Super-horizon perturbations and preheating

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It has recently been claimed by Bassett et al. that preheating after inflation may affect the amplitude of curvature perturbations on large scales, undermining the usual inflationary prediction. We analyze the simplest model, and confirm the results of Jedamzik and Sigl and of Ivanov that in linear perturbation theory the effect is negligible. However the dominant effect is second-order in the field perturbation and we show that this too is negligible, and hence conclude that preheating has no significant influence on large-scale perturbations in this model. We briefly discuss the likelihood of an effect in other models.

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I. INTRODUCTION

The standard inflationary paradigm is an extremely successful model in explaining observed structures in the Universe (see Refs. 1-2 for reviews). The inhomogeneities originate from the quantum fluctuations of the inflaton field, which on being stretched to large scales become classical perturbations. The field inhomogeneities generate a perturbation in the curvature of comoving hypersurfaces, and later on these inhomogeneities are inherited by matter and radiation when the inflaton field decays. In the simplest scenario, the curvature perturbation on scales much larger than the Hubble length is constant, and in particular is unchanged during the inflaton decay. This enables a prediction of the present-day perturbations which does not depend on the specific cosmological evolution between the late stages of inflation and the recent past (say, before nucleosynthesis).

It has recently been claimed 3 that this simple picture may be violated if inflation ends with a period of preheating, a violent decay of the inflaton particles into another field (or even into quanta of the inflaton field itself). Such a phenomenon would completely undermine the usual inflationary picture, and indeed the original claim was that large-scale perturbations would be amplified into the non-linear regime, placing them in conflict with observations such as measurements of microwave background anisotropies. Given the observational successes of the standard picture, these claims demand attention.

In a companion paper 4, we discuss the general criteria under which large-scale curvature perturbations can vary. As has been known for some time, this is possible provided there exist large-scale non-adiabatic pressure perturbations, as can happen for example in multi-field inflation models. 5-7 Under those circumstances a significant effect is possible during preheating, though there is nothing special about the preheating era in this respect and this effect always needs to be considered in any multi-component inflation model.

In this paper we perform an analysis of the simplest preheating model, as discussed in Ref. 3. We identify two possible sources of variation of the curvature perturbation. One comes from large-scale isocurvature perturbations in the preheating field into which the inflaton decays; we concur with the recent analyses of Jedamzik and Sigl 8 and Ivanov 9 that this effect is negligible due to the rapid decay of the background value of the preheating field during inflation. However, we also show that in fact a different mechanism gives the dominant contribution, which is second-order in the field perturbations coming from short-wavelength fluctuations in the fields. Nevertheless, we show too that this effect is completely negligible, and hence that preheating in this model has no significant effect on large-scale curvature perturbations.

II. PERTURBATION EVOLUTION

An adiabatic perturbation is one for which all perturbations $\delta x$ share a common value for $\delta x/\dot{x}$, where $\dot{x}$ is the time dependence of the background value of $x$. If the Universe is dominated by a single fluid with a definite equation of state, or by a single scalar field whose perturbations start in the vacuum state, then only adiabatic perturbations can be supported. If there is more than one fluid, then the adiabatic condition is a special case, but for instance is preserved if a single inflaton field subsequently decays into several components. However, perturbations in a second field, for instance the one into which the inflaton decays during preheating, typically violate the adiabatic condition.

We describe the perturbations via the curvature per-
turbation on uniform-density hypersurfaces, denoted $\zeta$. In linear theory the evolution of $\zeta$ is well known, and arises from the non-adiabatic part of the pressure perturbations. In any gauge, the pressure perturbation can be split into adiabatic and entropic (non-adiabatic) parts, by writing

$$\delta p = c_s^2 \delta \rho + \delta p_{\text{nad}}, \quad (1)$$

where $c_s^2 \equiv \dot{\rho}/\rho$ and the non-adiabatic part is

$$\delta p_{\text{nad}} \equiv \dot{\rho} \Gamma \equiv \dot{\rho} \left( \frac{\delta \rho}{\rho} - \frac{\delta \rho}{\dot{\rho}} \right). \quad (2)$$

The entropy perturbation $\Gamma$, defined in this way, is gauge-invariant, and represents the displacement between hypersurfaces of uniform pressure and uniform density.

On large scales anisotropic stress can be ignored when the matter content is entirely in the form of scalar fields, and in its absence the non-adiabatic pressure perturbation determines the variation of $\zeta$, according to the equation \[3\]

$$\frac{d\zeta}{dN} = -3Hc_s^2 \Gamma, \quad (3)$$

where $N \equiv \ln a$ measures the integrated expansion and $H$ is the Hubble parameter. The uniform-density hypersurfaces become ill-defined if the density is not a strictly decreasing function along worldlines between hypersurfaces of uniform density, and one might worry that this undermines the above analysis. However we can equally well derive this evolution equation in terms of the density perturbation on spatially-flat hypersurfaces, $\delta \rho_{s} \equiv - (d\rho/dN) \zeta$, which remains well-defined. Spatially-flat hypersurfaces are automatically separated by a uniform pressure perturbation at large scales, so the perturbed continuity equation in this gauge takes the particularly simple form

$$\frac{d\delta \rho_{s}}{dN} = -3 (\delta \rho_{s} + \delta p_{s}). \quad (4)$$

From this one finds that $\delta \rho_{s} \propto d\rho/dN$ for adiabatic perturbations and hence again we recover constant value for $\zeta$. However it is clearly possible for entropy perturbations to cause a change in $\zeta$ on arbitrarily large scales when the non-adiabatic pressure perturbation is non-negligible.

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- This is the notation of Bardeen, Steinhardt and Turner \[1\]. General issues of perturbation description and evolution are discussed in a companion paper \[1\]. The curvature perturbation of comoving spatial hypersurfaces, usually denoted by $\mathcal{R}$ \[1\], is practically the same as $\zeta$ well outside the horizon, since the two coincide in the large-scale limit.

### III. PREHEATING

During inflation, the reheat field into which the inflaton field decays possesses quantum fluctuations on small scales just like the inflaton field itself. As these perturbations are uncorrelated with those in the inflaton field, the adiabatic condition will not be satisfied, and hence there is a possibility that $\zeta$ might vary on large scales. Only direct calculation can demonstrate whether the effect might be significant, and we now compute this effect in the simplest preheating model, as analyzed in Ref. \[3\]. This is a chaotic inflation model with scalar field potential

$$V(\phi, \chi) = \frac{1}{2} m^2 \phi^2 + \frac{1}{2} g^2 \phi^2 \chi^2, \quad (5)$$

where $\phi$ is the inflaton and $\chi$ the reheat field. Slow-roll inflation proceeds with $\phi \simeq m_{\text{Pl}}$ and $g \phi \ll m$. The effective mass of the $\chi$ field is $g \phi$ and thus will be much larger than the Hubble rate, $H \simeq \sqrt{4\pi/3 m_{\text{Pl}}} g \phi$, for $g \gg m/m_{\text{Pl}} \sim 10^{-6}$. Throughout, we use the symbol `$\simeq$' to indicate equality within the slow-roll approximation.

This model gives efficient preheating, since the effective mass of $\chi$ oscillates about zero with large amplitude. In most other models of inflation, preheating is less efficient or absent, because the mass oscillates about a nonzero value and/or has a small amplitude. Any variation of $\zeta$ during preheating will be driven by the (non-adiabatic part of) the $\chi$ field perturbation. Our calculation takes place in three steps. The first is to compute the perturbations in the $\chi$ field at the end of inflation. The second is to compute how these perturbations are amplified during the preheating epoch by the strong resonance. Finally, the main part of the calculation is to compute the change in $\zeta$ driven by these $\chi$ perturbations.

#### A. The initial quantum fluctuation of the $\chi$-field

Perturbations in the $\chi$ field obey the wave equation

$$\ddot{\chi} + 3H \dot{\chi} + \left( \frac{k^2}{a^2} + g^2 \phi^2 \right) \chi = 0. \quad (6)$$

The slow-roll conditions ensure that the $\chi$ field remains in the adiabatic vacuum state for a massive field

$$\delta \chi_k \simeq \frac{e^{-i\omega t}}{\sqrt{2\omega}}, \quad (7)$$

where $\omega^2 = k^2/a^2 + g^2 \phi^2$. This is a solution provided

$$\nu \equiv \frac{m_{\chi}}{H} \simeq \sqrt{\frac{3g m_{\text{Pl}}}{4\pi m}} \gg 1, \quad (8)$$

where $m_{\chi} \equiv g \phi$ is the effective mass of the $\chi$ field.
The power spectrum of a quantity $x$, decomposed into Fourier components $x_k$, is defined as
$$\mathcal{P}_x = \frac{k^3}{2\pi^2} \langle |x_k|^2 \rangle,$$  \hfill (9)
where $k = |k|$ and the average is over ensembles. Hence the power spectrum for long-wavelength fluctuations ($k \ll m_\chi$) in the $\chi$ field simply reduces to the result for a massive field in flat space
$$\mathcal{P}_\delta \chi \simeq \frac{1}{4\pi^2 m_\chi} \left(\frac{k}{a}\right)^3,$$  \hfill (10)
where $m_\chi$ is the mass of the field at the required time. Physically, this says that at all times the expansion of the Universe has a negligible effect on the modes as compared to the mass. In particular, at the end of inflation we can write
$$\left.\mathcal{P}_\delta \chi\right|_{\text{end}} \simeq \frac{1}{\nu} \left(\frac{H_{\text{end}}}{2\pi}\right)^2 \left(\frac{k}{k_{\text{end}}}\right)^3.$$  \hfill (11)
The power spectrum has a spectral index $n_\delta \chi = 3$. This is the extreme limit of the mechanism used to give a blue tilt in isocurvature inflation scenarios [11].

### B. Parametric resonance

After inflation, the inflaton field $\phi$ oscillates. Strong parametric resonance may now occur, amplifying the initial quantum fluctuation in $\chi$ to become a perturbation of the classical field $\chi$. The condition for this is
$$q \equiv \frac{g^2 m^2}{4m^2} \gg 1,$$  \hfill (12)
where $\Phi$ is the initial amplitude of the $\phi$-field oscillations.

We model the effect of preheating on the amplitude of the $\chi$ field following Ref. [12] as
$$\mathcal{P}_\delta \chi = \mathcal{P}_\delta \chi_{\text{end}} e^{2\mu_k m \Delta t},$$  \hfill (13)
and the Floquet index $\mu_k$ is taken as
$$\mu_k \simeq \frac{1}{2\pi} \ln \left(1 + 2 e^{-\kappa^2}\right),$$  \hfill (14)
with
$$\kappa^2 \equiv \left(\frac{k}{k_{\text{max}}}\right)^2 \equiv \frac{1}{18q} \left(\frac{k}{k_{\text{end}}}\right)^2.$$  \hfill (15)
For strong coupling ($q \gg 1$), we have $\kappa^2 \ll 1$ for all modes outside the Hubble scale after inflation ends ($k \leq k_{\text{end}}$). Therefore $\mu_k \approx \ln 3/2\pi \approx 0.17$ is only very weakly dependent on the wavenumber $k$. Combining Eqs. (13) and (15) gives
$$\mathcal{P}_\delta \chi \simeq \left(\frac{H_{\text{end}}}{2\pi}\right)^2 \left(\frac{k}{k_{\text{end}}}\right)^3 \delta \chi_{\text{end}} e^{2\mu_k m \Delta t}.$$  \hfill (16)

### C. Change in the curvature perturbation on large scales

In order to quantify the effect parametric growth of the $\chi$ field fluctuations during preheating might have upon the standard predictions for the spectrum of density perturbations after inflation, we need to estimate the change in the curvature perturbation $\zeta$ on super-horizon scales due to entropy perturbations on large-scales.

The density and pressure perturbations due to first-order perturbations in the inflaton field on large scales (i.e. neglecting spatial gradient terms) are of order $g^2 \phi^2 \delta \chi^2$. Not only are the field perturbations $\delta \chi$ strongly suppressed on large scales at the end of inflation [as shown in our Eq. (14)] but so is the background field $\chi$. We can place an upper bound on the size of the background field by noting that in order to have slow-roll chaotic inflation (dominated by the $m^2 \phi^2/2$ potential) when any given mode $k$ which we are interested in crossed outside the horizon, we require $\chi \ll m/g$. The large effective mass causes this background field to decay, just like the super-horizon perturbations, and at the end of inflation we require $\chi \ll m/g(k/k_{\text{end}})^{3/2}$ when considering preheating in single-field chaotic inflation. Combining this with Eq. (14) we find that the spectrum of density or pressure perturbations due linear perturbations in the $\chi$ field has an enormous suppression for $k \ll k_{\text{end}}$:
$$\mathcal{P}_\chi \delta \chi_{\text{end}} \ll \sqrt{\frac{4\pi}{3}} \left(\frac{m}{g m_{\text{Pl}}}\right)^3 \left(\frac{m_{\text{Pl}} H_{\text{end}}}{2\pi}\right)^2 \left(\frac{k}{k_{\text{end}}}\right)^6.$$  \hfill (17)
Effectively the density and pressure perturbations have no term linear in $\delta \chi$, because that term is multiplied by the background field value which is vanishingly small.

By contrast the second-order pressure perturbation is of order $g^2 \phi^2 \delta \chi^2$ where the power spectrum of $\delta \chi^2$ is given by [13]
$$\mathcal{P}_\delta \chi^2 \simeq \frac{k^3}{2\pi} \int_0^{k_{\text{cut}}} \mathcal{P}_\delta \chi(k') \left|\frac{\delta \chi(k - k')}{k^3}\right|^2 d^3k'.$$  \hfill (18)
We impose the upper limit $k_{\text{cut}} \sim k_{\text{max}}$ to eliminate the ultraviolet divergence associated with vacuum state. Substituting in for $\mathcal{P}_\delta \chi$ from Eq. (12), we can write
$$\left.\mathcal{P}_\delta \chi^2\right|_{\text{end}} = \frac{8\pi}{9} \left(\frac{m}{g m_{\text{Pl}}}\right)^2 \left(\frac{H_{\text{end}}}{2\pi}\right)^4 \left(\frac{k_{\text{cut}}}{k_{\text{end}}}\right)^3 \left(\frac{k}{k_{\text{end}}}\right)^3,$$  \hfill (19)
Noting that $H_{\text{end}} \sim m$ and $k_{\text{cut}} \sim k_{\text{max}} \sim q^{1/4} k_{\text{end}}$, it is evident that the second-order effect will dominate over the linear term for $k < g^{1/2} q^{1/4} k_{\text{end}}$.

The leading-order contributions to the pressure and density perturbations on large scales are thus
\[ \delta \rho = m^2 \phi \delta \phi + \dot{\phi} \delta \phi + \frac{1}{2} g^2 \phi^2 \dot{\delta \chi}^2 + \frac{1}{2} \dot{\chi}^2, \]
\[ \delta p = -m^2 \phi \delta \phi + \dot{\phi} \delta \phi - \frac{1}{2} g^2 \phi^2 \dot{\delta \chi}^2 + \frac{1}{2} \dot{\chi}^2. \]

We stress that we will still only consider first-order perturbations in the metric and total density and pressure, but these include terms to second-order in \( \delta \chi \). From Eqs. (2), (20) and (21) we obtain
\[ \delta \rho_{nad} = \frac{-m^2 \phi \delta \chi^2 + \dot{\phi} g^2 \phi^2 \delta \chi^2}{3H \phi}, \]
where the long-wavelength solutions for vacuum fluctuations in the field obey the adiabatic condition \( \delta \dot{\phi} / \dot{\phi} = \delta \phi / \phi \). Inserted into Eq. (13), this gives the rate of change of \( \zeta \).

Note that the non-adiabatic pressure will diverge periodically when \( \dot{\phi} = 0 \) as the comoving or uniform density hypersurfaces become ill-defined. Such a phenomenon was noted in the single-field context by Finelli and Brandenberger [14], who evaded it by instead using Mukhanov’s variable \( u = a \delta \phi \), which renders well-behaved equations. Linear perturbation theory remains valid as there are choices of hypersurface, such as the spatially-flat hypersurfaces, on which the total pressure perturbation remains finite and small. In particular, we can calculate the change in the density perturbation due to the non-adiabatic part of the pressure perturbation on spatially-flat hypersurfaces from Eq. (13), which yields
\[ \Delta \rho_{nad} = -3 \int \delta \rho_{nad} H dt. \]
Even though \( \delta \rho_{nad} \) contains poles whenever \( \dot{\phi} = 0 \), the integrated effect remains finite whenever the upper and lower limits of the integral are at \( \phi \neq 0 \). From this density perturbation calculated in the spatially-flat gauge one can reconstruct the change in the curvature perturbation on uniform density hypersurfaces
\[ \Delta \zeta = -H \frac{\Delta \rho_{nad}}{\rho}. \]
Substituting in our expression for \( \delta \rho_{nad} \) we obtain
\[ \Delta \zeta = \frac{1}{\phi^2} \int \left( 1 + \frac{2m^2 \phi^2}{3H \phi} \right) g^2 \phi^2 |\delta \chi^2| H dt, \]
where we have averaged over short timescale oscillations of the \( \chi \)-field fluctuations to write \( |\delta \chi^2| = g^2 \phi^2 |\delta \chi^2| \).
To evaluate this we take the usual adiabatic evolution for the background \( \phi \) field after the end of inflation
\[ \phi = \Phi \frac{\sin(m \Delta t)}{m \Delta t}, \]
and time-averaged Hubble expansion
\[ H = \frac{2m}{3(m \Delta t + \Theta)}, \]
where \( \Theta \) is an integration constant of order unity. The amplitude of the \( \chi \)-field fluctuations also decays proportional to \( 1 / \Delta t \) over a half-oscillation from \( m \Delta t = n \pi \) to \( m \Delta t = (n+1) \pi \), with the stochastic growth in particle number occurring only when \( \phi = 0 \). Thus evaluating \( \Delta \zeta \) over a half-oscillation \( \Delta t = \pi / m \) we can write
\[ \Delta \zeta = \frac{2g^2 |\delta \chi^2| a_n^4}{3m^2} \int_{x_n}^{x_{n+1}} \left( 1 + \Theta + \frac{s}{s'} \right) \frac{s^2}{x^2} dx, \]
where \( x = m \Delta t, s(x) = \sin x / x, x_n = n \pi \) and a dash indicates differentiation with respect to \( x \). The integral is dominated by the second term in the bracket which has a pole of order 3 when \( s' = 0 \). Although \( s / s' \) diverges, it yields a finite contribution to the integral which can be evaluated numerically. For \( x_n \gg 1 \) the integral is very well approximated by \( 24 / x_n^2 \), independent of the integration constant \( \Theta \).

This expression gives us the rate of change of the curvature perturbation \( \zeta \) due to the pressure of the field fluctuations \( \delta \chi^2 \) over each half-oscillation of the inflaton \( \phi \). Approximating the sum over several oscillations as a smooth integral and using Eq. (13) for the growth of the \( \chi \)-field fluctuations during preheating (neglecting the weak \( k \)-dependence of the Floquet index, \( \mu_k \), on super-horizon scales) we obtain
\[ \zeta_{nad} = \frac{16g^2}{2\pi \mu_s} \frac{|\delta \chi^2|_{end}}{m^2} \frac{e^{2\mu_m \Delta t}}{s^2}. \]

The statistics of these second-order fluctuations are non-Gaussian, being a \( \chi^2 \)-distribution. Both the mean and the variance of \( \zeta_{nad} \) are non-vanishing. The mean value will not contribute to density fluctuations, but rather indicates that the background we are expanding around is unstable as energy is systematically drained from the inflaton field. We are interested in the variance of the curvature perturbation, and in particular the change of the curvature perturbation power spectrum on super-horizon scales which is negligible if the power spectrum of \( \zeta_{nad} \) on those scales is much less than that of \( \zeta \) generated during inflation, the latter being required to be of order \( 10^{-10} \) to explain the COBE observations.

To evaluate the power spectrum for \( \zeta_{nad} \) we must evaluate the power spectrum of \( \delta \chi^2 \) which is given by substituting \( P_{\delta \chi} \), from Eq. (10), into Eq. (15). This gives
\[ P_{\delta \chi^2} = \frac{2}{3v^2} \left( \frac{H_{end}}{2 \pi} \right)^4 \left( \frac{k_{max}}{k_{end}} \right)^3 \left( \frac{k}{k_{end}} \right)^3 I(\kappa, m \Delta t), \]
where
\[ I(\kappa, m \Delta t) = \frac{3}{2} \int_0^{\kappa_{end}} \frac{d \kappa'}{\kappa'} \int_0^{\pi} d \theta e^{2(\mu_{\kappa'} + \mu_{\kappa' \kappa})m \Delta t} \kappa' \sin \theta, \]
\[ \kappa = k / k_{\text{max}} \] as defined in Eq. (15), and \( \theta \) is the angle between \( \mathbf{k} \) and \( \mathbf{k}' \). Note that at the end of inflation we have \( I(\kappa, 0) = \kappa_{\text{cut}}^3 \sim 1 \), and \( \mathcal{P}_{\delta \chi} \propto k^3 \). This yields

\[ \mathcal{P}_{\zeta_{\text{nad}}} \approx \frac{2^{9/2} 3}{\pi^2 \mu^2} \left( \Phi \left( \frac{H_{\text{end}}}{m} \right) \right)^2 \left( \frac{g^4}{k_{\text{end}}} \right)^3 I. \]  

(32)

One might have thought that the dominant contribution to \( \zeta_{\text{nad}} \) on large scales would come from \( \delta \chi \) fluctuations on those scales, and that is indeed the presumption of the calculation of Bassett et al. However, in fact the integral is initially dominated by \( k' \sim k_{\text{cut}} \), namely the shortest scales. The reason for this is the steep slope of \( \mathcal{P}_{\delta \chi} \); were it much shallower (spectral index less than 3/2), then the dominant contribution would come from large scales.

To study the scale dependence of \( I(\kappa, m \Delta t) \) and hence \( \mathcal{P}_{\zeta_{\text{nad}}} \) at later times, we can expand \( \mu_{\kappa - \kappa'} \) for \( \kappa \kappa' \ll 1 \) as

\[ \mu_{\kappa - \kappa'} = \mu_{\kappa'} + \frac{2 \kappa' \cos \theta}{2 + e^{\pi \kappa^2}} \kappa + O(\kappa^2). \]  

(33)

We can then write the integral in Eq. (32) as

\[ I(\kappa, m \Delta t) = I_0(m \Delta t) + O(\kappa^2), \]  

(34)

where first-order terms, \( O(\kappa) \), vanish by symmetry and

\[ I_0(m \Delta t) = \frac{3}{2} \int_0^{k_{\text{cut}}} e^{4 \mu_{\kappa'} m \Delta t} \kappa'^2 d\kappa'. \]  

(35)

Thus the scale dependence of \( \mathcal{P}_{\zeta_{\text{nad}}} \) remains \( k^3 \) on large-scales for which \( \kappa \ll 1 \).

At late times these integrals become dominated by the modes with \( \kappa'^2 \ll (m \Delta t)^{-1} \) which are preferentially amplified during preheating. These are longer wavelength than \( k_{\text{cut}} \), but still very short compared to the scales which give rise to large scale structure in the present Universe. From Eq. (14) we have \( \mu_{\kappa'} \approx \mu_0 - \kappa'^2 / 3 \), for \( \kappa'^2 \ll 1 \), where \( \mu_0 = (\ln 3)/2\pi \), which gives the asymptotic behaviour at late times

\[ I_0 \approx 0.86(m \Delta t)^{-3/2} e^{4 \mu_0 m \Delta t}. \]  

(36)

Thus although the rate of growth of \( \mathcal{P}_{\zeta_{\text{nad}}} \) becomes determined by the exponential growth of the long-wavelength modes, the scale dependence on super-horizon scales remains proportional to \( k^3 \) for \( \kappa \lesssim (m \Delta t)^{-1/2} \). This ensures that there can be no significant change in the curvature perturbation, \( \zeta \), on very large scales before back-reaction on smaller scales becomes important and this phase of preheating ends when \( m \Delta t \sim 100 \).

Numerical evaluation of Eq. (32) confirms our analytical results, as shown in Fig. 1. For \( k \ll k_{\text{max}} \), the spectral index remains \( k^3 \) during preheating. Observable scales have \( \log_{10} k / k_{\text{max}} \approx -20 \).

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![FIG. 1. The power spectrum of the non-adiabatic curvature perturbation \( \mathcal{P}_{\zeta_{\text{nad}}} \), shown at four different times: from bottom to top \( m \Delta t = 0, 50, 100 \) and 150. The parameters used were \( g = 10^{-2}, m = 10^{-7}m_{\text{Pl}} \) and \( k_{\text{cut}} = k_{\text{max}} \).](image)

Our result shows that because of the \( k^3 \) spectrum of \( \delta \chi \), which leads to a similarly steep spectrum for \( \zeta_{\text{nad}} \), there is a negligible effect on the large-scale perturbations before the resonance ceases. The suppression of the large-scale perturbations in \( \delta \chi \), discussed in Refs. [3, 4], means that large-scale perturbations in \( \delta \chi \) are completely unimportant. However, it turns out that they don’t give the largest effect, which comes from the short-scale modes which dominate the integral for \( \zeta_{\text{nad}} \). Nevertheless, even they give a negligible effect, again with a \( k^3 \) spectrum. Indeed, that result with hindsight can be seen as inevitable; it has long been known [15] that local processes conserving energy and momentum cannot generate a tail shallower than \( k^3 \) (with our spectral index convention) to large scales, which is the Fourier equivalent of realizing that in real space there is an upper limit to how far energy can be transported. Any mechanism that relies on short-scale phenomena, rather than acting on pre-existing large-scale perturbations, is doomed to be negligible on large scales.

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IV. CONCLUSIONS AND DISCUSSION

As discussed in detail in a companion paper [4], large-scale curvature perturbations can vary provided there is a significant non-adiabatic pressure perturbation. This is always possible in principle if there is more than one field or fluid, and since preheating usually involves at least one additional field into which the inflaton resonantly decays, such variation is in principle possible.

In this paper we have focussed on the simplest preheating model, as discussed in Ref. [3]. We have identified the non-adiabatic pressure, and shown that the dominant effect comes from second-order perturbations in the preheating field. Further, the effect is dominated by pert-
turbations on short scales, rather than from the resonant amplification of non-adiabatic perturbations on the large astrophysical scales. Nevertheless, we have shown that the contribution has a $k^3$ spectrum to large scales, rendering it totally negligible on scales relevant for structure formation in our present Universe by the time backreaction ends the resonance. Amongst models of inflation involving a single-component inflaton field, this model gives the most preheating, and so this negative conclusion will apply to all such models.

Recently Bassett et al. [16] have suggested large effects might be possible in more complicated models. They consider two types of model. In one kind, inflation takes place along a steep-sided valley, which lies mainly along the direction of a field $\phi$ but with a small component along another direction $\chi$. In this case, one can simply define the inflaton to be the field evolving along the valley floor, and the second heavy field lies orthogonal to it. Taking that view, there is no reason to expect the preheating of the heavy field to give rise to a bigger effect than in the simpler model considered in this paper.

In the second kind of model, the reheat field is light during inflation, and this corresponds to a two-component inflaton field. As has long been known, there can indeed be a large variation of $\zeta$ in this case, which can continue until a thermalized radiation-dominated universe has been established. Indeed, in models where one of the fields survives to the present Universe (for example becoming the cold dark matter), variation in $\zeta$ can continue right to the present. This variation is due to the presence on large scales of classical perturbations in both fields (properly thought of as a multi-component inflaton field) generated during inflation, and the effect of these must always be considered in a multi-component inflation model, with or without preheating.

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