Kinetic analysis of elastomeric lag damper for helicopter rotors

Yafang Liu\textsuperscript{1}, Jidong Wang\textsuperscript{1} and Yan Tong\textsuperscript{1}

\textsuperscript{1} Beihang University, Xueyuan Road No.37, Beijing 100191, China

Abstract. The elastomeric lag dampers suppress the ground resonance and air resonance that play a significant role in the stability of the helicopter. In this paper, elastomeric lag damper which is made from silicone rubber is built. And a series of experiments are conducted on this elastomeric lag damper. The stress-strain curves of elastomeric lag dampers employed shear forces at different frequency are obtained. And a finite element model is established based on Burgers model. The result of simulation and tests shows that the simple, linear model will yield good predictions of damper energy dissipation and it is adequate for predicting the stress-strain hysteresis loop within the operating frequency and a small-amplitude oscillation.

1. Introduction
Elastomeric lag dampers are vital components of helicopter. They can suppress the ground resonance and air resonance that play an significant role in the stability of the aircraft. They are used in well-known aircraft such as McDonnell AH-64 Apache, Bell Model 412 and Boeing Model 360 [1]. Due to the widespread employment of the lag damper as critical element of the rotor, the modeling of the elastomeric lag dampers have received increasing attention in recent years. Hu, Xiang and Zhang [2] analyzed elastomeric lag damper kinematic coupling and its effect on helicopter air resonance in hover. Chellil et.al [3] compared elastomeric damper to other hydraulic damper, and they presented that elastomeric damper produced better results to reduce the vibration level in the helicopter rotors. Byers and Gandhi [4] believed that the radial absorbers is more effective in transferring damping to the rotor blade lag mode than chordwise absorbers.

For many years, the complex modulus method has been applied in describing the stiffness and damping properties of elastomers. The method builds the model of elastomeric lag damper by operating a linear spring and linear damper in parallel. However, we can know that the elastomeric is a kind of nonlinear material through the relevant researches of elastomeric properties. An in-depth discussion that the nonlinear charateristic of elastomer have an effect on design of lag damper is presented in article writted by Hausmann [6].

In order to solve this problem, some researchers have put forward some solutions. Felker [5] propose the expression of elastomeric lag damper force by the superposition of spring force which is expressed by a nonlinear function of displacement and damper force which is expressed by a nonlinear function of displacement and velocity. Others make a break with the complex modulus approach. Gandhi and Chopra [7] place an additional nonlinear spring in series with the linear, parallel spring and dashpot. The method for representing the series spring by a quadratic function of displacement is purely to improve the precision of the static response. However, it does not account for nonlinear damping. Other researchers [8] also used some methods that different with the complex modulus approach to simulate the elastomeric materials. However, they almost have got the same model as Gandhi and Chopra proposed. This method is based on the elastic displacement fields, which is extended to model strain-dependent material behavior. The mechanical analogy of the model [9] is structurally the same as
the model of Gandhi and Chopra, except that both springs and dampers are now represented by nonlinear functions. These methods are only available to solve some problems aimed to specific design parameters, rather than a comprehensive and common damper model. In this paper, we establish the dynamic model in time-domain by finite element to obtain a more comprehensive and common damper model. Firstly, we analyze stress-strain relationship when shear force is subjected to elastomeric lag damper which is made from silicone rubber. Within the operating frequency and a small displacement amplitude range, stress-strain hysteresis loop of elastomeric lag damper, which is based on ABAQUS, can be simulated when harmonic displacement is excited to silicone rubber.

2. The time-domain modeling of Elastomeric Damper
In this paper, we study the stress-strain hysteresis curve of elastomeric material with small amplitude applied shear force at a certain frequency. In order to simplify the model more conveniently, we can use the linear modeling of the elastomeric material to approximate the real material. The simplest elastomeric model is a spring and a damper in series or in parallel, named Maxwell model and Kelvin model. Maxwell model can express the relaxation elastomeric material, but cannot represent the shadow of creep. It reflects the steady flow only. In contrast, Kelvin model can express the creep, but cannot reflect the stress relaxation.

2.1. Burgers model
In this paper, we used a four-parameter model with the Maxwell and Kelvin unit in series, named Burgers model. Although there are some big differences between this model and the real elastomeric material, it can represent two main characteristics of silicone rubber. Figure 1 is a Burgers model.

![Burgers model](image)

**Figure 1.** Burgers model

The deformation of Burgers model includes elastic deformation, viscous deformation and elastomeric deformation, the stress-strain relationship:

\[ \varepsilon_1 = \sigma / E_1 \]  
\[ \sigma = \eta_3 \dot{\varepsilon}_2 \]  
\[ \sigma = E_3 \varepsilon_3 + \eta_3 \dot{\varepsilon}_3 \]  
\[ \text{Total strain:} \quad \varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \]  

Using differential operator method, the constitutive equation of Burgers model is transformed into:

\[ \sigma + p_1 \dot{\sigma} + p_2 \ddot{\sigma} = q_1 \dot{\varepsilon} + q_2 \ddot{\varepsilon} \]  

In this equation,

\[ p_1 = \frac{\eta_3}{E_1} + \frac{\eta_3 + \eta_3}{E_3}; p_2 = \frac{\eta_3 \eta_3}{E_1 E_3}; q_1 = \eta_3; q_2 = \frac{\eta_3 \eta_3}{E_3}. \]

2.2. The parameters of Burgers model
Using Laplace transform with formula (3), we can infer the equation of the creep compliance in Burgers model:

\[ J(t) = \frac{1}{E_1} + \frac{t}{\eta_2} + \frac{1}{E_3}(1 - e^{-\frac{t}{\eta_3}}) \]  

(4)

With formula (4), if creep compliance value is known each time, the least squares method can be used to fit out parameters \( E_1, \eta_2, E_3, \eta_3 \) of elastomeric material. Specific steps: we choose the initial value of elastic modulus and viscosity coefficient. Furthermore, the approach that solute iteratively with MATLAB and optimize the value of four parameters can be used.

2.3. Transform the parameters of Burgers model into Prony series

Because the parameters from the fitting model cannot be read into finite element software directly. We want to use the software to simulate the mechanical characteristics of elastomeric material, we need transform the parameters of Burgers model into Prony series.

Supposing the volume strain is thoroughly recoverable and elastic deformation, so the normalized bulk modulus \( k_i \) in Prony series are 0. Supposing shear strain was rheological, we can obtain:

\[ S_\eta = Y'(t) * \varepsilon_\eta = 2G(t)\varepsilon_\eta \]  

(5)

\[ \sigma_{ii} = Y'(t) * \varepsilon_{ii} = 3K\varepsilon_{ii} \]

or

\[ \varepsilon_\eta = J(t) * dS_\eta \]  

\[ \varepsilon_{ii} = J(t) * d\sigma_{ii} \]

(6)

\( S_\eta \) and \( \varepsilon_\eta \) are the deviatoric tensor of stress and the deviatoric tensor strain. \( \sigma_{ii} \), \( \varepsilon_{ii} \) are the spherical tensor of stress and the spherical tensor of strain, * represents Stieltjes convolution.

In Burgers model:

\[ G(t) = \frac{Y'(t)}{2} = \frac{G_1}{\alpha - \beta} [\frac{G_2}{\eta_3} - \beta] e^{-\beta t} - (\frac{G_2}{\eta_3} - \alpha)e^{-\alpha t} \]  

(7)

\[ J'(t) = \frac{1}{2G_1} + \frac{t}{2\eta_2} + \frac{1}{2G_2}(1 - e^{-\frac{t}{\eta_3}}) \]  

(8)

\[ \alpha = p_1 + \sqrt{p_1^2 - 4p_2} \] 

\[ \beta = p_1 - \sqrt{p_1^2 - 4p_2} \]

According to the generalized Hooke's law and the corresponding expression of elastomeric material, the Laplace solution with uniaxial compression can be obtained:

\[ \sigma_{xx} = \frac{1}{E} \varepsilon_{xx} = \frac{s[2J(s) + J'(s)]}{3} \frac{\sigma_0}{s} = \frac{2}{3} J(s) + \frac{1}{3} J'(s) \sigma_0 \]  

(9)

Substituting equation(6), equation(8) and \( K = \frac{2(1 + \mu)}{3(1 - 2\mu)} \) into equation (9):

\[ J^* = \frac{2J(t) + J'(t)}{3} = \frac{1}{2(1 + \mu)G_1} + \frac{t}{3\eta_2} + \frac{1}{3G_2}(1 - e^{-\frac{t}{\eta_3}}) \]

(10)
In order to distinguish with one-dimensional elastomeric parameters, we replace \( \eta \) with \( n \). By comparing to equation (4), we can infer:

\[
G_i = \frac{E_i}{2(1 + \mu)} , \quad n_2 = \frac{\eta_2}{3} , \quad G_2 = \frac{E_3}{3} , \quad n_3 = \frac{\eta_3}{3}
\]  

(11)

Substituting equation (7), the shear modulus of Burgers model can be transformed into Prony series:

\[
G(t) = G_\infty + G_1 e^{-\frac{t}{\tau_1}} + G_2 e^{-\frac{t}{\tau_2}}
\]  

(12)

Where \( G_\infty = G_0 - G_1 - G_2 \), \( \tau_1 = 1/\alpha \), \( \tau_2 = 1/\beta \) and \( \tau_1, \tau_2 \) reflect the tao1 and tao2 in ABAQUS. In ABAQUS, \( g_1 = G_1 / G_0, g_2 = G_2 / G_0 \), the values of \( k1 \) and \( k2 \) are 0.

3. Finite element modeling of dynamics of elastomeric damper

This paper mainly focuses on the analysis and calculation of plate-formed elastomeric damper. The simplified model is as follows:

\[\text{Figure 2. Simplified model of plate elastomeric lag damper}\]

The upper, lower and the intermediate plate are all made by aluminum alloy, and filled by silicone rubber material. Each piece of aluminum alloy and silicone rubber is a separate component, and the five component combine into a whole entity which is spliced by tie-binding command. At first, we should define the materials parameters. A series of unified unit N-mm-MPa are employed. And the parameters are listed as follows:

\[\text{Table 1 basic materials parameters.}\]

| Materials                        | Aluminum alloy plate | Silicone rubber |
|----------------------------------|----------------------|----------------|
| Modulus of elasticity           | 70Gpa                | 4MPa           |
| Poisson's ratio                  | 2.7                  | 0.475          |

Though calculation, hyperelastic parameters are settled as: \( C_{10} = 0.176, C_{01} = 0.00432, D_1 = 0 \). And the viscoelastic creep parameters are from Bin Zhang. By fitting on creep compliance in MATLAB, we can obtain the Burgers model parameters:

\[ E_1 = 0.557\text{MPa}, \eta_1 = 291 \times 10^3\text{MPa/s}, E_2 = 0.438\text{MPa}, \eta_2 = 5.94 \times 10^3\text{MPa/s}. \]

Using the method as mentioned in the last part, we can transform the parameters of Burgers model into Prony series:

\[ g_1 = G_1 / G_0 = 0.1525, \quad g_2 = G_2 / G_0 = 0.2567, \quad k1 = k2 = 0, \quad \tau_1 = 1/\alpha = 0.0009507, \quad \tau_2 = 1/\beta = 6.0332. \]

We need to set the boundary conditions for the model. The upper and lower aluminum alloy plates are set the whole constraint at the left end to fix the two plates. Since the shear stress-strain hysteresis curves are dependent on the displacement, we apply different frequency(3Hz,6Hz,9Hz) to the
intermediate aluminum plate. The curves of stress and strain changed with time can be obtained. Then we export the data to EXCEL to draw the force-displacement curves.

![Figure 3. 3Hz Stress-Strain curve](image1) ![Figure 4. 6Hz Stress-Strain curve](image2) ![Figure 5. 9Hz Stress-Strain curve](image3)

4. Elastomeric Lag Damper dynamic response tests

4.1. Experimental subject
(1) Verify the force-displacement hysteretic curves of elastomeric lag damper; (2) Discover the variation discipline of elastomeric lag damper parameters

4.2. Geometric parameters of Elastomeric Lag Damper plate
The silicone rubber and metal plate are spliced by vulcanization at room temperature. It can be used when it is cured after 36 hours.

| Component parameters | thickness (mm) | length (mm) | width (mm) | elasticity modulus (MPa) | Poisson ’s ratio |
|----------------------|----------------|-------------|------------|--------------------------|-----------------|
| Intermediate aluminium plate | 6 | 56 | 41 | 70000 | 0.3 |
| Upper and lower aluminum plates | 4 | 41 | 41 | 70000 | 0.3 |
| Silicon rubber | 8 | 33 | 41 | 4 | 0.475 |

Experimental equipment diagram and photographs are shown in Figure 6 and Figure 7.
4.3. Experiment methods
Adopting the method of sinusoidal excitation, the experiment is loaded with displacement as the controlled quantity. By applying different frequency sinusoidal force, we measure the shear displacement and force of the elastomeric lag damper under the same shear strain amplitude, respectively. Therefore, we can obtain the resulting of laws of lag dampers with different frequency. During the experiments, force sensor and laser displacement sensors was used to measure and Coco80 dynamic data acquisition and real-time monitoring system is used to record the force and displacement. Prediction of shear deformation $U$ is about 0.4 mm. Vibration frequency is 3Hz, 6Hz and 9Hz, respectively. Sampling frequency is 2.56 KHz, sampling points is 2560.

The experiments steps are as follow:

1. Install the sample and ensure the specimens are installed correctly;
2. Install the force sensor and ensure that the surface of the force sensor was attached closely to the
sample to make sure the measuring accuracy. Then contacted the displacement sensor and Coco80 and use the channels 2.

(3) Install the laser displacement sensor and contact it with the 24V DC power and the YE5874 power amplifier. Then contact the YE5874 power amplifier and Coco80, using the channels 1.

(4) Contact the JBK-50 vibration exciter and YE5874 power amplifier.

(5) Contact Coco80 and YE5874 power amplifier.

(6) Set the Coco80 frequency to 3HZ. Adjust the shear displacement input and control the amplitude at around 0.8mm. Then start to apply a sinusoidal force and use laser displacement sensor and force sensor to measure the shear deformation and damping force. In the meantime, using data collection devices to record the data of force and shear deformation.

(7) Keep the shear displacement constant. Transform the Coco80 input frequency into 6HZ and 9HZ, respectively. Then repeat the experiment from step 2.

4.4. Experiment results

From the EXCEL analysis of collected data, we can find that the force-displacement curves (Figure 8, 9 and 10) are nearly an oval in all the cases, which shows that the specimens have a favorable energy-absorbing effect.

![Figure 8. 3Hz Force-Displacement curve](image8.png)

![Figure 9. 6Hz Force-Displacement curve](image9.png)
Figure 10. 9Hz Force-Displacement curve

The force-displacement curves show that the slope of the ellipse increases with the increasing of the vibration frequency when the temperature and shear displacement amplitude is a constant. Namely, the stiffness of the elastomeric lag damper model increases with the increasing of the vibration frequency.

4.5 Analysis

From the experiment we can get the conclusion as follow:

(1) Dynamic characteristics of the elastomeric lag damper will change with the vibration frequency and the amplitude of the shear displacement.

(2) When the elastomeric lag damper is subjected to alternating stress, the strain phase is lagging behind the stress phase from 0 and $\pi/2$. 

(3) Under the small shear displacement amplitude, elastomeric lag damper stress-strain hysteresis curve is an ellipse.

(4) As the frequency increased, the area of the stress-strain curves decreased. Namely, the energy consumption decreased.

5. Comparison

We can transform the force-displacement curve drawn by experimental results into stress-strain curve by the formula as follow:

$$\sigma = \frac{F}{A}$$

$$\varepsilon = \frac{u}{t}$$

(13)

where $F$ and $u$ are resilience and shear displacement measured by experiments, respectively; and $A$ that is shear area is equal to $2 \times 1353 \text{ mm}^2$, and $t$ that is the thickness of the silicone rubber is equal to 8. By comparing the results of simulation to the results of experiment, we can draw three pictures as follows:
Based on comparisons of the results of the experiments and the dynamics simulations of the various lag damper models, the following conclusions can be made:

1) The two hysteresis loops curves are coincided obtained by the experiment and ABAQUS
simulation. It is verified that the model used in the thesis is reliable.

2) Within the low frequency and a small displacement amplitude range, the stiffness is increasing while the frequency is amplifying.

6. Conclusions
In this study, well-known techniques, for the standard experimental modal analysis and correlation of finite element models with test data, combined with the Burgers method have been used to identify the time-dependent properties of an elastomeric material. The linear modeling approach that employed Burgers Models to evaluate the behavior of elastomeric dampers will produce accurate estimates of energy dissipation, but it is not sufficient for prediction of large deformation response. A more in-depth investigation of some aspects, important for actual applications, was postponed to future work, including a discussion of the effects of nonlinear properties of elastomeric lag dampers when they experienced a high frequency and a large deformation.

7. References
[1] McGuire D P 1976 The Application of Elastomeric Lead-Lag Dampers to Helicopter Rotors Lord Corporation Library of Technical Papers (Erie, Pennsylvania)
[2] Hu G C, Xiang J w, Zhang X G 2002 Analytical Model of Elastomeric lag Damper kinematic coupling and its effect on helicopter air resonance in hover Chinese J. Aeronautics 15 27-32
[3] Chellil A, Nour A, Lecheb S, Kebir H and Chevalier Y 2013 Impact of the fuselage damping characteristics and the blade rigidity on the stability of helicopter Aerospace Sci. Technol 29 235-52
[4] Byters Land Gandhi F 2009 Embedded absorbers for helicopter rotor lag damping J. Sound and Vibration 325 705-21
[5] Felker F F, Lau B H, McLaughlin S, et al. 1987 Nonlinear behavior of an elastomeric lag damper undergoing dual-frequency motion and its effect on rotor dynamics J. AHS.32 45-53
[6] Hausmann G 1986 Structure analysis and design considerations of elastomeric dampers with viscoelastic material behavior Proc. 12th. European Rotorcraft Forum (Federal Republic of Germany :Garmisch-Partenirchen)
[7] Gandhi F and Chopra I 1994 Elastomeric lag damper effects lag-lag stability in forward flight. Proc. 35th AIAA/ASME/ASCE/AHS/ASC Structures (Hilton Head: Structural Dynamics and Materials Conference) p 2726-38
[8] Tarzanin F J and Panda B 1995 Development and application of nonlinear elastomeric and hydraulic lag damper models. Proc. 36th AIAA/ASME/ASCE/AHS/ASC Structures (New Orleans: Structural Dynamics and Materials Conference) p 2534-44
[9] Kunz D L 1997 Influence of elastomeric damper modeling on the dynamic response of helicopter rotors AIAA Journal 35 349-54