Study on mechanical properties of thin-walled cavity structure with a large aspect ratio for ultra-precision optical instrument

Lin Li, Jun Zhong, Jie Sui, Yun-fang Zhang, Wei-zhen Lv, Xiao-yan Wang
Space Optoelectronic Measurement and Perception Lab., Beijing Institute of Control Engineering, Beijing 100190, China

Email: cast_lilin@163.com

Abstract. The maximum size scale of a Thin-walled Cavity Structure (TSC) with A Large Aspect Ratio (TCSA) for ultra-precision optical instrument used in space station is more than 500. The TCSA should be strong enough, and at the same time, its mass should be lighter. Firstly, the characteristics of the thin-walled structure with ring-rib and longitudinal rib cavity were discussed, and the optimization design method of the rib structure was put forward based on the rib contribution theory. Secondly, the mathematical model was established, and the sensitivity analysis of the design variables and objective functions was carried out. On this basis, the iterative formula of the optimization of the rib structure was derived, and the optimization design of the TCSA was carried out. Finally, the system level simulation analysis and mechanical test were carried out for the optimized structure. The results show that the maximum relative changing rate of the first-order frequency before and after the test is 2.05%, the maximum relative error of the first-order frequency between the calculation mode and the test mode is 1.23%, and the fundamental frequency of the whole instrument is greater than 120Hz, which proves that the finite element model is more accurate. The displacement of random response of TCSA after optimization is 0.61mm, the maximum stress is 39.2mpa, far less than the allowable stress, which meets the mechanical requirements, and the adopted optimization method was proved to be useful. At the same time, the high precision optical instrument with good mechanical properties was also proved.

1. Introduction

As an important instrument on the space station, the excellent performance of optical instruments (OIs) will provide primary support for the operation of the space station in-orbit [1]. In the process of rocket launching, the complex and alternating mechanical environment will be transmitted to the spacecraft by means of the satellite rocket adapter, so as to act on the extremely high-precision optical instruments, which will cause serious impact on OIs [2-4]. For ultra-precision OIs, their sensitivity is higher, and the system is more sensitive to the external environment. The primary problem to be solved is how to ensure that the instrument is not been damaged and keep its stability during launching period. The harsh mechanical environment of the rocket is usually implemented through the ground mechanical test to check whether the instruments and equipment meet the requirements.

Because of its special geometry and excellent mechanical properties, TCS is widely used in aerospace, buried pipeline, ship transportation, traffic tunnel, rocket and other fields. It is of great significance to study the mechanical properties in a specific environment. In 1969, Tennyson et al. [5]
have carried out research on the characteristics of TCS, pointing out that local defects will seriously reduce the bearing capacity of cylindrical thin shell structure. The boundary element method can be used to solve the elastic dynamic problems of TCS, which can be divided into three categories: time domain boundary element method [6], frequency-domain boundary element method [7] and basic solution of elastic mechanics [8]. Zhou [9] used the double reciprocity method to analyse the characteristics of TAS with free vibration condition. Liu [10] took the submarine bulkhead as the research object, and makes a theoretical analysis of the cylindrical shell structure with longitudinal rib and ring rib. Alaim [11] proposed a plate finite element method with high-order motion filed for free vibration analysis of stiffened TCS. Rahimi [12] analysed the buckling behaviour of composite stiffened shells under axial compression. Morozov [13] analysed the buckling behaviours of anisotropic stiffened conical shell and cylindrical shell under axial compression, transverse bending, pure bending and torsion. Ringgaard [14] proposed an optimization method to maximize the material removal rate in the milling of TCS without the constraints of forced vibration and flutter stability.

At present, the characters of TCS are mostly focused on the mechanical properties and machining parameters optimization. With the increasingly complex application environment of TCS, how to improve the stiffness and strength has become the focus of the researchers. The ratio of the maximum size to the minimum size of TCSA on the ultra-precision OI used in space station is more than 500. The inner part of the structure is in the form of cavity, and the minimum thickness is only 1mm. TCSA needs to have light weight, high rigidity and good strength. In this paper, the TCSA was studied, the characteristics of TCSA with ring stiffened cavity are discussed, the optimization model of ring stiffened cavity is established. Finally, the numerical analysis and mechanical test of the whole machine performance are carried out, the results show that the optimized TCSA performance is good, and the optimization method of ring rib longitudinal reinforcement is effective.

2. Structural characteristics and optimization design method of ring stiffeners

TCSA on the ultra-precision OI for the space station adopts the ring rib longitudinal bar structure, which is shown in Figure 1. The random acceleration to be borne is 19.6grms, as shown in Table 1.

2.1. Characteristics of ring stiffened cavity

The strength calculation method of ring rib longitudinal reinforcement usually adopts to spread the area of rib evenly on its attached plate structure. As shown in Figure 2, the design of ring rib longitudinal reinforcement on TCS will change the circumferential and longitudinal instability waveforms of the whole structure. In [15], based on the study on the contribution of ribs to the shell strength, an optimization design method based on ribs is proposed to optimize the section shape of each rib, so as to achieve the best structural strength with less material and play the best strengthening effect on the shell.
Table 1. Random acceleration power spectrum.

| Frequency (Hz) | Power spectrum      |
|----------------|---------------------|
| 10–250         | +6 dB/oct           |
| 250–800        | 0.4 g²/Hz           |
| 800–2000       | -9 dB/oct           |
| Grms           | 19.6 grms           |

In order to quantitatively measure the contribution of rib structure to shell, the strain energy of thin-walled shell with thickness \( t \) is expressed by formula (1).

\[
U_0 = u_0^T K_0(t) u_0
\]  

(1)

For a structure with volume \( V \), the structural strain energy with rib \( I \) is expressed by formula (2).

\[
U_i = u_i^T (K_0(t) + K_i(w_i, h_i)) u_i
\]  

(2)

Where, \( w_i, h_i \) is the width and height of the rib.

Assuming that the section of each rib is square, i.e. \( w_i = h_i \), and \( l_i w_i h_i = V \), where \( l_i \) is the length of rib \( i \), then the contribution of rib \( i \) to the thin wall is expressed as formula (3) [15]:

\[
C_i = 1 - \frac{U_i}{U_0}
\]  

(3)

According to formula (3), the more the total strain energy is reduced, the greater the contribution of rib structure is, eq. the more important it is in the overall structure.

2.2. Optimized design and analysis of TCSA

Combined with the theory of ring stiffened cavity and rib contribution, the optimal design of TCSA is carried out. The thickness, \( x_o \), of the outer wall of TCSA is eight longitudinal bars along the Z-direction, and the optimization design is carried out.

2.2.1. Optimization model. Based on the variable density SIMP difference method, the optimization mathematical model with the minimum flexibility of TCSA as the objective function and the volume fraction percentage as the constraint condition is expressed as follows:

\[
\begin{align*}
\min & : F(x) = U^T K U \\
& = \sum_{k=1}^{n} (x_k)^p u_k^T k_{0} u_k \\
\text{s.t.:} & : V(x) = \sum_{k=1}^{n} v_k x_k \leq f V_0 \\
& 0 < x_{\min} \leq x_k \leq 1
\end{align*}
\]  

(4)

Where, \( F(x) \) is structural flexibility, \( K \) is the total stiffness, \( U \) is the displacement vector, \( x_k \) represents the relative density of the \( k\)-th element, \( u_k \) is the displacement vector of the \( k\)-th element, \( k_{0} \) is the micro element stiffness matrix, \( v_k \) represents the solid material volume of the \( k\)-th element, \( f \) represents the volume ratio, and \( V_0 \) is the total volume.

2.2.2. Sensitivity analysis. In the optimization method, sensitivity analysis is often used to study the stability of the optimal solution when the original data is inaccurate or changes, and then determine which parameters have a greater impact on the system. In this paper, sensitivity analysis involves two aspects: design variable and objective function. By analyzing the relationship between the design variables and the intermediate variables, the sensitivity of the target to the intermediate variables is
obtained. Sensitivity analysis can provide important information of new design points for the optimization design to find the search direction.

Firstly, the displacement sensitivity is analyzed. The structural balance equation of the system with static load is expressed as follows:

$$\mathbf{K} \cdot \mathbf{U} = \mathbf{R}$$  \hspace{1cm} (5)

Where, \( \mathbf{K} \) represents the total stiffness matrix of the system, and \( \mathbf{R} \) represents the load matrix.

Equation (5) differential on both sides:

$$\frac{\partial \mathbf{K}}{\partial x_k} \cdot \mathbf{U} + \mathbf{K} \cdot \frac{\partial \mathbf{U}}{\partial x_k} = \frac{\partial \mathbf{R}}{\partial x_k}$$

Then, the displacement sensitivity is expressed by formula (7):

$$\frac{\partial \mathbf{U}}{\partial x_k} = \mathbf{K}^{-1} \cdot \left( \frac{\partial \mathbf{R}}{\partial x_k} - \frac{\partial \mathbf{K}}{\partial x_k} \cdot \mathbf{U} \right)$$

(7)

For stress sensitivity, after finite element discretization, the relationship between element stress and node displacement is expressed by equation (8):

$$\sigma_j = D_j B_j \delta_j$$

Where, \( \sigma_j \) represents stress, \( \delta_j \) represents displacement.

By further differentiation, the stress sensitivity can be obtained as follows:

$$\frac{\partial \sigma_j}{\partial x_k} = \frac{\partial (D_j B_j)}{\partial x_k} \delta_j + D_j B_j \frac{\partial \delta_j}{\partial x_k}$$

(9)

For thin-walled plate element, the relationship between independent variable and intermediate variable is analysed.

$$\mathbf{K}_j = x_i \cdot \mathbf{K}_{M_i} + x_i^3 \cdot \mathbf{K}_{B_i}$$

(10)

When tension and compression are the main loads in thin-walled structure:

$$\mathbf{U}_i = \frac{1}{x_i} \mathbf{K}_{M_i}^{-1} \cdot \mathbf{R}$$

(11)

As an intermediate variable, the displacement relationship in linear state can be obtained. When the main load is bending moment, the second item on the right side of formula (9) plays a leading role, and the displacement can be expressed as:

$$\mathbf{U} = \frac{1}{x_i} \mathbf{K}_{B_i}^{-1} \cdot \mathbf{R}$$

(12)

For the thin-walled beam element, its stiffness and thickness can be regarded as a direct proportion relationship, which is expressed by equation (13):

$$u_i \propto \frac{1}{x_i}$$

(13)

At this point, the reciprocal of the thickness, \( N_i \), can be selected as the optimization variable to establish the linear relationship between the node displacement and the optimization variable. By deriving the objective function from the design variable, the sensitivity of the model can be analysed.

In equation (4), objective function \( F(x) \) differentiates variable \( x_i \):

$$\frac{\partial F(x)}{\partial x_i} = \frac{\partial U^T}{\partial x_i} \cdot \mathbf{K} U + U^T \cdot \frac{\partial \mathbf{K}}{\partial x_i} U + U^T \cdot \frac{\partial U}{\partial x_i}$$

(14)

When the load \( \mathbf{R} \) is constant, equation (7) was rewritten as:

$$\frac{\partial \mathbf{U}}{\partial x_k} = - \mathbf{K}^{-1} \cdot \frac{\partial \mathbf{K}}{\partial x_k} \cdot \mathbf{U}$$

(15)
In the same way, the derivation of both sides of formula (5) after transposition can be obtained as follows:

\[
\frac{\partial U^T}{\partial x_k} = -U^T \frac{\partial K}{\partial x_k} K^{-1}
\]

(16)

From Eqs. (14), (15), and (16), we can get:

\[
\frac{\partial F(x)}{\partial x_k} = -U^T \frac{\partial K}{\partial x_k} U
\]

(17)

Combining formula (4), the sensitivity of the objective function to the design variable can be obtained as follows:

\[
\frac{\partial F(x)}{\partial x_k} = -P(x_k) v_k^T k_k \mu_k \quad (k = 1, 2, ..., n)
\]

(18)

2.2.3. Optimization algorithm. The mathematical model in equation (4) needs to choose the appropriate optimization algorithm to solve it, that is, under certain constraints, to solve the objective function optimally.

The Lagrangian function established first:

\[
L = F + \lambda_1 (V - fV_o) + \lambda_2^T (K U - F) + \sum_{x=1}^{n} \lambda_{3x} (x_{min} - x_{x}) + \sum_{x=1}^{n} \lambda_{4x} (x_{x} - 1)
\]

(19)

Where, \( \lambda_1, \lambda_2, \lambda_{3x}, \lambda_{4x} \) are Lagrange multipliers, all of which are larger than zero.

If the design variable in the function satisfies the extreme value, equation (19) must satisfy the K-T condition, which is expressed by equation (20):

\[
\begin{align*}
\frac{\partial L}{\partial x_k} &= 0 \\
F &= KU \\
V &= fV_o \\
\lambda_{3x} (x_{min} - x_{x}) &= 0 \\
\lambda_{4x} (x_{x} - 1) &= 0 \\
\lambda_{3x} > 0, \lambda_{4x} > 0
\end{align*}
\]

(20)

Combining formulas (4), (15), (18), (19) and (20), the final optimization iteration formula can be obtained, which is expressed by formula (21):

\[
x^{j+1}_k = \begin{cases} 
\max(x_{min}, x'_k - m), & \text{if } x'_k B^j_k \leq \max(x_{min}, x'_k - m) \\
x'_k B^j_k, & \text{if } \max(x_{min}, x'_k - m) \leq x'_k B^j_k \leq \min(1, x'_k + m) \\
\min(1, x'_k + m), & \text{if } \min(1, x'_k + m) \leq x'_k B^j_k
\end{cases}
\]

(21)

In order to avoid large design variables in the iteration process, \( m \) is introduced as the limit constant, which is a damping factor to ensure the stability of the iteration, and the value between 0 and 1 can be useful.

After 53 iterations, the thickness and width of TCSA are 3.05mm and 8.18mm respectively.

3. TCSA performance verification

In order to test the mechanical properties of the structure before and after random vibration tests, random vibration analysis and mechanical tests were carried out respectively.

3.1. Mechanical analysis of random vibration

The random vibration of the whole machine is analysed and the response (3-sigma) value is calculated. Under the z-direction random vibration excitation, the maximum stress position acceleration response...
of TCSA is 52grms, the displacement is 0.61mm, and the maximum stress is 39.2 MPa. The stress nephogram is shown in Figure 3.

![Stress nephogram @ z-axis](image)

**Figure 3.** Stress nephogram @ z-axis

![Measuring points](image)

**Figure 4.** Measuring points.

According to Figure 3, the maximum stress is distributed dispersedly, eq. it is distributed in the eight positions where the sidewall bar on the diaphragm is connected with the diaphragm, and the maximum stress value at this position is far less than the allowable stress value of the material. The simulation analysis results and random vibration test results of M3, M6 and M7 that shown in Figure 4 with the maximum relative error of 4.3%, far less than 10% of the overall test requirements.

### 3.2. Mechanical test

There are six acceleration sensors on the whole OI, among which M5, M6 and M7 are the positions of the sensors on the TCSA, which can monitor the acceleration response characteristics during the test. M3 and M4 are the measuring points on the outer surface of the lower structure. M1 and M2 are the measuring points of the interface accessories installed on the whole instrument, which can monitor the input excitation conditions of the whole instrument.

The first-order modal information of simulation and test is given in Table 2. In the simulation analysis results, the first-order natural frequency of X/Y/Z is close to the first-order natural frequency obtained from the test, and the maximum relative error is 1.23%. The first mode in X/Y direction of the whole machine is swaying along X/Y direction, and the first mode in Z direction is twisting around Z axis. This is consistent with the inherent characteristics of TCSA.

Figure 5 shows the sweep frequency comparison curve of M7 before and after random vibration test in X/Y/Z direction. It can be seen that the sweep frequency curve of M7 before and after random vibration test in X/Y direction is in good agreement. The first-order frequency before and after random vibration test in X direction is 126.52 Hz and 125.22 Hz respectively, the relative change rate before and after test is 1.03%, and the first-order frequency before and after random vibration test in Y direction is 126.52hz and 123.93hz respectively. The relative change rate before and after the test is 2.05%. The sweep frequency curve before and after the z-direction random vibration test is slightly different, the first-order frequency moves forward about 5Hz, and there are undulating characteristic points between 120Hz and 40Hz, indicating that there are some local modes near the longitudinal fundamental frequency of the structure.

| Axis | First order (Hz) analysis | First order (Hz) test | relative errors |
|------|--------------------------|-----------------------|----------------|
| X    | 126.85                   | 126.52                | 0.26%          |
| Y    | 127.26                   | 126.52                | 0.58%          |
| Z    | 284.35                   | 280.83                | 1.23%          |

**Table 2.** Natural frequency and mode shape of TCSA.
In conclusion, the optimized TCSA has better mechanical properties. The mechanical test proves the accuracy of the simulation analysis model, and the optimization method is effective.

4. Conclusion
In this work, mechanical properties of TCSA for Ultra-precision OI are studied.

1) The transverse sweep frequency curve of TCSA matches well before and after mechanical tests. The maximum relative rate of the first-order frequency is 2.05%. The structure is good and undamaged before and after the tests, which shows that TCSA with ring rib longitudinal rib cavity can meet the mechanical conditions.

2) Modal analysis and swept frequency test show that the fundamental frequency of the whole structure is greater than 120Hz, and the maximum relative error of the first-order frequency is 1.23%, which meets the demand of OI.

3) The optimization method used in this paper can effectively optimize the thin-walled structure with ring rib and longitudinal rib cavity, and sensitivity analysis is conducive to the selection of optimization algorithm. This method can be used in the design of the same type of TCS.

Acknowledgments
The authors gratefully acknowledge the financial supports by the National Natural Science Foundation of China (No. 51905034).

References
[1] Li L, Wang X Y, Zhong J. 2019 Optics and Precision Engineering 27(07) 1561-1568
[2] Liu K, Lv C M, Dang W. 2017 Manned Spaceflight 23(2) 222-227
[3] Li L, Wang D, Yang H B. 2016 Optics and Precision Engineering 24(7) 1677-1684
[4] Yang X F, Xin Q. 2016 SPACECRAFT ENVIRONMENT ENGINEERING 33(6) 581-586
[5] TENNYSON R C, MUGGERIDGE. 1969 AIAA J. 7(11) 2127-2131
[6] Aliehali H, Kamalian M, Adampira M. 2016 Acta Geotechnica 11(2) 391-413
[7] Wuense M, Sladek J. 2017 Engineering Analysis with Boundary Elements 84 141-153
[8] Zheng B J, Gao X W, Zhang C. Applied Mathematics and Computation, 2016, 277: 111-126.
[9] Zhou Q, Chen Y. 2019 Chinese Journal of Theoretical and Applied Mechanics 51(1) 146-158
[10] Liu C, Yang Y. 2011 Chinese Journal of Ship Research 06(5) 11-15
[11] A. Alaimo, C. Orlando, S. Valvano. 2019 European Journal of Mechanics 77 1736-1742
[12] Rahimi G H, Zandi M. 2013 Aerespace Science and Technology 24(1) 198-203
[13] Morozov E V, Lopatin A V, Nesterov V A. J. 2011 Composite Structures 93(2) 308-323
[14] Ringgaard K, Mohammadi Y. 2019 Journal of Machine Tools and Manufacture 145 103430
[15] WEI LI. 2018 University of Science and Technology of China