Comparison of equivalent model methods of joint surface based on modal analysis

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Abstract. The mechanical joint surface plays an important role in transmitting motion, load and energy during the normal operation of the mechanical system. Effective mechanical joint equivalent models are established to solve the practical problem of the dynamic model engineering. Equivalent models of joint surface in this study include spring damping method, virtual material method and Bond connection method. Firstly, the model is established by CATIA. Secondly, the physical properties of the model are assigned by ANSYS or Abaqus finite element simulation software. Then the modal analysis of the model carries out. By comparing the mode shapes and natural frequencies of the three methods, it is concluded that the mode shapes of the three methods are basically the same. Therefore, all three methods are reliable joint surface equivalent models. However, the natural frequency obtained by the virtual material method is significantly lower than the natural frequencies obtained by the other two methods. It is concluded by analysis that the natural frequency error obtained by the virtual material method is small, and the spring damping method and the finite element method have large errors.

Keywords: Joint equivalent model, Modal mode, Modal test.

1. Introduction

In terms of finite element-based joint surface dynamics modeling methods, with the popularity of computers and the increasing speed of computing, scholars are increasingly applying numerical methods to simulate direct calculations. The focus of their research is mainly on what units to replace the joint surface, how to identify the parameters of these joint surfaces, and how to equilibrate these parameters into the finite element simulation unit to establish an efficient and reliable universally used joint surface equivalent model.

The classical equivalent unit is a spring-damping unit. When the rigid plane is in contact with the rough surface, the dynamic system can be equivalently modeled as a single-degree-of-freedom spring damping system [1]. On the one hand, the stiffness of the spring is considered to be the volumetric...
stiffness of the surface. It should be pointed out that the spring is nonlinear in nature. On the other hand, the damping model is considered to be linear and viscous with a linear damping coefficient. However, the model has the following disadvantages: (1) The number and position of the spring-damper unit cannot be accurately determined; (2) The spring-damper model characterizes the nonlinear characteristics of the joint is difficult. It is also possible to assume the microscopic contact portion of the two contact faces of the joint as a virtual isotropic material. The theoretical modulus [2, 3, 4, 5] is used to obtain the elastic modulus, Poisson’s ratio, density of the virtual material, and then determine the thickness of the virtual material according to the actual gap between the two surfaces. Meyer [6] uses the partial-partial contact zero-thickness unit pair. The joint is subjected to finite element analysis. Jie Zhang [7] proposed the concept of ideal joint surface and joint surface element. On this basis, many people [8] draw on the basic idea of finite element and proposes new methods of joint surface dynamics modeling-finite element method. It was found that the natural frequency deviation between the simulation analysis and the experiment was large.

2. Establishment of joint surface equivalent model

2.1. Spring damping method

The spring damping equivalent method uses several concentrated spring-damping to simulate a pair of contact surfaces. The contact stiffness of the joint surface is replaced by the stiffness of the spring. The contact damping of the joint surface is equivalent to the damping value of the damper. In this study, four pairs of spring-damper units are used to characterize the joint surface of the two plates. The four sets of spring-damper units are located at the four corners of the joint surface respectively.

The equivalent normal stiffness value of the joint surface is \( K_n \) and the normal damping value is \( C_n \). The equivalent tangential stiffness of the joint surface is \( K_t \) and the tangential damping is \( C_t \). The normal stiffness value \( K_n \) of each spring is \( K_n / 4 \) and the tangential stiffness values \( K_{nt} \) and \( K_{yt} \) are \( K_t / 4 \). Since the damping has no effect on the natural frequency, the finite element analysis in this study will have a damping ratio of 0.5.

The equivalent normal stiffness value \( K_n \) is obtained from the literature [5].

\[
K_n = \frac{2.79 \lambda \sigma_y \alpha_e^{-\delta} (n+1)(D-1) a_i^{0.5D-0.5} (a_i - a_e)}{2^{0.5-1.5D} G^{D-1} (\ln \gamma)^{0.5} \alpha_e^{0.5D-1.5} (1.5 - 0.5D)n} + \frac{2(2-0.5D)(D-1)E\gamma^{0.5} a_i^{0.5D-0.5} (a_i - a_e)^{-0.5D}}{3(1.5-0.5D)(1-0.5D)}
\]  

(1)

Where \( \sigma_y \) is the yield strength; \( \lambda \) is the determined parameter which is the ratio of yield strength to hardness; \( D \) is the fractal dimension; \( \gamma \) is the range parameter which is 1.5, \( G \) is the fractal roughness parameter; \( E \) is the effective elastic modulus; \( a_e \) is the critical elastic deformation area; \( a_p \) is the critical plastic deformation area; \( a_i \) is the maximum contact area of the single Micro-bulge.

In Eq. (1), the value of \( n \) is

\[
n = \frac{(D-1) \ln \lambda}{\ln 110 - (D-1) \ln 2}
\]

(2)

According to the literature [9], \( D \) and \( G \) are determined by roughness. According to the literature [10], the equivalent tangential stiffness is 0.25-0.35 times the equivalent normal stiffness value. Without loss of generality, the relationship between \( K_n \) and \( K_t \) is equation (3).
The material used in this study is 45# steel, and its density, elastic modulus and Poisson's ratio are \( 7850 \text{ kg} / \text{ m}^3 \), 210GPa, 0.3, respectively. The surface roughness of the joint surface is 3.2 \( \mu \text{m} \) and the surface pressure is 0.125MPa. The total normal stiffness value and the total tangential stiffness value of the spring obtained by the formula (1), (3) respectively.

Joint surface model is modeled by CATIA software. The built model is then imported into Abaqus finite element simulation software. The stiffness damping value of the spring and the physical properties of the material are assigned to the constructed model. Finally, the boundary conditions are set for modal analysis to obtain the mode shape and natural frequency. The first three natural frequencies are shown in Table 1, and the first three modal modes are shown in Fig. 1.

| Table 1. The first five natural frequencies of the spring damping method |
|--------------------------|------------------|------------------|
| Order | Frequency (Hz) | 1 | 2 | 3 |
| 1 | 855.81 | 1243.3 | 1540.6 |

2.2. Virtual material method

The virtual material method assumes that the microscopic contact portion of the two contact bodies is a kind of virtual isotropic material. Elastic modulus, Poisson's ratio, density, the thickness of the virtual material are obtained by some theoretical methods. The formulas of the elastic modulus, shear modulus, Poisson's ratio, thickness and density of the virtual material obtained by the literature [5] are respectively

\[
 E_i = \frac{K_h h}{A_u} 
\]

\[
 G_i = \frac{K_h h}{A_u} 
\]

\[
 \nu_i = \frac{E_i}{2G_i} - 1 
\]

\[
 h \approx 1 \text{mm} 
\]

\[
 \rho = \rho_1 + \rho_2 \frac{1}{2} 
\]

Where \( h \) is the thickness of the virtual material layer; \( A_u \) is the nominal contact area of the bonding surface; \( \rho_i \) is the density of the solids forming the bonding surface.

The model is imported into ANSYS software. The parts outside the rough layer are assigned according to the original material properties. The virtual material part is assigned according to the above material properties. Finally, the boundary conditions are set for modal analysis to obtain the mode shape and natural frequency. When the surface pressure is 0.125MPa and the surface roughness is 3.2 \( \mu \text{m} \), the first three natural frequencies are shown in Table 2, and the first three modal modes are shown in Fig. 1.
Table 2. The first three natural frequencies of the virtual material method

| Order | Frequency (Hz) |
|-------|----------------|
| 1     | 713.52         |
| 2     | 1048.3         |
| 3     | 1227.7         |

2.3. Bond connection method
First of all, a three-dimensional measuring machine is used to measure the three-dimensional surface topography to obtain a point cloud of a rough surface. Then use the CATIA software to build the imported point cloud into a three-dimensional real surface, and then perform three-dimensional entity reconstruction and physical assembly operations in turn. Finally, the generated assembly is imported into ANSYS software, and the physical properties and boundary conditions of the two entities are set. The relevant material parameters are: elastic modulus $E = 2 \times 10^{11} \text{ Pa}$, Poisson's ratio $\nu = 0.3$, density $\rho = 7850 \text{ kg/m}^3$. The contact condition is set to Bond. This contact condition does not allow relative sliding or separation between faces or lines, and this area can be considered to be connected together. The modal analysis is performed to obtain the mode shape and the natural frequency. When the surface pressure is 0.125MPa and the surface roughness is 3.2 $\mu$m, the first three natural frequencies are shown in Table 3 and the first three modal modes are shown in Fig. 1.

Table 3. The first three natural frequencies of the Bond connection method

| Order | Frequency (Hz) |
|-------|----------------|
| 1     | 868.33         |
| 2     | 1271.4         |
| 3     | 1505.2         |

3. Analysis of modal comparison results of three joint surface equivalent models

3.1. Comparison of mode shapes
Through the finite element simulation software, the modal simulation analysis of the three combined surface equivalent models is carried out, and the first three modes of the mode shape and the natural frequency are obtained. The first three modes of the three methods are shown in Fig. 1.
By comparing the first three modes of the mode, it is found that the modal modes of the three joint equivalent models are basically the same. Therefore, all three methods can be used as a reliable joint surface equivalent model.

3.2. Comparison of natural frequency

Through the modal simulation analysis, the first three natural frequency values of the three joint surface equivalent methods are also obtained. The natural frequency values obtained by each method are shown in Table 1, Table 2, and Table 3. The comparison of the frequency values obtained by three equivalent models is shown in Fig. 2.

![Fig 2. Comparison of natural frequency values obtained by modal analysis of three joint surface equivalent models](image)

Through the comparison of the natural frequencies of different methods, it is found that the first three natural frequency values obtained by the spring damping method are similar to those obtained by the finite element method. The natural frequency method obtained by the virtual material method is 150 to 300 smaller than that obtained by the other two methods.

4. Conclusion

The modal analysis of virtual material method, Bond connection method and spring damping method is carried out by finite element simulation, and the natural frequency and modal modes of the equivalent model of three joint surfaces are obtained. Then by comparing the natural frequencies and the mode shapes of the first three orders, the corresponding conclusions are as follows:

The mode shapes of the three joint surface equivalent models are similar, which proves the feasibility of the three joint surface equivalent models. Thereby the simulation of the whole machine can be performed. The natural frequency values obtained the virtual material method differs greatly from those obtained by the other two methods. The reason is that the spring damping method cannot accurately select the position of the spring damping, and the spring damping method and the finite element method are difficult to characterize the nonlinear characteristics between the joint surfaces. Therefore, the obtained natural frequency value has a large error. The virtual material method can well characterize the nonlinear characteristics between the joint surfaces. Because the material properties of the equivalent gap layer of the virtual material method are uniform, it is impossible to accurately characterize the uneven stress distribution between the bonding surfaces. The virtual material method also has certain errors.
In summary, the virtual material method is more suitable for the equivalent joint surface model than the Bond connection method and the spring damping method, so as to solve the problem of the practicality of the dynamic model engineering.

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