Highlights:

- The local skin friction coefficient is directly related to the magnetic parameter.
- The local Nusselt number is inversely related to the Prandtl number.
- For the least value of the magnetic parameter, the angular velocity is higher than the velocity.
Analytical solution of MHD micropolar nanofluid flow and forced convection heat transfer with entropy generation analysis past a linearly stretching sheet

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Abstract

This perusal attempts to model and interpret the entropy generation analysis and the flow field of 2-D, steady, viscous, incompressible and laminar boundary layer and forced convection heat transport of micropolar ferrofluid past a stretching sheet including suction and normal magnetic field effects. The porous sheet’s velocity and temperature are presumed to change linearly. Exact explicit solutions of the velocity, angular velocity and temperature distributions have been derived. The impacts of physical parameters on the local skin friction coefficient, the local Nusselt number, the entropy generation number further the velocities and temperature distributions are analyzed by tables and graphs. The angular velocity has more value than velocity for the least value of the magnetic and material parameters. The entropy generation number has a direct relation with material parameter and Brinkman either Reynolds numbers. Moreover, an inverse relation with the Prandtl number.
Keywords: Analytical solutions; Micropolar ferrofluid; Entropy generation analysis; Stretching sheet.

1. Introduction

After introducing nanofluids by Choi [1], a great deal happened in science and technology and is still ongoing. Nanofluids have many biomedical, science and technology applications, such as cancer therapeutics, nuclear reactors, geothermal power extraction and coolant. Khan et al. [2] studied Jeffrey nanofluid past an inclined stretching sheet. Hayat et al. [3] probed the importance of magnetic field on second-grade nanofluid flow for a heated stretching sheet. Ghadikolaei et al. [4] showed that the chemical reaction parameter has a direct and inverse relation with mass transfer rate and concentration, respectively. Mondal et al. [5] showed the variable viscosity parameter plays different roles for the velocity and temperature distributions. ODEs calculations are performed under three methods: analytical [6-16], semi-analytical [17-27] and numerical [28-38]. Sheikholeslami et al. [39] demonstrated Lorentz and thermophoresis forces play different roles on concentration. Golafshan et al. [40] illustrated thermophoresis parameter change slightly on the temperature profile either the local Nusselt number. Derakhshan et al. [41] demonstrated there is a direct relation between the Nusselt number and the viscosity coefficient. Lund et al. [42] depicted an increasing trend in the skin friction and mass transfer for the angle parameter. Jahan et al. [43] made linear and quadratic regressions to estimate the local Nusselt numbers by the Brownian motion and thermophoresis parameters. Li et al. [44] found out the Nusselt number for the shrinking sheet case is more than the stretching sheet case. Dzulkifli et al. [45] reported the unsteadiness parameter extends the range of solution for stretching/shrinking parameter. Turkyilmazoglu [46] demonstrated the heat transfer rate has an inverse relation with the heat jump on the wall. Fereidoonimehr et al. [47] realized by enhancing the molecular weight the nanoparticle
volume fraction raises. Turkyilmazoglu [48] presented analytical solutions induced by nonlinearly deforming porous sheet. Turkyilmazoglu [49] proved magnetic interaction parameter and Prandtl number have different effects on the momentum and temperature boundary layer thicknesses. El-Mistikawy [50] solved related ODEs analytically by using Kummer’s equation for the temperature constituents. In this study, ODEs arising from physics have been solved analytically. Graphs and tables are presented and the variety of physical parameters are explained in detail. The explicit exact solutions for the dimensionless flow, velocity, angular velocity and temperature are reported in Section 3. In Section 4, the physical quantities of interest, for instance, the local skin friction coefficient, the local Nusselt number and the entropy generation number are calculated. Results are discussed in Section 5 and the final section deals with the conclusions.

2. Mathematical model

The two-dimensional flow of an incompressible, viscous, steady and laminar micropolar fluid over a stretching sheet that is stretched continuously in the x-direction is considered. The x-component of the velocity changes linearly as \( U_w = ax \), where \( a \) is a positive constant and the surface temperature varies as \( T_w(x) = T_x + bx \), where \( b \) is also a positive constant. The applied uniform magnetic field is supposed to be in the y-direction. The geometry is shown in figure 1.
The simplified governing equations of the two-dimensional flow with negligible viscous dissipation are as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\rho_{nf} \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \left( \mu_{nf} + \kappa \right) \frac{\partial^2 u}{\partial y^2} + \kappa \frac{\partial N}{\partial y} - \sigma_{nf} B_y^2 u
\]

\[
\rho_{nf} j \left( u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} \right) = \gamma_{nf} \frac{\partial^2 N}{\partial y^2} - \kappa \left( 2N + \frac{\partial u}{\partial y} \right)
\]

\[
u \left( \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} \right) = \left( \rho C_p \right)_{nf} \frac{\partial^2 T}{\partial y^2}
\]

where \( u \) and \( v \) are velocity components in the x- and y-directions, respectively, \( N \) is the micro-rotation vector, \( T \) is the fluid temperature and \( \rho, \nu, \kappa, j, k, \gamma \) and \( C_p \) are the fluid density,
kinematic viscosity, vortex viscosity, micro-inertia per unit mass, thermal conductivity, spin gradient viscosity and the specific heat at constant pressure, respectively. $K = \kappa / \mu$ is dimensionless viscosity ratio and is called the material parameter which represents Newtonian fluid and micropolar fluid when are equal to zero and positive, respectively. $\mu$ is dynamic viscosity, Ahmadi [51] demonstrated $\gamma$ as $\gamma = (\mu + \kappa / 2) j = \mu (1 + K / 2) j$ and $j = \nu / a$ as a reference length. The corresponding boundary conditions are:

$$u = u_w(x) = ax, \quad v = v_w, \quad N = -m \frac{\partial u}{\partial y}, \quad T = T_w(x) = T_\infty + bx \text{ at } y = 0 \quad (5)$$

$$u \to 0, \quad N \to 0, \quad T \to T_\infty \text{ as } y \to \infty, \quad (6)$$

where $v_w$ is a constant mass flux velocity, the positive and negative cases are related to suction and injection, respectively. $m$ is the micro-gyration constraint with $0 \leq m \leq 1$. To convert the PDE boundary layer Eqs. (1)-(4) into the coupled ordinary differential equations, the following similarity transformations are used:

$$\eta = y \sqrt{\frac{a}{v_f}}, \quad \psi = \sqrt{av_f} x f(\eta), \quad u = axf'(\eta), \quad v = -\sqrt{av_f} f(\eta), \quad N = ax \sqrt{\frac{a}{v_f}} g(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (7)$$

Substituting (7) into (2), (3) and (4) the following ordinary differential equations are acquired:

$$(c_1 + K) f'' + c_2 \left[ f'' - f'^2 \right] - Mc_3 f' + Kg' = 0 \quad (8)$$

$$(c_1 + K / 2) g'' + c_2 \left[ f'g' - f'g \right] - K (2g + f') = 0 \quad (9)$$

$$c_4 \theta'' + \Pr c_5 \left[ f \theta' - f' \theta \right] = 0 \quad (10)$$

Following the similarity transformation Eqs. (5) and (6) turn into
\[ f(0) = s, \quad f'(0) = 1, \quad g(0) = -mf''(0), \quad \theta(0) = 1 \]
\[ f'(\infty) \to 0, \quad g(\infty) \to 0, \quad \theta(\infty) \to 0 \]

(11)

(12)

Where \( c_i, \ i = 1 \) to 5 are defined as:

\[
c_1 = \frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\phi)^{2.5}}, \quad c_2 = \frac{\rho_{nf}}{\rho_f} = \left(1 - \phi + \phi \frac{\rho_s}{\rho_f}\right), \quad c_3 = \frac{\sigma_{nf}}{\sigma_f} = \left(1 + \frac{3(\sigma - 1)\phi}{\sigma + 2 - (\sigma - 1)\phi}\right),
\]

\[ c_4 = \frac{k_{nf}}{k_f} = \left(\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}\right), \quad c_5 = \frac{\left(\rho C_p\right)_{nf}}{\left(\rho C_p\right)_{f}} = \left(1 - \phi - \frac{\rho C_p}{\rho C_p}_f\right)
\]

(13)

Here prime denotes differentiation with respect to the similarity variable \( \eta \), \( f(\eta) \), \( g(\eta) \) and \( \theta(\eta) \) are the dimensionless stream function, the dimensionless angular velocity and the dimensionless temperature, respectively.

| Table 1. Thermo-physical properties of H\textsubscript{2}O and Fe\textsubscript{3}O\textsubscript{4} [52] |
|---|---|---|---|---|
| | \( C_p \) (J/kg K) | \( \rho \) (kg/m\textsuperscript{3}) | \( k \) (W/m K) | \( \sigma \) (\Omega m\textsuperscript{-1}) |
| H\textsubscript{2}O | 4179 | 997.1 | 0.613 | 0.05 |
| Fe\textsubscript{3}O\textsubscript{4} | 670 | 5180 | 9.7 | 25000 |

The non-dimensional parameters used in the above equations are the magnetic parameter

\[ M = \frac{\sigma B_0^2}{\rho a}, \] the Prandtl number \( Pr = \frac{\rho C_p v_f}{k} \) and the suction/injection parameter \( s = -\frac{v_w}{\sqrt{a v_f}} \).

3. Exact analytic solutions

According to research conducted by A. Chakrabarti et al. [53], the physical solutions for stretching sheet come from the exponential relation
\[ f(\eta) = s + \frac{1 - e^{-\lambda \eta}}{\lambda} \]

\[ g(\eta) = -m f''(\eta) = m \lambda e^{-\lambda \eta} \]  

(14)

\[ \theta(\eta) = f'(\eta) = e^{-\lambda \eta} \]

The solutions forthwith fulfill the boundary conditions and only the positive solutions are physically right. The Eqs. (8) - (10) produce the relations

\[ (-Km + c_1 + K) \lambda^2 - sc_2 \lambda - c_3 M - c_2 = 0 \]  

(15)

\[ \left( c_1 m + \frac{Km}{2} \right) \lambda^2 - m sc_2 \lambda - 2Km + K - c_2 m = 0 \]  

(16)

\[ \frac{c_4 \lambda^2}{Pr} - sc_5 \lambda - c_3 = 0 \]  

(17)

Working out the solutions of Eqs. (15) – (17) results in

\[ \lambda_1 = \frac{\sqrt{2} \sqrt{Pr c_6 c_7 \left( \sqrt{c_6 c_7} + (Mc_3 + 2K)c_7 \right)}}{2c_6 c_7} \]

\[ \lambda_2 = \frac{\sqrt{2} \sqrt{Pr c_6 c_7 \left( -\sqrt{c_6 c_7} + (Mc_3 + 2K)c_7 \right)}}{2c_6 c_7} \]  

(18)

\[ m = \frac{(-1 + (1 + K) Pr) \lambda^2 - M Pr}{Pr \lambda^2 K} \]

\[ s = \frac{-Pr + \lambda^2}{Pr \lambda} \]

where \( c_i, \ i = 6 \) to 8 are defined as:
\[ c_6 = c_5 (c_1 + K) \text{Pr} - c_2 c_4, \quad c_7 = c_5 \left( \frac{K}{2} \right) \text{Pr} - c_2 c_4, \]
\[ c_8 = \left[ 2K^3 + \left( -6Mc_3 + 4c_1 \right) K^2 - 4c_3 M \left( \frac{-Mc_3}{8} + c_1 \right) K + c_4 c_4^2 M^2 \right] c_5 \text{Pr} \]
\[ -c_2 c_4 \left( Mc_3 - 2K \right)^2 \] (19)

4. Parameters of engineering interests

Some important physical parameters include the entropy generation number, the local skin friction coefficient and the local Nusselt number.

\[ C_{fx} = -\frac{\tau_w}{\rho_m u_w^2}, \quad Nu_x = \frac{xq_w}{k_f \left( T_w - T_{\infty} \right)} \] (20)

Where \( q_w \) is the surface heat flux as given

\[ \tau_w = \left[ \left( \mu_{nf} + \kappa \right) \left( \frac{\partial u}{\partial y} \right) + \kappa N \right]_{y=0}, \quad q_w = -k_{nf} \left( \frac{\partial T}{\partial y} \right)_{y=0} \] (21)

By substituting Eq. (21) into Eq. (20), the reduced local skin friction coefficient \( \text{Re}^{1/2} C_{fx} \) and the local Nusselt number \( \text{Re}^{1/2} Nu_x \) as follow

\[ \text{Re}^{1/2} C_{fx} = \frac{1}{c_2} \left( \frac{1}{c_1} + (1-m)K \right) f'' \left( 0 \right), \quad \text{Re}^{1/2} Nu_x = -c_4 \theta' \left( 0 \right) \] (22)

To improve the efficiency of thermal systems, it is important to calculate entropy generation. The entropy analysis displayed the regions of the system with more energy dissipation. The volumetric rate of the local entropy generation of the micropolar fluid flow containing magnetite ferrofluid with a magnetic field past the stretching sheet stated as follows [54]:

6 5 1 2 4 7 5 1 2 4
3 2 2 2 3
3 1 3 1 1 3 5
8
2
2 4 3
Pr , Pr , 2
2 6 4 4 Pr 8
2
Kc c c K c c c c c c c
McK Mc c K c M c K c c M c
c
c c Mc K
\[ \frac{\partial}{\partial y} \]
\[
S_{\text{gen}}^m = \frac{k_{nf}}{T_x^2} \left( \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right) + \frac{\left( \mu_{nf} + \kappa \right)}{T_x} \left( \frac{\partial u}{\partial y} \right)^2 + \sigma_{nf} \frac{B_0^2}{T_x} u^2
\]

Eq. (23) contains four terms with physical effects; the first term is about heat transfer through a finite temperature difference. The second term describes the fluid friction irreversibility and the last term shows the magnetic effect in the form of joule dissipation irreversibility. The characteristic entropy transfer rate described as:

\[
S_0^m = \frac{k_f (\Delta T)^2}{L^2 T_x^2}
\]

The non-dimensional volumetric entropy generation rate \( N_G \) defined as

\[
N_G = \frac{S_{\text{gen}}^m}{S_0^m} = \frac{\text{Re}_L \text{Br}}{\Omega} \left( \frac{1}{(1-\phi)^{2.5}} + \frac{K}{(f^*)^2} \right) + \frac{k_{nf}}{k_f} \left( \frac{(\theta)^2}{\chi^2} + \text{Re}_L \left( \theta' \right)^2 \right) + \frac{\text{Re}_L \text{Br} \sigma_{nf} 2}{\sigma_f} M \left( f' \right)^2
\]

where \( \text{Re}_L = cL^2/\nu_f \) the characteristic length-based Reynolds number, \( \text{Br} = \mu_f u^2/\nu_f k_f \Delta T \) the Brinkman number, which is the ratio of heat conduction from the surface and the viscous heat produced by shear in the boundary layer and \( \Omega = \Delta T/T_x \) is the dimensionless temperature ratio.

By substituting the gained explicit exact solutions of velocity and temperature fields into the Eq. (25), the exact form of the entropy generation number is achieved as:

\[
N_G = \frac{\text{Re}_L \text{Br}}{\Omega} \left( e^{-\lambda \eta} \right)^2 \left( \left[ \frac{1}{(1-\phi)^{2.5}} + K \right] \left( \lambda \right)^2 + \frac{\sigma_{nf} 2}{\sigma_f} M \right) + \frac{\sigma_{nf} 2}{k_f} \left( \frac{e^{-\lambda \eta}}{\chi^2} + \text{Re}_L \left( \lambda e^{-\lambda \eta} \right)^2 \right)
\]

5. Results and discussion
Tiwari-Das model has been employed to probe the entropy generation analysis, the 2-D boundary layer flow and the heat transfer accompanied by the magnetic field effect with micropolar fluid. The ODEs are solved analytically by introducing the closed-form solution for the flow field and forced convection heat transfer, also the important parameters of engineering. Besides, the ODEs are calculated numerically by Runge-Kutta-Fehlberg using Maple software.

| M  | Analytic     | Numeric      |
|----|--------------|--------------|
| 2  | -1.642262788 | -1.642262831 |
| 6  | -4.551524264 | -4.551524302 |
| 10 | -7.696036304 | -7.696036335 |
| 14 | -11.37389277 | -11.37389281 |
| 16 | -14.21719116 | -14.21719121 |

The local skin friction coefficients were obtained and tabulated by analytical and numerical methods by changing the magnetic parameter and fixing other parameters such as Pr=6.2 and K=50. As can be seen, the magnitude of $Re_x^{1/2} C_{f_x}$ enlarged significantly through increasing M. More accurately, by changing the M from 2 to 16, the $Re_x^{1/2} C_{f_x}$ boosted 765.707%. With the increase of the Lorentz force, the micro-gyration constraint decreases and as a consequence, the $Re_x^{1/2} C_{f_x}$ will be enlarged.
According to Fig. 2, Prandtl number variation on \( f' (\eta) \), \( g (\eta) \), \( \theta (\eta) \) and \( N_g (\eta) \) are intangible. By increasing the Prandtl number, velocity, angular velocity and temperature increments. Velocities and thermal boundary layers thicknesses increase very slightly. On the other hand, over the sheet, the angular velocity is more than velocity.
The hydrodynamic and thermal boundary layers getting thicker through increment in the magnetic parameter (Fig. 3). The angular velocity profiles have particular behaviors. In Adjacent of the sheet, the least M value has the most $g(\eta)$ and almost from the middle of the boundary layer, their relation will be direct. Moreover, the most value of $N_s(\eta)$ belongs to the least M. In other words, boosting the Lorentz force raises the hydrodynamic and thermal boundary layer thicknesses.
Fig. 4. Impact of material parameter on velocity (a), angular velocity (b), temperature (c) and entropy generation number (d) when $\phi=0.03, \text{Pr}=6.2, M = 3, \text{Re}=Br=1, \Omega=0.01, X=0.5$

With the increase of the $K$, the hydrodynamic and thermal boundary layers become thinner based on Fig. 4. Plus, the most value of the $K$ refers to the most value of the angular velocity. The $N_s(\eta)$ grows significantly with the rising of the $K$. More precisely, in the adjacent of the sheet, the $N_s(\eta)$ increases 967.82% through enhancing the $K$ from 10 to 100.
**Fig. 5.** Impact of nanoparticle volume fraction variation on velocity (a), angular velocity (b) and temperature (c) and entropy generation number (d) when 

\[
\text{Pr} = 6.2, K = 10, M = 2, \text{Re} = \text{Br} = 1, \Omega = 0.01, X = 0.5
\]

**Fig. 5.** Illustrated the profiles mentioned above due to the variation in \( \phi \). By addition the magnetic nanoparticles to the micropolar fluid, the hydrodynamic boundary layer thickness augments. Because the thermal conductivity of the nanomagnetic particles is bigger than the thermal conductivity of the base fluid, the temperature gradient would rise by adding \( \phi \). The most values of \( g(\eta) \) and \( N_g(\eta) \) are in the absence of the nanoparticles.
The similar computational process to the local skin friction was done for the Nusselt number by varying the Prandtl number and fixing other parameters such as $K=10$ and $M=3$ according to Table 3. The magnitude of the $\text{Re}_{x}^{-\nicefrac{1}{2}} \text{Nu}_{x}$ reduced intangibly due to increased Pr. More precisely, by changing the Pr from 3 to 10, the $\text{Re}_{x}^{-\nicefrac{1}{2}} \text{Nu}_{x}$ diminished 0.0865955%.

**Table 3.** Comparison of the local Nusselt number with varying Prandtl number

| Pr  | Analytic       | Numeric        |
|-----|----------------|----------------|
| 3   | -1.281597334   | -1.281598545   |
| 4   | -1.281563337   | -1.281563965   |
| 6   | -1.281109592   | -1.281109760   |
| 8   | -1.280745695   | -1.280745738   |
| 10  | -1.280487528   | -1.280487539   |

**Fig. 6.** Impact of Brinkman number on entropy generation number when $\text{Pr}=6.2, K=10, M=2, \text{Re}=1, \Omega=0.01, X=0.5$. 
Fig. 6 represents the variation of the entropy generation number for several values of the Brinkman number. It is noteworthy that the temperature rises due to the enhance in the Br, thus the $N_g(\eta)$ boost. Accordingly, the maximum value of the $N_g(\eta)$ belongs to the maximum value of the Brinkmann number.

6. Conclusion

In this study, the fluid flow and heat transfer of Fe$_3$O$_4$ water-based micropolar ferrofluids over a stretching sheet has been analytically scrutinized. Exact explicit solutions for the $f'(\eta)$, $g(\eta)$ and $\theta(\eta)$ have been developed. The effects of Pr, $K$, $M$, $\phi$ and Br on the $f'(\eta)$, $g(\eta)$, $\theta(\eta)$ and the $N_g(\eta)$ have been stated and explained in detail. The $Re_x^{1/2} C_{fx}$ and the $Re_x^{-3/2} Nu_x$ were computed analytically and numerically for the several values of the $M$ and the Pr, respectively.

The notable findings of the study can be sum up as:

- Exact solutions are achieved for $f(\eta)$, $f'(\eta)$, $g(\eta)$, $\theta(\eta)$ and $N_g(\eta)$.
- The $Re_x^{1/2} C_{fx}$ increase with the augment of the $M$.
- The $Re_x^{-3/2} Nu_x$ decrease with the augment of the Pr.
- An increase in the Pr increases the hydrodynamic and thermal boundary layer thicknesses insignificantly.
- With an increase in the M, the hydrodynamic and thermal boundary layers get thicker.
- The $g(\eta)$ is more than the $f'(\eta)$ for the least value of the M.
The $N_s(\eta)$ has a direct relation with $K$ and $Br$ either $Re$; Moreover, an inverse relation with the $Pr$.

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