Dissipative quantum dynamics of bosonic atoms in a shallow 1D optical lattice

J. Ruostekoski$^{1,2}$ and L. Isella$^1$

$^1$Department of Physics, Astronomy and Mathematics, University of Hertfordshire, Hatfield, Herts, AL10 9AB, UK
$^2$Institute for Theoretical Atomic and Molecular Physics, Harvard-Smithsonian Center for Astrophysics, Cambridge MA 02135

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We theoretically study the dipolar motion of bosonic atoms in a very shallow, strongly confined 1D optical lattice using the parameters of the recent experiment [Fertig et al., Phys. Rev. Lett. 94, 220402 (2005)]. We find that, due to momentum uncertainty, a small, but non-negligible, atom population occupies the unstable velocity region of the corresponding classical dynamics, resulting in the observed dissipative atom transport. This population is generated even in a static vapor, due to quantum fluctuations which are enhanced by the lattice and the confinement, and is not notably affected by the motion of atoms or finite temperature.

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The rich phenomenology of the dynamics of an atomic Bose-Einstein condensate (BEC) in a periodic optical lattice potential has recently attracted considerable theoretical and experimental interest. In particular, the nonlinear mean-field interaction of the BEC may give rise to dynamical and energetic instabilities in the transport properties of the atoms [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Most experiments to date have been carried out in shallow lattices where the motion of the BEC can be accurately described by the classical mean-field theories. In the strongly interacting regime, with very high lattice potentials, the atoms were shown to undergo a quantum phase transition to the Mott insulator state [11, 12]. Recently strongly damped oscillations of bosonic atoms were experimentally observed by Fertig et al. [12] at NIST in a tight elongated 1D atom trap, even in the limit of a very shallow lattice potential. In this Letter we numerically investigate the damped oscillations of bosonic atoms in a shallow lattice using the parameters of Ref. [12] and show that the observed damping rate can be explained by the growing population in dynamically unstable velocity region, resulting in phase decoherence. This population appears even without the dipolar motion and is not notably affected in the studied case by the atom dynamics. Moreover, for fixed nonlinearity, both, the population at unstable velocity and the damping rate, have approximately exponential dependence on the atom number. For larger atom numbers the quantum fluctuations are suppressed and the finite-temperature effects can become observable.

In the recent NIST experiment [12] an array of independent 1D atom ‘tubes’ were generated by applying a strong transverse 2D optical lattice potential. The bosonic atoms in each tube were also trapped along the axial direction by a harmonic magnetic potential and by a very shallow periodic optical lattice. The dipole oscillations of atoms along the weak axial lattice were excited by suddenly displacing the harmonic trap by means of applying a linear magnetic field gradient. In the absence of the lattice, the dipolar motion is undamped and unaffected by the interactions. However, significantly inhibited dynamics was observed even in the presence of a very shallow lattice with the height of $0.25E_r$, where $E_r \equiv 2\pi^2\hbar^2/m\lambda^2$ is the lattice photon recoil energy with the wavelength $\lambda$. The strong effect of such a shallow lattice on dissipation is surprising, since the lattice modulates the atom density only by 6%. Due to the limited resolution of the imaging system, it was not possible to measure the damping of the oscillations in the center-of-mass (cm) position of the atoms, but the oscillations in the cm velocity were imaged after a time-of-flight, following the sudden turn-off of the trapping potentials.

Here we numerically study the dipole oscillations of bosonic atoms in a 1D lattice within the truncated Wigner approximation (TWA) for the complete multi-mode field operator, beyond the discrete tight-binding approximation. The previous theoretical studies of the quantum dynamics of bosonic atoms in a combined harmonic and lattice potential indicate that quantum fluctuations can result in strong dissipation [13, 14, 15, 16, 17]. In Ref. [14] the dynamics was studied within the tight-binding approximation to the TWA for deep lattices. This approach is much less accurate for very shallow lattices studied here in Ref. [13]. Moreover, we find that the tight-binding approximation to the TWA for shallow lattices strongly underestimates the generated noise and the effect of vacuum fluctuations on the dynamics.

In Refs. [13, 18] the atoms were confined in a 2D array of decoupled tight 1D tubes by means adiabatically loading the atoms into a strong 2D optical lattice. We write the atom density in a 1D tube at $(x_0, y_0)$:

$$\rho(r) = \frac{15N_o a^2}{8\pi^2 l_x^2 R^3} \left(1 - \frac{x_0^2}{R_x^2} - \frac{y_0^2}{R_y^2} - \frac{z^2}{R_z^2}\right) e^{-\rho^2/l_z^2}, \quad (1)$$

where $\rho^2 \equiv (x-x_0)^2 + (y-y_0)^2$, $R_i$ are the Thomas-Fermi radii of the 3D BEC, with $\bar{R} \equiv (R_x R_y R_z)^{1/3}$, $N_o$ denotes
the total atom number, and \(d = \lambda/2\) the lattice spacing. By expanding around the lattice site minimum we may estimate \(l_s \approx (h/m_\omega)^{1/2}\) by the harmonic trap frequency in the transverse direction \(\omega = 2\sqrt{\delta E_r}/\hbar\), where \(s\) denotes the lattice height in the units of \(E_r\). In Ref. \[13\], \(s = 30\) and \(\lambda = 810\) nm, yielding \(\omega \simeq 2\pi \times 38.8\) kHz. For \(N_\alpha = 1.4 \times 10^5\) atoms \[13\], \(R_x \simeq 14\) \(\mu\)m, \(R_y \simeq 20\) \(\mu\)m, \(R_z \simeq 10.6\) \(\mu\)m, and within the classical radius there are \(\pi R_z R_y/\hbar \simeq 5400\) atom tubes. From Eq. \[1\] we obtain the number of atoms in each tube \(N(x_\alpha, y_\alpha) = \int d^3r \rho = 5N_\alpha [\sqrt{1 - x_\alpha^2/R_x^2} - \sqrt{1 - y_\alpha^2/R_y^2}]^{3/2}/2\pi R_z R_y\). At the central tube, \(N(0,0) \simeq 65\) atoms.

When a shallow optical lattice is applied, the wave function Eq. \[1\] is modified along the \(z\) axis. In the experiment the displacement of the harmonic potential was about \(\delta \simeq 3\mu cm\simeq 2.2l\), where \(l \equiv (\hbar/m_\omega)^{1/2}\) and the trap frequency \(\omega \simeq 2\pi \times 60\) Hz. Within \(2R_z\), there are about 52 lattice sites.

We study the NIST experiment of dipolar oscillations within the TWA, by suddenly displacing the harmonic trap. In the TWA, in a tight elongated trap \(\omega \ll \omega_z\), the dynamics of the 1D classical stochastic field \(\psi_W(z,t)\) follows from the nonlinear equation \[13\]:

\[
\frac{\partial}{\partial t} \psi_W = \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} + V + g|\psi_W|^2\right) \psi_W ,
\]

which coincides with the Gross-Pitaevskii equation (GPE). Here the potential is a combined harmonic trap and a periodic optical lattice \(V(z) = m_\omega^2 z^2/2 + sE_r \sin^2(\pi z/d)\) and \(g = 2\hbar \omega_z a\), where \(a\) denotes the scattering length. The thermal and quantum fluctuations are included in the initial state of \(\psi_W\) in Eq. \[2\] which represents an ensemble of Wigner distributed wave functions. Our basic formalism of the TWA is similar to Ref. \[15\].

Initially, before displacing the trap, the gas is assumed to be in thermal equilibrium and we approximate the field operator \(\hat{\psi}(z,t = 0)\), within the Bogoliubov theory \[21\]:

\[
\hat{\psi}(z) = \psi_0(z) \hat{a}_j + \sum_{j > 0} [u_j(z) \hat{a}_j - v_j^*(z) \hat{a}^*_j].
\]

Here \(\psi_0\) is the ground state solution of the GPE, obtained by evolving the GPE in imaginary time, and \(\langle \hat{a}^\dagger \hat{a} \rangle = N_0\), the number of ground state atoms. The mode functions \(u_j(z)\) and \(v_j(z)\) \((j > 0)\), and the energies \(\epsilon_j\) for excited states are obtained by solving the corresponding Bogoliubov equations \[15\] in the harmonic trap, modulated by the shallow lattice potential. In the TWA the time evolution of the ensemble of Wigner distributed wave functions \[Eq. \[2\]\] is unraveled into stochastic trajectories, where the initial state of each realization for \(\psi_W\) is generated according to Eq. \[3\] with the quantum operators \((\hat{a}_j, \hat{a}^\dagger_j)\) replaced by complex Gaussian-distributed random variables \((\alpha_j, \alpha^*_j)\). These are obtained by sampling the corresponding Wigner distributions \[13\]. In particular, for the quasiparticles the Wigner function is that of the ideal harmonic oscillators in a thermal bath \[21\], with the width \(\tilde{n}_j + \frac{1}{2}\), where \(\tilde{n}_j \equiv \langle \hat{a}^\dagger_j \hat{a}_j \rangle = [\exp(\epsilon_j/k_B T) - 1]^{-1}\) and the 1/2 width at \(T = 0\) for each mode represents the quantum noise.

Because the Wigner function returns symmetrically ordered expectation values, \((\alpha^\dagger_j \alpha_j) = \tilde{n}_j + \frac{1}{2}\) and \((\alpha_j^* W = \langle \alpha_j^* \rangle W = \langle \alpha_j^2 \rangle W = 0\), for \(j > 0\), it is generally not possible to extract from the TWA simulations the correlation functions for the full multi-mode field operator. Consequently, we derive the desired normally ordered expectation values by defining the ground state operators \(a_j\) for each individual lattice site \(j \in \mathbb{N}\): \(a_j(t) = \int dz \psi^*_0(z) \psi_W(z,t)\), where the integration is over the \(j\)th lattice site, \(\psi_W(z,t)\) is the stochastic field, determined by Eq. \[4\], and \(\psi_0(z)\) is the ground state wave function, obtained by evolving the GPE in imaginary time. This provides us with the basis for \(\hat{\psi}(z) = \sum_j \hat{a}_j \phi_j(z)\), where the ground state functions \(\phi_j\) are restricted to the \(j\)th site, and the normally ordered expectation values are easily obtained with respect to \(\hat{\psi}\), e.g., for the cm position \(z_{cm}\), the position fluctuations \(\Delta z = \langle \hat{\psi}^2(z) - \hat{\psi}(z)^2 \rangle^{1/2}\), and for the normalized phase coherence between the central well and its nth neighbor \(C_i \equiv |\langle \hat{a}^\dagger_{i+1} \hat{a}_i \rangle|/\sqrt{m_\omega n}\).

We numerically study the dipole oscillations using the experimental parameters \[13\]. In the central atom tube \(N \simeq 65\), resulting in the nonlinearity \(Ng \simeq 320\) \(\text{h}\) \(\text{f}\) \(\text{w}\) for \(a \simeq 5.313\) \(\text{nm}\). In Fig. \[1\] we use the same fixed \(Ng\), but vary \(N\), and show the cm position \(z_{cm}\) of the atoms for \(s = 0.25\). The atoms are initially in thermal equilibrium at \(t = 0\) and at \(\omega_0 = 1\) we displace the center of the harmonic trap from \(z = 0\) to \(z = \delta = 2.16l \simeq 7.4d\). In the initial state \[14\], we generate the noise for 300 modes, corresponding to 4-5 lowest energy bands. Even though the atoms mainly remain in the lowest band, synthesizing the TWA noise for only 70 modes \((\sim \text{one band})\) can underestimate the damping and the phase decoherence several tens of percents. This emphasizes the importance of the multi-mode approach to the TWA, beyond the tight-binding approximation, and is consistent with our previous observations \[17\]. In 1D TWA, unlike in 3D \[22\], we did not find any divergence of the calculated results, when the mode number was increased. For each run, we typically use 600 stochastic realizations. The integration of the nonlinear evolution in Eq. \[4\] is obtained using the split-step method \[23\] on a spatial grid of 4096 points. The numerics becomes much faster for larger atom numbers, due to smaller quantum fluctuations.

As in Ref. \[13\], we model the cm motion as a damped harmonic oscillator \(z_{cm} = -kz/m^* - \gamma z\), so that underdamped motion \(z_{cm}(t') \equiv -e^{-\gamma t'}(\delta \cos \Omega t' + \sin \Omega t')\), with \(\gamma \equiv \gamma_\delta/\Omega_0 = \sqrt{\hbar/k/m^* - \gamma^2}\), where \(t' \equiv t - t_0\) and \(m^*\) denotes the effective mass. At \(t' = 0\) we have \(z_{cm} = -\delta\) with \(v_{cm} = 0\). Even in a very shallow lattice, \(s = 0.25\), we find a significant damping for \(N = 75\) and 90 atoms, with \(\gamma/\omega \simeq 0.020\) and 0.013 \[Fig. 1\]. The damp-
We can recognize any notable finite-T effects on
mal atoms on
ration to be negligible compared to quantum fluctuations.
In the experiments \[13\] it was not possible to de-
sum by an integral, we obtain \(\bar{\gamma}\) imaging the expanding atom cloud, indicating that

case has notable damping \((\gamma \approx 0.002\omega)\).

In the combined harmonic and lattice potential, the
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and no momentum broadening was observed, despite the

In the experiment \[13\] the dynamics was measured by
ng the expanding atom cloud, indicating that \(\gamma\) did not repre-
the central tube with \(N \approx 65\), which is close to the
merically observed value of \(\gamma \approx 0.03-0.04(\pm 0.01)\omega\)

In the experiment \[13\] the dynamics was measured by
ng the expanding atom cloud, indicating that \(\gamma\) did not repre-
the central tube with \(\Omega \approx \omega\), but the average
age value over all the tubes. Although in the experiment
 is constant in each tube (as \(N\) varies), we can obtain
an upper bound limit for the average damping rate by
xtrapolating our fitted \(\gamma(N)\) (for fixed \(N_g\)) to small \(N\),
so that for \(\gamma/\omega \ll 1\), \(\gamma = \sum_{i,j} \gamma(N)N(x_i, x_j)/N_a\). Here
\(N(x_i, x_j)\) is the atom number in a tube at \((x_i, y_j)\) and
the summation is over all the tubes. By replacing the
sum by an integral, we obtain \(\bar{\gamma} \approx 0.047\omega\).

We also studied thermal effects on the dipolar motion
[Fig. 2]. In the experiments \[13\] it was not possible to de-
rrive information about temperature and the effect of ther-
amal atoms on \(\gamma\). For small \(N\) we found thermal fluctua-
tions to be negligible compared to quantum fluctuations.
Only for ground state atom numbers clearly exceeding
000, we can recognize any notable finite-T effects on \(\gamma\).
This strongly suggests that the experimentally observed
damping \[13\] duly was due to quantum fluctuations.

The classical nonlinear evolution of a BEC in a peri-
odic lattice is susceptible to dynamical instabilities arising
from both constant velocity, larger than a critical value \[4, 13, 18\],
and acceleration or force, smaller than a critical value \[7, 8, 13\]. The onset of the instability is
associated with inhibited transport, fragmentation of the
density profile, and the significant broadening of quasi-
momentum distribution. In Ref. \[13\] the importance of the
dynamical instabilities on dissipation was not clear,
since the velocity was much smaller than the critical value
and no momentum broadening was observed, despite the
strongly damped motion.

In the combined harmonic and lattice potential, the
separation of the acceleration and the velocity insta-
labilities is not straightforward, since the trap induces a
local force \(m\omega^2 z\) and, e.g., applying a constant force
\(m\omega^2 F\) on atoms is the same as displacing the trap:
\(z^2/2 - zF = (z - F)^2/2 + \text{const}\). Moreover, a weakly
accelerating lattice does not equal to applying a constant
force, since the acceleration in a shallow lattice \((s = 0.25)\)
do not induce notable cm motion. We numerically in-
tegrated the classical GPE for a BEC (without quantum or
thermal noise) and found the onset of the dynamical instabil-
ity at \(\delta_c \approx 8.9\ell\), corresponding to a maximum
velocity \(v_{\text{cm}} \approx 8.9\omega\ell\). For smaller displacements, no insta-
bilities were observed on the time scale of the experiment
\[13\] e.g., due to the weak force induced by the trap.
With quantum fluctuations, the classically observed sharp
set of the dynamical instability is smeared out due to the
phase uncertainty of atoms between neighboring lattice sites
\[13\]. This is similar to a BEC in a double-well potential,
where the macroscopic self-trapping of the atom population
can decay due to dissipation \[21\].

In order to investigate the role of the classical dynamical
instabilities in the dissipative quantum dynamics, we
Calculations indicated that the distribution in the TWA
from \(\hat{\psi}\) and found atoms occupying the dynamically
stable velocity region of the corresponding classical sys-
tem; Fig. \[13\]. This population is larger and grows more
rapidly, the larger the observed damping: e.g., at \(v = 9\omega\ell\)
and \(\omega t = 8\), \(n(p)\ell/\ell \approx 0.02\) and 0.08, for \(N = 225\)
and 75, and both \(n(p)\) and \(\gamma\) show similar exponential
dependence on \(N\). For \((N, N_g/\hbar\omega) = (90, 450)\) (here
\(\gamma \approx 0.026\omega\)), the ratio of the population at the unstable
velocity \(v \approx 9\omega\ell\) to that at the peak of the distribution
was 0.07 at \(\omega t = 3\) and as high as 0.09 at \(\omega t = 6\). It is
this finite high velocity occupation that results in dissis-

\[\text{FIG. 2: Thermal effects on the coherence and the cm motion.}
The coherence \(C_5\), as in Fig. \[1\] (left); curves from top represent
\((N_0, k_B T/h\omega) = (1000, 0), (900, 21.3), (400, 0),\) and \((360, 10.5)\).
The cm (right) for the same cases. Only the finite-T, \(N_0 = 900\) case has notable damping \((\gamma \approx 0.002\omega)\).\]
displaced trap case (e.g., at static potential was approximately equal to that in the stable region in the quasimomentum distribution in the experiment. The occupation of the dynamically unlabile lattice, had significant effect on the phase coherence in indicating that the motion of neither the atoms, nor the systems was almost the same as for the displaced trap, \( z \). In all the cases \( z _ { \text{cm} } \) remained close to zero, without notable cm motion. The decrease in \( C _ { i } \) in all the three systems was almost the same as for the displaced trap, indicating that the motion of neither the atoms, nor the lattice, had significant effect on the phase coherence in the experiments. The occupation of the dynamically unstable region in the quasimomentum distribution in the static potential was approximately equal to that in the displaced trap case (e.g., at \( p = \pm 9 \hbar /l, n \approx 0.05 \hbar /l, \) resulting in a similar number of high velocity atoms in the two cases. Moreover, the variation of this atom number was very little affected by the dipole oscillations. Consequently, neither phase decoherence nor the population with unstable velocities, which result in the damping of the dipole oscillations, are here due to the motion of the atoms, but the combined effect of the strong transverse confinement and the lattice. This is contrary to the system studied in Ref. \[14\] where the atom motion itself was argued to generate the decoherence.

As expected, both the increase of the lattice height and the initial displacement enhance dissipation. We studied the dynamics for \( s = 1, \delta = 2.16 \lambda \), and obtained for \( (N, Ng/\hbar \omega l) = (330,350), (430,340), (830,330) \), \( \gamma / \omega \approx 0.019, 0.012, 0.003 \), at \( T = 0 \). Also with \( s = 1 \) the damping is notable for \( \delta \) much smaller than the classical critical displacement \( \delta _ { c } \approx 6-7 \lambda \). On the other hand, for \( N = 175 \) and \( Ng \approx 380 \hbar \omega l \), the increase of \( \delta / l \) from 2.16 to 4.32 increased \( \gamma / \omega \) from 0.004 to 0.009.

In summary, we studied dissipative atom transport in a shallow 1D lattice. At a fixed nonlinearity, smaller \( N \) results in larger phase and momentum uncertainty and, consequently, larger population in the dynamically unstable velocity region. Surprisingly, this population is generated due to the confinement and the lattice, even with neither the atoms nor the lattice moving. The qualitative agreement between the TWA and the experiment \[13\], despite the approximations involved in generating the initial state at small \( N \), also represents a considerable success of the TWA which is traditionally considered more suitable in the limit of large field amplitude. For instance, the time-dependent Hartree-Fock-Bogoliubov theory predicts much too small a damping \[16\]. This suggests the TWA could provide a powerful technique to study vacuum fluctuations also in several other related condensed matter and atomic systems, such as corrugated superconducting nanowires \[20\].

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\[\begin{align*}
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