The $J/\psi$ production in PbPb ultraperipheral collisions at $\sqrt{s_{NN}} = 2.76$ TeV

Ya-ping Xie

Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China
Department of Physics, Lanzhou University, Lanzhou 730000, China and
Key Laboratory of Quark and Lepton Physics (MOE) and Institute of Particle Physics,
Central China Normal University, Wuhan 430079, China

Xurong Chen

Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

Abstract

We calculate the coherent and incoherent production of $J/\psi$ in PbPb ultraperipheral collisions. The production of $J/\psi$ in ultraperipheral collisions is product of photon flux distributions and cross section of photon-nucleus scatterings. The distributions of photon flux is computed in light-cone perturbation theory and the cross section of photon-nucleus scatterings is calculated in dipole model, we assume that the two gluons exchange contribution is the coherent cross section and the large-$N_c$ contribution is the incoherent cross section in photon-nucleus scattering. The numerical result of the rapidity distributions of $J/\psi$ production in PbPb ultraperipheral collisions at $\sqrt{s_{NN}} = 2.76$ TeV are compared with the experimental data measured by the ALICE collaboration.

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I. INTRODUCTION

The ultraperipheral collisions of relativistic heavy ions provide an opportunity to study the nucleus-nucleus interaction at high energies \cite{1, 2}. The heavy quarkonia can be produced in ultraperipheral relativistic heavy ion collisions. The $J/\psi$ photon productions in PbPb and p-Pb ultraperipheral collisions have recently been measured by the ALICE collaboration at CERN Large Hadron Collision (LHC) and the experimental data have been published in Refs. \cite{3–5}. On the other hand, many theoretical groups have studied the production of heavy vector mesons in ultraperipheral collisions at high energies using different approaches \cite{6–18}.

In the nucleus-nucleus collisions, when the impact parameter of the two nucleus is larger than the sum of their radius, there is no hadronic interactions between the two nucleus. The nuclei can interact by photons exchange. As photons can be emitted from nuclei at high energies. This collisions are called ultraperipheral collisions. Two types of photon scattering can occur in the ultraperipheral collisions. The first process is photon-nucleus scattering. The second one is photon-photon scattering. We only consider the vector meson production in the photon-nucleus scattering in this work.

The process of photon-nucleus scattering is well described in dipole mode \cite{19–21} including the wave functions of photon and vector meson \cite{22, 23} in small-$x$ physics. In dipole model, the photon can fluctuate into a dipole of quark and antiquark, and the dipole scatters on the nucleons by gluons exchange. There are some models of parameterization for the cross section between the dipole and nucleons. For example GBW model \cite{24, 25}, IIM model \cite{26, 27} and IPsat model \cite{28–30}. The wave function of photon can be calculated in light-cone perturbation theory, and the wave function of the vector meson can be parameterized in Gaus-LC and boosted-Gauss mode \cite{31}.

In the process of photon-nucleus scattering, the nucleus can remain intact or break up. If the nucleus remains intact, it is coherent process. If the nucleus breaks up, it is incoherent process. The authors of Refs. \cite{17, 30} presented the coherent and incoherent cross section. But, the ratio of incoherent to coherent of Ref. \cite{17} is lower than the experimental ratio. In this work, we distinguish coherent and incoherent cross section in a new mechanism, and we think the contribution of our calculation also contribute the ratio of the incoherent to the coherent production.

This paper is organized as follows: In Sec II, we present the calculation of distributions of the equivalent photon flux. In Sec III, the coherent and incoherent cross section are considered from color-dynamics, the numerical results are presented in Sec IV. The conclusions of this paper is in
II. THE DISTRIBUTIONS OF EQUIVALENT PHOTON FLUX

In Jackson’s textbook [32], the distributions of equivalent photon flux were calculated in classical electrodynamics, we shall calculate the distributions of equivalent photon flux in light cone perturbation theory in this work. We start with a photon emitted from a nucleus with \( Z \) electric charges, the emission of photon from a nucleus is illustrated in Fig. 1. We can write down their momentums in light-cone conventions, they are

\[
p = (p^+ + \frac{p_\perp^2 + m^2}{2p^+}, p_\perp), \quad q = \left((1 - \chi)p^+, \frac{(p_\perp - k_\perp)^2 + m^2}{2(1 - \chi)p^+}, p_\perp - k_\perp\right), \quad k = (\chi p^+, \frac{k_\perp^2}{2\chi p^+}, k_\perp)(1)
\]

The variable \( p \) is the momentum of the initial charged nucleus, \( p^+ = \sqrt{2E} \), where \( E \) is the energy of the nucleus, and \( k \) is the momentum of the photon, \( k^+ = \sqrt{2\omega} \), where \( \omega \) is the energy of the photon, and \( q \) is the momentum of the final charged nucleus, \( m \) is the mass of the nucleus, \( \chi \) is the momentum fraction of the initial nucleus carried by the photon. The cross section of the nucleus-nucleus collisions by photon exchange can be written as

\[
\sigma(AA) = \sum_{\lambda\alpha\beta} \int \frac{d^3k}{2(2\pi)^3} |\psi_{\alpha\beta}^\lambda(p, k)|^2 \sigma(\gamma A),
\]

(2)

where \( \sigma(\gamma A) \) is the cross section of photon-nucleus scattering. The \( \psi_{\alpha\beta}^\lambda(p, k) \) is the splitting wave function of the nucleus and photon, the splitting wave function in momentum space can be written as

\[
\psi_{\alpha\beta}^\lambda(p, k) = \frac{Ze}{\sqrt{8p^+k^+(p-k)^+}} \bar{u}_\beta(p-k)\gamma_\mu \cdot e^\mu_k(k)u_\alpha(p)
\]

(3)
where $\lambda$ denotes indice of polarization vector, $\alpha$ and $\beta$ denote the helicities of incoming and outgoing nucleus, and $\epsilon_\mu(k)$ is the polarization vector of the photon, where $\epsilon_\lambda(k) = (0, \frac{k_\perp + \epsilon_\perp}{k}, \epsilon_\perp')$, with $\epsilon_\perp' = \frac{1}{\gamma^2}(\mp 1, i)$. With the help of the light-cone theory, we can write the splitting wave function in momentum space as \[33\]

$$\psi^\lambda_{\alpha\beta}(p, k) = \frac{Ze}{\sqrt{k^+ (k_\perp - \chi p_\perp)^2 + \chi^2 m^2}} \times \begin{cases} \sqrt{2(k_\perp - \chi p_\perp) \cdot \epsilon_\perp'} |[\delta_{\alpha-\delta_{\beta-}} + (1 - \chi)\delta_{\alpha+\delta_{\beta+}}] + m\chi^2 \delta_{\alpha+\delta_{\beta-}}, & \lambda = 1, \\ \sqrt{2(k_\perp - \chi p_\perp) \cdot \epsilon_\perp'} |[\delta_{\alpha+\delta_{\beta+}} + (1 - \chi)\delta_{\alpha-\delta_{\beta-}}] - m\chi^2 \delta_{\alpha-\delta_{\beta+}}, & \lambda = 2, \end{cases} \tag{4}$$

The splitting wave function can be written in the coordinate space, which is fourier transform of the wave function in momentum space,

$$\psi^\lambda_{\alpha\beta}(p, k^+; \mathbf{x}) = \int d^2 k_\perp e^{ik_\perp \cdot \mathbf{x}} \psi^\lambda_{\alpha\beta}(p, k^+). \tag{5}$$

The splitting wave function of coordinate space reads

$$\psi^\lambda_{\alpha\beta}(p, k^+; \mathbf{x}) = \frac{2Ze\pi m}{\sqrt{k^+}} e^{i\chi p_\perp \cdot \mathbf{x}} \times \begin{cases} i\sqrt{2}K_1(\chi m|\mathbf{x}|) \mathbf{x} \cdot \epsilon_\perp' |[\delta_{\alpha-\delta_{\beta-}} + (1 - \chi)\delta_{\alpha+\delta_{\beta+}}] + \chi^2 K_0(\chi m|\mathbf{x}|) \delta_{\alpha+\delta_{\beta-}}, & \lambda = 1, \\ i\sqrt{2}K_1(\chi m|\mathbf{x}|) \mathbf{x} \cdot \epsilon_\perp' |[\delta_{\alpha+\delta_{\beta+}} + (1 - \chi)\delta_{\alpha-\delta_{\beta-}}] - \chi^2 K_0(\chi m|\mathbf{x}|) \delta_{\alpha-\delta_{\beta+}}, & \lambda = 2, \end{cases} \tag{6}$$

where $K_0(x)$ and $K_1(x)$ are modified Bessel functions.

we can write down the splitting wave function in momentum space as fourier transform of splitting wave function in coordinate space,

$$\psi^\lambda_{\alpha\beta}(p, k^+) = \int \frac{d^2 \mathbf{x}}{(2\pi)^2} e^{-ik_\perp \cdot \mathbf{x}} \psi^\lambda_{\alpha\beta}(p, k^+; \mathbf{x}). \tag{7}$$

Substituting Eqs. (6) and (7) into Eq. (2), we get

$$\sigma(\gamma A \rightarrow J/\psi A) = \sum_{\lambda\alpha\beta} \int \frac{d^3 k}{(2\pi)^3} |\psi^\lambda_{\alpha\beta}(p, k)|^2 \sigma(\gamma A \rightarrow J/\psi A)$$

$$= \sum_{\lambda\alpha\beta} \int \frac{d^2 k_\perp}{16\pi^3} \int \frac{d^2 \mathbf{x}}{(2\pi)^2} e^{-ik_\perp \cdot \mathbf{x}} \psi^\lambda_{\alpha\beta}(p, k^+; \mathbf{x}) \int \frac{d^2 \mathbf{x}'}{(2\pi)^2} e^{ik_\perp \cdot \mathbf{x}'} \psi^\lambda_{\alpha\beta}(p, k^+; \mathbf{x}') \sigma(\gamma A \rightarrow J/\psi A)$$

$$= \int \frac{d^2 \mathbf{x} dk^+}{k^+} \frac{Z^2 \alpha(\chi m|\mathbf{x}|)^2}{2\pi^2} \left\{K_1^2(\chi m|\mathbf{x}|)[1 + (1 - \chi)^2] + \chi^2 K_0^2(\chi m|\mathbf{x}|)\right\} \sigma(\gamma A \rightarrow J/\psi A), \tag{8}$$

with $dk^+/k^+ = d\omega/\omega$, we get the cross section of the nucleus-nucleus scattering by photon exchange

$$\sigma(\gamma A \rightarrow J/\psi A) = \int \frac{d\omega d^2 \mathbf{x}}{\omega} N(|\mathbf{x}|, \omega) \sigma(\gamma A), \tag{9}$$
where the distribution $N(|x|, \omega)$ is

$$N(|x|, \omega) = \frac{Z^2\alpha (\chi m x)^2}{2\pi^2 x^2} \left\{ K^2_2(\chi m |x|)[1 + (1 - \chi)^2] + \chi^2 K^2_0(\chi m |x|) \right\}. \quad (10)$$

From the classical electrodynamics \cite{32}, it is supposed that a charge $ze$ passes the origin at speed $v$ and impact parameter $x$, the electric field are $E_T(t) = \frac{ze\gamma_{LV}v}{(x^2 + \gamma_{LV}^2v^2)^{1/2}}$ and $E_L(t) = \frac{-ze\gamma_{LV}v}{(x^2 + \gamma_{LV}^2v^2)^{1/2}}$, the fourier transform for frequency spectra is $E(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(t)e^{i\omega t}dt$, the frequency spectra of electric field are $E_T(\omega) = \frac{ze\gamma_{LV}v}{(\gamma_{LV})^2}a^{1/2}\omega x/\gamma_{LV}K_1(\omega x/\gamma_{LV})$ and $E_L(\omega) = -i\frac{ze\gamma_{LV}v}{(\gamma_{LV})^2}a^{1/2}/\gamma_{LV}K_0(\omega x/\gamma_{LV})$, where $v \approx c \approx 1$ in ultraperipheral collisions, the equivalent photon flux per unit area is calculated as

$$N(|x|, \omega) = \frac{1}{8\pi^2}[E_T^2(\omega) + E_L^2(\omega)] = \frac{Z^2\alpha (\omega|x|/\gamma_L)^2}{\pi^2 x^2} \left[ K_1^2(\omega|x|/\gamma_L) + \frac{1}{\gamma_L^2} K_0^2(\omega|x|/\gamma_L) \right]. \quad (11)$$

The lorentz boost factor $\gamma_L$ in the collision reads $\gamma_L = E/m$, with $\chi = \omega/E$, we can write $\chi m |x| = \omega x/\gamma_L$. In the ultraperipheral collisions, it is easy to get $\chi \ll 1$, and $\gamma_L \gg 1$, we can neglect $\chi^2 K_0^2(\chi m |x|)$ and $\frac{1}{\gamma_L^2} K_0^2(\omega|x|/\gamma_L)$ in Eq. (10) and Eq. (11). The dominant of distribution of $N(|x|, \omega)$ is

$$N(|x|, \omega) = \frac{Z^2\alpha (\omega|x|/\gamma_L)^2}{\pi^2 x^2} K_1^2(\omega|x|/\gamma_L). \quad (12)$$

As to get the usable photon flux $n(\omega)$, we integrate $N(x, \omega)$ over the $x$ and its angle $\theta$ as Ref. \cite{34}, in the ultraperipheral collisions, with $|x| > 2R_A$, $|x|_{\text{min}} = 2R_A$, and $R_A$ is the radius of the nucleus, we get

$$n(\omega) = \int_0^{2\pi} d\theta \int_{2R_A}^{\infty} |x|d|x|N(x, \omega) = \int_{2R_A}^{\infty} |x|d|x| \frac{2Z^2\alpha (\omega|x|/\gamma_L)^2}{\pi x^2} K_1^2(\omega|x|/\gamma_L), \quad (13)$$

the integration result is

$$n(\omega) = \frac{2Z^2\alpha}{\pi} [\xi K_1(\xi)K_0(\xi) - \frac{\xi^2}{2} [K_1^2(\xi) - K_0^2(\xi)]] \quad (14)$$

where $\xi = 2\omega R_A/\gamma_L$, thus, the Eq. (9) can be written as

$$\sigma(AA \rightarrow J/\psi AA) = \int d\omega \frac{n(\omega)}{\omega} \sigma(\gamma A). \quad (15)$$

With $\omega = \frac{M_V}{2} \exp(y)$, we get $d\omega/\omega = dy$, and $y$ is the rapidity of the vector meson, with mass $M_V$. Finally, we get the rapidity distributions of the vector meson \cite{17}

$$\frac{d\sigma^{A_1A_2}}{dy} = n^{A_1}(y)\sigma^{A_2}(y) + n^{A_2}(-y)\sigma^{A_1}(-y), \quad (16)$$
where $\sigma^{\gamma A}(y)$ is the cross section of the photon-nucleus scattering, and $n(y)$ is the distribution of the photon flux, we shall calculate the cross section of photon-nucleus scattering at the next section.

III. THE CROSS SECTION OF COHERENT AND INCOHERENT PROCESSES

A. dipole model

After we calculate the rapidity distributions of the vector mesons in the ultraperipheral collisions, we only calculate the photon-nucleus cross section $\sigma^{\gamma A}(k)$ of Eq. (16). The process of photon-nucleus scattering is illustrated in Fig. 2. In small-$x$ physics, the dipole model describes the scattering of the photon-nucleus successfully. It is shown in Fig. 2 that the process of $\gamma A \rightarrow VA$ can be viewed as three steps. The photon breaks up into a pair of quark and antiquark at first step, the quark and antiquark are called dipole, the dipole scatters on the nucleons of the nuclei by gluons exchange at the second step, finally, the dipole becomes a vector meson at third step.

The differential cross section of vector meson in the photon-nucleus scattering can be written

$$\frac{d\sigma^{\gamma A \rightarrow VA}}{dt} = \frac{R^2_s(1 + \beta^2)}{16\pi} \left| A_{T,L}^{\gamma A \rightarrow VA}(x_A, Q^2, \Delta) \right|^2,$$

where $T$ and $L$ denote the transverse and longitudinal amplitudes. The factor $\beta$ is the ratio of the real part to the imaginary part of amplitude. It reads

$$\beta = \tan\left(\frac{\pi}{2}\lambda\right),$$

where $\lambda$ is calculated as

$$\lambda = \frac{\partial \ln(\text{Im} A(s))}{\partial \ln s}.$$
The factor $R_g^2$ reflects the skewdness, it gives

$$R_g = \frac{\Gamma(\lambda + 5/2)}{\Gamma(\lambda + 3)}.$$  \hspace{1cm} (20)

The amplitude of $A_{T,L}^{\gamma A \rightarrow V A}(x_A, Q^2, \Delta)$ contains three parts, the light cone wave function of photon fluctuating into $q\bar{q}$ dipole, the differential cross section of the dipole scatter on the nucleons, and the wave function of dipole recombining a vector meson. The amplitude reads

$$A_{T,L}^{\gamma A \rightarrow V A}(x_A, Q^2, \Delta) = i \int d^2r \int_{0}^{1} \frac{dz}{4\pi} \int d^2b (\Psi_V^{*} \Psi_{\gamma})_{T,L}(r, z) e^{-ib \cdot \Delta} \frac{d\sigma_{q\bar{q}}}{db},$$  \hspace{1cm} (21)

where $t = -\Delta^2$, $\Delta$ is the transfer momentum between the dipole and nucleons. Integrating over $t$, we can get the cross section $\sigma^{\gamma A}(k)$. The $x_A = M_V \exp(-y)/\sqrt{s_{NN}}$ is Bjorken variable, and $-Q^2$ is the virtuality of the photon, $b$ is the impact parameter between the dipole and the nucleons, $r$ is the size of the dipole, and $z$ is the momentum fraction of the photon carried by the quark or antiquark. The $(\Psi_V^{*} \Psi_{\gamma})_{T,L}(r, z)$ is the overlap of the functions of vector meson and the photon, the wave function of photon can be computed in light cone perturbation theory, the scalar function of the vector meson in this work is Gaus-LC model which can be found in Ref. [31]. In this work, we only consider the transverse amplitude, the transverse overlap reads

$$(\Psi_V^{*} \Psi_{\gamma})_{T}(r, z) = e_f e \frac{N_c}{\pi z(1 - z)} \{m_f^2 K_0(\epsilon r) \phi_T(r, z) - (z^2 + (1 - z)^2) \epsilon K_1(\epsilon r) \partial_r \phi_T(r, z)\},$$  \hspace{1cm} (22)

where $m_f$ is the mass of charm quark, $e_f$ is the electric charge of the charm quark, and $\epsilon = \sqrt{z(1 - z)Q^2 + m_f^2}$, $N_c$ is the number of the colors. The scalar function $\phi_T(r, z)$ of Gaus-LC model reads

$$\phi_T(r, z) = N_T (z(1 - z))^2 \exp\left(-\frac{r^2}{2R_T^2}\right).$$  \hspace{1cm} (23)

The scalar function of Boosted Gaussian model is

$$\phi_T(z, r) = N_T z(1 - z) \exp\left(-\frac{m_f^2 R^2}{8z(1 - z)} - \frac{2z(1 - z)r^2}{R^2} + \frac{m_f^2 R^2}{2}\right).$$  \hspace{1cm} (24)

The parameters of $N_T$, $R_T^2$ and $R^2$ we used are the same as Ref. [31].

**B. coherent cross section**

In the photon-nucleus scattering, the nucleus can remain intact or break up. If the nucleus remains intact, it is coherent process. If the nucleus breaks up, it is incoherent process. We consider
the coherent and incoherent contribution to the vector meson production using dipole model. We assume that the nucleus is made of \( A \gg 1 \) nucleons, and the dipole scatters off the nucleons by gluons exchange. The interaction is assumed perturbative, therefore, the two gluons exchange contribution is the leading order contribution. The cross section between dipole and nucleons is illustrated in Fig. 3. It is shown in Fig. 3 that the dipole scatters off the nucleons by two-gluon exchange. From QCD color dynamics, the color of two gluons can be color-singlet. The nucleus can remain color-singlet in the two-gluon exchange scattering. If the transfer momentum satisfies \( \Delta < 1/R_A \), the nucleus can remain intact. Thus, we can treat the two gluons exchange contribution as coherent cross section.

\[ \sigma_{qq}(x, r) = \sigma_0(1 - e^{-r^2 Q_S^2(x)/4}), \]  

(25)

with \( Q_S^2(x) = Q_{S0}^2(x_0/x)^\lambda \), \( Q_{S0}^2 = 1 \text{GeV}^2 \), the parameters \( \sigma_0, x_0, \lambda \) are presented in Ref. [24]. In this work, we use the GBW model as impact parameter dependent [35]. The differential cross section reads

\[ \frac{d\sigma_{qq}}{d^2b} = 2(1 - S_{x_A}(x_1, x_2, b)), \]  

(26)

where \( S_{x_A}(x_1, x_2, b) \) is the element of the \( S \)-matrix, we assume it is real. The \( x_1, x_2 \) are coordinates of the quark and antiquark in the coordinate space, where \( r = x_1 - x_2 \). The \( S_{x_A}(x_1, x_2, b) \) is
\[ S_{xA}(x_1, x_2, b) = e^{-Q^2_S(x_A, b)(x_1-x_2)^2/4}, \]  

(27)

where we use GBW model for the dipole amplitude, the saturated momentum of proton is

\[ Q^2_S(x_p) = Q^2_{S0}(x_0/x_p)^\lambda, \]

and\n
\[ Q^2_S(x_A) = A^{1/3} c(b) Q^2_{S0}(x_0/x_A)^\lambda \]

for A nucleons \[36\], if we consider the influences of impact parameter. It is modified as

\[ Q^2_S(x_A, b) = A^{1/3} Q^2_{S0} c(b) \sqrt{1 - b^2/R^2_A(x_0/x_A)^\lambda}, \]

(28)

where \( c(b) \) is a parameter. Therefore, we can write \( S_{xA}(x_1, x_2, b) \) as

\[ S_{xA}(x_1, x_2, b) = \exp\left(-\frac{Q^2_{S0} A^{1/3} c(b) \sqrt{1 - b^2/R^2_A(x_0/x_A)^\lambda}(x_1-x_2)^2}{4}\right). \]

(29)

where \( c(b), \lambda \) and \( x_0 \) are parameters to be fit from \( F^2 \). They are presented in next section. Then, we can write down the coherent differential cross section

\[ \frac{d\sigma}{dt} = \frac{R^2_g(1 + \beta^2)}{16\pi} \left| \int d^2r \int_0^1 dz \int d^2b (\Psi^*_V \Psi^*_\gamma)_T(r, z) e^{-ib\Delta_2}(1 - S_{xA}(x_1, x_2, b)) \right|^2, \]

(30)

where the \( S_{xA}(x_1, x_2, b) \) is defined in Eq. (29) and we only consider the real photon contribution in the ultraperipheral collisions.

C. incoherent cross section

Now, we consider double single gluon exchange cross section of the photon-nucleus scattering. The process with two single-gluon exchange is shown in Fig. 4. We can see that there are more two single gluon exchange in Fig. 4.

FIG. 4. The diagram of the scattering between the dipole and nucleons exchanging two-gluons including amplitude and conjugate amplitude, \( L \) is the length that the dipole penetrates through the nucleus, \( x_1 \) and \( x_2 \) are the coordinates of the quark and antiquark. \( x'_1 \) and \( x'_2 \) are the conjugate coordinates.
The $\zeta$ or $\eta$ are the ratios between length from initial scattering to first or second single gluon exchange scattering and the total length $L$. As to ensure that the final state vector meson is color-singlet, one single gluon exchange is impossible. Two single-gluon exchange can ensure that the final state vector meson and the nucleus are color-singlet, but the nucleons are not all color-singlet. In the high energy limits, the size of the nucleus is $r \approx \frac{m_p R_A}{p}$, the parton’s interaction radius is $r' \approx \frac{1}{x_p}$, in the ultraperipheral collision $x \ll \frac{1}{m_p R_A}$. The nucleons can exchange gluons easily after they emitted one gluon, the process is illustrated in the Fig. 4, the two nucleons exchange a gluon each other. The nucleons can ensure color singlet in the preocess, we think the contribution of the process depicted in Fig. 4 is part of the contribution of incoherent cross section in high energy limits.

The formulas of the differential cross section with double single gluon exchange can be written as [37]

$$d\sigma^{A \rightarrow V X} = \frac{R_y^2(1 + \beta^2)}{4\pi} \int d^2r \int_0^1 \frac{z}{4\pi} \int d^2b \int d^2r' \int_0^1 \frac{z'}{4\pi} \int d^2b' (\Psi_V \Psi_\gamma)_T (z, r) (\Psi_V^* \Psi_\gamma)_T (z', r') \times e^{-i(b-b') \cdot \Delta} \left(1 - S_{x_A} (x_1, x_2, b) - S_{x_A} (x_2', x_1', b') + \langle S_{x_A} (x_1, x_2, b) S_{x_A} (x_2', x_1', b') \rangle, \right)$$

(31)

where $\langle S_{x_A} (x_1, x_2, b) S_{x_A} (x_2', x_1', b') \rangle$ is the dipole-dipole correlator [38, 39]. It is also presented in Appendix of [37], which can be factorized into the product of $S_{x_A} (x_1, x_2, b) S_{x_A} (x_2', x_1', b')$ in MV model [40–42]. The detail calculation can be found in Refs. [38, 39]. We can write the dipole-dipole correlator as double integration of $\zeta$ and $\eta$ as follows:

$$\langle S_{x_A} (x_1, x_2, b) S_{x_A} (x_2', x_1', b') \rangle = S_{x_A} (x_1, x_2, b) S_{x_A} (x_2', x_1', b') + S_{x_A} (x_1, x_2, b) S_{x_A} (x_2', x_1', b')$$

$$\times \frac{1}{N_c^2} \left[ \frac{\mu^2 N_c}{2} F(x_1, x_2; x_2', x_1') \right]^2 \int_0^1 d\eta \int_0^{\eta} d\zeta e^{-\frac{\mu^2 N_c}{2} F(x_1, x_2; x_2', x_1')},$$

(32)

where the function $F(x, y; u, v)$ is defined in Ref. [39]. It reads

$$\mu^2 F(x, y; u, v) \equiv \frac{Q_s^2}{2C_F} (x - y) \cdot (u - v).$$

(33)

The relationships of $x_1, x_2$ and $r, b$ are easy to get. They can be written as

$$x_1 = b + r/2, \quad x_2 = b - r/2,$$

$$x_1' = b' + r'/2, \quad x_2' = b' - r'/2.$$  

(34)
We can get the two functions as
\[
\mu^2 F(x_1, x_2; x'_1, x'_2) = -\frac{Q_S^2}{2C_F} r \cdot r',
\] (35)
and
\[
\mu^2 F(x_1, x'_2; x_2, x'_1) = -\frac{Q_S^2}{8C_F} [(r + r')^2 - 4(b - b')^2].
\] (36)

Therefore, we can calculate the second line of Eq. (32) as
\[
\frac{1}{N_c^2} \left(\mu^2 F(x_1, x_2; x'_2, x'_1)\right)^2 \int_0^1 d\eta \int_0^\pi d\zeta e^{-\xi \mu^2 N_c F(x_1, x'_2; x_2, x'_1)}
\]
\[
= \frac{16(r \cdot r')^2}{N_c^2((r + r')^2 - 4(b - b')^2)} \left[ e^{\frac{Q_S^2((r + r')^2 - 4(b - b')^2)}{8}} - \frac{Q_S^2((r + r')^2 - 4(b - b')^2)}{8} - 1 \right].
\] (37)

If \( b - b' = 0 \), we get the same result as Eq. (49) of Ref. [39]. Then, the differential cross including large-\(N_c\) contribution can be written as
\[
\frac{d\sigma^{\gamma A \rightarrow V X}}{dt} = \frac{R^2_g(1 + \beta^2)}{4\pi} \int d^2 r \int_0^1 \frac{z}{4\pi} d\phi \int d^2 b \int d^2 r' \int_0^1 \frac{z'}{4\pi} d\phi' \int d^2 b' (\Psi_V^* \Psi_\gamma)_T(z, r)(\Psi_V^* \Psi_\gamma)_T(z', r')
\]
\[
\times e^{-i(b - b')} \Delta \left[ 1 - S_{xA}(x_1, x_2, b) - S_{xA}(x'_2, x'_1, b') + S_{xA}(x_1, x_2, b)S_{xA}(x'_2, x'_1, b') \right.
\]
\[
\left. + S_{xA}(x_1, x_2, b)S_{xA}(x'_2, x'_1, b') \right] \frac{16(r \cdot r')^2}{N_c^2((r + r')^2 - 4(b - b')^2)}
\]
\[
\times \left[ e^{\frac{Q_S^2((r + r')^2 - 4(b - b')^2)}{8}} - \frac{Q_S^2((r + r')^2 - 4(b - b')^2)}{8} - 1 \right].
\] (38)

We can see that the first two line of Eq. (38) is the just the coherent cross section. The rest is the large-\(N_c\) contributions. We think the large-\(N_c\) contribution should contribute incoherent cross section in the high energy limits. The large-\(N_c\) differential cross section reads
\[
\frac{d\sigma^{\gamma A \rightarrow V X}}{dt} = \frac{R^2_g(1 + \beta^2)}{4\pi} \int d^2 r \int_0^1 \frac{z}{4\pi} d\phi \int d^2 b \int d^2 r' \int_0^1 \frac{z'}{4\pi} d\phi' \int d^2 b' (\Psi_V^* \Psi_\gamma)_T(z, r)(\Psi_V^* \Psi_\gamma)_T(z', r')
\]
\[
\times e^{-i(b - b')} S_{xA}(x_1, x_2, b)S_{xA}(x'_2, x'_1, b') \frac{16(r \cdot r')^2}{N_c^2((r + r')^2 - 4(b - b')^2)}
\]
\[
\times \left[ e^{\frac{Q_S^2((r + r')^2 - 4(b - b')^2)}{8}} - \frac{Q_S^2((r + r')^2 - 4(b - b')^2)}{8} - 1 \right].
\] (39)

Using Eqs. (39) and (29), we get the incoherent cross section. Finally, with Eq. (16), we get the \( J/\psi \) rapidity distributions of coherent and incoherent production in PbPb ultraperipheral collisions.
IV. NUMERICAL RESULTS

The ALICE collaboration had measured the coherent and incoherent production in PbPb ultraperipheral collision at \( \sqrt{s_{NN}} = 2.76 \) TeV. The coherent and incoherent \( J/\psi \) production in the 
\(-0.9 < y < 0.9\) are \( d\sigma^{coh}/dy = 2.38^{+0.34}_{-0.24} \) mb and \( d\sigma^{inc coh}/dy = 0.98^{+0.19}_{-0.17} \) mb \[4\]. In the rapidity region \(-3.6 < y < 2.6\), the coherent production is \( d\sigma^{coh}/dy = 1.00 \pm 0.18^{+0.24}_{-0.26} \) mb \[3\]. The ratio of the incoherent to coherent is about \( 41.2^{+3.90}_{-2.1} \) at midrapidity. In the Ref. \[17\], the authors had calculate the production of the coherent and the incoherent in PbPb ultraperipheral collision. The prediction of Ref. \[17\] of coherent production is upper than the measurement of ALICE. The ratio of incoherent to coherent is \( 23\% \) using IPsat and Boosted Gaussian model, which is lower than the ratio of ALICE at midrapidity. We calculate the rapidity distributions of \( J/\psi \) production in PbPb ultraperipheral collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV, the following parameters are evolved in the calculations, the lorentz boost factor is \( \gamma_L = \sqrt{s_{NN}}/2m_p = 1482 \), the radius of Pb nucleus is \( R_A = 1.2A^{1/3} = 7 \) fm, with \( A = 208 \). The \( Q^2_{S0} = 1.0 \) GeV\(^2\), \( x_0 = 3.04 \times 10^{-4} \), \( \lambda = 0.229 \), \( c(b) = 0.312 \). The mass of \( J/\psi \) is \( M_V = 3.097 \) GeV, the mass of the charm quark is \( m_f = 1.4 \) GeV, and \( Q^2 = 0 \) GeV for the overlap of wave functions.

The parameters of GBW model are not very reliable because they are fit from the inclusive production of \( \gamma^* + p \rightarrow \gamma^* + p \). As there are no experimental data in the \( \gamma^* + A \rightarrow \gamma^* + A \). In the following calculation, the ratio between the incoherent and coherent is reliable.

We present the theoretical results and experimental data of \( J/\psi \) coherent and incoherent rapidity distributions in PbPb ultraperipheral collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV in Fig. 5 and Fig. 6.

The prediction of coherent production of \( J/\psi \) are shown in Fig. 5. The circle are experimental data measured by the ALICE collaboration \[3, 4\]. The solid curve and the dashed curve are predictions calculated in Boosted Gaussian and Gaus-LC model. The range of integrated transfer momentum is \( 0 < |t| < 0.1 \) GeV\(^2\). We can see the prediction of ours is larger than the experimental data. Because the parameters of our model is not reliable.

Now, let’s do the calculation of incoherent production from Eq. (39). In the calculation, we define \( \Delta b = b - b' \), and we suppose that \( \Delta b \ll b \). Then, we can take the approximation \( S(x_1', x_2', b') \approx S(x_1', x_2', b) \) in the calculation of incoherent production. The prediction of incoherent production of \( J/\psi \) are shown Fig. 6. The circle is the experimental data measured by the ALICE collaboration \[4\]. The range of integrated momentum is \( 0.1 \) GeV\(^2\) < \( |t| < 0.3 \) GeV\(^2\). We can see that the prediction of the incoherent give a good description to the data of ALICE. But the
The compared rapidity distributions of $J/\psi$ coherent production in PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, the circle are the experimental data of the ALICE collaboration [3, 4], the solid curve is used the Boosted Gaussian model, and dashed curve is used the Gaus-LC model. The coherent prediction is not a good description of the data of ALICE.

At the end of the day, let’s consider the ratio of incoherent to the coherent production, because the ratio of incoherent to the coherent is reliable. In our calculation, the ratio of incoherent to coherent prediction at midrapidity is 15% using Gaus-LC model and 16% using Boosted Gaussian at midrapidity. The results of ratio of incoherent to the coherent are presented in Table. I, where we add the ratio of Ref. [17]. We can see that the sum of the IPsat in Ref. [17] and this work give a good description of the ratio to the experimental ratio.

|               | IPsat model | IIM model | This work | IPsat+This work | IIM+This work |
|---------------|-------------|-----------|-----------|----------------|---------------|
| BG            | 22%         | 14%       | 16%       | 38%            | 30%           |
| Gaus-LC       | 21%         | 13%       | 15%       | 36%            | 28%           |
| ALICE         | 41.2$^{\pm3.0\%}_{-2.1\%}$ | 41.2$^{\pm3.0\%}_{-2.1\%}$ |

TABLE I. The prediction of ratio of incoherent to coherent at midrapidity at PbPb ultraperipheral collision, the prediction of IPsat and IIM model are taken from Ref. [17]. BG is the Boosted Gaussian wave function.
FIG. 6. The compared rapidity distributions of $J/\psi$ incoherent production in PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, the circle is the experimental data of the ALICE collaboration [3, 4], the solid curve is used the Boosted Gaussian model, and dashed curve is used the Gaus-LC model.

V. CONCLUSIONS

In summary, in this manuscript, we calculate the photon flux in the light-cone perturbative theory, and we get the same result as the classic electrodynamics. We calculate the production of $J/\psi$ in PbPb ultraperipheral collision at $\sqrt{s_{NN}} = 2.76$ TeV. The calculation is in the dipole picture. The Gaus-LC and Boosted Gaussian model are used in the forward wave function, and the GBW model is implemented in dipole cross section. We distinguish the coherent and incoherent cross section in the double gluon exchange and one gluon exchange. In the process where the nucleon exchange two gluon with the dipole, which is coherent process. In the process where the nucleon exchange one gluon with the dipole, we think it is incoherent process. In this work, the prediction of coherent is larger than the experimental data, but the ratio of the incoherent and coherent is reliable. We compute the ratio of the incoherent to the coherent, which is about 15% and 16%. We think the production of this work and Ref. [17] both contribute the incoherent production in PbPb ultraperipheral collision.
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