Particle tracking velocimetry in liquid gallium flow around a cylindrical obstacle

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Abstract
This paper demonstrates particle tracking velocimetry performed for a model system wherein particle-laden liquid metal flow around a cylindrical obstacle was studied. We present the image processing methodology developed for particle detection in images with disparate and often low signal- and contrast-to-noise ratios, and the application of our MHT-X tracing algorithm for particle trajectory reconstruction for the wake flow around the obstacle. Preliminary results indicate that the utilized methods enable consistent particle detection and recovery of long, representative particle trajectories with high confidence. However, we also underline the necessity of implementing a more advanced particle position extrapolation approach for increased tracking accuracy. Satisfactory tracking accuracy can be inferred from the fact that the fluctuations in the measured particle velocity are dominated by frequencies that agree sufficiently well with the expected frequencies of the cylinder wake.

Graphical abstract

1 Introduction
Liquid metal stirring, continuous casting, chemical reactors, etc. in many cases involve bubble flow in liquid metal. Some of these processes are, or potentially can be, controlled using applied magnetic field (MF) (Baake et al. 2017; Timmel et al. 2010, 2015; Wondrak et al. 2011; Birjukovs et al. 2020). However, control and optimization of these processes require a deep understanding and comprehensive knowledge of multi-phase flows in metallic melts.

Liquid metal multi-phase flow has been extensively studied by means of ultrasound Doppler velocimetry (UDV) (Strumpf 2017; Zhang et al. 2005; Wang et al. 2017), ultrasound transit time technique (Shew et al. 2006; Richter et al. 2018), X-ray imaging (Richter et al. 2018) and numerical simulations (Schwarz and Fröhlich 2014; Jin et al. 2016; Zhang and Ni 2014; Zhang et al. 2016; Gaudlitz and Adams 2009; Wang et al. 2016; Roig et al. 2012; May et al. 2018; Kroll-Rabotin et al. 2020; Bellot et al. 2018; Gisselbrecht et al. 2019), and many of its characteristics and mechanisms are presently sufficiently clear (Mougin and Magnaudet
known that the wake of bubbles (i.e., the flow pattern that the agglomeration of particles in a turbulent flow. It is well objective of the work presented herein. thus transported with the bubbles to the free surface. Inves-

bles generate turbulent flows that promote agglomeration of particles, leaving the latter at the gas-liquid interface and are otherwise very little experimental work (in contrast to many available simulations (Schwarz 2020; Schwarz and Fröhlich 2014; Jin et al. 2016; Zhang and Ni 2014; Zhang et al. 2016; Gaudlitz and Adams 2009; Wang et al. 2016; Roig et al. 2012) where bubble wakes or bubble/particle interactions are directly visualized in liquid metal (Lappan et al. 2020). One of the main reasons for this is the lack of suitable measurement techniques to perform such measurements in opaque liquids (in this case metals), where obviously powerful optical methods cannot be used. UDV has been applied to bubble wake flow characterization (Zhang 2021; Zhang et al. 2007), but, despite the possibility of fast measurements, the identification of individual particles is very difficult due to limited spatial resolution. The potential of flow analysis via particle tracking in liquid metal using positron emission particle tracking (PEPT) has been explored (Sommer et al. 2020; Burnard et al. 2014, 2011; Griffiths et al. 2011; Dybalska et al. 2020)—however, this method provides very low time resolution, making it difficult to use effectively in turbulent flows.

Recent studies and the advent of dynamic X-ray and neutron radiography of two-phase liquid metal flow (Akashi et al. 2019; Saito et al. 2005, 2005; Lappan et al. 2020; Sarma et al. 2015; Ščepanskis et al. 2017; Dzelme et al. 2015), though, fundamental investigation of systems with multi-phase flow containing gas bubbles and solid particles, with attempts to approach industrially relevant flow conditions, is underway (Schwarz 2020; Birjukovs et al. 2020, 2020, 2021; Bailey et al. 2017; Liu et al. 2018; Krull et al. 2017; Keplinger et al. 2019, 2018, 2017; Akashi et al. 2019). One must note, however, that the problem of proper neutron imaging of fast-moving gas bubbles has only just been solved for bubble flow without particles (Birjukovs et al. 2020, 2020, 2021)—this is another reason why, for now, we have restricted our first step to studying particle flow around a stationary cylindrical obstacle.

It was also proposed some time ago that neutron radiography could also be used to directly observe wake flow of bodies and particle flow within optically opaque systems (Cimbala et al. 1988). The first such benchmark study in the context of liquid metal flow with dispersed particles was recently done by Lappan et al. where gadolinium oxide particle flow around a cylindrical obstacle in a thin liquid metal channel was imaged dynamically with sufficient temporal resolution using high cold neutron flux (Lappan et al. 2020, 2021). The imaged turbulent particle-laden flow was investigated using particle image velocimetry (PIV) and the wake flow velocity field was measured and visualized.

In the present paper, we go one step further by performing particle tracking velocimetry (PTV). Unlike PIV which uses image feature correlations for velocimetry, PTV is explicit particle tracking where particles are treated as point-like bodies. Thus, PTV allows to quantify the dynamics of individual particles explicitly and enables wake flow analysis at finer length scales. Using PTV, we intend to measure not only the velocity field, but also to study maps of particle density versus time, the residence time of particles in the wake of an obstacle, and the correlation of particle motion due to interactions between particles and wake. As already mentioned above, initial tests have shown that visualizing the particles behind rapidly rising bubbles posed particular
challenges in terms of both spatial and temporal resolution. As a first step, we decided to focus on tracking the particles in the wake of a stationary obstacle. It is obvious to choose a cylindrical obstacle for this purpose. We are aware that the shape of larger ascending gas bubbles differs from that of a sphere and their ascent is not straight but follows zigzag or helical paths. This complexity of real bubble flow will be investigated in future studies.

Particle tracking in liquid metals is an important problem that can be solved using high-resolution neutron radiography. However, only a very limited number of papers address image processing required to successfully extract physically meaningful information from the acquired image. Notably, Heitkam et al. performed particle detection and tracking in froth using neutron imaging utilizing a particle-mask correlation approach (Heitkam et al. 2019). An original approach for detecting particles and tracking particle flow in the presence of bubbles was demonstrated by Sommer et al., although not in the context of liquid metal (Sommer et al. 2018). Another approach that is very promising for particle detection within flow with a high particle number density was developed by Anders et al. for optical measurements, but could potentially be generalized (Anders et al. 2019, 2020). However, the latter two do not seem to be readily applicable to low signal-to-noise ratio (SNR) images typically associated with high frame rate neutron imaging, and the former would be hard to generalize due to its reliance on preset particle masks. A very comprehensive overview of particle detection and tracking methods with an objective comparison is provided in Chenouard et al. (2014), but again, the showcased methodologies for particle detection are tested at SNR which, compared to the case in the present paper, is very high. In addition, as will be explained in Sect. 3.1, our case also exhibits correlated noise that produces “phantom” particles, which further complicates detection.

Furthermore, it is advantageous to combine a more noise-resilient image processing approach with a more general method for particle tracking based on detections. While advanced particle detection methods are shown, simple nearest neighbor tracking is used in Anders et al. (2019), Anders et al. (2020), which generally does not perform so well. Tracking methods used in Heitkam et al. (2019) and Sommer et al. (2018) each use a set of restrictions that make them problematic to use for particle flows with high number densities (i.e., inter-particle distances and particle sizes are of similar magnitudes) where detection is carried out under adverse imaging conditions. Once particles are detected, tracking in Heitkam et al. (2019) essentially relies on combined nearest neighbor and velocity-based predictions. Tracking in Sommer et al. (2018) is performed using a version of the shake-the-box algorithm Schanz et al. (2016), wherein tracks of particles are predicted based on preceding time steps and uncertainties between the predicted and actual positions are corrected by varying the predicted position in space until it matches the actual position. Newly entering particles are triangulated as in Wieneke (2012). While certainly tried and viable, this approach has many inherent constraints which, again, more often than not become a problem when particle number density is high, particle motion is highly irregular and the false positive and occlusion (i.e., overlaps of visible particle projections in images) rates are not negligible, as it is in our case. This is also true for the methods outlined in Chenouard et al. (2014). Even the cases where the most general and robust framework, multiple hypothesis tracking (MHT), is used, the imposed motion restrictions are quite severe. Specifically, near-constant position and/or velocity are assumed, i.e., no physics- or Kalman filter-based motion models are used. These approaches are therefore inapplicable to our case where particles may experience substantial acceleration and changes in position between consecutive frames due to interactions with the obstacle wake flow (Sect. 2). Note also that the experiment presented herein is a simplified setup, and we aim to develop methods that would also work under potentially more adverse conditions.

We have therefore developed an image processing methodology that reliably extracts particles of various sizes and visibility from sequences of images with a low SNR and spatially correlated noise. The particle positions determined over time are input into our previously developed MHT-X algorithm for object tracking (Zvejnieks et al. 2022) based on multiple hypothesis tracking (MHT), recently made much more computationally feasible in its offline (i.e., not real time) form. In this paper, we have augmented MHT-X with PIV-based motion prediction capabilities and adapted the previously developed physics-based motion models for particles instead of bubbles. As a result, we are able to reconstruct particle trajectories in the wake flow zone behind the cylindrical obstacle. We also point out the current limitations of our methods and propose future improvements that could increase the PTV quality, as well as the potential follow-up applications of the presented methods.

2 The experiment

This section gives a brief overview of the experiment and the dynamic neutron radiography setup used for flow imaging. The experimental setup is shown in Fig. 1 and more details can be found in our previous paper (Lappan et al. 2020). Given that neutron transmission imaging yields particle projections, it is desirable to avoid three-dimensional motion, so this experiment is performed for a quasi two-dimensional geometry. The cylindrical obstacle with a 5 mm diameter is made from stainless steel (X5CrNi18-10) and is centered and fixed in the straight section of the flow channel.
The boundary of such an obstacle should obey the no-slip condition. However, note that even if this is not exactly the case, the differences in the wake shape are not expected to be significant (May et al. 2018). The flow channel has a uniform 30 mm × 3 mm rectangular cross section-flow was imaged through the 3 mm dimension. To generate continuous liquid metal flow around the cylinder, the flow channel was designed as a closed loop. The loop is made from the same material as the cylindrical obstacle. Liquid metal flow is driven by a disk-type electromagnetic induction pump equipped with permanent magnets Lappan et al. (2020).

Flow measurements were performed at room temperature without additional heating using a low melting point gallium-tin alloy. Compared to pure gallium with the 30 °C melting point, the binary alloy with a 0.07 tin mass fraction is liquid at a slightly lower temperature of 25 °C Anderson and Ansara (1992). In this experiment, we opted for small particles made of gadolinium oxide. Gadolinium has an extremely high neutron attenuation coefficient μGd = 1.5 × 10^3 cm⁻¹ compared to gallium with μGa = 0.5 cm⁻¹ and tin μSn = 0.2 cm⁻¹ (Sears 1992; Paul Scherrer Institut (PSI) 2020). Gadolinium oxide particles with a d_p ∈ (0.3;0.5) mm diameter have been shown to provide sufficient image contrast for dynamic neutron imaging with a short image exposure time Lappan et al. (2020). Note that at the 3 mm thickness in the neutron flux direction the liquid alloy is rather transparent (~ 87% transmission) to neutron radiation. Gadolinium oxide also has a lower paramagnetic susceptibility than gadolinium (Lide 2019; Martienssen and Warlimont 2005). Gadolinium particles have been found to be inapplicable to experiments like the one outlined here: they are strongly attracted by the magnetic field of the electromagnetic induction pump and thus tend to rapidly clog the flow channel. Using gadolinium oxide particles instead, liquid metal flow can be driven continuously without interruptions for cleanup. Other metal and particle properties relevant for subsequent analysis include: GaSn density ρ_0 = 6160 kg/m³; GaSn viscosity μ = 2.1 mPa.s; particle density ρ_p = 7410 kg/m³.

Imaging was carried out with cold neutrons at the ICON beamline (Kaestner et al. 2011) of the Swiss spallation neutron source SINQ (Blau et al. 2009). During the measurements, the neutron source was operated at a constant proton beam current of 1.3 mA. The neutron flux ϕ is an important parameter for dynamic neutron imaging and significantly depends on the neutron beam aperture D. Most measurements were performed with D = 40 mm, providing ϕ = 5 × 10⁷ n cm⁻²s⁻¹. For a few measurement runs, the neutron beam aperture was doubled to D = 80 mm, which increases the neutron flux to ϕ = 1.8 × 10⁸ n cm⁻²s⁻¹ (by a factor of >3) (Kaestner et al. 2011). The neutron beam aperture, the flow experiment and the neutron scintillator screen of the imaging detector were aligned at fixed positions. The distance L_exp between the beam aperture and the scintillator was ~ 6.9 m, yielding the collimation ratio L_exp/D ~ 172 for D = 40 mm. The distance L_exp between the scintillator and the center plane of the liquid metal loop was 50 m (Lappan et al. 2020). The resulting geometrical unsharpness is L_exp/(L_exp/D) ~ 0.3 mm which is about the size of the smallest gadolinium oxide particles.

We used a 6LiF:ZnS scintillator with a 200 μm thickness and a 150 × 150 mm observable area. The light emitted by the scintillator was acquired by a sCMOS camera (Hamamatsu ORCA Flash 4.0; photographic objective: Nikon AF-S Nikkor 50mm f/1.4G). The camera’s field of view was set to 100 mm × 100 mm. The camera has a pixel array of 2048 × 2048 in total. Applying 2 × 2 pixel binning (average) for image noise reduction, the effective pixel array was reduced to 1024 × 1024. These camera settings result in a 10 px/mm spatial resolution. We chose a 10 ms image exposure time equivalent 100 frames per second required to capture individual particles moving in the liquid metal flow.
3 Particle detection

3.1 Image characterization and considerations for image processing

The region of interest where particles must be detected and tracked explicitly is the wake flow area highlighted in Fig. 2. Note that the flow is most disordered here, as expected (Fig. 2a). The particle tracks, which can be visualized using the time projection of minimum luminance (Fig. 2b), are strongly affected by the turbulent wake in contrast to the very smooth tracks at the sides of the wake flow zone. The analyzed field of view (FOV) was equal to 408 × 161 pixels (16-bit single-channel images) corresponding to 37.8 mm × 14.9 mm.

It was shown in Heitkam et al. (2019) that the images exhibit correlated noise in the form of grain-like structures with ~ 3-pixel sizes—this is a considerable fraction of a typical particle size in images and thus images may contain “phantom” particles (false positives). Particle projections visible due to neutron transmission contrast have strongly varying sizes and signal-and contrast-to-noise ratios (SNR and CNR, respectively) that also change over time as particles travel through the FOV. In addition, the recorded image sequence exhibits a pronounced global luminance non-uniformity, which can be seen in Fig. 2b. Given these factors, it was decided to build the image processing procedure around a local filter applied to interrogation windows (IWs, not to be confused with PIV terminology) taken from the images. Due to the very high area density of particles in the images, instead of utilizing a combination of segment estimation via a global filter with subsequent iterative refinement using targeted local filtering as in Birjukovs et al. (2021), it was decided to scan images entirely with partially overlapping IWs. This approach has three main benefits: first, IW overlaps imply redundancy which should reduce the rate of instances where true particles are excluded; second, local filtering over the entire image means reduced sensitivity to large-wavelength variations in luminance over the image; third, this works well with the false positive exclusion algorithm (Algorithm 4) that we introduce later.

3.2 The algorithm

The idea of sweeping images with partially overlapping IWs is illustrated in Fig. 3. An initial square IW with a side length \( L' \) is fitted into the upper-left corner of the image. A set of IWs is then generated from the initial IW by creating an IW position lattice with horizontal/vertical stepping \( \delta x \) and \( \delta y \), respectively, where the latter are a fraction of \( L' \). Thus, a set of \( n \) by \( n \) partially overlapping IWs is created where the number of steps in each direction \( n_k \) is determined by

\[
N_k = \left\lfloor \frac{L_k - L'}{\delta x_k} \right\rfloor, \quad k = x, y
\]  

Fig. 2 Pixel-wise a standard deviation and b minima of luminance values within the imaged flow channel over all captured frames. The region of interest is indicated with a red dashed frame. Note the cylindrical obstacle in both figures. The images shown here were rotated 90 degrees left with respect to the originals (Fig. 1b)—as such, here the originally downward flow is directed from left to right. The white arrow in (a) indicates the gravitational acceleration \( g \).

Fig. 3 A schematic representation of the IW sweep for the FOV: the initial IW (light blue) and shifted IWs (red) in the \( x > 0 \) (dotted) and \( y < 0 \) (dashed) directions. An example a horizontal scan in the \( x > 0 \) direction for an area indicated with a red dashed circle is shown in sub-figures 1-6.
where $L_k$ are the FOV dimensions. The $L_k - L'$ term ensures that the IWs are not excessively out of FOV bounds, since out-of-bounds parts of IWs are cropped, and overly cropped IW images do not provide enough meaningful information for local filtering. IW bounds $IW_{km}$ and centroids $r_{km}$ are given by

$$IW_{km} : \left\{ \left[1 + k \cdot \delta x; L' + k \cdot \delta x\right], \left[1 + m \cdot \delta y; L' + m \cdot \delta y\right] \right\};$$

$$k \in [0, n_x], m \in [0, n_y]$$

(2)

$$r_{km} = e_x \cdot \left(\left(1 + L'\right)/2 + k \cdot \delta x\right) + e_y \cdot \left(\left(1 + L'\right)/2 + m \cdot \delta y\right);$$

$$k \in [0, n_x], m \in [0, n_y]$$

(3)

All image processing operations are performed in Wolfram Mathematica. The general framework for image processing is as shown in Algorithm 1. Note that in this case we use the following parameters: $L' = 50$ (in pixels), $\delta x = \delta y = 10$. The values for $\delta x_k$ and $L'$ were chosen such that $\delta x_k$ is greater than the characteristic particle size, but otherwise a small fraction of $L'$; the latter was set to roughly match the expected scale of particle clusters seen in the FOV images. This is to ensure sufficient redundant detection for particles—we observe that the selected values are optimal for our case in terms of detection accuracy. However, smaller $\delta x_k$ make such an approach more computationally expensive. For instance, with the current settings, a $37 \times 12$ grid of IWs is generated for a total of 444 local images. The degree of redundancy for IWs is determined by $1 - \delta x_k/L'$ which in this case is 80%. This effectively inflates the amount of data (total image area) by a factor of ~ 16.9. Therefore, one should take care to optimize the underlying image processing code in terms of memory utilization and parallelize as many of its elements as possible.

**Algorithm 1: Image processing framework**

**Input:** A sequence of normalized images (pixel luminance rescaled to $[0; 1]$) with subtracted mean dark current

**IW generation**

1. Generate an $n_x \times n_y$ grid of IW positions (3) based on (1), and the chosen $L'$, $\delta x$ and $\delta y$
2. Disassemble images into their projections onto IWs (2)

**Particle detection in IWs**

3. Normalize the IW images
4. Local filtering (Algorithm 2)
5. Particle segmentation (Algorithm 3)
6. Luminance map-based false positive filtering (Algorithm 4)

**Assembly of global detection masks**

7. Map the false positive-filtered IW particle masks onto the full FOV using (3)
8. Sum the particle masks over the FOV
9. Minimum area thresholding
10. Morphological opening
11. Compute centroids for the resulting segments

**Output:** Centroids for particles detected in every FOV image

**Algorithm 2: Local (IW) filtering**

**Input:** A normalized IW image
1. Invert the image luminance map
2. Non-local means masking (NMM)
3. Soft color tone map masking (SCTMM)
4. Non-local means (NM) filtering
5. Mean filtering

**Output:** A filtered IW image
The stages of local filtering are shown in Fig. 4 and the filter structure is outlined in Algorithm 2. The luminance maps are inverted to highlight particles, since by default, due to intense neutron flux absorption by particles, they appear in images as lower luminance zones. This results in an IW image as shown in Fig. 4b. Next, non-local means masking (NMM) is performed (Fig. 4c) to increase the contrast-to-noise ratio for particles and remove the “haze” (correlated noise due to unsharpness described in Sect. 2), which is especially important for tightly-packed particle clusters. NMM transforms the original image $x$ into output $y$ as follows:

$$y = 2 * x - w_{nm} * NM(x, r_1, r_p)$$

(4)

where $NM(x, r_1, r_p)$ is the non-local means (NM) filter (Coll and Morel 2005), $w_{nm}$ is the NM mask weight, and $r_1$ and $r_p$ are the filtering neighborhood and neighborhood comparison radii, respectively. This is similar in principle to unsharp masking, but utilizes the NM filter instead of the Gaussian filter. We observe that here NMM distinctly outperforms simple unsharp masking since the NM filter captures the correlated noise much better. Here we set $w_{nm} = 1.5$, $r_1 = 1$ (pixels), $r_p = 5$. The best result is achieved when a noise power factor $p_n = 0.5$ is specified as well (estimated from the normalized luminance values for particles and “haze” (Fig. 4b)). The utilized NM filter computes the normalized neighborhood weights $\tilde{w}$ for averaging as in Coll and Morel (2011):

$$\tilde{w}_{ij} = \frac{w_{ij}}{\text{max}(w_{ij})}$$

(5)

where $i$ and $j$ are the neighborhood indices, $E_{ij}$ is the Euclidean distance between neighborhoods and $k_x$ is the filtering parameter. Here $k_x = 0.75$.

Next, soft color tone map masking (SCTMM) was applied for background reduction (Fig. 4d). SCTMM works by transforming an original normalized image $x$ to output $y$ in the following way:

$$y = x * (1 - CTM(x, c))$$

(6)

where $CTM(x, c)$ is the color tone mapping operation and $c$ is the luminance compression factor. The motivation and principles behind SCTMM are explained in detail in Birjukovs et al. (2021). Here we set $c = 0.65$. Afterward, NM filtering is performed (Fig. 4e) with $r_1 = 2$ and $r_p = 10$ ($p_n$, automatically derived from the neighborhood squared standard deviation of luminance), followed by the mean filter (Fig. 4f) with

$$w_{ij} = \exp \left[ -\max \left( 0, \frac{1}{k_x^2} \cdot \frac{E_{ij}^2}{p_n} - 2 \right) \right]$$

where $i$ and $j$ are the neighborhood indices, $E_{ij}$ is the Euclidean distance between neighborhoods and $k_x$ is the filtering parameter. Here $k_x = 0.75$.
a 2-pixel radius. Note that throughout the IW filtering procedure, images are re-normalized after each filtering stage.

Filtered images are then subjected to the segmentation procedure outlined in Algorithm 3. Here local adaptive (LA) binarization (mean- and deviation-based) Sezgin and Sankur (2004) is used because global thresholding yields very unstable particle detection in filtered IWs due to their dissimilar SNR and CNR, and thus post-filtering luminance distributions. LA binarization, however, is susceptible to the edges of low-luminance particles many of which are potential false positives. For this reason, a special luminance-based false positive filtering procedure was used as in Birjukovs et al. (2021) with minor modifications. The underlying operations are stated in Algorithm 4, and its application is illustrated in Fig. 5.

**Algorithm 3:** Local (IW) segmentation

**Input:** A normalized filtered IW image (Algorithm 2 and Figure 4f):
1. Apply SCTMM
2. Local adaptive (LA) binarization
3. Remove border components

**Output:** Particle segment mask for the IW (Figure 4g)

**Algorithm 4:** Luminance map-based false positive filtering for IW particle detection masks

**Input:**
- Filtered IWs (Algorithm 2)
- IW particle masks (Algorithm 3)
1. Multiply filtered IWs by the corresponding particle binary masks
2. Normalize the images
3. Compute $\langle I \rangle \cdot \max(I)$ for all particles in IWs
4. Thresholding for all particles:
   - if $\langle I \rangle \cdot \max(I) < \eta$; $\eta \in [0; 1]$ (user-defined) then
     - Flag the particle as a false positive
   - else
     - Nothing
   end
5. Remove the identified false positives from the particle detection masks

**Output:** IW particle detection masks without the detected false positives

The motivation and principles behind Algorithm 4 are provided in Birjukovs et al. (2021). Note that here we set the LA binarization neighborhood radius to 10 (values above the mean within the radius are set to 1, while the rest are assigned 0), $c = 0.65$ for SCTMM, and $\eta = 0.1$ is used for false positive filtering. Border components are removed to avoid artifacts and artificial particle splitting.

Once image filtering, segmentation and luminance-based false positive filtering are complete for the IWs from the original images, the filtered IW particle masks must be assembled into full FOV masks (Algorithm 1). Figure 6 shows the stages of this process. IW particle masks for every FOV image are mapped into the FOV (black background) and summed (Fig. 6b). Then segment area thresholding and morphological opening (disk structural elements) Haralick et al. (1987) are performed (Fig. 6c). Here the minimum area threshold is set to 5 pixels and the opening radius was set to 2 pixels. Finally, particle centroids (Fig. 6d) are computed for the remaining particle segments (4-connectivity is used).

It is important to note that persistent artifacts within images may be a problem in that they might introduce systematic errors into trajectories output by a tracing algorithm. Notice that one such artifact is present in Fig. 2a–a black spot in the right part of the FOV, which is a particle stuck to the channel wall. In such cases, removing these artifacts with texture synthesis-based inpainting (Inpaint 2020) is an effective solution. The artifacts in the considered images are
readily segmentable from the mean projection over time for a sequence of images using Otsu binarization Otsu (1979).

Computational performance analysis and particle detection statistics are provided in Appendices A and B.

4 Particle tracking

Once particle centroids were obtained for all images, tracing was carried out with the MHT-X algorithm that we have previously developed Zvejnieks et al. (2022). Here we present extensions for MHT-X for tracking dense particle flow in the conditions as in the present experiment.

MHT-X requires the definitions of an extrapolation method, association conditions, association constraints and statistical functions. Since in this case splitting and merging of particles does not occur in the experiment, only particle translation, entry and exit events must be considered. This makes the association constraints redundant and the split/merge statistical functions obsolete.

The association condition is a logical expression that determines whether two trajectories can be associated. The conditions from the original paper Zvejnieks et al. (2022) are used with the following changes. First, the association constraints on linear acceleration and deflection from Zvejnieks et al. (2022) are used, since constraints on motion are still desirable. However, the parameters are adjusted for the system considered here. Second, the sphere of influence (SOI) used to restrict the association range (only objects with overlapping SOI can be associated) is modified. Instead of defining the SOI about particle locations in frames, a prediction model pinpoints the location \( r \) of the region that the particle is expected to move to within time \( \Delta t \) and defines the SOI about that point. The prediction consists of the spline extrapolation for particle velocity \( \mathbf{v}_s \) and extrapolation derived from projecting the particle image velocimetry (PIV) field computed in Lappan et al. (2020) onto particle centroids \( \mathbf{v}_{\text{piv}} \):

\[
r(t_0 \pm \Delta t) = r(t_0) \pm (\alpha \cdot \mathbf{v}_s + (1 - \alpha) \cdot \mathbf{v}_{\text{piv}}) \cdot \Delta t
\]

where \( \alpha \) determines the prediction component weights.

The SOI radius \( R \) is based on the velocity magnitude, with higher velocity magnitudes yielding a smaller SOI:

\[
R = R_{\text{max}} \cdot \exp\left(-\frac{1}{\lambda_{\text{SOI}}} \cdot \| \alpha \cdot \mathbf{v}_s + (1 - \alpha) \cdot \mathbf{v}_{\text{piv}} \| \right)
\]

where \( R_{\text{max}} \) is the upper limit for the SOI radius, and \( \lambda_{\text{SOI}} \) is a control parameter. If \( \mathbf{v}_s \) is undefined, it and \( \alpha \) are set to 0.

This effectively assumes that particles with higher velocities are more difficult to deflect and vice versa, emulating cones of vision for moving particles. If two such cones overlap, an association is formed.

Exit and entry event statistical functions are kept as in Zvejnieks et al. (2022), except the horizontal x-axis is now the primary one. A model closely resembling the association condition has been implemented for translational motion associations. The translation likelihood estimator consists of three components determined by the location, the linear acceleration and the change in the motion direction.

The location-based likelihood compares the predicted location to the hypothesized location:
$p_{\text{pos}} = \mathcal{N}(\delta r, 0, \sigma_{\text{pos}} \cdot \Delta t)$ \hspace{1cm} (9)

where $\mathcal{N}(\mu, \sigma)$ is a normalized Gaussian distribution with its mean $\mu$ and standard deviation $\sigma$; $\delta r$ is the absolute difference between positions due to the prediction and the hypothesis. The acceleration-based likelihood is calculated as follows:

$p_{\text{acc}} = \mathcal{N}(a, 0, \sigma_a)$ \hspace{1cm} (10)

The direction-based likelihood component is designed to penalize large changes in the motion direction. The penalty scales with velocity magnitude:

$p_{\text{dir}} = \mathcal{N}(\delta \phi, 0, \pi \cdot \exp(-\|v\|/\lambda))$ \hspace{1cm} (11)

where $\delta \phi$ is the change in direction, $v$ is velocity and $\lambda$ is a control parameter.

The overall likelihood is computed as a weighted sum of the above contributions.

Fig. 7 PIV field projected onto particle centroids at different time stamps. Note the scale bar in (b) and the velocity color bar (normalized for the entire image sequence) in (d).

Fig. 8 Snapshots of constructed particle trajectories (indicated with different colors) at different time stamps. Particle Entry nodes are indicated with blue dots, while Exit nodes are shown as green crosses. Each of the snapshots shows the last 15 segments of reconstructed trajectories. The scale is identical to that shown in Fig. 7.
where \( u_1 \) and \( u_2 \) are weights.

Note that while here we use PIV and in general it is suggested for better tracking, in principle it can be easily disabled in the code if the user does not have the data.

5 Preliminary results

Before proceeding with tracking, the PIV velocity field \( \mathbf{v}_{\text{piv}} \) obtained in Lappan et al. (2020) was interpolated and projected onto the positions of particles detected in each frame. Delaunay triangulation is performed for \( \mathbf{v}_{\text{piv}} \) point grid and cubic interpolation is used for particle centroids that are within triangles formed by nearby \( \mathbf{v}_{\text{piv}} \) grid points, while nearest neighbor interpolation is used otherwise (SciPy). Interpolation is performed independently for both velocity components. Particle flow images with \( \mathbf{v}_{\text{piv}} \) projections for particles are shown in Fig. 7. Note that, according to the \( \mathbf{v}_{\text{piv}} \) field, many particles within the wake often travel in directions opposite (and sometimes normal, as seen in Fig. 7d) to the mean flow direction. The obtained \( \mathbf{v}_{\text{piv}} \) for particles is used in (7) and (8) for motion prediction.

Figures 8 and 9 present the results of applying MHT-X to the output of image processing. Figure 8 shows some of the reconstructed trajectories within the FOV at four different time stamps. Note that only the last 15 segments of the constructed trajectories are shown. This limitation was introduced for visual clarity, but the trade-off is that the trajectories of slower particles in the wake flow zone are more difficult to show. Despite this, several things can be observed. First, note that trajectories are not broken near the right boundary of the FOV where an image artifact was located before it was removed by texture synthesis inpainting. Second, notice that even with the limitation on the number of segments visible at a time per trajectory, rather long particle tracks can be observed both within and outside the wake flow zone. Third, one can see, especially in Fig. 8a and d, that densely packed trajectories that cross one another in close temporal proximity are correctly resolved. However, it is also evident that there are quite a few significantly fragmented trajectories, especially within the wake flow zone.

Figure 7 demonstrates the issue of particles being caught within the oscillating wake flow area exhibit both relatively small velocity magnitudes and rapid changes in motion direction. This is critical for the current MHT-X implementation since closely packed trajectories with low velocity magnitudes, according to (8) and (11), result in many feasible associations for trajectory connections. With the current spline-based trajectory extrapolation method Zvejnieks et al. (2022) it is often the case that trajectory fragment mismatch is such that MHT-X opts to assign Exit nodes to trajectories prematurely rather than reconstruct longer tracks from fragments. However, MHT-X is still able to resolve quite a number of physically meaningful and long trajectories, examples of which are shown in Fig. 9.

Figure 9 shows some of the longer trajectories recovered by MHT-X. In Fig. 9a and b one can see trajectories of particles that passed by, interacted with and then departed from the wake flow zone. Notice in Fig. 9b that as the particle is briefly captured by the wake flow, its velocity becomes lower as indicated by significantly shorter trajectory segments seen in the middle of the FOV (\( x \) direction). This is also seen in Fig. 10.
Then, as the particle exits the wake flow, it is again accelerated by the channel flow. Figure 9a, on the other hand, shows that the particle was not entrapped in the wake flow and traversed the FOV much faster. The trajectory in Fig. 9c is the longest observed in terms of the number of segments. The underlying particle was first observed and entered the wake flow zone from the top of the FOV and then had a considerable residence time within the wake before the trajectory was broken off in the left part of the FOV. This particular trajectory is of note for several reasons: first, it

![Fig. 10](image-url) Velocity time series for the particle with the trajectory shown in Fig. 9b: velocity $a \, x$ and $b \, y$ components, pixels per frame. The gray dots are the MHT-X output and the red curves are the median-filtered (1-point radius) velocity components

![Fig. 11](image-url) a The likelihoods of all trajectory segments, b mean segment likelihoods for all trajectories, c standard deviation of segment likelihoods within trajectories, normalized by mean likelihoods, and d node count for constructed trajectories. In (a–c) trajectories with 4+ nodes are considered
clearly shows what is also seen in the PIV projection images—particles captured by the wake behind the obstacle are often diverted towards the center of the wake flow zone and then their direction is reversed such that it is opposite to that of the mean channel flow Sommer et al. (2018); second, note the fragment of this trajectory highlighted by a white dashed frame in Fig. 9c—one can observe the particle trajectory forming a small loop. It is important that such motion with a low velocity in presence of other potentially interfering particles in the wake flow zone is nonetheless resolved with a high degree of confidence—note the minimum segment likelihood is 0.93 (color bar to the left). Figure 9d shows a similar trajectory except that its motion direction is not reversed during the residence time.

To assess the quality of MHT-X output more quantitatively, several metrics were evaluated for recovered trajectories: segment likelihoods for all segments, mean segment likelihoods and normalized (with respect to mean) dispersion of likelihoods for trajectories, as well as the trajectory size distribution—these are given in Fig. 11. The first thing to note is that most of the segments for trajectories with 4+ nodes (trajectories with < 4 nodes are not usable even for local PTV) have likelihoods mostly in excess of 0.9 with a sharp maximum just below 1 (Fig. 11a). This is important since greater likelihoods generally imply less ambiguous trajectory reconstruction and greater confidence that the output is physically accurate. Very high mean segment likelihoods for constructed trajectories (Fig. 11b) and mean likelihood dispersion mostly within 10% of the mean values (Fig. 11c) also speak to the quality of the generated results. Finally, Fig. 11d indicates that MHT-X produced a few hundred trajectories with ~20 nodes and tens of trajectories with 20+ nodes. Note that the trajectory with 130+ nodes seen to the right of the bulk of the distribution is the one shown in Fig. 9c. While in-depth physical analysis of particle flow dynamics (residence time within different regions of the FOV, trajectory curvature, etc.) requires more tracks with 30+ nodes (more for slower particles captured by the wake) than are currently produced, shorter tracks can be used for local PTV. The latter could potentially provide insights about the velocity field at smaller length scales than in the case of PIV.

Finally, it was previously assumed in Lappan et al. (2020) that the utilized gadolinium oxide particles are mostly passive tracers. However, it is worth testing this assumption by estimating the particle Stokes number \( Stk \) range. We first evaluate the Reynolds number for the cylindrical obstacle

\[
Re_c = \frac{\rho_p U_0 d_c}{\mu_0}
\]

with material properties as stated in Sect. 2, and using the free-stream velocity from PIV \( U_0 \in (10;14) \text{ cm/s} \) Lappan et al. (2020). This yields \( Re_c \in (1466;2053) \), indicating a transitional or possibly turbulent cylinder wake. This is also in line with the observed vortex shedding and wake flow oscillations. We then assess the particle \( Re \)

\[
Re_p = \frac{\rho_p U_{rel} d_p}{\mu_0}
\]

where \( U_{rel} \) is the relative particle-flow velocity \( (d_p \text{ range stated in Sect. 2}) \). Assuming that most of the particles have \( U_{rel} \in (0;10) \text{ mm/s} \) (i.e., up to \( \sim 33d_p \text{ per second} \)), one has \( Re_p \lesssim 15 \). Given the very coherent (even visually) motion seen in the cylinder wake and around the cylinder, it is very unlikely that \( U_{rel} \sim 0.1 \text{ m/s} \) (stationary, i.e., a particle stuck in the free-stream flow, \( Re_p \sim 150 \)) can be expected except for rather rare instances. And even then, \( Re_p = 150 \) is well below the particle wake flow delaminarization threshold, while particles with \( Re_p \lesssim 15 \) should not even exhibit considerable flow separation Clift et al. (1978). Therefore, the drag force on particles should not deviate too much from the Stokes law, which means one may set \( Re_{c} \sim 1 \) as an initial assumption. This implies that the particle time scale should be close to that of the wake flow, where the dominant time scale is associated with the vortex shedding frequency. Using \( Re_{c} \) range, we estimate the Strouhal number \( (Sr) \) for the obstacle via

\[
Sr = 0.198 \cdot (1 - 1.97/Re_{c})
\]

which results in \( Sr \sim 0.1978 \). \( Stk = \tau_p/\tau_0 \) which is the ratio of the particle and flow time scales, respectively. At the same time, \( Sr = d_c/\tau_0 U_0 \) and given \( Re_p \sim 1 \) one has \( \tau_p = \rho_p d_p^2/18 \mu_0 \). This means that \( Stk \) can be expressed through \( Sr \) as

\[
Stk = \frac{Sr \cdot \rho_p d_p^2 U_0}{18 \mu_0 d_c}
\]

yielding \( Stk \in (0.067;0.262) \) and implying that the particles should be fairly good tracers under the above assumptions.

Fig. 12 Smooth normalized density histogram (Scott binning, 2-nd order interpolation) of the first 10 dominant frequencies aggregated for 200 longest (by graph node count) trajectories
Another approach to check this and also qualitatively validate the MHT-X performance is to consider the reconstructed trajectories, perform Fourier analysis of their velocity fluctuations, and check if the fundamental frequency of the cylinder wake oscillation $f_0$ corresponds to the frequency content extracted from trajectories $f_t$. $f_0$ can be estimated from $Sr$ as the inverse of $r_p$, here one has $f_0 \approx (3.95; 5.54)$ Hz.

On the other hand, taking the first 10 dominant frequencies for 200 longest trajectories and assessing the probability density of the encountered frequencies reveals that there is a distinct peak with a full width at half maximum spanning $f_t \in (2.42; 4.23)$ Hz (Fig. 12) which overlaps with $f_0$.

This indicates consistency between the reconstructed dynamics and the expected flow properties. Finally, consider that Stokes’ drag law underestimates the drag force for $Re_p$ that is significantly greater than one, which means that the $Stk$ range derived here is likely to be overestimated.

### 6 Conclusions and outlook

To summarize, we have developed and demonstrated an image processing methodology coupled with the modified MHT-X code for particle detection and tracking in images from dynamic neutron radiography of particle flow in a liquid metal channel. Preliminary results indicate that the proposed approach is feasible-for local PTV as is, and for a more in-depth physical analysis of fully reconstructed trajectories, and thus wake flow, after the existing extrapolation scheme (Zvejnieks et al. 2022) is replaced with a better solution.

In this regard, we see several ways to improve/extend the functionality of MHT-X-in addition to the points made in Zvejnieks et al. (2022), we propose the following:

- Use the Kalman filter for improved particle motion prediction by using three contributions: PIV- motion and PTV-predicted motion, as well as predictions derived by solving effective equations of motion for particles (Lagrangian approach).
- For higher-quality PIV-based predictions, it is planned to utilize divergence-free interpolation (Tapley et al. 2021; Wendland 2009) to minimize errors stemming from the currently used polynomial interpolation ignoring the flow continuity constraint.
- A multi-level approach for divergence-free interpolation (Farrell et al. 2017) seems prospective for further improvement of PIV-based predictions.
- Better PTV-based predictions could also be generated applying divergence-free interpolation to sparse PTV velocity fields. However, since the PTV field is not defined on a regular grid like in the case of PIV, the multi-level method does not seem to be readily applicable here and will have to be extended.
- Given the low particle Reynolds numbers seen in our case, one can use a linear drag model for effective equations of motion for particles. One could then solve these equations as proposed in Tapley et al. (2021) using phase space volume contractivity preservation via splitting methods.

We anticipate that the approach developed here, with the improvements outlined, will be able to generate many more longer trajectories. The latter can then be analyzed by computing residence time maps for the FOV, examining how trajectory curvature profiles over arc lengths are correlated with local flow dynamics, etc. It is likely that applying dynamic mode decomposition Klevs et al. (2021) to PIV and PTV velocity fields could assist physical interpretation.

This study clearly demonstrates that flow analysis can be performed under conditions similar to what is shown in this paper-as such, more complicated and realistic experiments of a similar nature can be conducted and successful extraction of physically meaningful information can be expected.

It is very important to stress that the application of image processing and tracking approaches to a quasi two-dimensional flow (3 mm liquid metal thickness) shown herein is not the limit in that it should also be applicable not only to specifically neutron imaging, but also in that particle detection should be possible in systems potentially very different from the one demonstrated in this paper. While the presented methodology should be readily usable for Hele-Shaw vessels, setups with vessel thickness of 10 mm as in Anders et al. (2019), Anders et al. (2020), where optical imaging is performed, could be treated equally well. We also expect the developed image processing and tracking methods to be applicable to imaging with thicker liquid metal layers using neutron radiography. To prove this, our next step will be to analyze the particle-laded liquid metal flow images obtained in Sarma et al. (2015), Ščepanskis et al. (2017) for a 30 mm-thick vessel (data already shared with the authors). Note that thus far only PIV was attempted for these experiments. Imaging of particles in froth using neutron transmission as in Heitkam et al. (2019) (a cylindrical vessel with a 67-mm diameter), which remains an active field of study, will surely benefit from the presented methods as well (some data has been shared with the authors). Finally, the results could be used to analyze particle dynamics during solidification in two-phase systems Baranovskis et al. (2020). It must also be noted that greater neutron flux density can and should be leveraged to perform measurements in liquid metal beyond the 30 mm thickness threshold at high frame rates-this is readily doable at the ICON beamline at the Paul Scherrer Institute by imaging with a larger aperture Kaestner et al. (2011).
Clearly, a major problem here is obtaining information about the particle position in the direction of transmission. To provide solutions similar to optical stereo PIV for this purpose, we unfortunately, for the time being, depend on further technical development of neutron sources and neutron optics in addition to adaptation of reconstruction algorithms.

Appendix

Appendix A: Parallelization and performance

The image processing pipeline as outlined above was implemented in *Wolfram Mathematica* using its parallel computing functionality. Due to a large number of IWs per image ($\sim 10^5$) and the small relative area of an IW ($\sim 3.8\%$), it was decided to parallelize IW processing for individual images, thus performing sets of parallel computations, as many as there are images in a sequence. IW generation is performed in serial mode, while local filtering and particle segmentation (Algorithm 1, Steps 4 and 5) are parallelized. To speed up false positive filtering (Algorithm 4), it is split into a sequence of steps, each individually parallelized, as outlined in Algorithm 5. Afterward, the assembly of the full FOV masks is performed in parallel (image composition and addition), while area thresholding and morphological opening are performed in serial mode.

The implementation as presented in this paper was tested on two machines (*Windows 10*):

- **Intel Core i9-10980XE** (18 cores/36 threads) with 256 GB 2933 MHz DDR4 RAM
- **Intel Core i7-10700K** (8 cores/16 threads) with 32 GB 3200 MHz DDR4 RAM

### Algorithm 5: Splitting Algorithm 4 into sequential parallelized stages

1. Generate masked luminance maps for all IWs – multiplication of filtered images and particle segment masks
2. Compute $\langle I \rangle \cdot \max(I)$ for all resulting segment intensity maps
3. Compare the output against $\eta$ and flag false positives for all IWs
4. Get segment masks for all IWs
5. Map the particle masks of the true positives to all respective IWs

### Table 1 The results of the image processing code benchmarks. $\tau$ denotes wall time. RAM utilization accounts for the system processes

| System    | Threads | RAM (GB) | Wall time ($\tau$) | Algorithm 2 and 3 | Algorithm 4 | Mask assembly |
|-----------|---------|----------|--------------------|-------------------|--------------|---------------|
| Core i9   | 36/36   | $< 70$   | 1.26 hrs           | 59% $\tau$        | 28% $\tau$   | 13% $\tau$    |
| Core i7   | 16/16   | $< 32$   | 1.80 hrs           | 80% $\tau$        | 13.5% $\tau$ | 6.5% $\tau$   |
| Core i9   | 18/36   | $< 40$   | 1.69 hrs           | 72% $\tau$        | 19% $\tau$   | 9.0% $\tau$   |

Fig. 13 Particle count per frame (100 FPS, black) over a 1500-frame image sequence. The red curve is the averaged trend obtained via Gaussian total variation (TV) filtering (regularization parameter equal to 2) Rudin et al. (1992) and the statistically significant ($q = 0.9$ quantile) value ranges about the averaged curve are indicated with the light-gray envelope. The envelope is derived by filtering the quantile spline envelopes Antonov (2014) for data with the same TV filter as the data
The code was first tested using a sequence of 1500 FOV images. Three test cases were run multiple times each: using all available parallel threads (hyperthreading was used since all the underlying operations for IWs are independent) on the Core i9 and Core i7 systems, and running the code on the Core i9 CPU using half of the available parallel threads. The results are summarized in Table 1.

In both cases with all available parallel threads used, the CPU utilization for Algorithms 2 and 3 was consistently at 100%. For the Core i9 system with all threads utilized, all stages of Algorithm 5 combined exhibit mean CPU utilization of ~ 39% on average, with ~ 28% at minimum and ~ 83% at maximum. The global mask assembly runs with ~ 81% CPU utilization on average. CPU utilization for the Core i7 system was greater for both Algorithm 5 and global mask assembly: ~ 59% and ~ 90% on average, respectively. The discrepancies in processing time proportions and CPU utilization between the Core i9 and Core i7 systems are largely due to a considerable difference in the number of cores in favor of the Core i9 machine and the better single-core performance of the Core i7 machine. The speedup factor between the two systems systems running all available threads is ~ 1.43.

**Appendix B: Particle detection density**

Figures 13 and 14 show the particle detection density over frames (equivalently, in time with 100 FPS) and space for a 1500-frame image sequence. Particle count per frame (Fig. 13) is on average ~ 105 with a ~ 10% deviation, indicating consistency in particle detection.

Figure 14 indicates that particle detection density is considerably greater within the wake of the cylindrical obstacle—this makes sense intuitively, since particles entrapped in or travelling through the wake flow zone are slower and have longer residence times within the FOV than the particles travelling with the mean flow around the obstacle, to the top and bottom of the FOV. Hence, more detection events per unit area are generated in the wake flow region of the FOV. This means that generated detection events are physically consistent with what one would expect from the studied system.

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**Author Contributions** MB developed and implemented the image processing algorithm used to detect particles. PZ is the main developer of MHT-X object tracking algorithm and incorporated special likelihood functions for particle motion and motion prediction using particle image velocimetry (PIV) into MHT-X. Mihails Birjukovs performed image processing and PZ performed particle tracking. Experimental data were obtained by TL, MS, SH, PT and DM. MB and SH analyzed the results. Visualization was done by MB and PZ. The first version of the manuscript was written by MB, TL and PZ. SE and AJ were responsible for funding acquisition and research supervision. All co-authors contributed to manuscript editing and review prior to submission.

**Data availability** Both input and output for the image processing code and MHT-X, as well as associated visuals are available on demand—please contact the corresponding authors.

**Code availability** The image processing code is available at GitHub: Mihails-Birjukovs/Low_C-SNR_Particle_Detection. MHT-X can be found at GitHub as well: Peteris-Zvejnioks/MHT-X.

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