LIMITS ON MASS AND RADIUS FOR THE MILLISECOND-PERIOD X-RAY PULSAR SAX J1808.4−3658

DENIS A. LEAHY, 1 SHARON M. MORSINK, 2, 3 AND COIRE CADEAU 2

Received 2007 July 10; accepted 2007 September 16

ABSTRACT

SAX J1808.4−3658 has a 2.5 ms neutron star rotation period and exhibits X-ray pulsations due to its rotating hot spot. Here we present an analysis of the pulse shapes of SAX J1808.4−3658 during its 1998 outburst. The modeling of the pulse shape includes several effects, including gravitational light bending, Doppler effects, and two spectral components with different emissivity. In addition, we include the new effects of light travel time delays and the neutron star’s oblate shape. We also consider two different data sets, with different selections in time period (1 vs. 19 days of data combined) and different energy binning and time resolution. We find that including time delays and oblateness results in a stronger restriction on allowed masses and radii. A second result is that the choice of data selection strongly affects the allowed masses and radii. Overall, the derived constraints on mass and radius favor compact stars and a soft equation of state.

Subject headings: pulsars: individual (SAX J1808.4−3658) — relativity — stars: neutron — stars: rotation — X-rays: binaries

1. INTRODUCTION

The discovery (Wijnands & van der Klis 1998) of 2.5 ms pulsations originating from SAX J1808.4−3658 (hereafter SAX J1808) provides strong evidence that neutron stars in low-mass X-ray binaries are the progenitors of millisecond-period pulsars. SAX J1808 is now one of seven known accreting millisecond X-ray pulsars (see Wijnands [2005] for an observational review of the properties of the first six pulsars and Poutanen [2006] for an updated review including the first seven pulsars).

The pulsed X-rays observed during outburst are most likely produced from the energy released from the accretion of plasma funneled onto the neutron star’s magnetic poles (see, e.g., Fig. 12 of Gierliński et al. 2002). Spectral models (Gilfanov et al. 1998; Gierliński et al. 2002) provide strong evidence that the X-rays correspond to blackbody emission from a spot on the star which is then Compton scattered by electrons above the hot spot. Since, in this model, the pulsed light is emitted from the neutron star’s surface (or from a region very close to the surface), the accreting millisecond X-ray pulsars are excellent targets for light-curve fitting in order to constrain the neutron star equation of state (EOS). The X-ray light curve depends on the intrinsic properties of the emission (spot shape, size, location, and emissivity), as well as the neutron star’s macroscopic properties (mass, radius, and spin).

If tight enough constraints on the star’s mass and radius can be made, it could be possible to constrain the EOS of supernovae density material.

The first pulse shape analysis (Poutanen & Gierliński 2003, hereafter PG03) for SAX J1808 provided interesting constraints on the neutron star’s mass and radius. However, this analysis did not take into account two effects that are potentially important for rapidly rotating neutron stars: variable time delays due to light travel time across the star (Cadeau et al. 2005) and the oblate shape of the star (Cadeau et al. 2007). One of the motivations for the reexamination of the pulse shapes for SAX J1808 is to include these effects in the analysis. In this paper we reanalyze this data in order to determine the importance of these effects. In order to isolate these effects, we have kept all other aspects of our data analysis as close as possible to those of PG03.

Additional aspects that we explore are (1) the effects of time and energy binning of the data on the fitted parameter values, and (2) the effect of data selection. Papitto et al. (2005) constructed a light curve for SAX J1808 using an subset of the data used by PG03, and also used a different binning in energy and time. Since SAX J1808 was quite variable during the time period analyzed by PG03 (see Figs. 1 and 2 of Gierliński et al. 2002), it is important to carry out an analysis on a data set from a shorter, less variable time interval. Thus, we present independent fits to the Papitto et al. (2005) light curve in order to explore the sensitivity of the fits to different types of binning and to choice of data interval.

The outline of our paper is as follows. In § 2 we explain in detail the method and types of models that we use to analyze the data. In § 3 we present the results of our fits for light curves separated into two energy bands and show the effects due to changes in spectral models, time delays, and the star’s oblateness. In § 4 we present the results of our fits for a bolometric light curve. We conclude with a discussion of the mass-radius constraints found in our analysis.

2. METHOD

SAX J1808 went into outburst in 1998 April. From the RXTE observations of the 1998 outburst, two groups have constructed light curves. We now describe the different types of binning that were done by each group in order to construct the light curves.

The two light curves constructed by PG03 are shown in their Figure 5a and reproduced in Figure 1. In their analysis, they constructed light curves in two energy bands, a low-energy band (3−4 keV) and a high-energy band (12−18 keV). For each band they folded data from April 11–29 into light curves with 16 time bins per period. We refer to the light curves constructed by PG03 as “two-band light curves.”

Papitto et al. (2005) constructed a light curve shown in the top panel of their Figure 2 and reproduced here as Figure 2. This light curve was constructed by combining the data from all energies in the range 2−60 keV collected during an approximately 22 hr
period starting on April 18. These data are folded into one light curve with 64 time bins per period. The error bars for the bolometric data are about 2 times larger than for the two-band data due to fewer counts per bin: this is partly due to more bins (64 compared to a total of 32 for the PG03 light curves) and partly due to a shorter time period for data selection. We refer to the light curve constructed by Papitto et al. (2005) as a bolometric light curve.

The data used in the two types of light curves overlap to a significant extent in energy and in time period selected. Thus, we expect that the conclusions deduced independently from each binning should be roughly consistent with each other. The two main differences between the two methods are (1) a shorter time period selection for the Papitto et al. (2005) light curve, and (2) the binning by energy. The two-band light curves constructed by PG03 make use of one narrow energy band and one broad energy band and do not use the data from 5 to 12 keV or above 18 keV. The bolometric light curve constructed by Papitto et al. (2005) makes use of all the energy channels, but does not provide information about the change in pulse shape with energy. However, the Papitto et al. (2005) light curve provides 4 times better time resolution. In our modeling we fit the data sets independently.

2.1. Two-Band Light Curves

Spectral models by Gierliński et al. (2002) show that the spectrum is well approximated by the sum of a blackbody and a Comptonized spectrum. In the low-energy band, the blackbody flux is about 30% of the Comptonized flux, while in the high-energy band the blackbody radiation is negligible. In our models for the two-band light curves, we use three spectral components to model the data. These components are (1) Comptonized flux in the high-energy band, (2) Comptonized flux in the low-energy band, and (3) blackbody flux in the low-energy band. Models for each spectral component in the star’s rest frame (see Fig. 3 of Gierliński et al. 2002) follow a power-law form if only the limited range of energies for the appropriate energy band are considered. For each component, we model emitted monochromatic flux with the function

\[ F_{i,\text{em}}(E_{\text{em}}, \mu) = A_i(\mu)E_{\text{em}}^{-\Gamma_{i}+1}, \]

where \(E_{\text{em}}\) is the energy in the “emission” frame that rotates with the star, and \(\mu\) is the cosine of the angle between the normal to the star’s surface and the initial photon direction, as measured in the emission frame. The subscript \(i\) takes on values of 1, 2, or 3 corresponding to the three spectral components. The functions \(A_i(\mu)\) describe the anisotropy of the emitted light.

In the observer’s frame, Doppler effects must be taken into account in order to find the observed flux. The transformation laws have been described in detail elsewhere (see PG03; Cadeau et al. 2007) and are briefly summarized here. The Doppler boost factor is defined by

\[ \eta = (1 - v^2)^{1/2}/(1 - v \cos \xi), \]

where \(\xi\) is the angle between the fluid’s velocity vector and the initial photon direction, \(v\) is the magnitude of the fluid velocity at the latitude \(\theta\) of emission, \(v = \Omega_c R (1 - 2 M/R)^{-1/2} \sin \theta\), and \(\Omega_c\) is the star’s angular velocity. The observed energy is given by \(E_{\text{obs}} = \eta E_{\text{em}}\), the specific intensity transforms as \(I_{\text{obs}}(E_{\text{obs}}) = \eta I_{\text{em}}(E_{\text{em}})\), and the solid angle subtended by the spot transforms as \(d\Omega_{\text{obs}} = \eta d\Omega_{\text{em}}\). Since the flux is given by \(F(E) = \eta \Omega_{\text{obs}} \cdot d\Omega_{\text{obs}} 1/\eta^4 F_{i,\text{obs}}(E_{\text{obs}})\), the observed flux at energy \(E_{\text{obs}}\) for spectral component \(i\) is given by

\[ F_{i,\text{obs}}(E_{\text{obs}}) = \eta^4 F_{i,\text{em}}(E_{\text{em}}). \]  

For the power-law components given by equation (1), the observed flux for each component is

\[ F_{i,\text{obs}}(E_{\text{obs}}) = \eta^{3+\Gamma_{i}} A_i(\mu) E_{\text{obs}}^{-\Gamma_{i}+1}. \]

The Doppler boost factor and \(\mu\) depend on phase, so the monochromatic flux also depends on phase. If the monochromatic fluxes are integrated over the appropriate observed energy ranges, the phase-dependent factors are unchanged, and the integrated flux for each component is

\[ F_i = I_i \eta^{3+\Gamma_i} A_i. \]

The quantity measured in an X-ray detector is not flux, but photon number counts, \(N(E)\) for a specified energy. The monochromatic flux is related to the photon number counts by \(F(E) = EN(E)\). Since the photon number counts in the emission frame (for a given component) are given by \(N_{i,\text{em}}(E_{\text{em}}) = E^{-\Gamma_{i}}\), arguments similar to those given above show that the observed integrated photon number counts have the same phase dependence as the observed integrated flux, \(N_{i,\text{obs}} \propto \eta^{3+\Gamma_i} A_i(\mu)\). Hence, it does not matter whether flux or photon number counts are used.

Our spectral model uses fixed values of \(\Gamma_i\) determined by the best-fit model spectrum used by PG03 (see their Fig. 3). The
values of $\Gamma$, are $\Gamma_1 = 2.0$ (high-energy band and Compton), $\Gamma_2 = 1.44$ (low-energy band and Compton), and $\Gamma_3 = 3.34$ (low-energy band and blackbody). The anisotropy function for the Compton components in both energy bands are assumed to have the form

$$A_{1,2} = 1 - a \mu,$$

where we assume that the beaming is independent of energy. This simple parameterization has been shown by PG03 to be a reasonable approximation to the calculations by Sunyaev & Titarchuk (1985). For the blackbody component, we assume that the emitted light is isotropic. In our initial models we included an exponential absorption factor ($e^{-\tau/\mu}$), but we found that including a nonzero optical depth $\tau$ did not significantly affect our fits, so we set $\tau = 0$. The constants $I_1$ and $I_2$ are free parameters in our models. The constant $I_3$ is defined after introducing the parameter $b$, the ratio of the phase-averaged blackbody flux to the phase-averaged Comptonized flux in the low-energy band. The constant $b$ is defined by $b = F_2/F_1$, where the bar refers to an average over phase. In our fits we restrict $b$ to within 15\% of the value based on the PG03 spectral model.

### 2.2. Bolometric Light Curve

The bolometric light curve is modeled using a method similar to that used for the two-band light curves. The Compton component is modeled as a power law with photon spectral index $\alpha$ over the entire energy range, and the anisotropy is modeled using the one-parameter function given by equation (5). The blackbody component is modeled as a power law with photon spectral index $\Gamma = 3.34$, and is normalized so that the blackbody-to-Compton ratio in the 2–60 keV band agrees with the PG03 spectral model.

### 3. RESULTS FOR TWO-BAND LIGHT CURVES

In this section we apply a series of different methods to compute models for the two-band light curves constructed by PG03. Our main goal is to test the importance of including time delays and oblateness in the analysis of data from SAX J1808. In order to test these effects we begin by reproducing the original fits of PG03 and then adding sequentially the different effects.

#### 3.1. Fiducial Models

We begin by reproducing the analysis of PG03 as closely as possible. To do this we use the spectral model described in §2.1. We keep the spot size fixed so that the radius in the star’s frame is 1.5 km. (Later we show that changes in the spot size do not significantly change the fits.) For the first set of fits (which we refer to as “fiducial”), the exact light-bending angle formula is used, time delays are omitted, and the surface of the star is assumed to be spherical. The results for the fiducial best-fit models are shown in Table 1.

For each row of Table 1, the ratio of $2M/R$ is fixed and the other parameters are allowed to vary. The eight free parameters are the following: $M$, the mass of the neutron star; $R$, the colatitude of the spot’s center; $i$, the inclination angle of the observer; the parameters $I_1$, $I_2$, $b$, and $a$ describing the spectral model; and $\phi$, a phase angle. Since there are 32 data points and there are eight free parameters, the fits have 24 degrees of freedom. Our best-fit parameter values are slightly different from those of PG03, and our lowest $\chi^2$ is larger than that of PG03. However, our best-fit models still agree with the best-fit models of PG03.

For our fits we fix the value of $2M/R$ in order to simplify computation of the bending angles and the time delays. In order to correctly include the relativistic bending of light rays we require a numerical solution for the relationship between $\alpha$, the angle between the initial photon direction and the radial vector pointing to the spot, and $\psi$, the angle between the final photon direction and the spot’s radial vector. This relationship requires solving the integral relationship given by Pechenick et al. (1983) and depends on $2M/R$. Similarly, the time delays also depend on the value $2M/R$. Given a value of $2M/R$, it is then possible to interpolate from the same numerically generated table relating $\alpha$ and $\psi$ for each trial value of the neutron star mass. We note that PG03 did their fits by first fixing a value of mass and then allowing $2M/R$ to vary.

#### Table 1

| $2M/R$ | $M$ (M$\odot$) | $R$ (km) | $\theta$ (deg) | $i$ (deg) | $a$ | $\chi^2$/dof |
|--------|----------------|---------|----------------|----------|----|-------------|
| 0.60... | 1.07           | 5.30    | 20.5           | 63.5     | 0.603 | 31.1/24     |
| 0.50... | 1.03           | 6.10    | 22.9           | 44.6     | 0.557 | 35.1/24     |
| 0.40... | 1.00           | 7.38    | 18.6           | 44.7     | 0.570 | 38.6/24     |
| 0.30... | 0.83           | 8.13    | 22.7           | 31.5     | 0.538 | 40.9/24     |
| 0.20... | 0.70           | 10.29   | 13.8           | 42.3     | 0.589 | 43.5/24     |

For each row of Table 1, the ratio of $2M/R$ is fixed and the other parameters are allowed to vary. The eight free parameters are the following: $M$, the mass of the neutron star; $R$, the colatitude of the spot’s center; $i$, the inclination angle of the observer; the parameters $I_1$, $I_2$, $b$, and $a$ describing the spectral model; and $\phi$, a phase angle. Since there are 32 data points and there are eight free parameters, the fits have 24 degrees of freedom. Our best-fit parameter values are slightly different from those of PG03, and our lowest $\chi^2$ is larger than that of PG03. However, our best-fit models still agree with the best-fit models of PG03.

In our models we do not restrict the values of the inclination angle for the system, although distance-dependent limits on $i$ have been derived by Chakrabarty & Morgan (1998) and Wang et al. (2001). The more recent distance determination by Galloway & Cumming (2006) suggests a small inclination angle, but since there is still a fairly large uncertainty in $i$, we choose to keep its value free.

We now compare the results of our fiducial fits with those of PG03, given in their Table 1 and labeled “Model 2.” First, consider the model with the lowest value of $\chi^2$ in Table 1, corresponding to $2M/R = 0.6$. This model is very similar to the best-fit model given in Table 1 of PG03, which has $M = 1.0$ M$\odot$, $R = 5.0$ km, and $2M/R = 0.599$. Our best-fit model has a radius that is about 10\% larger than the PG03 best-fit model. The best-fit inclination angle, spot colatitude, and anisotropy parameter $a$ are all smaller in our best-fit model compared to the PG03 best-fit model, but the differences are within the error limits given by PG03.

It should be remembered that neutron stars with $2M/R \geq 0.57$ have regions where light can be emitted in two different directions and still reach the observer. However, this region is very small for the case of $2M/R = 0.6$, and we have checked that in our best-fit solution the spot never enters the region where multiple images would occur. The largest value of this ratio used in our computations is $2M/R = 0.6$, since properly taking multiple images into account is computationally difficult. Since the light curves are very close to sinusoidal, it would be very unlikely for a geometry producing multiple images to fit the data.

As the mass of the star is increased, PG03 find that the best-fit $\chi^2$ increases rapidly. In their models, the 1.4 M$\odot$ model has $\delta \chi^2 = 10$ compared to their model with 1.0 M$\odot$. This means that the 1.4 M$\odot$ star is a significantly worse fit (by about 3 $\sigma$) than the 1.0 M$\odot$ star. We attempted to fit the data with 1.4 M$\odot$ stars while allowing other parameters to vary. The resulting minimum value of $\chi^2 = 40.3$ for a 1.4 M$\odot$ star (with a radius of 8.3 km) is larger than our best-fit value of $\chi^2 = 31.1$ obtained with $M$ as free parameter: a result similar to that obtained by PG03.

The reason why only very small radius (and small mass) stars are allowed by the data can be understood by examining the low-energy band light curve reproduced in Figure 1. The low-energy light curve has a shape that is very close to sinusoidal. The main effect of the Doppler factor $\eta$ is to increase asymmetry in the light
curve. The maximum value of $\eta$ is related to the speed of the fluid at the emission region, which is proportional to $R \sin \theta \sin i$. In the low-energy band the model spectrum includes a blackbody and Comptonized components. The resulting best-fit spectral model is a power law with a fixed value of $b$ in MeV and $B$ is the bag constant (Glendenning 2000).

In Figure 3 we show mass-radius curves for neutron stars spinning at 401 Hz as shown in solid curves. The EOS plotted are those for models assuming a spherical surface and including time delays are shown as bold dot-dashed curves. Best-fit models computed by PG03 are shown as squares with error bars. Mass vs. radius curves for compact stars rotating at 401 Hz are shown as solid curves. The maximum value of $\eta$ is related to the speed of the fluid at the emission region, which is proportional to $R \sin \theta \sin i$. In the low-energy band the model spectrum includes a blackbody and Comptonized components. The resulting best-fit spectral model is a power law with a fixed value of $b$ in MeV and $B$ is the bag constant (Glendenning 2000).

### 3.3. Uncertainty in the Spectral Model

There is some uncertainty in the underlying spectral model for the emitted light. The spectrum is measured in the inertial frame, and in order to infer the spectrum in the frame moving with the star, some assumptions about the geometry of the spot and the observer and the distance to the source must be made. In order to test the dependence of the fitted parameters on the details of the spectral model we now vary some of the parameters that are fixed in our fiducial model. For the following cases we fix $2M/R = 0.6$ and present the results in Table 3. For each row in Table 3 we provide the parameter that has been changed from its value in the fiducial model. For reference, the first column of Table 3 is the fiducial model shown in Table 1.

The simplest spectral model is a power law with a fixed value of $\Gamma$ for all components. Gierliński et al. (2002) showed that the best fit for the photon spectral index is $\Gamma = 1.8$, although their Figure 2 shows $\Gamma$ on individual days varying between 1.9 and 2.1. In the row labeled $\Gamma = 1.8$ in Table 3 we show the results of fits with the spectral photon index fixed at this value for both the blackbody and Comptonized components. The resulting best-fit model has parameters that are very close to the fiducial model, but with a larger value of $\chi^2$.

### 3.2. Approximate Light-Bending Formula

Beloborodov (2002) derived a simple approximate formula for light bending,

$$\cos \alpha \sim \frac{2M}{R} + \left(1 - \frac{2M}{R}\right) \cos \psi.$$  

\[\text{(6)}\]  

\[\begin{array}{cccccc}
2M/R & M (M_\odot) & R (\text{km}) & \theta (\text{deg}) & i (\text{deg}) & a \\
0.60 & 1.16 & 5.69 & 16.6 & 77.6 & 0.671 & 32.6/24 \\
0.50 & 1.03 & 6.08 & 23.9 & 42.7 & 0.552 & 35.1/24 \\
0.40 & 0.95 & 7.04 & 24.4 & 34.5 & 0.540 & 38.3/24 \\
0.30 & 0.88 & 8.67 & 18.7 & 36.7 & 0.551 & 42.9/24 \\
0.20 & 0.64 & 9.42 & 19.1 & 32.1 & 0.544 & 42.8/24 \\
\end{array}\]
In the spectral models used by PG03, an absorption factor was included for the blackbody component with the form \( \exp(-\tau/\mu) \). In the row labeled \( \tau = 0.1 \) we consider the effect of a nonzero \( \tau \). For this test we fix \( \tau \) to a value consistent with the best-fit values found by PG03. The result is a better fit, at low significance (1.2 \( \sigma \)), but the best-fit parameters are within a few percent of the fiducial values (with \( \tau = 0 \)).

In all of the fits so far we have kept the ratio \( b \) of the average blackbody to Compton flux in the low-energy band fixed to within 15% of the model used by PG03. We now allow this ratio to be free while keeping the values of the photon spectral indices the same as in the fiducial model. The results of this fit are shown in the row labeled “\( b \) free” of Table 3. Allowing \( b \) to be free gives the most significant decrease in \( \chi^2 \). However, the changes in the best-fit parameters are again very small.

In the spectral model used by PG03, the radius of the star as measured at infinity was fixed at \( r_{\infty} = 2.4 \) km, corresponding to a spot size on the star ranging from 3.0 to 3.7 km depending on the star’s assumed mass. In the case of the model with \( 2M/R = 0.6 \) the spot size on the star is 3.0 km. For rows labeled \( r_{\infty} = 1-3 \) km in Table 3 we now allow for different values of the spot radius. (For reference, the spot size for the fiducial model is 1.5 km.) Allowing for a larger spot (3.0 km) increases the radius by 3% and decreases the observer’s inclination angle by 13° while increasing \( \chi^2 \). The dependence of the inclination angle on the spot size suggests that the best-fit inclination angle should not be considered to be accurate.

### 3.4. Time Delays

Two photons emitted simultaneously from the front and back of the star will arrive at the detector at different times separated by an interval of order \( \Delta t \sim 2R/c \). If the star has spin period \( P \) and the data are binned into \( N \) bins per period, then when the dimensionless ratio, \( \kappa \), of the time delay to the time per bin

\[
\kappa = \frac{2RN}{cP}
\]

is of order unity it becomes important to correctly bin the photons based on the variable times of arrival. For SAX J1808, \( \kappa \sim 0.025N(R/10 \text{ km}) \), so for 16 time bins (as in the PG03 data) and a 10 km star this ratio is \( \kappa \sim 0.4 \), so we expect that the variable time delays may be important for the larger stars. For the light curves constructed by Papitto et al. (2005) with 64 time bins, \( \kappa \sim 1.7 \) and the inclusion of the time delays is crucial in order that the light curve be modeled correctly.

When the variable time delays are included in a model light curve, the time delays tend to increase the asymmetry of the resulting light curve (compared to a light curve where the time delays are ignored). Since the addition of variable time delays creates an extra asymmetry, the asymmetry created by the Doppler boost factors must be reduced, resulting in a smaller radius. Hence, we expect that the addition of time delays to the light curves results in a reduction in the size of the best-fit stars, as discussed by Cadeau et al. (2005).

In Table 4 the best-fit models including time delays are shown. All other aspects of the models are identical to the fiducial models computed in § 3.1. Comparing Tables 1 and 4, we see that for fixed values of \( 2M/R \) the radius (and mass) decreases by about 10% when time delays are included.

The most important effect of including time delays is a large increase in \( \chi^2 \) (for fixed values of \( 2M/R \)) that can be seen by comparing Tables 1 and 4. This effect occurs because the inclusion of time delays in the model changes the shape of the light curves, making it harder to fit the data. This increase in \( \chi^2 \) has the effect of narrowing the allowed values of \( 2M/R \) to a smaller range.

### 3.5. Oblate Shape of Star

A rotating star has an oblate shape. The most important effect that the oblate shape has on the light curves is the change in the visibility condition (Cadeau et al. 2007; Morsink et al. 2007) for photons. The effect on light curves is most pronounced for large stars, since they are more oblate. However, if the emission-observer geometry is such that some of the photons must be emitted close to the tangent to the star, then the light curves constructed from oblate and spherical stars can be very different.

In cases in which both the observer and the spot are located in the same hemisphere (as defined by the star’s spin equator), our previous calculations (Cadeau et al. 2007) have shown that if the inclination angle, spot latitude, and ratio \( 2M/R \) at the spot’s location are kept fixed, the light curve constructed from an oblate star will be less modulated than the light curve constructed from a spherical star. The oblate shape makes it easier to see the spot when it is at the back of the star. The opposite effect occurs if spot and observer are located in opposite hemispheres.

We have developed a simple approximation (Morsink et al. 2007) that captures the essential features of oblate stars. In this approximation, we find that the deflection angle and times of arrival are approximated very well by the Schwarzschild metric. However, initial conditions for the directions that the photons can be emitted into must take the oblate shape into account. We have found a simple formula (Morsink et al. 2007) for the oblate shape that depends only on the ratios \( M/R \) and \( \Omega^2R^3/M \) and has very little EOS dependence.

Since the inclusion of an oblate shape decreases the modulation of a light curve, we can make some predictions about the effect of oblateness on the fitted radius. Suppose that the ratio \( M/R \) is kept fixed, as is done in our fitting procedure. Adding oblateness while keeping \( \theta \) and \( i \) fixed will result in a light curve that is not modulated enough. In order to fit the modulation correctly a fit with \( i + \theta \) larger than in the spherical case will be preferred. (This is because we need to increase the angular separation of the spot and the observer when the spot is at the back of the star.) An increase in \( i + \theta \) will change the magnitude of the quantity \( R \sin \theta \sin i \) which controls the asymmetry of the light curve. Since \( \sin \theta \sin i = |\cos(\theta - i) - \cos(\theta + i)|/2 \), there are two possible cases when \( \theta + i \) increases. In the first case, the quantity \( |\theta - i| \) decreases, which leads to an overall increase in the quantity \( \sin \theta \sin i \). In order to keep the asymmetry the same, the best-fit star must have a smaller value of \( R \) to compensate. In the second case, the quantity \( |\theta - i| \) increases, in which case the change in \( \sin \theta \sin i \) is indefinite and the star’s radius could either increase or decrease when oblateness is added. Although it is possible in principle for either case to occur, we have found that only the first case occurs for the models computed in this paper.
The general trend in the best-fit angles and stellar radius can be seen by comparing the results of the oblate fits in Table 5 with the results of the fits for spherical stars in Table 4. In each case (of fixed $M/R$) the addition of oblateness increases the combination of angles $\theta + i$ while decreasing the magnitude of their difference $|\theta - i|$, leading to a decrease in the best-fit radius as given in the first case as described in the previous paragraph. However, only in the case of the largest stars does the decrease in radius due to oblateness rival the decrease in radius due to the inclusion of time delays.

We again attempted to fit the data with 1.4 $M_\odot$ stars while allowing all other parameters to vary using the oblate models with time delays. The resulting minimum value of $\chi^2 = 237$ for a 1.4 $M_\odot$ star is unacceptably large and strongly excludes larger mass stars.

In Figure 3 we plot the 2 and 3 $\sigma$ confidence contours arising from the fits including time delays and oblateness as bold dotted-dashed curves. The inclusion of time delays and oblateness shrinks the allowed values of mass and radius to a very small region of Figure 3 compared to the allowed region when time delays and oblateness are not included. The shrinkage of the allowed region is mainly due to the increase in $\chi^2$ introduced when the time delays are included, as noted in $\S$ 3.4.

The allowed region of the mass-radius plane at the 3 $\sigma$ confidence level only allows compact stars with very small radius and mass when time delays and oblateness are included. The largest radius star in this region has a radius of 6.6 km and a mass of 0.9 $M_\odot$. The largest mass star in this region has a radius of 5.4 km and a mass of 1.1 $M_\odot$. These values are inconsistent with any known neutron star EOS, but could be described by a quark star EOS if the bag constant is larger than usually considered ($B^{1/4} \sim 200$ MeV).

4. BOLOMETRIC LIGHT CURVE

The bolometric light curve constructed by Papitto et al. (2005) presents an alternative method for binning the data than that used by PG03. There are advantages and disadvantages to both methods. The PG03 method has the advantage of explicitly separating the effects of the different contributions from blackbody and Comptonized components. The bolometric light curve has the advantage of including photons of all energies and also has 4 times better time resolution, which is important when modeling the asymmetry in the light curve. The time period selected by Papitto et al. (2005) is much shorter, which has the advantage of avoiding mixing time periods from SAX J1808 where the pulse shape is different due to variability in the parameters of the emission region.

In our models of the bolometric light curve, the ratio $2M/R$ is kept fixed as in the case of the two-band light curves. The free parameters are $M$, $I$, $\theta$, $i$, $b$, and $\phi$. The parameter $I$ is the normalization of the Comptonized component of the flux. Similar to the definition earlier in $\S$ 2.1, the parameter $b$ is the ratio of the black-body flux to the Comptonized flux in the energy range 2–60 keV and is allowed to vary within 15% of the PG03 spectral model. As in the two-band models, the spot radius is kept fixed at 1.5 km. Since there are 64 time bins, the degrees of freedom in the bolometric fits are 58.

4.1. No Time Delays and Spherical Surface

In Table 6 we show the results of fitting the bolometric light curve using a model that assumes a spherical surface and omits time delays. This table should be compared to the results of the fiducial two-band results shown in Table 1. In the case of the bolometric data, there is very little variation in the minimum value of $\chi^2$ for fixed values of $2M/R$, in contrast to the two-band data which favors large values of $2M/R$. As a result, the bolometric light curve is consistent with a larger range of masses and radii than allowed by the two-band light curves.

The region of the mass-radius plane allowed at the 3 $\sigma$ confidence level is shown in Figure 4 as a dotted curve. This allowed region includes the corresponding region allowed by the two-band fits. However, the region consistent with the bolometric light curve includes a much larger range of masses and radii than the two-band light curves allow. When time delays are omitted and a spherical surface is used, the bolometric data is consistent with very stiff EOS such as those including hyperons.

4.2. Model Dependence of Bolometric Data

We now test the dependence of the fits to the bolometric light curve on the assumed model. In Table 7 we show the results of
these tests, where the ratio $2M/R = 0.6$ is kept fixed. All models are computed using exact light bending, no time delays, and a spherical surface. The first row of Table 7 is repeated from Table 6, and in each subsequent row one parameter in the model is changed. The rows labeled $r_{sp} = 1–3$ km show the dependence of the best-fit parameters on the assumed spot size. The best-fit stellar radius varies by less than 5% as the spot size is changed, similar to the dependence on spot size seen in the two-band fits. The row labeled $\Gamma = 1.8$ shows that the star’s radius increases by 3% when the photon spectral index is decreased from 2.0 to 1.8.

### 4.3. Time Delays and Oblateness

In Table 8 we show the results of fitting the bolometric data when we include the effects of variable time delays and the oblate shape of the star in the theoretical models. As in the case of the two-band light-curve fits, low values of mass and radius are found. The resulting best fits to the bolometric data have slightly larger (by about 10%) radius than the fits to the two-band data. The inferred inclination angle is larger for the bolometric fits while the spot appears to be closer to the pole in the bolometric fits.

The region of the mass-radius plane consistent with the bolometric light curve (at the 2 and 3 $\sigma$ level) is shown in Figure 4. Comparison of Figures 3 and 4 shows that the allowed region of the mass-radius plane for the two-band data is inside the allowed region for the bolometric data. The range of masses and radii allowed at the 3 $\sigma$ level is much wider than the similar allowed region for the two-band light curves (for oblate stars including time delays) shown in Figure 3. This may be due to the larger error bars in the bolometric light curve compared to the two-band light curve, or it may be due to the two-band light curve including a long enough time interval that real variability in the light-curve results in a false bias in the resulting fitted parameters. More observations of SAX J1808 will be needed to determine which is the correct case.

The bolometric light curve is consistent with many neutron star and quark-hadron hybrid stars, as well as pure quark stars, as can be seen in Figure 4. At the 3 $\sigma$ level, the largest radius star has a radius of 12.1 km and a mass of 1.2 $M_\odot$. The largest mass star allowed has a radius of 10.6 km and a mass of 1.4 $M_\odot$.

### 5. DISCUSSION

In this paper we have revisited light curves constructed from SAX J1808’s 1998 outburst. Through our analysis, we have derived constraints on the mass and radius of the compact star which depend on the data analysis method and the selection of data. In this paper we investigated the effect on the best-fit mass and radius of the inclusion of phase-dependent time delays, the inclination of oblateness, the assumed spectral model, and the binning and selection of data.

Our results can be summarized as follows. The inclusion of phase-dependent time delays and the star’s oblate surface both tend to force the best-fit stellar models to have a smaller radius. The inclusion of these effects on the mass-radius confidence contours causes a significant shrinkage of the allowed values of SAX J1808’s mass and radius. The magnitude of these changes is larger than would occur for reasonable changes in the spectral model.

We found that the binning and selection of the data has a very large effect on the fitted parameter values. The data published by PG03 corresponds to 19 days of data binned into two energy bands (3–4 and 12–18 keV) where photons with energies outside of these two bands are excluded from the analysis. Our analysis of this data agrees with the analysis of the same data by PG03 when time delays are omitted and a spherical surface is used. When we include the time delays and add an oblate surface the allowed values of mass and radius are only consistent with quark stars with a very large bag constant.

The compact stars allowed by the two-band data are very small in both size and mass. If SAX J1808 is described by such a small star, it would suggest that a phase transition between hadronic neutron stars and bare quark stars exists. However, SAX J1808 is accreting matter, so it cannot be described by a truly bare quark star. We are unaware of any models for quark stars with accreted hadronic matter that have stellar parameters falling into the region of the mass-radius plane allowed by the two-band data.

The alternative data selection used by Papitto et al. (2005) used only 1 day of data (in the same 19 day period used by PG03). Since SAX J1808’s pulse shape was variable during the time period analyzed by PG03 (see Hartman et al. 2006), use of data from a single day of observation alleviates this significant problem of mixing data with variable properties. All photons in this data set in the range 2–60 keV were combined into one bolometric light curve by Papitto et al. (2005). We used a similar spectral model and assumptions to fit the bolometric light curve and found much less restrictive results. Many neutron star and hybrid quark-hadron EOS are allowed at the 3 $\sigma$ level by fits to the bolometric data. The largest allowed radius is 12.1 km (see Fig. 4).

In all of our models we have restricted our attention to circular spots with uniform surface brightness. More complicated spot patterns have been predicted by MHD simulations of accretion (Kulkarni & Romanova 2005). However, at the current level of accuracy in the data there is no need to consider more complicated spot shapes. The spots in our models have a constant size as measured at the star’s surface. When we varied the spot size, we found that the changes in the best-fit values of the star’s mass and radius were not very large. However, the best-fit values of the inclination angle did vary by a fairly large amount. For this reason we do not quote any best-fit values for the inclination angle for SAX J1808.

The two-band data have a couple of features that suggest that the more conservative limits set by the bolometric light curve should be preferred. The two-band data light curves have omitted the data in the 4–12 and 18–60 keV ranges. This results in fits

### Table 7

| Model | 2M/R ($M_\odot$) | R (km) | $\theta$ (deg) | $\iota$ (deg) | $\alpha$ | $\chi^2$/dof |
|-------|-----------------|-------|---------------|--------------|---------|-------------|
| $r_{sp} = 1.5$ km | 0.60 | 0.98 | 4.75 | 19.5 | 63.9 | 0.604 | 20.4/58 |
| $r_{sp} = 1.0$ km | 0.60 | 0.94 | 4.60 | 19.0 | 63.5 | 0.603 | 20.1/58 |
| $r_{sp} = 2.0$ km | 0.60 | 0.93 | 4.59 | 24.1 | 53.0 | 0.569 | 20.6/58 |
| $r_{sp} = 3.0$ km | 0.60 | 0.97 | 4.79 | 24.2 | 50.6 | 0.565 | 21.7/58 |
| $\Gamma = 1.8$ | 0.60 | 0.99 | 4.89 | 19.5 | 63.7 | 0.605 | 20.4/58 |

### Table 8

| Model | 2M/R ($M_\odot$) | R (km) | $\theta$ (deg) | $\iota$ (deg) | $\alpha$ | $\chi^2$/dof |
|-------|-----------------|-------|---------------|--------------|---------|-------------|
| 0.60 | 0.99 | 4.87 | 17.7 | 70.7 | 0.630 | 21.7/58 |
| 0.50 | 1.05 | 6.21 | 13.9 | 68.4 | 0.667 | 22.1/58 |
| 0.40 | 1.04 | 7.73 | 11.2 | 67.4 | 0.720 | 22.6/58 |
| 0.30 | 0.85 | 8.56 | 10.8 | 61.0 | 0.714 | 23.0/58 |
| 0.20 | 0.60 | 9.17 | 10.6 | 57.4 | 0.727 | 23.5/58 |
that may be skewed in favor of the two energy ranges that were selected, resulting in very small stars. The bolometric data include the photons in the entire range of collected data. However, the main advantage of the Papitto et al. (2005) light curve is that the data are selected from 1 day only, which avoids the complication of combining variable pulse shapes over the 19 day period used by PG03.

Our results favor a soft EOS for SAX J1808. The 2 $\sigma$ confidence limits for the bolometric light curve only allow soft EOS which (with two exceptions) have low maximum masses below $1.6 M_\odot$. (The exceptions correspond to quark star models with a low bag constant, such as the Q140 EOS and the hadronic EOS of the type calculated by Baldo et al. 1997.) Our 3 $\sigma$ confidence limits allow stiffer EOS (such as APR) with maximum masses above 2.0 $M_\odot$.

In contrast, recent measurements of the quiescent flux from SAXJ1808.4–3658 (Heinke et al. 2007) suggest a stiff EOS due to the very low inferred luminosity. While the measurements by Heinke et al. (2007) are quite robust, further exploration of cooling processes in quark and other soft EOS are probably required to truly rule out a soft EOS solely on the basis of observations during quiescence.

Another calculation of the radius based on magnetospheric arguments (Li et al. 1999) also suggests a soft EOS. However, Rappaport et al. (2004) have shown that the standard magnetospheric description of accretion onto a fast pulsar may not hold.

The bolometric results are consistent with a number of other EOS constraints derived for other neutron stars. A pulse shape analysis for the slowly rotating X-ray pulsar Her X-1 (Leahy 2004) also favored a softer EOS. The bounds on Her X-1 are more restrictive than ours in that all EOS allowed (at the 3 $\sigma$ level) by the Her X-1 analysis are also allowed by our analysis of SAX J1808.4–3658. The softest quark EOS allowed by our analysis is not allowed by the Her X-1 data.

An analysis of X-ray bursts originating from EXO 0748-676 by Özel (2006) predicts a stiff EOS at the 1 $\sigma$ confidence level. However, at the 2 $\sigma$ confidence level the analysis allows softer EOS (see, e.g., Alford et al. 2007) compatible with the bolometric results.

The fact that SAX J1808.4–3658 is an accreting neutron star and has a very rapid rotation rate suggests that it may have accreted a large amount of mass. However, it should be remembered that a high spin rate does not necessarily require a large accretion of mass. Cook et al. (1994) showed that it is possible to spin a $1.4 M_\odot$ star up to 640 Hz by accreting as little as 0.1 $M_\odot$ if the EOS is soft. Smaller mass stars have lower moments of inertia and are easier to spin up, so it is plausible that the models (allowed by the bolometric data) with $M = 1.3 M_\odot$ could have been born with a mass as high as 1.2 $M_\odot$, consistent with the masses of neutron stars in binary pulsar systems (Stairs 2004).