Energy Efficiency Improvement in Mobile Communication System by Reducing the PAPR

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Abstract. Peak to average power ratio (PAPR) is a great issue in multicarrier systems especially in the LTE/LTEA communication system, so there is still a wide range that handles the PAPR problem. Orthogonal frequency division multiple access (OFDMA) technique is used in the downlink LTE system as it has many advantages over multipath propagation system. On the other side it suffers from a high PAPR that will affect the energy efficiency. In this paper, a new hybrid scheme is proposed in the downlink mobile communication system to overcome this issue. Besides reducing the PAPR, the proposed scheme did not have a significant increase in the system complexity but also it had nearly the same bit error rate (BER) system performance as OFDM system. Also we introduce a combination of different pre-coding techniques in OFDM system that improve the BER such as (Discrete Hartley Transform, Discrete Cosine Transform and Discrete Sine Transform) with the companding techniques that have a good ability in reducing the PAPR with a little increase in system complexity such as (Absolute exponential, Tangent rooting and Logarithmic) companding techniques. We tested the system performance for different techniques combination, and according to the analysis and the simulations done, it was found that the hybrid technique that is composed from the Discrete Hartley transform pre-coding technique and the Logarithmic companding technique in the downlink system outperforms all other hybrid techniques combinations.

1. Introduction
In LTE advanced mobile communication system, the orthogonal frequency division multiple access (OFDMA) was implemented in the downlink and the SC-FDMA in the uplink [1]. The OFDM has many advantages over the multipath propagation systems [2] with one main disadvantage, which is the high peak to average power ratio (PAPR). Highly linear power amplifiers and Digital to Analog (D/A) converters were required to meet the large PAPR demands. And so, a special power amplifier with larger size and higher cost was needed to be implemented, but unfortunately the power efficiency will be decreased as we increase the linear region of power amplifier, which will require an additional complexity. Also at the same time the mobile terminals are limited in the power, so this problem is a critical issue in the downlink transmission [3].

In the literature review, several methods are explored that minimize PAPR from different categories such as coding techniques, probabilistic techniques, signal distortion and multiple signalling
techniques [4]. Each of these techniques achieves a good PAPR reduction when testing their performance one technique at a time in OFDM downlink system. However, when we apply a hybrid technique that combines two reduction techniques together in the system, it obviously achieves much better reduction in PAPR with a slight increase in the system complexity according to the appropriate chosen methods.

The pre-coding technique has a good ability in the PAPR reduction as it reduces the autocorrelation of the input stream. Also its main advantage is to improve the BER in comparison to the conventional OFDM system because of the diversity gain obtained due to spreading the data symbol on more than one subcarrier.

The companding technique, which is the most attractive scheme in reducing the PAPR, is also considered as one of the simplest techniques with less complexity. And there is no need for side information and no expansion in the bandwidth with very good ability in minimizing the PAPR.

In this paper, a new hybrid technique is proposed and evaluated in downlink mobile communication system that combines different pre-coding techniques such as: Discrete Hartley Transform (DHT), Discrete Cosine Transform (DCT) and Discrete Sine Transform (DST) with different advanced companding techniques such as: Tangent Rooting companding (TanhR), Logarithmic companding (LogR) and Absolute Exponential companding technique (AEXP). It was proved that the proposed hybrid system outperforms the conventional OFDM system, pre-coding techniques (DHT, DST and DCT) and companding techniques (AEXP, LogR and TanhR) in both of PAPR reduction and BER, as will be shown in the simulation outcomes.

The rest of the paper is organized as follows: Section two includes the PAPR problem, (OFDMA) system block diagram, a brief discussion about the advanced companding techniques and the pre-coding techniques. In Section three, the proposed hybrid techniques in downlink LTE system are presented and simulated. Simulation outcomes are analysed and illustrated in Section four. Finally, the conclusions are stated in Section five.

2. OFDMA System Model
In this Section, the PAPR definition and the basics of OFDM transmitters were reviewed. The ordinary OFDM block diagram is presented as shown in figure 1. The input stream of bits (X) at the transmitter of the OFDMA system was mapped by applying QPSK and M-QAM modulation techniques separately. Whereas, the output from the mapper X(s) is a sequence of a complex numbers, where X(s) is the data symbol and s is the sample index. After that, it was subjected to serial to parallel converter (S/P) block to get data blocks Xn. Finally, it was applied to the IFFT block that converted N subcarriers into Xm time domain signals as represented in equation (1).

\[
X_m = \frac{1}{M} \sum_{L=0}^{M-1} X_L e^{j2\pi t_m \frac{L}{M}}, \quad m = \{0, 1, ..., (N-1)\}
\]
After the mentioned process, $X_m$ was converted from parallel form into serial form to send the data sequentially, and then the cyclic prefix was added. Inverse steps were done at the receiver to extract the data. The current work will try to develop the LTE downlink system performance by improving the PAPR.

PAPR is defined as a ratio between the peak signal amplitude to the average power. Peak signal power occurs when $N$ modulated symbols with the same phase are added together [5]. PAPR of the OFDMA system is a critical issue that needs to be solved, because of the limited power at the user mobile terminal in LTE mobile communication system and also for operating the power amplifier in the linear region. PAPR of $X_m$ can be mathematically represented by equation (2).

$$PAPR(X_m) = \sum_{m=0}^{N-1} \frac{\max[|x_m^2|]}{E[|x_m^2|]}$$ (2)

Where, $(E)$ stands for the expectation value which calculates the mean square value. Further, some pre-coding and companding techniques will be mentioned briefly.

### 2.1. Tangent Rooting Companding Technique (TanhR)

TanhR is one of the companding techniques that work on the transformation of the signal distribution into tangent distribution using the hyperbolic function (tanh) [6] as in equation (3).

$$Z(x) = tanh\left( (|x| \cdot k^y) \right) \cdot sgn(x)$$ (3)

Where, the positive number $k$ represents the companding degree level for the signal envelope (x) which ranges from 5 to 25. $y$ is also a positive number ranges from 0.2 to 1, that controls effectively the PAPR. It controls the PAPR reduction level by adjusting the two factors $k$ and $y$ at the transmitter [7]. As the more $k$ increases and $y$ decreases, the more reduction is achieved. The recovered signal at the receiver after the expanding process can be extracted using equation (4).

$$z^{-1}(x) = \left( atanh\left( \frac{|x|}{k} \right) \right)^{\frac{1}{y}} \cdot sgn(x)$$ (4)

### 2.2. Logarithmic Companding Technique (LogR)

LogR companding is a low complexity companding technique with good system performance in PAPR reduction [8]. The logarithmic companding function can be represented as in equation (5).
\[ f(x) = \log (M|\alpha| + 1) \text{sgn}(\alpha) \]  

(5)

Where, M is a positive number that represents the companding degree level for the signal envelope (\( x \)). At the receiver end, the de-companding function is represented in equation (6).

\[ f^{-1}(x) = \left| \left( \exp \left( \frac{|\alpha|}{M} \right) - 1 \right) \right| * \text{sgn}(\alpha) \]  

(6)

2.3. **Absolute Exponential Companding Technique (AEXP)**

AEXP has an effective role, not only in reducing the PAPR for different modulation techniques of transmitted companded signal, but also BER will be improved by keeping the average power of the transmitted companded signal at the same level with the input signal [9]. At the transmitter, the companding function \( h(\alpha) \) is expressed in equation (7).

\[ h(\alpha) = \text{sgn}(\alpha) \sqrt{\alpha \left[ 1 - \exp \left( \frac{|\alpha|^2}{\sigma^2} \right) \right]} \]  

(7)

Where (d) is the degree of the exponential companding that controls the signal companded factor (\( \alpha \)). \( \alpha \) is the positive constant value that controls the average power of the output companded signal which is mathematically given as in equation (8). Where \( \sigma^2 \) is the input modulated signal variance, and \( x \) is the IFFT output signal.

\[ \alpha = \left( \frac{\mathbb{E}[|\alpha|^2]}{\mathbb{E}\left[ \frac{1}{N-1} \right] \left( 1 - \exp \left( \frac{|\alpha|^2}{\sigma^2} \right) \right)^2} \right)^{d/2} \]  

(8)

2.4. **Pre-coding Techniques**

Pre-coding matrix \( P \) was used with dimension \( N \times N \) before the IFFT so we can reduce the PAPR. This process will multiply the input data with the pre-coding matrix \( P \), described in equation (9), in order to distribute the energy of data symbols over the subcarriers for reducing the PAPR.

\[ P = \begin{bmatrix} P_{00} & \cdots & P_{0(N-1)} \\
\vdots & \ddots & \vdots \\
P_{(N-1)} & \cdots & P_{(N-1)(N-1)} \end{bmatrix} \]  

(9)

Where \( P \) is a pre-coding Matrix of size \( N \times N \).

2.4.1. **Discrete Hartley Transform (DHT) Technique.** The Discrete Hartley Transform (DHT) is a linear transform, According to [10] the N-point DHT can be defined as in equation (10).

\[ H_k = \sum_{n=0}^{N-1} x_n \left[ \cos \frac{2\pi nk}{N} + \sin \frac{2\pi nk}{N} \right] \quad k = 0, 1 \ldots N - 1 \]  

(10)

The DHT is also an invertible transform which allows us to recover the signal.

2.4.2. **Discrete Cosine Transform (DCT) Technique.** The Discrete Cosine Transform DCT is a technique that transforms a signal into the frequency domain, using only real numbers without any imaginary components. The idea of this technique is based on reducing the autocorrelation of the
input row for the reduction of PAPR [11]. The \( N \times N \) DCT matrix \( P_{ij} \) can be created using equation (11).

\[
P_{ij} = \begin{cases} 
\frac{1}{\sqrt{N}} & i = 0, \quad 0 \leq j \leq N - 1 \\
\frac{\sqrt{2}}{\sqrt{N}} \cos \frac{\pi (2j+1)i}{2N} & 1 \leq i \leq N - 1 \\
0 & 1 \leq j \leq N - 1
\end{cases}
\] (11)

And DCT can be defined as in equation (12).

\[
X_k = \sum_{n=0}^{N-1} x_n \cdot \cos \left( \frac{\pi}{N} \left( n + \frac{1}{2} \right) k \right) \quad k = 0,1 \ldots N - 1
\] (12)

2.4.3 Discrete Sine Transform (DST) Technique. Discrete Sine Transform (DST) is used to reduce the PAPR by reducing the autocorrelation of the data sequence [12]. For an input sequence \( b_n \), the output of using DST can be defined as in equation (13).

\[
y_n = \sum_{k=0}^{N-1} b_n B_n \cdot \sin \left( \frac{n \pi t}{T_s} \right) \quad k = 1 \ldots N - 1
\] (13)

Where, \( b_n \) is the input to the pre-coding transform, \( y_n \) is the output of pre-coding transform and \( B_n \) is the coefficient of the Discrete Sine Transform technique which is defined as in equation (14).

\[
B_n = \begin{cases} 
\frac{1}{\sqrt{2}} & , n = 0 \\
1 & , n = 1,2,3, \ldots, N - 1
\end{cases}
\] (14)

3. Proposed Technique

In this section, a new hybrid scheme in the downlink mobile communication system is introduced instead of the conventional OFDMA system. This hybrid technique is done by combining different pre-coding techniques like DHT, DCT and DST with different companding techniques like TanhR, AEXP and LogR to reduce the PAPR, with minimum BER system degradation and with a little increase in system complexity [13] [14]. In order to try to resolve the trade-off between the reduction of PAPR and other system performance parameters, such as increasing in BER, needing for extra side information data and increasing in system complexity, a pre-coding is used to compensate the degradation of BER caused by inserting a companding technique in OFDM system. The reason for using the companding technique is that, it is considered as one of the simplest reduction techniques, with less complexity, which can overcome the disadvantage of the pre-coding technique.

So the proposed hybrid technique improves the OFDM system as it has a very good ability in PAPR reduction with a little increase in complexity and very low BER system degradation. The proposed method block diagram is illustrated in figure 2.
Figure 2. Proposed technique in downlink mobile communication system.

Figure 2 shows the proposed block diagram, which is the same as the conventional OFDM system that illustrated in figure 1 but differs from it in the part where the proposed pre-coding and companding techniques, at both the transmitter and the receiver, were added.

4. Simulation Results and Analysis

Excessive simulations are done for evaluating the system performance and finding the best hybrid technique in reducing PAPR. The results showed that the proposed hybrid technique outperforms the conventional system, the companding technique separately and the pre-coding technique alone. The proposed model does not only provide better PAPR reduction, but also it improves the obtained BER system performance more than the conventional OFDM system.

The simulation parameters for analysing the PAPR and BER system performance are illustrated in Table 1.

Table 1. Simulation Parameters

| Parameter | Companding techniques | Number Of Symbols | Number Of Bits (n) | Modulation Technique | FFT size (N) | SNR | Cyclic prefix length | Channel model | Pre-coding techniques |
|-----------|-----------------------|-------------------|-------------------|----------------------|-------------|-----|---------------------|---------------|----------------------|
| Description | AEXP, LogR and TanhR | 1000              | 256000, 512000    | QPSK                 | 256,512     | 0-30 dB | 0.25*FFT size       | Rayleigh fading channel | DCT, DST and DHT |

The Complementary Cumulative Distribution Function (CCDF) of the proposed technique is also introduced in order to study the effect of PAPR. Figure 3 illustrates the variation of companding degree factor (d) for the AEXP companding to get the semi optimal value for d.
Figure 3 shows that, the \( d \) value is inversely proportional to the PAPR reduction, so as the \( d \) value decreases, the more reduction will be achieved. At the same time the BER will degrade as there is a trade-off between BER and PAPR. So, the best value for \( d \), according to the results shown in Figure 3 and figure 4, is \( d=1 \). Then, the system was tested for each pre-coding technique separately to get the best one in the PAPR reduction and with acceptable system BER.

Figure 3. PAPR at different \( d \) values for AEXP

Figure 5, firstly shows the PAPR results for each pre-coding technique separately and secondly, it shows a combination between these pre-coding techniques and the AEXP companding technique at \( d=1 \). Figure 6 illustrates the BER waveforms for the pre-coding techniques only and for the hybrid techniques.

Figure 4. BER at different \( d \) values for AEXP.
Figure 5. PAPR for pre-coding and hybrid pre-coding with AEXP companding.

Figure 6. BER for pre-coding and hybrid pre-coding with AEXP companding.

From figure 5 and 6, the best technique in achieving both PAPR reduction and less system BER degradation is the proposed DHT pre-coding with AEXP companding (DHT+AEXP) as it has SNR = 10.85 dB at BER = 10^{-3} which is greater than the conventional OFDM by just 0.75 dB and also it reduced the PAPR by 8.3 dB more than the reduction value of OFDM conventional system.

Figure 7 and figure 8 illustrate the mentioned hybrid techniques, with different combinations of different pre-coding techniques and LogR companding technique with a change in the values of the companding parameters K and y.
From figure 7 and 8, the best hybrid technique was found to be (DHT+ LogR) at K=10, y=0.8 as it had SNR= 10.45 dB at BER=10^{-3} which is greater than the conventional OFDM by just 0.35 dB and it reduced the PAPR by 8.34dB more than the reduction value of OFDM conventional system.

Figure 9 and figure 10 illustrate different hybrid techniques between different pre-coding techniques and TanhR companding technique with a change in the value of the companding parameters K and y. From these figures, it was observed that the best hybrid technique was (DHT+ TanhR) at K=5, y=1 as it has SNR= 11.15 dB at BER=10^{-3} which is approximately the same as conventional OFDM by just 1.053 dB and it reduced the PAPR by 9.24dB more than the OFDM conventional system.
A comparison was done between our proposed hybrid technique, which combines TanhR and DHT pre-coding, with the hybrid technique illustrated in [6] that combines TanhR and clipping. Table 2 shows the simulation analysis results for the two hybrid techniques.

Table 2. A comparison between our proposed technique and another hybrid technique.

|                              | (TanhR + DHT) at k=5, y=1 | (TanhR + clipping) at k=5, y=1 |
|------------------------------|----------------------------|---------------------------------|
| PAPR at 10-3                 | 1.45 dB                    | 3.05 dB                         |
| BER at 10-3                  | 11.15 dB                   | 11.6 dB                         |

Table 2 shows that our proposed hybrid technique (TanhR + DHT) outperforms the hybrid technique mentioned in [6] in PAPR reduction and system BER performance by 1.6 dB and by 0.45 dB respectively.
5. Conclusion
In this paper, a hybrid technique between many pre-coding and companding techniques was introduced in order to combine the advantages of both of them, which are achieving better PAPR reduction with accepted BER system performance and a little increase in system complexity. Different pre-coding techniques such as DCT, DHT and DST were explored in the downlink mobile communication system as they have a good ability in PAPR reduction and they also improve the system BER. Also different advanced companding techniques were explored as they have both the ability in PAPR reduction with less system complexity. Our results showed that the hybrid methods achieved better PAPR reduction but at the expense of a little increase in system complexity and all the proposed hybrid techniques outperform the conventional OFDM system. The worst technique was the DST pre-coding with TanhR while the best proposed technique was the DHT pre-coding with TanhR. Finally we compared our proposed method with a state of the art hybrid technique that combines TanhR with clipping and the results showed that our proposed technique gave better rates than the state of the art hybrid technique with respect to BER and PAPR reduction.

6. References
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