Modulational instability of ion-acoustic wave packets in quantum pair-ion plasmas

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Abstract Amplitude modulation of quantum ion-acoustic waves (QIAWs) in a quantum electron-pair-ion plasma is studied. It is shown that the quantum coupling parameter $H$ (being the ratio of the plasmonic energy density to the Fermi energy) is ultimate responsible for the modulational stability of QIAW packets, without which the wave becomes modulational unstable. New regimes for the modulational stability (MS) and instability (MI) are obtained in terms of $H$ and the positive to negative ion density ratio $\beta$. The growth rate of MI is obtained, the maximum value of which increases with $\beta$ and decreases with $H$. The results could be important for understanding the origin of modulated QIAW packets in the environments of dense astrophysical objects, laboratory negative ion plasmas as well as for the next generation laser solid density plasma experiments.

Keywords Pair-ion plasma · Modulational instability · Ion-acoustic waves · Quantum plasma

In the recent years, there has been a growing interest in investigating various collective modes and their properties in pair-ion plasmas (see, e.g., Hasegawa and Shukla 2005; Cramer and Yung 1986; Misra 2009; Samanta and Misra 2009). Such plasmas are believed to be ubiquitous in most space and laboratory plasmas (Amemiya et al. 1999; Franklin 2002). Moreover, it has been investigated that the pair-ion plasmas have potential applications in the atmosphere of D-region of Earth’s ionosphere, Earth’s mesosphere, the solar atmosphere, as well as in the microelectronic plasma processing reactors (Kim and Merlino 2007). Recent investigations indicate that such pair-ion plasmas could also be important with regard to the diagnostic point of view, since the dispersion properties of wave modes can be used to deduce the plasma parameters (Samanta and Misra 2009). On the other hand, in view of wide applications in dense astrophysical environments as well as in intense laser produced plasmas, Misra studied the formation of ion-acoustic shock-like oscillations in quantum pair-ion plasmas (Misra 2009). Again, the propagation of wave packets in a dispersive nonlinear plasma medium (where the dispersion is due to quantum tunneling associated with the Bohm potential) has been known to be subjected to the amplitude modulation, i.e., a slow variation of the wave packet’s envelope due to nonlinearity. The system’s evolution is then governed through the modulational instability (MI). A number of works can be found in the literature to study the MI in various quantum plasma systems (e.g., see Misra and Shukla 2007, 2008a, 2008b; Sabry et al. 2009; Misra et al. 2009).

In this work, we study the important effect of quantum tunneling associated with the Bohm potential on the MI of quantum ion-acoustic waves (QIAWs) in an electron-pair-ion plasma. We show that the quantum coupling parameter $H$ is ultimate responsible for the stability of modulated QIAW packets, without which the wave becomes modulational unstable. New regimes for the MI are obtained with...
the variation of $H$ along with the positive to negative ion density ratio $\beta$. We also obtain the growth rate of MI in terms of the system parameters.

In what follows, we consider the nonlinear propagation of QIAWs in an unmagnetized quantum plasma composed of electrons and both positive and negative ions. The basic normalized equations read (Misra 2009)

$$\phi = -\frac{1}{2} + \frac{1}{2} e^2 \frac{H^2}{2\mu} - \frac{\hbar^2}{2\mu} \frac{\partial^2 \sqrt{\mu}}{\partial x^2},$$

(1)

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial (n_\alpha u_\alpha)}{\partial x} = 0,$$

(2)

$$\frac{\partial u_\alpha}{\partial t} + u_\alpha \frac{\partial u_\alpha}{\partial x} = -\frac{\partial \phi}{\partial x},$$

(3)

$$\frac{\partial^2 \phi}{\partial x^2} = (\beta - 1)n_e - n_+ + n_-,$$

(4)

where the suffix $\alpha = +, -$ indicate the quantities for positive and negative ions respectively. Also, $\phi$ is the electrostatic potential normalized by $k_B T_F/e$ with $k_B$ denoting the Boltzmann constant, $e$ the elementary charge and $T_F \equiv h^2(3\pi^2 n_0)^{2/3}/2k_B m_e$ the electron Fermi temperature. Here $\hbar$ is the Planck’s constant divided by $2\pi$, $m_e$ is the electron mass and $n_0$ is its equilibrium number density. Moreover, $n_\alpha$ denotes the $\alpha$-particle perturbed number density normalized by its equilibrium value $n_{\alpha0}$, $H \equiv h\omega_{pe}/k_B T_F$ is the nondimensional quantum parameter describing the ratio of the plasmonic energy to the Fermi energy, $\omega_{pe} \equiv \sqrt{n_{\alpha0} e^2/\epsilon_0 m_\alpha}$ is the $\alpha$-particle plasma frequency and $m_\alpha$ is the mass. The speed of the $\alpha$-species particle $n_\alpha$ is normalized by the quantum ion-acoustic speed $c_s \equiv \sqrt{k_B T_F/m_-}$, the space $(x)$ and time $(t)$ variables are normalized by $c_s/\omega_{pe}$ and $t/\omega_{pe}^{-1}$ respectively. In (3), $\xi, \tau \equiv (m, -1)$ with $m \equiv Z_+ m_+ / Z_- m_-$. The parameter $\beta = Z_+ n_+ / Z_- n_-$. The positive to negative ion density ratio with $Z_+, -$ denoting the positive (negative) ion charge states. The second term in the right-hand side of (1) is due to the three-dimensional (3D) Fermi-Dirac pressure for electrons given by (Landau and Lifshitz 1980; Manfredi and Haas 2001; Manfredi 2005)

$$p_e = \frac{1}{5} \frac{m_e V_F^2}{n_0^{2/3}} n_e^{5/3}. $$

(5)

Since the equilibrium distribution is always 3D even in one-dimensional (1D) geometry (we can project the 3D Fermi-Dirac distribution over $x$-direction), the equilibrium pressure must indeed be given by its 3D expression (5) (Manfredi 2005).

In order to obtain an evolution equation describing the propagation of modulated QIAW envelopes we employ the standard reductive perturbation technique (RPT) in which the independent variables $x$ and $t$ are stretched as (see, e.g., Misra and Shukla 2007, 2008a, 2008b; Misra et al. 2009) and substituting all the above derived expressions from (1)–(4) and expressing the variables $(n_\alpha, u_\alpha, \phi)$ into the components for the carrier waves.

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$$n_\alpha, u_\alpha, \phi = (1, 0, 0)$$

(6)

$$+ \sum_{n=1}^{\infty} \sum_{l=-\infty}^{\infty} [n^{(n)}_\alpha(\xi, \tau), u^{(n)}_\alpha(\xi, \tau), \phi^{(n)}_\alpha(\xi, \tau)]$$

$$\times \exp[i(kx - \omega t)l],$$

(6)

where $\omega, k$ are the normalized wave frequency and wave number respectively. The state variables $n^{(n)}_\alpha$, etc., satisfy the reality condition $A^{(n)}_\alpha = A^{(n)*}_\alpha$ with asterisk denoting the complex conjugate. Now, substituting the expansion (6) into (1)–(4) and expressing the variables $x$ and $t$ in terms of the stretched coordinates $\xi, \tau$, and then collecting the terms in different power of $e$ we obtain for $n = 1, l = 1$ the first order quantities,

$$n^{(1)}_\alpha = \phi^{(1)}_\alpha / \Lambda, \quad n^{(1)} u^{(1)}_\alpha = \frac{\omega k^2}{\Lambda} \phi^{(1)}_\alpha, \quad u^{(1)}_\alpha = \frac{\omega \phi^{(1)}_\alpha}{\Lambda}.$$