Nonlinear sigma model approach for chiral fluctuations and symmetry breakdown in Nambu–Jona-Lasinio model.

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This paper is organized in two parts. We start with an observation that the recent claim that the chiral symmetry in NJL model is necessarily restored by violent chiral fluctuations at \( N_c = 3 \) (H. Kleinert and B. Van den Bossche, Phys. Lett. B 474 336 (2000)) appears to be incorrect since the critical stiffness of the effective nonlinear sigma model used in the above reference is not an universal quantity in 3+1-dimensions. In the second part we discuss a modified NJL model, where the critical stiffness is expressed via an additional cutoff parameter. This model displays a symmetry breakdown and also under certain conditions the chiral fluctuations give rise to a phase analogous to pseudogap phase of strong-coupling and low carrier density superconductors.

I. INTRODUCTION

Many concepts of particle physics have a close relation to superconductivity, for example Nambu–Jona-Lasinio model \[1\]–\[3\] was proposed in analogy to BCS theory and is considered to be a low-energy effective theory of QCD. Recently there was made a substantial progress in the theory of superconductivity in systems with strong attraction and low carrier density. Namely it was observed that away from the limits of infinitesimally weak coupling strength or very high carrier density, the BCS-like mean-field theories are qualitatively wrong and these systems possess along with a superconductive phase an additional phase where there exist Cooper pairs but no symmetry is broken due to violent fluctuations \( (\text{pseudogap phase}) \). What may be regarded as an indication of the importance of this concept to particle physics is that recently we have found a formation of the pseudogap phase due to dynamic quantum fluctuations at low \( N \) in the chiral Gross-Neveu model \[7\] in 2 + \( \epsilon \) dimensions \[8\]. The chiral GN model at low \( N \) exhibit two phase transition at two characteristic values of renormalized coupling constant \( g \). At a certain value \( g^* \) a gap modulus forms locally, but there exists as well another characteristic coupling value \( g_{KT} > g^* \) where the system undergoes a Kosterlitz-Thouless transition into a quasiordered state. The region between \( g_{KT} \) and \( g^* \) is analogous to the pseudogap phase in superconductors. At very large \( N \), \( g_{KT} \) merges with \( g^* \) thus recovering BCS-like scenario for the phase transition in the chiral GN model. Recently, an attempt was made \[8\] to generalize this result to the NJL model that lead the authors of \[8\] to the conclusion that at \( N_c = 3 \) the NJL model does not display a spontaneous breakdown of the chiral symmetry due to fluctuations. The paper \[8\] is at the moment a subject of numerous controversial discussions. Below we present a "no-go" result that one can not prove in principle in analogy to \[8\] the necessary restoration of the chiral symmetry at low \( N_c \) in the NJL model. As it is discussed in details below, the reason is the nonuniversality of the critical stiffness of the effective \( O(4) \)- nonlinear sigma model (NLSM) in 3 + 1-dimensions. This results in the fact that the 3 + 1-dimensional theory possesses an additional nonspecified fitting parameter that should be fixed from phenomenological considerations. This is in contrast to the chiral GN model in 2 + \( \epsilon \)-dimensions where one can prove that the model does not exhibit a quasi-long range order when the number of field components \( N \) drops below 8 \[8\].

In the last section we discuss possibility of formation of a phase analogous to the pseudogap phase in a modified NJL model taking special care of the existence of an additional cutoff parameter.

As it was mentioned above the subject of the discussion, which is the possibility of the restoration of chiral symmetry by directional fluctuations in a degenerate valley of an effective potential while preserving nonzero gap modulus locally, is closely related to the pseudogap phenomena in superconductors. Separation of the temperatures of the pair formation and of the onset of the phase coherence (pair condensation) in strong-coupling superconductors is in fact known already for many years \( (\text{Crossover from BCS superconductivity to – Bose-Einstein Condensation \ (BEC) of tightly bound fermion pairs}) \[6\]. Intensive theoretical studies of these phenomena in the past several years \( (\text{see for example}) \[6\] \[8\]), were sparked by experimental results on underdoped \( (\text{low carrier density}) \) cuprates that display a "gap-like" (pseudogap) feature \( \text{above} \ T_c \). The pseudogap disappears only at the substantially higher temperature \( T^* \). There is experimental evidence that this phenomenon in high-\( T_c \) superconductors may be connected with precritical pairing fluctuations \( \text{above} \ T_c . \)
Due to intimate relation of many problems in particle physics to superconductivity it seems to be natural to guess that pseudogap may become a fruitful concept in high energy physics too. We start with a brief introduction to this phenomena in superconductors and discuss its possible implications for QCD.

It is a well known fact that the BCS theory describes perfectly metallic superconductors. However it failed to describe even qualitatively superconductivity in underdoped High-$T_c$ compounds. One of the most exotic properties of the later materials is the existence of a pseudogap in the spectrum of the normal state well above critical temperature. From experimental point of view this manifests itself as an essential suppression of low frequency spectral weight thus being in contrast to exactly zero spectral weight in the case of superconductive gap. Moreover spectroscopy experiments show that superconductive gap evolves smoothly by magnitude and wave vector dependence to the pseudogap in the normal state. Except for it NMR and tunneling experiments indicate existence of incoherent Cooper pairs well above $T_c$. In principle it is easy to guess what is hidden behind this circumstances, and why BCS theory is incapable to describe it. Let us imagine for a moment that we were able to bind electrons in Cooper pairs infinitely tightly - obviously this implies that characteristic temperature of thermal pair decomposition will be also infinitely high, however this does not imply that the long-range order will survive at infinitely high temperatures. As it was first observed in $^4$He, the long-range order will be destroyed in a similar way as it happens say in superfluid $^4$He i.e. tightly bound Cooper pairs, at certain temperature will acquire a nonzero momentum and thus we will have a gas of tightly bound Cooper pairs but no macroscopic occupation of zero momentum level $q = 0$ and with it no long-range order. Thus a phase diagram of a strong coupling superconductor has three regions:

- **Superconductive phase** where there are condensed fermion pairs,
- **Pseudogap** phase where there exist fermion pairs but there is no condensate and with it there is no symmetry breakdown and no superconductivity,
- **Normal phase** with thermally decomposed Cooper pairs.

Of course, the existence of bound pairs above critical temperature will result in deviations from Fermi-liquid behavior that makes pseudogap phase to be a very interesting object of study. In order to describe superconductivity in a such system theory should incorporate pairs with nonzero momentum. Thus *the BCS scenario is invalid for description of spontaneous symmetry breakdown in a system with strong attractive interaction or low carrier density* (see [8]-[12]). So in principle in a strong-coupling superconductor the onset of a long range order has nothing to do with a pair formation transition. Existence of the paired fermions is a necessary but not sufficient condition for symmetry breakdown. BCS limit is a rather exotic case of infinitesimally weak coupling strength and high carrier density when the disappearance of superconductivity can be *approximately* described as a pairbreaking transition. Strong-coupling limit is another exotic case when temperatures of the pair decomposition and symmetry breakdown can be arbitrarily separated. There is nothing surprising in it: formally in the case of Bose condensation of $^4$He we can also introduce a characteristic temperature of thermal decomposition of a He atom, however this would not mean that this temperature will be somehow related to the temperature of the Bose condensation of the gas of these atoms.

Schematic phase diagram of a superconductor is shown in Fig 1.

![Schematic phase diagram of a superconductor](image)

**FIG. 1.** Schematic phase diagram of a superconductor with arbitrary coupling strength. In the strong coupling limit, temperature of superconductive phase transition tends to a plateau value corresponding to temperature of Bose condensation of the gas of tightly bound fermion pairs, whereas characteristic temperature of thermal pair decomposition grows monotonously as a function of the coupling strength.
One can obtain a pseudogap phase starting from BCS Hamiltonian. This was first done in a pioneering work by Nozieres and Schmitt-Rink and in a formalism of functional integral by Sa de Melo, Ranerdia and Engelbrecht. In order to study behavior of $T_c$ one should solve a set of number and gap equations including fluctuation corrections. In the BCS limit $T_c$ is not affected substantially by the gaussian corrections and superconductive transition can be described by a mean-field theory and correspondingly $T_c \approx T^*$. In the opposite limit numerical solution, and analytic perturbative treatment shows that the temperature of the superconductive phase transition tends to a constant sigma approach for description of chiral fluctuations proposed in. The authors claimed that at $T_c$ is not true gap even though a substantial depletion of low-frequency spectral weight is observed in this region experimentally - there it as interesting object of theoretical and experimental study as superconductive phase itself. One should stress in the pseudogap phase it exhibits reach exotic non-Fermi-liquid behavior due to pairing correlations that makes it as interesting object of theoretical and experimental study as superconductive phase itself. One should stress however that the term "pseudogap", originated in early experimental papers, may seem somewhat misleading since even though a substantial depletion of low-frequency spectral weight is observed in this region experimentally - there is no true gap in the spectrum.

II. CHIRAL FLUCTUATIONS IN THE NJL MODEL AT ZERO TEMPERATURE

As it was mentioned above, recently it was made an attempt of generalization to the NJL model the nonlinear-sigma approach for description of chiral fluctuations proposed in. The authors claimed that at $N_c = 3$ the NJL model does not display spontaneous symmetry breakdown due to chiral fluctuations. We show below that NLSM approach does not allow to prove that the chiral symmetry is always restored by fluctuations in the NJL model at $N_c = 3$. Below we also discuss difference with the chiral GN model where NLSM approach allows one to reach a similar conclusion at low $N_c$.

The Lagrangian of the NJL model reads:

$$\mathcal{L} = \bar{\psi}i\gamma^0 \psi + \frac{g_0}{2N_c} \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} \lambda_a i\gamma_5 \psi)^2 \right].$$

The three $2 \times 2$-dimensional matrices $\lambda_a/2$, generate the fundamental representation of flavor $SU(2)$, and are normalized by $\text{tr}(\lambda_a \lambda_b) = 2\delta_{ab}$. One can introduce Hubbard - Stratonovich fields $\sigma$ and $\pi_a$:

$$\mathcal{L} = \bar{\psi} (i\gamma^0 - \sigma - i\gamma_5 \lambda_a \pi_a) \psi - \frac{N_c}{2g_0} (\sigma^2 + \pi^2_a).$$

After integrating out quark fields, following to a standard mean-field variation procedure one can choose pseudoscalar solution $\pi_a$ to be vanishing and scalar solution $\varphi \equiv M$ to be given by a gap equation:

$$\frac{1}{g_0} = i(\text{tr} \gamma_1)(\text{tr} \gamma_1) \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2 - M^2}$$

The momentum integral is regularized by means of a cutoff $\Lambda$. The constituent quark mass $M$ in the limit $N_c \to \infty$ is analogous to superconductive gap in the BCS limit of the theory of superconductivity.

At finite $N_c$ one can study fluctuations around the saddle point solution. The quadratic terms of expansion around the saddle point are:

$$\mathcal{A}_0[\sigma', \pi'] = \frac{1}{2} \int d^4q \left[ \begin{pmatrix} \pi_a'(q) \\ \sigma'(q) \end{pmatrix} ^T \begin{pmatrix} G^{-1}_{\pi,\sigma} & 0 \\ 0 & G^{-1}_{\sigma,\sigma} \end{pmatrix} \begin{pmatrix} \pi_a'(-q) \\ \sigma'(-q) \end{pmatrix} \right],$$

where $(\sigma', \pi') \equiv (\sigma - M, \pi_a)$ and $G^{-1}_{\sigma,\sigma}$ are the inverse bosonic propagators. Implementing a momentum cutoff $\Lambda$, we can write $G^{-1}_{\pi,\sigma}$ for small $q_\perp$ as:

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1 As it was first discussed in the sixties, even in the BCS superconductors there is a narrow fluctuation region near $T_c$ that gives rise e.g. to a so-called paraconductivity effect. In particle physics, it was pointed on this phenomenon by Hatsuda and Kunihiro.
In analogy to 3D XY-model approach to strong-coupling superconductivity \(^2\) the authors of \(^3\) introduced a unit vector field \(n_i \equiv (n_0, n_a) \equiv (\sigma, \pi_a)/\rho\) and set up an effective nonlinear sigma-model

\[
\mathcal{A}_0[n_i] = \frac{\beta}{2} \int d^4x [\partial n_i(x)]^2. \tag{6}
\]

The prefactor \(\beta = M^2 Z(M/\Lambda)\), that follows from \((4), (5)\) is playing the role of a stiffness of the unit field fluctuations.

Now let us observe that from the arguments given in \((4), (5)\) it does not follow that NJL model necessarily remains in a chirally symmetric phase at \(N_c = 3\). At first, in contrast to discussed in \(2 + \epsilon\)-dimensional case, one unfortunately, can not make any similar calculations in a closed form in \(3+1\)-dimensions because this theory is not renormalizable. It was already observed in \((15)-(17)\) that cutoff of meson loops can not be set equal to cutoff for quark loops and thus the \(1/N_c\) corrected theory \((19)\) possesses two independent parameters that may be adjusted at will. We present another arguments of a different nature rooted in a nonuniversality of a critical stiffness of a NLSM in four dimensions, that does not allow in the framework of the NLSM approach to reach the conclusion of \((6)\). Our observation also applies to NLSM description of precritical fluctuations in general systems. It also allows us to show that the discussed below additional cutoff can not be related to the inverse coherence length of the radial fluctuations in the effective potential as it was suggested in \((19)\).

Basically the authors of \((6)\) by deriving \(G_{\sigma, \pi}\) have extracted two characteristics from the initial system: stiffness of the phase fluctuations in the degenerate minimum of the effective potential and the mass of the radial fluctuation. However knowledge of these characteristics does not allow in principle to judge if directional fluctuations will destroy long range order or system will possess a BCS-like phase transition. The reason is that a critical stiffness of the nonlinear sigma model is not an universal quantity in \(3 + 1\)-dimensions. So in principle knowledge of the stiffness of NJL model is not sufficient for finding the position of the phase transition in the effective nonlinear sigma model. The situation is just like in a Heisenberg magnet where the critical temperature depends on the stiffness along with lattice spacing and lattice structure. Thus if one is given only a stiffness coefficient one can not determine temperature of the phase transition \((15)\). The situation is in contrast to 2D case when a position of a KT-transition can be deduced from the stiffness coefficient \((18)\). In two dimensions the critical stiffness of O(2) nonlinear sigma model is an universal quantity and is given by \(\beta_{KT} = 2/\pi\) \((18)\), so comparing it with the stiffness coefficient derived from the initial theory (phase stiffness of the chiral GN model in \(D = 2\) is \(\beta = N/4\pi\)) one can judge if the system has enough stiffen phase in order to preserve quasi-long range order as we have shown in \((3)\). I.e. one can determine the number of field components \(N\) that is needed to remain below the position of Kosterliz-Thouless transition. This is in contrast to discussed here \(D = 3 + 1\) case.

Let us first recall a procedure how one can express a critical stiffness of the O(4)-nonlinear sigma model via an additional parameter: one can relax constrain \(n_i^2 = 1\) and introduce an extra integration over the lagrange multiplier \(\lambda\) rewriting \((4)\) as: \((\beta/2) \int d^4x \{[\partial n_i(x)]^2 + \lambda \left[n_i^2(x) - 1 \right] \}.\) Integration out the \(n_i(x)\)-fields, yields:

\[
\mathcal{A}_0[\lambda] = -\beta \int d^4x \frac{\lambda(x)}{2} + \frac{N_n}{2} \text{Tr} \ln \left[-\partial^2 + \lambda(x)\right], \tag{7}
\]

where \(N_n\) is the number of components of \(n_i(x)\), and \(\text{Tr}\) denotes the functional trace. This yields a gap equation:

\[
\beta = N_n \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + \lambda}. \tag{8}
\]

The model has a phase transition at a critical stiffness that depends on an unspecified additional cutoff parameter that should be applied to the gap equation:

\[
\beta^{cr} = N_n \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2}. \tag{9}
\]

\(^2\)Due to this reason one can not refer to lattice simulation for finding the value of the critical stiffness as it was done in \((19)\) since implicitly these numerical values contain information of lattice structure and are not universal.

\(^3\)There is a misleading statement about three dimensional case in \((19)\).
For example in the case of magnets the additional cutoff needed in (11) is naturally related to the lattice spacing. In the paper [15] it was proposed a criterion that states that one can relate the inverse coherence length extracted from radial fluctuations in an effective potential of an initial theory to the cutoff in the integral (10) so that all the parameters in the theory would be expressed from quantities derived from an initial model and thus this modified model would possess an universal critical stiffens. However there is no reason for relating the cutoff needed in (10) to the coherence length of the modulus fluctuations and moreover we show that this procedure leads in general to unphysical consequences. It was supposed in [19] that relation of coherence length to cutoff in the equation (9) yields an universal criterion for judgement of nature of symmetry breakdown in general physical systems. There is a simple counterexample: in the case of a strong-coupling superconductor, the characteristic nonlinear sigma model that describes fluctuations in a degenerate valley of the effective potential is a 3D XY-model. In the continuous case it is a free field theory and has no phase transition at all. The phase transition appears only in the lattice theory and of course its temperature depends on the lattice spacing. With increasing coupling strength the low-temperature phase stiffness of the effective 3D XY model tends to a plateau value $J = n/4m$, where $n$ and $m$ are density and mass of fermions [10]. Whereas the temperature of the phase transition of the 3D XY-model is

$$T^\text{3DXY}_c \propto \frac{n}{m},$$

where $a$ is the lattice spacing. To be careful one should remark that accurate analysis shows that a strong coupling superconductor possesses two characteristic length scales: size of the Cooper pairs that tends to zero with increasing coupling strength and a coherence length that tends to infinity with increasing coupling strength as the system evolves towards a weakly nonideal gas of true composite bosons [10-12]. At first if one relates the constant $a$ in (10) to the size of the Cooper pairs following to the arguments of [10] one would come to an incorrect conclusion of absence of the superconductivity in strong-coupling superconductors in the way similar as the authors of [10] came to a conclusion of inexistence of symmetry breakdown in the NJL model. This is in a direct contradiction with behavior of the strong coupling superconductors (see references mentioned in the Introduction). Second, if one attempts to relate $a$ in (10) to the second length scale of the theory - namely true coherence length, that tends to infinity with increasing coupling strength, then one would come to a qualitatively incorrect conclusion too [20]. Thus the existence of an universal NLSM-based fluctuations criterion [19] appears to be incorrect.

So in general the nonlinear sigma model approach for precritical fluctuations possesses an additional fitting parameter which is the cutoff in the gap equation (9) that can not be related to inverse coherence length extracted from radial fluctuations in an effective potential. Thus within the NLSM approach one can not proove if the NJL model displays necessarily the directional fluctuations driven restoration of the chiral symmetry at low $N_c$.

III. CHIRAL FLUCTUATIONS AT FINITE TEMPERATURE AND A MODIFIED NJL MODEL WITH A PSEUDOGAP

The authors of [10] employed NLSM arguments in attempt to show that the NJL model can not serve for the study of the chiral symmetry breakdown. We have shown above that this conclusion appears to be incorrect since the critical stiffness in 3+1-dimensions is not an universal quantity and one has an additional fitting parameter. This is an inborn feature of the discussed NLSM approach in 3+1 dimensions (compare with the mentioned above cutoffs discussions in nonrenormalizable models in a different approach [15-17], and also [21]). The above circumstance allows one to fix the critical stiffness from phenomenological considerations. However, we argue below that, what is missed in [10] is that, in principle, the low-$N_c$ fluctuation instabilities, when properly treated, have a clear physical meaning. Moreover we argue that one can employ a NLSM for descriptions of the chiral fluctuations, providing that special care is taken of the additional cutoff parameter. It was indeed already discussed in literature that at finite temperatures the chiral phase transition should be accompanied by developed fluctuations (3,4 and references therein). We argue that this process at low $N_c$ should give rise to a phase analogous to the pseudogap phase that may be conveniently described within a nonlinear sigma model approach. There are indeed other ways to describe these phenomena [11]. However the NLSM approach seems to be especially convenient in the case of a nonrenormalizable theory. The descrition of the two-step chiral phase transition and appearence of the intermediate phase requires to study the system at the next-to-mean-field level. Unfortunately the NJL model is not renormalizable and does not allow to make any conclusions about importance of fluctuations in a closed form [17]. On the other hand a pseudogap phase is a general feature of fermi system with attraction. The NLSM construction discussed below, due to its nonperturbative nature can not be regarded as a regular approximation but may be considered as a tractable modification of the NJL model that has a pseudogap. One can also find an additional motivation for employing these
arguments in the fact that NLSM allows one to prove an existence of a phase analogous to pseudogap phase in the chiral GN model [3], which is the closest relative to NJL model. Also NLSM approach works well for the description of precritical fluctuations in superconductors [3] - where the theory is renormalizable and essentially the same results can be obtained perturbatively. We stress that these phenomena is a general feature of any Fermi system with attraction. Also, to certain extend similar crossovers are known in a large variety of condensed matter systems. In particular, besides superconductors we can mention the exitonic condensate in semiconductors, the Josephson junction arrays, the itinerant and local-momentum theories of magnetism and the ferroelectrics.

Let us now consider the chiral fluctuations in NJL model at finite temperature. Then following standard dimensional reduction arguments [22], the chiral fluctuations should be described by a 3D $O(4)$-sigma model. Thus one has the following gap equation for the effective NLSM (i.e. finite temperature analogue of (1)):

$$\frac{J_T}{T} = N_c \int \frac{d^3 k}{(2\pi)^3} \frac{1}{k^2 + \lambda}$$  \hspace{1cm} (11)

The temperature of the phase transition of the three dimensional classical $O(4)$ sigma model with stiffness $J_T$ is expressed via an additional parameter $\tilde{\Lambda}_T$ needed in (11) as:

$$T_c = \frac{\pi^2 J_T}{2 \tilde{\Lambda}_T}$$  \hspace{1cm} (12)

The stiffness of thermal fluctuations $J_T$ can be readily extracted from the NJL model. At finite temperature the inverse bosonic propagator of the collective field $\pi$ for small $q$ can be written as:

$$G_\pi^{-1} = -2^{D/2} N_c \int \frac{d^3 p}{(2\pi)^3} \sum_n \left[ \frac{T}{(p^2 + M^2 + \omega_n^2)^2} \right] q^2 =$$

$$-2^{D/2} N_c \int \frac{d^3 p}{(2\pi)^3} \left[ \frac{1}{8 (p^2 + M^2)^{3/2}} \tanh \left( \frac{\sqrt{p^2 + M^2}}{2T} \right) - \frac{1}{16 T} \frac{1}{p^2 + M^2} \cosh^{-2} \left( \frac{\sqrt{p^2 + M^2}}{2T} \right) \right] q^2 =$$

$$-K(T, \Lambda_T, M, N_c) q^2,$$  \hspace{1cm} (13)

where $\Lambda_T$ is a momentum cutoff. The propagator (13) renders the gradient term that allows one to set up an effective classical 3D $O(4)$-nonlinear sigma model:

$$E = \frac{J_T(T, \Lambda_T, M, N_c)}{2} \int d^3 x [\partial n_i(x)]^2,$$  \hspace{1cm} (14)

where

$$J_T(T, \Lambda_T, M, N_c) = K(T, \Lambda_T, M, N_c) M^2(T, \Lambda_T)$$  \hspace{1cm} (15)

is the stiffness of the thermal fluctuations in the degenerate valley of the effective potential. The temperature-dependent quark mass $M$ that enters this expression is given by a standard mean-field gap equation that also should be regularized with the cutoff $\Lambda_T$:

$$\frac{1}{g_0} = 2 \times 2^{D/2} \sum_n \int \frac{d^3 p}{(2\pi)^3} \frac{T}{p^2 + M^2 + \omega_n^2}$$  \hspace{1cm} (16)

It can be easily seen that when we approach the temperature $T^*$ where the mass the $M(T)$ becomes zero, the stiffness $J(T, \Lambda_T, M, N_c)$ also tends to zero. Formula (14) defines a generalized Heisenberg model with a temperature dependent stiffness coefficient. Position of the phase disorder transition in a such system should be determined self-consistently by solving the system of the equations for $T_c$ and $M(T_c)$. Apparently just like in a superconductor with a pseudogap the phase transition in a such system is a competition between a thermal depletion of the gap modulus (this rudely corresponds to thermal pairbreaking in a superconductor) and a process of thermal excitations of the directional fluctuations in the degenerate minimum of the effective potential. ”BCS” limit corresponds to the situation when $T^*$ merges with $T_c$ and it is easily seen that this scenario always holds true at $N_c \to \infty$. I.e. at infinite $N$ the mean-field theory is always accurate just like BCS theory works well in the weak coupling superconductors. In the framework of this NLSM construction, at low $N_c$ the scenario of the phase transition depends on the choice of $M(0), \Lambda_T$ and $\tilde{\Lambda}_T$, that should be fixed from phenomenological considerations.
IV. CONCLUSION

In the first part of this paper we presented a no-go result that within a framework of the nonlinear sigma model approach one can not answer the question if the chiral symmetry in NJL model is always restored by quantum fluctuations at $N_c = 3$. The reason is the nonuniversality of the critical stiffness of 4D $O(4)$ nonlinear sigma model. This, along with the discussed above observations made in a framework of a different approach in [17] resolves numerous controversial discussions initiated by a recent paper [6] where it was argued that there is no spontaneous breakdown of the chiral symmetry in the NJL model, which appears to be incorrect.

In the second part of the paper we discussed a NLSM approach for precritical fluctuations in a modified NJL model where a critical stiffness is expressed via an additional cutoff parameter that should be fixed from phenomenological consideration. We discuss a formation in the above model of a phase analogous to the pseudogap phase in strong-coupling and low-carrier density superconductors. Appearance of this phase may accompany the chiral phase transition in QCD. Since the precursor pairing fluctuations is a general feature of any Fermi system with attraction and moreover it is a dominating region of a phase diagram of strong-coupling and low carrier density superconductors, the interesting question is to what extend the discussed above phase is developed in QCD and color superconductors.

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In fact, in a nonperturbative NLSM approach to the condensation from pseudogap phase in strong-coupling superconductors one can reach an excellent agreement with the perturbative results if one considers an effective sigma model on the lattice with the spacing given by $a \propto 1/n_b^{1/3}$, where $n_b$ is density of Cooper pairs. Then the model is related to the condensation of hard-core composite bosons on a lattice.

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