Spin polarized neutron matter within the Dirac-Brueckner-Hartree-Fock approach

F. Sammarruca

Physics Department, University of Idaho, Moscow, ID 83844, U.S.A

P. G. Krastev

Physics Department, Texas A&M University – Commerce, Commerce, TX 75429-3011, U.S.A
(Dated: December 12, 2021)

The relation between energy and density (known as the nuclear equation of state) plays a major role in a variety of nuclear and astrophysical systems. Spin and isospin asymmetries can have a dramatic impact on the equation of state and possibly alter its stability conditions. An example is the possible manifestation of ferromagnetic instabilities, which would indicate the existence, at a certain density, of a spin-polarized state with lower energy than the unpolarized one. This issue is being discussed extensively in the literature and the conclusions are presently very model dependent. We will report and discuss our recent progress in the study of spin-polarized neutron matter. The approach we take is microscopic and relativistic. The calculated neutron matter properties are derived from realistic nucleon-nucleon interactions. This makes it possible to understand the properties of the equation of state in terms of specific features of the nuclear force model.

1. INTRODUCTION

The properties of dense and/or highly asymmetric nuclear matter, where asymmetric may refer to isospin or spin asymmetries, are of great current interest in nuclear physics and astrophysics. This topic is broad-scoped since it reaches out to exotic systems on the nuclear chart as well as, on a dramatically different scale, exotic objects in the universe such as compact stars.

In this paper, we investigate the bulk and single-particle properties of spin-polarized neutron matter. The study of the magnetic properties of dense matter is of considerable interest in conjunction with the physics of pulsars, which are believed to be rapidly rotating neutron stars with strong surface magnetic fields. The polarizability of nuclear matter can have strong effects on neutrino diffusion and, in turn, variations of the neutrino mean free path due to changes in the magnetic susceptibility of neutron matter can impact the physics of supernovae and proton-neutron stars.

The magnetic properties of neutron/nuclear matter have been studied extensively since a long time by many authors and with a variety of theoretical methods. Predictions based on realistic NN potentials and the BHF approach is important if we are to interpret our conclusions in terms of the underlying nuclear force. This is precisely our focus, namely to understand the in-medium behavior of specific components of the nuclear force (in this case, the spin dependence). Different NN potentials can have comparable quality as seen from their global description of NN data and yet differ in specific features. Thus, it will be interesting to explore how, for a given many-body approach, predictions for spin-polarized neutron matter depend upon specific features of the NN potential. Second, it will be insightful to compare with predictions based on a realistic NN potential and the BHF method, especially at the higher densities, where the repulsive Dirac effect can have a dramatic impact on the short-range nature of the force.

This work is organized in the following way: after the introductory notes in this section, we briefly review our theoretical framework (section 2); our results are presented and discussed in section 3; we conclude in section 4 with a short summary and outlook.

2. BRIEF DESCRIPTION OF THE CALCULATION

The starting point of any microscopic calculation of nuclear structure or reactions is a realistic free-space NN interaction. A realistic and quantitative model for the

Reid hard-core potential as well as a non-local separable potentials. Relativistic calculations based on effective meson-nucleon Lagrangians predict the ferromagnetic transition to take place at several times nuclear matter density, with its onset being crucially determined by the inclusion of the isovector mesons. Clearly, the existence of such phase transition depends sensitively on the modeling of the spin-dependent part of the nuclear force and its behavior in the medium. Thus, this unsettled issue goes to the very core of nuclear physics.
clear force with reasonable theoretical foundations is the one-boson-exchange (OBE) model \[30\]. Unless otherwise specified, our standard framework consists of the Bonn B potential together with the Dirac-Brueckner-Hartree-Fock (DBHF) approach to nuclear matter. A detailed description of our application of the DBHF method to nuclear, neutron, and asymmetric matter can be found in our earlier works \[31\, 32\, 33\].

Similarly to what we have done to describe isospin asymmetries of nuclear matter, the single-particle potential is the solution of a set of coupled equations

\[
U_u = U_{ud} + U_{uu} \tag{1}
\]
\[
U_d = U_{du} + U_{dd} \tag{2}
\]

where the second summation indicates integration over the two Fermi seas of spin-up and spin-down neutrons, and

\[
< \sigma, \sigma'|G(\vec{p}, \vec{q})|\sigma, \sigma' > = \sum_{L, L', S, J, M, M_L} \frac{1}{2} \frac{1}{2} \frac{1}{2} |S(\sigma + \sigma') > < \frac{1}{2} \frac{1}{2} \frac{1}{2} S(\sigma + \sigma') > \times < LM_L; S(\sigma + \sigma')|JM > < L'M_L; S(\sigma + \sigma')|JM > \times \frac{i}{2} - L Y_{L', M_L}^*(\vec{k}_{rel}) Y_{L, M_L}(\vec{k}_{rel}) < \sigma \sigma' |G(\vec{k}_{rel}, K_{c.m.})| \sigma' \sigma > \tag{3}
\]

The notation $< j_1 m_1; j_2 m_2 | j_3 m_3 >$ is used for the Clebsh-Gordon coefficients. Clearly, the need to separate the interaction by individual spin components brings along angular dependence, with the result that the single-particle potential depends also on the direction of the momentum. Notice that the $G$-matrix equation is solved using partial wave decomposition and the matrix elements are then summed as in Eq. (4) to provide the new matrix elements in the uncoupled-spin representation needed for Eq. (3). The three-dimensional integration in Eq. (3) is performed in terms of the spherical coordinates of $\vec{q}$, $(q, \theta, \phi)$, with the final result depending upon both magnitude and direction of $\vec{p}$. On the other hand, the scattering equation is solved using relative and center-of-mass coordinates, $k_{rel}$ and $K_{c.m.}$. These are easily related to the momenta of the two particles in the nuclear matter rest frame through the standard definitions $K_{c.m.} = \vec{p} + \vec{q}$ and $k_{rel} = \vec{p} - \vec{q}$. (The latter displays the dependence of the argument of the spherical harmonics upon $\vec{p}$ and $\vec{q}$.)

Solving the $G$-matrix equation requires knowledge of the single-particle potential, which in turn requires knowledge of the interaction. Hence, Eqs. (1-2) together with the $G$-matrix equation constitute a self-consistency problem, which is handled, technically, exactly the same way as previously done for the case of isospin asymmetry \[31\]. The Pauli operator for scattering of two particles with unequal Fermi momenta, contained in the kernel of the $G$-matrix equation, is also defined in perfect analogy with the isospin-asymmetric one \[31\].

\[
Q_{\sigma \sigma'}(p, q, k_F^p, k_F^q) = \begin{cases} 
1 & \text{if } p > k_F^p \text{ and } q > k_F^q \\
0 & \text{otherwise}.
\end{cases} \tag{5}
\]

Notice that, although a full three-dimensional integration is performed in Eq. (3), the usual angle-average procedure is applied to the Pauli operator (when expressed in terms of $k_{rel}$ and $K_{c.m.}$) and to the two-particle propagator in the kernel of the $G$-matrix equation.

Once a self-consistent solution is obtained for the single-particle spectrum, the average potential energy for particles with spin polarization $\sigma$ is obtained as

\[
< U_\sigma > = \frac{1}{2} \frac{1}{2} \rho \int_0^{2\pi} dp \int_0^{2\pi} d\phi \int_0^{\pi} \sigma \sigma' |G(\vec{p}, \vec{q})|\sigma, \sigma' > \tag{6}
\]

The average potential energy per particle is then

\[
< U > = \frac{\rho u < U_u > + \rho d < U_d >}{\rho} \tag{7}
\]

The kinetic energy (or, rather, the free-particle operator in the Dirac equation when using the DBHF framework), is also averaged over magnitude and direction of the momentum. In particular, we calculate the average free-particle energy for spin-up(down) neutrons as

\[
< T_\sigma > = \frac{\int d\Omega \bar{T}(m^*_\sigma(\theta))}{\int d\Omega} \tag{8}
\]

where $\bar{T}$ is the average over the magnitude of the momentum. Notice that the angular dependence comes in through the effective masses, which, being part of the parametrization of the single-particle potential, are themselves direction dependent (and of course different for spin-up or spin-down neutrons).

Finally

\[
< T > = \frac{\rho u < T_u > + \rho d < T_d >}{\rho} \tag{9}
\]

and the average energy per neutron is

\[
\bar{e} = < U > + < T > \tag{10}
\]
As in the case of isospin asymmetry, it can be expected that the dependence of the average energy per particle upon the degree of polarization \[ \bar{\epsilon}(\rho, \beta) = \bar{\epsilon}(\rho, \beta = 0) + S(\rho)\beta^2 \] will follow the law

$$
\bar{\epsilon}(\rho, \beta) = \bar{\epsilon}(\rho, \beta = 0) + S(\rho)\beta^2
$$

(11)

where $\beta$ is the spin asymmetry, defined by $\beta = \rho_u - \rho_d$. A negative value of $S(\rho)$ would signify that a polarized system is more stable than unpolarized neutron matter. From the energy shift,

$$
S(\rho) = \bar{\epsilon}(\rho, \beta = 1) - \bar{\epsilon}(\rho, \beta = 0),
$$

(12)

the magnetic susceptibility can be easily calculated. If the parabolic dependence is assumed, then one can write the magnetic susceptibility as \[ \chi = \frac{\mu^2\rho}{2S(\rho)}, \]

(13)

where $\mu$ is the neutron magnetic moment. The magnetic susceptibility in often expressed in units of $\chi_F$, the magnetic susceptibility of a free Fermi gas,

$$
\chi_F = \frac{\mu^2m}{\hbar^2\pi^2}k_F^3
$$

(14)

where $k_F$ denotes the average Fermi momentum which is related to the total density by

$$
k_F = (3\pi^2\rho)^{1/3}.
$$

(15)

The Fermi momenta for up and down neutrons are

$$
k_F^u = k_F(1 + \beta)^{1/3} \quad k_F^d = k_F(1 - \beta)^{1/3}.
$$

(16)

For the most general case, it will be necessary to combine isospin and spin asymmetry. With twice as many degrees of freedom, the coupled self-consistency problem schematically displayed in Eqs.(1-2) is numerically more involved but straightforward. This is left to a future work.

3. RESULTS AND DISCUSSION

We begin by showing the angular and momentum dependence of the single-neutron potential, see Figs. 1-2. The angular dependence is rather mild, especially at the lowest momenta. As can be reasonably expected, it becomes stronger at larger values of the asymmetry, see Fig. 2. In Fig. 3, the asymmetry dependence is displayed for fixed density and momentum (here the angular dependence is averaged out). As the density of $u$ particles
The average energy per particle at various densities and as a function of the asymmetry parameter is shown in the third frame of Fig. 5. The first two frames display the contribution from the average potential energy and the average kinetic energy, respectively. The parabolic dependence on $\beta$, or linear on $\beta^2$, is obviously verified. In Fig. 6 we show the corresponding predictions obtained with the conventional Brueckner-Hartee-Fock approach. This comparison may be quite insightful, as we further discuss next. We notice that the Dirac energies are overall more repulsive, but the parabolas predicted with the BHF prescription appear to become steeper, relative to each other, as density grows. The energy difference between the totally polarized state and the unpolarized one for both the relativistic and the non-relativistic calculations is shown in Fig. 7. Although initially higher, the growth of the DBHF curve shows a tendency to slow down and the two sets of predictions cross over just above $3\rho_0$.

Before leaving this detour into the non-relativistic model, we observe that the predictions shown in Fig. 6...
are reasonably consistent with those from previous studies which used the Brueckner-Hartree-Fock approach and the Nijmegen II and Reid93 NN potentials \cite{18}. In fact, comparison with that work allows us to make some useful observations concerning the choice of a particular NN potential, for a similar many-body approach (in this case, BHF). We must keep in mind that off-shell differences exist among NN potentials (even if nearly equivalent in their fit of NN scattering data) and those will impact the \( G \)-matrix (which, unlike the \( T \)-matrix, is not constrained by the two-body data). Furthermore, off-shell differences will have a larger impact at high Fermi momenta, where the higher momentum components of the NN potential, (usually also the most model dependent), play a larger role in the calculation. Accordingly, the best agreement between our BHF predictions and those of Ref. \cite{18} is seen at low to moderate densities. Furthermore, as far as differences based on the choice of the NN potential are concerned, we would expect them to be more pronounced for nuclear matter than for pure neutron matter, since the largest variations among modern realistic potentials are typically found in the strength of the tensor force, which is stronger in \( T=0 \) partial waves (obviously absent in the \( nn \) system). This point will be explored in a later investigation.

In the remainder of this paper, we will focus on the DBHF model, which is our standard operational approach. To further explore the possibility of a ferromagnetic transition, we have extended the DBHF calculation to densities as high as \( 10\rho_0 \). The same method as described in Ref. \cite{33} is applied to obtain the energy per particle where a self-consistent solution cannot be obtained (see Section III of Ref. \cite{33} for details). The (angle-averaged) neutron effective masses for both the unpolarized and the fully polarized case are shown in Fig. 8 as a function of density.

DBHF predictions for the average energy per particle are shown in Fig. 9 at densities ranging from \( \rho=0.5\rho_0 \) to \( 10\rho_0 \). What we observe is best seen through the spin-symmetry energy, which we calculate from Eq. (12) and show in Fig. 10. We see that at high density the energy shift between polarized and unpolarized matter continues to grow, but at a smaller rate, and eventually appear to saturate. Similar observations already made in conjunction with isospin asymmetry were explained in terms of stronger short-range repulsion in the Dirac model \cite{33}. It must be kept in mind that some large contributions, such as the one from the \( ^1S_0 \) state, are not allowed in the fully polarized case. Now, if such contributions (typically attractive at normal densities) become more and more repulsive with density (due to the increasing importance of short-range repulsive effects), their absence will amount to less repulsive energies at high density. On the other hand, if large and attractive singlet partial waves remain attractive up to high densities, their suppression (demanded in the totally polarized case) will effectively amount to increased repulsion.

In conclusion, although the curvature of the spin-symmetry energy may suggest that ferromagnetic instabilities are in principle possible within the Dirac model, inspection of Fig. 10 reveals that such transition does...
not take place at least up to 10ρ₀. Clearly it would not be appropriate to explore even higher densities without additional considerations, such as transition to a quark phase. In fact, even on the high side of the densities considered here, softening of the equation of state from additional degrees of freedom not included in the present model may be necessary in order to draw a more definite conclusion.

Finally, in Fig. 11 we show the ratio χ_F/χ, whose behavior is directly related to the spin-symmetry energy, see Eq. (13). Clearly, similar observations apply to both Fig. 11 and Fig. 10. (The magnetic susceptibility would show an infinite discontinuity, corresponding to a sign change of S(ρ), in case of a ferromagnetic instability.)

![Graph showing density dependence of the ratio χ_F/χ](image)

**4. CONCLUSIONS**

We have calculated bulk and single-particle properties of spin-polarized neutron matter. The EOSs we obtain with the DBHF model are generally rather repulsive at the larger densities. The energy of the unpolarized system (where all nn partial waves are allowed) grows rapidly at high density with the result that the energy difference between totally polarized and unpolarized neutron matter tends to slow down with density. This may be interpreted as a precursor of spin-separation instabilities, although no such transition is actually seen up to 10ρ₀. Our analysis allowed us to locate the origin of this behavior in the contributions to the energy from specific partial waves and their behavior in the medium, particularly increased repulsion in the singlet states.

In future work, the impact of further extensions will be considered, such as: examining the effects of contributions that soften the EOS (especially at high density); extending our framework to incorporate both spin- and isospin-asymmetries; examining the temperature dependence of our observations for spin- and isospin-asymmetries of neutron and nuclear matter.

**Acknowledgements**

The authors acknowledge financial support from the U.S. Department of Energy under grant number DE-FG02-03ER41270.

---

[1] D.H. Brownell and J. Callaway, Nuovo Cimento 60 B, 169 (1969).
[2] M.J. Rice, Phys. Lett. 29A, 637 (1969).
[3] J.W. Clark and N.C. Chao, Lettere Nuovo Cimento 2, 185 (1969).
[4] J.W. Clark, Phys. Rev. Lett. 23, 1463 (1969).
[5] S.D. Silverstein, Phys. Rev. Lett. 23, 139 (1969).
[6] E. Östgaard, Nucl. Phys. A 154, 202 (1970).
[7] J.M. Pearson and G. Saunier, Phys. Rev. Lett. 24, 325 (1970).
[8] V.R. Pandharipande, V.K. Garde, and J.K. Srivastava, Phys. Lett. 38B, 485 (1972).
[9] S.O. Bäckman and C.G. Källman, Phys. Lett. 43B, 263 (1973).
[10] P. Haensel, Phys. Rev. C 11, 1822 (1975).
[11] A.D. Jackson, E. Krotscheck, D.E. Meltzer, and R. Smith, Nucl. Phys. A 386, 125 (1982).
[12] J. Dabrowski, Can. J. Phys. 62, 400 (1984).
[13] S. Marcos, R. Niembro, M.L. Quelle, and J. Navarro, Phys. Lett. B 271, 277 (1991).
[14] M. Kutsera and W. Wojcik, Phys. Lett. B 223, 11 (1989).
[15] P. Bernardos, S. Marcos, R. Niembro, M.L. Quelle, Phys. Lett. B 356, 175 (1995).
[16] S. Fantoni, A. Sarsa, and K.E. Schmidt, Phys. Lett. B 87, 181101 (2001).
[17] T. Frick, H. Müther, and A. Sedrakian, Phys. Rev. C 65, 061303 (2002).
[18] I. Vidaña, A. Polls, and A. Ramos, Phys. Rev. C 65, 035804 (2002).
[19] I. Vidaña and Ignazio Bombaci, Phys. Rev. C 66, 045801 (2002).
[20] A.A. Isayev and J. Yang, Phys. Rev. C 69, 025801 (2004).
[21] Fabio L. Braghin, Phys. Rev. C 71, 064303 (2005).
[22] N. Kaiser, Phys. Rev. C 70, 054001 (2004).
[23] A. Rios, A. Polls, and I. Vidaña, Phys. Rev. C 71, 055802 (2005).
[24] M. Kutsera and W. Wojcik, Phys. Lett. B 325, 271 (1994).
[25] A. Vidaurre, J. Navarro, and J. Bernabéu, Astron. Astrophysics 135, 361-364 (1984).
[26] I. Bombaci, A. Polls, A. Ramos, A. Rios, and I. Vidaña, Phys. Lett. B 632, 638-643 (2006).
[27] V. S. Uma Maheswari, D. N. Basu, J. N. De, and S. K. Samaddar, Nucl. Phys. A 615, 516-536 (1997).
[28] W. Zuo, U. Lombardo, and C. W. Shen, in Quark-Gluon Plasma and Heavy Ion Collisions, edited by W. M. Alberico et. al. (World Scientific, Singapore, 2002), p. 192; nucl-th/0204056.
[29] W. Zuo, Caiwan Shen, and U. Lombardo, Phys. Rev. C 67 037301 (2003).
[30] R. Machleidt, Adv. Nucl. Phys. 19, 189 (1989).
[31] D. Alonso and F. Sammarruca, Phys. Rev. C 67, 054301 (2003).
[32] F. Sammarruca, W. Barredo, and P. Krastev, Phys. Rev. C 71, 064306 (2005).
[33] P. Krastev and F. Sammarruca, Phys. Rev. C 74, 025808 (2006).