Modification of Prim’s algorithm on complete broadcasting graph

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Abstract. Broadcasting is an information dissemination from one object to another object through communication between two objects in a network. Broadcasting for \(n\) objects can be solved by \(n - 1\) communications and minimum time unit defined by \(\lceil 2\log n \rceil\). In this paper, weighted graph broadcasting is considered. The minimum weight of a complete broadcasting graph will be determined. Broadcasting graph is said to be complete if every vertex is connected. Thus to determine the minimum weight of complete broadcasting graph is equivalent to determine the minimum spanning tree of a complete graph. The Kruskal’s and Prim’s algorithm will be used to determine the minimum weight of a complete broadcasting graph regardless the minimum time unit \(\lceil 2\log n \rceil\) and modified Prim’s algorithm for the problems of the minimum time unit \(\lceil 2\log n \rceil\) is done. As an example case, here, the training of trainer problem is solved using these algorithms.

1. Introduction

Let \(A\) be a set of \(n\) objects in a communication network. Every communication happens between two objects. The spreading of an information from an object to other objects in the set \(A\) is called as broadcasting [1]. Broadcasting can be solved by developing an unordered sequence \((i, j)\) where \(i, j \in A\) as many as \(k(k \in \mathbb{Z}^+)\) which causes a complete spread. Broadcasting is called complete if every member of \(A\) knows the information coming from a source [2]. For example, for \(n = 8\) it requires seven calls (communications) such that the broadcasting becomes complete. Complete broadcasting for \(n = 8\) is presented in table 1.

Table 1 presents a complete broadcasting where the information (*) comes from object 1. Every rows in the table 1 displays the time-unit stage which should be traversed such that every object knows the information. Unordered sequences of a complete broadcasting can be depicted in to a graph. A simple graph \(G = (V, E)\) consists of a non-empty set \(V\) of vertices (or nodes) and a set \(E\) of edges where every edge connects two different vertices and no two edges connect the same pair of vertices [3]. By applying edge coloring method, complete broadcasting problem can be solved in the minimum time unit as displayed in figure 1. Consequently, for \(n = 8\) it
Table 1. Complete broadcasting for 8 objects (n = 8).

| Pairs of objects | Informed objects |
|------------------|-----------------|
| (1, 2)           | * *             |
| (2, 3)           | * * *           |
| (1, 4)           | * * * *         |
| (1, 5)           | * * * * *       |
| (2, 6)           | * * * * * *     |
| (3, 7)           | * * * * * * *   |
| (4, 8)           | * * * * * * * * |

only requires 3 time unit in obtaining a complete broadcasting, where every time unit shows a grouping of some pairs of communication.

Generally, the minimum time unit required for a complete broadcasting is $\lceil 2\log n \rceil$. It can be shown that for $n$ objects, it requires $(n - 1)$ calls (communications) to obtain a complete broadcasting [4]. In this case, the time unit is not the total time weight of the formed graph but as the grouping of some edges which are traversed in the same time.

Figure 1. Complete broadcasting graph.

Normally, the broadcasting term is used in entertainment sector such as television, radio, etc. The term broadcasting has been known for long and a topic in some researches [5], [6], [7], [8] and [9]. Broadcasting graphs which were considered in some researches usually are not weighted. However, in the real application, problems often comes as weighted graphs, for example the weight can represent distance, cost of transportation, traveling time, etc. Therefore, weighted broadcasting graph is an interest subject to observe. This paper concerns on determining the minimum weight of a complete broadcasting graph.

2. Methodology

On broadcasting, delivering information is carried out by an information giver to uninformed ones such that every existing node is connected. Therefore, the initial graph before complete broadcasting is obtained is a complete graph. For any $n(n \in Z^+)$, a complete graph is a graph with $n$ vertices where every two vertices are connected [10].

Here, it will be considered a weighted complete graph. Finding a complete broadcasting graph from a complete graph is actually the same as determining its spanning tree, because the condition for a broadcasting graph to be complete is if all members know the information. A tree is defined as an acyclic graph. A tree connecting all the vertices in a graph G is called as spanning tree of the graph G [11]. Hence, the output that will be generated in its relation
to the minimum weight of a complete broadcasting graph is a minimum spanning tree. The method that will be applied is the algorithms of minimum spanning tree. Kruskal’s and Prim’s algorithms are considered.

In Kruskal’s algorithm, the edges are ordered based on their weight in ascending order. The edges that will be input to a set T are the edges of graph G such that T forms a tree. An edge of a graph G is added to T if it does not form a cycle [12]. The algorithm is presented as follow,

- T is empty
- Choose edge \((i, j)\) with minimum weight
- Choose the next edge \((i, j)\) with minimum weight, which does not form a cycle in T, add \((i, j)\) to T
- Repeat the step 3 for \((n - 1)\) times
- The total steps are \((n - 1)\) times

Prim’s algorithm starts at a node which has edge with smallest weight. The edges which are added to the set T is the edges of G which is incident with a node in T, such that T is a tree. An edge of a graph G is added to T if it does not form a cycle. Two of more edges may have the same weight such that there are alternative nodes. In this case, any one of them can be chosen [12]. The algorithm is as follow,

- Take an edge with the minimum weight, add it to T
- Choose edge \((i, j)\) with minimum weight which is incident with the node in T but does not form a cycle in T, add it to T
- Repeat procedure 2 for \((n - 1)\) times

There is minimum time unit in solving a complete broadcasting graph. Therefore, the algorithms will be also analyzed in term of their minimum time unit.

3. Results and discussion

3.1. Problem category

There are several problems that appear in the information spread by using broadcasting method. Figure 2 displays complete broadcasting graph which are obtained from a complete graph with

![Figure 2](attachment:image.png)

**Figure 2.** (a) Time unit priority and (b) weight (cost) priority.

\(n = 6\). In figure 2(a), the information spread prioritizes the minimum time unit which is \(\lceil 2 \log 6 \rceil = 3\). This means that the spreading must complete in 3 time unit (grouping). In the 3 time unit, it achieves the minimum weight as many as 11. Meanwhile, figure 2(b) presents the spreading which prioritizes the weight. Here the weight is 9. It requires 4 time unit in finishing this spreading. From the two cases, broadcasting method can be categorized in to two:

- Minimum weight search for a complete broadcasting graph with prioritizing minimum time unit
- Minimum weight search for a complete broadcasting graph without prioritizing minimum time unit
3.2. Modification of Prim’s Algorithm
In Prim’s Algorithm, the weight calculation of a graph starts at a node, then continue to an adjacent node whose edge connecting it to the previous graph has the smallest weight. This process is repeated until all nodes are connected without containing any cycle. This concept can be applied to the problem of weighted broadcasting graph with considering the minimum time unit. In the Prim’s algorithm, there is only one node that can move to search the minimum weight in every step, while in this problem, it is allowed that more than one node to search nodes with minimum weight such that the output time unit is less than or equal to $\lceil 2\log n \rceil$.

By modifying a part of Prim’s algorithm, it can find the minimum weight of a complete broadcasting graph which prioritizes the minimum time unit. The modified Prim’s algorithm is presented as follow,

- Determine an initial node as an information giver in $G=(V,E)$. Assume the initial node as $v_s = 1 \in V$
- $U \leftarrow \{1\}, E' \leftarrow \emptyset, W \leftarrow \{V\setminus U\}$
- If $|U| \leq |W|$, then choose an edge with minimum weight $e = \{i,j\} \in E$, for $\forall i \in U, j \in W$
  - $E' \leftarrow E' \cup \{e\}, U \leftarrow U \cup \{j\}$ and $W \leftarrow \{V\setminus U\}$
- If $|U| > |W|$, then choose an edge with minimum weight $e = \{i,j\} \in E$, for $i \in U, \forall j \in W$
  - $E' \leftarrow E' \cup \{e\}, U \leftarrow U \cup \{j\}$ and $W \leftarrow \{V\setminus U\}$
- If $|W| = 0$, then stop. $M = (V, E')$ if a broadcasting graph with minimum weight $E'$ and time unit $\lceil 2\log n \rceil$. Otherwise, return to the step 3.

3.3. Minimum weight search
Let a district A consisting of seven towns is holding a Training of Trainer (TOT). Figure 3 illustrates the location of each town with the cost for assigning a trainer from a town to other towns. Each weight is in the unit of million Indonesian Rupiahs (IDR). It is assumed that the initial trainer is from the first town (K1) and each town can be traversed from any town.

![Figure 3](image-url)

**Figure 3.** Sketch of seven towns where Training of Trainer (TOT) is held.

3.3.1. Solution without prioritizing the minimum time unit

- **Kruskal’s algorithm implementation**
  The total weight obtained is 10 and the spread can be solved in 5 time unit for the fastest. It means that the TOT requires cost as much as 10 million IDR in 5 assigning stages. Figure 4(b) is the sketch of the route which needs to be taken to hold the TOT.

- **Prim’s algorithm implementation**
  According to the last step, the total weight obtained by Prim’s algorithm is the same as the one obtained from Kruskal’s algorithm. The route in figure 5(f) is also the same as the route in the figure 4(b).
Figure 4. Kruskal’s algorithm in solving the TOT problem.

Figure 5. Prim’s Algorithm in solving the TOT problem.

3.3.2. Solution with prioritizing the minimum time unit

The total weight which is obtained by implementing the modified algorithm is 14, greater than the total weight which is obtained from the Kruskal’s and Prim’s algorithm. However, the time unit is smaller, which is ⌈2log 7⌉ = 3. It represents that there are only three stages in assigning the trainer but requires cost as much as 14 million IDR. Figure 6(c) is the sketch of the route which needs to be taken in holding the TOT.

Figure 6. Modified Prim’s algorithm in solving TOT problem.

Modified Prim’s algorithm gives the total weight as much as 14. This value is not the minimum for the weighted complete broadcasting graph because it is obtained that the weight is 10 by applying Kruskal’s and Prim’s algorithm. However, the time unit from the modified algorithm is definitely less than or equal to ⌈2log n⌉, while Kruskal’s and Prim’s algorithm may requires more time unit.

A complete broadcasting graph considering the both measures, time unit and weight (cost), can be further analyzed. In this case, the minimum weight satisfies ⌈2log n⌉ time unit.

Figure 7. (a) A complete graph with n = 4. (b) First complete broadcasting graph model. (c) First complete broadcasting graph model. (d) First complete broadcasting graph model.
Modified Prim’s algorithm will give a complete broadcasting graph as illustrated in figure 7(b). Initially, the node 1 gives an information to the node 4 because the edge has the minimum weight. Hereinafter, at the same time the node 1 gives information to the node 3 and node 4 gives information to the node 2. The total weight obtained is $1 + 3 + 8 = 12$. If at the second stage the node 1 chooses the node 2 and no 4 chooses the node 3 as depicted in figure 7(c), the total weight is smaller, $1 + 5 + 5 = 11$. If the node 1 chooses node 3 at the first stage, then the complete broadcasting graph is obtained as presented in figure 7(d), and the total weight is even smaller, $3 + 1 + 1 = 5$. Therefore, to obtain the minimum weight of a complete broadcasting graph with prioritizing the minimum time and weight, it needs to be considered the total weight of each possible complete broadcasting graph.

4. Conclusion
It can be conclude that a complete broadcasting graph can be generated without considering the minimum time unit. The minimum weight search of the complete broadcasting graph without prioritizing the time unit can be solved by using the Kruskal’s and Prim’s algorithm. Moreover, to find the minimum weight with prioritizing the minimum time unit can be solved by implementing the modified Prim’s algorithm. The minimum weight search of the complete broadcasting graph with prioritizing the time unit and weight can be conducted by checking all possible complete broadcasting graphs which satisfy time unit $\lceil 2\log n \rceil$.

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