Pseudorandom amplitude-phase-shift keyed signals with irregular bi-level amplitude spectrum for moving target detection

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Abstract. This paper considers the target locating against the clutter background in radars with quasi-continuous mode of transmission and reception on common aerial of signals with pseudorandom amplitude- and phase-shift keying. Pseudorandom amplitude-phase-shift keyed signals with irregular bi-level amplitude spectrum are proposed for use. The high-level spectrum components of returns from the moving and stationary targets can be partially overlapped. It is proposed to perform the frequency rejection of high-level components of the stationary targets before the correlation signal processing. As a result, significant part of the energy of interfering reflections is cut out and just a small part of the energy of the useful signal is lost. The signal-to-clutter ratio increases. The synthesis method of pseudorandom amplitude-phase-shift keyed signal with irregular bi-level amplitude spectrum is considered. The spectrum of the synthesized signals is analyzed. Increase of the signal-to-clutter ratio was obtained in the processing of signals with frequency interference rejection. The result of detection modeling of a moving target against the background of reflections from a stationary object is presented.

1. Introduction
The most significant property of moving targets which helps to distinguish them from stationary clutter is the Doppler shift of return signal frequency. There are a lot of methods that are already known to use this feature. The moving targets indication (MTI) radars and the pulse-Doppler (PD) radars [1] are well-known. The key feature of MTI radars is to use relatively long pulses with low repetition frequency (PRF) for probing targets whereas PD class radars in general uses a large amount of short pulses with high PRF for same purpose. Both of them have an essential weakness: blind zones and range ambiguity when measuring distance or Doppler frequency shift. All efforts to eliminate ones inevitably make a deterioration of others.

The probing signals with randomized pulse repetition period are often used to mitigate the distance and velocity ambiguities [2–5] (pulse repetition jittering, staggered pulse repetition interval etc.). The sequential transmission of multiple repetition frequencies in dwell time could also be used for the same purpose [6–8]. Signals with pseudorandom amplitude and phase-shift keying [9–11] reveal an alternative approach and will be offered in present paper. The key feature of such class of signals is an irregular interval between modulated (coded) pulses not adjusted with the instrumental radar range, much shorter then coherent processing interval (CPI). Hereinafter we will refer such type of signals as “quasi-continuous signals”.
The envelope of a quasi-continuous signal is given by a binary pseudorandom sequence. Two or more sequential pulses may be radiated back to back without delay in such a way that probability of pulse transmission $C=0.1$. The law of phase-shift keying changes from pulse to pulse.

Use this type of signals makes it possible to completely eliminate both ambiguities on distance and Doppler shift due to their thumbtack shaped ambiguity function and stable side lobes level on time delay and Doppler shift cross-sections in $\Delta re[-T/2, T/2]$, $\Delta v e[-1/2t_b, 1/2t_b]$, where $T$ is the overall length of quasi-continuous signal and thus the CPI. $t_b$ is the duration of the coded bit.

Moving target detection with the target’s range and speed estimation is conducted in multi-channel devices. Each channel detects a signal with a fixed delay and Doppler frequency shift. Since the pulse repetition interval is random, the spectrum of the probing quasi-continuous signal is continuous. Signals reflected from a moving target and stationary objects overlap in frequency.

Clutter immunity depends on the time-bandwidth product of the probing signal. Traditional correlation processing suffers from high side lobe level of the ambiguity function. Because of this, the probability of moving target detection and frequency-time measurements of the reflected signal parameters is limited.

The literature describes methods to improve clutter immunity for various random pulse trains. So, in [12, 13] compressed sensing method based on iterative grid optimization has been proposed for clutter suppression for airborne random pulse repetition interval radar. In [14, 15] new approach of the clutter filtering for Doppler weather radars using staggered pulse repetition time proposed. The effectiveness of the above methods in radars with quasi-continuous signal is not known.

In [16] it is proposed to limit the amplitude of the received additive sum of returns. Clipping of input signal normalizes the high-power signal level that interferes with weak valid signal. A part of valid signal pulses is also suppressed during clipping. However, valid signal pulses that are not clipped can be coherently processed. As a result of the clipping, the signal-to-clutter-plus-noise ratio after the correlation processing increases.

However, this method is effective only when there are a few powerful point targets or they are located in a limited range of delays. When the delay of a weak signal is equal to the delay of one of the high-power signal, such weak signal is completely suppressed. An effective algorithm for detecting a week quasi-continuous signal of a moving target at its time overlap is required.

We propose our method how to improve the detection of high-speed target on the background of passive reflections: use of the quasi-continuous signals with a two-level pseudorandom envelope of the amplitude spectrum (the signal generation algorithm is described in this paper) and do frequency rejection before the processing of the return signal. Let us explain the idea of our proposals.

Let the amplitude spectrum of the probing quasi-continuous signal have a two-level random envelope. The shape of the spectrum envelope (the location on the frequency axis of high-level and low-level components of the spectrum) is described by a pseudo-random sequence. Doppler frequency shift between signals reflected from a moving target and stationary objects leads to the shift of their spectrums. The high-level components of the spectrum of these signals overlap partially.

During processing it is necessary to calculate the spectrum of the received signal. The location on the frequency axis of the high-level components of the signal spectrum of stationary objects is known. Therefore, they must be cut out of the spectrum of the return signal. The resulting spectrum should be multiplied by the spectrum of the reference signal with frequency shift $F_v=v/T$, where $v=0,1,2,...$ is the Doppler frequency shift of the receiver, and calculate the inverse Fourier transform [17]. The result of the calculations is the correlation function between the reference signal and the signal after frequency rejection.

Frequency rejection during processing changes the signal-to-clutter ratio. When the high-level components of the reflection spectrum from a moving target and stationary objects overlap slightly, a small part of the useful signal energy is lost, but a significant part of the interference energy is cut out. As a result, low-level components of the signal spectrum of stationary objects and a significant part of the high-level and low-level components of the useful signal remain in the spectrum of the processed signal after frequency rejection. The signal-to-clutter ratio increases, which helps to detect a weak signal of a moving target.
In [18], an algorithm for the synthesis of continuous signals with multi-position phase manipulation and a two-level pseudorandom envelope of the amplitude spectrum is proposed. When the radar uses a common aerial for transmission and reception, its probing signal must consist of pulses. Then, in the pauses between radiations it is possible to receive reflections. In order to turn a continuous signal into a pulse signal, we propose a modification of the algorithm from [18]. We introduced a binary sequence into the synthesis algorithm to set the law of amplitude manipulation of a quasi-continuous signal. As a result, a signal with pseudorandom amplitude manipulation, multi-position phase manipulation and bi-level pseudorandom envelope of the amplitude spectrum is synthesized.

The shape of the spectrum of the synthesized signals is analyzed. The results of modeling the detection of a moving target against the background of reflections from a stationary object are presented. Reflections from a stationary object are represented by a linear sum of returns with different delays and the same, close to zero, Doppler frequency shift. An increase in the signal-to-clutter ratio at frequency rejection during signal processing is estimated.

2. Mathematical definition of the quasi-continuous signal with bi-level irregular structure of the amplitude spectrum and it’s correlation processing with frequency rejection

The complex envelope \( u(t) \) of the probing quasi-continuous signal of duration \( T \) is given by

\[
 u(t) = \frac{1}{\sqrt{t_b}} \sum_{i=0}^{N_x-1} x_i \sum_{k=0}^{N_k-1} \exp \left( j \phi_{k_x,k} \right) \text{rect} \left( \frac{t - (i \cdot k_x + i_x) t_b}{t_b} \right) \quad 0 \leq t < T,
\]

where \( x_i \{1,0\}, \ i=0..N_x-1 \) the amplitude code, \( \phi_n \in [0, \pi] \), \( n=0..N-1 \), \( N=N_xk_x \) – the multi-level phase code, \( t_b \) is the duration of the coded bit. Then \( \omega_{i,k_x,k} = x_i \exp \left( j \phi_{k_x,k} \right) \), \( i=0..N_x-1 \), \( k_x=0..k_x-1 \) is the amplitude-phase code, associated with \( u(t) \). The duration of the signal can be defined as \( T=N t_b \), or \( T=N_x t_x \), \( t_x=k_x t_b \).

The parameter \( C_x = \sum_{i=0}^{N_x-1} x_i / N_x \) is the probability of the phase coded pulse to be transmitted. The quasi-continuous signal energy can be expressed as

\[
 E = \int_0^T |u(t)|^2 \, dt = \sum_{n=0}^{N-1} |\omega_n|^2 = k_x \sum_{i=0}^{N_x-1} x_i = k_x C_x N_x = C_x N.
\]

Figure 1 shows the structure of the quasi-continuous signal, which consists of the random train of the phase coded pulses.

![Figure 1. The structure of the quasi-continuous signal.](image-url)
where \( b_i \in \{1,0\}, i=0..N_b-1 \) - is pseudorandom binary sequence with length \( N_b=N/k_b \). The parameter \( k_b/T \) determines the minimum frequency interval with almost the same value of the amplitude spectrum of the signal. The parameter \( C_b = \sum_{i=0}^{N_b-1} b_i / N_b \) - is the probability that the \( i \)-th bit of sequence is 1, \( b=1 \). The value \( C_b \) is also the probability that at the frequency range \( k_b/T \) the amplitude spectrum of the \( u(t) \) signal will be high-level. The value of \((1-C_b)\) is the probability that the amplitude spectrum of the \( u(t) \) signal will be low-level.

Synthesis algorithm of signal with spectrum shape like this is given in table 1.

**Table 1.** Synthesis algorithm of multiphase signals with bi-level irregular structure of amplitude spectrum.

| Step | Description |
|------|-------------|
| 1.   | Form as in (1) \( N \) signal sample \( u(t_n), t_n=nT/N, n=0..N-1 \) (The signal envelope is determined by a given sequence \( x \). The instantaneous value \( \varphi \) of the signal phase has arbitrary values); form according to (2) function counts \( B(f_k), f_k = k/T, k=0..N-1 \); |
| 2.   | Calculate the Discrete Fourier Transform (DFT) \( U(f_k) = \sum_{n=0}^{N-1} u(t_n) \exp(-j2\pi kn/N) \); |
| 3.   | Calculate the inverse DFT \( y(t_n) = \sum_{k=0}^{N-1} Y(f_k) \exp\left( j\frac{2\pi kn}{N} \right) \); |
| 4.   | Calculate the inverse DFT \( u(t) = \sum_{k=0}^{N-1} \sum_{i=0}^{k-1} \frac{1}{\sqrt{k_b}} \sum_{i=0}^{k_i} \text{arg}\left[ y(t, x_i) \right] \cdot \text{rect}\left[ -\frac{t-(i+k_i)t_i}{t_b} \right] \), where the \( \text{arg} \) function returns the argument of a complex number; |
| 5.   | If \( l \) exceeds the \( L \), the signal synthesis is terminated, otherwise return to 2); |
| 6.   | Perform quantization of the phase of the synthesized signal, \( \varphi_b = \arctan[\arg[u(t_n)/\Delta_n]\Delta_n] \) is integer division operation. |

The synthesis algorithm includes a cyclic procedure. The amplitude spectrum of the signal is calculated and then corrected at each iteration of the synthesis algorithm. In addition, a new \( u(t) \) signal with a spectrum shape approaching the \( B(f) \) function is calculated and then corrected. An example of the synthesized signal and its amplitude spectrum is shown in figure 2.

The Doppler frequency shift causes a shift in the signal spectrum. In figure 3a line 1 shows the spectrum of the linear sum, \( s(t)=s_c(t)+\tilde{s}(t) \), of the useful signal, \( s_c(t)=A_c w(t-\tau_c) \exp[j(2\pi F_c t+\varphi_c)] \), and the interference signal, \( \tilde{s}(t)=A w(t-\tau_c) \exp[j(2\pi F_t+\varphi_t)] \). The \( s(t) \) and \( \tilde{s}(t) \) signals have the same delay \( \tau_c \) and different Doppler frequency shift: \( F_c=500/T \) and \( F_t=0 \). Initial phases, \( \varphi_c, \varphi_t \), of the signals are random variables with uniform distribution law in the range \([0,2\pi]\). In figure 3a line 2 shows useful signal \( s_c(t) \) spectrum. The power \( A_c^2 \) of the signal \( s_c(t) \) is less than the power \( A_t^2 \) of the signal \( \tilde{s}(t) \) on 33dB.
Figure 2. The quasi-continuous signal of the length \( N=6400, k_N=4 \).
1 – Envelope, \( N_e=40, k_e=160, C_e=0.325 \);
2 – Phase, \( K_\phi=32 \);
3 – Amplitude spectrum;
4 – The \( B(f) \) function, \( N_b=40, k_b=160, C_b=0.325 \).

Figure 3. Amplitude spectrum of signals \( s(t) \) (line 1) and \( \xi(t) \) (line 2).
a) before the frequency rejection.
b) after the frequency rejection.

The high-level components in the amplitude spectrum of the \( s(t) \) and \( \xi(t) \) signals are biased relative to each other. The product of the \( s(t) \) linear sum spectrum by the function \([1 - B(f)]\) removes the high-level components of the high-power signal with the Doppler shift \( F_\xi=0 \).

The amplitude spectrum from figure 3a after frequency rejection is shown in figure 3b.

The power of the \( \xi(t) \) signal decreased more than the power of the \( s(t) \) signal. The power ratio of these signals equals 17.3 dB after frequency rejection.

If the Doppler frequency of the \( \xi(t) \) signal lies in the known frequency range \( f \in [v_{min}/T, v_{max}/T] \), then the frequency rejection function is described by the expression

\[
B_R(f) = \prod_{\nu=v_{min}}^{v_{max}} \left[ 1 - B(f - \nu/T) \right],
\]
The response of a multichannel correlation receiver with frequency interference rejection to the received signal $s(t)$ is described by the expression

$$|X(\tau_m, F_v)| = \left|\frac{1}{2\pi} \int_{-\tau_{2\nu}}^{\tau_{2\nu}} S(f) R_{12}(f) U^*(f - F_v) \exp(i2\pi f \tau_m) df\right|, \quad (4)$$

where:

- $S(f) = \int_0^T s(t) u_b(t) \exp(-j2\pi f t) dt$ is the spectrum of the received signal after blanking the receiver for the time of emission of phase-shift keying pulses by the signal $u_b(t) = \sum_{k=0}^{N-1} (1 - x_k) \cdot \text{rect}\left(\left|t - i/T_1\right|/T_1\right)$;

- $U(f) = \int_0^T u(t) \exp(-j2\pi f t) dt$ is the reference signal spectrum;

- $\tau_m=m\cdot\tau_b$, $m=1,2,3,\ldots$ is the delay shift and $F_v=v/T$, $v=0,\pm1,\pm2,\ldots$ is the Doppler frequency shift of the receiver;

- * - the complex conjugation sign.

When $B_R(f)=1$ the expression (4) describes the traditional correlation processing.

Figure 4a shows the frequency-time response normalized to its maximum value. At the same time, $B_R(f)=1$, $S(f)$ is the spectrum of the linear sum of the $s_c(t)$ and $\xi(t)$ signals considered above. Frequency rejection is not performed. The $s_c(t)$ signal is not detected. Note that we also used the maximum of the function $|X(\tau_m, F_v)|$ of this example to normalize the responses in figures 4b and 4c.

The normalized response to the $s_c(t)$ signal is shown in figure 4b, and there is no $\xi(t)$ signal. Frequency rejection is not performed. The maximum response is -33 dB, which is comparable to the standard value of the $|X_{\text{norm}}(\tau_m, F_v)|$ function from figure 4a. Correlation processing increased the signal-to-clutter ratio $N$ times from $q_{\text{input}}=A_s^2/A_s^2=-33$ dB to $q_{\text{without rejection}}=0$ dB. However, this is not enough to detect the target.

When the linear sum of the $s_c(t)$ and $\xi(t)$ signals is processed and frequency rejection is performed, the RMS value of the $|X_{\text{norm}}(\tau_m, F_v)|$ function decreases (figure 4c). It becomes equal to -52 dB. The $s_c(t)$ signal is detected. The signal-to-clutter ratio is $q_{\text{with rejection}}=17.5$ dB.

Value $|X_{\text{norm}}(\tau_m, F_v)|$ is -35 dB, which is 2 dB less than the response of figure 4b. This is the loss in the $s_c(t)$ signal spectrum during frequency rejection. Rest part of the spectrum of the $s_c(t)$ signal coincide with the high-level components of the signal $\xi(t)$.
3. The quality of the synthesized signals

The quality of the synthesized signals is characterized by the ratio of the RMS values of high-level and low-level components of the discrete amplitude spectrum $|U(f_k)|$.

$$\eta = \frac{\sum_{k=0}^{N-1} (1-B(f_k)) \sum_{k=0}^{N-1} (B(f_k)|U(f_k)|^2)}{\sum_{k=0}^{N-1} B(f_k) \cdot \sum_{k=0}^{N-1} ((1-B(f_k))|U(f_k)|^2)} = \frac{1-C_b}{C_b} \frac{\sum_{k=0}^{N-1} B(f_k)|U(f_k)|^2}{\sum_{k=0}^{N-1} ((1-B(f_k))|U(f_k)|^2)}$$

(5)

The $\eta$ value is the dynamic range of the amplitude spectrum levels. The signal spectrum from figure 2 is characterized by $\eta=20$ dB.

The $\eta$ value determines how much the RMS value of the $|X_{\text{norm}}(\tau_n,F_v)|$ function will decrease after frequency rejection. The greater the $\eta$, the less energy is contained in the low-level components of the signal spectrum, and the greater part of the interference energy can be removed during frequency rejection, and the less will be the loss of the valid signal, and the higher the probability of its detection.

The synthesis algorithm does not check the quality of the synthesized signal. Therefore, an increase in the number of synthesis iterations does not lead to a significant increase in the $\eta$ value. Studies have shown that the value of $L=100$ is enough.

The quantization phase of the synthesized signal is produced when $L$ iterations are performed. The dynamic range of the amplitude spectrum levels increases with the number of phase $K$, figure 5. It reaches its maximum value at $K_c=32$. Further increase in the number of levels of phase quantization does not make sense.

![Figure 5](image)

**Figure 5.** Dependence of the dynamic range of amplitude spectrum levels on the number of quantization levels.

The dynamic range of the spectrum levels of the synthesized signal is determined by the parameters of its $|u(t)|$ envelope: $N_x$, $k_x$, $C_x$, as well as the parameters of the $B(f)$ function: $C_b$, $N_b$, $k_b$. If they are constant, the value of $\eta$ depends on the coefficient $k_N=N/(N,N_b)$.

The amplitude spectrum of the synthesized signal $u(t)$ with duration $T=N_x k_x t_{b}$ is the sum of the amplitude spectrum of $K_x=C_x N_x$ delayed phase-shift keyed pulses with duration $t_{c}=k_x t_{b}$. The random binary sequence $x_i$, $x_i \in \{1,0\}$, $i=0…N_c-1$, determines the phase coded pulse position along time axis 0..T.

When $k_N=1$ ($N=N_x N_b$), the length of the sum of phase-shift keyed pulses is $k_c=N_b$. Then $N_b$ samples of the phase-shift keyed pulses with duration $t_{c}=k_c t_{b}$ uniquely determine $N_b$ values of the discrete spectrum of this pulse.

When $k_N<1$ ($N<N_x N_b$), the length of the phase-shift keyed pulses is less than $N_b$, $k_c<N_b$. Then $k_c<N_b$ of samples of the phase-shift keyed pulses must uniquely determine $N_b$ of different values of the discrete spectrum of this pulse, which contradicts the Nyquist theorem. This explains the decrease in the high and
low level components difference in the amplitude spectrum of the synthesized signal at \( N < N_b \). The envelope of the amplitude spectrum of the synthesized signal is less similar to the function \( B(f) \), the less \( k_N \).

When \( k_N > 1 \) (\( N > N_b \)), the length of the phase-shift keyed pulses is more than \( N_b \), \( k_N > N_b \). For example \( k_N = 2 \). Then \( 2N_b \) samples of the phase-shift keyed pulses determine \( N_b \) samples of the discrete spectrum.

Figure 6 shows the change in the envelope of the amplitude spectrum of signals with parameters \( N_x = 40, C_x = 0.325, N_b = 40, C_b = 0.325, K_v = 32 \) depending on \( k_N \). The dynamic range of the components of the signal amplitude spectrum increases with increasing \( k_N \). It approaches the \( \eta_{\text{max}} \) maximum value obtained for the \( u(t) \) signal with constant amplitude, \( x_n = 1, n = 0..N-1 \).

![Figure 6](https://example.com/figure6.png)

**Figure 6.** Signal amplitude spectra for different \( k_N \).

Figure 7a shows the dependencies \( \eta(k_N) \) for \( N_b/N_x = 1, Q_x = 2 \) and different \( Q_b \) values. Figure 7b shows the dependencies \( \eta(k_N) \) for \( Q_b = 2, Q_x = 5 \) and different values of the ratio \( N_b/N_x \). Figure 7c shows the dependencies \( \eta(k_N) \) for \( N_b/N_x = 1 \) and different \( Q_x \) values.

![Figure 7](https://example.com/figure7.png)

**Figure 7.** The dependencies \( \eta(k_N) \) for different signal parameters.

The dependencies show:

1) The value of \( \eta \) is close to the maximum value at \( k_N > 16 \). The parameters \( C_x, N_x, C_b, N_b \) are arbitrary.

2) The value of \( \eta \) is larger for larger values of \( C_b \) and \( C_x \). The choice of a \( C_b \) is more important and changes the \( \eta \) more than the choice of \( C_x \). But the value \( \eta \) does not exceed 27dB.

3) When \( N_b > N_x \), the same value of \( \eta \) can be obtained at lower value of \( k_N \).

The absolute maximum of \( \eta \) equals to 27dB when \( C_b = 0.5 \). It is achievable for any \( C_x \) due to the increase in \( k_N \), and therefore the signal time-bandwidth product.
4. Estimation of gain in signal-to-clutter ratio on the results of simulation of generation and processing of signals with frequency rejection

The efficiency of frequency rejection is characterized by a gain $g$ in the signal-to-clutter ratio, $g=q_{\text{with rejection}}/q_{\text{without rejection}}$. Computer simulation of the generation and processing of the additive sum $s(t)$ of the useful signal $s_{c}(t)$ and the interference signal $\xi(t)$ allows us to estimate the value of $g$ for different distributions of interference on the delay and Doppler frequency.

During the simulation, the interference signal was formed by the expression

$$\xi(t) = \int A_k(t)u(t-t)\exp\left[i(2\pi F_k t + \varphi_k)\right]dt,$$

where $M_b$ defines the delay range of the interference, $F_k$ and $\varphi_k$ - Doppler frequency and the initial phase of the interference, respectively. The presented simulation results are limited to the case when the value of $F$ is a random variable from range $[-1/T; 1/T]$, $\varphi_k$ is a random variable from the range $[0, 2\pi]$.

When $M=1$, the signal $\xi(t)$ is reflected by a point object. When $M>1$, the source of interference is an extended object. Function $A_k(t)$ specifies the delay change in the amplitude of the interference components.

In the previous section of this paper, we presented the results of modeling the processing of the linear sum of two signals: the useful signal $s_{c}(t)$ and the interference signal $\xi(t)$. Signals were reflected from point objects located at the same distance. The $s_{c}(t)$ signal is reflected by a moving object. The $\xi(t)$ signal is reflected by a stationary object. The signals $s_{c}(t)$ and $\xi(t)$ had the same delay and different Doppler frequency shift. The signals differed in power. The signal-to-clutter ratio $q_{\text{input}}$ at the input of the correlation processing was $1/N$. This value was chosen specifically. The value of $N$ determines the integrated sidelobe level of the ambiguity function of the quasi-continuous signal [10]. Therefore, the power of the useful signal and the interference signal will be the same after the correlation processing. The signal-to-clutter ratio $q_{\text{without rejection}}$ after correlation processing is 1. When $q_{\text{input}}=N$ is retained, the value of $q_{\text{without rejection}}$ remains equal to 1 also if the interference is caused by a set of point reflectors or by an object with a length greater than the range resolution. The signal-to-clutter ratio increases after frequency rejection of high-level components of the interference spectrum. Its value determines the gain $g=q_{\text{with rejection}}/q_{\text{without rejection}}$. For the example above, $g=17.5 \text{ dB}$. The gain value $g$ is less than the dynamic range of signal of spectrum levels. There are several reasons for this.

The first reason: blanking the receiver with the signal $u_{th}(t)$ (see expression (4)) degrades the received signal for the time of pulse emission.

The return signals are received only in pauses between probing signal pulses. The probability that the receive path is opened in the $n$-th time delay equals $(1-C_s)$. So part of the return signal energy is lost. $(1-C_s)$ characterizes the probability of energy loss. If $E_s$ and $E_\xi$ are the energy of the valid signal and interference, respectively, then after blanking the receiver they are on average equal to $(1-C_s)E_s$ and $(1-C_s)E_\xi$, respectively.

The dynamic range $\eta_h$ of the signal spectrum levels after blanking is also decreased.

$$\eta_h = \sqrt{\frac{1-C_s}{C_s} \sum_{k=0}^{N-1} \left|\frac{B(f_k)}{S(f_k)}\right|^2} < \eta.$$

If delay of the point reflector signal is greater than $k_d h_b$, the shape of the envelope spectrum $S(f)$ is preserved. If $\tau < k_d h_b$, the shape of the envelope of the spectrum $S(f)$ is distorted. The dynamic range $\eta_h$ is reduced. When $\tau = h_b$, then after blanking of the receiving path from each phase-shift key pulse remains one elementary pulse with duration $h_b$. The spectrum of such a signal completely loses its two-level structure. The value $\eta_h=0 \text{ dB}$. Envelope and amplitude spectrum of signals with delays after blanking are
shown in figure 8. The dependence of $\eta_0$ on the delay of the interference signal is shown in figure 9. When $\tau > k_x t_b$, the dynamic range of $\eta_0$ decreases compared to the average of $(1-C_x)^2$ times.

![Figure 8. Dynamic range components of the signal spectrum with delay t after blanking](image1)

The value of $\eta_0$ decreases in average $(1-C_x)^2$ times for delay-distributed interference signals. The dependence of $\eta_0(\tau)$ is changed only at the site $\tau < k_x t_b$. If the amplitudes of $A_0(\tau)$ in the delay range $\tau \in [\tau, \tau + M t_b]$ have approximately the same value, and $M$ increases, then the value of $\eta_0(0)$ also increases and approaches $(1-C_x)^2$. This can be seen in the comparison of curves 1 ($M=1$), 2 ($M=10$) and 3 ($M=160$) in figure 8. If the distributed interference has strong point reflectors with $\tau < k_x t_b$, the change in $\eta_0(\tau)$ approaches curve 1.

![Figure 9. Gain in signal-to-clutter ratio in frequency rejection](image2)

The second reason why $g < \eta$: the energy loss of the useful signal during frequency rejection. Part of the spectral components of the useful signal is cut from the input sum of the signals. The value of losses depends on the overlap of high-level components of the amplitude spectrum of the useful signal and interference. It is determined by the $C_b$ parameter.

As a result, the gain in the signal-to-clutter ratio from frequency rejection is estimated as an average value, $g = \eta (1-C_x) C_b$.

For a signal with $\eta = 20$ dB, the gain value in the signal-to-clutter ratio will lie in the range of 7 to 17 dB depending on the $C_x$ and $C_b$. You can get $g = 20$ dB for a signal with $\eta = 27$ dB and $C_x = C_b = 0.5$.

5. Conclusion
One of the ways to improve the ability to detect a moving target of radar with transmission and reception on common aerial of quasi-continuous signals is the frequency rejection of reflections from stationary objects. This way is possible in the presence of probing signals providing minimization of valid and interfering signals’ power spectrum overlay.
This article presented wideband signals with pseudorandom bi-level amplitude-, multi-level phase-shift keying and irregular bi-level structure of amplitude spectrum, considered their synthesis. Signals with such amplitude spectrum structure allow using the clutter frequency rejection distributed along the delay axis, but localized at a relatively small range of Doppler frequency shifts. The gain in the signal-to-clutter ratio obtained after interference frequency rejection depends on the dynamic range of the levels of the components of the spectrum of the probing signal.

A good choice of quasi-continuous signal parameters increases the gain in the signal-to-clutter ratio up to 20dB. If the power of interference is so large that the frequency rejection does not allow to detect a moving target, only the increase of the time-bandwidth product of the probing signal helps to solve this problem.

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