Design of problem-solving questions to measure mathematical thinking type abstraction

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Abstract. The purpose of this study was to describe the design results of problem-solving question used to measure mathematical thinking type abstraction for junior high school students. The data analysis that used in this study is qualitative. This is design research which is divided into five stages, namely: Preliminary design, focus group discussion, trial, interview, and retrospective analysis. Based on the results of FGD, trial and interview, it was found that the question classified as problem-solving question, and could be used to measure mathematical thinking types abstraction, seen from the results of the subject answers that give rise the activities of abstraction which are observation of patterns, specialization, generalization, conjecturing, and testing conjectures. Observation of patterns can be seen from the subject that resolves the problem by observing the patterns. Specialization appears from the completion of the subject by looking at a specific example. Generalization is seen from the solution that describes a pattern into a general form. Conjecturing is composing allegations in solving problems. The last testing conjectures which is a process of checking and testing of guesses, whether the assumptions taken are right and correct.

1. Introduction
Mathematics is basic knowledge that is taught in every student, starting from primary school until higher education. It has an important role and invention and development of many knowledge field, it enhances human thinking, also solving daily problem. One of the important result in mathematics learning which is the students are able to solve problem or they have ability to solve problem, because problem solving is the important cognitive activity and with the ability of problem solving, the student will understand mathematics and other subject easily, and it will make them easier to understand the daily problems [1,2]. Problem solving is a situation of removing problems or finding solutions to real-life problems using logic [3,4].

But in reality, the student ability to solve problem is still deficient or low. This thing is related with PISA and TIMSS result that says Indonesia is still in low rank and got the score below the average [5,6]. PISA 2015 result focused on science, reading, mathematics and problem solving [5]. That means the problem solving ability became one of the ability that is included in PISA. Problem solving ability is the center to apply the domain in the dimension of TIMSS cognitive [6].

Based on the fact above, it can be concluded that there is a big problem in education that has not been fixed by Indonesian government. There are many causes that underlie it, one of them is the problem solving ability that has not made to be main focus yet in learning and lack of teachers’ concern in mathematical thinking process.

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The thinking process in problem solving still lacks of teacher’s concern especially in mathematics learning. This thing means that the learning in school, the students are exercised more to do emphirical thinking activity compared with doing mathematical thinking activity.

In general, mathematics learning process in school has two activities that should be done by teacher, they are the emphirical and mathematical thinking [7]. The emphirical thinking is more related with emphirical phenomena that soon will be developed and become basic science which is related with the use of fact, concept, and math procedure. Whereas, mathematical thinking is developed more flexible, not related with phenomena. Mathematical thinking is more related with thinking structure or logical thinking. Although, there is a possibility that it cannot be illustrated emphirically.

To achieve the math learning better, a student should not be only exercised in their emphirical thinking, but also should be exercised to think mathematically. Mathematical thinking is one of the important things that is useful to solve many problems in mathematics and it is one of the thinking activities about the detail mathematical understanding [8-10]. There are some thinking processes in mathematical thinking, one of it is abstraction [11]. Mathematical thinking is a mental activity that is included in abstraction and generalization of mathematics idea [12]. Abstraction means to illustrate from the result of a process or situation [9,13]. Abstraction is an important thing and such a basic step in a learning process and problem solving [14]. So that is why, the abstraction ability is really needed for the students.

Abstraction has some types and each of it take turns in settlement step, which are specialization, generalization, observation of patterns, conjecturing and testing conjecturing [11]. Observation of patterns is the to solve problems by finding out and deciding the pattern with looking at a problem [15,16]. Specialization is a way to solve a problem with looking at the special cases or the the same example [9,17]. Generalization is an activity that is used to see and illustrate the pattern in to general form or wider in order to see a view from specific things [8,9,17]. Conjecturing is an activity to make or compile an assumption to solve a problem [9,15]. After compiling the assumption, one needs to test it or check it, to see whether the allegation is true or not. Testing conjecturing is also part of mathematical thinking type of abstraction. Conjecturing testing is based on the explanation above regarding conjecturing, so testing conjecturing is checking and testing the allegations, whether the assumptions taken are correct and correct.

From the above explanations, it can be concluded that mathematical thinking is the type of abstraction and problem solving interconnected. When someone tries to solve a problem, then automatically the problem solver also performs some mathematical thinking activities or mathematical thinking which includes the abstraction process, that is: specialization, generalization, conjecturing and testing conjectures. These types of abstractions do not all appear or do not appear simultaneously.

Mathematical Thinking types of student abstraction on problem solving problems need to be studied and analyzed further to find out how mathematical thinking processes students in solving problems so that they can be known well and more clearly, and to analyze mathematical thinking ability, the abstraction type requires a problem in accordance with various theories of abstraction in mathematical thinking, problem solving characteristics, and in accordance with the ability of students in junior high school. Based on this description, this article is entitled "Design of Problem-Solving Questions to Measure Mathematical Thinking Type Abstraction".

2. Research methodology

This is design research, the data analysis that used in this research is qualitative and instruments that used is test and direct interview. In this case researcher made a question to measure mathematical thinking type abstraction in junior high school students. Students selected for the study subject were 2 students white criteria that that students are willing to solve the problem and can solve the problem. The subject selection is based on the recommendation of the supervising teacher. In this study there were five stages, namely preliminary design, focus group discussion (FGD), Trials (Test), Interview, and retrospective analysis.
3. Result

3.1. Preliminary design
At the preliminary design stage, researchers seek and understand the study of theories about mathematical thinking, abstraction, specialization, generalization, pattern observation, guessing, testing suspicion, and solving problems. From the search and understanding of theoretical studies, there are several problem solving questions that can be used to look at mathematical thinking of the types of abstraction of junior high school students. The questions are adopted from a book containing strategies and solutions to problem solving and from journals that contain analytic thinking algebra [18,19]. Then the question is selected and changed to fit the abilities of middle school students.

3.2. Focus Grup discussion (FGD)
The researcher discussed the question that has been designed with 2 math lecturers and a math teacher. From the results of the discussion with the lecturer and teacher, there were several suggestions given to the researcher, so the researcher decided on question number 1, the final sentence of the question was added "then check your answer" and the sentence sentence was changed to be simpler so students easily understood the problem. For question number 2, the researcher decides if the end of the question is also added to the sentence "then check your answer" and the context that was used previously was changed to the context of a square arrangement. The following are the results of discussion with two lecturers and a math teacher:

| No | Question | after FGD |
|----|----------|-----------|
| 1  | Calculate the number of multiples 4 from the first 200 even numbers! Prove that the answer is the right answer! |
| 2  | ![Pattern Image] In the picture above there are several black and white squares arranged and forming a pattern like the picture above! How many white squares are arranged if there are 350 black squares and prove that your answer is the right answer! |

3.3. Trial results
From the results of the test and interview questions showed that the two subjects were able to perform several activities from abstraction namely observing patterns, specialization, conjecturing, testing conjectures, but only S2 could do 5 complete abstraction activities while S1 could not make generalizations, when S1 was interviewed by researchers, s1 says if he can’t get generalization because it’s very difficult so S1 can only do 4 abstraction activities.

3.4. Retrospective analysis
3.4.1. Analysis of S1 answer of number 1. Based on the results of the test and interview S1 in question number 1, it can be seen that in solving the problem done by S1 is observing the pattern, it can be seen from s1, sorting the pattern first, then s1 guesses that the completion can be done by multiplying the sum of numbers first and last with 50 (lots of line pairs). When asked if there was another solution he could do, S1 answered if he could only finish it that way, then the abstraction activity that arises is observation of patterns and conjecturing with indicators analyzing a pattern and making predictions of completion. This can be seen from figure 1.
3.4.2. Analysis of S1 answer of number 2. Based on the results of the trial and interview 1 in question number 2, it can be seen that at the beginning of the settlement S1 made a pattern observation first seen from S1 who can determine the 4th, 5th and 6th term, then S1 resolves the problem by finding a new formula pattern by looking at the values in terms of terms 1 to 6th tribe. From looking at the example in tribe 1 through 6, s1 then makes a guess that the resolution of the problem can be done by adding lots of black squares with lots of white squares which have been reduced by many black squares, or 1 \((3 + 0)\), 2 \((3 + 1)\), 3 \((3 + 2)\), 4 \((3 + 3)\), 5 \((3 + 4)\), 6 \((3 + 5)\). From the allegations obtained from looking at this particular example, it shows that S1 also checks the formula for the 1st to 6th term. Next the researcher returns to the interview stage whether there is another way to solve this problem, S1 answers that only that way he understands, and when asked if he can generalize. S1 answers if he doesn't understand and says he can't make generalizations.

From the explanation, it was found that the abstraction activity that emerged was obsession with patterns, specialization, conjecturing and conjecturing testing with emerging indicators, namely analyzing a pattern, looking at and trying special cases, making guesswork for completion, and testing and communicating that the answer was correct. This can be seen from figure 2.

3.4.3. Analysis of S2 answer of number 1. For number 1, S2 has made a series of sequences with patterns according to the problem. This shows that the abstraction activity that appears in the S2 answer is the observation of patterns with indicators that dig up information or find information contained in the
pattern. Furthermore, after S2 explores information from a pattern, the S2 then solves the problem by multiplying it normally but is made simpler and simpler. Even though the answer is wrong, S2 has been able to make a settlement. This shows that in the type of answer conjecture S2 appears with an indicator that makes a guess of completion. This can be seen from figure 3.

**Figure 3.** Result of Answer S2 in Question number 1.

3.4.4. Analysis of S2 answer of number 2. S2 resolves the problem by observing the first pattern, S2 knows that every increase in one black box, then the white box will increase by 2, which is $4 = (1 \times 2) + 2$, $6 = (2 \times 2) + 2$, $8 = (3 \times 2) + 2$. From this description, it means that the S2 has been able to observe patterns and specializations with indicators that are gathering information or finding information contained in the pattern and seeing and trying special cases. Then from looking at this specific example, S2 writes the general formula, $2x + 2$. This shows that S2 can do form. From the explanation above, it is clear that to complete the answer S2 makes a presumption of completion, namely looking at a specific example that leads to the term-to the formula multiplied by 2 then added 2. In the answer S2 appears the conjuncture type with the indicator making guesses. Students' answers also showed that S2 tested the allegations that they thought were right. This means that a type of suspected test has emerged with testing indicators and communicating that the answer is correct.

4. Discussion
Solving the problems there will be several related activities but not always related, this is in accordance with the theory of Karadag which states that there are several activities of the type of mathematical thinking type abstraction namely generalization, specialization, observation of patterns, conjecturing and testing conjectures [11]. The results of the analysis above also show that the activities that arise in solving problems do not have to appear all, and some activities do not appear in the order of this according to the description of activities from mathematical thinking type abstraction which shows that the activity is not in order but conjecturing activities appear before the activity testing conjectures [11]. Beside that the problem solving also arises abstraction activities, namely observation of patterns, specialization, generalization, conjecturing conjecturing testing. This is in line with Mason and Burton which states that there are 4 thought processes namely specializing, generalizing, conjecturing and convincing the convincing meaning is the same as testing conjecture activities [8].

5. Conclusion
Based on the results of data analysis, it can be concluded that the problem solving questions can be used to measure abstraction in junior high school mathematical thinking type, this can be seen from the two answers the subject raises abstraction activities. in answers to S1 and S2 number 1 observation of patterns and conjecturing activities appear, for S1 answers number 2 observation of patterns, specialization, conjecturing, and testing conjectures appear. answer S2 number 2 appears all activities of abstraction, namely observation of patterns, specialization, generalization, conjecturing, and testing conjectures.
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