Chapter 1

Constraining Models of Dark Energy

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1.1 Measuring Dark Energy

Acceleration of the cosmic expansion opens new frontiers in our understanding of the quantum vacuum, gravitation, high energy physics, extra dimensions, or some combination of these research areas. Although we give the name “dark energy” to the accelerating mechanism, we know very little about its characteristics or even the area of physics from which it arises. One of the great endeavors of the past decade has been the evaluation of different observational techniques for measuring dark energy properties and of theoretical techniques for constraining models of cosmic acceleration given cosmological data. This chapter reviews a few of the key developments, promises, and cautions for revealing dark energy. We also present a few new calculations, on direct detection of acceleration through redshift drift, the minimum uncertainty in the equation of state, and testing gravity. For other recent reviews, see Caldwell & Kamionkowski (2009); Frieman, Turner, & Huterer (2008); Leon et al. (2010); Linder (2008); Silvestri & Trodden (2009).

One can probe dark energy through 1) its effect on the cosmic expansion, 2) its indirect effect on the growth of observed structure from the dark energy influence on the expansion, and 3) any direct contribution of it to the growth of structure. The first includes geometric probes and involves distances and volumes, coming directly from the metric. The second and third are growth probes, involving the growth
factor and growth rate of matter density perturbations or the overall gravitational potential. The third, direct contribution of dark energy to growth predominantly occurs when the dark energy represents a modification of the gravitational equations for growth, as dark energy tends to be smooth on subhorizon scales in most other models and so dark energy density perturbations contribute little to the Poisson equation.

The existence of dark energy was first discovered through the geometric probe of the distance-redshift relation of Type Ia supernovae (Perlmutter et al., 1999; Riess et al., 1998). Such data have been greatly expanded and refined so that now the analysis of the Union2 compilation of supernova data [Amanullah et al. (2010)], together with other probes, establishes that the energy density contribution of dark energy to the total energy density is $\Omega_{de} = 0.281 \pm 0.017$ and the dark energy equation of state, or pressure to density ratio, is $w = -1.03 \pm 0.09$.

Note that measurement of distances relative to low redshift, e.g. the Hubble diagram of supernovae, is the most sensitive probe of cosmic acceleration to date, i.e. it probes the deceleration parameter or $\ddot{a}$ most directly. Recall that the luminosity distance follows immediately from the metric, with $d_L = a^{-1} \int dt/a = a^{-1} \int da/(a^2 H)$ where $a$ is the expansion factor and $H = \dot{a}/a$ is the Hubble expansion rate.

Is there a more direct measure of acceleration than the distance (all distances being similar to the luminosity distance), which involves at best $\dot{a}$, rather than $\ddot{a}$ itself? One can devise a quantity involving the finite difference between expansion rates: the redshift drift $dz/dt_0$ as the universe ages, i.e. as we observe a given source at different epochs (Sandage, 1962; McVittie, 1962; Linder, 1997). The drift is of order the Hubble time, $\dot{z} \sim H$, so this is beyond present observational capabilities. Since it is an intriguing idea, though, let us pursue it a little further. For one thing, other effects intrude due to not living in a perfectly smooth universe.

The redshift of a source seen by an observer is

$$1 + z = \frac{(g_{\mu\nu} k^\mu u^\nu)^e}{(g_{\mu\nu} k^\mu u^\nu)^o}, \quad (1.1)$$

where subscript $e$ denotes the emitter frame, $o$ denotes the observer frame, and $g_{\mu\nu}$ is the metric, $k^\mu$ is the photon four-momentum, and $u^\nu$ is the source or observer four-velocity. If we want to find $dz/dt_0$ then we must take into account three contributions: 1) peculiar accelerations in the form of $\dot{u}$, 2) the homogeneous and inhomogeneous evolution of the...
metric, involving the scale factor \(a(t)\) and the metric potentials \(\psi\) and \(\phi\), and 3) the geodesic equation of the photon through the gravitational potentials along the path, i.e. \(k^\mu(x^\mu)\).

To estimate the impact of peculiar accelerations \(\dot{u}\), consider that they involve a spatial derivative of the potential, i.e. \(\dot{\psi} \sim \dot{u}\). Thus one can write the order of magnitude as \(\dot{u} \sim (k/H)H\psi\), where \(k\) is the characteristic inverse length scale. This contribution could be of order \(H\) for sources living in a galactic potential, just like the expansion factor contribution. One might be able to reduce this “noise” by using an array of redshift drift sources across the sky.

The metric and geodesic effects on the redshift drift are given by solving the geodesic equation, yielding

\[
\frac{dz}{dt_o} = \frac{\dot{a}_o - \dot{a}_e}{a_e} + 2[\dot{\psi}_e - (1 + z)\dot{\psi}_o] + \frac{2}{a_e} \partial_1(\psi_e - \psi_o) - (\dot{\psi}_e - \dot{\phi}_e) \\
+ (1 + z)(\dot{\psi}_o - \dot{\phi}_o) - H(z)(\phi_o - \phi_e) + H_0 (1 + z)\{a_o k_0^{(1)}\}
\]

where \(k_0^{(1)}\) is the first order correction to the observed photon frequency (and can be defined to be zero). Note that some terms only arise in modified gravity where the metric potentials \(\psi \neq \phi\). The first term, discussed below, is of order \(H\); all remaining terms are at most of order \(k\phi\), or \((k/H)\phi\) relatively, and so should be smaller by at least two orders of magnitude. A possible exception might involve supermassive binary black holes, where \(\dot{\phi}\) arises from inspirals; see Appendix B of Yunes, Pretorius, & Spergel (2010).

The first term is the standard McVittie-Sandage result (McVittie, 1962; Sandage, 1962), and involves the difference of the expansion rate between two redshifts, i.e. it provides some measure of the acceleration of the expansion. Even if this could be cleanly separated and measured, it provides only an order unity precision measurement of the acceleration for observations accurate to \(\delta z \sim 10^{-9}/(10\text{ years})\). To determine an assumed constant equation of state \(w\) to 1% would require \(\delta z \sim 10^{-12}/\text{year}\). Thus direct measurement of dark energy seems infeasible in the next generation. Instead we return to the geometric and growth probes.

1.2 Current Data

The response of the experimental cosmology community to the dark energy puzzle has been gratifyingly strong and diverse. As of mid
2010, experiments are underway using Type Ia and Type II supernovae, baryon acoustic oscillations, cosmic microwave background measurements, weak gravitational lensing, and galaxy clusters with the Sunyaev-Zel’dovich effect and X-rays. Even within the next year, more data will be released with continuing impact.

This activity is valuable, especially because currently the only technique that by itself unambiguously detects cosmic acceleration – let alone characterizes it precisely – is Type Ia supernovae. Other techniques need to be combined with external information, such as the Hubble constant, large scale structure data, or each other. X-ray clusters (Vikhlinin et al., 2009) and weak gravitational lensing (Schrabback et al., 2009) see acceleration at the 1-1.5σ level, if systematics do not enter in excess of the reports. Of course when data for various probes are combined together, one has strong concordant evidence for acceleration and moderate constraints on the dark energy equation of state viewed in a time-averaged, i.e. assumed constant, sense.

It is worthwhile to get an overview of the current results going beyond a constant equation of state. All the published Type Ia supernova data has been brought together and uniformly analyzed, employing-systematics tests and blinded cosmology fitting, in the Union2 compilation (Amanullah et al., 2010). In addition to results in terms of a constant equation of state $w$, time variations $w(a) = w_0 + w_a (1-a)$, $w_i(z)$ binned in redshift, and dark energy density $\rho_i(z)$ binned in redshift have been employed as well. This has also included other cosmological probes. All cases are consistent with the cosmological constant value of $w = -1$, however the results agree as well as with many other, dynamical models. Figure 1.1 illustrates the current state of our knowledge, viewed in terms of binned $w$.

Many different physical origins for acceleration are viable according to current data. Several have been explored in detail for the earlier Union1 compilation (Kowalski et al., 2008), showing that physical models quite distinct from the cosmological constant $\Lambda$ are acceptable (Rubin et al., 2009). One of the interesting results is that for some dark energy origins not all probes exhibit the complementarity familiar from the $\Omega_m-\Omega_\Lambda$ or $\Omega_m-w$ planes. For example, BAO and CMB basically give degenerate information for the linear potential (cosmic doomsday) model. Supernovae, because of their link to expansion through the metric, always give the most direct constraint on acceleration.
Fig. 1.1 Constraints from the Union2 supernova compilation, WMAP7 CMB, SDSS DR7 baryon acoustic oscillation, and Hubble constant data on the dark energy equation of state $w(z)$, in redshift bins. Top left plot appears to show that data have zeroed in on the cosmological constant value of $w = -1$, but this assumes $w$ is constant. When one allows for the values of $w$ to be different in different redshift bins, our current knowledge of dark energy is seen to be far from sufficient.

Top right plot shows that we do not yet have good constraints on whether $w(z)$ is constant. Bottom left plot (note change of scale) shows we have little knowledge of dark energy behavior, or even existence, at $z > 1$. Bottom right plot shows we have little detailed knowledge of dark energy behavior at $z < 1$. Outer (inner) boxes show 68% confidence limits with (without) systematics. The results are consistent with $w = -1$, but also allow considerable variation in $w(z)$. Adapted from Amanullah et al. (2010).

### 1.3 Future: Constraining What?

As Figure 1.1 demonstrates, we clearly need to learn considerably more about the nature of dark energy. What is its dynamics, does it have further degrees of freedom beyond $w$ (clustering, interaction, etc.), how accurate is an effective description in terms of a few parameters for interpreting next generation observations?

For the future, what type of constraints can we expect? How many
handles will we have on the physics behind cosmic acceleration? Recall
that originally the early cosmic acceleration of inflation was thought to
be untestable but now we have a rich variety of observational signatures
such as scalar perturbation tilt and running, spatial curvature, tensors,
nonGaussianity, and topological defects. Dark energy, although occur-
cing at an epoch more amenable to direct observation, has not yet
revealed so many tests.

In most cases we do not expect a perturbation generation mecha-
nism from the underlying physics because dark energy neither com-
pletely dominates the expansion nor has the acceleration ended, e.g. in
a reheating epoch. On the plus side, one observational window is the
expansion itself (inaccessible for inflation). That is why the expansion
equivalents of tilt and running – \( w \) and \( w' \) – play such a large role. As
we will see later in this section, another potential signature involves
perturbations within the dark energy itself.

1.3.1  **Limits on Measuring** \( w \)

Many models have a region or limit in parameter space in which their
equation of state closely approaches the cosmological constant value of
\( w = -1 \). As \( w \) nears \(-1\), it becomes increasingly difficult to achieve ac-
curate enough observations to distinguish the equation of state from \(-1\),
and potentially recognize these models as distinct from \( \Lambda \). Moreover,
there is a theoretical difficulty as well due to the covariance between \( w \)
and other parameters entering the expansion history.

Consider the simple flat model with constant \( w \); the Hubble expan-
sion is given by

\[
H^2 / H_0^2 = \Omega_m (1 + z)^3 + (1 - \Omega_m) (1 + z)^{3(1+w)}. \tag{1.3}
\]

The error on \( w \) from a measurement can be written as

\[
\delta w = \frac{y^{-3(1+w)}}{3(1-\Omega_m) \ln y} \left[ \delta(H^2 / H_0^2) - \delta \Omega_m (y^3 - y^{3(1+w)}) \right], \tag{1.4}
\]

where \( y = 1 + z \). Take the idealized situation of a perfect measurement
of \( H^2 / H_0^2 \). Then

\[
\sigma(w) \approx (3-5) \sigma(\Omega_m) \approx 0.016 \frac{\sigma(\Omega_m)}{0.004}, \tag{1.5}
\]

for the idealized measurement at \( z = 0.5 - 1 \).
One can carry out a similar estimation regarding determination of whether the dark energy density $\rho_{de}$ is constant. Here one finds

$$\sigma(\rho/\rho_0) \approx \frac{y^3 - 1}{1 - \Omega_m} \sigma(\Omega_m) \approx (3 - 10) \sigma(\Omega_m) \approx (0.01 - 0.04) \frac{\sigma(\Omega_m)}{0.004}.$$ 

These expressions give practical bounds on the accuracy of distinction from $\Lambda$, since $\Omega_m$ (and other covariant parameters) will be imperfectly known. (Not allowing for uncertainty in, e.g., $\Omega_m$ can give larger, bias effects. See, e.g., Figure 6 of Hlozek et al. (2008).)

1.3.2 Mapping Dynamics

Observables such as the distance-redshift relation and Hubble parameter-redshift relation can be used to test specific models of dark energy, but it is frequently useful to have a more model independent method of constraining dark energy properties. A calibration relation exists between the dark energy equation of state value and its time variation that defines homogeneous families of dark energy physics (de Putter & Linder, 2008b). This calibration provides a physical basis for the $w_0-w_a$ parametrization devised to fit the exact solutions for scalar field dynamics (Linder, 2003).

The resulting parameters $w_0$, $w_a$ give a highly accurate match to the observable relations of distance $d(z)$ and Hubble parameter $H(z)$, and the constraints imposed on them by data allow robust exploration of the nature of dark energy in a model independent manner. (Note that $w_0$, $w_a$ are thus defined in terms of this calibration as opposed to a fit to the unobservable $w(z)$.)

The accuracy with respect to the observables for a diverse group of models is exhibited in Table 1.1 for models near the extreme of current viability; less extreme models will be fit even better. Such 0.1% or better accuracy is more than sufficient for next generation experiments. While constant $w$ overcompresses (loses important information from) the expansion history information, $w_0-w_a$ faithfully preserves the information to better than the precision level of the data. Attempts to use further parameters generically lack additional leverage on the data (e.g. see the next subsection), giving no real benefit (save in the possible exception of early dark energy models). Thus, $w_0$, $w_a$ provide

\[ \text{I thank Bob Scherrer for suggesting that when } w \approx -1, \text{ one might do this analysis in terms of } \rho_{de}. \]
an excellent parameter space for the cosmic expansion history.

Table 1.1  Accuracy of \( w_0 - w_a \) in fitting the exact distances and Hubble parameters for various dark energy models. These numbers represent global fits over all redshifts (except for the last three cases, where the fit covers \( z = 0-3 \), due to early dark energy). Better fits can be found over finite redshift ranges. From [de Putter & Linder (2008a)].

| Model                  | \( \delta d/d \) | \( \delta H/H \) |
|------------------------|------------------|------------------|
| PNGB \( (w_0 = -0.85) \) | 0.05%            | 0.1%             |
| PNGB \( (w_0 = -0.75) \) | 0.1%             | 0.2%             |
| Linear Pot. \( (w_0 = -0.85) \) | 0.05%            | 0.1%             |
| Linear Pot. \( (w_0 = -0.75) \) | 0.1%             | 0.3%             |
| \( \phi^4 \) \( (w_0 = -0.85) \) | 0.01%            | 0.04%            |
| \( \phi^4 \) \( (w_0 = -0.75) \) | 0.02%            | 0.06%            |
| Braneworld \( (w_0 = -0.78) \) | 0.03%            | 0.07%            |
| SUGRA \( (n = 2) \)     | 0.1%             | 0.3%             |
| SUGRA \( (n = 11) \)    | 0.1%             | 0.3%             |
| Albrecht-Skordis \( (\Omega_e = 0.03) \) | 0.01%            | 0.02%            |
| Albrecht-Skordis \( (\Omega_e = 0.26) \) | 0.1%             | 0.4%             |

1.3.3  Further Dynamics?

The previous subsection showed that the two parameters of \( w_0, w_a \) for the dark energy equation of state provide information more than sufficient to match the data of next generation experiments. Their calibration of the expansion history data is accurate down to the \( \sim 0.1\% \) level. Nevertheless, there is the temptation to look for information about \( w(a) \) in some other form.

Principal component analysis has been suggested as an alternate view of the equation of state, with each specific experiment determining the combinations of redshift-dependent functions carrying the most information (see, e.g., Huterer & Starkman (2003); de Putter & Linder (2008a); Mortonson, Hu, & Huterer (2009) for discussions of the method). Here too, however, two parameters describe the vast majority of information. If we want to distinguish a model from the cosmological constant, say, the \( \chi^2 \), or signal to noise, is

\[
S/N = \left[ \sum \frac{\alpha_i^2}{\sigma_i^2} \right]^{1/2}, \tag{1.7}
\]
where $\sigma_i$ is the uncertainty on the coefficient $\alpha_i$ of the $i$th principal component, where $w(a) - w_b(a) = \sum_i \alpha_i e_i(a)$. (Note $\sigma_i$ itself captures no physics; it is the ratio $\sigma_i/\alpha_i$ that is important.)

Each principal component contributes a certain amount to the statistical significance, and we can quantify how much the modes beyond the first two matter. Taking a representative of the freezing class of dark energy, we can scan over every possible model parameter value, determine the corresponding $\alpha_i$ and $\sigma_i$, and weigh the contribution of higher modes. We then repeat this for the thawing class, and an oscillating case (see de Putter & Linder (2008c) for details). Table 1.2 lists the fraction of the total $S/N$ covered by the first two modes – for the case for each dark energy class where higher modes contribute the most.

| Model                | $(S/N)_2/(S/N)_{all}$ | $(S/N)_3/(S/N)_{all}$ |
|----------------------|-----------------------|-----------------------|
| Freezing ($w_a = 0.7$) | 0.972                 | 0.995                 |
| Thawing ($w_a = -0.5$) | 0.997                 | 0.9998                |
| Oscillating ($A = 0.5$) | 0.862                 | 0.922                 |

For the thawing class the higher modes contribute less than 0.3% to the total, and for the freezing class less than 2.8% in the most sensitive case, dropping to less than 0.5% for modes above the third. If we allow the oscillating case to reach $w = 0$, then the additional contribution can be up to 14% (actually much less because appropriate marginalization over the equation of state beyond where the distance data lie reduces this by a factor of several). These are the upper limits on contribution by modes beyond the first two, for a highly idealized experiment with 0.3% distance determination over $z = 0 - 3$. This is not to say that modes beyond the first two cannot contribute $S/N > 1$, only that the vast majority of the information contained in the data comes from two modes. This information is found to be at essentially the same level as from using $w_0$, $w_a$ directly.
1.3.4 Microphysics

Although the equation of state information is represented accurately by \( w_0, w_a \), there is further information on the dark energy nature that can be extracted. Even a perfectly measured \( w(z) \) does not generically tell us whether dark energy arises from a canonical, minimally coupled scalar field, a more complicated fluid description, or modification of gravitational theory on large scales. The properties of the perturbations to the dark energy, which must exist unless it is simply a cosmological constant, do carry such extra information. We can consider this information in terms of the sound speed.

A dark energy sound speed below the speed of light enhances the spatial variations of the dark energy. Clustering dark energy influences the growth of density fluctuations in the matter, and the pattern of large scale structure, and an evolving gravitational potential generates the Integrated Sachs-Wolfe (ISW) effect in the cosmic microwave background. These effects are suppressed while the equation of state is near \(-1\), so knowledge of the sound speed is strongest for models that have a period where \( w \) is far from \(-1\), and in particular for early dark energy models.

One quite natural model in this class, and possessing the interesting properties of “predicting” that \( w \approx -1 \) today and solving the coincidence problem, is the barotropic aether model of Linder & Scherrer (2009). The constraints on the sound speed, shown in Figure 1.2, due to current data are not definitive but show a slight preference for a low sound speed (de Putter, Huterer, & Linder, 2010).

Future data using a large galaxy sample for auto- and cross-correlations will provide a much clearer picture. Models with extended gravity can also be treated in terms of an effective sound speed. The potential for future data to explore new degrees of freedom for dark energy, in terms of sound speed and early dark energy, is quite exciting.

1.3.5 Extended Gravity

If dark energy is not a new physical component but a modification of the equations of motion, e.g. gravity beyond general relativity (GR), then we need a clear way of parameterizing the changes. This is most commonly accomplished through the relationship between the metric potentials \( \psi \) and \( \phi \) (which are equal within GR) and through the form of the Poisson equation or effective gravitational constant. See Table 1 of
Fig. 1.2 68.3, 95.4 and 99.7% confidence level contours in the early dark energy model with constant sound speed $c_s$ and early dark energy density fraction $\Omega_e$. The constraints are based on current data including CMB, supernovae, LRG power spectrum and crosscorrelation of CMB with matter tracers. From de Putter, Huterer, & Linder (2010).

Daniel et al. (2010) for a translation table among common approaches. Testing gravity on cosmic scales is an area of intense interest at the moment; using the most current data Daniel et al. (2010); Bean & Tangmatitham (2010); Zhao et al. (2010); Reyes et al. (2010) find consistency with GR, although again deviations are certainly allowed. See for example Figure 1.3.

The amplitude of the permitted deviations from GR depends on the functional forms assumed (e.g. time and scale dependence) and the covariances between them. Better data from growth probes could play a key role in tightening constraints or uncovering new physics. A particularly exciting prospect is comparing the density, velocity, and potential field information through combining imaging and spectroscopic surveys (Jain & Zhang, 2008; Reyes et al., 2010; Jain & Khoury, 2010).
Fig. 1.3 Current data constraints on the fractional deviation of the gravitational coupling from Newton’s constant, $G_{\text{eff}} - 1$, where this is fit for two bins in redshift $z$ and two bins in wavenumber $k$. (Zero deviation is assumed outside the bins.) Solid (dashed) contours give 95% (68%) cl and are marginalized over the other extended gravity parameter $\Sigma$ entering the matter perturbation growth equation. From [Daniel et al.] (2010b).

1.4 From Here to Eternity: 50 Ways to Leave $\Lambda$

To understand the nature of cosmic acceleration, one must know not only that $w \approx -1$ today, but the physics behind it. This is essential to comprehend its origin, to understand the areas to probe for distinction from $\Lambda$, and to predict the fate of the universe. We present examples of three diverse models that are emphatically not $\Lambda$, but give $w \approx -1$, as illustrations of very different physics that can cause the behavior to approach that of $\Lambda$. There are many more, and in addition there exist thawing models that for most of the history of the universe have $w \approx -1$ but leave $\Lambda$. These cases exemplify that current observations that appear as $w \approx -1$ are very far from indicating that dark energy is the cosmological constant.

The following three model examples are consistent with current data, $w \approx -1$ in an averaged sense, yet with completely distinct physics from

\[2\text{As Paul Simon almost said: "The answer is easy if you take it logically / I'd like to help you in the puzzle of dark energy / There must be 50 ways to leave your Lambda."} \]
A and from each other. One model looks like $\Lambda$ through microphysics, one through high energy physics, and one through gravity.

1.4.1 Microphysics

The equation of state ratio $w = p/\rho$ is an implicit relation between pressure and density but one can instead impose an explicit equation of state $p = p(\rho)$. These are called barotropic fluids and their dynamics is determined in terms of their microphysics, e.g. the sound speed of perturbations $c_s = \sqrt{dp/d\rho}$, by

$$w' = -3(1 + w)(c_s^2 - w).$$

By inspection this will have an attractor at $w = -1$ and so is an attractive class of models for understanding why $w$ may be close to $-1$.

Using only a stability/causality condition $0 \leq c_s^2 \leq 1$, and that in the past dark energy does not dominate over matter, one finds that in the past $w \to 0$, $c_s^2 \to 0$ (Linder & Scherrer, 2009). In the future, as mentioned there is an attractor to $w = -1$. Thus in both the past and the future the model looks like $\Lambda$CDM but can have some deviation in between. Because the dark energy traces the matter in the past, there is no fine tuning problem, and because of the rapid evolution between asymptotic behaviors, there is no coincidence problem. Over the more recent half of cosmic history, $w \approx -1$ naturally (see Figure 1.4). So this is quite an interesting model.

1.4.2 High Energy Physics

Naturalness is an issue with many high energy physics explanations of dark energy. Why is the energy scale associated with the dark energy density so small compared to the initial conditions characteristic of the high energy early universe? How is the scale (amplitude) and the form of the potential protected against quantum corrections? The cosmological constant suffers both problems.

Some quintessence models avoid the first through attractor solutions (e.g. inverse power law, exponential, or other tracker potentials (Ratra & Peebles, 1988; Wetterich, 1988; Zlatev, Wang, & Steinhardt, 1999)), while some avoid the second through symmetries (e.g. pseudo-Nambu Goldstone boson potentials (Frieman et al., 1995)). One model that accomplishes both uses the Dirac-Born-Infeld action and its rela-
Fig. 1.4 Barotropic models make a rapid transition from $w = 0$ at high redshift ($a \ll 1$) to $w \approx -1$ more recently (the transition from $w = -0.1$ to $w = -0.9$ always takes less than 1.5 e-folds). This is forced by the physics of Eq. (1.8) and in distinction to quintessence gives a prediction that observations of the recent universe should find $w \approx -1$.

tivistic kinematics. The DBI action

$$\mathcal{L} = -T \sqrt{1 - \dot{\phi}^2/T} + T - V$$

(1.9)
can be thought of as a relativistic generalization of quintessence with a Lorentz boost factor $\gamma = (1 - \dot{\phi}^2/T)^{-1/2}$. This action can arise from the world volume traced out by a 3-brane in a 10-dimensional spacetime. See Silverstein & Tong (2004); Alishahiha, Silverstein, & Tong (2004) for more details and links to string theory.

The two functions entering are the brane tension $T(\phi)$ and the interaction potential $V(\phi)$, but for our purposes all that will be important is that the general dynamics depends primarily on simply the asymptotic value of the ratio $V/T$ rather than the specific forms of the functions.

As described in Ahn, Kim, & Linder (2010, 2009), several new classes of attractors become enabled in the ultrarelativistic limit $\gamma \rightarrow \infty$, solving the first problem mentioned at the beginning of this subsection,
while the geometric origin of the action protects against the second problem.

In particular, an attractor to \( w = -1 \) exists for \( V/T \to \infty \), such as for \( V \sim \phi^2 \) in the simplest \( T \sim \phi^4 \) case (pure AdS\(_5\) geometry), or even for the canonical \( V = m^2 \phi^2 \) (mass term), \( T \sim \phi^4 \) case for large mass \( m \). Because the field is frozen in the past, and attracted to a \( \Lambda \)-like state in the future (although possessing no nonzero minimum of the potential), then \( w \approx -1 \) always holds. So this model too agrees with observations though originating from very different physics than the cosmological constant. Figure 1.5 illustrates the concordance.

![DBI model graph](image)

**Fig. 1.5** The DBI model has \( w \approx -1 \) naturally throughout cosmic history, for a simple condition on \( V/T \). In the past, the field is frozen, then thaws as its dark energy starts to dominate, but is attracted back to \( w = -1 \). So the deviation from \( w = -1 \) never gets large, despite the radically different physics from a cosmological constant vacuum energy. Adapted from Ahn, Kim, & Linder (2010).

### 1.4.3 Extended Gravity

The cosmological constant enters as a constant term added to the Ricci scalar in the Einstein-Hilbert action. Since the spacetime cur-
vature plays such an essential role in gravity, it might seem strange to add in an independent constant. Instead, one could promote the Ricci scalar to a function, \( f(R) \). For the extensive literature on such models, see Durrer & Maartens (2008); Sotiriou & Faraoni (2010); De Felice & Tsujikawa (2010).

Such models not only change the expansion history but add scale dependence to spacetime quantities such as light deflection (e.g. in the parametrized post-Newtonian formalism) as well as the growth of density perturbations. The strong coupling regime in regions of high density gradient restore general relativity, but to make this happen sufficiently quickly to accord with observations motivates a steep dependence on \( R \). Using an additional term in the action of the form

\[
 f(R) = -cr (1 - e^{-R/r}),
\]

where \( c \) is a fit parameter and \( r \) is determined by \( c \) and \( \Omega_m \), Linder (2009) found good agreement with both expansion and growth probes. (Many other \( f(R) \) do also, but this model allows relaxation of fine tuning, basically opening up the region \( c \lesssim 15 \).) The effective dark energy equation of state (without any physical dark energy) possesses \( w \approx -1 \) for \( c > 1 \), as seen in Figure 1.6.

Again, both the past and future appear as a cosmological constant universe despite there being no actual cosmological constant. The effect of the modified gravity on growth of structure can provide observational distinction from \( \Lambda \)CDM with general relativity. Current measurements, however, are not sufficiently precise to impose significant constraints (cf. Lombriser et al. (2010), and the model independent bounds in Figure 1.3).

### 1.5 Conclusions

While current data are consistent with a cosmological constant as a source for dark energy, a cornucopia of other physical origins are in agreement as well. There are many ways to leave \( \Lambda \) as an explanation for cosmic acceleration, some without the fine tuning and other issues. We briefly outlined approaches based on the microphysical properties of the dark energy, on a high energy physics origin, and on extending gravity beyond general relativity. All are valid possibilities.

The exciting goal of future observations is to explore this wonderland of physics. We have seen that for the dynamical aspects, next gener-
Fig. 1.6 For this extended gravity $f(R)$ model, the effective dark energy equation of state naturally has $w \approx -1$ throughout cosmic history. The larger the value of $c$, the more indistinguishable is the expansion history from $\Lambda$CDM. However the growth history can have observational signatures. From Linder (2009).

Comparison measurements of the equation of state and its time variation, $w$ and $w'$, in the calibrated form of $w_0$ and $w_a$ describe the experimental reach to better than observational accuracy. Comparison of tests of growth and expansion could give key clues to the underlying physics, as can contrasting the density, velocity, and gravitational potential fields of large scale structure. These should be enabled by future wide field imaging and spectroscopic surveys.

To give a more speculative view, the rich variety of information within the CMB, to be revealed by Planck and ground based polarization experiments, can explore signatures of early dark energy. If the early dark energy density at CMB last scattering is at much higher levels than the part in a billion in the cosmological constant model, then this would be a major clue to the physical origin (note percent level contributions can be accommodated within the barotropic model of Sec. 1.4.1). Lensing of the CMB, and weak lensing of galaxies, can probe aspects of dark energy clustering and interaction. Eventually we
can hope to have as wide an array of aspects of dark energy to probe as have been developed for inflation. We are very much at the beginning of our explorations of the physics behind cosmic acceleration.

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