On the possibility of metamaterial properties in spin plasmas

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Abstract. The fluid theory of plasmas is extended to include the properties of electron spin. The linear theory of waves in a magnetized plasma is presented, and it is shown that the spin effects cause a change of the magnetic permeability. Furthermore, by changing the direction of the external magnetic field, the magnetic permeability may become negative. This leads to instabilities in the long wavelength regimes. If these can be controlled, however, the spin plasma becomes a metamaterial for a broad range of frequencies, i.e. above the ion cyclotron frequency but below the electron cyclotron frequency. The consequences of our results are discussed.

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the consequences of a negative group velocity, were moreover given attention by Pafomov [4] and Agranovich and Ginzburg [5], and have since then been discussed by several authors, most notably by Veselago [6], who gave a detailed analysis of the consequences of such material properties (see also [3], [7]–[9]). Although not known to be found naturally, such materials have recently been realized in laboratory environments [10, 11], and the experimental development in conjunction with theoretical insights (see [12, 13]) has spawned a rapidly growing interest in these materials (see, e.g. [14, 15] for a review). From a sharp resonance in the material response to the applied external field, one may obtain negative $\varepsilon$ and $\mu$. The normal procedure for obtaining negative-index-of-refraction materials is to put together two structured materials that both have negative permittivity and negative permeability, such that the resulting composite material has a negative refractive index [15, 16]. A nonlinear metamaterial can also be constructed through the nonlinear properties of the constituent materials [17, 18], admitting new types of solitary wave structures [19]–[21].

The field of quantum plasmas is a rapidly growing field of research. From the non-relativistic domain, with its basic description in terms of the Schrödinger equation, to the strongly relativistic regime, with its natural connection to quantum field theory, quantum plasma physics provides promises of highly interesting and important applications, fundamental connections between different areas of science, as well as difficult challenges from a computational perspective. The necessity to thoroughly understand such plasmas motivates a reductive principle of research, for which we successively build more complex models based on previous results. The simplest lower order effect due to relativistic quantum mechanics is the introduction of spin, and as such thus provides a first step towards a partial description of relativistic quantum plasmas.

Already in the 1960s, Pines studied the excitation spectrum of quantum plasmas [22, 23], for which we have a high density and a low temperature as compared with normal plasmas. Recently, there has been increased interest in the properties of quantum plasmas [24]–[40], [42]–[44]. The studies have been motivated by the development in nanostructured materials [45] and quantum wells [46], the discovery of ultracold plasmas [47, 48], or a general theoretical interest. The list of quantum mechanical effects that can be included in a fluid picture includes the dispersive particle properties accounted for by the Bohm potential [24]–[36], the zero temperature Fermi pressure [24]–[28], spin properties [37]–[41] as well as certain quantum electrodynamical effects [49]–[52]. Within such descriptions, [24]–[28], [37, 38], [50]–[52] quantum and classical collective effects can be described within a unified picture, sometimes even showing a surprising overlap between classical and quantum behaviour [41].

Here, we study the linear theory of electromagnetic wave propagation in a magnetized plasma, with a special focus on the properties caused by the electron spin. We are then able to present a scheme for such a system to display metamaterial behaviour. Specifically this is induced by exposing a low temperature high density plasma to an external magnetic field, which creates a magnetization in the plasma due to the electron spin. By changing the direction of the external magnetic field, the magnetic permeability may become negative. It should be noted that the above procedure induces instabilities in the long wavelength regime. A number of ways to control these instabilities are pointed out. Assuming that this can be done successfully, the spin plasma becomes a metamaterial for a broad range of frequencies, i.e. above the ion cyclotron frequency but below the electron cyclotron frequency. The conditions needed to create a sufficient magnetization are discussed in the final section of the paper.
2. Basic equations

The theory for quantum plasmas including the effects of particle dispersion \([24]–[36]\), the Fermi pressure \([24]–[28]\) and effects due to the electron spin \([37]–[39]\) has been described in a number of recent papers. For our purposes it will be sufficient to include the spin effects, as the Fermi pressure and the Bohm–de Broglie potential will not affect whether the plasma is a metamaterial or not\(^2\). Furthermore, neglecting terms that are quadratic in the spin vector\(^3\), the governing spin plasma equations can be written

\[
\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s v_s) = 0, \quad (1)
\]

\[
m_s n_s \left( \frac{\partial}{\partial t} + v_s \cdot \nabla \right) v_s = q_s n_s (E + v_s \times B) + n_s \frac{2 \mu_s}{\hbar} s^a \nabla B_a - \nabla P_s, \quad (2)
\]

together with the spin evolution equation

\[
\left( \frac{\partial}{\partial t} + v_s \cdot \nabla \right) s_s = \frac{2 \mu_s}{\hbar} s_s \times B. \quad (3)
\]

Here \(q_s\) and \(m_s\) are the charge and mass of species \(s\), \(\mu_s = q_s \hbar / 2m_s\) is the magnetic moment, \(P_s = k_B T n_s\) is the pressure (for simplicity we use an isothermal pressure model), \(n_s\) is the number density, \(v_s\) is the velocity and \(s_s\) is the spin vector.

Next we concentrate on the linear wave modes in a magnetized plasma described by equations (1)–(3) together with Maxwell’s equations

\[
\nabla \times E = -\frac{\partial B}{\partial t}, \quad (4)
\]

and

\[
\nabla \times B = \mu_0 J + \frac{1}{c^2} \frac{\partial E}{\partial t}, \quad (5)
\]

where, in addition to the free current density, we include the magnetization current source

\[
J_m = \nabla \times M_s = \nabla \times \left( \frac{2 \mu_s n_s}{\hbar} s_s \right), \quad (6)
\]

with \(J = \sum_s q_s n_s v_s + \sum_s J_m\). For this purpose we chose a coordinate system such that the unperturbed magnetic field is \(B_0 = B_0 \hat{z}\), the wavevector is \(k = k_\perp \hat{x} + k_z \hat{z}\), and the variables are divided into an equilibrium value (index 0) and a perturbed part (index 1). For simplicity the index 1 is omitted on the electric field and the velocity, since these variables have a zero unperturbed part. As a preparation, we first consider the linear theory without the spin terms. Using the continuity equation to express the density in terms of the velocity, the momentum equation relates the velocity to the electric field. Then from the momentum equation we solve

\(^2\) This is clear from the dispersion relation presented in \([52]\), where the effect of the Fermi pressure and the Bohm–de Broglie potential is incorporated in an effective thermal velocity, as far as the linear wave modes are concerned.

\(^3\) The terms that are quadratic in the spin vector can be omitted if the characteristic spatial scale is longer than the thermal de Broglie wavelength.
for the velocity in terms of the electric field to find the susceptibility tensor for each particle species. Combining this with Maxwell’s equations, the result becomes

\[
\left( \delta_{ij} \left( 1 - \frac{k_j^2 c^2}{\omega^2} \right) + \frac{k_i k_j c^2}{\omega^2} + \chi_{ij} \right) E^j = 0
\]

(7)

where \( \delta_{ij} \) is the Kronecker delta and the susceptibility tensor is

\[
\chi = \sum_s \begin{pmatrix}
-\frac{\omega_p^2 (\omega^2 - k_z^2 v_{ts}^2)}{\omega_d^2} & -\frac{\omega_p^2 \omega_{cs} (\omega^2 - k_z^2 v_{ts}^2)}{\omega_d^2} & -\frac{\omega_p^2 k_z^2 v_{ts}^2}{\omega_d^2} \\
\frac{\omega_p^2 \omega_{cs} (\omega^2 - k_z^2 v_{ts}^2)}{\omega_d^2} & -\frac{\omega_p^2 (\omega^2 - k_z^2 v_{ts}^2)}{\omega_d^2} & i \frac{\omega_p^2 \omega_{cs} k_z v_{ts}^2}{\omega_d^2} \\
-\frac{\omega_p^2 k_z v_{ts}^2}{\omega_d^2} & -i \frac{\omega_p^2 \omega_{cs} k_z v_{ts}^2}{\omega_d^2} & -\frac{\omega_p^2 (\omega^2 - k_z^2 v_{ts}^2 - \omega_{cs}^2)}{\omega_d^2}
\end{pmatrix}
\]

where \( v_{ts} = (k_B T / m_s)^{1/2} \) is the thermal velocity, \( \omega_p = (n_0 q_s^2 / \varepsilon_0 m_s)^{1/2} \) the plasma frequency, \( \omega_{cs} = q_s B_0 / m_s \) the gyrofrequency and \( \omega_d^4 = (\omega^2 - k_z^2 v_{ts}^2) (\omega^2 - k_z^2 v_{ts}^2) - \omega_{cs}^4 (\omega^2 - k_z^2 v_{ts}^2 - k_z^2 v_{ts}^4) \).

The next aim is to add the electron spin contribution, where the spin effects due to the ions are neglected due to their small magnetic moment. In general the spin vector is a dynamical variable whose relation to the EM field is complex already in the linear theory. Thus from now on, we limit ourselves to the case where the dynamics is slow compared to the spin precession period, which is equivalent to limiting ourselves to frequencies \( \omega \ll |\omega_{cs}| \). In that case we can write the spin vector (for electrons, where we let index \( s = e \)) as

\[
s_e = -\frac{\hbar}{2} \mathbf{B} \tanh \left( \frac{\mu_B B_0}{k_B T} \right),
\]

(9)

where \( \mu_B \) is the Bohr magneton and the tanh-factor is due to thermodynamic considerations\(^4\).

The expression (9) applies when the wave dynamics takes place at a timescale much faster than the spin relaxation frequency, such that the fractions of particles in the two spin states remain constant. For the opposite limit, we should make the substitution \( \tanh(\mu_B B_0 / k_B T) \to \tanh(\mu_B B / k_B T) \) in equation (9). (Note here that we use an isothermal approximation with a constant temperature.) The linearized magnetization current then becomes

\[
\mathbf{J}_m = -\mu_B \tanh \left( \frac{\mu_B B_0}{k_B T} \right) \left( \nabla n_1 \times \hat{z} + n_0 \nabla \times \left( \frac{\mathbf{B}_1 - B_{1z} \hat{z}}{B_0} \right) \right).
\]

(10)

The magnetic field \( \mathbf{B}_1 \) can be expressed in terms of \( \mathbf{E} \) from Faraday’s law, and the density is given in terms of the velocity from the continuity equation, which is expressed in terms of \( \mathbf{E} \) from the given susceptibility of each species. Thus the magnetization current (10) is also expressed in terms of \( \mathbf{E} \). However, before that procedure is implemented, we must also modify the standard susceptibility equation (8) to account for the spin dependent force in the momentum equation. This can be achieved by noting that when solving the momentum equation for the velocity, the magnetic dipole spin force can simply be accounted for by including the different

\(^4\) Using Fermi–Dirac statistics, the slight overweight of particles with spin orientation in the lower energy state results in a macroscopic spin-vector proportional to \( \tanh(\mu_B B_0 / k_B T) \).
components as ‘effective electric fields’. Thus when solving for the electron velocity in terms of the electric fields, the spin force is included simply by making the substitutions

\[
\begin{align*}
\vec{E}_x &= E_x + \tanh \left( \frac{\mu_B B_0}{k_B T} \right) \frac{i\hbar k_x}{m_e} B_{1z}, \\
\vec{E}_y &= E_y \\
\vec{E}_z &= E_z + \tanh \left( \frac{\mu_B B_0}{k_B T} \right) \frac{i\hbar k_z}{m_e} B_{1z}.
\end{align*}
\]

Again expressing \( B_{1z} \) in terms of \( \mathbf{E} \) through Faraday’s law, these alterations can be expressed as a spin modification of the free current susceptibility. Thus formally we can write

\[
\vec{j}^i = j^i_{\text{free}} + j^i_{\text{sp}} = \chi^i_{\text{free}} E_j + \chi^i_{\text{sp}} E_j,
\]

where the direct spin magnetization contained in \( \chi^i_{\text{sp}} \) can be determined from (10) (by expressing \( \mathbf{B}_j \) and \( n_1 \) in terms of \( \mathbf{E} \)), and the free part of the susceptibility can be divided as

\[
\chi^i_{\text{free}} = \chi^i_{\text{L}} + \chi^i_{\text{md}},
\]

where \( \chi^i_{\text{L}} \) is the (free current) susceptibility due to the Lorentz force given by (8), and the contribution from the magnetic dipole force \( \chi^i_{\text{md}} \) can be found from (8) combined with the substitution in (11) and the \( z \)-component of (4). The theory outlined here is straightforward, but results in rather cumbersome formulae. To reduce the complexity and arrive at more transparent expressions, we introduce the following simplifications:

1. The plasma is quasi-neutral, which is a valid approximation provided \( \omega^2_{pi} \gg \omega^2_{ci} \).
2. The perpendicular (to the magnetic field) free electron (i.e. non-spin) part of the current can be approximated by the \( \mathbf{E} \times \mathbf{B} \)-drift, which is valid for frequencies well below the electron gyro frequency.
3. The displacement current in Maxwells equations is small. This is valid when point 1 applies together with \( \omega^2_{pi} \gg \omega^2 \), and amounts to neglecting the Kronecker delta term in (7).
4. Only electron thermal motion is of significance, which is valid if \( k^2 v_{ti}^2 \ll \omega^2 \).

The theory outlined above now reduces to equation (7) with \( \chi_{ij} \) given by

\[
\chi = \begin{pmatrix}
-\frac{\tilde{\omega}^2_{pi}}{\omega^2 - \omega^2_{ci}} & -i\frac{\tilde{\omega}^2_{pi}}{\omega_{ci}(\omega^2 - \omega^2_{ci})} & -i\frac{\tilde{\omega}^2_{pi} k_{1} k_{i} \tilde{c}^2_{s}}{\omega_{ci} \omega_{ci}} \\
-i\frac{\tilde{\omega}^2_{pi}}{\omega_{ci}(\omega^2 - \omega^2_{ci})} & -\frac{\tilde{\omega}^2_{pi} \tilde{c}^2_{s}}{\omega^2} & 1 - \frac{\tilde{\omega}^2_{pi} k_{1} k_{i} \tilde{c}^2_{s}}{\omega_{ci} \omega_{ci}} \\
-i\frac{\tilde{\omega}^2_{pi} k_{1} k_{i} \tilde{c}^2_{s}}{\omega_{ci} \omega_{ci}} & 1 - \frac{\tilde{\omega}^2_{pi} k_{1} k_{i} \tilde{c}^2_{s}}{\omega_{ci} \omega_{ci}} & -\frac{\omega^2}{\omega^2} \left( 1 - \frac{\omega^2}{k^2 \tilde{c}^2_{s}} \right)
\end{pmatrix},
\]

where \( \tilde{\omega}_{pi} = \omega_{pi} (1 - \mu_0 M_0 / B_0)^{-1/2} \) is the spin modified ion plasma frequency, \( \tilde{c}_s = [c_s^2 - (\mu_B B_0 / m_i) \tanh(\mu_B B_0 / k_B T)]^{1/2} \) is the spin modified ion-acoustic velocity and \( c_s = v_b (m_e / m_i)^{1/2} \) is the standard ion-acoustic velocity. Furthermore, the unperturbed magnetization \( M_0 \) is given by \( M_0 = n_0 \mu_B \tanh(\mu_B B_0 / k_B T) \). We note that in contrast to the other tensor

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components, $\chi_{33}$ is not modified by the spin effects. As a specific example, we can investigate the magnetohydrodynamic limit $\omega \ll \omega_{ci}$, in which case we get the dispersion relation

$$\left(\omega^2 - k^2 \tilde{C}_A^2\right) \left[(\omega^2 - k^2 \tilde{C}_A^2 - k^2 \tilde{c}_s^2) (\omega^2 - k^2 \tilde{c}_i^2) - k^2 \tilde{c}_s^2 \tilde{c}_i^2\right] = 0,$$

by putting the determinant of (7) to zero. Here, $\tilde{C}_A = C_A(1 - \mu_0 M_0 / B_0)^{1/2}$ is the spin-modified Alfvén velocity, and $C_A = (B_0^2 / \mu_0 m_i n_0)^{1/2}$ is the standard Alfvén velocity. Equation (13) agrees with the results of [40] in the appropriate limiting cases, provided a sign error is corrected in the last term of their dispersion relation. The first factor of equation (13) describes the shear Alfvén mode, whereas the second factor has two roots, describing the fast and slow magnetoosonic modes.

3. The possibility of a plasma as a metamaterial

Next, we are interested in the possibilities to get metamaterial properties. As a starting point we note from (12) and (7) that the (relative) dielectric component $\varepsilon_{xx}$ is given by

$$\varepsilon_{xx} = -\frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2}$$

which becomes negative for frequencies above the ion cyclotron frequency. Here the first Kronecker delta term in (7) has been neglected due to point 3 above. Next, we note that the relative magnetic permeability is given by $\mu_r = B_0 / (B_0 - \mu_0 M_0)$, which causes the transition from $\omega_{pi}^2 \rightarrow \tilde{\omega}_{pi}^2$ in (12). In thermodynamic equilibrium, the spin magnetization $M_0 = \mu_B n_0 \tanh(\mu_B B_0 / k_B T)$ enhances the external field, which corresponds to a paramagnetic situation where $\mu_r > 1$. However, let us now assume that we have a laboratory plasma immersed in an external magnetic field, where the internal spin magnetization gives a significant contribution to $B_0$. In particular, we can write $B_0 = B_{0\text{ext}} + B_{0\text{sp}}$, where $B_{0\text{ext}}$ is the field due to the external sources and the spin contribution is $B_{0\text{sp}} = \mu_0 M_0$. Then consider what happens if we rapidly switch the direction of the external field by 180°, and study the properties before the spin state has time to reach a new thermodynamic equilibrium state. In particular, we are interested in the case where the external contribution to $B_0$ is smaller than the internal contribution $\mu_0 M_0$, and directed in the opposite direction. The above linearized theory then still applies, but with the difference that $\mu_r = B_0 / (B_0 - \mu_0 M_0) = (B_{0\text{ext}} + B_{0\text{sp}}) / B_{0\text{ext}} > 1$ before the switching of the external field, but $\mu_r = -(B_{0\text{sp}} - B_{0\text{ext}}) / B_{0\text{ext}} < 0$ after the switching, where the last inequality holds provided $B_{0\text{sp}} > B_{0\text{ext}}$. A first observation of the changed properties of this system is that long wavelength waves (i.e. $kC_A < \omega_{ci}$) described in (13) are now unstable, since $\tilde{C}_A^2$ changes sign with $\mu_r$ and becomes negative for the above scenario. We shall assume that it is still useful to study the stable waves with shorter wavelengths, however. In particular this is of interest in case one of the following conditions apply:

1. The plasma system is of a rather small size, and the long wavelength waves (i.e. $kC_A < \omega_{ci}$) are stabilized by inhomogeneities not included in the model.
2. The growth rate $\gamma$ of the long wavelength (at most of the order $\gamma \sim \omega_{ci}$) is slow enough such that there is still time to study the physics on a timescale much shorter than $\omega_{ci}^{-1}$.
3. Some dissipative mechanism not included in our model will be sufficient to stabilize the instability (however, note that the addition of a finite resistivity will not suffice for this purpose).

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As a specific example, we study transverse waves with \( \omega > \omega_{ci} \) propagating along \( B_0 \) (that is in the opposite direction of \( B_0 - \mu_0 M_0 \)). The dispersion relation then reduces to:

\[
\frac{\omega^2}{\omega_{ci}} \pm \omega = k^2 \tilde{C}_A^2
\]

(15)

where the \( + \) (\( - \)) sign corresponds to a right hand (left hand) circular polarization. As described above, both \( \mu_r \) and \( \tilde{C}_A \) change sign with \( B_0 - \mu_0 M_0 \), and thus (15) confirms that waves are unstable in the long wavelength regime. However, for shorter wavelengths, \( |k\tilde{C}_A| > 2\omega_{ci} \), the waves are stable regardless of the sign of \( \tilde{C}_A^2 \). Next, considering this special case such that \( k \) and \( \omega \) are real, we find that the time-averaged Poynting vector \( \langle S \rangle \) is

\[
\langle S \rangle = \langle E \times H \rangle = |E|^2 \frac{k}{\omega} \frac{(B_0 - \mu_0 M_0)}{B_0}
\]

(16)

which apparently changes sign with \( B_0 - \mu_0 M_0 \) such that the above system shows the characteristics of a metamaterial.

4. Summary and conclusion

Previous theories \([37, 38]\) on magnetized spin plasmas have been extended. In particular, the linear theory has been generalized to cover the frequency range with frequencies comparable to the ion cyclotron frequency, but still much less than the electron cyclotron frequency. The purpose has been to investigate whether it is possible to produce the characteristics of a metamaterial. It is found that in principle this is possible by switching the direction of an external magnetic field by 180°, provided the spin magnetization of the plasma is sufficiently large. However, it is a great challenge to produce the desired plasma conditions in the laboratory. In particular, we need to combine low temperatures with high densities to obtain a sufficient magnetization. Ways to reach high plasma densities have been known for a rather long time, and methods to reach extremely low plasma temperatures have recently been found \([47, 48]\). On the other hand, ultra cold plasmas are still of too low density to be useful for the above purpose. Laser produced plasmas can reach sufficient densities, and it is possible that experiments can be designed to keep the temperature sufficiently low. A challenge with such a setting might be that the plasma background dynamics is too fast for successful experiments of this kind to be done. In conclusion, the production of a plasma metamaterial consists of two challenges. Firstly, to produce a plasma with a sufficient magnetization, and secondly to master the long wavelength instabilities that are introduced when the direction of the external field is changed.

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