Characteristics of students in comparative problem solving

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Abstract. Often teachers provided examples and exercised to students with regard to comparative problems consisting of one quantity. In this study, the researchers gave the problem of comparison with the two quantities mixed. It was necessary to have a good understanding to solve this problem. This study aimed to determine whether students understand the comparison in depth and be able to solve the problem of non-routine comparison. This study used qualitative explorative methods, with researchers conducting in-depth interviews on subjects to explore the thinking process when solving comparative problems. The subject of this study was three students selected by purposive sampling of 120 students. From this research, researchers found there were three subjects with different characteristics, namely: subject 1, he did the first and second questions with methods of elimination and substitution (non-comparison); subject 2, he did the first question with the concept of comparison although the answer was wrong, and did the second question with the method of elimination and substitution (non-comparison); and subject 3, he did both questions with the concept of comparison. In the first question, he did wrong because he was unable to understand the problem, while on the second he did correctly. From the characteristics of the answers, the researchers divided into 3 groups based on thinking process, namely: blind-proportion, partial-proportion, and proportion thinking.

1. Introduction
In order for students to more easily understand mathematics, teachers need to present real situations during the lesson. Real situations are usually presented when learning in the classroom, but only in words. For example, on a broad topic area. The teacher gives an example of the problem of determining the area of the pool, frame, or other objects that are often encountered. Teachers seldom invite students to take advantage of the school environment for learning. For example, students are invited to the school garden, then given the task to determine the area of various types of leaves, tile fragments, or other objects. Teachers are more frequently present a problem with the word (phrase) rather than inviting students into the real world that exists around the school [1]. They argue that by presenting a contextual problem, students can visualize the problem in real situations. Presenting math in everyday life increasingly necessary as technological developments, environmental, social and cultural constantly [2]. Students need to be exposed to real situations to apply their mathematical skills [3]. Mathematical problems that are often encountered in everyday life is a comparison.

Comparisons are always taught at every level of education from elementary to high school. In SD, the concept of comparison is taught about the scale and ratio. Then the material is taught again in junior high school and followed by a comparison of value and turning value. In general, the problem presented in the comparison material is only a comparison of one quantity. For example, "Rafi requires 10 workers
to complete home construction in a month. He wants the house construction to finish faster and add 3 workers. How many days the house was completed? " Or "Aaron needs 1 liter of oil to fry 50 chickens, how many liters does Aaron need to fry 150 chickens?". In reality, we often encounter comparative problems with more than one quantity. Suppose that at the time of building a house, not only the workers and the duration of the course are compared. Let's say the comparison of cement, water, and sand; comparison of workers, length of work, and money spent; iron, wire, and cement; and so forth. In this study, the researchers tried to present the problem of comparison with two objects: coffee and sugar on problem A and meatballs and noodles on problem B. Researchers got the idea after looking at research conducted by Van Dooren [4]. In the study, he discovered the phenomenon that students used comparative methods to solve arithmetic problems [5] and to solve the problem of finding unknown value [4].

Other studies discussing the problem of comparison [6,7,8,9]. Research conducted by Fernandez [9] shows that there is an increase in the use of addition method in elementary students, and there is a decrease in the use of the method of addition to junior high school students. The use of proportional methods in the summing situation increases in elementary and junior high. In addition, the presence or absence of integer ratios strongly influences this behavior but the nature of the quantity has little effect on the use of the comparison method. Comparative problems that allow for multiple representations make it difficult for students to understand and solve comparative problems [7]. While research Van Dooren [6] found that in the early years of elementary school, there was an improvement from the application of the sum method wherever the situation. In the following year, they use proportional methods wherever the situation is. Between these two stages, many students experience a transitional stage in which they simultaneously use the sum method for proportional problems and proportional methods of addition problems, switching between them based on the numbers given in the problem. This means that the type of number used in the problem also determines the selection of the settlement method. Another case is that students use the concept of comparative comparison to solve the problem of value-turning ratio and the students use the concept of reversing the value of the comparison to solve the problem of comparative worth. This shows that students' understanding of the problem of comparison is still not good.

The level of student understanding determines the truth of the settlement. In order for students to understand, teachers need to relate problems with everyday situations. Many teachers teach comparison only to solve the problem, i.e. by multiplying cross to find the unknown value. To understand the problem of comparison, one thing we have to understand is the sum and its relation to multiplication [10,11]. When students are wrong in solving comparative problems, there are two possibilities. First, the student has not understood the concept of complete comparison, so that he is wrong in choosing the method or wrong in the calculation. Second, teachers are unable to communicate the knowledge they have for students to understand.

Research with student subjects with comparative problems has been widely studied [6,9,12]. While research with subject teachers has been investigated by [1,2]. Lemonidis [13] found that prospective students experienced pseudo-proportion when making samples of comparative problems. In his research, there are students who make examples of non-comparison questions that are considered comparisons. Based on these problems, researchers are interested to describe the characteristics of students in solving the problem of comparison with two quantities mixed.

2. Methods and Material
This research was conducted on students of 4th semester academic year 2016/2017 in Yogyakarta. Of the 120 students divided into 3 classes, it is given the problem of comparison problem, as in Table 1.
### Problem

| Rafi every day always makes coffee with the same dosage. For 30 days, Rafi spends 600 grams of coffee and 1 kg of sugar. If today he bought 500 grams of coffee and ¾ kg of sugar, then how many days would Rafi be able to make coffee with existing supplies? |
| Bu Tumini requires 90 grains of meatballs and 3 kg of noodles to make 30 servings of meatballs. How many grains of meatballs and noodles are needed to make 100 servings of meatballs? |

A

B

The matter is a matter of comparison with two objects compared in one question. Researchers assume that there are students who solve the problem by using two linear equations. The reason, on the matter, each of the two things. In question A, there is sugar and coffee; on problem B, there are meatballs and noodles. This is not very familiar to students, however, the problem can be found in everyday life. From the students' work, the researchers grouped into two groups, the group of students who completed correctly each of the questions and groups of students who completed with the wrong answers. The researcher chose the research subject of the group of students who answered wrongly by the purposive method. Researchers consider student completion and communication skills. Finally, selected three students who became the subject of research. Selected students are then interviewed for data. After the data collected, then performed the analysis, then interpreted. The process of data analysis in this study contains the following steps: (1) transcribing the collected interview data, (2) understanding all existing data from various sources (interviews, photos, videos, field notes, student answers), (3) categorize / organize the collected data by making the coding, (5) describe the student's thinking structure, (6) analyze the students' thinking process, (7) analyze the unique things (if any), and (8) Make a conclusion.

### 3. Results and Discussion

#### 3.1. Results

After examining the students' work, the researcher found that 15% of students were able to answer correctly about A and 45% for problem B. From that fact, it showed that students had difficulty in solving the problem of comparison of two objects in one problem. Uniquely, there are 5 students who use the method of elimination and substitution to solve the problem. We know that both methods are found on the subject of a two-variable linear equation system (SPLDV). It is possible that students use SPLDV because they have studied SPLDV before. In SPLDV learning, students will encounter problems with two variables and three conditions (including those asked). For example: "Rafi bought 5 packs of coffee and 1 kg of sugar, he had to pay Rp 25,000.00. At another time, Rafi bought 15 packs of coffee and 1.5 kg of sugar and had to pay Rp 50,000.00. What is the price of 1 pack of coffee and 2 kg of sugar?". From the example of the problem, there are two variables, namely coffee, and sugar. The first condition, 5 packs of coffee and 1 kg of sugar, he must pay Rp 25,000.00, the second condition, 15 packs of coffee and 1.5 kg of sugar and must pay Rp 50,000.00, the third condition, the price of 1 pack of coffee and 2 kg of sugar. Unlike the A question: For 30 days, Rafi spends 600 grams of coffee and 1 kg of sugar. If today he bought 500 grams of coffee and ¾ kg of sugar, then how many days would Rafi be able to make coffee with existing supplies? This problem is not a problem that can be solved by the method of elimination or substitution. This problem is a matter of determining a value that does not yet exist, i.e. the number of days. Facts in the field, the researchers found there are three subjects with different characteristics different, namely: subject 1, he answered the first and second questions with methods of elimination and substitution (non-comparison); Subject 2, he answered the first question with the concept of comparison although the answer was wrong, and answered the second problem with the method of elimination and substitution (non-comparison); And subject 3, he answered both questions
with the concept of comparison. On the first question, he answered wrong because he was unable to understand the problem, while on the second question he answered correctly. From the characteristics of the answer, the researchers divide into 3 groups based on thinking process, namely: blind-proportion, partial-proportion, and proportion thinking.

3.2. Discussion

Comparative problem is a complex problem. This makes students have to think deeply to solve it. Typically, the comparative problems that teachers present in schools only compare one object only. Suppose the mother needs 1 kg of sugar to make 40 cups of tea. How much tea is made if the supply of sugar as much as 750 grams? Of course, such problems will easily be solved. In this study, researchers present the problem of comparison with two objects at once. On problem A, the objects that are compared are coffee and sugar, while the problem B is meatballs and noodles.

Of all students, there are students (subject 1) who answered similarly to the answers written researcher. Actually, the subject has found that the dose of Rafi makes coffee per day is 20 grams of coffee and 33.3 grams of sugar. He also found that 500 grams of coffee would run out for 25 days, and 750 grams of sugar would run out for 22.5 days. However, he did not understand the sentence: Rafi every day always makes coffee with the same dosage. The same dosage means that the coffee and sugar ratio is always the same. Therefore, he wrote, "So, for 500 g coffee can be used for 25 days and for 750 grams sugar can be used for 22.5 days".

![Figure 1. Subject 1 on Problem A](image)

To explore further information, researchers conducted interviews on the subject of answers to the problem given. The interview snippet as presented in Table 2.

**Table 2. Interview Researcher with Subject**

| Researcher | From the answers, you have obtained, how many days Rafi can make coffee? |
|------------|-------------------------------------------------------------------------|
| Subject    | If the coffee will run out for 25 days if the sugar will run out in 22.5 days. |
| Researcher | So, there are two answers? Which one for coffee, the other for sugar? |
| Subject    | Yes, because there are two objects, then there are two answers. |

From the interview transcript in Table 2, it shows that the Subject does not understand the situation in everyday life that if making coffee requires coffee and sugar. However, he considers coffee and sugar to be something separate. Thus, he found two answers, for coffee and sugar. If the job is seen, the Subject...
has actually used the comparison method correctly. He also found the time coffee and sugar will run out. But he was not able to interpret the phrase "Rafi every day always make coffee with the same dose".

In contrast to problem A, on problem B, the subject does not use the comparison method to solve the problem. Instead, he uses the methods of elimination and substitution to solve comparative problems. This is a unique phenomenon. From the Subject's work, he gets the answer \( x = -3.1 \) and \( y = 103 \) (see Figure 2). At the beginning of the work, the subject did the example, namely by using variables \( x \) for meatballs and \( y \) variables for noodles. So, 90 grains of meatballs and 3 kg of noodles to make 30 servings of meatballs were changed to \( 90x + 3y = 30 \), and the meatballs and noodles needed to make 100 servings of meatballs were changed to \( x + y = 100 \). Then, from the equation, two linear equations (SPLDV). After using the method of elimination and substitution found \( x = -3.1 \) and \( y = 103 \). According to him, the interpretation of \( x = -3.1 \) and \( y = 103 \) is to make 100 servings of meatballs require -3.1 meatballs and 103 noodles. From the interpretation of the answers obtained, it appears that the subject just answers the problem by performing imitation procedures. He does not reflect on what he has written. Could meatballs amount to -3.1?

\[
\begin{align*}
\text{Table 3. Interview Researcher with Subject} \\
\text{Researcher} : & \text{This second question, included in what subject?} \\
\text{Subject} : & \text{System of linear equations of two variables, sir.} \\
\text{Researcher} : & \text{Why?} \\
\text{Subject} : & \text{Because there are two variables, meatballs, and noodles.} \\
\text{Researcher} : & \text{Because there are two variables, then use SPLDV?} \\
\text{Subject} : & \text{Yes, as far as I know, sir.} \\
\text{Researcher} : & \text{Why is the first problem not using SPLDV?} \\
\text{Subject} : & \text{Mmm, which is the first question of comparison, sir.} \\
\end{align*}
\]

To explore further information, researchers conducted interviews on the subject of answers to the problem given. The interview snippet as presented in Table 3.
Researcher : The intent of x = -3.1 and y = 103 what is it?
Subject : X the number of meatballs, and the number of noodles needed for 100 servings.
Researcher : The number of meatballs is negative?
Subject : Mmm, yes sir.
Researcher : Can?
Subject : No sir
Researcher : then?
Subject : Confused sir.

From the interview in Table 3, the subject understood the problem of comparison as a two-variable linear equation system problem. It uses the method of elimination and substitution to find the values of x and y. Having obtained the values x = -3.1 and y = 103, the subject does not reflect his work. Is the value of x = -3.1 and y = 103 satisfying the existing equation? Could x be negative? When the researcher asks why x is negative, it cannot answer correctly.

In contrast to subject 1 work, the subject’s work produces the wrong answer to both questions. On the first question, he understood the problem as a matter of comparison. He writes the comparison relationship of coffee, sugar, and the day by writing \( \frac{600}{500} = \frac{1}{3/4} = \frac{30}{x} \). From the equation, it should be split into two forms of the equation, i.e. \( \frac{600}{500} = \frac{30}{x} \) to find the number of days to spend coffee and \( \frac{1}{3/4} = \frac{30}{x} \) to find the number of days to spend sugar. Subject 2 uses cross product at once from three comparisons (see Figure 3).

Figure 3. Subjek 2 on problem A

To explore further information, researchers conducted interviews on the subject of answers to the problem given. Interview fragments as presented in Table 4.

Table 4. Interview Researcher with Subject

| Researcher | Subject | Researcher | Subject | Researcher | Subject | Researcher | Subject |
|------------|---------|------------|---------|------------|---------|------------|---------|
| : The intent of x = -3.1 and y = 103 what is it? | : A comparison of 600 grams of coffee with 500 grams of coffee equals 1 kg of sugar compared to \( \frac{1}{3/4} \) kg of sugar equal to 30 days compared with x days. | : Why use 'equal to' sign? | : Because the same proportion, sir. | : Then, the next step, why are you streaking 600 and 3? | : Let it be simpler sir. | : obtained 8x = 150 from where? | : 200 equals 500 simplified, then 2 multiplied by 4 and 5 multiplied by 30. | : Why is that? |
Subject: This is a comparison, sir, so it is way with cross product.

From the conversation quote in Table 4, it can be observed that the Subject at the beginning of the job he wrote down the formula correctly. However, he did not understand the meaning of the comparison in depth. Thus, he only runs the procedure of completing the comparison by means of cross product. The cross-product he's working on is not right. If it is correct, it certainly will not multiply 2 by 4 and 5 by 30 simultaneously.

In the second question, Subject 2 is working like subject 1 in the same question. It uses a two-variable linear equation system to answer questions. What distinguishes Subject 1’s answer from subject 2 is on the value of x. If subject 1 gets the value of x = -3.1 in subject 2 the value of x is 3.1 ≈ 3. In fact, as in Figure 4, the subject writes 3x + 3y = 300 - (90x + 3y = 30). Supposedly, it gets -87x = 270, and x = -3.1. This means, in addition to the notion of comparisons that have not been good, subject 2 also has an understanding of the operation of algebraic forms that have not been good.

The researcher conducted an interview with Subject 2 concerning the work on the second problem. As for the interview quote as in Table 5.

Table 5. Interview Researcher with Subject

| Researcher | Subject | Researcher | Subject | Researcher | Subject |
|------------|---------|------------|---------|------------|---------|
| Why are you using SPLDV? | Because there are two objects, sir, meatballs and noodles. | 87x = 270 obtained from where? | From 3x + 3y = 300 minus 90x + 3y = 30. | Already correct? | Mmm, it should be -87x, sir. |
| Why are you writing 87x? | So x is not negative, sir. |

Subject 2 is aware that he made a mistake while writing 87x = 270. He wrote 87x = 270 to avoid negative x values. He realizes that the value of x is unlikely to be negative. However, he is like subject 1, using SPLDV to solve a comparison problem with two objects.

In subject 3, he worked on A and B with the concept of comparison. On problem A, he answers wrongly, and on problem B he answers correctly. In problem A, he initially makes two equations: 600x + y = 30 and 500x + 0.75y = a. It exemplifies coffee as x, and sugar as y, as in Figure 5.
Figure 5. Subjek 3 on problem A

To get more information, the researcher conducted an interview with Subject 3 on the job on the first problem. As for the interview quote as in Table 6.

Table 6. Interview Researcher with Subject

| Researcher | : What is the meaning of 600x + y = 30 and 500x + 0.75y = a? |
| Subject | : 600 grams of coffee and 1 kg of sugar can make coffee for 30 days. The second, 500 grams of coffee and ¾ kg of sugar to make coffee during ‘a’ day. |
| Researcher | : Why did you make those two equations? |
| Subject | : Because there are two variables. |
| Researcher | : Then, you use what way to solve it? |
| Subject | : Comparison. |
| Researcher | : Your purpose to write the two equations for what? |
| Subject | : To make it easy. |
| Researcher | : What does \( \frac{600}{500} = \frac{30}{a} \) mean? |
| Subject | : Comparison of coffee and day. |
| Researcher | : So, that’s it? |
| Subject | : Yeah, because when the coffee is gone, can not make more coffee. |

From the conversation, it can be seen that the subject understands if the given problem is a matter of comparison, although at first, he makes two equations. He makes two similarities in order to facilitate workmanship. In contrast to subject 1 which makes two equations to be used as a system of two linear equations. Subject 3 means that if the coffee has run out, it can not make more coffee. He does not pay attention to the sugar and the dosage. In fact, supplies of sugar and coffee are not balanced, 500 grams of coffee and ¾ kg of sugar. Of course one will be exhausted first.

On the question of B, it starts as in A. It tells meatballs as x and noodles as y. After that, he makes two equations 90x + 3y = 30 and ax + by = 100 which is used to facilitate the workmanship. In contrast to A’s work, he makes a comparison relationship \( \frac{100}{30} = \frac{ax}{90x} = \frac{by}{3y} \) as in Figure 6. After that, it separates into \( \frac{100}{30} = \frac{ax}{90x} \) and \( \frac{100}{30} = \frac{by}{3y} \). Then, he got the answer 300 grains of meatballs and 10 kg of noodles to make 100 servings of meatballs.
To get more information, the researcher conducted an interview with Subject 3 on the job on the first problem. As for the interview quote as in Table 7.

**Table 7. Interview Researcher with Subject**

| Researcher | Subject |
|------------|---------|
| **Researcher**: What is the meaning of $90x + 3y = 30$ and $ax + by = 100$? | 90 but meatballs and 3 kg noodles to make 30 servings of meatballs. And how many meatballs and noodles for 100 servings. |
| **Subject**: Why did you make those two equations? | Because there are two variables. |
| **Researcher**: Then, you use what way to solve it? | Comparison. |
| **Researcher**: Your purpose to write the two equations for what? | To make it easy. |
| **Researcher**: What the purpose of $\frac{100}{30} = \frac{ax}{90x} = \frac{by}{3y}$? | Comparison of 100 portions versus 30 servings and ‘a’ grain of meatballs with 90 grains of meatballs, and ‘b’ kg of noodles and 3 kg of noodles. |
| **Researcher**: After that, why do you split up into $\frac{100}{30} = \frac{ax}{90x}$ with $\frac{100}{30} = \frac{by}{3y}$. | |
| **Subject**: Because of the different, one for meatballs and the other for noodles. | |

From the conversation, it can be seen that the subject understands if the given problem is a matter of comparison, although at first he makes two equations. He makes two similarities in order to facilitate workmanship. Unlike the work on the first problem, that he interpreted that if the coffee is up, it can not make more coffee. In the second work, the subject separates the comparisons into two parts, namely the ratio of meatballs to the portion and the ratio of noodles to the serving. He realized that there were two types of things that needed to be determined.

**4. Conclusion**

The problem of comparison is closely related to the problems in everyday life such as those expressed by Silvestre and da Ponte, and Boyer and Levine [7,12]. To understand the problem of comparison is not something easy. Comparative problem is a complex and basic problem for understanding algebra. The complexity of the problem in comparison because in comparison can be attributed to other material. The findings of this study may be one of the reasons why students have difficulty in solving comparative problems [6,8,9]. Student prospective teachers are the next generation of teachers who will be teaching students at the school. If teachers have an understanding of the concept of superficial comparison, of course the students also experience the same thing. Therefore, prospective teachers need to review lessons in elementary, junior and senior high schools.
References

[1] Verschaffel L, Greer B and De Corte E 2000 Making Sense of Word Problems (Netherlands: Swets & Zeitlinger) 62

[2] Rosa M and Orey D C 2015 A trivium curriculum for mathematics based on literacy, matheracy, and technoracy: an ethnomathematics perspective 4 587-598

[3] Rosa M and Orey D C 2010 Revista Latinoamericana de Etnomatemática 3 2

[4] Van Dooren W, De Bock D, Evers M and Verschaffel L 2009 Journal for Research in Mathematics Education 187-211.

[5] Van Dooren W, De Bock D, Hessels A, Janssens D and Verschaffel L 2004 Students' Overreliance on Proportionality: Evidence from Primary School Pupils Solving Arithmetic Word Problems International Group for the Psychology of Mathematics Education

[6] Dooren W V, Bock D D and Verschaffel L 2010 Cognition and Instruction 3 360-381

[7] Silvestre A I and da Ponte J P 2012 Missing value and comparison problems: What pupils know before the teaching of proportion PNA 3 73-83 retrieved from http://hdl.handle.net/10481/19500

[8] Irfan M 2017 Jurnal Penelitian Pendidikan dan Pengajaran Matematika (JP3M) 1 4

[9] Fernández C, Llinares S, Van Dooren W, De Bock D and Verschaffel L 2012 European journal of psychology of education 3 421-438

[10] Son J W 2013 Educational studies in mathematics 1 49-70

[11] Vera F and Lloyd P 2016 Proportional reasoning ability of school leavers aspiring to higher education in South Africa Pythagoras 1 1-10

[12] Boyer T W and Levine S C 2012 Journal of Experimental Child Psychology 111 516-533 doi:10.1016/j.jecp.2011.11.001

[13] Lemonidis C 2008 Prospective teachers’ application of the mathematical concept of proportion in real life situations Research in Mathematics Education p 163-172

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