A SCREENING MECHANISM
FOR EXTRA W AND Z GAUGE BOSONS

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Abstract

We generalize a previous construction of a fermiophobic model to the case of more than one extra $W$ and $Z$ gauge bosons. We focus in particular on the existence of screening configurations and their implication on the gauge boson mass spectrum. One of these configurations allows for the existence of a set of relatively light new gauge bosons, without violation of the quite restrictive bounds coming from the $\rho_{NC}$ parameter. The links with Bess and degenerate Bess models are also discussed. Also the signal given here by this more traditional gauge extension of the SM could help to disentangle it from the towers of Kaluza-Klein states over $W$ and $Z$ gauge bosons in extra dimensions.
Introduction

It has been argued several times in the literature that the Standard Model (SM) should be considered as a low-energy realization of a more fundamental theory, but the lack of evidence for the existence of new particles at LEP1 energies, apart from those predicted by the SM (with the exception of the SM Higgs boson), imposes severe restrictions on its extensions. Moreover, the precision in the measurement of the ratio between the strength of the neutral and charged current couplings, the so-called \( \rho_{NC} \) parameter (NC stands for neutral current) \[1, 2\], rules out models with a too large contribution to this quantity.

At one loop in perturbation theory the contributions to \( \rho_{NC} \) grow at most quadratically in the masses of the scalar particles running in the loop. The quadratic terms can rapidly lead to a disagreement of the prediction with the experimental data \[3\]. Therefore the phenomenon of “screening”, i.e. the exact cancellation at one loop of the contributions to \( \rho_{NC} \), which are quadratic in the mass of the scalar particles, becomes an important ingredient to be fulfilled by any viable alternative to the SM. In general the suppression of the new contributions could also occur numerically, via a fine-tuning of the parameter of the model, but we will not consider this possibility. We want instead to investigate in detail the occurrence of a screening phenomenon, i.e. the conditions to obtain an exact analytical cancellation of the terms quadratic in the masses of the scalar particles. Then we will analyse the implications of this requirement in the gauge boson mass spectrum.

An extension of the SM, based on an \( SU(2)_L \times SU(2)_R \times U(1) \) gauge group, has been presented in a previous paper \[4\]. It has been called fermiophobic (FP) because of the absence of coupling between the fermions and the gauge particles of the extra \( SU(2)_R \) group. The scalar sector of this model is the minimal one sufficient to give mass to the vector bosons and to introduce a non-trivial interaction among them: two doublets and one bidoublet. No triplet is needed owing to the absence of right-handed neutrinos in the model. Although, in general, \( SU(2) \) gauge extensions of the SM are characterized by large deviations in the \( \rho_{NC} \) parameter, two sets of configurations of the parameter space have been found explicitly \[4\], which yield a screening of scalar quadratic contributions. Moreover, the model in \[4\], apart from being anomaly-free and without FCNC at tree level, passes successfully all possible phenomenological checks allowing for the existence of relatively light new gauge bosons. A clear signature of this model would be the discovery of a relatively light extra \( Z' \) mainly coupled to the hypercharge.

Motivated by the good properties shown by the fermiophobic model proposed in \[4\] in passing all phenomenological tests, we have studied a more general extension, based on an \( SU(2)_L \times SU(2)^n \times U(1)_Y \) (with \( n \geq 1 \)) gauge group, focusing in particular on the existence of screening configurations and their implications on the gauge boson mass spectrum.

Although the relation between custodial symmetry and screening is known \[1, 5\], we have preferred to investigate the general expression of \( \rho_{NC} \) directly. We have not invoked \textit{a priori} any extra global symmetry, but we have studied the conditions under which the screening phenomenon occurs and found the corresponding configurations in the parameter space of the model. We will see, in detail, how the requirements imposed by these configurations are so strong that they allow us to get the exact mass spectrum of the new gauge particles. The most remarkable consequence of this model is the possibility of having sets of relatively \textit{light} gauge particles without contradiction with the experimental data regarding \( \rho_{NC} \).

In a series of papers, Casalbuoni et al. \[6\] have constructed a gauge extension of the SM that includes, besides the standard \( \gamma, Z \) and \( W^\pm \) gauge vector bosons, also two new triplets of spin-1
particles. They have called the model degenerate Bess. The interest of that model is mainly in its decoupling properties [4]. The latter originate from an extra global symmetry that the model has when the gauge couplings are turned off and that is also responsible for the degeneracy of the vector bosons masses. The non-linear realization of this model is able to reproduce the Higgs-less SM in the limit of infinite mass of the new vector bosons. A recently proposed linear realization of this model [8] has two interesting limits depending on the choice of the parameters of the scalar potential: in one case it coincides with its non-linear version (degenerate Bess) and, for a different choice, with the SM with a light Higgs field. Obviously this implies that at least at low energies the model shares the phenomenological success of the SM. We have studied the choices in the parameter space necessary to find a contact between our model, Bess and degenerate Bess.

Similar extensions of the SM have already been proposed in the literature [9]. Some of these extensions (with n = 1) appeared in the framework of composite [10] or supersymmetric models [11, 12]. More recently, in the context of extra dimensions [13], the tower of Kaluza-Klein states over the W and Z gauge bosons would resemble an infinite tower of $W^{(n)}$ and $Z^{(n)}$ gauge bosons. The different type of contributions of these states to $\Delta \rho$ and the $\epsilon$ parameters as compared to the ones given here for a more traditional gauge extension of the SM, could help to disentangle both cases.

The paper is organized in the following way. In the first section we present the Lagrangian of the model. In section 2 we give the gauge boson mass matrices in the interacting basis for the charged and neutral sectors where the photon is identified. We then discuss in section 3 the spectrum of scalars: Goldstones and Higgses. Section 4 is devoted to the computation of $\rho_{NC}$. In section 5 we describe the screening configurations and the corresponding mass spectrum for the gauge particles and we also compute the whole set of $\epsilon$ parameters at tree level for the second configuration. We discuss in section 6 the link with degenerate Bess and finally we comment on other phenomenological constraints and give our conclusions.

We will adhere in this paper to most of the conventions used in [4].

1 The model

We consider a Lagrangian based on a gauge group $G = SU(2)_L \times SU(2)^n \times U(1)_Y$. The kinetic and self-interaction terms of the vector bosons are given, for each sub-group of $G$, by the usual field-strength tensor. We introduce a complex scalar doublet $\Phi_j$ for each $SU(2)$ group, and a complex bidoublet $\Psi_{jk}$ for every combination of two distinct $SU(2)$ groups present in $G$ (where $j, k = 0, 1, \ldots, n$ label the corresponding groups). We need at least one complex doublet for each $SU(2)$ group, to give mass to the new gauge vector bosons, and to break completely $G$ down to $U(1)_{em}$. We introduce also the bidoublets because they induce a non-zero mixing among the different $SU(2)$ groups of the theory, also when these groups are characterized by sensibly different energy scales. As in the case of $n = 1$, triplets are not needed.

This choice seems to be a good compromise between the idea of studying the screening in a general kind of scalar sector and the need of introducing the minimal extension sufficient to get an acceptable mass spectrum for all gauge and fermionic particles, and also to generate an interaction between the different $SU(2)$ groups.

The Lagrangian of the Yang-Mills sector is:

$$L_{YM} = -\frac{1}{4} \sum_{j=0}^{n} \sum_{i=1}^{3} \sum_{\mu\nu} G_{j\mu\nu}^{i} G_{ji\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu},$$

(1)
where
\[ G^i_{j\mu\nu} = \partial_\mu W^i_{j\nu} - \partial_\nu W^i_{j\mu} + g_j e^{ilm} W^l_{i\mu} W^m_{j\nu} \quad i, l, m = 1, 2, 3; \quad j = 0, 1, 2, \ldots, n \]
\[ B^\mu_\nu = \partial_\mu B_\nu - \partial_\nu B_\mu \] (2)
are the usual field-strength tensors, \( W^i_{j\mu} \) is the gauge field of the \( j \)th \( SU(2) \) group, while \( B_\mu \) corresponds to the \( U(1)_Y \) group.

The quantum numbers of the scalar multiplets are indicated by a list of \( n+2 \) elements corresponding to the different groups:
\[(0, 1, 2, \ldots, n, Y) \] (3)
and are:
\[(1, \ldots, \frac{j}{2}, \ldots, 1, -\frac{1}{2}) \] for each doublet \( \Phi_j \)
\[(1, \ldots, \frac{j}{2}, \ldots, \frac{k}{2}, \ldots, 0) \] for each bidoublet \( \Psi_{jk} \)
\[j < k \quad \text{and} \quad j, k = 0, 1, 2, \ldots, n, \] (4)
where 1 and the ellipses stand for a singlet behaviour with respect to the corresponding \( SU(2) \) group of the list and the 2 for a doublet. The values \(-\frac{1}{2} \) and 0 are the hypercharge of doublets and bidoublets, respectively.

Starting from these assignments, and therefore from the transformation rules of each field under a gauge transformation, we write the covariant derivatives of the scalar multiplets.
\[ D^\mu \Phi_j \equiv \partial^\mu \Phi_j - ig_j \tilde{\tau} \cdot \tilde{W}^\mu_{j\Phi} + i \tilde{g} B^\mu \Phi_j \] (5)
with \( \Phi_j = \begin{pmatrix} \phi^0_j \\ \phi^j \end{pmatrix} \)
is the covariant derivative of the complex doublet \( \Phi_j \), where \( \tilde{g} \) is the hypercharge coupling constant, \( g_j \) is the coupling constant of the \( j \)th \( SU(2) \) group, and \( \tilde{\tau} \) are the Pauli matrices.
\[ D^\mu \Psi_{jk} \equiv \partial^\mu \Psi_{jk} - ig_j \tilde{\tau} \cdot \tilde{W}^\mu_{jk} \Psi_{jk} + ig_k \Psi_{jk} \tilde{\tau} \cdot \tilde{W}^\mu_{k} \] (6)
with \( \Psi_{jk} = \begin{pmatrix} \varphi^0_{(jk)} \\ \varphi^+_{(jk)} \\ \varphi^-_{(jk)} \\ \varphi^0_{(jk)} \\ \varphi^+_{(jk)} \\ \varphi^-_{(jk)} \end{pmatrix} \)
is the covariant derivative of the complex bidoublet \( \Psi_{jk} \), \( g_j \) and \( g_k \) being respectively the coupling constants of the \( j \)th and of the \( k \)th \( SU(2) \) groups. The ratio between a gauge coupling \( g_j \) and \( g_0 \) will be denoted by
\[ x_j = \frac{g_j}{g_0}, \] (7)
where also \( x_0 = 1 \) and \( x_{n+1} \equiv y = \tilde{g}/g_0 \) is introduced to shorten the notation in the following.

Starting from the definition of these derivatives, we explicitly write the kinetic and the mass terms of the Lagrangian of the scalar sector, leaving the scalar potential generically indicated with \( V(\phi) \). All the fields are given in the interaction basis:
\[ \mathcal{L}_{\text{sc}} = \sum_{j=0}^{n} (D^\mu \Phi_j)^\dagger (D^\mu \Phi_j) + \sum_{j,k=0 \ j<k}^{n} \text{Tr} \left \{ (D^\mu \Psi_{jk})^\dagger (D^\mu \Psi_{jk}) \right \} - V(\phi). \] (8)
We must introduce a gauge-fixing term, and we choose it in the usual way, in order to remove the mixing terms between Goldstone and gauge vector bosons:

\[
\mathcal{L}_{gf} = -\frac{1}{2\xi} \sum_{j=0}^{n} \sum_{i=1}^{3} \left[ \partial^{\mu} W_{j\mu}^i + ig_j \xi \left( \Phi_j^\dagger \frac{\tau^i}{2} \langle \Phi_j \rangle_0 - \langle \Phi_j^\dagger \rangle_0 \frac{\tau^i}{2} \Phi_j \right) \right] + \sum_{k=j+1}^{n} \text{Tr} \left( \Psi_{jk}^\dagger \frac{\tau^i}{2} \langle \Psi_{jk} \rangle_0 - \langle \Psi_{jk}^\dagger \rangle_0 \frac{\tau^i}{2} \Psi_{jk} \right) \right) \right] + \frac{1}{2\xi} \sum_{j=0}^{n} \left[ \partial^{\mu} B_\mu - \frac{i\tilde{g} \xi}{2} \left( \Phi_j^\dagger \langle \Phi_j \rangle_0 - \langle \Phi_j^\dagger \rangle_0 \Phi_j \right) \right] \right] \right]
\]

where

\[
\langle \Phi_j \rangle_0 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} v_j \\ 0 \end{array} \right) \text{ and } \langle \Psi_{jk} \rangle_0 = \frac{1}{\sqrt{2}} \left( \begin{array}{cc} v_{ij} & 0 \\ 0 & v_{jk} \end{array} \right) \]

are the vacuum expectation values (VEVs) of the various doublets and bidoublets and

\[
\Phi_j^\dagger = \Phi_j - \langle \Phi_j \rangle_0, \quad \Psi_{jk}^\dagger = \Psi_{jk} - \langle \Psi_{jk} \rangle_0.
\]

We choose to work in the Landau gauge (\(\xi = 0\)). With respect to fermions, since we assume this model to be fermiophobic as in \([4]\), the unique gauge-invariant coupling that can be constructed between fermions and scalars involves only the \(\Phi_0\) doublet, so the couplings are the same as in the SM. A first positive consequence is that one automatically avoids having FCNC problems at tree level, in contrast to the usual left-right models. Also, no mass term is introduced for the right-handed neutrino.

Finally, the Lagrangian of our model can be obtained by combining these various terms, i.e.

\[
\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_{sca} + \mathcal{L}_{gf}.
\]

2 Gauge boson mass matrices

The kinetic term of the scalar Lagrangian in eq.(8) generates, after the spontaneous symmetry breaking, a mass term for the charged (\(W_j^\pm\) with \(j = 0, 1, \cdots, n\)) and neutral gauge bosons (\(Z_j\) with \(j = 0, 1, \cdots, n\)).

2.1 Charged sector

The charged \(W_j^\pm\) boson mass matrix is an \((n + 1) \times (n + 1)\) symmetric matrix. In a compact way it can be written as

\[
M_{C}^{ij} = \frac{g_{i}^{2}}{4} x_{i-1} x_{j-1} \left( \delta_{ij} v_{i-1}^2 + s(i) - \theta(j - i - 1)2v_1^{(i-1)j-1} v_2^{(i-1)j-1} \right) \quad \text{with} \quad i \leq j
\]

where \(\theta(i)\) stands for the Heaviside function (with \(\theta(0) = 1\), \(u_{ij}^2 = v_{1}^{(i)j} + v_{2}^{(i)j} \) and

\[
s(k) = \sum_{i=0}^{n} u_{ik}^2.
\]

1 \(u_{ij}\) is obviously symmetric under the interchange of its indices.
The mass eigenvalues can be obtained from eq. (13) by performing an orthogonal transformation

$$M^2_C = R_C M^2_C R_T$$

where $M^2_C$ is the charged diagonal mass matrix and $R_C$ is the rotation matrix that transforms the interacting charged $W^\pm_j$ fields into the physical mass eigenstates $W^{P\pm}_j$

$$
\begin{pmatrix}
W^P_0 \\
W^P_1 \\
W^P_2 \\
\vdots \\
W^P_n
\end{pmatrix}
= R_C
\begin{pmatrix}
W^\pm_0 \\
W^\pm_1 \\
W^\pm_2 \\
\vdots \\
W^\pm_n
\end{pmatrix}

(16)

This rotation matrix $R_C$ involves $\frac{n(n+1)}{2}$ mixing angles. The specific form of the rotation matrix $R_C$ and the mass eigenvalues will be given in section 5 for the screening configurations.

### 2.2 Neutral sector

The neutral mass matrix in the interacting basis $(W^3_0, W^3_1, W^3_2, \ldots, W^3_n, B)$ is an $(n+2) \times (n+2)$ matrix given by

$$M^2_{Nij} = \frac{g_0^2}{4} x_{i-1} x_{j-1} \left( \delta_{ij} (v^2_i + s(i)) - \theta(j - i - 1) u^2_{i-1,j-1} \right) \quad \text{with} \quad i \leq j$$

(17)

where, to keep the notation as compact as possible, we have defined

$$u^2_{n+1,i} \equiv v^2_i \quad \text{and} \quad v^2_{n+1} \equiv 0.$$  

(18)

The diagonalization of this mass matrix is done in two steps. First, we perform a finite rotation with a unitary matrix $U$ to identify the photon field. This transforms the matrix of eq. (17) in a block form with the first row and column filled with zeros corresponding to the zero mass eigenvalue of the photon. Afterwards, a second rotation $R_N$ is performed to obtain the mass eigenstates of the $Z_i$ fields.

Accordingly, the transformations from the interacting $(W^3_0, W^3_1, W^3_2, \ldots, W^3_n, B)$ to the mass eigenstates basis $(A, Z^P_0, Z^P_1, Z^P_2, \ldots, Z^P_n)$ are

$$
\begin{pmatrix}
A \\
Z_0 \\
Z_1 \\
\vdots \\
Z_{n-1} \\
Z_n
\end{pmatrix}
= U
\begin{pmatrix}
W^3_0 \\
W^3_1 \\
W^3_2 \\
\vdots \\
W^3_n
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
A \\
Z^P_0 \\
Z^P_1 \\
\vdots \\
Z^P_{n-1} \\
Z^P_n
\end{pmatrix}
= R_N
\begin{pmatrix}
A \\
Z_0 \\
Z_1 \\
\vdots \\
Z_{n-1} \\
Z_n
\end{pmatrix}

\text{where} \quad R_N =
\begin{pmatrix}
1 & \cdots & 0 & \cdots & 0 \\
\vdots & & \ddots & & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & \cdots
\end{pmatrix}

The matrix $\tilde{R}_N$ has the same structure as the matrix $R_C$.

The finite matrix has been built in two steps. In the first step, the first row has been constructed with the constraint of being orthogonal to the mass matrix of eq. (17). In a second step, all the other rows have been constructed to be orthogonal to the first one. A different choice of the orthogonal rows simply implies a redefinition of the mixing angles of the matrix $R_N$. The resulting matrix $U$ is

$$U_{ik} = \delta_{i1} \frac{P(x_{k-1})}{f(-1)} + \delta_{k,i-1} \frac{f(k-1)}{f(k-2)} - \theta(k - i) \theta(i - 2) \frac{P(x_{k-1}) P(x_{i-2})}{f(i-2) f(i-3)}$$

(19)
where the functions $P(x_i)$ and $f(i)$ are defined as

$$P(x_i) = \frac{x_1 \cdots x_n}{x_i} \quad \text{and} \quad f(i) = \sqrt{\sum_{s=i+1}^{s=n+1} P(x_s)^2}$$

(20)

Notice that $f(n+1) = 0$.

This is a generalization for arbitrary $n$ of the $U$ matrix introduced in [14] for $n = 1$. Applying the transformation matrix $U$ to $M_N^2$

$$\tilde{M}_N^2 = U M_N^2 U^T$$

(21)

we obtain the following matrix elements:

$$\tilde{M}_{N,ij}^2 = \frac{g_0^2}{4} \theta(i-2)\theta(j-2) \left\{ F_{1}^{ij} \left[ \delta_{ij} (v_{j-2}^2 + s(j-1)) - \theta(j-i-1) u_{j-2}^2 \right] 
+ F_{2}^{ij} \left[ \sum_{r=j}^{n+2} \left( \sum_{s=0}^{i-2} u_{s r-1}^2 \right) + \frac{f(i-2)^2}{P(x_{i-2})^2} u_{r-1 i-2}^2 + \frac{f(j-2)^2}{P(x_{j-2})^2} u_{r-1 j-2}^2 \right] 
+ \frac{f(j-2)^2}{P(x_{j-2})^2} \left[ \left( \sum_{r=i}^{j-2} u_{r-1 j-2}^2 \right) - \theta(j-i+1)(v_{j-2}^2 + s(j-1)) \right] \right\} \quad \text{with} \quad i \leq j$$

(22)

with

$$F_{1}^{ij} = \frac{x_{i-2} x_{j-2} f(i-2) f(i-3)}{f(j-2) f(i-3)}$$

$$F_{2}^{ij} = \frac{P(x_0)^4}{x_{i-2} x_{j-2} f(i-2) f(i-3) f(j-2) f(j-3)} \cdot$$

(23)

The matrix elements of $M_N$ have been denoted by $\tilde{M}_N$ to recall that they are not the final result, because it is still necessary to make the second rotation on the $Z$ fields in order to get the mass eigenvalues:

$$M_{N,d}^2 = R_N \tilde{M}_N^2 R_N^T.$$  

(24)

The complete rotation matrix is denoted by

$$\mathcal{R} = R_N \times U.$$  

(25)

The precise form of the mass eigenvalues and the explicit dependence on the VEVs and coupling constants of the rotation matrix $R_N$ or $\mathcal{R}$ will be given in section 5 for the screening configurations.

3 Scalars

In this section we analyse the spectrum of scalar particles of the theory, Goldstones and Higgses. In the interacting basis of scalar fields we have $4(n+1)$ degrees of freedom (d.o.f.) coming from the doublets together with $4n(n+1)$ d.o.f. from the bidoublets. Half of them are charged, while the others are neutral.
3.1 Charged Goldstone bosons and Higgses

The charged d.o.f. combine in $2(n+1)$ charged Goldstone bosons $G_i^\pm$ ($i = 0, \ldots, n$) and $2n(n+1)$ states corresponding to the physical charged Higgses $H_i^\pm$ ($i = n+1, \ldots, n(n+2)$).

We will start by identifying the Goldstone bosons that will be “eaten” up to give masses to their corresponding gauge bosons.

From the Lagrangian in eq. (8), after performing the rotations of the gauge fields from the interacting to the mass eigenstates, eq. (14), we identify each Goldstone boson as the coefficient of the term linear in the corresponding physical gauge boson.

Then we introduce a unitarity matrix $A$ that gives the projection of the interacting fields onto the mass eigenstates

$$
\begin{pmatrix}
\phi_0^+ \\
\phi_1^+ \\
\vdots \\
\phi_n^+ \\
\varphi_1^+(01) \\
\varphi_2^+(01) \\
\vdots \\
\varphi_1^+(j l) \\
\varphi_2^+(j l) \\
\vdots \\
\varphi_1^+(n-1n) \\
\varphi_2^+(n-1n)
\end{pmatrix} = A
\begin{pmatrix}
G_0^+ \\
G_1^+ \\
\vdots \\
G_n^+ \\
H_{n+1}^+ \\
H_{n+2}^+
\end{pmatrix}
$$

$$
C_A^j = \begin{pmatrix}
g_0(R_C)_{11}v_0 \\
g_1(R_C)_{12}v_1 \\
\vdots \\
g_n(R_C)_{n,n+1}v_n \\
\frac{g_0(R_C)_{11}v_0 - g_1(R_C)_{21}v_1}{N_i} \\
\frac{-g_0(R_C)_{11}v_0 + g_1(R_C)_{21}v_1}{N_i} \\
\vdots \\
\frac{g_{j-1}(R_C)_{j+1,j}v_{j-1} - g_j(R_C)_{j+1,j+1}v_j}{N_i} \\
\frac{-g_{j-1}(R_C)_{j+1,j}v_{j-1} + g_j(R_C)_{j+1,j+1}v_j}{N_i} \\
\vdots \\
\frac{-g_{n-1}(R_C)_{n+1,n}v_{n-1} - g_n(R_C)_{n+1,n+1}v_n}{N_i} \\
\frac{-g_{n-1}(R_C)_{n+1,n}v_{n-1} + g_n(R_C)_{n+1,n+1}v_n}{N_i}
\end{pmatrix}
$$

(26)

where the normalization factor $N_i$ is given by

$$
N_i = g_0 \sqrt{\sum_{s=0}^{n} x_s^2 v_s^2 (R_C)_{s,s+1}^2 + \sum_{j=0}^{n-1} \sum_{l=j+1}^{n} \left( x_j (R_C)_{i,j+1} v_1^{(j)} - x_l (R_C)_{i,l+1} v_2^{(j)} \right)^2 + v_1 \leftrightarrow v_2 }.
$$

(27)

The rotation matrix $A$ can be split into two sets of columns $A = \{ C_A^i, C_A^\alpha \}$. The first set $C_A^i$ (given in eq. (26)), with $i$ running from 1 to $n+1$, corresponds precisely to the decomposition of the charged Goldstone boson $G_i$ with mass $m_A^i$ in terms of the interacting basis. The second set $C_A^\alpha$ with $\alpha = n+2, \ldots, (n+1)^2$ is the corresponding projection of the mass eigenstate $H_{a-1}$ whose mass is denoted by $m_A^a$. We do not give here the explicit form of $C_A^\alpha$ since we do not need it to find the screening configurations. In order to construct $C_A^\alpha$ it is just necessary to find a set of states orthonormal among themselves and to the Goldstone bosons. For an explicit example in the case $n = 1$, see [4]. Notice that the choice of this basis of orthonormal states leaves extra freedom to introduce $n(n+1)(n(n+1) - 1)/2$ angles.

Notice that we use a different index notation for the $H_i^\pm$ and their mass than in [4]. Our first physical charged Higgs is named $H_{n+1}^\pm$ instead of $H_1^\pm$ of [4], to be able to get a compact expression for $\rho_{NC}$ in the next section.
3.2 Neutral Goldstone bosons and Higgses

The neutral sector is organized in a set of \( (n+1) \) neutral Goldstone bosons and \((2n+1)(n+1)\) neutral physical Higgses. We assume here, as in [3], that there are no other sources of CP-violation apart from the phase of the Kobayashi-Maskawa matrix. This hypothesis allows us to split the neutral scalars in a CP-odd set \((n(n+1)\) neutral physical Higgses \(H^0_i\) \(i = n+1, \ldots, n(n+2)\) and \(n+1\) Goldstone bosons \(G^0_i\) \(i = 0, \ldots, n\) and a CP-even set \(((n+1)^2\) neutral Higgses) without mixing between the two sets.

In a similar way to the charged case, the rotation of the neutral gauge bosons from the interacting to the mass eigenstate basis in the Lagrangian of eq.(8), using eq.(25), provides us with the definition of the neutral Goldstone bosons in terms of the interacting fields.

Also here we introduce a matrix for the physical CP-odd states and the neutral would-be Goldstone \(G\) that gives the projection of the interacting fields onto the mass eigenstates basis:

\[
\begin{bmatrix}
\text{Im} \phi^0_0 \\
\text{Im} \varphi^0_0 (0, 1) \\
\vdots \\
\text{Im} \phi^0_n \\
\text{Im} \varphi^0_0 (n-1, n) \\
\text{Im} \varphi^0_2 (n-1, n)
\end{bmatrix}
= C
\begin{bmatrix}
G^0_0 \\
G^0_1 \\
\vdots \\
G^0_n \\
H^0_{n+1} \\
H^0_{n+2}
\end{bmatrix}
C^\dagger =
\begin{bmatrix}
(g_0 R_{i+1+1} - \tilde{g} R_{i+1 n+2} v_0) \\
(g_1 R_{i+1 + 2} - \tilde{g} R_{i+1 n+2} v_1) \\
\vdots \\
(g_n R_{i+1 n+1 - \tilde{g} R_{i+1 n+2} v_n} \\
(g_0 R_{i+1 + 1})^{(0)} \\
\vdots \\
(g_n R_{i+1 n+1 - \tilde{g} R_{i+1 n+2} v_n})^{(n)}
\end{bmatrix}
\tag{28}
\]

where

\[
\mathcal{R}_{ij} = \delta_{ij} \frac{P(x_{j-1})}{f(-1)} + (R_N)_{i,j+1} \frac{f(j-1)}{f(j-2)} - \sum_{s=2}^{n+2} (R_N)_{i,s} \frac{P(x_{j-1}) P(x_{s-2})}{f(s-2) f(s-3)}
\tag{29}
\]

with \(i, j = 1, \ldots, n+2\) and the normalization factor is

\[
\tilde{N}_i = g_0 \sqrt{\sum_{k=0}^{k=n} (x_k R_{i+1 k+1} - y R_{i+1 n+2})^2 v_k^2 + \sum_{j=0}^{j=n-1} \sum_{l=j+1}^{l=n} (x_j R_{i+1 j+1} - x_l R_{i+1 l+1})^2 u_{jl}^2.}
\tag{30}
\]

We can split the matrix into two sets of columns \(C = \{C_C^i, C_C^\alpha\}\), with the same conventions for the column indices as in the charged case. The column \(C^i_C\) given in eq.(28) corresponds to the neutral Goldstone boson \(G^0_{i-1}\) with mass \(m^C_{i-1}\) \((i = 1, \ldots, n+1)\). The column index \(\alpha\) runs from \(n+2\) to \((n+1)^2\) and labels the set of physical CP-odd neutral Higgses \(H^0_{\alpha-1}\) with mass \(m^\alpha_{\alpha-1}\). The construction of the columns \(C^\alpha_C\) follows the same rules as in the charged case. Also in this case one has the freedom to define \(n(n+1)(n(n+1) - 1)/2\) angles.
There is also a third matrix, which we have called $B$, that relates the real part of the neutral fields with a set of CP-even $(n+1)^2$ physical Higgses with mass $m_{i-1}^B (i = 1, \ldots, (n+1)^2)$. We do not give an explicit form for this matrix since in the screening configurations it is always taken to be equal to $A$ and $C$, except for the rows concerning the fields $\phi_2^{(j)}$ whose sign in matrix $B$ is reversed with respect to the other matrices due to the conventions used in the bidoublets sector. This requirement simply fixes the rotation angles in the matrix $B$, and we always have enough freedom to choose them. The first column $C_1^B$ corresponds to the SM Higgs-like with a mass $m_0^B$.

4 Computation of $\rho_{NC}$

In this section we give the explicit expression of the leading contribution (quadratic in the scalar masses) to $\Delta \rho_{NC} \equiv \rho_{NC} - 1$ for arbitrary $n$.

In a renormalizable theory, at one loop, the contribution to $\Delta \rho_{NC}$ can grow, by power counting, at most quadratically. As we have said before, we are interested in models for which there are natural configurations of the VEVs and of the masses of the scalars such that the potentially large $m^2$ contributions to $\Delta \rho_{NC}$ cancel and we have a screening phenomenon.

The $\rho_{NC}$ parameter is defined following [2] as the ratio between the neutral and the charged current couplings, and therefore the expression of $\Delta \rho_{NC}$, evaluated at $q^2 = 0$, is:

$$\Delta \rho_{NC} = \frac{\Sigma_W(0)}{M_W^2} - \frac{\Sigma_Z(0)}{M_Z^2}$$

where $\Sigma_V(q^2)$ (with $V = Z, W$) stands for the transverse part of the self-energy corrections to the propagator of the vector boson $V$. The contribution coming from $\Sigma_{\gamma Z}(0)$ is absent since we have checked that it is exactly zero by the orthogonality properties of matrix $A$.

We use this definition of $\rho_{NC}$, at $q^2 = 0$, since we are interested in corrections that are leading, universal and independent of the value of $q^2$. The kind of diagrams that can give a quadratic contribution, in the masses of the scalar fields, to eq.(31) are shown in Fig.1. Their explicit expression is:

$$z[m_i] = -\frac{m_i^2}{16\pi^2} \left( C_{UV} + 1 - \ln m_i^2 \right)$$

$$t[m_i, m_j] = \frac{1}{16\pi^2} \left[ (C_{UV} + 3/2) \left( m_i^2 + m_j^2 \right) - \left( \frac{m_i^4 \ln m_i^2 - m_j^4 \ln m_j^2}{m_i^2 - m_j^2} \right) \right]$$

where $C_{UV} = 1/\epsilon - \gamma + \log 4\pi$ and any symmetry factor or coupling constant has been factorized out.

If one expands the Lagrangian in eq.(8), keeping the vertices involving two gauge bosons ($W$, $Z$ and $A$) and two scalars and those involving one gauge and two scalars, those involving one gauge and two scalars, the result is

$$\Delta \rho_{NC} = \frac{1}{4M_W^2} \left\{ \sum_{l,m=0}^{n(n+1)} \left[ \left( \sum_{j=0}^{n} A_{j+1l+1} B_{j+1m+1} P_j \right) + \sum_{j=0}^{n-1} \sum_{k=j+1}^{n} \left( (A_{s1l+1} B_{s1m+1} - A_{s2l+1} B_{s2m+1}) P_j \right) \right]^2 f[m_i^A, m_i^B] \right\}$$
Figure 1: Feynman diagrams contributing to ρNC; a) z[m_i] and b) t[m_i, m_j]. The tadpole diagram (not included) coming from the scalar potential only renormalizes the VEVs and is not relevant for our computation [1, 2].

\[ + \sum_{l,m=0}^{n(n+2)} \left( \sum_{j=0}^{n} A_{j+1t+1C_{j+1m+1}P_{j}} + \sum_{j=0}^{n-1} \sum_{k=j+1}^{n} \sum_{f=1}^{2} A_{s_{f}t+1C_{s_{f}m+1}P_{j}} \right)^{2} f[m_{i}, m_{j}] \]

\[ - \left( A_{s_{2}t+1C_{s1m+1} + A_{s1t+1C_{s2m+1}}P_{k}} \right)^{2} f[m_{i}, m_{j}] \]

\[ - \frac{1}{4M_{Z}^{2}} \sum_{l,m=0}^{n(n+2)} \left( \sum_{j=0}^{n} A_{j+1t+1A_{j+1m+1}N_{1_{j}}} + \sum_{j=0}^{n-1} \sum_{k=j+1}^{n} \sum_{f=1}^{2} A_{s_{f}t+1A_{s_{f}m+1}N_{2_{jk}}} \right)^{2} f[m_{i}, m_{j}] \]

\[ + \left( \sum_{j=0}^{n} C_{j+1t+1B_{j+1m+1}N_{1_{j}}} + \sum_{j=0}^{n-1} \sum_{k=j+1}^{n} (C_{s_{1}t+1B_{s1m+1} - C_{s_{2}t+1B_{s2m+1}}N_{2_{jk}}) \right)^{2} f[m_{i}, m_{j}] \]

where we have used that

\[ t[m_{i}, m_{j}] = -z[m_{i}] - z[m_{j}] + f[m_{i}, m_{j}] \] (33)

with

\[ f[m_{i}, m_{j}] = \frac{1}{16\pi^{2}} \left[ \frac{1}{2} (m_{i}^{2} + m_{j}^{2}) - \frac{m_{i}^{2}m_{j}^{2}}{m_{i}^{2} - m_{j}^{2}} \right] \log \frac{m_{i}^{2}}{m_{j}^{2}} \] (34)

We have introduced, for convenience, the indices \( s_{f} \ (f = 1, 2) \):

\[ s_{1} = n + 2 \left[ \left( n - \left( \frac{j+1}{2} \right) \right) j + k \right] \quad s_{2} = s_{1} + 1; \] (35)

\( s_{1} \) labels the fields \( \phi_{1}^{\pm} (jk) \), \( Re\phi_{1}^{0} (jk) \) and \( Im\phi_{1}^{0} (jk) \), while \( s_{2} \) does the same for \( \phi_{2}^{\pm} (jk) \), \( Re\phi_{2}^{0} (jk) \) and \( Im\phi_{2}^{0} (jk) \). The functions \( P, N_{1}^{\pm} \) and \( N_{2}^{\pm} \) are defined as

\[ P_{x} = g_{x} (R_{C})_{1x+1}, \quad N_{1_{j}}^{\pm} = g_{j} R_{2j+1} \pm \tilde{g} R_{2n+2}, \quad N_{2_{jk}}^{\pm} = g_{j} R_{2j+1} \pm g_{k} R_{2k+1}. \] (36)
All the masses of the Goldstone bosons ($m_A^i$ and $m_C^i$, $i = 0, 1, \ldots, n$) should be taken equal to zero, since we are working in the Landau gauge.

The expression of $\Delta \rho_{NC}$ remarkably simplifies after the use of eq. (33) and of the orthogonality properties of the matrices $A$, $B$ and $C$, independently of their explicit expression. From eq. (32) we notice that all the divergences cancel, indeed they cancel separately inside each self-energy; furthermore also all the finite contributions from $z[m_i]$ cancel. The function $f[m_i, m_j]$ gives a measure of the isospin breaking and its properties serve as a guideline to find the conditions to obtain the screening. In fact $f[m_i, m_j] = 0$ helps to find the conditions concerning the degeneracy of the scalar masses, while $f[m_i, 0] = m_i^2/(32\pi^2)$ gives us the constraints on the VEVs in order to compensate the scalar contributions between the two self-energies. The particular conditions will be discussed in the next section.

Also, we have verified that the previous expression reduces for the case $n = 1$ to the one of [4], in all the cases that are explicitly given there.

From a direct inspection of eq. (32), one notices the absence of terms proportional to $f[m_i^B, m_j^B]$ and to $f[m_i^C, m_j^C]$, i.e. the absence of self-energy diagrams with two neutral scalars or two neutral physical pseudo-scalars running in the loop. On the other side, in the unitary gauge, the self-energy diagrams involving one physical scalar and one Goldstone boson are obviously absent and their contribution is shifted to the diagrams with the Goldstone boson replaced by the gauge field. From the previous two observations we learn that the contribution from the diagrams in Fig.1 vanishes in the unitary gauge, if the scalar sector contains either only physical scalars or only physical pseudoscalars (cf. e.g. [8]).

5 Screening Configurations

It was observed in [4] for the case $n = 1$ that under certain conditions the contributions to $\rho_{NC}$, which are quadratic in the masses of the scalar particles, vanish.

We have found that also when the gauge group is enlarged to $SU(2)_L \times SU(2)^n \times U(1)_Y$, there are at least two configurations in which these contributions to $\rho_{NC}$ vanish. The requirements imposed on the VEVs and consequently the predicted mass spectrum of the gauge bosons are one of the differences between the configurations. In the first one, which we will refer to as case I, the masses of all the new gauge bosons are sent to infinity, while in the second one, referred to as case II, a finite-mass spectrum for the new gauge particles is allowed.

A common condition to both configurations is the requirement of alignment of the scalar transformation matrices $A$, $B$ and $C$ or, to be more precise:

$$A_{\alpha \beta} = B_{\alpha \beta} = C_{\alpha \beta} \quad \text{except for} \quad B_{s2 \beta} = -A_{s2 \beta} = -C_{s2 \beta}. \quad (37)$$

The difference in sign is just due to the convention chosen to write the $\phi_2^{(jk)}$ field inside the bidoublet $\psi_{jk}$.

In order to understand the implications of this first condition on the parameter space, we should count the number of free parameters of the gauge and scalar sector (without the potential). We have on one side $(n + 1)^2$ VEVs and $n + 2$ coupling constants and, on the other, $n(n + 1)(n(n + 1) - 1)/2$ mixing angles among the charged scalars. In the neutral sector, we have $n(n + 1)(n(n + 1) - 1)/2$ and $n(n + 2)(n(n + 1))^2/2$ mixing angles among the neutral pseudoscalar and the neutral scalar bosons, respectively. Of course, we also have the set of masses of the physical Higgses, already discussed.
This condition eq. (37) imposes constraints on the VEVs and requires each mixing angle of matrix \( A \) to be equal to the corresponding mixing angle in matrix \( B \) and \( C \). So, at the end, in total, only \( n(n+1)(n(n+1)-1)/2 \) free angles are left. Notice that the remaining mixing angles between the neutral scalars are completely determined by eq. (37).

A second condition will be required on the masses of the physical scalars.

The particular requirements on the VEVs and on the scalar masses depend on the case and will be discussed in the next subsections.

5.1 Case I

In this first scenario, the set of equations given by (37) are satisfied by taking the large-\( v_i \) limit (with \( i > 0 \)) while keeping \( v_0, v_1^{(i-1)j-1}, \) and \( v_2^{(i-1)j-1} \) fixed and small with respect to the \( v_i \)'s. When a hierarchy between the \( v_i \)'s is introduced

\[
v_0 < v_1 < v_2 < \ldots < v_n
\]

we find the expected alignment of the matrices \( A, B \) and \( C \). In that limit the leading-\( v_i \) dependence of the charged and neutral rotation matrices is, respectively,

\[
(R_C)_{ij} \sim \frac{1}{v_j^{(i-1)j-1}} \quad \text{and} \quad (R_N)_{ij} \sim \theta(i-2)\theta(j-2)\frac{v_i^{(i-2)}}{v_j^{(j-2)}}, \quad i < j,
\]

where the elements along the diagonal are equal to 1 and the off-diagonal ones change sign when \( i > j \).

The leading term of the normalization factors is

\[
N_i \sim g_0 \sqrt{\delta_{i1} [v_0^2 + s(1)] + \theta(i-2)x_{i-1}^2 v_i^{(i-2)}}
\]

\[
\tilde{N}_i \sim g_0 \sqrt{\delta_{i1} \frac{f(-1)^2}{f(0)^2} [v_0^2 + s(1)] + \theta(i-2)x_{i-1}^2 v_i^{(i-2)} \frac{f(i-2)^2}{f(i-1)^2}}.
\]

Now it is easy to see, by looking at the charged matrix \( A \) and at eq. (40), that in the large \( v_k \) limit (\( k = 1, \ldots \)) the only non-vanishing term of the column \( i \) with \( i > 1 \) is the element in row \( i \), which is equal to 1. All the other terms are suppressed either by a mixing angle or by the normalization factor. The first column \( i = 1 \) is exceptional since its \( v_0 \) is kept finite and the normalization factor does not grow. In that case all the terms proportional to a mixing angle are still vanishing while the others remain. Then this column turns out to be

\[
C_A^T = \frac{1}{\sqrt{v_0^2 + s(1)}} \begin{pmatrix} v_0, 0, \ldots, 0, v_1^{(01)}, -v_2^{(01)}, \ldots, v_1^{(0n)}, -v_2^{(0n)}, 0, \ldots, 0 \end{pmatrix}.
\]

It is not difficult to check, using eqs. (28) and (40), that the same holds exactly for the neutral sector. Notice that it is precisely the hierarchy of eq. (38) that guarantees that all the terms with a mixing angle vanish, since the VEVs in the numerator cannot be larger than the ones in the denominator. The standard gauge boson masses are in this case

\[
M_{W_P}^2 = \frac{g_0^2}{4} (v_0^2 + s(1)), \quad M_{Z_P}^2 = \frac{g_0^2}{4} \frac{f(-1)^2}{f(0)^2} (v_0^2 + s(1)),
\]
where we have used that \( f(0)^2 = f(-1)^2 - P(x_0)^2 \). The new gauge boson masses are, at first order in the large \( v_i \) expansion:

\[
M_{W_i}^2 = \frac{g_0^2}{4 v_i^2} \left( v_i^2 + s(i + 1) \right)
\]

\( M_{Z_i}^2 = \frac{g_0^2}{4 v_i^2} \frac{f(i-1)^2}{f(i)^2} \left\{ v_i^2 + s(i + 1) + \frac{P(x_i)}{f(i-1)^2} \sum_{s=0}^{s=n-1} \left( v_s^{'2} + \sum_{r=i+1}^{s=n} u_{sr}^2 - \left( \frac{f(i)^2 + f(i-1)^2}{P(x_i)^2} \right) u_{is}^2 \right) \right\}
\]

and go to infinity when the large \( v_i, i > 0 \) limit is taken. From the request that the coefficient in front of the electromagnetic current be the electric charge, one finds, once the matrix \( U \) has been applied on the interacting fields, that

\[
e = g_0 P(x_0) / f(-1).
\]

Using eq. (44) and the relation \( e = g_s W \), the sine and cosine of the weak mixing angle are respectively given by

\[
s_W = P(x_0) / f(-1) \quad \text{and} \quad c_W = f(0) / f(-1).
\]

Therefore, in this first configuration, it follows from eqs. (42) and (43) that \( \rho \) defined as

\[
\rho = M_{W}^2 / c_W^2 M_{Z}^2
\]

is exactly 1.

This configuration gives us the possibility to interpret in a simple way the alignment condition of eq. (37). If one writes the Lagrangian of the model in this particular configuration, by using matrices \( A, B \) and \( C \), which now turn out to be equal and very simple, one sees that the Goldstones and physical scalar fields are organized in a transparent way:

\[
\mathcal{L} = \sum_{i=0}^{i=n} (D_\mu \Upsilon_i)^\dagger D_\mu \Upsilon_i + \sum_{i=1}^{i=n(n+1)} (D_\mu \Omega_i)^\dagger D_\mu \Omega_i
\]

where

\[
\Upsilon_i = \begin{pmatrix} G^0_i + i h^0_i \\ G^-_i \end{pmatrix} \quad \text{and} \quad \Omega_i = \begin{pmatrix} H^0_i + i h^0_{i+n+1} \\ H^-_i \end{pmatrix}.
\]

The set of \( \Upsilon_i \) are complex doublets made of Goldstone bosons and a singlet Higgs, and \( \Omega_i \) are complex doublets of matter. Thanks to the alignment, \( \Delta \rho_{NC} \) is nothing else than the sum of the contributions of a set of complex doublets. Therefore it, automatically, tells us which are the conditions on the scalar masses to obtain the screening configuration:

\[
m_{l}^A = m_{l'}^B \quad \text{or} \quad m_{l}^A = m_{l'}^C
\]

where \( l, l' = n+1, ..., n(n+2) \). We can now resume the main points of case I. In order to get the alignment condition eq. (37), we perform the large \( v_i \) (\( i > 0 \)) limit, with a hierarchy among the VEVs, and we set equal each angle of matrix \( A \) to the corresponding of matrix \( B \) and \( C \) (so at the end only \( n(n+1)(n(n+1) - 1)/2 \) common angles are left free). Then we see that the only remaining source of scalar quadratic terms is given by the isospin breaking, i.e. the mass splitting in the doublets of eq. (49). Once eq. (49) is fulfilled we have screening.
5.2 Case II

This is the most interesting screening configuration. In fact a finite light mass spectrum is allowed for the gauge bosons. The result is obtained exactly, without the need of any approximation or limiting procedure. In this second scenario all the VEVs are kept finite. Obviously this makes it much more involved to find the solution of eq.(57), so we will give some details of the way to proceed. We split the set of equations $A = C$ into two sets. A first set is

$$\frac{x_{k-1}R_{i+1k} - yR_{i+1n+2}}{x_{k-1}(RC)_{i+k}} = \frac{\tilde{N}_i}{N_i}, \quad (50)$$

with $k = 1, .., n + 1$, while the second is

$$\frac{x_{j}\tilde{R}_{i+1j+1} - x_l\tilde{R}_{i+1l+1}}{x_jv_1^{(j)}(RC)_{i+j+1} - x_lv_1^{(l)}(RC)_{i+l+1}}v_1^{(j)} = \frac{\tilde{N}_i}{N_i}, \quad (51)$$

$$\frac{x_{j}\tilde{R}_{i+1j+1} - x_l\tilde{R}_{i+1l+1}}{x_jv_2^{(j)}(RC)_{i+j+1} - x_lv_2^{(l)}(RC)_{i+l+1}}v_2^{(j)} = \frac{\tilde{N}_i}{N_i}$$

From the second set of equations one immediately obtains the condition

$$v_1^{(j)} = v_2^{(j)}. \quad (52)$$

Notice that our VEVs are taken to be real. Moreover, once this condition is imposed the fulfilment of eq.(51) is automatically ensured by the fulfilment of eq.(50). This is easily seen by making the difference between eq.(50), evaluated at $k = j + 1$, and the same equation evaluated at $k = l + 1$. It means that we can concentrate just on eq.(50). We write eq.(50) in the following way:

$$\frac{x_{k-1}(RC)_{i+k}}{x_{k-1}R_{i+1k} - yR_{i+1n+2}} = \frac{x_{l-1}(RC)_{i+l}}{x_{l-1}R_{i+1l} - yR_{i+1n+2}} \quad \text{with} \quad k < l. \quad (53)$$

From the solution of the previous equation, we obtain the constraints on the VEVs,

$$v_i = \frac{v_0}{x_i} \quad \text{and} \quad u_{i,j}^2 = u_{i+1}^2 \left(\frac{x_{i+1}}{x_j}\right)^2, \quad j > i. \quad (54)$$

The exact form of the rotation matrices (charged and neutral) corresponding to this screening configuration is the cornerstone of the calculation. The charged matrix becomes

$$(RC)_{i,k} = N_i^C \left(\frac{\theta(k - i)}{x_{k-1}} - \theta(i - 2)\delta_{k,i-1}x_{i-2} \sum_{j=k}^n \frac{1}{x_j}\right), \quad (55)$$

where

$$N_i^C = \frac{1}{\sqrt{\sum_{k=i-1}^n \frac{1}{x_k} \left(1 + \theta(i - 2)x_{i-2}^2 \sum_{j=i-1}^n \frac{1}{x_j}\right)}}. \quad (56)$$
while the neutral one is

\[ R_{i,k} = \delta_i P_{k-1} \frac{S_{k,k}}{f(1)} + \theta(i-2) S_{i,k} \]  

(57)

where

\[ S_{i,k} = \frac{\theta(n + 1 - k)}{x_{k-1}} \left[ \delta_i N_i^N + \theta(i-2) \theta(k-i) N_i^N \right] - \sum_{j=1}^n \frac{1}{x^2_j} \left[ y \delta_{i1} \delta_{k,n+2} N_i^N + x_{i-2} \delta_{k,1-2} \theta(i-2) N_i^N \right] \]

with

\[ N_i^N = \frac{1}{\sqrt{\left( \sum_{k=0}^n \frac{1}{x^2_k} \left( 1 + y^2 \sum_{j=0}^n \frac{1}{x^2_j} \right) \right)^{-1}}} \]

\[ N_i^N = N_i^C \quad i \geq 2. \]  

(58)

If we introduce these results into the general expression of \( \Delta \rho_{NC} \) displayed in eq. (62), the latter greatly simplifies since now \( P_j, N_{1j}^\pm \) and \( N_{2jk}^\pm \) are constants, independent of \( j, k \). It is then immediate to find the extra conditions necessary to get a vanishing \( \Delta \rho_{NC} \).

a) The projection of the bidoublet fields on the physical set of Goldstones and Higgses should satisfy, for fixed \( l \) and any value of \( s \):

\[ A_{s,1+l}^2 = A_{s,2+l+1}^2, \]  

(59)

where \( l = n + 1, \ldots, n(n + 2) - 1 \), with the constraint that the sign of \( A_{s,1+l}/A_{s,2+l+1} \) should be the same for the whole column \( l \). Notice that the constraint of eq. (63) is also satisfied in this configuration when \( l = 0, \ldots, n \). A final important remark concerning this new constraint is that it corresponds to \( n(n + 1)(n(n + 1) - 1)/2 \) independent equations, exactly the same number of free scalar mixing angles as we have. Again, as happens for \( n = 1 \), this condition fixes the value of these angles.

b) A second condition concerns, as in case I, the required degeneracy of the masses of the scalar particles in order to cancel \( \Delta \rho_{NC} \):

\[ m_i^A = m_i^B \quad \text{if} \quad A_{s,1+l}/A_{s,2+l+1} = +1 \]  

(60)

and

\[ m_i^A = m_i^C \quad \text{if} \quad A_{s,1+l}/A_{s,2+l+1} = -1 \]  

(61)

where \( l = n + 1, \ldots, n(n + 2) \). Notice that no restriction is imposed on the masses \( m_j^B \) with \( j = 0, \ldots, n \) \((m_0^B \) is the SM-Higgs-like).

The conditions of case II are more restrictive than those of case I. In order to obtain the screening, here, we need not only to put constraints on the VEVs eq. (54) but also to fix the common mixing angles in the matrices \( A, B \) and \( C \) eq. (54), without any arbitrariness left. Also, as in the previous case, a constraint on the masses eq. (60) and (61) is required.

Finally the spectrum of gauge boson masses turns out to be very simple in this second configuration,

\[ M_{W_0}^2 = \frac{g_0^2}{4} v_0^2 \]

\[ M_{Z_0}^2 = \frac{g_0^2}{4} v_0^2 \left( y^2 \sum_{a=0}^n \frac{1}{x^2_a} + 1 \right) \]

\[ M_{W_i}^2 = M_{Z_i}^2 = \frac{g_0^2}{4} \left( v_0^2 + \sum_{i=0}^{i-1} x_i^2 u_{i+1} + x_{i-1}^2 u_i + \sum_{r=i}^n \frac{1}{x^2_r} \right) \]

\[ i > 0. \]  

(62)
It implies that, at tree level, \( \rho \) is not 1 but

\[
\rho_{\text{tree}} = 1 - t_W^2 \sum_{s=1}^{n} \frac{1}{x_s^2},
\]  

(63)

where \( t_W \) stands for the tangent of the weak mixing angle. As a consequence of eq.(63) it is clear that the model requires the extra gauge couplings to be large (as in [4]) so as to obtain an acceptable value of \( \rho \) at tree level. However, large couplings do not necessarily imply large effects in the observables. It was noted in [4] that all the potentially large couplings in the charged sector are always suppressed by a small mixing angle (see eq.(55) in the limit of large coupling with \( x_i > x_j \) when \( i > j \)). On the other hand, in the neutral sector the terms proportional to a large coupling constant cancel in the sum of diagrams with light gauge bosons and heavy scalars together with the diagrams with only heavy gauge fields, leaving only subleading corrections to \( \Delta \rho_{\text{NC}} \). Similar scenarios were described in [6] and [15].

5.2.1 The \( \epsilon \) parameters: tree level

Given the interesting spectrum of masses and properties of this second case, it is worthwhile to analyze it in more detail and compute the full set of \( \epsilon \) parameters[18]. We will follow closely the discussion in [8] and the conventions for the parameters \( \Delta \tilde{\rho}, \Delta r_W \) and \( \Delta k \). Similarly to what happen in [8] also our model exhibits a decoupling property in all \( \epsilon \) parameters.

At tree level the Fermi constant \( G_F \) is given in our model in case II by

\[
G_F \sqrt{2} = \frac{\pi \alpha}{2s_W^2} \frac{1}{\sum_{s=0}^{n} x_s^2} \left( \frac{1}{M_{W_0}^2} + \sum_{s=1}^{n} \frac{1}{x_s^2} \right) = \frac{1}{2t_W^2} \left( 1 - \frac{n}{x_0^2} \left( 1 - \frac{M_{W_0}^2}{M_{W_1}^2} \right) \right) + \mathcal{O} \left( \frac{1}{x_i^4} \right),
\]  

(64)

where the first term in the large \( x_i \) expansion \((1/2x_0^2)\) would give the exact result if \( M_{W_0} = M_{W_1} \).

Following [8] we can also relate the cosinus of the weak mixing angle of our model with the corresponding one in the Standard Model denoted by \( c_0 \) in the limit of large \( x_i \)

\[
c_W^2 = c_0^2 \left( 1 - t_W^2 \sum_{s=1}^{n} \frac{1}{x_s^2} + \sum_{s=1}^{n} \frac{1}{x_s^2} \left( 1 - \frac{M_{W_0}^2}{M_{W_1}^2} \right) \right),
\]  

(65)

where

\[
c_0^2 = \frac{1}{2} + \frac{\pi \alpha}{2 \sqrt{2} G_F M_{Z_0}^2} \left( 1 - \frac{s_W^2}{c_W^2 - s_W^2} \Delta r_W \right)
\]  

(66)

Comparing the SM definition of \( \Delta r_W \)

\[
\frac{M_{W_0}^2}{M_{Z_0}^2} = c_0^2 \left( 1 - \frac{s_W^2}{c_W^2 - s_W^2} \Delta r_W \right)
\]  

(67)
with eq.(62) and eqs.(64-66), we find
\[ \Delta r_W = 2 c_W^2 \sum_{s=1}^n \frac{1}{x_s^2} \left( \frac{c_W^2 - s_W^2}{s_W^2} \right) \sum_{s=1}^n \frac{1}{x_s^2} \left( 1 - \frac{M_{W_0}^2}{M_{W_1}^2} \right) \]
(68)
where we have used that \( M_{Z_0}^2 \) of eq.(62) can be rewritten in a more convenient way
\[ M_{Z_0}^2 = \frac{g_0^2 v_0^2}{4 e_W} \left( 1 + \frac{\Delta \rho}{2} \right) \sum_i \left( T_{3L}^i - s_0^2(1 + \Delta k) Q_i^i \right) \bar{f}_i \gamma^\mu f_i \]
(69)
In order to obtain \( \Delta k \) and \( \Delta \tilde{\rho} \) (current) we will compare with the standard definition of the \( Z \) current
\[ J_{Z_0}^{\mu} = \left( J_{W_0}^{\mu} 0 \cdots J_{B}^{\mu} \right) \left( \begin{array}{c} W_{0\mu}^3 \\ \vdots \\ W_{n\mu}^3 \\ B_{\mu} \end{array} \right) \]
(72)
that can be further decomposed using eq.(19) in
\[ J_{Z_0}^{\mu} = (R_N^T)_{22} J_{Z_0}^{\mu} + \sum_{s=1}^n (R_N^T)_{2+s,2} J_{Z_s}^{\mu} \]
(72)
that can be further decomposed using eq.(19) in
\[ J_{Z_0}^{\mu} = c_W J_{W_0}^{\mu} - \frac{s_W^2}{c_W} \frac{1}{y} J_{B}^{\mu} \]
(73)
where
\[ J_{W_0}^{\mu} = g \sum_i T_{3L}^i \bar{f}_i \gamma^\mu f_i \quad \text{and} \quad J_{B}^{\mu} = \tilde{g} \sum_i \bar{Y}_i \bar{f}_i \gamma^\mu f_i \]
(74)
Comparing eq. (72-74) and eq. (70) one finally obtains

\[ 1 + \frac{\Delta \tilde{\rho}}{2} = \left( 1 - y^2 \sum_{s=1}^{n} \frac{1}{x_s^2} + \sum_{s=1}^{n} \frac{1}{x_s^2} \left( 1 - \frac{M^2_{W^p}}{M^2_{W^1}} \right) \right) \left( (R_{N}^T)_{22} + \sum_{s=1}^{n} \frac{c_{W}P(x_0)^2(R_{N}^T)_{2+s+2}}{x_s f(s)f(s+1)} \right) \]

\[ \Delta k = \sum_{s=1}^{n} \frac{c_{W}^3}{s_{W}^2 x_s f(s)f(s+1)} \frac{P(x_0)^2(R_{N}^T)_{2+s+2}}{(R_{N}^T)_{22} + \sum_{s=1}^{n} \frac{c_{W}P(x_0)^2(R_{N}^T)_{2+s+2}}{x_s f(s)f(s+1)}} \]  

(75)

where \((R_{N}^T)_{2+s+2}\) can be derived from eq. (19) and eq. (57)

\[ (R_{N}^T)_{2+s+2} = N_1^N \frac{f(-1)^2}{f(s-1)f(s)x_s} \]  

(76)

In the large \(x_i\) expansion one finally gets

\[ \Delta \tilde{\rho} = -\frac{1}{c_{W}^2} \sum_{s=1}^{n} \frac{1}{x_s^2} + 2 \sum_{s=1}^{n} \frac{1}{x_s^2} \left( 1 - \frac{M^2_{W^p}}{M^2_{W^1}} \right) \]

\[ \Delta k = \sum_{s=1}^{n} \frac{1}{x_s^2} \]

(77)

Recalling the definition for the \(\epsilon\) parameters in terms of the oblique corrections \(\Delta \tilde{\rho}, \Delta k\) and \(\Delta r_W\) [18]

\[ \epsilon_1 = \Delta \tilde{\rho} \]

\[ \epsilon_2 = c_{W}^2 \Delta \tilde{\rho} + \frac{s_{W}^2}{c_{W}^2 - s_{W}^2} \Delta r_W - 2s_{W}^2 \Delta k \]

\[ \epsilon_3 = c_{W}^2 \Delta \tilde{\rho} \left( c_{W}^2 - s_{W}^2 \right) \Delta k \]  

(78)

and using eq. (18) and eq. (77) we obtain

\[ \epsilon_1 = -\frac{1}{c_{W}^2} \sum_{s=1}^{n} \frac{1}{x_s^2} + 2 \sum_{s=1}^{n} \frac{1}{x_s^2} \left( 1 - \frac{M^2_{W^p}}{M^2_{W^1}} \right) \]

\[ \epsilon_2 = \frac{1}{2} \left( 2 \sum_{s=1}^{n} \frac{1}{x_s^2} \right) \left( \frac{c_{W}^2 - s_{W}^2}{c_{W}^2} \right) \left[ -3 + \left( \frac{c_{W}^2 - s_{W}^2}{c_{W}} \right) + 2 \left( 1 - \frac{M^2_{W^p}}{M^2_{W^1}} \right) c_{W}^2 \right] \]

\[ \epsilon_3 = 2 \sum_{s=1}^{n} \frac{1}{x_s^2} \left[ -s_{W}^2 + \left( 1 - \frac{M^2_{W^p}}{M^2_{W^1}} \right) c_{W}^2 \right]. \]  

(79)

We find that similarly to what happens in the model described in [8] (and, of course, also in [4]) there is a decoupling property in the whole set of \(\epsilon\) parameters in the large \(x_i\) limit.

6 Links with Bess and degenerate Bess models

It has been shown in [4] that the FP model \((n = 1)\) in the configuration II \((v_1^{(01)} = v_2^{(01)} = v, v_1 = v_0/x_1)\) is able to reproduce the masses and mixings of the gauge boson particles of the Bess model [17] in a subset of the parameter space of both theories.
The translation table in our present notation is

\[
\begin{align*}
\alpha & \rightarrow 1/(1 + 2x_1^2) \\
f & \rightarrow v\sqrt{1 + 2x_1^2} \\
g'' & \rightarrow 2g_0x_1 \\
g & \rightarrow g_0 \\
g' & \rightarrow g_0y.
\end{align*}
\]

In the left column, we have the 5 parameters of the Bess model (we use the notation of [17]), which reduce to 4 free parameters with the extra constraint \(\alpha = 2g^2/(2g^2 + g''^2)\). On the right, we have the 4 free parameters \((g_0, x_1, y, v)\) of our model after imposing the extra constraint \(v_1 = \sqrt{2}v\). In this particular configuration the model shares all the good properties of the Bess model plus a screening of quadratic scalar contributions to \(\rho_{NC}\).

The link with the degenerate Bess [6], on the contrary, turns out to be more complicated, and we need to impose several constraints on both models. Beside the presence of an extra family of gauge bosons, the degenerate Bess model has also two important differences with respect to the Bess model:

i) while the Bess does not decouple in the large mass limit, the degenerate Bess does;

ii) the \(\rho\) parameter at tree level (eq.(46)) is different from 1 in the Bess model but equal 1 in the degenerate Bess.

The degenerate Bess model has, concerning the gauge sector, 5 parameters \((r, x, \tilde{g}, \tilde{\theta} \text{ or } \tilde{g}', v)\) in the notation of [6]. Our model has on the contrary 13 free parameters in case I (although some of them become infinitely large), and 7 free parameters in case II.

It is not difficult to see that case I cannot overlap with the degenerate Bess model from the condition that \(M_{W_2} = M_{W_1}\). This is equivalent to imposing that \(x_2^2v_2^2 = x_1^2v_1^2\), which is inconsistent in the large \(v_i\) limit with the necessary hierarchy found in case I. Even if one imposes an unnatural inverse hierarchy for the couplings, the rotation matrices of case I are no longer valid, breaking the alignment condition of eq.(37). On the contrary, case II fulfills this condition automatically.

The imposed degeneracy between the new charged gauge bosons is translated into strong constraints in both models. In the degenerate Bess we need to impose

\[x \rightarrow 0.\] (81)

This condition implies a complete degeneracy of all new \(Z\) and \(W\) gauge bosons. On the other side, apart from the constraints of case II, i.e.

\[
\begin{align*}
&v_1^{(ij)} = v_2^{(ij)} \quad \text{with } ij = 01, 02, 12 \\
&v_2 = \frac{v_0}{x_2} \quad v_1 = \frac{v_0}{x_1} \\
&u_{02}^2 = \frac{u_{01}^2 x_1^2}{x_2^2}
\end{align*}
\]

one should require in addition, in our model:

\[
v_{12}^2 = \frac{u_{01}^2}{x_2^2} \quad x_2 \gg x_1 \gg 1,
\] (83)
that is the corresponding of condition \([81]\). The second condition of eq.(\[83\]) is necessary to ensure the absence of mixing between \(W_2^\pm\) and the others \(W\) gauge bosons and also to get a tree-level \(\rho\) parameter equal to 1.

The translation table of the remaining parameters of the degenerate Bess in the small-\(r\) limit is

\[
\begin{align*}
\tilde{g}v &\rightarrow g_0 v_0 \\
\tilde{c}_\theta &\rightarrow \frac{1}{\sqrt{1 + y^2}} = \tilde{c}_W \\
r &\rightarrow \frac{v_0^2}{v_0^2 + u_{01}^2 (1 + x_i^2)},
\end{align*}
\]

where all terms of order \(1/x_i^2\) \((i \geq 1)\) have been thrown away, and \(\tilde{c}_W\) stands for the large \(x_i\) \((i = 1, 2)\) limit of eq.\((45)\) according to eq.\((83)\). The gauge boson masses are, according to eqs.\((62)\), \((83)\) and \((84)\):

\[
\begin{align*}
M_{W_0}^2 &= \frac{g_0^2}{4} v_0^2 \\
M_{Z_0}^2 &= \frac{g_0^2}{4 c_W} v_0^2 \\
M_{W_1}^2 &= M_{W_2}^2 = M_{Z_1}^2 = M_{Z_2}^2 = \frac{g_0^2}{4} (v_0^2 + u_{01}^2 (1 + x_1^2)).
\end{align*}
\]

The new gauge boson masses are all degenerate and larger than the \(M_{W_0}\) mass in the small \(r\) limit.

7 Discussion and Conclusions

In this paper we have presented an extension of the SM based on a gauge group \(SU(2)_L \times SU(2)_n \times U(1)_Y\) \((n \geq 1)\), with a non-trivial scalar sector. We have investigated in detail the \(\rho\) parameter, with particular care for the scalar sector. Imposing the exact cancellation of the quadratic terms in the masses of the scalar fields leads to what we call a “screening” configuration. We have found two different possibilities for the screening to occur and we have calculated the respective mass spectra of the new gauge bosons. While in one case the screening phenomenon is obtained at the price of sending all the masses of the new gauge bosons to infinity, the second one is much more phenomenologically interesting, because a finite, relatively light mass spectrum is allowed. \(\rho\) severely constrains the parameter space of the model and we consider as a very good indication the fact that the screening occurs for arbitrary values of \(n\). Indeed by analyzing the set of \(\epsilon\) parameters we have found that the model in case II exhibits a decoupling property in the large \(x_i\) limit.

Of course, one should take into account other constraints coming from the precision measurements at LEP and Fermilab. Indeed in a previous paper [4], the phenomenology of this type of fermiophobic models was analyzed in detail in the case \(n = 1\). A tree-level fit was done to the electroweak data, also the one-loop contributions to \(R_b\), \(\Delta \rho\) and to all the flavour-changing processes that could be affected significantly by this model were computed. In that respect this kind of fermiophobic models (for any \(n\)) has a clear advantage with respect to other gauge extensions due to the absence at tree level of FCNC and to the suppressed one-loop contributions to the process \(b \rightarrow s\gamma\) and to the \(B_0 - \bar{B}_0\) and \(K_0 - \bar{K}_0\) mixing. All these analysis show that relatively light \(W'\) and \(Z'\) are not excluded provided that \(x\) is sufficiently large. The Tevatron data could have changed this conclusion. However, analyzing the production cross section of the \(W'\) and \(Z'\), it was shown that the previous statement remains valid. Thanks to the fermiophobic nature of the new gauge bosons, the \(W'\) couples to the fermions only through the charged mixing angle while the \(Z'\) has a direct coupling that scales as \(1/x\). Therefore the
weakness of these couplings that induce small cross sections allow the existence of relatively light new
gauge bosons with no contradiction with the Tevatron data.

All these conclusions can be applied also to our model, that reduces to the one studied in [4] in the
case \( n = 1 \). One can see looking to eq. (79) that the contributions to the electroweak observables enter
in an additive way. These contributions are organized in a tower structure, i.e. the lowest order \( W \) and
\( Z \) families fix the bounds on the next ones. The inclusion of extra \( W \)'s and \( Z \)'s receives then just more
stringent bounds on the extra coupling constants \( x_i \) (with \( i > 1 \)), mixing angles and masses. The good
behaviour of the case \( n = 1 \) is not affected by the introduction of new gauge bosons; in the allowed
region of the parameter space we can reach two different limit configurations: we can choose to have
a very light new extra boson, letting all the others to be heavy, or to have a more balanced situation,
with all the bosons in an intermediate mass range.

The keypoint is indeed, as it was observed in [4], the fermiophobic nature of these models that
automatically allows to introduce a milder modification of the whole SM phenomenology, in contrast to
other kind of gauge extensions of the SM. More precise and quantitative predictions can be obtained, as
in the case \( n = 1 \) [4], only through a complete phenomenological analysis, which is nevertheless beyond
the scope of this paper.

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