Study on the mixing among the $0^{++}$ mesons around $1 \sim 2$ GeV with the QCD sum rules

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We calculate the correlation functions of $0^{++}$ $q\bar{q}$, $s\bar{s}$ and glueball in the QCD sum rules and obtain the mass matrix where non-diagonal terms are determined by the cross correlations among the three states. Diagonalizing the mass matrix and identifying the eigenstates as the physical $0^{++}$ scalar mesons, we can determine the mixing. Concretely, our calculations determine the fractions of $q\bar{q}$, $s\bar{s}$ and glueball in the physical states $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$, the results are consistent with that gained by the phenomenological research.

I. INTRODUCTION

Existence of glueball is a long-standing puzzle in the QCD theory. Searching for it becomes the most challengeable task for high energy physics society. The QCD theory predicts its existence and the lattice QCD almost determines the mass spectra of glueballs with various quantum numbers[1–9]. It is believed that the mass of the lighter glueballs should be at around 1 to 2 GeV. But where are they, can we pin down them? Several bound states near 2 GeV have been found in recent experiments[10]. People believe that the number of these states indeed exceeds that predicted by the simple symmetry analysis. One natural explanation is that there exist exotic states and the newly observed resonances are either such exotic states, glueball, hybrid and multi-quark states, or their mixtures. In fact, none of the resonances which are newly observed at BES and BELLE can be identified as glueballs, so that one is tempted to conclude that glueballs mix with the regular quark states. The lattice and other model-dependent calculations all predict the mass of the $0^{++}$ glueball falling within the range of about $1.7\text{GeV}$[11–13]. Meanwhile the mass of the state made of pure light quarks $q\bar{q}$, where, the $q$ refers to $u$, $d$ and $s$ quark, is also near $1.3 \sim 1.7\text{GeV}$[14–16], therefore it is very possible that the scalar glueball and the quark states mix to constitute physical
states. The observed resonances $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ which have masses close to $[1.3 \sim 1.7]$GeV, can really be such mixtures.

For this mixing, it implies that the scalar glueball does not independently exist as a physical state which people explore in experiment, but the three physical states: $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ possess glueball components. In fact, many authors have discussed the mixing of these three physical states[17–21]. Generally, this issue was discussed based on phenomenology, namely by fitting data of various reactions, the mixing parameters are fixed. It would be interesting to investigate this problem from a more fundamental theory. However, the energy scale for the mixing is low and the non-perturbative QCD effects may dominate, therefore the regular perturbative theory does not apply. By contrast, the QCD sum rules may be the bridge between perturbative quantum field theory and the non-perturbative phenomena[22], thus should be a reasonable approach for this research. Two groups have done the significant work[23–25]. Narison et al’s work fixed the mixing of the three states: $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ through the decays of the light-quark meson and the glueball. In their work, the masses of the scalar light-quark states and glueball are determined in the QCD sum rules and by using them to estimate the decay rates of the corresponding processes they fix the mixing parameters. By contrast, we assume that the scalar light-quark states $|N, S\rangle$ and the glueball $|G\rangle$ are un-physical, therefore the masses independently determined in the QCD sum rules cannot be used to estimate the decay rates. In another work, Steele et al. predicted that the mixing states should involve mixing of $f_0(980)$ with the $f_0(1500)$ and $f_0(1710)$ in terms of the Gaussian QCD sum rule. Instead, in our work, we are going to investigate the mixing of the three states all near $2\text{GeV}$ in the QCD sum rules.

The first step of our work is to define the currents for the un-physical states: glueball $|G\rangle$, light-quark states $|N\rangle$ and $|S\rangle$ (N is for u, d quarks, and S is for s quark), then find their relations to the three physical states: $|f_1\rangle$, $|f_2\rangle$ and $|f_3\rangle$ via a mixing matrix $V$.

The work is organized as follows. After this introduction, we calculate the correlation functions in terms of the QCD sum rules, in Section III, we formulate the mixing matrix and show the relations between the unphysical states and the physical scalar mesons. In Section IV, we present our numerical results and the last section is devoted to our conclusion and discussion.
II. THE CORRELATION FUNCTION

In the scenario of the QCD sum rules, the correlation function $\Pi(q^2)$ is defined as:

$$\Pi(q^2) = i \int dx e^{iqx} \langle 0 | T \{ J(x), J(0) \} | 0 \rangle.$$  \hspace{1cm} (1)

By the dispersion relation, at the hadron hand, the correlation function can be written as:

$$\Pi(q^2) = \frac{1}{\pi} \int ds \frac{\text{Im} \Pi(s)}{s - q^2}. \hspace{1cm} (2)$$

After the Borel transformation and considering the quark-hadron duality, we obtain the “Moment” $\mathcal{R}$ as:

$$\mathcal{R}_k = \frac{1}{\pi} \int_{s_0}^{s_0} ds s^k \text{Im} \Pi(s) e^{-s\tau} \hspace{1cm} (3)$$

where $\tau$ is the Boral parameter and $s_0$ is the threshold for the continuity.

So in our work, the relevant correlation functions are defined as:

$$\Pi^{qq}(q^2) = i \int dx e^{iqx} \langle 0 | T \{ J_q(x), J_q(0) \} | 0 \rangle$$

$$\Pi^{qs}(q^2) = i \int dx e^{iqx} \langle 0 | T \{ J_s(x), J_q(0) \} | 0 \rangle$$

$$\Pi^{qs}(q^2) = i \int dx e^{iqx} \langle 0 | T \{ J_g(x), J_g(0) \} | 0 \rangle$$

$$\Pi^{qs}(q^2) = i \int dx e^{iqx} \langle 0 | T \{ J_q(x), J_g(0) \} | 0 \rangle$$

$$\Pi^{qs}(q^2) = i \int dx e^{iqx} \langle 0 | T \{ J_s(x), J_g(0) \} | 0 \rangle$$  \hspace{1cm} (4)

where $J_g(x)$ is

$$J_g(x) = \alpha_s G^a_{\mu\nu}(x) G^{a\mu\nu}(x), \hspace{1cm} (5a)$$

and $J_{q,s}(x)$ is:

$$J_{q,s}(x) = m_{q,s} \psi_{q,s}(x) \bar{\psi}_{q,s}(x). \hspace{1cm} (5b)$$
FIG. 1: The Feynman Diagrams for $\Pi_{QCD}^{qg,sg}$: (a) perturbative part; (b-c) with quark condensates; (d) with gluon condensates; (e) with quark-gluon condensates

The “Moments” $\mathcal{R}$ are defined as:

$$
\mathcal{R}_{k}^{qq} = \frac{1}{\pi} \int_{s_0}^{s} dss k \text{Im} \Pi^{qq}(s)e^{-st} ;
$$

$$
\mathcal{R}_{k}^{ss} = \frac{1}{\pi} \int_{s_0}^{s} dss k \text{Im} \Pi^{ss}(s)e^{-st} ;
$$

$$
\mathcal{R}_{k}^{gg} = \frac{1}{\pi} \int_{s_0}^{s} dss k \text{Im} \Pi^{gg}(s)e^{-st} ;
$$

$$
\mathcal{R}_{k}^{qg} = \frac{1}{\pi} \int_{s_0}^{s} dss k \text{Im} \Pi^{qg}(s)e^{-st} ;
$$

$$
\mathcal{R}_{k}^{sg} = \frac{1}{\pi} \int_{s_0}^{s} dss k \text{Im} \Pi^{sg}(s)e^{-st} ,
$$

(6)

where $\mathcal{R}_{k}^{gg}$ can be found in Refs. [12, 13, 26] and $\mathcal{R}_{k}^{qq,ss}$ is given in Refs. [14, 27]. For the mixing current, we calculate the correlation functions and the “Moments” $\mathcal{R}_{k}^{qg,sg}$ are obtained from the Feynman Diagrams in Fig-1

With the Operator Product Expansion (OPE), the correlation function $\Pi^{qg,sg}(q^2)$ is decomposed as:

$$
\Pi^{qg,sg}(q^2) = C_0 \hat{O}_0 + C_3 \langle qq, ss \rangle + C_4 \langle \alpha_s G^2 \rangle + C_5 \langle g_s O_5 \rangle + \cdots ,
$$

(7)

where $C_i (i = 1, 3, 4, 5, \cdots)$ are the Wilson coefficients, and the operator $\hat{O}_0$ is the unit operator.
In the fixed-point gauge\cite{28}, we calculate the two-loop diagram in Fig-1(a) and then we have:

\[
C_0 = -\frac{1}{\epsilon} \frac{3\alpha_s^2}{\nu^2} m_{q,s}^2 \log \frac{Q^2}{\nu^2} Q^2 + \frac{\alpha_s^2}{\pi^3} m_{q,s}^2 Q^2 \left[ \frac{3\log^2 Q^2}{\nu^2} \right. \\
+ \log \frac{Q^2}{\nu^2} \left( -3 \log 4\pi + 3\gamma_E - \frac{35}{4} \right) \right] + \cdots,
\]  
(8)

where, \(Q^2 = -q^2\). In Eq.(8), we drop out the terms which are not proportional to \(\log[Q^2/\nu^2]\) because they do not contribute to the moment \(\mathcal{R}_{k}^{q,s}\) and disappear after the Borel transformation. The Feynman diagrams related to the counter terms are presented in Fig-2.

In the \(\overline{MS}\) scheme, we find:

\[
C_0^{(o)} = 0,
\]  
(9a)

and

\[
C_0^{(b)} = \frac{1}{\epsilon} \frac{3\alpha_s^2}{\nu^2} m_{q,s}^2 \log \frac{Q^2}{\nu^2} Q^2 - \frac{\alpha_s^2}{\pi^3} m_{q,s}^2 Q^2 \left[ \frac{3}{4} \log^2 \frac{Q^2}{\nu^2} \right. \\
+ \log \frac{Q^2}{\nu^2} \left( -3 \log 4\pi + 3\gamma_E - \frac{9}{4} \right) \right] + \cdots
\]  
(9b)

Eventually we have the coefficient \(C_0\) at the two-loop order as:

\[
\overline{C}_0 = C_0 + C_0^{(a)} + C_0^{(b)}
\]  
(10)

\[
= \frac{\alpha_s^2}{\pi^3} m_{q,s}^2 Q^2 \left[ \frac{3}{4} \log^2 \frac{Q^2}{\nu^2} - \frac{13}{2} \log \frac{Q^2}{\nu^2} \right],
\]

which corresponds to the perturbative contribution to the moments. The other Wilson coefficients are calculated from Fig.1(b-e) as:

\[
C_3 = -4\pi \frac{\alpha_s^2}{\nu^2} m_{q,s} \log \frac{Q^2}{\nu^2}; \\
C_4 = \frac{m_{q,s}^2}{Q^2} \left[ -\frac{\alpha_s}{\pi} \log \frac{Q^2}{\nu^2} + 3\frac{\alpha_s}{\pi} \right]; \\
C_5 = -\frac{2}{Q^2} m_{q,s} \alpha_s.
\]  
(11)
With the correlation function Eq.(7), the moment is:

\[ R_{qg, sg}^0 = \frac{1}{\tau^2} (1 - \rho_1(s_0 \tau)) a_0^{q,s} - \frac{2a_1^{q,s}}{\tau^2} \left[ \gamma_E + E_1(s_0 \tau) + \log s_0 \tau + e^{-s_0 \tau} - 1 \right. \]

\[-(1 - \rho_1(s_0 \tau)) \log \frac{s_0}{\nu^2} - \frac{b_1^{q,s}}{\tau} (1 - \rho_0(s_0 \tau)) m_q \langle q \bar{q} \rangle \]

\[ + \left[ c_0^{q,s} - c_1^{q,s} (\gamma_E + \log \tau \nu^2 + E_1(s_0 \tau)) \right] \langle \alpha_s G^2 \rangle + d_0^{q,s} \langle g_s O_5 \rangle \]

where,

\[ a_0^{q,s} = -\frac{13\alpha_s^2}{4\pi^3} m_{q,s}^2 \quad a_1^{q,s} = \frac{3\alpha_s^2}{4\pi^3} m_{q,s} \]

\[ b_1^{q,s} = -\frac{4\alpha_s^2}{\pi} \quad c_0^{q,s} = \frac{3\alpha_s}{\pi} m_{q,s} \]

\[ c_1^{q,s} = -\frac{\alpha_s}{\pi} m_{q,s} \quad d_0^{q,s} = -2\alpha_s m_{q,s} \]

(12a)

and \( \rho_1, \rho_2, \ldots \), \( \gamma_E \) and \( E_1(x) \) are already given in [12, 13, 26].

It is noted that our result is different from that given in [25]. This is understood since different subtraction schemes are employed in the two works.

### III. EQUATIONS FOR MIXING MATRIX \( V \)

We define the physical states as \(|f_1\rangle|f_2\rangle \) and \(|f_3\rangle \), whereas the un-physical states as \(|N\rangle = |qq\rangle \), \(|S\rangle = |ss\rangle \) and \(|G\rangle \). The mixing matrix connecting them is:

\[
\begin{pmatrix}
|f_1\rangle \\
|f_2\rangle \\
|f_3\rangle
\end{pmatrix}
= \begin{pmatrix}
V_{11} & V_{12} & V_{13} \\
V_{21} & V_{22} & V_{23} \\
V_{31} & V_{32} & V_{33}
\end{pmatrix}
\begin{pmatrix}
|N\rangle \\
|S\rangle \\
|G\rangle
\end{pmatrix}
\]

(13)

According to the first approximation, it is assumed that \(|N\rangle, |S\rangle \) and \(|G\rangle \) constitute a complete basis[18], but as a matter of fact, when the other resonances \( f_0(1790) \) and \( f_0(1812) \) were observed by the BES collaboration [30, 31], we suggested that the hybrids might join the game and mix with the aforementioned states [21, 32]. But it seems that one can first ignore the hybrids which might be heavier than the other three, and assume that the three physical mesons are only composed of the regular quark and glueball components. We will discuss this issue in the last section. So, the mixing matrix \( V \) transforms the flavor representation into the physical representation, i.e. the mass
representation, so it must be unitary, thus we have:

\[
\begin{align*}
V_{11}^2 + V_{21}^2 + V_{31}^2 &= 1; \\
V_{12}^2 + V_{22}^2 + V_{32}^2 &= 1; \\
V_{13}^2 + V_{23}^2 + V_{33}^2 &= 1,
\end{align*}
\]

(14)

and the conditions are enforced

\[
\begin{align*}
V_{11}V_{12} + V_{21}V_{22} + V_{31}V_{32} &= 0; \\
V_{11}V_{13} + V_{21}V_{23} + V_{31}V_{33} &= 0; \\
V_{11}V_{13} + V_{21}V_{23} + V_{31}V_{33} &= 0.
\end{align*}
\]

(15)

Next, we will build the equations to solve this mixing matrix \( V \) in terms of the QCD sum rules.

In QCD sum rules, the integrand of the dispersion integral includes the imaginary part of the correlation function \( \Pi \) at \( q^2 > 0 \) and then one inserts a complete set of physical states of \( 0^{++} \) hadrons between the currents \[33\]. In the quark-hadron duality the lowest states’ contributions dominate and the contributions of the higher exited states and the continuum should be dropped out by introducing the threshold \( s_0 \) as the lower bound of the integration. Since we are investigating the mixing, we insert all the three lowest states \( |f_1\rangle, |f_2\rangle \) and \( |f_3\rangle \) into Eq.(6) and then we have:

\[
\frac{1}{\pi} \text{Im} \Pi^{ij}(s) = \sum_{n=1,2,3} \langle 0|J_i|f_n\rangle \langle f_n|J_j|0\rangle \delta(s - m_n^2) + \rho^h(s) \theta(s - s_0^h),
\]

(16)

where \( i, j = q, s, g \) stand for the different currents (see Eq.(4)), \( n \) labels the state in the complete set, \( \rho^h(s) \) represents all the higher exited states and the continuum and \( s_0^h \) is the threshold for these higher states.

Putting Eq.(16) back into Eq.(6) and with the quark-hadron duality, we finally have the moments as:

\[
\begin{align*}
\mathcal{R}_0^{qq} &= \langle 0|J_q|f_1\rangle^2 e^{-m_1^2 \tau} + \langle 0|J_q|f_2\rangle^2 e^{-m_2^2 \tau} + \langle 0|J_q|f_3\rangle^2 e^{-m_3^2 \tau} \\
&= \left( V_{11}^2 e^{-m_1^2 \tau} + V_{21}^2 e^{-m_2^2 \tau} + V_{31}^2 e^{-m_3^2 \tau} \right) \langle 0|J_q|N\rangle^2 \\
\mathcal{R}_0^{ss} &= \langle 0|J_s|f_1\rangle^2 e^{-m_1^2 \tau} + \langle 0|J_s|f_2\rangle^2 e^{-m_2^2 \tau} + \langle 0|J_s|f_3\rangle^2 e^{-m_3^2 \tau} \\
&= \left( V_{12}^2 e^{-m_1^2 \tau} + V_{22}^2 e^{-m_2^2 \tau} + V_{32}^2 e^{-m_3^2 \tau} \right) \langle 0|J_s|S\rangle^2 \\
\mathcal{R}_0^{gg} &= \langle 0|J_g|f_1\rangle^2 e^{-m_1^2 \tau} + \langle 0|J_g|f_2\rangle^2 e^{-m_2^2 \tau} + \langle 0|J_g|f_3\rangle^2 e^{-m_3^2 \tau} \\
&= \left( V_{13}^2 e^{-m_1^2 \tau} + V_{23}^2 e^{-m_2^2 \tau} + V_{33}^2 e^{-m_3^2 \tau} \right) \langle 0|J_g|G\rangle^2
\end{align*}
\]

(17a)  

(17b)  

(17c)
\[\mathcal{R}_{0}^{sg} = \langle 0 | J_{q} | f_{1} \rangle \langle f_{1} | J_{g} | 0 \rangle e^{-m_{1}^{2} \tau} + \langle 0 | J_{q} | f_{2} \rangle \langle f_{2} | J_{g} | 0 \rangle e^{-m_{2}^{2} \tau} + \langle 0 | J_{q} | f_{3} \rangle \langle f_{3} | J_{g} | 0 \rangle e^{-m_{3}^{2} \tau} \]
\[= \left( V_{11} V_{13} e^{-m_{1}^{2} \tau} + V_{21} V_{23} e^{-m_{2}^{2} \tau} + V_{31} V_{33} e^{-m_{3}^{2} \tau} \right) \langle 0 | J_{q} | N \rangle \langle G | J_{g} | 0 \rangle \] (17d)

\[\mathcal{R}_{0}^{gq} = \langle 0 | J_{g} | f_{1} \rangle \langle f_{1} | J_{q} | 0 \rangle e^{-m_{1}^{2} \tau} + \langle 0 | J_{g} | f_{2} \rangle \langle f_{2} | J_{q} | 0 \rangle e^{-m_{2}^{2} \tau} + \langle 0 | J_{g} | f_{3} \rangle \langle f_{3} | J_{q} | 0 \rangle e^{-m_{3}^{2} \tau} \]
\[= \left( V_{12} V_{13} e^{-m_{1}^{2} \tau} + V_{22} V_{23} e^{-m_{2}^{2} \tau} + V_{32} V_{33} e^{-m_{3}^{2} \tau} \right) \langle 0 | J_{g} | S \rangle \langle G | J_{q} | 0 \rangle , \] (17e)

where \(m_{1}, m_{2}\) and \(m_{3}\) are the masses of \(| f_{1} \rangle, | f_{2} \rangle\) and \(| f_{3} \rangle\). In Eq. (17), and the relationship of the physical and the un-physical states is involved in the calculations: such the concerned current only couples to the certain un-physical state with the right quantum number and flavor. For example, the current of the glueball cannot couple to the state of the light-quark, vice versa. For the physical state \(| f_{1} \rangle, | f_{2} \rangle\) and \(| f_{3} \rangle\), we have:

\[
\begin{align*}
\langle 0 | J_{q} | f_{i} \rangle &= \langle 0 | J_{q} | V_{i1} | N \rangle = V_{i1} \langle 0 | J_{q} | N \rangle ; \\
\langle 0 | J_{g} | f_{i} \rangle &= \langle 0 | J_{g} | V_{i2} | S \rangle = V_{i2} \langle 0 | J_{g} | S \rangle ; \\
\langle 0 | J_{g} | f_{i} \rangle &= \langle 0 | J_{g} | V_{i3} | G \rangle = V_{i3} \langle 0 | J_{g} | G \rangle ,
\end{align*}
\] (18)

where \(i = 1, 2, 3\) for the three physical states. The un-physical states \(| N \rangle, | S \rangle\) and \(| G \rangle\) directly couple to the certain currents, but do not correspond to any physical values. Thus we need to relate them to the physical states in terms via the moments in Eq. (17). Thus we are able to establish the equations for the ratios among the moments:
FIG. 3: The Gray Area is the value of the lhs. of the first three equations in Eq.(19): (a) $s_{0qq} \in [3.1, 3.7]\text{GeV}^2$; (b) $s_{0ss} \in [4.2, 4.8]\text{GeV}^2$; (c) $s_{0gg} \in [3.5, 4.1]\text{GeV}^2$. The Area between the Dashed line and the Dotted line is the value of the rhs. of the first three equations in Eq.(19): (a) $|V_{11,21}| \in [0.6, 0.8]$; (b) $|V_{12,22}| \in [0.01, 0.5]$; (c) $|V_{13,23}| \in [0.5, 0.8]$. So the overlapping region is the proper parameter area for the mixing matrix $V$.

Totally we have eight equations in Eq.(19) and Eq.(14) for determining the mixing matrix. Supposing the matrix is real, there should be nine independent elements, but we only have eight equations, so that this equation group is not enough to directly determine the whole matrix. However, as we know, the matrix is unitary (as the matrix is real as assumed, it is an orthogonal matrix), thus we may gain an extra equation to fix all elements of the matrix. Namely, on the other hand, if we fix one element of the matrix $V$, in our work, for example, $V_{23}$, then all other elements of the matrix $V$ can be obtained by solving these eight equations. Sequently, let $V_{23}$ run in the region $[-1, 1]$, the unitarity condition may help to eventually fix its value and the best fitting of the $V$ is expected.

IV. NUMERICAL RESULTS

In Eq.(19), it needs the values of the condensates and some other parameters as inputs. From [33], we set them as:

$$
m_q = 0.008\text{GeV}, \quad m_s = 0.14\text{GeV},
$$

$$
m_0 = \sqrt{0.8}\text{GeV}, \quad \langle \bar{q}q \rangle = -0.24^3\text{GeV}^3,
$$

$$
\langle \alpha_s G^2 \rangle = 0.06\text{GeV}^4, \quad \langle g_s O_5 \rangle = m_0^2 \langle \bar{q}q \rangle \text{GeV}^5.
$$

(20)

The other parameters are related to the QCD sum rules: the Borel parameter $\tau$ and the threshold of $s_0^{qq}$, $s_0^{ss}$, $s_0^{gg}$, $s_0^{qg}$ and $s_0^{sq}$ defined in Eq.(19). By the general strategy, one should search for plateaus in the diagrams of the correlation versus the Borel parameter and the threshold $s_0$. Only the parameters fall in a certain region, the plateaus can appear, namely within the plateaus the
FIG. 4: The Gray Area is the value of the lhs. of the last two equations in Eq. (19): (d) $s_{0qg} \in [3.6, 4.2]\text{GeV}^2$; (e) $s_{0sg} \in [4.2, 4.8]\text{GeV}^2$. The Area between the Dashed line and the Dotted line is the value of the rhs. of the last two equations in Eq. (19): (d) $V_{11,21} \in [-0.7, -0.6]$, $V_{31} \in [0.6, 0.72]$, and $V_{32} \in [-0.72, -0.6]$; (e) $V_{21} \in [0.1, 0.3]$, $V_{22} \in [0.45, 0.47]$, $V_{31} \in [0.6, 0.7]$ and $V_{32} \in [-0.69, -0.60]$

results are not sensitive to the choice of Borel parameter and $s_0$, then are trustworthy. In this work, there are six correlation functions in total, so we require all of them to have a common plateau region for the Borel parameter, where all the six moments are relatively independent of the Borel parameter. Obviously this condition is not easy to be satisfied. Once such a region is found, we would be able to conclude that the results based on the QCD sum rules make sense. The dependence of all six moments on the Borel parameter are presented in Fig-3 and Fig-4. And we can see obvious appearance of plateaus.

We first have to check if in the parameter regions Eqs. (19) have real solutions. We find that there are indeed. As we require the matrix $V$ to be real, only a very narrow parameter space is available. The Fig-3 and Fig-4 show the values of the right-hand side (rhs) and the left-hand side (lhs) of the equations in Eq. (19), where the Fig-3 is for the first three equations, and the Fig-4 is for the last two equations. Taking the error tolerance into account, the lines would be widened into bands, in the Fig-3 and Fig-4, the region between the Dashed line and the Dotted line is for the rhs of the Eq. (19) and the Gray one is for lhs. It is clear that, only in the overlapping region, rhs and lhs can be equal, and appearance of the overlapping region implies that a solution of the Eq. (19) may exist.

Searching for such an overlapping region in Fig-3 and Fig-4 one needs to find a proper parameter space. Eventually, we have found a satisfactory region where the best-fitted parameters are: the Borel parameter $\tau \in [1/1.8^2, 1/2.1^2]\text{GeV}^{-2}$ and the five thresholds which must be close to $s_{0qg}^{eg} = 3.4\text{GeV}^2$, are $s_{0g}^{eg} = 4.5\text{GeV}^2$, $s_{0g}^{eg} = 3.8\text{GeV}^2$, $s_{0g}^{eg} = 3.9\text{GeV}^2$ and $s_{0g}^{eg} = 4.5\text{GeV}^2$. At the same time, the allowed value ranges of the matrix elements $V_{ij}$ are also set. From Fig-3 one
FIG. 5: The dependence of the matrix elements $V_{ij}$ on the Borel parameter $\tau$: in (a,b,c) the line is for $V_{11,12,13}$, the dashed line is for $V_{21,22,23}$ and the Dotted line is for $V_{31,32,33}$

notices that only as the matrix elements fall in the following regions:

\[
\begin{align*}
V_{11} &\in [-0.6, -0.8] \quad V_{12} \in [0.01, 0.5] \quad V_{13} \in [0.5, 0.8] \\
V_{21} &\in [-0.6, -0.8] \quad V_{22} \in [0.3, 0.6] \quad V_{23} \in [0.5, 0.8] \\
\end{align*}
\]

(21)

all the requirements are satisfied. It is also noted that due to the unitarity condition (14), $V_{3i}$ ( $i = 1, 2, 3$) depend on other elements $V_{1i}$ and $V_{2i}$, thus their value-ranges would be uniquely determined (there might be a sign difference), once the others are fixed.

Fig-4 corresponds to the last two equations in Eq.(19), and apparently overlapping regions exist when the matrix elements of $V$ reside in the ranges (21). Moreover, for the last two equations in Eq.(19), we set $k = 3$. The reason is that, only when $k = 0$ or 3, the equations Eq.(19) have real solutions. However, $k = 0$ is not proper since when $k = 0$, the lhs does not appear in the plateau.

We solve the equations Eq.(19) together with the three equations in Eq.(14). Our strategy is to set $V_{23}$ as a free parameter and let it run within a range. We find that only when $V_{23} \sim -0.69$, the matrix $V$ is real and orthogonal. The numerical solution is given in Tab-I and the dependence of the matrix elements $V_{ij}$ on the Borel parameter $\tau$ is shown in Fig.5.

Since such terms $V_{ij}^2$ exist in Eq.(19), the solution may not be unique. As a matter of fact, we obtain eight independent groups of solutions. However, enforcing the unitary condition to the matrix $V$, we find that several groups are practically identical (i.e. they deviate from each other by just a common phase) and others must be dropped out because they do not satisfy the orthogonal condition. Finally only one group of solutions remains which is presented in the following table.
Our numerical results show that \( \tau = 1/2^2 \text{GeV}^2 \) is the center of the common plateau, and the mixing matrix \( V \) is

\[
V = \begin{pmatrix}
-0.74^{+0.02}_{-0.02} & 0.09^{+0.22}_{-0.26} & 0.63^{+0.08}_{-0.04} \\
-0.60^{+0.08}_{-0.08} & 0.47^{+0.03}_{-0.05} & -0.69^* \\
0.30^{+0.08}_{-0.19} & 0.88^{+0.02}_{-0.02} & 0.35^{+0.07}_{-0.13}
\end{pmatrix}
\]  

(22)

The numerical analysis indicates that the matrix elements \( V_{11}, V_{22}, V_{32} \) and \( V_{13} \) do not change much when the Borel parameter runs from \( 1/1.8^2 \text{GeV}^2 \) to \( 1/2.15^2 \text{GeV}^2 \), but it is also noted that the errors of \( V_{31}, V_{12} \) and \( V_{33} \) are relatively larger.

The ratio of the contribution of the perturbative part to the “Moments” \( R \) and the lowest state below the threshold is given in Fig-6.

V. CONCLUSION AND DISCUSSION

From the FIG-6, we find that, within the range of \( \tau \in [1/2.15^2, 1/1.8^2] \text{GeV}^2 \), the fraction of the perturbative part in the total contribution is over 60\%. By the general principle of the QCD sum rules, after performing the Borel transformation, the perturbative contribution should dominate, and it is a criterion for judging the reliability of the results. 60\% is not too bad at all.
For a comparison let us write down the mixing matrix given by Close et al. [18]:

$$V_C = \begin{pmatrix} -0.79 & -0.13 & 0.60 \\ -0.62 & 0.37 & -0.69 \\ 0.14 & 0.91 & 0.39 \end{pmatrix}$$  \hspace{1cm} (23)

In this work, we calculate the mixing of the $|N\rangle$, $|S\rangle$ and $|G\rangle$ to result in the physical resonances $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. The mixing matrix Eq.\hspace{1cm} (22) which we obtained in the QCD sum rules is consistent with $V_C$ [18] which was achieved based on phenomenological studies.

This work is based on the conjecture of Close and Kirk [18] that only the mesons heavier than 1GeV are mixtures of $q\bar{q}$, $s\bar{s}$ and glueball $G$, because the lattice results indicate that the mass of $0^{++}$ is around $1.5 \sim 1.7\text{GeV}$. This was also suggested by Narison et al. in their earlier papers [35–38].

With this picture we calculate the mixing off-diagonal correlators which result in the physical resonances $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. Narison and his collaborators computed the off-diagonal correlators in their pioneer work when they considered a mixing between meson and glueball [39, 40].

Today, thanks to the progress of experimental facilities and innovation of the data-analysis, many new resonances have been observed and data are updated. The available new data enable us to re-study the mixing effects, even though the basic techniques have been provided in those pioneer papers. That is the aim of this work. We are indeed very encouraged by the consistency between the numerical results obtained in terms of the QCD sum rules and that gained by the phenomenological research. It implies that the QCD sum rules are really a good approach for studying hadron physics even though certain uncertainties unavoidably exist.
Moreover, as we indicated above, the another two resonances $f_0(1790)$ and $f_0(1812)$ were observed and they also reside in the range of 1 to 2 GeV, therefore we do not have reason to ignore a possibility that all the five physical states $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, $f_0(1790)$ and $f_0(1812)$ are mixtures of $q\bar{q}$, $s\bar{s}$, $q\bar{q}G$ and $s\bar{s}G$ and glueball of $0^{++}$. But it would be much more difficult to calculate the mixing not only because then we have to deal with a five-dimensional matrix, but also the leading order of the perturbative part of the correlation function is two-loop feynman diagrams. But if it is the real physics, we need to carry out the calculations, and it will be the task of our next work.

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