Majorana edge states and topological properties in 1D/2D Rashba semiconductor proximity coupled to iron-based superconductor

Hiromi Ebisu¹, Keiji Yada², Hideaki Kasai¹,³ and Yukio Tanaka²

1 Department of Applied Physics, Osaka University, 2-1 Yamadaoka, Suita, Osaka 565-0871, Japan
2 Department of Applied Physics, Nagoya University, Nagoya 464-8603, Japan
3 Center for Atomic and Molecular Technologies, Osaka University, 2-1 Yamadaoka, Suita, Osaka 565-0871, Japan

E-mail: ebisu@dyn.ap.eng.osaka-u.ac.jp

Received 26 May 2014, revised 22 August 2014
Accepted for publication 23 September 2014
Published 26 November 2014

Abstract

We study Majorana edge states and their topological properties in one-dimensional (1D) and two-dimensional (2D) Rashba semiconductors deposited on iron-based superconductors under the applied Zeeman field for various directions. Using the recursive Green’s function method, we calculate the local density of states (LDOS) both for $s_-$ and $s_+$-wave pairings. We elucidate that it shows anisotropic response to the applied Zeeman field specific to Majorana edge states. This anisotropy can be understood by the winding number, which shows whether the present system is topological or not. The resulting LDOS and winding numbers for $s_-$ and $s_+$-wave pairings are significantly different at the lower Zeeman field. These results serve as a guide to determine the pairing symmetry of iron-pnictide.

Online supplementary data available from stacks.iop.org/sust/28/014001/mmedia

Keywords: topological superconductor, iron-based superconductor, mirror reflection symmetry

(Some figures may appear in colour only in the online journal)

1. Introduction

Topological superconducting systems with gapless surface Andreev bound states (SABSs) are currently a popular focus in condensed matter physics. [1–4] In these systems, Majorana fermions [1, 3, 5, 6] appear as SABSs or vortex core states. For the future application of a fault tolerant quantum computation, Majorana fermions are important ingredients [7] since they obey non-Abelian statistics. Topological superconductivity with Majorana fermions was originally discussed in spinless triplet $p$-wave superconductors [1, 6, 8]. However, it is not easy to realize spinless triplet superconductor in real solid state systems.

Fu and Kane proposed that topological superconductivity is possible in a ferromagnet/spin–singlet $s$-wave superconductor hybrid system deposited on the surface of a topological insulator [9]. The elucidation of the physical properties of Majorana fermions in this system has become a hot topic and several theoretical [10–13] and experimental studies [14, 15] have been presented. The key concept for the generation of topological superconductivity from conventional spin–singlet $s$-wave superconductor is the simultaneous existence of spin–orbit coupling and the broken time reversal symmetry such as the Zeeman field [16, 17]. In the presence of the Rashba spin–orbit coupling, the energy dispersion of free electrons splits into two. Further, by introducing Zeeman field and tuning chemical potential, one of the spin-polarized Fermi surfaces disappears. Then, exotic spinless metallic state, i.e. helical metallic state, is attainable where the direction of the spin of an electron is rocked to the momentum.
A semiconductor quantum well coupled to an s-wave superconductor and a ferromagnetic insulator is a possible candidate [18, 19] in which one-dimensional chiral Majorana edge state is generated as a SABS. Majorana edge states were also proposed in semiconductor nano-wires deposited on the surface of spin-singlet s-wave superconductor in the presence of an external Zeeman field [20, 21]. Since the Majorana edge state is generated as the end state of the nano-wire, a braiding operation might be possible by making a network of wires [22]. It is noted that several experiments supported the existence of Majorana edge state [23–28] through zero bias conductance peak [29–34] or anomalous Josephson current [35–37].

In addition to these streams, there have been many studies on topological superconductivity in time-reversal invariant (TRI) systems without the Zeeman field. [38–52] These systems belong to so-called DIII class in the periodic table [38] of topological materials. Doped topological insulator Bi2Se3 is one of the candidate materials of the DIII class superconductor [53, 54]. Although several theoretical and experimental works support the realization of TRI topological superconductivity in this material, pairing symmetry and resulting Majorana modes are not fully determined yet.

There are several proposals to realize TRI topological superconductors (SCs) based on hybrid systems, proximity coupled to unconventional superconductors [50, 55, 56]. Recently, Zhang, Kane and Mele proposed hybrid systems combining doped Rashba semiconductors (RSs) and iron-based SCs [57]. They have assumed that spin–singlet s±-wave pairing is realized in iron-based SCs [58–60]. Then, the resulting pair potential in semiconductor has a sign change between Γ and M points. A Kramers pair of Majorana edge states is generated at the boundary of the 2D (1D) TRI hybrid superconducting system due to the sign change of the pair potential between the two Fermi surfaces. However, the pairing symmetry of the iron based superconductors is still the subject of hot debate. A so-called s±+wave pairing with no sign change of the pair potential is also possible by the orbital fluctuation [61–63]. Although the tunneling spectroscopy of iron-based superconductors has been calculated [64, 65], the significant qualitative difference of the line shape of the tunneling conductance between s±-wave and s±+wave pairing does not exist. The tunneling conductance of normal metal / s±-wave superconductor junctions is not distinct as compared to d-wave [29, 30] or p-wave [66–69], superconductor junctions. Thus, it is highly encouraged to present a new idea to distinguish the above two pairings at a qualitative level.

In the present paper, we study 1D/2D edge states of doped Rashba semiconductor deposited on iron-based superconductors by applying the Zeeman field for various directions. Using the recursive Green’s function method, we calculate local density of states on the edge and the angle resolved local density of states in two-dimensional case both for s± and s±+wave pairing cases. We concentrate on the Majorana edge modes as an ABS and relevant local density of state (LDOS) including zero energy peaks (ZEP). We elucidate that the anisotropic response to the Zeeman field stems from the mirror reflection symmetry [70–74]. We also calculate the winding numbers of the Hamiltonian and analyze topological properties. The resulting LDOS and winding numbers are seriously different between these two pairing cases. It can be concluded that Rashba semiconductor/iron-based superconductor hybrid junctions are useful to determine the pairing symmetry of iron-pnictide.

The organization of this paper is as follows. In section 2, we explain the model, Hamiltonian, and the formulation. In section 3.1, we study a 1D Rashba semiconductor nanowire / iron-based superconductor hybrid system. We calculate the energy spectrum of the bulk nanowire and LDOS on the edge for various directions of the applied Zeeman field. In section 3.2, we study a 2D Rashba semiconductor layer / iron-based superconductor hybrid system. Angle resolved LDOS for fixed momentum k∥ parallel to the surface and angle averaged LDOS on the edge are calculated. In figures 3(a) and (b), we compare the results of s±-wave and s±+wave case. In section 4, we interpret the calculated results in section 3 based on the winding number of the system. We also discuss the reason why LDOS is sensitive to the direction of the applied Zeeman field in terms of the Ising-like spin of Majorana edge state. In section 5, we summarize our results.

2. Formulation

In this section, we introduce the model Hamiltonian of 1D and 2D Rashba semiconductors deposited on iron-based superconductors as shown in figure 1(a) and write a formula for the recursive Green’s function to calculate LDOS on the edge.
2.1. Model

The model Hamiltonian of 1D Rashba doped semiconductor nanowire in momentum space reads [57]

\[
H_{0}^{1D}(k_{x}) = (-2t \cos k_{x} - \mu)\sigma_{0}\tau_{x} + 2\lambda_{R} \sin k_{x}\sigma_{y}\tau_{z} - (\Delta_{0} + 2\Delta_{1} \cos k_{x})\sigma_{x}\tau_{y},
\]

(1)

where \(t\) is the nearest neighbor hopping, \(\mu\) is a chemical potential, \(\lambda_{R}\) is Rashba spin–orbit coupling. We have chosen the value of chemical potential \(\mu\), so that the charge transport properties of Rashba semiconductor become metallic by doping effect. \(\Delta_{0}\) and \(\Delta_{1}\) are proximity induced pair potentials in the 1D Rashba semiconductor. In general, these values are less than those in the bulk iron-based superconductor. \(\sigma\) and \(\tau\) are the Pauli matrices in spin space and electron–hole space, respectively. When \(|\mu - t\Delta_{0}/\Delta_{1}| < 2\lambda_{R}\sqrt{1 - \Delta_{0}^{2}/(4\Delta_{1}^{2})}\), the quasiparticle yields a different sign from the pair potential at the inner and the outer Fermi points. Then, we verify that \(s_{\pm}\)-wave pairing is realized in the 1D Rashba semiconductor.

The corresponding model Hamiltonian of the 2D Rashba semiconductor in momentum space is given by

\[
H_{0}^{2D}(k_{x}, k_{y}) = (-2t \cos k_{x} \pm \cos k_{y} - \mu)\sigma_{0}\tau_{x} - 2\lambda_{R} \sin k_{x}\sigma_{y}\tau_{0} + 2\lambda_{R} \sin k_{y}\sigma_{0}\tau_{y} - (\Delta_{0} + 2\Delta_{1} \cos k_{x} \pm \cos k_{y})\sigma_{x}\tau_{y}.
\]

(2)

When we choose \(|\mu - t\Delta_{0}/\Delta_{1}| < 2\lambda_{R}\sqrt{2 - \Delta_{0}^{2}/(8\Delta_{1}^{2})}\), the \(s_{\pm}\)-wave pairing is realized.

The nodal lines for the \(s_{\pm}\)-wave case in 1D and 2D are shown in figures 1(b) and (c), respectively. For the \(s_{\pm}\)-wave pairing, Majorana Kramers doublets emerge as edge states, and a DIII class topological superconducting state is realized [57]. The applied Zeeman field is given by

\[
\tilde{V}_{i} = \begin{cases} 
V_{i}\sigma_{x}\tau_{x} & (i = x, \ x\text{-direction}) \\
V_{i}\sigma_{y}\tau_{0} & (i = y, \ y\text{-direction}) \\
V_{i}\sigma_{z}\tau_{z} & (i = z, \ z\text{-direction})
\end{cases}
\]

(3)

Thus, the total Hamiltonian in 1D and 2D with the Zeeman field is given by

\[
H_{i}^{1D(2D)}(k_{x}, k_{y}) = H_{0}^{1D(2D)}(k_{x}, k_{y}) + \tilde{V}_{i}.
\]

(4)

In the presence of the Zeeman field, the time-reversal symmetry is broken and topological nature of the Majorana edge states changes.

2.2. Recursive Green’s function

First, we derive a general formula of the recursive Green’s function in 1D. Consider a system which has finite length in the \(x\)-direction. Suppose the number of the sites is \(N_{s}\), and add one more site in the \(x\)-direction (figure 2(a)). Based on the

\[
\rho^{1D}(E) = -\frac{1}{\pi} \text{Im} \left\{ \text{Tr} \left[ G_{N_{s}+1,N_{s}+1}(E + i0^{+}) \right] \right\},
\]

(13)

where \(\text{Im}\) denotes to choose imaginary part. In the evaluation of the trace of \(G_{N_{s}+1,N_{s}+1}(E + i0^{+})\), we sum up only the electronic part of \(G_{N_{s}+1,N_{s}+1}(E + i0^{+})\).

In the 2D model, we assume that the boundary is flat as shown in figure 2(b). In this case, momentum \(k_{y}\) is a good quantum number. Then, we can calculate the recursive

\[
G = G_{0} + G_{0}TG
\]

(5)

for Green’s function \(G\) of this system, where

\[
G_{0} = \left[ N_{i} \right] G_{N_{s},N_{i}} \left( N_{i} \right) + \left[ N_{i} + 1 \right] g_{\text{iso}} \left( N_{i} + 1 \right)
\]

\[
T = \left[ N_{i} + 1 \right] T_{N_{i}+1,N_{i}} \left( N_{i} + 1 \right) + h. c.
\]

\[
G_{N_{i},N_{i}} \text{ is the Green’s function at } N_{i}\text{-site, } g_{\text{iso}} \text{ is the isolated Green’s function at } N_{i} + 1\text{-site, and } T_{N_{i}+1,N_{i}(N_{i}+1)} \text{ is a transfer integral from } N_{i}\text{-th to } N_{i} + 1\text{-th } (N_{i} + 1 \to N_{s}) \text{ site. We derive following two equations from equation } (5),
\]

\[
G_{N_{i}+1,N_{i}+1} = g_{\text{iso}} + g_{\text{iso}} T_{N_{i}+1,N_{i}} G_{N_{i},N_{i}+1}
\]

\[
G_{N_{i},N_{i}+1} = G_{N_{i},N_{i}} T_{N_{i},N_{i}+1} G_{N_{i}+1,N_{i}+1}.
\]

(6)

(7)

(8)

By substituting \(G_{N_{i},N_{i}+1}\) in (7) for (8), we obtain

\[
G_{N_{i}+1,N_{i}+1} = \left( g_{\text{iso}}^{-1} - T_{N_{i}+1,N_{i}} G_{N_{i},N_{i}+1} T_{N_{i},N_{i}+1} \right)^{-1}.
\]

(9)

Once we know \(G_{N_{i},N_{i}}\), we obtain \(G_{N_{i},N_{i}+1}\). In order to calculate the LDOS, we evaluate a retarded Green’s function \(G_{i}(E + i0^{+})\) with site index \(j\) and energy \(E\) measured from the Fermi level, where \(0^{+}\) is an infinitesimal number. In the present model, the following relations

\[
T_{j+1,j} = -i\sigma_{0}\tau_{z} - i\lambda_{R}\sigma_{y}\tau_{0} - \Delta_{1}\sigma_{y}\tau_{y} \quad (j = 1 \sim N_{i})
\]

(10)

\[
g_{\text{iso}}(E) = (E_{k} \times 4 - H_{\text{iso}})^{-1}
\]

(11)

\[
H_{\text{iso}} = -\mu\sigma_{0}\tau_{z} - \Delta_{0}\sigma_{y}\tau_{y} + \tilde{V}_{i},
\]

(12)

are satisfied. After calculating \(G_{1,E}(E) = g_{\text{iso}}(E)\), we can obtain the Green’s function of any number of sites in the \(x\)-direction. Finally, LDOS on the edge is given by

\[
\rho^{1D}(E) = -\frac{1}{\pi} \text{Im} \left\{ \text{Tr} \left[ G_{N_{s}+1,N_{s}+1}(E + i0^{+}) \right] \right\},
\]
Green’s function for each $k_y$ in a similar way to the 1D case. We obtain

$$
T_{i+1,i} = -t_{0} \tau - i \lambda \sigma_{z} \tau_{z} - \Delta_{1} \sigma_{y} \tau_{y}, \quad (i = 1 \sim N) \tag{14}
$$

$$
g_{iso}(E, k_y) = \left( E I_{k_y} - H_{iso}(k_y) \right)^{-1}, \tag{15}
$$

$$
H_{iso}(k_y) = \left( -2t \left( \cos k_{x} + \cos k_{z} \right) - \mu \right) \sigma_{y} \tau_{y}
- 2 \lambda_{R} \sin k_{x} \sigma_{x} \tau_{z}
- \left( \Delta_{0} + 2 \Delta_{1} \cos k_{z} \right) \sigma_{y} \tau_{y} + \tilde{V}. \tag{16}
$$

Angle resolved local density of states (ARLDOS) for each $k_y$ is given by

$$
D^{2D}(E, k_y) = -\frac{1}{\pi} \text{Im} \left\{ \text{Tr} \left[ G_{N_{y}+1,N_{y}+1}(E + i\delta, k_y) \right] \right\}. \tag{17}
$$

To obtain the LDOS, we calculate

$$
\rho^{2D}(E) = \frac{1}{N_{y}} \sum_{k_y} D^{2D}(E, k_y). \tag{18}
$$

where, $N_{y}$ is a number of sites in the $y$-direction. Using equations (13), (17), and (18), we obtain LDOS on the edge of the 1D model and ARLDOS and LDOS in the 2D model in the presence of the Zeeman field ($x$-, $y$-, and $z$-directions) for both the $s_{z}$ and $s_{++}$-wave pairings.

3. Calculated results of LDOS

In this section, we show the results of LDOS in 1D and ARLDOS and LDOS in 2D. In this paper, we fix parameters as $t = 1$, $\lambda_{R} = 0.5$, $\mu = -1$. We choose $\Delta_{0} = -0.2$ and $\Delta_{1} = 0.2$ for $s_{z}$-wave pairing and $\Delta_{0} = 0.2$ and $\Delta_{1} = 0$ for $s_{++}$-wave pairing.

3.1. 1D Rashba semiconductor

First, we show the LDOS on the edge of a 1D Rashba semiconductor nanowire deposited on iron-based superconductors with the Zeeman field in the $x$-, $y$-, and $z$-directions for $s_{z}$-wave case (figure 3). Due to the presence of Majorana Kramers doublets realized without the Zeeman field, we can clearly see the zero energy peak of the LDOS of the edge state for $V_{x} = 0$ ($i = x, y, z$) (figure 3). Since this Kramers doublet is topologically protected by the time-reversal symmetry, a pair of Kramers doublet is lifted by the applied Zeeman field in general. Here, we show the LDOS under the applied Zeeman field in the $x$-, $y$- and $z$-direction in figures 3(a), (b), and (c), respectively. A zero energy peak in the LDOS remains in the presence of the Zeeman field in the $x$- and $z$-directions while it disappears for the $y$-direction. This suggests that remaining symmetry of the Hamiltonian, which is discussed below, protects the Majorana edge states under the Zeeman field in the $x$- and $z$-directions. Next, we see LDOS at the edge under the Zeeman field in the $x$-direction.

We can also see a closing of the bulk energy gap at $V_{x}/t \sim 1$ and 3. A zero energy peak survives after the first gap closing at $V_{x}/t \sim 1$ as shown in figure 3(d). However, this zero energy peak disappears after the second gap closing at $V_{x}/t \sim 3$. Similar features are obtained when the Zeeman field is along the $z$-direction.

In the case of the $s_{++}$-wave pairing, no zero energy state (ZES), i.e. (a) $x$-, (b) $y$- Majorana edge state, appears on the edge without the Zeeman field, but if we apply the Zeeman field in the $x$-, and $z$-directions, LDOS has a ZEP at $V_{x}/t = 1$ (figure 4(a)–(c)) [20, 21] which is quite different from the $s_{z}$-wave pairing case. When, we apply the Zeeman field in the $y$-direction, ZEP is not seen at all (figure 4(b)). These features are essentially the same with those in conventional $s$-wave superconductor / 1D Rashba semiconductor hybrid systems. For both $s_{z}$ and $s_{++}$ cases, LDOS is sensitive to the direction of the Zeeman field. In the presence of the sufficient large magnitude of $V_{x}$ ($i = x, y, z$), LDOS has a ZES for $i = x$ and $i = z$ but not for $i = y$.

3.2. 2D Rashba semiconductor

ARLDOS of the 2D Rashba semiconductor on iron-based superconductor is shown in figures 5–10 for both $s_{z}$ and $s_{++}$-wave pairing cases. The resulting LDOS of the $s_{z}$-wave and $s_{++}$-wave are shown in figures 11 and 12, respectively. Without the Zeeman field, helical edge modes crossing at $k_{y} = 0$ appear in the $s_{z}$-wave pairing (figures 5(a), 6(a) and 7(a), while they do not in the $s_{++}$-wave case (figures 8(a), 9(a) and 10(a)). The resulting angular averaged LDOS $\rho^{2D}(E)$ at $E = 0$ has a nonzero value for the $s_{z}$-wave pairing.
If we apply the Zeeman field, ARLDOS changes seriously. In the case of the $s_+^-$-wave pairing, ARLDOS is very sensitive to the direction of the Zeeman field. When the applied Zeeman field is along the $x$-direction, the helical edge mode disappears and ARLDOS at $\kappa_y = k_y = 0$ with $V_x = V_x^t$ (solid line), $V_y = 0.5$ (dashed line), $V_y = 2$ (dot-dashed line). ARLDOS is normalized by its value in the normal state at $E = 0$, and the energy is normalized by $\Delta_M = \Delta_0 + 2\Delta_1$.

Figure 4. (a)–(c) Intensity plot of LDOS of a 1D Rashba semiconductor nanowire on $s_+^+$-wave superconductor with the Zeeman field in (a) $x$-, (b) $y$-, and (c) $z$-direction. (d) Normalized LDOS of a Rashba semiconductor nanowire on $s_-$-wave superconductor under the Zeeman field in the $x$-direction, for $V_y/t = 0$ (solid line), $V_y/t = 0.5$ (dashed line), $V_y/t = 2$ (dot-dashed line). LDOS is normalized by its value in the normal state at $E = 0$, and the energy is normalized by $\Delta_M = \Delta_0 + 2\Delta_1$.

Figure 5. ARLDOS of a 2D Rashba semiconductor layer in the case of $s_+^-$-wave pairing under the Zeeman field in the $x$-direction for $V_y/t = (a) 0$, (b) 0.5, (c) 1 and (d) 2. ARLDOS is normalized by its value in the normal state at $k_y = 0$ and $E = 0$. Energy is normalized by $\Delta_M = \Delta_0 + 4\Delta_1$.

Figure 6. ARLDOS of a 2D Rashba semiconductor layer in the case of $s_+^-$-wave pairing under the Zeeman field in the $y$-direction for $V_x/t = (a) 0$, (b) 0.5, (c) 1 and (d) 2. ARLDOS is normalized by its value in the normal state at $k_y = 0$ and $E = 0$. Energy is normalized by $\Delta_M = \Delta_0 + 4\Delta_1$.

Next, we focus on the $s_+^-$-wave pairing case where a chiral Majorana edge mode is generated for the Zeeman field along the $x$- and $z$-directions. In this case, the Majorana edge
mode appears only after the applied Zeeman field exceeds a critical value $[18-21]$. When the applied field is along the $x$-direction, although the edge mode appears, it is very near the continuum levels (figure 8(d)) $[75]$. The line shape of the resulting LDOS does not have a sufficient change as a function of $E$ shown in (dot-dashed line in figure 12(a)). On the other hand, when the Zeeman field is applied in the $y$-direction, the bulk energy gap closes without any generation of the edge mode shown in figure 9. For the $z$-direction case, two chiral edge modes are generated at $k_y = 0$ and $k_y = \pi$ (figure 10(d)) and then one of the chiral edge mode vanishes (figure 10(e)). These features are similar to those in the $s_\pm$ -wave pairing case (figure 7).

To summarize, the resulting ARLDOS and LDOS are sensitive to the direction of the applied Zeeman field for both the $s_\pm$ -wave and $s_\pm$ -wave pairings. The difference between the $s_\pm$ -wave and $s_\pm$ -wave pairings becomes clear when the applied Zeeman field is along the $z$-direction with $0 < V_z/t < 1$ as seen from figures 7 and 10.

4. Symmetry of the Hamiltonian and topological invariants

In this section, we elucidate that the anisotropic response of LDOS and LDOS to the Zeeman field stems from the
mirror reflection symmetry in 1D by introducing a winding number.

4.1. 1D case

In the 1D case, the Hamiltonian belongs to the DIII class in the periodic table [38] and has time-reversal symmetry and particle–hole symmetries. In addition to these two, the system has a mirror reflection symmetry.

These symmetries are given as follows:

(i) time-reversal symmetry

$$\Theta^\dagger H_0^{1D}(k_x)\Theta = (H_0^{1D}(-k_x))^*, \quad \Theta = -i\sigma_z\tau_0, \quad (19)$$

(ii) particle-hole symmetry

$$C^\dagger H_0^{1D}(k_x)C = -(H_0^{1D}(-k_x))^*, \quad C = \sigma_0\tau_x, \quad (20)$$

Figure 10. ARLDOS of a 2D Rashba semiconductor layer in the case of $s_+-$wave pairing under the Zeeman field in the z-direction for $V_z/t =$ (a) 0, (b) 0.5, (c) 1, (d) 2 and (e) 3. ARLDOS is normalized by its value in the normal state at $k_x = 0$ and $E = 0$. Energy is normalized by $\Delta_M = \Delta_0 + 4\Delta_1$.

Figure 11. (a)–(c) Normalized LDOS of a 2D Rashba semiconductor layer on $s_+-$wave superconductor with the Zeeman field $V_z/t = 0$ (solid line), $V_z/t = 0.5$ (dashed line), and $V_z/t = 2$ (dot-dashed line) in (a) $x$-direction, (b) $y$-direction, and (c) $z$-direction respectively. LDOS is normalized by its values in the normal state at $E = 0$.

Figure 12. (a)–(c) Normalized LDOS of a 2D Rashba semiconductor layer on $s_+-$wave superconductor with the Zeeman field $V_z/t = 0$ (solid line), $V_z/t = 0.5$ (dashed line), and $V_z/t = 2$ (dot-dashed line) in (a) $x$-direction, (b) $y$-direction, and (c) $z$-direction, and respectively. LDOS is normalized by its values in the normal state at $E = 0$. 
(iii) mirror reflection symmetry (mirror plane is xz-plane)

\[ M_{xz}^i H_0^D(k_x) M_{xz} = H_0^D(k_x), \quad M_{xz} = i\sigma_y \tau_0. \]  

Combining three symmetries together, we can define operator \( \Gamma^{1D} = M_{xz} \Theta \mathcal{C} \) which anti-commutes with \( H_0^{1D}(k_x) \).

\[ \left\{ \Gamma^{1D}, H_0^{1D}(k_x) \right\} = 0. \]

Now, we consider the effect of the Zeeman field term \( \tilde{V}_z \) in equation (4). When the Zeeman field is applied in the x- and z-directions, \( \tilde{V}_z \) anti-commutes with \( \Gamma^{1D} \).

\[ \left\{ \Gamma^{1D}, H_0^{1D}(k_x) \right\} = 0, \quad i = x, z. \]

However, \( \tilde{V}_z \) does not anti-commute with \( \Gamma^{1D} \) when the direction of the Zeeman field is along the y-direction. If we find an operator which anti-commutes with the Hamiltonian, i.e. the system has a chiral symmetry [38], we can define the winding number by the following procedures [43, 76, 77]. First, we diagonalize \( \Gamma^{1D} \) by the unitary matrix \( U_1 \)

\[ U_1^\dagger \Gamma^{1D} U_1 = \begin{pmatrix} I_{2\times2} & 0 \\ 0 & -I_{2\times2} \end{pmatrix}. \]

Using this \( U_1 \), the Hamiltonian is transformed as

\[ U_1^\dagger H_1(k_x) U_1 = \begin{pmatrix} 0 & q(k_x) \\ q(k_x)^\dagger & 0 \end{pmatrix}, \]

with

\[ q(k_x) = (2i \cos k_x - \mu)\sigma_0 + 2i \sin k_x \sigma_y \\
- i(\Delta_0 + 2\Delta_1 \cos k_x) \sigma_z. \]

The winding number \( w_{1d} \) is defined as

\[ w_{1d} = \frac{1}{2\pi i} \oint D^{-1} dD, \quad D = \text{det} q(k_x), \]

where \( \oint \) represents the line integral over the closed loop in the 1D Brillouin zone [76]. This quantity means how many times the trajectory of the \( \text{det} q(k_x) = m_1 + im_2 \) wraps around the origin in complex space, where \( m_1 \) and \( m_2 \) are real quantities. The number \( w_{1d} \) is a topological invariant, which keeps its value unless the energy gap of the Hamiltonian is closed. If \( w_{1d} \) has a nonzero value due to the bulk-edge correspondence, there are ZESs [76].

As an example, we plot trajectories of \( \text{det} q(k_x) \) in complex \( m_1 - m_2 \) plane and the energy dispersion of \( H_0^{1D}(k_x) \) in the case of \( \xi_z \)-wave pairing without the Zeeman field (figures 13(a) and (b)) and with it (figures 13(c)–(f)). We can easily find \( w_{1d} = 2 \) without the Zeeman field as shown in figure 13(a). \( w_{1d} \) remains as 2 up to \( V_z = V_{c1} (V_{c1}/t \sim 1.02) \) which is the critical value of the bulk energy gap closure. At \( V_z = V_{c1} \), the trajectory just crosses \( m_1, m_2 = (0, 0) \) (figure 13(c)) and the bulk energy gap closes at \( k_x = 0 \) (figure 13(d)). If \( V_z \) exceeds \( V_{c1} \), the resulting \( w_{1d} \) becomes 1. At \( V_z = V_{c2} \), with \( V_{c2}/t \sim 3.06 \), the trajectory crosses \( m_1, m_2 = (0, 0) \) (figure 13(e)) and the bulk energy gap closes at \( k_x = \pm \pi \) (figure 13(f)). Above \( V_{c2} \), \( w_{1d} \) becomes zero. These properties are consistent with the fact that the topological invariant \( w_{1d} \) changes when the bulk energy gap is closed. We also calculate \( w_{1d} \) under the Zeeman field in the \( z \)-direction for the \( \xi_z \)-wave pairing. The change of \( w_{1d} \) and critical values of \( V_{c1} \) and \( V_{c2} \) are the same as the in case with the Zeeman field in the \( x \)-direction. As a reference, we obtain \( w_{1d} \) for the \( \xi_{+z} \)-wave pairing with the Zeeman field both in \( x \)- and \( z \)-directions. In these directions, \( w_{1d} \) changes 0, 1, 0 with the increase of the Zeeman field. The critical values \( V_{c1} \) and \( V_{c2} \) are given as \( V_{c1} \sim 1.02 \) and \( V_{c2} \sim 3.00 \), respectively. The above results are shown in table 1. The arrow in the table 1 shows the change of the topological numbers with the increase of the Zeeman field.

From this table, it is clear that the difference between \( \xi_z \)- and \( \xi_{+z} \)-wave pairings is whether the state with \( w_{1d} = 2 \) is realized or not at the lower Zeeman field in the \( x \)- and \( z \)-directions. This point is also clearly seen in figures 4 and 5. Up to \( V_z = V_{c1} \), ZES and resulting ZEP appear in the LDOS on the edge only for the \( \xi_z \)-wave pairing. The response of the
1D case

The Zeeman field in the z-direction also has this combined symmetry. In the presence of this symmetry, we can define the topological number and understand the origin of the edge state in the context of the bulk-edge correspondence.

In addition to the above discussions, we can provide another explanation about this anisotropic response to the Zeeman field using the index theorem. We focus on the zero energy edge state and analyze it using the eigenstate of the time-reversal symmetry and the mirror reflection symmetry.

\[ \left( \mathcal{M}_z \Theta \right)^\dagger V_\sigma \sigma_x \tau_x, \mathcal{M}_z = V_\sigma \sigma_x \tau_x. \]

The Zeeman field in the z-direction also has this combined symmetry. In the presence of this symmetry, we can define the topological number and understand the origin of the edge state in the context of the bulk-edge correspondence.

In addition to the above discussions, we can provide another explanation about this anisotropic response to the Zeeman field using the index theorem. We focus on the zero energy edge state and analyze it using the eigenstate of the time-reversal symmetry and the mirror reflection symmetry.

\[ \left( \mathcal{M}_z \Theta \right)^\dagger V_\sigma \sigma_x \tau_x, \mathcal{M}_z = V_\sigma \sigma_x \tau_x. \]

The Zeeman field in the z-direction also has this combined symmetry. In the presence of this symmetry, we can define the topological number and understand the origin of the edge state in the context of the bulk-edge correspondence.

In addition to the above discussions, we can provide another explanation about this anisotropic response to the Zeeman field using the index theorem. We focus on the zero energy edge state and analyze it using the eigenstate of the time-reversal symmetry and the mirror reflection symmetry.

\[ \left( \mathcal{M}_z \Theta \right)^\dagger V_\sigma \sigma_x \tau_x, \mathcal{M}_z = V_\sigma \sigma_x \tau_x. \]

The Zeeman field in the z-direction also has this combined symmetry. In the presence of this symmetry, we can define the topological number and understand the origin of the edge state in the context of the bulk-edge correspondence.

In addition to the above discussions, we can provide another explanation about this anisotropic response to the Zeeman field using the index theorem. We focus on the zero energy edge state and analyze it using the eigenstate of the time-reversal symmetry and the mirror reflection symmetry.

\[ \left( \mathcal{M}_z \Theta \right)^\dagger V_\sigma \sigma_x \tau_x, \mathcal{M}_z = V_\sigma \sigma_x \tau_x. \]

The Zeeman field in the z-direction also has this combined symmetry. In the presence of this symmetry, we can define the topological number and understand the origin of the edge state in the context of the bulk-edge correspondence.

In addition to the above discussions, we can provide another explanation about this anisotropic response to the Zeeman field using the index theorem. We focus on the zero energy edge state and analyze it using the eigenstate of the time-reversal symmetry and the mirror reflection symmetry.

\[ \left( \mathcal{M}_z \Theta \right)^\dagger V_\sigma \sigma_x \tau_x, \mathcal{M}_z = V_\sigma \sigma_x \tau_x. \]

The Zeeman field in the z-direction also has this combined symmetry. In the presence of this symmetry, we can define the topological number and understand the origin of the edge state in the context of the bulk-edge correspondence.

In addition to the above discussions, we can provide another explanation about this anisotropic response to the Zeeman field using the index theorem. We focus on the zero energy edge state and analyze it using the eigenstate of the time-reversal symmetry and the mirror reflection symmetry.

\[ \left( \mathcal{M}_z \Theta \right)^\dagger V_\sigma \sigma_x \tau_x, \mathcal{M}_z = V_\sigma \sigma_x \tau_x. \]

The Zeeman field in the z-direction also has this combined symmetry. In the presence of this symmetry, we can define the topological number and understand the origin of the edge state in the context of the bulk-edge correspondence.
Table 3. Changes of \((w_1d(0), w_1d(\pi))\) with the Zeeman field in x-direction in 2D. The critical values of the Zeeman field where the winding number is changed are mentioned in supplementary data.

| Pairing | \((w_1d(0), w_1d(\pi))\) |
|---------|--------------------------|
| \(s_z\) | \((2, 0) \rightarrow (1, 1) \rightarrow (0, 1) \rightarrow (0, 0)\) |
| \(s_{++}\) | \((0, 0) \rightarrow (1, 1) \rightarrow (0, 1) \rightarrow (0, 0)\) |

Table 4. Changes of \((w_1d(0), w_1d(\pi))\) with the Zeeman field in z-direction in 2D. The critical values of the Zeeman field where the winding number is changed are mentioned in supplementary data.

| Pairing | \((w_1d(0), w_1d(\pi))\) |
|---------|--------------------------|
| \(s_z\) | \((2, 0) \rightarrow (1, 1) \rightarrow (0, 1) \rightarrow (0, 0)\) |
| \(s_{++}\) | \((0, 0) \rightarrow (1, 1) \rightarrow (0, 1) \rightarrow (0, 0)\) |

For \(s_z\)-wave pairing, there are three critical fields \(V_{c1}, V_{c2},\) and \(V_{c3}\), where the energy gap at \(k_y = 0\) or \(k_y = \pi\) closes between \((w_1d(0), w_1d(\pi)) = (2, 0)\) and \((1, 1), (1, 1)\) and \((0, 1), (0, 0)\), respectively. These are \(V_{c1} \sim 1.02,\) \(V_{c2} \sim 3.06\) and \(V_{c3} \sim 5.10\) for both the x-direction and z-direction cases. For the \(s_{++}\)-wave pairing, the corresponding \(V_{c1}, V_{c2},\) and \(V_{c3}\) are given by \(V_{c1} \sim 1.02,\) \(V_{c2} \sim 3.00\) and \(V_{c3} \sim 5.00\) for both the x-direction and z-direction cases. The \(w_1d\) changes \((0, 0), (1, 1), (0, 1)\) and \((0, 0)\) with the increase of Zeeman field. We can see that the results shown in table 3 (4) are consistent with the change of the number of the chiral edge modes at \(k_y = 0\) and \(k_y = \pi\) shown in figures 5 (7) and 8 (10). The difference between the \(s_z\)- and \(s_{++}\)-wave only appears at the lower Zeeman field whether the topological state with \((w_1d(0), w_1d(\pi)) = (2, 0)\) is realized or not. In the higher Zeeman field, since the number of the Fermi surface becomes one and the sign change of the pair potential no longer occurs. Thus, there is no essential difference between the \(s_z\)- and \(s_{++}\)-wave pairings.

5. Conclusion

In this paper, we have studied the Majorana edge states and their topological properties in one-dimensional (1D) and two-dimensional (2D) Rashba semiconductors deposited on iron-based superconductors under the applied Zeeman field for various directions. Using the recursive Green’s function method, we have calculated the LDOS and ARLDOS for both \(s_z\)- and \(s_{++}\)-wave pairings. We have discussed whether the Majorana edge state emerges or not based on the symmetry of the Hamiltonian. We have shown that Majorana edge states are protected by mirror reflection symmetry and therefore show an anisotropic response to the directions of the applied Zeeman field. The resulting LDOS and winding numbers for \(s_z\)- and \(s_{++}\)-wave pairings are essentially different at the lower Zeeman field. These results serve as a guide to determine the pairing symmetry of iron-pnictide. Finally, we make a brief comment on the effect of the disorder. In the present model calculation, we assume there is no mismatch of the lattice constant and wavelength between the pnictides and Rashba semiconductor. In a more realistic case, however, this mismatch results in the disorder which may affect the induced gap in the Rashba semiconductor. The disorder’s effect on charge transport has been studied in nanowire/superconductor junctions with conventional spin–singlet s-wave pairings [80]. This situation corresponds to the \(s_{++}\)-wave case in our model. It has been shown that the Majorana edge state clearly shows up even in the presence of the disorder and the disorder in the superconductor has little influence on the Majorana edge state. It is an interesting issue to study the effects of the disorder on the Majorana fermion realized in the nanowire /s\(s_z\)-wave pairing case. Although the induced gap may be reduced, we expect that the essence of the physics, i.e. the topological nature of the system, does not change even in the presence of the disorder.

Acknowledgments

We would like to thank M Sato for valuable discussions. This work was supported in part by a Grant-in-Aid for Scientific Research from MEXT of Japan, ‘Topological Quantum Phenomena’, Grants No. 22103005 and the EU–Japan program IRON-SEA.

References

[1] Read N and Green D 2000 Phys. Rev. B 61 10267
[2] Qi X-L and Zhang S-C 2011 Rev. Mod. Phys. 83 1057
[3] Alicea J 2012 Rep. Prog. Phys. 75 076501
[4] Tanaka Y, Sato M and Nagaosa N 2012 J. Phys. Soc. Jpn. 81 011013
[5] Wilczek F 2009 Nature Phys. 5 614
[6] Ivanov D A 2001 Phys. Rev. Lett. 86 268
[7] Nayak C, Simon S H, Stern A, Freedman M and das Sarma S 2008 Rev. Mod. Phys. 80 1083
[8] Kitaev A Y 2001 Uspekhi fizicheskikh nauk (Suppl.) 171 131
[9] Fu L and Kane C L 2008 Phys. Rev. Lett. 100 096407
[10] Fu L and Kane C L 2009 Phys. Rev. Lett. 102 216403
[11] Akhmerov A R, Nilsson J and Beenakker C W J 2009 Phys. Rev. Lett. 102 216409
[12] Tanaka Y, Yokoyama T and Nagaosa N 2009 Phys. Rev. Lett. 103 107002
[13] Linder J, Tanaka Y, Yokoyama T, Sudbo A and Nagaosa N 2010 Phys. Rev. Lett. 104 067001
[14] Veldhorst M et al 2012 Nat. Mat. 11 417
[15] Williams J R, Bestwick A J, Gallagher P, Hong S S, Cui Y, Bleich A S, Analytis J G, Fisher I R and Goldhaber-Gordon D 2012 Phys. Rev. Lett. 109 056803
[16] Sato M and Fujimoto S 2009 Phys. Rev. B 79 094504
[17] Sato M, Takahashi Y and Fujimoto S 2009 Phys. Rev. Lett. 103 020401
[18] Sau J D, Lutchyn R M, Tewari S and Das Sarma S 2010 Phys. Rev. Lett. 104 040502
[19] Alicea J 2010 Phys. Rev. B 81 125318
[20] Oreg Y, Refael G and von Oppen F 2010 Phys. Rev. Lett. 105 177002
[21] Lutchyn R M, Sau J D and Das Sarma S 2010 Phys. Rev. Lett. 105 077001
[22] Alicea J, Oreg Y, Refael G, von Oppen F and Fisher M 2011 Nat. Phys. 7 412
