MONEY IN AN ESTIMATED BUSINESS CYCLE MODEL OF THE EURO AREA*

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This article examines the role of money in a small-scale dynamic general equilibrium model of the euro zone estimated by maximum likelihood. The model allows for both intertemporal and intratemporal non-separability in preferences. We find, first, that real balances do not affect the marginal utility of consumption. Second, money demand shocks mainly help to forecast real balances while real shocks explain the bulk of price, output and interest rates fluctuations. Third, the calculation of the natural rate of interest reveals that the evolution of the interest rate is mostly accounted for by the real sources of fluctuations.

The European Central Bank conducts its policy for the euro area giving monetary aggregates a relevant weight as indicators of future inflation; for example, the ECB uses the deviations of M3 growth from a reference value as an indicator of inflationary pressures. The merits of the so-called ‘two pillar’ strategy has generated a lively debate among scholars and practitioners. Some papers have used the $P^*$ model to assess the ability of monetary aggregates to help forecast inflation (Gerlach and Svensson, 2000). Nevertheless, much of the previous research has not been addressed in a fully articulated equilibrium business cycle model. This article assesses the historical role played by money in shaping the joint evolution of output, interest rates and prices in euroland.

We aim at contributing to bridging the gap between evidence and theory in this field and, to that end, we present a maximum likelihood estimation of a small-scale dynamic general equilibrium model. An essential feature of this approach is that we make use of all the cross-equation restrictions implied by the theoretical model, which affect the dynamics of the endogenous variables and the structural sources of fluctuations.

Our article is akin to other empirical work that has studied the macroeconomic performance in the euro area. Smets and Wouters (2003) estimate a dynamic general equilibrium model with nominal and real frictions but in their setup there is no role for money. In the work by Coenen et al. (2001) the role of money is limited to its information content, through the money demand equation, when output is subject to measurement error. Unlike these papers, though, our model makes explicit several channels through which real balances might affect output and inflation.

The model can be described in terms of three equations that stem from the optimal agent’s decisions: the intertemporal allocation of consumption, the New Keynesian Phillips curve and a money demand equation. Following Ireland (2004), a special characteristic of that set of equations is that the marginal utility of consumption depends upon real balances so making both output and inflation depend on the level of real balances. The model is augmented with an interest rate rule,

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which does not admit a truly structural interpretation (since the ECB did not exist before 1999) but which is a necessary device to close the model. Alternative monetary channels exist that may be empirically relevant but we do not consider them here. Nelson (2002) discusses how real money growth may be a valuable statistic when both the aggregate demand and the money demand relationships depend on long term yields. Christiano and Rostagno (2001) consider that monetary monitoring may help to avoid the indeterminacy of the equilibrium associated with the nominal interest rate rule.

The optimal choices made by households and firms result in forward-looking demand and supply schedules, which makes it difficult to account for the inertia observed in output and prices. Authors like Fuhrer and Moore (1995) advocate mechanisms that add lags both in the demand and supply relationships to match the persistence observed in the data. Other papers, like Rotemberg and Woodford (1997), emphasise the importance of allowing for higher order autocorrelations in the unobservable variables as well as for cross correlations among them. As a compromise, in this article we allow for habit formation in consumer preferences (Fuhrer, 2000) as well as to autocorrelated shocks to capture the dynamics of the variables in the economy.

This article extends previous results in the literature, estimating a model that incorporates non-separability in preferences both at the intertemporal margin (i.e., habits) as well as at the intratemporal one (i.e., money in the marginal utility of consumption). Our main result is that the business cycle influence of money balances seems to have been very limited during the sample period. Preferences are separable between consumption and real balances, and money demand shocks explain very little of the variability of output, inflation and nominal interest rates, while accounting for most of the real balance fluctuations. We also compute the natural rate of interest consistent with flexible prices and compare it with the real interest rate estimated in the model (with sticky prices). The estimated monetary policy rule, albeit with a significant response of the nominal rate to money growth, implies a real rate close to the natural rate. These results are obtained in models whose estimated parameters are in general both reasonable and similar to others available in the literature.

The rest of the article is structured as follows. Section 1 presents the model economy. Section 2 describes the maximum likelihood estimates of alternative models. Section 3 presents some counterfactual exercises to assess further the role of money in setting the monetary policy, including the computation of the natural rate of interest consistent with the estimated model. Section 4 concludes.

1. Money in a Sticky Price Model

In this Section we set out the basic equations of the model, whose predecessors can be found in Rotemberg and Woodford (1997), McCallum and Nelson (1999), Woodford (1996), and more recently Ireland (2004) among others. The economy consists of a representative household, a continuum of producing firms indexed by \( j \in [0, 1] \) and a monetary authority. The model displays sufficient symmetry for the analysis to focus on the behaviour of a representative goods-producing firm.

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1.1. Households

The representative household of the economy maximises the following expected stream of utility:

$$\max_{C_t, N_t, M_t, B_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t a_t \left[ \Psi \left( \frac{C_t^*}{\epsilon_t P_t} , \frac{M_t}{\epsilon_t P_t} \right) - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

(1)

where $C_t^* = C_t/C_{t-1}^h$ and $C_t$ is the CES aggregator of the quantities of the different goods consumed,

$$C_t = \left( \int_0^1 C_t(j)^{1-\epsilon} \, dj \right)^{1/\epsilon}$$

and $P_t(j)$ is the price of the variety $j$. $M_t/P_t$ and $N_t$ represent real balances and hours, respectively; $a_t$ is a preference shock and $\epsilon_t$ represents a velocity shock. The parameter $\beta \in (0, 1)$ is a discount factor and $\varphi \geq 0$ represents the inverse of the Frisch labour supply elasticity. The marginal utility of consumption depends upon real balances but it is independent of labour supply decisions. In addition, the assumption of separability between a consumption-real balances basket and hours implies that aggregate demand relationships are invariant to the specification of the firm’s problem. Notice that through the function $\Psi$, we allow for non-separability among consumption and real balances in preferences, as well as for habits in consumption (i.e. $h \neq 0$). The presence of habits has been emphasised by Fuhrer (2000) and Christiano et al. (2005) as a potentially important component of the monetary transmission mechanism that helps to account for the observed serial correlation in the response of output to monetary policy shocks. Intratemporal non-separability makes it possible to test the relevance of a direct effect of money balances on supply and demand decisions through a set of parameter restrictions.

The budget constraint is:

$$\frac{M_{t-1} + B_{t-1} + W_t N_t + T_t + D_t}{P_t} = C_t + \frac{B_t/\epsilon_t + M_t}{P_t}.$$ 

(2)

Households enter period $t$ with money holdings $M_{t-1}$ and bonds $B_{t-1}$. At the beginning of the period they receive lump-sum nominal transfers $T_t$, labour income $W_t N_t$, where $W_t$ denotes the nominal wage, and a nominal dividend $D_t$ from the firms. They use some of these funds to purchase new bonds at nominal cost $B_t/\epsilon_t$, where $\epsilon_t$ denotes the gross nominal interest rate between $t$ and $t+1$. The household carries $M_t$ units of money into the period $t+1$.

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1 When $\varphi = 0$, preferences are linear in labour, and thus the labour-supply elasticity is infinite.
1.2. Firm Behaviour and Price Setting

The production function for firm \( j \) is,

\[
Y_t(j) = z_t N_t(j)^{1-\varepsilon}
\]

where \( Y_t(j) \) is output, \( N_t(j) \) represents the number of hours hired from the household (i.e. \( N_t = \int_0^1 N_t(j) \, dj \)), \( z_t \) is a common technology shock and \((1 - \varepsilon)\) is the elasticity of labour with respect to output. Letting

\[
Y_t = \int_0^1 Y_t(j)^{1-\varepsilon} \, dj
\]

the market clearing condition implies \( Y_t = C_t \). The representative firm sells its output in a monopolistically competitive market and sets nominal prices on a staggered basis, as in Calvo (1983). Each firm resets its price with probability \( 1 - \theta \) each period, independently of the time elapsed since the last adjustment. Thus, each period a measure \( 1 - \theta \) of producers reset their prices, while a fraction \( \theta \) simply adjust prices at the pace of steady-state inflation, \( \pi \), (i.e. non-adjusting firms simply set: \( P_t(j) = P_{t-1}(j) \pi \)). Hence, \( \theta^k \) will be the probability that the price set at time \( t \) will still hold at time \( t + k \). Notice that, if there were no constraints on the adjustment of prices, firms would set prices as \( P_t(j) = \left[ \varepsilon / (\varepsilon - 1) \right] MC_t(j) \), where \( MC_t(j) = W_t / [\partial Y_t(j) / \partial N_t(j)] \) is the nominal marginal cost and \( \varepsilon / (\varepsilon - 1) \) is the steady-state price markup.

This framework implies that inflation is a purely forward-looking variable. Nevertheless, recent research has pointed out the importance of allowing for a hybrid specification in which part of the inflation dynamics is explained by some backward-looking component in order to account for the inertia observed in inflation time series. To account for this formally, we follow Gali and Gertler (1999) by assuming that only a fraction \( (1 - \omega) \) of firms behave on a staggered basis when setting prices at each point of time. We denote by \( P^f_t \) the prices set by these forward-looking firms. The remaining firms, of measure \( \omega \), instead use a simple rule of thumb (backward-looking) when setting prices \( (P^b_t) \). In logs, the price index of newly set prices is:

\[
p^*_t = (1 - \omega)p^f_t + \omega p^b_t.
\]

The aggregate price level evolves as follows:

\[
P_t = \left[ \theta P^1_{t-1} + (1 - \theta)(1 - \omega) \left( P^f_t \right)^{1-\varepsilon} + (1 - \theta)\omega \left( P^b_t \right)^{1-\varepsilon} \right]^{1/1-\varepsilon}
\]

and we shall further assume that the backward-looking firms set their prices according to the following rule of thumb:

\[
P^b_t = P_{t-1} \pi_{t-1}
\]

where \( \pi_{t-1} = P_{t-1} / P_{t-2} \).

1.3. Central Bank Reaction Function

Whereas the assumption of a representative agent in the private sector poses no particular problems, it is harder to think of a single monetary policy rule for the euro area, in which a
where the innovation $\varepsilon_t$ is normally distributed with standard deviation $\sigma_\varepsilon$; and $\mu_t = M_t/M_{t-1}$ is the rate of money growth.

Rudebusch and Svensson (2002) show, in the context of a backward-looking model, that an interest rate rule that depends on money growth (or the changes in real balances given the expression (8) presented below) can be rationalised as a result of an optimal reaction function when the central bank aims to minimise the variance of money growth. In the context of this forward-looking model with price rigidities and money in the utility function, the rationale for this rule is that, given the potential importance of money in affecting the path of inflation, it may have some value as a monetary policy indicator to try to stabilise inflation. More generally, it allows us to test whether money plays an independent role in setting interest rates.

1.4. Equilibrium

The symmetric equilibrium can be log-linearised to yield the following set of equations:

$$
\tilde{y}_t = \frac{\phi_1}{\phi_1 + \phi_2} \tilde{y}_{t-1} + \beta \phi_1 + \phi_2 E_{i \tilde{y}_{t+1}} - \frac{1}{\phi_1 + \phi_2} (\tilde{r}_t - E_{i \tilde{r}_{t+1}}) \\
- \frac{\beta \phi_1}{\phi_1 + \phi_2} E_{i \tilde{y}_{t+2}} + \frac{\psi_2}{\psi_1 (1 - \beta h)} \frac{1}{\phi_1 + \phi_2} \left( \frac{1}{\phi_1 + \phi_2} \right) (\tilde{m}_t - \tilde{e}_t) \\
- \frac{\psi_2}{\psi_1 (1 - \beta h)} \left( \frac{\beta h}{\phi_1 + \phi_2} E_{i \tilde{m}_{t+1}} - \tilde{e}_{t+1} \right) \\
+ \frac{\psi_2}{\psi_1 (1 - \beta h)} \left( \frac{\beta h}{\phi_1 + \phi_2} E_{i \tilde{m}_{t+2}} - \tilde{e}_{t+2} \right) + \left( \frac{1 - \beta h \rho_a}{1 - \beta h} \right) \left( \frac{1 - \rho_a}{\phi_1 + \phi_2} \right) \tilde{a}_t
$$

(5)

$$
\tilde{m}_t = \gamma_1 \tilde{y}_t - \gamma_2 \tilde{r}_t + \left[ \gamma_2 (r - 1) (h \phi_2 - \phi_1) - h \gamma_1 \right] \tilde{y}_{t-1} \\
- \left[ \gamma_2 (r - 1) \beta \phi_1 \right] E_{i \tilde{y}_{t+1}} + \frac{\psi_2}{\psi_1 (1 - \beta h)} \left[ \frac{1}{\phi_1 + \phi_2} \right] (r - 1) \gamma_2 \tilde{e}_{t+1} \\
- \left( \frac{r - 1) \beta h (1 - \rho_a)}{1 - \beta h} \right) \left( \frac{1}{\phi_1 (1 - \beta h)} + 1 \right) \tilde{a}_t
$$

(6)

$^2$ The symbol ^ represents percentage deviations of a variable from its steady-state value.

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\[
\hat{r}_t = \rho_r \hat{r}_{t-1} + (1 - \rho_r) \rho_y \hat{y}_t + (1 - \rho_r) \rho_z \hat{z}_t + (1 - \rho_r) \rho \hat{\mu}_t + \epsilon_t \tag{7}
\]

\[
\hat{\mu}_t = \hat{m}_t - \hat{m}_{t-1} + \hat{\Pi}_t \tag{8}
\]

\[
\hat{\Pi}_t = \gamma_f E_t (\hat{\Pi}_{t+1}) + \gamma_b \hat{\Pi}_{t-1} + \lambda \hat{m}_t \tag{9}
\]

\[
\hat{m}_t = (\chi + \phi_2) \hat{y}_t - \phi_1 \hat{y}_{t-1} - \beta \phi_1 E_t \hat{y}_{t+1} - \frac{1}{\psi_1 (1 - \beta h)} (\hat{m}_t - \hat{\alpha}_t) + \frac{\psi_2}{\psi_1 (1 - \beta h)} E_t (\hat{m}_{t+1} - \hat{e}_{t+1}) - \frac{\beta h (1 - \rho_a)}{(1 - \beta h)} \hat{a}_t - (1 + \chi) \hat{z}_t \tag{10}
\]

\[
\hat{\alpha}_t = \rho_a \hat{\alpha}_{t-1} + \epsilon_{\alpha_t} \tag{11}
\]

\[
\hat{e}_t = \rho_s \hat{e}_{t-1} + \epsilon_{\epsilon_t} \tag{12}
\]

\[
\hat{z}_t = \rho_z \hat{z}_{t-1} + \epsilon_{z_t} \tag{13}
\]

where \(\psi_1\) and \(\psi_2\) represent the partial derivatives of the function \(\Psi\) and the following relationships hold between structural parameters, the steady-state, and the reduced-form parameters of (5)–(10),

\[
\psi_1 = -\left[ \frac{\Psi_1}{(Y)^{(1-h)} \Psi_{11}} \right] \quad \lambda = (1 - \theta)(1 - \beta \theta)(1 - \omega) \xi
\]

\[
\psi_2 = -\left[ \frac{\Psi_2}{(Y)^{(1-h)} \Psi_{11}} \right] (\frac{m}{r}) \quad \gamma_f = \beta \theta \{ \theta + \omega [1 - \theta (1 - \beta)] \}^{-1}
\]

\[
\gamma_1 = \left[ \frac{r (Y)^{(1-h)} \psi_2}{m \psi_1 + (r - 1) \frac{1}{\psi_1}} \right] \gamma_2 \quad \gamma_h = \omega \{ \theta + \omega [1 - \theta (1 - \beta)] \}^{-1}
\]

\[
\gamma_2 = \frac{r}{(r - 1) m} \left[ \frac{\psi_2}{(r - 1) e \psi_{12} - r \psi_{22}} \right] \xi = \frac{(1 - \alpha)}{1 + \alpha (\epsilon - 1)} \{ \theta + \omega [1 - \theta (1 - \beta)] \}^{-1}
\]

\[
\phi_1 = \frac{(\psi_1 - 1) h}{1 - \beta h} \quad \phi_2 = \frac{\psi_2 - 1 + (\psi_1 - 1) \beta h^2 - \beta h}{1 - \beta h}
\]

Equation (5) represents the optimal intertemporal allocation of wealth. The assumption of non-separability between consumption and real balances makes the marginal utility of consumption a function of the amount of real balances optimally demanded by the households. The presence of habits makes the marginal utility of consumption also dependent on lags of output and further leads of money and output. Hence, in equilibrium, output will depend on current and expected real balances after
accounting for the money demand shock. Notice that as \( h \to 0 \), (5) approaches the usual Euler equation for consumption under time-separable preferences. The effect of real balances on demand will disappear when \( \psi_2 = 0 \), i.e. as long as the cross-derivative between consumption and real balances (\( \Psi_{12} \)) is zero in the utility function. Demand also depends upon the present discounted value of future short-term interest rates; so the sensitivity of output to interest rate movements depends upon the coefficient \( \psi_1 \), which is related to the inverse of risk aversion.

Expressions (6), (7) and (8) describe the money market. Equation (6) is a generalised money-demand equation, where the coefficients \( \gamma_1 \) and \( \gamma_2 \) are the money-income and money-interest rate elasticities. Again the presence of habits in the utility function generates a dynamic equation in which money demand depends also on future output and real balances as well as on the preferences shock \( a_u \). It is worth pointing out that what matters for the dynamics of output and inflation are fluctuations in real balances once fluctuations in velocity shocks, \( e_e \), have been taken into account (i.e., \( \hat{m}_i - \hat{e}_i \)). Equation (8) is an identity that specifies nominal money growth in terms of real balances and inflation.

The supply side of the model is characterised by two equations: first, a New Keynesian Phillips curve, (9) that allows for both expected and past inflation terms, as well as real marginal costs to affect current inflation; and second, a linear relationship between the real marginal costs with detrended output, real balances and the technology shock (10). Notice that, if we assume that all new prices (\( p_t^* \)) are set on a profit maximising basis, i.e. \( \omega = 0 \), then inflation becomes a purely forward-looking variable. Moreover, the assumption of decreasing returns to labour implies that the effect of output on inflation depends not only on the degree of nominal rigidities but also on the elasticity of output to employment (\( 1 - \zeta \)) and the labour supply elasticity (\( \phi \)) through the coefficient \( \chi \). The non-separability in preferences between real balances and consumption generates a direct effect of the former variable on marginal costs and then on inflation. In presence of habits, the marginal cost also depends on leads and lags of output, money balances and the preference shock \( a_u \). To close the model we include the AR(1) distribution for the aggregate demand shock (11), the money demand shock (12) and the technology shock (13), with innovations \( e_a, e_e \) and \( e_z \) respectively.

Allowing for non-separability between real balances and consumption unveils a direct effect of real balances on both output and inflation equilibrium relationships. This can be interpreted as providing microfoundations for the direct effect of the real balances gap (approximated here by \( \hat{m}_i - \hat{e}_i \)) on inflation, an effect that has been discussed by some authors in a reduced form framework such as the \( P^* \) model.\(^3\) While the \( P^* \) model has been used in the literature to look at the inflation forecasting properties of monetary aggregates, Gerlach and Svensson (2000) go beyond that and find that the real balances gap (i.e. the difference between the current level of real balances and its long-run equilibrium level) rather than the \( P^* \) gap is the reliable leading indicator for future inflation in the Euro area. Our structural model imposes parameter restrictions that help in assessing the empirical relevance of such hypotheses.

\(^3\) \( P^* \) is the price level prevailing at the current money stock with velocity and output at their long-run equilibrium level. Hallmann et al. (1991) provide evidence of how that model explains the observed US inflation.

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The maximum likelihood estimation follows Hansen and Sargent (2000) and recent applications can be found in Kim (2000) and Ireland (2001). To that end the stationary solution of the model (5)–(13) is expressed in a state-space form and it is estimated using a recursive Kalman filter procedure (see Appendix). Since we are interested in estimating the parameters that characterise the four stochastic processes jointly with the other structural parameters, exploiting the cross-equation restrictions, we see this strategy as a suitable alternative to the estimation based on impulse-response functions such as that in Rotemberg and Woodford (1997) and Christiano et al. (2005).

We use euro area quarterly data from 1980:1 to 1999:4 for the logs of detrended output, detrended real balances, inflation and gross nominal interest rates. The output, inflation and nominal interest rate data are from the Area Wide Model data set which has aggregated the individual country data using an index method (Fagan et al., 2001). Real output is measured through real GDP, inflation is defined as the change in the log of GDP deflator and the interest rate is the three-month money market rate. Real balances are measured dividing M3 by the GDP deflator, and are obtained from the work by Brand and Cassola (2000). Figure 1 displays the data used in the estimation process. A preliminary unconditional analysis of the data suggests a limited role for monetary aggregates. There is a positive and very high correlation between output and real balances but there is no evidence of real balances leading output. Real balances lead inflation two quarters ahead but with a low correlation coefficient (0.24).4

In general we assume that there is a deterministic trend affecting only output and real balances but a look at the data on inflation and interest rates show that we are facing two additional borderline non-stationary variables. In particular, the decline in these variables might well reflect the nominal convergence, i.e. changes in monetary policy strategy, experienced by these economies prior to the EMU. Hence, in our empirical implementation we will compare the estimates with and without detrending inflation and nominal interest rates.

2. Maximum-likelihood Estimates

2.1. Non-separable Preferences

The first column of Table 1 reports the maximum likelihood estimates of the general model – equations (5)–(13) – without any filtering on inflation and nominal interest rates. The next two columns present the estimates with two alternatives detrending methods of inflation and interest rates. In particular, we have approximated their low frequency movements by a second order polynomial in time (column (2)) and a fifth order polynomial in time (column (3)).5 The most interesting result concerns the parameter $\psi_2$, which governs the separability of the utility function on real balances. The t-ratio of the null $\psi_2 = 0$ implies that the real balances effect in the dynamics of output (5) and inflation (9) is non-significantly different from zero.6 Looking across

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4 See Nicoletti Altimari (2001) for recent evidence of the forecasting properties of monetary aggregates in the Euro area.

5 The fifth order polynomial in time produces virtually the same business cycle as a band-pass filter that discards fluctuations outside a frequency range between 2 and 32 quarters. Alternative detrending methods yield extremely similar results that are available from the authors upon request.

6 Ireland (2001) obtains also a non-significant effect for the US.

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columns, we see that this result does not depend upon the filtering of inflation and nominal interest rates.

As regards the other parameters of interest, we find clear evidence of habit formation; the estimated $h$ is close to 0.9, slightly higher than the value of 0.8 obtained by Fuhrer (2000) for the US. Nevertheless, once we control for the long-run fluctuations in inflation and nominal interest rates, the estimated value of the parameter $h$ is closer to one.
Thus, the model displays strong inertia in consumption, so that utility is mostly derived from the increase in consumption, rather than for its level. The interest rate elasticity in the Euler equation ($\psi_1$), is significantly positive, suggesting an intertemporal elasticity of substitution slightly above (but not significantly different from) one.

Besides the direct real balance effects on demand and supply, money is also related to the behaviour of output and prices through the money demand and the policy reaction function equations. The elasticity of money demand with respect to output ($\gamma_1$) and interest rates ($\gamma_2$) is poorly estimated. In the benchmark model, the interest

Table 1

Maximum Likelihood Estimates

| Estimated Parameters | (No Detrending $\{\pi_t, r_t\}$) | Detrending $\{\pi_t, r_t\}$ |
|----------------------|----------------------------------|-----------------------------|
|                     | (1)                              | P(2)                        | P(5)                        |
| $\beta$             | 0.988 (0.0025)                   | 0.988 (0.0012)              | 0.988 (0.0001)              |
| $\psi_1$            | 0.952 (0.037)                    | 0.942 (0.076)               | 0.929 (0.053)               |
| $\psi_2$            | 0.0000 (0.0006)                  | 0.0000 (0.0007)             | 0.0000 (0.0039)             |
| $\kappa$            | 0.906 (0.037)                    | 0.907 (0.049)               | 0.865 (0.034)               |
| $\gamma_1$          | 0.118 (0.092)                    | 0.076 (0.140)               | 0.037 (0.043)               |
| $\gamma_2$          | 0.366 (0.327)                    | 0.000 (0.862)               | 0.222 (0.062)               |
| $\gamma_f$          | 0.988 (0.507)                    | 0.988 (0.409)               | 0.988 (0.025)               |
| $\chi$              | 0.682 (0.688)                    | 0.286 (0.206)               | 0.988 (0.051)               |
| $\lambda$           | 1.063 (0.507)                    | 19.32 (0.409)               | 9.045 (0.025)               |
| $\rho_r$            | 0.5                              | 0.0                         | 0.0                         |
| $\rho_y$            | 0.072 (0.045)                    | 0.000 (0.000)               | 0.000 (0.031)               |
| $\rho_a$            | 1.175 (0.144)                    | 2.057 (0.492)               | 1.812 (0.392)               |
| $\rho_u$            | 0.705 (0.184)                    | 0.689 (0.371)               | 0.530 (0.065)               |
| $\rho_\sigma$       | 0.984 (0.017)                    | 0.930 (0.042)               | 0.846 (0.072)               |
| $\rho_\epsilon$     | 0.963 (0.021)                    | 0.957 (0.024)               | 0.962 (0.022)               |
| $\rho_z$            | 0.969 (0.021)                    | 0.903 (0.123)               | 0.965 (0.024)               |
| $\sigma_a$          | 0.052 (0.056)                    | 0.012 (0.007)               | 0.005 (0.002)               |
| $\sigma_\epsilon$   | 0.0046 (0.0004)                  | 0.0044 (0.0005)             | 0.0044 (0.0003)             |
| $\sigma_z$          | 0.0045 (0.0009)                  | 0.0042 (0.0018)             | 0.0054 (0.0010)             |
| $\sigma_\epsilon$   | 0.0019 (0.0002)                  | 0.0040 (0.0012)             | 0.0036 (0.0006)             |

Log-Likelihood

1429.66 1452.33 1452.15

Note. The unconditional means of $\pi$ and $r$ have been used to obtain $\beta$. $P(i)$ means detrended using an $i$th polynomial in time.

Thus, the model displays strong inertia in consumption, so that utility is mostly derived from the increase in consumption, rather than for its level. The interest rate elasticity in the Euler equation ($\psi_1$), is significantly positive, suggesting an intertemporal elasticity of substitution slightly above (but not significantly different from) one.

Besides the direct real balance effects on demand and supply, money is also related to the behaviour of output and prices through the money demand and the policy reaction function equations. The elasticity of money demand with respect to output ($\gamma_1$) and interest rates ($\gamma_2$) is poorly estimated. In the benchmark model, the interest
rate elasticity, $\gamma_2 = 0.36$, is very much in line with other estimated values in the literature but the income elasticity is small, $\gamma_1 = 0.12$, in contrast with the usual calibration of this parameter at its long-run value (1.0) (Chari et al., 2000). This result does not change once we consider the high frequency movements of inflation and nominal interest rates.\(^7\)

The estimates for the supply side of the economy reveal the importance of the forward-looking component of inflation, since $\gamma_f$ is close to one. In fact, the data do not reject the restriction $\gamma_f = \beta$ since the unrestricted estimated of $\gamma_h$ is not significantly different from zero.\(^8\) This means that the proportion of price setters that use a backward-looking rule of thumb ($\omega$) is zero. This result is very robust to the detrending procedure of inflation. The most drastic changes affect the slope coefficient of the Phillips curve. Hence, for the benchmark model estimation, the parameter $\lambda$ is significant reflecting the fact that marginal costs are an important determinant of inflation, whereas the elasticity of labour supply ($\phi$) is estimated with less precision. The estimated value of $\lambda$ (1.06) is consistent with a high degree of flexibility in price formation.

As regards the interest rate rule, in the first column we report that the response of interest rates to output is low ($\rho_y = 0.07$) and even lower once we drop the medium run fluctuations in nominal rates. The inflation response ($\rho_z = 1.17$) is above one, and it tends to increase once we consider only the business cycle fluctuations in the nominal interest rates and inflation. Finally, the smoothing parameter is set to 0.5, a restriction that is easily accepted by the data, while it is zero in models with detrended interest rate. Interestingly, the model displays a significant response of the interest rate to the rate of growth of money ($\rho_m \in (0.5, 0.7)$); this effect is less common in the literature and opens up a channel of influence of money balances that may be potentially important.

Most of the inertial behaviour of supply and demand is inherited through the high persistence presented especially in both preferences ($\rho_a$) and supply ($\rho_z$) shocks. As can be seen from columns two and three this is partly explained by the fact that inflation and interest rate are not detrended while they display highly persistent movements over the sample period. Finally, the effects of detrending inflation and nominal interest rates translate into a significant reduction in the volatility of the preference shock, $a_o$, as well as an increase in the volatility of the interest rate shock.

2.2. Money Under Separability

We now formally test for non-separability in preferences between consumption and real balances. Hence, the first column of Table 2 reproduces the estimates of the first column in Table 1, while in column two we present the estimates imposing $\psi_2 = 0$, i.e. money matters only through its effects on the policy rule. A likelihood ratio test accepts the null of separability at the 1% significance level. Moreover, the point estimates of the

\(^7\) We have estimated the model imposing a unit income elasticity. The results were discouraging. A likelihood ratio test rejects the restriction, and the other estimated parameters took values that were far away from the unrestricted model. Moreover, in this case the estimation results became very sensitive to the initial conditions.

\(^8\) Thus, the model is estimated assuming $\gamma_h = 1 - \gamma_f$. This constraint is not exactly satisfied in the theoretical model, but the error is very small for values of $\beta$ close to one; see also Boivin and Gianonni (2001).
remaining parameters are unaffected. The main difference among columns (1) and (2) lies in the size of the standard errors. Imposing separability enables us to obtain much more precise estimates.

Given the previous results, we concentrate in this Section on a standard constant relative risk aversion (CRRA) utility function,
\[ U \left( \frac{C_t}{e_t P_t}, \frac{M}{e_t P_t} \right) = \left( \frac{1}{1 - \sigma} \right) \left( \frac{C_t}{C_{t-1}} \right)^{1-\sigma} + \frac{1}{1 - \delta} \left( \frac{M_t}{e_t P_t} \right)^{1-\delta} \] (14)

which allows us to estimate the parameter of the curvature of the utility function, \( \sigma \), i.e. the inverse of the intertemporal elasticity of substitution, and the parameter \( \delta \) which is related to the interest rate elasticity of the money demand. This specification imposes a tight restriction between the interest rate elasticity and the output elasticity of money demand through the parameter \( \delta \). Formally, the output, real balances, and marginal cost equations are now as follows:

\[
\hat{y}_t = \frac{\phi_1}{\phi_1 + \phi_2} \hat{y}_{t-1} + \frac{\beta \phi_1}{\phi_1 + \phi_2} E_t \hat{y}_{t+1} - \frac{1}{\phi_1 + \phi_2} (\hat{r}_t - E_t \hat{r}_{t+1}) - \beta \phi_1 \frac{E_t \hat{y}_{t+2}}{(1 - \beta h) \phi_1 + \phi_2} \hat{a}_t
\] (15)

\[
\hat{m}_t = -\frac{\phi_1}{\delta} \hat{y}_{t-1} + \frac{\phi_2}{\delta} \hat{y}_t - \frac{\beta \phi_1}{\delta} E_t \hat{y}_{t+1} - \frac{1}{\delta (r - 1)} \hat{r}_t + \frac{\beta h (1 - \rho_a)}{(1 - \beta h) \delta} \hat{a}_t + \frac{\delta - 1}{\delta} \hat{e}_t
\] (16)

\[
\hat{m}_c_t = -\phi_1 \hat{y}_{t-1} + (\chi + \phi_2) \hat{y}_t - \beta \phi_1 E_t \hat{y}_{t+1} - (1 + \chi) \hat{z}_t - \frac{\left[ \beta h (1 - \rho_a) \right]}{1 - \beta h} \hat{a}_t
\] (17)

where now \( \phi_1 = \frac{\sigma - 1}{h} \), \( \phi_2 = \frac{\sigma + (\sigma - 1) h}{1 - \beta h} \).

The estimated parameters appear in the third column of Table 2. The risk aversion \( \sigma \) is slightly above one, consistent with the estimates of \( \psi_1 \) in columns (1) and (2). The parameter \( \delta \) is precisely estimated, implying strongly significant elasticities of the demand for money. The implied point estimate of the interest rate elasticity is around 0.4 with a standard error of 0.09. Notwithstanding, these estimates continue yielding a highly significant but very small income elasticity (i.e. \( c_1 = \phi_2 / \delta \)) of 0.018, which might be interpreted as a short-run elasticity, since we are working at business cycle frequency. Our estimate is close to those obtained in single equation estimation that does not restrict that elasticity in the short run e.g., Coenen and Vega (2001) for the euro area. There is also evidence of small interest rate and income elasticities in general equilibrium models. Thus, Lippi and Neri (2003) estimate a model under optimal policy by maximum likelihood for the euro area and obtain significant but close to zero elasticities. The values obtained by Ireland (2004) for the US in a dynamic general equilibrium model with money in the utility function are of the same order of magnitude as those reported here (\( \gamma_2 \in [0.15, 0.7], \gamma_1 \approx 0.01 \)). Christiano et al. (2005) estimate somewhat higher elasticities for the US, but still well below the values used in calib-

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9 These results are obtained detrending real balances and output. The estimations in Table 2 do not change (except for the parameter \( \lambda \)) when inflation and interest rates are also detrended.

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ration, and they interpret these small elasticities as capturing the short-run response of money demand. Later, we will discuss the implications that a unit income elasticity of the money demand might have on the business cycle properties of the model.

The estimated values of $\lambda$ and $\chi$ are similar to the ones displayed in columns (1) and (2) and are highly significant. Since the measure of backward-looking firms is zero ($\omega = 0$) we may recover the degree of price stickiness ($\theta$) from the estimated slope coefficient, $\lambda = 1.19$. Assuming that the average labour share, $(1 - z)$ is $3/4$, and that the steady-state markup, $e/(e - 1)$, is $1.20$ we obtain that $\theta = 0.2$, which implies that prices change every 1.25 quarters. Moreover, the low value of $\chi$ implies that the implicit labour supply elasticity is around $6$, quite close to the assumption of a perfectly elastic labour supply.

These estimates are above the range of the ones obtained by Galí et al. (2001) using only information about marginal costs. To gain further insight into this result we have carried out a sensitivity exercise. Table 3 shows that there is a trade-off between the values of the labour supply elasticity $(1/\varphi)$ and the corresponding estimates of the slope inflation coefficient $\lambda$. As we move towards models in which lower values of the labour supply elasticity (high $\chi$) are imposed, we estimate a lower value of $\lambda$. In all cases the data accept the restriction on $\varphi$ at the 5% significance level. Although this points towards some lack of precision in the joint estimation of nominal and real rigidities, this do not alter the main message about the small degree of price inertia. Thus, the largest value of $\chi$ (6.0) accepted by the data is consistent with firms changing prices almost every 6 months (1.7 quarters). This low degree of price inertia is not far from what recent microeconometric evidence suggests; thus, Bils and Klenow (2004) and Golosov and Lucas (2003) estimate an expected duration of around 1.8 or 1.9 for the US.

The interest rate rule in this restricted model is largely unchanged with respect to the estimates in columns (1) and (2). The significant response to money growth is consistent with recent general equilibrium model estimates for the US (Ireland, 2004), but

| CRRA Model | $1/\varphi$ | $\chi$ | $\lambda$ | $\theta$ | Dur | p-value |
|------------|-------------|--------|-----------|---------|-----|---------|
| **Unrestricted Estimates** | | | | | | |
| 5 | 0.596 | 1.18 | 0.2 | 1.2 |
| (0.09) | | | | | |
| **Restricted Estimates** | | | | | | |
| 2 | 1.0 | 0.95 | 0.22 | 1.3 | 0.36 |
| (0.05) | | | | | |
| 1 | 1.67 | 0.92 | 0.22 | 1.3 | 0.17 |
| (0.01) | | | | | |
| 0.5 | 3.0 | 0.51 | 0.31 | 1.4 | 0.17 |
| (0.16) | | | | | |
| 0.235 | 6.0 | 0.30 | 0.40 | 1.7 | 0.18 |
| (0.01) | | | | | |

Note. $\chi = (\varphi + z)/(1 - z)$ was obtained by setting $z = 0.25$. The p-value column reports the marginal significance level of the likelihood ratio test. Duration (Dur) is defined as $1/(1 - \theta)$.

For values of $\chi$ above 6 some parameter estimates display implausible values.

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depart slightly from the single equation estimates provided by Clarida et al. (1998) for the Bundesbank during a similar sample period. While those authors found a significant response of nominal interest rates to output, the response to monetary aggregates was negligible. The (aggregate of the) European monetary authorities have followed a flexible monetary targeting strategy or inflation targeting in which current money growth was a leading indicator of future inflation. This is striking given the poor role assigned to money in our estimates of output and inflation equations. We will turn to this issue below when examining the role played by the structural shocks.

3. Money and Monetary Policy in the Euro Area

The estimated models have features that give somewhat conflicting views about the influence of real balances on output and inflation in the euro area. First, money does not seem to play a direct role in the demand and supply sides of the economy. Second, a very low income elasticity of money demand raises legitimate doubts as to whether that relationship is adequately specified. And finally, money growth appears in the policy rule, which is puzzling given the above-mentioned results.

In this Section we use the estimated models to take a closer look at the role of monetary aggregates conditioning on the estimated sources of fluctuations. We assess the role of money demand shocks in the historical decomposition of inflation and real balances. Next we analyse the implications of alternative specification of money demand for the dynamics of the variables. As a further check to assess the role of money we compare the implications of our estimated model with two alternatives where money plays no role.

Table 4 analyses the importance of the structural shocks quantitatively through the forecast error variance decomposition. The first row in Table 4 refers to the model with

| Table 4 |
|---------|
| Variance Decomposition (one year ahead) (%) |

Separable preferences \((\psi_2 = 0)\)

|             | \(\varepsilon_{o_t}\) | \(\varepsilon_{r_t}\) | \(\varepsilon_{b}\) | \(\varepsilon_{i}\) |
|-------------|------------------------|------------------------|---------------------|---------------------|
| Output      | 51.8                   | 0.10                   | 47.8                | 0.10                |
| Real balances | 1.70                   | 97.8                   | 0.40                | 0.10                |
| Inflation   | 43.2                   | 13.8                   | 15.5                | 27.3                |
| Interest rate | 77.4                   | 3.90                   | 11.1                | 7.50                |

Income elasticity equals 1 \((\gamma_1 = 1)\)

|             | \(\varepsilon_{o_t}\) | \(\varepsilon_{r_t}\) | \(\varepsilon_{b}\) | \(\varepsilon_{i}\) |
|-------------|------------------------|------------------------|---------------------|---------------------|
| Output      | 51.9                   | 0.10                   | 47.8                | 0.10                |
| Real balance | 22.0                   | 45.5                   | 31.5                | 0.80                |
| Inflation   | 11.3                   | 12.1                   | 52.6                | 23.8                |
| Interest rate | 87.1                   | 2.10                   | 9.30                | 1.20                |
| Policy rule without money growth \((\rho_m = 0)\)

|             | \(\varepsilon_{o_t}\) | \(\varepsilon_{r_t}\) | \(\varepsilon_{b}\) | \(\varepsilon_{i}\) |
|-------------|------------------------|------------------------|---------------------|---------------------|
| Output      | 51.4                   | 0.00                   | 48.3                | 0.10                |
| Real balances | 1.00                   | 98.2                   | 0.50                | 0.20                |
| Inflation   | 46.9                   | 0.00                   | 18.2                | 34.7                |
| Interest rate | 80.5                   | 0.00                   | 16.4                | 2.90                |
separable preferences and CRRA in consumption (column 3 in Table 2). Both real demand (preference) and supply (technology) shocks are the most important sources of fluctuations in output and inflation and, therefore, in interest rates. Although money demand shocks \((e_t)\) mainly help to forecast real balances and have almost no predictive power of the movements of output and interest rates, they explain almost 15% of inflation fluctuations. This result could make a case for money as an indicator of future inflation.\(^{11}\)

A first indication that this is not necessarily the case is given by Figure 2 that depicts the historical contribution of both preference and money demand shocks to inflation in the model.\(^{12}\) Money demand shocks seem to have played a very limited role in explaining the high inflation rates of the early 1980s or the low inflation rates during the late 1990s. On the contrary, the top panel confirms that inflation fluctuations are mainly explained by preference shocks (as well as by technology shocks). These results are also consistent with the VAR evidence for the euro area showing the small

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\(^{11}\) Allowing for a direct real balance channel on inflation (model with \(\psi_2 \neq 0\) in Table 2) does not substantially change that result.

\(^{12}\) The historical decomposition was constructed using the moving average representation of the endogenous variables for the estimated model parameters. Inflation is plotted in deviations from its unconditional mean.

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importance of monetary policy shocks to explain the fluctuations of output and inflation (Peersman and Smets, 2001).

As noted above the low income elasticity of money demand might be a sign of an inadequate specification that could blur the assessment of the role played by the monetary aggregates during the sample period. A simple counterfactual exercise helps to gauge the relevance of this parameter. Our estimated short-run income elasticity is lower than one, so making velocity a function of output. This contrasts with the usually calibrated value of 1.0, which implies that velocity only responds to movements in the interest rates. We have analysed the differences between these two values in the model with CRRA in consumption. As expected, setting a unit income elasticity leads to an increase in the unconditional variability of real balances generating a much larger response of real balances to different shocks. Despite these changes, Table 4 shows that although money demand shocks account for a lower proportion (45%) of the forecast error variance decomposition of real balances, the relative contribution of each source of fluctuations to inflation and output remains unchanged.

A remaining question is how to explain the contribution of money demand shocks to the fluctuations of inflation (13.8% in Table 4). More precisely what we want to assess is whether this value indicates that money has affected inflation and output in a way that makes it a crucial statistic for monetary policy. To that end, we perform two additional exercises, by comparing the implications of our model with two alternatives in which the fluctuations of output are not affected by shocks to money demand: a Neo-Wicksellian sticky-price model and a frictionless model.

In a recent article, Rudebusch and Svensson (2002) have shown, in the context of a backward-looking model, that the difference between monetary targeting and flexible inflation targeting is that the interest rate policy rule in the first case depends on current and lagged real balances. Similarly, our estimated reaction function can be rewritten as an inflation targeting rule with a real balances term. Formally, consider the estimated model with CRRA in consumption, replacing money growth in (7) by (8), yields:

\[
\hat{r}_t = 0.5\hat{r}_{t-1} + (1 - 0.5)0.05\hat{y}_t + (1 - 0.5)(1.18 + 0.68)\hat{x}_t \\
+ (1 - 0.5)0.68(\hat{m}_t - \hat{m}_{t-1}) + \hat{\epsilon}_t.
\]  

We focus on how much the response of the interest rate to real balances has affected the shape of business fluctuations in the euro area. To address that issue we have repeated the variance decomposition exercise setting the coefficient of real balances in (18) to zero. Under this restriction, the model corresponds to the Neo-Wicksellian framework advocated by Woodford (2003) augmented with habits formation (Amato and Laubach, 2004). This model is recursive and in this setup the variability of real variables is orthogonal to money demand shocks.

As can be seen in Table 4, the variance decomposition of output, nominal interest rates and inflation to both demand and supply shocks remains unchanged once we impose the Neo-Wicksellian structure in our setting, as compared with the unrestricted model. As expected, the only change is that the contribution of velocity shocks to

\[\text{The value of the contemporaneous output elasticity } (\phi_2/\delta = 1 + \phi_1(1 + \beta)/\delta) \text{ has been restricted so that the long-run coefficients of output add up to one.}\]
inflation goes now to zero. This means that the estimated (small) contribution of money to inflation (13.8%) was mainly a consequence of the presence of real balances in the rule, with no interaction with the rest of the model.

An alternative way of assessing the importance of real balances for the monetary policy is to compare the evolution of the estimated real interest rate in our model with the natural interest rate in a frictionless, otherwise similar, economy in which again monetary shocks play no role. The natural rate is defined as the real interest rate consistent with flexible prices; it can be solved as a function of the exogenous demand and supply shocks, as well as of the previous period’s output, but it does not respond to innovations in real balances.\footnote{We assume that price rigidities are the only distortion that prevents the equilibrium from being optimal. Thus, we disregard the money distortion and the steady state distortions.} We solve for the equilibrium values of the real rate and output that satisfy the IS relation (15), as well as the zero marginal cost condition (17) in the model with separability and CRS in consumption.\footnote{We evaluate the natural rate in the cashless limit economy. The estimated model is used to obtain the moving average representation of output and the real rate under flexible prices.}

Figure 3 shows the estimated time series of the real interest rates ($r_t$) and the corresponding natural rate ($r^*_t$). The natural rate presents very persistent fluctuations around a 4% annual rate between 1980 and 1992, and after 1993 it has a progressive downward trend until the end of the sample period (2% annual rate). Although our estimated interest rate deviates somewhat from the natural rate, differences are only quantitatively important and persistent at the beginning of both the 1980s and 1990s, while during the second half of the 1990s both rates move very closely (the sample correlation between these variables is 0.93).
Overall, the standard deviations of both series are roughly the same: 1.64 for the real rate and 1.48 for the natural rate (annualised percentages). This standard deviation for the natural rate is a much lower number than that estimated by Rotemberg and Woodford (1997) for the US (27%). Instead, our estimates are closer to the simulations performed by Neiss and Nelson (2001) for the US. Our result is partly explained by the low estimated level of price rigidity. In fact, both the natural rate and the real rate show a similar pattern when instead of a price inertia of 1.2 we impose the estimates corresponding to a price inertia of 1.7 (i.e., $\chi = 6$ in Table 3).

The gap between the current level of the natural rate and the interest rate implied by the central bank monetary policy ($r_t - r^n_t$) may be a relevant indicator of inflationary pressures. For our sample period the contemporaneous correlation between inflation ($p_t$) and the interest rate gap is $-0.45$.

The fact that the estimated real rate and the natural rate have moved so closely means that monetary policy in the euro area has not been too far from what would have been an optimal response to real (preferences and technology) shocks, and that the estimated policy rule is consistent with a nominal rate that mainly responds to non-monetary shocks (those behind the real rate and output). Moreover, another (not reported) counterfactual exercise shows that the estimated real rate would have been very similar under a policy rule that were not responding to changes in monetary aggregates. Overall, these results confirm that changes in real balances have played a minor role in explaining the dynamics of output and inflation in the euro area over the estimated sample period.

4. Conclusions

In this article we have estimated a small scale dynamic general equilibrium model for the euro area specified at the level of preferences and technology. A novel feature of our model is that it includes an effect of real balances on the marginal utility of consumption while allowing for intertemporal nonseparability too. This generates a direct effect of money on private agents’ decisions that, coupled with the presence of money growth in the policy rule, opens up alternative channels through which money may affect the business cycle properties of output and inflation in the euro area. The data accept the separability between consumption and real balances in the preferences and therefore reject the channel of real balances on agents’ decisions; the policy rule assigns a large weight to the interest rate responses not only to inflation but also to money growth, whereas there is a minor role for output responses. The different specifications considered in this article imply that the fluctuations in output and inflation are mainly driven by real shocks whereas money demand shocks affect only real balances. Moreover, the calculation of the model consistent natural rate of interest reveals that the evolution of the interest rate in the eurozone is mostly accounted for by the real sources of fluctuations. Hence, the estimated monetary policy rule is consistent with a path of real interest rates close to a natural rate that mainly responds to non-monetary and non-velocity shocks. However, it is possible the existence of alternative

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16 The reason is that they estimated a large variance for the technology shocks that must be counteracted by very large swings in the interest rate to avoid movements in the inflation rate.

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direct channels through which real balances may affect output and inflation fluctuations different from the ones considered here.

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Appendix. Maximum-likelihood Computation

This Appendix restates the setup of Ireland (2004) for the log-linearised set of equations (5)–(13). That may be written in the following form

\[ E_t \hat{X}_{t+1} = \Gamma_1 \hat{X}_t + \Gamma_2 e_t \]  

(A1)

where \( \hat{X}_{t+1} = [\hat{r}_{t-1}, \hat{\mu}_{t-1}, \hat{\pi}_{t-1}, \hat{m}_{t-1}, \hat{\pi}_t, \hat{\gamma}_{t+1}]' \) and \( e_t = [e_{at}, e_{et}, e_{zt}, e_{rt}]' \). The matrices of coefficients \( \Gamma_1 \) and \( \Gamma_2 \) are nonlinear functions of the structural parameters and the steady state.

The system is solved for the region of parameter values that guarantee a unique equilibrium. Thus, the solution of the system (A1) has three eigenvalues greater than one corresponding to the subset of predetermined variables in the vector \( \hat{X}_t \).

Given that the rank of the matrix \( E_\epsilon e_t' = V \) is the same as the rank of the matrix of the variance covariance of the data, the construction of the likelihood is relatively simple. We make use of the Kalman filter to construct an updating procedure of such a likelihood. The solution of the system has the following state-space representation

\[ S_t = AS_{t-1} + B e_t \]

\[ d_t = CS_t \]

where \( d_t = [\hat{y}_t, \hat{\mu}_t, \hat{\pi}_t, \hat{r}_t] \) is the vector of observable variables. Given a vector of initial values we can compute \( E(S) = 0 = S_{t=0} \) and \( E(SS') = \Sigma_{S} = (I - A \otimes A')^{-1} BV B' \). Thus, the update procedure of the likelihood function is

\[ u_t = d_t - \hat{d}_{t|t-1} = d_t - CS_{t|t-1} \]

\[ \Omega_t = E(u_t u_t') = C \Sigma_{S} C' \]

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