Number-resolved photocounter for propagating microwave mode

R. Dassonneville,¹ R. Assouly,¹ T. Peronnin,¹ P. Rouchon,² and B. Huard¹

¹Univ Lyon, ENS de Lyon, Univ Claude Bernard, CNRS, Laboratoire de Physique, F-69342 Lyon, France
²Centre Automatique et Systèmes, Mines-ParisTech, PSL Research University, 60 bd Saint-Michel, 75006 Paris, France.
QUANTIC team, INRIA de Paris, 2 rue Simone Iff, 75012 Paris, France.
(Dated: April 13, 2020)

Detectors of propagating microwave photons have recently been realized using superconducting circuits. However a number-resolved photocounter is still missing. In this letter, we demonstrate a single-shot counter for propagating microwave photons that can resolve up to 3 photons. It is based on a pumped Josephson Ring Modulator that can catch an arbitrary propagating mode by frequency conversion and store its quantum state in a stationary memory mode. A transmon qubit then counts the number of photons in the memory mode using a series of binary questions. Using measurement based feedback, the number of questions is minimal and scales logarithmically with the maximal number of photons. The detector features a detection efficiency of 0.96 ± 0.04, and a dark count probability of 0.030 ± 0.002 for an average dead time of 4.5 µs. To maximize its performance, the device is first used as an in situ waveform detector from which an optimal pump is computed and applied. Depending on the number of incoming photons, the detector succeeds with a probability that ranges from 56% to 99%.

IntroductionPhoton detectors are an important element in the quantum optics toolbox. At optical frequencies, detectors such as single-photon avalanche photodiodes or superconducting nanowire single-photon detectors are readily available [1]. In contrast, at GHz frequencies, this kind of absorptive detectors are harder to realize due to the low energy of the microwave photons, roughly 5 orders of magnitude lower in energy compared to their optical counterparts. Detecting the microwave photons of a stationary mode is nowadays routinely performed using the dispersive interaction with a qubit [2–6]. Detecting propagating microwave photons remains challenging because the light-matter interaction is smaller. Yet some photon detectors for propagating modes have been developed based on various approaches [7]: direct absorption [8, 9], encoding parity in the phase of a qubit [10, 11], encoding the probability to have a single photon in a qubit excitation [12] or reservoir engineering [13]. Several implementations of a photocounter – a microwave photodetector able to resolve the number of photons – for a propagating mode have been proposed [11, 14–16]. However, such a photocounter has yet to be demonstrated. Refs. [10, 11] only distinguish the parity of the number of photons. Refs. [12, 13] only distinguish Fock state |1⟩ from the rest while Refs. [8, 9] distinguish 0 photon from at least 1.

Here, we demonstrate a photocounter, i.e. a photon detector that can resolve the number of photons in a given propagating mode. To optimize the efficiency of our counter, we devised a way to calibrate in situ the arrival time and envelope of the propagating mode. The device can detect photons in a 20 MHz band around 10.22 GHz and is able to distinguish between 0, 1, 2 and 3 photons using active reset by measurement based feedback. Finally, we propose a parameter free model that predicts accurately the behavior of the counter as demonstrated by coherent state photocounting and Wigner tomography.

Setup and protocolThe purpose of a photocounter is to count the number of photons in a propagating mode with state |ψ⟩ by providing an integer outcome n with probability |⟨ψ|n⟩|². Our photocounter proceeds in three steps (Fig. 1.a). In step 1, it catches the incoming wavepacket and converts it into a high-Q sta-
tionary mode (memory). Then, in step 2, it counts the number of photons in the memory using an ancillary qubit. Finally (step 3), it resets the memory and qubit in their ground state. The catch and memory reset operations (1,3) are performed by frequency conversion using a Josephson Ring Modulator (JRM) [17, 18]. The input transmission line is coupled to a buffer mode at frequency \( \omega_b/2\pi = 10.220 \text{GHz} \), which sets the operating bandwidth of the counter to \( \kappa_b/2\pi = 20 \text{MHz} \). When pumped by a coherent tone of amplitude \( p(t) \) at \( \omega_b - \omega_m \), the JRM introduces a frequency conversion term \( \hat{H}_{\text{JRM}} = g_{d=0}p(t) b \hat{m}^\dagger + \text{h.c.} \) between the buffer \( b \) and the memory \( \hat{m} \). The memory is a high-Q mode at frequency \( \omega_m/2\pi = 3.74527 \text{GHz} \) with a relaxation time \( T_{1,\text{in}} = 4 \mu\text{s} \). When the memory is initially empty, this term enables us to catch the incoming wavepacket onto the buffer by storing its quantum state in the memory. Conversely, when the counting operation is over, we use it to release the photons from the memory into an arbitrary outgoing wavepacket.

**Catch efficiency and envelope detection.** From the point of view of the memory, the pumped JRM induces a tunable coupling to a transmission line [19]. It is thus possible to catch or release an arbitrary wavepacket into and from the memory [20–26]. Besides, the parasitic non-linearities induced by the Josephson junctions of the JRM can be canceled by setting the flux through the JRM optimally, which we did [27]. Using input-output formalism in the rotating frame, and neglecting the relaxation of the memory, the dynamics is captured by

\[
\frac{d\hat{b}}{dt} = -\frac{\kappa_b}{2} \hat{b} - g_{d=0}p(t) \hat{m} + \sqrt{\kappa_b} \hat{b}_{\text{in}}(t) \\
\frac{d\hat{m}}{dt} = g_{d=0}^* p(t) \hat{b}.
\]

(1)

For any given envelope \( \langle b_{\text{in}}(t) \rangle \) of the incoming wavepacket, there exists an optimal pump \( p_{\text{opt}}(t) \) for which the incoming quantum state is perfectly swapped into the memory [28]. For instance if the incoming wavepacket is \( b_{\text{in}}(t) \propto 1/\cosh\left(\sqrt{\pi^2t/\sigma}\right) \) (Fig. 4.a), the optimal catching pump is given by \( p_{\text{opt}}(t) \propto (1 + 2^{\lambda}\tanh(\lambda\kappa_b t/4))(e^{\lambda\kappa_b t/2} + 1 - \lambda/2)^{-1/2} \) where \( \lambda = 1/\sqrt{8\pi/\kappa_b \sigma} \). Note that even at non-optimal flux through the JRM, an optimal pump can be found to obtain a complete-catch [27].

To measure the catch efficiency \( \eta \), we follow a catch-wait-release protocol as in Ref. [22], where we compare the coherent energy of an input wavepacket to the one of the released wavepacket after a waiting time \( t_w \) between the catch and the release events. This ratio of energy is then given by \( \eta^2e^{-t_w/\tau_{1,m}} \) assuming that the catch and the release have the same efficiency. We could measure a lower bound \( \eta \geq 0.92 \) but a parasitic reflection in the measurement setup prevented us from providing a more accurate figure.

**In situ calibration of the incoming wavepacket envelope.** a) Amplitude of an arbitrary incoming wavepacket sent onto the buffer and of the sampling pump pulse. A following measurement of the mean photon number \( \langle n(t_d) \rangle \) in the memory is performed using the qubit. b) Left panels: various incoming waveforms. Right panels: Solid blue lines (grey shadows) show the measured (predicted using Eq. (1)) mean photon number \( \langle n \rangle \) normalized by its maximum \( n_{\text{max}} \).

In order to generate the optimal pump \( p_{\text{opt}}(t) \) for an arbitrary incoming wavepacket at \( \omega_m \), one needs to determine the envelope \( \langle b_{\text{in}}(t) \rangle \). Interestingly, the envelope of any incoming waveform with a bandwidth smaller than \( \kappa_b \), can be determined in situ. The photocounter can indeed operate as a sample and hold power meter. Turning on the pump for a short sampling time of 20 ns after a variable delay \( t_d \) and counting the mean number of photons in the memory, using the coupled transmon qubit [27], enables us to directly probe \( \langle b_{\text{in}}(t_d) \rangle \) up to global prefactor (Fig. 2.a). We demonstrate this functionality on a variety of generated waveforms displayed in Fig. 2.b (left panel). The distortion of the waveforms introduced by the finite bandwidth \( \kappa_b \) of the counter and the non zero sampling time can be seen in the measured mean photon number \( \langle n \rangle \) as a function of \( t_d \) (right panel). The simple model Eq. (1) accurately reproduces the measured envelopes, where the only free parameter is the difference of propagation time of 15 ns between the buffer and the pump lines.

**Optimal counting** Once the incoming wavepacket is characterized and efficiently caught, step 2 consists in measuring the number of photons present in the memory in a single shot manner. To do so, we use a transmon qubit at frequency \( \omega_q/2\pi = 4.32731 \text{GHz} \) dispersively coupled to the memory such that \( \hat{H}_{\text{cm}} = -\chi \hat{m}^\dagger \hat{m} [e]e \). Owing to a dispersive shift \( \chi/2\pi = 3.28 \text{MHz} \) much larger than the qubit decoherence rate \( T_2 = 13.6 \mu\text{s} \), the device operates in the photon-number resolved regime [30]. It is thus possible to access information about the photon number by entangling the memory mode with the qubit.
FIG. 3. Binary decomposition. a) Pulse sequence used to extract $u_k$ experimentally. The green corresponds to taking the remainder modulo $2^k$ of the photon number, the red to the subtraction of the previously found digits $n_{k-1} = \lfloor u_{k-1} \cdots u_1 \rfloor_2$ and the blue to the extraction of the result via a measurement of the qubit. The $\pi/2$ pulses consist of sech waveforms with $\sigma = 4\text{ ns}$ truncated at $4\sigma$ further optimized to mitigate the effect of the transmon qubit anharmonicity of $-98\text{ MHz}$ [29]. b) Trajectory of the qubit on the Bloch sphere when the cavity is in a Fock state $|n\rangle$ with yet unknown bit $u_k$. Left ①: the qubit is prepared in $(|g\rangle + i|e\rangle)/\sqrt{2}$ with an unconditional $\pi/2$ pulse around $x$. Middle ②: Trajectory of the qubit states $|u_k = 0\rangle$ and $|u_k = 1\rangle$ corresponding to the two possible values of the $k$-th bit of the photon number during the waiting time $T_k$. Right ③: The last $\pi/2$ pulse around an axis shifted by an angle $\phi(n_{k-1})$ from the $x$ axis maps $u_k$ onto ground or excited states.

and reading out its state. It is made possible by another resonator (the readout) dispersively coupled to the qubit. We optimized the readout fidelity up to $97\%$ in $252\text{ ns}$, using a CLEAR-like sequence [27, 31], mostly limited by the finite qubit relaxation time $T_1 = 7.1\text{ \mu s}$. The actual counting uses an optimal scheme that measures the number of photons bit by bit [32, 33]. We denote $u_k$ the $k$-th least significant bit of $n = [u_N u_{N-1} \ldots u_1]_2$. Starting from $u_1$, each value of $u_k$ is encoded into the transmon qubit state and then read out. The main difficulty in implementing this scheme comes from the need to know the value of $n_{k-1} = [u_{k-1} \cdots u_1]_2$ in order to extract $u_k$. Each step $Q_k$ (Fig. 3.a) of the recursive determination of the $u_k$’s is based on the relation

$$2^k u_k = n - n_{k-1} \mod 2^k.$$  

The transmon qubit is prepared in $|g\rangle + i|e\rangle/\sqrt{2}$ with a $\pi/2$ pulse (Fig. 3.b①). Then, the memory and qubit interact dispersively for a time $T_k = \frac{2\pi}{\sqrt{\sigma}}$. $T_k$ is chosen such that the qubit ends up in one of two orthogonal states $|u_k = 0\rangle$ and $|u_k = 1\rangle$ that only depend on the value $u_k$ (Fig. 3.b②). Precisely, the phase of the qubit states picks up an offset $\phi(n_{k-1}) = -\frac{n_{k-1} \pi}{2^k}$ for $u_k = 0$. Finally, using the knowledge of $n_{k-1}$, it is possible to map $|u_k = 0\rangle$ and $|u_k = 1\rangle$ onto $|e\rangle$ and $|g\rangle$ using a second $\pi/2$ pulse around the right axis (Fig. 3.b③). Reading out the qubit state thus provides $u_k$ directly. This scheme is optimal in the sense that each binary question $Q_k$ is able to extract one bit of new information about the photon number. The number of binary questions that are required to determine a photon number $n$ is $N = \lceil\log_2 n\rceil$. Note that it is possible to avoid the feedforward for $n_{k-1}$ using optimal quantum control algorithm [34], although it would lead to degraded counting fidelities with the device we use.

**Single-shot photocounting**

We now demonstrate the number resolved photocounting using 2 questions $Q_1$ and $Q_2$. The device thus resolves photon numbers from 0 to 3. The feedforward of $n_1$ is performed with minimal added latency ($200\text{ ns}$) using Quantum Machines’ FPGA-based control system (OPX). To benchmark the photocounter, we send at its input a sech waveform in a coherent state of complex amplitude $\alpha$ using a microwave source (Fig. 4.a). This state is caught in the memory using an optimal pump followed by the two binary questions $Q_1$ and $Q_2$ that reveal a number $n_2 = [u_2 u_1]_2$ between 0 and 3. For an ideal photocounter, the distribution of $n_2$ would follow a Poisson distribution modulo 4, $P_{n_2} = e^{-|\alpha|^2} \sum_j \frac{|\alpha|^{2(j+1)}}{(j+1)!}$ (dashed lines in Fig. 4.b). The measured probabilities $P_{n_2}$ (green diamonds) qualitatively follow the ideal Poisson distribution. However we obtain a more quantitative agreement by solving a master equation that takes into account imperfections like the finite lifetimes of the memory and qubit, the non-zero effective temperature, and nonlinear terms [35] $\hat{H}_K = -K \hat{m}^2 \hat{m}^2 - K_e |\alpha|\langle \alpha | \hat{m}^2 \hat{m}^2$ with $K/2\pi = 27\text{ kHz}$ and $K_e/2\pi = 75\text{ kHz}$. All the above parameters are calibrated using independent measurements [27].

Using this model, we compute the probabilities $P_{n_2}(m)$ that the counter would measure $m \mod 4$ if a Fock state $|n\rangle$ was sent at the input. They are shown in Table I. If the detector was giving totally random outcomes, the probabilities would all be equal to $25\%$ since there are 4 possible answers. Here, we obtain fidelities $P_{\{n\}}(n)$ well above $25\%$ and infidelities $P_{\{n\}}(m \neq n)$ smaller or of the same order of $25\%$. Interestingly, degraded to a photodetector that clicks when $m \neq 0$, these figures imply a detection fidelity of $1 - P_{\{\downarrow\}}(0) > 96\%$ for a single photon. It is possible to estimate the various sources of infidelities using the model. We discuss below the 3 main sources of errors, which are the finite lifetimes of the memory and qubit and the correction $K_e$ to the memory anharmonicity when the qubit is excited (see supplemental material [27]). The finite qubit lifetime affects the various $n_2$ values differently owing to the choice of encoding in the qubit state during questions $Q_k$’s. It is possible to choose which photon number to affect the least by swapping the roles of $|g\rangle$ and $|e\rangle$ using a $\pi$-shift in the phase of the second pulse of questions $Q_k$’s. The photon number corresponding to the qubit being in the excited state after each question is the one
with maximum error. Here, we choose to minimize the error on \( n_2 = 0 \) and thus minimize the dark count of the counter to a measured probability of 3\% (measurement of \( 1 - P_3 \) at \( \alpha = 0 \) in Fig. 4.b). When the incoming number of photons increases, the memory relaxation starts to limit the fidelity since the loss rate of the memory is proportional to the number of photons that are stored. It explains most of the decrease of fidelity with photon number from 99\% down to 56\%. Finally, because of the non-zero \( K_\alpha \), during the time \( T_k \) of the question \( Q_k \), the qubit acquires an additional parasitic phase that rapidly increases with the photon number resulting in larger infidelities for higher \( n \).

Beside, the counter presents a short non-deterministic dead time of 4.5 µs on average thanks to the reset of both the memory and the qubit (Fig. 4.a). The memory is reset to its ground state by a release pump that empties its photons into the transmission line. The qubit is reset to its ground state using a measurement based feedback loop.

### Wigner tomography

As for any measurement device, the counter exerts a backaction on the quantum state of the incoming mode. Using the qubit, it is possible to perform a Wigner tomography of the collapsed quantum state of the memory conditioned on the outcome \( n_2 \) of the counter. The Wigner function \( W(\beta) \) is given by the average outcome of the parity measurement after a calibrated coherent displacement of the memory with amplitude \( -\beta \) [36–38]. The top panels of Fig. 4.c show the Wigner functions for \( n_2 \) from 0 to 3 after catching a coherent state of amplitude \( |\alpha|^2 = 0.5 \). They are heralded on the result of the counter with a total of 44000 realizations per pixel. The bottom panels show the computed Wigner functions using our model above. For an outcome \( n_2 \), an ideal photocounter would project the incoming state \( |\psi\rangle \) into \( |\psi_{n_2}\rangle \propto \sum_j (n_2 + 4j) |n_2 + 4j\rangle |\psi\rangle \).

Given the small mean photon number \( |\alpha|^2 = 0.5 \), the ideal state is close to Fock state \( |n_2\rangle \). The measured Wigner functions \( W(\beta) \) for \( n_2 = 0, 1 \) or 2 are indeed close to what would be obtained for pure Fock states \( |n_2\rangle \). However for \( n_2 = 3 \), the finite lifetimes of both memory and qubit induce a mixture of various Fock states, and the Wigner function does not exhibit the expected fringes. To quantify this agreement, we compute the fidelity \( \mathcal{F}(\rho, \rho') \) between the collapsed quantum state of the memory \( \rho \) and the ideal projected quantum state \( \rho_{n_2} = |\psi_{n_2}\rangle\langle\psi_{n_2}| \). Since we have direct experimental access to the Wigner functions, we chose the following definition of the fidelity [39, 40] 

\[
\mathcal{F}(\rho, \rho') = \text{Tr}(\rho\rho') + \sqrt{(1 - \text{Tr}(\rho^2))(1 - \text{Tr}(\rho'^2))}.
\]

It can thus be computed directly from the Wigner functions using 

\[
\text{Tr}(\rho_1\rho_2) = \pi \int W_{\rho_1}(\beta)W_{\rho_2}(\beta) d^2\beta.
\]

![FIG. 4. Photocounting coherent states. a) Pulse sequence showing an incoming mode on the buffer with a coherent state of amplitude \( \alpha \) and the optimal shape of the pump to catch the wavepacket with minimal distortion. The qubit performs photocounting bit by bit with pulse sequences \( Q_k \)'s described in Fig. 3. \( Q_2 \) uses the outcome of \( Q_1 \) in a feedback protocol that adds as little as 200 ns delay. Finally, a direct Wigner tomography of the memory can be performed [36–38] before the memory and qubit are reset. b) Green diamonds: measured probabilities \( P_{\alpha} \) of finding a number \( n_2 = n \bmod 4 \) photons as a function of the mean photon number \( |\alpha|^2 \) of the incoming coherent state after 200000 runs of the sequence. Dashed lines: modulo 4 Poisson distribution. Solid lines: Master equation solution without any free parameter. c) Corresponding measured (top) and simulated (bottom) Wigner functions for \( \alpha = \sqrt{0.5} \) mean photons [27]. From left to right, the Wigner function is heralded on the counter outcome \( n_2 = 0, 1, 2 \) and 3.

---

**TABLE I.** Probabilities of getting the outcome \( m \) if the incoming mode is in Fock state \( |n\rangle \). The probabilities are computed using the master equation validated by Fig. 4. Diagonal terms are all above 25\%, which would correspond to a completely random counter with 4 possible outcomes.

| \( P_{\alpha_0}(m) \) | \( 0 \) | \( 1 \) | \( 2 \) | \( 3 \) |
|---|---|---|---|---|
| \( m = 0 \) | 99\% | 3.6\% | 21\% | 5.4\% |
| \( m = 1 \) | 0.3\% | 79\% | 4.4\% | 29\% |
| \( m = 2 \) | 0.6\% | 1.0\% | 73\% | 10\% |
| \( m = 3 \) | 0.3\% | 16\% | 1.5\% | 56\% |
sured Wigner functions in Fig. 4c (top panels), we obtain fidelities of 86%, 52%, 32% and 4.9% for \( n_2 = 0, 1, 2 \) and 3 respectively. This deviation from the ideal case is well captured by our model, which predicts the measured collapsed quantum states with fidelities between top and bottom panels of Fig. 4c above 97% for the four outcomes of the counter. From the model, it appears that the dominant origin for the non-idealities are the finite lifetimes of the qubit and memory [27].

**Conclusion**  We have developed a photocounter using measurement based feedback that is able to resolve the photon number from \( n = 0 \) up to \( n = 3 \) in a propagating microwave mode. It has a low dark count probability of 3%, and a short dead time of 4.5\( \mu \)s. The counter also features a time-resolved power-meter that can be used to determine the envelope of the incoming waveform in situ, and to optimize the detection efficiency up to \( \eta = 0.96 \pm 0.04 \). Interestingly, during reset, the collapsed quantum state of the memory is released back into an arbitrary propagating mode. Future devices with longer lifetimes would considerably improve the fidelities \( \mathcal{F} \) above, which would then turn our photocounter Quantum Non Demolition. The counter would then quickly scale up to resolve higher photon number thanks to its logarithmic complexity. The photocounter can also be used in a degraded mode to measure parity by asking a single question \( Q_1 \) as in Refs. [10, 11], and thus perform propagating Wigner tomography [41]. Microwave photodetection and photocounters enable quantum optics-like experiments in the microwave range and facilitate the implementation of a quantum network. For instance, photodetection has made possible the entanglement between remote stationary qubits [12, 25, 26]. However, a photocounter is required for some protocols, such as quantum illumination [42–45], that rely on the ability to resolve the photon number. Our device could then demonstrate quantum illumination in the microwave domain [46].

We are grateful to Olivier Buisson, Michel Devoret, Zaki Leghtas, Danijela Marković, Mazyar Mirrahimi, Alain Sarlette for discussions. We acknowledge IARPA and Lincoln Labs for providing a Josephson Traveling-Wave Parametric Amplifier. The device was fabricated in the cleanrooms of Collège de France, ENS Paris, CEA Saclay, and Observatoire de Paris. The feedback code was developed in collaboration with Quantum Machines. This work is part of a project that has received funding from the European Union’s Horizon 2020 research and innovation program under grant agreement No 820565.

---

[1] R. H. Hadfield, Single-photon detectors for optical quantum information applications, Nat. Photonics 3, 696 (2009).

[2] S. Gleyzes, S. Kuh, C. Guerlin, J. Bernu, S. Delégilse, U. Busk Hoff, M. Brune, J. M. Raimond, and S. Haroche, Quantum jumps of light recording the birth and death of a photon in a cavity, Nature 446, 297 (2007).

[3] C. Guerlin, J. Bernu, S. Delégilse, C. Sayrin, S. Gleyzes, S. Kuhr, M. Brune, J. M. Raimond, and S. Haroche, Progressive field-state collapse and quantum non-demolition photon counting, Nature 448, 889 (2007).

[4] B. R. Johnson, M. D. Reed, A. A. Houck, D. I. Schuster, L. S. Bishop, E. Ginossar, J. M. Gambetta, L. Dicarlo, L. Frunzio, S. M. Girvin, and R. J. Schoelkopf, Quantum non-demolition detection of single microwave photons in a circuit, Nat. Phys. 6, 663 (2010).

[5] P. J. Leek, M. Baur, J. M. Fink, R. Bianchetti, L. Steffen, S. Filipp, and A. Wallraff, Cavity quantum electrodynamics with separate photon storage and qubit readout modes, Phys. Rev. Lett. 104, 6 (2010).

[6] L. Sun, A. Petreno, Z. Leghtas, B. Vlastakis, G. Kirchner, K. M. Sliwa, A. Narla, M. Hatridge, S. Shankar, J. Blumoff, L. Frunzio, M. Mirrahimi, M. H. Devoret, and R. J. Schoelkopf, Tracking photon jumps with repeated quantum non-demolition parity measurements, Nature 511, 444 (2014).

[7] X. Gu, A. Frisk, A. Miranowicz, Y.-x. Liu, and F. Nori, Microwave photonics with superconducting quantum circuits, Phys. Rep. 718–719, 1 (2017).

[8] Y. F. Chen, D. Hover, S. Sendelbach, L. Maurer, S. T. Merkel, E. J. Pritchett, F. K. Wilhelm, and R. McDermott, Microwave photon counter based on josephson junctions, Phys. Rev. Lett. 107, 1 (2011).

[9] K. Inomata, Z. Lin, K. Koshino, W. D. Oliver, J. S. Tsai, T. Yamamoto, and Y. Nakamura, Single microwave-photon detector using an artificial A-type three-level system, Nat. Commun. 7, 1 (2016).

[10] J. C. Besse, S. Gasparinetti, M. C. Collodo, T. Walter, P. Kurpiers, M. Pechal, C. Eichler, and A. Wallraff, Single-Shot Quantum Nondemolition Detection of Individual Itinerant Microwave Photons, Phys. Rev. X 8, 21003 (2018).

[11] S. Kono, K. Koshino, Y. Tabuchi, A. Noguchi, and Y. Nakamura, Quantum non-demolition detection of an itinerant microwave photon, Nat. Phys. 14, 546 (2018).

[12] A. Narla, S. Shankar, M. Hatridge, Z. Leghtas, K. M. Sliwa, E. Zalys-Geller, S. O. Mundhada, W. Pfaff, L. Frunzio, R. J. Schoelkopf, and M. H. Devoret, Robust concurrent remote entanglement between two superconducting qubits, Phys. Rev. X 6 (2016).

[13] R. Lescanne, S. Delégilse, E. Albertinale, U. Réglade, T. Capelle, E. Ivanov, T. Jacqmin, Z. Leghtas, and E. Flurin, Detecting itinerant microwave photons with engineered non-linear dissipation (2019), arXiv:1902.05102 [quant-ph].

[14] G. Romero, J. J. García-Ripoll, and E. Solano, Microwave photon detector in circuit QED, Phys. Rev. Lett. 102, 1 (2009).

[15] B. Royer, A. L. Grimsso, A. Choquette-poitevin, and A. Blais, Itinerant Microwave Photon Detector, Phys. Rev. Lett. 120, 203602 (2018).

[16] A. M. Sokolov and F. K. Wilhelm, A superconducting detector that counts microwave photons up to two (2020), arXiv:2003.04625 [quant-ph].

[17] N. Bergeal, F. Schackert, M. Metcalfe, R. Vijay, V. E. Manucharyan, L. Frunzio, D. E. Prober, R. J. Schoelkopf, S. M. Girvin, and M. H. Devoret, Phase-preserving am-
plification near the quantum limit with a Josephson ring modulator, Nature 465, 64 (2010).

[18] N. Roch, E. Flurin, F. Nguyen, P. Morfin, P. Campagne-Ibarcq, M. H. Devoret, and B. Huard, Widely Tunable, Nondegenerate Three-Wave Mixing Microwave Device Operating near the Quantum Limit, Phys. Rev. Lett. 108, 147701 (2012).

[19] T. Peronnin, D. Marković, Q. Ficheux, and B. Huard, Sequential measurement of a superconducting qubit (2019), arXiv:1904.04635 [quant-ph].

[20] Y. Yin, Y. Chen, D. Sank, P. J. J. O’Malley, T. C. White, R. Barends, J. Kelly, E. Lucero, M. Mariotani, A. Megrant, C. Neill, A. Vainsencher, J. Wenner, A. N. Korotkov, A. N. Cleland, and J. M. Martinis, Catch and release of microwave photon states, Phys. Rev. Lett. 110, 1 (2013).

[21] J. Wenner, Y. Yin, Y. Chen, R. Barends, B. Chiaro, E. Jeffrey, J. Kelly, A. Megrant, J. Y. Mutus, C. Neill, P. J. J. O’Malley, P. Roushan, D. Sank, A. Vainsencher, T. C. White, A. N. Korotkov, A. N. Cleland, and J. M. Martinis, Catching time-reversed microwave coherent state photons with 99.4% absorption efficiency, Phys. Rev. Lett. 112, 1 (2014).

[22] E. Flurin, The Josephson Mixer, a Swiss army knife for microwave quantum optics, Ph.D. thesis, École Normale Supérieure (2014).

[23] C. J. Axline, L. D. Burkhart, W. Pfaff, M. Zhang, K. Chou, P. Campagne-Ibarcq, P. Reinhold, L. Frunzio, S. M. Girvin, L. Jiang, M. H. Devoret, and R. J. Schoelkopf, On-demand quantum state transfer and entanglement between remote microwave cavity memories, Nat. Phys. 14, 705 (2018).

[24] Y. P. Zhong, H. S. Chang, K. J. Satzinger, M. H. Chou, A. Bienfait, C. R. Comner, Dumar, J. Grebel, G. A. Pears, R. G. Povey, D. I. Schuster, and A. N. Cleland, Violating Bell’s inequality with remotely connected superconducting qubits, Nat. Phys. 15, 4 (2019).

[25] P. Campagne-Ibarcq, E. Zalyss-Geller, A. Narla, S. Shankar, P. Reinhold, L. Burkhart, C. Axline, W. Pfaff, L. Frunzio, R. J. Schoelkopf, and M. H. Devoret, Deterministic Remote Entanglement of Superconducting Circuits through Microwave Two-Photon Transitions, Phys. Rev. Lett. 120, 200501 (2018).

[26] P. Kurpiers, P. Magnusd, T. Walter, B. Royer, M. Pechal, J. Heinsoo, Y. Salathé, A. Akin, S. Storj, Z. C. Besse, S. Gasparinetti, A. Blais, and A. Wallraff, Deterministic quantum state transfer and remote entanglement using microwave photons, Nature 558, 264 (2018).

[27] See Supplementary Material.

[28] A. N. Korotkov, Flying microwave qubits with nearly perfect transfer efficiency, Phys. Rev. B - Condens. Matter Mater. Phys. 84, 1 (2011).

[29] F. Motzoi, J. M. Gambetta, P. Rebentrost, and F. K. Wilhelm, Simple Pulses for Elimination of Leakage in Weakly Nonlinear Qubits, Phys. Rev. Lett. 103, 110501 (2009).

[30] D. I. Schuster, A. A. Houck, J. A. Schreier, A. Wallraff, J. M. Gambetta, A. Blais, L. Frunzio, J. Majer, B. Johnson, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Resolving photon number states in a superconducting circuit, Nature 445, 515 (2007).

[31] D. T. McClure, H. Paik, L. S. Bishop, M. Steffen, J. M. Chow, and J. M. Gambetta, Rapid Driven Reset of a Qubit Readout Resonator, Phys. Rev. Applied 5, 11001 (2016).

[32] S. Haroche, M. Brune, and J. Raimond, Measuring photon numbers in a cavity by atomic interferometry: optimizing the convergence procedure, Journal de Physique II 2, 659 (1992).

[33] R. Heeres, P. Reinhold, and R. Schoelkopf, Private communication (2016).

[34] C. S. Wang, J. C. Curtis, B. J. Lester, Y. Zhang, Y. Y. Gao, J. Freeze, V. S. Batista, P. H. Vavcaro, I. L. Chuan, L. Frunzio, L. Jiang, S. M. Girvin, and R. J. Schoelkopf, Efficient multiphoton sampling of molecular vibronic spectra on a superconducting bosonic processor (2019), arXiv:1908.03598 [quant-ph].

[35] M. Khezri, E. Mlinar, J. Dressel, and A. N. Korotkov, Measuring a transmon qubit in circuit QED: Dressed squeezed states, Phys. Rev. A 94, 12347 (2016).

[36] L. G. Lutterbach and L. Davidovich, Method for Direct Measurement of the Wigner Function in Cavity QED and Ion Traps, Phys. Rev. Lett. 78, 2547 (1997).

[37] P. Bertet, A. Auffèves, P. Maiti, S. Osnaghi, T. Meunier, M. Brune, J. M. Raimond, and S. Haroche, Direct Measurement of the Wigner Function of a One-Photon Fock State in a Cavity, Phys. Rev. Lett. 89, 200402 (2002).

[38] B. Vlastakis, G. Kirchmair, Z. Leghtas, S. E. Nigg, L. Frunzio, S. M. Girvin, M. Mirrahimi, M. H. Devoret, and R. J. Schoelkopf, Deterministically encoding quantum information using 100-photon Schrödinger cat states, Science 342, 607 (2013).

[39] P. E. M. F. Mendonça, R. d. J. Napolitano, M. A. Marchiolli, C. J. Foster, and Y.-C. Liang, Alternative fidelity measure between quantum states, Phys. Rev. A 78, 052330 (2008).

[40] J. A. Miszczak, Z. Puchała, P. Horodecki, A. Uhlmann, and K. Zyczkowski, Sub- and super-fidelity as bounds for quantum fidelity, Quantum Info. Comput. 9, 103130 (2009).

[41] J.-c. Besse, S. Gasparinetti, M. C. Cillodo, T. Walter, A. Remm, J. Krause, C. Eichler, and A. Wallraff, Parity Detection of Propagating Microwave Fields, Phys. Rev. X 10, 11046 (2019).

[42] S.-H. Tan, B. I. Erkmen, V. Giovannetti, S. Guha, S. Lloyd, L. Maccione, S. Pirandola, and J. H. Shapiro, Quantum Illumination with Gaussian States, Phys. Rev. Lett. 101, 253601 (2008).

[43] S. Lloyd, Enhanced Sensitivity of Photodetection via Quantum Illumination, Science 321, 1463 (2008).

[44] S. Guha and B. I. Erkmen, Gaussian-state quantum-illumination receivers for target detection, Phys. Rev. A 80, 052310 (2009).

[45] Z. Zhang, S. Mouradian, F. N. C. Wong, and J. H. Shapiro, Entanglement-Enhanced Sensing in a Lossy and Noisy Environment, Phys. Rev. Lett. 114, 110506 (2015).

[46] J. H. Shapiro, The quantum illumination story (2019), arXiv:1910.12277 [quant-ph].
Supplementary Material
Number-resolved photocounter for propagating microwave mode

R. Dassonneville,¹ R. Assouly,¹ T. Peronnin,¹ P. Rouchon,² and B. Huard¹

¹Univ Lyon, ENS de Lyon, Univ Claude Bernard, CNRS, Laboratoire de Physique, F-69342 Lyon, France
²Centre Automatique et Systèmes, Mines-ParisTech, PSL Research University, 60 bd Saint-Michel, 75006 Paris, France.
QUANTIC team, INRIA de Paris, 2 rue Simone Iff, 75012 Paris, France.
(Dated: April 13, 2020)
I. MEASUREMENT SETUP

The sample and its fabrication are described in Ref. [1]. The sample is cooled down to 24 mK in a BlueFors LD250 dilution refrigerator. The diagram of the microwave wiring is given in Fig. S1. The buffer, memory, qubit and readout pulses are generated by modulation of continuous microwave tones produced respectively by generators E8752D from Keysight, SGS100A from Rhodes & Schwarz, SGS100A from Rhodes & Schwarz, and SynthHD PRO from Windfreak set respectively at the frequencies $f_b + 50$ MHz, $f_m - 120$ MHz, $f_q + 200$ MHz, and $f_e + 51$ MHz. The pump pulses are also generated by modulation of continuous microwave tone, however the local oscillator at $f_b - f_m + 170$ MHz is produced by mixing the buffer and the memory RF sources for phase stability. The readout is modulated through a single sideband mixer while the others are modulated via IQ-mixers. The IF modulation pulses are generated by 9 channels of an OPX from Quantum Machines with a sample rate of 1 GS/s. The acquisition is performed, after down-conversion by their local oscillators, by digitizing a 51 MHz (readout) or a 50 MHz (buffer) signal with the 1 GS/s ADC of the OPX from Quantum Machines. The signals coming out of the buffer mode and of the readout mode are multiplexed into a single transmission line using a diplexer before getting amplified by a Traveling Wave Parametric Amplifier [2] (TWPA provided by IARPA and the Lincoln Labs). The TWPA is pumped at a frequency $f_{TWPA} = 7.636$ GHz and at a power that allowed the TWPA to reach a system efficiency of 18% from the buffer output to the ADC. The signal coming out of the buffer mode is filtered using a 20 cm waveguide WR62 with a cutoff frequency at 9.8 GHz in order to avoid the strong pump of the JRM to reach the TWPA and reciprocally. The next stage of amplification is performed by a High Electron Mobility Transistor (HEMT) amplifier (from Caltech) at 4 K and by two room temperature amplifiers.

II. SYSTEM CHARACTERIZATION AND FLUX DEPENDENCE

Using a vector network analyzer we measure the buffer resonance frequency as a function of the current running through a superconducting coil directly above the sample. The extracted buffer frequency $\omega_b$ is displayed in Fig. S2.a. The current is generated by applying a voltage $V_{coil}$ to a resistor in series with the coil. The periodicity of the buffer frequency allow us to convert the voltage $V_{coil}$ into a flux $\Phi_{ext}$ through the 4 inner loops of the JRM.

Even though the qubit consists in a single junction transmon, its frequency $\omega_q$ has a slight flux dependence due to its coupling with the memory. The qubit frequency, as a function of the flux, is extracted from Ramsey oscillations (Fig. S2.c). With these measurements, we are also able to extract the qubit coherence time $T_2$ as a function of flux $\Phi_{ext}$ (solid line in Fig. S2.d).

The memory cannot be probed directly in reflection nor in transmission with the measurement setup. To measure its frequency $\omega_m$ (Fig. S2.b), we use the qubit to determine at what excitation frequency the memory gets populated. We send a probe pulse on the memory via its weakly coupled port followed by a conditional $\pi$-pulse on the qubit at $\omega_q$. The qubit is thus excited only if the memory has zero photons. Measuring the qubit average excitation as a function of probe frequency leads to determining the frequency $\omega_m$ at which the state $|0\rangle$ is most depleted. We also measure the relaxation times of the qubit $T_{1,q}$ (see Fig. S2.d). The qubit decoherence time is limited by the relaxation since $T_2$ is close to $2T_{1,q}$.

We extract the buffer self-Kerr rate $K_{bb}$ from the dependence of its frequency $\omega_b$ as a function of probe power (Fig. S3.a). To measure the pump-buffer cross-Kerr rate $K_{bp}$ (Fig. S3.b), we measure $\omega_b$ while driving the pump at various powers. The pump is driven off resonance from $\omega_b - \omega_m$ to avoid frequency conversion. The buffer self-Kerr and buffer-pump cross-Kerr rates both vanish at the same flux point [3], which we hence choose as our working point. A non-zero cross-Kerr rate would indeed make the pump optimization more challenging for catch and reset operations.

The measurement of the memory self-Kerr rate $K$ and the qubit-dependent non linear rate $K_c$ were done in a previous cool down by monitoring the average phase acquired by a coherent state in the memory mode as a function of time while varying the mean photon number and the initial qubit state. Having prepared the qubit in either $|g\rangle$ or $|e\rangle$, we load the memory with a coherent state of amplitude $\alpha = \sqrt{n}$. We then wait for a time $t_{int}$. Finally, we release the state of the memory into the transmission line and record the average phase $\phi(t_{int})$ of the released pulse. The detuning $\delta\omega_m$ between the resonant frequency of the memory $\omega_m$ and a reference resonant frequency (when the memory is in the vacuum state and the qubit in $|g\rangle$) can be determined as $\delta\omega_m = \frac{\phi(t_{int})}{t_{int}}$. The slope of $\delta\omega_m$ as a function of mean photon number $n$ then gives the self-Kerr rate $K$ ($K_c + K$) when the qubit is prepared in $|g\rangle$ ($|e\rangle$). The rates $K$ and $K_c$ are plotted as a function of flux in Fig. S3.c.

Using a populated Ramsey protocol (see details in Fig. S6) as function of flux, we also extract the qubit-memory dispersive coupling $\chi$ (Fig. S3.d). It was also performed in a previous cool down.
FIG. S1. Scheme of the measurement setup. The RF sources color refers to the frequency of the matching element in the device up to a modulation frequency. Identically colored sources represent a single instrument with a split output.
III. READOUT OPTIMIZATION

The readout strategy is a compromise between readout speed, fidelity and QNDness. Note that the feedback protocol of the photocounter requires a QND measurement so that non-QNDness limits the counter fidelity. In order to make fast and faithful qubit measurements, we have implemented a CLEAR-like sequence [4] with amplitude $r_m(t)$ shown in Fig. S4.a. The QNDness of the readout is limited by the possible ionization of the transmon out of the qubit subspace [5, 6]. We found that not only this constraint limits the amplitude of the readout pulse but also that the ionization probability increases with the occupation of the memory mode (Fig. S4.b). In future design, the efficiency of the photocounter could be improved by using less sensitive coupling schemes [7–9].

In order to determine the state of the qubit as a function of the reflected signal with the best fidelity, we used a set of optimized demodulation weights that we computed to maximize the complex signal difference between the ground and excited states as shown in Ref. [10]. It is convenient to quantify the readout error using the overlap $\epsilon_0$ between the two Gaussian distributions corresponding to the two qubit states [11].

The qubit temperature was measured by repeatedly measuring the qubit, recording the demodulated signal from the readout into a complex histogram (such as the one shown in Fig. S4b) and fitting it with a set of 2 2D-gaussians of equal width. The temperature is then extracted by taking the ratio of the amplitudes of the 2 gaussians. For additional precision, the center of the gaussian corresponding to the qubit being in the excited state $|e\rangle$ was estimated by doing the same measurements after performing a $\pi$-pulse such that the final fit only had 2 free parameters: the center of the gaussian corresponding to $|g\rangle$ and the qubit temperature. We found an effective temperature of 33 mK.

FIG. S2. a) Buffer frequency, b) memory frequency, c) qubit frequency, d) qubit decoherence time $T_2$ (dashed line) and lifetime $T_{1,q}$ (solid line) as a function of flux $\Phi_{ext}$ in the inner loops of the JRM. Notice that the flux range is different in a) compared to b-d). Vertical dashed line: working point for the main text.
FIG. S3. Rates of nonlinear terms in the device as a function of the external flux $\Phi_{\text{ext}}$. Notice that the flux range is different for each panel. a) Buffer self-Kerr rate $K_{bb}$, b) Pump-buffer cross-Kerr rate $K_{bp}$, c) dots: memory self-Kerr rate $K_e$, stars: nonlinear rate $K_e$, d) dispersive shift $\chi$ between qubit and memory.

FIG. S4. Readout optimization. a) CLEAR-like readout pulse sequence. Driving amplitude $r_{\text{in}}$ of the readout as a function of time $t$. Blue: readout excitation. Orange: readout reset. b) Histogram of the two demodulated quadratures $I$ and $Q$ of the reflected readout pulse for $10^4$ realizations after applying a $\pi/2$-pulse on the qubit. The 2 peaks correspond to the $|g\rangle$ and $|e\rangle$ states of the qubit. The few points in the upper left corner correspond to the transmon in an ionized state. c) Probability to observe the transmon outside of its qubit subspace as a function of the mean number of photons inside of the memory for the readout power used in the main text.

IV. OPTIMAL CATCHING PUMP

In this section, we derive the optimal pump to catch an arbitrary wavepacket with a bandwidth smaller than the bandwidth of the buffer $\kappa_b = 2\pi \cdot 20\,\text{MHz}$. We first derive the optimal pump to catch an incoming wavepacket assuming $\kappa_m = 0$ and we then show that a small memory relaxation rate $\kappa_m$ and a cross-Kerr rate $K_{bp}$ do not prevent the catch from being complete.
a. Ideal case

Let us consider the Langevin equations for the buffer $b$ and memory $m$ with a conversion pump $p$ in the frame rotating with $b_{in}$ and $m$

\[
\frac{db}{dt} = -\frac{\kappa_b}{2}b(t) - g_3p^*(t)m(t) + \sqrt{\kappa_b}b_{in}(t)
\]
\[
\frac{dm}{dt} = g_3p(t)b(t),
\]

where, for simplicity, we assumed that the external flux used is chosen such that all the self-Kerr and cross-Kerr terms cancel out. Note that an arbitrary choice of phase reference allows us to constrain $b$ to be a real function.

We start by parametrizing the equations with dimensionless variables using $\tau = \frac{2\kappa_b}{\lambda}t$, $u = \frac{2g_3p}{\kappa_b}$

\[
\dot{b} = -b - u^*m + \frac{2}{\sqrt{\kappa_b}}b_{in}
\]
\[
\dot{m} = ub
\]

where the dots denote the derivatives with respect to $\tau$.

Catching the incoming wavepacket $b_{in}$ perfectly comes down to finding the pump $u(\tau)$ such that $b_{out} = 0$ uniformly. Since $b_{in} + b_{out} = \sqrt{\kappa_b}b$, $u$ is the solution of the following differential equations

\[
um^* = b - \dot{b}
\]
\[
\dot{m} = ub.
\]  

(S1)

(S2)

For any signal with a bandwidth lower than the buffer coupling rate $\kappa_b$, these equations can be solved numerically. In the following subsection, we focus on the case of a sech input waveform, where the calculation can be carried on analytically.

Case of an incoming hyperbolic secant waveform

In the experiment, we frequently used an incoming hyperbolic secant waveform $b(\tau) = \sqrt{\frac{\lambda}{2}} \sech(\lambda \tau/2)$. To do so, we remark that

\[y = |m|^2 + b^2\]

is a flat output [12], meaning that $m$, $u$ and $b$ can be expressed as functions of $y$, $\dot{y}$ and $\ddot{y}$. Combining Eq. (S1) and Eq. (S2), we get $m^*\dot{m} = (b - \dot{b})b$. Taking the real part and using the limited bandwidth ($\dot{y} \leq 2y$) and the assumption that there is no loss ($0 \leq \dot{y}$), we get

\[b^2 = \dot{y}/2, \quad |m|^2 = y - \dot{y}/2.\]  

(S3)

Setting $y = \frac{1}{1 + e^{-\lambda \tau}}$ with $0 \leq \lambda \leq 2$, using Eq. (S3), we get $b(\tau) = \sqrt{\frac{\lambda}{2}} \sech(\lambda \tau/2)$ as desired and $|m| = \sqrt{\frac{\lambda e^{2\lambda \tau} - 1}{2} \sech(\lambda \tau/2)}$. Multiplying Eq. (S1) by its complex conjugate, we get $|u| = \frac{b - \dot{b}}{|m|}$. From Eq. (S1), we can also see that $\arg(u) = \arg(m)$. Hence, there is a function $\theta$ such that $m = |m|e^{i\theta}$ and $u = |u|e^{i\theta}$. By multiplying Eq. (S2) by $m^*$ and using Eq. (S1) one gets $m^*m = (b - \dot{b})b$. Since $b$ is real, the imaginary part, yields $\dot{\theta} = 0$. For simplicity, we choose $\theta(\tau) = 0$, which leads to

\[u = \frac{b - \dot{b}}{|m|}.\]  

(S4)

Finally we find

\[u(\tau) = \sqrt{\frac{\lambda}{e^{2\lambda \tau} + 1 - \lambda/2}} \left(1 + \frac{\lambda}{2} \tanh(\lambda \tau/2)\right).\]  

(S5)

Going back to the original time variable $t$, we conclude that an incoming wavepacket with a shape $b_{in}(t) = \sqrt{\frac{\lambda}{8\kappa_b}} \sech(\lambda \kappa_b t/4)$ is perfectly caught by a pump $p_{opt}(t) = \frac{2g_3}{\kappa_b} \sqrt{\frac{\lambda/2}{e^{2\lambda \kappa_b t/4} + 1 - \lambda/2}} \left(1 + \frac{\lambda}{2} \tanh(\lambda \kappa_b t/4)\right)$. 

b. Finite memory lifetime

In order to account for the memory relaxation rate $\kappa_m$, the Langevin equations become

$$\frac{db}{dt} = -\frac{\kappa_b}{2} b(t) - g_3 p^*(t) m + \sqrt{\kappa_b} b_{in}(t)$$
$$\frac{dm}{dt} = -\frac{\kappa_m}{2} m(t) + g_3 p(t) b(t).$$

Without loss of generality, we assume that $b_{in}$ and $p$ are real, hence $m$ and $b$ are also real. Using the same definition for $y$ and introducing $\varepsilon = \kappa_m/\kappa_b$, we get the following modified version of Eq. (S3) to derive $b$ and $m$ as algebraic functions of $y$ and $\dot{y}$.

$$(1 + \varepsilon)b^2 = \dot{y}/2 + \varepsilon y, \quad (1 + \varepsilon)|m|^2 = y - \dot{y}/2$$

(S6)

Given Eq. (S4), $u$ can be expressed as an algebraic function of $y$, $\dot{y}$ and $\ddot{y}$. In this case the no-loss assumption is replaced by the weaker constraint that the ratio between the outgoing power $-\frac{dy}{dt}$ and the total energy $y$ is smaller than $\kappa_m$ i.e. $\frac{dy}{dt} \geq -\kappa_m y$ (i.e. $\dot{y}/2 + \varepsilon y \geq 0$). The bandwidth limit $\frac{dy}{dt} \leq \kappa_b y$ remains valid (i.e. $\dot{y} \leq 2y$).

To carry on the calculation analytically, we set $y = \frac{1}{\varepsilon + e^{-\lambda\tau}}$ so that

$$b(\tau) = \sqrt{\frac{\lambda/2 + \varepsilon}{1 + \varepsilon}} \frac{1}{e^{(\lambda/2 + 2\varepsilon)\tau} + e^{-\lambda\tau/2}}.$$

We also get

$$m(\tau) = \sqrt{e^{(\lambda + 2\varepsilon)\tau}} \frac{1 - \lambda/2}{1 + \varepsilon} \frac{1}{e^{(\lambda/2 + 2\varepsilon)\tau} + e^{-\lambda\tau/2}}.$$

From the above expressions for $b$ and $m$, we can then compute $u$ using Eq. (S4). Given the small value of $\varepsilon \approx 0.002$ in the device of the main text, we have chosen to neglect the memory relaxation and to use the results from the ideal case above.

c. Finite cross-Kerr rate

Even in the presence of a small cross-Kerr rate $K_{bp}$ between the buffer and the pump, an optimal catch pump can be found which guarantees that no signal is reflected i.e. $b_{out} = 0$. The modified Langevin equations are as follows

$$\frac{db}{dt} = -\left(\frac{\kappa_b}{2} + iK_{bp}|p(t)|^2\right) b(t) - g_3 p^*(t) m(t) + \sqrt{\kappa_b} b_{in}(t)$$
$$\frac{dm}{dt} = -\frac{\kappa_m}{2} m(t) + g_3 p(t) b(t).$$

Introducing the dimensionless cross-Kerr rate $k = K_{bp}\kappa_b/g_3^2$, we get a modified version of equations (S1) and (S2)

$$um^* = b - \dot{b} + ik|u|^2 b$$
$$\dot{m} = -\varepsilon m + ub.$$

Since $b$ is real, the real quantity $y = |m|^2 + b^2$ can still be used to parametrize the system, despite the fact that $m$ and $u$ are now complex. The values of $b$ and $|m|$ can still be expressed as functions of $y$ and $\dot{y}$ by Eq. (S6). The modulus $|u|$ of the pump is obtained by solving

$$|u|^2 |m|^2 = (b - \dot{b})^2 + k^2 |u|^4 b^2.$$

The argument $\theta_m$ of $m$ results from the integration

$$\theta_m(\tau) = \theta_m(0) + k \int_0^\tau \frac{|u(s)|^2 b(s)^2}{|m(s)|^2} ds,$$

where $|u|$, $|m|$ and $b$ are algebraic functions of $y$, $\dot{y}$ and $\ddot{y}$. The argument $\theta_u$ of $u$ is given by the argument of $m(b - \dot{b} + ik|u|^2 b)$ which coincides then with the argument of $(\dot{m} + \varepsilon m)/b$.

Using the above derivation, one sees that finding the optimal pump in case of cross-Kerr effect requires not only to adjust the envelope of the pump, as done in the main text, but also adjusting the phase of the pump $\theta_u$ dynamically to compensate for the time dependent buffer frequency shift.
V. CATCH EFFICIENCY MEASUREMENT

To access the catch efficiency, we measure the ratio of energy \( R_c(t_w) \) between the released signal after a catch and a waiting time \( t_w \) and the signal reflected on the buffer when the pump is turned off as in Refs. [3, 13]. Assuming the same efficiency \( \eta \) for the catch and release operations, the energy ratio is given by \( R_c(t_w) = \eta^2 \exp\left(-t_w/T_{1,m}\right) \).

In practice, due to the finite directivity of a directional coupler, there are interferences between the signal parasitically bypassing the coupler towards the output line and the desired signal coming from the buffer. This problem exclusively affects the denominator of the measured energy ratio since the parasitic signal does not spatially overlap with the signal that is released after \( t_w \). In our case, the interferences were destructive, which leads to an underestimation of the denominator. We were thus able to measure raw energy ratios in excess of 100%.

It is however possible to get a lower bound on the actual efficiency \( R_c(t_w) \) by measuring the coupler directivity. Right after the run, we measured the directivity of at room temperature using a calibrated vector network analyzer. On Fig. S5 are shown the lowest possible values of \( R_c(t_w) \) (dots) assuming fully destructive interferences in the denominator (correction by a factor 0.746 on the raw ratio). Fitting these lower values by an exponential decreasing function at rate \( 1/T_{1,m} \), we get a lower bound on the catch efficiency \( \eta = \sqrt{R_c(t_w = 0)} \geq 0.92 \).

VI. DIFFERENT METHODS FOR MEASURING THE MEAN PHOTON NUMBER

We use several methods to measure the mean photon number \( \langle n \rangle \) in the memory in order to calibrate the buffer and memory displacement pulses (Fig. S6). The experiment begins by a displacement pulse on the memory mode with a driving voltage \( \mu \alpha \), where \( \mu \) is a conversion factor between voltages and amplitudes to be determined. The following procedures then determine the mean photon number \( \langle n \rangle = |\alpha|^2 + n_{th} \) as a function of the driving voltage by different ways and thus calibrate \( \mu \). \( n_{th} \) is the residual equilibrium thermal photon number in the memory.

a. Photon number selective \( \pi \)-pulse

The first method relies on the possibility to perform a \( \pi \)-pulse \( \Pi_{|n\rangle} \) conditionally on the photon number \( n \). It is done by driving the qubit at frequency \( \omega_q - \chi n \) with a long enough pulse so that the frequency spreading is smaller than \( \chi/2 \). The pulse maps the probability to have \( n \) photons \( P(n, \alpha) \) into the measured probabilities \( P_{|n\rangle,\alpha}(\epsilon) \) for the qubit to be found in its excited state (Fig. S6.a). Fitting the distribution \( P(n, \alpha) \) for each \( \alpha \) by a Poisson distribution, we calibrate \( \mu \) neglecting the thermal population. A limitation of this method occurs at high photon number. Indeed, the dispersive shift \( \chi \) slightly depends on photon number \( n \), so that the qubit drive frequency is off resonant.
FIG. S6. Three methods for calibrating the memory displacement amplitude. The measurements were performed on a previous cooldown. a) Photon-number selective $\pi$-pulse. Dots: measured probability to have $n$ photons in the memory as a function of $|\alpha|^2$. Solid lines: Poisson distribution fitted to calibrate the mean photon number on the x-axis. b) Vacuum detector. Dots: probability $P_{[0],\alpha}(e)$ that the memory is empty as a function of waiting time for various preparation amplitudes. Solid lines: fit of the measured probabilities using the expression for memory relaxation in the text. c) Populated Ramsey. Dots: signal difference $S_+ - S_-$ between two encodings of the Ramsey-like interferences in presence of various mean photon numbers $\langle n \rangle$. Solid lines: theoretical prediction allowing to calibrate the displacement amplitude and the thermal occupancy $n_{th}$. d) Result of the calibration using the three methods: photon number selective $\pi$-pulse in orange diamonds, vacuum detector in blue dots and populated Ramsey in green stars. The black dashed line represents the overall fitted value for $\mu$.

b. Vacuum detector

To calibrate the conversion factor $\mu$ at high photon numbers $|\alpha|^2 \gg 1$, we perform another method, which is to use the qubit as a vacuum detector [1]. Applying a $\pi$-pulse $\Pi[0]$ encodes the probability that the memory is empty into the probability for the qubit to be in the excited state. Now, after a waiting time $t$, the memory has relaxed and, neglecting $n_{th}$ for the large $|\alpha|^2$, the measured probability $P_{[0],\alpha}(e)$ evolves following $\exp(-|\alpha|^2 e^{-t/T1_m})$ (Fig. S6.b). Fitting the value of $\mu$ for each value of $\mu\alpha$ to match this expression with the measured $P_{[0],\alpha}(e, t)$ leads to an accurate determination of the conversion factor $\mu$ as a function of $\alpha$. This photon number calibration has a higher range than the previous one but is less sensitive for low average photon numbers.

c. Populated Ramsey oscillations

Our last method to calibrate the conversion factor $\mu$ relies on a Ramsey-like sequence [14] (Fig. S6.c). After the coherent displacement of the memory, we prepare the qubit in an equal superposition of ground and excited states by applying an unconditional $\frac{\pi}{2}$−pulse. After a waiting time $t$, the phase of the superposition increases by $\chi nt$ for each Fock state $|n\rangle$. We then apply a second unconditional $\pm\frac{\pi}{2}$−pulse giving the signal $S_{\pm}$. The signal difference is
given by \( S_+ - S_- = \cos(\langle n \rangle \sin(\chi t)) \exp(\langle n \rangle (\cos(\chi t) - 1) - t/T_2) \) from which we extract the mean photon number \( \langle n \rangle \). Without driving the memory, the measured mean number gives the thermal population of the memory \( n_{\text{th}} = 0.014 \) corresponding to an effective temperature of 44 mK. Offsetting the measured \( \langle n \rangle \) by this thermal occupation leads to a calibration of \( \mu \). This last method has a good sensitivity at low photon numbers however it cannot be used for large photon numbers where the pattern becomes insensitive to \( \langle n \rangle \).

### VII. NUMERICAL MODEL

We simulated our system using the QuantumOptics.jl library\(^{[1]}\).

The device Hamiltonian reads\(^{[1]}\)

\[
\hat{H}/\hbar = \omega_b \hat{b}^\dagger \hat{b} + \omega_m \hat{\hat{m}}^\dagger \hat{\hat{m}} + \frac{\omega_d}{2} \sigma_z
+ g_3 \hat{\hat{m}}^\dagger \hat{b} + g_5^* \hat{b}^\dagger \hat{\hat{m}}^\dagger
- \chi \hat{\hat{m}}^\dagger |e\rangle \langle e| - K \hat{\hat{m}}^2 \hat{\hat{m}}^2 - K_c |e\rangle \langle e| \hat{\hat{m}}^2 \hat{\hat{m}}^2.
\]

To simplify the model, we restrict the transmon to its first two levels and we do not consider the readout resonator and its dispersive coupling to the qubit. We simulate the readout of the qubit by an instantaneous projective measurement taking place at half of our experimental readout duration. During the readout time, before and after the projection, the system evolves freely. We also take into account the overlap error \( \varepsilon_o \)\(^{[11]}\) in the readout which we measured to be below 1%. Moreover, we consider the catch of the wavepacket incoming onto the buffer to be optimal (Section IV). Thus, we further reduce the numerical Hilbert space by putting aside the buffer and the pump. The catch is then simulated by an instantaneous displacement on the memory field.

Finally, we model our system in the memory and qubit rotating frame using the following Hamiltonian.

\[
\hat{\hat{H}}/\hbar = -\chi \hat{\hat{m}}^\dagger \hat{\hat{m}} |e\rangle \langle e| - K \hat{\hat{m}}^2 \hat{\hat{m}}^2 - K_c |e\rangle \langle e| \hat{\hat{m}}^2 \hat{\hat{m}}^2 + \text{Re}(f(t)) \hat{\hat{\sigma}}_x + \text{Im}(f(t)) \hat{\hat{\sigma}}_y
\]  

(7)

with \( f(t) \) the complex envelope containing all the qubit drives. Using a time-dependent Hamiltonian allows us to simulate the optimal counting with the questions \( Q_0 \) and \( Q_1 \). For instance, we can thus accurately take into account the finite duration of the \( \frac{\pi}{2} \) pulses. A Lindblad master equation enables to take into account the qubit relaxation time \( T_{1,q} \) and pure dephasing time \( T_\sigma \) as well as temperatures of qubit and memory. We restrict the Hilbert space of the memory mode between 0 and 29 photons.

### VIII. WIGNER TOMOGRAPHY

We use the method of Refs.\(^{[16–18]}\) to directly measure the Wigner function \( W(\beta) = \frac{2}{\pi} \text{Tr} \left( D_\beta \mathcal{P} D_\beta^\dagger \right) \) of the memory mode. We perform a displacement \( D_\beta^\dagger \) of amplitude \( -\beta \) (sech-shape with \( \sigma = 13 \) ns) followed by a parity measurement. \( \mathcal{P} = \exp(i \pi \hat{m} \hat{m}) \) is the photon parity operator. The Wigner functions are measured on a 51x51 square matrix of amplitudes \( \beta \) where \( |\text{Re}(\beta)|, |\text{Im}(\beta)| \leq 2.2 \). The measured Wigner functions for mean photon numbers \( |\alpha|^2 = 0, 1, 1.5 \) and 2 are shown in Fig. S7.a. Each column corresponds to postselected measurements for a given detected photon number \( n_2 \).

Our numerical model above allows us to compute the predicted Wigner functions for each panel of the figure. The predictions are shown in Fig. S7.b. Note that these figures are obtained by computing the Wigner function directly without modeling the readout of the parity photon number after displacement.

For an arbitrary outcome \( n_2 \), the photocounter would ideally project the incoming state \( |\psi\rangle \) into \( |\psi_{n_2}\rangle \propto \sum_j |n_2 + 4j\rangle \langle n_2 + 4j|\psi\rangle \). We discuss non-idealities in the measurement backaction in the main text. They are mainly due to the finite lifetimes of the qubit and memory for low mean photon numbers \( |\alpha|^2 \).
FIG. S7. Measured (a) and computed (b) Wigner functions after catching a coherent state with a mean photon number $|\alpha|^2 = 0.5$, 1, 1.5 and 2 from top to bottom respectively and heralding on a detected number $n_2 = 0$, 1, 2 or 3 from left to right respectively. For each panel, the fidelity between the measured Wigner function and the predicted one does not get below 95%.

| $|\alpha|^2$ | $n_2 = 0$ | $n_2 = 1$ | $n_2 = 2$ | $n_2 = 3$ |
|-------------|-----------|-----------|-----------|-----------|
| 0.5         | 86%       | 52%       | 32%       | 4.9%      |
| 1           | 77%       | 50%       | 34%       | 11%       |
| 1.5         | 58%       | 48%       | 38%       | 18%       |
| 2           | 39%       | 42%       | 37%       | 22%       |

TABLE SI. Fidelities $\mathcal{F}$ between the measured collapsed quantum states $\rho$ and the ideal quantum states $\rho_{n_2} = |\psi_{n_2}\rangle\langle\psi_{n_2}|$ for various outcomes $n_2$ and various mean photon numbers $|\alpha|^2$.

In Fig. S7, some Wigner functions are not invariant by a phase shift as one could expect from mixtures of Fock states. These patterns in the figure indicate coherences between Fock states. Our simulations show that the coherences originate from two main phenomena. First, the photon number measurement is performed modulo 4, which preserves coherences between different photon numbers modulo 4 by projection. Second, due to the finite duration of the $\pi/2$ pulses in the pulse sequence that performs question $Q_k$, the encoding of the $k$-th bit of the photon number in the qubit state is imperfect. Therefore, postselecting on the measured binary code $n_2$ preserves some coherence between the Fock states that compose the initial coherent state $|\alpha\rangle$. Finally, the Wigner functions appear distorted due to the memory nonlinear rates $K$ and $K_e$.

The deviations from the ideal projected quantum state (fidelities in Table SI) are further investigated in the Section IX.

### IX. ERROR BUDGET OF THE PHOTOCOUNTER

In this section we numerically investigate the origin of the errors on the success probabilities $\mathcal{P}_{\langle n\rangle}(n)$ to find $n$ photons when the incoming wavepacket is in a Fock state $|n\rangle$ and on the QNDness, which is characterized by the fidelities $\mathcal{F}$ above. We study the error budget by sweeping one (or more) parameters independently of the others in our model.

- The finite qubit relaxation time $T_{1,q}$ entails different errors depending on the choice of encoding the outcome
12

$n^2$ in the qubit state during questions $Q_k$’s. This choice is done by the sign of the second $\pi/2$ pulse in the sequence of Fig. 3. For each question $Q_k$, the outcome on the $k$-th bit of the photon number corresponding to the qubit excited state will get mixed with the outcome corresponding to the qubit ground state. These errors scale exponentially with $1/T_{1,\text{q}}^{\text{model}}$ (Fig. S8.a and Fig. S9.a).

- The finite memory relaxation time $T_{1,\text{m}}$ causes errors except for $|n = 0\rangle$ (Fig. S8.b and Fig. S9.b). The dominant source of error is then the mixing of the outcome $n^2$ with $n^2 - 1$.

- The finite lifetimes $T_{1,q}$ and $T_{1,m}$ are our main sources of errors as the counting probabilities $P_{|n\rangle}(n)$ (Fig. S8.c) and state fidelities $F$ (Fig. S9.c) get close to 1 when both $T_{1,q}$ and $T_{1,m}$ increase. If both $T_{1,q}$ and $T_{1,m}$ increase by an order of magnitude, the success probability will not get below 85% for all outcomes (Fig. S8.c). The QNDness is more demanding and one would need to increase by more than two orders of magnitude the lifetimes in order to get fidelities beyond 80% (insets of Fig. S9.a-c). Note that current state-of-the-art in 3D cavities and new materials demonstrates lifetimes indeed larger than two orders of magnitude [19, 20].

- Our device does not seem to be limited by thermal excitations (Fig. S8.d and Fig. S9.d).

- A more faithful qubit readout would not bring significant improvements in the success probabilities and QNDness (Fig. S8.f and Fig. S9.f).

- The memory self-Kerr rate $K$ does not seem to affect the success probabilities and QNDness (not shown). Indeed, the Fock states are eigenstates of the self-Kerr term. However, the additional self-Kerr rate $K_e$ when the qubit is in $|e\rangle$ has an important impact (Fig. S8.e and Fig. S9.e). During the interaction time $T_k$ of question $Q_k$, the qubit acquires an additional parasitic phase $n^2 K_e T_k$ for each Fock state $|n\rangle$. Therefore, for $n \geq 1$ and each question $Q_k$, the qubit phase does not end up in the right value, which undermines the photon number encoding. As long as $n^2 K_e 2\pi/(\chi^2 K_q) \ll 1$, this effect can be neglected. For our device, it translates into $n \ll 3.7$. This square dependence on the photon number $n$ is the main limitation of this scheme for increasing the maximal number of photons the detector can resolve.

Similar to Ref [21], we compute the rate $K_e$ using perturbation theory to the fourth order in the transverse coupling strength

$$g = \sqrt{\chi \Delta (\Delta - K_q)/(2K_q)}.$$  

It is obtained as a function of the detuning $\Delta = \omega_m - \omega_q$, transmon anharmonicity $-K_q = -E_C/\hbar$ and dispersive shift $\chi$

$$K_e = \frac{\chi^2}{K_q} \frac{(2\Delta^3 - (\Delta - K_q)^3)}{2\Delta(\Delta - K_q)(\Delta + K_q)}$$  \hspace{1cm} (S8)

It is then possible to reduce $K_e$ considerably while preserving the behaviour of the device for large photon numbers by careful optimisation of the device parameters. For example setting the detuning accurately to $\Delta = \frac{K_e}{(1 - \sqrt{2})}$ cancels the rate $K_e$ completely.
FIG. S8. Success probabilities $P_{|0\rangle}(0)$ (blue), $P_{|1\rangle}(1)$ (orange), $P_{|2\rangle}(2)$ (green) and $P_{|3\rangle}(3)$ (red) as a function of the ratio between the parameter in the model and the same parameter in experiment. All curves are calculated in the case of an initial coherent state of amplitude $|\alpha| = \sqrt{0.5}$. Vertical dashed lines indicate the result of the model for the actual experiment. Each panel probes the errors coming from a) the qubit relaxation time $T_{1,q}$, b) the memory relaxation time $T_{1,m}$, c) both qubit and memory relaxation times $T_1 = (T_{1,q}, T_{1,m})$, d) qubit and memory thermal population, e) additional Kerr rate $K_e$ when the qubit is excited, and f) readout error $\epsilon_0$. 
FIG. S9. QNDness of the detector. Fidelity $F$ between the quantum state $\rho$ predicted by our model and the ideal projected state $\rho_{n_2}$ after catching a coherent state of amplitude $|\alpha| = \sqrt{0.5}$ for the outcomes $n_2 = 0$ (blue), $n_2 = 1$ (orange), $n_2 = 2$ (green) and $n_2 = 3$ (red). Each panel addresses the same parameter as in Fig. S8. Insets are the Wigner functions heralded on the counter outcome $n_2 = 3$ for the maximal value of the model parameter. Note that on the top panels, the maximal value improves QNDness while it deteriorates it for bottom panels.
1. T. Peronnin, D. Marković, Q. Ficheux, and B. Huard, Sequential measurement of a superconducting qubit (2019), arXiv:1904.04635 [quant-ph].

2. C. Macklin, K. O’Brien, D. Hover, M. E. Schwartz, V. Bolkhovsky, X. Zhang, W. D. Oliver, and I. Siddiqi, A near–quantum-limited josephson traveling-wave parametric amplifier, Science 350, 307 (2015).

3. E. Flurin, The Josephson Mixer, a Swiss army knife for microwave quantum optics, Ph.D. thesis, École Normale Supérieure (2014).

4. D. T. McClure, H. Paik, L. S. Bishop, M. Steffen, J. M. Chow, and J. M. Gambetta, Rapid Driven Reset of a Qubit Readout Resonator, Phys. Rev. Applied 5, 11001 (2016).

5. D. Sank, Z. Chen, M. Khezri, J. Kelly, R. Barends, B. Campbell, Y. Chen, B. Chiaro, A. Dunsworth, A. Fowler, E. Jeffrey, E. Lucero, A. Megrant, J. Matus, M. Neeley, C. Neill, P. J. J. O’Malley, C. Quintana, P. Roushan, A. Vainsencher, T. White, J. Wenner, A. N. Korotkov, and J. M. Martinis, Measurement-Induced State Transitions in a Superconducting Qubit: Beyond the Rotating Wave Approximation, Phys. Rev. Lett. 117, 190503 (2016).

6. R. Lescanne, L. Verney, Q. Ficheux, M. H. Devoret, B. Huard, M. Mirrahimi, and Z. Lehtas, Escape of a Driven Quantum Josephson Circuit into Unconfined States, Phys. Rev. Applied 11, 14030 (2019).

7. S. Touzard, A. Kou, N. E. Frattini, V. V. Sivak, S. Puri, A. Grimm, L. Fruenzi, S. Shankar, and M. H. Devoret, Gated Conditional Displacement Readout of Superconducting Qubits, Phys. Rev. Lett. 122, 80502 (2019).

8. J. Ikonen, J. Goetz, J. Ilves, A. Keränen, A. M. Gunyho, M. Partanen, K. Y. Tan, D. Hazra, L. Grönberg, V. Vesterinen, S. Simbierowicz, J. Hassel, and M. Möttönen, Qubit Measurement by Multichannel Driving, Phys. Rev. Lett. 122, 80503 (2019).

9. R. Dassonneville, T. Ramos, V. Milchakov, L. Planat, É. Dumur, F. Foroughi, J. Puertas, S. Leger, K. Bharadwaj, J. Delaforce, C. Naud, W. Hasch-Guichard, J. J. Garc¨ıa-Ripoll, N. Roch, and O. Buisson, Fast High-Fidelity Quantum Nondemolition Qubit Readout via a Nonperturbative Cross-Kerr Coupling, Phys. Rev. X 10, 11045 (2020).

10. C. A. Ryan, B. R. Johnson, J. M. Gambetta, J. M. Chow, M. P. da Silva, O. E. Dial, and T. A. Ohiki, Tomography via correlation of noisy measurement records, Phys. Rev. A 91, 22118 (2015).

11. T. Walter, P. Kurpiers, S. Gasparinetti, P. Magnard, A. Potońciuk, Y. Salathé, M. Pechal, M. Mondal, M. Oppliger, C. Eichler, and A. Wallraff, Rapid high-fidelity single-shot dispersive readout of superconducting qubits, Phys. Rev. Applied 7, 054020 (2017).

12. M. Fliss, J. Levine, P. Martin, and P. Rouchon, Flatness and defect of non-linear systems: introductory theory and examples, International Journal of Control 61, 1327 (1995).

13. E. Flurin, N. Roch, J. D. Pillet, F. Mallet, and B. Huard, Superconducting quantum node for entanglement and storage of microwave radiation, Phys. Rev. Lett. 114, 1 (2015).

14. P. Campagne-Ibarcq, Measurement back action and feedback in superconducting circuits, Ph.D. thesis, École Normale Supérieure (ENS) (2015).

15. S. Krämer, D. Plankensteiner, L. Ostermann, and H. Ritsch, QuantumOptics.jl: A Julia framework for simulating open quantum systems, Computer Physics Communications 227, 109 (2018).

16. L. G. Lutterbach and L. Davidovich, Method for Direct Measurement of the Wigner Function in Cavity QED and Ion Traps, Phys. Rev. Lett. 78, 2547 (1997).

17. P. Bertet, A. Auffeves, P. Maioli, S. Osanghi, T. Meunier, M. Brune, J. M. Raimond, and S. Haroche, Direct Measurement of the Wigner Function of a One-Photon Fock State in a Cavity, Phys. Rev. Lett. 89, 200402 (2002).

18. B. Vlastakis, G. Kirchmair, Z. Lehtas, S. E. Nigg, L. Fruenzi, S. M. Girvin, M. Mirrahimi, M. H. Devoret, and R. J. Schoelkopf, Deterministically encoding quantum information using 100-photon Schrödinger cat states, Science 342, 607 (2013).

19. M. Reagor, W. Pfaff, C. Axline, R. W. Heeres, N. Ofek, K. Sliwa, E. Holland, C. Wang, J. Blumoff, K. Chou, M. J. Hatridge, L. Fruenzi, M. H. Devoret, L. Jiang, and R. J. Schoelkopf, Quantum memory with millisecond coherence in circuit qed, Phys. Rev. B 94, 014506 (2016).

20. A. P. M. Place, L. V. H. Rodgers, P. Mundada, B. M. Smitham, M. Fitzpatrick, Z. Leng, A. Premkumar, J. Bryon, S. Sussman, G. Cheng, T. Madhavan, H. K. Babla, B. Jaeck, A. Grenis, N. Yao, R. J. Cava, N. P. de Leon, and A. A. Houck, New material platform for superconducting transmon qubits with coherence times exceeding 0.3 milliseconds (2020), arXiv:2003.00024 [quant-ph].

21. M. Elliott, J. Joo, and E. Ginossar, Designing Kerr interactions using multiple superconducting qubit types in a single circuit, New Journal of Physics 20, 023037 (2018).