We re-examine the connection between interferometry and the Wigner representation for source freeze-out, and continuous emission. At the operator level, two equivalent representations of the two-particle spectrum are found, which contradict the standard expression of kinetic theory. The discrepancy is resolved using two toy models. Further, we revisit interferometry in scale-invariant one-dimensional hydrodynamics, and argue that recent experimental results are evidence for a short kaon emission time. Using two exactly calculable models of two- and three-dimensional flow, it is shown that the saddle point approximation, which is reasonable for one-dimensional flow, is no longer adequate. In these models the scaling law is altered, and we argue that such qualitative trends, together with other observables, are vital if one is to draw conclusions about the unknown source parameters.

1. Introduction

The hope of discovering a quark–gluon plasma (QGP) in heavy ion collisions is to some extent connected to possibility of measuring the geometric size of the region of secondary particle production. A quantitative estimate of this size is necessary to obtain the energy density, an important quantity in the discussion of the deconfinement phase transition. An important tool in accomplishing such a size measurement is interferometry.

Recent experiments at the CERN SPS indicate that interferometry is sensitive to hydrodynamic motion in the emitting source [1,2]. At RHIC energies one may expect the signature of this motion to be even more pronounced. In this paper, we therefore wish to reexamine various techniques to calculate the two-boson spectra for expanding sources.

Intensity interferometry was proposed by Hanbury-Brown and Twiss to measure stellar sizes. The nature of light emission in stars is well understood; the thermal mechanism guarantees that photons are emitted independently from different parts of the photosphere, while the small angular size of the star makes it impossible to obtain an optical image. These two factors create the conditions necessary for interference between the two-particle amplitudes of photons emitted by different parts of the stellar surface. Under certain assumptions which will be discussed later, the two-particle detection probability is

\[
W(k_1, k_2) = \int d^4x_1 d^4x_2 \rho(x_1) \rho(x_2) \left[ 1 + \cos(k_1 - k_2) \cdot (x_1 - x_2) \right]
\]  

(1.1)

The nature of particle sources in nucleus–nucleus collisions [3] is less well understood. In particular, the correspondence between measured quantities and parameters of the emitting system is less clear. One possible reason Eq. (1.1) may be inapplicable is the presence of correlations on the same scale as the size of the emitting system. An extreme case – when interferometry is completely unrelated to the total source size – is the scattering of coherent light in fluids; the rate of two–photon coincidence depends on the three-particle distribution function in the fluid [4]. For A–A collisions, even in the simplest scenario of hydrodynamic evolution, the relation between the inclusive one– and two–particle spectra and the parameters of the emitting system does not follow the classical scheme of interferometry. The width of correlator is not directly connected to the geometric size of the source, an effect first discussed in Ref. [5]. For an expanding spherical shell, the apparent (or “visible”) source size is smaller, the larger the total pion pair momentum \( P_\perp \). In nuclear collision models, the dependence of the apparent size on \( P_\perp \) may reflect a more complicated interplay between the collision dynamics and the true source size [6].

A relativistically covariant theory for interferometry in the hydrodynamic model of nuclear collisions was developed in Ref. [7], with the main focus on freeze–out as a realistic mechanism for final state hadron production. In a subsequent study [8], it was shown that if the dependence of the correlation function on the difference of the longitudinal momenta is rescaled by \( \sqrt{m_\perp} \), the width of the correlator becomes independent of \( m_\perp \). This scaling behavior has also been discovered in parallel studies [9] based on the intra-nuclear cascade approach [10], where classical currents are the sources of final–state pions. The \( m_\perp \) scaling dependence has been observed at SPS energies by the NA35 and NA44 collaborations [11,12].
A covariant theory of interference for continuous emission has also been applied to dilepton \[1\] and photon \[2; 3\] emission from a quark–gluon plasma. Analysis of polarization effects in photon interference \[2\] indicates that there is a dilemma in the proper choice of formalism: the languages of locally defined states, and Wigner phase-space distributions lead to different answers.

Events generators such as RQMD \[4\] suggest a classical description of the space–time evolution of high energy heavy ion collisions. Other models, based on the solution of semi–classical kinetic equations, often provide descriptions in terms of one–particle distributions. Interferometry is a manifestly quantum phenomenon, and we shall therefore begin in Section 2 by examining its connection to the widely used Wigner representation, for the cases of source freeze–out (initial data problem) and photons (continuous emission). At the operator level, the Wigner representation is simply a formal re–expression of the one– and two–particle spectra, and we obtain two equivalent expressions for the two–particle spectrum. These are in contradiction with the standard expressions of kinetic theory. In Section 3, we attempt to resolve the issue by way of two toy models, viz., the emission of particles from one and two cavities. In an appendix, we reanalyze the derivation of the two–particle spectrum in classical source models.

In Section 4, we shall revisit interferometry in scale–invariant one–dimensional hydrodynamics, deriving the \(m_\perp\)–dependence of the longitudinal effective size in the case of the “extended freeze–out.” We argue that recent experimental results \[2\] confirming this behavior are evidence for short kaon emission times, and in disagreement with the prediction of kaon distillation \[15\].

Unlike the inverse scattering problem, where an analysis of the phase shifts allows one to obtain the shape of the potential, the inverse problem of interferometry has no exact mathematical formulation. The interpretation of the two–particle spectra requires a model that is determined \textit{a priori}, up to the numerical values of its parameters. We therefore present in Section 5 some results for two– and three–dimensional flow in exactly calculable examples: the transverse explosion of a long filament, and the spherical explosion of a point–like source. In both cases the saddle point approximation, which is reasonable for one–dimensional flow, is no longer adequate. The scaling law is altered, and we argue that such qualitative trends, together with other observables, are important in establishing the type of model one is dealing with. Only once the model is given, can one draw conclusions about the unknown source parameters. We conclude in Section 6.

2. Initial data and continuous emission for interferometry

The collision dynamics in heavy ion physics is rather complicated. However, some stages of the collision may be described in terms of one–particle distributions, or even by a few macroscopic parameters. A strong, yet attractive, simplification occurs if we assume that the hot expanding matter freezes out at some critical temperature \(T_c\). Generally, such models rely on (semi–) classical assumptions, and produce (semi–) classical final distributions. On the other hand, interferometry is a manifestation of the quantum nature of the constituents of the system, and it is therefore necessary to examine carefully whether these distributions can serve as an adequate input.

A natural connection between quantum mechanics and (semi–) classical descriptions is provided by the density matrix \(\rho\). In a formal language, the problem we wish to solve in interferometry is to determine judiciously chosen parameters of \(\rho\) by measuring the inclusive cross-sections \(dN_1/d\vec{k}\), \(dN_2/d\vec{k}_1 d\vec{k}_2, \ldots\).

2.1 Source freeze–out

Let \(|i\rangle\) be one of the possible initial states of the system emitting a pion field \(\hat{\phi}(x)\). At this stage, we do not wish to specify the exact nature of the initial state, but study instead the observables relevant for interferometry at the operator level.

The field \(\hat{\phi}(x)\) is to be detected later by some device, or analyzer. Let it be tuned to the measurement of the momentum of the free pion. Thus, the eigenfunctions of the analyzer are the free pion wave functions

\[
f_{\vec{k}}(x) = (2\pi)^{-3/2}(2k_0)^{-1/2} e^{-ik \cdot x}.
\]

These functions are defined for \(x^0 > t_c\), where \(t_c\) is the time when the system “prepared” the final pion. The corresponding annihilation operator \(\hat{A}_{\vec{k}}\) for momentum \(\vec{k}\) in the final state is given by

\[
\hat{A}_{\vec{k}} = \int d^3x f_{\vec{k}}(x) i \frac{\partial^0}{\partial x^0} \hat{\phi}(x),
\]

where \((a \partial^0_x b) \equiv a(\partial^0_x b) - (\partial^0_x a)b\). The operator \(\hat{A}_{\vec{k}}\) describes the effect of a detector (analyzer) far from the point of emission, so, by definition, the pion is detected on mass–shell, \(k^0 = (\vec{k}^2 + m^2)^{1/2}\). The inclusive amplitude to find one pion with momentum \(\vec{k}\) in the final state is
\[ \langle X | \hat{A}_k \hat{S} | \text{in} \rangle , \]  
(2.3)

where \( \hat{S} \) is the evolution operator after freeze–out, and the states \( |X\rangle \) form a complete set of all possible secondaries. Summing the squared modulus of this amplitude over all (undetected) states \( |X\rangle \), and averaging over the initial ensemble, we find the one–particle inclusive spectrum

\[ \frac{dN_1}{d^4k} = \text{Tr} \hat{\rho}_\text{in} \hat{S}^\dagger \hat{A}_k^\dagger \hat{A}_k \hat{S} \ , \]  
(2.4)

where the density operator \( \hat{\rho}_\text{in} \) will describe the emitting system. For now, we shall study only the operator part of Eq. (2.4), \( \hat{N}_k = \hat{A}_k^\dagger \hat{A}_k \), which gives the number of pions detected by an analyzer tuned to momentum \( \vec{k} \). In the same way one may obtain the inclusive two–pion spectrum

\[ \frac{dN_2}{d^2k_1 d^2k_2} = \text{Tr} \hat{\rho}_\text{in} \hat{S}^\dagger \hat{A}_k^\dagger_{k_1} \hat{A}_k^\dagger_{k_2} \hat{A}_k_{k_1} \hat{A}_k_{k_2} \hat{S} \ , \]  
(2.5)

and the “device” operator which counts the number of pions pairs is

\[ \hat{A}_k^\dagger_{k_1} \hat{A}_k^\dagger_{k_2} \hat{A}_k_{k_1} \hat{A}_k_{k_2} = \hat{N}_k (\hat{N}_k - \delta(\vec{k}_1 - \vec{k}_2)) \ . \]  
(2.6)

The pion field \( \hat{\varphi}(x) \) reaches the detector after free propagation. This simplest kind of evolution is described by the retarded Green function

\[ \hat{\varphi}(x) = \int d\Sigma_\mu(y) G_{\text{ret}}(x - y) \overset{\leftrightarrow}{\partial^\mu} \hat{\varphi}(y), \]  

\[ \hat{\varphi}^\dagger(x) = \int d\Sigma_\mu(y) \hat{\varphi}^\dagger(y) \overset{\leftrightarrow}{\partial^\mu} G_{\text{adv}}(y - x) \ , \]  
(2.7)

where the pion field \( \hat{\varphi}(y) \) is given on the 3-dimensional freeze–out hypersurface (a Cauchy hypersurface). The space of states in which the density matrix acts is also defined on this surface. Substituting Eqs. (2.7) in (2.2), we find the one–particle inclusive spectrum

\[ \hat{S}^\dagger \hat{A}_k \hat{S} = \int d^3x \ f_\vec{k}(x) i \overset{\leftrightarrow}{\partial^0} \int d\Sigma_\mu(y) G_{\text{ret}}(x - y) \overset{\leftrightarrow}{\partial^\mu} \hat{\varphi}(y) \ , \]  

\[ \hat{S}^\dagger \hat{A}_k^\dagger \hat{S} = \int d^3x \int d\Sigma_\mu(y) \hat{\varphi}^\dagger(y) \overset{\leftrightarrow}{\partial^0} G_{\text{adv}}(y - x) i \overset{\leftrightarrow}{\partial^\mu} f_\vec{k}(x) \ . \]  
(2.8)

These equations may be simplified using the explicit form of the free pion propagators:

\[ G_{\text{ret}}(x - y) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-i p \cdot (x - y)}}{(p^0 \pm i\epsilon)^2 - \vec{p}^2 - m^2} \]  
(2.9)

Substituting Eqs. (2.9) into (2.8) and integrating, first over \( d^3x \) which sets \( \vec{p} \) equal to \( \vec{k} \), and then over \( p^0 \), we obtain

\[ \hat{S}^\dagger \hat{A}_k \hat{S} = \theta(x^0 - y^0) \int d\Sigma_\mu(y) f_\vec{k}(y) i \overset{\leftrightarrow}{\partial^\mu} \hat{\varphi}(y) \ , \]  

\[ \hat{S}^\dagger \hat{A}_k^\dagger \hat{S} = \theta(x^0 - y^0) \int d\Sigma_\mu(y) \hat{\varphi}^\dagger(y) i \overset{\leftrightarrow}{\partial^\mu} f_\vec{k}(y) \ . \]  
(2.10)

As before, the pion four–momentum \( k \) is on mass–shell. This is not surprising, as the propagation is free and our analyzers perform an on–shell Fourier expansion of the initial data. The \( \theta \)–function appears because the propagation is retarded. Using (2.10), we may rewrite the number operators for single pions and pairs of pions as

\[ \hat{N}_k = \theta(x^0 - y^0) \int d\Sigma_\mu(y_1) d\Sigma_\nu(y_2) \left[ f_\vec{k}(y_1) i \overset{\leftrightarrow}{\partial^\mu} \hat{\varphi}^\dagger(y_1) \right] \left[ \hat{\varphi}(y_2) i \overset{\leftrightarrow}{\partial^\nu} f_\vec{k}(y_2) \right] \]  
(2.11)
\[ \hat{N}_{\vec{k}_1} \left( \hat{N}_{\vec{k}_2} - \delta(\vec{k}_1 - \vec{k}_2) \right) = \theta(x^0 - y^0) \int d\Sigma_\rho(y_1) \frac{d\Sigma_\nu(y_3)}{y_3} \frac{d\Sigma_\lambda(y_4)}{y_4} d\Sigma_\mu(y_2) \times \left[ f_{k_1}^\dagger(y_1) f_{k_2}^\dagger(y_3) \frac{\partial}{\partial y_1} \frac{\partial}{\partial y_3} \phi^\dagger(y_1) \phi(y_3) \right] \left[ \phi(y_4) \phi^\dagger(y_2) \frac{\partial}{\partial y_4} \frac{\partial}{\partial y_2} f_{k_2}^*(y_4) f_{k_1}^*(y_2) \right] , \] (2.12)

respectively.

We shall now attempt to incorporate the technique used in kinetic studies by passing to the Wigner representation at the operator level, i.e., without specifying any physical information about emitting system. Firstly, for \( \hat{N}_{\vec{k}} \), we choose new variables

\[ \vec{y}_1 = \vec{R} + \frac{\vec{z}}{2}, \quad \vec{y}_2 = \vec{R} - \frac{\vec{z}}{2} . \] (2.13)

This transformation is motivated by a simple idea: we expect that \( \vec{R} \) will label the sources, while the Fourier transform over \( \vec{z} \) will yield the local spectrum. We assume that the 3–vectors \( \vec{R} \) and \( \vec{z} \) lie in the Cauchy hypersurface, which is obviously not planar, in general. The local time–like direction coincides with the normal. As the equation for \( \phi \) is second order, the initial data includes the derivatives \( \frac{\partial}{\partial y_i} \), which lie along the local normal. To adjust the variables, we write

\[ \frac{\partial}{\partial y_1} = \frac{1}{2} \frac{\partial}{\partial R} + \frac{\partial}{\partial z} , \quad \frac{\partial}{\partial y_2} = \frac{1}{2} \frac{\partial}{\partial R} - \frac{\partial}{\partial z} , \] (2.14)

allowing us to introduce a compact form of the Wigner representation of the operator \((2.11)\)

\[ \hat{N}(R, \frac{k_1 + k_2}{2}) = \int d^3 z e^{-i(\vec{k}_1 + \vec{k}_2) \cdot \vec{z}/2} e^{-i k_0^0 (R^0 + z^0 / 2)} \frac{1}{2} \left[ \frac{1}{2} \frac{\partial}{\partial R} + \frac{\partial}{\partial z} \right] \phi^\dagger(R^0 - \frac{z^0}{2}, \vec{R} + \frac{\vec{z}}{2}) \times \phi(R^0 - \frac{z^0}{2}, \vec{R} - \frac{\vec{z}}{2}) \frac{1}{2} \left[ \frac{1}{2} \frac{\partial}{\partial R} - \frac{\partial}{\partial z} \right] e^{i k_0^0 (R^0 + z^0 / 2)} , \] (2.15)

which may be shown to depend only on the combination \((k_0^0 + k_0^2)/2\). If the Cauchy surface is planar, all the derivatives above will be along the common normal. Then, the one particle spectrum may be represented in a compact symbolic form

\[ \hat{N}_{\vec{k}} = \frac{1}{2k^0} \int d^3 R \hat{N}(R, k) . \] (2.16)

For the two–particle operator we have four arguments in the integrand of \((2.12)\), which allows us to choose the variables for the Wigner transformation in two ways:

\[ \vec{y}_1 = \vec{R}_1 + \frac{\vec{z}_1}{2}, \quad \vec{y}_2 = \vec{R}_1 - \frac{\vec{z}_1}{2}, \]
\[ \vec{y}_3 = \vec{R}_2 + \frac{\vec{z}_2}{2}, \quad \vec{y}_4 = \vec{R}_2 - \frac{\vec{z}_2}{2} ; \] (2.17)

or, equally,

\[ \vec{y}_1 = \vec{R}_1 + \frac{\vec{z}_1}{2}, \quad \vec{y}_4 = \vec{R}_1 - \frac{\vec{z}_1}{2}, \]
\[ \vec{y}_3 = \vec{R}_2 + \frac{\vec{z}_2}{2}, \quad \vec{y}_2 = \vec{R}_2 - \frac{\vec{z}_2}{2} . \] (2.18)

Correspondingly, the operators for the two–particle spectrum may be rewritten in the two identical forms

\[ \hat{N}_{\vec{k}_1} \left( \hat{N}_{\vec{k}_2} - \delta(\vec{k}_1 - \vec{k}_2) \right) = \frac{1}{4k_1^0 k_2^0} \int d^3 R_1 d^3 R_2 \hat{N}(R_1, k_1) \hat{N}(R_2, k_2) \] (2.19)

and

\[ \hat{N}_{\vec{k}_1} \left( \hat{N}_{\vec{k}_2} - \delta(\vec{k}_1 - \vec{k}_2) \right) = \frac{1}{4k_1^0 k_2^0} \int d^3 R_1 d^3 R_2 \hat{N}(R_1, \frac{k_1 + k_2}{2}) \hat{N}(R_2, \frac{k_1 + k_2}{2}) e^{-i (\vec{k}_1 - \vec{k}_2) \cdot (\vec{R}_1 - \vec{R}_2)} , \] (2.20)
2.2 Continuous emission

We shall now consider continuous emission from an evolving system, using the case of photons as an example [12]. Unlike for pion emission, where the (transition) currents which might emit the pions are not conserved, this problem has a clearer footing. Also, technically, the Wigner representation for continuous emission is simpler, as there are no derivatives in the equations that will replace (2.11) and (2.12). As in the previous section, we define the inclusive probabilities for one and two photons

\[ \frac{dN_\gamma}{dk} = \sum_\lambda \text{Tr} \hat{\rho}_\text{in} \hat{S}^\dagger \hat{c}^\dagger (\vec{k}, \lambda) \hat{c} (\vec{k}, \lambda) \hat{S} , \]  

(2.21)

and

\[ \frac{dN_\gamma}{dk_1 dk_2} = \sum_{\lambda_1, \lambda_2} \text{Tr} \hat{\rho}_\text{in} \hat{S}^\dagger \hat{c}^\dagger (\vec{k}_1, \lambda_1) \hat{c}^\dagger (\vec{k}_2, \lambda_2) \hat{c} (\vec{k}_1, \lambda_1) \hat{S} , \]  

(2.22)

respectively. The operators \( \hat{c}(\vec{k}) \) appear in the decomposition of the electromagnetic field operator,

\[ \hat{A}_\mu (x) = \sum_\lambda \int d^3k \frac{\epsilon^{\lambda(\mu)}(\vec{k}) e^{\nu, \lambda}(\vec{k})}{(2\pi)^3 (2\omega_\vec{k})^{1/2}} \left[ \hat{c}(\vec{k}, \lambda) e^{-ik \cdot x} + \hat{c}^\dagger (\vec{k}, \lambda) e^{ik \cdot x} \right] , \]  

(2.23)

where \( \lambda \) runs over the two physical polarizations of the photon. Since the quantum equation of motion for the field \( \hat{A}_\mu (x) \) has the classical form,

\[ \hat{A}^\mu (x) = \int d^4y \, D^{\mu \nu}_{\text{ret}} (x - y) \, \hat{j}_\nu (y) , \]  

(2.24)

it is easy to show that the operator of the number of emitted photons is

\[ \hat{N}_\gamma = \sum_\lambda \epsilon^{\mu, \lambda}(\vec{k}) \epsilon^{\nu, \lambda}(\vec{k}) \int \frac{d^4x d^4y}{2\hbar \omega_\vec{k}^2} e^{-ik \cdot (x - y)} \, \hat{j}_\mu (x) \hat{j}_\nu (y) . \]  

(2.25)

Now we may safely perform a full 4–dimensional Wigner transformation

\[ x^\mu = R^\mu + \frac{z^\mu}{2} , \quad y^\mu = R^\mu - \frac{z^\mu}{2} . \]  

(2.26)

Introducing

\[ \hat{\Pi}_{\mu \nu} (R, p) = \frac{1}{2\pi^2} \int \frac{d^4y}{(2\pi)^4} e^{-ip \cdot x} \, \hat{j}_\mu (R + z/2) \hat{j}_\nu (R - z/2) , \]  

(2.27)

we may rewrite

\[ \hat{N}_\gamma = \sum_\lambda \epsilon^{\mu, \lambda}(\vec{k}) \epsilon^{\nu, \lambda}(\vec{k}) \int d^4R \, \hat{\Pi}_{\mu \nu} (R, k) . \]  

(2.28)

For the operator that counts the number of photon pairs,

\[ \hat{N}_{\gamma_1}(\hat{N}_{\gamma_2} - \delta(\vec{k}_1 - \vec{k}_2)) = \sum_{\lambda_1, \lambda_2} \epsilon^{\mu, \lambda_1}(\vec{k}_1) \epsilon^{\nu, \lambda_1}(\vec{k}_1) \epsilon^{\rho, \lambda_2}(\vec{k}_2) \epsilon^{\sigma, \lambda_2}(\vec{k}_2) \times \int \frac{d^4x d^4y d^4x' d^4y'}{4k_1^0 k_2^0 (2\pi)^6} e^{-ik_1 \cdot (x - y) - ik_2 \cdot (x' - y')} \hat{j}_\mu (x) \hat{j}_\nu (x') \hat{j}_\rho (y) \hat{j}_\sigma (y') , \]  

(2.29)

The two–photon detection function can be rewritten two ways, by changing variables either as

\[ x^\mu = R_1^\mu + \frac{z_1^\mu}{2} , \quad y^\mu = R_1^\mu - \frac{z_1^\mu}{2} , \quad x'^\mu = R_2^\mu + \frac{z_2^\mu}{2} , \quad y'^\mu = R_2^\mu - \frac{z_2^\mu}{2} , \]  

(2.30)
or,
\[ x^\mu = R_1^{\mu} + \frac{z_1^\mu}{2}, \quad y^\mu = R_1^{\mu} - \frac{z_1^\mu}{2}, \quad x'^\mu = R_2^{\mu} + \frac{z_2^\mu}{2}, \quad y'^\mu = R_2^{\mu} - \frac{z_2^\mu}{2}. \] (2.31)

These transformations lead to two equivalent representations of the two–photon spectrum:
\[ \hat{N}_{k_1}(\hat{N}_{k_2} - \delta(k_1 - k_2)) = \sum_{\lambda_1, \lambda_2} \epsilon_{\mu}^{(\lambda_1)}(\hat{k}_1) \epsilon_{\nu}^{(\lambda_1)}(\hat{k}_1) \epsilon_{\sigma}^{(\lambda_2)}(\hat{k}_2) \epsilon_{\tau}^{(\lambda_2)}(\hat{k}_2) \int d^4 R_1 d^4 R_2 \, \hat{\Pi}^{\mu\nu}(R_1, k_1) \hat{\Pi}^{\sigma\tau}(R_2, k_2) \] (2.32)
and
\[ \hat{N}_{k_1}(\hat{N}_{k_2} - \delta(k_1 - k_2)) = \sum_{\lambda_1, \lambda_2} \epsilon_{\mu}^{(\lambda_1)}(\hat{k}_1) \epsilon_{\nu}^{(\lambda_1)}(\hat{k}_1) \epsilon_{\sigma}^{(\lambda_2)}(\hat{k}_2) \epsilon_{\tau}^{(\lambda_2)}(\hat{k}_2) \times \int d^4 R_1 d^4 R_2 \, \hat{\Pi}^{\mu\nu}(R_1, \frac{k_1 + k_2}{2}) \hat{\Pi}^{\sigma\tau}(R_2, \frac{k_1 + k_2}{2}) \cos(R_1 - R_2) \cdot (k_1 - k_2). \] (2.33)

respectively.

2.3 Discussion

We have obtained three different expressions for the two–particle spectra, for both the initial data (freeze–out) problem and for continuous emission, viz. Eqs. (2.12), (2.19) and (2.20), and (2.29), (2.32) and (2.33), respectively. An apparent paradox appears if one compares these expressions with those commonly used. For the freeze–out mechanism of the pion production these expressions have the form
\[ 4k_1^0 k_2^0 \frac{dN_\pi}{dk_1 dk_2} = \int d^3 R_1 d^3 R_2 \, N(R_1, k_1) N(R_2, k_2) \] (2.34)
+ \int d^3 R_1 d^3 R_2 \, N(R_1, \frac{k_1 + k_2}{2}) N(R_2, \frac{k_1 + k_2}{2}) \, e^{-i (\vec{k}_1 - \vec{k}_2) \cdot (\vec{R}_1 - \vec{R}_2)},

while for continuous photon emission
\[ 4k_1^0 k_2^0 \frac{dN_\gamma}{dk_1 dk_2} = \sum_{\lambda_1, \lambda_2} \epsilon_{\mu}^{(\lambda_1)}(\vec{k}_1) \epsilon_{\nu}^{(\lambda_1)}(\vec{k}_1) \epsilon_{\sigma}^{(\lambda_2)}(\vec{k}_2) \epsilon_{\tau}^{(\lambda_2)}(\vec{k}_2) \int d^4 R_1 d^4 R_2 \times \] (2.35)
\[ \times \left[ \hat{\Pi}^{\mu\nu}(R_1, k_1) \hat{\Pi}^{\sigma\tau}(R_2, k_2) + e^{-i (R_1 - R_2) \cdot (k_1 - k_2)} \hat{\Pi}^{\mu\nu}(R_1, \frac{k_1 + k_2}{2}) \hat{\Pi}^{\sigma\tau}(R_2, \frac{k_1 + k_2}{2}) \right]. \]

(For references and a detailed derivation of such formulae, see Ref. [16]; there, the off–mass–shellness of the distributions in the cross–term is emphasized as a physical result.) These expressions appear to be in line with kinetic theory: they seem to allow one to use the (semi–) classical phase–space distributions as the input for subsequent studies of the interferometry problem. Below, we shall resolve the manifest contradiction between Eqs. (2.34) and (2.35) on the one hand, and the expressions given in the previous sections on the other. For now, we emphasize that the phase–space densities with argument \((k_1 + k_2)/2\) cannot be considered as distributions of the single particles in definite quantum states. For the photons, there appears an additional discrepancy between the transversality of the polarization tensor, and the physical polarization of the detected photons [12].

3. Interferometry

Up to this point, we have only discussed how the particles are detected, and have yet to describe the states in which particles were created. We have seen that the formal Wigner transformation puts the one–particle operators into a form resembling classical phase–space distributions, but that the representation for the two–particle “detection” operators is not uniquely defined. It allows for two equivalent forms, each of which, by coincidence, resembles one of the terms of the familiar interferometric formula, Eq. (2.34), or Eq. (2.35). Each Wigner operator representation is supposed to yield the two term interferometric formula as a final result.

To proceed with a discussion of interferometry, we require physical input. This input is already present in equations like (2.4) and (2.7), where the operators are averaged with the density matrix of the initial state. It is not evident that any arbitrary input will make interferometry which yields physically useful information possible.
Interference of quantum mechanical amplitudes appears when for identical initial states and identical final states (of one or more particles) there exists more than one intermediate transition amplitude. Interference in one–particle propagation exists, and is used in optical devices to obtain an optical image of the source. Interference in the propagation of the two–particle amplitude does not necessarily exist, because there may not be two alternative paths for the evolution of the two–particle state. An example is two–photon emission from a single atomic transition, and for each different case, we must examine the initial state of the system, i.e., the density matrix \( \rho_{in} \).

### 3.1 Toy model. Emission from a one–dimensional cavity

Consider the case of an initial state which is a one–dimensional cavity filled with hot bosonic radiation. For the sake of simplicity, we shall impose periodic boundary conditions on the walls of the cavity. The cavity eigenfunctions are

\[
\phi_p(y) = (2a)^{-1/2}(2p^0)^{-1/2}e^{-ipy}, \quad \text{for } -a \leq y \leq a,
\]

where \( 2a \) is the distance between the walls, and the allowed values of the momentum \( p \) are

\[
p_n = \frac{n\pi}{a}, \quad n = 0, \pm 1, \pm 2, \ldots
\]

The boson annihilation and creation operators in the cavity are defined by their field decomposition:

\[
\hat{\varphi}(y) = \sum_p \hat{a}_p \phi_p(y),
\]

so that, if the walls are removed at time \( y^0 = 0 \), the average number of particles with the momentum \( k \) is given by (see Eq. (2.11))

\[
\langle \hat{N}_k \rangle = \int_{-a}^{a} dy_1 \int_{-a}^{a} dy_2 \sum_p n(p) \left[ f_k(y_1) \right] \left[ \phi_p(y_1) \right] \left[ \phi_p(y_2) \right] \left[ f_k(y_2) \right],
\]

where \( n(p) = (\hat{a}_p^\dagger \hat{a}_p) \) is the boson occupation number. The integrals may be evaluated to yield

\[
\langle \hat{N}_k \rangle = \sum_p n(p) \frac{(\omega_k + \omega_p)^2}{4\omega_k\omega_p} \frac{\sin^2(p-k)a}{\pi a(p-k)^2}.
\]

Using Eq. (2.12), we may similarly write for the two–particle spectrum

\[
\langle \hat{N}_{k_1}, \hat{N}_{k_2} \rangle = \sum_{p_1, p_2, p_3} \frac{(\omega_{k_1} + \omega_{p_1})(\omega_{k_1} + \omega_{p_2})(\omega_{k_2} + \omega_{p_3})(\omega_{k_2} + \omega_{p_4})}{4\pi^2 a^2 \omega_{k_1}\omega_{k_2}\omega_{k_3}\omega_{k_4}} \times \langle \hat{a}_{p_1}^\dagger \hat{a}_{p_1}^\dagger \hat{a}_{p_3}^\dagger \hat{a}_{p_4} \rangle \int_{-a}^{a} dy e^{-i(k_1-1)y} \int_{-a}^{a} dy e^{-i(k_2-2)y} \int_{-a}^{a} dy e^{-i(k_3-3)y} \int_{-a}^{a} dy e^{-i(k_4-4)y}.
\]

In case of the Gibbs ensemble, the average of the four Fock operators is easily calculated

\[
\langle \hat{a}_{p_1}^\dagger \hat{a}_{p_1}^\dagger \hat{a}_{p_4} \hat{a}_{p_4} \rangle = n(p_1)n(p_3) \left[ \delta_{p_1,p_2} \delta_{p_3,p_4} + \delta_{p_1,p_4} \delta_{p_3,p_2} \right],
\]

so that

\[
\langle \hat{N}_{k_1}, \hat{N}_{k_2} \rangle = \langle \hat{N}_{k_1} \rangle \langle \hat{N}_{k_2} \rangle + \sum_{p_1, p_3} n(p_1)n(p_3) \frac{(\omega_{k_1} + \omega_{p_1})(\omega_{k_1} + \omega_{p_3})(\omega_{k_2} + \omega_{p_1})(\omega_{k_2} + \omega_{p_3})}{4\pi^2 a^2 \omega_{k_1}\omega_{k_2}\omega_{k_3}\omega_{k_4}} \times \frac{\sin(p_1 - k_1)a}{(p_1 - k_1)} \frac{\sin(p_1 - k_2)a}{(p_1 - k_2)} \frac{\sin(p_3 - k_1)a}{(p_3 - k_1)} \frac{\sin(p_3 - k_2)a}{(p_3 - k_2)}.
\]
Let us first discuss the single particle spectrum, \(\langle \hat{N}_k \rangle\). From Eq. (3.2), it is clear that \(\langle \hat{N}_k \rangle\) has an infinite sequence of alternating zeros and peaks, in analogy to an optical Fabry–Perot interferometer, where the oscillations arise from the first order (one-particle) multi-ray interference [17]. Here, we rely on the interference between all possible histories of a single particle, and, in principle, the distance between two neighboring zeros of \(\langle \hat{N}_k \rangle\) will determine the size of the cavity.

The two-particle spectrum, Eq. (3.8), has a rather complex structure. It arises because any given two-particle final state can be traced back to an infinite number of initial states; when the cavity is opened, every eigenstate of the finite-sized cavity has, in addition to its “fundamental harmonic,” an infinite sequence secondary peaks, or “satellites.” A given plane wave particle may thus have originated from any single–particle level in the cavity. The resulting unwanted structure is an artifact of our choice of boundary conditions, and would disappear if the zeros in \(\langle \hat{N}_k \rangle\) were not equidistant.

As long as the detailed behavior inherent in this model is not expected to appear in a real physical application of interferometry, it is reasonable to modify it by eliminating the rapid oscillations in the two–particle spectrum Eq. (3.8). As artificial as the model is, any procedure will do. Ad hoc, one could remove the secondary spectral lines by hand, restricting the sum in (3.8) to \(|p_i - k_j| \leq \pi/a\). This scheme is essentially equivalent to taking a sufficiently large cavity (for a given momentum) – two of the four integrals in Eq. (3.7) become delta–functions, and the other two can be written as

\[
4 \frac{\sin^2(k_1 - k_2)a}{(k_1 - k_2)^2}
\]

This factor is strongly peaked at \(k_1 = k_2\), with a corresponding suppression of the satellites; physically, for reasonable \(a\), the detected particles can be traced back to their parent levels in the closed cavity, and no interference is possible. For \(k_1 = k_2\), we recover the two particle normalization.

### 3.2 Toy model 2. Emission from two one-dimensional cavities

Next, we consider two cavities, defined by walls at \(L \pm a\) and \(-L \pm a\). There are now two sets of the eigenstates, defined separately for each cavity:

\[
\phi_{p,N}(y) = (2a)^{-1/2}(2k_0)^{-1/2}e^{-iy}e^{-iyp} , \quad \text{for } L_N - a \leq y \leq L_N + a \\
= 0 , \quad \text{otherwise.}
\]

(3.10)

The index \(N\) runs over the two cavities, and \(L_N = \pm L\). The spectrum of states in each cavity is defined by Eq. (3.5), and the field decomposition is

\[
\hat{\varphi}(y) = \sum_{p,N} \hat{a}_{p,N} \phi_{p,N}(y) ,
\]

(3.11)

where the Fock operators have acquired an additional index enumerating the cavities. States belonging to different cavities are independent, as implied by the commutation relations:

\[
[a_{p,N}, a_{p',N'}^+] = \delta_{pp'} \delta_{NN'} .
\]

(3.12)

With the agreement that there is a one-to-one correspondence between the states in any one cavity and states in which we detect the particles, the one–particle distribution reads

\[
\langle \hat{N}_k \rangle = \sum_N n(k, N) = \sum_N \langle \hat{a}_{k,N}^\dagger \hat{a}_{k,N} \rangle .
\]

(3.13)

We now consider the two-particle spectrum, and our intention to resolve the discrepancy between Eqs. (2.12), (2.19) and (2.20). In our model, there is interference between four amplitudes: two amplitudes when each particle is emitted from a different cavity, and two more when both particles originate from the same cavity. The general expression for the two-particle spectrum reads

\[
\langle \hat{N}_k \langle \hat{N}_k - \delta k_1 - k_2 \rangle \rangle = \sum_{p_1 , p_2} \langle \hat{a}_{p_1,N_1}^\dagger \hat{a}_{p_2,N_2}^\dagger \rangle \langle \hat{a}_{p_1,N_1} \hat{a}_{p_2,N_2} \rangle \frac{\omega_{k_1} + \omega_{p_1}}{4\pi^2 a^2} \frac{\omega_{k_2} + \omega_{p_2}}{4\pi^2 a^2} \frac{\omega_{k_1} + \omega_{p_3}}{4\pi^2 a^2} \frac{\omega_{k_2} + \omega_{p_4}}{4\pi^2 a^2} \\
\times \int_{L_1 - a}^{L_1 + a} \int_{L_2 - a}^{L_2 + a} \int_{L_3 - a}^{L_3 + a} \int_{L_4 - a}^{L_4 + a} dy \, e^{-i(k_1 - p_1)y} \, dy \, e^{-i(k_1 - p_2)y} \, dy \, e^{-i(k_2 - p_3)y} \, dy \, e^{-i(k_2 - p_4)y} ,
\]

(3.14)
where \( L_j = L(N_j) = \pm L \), and momentum \( p_j \) originates from the cavity \( N_j \). The statistical average may be found using (3.12):

\[
\langle \hat{a}^\dagger_{p_1, N_1} \hat{a}^\dagger_{p_3,N_3} \hat{a}_{p_4,N_4} \hat{a}_{p_2,N_2} \rangle = n(p_1, N_1) n(p_3, N_3) \left[ \delta_{p_1 p_2} \delta_{N_1, N_2} \delta_{p_3 p_4} \delta_{N_3, N_4} + \delta_{p_1 p_4} \delta_{N_1, N_4} \delta_{p_3 p_2} \delta_{N_3, N_2} \right].
\] (3.15)

Eq. (3.14) becomes

\[
\langle \hat{N}_{k_1} (\hat{N}_{k_2} - \delta_{k_1 k_2}) \rangle = \langle \hat{N}_{k_1} \rangle \langle \hat{N}_{k_2} \rangle + \sum_{p_1, p_3, N_1, N_3} n(p_1, N_1) n(p_3, N_3) \frac{\omega_{k_1} + \omega_{p_1} (\omega_{k_2} + \omega_{p_2}) (\omega_{k_3} + \omega_{p_3})}{4 \pi^2 4a^2 4 \omega_{k_1} \omega_{k_2} \omega_{p_1} \omega_{p_3}} \sin(p_1 - k_1) a \sin(p_1 - k_2) a \sin(p_3 - k_1) a \sin(p_3 - k_2) a \times e^{-i(L_1 - L_3)(k_1 - k_2)}. \]

The two terms in the sum with \( N_1 \neq N_3 \) (i.e., \( L_1 \neq L_3 \)) lead to the usual interference term, with periods of oscillation \( \Delta k \sim 1/(2L) \).

The resolution of the detectors should not allow us to determine which cavity the particle originated from, i.e., we do not have

\[
|k_1 - k_2| L \gg 1.
\]

This condition corresponds to the Rayleigh criterion, i.e., different emission points cannot be resolved sufficiently to construct an “optical” image of source. Further, in most physical situations, \( L \gg a \), so that

\[
|k_1 - k_2| a \ll 1.
\]

This last inequality effectively resolves the artificial problem discussed at the end of the last section. The level spacing is sufficiently large that the analyzers do not detect particles coming from two different levels simultaneously. In other words, a measurement of the one-particle spectrum does not allow for a determination of the cavity size.

We now consider the two-particle spectrum starting from its Wigner operator representations (2.19) or (2.20). An optimistic expectation is that the mean coordinates \( R = \pm L \) will label the positions of the cavities, while coordinates \( z \) will be responsible for the internal dynamics of each of them, even after the operators \( \hat{N}_k \) and \( \hat{N}_{k_1} (\hat{N}_{k_2} - \delta_{k_1 k_2}) \) have been averaged. We recall that the introduction of Wigner distributions depending on coordinates and momenta assumes that we have foregone a full quantum description of the particles. However, any quantum interference problem cannot even be formulated unless both final and initial states of the particles are described in terms of their quantum numbers.

Both forms (2.19) and (2.20) of the Wigner operator reproduce all four terms in (3.16), after averaging over the set of quantum states of the two cavities. For example, consider Eq. (2.19). Substituting Eqs. (3.10) and (3.11) into Eq. (2.15), we obtain

\[
\langle \hat{N}_{k_1} (\hat{N}_{k_2} - \delta_{k_1 k_2}) \rangle = \sum_{p_1, \ldots, p_4, N_1, \ldots, N_4} \langle \hat{a}^\dagger_{p_1, N_1} \hat{a}^\dagger_{p_3,N_3} \hat{a}_{p_4,N_4} \hat{a}_{p_2,N_2} \rangle \frac{\omega_{k_1} + \omega_{p_1} (\omega_{k_2} + \omega_{p_2}) (\omega_{k_3} + \omega_{p_3})}{4 \pi^2 4a^2 4 \omega_{k_1} \omega_{k_2} \omega_{p_1} \omega_{p_3}} \times \int dR_1 dR_2 dz_1 dz_2 e^{-i k_1 z_1 - i k_2 z_2} \left[ e^{-i p_1 (R_1 + \frac{z_1}{2})} \right]_{N_1} \left[ e^{i p_2 (R_2 + \frac{z_2}{2})} \right]_{N_2} \left[ e^{-i p_3 (R_2 - \frac{z_2}{2})} \right]_{N_3} \left[ e^{i p_4 (R_1 - \frac{z_1}{2})} \right]_{N_4}.
\]

(3.19)

The average of the product of Fock operators has four terms, given by Eq. (3.13), each corresponding to a specific combination of cavity states. The exponentials in (3.19) originate from the wave functions of the cavities, which set the limits of integration depending of the choice of cavity. For example, if \( N_1 = N_4 \to -L \) and \( N_2 = N_3 \to +L \) then the limits of integration are

\[
-L - a < R_1 + \frac{z_1}{2} < -L + a, \quad L - a < R_1 - \frac{z_1}{2} < L + a;
\]

\[
-L - a < R_2 - \frac{z_2}{2} < -L + a, \quad L - a < R_2 + \frac{z_2}{2} < L + a.
\]

(3.20)

If Eq. (2.19) had represented only the first term of (2.34), then the first line of inequalities above would have contained only the cavity coordinate \(-L\), while the second line would have contained only \(+L\). This is not the case, and we
see that the coordinates $R_i$ themselves fail to enumerate the cavities, contrary to our naive assumption. Thus, the requirement that the initial state be defined in terms of quantum states makes the Wigner distribution, which was introduced in a formal manner in Sections 2.1 and 2.2, an inconvenient tool for handling interferometry problems. If we consider all terms in the sum over $N_i$ in Eq. (3.19), we recover the two–term formula (3.16) which was obtained from the original representation of the two–particle spectrum in terms of the locally defined quantum states.

3.3 Hydrodynamics and interferometry

The generalization of the two–cavity model to the freeze–out of an extended hydrodynamic system is evident: we consider a continuous set of decaying cells, taking into account the curvature of the freeze–out surface $T(x) = T_c$ and the Doppler shift of the thermal spectrum of moving cells [7,8]. In addition, we neglected the effects of the multiple local emission, i.e., the possibility that two particles are emitted from the same elementary fluid cell. For the one– and two–particle spectra we then have [7]

$$ k^0 \frac{dN_1}{dk} = J(k, k) , \quad (3.21) $$

and

$$ k^0_1 k^0_2 \frac{dN_2}{dk_1 dk_2} = J(k_1, k_1) J(k_2, k_2) + \text{Re} \left[ J(k_1, k_2) J(k_2, k_1) \right] , \quad (3.22) $$

respectively, where the emission function $J(k_1, k_2)$ is given by

$$ J(k_1, k_2) = \int_{\Sigma_c} d\Sigma u(x) \frac{k_1^\mu + k_2^\mu}{2} n(k_1 \cdot u(x)) e^{-i(k_1-k_2)x} . \quad (3.23) $$

In Ref. [8], these equations were used to study interferometry for several types of one–dimensional flow.

4. One–dimensional interferometry

We shall analyze in this Section only the scale–invariant one–dimensional hydrodynamic regime, and neglect the boundary effects which were examined in Ref. [8]. In the central rapidity region, the scale–invariant solution does not differ much from the Landau model, provided that freeze–out takes place after sufficiently long evolution, so that the initial longitudinal size of the Lorentz–contracted nuclei in the c.m.s. is negligible.

First, some notation: The parameters of the model are the critical temperature, $T_c (\sim m_\pi)$, and the space–like freeze–out hypersurface, defined by $t^2 - x_\perp^2 = \tau^2 = \text{const}$. The rapidity $y$ of a fluid cell is restricted to $\pm Y$ in the c.m.s. We assume a Gaussian transverse distribution of hot matter in a pipe with the effective area $S_\perp = \pi R_\perp^2$. The particles are described by their momenta

$$ k_i^\mu = (k_i^0, k_i^1, \vec{p}_i) \equiv (m_i \cos \theta_i, m_i \sin \theta_i, \vec{p}_i) \quad (4.1) $$

where $\vec{p}_i$ is the transverse momentum, $\theta_i$ the particle rapidity in $x_i$–direction, and $m_i^2 = m_\pi^2 + \vec{p}_i^2$ is the transverse mass. Let

$$ 2\alpha = \theta_1 - \theta_2 , \quad 2\theta = \theta_1 + \theta_2 \quad (4.2) $$

be the difference and the sum of particle rapidity, and $\vec{q}_\perp = \vec{p}_1 - \vec{p}_2$. The one–particle distribution has the form of a Bose thermal distribution of pions at temperature $T = T_c$. Since the $m_\perp$ values of interest are larger than $T_c$ we may approximate the Bose distribution by a Boltzmann form, and use the saddle point method to estimate the integrals (4.3).

For the one–particle distribution these two steps yield

$$ \frac{dN}{d\theta_1 d\vec{p}_1} \approx \tau S_\perp m_1 \int_{-Y}^{Y} dy \ \cosh(\theta_1 - y) e^{-m_1 \cosh(\theta_1-y)/T_c} \approx \tau S_\perp m_1 \sqrt{\frac{2\pi T_c}{m_1}} e^{-m_1/T_c} . \quad (4.3) $$

The general expression for $J(k_1, k_2)$ is
\[ J(k_1, k_2) = \frac{1}{2} \frac{1}{\tau S_{\perp}} e^{-\bar{q}^2 R_{\perp}^2/2} \int_{-Y}^{Y} dy \left[ (m_1 + m_2) \cosh(\theta - y) \cosh \alpha - (m_1 - m_2) \sinh(y - \theta) \sinh \alpha \right] \]

\[ \times \exp \left\{ -\frac{1}{T_c} \left[ (m_1 + iF(m_1 - m_2)) \cosh(y - \theta) \cosh \alpha - (m_1 + iF(m_1 + m_2)) \sinh(y - \theta) \sinh \alpha \right] \right\} \]

where \( F = \tau T_c \). After some tedious algebra, using the saddle-point approximation, we obtain

\[ R(k_1, k_2) = \text{Re} \left[ J(k_1, k_2)J(k_2, k_1) \right] \]

\[ = \frac{1}{4} 2\pi \mu \tau^2 S_{\perp}^2 e^{-\bar{q}^2 R_{\perp}^2} \frac{g(z) g(\frac{1}{z})}{h(z) h(\frac{1}{z})}^3 \]

\[ \times \exp \left\{ -\frac{\mu}{T_c} \left[ h(z) \cos \frac{H(z)}{2} + h(\frac{1}{z}) \cos \frac{H(\frac{1}{z})}{2} \right] \right\} \]

\[ \times \cos \left\{ \frac{\mu}{T_c} \left[ h(z) \sin \frac{H(z)}{2} + h(\frac{1}{z}) \sin \frac{H(\frac{1}{z})}{2} \right] + \frac{3}{4} \left[ H(z) + H(\frac{1}{z}) \right] + G(z) + G(\frac{1}{z}) \right\} \]

(4.6)

where \( \mu = (m_1 m_2)^{1/2} \) and \( z = (m_1/m_2)^{1/2} \), and we have introduced the functions

\[ h(z) = \left\{ z^2 - F^2 (z - \frac{1}{z})^2 + 4F^2 \sinh^2 \alpha \right\}^2 + 4F^2 (z^2 - \cosh 2\alpha)^2 \right\}^{1/4} ; \]

\[ \tan H(z) = \frac{2F \cosh 2\alpha - z^2}{z^2 - F^2 (z - \frac{1}{z})^2 + 4F^2 \sinh^2 \alpha} ; \]

\[ g(z) = \left[ z^2 + \cosh 2\alpha \right] z^2 + F^2 (z^2 - \frac{1}{z^2})^2 \right\}^{1/2} ; \]

\[ \tan G(z) = \frac{F(z^2 - \frac{1}{z^2})}{z^2 + \cosh 2\alpha} \]

(4.7)

The formulae (4.6) and (4.7) are useful for a measurement of the longitudinal size in the case that the pions have unequal transverse momenta; the case of equal transverse masses \( m_1 = m_2 \equiv m_{\perp} \) was derived in Ref. \[8\]. The main result in that study was that the full longitudinal size of the freeze–out domain is not seen in the correlation function. Since 1–d expansion typically has large velocity gradients, the local thermal spectra of the slices with different rapidities do not overlap. The correlator measures the effective size of the fluid slice which forms the observable spectrum at a given rapidity \( \theta \) (or longitudinal momentum \( k_{\parallel} \))[8]:

\[ \Delta y = \sqrt{\frac{T_c}{m_{\perp}}} \quad \text{or} \quad \Delta x_{\parallel} = \frac{\tau}{\cosh \theta} \sqrt{\frac{T_c}{m_{\perp}}} \].

(4.8)

These results seem to be in agreement with recent data [12], where the \( m_{\perp} \)-dependence of the effective longitudinal size was examined. The physical origin of the dependence \( \Delta = \frac{1}{2} \) is very simple: the larger the transverse momentum of a particle in the fluid, the more it is frozen into collective longitudinal motion, and the less the spectral pattern is broadened by thermal motion. Later we shall discuss what kind of parameters replace \( m_{\perp} \) in the case of transverse hydrodynamic motion.

The details of the correlator [3, 23], in particular whether or not oscillatory behavior around unity may occur, have caused some controversy [18, 19]. The correlator derived in the formalism of the Wigner distributions (see Eq. (2.34)) does not reveal this behavior, while Eq. (3.22) permits such oscillations. We have already discussed why a naive usage of the Wigner formalism may distort interference effects. There is a simple argument why oscillations are unavoidable in this class of hydrodynamic models: mathematically, only very restrictive conditions on the distribution functions will yield a positive definite result. Physically, when the velocity gradients are large (corresponding to small \( \tau \) in 1–d expansion), two fluid cells with large relative velocity may still be close in coordinate space, even though the bulk of their spectra are Doppler–separated. We then effectively have two point–like sources, individually not resolvable, and the oscillations in the correlator are of the same origin as in HBT interferometry of a double stellar source.

Up to now, we have assumed an instantaneous common proper time of emission for all particle types. However, it has been conjectured [13] that the hot expanding system may undergo a so–called “strangeness distillation,” i.e. a
5. Two– and three–dimensional interferometry

A picture of sharp common freeze–out for all particles leads to the conclusion that \( \gamma \equiv m_\perp^{1/2} / m_\perp \) is independent of \( m_\perp \), and that \( \gamma = \gamma_\pi \). This is indeed observed experimentally [3], suggesting that the kaon fluid remains coupled to the pions right until their common (sharp) freeze–out. If we allow for a dynamical decoupling of the kaons from the pionic fluid, then the full kaon spectra will result from a gradual emission of the kaons over some interval \( \tau_0 < \tau < \tau_\pi \). For simplicity, we shall assume that the emission function is given by the unweighted average

\[
\langle J \rangle_\tau = \frac{1}{\tau_\pi - \tau_0} \int_{\tau_0}^{\tau_\pi} d\tau J(\tau) ,
\]

where \( J \) is calculated numerically from Eq. (3.23), and \( \tau_\pi \) is the pion freeze–out time. In the integral (4.9), we assume that the temperature is a function of \( \tau \), according to \( \tau T^3 = \text{const} \). In subsequent calculations we shall take \( T_\pi = 130 \text{ MeV} \) at \( \tau_\pi = 30 \text{ fm} \).

In Fig. 1 we show the correlator as a function of \( \Delta k_\parallel / m_\perp^{1/2} \). The various curves correspond to different values of \( m_\perp \). The curves marked “s” are the result if we take \( \tau_0 = \tau_\pi \) for the emission function. They coincide almost perfectly with each other, in agreement with Eq. (4.8). The curves marked “g” correspond to a gradual emission with \( \tau_0 = 7 \text{ fm} \) (\( T_0 = 180 \text{ MeV} \)). Clearly, the \( m_\perp^{1/2} \)–scaling is violated for this type of emission. NA44 data gives no indication of such scaling violation. A systematic and parallel study of pion and kaon interferometric source sizes as a function of \( m_\perp \) will provide a sensitive test for strangeness distillation in hadronic matter at RHIC.

5. Two– and three–dimensional interferometry

In this Section, we discuss the interferometric measurements for two– and three–dimensional hydrodynamic flow. To motivate this discussion, we begin by recalling that the most important lesson from the case of a one–dimensionally expanding source is that there is no universal prescription for decoding interferometric data. Thus, in interpreting this data, we must first determine which model is applicable to the data. The model should depend on a minimal number of parameters, and exhibit some specific and clear behavior that is qualitatively recognizable in the data. Only then may we hope to extract these unknown parameters from interferometry. For example, in 1–d hydrodynamic motion of the Bjorken or Landau type, the signatures are the plateau in the central rapidity region, an increase of the correlator on the physical parameters. This is an important requirement for any model calculation.

Unfortunately, this attractive feature of the boost–invariant solution for one–dimensional flow is absent in two– and three–dimensional flows, even if the radial motion is very strong. For radial flow, the angular dependence of the Cartesian velocity components is weak, and the localization of the momentum spectrum is far less than in one–dimensional expansion. These small gradients mean that a saddle point integration will be a bad approximation, and, consequently, we have no simple dependence of the correlator upon the flow parameters. While even moderate radial flow does obscure the true transverse source size, the spectrum is not sufficiently localized to allow one to obtain a simple formula for the correlator (at least in terms of standard variables).

Let us illustrate this using two unphysical, but exactly calculable examples. The first is a scale–invariant two–dimensional expansion in the absence of longitudinal flow, as in the expansion of a long, thin filament. The second example is a scale–invariant three–dimensional expansion, corresponding to a point–like explosion. An advantage of these models is that the hydrodynamic equations are exactly solvable, and we are guaranteed dynamical consistency between the velocity field and the shape of freeze–out surface. This is an important requirement for any model calculation.

5.1 The explosion of a long filament
We consider the transverse expansion of a filament of length $L$, much larger than the transverse freeze–out radius. For this case of purely cylindrical expansion, a convenient parametrization of the coordinates is

$$x^\mu = (\tau \cosh \beta, \tau \sinh \beta \cos \psi, \tau \sinh \beta \sin \psi, z) \ ,$$

while the temperature and the velocity fields may be written as

$$\tau^2 T^3 = \text{const}$$

$$u^\mu = (\cosh \beta, \sinh \beta \cos \psi, \sinh \beta \sin \psi, 0) \ ,$$

where $\beta$ turns out to be the radial rapidity of the fluid element. An element of the freeze–out hypersurface is given by $d\Sigma^\mu = u^\mu \tau^2 \sinh \beta \, dz \, d\beta \, d\psi$.

The emission function $J(k_1, k_2)$ may be expressed using the integral

$$J = \int_0^\infty \sinh \beta d\beta \int_0^{2\pi} d\psi \exp \left[ -a \cosh \beta + b_1 \sinh \beta \cos \psi + b_2 \sinh \beta \sin \psi \right]$$

$$= 2\pi \int_0^\infty \sinh \beta d\beta \, e^{-a \cosh \beta} I_0(\sqrt{b_1^2 + b_2^2} \sinh \beta) = 2\pi \frac{e^{-\sqrt{a^2 - b_1^2 - b_2^2}}}{\sqrt{a^2 - b_1^2 - b_2^2}} ,$$

and derivatives of $J$ with respect to its parameters. Introducing the shorthand notation

$$m_z = \sqrt{m^2 + k_z^2}, \quad Q^2 = -(k_1 - k_2)^2, \quad H = \left[ 1 + T^2 \tau^2 \frac{Q^2}{m_z^2} - i \frac{T \tau Q^2}{m_z^2} \right] ,$$

we find

$$J(k_1, k_2) = \frac{2\pi L^2 m_z^2}{T} \frac{e^{-m_z H/T}}{(m_z H/T)^3} \left[ \frac{m_z H}{T} + 1 \right] \left[ 1 + \frac{Q^2}{8m_z^2} \right] = J(k_2, k_1) \ ,$$

which immediately gives us the one–particle spectrum of the model:

$$k^3 \frac{dN_k}{dk} = J(k, k) = 2\pi L^2 T \frac{e^{-m_z H/T}}{(m_z H/T)} \left[ \frac{m_z}{T} - 1 \right] .$$

Similar to one–dimensional boost invariant flow, we obtain a plateau for cylindrical boost invariant expansion, but now in the radial rapidity distribution. The dependence of the spectral density on $m_z$ is noteworthy: the localization of the spectrum due to radial flow is more pronounced for greater $m_z$. Particles with $m_z/T \gg 1$ are strongly frozen into collective radial flow. For this reason we may expect the width of the correlator in the transverse direction to be defined by $m_z$, rather than $m_\perp$.

The expression for the normalized two-particle spectrum is

$$C(k_1, k_2) - 1 = \text{Re} \frac{e^{-2m_z H(-1)/T}}{H^6} \left[ \frac{m_z H/T + 1}{m_z H/T + 1} \right]^2 \left[ 1 + \frac{Q^2}{8m_z^2} \right]^2 .$$

Its explicit real form is of little practical value, except to estimate the reliability of the saddle–point approximation for the two– and three–dimensional flows. We shall return to this point later. Instead, let us first approximate (5.8) in the case of small differences in radial rapidities of the particles. For the sideways and outward directions we obtain:

$$C(Q) - 1 = e^{-T \tau^2 Q_{side}^2/m_z} \cos \left( \frac{T \tau}{2m_z^2} Q_{side}^2 \right)$$

for $Q_{side} = 0$, and

$$C(Q) - 1 = e^{-T \tau^2 Q_{out}^2/m_z} \cos \left( \frac{T \tau}{2m_z^2} Q_{out}^2 \right)$$

for $Q_{out} = 0$. We then obtain the same radii for the sideways and outward directions:

$$R_{exp} = \tau \sqrt{\frac{T}{m_z}}, \quad R_{cos} = \frac{1}{T \tau} R_{exp} .$$
From symmetry, it is not unexpected that the radii should be the same for sideways and outward directions. The radius $R_{\text{exp}}$ is defined by the exponential, and dominates the shape of the correlator for $T\tau > 1$, while $R_{\cos}$, defined by the zero of the cosine function, dominates for $T\tau < 1$. Formally, the expressions (5.9) and (5.10) are valid at small relative transverse rapidities of the pions. They very much resemble the basic formula for longitudinal expansion. However, one should keep in mind that the latter are obtained without the saddle–point approximation. This approximation fails in the cylindrical case because the integrand of Eq. (5.4) varies only slowly with the azimuthal angle $\psi$ over the entire domain of the integration. To clarify this statement, we have estimated numerically by how much the integral fails in the cylindrical case because the integrand of Eq. (5.4) varies only slowly with the azimuthal angle $\psi$ over the entire domain of the integration. To clarify this statement, we have estimated numerically by how much the integral

$\int_0^{\infty} \sinh^2 \beta d\beta \int_0^{2\pi} d\psi e^{-\cosh \beta + \frac{2}{3}\sinh \beta} \sin \theta d\theta d\psi \sin \theta$.

were $K_1$ is the modified Bessel function. It is now straightforward to find the one– and two–particle spectra:

$C(k_1, k_2) - 1 = \text{Re} \left[ \frac{K_2(H)}{H^2 K_2(m/T)} \right]^2 \left[ 1 + \frac{Q^2}{8m^2} \right]^2 \frac{1 + Q^2/4m^2}{1 + 5T^2/4Q^2/2m^2} \cos \left( \frac{Q^2}{m^2} \left( \frac{5T\tau}{2} + \frac{m\tau}{2} \right) \right)$.

The radius which may be extracted from this correlator is

$R_{\text{exp}} = \tau \sqrt{\frac{T}{m}}$.

and the model has no large parameter which could allow an asymptotic estimate of the emission function.

6. Conclusion

Preliminary results recently reported by the NA35 and NA44 collaborations seem to confirm the main predictions of interferometry for a hydrodynamic one–dimensional flow scenario of the heavy ion collision. The intensity correlations provide a multidimensional test of the source, and the fact that so many parameters coincide can hardly be accidental. Thus, we have strong evidence that even at SPS energies ($s^{1/2} \sim 10$ A · GeV), a hydrodynamic regime develops, and that the freeze–out takes place during a short interval. This collective behavior can be expected to occur at RHIC energies, and thus it is highly desirable to continue developing the formalism.
In this paper, we have emphasized the most important physical aspects of the theory, analyzed the reliability of its different modifications known in the literature, and tried to isolate some controversial points. We argue that, strictly speaking, interferometry does not permit the initial data to be given in terms of semi–classical distributions, unless this description is augmented by a clear indication of the length scale that defines the quantum states of the particles. Our conclusion here is that the traditional operator approach, based on the precise definition of the particle states, provides a firm footing for the calculation of the two–particle spectra. In this case, distributions such as \( n(p,N) \) in Eq. (3.15), and \( n(k,x) \) in Eq. (3.23) can be thought of as on–shell Wigner phase–space distributions of the preceding kinetic stage. For any type of hydrodynamic expansion, no principal or technical problems are anticipated.

We have extended previous calculations \([8]\) for the longitudinally expanding system to the case of unequal transverse momenta, and have shown that the HBT correlator may carry information about strangeness distillation. From the NA44 data analysis, it seems that such distillation does not occur at SPS energies. We further considered transversally expanding sources, and demonstrated that in this case one has to change the parameterization of the correlator.

We emphasize that, while HBT for hydrodynamic sources is well understood as a physical phenomenon, it is always a problem to choose the “correct” model for fitting the data. We impose the condition that the model should allow one to recognize it via a qualitative analysis. Only then can one hope to understand what parameters are responsible for the correlator behavior, and find their values by fitting the data. Only after such an analysis has been performed, does it become possible to use the dynamical equations of the model, and trace the freeze–out parameters back to the earlier stages of evolution and, eventually, to an estimate of the energy density. Practically, this requirement means that we must begin with an analytic expression for a solution of the relativistic hydrodynamic equations. This guarantees the consistency between the shape of the freeze–out surface and the velocity field. Unfortunately, analytic solutions for the case of three–dimensional expansion are not yet known (the equations are nonlinear, and the initial data is not well known). A reasonable analytic approximation will do, but to our knowledge, a suitable expression has not yet been derived. An approximate formula describing a realistic, expanding system at the freeze–out stage is an important problem for boson interferometry at RHIC.

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Appendix

Descriptions of nuclear collisions in terms of semi–classical kinetic equations rely heavily on the classical nature of the source that emits the final–state particles. Such sources may be expressed naturally in terms of Wigner functions, and it is therefore of interest to what extent these functions may be used directly to solve the interferometry problem. To our knowledge, equations like (2.34) and (2.35) were originally derived in a model with classical sources. Here we revisit this derivation.

Let \( \varphi(x) \) be a quantum pion field that is emitted by a classical current \( j(x) \). The corresponding solution of the Schrödinger equation for the state vector is a coherent state \( |\Phi\rangle \), with the property that

\[
\langle \Phi|\hat{\varphi}(x)|\Phi\rangle = \varphi_{cl}(x) ,
\]

where the classical field \( \varphi_{cl}(x) \) obeys the inhomogeneous wave equation with source \( j(x) \):

\[
(\Box + m^2) \varphi_{cl}(x) = j(x) .
\]

For space–time regions outside of where the currents \( j(x) \) are localized, \( \varphi_{cl}(x) \) obeys the homogeneous wave equation, and may be expanded in the plane–wave modes of Eq. (2.3):

\[
\varphi_{cl}(x) = \int d^3k \alpha_k f_k(x) .
\]

The coherent state \( |\Phi\rangle \) may be obtained by solving the following equation for each mode:

\[
A_k^{-1}|\Phi_k\rangle = \alpha_k|\Phi\rangle ,
\]

and, after normalizing to \( \langle \Phi|\Phi\rangle = 1 \), one obtains the representation
The two-particle spectrum now reads

\[ |\Phi_{\vec{k}}\rangle = e^{-|\alpha_{\vec{k}}|^2/2} \sum_{n_k=0}^{\infty} \frac{\alpha_{\vec{k}}^{n_k}}{(n_k!)^{1/2}} |n_k\rangle = e^{-|\alpha_{\vec{k}}|^2/2} \sum_{n_k=0}^{\infty} \frac{\alpha_{\vec{k}}^{n_k} A_{\vec{k}}^{\dagger n_k}}{n_k!} |0\rangle . \]  

(A.5)

From the equation of motion for the classical field, (A.3), one immediately obtains

\[ \alpha_{\vec{k}} = i(2\pi)^{-3/2}(2\omega_k)^{-1/2} j(\omega_k, \vec{k}) , \]  

(A.6)

so that the coherent state \( |\Phi\rangle \) may be rewritten as

\[ |\Phi\rangle = \exp \left[ -\int d^3k \frac{|j(\omega_k, \vec{k})|^2}{4\omega_k(2\pi)^3} \right] \exp \left[ -i \int d^3k \frac{j(\omega_k, \vec{k})}{(2\pi)^{1/2}(2\omega_k)^{1/2}} A_{\vec{k}}^{\dagger} \right] |0\rangle \]  

(A.7)

We may also write \( |\Phi\rangle = S|0\rangle \), and it can be shown explicitly that the expression (A.7) emerges if the \( S \)-matrix is taken in the form of a normal product.

Evaluating the one- and two-particle spectra is now a question of determining the right hand sides of

\[ \langle \langle N_{\vec{k}} \rangle \rangle = \langle \langle \alpha_{\vec{k}}^{\dagger} \alpha_{\vec{k}} \rangle \rangle = \langle \langle j^*(\omega_k, \vec{k}) j(\omega_k, \vec{k}) \rangle \rangle \]  

(A.8)

\[ \langle \langle N_{\vec{k}_1}(N_{\vec{k}_2} - \delta(\vec{k}_1 - \vec{k}_2)) \rangle \rangle = \langle \langle \alpha_{\vec{k}_1}^{\dagger} \alpha_{\vec{k}_2}^{\dagger} \alpha_{\vec{k}_2} \alpha_{\vec{k}_1} \rangle \rangle \]  

\[ = \langle \langle j^*(\omega_{k_1}, \vec{k}_1) j^*(\omega_{k_2}, \vec{k}_2) j(\omega_{k_1}, \vec{k}_1) j(\omega_{k_2}, \vec{k}_2) \rangle \rangle / 4\omega_{k_1} \omega_{k_2} (2\pi)^6 \]  

(A.9)

The Fourier transform of the currents is defined in a standard way:

\[ j(k) = j(k_0, \vec{k}) = \int d^4x j(x) e^{i\vec{k}\cdot\vec{x}} , \]  

(A.10)

and we therefore obtain our previous answer for the one-particle spectrum

\[ \langle \langle N_{\vec{k}} \rangle \rangle = \int \frac{d^4x d^4y}{2\omega_k (2\pi)^3} e^{-ik \cdot (x-y)} \langle \langle j^*(x) j(y) \rangle \rangle . \]  

(A.11)

The two-particle spectrum now reads

\[ \langle \langle N_{\vec{k}_1}(N_{\vec{k}_2} - \delta(\vec{k}_1 - \vec{k}_2)) \rangle \rangle = \int \frac{d^4x d^4y d^4x' d^4y'}{4\omega_{k_1} \omega_{k_2} (2\pi)^6} e^{-i(k_1 \cdot (x-y) - k_2 \cdot (x'-y'))} \langle \langle j^*(x) j^*(x') j(y) j(y') \rangle \rangle , \]  

(A.12)

\[ i.e., \text{we recover the well known result that a coherent source does not lead to any quantum interference effects.} \]

In other words, quantum evolution follows a single trajectory, which ends up as a pure coherent state of the pion field. After we have averaged over this state, further consideration of the two-pion amplitudes and their interference is impossible. However, we note that there remains the opportunity to account for statistical effects inherent in the distribution of the classical currents. An example is given in Ref. [4]: coherent light propagating in a fluid induces (classical) polarization currents in the molecules. The latter become sources for a secondary (scattered) field. Intensity correlations of this field allow one to study the multi-particle distributions in the fluid. For simple fluids, the exact solution of the inverse problem is possible [4].

One may formally consider the \( S \)-matrix in Eq. (A.10) as the perturbation series (assuming, \textit{e.g.}, that the currents are weak). For such an expansion it is convenient to use the \( S \)-matrix in the form of a \( T \)-ordered exponent:

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\[ S = T \exp \left\{ -i \int d^4x [j^*(x) \hat{\varphi}(x) + j(x) \hat{\varphi}^+(x)] \right\}. \quad (A.16) \]

If only terms up to second order in the currents are retained, then the coherent state is effectively replaced by a two-particle state. This state is no longer the eigenstate of the annihilation operator, and the matrix element that must be evaluated now reads
\[ \int d^4xd^4y d^4x' d^4y' \langle j^*(x) j^*(x') j(y) j(y') \rangle \langle 0 | \hat{\varphi}(x') \hat{\varphi}^+(x') \hat{A}_{k_1}^\dagger \hat{A}_{k_2} \hat{A}_{k_1} \hat{\varphi}^+(y) \hat{\varphi}^+(y') | 0 \rangle. \quad (A.17) \]

Commuting the creation and annihilation operators with the field operators we arrive at
\[ \int d^4xd^4y d^4x' d^4y' \langle j^*(x) j^*(x') j(y) j(y') \rangle \frac{e^{ik_1x+ik_2x'} + e^{ik_1x'+ik_2x}}{4\omega_{k_1}\omega_{k_2}(2\pi)^6}. \quad (A.18) \]

Choosing instead of \((x, x')\) the coordinates \((R, z)\), we obtain the form
\[ 4k_1^{0}k_2^{0} d\Omega \frac{dN}{dk_1dk_2} = \int d^4R_1d^4R_2 \left[ F(R_1, k_1)F(R_2, k_2) + e^{-i(R_1-R_2)-(k_1-k_2)} F(R_1, \frac{k_1+k_2}{2})F(R_2, \frac{k_1+k_2}{2}) \right], \quad (A.19) \]

where
\[ F(R, p) = \int \frac{d^4z}{(2\pi)^4} e^{-ipz} j^*(R+z/2) j(R-z/2). \quad (A.20) \]

is the Wigner representation of the current product. In calculating the average \( \langle \ldots \rangle \), one usually assumes that the product of four currents factorizes into the binary products.

Clearly, this way of reasoning contradicts the concept of the classical current itself: if the field is classical, then the number of quanta is undefined, in principle. The selection of fluctuations with the exactly two emitted quanta is an additional measurement of the intermediate state of the system. This measurement prepares a new state of the system and removes all information about the classical nature of the emitting currents. The further evolution can now follow two different trajectories. This is exactly why Eq. (A.18) has acquired a typical interference structure, even before the particles histories were traced back to their sources.

On the other hand, if we can select a state with two quanta in the emission field, then the current is essentially a quantum mechanical transition current. In this case we must take into account an explicit backward reaction in the emitting system, and we arrive naturally at the picture which has been already considered in connection with photon emission. We conclude that there is no consistent way to express the correlations in terms of off-shell Wigner functions.

* E-mail: makhlin, gene, and welke@rhic.physics.wayne.edu, respectively.

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**Figure Caption**

FIG. 1. The correlation function $C_2$, as a function of the rescaled longitudinal momentum difference, for different values of $m_\perp$. The curves marked “s” correspond to sharp emission, and those marked “g” are for gradual emission.
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