Stability of DC transport in HTS conductor with local critical current reduction

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Abstract

A common feature of commercially available conductors based on high-temperature superconducting compounds is the fluctuation of critical current along the length. Fortunately, the practice adopted by manufacturers nowadays is to supply the detailed I_c(x) data with the conductor. Compared to knowing just the average of critical current, this should also allow a much better prediction of the conductor performance. Statistical methods are suitable for this purpose in the case when the fluctuations are regular at the low end of critical current distribution. However, a different approach is necessary at the presence of ‘weak spots’ that drop out of any statistics. Because of the strong nonlinearity of the current–voltage curve, such a location could transform into a ‘hot spot’ at transporting direct current (DC), with an abrupt increase of temperature endangering the conductor operation. We present a set of analytical formulas including the prediction of the maximum DC that could be carried sustainably before the thermal runaway appears. It is necessary to know the cooling conditions as well as the properties of the conductor constituents and their architecture. A formula for the voltage appearing on a weak spot, and its dependence on the DC, is also proposed. For this purpose the result of previous theoretical work has been slightly modified after comparing it with numerical iterative computations and finite element modeling. We demonstrate that the derived model allows a powerful analysis of experimental data comprising an estimation of the weak spot parameters i.e. its critical current and the length of the defect zone.

Keywords: HTS conductors, coated conductors, critical current, hot spot, stability

(Some figures may appear in colour only in the online journal)

1. Introduction

Taking advantage of superconductivity as a resistance-less state of some materials requires minimizing any energy impacts that could drive the superconductor into a normal state. Studies of these processes with a particular focus on the coil windings for magnets resulted in the now standard composition of conductor that, along with the functional superconducting material, also contains metallic parts that usually occupy more than 50% of its cross-section [1]. A wealth of research has been dedicated to the development of rules and models for optimal use of such auxiliary materials with regard to the stability of operation, i.e. the tolerance with respect to various disturbances leading to a temperature rise [2–5]. Experimental testing of conductor stability by applying a local heat pulse became a standard methodology for low-temperature superconductor wires and magnets [6–13], and
has been transferred to the study of conductors based on high-temperature superconductor (HTS) materials too [13–25]. In some investigations of HTS tapes and coils, the quench was induced by other means like short overcurrent pulse [26], by applying a magnetic tip [27], or by laser irradiation [28].

Normally in these studies, the position where the temperature starts to rise was known a priori. However, in several experiments on HTS magnets, unexpected locations have been detected, leading to the suspicion that a defect at a particular point of the tape causes local overheating [29, 30]. A possible explanation of this observation is a local reduction in the critical current, $I_c$. It could be the consequence of mechanical damage during coil winding that produces a series of cracks [31–34]. Nevertheless, in the second generation of HTS tapes prepared in form of coated conductors (CCs), a fluctuation of the conductor’s critical current has been commonly detected just after its production [35–43]. This phenomenon, nicknamed the ‘$I_c(\alpha)$ feature’, attracted a lot of interest in the research focused on the development of a resistive fault current limiter (FCL) device [44–48]. Dedicated numerical modeling methods were developed to reproduce all the essential features of the electro-thermal problem in the composite CC tape [40, 49–51]. Also, in the HTS magnet design, the observed volatility in the local critical current attracted growing attention. For example, in the coil stability tests, detailed knowledge of the $I_c$ variability along the conductor length is essential to ensure that the quenching will be initiated by the heater and not in the sections with a low local critical current [52]. Hot spots originating from inhomogeneity have been identified in some cases as the limitation of high-field magnet performance [30, 53–55], and contributed to the conservative design concept, substantially limiting the exploitation of the conductor transport capability [56]. Delivering the information about the $I_c(\alpha)$ distribution together with the supplied tape has become a common practice thanks to the development of scanning techniques [37, 57–59] suitable for long-length reel-to-reel inspection [60–63].

With the help of statistical methods, one can implement the fluctuations of critical current into the design of a superconducting device, provided the conductor properties follow a regular pattern [64–68]. Nevertheless, the focus of this paper is on an ‘individual’ inhomogeneity like the one shown in figure 1. From all the data provided for the purchased 50 m of tape, only the portion around the lowest value of $I_c$ is shown. In the case of selecting for a short-sample test, for any of the 10 cm long sections marked A, B, C, E a critical current of at least 160 A would be determined. However, there is a location in section D, with the critical current reduced by ~15%. The properties of this ‘weak spot’ fall outside of the statistics characterizing the rest of the tape. Fortunately, the supplier’s information allows investigation of the electro-thermal process in this location when transporting an electrical current. This is the main topic discussed here. Our analysis is similar to the stability study of HTS coils [69–71]; however, the heated region is not the whole coil winding but only the weak spot and its surroundings. Longitudinal heat flow along the conductor length is considered relevant in the cooling of defects with the length of a few millimeters [72, 73]. On the other hand, we disregard a lateral non-uniformity, and assume that the local heating happens in a strip stretched across the tape width [74–76].

![Figure 1](image1.png)

**Figure 1.** Detail of $I_c(\alpha)$ data of an industrially produced coated conductor tape.

![Figure 2](image2.png)

**Figure 2.** Evolution of local temperature in a weak spot at transporting the DC $I$.

Our main concern is illustrated in figure 2, showing the possible scenarios of temperature evolution in the weak spot after switching on the DC, $I$. For currents up to a certain limit, marked $I_{tr}$, the temperature would rise for some time, and then attain a stable value. When the supplied current is above $I_{tr}$, a fast rise of temperature occurs called the ‘thermal runaway’ [16, 30, 77–79], and the weak spot converts to the ‘hot spot’ [44, 49, 73, 80, 81]. A reliable prediction of the thermal runaway current, $I_{tr}$, is therefore of utmost importance. The significance of understanding the transformation from a weak spot to a hot spot for devices operating at liquid nitrogen temperatures is underlined by a very slow to non-existent spread of the normal zone in HTS tapes [14, 50, 63, 82–85], which even led to proposals for changing the tape architecture [86].

The paper is organized as follows: in section 2, the formulation of the problem we intend to solve i.e. predicting the thermal runaway current for a conductor containing a weak...
spot with certain characteristics, is presented. In section 3, we first mention a simple iterative numerical procedure allowing us to find the conditions for the thermal runaway that at the same time represent the limits of a stable operation. Then, we show that after some simplifications it is possible to derive the analytical expressions for predicting \( I_{tr} \) from known parameters of the weak spot, the conductor architecture, and the cooling conditions. After checking the agreement with the results of the numerical iterative procedure, we examine in section 4 the analytical formula for \( I_{tr} \) and discuss the impact of various parameters on the conductor stability. Then, in section 5 we present the verification of our previous findings by measurements. Experimental identification of a thermal runaway is rather challenging. Fortunately, we have found a way to widen the range of the analyzed current-voltage data adopting the formula proposed in [70] in a slightly altered form. This empirical modification provided results in excellent agreement with two numerical procedures: in addition to the numerical iterative solution of the heat balance, we have utilized a finite element code to model the process of hot spot creation. As a result, we can now propose an analytical expression capable of predicting the current-voltage curve when approaching \( I_{tr} \) in a quasi-static DC test. By using this procedure of analyzing experimental data we have found that the power-law approximation of the current-voltage curve measured on the sample containing a weak spot is distorted due to the elevation of temperature that is varying for each data point. The finite element computation, in which a realistic composite architecture as well as the heat transfer characteristic including the hysteresis at critical heat flux was implemented, gave identical results.

The theory presented here is not limited strictly to the second generation of HTS conductors; however, several examples will be presented having in mind a CC tape. We will refer to two hypothetical CC tapes (see table 1) based on the rare-earth superconducting compound REBCO (where RE represents a Rare Earth element like Y, Sm, Eu, Gd,... and B replaces Ba, C replaces Cu and O stands for oxygen) with the properties essential for the stability shown in table 1. The calculation of thermal conductivity is described in detail in section 3, formula (16). The specific heat is obtained by summing up the heat capacities of all the tape layers. Properties of materials at 77 K used in these calculations are reported in the appendix. The Tape MAG resembles the standard tape utilized for coil winding [87] and Tape FCL has the characteristics dedicated for the use in a resistive FCL [88]. We disregard the possibility of current transfer to parallel metallic paths [89, 90], which nevertheless could push thermal runaway to higher currents for some conductors or operating conditions.

In our considerations, we deal with only one single weak spot in an otherwise ‘healthy’ tape. In the case of a number of hot spots in a long conductor, this could seem to be an exaggeration. However, due to a highly nonlinear current–voltage curve of HTS material, after the starting of thermal runaway in one place, the rest of the conductor carrying the same current will not reach the runaway temperature. Thus, there is always one particular location relevant to the stability examination. Not surprisingly it is the value of critical current in the weak spot that is essential for predicting \( I_{tr} \). Nonetheless, our analysis reveals the role of other factors, in particular, the defect dimension. In the applications where the tape is exposed to a magnetic field with a varying strength like electromagnets, the original low-field TapeStar data should be modified accordingly. This would probably re-order the expected \( I_{tr} \) ranking, and shift attention to the locations experiencing the maximum magnetic field. However, also in large coils, the thermal runaway will start in a weak spot with the lowest \( I_{tr} \), and the other weak spots (with higher \( I_{tr} \)) will still be in the conditions of a stable temperature.

### Table 1. Architecture and some thermal properties at 77 K of two hypothetical tapes used in the illustrative computations presented throughout this paper.

| Tape         | Tape MAG | Tape FCL |
|--------------|----------|----------|
| Tape width, \( w_t \) (mm) | 4        | 12       |
| Hastelloy (\( \mu m \)) | 50       | 100      |
| REBCO (\( \mu m \)) | 1        | 3        |
| Ag (\( \mu m \))      | 2        | 2        |
| Cu (\( \mu m \))      | 40       | 0        |
| Longitudinal thermal conductivity (W K \(^{-1}\) m \(^{-1}\)) | 242      | 16.8     |
| Specific heat of 1 m of tape (J K \(^{-1}\) m \(^{-1}\)) | 1.62     | 1.91     |

2. Formulation of the problem

The problem to solve is schematically illustrated in figure 3. A superconductor wire carrying DC electrical current, \( I \), exhibits the following property: critical current is \( I_{c0} \) everywhere except the location with the length \( \Delta x_{ws} \) where it is reduced to \( I_{cmin} \). This is the ‘weak spot’ in which an appreciable electric field \( E_{ws} \) would appear, while the rest of the conductor remained resistance-less at \( I_{c0} > I > I_{cmin} \). As a consequence, the local power dissipation \( P_{ws} = E_{ws} \Delta x_{ws} \) will start. We focus here on investigating a small-scale non-uniformity with \( \Delta x_{ws} \) in the millimeter range [37, 42, 73]. Then, the weak spot temperature, \( T_{ws} \), can be considered uniform in the whole volume of the conductor within \( \Delta x_{ws} \). Such simplification, essential for the development of an analytical model, differs from the formulation used in numerical modeling where the composite structure has been implemented [27, 41, 73, 80, 91, 92]. The local rise of temperature triggers a heat flow towards the ambient, while the latter is assumed to keep the baseline temperature \( T_0 \) all the time. The expression controlling the heating process is

\[
C_w \frac{\partial T_{ws}}{\partial t} = P_{ws} - P_{cool}
\]  

(1)

with \( C_w \) being the heat capacity of the weak spot (J K \(^{-1}\)), and \( P_{cool} \) the power of heat removal [W]. At the ambient temperature \( T_0 \) one could expect in the first approximation that

\[
P_{cool} = K_{ws} (T_{ws} - T_0)
\]

(2)
where $K_{ws}$ is the thermal conductance [93] between the weak spot and its ambient [W K$^{-1}$]. The basic arrangement we have in mind here is the HTS wire immersed in liquid nitrogen with $T_0 = 77.3$ K. We also assume that the rest of the conductor, where the critical current is equal to $I_{c0}$, is kept at $T_0$. In order to allow a variation of cooling efficiency with temperature, we introduced the linear approximation

$$K_{ws} = K_0 + K_1 (T_{ws} - T_0)$$  \hspace{1cm} (3)

with $K_0, K_1$ derived from the wire geometry, its thermal properties, and the cooling conditions. The method of estimating these parameters will be described in more detail later.

Taking into account that the REBCO layers in CC tapes carry $>10^{10}$ A m$^{-2}$, at electric fields in the $10^{-4}$–$10^{-2}$ V m$^{-1}$ range there will be no significant flow of current in parallel metallic layers of the CC tape at 77 K, in contrast to the studies exploring current sharing [41, 94, 95]. Current–voltage characteristic of the weak spot in our analysis is therefore controlled by the intrinsic properties of the superconductor. From the possible approximations [96] we have chosen the power-law

$$E_{ws} = E_c \left( \frac{I}{I_{c_{min}}} \right)^n$$  \hspace{1cm} (4)

where $E_c$ is the critical current criterion usually taken as $10^{-4}$ V m$^{-1}$, and the exponent $n \gg 1$. Further, we assume that the critical current at the weak spot changes with temperature as

$$I_{c_{min}}(T) = I_{c_{min}} \frac{T_c - T}{T_c - T_0}$$  \hspace{1cm} (5)

i.e. decreasing from the value $I_{c_{min}}$ at $T_0$ to vanish completely at the temperature $T_c$. Disregarding a possible variation of $n$ with temperature is another simplification we adopted similarly to some other studies [80, 97–99]. Inserting the relations (2–5) into (1) results in the basic equation analyzed further in this paper

$$\frac{\partial T_{ws}}{\partial t} = \frac{1}{C_{ws}} \left[ P_0(I) \left( \frac{\Delta T_c}{\Delta T_c - \Delta T} \right)^n - K_0 \Delta T - K_1 (\Delta T)^2 \right].$$  \hspace{1cm} (6)

Here, $\Delta T_c = T_c - T_0$, the auxiliary variable characterizing the weak spot warming is $\Delta T = T_{ws} - T_0$, and

$$P_0(I) = P_{ws}(T_0, I) = I \Delta T_{ws} E_c \left( \frac{I}{I_{c_{min}}} \right)^n$$  \hspace{1cm} (7)

is the dissipation at the conditions when the weak spot temperature is equal to the coolant temperature, $T_0$. This happens for the instant of switching the current $I$ on.

The weak spot temperature tends to rise because of the first term in square brackets in (6). Warming of the weak spot, resulting in the $\Delta T$ growth, will turn on the cooling power given by the two remaining terms. As we can see, the expression (6) is an equation for variable $\Delta T$ with all the remaining quantities being constants. The main concern is that the weak spot temperature remains stable i.e. both sides of (6) are equal to zero. Then, the dissipation term is to be exactly balanced by the cooling terms:

$$P_0(I) \left( \frac{\Delta T_c}{\Delta T_c - \Delta T} \right)^n = K_0 \Delta T + K_1 (\Delta T)^2.$$  \hspace{1cm} (8)

This is illustrated in figure 4, where the dependence of both sides on $\Delta T$ at three DC currents is plotted as calculated for a weak spot in the tape with Tape MAG architecture. It is immersed in a liquid nitrogen bath with the thermal conductance parameters $K_0 = 0.03664$ W K$^{-1}$ and $K_1 = 0.0002$ W K$^{-2}$ obtained from the considerations explained in the following section.

In analogy to figure 2, there are two possible scenarios: at currents up to $I = 180$ A, which in this case is the value of thermal runaway current, $I_{tr}$, the solution of (8) exists, resulting in a stable temperature elevated by $\Delta T$ over the ambient temperature $T_0$. At higher currents, the increase of dissipation with temperature is too sharp to achieve a solution, and the weak spot temperature starts to grow without a limitation.

What could then happen if the existence of a weak spot had remained undetected because the critical current was established by testing the sample from a ‘healthy’ portion of the tape? Let us imagine the case when $I_{c0} = 181$ A; thus, the weak spot in our example represents a local reduction of critical current by ~17%. Supplying such ‘nominal’ critical current will transform the weak spot into a hot spot exposing the conductor to serious danger of irreparable local damage. Therefore, if one knows that there have been weak spots revealed e.g. in the reel-to-reel procedure [100], it is important to estimate the value of $I_{tr}$, and take it into account further considerations. This is the topic of the next section.

3. Triggering of a thermal runaway

In order to identify the maximum value of current, below which the thermal runaway will not occur, we adopted an iterative numerical procedure [68, 69, 101] solving the nonlinear equation (8): here, testing the evolution with the temperature of both sides for different values of the transported current $I$ allows us to find $I_{tr}$. In the same computation, one also finds the maximum achievable stable temperature of the weak spot, $T_{ws}$, fulfilling the condition

$$P_0(I) \left( \frac{\Delta T_c}{\Delta T_c - \Delta T} \right)^n = K_0 \Delta T_{ws} + K_1 (\Delta T_{ws})^2.$$  \hspace{1cm} (9)

where $\Delta T_{ws} = T_{ws} - T_0$.

The outcome of such a numerical procedure is rather straightforward. But, for understanding the influence of various conductors and cooling parameters, the possibility of finding $I_{tr}$ and $\Delta T_{tr}$ from (9) in the form of an analytical expression would be helpful. Fortunately, this can be achieved taking into consideration that not only must the two sides of (9) be equal, but also the derivatives with respect to $\Delta T$ [69, 70]. In other words, at the intersection of the dissipation curve plotted for $I_{tr}$ and the cooling curve in figure 4, we also observe that the
Combining the expressions (9) and (10) leads to a quadratic equation for the unknown $\Delta T_w$:

$$n \frac{\Delta T_c - \Delta T_w}{\Delta T_i} \left( \frac{\Delta T_c - \Delta T_i}{\Delta T_i} \right)^n = K_0 + 2K_1\Delta T_i. \quad (11)$$

It has the solution

$$\Delta T_w = \frac{\Delta T_i}{n+2} \left[ 1 - \frac{1}{\delta} + \frac{1}{\delta} \sqrt{1 + \frac{2\delta}{(n+1)}} \right] \quad (12)$$

in which the auxiliary variable is

$$\delta = \frac{2\Delta T_c K_1}{(n+1)K_0}. \quad (13)$$

As we can see, the properties of superconducting material with repercussions on $\Delta T_w$ are the $n$-value and the separation of temperature at which, according to (5), the supercurrents vanish, $T_c$, from the ambient temperature $T_0$. Surprisingly, the property defining the weak spot, namely its critical current $I_{cmin}$ that is lower with respect to the rest of the conductors, does not play any role in predicting $\Delta T_w$.

In many practical situations, including CC tape operating in liquid nitrogen, the variation of the thermal conductance with temperature is rather weak, and one will find that $\Delta \ll 1$. Then, with the help of approximating $\sqrt{1+x} \approx 1 + \frac{x}{2}$ where $x = \frac{2\Delta T_c}{n+1} + \frac{\delta^2}{2}$, the simplified expression can be derived:

$$\Delta T_w = \frac{\Delta T_i}{n+1} \left[ 1 + \frac{\Delta T_c K_1}{(n+2)K_0} \right]. \quad (14)$$

In the case of disregarding the dependence of the thermal conductance on temperature, the $K_1$ term would be zero, and the formula (14) will attain the form identical to the prediction...
derived in [70] dealing with a coil made from a uniform conductor. It would simply state that a lower $n$, as well as the operating temperature providing a sufficient reserve with respect to $T_c$, are the main factors for lowering the probability of a thermal runaway. Thus, if the weak spot is caused by an imperfection in the superconducting microstructure resulting in a reduced critical temperature or weaker flux pinning [42, 102], the thermal runaway would appear sooner. However, improvements in the industrial production of CCSs make macroscopic geometrical defects probable causes of weak spots. Therefore, here we undertake the alternative that the superconducting material itself keeps the undisturbed values of $T_c$ and $n$ and the reduction of local critical current from $I_{c0}$ to $I_{cmin}$ is due to constrictions in the path available for the flow of current [32, 62, 103–105].

The second term in square brackets in (14) is an improvement with respect to the previous studies on the stability of the superconducting conductors and magnets where the hot spot was determined by local electromagnetic fields interacting with a uniform conductor. For demonstrating how the weak spot parameters $I_{cmin}$ and $\Delta \tau_{ws}$ influence the thermal runaway and hot spot creation, we must first analyze the role of parameters controlling the heat exchange between the conductor and its environment. The heat removal along the conductor’s length to the portions kept at the ambient temperature, $T_0$, should be analyzed separately from the heat flow in a direction transversal to its surface (still at $T_0$). This is schematically indicated in figure 3 by the symbols $Q_\perp$ and $Q_{\parallel}$, respectively.

Numerous studies of cooling revealed that the efficiency of heat exchange between a heated object and its environment is suggested in the dedicated heat transfer studies [42]. Like the object shape and orientation [111–113] and its surface [114–117]. A rigorous treatment of the subject goes beyond the scope of this work. For practical reasons, we assume that, in the limited range of temperatures relevant for the present analysis, the transversal heat removal from the weak spot surface (in Watt) at the temperature difference $\Delta T = T_{ws} - T_0$ can be expressed as

$$Q_{\perp} = 2 w_T (h_0 + h_1 \Delta T) \Delta T.$$  \hspace{1cm} (15)

Here, for the cooled surface of the weak spot, we took the product $2 w_T \Delta x$; i.e. the tape is wetted on both flat faces ignoring the side edges. Also, the temperature in the weak spot is considered uniform across the tape width, $w_T$, as well as its thickness, $t_T$. Regarding the heat transfer coefficient, $h_0$, one can find rather wide-ranging data in the literature, from 100 to 2000 W m$^{-2}$ K$^{-1}$ [22, 27, 118, 119]. The linear term, $h_1$, is the first approximation of more complex dependences suggested in the dedicated heat transfer studies [112, 120]. In our experiments, for the Tape FCL suspended in liquid nitrogen, we have found the values of $h_0 = 350$ W m$^{-2}$ K$^{-1}$ and $h_1 = 25$ W m$^{-2}$ K$^{-2}$. When the same tape was attached to a plate from machinable aluminum nitride (that is commercially supplied under the abbreviation BNP), a better heat transfer was observed, characterized by $h_0 = 900$ W m$^{-2}$ K$^{-1}$ and $h_1 = 50$ W m$^{-2}$ K$^{-2}$. Throughout the rest of the paper we refer to these two sets of parameters as characterizing the ‘standard’ and the ‘enhanced’ cooling in liquid nitrogen bath, respectively.

The tape property that controls the longitudinal heat transfer, $Q_{\parallel}$, is the tape’s (longitudinal) thermal conductivity. It is calculated from the thermal conductivity $k_l$ and the geometry (thickness of the $i$th layer is $t_i$, $t_T = \sum t_i$) of its constituents:

$$k_i = \frac{\sum k_i \cdot t_i}{\sum t_i}.$$  \hspace{1cm} (16)

The longitudinal heat transfer from the weak spot [in Watt] is governed by the equation

$$Q_{\parallel} = w_T t_T k_i \frac{\partial T}{\partial x}$$  \hspace{1cm} (17)

in which the temperature gradient would be, in reality, rather complex because a transversal escape of heat influences the longitudinal heat flow. Smart simplification is achieved by using the concept of ‘thermal length’ expressed by the formula [4, 70]

$$l_{th} = \sqrt{\frac{t_T k_i}{2 \cdot h_0}}$$  \hspace{1cm} (18)

allowing us to estimate that

$$\frac{\partial T}{\partial x} \approx \frac{\Delta T}{l_{th}},$$  \hspace{1cm} (19)

i.e. the quantity $\Delta T = T_{ws} - T_0$ is again controlling the heat transfer. Using this approximation in (17), and taking into account that from the weak spot location the heat flows in both $+x$ and $-x$ directions, leads to the final formula

$$Q_{\parallel} = 2 w_T t_T k_i \frac{T_{ws} - T_0}{l_{th}}.$$  \hspace{1cm} (20)

Now, we can combine the expressions (15) and (20) to find the total cooling power $P_{cool} = Q_{\perp} + Q_{\parallel}$, and derive the formulas for two parameters controlling the heat removal that were previously introduced in (3):

$$K_0 = 2 w_T \left( \Delta x_{ws} h_0 + t_T \frac{k_i}{l_{th}} \right)$$  \hspace{1cm} (21)

$$K_1 = 2 w_T \Delta x_{ws} h_1$$  \hspace{1cm} (22)

It is worth noticing that the only property of the weak spot entering these expressions is its length, $\Delta x_{ws}$. This is consistent with the comment below the original $T_{tr}$ formula (12) that the local critical current $I_{cmin}$ does not affect the maximum stable temperature.

It is no surprise that the heat removal along the tape length plays a significant role for short defects, because at $\Delta x_{ws} \to 0$ only the second term in the bracket of (21) survives. In contrast, for long defects with the dimension $\Delta x_{ws} \gg \frac{2 \cdot l_{th}}{t_T}$, the transversal heat flow through the surface is essential. This is
Figure 5. Example of thermal conductance coefficients $K_0$ (full lines, axis on the left) and $K_1$ (dashed lines, axis on the right) computed for Tape FCL in two different cooling regimes: standard cooling by liquid nitrogen bath of freely suspended tape (open symbols), and enhanced cooling due to tape touching from one side a ceramic BNP plate with good thermal conductivity (full symbols).

Figure 6. Weak spot temperature elevation with respect to the ambient temperature, triggering thermal runaway, $\Delta T_{tr} = T_{tr} - T_0$, as predicted by the exact and simplified formulas (lines) and from the numerical iterative computation (empty symbols) for weak spots of various lengths in Tape FCL. $T_c = 87$ K, $n = 35$, cooling in standard conditions characterized by $h_0 = 350$ W m$^{-2}$ K$^{-1}$ and $h_1 = 25$ W m$^{-2}$ K$^{-2}$. The noticeable point is the rather narrow range of values on the vertical axis limited by two theoretical predictions for the short and long weak spot.
Figure 7. Dissipation in the weak spots of various lengths but with the same $I_{c\text{min}} = 450 \, \text{A}$ in Tape FCL, predicted by exact and simplified formulas (lines) and from numerical iterative computation (empty symbols). $T_c = 87 \, \text{K}$, $n = 35$, cooling in standard conditions characterized by $h_0 = 350 \, \text{W m}^{-2} \text{K}^{-1}$ and $h_1 = 25 \, \text{W m}^{-2} \text{K}^{-2}$. Negligible difference and fair reproducing of numerical results justifies using simplified expression in further analysis.

illustrated in figure 5 where the dependence of $K_0$ and $K_1$, respectively, on the weak spot length in Tape FCL is compared for two cooling regimes; the standard and the enhanced one.

With the $K_0$ and $K_1$ known, we can now calculate the thermal runaway temperature $T_r$ that at the same time is the maximum temperature at which the weak spot remains stable. Let us find $K_0$ and $K_1$ for various weak spot lengths $\Delta x_{ws}$ in the Tape FCL immersed in liquid nitrogen, and then with the help of (21) and (22) compute the maximum allowed weak spot temperature, either using the formula (12) directly or in its simplified version (14). The result is shown in figure 6 in terms of the difference $\Delta T_r = T_r - T_0$ together with several data points obtained by the iterative numerical procedure. As we can see, the analytical expression (12) agrees perfectly with the results of much more time-consuming numerical computation. The simplified expression (14) overestimates the exact solution, but this difference is negligible for short defects and does not exceed 0.2 mK for long defects. In the same plot the limiting cases of $\Delta x_{ws} \to 0$ and $\Delta x_{ws} \to \infty$, respectively, are also indicated. Notice that the variation of $\Delta T_r$ due to the spot length is the result of including the longitudinal heat removal into our considerations. Still, in the whole range presented, the observed span of $\Delta T_r$ is 5 mK only.

Once we know the value of $\Delta T_r$ it is possible to calculate the maximum dissipation in the weak spot at starting thermal runaway:

$$P_{tr} = K_0 \Delta T_r + K_1 (\Delta T_r)^2$$  \hspace{1cm} (23)

An exact solution requires inserting $\Delta T_r$ here from (12). In pursuit of understanding the influence of various parameters, we have simplified (23) in the following way: because usually $K_1 \Delta T_r \ll K_0$, it is thinkable to neglect the second right-hand term in (23). Then, using (14) results in the approximation

$$P_{tr} = \frac{\Delta T_c n}{n+1} \left[ K_0 + \frac{\Delta T_c K_1}{(n+2)} \right]$$  \hspace{1cm} (24)

which could be further simplified for the same reason to the form

$$P_{tr} = \frac{\Delta T_c K_0}{n+1}.$$  \hspace{1cm} (25)

The validity of these approximations can be judged from figure 7, where the three expressions are plotted together with the results of the numerical iterative computation. All three curves converge to the same value at $\Delta x_{ws} \to 0$. This is not surprising, because we assumed that in the limited interval of temperatures coming into consideration the thermal conductivities can be taken as constant. Notice that the critical current at the weak spot, $I_{c\text{min}}$, has again no influence on the local dissipation power at the thermal runaway, $P_{tr}$.

Now coming to the testing with DC $I$, the power $P_u$ is the result of the temperature rise that started at the same current but at temperature $T_0$. By comparing (23) and (9), we can find that the starting dissipation is
operation and testing of the conductor with known distribution of the weak spots: the tolerable value of current is limited by the weak spot that, to a great extent, is distinguished by the lowest $I_{cmin}$. This justifies the approach of some tape manufacturers that also report, together with the average and standard deviation of $I_c(x)$ data, the information about the minimum $I_c$.

It would be nice to have a more explicit expression showing the relevance of various parameters in (29). Therefore, we have examined what happens when taking the more simplified version of (14) which is $\Delta T_r = \Delta T_c / (n + 1)$. Then, one can find that

$$I_s = \frac{1}{e} \left( K_0 + K_1 \frac{\Delta T_r}{\Delta T_c} \right) \frac{\Delta T_c}{\Delta x_{ws} E_n}$$

(30)

where the base of natural logarithms, $e$, is the approximation of the term $(\frac{n}{n+1})^{n+1}$ for $n \gg 1$. Taking into consideration that $K_1 \Delta T_r \ll K_0$, further allows us to simplify this expression to the form

$$I_s = \frac{K_0 \Delta T_c}{e \Delta x_{ws} E_n}$$

(31)

Surprisingly the prediction of $I_r$ with this simplified formula is nearly identical with the result of using the exact expression (29), as demonstrated in figure 8. Longitudinal heat flow helps small weak spots to avoid the thermal runaway; therefore, at $\Delta x_{ws} \to 0$, the value of $I_r/I_{cmin}$ would grow to infinity. Asymptotic value for a long weak spot, that can be obtained in the limit $\Delta x_{ws} \to \infty$ in (21), is presented as well. The important point here is that the simplified prediction (31) is in excellent agreement with the results of iterative computations. Thus, instead of solving the problem formulated by equation (6) by numerical means, there is a rather simple analytical expression available for estimating the DC at which the thermal runaway would be triggered:

$$I_r = I_{cmin} \left( \frac{K_0 \Delta T_c}{I_{cmin} \Delta x_{ws} E_n} \right)^{1/n}$$

(32)

Thus, the main factor here is the local critical current $I_{cmin}$ which is the proportionality factor in (32). The rest of the parameters affect $I_r$ much less, due to the exponent $1/(n + 1)$. As one can see, reducing the power-law exponent, $n$, and the defect dimension, $\Delta x_{ws}$, would push $I_r$ up. A positive effect could be also achieved by an increase in the numerator. This means a better cooling—characterized by the constant term of thermal conductance, $K_0$—as well as a bigger gap between the operating temperature, and the temperature signaling the complete loss of critical current, $\Delta T_c = T_c - T_0$.

Traditionally, the critical current defined by the $1 \mu V \text{ cm}^{-1}$ criterion is considered a safe level of current that should be used in short sample testing. Imagine now, as an example, that the weak spot with $I_{cmin} = 450$ A is located in the conductor which, outside that location, exhibits the critical current $I_{c0} = 530$ A. Then, $I_{c0}/I_{cmin} = 1.18$ i.e. the weak spot represents a 15% local reduction of the critical current. An attempt to check the ‘undisturbed’ value of the conductor critical current $I_{c0}$ in an experiment on a sample containing such

![Figure 8. Thermal runaway current computed for weak spots of various lengths with the same $I_{cmin} = 450$ A in Tape FCL, predicted by an exact formula (full line) compared with the result of numerical iterative computation (empty symbols) and prediction of a simplified formula (crosses). The weak spot and cooling parameters are the same as in figure 7. Differences cannot be seen with the current resolution.](image-url)
This is illustrated in figure 6, where the weak spot would convert to a hot spot motivated us to discuss how various tape properties influence the thermal runaway. It is considered that $n = 30$ for Tape MAG and $n = 35$ for Tape FCL.

It is reassuring that the predictions presented in figures 6–8, respectively, show that the analytical expressions for the thermal runaway temperature (14) and the power (24), as well as the maximum current transportable before entering the thermal runaway (32), are in excellent agreement with the results obtained by the iterative numerical computations. We take advantage of having at our disposal these analytical expressions in the following section.

4. Factors influencing the runaway current

The practical importance of predicting the current at which the weak spot will most probably be hindered by the thermal runaway, as we can see in figure 8 for nearly all the considered weak spot dimensions it will transform into a hot spot at $I = I_{c0} = 1.18I_{c_{min}}$.

In the illustrations of our theory presented here, we typically consider a short piece of an HTS tape immersed in liquid nitrogen. Now, we briefly discuss if the prediction (32) could also be useful in the assessment of an HTS coil operating at low temperature (4 or 20 K) achieved by a cryocooler or a liquid helium bath. First of all, the $I_{c_{min}}$ values should be taken at respecting such conditions. Let us suppose that the weak spot is caused by an obstacle for the flow of current in the superconductor. Then, we can modify the Tapestar data with the help of the temperature and field dependence obtained on the short sample(s) of the tape, and eventually identify the absolute minimum of $I_{c_{min}}$ in the whole coil winding at the operating conditions. A clear advantage of low temperatures is in the increase of $T_c$ in (5) towards the critical temperature of the utilized REBCO compound. Success in such development may have a positive albeit marginal effect on $I_{tr}$.

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Figure 11 nicely illustrates that adding the stabilizing Cu layer enhances the $I_a/I_{c_{\text{min}}}$ ratio \([121-123]\): it is the additional heat conduction along such a metallic sheath that causes higher thermal conductance resulting in a larger thermal length \(l_h\). Then, for example, a weak spot with a dimension of 2 mm is considered ‘long’ in the Tape FCL but ‘short’ in the Tape MAG. A similar influence is expected also when adding a non-metallic cover layer, provided it has good thermal conductivity \([124, 125]\).

The plots in figure 11 also give an idea of how much larger currents could withstand short weak spots compared to this conservative estimate: up to \(\sim 10\%\) in the weakly stabilized Tape FCL and up to \(\sim 25\%\) in the Tape MAG with standard Cu stabilization. Probably the most significant finding of this analysis is that at a rather significant difference in the tape architecture the ratio $I_a/I_{c_{\text{min}}}$ has been changed by less than 10%.

5. Experimental verification

We have tested the validity of formulas predicting the thermal runaway conditions on a series of samples cut from the 12 mm wide tape from recent production \([88]\) that exhibits an impressive critical current close to 800 A, and its architecture is identical with the Tape FCL. We knew from the Tapestar data that up to \(\sim 10\%\) dropouts of critical current are to be expected randomly distributed among the samples. Taking into consideration the studies showing that local defects can be induced in HTS conductors by mechanical stress \([31-34, 126]\) we attempted to create additional, more prominent, weak spots by bending the tape by hand to a diameter as low as 20 mm. Indeed, the existence of weak spots with a wide range of critical currents was confirmed in DC transport tests when the electric field distribution along the sample was recorded with the help of the multitap device shown in figure 12. There are 19 flexible reed contacts, separated by ~3 mm, touching the sample surface gently and connected by the electrical wiring to the input of a multichannel DC voltmeter. Pressing contacts commonly exhibit larger noise compared to the soldered voltage taps, but for our investigation, it is crucial to minimize a possible modification of local cooling conditions. From the total sample length of 100 mm, the current terminations occupied 20 mm each, leaving the central ~60 mm part for examination. The experiment was performed in two steps, and in each of them the DC was raised under manual control: first, at a continuous current ramp, the location of the weak spot was identified by noticing the pair of contacts showing the first appearance of an electrical signal from \(\sim 0.2 \mu V\) of background noise. The position of the ‘active’ pair of contacts was fully reproducible but individual for each of the samples. Only this signal was registered in the subsequent step when the current ramp was repeated with small increments, always waiting for the voltage stabilization that we considered a sign of reaching the thermal equilibrium. Eventually, at some value of current, the voltage signal continued to rise—with a growing rate—also at a constant current. At this point, we cut the current in order to avoid damaging the sample. In some cases, we noticed in the

![Graph](image-url)

Figure 11. The $I_a/I_{c_{\text{min}}}$ ratio computed for the weak spots of different lengths in two tapes of different architecture. We have considered that $I_{c_{\text{min}}}=150$ A and $n=30$ in the 4 mm wide Tape MAG while $I_{c_{\text{min}}}=450$ A and $n=35$ in the 12 mm wide Tape FCL, respectively. Results obtained for standard and enhanced cooling conditions are compared. Vertical arrows indicate the thermal lengths calculated from \((18)\) for each of the curves.

One could have already noticed in figure 8 that the length of a weak spot, \(\Delta x_{ws}\), plays an important role. It appears in the right-hand term of \((32)\) explicitly in the denominator but it is also implicitly involved in $K_0$ through \((21)\). Then, the relevant parameter is the ratio $K_0/\Delta x_{ws}$, that taking into account \((18)\) can be found in the form

$$K_0/\Delta x_{ws} = 2w_7h_0\left(1 + \frac{2l_h}{\Delta x_{ws}}\right) \tag{33}$$

where the thermal length, \(l_h\), is defined by \((18)\). This quantity controls the character of the $I_a/I_{c_{\text{min}}}$ dependence on $\Delta x_{ws}$, dividing the $\Delta x_{ws}$ range into an interval of short defects smaller than \(l_h\) where the rapid change of $I_a/I_{c_{\text{min}}}$ is observed, and an interval of long defects where $I_a/I_{c_{\text{min}}}$ approaches a roughly constant value. It is illustrated in figure 11, where in the same plot the curves computed for two tapes and two different cooling conditions are presented. For each of the dependences the calculated value of \(l_h\) is indicated by a vertical arrow. From \((33)\) it is easy to comprehend the rapid growth of $I_a/I_{c_{\text{min}}}$ at shortening the weak spot below \(l_h\), and its divergence when approaching $\Delta x_{ws} \rightarrow 0$ observed in the graph. It is also understandable that for the long defects i.e. with $\Delta x_{ws} \gg l_h$ this expression tends towards a constant value $K_0/\Delta x_{ws} \approx 2w_7h_0$, resulting eventually in the saturation of the $I_a/I_{c_{\text{min}}}$ ratio. It also allows us to find the ‘secure’ estimate of thermal runaway current for the weak spot with known critical current $I_{c_{\text{min}}}$, but unknown dimensions, as

$$I_\tau,s = I_{c_{\text{min}}}\left(\frac{2w_7h_0\Delta T_c}{I_{c_{\text{min}}}}\right)^{1/3} \tag{34}$$

where $T_c$ is the critical temperature of the tape and $\Delta T_c = T_c - T_{\text{ambient}}$.
Figure 12. Multitap device used for identifying the weak spot locations in CC tapes.

Figure 13. Theoretical predictions (curves) for the voltage appearing at the starting thermal runaway computed for three different weak spot lengths in Tape FCL. These are compared with the current–voltage curves measured in standard cooling conditions on the weak spots with various $I_{c\text{min}}$ detected in five different samples (symbols—empty squares). One can notice that the measured curves enter an instability at the voltage levels in, roughly, an inverse proportion to the critical currents, in agreement with the expectation deduced from the expression (35).

Subsequent test that a reduction of the critical current or even its complete loss occurred.

Following this procedure, we were able to record for each of the samples tested the voltage just on the weak spot with the lowest $I_{c\text{min}}$. These are then the data we used in comparison with the theoretical predictions. The voltage signal, $U_{tr}$, detected on the weak spot [127] at the moment of starting thermal runaway fulfills the condition $P_{tr} = U_{tr} I_{tr}$. Thus, its value can be determined with the help of (25) and (28) as

$$U_{tr} = \frac{\Delta T_c K_0}{(n + 1) I_{tr}} = \frac{\Delta T_c K_0}{(n + 1) I_{c\text{min}} \left( K_0 \Delta x \Delta x_{c\text{ne}} \right)}$$

(35)
Then, both the parameters characterizing the local weak spot, $\Delta x_{ws}$ and $I_{cmin}$, respectively, have an impact. In figure 13 the set of $U_tr(I_{tr})$ dependences computed using (28) and (35) at a standard cooling regime for three hypothesised values of $\Delta x_{ws} = 0.7, 1.4$ and $2.3 \text{ mm}$, respectively, assuming $n = 25$ and $I_{cmin}$ ranging from 100 to 1000 A, is plotted. In the same plot the current–voltage curves measured on the weak spot locations in five samples at the same cooling conditions i.e. immersed in liquid nitrogen and suspended only by current terminations, are inserted. As one can observe only in one case (the tape having the weak spot with ~600 A of critical current), several points have been caught during the phase of a rapid voltage growth, too. However, in this experiment, the sample was damaged irreparably, and therefore in other trials, we did break the current at an earlier stage of recognizing a rapid signal growth.

It is then rather problematic to define an exact criterion for identifying the thermal runaway in the recorded voltage in a reproducible way. As predicted by (35), the value of $U_tr$ is inversely proportional to $I_{tr}$, and thus a weak spot with low $I_{tr}$ would enter the thermal runaway at a higher voltage than the weak spots with higher critical currents. In principle, because $T_{tr}$ should be the same for all samples with an identical architecture, the thermal runaway could be well defined in the terms of temperature. This approach would work for coils, where placing a thermometer does not represent a significant modification [69]. However, in our case, we need to determine the local weak spot temperature, and such measurement would be prone to several experimental errors. For this reason, we limit our analysis to the inspection of the voltage signals: one should observe a rapid growth of the recorded voltage, and an increasing separation between the data points at approaching the thermal runaway. With this approach, we consider the data in figure 13, recorded on the samples with weak spots characterized by very diverging values of $I_{cmin}$, as confirming our predictions. In particular, one can observe that the weak spots with lower critical currents enter into a thermal runaway at higher voltages.

The qualitative agreement with the theoretical prediction (35) is also rather decent. It is proper to notice that in our model neither the detailed defect geometry is taken into consideration, nor is the possible anisotropy of the superconductor properties [128–130]. The only relevant parameter is the longitudinal dimension at which the reduction of the critical current is observed. Theoretical curves in figure 13 assumed that plausible $\Delta x_{ws}$ values are in the range of a few millimeters. This is consistent with the findings of the dedicated studies [37, 42, 73, 105], and also with the Tapestar data illustrated in figure 1.

Four of the samples from the first testing round, together with two new samples, then underwent the same procedure of the current–voltage dependence registration, but at enhanced cooling conditions i.e. with one surface attached to the BNP ceramic plate. The outcome of this round is shown in figure 14 together with the result of the computation where the only modification has been the insertion of a higher $K_0$ corresponding to the better cooling conditions. We have also succeeded now in catching some data at the voltage take-off, recognized by the sudden increase of the distance between data points, for four samples. As one can see, because of better cooling
the weak spots now sustain higher voltages before entering the thermal runaway. This is in agreement with a theoretical prediction obtained in the same way as in the previous case of standard cooling.

Until now we have focused on only one point in the current–voltage curve, namely the one marking the thermal runaway, in order to compare with the formulas enabling us to find the limit of a safe operation. As we have found, exact identification of the thermal runaway in experimental voltage data is rather vague. Fortunately, the impact of enhanced cooling is more profound than just an increase in $I_u$ expected from (32): one can notice that the shape of these curves has changed too. We can again utilize an iterative numerical procedure, and for any $I < I_u$ find the intersection of two curves in figure 4 i.e. the solution $\Delta T_f$ at which the two sides of equation (8) are balanced. The index ‘f’ has been introduced because, after setting the current $I$, it requires some time until the temperature will reach its final, equilibrium value. For this process an elegant theory has been developed by Rakhmanov et al [70] by the means of expanding the physical quantities around the thermal runaway point. In this way, they have derived the analytical expression predicting $T_f$ for any DC. However, we have found that to reach good agreement with the results of the iterative procedure in a wider range of temperatures, the formula has to be slightly modified to the form

$$T_f(I) = T_0 + \Delta T_f \sqrt{\frac{|I - I_u|}{(n + 1)I_u}}$$

The impact of this modification is illustrated in figure 15 for the weak spot with properties we have identified in one of the tested samples. The original formula predicts, in contrast to the iterative numerical procedure, a sharper reduction of $T_f$ at lowering the current below $I_u$. We have also performed a finite element modeling by Comsol Multiphysics [27] to assess the validity of both predictions. Afterward, we decided to drop out the factor 2 that is in the numerator of the original formula otherwise identical to (36). We have no other justification for this step except for a better agreement with the results of numerical computations performed by the two independent methods.

With the help of (36) it is easy to determine the voltage appearing at the hot spot warmed up to $T_f$:

$$U(I, T_f) = E_c \Delta x_{ws} \left( \frac{I}{I_{c_{\text{min}}}} \frac{T_c - T_0}{T_c - T_f} \right)^n$$

Figure 15. Evolution of the hot spot temperature due to increasing DC computed by the iterative numerical procedure (empty symbols), compared with the prediction obtained by adopting the formula from [70] (full squares). After its modification to the expression (36), the result plotted by the empty squares is obtained. Outcome of finite element modeling is presented by the short-dashed curve. All the models coincide at the current reaching $I_u$ (dashed vertical line).
Figure 16. Voltage on the weak spot computed, for slowly increasing DC, by the iterative numerical procedure (empty symbols) compared to the prediction obtained by adopting the formula from [70] (full squares), and its modified expression (36) (empty squares). The result of finite element modeling is presented by the short-dashed curve. Predictions from all these models substantially differ from the expectation of the intrinsic current–voltage curve disregarding the weak spot heating (dot-dashed curve).

Table 2. Thermal conductivity and specific heat of materials used in this paper.

| Material   | Thermal conductivity at 77 K (W K$^{-1}$ m$^{-1}$) | Specific heat at 77 K (J K$^{-1}$ cm$^{-3}$) |
|------------|---------------------------------------------------|---------------------------------------------|
| Hastelloy  | 7.75                                              | 1.53                                        |
| REBCO      | 8.8                                               | 1                                           |
| Ag         | 479                                               | 1.72                                        |
| Cu         | 517                                               | 1.75                                        |

In figure 16 one can see that the current–voltage curve computed using formula (37) coincides with the results of the two numerical methods, and is better than the prediction obtained using the original expression for $T_f$ from [70]. Parameters of the weak spot used in the computation were $\Delta x_{ws} = 2$ mm, $I_{c_{\min}} = 157.4$ A and $n = 25$. Neglecting the warming of the weak spot, the current–voltage curve plotted by the dot-dashed line is to be expected. The combination of formulas (36) and (37) provides a powerful analytical tool for interpreting the experimental data in a wider range of currents. In figure 17 the current–voltage curves registered for the sample with the highest value of $I_{tr}$ in two different cooling regimes are plotted, together with the predictions obtained by the analytical formula (37) and by the iterative computations assuming $\Delta x_{ws} = 2$ mm, $I_{c_{\min}} = 685$ A and $n = 25$. As expected, better cooling allows a higher $I_{tr}$ to be reached. Both the iterative numerical procedure and the analytical model provide predictions in the data range of interest that are in good agreement with the experiment. One can see that it is not only a higher $I_{tr}$ that the better cooling allows; plotting the same data in log–log graph presented in figure 18 also reveals that in both experiments the dependence does not follow the power law (4) with intrinsic $n = 25$ indicated by the dashed line. The simplistic approach of fitting the measured curves at 10 $\mu$V—this corresponds to the 1 $\mu$V cm$^{-1}$ criterion in 10 cm long sample—would result in evaluating $n = 36$ in the experiment with enhanced cooling, and $n = 52$ in the standard experiment. These values substantially exceed the true property of the superconducting material used in theoretical modeling. Similar behavior has also been observed in low temperature wires [131] and HTS coils [132].

A reasonable coherence between the theory and the experiment, including the vertical shape of $U(I)$ dependence [133] at $I_{tr}$, was found for all the experiments presented in figure 13 when the measured samples were suspended in liquid nitrogen.
Figure 17. Current–voltage curves measured on one of the tested samples in two different cooling conditions compared with the theoretical prediction (37) assuming the hot spot length 2 mm and its critical current of 685 A and $n = 25$ (dotted lines). The results of numerical iterative computation using the same parameters (full lines) exhibit good agreement as well. Squares plot the data registered in the standard cooling conditions, while triangles show the results obtained at enhanced cooling. One can see that the theory nicely predicts the observed impact of cooling.

Figure 18. The same data as in figure 17 plotted in log-log scales. The dashed line represents the current–voltage curve of the superconductor at the coolant temperature $T_0$ considered in computations.
Figure 19. Current–voltage curves measured at two different cooling conditions on the sample with the lowest $I_{\text{cmin}}$, compared with the theoretical expression (37) assuming $\Delta x_{ws} = 1$ mm, $I_{\text{cmin}} = 154$ A and $n = 25$ (dotted lines). The results of numerical iterative computation using the same parameters are shown by full lines. One can observe a good quality of the prediction for the measurement in the standard cooling conditions (squares), but there is some discrepancy in the prediction for the experiment with enhanced cooling (triangles).

Nevertheless, in some trials with enhanced cooling, when the ceramic plate touched the measured sample from the bottom, a behavior deviating from theoretical predictions was observed. As an example, the experimental data for the defect with the lowest critical current are plotted in figure 19, together with the theoretical results obtained in the same way as in the previous case. The predictions for the thermal runaway in standard conditions, as well as the whole shape of the current–voltage curve, are plausible when assuming the weak spot parameters $\Delta x_{ws} = 1$ mm, $I_{\text{cmin}} = 154$ A and $n = 25$. On the other hand, the value of $I_u$ determined in the experiment with the enhanced cooling conditions the predictions of both theoretical models is exceeded by $\sim 3$ A. An obvious candidate for explaining this discrepancy—save an experimental error—is an increase in the cooling mechanism stronger than the one observed in other cases, when the ceramic plate is put in contact with the tested sample. We intend to study this phenomenon systematically in the future; here it simply serves more as an illustration of how the analytical model presented in this paper could help in the investigation of a hot spot appearance in HTS tapes.

6. Conclusions

The local reduction of critical currents in some spots of an HTS conductor is a phenomenon with a significant impact on the design and operation of devices like high-field magnets or FCLs for the DC grid. In contrast to low-temperature superconducting wires, the spread of the normal zone is slow, and the conversion of a weak spot into a hot spot is local and rather quick. In this paper, we present a complete theory that allows us to predict the limits of a safe operation of an HTS conductor with a local weak spot. Besides the weak spot length and its critical current, the conductor architecture, in particular the amount of stabilizing metallic layers, and the conditions of cooling play an important role. It is obvious from formulas (14) and (25) that the value of local critical current influences neither the maximum stable temperature, $T_{tr}$, nor the maximum of local dissipation, $P_{tr}$, before entering into the thermal runaway. On the other hand, the weak spot dimension could have an impact: we have found that a very small weak spot will never convert to a hot spot because of the heat removal along the conductor length. In this regard, the main role of the stabilizing layers is in thermal conduction, which could be improved also by adding non-metallic layers.

The set of formulas, allowing to identify the conditions of the thermal runaway, is finalized with the prediction (32) of the maximum DC that the weak spot allows to tolerate. Surprisingly, for the typical architectures of CC tapes designed for the magnet coil and for the FCL applications, this prediction differs less than what we have expected. From figure 11, where the results of our theory are presented for two realistic
cases of cooling in a liquid nitrogen bath, one can see that the thermal runaway could appear at the DC exceeding the critical current in the weak spot, \( I_{\text{cmin}} \), by roughly 10%–25%. The preliminary analysis which we have carried out indicates that this conclusion is rather robust and remains true for the HTS coils operating at low temperatures too. The most difficult task, however, would be a valid estimation of \( I_{\text{cmin}} \) at the lack of reel-to-reel conductor characterization in the operating conditions.

The validity of theoretical expressions has been verified in experiments on a set of samples with weak spots characterized by a wide range of critical currents. In the search for a tool that allows us to analyze not only the instant when the thermal runaway starts but the whole process of quasistatic increase of temperature at the current transport, we have modified the result derived in [70], and obtained the working formula (37). It predicts the shape of the current–voltage curve in realistic experimental conditions i.e. with the temperature rise caused by the local dissipation. A detailed analysis of the experimental data revealed several important features. The impact of hypothesized values of weak spot parameters \( \Delta \), \( I_{\text{cmin}} \) and \( n \) can now be verified in a much wider range of currents. As a result, the sensitive fitting of theory to experimental data allows a rather convincing deduction of the possible weak spot properties, also in case of a missing \( I_c(x) \) information. Likewise, our formula allows a quantitative prediction of the change of slope in the measured current–voltage dependence because of the sample heating due to the DC transport. Rather disturbing is the finding that the power exponent derived through the processing of experimental data without the examination of a possible sample heating—which is probably a quite common case—could be wrong by the factor as large as 2. A detailed experimental work combining the current–voltage measurement with the determination of temperature in the locations with the lowest critical current would be very helpful for a better understanding of this phenomenon.

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Appendix

Properties of materials at cryogenic temperatures have been collected in several comprehensive sources [134, 135]. However, particularly for metals, these depend strongly on the processing and on the microstructure. Therefore, presenting the actual values used in computations [136, 137] is necessary for a cross-check verification. For this purpose, we report which data were used in the computations presented in this paper in the table 2.

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