Influential Parameter Analyses for Sinking Velocity with Measured Data

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Abstract. To study the effect of operational parameters on the sinking velocity of the landing gear of carrier-based aircraft, the influential analyses of various related parameters were performed based on the measured data of F/A-18. The probabilistic statistical characteristics of the sinking velocity were firstly investigated. And then correlation coefficient method and Levenberg-Marquardt (LM) algorithm were applied to finish influential parameter analyses. The analysis results show that the important influential parameters are aircraft glide angle, deck pitch angle, approaching velocity and engaging velocity; the mathematical model of sinking velocity is established by LM algorithm, and the determination coefficient is calculated via the variance analysis, which reveals that the proposed regression model is good. Compared with the measured data, the average errors for the predicted values of the established model of sinking velocity are less than 5%, which validates the effectiveness and feasibility of the method in engineering practice. The present efforts provide a way to study the relationships of the sinking velocity and the related influential parameters. And this work was proved to be a useful method for the study of sinking velocity.

1. Introduction

Sinking velocity is defined as the component of the vertical direction in the deck landing process, which is an important parameter for the landing gear design of carrier-based aircraft [1]. The sinking velocity not only reflects the impact severity of the aircraft, but also seriously influences the weight of the landing gear and airframe structure. The design parameter value of sinking velocity is too small, which leads to the weak strength of aircraft structure and is unable to reach the requirement of reliability. Therefore, it is urgent to investigate the sinking velocity to design the relevant parameters of landing gear and guarantee the safety of carrier-based aircraft.

There have been quite a large body of literature on carrier-based aircraft, and various techniques have been developed. References [2, 3] studied the probabilistic statistical features of the sinking velocity for carrier-based aircraft landing. Considering different operational conditions, the running state of carrier-based aircrafts was investigated by Micklos [4]. Flight dynamics model was applied to study the effect of influential factors on the sinking velocity [5]. An improved linearization method was proposed to emulate aircraft groundspeed variations [6]. Landing dynamics model was established to study the aircraft landing on the deck [7]. An actual model of arresting hook was derived...
to investigate the influence on collision process [8]. Realistic mechanisms and strategies were employed to establish a model for carrier landing operations [9]. The carrier-based aircraft landing laws landed on the carrier was discussed by using the dynamics model [10]. The safety carrier-based aircraft ski-jump take off was evaluated by the integrated dynamic simulation models [11, 12]. The flow field of exhaust jets and its impact on the flight deck were studied [13]. Simulated model was used to verify the deck movement and ship wake in the landing process of carrier-based aircraft [14]. An adaptive disturbance rejection algorithm was developed to discuss the carrier-based aircraft dynamics and the linearized longitudinal model under turbulence conditions [15]. Although the above efforts investigated the carrier-based aircraft from different perspectives, some shortcomings are still existence: ① only provide the measured data of different kinds of carrier-based aircrafts under considering different working conditions without studying the influence of relevant factors; ② only finish the deterministic analysis of carrier-based aircraft without considering the randomness of influential factors; ③ most of works investigate the sinking velocity of carrier-based aircraft on basis of theoretical methods without verifying the feasibility in engineering. Additionally, multiple disciplines need to be considered in the landing process of carrier-based aircraft, which include flight dynamics, aircraft control, and structural design and so forth. Hence, it is urgent to study the influence of related parameters on the sinking velocity.

To address the above issues, correlation coefficient analysis method and multivariate statistical analysis method are developed to implement the influential parameter analyses for the sinking velocity of carrier-based aircraft. According to the measured data from the deck landing parameters of the typical serving aircraft F/A 18, the numerical features of the sinking velocity are obtained via statistical analysis principles; and then the correlation analysis of influencing factors is carried out based on the correlation coefficient analysis method; the mathematical model of the sinking velocity and the related variables is built with Levenber-Marquardt (LM) algorithm and the influential parameter analysis is finished.

2. Basic theory
2.1. Procedure of influential parameter analyses
To design the parameters of landing gear of carrier-based aircraft, it is important to investigate the relationship of the sinking velocity and influencing factors. The procedure of influential parameter analyses is shown in Figure 1. From Figure 1, the detailed ideas of influential parameter analyses for the sinking velocity of carrier-based aircraft are listed as follows:
Start
Define the study objective
Collect the measured data of the sinking velocity
Obtain the numerical features
Investigate influential factors of the sinking velocity
Perform correlation analysis
Select important influential factors
Build mathematical model
Does it meet the requirement?
Implement influential parameter analyses
End

Figure 1. Procedure of influential parameter analyses of the sinking velocity.

(1) Ensure the study objective and collect the measured data of the sinking velocity. The carrier-based aircraft F/A 18 is selected as the research object, the measured data are chosen from Reference [4].

(2) Obtain the numerical characteristics of the deck landing sinking velocity of carrier-based aircraft including the mean, standard deviation and distribution feature by using statistical analysis method [16, 17].

(3) Study the influential factors on the sinking velocity. The influential factors mainly contain three types: landing environmental parameters, flight attitude parameters and landing operational parameters.

(4) Perform the correlation analyses between the sinking velocity and influential factors by using the principles of partial correlation analysis [18, 19], and further calculate the correlation coefficient.

(5) Chose important influential factors based on the results of correlation analyses, and establish the mathematical model of the sinking velocity in light of LM algorithm [20], and implement the influential parameter analyses of the sinking velocity.

2.2. Correlation analysis method
Correlation analysis is firstly applied to study the correlation relationship among natural phenomena and explore the related direction and degree among the related stochastic phenomena. Subsequently, the relationship of multiple parameters is performed by correlation analysis. The correlation coefficient analysis is selected to finish the correlation analyses of the sinking velocity of carrier-based aircraft. For multiple parameters involved in the process of influential parameter analyses for the sinking velocity of carrier-based aircraft, partial correlation analysis method is utilized to perform the correlation analysis.

The correlation coefficient is the statistic index for the correlative degree between two variables, whose range is [-1, 1]. Here, a positive value reveals that the output has a positive variety that changes with the change of the input variable, and vice versa. The formula of correlation coefficient is

$$r_{xy} = \frac{s_{xy}}{s_x s_y}$$

(1)
in which \( r_{xy} \) is the correlation coefficient; \( s_{xy} \) the covariance of samples; \( s_x \) and \( s_y \) the standard deviations. \( r_{xy} = 1 \) illustrates that the two variables are completely linear correlation, \( r_{xy} = -1 \) reveals that the two variables are perfectly negative correlated, and \( r_{xy} = 0 \) expresses that the two variables are completely uncorrelated. |\( r_{xy} | \geq 0.8 \) indicates the variables are highly correlated, \( 0.5 \leq |\( r_{xy} | < 0.8 \) means the variables are moderately correlated, \( 0.3 \leq |\( r_{xy} | < 0.5 \) explains the variables are lowly correlated, and |\( r_{xy} | < 0.3 \) denotes the variables are weakly correlated. \( s_{xy} \), \( s_x \) and \( s_y \) are

\[
\begin{align*}
    s_x &= \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2, \\
    s_y &= \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2, \\
    s_{xy} &= \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})
\end{align*}
\]

(2)

where \( \bar{x} \) and \( \bar{y} \) are the mean values of the variables of \( x \) and \( y \), respectively. The formula of \( \bar{x} \) and \( \bar{y} \) are

\[
\begin{align*}
    \bar{x} &= \frac{1}{n} \sum_{i=1}^{n} x_i; \\
    \bar{y} &= \frac{1}{n} \sum_{i=1}^{n} y_i
\end{align*}
\]

(3)

Partial correlation analysis is utilized to study the relationship of multiple variables, the measure indexes of partial correlation analysis are first order partial correlation coefficient, second order partial correlation coefficient and higher order partial correlation coefficient, and the theories of these coefficients are listed as follows

(1) First order partial correlation coefficient reflects the correlative degree between two variables, which is

\[
r_{ij-h} = \frac{r_{ij} - r_{ih} r_{jh}}{\sqrt{(1-r_{ih}^2)(1-r_{jh}^2)}}
\]

(4)

in which \( r_{ij} \) is the correlation coefficients between \( x_i \) and \( x_j \); \( r_{ih} \) is the correlation coefficients between \( x_i \) and \( x_h \); \( r_{jh} \) is the correlation coefficients between \( x_j \) and \( x_h \).

(2) Second order partial correlation coefficient indicates correlative degree between two variables, i.e.,

\[
r_{ij-hm} = \frac{r_{ij-h} - r_{im-h} r_{jm-h}}{\sqrt{(1-r_{im-h}^2)(1-r_{jm-h}^2)}}
\]

(5)

(3) Higher order partial correlation coefficient is applied to study the correlative degree between two variables. Assuming that there are \( k \) variables, namely \( x_1, x_2, \ldots, x_k \), the \( g \) (\( g \leq k-2 \)) order partial correlation coefficient of between \( x_i \) and \( x_j \) can be expressed as

\[
r_{ij-l_1 \ldots l_g} = \frac{r_{ij-l_1 \ldots l_g} - r_{il_1 \ldots l_g} r_{jl_1 \ldots l_g}}{\sqrt{1-r_{il_1 \ldots l_g}^2(1-r_{jl_1 \ldots l_g}^2)}}
\]

(6)

where \( r_{ij-l_1 \ldots l_g} \), \( r_{il_1 \ldots l_g} \) and \( r_{jl_1 \ldots l_g} \) are the \( g \) order partial correlation coefficients.

The correlation between two variables can be determined by the hypothesis test, i.e.,

\[
H_0 : \rho = 0 \iff H_1 : \rho \neq 0
\]

(7)

\[
t = r \sqrt{\frac{n-k-2}{1-r^2}}
\]

(8)

\[
p = P \left\{ \left| z \right| > t \right\}
\]

(9)

where \( n \) is the number of sample; \( n-k-2 \) is the degree of freedom; \( r \) is the partial correlation coefficient.
2.3. Multivariate statistical analysis method

To accomplish the influential parameter analyses of sinking velocity of carrier-based aircraft, nonlinear regression method is selected to deal with multivariate statistical analysis. LM algorithm is utilized to build the mathematical model of the sinking velocity, which can address the problems of abnormal convergence and strong dependence on initial values. The form of the relationship is written as follows

\[ y = f(x, \beta) \] (10)

in which \( y \) indicates the output response; \( x \) expresses the vector of input variables; \( \beta \) represents the vector of regression coefficients.

Eq. (10) is rewritten as first order Taylor function, which takes \( \beta \) as the variable, i.e.,

\[
\begin{pmatrix}
\frac{\partial f(x_1)}{\partial \beta_1} & \frac{\partial f(x_1)}{\partial \beta_2} & \cdots & \frac{\partial f(x_1)}{\partial \beta_m} \\
\frac{\partial f(x_2)}{\partial \beta_1} & \frac{\partial f(x_2)}{\partial \beta_2} & \cdots & \frac{\partial f(x_2)}{\partial \beta_m} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f(x_n)}{\partial \beta_1} & \frac{\partial f(x_n)}{\partial \beta_2} & \cdots & \frac{\partial f(x_n)}{\partial \beta_m}
\end{pmatrix}
\]

\[ J = \begin{pmatrix}
\frac{\partial f(x_1)}{\partial \beta_1} & \frac{\partial f(x_1)}{\partial \beta_2} & \cdots & \frac{\partial f(x_1)}{\partial \beta_m} \\
\frac{\partial f(x_2)}{\partial \beta_1} & \frac{\partial f(x_2)}{\partial \beta_2} & \cdots & \frac{\partial f(x_2)}{\partial \beta_m} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f(x_n)}{\partial \beta_1} & \frac{\partial f(x_n)}{\partial \beta_2} & \cdots & \frac{\partial f(x_n)}{\partial \beta_m}
\end{pmatrix} \] (11)

Eq. (11) can be rewritten as follows

\[ Y = F + J \Delta \beta \] (12)

Here \( Y \) and \( F \) are displayed by

\[ Y = \begin{bmatrix} y_1 & y_2 & \cdots & y_n \end{bmatrix}^T \] (13)

\[ F = \begin{bmatrix} f(x_1, \beta_0) & f(x_2, \beta_0) & \cdots & f(x_n, \beta_0) \end{bmatrix}^T \] (14)

Based on Eq. (12), \( \Delta \beta \) is

\[ \Delta \beta = (J^T J)^{-1} J^T (Y - F) \] (15)

The rank of Hessen matrix \( J^T J \) is required to be full rank. Subsequently, damping factor \( \lambda \) is introduced into Eq. (15) for the LM algorithm, i.e.,

\[ \Delta \beta = -(J^T J + \lambda I)^{-1} J^T (Y - F) \] (16)

The procedure of LM algorithm is shown as follows: define the initial value \( c_0 \) and ensure \( n \) group samples \((x_i, y_i)\); and obtain Jacobian matrix \( J \), \( Y \) and \( F \), and select the initial value of damping factor \( \lambda \); compute Hessen matrix \( J^T J \) and \( \Delta \beta \); check the calculation result meet the convergence condition (as shown in Eq. (17)), if the calculation result satisfies the convergence condition and \( \Delta \beta \) meets the precision requirement, the regression coefficients are acquired, conversely, the initial value of damping factor \( \lambda \) needs to be reset.

\[
(F_i - Y_i)^T \begin{pmatrix} \beta + \Delta \beta \end{pmatrix} (F_i - Y_i) < (F_i - Y_i)^T \begin{pmatrix} \beta + \Delta \beta \end{pmatrix} (F_i - Y_i) < (F_i - Y_i)^T \begin{pmatrix} \beta \end{pmatrix} (F_i - Y_i) \]

2.4. Statistical characteristics of sinking velocity

As for naval aircraft, the Joint Service Specification Guide-JSSG-2006 from United States Department of Defense (USDD) reveals that the sinking velocity of carrier-based aircraft is affected by Frensol lens device, aircraft velocity, carrier size and characteristics, and sea conditions as well. Therefore, it is necessary to investigate the statistical characteristics of sinking velocity of carrier-based aircraft on basis of the measured data.

From Reference [4], we know that the measured data used in this paper come from the report of the deck landing measured parameters on F/A-18 and other carrier-based aircrafts of United States Navy in 1991. The measured data of sinking velocity of each flight for F/A-18 are listed in Table 1.

**Table 1. F/A-18 sinking velocity measured data.**
2.5. Distribution characteristics of sinking velocity for F/A-18

In light of the measured data, the distribution histogram method is applied to study the distribution characteristics of sinking velocity based on the frequency statistics in a certain range. The histogram of the distribution characteristics is shown in Figure 2. As shown in Figure 2, it is demonstrated that the distribution characteristics of sinking velocity for F/A-18 basically obeys a normal distribution.

![Figure 2. Histogram of sinking velocity for F/A-18 carrier-based aircraft.](image)

2.6. Distribution hypothesis test of sinking velocity for F/A-18

To further verify the distribution characteristics of sinking velocity, K-S hypothesis test method is used to check whether the distribution characteristics of sinking velocity for F/A-18 obey a normal distribution or not. The analysis results are shown in Table 2.

| Statistic | Day     | Night   |
|-----------|---------|---------|
| Mean, m/s | 3.713   | 3.331   |
| Std. dev., m/s | 0.665 | 0.828   |
| Max. abs. err., m/s | 0.016 | 0.028   |
From Table 2, the asymptotic significant values of sinking velocity are greater than the given significance level 0.05 in the process of deck landing of carrier-based aircraft during day or night, which explains that the distribution characteristics of sinking velocity obeys normal distribution.

2.7. Numerical features of sinking velocity for F/A-18

According to the above statistical results, the numerical features of sinking velocity are achieved with measured data. Hereinto, the sinking velocity boundary is determined by using the mean plus and minus 3.09 times standard deviation. The results are displayed in Table 3.

| Statistic                  | Sinking velocity |
|----------------------------|------------------|
|                            | Day              | Night            |
| Mean, m/s                  | 3.801            | 3.434            |
| Std. dev., m/s             | 0.671            | 0.793            |
| Range, m/s                 | 1.728-5.874      | 0.984-5.888      |
| Max., m/s                  | 5.874            | 5.888            |

As illustrated in Table 3, the ranges of sinking velocity of F/A-18 during day and night are [1.728, 5.874] and [0.984, 5.888]. The maximum sinking velocity of F/A-18 based on the measured data is 5.888 m/s. The mean of F/A-18 deck landing sinking velocity in the night is smaller than that of the day. According to the calculation method in the U.S. military standard MIL-A-8863C, the maximum sinking velocity at night is greater than that of the day. It is revealed the landing time indeed influences the sinking velocity.

3. Correlation analysis of sinking velocity for F/A-18

According to the deck landing parameters measured results, the scatter charts between the important parameters of deck landing and the sinking velocity are given. However, it is necessary to evaluate the correlation degree between the deck landing parameters and the sinking velocity. Moreover, multiple factors are involved in the process of correlation analyses, the important influential factors are selected to indicate the procedure of partial correlation analysis.

The scatter charts between the aircraft glide angle and sinking velocity of the F/A-18 carrier-based aircraft during day and night are shown in Figure 3. As shown in Figure 3, there is a strongly linear relationship between aircraft glide angle and sinking velocity for the day and the night.

![Figure 3](image)

Based on the basic theory of partial correlation analysis, the results of aircraft glide angle and sinking velocity are shown in Table 4.

| Variable | statistic | Day       | Night      |
|----------|-----------|-----------|------------|

Table 4. Partial correlation analysis results of aircraft glide angle and sinking velocity.
As shown in Table 4, the partial correlation coefficient and significant of aircraft glide angle and sinking velocity during day are 0.988 and 0. The significant value of aircraft glide angle and sinking velocity during day is less than 0.01, which indicates the aircraft glide angle and the sinking velocity are highly positive correlated. Additionally, the partial correlation coefficient of aircraft glide angle and sinking velocity is 0.992 with the significant of 0, which reveals that aircraft glide angle and sinking velocity are also highly positive correlated.

| Variable            | Day                  | Night                |
|---------------------|----------------------|----------------------|
|                     | Statistic            | Approach velocity, m/s | Sinking velocity, m/s | Approach velocity, m/s | Sinking velocity, m/s |
| Aircraft glide angle, deg | Correlation          | 1.000                | 0.988               | 1.000                | 0.992               |
|                     | Significant (bilateral) | 0                    | 0                   | 0                    | 0                   |
|                     | df                   | 0                    | 165                 | 0                    | 60                  |
| Sinking velocity, m/s | Correlation          | 0.545                | 1.000               | 0.656                | 1.000               |
|                     | Significant (bilateral) | 0                    | 0                   | 0                    | 0                   |
|                     | df                   | 0                    | 165                 | 0                    | 60                  |

Figure 4. Aircraft approach velocity-sinking velocity scatter chart. The scatter chart between the aircraft approach velocity and sinking velocity of F/A-18 during day and night is shown as Figure 4. From Figure 4, the relationship between approach velocity and sinking velocity is not clear. The partial correlation analysis results of aircraft approach velocity and sinking velocity are listed in Table 5.

Table 5. Correlation analysis results of aircraft approach velocity and sinking velocity.

| Variable            | Statistic            | Day                  | Night                |
|---------------------|----------------------|----------------------|----------------------|
|                     | Approach velocity, m/s | Sinking velocity, m/s | Correlation          | Significant (bilateral) | df | Approach velocity, m/s | Sinking velocity, m/s | Correlation          | Significant (bilateral) | df |
| Aircraft glide angle, deg | Correlation          | 1.000                | 0.988               | 1.000                | 0.992               |
|                     | Significant (bilateral) | 0                    | 0                   | 0                    | 0                   |
|                     | df                   | 0                    | 165                 | 0                    | 60                  |
| Sinking velocity, m/s | Correlation          | 0.545                | 1.000               | 0.656                | 1.000               |
|                     | Significant (bilateral) | 0                    | 0                   | 0                    | 0                   |
|                     | df                   | 165                  | 0                    | 60                  | 0                   |

(a) Day

(b) Night
Figure 5. Aircraft engaging velocity-sinking velocity scatter chart.

As shown in Table 5, the partial correlation analysis results show that the partial correlation coefficient of the aircraft approach velocity and sinking velocity is 0.545, the significant is 0 during day, thus the aircraft approach velocity and the sinking velocity are moderately positive correlated; the partial correlation coefficient of the aircraft approach velocity and sinking velocity is 0.656, the significant is 0 during night, so the aircraft approach velocity and the sinking velocity are moderately positive correlated. Therefore, it is obvious that the aircraft approach velocity has a certain influence on the sinking velocity.

The scatter chart between the engaging velocity and sinking velocity of the F/A-18 carrier-based aircraft during day and night is shown as Figure 5. As displayed in Figure 5, it is illustrated that the relationship between engaging velocity and sinking velocity is not clear when the landing time is during day or night. The partial correlation analysis results of the engaging velocity and sinking velocity are shown in Table 6.

Table 6. Partial correlation analysis results of aircraft engaging velocity and sinking velocity.

| Variable                  | Statistic         | Day |            |            | Night |            |            |
|---------------------------|-------------------|-----|------------|------------|-------|------------|------------|
| Engaging velocity, (m/s)  | Correlation       | 1.000 | 0.561     | 1.000     | 0.321 |            |            |
|                           | Significant (bilateral) | 0 | 0          | 0 | 0.011 |            |            |
|                           | df                | 0   | 165        | 0 | 60    |            |            |
| Sinking velocity, m/s     | Correlation       | 0.561 | 1.000     | 0.321     | 1.000 |            |            |
|                           | Significant (bilateral) | 0 | 0          | 0.011 | 0 |            |            |
|                           | df                | 165 | 0          | 60 | 0 |            |            |

From Table 6, the partial correlation coefficient and significant of the aircraft engaging velocity and sinking velocity are 0.561 and 0 during day, the aircraft engaging velocity and sinking velocity are thus moderately positive correlated; at night, the correlation coefficient of the aircraft engaging velocity and sinking velocity is 0.321, the significant is 0 and is less than 0.01, it is considered that the aircraft engaging velocity and the sinking velocity are lowly positive correlated. Therefore, it can be seen that the aircraft engaging velocity has a certain influence on the sinking velocity.

The scatter chart between the deck pitch angle and sinking velocity during day and night is shown as Figure 6. It can be seen that the relationship between deck pitch angle and sinking velocity is not clear when the landing is during day or night from Figure 6. The partial correlation analysis results of the deck pitch angle and sinking velocity are shown in Table 7.

Table 7. Partial correlation analysis results of deck pitch angle and sinking velocity.

| Variable                  | Statistic         | Day |            |            | Night |            |            |
|---------------------------|-------------------|-----|------------|------------|-------|------------|------------|
| Deck pitch angle, deg     | Correlation       |            |            |            | 0.165 |            |            |
|                           | Significant (bilateral) |            |            | 0.023 | 0 |            |            |
|                           | df                |            | 60 |            | 0 |            |            |
Table 7. Partial correlation analysis results for F/A-18 landing parameters and sinking velocity.

| Landing Parameter                  | Day Correlation coefficient | Day Correlation degree | Night Correlation coefficient | Night Correlation degree |
|-----------------------------------|----------------------------|------------------------|-------------------------------|-------------------------|
| Pitch angle                       | 0.058                      | 0.453                  | No                            | 0.246                   |
| Roll angle                        | -0.093                     | 0.230                  | No                            | 0.997                   |
| Pitch rate                        | -0.225                     | 0.003                  | No                            | 0.140                   |
| Roll rate                         | -0.084                     | 0.278                  | No                            | 0.478                   |
| F.P.A                             | -0.064                     | 0.413                  | No                            | 0.613                   |
| Yaw                               | -0.041                     | 0.601                  | No                            | 0.499                   |
| Off-center engaging distance      | -0.001                     | 0.985                  | No                            | 0.992                   |
| Ramp to touchdown distance        | -0.041                     | 0.600                  | No                            | 0.205                   |
| Landing weight                    | 0.196                      | 0.011                  | No                            | 0.674                   |
| Carrier velocity                  | 0.215                      | 0.005                  | No                            | 0.005                   |
| Deck roll angle                   | -0.094                     | 0.228                  | No                            | 0.051                   |

Table 8. Correlation classification for F/A-18 influential factors.

In accordance with the correlation analysis, the classification results of influential factors for F/A-18 are shown in Table 9.

As shown in Tables 8 and 9, the correlation analyses between influential parameters and sinking velocity exist differences during day and night. Aircraft glide angle and deck pitch angle are highly correlated with the sinking velocity, and approaching velocity is moderately correlated with sinking velocity. Engaging velocity and sinking velocity is moderately correlated during day.

Through the correlation analyses, the parameters of aircraft glide angle, deck pitch angle, aircraft engaging velocity and approach velocity are related to sinking velocity. However, it is still unable to
describe the influencing degree. Therefore, the sinking velocity calculation formula should be fitted to determine influence of the above deck landing parameters.

4. Regression analysis of sinking velocity for F/A-18

The influential parameters are determined by the partial correlation analysis. The mathematical model of sinking velocity is fitted by the LM algorithm, to determine the influence degree of the related parameters.

Based on the ideal condition of deck landing, the relationships of sinking velocity and glide angle, deck pitch angle, engaging velocity are shown as Fig. 7.

![Figure 7. Landing parameter vector relationship.](image)

The mathematical model of sinking velocity is fitted with 267 group data of F/A-18. The initial values of parameters are estimated, the LM algorithm is adopted to gain undetermined coefficients.

The details of establishing the mathematical model of sinking velocity are:

(1) The first regression analysis

The formula of the mathematical model is

\[ V_v = \beta_1 V_e \tan (\beta_2 \gamma_p - \beta_3 \delta_p) + \beta_4 \]  

(18)

The initial values of undetermined coefficients are set up. The undetermined coefficients are acquired using the LM algorithm. Because the 95% confidence intervals of \( \beta_1, \beta_2, \beta_3 \) and \( \beta_4 \) do not contain 0; the correlation coefficient is -0.996 and its absolute value is close to 1. Therefore, the mathematical model should be updated.

(2) The second regression analysis

Based on Eq. (18), the mathematical model is

\[ V_v = \beta_1 V_e \tan (\gamma_p - \beta_4 \delta_p) + \beta_4 \]  

(19)

Furthermore, the undetermined coefficients are obtained. However, \( \beta_4 \) is not significant for the 95% confidence interval contains 0; the relationship between \( \beta_1 \) and \( \beta_4 \) can be expressed by an identical parameter owing to the absolute value of correlation coefficient between \( \beta_1 \) and \( \beta_4 \) is 0.914 and is close to 1, the regression model should be further modified.

(3) The third regression analysis

On basis of first and second regression analyses, the mathematical model is simplified as follows:

\[ V_v = \beta_1 V_e \tan (\gamma_p - \beta_3 \delta_p) \]  

(20)

Similarly, the LM algorithm and the measured data are used to determine these related coefficients. The third regression analysis results are displayed in Table 10.

| Undetermined coefficient | Estimate | Std. error | 95% Confidence interval | Correlation coefficient |
|-------------------------|----------|------------|-------------------------|-------------------------|
| \( \beta_1 \)           | 0.998    | 0.004      | 0.989 - 1.006           | \( \beta_1 \)           |
| \( \beta_3 \)           | 1.003    | 0.033      | 1.060 - 1.073           | -0.871                  |

As shown in Table 10, the 95% confidence interval of parameters \( \beta_1 \) and \( \beta_3 \) is not contain 0, which indicates that \( \beta_1 \) and \( \beta_3 \) are statistically significant. The formula of the mathematical model of sinking velocity is

\[ V_v = 0.998 V_e \tan (\gamma_p - 1.003 \delta_p) \]  

(21)

The variance analysis (as shown in Eq. (22)) is implemented to validated the mathematical model combined with the related parameters shown in Table 13.
where \( S_r \) is the residual square sum, \( S_u \) is the updated residual square sum.

\[
R^2 = 1 - \frac{S_r}{S_u}
\]  \hspace{1cm} (22)

The determination coefficient \( R^2 = 0.971 \) is acquired from Eq. (22) and Table 11, which indicates that the proposed model is good. Additionally, the predicted values and the measured values of sinking velocity for F/A-18 carrier-based aircraft are applied to validate the effectiveness and applicability of the proposed method. The computing errors are shown in Figure 8.

**Figure 8.** Computational errors of sinking velocity.

From Figure 8, the absolute errors of mathematical model of sinking velocity with LM algorithm are less than 5% during day and night; the fitting accuracies of the mathematical model of sinking velocity with LM algorithm during day and night reach 98.4% and 98.7%. Therefore, the established mathematical model of sinking velocity is feasible in engineering practice.

## 5. Conclusions

In this paper, the correlation analysis and LM algorithm are utilized to study the relationships of multiple influential parameters and establish the mathematical model of the sinking velocity and the related variables with important influence degree by the measured data of F/A-18 carrier-based aircraft. The results of influential parameter analyses of sinking velocity for carrier-based aircraft are summarized as follows:

1. Through the statistical analysis with the measured data, the numerical feature of sinking velocity of carrier-based aircraft obeys normal distribution in the process of deck landing during day and night.

2. Aircraft glide angle and deck pitch angle are highly correlated, and the approaching velocity is moderately correlated, the engaging velocity is moderately correlated during day and lowly correlated during night.

3. The LM algorithm is applied to build the mathematical model of sinking velocity with updated modelling by considering the effects of influential factors. And then the determination coefficient \( R^2 = 0.971 \) is calculated via the variance analysis.

4. The predicted values of the mathematical model of sinking velocity for F/A-18 are basically consistent with the measured data, which validates the effectiveness and feasibility of the method in engineering practice.

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