TWO-DIMENSIONAL GALAXY-GALAXY LENSING: A DIRECT MEASURE OF THE FLATTENING AND ALIGNMENT OF LIGHT AND MASS IN GALAXIES

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ABSTRACT

We propose a new technique to directly measure the shapes of dark matter halos of galaxies using weak gravitational lensing. Extending the standard galaxy-galaxy lensing method, we show that the shape parameters of the mass distribution of foreground galaxies can be measured from the two-dimensional shear field derived from background galaxies. This enables the comparison of the ellipticity of the mass distribution with that of the light in galaxies as well as an estimate of the degree of alignment between the stellar component and dark matter. We choose the specific case of an elliptical, isothermal profile and estimate the feasibility and significance of the detection of this signal. The prospects for applying this technique are excellent with large ongoing surveys like the Sloan Digital Sky Survey. The expected signal is smaller but comparable in significance to that of the mass in standard galaxy-galaxy lensing analyses. Since shapes of halos depend on the degree of dissipation and the transfer of angular momentum during galaxy assembly, constraints obtained from the analysis will provide important input to models of galaxy formation.

Subject headings: galaxies: fundamental parameters — galaxies: halos — gravitational lensing — methods: numerical

1. INTRODUCTION

Current observations require the existence of dark matter halos for galaxies. However, fundamental parameters such as their total mass and spatial extent are not well constrained. The mass distribution in galaxies is probed primarily via dynamical tracers of the galactic potential on various scales: stars in the inner regions, H I gas in regions outside the optical radius, and orbital motions of bound satellites in the outermost regions (e.g., Zaritsky & White 1994). Probes of halo structure at radii devoid of any luminous tracers are therefore needed; weak gravitational lensing offers precisely that.

Galaxy-galaxy lensing, the preferential tangential alignment of the images of background galaxies around bright foreground ones, is detected statistically by stacking galaxies. The first observational attempt to look for galaxy-galaxy lensing was made by Tyson et al. (1984). Recent studies have been very successful, and a signal at the 99.5% confidence level was first reported by Brainerd, Blandford, & Smail (1996) using deep ground-based CCD data. Several subsequent studies using Hubble Space Telescope images and ground-based data (Griffiths et al. 1996; Dell’Antonio & Tyson 1996; Hudson et al. 1998; T. M. D. Ebbels et al. 2000, in preparation; Fischer et al. 1999: Sloan Digital Sky Survey [SDSS] commissioning data) report unambiguous detection of a galaxy-galaxy lensing signal.

In the method proposed in this Letter, additional information that is available but not exploited in current galaxy-galaxy lensing studies is utilized, namely, the light distribution of galaxies selected to be foreground lenses. The general results derived by Schneider & Bartelmann (1997) are used to relate the shear field to the mass multipole moments. We show that the shapes and orientations of the foreground galaxies (probes of the light) can be compared statistically to that of the shear field (probe of the mass), thus providing a direct method to compare the ellipticity of the light to that of the mass as well as any potential misalignments between them reliably. These parameters offer an important clue to the galaxy formation process, since they provide a quantitative measure of the importance of dissipation in the assembly of galaxies. Any variation of the flattening of the total mass (predominantly the dark matter component) with radius is a probe of the efficiency of angular momentum transfer to the dark halo and might provide insight into the structure and composition of galaxy halos. With regard to the relative orientation of the light and mass in galaxies, the two components are likely to be aligned on average and any misalignments might occur transiently, for instance, after a violent merger.

The outline of this Letter is as follows. In § 2, the current status of modeling in galaxy-galaxy lensing studies is reviewed. In § 3 the formalism used to extract the shape of the mass distribution is presented, with the application to an elliptical isothermal mass model in § 4. The feasibility of detection of the flattening of the mass is examined in § 5, and a measure of the alignment of mass and light is discussed in § 6. We conclude in § 7 with a discussion of the importance of studying shapes of dark halos for understanding key issues in the formation and structure of galaxies and the relation between the luminous and dark component in galaxies.

2. CURRENT STATUS OF GALAXY-GALAXY LENSING

The galaxy-galaxy lensing signal is measured ideally from a deep image by selecting a population of brighter (assumed to be foreground) galaxies as lenses and measuring the induced shear distortion in the fainter (background) galaxies. The shear signal obtained from direct averaging in radial bins around the bright foreground galaxies is then stacked to obtain the radial profile of the shear γ as a function of distance from the lens. This provides reasonable constraints on the circular velocity of a fiducial halo, found to be in the range 210–250 km s$^{-1}$ for a typical $L^*$ galaxy but fairly insensitive to the radial extent of the halos (consistent with halo sizes in excess of 100 $h^{-1}$ kpc). In analyses to date, the errors are dominated by shot noise and error arising from the unknown redshift distribution of galaxies. However, with forthcoming surveys like the SDSS,
which plan to image over one hundred million galaxies in many bands and provide reliable photometric redshifts, the prospects for galaxy-galaxy lensing studies are extremely good (Fischer et al. 1999).

3. EXTRACTING SHAPE PARAMETERS FOR THE MASS DISTRIBUTION

The mass distribution of a lensing galaxy is described by the convergence field \( \kappa(x) \), defined to be the surface mass density \( \Sigma(x) \) expressed in units of the critical surface mass density \( \Sigma_{\text{crit}} \). The critical surface mass density depends on the precise configuration, i.e., on the angular diameter distances from the lens to the source \( D_{\text{ls}} \), observer to source \( D_{\text{os}} \), and observer to lens \( D_{\text{ol}} \). It is given by \( \Sigma_{\text{crit}} = (c^2/4\pi G) (D_{\text{ls}} D_{\text{os}} D_{\text{ol}}) \). Standard galaxy-galaxy lensing provides a measure of the mass \( M \) within an aperture, which is given by

\[
M = \int d^2x w(x) \kappa(x),
\]

where \( w(x) \) is a weight function normalized so that \( \int d^2x w(x) = 1 \). It is chosen to be continuous and differentiable and is required to fall off rapidly to zero outside the aperture scale \( \beta \). The shape parameters of the mass distribution are characterized by the quadrupole moments of the convergence \( \kappa(x) \) within the aperture (Schneider & Bartelmann 1997), which are defined as

\[
Q_{ij} = \int d^2x \kappa(x) w(x) x_i x_j.
\]

This tensor can be decomposed into a trace-free piece \( Q \) and a trace \( T \) defined as

\[
Q = Q_{11} - Q_{22} + 2iQ_{12}, \quad T = Q_{11} + Q_{22}.
\]

The ellipticity of the mass \( \epsilon \) is then simply

\[
\epsilon = \frac{Q}{T} = \frac{(a_x^2 - b_x^2)}{(a_x^2 + b_x^2)} e^{i\varphi_x},
\]

where \( a_x \) and \( b_x \) are, respectively, the major and minor axes of the mass distribution and \( \varphi_x \) is its position angle relative to the positive x-axis.

Schneider & Bartelmann (1997) have shown that multipole moments of \( \kappa(x) \) can be computed from the observed shear \( \gamma(x) \) field. In particular, the quadrupole moments (eq. [3]) can be expressed as

\[
Q = \int d^2x e^{2i\varphi} [g_2(x)\gamma_2(x) + ig_1(x)\gamma_1(x)],
\]

where the rotated shear components \( \gamma_1 \) and \( \gamma_2 \) correspond, respectively, to a tangential and curl-type shear pattern about the center of mass of the lens (see Rhodes, Refregier, & Groth 2000 for an illustration). They are related to the unrotated components by

\[
\gamma_i = -|\cos(2\varphi)\gamma_i + \sin(2\varphi)\gamma_j|, \\
\gamma_i = -|\sin(2\varphi)\gamma_i + \cos(2\varphi)\gamma_j|.
\]

where \( \varphi \) is the polar angle from the x-axis. The associated aperture functions \( g_1(x) \) and \( g_2(x) \) are given by

\[
g_1(x) = 2V_2(x) - x^2 w(x), \quad g_2(x) = -2V_2(x),
\]

where \( V_2(x) = x^{-3} \int_0^x dx' x'^{\alpha+1} w(x') \). Similarly, the trace part \( T \) and the mass \( M \) can also be written as

\[
T = \int d^2x g_i(x) \gamma_i(x), \quad M = \int d^2x h_i(x) \gamma_i(x),
\]

where \( h_i(x) = 2V_0(x) - w(x) \).

4. APPLICATION TO THE ELLIPTICAL ISOTHERMAL MODEL

We consider an isothermal model with concentric elliptical equipotentials (Natarajan & Kneib 1996). The projected potential for this model is \( \psi = \alpha r \), where \( \alpha \) is the Einstein radius and \( r \) is a generalized elliptical radius. If the x-axis is aligned with the major axis of the potential, the generalized radius is given by \( r^2 = x_x^2 (1 + \epsilon) + x_y^2 (1 - \epsilon) \), where \( \epsilon \) is the ellipticity of the equipotentials. The Einstein radius is related to the velocity dispersion of the galaxy \( \sigma_x \) by \( \alpha = 4\pi(\sigma_x/c)^2 (D_{\text{ls}} D_{\text{os}}) \) and is of the order of \( 1'' \) for galaxies.

For a weakly elliptical model (\( \epsilon \ll 1 \)), the potential has the form

\[
\psi(x) = \alpha x \left[ 1 - \frac{\epsilon}{2} \cos 2(\varphi - \varphi_0) \right],
\]

where \( \varphi_0 \) is the position angle of the potential, and reduces to that of a singular isothermal sphere in the circular limit (\( \epsilon = 0 \)). Current observational limits on ellipticities of halos have been compiled in a comprehensive recent review by Sackett (1999); however, there are no constraints on the dispersion in the shape parameters. Therefore, for the purposes of this calculation we have used the central value of \( \epsilon = 0.3 \). Besides, higher order terms will be approximately \( (0.3)^2 = 0.09 \), which are still corrections at only the 10% level, consistent with the assumption of small \( \epsilon \).

Restricting our analysis to the weak regime, to first order in \( \epsilon \), the associated convergence \( \kappa = \nabla^2 \psi/2 \), where \( \psi = \alpha r \), is given by

\[
\kappa(x) = \frac{\alpha}{2x} \left[ 1 + \frac{3\epsilon}{2} \cos 2(\varphi - \varphi_0) \right],
\]

and the complex shear \( \gamma = \gamma_1 + i\gamma_2 = |\alpha| - \beta^2 + \ldots \)
2i∂1∂2ψ/2 is

$$\gamma = - \frac{\alpha}{2x} \left[ 1 + \frac{3\epsilon}{2} \cos 2(\phi - \varphi_0) \right] e^{2i\epsilon}. \quad \text{(11)}$$

The rotated shear components are thus

$$\gamma_1 = \frac{\alpha}{2x} \left[ 1 + \frac{3\epsilon}{2} \cos 2(\phi - \varphi_0) \right], \quad \gamma_2 = 0, \quad \text{(12)}$$
yielding a tangential shear modulated by an elliptical pattern.

The ellipticity of the underlying mass distribution $\kappa(x)$ needs to be related to that of the projected (two-dimensional) potential $\phi(x)$. Using a normalized Gaussian as the weight function $w(x) = e^{-x^2/2}/(2\pi^{1/2})$, we evaluate the integral for the quadrupole (eq. [3]) and monopole (eq. [1]) moments of $\kappa(x)$,

$$M = \sqrt{\frac{8}{\pi} \frac{\alpha}{\beta}}, \quad |Q| = \sqrt{\frac{8}{\pi^2} \frac{\alpha}{\beta} \epsilon}, \quad T = \sqrt{\frac{8}{\pi^2} \frac{\alpha}{\beta}}. \quad \text{(13)}$$

The ellipticity of the mass (eq. [4]) is thus $\epsilon_\mu = 3\epsilon/4$. The ellipticity of the potential $\epsilon_\phi$, obtained similarly by taking moments of $\psi$, is $\epsilon_\phi = \epsilon/4$. Note that the ellipticity of the potential $\epsilon_\phi$ computed above is smaller than $\epsilon$ (by a factor of 4), as it is weighted by the circular Gaussian window function. Comparing the weighted ellipticities, we find that $\epsilon_\mu > \epsilon_\phi$, as expected, since equipotentials are always rounder than the mass contours.

5. MEASURING THE FLATTENING OF THE MASS DISTRIBUTION

We now show how these results can be used to measure the flattening of the mass distribution. As in ordinary galaxy-galaxy lensing, the galaxy catalog is separated into a foreground and a background subsample, using magnitude, colors, or photometric redshifts. The ellipticity of the galaxies in both subsamples is then measured by taking second moments of the light distribution. The ellipticities of the foreground sample yield the ellipticity of the light $\epsilon_\mu$ associated with each lens. While $\epsilon$ is ignored in ordinary galaxy-galaxy lensing, we instead align the foreground galaxies along their major axes before stacking. We then measure the average ellipticity of the mass $\epsilon_\mu$, as described above, by replacing the integrals in equations (5) and (8) by sums over the sheared background galaxies. This yields a measurement of the component of the average ellipticity of the mass $\epsilon_\mu$ parallel to the that of the light, i.e.,

$$\epsilon_\mu = \text{Re}(\epsilon_\phi^* \tilde{\epsilon}_l), \quad \text{(14)}$$

where the ellipticities are taken to be complex numbers with $\epsilon = \epsilon_1 + i\epsilon_2$, the asterisk denotes complex conjugation, and $\tilde{\epsilon}_l = \epsilon/l|\epsilon|$ is the unit ellipticity of the light.

We now compute the uncertainty in measuring $\epsilon_\mu$ by taking the square of the mean of the discrete estimators for $M$, $T$, and $Q_l = \text{Re}(Q)$ and converting back into integrals (Schneider & Bartelmann 1997). In the absence of lensing, we find

$$\sigma^2[M] = \frac{\sigma^2}{n_p n_A} \int d^2x h^2(x),$$
$$\sigma^2[T] = \frac{\sigma^2}{n_p n_A} \int d^2x g^2(x),$$
$$\sigma^2[Q_l] = \frac{\sigma^2}{2 n_p n_A} \int d^2x [g^2(x) + g^2(x)] \quad \text{(15)}$$

where $\sigma^2 = \langle \epsilon^2 \rangle = \langle \epsilon^2 \rangle$ is the variance of the intrinsic ellipticity distribution of galaxies ($\sim 0.3^2$), $n_p$ and $n_A$ are, respectively, the number density of background and foreground galaxies, and $A$ is the area covered by the survey.

For the elliptical isothermal model with the Gaussian weight function, we can evaluate these integrals and find

$$\sigma^2[M] = \frac{\sigma^2}{4 \pi n_p n_A \beta^2}, \quad \sigma^2[T] = \frac{\sigma^2}{2 \pi n_p n_A},$$
$$\sigma^2[Q_l] = \frac{3 \sigma^2 \beta^2}{4 \pi n_p n_A}. \quad \text{(16)}$$

By propagating these errors in the definition of the ellipticity of the mass (eq. [4]), we can compute the signal-to-noise ratio ($S/N)_\mu = \epsilon_\mu/\sigma(\epsilon_\mu)$ for measuring $\epsilon_\mu$ and find, to first order in $\epsilon$,

$$S/N)_\mu = 4.6 \frac{\epsilon_\phi}{0.3} \left( \frac{A}{1.5 \text{ arcmin}^{-2}} \right)^{1/2} \times \left( \frac{n_f}{0.035 \text{ arcmin}^{-2}} \right)^{1/2} \left( \frac{0.3}{\sigma} \right) \left( \frac{1000 \text{ deg}^2}{A} \right)^{1/2}. \quad \text{(17)}$$

Here we have chosen to scale $\epsilon_\phi$ in units of 0.3, which is reasonable given current observational limits on the flattening of dark matter halos (see Table 3 in the Sackett 1999 review and references therein; Buote & Canizares 1998). In addition, in these scalings we have used the survey specifications (ellipticity dispersion, number density of foreground lenses $n_f$, number density of background galaxies $n_b$, and approximate observed Einstein radius) quoted for the SDSS commissioning run provided by Fischer et al. (1999) with a modestly expanded area (1000 deg$^2$) from the current area of 225 deg$^2$. In the context of estimating the scatter arising in the mass estimates from galaxy-galaxy lensing due to halo shapes, Fischer et al. (1999) mention in passing that with 10 times more data than the commissioning run, halo shapes can be measured; this figure is comparable to our estimate of the signal-to-noise ratio.

Note that since these numbers have been taken from the SDSS commissioning run (which suffered from poor seeing), they are conservative and do take into account several observational errors. The dispersion of 0.3 in the ellipticity distribution includes a correction for seeing (see, for instance, weak-lensing observations of Rhodes et al. 2000 and Bacon, Refregier, & Ellis 2000 and references therein). The effect of seeing and noise are also reflected in the modest number density assumed for background sources. Note, however, that in contrast to the case of ordinary galaxy-galaxy lensing, foreground galaxies will need to be sufficiently resolved to measure their shapes prior to alignment and stacking. This could induce some
degradation in the signal, but since the foreground galaxies are typically brighter and larger, this effect is expected to be small.

The shape parameters of the mass will therefore be easily detectable with SDSS in the near future (T. McKay et al. 2000, in preparation). For the total SDSS area of $10^4$ deg$^2$, the significance rises to 15 σ. This, in fact, implies that potentially even the radial dependence of the flattening can be studied by considering annuli-shaped weight functions (for instance, the difference of two Gaussians). Moreover, the degree of flattening as a function of the morphological galaxy type can also be studied.

It is interesting to compare the $(S/N)_\epsilon$ expected for measuring $\epsilon_\ell$, estimated above with that of the usual galaxy-galaxy lensing $(S/N)_M = M\sigma[M]$, which measures the mass enclosed within an aperture. For the model considered here, we find the following relation:

$$\langle S/N_\epsilon \rangle = 0.17 \left( \frac{\epsilon_\ell}{0.3} \right) \langle S/N_\ell \rangle .$$

Therefore, shape parameters can be measured with a significance that is smaller but comparable to that of the enclosed mass. Note that for the current SDSS survey area of $A = 225$ deg$^2$, we find $\langle S/N_\ell \rangle \approx 13$, which agrees roughly with the significance of the reported Fischer et al. (1999) results when averaged over all radial bins.

6. MEASURING THE ALIGNMENT OF LIGHT AND MASS

Given the good prospects expected from the above results, one can be more ambitious and try to characterize the alignment of mass and light in more detail. We can make use of the amplitude of the light ellipticity $\epsilon_\ell$, which we have not used until now. This can be done by grouping the lens galaxies into several $\epsilon_\ell$ bins and computing $\epsilon_\ell$ separately for each bin. A more direct approach would be to consider the correlation function of the ellipticities of the mass and light, defined as

$$C_{\ell M} = \text{Re}(\epsilon_\ell \epsilon_\ell^*) ,$$

where the average is over all lens galaxies and $\epsilon_\ell$ is an estimate of the mass ellipticity of each lens derived from its associated background galaxies. While $\epsilon_\ell$ for an individual lens is rather noisy, a significant measurement of $C_{\ell M}$ can be obtained by averaging over a large number of lens galaxies. This correlation function could also be computed for several annuli and would, therefore, yield a direct measure of the radial dependence of the alignment of mass and light.

7. DISCUSSION

The shapes of dark matter halos (see Sackett 1999 for a more comprehensive review) have been probed via many techniques, and the consensus from these studies is that the precise shapes offer important clues to the galaxy formation process and perhaps to the nature of dark matter. Cosmological N-body simulations suggest that dark matter halos are triaxial and that dissipation determines their shape. High-resolution simulations find that the effect of dissipation (Katz & Gunn 1991; Dubinski 1994) is to transform an initially triaxial halo from prolate-triaxial to oblate-triaxial while preserving the degree of flattening, yielding on average rounder and more oblate dark halos than those in dissipationless simulations.

Comparing the shape of the mass profile inferred from X-ray data for a sample of ellipticals with that of the light, Buote & Canizares (1994, 1998) find that the dark matter is at least as flattened as the light and is definitely more extended. The origin of the X-ray isophotal twist in the case of NGC 720, they argue, reflects an intrinsic misalignment of the stars with the dark matter. Keeton, Kochanek, & Seljak (1997) incorporate the shape of the light distribution as a constraint in modeling individual lenses (that produce multiple images of background quasars) and find that an additional component of shear is required to match the observations. They speculate that this component could arise from a misalignment between the luminous galaxy and its dark matter halo. Our proposed technique will provide reliable measurements of the shape and orientation of light and mass in galaxies, thereby aiding in the understanding of the coupling of baryons with the dark matter.

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