First observations of $\psi(2S)$ and $\chi_{cJ}(1P)$ decays to four-body final states $h^+h^-K_S^0K_S^0$ §

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First observations of $\chi_{c0}$, $\chi_{c1}$, and $\chi_{c2}$ decays to $\pi^+\pi^-K_S^0K_S^0$ and $K^+K^-K_S^0K_S^0$, as well as $\psi(2S)$ decay to $\pi^+\pi^-K_S^0K_S^0$ are presented. The branching fractions of these decay channels are determined using $14 \times 10^6 \psi(2S)$ events collected at BESII/BEPC. The branching fractions of $\chi_{c0}$, $\chi_{c2} \rightarrow K^0_SK_S^0$ are measured with improved statistical precision.

PACS numbers: 13.25.Gv, 12.38.Qk, 14.40.Gx

§ The $h^\pm$ denote charged pions or kaons.
I. INTRODUCTION

Experimental data on charmonia and their decay properties are essential input to test QCD models and QCD based calculations. The importance of the Color Octet Mechanism (COM) \([1]\) in radiative decays of the \(\Upsilon\) \([2]\), \(J/\psi\) production in inclusive B decays \([2]\), as well as inclusive decays of P-wave charmonia \([3]\) has been emphasized for many years. Recently, QCD predictions of two-body exclusive decays of P-wave charmonium with the inclusion of the COM have been made \([2, 3]\) and compared to previous measurements \([4, 5]\). More experimental data of two- and four-body exclusive decays of P-wave charmonia with improved precision are important for further testing this new QCD approach including the effect of the COM.

In this paper, results on \(\psi(2S)\) and \(\chi_{cJ}\) \((J = 0, 1, 2)\) two- and four-body hadronic decays with inclusion of a pair of \(K_S^0\) mesons are presented. This analysis is based on \(14 \times 10^6\) \(\psi(2S)\) decays collected with BESII at the BEPC \(e^+e^-\) Collider. A sample of 6.42 pb\(^{-1}\) data taken at 3.65 GeV is used for continuum background studies.

II. BES DETECTOR

The BESII detector is described elsewhere \([6]\). Charged particle momenta are determined with a resolution of \(\sigma_p/p = 1.78\% \sqrt{T + p^2}\) \((p\) in GeV/c\) in a 40-layer main drift chamber (MDC). Particle identification is accomplished using specific ionization \((dE/dx)\) information in the drift chamber and time-of-flight (TOF) information in a barrel-like array of 48 scintillation counters. The \(dE/dx\) resolution is \(\sigma_{dE/dx} = 8\%\); the TOF resolution is \(\sigma_{TOF}=200\) ps for hadrons. A 12-radiation-length barrel shower counter (BSC) measures energies of photons with a resolution of \(\sigma_E/E = 21\% / \sqrt{E}\) \((E\) in GeV\).

III. MONTE CARLO SIMULATION

A Geant3 based Monte Carlo, SIMBES \([7]\), which simulates the detector response, including interactions of secondary particles in the detector material, is used to determine detection efficiencies and mass resolutions, as well as to optimize selection criteria and estimate backgrounds. Under the assumption of a pure E1 transition, the distribution of polar angle \(\theta\) of the photon in \(\psi(2S)\) \(\rightarrow \gamma \chi_{cJ}\) decays is given by \(1 + k \cos^2 \theta\) \([8]\) with \(k = 1, -\frac{3}{2}, \) and \(-\frac{1}{2}\) for \(J = 0, 1,\) and 2, respectively. The angular distributions for \(K_S^0\) mesons from \(\chi_{c0,2} \rightarrow K_S^0 K_S^0\) decays are produced according to the model of \(\chi_{cJ} \rightarrow M M\) \([9]\), where \(M\) stands for a 0\(^-\) meson. Angular distributions for daughters from other decays are generated isotropically in the center-of-mass system of the \(\psi(2S)\) or \(\chi_{cJ}\).

IV. DATA ANALYSIS

To be regarded as a good photon, a shower cluster in the BSC must have an energy deposit of more than 50 MeV and at least one hit in the first six layers of the BSC. To remove soft photons emitted by charged particles, the differences of azimuthal angles, \(d\phi\), and \(z\) coordinates at the first layer of the BSC, \(dz\), between good photons and each charged track must satisfy either a loose requirement (selection-A: \(d\phi > 10^\circ\) or \(dz > 0.3\) m\)) or a tight requirement (selection-B: \(d\phi > 20^\circ\) or \(dz > 1.0\) m\)). Here the \(z\) coordinate is defined to point in the positron direction.

Each charged track is required to have a good helix fit. For final states containing charged kaons, particle identification is required; usable particle identification information in one or both of the MDC \((dE/dx)\) and TOF subsystems is necessary. A particle identification \(\chi^2\) is calculated for each track for the pion, kaon or proton hypotheses using this information, and the associated probability \(prob\) is determined. A track is identified as a kaon, if the probability of the track being a kaon \(prob(K) > 0.01\); otherwise it is regarded as a pion. For final states containing only pions, no particle identification is done and all tracks are assumed to be pions.

Each event is required to contain two \(K_S^0\) mesons. The reconstruction of the decay \(K_S^0 \rightarrow \pi^+\pi^-\) and related checks are described in detail elsewhere \([10]\). A \(K_S^0\) candidate must satisfy \(|M_{\pi^+\pi^-} - M_{K_S^0}| < 20\) MeV and have a decay length transverse to the beam axis \(R_{xy} < 0.3\) cm. The \(K_S^0\) sideband sample, used for background estimation, is selected with one \(\pi^+\pi^-\) pair within the \(K_S^0\) mass window and the other pair in the \(K_S^0\) mass sideband region defined by \(40\) MeV < \(|M_{\pi^+\pi^-} - M_{K_S^0}| < 60\) MeV.

Four constraint \((4C)\) kinematic fits are performed on the selected events for the following decay modes: (1) \(\psi(2S) \rightarrow \gamma K_S^0 K_S^0\), (2) \(\psi(2S) \rightarrow \gamma \pi^+\pi^- K_S^0 K_S^0\), and (3) \(\psi(2S) \rightarrow \gamma K^+ K^- K_S^0 K_S^0\). The fits are made to each combination of a good photon and two \(K_S^0\) candidates in an event, the combination with the minimum \(\chi^2_{4C}\) is selected, and the \(\chi^2_{4C}\) is required to be less than 35. The associated probability \(prob_{4C}\) is calculated.
Background from $\psi(2S) \to \pi^+\pi^- J/\psi$ decay is removed by calculating the mass recoiling, $M_{\text{recoil}}$, against all pairs of oppositely charged tracks, assuming them to be pions, and requiring $|M_{\text{recoil}} - M_{J/\psi}| > 25$ MeV. Background contamination from continuum production is found to be negligible for all decay channels.

An unbinned maximum likelihood method is used in fitting the signal for all decay channels except $\psi(2S) \to h^+ h^- K^0_S K^0_S$. The branching fractions of $\psi(2S) \to \gamma \chi_{cJ}$ ($J = 0, 1, 2$) needed in the measurement are taken from Particle Data Group (PDG) tables [8].

A. $\psi(2S) \to \gamma K^0_S K^0_S$

The decay $\psi(2S) \to \gamma K^0_S K^0_S$ has one photon plus a pair of $K^0_S$ candidates. The event should have four charged tracks with total charge zero. The loose photon selection, selection-A, is applied because of the low background in the channel. The $K^0_S K^0_S$ invariant mass distribution of the selected events is shown in Fig. 1. A few $K^0_S$ sideband events survive the selection, which is consistent with the low background observed in Fig. 1 (a). No background is expected from $\psi(2S) \to \gamma \chi_{cJ}$ with $\chi_{cJ} \to 2(\pi^+\pi^-)$ for $J = 0, 1, 2$ and $\psi(2S) \to \gamma \chi_{c1}$ with $\chi_{c1} \to K^0_S K^{\pm}\pi^{\mp}$ according to the analysis of simulated MC events.

The $K^0_S K^0_S$ invariant mass distribution is fitted with two Breit-Wigner resonances for $\chi_{c0}$ and $\chi_{c2}$, each convoluted with Gaussian resolution functions, plus a second order polynomial background. The $\chi_{c0,2}$ widths in the fitting are fixed to their PDG values [8]. The resulting fit is shown in Fig. 1 (b). Including the $\chi_{c1}$ resonance in the fit yields zero events for the CP violating decay $\chi_{c1} \to K^0_S K^0_S$.

![Graph](image-url)

**FIG. 1**: Distribution of $K^0_S K^0_S$ invariant mass of $\psi(2S) \to \gamma K^0_S K^0_S$ candidates. (a) Points with error bars are data, and the histogram is sideband background. (b) Points with error bars are data, and the solid line is the fit described in the text.
B. $\psi(2S) \rightarrow \gamma \pi^+ \pi^- K^0_S K^0_S$

The $\psi(2S) \rightarrow \gamma \pi^+ \pi^- K^0_S K^0_S$ decay channel contains one photon and six charged tracks with total charge zero. The requirements here are similar to the previous case, but there are two additional pions. Background from $\pi/K$ misidentification is suppressed by the requirement $\text{prob}(\gamma \pi^+ \pi^- K^0_S K^0_S) > \text{prob}(\gamma K^- K^0_S K^0_S)$. The $\pi^+ \pi^- K^0_S K^0_S$ invariant mass distribution for selected events is shown in Fig. 2.

In Fig. 2 there are two kinds of background in the mass region between 3.0 and 3.64 GeV/c$^2$: (1) background corresponding to $K^0_S$ sidebands, and (2) $\psi(2S)$ decays and $\chi_{cJ}$ decays different from the signal channel, where the decays also include a pair of $K^0_S$ mesons. Studies with $K^0_S$ sideband events for both data and MC show that $K^0_S$ sideband background from wrong combinations of $\pi^+ \pi^-$ is slightly enhanced in the $\chi_{cJ}$ signal region. MC studies show that the smooth background spread over the whole mass region from (2) results mainly from the following decay channels: (a) $\psi(2S) \rightarrow \gamma \chi_{cJ}$ with $\chi_{cJ} \rightarrow 3(\pi^+ \pi^-)$ and $\chi_{cJ} \rightarrow K^+ K^- K^0_S K^0_S$, (b) $\psi(2S) \rightarrow \pi^0 \pi^+ \pi^- K^0_S K^0_S$, and (c) $\psi(2S) \rightarrow \omega K^0_S K^0_S$ with $\omega \rightarrow \pi^+ \pi^- \pi^0$. Background events in the high mass region above 3.64 GeV/c$^2$ in Fig. 2 are from $\psi(2S) \rightarrow \pi^+ \pi^- K^0_S K^0_S$ decays combined with an unassociated low energy photon.

The $\pi^+ \pi^- K^0_S K^0_S$ invariant mass distribution between 3.0 to 3.64 GeV/c$^2$ is fitted with three Breit-Wigner resonances $\chi_{cJ}$ ($J = 0, 1, 2$), convoluted with Gaussian resolution functions, plus a second order polynomial background. The widths of the $\chi_{c0,1,2}$ resonances in the fit are fixed to their PDG values. The fit is shown in Fig. 2. The numbers of events in the three peaks determined from the fit include signal and $K^0_S$ sideband background, which is somewhat enhanced in the regions of the peaks. The $K^0_S$ sideband sample for data is fitted with a fake signal shape, found by fitting the MC $K^0_S$ sideband sample, plus a second order polynomial background. The numbers of sideband background events, 5.3, 0.6 and 5.5 for $\chi_{c0}$, $\chi_{c1}$ and $\chi_{c2}$ respectively, are then subtracted from the total numbers of events in three peaks.

FIG. 2: Distribution of $\pi^+ \pi^- K^0_S K^0_S$ invariant mass for $\psi(2S) \rightarrow \gamma \pi^+ \pi^- K^0_S K^0_S$ candidates. Points with error bars are data. The light shaded area in (a) is background simulation, where some unknown branching ratios are normalized to agree with the overall $\chi_{cJ}$ background level, and the dark shaded area is $K^0_S$ sideband. The solid line in (b) is the fit.
C. $\psi(2S) \rightarrow \gamma K^+ K^- K^0_S K^0_S$

The $\psi(2S) \rightarrow \gamma K^+ K^- K^0_S K^0_S$ decay has the same topology as $\psi(2S) \rightarrow \gamma \pi^+ \pi^- K^0_S K^0_S$, and thus it is subject to similar event selection criteria except for the kaon identification requirement for two of the charged tracks. First, the $K^0_S K^0_S$ pair is searched for under the assumption that all charged tracks are pions. Kaon identification is only done for the two charged tracks remaining after reconstruction of the $K^0_S K^0_S$ pair. We also require $\text{prob}_{AC}(\gamma K^+ K^- K^0_S K^0_S) > \text{prob}_{AC}(\gamma \pi^+ \pi^- K^0_S K^0_S)$ for the 4C kinematic fit probabilities to suppress contamination from $\psi(2S) \rightarrow \gamma \pi^+ \pi^- K^0_S K^0_S$ decays. The $K^+ K^- K^0_S K^0_S$ invariant mass distribution for selected events is shown in Fig. 3.

As seen from Fig. 3, only one event survives from the $K^0_S$ sideband sample for data. MC events for the following possible background channels are generated: (1) $\psi(2S) \rightarrow \gamma \chi_{cJ}$ with $\chi_{cJ} \rightarrow 3(\pi^+ \pi^-)$ and $\pi^+ \pi^- K^0_S K^0_S$, (2) $\psi(2S) \rightarrow \pi^+ \pi^- K^0_S K^0_S$, and (3) $\psi(2S) \rightarrow \omega K^0_SK^0_S$ with $\omega \rightarrow \pi^+ \pi^- \pi^0$. However, no event from these background channels survives the selection criteria. Another study with a large sample of simulated $\psi(2S) \rightarrow \text{anything}$ shows that negligible background comes from decays of $\psi(2S) \rightarrow \phi K^0_SK^0_S \rightarrow \pi^0 K^+ K^- K^0_SK^0_S$.

The $K^+ K^- K^0_S K^0_S$ invariant mass distribution is fitted with three Breit-Wigner resonances, $\chi_{cJ}$ ($J = 0, 1, 2$), convoluted with Gaussian resolution functions, plus a flat background. Because of low statistics in the signal region, not only the widths and mass resolutions for the $\chi_{cJ}$ ($J = 0, 1, 2$), but also the masses of the $\chi_{c1}$ and $\chi_{c2}$ in the fitting are fixed to their PDG values. The fitting results are shown in the Fig. 4.

![FIG. 3: Distribution of $K^+ K^- K^0_S K^0_S$ invariant mass of $\psi(2S) \rightarrow \gamma K^+ K^- K^0_S K^0_S$ candidates. Points with error bars are data, and the histogram is sideband background. The solid line is the fit.](image)

D. $\psi(2S) \rightarrow h^+ h^- K^0_S K^0_S$

The selection of $\psi(2S) \rightarrow h^+ h^- K^0_S K^0_S$ decays requires six charged tracks with total charge zero and no good photon in the event, as defined above. Good photons are rejected with the tight selection, selection-B, in order to gain higher detection efficiency for signal events. The $K^0_S$ reconstruction uses all combinations of oppositely charged tracks assuming all tracks are pions. To further suppress background of $\psi(2S)$ radiative decays, a requirement on the missing momentum of six charged tracks is employed: $P_{\text{miss}} < 80$ MeV. The two charged tracks $h^+$ and $h^-$ recoiling against the $K^0_S$ pair are assumed to have the same mass $m$. Using energy-momentum conservation, the mass squared $m^2$ is calculated from

$$m^2 = \frac{E^4 + (P_{h^+}^2 - P_{h^-}^2)^2 - 2E^2(P_{h^+}^2 + P_{h^-}^2)}{4E^2}$$

(1)

where $E = M_{\psi(2S)} - E_{K^0_S K^0_S}$, and $P_{h^\pm}$ is the momentum of $h^+$ or $h^-$. The distribution of $m^2$ for selected events is shown in Fig. 4. The peak at low mass is consistent with $\pi^+ \pi^-\pi^0$; there is no evidence for $K^+ K^-$. Two events from the continuum data sample survive the above selection and their effect will be included in the systematic error. No background is found in MC studies of the following decay channels: (1) $\psi(2S) \rightarrow \gamma \chi_{cJ}$
with \( \chi_{cJ} \rightarrow 3(\pi^+\pi^-) \), \( \pi^+\pi^-K^0_SK^0_S \), and \( K^+K^-K^0_SK^0_S \) and (2) \( \psi(2S) \rightarrow \omega K^0_SK^0_S \) with \( \omega \rightarrow \pi^+\pi^-\pi^0 \). Background estimated using the \( K^0_S \) sideband data is subtracted from the observed number of signal events. A MC study shows that the shape of the charged pion signal in the \( m^2 \) spectrum is well described by a Gaussian function, and its mean and resolution are consistent with data. The spectrum is fitted with a Gaussian signal function and a flat background using a binned maximum likelihood fit where the resolution is fixed to the MC determined value. The fitting result is shown in the Fig. 4.

**FIG. 4:** Distribution of invariant mass squared of the two remaining charged particles after \( K^0_SK^0_S \) selection for \( \psi(2S) \rightarrow h^+h^-K^0_SK^0_S \). (a) Points with error bars are data. The histogram is the \( K^0_S \) sideband background. (b) Points with error bars are the data with the \( K^0_S \) sideband background subtracted. The solid line is the fit.

### E. Systematic Errors

Systematic errors for the efficiency are caused by differences between data and MC simulation. Our studies have determined these errors to be 2% per track for the tracking efficiency, 2% for photon identification, 5% for the 4C kinematic fit, and 2.1% for the \( K^0_S \) reconstruction efficiency. A correction factor due to the overestimate of the \( K^0_S \) reconstruction efficiency of the MC relative to data is determined to be 95.8%. The change of fitting range and background shape function contributes a difference of final results less than 3%. Other systematic errors arise from the uncertainties in the total number of \( \psi(2S) \) events, \((14.00 \pm 0.56) \times 10^6 \) \([15]\), and in the branching fractions for \( K^0_S \rightarrow \pi^+\pi^- \) and \( \psi(2S) \rightarrow \gamma\chi_{cJ} \) \((J=0,1,2)\). In \( \psi(2S) \rightarrow \pi^+\pi^-K^0_SK^0_S \) decay, with two events found in continuum data, an additional error of 7.7% is added.
Possible resonance structures have been searched for the $\chi_{c0} \to \pi^+\pi^- K_0^0 K_S^0$ final state which is the channel with the highest number of observed events. Some excess for inclusive decays of $K^+(892)^+ \to K_S^0 \pi^+$, $f_0(1710) \to K_S^0 K_S^0$, $\rho(770) \to \pi^+\pi^-$ and $f_0(980) \to \pi^+\pi^-$ can be seen from the selected events. Insufficient statistics and complicated structures in these decay modes make it difficult to identify clear signals for two-body decays with intermediate resonances. Efficiencies for final states with resonances, such as $K^+(892)^+ K^+(892)^-$, $K_0^{*+}(1430)^+ K_0^{*-}(1430)^-$, $K_0^{*+}(1430)^+ K_0^{*-}(1430)^-$, $f_0(1370)f_0(1710)$, $f_0(980) f_0(980)$, $f_0(980) f_0(2200)$ and $K_1(1270)^0 K_0^0$ \cite{10} are studied using phase space MC events. The averaged difference in efficiency between final states with and without intermediate resonance is estimated to be 7.7\%, which is regarded as systematic error in the measurements of the branching fractions for the four-body final states. The results of four-body final states $h^+h^- K_0^0 K_S^0$ in our measurements include those of both non-resonance and intermediate resonance.

Final results of signal yield and branching fractions for the $\chi_{cJ}(1P)$ and $\psi(2S)$ two- and four-body hadronic decays involving $K_{S}^{0}$ pair production are summarized in Table IV. The masses of the $\chi_{cJ}$ ($J=0,1,2$) extracted from the fits are also listed. The 90% confidence level (CL) upper limits on the branching fractions in the table are obtained using the Feldman-Cousins method \cite{17}. The branching fractions of $\chi_{cJ}$ $(J=0,1,2)$ decays to $\pi^+\pi^- K_0^0 K_S^0$ and $K^+K^- K_0^0 K_S^0$, as well $\psi(2S)$ decay to $\pi^+\pi^- K_0^0 K_S^0$ are observed for the first time. The branching fractions of $\chi_{c0}$ and $\chi_{c2}$ decays to $K_0^0 K_S^0$ are measured with improved precision.

Decay rates, determined using updated $\chi_{cJ}$ total widths \cite{8} and branching fractions for $\chi_{cJ} \to \pi^0\pi^0$, $\pi^+\pi^-$ \cite{2} and $\chi_{cJ} \to p\overline{p}$ $(J=1,2)$ decays \cite{8}, provide support for the COM (see Table III). According to isospin symmetry, the
TABLE III: Comparison of partial widths for $\chi_{cJ} \rightarrow \pi \pi, K \overline{K}$ and $p\overline{p}$ decays between PDG [8] and the COM predictions. Also shown is the result based on this analysis.

| Decay          | $\Gamma_i$(PDG) in KeV/c² | $\Gamma_i$(COM) in KeV/c² |
|----------------|----------------------------|---------------------------|
| $\chi_{c0} \rightarrow \pi^+\pi^-$ | 49.5 ± 6.7               | 45.4                      |
| $\chi_{c2} \rightarrow \pi^+\pi^-$ | 3.73 ± 0.64              | 3.64                      |
| $\chi_{c0} \rightarrow \pi^0\pi^0$ | 25.3 ± 3.3               | 23.5                      |
| $\chi_{c2} \rightarrow \pi^0\pi^0$ | 2.3 ± 1.5                | 1.93                      |
| $\chi_{c1} \rightarrow p\overline{p}$ | 0.066 ± 0.016           | 0.0562                   |
| $\chi_{c2} \rightarrow p\overline{p}$ | 0.143 ± 0.018           | 0.15419                  |
| $\chi_{c0} \rightarrow K^+K^-$ | 61 ± 10                   | 38.6                      |
| $\chi_{c2} \rightarrow K^+K^-$ | 1.98 ± 0.47              | 2.89                      |
| $\chi_{c0} \rightarrow K^0\overline{K}^0$ | 71 ± 12 (this paper)    | 71 ± 0.80 (this paper)   |
| $\chi_{c2} \rightarrow K^0\overline{K}^0$ | 3.76 ± 0.80             |                           |

$\chi_{cJ} \rightarrow K^0\overline{K}^0$ and $K^+K^-$ decays should have the same partial width. Assuming equal decay widths for $\chi_{cJ} \rightarrow K^0\overline{K}^0_S$ and $K^0\overline{K}^0_L$, we find that the partial width of the $\chi_{c0} \rightarrow K^0\overline{K}^0_S$ decay estimated using the result obtained in this paper is not consistent ($2.7\sigma$) with the COM prediction for $\chi_{c0} \rightarrow K^+K^-$, while the agreement between them for the corresponding $\chi_{c2}$ decay is within 1.1σ. A comparison for the $\chi_{cJ} \rightarrow K^+K^-$ ($J = 0, 2$) decays shows that the discrepancy between PDG values and the COM predictions is 2.2σ and 1.9σ for $\chi_{c0}$ and $\chi_{c2}$ decays, respectively.

Furthermore, the sum of all known $\chi_{c0}$ two-body branching fractions is less than 2%. It therefore is important to measure more $\chi_{cJ}$ decay modes, including two-body modes with intermediate resonance and many-body modes, because of their large contribution to the hadronic decay width. Theoretical predictions with inclusion of the COM for $\chi_{cJ}$ decays to many-body final states are required for comparison with data.

Acknowledgments

The BES collaboration thanks the staff of BEPC for their hard efforts and the members of IHEP computing center for their helpful assistance, and also K. T. Chao and J. X. Wang for helpful discussions on the COM. This work is supported in part by the National Natural Science Foundation of China under contracts Nos. 19991480,10225524,10225525, the Chinese Academy of Sciences under contract No. KJ 95T-03, the 100 Talents Program of CAS under Contract Nos. U-11, U-24, U-25, and the Knowledge Innovation Project of CAS under Contract Nos. U-602, U-34(IHEP); by the National Natural Science Foundation of China under contracts Nos. 19991480,10225524,10225525, the 100 Talents Program of CAS under Contract Nos. U-602, U-34(IHEP); by the National Natural Science Foundation of China under Contract No.10175060(USTC),and No.10225522(Tsinghua University); and by the Department of Energy under Contract No.DE-FG02-04ER41291 (U Hawaii).

[1] G.T. Bodwin, E. Braaten and G.P. Lepage, Phys. Rev. D46, 1914(1992); Phys. Rev. D51, 1125(1995);
[2] F. Maltoni and A. Petrelli, Phys. Rev. D59, 074006(1999);
[3] M. Beneke, F. Maltoni and I.Z. Rothstein, Phys. Rev. D59, 054003(1999);
[4] H.W. Huang and K.T. Chao, Phys. Rev. D54, 6850(1996); A. Petrelli, Phys. Lett. B380, 159(1996).
[5] J. Bolz, P. Kroll and G.A. Schuler, Eur. Phys. J. C2, 705(1998); Phys. Lett. B392, 198(1997).
[6] S.M.H. Wong Nucl. Phys. A674, 185(2000); Eur. Phys. J. C14, 643(2000).
[7] BES Collab., J.Z. Bai et al., Phys. Rev. D67, 032004(2003); Phys. Rev. D67, 112001(2003); Phys. Rev. D60, 072001(1999); Phys. Rev. Lett., 81, 3091(1998). CLEO Collab., B.I. Eisenstein et al., Phys. Rev. Lett., 87, 061801(2001). E835 Collab., M. Andreotti et al., Phys. Rev. Lett., 91, 091801(2003); S. Bagnasco et al., Phys. Lett. B533, 237(2002); M. Ambrogiani et al., Phys. Rev. Lett., 83, 2920(2002).
[8] S. Eidelman et al. (Particle Data Group), Phys. Lett. B592, 1(2004)(URL: http://pdg.lbl.gov).
[9] BES Collab., J.Z. Bai et al., Nucl. Instr. Meth. A458, 627(2001); J.Z. Bai et al., Nucl. Instr. Meth. A344, 319(1994).
[10] H.M. Liu et al., 'The BESII detector simulation', to be published to NIM.
[11] G. Karl et al., Phys. Rev. D13, 1203(1976); L.S. Brown and R.N. Cahn, Phys. Rev. D13, 1195(1976).
[12] P.K. Kabir and A.J.G. Itey, Phys. Rev. D13, 3161(1976).
[13] Zhe Wang et al., HEP&NP, 27, 1 (2003).
[14] J.C. Chen, Phys. Rev. D62, 034003(2000).
[15] X.H Mo et al., HEP&NP, 28 (2004) 455, hep-ex/0407055
[16] BES Collab., M. Ablikim et al., Phys. Rev. D70, 092003 (2004); M. Ablikim et al., contributed paper to LP2005, paper-122.
[17] G. Feldman and R. Cousins, Phys. Rev. D57, 3873(1998); J. Conrad, et al., Phys. Rev. D67, 012002(2003).