Numerical modelling of mudcrack growth

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ABSTRACT

The fracture morphologies of mud pastes show significantly complex patterns in nature. The mud pastes initially have fluid-like properties, but gradually change from “fluid” to porous “solid” in their drying process. The mudcrack phenomena in nature, therefore, is one of the complex physical phenomena of interest in material sciences. However, although it has been considered that mudcracks may be induced by the friction caused by the differences between shrinkage ratios of soil skeletons, the mechanical details remain unknown. In this work, we attempt to develop a novel numerical model based on three dimensional finite element method for mudcrack growth in Bentonite pastes. This model is the first description for mudcrack simulation based on three dimensional solid analysis. In order to validate the proposed model, Hausdorff fractal dimension of the numerical fracture patterns is compared with those of the experiments. As a result, the numerical results showed fracture patterns reasonably, and the fractal dimension of the cracking pattern by numerical simulation was almost consistent with the experimental results.

Keywords: mudcrack growth, three dimensional simulation, finite element method, fractal dimension

1. INTRODUCTION

The dry shrinkage cracking of mud pastes (hereinafter, “mudcrack”) is a fracture process commonly observed on surfaces of farms, tidelands, etc. in nature as shown in Fig. 1. Under some multiple physical processes such as wet-drying interactions or changing of material properties, the fracture morphologies show significantly complex patterns (e.g., Aydin and Degraff, 1988). Additionally, the mud pastes composed of numerous microscopic soil-particles and solvents initially have fluid-like properties, but gradually change from “fluid” to porous “solid” in their drying process. The mudcrack phenomena in nature, therefore, is one of the complex physical phenomena of interest in material sciences.

Recently, it has been clarified that mudcracks generated in the ancient Earth may affect the modern geological structures (Zhao, et al., 2014). Therefore, to establish some evaluation method for mudcrack patterns may be useful for assessing the natural rock failures such as block or flexural toppling in nature, which is one of the natural disasters. Hence, understanding the mechanism of mudcrack patterns become increasingly important for the geomechanical fields.

Fig. 1. Example of mudcrack pattern in nature.

However, although it has been considered that mudcracks may be induced by the friction caused by the differences between shrinkage ratios of soil skeletons...
(Müller and Dahm, 2000), the mechanical details remain unknown. In terms of geometric morphologies, we often observe geometric patterns of mudcracks having many T- and Y-type joints in nature (e.g., Sletten, et al., 2003). Interestingly, the crack propagation speed decreases as the mud-pastes thickness becomes less.

The analysis of fractal dimensions is an effective evaluation method for the fracture patterns of mudcracks, and many researchers have attempted to understand the patterns by using this geometric approach (e.g., Velde, 1999; Baer, et al., 2009). In the most recent literature, researchers have proposed a three dimensional fractal evaluation approach that uses digital images captured by computed tomography and laser scan systems (Perret, et al., 2003). Also, in numerical approaches, the fracture pattern of mud pastes were described by using a simple three dimensional model called as spring network model (Nishimoto, et al., 2007). However, we have not obtained such complicated patterns in modern three dimensional solid simulations.

In this work, we attempt to develop a novel numerical model based on three dimensional finite element method for mudcrack growth in Bentonite pastes. This model is the first description for mudcrack propagation method for mud pastes cracking system because it has useful for mud pastes cracking system because it has been found that the crack propagations in paste-like materials may be suppressed by the stress relaxation occurring around crack tips rather than by singularity of crack tip stress (Kitsunezaki, 2009).

The smeared crack model can be categorized into fixed and rotating crack models. With a fixed crack model, the orientation of the crack is fixed during the entire computational process, whereas a rotating crack model allows the orientation of the crack to co-rotate with the axes of the principal strain. In this work we used the fixed crack model, and employed the following equation for fractured finite elements:

\[ \int_{\Omega} B^T D_1 B u d\Omega = \int_{\Omega} B^T D_1 \varepsilon' d\Omega \]

where \( T \) is the transformation matrix for the stress and the strain, which are reflecting the orientation of the crack.

Fig. 2. Assumed shrinkage strain distributions.

2.1 Finite element formulation

The following governing equation for non-fractured finite element is employed.

\[ \int_{\Omega} B^T D_1 B u d\Omega = \int_{\Omega} B^T D_1 \varepsilon' d\Omega \]

where \( B \) is the strain-displacement matrix, \( D_1 \) is the elastic stress-strain matrix, \( u \) is the nodal displacement, \( \Omega \) is the volume of an element and \( \varepsilon' \) is the shrinkage strain. In this work, normal element of \( \varepsilon', \varepsilon''_s \), is assumed by the following equations:

\[ \varepsilon''_s = -\left( \frac{\exp(-\beta t) - 1}{\exp(-\beta) - 1} \right); t = \frac{z}{z_{\text{max}}} \]

where \( \beta \) is the positive coefficient, \( t \) is the normalized thickness of mud pastes, \( z_{\text{max}} \) is the thickness of mud pastes and \( z \) is the distance from bottom. The distribution of \( \varepsilon''_s \) is shown in Fig. 2. The distribution shows that the surface drying becomes strong as the \( \beta \) becomes large, or vice versa.

Regarding the stress-strain matrix, \( D \), we used the smeared crack model (Rots, 1970). This model is conceptually and computationally quite simple, but this model is very effective in capturing essential fracture behavior in various materials. This model would be useful for mud pastes cracking system because it has
Fractal geometry provided exact dimensions to morphologies that could not be quantified in Euclidean geometric analyses (Mandelbrot, 1982). A fractal is generally defined as having a morphology exhibiting self-similarity, such as a coastline, rock surface or crack. Also, it is well known that pattern analysis by fractal dimension is useful for characterizing the crack propagation phenomena in wetted pastes (e.g., Preston, et al., 1997; Baer, et al., 2009).

In this work, the fractal dimensions (Hausdorff dimensions, \( F_d \)) of mudcrack are determined by the box counting probability method (Baer, et al., 2009). Specifically, to calculate \( F_d \), a digital image of the analytical objects is overlaid with boxes of side length \( d \), and the number \( N(d) \) of boxes required to cover the object is counted and plotted against \( d \) on a double-logarithmic graph. The slope of tangent to this plot is taken as the fractal dimension, \( F_d \). That is,

\[
\log(N(d)) = F_d \log(d)
\]

In this study we assumed the following four cases of \( d \): 2, 3, 4, 6, 8, 12, 16, 32 and 64 pixels.

![Fig. 3. Crack propagation pattern (\( \beta = 3.0 \)).](image1)

![Fig. 4. Crack propagation pattern (\( \beta = -3.0 \)).](image2)

### 3 FRACTAL DIMENSION

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### 4 NUMERICAL SIMULATION RESULTS AND DISCUSSION

The three dimensional models with a 50×50×2 (long×wide×height) specimen was simulated. In this numerical example the Poisson’s ratio \( \nu \) of 0.3, Young’s modulus \( E \) of 40 ± 40 N/mm² with random manner, and the tensile strength \( f_t \) of 0.02 ± 0.025 N/mm² with random manner were set up as the material
constants. In the smeared crack model we employed the 4-node tetrahedral element, and the number of elements and nodes were 481,107 and 96,390 respectively. In advance of the calculation, we prepared two analytical models with a different $\beta$ in equation (2); $\beta = 3.0$ and $\beta = -3.0$.

The crack propagation results for each case are shown in Figs. 3 and 4. From these figures, we can observe that some “crack seeds” that initially occurred in the entire analytical region are interlocked with one another, and some grown-up cracks propagated locally to the various direction with increasing analytical steps. Finally, these cracks connected each other with angles close to 90° or 120° regardless of the value of $\beta$. Also, these T- and Y-pattern joints can be seen in nature as shown in Fig. 1.

The cracks in the case of $\beta = 3.0$ is concentrated more than those of $\beta = -3.0$. These results indicate that many sparse cracks may be generated under the strong drying condition, whereas many dense cracks may be generated under the gentle drying condition. In nature, we also observe many sparse cracking patterns and many dense cracking patterns, so that the differences may be caused by atmospheric temperature and humidity.

Fig. 5 shows the relationship between fractal dimension and analytical step. The fractal dimensions of numerical cracking pattern increased gradually as the analytical step increased, and they converged towards approximately 1.5, regardless of the value of $\beta$. Figs. 6(a) and 6(b) show the fracture pattern and the relationship between fractal dimension and elapsed time in drying Bentonite, respectively. In the experiment, the sample was set in a thermostatic device for 24 hours to remove the influence of a changing temperature. In the device, the temperature and humidity were kept constant at 25 °C and 37%, respectively. From Fig. 6(b), we understand that the fractal dimensions of Bentonite paste converged towards approximately 1.46. This tendency and the fractal dimension value is almost consistent with the numerical results in Fig. 5.

Consequently, the proposed method is qualitatively effective for mudcrack propagation analysis.

5 CONCLUSIONS

The three dimensional numerical simulation for mudcrack pattern were proposed and validated. Our interpretation of the results is summarized as follows:

(1) The simplified crack propagation algorithm is a powerful computational technique for solving the crack propagation phenomena of mud pastes. By using this technique, we can determine the various fracture patterns of mud pastes.

(2) The cracks in numerical simulations connected each other with angles close to 90° or 120°. These joint-patterns can be seen in nature.

(3) From numerical simulation results, we found that the differences fracture pattern in nature may be caused by atmospheric temperature and humidity.

(4) The fractal dimensions of numerical cracking pattern converged towards approximately 1.5 that
was almost consistent with the experimental value of Bentonite, 1.46.

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