DECAY RATES OF MEDIUM-HEAVY Λ-HYPERNUCLEI WITHIN
THE PROPAGATOR METHOD

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The Λ decay rates in nuclei has been calculated in ref. 1 using the Propagator
Method in Local Density Approximation. We have studied the dependence of the
widths (including the one for the two-body induced process \(ΛNN \rightarrow NNN\)) on
the \(NN\) and \(ΛN\) short range correlations. Using a reasonable parametrization
of these correlations, as well as realistic nuclear densities and Λ wave functions, we
reproduce, for the first time, the experimental non-mesonic widths from medium
to heavy hypernuclei.

1 Introduction

The study of hypernuclear physics may help in understanding some present pro-
blems related, for instance, to aspects of weak interactions in nuclei, or to the
origin of the spin-orbit interaction in nuclei. Besides, it is a good instrument to
study the role of quark degrees of freedom in the hadron-hadron interactions at
short distances and the renormalization properties of pions in the nuclear medium.

The weak decay of hypernuclei occurs via two channels; the so called mesonic
channel:

\[
Λ \rightarrow πN \ (Γ_M),
\]

and the non-mesonic one, in which the pion emitted from the weak hadronic vertex
is absorbed by one or more nucleons in the medium, for example

\[
ΛN \rightarrow NN \ (Γ_1),
\]

\[
ΛNN \rightarrow NNN \ (Γ_2).
\]

Obviously, the non-mesonic processes \((Γ_{NM} = Γ_1 + Γ_2 + ...)\) can also be mediated
by the exchange of more massive mesons. The non-mesonic decay is only possible
in nuclei and, nowadays, the study of the hypernuclear decay is the only practical
way to get information on the weak process \(ΛN \rightarrow NN\), especially on its parity
conserving part.

The free Λ decay is compatible with the \(ΔI = 1/2\) isospin rule, which is also
valid in other non-leptonic strangeness changing processes. This rule is based on the
experimental observation that \(Γ_{Λ → π^-p}/Γ_{Λ → π^0n} \sim 2\), but it is not yet understood
on theoretical grounds. From theoretical calculations like the one in ref. 2 and
from experimental measurements 3 there is some evidence that the \(ΔI = 1/2\) rule
is broken in the nuclear mesonic decay. However, this is essentially due to shell
effects and might not be directly related to the weak process. A recent estimate of
\[ \Delta I = 3/2 \] contributions to the \( \Lambda N \to NN \) reaction found moderate effects on the hypernuclear decay rates. In the present calculation of the decay rates in nuclei we will assume this rule as valid. The momentum of the final nucleon in \( \Lambda \to \pi N \) is about 100 MeV for \( \Lambda \) at rest, so this process is suppressed by the Pauli principle, particularly in heavy nuclei. It is strictly forbidden in infinite nuclear matter (where \( k_F^0 \approx 270 \) MeV), but in finite nuclei it can occur because of three important effects: 1) in nuclei the Pauli blocking is less effective and the hyperon has a momentum distribution which allows larger momenta for the final nucleon, 2) the final pion feels an attraction by the medium such that for fixed momentum it has an energy smaller than the free one and consequently, the final nucleon again has more chance to come out above the Fermi surface, and 3) on the nuclear surface the local Fermi momentum is smaller than \( k_F^0 \) and favours the decay. Nevertheless, the mesonic width decreases fast as the mass number \( A \) of the hypernucleus increases. The mesonic rate is very sensitive to the pion self-energy, so that from the study of the mesonic decays it could be possible to extract important information on the pion-nucleus optical potential, which we do not know, nowadays, in a complete form.

The final nucleons in the non-mesonic process \( \Lambda N \to NN \) come out with a momentum \( \approx 420 \) MeV, so that this decay is not forbidden by the Pauli principle. On the contrary, apart from the \( s \)-shell hypernuclei, it dominates over the mesonic decay. The non-mesonic channel is characterized by large momentum transfers, so that the details of the nuclear structure do not have a substantial influence while the \( NN \) and \( \Lambda N \) short range correlations turn out to be very important. There appears to be an anticorrelation between mesonic and non-mesonic decay modes: indeed the total lifetime is fairly constant from light to heavy hypernuclei. \( \tau_{\text{exp}} = (0.5 \pm 1) \tau_{\text{free}}. \)

Nowadays, the main problem concerning the weak decay rates is to reproduce the experimental values for the ratio \( \Gamma_n/\Gamma_p \) between the neutron and the proton induced widths \( \Lambda n \to nn \) and \( \Lambda p \to np \). All theoretical calculations underestimate the experimental data for all the considered hypernuclei:

\[
\left\{ \frac{\Gamma_n}{\Gamma_p} \right\}_{\text{Th}} \ll \left\{ \frac{\Gamma_n}{\Gamma_p} \right\}_{\text{Exp}} \quad 0.5 \leq \left\{ \frac{\Gamma_n}{\Gamma_p} \right\}_{\text{Exp}} \leq 2. \quad (4)
\]

In the One Pion Exchange (OPE) approximation the values for this ratio are 0.1 \( \div \) 0.2. On the other hand, the OPE model has been able to reproduce the 1-body stimulated non-mesonic rates \( \Gamma_1 = \Gamma_n + \Gamma_p \) for light and medium hypernuclei. In order to solve this puzzle many attempts have been made up to now, but without success. Among these we recall the inclusion in the \( \Lambda N \to NN \) transition potential of mesons heavier than the pion \( \pi \), the inclusion of contributions that violate the \( \Delta I = 1/2 \) rule and the description of the short range baryon-baryon interaction in terms of quark degrees of freedom. This last calculation is the only one which has found a fairly large (but not sufficient) increase of the neutron to proton ratio with respect to the OPE one. However, this calculation is carried out only for \( s \)-shell hypernuclei; moreover, the employed quark-lagrangian does not reproduce the experimental ratio between the \( \Delta I = 1/2 \) and \( \Delta I = 3/2 \) transition amplitudes for the \( \Lambda \) free decay.
The analysis of the ratio $\Gamma_n/\Gamma_p$ is influenced by the 2-nucleon induced process $\Lambda NN \rightarrow NNN$. By assuming that the meson produced in the weak vertex is mainly absorbed by a strongly correlated neutron-proton pair, the 3-body process turns out to be $\Lambda np \rightarrow nmp$, so that a considerable fraction of the measured neutrons could come from this channel and not only from the $\Lambda n \rightarrow nn$ and $\Lambda p \rightarrow np$ ones. In this way it might be possible to explain the large experimental $\Gamma_n/\Gamma_p$ ratio, which originally has been analyzed without taking into account the 2-body stimulated process. Nevertheless, the situation is far from being clear and simple. In fact, $\Gamma_n/\Gamma_p$ is sensitive to the energy spectra of the emitted nucleons, whose calculation also requires a careful treatment of the final state interactions of the nucleons. In ref. the energy distributions were calculated using a Monte Carlo simulation to describe the final state interactions. A direct comparison of those spectra with the experimental ones seems to favour large $\Gamma_n/\Gamma_p$ ratios, in strong disagreement with the OPE predictions. However, it was also pointed out the convenience of measuring the number of protons per decay event. In fact, this observable gives a more reliable $\Gamma_n/\Gamma_p$ ratio and it is less sensitive to details of the Monte Carlo simulation accounting for the final shape of the spectra.

2 Decay widths

We present here the results of a new evaluation of the decay rates for medium to heavy $\Lambda$-hypernuclei based on the Propagator Method introduced in ref. The parameters of the model were adjusted to reproduce the non-mesonic width of $^{12}\Lambda C$. Then the decay rates for heavier hypernuclei have been predicted and compared, when possible, with the experiment.

2.1 The model

The propagator technique, extensively explained in refs., provides a unified picture of the different decay channels and it is equivalent to the standard Wave Function Method. The calculation of the widths is performed in nuclear matter, and then extended to finite nuclei via the Local Density Approximation (LDA).

The starting point for the calculation is the $\Lambda \rightarrow \pi N$ effective lagrangian, parametrized in the form:

$$L_{\Lambda\pi N} = Gm_\pi^2\bar{\psi}_N(A + B\gamma_5)\vec{\sigma} \cdot \vec{\phi}_\pi \psi_\Lambda + h.c.,$$

where the values of the weak coupling constants $G$, $A$, $B$ are fixed on the observables of the free $\Lambda$ decay. In order to enforce the $\Delta I = 1/2$ rule, in eq. the hyperon is assumed to be an isospin spurion with $I_z = -1/2$. The decay rate of a $\Lambda$ in nuclear matter is obtained from the imaginary part of the $\Lambda$ self-energy through the relation:

$$\Gamma_\Lambda = -2\text{Im}\Sigma_\Lambda.$$
different decay channels when they are cut in all the possible ways. The structure of the decay width in nuclear matter is the following:

\[ \Gamma_\Lambda(\vec{k}) \propto \int d\vec{q} \cdots \theta(\mid \vec{k} - \vec{q} \mid - k_F) \theta(k_0 - E_N(\vec{k} - \vec{q}) - V_N) \text{Im} G_\pi(q) \mid_{q_0 = k_0 - E_N(\vec{k} - \vec{q}) - V_N}, \]

where \( G_\pi(q) = \frac{1}{q_0^2 - \vec{q}^2 - m_\pi^2 - \Sigma_\pi^*(q)}, \)

is the pion propagators in nuclear matter, \( \Sigma_\pi^* \) being its proper self-energy; \( k_F \) is the Fermi momentum, \( E_N \) is the nucleon total free energy and \( V_N \) is the nucleon binding energy. Moreover, \( k_0 = E_\Lambda(\vec{k}) + V_\Lambda \), is the \( \Lambda \) energy, containing a binding term (which we take from the experiment).

The decay widths in finite nuclei are obtained in LDA; a local Fermi sea of nucleons, related to the nuclear density by

\[ k_F(\vec{r}) = \left\{ 3\pi^2 \rho(\vec{r}) \right\}^{1/3}, \]

is introduced. Besides, the nucleon binding potential \( V_N \) also becomes \( r \)-dependent in LDA. The decay width in finite nuclei is then obtained through the relations:

\[ \Gamma_\Lambda(\vec{k}) = \int d\vec{r} \mid \psi_\Lambda(\vec{r}) \mid^2 \Gamma_\Lambda(\vec{k}, \rho(\vec{r})), \quad \Gamma_\Lambda = \int d\vec{k} \mid \tilde{\psi}_\Lambda(\vec{k}) \mid^2 \Gamma_\Lambda(\vec{k}), \]

where \( \psi_\Lambda (\tilde{\psi}_\Lambda) \) is the \( \Lambda \) wave function in the coordinate (momentum) space.

### 2.2 Results

In ref. \(^1\), we have studied the influence of the short range correlations and the \( \Lambda \) wave function on the decay width of \(^{12}\Lambda C\), which we have used as a testing ground for the theoretical framework in order to fix the parameters of our model.

The baryon-baryon correlations are governed by the Landau parameters \( g' \) and \( g'_\Lambda \). The choice of these parameters which reproduce the experimental non-mesonic rate of \(^{12}\Lambda C\) is \( g' = 0.8, \ g'_\Lambda = 0.4 \). Neglecting the contribution of the 2-body induced decay channel (namely in ring approximation), with the same \( g'_\Lambda \), we find that \( g' = 0.7 \) should be used. This value is in agreement with the fenomenology of the \( (e, e') \) quasi-elastic scattering \(^4\).

The \( \Lambda \) wave function is obtained from a Wood-Saxon well which exactly reproduces the first two experimental \( \Lambda \) single particle eigenvalues \( (s \text{ and } p \text{ levels}) \). The numerical results for \(^{12}\Lambda C\) are shown in table \(^2\), where they are compared with the experimental data from BNL \(^3\) and KEK \(^4\). In addition to the Wood-Saxon wave function that reproduces the \( s \) and \( p \) levels, we also made a test using the harmonic oscillator wave function with an "experimental" frequency \( \omega \) obtained from the \( s-p \) experimental energy shift, the Wood-Saxon wave function of Dover \textit{et al.} \(^5\), and the microscopic wave function calculated from a non-local self-energy using a realistic \( \Lambda N \) interaction in ref. \(^6\). We have found that all these different \( \Lambda \) wave functions give rise to total decay widths which may differ at most by 15%.
Then, using Wood-Saxon wave functions that reproduce the $s$ and $p$ $\Lambda$-levels and the Landau parameters $g' = 0.8$, $g'_\Lambda = 0.4$, we have extended the calculation to heavier hypernuclei. We note that, in order to reproduce the experimental $s$ and $p$ levels for the hyperon we must use potentials with a nearly constant depth, around $28 \div 32$ MeV, from medium to heavy hypernuclei.

Our results are shown in table 2. The mesonic rate rapidly vanishes by increasing the mass number $A$. This is well known and it is related to the decreasing phase space allowed for the mesonic channel, and to smaller overlaps between the $\Lambda$ wave function $\psi_\Lambda$ and the nuclear surface, as $A$ increases. The 2-body induced decay is rather independent of the hypernuclear dimension and it is about 15% of the total width. The total width is also nearly constant with $A$, as we already know from the experiment. In fig. 1 we compare the results from table 2 with recent (after 1990) experimental data for non-mesonic decay. We remind that the data for nuclei heavier than $^{12}_\Lambda$C refer to the total width. The theoretical results are in good agreement with the data over the whole hypernuclear mass range explored. Moreover, we also see that the saturation of the $\Lambda N \rightarrow NN$ interaction in nuclei appears to be well reproduced.

The main open problem in the study of weak hypernuclear decays is to understand the large experimental value of the ratio $\Gamma_n/\Gamma_p$. However, we have to remind that the data for $\Gamma_n/\Gamma_p$ have a large uncertainty and they have been analyzed without taking into account the 3-body decay mechanism. The study of ref. 10 showed that, even if the three body reaction is only about 15% of the total decay rate, this mechanism influences the analysis of the data determining the ratio $\Gamma_n/\Gamma_p$. Energy
spectra of neutrons and protons from the non-mesonic decay mechanisms were calculated in ref. \cite{ref11}. The momentum distributions of the primary nucleons were determined from the Propagator Method and a subsequent Monte Carlo simulation was used to account for the final state interactions. It was shown that the shape of the proton spectrum is sensitive to the ratio $\Gamma_n/\Gamma_p$. In fact, the protons from the three-nucleon mechanism appeared mainly at low energies, while those from the two-body process peaked around 75 MeV. Since the experimental spectra show a fair amount of protons in the low energy region they would favour a relatively large three-body decay rate or, conversely, a reduced number of protons from the two-body process. Consequently, the experimental spectra were compatible with values for $\Gamma_n/\Gamma_p$ around $2 \div 3$, in strong contradiction with the present theories.

Within the Propagator Method with modified parameters and realistic $\Lambda$ wave functions we have then generated new nucleon spectra for various hypernuclei using the Monte Carlo simulation of ref. \cite{ref11}. Although the new non-mesonic widths are sizably reduced (by about 35%) with respect to those of refs. \cite{ref10,ref11}, the resulting nucleon spectra are practically identical. The reason is that the ratio $\Gamma_2/\Gamma_1$ is essentially the same in both models, and the momentum distributions for the primary emitted nucleons are also very similar. As a consequence, the conclusions drawn in ref. \cite{ref11} still hold.

Therefore, the origin of the discrepancy between theory and experiment for the ratio $\Gamma_n/\Gamma_p$ still needs to be resolved. However, we must notice that on the experimental side, although new spectra are now available \cite{ref16,ref17}, they have not been corrected for energy losses inside the target and detector, so a direct comparison with the theoretical predictions is not yet possible. Attempts to incorporate these corrections by combining a theoretical model for the nucleon rescattering in the nucleus with a simulation of the energy losses in the experimental set-up are now being pursued \cite{ref24}. On the other hand, a forward step towards a clean extraction of the ratio $\Gamma_n/\Gamma_p$ would be obtained if the nucleons which come out from the different non-mesonic processes, $\Lambda N \rightarrow NN$ and $\Lambda NN \rightarrow NNN$ were disentangled \cite{ref25}.

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Figure 1. $\Lambda$ decay widths in finite nuclei as a function of the mass number $A$. 