Cache-Aided Data Delivery over Erasure Broadcast Channels

Mohammad Mohammadi Amiri and Deniz Gündüz

Abstract

A cache-aided broadcast network is studied, in which a server delivers contents to a group of receivers over an erasure broadcast channel. The receivers are divided into two sets with regards to their channel qualities: the weak and strong receivers, where all the weak receivers have statistically worse channel qualities than all the strong receivers. The weak receivers, in order to compensate for the high erasure probability, are equipped with cache memories of equal size, while the receivers in the strong set have no caches. Data can be pre-delivered to weak receivers’ caches over the off-peak traffic period before the receivers reveal their demands. It is first assumed that the receivers in the same set all have the same erasure probability, and a joint caching and channel coding scheme, which divides each file into several subfiles, and applies a different caching and delivery scheme for each subfile, is proposed. It is shown that all the receivers, even those without any cache memories, benefit from the presence of caches across the network. An information-theoretic trade-off between the cache size and the achievable rate is formulated, and it is shown that the proposed scheme improves upon the state-of-the-art in terms of the achievable trade-off. The proposed scheme is then extended to the scenario where the receivers in both sets of weak and strong receivers have different erasure probabilities.

I. Introduction

Wireless content caching is a promising technique to flatten the traffic over the backhaul network by shifting part of the traffic from the peak to the off-peak periods [1]–[5]. Video-on-demand services for mobile users would particularly benefit from content caching due to highly skewed popularity distribution across files. Popular contents that are likely to be requested by a majority of the users can be proactively cached at the network edge during a period of low network traffic, known as the placement phase. The delivery phase is performed during a peak traffic period, when the users reveal their demands, and the cached contents can be exploited to reduce both the load over the backhaul links and the latency in delivery [6].
An information-theoretic approach to proactive content caching and delivery has been initiated by Maddah-Ali and Niesen in [6], [7], where they consider a library of same-size files to be delivered over a noiseless shared channel, while the receivers are equipped with cache memories of equal size. They characterize a trade-off between the cache size and the minimum rate required during the delivery phase to serve all the receivers for all demand combinations, and show that coding can significantly reduce the required delivery rate. Several improved coded caching schemes and information theoretic performance bounds have been introduced since then [8]–[18].

Conventional coded caching schemes have been employed in various applications with different network settings, including device-to-device caching [19], [20], femtocaching [21], [22], online cache placement [23], coded caching of files with non-uniform popularities [24]–[26] and distinct lengths [27], hierarchical coded caching [28], [29], caching files to users with non-uniform cache sizes [30], [31], multi-layer caching [28], coded caching in distributed computation [32], [33], and coded caching for wireless communications with delayed channel state information feedback [34], [35].

In contrast to the setting introduced in [6], a noisy channel is considered for the delivery phase in [36]–[44]. Here, we follow the model considered in [37], [38], and assume that the delivery phase takes place over a memoryless packet erasure broadcast channel, and the receivers are grouped into two disjoint sets of weak and strong receivers. All the weak receivers have statistically worse channels compared to all the strong receivers. To compensate for the worse channel quality, each weak receiver is equipped with a cache memory of equal size. Assuming equal-rate files in the library and the same erasure probability for all the receivers in the same set, we derive a trade-off between the size of the caches provided to the weak receivers and the rate of the files, for which any demand combination can be reliably satisfied over the erasure broadcast channel. We show that, for a given cache size, the proposed scheme can achieve a higher rate than that is achieved in [38]. We then consider different erasure probabilities for the receivers in both sets of weak and strong receivers, and extend the achievable scheme to this scenario.

The rest of this paper is organized as follows. We introduce the system model in Section II. In Section III, our main result for the case of uniform erasure probabilities for all the receivers in the same set is summarized and compared with the state-of-the-art. The proposed scheme for equal erasure probabilities among the receivers in the same set is first illustrated through an example and then described for the general setting in Section IV. The main result together
with the achievable scheme for the scenario of non-uniform erasure probabilities across different receivers in each set is presented in Section VI. We conclude the paper in Section VI.

II. System Model and Preliminaries

We consider a server with a library of $D$ popular files $\mathcal{W} \triangleq \{W_1, \ldots, W_N\}$. Each file $W_i$ is distributed uniformly over the set $\lfloor 2^{nR} \rfloor$, $\forall i \in [N]$, where $R$ denotes the rate of each file, and $n$ is the number of channel uses during the delivery phase. Number of receivers demanding files from the server is $K \leq N$, and $d_k$ represents receiver $k$’s demand, where $d_k \in [N]$, $\forall k \in [K]$. The server needs to deliver file $W_{d_k}$ to receiver $k$, $\forall k \in [K]$, simultaneously.

Following [38], the channel between the server and the receivers is modeled as a memoryless packet erasure channel. For each channel use, the server transmits an $F$-bit codeword from the alphabet $\mathcal{X} \triangleq \{0, 1\}^F$, and the output alphabet at each receiver $k \in [K]$ is $\mathcal{X} \cup \{\Delta\}$, where the erasure symbol $\Delta$ corresponds to a packet that is not received at the receiver. Receiver $k$, for $k \in [K]$, receives the transmitted codeword $x \in \mathcal{X}$ correctly with probability $1 - \delta_k$, and

1For two integers $K_1 < K_2$, we will denote the set of integers $\{K_1, K_1 + 1, \ldots, K_2\}$ by $[K_1 : K_2]$. The set $[1 : K]$ will shortly be denoted by $[K]$. 

Fig. 1. Cache-aided packet erasure broadcast channel with $K$ receivers requesting files from the library $\mathcal{W} = \{W_1, \ldots, W_N\}$. The first $K_w$ receivers have statically worse channels but they are equipped with a cache of size $M$, while the next $K_s$ receivers have better channel qualities, but they are not provided with any cache memories.
the erasure symbol $\Delta$ with probability $\delta_k$. Thus, given the transmitted content $x \in \mathcal{X}$, receiver $k \in [K]$ observes the output $y_k \in \mathcal{Y}$ with the probability of

$$P(Y_k = y_k | X = x) = \begin{cases} 
1 - \delta_k, & \text{if } y_k = x, \\
\delta_k, & \text{if } y_k = \Delta, \\
0, & \text{otherwise}.
\end{cases} \tag{1}$$

Two disjoint sets of receivers are considered with regards to the receivers’ channel qualities, sets of weak and strong receivers located in an area with relatively bad and good network coverage, respectively. Thus, the channel condition of each strong receiver is statistically better than that of each weak receiver. Without loss of generality, the first $K_w$ receivers in $[K_w]$ are considered as the set of weak receivers, and the next $K_s = K - K_w$ receivers in $[K_w + 1 : K]$ form the set of strong receivers. To compensate, as depicted in Fig. 1, each weak receiver is equipped with a cache memory of size $nM$ bits. The two following scenarios are studied in this paper:

1) **Homogeneous scenario:** It is assumed that all the receivers in the same set have the same erasure probability; that is, all the weak receivers have erasure probability $\delta_w$, and all the strong receivers have erasure probability $\delta_s$, where $\delta_s < \delta_w$:

$$\delta_k = \begin{cases} 
\delta_w, & \text{if } k \in [K_w], \\
\delta_s, & \text{if } k \in [K_w + 1 : K].
\end{cases} \tag{2}$$

2) **Heterogeneous scenario:** The strong and weak receivers can have arbitrary erasure probabilities, with the condition that any strong receivers has a lower erasure probability than any weak receiver. Without loss of generality, we enumerate the receivers in the order of improving channel quality, that is, we have $\delta_1 \geq \delta_2 \geq \cdots \geq \delta_K$.

The transmission is performed in two phases $[6]$. It starts with the placement phase which takes place during the off-peak traffic period, and the caches of the weak receivers are filled by the server without knowing their demands. Thus, only the weak receivers take part in the placement phase, and the contents of the cache of receiver $k$, for $k \in [K_w]$, at the end of this phase is denoted by $Z_k$. Since the demands are not known during the placement phase, $Z_k$ does not depend on any specific demand combination, and it is instead a function of the library $\mathcal{W}$,
for $k \in [K_w]$. The caching function at receiver $k \in [K_w]$ is given by
\[
\phi_k : \left(\left\lfloor 2^{nR} \right\rfloor \right)^N \rightarrow \left(\left\lfloor 2^{nM} \right\rfloor \right),
\] (3)
which maps the entire library $W$ to the cache content $Z_k$, i.e., $Z_k = \phi_k (W_1, ..., W_N)$. It is to be noted that, since the placement phase is performed over a low-congestion period, it is assumed that no erasure occurs during this phase. The delivery phase follows once the demands of the receivers are revealed to the server, which transmits a length-$n$ codeword $X^n$ over the erasure broadcast channel. For a demand vector $(d_1, ..., d_K)$, a coded delivery function
\[
\psi : \left(\left\lfloor 2^{nR} \right\rfloor \right)^N \times [N]^K \rightarrow X^n
\] (4)
generates a common message $X^n$, where $n$ corresponds to the blocklength, as a function of entire library $W$ and the receiver demands, i.e., $X^n = \psi(W_1, ..., W_N, d_1, ..., d_K)$, to be delivered over the erasure broadcast channel. Each receiver $k \in [K]$ observes $Y^n_k$ according to (1). For a specific demand combination $(d_1, ..., d_K)$, each weak receiver $k \in [K_w]$ tries to decode $W_{d_k}$ from its output $Y^n_k$ along with the content available locally in its cache and the demand vector, utilizing the decoding function
\[
\mu_k : Y^n \times \left(\left\lfloor 2^{nM} \right\rfloor \right) \times [N]^K \rightarrow \left(\left\lfloor 2^{nR} \right\rfloor \right),
\] (5)
i.e., the reconstructed file by each weak receiver $k \in [K_w]$ is
\[
\hat{W}_{d_k} = \mu_k (Y^n_k, Z_k, d_1, ..., d_K).
\] (6)
On the other hand, each strong receiver $k \in [K_w + 1: K]$ aims to reconstruct $W_{d_k}$ from its output $Y^n_k$ and demand vector $(d_1, ..., d_K)$ through the decoding function
\[
\mu_k : Y^n \times [N]^K \rightarrow \left(\left\lfloor 2^{nR} \right\rfloor \right),
\] (7)
which generates the reconstructed file
\[
\hat{W}_{d_k} = \mu_k (Y^n_k, d_1, ..., d_K),
\] (8)
at each strong receiver $k \in [K_w + 1: K]$.

**Definition 1.** An error occurs if $\hat{W}_{d_k} \neq W_{d_k}$ for any $k \in [K]$, where $\hat{W}_{d_k}$ is the reconstructed
file at receiver $k$, and the probability of error is defined as

$$P_e \triangleq \max_{(d_1, \ldots, d_K) \in [N]^K} \Pr \left\{ \bigcup_{k=1}^{K} \{ \hat{W}_{d_k} \neq W_{d_k} \} \right\}. \quad (9)$$

**Definition 2.** A memory-rate pair $(M, R)$ is said to be achievable, if for $n$ large enough, there exists a caching function (3), coded delivery function (4), and decoding functions (5) and (7) at weak and strong receivers, respectively, such that for every $\varepsilon > 0$, $P_e < \varepsilon$.

**Definition 3.** For a given cache size $M$ at the weak receivers, the capacity of the network is defined as

$$C(M) \triangleq \sup \{ R : (M, R) \text{ is achievable} \}. \quad (10)$$

We note that, the capacity of the above caching network remains an open problem even when the delivery channel is an error-free shared bit pipe except for the uncoded cache placement phase [14]. Here, our goal is to provide an achievable coding scheme that improves upon the state-of-the-art.

The system model considered in this paper is inspired by those studied in [36]–[38]. It is reasonable that cache memories are placed at receivers at which the coverage is relatively weak. Indeed, it is shown in [38] that placing cache memories at the strong receivers, which already have good coverage, results in a lower capacity. This is mainly due to the definition of the capacity in this framework. Note that, the capacity here characterizes the highest rate of the messages to be delivered to all the receivers in the network. Since a symmetry is imposed across the receivers regarding their requests and file delivery, the system performance is determined by the worst receivers. Therefore, the goal of the cache placement should be to improve the performance of the weak receivers to increase the network capacity. Accordingly, by equipping weak receivers with cache memories, and exploiting the coding scheme proposed in this paper, the strong receivers also benefit from the presence of cache memories at the weak receivers.

Following well-known results from multi-user information theory are included here for completeness as they will be instrumental in deriving our results later in the paper.

**Proposition 1.** The capacity region of a packet-erasure broadcast channel with $K$ receivers, where file $W_i$ with rate $R_i$ is targeted for receiver $i$ with erasure probability $\delta_i$, for $i = 1, \ldots, K$,
is the closure of the set of non-negative rate tuples \((R_1, \ldots, R_K)\) that satisfy

\[
\sum_{k=1}^{K} \frac{R_k}{(1 - \delta_k) F} \leq 1, \quad (11)
\]

where \(F\) denotes the length of the binary channel input.

**Proposition 2.** Consider a packet-erasure broadcast channel with two disjoint sets of receivers \(S_1\) and \(S_2\), where the channels of the receivers in set \(S_i\) have erasure probability \(\delta_i\), for \(i = 1, 2\). The transmitter aims to send content \(A_i\) at rate \(R_i\) to the receivers in set \(S_i\), for \(i = 1, 2\). If content \(A_2\) is known by all the receivers in set \(S_1\) as side information, then using the joint encoding scheme proposed in [46], rate pairs \((R_1, R_2)\) satisfying the following conditions can be achieved [38]

\[
R_1 \leq (1 - \delta_1) F, \quad (12)
\]
\[
R_1 + R_2 \leq (1 - \delta_2) F, \quad (13)
\]

which is equivalent to

\[
\max \left\{ \frac{R_1}{(1 - \delta_1) F}, \frac{R_1 + R_2}{(1 - \delta_2) F} \right\} \leq 1. \quad (14)
\]

We note that Proposition 2 is a simple application of the joint source-channel coding scheme in [46], where we can consider \((A_1, A_2)\) as the common source to be broadcasted to all the receivers, and \(A_2\) as the side information available at the receivers in set \(S_1\).

For notational convenience, in the rest of the paper we use

\[
\text{JE} \left( (A_1)_{S_1}, (A_2)_{S_2} \right) \quad (15)
\]

to represent the transmission of content \(A_1\) to the receivers in \(S_1\), and content \(A_2\) to the receivers in set \(S_2\) through joint encoding, where \(S_1 \cap S_2 = \emptyset\), and \(A_2\) is available at all the receivers in \(S_1\) as side information.

**III. Achievable Rate-Memory Pairs: Homogeneous Scenario**

A coding scheme as well as an information theoretic upper bound on the capacity of the above system model were proposed in [38] for the homogeneous scenario. Here, we present a new coding scheme, called the successive cache-channel coding (SCC) scheme, that generalizes and improves upon the one in [38]. We present the \((M, R)\) pairs achieved by this scheme in
Theorem 1 below for the homogeneous scenario. The details of the scheme will be presented in Section IV.

Theorem 1. Consider cache-aided content delivery over the homogeneous packet-erasure broadcast channel with $N$ files, and $K_w$ weak and $K_s$ strong receivers with erasure probabilities $\delta_w$ and $\delta_s > \delta_w$, respectively, where each weak receiver has access to a cache of capacity $M$. Memory-rate pairs $(M_{(p,q)}, R_{(p,q)})$ are achievable for any $p \in [0 : K_w]$ and $q \in [p : K_w]$, where

$$R_{(p,q)} \triangleq \frac{F \sum_{i=p}^{q} (\gamma(p,i))}{1 - \delta_w \sum_{i=p}^{q} \left( \frac{K_w - i}{i+1} \gamma(p,i) \right) + \frac{K_s}{1 - \delta_s}},$$

(16a)

$$M_{(p,q)} \triangleq \frac{N \sum_{i=p}^{q} i \gamma(p,i)}{K_w \sum_{i=p}^{q} \gamma(p,i)},$$

(16b)

with $\gamma(p,i)$ defined as follows:

$$\gamma(p,i) \triangleq \frac{(K_w)}{p^i} \left( \frac{1 - \delta_s}{1 - \delta_w} - 1 \right)^{i-p}, \text{ for } i = p, \ldots, q.$$

(16c)

The upper convex hull of these $(K_w + 1)(K_w + 2)/2$ memory-rate pairs can also be achieved through memory-sharing.

Next, we compare the achievable rate of the SCC scheme for the homogeneous scenario with the scheme of [38], which we will refer to as the STW scheme. In Fig. 2, the achievable memory-rate trade-off of the SCC scheme is compared with the STW scheme when $K_w = 2$, $K_s = 2$, $N = 20$, $F = 10$, $\delta_s = 0.2$, and $\delta_w = 0.8$. The upper bound on the capacity of the cache-aided packet-erasure broadcast channel derived in [38, Theorem 7] is also included. The SCC scheme outperforms the STW scheme due to the improved achievable memory-rate pair $(M_{(0,2)}, R_{(0,2)})$, which is not achievable by the STW scheme. This improvement can be extended to a wider range of cache sizes through memory-sharing, reducing the gap to the upper bound.

In Fig. 3 we plot the achievable rates for both schemes in the homogeneous scenario with $K_w = 7$, $K_s = 10$, $N = 50$, $F = 20$, $\delta_s = 0.2$, and $\delta_w = 0.9$. The upper bound on the capacity derived in [38, Theorem 7] is also included. Observe that, for relatively small cache sizes, where the best memory-rate trade-off is achieved by time-sharing between $(M_{(0,0)}, R_{(0,0)})$
Fig. 2. Capacity characterization for the homogeneous scenario with $K_w = 2$, $K_s = 2$, $N = 20$, $F = 10$, $\delta_s = 0.2$, and $\delta_w = 0.8$.

and $(M_{(0,1)}, R_{(0,1)})$, and for relatively large cache sizes, where the best memory-rate trade-off is achieved by time-sharing between $(M_{(6,7)}, R_{(6,7)})$ and $(M_{(7,7)}, R_{(7,7)})$, both schemes achieve the same rate; however, the proposed SCC scheme achieves a higher rate than STW scheme for intermediate cache sizes, and reduces the gap to the upper bound. For a cache capacity of $M = 30$, the SCC scheme provides approximately 15% increase in the achievable rate compared to the STW scheme.

In Fig. 4, the achievable rates of the SCC and STW schemes in the homogeneous scenario are compared for different values of $\delta_w$. System parameters considered in this comparison are $K_w = 20$, $K_s = 10$, $N = 100$, $F = 50$, $\delta_s = 0.2$, and $\delta_w = 0.7, 0.8, 0.9$. Observe that, unlike the STW scheme, the performance of the SCC scheme does not deteriorate notably for intermediate and relatively high cache capacities by increasing $\delta_w$, i.e., having worse channel qualities for the weak receivers. This is because the SCC scheme successfully exploits the available cache capacities, and there is little to gain from decreasing $\delta_w$ when $M$ is sufficiently large. Moreover, the superiority of the SCC scheme over the STW scheme is more pronounced for higher values of $\delta_w$, in which case, exploiting the cache memories available at the weak receivers more effectively
IV. THE SUCCESSIVE CACHE-CHANNEL CODING (SCC) SCHEME FOR THE HOMOGENEOUS SCENARIO

Before presenting the SCC scheme for the general homogeneous scenario, the main ideas behind this scheme are illustrated on an example.

For notational convenience, \( \binom{K_w}{i} \) different subsets of set \([K_w]\) each of size \(i\) are represented by \(S_1^{(i)}, S_2^{(i)}, \ldots, S_{\binom{K_w}{i}}^{(i)}\), i.e.,

\[
S_l^{(i)} \subset [K_w]; \quad |S_l^{(i)}| = i, \quad \text{for } l = 1, \ldots, \binom{K_w}{i},
\]

where for a set \(S\), \(|S|\) represents the cardinality of set \(S\).

Example 1. Consider the cache-aided packet-erasure broadcast network depicted in Fig. I for the homogeneous scenario with \(K_w = 3\) weak and \(K_s = 2\) strong receivers. In the following, we investigate the achievable memory-rate pair \((M_{(0,2)}, R_{(0,2)})\), which corresponds to the memory-
rate pair in (16) for \( p = 0 \) and \( q = 2 \), for an arbitrary demand combination \((d_1, ..., d_5)\). Each file \( W_d, d \in [N] \), is divided into three subfiles \( W_d^{(0)}, W_d^{(1)} \) and \( W_d^{(2)} \), where subfile \( W_d^{(i)} \) has a rate of \( R^{(i)} \), for \( i = 0, 1, 2 \), given by

\[
R^{(i)} \triangleq \frac{\gamma(0,i)}{\sum_{j=0}^{2} \gamma(0,j)} R,
\]

where \( \gamma(0,i) \) is as defined in (16c). Therefore, we have \( \sum_{i=0}^{2} R^{(i)} = R \).

In the placement phase, subfiles \( W_1^{(i)}, ..., W_N^{(i)} \) are placed in the caches of \( K_w = 3 \) weak receivers through the procedure proposed in [6, Algorithm 1] for cache capacity \( iN/K_w \), for \( i = 0, 1, 2 \). In this cache placement procedure, each subfile \( W_d^{(i)} \) is first divided into \( \binom{3}{i} \) non-overlapping pieces, each at a rate of \( R^{(i)}/\binom{3}{i} \).

\[
W_d^{(i)} = \left( W_{d,S_1^{(i)}}^{(i)}, W_{d,S_2^{(i)}}^{(i)}, ..., W_{d,S_{\binom{3}{i}}^{(i)}}^{(i)} \right), \forall d \in [N], \forall i \in [0:2],
\]

Fig. 4. Capacity characterization for the homogeneous scenario with \( K_w = 20, K_s = 10, N = 100, F = 50, \delta_s = 0.2 \), and different values for \( \delta_w \) given by \( \delta_w = 0.7, 0.8, 0.9 \).
For the example under consideration, we have, \( \forall d \in [N] \),

\[
W_d^{(0)} = \left\{ W_{d,\{0\}} \right\},
\]

\[
W_d^{(1)} = \left\{ W_{d,\{1\}}, W_{d,\{2\}}, W_{d,\{3\}} \right\},
\]

\[
W_d^{(2)} = \left\{ W_{d,\{1,2\}}, W_{d,\{1,3\}}, W_{d,\{2,3\}} \right\}.
\]

The piece \( W_{d,S_l}^{(i)} \) is placed in the cache of each receiver \( k \in S_l^{(i)} \), for \( l = 1, \ldots, {i \choose 3} \). Therefore, the cache contents of the weak receivers after the placement phase are as follows:

\[
Z_1 = \bigcup_{d \in [N]} \left( W_{d,\{1\}}, W_{d,\{1,2\}}, W_{d,\{1,3\}} \right),
\]

\[
Z_2 = \bigcup_{d \in [N]} \left( W_{d,\{2\}}, W_{d,\{1,2\}}, W_{d,\{2,3\}} \right),
\]

\[
Z_3 = \bigcup_{d \in [N]} \left( W_{d,\{3\}}, W_{d,\{1,3\}}, W_{d,\{2,3\}} \right),
\]

where the required cache capacity for each weak receiver is:

\[
M = \left( \frac{R^{(1)}}{3} + \frac{2R^{(2)}}{3} \right) N = \frac{\gamma(0,1) + 2\gamma(0,2)}{3} \sum_{j=0}^{2} \gamma(0,j) NR.
\]

The server tries to satisfy all the demands in the delivery phase by sending 4 distinct messages in an orthogonal fashion, i.e., by time division multiplexing, where the codewords corresponding to the \( i \)-th message, \( i = 1, \ldots, 4 \), are of length \( \beta_i n \) channel uses, such that \( \sum_{i=1}^{4} \beta_i = 1 \).

The first message is targeted only for the weak receivers, and its goal is to deliver the missing subfiles of file \( W_{d_k}^{(2)} \) to receiver \( k \in [K_w] \), that is, having received this message, each weak receiver should be able to decode the third subfile of its desired file. Exploiting the delivery phase algorithm of [6, Algorithm 1] for cache capacity \( 2N/K_w \), the following content is sent to the weak receivers:

\[
\left\{ W_{d_1,\{2,3\}}^{(2)} \oplus W_{d_2,\{1,3\}}^{(2)} \oplus W_{d_3,\{1,2\}}^{(2)} \right\},
\]

where \( \oplus \) denotes the bitwise XOR operation. Observe that the content (23) has a rate of \( R^{(2)}/3 \). The capacity region of the standard packet erasure broadcast channel presented in Proposition...
1 suggests that all the weak receivers can decode (23), for \( n \) large enough, if
\[
\frac{R^{(2)}}{3(1 - \delta_w)} F \leq \beta_1.
\] (24)

Having received (23), each receiver \( k \in [K_w] \) can recover its missing piece \( W^{(2)}_{d_k,3} \) of subfile \( W^{(2)}_{d_k} \) using its cache contents \( Z_k \). Thus, together with its cache content, each receiver \( k \in [K_w] \) can recover subfile \( W^{(2)}_{d_k} \).

Through the second message of the delivery phase, the server simultaneously delivers subfile \( W^{(2)}_{d_k} \) to each strong receiver \( k \in [K_w + 1 : K] \), and the missing bits of subfile \( W^{(1)}_{d_k} \) to each weak receiver \( k \in [K_w] \). The content targeted to the weak receivers delivered by using the delivery phase algorithm proposed in [6, Algorithm 1] corresponding to cache capacity \( N/K_w \); that is, the contents
\[
\left\{ W^{(1)}_{d_1,\{2\}} \oplus W^{(1)}_{d_2,\{1\}}, W^{(1)}_{d_1,\{3\}} \oplus W^{(1)}_{d_3,\{1\}}, W^{(1)}_{d_2,\{3\}} \oplus W^{(1)}_{d_3,\{2\}} \right\}
\] (25)

are transmitted to the weak receivers. Therefore, the goal is to deliver \( W^{(2)}_{d_k} \) to strong receiver \( k \), while delivering the contents in (25) to the weak receivers in parallel. The transmission is performed over 3 sub-messages, transmitted over orthogonal time periods. With the first sub-message,
\[
\left\{ W^{(2)}_{d_4,\{1,2\}}, W^{(2)}_{d_5,\{1,2\}} \right\}
\] (26)

are delivered to strong receivers 4 and 5, while
\[
\left\{ W^{(1)}_{d_1,\{2\}} \oplus W^{(1)}_{d_2,\{1\}} \right\}
\] (27)

is delivered to weak receivers 1 and 2. Recall that, both receivers 1 and 2 already have the contents in (26) in their caches, which act as side information. Hence, the transmission can be carried out by using the joint encoding scheme of [46]. Using the notation introduced in (15), the first sub-message of the second message of the delivery phase is obtained by performing the following joint encoding:
\[
JE \left( \left( W^{(1)}_{d_1,\{2\}} \oplus W^{(1)}_{d_2,\{1\}} \right)_{\{1,2\}} \right), \left( W^{(2)}_{d_4,\{1,2\}}, W^{(2)}_{d_5,\{1,2\}} \right)_{\{4,5\}} \right).
\] (28)

With the second sub-message,
\[
\left\{ W^{(2)}_{d_4,\{1,3\}}, W^{(2)}_{d_5,\{1,3\}} \right\}
\] (29)
which are available in the caches of receivers 1 and 3 as side information, are delivered to receivers 4 and 5, while
\[ \left\{ W_{d_1,\{3\}}^{(1)} \oplus W_{d_3,\{1\}}^{(1)} \right\} \]

is delivered to receivers 1 and 3. Hence, the second sub-message is obtained by joint encoding:
\[ \text{JE} \left( \left( W_{d_1,\{3\}}^{(1)} \oplus W_{d_3,\{1\}}^{(1)} \right)_{\{1,3\}}, \left( W_{d_4,\{1,3\}}^{(2)}, W_{d_5,\{1,3\}}^{(2)} \right)_{\{4,5\}} \right). \]
Finally, with the third sub-message, receivers 4 and 5 are delivered
\[ \left\{ W_{d_4,\{2,3\}}^{(2)}, W_{d_5,\{2,3\}}^{(2)} \right\}, \]
and receivers 2 and 3, while having (32) as side information, are delivered
\[ \left\{ W_{d_2,\{3\}}^{(1)} \oplus W_{d_3,\{2\}}^{(1)} \right\}, \]
by joint encoding as follows:
\[ \text{JE} \left( \left( W_{d_2,\{3\}}^{(1)} \oplus W_{d_3,\{2\}}^{(1)} \right)_{\{2,3\}}, \left( W_{d_4,\{2,3\}}^{(2)}, W_{d_5,\{2,3\}}^{(2)} \right)_{\{4,5\}} \right). \]
Observe that, with each sub-message, contents of rate \(2R^{(2)}/3\), available at the weak receivers as side information, are delivered to the strong receivers; while, simultaneously, content at rate \(R^{(1)}/3\) is transmitted to the weak receivers. Overall, \(W_{d_4}^{(2)}, W_{d_5}^{(2)}\) and the contents in (25) with a total rate of \(2R^{(2)}\) and \(R^{(1)}\) are delivered to the strong and weak receivers, respectively, through three different sub-messages by using the joint encoding scheme of [46] that exploits the side information at the weak receivers. Using the achievable rate region of the joint encoding scheme for the packet erasure channels stated in Proposition [2] \(W_{d_4}^{(2)}, W_{d_5}^{(2)}\) and the contents in (25) can be simultaneously decoded by the strong and weak receivers, respectively, for \(n\) large enough, if
\[ \max \left\{ \frac{R^{(1)}}{(1 - \delta_w) F}, \frac{R^{(1)} + 2R^{(2)}}{(1 - \delta_s) F} \right\} \leq \beta_2. \]
From the expressions for \(R^{(1)}\) and \(R^{(2)}\) in (18), it can be verified that the two terms in the maximization in (35) are equal. Thus, the condition in (35) can be simplified as
\[ \frac{R^{(1)}}{(1 - \delta_w) F} \leq \beta_2. \]
Observe that, receiver 1 can obtain contents $W^{(1)}_{d_1,\{2\}}$ and $W^{(1)}_{d_1,\{3\}}$ after receiving (27) and (30), respectively. Receiver 2 can recover $W^{(1)}_{d_2,\{1\}}$ and $W^{(1)}_{d_2,\{3\}}$ after receiving (27) and (33), respectively. Finally, decoding (30) and (33) allows receiver 3 to recover $W^{(1)}_{d_3,\{1\}}$ and $W^{(1)}_{d_3,\{2\}}$, respectively. Thus, having received the second message, each weak receiver $k \in [K_w]$ has received all the missing bits of subfile $W^{(1)}_{d_k}$ of its request, while each strong receiver $l \in [K_w + 1 : K]$ has recovered subfile $W^{(2)}_{d_l}$ of its request.

The third message of the delivery phase is designed to deliver $W^{(1)}_{d_k}$ to each strong receiver $k \in [K_w + 1 : K]$, and $W^{(0)}_{d_k}$ is delivered to each weak receiver $k \in [K_w]$. Third message is also divided into three sub-messages, transmitted over orthogonal time periods. In the first period,

\[
\left\{ W^{(1)}_{d_4,\{1\}}, W^{(1)}_{d_5,\{1\}} \right\},
\]  

which are available locally at receiver 1 as side information, are delivered to receivers 4 and 5, while

\[
\left\{ W^{(0)}_{d_1,\{0\}} \right\}
\]  

is delivered to receiver 1. Therefore, the first sub-message is generated by joint encoding:

\[
JE\left( \left( W^{(0)}_{d_1,\{0\}} \right)_{\{1\}}, \left( W^{(1)}_{d_4,\{1\}}, W^{(1)}_{d_5,\{1\}} \right)_{\{4,5\}} \right).
\]  

In the second period, the goal is to send

\[
\left\{ W^{(1)}_{d_4,\{2\}}, W^{(1)}_{d_5,\{2\}} \right\}
\]  

to receivers 4 and 5, while delivering

\[
\left\{ W^{(0)}_{d_2,\{0\}} \right\}
\]  

to receiver 2, which has access to (40) as side information. The second sub-message is generated by joint encoding:

\[
JE\left( \left( W^{(0)}_{d_2,\{0\}} \right)_{\{2\}}, \left( W^{(1)}_{d_4,\{2\}}, W^{(1)}_{d_5,\{2\}} \right)_{\{4,5\}} \right).
\]  

In the last period, the sub-message obtained through the following joint encoding is transmitted:

\[
JE\left( \left( W^{(0)}_{d_3,\{0\}} \right)_{\{3\}}, \left( W^{(1)}_{d_4,\{3\}}, W^{(1)}_{d_5,\{3\}} \right)_{\{4,5\}} \right),
\]
where
\[
\left\{ W_{d_4,\{3\}}, W_{d_5,\{3\}} \right\}
\] (44)
are targeted for strong receivers 4 and 5, while receiver 3 decodes
\[
\left\{ W_{d_3,\{0\}} \right\},
\] (45)
having (44) as side information. From (39), (42) and (43), in each period of this part of the delivery phase, content at rate \(R^{(0)}\) is targeted for the weak receivers, while content at rate \(2R^{(1)}/3\), available locally at the weak receivers, is aimed for the strong receivers. Therefore, through joint encoding over three periods, contents with a total rate of \(3R^{(0)}\) are delivered to the weak receivers, while the strong receivers receive contents at a total rate of \(2R^{(1)}\). According to Proposition 2, all the weak and strong receivers can decode their messages, for \(n\) large enough, if
\[
\max\left\{ \frac{3R^{(0)}}{(1 - \delta_w) F}, \frac{3R^{(0)} + 2R^{(1)}}{(1 - \delta_s) F} \right\} \leq \beta_3.
\] (46)
Again, from the expressions of \(R^{(0)}\) and \(R^{(1)}\) in (18), it can be verified that (46) can be simplified as
\[
\frac{3R^{(0)}}{(1 - \delta_w) F} \leq \beta_3.
\] (47)
Therefore, if (47) holds, each weak receiver \(k \in [K_w]\) obtains \(W_{d_k,\{0\}}\), while each strong receiver \(l \in [K_w + 1 : K]\) obtains \(W_{d_l}^{(1)}\). Thus, with the third message of the delivery phase, the demands of the weak receivers are fully satisfied.

the last and fourth message of the delivery phase is generated only for the strong receivers with the goal of delivering them the missing bits of their demands, in particular, subfile \(W_{d_k,\{0\}}^{(0)}\), of rate \(R^{(0)}\), is delivered to each strong receiver \([K_w + 1 : K]\). From the capacity region of the standard erasure broadcast channel given in Proposition 1, each receiver \(k \in [K_w + 1 : K]\) can decode \(W_{d_k,\{0\}}^{(0)}\) successfully for \(n\) sufficiently large, if
\[
\frac{2R^{(0)}}{(1 - \delta_s) F} \leq \beta_4.
\] (48)
Combining (24), (36), (47), (48), and the fact that \(\sum_{i=1}^{4} \beta_i = 1\), we have the condition
\[
\frac{R^{(2)}}{3(1 - \delta_w) F} + \frac{R^{(1)}}{(1 - \delta_w) F} + \frac{3R^{(0)}}{(1 - \delta_w) F} + \frac{2R^{(0)}}{(1 - \delta_s) F} \leq 1.
\] (49)
By replacing $R^{(i)}$ with the expressions from (18), for $i = 0, 1, 2$, and using the fact that $\gamma(0, 0) = 1$, (49) corresponds to

$$R \leq \frac{2}{1-\delta_w} \left( 3 + \gamma(0, 1) + \frac{1}{2} \gamma(0, 2) \right) + \frac{2}{1-\delta_w}. \quad (50)$$

Observe that, the term in the right hand side of inequality (50) is $R(0, 2)$, which is given by (16b). The cache size of each weak receiver exploited by our coding scheme is given by (22), and we have $M = M_{(0,2)}$, where $M_{(0,2)}$ is defined as in (16a). Thus, the memory-rate pair $(M_{(0,2)}, R_{(0,2)})$ given by (16) is achievable for the setting under consideration. \hfill \Box

Next, we present the SCC scheme for a general setting of the homogeneous scenario, achieving the memory-rate pair $(M_{(p,q)}, R_{(p,q)})$ given by (16), for any $p \in [0:K_w]$, and $q \in [p:K_w]$. For a given $(p,q)$ pair, where $p \in [0:K_w]$, and $q \in [p:K_w]$, each file $W_d$, $d \in [N]$, is divided into $(q-p+1)$ non-overlapping subfiles, represented by

$$W_d = \left( W_d^{(p)}, \ldots, W_d^{(q)} \right), \quad (51)$$

where subfile $W_d^{(i)}$, for $i \in [p:q]$, has a rate of

$$R^{(i)} = \frac{\gamma(p,i)}{\sum_{j=p}^{q} \gamma(p,j)} R, \quad (52)$$

such that $\sum_{i=p}^{q} R^{(i)} = R$.

In the sequel, the placement and delivery phases of the SCC scheme are explained, and the achievability of the memory-rate pair in (16) is proven.

A. Placement Phase

In the placement phase, for each set of subfiles $W_1^{(i)}, \ldots, W_N^{(i)}$ a cache placement procedure, corresponding to the one proposed in [6, Algorithm 1] for a cache capacity of $iN/K_w$, is performed, $\forall i \in [p:q]$; that is, each subfile $W_d^{(i)}$ is partitioned into $\binom{K_w}{i}$ independent equal-rate pieces,

$$W_d^{(i)} = \left( W_{d,S_1^{(i)}}^{(i)}, W_{d,S_2^{(i)}}^{(i)}, \ldots, W_{d,S_{\binom{K_w}{i}}^{(i)}}^{(i)} \right), \forall d \in [N], \forall i \in [p:q]. \quad (53)$$
The piece $W_{d,S}^{(i)}$ of rate $R^{(i)}/(K_w^i)$ is cached by receivers $k \in S_l^{(i)}$, for $l = 1, \ldots, (K_w^i)$. Thus, the content placed in the cache of each weak receiver $k \in [K_w]$ is given by

$$Z_k = \bigcup_{d \in [N]} \bigcup_{i \in [p:q]} \bigcup_{l \in [K_w^i]} W_{d,S}^{(i)}.$$

(54)

Accordingly, $(K_w - 1)_{i=1}^{q}$ pieces, each of rate $R^{(i)}/(K_w^i)$, corresponding to each subfile $W_{d}^{(i)}$ are cached by each weak receiver $k \in [K_w]$ which requires a total cache capacity of

$$M = N \sum_{i=p}^{q} (K_w - 1) R^{(i)} / (K_w^i) = N K_w \sum_{i=p}^{q} i R^{(i)} = \frac{N \sum_{i=p}^{q} i \gamma (p, i)}{K_w \sum_{i=p}^{q} \gamma (p, i)} R.$$

(55)

An example of this cache placement phase for $K_w = 3$, $p = 0$, and $q = 2$ is given in (21).

B. Delivery Phase

In the delivery phase, the goal is to satisfy all the demands for an arbitrary demand combination $(d_1, \ldots, d_K)$. The delivery phase consists of $(q - p + 2)$ different messages, transmitted over orthogonal time periods, where the codewords of the $i$-th message are of length $\beta_i n$ channel uses, for $i = 1, \ldots, q - p + 2$, such that $\sum_{i=1}^{q-p+2} \beta_i = 1$.

In the first message of the delivery phase, only the set of weak receivers are targeted, and the server delivers the missing bits of subfile $W_{d_k}^{(q)}$ to each receiver $k \in [K_w]$. This part of the delivery phase is performed using the coded delivery procedure proposed in [6, Algorithm 1] for cache capacity $qN/K_w$. The first message is delivered over $(K_w q + 1)$ orthogonal periods, where the coded content

$$X_l^{(q)} = \bigoplus_{k \in S_l^{(q+1)} \setminus \{k\}} W_{d_k,S_l^{(q+1)}}^{(q+1)}(k),$$

(56)

which has a rate of $R^{(q)}/(K_w q)$, is delivered to receivers $k \in S_l^{(q+1)}$ in $l$-th period, for $l = 1, \ldots, (K_w q + 1)$. Since a total of $(K_w q + 1)$ XOR-ed contents are delivered through the first message, each of which corresponds to $X_l^{(q)}$ given in (56), the total rate delivered to the weak receivers with the first message of the delivery phase is

$$\frac{(K_w)}{(K_w q + 1)} R^{(q)} = \frac{K_w - q}{q + 1} R^{(q)}.$$

(57)
Observe that, for \( q = K_w \) in the placement phase presented in Section IV-A, all the weak receivers have access to all the bits of the subfiles \( W^{(q)}_d \), \( \forall d \); therefore, as it can also be seen from (57), no bits need to be delivered over the shared link.

Having received (56) along with their cache contents, each weak receiver \( k \in [K_w] \) can recover

\[
\left\{ W^{(q)}_{d_k, S} : S \subset [K_w] \setminus \{k\}, |S| = q \right\},
\]

i.e., all the missing bits of subfile \( W^{(q)}_d \) of its desired file. Using the capacity region of the standard packet erasure broadcast channel described in Proposition 1, (56) can be decoded with error probability close to 0 for \( n \) sufficiently large, if

\[
\frac{K_w-q}{q+1} R^{(q)} \leq (1 - \delta_w) F \leq \beta_1.
\]

An example of the contents delivered with the first message of the delivery phase for \( K_w = 3 \), and \( q = 2 \) is given in (23).

The delivery procedures performed from the second message up to message \( q - p + 1 \) are similar, as we explain next. With the \( (q - i + 1) \)-th message, the server delivers \( W^{(i+1)}_{d_k} \) to each strong receiver \( k \in [K_w + 1 : K] \), while at the same time, delivering the missing bits of subfile \( W^{(i)}_{d_k} \) to each weak receiver \( k \in [K_w] \), for \( i = q - 1, \ldots, 2 \). The \( (q - i + 1) \)-th message is delivered through \( (K_w)_{i+1} \) sub-messages over different orthogonal time periods. During each time period \( l \), using the coded delivery procedure proposed in [6, Algorithm 1], sub-message \( l \) delivers coded content

\[
X^{(i)}_l \Delta \equiv \left( \bigoplus_{k \in S^{(i+1)}_l \setminus \{k\}} W^{(i)}_{d_k, S^{(i+1)}_l \setminus \{k\}} \right)
\]

to weak receivers in set \( S^{(i+1)}_l \), and in parallel, contents

\[
\left\{ W^{(i+1)}_{d_{K_w+1}, S^{(i+1)}_l}, W^{(i+1)}_{d_{K_w+2}, S^{(i+1)}_l}, \ldots, W^{(i+1)}_{d_K, S^{(i+1)}_l} \right\}
\]

to strong receivers \( [K_w + 1 : K] \), for \( l = 1, \ldots, (K_w)_{i+1} \). Observe that, after receiving (60) correctly, each weak receiver \( k \in [K_w] \) can recover the missing bits of subfile \( W^{(i)}_{d_k} \) of its request, given

---

\^2For example, in the second message, subfile \( \left\{ W^{(q)}_{d_k} \right\} \) is delivered to each strong receiver \( k \in [K_w + 1 : K] \), and subfile \( \left\{ W^{(q-1)}_{d_k} \right\} \) to each weak receiver \( k \in [K_w] \). With the third message, subfile \( \left\{ W^{(q-1)}_{d_k} \right\} \) is delivered to each strong receiver \( k \in [K_w + 1 : K] \), and subfile \( \left\{ W^{(q-2)}_{d_k} \right\} \) to each weak receiver \( k \in [K_w] \), and so on so forth.
by
\[
\left\{ W^{(i)}_{d_k, S} : S \subset [K_w] \setminus \{k\}, |S| = i \right\}.
\] (62)

Note that, the weak receivers in set $S^{(i+1)}_l$ have access to contents in (61) targeted to the strong receivers. Therefore, for $i = q - 1, \ldots, p$, the $l$-th sub-message of message $(q - i + 1)$ is generated by performing the following joint encoding:
\[
\text{JE} \left( \left( X^{(i)}_l \right)_{S^{(i+1)}_l}, \left( W^{(i+1)}_{d_{K_w+1}, S^{(i+1)}_l}, \ldots, W^{(i+1)}_{d_{K}, S^{(i+1)}_l} \right)_{[K_w+1:K]} \right), \quad \text{for } l = 1, \ldots, (K_w^i)_{i+1},
\] (63)

which delivers a coded content at rate $R^{(i)}/(K_w^i)$ to the weak receivers and the contents of rate $K_s R^{(i+1)}/(K_w^i)$, available locally to the weak receivers, to the strong receivers. Since message $(q - i + 1)$ is delivered by $(K_w^i)_{i+1}$ sub-messages, transmitted over orthogonal slots through time-sharing, coded contents at a total rate of
\[
\frac{(K_w^i)_{i+1}}{K_w^i} R^{(i)} = \frac{K_w - i}{i + 1} R^{(i)}
\] (64)

are sent to the weak receivers, and the contents of total rate $K_s R^{(i+1)}$ known by the weak receivers as side information, are delivered to the strong receivers through message $(q - i + 1)$, for $i = q - 1, \ldots, p$. Capacity region of the joint encoding scheme presented in Proposition 2 suggests that each weak receiver and each strong receiver can decode their targeted contents after receiving message $(q - i + 1)$ with an error probability close to 0 for $n$ large enough, if, for $i = q - 1, \ldots, p$,
\[
\max \left\{ \frac{K_w - i}{i + 1} R^{(i)} (1 - \delta_w) F, \frac{K_w - i}{i + 1} R^{(i)} + K_s R^{(i+1)} (1 - \delta_s) F \right\} \leq \beta_{q-i+1}.
\] (65)

By choice of (52), it can be verified that the two terms in the maximization (65) are equal. So, (65) can be simplified to
\[
\frac{K_w - i}{i + 1} R^{(i)} (1 - \delta_w) F \leq \beta_{q-i+1}, \quad \text{for } i = q - 1, \ldots, p.
\] (66)

With messages $2, 3, \ldots, q - p + 1$ of the delivery phase, each strong receiver $k \in [K_w + 1 : K]$ can recover all subfiles $\left( W^{(q)}_{d_k}, W^{(q-1)}_{d_k}, \ldots, W^{(p+1)}_{d_k} \right)$; while each weak receiver $k \in [K_w]$ can recover all the subfiles $\left( W^{(q-1)}_{d_k}, W^{(q-2)}_{d_k}, \ldots, W^{(p)}_{d_k} \right)$. Together with the contents delivered with message 1, the demands of the weak receivers are all completely satisfied, while each strong receiver
k ∈ [K_w + 1 : K] only requires the subfile W_{d_k}^{(p)}.

In the last message of the delivery phase, only the strong receivers are targeted, and the content
\[ \left\{ W_{d_k}^{(p)} \right\} \]
\[ (67) \]
of rate \( R^{(p)} \) is delivered to each strong receiver \( k ∈ [K_w + 1 : K] \). According to (61), each strong receiver \( k ∈ [K_w + 1 : K] \) can decode (67) successfully, for \( n \) large enough, if
\[ \frac{K_s R^{(p)}}{(1 - \delta_s) F} ≤ \beta_{q-p+2}. \]
\[ (68) \]

Remark 1. It is to be noted that, the achievable STW scheme studied in [38] is a special case of the SCC scheme for the homogeneous scenario with \( q = p + 1 \). The delivery phase of the STW scheme is performed through delivering 3 messages in different orthogonal time periods. The SCC scheme utilizes a more flexible caching and coding scheme which applies a finer subpacketization compared to [38], together with the joint encoding scheme of [46], which is also used in [38] enabling all the receivers to exploit the cache capacities of the weak receivers.

C. Achievable Memory-Rate Pair Analysis (Proof of Theorem 1)

By combining (59), (66), and (68), we have
\[ \sum_{i=p}^{q} \left( \frac{K_w - i}{i+1} R^{(i)} \right) + \frac{K_s R^{(p)}}{(1 - \delta_s) F} ≤ \sum_{i=p-1}^{q} \beta_{q-i+1} = 1. \]
\[ (69) \]
Replacing \( R^{(i)} \), for \( i = p, ..., q \), with the expression in (52) results in
\[ R ≤ \frac{F \sum_{i=p}^{q} (\gamma(p,i))}{1 - \delta_w} \sum_{i=p}^{q} \left( \frac{K_w - i}{i+1} \gamma(p,i) \right) + \frac{K_s}{1 - \delta_s}, \]
\[ (70) \]
which together with the cache capacity of each weak receiver given in (55), the achievability of the memory-rate pair in (16) is proven.

V. ACHIEVABLE MEMORY-RATE PAIRS: HETEROGENEOUS SCENARIO

Next, we extend the SCC scheme to the heterogeneous scenario, in which we allow the weak and strong receivers to have distinct erasure probabilities. Our main result, which provides a general achievable memory-rate trade-off in this scenario is stated in the following theorem.
Theorem 2. For the heterogeneous scenario of the cache-aided packet erasure broadcast network with \( N \) files, and \( K_w \) weak and \( K_s \) strong receivers, where each weak receiver has access to a cache of capacity \( M \) bits, memory-rate pairs \( \left( \tilde{M}_{(p,q)}, \tilde{R}_{(p,q)} \right) \) are achievable for any \( p \in [0 : K_w] \), \( q \in [p : K_w] \), where

\[
\tilde{R}_{(p,q)} \Deltaq \frac{F \sum_{i=p}^{q} \tilde{\gamma} (p, i)}{\sum_{i=p}^{q} \left( \tilde{\gamma} (p, i) \right) \left( \frac{K_w-i}{K_w} \right) \sum_{j=1}^{K_w-i} \left( \frac{1-\delta_j}{K_w} \right)} + \frac{K_s}{K_w} \sum_{j=K_w+1}^{\infty} \frac{1}{1-\delta_j},
\]

(71a)

\[
\tilde{M}_{(p,q)} \Deltaq \frac{N \sum_{i=p}^{q} i \tilde{\gamma} (p, i)}{K_w \sum_{i=p}^{q} \tilde{\gamma} (p, i)},
\]

(71b)

with \( \tilde{\gamma} (p, i) \) defined as follows:

\[
\tilde{\gamma} (p, i) \Deltaq \begin{cases} \frac{K_w}{K_w-p} \prod_{j=p}^{i-1} \eta (j), & i = p+1, \ldots, q, \\ 1, & i = p, \end{cases}
\]

(71c)

where

\[
\eta (i) \Deltaq \frac{K_s}{(1-\delta_{K_w-i})} \sum_{j=K_w+1}^{\infty} \frac{1}{1-\delta_j} - 1, \quad \text{for } i = p, \ldots, q-1.
\]

(71d)

The upper convex hull of \((K_w+1)(K_w+2)/2\) memory-rate pairs given in (71) can also be achieved through memory-sharing.

For the heterogeneous scenario, the capacity of the network under consideration is upper bounded by

\[
\min_{\mathcal{S} \subseteq [K]} \left\{ F \left( \sum_{k \in \mathcal{S}} \frac{1}{1-\delta_k} \right)^{-1} + \frac{M}{N} |\mathcal{S} \cap [K_w]| \right\}.
\]

(72)

We observe from the figure that the gap between the achievable rate and the upper bound is negligible for a wide range of cache sizes.

In Fig. 5, the effect of \( K_w \) is considered for the heterogeneous scenario with the system parameters \( K = 15, N = 100, F = 10, \delta_k = 0.9-0.01k, \) for \( k = 1, \ldots, 5, \) and \( \delta_l = 0.2-0.01l, \) for \( l = 6, \ldots, 15. \) In this figure, the achievable rates are plotted with respect to the total cache capacity.
of $K_w M$ for 4 different values for the number of weak receivers in the system, $K_w = 4, 5, 10, 15$. Note that the erasure probabilities are set such that the first 5 receivers have significantly worse channels than the remaining 10 receivers. Note also that the parameter $K_w$ determines which receivers are provided with cache memories. As it can be seen, the setting with $K_w = 5$ achieves significantly higher rates over a wide range of total cache capacities compared to the other settings under consideration. If receiver 5, which has a relatively bad channel quality, is not provided with any cache memory, and only the first 4 receivers are equipped with cache memories, i.e., $K_w = 4$, the performance degrades except for very small values of total cache size. Furthermore, equipping receivers with relatively good channel qualities with cache memories deteriorates the performance of the system in terms of the achievable rate. This result confirms that it is more beneficial to allocate cache memories to the receivers with relatively worse channel qualities. The upper bound on the achievable rate for the setting with $K_w = 5$ and $K_s = 10$ is also included in this figure. We observe that the gap between the upper bound and the achievable rate for the same setting is negligible for a wide range of cache sizes.
A. Proof of Theorem 2

In order to extend the SCC scheme from the homogeneous to the heterogeneous scenario, the rate of the coded content targeted to a group of weak receivers for each message of the delivery phase is modified such that it can be decoded by the weakest receiver among the intended group of weak receivers.

In the following, for a given \((p, q)\) pair, where \(p \in [0 : K_w]\) and \(q \in [p : K_w]\), the SCC scheme achieving the memory-rate pair \((\tilde{M}_{(p, q)}, \tilde{R}_{(p, q)})\) is presented for the heterogeneous scenario. We divide file \(W_d, d \in [N]\), into \((q - p + 1)\) distinct subfiles:

\[
W_d = \left(\tilde{W}_d^{(p)}, ..., \tilde{W}_d^{(q)}\right),
\]

where \(W_d^{(i)}, i \in [p : q]\) is of rate

\[
\tilde{R}^{(i)} \triangleq \frac{\tilde{\gamma}(p, i)}{\sum_{j=p}^{q} \tilde{\gamma}(p, j)} R,
\]

such that \(\sum_{i=p}^{q} \tilde{R}^{(i)} = R\).

**Placement Phase:** Each subfile \(\tilde{W}_d^{(i)}\) is partitioned into \(( \binom{K_w}{i} )\) equal rate pieces, denoted by

\[
\tilde{W}_d^{(i)} = \left(\tilde{W}_{d,S_1^{(i)}}^{(i)}, \tilde{W}_{d,S_2^{(i)}}^{(i)}, ..., \tilde{W}_{d,S_{\binom{K_w}{i}}^{(i)}}^{(i)}\right), \forall d \in [N], \forall i \in [p : q],
\]

where each piece \(\tilde{W}_{d,S_l^{(i)}}^{(i)}\), \(l \in [\binom{K_w}{i}]\), is of rate \(\tilde{R}^{(i)}/( \binom{K_w}{i} )\). Similarly to the placement phase of the SCC for the homogeneous scenario presented in Section IV-A, each receiver \(k \in [K_w]\) caches piece \(\tilde{W}_{d,S_l^{(i)}}^{(i)}\) if \(k \in S_l^{(i)}\). Thus, the receiver \(k\)’s cache content is given by

\[
Z_k = \bigcup_{d \in [N]} \bigcup_{i \in [p : q]} \bigcup_{l \in [\binom{K_w}{i}]} \tilde{W}_{d,S_l^{(i)}}^{(i)},
\]

and the corresponding cache capacity is

\[
M = N \sum_{i=p}^{q} \binom{K_w-1}{i-1} \frac{\tilde{R}^{(i)}}{\binom{K_w}{i}} = N \frac{q}{K_w} \sum_{i=p}^{q} i \tilde{R}^{(i)} = \frac{N \sum_{i=p}^{q} i \tilde{\gamma}(p, i)}{K_w \sum_{i=p}^{q} \tilde{\gamma}(p, i)} R.
\]

**Delivery Phase:** During the delivery phase, \((q - p + 2)\) messages are delivered over orthogonal time periods, where the codewords of the \(i\)-th message are of length \(\tilde{\beta}_i n\) channel uses, for
\[ i = 1, \ldots, q - p + 2, \text{ such that } \sum_{i=1}^{q-p+2} \beta_i = 1. \]

The first message of the delivery phase is only targeted for the set of weak receivers, and the goal is to deliver the missing bits of subfile \( \tilde{W}_d^{(q)} \) to receiver \( k \in [K_w] \). It is to be noted that, for \( q = K_w \), based on the cache contents in (76), all the weak receivers have all the subfiles \( \tilde{W}_d^{(q)} \), \( \forall d \in [N] \); therefore, no message needs to be delivered. In the sequel, we consider \( q < K_w \).

The first message of the delivery phase is transmitted over \( \binom{K_w}{q+1} \) orthogonal time slots, where in each slot, a sub-message is delivered to a group of \( q + 1 \) weak receivers. Sub-message \( l \) is a codeword of length \( \beta_{1,l,n} \) channel uses and is targeted for the receivers in set \( S_l^{(q+1)} \), for \( l = 1, \ldots, \binom{K_w}{q+1} \), such that \( \sum_{l=1}^{\binom{K_w}{q+1}} \beta_{1,l} = \tilde{\beta}_1 \). Following the procedure proposed in \([6, \text{Algorithm 1}]\), the content delivered as sub-message \( l \) is given by

\[
\tilde{X}_l^{(q)} \triangleq \left( \bigoplus_{k \in S_l^{(q+1)}} \tilde{W}_d^{(q)} \right)_{d_k, S_l^{(q+1)} \setminus \{k\}} , \quad \text{for } l = 1, \ldots, \binom{K_w}{q+1}. \tag{78}
\]

After receiving \( \tilde{X}_l^{(q)} \) of rate \( \tilde{R}^{(q)}(q) / \binom{K_w}{q} \), each receiver \( k \in S_l^{(q+1)} \) can obtain \( \tilde{W}_d^{(q)} \) of its desired file, having access to \( Z_k \) given in (76). Thus, together with its cache content, receiver \( k \in [K_w] \) can get \( \tilde{W}_d^{(q)} \). The rate of \( \tilde{X}_l^{(q)} \) is adjusted such that the weakest receiver in \( S_l^{(q+1)} \) can decode it successfully, i.e.,

\[
\frac{\tilde{R}^{(q)}}{\binom{K_w}{q}} \leq \beta_{1,l} \left( 1 - \max_{j \in S_l^{(q+1)}} \{ \delta_j \} \right) F, \quad \text{for } l = 1, \ldots, \binom{K_w}{q+1}, \tag{79}
\]

which after summing over all the sets \( S_l^{(q+1)} \), for \( l = 1, \ldots, \binom{K_w}{q+1} \), one can obtain

\[
\frac{\tilde{R}^{(q)}}{\binom{K_w}{q}} \sum_{j=1}^{K_w-q} \binom{K_w-j}{q} \frac{1 - \delta_j}{1 - \delta_j} \leq \sum_{l=1}^{\binom{K_w}{q+1}} \beta_{1,l} F = \tilde{\beta}_1 F. \tag{80}
\]

The delivery techniques performed to send messages \( 2, 3, \ldots, q - p + 1 \) follow the same procedure. With message \( (q - i + 1) \) of length \( \tilde{\beta}_{q-i+1,n} \) channel uses, the server delivers the missing bits of subfile \( \tilde{W}_d^{(i)} \) to each weak receiver \( k \in [K_w] \), and \( \tilde{W}_d^{(i+1)} \) to each strong receiver \( k \in [K_w+1 : K] \), for \( i = q-1, \ldots, p \). Message \( (q-i+1) \) is delivered through \( \binom{K_w}{i+1} \) sub-messages, transmitted over orthogonal time periods, where sub-message \( l \) is of length \( \beta_{q-i+1,l,n} \) channel
uses, such that \( \sum_{l=1}^{(K_w)} \beta_{q-i+1,l} = \beta_{q-i+1} \). With \( l \)-th sub-message, the coded content

\[
\tilde{X}_l^{(i)} \triangleq \left( \bigoplus_{k \in S_l^{(i+1)}} \tilde{W}_d^{(i)} \right)_{d_k S_l^{(i+1)} \setminus \{k\}},
\]

is delivered to the weak receivers in set \( S_l^{(i+1)} \), while

\[
\left\{ \tilde{W}_d^{(i+1)}_{d_{K_w+1},S_l^{(i+1)}}, \ldots, \tilde{W}_d^{(i+1)}_{d_{K},S_l^{(i+1)}} \right\},
\]

is delivered to the strong receivers, for \( l = 1, \ldots, (K_w) \). Observe that, after receiving sub-message \( \tilde{X}_l^{(i)} \), each receiver \( k \in S_l^{(i+1)} \) can obtain \( \tilde{W}_d^{(i)} \) for \( l = 1, \ldots, (K_w) \), i.e., the missing bits of subfile \( \tilde{W}_d^{(i)} \) of its desired file. Note that, \( \tilde{X}_l^{(i)} \) is of rate \( \tilde{R}^{(i)} / (K_w) \), and the content in (82) has a total rate of \( K_s \tilde{R}^{(i+1)}/(K_w) \) and is known by each weak receiver in set \( S_l^{(i+1)} \). Therefore, the following content can be sent as \( l \)-th sub-message by performing joint encoding:

\[
\text{JE} \left( \left( \tilde{X}_l^{(i)} \right)_{S_l^{(i+1)}}, \left( \tilde{W}_d^{(i+1)} \right)_{d_{K_w+1},S_l^{(i+1)}}, \ldots, \left( \tilde{W}_d^{(i+1)} \right)_{d_{K},S_l^{(i+1)}} \right)_{[K_w+1,K]},
\]

However, to increase the efficiency of the delivery phase, the \( l \)-th sub-message is delivered via \( K_s \) orthogonal time periods, such that in the \( m \)-th period a codeword of length \( \beta_{q-i+1,l,m} \) channel uses is transmitted, where

\[
\sum_{m=1}^{K_s} \beta_{q-i+1,l,m} = \beta_{q-i+1,l}.
\]

Coded content \( \tilde{X}_l^{(i)} \), targeted for receivers in set \( S_l^{(i+1)} \), is divided into \( K_s \) non-overlapping equal-rate pieces

\[
\tilde{X}_l^{(i)} = \left( \tilde{X}_l^{(i)}_{l,1}, \ldots, \tilde{X}_l^{(i)}_{l,K_s} \right),
\]

and the delivery over the \( m \)-th time period is performed by joint encoding:

\[
\text{JE} \left( \left( \tilde{X}_l^{(i)} \right)_{S_l^{(i+1)}}, \left( \tilde{W}_d^{(i+1)} \right)_{d_{K_w+m},S_l^{(i+1)}} \right), \quad \text{for } m = 1, \ldots, K_s.
\]

Recall that, \( \tilde{W}_d^{(i+1)}_{d_{K_w+m},S_l^{(i+1)}} \), for \( m = 1, \ldots, K_s \), is known by each receiver in \( S_l^{(i+1)} \). Proposition 2 suggests that the codeword (85) can be decoded correctly by the intended receivers if

\[
\max \left\{ \frac{\tilde{R}^{(i)} / (K_s F / (K_w))}{1 - \max_{j \in S_l^{(i+1)}} \{ \delta_j \} F}, \frac{\tilde{R}^{(i)} / (K_s F / (K_w)) + \tilde{R}^{(i+1)} / (K_w)}{(1 - \delta_{K_w+m} F)} \right\} \leq \beta_{q-i+1,l,m}, \quad \text{for } m = 1, \ldots, K_s,
\]

(86)
where the rate of $\tilde{X}_{i,m}^{(i)}$, i.e., $\tilde{R}(i) / (K_w i)$, is limited by the weakest receiver in $S_i^{(i+1)}$. By summing up all the $K_w$ inequalities in (86), we have

$$
\max \left\{ \frac{\tilde{R}(i) / (K_w i)}{1 - \max_{j \in S_i^{(i+1)}} \{ \delta_j \}} F, \left( \frac{1}{K_w} \tilde{R}(i) + \frac{1}{K_w} \tilde{R}(i+1) \right) \sum_{m=1}^{K_w} \frac{1}{(1 - \delta_{K_w+m}) F} \right\} \leq \tilde{\beta}_{q-i+1,l}. \tag{87}
$$

By choice of (74), it can be determined that inequality (87) is equivalent to

$$
\frac{\tilde{R}(i) / (K_w i)}{1 - \max_{j \in S_i^{(i+1)}} \{ \delta_j \}} F \leq \tilde{\beta}_{q-i+1,l}, \tag{88}
$$

which holds for every $l \in [(K_w i_{i+1})]$, each corresponding to a set of weak receivers $S_i^{(i+1)}$. After summing over all values of $l$, one can obtain

$$
\frac{\tilde{R}(i)}{K_w} \sum_{j=1}^{K_w-i} \frac{K_w-j}{1 - \delta_j} \leq \sum_{l=1}^{K_w} \tilde{\beta}_{q-i+1,l} F = \tilde{\beta}_{q-i+1} F, \quad \text{for } i = q - 1, \ldots, p. \tag{89}
$$

Note that, with messages 2 to $q - p + 1$, each weak receiver $k \in [K_w]$ can obtain subfiles $(\tilde{W}_{d_k}^{(q-1)}, \tilde{W}_{d_k}^{(q-2)}, \ldots, \tilde{W}_{d_k}^{(p)})$, while each strong receiver $k \in [K_w + 1 : K]$ can decode subfiles $(\tilde{W}_{d_k}^{(q)}, \tilde{W}_{d_k}^{(q-1)}, \ldots, \tilde{W}_{d_k}^{(p+1)})$; therefore, together with message 1, the demand of weak receivers are fully satisfied. However, strong receiver $k \in [K_w + 1 : K]$ only requires to receive subfile $\tilde{W}_{d_k}^{(p)}$.

The last message delivers subfile $\tilde{W}_{d_k}^{(p)}$ to strong receiver $k \in [K_w + 1 : K]$ using the channel coding scheme for standard packet-erasure broadcast channels. According to Proposition 1, each receiver $k \in [K_w + 1 : K]$ can decode subfile $\tilde{W}_{d_k}^{(p)}$ of rate $\tilde{R}(p)$ correctly, if

$$
\tilde{R}(p) \sum_{j=K_w+1}^{K} \frac{1}{1 - \delta_j} \leq \tilde{\beta}_{q-p+2} F. \tag{90}
$$

By summing up inequalities (80), (89) and (90), we have

$$
\sum_{i=p}^{q} \left( \frac{\tilde{R}(i)}{K_w i} \sum_{j=1}^{K_w-i} \frac{1}{1 - \delta_j} + \tilde{R}(p) \sum_{j=K_w+1}^{K} \frac{1}{1 - \delta_j} \right) \leq \sum_{i=p-1}^{q} \tilde{\beta}_{q-i+1} F = F. \tag{91}
$$
Finally, by replacing $\tilde{R}^{(i)}$, for $i = p, \ldots, q$, with the expression in (74), one can obtain

\[
R \leq F \sum_{i=p}^{q} \tilde{\gamma}(p, i) - \sum_{i=p}^{q} \left( \frac{\tilde{\gamma}(p, i)}{K_{w}} \right) \sum_{j=1}^{K_{w}-i} \frac{1}{1-\delta_{j}} + \sum_{j=K_{w}+1}^{K_{s}} \frac{1}{1-\delta_{j}}
\]

(92)

which, together with the cache capacity of each weak receiver, $M$, given in (77), proves the achievability of the memory-rate pairs $(\tilde{M}_{(p,q)}, \tilde{R}_{(p,q)})$ in (71).

VI. CONCLUSIONS

We have studied the cache-enabled packet-erasure broadcast channel introduced in [38]. In this model a server delivers contents simultaneously to a set of receivers, each demanding one content from a finite library. We have two disjoint sets of receivers: a set of weak receivers placed in an area with relatively bad coverage, and a set of strong receivers with relatively good network coverage. To compensate for the weak channel, each weak receiver is equipped with a cache of equal size. Assuming that the receivers in the same set all have the same erasure probability, we have characterized a lower bound on the capacity which denotes the highest rate of the contents in the library that can be delivered reliably to all the receivers under any demand combination. We have proposed an improved joint caching and channel coding scheme that achieves a higher rate by finer subpacketization of the files in the library, which enables each receiver, even the strong receivers without a cache memory, to benefit further from the cache memories available at the weak receivers. We have then extended the proposed achievable scheme to the scenario of non-uniform erasure probabilities among the receivers within set of weak and strong receivers. This model illustrates that storage can be converted into spectral efficiency, benefiting the whole network, if it is placed strategically across the network, and exploited intelligently.

REFERENCES

[1] L. W. Dowdy and D. V. Foster, “Comparative models of the file assignment problem,” ACM Comput. Surv., vol. 14, pp. 287–313, Jun. 1982.

[2] K. C. Almeroth and M. H. Ammar, “The use of multicast delivery to provide a scalable and interactive video-on-demand service,” IEEE J. Sel. Areas Commun., vol. 14, no. 6, pp. 1110–1122, Aug. 1996.

[3] A. Meyerson, K. Munagala, and S. Plotkin, “Web caching using access statistics,” in Proc. ACM-SIAM SODA, Washington, D.C., USA, Jan. 2001, pp. 354–363.

[4] I. Baev, R. Rajaraman, and C. Swamy, “Approximation algorithms for data placement problems,” SIAM J. Comput., vol. 38, no. 4, pp. 1411–1429, Jul. 2008.
[5] S. Borst, V. Gupta, and A. Walid, “Distributed caching algorithms for content distribution networks,” in *Proc. IEEE INFOCOM*, San Diego, CA, Mar. 2010, pp. 1–9.

[6] M. A. Maddah-Ali and U. Niesen, “Fundamental limits of caching,” *IEEE Trans. Inform. Theory*, vol. 60, no. 5, pp. 2856–2867, May 2014.

[7] ——. “Decentralized caching attains order optimal memory-rate tradeoff,” *IEEE/ACM Trans. Netw.*, vol. 23, no. 4, pp. 1029–1040, Apr. 2014.

[8] Z. Chen, P. Fan, and K. B. Letaief, “Fundamental limits of caching: Improved bounds for users with small buffers,” *IET Communications*, vol. 10, no. 17, pp. 2315–2318, Nov. 2016.

[9] M. Mohammadi Amir and D. Gündüz, “Fundamental limits of coded caching: Improved delivery rate-cache capacity trade-off,” *IEEE Trans. Commun.*, vol. PP, no. 99, pp. 1–1, Dec. 2016.

[10] M. Mohammadi Amir, Q. Yang, and D. Gündüz, “Coded caching for a large number of users,” in *Proc. Information Theory Workshop (ITW)*, Cambridge, UK, Sep. 2016, pp. 171–175.

[11] K. Wan, D. Tuninetti, and P. Piantanida, “On caching with more users than files,” in *Proc. IEEE Int’l Symp. on Inform. Theory (ISIT)*, Barcelona, Spain, Jul. 2016, pp. 135–139.

[12] C. Tian and J. Chen, “Caching and delivery via interference elimination,” *arXiv:1604.08600v1 [cs.IT]*, Apr. 2016.

[13] S. Sahraei and M. Gastpar, “K users caching two files: An improved achievable rate,” in *Proc. CISS*, Princeton, NJ, Mar. 2016, pp. 620–624.

[14] Q. Yu, M. A. Maddah-Ali, and A. S. Avestimehr, “The exact rate-memory tradeoff for caching with uncoded prefetching,” *arXiv:1609.07817v1 [cs.IT]*, Sep. 2016.

[15] C. Tian, “Symmetry, outer bounds, and code constructions: A computer-aided investigation on the fundamental limits of caching,” *arXiv:1611.00024v1 [cs.IT]*, Oct. 2016.

[16] A. Sengupta, R. Tandon, and T. C. Clancy, “Improved approximation of storage-rate tradeoff for caching via new outer bounds,” in *Proc. IEEE Int’l Symp. on Inform. Theory (ISIT)*, Hong Kong, Jun. 2015, pp. 1691–1695.

[17] H. Ghasemi and A. Ramamoorthy, “Improved lower bounds for coded caching,” in *Proc. IEEE Int’l Symp. on Inform. Theory (ISIT)*, Hong Kong, Jun. 2015, pp. 1696–1700.

[18] C.-Y. Wang, S. H. Lim, and M. Gastpar, “A new converse bound for coded caching,” *arXiv:1601.05690v1 [cs.IT]*, Jan. 2016.

[19] M. Gregori, J. Gomez-Vilardebo, J. Matamoros, and D. Gündüz, “Wireless content caching for small cell and D2D networks,” *IEEE J. Sel. Areas Commun.*, vol. 34, no. 5, pp. 1222–1234, May 2016.

[20] M. Ji, G. Caire, and A. F. Molisch, “Fundamental limits of caching in wireless D2D networks,” *IEEE Trans. Inform. Theory*, vol. 62, no. 2, pp. 849–869, Dec. 2015.

[21] K. Shamugam, N. Golrezaei, A. G. Dimakis, A. F. Molisch, and G. Ciare, “Femtocaching: Wireless content delivery through distributed caching helpers,” *IEEE Trans. Inform. Theory*, vol. 59, no. 12, pp. 8402–8413, Sep. 2013.

[22] N. Golrezaei, A. F. Molisch, A. G. Dimakis, and G. Caire, “Femtocaching and device-to-device collaboration: A new architecture for wireless video distribution,” *IEEE Commun. Mag.*, vol. 51, no. 4, pp. 142–149, Apr. 2013.

[23] R. Pedarsani, M. A. Maddah-Ali, and U. Niesen, “Online coded caching,” in *Proc. IEEE Int’l Conf. Commun. (ICC)*, Sydney, Australia, Jun. 2014, pp. 1878–1883.

[24] U. Niesen and M. A. Maddah-Ali, “Coded caching with nonuniform demands,” in *Proc. IEEE Conf. Comput. Commun. Workshops (INFOCOM WKSHPS)*, Toronto, ON, Apr. 2014, pp. 221–226.

[25] M. Ji, A. M. Tulino, J. Llorca, and G. Caire, “Order-optimal rate of caching and coded multicasting with random demands,” *arXiv: 1502.03124v1 [cs.IT]*, Feb. 2015.
[26] J. Zhang, X. Lin, and X. Wang, “Coded caching under arbitrary popularity distributions,” in Proc. Infor. Theory Applications Workshop (ITA), San Diego, CA, Feb. 2015, pp. 98–107.

[27] J. Zhang, X. Lin, C. C. Wang, and X. Wang, “Coded caching for files with distinct file sizes,” in Proc. IEEE Int’l Symp. on Inform. Theory (ISIT), Hong Kong, Jun. 2015, pp. 1686–1690.

[28] N. Karamchandani, U. Niesen, M. A. Maddah-Ali, and S. Diggavi, “Hierarchical coded caching,” in Proc. IEEE Int’l Symp. on Inform. Theory (ISIT), Honolulu, HI, Jun. 2014, pp. 2142–2146.

[29] K. Poularakis and L. Tassiulas, “On the complexity of optimal content placement in hierarchical caching networks,” IEEE Trans. Commun., vol. 64, no. 5, pp. 2092–2103, May 2016.

[30] S. Wang, W. Li, X. Tian, and H. Liu, “Coded caching with heterogeneous cache sizes,” arXiv:1504.01123v3 [cs.IT], Apr. 2015.

[31] M. Mohammadi Amiri, Q. Yang, and D. Gündüz, “Decentralized coded caching with distinct cache capacities,” arXiv:1601.05690v1 [cs.IT], Nov. 2016.

[32] S. Li, M. A. Maddah-Ali, Q. Yu, and A. S. Avestimehr, “A fundamental tradeoff between computation and communication in distributed computing,” arXiv:1604.07086v1 [cs.IT], Apr. 2016.

[33] ———, “A scalable framework for wireless distributed computing,” arXiv:1608.05743v1 [cs.IT], Aug. 2016.

[34] J. Zhang, F. Engelmann, and P. ELia, “Coded caching for reducing CSIT-feedback in wireless communications,” in Proc. Allerton Conf., Monticello, IL, Sep. 2015, pp. 1099–1105.

[35] J. Zhang and P. ELia, “Fundamental limits of cache-aided wireless bc: Interplay of coded-caching and CSIT feedback,” arXiv:1511.03961v2 [cs.IT], Apr. 2016.

[36] W. Huang, S. Wang, L. Ding, F. Yang, and W. Zhang, “The performance analysis of coded cache in wireless fading channel,” arXiv:1504.01452v1 [cs.IT], Apr. 2015.

[37] R. Timo and M. Wigger, “Joint cache-channel coding over erasure broadcast channels,” in Proc. IEEE Int’l Symp. on Wirel. Commun. Systems (ISWCS), Brussels, Belgium, Aug. 2015, pp. 201–205.

[38] S. Saeedi Bidokhti, R. Timo, and M. Wigger, “Noisy broadcast networks with receiver caching,” arXiv:1605.02317v1 [cs.IT], May 2016.

[39] M. Maddah-Ali and U. Niesen, “Cache-aided interference channels,” arXiv: 1510.06121v1 [cs.IT], Oct. 2015.

[40] N. Naderializadeh, M. Maddah-Ali, and A. S. Avestimehr, “Fundamental limits of cache-aided interference management,” arXiv:1602.04207v1 [cs.IT], Feb. 2016.

[41] Q. Yang and D. Gündüz, “Centralized coded caching for heterogeneous lossy requests,” arXiv:1604.08178v1 [cs.IT], Apr. 2016.

[42] P. Hassanzadeh, E. Erkip, J. LLorca, and A. Tulino, “Distortion-memory tradeoffs in cache-aided wireless video delivery,” arXiv:1511.03924v1 [cs.IT], Nov. 2015.

[43] R. Timo, S. B. Bidokhti, M. Wigger, and B. Geiger, “A rate-distortion approach to caching,” in Proc. International Zurich Seminar on Communications, Zurich, Switzerland, Mar. 2016.

[44] J. Zhang and P. ELia, “Wireless coded caching: A topological perspective,” arXiv:1606.08253v1 [cs.IT], Jun. 2016.

[45] R. L. Urbanke and A. D. Wyner, “Packetizing for the erasure broadcast channel with an internet application,” in Proc. Int’l. Conf. Combinatorics, Information Theory and Statistics, Portland, ME, May 1997, p. 93.

[46] E. Tuncel, “Slepian-Wolf coding over broadcast channels,” IEEE Trans. Inform. Theory, vol. 52, no. 4, pp. 1469–1482, Apr. 2006.