Three-dimensional ray propagation in a toy sunspot

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Abstract. In time-distance helioseismology wave travel times are measured from the cross-correlation between Doppler velocities recorded at any two locations on the solar surface. However, one of the main uncertainties associated with such measurements is how to interpret observations made in regions of strong magnetic field. Isolating the effects of wave anisotropies produced by the magnetic field from those thought to be associated with temperature and flow perturbations has proved to be quite complex and has yet to yield results when extracting acoustic travel times from the cross-correlation function. One possible way to decouple these effects is by using a three-dimensional toy sunspot with a surrounding stratified field-free Model S atmosphere to model the magneto-acoustic ray propagation and produce artificial travel time perturbation maps that directly account for wave speed anisotropies produced by the magnetic field.

1. Introduction

Time-distance helioseismology is a powerful diagnostic tool used in local helioseismology to probe the subsurface structure and dynamics of the solar interior, in particular in and around solar active regions. To date however, results obtained by time-distance helioseismology have not directly accounted for the effects of the magnetic field on the wave speed in travel time perturbation maps or inversions, but have indirectly included magnetic effects only through their influence on the acoustic properties of the medium (e.g. the sound speed). However, recent work in sunspot seismology has pointed to the significant influence of near-surface magnetic fields and possible contamination due to their effects in helioseismic inversions for sound speed beneath sunspots [1]. Prior to this, a number of other very important results have highlighted the complications of interpreting helioseismic observations in regions of strong magnetic fields (e.g. [2, 3, 4, 5, 6]). The key issue is how to isolate the effects of wave anisotropies produced by the magnetic field and efforts to do this both observationally and computationally have proved to be quite difficult mainly because of a general lack of understanding of the process involved. We aim to address some of these issues by performing “forward modeling” of rays in a simulated sunspot atmosphere, with the aim of isolating and analysing the direct effects of the magnetic field on the wave-speed anisotropies observed.

2. The sunspot model

The “toy” sunspot model created for our analysis was chosen in such a way that it models the observed surface magnetic field profile of an actual sunspot as closely as possible, primarily because the main effects of the magnetic field on wave propagation occur in the top ~ 2 Mm
of the sunspot atmosphere, so it is critical that the toy sunspot employed models the field configuration (e.g. field strength, inclination etc.) as accurately as possible for one to be able to confidently draw conclusions from any analysis undertaken.

The sunspot model chosen essentially consists of an untwisted, axisymmetric flux tube in magnetostatic and hydrostatic equilibrium with a surrounding stratified GONG Model S [7] atmosphere (obtained from the L5BL.D.15C.PRES.960126.AARHUS Model S package). The flux tube itself was derived from polynomial fits to the observed scatter plots of the radial ($B_r$) and vertical ($B_z$) surface magnetic field profiles (see Figure 1) obtained from IVM (Imaging Vector Magnetograph) vector magnetograms of AR 9026 - a fairly isolated and symmetrical sunspot near disk centre, ideal for helioseismic analysis - on 5 June 2000.

![Figure 1](image_url)

**Figure 1.** Plots of the radial ($B_r$, left) and vertical ($B_z$, right) IVM surface magnetic field profiles of AR 9026 on 5 June 2000, shown as a function of sunspot radius (r, Mm). Values of $B$ are shown Tesla (T). Solid lines indicate the polynomial fits

The fits of $B_r$ and $B_z$ are then used to derive an analytical form for the potential field:

$$
\psi(c, \lambda)(r, z) = \alpha(c, \lambda)(z)\psi(r, 0) + \beta(c, \lambda)(z)rB_r(r, 0)
$$

where

$$\alpha(c, \lambda)(z) = c + (1 - c)(1 + \frac{z}{\lambda})e^{-\frac{z}{\lambda}},
$$

$$\beta(c, \lambda)(z) = -z(c + (1 - c)e^{-\frac{z}{\lambda}}),$$

and $c$ and $\lambda$ are free parameters which are adjusted to enable us to achieve the correct form for $B_r$ and $B_z$ at the surface. Once a form for the surface potential field is obtained (see Figure 2 for the resulting potential field configuration), and the values of $B_r$ and $B_z$ are known, the next step essentially involves solving the standard equations of magnetohydrostatics (MHS), using the GONG Model S atmosphere and it’s variables as the unperturbed background or external environment.

First, the pressure $P(r, z)$, is calculated using horizontal force balance

$$
P_i(r, z) = P_e(r, z) + \Delta P(r, z)
$$

where $P_i(r, z)$ and $P_e(r, z)$ denote internal and external pressure respectively and the change in pressure is therefore

$$
\Delta P(r, z) = -P_{mag}(r_b, z) + \int_{r_0}^{r_b} F_r dr - \int_{r_0}^{r_b} F_{rb} dr_b,
$$

where $P_{mag}(r_b, z)$ refers to the magnetic pressure along the boundary layer (i.e current sheet situated at $r_b$) between the sunspot and external atmosphere, and $F_r$ and $F_{rb}$ are the radial
components of the Lorentz Force inside and along the boundary of the sunspot atmosphere respectively. Once the pressure inside the sunspot and along the current sheet is known, the density \( \rho(r, z) \), can similarly be calculated using vertical force balance,

\[
\rho_i(r, z) = \rho_e(r, z) + \Delta \rho(r, z) \tag{6}
\]

where the change in density is given by

\[
\Delta \rho(r, z) = \frac{1}{g} \left[ F_z - \frac{\partial \Delta P(r, z)}{\partial z} \right] \tag{7}
\]

This is essentially all that is required to then compute the modified sound speed \( C_s(r, z) \) or thermal profile of the sunspot atmosphere,

\[
C^2_{s_i}(r, z) = C^2_{s_e}(r, z) + \Gamma(z) \frac{P_i(r, z)}{\rho_i(r, z)} - \frac{P_e(r, z)}{\rho_e(r, z)} \tag{8}
\]

while for the sake of simplicity, assuming the ratio of specific heat \( \Gamma^1 \) that appears in the sound speed is the same function of height as it is in the external Model S atmosphere. Figure 3 shows the resulting thermal structure in the top 1 Mm of the toy sunspot.

Figure 2. Potential field configuration for the toy sunspot flux tube as a function of radius \( r \) and depth \( z \) in Mm.
Figure 3. Contour plot of the thermal profile ($C_2^2$) of the toy sunspot. Lighter colours (yellow/green) indicates a cooler profile. Solid black lines indicate magnetic field lines, dashed line indicates the location of the $a = c$ layer.

3. Calculating ray paths for acoustic waves in a model sunspot atmosphere

The ray paths are calculated in Cartesian geometry, in the realm of frequency dependent ray paths described by [8], with the complete form of the three-dimensional dispersion relation:

$$D = \omega^2 \omega_c^2 k_h^2 + (\omega^2 - a^2 k_{\parallel}^2) \times [\omega^4 - (a^2 + c^2)\omega^2 k^2 + a^2 c^2 k^2 k_{\parallel}^2 + c^2 N^2 k_h^2 - (\omega^2 - a^2 k^2) \omega_c^2] = 0,$$

where

$$\omega_c^2 = \frac{c^2}{4H\rho^2} (1 - 2H')$$

is the square of the acoustic-cutoff frequency, with $H\rho(z)$ the density scale height, and $H' = dH\rho/dz$, $a$ is the three-component Alfvén velocity, $k_h$ and $k_{\parallel}$ are the horizontal and parallel components of the wave-vector $k$ and

$$N^2 = \frac{g}{H\rho} - \frac{g^2}{c^2}$$

is the squared Brunt-Väisälä frequency, with $g$ being the gravitational acceleration. The construction of $k$ is completed by specifying the governing equations of the ray paths

$$\frac{dx}{d\tau} = \frac{\partial D}{\partial k},$$

$$\frac{dk}{d\tau} = -\frac{\partial D}{\partial x},$$

$$\frac{dt}{d\tau} = -\frac{\partial D}{\partial \omega},$$

$$\frac{d\omega}{d\tau} = -\frac{\partial D}{\partial \tau}.$$
where $\tau$ parameterizes the progress of a disturbance along the ray path. The phase function $S$ evolves according to

$$\frac{dS}{d\tau} = k \frac{dx}{d\tau} \quad (16)$$

We iteratively find the initial wave-vector ($k_{\text{init}}$) by using an initial “guess” which comes from solving $D = 0$ for the wave-number, assuming the wave-vector is in the directions $\alpha$, $\beta$ - where $\alpha$ and $\beta$ are angles from the vertical and the $x$-$z$ plane respectively of the initial shot. In order to avoid unnecessary complications, we initialize the rays from the bottom of the ray-paths (essentially the lower turning point of the ray, $z_{\text{bot}}$), as this avoids initializing rays from evanescent regions in the very sensitive near-surface region. Furthermore, the value of $\alpha$ is fixed at $\alpha = 90^\circ$, this allows us to adjust the the initial shooting depth $z_{\text{bot}}$ to obtain the desired range of skip distances. The value of $\beta$ also remains constant at $\beta = 0^\circ$, which essentially means that we are exclusively treating horizontal rays (in the $x$-$z$ plane only). Finally, by ensuring that the rays remain on the fast-wave branch at all times, we essentially avoid any mode conversion effects near the $a = c$ layer.

**Table 1.** Annuli/pupil geometry used to bin ray travel time measurements.

| $\Delta$ | Pupil Size (Mm) |
|----------|------------------|
| 1        | 3.7 - 8.7        |
| 2        | 6.2 - 11.2       |
| 3        | 8.7 - 14.5       |
| 4        | 14.5 - 19.4      |
| 5        | 19.4 - 29.3      |
| 6        | 26.0 - 35.1      |
| 7        | 31.8 - 41.7      |
| 8        | 38.4 - 47.5      |
| 9        | 44.2 - 54.1      |
| 10       | 50.8 - 59.9      |
| 11       | 56.6 - 66.7      |

Of course, as [5] and [9] have shown, mode transmission and conversion between fast and slow magneto-acoustic waves have distinct effects on helioseismic waves that should not be ignored. Mode conversion occurs, but we choose not to directly account for it, as a result the complexities of the ray calculations are greatly reduced and also, more importantly, it means that the resulting simulations can directly be compared with actual helioseismic inferences obtained from local helioseismic analysis techniques (such as time-distance, which does not include any mode conversion effects in either observations or forward modeling) as closely as possible.

The computational ray propagation grid extends across the 16 Mm radius of the toy sunspot in regular 0.5 Mm spatial increments in the horizontal $x$-direction and down to a depth of 25 Mm in the vertical $z$-direction, employing a much finer grid spacing in the top 1.5 Mm, followed by 0.5 Mm increments down to a depth of 25 Mm. This extensive computational grid allows us to obtain the desired range of skip distances required to replicate the “centre-to-annulus” skip distance geometry (i.e. averaging rays from a central point/pixel to a surrounding annulus of different sizes to probe varying depths beneath the solar surface) often employed in time-distance helioseismology for the derivation of mean travel time perturbation maps (see [10] for a more comprehensive description of this process). The 11 standard pupil/annuli ($\Delta$) sizes measured for are detailed in Table 1.
4. Results

4.1. Skip distances and timings

The ray propagation grids were computed for the frequency range $\omega = 3 - 5$ mHz, with increments of 1 mHz. Figure 4 shows the output of the resulting simulation for two particular positions on the toy sunspot, $r = 5$ (left) and 10 Mm (right) respectively. The phase travel time perturbation ($\delta \tau_p$) profiles (ray travel time differences between rays in the toy sunspot and non-magnetic Model S atmosphere) are shown as a function of ray skip distance for $\omega = 3$ (black), 4 (red) and 5 mHz (blue). A certain amount of unsmoothness (“wiggles”) is apparent in the $\delta \tau_p$ profiles, mainly at 3 mHz. Unfortunately these effects are primarily due to inherent unsmoothness in the Model S atmosphere in the top 1-2 Mm. Although a certain level of averaging has been applied to the Model S and toy sunspot atmospheres, the issue is also somewhat exacerbated when adopting the rigid form of the acoustic cutoff frequency in Equation 10.

![Figure 4](image-url)

**Figure 4.** Travel time perturbations ($\delta \tau_p$) as a function of skip distance ($x$) for two isolated positions on the sunspot, $r = 5$ Mm (left) and $r = 10$ Mm (right), plotted for three separate frequencies: $\omega = 3$ mHz (black), $\omega = 4$ mHz (red) and $\omega = 5$ mHz (blue).

However, regardless of these small effects, a clear frequency dependence of $\delta \tau_p$ is apparent in the sunspot - particularly for rays with short skip distances (i.e. surface skimmers with very shallow lower turning points) - with the magnitude of $\delta \tau_p$ decreasing as the frequency increases from 3 mHz. Frequency dependence of travel time perturbations in active regions has also been observed by both acoustic holography [6] and time-distance [1], but the magnitude of $\delta \tau_p$ appear to increase with increasing $\omega$ in these observations - a discrepancy for which we cannot account for at the present time with these preliminary results. [9] also observed a similar behaviour when modeling rays in inclined fields and described several related but distinct effects that strong magnetic fields appear to have on seismic waves, with an important “dual effect” that the magnetic field has on individual ray paths (that is, increasing their skip distances while at the same time, speeding them up considerably) being one of these effects.

A comparison between rays propagated inside the toy sunspot with rays propagated in a purely unperturbed Model S atmosphere clearly reveals these effects to the naked eye. All rays shown in Figure 5 are initialized at a depth of $z = 2$ Mm, with the rays in the sunspot model also initialized at a distance of $r = 5$ Mm from the spot centre. While the rays in the Model S atmosphere appear quite symmetrical about their lower turning points (as expected), strong asymmetries (at both turning points) are associated with the same rays when initialized inside the sunspot. Furthermore, the rays at all three frequencies appear to have undergone a longer skip distance, in a slightly shorter amount of time. Figure 5 is purely one example of what happens to a number of rays in the toy sunspot atmosphere, but it is also indicative of the large-scale effects of the magnetic field on ray propagation - effects which are more pronounced as we approach the spot centre and in regions of significantly inclined magnetic fields.
4.2. Binned travel time perturbations

The mean ray $\delta \tau_p$ (for $\omega = 3 - 5$ mHz) were calculated and binned into 11 annuli of increasing sizes (outlined in Table 1). The travel time perturbation profiles of 8 of the 11 bins are shown in Figure 6. Once again, the clear frequency dependence of $\delta \tau_p$ is evident in all bins, with perturbations decreasing with increasing frequency as before. Also, all $\delta \tau_p$ maps exhibit similar perturbation profiles, where the magnitude of $\delta \tau_p$ decreases as we move away from the centre of the sunspot (i.e. decreasing field strength). The sign of $\delta \tau_p$ appears to be exclusively negative for the first three pupil geometries, with a transition occurring to slightly positive $\delta \tau_p$ close to the spot centre and beyond as the bins (i.e. skip distances) become larger.

For certain pupil geometries (and for the smaller ones in particular), there were problems associated with obtaining a sufficient amount of rays near the center of the flux tube, which tends to suggest that strong near-surface magnetic fields severely restrict rays with short skip distances (or very shallow lower turning points). As such, we chose not to interpolate data points across such regions to maintain the integrity of the observations.

4.3. Isolating the thermal component of travel time perturbations

One of the keys to understanding the role played by the magnetic field in local helioseismology, is to be able to isolate it from effects thought to be produced by thermal or flow perturbations. The simplest method which one can undertake to isolate such effects is to essentially switch the magnetic field “off” when calculating the ray paths in the simulations - that is, set $a = 0$, but maintain the modified sound speed profile obtained in the model sunspot (seen in Figure 3). The resulting $\delta \tau_p$ measured would then be purely a result of “thermal variations” along the ray path. A sample of the resulting $\delta \tau_p$ calculations are shown below in Figure 7.

From these preliminary results, one can clearly see that regardless of frequency, the magnitude of the thermal component of the measured $\delta \tau_p$ (dashed lines) is much smaller than the magnetic component (solid lines, colours represent frequencies). The result is particularly obvious in the smaller pupils, however less so when skip distances are becoming large. But the general pattern remains the same in all 11 pupils, with the thermal $\delta \tau_p$ exhibiting a homogeneous behaviour across the sunspot, while the $\delta \tau_p$ measured in the toy sunspot are clearly affected by both frequency and position on the spot. These preliminary results tend to strongly suggest that the
vast majority of wave-anisotropies observed in active regions are a direct result of the magnetic field and not thermal effects, particularly in the near-surface layers.

Figure 6. Mean phase travel time perturbation ($\delta\tau_p$, in minutes) profiles calculated in the toy sunspot atmosphere at three frequencies (same as in Figure 4). Pupil sizes are indicated on the top frame of each profile.
5. Conclusions

Whether it be through direct observations, forward modeling or inversions, in order to be able to confidently interpret helioseismic observations and inferences made in regions of strong magnetic field, the actual physical effects of near-surface magnetic fields on ray propagation must be better understood and taken into account when analyzing or modeling solar active regions.

Our approach here is akin to forward modeling of rays, but in a simulated sunspot atmosphere, to isolate and understand the effects of the wave-anisotropies produced by the magnetic field from those thought to be produced from thermal or flow perturbations. Preliminary results tend to suggest that the magnetic field is the most likely cause of observed travel time perturbations in sunspots. The frequency dependence of these measurements, along with the sign and magnitude of the simulated $\delta \tau_p$ profiles, and the small thermal component extracted from the $\delta \tau_p$, indicates that strong near-surface magnetic fields (as observed in solar active regions) may be seriously altering the magnitude and lateral extent of sound-speed inversions made by time-distance helioseismology. For our next step, we aim produce actual two-dimensional “maps”
of artificial travel time perturbations from these simulations and use standard time-distance inversion procedures to derive the three-dimensional sound-speed perturbation inversions of the toy sunspot and compare the results.

Acknowledgments
This work was supported by the European Helio- and Asteroseismology Network (HELAS), a major international collaboration funded by the European Commission’s Sixth Framework Programme.

6. References
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