Effective Restoration of the $U_A(1)$ symmetry in the SU(3) linear $\sigma$ model

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The effective restoration of the chiral $U_A(1)$ symmetry in strong interactions is studied using the linear chiral SU(3)$\times$SU(3) model at finite temperatures. We find that the disappearance of the chiral anomaly causes a considerable change in the meson mass spectrum. We propose several signals for detecting this chiral phase in ultrarelativistic heavy-ion collisions: The $\eta/\pi^0$ ratio is enhanced by an order of magnitude, the $a_0$ is suppressed in the $K\bar{K}$ mass spectrum, and the scalar $\kappa$ meson appears as a peak just below the $K^-(892)$ in the invariant $\pi K$ mass spectrum.

25.75.-q, 12.39.Fe, 11.30.Rd

Probing the phase transition of strongly interacting matter at finite temperature is one of the primary goals of ultrarelativistic heavy-ion collisions. At BNL’s Relativistic Heavy-Ion Collider (RHIC) the central collision zone of the two gold ions may create temporarily hot matter, presumably similar to what is studied numerically in lattice gauge calculations. Present simulations indicate that the phase transition seen in hot quantum chromodynamics (QCD) coincides with the chiral phase transition. But the fundamental question of which chiral symmetry is restored in hot QCD is not settled yet. There are two scenarios according to Shuryak: only SU(2) chiral symmetry is restored in hot QCD is probably not fully restored at $T_c$ but that the $a_0 - \pi$ mass splitting drops drastically. Simulations with domain wall fermions demonstrate that anomalous chiral symmetry breaking effects are at or below the 5% level above but close to $T_c$. Note that all the above results consistently indicate that effects from the $U_A(1)$ breaking are strongly suppressed, i.e. an effective restoration of the $U_A(1)$ symmetry close to $T_c$.

Recently, the issue of finding signals for the restoration of chiral symmetry in ultrarelativistic heavy-ion collisions has received considerable attention. E.g. signals for the restoration of the SU(2) chiral symmetry associated with the $\sigma$ meson have been proposed in $[21,22]$. In particular, signals for the partial restoration of the $U_A(1)$ symmetry in connection with the $\eta'$ meson have been invoked in $[10,24,25]$. Effects of the $U_A(1)$ anomaly on meson masses have been also studied within the SU(3) Nambu–Jona-Lasinio model by Kunihiro $[13,17]$.

Here, we are going to study the full SU(3) linear $\sigma$ model at finite temperature including effects from the effective restoration of the $U_A(1)$ symmetry. Our aim is to extract signals from the strong suppression of $U_A(1)$ breaking effects in a chirally restored phase. The additional scalar mesons in the SU(3) $\sigma$ model compared to the standard SU(2) $\sigma$ model provides novel signals for the effective restoration of the $U_A(1)$ symmetry: there will be an enhancement of the number of $\eta$ mesons due to a feedback from the decay of $a_0$ mesons, its chiral partner, which will be suppressed then in the $K\bar{K}$ mass spectrum, and there appears a new scalar resonance in the invariant $\pi K$ mass spectrum, stemming from the chiral partner of the kaon, the $\kappa$ meson.

The inclusion of the strangeness degree of freedom, i.e. going from SU(2) to SU(3), finds its justification in the recent findings that the strange quark mass is about $m_s = 100 - 150$ MeV, close to half the expected temperatures at RHIC of $T \approx 200 - 300$ MeV in the initial stage of the collision. Therefore, strangeness has to be included in the linear $\sigma$ model for studying hot matter relevant to physics at RHIC.

The SU(3) linear $\sigma$ model is known for a long time $[20]$. Only recently, more than thirty years later, there
has been a renaissance of this chiral Lagrangian. The
resurrection of the $\sigma$ meson \cite{30} and the finding of the
$\kappa(900)$ resonance in the $\pi K$ scattering data \cite{31,32}
leads to the conclusion that there is a low-mass scalar nonet \cite{33}. Jaffe predicted this scalar nonet long ago \cite{34}
as being built out of $(qqqq)$ states with an inverted mass spec-
trum compared to the pseudoscalar meson spectrum. Recent
lattice simulations are supporting this picture \cite{36}. It is
interesting to note that the inversion of the mass spectrum is
implemented in the SU(3) linear $\sigma$ model by the
anomaly term which has the opposite sign for the
pseudoscalar and scalar masses. The model Lagrangian
gives a surprisingly good agreement with the data already
at tree-level \cite{33,34}. The model was also extended to fi-
nite temperatures \cite{35,36} without implementing effects
from the effective $U_A(1)$ symmetry restoration.

Let us now write down the SU(3)$\times$SU(3) chiral La-
grangian:

$$
\mathcal{L} = \frac{1}{2} \Tr \left( \partial_{\mu} \Phi^\dagger \partial^{\mu} \Phi + \mu^2 \Phi^\dagger \Phi \right) - \lambda \Tr (\Phi^\dagger \Phi)^2
- \lambda' (\Tr \Phi^\dagger \Phi)^2 + c \cdot \left( \det \Phi + \det \Phi^\dagger \right) + \epsilon \cdot \sigma + \epsilon' \cdot \zeta .
$$

(1)

where $\Phi$ is a 3x3 complex matrix describing the pseudo-
scalar and scalar nonet. The term proportional to the
determinant breaks the $U_A(1)$ symmetry. The last two
terms break chiral symmetry explicitly, which are sim-
ulating effects from a finite light quark and a strange
quark mass, respectively. There are two order param-
ters corresponding to the light quark condensate $\sigma$ and
the strange quark condensate $\zeta$. Chiral symmetry is re-
stored at a certain temperature as the $\sigma$ order parameter
drops towards zero (and effects from the explicit symme-
try breaking term $\epsilon \cdot \sigma$ can be ignored). Then the masses
of the pion and the sigma mesons and the masses of the
$\eta$ and the $a_0$ meson are the same separately. Their mass
gap is proportional to the coefficient $c$ of the anomaly
term times the strange order parameter $\zeta$:

$$
m_\pi = m_\sigma < m_{a_0} = m_\eta , \quad \Delta m = 4c \cdot \zeta .
$$

(2)

If the chiral $U_A(1)$ symmetry is effectively restored, then
this mass gap vanishes and all four meson masses are the
same irrespective of the value of $\zeta$:

$$
m_\pi = m_\sigma \approx m_{a_0} = m_\eta \quad \text{for} \ c \approx 0 .
$$

(3)

Note that this is only the case for a (effective) restora-
tion of the $U_A(1)$ symmetry ($c \approx 0$) as the strange
order parameter $\zeta$ will not decrease strongly at $T_c$ due to the
finite strange quark mass \cite{34}.

We assume now that the coefficient of the anomaly term $c$
drops as a function of temperature. As a guide-
line we take the temperature effects to be proportional
to the topological susceptibility in pure glue theory as
suggested by the Witten-Veneziano formula \cite{17}. We
use the lattice data as published in \cite{12} so that the co-
efficient is nearly constant until $T_c$ and drops then by

an order of magnitude (but is not vanishing). We take
the critical temperature to be $T_c = 150$ MeV as deduced
from recent investigations on the lattice (see \cite{2} for a
summary). Thermal excitations for all pseudoscalar and
scalar fields are taken into account in a Hartree scheme in
a selfconsistent way. The gap equations for the two order
parameters are solved together with the expressions for
the meson masses and the thermal excitations iteratively
until convergence is achieved.

The meson masses as a function of temperature are
shown in Figs. 1 and 2. The most striking feature com-
pared to a calculation with a constant anomaly coefficient
is that the phase transition is shifted to lower tempera-
tures. For a constant coefficient, the phase transition is
much less pronounced and happens at $T_c \approx 210$ MeV
\cite{10}. Here, the chiral phase transition happens precisely
at the same temperature at which the anomaly coefficient
is put to drop down, i.e. at $T_c = 150$ MeV. Remarkably,
the restoration of $U_A(1)$ symmetry shifts the value of
the critical temperature and strengthens the chiral phase
transition which was also seen within the Nambu–Jona-
Lasinio model \cite{26}. The $U_A(1)$ symmetry is restored at
$T \approx 250$ MeV which can be read off of Fig. 1 by the
degeneracy of the chiral partners $\pi - \sigma$ and $\eta - a_0$. The
overall meson mass spectrum changes significantly across
the phase transition. The mass of the $\eta'$ drops consid-
ernably to $m_{\eta'} \approx 650$ MeV and is then approxi-
mately degenerate with the kaon mass above $T_c$. The
pseudoscalar mixing angle is ideal above $T_c$ due to the
smallness of the anomaly term. This means in principle
that the $\eta'$, as the chiral partner of the $a_0$, is a purely
light quark system while the $\eta$ becomes purely strange.
Nevertheless, it is apparent from Fig. 2 that a level cross-
ing of the $\eta$ and the $\eta'$ masses occurs around $T_c$ (see also
\cite{2,26}). Hence, the $\eta$ and $\eta'$ are switching their identity
at $T_c$. The $\eta$ is now the chiral partner of the $a_0$ and is
the nonstrange state while the $\eta'$ is the pure strange
quark state. Surprisingly, the $\eta'$ mass is even slightly smaller
than the single strange quark state, the kaon, which is an
interesting nontrivial effect originating from the different
thermal contributions for the kaon and the $\eta$ masses.
The number of $\eta'$ mesons will then be enhanced in the
hot medium. In return, the number of $\eta$ mesons will in-
crease then by the decay $\eta' \rightarrow \eta \pi \pi$ at freeze-out which
will be seen by a modified slope parameter in the low
transverse momentum spectra.

A more dramatic effect is associated with the scalar
isosvector meson $a_0$. The free $a_0$ decays mainly to $\eta + \pi$
and to two kaons. The decay to two pions is forbidden
as it violates isospin. As evident from Fig. 1 the $a_0$
mass decreases strongly with temperature, as is it also
seen on the lattice \cite{17,24}. As chiral SU(3) symmetry
is restored at $T_c$, its mass gets degenerate with the $\eta$
mass, its chiral partner. In addition, as $U_A(1)$ is effec-
tively restored, the $a_0$ and $\eta$ masses will be close to the
pion mass just above $T_c$ (as seen in Fig. 1). Hence, already
below $T_c$ the decay $a_0 \rightarrow \eta + \pi$ must be blocked by phase space just when the mass difference of the $a_0$ and the $\eta$ equals the thermal pion mass. Also, the decay matrix element of $a_0 \rightarrow \eta + \pi$ is considerably reduced above $T_c$ as it is proportional to the $\sigma$ order parameter and the anomaly term. The decay to two kaons is heavily suppressed as the $a_0$ is actually lighter than one kaon alone if $U_A(1)$ is effectively restored (see Fig. 1) compared with Fig. 2. Hence, the inelastic channels for the $a_0$ are closed above $T_c$. The elastic channels are still large as they are proportional to the coupling constant $\lambda$ which is of the order of 10. We conclude that there is chemical equilibrium between the $a_0$, $\eta$, $\sigma$ and pion in the chiral phase as it is $U(2) \times U(2)$ symmetric. This argument is supported by the work of Song and Koch [21] who find that the $\sigma$ and pion mesons are in chemical equilibrium in the SU(2) linear $\sigma$ model.

For detecting this chiral symmetric phase in ultrarelativistic heavy–ion collisions, the expansion stage of the hot and chemically equilibrated matter must be short and/or out of equilibrium. If the expansion is adiabatically, the system adjusts itself, freezes out like a free gas and no effect will be visible. A signal will show up, if either the system expands from the chiral phase above $T_c$ until the freeze-out temperature $T_f$ faster than the lifetime of the $a_0$ of about $2 - 4$ fm or if the chemical freeze-out happens before the thermal freeze-out [22]. Then the number of $a_0$'s is approximately conserved during the expansion and the numbers of $a_0$'s at freeze-out will be approximately equal to the numbers of pions and three times the number of $\eta$ mesons (due to isospin counting). The $a_0$ will then mainly decay to $\eta + \pi$ increasing the numbers of the $\eta$ mesons drastically. Taking into account the decays $a_0 \rightarrow \eta + \pi$ and $\sigma \rightarrow 2\pi$, we get the ratio $\eta/\pi^0 = (3n_{a_0} + n_\eta)/(n_{a_0} + 2/3n_\sigma + n_{\pi^0}) = 3/2$ as a signal of the formation of an $U(2) \times U(2)$ symmetric phase. Hence, it is even possible to produce more $\eta$ mesons than $\pi^0$'s. The two-kaon decay channel is suppressed at finite temperature due to the larger mass of the kaon and the smaller mass of the $a_0$ at $T_f$ [22]. This decay is a sub-threshold process even at $T = 0$ so that a slight change in the masses will reduce the branching ratio.

We discuss now another observable related to the effective restoration of the $U_A(1)$ which is associated with the $\kappa(900)$ resonance. The $\kappa$ meson is very broad similar to the $\sigma$ meson [24] and decays to a pion and a kaon. Its mass depends strongly on the $U_A(1)$ anomaly and is decreasing with temperature as shown in Fig. 2. The decay width depends on the coefficient of the anomaly term and decreases therefore in the chiral $U_A(1)$ phase. We find at tree-level that the width changes from values around $\Gamma \approx .8$ GeV to $\Gamma \approx .2$ GeV when setting the contribution from the anomaly term to zero. Hence, the barely visible broad resonance gets a much smaller width in the chiral $U_A(1)$ phase and can possibly be seen. As indicated by Fig. 2 the mass of the $\kappa$ approaches that of the kaon towards chiral symmetry restoration so that the strong decay $\kappa \rightarrow K + \pi$ is blocked by phase space. This happens already below $T_c$ similar as for the $a_0$. The $\kappa$ meson can then be visible in the invariant $\pi K$ mass spectrum, if the system freezes out dominantly around $T_c$. Chiku and Hatsuda have demonstrated this effect in connection with the $\sigma$ meson appearing in the $\pi \pi$ channel [22]. The decay channel $\kappa \rightarrow \pi + K$ opens just below $T_c$ so that there will be a pronounced cusp structure in the corresponding spectral function as depicted in fig. 3. The $\kappa$ resonance will then emerge in the $\pi K$ invariant mass spectrum around 850 MeV. For the $\pi K$ mass spectrum, there is a only one background in that mass region, which is from the vector kaon, the $K^*(892)$. Vector meson masses go up with temperature if studied in a SU(2) gauged chiral Lagrangian [25] as they have to be degener-
erate with their heavier axial vector chiral partners. A study in a SU(3) linear σ model with vector mesons shows that the $K^*(892)$ mass stays approximately constant until $T_c$ and effectively rises then for higher temperatures [44]. Hence, the background from $K^*(892)$ should be comparably low.

The appearance of the $\kappa$ in the $\pi K$ spectra can be detected by two particle correlation at BNL’s RHIC. Here, the same techniques employed for reconstructing the $\rho$ in the $\pi\pi$ mass spectrum [43] can be utilized. The STAR group, as well as BRAHMS and PHOBOS, will reconstruct $\phi$ mesons in the $KK$ spectra [45] where the $a_0$ will be seen, too. The $\eta$ meson will be measured in the diphoton spectra at the PHENIX detector [46] and the enhancement proposed here can be checked.

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