Neutrino - Photon Interactions at Low Energy

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Abstract

We discuss $\nu - \gamma$ interactions in the presence of a homogeneous magnetic field with energies less than pair production threshold. The neutrinos are taken to be massless with only standard-model couplings. The magnetic field fulfills the dual purpose of inducing an effective neutrino-photon vertexes and of modifying the photon dispersion relation. Our conclusion is $\nu - \gamma$ interactions are too weak to be of importance for pulsar physics.

Introduction

By now it is well known that in many astrophysical environments the absorption, emission, or scattering of neutrinos occurs in the presence of strong magnetic fields [1]. Of particular conceptual interest are $\nu - \gamma$ interactions. These interactions do not occur in vacuum because neutrinos do not couple to photons and they are kinematically forbidden. In the presence of an external field, neutrinos acquire an effective coupling to photons by virtue of intermediate charged particles. In addition, external fields modify the dispersion relations of all particles so that phase space is opened for neutrino-photon reactions of the type $1 \rightarrow 2 + 3$.

The processes, which we’ll discuss here, are related to the process of photon splitting that may occur in magnetic fields [2]. In photon splitting the magnetic field plays the dual role of providing an effective three-photon vertex which does not exist in vacuum, and of modifying the dispersion relation of the differently polarized modes such that $\gamma \rightarrow \gamma \gamma$ becomes kinematically allowed for certain polarizations of the initial and final states.

We would like to stress that $\nu - \gamma$ interactions are important when typical energies of the particles involving in the reactions are less than $2 m_e \approx 1 MeV$. If $E > 1 MeV$ it becomes important reactions where involve real electron-positrons and $\nu - \gamma$ interactions become less important. In the last case one may consider $\nu - \gamma$ processes as radiational corrections to those where photons do not exist.

Therefore, we are interested to $\nu - \gamma$ processes with typical energies $E << 1 MeV$ (numerically our results work for $E < 0.5 MeV$).

Energy limitation allows us to use the limit of infinitely heavy gauge bosons and thus an effective four-fermion interaction,

$$L_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu \bar{E} \gamma^\mu (g_V - g_A \gamma_5) E.$$  \hspace{1cm} (1)

Here, $E$ stands for the electron field, $\gamma_5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3$, $g_V = 2 \sin^2 \theta_W + \frac{1}{2}$ and $g_A = \frac{1}{2}$ for $\nu_e$, and $g_V = 2 \sin^2 \theta_W - \frac{1}{2}$ and $g_A = -\frac{1}{2}$ for $\nu_{\mu, \tau}$. In our subsequent calculations we will always use $\sin^2 \theta_W = \frac{1}{2}$ for the weak mixing angle so that the vector coupling will identically vanish for $\nu_\mu$ and $\nu_\tau$. 
We'll consider following amplitudes $\bar{\nu}\nu, \bar{\nu}\nu\gamma, \bar{\nu}\nu\gamma\gamma$ and related processes.

In the Standard Model neutrino current couple to the electron via vector or axial-vector couplings.

For vector coupling, one may get $\nu - \gamma$ interaction amplitudes using Euler-Heisenberg effective lagrangian of constant electromagnetic field.

$$L = -\mathcal{F} - \frac{1}{8\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-m^2 s} [(es)^2 L - c.t.]$$ (2)

$$L = ig \frac{\cosh[es\sqrt{2(\mathcal{F} + iG)}] + \cosh[es\sqrt{2(\mathcal{F} - iG)}]}{\cosh[es\sqrt{2(\mathcal{F} + iG)}] - \cosh[es\sqrt{2(\mathcal{F} - iG)}]}$$ (3)

$$\mathcal{F} = \frac{1}{4} F_{\mu\nu}^2, G = 4 i F_{\mu\nu} F_{\mu\nu}^\ast, F^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$ (4)

In the Euler-Heisenberg lagrangian we'll assume

$$F_{\mu\nu} = \bar{F}_{\mu\nu} + N_{\mu\nu} + f_1^{\mu\nu} + f_2^{\mu\nu} + ...$$ (5)

here $\bar{F}_{\mu\nu}$ is the external constant electromagnetic field, $N_{\mu\nu} = g_{\nu} \frac{G}{\sqrt{2}} \{\partial_\mu \bar{\nu}\gamma_\nu (1 - \gamma_5) \nu - \partial_\nu \bar{\nu}\gamma_\mu (1 - \gamma_5) \nu\}$ and $f_1, f_2$ are the fields of real photons involving in the process. We'll get $\nu - \gamma$ interaction amplitudes for vector coupling expanding the Euler-Heisenberg lagrangian to the Taylor series with respect to the weak fields ($N_{\mu\nu}, f_1^{\mu\nu}$).

Unfortunately we could not use the same procedure for axial-vector coupling. Therefore we calculated each amplitude separately for axial-vector coupling.

The $\bar{\nu}\nu$ effective term

This amplitude will allow us to get dispersion relation of neutrinos in an external field. One may expand to the Taylor series the Heisenberg-Euler lagrangian and get vector part of this amplitude. We have calculated\(^1\) axial-vector part of this amplitude \(\bar{\nu}\nu\gamma\gamma\) effective vertex

$$L_A = \frac{g_A G_F}{\sqrt{2\pi^2}} \int_0^\infty \frac{ds}{s^2} e^{-m^2 s} \{A \partial_\nu + B_{\mu\nu} \partial_\mu \bar{\nu}\gamma_\nu (1 - \gamma_5) \nu\}$$ (6)

$$A = -4e^2 s^2 G$$

$$B_{\mu\nu} = \frac{4i}{e^2 s^2} \frac{dC_1}{dF_{\mu\lambda}} \frac{dC_2}{dF_{\nu\lambda}}$$ (7)

$$C_1 = \frac{2ie^2 s^2 G}{\cosh[es\sqrt{2(\mathcal{F} + iG)}] - \cosh[es\sqrt{2(\mathcal{F} - iG)}]}$$ (8)

$$C_2 = -2[\cosh[es\sqrt{2(\mathcal{F} + iG)}] - \cosh[es\sqrt{2(\mathcal{F} - iG)}]]$$ (9)

It is easy to see that the effect is very small for external fields $E, B \ll \frac{m^2_{\bar{\nu}\nu}}{e}$.

The $\bar{\nu}\nu\gamma$ effective vertex

It is shown in \(\bar{\nu}\nu\gamma\) effective vertex that the vector part of this matrix element is proportional to the effective charge radius and may be neglected. Axial vector part of the amplitude has the form

$$M_5 = \frac{g_A e^3 G_F}{\sqrt{24\pi^2 m^2_{\bar{\nu}\nu} e}} Z \varepsilon_\mu \bar{\nu}\gamma_\nu (1 - \gamma_5) \nu \left\{ -C_{\parallel} k_\nu \bar{F} (F^\ast k)^\mu + C_{\perp} \left[ k_{\parallel}^\nu (k\bar{F})^\mu + k_{\perp}^\nu (k\bar{F})^\nu - k_{\perp}^\mu \bar{F}^{\mu\nu} \right] \right\},$$ (10)
where $\varepsilon$ is the photon polarization vector. At low energy $C_{\perp}$ and $C_{\parallel}$ are real functions on $B$.

For $E_\gamma < 2m_e$ the photon refractive index always obeys the Cherenkov condition $n > 1$ \cite{4}. Therefore only the Cherenkov process $\nu \to \nu \gamma$ is kinematically allowed.

The four-momenta conservation constrains the photon emission angle to have the value

$$\cos \theta = \frac{1}{n} \left[ 1 + (n^2 - 1) \frac{\omega}{2E} \right],$$

where $\theta$ is the angle between the emitted photon and incoming neutrino. It turns out that for all situations of practical interest we have $|n - 1| \ll 1$ \cite{2}. This reveals that the outgoing photon propagates parallel to the original neutrino direction.

It is interesting to compare this finding with the standard plasma decay process $\gamma \to \bar{\nu} \nu$ which is dominated by the $\Pi^{\mu \nu}$. Therefore, in the approximation $\sin^2 \theta_W = \frac{1}{4}$ only the electron flavor contributes to plasmon decay. Here the Cherenkov rate is equal for (anti)neutrinos of all flavors.

For neutrinos which propagate perpendicular to the magnetic field, a Cherenkov emission rate can be written in the form

$$\Gamma \approx 2.0 \times 10^{-9} \, s^{-1} \left( \frac{E_\nu}{2m_e} \right)^{5/2} \left( \frac{eB}{m_e^2} \right)^2 \left( \frac{h(B)}{} \right),$$

where

$$h(B) = \begin{cases} (4/25) \left( \frac{eB}{m_e^2} \right)^4 & \text{for } eB \ll m_e^2, \\ 1 & \text{for } eB \gg m_e^2. \end{cases}$$

Turning next to the case $E > 2m_e$ we note that in the presence of a magnetic field the electron and positron wavefunctions are Landau states so that the process $\nu \to e^+e^-$ becomes kinematically allowed. Therefore, neutrinos with such large energies will lose energy primarily by pair production rather than by Cherenkov radiation \cite{3}.

### The $\bar{\nu}\nu\gamma\gamma$ effective vertex

In the vacuum this amplitude is highly suppressed. The axial-vector coupling is zero due to Landau-Yang theorem (two photons cannot have total spin equal to one) in four-Fermi limit. Beyond four-Fermi limit this amplitude is double Fermi suppressed. In \cite{3} it was found that $\bar{\nu}\nu \to \gamma\gamma$ (or crossed versions) cross-section is $\sigma \sim 10^{-68} \left( \frac{\omega}{M_{\text{MeV}}} \right)^6 \text{cm}^2$.

However, this amplitude is not double Fermi suppressed in a magnetic field. In \cite{3} it was shown that in a weak magnetic field ($B \ll \frac{m_e^2}{e}$) vector coupling is dominant and cross section of the process is $\sigma \sim 10^{-50} \left( \frac{\omega}{M_{\text{MeV}}} \right)^6 \left( \frac{eB}{m_e^2} \right)^2 \text{cm}^2$.

In a strong magnetic field \cite{3} contribute both vector and axial vector couplings. Since $B \gg \frac{m_e^2}{e}$ in the amplitude survive only lowest Landau levels and we lost dependence on magnetic field $\sigma \sim 10^{-50} \left( \frac{\omega}{M_{\text{MeV}}} \right)^6 \text{cm}^2$.

### The $\bar{\nu}\nu\gamma\gamma$ effective vertex

In this amplitude contribute only vector coupling \cite{3}. The cross section, \cite{3} of the process $\bar{\nu}\nu \to \gamma\gamma\gamma$ (or crossed processes $\nu\gamma \to \nu\gamma\gamma$ and $\gamma\gamma \to \bar{\nu}\nu\gamma$), is $\sigma \sim 10^{-50} \left( \frac{\omega}{M_{\text{MeV}}} \right)^{10} \text{cm}^2$. 
Conclusions

The strongest magnetic fields known in nature are near pulsars. However, they have a spatial extent of only tens of kilometers. Therefore, even if the field strength is as large as the critical one, most neutrinos escaping from the pulsar or passing through its magnetosphere will not interact with photons. Thus, the magnetosphere of a pulsar is quite transparent to neutrinos. Therefore our main conclusion is $\nu - \gamma$ interactions are too weak to be of practical importance for pulsar physics.

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