu)}} \]  

(32)

\[ \langle O_D^{\nu = \frac{d}{2}}(\vec{x}) O_D^{\nu = \frac{d}{2}}(\vec{y}) \rangle = \frac{d}{\pi^\frac{d}{2}} \frac{\Gamma(d)}{\Gamma\left(\frac{d}{2}\right)} \frac{1}{|\vec{x} - \vec{y}|^{2d}} \]  

(33)

\[ \langle O_D^{\nu = 0}(\vec{x}) O_D^{\nu = 0}(\vec{y}) \rangle = \frac{\Gamma\left(\frac{d}{2}\right)}{2\pi^\frac{d}{2}} \frac{1}{|\vec{x} - \vec{y}|^d} \]  

(34)

For Neumann boundary conditions:

\[ \langle O_N^{\nu \neq 0, \frac{d}{2}}(\vec{x}) O_N^{\nu \neq 0, \frac{d}{2}}(\vec{y}) \rangle = \frac{1}{\sigma^2(\nu)} \langle O_N^{\nu \neq 0}(\vec{x}) O_N^{\nu \neq 0}(\vec{y}) \rangle \]  

(35)

\[ \langle O_N^{\nu = \frac{d}{2}}(\vec{x}) O_N^{\nu = \frac{d}{2}}(\vec{y}) \rangle = \frac{\Gamma\left(\frac{d}{2}\right)}{2\pi^\frac{d}{2}} \frac{1}{|\vec{x} - \vec{y}|^{2d}} \]  

(36)

\[ \langle O_N^{\nu = 0}(\vec{x}) O_N^{\nu = 0}(\vec{y}) \rangle = \frac{1}{\sigma^2(0)} \langle O_N^{\nu = 0}(\vec{x}) O_N^{\nu = 0}(\vec{y}) \rangle \]  

(37)

And for mixed boundary conditions:

\[ \langle O_M^{\beta = 0, \nu < 1}(\vec{x}) O_M^{\beta = 0, \nu < 1}(\vec{y}) \rangle = \sigma^2(\nu) \frac{1}{2\pi^\frac{d}{2}} \frac{\Gamma\left(\frac{d}{2} - \nu\right)}{\Gamma(1 - \nu)} \frac{1}{|\vec{x} - \vec{y}|^{2(\frac{d}{2} - \nu)}} \]  

(38)

\[ \langle O_M^{\beta = 0, \nu > 1}(\vec{x}) O_M^{\beta = 0, \nu > 1}(\vec{y}) \rangle = \sigma^2(\nu) \frac{1}{2\pi^\frac{d}{2}} \frac{\Gamma\left(\frac{d}{2} - \nu\right)}{\Gamma(1 - \nu)} \frac{1}{|\vec{x} - \vec{y}|^{2(\frac{d}{2} - \nu)}} \]  

(39)

\[ \langle O_M^{\beta = 0, \nu < 1}(\vec{x}) O_M^{\beta = 0, \nu > 1}(\vec{y}) \rangle = \frac{1}{\beta^2(\alpha, \nu)} \langle O_M^{\nu = 0}(\vec{x}) O_M^{\nu = 0}(\vec{y}) \rangle \]  

(40)

\[ \langle O_M^{\nu = \frac{d}{2}}(\vec{x}) O_M^{\nu = \frac{d}{2}}(\vec{y}) \rangle = \langle O_D^{\nu = \frac{d}{2}}(\vec{x}) O_D^{\nu = \frac{d}{2}}(\vec{y}) \rangle \]  

(41)
Here the coefficients $\nu$, $\beta$ and $\sigma$ are defined as $\nu = \sqrt{\frac{d^2}{4} + m^2}$, $\beta(\alpha, \nu) = 1 + \alpha \left(\frac{d}{2} - \nu\right)$ and $\sigma(\nu) = \frac{d}{2} - \nu$ respectively.

We then find that different boundary conditions in the AdS/CFT correspondence allow us to derive boundary two-point functions which are consistent with the unitarity bound. In the Neumann case the unitarity bound is obtained for $m = 0$ while with mixed boundary conditions it is reached when $\beta = 0$ and $m^2 > -d^2/4 + 1$. The corresponding two-point functions have different normalizations. The conformal dimension $\frac{d}{2} - \nu$ is obtained in the case of mixed boundary condition with $\beta = 0$ and $-d^2/4 < m^2 < -d^2/4 + 1$, and the normalization of the corresponding boundary two-point function differs from the one found in [15].

Another important point is the interpretation of the new boundary conditions in the string theory context. Dirichlet boundary conditions are natural when thinking of the asymptotic behavior of the supergravity fields reaching the border of the AdS space. Possibly Neumann and mixed boundary conditions are related to more complex solutions involving strings and membranes reaching the border in more subtle ways.

4 ABELIAN GAUGE THEORY

For the Abelian gauge field in a fixed gauge the corresponding conformal correlators were found in [3]. Now we will consider the gauge dependence of this result. To do that we take the formulation of electrodynamics in an arbitrary gauge when the space-time background is $AdS_{d+1}$. As in the case of flat Minkowski space it will prove convenient to start from the St"uckelberg action

$$I_S = \int d^{d+1}x \sqrt{g} \left[ \frac{1}{4} F_{\alpha\beta} g^{\alpha\rho} g^{\beta\sigma} F_{\rho\sigma} + \frac{m^2}{2} A_{\alpha} g^{\alpha\beta} A_{\beta} + \frac{1}{2a} \left( g^{\alpha\beta} \nabla_\alpha A_\beta \right)^2 \right],$$

where $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$, $\nabla_\alpha$ is the covariant derivative and $a$ is a real positive constant. Electrodynamics in an arbitrary covariant gauge, specified by the constant $a$, is defined as the limit $m^2 \to 0$ of St"uckelberg theory. On the other hand the limit $a \to \infty$, while keeping $m^2 > 0$, results in the Proca theory. The mass term in (44) will help us to control the infrared divergent terms which will arise along the calculation.
The Lagrange equations of motion arising from (44) are
\[ \nabla_{\mu} F^{\mu\nu} + \frac{1}{a} \nabla_{\mu} L^{\mu\nu} - m^2 A^\nu = 0 , \] (45)
where \( L^{\mu\nu} \equiv g^{\mu\nu} g^{\alpha\beta} \nabla_{\alpha} A_{\beta} \). We now use a decomposition of \( A^\nu \) into a scalar field \( \Phi \equiv \nabla_\nu A^\nu \), and a vector field \( U^\nu \equiv A^\nu - \frac{1}{am^2} \nabla^\nu \Phi \). These new fields satisfy, respectively, the equations of motion
\[ \left( g_{\mu\nu} \nabla^\mu \nabla^\nu - am^2 \right) \Phi = 0 , \] (46)
and
\[ \nabla_{\mu} U^{\mu\nu} - m^2 U^\nu = 0 . \] (47)
Clearly \( U^\mu \) is a Proca field with mass \( m \) since \( \nabla_{\mu} U^{\mu} = 0 \).

As usual we shall look for a solution written in terms of boundary field values specified at the near-boundary surface \( x^0 = \epsilon \) the limit \( \epsilon \to 0 \) being performed at the very end of the calculations. The details are given in [14]. First of all we must make sure that the massless limit can be taken. There are infrared divergences in \( \Phi/am^2 \) and \( U_\nu \) but a careful analysis shows that they cancel out when the field \( A_\mu \) is recomposed. This shows that \( A_\mu \) is indeed an analytic function of \( m^2 \) in the vicinity of \( m^2 = 0 \).

In the computation of the two-point function we find that all potentially dangerous powers of \( \epsilon \) cancel out among themselves. We then find that the gauge dependence concentrates on the contact terms while the non-trivial part of the boundary conformal theory correlators turns out to be that already found by working in a completely fixed gauge[3]. Another important feature is that although we have fixed all components of the potential at the border the pieces containing \( \hat{A}_{\epsilon,0} \) give only contact terms and the only non-trivial pieces are those containing \( \hat{A}_{\epsilon,i} \). Therefore the boundary theory still retains information on the gauge degrees of freedom of the bulk theory. This then lends further support to the holographic principle.

5 ACKNOWLEDGEMENTS

This work is partially supported by CNPq and FAPESP.

References
[1] G. ’tHooft, “Dimensional Reduction in Quantum Gravity”, gr-qc/9310026; L. Susskind, “The World as a Hologram”, hep-th/9409089. J. Math. Phys. 36 (1995) 6377.

[2] J. Maldacena, “The Large N Limit of Superconformal Field Theories and Supergravity”, hep-th/9711200. Adv. Theor. Math. Phys. 2 (1998) 231.

[3] S. Gubser, I. Klebanov and A. Polyakov, “Gauge Theory Correlators from Non-Critical String Theory”, hep-th/9802109. Phys. Lett. B428 (1998) 105; E. Witten, “Anti-de Sitter Space and Holography”, hep-th/9802150. Adv. Theor. Math. Phys. 2 (1998) 253.

[4] For a review see O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, “Large $N$ Field Theories, String Theory and Gravity”, hep-th/9905111.

[5] S. Deser and R. Jackiw, “Self-Duality of Topologically Massive Gauge Theories”, Phys. Lett. B139 (1984) 371.

[6] S. Deser, R. Jackiw and S. Templeton, “Topologically Massive Gauge Theories”, Ann. Phys. (NY) 140 (1982) 372.

[7] P. K. Townsend, K. Plich and P. van Nieuwenhuizen, “Self-Duality in Odd Dimensions”, Phys. Lett. B136 (1984) 38.

[8] P. Minces and V. O. Rivelles, “Chern-Simons Theories in the AdS/CFT Correspondence”, hep-th/9902123. Phys. Lett. B455 (1999) 147.

[9] P. Breitenlohner and D. Freedman, “Stability in Gauged Extended Supergravity”, Ann. Phys. 144 (1982) 249.

[10] P. Minces and V. O. Rivelles, “Scalar Field Theory in the AdS/CFT Correspondence Revisited”, hep-th/9907079.

[11] H. O. Girotti and V. O. Rivelles, “Gauge Dependence in the AdS/CFT Correspondence”. hep-th/9910017.

[12] W. Mück and K. S. Viswanathan, “Conformal Field Theory Correlators from Classical Field Theory on Anti-de Sitter Space II. Vector and Spinor Fields”, hep-th/9805145. Phys. Rev. D58 (1998) 106006.

[13] M. Henneaux, “Boundary Terms in the AdS/CFT Correspondence for Spinor Fields”, hep-th/9902137.
[14] P. Breitenlohner and D. Freedman, “Stability in Gauged Extended Supergravity”, Ann. Phys. 144 (1982) 249.

[15] I. Klebanov and E. Witten, “AdS/CFT Correspondence and Symmetry Breaking”, hep-th/9905104.

[16] V. K. Dobrev, “Intertwining Operator Realization of the AdS/CFT Correspondence”, hep-th/9812194.