Left-handed color-sextet diquark in Kaon system

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Abstract

We investigate whether a color-sextet scalar diquark ($H_6$) coupling to the left-handed quarks contributes to the $\Delta S = 2$ process. It is found that the box diagrams mediated by $W$ and $H_6$ bosons have no contributions to $\Delta S = 2$ when the limit of $m_t = 0$ is used, and the flavor mixing matrices for diagonalizing quark mass matrices are introduced at the same time. When the heavy top-quark mass effects are taken into account, it is found that in addition to the $W - H_6$ box diagrams significantly contributing to $\Delta S = 2$, their effects can be as large as those from the $H_6 - H_6$ box diagrams. Using the parameters that are constrained by the $K^0 - \bar{K}^0$ mixing parameter $\Delta M_K$ and the Kaon indirect CP violation $\epsilon_K$, we find that the left-handed color-sextet diquark can lead to the Kaon direct CP violation being $Re(\epsilon'/\epsilon) \sim 0.4 \times 10^{-3}$. In the chosen scheme, although the diquark contribution to $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is small, the branching ratio of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ can reach the current experimental upper bound.

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I. INTRODUCTION

Despite the success of the standard model (SM) in explaining most experimental data, the SM is an effective theory only at the electroweak (EW) scale because some long standing phenomena are still puzzling, such as baryogenesis, neutrino mass, and the muon anomalous magnetic dipole moment. More recently, from the RBC-UKQCD collaboration [1, 2] using the lattice QCD and the group using a large $N_c$ dual QCD [3–5], it was found that the predicted Kaon direct CP violation in the SM is less than the experimental data by more than a $2\sigma$. If this inconsistency is finally confirmed, it indicates the necessity of a new physics effect.

A new mechanism with a colored scalar (e.g., diquark), which was used to resolve the strong CP problem and can contribute to the Kaon indirect ($\epsilon_K$) and direct ($\epsilon'/\epsilon$) CP violation, was originally proposed in [6, 7]. Although a diquark can originate from grand unified theories (GUTs) [6, 8], it can be an unobserved particle with a mass of order of TeV in the SM gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$.

If we take the diquark as the particle in the SM gauge symmetry, the possible representations can be: $(3, 1, -1/3), (6, 1, 1/3), (3, 3, -1/3)$, and $(3, 1, 2/3)$, where $(3, 1, -1/3)$ and $(6, 1, 1/3)$ can couple to the $SU(2)_L$ quark doublets and singlets. Moreover, it can be verified that before EW symmetry breaking (EWSB), the Yukawa couplings of $H_3 = (3, 1, -1/3)$ and $H_6 = (6, 1, 1/3)$ to the left-handed quark doublets are flavor symmetric and antisymmetric, respectively. Although the $H_3$ Yukawa coupling matrix in flavor space is symmetric, there are six independent Yukawa matrix elements; thus, the symmetric property, similar to a generic case, may not exhibit unique behavior in the flavor physics. The detailed analysis of $H_3$ can be found in [9]. However, the antisymmetric $H_6$ Yukawa matrix only has three independent elements; thus, if we assume that the new Yukawa couplings are real parameters, there are only four new parameters involved, including the diquark mass.

In addition to involving fewer parameters, $H_6$ has some interesting characteristics. It was argued in [7] that the box diagrams mediated by one W gauge boson and one $H_6$ for $\Delta S = 2$ vanish. We revisit the issue and find that the conclusion is only correct in the limit of $m_t = 0$; here, we still take the light quarks to be massless, i.e. $m_{u,c} = 0$. Moreover, to achieve the vanished result, the unitary flavor mixing matrices introduced to diagonalize quark mass matrices have to be simultaneously included. The situation is very different
from the case of $H_3$, where due to the antisymmetric property in color space, the same box diagrams using $H_3$ instead of $H_6$ vanish, even when the $m_t$ effect is taken into account \cite{7, 9}.

In order to show the importance of the heavy top-quark mass effect, we study the $\Delta S = 2$ processes with and without the limit of $m_t = 0$ in detail. For the massive top-quark, in addition to the $W$-mediated box diagrams, we have to consider the charged-Goldstone-boson ($G$) contributions when the 't Hooft-Feynman gauge is used. It is found that the contribution to $\epsilon_K$ from one $W(G)$ and one $H_6$ box diagrams can be as large as that from the pure diquark box diagrams, which are insensitive to $m_t$.

When the parameters in the $H_6$ model are constrained by the $K^0 - \bar{K}^0$ mixing parameter $\Delta M_K$ and the indirect CP violation $\epsilon_K$, we consider the implications on $\epsilon'/\epsilon$ and $K \to \pi \nu \bar{\nu}$. It is found that if we take the antisymmetric $H_6$ Yukawa matrix to be real, $\epsilon'/\epsilon$ from the diquark contribution can reach a value of around $0.4 \times 10^{-3}$, which is comparable to the SM result. Although there is no new CP violating effect contributing to $K_L \to \pi^0 \nu \bar{\nu}$, we find that the branching ratio (BR) for the $K^+ \to \pi^+ \nu \bar{\nu}$ decay induced by the $H_6$-mediated $Z$-penguin can reach the upper limit of the current experimental data. The NA62 experiment at CERN plans to measure $BR(K^+ \to \pi^+ \nu \bar{\nu})$ with a 10% precision \cite{10, 11}. If an unexpected large BR for $K^+ \to \pi^+ \nu \bar{\nu}$ is found in NA62, the diquark may be a potential candidate explaining the anomaly. On the other hand, if the measured $BR(K^+ \to \pi^+ \nu \bar{\nu})$ is close to the SM result, then we can use it to bound the parameter space of the diquark coupling.

The paper is organized as follows: We introduce the Yukawa couplings and EW gauge couplings of the color-sextet diquark in Sec. II. We calculate the diquark box diagrams for $\Delta S = 2$ with and without the limit of $m_t = 0$ in Sec. III. In addition, assuming that the $H_6$ Yukawa matrix is a real matrix, we study the $\epsilon_K$ and $\Delta M_K$ constraints on the free parameters. In Sec. IV, using the constrained parameters, we study the diquark contributions to the $\epsilon'/\epsilon$ and $K \to \pi \nu \bar{\nu}$. A summary is given in Sec. VI.

II. COLOR-SEXTET DIQUARK YUKAWA AND GAUGE COUPLINGS

In order to study the color-sextet diquark effects, in this section, we introduce the diquark Yukawa couplings to the SM quarks and its gauge couplings to the EW gauge bosons. With the real diquark Yukawa matrix, the loop contributions to $\epsilon'/\epsilon$ and $K \to \pi \nu \bar{\nu}$ are dominated by the $Z$-penguin \cite{9}, so, we skip discussions of the diquark coupling to the gluons.
A. Yukawa couplings

The SM gauge invariant Yukawa couplings of $H_6(6,1,1/3)$ to the left-handed quarks can be written as:

$$- \mathcal{L}_Y = Q^T C y \varepsilon H_6^\dagger P_L Q + H.c.,$$  \hspace{1cm} (1)

where the flavor indices are suppressed; $y$ is the Yukawa matrix, and $\varepsilon$ is a $2 \times 2$ antisymmetric matrix with $\varepsilon_{12} = -\varepsilon_{21} = 1$. The $H_6$ representation in $SU(3)_C$ can be expressed as $H_6 = S^a H_6^a$ ($a = 1-6$), where the symmetric matrix forms of $S^a = (S^a)^T$ can be found in [12].

For the complex conjugate state, we use $(\bar{S}^a)_{\alpha\beta} = (S^a)_{\beta\alpha}$, i.e., $H_6^\dagger = \bar{S}^a H_6^*$; thus, we obtain $Tr(S^a \bar{S}^b) = \delta^a_b$ and $(S^a)^{\beta\alpha}(\bar{S}^a)_{\rho\sigma} = 1/2(\delta^\beta_\rho \delta^\alpha_\sigma + \delta^\beta_\sigma \delta^\alpha_\rho)$. Using the fermion anticommutation relations, the antisymmetric $C$ and $\varepsilon$, and the symmetric $P_L$ and $H_6$, we can verify that $y$ is an antisymmetric matrix, e.g., $y^T = -y$. Since our purpose is to study the antisymmetric effects of $y$, we only concentrate on the $H_6$ couplings to the left-handed quarks and assume that the couplings to the right-handed quarks are small.

Basically, we can use $y$ to investigate the relevant phenomena [7]; however, the flavor mixings induced from the electroweak symmetry breaking (EWSB) may change the antisymmetric property of the Yukawa matrix that originally arises from $y$. To see the influence of the quark-flavor mixings, we introduce the unitary matrices $V_{u,d}^L$ to diagonalize the quark mass matrices, and Eq. (1) in terms of quark mass eigenstates can then be expressed as:

$$- \mathcal{L}_Y = u^T_L C \left[ (V^u_L y V^d_L)^T - (V^d_L y V^u_L)^T \right] H_6^\dagger d_L + H.c.$$  \hspace{1cm} (2)

Using $\left( V^d_L y V^u_L \right)^T = -V^u_L y V^d_L$, we obtain:

$$- \mathcal{L}_Y = u^T_L C g H_6^\dagger d_L + H.c., \quad g = 2V^u_L y V^d_L.$$  \hspace{1cm} (3)

It can be seen that when the flavor mixings are introduced, the new Yukawa matrix $g$ generally is not an antisymmetric matrix. If we link $V^u_{L,d}$ to the Cabibbo-Kobayashi-Maskawa (CKM) matrix, defined by $V \equiv V^u_L V^d_L$, the new Yukawa matrix $g$ can be expressed as:

$$g = 2y V,$$  \hspace{1cm} (4)

where $V^u_L = 1$ is taken. According to the Wolfenstein’s parametrization [13], the off-diagonal elements of the CKM matrix can be parametrized using a small parameter of $\lambda \approx 0.225$. 

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Based on the characteristic of the CKM matrix, we can take some useful approximations from the phenomenological viewpoint; then, $\mathbf{g}$ can be approximately regarded as $\mathbf{y}$.

In order to understand the similarity and difference between $\mathbf{y}$ and $\mathbf{g}$, we first analyze the diagonal elements of $\mathbf{g}$ as:

\begin{align}
&g_{11} = 2y_{12}V_{21} + 2y_{13}V_{31} \sim -2y_{12}\lambda, \\
&g_{22} = 2y_{21}V_{12} + 2y_{23}V_{32} \sim -2y_{12}\lambda, \\
&g_{33} = 2y_{31}V_{13} + 2y_{32}V_{23} \sim 0, \\
\end{align}

where we have dropped the $\lambda^2$ and $\lambda^3$ terms that are from the CKM matrix elements as the approximation. If $|y_{12}| \sim |y_{31}|\lambda^2$ or $|y_{12}| \sim |y_{32}|\lambda$ is satisfied, we can take $g_{11} \sim g_{22} \sim 0$. Under these circumstances, $g_{ii} \approx 0$ is similar to $y_{ii} = 0$. Using the same approximation, we can analyze the off-diagonal elements of $\mathbf{g}$ as:

\begin{align}
&g_{12} = 2y_{12}V_{22} + 2y_{13}V_{32} \sim 2y_{12}, \\
&g_{21} = 2y_{21}V_{11} + 2y_{23}V_{31} \sim 2y_{21}, \\
&g_{13} = 2y_{12}V_{23} + 2y_{13}V_{33} \sim 2y_{13}, \\
&g_{31} = 2y_{31}V_{11} + 2y_{32}V_{21} \sim 2y_{31} - 2y_{32}\lambda \\
&g_{23} = 2y_{21}V_{13} + 2y_{23}V_{33} \sim 2y_{23}, \\
&g_{32} = 2y_{31}V_{12} + 2y_{32}V_{22} \sim 2y_{31}\lambda + 2y_{32}.
\end{align}

From Eq. (6), it can be clearly seen that with the exception of $g_{31}$ and $g_{32}$, other elements approximately obey $g_{ij} \approx -g_{ji}$, which is posseed by $y_{ij}$. If $|y_{32}| < |y_{31}|\lambda$ is satisfied, we can have $g_{31} \approx -g_{13}$; however, we cannot obtain $g_{23} \approx -g_{32}$. We note that above analysis is simply to show the conditions under which $g_{ij} \approx -g_{ji}$ can be achieved. In general, it is not necessary to take $\mathbf{g}$ as an approximately antisymmetric matrix.

**B. EW gauge couplings to $H_6$**

In order to obtain the photon and $Z$-boson gauge couplings to the $H_6$ diquark, we write the $U(1)_Y$ covariant derivative of $H_6$ as:

\[ D_\mu H_6 = (\partial_\mu + ig'Y_{H_6}B_\mu)H_6, \]

where $g'$ is the $U(1)_Y$ gauge coupling constant; $Y_{H_6}$ is the $H_6$ hypercharge, and $B_\mu$ is the $U(1)_Y$ gauge field. Since $H_6$ is an $SU(2)$ singlet, its hypercharge is equal to its electric charge, i.e., $Y_{H_6} = e_{H_6}$. Using $B_\mu = \cos \theta_W A_\mu - \sin \theta_W Z_\mu$, the EW gauge couplings to the
diquark can be obtained from the $U(1)_{Y}$ gauge invariant kinetic term of $H_6$ and can be written as:

$$\mathcal{L}_{VH_6H_6} = i e_{H_6} e (\partial_\mu H_{6a}^* H_6^a - H_{6a}^* \partial_\mu H_6^a) A^\mu$$

$$- ig_{H_6} \frac{\sin^2 \theta_W}{\cos \theta_W} (\partial_\mu H_{6a}^* H_6^a - H_{6a}^* \partial_\mu H_6^a) Z^\mu,$$

where $\theta_W$ is the Weinberg’s angle; $e = g' \cos \theta_W = g \sin \theta_W$ and $g'/g = \tan \theta_W$ are applied; $g$ is the $SU(2)_L$ gauge coupling constant, and $e_{H_6} = Y_{H_6} = 1/3$ is the $H_6^a$ electric charge. The associated Feynman rules can be obtained as:

$$A_\mu H_{6a}^* H_6^b : - i e_{H_6} e (p_b + p_a) \mu \delta^b_a,$$

$$Z_\mu H_{6a}^* H_6^b : i g_{H_6} \frac{\sin^2 \theta_W}{\cos \theta_W} (p_b + p_a) \mu \delta^b_a.$$

### III. $H_6$-MEDIATED BOX DIAGRAMS FOR $\Delta S = 2$

Based on the diquark Yukawa couplings in Eq. (3), we discuss the diquark effects on the $\Delta S = 2$ process, where the Feynman diagram arises from the intermediates of $W(G)$ and $H_6$ are sketched in Fig. 1. The other types of box diagrams can be obtained using $H_6$ instead of $W(G)$. In order to understand the effects of the massive top-quark, we distinguish the $m_t = 0$ case from the $m_t \neq 0$ case.

![Box diagram](image)

**FIG. 1:** Box diagram mediated by one $W(G)$ and one $H_6$ for $\Delta S = 2$, where $G$ denotes the charged-Goldstone boson.
1. In the limit of $m_t = 0$

Using the $H_6$ Yukawa couplings in Eq. (3), the effective Hamiltonian for $\Delta S = 2$ from one $W$ and one $H_6$ box diagram shown in Fig. [1] can be written as:

$$
H_{\text{Box}}^{W,H_6} = -i g^2 4 \sum_{i,j} (V^*_i V_j) g_{j2} g_{i1} (S_\alpha)_{\rho\beta} (S^\alpha)_{\rho'\beta'} \int \frac{d^4q}{(2\pi)^4} \frac{q^{\mu'\nu'}}{(q^2 - m^2_{H_6})(q^2 - m^2_W)q^4} \times \left( \bar{S}_a \rho \beta P_L d^\beta \right) \left( S_a \rho' \beta' P_R \gamma^\mu P_L s^\mu \right),
$$

where $i, j$ denote the flavor indices of up-type quarks. Due to the limit of $m_{u,c,t} = 0$, the charged-Goldstone-boson has no contributions to the $\Delta S = 2$ process. Using $\gamma_\mu \gamma_\nu = g_{\mu\nu} - i\sigma_{\mu\nu}$ and the loop integral

$$
\int \frac{d^4q}{(2\pi)^4} \frac{q^{\mu'\nu'}}{(q^2 - m^2_{H_6})(q^2 - m^2_W)q^4} = i \frac{g^{\mu'\nu'}}{4(4\pi)^2 m^2_{H_6}} \frac{\ln y_W}{1 - y_W},
$$

the $\Delta S = 2$ effective Hamiltonian can be expressed as:

$$
H_{\text{Box}}^{W,H_6} = \frac{G_F^2 (V^*_i V_j)^2}{16\pi^2} m^2_W C_{WH_6} \bar{s} \gamma_\mu P_L d \bar{s} \gamma^\mu P_L d,
$$

where the Fierz transformation is applied and $y_X \equiv m^2_X / m^2_{H_6}$ for any particle $X$; $G_F / \sqrt{2} = g^2/(8m^2_W)$ is used, and the effective Wilson coefficient $C_{WH_6}$ at the $\mu = m_{H_6}$ scale is given as:

$$
C_{WH_6} = -\frac{V^*_j V_i}{V^*_i V_j} \frac{g_{j2} g_{i1}}{4y_W \ln y_W y_W - 1}.
$$

Unlike the case in the color-triplet diquark [7, 9], the symmetric color factor of $\delta_\rho^\beta \delta_\rho'^\beta + \delta_\rho'^\beta \delta_\rho \beta'$ from the color-sextet diquark does not lead to a cancellation. From Eq. (14), at first sight, the box diagrams mediated by one $W$ and one $H_6$ do not vanish in the limit of $m_t = 0$. However, it is found that Eq. (14) indeed vanishes. We can verify the result as follows:

$$
\sum_i V_{id} g_{i1} = \sum_i V^T_{di} (2y V)_{i1} = 2(V^T y V)_{11} = 0,
$$

where we use the quark-flavor indices to label the CKM matrix elements and use Arabic numerals to show the Yukawa couplings. The null result arises from the antisymmetric property of $(V^T y V)$; hence, $C_{WH_6} = 0$. The result has nothing to do with the structure of $g$ and is also suitable for $B$- and $D$-meson systems. Intriguingly, our conclusion is the same as that in [7] using a different viewpoint. We should emphasize that it is $y$, which was used in [7]. It can be seen that if we use $y$ instead of $g$, $\sum_i V_{id}g_{i1}$ does not vanish.
In the limit of $m_t = 0$, the nonvanished contributions to $\Delta S = 2$ are from the pure diquark box diagrams, and the effective Hamiltonian from the $H_6$ box diagrams can be simply written as:

$$H_{\text{Box}}^{H_6H_6} = \frac{G_F^2}{16\pi^2} m_W^2 \left[ 6y_W \left( \frac{(g^g g_{21})}{g^2} \right)^2 \right] \bar{s} \gamma_{\mu} P_L d \bar{s} \gamma_{\mu} P_L d. \quad (16)$$

Following the hadronic matrix elements for $\Delta S = 2$, which were obtained in [14], the matrix element of $K^0 - \bar{K}^0$ mixing can be formulated as:

$$M_{12}^{K^*} = \langle K^0 | H_{\text{Box}}^{H_6H_6} | K^0 \rangle = \frac{G_F^2}{48\pi^2} m_K^2 \frac{f_K^2 V_{ts}^* V_{td}}{P_{1VLL}^{VLL}} \alpha_s \left( \frac{m_H_6}{\alpha_2(m_t)} \right)^{6/21} 6y_W \left( \frac{(g^g g_{21})}{g^2 V_{ts}^* V_{td}} \right)^2, \quad (17)$$

where $f_K$ is the $K$-meson decay constant; $P_{1VLL}^{VLL} \approx 0.48$ is the nonperturbative QCD effects, and the factor $(\alpha_s(m_H_6)/\alpha_2(m_t))^{6/21}$ is the renormalization group (RG) evolution from the $m_{H_6}$ scale to the $m_t$ scale. Since the Kaon indirect CP violation parameter $\epsilon_K$ is associated with $V_{ts}^* V_{td}$ in the SM, we scale the parameters $(g^g g_{21})$ with the $(V_{ts}^* V_{td})^{-1}$ factor. Thus, the mass difference between $K_L$ and $K_S$ and $\epsilon_K$ can be obtained as:

$$\Delta M_K = 2 Re M_{12}^{K^*}, \quad \epsilon_K = \frac{\exp(i\pi/4)}{\sqrt{2} \Delta M_K} I m M_{12}^{K^*}. \quad (18)$$

The predicted $\Delta M_K$ in the SM is close to the experimental data; therefore, we use $\Delta M_K^{\exp}$ instead of $\Delta M_K$, which appears in the denominator of $\epsilon_K$. In addition, the SM contribution to $\epsilon_K$ is also consistent with the experimental data, so the room for new physics actually is very limited. In our numerical analysis, we require that the new physics effects should satisfy [19]:

$$|\Delta M_K^{\text{NP}}| \leq 0.2 \Delta M_K^{\exp}, \quad |\epsilon_K^{\text{NP}}| \leq 0.4 \times 10^{-3}. \quad (19)$$

Generally, $y$ can carry two physical CP phases, for which a detailed discussion is given in the Appendix. If we require $y$ to be a real matrix, from $g = 2yV$, it can be found that the origin of the CP violation in the diquark model arises from the CKM matrix. Since it is not our purpose to show the generic CP phase effects of $y$ in this study, we assume that $y$ is a real matrix in the following numerical analysis, unless stated otherwise.
Wolfenstein’s parametrization [13] as an input, the CP phase of the CKM matrix appears in \( V_{td} \) and \( V_{ub} \), and the Kaon CP violation in the SM arises from \( \text{Im}(V_{ts}V_{td}) \); therefore, when the \( \epsilon_K \) constraint is considered, we can simply focus on the \( V_{td} \) related effects in \((g^\dagger g)_{21}\). In addition, because \( \Delta M_K \) may cause a strict constraint on the free parameters, we should also pay attention to the effects, which arise from a large CP-conserving CKM factor. Accordingly, to study the \( \Delta M_K \) and \( \epsilon_K \) constraints, we decompose \((g^\dagger g)_{21}\) to be CP-even and CP-odd parts as:

\[
(g^\dagger g)_{21}^{\text{CP-even}} = 4(V^\dagger y^\dagger y V)_{21}^{\text{CP-even}} = 4V_{ks}^*V_{ud}(y^\dagger y)_{k1} + V_{ks}^*V_{cd}(y^\dagger y)_{k2},
\]

\[
(g^\dagger g)_{21}^{\text{CP-odd}} = 4(V^\dagger y^\dagger y V)_{21}^{\text{CP-odd}} = 4V_{ks}^*V_{td}(y^\dagger y)_{k3},
\]

where the sum of all \( k \) flavors is implied, and \((y^\dagger y)\) is a symmetric matrix. It can be seen that if we use \((y^\dagger y)_{ij}\) as the free parameters, there are nine components in \((g^\dagger g)_{21}\); six terms are from \((g^\dagger g)_{21}^{\text{CP-even}}\), and three terms are from \((g^\dagger g)_{21}^{\text{CP-odd}}\). We note that without any approximation, indeed \( \text{Im}(V_{cs}^*V_{cd}) = -\text{Im}(V_{ts}^*V_{td}) \); however, since the associated new physics effects are small, we neglect the \( \text{Im}(V_{cs}^*V_{cd}) \) contributions.

From Eq. (20), each \((y^\dagger y)_{ij}\) is associated with a CKM factor \( V_{is}^*V_{jd} \). It is known that the CKM matrix elements have a hierarchical structure; therefore, when the CKM factor is larger, the allowed \((y^\dagger y)_{ij}\) is smaller. For clarity, in terms of the \( \lambda \) parameter, we show the corresponding CKM factor for the associated \((y^\dagger y)_{ij}\) in Table I. It can be seen that although \((y^\dagger y)_{13}\) and \((y^\dagger y)_{33}\) can contribute to \( \epsilon_K \), because the associated CKM factors are small, their allowed values are much larger than the others. Nevertheless, due to \( y^T = -y \), we can obtain:

\[
(y^\dagger y)_{13} = y_{12}^*y_{23},
\]

\[
(y^\dagger y)_{33} = |y_{13}|^2 + |y_{23}|^2 \geq 0,
\]

where the same parameters also appear in \((y^\dagger y)_{31} = y_{12}^*y_{23}, (y^\dagger y)_{11} = |y_{12}|^2 + |y_{13}|^2 \geq 0, \) and \((y^\dagger y)_{22} = |y_{12}|^2 + |y_{23}|^2 \geq 0, \) which are strictly bounded by \( \Delta M_K \). Hence, due to the antisymmetric nature of the \( y \) matrix, we cannot take all \( g_{j2}^*g_{j1} \) as independent parameters.

To determine the magnitude of each involved \((y^\dagger y)_{ij}\) when the \( \Delta M_K \) and \( \epsilon_K \) constraints are satisfied, we take all the six terms of \((g^\dagger g)_{21}^{\text{CP-even}}\) and the three terms of \((g^\dagger g)_{21}^{\text{CP-odd}}\) in Eq. (20) as independent effects. At the moment, we do not consider the possible cancellations among different parameters. To avoid the collider constraint, in our following numerical
TABLE I: \(\lambda\)-parameter dependence of the CKM factor, which is associated with the parameter \((y^\dagger y)_{ij}\).

| CKM | \(\lambda\) | \(\lambda^0\) | \(\lambda^2\) | \(\lambda^3\) | \(\lambda^4\) | \(\lambda^5\) |
|------|------------|------------|------------|------------|------------|------------|
| \((y^\dagger y)_{11}\) | \((y^\dagger y)_{21}\) | \((y^\dagger y)_{31}\) | \((y^\dagger y)_{12}\) | \((y^\dagger y)_{22}\) | \((y^\dagger y)_{23(32)}\) | \((y^\dagger y)_{13}\) | \((y^\dagger y)_{33}\) |

estimates, we always fix \(m_{H_6} = 1.5\) TeV. Thus, each diquark contribution to \(\Delta M_K\) from \((g^\dagger g)_{21}^{CP-even}\) is shown as a function of \((y^\dagger y)_{ij}\) in Fig. 2(a). The solid, dashed, and dotted lines denote the contributions from \((y^\dagger y)_{11}\), \((y^\dagger y)_{21}\), and \((y^\dagger y)_{31}\), whereas the thick solid, dashed, and dotted are the contributions from \((y^\dagger y)_{12}\), \((y^\dagger y)_{22}\), and \((y^\dagger y)_{32}\), respectively. Due to the same CKM factor, the lines of \((y^\dagger y)_{11}\) (solid) and \((y^\dagger y)_{22}\) (thick dashed) overlap.

The results with the \(\epsilon_K\) constraint are shown in Fig. 2(b), where the solid, dashed, and dotted lines denote the effects from \((y^\dagger y)_{13}\), \((y^\dagger y)_{23}\), and \((y^\dagger y)_{33}\), respectively. Due to \((y^\dagger y)_{ij} = (y^\dagger y)_{ji}\), we see that the constraint on \((y^\dagger y)_{13}\) from \(\Delta M_K\) is stricter than that from \(\epsilon_K\). As mentioned earlier, \((y^\dagger y)_{33} = |y_{12}|^2 + |y_{13}|^2\) is correlated to \((y^\dagger y)_{11} = |y_{13}|^2 + |y_{23}|^2\) and \((y^\dagger y)_{22} = |y_{12}|^2 + |y_{23}|^2\), so, although large ranges of \(y_{13}\) and \(y_{23}\) in \((y^\dagger y)_{33}\) are allowed under the \(\epsilon_K\) constraint, they indeed have been bounded by \((y^\dagger y)_{11,22}\) through \(\Delta M_K\). Hence, with the exception of \((y^\dagger y)_{23}\), it is found that the constraint from \(\Delta M_K\) is more serious than that from \(\epsilon_K\), and the resulting upper values of \(|(y^\dagger y)_{ij}|\) are generally below 0.12.

FIG. 2: (a) \(\Delta M_{H_6}^K\) (in units of \(10^{-15}\)) and (b) \(\epsilon_{H_6}^K\) (in units of \(10^{-3}\)) as a function of \((y^\dagger y)_{ij}\), where \(m_{H_6} = 1.5\) TeV is used, where the horizontal dashed line in (a) is the upper limit of \(\Delta M_{NP}^K \leq 0.2\Delta M_{exp}^K\), and the shaded area in (b) denotes \(|\epsilon_{NP}^K| \leq 0.4 \times 10^{-3}\).
We have discussed that in the limit of \( m_t = 0 \), the \( W - H_6 \) box diagrams for \( \Delta S = 2 \) vanish. It is of interest to see if the box diagrams can have a sizable effect when \( m_t(\bar{m}_t) \approx 165 \text{ GeV} \) is used. In addition to the \( W \)-mediated box diagrams, in the ’t Hooft-Feynman gauge, we now have to include the charged-Goldstone-boson contribution. Since the Goldstone-boson Yukawa coupling to a quark is proportional to the quark mass, we only need to calculate the top-quark contribution. Hence, the effective Hamiltonian for \( \Delta S = 2 \) can be written as:

\[
\mathcal{H}_{\text{Box}}^{(W^+G)H_6} = \mathcal{H}_{\text{Box}}^{WH_6} + \mathcal{H}_{\text{Box}}^{GH_6} = \frac{C_F^2 (V_{ts}^* V_{td})^2}{16\pi^2} m_W^2 \left( C_{WH_6} + C_{GH_6} \right) \bar{s}\gamma_\mu P_L d \bar{s}\gamma^\mu P_L d, \\
C_{WH_6} = -8 y_W h_{ij} V_{j2}^* V_{id} I_{WH_6}(y_W, y_u, y_u), \\
C_{GH_6} = -4 y_t^2 h_{33} I_{GH_6}(y_W, y_t),
\]

(22)

where \( h_{ij} = g_{r2} g_{i1} / (g^2 V_{ts}^* V_{td}) \), the sum of \( i, j \) is implied, and the loop functions are shown as:

\[
I_{WH_6}(y_W, y_u, y_u) = \int_0^1 dx_1 \int_0^{x_1} dx_2 \frac{x_2}{1 + (y_W - 1)x_1 + (y_u - y_W)x_2 + (y_u - y_u)x_3}, \\
I_{GH_6}(y_W, y_t) = \int_0^1 dx_1 \int_0^{x_1} dx_2 \frac{x_2}{1 + (y_W - 1)x_1 + (y_t - y_W)x_2^2}.
\]

(23)

Because \( C_{WH_6} = 0 \) when \( m_{u,c,t} = 0 \), it is expected that \( C_{WH_6} \) with \( m_t \neq 0 \) can be expressed in terms of the difference between the \( m_t = 0 \) and \( m_t 
eq 0 \) cases. From the identity of \((V^T g)_{ii} = 0\), we can see that \( \sum_{k'} v_{k'd} g_{k'1} = -v_{td} g_{31} \), and \( \sum_{k'} V_{k's} g_{k'2} = -V_{ts} g_{32} \), where \( k' \) denotes the flavors of the first two generations. Accordingly, \( C_{WH_6} \) in Eq. (22) can be formulated as:

\[
C_{WH_6} = -8 y_W h_{33} \left( I_{WH_6}^0(y_W) + I_{WH_6}^t(y_W, y_t) - 2 I_{WH_6}^t(y_W, y_t) \right),
\]

(24)

with

\[
I_{WH_6}^0(y_W) = I_{WH_6}(y_W, 0, 0), \\
I_{WH_6}^t(y_W, y_t) = I_{WH_6}(y_W, y_t, 0) = I_{WH_6}(y_W, 0, y_t), \\
I_{WH_6}^{tt}(y_W, y_t) = I_{WH_6}(y_W, y_t, y_t).
\]

(25)
It can be seen that in the case of $m_t \neq 0$, the nonvanished $C_{WH_6}$ is related to $g_{32}^* g_{31}$, and the relationship to $(y^+ y)_{ij}$ can be obtained as:

$$
g_{32}^* g_{31} = 4(yV)^*_{32}(yV)_{31}
\approx 4 \left( (|y_{31}|^2 - |y_{32}|^2) \lambda + y_{31}^* y_{32} \lambda^2 + y_{32}^* y_{31} \right)
\approx 4(y^+ y)_{21} + 4[(y^+ y)_{11} - (y^+ y)_{22}] \lambda,
$$

where $V_{ud, cs} \approx 1$ and $V_{cd} \approx -V_{ud} \approx -\lambda$ are used, and the $\lambda^2$ term has been neglected. If we drop the $\lambda$-related subleading effects, we can obtain $g_{32}^* g_{31} \approx 4(y^+ y)_{21}$.

Using the results in [14], the $K^0 - \bar{K}^0$ mixing matrix element in the diquark model can be expressed as:

$$
\langle \bar{K}^0 | H^{(W+G)H_6}_{\Box} | K^0 \rangle = \frac{G_F^2 (V_{ts} V_{td})^2}{48 \pi^2} m_{W}^2 m_{K}^2 f_{K}^2 \frac{P_{1}^{VLL}}{\alpha_{2}(m_{t})} \left( \frac{\alpha_{s}(m_{H_6})}{\alpha_{2}(m_{t})} \right)^{6/21} (C_{WH_6} + C_{GH_6}),
$$

$$
C_{WH_6} + C_{GH_6} \approx -16 \frac{(y^+ y)_{21}}{g^2 V_{ts} V_{td}} \left[ 2y_W \left( I_{W,H_6}^0 (y_W) + I_{W,H_6}^H (y_W, y_t) - 2 I_{W,H_6}^H (y_W, y_t) \right)
\right. 
\left. + y_t^2 I_{GH_6} (y_W, y_t) \right].
$$

With $m_{H_6} = 1.5$ TeV, the values of the loop functions are estimated to be $I_{W,H_6}^0 \approx 2.93$, $I_{W,H_6}^t \approx 2.02$, $I_{W,H_6}^{tt} \approx 1.67$, and $I_{GH_6} \approx 56.51$. Although $I_{GH_6}$ is much larger than the others, when including the $y_t^2$ factor, the charged-Goldstone-boson contribution is close to that derived from the $W-H_6$ box diagrams. To see the influence of the heavy top-quark, we plot $\epsilon^N_K$ (in units of $10^{-3}$) as a function of $(y^+ y)_{21}$ (in units of $10^{-3}$) in Fig. 3, where the dashed and dotted lines denote the results from $H_{\Box}^{WH_6}$ and $H_{\Box}^{GH_6}$, respectively, and the solid line is the combined results. A comparison with the results in Fig. 2(a) shows that the constraint on $(y^+ y)_{21}$ from $H_{\Box}^{(W+G)H_6}$ is as strict as that from $H_{\Box}^{H_6}$ through $\Delta M_K$.

That is, $W(G) - H_6$ box diagrams indeed are important in the color-sext diquark model when the heavy top-quark mass effects are included. We noted that the pure $H_6$-mediated box diagrams are insensitive to the $m_t$ effects. We checked that the difference between $m_t = 0$ and $m_t = 165$ GeV is only around 5%; therefore, we did not reanalyze the $H_{\Box}^{H_6}$ contributions with $m_t \neq 0$.

IV. $\epsilon'/\epsilon$ AND $K \rightarrow \pi \nu \bar{\nu}$

In this section, we study the $H_6$ contributions to the Kaon direct CP violation and to the rare $K \rightarrow \pi \nu \bar{\nu}$ decays, where the relevant Feynman diagrams are sketched in Fig. 4 and
FIG. 3: \( W\cdot H_6 \) (dashed) and \( G\cdot H_6 \) (dotted) box diagrams contributing to \( \epsilon_K \) (in units of \( 10^{-3} \)), where the solid line denotes the results of \( \mathcal{H}_{\text{Box}}^{(W+G)H_6} \), and \( m_t = 165 \text{ GeV} \) and \( m_{H_6} = 1.5 \text{ TeV} \) are used.

The current experimental data are:

\[
Re(\epsilon'/\epsilon)_{\text{exp}} = (1.62 \pm 2.3) \times 10^{-4} \quad \text{and} \quad BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{exp}} = (1.7 \pm 1.1) \times 10^{-10}
\]

Since the SM prediction on \( \epsilon'/\epsilon \) is inconsistent with the experimental data by a 2\( \sigma \) deviation, studies of the anomaly using the new physics effects can be found in [9, 16–38].

In addition to the tree Feynman diagram, the \( K \rightarrow \pi\pi \) decays can be induced from the \( Z \) penguins. Since the involved Yukawa couplings for the penguin diagrams are associated with \( g_{12}^* g_{11} \), in which \( g_{12}^* g_{11} \) is the same as that appearing at the tree level, and \( g_{32}^* g_{31} \) is dominated by the real part, it is expected that the \( Z \)-penguin contribution to \( \epsilon'/\epsilon \) will be smaller than the tree Feynman diagram; therefore, we neglect the loop contributions to \( \epsilon'/\epsilon \).

Because neutrinos only couple to the \( Z \) boson, the \( Z \)-penguin diagram can have a significant influence on the rare \( K \rightarrow \pi\nu\bar{\nu} \) decays. Since light quarks with \( m_q \approx 0 \) have no contributions to the \( Z \)-penguin, we only need to consider the top-quark contribution to
FIG. 4: (a) $H_6$-mediated Feynman diagram for the $d \to s\bar{u}u$ decay. (b) $Z$-penguin diagram for $d \to sZ^*$ through the intermediate of $H_6$.

As a result, the involved Yukawa coupling is $g^*_5g_{31} \approx 4(y^\dagger y)_{21}$ and can only have a sizable influence on the CP-conserving $K^+ \to \pi^+\nu\bar{\nu}$ decays. Hence, in the following analysis, we numerically show the $H_6$ contributions to $\epsilon'/\epsilon$ and $K^+ \to \pi^+\nu\bar{\nu}$, which arise from Fig. 4(a) and (b), respectively.

1. $\epsilon'/\epsilon$ from the tree diagram

Based on the Yukawa interactions in Eq. (3), the effective Hamiltonian for $d \to s\bar{u}u$ can be expressed as:

$$H_{H_6}(d \to s\bar{u}u) = -G_F V^*_t s V^*_d \sqrt{2} C_{H_6}^T (Q_1 + Q_2),$$

where the effective Wilson coefficient $C_{H_6}^T$ at the $\mu = m_{H_6}$ scale and the effective operators are given as:

$$C_{H_6}^T = \frac{y_W}{2} \frac{g^*_5 g_{11}}{g^2 V^*_t s V^*_d},$$

$$Q_1 = (\bar{s}d)_{V-A}(\bar{u}u)_{V-A},$$

$$Q_2 = (\bar{s}u)_{V-A}(\bar{d}u)_{V-A}.$$

Using $g = 2yV$, we obtain:

$$g^*_5g_{11} = |y_{12}|^2 V^*_{ts} V_{td} + |y_{13}|^2 V^*_{ts} V_{td} + (y^\dagger y)_{23} V^*_{cs} V_{td} + (y^\dagger y)_{23} V^*_{ts} V_{cd}.$$

It can be seen that if we drop the small $Im(V^*_{ts} V_{td})$ effects, the sizable CP violating effect arises from $(y^\dagger y)_{23} V^*_{cs} V_{td}$.

The direct CP violating parameter from new physics in $K$ system can be estimated by [4]:

$$Re\left(\frac{\epsilon'}{\epsilon}\right) \approx -\frac{\omega}{\sqrt{2}|\epsilon_K|} \left[ ImA_0 - \frac{ImA_2}{ReA_0} \right],$$

(33)

14
where $\omega = ReA_2/ReA_0 \approx 1/22.35$ denotes the $\Delta I = 1/2$ rule, and Eq. (33) is only related to the ratios of hadronic matrix elements. In order to consider the hadronic effects, we employ the SM results as [4]:

$$ReA^\text{SM}_0 \approx \frac{G_F V_{ts} V_{td}^{\ast}}{\sqrt{2}} z_-(Q_-)_0 (1 + q_T),$$

$$ReA^\text{SM}_2 \approx \frac{G_F V_{ts} V_{td}^{\ast}}{\sqrt{2}} z_+(Q_+)_{2},$$

(34)

where the operators $Q_+$ and $Q_-$ are defined as:

$$Q_+ = \frac{1}{2} (Q_2 + Q_1), \quad Q_- = \frac{1}{2} (Q_2 - Q_1);$$

(35)

$q_T = z_+(Q_+)_{0}/(z_-(Q_-)_0) \lesssim 0.1$ [4]; $z_\pm = z_2 \pm z_1$, and the values of $z_{1,2}$ at $\mu = m_c$ are $z_1 = -0.409$ and $z_2 = 1.212$.

According to Eq. (33), the isospin decay amplitudes for $K \rightarrow \pi\pi$ through the intermediate of $H_6$ can be written as:

$$A_{H_6}^{0(2)} = -\frac{G_F V_{ts} V_{td}^{\ast}}{\sqrt{2}} C_{H_6}^{T} 2(Q_+)_{0(2)}.$$

(36)

If we take $ReA^\text{SM}_{0(2)} \approx ReA^\text{exp}_{0(2)}$ from Eq. (33), the Kaon direct CP violation can then be expressed as:

$$Re \left( \frac{\epsilon'/\epsilon}{\epsilon} \right)^T_{H_6} \approx T^{(1/2)}_{H_6} - T^{(3/2)}_{H_6}$$

(37)

$$T^{(1/2)}_{H_6} = 0.705 \frac{r_1 w y_{\mu} q_T}{z_+ (1 + q_T)} \text{Im} \left( \frac{4(y^{\dagger}y)_{23} V_{td}}{g^2} \right),$$

$$T^{(3/2)}_{H_6} = 0.705 \frac{r_1 w y_{\mu}}{z_+} \text{Im} \left( \frac{4(y^{\dagger}y)_{23} V_{td}}{g^2} \right),$$

(38)

where the factor of 0.705 is the renormalization group running effect from $\mu = m_{H_6}$ to $\mu = m_c$. Due to the small $q_T$ factor, the $T^{(1/2)}_{H_6}$ contribution is smaller than $T^{(3/2)}_{H_6}$. Since $Re(\epsilon'/\epsilon)^T_{H_6}$ is linearly proportional to $(y^{\dagger}y)_{23}$, we show the values of $Re(\epsilon'/\epsilon)^T_{H_6}$ (in units of $10^{-3}$) with the benchmarks referenced in Table II. It can be seen that the results can be as large as the SM prediction.

Examining the results in Fig. 2(b), it can be found that the upper bound of $|(y^{\dagger}y)_{23}|$ is around 0.05, where the constraint is singly from $(y^{\dagger}y)_{23}$ in $H_{Box}^{H_6H_6}$. Indeed, a larger $|(y^{\dagger}y)_{23}|$ is allowed when we combine the contributions of $H_{Box}^{H_6H_6}$ and $H_{Box}^{W+G}$ to $\epsilon_K$. For clarity, we show the contours for $\epsilon^H_{K}$ (in units of $10^{-3}$) as a function of $(y^{\dagger}y)_{23}$ and $(y^{\dagger}y)_{21}$ in Fig. 5(a).
TABLE II: $Re\left(\frac{\epsilon'}{\epsilon}\right)^T_{H_6}$ with the taken benchmarks of $(y^\dagger y)_{23}$.

| $(y^\dagger y)_{23}$ | 0.05 | 0.07 | 0.09 | 0.10 |
|---------------------|------|------|------|------|
| $Re\left(\frac{\epsilon'}{\epsilon}\right)^T_{H_6}(10^{-3})$ | 0.25 | 0.34 | 0.44 | 0.49 |

It can be seen that when $|\epsilon_K^{NP}| \leq 0.4 \times 10^{-3}$ is satisfied, $(y^\dagger y)_{23}$ can be slightly larger than 0.05. However, from the plot, we also find that the range of $|(y^\dagger y)_{21}|$ is enhanced and exceeds the limit that is shown in Fig. 2(a). In order to examine that if the allowed range of $(y^\dagger y)_{21}$ can be extended when the $(y^\dagger y)_{21,11,22,31}$ effects are simultaneously considered, we scan the parameters to find the allowed values, which satisfy the condition of $\Delta M^{NP} \leq 0.2\Delta M^{exp}_K$, in the regions:

$$(y^\dagger y)_{11,22} = [0, 0.08], (y^\dagger y)_{21} = [-0.01, 0.01], (y^\dagger y)_{31} = [-0.05, 0.05].$$ (39)

Using $3 \cdot 10^4$ sampling data points, the correlation between the constrained $(y^\dagger y)_{11}$ and $(y^\dagger y)_{21}$ is given in Fig. 5(b). From the plot, it can be seen that $(y^\dagger y)_{21} < -0.005$ is allowed.

![Contour Plot](image_a.png)  ![Correlation Plot](image_b.png)

FIG. 5: (a) Contours for $\epsilon_K^{H_6}$ (in units of $10^{-3}$) as a function of $(y^\dagger y)_{21}$ and $(y^\dagger y)_{23}$. (b) Correlation between the constrained $(y^\dagger y)_{21}$ and $(y^\dagger y)_{11}$ when the condition of $\Delta M^{NP}_K \leq 0.2\Delta M^{exp}_K$ is satisfied.
2. $K \rightarrow \pi \nu \bar{\nu}$ from the $Z$-penguin

Using the $d \rightarrow sZ^*$ result, which is induced from the color-triplet quark and was obtained in [9], the Lagrangian for $d \rightarrow sZ^*$ through the $H_6$ loop can be given as:

$$
\mathcal{L}_{d \rightarrow sZ^*} = -\frac{g}{\cos \theta_W} C_{dZ}^sd \tilde{s} \gamma_\mu P_L d Z^\mu + \text{H.c.} ,
$$

where $\theta_W$ is the Weinberg’s angle and $C_{dZ}^sd$ is given as:

$$
C_{dZ}^sd = \frac{g^2 g_3^1}{(4\pi)^2} I_Z(y_t) ,
$$

$$
I_Z(y_t) = -\frac{y_t}{1-y_t} - \frac{y_t \ln y_t}{(1-y_t)^2} .
$$

According to the SM $Z$-boson coupling to the neutrinos, the effective Hamiltonian for $d \rightarrow s\nu \bar{\nu}$ can be expressed as:

$$
\mathcal{H}_{d \rightarrow s\nu \bar{\nu}} = \frac{G_F \lambda_t^{sd} \alpha}{\sqrt{2}} C_{\nu}^ds \tilde{s} \gamma_\mu P_L d \bar{\nu} \gamma_\mu (1-\gamma_5) \nu + \text{H.c.} ,
$$

$$
C_{\nu}^sd = \frac{X_t^{H_6}}{\sin^2 \theta_W} = \frac{1}{2 \sin^2 \theta_W} \frac{g^2 g_3^1}{g^2 X_t^{sd}} I_Z(y_t) ,
$$

with $\lambda_t^{qs} = V_{q's}^d V_{q'd}$.

Using the formula shown in [29], the BR for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ from the $H_6$ and SM contributions can be formulated as:

$$
BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \frac{\kappa_+ (1+\Delta_{EM})}{\lambda_{10}} \left[ |Re \left( \lambda_t^{cs} X_c + \lambda_t^{ds} X_t \right) |^2 + |Im \left( \lambda_t^{ds} X_t \right) |^2 \right] ,
$$

where $\Delta_{EM} = -0.003$; $X_c = \lambda_t^{cs} P_c(X) \approx 0.404 \lambda^4$ denotes the charm-quark contribution [39, 40]; $X_t = X_t^{SM} + X_t^{H_6}$ with $X_t^{SM} \approx 1.481$, and $\kappa \approx 5.173 \times 10^{-11}$ [16]. Accordingly, we show $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ as a function of $(y^t y)_{21}$ in Fig. [5]. Since a negative $(y^t y)_{21}$ leads to constructively interfere with the SM contribution due to $V_{ts} < 0$, it can be seen that $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ is sensitive to the negative $(y^t y)_{21}$ and it can reach the value of current experimental measurement. Although $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ can be also sensitive to the positive $(y^t y)_{21}$, its magnitude has to be somewhat large and $(y^t y)_{21} > 7 \times 10^{-3}$, where according to Fig. [5](a), the region is excluded by the $\epsilon_K$ constraint.

V. SUMMARY

We studied the diquark contributions to the rare $K$ processes, including $\Delta M_K$, $\epsilon_L$, $\epsilon' / \epsilon$, and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. We showed that the box diagrams mediated by the $W$ and $H_6$ bosons
vanish when the limit of $m_t = 0$ is taken and the flavor mixings $V_{L}^{u,d}$ are introduced. However, when the heavy top-quark mass is applied, it was found that the nonvanished $W(G) - H_6$ box diagrams can be significantly enhanced and can be as large as the pure $H_6$ box diagrams.

Due to the CKM hierarchical structure, it was found that $\Delta M_K$ gives the most strict constraints on the parameters with the exception of $(y^\dagger y)_{23}$, which the $\epsilon_K$ gives the strongest constraint.

With the bounded parameters, the Kaon direct CP violation $\epsilon'/\epsilon$ mediated by the $H_6$ at the tree level can reach $Re(\epsilon'/\epsilon) \sim 0.4 \times 10^{-3}$. Although the $H_6$-mediated $Z$-penguin contribution to $K_L \to \pi^0 \nu \bar{\nu}$ is small in our scheme, the contribution to $K^+ \to \pi^+ \nu \bar{\nu}$ can fit the current experimental upper bound.

**Appendix A: CP phases of the diquark Yukawa couplings**

We studied the $H_6$ effects with the assumption of a real $y$ matrix. However, it is of interest to examine how many physical CP phases involve in $y$ when $y$ is taken as a complex matrix. To explore the question, before EWSB, we can choose the quark states in a way that the SM Higgs Yukawa couplings to the up-type quarks are diagonalized and expressed as:

$$Y_{\text{dia}}^u = V_L^u Y_u V_R^{u\dagger},$$  \hspace{1cm} (A1)
where $Y^u$ denotes the Yukawa matrix; $Y^u_{\text{dia}}$ is a real and diagonal matrix, and $V^u_{L,R}$ are the unitary matrices. Since $Y^u$ is an arbitrary $3 \times 3$ complex matrix, we have freedom to choose the quark basis such that $Y^u$ is diagonalized before EWSB.

In this basis, Eq. (1) can be written as:

$$-\mathcal{L}_Y = u^T_L C f H^\dagger_6 d_L + H.c.,$$  \hspace{1cm} (A2)$$

with $f = \begin{pmatrix} 2V^u_L y V^u_L^\dagger \end{pmatrix}$. Since the down-type quarks are not the physical eigenstates, we can rotate the down-type quark states through the $d_L \to V^u_L^d d_L$ transformation; then, $f$ is an antisymmetric matrix and can represent $y$. From above analysis, it can be seen that choosing a proper basis, $y$ is still an antisymmetric matrix when $Y^u$ is diagonalized before EWSB. In the following discussions, we use $f$ and $Y^u_{\text{dia}}$ instead of $y$ and $Y^u$.

Next, we analyze how many physical CP phases exist in $f$. Set the complex elements of $f$ as: $f_{12} = |f_{12}| e^{i\theta_{12}}$, $f_{13} = |f_{13}| e^{i\theta_{13}}$, and $f_{23} = |f_{23}| e^{i\theta_{23}}$, where $\theta_{12,13,23}$ are independent CP phases. In order to rotate the unphysical CP phases away, we transform the quark states as $u_{L,R} \to V u_{L,R},$ and $d_L \to V d_L$ with $V(\theta) = \text{diag}(1, e^{i\theta_2}, e^{i\theta_3})$, where we remove one overall phase from $V_{11}$. Then, the $Y^u_{\text{dia}}$ and $y$ are respectively transformed as $Y^u_{\text{dia}} \to Y^u_{\text{dia}}$ and

$$f' = V^*(\theta) f V(\theta) = \begin{pmatrix} 0 & f_{12} e^{-i\theta_2} & f_{13} e^{-i\theta_3} \\ f_{21} e^{-i\theta_2} & 0 & f_{23} e^{-i(\theta_2 + \theta_3)} \\ f_{31} e^{-i\theta_3} & f_{32} e^{-i(\theta_2 + \theta_3)} & 0 \end{pmatrix}.$$  \hspace{1cm} (A3)$$

Since $\theta_{2,3}$ are free parameters, we can rotate away the phases of $f_{12}$ and $f_{23}$ requiring $\theta_{12} = \theta_2$ and $\theta_{13} = \theta_3$. Since $\theta_2$ and $\theta_3$ are fixed, the phase of the $f'_{23}$ element is $f'_{23} = e^{-i(\theta_2 + \theta_3)} = |f_{23}| e^{i(\theta_{23} - \theta_{2} - \theta_{3})}$ and cannot be rotated away. Hence, we conclude that before EWSB, there is only one unrotated CP phase in $f$.

Based on the basis that leads to $Y^u_{\text{dia}}$, we now consider the flavor mixings after EWSB. We can introduce the $U^d_{L,R}$ unitary matrices to diagonalize the down-type quark Yukawa matrix as:

$$Y^d \rightarrow U^d_{L} Y^d U^d_{L}^\dagger = Y^d_{\text{dia}}.$$  \hspace{1cm} (A4)$$

Since up-type quark Yukawa matrix has been diagonalized, we have $U^u_{L,R} = 1$. Accordingly, the CKM matrix can be expressed as $V_{\text{CKM}} = U^u_L U^d_L = U^d_L$. Thus, in terms of mass eigenstates, Eq. (A2) can be written as:

$$-\mathcal{L}_Y = u^T_L C f V_{\text{CKM}} H^\dagger_6 d_L + H.c.$$  \hspace{1cm} (A5)$$
Clearly, $fV_{\text{CKM}}$ is not an antisymmetric matrix. So far, $V_{\text{CKM}}$ is a generic unitary matrix and the unphysical phases have not yet been removed. From the SM charged weak interactions, which is written as $\bar{u}_L\gamma_\mu V_{\text{CKM}}d_LW^{+\mu}$, we can see that five of six phases in $V_{\text{CKM}}$ can be removed by redefining the phases of up- and down-type quarks. If we choose that up- and down-type quarks absorb two and three CP phases from $V_{\text{CKM}}$, respectively, from Eq. (A5), it can be seen that the two absorbed CP phases from the up-type quarks will flow to the diquark Yukawa coupling as $f'' = V^*(\phi)fV^\dagger(\phi)$ with $V(\phi) = (1, e^{i\phi_2}, e^{i\phi_3})$, where the matrix form of $f''$ is the same as that in Eq. (A3) and has three CP phases, i.e., $\phi_2$, $\phi_3$, and $\theta_{23} - \theta_2 - \theta_3$. Again, one of the three CP phases is a global phase and can be absorbed to the up-quark states. Hence, if $V_{\text{CKM}}$ in Eq. (A5) carries one physical CP phase, the associated $f$ matrix in principle can have two independent CP phases.

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