Implications of an Improved Neutron-Antineutron Oscillation Search for Baryogenesis: A Minimal Effective Theory Analysis

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Future neutron-antineutron (n-¯n) oscillation experiments, such as at the European Spallation Source (ESS), aim to find first evidence of baryon number violation. We investigate implications of an improved n-¯n oscillation search for baryogenesis via interactions of n-¯n mediators, parameterized by an effective field theory (EFT). We find that even in a minimal EFT setup, there is overlap between the parameter space probed by the ESS and the region that can realize the observed baryon asymmetry of the universe. The mass scales of exotic new particles are in the TeV-PeV regime, inaccessible at the LHC or its envisioned upgrades. Given the innumerable high energy theories that can match to, or resemble, the minimal EFT that we discuss, future n-¯n oscillation experiments could probe many viable theories of baryogenesis beyond the reach of other experiments.

Introduction — The search for physics beyond the Standard Model (BSM) requires efforts at both high energy and intensity frontiers. In this regard, a particularly powerful probe is offered by rare processes that violate (approximate) symmetries of the Standard Model (SM), such as baryon and lepton numbers (B and L), which can be inaccessible to high energy colliders but within reach of low-energy experiments. A well-known example is proton decay, whose non-observation leads to strong constraints on ΔB = ΔL = ±1 new physics even at the scale of Grand Unified Theories (GUTs), ~10^{16} GeV [1, 2].

Baryon and lepton number violation are intricately tied to one of the outstanding puzzles in fundamental physics, the origin of the baryon asymmetry in the universe. If baryogenesis occurs at temperatures above the weak scale, B − L violation is required to avoid washout by electroweak sphalerons. In this regard, constraints from proton decay (which conserves B − L) are not applicable. Here we consider instead B-violating, L-conserving new physics at an intermediate (sub-GUT) scale, so that baryogenesis may proceed both above and below weak scale temperatures. From the low energy point of view, effects of heavy new particles are encoded in higher dimensional operators in an effective field theory (EFT), where B-violating, L-conserving interactions can appear first at the dimension-nine level [3], in the form of |ΔB| = 2, ΔL = 0 operators. In this case, neutron-antineutron (n-¯n) oscillation (see [4] for a recent review) is well placed to search for B violating phenomena and shed light on baryogenesis.

Current measurements constrain the free neutron oscillation time to be τn-¯n > 10^6 s [5, 6]. Upcoming experiments, in particular at the European Spallation Source (ESS) [7,8], are poised to improve on this limit by roughly three orders of magnitude. As we will see in detail below, such numbers translate into new physics scales of roughly (τn-¯nΔGCD)^{1/5} ~ O(10^2 GeV), well above the energies directly accessible at existing or proposed colliders. Discussions of the physics implications of a potential n-¯n oscillation discovery, in particular for baryogenesis, are therefore both important and timely.

The purpose of this letter is to explore the connection between n-¯n oscillation and baryogenesis in the context of a minimal EFT extension of the SM that realizes direct low scale baryogenesis from B violating decays of new particles mediating n-¯n oscillation. While there exist numerous baryogenesis frameworks, such as electroweak baryogenesis [9], Affleck-Dine baryogenesis [10], and leptogenesis [11] (see e.g. [12–20] for reviews), the choice of our minimal EFT is motivated from the bottom-up by imminent improvements in n-¯n oscillation searches. Despite being simplistic, this minimal setup provides a useful template to identify viable baryogenesis scenarios, which may be realized in a similar manner in more complex and realistic theories, that are compatible with an n-¯n oscillation signal within reach of the ESS (for discussions of some other baryogenesis scenarios that can also involve n-¯n oscillation signals, see e.g. [21–35]).

Improved n-¯n oscillation searches and sensitivity to the scale of new physics — Neutron-antineutron oscillation has been searched for in the past with both free neutrons [5,39,40] and neutrons bound inside nuclei [6,41–45]. Among free neutron oscillation searches, the Institut Laue-Langevin (ILL) experiment [5] sets the best limit to date on the oscillation time, τn-¯n > 0.86 × 10^8 s at 90% C.L. Among intranuclear searches, Super-Kamiokande (Super-K) [0] provides the best limit, which, after correcting for nuclear effects, corresponds to τn-¯n > 2.7 × 10^8 s at 90% C.L. for the free neutron oscillation time. Improved n-¯n oscillation searches are under consideration with both free and bound neutrons [7,8,46]. In particular, the upcoming European Spallation Source (ESS) experiment can extend the reach on τn-¯n by three orders of magnitude with respect to the ILL [7,8].
We now elucidate the connection between $\tau_{n\bar{n}}$ and the new physics scale in the EFT context. The lowest dimension effective operators contributing to $n-\bar{n}$ oscillation at tree level are dimension-nine operators of the form $\mathcal{O}_{n\bar{n}} \sim (uudddd)$. The classification of these operators dates back to the 1980s [37, 38] and was refined recently in [52], which established an alternative basis more convenient for renormalization group (RG) running. A concise review of the full set of tree-level $n-\bar{n}$ oscillation operators is provided in the Appendix. In what follows, we focus on one of these operators for illustration,

$$\mathcal{L} \supset c_1 \frac{1}{2} \epsilon_{i j k} \epsilon_{\nu j k} \overline{\nu} P_R d_i \overline{h} \nu P_R d_i + \text{h.c.},$$

with $c_1 \equiv (\Lambda_{n\bar{n}}^{(1)})^{-5}$. (1)

Here $u, d$ are SM up and down quark fields, respectively, and $\nu^i, \overline{\nu}^i$ are their charge conjugates. $i^{(j)}, j^{(j)}, k^{(j)}$ are color indices, and “h.c.” denotes hermitian conjugate. The operator suppression scale $\Lambda_{n\bar{n}}^{(1)}$ is generally a weighted (geometric) average of new particle masses, modulo appropriate powers of couplings and loop factors.

If the operator is generated by integrating out new particles at a high scale $M$, computing $\tau_{n\bar{n}}$ requires RG evolving the EFT down to a low scale $\mu_0$ (usually chosen to be 2 GeV), where it can be matched onto lattice QCD. The leading contribution to RG rescaling reads [51, 52]

$$\frac{c_1(\mu_0)}{c_1(M)} = \left[ \frac{\alpha_s^{(4)}(m_b)}{\alpha_s^{(4)}(\mu_0)} \right]^{\bar{\alpha}_s} \left[ \frac{\alpha_s^{(5)}(m_t)}{\alpha_s^{(5)}(\mu_t)} \right]^{\bar{\alpha}_s} \left[ \frac{\alpha_s^{(6)}(M)}{\alpha_s^{(6)}(m_t)} \right]^{2 \bar{\alpha}_s}$$

$$= \{0.726, 0.684, 0.651, 0.624\},$$

for $M = \{10^3, 10^4, 10^5, 10^6\}$ GeV. (2)

Here $\alpha_s^{(n)}$ is the effective strong coupling with $n_f$ light quark flavors, whose value is obtained with the RunDec package [53]. Corrections from two-loop running as well as one-loop matching onto lattice QCD operators were recently computed [52] and are small, and will be neglected in our calculations. No additional operators relevant for $n-\bar{n}$ oscillation are generated from RG evolution.

The $n \rightarrow \bar{n}$ transition rate is determined by the matrix element of the low-energy effective Hamiltonian between the neutron and antineutron states, $\tau_{n\bar{n}}^{-1} = |\langle \bar{n} | H_{\text{eff}} | n \rangle|^2$. Thus, once $\langle \bar{n} | \mathcal{O}_{n\bar{n}}(\mu_0) | n \rangle$ are known, we can relate $\tau_{n\bar{n}}$ to the six-quark operator coefficients. Recent progress in lattice calculations [54, 55] has greatly improved the accuracy and precision on $\langle \bar{n} | \mathcal{O}_{n\bar{n}}(\mu_0) | n \rangle$ compared to previous bag model calculations [49, 50] often used in the literature. Using the results in [55], and assuming the operator in Eq. (1) gives the dominant contribution to $n-\bar{n}$ oscillation, we can relate $\tau_{n\bar{n}}$ to the operator suppression scale $\Lambda_{n\bar{n}}^{(1)}$, as shown in Fig. 1 (for a representative RG rescaling factor of 0.7). The current best limit from Super-K translates into $\Lambda_{n\bar{n}}^{(1)} \gtrsim 4 \times 10^5$ GeV, while the improved free neutron oscillation search at the ESS will probe up to $\Lambda_{n\bar{n}}^{(1)} \sim 1.3 \times 10^6$ GeV. These numbers are representative of the whole set of $n-\bar{n}$ oscillation operators, and do not vary significantly with the starting point of RG evolution $M$ (see Appendix for details).

**A minimal EFT for $n-\bar{n}$ oscillation and baryogenesis** — One of the simplest possibilities for generating the operator in Eq. (1) at tree level is with a Majorana fermion $X$ of mass $M$ that couples to the SM via a dimension-six operator of the form $\frac{1}{M} X uuddd$, which originates at an even higher scale $\Lambda \gg M$ via some UV completion that we remain agnostic about. A familiar scenario that realizes this EFT setup is supersymmetry (SUSY) with $R$-parity violation (RPV), where the bino plays the role of $X$ and the dimension-six operator is obtained by integrating out squarks at a heavier scale. However, this simple EFT with a single BSM state does not allow for sufficient baryogenesis due to unitarity relations: in the absence of $B$-conserving decay channels, $X$ decay cannot generate a baryon asymmetry at leading order in the $B$-violating coupling, a result known as the Nanopoulos-Weinberg theorem [56] (see [57] for a recent discussion); meanwhile, $2 \rightarrow 2$ processes $uX \rightarrow d\bar{d}$ and $\bar{u}X \rightarrow d\bar{d}$ are forced to have the same rate and thus do not violate $CP$.

A minimal extension that can accommodate both $n-\bar{n}$ oscillation and the observed baryon asymmetry involves two Majorana fermions $X_1, X_2$ (with $M_{X_1} < M_{X_2}$), each having a $B$ violating interaction $\frac{1}{\Lambda} X uudd$. In addition, a $B$ conserving coupling between the two is necessary to evade constraints from unitarity relations. In the context
of RPV SUSY, this corresponds to the presence of a wino or gluino in addition to the bino, which is known to allow for sufficient baryogenesis \cite{27,59}.

Guided by minimality, we assume $X_{1,2}$ are both SM singlets, and consider just one of the many possible $B$ conserving operators in addition to the two $B$ violating ones. Our minimal EFT thus consists of the following dimension-six operators that couple $X_{1,2}$ to the SM\footnote{Our minimal EFT bears similarities with the models studied in \cite{60,61}. However, these papers focused on baryogenesis using operators of the form $(\bar{d}^c P_R d)(\bar{u}^c P_R u)$, which, upon Fierz transformations, are equivalent to generation-antisymmetric components of the $(\bar{u}^c P_R d)(\bar{d}^c P_R u)$ operators in Eq. (3), and thus do not mediate $n\bar{n}$ oscillation at tree level.}:

\[
\mathcal{L} \supset \eta_{X_1} \epsilon^{ijk}(\bar{u}^c P_R d_i)(\bar{d}_k^c P_R X_j) \\
+ \eta_{X_2} \epsilon^{ijk}(\bar{u}^c P_R d_i)(\bar{d}_k^c P_R X_j) \\
+ \eta_c (\bar{u}^c P_L X_1)(\bar{X}_2 P_R u_i) + \text{h.c.},
\]

with $|\eta_{X_1}| \equiv \Lambda_{X_1}^{-2}$, $|\eta_{X_2}| \equiv \Lambda_{X_2}^{-2}$, $|\eta_c| \equiv \Lambda_c^{-2}$.

Eq. (3) to capture the generic qualitative features possible in a two $B$ mediators setup, which can be realized in more complicated and realistic frameworks.

**Calculation of the baryon asymmetry** — The relevant processes for baryogenesis include:

- $B$ violating processes: single annihilation $uX_{1,2} \rightarrow \bar{d}\bar{d}$, $dX_{1,2} \rightarrow \bar{u}\bar{u}$, decay $X_{1,2} \rightarrow u\bar{d}$, and off-resonance scattering $udd \rightarrow \bar{u}\bar{d}d$;

- $B$ conserving processes: scattering $uX_1 \rightarrow uX_2$, co-annihilation $X_1 X_2 \rightarrow \bar{u}\bar{u}$, and decay $X_2 \rightarrow X_1 u\bar{u}$;

as well as their inverse and $CP$ conjugate processes. $CP$ violation arises from interference between tree and one-loop diagrams in $uX_{1,2} \leftrightarrow \bar{d}\bar{d}$, $uX_1 \leftrightarrow uX_2$ and $X_2 \leftrightarrow uud$, and additionally from $udd \leftrightarrow \bar{u}\bar{d}d$ (in a way that is related to $X_2 \leftrightarrow uud$ by unitarity). In each case, $CP$ violation is proportional to $\text{Im}(\eta_{X_1}^* \eta_{X_2} \eta_c) \sim \Lambda^{-6}$. We work at leading order in the EFT expansion, i.e. $\mathcal{O}(\Lambda^{-4})$ for the rates of $CP$-conserving processes and the $CP$-symmetric components of $CP$-violating processes, and $\mathcal{O}(\Lambda^{-6})$ for the $CP$-violating rates. We choose a mass ratio $M_{X_2}/M_{X_1} = 4$, which maximizes $\Gamma(X_2 \rightarrow u\bar{d}) = \Gamma(X_2 \rightarrow \bar{u}\bar{d}d)$ for fixed $M_{X_2}$ (see Eq. (A.32)).

We calculate the baryon asymmetry by numerically solving a set of coupled Boltzmann equations to track the abundances of $X_{1,2}$ and $B - L$ (above) $T = 140 \text{ GeV}$ (we assume sphalerons are active when $T > 140 \text{ GeV}$, resulting in $Y_B = \frac{32}{79} Y_{B-L}$). Our aim is to find regions of parameter space that can achieve the observed $Y_B = 8.6 \times 10^{-11}$ \cite{62,63}, with suitable choice of $CP$ phases. Technical details of this calculation can be found in the Appendix.

If all three operator coefficients in Eq. (3) have similar sizes, $\Lambda_{X_1} \sim \Lambda_{X_2} \sim \Lambda_c$, it is difficult to obtain the observed baryon asymmetry in the region of parameter space to be probed by the ESS. For $M_{X_{1,2}} \gtrsim 10^3 \text{ GeV}$, the $\Lambda$’s are sufficiently low in the ESS region that $X_{1,2}$ remain close to equilibrium until their abundances become negligible, while efficient washout suppresses $B(-L)$ generation. For lower masses and higher $\Lambda$’s, on the other hand, $X_2$ may freeze out with a significant abundance, and decay out of equilibrium at later times when washout has become inefficient, so that both limitations from the higher mass regime are overcome. However, its $CP$ violating branching fraction $\epsilon_{CP} \sim M_{X_2}^2/\Lambda^6$ is too small to generate the desired $Y_B$. We find that for $\Lambda_{X_1} = \Lambda_{X_2} = \Lambda_c$, the maximum $Y_B$ possible in the ESS sensitivity region is $\mathcal{O}(10^{-13})$, well below the observed value.

Achieving the desired baryon asymmetry in the ESS reach region therefore requires hierarchical $\Lambda$’s; such scenarios can arise if new particles in the UV theory that mediate the corresponding operators have hierarchical masses and/or couplings, or if the EFT operators are generated at different loop orders. We find compatible
regions of parameter space in two distinct scenarios, one with late decays of $X_2$ and the other with earlier decays. These are schematically illustrated in Fig. [2] and discussed in turn below (a detailed analysis with benchmark numerical solutions is presented in the Appendix).

**Late decay scenario** — For $\Lambda_{X_2} \sim \Lambda_c \gg \Lambda_{X_1}$, $n\bar{n}$ oscillation is dominated by $X_1$ exchange and probes the $M_{X_1}$-$\Lambda_{X_1}$ parameter space (see Fig. [3]). This hierarchy leads to weaker interactions for $X_2$ compared to the degenerate case, causing it to freeze out with a higher abundance $Y_{X_2}^{eq}$. Also, $X_2$ becomes long-lived and decays after washout processes have become ineffective, thereby creating substantial baryon asymmetry (see Fig. [2]). In this case, its $CP$-violating branching fraction scales as $\epsilon_{CP} \sim M_{X_2}^2 \eta_{X_2} \eta_{X_1} / \max(\eta_{X_2}^2, \eta_{X_1}^2) \sim \Lambda_{X_2}^2 / \Lambda_{X_1}^2$, and does not decouple as $\Lambda_{X_2}$ and $\Lambda_c$ are both increased, enabling $Y_B \sim Y_{X_2}^{eq} \epsilon_{CP}$ to reach the observed value.

Numerically, we find that this baryogenesis scenario is viable with $\Lambda_{X_2}, \Lambda_c \gtrsim 20 \Lambda_{X_1}$ in the parameter space to be probed by the ESS. In Fig. [3] we show regions in the $M_{X_1}$-$\Lambda_{X_1}$ plane that can accommodate the observed baryon asymmetry for various choices of $\Lambda_{X_1}/\Lambda_{X_1} = \Lambda_c/\Lambda_{X_1}$. In each case, the lower boundary of the viable region is effectively determined by the requirement that $X_2$ freezes out with sufficient abundance. As we move upward from this lower boundary, increasing all three $\Lambda$’s while keeping their ratios fixed, at some point we enter a regime where $X_2$ decouples from the SM bath while relativistic, and $Y_{X_2}^{eq}$ saturates at $Y_{X_2}^{eq}(T \gg M_{X_2}) = \frac{1}{\pi^2} T^2$, so that further increasing the $\Lambda$’s only reduces $\epsilon_{CP}$ and hence the final $Y_B$. Furthermore, for sufficiently high $\Lambda_{X_2}$ and $\Lambda_c$, $X_2$ dominates the energy density of the universe before it decays (this does not happen for $X_1$ in the parameter space we consider), so that its decay injects significant entropy into the plasma, diluting the baryon asymmetry. Both of these effects—saturation and dilution—determine the upper boundary of the viable region.

**Early decay scenario** — For the opposite hierarchy $\Lambda_{X_1} \gg \Lambda_{X_2}$, $n\bar{n}$ oscillation is dominated by $X_2$ exchange and probes the $M_{X_2}$-$\Lambda_{X_2}$ parameter space (see Fig. [4]). In this case, $X_2$ is short-lived, and its abundance closely follows the equilibrium curve. However, small departures from equilibrium, always present in an expanding universe because interaction rates are finite, can be sufficient for baryogenesis if washout can be suppressed. The rates for washout processes involving $X_1$ and $X_2$ are proportional to $n_1 \Lambda_{X_1}^{-4}$ and $n_2 \Lambda_{X_2}^{-4}$, respectively, where $n_{1,2}$ are the number densities of $X_1, X_2$. If $\Lambda_{X_1} \sim \Lambda_{X_2}$, washout would be efficient until $T \sim M_{X_1}$, i.e., until $n_1$ starts to fall exponentially. In contrast, by increasing $\Lambda_{X_1}$, we enter a regime where washout is dominated by $X_2$ processes at high temperatures and becomes inefficient as

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**FIG. 3.** Parameter space excluded by Super-K (below the solid line) and to be probed by the ESS (up to the dashed line) for the late decay scenario, assuming $M_{X_1} = 4 M_{X_2}$. For $\Lambda_{X_2} = \Lambda_c = 50 \Lambda_{X_1}$, the green shaded region can accommodate $Y_B = 8.6 \times 10^{-11}$. For $\Lambda_{X_2} = \Lambda_c = 25 \Lambda_{X_1}$ (100 $\Lambda_{X_1}$), the viable region is between dashed red (dot-dashed blue) lines. The gray shaded region marks $\Lambda_{X_1} < M_{X_2}$, where EFT validity requires greater than $O(1)$ coupling.

**FIG. 4.** Parameter space excluded by Super-K (below the solid line) and to be probed by the ESS (up to the dashed line) for the early decay scenario, assuming $M_{X_1} = 4 M_{X_2}$. Points represent solutions with $Y_B = 8.6 \times 10^{-11}$ found in a scan over $\Lambda_{X_1} < \Lambda_{X_2} < 100 \Lambda_{X_1}$, $M_{X_2} < \Lambda_c < \Lambda_{X_2}$. For all these points, $\Lambda_{X_1} \sim 10 \Lambda_{X_2}$ is needed to suppress washout. The gray shaded region marks $\Lambda_{X_2} < M_{X_2}$, where EFT validity requires greater than $O(1)$ coupling.
soon as the temperature falls below $M_{X_2}$ (washout due to $udd \leftrightarrow udd$, whose rate $\sim T^{11}/M^{2A^8}$ falls steeply with $T$, is also irrelevant at this point), resulting in a short period of baryon asymmetry generation from $X_2$ decays (see Fig. 2). Note that increasing $\Lambda_X$, with respect to $\Lambda_{X_2}$ also helps to increase departures from equilibrium compared to the degenerate case.

Fig. 4 shows points in the $M_{X_2}$-$\Lambda_{X_2}$ plane that can realize the observed $Y_B$ through this early decay process, based on a numerical scan over the region $\Lambda_{X_2} < \Lambda_{X_1} < 100 M_{X_2}$, $M_{X_2} < \Lambda_c < \Lambda_{X_2}$. For the majority of these points, $\Lambda_{X_1}$ is within a factor of two from $10 \Lambda_{X_2}$, while $\Lambda_c \lesssim 3 M_{X_2}$. The results can be understood from the competing effects of baryon asymmetry generation and washout, $\Gamma_{\Delta B \neq 0}/\Gamma_{\text{wo}} \sim M^2 n_2 (\Lambda_{X_2}^5/\Lambda_{X_1}^2 \Lambda_0^2)^{-1}/(n_1 \Lambda_{X_1}^{-4} + n_2 \Lambda_0^{-4}) \sim (M^2/\Lambda_0^2) \cdot \min\{\Lambda_{X_2}^4/\Lambda_{X_1}^2, \Lambda_{X_1}/\Lambda_{X_2}, e^{-(M_{X_2} - M_{X_1})/T}\}$, where the rate of baryon asymmetry generation $\Gamma_{\Delta B \neq 0}$ is calculated from $CP$-violating $X_2$ decays. First of all, a lower ratio $\Lambda_c/M_{X_2}$ is always preferable (within the range of EFT validity), while the ratio $\Lambda_{X_2}/\Lambda_{X_1}$ has an optimal value of $\sim 1/10$ as a result of balancing between faster baryon asymmetry generation at higher temperatures (which favors higher $\Lambda_{X_2}/\Lambda_{X_1}$) and later transition to $X_1$-dominated washout (which favors lower $\Lambda_{X_2}/\Lambda_{X_1}$).

The requirement of sufficient departure from equilibrium precludes arbitrarily low $\Lambda_c$ and leads to a minimum $M_{X_2}$ for this scenario to work, which we see from Fig. 4 is a few $\times 10^4$ GeV. Finally, the overall size of $\Lambda_{X_1,2}$ is essentially determined by the requirement that $Y_B$ freezes out around the time $\Gamma_{X_2}^{\Delta B \neq 0}/\Gamma_{\text{wo}}$ reaches its maximum, and is higher for higher $M_{X_2}$.

**Complementary probes** — In the region of parameter space that is allowed by existing $n\bar{n}$ searches, within reach of the ESS, and realizes the observed baryon asymmetry, we find $M_{X_{1,2}} \gtrsim 10^4 (10^5)$ GeV in the late (early) decay scenario. Given that $X_{1,2}$ are SM singlets that only couple to the SM via higher dimensional operators, it is unlikely that they can be detected at the LHC or its envisioned upgrades. Likewise, there are no strong flavor physics constraints on our minimal EFT with just the operators in Eq. (3). We note, however, that this outlook can change in a more complicated model that preserves the general features of baryogenesis of our minimal EFT if at least one of $X_{1,2}$ carries SM charges or couples to other fermion species. For example, colored particles at the TeV scale, such as the gluino in RPV SUSY, could be within LHC reach. Likewise, extending the exotic fermion couplings to other quark flavors can introduce potential constraints from flavor violation considerations such as $K^0 - \bar{K}^0$ mixing [60]. Nevertheless, our minimal EFT study illustrates that $n\bar{n}$ oscillation might be uniquely placed to probe realistic baryogenesis scenarios that are otherwise inaccessible via other searches.

**Conclusions** — Establishing baryon number violation (or the absence thereof up to a certain scale) will have far-reaching implications on our understanding of fundamental particle interactions, in particular on the mechanism that generates the observed baryon asymmetry in our universe. Motivated by the unprecedented sensitivity to the $n\bar{n}$ oscillation time that can be achieved at the ESS, which offers new opportunities to probe $|\Delta B| = 2, \Delta L = 0$ interactions, we studied implications of a potential discovery for baryogenesis scenarios involving $n\bar{n}$ mediators. We took a bottom-up EFT approach with a minimal set of four-fermion operators coupling the $n\bar{n}$ mediators to the SM, which, despite being simplistic, sets a useful template that more sophisticated theories can build upon. We identified two viable baryogenesis scenarios – one involving late out-of-equilibrium decays of a heavy Majorana fermion, and another involving earlier decays assisted by a suppressed washout rate – that can be realized in the parameter space to be probed by the ESS, with no corresponding collider or flavor signals. These results highlight the capability of $n\bar{n}$ oscillation experiments to probe an important BSM phenomenon, that of baryogenesis, beyond the scope of other searches.

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showing the Super-K exclusion and ESS reach on $\Lambda_{\text{180 MeV}}$, we have

As we will see explicitly below, $\Lambda_{\text{180 MeV}}$ is defined here roughly coincides with suppression scales of dimension-nine operators mediating $n\bar{n}$ oscillation. This is because the nuclear matrix elements $\langle \bar{n}|\mathcal{O}_{n\bar{n}}|n\rangle \sim \mathcal{O}(\Lambda_{\text{QCD}}^0)$. Taking $\Lambda_{\text{QCD}} = 180 \text{ MeV}$, we have

$$\Lambda_{n\bar{n}} = 4.25 \times 10^5 \text{ GeV} \left( \frac{\tau_{n\bar{n}}}{2.7 \times 10^8 s} \right)^{1/5} = 1.35 \times 10^6 \text{ GeV} \left( \frac{\tau_{n\bar{n}}}{0.86 \times 10^{11} s} \right)^{1/5},$$

showing the Super-K exclusion and ESS reach on $\Lambda_{n\bar{n}}$, respectively. There are 12 independent operators that contribute to $n\bar{n}$ oscillation at tree level. Using the basis of [52], we write

$$\mathcal{L}_{\text{eff}} \supset \sum_{i=1}^{6} c_i \mathcal{O}_i + \bar{c}_i \bar{\mathcal{O}}_i + h.c.,$$
Here we have chosen the corresponding operators have larger (positive) anomalous dimensions, hence more suppressed effects at low energy.

The operator basis of Eq. (A.4) is particularly convenient because different operators do not mix as they are evolved (when $\mu_i$ and $\bar{\mu}_i$ are renormalized at $\mu_i$ and $\bar{\mu}_i$, respectively, rather than at $10^5$ GeV).

In the special case that the RHS of Eq. (A.6) is dominated by a single term, say the one proportional to $c_i \equiv \left(\Lambda_{n\bar{n}}^{(i)}\right)^{-5}$, we can establish a correspondence between $\tau_{n\bar{n}}$ (equivalently $\Lambda_{n\bar{n}}$) and $\Lambda_{n\bar{n}}^{(i)}$. This is shown in Fig. 3. As mentioned above, all $\Lambda_{n\bar{n}}^{(i)}$ are somewhat lower because the corresponding operators have larger (positive) anomalous dimensions, hence more suppressed effects at low energy.

2. Details of baryogenesis calculation

Boltzmann equations

The Boltzmann equations to be solved for our minimal EFT are

$$\frac{dn_a}{dt} + 3Hn_a = C_a \quad (a = 1, 2, 3),$$

where $n_{1,2}$ are the number densities of $X_{1,2}$, and $n_3$ represents $n_{B-L}$ ($n_B$) for $T > 140$ GeV ($T < 140$ GeV) when electroweak sphalerons are assumed to be active (inactive). We define

$$W_{i_1...i_m\rightarrow f_1...f_n} \equiv \int (d\Pi_{i_1}...d\Pi_{i_m})(d\Pi_{f_1}...d\Pi_{f_n}) (2\pi)^4\delta^4\left(\sum_{\alpha} p_{i_\alpha} - \sum_{\beta} p_{f_{\beta}}\right) \left|f_{i_1}^{eq}...f_{i_m}^{eq}\right| \left|\mathcal{M}_{i_1...i_m\rightarrow f_1...f_n}\right|^2.$$
FIG. 5. Suppression scale $\Lambda_{\text{fin}}^{(i)} \equiv c_i^{-1/5}$ of the $|\Delta B| = 2$ six-quark operators $\mathcal{O}_i$ in Eq. (A.4) that can be probed with free neutron oscillation time $\tau_{\text{n}}$ (corresponding to new physics scale $\Lambda_{\text{n}} \equiv (\tau_{\text{n}} \Lambda_{\text{QCD}})^{1/5}$ as defined in Eq. (A.1)) when each operator is considered individually. The widths of the bands arise from variations of $\langle \bar{n}| \mathcal{O}_i(\mu_0)|n \rangle$ within current lattice calculation uncertainties and of $\mu_i$ between $10^3$ and $10^6$ GeV. The results apply equally to the parity-conjugate operators $\bar{\mathcal{O}}_i$.

$$= \frac{1}{(d^n_i \ldots d^n_m)(d^n_f_1 \ldots d^n_f_n)(2\pi)^4 \delta^4 \left( \sum_\alpha p_i^\alpha - \sum_\beta p_f^\beta \right)} \left( f_{eq}^{f_1} \ldots f_{eq}^{f_n} \right) |\mathcal{M}_{i_1 \ldots i_m \rightarrow f_1 \ldots f_n}|^2,$$  \hspace{0.5cm} (A.8)

where $f_{eq}^a$ is the equilibrium distribution at zero chemical potential for species $a$. Assuming a common temperature is maintained for all species, we have

$$f_a = e^{\mu_a/T} f_{eq}^a \equiv r_a f_{eq}^a \equiv (1 + \Delta_a) f_{eq}^a,$$  \hspace{0.5cm} (A.9)

for the actual distribution of species $a$, with $\Delta_a$ characterizing the amount of departure from equilibrium. The collision terms can then be written in terms of the $W$'s and $r$'s,

$$-C_1 = \left( r_u r_1 - r_d^2 \right) W_{uX_1 \rightarrow \bar{d}d} + \left( r_\bar{u} r_1 - r_d^2 \right) W_{\bar{u}X_1 \rightarrow d\bar{d}} + \left( r_d r_1 - r_\bar{u} r_d \right) W_{dX_1 \rightarrow \bar{u}\bar{d}} + \left( r_\bar{d} r_1 - r_u r_\bar{d} \right) W_{\bar{d}X_1 \rightarrow ud}$$

$$+ \left( r_u r_1 - r_\bar{u} r_d \right) W_{uX_1 \rightarrow uX_2} + \left( r_\bar{u} r_1 - r_d r_\bar{d} \right) W_{\bar{u}X_1 \rightarrow \bar{d}d} + \left( r_d r_2 - r_\bar{u} r_d \right) W_{dX_1 \rightarrow \bar{u}\bar{u}} + \left( r_\bar{d} r_2 - r_u r_\bar{d} \right) W_{\bar{d}X_1 \rightarrow ud} \hspace{0.5cm} \text{(A.10)}$$

$$-C_2 = \left( r_u r_2 - r_d^2 \right) W_{uX_2 \rightarrow \bar{d}d} + \left( r_\bar{u} r_2 - r_d^2 \right) W_{\bar{u}X_2 \rightarrow d\bar{d}} + \left( r_d r_2 - r_\bar{u} r_d \right) W_{dX_2 \rightarrow \bar{u}\bar{d}} + \left( r_\bar{d} r_2 - r_u r_\bar{d} \right) W_{\bar{d}X_2 \rightarrow ud}$$

$$+ \left( r_u r_2 - r_\bar{u} r_d \right) W_{uX_2 \rightarrow uX_2} + \left( r_\bar{u} r_2 - r_d r_\bar{d} \right) W_{\bar{u}X_2 \rightarrow \bar{d}d} + \left( r_d r_1 - r_\bar{u} r_d \right) W_{dX_2 \rightarrow \bar{u}\bar{u}} + \left( r_\bar{d} r_1 - r_u r_\bar{d} \right) W_{\bar{d}X_2 \rightarrow ud} \hspace{0.5cm} \text{(A.11)}$$

$$-C_3 = \left( r_u r_1 - r_d^2 \right) W_{uX_1 \rightarrow \bar{d}d} - \left( r_\bar{u} r_1 - r_d^2 \right) W_{\bar{u}X_1 \rightarrow d\bar{d}} - \left( r_d r_1 - r_\bar{u} r_d \right) W_{dX_1 \rightarrow \bar{u}\bar{d}} - \left( r_\bar{d} r_1 - r_u r_\bar{d} \right) W_{\bar{d}X_1 \rightarrow ud}$$

$$+ \left( r_u r_2 - r_d^2 \right) W_{uX_2 \rightarrow \bar{d}d} - \left( r_\bar{u} r_2 - r_d^2 \right) W_{\bar{u}X_2 \rightarrow d\bar{d}} - \left( r_d r_2 - r_\bar{u} r_d \right) W_{dX_2 \rightarrow \bar{u}\bar{d}} - \left( r_\bar{d} r_2 - r_u r_\bar{d} \right) W_{\bar{d}X_2 \rightarrow ud}$$

$$+ 2 r_u r_1^2 W_{u\bar{d}d}^\prime + 2 r_\bar{u} r_1^2 W_{\bar{u}ud}^\prime \hspace{0.5cm} \text{(A.12)}$$

where $W_{u\bar{d}d}^\prime$ and $W_{\bar{u}ud}^\prime$ are computed from the corresponding matrix elements with contributions from on-shell $X_{1,2}$ exchange subtracted. We have grouped together terms that are identical as dictated by CPT invariance, $W_{i \rightarrow j} = W_{j \rightarrow -i}$ (where bar denotes CP conjugate state).

To further simplify, we note that several processes conserve CP up to one-loop level, and as a result

$$W_{dX_1 \rightarrow \bar{u}d} = W_{dX_1 \rightarrow ud}, \quad W_{dX_2 \rightarrow \bar{u}d} = W_{dX_2 \rightarrow ud}, \quad W_{X_1 \rightarrow u\bar{d}d} = W_{X_1 \rightarrow \bar{u}d\bar{d}}.$$

For the CP-violating processes, on the other hand, we define their CP-symmetric and CP-asymmetric components,

$$W_{uX_1 \rightarrow \bar{d}d}^{(0)} = \frac{1}{2} (W_{uX_1 \rightarrow \bar{d}d} + W_{\bar{u}X_1 \rightarrow dd}), \quad \varepsilon W_{uX_1 \rightarrow \bar{d}d} = \frac{1}{2} (W_{uX_1 \rightarrow \bar{d}d} - W_{\bar{u}X_1 \rightarrow dd}).$$
As using these relations and noting that $r$, solutions. In order to determine viable parameter space regions for baryogenesis, we set $\arg\left(\begin{array}{c}
\end{array}\right)$

Meanwhile, the chemical potentials $\mu_{u,d}$ are related to $n_{B(-L)}$ (see e.g. [65]): for $T > 140$ GeV,

$$\Delta_u = -\frac{10 n_{B(-L)}}{79} \frac{M_u^2}{T^3}, \quad \Delta_d = \frac{38 n_{B(-L)}}{79} \frac{M_d^2}{T^3},$$

as follows from equilibration of Yukawa interactions and SU(3) and SU(2) sphalerons, and conservation of hypercharge; for $T < 140$ GeV,

$$\Delta_u = \left[2 \frac{n_{u}}{T^3} + (2 + N_d^{-1} + N_c^{-1}) \frac{n_{B}}{T^3}\right] \left[1 + (2 + N_d^{-1} + 3 N_c^{-1}) N_u\right]^{-1}, \quad \Delta_d = N_d^{-1} \left[3 \frac{n_{B}}{T^3} - N_u \Delta_u\right],$$

as follows from equilibration of charged current interactions, and conservation of electric charge and lepton number. Here $N_u$ ($N_d$, $N_c$) is the number of generations of relativistic up-type quarks (down-type quarks, charged leptons), and $n_{L}/T^3$ is a constant fixed by $-\frac{51}{79} n_{L}/T^3$ at $T = 140$ GeV.

Following the standard change of variables $x = M_{X_2}/T$, $Y_a = \frac{2a}{s} = (2\frac{\pi^2}{45} \epsilon_{\text{eff}})^{-\frac{1}{2}} \frac{n_{a}}{T^3}$, we have

$$\frac{dY_a}{dx} = \left(\frac{\pi}{45}\right)^{1/2} \frac{M_{X_2}}{s^2 x^2} g_a^{1/2} C_a \quad (a = 1, 2, 3),$$

where $g_a^{1/2} = \frac{N_u}{g_{\text{eff}}} \left(1 + \frac{1}{3} \frac{T}{n_{u}} \frac{\partial n_{u}}{\partial T}\right)$. This is the final form of the Boltzmann equations that we use in our numerical solutions. In order to determine viable parameter space regions for baryogenesis, we set $\arg(\eta_{X_1}^{a} \eta_{X_2}^{a} \eta_{e}^{a} = \pi/2 to
maximize CP violation, and look for solutions with the final $Y_B \geq 8.6 \times 10^{-11}$; for such parameter choices, the exact amount of observed baryon asymmetry can then be achieved with some suitable choice of $\arg(\eta_X^c, \eta_X^a, \eta_c) \leq \pi/2$.

As mentioned in the letter, if $X_2$ is sufficiently long-lived, its decay may dump significant entropy into the plasma, diluting the baryon asymmetry. We account for this effect by dividing the final $Y_B$ from solving the Boltzmann equations by a dilution factor $d_s = 1.83 h_{\text{eff}}^{1/4} M_{X_2} Y_{X_2}^d (\Gamma_{X_2} M_{p_1}^{-1})^{-1/2} = 1.42 x_d Y_{X_2}^d$, if $d_s > 1$. Here $\Gamma_{X_2}$ is the total decay width of $X_2$, and $x_d$ and $Y_{X_2}^d$ are the values of $M_{X_2}/T$ and $Y_{X_2}$ at the time of $X_2$ decay, determined by $\Gamma_{X_2} = H$.

**Interaction rates**

We now provide analytical expressions for the interaction rates $W$ that appear in the collision terms. For a $2 \to 2$ process $ab \to cd$,

$$W_{ab \to cd} = n_a^{eq} n_b^{eq} \langle \sigma v \rangle_{ab \to cd} = \frac{T}{512 \pi^5 S_i S_f} \int_{s_{\min}}^{\infty} \frac{p_i p_f}{s} \langle |\mathcal{M}|^2 \rangle_{ab \to cd} K_1(\sqrt{s}/T) \, ds,$$

where $S_i$, $S_f$ are symmetry factors for the initial and final states (e.g. $S_i = 2$ if $a$ and $b$ are identical particles) and $p_i = |\vec{p}_i| = |\vec{p}_b|$, $p_f = |\vec{p}_c| = |\vec{p}_d|$ in the center of mass frame. The sum is over initial and final state spins and colors, while “( )” means averaging over $\cos \theta$, with $\theta$ being the scattering angle in the center of mass frame. We take the upper limit of integration to $\infty$ for simplicity since the integrand is exponentially suppressed for center-of-mass energies above the EFT cutoff $\Lambda$ for temperatures where the EFT is valid ($T \ll \Lambda$). Computing the scattering amplitudes at tree level, we find

$$p_i p_f \langle |\mathcal{M}|^2 \rangle_{uX_a \to \bar{d}d} = \frac{1}{2} |\eta_c|^2 (s - M_{X_a}^2)^2 (s + 2M_a^2),$$

$$p_i p_f \langle |\mathcal{M}|^2 \rangle_{dX_a \to \bar{d}d} = \frac{7}{2} |\eta_c|^2 (s - M_{X_a}^2)^2 (s + 1/11 M_a^2),$$

$$p_i p_f \langle |\mathcal{M}|^2 \rangle_{X_1 \to uX_a} = \frac{1}{2} |\eta_c|^2 \left[ s^2 - \frac{3}{2} (M_{X_1}^2 + M_{X_2}^2) s + \frac{1}{2} M_{X_1}^2 M_{X_2}^2 + \frac{3}{4} \cos 2\phi_c M_{X_1} M_{X_2} s \right],$$

$$p_i p_f \langle |\mathcal{M}|^2 \rangle_{X_2 \to \bar{u}u} = \frac{1}{2} |\eta_c|^2 \left[ s^2 - \frac{1}{2} (M_{X_1}^2 + M_{X_2}^2) s - \frac{1}{2} (M_{X_1}^2 - M_{X_2}^2)^2 - \frac{3}{2} \cos 2\phi_c M_{X_1} M_{X_2} s \right],$$

where $\phi_c = \arg \eta_c$. We have seen above that all the CP violation in $2 \to 2$ processes can be encoded in $\epsilon W_{uX_a \to uX_a}$. Computing also one-loop diagrams for this process, we find

$$p_i p_f \langle |\mathcal{M}|^2 \rangle_{uX_a \to uX_a} - p_i p_f \langle |\mathcal{M}|^2 \rangle_{\bar{u}X_a \to \bar{u}X_a} = -\text{Im}(\eta_X^c, \eta_X^a, \eta_c) \frac{3}{32 \pi^2 s} M_{X_1} M_{X_2} (s - M_{X_1}^2)^2 (s - M_{X_2}^2)^2.\tag{A.28}$$

We have explicitly checked that CP violation in $uX_a \to \bar{d}d$ satisfy expectations from unitarity relations Eq. (A.15).

For decay processes,

$$W_{i \to abc} = n_i^{eq} \frac{K_1(M_i/T)}{K_2(M_i/T)} \Gamma_{i \to abc} = \frac{M_{X_a}^2 T}{\pi^2} \frac{K_1(M_{X_a}/T)}{K_2(M_{X_a}/T)} \Gamma_{i \to abc},\tag{A.29}$$

where $i$ is the rest frame decay rate, summed over final state spins and colors, and averaged over the initial state spin. At tree level, we have

$$\Gamma_{i \to udd} = \frac{3}{4096 \pi^3} |\eta_a|^2 M_a^4,$$

$$\Gamma_{X_2 \to \bar{X}_1 uu} = \frac{1}{2048 \pi^3} |\eta_c|^2 M_{X_2}^2 \left[ (1 - \rho^2) \left[ (1 + \rho^2)(1 - 8\rho^2 + \rho^4) + 2 \cos 2\phi_c \rho (1 + 10\rho^2 + \rho^4) \right] -24\rho^3 \left[ \rho - \cos 2\phi_c (1 + \rho^2) \right] \log \rho \right],$$

where $\rho = M_{X_1}/M_{X_2}$. The CP-violating decay rate at one-loop level reads

$$\Gamma_{X_2 \to udd} - \Gamma_{X_2 \to \bar{u}d} = \frac{1}{16384 \pi^4} \text{Im}(\eta_X^c, \eta_X^a, \eta_c) M_{X_2}^4 \rho \left[ (1 - \rho^4)(1 - 8\rho^2 + \rho^4) - 24\rho^4 \log \rho \right].\tag{A.32}$$

This function is maximized at $\rho = 0.265 \simeq 1/4$. 
**Benchmark solutions**

To have a more detailed understanding of the baryogenesis scenarios discussed in the letter, let us examine a few benchmark solutions to the Boltzmann equations. We choose \( M_{X_2} = 4M_{X_1} = 2 \times 10^6 \text{GeV} \), which can accommodate solutions in both the late and the early decay scenarios, and consider the following three benchmarks:

- **Degenerate**: \( \Lambda_{X_1} = \Lambda_{X_2} = \Lambda_c = 1.5 \times 10^6 \text{GeV} \).
- **Late decay**: \( \Lambda_{X_1} = 1.5 \times 10^6 \text{GeV}, \Lambda_{X_2} = \Lambda_c = 80 \Lambda_{X_1} \).
- **Early decay**: \( \Lambda_{X_2} = 1.5 \times 10^6 \text{GeV}, \Lambda_{X_1} = 10 \Lambda_{X_2}, \Lambda_c = 0.2 \Lambda_{X_2} \).

All three benchmarks induce \( n \bar{n} \) oscillation at a level consistent with current constraints and within reach of the ESS, as we can see from Figs. 3 and 4 (the degenerate case has \((\Lambda_{n\bar{n}}^{(1)})^{1/5} \sim M_{X_1}A_X^{1/3}\)), so the \( n \bar{n} \) oscillation reach can be read off from Fig. 3 as in the late decay scenario).

We plot the evolution of various quantities from solving the Boltzmann equations in Fig. 6. The upper-left panel shows the amount of departure from equilibrium for \( X_1 \) (dashed) and \( X_2 \) (solid), quantified by \( \Delta_a = (Y_a - Y_a^{\text{eq}})/Y_a^{\text{eq}} \), while the solid curves in the upper-right panel show the baryon asymmetry \( Y_B \).

We first note that, with the exception of \( \Delta_2 \) in the late decay benchmark, departures from equilibrium are very small due to efficient depletion of \( X_{1,2} \) number densities by rapid decays once they become nonrelativistic. As a rough estimate, assuming radiation domination, we have \( \Gamma_{1,2}/H \sim 10^{-5} M_{X_{1,2}}^5 \frac{m_a}{T^3} \gg 10^{-5} M_{X_{1,2}}^3 \frac{m_a}{T^3} \) when \( T < M \), where \( 10^{-5} \) is the size of the phase space factor. For \( M \sim 10^5 \text{GeV} \) and \( \Lambda \sim 10^6 \text{GeV} \), we have \( \Gamma_{1,2}/H \gtrsim 10^5 \) and thus efficient decays that keep \( \Delta_a \ll 1 \). On the other hand, \( \Delta_2 \) in the late decay benchmark evades this pattern with much higher values for \( \Lambda_{X_2} \sim 10^8 \text{GeV} \), which result in \( \Gamma_{1,2}/H \sim 10^{-3}(M_{X_2}/T)^2 \), and thus later decay, for \( X_2 \). In this case, \( \Delta_2 \) starts to grow exponentially once the most efficient \( 2 \rightarrow 2 \) process \( dX_2 \rightarrow u \bar{u} \) freezes out, which happens when \( \Gamma_{1,2}/H \sim n_{eq}^2(\sigma v)/H \sim g_{eq} T^3 \cdot 10^{-2} M_{X_2}^2 \frac{m_a}{T^3} \sim 10 (M_{X_2}/T) \sim 1 \), i.e. when \( x = M_{X_2}/T \sim 10 \).

It is also worth noting that departures from equilibrium \( \Delta_{1,2} \) are nonzero even when \( X_{1,2} \) are relativistic, as a result of Hubble expansion. To see this, we write both sides of the Boltzmann equation for \( X_{a} \) schematically as

\[
\text{LHS} = \frac{dY_a}{dx} = Y_a^{\text{eq}} \frac{d\Delta_a}{dx} + (1 + \Delta_a) \frac{dY_a^{\text{eq}}}{dx} \sim \frac{dY_a^{\text{eq}}}{dx} \sim \frac{M}{T},
\]

\[
\text{RHS} \sim \frac{M_{pl}}{MT^4} \frac{dY_a}{dx} = \frac{M_{pl}}{MT^4} \left( C_a^{2+2} + C_a^{1+3} \right) \sim -\frac{M_{pl}}{MT^4} \left( \frac{T^8}{A^4} + \frac{M^6T^2}{A^4} \right) \Delta_a \sim -\frac{M_{pl} T^4}{M A^4} \Delta_a.
\]

Setting them equal, we have

\[
\Delta_a \sim \frac{M^2 A^4}{M_{pl} T^8} \sim x^5 \quad (x \ll 1).
\]

This power law dependence is clearly seen from the upper-left panel of Fig. 6. Also note that \( \Delta_a \) is larger for higher \( \Lambda \), as it is harder to catch up with Hubble expansion when interactions are weaker. When nondegenerate \( \Lambda \)'s are involved in the \( X_{1,2} \) number changing processes, the lowest of them tends to determine the total interaction rate, and thus \( \Delta_a \). For example, at high temperatures, \( \Delta_{1,2} \) are lower in the early decay benchmark compared to the degenerate case because of a lower \( \Lambda_c \). They exceed the degenerate curves later when coannihilation becomes Boltzmann suppressed; from here on, \( \Delta_1 \) tends to grow faster due to a higher \( \Lambda_{X_1} \), while the lower \( \Lambda_c \) maintains \( \Delta_1 \approx \Delta_2 \) via \( u X_1 \leftrightarrow u X_2 \).

Next, to understand the trend of the \( Y_B \) curves in the upper-right panel of Fig. 6 it is useful to note that the Boltzmann equation for \( Y_B \) has the following form,

\[
\frac{dY_B}{d\log x} = j(x) - w(x) Y_B.
\]

Here \( j(x) \) is a source function that is proportional to \( CP \) violating interaction rates and departures from equilibrium,

\[
j(x) \propto \left( 2 \epsilon W_{u X_1 \leftrightarrow u X_2} \right) (\Delta_2 - \Delta_1) - \left( 2 \epsilon W_{X_2 \leftrightarrow u \bar{u} \bar{d}} \right) \Delta_2,
\]

see Eq. (A.18). \( w(x) \) is a washout function that tends to erase the baryon asymmetry. At high temperatures, both \( j(x) \) and \( w(x) \) vary slowly (as powers of \( x \) as we will see below), so that the evolution of \( Y_B \) is approximately adiabatic,

\[
Y_B \approx Y_B^{\text{adiabatic}} = j(x)/w(x) \quad (\text{high } T).
\]
FIG. 6. Evolution of $\Delta_1 = (Y_{X_1} - Y_{X_1}^{\text{eq}})/Y_{X_1}^{\text{eq}}$ (upper-left, dashed), $\Delta_2 = (Y_{X_2} - Y_{X_2}^{\text{eq}})/Y_{X_2}^{\text{eq}}$ (upper-left, solid), and $Y_B$ (upper-right, solid) with $x = M_{X_2}/T$, in the three benchmark scenarios. $Y_B$ follows the adiabatic curve $Y_B^{\text{adiabatic}}(x) = j(x)/w(x)$ (upper-right, dashed) until $B$ violating interaction rates become Boltzmann suppressed. The source and washout functions $j(x)$ and $w(x)$ in the Boltzmann equation for $Y_B$, Eq. (A.36), are plotted in the lower panels (solid). The source function $j(x)$ is dominated by contributions from $2 \leftrightarrow 2$ processes (lower-left, dashed) at early times, and by contributions from decays at the time of baryon asymmetry generation.

The adiabatic solutions are shown by the dashed curves in the upper-right panel of Fig. 6. The true solutions follow the adiabatic approximation as long as $|\frac{d(j/w)}{d \log x}| \ll j$, which is seen to be the case until $x \sim 10^{1+2}$; after that, $j/w$ varies too fast for $Y_B$ to follow, and $Y_B$ freezes out.

We see from the plot that the key to generating sufficient baryon asymmetry in both late and early decay scenarios is the appearance of a sharp peak in $j(x)/w(x)$ of $j(x)$, which allows $Y_B$ to rise to significantly higher values compared to the degenerate case, and freeze out just before $j/w$ turns over. To see how the peak arises in each case, we plot the functions $j(x), w(x)$ in the lower panels of Fig. 6 (solid curves). In addition, to compare contributions from $2 \leftrightarrow 2$ vs. $1 \leftrightarrow 3$ processes, we plot the former (corresponding to the first term on the RHS of Eq. (A.37)) with dashed curves in the lower-right panel. Quite generally, the ratio of the two scales as $T/M$ to some positive power, so $2 \leftrightarrow 2$ ($1 \leftrightarrow 3$) processes dominate at high (low) temperatures. This makes it clear that $1 \leftrightarrow 3$ processes are responsible for sufficient baryon asymmetry generation in both late and early decay scenarios.

To have a more detailed understanding of these plots, we first note that at high temperatures,

$$T \gg M : \quad j \sim \frac{M_{\text{pl}}}{T^5} (\epsilon W_{uX_1 \rightarrow uX_2})(\Delta_2 - \Delta_1) \sim \frac{M_{\text{pl}}}{T^5} \frac{M^2 T^8}{\Lambda^6} \sim \frac{M^4}{\Lambda^2 T^2} \sim x^2,$$

(A.39)

$$w \sim \frac{M_{\text{pl}}}{T^5} W_{2 \rightarrow 2} \sim \frac{M_{\text{pl}} T^8}{T^5 \Lambda^4} \sim \frac{M_{\text{pl}} T^3}{\Lambda^4} \sim x^{-3},$$

(A.40)
and therefore,

$$Y_B \simeq j/w \sim \frac{M^4 \Lambda^2}{M_{pl} T^5} \sim x^5 \quad (x \ll 1).$$  \tag{A.41}$$

These power law behaviors can be clearly seen in Fig. 6. Note that in the parameter space probed by $n$-$\bar{n}$ oscillation, it is not possible to fully generate the observed baryon asymmetry while $X_{1,2}$ are relativistic (however, viable baryogenesis via $2 \rightarrow 2$ processes at $T > M$ is possible with higher $\Lambda$’s, as demonstrated in [61]).

As the temperature falls below the $X_{1,2}$ masses, interaction rates become Boltzmann suppressed. From Eq. (A.18),

$$T \ll M : \quad j \sim \frac{M_{pl}}{T^5} n_{eq}^2 \frac{M^7}{\Lambda X_1 \Lambda X_2 \Lambda^2_c} \cdot \Delta_2,$$

$$w \sim \frac{M_{pl}}{T^5} \left[ n_{eq} \frac{M^5}{\Lambda X_1} \max(1, \Delta_1) + n_{eq}^2 \frac{M^5}{\Lambda X_2} \max(1, \Delta_2) \right].$$  \tag{A.43}$$

In the absence of exponential growth of $\Delta_{1,2}$, $j(x) \propto n_{eq}^2 \propto e^{-M X_2/T}$ simply falls exponentially when $T < M_{X_2}$. Meanwhile, when $\Lambda X_1 \sim \Lambda X_2$, $w(x)$ is dominated by the term proportional to $n_{eq}^2 \propto e^{-M X_1/T}$, and so becomes exponentially suppressed at a later time when $T < M_{X_1}$. This results in a period of efficient washout of the baryon asymmetry generated previously, and ultimately, $j(x)/w(x) \propto e^{-(M X_2-M X_1)/T}$. As we see from Fig. 6, the degenerate benchmark curves indeed follow these expectations.

In contrast, the late decay scenario features an exponentially growing $\Delta_2$, due to freeze-out of the $X_2$ abundance discussed above. This enhanced departure from equilibrium induces a plateau in the source function $j(x)$, before $X_2$ finally decays. Meanwhile, washout is still dominated by processes involving $X_1$, and $w(x)$ is similar to the degenerate case. The overall effect is thus a much higher $j(x)/w(x)$ than the degenerate case, peaked around $x \sim 10^{2-1}$, allowing for sufficient baryon asymmetry generation.

The early decay scenario, on the other hand, features a dip in the washout function. With $\Lambda X_1 \gg \Lambda X_2$, $w(x)$ is now dominated by the term proportional to $n_{eq}^2 \propto e^{-M X_2/T}$ at high temperatures, which falls off exponentially earlier than the $n_{eq}^1$ term. Baryon asymmetry generation is further assisted by the delayed (eventual) fall-off of the source function $j(x)$ due to the growth of $\Delta_2$ around $x \sim 10$ explained previously. This also results in a gap between the “1 $\leftrightarrow$ 3 peak” and the “2 $\leftrightarrow$ 2 peak” of $j(x)$, as the latter falls off around the same time as in the degenerate benchmark because $\Delta_2 - \Delta_1$ remains small. Overall, $j(x)/w(x)$ is significantly boosted compared to the degenerate case, with a peak around $x \sim 10^{1.3}$, corresponding to efficient baryogenesis at a time earlier than the late decay scenario.