Leptogenesis at the TeV scale

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Abstract

We present a general description of the problems encountered when attempting to build a simple model of leptogenesis and hence of baryogenesis at an energy scale as low as 1-10 TeV. We consider three possible lepton asymmetry enhancement mechanisms in the out-of-equilibrium decay scenario, emphasizing the three body decay mechanism as most natural. A new model based on the three body decays of right-handed neutrinos is proposed. It naturally allows both leptogenesis and neutrino mass generation at low scale. Also discussed is the possibility of inducing leptogenesis at low scale in existing neutrino mass models: Fukugita-Yanagida model, Higgs triplet model, Zee model and models with R-parity violation.

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1 Introduction

The baryon asymmetry of the universe is usually expressed in terms of the ratio of the baryon density \( n_B \) to the entropy density \( s \) of the universe. From nucleosynthesis constraints this ratio is determined to be [1]:

\[
\frac{n_B}{s} \simeq (6 - 10) \cdot 10^{-11}
\]

in good agreement with the latest results from Cosmic Microwave Background data [2]. To explain this asymmetry the electroweak baryogenesis mechanism is certainly the most attractive, especially because it is testable. However, in the Standard Model this mechanism is now excluded [3, 4]. In the minimal supersymmetric model this mechanism could be operative but only if a certain number of rather restrictive conditions are satisfied [5]. In this context it is important to look for other alternatives. Beside the electroweak baryogenesis mechanism in the context of more complicated models, leptogenesis [6-15] is probably the most attractive and simple alternative. It is based on a two step process. First, at a certain temperature in the thermal evolution of the universe, a lepton asymmetry is produced. Secondly, once this asymmetry has been produced, it is partly converted to a baryon asymmetry by the sphaleron processes [16, 17] which are very fast \( B + L \) violating processes in thermal equilibrium at temperature above \( \sim 100-200 \) GeV [3, 18, 19]. Probably the most attractive model of leptogenesis is the one of Fukugita-Yanagida [6] based on heavy right-handed neutrinos. The Heavy Higgs triplet model of Ma-Sarkar [20, 21] is also very interesting and simple. Both models have the very attractive feature that the interactions at the origin of the lepton asymmetry also induce naturally small neutrino masses via the seesaw mechanism. However, these models of leptogenesis and neutrino masses have an intrinsic problem for phenomenology: their lack of testability. In those models the natural scale of the interactions at the origin of both leptogenesis and neutrino masses (in agreement with atmospheric and solar neutrino data) lies from \( 10^{10} \) GeV to \( 10^{15} \) GeV. It is thus particularly interesting to look for alternatives at lower energy scales of order 1-10 TeV which would be directly testable in a relatively near future. In this paper the question of how to build a successful leptogenesis mechanism at the 1-10 TeV scale is addressed in detail.

In section 2 we begin with introducing the usual Fukugida-Yanagida leptogenesis model. This is useful to illustrate the various problems encountered when we attempt to build at low scale a leptogenesis mechanism in the out-of-equilibrium decay scenario.\footnote{The possibility of generating baryogenesis at low energy from other scenarios such as the Affleck-Dine} These problems will be listed in section 3. In section 4 we consider three possible mechanisms for...
the enhancement of the asymmetry, which could be used to remedy these problems. We emphasize the fact that 3 body decays can lead naturally to a sufficiently large lepton asymmetry without the need for unnatural hierarchies of couplings or finely tuned mass degeneracies (as the other enhancement mechanisms do). At the end of section 4 we briefly review whether the three enhancement mechanisms introduced may lead to a successful leptogenesis mechanism at low energy in the framework of existing neutrino mass models: the Fukugita-Yanagida model, the Higgs triplet model, the Zee model and the models with R-parity violation. We stress that, except for the debatable ”mass degeneracy” enhancement mechanism in the Fukugita-Yanagida model, it is very difficult if not impossible to build a successful low scale mechanism in these frameworks. After this general description of the problems and mechanisms, a minimal model is presented in section 5 which, using the 3 body decay mechanism, avoids all these problems. It is based on the decay of heavy right-handed Majorana neutrinos and the existence of 2 scalar charged singlets. This model allows for a consistent generation of both baryogenesis and neutrino masses at the 1-10 TeV scale. A reader familiar with this topics may choose to skip the general discussions of sections 2-4 and go directly to the discussion on three body decays in section 4 and then to section 5 where the original model is presented. Our conclusions are contained in Section 6.

2 The heavy right-handed neutrino mechanism

We begin with introducing briefly the Fukugita-Yanagida model [6] which is useful to illustrate in the next sections the various problems encountered when trying to build a simple leptogenesis model at the 1-10 TeV scale. This model is based on the existence of 3 self-conjugate (i.e. Majorana) neutrinos $N_i$ which are singlets under $SU(2)_L$. These neutrinos are expected to have Majorana masses $M_{N_i}$ much larger than the electroweak scale since these masses are not protected by the $SU(2)_L \times U(1)$ gauge symmetry. They naturally couple to one lepton doublet and the standard model scalar doublet via the usual Yukawa interactions. The lagrangian of the model is therefore:

$$L_{FY} = L_{SM} + \bar{\psi}_{Ri}i\not{\partial}\psi_{Ri} - \frac{M_{N_i}}{2}(\bar{\psi}_{Ri}\psi_{Ri}^c + h.c.) + (h_{ij}\bar{L}_j\psi_{Ri}\Phi + h.c.),$$

where the $\psi_{Ri}$ are the two component Majorana spinors which in terms of the 4-components self-conjugated Majorana spinor $N_i$ are given by $N_i = \psi_{Ri} + \psi_{Ri}^c$ (with $\psi_{Ri} = \frac{1}{2}(1 + \gamma_5)N_i$). The scenario [22] will not be considered here. Similarly the possibility of generating leptogenesis by assuming large extra dimensions will also not be considered.
Figure 1: Tree level and loop diagrams interfering together.

and $\psi^c_R = P_L C \bar{\psi}^T). L_i = (\nu_{L_i} L_i)^T$ and $\Phi = (\phi^0 \phi^-)^T$. The Yukawa interactions in Eq. (2) are at the origin of seesaw induced neutrino masses \[23\]:

$$(m_\nu)_{ij} = (h)_{ik} M^{-1}_{N_k} (h^\dagger)_kj v^2/2,$$  \hspace{1cm} (3)

with $v = \sqrt{2}\langle\phi^0\rangle = 246$ GeV. As is well-known these masses are naturally small if the masses of the right-handed neutrinos are heavy. To have a neutrino mass of order the SuperKamiokande bound ($\sim 0.1$ eV) with for example Yukawa couplings $h$ of order $10^{-2}$ we need $M_{N_i} \sim 10^{10} - 10^{11}$ GeV.

Beside inducing the $\nu$ masses, the Yukawa couplings are also at the origin of the right-handed neutrino decays $N_k \to L_j + \Phi^*$. The decay width is:

$$\Gamma_{N_k} = \frac{1}{8\pi} \sum_j |h_{kj}|^2 M_{N_k}.$$  \hspace{1cm} (4)

From these decays a lepton asymmetry can be created. The Yukawa couplings $h_{ij}$ provide the source of CP-violation which is necessary for the creation of the asymmetry. In the decay, this CP-violation can manifest itself only at one loop level, where it is associated with the imaginary part of one loop diagrams. The lowest order non-trivial asymmetry comes from the interference of the tree level diagrams with the imaginary part of the one loop diagrams of Fig. 1. There are two types of one-loop diagrams: vertex diagrams as considered originally by Fukugita and Yanagida and self-energy diagrams as first introduced and discussed in Ref. [10]. The lepton asymmetry induced by these diagrams,

$$\varepsilon_{N_k} = \frac{\Gamma(N_k \to L_j + \Phi^\dagger) - \Gamma(N_k \to L_j^\dagger + \Phi)}{\Gamma_{N_k}},$$  \hspace{1cm} (5)

which is nothing but the averaged amount of lepton number which is created per decay of right-handed neutrinos, is then obtained as:

$$\varepsilon_{N_k}^V = -\frac{1}{8\pi} \sum_l \text{Im}[h_{kl}^* h_{kj}^* h_{li}^* h_{lj}] \frac{\sqrt{x_l}}{\sum_i |h_{ki}|^2} \left[\log(1 + 1/x_l) - 1\right].$$  \hspace{1cm} (6)
\[ \varepsilon_{N_k}^S = -\frac{1}{8\pi} \sum_{l} \sum_{i,j} \text{Im}[h_{ki}^* h_{kj}^* h_{li} h_{lj}] \frac{\sqrt{x_l(x_l - 1)^{-1}}}{\sum_i |h_{ki}|^2}, \]  

(7)

where \( \varepsilon_{N_k}^V \) and \( \varepsilon_{N_k}^S \) are the contribution of the vertex and self-energy diagrams respectively, with \( x_l = (M_{N_l}/M_{N_k})^2 \). This leads to a ratio \( n_L/s \simeq \sum_k (\varepsilon_{N_k}^V + \varepsilon_{N_k}^S)n_\gamma/2s \sim \sum_k (\varepsilon_{N_k}^V + \varepsilon_{N_k}^S)/g_* \) with \( n_\gamma \) the photon number density and \( g_* \) the number of active degrees of freedom at these temperatures. The sphalerons which are active at temperatures between \( \sim 100 - 200 \text{ GeV} \) and \( \sim 10^{12} \text{ GeV} \) will convert approximately one third of this lepton asymmetry to a baryon asymmetry. \( n_B/s \) is given by \[8\] :

\[
\left( \frac{n_B}{s} \right)_{\text{fin}} = \left( \frac{8N_F + 4N_H}{22N_F + 13N_H} \right) \left( \frac{n_B - n_L}{s} \right)_{\text{init}},
\]

(8)

where \( N_H \) is the number of Higgs doublets and \( N_F \) the number of fermionic families. However, in order for this lepton asymmetry to be effectively created in the thermal evolution of the universe, the decays must take place out-of-equilibrium. This will be the case if the decay rate is smaller than the expansion rate of the universe, parametrized in terms of the Hubble constant \( H \) \[24\] :

\[
\Gamma_{N_k} < H(T = M_{N_k}) = \sqrt{\frac{4\pi^3 g_* T^2}{45 M_{\text{Planck}}}} \bigg|_{T = M_{N_k}}.
\]

(9)

Since this inequality is mediated by the Planck scale \( M_{\text{Planck}} \sim 10^{19} \text{ GeV} \), it is easier to satisfy it at scales not many orders of magnitude smaller than this scale. To satisfy it at even smaller scales, values of the Yukawa couplings much smaller than unity will have to be considered.

If Eq. (9) is not satisfied or in presence of scattering effects which may damp the asymmetry, an explicit calculation of the evolution of the asymmetry from the Boltzmann equations is required. Since for most of the realistic models of leptogenesis at the 1-10 TeV scale, discussed below, such a detailed calculation is necessary, we explicitly outline the calculation in appendix A.
3 Why it is difficult to construct a simple leptogenesis model at the TeV scale

Trying to build a leptogenesis model at the 1-10 TeV scale the main difficulties we have to face are the following:

- First the out-of-equilibrium condition for the decay width, Eq. (9), imposes the general condition that the couplings are very tiny. This is due to the facts that first, as was just mentioned, this condition is mediated by the Planck scale and secondly, the decay width is in general only linear in the mass of the decaying particle in contrast to the Hubble constant which depends quadratically on this mass. This means that the product of couplings entering the decay width has to be as much as 10 orders of magnitude smaller at the TeV scale than at the $10^{13}$ GeV scale. Beyond the fact that the naturality of such tiny couplings can be questionable, the major problem is that it generally induces a very tiny asymmetry due to the fact that the asymmetry in most possible models is proportional to the same tiny couplings. For example in the Fukugita-Yanagida model, to naturally satisfy Eq. (9) together with getting $\varepsilon_N/g_*$ of order $10^{-10}$, we need typically $M_N$ above $10^{10}$ GeV. With smaller value of $M_N$, for example with $M_N \sim 10$ TeV, the condition of Eq. (9) imposes that the Yukawa couplings have to be smaller than $\sim 10^{-6}$. Inserting this value of the couplings in Eqs. (6)-(7) we obtain from the couplings alone a factor of $\sim 10^{-12}$ in the asymmetry, which makes the asymmetry at least $\sim 6$ orders of magnitude too small. By taking larger couplings we could obtain a larger asymmetry $\varepsilon_N$ but the final value of $n_L/s$ will be damped by an extra factor (which is larger than the factor gained in $\varepsilon_N$) from the inverse decay processes. Consequently we don’t gain anything. Possible mechanisms to remedy this problem are discussed in section 4.

- At the TeV scale, various scatterings can also be very fast with respect to the Hubble constant. The scatterings directly proportional to the couplings which are constrained by the out-of-equilibrium condition on the decay have in general a relatively small damping effect. However, other scatterings can be very fast and damp largely the asymmetry. This is particularly the case with gauge scatterings if the particles producing the asymmetry are not neutral or $SU(2)_L$ singlets. To illustrate this fact let us take the example of a charged $SU(2)_L$ singlet scalar $S^+$ whose decay would be at the origin of a large asymmetry. The lepton number conserving $S^+S^- \leftrightarrow \gamma\gamma$ scattering is a very fast process which would damp the asymmetry by several orders of magnitude. Integrating the Boltzmann equation (Eq. (25) in appendix A), with a
mass $m_S \sim 1$ TeV, we observe that, due to this scattering, there is no substantial departure from equilibrium down to $T \sim 50$ GeV.\footnote{As well known this freeze-out temperature can also be obtained by looking for the temperature at which the scattering rate $\Gamma_{\text{scatt}} = \gamma_{\text{scatt}}/n_S$ is of order the Hubble constant [see Eqs. (27) and (24)].} At this freeze-out temperature, the ratio of the number density of $S$ particles to the entropy (given by Eqs. (24) and (27) in appendix A) is strongly Boltzmann suppressed: we get $n_S^{eq}/s \sim 3 \cdot 10^{-10}$ to be compared with $n_S^{eq}/s \sim 2 \cdot 10^{-3}$ at $T = m_S \sim 1$ TeV. We therefore expect that the asymmetry is suppressed by about six-seven orders of magnitude!\footnote{Actually this calculation gives an order of magnitude estimate but the exact suppression factor depends on the value of $K_S = \Gamma_S/H(T = m_S)$. For large value of $K_S$ (i.e. $K_S >> 1$), some scalar singlets will have the time to decay before thermalizing well above $T \sim 50$ GeV, so that the suppression factor will be less important. However in this case the inverse decay processes will damp largely the asymmetry so that we also end up with a several orders of magnitude suppression of the lepton asymmetry.} To compensate this suppression we would need a large enhancement in addition to the one already required to solve the problem of the previous paragraph. We conclude from this fact that, at such low scales, the particle at the origin of the asymmetry be better neutral and a gauge singlet of any low-energy non-abelian gauge symmetry. In this respect a right-handed neutrino (with heavier right-handed W if this model is embedded in a left-right model) is a particularly suitable candidate. A neutral gauge singlet scalar would also be a candidate although it is in general not so easy to introduce a lepton number violation with such a candidate.\footnote{Except at least with right-handed sneutrinos in supersymmetric models. For a leptogenesis model based on sneutrino decay at higher temperatures see Ref. [24].}

- In the more ambitious and more interesting case where the source of lepton number violation at the origin of the asymmetry is also at the origin of the neutrino oscillations, an other problem could in general occur. Two cases have to be distinguished:

\begin{itemize}
  \item First in the case where the neutrino masses are produced at tree level, as in the seesaw mechanism with right-handed neutrinos, the values of the couplings which are needed to generate neutrino masses are generally slightly larger than the ones allowed by the out-of-equilibrium condition. For example, with right-handed neutrinos with masses of order 10 TeV the typical value of the $h_{ij}$ Yukawa couplings which is needed to have a neutrino mass of order $10^{-1}$ eV is $\sim 5 \cdot 10^{-6}$, that is to say about 1 order of magnitude larger than the bound from the out-of-equilibrium condition on the decay width of Eq. (3). This will induce a relatively large damping effect from inverse decays [$K_N$ will be of order 100 in Eq. (25)-(26)]. This effect could be possibly compensated by an enhancement
mechanism if any but this is in general not possible (except possibly with the "mass degeneracy" mechanism introduced in section 4 below).

– Secondly in the case of neutrino masses generated by radiative processes, as in the Zee model or in R-parity violating supersymmetric models, it is quite difficult to generate the neutrino masses and the lepton asymmetry from the same interactions. The reason is that to generate in those models a neutrino mass of at least the SuperKamiokande bound we need couplings several orders of magnitude larger than the upper bound on these couplings from the out-of-equilibrium condition. For example the trilinear R-parity violating couplings needed for neutrino masses are typically of order $10^{-4}$ \[26\] while the out-of-equilibrium condition on the associated L-violating 2 body scatterings requires couplings of order $10^{-7}$ \[27, 28\]. The out-of-equilibrium condition for these scatterings is then violated by 6 orders of magnitude which induces a huge suppression of the associated asymmetry, which would be very difficult if not impossible to compensate by any enhancement effect \[28].\footnote{In the case where the neutrino masses are not generated by the trilinear R-parity violating terms but by the bilinear R-parity violating terms the discussion is different but, as explained in Ref. \[29\], this leads to a far too small lepton asymmetry.} There are nevertheless some ways for avoiding those suppressions due to neutrino constraints as will be explained below.

4 Three possible enhancement mechanisms

To satisfy approximately the out-of-equilibrium condition on the decay width together with inducing a large enough asymmetry we will consider three possible enhancement mechanisms:

1. **Mass degeneracy**: It has been observed in Ref. \[10\] that asymmetries induced by self-energy diagrams (as the third diagram of Fig. 1) display an interesting resonant behavior when the masses of the decaying particles are nearly degenerate. In the Fukugita-Yanagida model this resonant behavior occurs when at least 2 right-handed neutrinos are nearly degenerate [see Eq. \[7\]]. This resonance effect turns out to be maximum for a mass difference of order the decay width.\footnote{A more careful calculation of the asymmetry based on a resummed calculation gives approximately the same result \[11\].} In this way, starting with right-handed neutrinos with masses of order $\sim 10$ TeV, and with Yukawa couplings of
order $10^{-6}$, i.e. satisfying approximately the out-of-equilibrium condition of Eq. (7), one can in principle get a many orders of magnitude enhancement by requiring enough degeneracy. The degree of degeneracy which is required at this scale, if smaller than the one corresponding to the resonance condition, is nevertheless huge [11]:

$$\frac{(m_{Ni} - m_{Nj})}{(m_{Ni} + m_{Nj})} \sim 10^{-7}$$

The naturality of such a degeneracy is highly debatable. Note also that the perturbativity of such a huge enhancement has been questioned in Ref. [12].

2. **Hierarchy of couplings:** Another possibility assumes two particles decaying to the same decay products. The lighter one (which we denote by "A") and the heavier one ("B") couple to these decay products with couplings $g_A$ and $g_B$ respectively. The $g_A$ couplings are taken very suppressed in order that $A$ decays out-of-equilibrium at the 1-10 TeV scale. It is this decay which is at the origin of the asymmetry. The couplings $g_B$ of the heavier particle are on the other hand taken unsuppressed (and are eventually large enough to be at the origin of the neutrino masses). At temperature of order $m_A$ we can take $m_B$ large enough for all particles $B$ to have decayed away (a factor $m_B/m_A$ of $\sim 3 - 10$ is in general enough because the number of particle "B" is fastly Boltzmann suppressed at temperature below its mass). With these assumptions a large asymmetry can be produced from the fact that the one loop diagrams similar to the ones of Fig. 1 for the decay of $A$ with a virtual $B$ will give an asymmetry proportional to $(g_A g_B)^2/g_A^2 = g_B^2$ which is unsuppressed. This simple and attractive mechanism has been discussed in the framework of a R-parity violating model in Ref. [30, 29]. The naturality of the hierarchy of couplings between two particles $A$ and $B$ which are very similar since they couple to the same decay products is nevertheless debatable. The typical value of the ratio $g_A/g_B$ which is needed is of order $10^{-3}$ [29].

3. **Three body decays:** Three body decays appear to be very interesting and more natural for generating a lepton asymmetry at the 1-10 TeV scale. First because, to satisfy the out-of-equilibrium condition on the decay width, three body decays are naturally smaller than two body decays due to phase space suppressions and the fact that they naturally involve more couplings. Secondly because, to induce a large enough asymmetry, they don’t require any special hierarchy of couplings or mass degeneracy. The mechanism is the following: let us assume a trilinear coupling $g_1$ between three particles $A$, $B$, $C$ with $m_B < m_A < m_C$ and let us assume that $C$ in addition to its decay to $A + B$ can also decay to lighter particles $D + E$ through some couplings $g_2$. Then $A$ can decay only to the three body decay $A \to B + C^* \to B + D + E$
through a virtual $C$. In this context it is easy to see that a large asymmetry can be produced. The point is that the out-of-equilibrium condition on the decay width will give an upper bound on a quartic expression in the couplings (i.e. on $g_1^2 g_2^2$) but the asymmetry will be in general only quadratic in these couplings. This has to be compared with the usual two body decays where both the asymmetry and the decay width are quadratic in the couplings. For example if the asymmetry comes from loop diagrams not involving the $A-B-C$ couplings $g_1$ but various $C-D-E$ couplings $g_2$ the asymmetry will be proportional to $g_1^2 g_2^4 / (g_1^2 g_2^2) \sim g_2^2$ and can be very large even if $g_1^2 g_2^2$ has to be small to satisfy the out-of-equilibrium decay condition. Note that no special hierarchy of couplings has to be assumed. For example, taking all couplings $g_1$ and $g_2$ of order $10^{-3}-10^{-4}$ the asymmetry can be naturally large enough ($g_2^2 \sim 10^{-6}-10^{-8}$) with a sufficiently suppressed decay ($g_1^2 g_2^2 \sim 10^{-12}-10^{-16}$). Therefore, if as explained above the natural scale for producing a large asymmetry with the usual two body decays is around $\sim 10^{10}$ GeV or above, with three body decays the 1-10 TeV scale is a perfectly natural scale for producing such a large asymmetry! No special "trick" has to be used to enhance the asymmetry at low scale as with the usual two body decays and the two other mechanisms. Note in addition that, if a hierarchy of couplings is in general not necessary, an even much larger asymmetry can be obtained by taking the hierarchy $g_2 > g_1$. Moreover as we will see with an example in the next section the values of the $g_1$ and/or $g_2$ couplings which are required for leptogenesis have typically the size required to generate radiatively neutrino masses with the right orders of magnitude. Note however that in this framework it must be still checked that no two body scatterings, which can be naturally large with such values of the couplings, can erase the asymmetry. In the next section we will see with an explicit example that this problem can be in general avoided easily. The point is that, with this mechanism, even if at $T \sim m_C$ there exist in general very stringent bounds on the coupling $g_2$ from imposing that associated scatterings don't erase any lepton asymmetry \cite{27, 31, 32}, those bounds can be considerably relaxed. The asymmetry produced by the 3 body decay of particle $A$ will not be erased by those scatterings even for much larger values of $g_2$ because it is produced at $T \sim m_A$ which is below $m_C$ (i.e. when the $C$ particles have already decayed away).

We close this section by briefly reviewing the possibility to implement these mechanisms in the framework of well-known neutrino mass models such as the Fukugita-Yanagida

\footnote{In other words, if the lepton number violation lies in the $g_2$ couplings this means that the amount of lepton number violation is not constrained to be small anymore by the out-of-equilibrium condition.}
model, the triplet Higgs model \[20, 21\], the Zee model \[32-36\] and the models with R-parity violation \[27\].

- **The Fukugita-Yanagida model**: here, as said above, the mass degeneracy mechanism has been extensively discussed in the literature \[10-12\]. On the other hand the three body decay mechanism and the hierarchy of coupling mechanism don’t appear to be very helpful here. The former mechanism would require the existence of heavier particle which are not present in this model. The later mechanism would require a neutrino $N_1$ with mass $M_1$ and suppressed Yukawa couplings $h_1$ which is lighter than another one $N_2$ with mass $M_2$ and unsuppressed Yukawa couplings $h_2$. In this case, from Eqs. (1)-\(\bar{6}\), $\varepsilon_{N_1}$ would be proportional to $(1/8\pi)h_2^2M_1/M_2$ (neglecting higher order terms in $(M_1/M_2)^2$). However, assuming that the neutrino masses cannot be much larger than $\sim 1$ eV, the combination $h_2^2v^2/M_2$ cannot be much larger than 1 eV. This gives an upper bound on $\varepsilon_{N_1}$ which is given by:

$$\varepsilon_{N_1} \lesssim \frac{1}{4\pi} \frac{m_\nu M_{N_1}}{v^2}$$

(10)

and which for example with $M_1 \sim 10$ TeV, is at most of order $10^{-11}$, that is to say anyway at least 3 orders of magnitude too small. To obtain a value of $\varepsilon_{N_1}$ at least of order $\sim 10^{-8}$, which is necessary to have $n_L/s \sim 10^{-10}$, we need a value of $M_{N_1}$ above the bound:

$$M_{N_1} \gtrsim 4\pi v^2 \varepsilon_{N_1}/m_\nu \sim 10^7\text{GeV}.$$  

(11)

- **The Higgs triplet model**: in this model the neutrino masses are also generated through a seesaw mechanism. Therefore the hierarchy of couplings mechanism is not helpful in the same way as for the Fukugita-Yanagida model. The three body decay mechanism is also not possible because here too it would require additional particles which are not present in this model. The mass degeneracy mechanism on the other hand could be useful for the triplets as for the right-handed neutrinos. However, in contrast with the right-handed neutrinos, the Higgs triplets are not $SU(2)_L$ singlets. Therefore, as discussed in Ref. \[27\], if the leptogenesis mechanism is quite natural for triplet masses above $10^9 - 10^{10}$ GeV, for lower masses the damping effect of the associated gauge scatterings (i.e. of a triplet pair to a gauge boson pair) becomes important and it appears to be impossible to have a successful mechanism at scales as low as 1-10 TeV. The degree of degeneracy required would be larger than the resonance condition would allow.  

\[8\] Of course a low value of the triplet mass would be welcome to avoid large corrections to the mass of the Higgs doublet coming from their self-energy with an internal heavy triplet. This is however not
• **The Zee model:** in the Zee model the three body decay mechanism cannot be used because here too it requires additional particles which are not present in this model. To avoid the problems related to the neutrino masses explained in section 3, the hierarchy of couplings mechanism could be used. However similar to the triplet case the gauge scatterings will considerably damp the asymmetry. In the Zee model the main effect comes from the $S^+S^- \leftrightarrow \gamma\gamma$ scatterings discussed previously in section 3.

• **The supersymmetric models with R-parity violation:** in these models the neutrino masses can be induced by the R-parity and L violating terms. To induce the leptogenesis from the same terms we could use the hierarchy of couplings mechanism or the three body decay mechanism. By using them we could avoid the problems related to the neutrino masses explained in section 3. However here too the gauge scatterings would damp the asymmetry largely by rendering the decaying particles in thermal equilibrium down to temperature far below their mass. For example the decays of the sfermions or of the charginos couldn’t produce a large enough asymmetry due to the effect of the scatterings of a sfermion pair or of a chargino pair to a gauge boson pair. For a neutralino, if this neutralino is essentially a wino $\tilde{W}_3$ or a Higgsino, the scattering of 2 neutralinos going to 2 $W$ mediated by a chargino will have a very large damping effect in the same way. A similar comment applies to the case of a bino (as well as to the case of a wino) through the squark mediated scatterings of 2 binos to a quark pair.

In summary except for the debatable mass degeneracy mechanism in the Fukugita-Yanagida model, there is no simple leptogenesis solutions at low scale in all these models.

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9 This would require the introduction of a second charged scalar singlet in order to have CP-violation, both scalar singlets playing the role of particle "A" and "B" in the hierarchy of coupling mechanism explained in section 3. The one loop (self-energy) diagram responsible for the asymmetry would be the same as in the triplet model replacing triplets by singlets.

10 We thank S. Davidson for pointing us the potential effect of this scattering. Calculating explicitly its effect, which was not taken into account in Ref. [38], it can be checked that it changes drastically the results of this reference. It also turns out that this effect was underestimated in Ref. [30, 29] for which a reanalysis of the parameter space in this model should be performed.
5 An explicit model based on three body decays

In the following, based on the most natural 3 body decay mechanism, we build an alternative and successful model which avoids the problems of the existing neutrino mass models just explained.

5.1 The model

Probably the 3 body decay enhancement mechanism explained in section 4 could be implemented in many different contexts. However as explained above to avoid large damping effects of gauge scatterings, the decay should be from a neutral gauge singlet, which restricts the possibilities. The most natural candidate is the right-handed neutrino which we will consider. To implement the 3 body decay mechanism, a heavier particle has to be introduced. One simple and minimal possibility is to assume additional charged scalar singlets. In the following we will assume two scalar singlets $S_1^+, S_2^+$. We will also assume two Higgs doublets $H_k \equiv (\phi_k^0, \phi_k^-)^T$, $k = 1, 2$ as in the Zee model [32-36]. In this context, beside the usual Yukawa couplings of the right-handed neutrinos of Eq. (2) and beside the Zee couplings of these singlets to two leptons or to two scalar doublets, the right-handed neutrinos can couple to a charged scalar singlet and a right-handed lepton. In a general way we have therefore the following interactions:

$$L_Y \ni h^k_{L_{ij}} \bar{l}_i \psi_{Ri} H_k + h^k_{R_{ij}} l_{Rj}^T C^{-1} \psi_{Ri} S_k^+ + f^k_{ij} l_i^T C^{-1} i \tau_2 l_j S_k^+ + \lambda^k_5 H_1^T i \tau_2 H_2 S_k^+ + h.c. \quad (12)$$

where $L_i \equiv (\nu_i, l_i^-)^T$. From this it is easy to build a successful model of leptogenesis.

5.2 Leptogenesis

For leptogenesis we assume that the right-handed neutrinos decay to a Higgs doublet and a left-handed lepton with suppressed couplings $h^k_{L_{ij}}$ in order to satisfy approximately the out-of-equilibrium condition on the associated two body decay widths. Assuming right-handed neutrinos at low scales these couplings have anyway to be tiny not to induce too large masses for the light neutrinos. The right-handed neutrinos can also decay to a right-handed anti-lepton plus 2 left-handed leptons or 2 scalar doublets via a scalar singlet as shown in Fig. 2. We assume that these three body decay partial widths also satisfy the out-of-equilibrium condition due to three body decay suppression and the fact that we

\footnote{See also Ref. [38] for a non-leptogenesis baryogenesis model based on the R-parity violating 3 body decays of a neutralino and Refs. [14, 39] for leptogenesis models at the electroweak phase transition based also on R-parity violating 3 body decays of neutralinos.}
assume that the couplings $h_{Rij}^k$, $f_{ij}^k$ and $\lambda_S^k$ are small enough (without necessarily being tiny). In this way the asymmetry is obtained naturally large as explained in section 4. To see that let us first write down the $N_i$ decay widths:

$$\Gamma_{N_i} \simeq \frac{1}{8\pi} \sum_{jk} |h_{Lij}^k|^2 M_{N_i}$$

$$+ \frac{1}{(2\pi)^3} \frac{1}{48} \sum_{jkl} h_{Rij}^k h_{Rij}^{l*} \frac{M_{N_i}^3 \lambda_S^k \lambda_S^l}{m_{S_k}^2 m_{S_l}^2}$$

$$+ \frac{1}{(2\pi)^3} \frac{1}{96} \sum_{jklmn} h_{Rij}^k h_{Rij}^{l*} f_{mn} f_{mn}^{l*} \frac{M_{N_i}^5}{m_{S_k}^2 m_{S_l}^2}. \quad (13)$$

In this equation we have neglected all light fermions and scalar masses and we kept only the highest term in $M_{N_i}^2/m_{S_k}^2$ (k=1,2; i=1,2,3). The asymmetry induced by the three body decays is given by the diagrams of Fig. 2. Calculating the interference of tree level and one-loop diagrams we get:

$$\varepsilon_{N_i} \simeq A_{N_i} \sum_{j,m,n} \left[ \text{Im}[h_{Rij}^{2*} h_{Rij}^1 \lambda_S^1 \lambda_S^2] \left( \frac{|f_{mn}^1|^2}{m_{S_1}^2} - \frac{|f_{mn}^2|^2}{m_{S_2}^2} \right) \right. $$

$$+ \text{Im}[h_{Rij}^2 h_{Rij}^{1*} f_{mn} f_{mn}^{2*}] \left( \frac{|\lambda_S^1|^2}{m_{S_1}^2} - \frac{|\lambda_S^2|^2}{m_{S_2}^2} \right) $$

$$+ \text{Im}\left[f_{mn}^2 f_{mn}^{1*} \lambda_S^1 \lambda_S^2 \left( \frac{|h_{Rij}^1|^2}{m_{S_1}^2} - \frac{|h_{Rij}^2|^2}{m_{S_2}^2} \right) \right], \quad (14)$$

with:

$$A_{N_i} = \frac{1}{\Gamma_{N_i}} \frac{1}{(2\pi)^3} \frac{1}{12} \frac{\pi}{(4\pi)^2} \frac{M_{N_i}^5}{m_{S_1}^2 m_{S_2}^2}. \quad (15)$$

In this asymmetry there are several combinations of couplings which provide the CP-violating phases. Those phases cannot be absorbed in a redefinition of couplings. Note that with only one scalar singlet these phases would vanish. We need therefore at least two
scalar singlets. Similarly we need two scalar doublets because with only one we wouldn’t have any couplings of the scalar singlets with two scalar doublets, hence no asymmetry.

Actually in this model the three body decay asymmetry of Eq. (14) doesn’t give the full result. We have also to take into account the effect of the small mixings between the light charged scalars $\phi_{1,2}^\pm$ and the heavy charged singlets induced by the vacuum expectation values of the neutral components of both Higgs doublets. Due to these vevs and associated mixings the three body decays of Fig. 3 lead also to the $N_i \rightarrow l_j^c + l_l^c$ two body decays as shown in Fig. 4. Taking into account the contribution of these two body decays to $\Gamma_{N_i}$ we have to add in Eq. (13) the following term:

$$\Gamma_{(2)}^{N_i} \approx \frac{1}{8\pi} \sum_{jkl} h_{Rij}^k h_{Rlj}^l \frac{M_{N_i} v^2 \lambda^*_k \lambda^l_S}{m^2_S k m^2_S l}, \quad (16)$$
and, for the $\varepsilon_N$ asymmetries, $A_{N_i}$ in Eq. (14) becomes now:

$$A_{N_i} \simeq \frac{1}{\Gamma_{N_i}} \frac{1}{(2\pi)^3} \frac{1}{12} \frac{\pi}{(4\pi)^2} \frac{M_{N_i}^5}{m_{S_1}^2 m_{S_2}^2} + \frac{1}{\Gamma_{N_i}} \frac{1}{(4\pi)^2} \frac{M_{N_i} m_{h^+} v^2}{m_{S_1}^2 m_{S_2}^2},$$

(17)

with $m_{h^+}$ the mass of the physical light charged scalar which we will take typically around 300-400 GeV. These two body contributions to the decay width and the asymmetry have about the same magnitude than the three body decay contributions because as can be seen from Fig. 4 they involve the same couplings. With respect to the three body decays, the two body decays are enhanced by a smaller phase space suppression but are suppressed by $v^2/M_{N_i}^2$ and $m_{h^+}^2/M_{N_i}^2$ factors, so that they have about the same order of magnitude.

Collecting all these results, to satisfy the out-of-equilibrium condition on the decay width, we need (for example with $M_N \sim 3$ TeV and $M_{S_{1,2}} \sim 10$ TeV):

$$|h_{Lij}^k| < 4 \times 10^{-7}, \tag{18}$$

$$|h_{Rij}^{f_m^*}| < 1 \times 10^{-4}, \tag{19}$$

$$|h_{Rij}^k \lambda_{S^*}^i| < 2 \times 10^{-4} \text{ TeV}. \tag{20}$$

Comparing Eqs. (14) and (17) with Eqs. (18)-(20), it is not difficult to see that very large values of $\varepsilon_N$ (as large as $10^{-2}$) can be obtained with values of the couplings satisfying Eqs. (18)-(20). However, as discussed below some care has to be taken with the potentially large damping effect of various scatterings.

### 5.3 Scattering effects

Specially important are the scatterings with an intermediate scalar singlet:

- First the very fast L-violating scatterings proportional to $\lambda_S^2 f^2$ (such as $l_L + l_L \leftrightarrow S \leftrightarrow \phi + \phi$) can wash out very strongly the asymmetry. However, and this is an important point, the bounds on $\lambda_S^2 f^2$ we usually obtain by requiring that these scatterings don’t erase any preexisting asymmetry (at $T \sim m_{S_{1,2}}$) don’t apply here. Much larger couplings can be taken here without large wash-out of the asymmetry because the asymmetry is produced at $T \sim M_N$ which is below $m_{S_{1,2}}$. The large scattering effects at $T \sim m_{S_{1,2}}$ come mostly from the $S_{1,2}$ resonance region. For $T \sim M_N < m_{S_{1,2}}$ these contributions are significantly Boltzmann suppressed in Eq. (29).

\textsuperscript{12}To have a large enough Boltzmann suppression we typically need that $m_S$ is larger than $m_N$ by a factor from $\sim 3$ to 15 depending on the values of the parameters.
• Second scatterings proportional to \( h_R^4 \), such as \( N + N \leftrightarrow l_R + \bar{l}_R \), which conserve lepton number but bring the \( N \) to thermal equilibrium if they are too fast. These scatterings put an upper bound on the \( h_R \) couplings under which we will remain. The dependence of this bound in the values of various other parameters is non-trivial. By taking \( h_R \) below \( 10^{-3} \) we are in general safe.

• Third the scatterings proportional to \( h_R^2 f^2 \) (such as \( N + l_R \leftrightarrow S \leftrightarrow l_L + l_L \)) or \( h_R^2 \lambda_S^2 \) (such as \( N + l_R \leftrightarrow S \leftrightarrow \phi + \phi \)) which both violate lepton number and can render the \( N \) in thermal equilibrium down to temperature below its mass. They are more dangerous than the \( N + N \leftrightarrow l_R + \bar{l}_R \) scatterings because their effect is enhanced by the charged scalar resonance region contribution. However this resonant region contribution is also significantly Boltzmann suppressed at temperature below the charged scalar mass when the asymmetry is produced.

Other scatterings involving the large top Yukawa coupling (such as \( N + l \leftrightarrow \phi \leftrightarrow \bar{t} + b \)) have in general a small effect. A set of values for which all these scatterings have moderate effects when integrating the Boltzmann equations\(^\text{13}\) and which gives rise to \( n_L/s \) of order \( 10^{-10} \) at \( T \sim 100-200 \) GeV is for example the following:

\[
\begin{align*}
M_N &= 4 \text{ TeV}, \\
m_{S_1} &\sim 25 \text{ TeV}, \\
m_{S_2} &\sim 30 \text{ TeV}, \\
f &\sim 2 \cdot 10^{-2}, \\
\lambda_S &\sim 50 \text{ GeV}, \\
h_R &\sim 1.5 \cdot 10^{-4}, \\
h_L &\sim 10^{-8}.
\end{align*}
\]  

(21)

Here for simplicity to obtain this result we took only the decay of one right-handed neutrino assuming the other ones are heavier. We also took all \( f_{i j}^k, h_{R_{i j}}^k, h_{L_{i j}}^k, \lambda_S^k \) couplings with a same value \( f, h_R, h_L, \lambda_S \) respectively. In Eq. (24) maximal and positive CP-violating phases were taken.

### 5.4 Neutrino masses

In this model the neutrino masses can come either from the usual radiative effects in the Zee model sector of the model or from the usual Yukawa couplings of the right-handed neutrinos via the seesaw mechanism. Particularly interesting is the possibility that the neutrino masses in this model could be mostly due to the radiative Zee contribution (Fig. 5) which gives:

\[
(m_\nu)_{ij} = \sum_k \frac{\lambda_S^k}{m_{S_k}^2 - m_{S_1}^2} \frac{v_2}{v_1} \frac{1}{(4\pi)^2} \ln \frac{m_{S_k}^2}{m_{S_1}^2} f_{i j}^k (m_{i j}^2 - m_{i i}^2),
\]

\(^\text{13}\)This and other details will be given in a subsequent publication \[40\].
where $v_{1,2}$ are the vacuum expectation values of both neutral Higgs bosons (for which we took $v_2 \sim v_1$). In fact interestingly it turns out that the leptogenesis constraints on the various couplings (i.e. to have an asymmetry of order $10^{-10}$) lead naturally to a value of the largest neutrino mass of the order of the SuperKamiokande bound ($\sim 0.1$ eV) from atmospheric neutrino data. This is the case for example with the values of Eq. (21). To fit in addition the solar data, unlike in Eq. (21), a hierarchy among the $f^k_{ij}$ couplings has to be assumed. Assuming for simplicity $f^1_{ij} \sim f^2_{ij} \sim f_{ij}$, for example the LOW solution of solar neutrino experimental data can be accommodated by assuming in Eq. (22) as in the Zee model the following hierarchy: $|f_{e\mu}| \sim 3 \cdot 10^2 |f_{\tau\tau}| \sim 10^7 |f_{\mu\tau}|$. A $\nu$ mass of order $0.1$ eV requires in addition that $f^k_{e\mu} \lambda^k S^2_S / m^2_S$ in Eq. (22) is of order $\sim 10^{-8}$ GeV. With such a value of this combination of couplings it turns out that the $l_\mu + l_\tau \leftrightarrow \phi + \phi$ scatterings are fast enough to be in thermal equilibrium down to temperature below $M_N$. However it will not erase all lepton asymmetries. In fact it has been shown in Ref. [41] that for the LOW solution [as well as for the vacuum oscillation (VO) solution] all preexisting lepton asymmetries will not be erased in the Zee model: the $f_{\mu\tau}$ coupling is tiny enough to prevent the erasure of a preexisting $L_e - L_\mu - L_\tau \equiv L_1$ asymmetry. This quantum number will be violated only by $l_\mu + l_\tau \leftrightarrow \phi + \phi$ and $l_\mu + l_\tau \leftrightarrow l_e + l_\mu$ processes which are slow enough to be out-of-equilibrium (in particular at $T \sim m_S$ and below). In our case, since we need these processes to be out-of-equilibrium only at $T \sim m_N$ which is below $m_S$, these constraints are even less stringent and it turns out that not only the LOW and VO solutions but also the large mixing angle (LMA) solution can be accommodated easily to create large asymmetries.
With the LOW solution, for example the following set of values,

\[ M_N = 1 \text{ TeV}, \quad m_{S_1} \sim 4 \text{ TeV}, \quad m_{S_2} \sim 5 \text{ TeV}, \quad f_{e\mu} \sim 2 \cdot 10^{-1}, \]
\[ \lambda_S \sim 50 \text{ GeV}, \quad h_R \sim 5 \cdot 10^{-5}, \quad h_L < 10^{-8}. \] (23)

leads to a large \( L_1 \) asymmetry together with two neutrino masses of order 0.1 eV.\(^{14}\) In this case, the \( L_1 \) asymmetry is not erased at all, neither by the \( l_\mu + l_\tau \leftrightarrow \phi + \phi \) scatterings, nor by the \( l_\mu + l_\tau \leftrightarrow l_e + l_\mu \) scatterings (including their scalar singlet on-shell part which are Boltzmann suppressed). We obtain \( n_{L_1}/s \sim 10^{-9} \) (at \( T \sim 100 - 200 \text{ GeV} \) taking maximal CP-violating phases (and \( n_{L_1}/s \sim 10^{-10} \) if these phases are reduced by one order of magnitude).\(^{15}\) Note that in this case also a \( L_2 \equiv L_\mu - L_e - L_\tau \) asymmetry will be created. This asymmetry will be erased significantly because \( f_{e\tau} \) is not tiny enough to avoid the associated damping effect, but still we get \( n_{L_2}/s \sim 10^{-11} \). Note that for such a value of \( f_{e\tau} \) the produced \( L_2 \) asymmetry turns out to be rather sensitive to the exact value of \( f_{e\tau} \). Other sets of values could lead to a larger \( L_2 \) asymmetry. The produced \( L_3 \equiv L_\tau - L_e - L_\mu \) asymmetry on the other hand will be completely negligible because \( f_{e\mu} \) is large.

The LMA solution requires \( |f_{e\mu}/f_{e\tau}| \sim m_\tau^2/m_\mu^2 \sim 3 \cdot 10^2 \) together with \( |f_{e\tau}/f_{\mu\tau}| \sim \sqrt{2} \Delta m^2_{\text{atmos}}/\Delta m^2_{\text{solar}} \sim 10^2 \) (see e.g. Refs. \([35, 36]\)). Assuming for example the set of values of Eq. (23), this gives \( f_{\mu\tau} \sim 3 \cdot 10^{-5} \) which is tiny enough to prevent any sizable erasure of the \( L_1 \) lepton asymmetry from \( l_\mu + l_\tau \leftrightarrow \phi + \phi \) or \( l_\mu + l_\tau \leftrightarrow l_e + l_\mu \) scatterings as with the LOW solution. We also get in this case \( n_{L_1}/s \sim 10^{-9} \) and \( n_{L_2}/s \sim 10^{-11} \) for maximal CP-violating phases. Note that the LMA solution leads nevertheless to nearly bi-maximal mixing and \( \nu_e \) survival probability which become to be disfavoured by the data (see e.g. \([42, 37, 43]\)). There are however ways to avoid this problem \([36, 37, 44]\), in particular by allowing both Higgs doublets to have Yukawa couplings with leptons, in case the second mixing angle can be reduced \([30]\).

Note that in this model not only a lepton asymmetry is created, but any preexisting asymmetries (if there are any) will be practically erased, so that the physics of higher energies would be irrelevant for baryogenesis. A preexisting \( L_3 \) asymmetry will be erased in the same way as the produced \( L_3 \) asymmetry mentioned above. A preexisting \( L_2 \) or \( L_1 \) asymmetry will be erased to a large extend at \( T \sim m_S \) by the effect of the \( S \) mediated

\(^{14}\)The neutrino mass hierarchy in this case is that the two heaviest neutrinos have masses differing by the small solar \( \Delta m^2_{\text{solar}} \) and the lightest neutrino differ from the other two by the larger atmospheric \( \Delta m^2_{\text{atmos}} \).

\(^{15}\)Note that we could naively believe that if the \( f_{e\tau} \) couplings are small the produced \( L_1 \) asymmetry will also be small. However the situation is more subtle: this is not the case because in \( \varepsilon_N, L_1 \) is also violated in the contribution proportional to the large \( f_{e\mu} \) couplings via the \( h_R \) couplings.
$N + l_{e,\mu,\tau} \leftrightarrow l_e + l_\mu$ and the $N + l_{e,\mu,\tau} \leftrightarrow \phi + \phi$ scatterings as well as for $L_2$ by the $l_e + l_\tau \leftrightarrow \phi + \phi$ scatterings. These scatterings erase the $L_{1,2}$ preexisting asymmetries much more than the subsequently produced $L_{1,2}$ asymmetries because around $T \sim m_S$, unlike at $T \sim M_N$ and below, they are not Boltzmann suppressed. The damping factor turns out to be very sensitive to the exact values of the $f$ and $h_R$ couplings but for example for the set of values of Eq. (23) we observe that these eventual preexisting asymmetries are erased by several orders of magnitude [40].

Note also that in this case, where the neutrino masses are induced in the Zee radiative way, neither for leptogenesis nor for neutrino masses, do we need the $h_{Lij}$ couplings. These couplings could all vanish without changing anything for neutrino masses and leptogenesis. In this case all tree level couplings are relatively large. This is a unique feature of this case that, unlike all other models of leptogenesis at low scale, no special mass degeneracies or large hierarchies between the couplings is needed (except for the flavor structure associated to the neutrino masses).

Note however that if the $h_L$ couplings are non-vanishing at tree level there is also the possibility that they contribute non-negligibly to the neutrino masses. A neutrino mass of order $\sim 0.1$ eV couldn’t be generated in this model by the lighter right-handed neutrino $N_1$, whose decay is responsible for the asymmetry [17], but it could come from the two heavier neutrinos $N_2,3$. This could even be an alternative solution to explain that the second mixing angle is not as maximal as expected in the Zee model but we will not enter into these details. Here we just assumed that the $h_L$ couplings are tiny enough for the seesaw contribution to be negligible and focussed on the radiative contribution.

5.5 Discussion

A very attractive property of this model is that it could be tested at future accelerators. Heavy charged scalars could be observed through electromagnetic interactions (e.g. from the large $l + \bar{l} \rightarrow S^+ + S^-$ or $q + \bar{q} \rightarrow S^+ + S^-$ cross sections [35]). A charged scalar pair can also be produced from $e^+e^-$ or $e^-e^-$ annihilation with an intermediate right-handed neutrino in the t-channel [42] which also allows to observe indirectly the right-handed neutrinos. The charged scalar can then decay to two left-handed leptons, two scalars or a right-handed neutrino and a right-handed charged lepton. They can also interact

---

Note that if at tree level $h_L = 0$, an effective $h_L$ coupling is induced at the one loop level from the $h_R$, $f$ and the lepton Yukawa couplings. With the values of Eq. (23) we get $h_L$ below $10^{-9}$.

As explained in section 3 above that would imply that $K_N$ is of order 100 which would give an inverse decay damping effect very difficult to compensate by an appropriate choice of the parameters.
with an electron of the detector to produce a right-handed (virtual) neutrino. Note that right-handed neutrinos can be observed through their relatively large coupling to a right-handed fermion and a charged scalar singlet. This has to be contrasted with leptogenesis models where the right-handed neutrinos couple only to left-handed doublet, in case they should have either a mass far beyond the reach of any foreseeable future accelerator or tiny couplings, which makes them difficult to observe. To produce right-handed neutrinos, the \( S \rightarrow l^+_R + N \) decay and \( l^+_R + \bar{l}^+_L \rightarrow N + l^-_L \) scatterings (with a charged singlet scalar in the t-channel) are particularly interesting here. With for example the values of Eq. (23), the \( S \)-mediated cross section is larger than the usual \( Z \)-mediated \( e^+ + e^- \rightarrow \nu + N \) cross section \([46]\) induced by neutrino mixing. Above the \( S \) production threshold the production of \( N \) from \( S \rightarrow l^+_R + N \) is completely dominant. Once it is produced the \( N \) will decay very slowly (but still inside the detector) or it will interact with an electron of the detector. Note also that a value of \( f_{e\mu} \) of order \( 10^{-1} \) as in Eq. (23) is about one order of magnitude smaller than the experimental upper bounds on it from electroweak data \([47, 48]\). The values of the \( h_R \) and \( f \) couplings we have considered are in particular small enough not to violate the bound on \( \Gamma(\mu \rightarrow e\gamma) \) \([47, 48]\). The contribution of the \( f \) couplings to the muon anomalous magnetic moment \([48]\) doesn’t exceed \( a_\mu \sim 10^{-13} \), i.e. is 3-4 orders of magnitude smaller than the present experimental sensitivity.

In a more speculative vein note also that, if we add two additional ”complex” neutral scalar singlets, this model displays the puzzling ”symmetry” that for every left-handed doublet there are two associated ”right-handed” \( SU(2)_L \) singlets (i.e. for the scalars as well as for the leptons). This could be due to the breaking of a right-symmetry in some left-right symmetric model. Once the right symmetry is broken from every right doublet remain just two singlets.

Before concluding note finally that in appendix B we comment on the leptogenesis model at the TeV scale recently proposed in Ref. \([45]\). We explain why we disagree with the statement that phase space suppressed two body decay could lead to an enhancement of the asymmetry.

6 Summary

In summary after discussing the various issues associated with low scale leptogenesis, and in particular the problems of existing neutrino mass models, we showed how a leptogenesis mechanism based on three body decays could avoid easily these problems. In general three body decays require models with a particle content which is more elaborate than
in models with two body decays. But in contrast with the usual two body decays where the natural scale for producing a large asymmetry is around $\sim 10^{10}$ GeV (or above), for three body decays the 1-10 TeV scale is a perfectly possible scale. This does not require unnatural large coupling hierarchies or mass degeneracies like the other two mechanisms with two body decays do. Moreover the values of the couplings in the three body decays which are required for leptogenesis have typically the size required to induce the correct neutrino masses radiatively. Three body decay induced leptogenesis and one loop induced neutrino masses constitute therefore a TeV scale alternative to the usual leptogenesis and neutrino mass framework with masses in the $10^{10} - 10^{15}$ GeV range; in the later case the leptogenesis is induced by two body decays and the neutrino masses are induced in the seesaw mechanism. This three body decay mechanism could be operative in many different contexts. We implemented it in a simple and minimal model with right-handed neutrinos and charged scalar singlets. Beyond the fact that this model leads to a large enough lepton asymmetry at low scale, it displays a number of attractive properties:

- Neutrino masses and mixings in agreement with the solar and atmospheric neutrino experimental data can be produced easily in this model through the radiative Zee mechanism. The radiative Zee contribution to the largest neutrino mass is naturally of order the SuperKamiokande bound.

- Instead of, as usually discussed in the literature, only erasing some or all possible preexisting lepton asymmetries, the scalar singlet Zee couplings here can lead both: a large erasure of all preexisting asymmetries and the subsequent creation of a new lepton asymmetry. This is related to the fact that three body decays involve naturally two scales, the mass of the decaying particle $M_N$ and the mass of the virtual particle $m_S$. At $T \sim m_S$ the preexisting asymmetries are washed out and at $T \sim M_N$ a new asymmetry is created.

- Beside the scalar singlet Zee couplings, the leptogenesis in this model is not based on the couplings of the right-handed neutrinos to the left-handed leptons, which we could take as vanishing, but on their couplings to the right-handed leptons. These later couplings are relatively large, which could allow to observe the right-handed neutrinos much more easily than in the usual Fukugita-Yanagida case where the right-handed neutrinos couple only to left-handed leptons.

- In this model the right-handed neutrinos could have a mass as low as $\sim 1$ TeV and the charged scalar singlets could have a mass as low as a few TeV.
The origin of the baryon asymmetry and of the related neutrino masses could be therefore directly tested at future accelerators running around the 10 or 20 TeV scale.

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A  Boltzmann equations

It is customary to express the number of particles "i" in terms of

\[ X_i = \frac{n_i}{s} \]

which gives the number of particle "i" per comoving volume \( n_i \) divided by the entropy per comoving volume:

\[ s = g_i \frac{2 \pi^2}{45} T^3. \]  

(24)

Similarly the total lepton number per comoving volume is parametrized by \( X_L = n_L/s = (n_l - n_\bar{\nu})/s \). As a function of \( z = M/T \) with \( M \) an arbitrary mass scale (for example \( M = M_a \), the mass of the particle "a" whose decay is at the origin of the asymmetry), \( X_a(z) \) and \( X_L(z) \) are given by \([19]\):

\[
\frac{dX_a}{dz} = -z K_a \frac{K_1(z)}{K_2(z)} \left( \frac{M_a}{M} \right)^2 \left[ X_a - X_{eq} \right] + \frac{1}{sH(M)} \Delta n_a \frac{X_{i_1} X_{i_2} \cdots}{X_{eq} X_{eq} X_{eq}} \gamma_{scatt} (i_1 + i_2 + \cdots \rightarrow f_1 + f_2 + \cdots),
\]

(25)

\[
\frac{dX_L}{dz} = \sum_{a,k} z K_a \frac{K_1(z)}{K_2(z)} \left( \frac{M_a}{M} \right)^2 \left[ \epsilon_a (X_a - X_{eq}) - \frac{1}{2} \frac{X_{eq}}{X_{eq} X_{eq}} X_L \right] + \frac{z}{sH(M)} \left[ \Delta n_i \frac{X_{i_1} X_{i_2} \cdots}{X_{eq} X_{eq} X_{eq}} \gamma_{scatt} (i_1 + i_2 + \cdots \rightarrow f_1 + f_2 + \cdots) \right] - \Delta n_i \frac{X_{i_1} X_{i_2} \cdots}{X_{eq} X_{eq} X_{eq}} \gamma_{scatt} (i_1 + i_2 + \cdots \rightarrow f_1 + f_2 + \cdots).
\]

(26)

In those equations \( K_{1,2} \) are the usual modified Bessel functions and \( X_{i}^{eq}(z) = n_{i}^{eq}(z)/s \) gives the number density of the particle "i" in thermal equilibrium. It is a good approximation to use Maxwell-Boltzmann statistics:

\[
n_{i}^{eq} = g_i \frac{M_i^2}{2 \pi^2} T K_2(M_i/T),
\]

(27)
with \( g_i \) the number of degree of freedom of the particle "i". For massless particle the number density is:

\[
n_{eq}^i = \frac{g_i T^3}{\pi^2} \tag{28}
\]

with for the photon \( g_\gamma = 2 \). In Eqs. (25)-(26), \( K_a \equiv \Gamma_a / H(M_a) \) parametrizes the effect of the decays and inverse decays of \( N_i \). If the out-of-equilibrium decay condition of Eq. (4) is not satisfied, the inverse decay term of Eqs. (25)-(26) (i.e. the terms proportional to \( K_a X^eq_a \)) will compensate the decay term (i.e. the terms proportional to \( K_a X_a \)). In this way the decaying particle remains at equilibrium and no asymmetry is produced. In Eqs. (25)-(26), the \( \gamma_{scatt}^eq \) are the scattering reaction densities which can be obtained from the scattering cross sections in the following way:

\[
\gamma_{scatt}^eq = \frac{T}{64\pi^4} \int_{s_0}^\infty ds \hat{\sigma}(s) \sqrt{s} K_1(\sqrt{s}/T), \tag{29}
\]

where \( \hat{\sigma}(s) \) is the reduced cross section and is given by \( 2[s-(m_1+m_2)^2][s-(m_1-m_2)^2] \sigma(s)/s \) with \( \sigma(s) \) the cross section. \( m_{1,2} \) are the masses of the particles in the scattering initial state and \( s_0 \) is the scattering threshold. \( \Delta n_i \) is the net number of particle "i" which were created in one scattering \( i_1 + i_2 + \cdots \rightarrow f_1 + f_2 + \cdots \). The scatterings can damp the asymmetry in two different ways. First if they don’t conserve lepton number they are present in Eq. (26) and, if fast enough, can directly reequilibrate the lepton number to 0. Secondly if they change the number of particle "a" they are present in Eq. (25) and if fast enough they will damp the asymmetry by imposing the species "a" to remain in equilibrium with the thermal bath down to temperature much below its mass. These scattering effects in Eq. (25) appear to be especially important in most of the possible leptogenesis model candidates at the TeV scale.

B Comments on the model of Ref. [45]

The model recently proposed in Ref. [45] is based on a particle content which is similar to the one proposed in section 5. However quite different assumptions on the various possible couplings and on the leptogenesis mechanism have been made. It is claimed that, if the Fukugita-Yanagida model cannot lead easily to a large enough asymmetry, the self-energy diagram of Fig. 1 above with \( h_R \) couplings in the loop instead of \( h_L \) couplings (i.e. with a \( l_R \) and a charged scalar singlet \( S \) in the loop instead of a \( l_L \) and a \( \phi \)), can lead to a much larger asymmetry by taking large \( h_R \) couplings. It is also said that this requires a large phase space suppression of the \( S \) decay width to \( l_R \) and \( N \) (a value of \( y = 1 - (m_S^2/m_N)^2 \) as
small as $10^{-4}$-$10^{-5}$ has been assumed) to avoid associated wash out of the asymmetry. Note however that in this case the asymmetry is proportional to $(1/8\pi) \cdot (h_L^2 h_R^2 y^2)/(h_L^2 + h_R^2 y^2)$ (assuming universal $h_L$ and $h_R$ couplings for all right-handed neutrinos as it has been done).

From this result we agree that the numerator can be enhanced only if $h_R y$ is taken large since $h_L$ is anyway constrained to be small. By unitarity this increase of the numerator will be nevertheless compensated by a same increase of the denominator. Therefore with respect to the Fukugita-Yanagida model we don’t gain anything by taking $h_R$ larger than $h_L$ in this self-energy diagram (and this for any value of $y$). The large asymmetry obtained in Ref. [45] has been obtained from a large right-handed neutrino mass degeneracy (as can be done in the Fukugita-Yanagida model) and from omitting the $h_R^2 y^2$ term in the $N$ decay width, hence in the denominator of the asymmetry. To take into account this term in the decay width implies a decrease of the asymmetry and in addition an enhancement of the damping factor from inverse decay processes. Note also that the $h_R$ couplings have been assumed in Ref. [45] to be of order $\sim 1$. This would induce an additional large damping effect from the $N + N \leftrightarrow l_R + \bar{l}_R$ scatterings which has not been taken into account. The suppression factor due to these scatterings will not be as large as $\sim 10^6$ as in the example given in section 3 above because $K_N \sim 100$ and $m_N \sim 10$ TeV have been taken here (instead of $K_S \sim 1$ and $m_S \sim 1$ TeV in the example of section 3) but still we estimate it to be at least of order $10^2$-$10^3$. Taking into account these various suppression effects, it would be very difficult to induce a large enough asymmetry in the way of Ref. [45], unless we assume a huge degeneracy of the right-handed neutrinos as can be done in the usual Fukugita-Yanagida model.
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