Design of Decoupling Fuzzy Logic Controller for Quadrotor UAV

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Abstract - This article first briefly introduces the quadrotor UAV model based on the Newton-Euler method, including kinematics model, dynamic model and force and moment model, and then proposes a decoupling fuzzy controller, which can effectively decouple the position and the orientation of the quadrotor UAV, i.e., the inner loop uses the input and output linearization method to control orientation, and the outer loop uses a fuzzy logic controller to control the position of the quadrotor UAV. Finally, we use Matlab to model the quadrotor UAV and design the decoupling fuzzy controller to control the quadrotor UAV. The results show that the tracking performance is good, which proves the feasibility of the controller algorithm.

1. INTRODUCTION
Today, UAV has been applied in military and civilian fields. With the development of communication technology, sensor technology and manufacturing technology, UAV has become more and more intelligent and have become an indispensable part of human life.

There are many types of UAVs, such as: (1) rotorcraft UAV; (2) quadrotor UAV; (3) flapping-wing UAV. In practical applications, rotorcraft UAV and quadrotor UAV are more common and mature, although flapping-wing UAV has received a lot of attention, but it is still under research and exploration, and its practical application may take some time. Compared with the rotorcraft UAV, the quadrotor UAV has a greater advantage, mainly because the quadrotor UAV model is simpler than the rotorcraft model and its mechanical structure is simpler, while the rotorcraft UAV still retains a variable pitch rotor and has very complex structure [1].

Regarding the design of quadrotor UAV controllers, many scholars have conducted research in the past ten years. They are roughly divided into four categories. The first category is the design of classical PID controller, the second category is linear controller, the third category is nonlinear controller and the fourth category is intelligent controller. The first type of control method is to use the classical control idea to control the corresponding controlled output by adjusting the PID parameters of the controller. This is a non-model-based controller design method. The idea of the second type of control method is to linearize the UAV model near the equilibrium point, and then use linear control methods such as LQG, RPT, to stabilize the UAV at this equilibrium point. The idea of the third type of control method is to directly use the UAV nonlinear model and directly use the nonlinear control method to control the UAV. The fourth type of control method is a non-model-based method that uses data or expert experience to design a learning controller.
For the classical PID controller design, 4 PID controllers are designed in literature [2] to control the roll angle, pitch angle, yaw angle and altitude of the quadrotor respectively. The simulation results show that the control method has low error overshoot, small adjustment time and the steady error is almost 0. For the linear controller design, the quaternion method and LQR controller are used in literature [3] to control the attitude of the quadrotor UAV, and two gains are designed, one feedback gain is used when the quadrotor UAV is far away from the reference point, and the other feedback gain is used when UAV has tracked the reference trajectory. In literature [4], RPT, namely Robust and Perfect Tracking Control is used to control the outer loop of UAV to make the unmanned system move along a given trajectory. For nonlinear controller design, the input-output linearization method is used in literature [5] to make the closed loop system of the quadrotor UAV a linear system, and then use linear control theory to control the quadrotor UAV. In the literature [6], the sliding mode control method is used to track the position and yaw angle of the quadrotor UAV and to stabilize its roll angle and pitch angle. The simulation shows that the performance is good. For the design of intelligent controller, the author in the literature [7] used fuzzy control method to stabilize the attitude of quadrotor UAV, so that the roll angle, pitch angle and yaw angle of UAV are stable at the given value. Literature [8] uses the combination of ANN and PID controller. The quadrotor UAV continuously learns and updates the parameter values of the PID controller by analyzing historical data and online data to optimize control performance.

Each of these four types of control methods has advantages and disadvantages. The advantage of classical PID control lies in its simplicity, but its disadvantage lies in that it can only perform simple control schemes that stabilize at a given value and the parameter adjustment depends on experience. The advantage of linear control is that the model is simple and the online calculation is fast, but the disadvantage is that the scope of application is small. In practical applications, most of the objects are nonlinear models. Therefore, the linear control method can only be applied to the neighborhood of the interest point, which will lead to model error. The advantage of nonlinear control lies in its accuracy. It directly adopts the nonlinear model of the object, which is closer to the real object, but the disadvantages are also very obvious. One is that the amount of calculation is large so it is difficult to achieve online control. The other is that nonlinear control is still in theoretical development, therefore, there is still some distance to apply nonlinear control in actual world. Intelligent control is mainly fuzzy logic control and neural network control. Their advantage is that they control the system directly based on empirical inference engines or big data. The generation of control commands does not need to be based on models, thus speeding up the online control rate. However, it also has disadvantages. The major disadvantage is that there is still a lack of theoretical research on intelligent methods, and the lack of theoretical support in application will cause a "black box" phenomenon.

Although most quadrotor UAVs in the business still use PID control, the research on other control algorithms will inevitably promote the development of quadrotor UAV control. Especially with the introduction of the concept of intelligence, the research on intelligent control is more important. This article improves the fuzzy logic controller proposed in literature [9] and uses decoupling fuzzy controller to control the UAV.

This article is organized in the following sections. The second section describes the kinematics and dynamics model of the quadrotor, the third section describes the application of the fuzzy logic controller in the quadrotor UAV and shows the simulation results, and the last section is a summary.

2. MODEL OF QUADROTOR UAV
The Quadrotor UAV model is roughly divided into kinematic model and dynamic model. The kinematic model describes the relationship between the position and velocity of the UAV, and the dynamic model describes the relationship between the force of the UAV and the resulting acceleration.

2.1. Adopted Coordinate System
In this article, three coordinate systems are mainly used, namely the inertial coordinate system, the vehicle coordinate system and the body coordinate system.
1. The inertial coordinate system, also called the basic coordinate system, is a coordinate system fixed at a certain position in space, expressed by $F_{\text{inertial}} = (x_{\text{inertial}}, y_{\text{inertial}}, z_{\text{inertial}})$.

2. Vehicle coordinate system is a coordinate system in which the origin is fixed on the quadrotor UAV, and the x-axis, y-axis and z-axis are aligned with the corresponding coordinate axes of the inertial coordinate system, represented by $F_{\text{vehicle}} = (x_{\text{vehicle}}, y_{\text{vehicle}}, z_{\text{vehicle}})$.

3. Body coordinate system is a right-hand coordinate system with an origin fixed on the quadrotor UAV, and the x-axis is aligned with the arm of front motor, the y-axis is aligned with the arm of left motor. It is represented by $F_{\text{body}} = (x_{\text{body}}, y_{\text{body}}, z_{\text{body}})$.

2.2. Quadrotor UAV Kinematics Model

The quadrotor UAV kinematics model is roughly divided into a translational motion model and a rotational motion model.

The speed relationship of the translational motion of quadrotor UAV between the inertial coordinate system, the vehicle coordinate system and the body coordinate system are as follows.

\[
F_{\text{vehicle}} = \begin{bmatrix} x_{\text{vehicle}} \\ y_{\text{vehicle}} \\ z_{\text{vehicle}} \end{bmatrix}, \quad F_{\text{body}} = \begin{bmatrix} x_{\text{body}} \\ y_{\text{body}} \\ z_{\text{body}} \end{bmatrix}
\]

Where $F_{\text{vehicle}}$ is the rotation matrix, which is defined as follow.

\[
F_{\text{vehicle}} = \begin{bmatrix}
\cos \phi & \cos \psi \sin \phi & \sin \psi \sin \phi \\
\cos \theta \sin \phi & -\sin \theta \cos \phi & -\sin \phi \cos \theta \\
\sin \theta \sin \psi & \cos \theta \cos \psi & \cos \theta \sin \psi \\
\end{bmatrix}
\]

Where $\phi$, $\theta$, $\psi$ represent roll, pitch and yaw angle respectively.

The speed relationship of the rotational motion of quadrotor UAV between the vehicle coordinate system and the body coordinate system is as follow.

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi} \\
\end{bmatrix} = \mathbf{T} \begin{bmatrix}
\dot{\mathbf{r}} \\
\dot{\mathbf{e}} \\
\end{bmatrix}
\]

Where $\mathbf{T}$ is transformation matrix, represented by the following matrix.

\[
\mathbf{T} = \begin{bmatrix}
1 & \sin \theta \sin \phi & \cos \theta \sin \phi \\
\cos \theta & \cos \theta \cos \phi & -\sin \phi \\
-\sin \theta & \sin \theta \cos \phi & \cos \phi \\
\end{bmatrix}
\]

Where $\phi$, $\theta$, $\psi$ are roll, pitch and yaw angle, $\mathbf{r}$ is the angular velocity relative to the body frame.

2.3. Quadrotor UAV Dynamics Model

The dynamic model of quadrotor UAV can be obtained in two ways, one is Newton-Euler formalism and the other is Euler-Lagrangian formalism. This article uses Newton-Euler formalism to build the quadrotor UAV model.

By applying the Newton-Euler method, the following dynamic model equation based on the body frame can be obtained.

\[
\begin{bmatrix}
M_{\text{body}} & 0 \\
0 & J_{\text{body}} \\
\end{bmatrix} \begin{bmatrix}
\dot{p}_x \\
\dot{p}_y \\
\dot{p}_z \\
\dot{q}_\psi \\
\dot{q}_\theta \\
\dot{q}_\phi \\
\end{bmatrix} + \begin{bmatrix}
\omega_{\text{body}} \times M_{\text{body}} \\
\omega_{\text{body}} \times J_{\text{body}} \dot{\omega}_{\text{body}} \\
\end{bmatrix} = \begin{bmatrix}
\tau_x \\
\tau_y \\
\tau_z \\
\end{bmatrix}
\]

After simplification, Newton dynamics equation and Euler dynamics equation can be obtained respectively as follows.

\[
\begin{bmatrix}
\dot{p}_x \\
\dot{p}_y \\
\dot{p}_z \\
\dot{q}_\psi \\
\dot{q}_\theta \\
\dot{q}_\phi \\
\end{bmatrix} = \begin{bmatrix}
\phi \dot{p}_y - \theta \dot{p}_z \\
\theta \dot{p}_x - \phi \dot{p}_z \\
\psi \dot{p}_x + \theta \dot{p}_x \\
\phi \dot{q}_\psi - \theta \dot{q}_\theta \\
\theta \dot{q}_\theta + \phi \dot{q}_\phi \\
\psi \dot{q}_\theta + \theta \dot{q}_\phi \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{q}_\psi \\
\dot{q}_\theta \\
\dot{q}_\phi \\
\end{bmatrix} = \begin{bmatrix}
\frac{1}{I_\psi} \tau_z \\
\frac{1}{I_\theta} \tau_y \\
\frac{1}{I_\phi} \tau_x \\
\end{bmatrix}
\]
2.4. Quadrotor UAV Force and Moment Model
The control input of the quadrotor UAV model is the lift perpendicular to the UAV plane generated by the rotation of the four rotors at the end of each arm. The moments generated by these four lift forces at the same time make the quadrotor UAV complete the rotation motion.

Specifically, they are upward thrust, rolling torque, pitching torque and yawing torque. Their expressions are as follows.

\[
T = T_f + T_b + T_l + T_r \quad (8)
\]

\[
\tau_x = (T_f - T_l) \quad (9)
\]

\[
\tau_y = (T_b - T_r) \quad (10)
\]

\[
\tau_z = K(T_f + T_b - T_l - T_r) \quad (11)
\]

Equation (8) is the calculation formula of the upward thrust, equation (9) is the calculation formula of the rolling torque, equation (10) is the calculation formula of the pitching torque and equation (11) is the calculation formula of the yawing torque.

3. Design of Decoupling Fuzzy Logic Controller on Quadrotor UAV

3.1. Fuzzy Logic Control
Fuzzy logic controllers are roughly divided into three types of fuzzy inference controllers, namely Mamdani, Sugeno and Tsukamoto fuzzy logic controller [10]. The principles of these three types of fuzzy controllers are similar. They all use a fuzzy rule to determine the output. The difference between them is their different membership functions. This article mainly adopts the Mamdani fuzzy logic controller.

The working principle of Mamdani fuzzy logic controller is described as follow.

If \( x_1 \) is \( A^K_i \) and \( x_2 \) is \( A^K_j \), then \( y \) is \( B^K \).

Its membership function is shown below.

![Figure 1 membership function of Mamdani fuzzy logic controller](image)

3.2. Fuzzy Rule
The key of the fuzzy controller is the fuzzy rule, and the corresponding control strategy is carried out according to the fuzzy rule. This article adopts the fuzzy rules as shown in the figure below.

![Figure 2 fuzzy rules](image)

In the above figure, \( \varepsilon \) is the error between the reference position and the actual position, and \( \dot{\varepsilon} \) is the error between the reference speed and the actual speed. N, Z, and P respectively represent the corresponding membership function.

In this article, defuzzification method uses the area-weighted operator, which calculates the area occupied by the output of each of the nine rules in the output curve, and then divides it by the output of all of the nine rules in the output curve. In this way, corresponding weight can be obtained.
The three-dimensional surface diagram in the figure below shows the relationship between the output of the fuzzy controller and the error $\varepsilon, \dot{\varepsilon}$.

![Surface diagram of fuzzy controller](image)

**Figure 3** surface diagram of fuzzy controller

### 3.3. Controller Algorithm and Architecture

The controller consists of three fuzzy controllers in the outer loop and input-output linearization feedback control in the inner loop. The three fuzzy controllers respectively control the acceleration $\ddot{p}_x, \ddot{p}_y, \ddot{p}_z$ of the quadrotor UAV along the x, y, and z axes of its vehicle frame. Linearization feedback is to eliminate the nonlinear term in the quadrotor UAV, making the closed-loop system a chain integrator system. Now we propose the calculation steps of the decoupled fuzzy controller algorithm.

**Procedure:**

**Step 1:** Calculating the error between the given position and the real position and the error between the given speed and the real speed, as shown in the following formula.

$$
\begin{align*}
\varepsilon_x &= \text{Position} - \text{Punctual} \\
\varepsilon_y &= \text{Position} - \text{Punctual} \\
\varepsilon_z &= \text{Position} - \text{Punctual}
\end{align*}
$$

**Step 2:** Designing three fuzzy controllers separately to control the acceleration $\ddot{p}_x, \ddot{p}_y, \ddot{p}_z$ of the quadrotor UAV along the x, y and z axis of vehicle frame. The inputs of these three fuzzy controllers are the position error $\varepsilon_x$ and the velocity error $\dot{\varepsilon}_x$ along the x axis of the vehicle frame, the position error $\varepsilon_y$ and the velocity error $\dot{\varepsilon}_y$ along the y axis of the vehicle frame, and the position error $\varepsilon_z$ and the velocity error $\dot{\varepsilon}_z$ along the z axis of the vehicle frame. The outputs are the acceleration $\ddot{p}_x$ along the x axis of the vehicle frame, the acceleration $\ddot{p}_y$ along the y axis of the vehicle and the acceleration $\ddot{p}_z$ along the z axis of the vehicle frame. The structure of the fuzzy controller is shown in the figure below.

![Structure of fuzzy logic controller](image)

**Figure 4** structure of fuzzy logic controller

**Step 3:** After obtaining the accelerations $\ddot{p}_x, \ddot{p}_y, \ddot{p}_z$ along the three orthogonal directions, we view them as the given values and then use the space mechanics balance analysis method to obtain the required roll angle $\phi_{\text{reference}}$, pitch angle $\theta_{\text{reference}}$, the calculation formula is as follow.

$$
\phi_{\text{reference}} = \arctan\left(\frac{\ddot{p}_x}{\ddot{p}_y}\right)
$$

(14)
And the reference value of yaw angle is given.

Step 4: Taking the $\phi_{\text{reference}}$, $\theta_{\text{reference}}$, and $\psi_{\text{reference}}$ obtained in step 3 as the given value of the inner loop system, so that the roll angle $\phi$, pitch angle $\theta$ and yaw angle $\psi$ of the quadrotor UAV respectively track $\phi_{\text{reference}}$, $\theta_{\text{reference}}$, and $\psi_{\text{reference}}$. The inner loop system is a model-based nonlinear control system, and the state space equation is shown below.

$$
\begin{align*}
I_x \dot{\phi} + (I_z - I_y) \dot{\psi} \dot{\phi} &= \tau_\phi \\
I_y \dot{\theta} + (I_x - I_z) \dot{\phi} \dot{\theta} &= \tau_\theta \\
I_z \dot{\psi} + (I_y - I_x) \dot{\phi} \dot{\psi} &= \tau_\psi
\end{align*}
$$

Where $\tau_\phi$, $\tau_\theta$, $\tau_\psi$ are rolling torque, pitching torque and yawing torque respectively.

Let

$$
\begin{align*}
\dot{\phi} &= \phi_{\text{reference}} - \phi \\
\dot{\theta} &= \theta_{\text{reference}} - \theta \\
\dot{\psi} &= \psi_{\text{reference}} - \psi
\end{align*}
$$

Then we can get

$$
\begin{align*}
\dot{e}_\phi &= -\phi \\
\dot{e}_\theta &= -\theta \\
\dot{e}_\psi &= -\psi
\end{align*}
$$

Let $x_1 = e_\phi$, $x_2 = \dot{e}_\phi$, $x_3 = e_\theta$, $x_4 = \dot{e}_\theta$, $x_5 = e_\psi$, $x_6 = \dot{e}_\psi$.

Then we can get the following state space equation.

$$
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{I_x - I_y}{I_x} x_3 x_6 - \frac{1}{I_x} \tau_\phi \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{I_x - I_z}{I_x} x_5 x_6 - \frac{1}{I_x} \tau_\theta \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= \frac{I_y - I_x}{I_x} x_3 x_4 - \frac{1}{I_x} \tau_\psi
\end{align*}
$$

Then using input-output linearization feedback and let the input $\tau_\phi$, $\tau_\theta$, $\tau_\psi$ are respectively

$$
\begin{align*}
\tau_\phi &= (I_z - I_y)x_4 x_6 + x_1 + x_2 \\
\tau_\theta &= (I_x - I_z)x_5 x_6 + x_3 + x_4 \\
\tau_\psi &= (I_y - I_x)x_3 x_4 + x_5 + x_6
\end{align*}
$$

Then the following linear closed loop system can be obtained.

$$
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{1}{I_x} x_1 - \frac{1}{I_x} x_2 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -\frac{1}{I_y} x_3 - \frac{1}{I_y} x_4 \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= -\frac{1}{I_x} x_5 - \frac{1}{I_x} x_6
\end{align*}
$$

From the state space equation, the linear system is exponentially convergent, so the roll angle, pitch angle and yaw angle of the quadrotor UAV will exponentially converge to the given value $\phi_{\text{reference}}$, $\theta_{\text{reference}}$, $\psi_{\text{reference}}$.

Step 5: Calculating the lift of the four rotors respectively. According to the force and moment model of section 2.4, the lift forces of the four rotors can be calculated separately and the calculation formulas are as follows.
In the above formulas, $M$ is the total weight of the quadrotor UAV, $l$ is the arm length of the UAV, and $K$ is the yawing torque constant.

So far, the decoupling fuzzy controller algorithm is over, and the following is the structure diagram of the controller.

Figure 5 structure diagram of decoupling fuzzy controller

From the above algorithm process and controller structure diagram, it can be seen that the fuzzy controller can control the position and attitude of the quadrotor UAV well. The outer loop controls the position of the quadrotor UAV and the inner loop controls its attitude, which has strong coupling. In this way the strong coupling quadrotor UAV model is decoupled and its six controlled outputs are respectively controlled, so it is a decoupling control method.

3.4. Simulation Result

This article uses Matlab to simulate the quadrotor UAV model and the fuzzy logic controller, and then selects two reference space trajectories to test the performance of this fuzzy logic controller. The first trajectory is a horizontal circular motion, and the second trajectory is a three-dimensional elliptical motion, and the direction of the quadrotor UAV is always tangent to the circular direction, namely the given trajectory of yaw angle is

$$\psi_{\text{reference}} = 1$$

The two reference trajectories are as follows.

Figure 6 reference trajectory

Part of the Matlab code is attached to appendix. The following figure shows the simulation result (the initial position of the quadrotor UAV is (0, -10, 0)).
The left graph in Figure 7 represents the tracking performance of the quadrotor UAV along the first reference space trajectory, the right graph represents the tracking performance along the second reference space trajectory, and Figure 8 represents the yawing angle error $\theta_{\text{reference}}$ between the reference yawing angle $\theta_{\text{reference}}$ and actual yawing angle $\theta_{\text{actual}}$ around the z axis of vehicle frame.

It can be concluded from Figure 7 that the tracking performance of the decoupled fuzzy control algorithm is great. No matter where the initial position is, it can eventually converge to the reference trajectory and keep moving along the trajectory. Besides, it can be concluded from Figure 8 that the control algorithm can control the direction of the quadrotor UAV, so that the direction of the quadrotor UAV can be changed according to the reference value.

4. CONCLUSION
This article generally introduces the quadrotor UAV model and the design algorithm of the decoupled fuzzy logic controller. Regarding of quadrotor UAV models, three parts including kinematics model, dynamics model and force and moment model are introduced. In terms of controller design, the concept of fuzzy controller and the application of fuzzy controller on quadrotor UAV are introduced. The main contribution of this article is to propose a controller algorithm that can decouple the position and attitude of the quadrotor UAV model. The controller algorithm mainly uses fuzzy control method and input-output linearization method. The simulation example shows that the controller can make the quadrotor UAV move along the reference trajectory, and the convergence effect is good, which verifies the feasibility of the designed controller algorithm.

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