New method for non-standard invisible particle searches in tau lepton decays

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Motivated by models proposed to explain the Standard Model anomalies, and the unprecedented \( \tau^+\tau^- \) data to be collected by the Belle II experiment during the next years, we study the kinematics of tau pair decays and propose a new method to search for lepton flavor violation processes in tau lepton decays to invisible beyond Standard Model particles, such as \( \tau \rightarrow \ell\alpha \), where \( \ell \) is either an electron or a muon, and \( \alpha \) is a massive particle that escapes undetected. The new method improves by one order of magnitude the expected upper limit on the \( \tau \rightarrow \ell\alpha \) production in 3x1 prong tau decays and establishes the possibility of performing this search in 1x1 prong tau decays which has not been previously considered.

I. INTRODUCTION

The Standard Model of particles physics (SM) has been incredibly successful in explaining all observed data up today, with only a few remaining tensions between prediction and experiment for instance, the long-standing 3.7 \( \sigma \) discrepancy in the anomalous magnetic moment of the muon \( a_\mu = (g - 2)_{\mu}/2 \) [1, 3]. However, observed phenomena such as the predominance of matter over antimatter in the universe, or the mass of the neutrinos, point us to the needs for physics beyond the SM (BSM). Due to the lack of clear indications of the SM applicability boundaries, we have to follow a broad search strategy for BSM physics and one of these strategies involves the search for the leptonic flavour violating (LFV) processes, which are highly suppressed in the SM and that, if observed, will be a clear indication of new physics.

In the search for LFV processes, the tau lepton is a unique laboratory with an indirect probe to energies not directly accessible by accelerators. With the upcoming data from the Belle II experiment [6, 7], we expect an unprecedented statistics of around \( \sim 5 \times 10^{10} \) tau lepton pairs, where several interesting LFV searches will become accessible to the tau lepton sector [8, 9]. Of particular importance is the search for LFV tau lepton decays to invisible BSM particles, motivated by models proposed to explain SM anomalies like the \( a_\mu \) discrepancy and which contain new \( Z' \) gauge bosons [10, 12] or axion-like particles [13, 15]. One possibility of such searches is \( \tau \rightarrow \ell\alpha \), where \( \ell \) is either an electron or a muon, and \( \alpha \) is a massive particle that escapes to detection. This decay is not present in the SM, but appears in different new physics models [16, 18]. Nevertheless, it will take several years for Belle II to accumulate the expected \( \tau^+\tau^- \) statistics needed to exclude some BSM processes, and the experimental side can profit from new methods to expand the output of the available data.

Inspired by searches for dark matter or invisible heavy particles in \( X \bar{X} \rightarrow (Y\bar{a} + N)(Y\bar{b} + N) \) processes [19, 22], with \( Y\bar{a} \) and \( Y\bar{b} \) being the only detectable products, we generalize the idea to \( X \bar{X} \rightarrow (\sum_{i=1}^a Y_{ai} + N_1)(\sum_{j=1}^m Y_{bj} + N_2) \) decays where \( Y \) represents visible particles and \( N \) particles that evade detection. This generalization allows us to study \( X \bar{X} \) pair decays with a BSM process in one side and an SM process in the other side, increasing the possibility of production of a non-SM particle compared to requiring a double creation of the unknown invisible particle as in previous studies.

By studying the generalized case of the \( X \bar{X} \) pair decay, we determine a kinematic constraint that relates the masses of the mother particle \( X \) and the undetectable particles \( N_1 \) and \( N_2 \). We use this relationship to propose new searching variables for non-standard invisible particles in tau lepton decays from collisions with initial state energy and momentum well defined. B-Factories such as BaBar, Belle, and Belle II provide an ideal environment with these characteristics, colliding electrons and positrons with center-of-mass energy known.

We apply our findings to the search for LFV decays \( \tau \rightarrow \ell\alpha \), emulating the Belle II experiment conditions. We propose a new two-dimensional method that reduces by one order of magnitude the Belle II expected upper limit on the production of this BSM decay, when searched for in 3x1 prong decays, and which opens the possibility of an additional search in 1x1 prong decays.

II. KINEMATIC CONSTRAINTS

Let us consider the pair decay

\[
X \bar{X} \rightarrow (\sum_{i=1}^a Y_{ai} + N_1)(\sum_{j=1}^m Y_{bj} + N_2)
\]  (1)

at an center-of-mass-energy (CMS) \( \sqrt{s} \). Here \( N_1 \) and \( N_2 \) are invisible particles that elude detection, and \( Y_{ai} \) and \( Y_{bj} \) are the \( i \)-th and \( j \)-th visible particles from the \( X \) and \( \bar{X} \) decays, respectively. To facilitate calculations, we define \( h_a = \sum_{i=0}^a Y_{ai} \) and \( h_b = \sum_{j=0}^m Y_{bj} \), and we will treat the \( X \bar{X} \) pair decay as illustrated in Figure [1]...
with $p_1 = (E_1, p_1)$, $p_2 = (E_2, p_2)$, $p_3 = (E_3, p_3)$, and $p_4 = (E_4, p_4)$ being the four-momenta at CMS for $h_a$, $h_b$, $N_1$, and $N_2$, respectively. We follow a similar approach as in Ref. [20], but allowing both decays to produce different missing particles. The kinematic equations for the process are

$$q^\mu = p_1^\mu + p_2^\mu + p_3^\mu + p_4^\mu, \quad \mu = 0, 1, 2, 3,$$  
(2)

$$p_1^2 = m_1^2,$$  
(3)

$$p_2^2 = m_2^2,$$  
(4)

$$(p_a + p_1)^2 = (p_b + p_2)^2 = m_X^2,$$  
(5)

where $m_1$, $m_2$, and $m_X$ are the masses of $N_1$, $N_2$, and $X$, respectively. By defining the normalized variables $\mu_i = m_i/\sqrt{s}$, $z_i = E_i/\sqrt{s}$, $a = p_a/\sqrt{s}$, $b = p_b/\sqrt{s}$, $k_1 = p_1/\sqrt{s}$, $k_2 = p_2/\sqrt{s}$, from Eq. (2) we have $k_1 + k_2 + a + b = 0$ and $z_1 + z_2 + z_a + z_b = 1$. Then we can rewrite Eqs. (6)–(8) as

$$|k_1|^2 + \mu_1^2 = z_1^2,$$  
(6)

$$|k_1 + a + b|^2 + \mu_2^2 = (1 - z_a - z_b - z_1)^2,$$  
(7)

$$|k_1 + a|^2 + \mu_X^2 = \frac{1}{4},$$  
(8)

where we have used $z_X = 1/2$. From Eq. (3) we have

$$k_1 \cdot k_1 = k_1^2 = \left(\frac{1}{2} - z_a\right)^2 - \mu_1^2,$$  
(9)

and from Eq. (7) and Eq. (8) we obtain

$$a \cdot k_1 = A,$$  
(10)

and

$$b \cdot k_1 = B,$$  
(11)

where

$$A = \frac{1}{2} (z_a - z_b - \mu_X^2 + \mu_1^2 - |a|^2),$$  
(12)

$$B = \frac{1}{2} (z_a^2 - z_b + \mu_X^2 - \mu_2^2 - |b|^2) - a \cdot b,$$  
(13)

In addition, $k_1$, $a$, and $b$, must comply with

$$|k_1 \times a \times b|^2 = (|b \cdot k_1| a - (a \cdot k_1) b)^2,$$  

$$= |k_1|^2 |a \times b|^2 \sin^2 \theta,$$  

$$\leq |k_1|^2 |a \times b|^2,$$  

(14)

and by using Eqs. (9)–(11) this transforms to

$$|B a - A b|^2 - \left(\frac{1}{2} - z_a\right)^2 - \mu_1^2 \right) |a \times b|^2 \leq 0,$$  

(15)

From Eqs. (12) and (13) it is straightforward to show that

$$B a - A b = \frac{1}{2} \left((\mu_X^2 - \mu_2^2) a + (\mu_X^2 - \mu_1^2) b + H\right).$$  

(16)

Then Eq. (15) transforms to

$$A_1 (\mu_X^2 - \mu_2^2) + A_2 (\mu_X^2 - \mu_1^2) + A_3 (\mu_X^2 - \mu_1^2) (\mu_X^2 - \mu_2^2) + B_1 (\mu_X^2 - \mu_1^2) + B_2 (\mu_X^2 - \mu_2^2) + C_1 \mu_1^2 + D_1 \leq 0,$$  

(18)

where

$$A_1 = |b|^2,$$  

(19)

$$A_2 = |a|^2,$$  

(20)

$$A_3 = 2 (a \cdot b),$$  

(21)

$$B_1 = 2 (b \cdot H),$$  

(22)

$$B_2 = 2 (a \cdot H),$$  

(23)

$$C_1 = 4 |a \times b|^2,$$  

(24)

$$D_1 = H \cdot H - 4 |a \times b|^2 \left(\frac{1}{2} - z_a\right)^2.$$  

(25)

Equation (15) is our main result and contains the available kinematics information of the $XX \rightarrow (h_a + N_1)(h_b + N_2)$ decay.

### III. SEARCH FOR $\tau \rightarrow \ell \alpha$ DECAYS

The last search for $\tau \rightarrow \ell \alpha$ decays was performed by the ARGUS Collaboration [22] in $\tau \rightarrow \ell + \text{anything}$ data, with $\ell$ being an electron or a muon. The main challenge in the $\tau \rightarrow \ell \alpha$ search is to separate these signal decays from the same-signature SM process $\tau \rightarrow \ell \nu \nu \tau$. Since the signal is a two-body decay, ARGUS used the fact that, in contrast to the three-body decay, in the tau rest frame the momentum of the lepton is a constant value given by

$$b(m_\alpha) = \frac{m^2 - m_\alpha^2 + m_\tau^2}{m_\tau^2},$$  

(26)

where $m_\tau$ is the mass of the tau; $m_\alpha$ the mass of the $\alpha$ boson; and $m_\ell$ the mass of lepton. This feature would
allows us to separate the two decays and to determine \( m_\alpha \) if the reconstruction of the tau rest frame were possible. Unfortunately, each tau decay involves a missing particle, making impossible a full reconstruction of the tau.

To overcome this problem, ARGUS required the other tau in the \( \tau^+\tau^- \) production to decay to three pions and developed the so-called pseudo-rest-frame technique in which the lepton in the one-prong side is boosted to the tau rest frame by approximating: a) the momentum direction of the tau in the one-prong side as the opposite direction of the momentum of the three pions in the three-prong side; and b) the tau energy by \( E_\tau = \sqrt{s}/2 \). From now on we will refer to these approximations as the ARGUS method.

By using Eq. 18 we can construct other methods to search for \( \tau \to \ell\alpha \) decays. For simplicity, and in order to compare to the ARGUS method, let us consider the process \((\tau^+ \rightarrow \pi^+\pi^-\pi^+\bar{\nu}_\tau)(\tau^- \rightarrow e^-\alpha)\). For the decays studied in Section II this is a particular case where \( \mu_1 = \mu_\alpha, \mu_2 = \mu_\nu, \) and \( \mu_X = \mu_\tau. \) Assuming \( \mu_\nu = 0, \) Eq. 18 reduces to

\[
A_0(\mu_\alpha^2)^2 + B_0\mu_\alpha^2 + C_0 \leq 0, \tag{27}
\]

where

\[
A_0 = A_1, \tag{28}
\]

\[
B_0 = -B_1 + C_1 - (2A_1 + A_3)\mu_\tau^2, \tag{29}
\]

\[
C_0 = (A_1 + A_2 + A_3)\mu_\tau^4 + (B_1 + B_2)\mu_\tau^2 + D_1. \tag{30}
\]

Then, Eq. 27 translates to

\[
M_{\min}^2 \leq m_\alpha^2 \leq M_{\max}^2, \tag{31}
\]

where

\[
M_{\min}^2 = (\sqrt{s})^2 \left( \frac{-B_0 - \sqrt{B_0^2 - 4A_0C_0}}{2A_0} \right), \tag{32}
\]

\[
M_{\max}^2 = (\sqrt{s})^2 \left( \frac{-B_0 + \sqrt{B_0^2 - 4A_0C_0}}{2A_0} \right). \tag{33}
\]

According to Eq. 31 the distribution of the square value of these new variables, \( M_{\min} \) and \( M_{\max} \), must show endpoints at the value of \( m_\alpha^2 \). These endpoints can be used to untangle \( \tau \to \ell\alpha \) decays from the SM processes and to measure the mass of the \( \alpha \) particle in case of observation. Also, if these new variables are not highly correlated, they could be combined in a two-dimensional distribution to increase the statistical power of the method. In the rest of the paper we will refer to these as the \( M_{\min}, M_{\max}, \) and 2D methods, respectively.

### A. Simulated data selection

To study the kinematic bounds in Eq. 31 at the energies of the Belle II experiment, we simulate \( e^+e^- \rightarrow \tau^+\tau^- \) and \( e^+e^- \rightarrow q\bar{q} \) processes at \( \sqrt{s} = 10.58 \text{ GeV} \) using Pythia8 [23] implemented in ROOT 6.20 [24], where we added a new stable \( \alpha \) spin-0 particle to account for the \( \tau \to e\alpha \) decay which is simulated using a phase-space model. We estimate the number of simulated events for \( \tau^+\tau^- \) and \( q\bar{q} \) decaying to SM processes from the cross-sections reported by Belle II [7]. For particles with transverse momentum \( p_T \), the momentum precision \( \sigma \) in the Belle II experiment [25], varies from \( \sigma/p_T \approx 5\% \) for very low \( p_T \) particles, to 0.3\% for \( p_T \geq 0.5 \text{ GeV} \). To have more realistic simulated data, we apply a Gaussian smearing to the momentum components of the final state particles for an average precision of \( \sigma/p_T = 1\% \).

To select tau pairs in 3x1 prong decays, per event, we require four charged particles in the final state with no more than one photon with an energy greater than 0.05 GeV; the latter to suppress decays with neutral pions decaying to photons in the kinematic regime of the photon reconstruction in the Belle II detector. Tau pair decays are produced back-to-back in the CMS and their decay produce jet-like events, with two cones of collimated particles around the thrust axis \( n_T \), defined as the vector that maximize the thrust magnitude \( T \):

\[
T = \frac{\sum |p_i \cdot n_T|}{\sum |p_i|}, \tag{34}
\]

where \( p_i \) is the momentum of the \( i \)-th particle in the CMS. To enhance the selection of \( (\tau^+ \rightarrow \pi^+\pi^-\pi^+\bar{\nu}_\tau)(\tau^- \rightarrow e^-\alpha + \text{anything}) \), 3-prong candidates are reconstructed in combinations of three pions on the same side of a plane perpendicular to the thrust axis, while the 1-prong candidate requires one electron on the opposite side.

Figure 2 and Fig. 3 show the distributions for \( M_{\min} \) and \( M_{\max} \) in the simulated data before the momentum smearing. To illustrate, the number of \( \tau \to e\alpha \) decays for \( m_\alpha = 1 \text{ GeV} \) has been set equal to the number of SM background events. In both variables, the signal distribution shows clear endpoints and a peaking structure at \( m_\alpha^2 \), and the background extends all over the kinematic allowed region without significantly peaking at any point. These striking differences between signal and background data distributions will allow us to disentangle one from each other. In the two-dimensional distribution of \( (M_{\min}^2, M_{\max}^2) \), we do not observe a direct correlation for the signal events. However, background events appear to be correlated.

### B. Production measurement

The production of a BSM decay is usually measured relative to a similar SM process; this to cancel out in the ratio systematic effects related to luminosity, cross-section, and branching ratios. The \( \tau \to e + \text{anything} \) data is dominated by \( \tau \to e\bar{\nu}_e\nu_\tau \) decays, and this is used as the normalization process in the \( \tau \to \ell\alpha \) production measurement. Then, to estimate this relative production we need to identify three components in the \( \tau \to e + \text{anything} \) data: the \( \tau \to e\alpha \) decays; the SM process \( \tau \to e\bar{\nu}_e\nu_\tau \); and anything else is considered as background.
For this, the data should follow a probability distribution given by

\[
f(x) = \frac{N_\alpha S_\alpha(x) + N_\nu S_\nu(x) + N_b B(x)}{N_\alpha + N_\nu + N_b},
\]

\[= \frac{N_\nu \mu \epsilon_\nu S_\nu(x) + N_\nu S_\nu(x) + N_b B(x)}{N_\nu \mu \epsilon_\nu + N_\nu + N_b},
\]

where \(N_\alpha\), \(N_\nu\), and \(N_b\) are number of \(\tau \to e\alpha\) decays, the number of \(\tau \to e\bar{\nu}_\tau\nu_\tau\) decays, and the number of background events, respectively. These components are described by the probability density functions \(S_\alpha(x)\), \(S_\nu(x)\) and \(B(x)\). Here \(\epsilon_\alpha/\epsilon_\nu\) is the relative observation efficiency of the first two components, and \(\mu\) is the relative branching ratio

\[
\mu = \frac{Br(\tau \to e\alpha)}{Br(\tau \to e\bar{\nu}_\tau\nu_\tau)}.
\]

This is the parameter of interest in which a non-zero value indicates the presence of signal in data. Then the measurement of the \(\tau \to e\alpha\) production reduces to estimate the value of \(\mu\). For the search of tiny signals, it is better to formulate the \(\mu\) determination in terms of a hypothesis test to exclude the presence of a possible signal at a desired confidence level (CL).

To test the performance of the \(M_{\text{min}}, M_{\text{max}}\), and 2D methods in the determination of \(\mu\), we use simulated data samples of \(\tau \to e + \text{anything}\) composed of SM-only processes that follow the selection criteria described in Section IIIA. Then by using the model in Eq. (35), C.L. upper limits on \(\mu\) are estimated with an asymptotic CLs technique [24] implemented in RooStats [27]. For the data modeling, the probability density distributions \(S_\alpha(x)\), \(S_\nu(x)\) and \(B(x)\), are extracted as templates from independently simulated data samples, where the relative efficiency is found to be \(\epsilon_\alpha/\epsilon_\nu = 1.17\). Four methods are studied for the upper limit estimate,
1. The ARGUS method, using the normalized electron energy in the pseudo-rest-frame, \(x = 2E_{e}/m_{\tau}\), as the discriminating variable.

2. The \(M_{\text{min}}\) method, using \(M_{\text{min}}^{2}\) as the discriminating variable.

3. The \(M_{\text{max}}\) method, using \(M_{\text{max}}^{2}\) as the discriminating variable.

4. The 2D method. A combination of \(M_{\text{min}}^{2}\) and \(M_{\text{max}}^{2}\) in a two-dimensional density distribution.

Figure 4 summarizes the results on the upper limit estimate for masses of the \(\alpha\) particle between 0 and 1.6 GeV for an integrated luminosity of 50 ab\(^{-1}\); the data Belle II expects to collect during the next decade. The ARGUS and the \(M_{\text{min}}\) methods present similar performance for the upper limit estimate. However, for lower \(m_{\alpha}\) values, we obtain better upper limits when using the \(M_{\text{max}}\) variable than with these two methods. This improvement is not negligible at all; for \(m_{\alpha} = 0\), the upper limit in the \(M_{\text{max}}\) method is half the one achieved with the ARGUS technique. If we use a simple scaling of \(1/\sqrt{N}\) for the limit estimate as data increase, this translates to four times more data in the ARGUS or the \(M_{\text{min}}\) method to perform as good as the \(M_{\text{max}}\) variable. However, the 2D method produces a better upper limit than the other three methods alone, improving the expected upper limit by one order of magnitude compared to the ARGUS technique. For the \(m_{\alpha} = 0\), the ARGUS method upper limit is 15 times larger than the upper limit in the 2D method. Using the simple data scaling, this means the ARGUS technique requires 225 times more data to perform as well as the 2D method.

For the ARGUS and the 2D method, Fig. 5 shows for \(m_{\alpha} = 0\) the upper limit on the relative branching ratio as a function of the integrated luminosity. We note that to reach an upper limit below \(10^{-4}\), 1 ab\(^{-1}\) of data is necessary for the 2D method. However, an order of magnitude more statistics is required for the ARGUS technique. It is clear that with the proposed 2D method, Belle II could reach the level of the expected upper limit in the ARGUS technique for the full data sample, but with only a fraction of the data, which could be collected during the first years of operation.

IV. CONCLUSIONS

We have studied the kinematics of the decay of a particle-antiparticle pair for known center-of-mass energy when in each decay, one of the produced particles escapes detection. This study led us to determine kinematic bounds on the mass of the new \(\alpha\) particle in the search for LFV \(\tau \rightarrow \ell\alpha\) decays. We propose using these bounds in a two-dimensional method for the production measurement of this BSM process in tau pair decays at electron-positron colliders.

For the upper limit estimate on the relative production of the \(\alpha\) particle, we apply the method to simulated data that emulates some of the Belle II experiment conditions. The proposed variables, \(M_{\text{min}}\) and \(M_{\text{max}}\), show similar or better performance than the commonly used ARGUS technique. And when we combine both variables in the 2D method, the upper limit estimate is one order of magnitude lower than the one obtained by the ARGUS, \(M_{\text{min}}\) or \(M_{\text{max}}\) methods alone. With the 2D method, Belle II could attain the expected upper limit in the ARGUS technique with only a fraction of the full data sample to be collected during its operation.

For the performance comparison of the methods presented in this work, the upper limit estimate lacks several experimental effects, such as trigger efficiencies, beam
backgrounds, or particle identification efficiency. However, they will similarly affect any of the methods considered. More importantly, they should not change the relative performance of the discriminant variables, in which the 2D approach achieves the best upper limit estimate. In most cases, these effects will cancel out in the ratio of signal to background processes, or they will be a global scale factor in the data distribution model, which does not alter the upper limit comparison for the different methods.

One important difference between the proposed approach and the ARGUS method, is that we do not need a 3-prong tau decay as required in the pseudo-rest-frame technique, then our methods can be implemented in 1x1 prong decays such as $\tau^+ \rightarrow \pi^+\nu_\tau)(\tau^- \rightarrow e^-\alpha)$, or any nx1 prong tau decay. Since $Br(\tau^- \rightarrow \pi^-\nu_\tau) = 1.16 \times Br(\tau^- \rightarrow \pi^-\pi^-\pi^-\nu_\tau)$ [28], we expect an upper limit of the same order of magnitude for 1x1 prong decays as the obtained from 3x1 prong decays, and combined will increase even more the reach of Belle II on the production search for LFV $\tau \rightarrow e\alpha$ decays.

Although it is beyond the scope of the present work, we should mention that the proposed methods could be applied for studies on the tau neutrino mass upper limit from colliders [29], or for heavy neutrinos searches in the tau lepton sector [30]. Also, when in Eq. 13 we take $m_X = m_\tau$ as the parameter of interest, and SM processes are required in the two tau decays, endpoints can be found for the mass measurement of the tau lepton.

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