An upper bound on the number of e-foldings

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ABSTRACT: If the present acceleration of the universe is due to an asymptotically de Sitter universe with small cosmological constant, and the principle of Cosmological Complementarity is valid, then the number of e-foldings during inflation is bounded.

KEYWORDS: Inflation, de Sitter space, Cosmology.
1. Introduction

A remarkable series of observations [1]-[8] (for a review see [9]), indicate that we live in an accelerating universe. It is quite tempting to speculate that the source of this acceleration is a small positive cosmological constant, leading to an asymptotically de Sitter (dS) space. If this is indeed the case, there is a restriction on the number of e-foldings during inflation.

We emphasize that when we say the observed acceleration is due to a cosmological constant, we mean that space-time is asymptotically de Sitter, with the value of the cosmological constant that fits the data. Our bound would not apply to a model of a meta-stable dS minimum, which decayed into an asymptotic space-time with vanishing cosmological constant. More generally, it is important to realize that theories of quantum gravity are defined with fixed asymptotic conditions. The same effective Lagrangian may have solutions which describe the physics of completely different quantum models of gravity. In the second half of this paper we will argue that inflation models with large numbers of e-foldings and a positive “true” cosmological constant, are not asymptotically dS, and have only a temporary dS phase. Their true asymptotic behavior depends on issues in quantum gravity which are not yet resolved. In a brief comment at the conclusion of this paper, we will take up the difficult problem of how to differentiate a truly asymptotically dS space-time from such a meta-stable scenario, by current observations.

The existence of a small cosmological constant, $\lambda^1$, makes the universe eventually appear to a local observer as a finite cavity of size $\lambda^{-1/2}$ [10]. Second, as was shown

\[1\)When equations do not appear dimensionally homogeneous, the reader should understand that we are using Planck units.
in a recent paper [11], this finite size cavity can only accommodate a limited amount of entropy stored in field theoretical degrees of freedom. It was shown in [11] that this limited amount of entropy scales like \( \lambda^{-3/4} \). Any excess entropy beyond this bound has to be encoded into black holes or imprinted onto the walls of the cavity. That excess entropy in turn is limited to be smaller than the entropy of empty de Sitter space [14, 15].

This third version of our paper is written to clarify the basic assumption which underlies our bound. It was left implicit in the first version and explained only cursorily in the second. We have found that many people have been confused about our claim, because they did not understand this basic assumption.

In [31] we announced a principle we called *Cosmological Complementarity*. According to this principle, the physics of an asymptotically de Sitter space-time is completely described by a pure state in a Hilbert space of finite dimension, determined by the value of the cosmological constant. This principle cannot be derived from the covariant entropy bound [19] or any simple generalization of it. It was meant to parallel the notion of Black Hole Complementarity [32], which is necessary to the unitary resolution of the black hole information problem. Complementarity for dS space does not however follow from Complementarity for black holes. Cosmological Complementarity was meant to be a guide in formulating a quantum theory of asymptotically dS space-times. It is the basic assumption on which our bound on the number of e-foldings of inflation rests.

The other important assumption we will make is that at early times, the standard description of the inflationary universe in terms of quantum field theory in curved space-time makes sense over the entire inflationary patch. There are several reasons for this. One is that there are no obvious inconsistencies in the field theoretic treatment. Indeed, if the true cosmological constant were zero, inflation theorists would use the predictions for fluctuations on the scale of this patch to try to fit the data that would be seen as more and more of the CMB fluctuations came into our horizon. However the most important reason for making this assumption is that it affects the state of an observer inside the cosmological horizon volume. The Bunch Davies (BD) wave function is typically written in terms of momentum modes in the flat coordinate system for the expanding half of the approximate dS space of the inflationary epoch. It contains no correlations between different modes. However, if we rewrite it in terms of wave functions concentrated in the cosmological horizon volume of a given observer in the asymptotically dS space, and wave functions with support outside that volume, then there are correlations between the inside and outside modes. Thus, the standard inflationary picture contains correlations between the observables in a single cosmological horizon volume, and those of a potentially larger set of degrees of freedom. Indeed, one of the prime virtues of inflation is its ability to explain correlations between widely
separated objects that might otherwise appear causally disconnected.

We will count the entropy implied by the inflationary picture, with a variety of assumptions about what the products of reheating are. We argue that if it is larger than the dS entropy of asymptotic dS space, that the density matrix on the finite dimensional dS Hilbert space cannot be pure. This contradicts the assumption of Cosmological Complementarity.

The reader is perfectly justified in concluding that he can avoid having to think about a bound on the number of e-foldings by rejecting our assumption about purity of the density matrix. This is an assumption which is virtually impossible to check in any experimental way. We remind the reader who is inclined to do this, that similar remarks could have been made about unitarity of the S-matrix for black hole production and decay in asymptotically flat space-time. It is only because examples exist of exactly unitary theories of quantum gravity with asymptotically flat and AdS geometries that we have strong arguments for a unitary S-matrix in this case\textsuperscript{2}. Once we allow for the possibility of violation of unitarity for a single causally connected patch in any space-time we must ask ourselves why it doesn’t occur in general.

Indeed, models with a very large number of e-foldings often appear to lead to the phenomenon of eternal or self-reproducing inflation, when analyzed by the methods of quantum field theory in curved space-time. Such models appear to violate unitarity for a single observer, even when there is no cosmological horizon. Indeed they can imply the existence of infinite numbers of causally disconnected regions in the future, which were all correlated in the past. In the landscape scenario for string theory [33] there are non-accelerating FRW cosmologies in many of these regions, which have different degrees of freedom and different space-time dimensions. If one tries to think of these models in terms of an S-matrix\textsuperscript{3}, then the S-matrix for a given type of asymptotic region is non-unitary. Similarly [34] there are treatments of cosmology in tree level string theory which seem to imply multiple asymptotic regions, with the S matrix for any one region violating unitarity. If there are valid quantum theories of this type, then one might equally well contemplate a similar treatment of cosmologies with a number of e-foldings violating our bound. We will argue however that the dS region is unlikely to be stable in such models, so our bound does not apply.

In this paper we will argue that the limited entropy that can fit into asymptotic de Sitter space puts an upper bound on the amount of inflation. On the other hand, there is a minimum number of e-foldings required in order to reconcile the isotropy of the microwave background on large scales with causality \textsuperscript{4}. We will see that these

\textsuperscript{2}The arguments are not so strong as to have convinced everyone in the field.

\textsuperscript{3}This proposal is due to L. Susskind.

\textsuperscript{4}For an alternative, see work by Banks and Fischler on "holographic cosmology" [16][17]
numbers are rather close.

It is quite remarkable that a small cosmological constant, seemingly irrelevant in magnitude when compared to the energy density during inflation, has such an important impact. The essential ingredient is that because of the UV-IR connection, entropy requires storage space. The existence of a small cosmological constant restricts the available storage space.

In the first section we present the details of how the bound on the number of e-foldings is obtained. This will be followed by a section where this bound is discussed in a different "holographic gauge". We end by offering some conclusions.

2. The Number of e-Foldings is Bounded

We begin by making a precise statement of the physical assumptions that go into the derivation of our bound. We first assume the standard picture of inflationary cosmology, that is, that inflation produces a large volume of space, of order \( e^{3N_e} \Lambda_I^{-\frac{2}{3}} \) in which all the degrees of freedom are well described by local field theory, and are correlated with each other. This system has a minimal entropy, which is needed to describe the fluctuations in the energy density. We will study models in which the classical gravitational equations predict that long after the inflationary period the universe enters an asymptotically dS phase, with cosmological constant \( \lambda \). We then assume\[13\], that the asymptotically static dS observer has a complete, unitary description of all the physics that she can measure\[^5\], using a Hilbert space of finite dimension, determined by \( \lambda \). If the entropy needed to describe the large inflationary patch exceeds the logarithm of the dimension of this Hilbert space, then we have a contradiction. Either we must give up the assumption of unitary evolution in causal patches, or we must conclude that quantum fluctuations in the model, destroy its classical asymptotic dS geometry.

Next, we briefly review the physics that leads to the bound on the entropy that a fluid described by an equation of state \( p = \kappa \rho \) can encode. This physics was discussed in a recent paper [11]. For the case of radiation, a hand-waving derivation of the bound was also presented in [12].

For any such fluid contained in a finite cavity of radius \( R \), it was shown in [11] that there is a threshold on the amount of entropy stored in the fluid, beyond which black holes are formed. The entropy \( S_0 \) at the verge of black hole formation scales like

\[
S_0 \sim R^{3-\frac{2}{1+\kappa}}
\]

[^5]: This means that her density matrix is pure, if she makes all possible observations of the states on her horizon.

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If we now imagine that the cavity is the region that is causally accessible to the local observer living in asymptotic de Sitter space-time with cosmological constant $\lambda$, then the length scale in the previous equation $R$ is replaced by $\lambda^{-1/2}$.

This implies an upper bound on the entropy that scales parametrically with $\lambda$ as follows

$$S_0 \sim \lambda^{-\frac{1+3\kappa}{2(1+\kappa)}}$$

(2.2)

We will consider the bound in the context of a Lemaître-Friedmann-Robertson-Walker (LFRW) cosmology where the geometry can be described by the metric

$$ds^2 = dt^2 - a^2(t)(dr^2 + r^2d\Omega)$$

(2.3)

where we choose a spatially flat universe since we are interested in using the upper bound on the entropy at the exit from inflation.

The entropy $S_0$ carried by the dominant fluid in such a universe can then be rewritten, using equilibrium thermodynamics, as

$$S_0 = a_0^3 \rho_0 \frac{1}{1+\kappa}$$

(2.4)

The thermodynamical relation\(^6\) that is used, relates the energy density $\rho$ to the entropy density $\sigma$

$$\sigma \sim \rho^{\frac{1}{1+\kappa}}$$

(2.5)

In what follows, we will apply these ideas in the context of inflation. Our approach is to first express the entropy according to equation (4.2), where $a_0$ and $\rho_0$ are evaluated at the exit of inflation. At the exit of inflation it is reasonable to take for the energy density $\rho_0$ a value representative of the energy density during inflation.

$$\rho_0 \sim \Lambda_I$$

(2.6)

where $\Lambda_I$ is the value of the energy density during inflation.

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\(^6\)In principle there is a multiplicative constant of order one (in Planck units) in this relation (in 1+1 CFT it is related to $c$). It would weaken our bound on the number of e-foldings only if the constant were much less than one. In CFT this constant is bounded from below by a number of order one. It affects the bound on $N_e$ by an additive constant $\sim \ln c$. For $\kappa = 1$ this constant reflects the typical distance in Schwarzschild radii between black holes in the black hole fluid, and the right equation of state is only achieved when it is of order one. For non-relativistic matter of mass $M$ the coefficient is of order $\frac{M}{M_p}$ and could increase the bound on $N_e$ by $\sim 10$, if the Compton wavelength is inside the Hubble radius. This is still stronger than the bounds for stiffer fluids.
For the scale factor $a_0$ at the exit of inflation, we will take the apparent horizon during inflation multiplied by the number of e-foldings, $N_e$.

$$a_0 \sim \Lambda_I^{-1/2} e^{N_e} \quad (2.7)$$

Substituting the expressions for $a_0$ and $\rho_0$ into equation (2.4) leads, using equation (2.2) to the upper bound on the number of e-foldings, $N_e$

$$N_e < \frac{1 + 3\kappa}{6(1 + \kappa)} \log \frac{\Lambda_I}{\lambda} \quad (2.8)$$

Let us recapitulate what the physics involved in obtaining this bound is. At the end of inflation, the energy density in the universe becomes dominated by a fluid with an equation of state $p = \kappa \rho$. The upper bound on the number of e-foldings for a given $\kappa$, is the threshold beyond which black holes form and some of the excess entropy gets imprinted onto the asymptotic de Sitter horizon. The biggest value for the number of e-foldings occurs for the stiffest equation of state, $\kappa = 1$.

$$N_e < \frac{1}{3} \log \frac{\Lambda_I}{\lambda} \quad (2.9)$$

This case is one that corresponds to a universe filled with black holes [16][18]. For illustrative purpose, we will estimate the value of $N_e$ for the case where after inflation ends, the energy is dominated by a $\kappa = 1$ fluid. We will assume a value for $\Lambda_I \sim (10^{16}GeV)^4$, which is consistent with having not observed yet a background of gravitons. We obtain the upper bound on $N_e$

$$N_e \sim 85 \quad (2.10)$$

where we took $\lambda$ to be of $O((10^{-3}eV)^4)$.

For the sake of comparison, the case $\kappa = 1/3$ yields with the same value for $\Lambda_I$

$$N_e \sim 65 \quad (2.11)$$

This value for the maximum number of e-foldings is close to the value necessary to solve the "horizon problem". It is interesting to note that a small value for the number of e-foldings may have observable implications for the low values of $l$ in the spectrum of fluctuations of the microwave background.

At this point, a field theorist may be puzzled because he can write models of inflation where the inflaton is moving so slowly that the number of e-foldings becomes enormous, that nonetheless have solutions which appear to asymptote to a dS universe with arbitrary value of the cosmological constant. The next section is devoted to answering the legitimate concerns of such a field theorist.
3. The View from a Different "Holographic Gauge"

The work of Bousso [19] shows that there are many different ways to project the information in a space-time onto a collection of holographic screens. Different coordinate systems often suggest different natural screens. Thus, for example, in the presence of a black hole, an external observer finds it natural to project the information in the hole onto the horizon, at least until the black hole begins to evaporate significantly. An infalling observer chooses a different sets of screens. Similarly, in dS space, each static observer uses his cosmological horizon for a screen. In [20] and [21] we suggested that for certain states of the system there is an alternative description, which approximates quantum field theory in global coordinates better and better as the cosmological constant goes to zero. The states in question have only black holes which are much smaller than the cosmological horizon, and the state in each horizon volume is well described by local field theory. The holographic screens are chosen to be those of small causal diamonds distributed throughout the volume of dS space-time.

The motivation for this suggestion was the observation that such field theoretic states have an entropy per horizon volume of order $R^{3/2}$, while the total dS entropy scales like $R^2$, where $R$ is the dS radius. This indicates the possibility of a description of field theoretic states by of order $R^{3/2}$ commuting copies of the field theory degrees of freedom in a single horizon.

In quantum field theory in curved space time, dS space is, at late times, an exponentially expanding sphere. The number of copies of a given horizon volume grows to infinity as the global time goes to infinity. Even if we impose a UV cutoff on the system, the number of field theoretic degrees of freedom appears to go to infinity with the global time. The counting of degrees of freedom above suggests that this picture is not valid for finite $\lambda$, but requires an IR cutoff when the entropy implied by the field theory picture exceeds the holographic bound in dS space. Field theory states with more entropy will back-react on the geometry and, when the back reaction is taken into account, the system will no longer be asymptotically dS. As we noted above, these solutions with back reaction do not refer to the quantum theory of asymptotically dS space.

This new holographic gauge allows us to understand why the conventional treatment of inflation in a universe which is asymptotically matter or radiation dominated is compatible with the holographic principle. This is true no matter how many e-foldings there are. We simply use the global gauge for the inflationary phase of expansion. Conventional restrictions on inflationary models ensure that the states of the system are well described by field theory. Eventually, when we enter the non-accelerating phase, the particle horizon can grow indefinitely and there is no contradiction with the large
entropy implied by the field theoretic treatment of the inflationary stage.

However, if we truly have an asymptotically dS future, with cosmological constant $\lambda$, there will be such a contradiction. Let us consider the case of radiation. As in the previous section the entropy implied by conventional field theory (which we can now understand as a holographic entropy in the global gauge) is

$$S_{inf} = e^{3N_e} \Lambda_f^{-3/4}$$

We can also view this from the global holographic gauge of the asymptotic dS universe, with the true cosmological constant. Since the inflationary model has been tuned so that horizon scale black holes do not form within the present cosmological horizon, the state of the system qualifies as a field theoretic state over scales larger than the horizon. However, we have claimed that quantum gravity will put an IR cutoff on this picture, determined by requiring that the volume of space over which the global gauge is valid has field theoretic entropy less than or equal to the total dS entropy. This is a refined version of the bound on the number of e-foldings announced in the previous section. We believe that the numerical value of the bound obtained by this method, is more accurate than the estimates in the first section. Its form is

$$N_e \leq \frac{1}{4} \ln(\Lambda_I/\lambda) + \frac{1}{12} \ln(M_P^4/\lambda)$$

This gives $N_e \sim 88$, for $\Lambda_I$ of order $(10^{16} GeV)^4$.

It is easy to write down models of inflation with an arbitrarily large number of e-foldings, which appear to asymptote to a dS universe with fixed cosmological constant independent of $N_e$. Consider for example a canonical chaotic inflation model \cite{22}, a single scalar, with potential

$$V = g\phi^4 + \lambda$$

To fit the amplitude of density fluctuations observed in the CMB, $g$ is chosen to be $g \sim 10^{-15}$. The slow roll condition and the condition that the energy density be less than the Planck scale are satisfied for a range of $\phi/M_P$ of order $10^3$, and the number of e-foldings is of order $10^3$. It is hard to understand how such a fine tuned model could emerge from a fundamental theory like string theory. Nonetheless, one would like to understand whether such a model is inconsistent.

The spectrum of density fluctuations in models of this type is not scale invariant over the whole range of scales which are generated during inflation. The parameter $g$ is tuned so that $\frac{\delta \rho}{\rho} \sim 10^{-5}$, the experimental number, during the last 25 or so e-foldings of inflation (which generate fluctuations on the scales observed in the CMB). However, during most of the inflationary history, and thus on most scales larger than our horizon
within the current inflationary patch, the amplitude of fluctuations is much larger. This leads to the following paradoxical situation. We generally imagine that fluctuations on scales much larger than our horizon scale cannot effect us. However, in inflationary models, the true particle horizon is at least as large as the inflationary patch. Otherwise field theory would not be able to generate correlations between fluctuations on these large scales (assuming no correlations in the initial state). Although they will not affect us for a long time, these large amplitude, large scale fluctuations do affect the answer to the question of whether the universe is asymptotically dS. We conjecture that these large amplitude “superhorizon” fluctuations can gravitationally collapse, and our horizon volume will in fact be contained in a huge black hole. An observer inside a black hole can remain oblivious to that fact for times of order the Schwarzschild radius of the hole. Since, in the present situation, this radius is much larger than our current horizon, there is no real contradiction with conventional predictions of the model for experiments done today. We are simply saying that, in models of this sort, at some time in the distant future, the currently observed accelerated expansion will be replaced by a Big Crunch. Thus, the hypothesis that a model of this type asymptotes to a dS universe is false, but it may still give rise to a period of accelerated expansion from the point of view of local observers. We emphasize that this is a conjectural resolution of the contradiction posed by classical solutions with many e-foldings and dS asymptotics. Our bound is based on quantum mechanical reasoning and cannot be evaded without giving up one of the assumptions we have made. We suggest that, in these models, it is the assumption of asymptotic dS space which must be relaxed, and are proposing a tentative mechanism for changing the asymptotic behavior.

The global structure of inflationary models has been the subject of intensive investigation [23], with results that the present authors find confusing. The standard field theory arguments lead to a picture of eternal or self-reproducing inflation. We are not convinced that, in the absence of a completely understood theory of quantum gravity, we are in a position to understand these issues, but they do appear to imply violations of unitarity for observers in causal patches. This would remove the basic assumption underlying our bound. However, we think it is very unlikely that a dS observer would survive forever in such a universe.

The question of finding global time slices in a self-reproducing universe is a vexing one, but if the model is to be fit into the conventional framework of quantum mechanics, it must have a unitary time evolution operator $U(t, t_0)$. The Hilbert space of the asymptotically dS local observer is a finite dimensional sub-factor of the vastly larger (perhaps infinite dimensional) Hilbert space of the self-reproducing universe. If our bound is violated, then at the end of inflation, this factor is correlated with a much larger sub-factor of the full space. We know of no example in quantum mechanics of
a similar situation where the small system does not decay and become mixed with the larger one. Sometimes the decay proceeds by tunnelling, and takes a long time, but it always occurs. Indeed, in all semi-classical contexts in which a dS minimum can interact with a system with a larger number of degrees of freedom, it decays.

Finally, we want to mention an alternative to our bound which seems to be what is assumed by various colleagues who have raised objections to it [35]. Models with a number of e-foldings violating our bound do not violate the covariant entropy bound. Nonetheless, no observer in such a universe can access more than a finite amount of information, given by the dS entropy. This is taken to suggest the existence of a quantum model, with a finite dimensional Hilbert space which describes such universes. How can this be compatible with the evolution of the restricted state vector of the causal patch from a pure to a mixed state? The suggestion is that this is an artifact of the field theory approximation. The real state of this system is pure, but approximates the mixed state well. It is surely true that there are states like this in the causal patch Hilbert space. What is less than obvious is what the dynamics might be in this space, which naturally evolves the system into this state. To find it one would have to construct the hypothetical quantum gravity model which describes this space-time. We can only admire the confidence with which this opinion is stated, but we cannot share it.

4. Conclusions

We have argued that a universe which is truly asymptotically dS is only consistent with inflationary models which satisfy the bound on the number of e-foldings. However, our explanation of how this is consistent with lagrangian models of inflation which can incorporate an arbitrarily large number of e-foldings and an arbitrary cosmological constant, implies that it might be difficult to distinguish the two scenarios by observation. Namely, we have argued that such models do not truly give rise to asymptotically dS universes, but that the Big Crunch or other instability which ends the dS era, might occur only after many times the current age of the universe.

In this sense, models with a large number of e-foldings and a positive cosmological constant are similar to quintessence models like those of [24], in which the current phase of acceleration is modelled by something like old inflation: a temporary sojourn in a meta-stable minimum with positive cosmological constant. This is followed by tunnelling out to a region of rolling scalar field with no acceleration. Such models might arise naturally if string theory truly has a discretuum of meta-stable dS vacua [25]
Models of this type raise the spectre of non-falsifiability. One could imagine that no experiment in the foreseeable future could distinguish between them and a model with a true asymptotic dS space.

We can imagine two ways out of this uncomfortable situation. If the hypothesis of Cosmological SUSY Breaking [26] is correct, then radiative corrections to the long distance effective lagrangian may depend on the physics at large space-like distance, in a significant way. These virtual effects might then be able to distinguish between different models.

A more exciting possibility is raised by the low $L$ anomalies in the WMAP data. The WMAP data [30] shows weak evidence of a problem for the theoretical predictions of inflationary models for low values of angular momentum. Theoretical calculations of the amplitudes of the fluctuations at fixed and small spherical harmonic, depends somewhat on the theoretical predictions outside our current horizon [28]. By introducing a break in the spectrum at the horizon scale, one can fix the “problem”. Indeed, in Sarkar’s model one can use the extra freedom to obtain a fit to the data without a cosmological constant. While we are (for religious reasons) not particularly sympathetic to the latter attempt, we would like to suggest that this aspect of the data may eventually be viewed as evidence that inflationary models which predict a scale invariant spectrum of fluctuations far outside the current horizon are ruled out. Our bound on the number of e-folds would then suggest that a true asymptotic dS behavior was the most likely explanation of this.

Inflationary models with small numbers of e-foldings exist. They are models with potentials of the form

$$V = \frac{M^6}{M_P^2} U(\phi/M_P)$$

with $M$ of order the GUT scale. They arise naturally in brane world models derived from string theory, which use a mild difference between the Kaluza-Klein scale and the higher dimensional Planck scale to explain the ratio between the scales of gravity and coupling unification. The natural prediction of models of this type is $O(1)$ e-folds. One might imagine mild fine tuning of parameters, or perhaps the sort of dynamical friction in multi-field models that was discussed in [29], could increase this to $10 - 100$, but more e-folds than that are implausible.

Alternatively, one could turn to holographic cosmology [16]. The current version of these models [17] predicts a scale invariant spectrum, but only over a very limited

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7This was emphasized to one of us (TB) at the Middle East String Theory Conference in Crete [27].
range of scales. The range depends on parameters we can not yet calculate but the largest scale is very unlikely to exceed the present horizon by very much.

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