Muonless events in ICAL at INO

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ABSTRACT: The primary physics signal events in the Iron Calorimeter at India-based Neutrino Observatory are the $\nu_\mu$ charged current (CC) interactions with a well defined muon track. Apart from these events, the Iron Calorimeter can also detect other types of neutrino interactions, i.e. the electron neutrino charged current interactions and the neutral current events. It is possible to have a dataset containing mostly $\nu_e$CC events, by imposing appropriate selection cuts on the events. The $\nu_\mu$CC and the neutral current events form the background to these events. This study uses Monte Carlo generated neutrino events, to design the necessary selection cuts to obtain a $\nu_e$CC rich dataset. An optimized set of constraints are developed which balance the need for improving the purity of the sample and having a large enough event sample. Depending on the constraints used, one can obtain a neutrino data sample with the purity of $\nu_e$ events varying between 55\% to 70\%.

KEYWORDS: Performance of High Energy Physics Detectors; Neutrino detectors

ArXiv ePrint: 1501.03252
1 Introduction

India-based Neutrino Observatory or the INO, is an experimental facility to be set up in the southern part of India. An important component of INO will be the Iron Calorimeter (ICAL). The ICAL aims to study the interactions of atmospheric neutrinos and antineutrinos. The determination of the neutrino mass hierarchy is one of its prime goals, apart from adding to the precision of the oscillation parameters [1].

ICAL is a giant magnetized neutrino detector, with Resistive Plate Chambers (RPCs) as the active detector elements [2–4]. Efficient tracking abilities, good energy and timing resolutions, good identification of the charge of the particles are the essential capabilities of this detector [5, 6].

ICAL comprises of three modules, with ~30,000 RPCs, and 151 iron layers weighing about 50 ktons in total. A sketch of ICAL is shown in figure 1. Each module contains $8 \times 8$ RPCs in a
layer, each of which spans a surface of $1.84\,\text{m} \times 1.84\,\text{m}$ and is $2.5\,\text{cm}$ in width (height) [7]. The steel structures to support them, however, occupy gaps of 16 cm in between the RPCs. An additional pair of slits is also present in each module, making way for the current carrying coils of the magnet. The RPC layers are interspaced with iron plates of 5.6 cm thickness and a gap of 4 cm, thus making the successive RPCs 9.6 cm apart from each other in the vertical direction. The 4 cm gap, in addition to housing the RPC units, also provides air circulation to cool the current carrying coils.

The iron layers not only serve as the heavy target for the neutrinos but also carry the solenoidal magnetic field in the detector [8]. The neutrinos produce charged/neutral particles during interactions, which propagate through the detector. The iron layers are magnetised to $\sim 1.3\,\text{T}$. The reconstruction of the trajectory of a charged particle in the magnetic field provides both their momentum measurement and charge determination. The kilometre-thick rock covering the INO cavern cuts down the cosmic ray particles. Hence, most of the events in the detector are produced by neutrino interactions only. The analysis of the properties of the particles, produced by the neutrino interaction in the detector, allows the study of the properties of atmospheric neutrinos [9].

The primary cosmic ray particles (like protons) interact with the atmospheric nuclei in the upper layers. To a large extent, these interactions produce pions, which eventually decay into muons and muon neutrinos. The muons can further decay, leading to both muon neutrinos and electron neutrinos in the final state. Effects of oscillations can be seen on the upward going neutrinos which pass through the Earth [10]. Oscillations lead to the creation of the tau neutrinos in the neutrino flux.

The neutrinos and anti-neutrinos undergo two types of interactions, depending on the mediating particle. The charged current (CC) interactions are mediated by the $W^\pm$ particles and the neutral current (NC) interactions by the $Z^0$. The CC events are different for each flavour and can be distinguished by the charged lepton in the final state. If the charge of this lepton can be determined, then a distinction between neutrino and anti-neutrino interactions can be made. The NC events have a neutrino/antineutrino in the final state hence it is not possible to distinguish either the flavour or neutrino from anti-neutrino [11].
The CC interactions can be quasi elastic (QE), resonance scattering (RS) or deep-inelastic scattering (DIS). The QE interaction gives a lepton corresponding to the neutrino flavor. The RS interaction produces one/two pions too in addition to the lepton. The DIS interaction, which dominates beyond the neutrino energy of 4 GeV, gives a shower of hadrons in addition to the charged lepton.

The detectable NC interactions can be of RS or DIS type only. The RS and DIS interactions produce hadrons and a neutrino which goes undetected. So, only the hadron showers are visible in the detector in these cases. In the case of NC elastic scattering, because of no hadron shower in the final state, the event cannot be detected.

The presence of the thick iron layers puts a lower threshold on the minimum energy of a detectable particle. Therefore, the detection probability of sub-GeV neutrinos is very small, due to the trigger criteria, which is added to remove random noise events.

The ICAL is more efficient in the study of muon neutrinos in the GeV range. The presence of the magnetic field further enables us to tell apart the $\mu^-$ tracks from the $\mu^+$ tracks in the detector. However, apart from these well-recognizable muon track events, i.e., $\nu_\mu/\bar{\nu}_\mu$CC interactions, the ICAL will also contain $\nu_e/\bar{\nu}_e$CC interactions and also the NC interactions of all three flavors [12]. These interactions do not give clear muon tracks like the $\nu_\mu$CC events. Therefore, apart from the muon track containing events, we should also focus on these “muonless” events, in order to extract maximum possible information from the ICAL detector.

The paper is organized as follows. In section 2, the details about the generated data which we have used in the study are explained, along with the details of how an event is detected and characterised. Section 3 deals with the application of various selection criteria which aim to increase $\nu_e$CC purity (fraction of $\nu_e$CC events) in the sample. The criteria are explained in detail, along with their physics justification. Their effects on the total dataset are included in the corresponding subsections. Some of the criteria are applied on the total dataset to check if the percentage of NC events can be enhanced, in section 4. Section 5 discusses the background contribution of the $\nu_e$CC and the NC events to the $\nu_\mu$CC events sample. In section 6, we have studied the contribution of the muonless events in determining the neutrino mass hierarchy. Section 7 concludes with a summary of the results and a few important comments.

Very few experiments have been able to study $\nu_e/\bar{\nu}_e$ events at higher energies. IMB, Kamioka, Super-Kamioka and very recently MINOS and T2K data are perhaps the only examples. ICAL is sensitive to neutrinos in the energy range of GeVs. Therefore, obtaining a source of data rich in electron neutrinos in this higher energy range is an important aspect for the whole neutrino community. One will be able to study the characteristics of these high energy $\nu_e/\bar{\nu}_e$s from the atmosphere. Other possible physics studies are the contribution of these events in the determination of the neutrino Mass Hierarchy, and also the use of NC events in searches for the sterile neutrinos [13, 14].

2 Simulations

The following analysis is done with neutrino events generated by the Nuance neutrino generator [15]. To reduce the influence of statistical fluctuations, we have generated 500 years of data. The relative error due to fluctuations in the simulations is about 1% of the corresponding error in the actual data recorded in about 10 years.
To begin with, we use normal hierarchy (NH) parameters. Simulations with inverted hierarchy parameters (IH) have also been checked, but not mentioned here, to avoid repetition. The generated events are then simulated in the ICAL detector using GEANT4 [16].

2.1 Signal-detection

Charged particles produced by the neutrino interactions pass through one or more RPCs and generate hits. These hits are our primary signals. The RPC layer number gives the z-coordinate of the hit. The x and y-coordinates are given by the copper-strips of the pick-up panels which are orthogonally oriented at the top and the bottom of the RPCs. The number of strips in x-direction, with a signal, gives x-strips and similarly in y-direction gives y-strips. The maximum of the number of x-strips or y-strips is defined to be the number of “strips-hit” in that layer. This number of strips-hit in a layer, when summed over all the layers which have received hits in the event, gives the number of strips-hit. The hits distribution mentioned hereafter refers to this value.

2.2 Types of events and their signatures

Vertical and high energy muons travel through large number of layers before stopping/decaying. Therefore, the $\nu_\mu$s which have high energy and are incident mostly along the vertical direction give hits in large number of layers, in case of CC interactions. In fact, the muons thus produced, form clear tracks in the detector and their momentum can be reconstructed. The curvature induced along the muon path due to the magnetic field, leads to the charge identification. On the contrary, the $\nu_\mu$s which have lower energy or are incident mostly along the horizontal direction [17] are confined to a smaller number of layers. They can hardly be distinguished from the hadron showers which also emanate from the event vertex.

The $\nu_e$CC events produce electrons which can give rise to em showers, but no track can be seen. The NC events have no charged lepton in the final state and hence have lower number of hits. In these events, only the final state hadrons are visible in the detector. They are indistinguishable from $\nu_e$CC events [18, 19].

3 Selection criteria to obtain $\nu_e$CC rich sample

We devise certain conditions to ensure that the selected event sample contains mostly $\nu_e$CC events, with minimum possible background of $\nu_\mu$CC and NC events.

The total number of hits created by the produced particles in the detector and the number of layers with hits are first studied.

In order to understand the behaviour of the different neutrino events in the detector, we first look at the hits distributions of all the three event types: the NC, the $\nu_e$CC and the $\nu_\mu$CC, in different energy ranges of the incident neutrinos.

The distributions in figure 2 show that, for $\nu_\mu$CC events, the number of hits increases with the energy of the incoming neutrino. This increase is much less for $\nu_e$CC events and hardly exists in case of the NC events. The figure also clearly suggests a lower threshold of $\sim 10$ hits to suppress a large fraction of NC events and low energy $\nu_e$, $\nu_\mu$CC events. With this cut, 12% of the total NC events are retained. The survival fraction for $\nu_e$CC events is about 18%. So, out of the total set of survived $\nu_e$CC and NC, more than 60% are $\nu_e$CC events, as seen in table 1. The events containing
Figure 2. Hits distribution in the neutrino energy bins (from top to bottom in order): $E_\nu = \{0.1, 0.8\}$ GeV; $E_\nu = \{0.8, 5.0\}$ GeV; $E_\nu = \{5.0, 20.0\}$ GeV; $E_\nu = \{20.0, 100.0\}$ GeV, in case of the three types of neutrino events (from left to right in each row): others (all NC + $\nu_\tau$CC); $\nu_e$CC; $\nu_\mu$CC events. The x-axis gives the number of hits, and the y-axis the events count.
high energy muons can be separated by restricting the number of layers or by identifying the muon track in ICAL.

3.1 Hits and layers

The electrons/positrons, in general, travel a shorter distance than the hadrons. On the other extreme, the muons of the $\nu_\mu$ CC events travel through several layers. A primary observation of the layer distribution shows that a cut on the number of layers hit in an event is an effective criterion. Here the “layers” refer to the number of layers which receive one or more hits in an event. However, the layer cut is a very sensitive cut, owing to the thick iron layer in between two RPC layers.

As mentioned earlier, $\nu_\mu$ CC events either with low energy muons and/or in horizontal direction, do not have any identifiable muon track. Such events have been found to be a significant contribution in the selected events sample. So, a separate count is maintained for them. The rest of the backgrounds, i.e. the $\nu_e$ NC, $\nu_\mu$ NC, $\nu_\tau$ CC and NC are all contained in the “others”. The $\nu_\tau$ CC events being pretty small in number, are not separately counted.

The $\nu_e$ CC events layer distribution, after a minimal hits cut (typically 10), peaks around 5 while that of the $\nu_\mu$ CC events peaks around 10, as seen in figure 3. So, by selecting events which are confined in 5 layers or less, we can reject $\nu_\mu$ CC events which are energetic and/or vertical. However, low energy $\nu_\mu$ CC events, especially those in the horizontal direction, do pass this cut and give rise to the events listed below. Various cuts on the number of hits and layers have been imposed on the set of events $E_\nu = \{0.1, 100\}$ GeV. A few significant ones among them are listed in table 1.

As shown in table 1, requiring the number of layers with hits in the event to be below cut value, leads to a reduction in the fraction of $\nu_\mu$ CC events in the sample.

The average number of hits per layer has been studied as a mean to eliminate events containing tracks. The muon tracks give mostly 2–3 hits in a layer. So, applying a lower cut on the average hits per layer (hpl) seems to be quite reasonable in rejecting most $\nu_\mu$ CC events. The hits per layer cut is useful in studying vertical, high energy muon events. Here, given that the number of layers is constrained to be smaller than or equal to 5, it is not very effective. The addition of this cut leads only to a marginal improvement.

![Figure 3. Distribution of number of layers for all events with more than 10 hits, for the 500 years NH data in $E_\nu = \{0.1, 100\}$ GeV.](image)
Table 1. Events counts after applying the selection cuts on the Geant output for the NH 500 years data in $E_\nu = \{0.1, 100\}$ GeV.

| Selection criteria | $\nu_e$CC | others | $\nu_\mu$CC |
|--------------------|----------|--------|-------------|
| hits > 0           | 1106742  | 1050814| 1682527     |
| hits > 10 (only)   | 202838   | 130642 | 672566      |
| hits > 15 (only)   | 97535    | 69340  | 445977      |
| hits > 20 (only)   | 52398    | 42476  | 314597      |
| hits > 15; layers ≤ 4 | 47711   | 19390  | 19875       |
| hits > 15; layers ≤ 5 | 68702   | 32953  | 36211       |
| hits > 15; layers ≤ 7 | 89614   | 52550  | 76194       |
| hits > 10; layers ≤ 4 | 125321  | 56177  | 62113       |
| hits > 10; layers ≤ 5 | 163807  | 82717  | 107350      |

Applying the above or similar cuts, it is seen that the fraction of $\nu_e$CC events in the sample increases, but at the cost of sample size and fraction of vertical events. Hence, an optimized set of criteria needs to be chosen.

3.2 Distribution pattern of the hits in the layers

The cuts on the basis of hits and layers are indeed the simplest and very effective selection criteria. However, a number of various other parameters have also been studied, to ensure how much they can contribute to improve the purity of the $\nu_e$CC events in the sample. The purity of the $\nu_e$CC events in a sample is the ratio of the number of $\nu_e$CC events to the total number of events in the selected sample.

The behaviour of a $\nu_e$-interaction is certainly different from the other two types of interactions, as far our physics knowledge is concerned. The presence of the electron/positron makes it stand apart from the NC interactions. The way electrons/positrons lose their energies in the detector is different from the way $\mu^+/\mu^-$ do. The challenge is to utilize these characteristics in distinguishing $\nu_e$ events from $\nu_\mu$ events in the data from the ICAL detector.

3.2.1 Maximum hits difference

The $\nu_e$CC events contain em showers. They should generate a huge number of hits, but most of them are absorbed by the thick iron layers. However, in some events the shower may start at the edge of the iron layer. In such cases, a sudden and significant increment in the number of hits in the following layer is expected.

The difference in the number of hits in two adjacent layers in an event is calculated. This difference is maximized over all such pairs in that event. The value of the maximum difference in hits thus obtained forms our present selection criteria. The effect of this cut is shown in table 2.

This selection criteria helps in improving the $\nu_e$CC events ratio by about 3–4%. However, a simultaneous study of the Nuance MC information shows that the larger hits difference is given by mostly horizontal $\nu_e$CC events.
Table 2. Events counts after applying the hits-layers selection criteria and adding the cut on maximum difference in the number of hits in adjacent layers. (500 years NH data in $E_\nu = \{0.1, 100\}$ GeV.) [“h” = # hits; “L” = # Layers.]

| Selection criteria | $\nu_e$CC | others | $\nu_\mu$CC |
|--------------------|----------|--------|-------------|
| $h > 10; L \leq 5$ | 163807   | 82717  | 107350      |
| $h > 10; L \leq 5$; max hits diff. $> 5$ | 82500    | 34701  | 38824       |
| $h > 15; L \leq 5$ | 68702    | 32953  | 36211       |
| $h > 15; L \leq 5$; max hits diff. $> 5$ | 50295    | 21844  | 23991       |

Table 3. Events counts after applying the hits-layers selection criterion and demanding (i) 5 additional hits in adjacent layers ($h_{L}, h_{L+1}$); (ii) 50–60% of total number of hits in one layer. (500 years NH data in $E_\nu = \{0.1, 100\}$ GeV.) [“hits” = total # hits; “$h_{L}$” = hits in any of the layers, say the Lth layer.]

| Selection criteria | $\nu_e$CC | others | $\nu_\mu$CC | $\nu_e$CC purity % |
|--------------------|----------|--------|-------------|---------------------|
| hits $> 15$; layers $\leq 5$ | 68702    | 32953  | 36211       | 50                  |
| hits $> 15$; layers $\leq 5$; $h_{L} > h_{L\pm 1} + 5$ | 47009    | 21191  | 22934       | 52                  |
| hits $> 15$; layers $\leq 5$; $h_{L} > 50$% hits | 38479    | 13745  | 16934       | 56                  |
| hits $> 15$; layers $\leq 5$; $h_{L} > 60$% hits | 29123    | 9038   | 11948       | 58                  |
| hits $> 15$; layers $\leq 4$ | 47711    | 19390  | 19875       | 55                  |
| hits $> 15$; layers $\leq 4$; $h_{L} > h_{L\pm 1} + 5$ | 34399    | 13308  | 13868       | 56                  |
| hits $> 15$; layers $\leq 4$; $h_{L} > 50$% hits | 32737    | 10931  | 12679       | 58                  |
| hits $> 15$; layers $\leq 4$; $h_{L} > 60$% hits | 26006    | 7735   | 9690        | 60                  |

3.2.2 Comparing the hits in each layer

The number of hits in every individual layer in an event is studied. This criterion, in a way seeks a pattern in the number of hits in adjacent layers. A variety of patterns are assumed and checked with the set of events. Two of them are stated below. The underlying logic still rests on the concept of the em shower.

Additional hits in the next layer. One of the layers hit is chosen and additional 5 or 6 hits are demanded in the very next layer. All the layers in the event are checked. The event to be selected must have at least one such pair of layers. A lower threshold of 2 layers becomes inherent.

Majority of hits in one layer. One can call this criterion a modified version of the earlier one. According to this criteria, the event must contain 50% or 60% of the total number of hits in a single layer. Therefore, no lower cut on the number of layers is required here.

The effect of the selection cuts are shown in table 3.

If the statistical errors are considered for the event counts, then the corresponding variations in the calculated values of the purity are far less than unity (the value of this purity, as a percentage, varies by at most 1.5 when the sample size falls to 10 events/year).
Figure 4. Schematic diagram of hits in the RPC layers.

Table 4. Events counts after applying the hits-layers selection criteria; adding the cut on the variance of the mean of the vertical distribution of hits in layers, i.e. rms; the criteria of max hits diff. is included for a further improvement. (500 years NH data in $E_\nu = \{0.1, 100\}$ GeV.) [$\text{“h”} =$ # hits; “L” = # Layers; “hpl” = avg hits/layer.]

| Selection criteria | $\nu_e$CC | others | $\nu_\mu$CC | $\nu_e$CC purity % |
|--------------------|-----------|--------|-------------|-------------------|
| $h > 15; L \leq 5$ | 68702     | 32953  | 36211       | 50                |
| $h > 15; L \leq 5; \text{rms} < 1.2$ | 56254     | 24916  | 25431       | 53                |
| $h > 15; L \leq 5; \text{rms} < 1.2; \text{max hits diff.} > 4$ | 48248     | 20452  | 21241       | 54                |
| $h > 15; L \leq 5; \text{rms} < 1.2; \text{max hits diff.} > 6$ | 39610     | 15585  | 16969       | 55                |
| $h > 10; L \leq 4$ | 125321    | 56177  | 62113       | 51                |
| $h > 10; L \leq 4; \text{rms} < 1.2$ | 111858    | 47961  | 52860       | 53                |
| $h > 10; L \leq 4; \text{rms} < 1.2; \text{max hits diff.} > 3$ | 86157     | 35115  | 37026       | 54                |
| $h > 10; L \leq 5; \text{rms} < 1.2; \text{max hits diff.} > 3$ | 99814     | 43409  | 46455       | 56                |
| $h > 10; \text{mean} < 2; \text{rms} < 1.2; \text{max hits diff.} > 3$ | 83954     | 35130  | 36127       | 54                |
| $h > 10; \text{mean} < 2; \text{rms} < 1.2; \text{max hits diff.} > 5$ | 60959     | 23063  | 24129       | 56                |
| $h > 10; \text{mean} < 2; \text{rms} < 1.2; \text{max hits diff.} > 5; \text{hpl} > 4$ | 51249     | 18247  | 18922       | 58                |

3.2.3 The overall distribution pattern of hits from the average number of hits among the layers

The hits in different layers of the $\nu_e$CC events are non-uniform. The hits are mostly over concentrated in some layers, while entirely sparse in the rest. Figure 4 shows the hit pattern among various layers in an event (left panel) and the number of hits vs layer number (right panel). For the plot in the right panel, the lowest layer hit is labelled to be 0, the next 1 and so on. In such a plot, the $\nu_\mu$CC gives a broader peak than the $\nu_e$CC/NC. Hence, events selected with such sharper peaks should reduce the fraction of $\nu_\mu$CC events in our sample. This property can be parametrized as either the mean or RMS value of the layerwise hits distribution of each event. However, having studied both the quantities, the cut on the RMS value appeared comparatively more effective. Some of the results are shown in table 4.

The RMS cut appears quite effective in improving the ratio. In fact, the cut on the maximum difference in the hits further helps in improving the results. Therefore, one can obtain a $\sim 55\%$ $\nu_e$CC events purity, with a moderately large sample size.
Figure 5. Distribution of number of hits for all non-zero hit events with $E_\nu = \{0.1, 100\}$ GeV for 500 years of NH data.

Figure 6. Distribution of number of layers which received one or more hits in an event, $E_\nu = \{0.1, 100\}$ GeV for 500 years of NH data.

4 The NC events fraction

Our primary/main focus in this paper is to obtain a $\nu_e$CC rich sample of neutrino events at ICAL. So, the selection cuts so far have been favoring $\nu_e$CC events. However, the NC events fraction can also be enhanced comparatively.

The cuts are based on the simplest criteria of hits and layers. However, the hardware threshold to be put in ICAL for accepting NC events must be taken into account. This requires a trigger algorithm different from that for the muon track-containing events.

The NC events give mostly very few hits, and are confined in very few layers, see figures 5 and 6. So, to obtain a NC events rich sample, one might be tempted to put an upper threshold on the number of hits. But, to deal with such a small number of hits in an event, sub-GeV neutrinos should be accounted too. So, the dataset including neutrinos with $E_\nu = \{0.1,100\}$ GeV is the appropriate one.
Table 5. Enhancing NC fraction: events counts after applying the selection cuts on the Geant output of the NH 500 years data files in $E_\nu = \{0.1,100\}$ GeV.

| Selection criteria       | $\nu_e$ CC (NH evts) | others (NH evts) | $\nu_\mu$ CC (NH evts) | NC purity % |
|--------------------------|----------------------|------------------|-------------------------|-------------|
| $0 < \text{hits} \leq 10$ (only) | 903904              | 920172           | 1009961                 | 32          |
| $0 < \text{hits} \leq 10$; layers $\leq 2$ | 659926              | 724065           | 478480                  | 39          |
| $0 < \text{hits} < 4$ (only) | 406705              | 568177           | 345322                  | 43          |
| $0 < \text{hits} < 4$; layers $\leq 2$ | 397895              | 558109           | 321648                  | 44          |
| $0 < \text{hits} < 4$; layers $= 1$ | 287799              | 436263           | 198539                  | 47          |
| $4 \leq \text{hits} \leq 10$; layers $= 1$ | 70330               | 38360            | 31046                   | 27          |

Table 6. Events counts before applying the selection cuts on the Geant output of the NH 500 years data files in $E_\nu = \{0.8,20\}$ GeV.

| Selection criteria       | $\nu_e$ CC | others (NC+$\nu_e$ CC) | $\nu_\mu$ CC |
|--------------------------|------------|-------------------------|--------------|
| all generated events     | 676014     | 820854                  | 1103263      |
| # events with hits $> 0$ in ICAL | 649487     | 678590                  | 1087709      |

The NC counts in the selected sample are almost equal to the sum of the selected $\nu_e$ CC and the $\nu_\mu$ CC events. The NC events have a very small number of hits in general. The cuts used earlier demanded a minimum of 10 hits and hence discriminated against the NC events. The dominance of the NC events can be gradually realized in case of events having 10 hits or less, as shown in table 5.

If we demand number of hits $\leq 3$ and 1 or 2 layers, we get the event samples shown in table 5, which are quite rich in NC events. If the noise is kept under control, such events can be used to study mixing with sterile neutrinos. Trigger efficiency will play a major role in selecting such events. In fact, it has been checked that a sample of single hit events has more than 50% of NC events. But obviously, just one-hit is an unacceptable criteria. Therefore, the selection cuts will have to be redesigned entirely, to obtain a NC events sample with a significant purity.

5 $\nu_e$ CC and NC events as background to $\nu_\mu$ CC events

Events with a muon track are the primary data for the ICAL, especially those within the range $E_\nu = \{0.8,20\}$ GeV. The $\nu_\mu$ CC events detected at ICAL must pass through (i.e. give hits in) a minimum number of layers (5 or 6), so that the muon track can be reconstructed. This layer cut will undoubtedly select mostly $\nu_\mu$ CC events. However, some $\nu_e$ CC and NC events also will pass this cut and be background to the $\nu_\mu$ CC events sample.

As shown in table 6, out of all the generated events, about 20% of the “others” do not give any hit in the ICAL. For $\nu_e$ CC and $\nu_\mu$ CC events, this fraction of “undetectable” events is about 5%. The layers distribution of each type of detectable events (for the energy range $E_\nu = \{0.1,100\}$ GeV) is shown in figure 6.
Table 7. Events counts after applying the selection cuts on the Geant output for the 500 years of NH data with $E_\nu = \{0.8, 20\}$ GeV.

| Selection criteria | $\nu_e$CC | others (NC+$\nu_e$CC) | $\nu_\mu$CC |
|--------------------|----------|-----------------------|-------------|
| # events: $L \geq 5$ | 84115 | 73849 | 683635 |
| ~ 10% | ~ 9% | ~ 81% |
| # events: $L \geq 6$ | 35678 | 37031 | 579760 |
| ~ 5% | ~ 6% | ~ 89% |

Since reconstructable $\nu_\mu$CC events demand a minimum number of layers to be hit, the distributions for two different layer-cuts are shown in table 7. This feature of large suppression of the $\nu_e$CC and NC events with this cut is evident in figure 6.

6 Contribution of the muonless events to $\nu$ mass hierarchy

In this section, we study the effect of muonless events in the mass hierarchy determination. This is expected to be much smaller compared to the muon events case. Nevertheless, we pursued this study with the hope of improving the hierarchy sensitivity of ICAL.

6.1 Physics motivation and application

The matter effect modifies neutrino oscillation probabilities. For long pathlengths ($L \geq 5000$ km) and moderately large energies ($5 \text{ GeV} \leq E_\nu \leq 10 \text{ GeV}$), matter effects lead to large changes in both $P(\nu_\mu \rightarrow \nu_\mu)$ ($P_{\mu\mu}$) and $P(\nu_e \rightarrow \nu_\mu)$ ($P_{e\mu}$). These changes can lead to an observable change in the muon event rate. By measuring this change, it is possible to determine the neutrino mass hierarchy. The oscillation probabilities involving $\nu_e$, $P(\nu_e \rightarrow \nu_e)$ ($P_{ee}$) and $P(\nu_\mu \rightarrow \nu_e)$ ($P_{\mu e}$), also undergo large changes due to matter effects. The spectrum of the electron event is given by

$$
\frac{dN_e}{dE_\nu} = \left[ \frac{d\Phi_e}{dE_\nu} P_{ee} + \frac{d\Phi_\mu}{dE_\nu} P_{\mu e} \right] \sigma_\nu.
$$

Since the muon neutrino flux $d\Phi_\mu/dE_\nu$ is twice the electron neutrino flux $d\Phi_e/dE_\nu$ and the change in $P_{\mu e}$ is half the change in $P_{ee}$, the effect of these large changes mostly cancel each other out in the electron event sample. This fact makes finding matter effects in muonless events even more challenging.

6.2 The generated events sample

The data files from Nuance, in the energy range $E_\nu = \{0.1, 100\}$ GeV are fed into the Geant4 INO ICAL code to get the events sample for the following studies. The neutrino oscillations have been applied using the normal and the inverted mass hierarchy parameters, which are denoted as NH and IH respectively. The oscillation parameters used are as follows: $\Delta m^2_{21} = 7.5 \times 10^{-5} \text{ eV}^2$, $\Delta m^2_{31}(\text{NH}) = 2.51 \times 10^{-3} \text{ eV}^2$, $\Delta m^2_{31}(\text{IH}) = -2.43 \times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{12} = 0.31$, $\sin^2 2\theta_{13} = 0.09$, $\sin^2 \theta_{23} = 0.5$ and $\delta_{CP} = 0$. 
Table 8. The $\nu_e$CC and $\nu_\mu$CC events count for 500 years of Nuance data, before interacting with the ICAL detector. Here, only the energy range $E_\nu = \{0.8, 20\}$ GeV is considered, since it makes the major contribution to the value of $\chi^2$.

| Sample ID | NH $\nu_e$CC | NH $\nu_\mu$CC | IH $\nu_e$CC | IH $\nu_\mu$CC |
|-----------|---------------|----------------|--------------|----------------|
| seed 1    | 676014        | 1103263        | 671309       | 1103667        |
| seed 2    | 674971        | 1103879        | 670827       | 1105891        |
| seed 3    | 675963        | 1102817        | 669664       | 1104746        |

6.3 The average mass hierarchy (MH) $\chi^2$

Due to MC statistical fluctuations, if we simulate the NH events twice, with two different seeds, the $\chi^2$ between these two event samples will be non-zero. In fact, $\chi^2_{\text{true}} = \chi^2(\text{NH1} - \text{NH2})$ will be approximately twice the number of bins. In addition, we calculate $\chi^2_{\text{false}} = \chi^2(\text{IH} - \text{NH})$. If the NH is the true hierarchy, then we expect $\chi^2_{\text{false}}$ to be appreciably greater than $\chi^2_{\text{true}}$.

To minimize the overall effect of MC fluctuations, we do our calculations for very large statistics and scale them down to 10 years. Here we consider data for 500 years. We have simulated the data for NH with three different seeds and similarly for IH. Thus, we have six values of $\chi^2_{\text{true}}$ and nine values of $\chi^2_{\text{false}}$. We take the average of each and define the average $\chi^2$ as:

$$\langle \chi^2 \rangle = \langle \chi^2_{\text{false}} \rangle - \langle \chi^2_{\text{true}} \rangle.$$  

The numbers of the $\nu_e$CC and the $\nu_\mu$CC events generated for each of the seeds are listed in table 8.

6.4 Calculation of average $\chi^2$ assuming normal hierarchy (NH)

The events are simulated in the energy range $E_\nu = \{0.1, 100\}$ GeV, for both NH and IH, each with three different seeds. For analysing the hierarchy information inherent in the muonless events, we need a selected sample rich in mostly vertical $\nu_e$CC events. A further study of the above selection criteria has shown that efforts to increase the purity of the $\nu_e$CC events results in a simultaneous depletion in the fraction of vertical events in the sample. Hence, we apply some straightforward cuts to optimize between the purity and the vertical events fraction of the $\nu_e$CC events. We select a sample with hits (h), layers (L), and hits/layer (hpl) cuts as stated here: 11 ≤ h ≤ 14, hpl ≥ 3.5; 15 ≤ h ≤ 16, hpl ≥ 4; 17 ≤ h ≤ 18, L ≤ 4; 19 ≤ h ≤ 20, L ≤ 5; 21 ≤ h ≤ 25, hpl ≥ 4; 26 ≤ h ≤ 30, hpl ≥ 5; 31 ≤ h ≤ 35, hpl ≥ 6; 36 ≤ h ≤ 40, hpl ≥ 7; h > 40, hpl ≥ 11.5.

To compare the distributions of these events, we sort them into a number of bins. We consider four different binning schemes. They are:

- 1-bin scheme: The events are all contained in one single bin. Each of these events must have a minimum of 11 hits and be confined in 4 or less layers.

- 3-bin scheme: The selected sample is divided into 3 bins, based on the number of hits. The events in the first bin should have a minimum of 11 hits but ≤ 20 hits. The second bin covers the range of 21 to 40 hits, while events with 40 to 100 hits are put in the third bin.

- 10-bin scheme: The events are classified into 10 uniformly divided bins in the hits range 11 to 100.
Table 9. Values of $\chi^2_{\text{true}}$ or $\chi^2_{t}$ from the three+three possible combinations of NH and IH datasets, using four different binning schemes. The number of bins is indicated in the first row.

| Sample pairs | $\chi^2_{(1)}$ | $\chi^2_{(3)}$ | $\chi^2_{(10)}$ | $\chi^2_{(15)}$ |
|--------------|----------------|----------------|----------------|----------------|
| NH2-NH1      | 2              | 6              | 10             | 30             |
| NH3-NH1      | 1              | 3              | 9              | 27             |
| NH3-NH2      | 1              | 9              | 18             | 17             |
| IH2-IH1      | 3              | 9              | 14             | 44             |
| IH3-IH2      | 0              | 14             | 23             | 22             |
| IH3-IH1      | 2              | 7              | 19             | 47             |
| **Average**  | 2              | 8              | 16             | 31             |

Table 10. Values of $\chi^2_{\text{false}}$ or $\chi^2_{f}$ from the nine possible combinations of NH and IH datasets. The number of bins is indicated in the first row.

| Sample pairs | $\chi^2_{(1)}$ | $\chi^2_{(3)}$ | $\chi^2_{(10)}$ | $\chi^2_{(15)}$ |
|--------------|----------------|----------------|----------------|----------------|
| NH1-IH1      | 44             | 55             | 72             | 82             |
| NH1-IH2      | 26             | 29             | 46             | 62             |
| NH1-IH3      | 39             | 39             | 50             | 70             |
| NH2-IH1      | 27             | 29             | 43             | 50             |
| NH2-IH2      | 13             | 14             | 31             | 63             |
| NH2-IH3      | 23             | 28             | 39             | 54             |
| NH3-IH1      | 35             | 53             | 80             | 74             |
| NH3-IH2      | 19             | 23             | 45             | 67             |
| NH3-IH3      | 30             | 33             | 50             | 60             |
| **Average (all)** | 28         | 34             | 51             | 65             |

- 15-bin scheme: The hits range 11 to 20 is divided into 10 bins. The events giving hits from 21 to 40 are grouped under 4 uniform bins. The fifteenth bin comprises the events giving more than 40 hits.

We expect the $\langle \chi^2 \rangle$ to increase as the number of bins increases, until a saturation value is reached.

The values for $\chi^2_{\text{true}}$ are shown in table 9, whereas table 10 contains the values of $\chi^2_{\text{false}}$.

In table 11, we have listed $\langle \chi^2 \rangle = \langle \chi^2_{\text{false}} \rangle - \langle \chi^2_{\text{true}} \rangle$ and the standard deviation in $\langle \chi^2 \rangle$. This standard deviation is simply the sum of the standard deviations from the mean values of $\chi^2_{\text{true}}$ and $\chi^2_{\text{false}}$.

A cos $\theta$ binning has been attempted, but it appears to be of no additional help in the present study.

In the above calculations, we assumed that NH is true. The results for the case where IH is true are very similar.
Table 11. Average $\chi^2$ and standard deviation for different binning schemes.

| Hits binning | $\langle \chi^2 \rangle$ 500 yrs | $\sigma(\chi^2)$ 500 yrs | $\langle \chi^2 \rangle$ 10 yrs |
|--------------|----------------------------------|--------------------------|-------------------------------|
| 1            | 26                               | 10                       | 0.5                           |
| 3            | 26                               | 16                       | 0.5                           |
| 10           | 35                               | 20                       | 0.7                           |
| 15           | 34                               | 20                       | 0.7                           |

Figure 7. Distribution of $\chi^2_{\text{false}}$ and $\chi^2_{\text{true}}$ calculated with 10 years of NH data and IH data, assuming NH ordering to be true.

6.5 The frequentist approach with 10 years events samples

Statistical fluctuations are intrinsic in data. The Monte Carlo simulations mimic these fluctuations but every different simulation carries a different set of fluctuations. To properly estimate the effect of statistical fluctuations on $\langle \chi^2 \rangle$, one should do a number of NH and IH simulations. From these, one can obtain the average value and the standard deviation of $\chi^2_{\text{true}}$ and $\chi^2_{\text{false}}$. The difference in the average values of $\chi^2_{\text{true}}$ and $\chi^2_{\text{false}}$ gives the expected mean value of the $\langle \chi^2 \rangle$ and the sum of their standard deviations gives us the probable range of $\langle \chi^2 \rangle$ [20].

We have divided the 500 years of data into fifty 10 years data samples, for both NH and IH. Using these, we computed 1225 values of NH-NH $\chi^2_{\text{true}}$ and 2500 values of NH-IH $\chi^2_{\text{false}}$. The plots in figure 7 show these $\chi^2_{\text{false}}$ and $\chi^2_{\text{true}}$ distributions, in the 15-bin scheme. The $\langle \chi^2_{\text{false}} \rangle$ is higher than the $\langle \chi^2_{\text{true}} \rangle$ by almost 3 units (i.e. $\langle \chi^2 \rangle \sim 3$).

However, both the $\chi^2_{\text{false}}$ and the $\chi^2_{\text{true}}$ have very broad distributions. Hence, no conclusive statement is possible.

7 Summary and conclusion

The hits and layers cuts have been confirmed to be the most important criteria for selecting a $\nu_e$CC rich events sample. These give a sample containing around $\sim 50\%$ $\nu_e$CC events. The selection criteria to be finally chosen depend on the requirements of the physics study. One might insist on
Table 12. The best $\nu_{e}$CC purity in the total sample and the corresponding sample size for 500 years of NH data.

| Selection criteria          | Best $\nu_{e}$CC purity | Sample size | Remarks                       |
|-----------------------------|--------------------------|-------------|-------------------------------|
| Maximum hits diff.          | 53%                      | 156,000     | large sample size             |
| Overall pattern: hits in layers | 58%                      | 88,000     |                               |
| Comparison: hits in layers  | 60%                      | 43,000      |                               |
| Single layer hits           | 68%                      | 6,500       | very small sample size; single layer more prone to noise |

the maximum possible purity of the $\nu_{e}$CC events, even compromising the vertical events fraction or the sample size.

The effects of various selection criteria are summarized in table 12.

The results of optimizing the selection criteria may also be summed up in the following manner:

- **Maximum obtainable $\nu_{e}$CC purity:** $\sim 60\%$ with $\sim 100$ events per year.
- **Maximum obtainable NC purity:** $\sim 47\%$ with $\sim 1800$ events per year, provided noise is under control.
- The selection criteria described in this report retain a majority of horizontal or near horizontal events. Any efforts to increase the percentage of vertical or near vertical events requires a compromise on the purity of the $\nu_{e}$CC events.

We have also studied the effect of our selection cuts on different types of neutrino interactions. Using the information from Nuance, we have separated the events into three types: QE events, RS events and DIS events. By applying our selection cuts to the QE events, we obtain a sample containing 75% $\nu_{e}$CC events. The corresponding numbers for RS and DIS events are 67% and 50% respectively. The events sample is dominated by RS and DIS events. Hence, the purity of $\nu_{e}$CC events in the selected sample varies between 50–60%, depending on the selection cuts used.

In the selected $\nu_{\mu}$CC sample, $\nu_{e}$CC + NC events form a 20% background, if the events have a minimum of 5 layers hit. If this minimum is raised to 6 layers, the background contribution comes down to 10%.

We also conclude that there is a small contribution from the muonless events in determining the neutrino mass hierarchy. However, the statistical fluctuations in the data are too large for this contribution to have a significant effect.

**Acknowledgments**

We express our deep gratitude to all our co-members of the INO Collaboration. We particularly thank Prof. Gobinda Majumder and Prof. Indumathi for their invaluable suggestions. We also specially thank Lakshmi S. Mohan for the useful discussions involving her work on separating $\nu_{\mu}$CC events from the NC events. We extend our thanks to Tarak Thakore for numerous discussions.
We are grateful for the worthy suggestions from Prof. Amol Dighe regarding the mass hierarchy contribution of the muonless events. We also thank the Department of Atomic Energy (DAE) and the Department of Science and Technology, Government of India, for financial support.

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