Supersymmetric nonlinear sigma models as anomalous gauge theories

Aya KONDO and Tomohiko TAKAHASHI

Department of Physics, Nara Women’s University,
Nara 630-8506, Japan

December 30, 2019

Abstract

We revisit supersymmetric nonlinear sigma models on the target manifold $CP^{N-1}$ and $SO(N)/SO(N-2) \times U(1)$ in four dimensions. These models are formulated as gauged linear models, but it is indicated that the Wess-Zumino term should be added to the linear model since the hidden local symmetry is anomalous. Applying a procedure used for quantization of anomalous gauge theories to the nonlinear models, we determine the form of the Wess-Zumino term, by which a global symmetry in the linear model becomes smaller than the conventional one. Moreover, we analyze the resulting linear model in the $1/N$ leading order. Consequently, we find that the model has a critical coupling constant similar to bosonic models. In the weak coupling regime, the $U(1)$ local symmetry is broken but supersymmetry is never broken. In contrast to the bosonic case, it is impossible to find stable vacua in the strong coupling regime as far as in the $1/N$ leading order. These results are straightforwardly generalized to the case of the hermitian symmetric space.
1 Introduction

A nonlinear sigma model is regarded as a low energy effective field theory, where the relevant
degrees of freedoms are massless Nambu-Goldstone (NG) bosons associated with broken global
symmetries. Interestingly, any nonlinear sigma model based on the coset manifold is gauge
equivalent to a linear model with a so-called hidden local symmetry [1]. Although the gauge
fields for the hidden local symmetry are redundant variables, dynamical vector bosons may be
generated by quantum corrections even in four dimensions.

In supersymmetric field theories, nonlinear realization was extensively studied in [2] and
general methods to construct a nonlinear Lagrangian are provided. The characteristic feature
is that massless fermions appear as supersymmetric partners of NG bosons. These NG bosons
and their fermionic partners are described by chiral superfields in four dimensions with
supersymmetry. Then, the target space must be the Kähler manifold since chiral superfields
are complex.

Supersymmetric nonlinear sigma models with hidden local symmetries were studied on some
Kähler manifolds in [3 4 5 6] and then have been generalized by Higashijima-Nitta about
twenty years ago [7]. They showed that a supersymmetric nonlinear sigma model is formulated
as a linear gauge theory, if its target manifold is the hermitian symmetric space. However,
importantly, this is a classical correspondence between both models.

In the case of quantum field theories, nonlinear sigma models are nonrenormalizable in four
dimensions. So they are defined by the theory with ultra-violet momentum cutoff as well as
Nambu-Jona-Lasinio (NJL) model [8], or by some other non-perturbative methods. Although supersymmetry increases difficulties in analyzing the quantum dynamics, they seem not to be physical but to be technical, similar to an ambiguity of subtraction in NJL model, and so a relatively tractable problem.

Most crucially, a hidden local symmetry is generically anomalous in supersymmetric non-linear models in four dimensions, since the symmetry acts on chiral superfields. For example, let us consider the following Kähler potential as a gauged linear model:

$$K(\phi, \phi^\dagger) = \phi^\dagger e^{2V} \phi - \frac{2}{g^2} V,$$

where $\phi_i (i = 1, \cdots, N)$ is a chiral superfield and $V$ is a $U(1)$ gauge vector superfield. The last term is a Fayet-Illiopoulos (FI) term with a coupling constant $g$. The model has the global symmetry $SU(N)$ and the local one $U(1)$. In order to see this model to be equivalent to the $CP^{N-1}$ model, it has been thought that one has only to take $\phi_N = 1$ as a gauge fixing condition [6, 7]. Eliminating $V$ by the equation of motion, one may found the Kähler potential of the $CP^{N-1}$ model, the target manifold of which is parameterized by the remaining chiral superfields. However, the important point is that the anomalous hidden local symmetry does not allow us to take arbitrary gauge fixing condition. In this example, $U(1)$ is anomalous and so it is impossible to transform to the $CP^{N-1}$ model.

For one thing, we can avoid the anomaly problem by considering non-anomalous hidden local symmetries in the gauged linear model. Alternatively, one can add additional chiral superfields coupled to the vector superfield in order to cancel the anomaly. However, both methods are not helpful for formulating the nonlinear sigma model based on the hermitian symmetric space.

In this paper, we will start with the supersymmetric nonlinear sigma model, which includes only the chiral superfields and so is a well-defined theory without the anomaly. Then, we will rewrite the model by introducing an auxiliary vector superfield and performing a Legendre transformation. At this stage, the vector superfield is not a gauge field since the original Lagrangian is not gauge invariant and the path integral measure is not divided by the gauge volume. Next, we will insert the Fadeev-Popov determinant to the partition function by following the technique which was used for the quantization of anomalous gauge theories in [9]. As a result, we obtain the gauged linear model with a Wess-Zumino term which is equivalent to the original nonlinear sigma model.

We will explicitly deal with $CP^{N-1}$ and $SO(N)/SO(N-1) \times U(1)$ models, but our results can be generalized straightforwardly to other target manifolds, because these models capture typical features of the models without or with F-term constraint[7]. Both nonlinear models will be formulated as anomalous gauged linear models. Importantly, the symmetry in the gauged linear model is different from a conventional symmetry due to the effect of the Wess-Zumino term. For instance, we will show that the gauged linear model for the $CP^{N-1}$ model has the symmetry $SU(N - 1)_{\text{global}} \times U(1)_{\text{local}}$, which is smaller than the conventional symmetry.
$SU(N)_{\text{global}} \times U(1)_{\text{local}}$.

This paper is organized as follows. First, we will show the details about the supersymmetric $CP^{N-1}$ model. In the section 2.1, we will explain the quantum equivalence between this model and an anomalous gauged linear model with a Wess-Zumino term, which is derived from the Jacobian factor for chiral superfields. In the section 2.2, we will calculate a three-point vertex function given by triangle diagrams and exactly determine the form of the Wess-Zumino term in the theory including the momentum cut-off $\Lambda$. For renormalizable theories, the Feynman integral for the triangle diagram is expanded by the powers of $1/\Lambda$ and only finite terms for $\Lambda \to \infty$ contribute to the anomaly \[10\]. Here, we will provide an exact anomalous term depending on $\Lambda$, which includes higher power terms of $1/\Lambda$. In the section 2.3, we will analyze the effective potential of the linear model in the $1/N$ leading order. We find that the model has the critical coupling, below which the $U(1)_{\text{local}}$ symmetry is broken and supersymmetry is unbroken. Remarkably, in contrast to the bosonic $CP^{N-1}$ model \[1\], we will show that there is no stable vacuum beyond the critical coupling in the $1/N$ leading order. In the section 2.4, we will discuss the vector supermultiplet which is dynamically generated but unstable as similar to the bosonic case\[1\]. Interestingly, we observe that, when approaching the critical point, the vector multiplet tends to become massless. This behavior suggests the possibility that the $U(1)_{\text{local}}$ symmetry is restored at the critical coupling. Next, we will consider $SO(N)/SO(N-2) \times U(1)$ model in the section 3.1 and 3.2 as an example of the nonlinear model with F-term constraint. Although an F-term is added to the model, the qualitative features are unchanged. Finally, we will give concluding remarks in the section 4. In the appendix A, we present details of calculation of Feynman integrals in the cut-off theory.

2 Supersymmetric $CP^{N-1}$ model

2.1 Anomalous gauged linear models

The supersymmetric $CP^{N-1}$ model is defined by the Lagrangian

$$\mathcal{L} = \int d^2 \theta d^2 \bar{\theta} K_0(\varphi, \varphi^\dagger),$$

where $\varphi_i$ ($i = 1, \cdots, N-1$) are chiral superfields and $K_0$ is the Kähler potential given by

$$K_0(\varphi, \varphi^\dagger) = \frac{1}{g^2} \log \left( \frac{1}{g^2} + \varphi^\dagger \varphi \right).$$

As well-known, this Kähler potential provides the Fubini-Study metric for $CP^{N-1}$ manifold, which is parameterized by the complex fields $\varphi_i$, $\varphi_i^\dagger$. The parameter $g$ is a coupling constant with the dimension of mass inverse. The Kähler potential can be expanded at $\varphi = 0$ as

$$K_0(\varphi, \varphi^\dagger) = \frac{1}{g^2} \log \frac{1}{g^2} + \varphi^\dagger \varphi - \frac{g^2}{2}(\varphi^\dagger \varphi)^2 + \cdots,$$
where the first term has no effect on the Lagrangian, and so we find that the chiral field $\varphi$ is canonically normalized in (2.2).

By introducing an auxiliary vector superfield $V$, we can change the Kähler potential into

$$K'_0(\varphi, \varphi^\dagger, V) = e^{2V} \left( \frac{1}{g^2} + \varphi^\dagger \varphi \right) - \frac{2}{g^2} V,$$

where the last term is a FI D-term. The equation of motion of $V$ leads to

$$\frac{\delta K'_0}{\delta V} = 2e^{2V} \left( \frac{1}{g^2} + \varphi^\dagger \varphi \right) - \frac{2}{g^2} = 0 \Rightarrow -2V = \log \frac{1}{g^2} + \varphi^\dagger \varphi, \quad (2.5)$$

Substituting this back into (2.4), we obtain the same Kähler potential (2.2) for the $CP^{N-1}$ model up to irrelevant constant terms.

In (2.4), we perform change of variables such as

$$2V \rightarrow 2V - i(\lambda - \bar{\lambda}) \quad (2.6)$$

$$\varphi_i \rightarrow e^{i\lambda} \varphi_i, \quad (2.7)$$

$$\bar{\varphi}_i \rightarrow e^{-i\bar{\lambda}} \bar{\varphi}_i, \quad (2.8)$$

where $\lambda$ is a chiral superfield. Then, we find the Kähler potential to become

$$K(\phi, \phi^\dagger, V) = \phi^\dagger e^{2V} \phi - \frac{2}{g^2} V, \quad (2.9)$$

where $\phi_i (i = 1, \cdots, N)$ are chiral superfields:

$$\phi_i = \varphi_i \quad (i = 1, \cdots, N - 1), \quad \phi_N = \frac{1}{g} e^{-i\lambda}. \quad (2.10)$$

This Kähler potential gives a gauged linear model with the global symmetry $SU(N)$ and the local symmetry $U(1)_{\text{local}}$. If we take $\phi_N = 1/g$ as a gauge fixing condition for $U(1)_{\text{local}}$, the Kähler potential (2.9) reproduces the expression (2.4) and then the first one (2.2) by eliminating $V$. Hence, it was claimed that the supersymmetric $CP^{N-1}$ model can be obtained from a gauged linear model.

However, it should be noticed that $U(1)_{\text{local}}$ is an anomalous symmetry and this anomaly is an obstruction in proving the equivalence between both models. In order to include the anomaly, we have to deal with contributions from path integral measures. The idea is basically same as the quantization of anomalous gauge theory[9], although the original Lagrangian (2.1) is not gauge invariant in our case.

At first, we introduce the auxiliary vector superfield $V$ to the partition function of the $CP^{N-1}$ model:

$$Z = \int d\varphi d\varphi^\dagger \exp \left( i \int d^8z K_0(\varphi, \varphi^\dagger) \right) = \int d\varphi d\varphi^\dagger dV \exp \left( i \int d^8z K'_0(\varphi, \varphi^\dagger, V) \right). \quad (2.11)$$
where the superspace coordinate is denoted by $z = (x, \theta, \bar{\theta})$, integration measures by $d^8z = d^4xd^2\theta d^2\bar{\theta}$. In general, the $V$ integration leads to not only $K_0$ as a saddle point, but also higher order quantum corrections. However, in supersymmetric theories, we have no quantum corrections as proved by Higashijima-Nitta[11] and so this is an exact rewriting.

Let us define the Fadeev-Popov determinant $\Delta_f[V]$ for the gauge fixing condition $f[V] = 0$:

$$\Delta_f[V] \int d\lambda d\bar{\lambda} \delta \left( f[V^{(\lambda, \bar{\lambda})}] \right) = 1,$$

(2.12)

where $d\lambda d\bar{\lambda}$ is a gauge invariant measure and $V^{(\lambda, \bar{\lambda})}$ is a gauge transformation of $V$:

$$2V^{(\lambda, \bar{\lambda})} = 2V + i(\lambda - \bar{\lambda}).$$

(2.13)

Inserting (2.12) into (2.11) and changing an integration variable as $V \rightarrow V(-\lambda, -\bar{\lambda})$, the partition function (2.11) is expressed in terms of the functional integral over $\lambda$, $\bar{\lambda}$ and the original fields:

$$Z = \int d\phi d\phi^\dagger D\!V d\lambda d\bar{\lambda} \exp \left( i \int d^8z K'(\varphi, \varphi^\dagger, \lambda, \bar{\lambda}, V) \right),$$

(2.14)

$$D\!V \equiv dV \Delta_f[V] \delta(f[V]),$$

(2.15)

where $dV$ is assumed to be gauge invariant and so $D\!V$ corresponds to a gauge invariant measure divided by the gauge volume. The Kahler potential $K'$ is given by

$$K'(\varphi, \varphi^\dagger, \lambda, \bar{\lambda}, V) = e^{2V} \left\{ \frac{1}{g^2} e^{i\bar{\lambda}} e^{-i\lambda} + (\varphi^\dagger e^{i\bar{\lambda}})(e^{-i\lambda} \varphi) \right\} - \frac{2}{g^2} V.$$

(2.16)

If we take the chiral superfields $\varphi' = e^{-i\lambda} \varphi$ as integration variables, the functional measure produces the Jacobian factor derived from the relation [12, 13]

$$\frac{\delta \varphi'_j(z)}{\delta \varphi_k(z')} = \delta^8 \left( \frac{1}{4} \delta^{\bar{\lambda}}(z - z') \right).$$

(2.17)

Moreover, we change the variable from $\lambda$ to $\phi_N = e^{-i\lambda}/g$. Since $\lambda$ is a chiral field, we have a similar relation to (2.17):

$$\frac{\delta \phi_N(z)}{\delta \lambda(z')} = -i \frac{1}{g} e^{-i\lambda(z)} \frac{\bar{D}^2}{4} \delta^8(z - z').$$

(2.18)

So, in the partition function integrated over the new variables, we have the Wess-Zumino term with the factor $N$, in which $N-1$ and 1 are coming from the measures of $\varphi_j$ and $\lambda$, respectively. Finally, we can rewrite the partition function of the $CP^{N-1}$ model as follows,

$$Z = \int d\phi d\phi^\dagger DV \exp \left( i \int d^8z K(\phi, \bar{\phi}, V) + i \alpha[V, \phi_N, \bar{\phi}_N] \right),$$

(2.19)

$$\alpha[V, \phi_N, \bar{\phi}_N] = \frac{N}{16\pi^2} \int d^4xd^2\theta \log(g \phi_N) W^\alpha W_\alpha + \text{h.c.} + O(1/\Lambda^2),$$

(2.20)
where the Kähler potential is given by (2.9). $\alpha[V, \phi_N, \bar{\phi}_N]$ is the anomalous term generated by the Jacobian factor. $\Lambda$ is the ultraviolet cut-off parameter to regularize the functional measure \[12,13\], in which the leading term is given by the Wess-Zumino term for $U(1)_{\text{local}}$.

Consequently, we show that the supersymmetric $CP^{N-1}$ model is quantumly equivalent to the theory given by the Kähler potential (2.9) and the F term (2.20). This $F$ term reduces the flavor symmetry to $SU(N-1)$ and so this gauged linear model has the symmetry $SU(N-1) \times U(1)_{\text{local}}$.

### 2.2 Exact anomalous terms in cut-off theories

The $CP^{N-1}$ model in four dimensions is nonrenormalizable and it is regarded as a low energy effective field theory with a ultraviolet cutoff. So, we have to evaluate the anomalous contribution in the gauged linear model by keeping the cutoff finite. In this section, we consider the cutoff dependence of the anomalous term by calculating the triangle diagram.

First, we consider the vacuum functional

$$e^{i\Gamma[V]} = \int d\phi d\phi^\dagger \exp \left( i \int d^8z K(\phi, \bar{\phi}, V) \right). \quad (2.21)$$

Since $U(1)_{\text{local}}$ is anomalous, $\Gamma[V]$ is not gauge invariant due to the triangle diagram. On the other hand, since the partition function (2.19) is gauge invariant, the anomaly from the gauge transformation of $\Gamma[V]$ is canceled by the gauge transformation of $\alpha[V, \phi_N, \bar{\phi}_N]$:

$$\delta \alpha[V, \phi_N, \bar{\phi}_N] = -\delta \Gamma[V]. \quad (2.22)$$

Therefore, $\alpha[V, \phi_N, \bar{\phi}_N]$ can be determined by solving this equation for given $\delta \Gamma[V]$.

Here let us explain in detail the calculation of $\delta \Gamma[V]$ in the cutoff theory. The Lagrangian for the chiral spinor is given by

$$\int d^2\theta d^2\bar{\theta} \phi^\dagger e^{2V} \phi = i \bar{\Psi} \gamma^\mu P_R \Psi + v_\mu \bar{\Psi} \gamma^\mu P_R \Psi + \cdots, \quad (2.23)$$

where we have used four-component notation for the spinor, and $v_\mu$ denotes a vector field in $V$. $P_R$ is a projection operator on the right-handed fermion field: $P_R = (1 + \gamma_5)/2$. The famous two triangle diagrams contribute to the three-point vertex function of $v^\mu$ [10]:

$$\Gamma^{(3)}_{\mu\nu\rho}(k_1, k_2) \equiv -N \int \frac{d^4k}{i(2\pi)^4} \left\{ \text{tr} \left[ \frac{1 + \gamma_5}{2} \frac{1}{-\bar{k} - \bar{\phi} + k + \bar{k}} \gamma_\mu \frac{1}{-k - \phi - k} \gamma_\nu \frac{1}{k - \phi - k} \gamma_\rho \right] 
+ \text{tr} \left[ \frac{1 + \gamma_5}{2} \frac{1}{-\bar{k} + \bar{\phi} - k - \bar{k}} \gamma^\rho \frac{1}{-\bar{k} + \bar{\phi} + k + \bar{k}} \gamma^\nu \frac{1}{k + \phi - k} \gamma^\mu \right] \right\}, \quad (2.24)$$

where $N$ component fermions yield the factor $N$. As in the NJL model, this integral is divergent and so we introduce the ultra-violet cutoff $\Lambda$ after Wick rotation. It is noted that the cutoff...
is different from the previous one in (2.20) and there is no simple relation between them. The four-vector $a^\mu$ is introduced due to arbitrariness of the momenta carried by internal lines.

More precisely, we can introduce two four-vectors $a^\mu$ and $b^\mu$ independently to each triangle diagram. In this case, we have to choose $a^\mu = -b^\mu$ for avoiding non-chiral anomalies for all three currents as explained in [10]. Actually, the charge conjugation matrix $C$ satisfies $C^{-1} \gamma^\mu C = -\gamma^\mu T$ and we have

$$\text{tr} \left[ \frac{1}{-k - d} \gamma^\mu - \frac{1}{-k + d + k_1} \gamma^\nu - \frac{1}{k_2} \gamma^\rho \right] = -\text{tr} \left[ \frac{1}{k + d} \gamma^\mu \frac{1}{k + d + k_2} \gamma^\nu \frac{1}{k_1} \gamma^\rho \right].$$

So, the traces which contain no $\gamma_5$ in (2.24) cancel to each other if a momentum variable is flipped in one diagram: $k^\mu \to -k^\mu$. Therefore, only the traces involving $\gamma_5$ are left and this justifies a choice of $a^\mu = -b^\mu$.

Now, we evaluate the anomaly term $\delta \Gamma[V]$, which corresponds to the Fourier transformation of the divergence of (2.24):

$$(k_1 + k_2) \gamma^\nu \Gamma^{(3)}_{\mu\nu\rho}(k_1, k_2)$$

$$= 4N i \epsilon_{\nu\mu\lambda\rho} \int_{k^2 \leq \Lambda^2} \frac{d^4 k}{(2\pi)^4} \left\{ \frac{(k + a)^\nu k_2^\lambda}{(k + a)^2 (k + a + k_2)^2} - \frac{- (k + a)^\nu k_1^\lambda}{(k + a)^2 (k + a - k_1)^2} \right\}. \tag{2.25}$$

These integrals can be calculated straightforwardly by picking up anti-symmetric parts on the two indices $\nu, \lambda$. Combining the denominator by the Feynman parameter technique, we perform the $k$ integration by using the formula in the appendix. Then if one rotates back to the Minkowski space, the resulting function is given by

$$i(k_1 + k_2) \gamma^\nu \Gamma^{(3)}_{\mu\nu\rho}(k_1, k_2)$$

$$= -\frac{N}{8\pi^2} \epsilon_{\nu\mu\lambda\rho} \int_0^1 dx \left\{ a^\nu k_2^\lambda g(-(a + xk_2)^2, -a^2 - 2xa \cdot k_2 - xk_2^2) + a^\nu k_1^\lambda g(-(a - xk_1)^2, -a^2 + 2xa \cdot k_1 - xk_1^2) \right\}, \tag{2.26}$$

where $g(p^2, m^2)$ is defined by (A.6). This is the exact result for the anomalous vertex function in the cut-off theory.

Suppose that the currents for the $\mu, \rho$ directions are conserved, we have to choose $a = k_1 - k_2$ as explained in [10]:

$$i(k_1 + k_2) \gamma^\nu \Gamma^{(3)}_{\mu\nu\rho}(k_1, k_2) = -\frac{N}{4\pi^2} \epsilon_{\nu\mu\lambda\rho} k_1^\nu k_2^\lambda f(k_1, k_2), \tag{2.27}$$

where $f(k_1, k_2)$ is given by

$$f(k_1, k_2) = \frac{1}{2} \int_0^1 dx \left\{ g(-(k_1 - (1 - x)k_2)^2, -k_1^2 + 2(1 - x)k_1 \cdot k_2 - (1 - x)k_2^2) + g(-(1 - x)k_1 - k_2)^2, -(1 - x)k_1^2 + 2(1 - x)k_1 \cdot k_2 - k_2^2) \right\}. \tag{2.28}$$
This result is expressed in terms of the chiral current $J^\mu \equiv \bar{\psi} \sigma^\mu \psi = \bar{\Psi} \gamma^\mu P_R \Psi$:

$$\partial_\nu \langle J^\nu(x) \rangle = -\frac{N}{32\pi^2} \epsilon_{\nu \mu \lambda \rho} F^{\nu \mu} f \left( -i \frac{\partial}{\partial x}, -i \frac{\partial}{\partial x} \right) F^{\lambda \rho}. \quad (2.29)$$

The expansion in powers of $1/\Lambda$ is evaluated as

$$\partial_\nu \langle J^\nu(x) \rangle = -\frac{N}{32\pi^2} \epsilon_{\nu \mu \lambda \rho} F^{\nu \mu} F^{\lambda \rho} + \frac{N}{96\pi^2 \Lambda^2} \epsilon_{\nu \mu \lambda \rho} F^{\nu \mu} \Box F^{\lambda \rho} + O(1/\Lambda^4), \quad (2.30)$$

where the first term agrees with the conventional chiral anomaly.

Since the operator $f$ consists of space-time derivatives, we can easily provide $\delta\Gamma[V]$ in the supersymmetric model. Finally, from $\delta\Gamma[V]$ and (2.22), the resulting anomalous term can be obtained as

$$\alpha[V, \phi_N, \bar{\phi}_N] = -\frac{N}{16\pi^2} \int d^4x d^2\theta \log(g^2 \phi_N) W^{\alpha f} f \left( -i \frac{\partial}{\partial x}, -i \frac{\partial}{\partial x} \right) W_\alpha + \text{h.c.}. \quad (2.31)$$

This is an exact result for (2.20) including all orders of $\Lambda$.

**2.3 Effective potentials in the $1/N$ leading order**

Now that the $CP^{N-1}$ model is formulated as the consistent linear model, we can consider the effective potential of this model in the $1/N$ expansion. In the Wess-Zumino gauge, the scalar components are the D-term $-D$ of the vector superfield $V$ and the first component of $\phi_N$. As in [14], we take negative sign convention for the D-term of $V$. The F-term of $\phi_N$ are irrelevant to the effective potential.

In order to perform the $1/N$ expansion, we define the coupling $g^2$ by

$$g^2 \equiv \frac{G}{N}, \quad (2.32)$$

and we study the limit of large $N$ with fixed $G$. This is a conventional choice used in the $CP^{N-1}$ model. Moreover, since $g\phi_N$ should be order one for the anomalous term to be leading order, the vacuum expectation value of $\phi_N$ should be defined as

$$\langle \phi_N \rangle \equiv \sqrt{N}z, \quad (2.33)$$

where $z$ is a fixed complex number in the $1/N$ expansion.

Substituting these component fields to (2.31), we can calculate an anomalous contribution to the effective action:

$$\alpha[V, \phi_N, \bar{\phi}_N] = -\frac{N}{16\pi^2} \int d^4x \log(G|z|^2) D f \left( -i \frac{\partial}{\partial x}, -i \frac{\partial}{\partial x} \right) D. \quad (2.34)$$
For constant $D$, the operator $f$ becomes one and so a quadratic term of $D$ is generated in the effective potential.

We notice that for constant $W_\alpha$, higher order correction terms may arise from other diagrams (square, pentagon and so on) in the superpotential as

$$\log(g\phi_N) \Lambda^3 F \left( \frac{W^\alpha W_\alpha}{\Lambda^3} \right), \quad (2.35)$$

where $F(\cdots)$ denotes a certain function. If we expand it in the power series of $W^\alpha W_\alpha/\Lambda^3$, since the constant fields are included as $W^\alpha W_\alpha = \theta \theta D^2 + \cdots$ and $\log(g\phi_N) = \log(Gz) + \cdots$, the quadratic and higher powers do not contribute to the effective potential. So, (2.34) leads to an exact result of the anomalous effective potential.

Consequently, we can provide the effective potential in the leading order in the $1/N$ expansion:

$$\frac{1}{N} V(z, D) = -\frac{1}{G} D + D|z|^2 + \frac{1}{16\pi^2} D^2 \log(G|z|^2) + \frac{1}{32\pi^2} \left[ \Lambda^4 \log \left( 1 + \frac{D}{\Lambda^2} \right) - D^2 \log \left( 1 + \frac{\Lambda^2}{D} \right) + D\Lambda^2 \right]. \quad (2.36)$$

Here, the first and second terms arise from the tree level action, where we note again the negative sign convention of the D-term. The third term is the anomalous potential from (2.34). The fourth term is given by one-loop calculation, which is performed in a supersymmetric NJL model in [14]. In the calculation, $D$ is a mass square parameter for the scalar component of $\phi$ and so $D$ must be positive for a consistent vacuum.

The stationarity condition with respect to $z$ is

$$\frac{\delta V}{\delta z} = \frac{D}{z} \left( \frac{1}{16\pi^2} D + |z|^2 \right) = 0. \quad (2.37)$$

Then, we conclude $D = 0$ and so supersymmetry is never broken in the leading order.

Another stationarity condition leads to

$$\frac{\delta V}{\delta D} = 0 \Rightarrow -\frac{1}{G} + |z|^2 + \frac{1}{32\pi^2} \left[ 2\Lambda^2 - 2D \log \left( 1 + \frac{\Lambda^2}{D} \right) + 4D \log(G|z|^2) \right] = 0. \quad (2.38)$$

Substituting $D = 0$ into the above, we find

$$|z|^2 = \frac{1}{G} - \frac{\Lambda^2}{16\pi^2} \quad (2.39)$$

The model becomes inconsistent if $G$ is larger than $G_{cr} = 16\pi^2/\Lambda^2$.

Accordingly, we conclude that, in the $1/N$ leading order, the model has a stable vacuum only for the weak coupling $G < G_{cr}$, and supersymmetry is unbroken in this vacuum.

Here it should be noted that the anomalous potential has an important role on the robustness of supersymmetry. If we naively quantize the gauged linear model without the anomalous term,
the stationarity condition with respect to $z$ becomes $Dz^* = 0$ instead of (2.37) and so we have $D = 0$ or $z = 0$. The stationarity condition with respect to $D$ implies the gap equation

$$|z|^2 - \int \frac{d^4k}{(2\pi)^4} \left( \frac{1}{k^2} - \frac{1}{k^2 + D} \right) = \frac{1}{G} - \frac{1}{G_{cr}};$$

(2.40)

which is the same as that of a bosonic $CP^{N-1}$ model\[1\]. Then, we might have two phases: (i) $G < G_{cr}$, $|z| \neq 0$, $D = 0$ and (ii) $G > G_{cr}$, $|z| = 0$, $D \neq 0$. While the first case corresponds to the above supersymmetric model, the second is appeared as a new phase. If there were no anomaly, $D$ would acquire a vacuum expectation value in strong coupling region and so supersymmetry would be spontaneously broken. But it is not the case and so it is regarded that the anomalous term keeps supersymmetry unbroken.

We note that, although there is no vacuum in the strong coupling region in the $1/N$ leading order, there still remains a possibility of finding a vacuum in higher order or by considering some nonperturbative effects.

### 2.4 Dynamical vector supermultiplets

We showed that $z$ has the vacuum expectation value (2.39) in the weak coupling region. On this vacuum, the anomalous term (2.41) induces the kinetic term for the vector superfield:

$$-\frac{N}{16\pi^2} \int d^4x d^2\theta \log(\sqrt{G}z) W^\alpha f \left( -i \frac{\partial}{\partial x}, -i \frac{\partial}{\partial x} \right) W_\alpha + h.c..$$

(2.41)

It is well-known that, in general, vector bosons are dynamically generated in the model with hidden local symmetries \[1\]. Also in this model, loop diagrams of components of $\phi$ generate the kinetic term for a vector boson. In addition, the anomalous term (2.41) supplies the kinetic term, which however enhances the possibility of wrong sign due to the logarithmic function. If the logarithmic function is positive, the anomalous term encourages the appearance of negative metric states.

Fortunately, it can be easily seen that the large $N$ dynamics prohibits such a negative metric state. For the vacuum expectation value (2.39), we find

$$G|z|^2 = 1 - \frac{G}{G_{cr}} < 1 \quad (G < G_{cr}).$$

(2.42)

Therefore, the kinetic term of the vector superfield is well behaved since the logarithmic function becomes negative for $G < G_{cr}$. Then, the anomalous term (2.41) leads to the vertex function of the vector field:

$$\Gamma_{\mu\nu}^{(2)}(p) = (p^2 \eta_{\mu\nu} - p_\mu p_\nu) \frac{N}{32\pi^2} \log \left( 1 - \frac{G}{G_{cr}} \right) f(p, -p),$$

(2.43)
where \( f(p, -p) \) can be evaluated explicitly from (2.28). For \( p^2 > 0 \), we find
\[
f(p, -p) = \frac{1 + 7p^2/3\Lambda^2}{1 + 2p^2/\Lambda^2}.
\] (2.44)

Now, we calculate all of the two-point vertex function of the vector field for the time-like momentum. For loop integrations with a cut-off, we have the freedom to choose a momentum shift carried by internal lines, as well as the anomaly calculation in the section 2.2. Here, by adopting a symmetric momentum shift \((a^\mu = -p^\mu/2)\), the vertex function \( \Gamma^{(2)}_{\mu\nu}(p) \) for the vector component is given by
\[
\Gamma^{(2)}_{\mu\nu}(p) = \Gamma^{f(2)}_{\mu\nu}(p) + \Gamma^{b(2)}_{\mu\nu}(p),
\] (2.45)
\[
\Gamma^{f(2)}_{\mu\nu}(p) = -N \int_0^1 dx \int_{k^2 \leq \Lambda^2} \frac{d^4k}{(2\pi)^4} \{ k^2 + 2(1/2 - x)p \cdot k + p^2/4 \} \{ -k^2 + 2(1/2 - x)p \cdot k + p^2/4 \}^2,
\] (2.46)
\[
\Gamma^{b(2)}_{\mu\nu}(p) = N \int_0^1 dx \int_{k^2 \leq \Lambda^2} \frac{d^4k}{(2\pi)^4} \{ k^2 + 2(1/2 - x)p \cdot k + p^2/4 \} \{ -k^2 + 2(1/2 - x)p \cdot k + p^2/4 \}^2,
\] (2.47)

where \( \Gamma^f \) and \( \Gamma^b \) are coming from fermion and boson one-loop diagrams, respectively. After the \( k \) integration by using the formula in the appendix, we find that each vertex function includes a quadratic term of \( \Lambda \), which corresponds to the vector self-energy. It implies that gauge symmetry is broken by introducing the cut-off parameter. However, the quadratic terms cancel to each other in the total vertex function owing to supersymmetry. As a result, the vertex function is expressed in the conventional gauge invariant form: for \( 0 < p^2 < 4\Lambda^2 \) in the Minkowski space,
\[
\Gamma^{(2)}_{\mu\nu}(p) = -(p^2 \eta_{\mu\nu} - p_\mu p_\nu) \frac{N}{16\pi^2} \left( 1 + \log \frac{4\Lambda^2 - p^2}{4p^2} + i\pi \right).
\] (2.48)

The integral is calculated as a real number in the Euclidean space, but the imaginary part appears in the Minkowski space due to the logarithm function.

Combining these results with tree level terms, the resulting vertex function for the time-like momentum is given by
\[
\Gamma^{(2)}_{\mu\nu}(p) = -(p^2 \eta_{\mu\nu} - p_\mu p_\nu) F(p^2) + m^2 \eta_{\mu\nu},
\] (2.49)
\[
F(p^2) = \frac{N}{16\pi^2} \left\{ 1 + \log \frac{4 - p^2/\Lambda^2}{4p^2/\Lambda^2} - \frac{1 + 7p^2/3\Lambda^2}{2(1 + 2p^2/\Lambda^2)} \log \left( 1 - \frac{G}{G_{\text{cr}}} \right) \right\} + i \frac{N}{16\pi},
\] (2.50)
\[
m^2 = N \left( \frac{2}{G} - \frac{2}{G_{\text{cr}}} \right).
\] (2.51)

From this vertex function, we could expect that a massive vector particle appears dynamically, however it includes the nonzero imaginary part and so the “would be” vector particle is unstable. Actually, the vector particle has couplings with the scalar and spinor components of \( \phi \), which remain massless in the \( 1/N \) leading order and so the vector state decays into these massless particles.
Finally, we elucidate the behavior of the unstable vector state in terms of the spectral function. The propagator can be derived from the vertex function (2.43):

$$\Delta_{\mu\nu}(p) = i \Delta'(p) \left\{ \eta_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{m^2} F(p^2) \right\}, \quad \Delta'(p) = \frac{1/F(p^2)}{m^2/F(p^2) - p^2}. \quad (2.52)$$

Here we forget for a moment that $F(p^2)$ is divergent for $\Lambda^2 \to \infty$ as in [8]. If so, the spectral function $\rho(\sigma^2)$ is given by the imaginary part of $\Delta'(p)$ and then $\Delta'(p)$ is expressed by $\rho(\sigma^2)$:

$$\Delta'(p) = \int_0^{\Lambda^2} d\sigma^2 \frac{\rho(\sigma^2)}{\sigma^2 - p^2 - i\epsilon}, \quad (2.53)$$

where a new cutoff is introduced as in [8], although there are no simple relation between both cutoffs. By using (2.50) and (2.51), we can evaluate $\rho(\sigma^2)$ numerically and the resulting plots are depicted in Fig. 1. We note that $\rho(\sigma^2)$ is given by order $1/N$.

From these plots, we find a peak in the region $\sigma^2 \lesssim \Lambda^2$ for the coupling $G \gtrsim 0.5 G_{cr}$, but the width is large and the peak is hard to distinguish for $0.7 G_{cr} \gtrsim G \gtrsim 0.5 G_{cr}$. Near the critical coupling, the position of the peak approaches to $\sigma^2 \sim 0$ and the width becomes gradually narrower.

The position of the peak can be evaluated numerically by using the numerical results of $\rho(\sigma^2)$. The resulting plots are shown in Fig. 2. We find that the “mass” of the unstable vector state decreases to zero for the coupling $G$ approaching to $G_{cr}$. Since supersymmetry is not broken in this vacuum, the vector supermultiplet is dynamically generated for $G \gtrsim 0.7 G_{cr}$, but it is unstable.

Most interestingly, we find that the spectral function approaches rapidly to a delta function for $G \to G_{cr}$, namely $\rho(\sigma^2) \to \delta(\sigma^2)$. This behavior suggests that a massless vector supermultiplet is dynamically generated and the $U(1)$ gauge symmetry is restored at the critical
Figure 2: The plots of the position of the peak of $\rho(\sigma^2)$, $\sigma = M$. It corresponds to the mass of the unstable vector multiplet.

coupling. Unfortunately, the analysis just at $G = G_{cr}$ seems to be subtle in the leading order, because $|z|^2$ becomes zero and so the logarithmic term in the effective potential diverges.

3 Nonlinear sigma models with F-term constraint

3.1 $SO(N)/SO(N - 2) \times U(1)$ model

We consider a supersymmetric nonlinear sigma model based on the manifold $SO(N)/SO(N - 2) \times U(1)$ [7]. The model is formulated by a gauged linear sigma model as well as the $CP^{N-1}$ model. We introduce the chiral superfields $\phi_i$ ($i = 1, \cdots, N$) and the Kähler potential is the same as (2.4). In addition, the linear model has the superpotential by using an extra chiral superfield:

$$W(\phi_0, \phi) = \frac{1}{2} \phi_0 \phi^2,$$

(3.1)

where the chiral superfield $\phi_0$ corresponds to a Lagrange multiplier and then it induces the constraint $\phi^2 = 0$. For the $U(1)$ symmetry, $\phi$ and $\phi_0$ has the charge $+1$ and $-2$, respectively.

In order to transform back to the nonlinear model, we have to fix the gauge of the $U(1)$ symmetry as $\phi_N = 1/g$ similar to the case of the $CP^{N-1}$ model. Here, we should notice that this rewriting also suffers from the anomaly. Since the total $U(1)$ charge for $\phi_0$ and $\phi_i$ equals to $N - 2$, the anomalous term turns out to be given by

$$\alpha[V, \phi_N, \bar{\phi}_N] = -\frac{N - 2}{16\pi^2} \int d^4x d^2\theta \log(g \phi_N) W^a f \left(-i \frac{\partial}{\partial x}, -i \frac{\partial}{\partial x}\right) W_a + \text{h.c.}$$

(3.2)

As a result, the symmetry of the linear model is reduced to $SO(N - 1) \times U(1)_{\text{local}}$, while the Kähler potential has the symmetry $SO(N) \times U(1)_{\text{local}}$. 13
In the background $\langle \phi_0 \rangle = [w, 0, h]$, the part of the Lagrangian derived from (3.1) is expanded by the component fields $\phi^i = [A^i, \psi^i, F^i]$ as

$$
\int d^2 \theta W(\phi_0, \phi) + \text{h.c.} = w F^i A^i + \frac{1}{2} h A^i A^i - w \psi^i \psi^i + \text{h.c.}.
$$

(3.3)

Eliminating the auxiliary fields $F_i$ by the equations of motion $F_i^* + w A^i = 0$, (3.3) yields mass terms for component fields. By including the contribution from the Kähler potential, the mass terms in this background are given as

$$
\mathcal{L}_{\text{mass}} = -(D + |w|^2) A^i A^i + \left( \frac{1}{2} h A^i A^i - w \psi^i \psi^i + \text{h.c.} \right).
$$

(3.4)

### 3.2 Effective potentials including F-terms

The mass term (3.4) is essentially same as that of the supersymmetric NJL model analyzed in [14]. For the scalar, the mass square eigenvalues are given by $D + |w|^2 \pm |h|$. According to [14], the effective potential in the $1/N$ leading order can be calculated as

$$
\frac{1}{N} V(z, D, w, h) = -\frac{1}{g^2} D + N(D + |w|^2 - |h| \cos \theta)|z|^2 + \frac{1}{16\pi^2} D^2 \log(G|z|^2)
$$

$$
+ \frac{1}{16\pi^2} \left\{ F(D + |w|^2 + |h|) + F(D + |w|^2 - |h|) - 2F(|w|^2) \right\},
$$

(3.5)

where $\theta$ is the phase of $h A^i A^i$ and the function $F(x)$ is defined by

$$
F(x) = \frac{1}{2} \left[ \log(1 + x) - x^2 \log \left( 1 + \frac{1}{x} \right) + x \right].
$$

(3.6)

We set the cutoff $\Lambda$ equal to one for simplicity. The potential (3.5) reduces to a similar expression to the previous one (2.36) if taking the limit $h, w \to 0$. We note that the factor of the anomalous term $N - 2$ is approximated as $N$ for large $N$.

Differentiating the potential (3.5), the stationarity conditions are given by

$$
\frac{\delta V}{\delta \theta} = 0 \Rightarrow |h||z|^2 \sin \theta = 0,
$$

(3.7)

$$
\frac{\delta V}{\delta |h|} = 0 \Rightarrow I(D + |w|^2 + |h|) - I(D + |w|^2 - |h|) = 16\pi^2 |z|^2 \cos \theta,
$$

(3.8)

$$
\frac{\delta V}{\delta w} = 0 \Rightarrow w^* \left\{ I(D + |w|^2 + |h|) + I(D + |w|^2 - |h|) \right\} = -16\pi^2 w^* |z|^2,
$$

(3.9)

$$
\frac{\delta V}{\delta z} = 0 \Rightarrow \frac{1}{z} \left( \frac{1}{16\pi^2} D^2 + (D - |h| \cos \theta)|z|^2 \right) = 0,
$$

(3.10)

$$
\frac{\delta V}{\delta D} = 0 \Rightarrow -\frac{1}{G} + |z|^2 + \frac{1}{8\pi^2} D \log(G|z|^2)
$$

$$
+ \frac{1}{16\pi^2} \left\{ I(D + |w|^2 + |h|) + I(D + |w|^2 - |h|) \right\} = 0,
$$

(3.11)
where \( I(x) \) is defined by

\[
I(x) \equiv F'(x) = 1 - x \log \left(1 + \frac{1}{x}\right).
\] (3.12)

The stationarity condition (3.7) implies that \( \theta = 0 \) or \( \pi \), or \( |h| = 0 \). Note that \( |z| \) must not be zero since the potential includes \( \log |z| \). Since \( I(x) \) is a monotonically decreasing function \[14\], we find, if \( |h| \neq 0 \),

\[
I(D + |w|^2 + |h|) - I(D + |w|^2 - |h|) < 0.
\] (3.13)

So, from (3.8), it follows that \( \theta = \pi \) if \( |h| \neq 0 \). However, these values do not satisfy the stationarity condition (3.11) and so \( |h| \) must be zero. Then, from (3.8) and (3.10), it follows that \( \theta \) must be \( \pi/2 \) and \( D \) must be zero. At this stage, we conclude that supersymmetry is unbroken in this model since \( D = 0 \) and \( h = 0 \).

From (3.9) and (3.11), we find that if \( w \neq 0 \),

\[
-\frac{1}{G} + \frac{1}{8\pi^2} D \log(G|z|^2) = 0.
\] (3.14)

It is inconsistent for \( D = 0 \) and so \( w \) must be zero.

After all, \( D, h \) and \( w \) are zero, and \( |z| \) is given by the same expression of (2.39). At this vacuum, the effective action is essentially same as that of the \( CP^{N-1} \) model in the 1/N leading order. Therefore, the analysis of the vector boson is also the same and so one massive vector particle appears in this model, but it decays to massless components.

4 Concluding Remarks

We have shown that the supersymmetric \( CP^{N-1} \) and \( SO(N)/SO(N - 1) \times U(1) \) models are formulated as anomalous gauge theories. By the anomalous term, the gauged linear models have smaller symmetries than conventional ones: the remaining symmetry is \( SU(N - 1)_{\text{global}} \times U(1)_{\text{local}} \) for \( CP^{N-1} \), and \( SO(N - 1)_{\text{global}} \times U(1)_{\text{local}} \) for \( SO(N)/SO(N - 2)_{\text{global}} \times U(1)_{\text{local}} \).

In the 1/N leading order, the linear model has a vacuum for \( G < G_{\text{cr}} \), where the \( U(1)_{\text{local}} \) symmetry is broken but supersymmetry is unbroken. It is a remarkable feature of both models that there is no stable vacuum for \( G > G_{\text{cr}} \) in the 1/N leading order.

From the analysis of the spectral function, we expect that the dynamical gauge boson becomes massless at the critical coupling and so the \( U(1)_{\text{local}} \) symmetry is restored. To show this, it is necessary to study the models in the strong coupling regime by other methods than the 1/N leading order. In particular, it is interesting to clarify the fate of supersymmetry for \( G > G_{\text{cr}} \).

It has been shown that all supersymmetric nonlinear sigma models for the hermitian symmetric space are formulated as gauge theories, although the anomaly is not included in \[7\]. In
this paper, we deal with the two models for the hermitian symmetric space and show that the anomaly should be taken into account in the models. Then, it is natural to ask whether the anomalous term is required for analyzing the model for other hermitian symmetric space.

In the case of the Grassmann manifold $G_{M,N}$, the linear model is described by a chiral superfield of the $(N, \bar{M})$ representation of $U(N)_L \times U(M)_R$ and the model has no F-term constraint. Since $U(M)_R$ is gauged in this model, the anomalous term should be added in the nonlinear sigma model for $G_{M,N}$.

For $Sp(N)/U(N)$ and $SO(2N)/U(N)$, we have similarly a chiral superfield $\phi$ and an additional chiral field $\phi_0$ to impose the F-term constraint. Although the gauge symmetry is non-abelian, it can be easily seen that the anomalous term is required also in this case by considering $U(1)_D$, which is a subgroup of $U(N)$ [7]. For $U(1)_D$, $\phi$ and $\phi_0$ have 1 and $-2$ charge, respectively. Counting the total charge, the anomalous factor for $U(1)_D$ is given by $N(N + 1)$ for $Sp(N)/U(N)$, and $N(N - 1)$ for $SO(2N)/U(N)$. Since these factors are nonzero, we should include the anomalous term in the linear model for these target manifolds.

Similarly, we can deal with $E_6/SO(10) \times U(1)$ and $E_7/E_6 \times U(1)$ in terms of the $U(1)_D$ charge. In the case of $E_6/SO(10) \times U(1)$, there are two chiral superfields of the 27 representation of $E_6$ and they have 1 and $-2$ charge. So, we need the anomalous term in the linear model. For $E_7/E_6 \times U(1)$, we have two chiral superfields of the 56 representation of $E_7$, which have 1 and $-3$ charge for $U(1)_D$ and so the anomalous term is required. Consequently, we conclude that it is necessary to include the anomalous term in all linear models corresponding to the nonlinear sigma model whose target manifold is the hermitian symmetric space.

Finally, we comment on a supersymmetric NJL model proposed by Cheng, Dai, Faisei and Kong[15, 16]. The model is given by the K"ahler potential truncating higher-order terms of (2.3). One analysis of the model was performed in [14] by introducing an auxiliary vector superfield and calculating an effective potential in the $1/N$ leading order. Relating to an auxiliary vector superfield, the model has hidden $U(1)$ local symmetry with the anomaly, as well as in the $CP^{N-1}$ model. However, the anomalous term was not included in the effective potential in the previous analysis. The result including the anomaly will be reported in the near future[17].

Acknowledgments

The authors would like to thank H. Itoyama, T. Kugo, N. Maru and H. Ohki for valuable discussions. The research of T. T. was supported in part by Nara Women’s University Intramural Grant for Project Research.
A Feynman integrals in cut-off theories

First, let us consider the Feynman integral

\[ I = \int_{k^2 \leq \Lambda^2} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + 2k \cdot p + m^2}, \]  

(A.1)

where \( k^\mu \) and \( p^\mu \) are Euclidean momenta. The dot product for the two momenta is written by \( k \cdot p = |k||p|\cos \theta \), where \( \theta \) is the angle between the two vectors and \( |k| \) is the norm. Writing \( k = |k| \) and \( p = |p| \), the Feynman integral is expressed as

\[ I = \frac{4\pi}{16\pi^4} \int_0^\Lambda dkk^3 \int_0^\pi d\theta \frac{\sin^2 \theta}{k^2 + m^2 + 2kp \cos \theta}, \]  

(A.2)

where we have used \( d^4k = dk \, d\theta \, 4\pi k^3 \sin^2 \theta \) in four dimensions.

Here, the \( \theta \) integration can be performed by the formula,

\[ \int_0^\pi \frac{d\theta}{a + 2b \cos \theta} = \frac{\pi}{4b^2} \left( a - \sqrt{(a + 2b)(a - 2b)} \right) \quad (a > 2|b|, \ b \neq 0). \]  

(A.3)

In the case of \( m > p \), we have \( k^2 + m^2 > 2kp \) and so the Feynman integral becomes

\[ I = \frac{1}{16\pi^2} \int_0^\Lambda dkk^3 \left\{ k^2 + m^2 - \sqrt{(k^2 + m^2 + 2kp)(k^2 + m^2 - 2kp)} \right\}. \]  

(A.4)

Then, the \( k \) integration can be easily performed. The resulting integral is

\[ I = \frac{1}{16\pi^2} \left\{ \frac{\Lambda^4 + \Lambda^2 m^2 - \Lambda^2 p^2}{\Lambda^2 + m^2} + \frac{p^2}{2} \left( 1 - \frac{2p^2}{\Lambda^2 + m^2} \right) g(p^2, m^2) + (p^2 - m^2) h(p^2, m^2) \right\}, \]  

(A.5)

where \( h(p^2, m^2) \) and \( g(p^2, m^2) \) are defined by

\[ g(p^2, m^2) = \frac{\Lambda^4}{2p^4} \left( 1 + \frac{m^2}{\Lambda^2} \right) \left( 1 + \frac{m^2}{\Lambda^2} - \sqrt{\left( 1 + \frac{m^2}{\Lambda^2} \right)^2 - \frac{4p^2}{\Lambda^2}} - \frac{2p^2}{\Lambda^2} \right), \]  

(A.6)

\[ h(p^2, m^2) = \log \frac{\Lambda^2 + m^2 - 2p^2 + \sqrt{(\Lambda^2 + m^2)^2 - 4\Lambda^2 p^2}}{2(m^2 - p^2)}. \]  

(A.7)

Next, we illustrate the integration with a momentum in the numerator of the integrand:

\[ \int_{k^2 \leq \Lambda^2} \frac{d^4k}{(2\pi)^4} \frac{k_\mu}{(k^2 + 2k \cdot p + m^2)^2} = -\frac{1}{2} \frac{\partial}{\partial p^\mu} \int_{k^2 \leq \Lambda^2} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + 2k \cdot p + m^2} \]

\[ = \frac{4\pi}{16\pi^4} \int_0^\Lambda dkk^3 \int_0^\pi d\theta \frac{k \sin^2 \theta \cos \theta}{(k^2 + m^2 + 2kp \cos \theta)^2} \frac{p_\mu}{p}. \]  

(A.8)

By using the formula

\[ \int_0^\pi d\theta \frac{\sin^2 \theta \cos \theta}{a + 2b \cos \theta} = \frac{\pi(-a^2 + 2b^2)}{8b^3} + \frac{\pi a}{8b^2} \sqrt{(a + 2b)(a - 2b)}, \]  

(A.9)
the $\theta$ integration is performed and then we find that the result of the $k$ integration is given by

$$\int_{k^2 \leq \Lambda^2} d^4k \frac{k_\mu}{(2\pi)^4 (k^2 + 2k \cdot p + m^2)^2} = \frac{p_\mu}{16\pi^2} \left\{ \frac{\Lambda^2}{\Lambda^2 + m^2} + \frac{1}{2} \left( \frac{2p^2}{\Lambda^2 + m^2} \right) g(p^2, m^2) - h(p^2, m^2) \right\}.$$  \hfill (A.10)

Other Feynman integrals can be calculated by similar procedure. We give the results of calculation of other Feynman integrals used in this paper:

$$\int_{k^2 \leq \Lambda^2} d^4k \frac{1}{(2\pi)^4 (k^2 + 2k \cdot p + m^2)^2} = \frac{1}{16\pi^2} \left\{ - \frac{\Lambda^2}{\Lambda^2 + m^2} - \frac{p^2}{\Lambda^2 + m^2} g(p^2, m^2) + h(p^2, m^2) \right\},$$  \hfill (A.11)

$$\int_{k^2 \leq \Lambda^2} d^4k \frac{k_\mu k_\nu}{(2\pi)^4 (k^2 + 2k \cdot p + m^2)^2} = \frac{1}{16\pi^2} \left\{ \frac{p_\mu p_\nu}{p^2} \left( \frac{\Lambda^2 (\Lambda^2 + m^2 - 3p^2)}{2(\Lambda^2 + m^2)} - \frac{1}{4} \left( \frac{\Lambda^2 (\Lambda^2 + m^2 - 3p^2)}{2(\Lambda^2 + m^2)} - \frac{6p^4}{\Lambda^2 + m^2} \right) g(p^2, m^2) 
+ \frac{3p^2 - m^2}{2} h(p^2, m^2) \right\} 
+ \frac{1}{16\pi^2} \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \left( \frac{\Lambda^2 (\Lambda^2 + 3m^2 - 3p^2)}{3(\Lambda^2 + m^2)} \right) 
- \frac{1}{6} \left( \frac{\Lambda^2 (\Lambda^2 + m^2 - p^2 + \frac{(4m^2 - 6p^2)p^2}{\Lambda^2 + m^2})}{3(\Lambda^2 + m^2)} \right) g(p^2, m^2) \right\}. \hfill (A.12)

It is noted that the consistency of (A.5), (A.10), (A.11) and (A.12) can be checked by the relation

$$\frac{1}{k^2 + 2k \cdot p + m^2} = \delta^{\mu\nu} \frac{k_\mu k_\nu}{(k^2 + 2k \cdot p + m^2)^2} + 2p^{\mu} \frac{k_\mu}{(k^2 + 2k \cdot p + m^2)^2} + m^2 \frac{1}{(k^2 + 2k \cdot p + m^2)^2}. \hfill (A.13)$$

References

[1] M. Bando, T. Kugo and K. Yamawaki, “Nonlinear Realization and Hidden Local Symmetries,” Phys. Rept. 164, 217 (1988).

[2] M. Bando, T. Kuramoto, T. Maskawa and S. Uehara, “Nonlinear Realization in Supersymmetric Theories,” Prog. Theor. Phys. 72, 313 (1984).

[3] S. Aoyama, “The Supersymmetric $U_{N,\sigma}$ Model and Its $O_2$ Extended Supersymmetry,” Nuovo Cim. A 57, 176 (1980).

[4] U. Lindstrom and M. Rocek, “Scalar Tensor Duality and $N=1$, $N=2$ Nonlinear Sigma Models,” Nucl. Phys. B 222, 285 (1983).
[5] N. J. Hitchin, A. Karlhede, U. Lindstrom and M. Rocek, “Hyperkahler Metrics and Supersymmetry,” Commun. Math. Phys. 108, 535 (1987).

[6] T. Kugo, “Supersymmetric Non-linear Realization”, Soryushiron Kenkyu (Kyoto) 95 (1997) C56; SCGT96 Proceedings (World scientific, 1996), ed. by J. Nishimura and K. Yamawaki, available in [http://ekenwww.phys.nagoya-u.ac.jp/Scgt/proc/].

[7] K. Higashijima and M. Nitta, “Supersymmetric nonlinear sigma models as gauge theories,” Prog. Theor. Phys. 103, 635 (2000)

[8] “Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I,” Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961), 345.

[9] K. Harada and I. Tsutsui, “On the Path Integral Quantization of Anomalous Gauge Theories,” Phys. Lett. B 183, 311 (1987).

[10] S. Weinberg, “The Quantum Theory of Fields, Volume II Modern Applications”, Cambridge University Press, (2005), section 22.

[11] K. Higashijima and M. Nitta, “Quantum equivalence of auxiliary field methods in supersymmetric theories,” Prog. Theor. Phys. 103, 833 (2000)

[12] K. Konishi and K. Shizuya, “Functional-Integral Approach to Chiral Anomalies in Supersymmetric Gauge Theories,” Nuovo Cim. 90A (1985) 111.

[13] T. E. Clark, O. Piguet and K. Sibold, “Absence of radiative corrections to the axial current anomaly,” Nucl. Phys. B159 (1979), 1.

[14] T. Kugo, “Spontaneous Supersymmetry Breaking, Negative Metric and Vacuum Energy,” arXiv:1703.00600 [hep-th].

[15] Y. Cheng, Y. M. Dai, G. Faisel and O. C. W. Kong, “A Simple Model of Dynamical Supersymmetry Breaking with the Generation of Soft Mass(es),” arXiv:1507.01514 [hep-ph].

[16] Y. Cheng, Y. M. Dai, G. Faisel and O. C. W. Kong, “Analysis on a Nambu–Jona-Lasinio Model of Dynamical Supersymmetry Breaking,” arXiv:1603.00724 [hep-th].

[17] A. Kondo, H. Ohki and T. Takahashi, in preparation.