On the scattering of $D$ and $D^*$ mesons off the $X(3872)$

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Abstract

Both the mass (just below the $D^{*0}\bar{D}^0$ threshold) and the likely quantum numbers ($J^{PC} = 1^{++}$) of the $X(3872)$ suggest that it is either a weakly-bound hadronic “molecule” ($X(3872) \sim 1/\sqrt{2}[D^{*0}\bar{D}^0 + \bar{D}^{*0}D^0]$) or a virtual state of charmed mesons. Assuming the $X(3872)$ is a weakly-bound molecule, the scattering of neutral $D$ and $D^*$ mesons off the $X(3872)$ can be predicted from the $X(3872)$ binding energy. We calculate the phase shifts and cross section for scattering of $D^0$ and $D^{*0}$ mesons and their antiparticles off the $X(3872)$ in an effective field theory for short-range interactions. This provides another example of a three-body process, along with those in nuclear and atomic systems, that displays universal properties. It may be possible to extract the scattering within the final state interactions of $B_c$ decays and/or other LHC events.
I. INTRODUCTION

In recent years many new and possibly exotic charmonium states have been observed at the B-factories at SLAC [1], at KEK [2] in Japan, and at the CESR collider at Cornell [3]. This has revived the field of charmonium spectroscopy [4, 5, 6, 7, 8, 9], which will be an important part of PANDA at the FAIR facility [10]. Because several of the new states exist very close to scattering thresholds, it is useful to interpret them as hadronic molecules, a concept introduced in Refs. [11, 12, 13] well before these most recent experiments. A particularly interesting example is the $X(3872)$, discovered by the Belle collaboration [14] in $B^\pm \to K^{\pm}\pi^+\pi^-J/\psi$ decays and quickly confirmed by CDF [15], D0 [16], and BaBar [17]. The state has likely quantum numbers $J^{PC} = 1^{++}$ and is very close to the $D^{*0}\bar{D}^0$ threshold.

As a consequence, the $X(3872)$ has a resonant S-wave coupling to the $D^{*0}\bar{D}^0$ system. Early examples of discussions of the possible molecular nature of the $X(3872)$ can be found in Refs. [18, 19, 20]. An extensive program (see a status report in Ref. [21]) provides predictions for its decay modes based on the assumption that it is a $D^{*0}\bar{D}^0$ molecule with even C-parity:

\begin{equation}
(D^{*0}\bar{D}^0)_+ \equiv \frac{1}{\sqrt{2}} (D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}) .
\end{equation}

The measured mass and width of the $X(3872)$ differ significantly in the $J/\psi\pi^+\pi^-$ and $D^{*0}\bar{D}^0$ decay channels. This effect can be understood from a line shape analysis which shows that the true mass and width of the $X(3872)$ are measured in the $J/\psi\pi^+\pi^-$ channel because the $D^{*0}\bar{D}^0$ channel is contaminated by a threshold enhancement [22, 23, 24, 25]. Using the latest measurements in the $J/\psi\pi^+\pi^-$ channel [26, 27, 28], the mass of the $X(3872)$ is [29]:

\begin{equation}
m_X = (3871.55 \pm 0.20) \text{ MeV},
\end{equation}

which corresponds to an energy relative to the $D^{*0}\bar{D}^0$ threshold [30] of

\begin{equation}
E_X = (-0.26 \pm 0.41) \text{ MeV}.
\end{equation}

The central value corresponds to a $(D^{*0}\bar{D}^0)_+$ bound state with binding energy $B_X = 0.26$ MeV (but a virtual state cannot be excluded from the current data in the $J/\psi\pi^+\pi^-$ and $D^{*0}\bar{D}^0$ channels [23, 31, 32]). The $X(3872)$ is also very narrow, with a width smaller than 2.3 MeV.

Because the $X(3872)$ is so close to the $D^{*0}\bar{D}^0$ threshold, it has universal low-energy properties that depend only on its binding energy [33]. Close to threshold, the coupling to charged $D$ mesons can be neglected because the $D^{*+}\bar{D}^-$ threshold is about 8 MeV higher in energy. Therefore, the properties of the $X(3872)$ can be described in a universal EFT with contact interactions only. This EFT is a pionless EFT because the pion degrees of freedom are not dynamical near threshold; they are integrated out and all effective interactions are short range. This EFT is widely used in low-energy nuclear physics [34, 35, 36, 37]. For an EFT of the $X(3872)$ including explicit pions, see Refs. [38, 39]. The study of the $X(3872)$ as a $(D^{*0}\bar{D}^0)_+$ molecule in the pionless EFT was initiated by Braaten and Kusunoki [40]. A number of predictions for production amplitudes [42], decays [43], formation [41], and line

\footnote{Note, however, that $J^{PC} = 2^{-+}$ cannot be experimentally excluded at present.}
shapes \[22,23\] within this framework followed. The interactions of the \(X(3872)\) with other hadrons, however, are not known.

In this paper, we extend these studies to three-body processes in the pionless EFT. Based on the assumption that the \(X(3872)\) is an S-wave \((D^0\bar{D}^0)_X\) molecule, we provide model-independent predictions for the scattering of \(D^0\) and \(D^{*0}\) mesons and their antiparticles off the \(X(3872)\) in the pionless EFT. We will refer to these reactions collectively as \(D^{(*)}X\) scattering. This reaction may contribute to the final state interaction in decays of \(B_c\) mesons into \(D\) and \(D^*\) mesons, in rare events in \(B\bar{B}\) production where one of the \(B\)'s decays into an \(X\) and the other one into a \(D\) or \(D^*\) meson, and in prompt events at colliders. In the next section we will provide the EFT for \(D^{(*)}X\) scattering. In section \(\text{III}\) we will present our results and discuss possible scenarios for observing this process.

### II. FORMALISM AND CALCULATION

In this section we set up the EFT for \(D^{(*)}X\) scattering, derive the integral equation for the scattering amplitude, and provide an expression for the total cross section. The formalism can be taken over from the pionless theory in low-energy nuclear physics, but we briefly describe the issues relevant for \(D^{(*)}X\) scattering. For a more detailed discussion and a bibliography of the original work, see the reviews of Refs. \[34,35,36,37\]. For the derivation of the three-body equations, it is convenient to introduce a non-dynamical auxiliary field \(X\) for the \(X(3872)\). The EFT is organized in an expansion around the non-trivial fixed point of the coupling between \(D^0\) and \(D^{0*}\) mesons corresponding to infinite scattering length or, equivalently, the \(X(3872)\) being a threshold bound state. The binding momentum, \(\gamma\), of the \(X(3872)\) is \(\sqrt{2\mu_X B_X}\), with the reduced mass \(\mu_X = m_{D^0} m_{D^{*0}} / (m_{D^0} + m_{D^{*0}})\). So an infinite scattering length corresponds to the \(\gamma \equiv 0\) limit. The EFT expansion is then in powers of \(\gamma / \Lambda_b\) and \(k / \Lambda_b\) where \(k\) is the typical momentum exchange and \(\Lambda_b\) is the breakdown scale of the pionless EFT. We will estimate \(\Lambda_b\) from one-pion exchange in the discussion of the errors below. To leading order in this expansion, the effective Lagrangian for the interaction of the \(X(3872)\) with neutral \(D\) and \(D^*\) mesons can be written as:

\[
\mathcal{L} = \sum_{j=D^0, D^{*0}, \bar{D}^0, \bar{D}^{*0}} \psi_j^\dagger \left(i\partial_t + \frac{\nabla^2}{2m_j}\right) \psi_j + \Delta X^\dagger X
\]

\[
- \frac{g}{\sqrt{2}} \left( X^\dagger (\psi_{D^0} \psi_{D^{*0}} + \psi_{D^{*0}} \psi_{D^0}) + \text{H.c.} \right) + \ldots ,
\]

where H.c. denotes the Hermitian conjugate and the dots indicate higher order terms with more derivatives and/or fields. The terms with more derivatives are suppressed at low energies. As shown in \[40\], there is no Efimov effect \[41\] in this system, so three-body terms will not contribute up to next-to-next-to-leading order (N2LO) in the expansion in \(\gamma / \Lambda_b\). Four- and higher-body forces do not contribute in three-body processes. The only corrections up to N2LO are effective range contributions and their inclusion is in principle straightforward \[45,46,47\]. However, the effective range for the \(X(3872)\) is not known. We will therefore restrict our calculation to leading order only, and estimate the size of higher order corrections. The \(D^0\), \(D^{*0}\), \(\bar{D}^0\), and \(\bar{D}^{*0}\) mesons are treated as distinguishable particles; charge conjugation invariance yields \(m_{D^0} = m_{\bar{D}^0}\) and \(m_{D^{*0}} = m_{\bar{D}^{*0}}\).

The parameters \(\Delta\) and \(g\) in Eq. \(4\) are not independent; only the combination \(g^2 / \Delta\) enters into physical observables. Since the theory is nonrelativistic, all particles propagate
forward in time and the tadpoles vanish. The propagator for the \( D^{(*)} \) mesons is

\[
i S_j(p_0, p) = \frac{i}{p_0 - p^2/(2m_j) + i\epsilon}, \quad j = D^0, D^{*0}, \bar{D}^0, \bar{D}^{*0},
\]

where \( p^2 \equiv p^2 \). The \( X \) propagator is more complicated because of its coupling to two-meson states. The bare \( X \) propagator is constant, \( iD_{X,0}(p_0, p) = i/\Delta \), but the full propagator is dressed by \( D^0 \) and \( D^{*0} \) meson loops to all orders (see Fig. 1). The bare and full \( X \) propagators are indicated by the double dashed and double lines, respectively. The \( D \) mesons are indicated by the solid (\( D^0 \) and \( \bar{D}^0 \)) and dashed (\( \bar{D}^{*0} \) and \( D^{*0} \)) lines. Note that each loop receives contributions of two combinations of \( D^{(*)} \) mesons: \( D^0 \bar{D}^{*0} \) and \( \bar{D}^0 D^{*0} \).

Summing the resulting geometric series leads to the full \( X \) propagator:

\[
iD_X(p_0, p) = iD_{X,0}(p_0, p) [1 - D_{X,0}(p_0, p)\Sigma(p_0, p)]^{-1},
\]

where \( \Sigma(p) \) is the self energy of the \( X \). Using the reduced mass of the \( D^0 \) and \( D^{*0} \) mesons \( \mu_X \) and their total mass \( M_X = m_{D^0} + m_{D^{*0}} \), the self energy can be written

\[
\Sigma(p_0, p) = -2\mu_X g^2 \int \frac{d^3q}{(2\pi)^3} \left[q^2 - 2\mu_X p_0 + \frac{p^2}{4} + \sqrt{1 - \frac{4\mu_X}{M_X} p_0 + \frac{4\mu_X}{M_X} p^2 - i\epsilon - 2\frac{2}{\pi}\Lambda + \mathcal{O}(1/\Lambda)}\right]^{-1},
\]

where the ultraviolet divergence was regulated with a momentum cutoff \( \Lambda \). Substituting this expression into Eq. (6) and dropping terms that vanish as \( \Lambda \to \infty \), we obtain the full \( X \) propagator:

\[
iD_X(p_0, p) = \frac{-i4\pi}{2\mu_X g^2} \left[-\gamma + \sqrt{-2\mu_X p_0 + \frac{\mu_X}{M_X} p^2 - i\epsilon}\right]^{-1},
\]

where we have matched the bound state pole position

\[
\gamma \equiv \frac{1}{a} = \frac{4\pi}{2\mu_X g^2} + \frac{2}{\pi}\Lambda,
\]

to the binding momentum of the \( X(3872) \): \( \gamma = \sqrt{2\mu_X B_X} \) and \( a \) is the \( D^0\bar{D}^{*0} \) scattering length. The combination of bare coupling constants \( g^2/\Delta \) must depend on the cutoff \( \Lambda \) as prescribed by Eq. (9) since \( a \) and \( \gamma \) are physical quantities.

Using the full \( X \) propagator we may calculate the scattering of a \( D^0 \) or \( D^{*0} \) meson (or their antiparticles) off the \( X(3872) \). Because of their different masses, the scattering of a \( D^0 \) versus a \( D^{*0} \) will lead to a different scattering amplitude and cross section even though the interaction strength \( g^2/\Delta \) is the same. The scattering amplitude is the solution of the
FIG. 2: Integral equation for scattering of a particle $S$ (single line) off the $X(3872)$ (double line). The dashed line indicates the particle $\bar{S}$ complementary to $S$ as explained in the text.

integral equation shown in Fig. 2. The $X$ and the scattered meson, denoted by $S$, are represented by a double line and a single line, respectively. If an $S$ particle is scattered, a second complementary particle type, $\bar{S}$ (represented by a dashed line in Fig. 2) appears in the loops. The masses of the $S$ and $\bar{S}$ particles are different. For a given scattered particle $S$, the corresponding particle $\bar{S}$ complementary to $S$ can be read off the flavor wave function of the $X(3872)$ in Eq. (11). For example, if $S = D^0$ then $\bar{S} = D^{*0}$.

We now formulate the scattering problem in the center-of-mass frame of the $SX$ system. With $k$ the relative momentum of $S$ and $X$, the total energy is

$$E = \frac{k^2}{2\mu_{SX}} - B_X,$$

where $\mu_{SX} = m_S M_X / (M_X + m_S)$ is the reduced mass of the $SX$ system and $S = D^0, D^{*0}, \bar{D}^0$, or $\bar{D}^{*0}$. The resulting integral equation for the off-shell $SX$ scattering amplitude is

$$T(k, p) = \frac{2\pi \gamma}{\mu_X} \left( p^2 + k^2 + \frac{2\mu_X}{m_S} p \cdot k - 2\mu_X E \right)^{-1}$$

$$+ \frac{1}{(2\pi)^2} \int d\Omega_q \int_0^\infty dq \frac{q^2 T(k, q)}{-\gamma + \sqrt{-2\mu_X (E - q^2/(2\mu_{SX})) - i\epsilon}}$$

$$\times \left( p^2 + q^2 + \frac{2\mu_X}{m_S} p \cdot q - 2\mu_X E \right)^{-1},$$

(11)

where $k$ and $p$ are the relative momenta in the incoming and outgoing $SX$ system, respectively. Performing a partial wave decomposition of $T(k, p)$,

$$T(k, p) = \sum_l (2l + 1) T_l(k, p) P_l(\cos \theta_{kp}),$$

(12)

where $\theta_{kp}$ is the angle between $k$ and $p$ and $P_l(\cos \theta_{kp})$ is a Legendre polynomial, and projecting onto the $l$-th partial wave, we obtain

$$T_l(k, p) = \frac{2\pi \gamma}{\mu_X} \frac{m_S}{2\mu_X pk} \left( -1 \right)^l Q_l \left( \frac{m_S}{2\mu_X pk} (p^2 + k^2 - 2\mu_X E) \right)$$

$$+ \frac{1}{\pi} \int_0^\infty dq \frac{q^2 T_l(k, q)}{-\gamma + \sqrt{-2\mu_X (E - q^2/(2\mu_{SX})) - i\epsilon}}$$

$$\times \frac{m_S}{2\mu_X pq} \left( -1 \right)^l Q_l \left( \frac{m_S}{2\mu_X pq} (p^2 + q^2 - 2\mu_X E) \right),$$

(13)
where

\[ Q_l(z) = \frac{1}{2} \int_{-1}^{1} dx \frac{P_l(x)}{z - x} \]  

is a Legendre function of the second kind. The integral equation for the S-wave amplitude \( T_0(k, p) \) reduces to

\[
T_0(k, p) = \frac{2\pi\gamma}{\mu_X} \frac{m_S}{4\mu_X pk} \ln \left( \frac{p^2 + k^2 + 2\mu_X pk - 2\mu_X E}{p^2 + k^2 - 2\mu_X pk - 2\mu_X E} \right) \\
+ \frac{1}{\pi} \int_0^\infty dq \frac{q^2 T_0(k, q)}{-\gamma + \sqrt{-2\mu_X (E - q^2/(2\mu_X)) - i\epsilon}} \\
\times \frac{m_S}{4\mu_X pq} \ln \left( \frac{p^2 + q^2 + 2\mu_X pq - 2\mu_X E}{p^2 + q^2 - 2\mu_X pq - 2\mu_X E} \right).
\]  

(15)

Solutions of the integral equations (13) and (15) can be obtained numerically using standard techniques.

The amplitudes \( T_l \) are related to the scattering phase shifts through the relation:

\[
T_l(k, k) = \frac{2\pi}{\mu_{SX}} \frac{1}{k \cot \delta_l - ik}.
\]  

(16)

Using the expression for the differential cross section in terms of the phase shifts:

\[
\frac{d\sigma}{d\Omega} = \left| \sum_l \frac{2l + 1}{k \cot \delta_l - ik} P_l(\cos \theta) \right|^2,
\]  

we obtain the total cross section for SX scattering:

\[
\sigma_{XS}(E) = \sum_l \frac{(2l + 1)\mu_{SX}^2}{\pi |T_l(k, k)|^2}.
\]  

III. RESULTS AND DISCUSSION

In this section we present our results for the SX scattering amplitude and the total cross section at leading order (\( S = D^0, D^{*0}, \bar{D}^0, \) or \( \bar{D}^{*0} \)). With the masses of the \( D^0 \) and \( D^{*0} \) mesons fixed, these quantities depend at this order only on the binding momentum \( \gamma \) of the \( X(3872) \) (or, equivalently, the \( D^{*0} \bar{D}^0 \) scattering length \( a = 1/\gamma \)). Our results are given in units of the scattering length and may be scaled to physical units once \( a \) is determined. At present the error in the experimental value for \( E_X \) in Eq. (3) implies a large error in the scattering length. In particular, we obtain the ranges \( \gamma = (0 - 36) \) MeV and \( a = (5.5 - \infty) \) fm with central values \( \gamma = 22 \) MeV and \( a = 8.8 \) fm.

In Fig. 3, we show our results for the S-wave scattering amplitude \( f_0(k) = 1/(k \cot \delta_0(k) - ik) \) for the scattering of \( D^0 \) and \( D^{*0} \) mesons off the \( X(3872) \) for center-of-mass momenta \( k \) from threshold up to 0.5\( \gamma \). These momenta are still well below the breakup threshold of \( k_B = 1.14\gamma \) for \( D^0X \) scattering and \( k_B = 1.17\gamma \) for \( D^{*0}X \) scattering. The scattering
FIG. 3: S-wave scattering amplitude $f_0(k) = 1/(k \cot \delta_0(k) - ik)$ for scattering of $D^0$ and $D^{*0}$ mesons off the $X(3872)$ in units of the scattering length $a$. The scattering amplitude is identical for particles and antiparticles.

amplitude of a particle is the same as that of its corresponding antiparticle. There is clearly an enhancement of the real part of the scattering amplitude at threshold by a factor of 10 to 17 depending on whether $D^0X$ or $D^{*0}X$ scattering is considered. This could lead to an enhancement of the scattering cross section by two orders of magnitude compared to the already large cross section for $D^0\bar{D}^{*0}$ scattering. The resulting scattering lengths for $D^0X$ and $D^{*0}X$ scattering are

$$a_{D^0X} = -9.7a, \quad \text{and} \quad a_{D^{*0}X} = -16.6a.$$  \tag{19}

The corresponding cross sections as a function of the center-of-mass momentum $k$ are shown in Fig. 4. The difference between the contribution of S-waves ($l = 0$) and the full cross section (including all partial waves up to $l = 6$) is negligible for momenta below $\gamma$. Using the central value of the scattering length $a$ estimated above, we obtain for the scale factor $a^2 = 0.78$ barn. This factor can become infinite if the $X(3872)$ is directly at threshold, while the lower bound from the error in $E_X$ would give a value of 0.3 barn. Even in this case the total cross section at threshold will be of the order 300 barns for $D^0X$ scattering and 1000 barns for $D^{*0}X$ scattering.

Next we will estimate the leading corrections to our results arising from the effective range of the $D^0\bar{D}^{*0}$ system. The effective corrections can in principle be calculated up to next-to-next-to-leading order in our expansion in a straightforward way \cite{45, 46, 47}, but since the effective range is not known for this system, an estimate will suffice.

Naively one expects the breakdown scale of the pionless theory (and the size of the effective range) to be set by the pion mass just as in nucleon-nucleon scattering since the longest range interaction not explicitly included is the one-pion exchange. The situation in the $D^0\bar{D}^{*0}$ system is potentially more interesting \cite{38} because the mass splitting of the $D^0$ and $D^{*0}$ mesons, $\Delta = 142$ MeV, is almost of the same size as the neutral pion mass,
FIG. 4: Total cross section for scattering of $D^0$ and $D^{*0}$ mesons off the $X(3872)$ for S-waves ($l = 0$) and including higher partial waves with $l < 7$, in units of the scattering length $a$. The cross section is the same for the scattering of particles as it is for the scattering of antiparticles.

$m_\pi = 135$ MeV. The range of the one-pion exchange interaction is set by the smaller scale $\mu = \sqrt{\Delta^2 - m_\pi^2} \approx 45$ MeV. The mass difference $\Delta$ appears in the propagator of the exchanged pion because it carries energy $q^0 = \Delta$, leading to the one-pion exchange amplitude for the $D^0\bar{D}^{*0}$ interaction [38]:

$$\frac{g^2}{2 f_\pi^2} \epsilon^* \cdot q \epsilon \cdot q, \quad (20)$$

where $g$ is the $D$-meson axial transition coupling, $f_\pi$ the pion decay constant, $\epsilon$ and $\epsilon^*$ the polarization vectors of the incoming and outgoing $D^{0*}$ mesons, and $q$ the three-momentum of the exchanged pion. However, the work of Ref. [38] shows that, in part because of the small size of the axial coupling, this contribution to NLO effects is very small; contact corrections will dominate, bringing us back to an estimate of the effective range of $r_0 \sim 1/m_\pi \sim 1.5$ fm ($\Lambda_b \sim m_\pi$). Since the leading corrections to our results are of order $kr_0$ and $r_0\gamma$, we expect the errors to remain less than 20% even at momenta close to breakup (using the central value of the binding energy of the $X(3872)$ in Eq. (3)). For larger momenta, the error is dominated by the $kr_0$ correction and will increase to 35% at momenta of order 45 MeV. Compared to the errors from effective range corrections the effects from the charged $D$ meson channel can safely be neglected; they only enter at much higher momenta since the energy difference between the neutral and charged thresholds of about 8 MeV corresponds to typical momenta of order 130 MeV or $6\gamma$.

To observe the three body interactions described here requires identifying an experimental process where, for example, two $D^0$ mesons and one $\bar{D}^{*0}$ are produced very near each other in space and time. One possibility is provided by the decay of the $B_c$ particle. The $B_c$ was discovered through its decays into $J/\psi$ in Run I at CDF [48, 49]. Particle Data Book (2007) averages are: $m_{B_c} = (6.286 \pm 0.005)$ GeV and $\tau_{B_c} = (0.46 \pm 0.07) \times 10^{-12}$ s. Several
analyses have been undertaken (see references in [50], Chapter 4) to determine the most likely mode by which the $B_c$ would decay; the $b$ quark decaying first, the $c$ quark decaying first, the two valence quarks annihilating, etc. For access to the three body neutral $D^{(*)}$ meson interactions, we require that the $B_c$ decay in a mode such as that in Fig. 5 yielding three $c(\bar{c})$ quarks in the final state. The mass total for the three body $D$ meson system will be 5.75 to 5.88 GeV (depending upon whether the third $D$ is a $D^0$ or a $D^{*0}$). Along with the additional meson in a P-wave required to balance the $B_c$ charge and spin, there is not much phase space available. The $q$ in the diagram could be the Cabbibo-favored strange quark or the Cabbibo-suppressed ($|V_{cd}/V_{cs}| \sim 1/4$) down quark. Relative suppression of both decay modes is caused by Pauli interference between the spectator $\bar{c}$ and the $\bar{c}$ from the (second vertex of the) weak decay of the $b$ quark. From Ref. [50]: the estimate for the quark level $B_c \rightarrow c\bar{c}s$ decay is about 1.4% [51]; the detection efficiency for a single $D^0$ is expected to be 11 to 31%; the production cross section at the LHC (not including feeddowns, which may provide an increase of more than a factor of five, but might also be harder to identify) of the $B_c$ is expected at the 30-60 nb level. At LHCb, the yield will be perhaps $10^7$ $B_c$ events per week of running. So the prospect of seeing the three body neutral $D^{(*)}$ meson interactions through $B_c$ decay may well be difficult, but is worth investigating.

Another possibility for observing $XD^{(*)}$ scattering would be in a $B\bar{B}$ production event where one $B$ decays to an $X(3872)$ and its partner $B$ decays to a $D^{(*)}$ [52]. Conditions at asymmetric $B$-factories do not favor the interaction of the decay products [53], but conditions at the LHC may. Heavy flavor production at the LHC has received extensive attention because of the need to correct for Standard Model background processes in the search for the Higgs or new physics. The theoretical and experimental prospects are reviewed in Ref. [54]. Production of the $X(3872)$ under CDF conditions is already dominated by prompt events [55], and heavy flavor production at the LHC is expected to be dominated by $gg$ fusion. The cross section for $b\bar{b}$ at the LHC is about 0.5 mb. Predictions of $b\bar{b}$ correlations indicate that there are events where the opening angle between them may be small (see Fig. 5 on pg. 266 in Ref. [54]); the $gg \rightarrow QQ$ cross section is dominated by rapidity differences less than one [56]. This is important if we expect their decay products to interact. The cross section for $c\bar{c}$ production at the LHC is larger (10 mb), but a minimum of $c\bar{c}c\bar{c}$ would be required to produce an $XD^{(*)}$ scattering event. The ALICE detector will be sensitive to quarkonia-type particles such as the $X(3872)$, as well as $D$ particles, which might be produced promptly, while the LHCb detector is optimized to look at $B$ decay products at larger rapidity, where an $XD^{(*)}$ interaction might be more likely. Another process that might yield smaller opening angles and an enhanced opportunity for final state interactions would be the production of $X$ and $D^{(*)}$ from $b\bar{b}b\bar{b}$, which is expected at the LHC at a cross section of $\sim 500$ fb [57].
Nucleus-nucleus collisions in the LHC will also produce \( X(3872) \)'s along with associated \( D^{(*)} \) mesons.

The effect of final state interactions involving the \( X \) and \( D^{(*)} \), along with the characteristics of the two-body resonance, may be reflected in the distribution in space and energy of the detected particles. Typically, (e.g., Ref. [58]) there will be an enhancement or de-enhancement of the total cross section relative to the situation where three body scattering does not occur in the final state. In particular, the behavior of the \( X(3872) \) produced in isolation should be distinguishable from its behavior when in the presence of \( S = D^0, D^{*0}, D^0, \) or \( D^{*0} \), a situation that may be accessible in the rich environment of the LHC. For example, if to leading order we can assume that the \( X \) and \( D^{(*)} \) are produced isotropically in their opening angle, we might attribute violations of isotropy at small opening angles to the effect of the large \( XD^{(*)} \) scattering in the final state.

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