Investigation of fluid transients in a penstock attached to a Francis turbine rig using seismic interferometry

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Abstract. Pressure fluctuations in hydropower plants contribute to fatigue loads. Pressure pulsations generated by a reaction turbine will create fluid transients in the attached system which can increase the load on structural parts if resonance occurs. Several factors govern transients in a pipe system where the main parameters are the length between reflection points and the propagation velocity. This paper investigates how to find the propagation velocity of pressure waves measured in the penstock at the Hydropower Laboratory at NTNU by applying techniques from seismic interferometry. A simple 1-D model has been developed using the method of characteristics to compare the experimental results to a known system. Seismic interferometry is primarily used in academic and research settings, the basic principle is to obtain the Green’s function between measured signals. The results from the experimental data and simulations are promising, but further research is needed to optimize both data acquisition and post processing.

1. Introduction
Reaction turbines operate by converting both pressure energy and kinetic energy in the flow to mechanical energy at the shaft [4]. For reaction turbines, there is a direct connection between the flow through the turbine and the flow in the conduit. Pressure fluctuations originating from a reaction turbine at steady state will propagate in the conduit and may cause a resonance condition in the power plant [3] [5] [6]. Resonance condition in a pipe system means that a disturbance in the system amplifies with time rather than being dampened. Consequences may be severe pressure and flow fluctuation [6].

The wave speed $a$ and pipe lengths $L$ govern if resonance condition can arise in a conduit system [1]. The water hammer effect describes how pressure pulses propagate in a pipe, i.e. a pipe with constant length and wave speed $a$ will have a fundamental period $T_f = 4L/a$. A cyclic pressure fluctuation with amplitude $A_p$ and frequency $f_p = 1/T_f$ will provoke resonance condition if the amplitude is large enough [6]. In addition to the fundamental harmonic higher harmonics can cause resonance condition. The total theoretical resonance frequency of a system is expected to be an odd harmonic [5].

Dörfler et al., presents a case where a power plant with four Francis turbines with separate penstocks experience resonance condition at high load. During tests for a new air injection system, significant differences in terms of power swings were detected. Dörfler et al. imply that variations in penstock length caused these differences [3]. Several cases of power plants experiencing resonance related to higher harmonic frequencies in the conduit are presented in [5].
To further improve the understanding of dynamic behavior in the penstock in a hydropower plant seismic interferometry has been applied to measurements conducted at the Hydropower laboratory at NTNU. The goal with the measurements and post-processing is to evaluate if seismic interferometry is applicable to determine the wave speed in a short pipe section. The level of noise in signals from pressure measurements in the conduit is high since the turbine, generator and other sources generate fluctuations. Measured data with a high level of noise are challenging to interpret without well-designed processing methods.

Interferometry is based on studying the interference phenomena between a pair of signals and thereby obtain information about differences [8]. The term originates from radio astronomy, where it refers to the cross-correlation methods applied to radio signals from distant objects [9]. The first to derive seismic interferometry theory was Claerbout in 1968 [10]. Since then different methods have been developed, and a vast amount of research has been done to improve techniques [7] [8] [18]. Seismic interferometry is divided into two main groups called controlled-source and passive seismic interferometry. In all cases, the response recorded at one receiver can be treated as there were a source at the other. Seismic interferometry is also often called Greens function retrieval since a point source response is equal to a Greens function convolved with a wavelet [8].

The measured response at points along a pipe section should unveil how fast pressure waves propagate and how distorted the signal gets from one point to another [11].

2. Basic Theory

2.1. Direct wave interferometry:
Pipe flow is often treated as 1-D flow [1]. With this in mind direct wave interferometry is described as by Waapenar et al., [9] in the following manner:

Figure 1a illustrates how an impulse at location $x_s$ at time $t_0$ radiates a plane wave in friction less medium with propagation velocity $a$. The wave propagates to the right along the x-axis.

At location $x_A$ and $x_B$ there are receivers. Figure 1b shows the recorded response at $x_A$ at time $t_A$ and figure 1c shows the response at $x_B$ at time $t_B$. The responses are denoted $G(x_A, x_S, t)$ and $G(x_B, x_S, t)$. G stands for Green’s function as the responses represent the Green’s function between $x_A$ or $x_B$ with $x_S$. In figure 1 the source at $x_S$ is an impulse and the Green’s function at $x_i$ is $G(x_i, x_S, t) = \delta(t - t_i)$ where $t_i = (x_i - x_S)/a$. \(\delta\) is the dirac delta function.

Cross correlating the responses at $x_A$ and $x_B$ result in an impulse at $t_B - t_A$ shown in figure 1d. Since the responses at $x_A$ and $x_B$ have the same path from $x_S$ to $x_A$ the travel time $t_A - t_S$ cancels during cross-correlation. The result is that the impulse in figure 1d can be treated as there were a source at $x_A$ measured at $x_B$ i.e., the Green’s function $G(x_B, x_A, t)$. Note that the source and propagation velocity is not needed. The same is true for the time of $t_S$ since both responses are shifted the same amount.

More precisely the cross-correlation of the impulses at $x_A$ and $x_B$ is represented by $G(x_B, x_S, t) * G(x_A, x_S, -t)$ where the asterisk denotes temporal convolution. The time reversal of the second

Figure 1. 1-D Wave propagating in friction less homogeneous fluid. Taken from [9].
Green’s function turns the convolution into a correlation.

\[ G(x_B, x_A, t) = G(x_B, x_S, t) * G(x_A, x_S, -t) \]  

Equation (1) shows that the cross-correlation at two receivers gives the response at one of those receivers as if there were a source at the other. It also shows why seismic interferometry is often called Green’s function retrieval. The source does not necessarily need to be an impulse it can also be a signal. If the signal is a wavelet \( S(t) \) then the response at \( x_B \) and \( x_A \) can be written as

\[ u(x_A, x_S, t) = G(x_A, x_S, t) * S(t) \quad \text{and} \quad u(x_B, x_S, t) = G(x_B, x_S, t) * S(t). \]

The cross-correlation of the responses at two receivers gives the Green’s function between these receivers convolved with the auto correlation of the source function. This is expressed in equation (2).

\[ G(x_B, x_A, t) * S_S(t) = u(x_B, x_S, t) * u(x_A, x_S, -t) \]  

(2)

For retrieving the cross-correlation of two receivers in a field of random noise some additional steps are needed since the noise propagates in both directions. The steps are derived by Waapnar et al., in [8] and yields equation (3).

\[ \{G(x_B, x_A, t) + G(x_B, x_S, -t)\} * S_N(t) = \langle u(x_B, x_S, t) * u(x_A, x_S, -t) \rangle \]  

(3)

\( S_N(t) \) is the autocorrelation of the random noise source, \( \langle \ldots \rangle \) indicates ensemble averaging. Figure 2 illustrates the result of cross correlating the response at two receivers in a homogeneous distributed random noise field. The peak response in the cross-correlation plot acts as if it was a source at the other receiver.

![Figure 2](image)

**Figure 2.** Homogeneous distribution of random noise around two receivers. Taken from [17].

The cross-correlation between two receivers in a medium with a homogeneous distribution of noise should be nearly symmetrical around \( \tau = 0 \) as in figure 2. Meaning that the positive and negative parts correspond to the Green’s function and its anticausal counterpart [15] [16]. The causal and anticausal parts can be seen as the response peaks at the positive and negative \( \tau \)-axis in the cross-correlation plot in figure 2. In reality, the causal and anticausal response may differ a lot in amplitude. The difference in amplitude is caused by differences in the energy flux in the traveling waves between two receivers. The effect of disturbances can be seen in figure 3.
2.2. Wave speed
A homogenous fluid in a completely filled pipe has a wave speed as described by equation (4) [1] [2].
A calculated example is presented under to get an indication of the expected wave speed in the Laboratory. The diameter and thickness used to calculate $a$ are equal to the values in the hydropower laboratory at NTNU. The other values are as proposed in examples written by Wylie and Streeter [1].

The variation in $a$ is due to different cases of pipe support. Well-fastened pipes have a lower wave speed than pipes only fastened in one end. Different ways to model $\psi$ are proposed in [1] [2].

$$a^2 = \frac{K}{\rho(1 + \frac{D}{e} \frac{K}{E} \psi)}$$

**Calculated wave speed using equation (4).** $\rho$ = Density of liquid (999 kg/m$^3$), $K$ = Volumetric compressibility modulus of liquid (2.2 GPa), $D$ = Diameter of pipe (0.35 m), $e$ = Thickness of pipe (0.01 m), $E$ = Modulus of elasticity pipe (207.7 GPa), $\psi$ = Pipe support parameter [1], $a$ = theoretical wave speed (1261-1288 m/s)

As expressed in equation (4) the wave speed is dependent on several parameters. In addition to these, the effect of air entrainment is dominant as seen in figure 4. Wylie and Streeter recommend new expressions for both compressibility and density if air is present. The new expressions for $K$ and $\rho$ are functions of the amount of air in the mixture as well as the related parameter values, i.e. $K_{\text{water}}, K_{\text{air}}, \rho_{\text{water}}, \rho_{\text{air}}$. Variables related to the pipe are neglected in the new expression for the wave speed since the effect of air entrainment is dominant. figure 4 illustrates how a theoretical wave speed behaves as a function of the percentage of air per total volume [1]. The plot is recreated from [1] where it is validated with experimental data at a static pressure of 324 kPa.

3. Experimental Setup

3.1. Francis Test Rig
Measurements were conducted at the Waterpower laboratory at NTNU on the Francis model test rig. The rig was operated in open loop configuration and follows the steps (1-9) illustrated in figure 5. The open loop configuration used during the experiments maintained a head $H \approx 12.5$ (m). The guide
vanes were kept at maximum opening $\alpha = 14$ (degrees) to minimize obstruction of pressure fluctuations from the turbine, and to maximize rotor stator interaction [13].

3.2. Pressure Sensors

The placements of the pressure sensors are illustrated in figure 6 which is a 2D presentation of (4), (5), (6), (7) and (8) in figure 1. The pressure sensors were placed along the penstock between the pressure tank (4) and the turbine inlet (7). Ideally, all sensors should have been mounted flush with the inner wall of the pipe to prevent disturbances and measurement uncertainties. A recessed sensor will have an added water column which will disturb the measurements [12]. Table 1 presents sensor type, placement, and mounting method.

![Figure 5. Open loop configuration. (1) centrifugal pump, (2) and (3) open water channel, (4) upstream pressure tank, (5) flowmeter, (6) generator, (7) Francis turbine, (8) downstream pressure tank, (9) water outlet to basement.](image)

![Figure 6. Overview of sensor setup](image)

| Sensor | x   | y   | z   | ± $\mu$ | Type          | Mounting |
|--------|-----|-----|-----|---------|---------------|----------|
| PTIN2  | -880| 0   | 0   | 6       | Kuhlite KHM 375 | Flush    |
| IN3    | -1136| 0  | 0   | 10      | Unik5000      | recessed |
| IN2    | -6453| 0  | 0   | 30      | Unik5000      | recessed |
| PTIN1  | -7801| 0  | 0   | 3       | Kuhlite KHM 375 | flush    |
| IN1    | -13550*| 0 | 0   | ?       | Unik5000      | recessed |

Table 1. * indicates that the placement is not measured accurately. $\mu$ is the uncertainty related to the measured placement in mm.

Figure 7 illustrates how sensors PTIN2 and IN3 are mounted. IN3 which is recessed has a small column of water between the sensor and the main flow which may cause disturbances. Both sensor types have a sample rate more than 10 times the expected input signal. Static calibration of the transducers was performed following guidelines in IEC60193 [14]. Dynamic calibration should have been performed,
but necessary equipment was unavailable. The sensor IN1 is placed in a bend close to the pressure tank, accurate measurements of the placement were challenging to perform.

![Unik5000 and Kuhlite KHM 375](image)

**Figure 7.** Recessed and flush mounted pressure sensor. Picture is rotated 90 degrees to the right. (Sensors are mounted on the lower part of the pipe).

### 3.3. Measurement Procedure

The measurements were conducted with constant maximum guide vane opening set to 14 degrees. The speed of the turbine was regulated between 300 and 400 rpm with increments of 10 rpm. The pressure sensors were connected to a DAQ-bridge and a LabVIEW program developed at the Waterpower laboratory was used to log the data.

### 3.4. 1-D Simulation Method of Characteristics

A 1-D simulation model was developed using the method of characteristics in order to compare the measured data with a known system with a set wave velocity and length. The model was validated against an example in [1]. The region modeled is the pipe section between the pressure tank (4) and the turbine (7) in figure 5. Boundary conditions at the inlet (4) and the outlet (7) of the pipe were varied to evoke different results. The pipe section was also divided into several sections with different cross-sections to simulate the flow meter (5) in figure 5.

### 3.5. Data Post Processing

MATLAB was used to post-process the simulated and measured data. First, a spectral analysis was conducted to identify frequency regions with enough energy to perform interferometry, and also regions with frequencies related to noise which were removed. Data from all sensors were cross-correlated with PTIN2 as a reference using the `xcorr`-function in MATLAB. Further improvement of the visualization was attempted by using the `deconvwnr`-function in MATLAB. The results from both methods were filtered with a band-pass filter and plotted with the `wiggle`-function in MATLAB.

### 4. Results

Figure 8 shows 0.3 seconds of a 30 second sample of the experimental data measured at PTIN2 and PTIN1. PTIN2 and PTIN1 are the flush mounted pressure transducers.

![Experimental Data](image)

**Figure 8.** 0.3 (s) of 30 (s) sample measured at PTIN2 and PTIN1. 
*Turbine speed: 350 rpm.*
*Guide vane opening: 14 deg.*
Figure 9 presents the spectral analysis of the data from figure 8. The analysis shows that there is much noise in the measurements, but also distinct peaks related to pressure pulsations from the turbine. The blade passing frequency = 175 Hz is the most dominant peak. The peaks at 50 Hz and 300 Hz are related respectively to noise from electrical equipment and noise from machinery. Many other peaks remain unidentified, but could related to reflections in the conduit and higher harmonics.

Figure 10 shows the filtered Green’s function between each receiver and PTIN2. The responses are plotted vertically with the turbine to the left at Distance = 0. Each transducer is plotted with a distance from the turbine equal to the distances in Table 1. The dashed lines are where the response from an impulse given at PTIN2 at \( t = 0 \) would be largest, given a certain wave speed. The dashed line is fitted to the data by trial and error, the others are only shown for comparison.

Figure 11 shows the deconvolution interferometry of the experimental data. The responses and dashed lines are plotted as in figure 10.

Figure 12 shows the deconvolution interferometry of simulated data. The simulated data has a wave speed equal to 1000 (m/s). The location of where the pressure is retrieved from the simulated model ensures that the exact solution of the characteristic’s method is obtained.
Figure 13a shows a 5 second sampling of the numerical simulation. All points in time and space are included, time increment $dt = 0.0002$ (s), and space increment $dx = 0.1$ (m). Data from vertical lines at Distance = (1,2,6,8 and 9) would give the simulated pressure transducers used to produce figure 12. Figure 13b shows the Green’s function retrieved between the nodes at Distance = 0.1 (m) and the other nodes.

**Figure 11.** Deconvolution interferometry applied to the same experimental data as in figure 10.

Responses are located as in figure 10 and related to the same pressure sensors.

The dashed lines represent the same paths as in figure 10.

**Figure 12.** Deconvolution interferometry applied to simulated data. The simulation included a reflection point, friction and wave speed $a = 1000$ (m/s). Boundary condition at outlet was set to give an impulse at the start of the simulation and random noise throughout the simulation time. The inlet was set to constant head.
Discussion

Figure 11 and 12 shows how the propagation velocity is visualized by cross-correlation interferometry and deconvolution interferometry on a known simulated system. Both figures represent a case where the simulated pressure waves have a propagation velocity of 1000 (m/s), and one can see how the response peaks align on the dashed line. The cross-correlation of the experimental data in figure 10 show that the largest amplitudes are aligned with a straight line corresponding to a propagation velocity of 945 (m/s). The same is true for the amplitudes in figure 11. This is surprising since the wave speed is expected to be higher as the theoretical wave speed in equation (4) implies. One possible explanation for the low wave speed would be the presence of a tiny amount of air in the conduit. As figure 4 shows, the wave velocity is highly sensitive to the volumetric amount of air in water. Another possibility is that the data acquisition and post-processing is conducted in a way that loose vital information. Attempts to increase the frequency range to discover possible merged peaks have been made but higher resolution did not unveil a different result. It should also be noted that the Greens function between the recessed transducers (IN1-IN3) and PTIN2 needed a different filter than the flush mounted sensors. The recessed sensors gave unnaturally large amplitudes at high frequencies, as mentioned earlier a dynamic calibration was never performed. The experimental results are however promising since they show the same trends as the simulated data.

The results from the deconvolution interferometry show that the experimental and simulated data have similarities. The most significant difference is that the amplitudes in the experimental plot decrease and get smeared out while the amplitudes in the simulated plot are more constant throughout the pipe section. The simplicity of the simulated model is probably the reason for this. The simulated pipe sections are stiff, meaning that no energy is used to move or distort the pipe, and the friction term related to the acceleration of the fluid is neglected. In both plots the most significant peaks from the Green’s function between the receivers follow the dashed line corresponding to the lowest wave velocity. The wave speed in the simulated data is \( a = 1000 \) (m/s). Both figure 11 and figure 12 show that the noise distribution is non-homogeneous since the responses are most substantial for \( t > 0 \), this can be compared to the situation in figure 3.

If measurement techniques allowed continuous measurement of the pressure along pipes, better pictures of how reflection points influence the flow could be produced as illustrated in figure 13. figure 13a shows a lot of noise and it is difficult to determine anything about the flow from the picture. By applying cross-correlation interferometry distinct lines appear where the cross-correlation return the greatest values, i.e. the peaks in figure 10.

![Figure 13. Simulated data and cross-correlation](image-url)
6. Conclusion

Applying seismic interferometry as a post processing technique to evaluate experimental and simulated pipe flow in a penstock connected to a reaction turbine show interesting results. The technique is promising and can be used to identify the wave speed in experimental results and to find reflection points in both experimental measurements and simulated data.

The results from the experimental data used in this paper are difficult to analyse, but they show the same behavior as the simulated results. The results indicate that the wave velocity is lower than the expected theoretical value. Further study is needed to optimize how to acquire and post-process data.

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Reference list.

[1] Wylie E B and Streeter V L 1983 Fluid Transients (Thomson-Shore, Dexter, M1, fourth edition)
[2] Twyman J 2018 Wave speed calculation for water hammer analysis. Obras y Proyectos, (20), 86-92.
[3] Dörfler P, Sick M and Coutu A 2012. Flow-induced pulsation and vibration in hydroelectric machinery: engineer’s guidebook for planning, design and troubleshooting. Springer Science & Business Media.
[4] Brekke H 2003 Pumper & turbiner Vannkraftlaborriet NTNU.
[5] Jaeger C 1963 The theory of resonance in hydropower systems. Discussion of incidents and accidents occurring in pressure systems. Journal of Basic Engineering, 85(4), 631-640.
[6] Chaudhry M H 1979 Applied hydraulic transients (No. 627 C4) New York: Van Nostrand Reinhold.
[7] Haavik K E 2016 Source-Depth Diversity for Enhanced Marine Seismic Imaging. NTNU
[8] Curtis A Gerstoft P Sato H Snieder R and Wapenaar K 2006 Seismic interferometry—Turning noise into signal The Leading Edge, 25(9), 1082-1092.
[9] Wapenaar K Draganov D Snieder R Campman X & Verdel A 2010. Tutorial on seismic interferometry: Part I—Basic principles and applications. Geophysics, 75(5), 75A195-75A209.
[10] Claerbout J F 1968 Synthesis of a layered medium from its acoustic transmission response. Geophysics, 33(2), 264-269.
[11] Vaezi Y 2016 Application of Seismic Interferometry in Microseismic Monitoring. Department of Physics, University of Alberta.
[12] Franklin R and Wallace J M 2016 Absolute Measurements of Static-Hole Error Using Flush Transducers. J.Fluid Mech., 42(01), 33-48.
[13] Antonsen Ø 2007 Unsteady flow in wicket gate and runner with focus on static and dynamic load on runner. NTNU
[14] IEC 60193 1999 Hydraulic turbines, storage pumps and pump-turbines- Model acceptance tests, (1999-11-16)
[15] Lobkis O I and Weaver R L 2001 On the emergence of the Green’s function in the correlations of a diffuse field. The Journal of the Acoustical Society of America, 110(6), 3011-3017.
[16] Snieder R and Hagerty M 2004 Monitoring change in volcanic interiors using coda wave interferometry: Application to Arenal Volcano, Costa Rica. Geophysical research letters, 31(9).
[17] Garnier J and Papanicolaou G 2009 Passive sensor imaging using cross correlations of noisy signals in a scattering medium. SIAM Journal on Imaging Sciences, 2(2), 396-437.
[18] Vasconcelos I and Snieder R 2007 Interferometry by deconvolution, Part I: theory and numerical examples. Center for Wave Phenomena and Department of Geophysics, Colorado School of Mines, Golden, CO 80401