Voids distribution of pavement filters subjected to permeating fines: a coupled DEM-statistical inference

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Abstract. Granular soils in pavement systems, insidiously experience fines migration during their expected lifespan. Experimental monitoring of particles movement in a granular matrix is extremely challenging due to its concealed nature. Numerically modelling the permeation process demands a high computational power and requires additional conditions, which are frequently difficult to be obtained. This paper proposes an integrated numerical approach to estimate the conditional probability of observing a given voids ratio of a filter subjected to different fractions of permeating fines. The discrete element method (DEM) has been used for developing numerical samples of random voxels to estimate the void ratio distribution of the fines-free filter (i.e. virgin filter). Qualitative results obtained from a pilot scale experimental program have been presented to validate the hypotheses on which the novel methodology has been developed. The results further show that the distribution of the voids ratio in the filters subjected to permeating fines follow a gamma distribution, with a 0.9 probability of observing a voids ratio lower than the average value in the virgin filter.

1. Introduction
Seepage induced fine particle migration detrimentally affects the hydraulic performance and the life expectancy of the drainage systems in highway pavements. During the pavement construction, granular filter layers are designed using empirical criteria to minimise this fines migration [1-3]. The widely-used US Army Corps of Engineers filter design criteria (Eq. 1), for instance, consider ratios of determining sizes belonging to the particle size distributions (PSDs) of the filter and fine soil layers (i.e. fines). The left and the ride hand side of the criteria given in Eq. 1 are known as the clogging and permeability criterion respectively. However, even after designing the filter following these criteria, still, spatiotemporal factors, such as inadequate compaction or segregation during compaction, excessive cyclic traffic loads and extreme weather conditions can result in undesired particle migration during the lifetime of a pavement filter layer [4-6].

\[
\frac{D_{15,\text{filter}}}{D_{85,\text{fines}}} \leq 5 \leq \frac{D_{15,\text{filter}}}{D_{15,\text{fines}}} \tag{1}
\]

where \(D_{15,\text{filter}}\), \(D_{85,\text{fines}}\), \(D_{15,\text{filter}}\), and \(D_{15,\text{fines}}\) are the diameters of corresponding to cumulative weight percentages of 15%, 85% and 15% of the filter and soil PSDs, respectively.

Migrated fines occupy the void spaces in the virgin filter (i.e. filter fabric free of fines) following a highly randomised pattern. This affects the mechanical and hydraulic performance of the filter soil, such as the compressional behaviour, strength, critical state line, static liquefaction, undrained fragility, and also the hydraulic conductivity [7,8]. It is extremely challenging to monitor (or correlate) the
resulting real-time variations of mechanical and hydraulic performances of these filters due to the concealed nature of the fines migration phenomenon. Very little experimental information, therefore, is available on the real-time variations of the void size distribution in the filter under the continuous infiltration of fines.

Due to these experimental challenges, the conventional numerical modelling techniques, which assume the continuum nature of materials, cannot comprehensively model this migration problem [4]. On the contrary, the discrete element method (DEM) [9], which simulates the particle movements using Newton’s second law of motion, has shown potentials in modelling this phenomenon. However, DEM simulations are only possible for an assembly containing a limited number of particles [10]. The computational cost associated with DEM simulations limits them being applied to infinitely many numbers of particles, which ideally represent the large-scale applications. The scale issue can be one way addressed by coupling the DEM with continuum models; however, this significantly increases the demand for computational power. Additionally, improved algorithms are required to accurately couple the two techniques [11]. Therefore, a requirement exists for an alternative feasible robust inference technique that can be readily used in large-scale applications.

This study sets out to develop a probabilistic approach to infer the voids distribution in a filter-fines mixture after the migration of fines into the virgin filter. A pilot scale experimental program has been carried out to investigate the progression of fines migration in a well-designed (i.e. following Eq. 1) filter-soil assembly. A database of potential void ratios has been developed for the tested filter soil, which was free of fines. The void ratio distribution was then employed to infer the conditional probability distribution of the void ratio of the filter-fines mixture upon plausible infiltrations of different fines contents.

2. Methodology
This study comprises of a pilot scale experimental program and a numerical simulation using DEM, which was followed by statistical analysis. In the experimental approach, a constant hydraulic gradient was used to induce base particle migration through a uniform granular filter. The numerical tests were used to develop the void ratio distribution of the tested filter soil, which was free of fines. The void ratio distribution was then employed to infer the conditional probability distribution of the void ratio of the filter-fines mixture.

2.1 Unidirectional flow tests
A custom-made transparent acrylic cell was used to perform the unidirectional flow tests under a constant hydraulic gradient of 5 (fig. 1a). A comparatively extreme hydraulic gradient was employed, following the literature [8,13], to reduce the experimental duration and enable a rapid migration of fines. Filter and the fine soil was constituted to have uniform distributions spanning from 1.18 mm to 4.75 mm and 63 µm to 600 µm respectively (fig. 1b). Their respective PSDs were designed according to the design criteria specified in Eq. 1. The stainless steel sieve plate that holds the sample in the cell has uniform 2mm size apertures to retain the filter particles while allowing only the water and fines to pass through the assembly during the flow tests.

First, the filter soil was placed on the sieve plate, and the layer was saturated at 8% and compacted up to 90-95% of the optimum proctor compaction. The base material was carefully placed on top of the filter using a funnel, such that the particles are evenly and loosely distributed. The filter and base height were taken as 70 mm and 35 mm (2:1) respectively. Marbles with 16mm particle size were used to provide and maintain overburden stress of approximately 1kPa.
The cell was filled from the bottom-up with filtered de-ionised water at an average temperature of 26.2°C to avoid dispersing the sample. The top air release valve was used to minimise the presence of trapped air in the cell. Then it was allowed to saturate for a duration of 60 minutes. After saturation, the flow direction was reversed subjecting the sample to a downward flow. The hydraulic gradient was kept constant (at 5), and the volumetric flow was measured during the test. When the hydraulic flow stabilized (approximately 90 minutes for this sample), the test was terminated. After the test, EPO-TEK 301 resin was injected to the filter-fines mixture. A representative core sample was taken from this mixture after the resin got fully crystalized to observe its micro-structure from the scanning electron microscope (SEM). The remaining filter soil was equally divided (as top and bottom sections) and their respective post-test PSDs were estimated using sieve analysis (ASTM D422).

2.2 Statistical estimation of the voids distribution: DEM approach

Open source DEM software, YADE, was employed to generate the tested filter sample numerically. In this method, a cloud of 10000 grains belonging to the filter PSD was generated [12] with the simulation parameters specified in table 1. The particle cloud is then allowed to free-fall under gravity (as given in fig 2a and b) into a bounding box with 1000x1000x1000 mm dimensions (D x D x D in Figure 2c). When the particle assembly gets fully settled in the bounding box, an overburden stress of 1000 kPa was applied on the sample to mimic the experimental conditions. The compaction consolidates the sample (fig 2d). After completing the consolidation, the bounding box was divided into identical square voxels with 10x10x10 mm dimensions (Figure 2c and d). The voxel samples were used to create the database for estimating the void ratio distribution of the filter before the fines migration.

| Property | Number of grains | Grain shape | Density (kg/m³) | Young’s modulus (Pa) | Poisson’s ratio | Friction angle (deg) |
|----------|-----------------|-------------|-----------------|-----------------------|----------------|---------------------|
| Value    | 10000           | Spherical   | 2600            | 10¹                   | 0.3            | 28                  |

Figure 1. Experimental set-up for the unidirectional flow tests
Previous studies have proposed an index known as the skeleton void ratio \( (e_s) \) hypothesising that the fines content \( (f_c) \) in a soil mixture would only occupy the void space of the coarse grain material, which transfers the effective stress of the soil assembly [14]. This hypothesis can be also applied to the migration phenomenon discussed in this study since the virgin filter act as the load bearing skeleton of the resulting filter-fines mixture. Hence, the statistical void ratio distribution should be identical to the distribution of \( e_s \) pertinent to the filter-fines mixture. Using this analogy, the resulting void ratio of the mixture \( (e_F) \) can be therefore obtained using Eq. 3.

\[
  e_s = \frac{e_F + f_c}{1 - f_c} \\
  e_F = e_s(1 - f_c) - f_c
\]

The \( f_c \) in Eq. 3 is a real number that can have any positive value from 0 to 1. Hence, the random number generating algorithm in MATLAB 2017 software is used to generate a random sample of 10000 \( f_c \) values to perform a Monte-Carlo simulation using the Eq. 3. The cumulative probability distribution of possible different \( e_F \) values subjected to different fines contents \( (P(e_F | f_c)) \) (i.e. the conditional probability distribution [15]) has been generated using this approach.

3. Results
The post-grain size distribution of the filter reveals that the top and bottom section of the filter has received an approximately equal amount of fines (fig 3a). This also suggests that the duration of the experiments were sufficient for the fines to equally distribute throughout the filter. The fines fraction has increased approximately by 5% on average. The SEM image of the filter fabric shows that the fines have entered the coarse matrix and they have resided in the voids of the filter soil matrix (fig 3b). This reduces the void volume in the fabric and hence the void ratio of the mixture soil. Also this supports the validity of using eq. 3 in developing the mixture void distribution.

The void ratio distribution obtained using the voxel samples in the DEM simulations is shown in figure 4a. The void ratios follow a generalised extreme value distribution with a mean value of 0.798 and a standard deviation of 0.225. The location, scale and the shape parameters of this distribution are 0.692, 0.115 and 0.262 respectively. The estimated void ratio distribution of the post filter matrix was
found to follow a gamma distribution with shape parameters $\alpha$ and $\beta$ being equal to 1.56 and 0.26 (fig 4b). The probability of observing a void ratio lesser than the void ratio of the virgin filter was 0.9.

Figure 3. Experimental results: a) PSD of migrated fines fraction and the virgin filter; b) SEM image of the micro-structure of a sample taken from fines-filter mixture after the flow tests.

Figure 4. Statistical and probability distributions: a) void ratio distribution of the virgin filter; b) cumulative probability distribution of the void ratio of the fines-filter mixtures.

4. Future directions
Bayesian statistical inference (e.g. as given in [15]) can be employed to integrate the conditional probability distribution of the mixture void ratio, given in fig 4b, to probabilistically quantify the variations of hydraulic and mechanical responses of the pavement filters. This will enable a probabilistic lifetime assessment for the pavement filters. The proposed methodology simplifies the complex physics involved in the fines migration phenomenon in pavement filters. A comprehensive database of the void ratio distribution should be developed in the future using DEM with more emphasis on the typical relative compactions found in pavement applications. In addition, the particle shape effects and intrinsic material properties of the filter medium should be closely investigated in these DEM simulations to improve this technique to be a convenient tool for the practising engineers.
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