Perturbative and nonperturbative correlations in double parton distributions

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We argue that perturbative QCD correlations contribute dominantly to the double parton distributions being compared to the nonperturbative ones in the limit of large enough hard scales and at not parametrically small longitudinal momentum fractions.

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I. INTRODUCTION

Multiple parton interactions (MPI) are of great relevance for the LHC physics, where they represent an important background to the signals from the Higgs production and other interesting processes. Hence, there is nothing surprising in a big splash of research activity around MPI that has been increasing in recent years. Apparently, such a situation was provoked and supported by an experimental evidence for double parton scattering (DPS) processes producing two independently-supported hard particles in the same collision observed in $p + p$ and $p + \bar{p}$ measurements of final-states containing four jets, $\gamma + 3$ jets, and $W + 2$ jets, performed by the AFS, UA2, CDF, D0, ATLAS, and CMS Collaborations. Besides, the recent measurements of double $J/\psi$ production as well as of single $J/\psi$ production as a function of the event multiplicity in $p + p$-collisions at LHC have been also successfully interpreted in the context of DPS and MPI models, respectively.

The theoretical study and phenomenological analysis of DPS have a long history and date back to the early days of the parton model with subsequent extension to the perturbative QCD and an interest renewed in more recent years (see, for instance, the reviews with many references to prior work). The DPS importance for understanding the proton-nucleus and nucleus-nucleus collisions data was also only recently realized. Nevertheless, the MPI phenomenology relies on the models which are physically intuitive and involve substantial simplifying assumptions. Therefore, it is extremely desirable to unite and to strengthen theoretical efforts in order to achieve a better description of MPI and, in particular, DPS that is, very likely, most significant multiple scattering mode at the LHC.

The initial state of DPS is coded by double parton distribution functions (dPDFs) which quantify the joint distribution of two partons in a hadron, depending on their quantum numbers, their longitudinal momentum fractions and their relative transverse distance from each other. The starting cross section formula for DPS is somewhat similar to that commonly used for single parton scattering (SPS). It was derived by making use of the light-cone variables and the same approximations as those applied to the processes with a single hard scattering. The inclusive DPS cross section (in the momentum representation) in a hadron-hadron collision with the two hard parton subprocesses $A$ and $B$ may be written in the factorized form as

$$\sigma^{(A,B)}_{\text{DPS}} = \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2; q; Q_1^2, Q_2^2) \times \hat{\sigma}_{ik}^A(x_1, x'_1) \hat{\sigma}_{jl}^B(x_2, x'_2) \Gamma_{kl}(x_1, x_2; -q; Q_1^2, Q_2^2) \times dx_1 dx_2 dx'_1 dx'_2 \frac{d^2 q}{(2\pi)^2},$$

where $\hat{\sigma}_{ik}^A$ and $\hat{\sigma}_{jl}^B$ are the parton-level subprocess cross sections, $\Gamma_{ij}(x_1, x_2; q; Q_1^2, Q_2^2)$ are the generalized dPDFs, depending on the longitudinal momentum fractions $x_1$ and $x_2$ of the two partons $i$ and $j$ undergoing the hard processes $A$ and $B$ at the scales $Q_1$ and $Q_2$. The combinatorial factor $m/2$ accounts for indistinguishable ($m = 1$) and distinguishable ($m = 2$) final states. The dPDFs in the momentum representation depend on the transverse momentum $q$ which is equal to the difference of the momenta of partons from the wave function of the colliding hadrons in the amplitude and the amplitude conjugated. Such a dependence arises because the difference of parton transverse momenta within the parton pair is not conserved. This transverse momentum $q$ is the Fourier conjugated variable of the parton pair transverse separation which is used in the mixed (momentum and coordinate) representation. The dPDFs and the corresponding evolution equations were well-known only for $q = 0$ (in the other words, integrated over the parton pair transverse separation) in the collinear approximation. In that approximation the two-parton distribution functions,

$$\Gamma_{ij}(x_1, x_2; q = 0; Q^2, Q'^2) = D_{ij}^L(x_1, x_2; Q^2, Q'^2),$$
with the two hard scales set equal and satisfy the generalised Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations, derived initially in Refs. \[29, 30\]. The subsequent extension to two different hard scales was done in Ref. \[33\], whereas the single distributions satisfy very widely known the DGLAP equations \[34–37\]. The functions in question have a specific interpretation in the leading logarithm approximation of perturbative QCD. They are the inclusive probabilities which allow one to find two bare partons of types \(i\) and \(j\) with the given longitudinal momentum fractions \(x_1\) and \(x_2\) in a hadron \(h\).

These well-known collinear distributions were a starting point to derive the revised formula for inclusive cross section of the DPS process without the oversimplified additional factorization assumption (which, in general, is inconsistent with the QCD evolution) suggested in Ref. \[38\]. New formula absorbs three contributing terms: (i) a "traditional" factorization component, (ii) single and (iii) double perturbative splitting graphs induced by the QCD evolution. Afterwards, the similar results were also obtained in Ref. \[39\], with an emphasis on the differential cross sections, and were partly corroborated in Refs. \[40, 41\], albeit with some distinctions dictated mainly by a terminology. It was found in Refs. \[38, 42\] that so-called single and double perturbative splitting graphs can meaningfully contribute to the inclusive cross section for a DPS process if compared to a "traditional" factorization component. It makes sense to mention here the discussion \[39, 45\] concerning the double perturbative splitting graphs. Formally, this contribution within the collinear approach in the region of not too small \(x\) should be considered as a result of the interaction of one parton pair with the 2 \(\rightarrow\) 4 hard subprocess \[33–44\], since the dominant contribution to the phase space integral comes from a large \(q^2 \sim \min(Q_1^2, Q_2^2)\). However, as it was argued in Refs. \[38, 42\] the contribution under discussion may be validly included in the DPS cross section at suitable low longitudinal momentum fractions.

In any case the numerical evaluation of single and double perturbative splitting graphs to the DPS cross section is desirable at the LHC kinematics, where the large available values of \(Q_1\) and \(Q_2\) (in comparison with the characteristic QCD small reference scale \(\mu\)), \(\ln(1/x_1)\) and \(\ln(1/x_2)\) (in comparison with 1) can provide the configurations with the Balitskii-Fadin-Kuraev-Lipatov (BFKL) \[46–49\] or DGLAP evolution in the ladders before and after splitting being dependant on the processes under consideration. Such estimates for the contribution of single perturbative splitting graphs to the DPS cross section have been recently done in Ref. \[50\] (three-parton interactions in terminology of that work) and it was pointed out that the relative contribution of these evolution effects increases with the hard scales increasing and may resolve the longstanding puzzle — the underestimation of observed effective DPS cross section by a factor of two in the independent parton approximation (taking into account a "traditional" factorization component only).

The main purpose of the present paper is to demonstrate analytically that the perturbative QCD correlations contribute dominantly to dPDFs in comparison with nonperturbative ones in the limit of large enough hard scales and at parametrically not small longitudinal momentum fractions what definitely supports the numerical observation of Ref. \[54\]. The paper is organized as follows. In Sec. \[II\] we discuss the two-parton distribution functions resulted from the perturbative QCD theory introducing the appropriate definitions and designations. The asymptotic behavior of dPDFs is discussed in Sec. \[III\] and we summarize in Sec. \[IV\].

II. DOUBLE PARTON DISTRIBUTIONS IN THE LEADING LOGARITHM APPROXIMATION

The enormously huge analysis performed in Refs. \[34–36\] for the hard processes (deep-inelastic electron-proton scattering and electron-positron annihilation into hadrons) in vector, pseudoscalar and QCD field theories has provided a community with a powerful instrument, a leading logarithm approximation, in terms of the parton model with a variable cutoff parameter with respect to the transverse momenta. The dependence of multiparton distribution and fragmentation functions on a value of this cutoff parameter is determined by the evolution equations. The most transparent method for deriving such equations in any renormalizable quantum field theory was formulated in an outstanding paper by Lipatov \[55\]. The value of hard scale (most frequently, the transfer momentum squared \(Q^2\) or its logarithm \(\xi = \ln(Q^2/\mu^2)\) or double logarithm (which takes into account explicitly the behavior of the effective coupling constant in the leading logarithm approximation) is treated as an evolution variable

\[
t = \frac{1}{2\pi\beta} \ln \left[ 1 + \frac{q^2(\mu^2)}{4\pi} \beta \ln \left( \frac{Q^2}{\mu^2} \right) \right] = \frac{1}{2\pi\beta} \ln \left( \frac{\ln(\frac{Q^2}{\Lambda_{QCD}^2})}{\ln(\frac{\mu^2}{\Lambda_{QCD}^2})} \right).
\]

Here \(\beta = (33 - 2n_f)/12\pi\) in QCD, \(g(\mu^2)\) is the running coupling constant at some characteristic scale \(\mu^2\) above which perturbative theory is applicable, \(n_f\) is the number of active flavors and \(\Lambda_{QCD}\) is the QCD dimensional parameter.

The DGLAP evolution equations \[34–37\] develop the simplest form if one employs the natural dimensionless evolution variable \(t\); that is,

\[
\frac{dD_i^j(x, t)}{dt} = \sum_{j'} \int_x^1 \frac{dx'}{x'} D_{i}^{j'}(x', t)P_{j' \rightarrow j}(\frac{x}{x'}) \cdot (3)
\]
They describe the evolution of single distributions, $D^i_t(x, t)$, of bare quarks and gluons within dressed partons (quarks and gluons, $i, j = q/g$) in response to the change of evolution variable $t$. The kernels, $P$, of these equations in the Lipatov’s method already include a regularization at $x \to x'$ unlike the regularization in Ref. [37] where it was introduced ad hoc by requiring the momentum conservation law to be valid. Moreover, the Lipatov’s method makes it possible to derive evolution equations for multiparton distribution (and for fragmentation) functions as well.

These equations were derived for the first time [29, 30] in the following form

$$
\frac{dD^{i_{j_1}j_2}(x_1, x_2, t)}{dt} = \sum_{j_1'j_2'} \int_{x_1}^{1-x_2} \frac{dx_1'}{x_1'} D^{i_{j_1}j_2}(x_1', x_2, t) P_{j_1'j_1} \left( \frac{x_1}{x_1'} \right) + \sum_{j_2'} \int_{x_2}^{1-x_1} \frac{dx_2'}{x_2'} D^{i_{j_1}j_2'}(x_1, x_2', t) P_{j_2'j_2} \left( \frac{x_2}{x_2'} \right) + \sum_{j'} D^{i_{j'}}(x_1 + x_2, t) \frac{1}{x_1 + x_2} P_{j'j_1} \left( \frac{x_1}{x_1 + x_2} \right),
$$

which appears in the nonhomogeneous part of the equations, does not involve a $\delta$-function regularizing term. The equations in question describe the dPDFs evolution of bare quarks and gluons in dressed partons (quarks and gluons) in response to the variation in the evolution variable $t$ — that is, for the cases where the scales for the two hard processes are commensurate ($Q^2_1 \simeq Q^2_2$), so that there is still not yet another large logarithm $\ln(Q_1^2/Q_2^2)$, which would require going beyond the leading logarithm approximation, in which, we recall, the most frequently variable $\xi$ is specified apart from a constant in view of an ambiguous choice of reference scale $\mu^2$.

It is readily verified by direct substitution that the solution of Eq. (4) can be written as the convolution of single distributions [29, 30]

$$
D^{i_{j_1}j_2}(x_1, x_2, t) = \sum_{j_1'j_2'} \int_{0}^{t} dt' \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} D^{i_{j_1}j_2'}(z_1 + z_2, t') \frac{1}{z_1 + z_2} \times P_{j'j_1} \left( \frac{z_1}{z_1 + z_2} \right) D^{i_{j_1'}j_2'}(z_1', t - t') D^{i_{j_2'}}(z_2', t - t').
$$

This expression coincides with the jet calculus rules [51, 52] proposed originally for the fragmentation functions and is a generalization of the Gribov-Lipatov relation installed for single functions [34, 36] (the distribution of bare partons inside a dressed constituent is identical to the distribution of dressed constituents in the fragmentation of a bare parton in the leading logarithm approximation). Nevertheless, the direct numerical integration of Eq. (4) occurs more effective [52] than the explicit solution (6) at a phenomenological handling because of the singular $\delta$-like initial conditions of single distributions $D^i_t(x, t)$ (the Green’s functions) incoming in this solution.

The solution presented in (6) shows that in the leading logarithm approximation dPDFs are strongly correlated at the parton level; that is,

$$
D^{i_{j_1}j_2}(x_1, x_2, t) \neq D^i_t(x_1, t) D^{j_1} j_2(x_2, t).
$$

The distributions of bare quarks and gluons in a hadron are more interesting for phenomenological applications. Apparently, it can be done within the well-known approach of factorizing soft and hard stages (physics of short and long distances) [54]. As a result, the equations (6) and (4) describe the evolution of parton distributions in a hadron with $t(Q^2)$, if one replaces the index $i$ by index $h$ only. However, the initial conditions for new equations at $t = 0(Q^2 = \mu^2)$ are unknown a priori and should be introduced phenomenologically being extracted from the experiments or taken from some models dealing with the physics of long distances [at the parton level, $D^i_t(x, t = 0) = \delta_{ij}(x - 1)$ and $D^i_{j_2 j_2}(x_1, x_2, t = 0) = 0$]. Nevertheless, the solutions of the generalized DGLAP evolution equations with the given initial conditions may be written as before via the convolution of single distributions and they read

$$
D^i_{h_{j_1}j_2}(x_1, x_2, t) = \sum_{j_1'j_2'} \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} D^{i_{j_1}j_2'}(z_1, z_2, 0) \times D^{j_1'}(z_1, t) D^{j_2'}(z_2, t),
$$

and

$$
D^i_{h_{j_1}j_2}(x_1, x_2, t) = \sum_{j_1'j_2'} \int_{0}^{t} dt' \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} D^{i_{j_1}j_2'}(z_1 + z_2, t') \frac{1}{z_1 + z_2} \times P_{j'j_1} \left( \frac{z_1}{z_1 + z_2} \right) D^{i_{j_1'}j_2'}(z_1', t - t') D^{i_{j_2'}}(z_2', t - t'),
$$

are the dynamically correlated distributions given by perturbative QCD (compare (6) to (10)).

The first term of generalized solution is a solution of homogeneous evolution equation (independent evolution
of two branches), where the input two-parton distribution is generally not known at the low scale $\mu(t = 0)$. For this non-perturbative two-parton function at low $z_1, z_2$ one may assume the factorization $D^{i_1 j_2'}(x_1, z_2, 0) \simeq D^{i_1 j_2'}(z_1, 0) D^{i_2 j_2'}(z_2, 0)$ neglecting the constraints imposed by the momentum conservation ($z_1 + z_2 < 1$). This leads to

$$D^{i_2 j_2'}(x_1, x_2, t) \simeq D^{i_1 j_2'}(x_1, t) D^{i_2 j_2'}(x_2, t)$$

(11) and justifies partly the factorization hypothesis for dPDFs usually applied in the practical calculations.

Surely, it is of great interest to know the magnitude of induced correlations with respect to the factorization component. Numerically, the contribution of these evolution-induced correlations was first estimated in Ref. \[32\]. The required (in these calculations) initial data for single parton distributions $D_1(x, 0)$ were specified at the scale of $Q_0 = \mu = 1.3$ GeV in accordance with the parametrization of the CTEQ Collaboration [52]. The ratio of gluon-gluon correlations in the proton that appear owing to the evolution to the factorized component,

$$R(x, t) = \left. \frac{D^{gg}_{QCD,corr}(x_1, x_2, t)}{D^{gg}_1(x_1, t)D^{gg}_2(x_2, t)(1-x_1-x_2)^2} \right|_{x_1 = x_2 = x}$$

(12) has been calculated and it has been shown that at the hard scale of the CDF measurement ($Q \sim 5$ GeV) the ratio $R$ is nearly 10% and increases right up to 30% at much higher scale ($Q \sim 100$ GeV) for the longitudinal momentum fractions $x \leq 0.1$ accessible to these measurements. The correlations may increase right up to 90% for the finite longitudinal momentum fractions $x \sim 0.2 \div 0.4$. They become important for almost all $x$ with $t$ increasing.

Here it should be underlined that the momentum conserving phase space factor $(1-x_1-x_2)^2$ is introduced in Eq. (12) instead of $(1-x_1-x_2)$ usually used. The reason is quite obvious again — this factor is introduced, generally speaking, ”by hand” in order to ”secure” the momentum conservation law, i.e. in order to make the product of two single distributions vanishing at $x_1 + x_2 = 1$. However, the generalized QCD evolution equations demand higher power of $(1-x_1-x_2)$ at $(x_1 + x_2) \to 1$: only the phase space integrals in Eqs. (6), (9) and (10) give

$$\int_{x_1}^{1-x_2} \int_{x_2}^{1-x_1} (1-x_1-x_2)^2/2.$$  

In fact, this exponent should depend on $t$ (getting larger with $t$ increasing) as it takes place for single distributions at $x \to 1$ [36]. The numerical calculations also support this assertion - the exponent of $(1-x_1-x_2)^2$ for the perturbative QCD gluon-gluon correlations is found to be larger than 2 and increasing with $t(Q)$. However, the introduced factor $(1-x_1-x_2)^2$ has practically no impact on the ratio under consideration in the region of small $x_1$ and $x_2$. But just this region, in which multiple interactions can contribute to the cross section noticeably, is especially interesting from experimental view point.

The properties of dPDFs in hadrons were studied in more detail [53, 57] by integrating directly the evolution equations (1) (in Ref. [52] only homogeneous evolution equations). This method demonstrates more efficiency since then one does not deal with the singular Green functions (single parton level functions satisfying singular delta-like conditions). These numerical investigations confirm also that the evolution effects are getting larger with increasing the hard scales.

Note that the particular solutions (10) of non-homogeneous equations contribute to the inclusive cross section of DPS with a larger weight (different effective cross section) [38, 37, 41, 42, 50, 57] being compared to the solutions (9) of homogeneous equations. The latter solutions are usually approximated by factorized form if initial nonperturbative correlations are absent. These initial conditions-correlations are a priori unknown, nevertheless they are not fully arbitrary but obey nontrivial sum rules [53, 56] which are esteemed by evolution equations. The problem of specifying the initial conditions-correlations for the evolution equations which obey exactly these sum rules and have the correct asymptotic behavior near the kinematical boundaries, has been extensively studied [53, 56, 57]. Fortunately, the explicit form of evolution equations solutions allows us to answer the question which correlations (perturbative (10) or nonperturbative (4)) are more significant in the regime of large enough hard scale.

### III. ASYMPTOTIC BEHAVIOR

Indeed, the evolution equations are explicitly solved by introducing the Mellin transformations

$$M^j_h(n, t) = \int_0^1 dx x^n D^j_h(x, t),$$

(13)

$$M^{j_1 j_2}_h(n_1, n_2, t) = \int_0^1 dx_1 dx_2 \theta(1-x_1-x_2)x_1^{n_1}x_2^{n_2}D^{j_1 j_2}_h(x_1, x_2, t),$$

(14)

which lead to a system of ordinary linear differential equations at the first order:

$$dM^j_h(n, t)/dt = \sum_{j'} M^{j'}_h(n, t) P_{j' \to j}(n),$$

(15)

$$dM^{j_1 j_2}_h(n_1, n_2, t)/dt = \sum_{j'_1} M^{j'_1 j_2}_h(n_1, n_2, t) P_{j'_1 \to j_1}(n_1) + \sum_{j'_2} M^{j_1 j'_2}_h(n_1, n_2, t) P_{j_2 \to j_2}(n_2) + \sum_{j'} M^{j'}_h(n_1 + n_2, t) P_{j' \to j_1 j_2}(n_1, n_2),$$

(16)
where the kernels,

\[ P_{j' \to j}(n) = \int_0^1 x^n P_{j' \to j}(x)dx, \quad (17) \]

\[ P_{j' \to j_1 j_2}(n_1, n_2) = \int_0^1 x^{n_1}(1 - x)^{n_2} P_{j' \to j_1 j_2}(x)dx, \quad (18) \]

are well-known and can be found in the explicit form, for instance, in Refs. 56 [52, 62].

In order to obtain the distributions in \( x \) representation, an inverse Mellin transformation should be performed

\[ xD^j_h(x, t) = \int \frac{dn}{2\pi i} x^{-n} M^j_h(n, t), \quad (19) \]

\[ x_1 x_2 D^{j_1 j_2}_h(x_1, x_2, t) = \int \frac{dn_1}{2\pi i} x_1^{-n_1} \int \frac{dn_2}{2\pi i} x_2^{-n_2} M^{j_1 j_2}_h(n_1, n_2, t), \quad (20) \]

where the integration runs along the imaginary axis to the right hand side from all \( n \) singularities. It can be done in a general case only numerically. However, the asymptotic behavior can be estimated in some interesting and particularly simple limits in the technique under consideration.

Note that the exact solution for single distributions in the moments representation can be written in a matrix form symbolically as

\[ M^j_t(n, t) = [\exp P(n)t]^j_t, \quad (21) \]

and the solutions of the generalized DGLAP evolution equations with the given initial conditions may be written again as a convolution of single distributions and, in the moments representation, they read

\[ M^{j_1 j_2}_h(n_1, n_2, t) = \sum_{j'_1 j'_2} M^{j_1 j'_2}_h(n_1, n_2, 0) M^{j'_1 j_2}_h(n_1, t) M^{j_2}_h(n_2, t) + M^{j_1 j_2}_{h(QCD)}(n_1, n_2, t), \quad (22) \]

\[ M^{j_1 j_2}_{h(QCD)}(n_1, n_2, t) = \sum_{j} M^{j}_h(n_1 + n_2, 0) M^{j_1 j_2}_h(n_1, n_2, t) \quad (23) \]

are the particular solutions of the complete equations with the zero initial conditions at the hadron level, and

\[ M^{j_1 j_2}_t(n_1, n_2, t) = \sum_{j'_1 j'_2} \int d\tau M^{j}_t(n_1 + n_2, \tau) \]

\[ \times P_{j' \to j_1 j_2}(n_1, n_2) M^{j_1}_t(n_1, t - \tau) M^{j_2}_t(n_2, t - \tau) \quad (24) \]

are the particular solutions of the complete equations with the zero initial conditions at the parton level. While the first term in the expression \( (22) \) represents the solutions of homogeneous evolution equations with the given initial conditions \( M^{j'_1 j'_2}_h(n_1, n_2, 0) \). These nonperturbative unknown two-parton initial conditions are just the reckoning for the unsolved confinement problem. If one assumes that there is the approximate factorization of these initial conditions in the moments representation

\[ M^{j'_1 j'_2}_h(n_1, n_2, 0) \approx M^{j'_1}_h(n_1, 0) M^{j'_2}_h(n_2, 0), \quad (25) \]

then the solutions of the homogeneous evolution equations will be approximately factorized in \( x \) representation as well.

Now we take up the initial condition effects and their impact on the asymptotic behavior \((t \to \infty)\) of the dPDFs. The equations \((22), (23)\) and \((24)\) show that the initial conditions are parts of the solutions with different dependence on evolution variable \( t \). In order to understand better the character of this dependence, at first we consider a toy model with only one type of partons (for instance, QCD theory with gluons only or six-dimensional \( \phi^3 \) theory \([51, 52]\)). In this case the dPDFs become simpler and look like

\[ M^{11}_h(n_1, n_2, t) = M^{11}_h(n_1, n_2, 0) \exp\{[P(n_1) + P(n_2)]t\} + P(n_1 + n_2) \frac{P(n_1 + n_2) - P(n_1) - P(n_2)}{P(n_1) + P(n_2) - P(n_1 + n_2)} \exp\{[P(n_1) + P(n_2)]t\}. \quad (26) \]

Thus, for a large enough \( t \) we have two different asymptotic regimes depending on the relation between kernels \( P(n_1 + n_2) \) and \( P(n_1) + P(n_2) \):

1. If \( P(n_1 + n_2) < P(n_1) + P(n_2) \), then

\[ M^{11}_h(n_1, n_2, t)|_{t \to \infty} = \left[ M^{11}_h(n_1, n_2, 0) + \frac{P(n_1 + n_2) - P(n_1) - P(n_2)}{P(n_1) + P(n_2) - P(n_1 + n_2)} \exp\{[P(n_1) + P(n_2)]t\} \right]. \quad (27) \]

2. If \( P(n_1 + n_2) > P(n_1) + P(n_2) \), then

\[ M^{11}_h(n_1, n_2, t)|_{t \to \infty} = \frac{P(n_1 + n_2) - P(n_1) - P(n_2)}{P(n_1 + n_2) + P(n_1) - P(n_2)} \exp\{[P(n_1) + P(n_2)]t\}. \quad (28) \]

The asymptotic behavior of dPDFs is independent of the initial conditions-correlations \( M^{11}_h(n_1, n_2, 0) \) at all for the second option and is specified by the correlations perturbatively calculated.

In the toy \( \phi^3 \)-model \([51, 52]\)

\[ P(n) = \frac{1}{(n + 2)(n + 3)} - \frac{1}{12}, \quad (29) \]

and both asymptotic regimes are realized. However, if we are interested in the dPDFs in the region of finite \( x_1 \)
and \( x_2 \), then we conclude that the dPDFs "forget" the initial conditions-correlations because their asymptotic behavior in this region is determined by large moments \( n_1 \) and \( n_2 \), where \( P(n_1 + n_2) > P(n_1) + P(n_2) \).

The presence of several parton types does not essentially complicate the analysis of the asymptotic behavior of the dPDFs. Indeed, in this case one has to express single parton distributions via the eigenfunctions of corresponding DGLAP equations (see, for instance, Refs. 33, 52, 63), to put them into Eqs. (22), (23) and (24) and to take the leading contributions into consideration only. As a result, the relation between maximum eigenvalues \( \Lambda(n_1 + n_2) \) and \( \Lambda(n_1) + \Lambda(n_2) \) will determine the asymptotic behavior regime of the dPDFs:

1. If \( \Lambda(n_1 + n_2) < \Lambda(n_1) + \Lambda(n_2) \), then the dPDFs are dependent on the initial conditions-correlations \( M^{(1)}(n_1, n_2, 0) \).
2. If \( \Lambda(n_1 + n_2) > \Lambda(n_1) + \Lambda(n_2) \), then the dPDFs are independent of the initial conditions-correlations \( M^{(1)}(n_1, n_2, 0) \).

The eigenvalues and the eigenfunctions for the single distributions in QCD have been thoroughly studied 34, 52, 63. The results of those calculations show that in QCD as well as in the case of a model example both asymptotic regimes are realized. Therefore, one needs to know the initial conditions-correlations (which, generally speaking, are arbitrary and should be extracted from the experiments) to determine even the asymptotic behavior of the dPDFs. However, we come again to the relation

\[
\Lambda(n_1 + n_2) > \Lambda(n_1) + \Lambda(n_2)
\]

for large moments \( n_1 \) and \( n_2 \) that determines the dPDFs in the region of not parametrically small \( x_1 \) and \( x_2 \), because \( \Lambda(n) \sim -\ln(n), n \gg 1 \) (the n-dependence of the eigenvalues \( \Lambda(n) \) can be found in detail, for instance, in Refs. 36, 52, 63).

IV. CONCLUSIONS

Using the explicit form of the solutions of evolution equations in the Mellin representation we conclude that

the dPDFs "forget" the initial conditions-correlations (unknown \textit{a priori}) at not parametrically small longitudinal momentum fractions and the correlations perturbatively calculated survive only in the limit of large enough hard scales. Such a dominance is the mathematical consequence of the relation between the maximum eigenvalues \( \Lambda(n) \) in the moments representation, \( \Lambda(n_1 + n_2) > \Lambda(n_1) + \Lambda(n_2) \), in QCD at large \( n_1 \) and \( n_2 \) (finite \( x_1 \) and \( x_2 \)). It is independent of the strength of the initial conditions-correlations at all.

The asymptotic behavior analysis "teaches" us a tendency only and tells nothing practical about the values of \( x_1, x_2 \) and \( t(Q^2) \) where the correlations start to be significant and the asymptotic behavior is a good approximation to the real one, but the numerical estimates 53, 63 share the conclusion that the perturbative correlation effects are meaningful in the kinematical region accessible to experimental measurements at the Tevatron and LHC energies.

The QCD dynamical correlations result effectively in the dependence 33, 50, 64 of the experimentally extracted effective cross section of DPS on the resolution scale unlike naively accepted expectations. The measurements covering a larger range of the resolution scale variation will demonstrate the evolution effects more distinctly in accordance with the numerical observation in Ref. 50 and the asymptotic QCD behavior considered above.

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