Interference effects for the top quark decays $t \rightarrow b + W^+/H^+(\rightarrow \tau^+\nu_\tau)$

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(Dated: May 14, 2019)

Applying the narrow-width approximation (NWA), we first review the NLO QCD predictions for the total decay rate of top quark considering two unstable intermediate particles: the $W^+$-boson in the standard model (SM) of particle physics and the charged Higgs boson $H^+$ in the generic type-I and II two-Higgs-doublet models, i.e. $t \rightarrow b + W^+/H^+(\rightarrow \tau^+\nu_\tau)$. We then estimate the errors arisen from this approximation at leading-order perturbation theory. Finally, we shall investigate the interference effects in the factorization of production and decay parts of intermediate particles. We will show that for nearly mass-degenerate states ($m_{H^+} \approx m_{W^+}$), the correction due to the interference effect is considerable.

PACS numbers: 14.65.Ha, 13.88.+e, 14.40.Lb, 14.40.Nd

I. INTRODUCTION

Since the discovery in 1995 by the CDF and D0 experiments at the $p\bar{p}$ collider Tevatron at Fermilab, the top quark has been in or near the center of attention in high energy physics. It is still the heaviest particle of the Standard Model (SM) of elementary particle physics and its short lifetime implies that it decays before hadronization takes place. The remarkably large mass implies that the top quark couples strongly to the agents of electroweak symmetry breaking, making it both an object of interest itself and a tool to investigate that mechanism in detail. The CERN Large Hadron Collider (LHC), producing a $t\bar{t}$ pair per second, is potentially a top quark factory which allows to perform precision tests of the SM and will enhance the sensitivity of beyond-the-SM effects in the top sector. In this regards, a lot of theoretical works has gone into firming up the cross sections for the $t\bar{t}$ pair and the single top production at the Tevatron and the LHC, undertaken in the form of higher order QCD corrections [1–4]. Historically, improved theoretical calculations of the top quark decay width and distributions started a long time ago [5]. Subsequent theoretical works leading to analytic derivations implementing the $\mathcal{O}(\alpha_s)$ corrections were published in [6, 7] and corrected in [8] (see also Refs. [9–11]). Moreover, in [12, 13] the $\mathcal{O}(\alpha_s)$ radiative corrections to the decay rate of an unpolarized top quark is calculated where the helicities of the W-gauge boson are specified as longitudinal, transverse-plus and transverse-minus.

On the other hand, in the dominant decay mode $t \rightarrow bl^+\nu_l$ the bottom quark hadronizes into the b-jet $X_b$ before it decays. Considering this hadronization process, in [13] the energy distribution of bottom-flavored hadrons (B-mesons) inclusively produced in the SM decay chain of an unpolarized top quark, i.e. $t \rightarrow bW^+ \rightarrow b(l^+\nu_l)$, was calculated some thirty years ago [5]. Subsequent theoretical works leading to analytic derivations implementing the $\mathcal{O}(\alpha_s)$ corrections were published in [6, 7] and corrected in [8] (see also Refs. [9–11]). Moreover, in [12, 13] the $\mathcal{O}(\alpha_s)$ radiative corrections to the decay rate of an unpolarized top quark is calculated where the helicities of the W-gauge boson are specified as longitudinal, transverse-plus and transverse-minus.

Charged Higgs bosons emerge in the scalar sector of several extensions of the SM and are the object of various beyond SM (BSM) searches at the LHC. Since the SM does not include any elementary charged scalar particle, thus the experimental observation of a charged Higgs boson would necessarily be a signal for a nontrivially extended scalar sector and a definitive evidence of new physics beyond the SM. In recent years, searches for charged Higgs have been done by the ATLAS and the CMS collaborations in proton-proton collision and numerous attempts are still in progress at the LHC. Among many proposed scenarios beyond the SM which motivate the existence of charged Higgs, a generic two-Higgs-doublet model (2HDM) [18–20] provides a greater insight of the SUSY Higgs sector without including the plethora of new particles which SUSY predicts. Within this class of models, the Higgs sector of the SM is extended by introducing an extra doublet of complex SU(2)$_L$ Higgs scalar fields. After spontaneous symmetry breaking, the two scalar Higgs doublets $H_1$ and $H_2$ yield three physical neutral Higgs bosons (h, H, A) and a pair of charged-Higgs bosons ($H^\pm$) [19]. Moreover, after electroweak symmetry breaking each doublet acquires a vacuum expectation value (VEV) $v_i$ such that $v_1^2 + v_2^2 = (\sqrt{2}G_F)^{-1}$ where $G_F$ is the Fermi’s constant and the $v_1$ and $v_2$ are the VEVs of $H_1$ and $H_2$, respectively. Furthermore, it is often useful to express the parameters $\tan\beta = v_2/v_1$ as the ratio of VEVs and the neutral sector mixing term $\sin(\beta - \alpha)$. In fact, the angles $\alpha$ and $\beta$ govern the mixing between mass eigenstates in the CP-even sector and CP-odd/charged sectors, respec-

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The dominant production and decay modes for a charged Higgs boson depend on the value of its mass with respect to the top quark mass and can be classified into three categories [21]. Among them, light charged Higgs scenarios are defined by Higgs-boson masses smaller than the top quark mass. In the 2HDM, the main production mode for light charged Higgses is through the top quark decay $t \to bH^+$. Therefore, at the CERN LHC the light Higgses can be searched in the subsequent decay products of the top pairs $tt \to H^+H^-bb$ and $tt \to H^+W^-bb$ when charged Higgses decay into the $\tau$-lepton and neutrino. In [22, 23], the $O(\alpha_t)$ QCD corrections to the hadronic decay width of a charged Higgs boson, i.e. $\Gamma(t \to bH^+)$, is calculated and in [24] the leading order contribution and the $O(\alpha_s)$ corrections to the polarized top quark decay into $bH^+$ is computed. In [25, 26], the energy distribution of B-hadrons in the unpolarized top decays through the 2HDM scenarios, i.e. $t \to bH^+ \to BH^+ + jets$. In [27], in the 2HDM framework the $O(\alpha_t)$ angular distribution of energy spectrum of $B/D$-mesons is studied considering the polar and the azimuthal angular correlations in the rest frame decay of a polarized top quark, i.e. $t(\tau) \to B/D + H^+ + X$ followed by $H^+ \to \tau^+\nu_{\tau}$. Note that, even though current ATLAS and CMS measurements exclude a light charged Higgs for most of the parameter ranges in the context of the minimal supersymmetric standard model (MSSM) scenarios, these bounds are significantly weakened in the Type II 2HDM (MSSM) once the exotic decay channel into a lighter neutral Higgs, $H^+ \to AW^\pm/HW^\pm$, is open. In [28], the production possibility of a light charged Higgs in top quark decay via single top or top pair production is examined with a subsequent decay as $H^\pm \to AW^\pm/HW^\pm$. There is shown that this decay mode can reach a sizable branching fraction at low tan $\beta$ once it is kinematically permitted. These results show that the exotic decay channel $H^+ \to AW^+/HW^+$ is complementary to the conventional $H^+ \to \tau^+\nu_\tau$ channel considered in the current MSSM scenarios.

Two considerable points about all mentioned works are; firstly, in all works authors have applied the narrow width approximation (NWA) in which an intermediate gauge boson ($W^\pm$-boson in the SM and $H^\pm$-boson in the 2HDM scenario) is considered as the on-shell particle and, secondly, the contribution of interference term in total decay rate of top quarks is ignored. In this work we shall examine how much these two approximations change the results. Note that, an important condition limiting the applicability of the narrow width approximation, however, is the requirement that there should be no interference of the contribution of the intermediate particle for which the NWA is applied with any other close-by resonance. It should be noted that, in a general case, if the mass gap between two intermediate particles is smaller than one of their total widths, the interference term between the contributions from the two nearly mass-degenerate particles may become large. In these cases, a single resonance approach or the incoherent sum of the contributions due to two resonances does not necessarily hold.

This work is organized as follows. In Sec. II, we calculate the Born rate of top quark decay in the SM through the direct approach and the narrow width approximation. We will also present the NLO QCD corrections to the tree-level rate of top decay. In Sec. III, the same calculations will be done by working in the general 2HDM. In Sec. IV, we present our results for the interference effects on the total top quark decay and show when this effect is considerable. In Sec. V, we summarize our conclusions.

II. TOP QUARK DECAY IN THE SM

$t \to bW^+ \to bl^+\nu_l$

For the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [29] one has $|V_{tb}| \approx 1$. Therefore, the top quark decays within the SM are completely dominated by the mode $t \to bW^+$ followed by $W^+ \to l^+\nu_l$ where $l^+ = e^+, \mu^+, \tau^+$. Since the top quark’s lifetime is much shorter than the typical strong interaction time, the top quark decay dynamics is controlled by the perturbation theory. Therefore, incorporating the QED/QCD perturbative corrections one has precise theoretical predictions for the decay width to be confronted with the experimental data. As a warm-up exercise we start to calculate the decay width of the following process

\[ t(p_t) \to b(p_b) + W^+(p_W) \to b(p_b) + l^+(p_l) + \nu_l(p_{\nu_l}), \]

to the Born approximation. The matrix element of this process at tree level is given by

\[ M_{SM}^{SM, Born} = M_0^{SM} = \frac{-g_W^2}{8} |V_{tb}| \left( \frac{-g^2 + E_W^2 m_W^2}{p_W^2 - m_W^2} \right) \times \left[ \bar{b}_s(p_b) \gamma_\alpha (1 - \gamma_5) v_l(p_l) \right] \times \left[ \bar{l}_\nu(p_{\nu_l}) \gamma_\beta (1 - \gamma_5) v_l(p_{\nu_l}) \right], \]

where $g_W^2 = 4\alpha_\text{em}^2/\sin^2\theta_W = 8m_W^2G_F/\sqrt{2}$ is the electroweak coupling constant, $\theta_W$ is the weak mixing angle and $m_W$ is the $W^+$ boson mass. Since the second term in the parenthesis (2) is proportional to the lepton mass due to the conservation of the lepton current then it can be omitted simply. Therefore, the matrix element squared reads

\[ |M_0^{SM}(t \to bl^+\nu_l)|^2 = \left( \frac{2g_W^2 |V_{tb}|}{p_W^2 - m_W^2} \right)^2 (p_b \cdot p_{\nu_l})(p_l \cdot p_l), \]

where, for the convenient scalar products in the top quark rest frame one has $2p_b \cdot p_{\nu_l} = m_b^2 + m_l^2 - 2m_bE_l$ and $p_l \cdot p_l = m_l E_l$ in which $E_l$ is the energy of lepton in the top quark rest frame. Technically, to obtain the matrix element squared for the polarized top decay one should replace $\sum_s s_l u(p_l, s_l) \bar{u}(p_l, s_l) = (\gamma_\ell^\mp + m_\ell)$ in the unpolarized Dirac string by $u(p_l, s_l) \bar{u}(p_l, s_l) = (1 - \gamma_\tau^\mp)(\gamma_\sigma + m_\tau)/2$.\]
Since the main contribution to the top quark decay mode (1) comes from the kinematic region where the $W^+$ boson is near its mass shell, one has to take into account its finite decay width $\Gamma_W$. For this reason, in Eq. (3) we employ the Breit-Wigner prescription of the $W^+$ boson propagator for which the propagator contribution of an unstable particle of mass $M_W$ and total width $\Gamma_W$ is given by

$$\frac{1}{p_W^2 - M_W^2} \rightarrow \frac{1}{p_W^2 - M_W^2 + iM_W\Gamma_W}.$$  \hfill (4)

Thus, the matrix element squared (3) reads

$$|M_0^{SM}(t \rightarrow bl^+\nu_t)|^2 = \frac{g_W^4 m_t^4 |V_{tb}|^2}{(p_W^2 - m_W^2)^2 + m_W^2 \Gamma_W^2} \times E_l \left\{ 1 + \left( \frac{m_l}{m_t} \right)^2 - \left( \frac{m_b}{m_t} \right)^2 - \frac{2E_l}{m_t} \right\}. \hfill (5)$$

The phase space element for the three particles final state is given by

$$dP_{SB} = (2\pi)^{-5} dp_1 dp_2 dp_3 d\Omega = \frac{1}{2^5 \pi^3} dE_0 dE_1 dE_2,$$  \hfill (6)

Ignoring the lepton mass ($m_l \approx 0$), the kinematic restrictions are: $m_t/2 - E_1 \leq m_b \leq m_t/2$ and $0 \leq E_2 \leq m_t/2(1 - (m_b/m_t)^2)$, where $E_j$ is the energy of b-quark. For the Born decay width, we use Fermi’s golden rule

$$d\Gamma_0 = \frac{1}{2m_t^2 |M_0|^2} dP_{SB},$$  \hfill (7)

where $|M_0|^2 = \sum_{spin} |M_0|^2 / (1 + 2s_t)$ and $s_t$ stands for the top quark spin. Thus, for the case of unpolarized top quark decay we obtain the decay rate at the lowest-order as

$$\Gamma_0^{SM}(t \rightarrow bl^+\nu_t) = \frac{m_t\alpha^2 |V_{tb}|^2}{192\pi \sin^4 \theta_W} \left\{ \frac{2(1 + R - 2\omega)}{R - 1}(1 + R - 2\omega) + \frac{3(\omega - 1)(R - \omega) + \omega^2}{m_t^2 \Gamma_W} \ln \frac{\omega \Gamma_W^2 + m_t^2(R - \omega)^2}{\omega \Gamma_W^2 + m_t^2(1 - \omega)^2} + \frac{1}{m_t\sqrt{\omega \Gamma_W}} \left[ 3\omega \Gamma_W^2(1 + R - 2\omega) + m_t^2(1 - R)^3 + m_t^2(R - \omega)^2(2 + R - 3\omega) - \frac{\tan^{-1} \frac{m_t(\omega - R)}{\sqrt{\omega \Gamma_W}} + \frac{\tan^{-1} m_t(1 - \omega)}{\sqrt{\omega \Gamma_W}}}{\tan^{-1} \frac{m_t(\omega - R)}{\sqrt{\omega \Gamma_W}} + \frac{\tan^{-1} m_t(1 - \omega)}{\sqrt{\omega \Gamma_W}}} \right] \right\}, \hfill (8)$$

where we defined $R = (m_b/m_t)^2$ and $\omega = (m_W/m_t)^2$. Concentrating on the case $l^+\nu_t = \tau^+\nu_\tau$ with $m_\tau = 1.776$ GeV and taking $m_b = 172.98$ GeV, $m_W = 80.339$ GeV, $m_b = 4.78$ GeV, $\Gamma_W = 2.085 \pm 0.042$ GeV, $\sin^2 \theta_W = 0.2312$, $\alpha = 0.0077$ and $|V_{tb}| \approx 1$ \cite{30}, one has

$$\Gamma_0^{SM}(t \rightarrow b\tau^+\nu_\tau) = 0.1543. \hfill (9)$$

Extension of this approach to higher orders of perturbative QED/QCD is complicated. For example, at the NLO perturbative QCD the phase space element contains four particles including a real emitted gluon so that this leads to cumbersome computations. For this reason, in all manuscripts authors have applied the narrow width approximation which will be described in the following.

## A. Narrow Width Approximation

The separation of a more complicated process into several subprocesses involving on-shell incoming and outgoing particles is achieved with the help of the narrow-width approximation (NWA). This approximation is based on the observation that the on-shell contribution is strongly enhanced if the total width is much smaller than the mass of the particle, i.e. $\Gamma \ll M$. Here, we briefly describe how the NWA works for the decay process of top quark.

On squaring the Born matrix element (2) and taking the Breit-Wigner prescription, one is led to the following Born contribution

$$|M_0^{SM}|^2 = \frac{1}{(p_W^2 - m_W^2)^2 + (m_W \Gamma_W)^2} \times |M_0^{Born}(t \rightarrow bW^+)|^2 \times |M_0^{Born}(W^+ \rightarrow l^+\nu_l)|^2,$$  \hfill (10)

where $|M_0^{Born}(t \rightarrow bW^+)|^2 = 2g_W^2 |V_{tb}|^2 (p_b \cdot p_t)$ and $|M_0^{Born}(W^+ \rightarrow l^+\nu_l)|^2 = 2g_W^2 (p_t \cdot p_\nu)$. We now factorize the three body decay rate (6) into the two body rates $\Gamma(t \rightarrow bW^+)$ and $\Gamma(W^+ \rightarrow l^+\nu_l)$ using the NWA for the $W^+$-boson for which the condition $\Gamma_W \ll m_W$ holds. First, we introduce the following identity

$$1 = \int dP_{SB} \int \frac{d^3 p_{W+}}{2E_{W+}} \delta^4 (p_{W+} - p_t - p_\nu), \hfill (11)$$

therefore, the decay rate (7) reads

$$d\Gamma_0 = 2m_W \int dP_{SB} \frac{d^2 p}{2\pi} |M_0^{SM}|^2 \times \left\{ \sin^2 \theta_W \right\} \times \left\{ \frac{d^3 p_b}{(2\pi)^3 2E_b} \frac{d^3 p_W}{(2\pi)^3 2E_W} \times \left\{ \frac{d^3 p_t}{(2\pi)^3 2E_t} \frac{d^3 p_\nu}{(2\pi)^3 2E_\nu} \delta^4 (p_{W+} - p_t - p_\nu) \right\} \right\} \times \left\{ \frac{d^3 p_W}{(2\pi)^3 2E_W} \delta^4 (p_{W+} - p_t - p_\nu) \right\}. \hfill (12)$$

Next, the phase space is nicely factorized so that by substituting $|M_0^{SM}|^2$ (10), one finds

$$d\Gamma_0 = \frac{m_W}{\pi} \int dP_{SB} \frac{1}{(p_W^2 - m_W^2)^2 + (m_W \Gamma_W)^2} \times \left\{ \frac{1}{2m_t} \left| M_{W \rightarrow l^+\nu_l} \right|^2 dP_{SB}(t \rightarrow bW^+) \times \frac{1}{2m_W} \left| M_{W \rightarrow l^+\nu_l} \right|^2 dP_{SB}(W^+ \rightarrow l^+\nu_l) \right\}. \hfill (13)$$
Adopting the NWA approach, the Breit-Wigner Resonance is replaced by a delta-function as [31]

\[
\frac{1}{(p_W^2 - m_W^2)^2 + (m_W \Gamma_W)^2} \approx \frac{\pi}{m_W \Gamma_W} \delta(p_W^2 - m_W^2). \tag{14}
\]

This approximation is expected to work reliably up to terms of \(O(\Gamma_W/m_W^2)\). As was discussed, a necessary condition limiting the applicability of this approximation is the requirement that there should be particles with a total decay width much smaller than their mass, otherwise the integral (13) is reduced to [31]

\[
\int \frac{1}{(p_W^2 - m_W^2)^2 + (m_W \Gamma_W)^2} - \frac{1}{m_W \Gamma_W} \tan^{-1} \left( \frac{m_W^2 - p_W^2}{m_W \Gamma_W} \right). \tag{15}
\]

Using the NWA, the three body decay \(t \to b \tau^+ \nu_\tau\) is factorized as

\[
\Gamma(t \to b \tau^+ \nu_\tau) = \Gamma(t \to b W^+) \frac{\Gamma(W^+ \to l^+ \nu_l)}{\Gamma_W} = \Gamma(t \to b W^+) Br(W^+ \to l^+ \nu_l), \tag{16}
\]

which is a result expected from physical intuition and it is expected to work reliably up to terms of \(O(\Gamma_W/m_W^2)\). For three individual leptonic branching ratios, one has \(Br(W^+ \to e^+ \nu_e) = 10.75 \pm 0.13, Br(W^+ \to \mu^+ \nu_\mu) = 10.57 \pm 0.15\) and \(Br(W^+ \to \tau^+ \nu_\tau) = 11.25 \pm 0.20\) in units \(10^{-2}\) [32].

In (16), the partial Born width of the decay \(t \to b W^+\) differential in the angle \(\theta_P\) enclosed between the top-quark polarization three-vector \(\vec{P}\) and the bottom quark three-momentum \(\vec{p}_b\), is given by [14, 16]

\[
\frac{d\Gamma^S_{t \to b W^+}}{d \cos \theta_P} = \frac{1}{2} \left( \Gamma_{t \to b W^+}^S + P \Gamma_{t \to b W^+}^S \cos \theta_P \right), \tag{17}
\]

where \(P = |\vec{P}|(0 \leq P \leq 1)\) is the degree of polarization and

\[
\Gamma_{t \to b W^+}^S = \frac{m_t \alpha \sqrt{S^2 - R}}{8 \sin^2 \theta_W} \left( 1 + R - 2 \omega + \frac{(1 - R)^2}{\omega} \right),
\]

\[
\Gamma_{t \to b W^+}^S = \frac{m_t \alpha}{4 \omega \sin^2 \theta_W} (1 - R - 2 \omega)(S^2 - R). \tag{18}
\]

Here, we defined \(S = (1 + R - \omega)/2\). Taking the input parameters as before, one has

\[
\Gamma_{t \to b W^+}^S(t \to b W^+) = 1.463, \quad \Gamma_{t \to b W^+}^S(t \to W^+ \tau^+ \nu_\tau) = 0.579. \tag{19}
\]

Considering the factorization (16), one has \(\Gamma_{t \to b \tau^+ \nu_\tau}^S = 1.463 \times Br(W^+ \to \tau^+ \nu_\tau) = 0.1645\) which is in agreement to the result obtained in the full calculation (9) up to the accuracy about 5%. One also has \(\Gamma_{t \to b \tau^+ \nu_\tau}^S(t \to b \tau^+ \nu_\tau) = 0.1543\) and \(\Gamma_{t \to b \tau^+ \nu_\tau}^S(t \to b \tau^+ \nu_\tau) = 0.1569\).

In [13], we have calculated the NLO QCD corrections to the differential decay rate of \(t \to b W^+\) in the massless (with \(m_b = 0\)) and massive (with \(m_b \neq 0\)) schemes. The result for the massless decay rate reads

\[
\Gamma_{t \to b W^+}^{SM} = \Gamma_{t \to b W^+}^{SM} \left( 1 + \frac{C_F \alpha_s}{2\pi} \left[ -\frac{2\pi^2}{3} - 4L_i^2(\omega) - \frac{5 + 4\omega}{1 + 2\omega} \ln(1 - \omega) - 2\omega(1 + \omega)(1 - 2\omega) \right] \right) \ln \omega + 2 \ln \omega \ln(1 - \omega) - \frac{1 + 3(1 + 2\omega)(\omega - 2)}{2(1 - \omega)(1 + 2\omega)}. \tag{20}
\]

In [13], using the NWA approach we have also calculated the NLO QCD corrections to the differential decay rate of \(t \to b W^+\) considering the helicity contributions of \(W^+\) boson. In [15–17], we have computed the differential decay width for the process \(t(\uparrow) \to b W^+\) up to the NLO accuracy. Our numerical results read

\[
\Gamma_{t \to b W^+}^{SM} = \Gamma_{t \to b W^+}^{SM} (1 - 0.0853), \quad \Gamma_{t \to b W^+}^{SM} = \Gamma_{t \to b W^+}^{SM} (1 - 0.1308). \tag{21}
\]

Thus, the \(O(\alpha_s)\) contribution to the unpolarized and polarized top width is \(-8.53\%\) and \(-13\%\), respectively, while the contribution of the finite \(W\)-width effect is about 5%. It should be noted that the \(O(\alpha)\) electroweak corrections contribute typically by +1.55% [33, 34].

### III. TOP QUARK DECAY IN THE 2HDM

In the theories beyond the SM with an extended Higgs sector one may also have the following decay mode

\[
t \to b H^+ \to b l^+ \nu_l, \tag{22}
\]

provided that \(m_t > m_{H^+} + m_b\). A model independent lower bound on the Higgs mass \(m_{H^+}\) arising from the nonobservation of the charged Higgs pair production at LEP II has yielded \(m_{H^+} > 79.3\) GeV at 95\% C.L. [35]. As is asserted in Ref. [36], a charged Higgs with a mass in the range \(80 \leq m_{H^\pm} \leq 160\) GeV is a logical possibility and its effect should be searched for in the process (22). A beginning along these lines has already been done at the Tevatron [37, 38], but a definitive search of the charged Higgs bosons over a good part of the \((m_{H^\pm} - \tan \beta)\) plane is a plan which still has to be done and this belongs to the CERN LHC experiments [39].

Here, we review some technical detail about the decay rate of unpolarized top quark in the process (22) by working in the general 2HDM [18–20] where \(H_1\) and \(H_2\) are the doublets whose VEVs give masses to the down and up type quarks. Moreover, a linear combination of the charged components of doublets \(H_1\) and \(H_2\) gives two physical charged Higgs bosons \(H^\pm\), i.e. \(H^\pm = H^\pm \cos \beta - H_1^\pm \sin \beta\).

In a general 2HDM in order to avoid tree level flavor changing neutral currents (FCNC), that can be induced by Higgs exchange, the generic Higgs boson coupling to
all types of quarks must be restricted. Fortunately, there are several classes of two-Higgs-doublet models which naturally avoid this difficulty by restricting the Higgs coupling. Imposing flavor conservation, there are four possibilities (models I-IV) for the two Higgs doublets to couple to the SM fermions so that each gives rise to rather different phenomenology predictions. In these four models, assuming massless neutrinos the generic charged Higgs coupling to the SM fermions can be expressed as a superposition of right- and left-chiral coupling factors [40], so that the relevant part of the interaction Lagrangian of the process (22) is given by

$$L_I = \frac{g_W}{2\sqrt{2}m_W} H^+ \left\{ V_{tb} \left[ \bar{u}_t(p_t) \{ A(1 + \gamma_5) + B(1 - \gamma_5) \} u_b(p_b) \right] + B(1 - \gamma_5) \bar{u}_t(p_t) \right\}$$

(23)

where A, B and C are three model dependent parameters. In the first possibility (called model I), the H2-doublet gives mass to all quarks and leptons so that the other one, i.e. doublet H1, essentially decouples from fermions. In this model, one has

$$A_I = m_t \cot \beta, \quad B_I = -m_b \cot \beta, \quad C_I = -m_\tau \cot \beta.$$  

(24)

In the second scenario (called model II), the H2-doublet gives mass to the right-chiral up-type quarks (and possibly neutrinos) and the H1-doublet gives mass to the right-chiral down-type quarks and charged leptons. In this possibility, the Lagrangian (23) contains

$$A_{II} = m_t \cot \beta, \quad B_{II} = m_b \tan \beta, \quad C_{II} = m_\tau \tan \beta.$$  

(25)

There are also two other scenarios (models III and IV) in which the down-type quarks and charged leptons receive masses from different doublets; in model III both up- and down-type quarks couple to the second doublet (H2) and all leptons to the first one, thus, one has

$$A_{III} = m_t \cot \beta, \quad B_{III} = m_b \tan \beta, \quad C_{III} = -m_\tau \cot \beta.$$  

(26)

and in the fourth scenario (model IV), the roles of two doublets are reversed with respect to the model II, i.e.

$$A_{IV} = m_t \cot \beta, \quad B_{IV} = -m_b \cot \beta, \quad C_{IV} = m_\tau \tan \beta.$$  

(27)

These four models are also known as type I-IV 2HDM scenarios. Note that, the type-II scenario is, in fact, the Higgs sector of the MSSM up to SUSY corrections [41, 42]. In other words, in the MSSM we have a type-II 2HDM sector in addition to the supersymmetric particles including the stops, charginos and gluinos.

After this description, we start to calculate the Born term contribution to the decay rate of the process $t \to b^+ \nu_t$ ($l^+ = e^+, \mu^+, \tau^+$). Considering the decay process

$$t(p_t) \to b(p_b) + H^+(p_{H^+}) \to b(p_b) + l^+(p_l) + \nu_l(p_\nu).$$

(28)

and using the couplings from the Lagrangian (23) one can write the matrix element of the process (28) as

$$M_{0}^{BSM}(t \to b^+ \nu_t) = \frac{\frac{g_W}{2\sqrt{2}m_W} |V_{tb}|}{p_{H^+}^2 - m_{H^+}^2 + im_{H^+}\Gamma_{H^+}} \times C[\bar{u}_t(p_t) \{ A(1 + \gamma_5) + B(1 - \gamma_5) \} u_b(p_b)] \times \left[ \bar{u}_b(p_b) \{ A(1 + \gamma_5) + B(1 - \gamma_5) \} u_t(p_t) \right].$$

(29)

Thus, the matrix element squared reads

$$|M_{0}^{BSM}(t \to b^+ \nu_t)|^2 = \left( \frac{g_W^2 |V_{tb}|^2}{2m_W^2} \right)^2 \frac{1}{[p_{H^+}^2 - m_{H^+}^2]^2 + m_{H^+}^2 \Gamma_{H^+}^2} \times C^2 (p_t \cdot p_b) \{ A^2 + B^2 \} p_t \cdot p_b + 2m_b m_t AB.$$  

(30)

The kinematic restrictions and the phase space element are as before, see Eq. (6). Thus defining $y = (m_{H^+}/m_t)^2$, for the unpolarized decay rate one has

$$\Gamma_{0}^{BSM}(t \to b^+ \nu_t) = \left( \frac{C \alpha |V_{tb}|}{16\sqrt{\pi} m_W^5 \sin^2 \theta_W} \right)^2 \times$$

$$\times \left\{ m_t(R - 1) \left[ 8AB \sqrt{R} + (A^2 + B^2)(3R - 4y) \right] + m_t \tan^{-1} \frac{m_b(y - R)}{\sqrt{y\Gamma_H}} + m_t \tan^{-1} \frac{m_t(1 - y)}{\sqrt{y\Gamma_H}} \times \left[ 4AB \sqrt{R}(\Gamma_H^2 + (1 - y)m_t^2) + (A^2 + B^2)[(2R - 3y)\Gamma_H^2 + m_b^2(1 - y)(1 - R - y)] \right]$$

$$+ m_t \left[ 4AB(2y - 1) \sqrt{R} + (A^2 + B^2)(\frac{y\Gamma_H^2}{m_t^2} - (1 + R) - 4y - 2Ry + 3y^2) \right] \times \ln \frac{y\Gamma_H^2 + m_b^2(R - y)^2}{y\Gamma_H^2 + m_t^2(1 - y)^2}. \right\}.$$  

(31)

Leaving this result and working in the NWA framework, where $p_{H^+}^2 = m_{H^+}^2$ is put from the beginning, we have

$$\Gamma_{0}^{BSM}(t \to b^+ \nu_t) = \Gamma_0(t \to bH^+) \times Br(H^+ \to t^+ \nu_t).$$  

(32)

where $Br(H^+ \to t^+ \nu_t) = \Gamma_0(H^+ \to t^+ \nu_t)/\Gamma_{H^+}^{Total}$, and the polarized and unpolarized tree-level decay widths read [25–27]

$$\Gamma_{0}^{BSM}(t \to bH^+) = \frac{m_t}{16\pi} \left\{ (a^2 + b^2)(1 + R - y) + 2(a^2 - b^2)\sqrt{R} \right\} \lambda^2(1, R, y),$$

$$\Gamma_{0}^{BSM}(t \to bH^+) = \frac{m_t}{8\pi} (ab)^2 \lambda(1, R, y).$$  

(33)

Here, $\lambda(x, y, z) = (x - y - z)^2 - 4yz$ is the Källén function (triangle function) and, for simplicity, we introduced the
coefficients $a$ and $b$ as
\[ a = \frac{g_W}{2\sqrt{2}m_W}|V_{tb}|(A + B), \]
\[ b = \frac{g_W}{2\sqrt{2}m_W}|V_{tb}|(A - B), \]
\[ c = \frac{g_W}{2\sqrt{2}m_W}C. \]

The advantage of this notation is that the coupling of the charged Higgs to the bottom and top quarks is expressed as a superposition of scalar and pseudoscalar coupling factors. The NLO QCD radiative corrections to the polarized and unpolarized rates are given in our previous works [25–27].

Since all current search strategies postulate that the charged Higgs decays either leptonically ($H^+ \rightarrow \tau^+ \nu_{\tau}$) or hadronically ($H^+ \rightarrow cs$), then following Ref. [43] we adopt the relevant branching fraction $Br(H^+ \rightarrow \tau^+ \nu_{\tau})$, as
\[ Br(H^+ \rightarrow \tau^+ \nu_{\tau}) = \frac{\Gamma(H^+ \rightarrow \tau^+ \nu_{\tau})}{\Gamma(H^+ \rightarrow \tau^+ \nu_{\tau}) + \Gamma(H^+ \rightarrow cs)}, \tag{35} \]
where, in the model I (and IV) one has
\[ \Gamma_0(H^+ \rightarrow \tau^+ \nu_{\tau}) = \frac{g_W^2m_{H^+}^2m_{\tau}^2\cot^2\beta}{32\pi m_W^2}, \]
\[ \Gamma(H^+ \rightarrow cs) = \frac{3g_W^2m_{H^+}^2|V_{cs}|^2(\cot^2\beta)\lambda^2(1 - \frac{m_s^2}{m_{H^+}^2} - \frac{m_c^2}{m_{H^+}^2})}{32\pi m_W^2}, \]
\[ \times \left[ (m_c^2 + m_s^2)(1 - \frac{m_c^2}{m_{H^+}^2} - \frac{m_s^2}{m_{H^+}^2}) + 4\frac{m_c^2m_s^2}{m_{H^+}^2} \right], \tag{36} \]
and for the model II (and III) one has
\[ \Gamma_0(H^+ \rightarrow \tau^+ \nu_{\tau}) = \frac{g_W^2m_{H^+}^2m_{\tau}^2\tan^2\beta}{32\pi m_W^2}, \]
\[ \Gamma(H^+ \rightarrow cs) = \frac{3g_W^2m_{H^+}^2|V_{cs}|^2\lambda^2(1 - \frac{m_s^2}{m_{H^+}^2} - \frac{m_c^2}{m_{H^+}^2})}{32\pi m_W^2}, \]
\[ \times \left[ (m_c^2\cot^2\beta + m_s^2\tan^2\beta)(1 - \frac{m_c^2}{m_{H^+}^2} - \frac{m_s^2}{m_{H^+}^2}) \right. \]
\[ \left. - 4\frac{m_c^2m_s^2}{m_{H^+}^2} \right]. \tag{37} \]

Both results are in complete agreement with Ref. [22]. In the limit of $m_s^2/m_{\tau}^2 \rightarrow 0$ ($i = c, s$), the branching fraction (35) in the type-I 2HDM is simplified as
\[ Br(H^+ \rightarrow \tau^+ \nu_{\tau}) = \frac{1}{1 + 3|V_{cs}|^2(\frac{m_s^2}{m_{\tau}^2} + (\frac{m_s}{m_{\tau}})^2)}, \tag{38} \]
which is independent of the tan $\beta$ and in the type-II reads
\[ Br(H^+ \rightarrow \tau^+ \nu_{\tau}) = \frac{1}{1 + 3|V_{cs}|^2((\frac{m_s}{m_{\tau}})^2 + (\frac{m_s}{m_{\tau}})^2\cot^2\beta)}. \tag{39} \]

Taking $m_s = 95$ MeV, $m_c = 1.67$ GeV, $m_{\tau} = 1.776$ GeV and $|V_{cs}| = 0.9734$, the branching ratio in the model I reads $Br = 0.284$. While in the model I, the branching ratio is independent of the tan $\beta$, in the type-II scenario it depends on the tan $\beta$. It is simple to prove that for tan $\beta > 5$ one has $Br \approx 1$ in the model II to a very high accuracy. Direct searches at the LHC, with the center-of-mass energy of 7 TeV [44–46] and 8 TeV [47, 48] set stringent constraints on the $m_{H^+} - \tan^2\beta$ parameter space. Taking $m_{H^+} = 95$ GeV and tan $\beta = 8$, from full calcu-
we studied the de-

one has
\[
\Gamma_{Model \, I}^{NLO}(t \rightarrow b H^+) = \Gamma_0^{Model \, I}(1 - 0.0879),
\]
\[
\Gamma_{Model \, II}^{NLO}(t \rightarrow b H^+) = \Gamma_0^{Model \, II}(1 - 0.4635),
\]
where \( \Gamma_0^{Model \, I} = 159 \times 10^{-4} \) and \( \Gamma_0^{Model \, II} = 700 \times 10^{-4} \). Also, for \( m_H^+ = 85 \) GeV and \( \tan \beta = 8 \), they read
\[
\Gamma_{Model \, I}^{NLO}(t \rightarrow b H^+) = \Gamma_0^{Model \, I}(1 - 0.0870),
\]
\[
\Gamma_{Model \, II}^{NLO}(t \rightarrow b H^+) = \Gamma_0^{Model \, II}(1 - 0.4627),
\]
where \( \Gamma_0^{Model \, I} = 149 \times 10^{-4} \) and \( \Gamma_0^{Model \, II} = 656 \times 10^{-4} \). As is seen the NLO QCD corrections to the decay rates are significant, specially when the type-II 2HDM scenario is concerned.

IV. INTERFERENCE EFFECTS OF AMPLITUDES

Considering the decay modes
\[
t \rightarrow b + W^+ / H^+ \rightarrow l^+ \nu^+ \nu
\]
the full amplitude for the top decay process is the sum of the amplitudes in the SM and BSM theories, i.e.
\[
M_{Total}^{SM}(t \rightarrow b l^+ \nu) = M_{t \rightarrow b l^+ \nu}^{SM} + M_{t \rightarrow b l^+ \nu}^{BSM}
\]
At the Born level, the matrix element squared is \( |M_0|^2 = |M_0^{SM}|^2 + |M_0^{BSM}|^2 + 2 Re(M_0^{BSM} \cdot M_0^{SM}) \) where the amplitudes \( M_0^{SM} \) and \( M_0^{BSM} \) are given in (2) and (29), respectively. Considering the Born decay width (7) and the phase space element (6) one can obtain the total Born decay width as
\[
\Gamma_t^{tot} = \Gamma_0^{SM}(t \rightarrow b l^+ \nu) + \Gamma_0^{BSM}(t \rightarrow b l^+ \nu) + \Gamma_0^{Int}.
\]
where \( \Gamma_0^{SM} \) and \( \Gamma_0^{BSM} \) are given in (8) and (31), respectively. In all manuscripts it is postulated that the contribution of interference term can be ignored while it needs some subtle accuracies. In this section we intend to estimate this contribution in the leading-order, i.e. \( \Gamma_0^{Int} \), to show when one is allowed to omit this contribution.

The contribution of interference amplitude squared is obtained as
\[
|\Delta M_0|^2 = \frac{1}{2} \sum_{spin} (2 Re|M_0^{BSM} \cdot M_0^{SM}|^2) = \left( \frac{g_W^2}{8 m_W} |V_{tb}|^2 \right)^2 \times \frac{(p_W^2 - m_W^2)^2 |p_H^2|^2}{(p_W^2 - m_W^2)^2 + m_W^4 \Gamma_W^2 \Gamma_H^2} \times \left( -m_0^2 C[A m_t(p_\nu, p_\nu) + B m_b(p_\nu, p_\nu)] \right),
\]
where \( p_\nu \cdot p_\nu = m_t E_\nu \) and \( 2 p_\nu \cdot p_\nu = m_t^2 + m_\nu^2 - m_0^2 - 2 m_t E_\nu \) in the top rest frame. The kinematic restrictions are
\[
0 \leq E_\nu \leq \frac{m_t(1 - R)}{2},
\]
\[
\frac{m_t}{2} (1 - R - 2E_\nu/m_t) \leq E_\nu \leq \frac{m_t}{2} (1 - \frac{m_t R}{m_t - 2E_\nu}).
\]
Finally, defining \( l = (m_t/m_b)^2 \), for the contribution of interference term in the top decay rate one has

\[
\Gamma_{t \to b l^+ \nu}^{int} = \frac{C \alpha^2 \sqrt{t}}{2 \pi m_t^2 \omega \sin^4 \theta_W} \left\{ 2m_t(R - 1)(A - B \sqrt{R}) + \frac{1}{m_t^2(y - \omega)^2 + (\sqrt{R} H + \sqrt{\omega} W)^2} \left[ f(y, \omega, \Gamma_H, \Gamma_W) + g(y, \omega, \Gamma_H, \Gamma_W) + h(y, \omega, \Gamma_H, \Gamma_W) + Q(y, \omega, \Gamma_H, \Gamma_W) \right] \right\},
\]

(47)

where \( f(y, \omega, \Gamma_H, \Gamma_W) = f(y \leftrightarrow \omega, \Gamma_H \leftrightarrow \Gamma_W) \) so that

\[
f(y, \omega, \Gamma_H, \Gamma_W) = m_t \ln \left( \frac{(R - 1)A - B \sqrt{R}}{(1 - y)^2 \omega + y \Gamma_H^2} \right) = m_t^2(y - \omega) \left[ A(1 - y)^2 + 2B \sqrt{R}(1 - y^2) \right] + 2y \Gamma_H^2 \left[ A(y + \omega - 2) - B(y + \omega) \sqrt{R} \right],
\]

and \( Q(y, \omega, \Gamma_H, \Gamma_W) = h(y \leftrightarrow \omega, \Gamma_H \leftrightarrow \Gamma_W) \) where

\[
h(y, \omega, \Gamma_H, \Gamma_W) = 2 \left[ \tan^{-1} \left( \frac{m_t(R - y)}{\sqrt{R} \Gamma_H^2} \right) - \tan^{-1} \left( \frac{m_t(1 - y)}{\sqrt{R} \Gamma_H^2} \right) \right] \times \left[ (B \sqrt{R} - A) \left( y \Gamma_W \Gamma_H^2 \sqrt{\omega} + (\Gamma_H \sqrt{y})^3 \right) + m_t^2 \Gamma_H(y^2 - 2y \omega + 1) \sqrt{y} - m_t^2(y^2 - 1) \Gamma_W \sqrt{\omega} \right] + A m_t \left[ (1 - \omega) \Gamma_H \sqrt{y} + (1 - y) \Gamma_W \sqrt{\omega} \right].
\]

(49)

In the above relations \( \Gamma_W = 2.085 \) [30] and concentrating on \( l^+ = \tau^+ \) one has \( \Gamma_H = \Gamma(H^+ \rightarrow \tau^+ \nu_{\tau}) + \Gamma(H^+ \rightarrow c \bar{s}) \) which are given in (36) and (37) for two models I and II. In Figs. 6-8 we studied the dependence of the interference term on \( \tan \beta \) considering \( m_{H^+} = m_{W^+} \) (in Fig. 6), \( m_{H^+} = 85 \) GeV (in Fig. 7) and \( m_{H^+} = 120 \) GeV (in Fig. 8). As is seen, for \( m_{H^+} = m_{W^+} \) this contribution in the model I is positive for all values of \( \tan \beta \) while this is negative in the model II. This behavior is vice-versa in Fig. 8 where we set \( m_H = 120 \) GeV. For \( \tan \beta > 2 \), the absolute value of interference contribution at the model II is always larger than the one in the model I.

In Figs. 9 and 10 we investigated the dependence of the interference term on the charged Higgs mass by fixing \( \tan \beta = 8 \) (in Fig. 9) and \( \tan \beta = 30 \) (in Fig. 10). As is seen the maximum value of the interference contribution occurs for \( m_{H^+} \approx m_{W^+} \) and it goes to zero when \( \tan \beta \) increases. To work out our conclusion, here, we concentrate on two following examples:

1. Taking \( m_{H^+} \approx m_{W^+} \) and \( \tan \beta = 8 \), in the type-I 2HDM one has

\[
\Gamma_{NLO}^{Total}(t \rightarrow b l^+ \nu_\tau) = \Gamma_0^{SM}(1 - 0.0853) + \Gamma_0^{BSM,I}(1 - 0.463) + \Gamma^{int},
\]

where \( \Gamma_0^{SM} = 0.1543 \), \( \Gamma_0^{BSM,I} = 4.53 \times 10^{-3} \) and the interference contribution at lowest order reads \( \Gamma^{int}_0 = 9.95 \times 10^{-5} \).

This result in the type-II 2HDM reads

\[
\Gamma_{NLO}^{Total}(t \rightarrow b l^+ \nu_\tau) = \Gamma_0^{SM}(1 - 0.0853) + \Gamma_0^{BSM,II}(1 - 0.463) + \Gamma^{int},
\]

where \( \Gamma_0^{BSM,II} = 6.94 \times 10^{-2} \) and the interference

![Figure 6](image_url)

Figure 6: The interference contribution as a function of \( \tan \beta \) for which we fixed the Higgs mass as \( m_{H^+} = m_{W^+} \).

![Figure 7](image_url)

Figure 7: As in Fig. 6 but for \( m_{H^+} = 85 \) GeV.

![Figure 8](image_url)

Figure 8: As in Fig. 6 but for \( m_{H^+} = 120 \) GeV.
contribution reads $\Gamma_{int}^{\text{total}} = -6.82 \times 10^{-3}$. This example shows that the interference contribution in the type-I and II scenarios is about 2% and −9% of the contribution from 2HDM at LO, respectively.

2. Taking $m_{H^+} = 85$ GeV and $\tan \beta = 8$, in the type-I 2HDM one has

$$\Gamma_{NLO}^{\text{total}}(t \to b\nu^+\nu_\tau) = \Gamma_{0}^{SM}(1 - 0.0853) + \Gamma_{0}^{BSM, I}(1 - 0.870) + \Gamma_{int}, \quad (52)$$

where $\Gamma_{0}^{BSM, I} = 4.2 \times 10^{-3}$ and the interference contribution at LO reads $\Gamma_{0}^{\text{int}} = 9.2 \times 10^{-6}$. This result in the type-II 2HDM reads

$$\Gamma_{NLO}^{\text{total}}(t \to b\nu^+\nu_\tau) = \Gamma_{0}^{SM}(1 - 0.0853) + \Gamma_{0}^{BSM, II}(1 - 0.462) + \Gamma_{int}, \quad (53)$$

where $\Gamma_{0}^{BSM, II} = 0.0651$ and the interference term is $\Gamma_{int}^{\text{total}} = -129 \times 10^{-6}$.

These two examples show that for $m_{H^+} \approx m_{W^+}$ the contribution of interference term, specifically in the type-II 2HDM, is considerable and its value cannot be ignored.

Therefore, our numerical results emphasis that if the mass gap between two intermediate particles ($W^+$ and $H^+$ in our work) is smaller than one of their total widths, the interference term between the contributions from the two nearly mass-degenerate particles may become considerable. In other words, the interference effects can be considerable if there are several resonant diagrams whose intermediate particles (in general, with masses $M_1$ and $M_2$ for two resonnances) are close in mass compared to their total decay widths: $|M_1 - M_2| \leq (\Gamma_1, \Gamma_2)$ [31]. In these situations, a single resonance approach or the incoherent sum of two resonance contributions does not necessarily hold and it needs more attention. In fact, if the mass difference is smaller than their total widths, the two resonances overlap. This can lead to a considerable interference term which was neglected in the standard NWA, but can be taken into account in the full calculation or in a generalized NWA [31].

V. CONCLUSIONS

In a general 2HDM, the main production mode of light charged Higgs bosons (with $m_{H^+} \leq m_t$) is through the top quark decay, $t \to bH^+$, followed by $H^+ \to \tau^+\nu_\tau$. In this work, we have calculated the total decay rate of top quark, i.e. $t \to b + W^+/H^+ \to b\nu_\tau$, at the standard model of particle physics as well as the 2HDM theory. In the first part of our work, we calculated the Born decay rate for the full process in which one deals with the stable intermediate bosons $W^+$ and $H^+$. Extension of this procedure to higher orders of perturbative QED/QCD is complicated but it would be possible using the narrow-width approximation for particles having a total width much smaller than their masses. Next, using the NWA we recalculated the aforementioned decay rates and showed that the accuracy of NWA is about $(2 - 5)\%$. Within this approach we presented our numerical analysis at NLO.

A necessary and important condition limiting the applicability of NWA is the requirement that there should be no interference of the contribution of the intermediate particle for which the NWA is applied with any other close-by resonance. While within the SM of particle physics this condition is usually valid for relevant processes at high-energy colliders such as the CERN LHC or a future Linear Collider, many models of physics beyond the SM have mass spectra where two or more states can be nearly mass-degenerate. If the mass gap between two intermediate particles is smaller than one of their total widths, their resonances overlap so that the interference contribution can not be neglected if the two states mix. In the last section of our paper, we investigated the interference effect of two intermediate particles in top decay, i.e. $W^+$ and $H^+$ bosons, and showed when this effect is considerable and the NWA is insufficient. Our results confirmed that when the mass of charged Higgs boson is considered equal or near to the $W^+$-mass (referred as nearly mass-degenerate particles), the interference effects are sizable and considerable, specifically for the type-II 2HDM. For larger values of $m_{H^+}$ this contribution can be omitted with high accuracy.
It should be pointed out that several cases have been already identified in the literature in which the NWA is insufficient due to sizable interference effects, e.g. in the context of two- and multiple-Higgs models and in Higgsless models in Ref. [51].

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