Modeling the stress-strain state of plates in a form of Reuleaux Triangle profile during grinding of plastic materials

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Abstract. The article deals with the mechanical characteristics that arise in the crushing zone during grinding plastic material in a crusher with type-setting plates in a form of Reuleaux Triangle Profile. It is described the methodology for calculating internal and external energy and the load in the crushing zone. Much attention is given to the modeling of the stress-strain state of the plates during grinding plastic materials. The analysis of deformations made it possible to determine the critical zones of the impact of loads on the plates.

1. Introduction
Flow plastic chips are formed in industrial production during the mechanical processing of plastic materials (low-carbon steel, aluminum, copper, brass). The processing of such chips is difficult due to the plastic properties of the material. One of the possible solutions to the problem of processing flow chips is grinding materials in a roller crusher with type-setting plates in the form of a Reuleaux Triangle Profile [1-3]. Constructive solutions in the manufacture of a crusher should take into account deformation loads and forces acting on the plates. It is important to calculate the loads arising in the crushing zone and make a model of the stress-strain state of the plates to find appropriate solutions.

2. Methodology for calculating loads in the crushing zone
A literature review of mechanics of plastic deformation does not make it possible to determine the optimal ways to solve the problems of calculating the forces and deformations between plates in the form of Reuleaux Triangle Profile during grinding plastic materials. The objectives of determining deformations with significant material broadening are quite complex, as the calculations of stresses are usually made on the basis of empirical formulas [4-6]. In the process of crushing plastic materials with a width to thickness ratio of more than 10, the broadening of the material occurs insignificantly and the task is reduced to the analysis of the plane-deformed state. The deformation process of the material occurs in a tapering channel formed by the arcs of the RK-profile plates and their lateral surfaces (figure 1).
In the process of rotation of the plates, the friction forces on the surfaces of the rolls change direction due to the advance and backward movement of the material relative to the neutral section of the roll. As a result, a pressure peak arises in the region of the neutral cross section. The magnitude of this peak depends on the deformation resistance of the crushed material and the coefficient of friction between the material and the rolls [7, 8].

The process of grinding plastic materials can be described with help of the energy balance method. However, the deformation work balance method gives approximate values of the deformation energy and the amount of force on the roll. This method does not allow to determine the distribution of stresses on the contact surface of the deformable material. Therefore, some assumptions were made [9]:

- there is no broadening (the outflow of material is limited by the side surfaces of the plates in the form of Reuleaux Triangle Profile);
- the deformation is uniform;
- there is no deformation of the rolls.

Then, equating the work of deformation during grinding specific internal energy, can be written:

$$U_i = \int_0^\infty \sigma d\varepsilon$$

(1)

where $\bar{\sigma}$ – constant stress causing plastic deformation; $d\varepsilon$ - infinitesimal effective deformation under plane stress conditions, which, is calculated with accepted assumptions by the formula:

$$d\varepsilon = \frac{2}{\sqrt{3}} d\varepsilon_y$$

(2)

For the accepted condition of uniform deformation

$$d\varepsilon_y = \frac{dt}{t}$$

(3)
With a constant value of stress $\bar{\sigma}$ and in the absence of mechanical hardening, equation 1, after substituting the values of quantities from expressions 2 and 3 into it, can be integrated, as a result:

$$u_i = \frac{2}{\sqrt{3}} \bar{\sigma} \ln \frac{t_b}{t}$$  \hspace{1cm} (4)

Equation 4 can also be used to mechanically hardening material if it is assumed that $\bar{\sigma}$ is the average stress causing plastic deformation. It is possible to obtain a value of the specific internal strain energy that is closer to the real value: By introducing the correction factor $C$ into equation 6, which takes into consideration additional shifts, it is possible to obtain a value of the specific internal strain energy that is closer to the real value:

$$u_i = \frac{2}{\sqrt{3}} C \bar{\sigma} \ln \frac{t_b}{t}$$  \hspace{1cm} (5)

External deformation energy. The roll torque $T$ shown in figure 1 determines the required external energy, which can be expressed as:

$$u_b = \frac{2T\alpha_0}{(t_b + t_a)\ell_d}$$  \hspace{1cm} (6)

where $\alpha_0$ is the angle of rotation of the rolls, during which the volume under the rolls is displaced, and can be approximately taken equal to $\alpha_0$.

Equating the external specific energy to the internal specific energy, provided there are no losses, we get:

$$T = \frac{C(t_b + t_a)\ell_d \bar{\sigma} \ln(t_b / t_a)}{\sqrt{3}\alpha_b}$$  \hspace{1cm} (7)

The force acting on the roll is applied at a distance $a$ from the centers of rotation of the rolls. If we assume that the torque of the rolls depends on this force, then it can be calculated by a simple equation:

$$F_s = T \frac{2\alpha}{2\alpha}$$  \hspace{1cm} (8)

For a small reduction, it can be assumed that

$$\alpha = \frac{\ell_d}{2}$$  \hspace{1cm} (9)

The force acting on the rolls in this way may not fully correspond to the actual force, since the torque of the roll also includes the horizontal component of the force acting on the rolls. The aluminum D16 alloy was chosen to calculate the load according to the presented method. The calculation result:

$$T = \frac{1(0.02 + 0.008)0.032 \cdot 440 \cdot \ln(0.02 / 0.008)}{\sqrt{3} \cdot 0.016} = 13.09[MPa]$$  \hspace{1cm} (10)

The force acting on the roll will be as follows:

$$F_s = \frac{13.09}{2 \cdot 0.016} = 409.6[MPa]$$  \hspace{1cm} (11)
3. Modeling the stress-strain state during grinding of plastic materials

During the grinding of plastic materials, shear stresses and compressive stresses arise. If the size of a piece of material is greater than the width of the plate, then shear deformations will be limited by the side surfaces of adjacent plates (excess material is cut off, the guillotine effect is realized) and directed tangentially to the arc of the roll profile (figure 2) [10-12].

![Figure 2. Distribution of stresses during grinding of plastic materials.](image)

The distribution of normal and tangential loads is uneven due to different angular velocities in different parts of the roll. The analysis of the stress-strain state of the plate in the form of Reuleaux Triangle Profile is carried out by the finite element method in SolidWorks Simulation. The analysis was realized taking into account the nature of the stresses arising in the slotted gap. The numerical values of the loads are determined by formulas for calculating the loads arising from the grinding of plastic materials. Steel X12M was chosen as the material of the plates, and the ultimate strength of the aluminum alloy D16 was chosen as the limiting load. The result of modeling the stress state during deformation of plastic materials with a constant slotted gap is shown in figure 3.

![Figure 3. Diagram of stresses during grinding of the aluminum alloy D16.](image)

A load corresponding to the stress distribution scheme was selected to analyze the stress-strain state (figure 2). The diagram shows that stresses are distributed over the volume of the plate in the range from
0.8 to 556.3 MPa, which does not exceed the threshold values of the strength of the plate material. The greatest stresses arise on the edges of the hexagonal bore as a result of the occurrence of tangential stresses directed in the opposite direction of plates rotation and amount to 556.3 MPa. A similar stress concentration was noted in the process of modeling the stress-strain state during grinding of brittle materials [3]. The design of the crusher is provided with rounded interfacing surfaces of the plate along the radius R0.5 mm to reduce the probability of occurrence and propagation of the main crack from the corner of the window. Similar radius roundings are made on the opposite seating surfaces of the hexagon shaft. Average values of stresses in the working zone of the plates are 180-320 MPa, which are much lower than the ultimate strength of the plate material. However, the loads are of a short-term nature, and the stress vectors are displaced along the plate surface.

The deformations arising under the influence of the load (figure 4) are insignificant and are compensated by the elastic properties of the plate material.

![Diagram of deformations during grinding of the aluminum alloy D16.](image)

Based on the results of modeling the stress-strain state, it can be concluded that during grinding plastic materials in the plates in the form of Reuleaux Triangle Profile, there are no critical stresses and deformations that can affect the reliability of the crusher. All calculations on the deformation of plastic materials during grinding with a constant slot gap were confirmed by the simulation.

4. Conclusion
The analysis of the work and stresses in the grinding zone showed that an effective combination of various destruction mechanisms (abrasion, crushing, cutting, alternating loads) increases the intensity of deformation processes and specific loads on the material. The analysis of the stress-strain state of the plates in the form of Reuleaux Triangle Profile showed that the maximum stress values do not exceed the safety margin of the plate material. It is identified zones of maximum stress concentration values. The necessary Constructive solutions have been taken to reduce the risk of defects in these zones.

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