Lopsided texture compatible with thermal leptogenesis in partially composite Pati–Salam unification

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Abstract

In this paper, we consider a lopsided flavor texture compatible with thermal leptogenesis in partially composite Pati–Salam unification. The Davidson–Ibarra bound for the successful thermal leptogenesis can be recast to the Froggatt–Nielsen (FN) charge of the lopsided texture. We found the FN charge $n_{\nu 1}$ of the lightest right-handed neutrino $\nu_R$ can not be larger than an upper bound, $n_{\nu 1} \lesssim 4.5$.

To realize these FN charges, we utilize the partial compositeness. In this picture, the hierarchies of the Yukawa matrices is a consequence of mixing between massless chiral fermions $f$ and heavy vector fermions $F$. That is induced by the linear mixing terms $\lambda^f f_L F_R$ and $\lambda^{f'} f'_L f'_R$. If the GUT breaking Higgs contributes these linear mixing terms, the resulting Yukawa interactions can be different between quarks and leptons.

For this purpose, we use the bi-fundamental Higgs $H_R(4,1,2)$ under the PS group $G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$. A particular set of non-renormalizable couplings between this GUT Higgs and fermions generates GUT breaking linear mixing.

As a result, the large tan $\beta$ case $n_{\nu i} = (1, 0, 0)$ in which leptons receive mass term seems to be natural. Moreover, it is found that heavy (composite) neutrino sector should have (almost) same flavor structure to reproduce the large mixing of neutrinos by the seesaw formula. If the VEV of GUT breaking Higgs mediates some flavor structure, they contribute to some mass term. Then, this statement can be hold for even in other Pati–Salam model, that does not assume the partial compositeness.
1 Introduction

The peculiar flavor structure of the Standard Model may be some hints of the theoretical origin of the Higgs boson and the flavor puzzle. On the other hand, treatments of the flavor structure strongly depend on the viewpoint on what the Higgs boson is. Then, study of model independent flavor texture is one of the dominant approach. For example, the Fritzsch texture \[1–8\], the Democratic texture \[9–22\], and the lopsided texture \[23–29\]. Among them, the lopsided texture is seems to be more natural in two reasons:

- If we assume the type-I seesaw mechanism \[30\], majorana mass matrix would be waterfall texture whether Yukawa matrix is cascade or waterfall \[31\] in Table 1. Then, waterfall is more desirable for the unified description of flavor.

- If the waterfall texture is symmetric matrix, quark Yukawa matrices should have approximate zero texture \[32\] in order to realize CKM matrix \[33,34\]. In some sense, zero texture in low energy is unnatural without a complicated symmetry. Then, the asymmetric waterfall texture is more desirable.

$$
\begin{pmatrix}
\varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \delta & \delta \\
\varepsilon & \delta & 1 \\
\end{pmatrix}
$$

Table 1: (a) The cascade and waterfall texture, with \(1 \gg \delta \gg \varepsilon\) \[35\]. (b) A example of symmetric Yukawa texture compatible with CKM matrix \[32\].

In this paper, we consider a lopsided flavor texture compatible with thermal leptogenesis in partially composite Pati–Salam unification. The Davidson–Ibarra bound \[36,37\] for the successful thermal leptogenesis can be recast to the Froggatt–Nielsen (FN) charge \[38\] of the lopsided texture. We found the FN charge \(n_{\nu_1}\) of the lightest right-handed neutrino \(\nu_{R1}\) can not be larger than a upper bound, \(n_{\nu_1} \lesssim 4.5\).

In order to explore the unified origin of flavor structure, it is reasonable to consider Grand Unified Theory (GUT) \[39\]. Since the proton decay have not been observed for a long time \[40\], it is somewhat reasonable to consider the Pati–Salam unification \[41\], a GUT model with no proton decay. From the viewpoint of unification, it is rational to consider the case \(n_{\nu_i} \simeq n_{q_i} = (3, 2, 0)\) or \(n_{\nu_i} \simeq n_{l_i} = (n+1, n, n)\).

To realize these FN charges, we utilize the partial compositeness \[42,43\]. In this picture, the hierarchies of the Yukawa matrices is a consequence of mixing between massless chiral fermions \(f\) and heavy vector fermions \(F\). That is induced by the linear mixing terms \(\lambda f^t L F_R\) and \(\lambda' F^t L f_R\). If the GUT breaking Higgs contributes these linear mixing terms, the resulting Yukawa interactions can be different between quarks and leptons.
For this purpose, we use the bi-fundamental Higgs $H_R(4,1,2)$ under the PS group $G_{PS} = SU(4)_C \times SU(2)_L \times SU(2)_R$. A particular set of non-renormalizable couplings between this GUT Higgs and fermions generates GUT breaking linear mixing.

As a result, the large tan $\beta$ case $n_{\nu_i} = (1, 0, 0)$ in which leptons receive mass term seems to be natural. Moreover, it is found that heavy (composite) neutrino sector should have (almost) same flavor structure to reproduce the large mixing of neutrinos by the seesaw formula. If the vacuum expectation value (VEV) of GUT breaking Higgs mediates some flavor structure, they contribute to some mass term. Then, this statement can be hold for even in other Pati–Salam model, that does not assume the partial compositeness.

In the construction of this model, several points have not been discussed enough: UV completion of the non-renormalizable couplings, precise values of the flavor structures and its origin, and so on. We leave it for our future work.

This paper is organized as follows. In the next and after next section, we review the Davidson–Ibarra bound, the lopsided texture, and the partial compositeness. In Sec. 4, we discuss a partially composite Pati–Salam Unification. Sec. 5 is devoted to conclusions and discussion.

## 2 Thermal Leptogenesis with Lopsided Texture

In this section, we discuss how the “Davidson–Ibarra” bound of the thermal leptogenesis [36, 37] restrict the Froggatt–Nielsen (FN) charge [38] of the lopsided texture. First of all, the Yukawa interactions of the Standard Model is defined as

$$\mathcal{L} \ni \sum_f -y_{fij} \bar{f}_L^i f_R^j H + h.c.,$$

for the SM fermions $f = q, l, f' = u, d, \nu, e$ and the Higgs field $H$. The lopsided texture (at the GUT scale $\Lambda_{GUT} = 2 \times 10^{16}$ GeV) is represented as

$$y_u \propto \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^0 \end{pmatrix}, \quad y_d \propto y_e^T \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^1 & \lambda^0 & \lambda^0 \end{pmatrix}, \quad m_\nu \propto \begin{pmatrix} \lambda^2 & \lambda^1 & \lambda^1 \\ \lambda^1 & \lambda^0 & \lambda^0 \\ \lambda^1 & \lambda^0 & \lambda^0 \end{pmatrix}.$$

Here, $\lambda$ is the Cabibbo angle $\lambda \simeq 0.22$. If the light neutrino mass is induced by the seesaw mechanism [30], the neutrino Yukawa and heavy majorana mass matrices should have the following form

$$y_\nu \propto \begin{pmatrix} \lambda^{n_{\nu_1}+1} & \lambda^{n_{\nu_2}+1} & \lambda^{n_{\nu_3}+1} \\ \lambda^{n_{\nu_1}} & \lambda^{n_{\nu_2}} & \lambda^{n_{\nu_3}} \\ \lambda^{n_{\nu_1}} & \lambda^{n_{\nu_2}} & \lambda^{n_{\nu_3}} \end{pmatrix}, \quad M_{\nu R} \propto \begin{pmatrix} \lambda^{2n_{\nu_1}} & \lambda^{n_{\nu_1}+n_{\nu_2}} & \lambda^{n_{\nu_1}+n_{\nu_3}} \\ \lambda^{n_{\nu_1}+n_{\nu_2}} & \lambda^{2n_{\nu_2}} & \lambda^{n_{\nu_2}+n_{\nu_3}} \\ \lambda^{n_{\nu_1}+n_{\nu_3}} & \lambda^{n_{\nu_2}+n_{\nu_3}} & \lambda^{2n_{\nu_3}} \end{pmatrix},$$

in order to realize the large mixing of MNS matrix [14]. These textures, realized by the $U(1)$ Froggatt–Nielsen (FN) charges in Table 2, seems to be natural by two reasons, as we mentioned at the introduction:
• If we assume the type-I seesaw mechanism, majorana mass matrix would be waterfall texture whether Yukawa matrix is cascade or waterfall [31] in Table 1. Then, waterfall is more desirable for the unified description of flavor.

• If the waterfall texture is symmetric matrix, quark Yukawa matrices should have approximate zero texture [32] in order to realize CKM matrix [33,34]. In some sense, zero texture in low energy is unnatural without a complicated symmetry. Then, the asymmetric waterfall texture is more desirable.

| Field | $10_1$ | $10_2$ | $10_3$ | $5_1$ | $5_2$ | $5_3$ | $1_1$ | $1_2$ | $1_3$ |
|-------|--------|--------|--------|------|------|------|------|------|------|
| $U(1)$ | 3      | 2      | 0      | $n+1$| $n$  | $n$  | $n_{\nu 1}$| $n_{\nu 2}$| $n_{\nu 3}$|

Table 2: The FN charge assignments of the SM fermions grouped into the representations of $SU(5)$, $10_i = (q_L, u_R, d_R, e_R), 5_i = (d_L, l_L), 1_i = \nu_{\ell_R}$.

The thermal leptogenesis with the lopsided texture is discussed in several papers [28, 43, 10]. They agree that larger $n_{\nu \ell}$ are incompatible with the Davidson–Ibarra bound. Here let us confirm this fact systematically.

In order to retain the room for adjustment of parameters, two Higgs doublet model (2HDM) is assumed. The fermion mass matrices are given by

$$m_{uij} = \frac{v}{\sqrt{2}} y_{uij} s_\beta, \quad m_{dij} = \frac{v}{\sqrt{2}} y_{dij} c_\beta, \quad (4)$$

$$m_{\nu_{\ell i j}} = \frac{v}{\sqrt{2}} y_{\nu_{\ell i j}} s_\beta, \quad m_{ei j} = \frac{v}{\sqrt{2}} y_{ei j} c_\beta, \quad (5)$$

where $\tan \beta \equiv v_u/v_d$, $v_u = v \sin \beta$, $v_d = v \cos \beta$.

In this case FN charges of leptons $n_{\ell i}, n_{ei}, n_{\nu i}$ have dependence of $\tan \beta$ through the above mass relations,

$$m_{\nu_{\ell i}} \simeq \frac{1}{\sqrt{2}} \lambda^{n_{\ell i}+n_{\nu \ell}} v c_\beta, \quad m_{ei j} \simeq \frac{1}{\sqrt{2}} \lambda^{n_{ei}+n_{\nu \ell}} v c_\beta. \quad (6)$$

Tentatively we treat $n_{\ell i}, n_{ei}, n_{\nu i}$ as free parameters, without fixing them like in Table 2. then the light neutrino mass is given by

$$m_{\nu} \equiv m \begin{pmatrix} \lambda^{2 n_{11}} & \lambda^{n_{11}+n_{12}} & \lambda^{n_{11}+n_{13}} \\ \lambda^{n_{11}+n_{12}} & \lambda^{2 n_{12}} & \lambda^{n_{12}+n_{13}} \\ \lambda^{n_{11}+n_{13}} & \lambda^{n_{12}+n_{13}} & \lambda^{2 n_{13}} \end{pmatrix}. \quad (7)$$

In the many model with lopsided textures, the mass eigenvalues of the lighter neutrinos $m_{\nu i}$ are roughly fixed as

$$m_{\nu \text{diag}} \sim m \begin{pmatrix} \lambda^{2 n_{11}} & 0 & 0 \\ 0 & \lambda^{2 n_{12}} & 0 \\ 0 & 0 & \lambda^{2 n_{13}} \end{pmatrix} \sim \begin{pmatrix} 0.002 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.05 \end{pmatrix} \text{[eV]}. \quad (8)$$
Then, the overall factor $m \sim \lambda^{-2n_{11}}0.002$ [eV] $\sim \lambda^{-2n_{12}}0.05$ [eV] also depends on $\tan\beta$ through the FN charge of the left-handed leptons.

By the seesaw mechanism, we can reconstruct the heavy majorana mass matrix as follows

$$M_{\nu R} = \frac{v^2 s^2_\beta}{2} y_{\nu}^T m^{-1}_{\nu} y_{\nu},$$

$$= \frac{v^2 s^2_\beta}{2m} \left( \begin{array}{ccc}
\lambda^{2n_{\nu_1}} & \lambda^{n_{\nu_1}+n_{\nu_2}} & \lambda^{n_{\nu_1}+n_{\nu_3}} \\
\lambda^{n_{\nu_1}+n_{\nu_2}} & \lambda^{2n_{\nu_2}} & \lambda^{n_{\nu_2}+n_{\nu_3}} \\
\lambda^{n_{\nu_1}+n_{\nu_3}} & \lambda^{n_{\nu_2}+n_{\nu_3}} & \lambda^{2n_{\nu_3}}
\end{array} \right) \times \frac{246^2 [\text{GeV}^2] s^2_\beta \lambda^{2n_{\nu_1}}}{2m} \left( \begin{array}{ccc}
1 & 0 & 0 \\
0 & \lambda^{2(n_{\nu_2}-n_{\nu_1})} & 0 \\
0 & 0 & \lambda^{2(n_{\nu_3}-n_{\nu_1})}
\end{array} \right).$$

(9)

If we assume the normal hierarchy for the $\nu_{Ri}$ and $n_{\nu_1} > n_{\nu_2} > n_{\nu_3}$, the FN charge of the lightest right-handed neutrino $n_{\nu_1}$ is bounded by the Davidson–Ibarra bound:

$$M_{\nu R1} \sim \frac{6 \times 10^4 [\text{GeV}^2] s^2_\beta \lambda^{2n_{\nu_1}}}{2\lambda^{-2n_{11}}0.002 [\text{eV}]} \gtrsim 10^9 [\text{GeV}],$$

(11)

$$6 \times 10^{14} [\text{GeV}] s^2_\beta \lambda^{2(n_{\nu_1}+n_{11}-1)} \gtrsim 10^9 [\text{GeV}],$$

(12)

$$1.5 \times 10^7 \lambda^{2(n_{\nu_1}+n_{11})} \gtrsim 1, \quad \Rightarrow \quad 5.5 \gtrsim (n_{\nu_1} + n_{11}).$$

(13)

The factor $6 \times 10^{14}$ is well-known result of the seesaw scale [17]. In the third line, we set $s_\beta \simeq 1$. The result shows that successful thermal leptogenesis requires larger $\tan\beta$ and smaller FN charges, in the model with lopsided textures. For example, $n_{\nu_1} \lesssim 4.5$ for $n_{11} = 1$ (large $\tan\beta$), and $n_{\nu_1} \lesssim 2.5$ for $n_{11} = 3$ ($\tan\beta \sim 1$). This result is consistent with the previous papers [15, 16].

However, this bound can be applied only for the strong hierarchical heavy neutrinos $M_{2,3} > 100 M_1$ [18, 19]. It corresponds to the case $n_{\nu_1} \gtrsim n_{\nu_2} + 1.5$, where the hierarchy of $M_{\nu R}$ is about as same as that of up-type quarks. We discuss on both cases in the following section.

### 3 Partial Compositeness in Composite Higgs Model

Construction of lopsided texture is basically classified two ways: FN mechanism, and mixing between SM and heavy fermions\footnote{1}\cite{24}. The typical example of the latter case is $E_6$ twist mechanism\,\cite{24}, universal seesaw\,\cite{52,53}, partial compositeness\,\cite{42,43}, and so on. Among them, a paper with $E_6$ twist naively fails thermal leptogenesis\,\cite{46}, because the model should have large FN charge $n_{\nu_1} = (3,2,0)$ and small $\tan\beta$. Here, we consider partial compositeness for realization of the lopsided texture.

The basic idea of partial compositeness is that the SM fields at low energy are the mixed states between elemental (light) fields and composite (heavy) fields, like $\rho - \gamma$ mixing. Flavor structures are induced from the mixing between heavy and light fermions with same quantum numbers. In this section, we will shortly review the composite Higgs model with partial compositeness.
3.1 Partial compositeness

The original context basically treats lower composite scale around TeV and some large global symmetry that contains the Standard Model group. The minimal model has the strong sector with a global symmetry $SO(5) \times U(1)_X$ that is broken down to $SO(4) \times U(1)_X$ at the scale $f$ \cite{4,13}. In this paper, we do not assume the compositeness and such symmetry.

The composite Higgs model (under the breaking scale $f$) can be described by a simplified two-site description \cite{13}, where the composite sector is replaced by the first resonances which mix with the SM fields. The Lagrangian is divided to three parts:

$$\mathcal{L} = \mathcal{L}_{\text{composite}} + \mathcal{L}_{\text{elementary}} + \mathcal{L}_{\text{mixing}}.$$  

(14)

The linear mixing terms $\mathcal{L}_{\text{mix}}$ represent mass terms between heavy (composite) and massless (elemental) fields. Due to this mixing, the massless eigenstates which are identified with the SM fields are superposition of elementary and composite states. For the simplicity, the Lagrangian only for fermions are described as

$$\mathcal{L}_{\text{composite}} = \bar{Q}_i (i \not{D} - M_{Qi}) Q_i + \bar{U}_i (i \not{D} - M_{Ui}) U_i + \bar{D}_i (i \not{D} - M_{Di}) D_i$$
$$+ \bar{L}_i (i \not{D} - M_{Li}) L_i + \bar{N}_i (i \not{D} - M_{Ni}) N_i + \bar{E}_i (i \not{D} - M_{Ei}) E_i$$
$$+ Y_{ij}^Q \bar{Q}_i \tilde{H} U_{Rj} + Y_{ij}^D \bar{Q}_i H D_{Rj} + Y_{ij}^N \bar{L}_i \tilde{H} N_{Rj} + Y_{ij}^E \bar{L}_i H E_{Rj}$$
$$+ \bar{Q}_{ij} \tilde{U}_{Li} H Q_{Rj} + \bar{Q}_{ij} \tilde{D}_{Li} H D_{Rj} + \bar{Q}_{ij} \tilde{N}_{Li} H L_{Rj} + \bar{Q}_{ij} \tilde{E}_{Li} H E_{Rj} + \text{h.c.},$$

(15)

$$\mathcal{L}_{\text{elementary}} = i \bar{q}_L \not{D} q_L + i \bar{u}_R \not{D} u_R + i \bar{d}_R \not{D} d_R + i \bar{\nu}_L \not{D} \nu_L + i \bar{e}_R \not{D} e_R,$$

(16)

$$\mathcal{L}_{\text{mixing}} = \lambda_{ij}^q \bar{q}_L Q_{Rj} + \lambda_{ij}^u \bar{U}_L q_{Rj} + \lambda_{ij}^d \bar{D}_L d_{Rj} + \lambda_{ij}^l \bar{L}_L \nu_{Rj} + \lambda_{ij}^e \bar{E}_L e_{Rj} + \text{h.c.}.$$  

(17)

Here, the small letter fields $q_L, u_R, d_R, l_L, \nu_R, e_R$ are the elemental quarks and capital letter fields $Q, U, D, L, N, E$ are vector-like (composite) heavy fields with same gauge charges of corresponding elemental fields. $\tilde{H} \equiv i\sigma^2 H^*$ is the conjugate field of the Higgs doublet $H$.

Redefining the physical states, one can obtain the SM Yukawa interactions. For the doublet quarks, the mass matrix is rewritten as follows;

$$\begin{pmatrix} \bar{q}_L \\ Q_L \end{pmatrix} \begin{pmatrix} \lambda_{ij}^q \\ M_{Qij} \delta_{ij} \end{pmatrix} Q_{Rj} + \text{h.c.}. $$

(18)

Under the assumption $M_{Qij} \gg \lambda_{ij}^q$, we can diagonalize this $3 \times 6$ component matrix perturbatively. At the leading order, the mass eigenstates of the doublets are,

$$\begin{pmatrix} q_L^\text{phys} \\ Q_L^\text{phys} \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2} \Delta \Delta^\dagger \\ 1 - \frac{1}{2} \Delta^\dagger \Delta \end{pmatrix} \begin{pmatrix} q_{Lj} \\ Q_{Lj} \end{pmatrix} \approx \begin{pmatrix} q_L - \lambda_{ij}^q M_{Q^{-1}} Q_{Lj} \\ \lambda_{ij}^q q_L + Q_{Lj} \end{pmatrix} + \mathcal{O} \left( \frac{\lambda_{ij}^q M_{Q^{-1}}}{M_Q} \right),$$

(19)

where $\Delta_{ij} \equiv \lambda_{ij}^q M_{Q^{-1}}$. Substituting Eq. (19) and similar equations for other fermions in
Eq. \((\ref{eq:16})\), the SM Yukawa interactions are represented by seesaw-like formulas

\[
y_u = \lambda_q M_Q^{-1} Y^U M_U^{-1} \lambda_u, \quad y_\nu = \lambda_l M_L^{-1} Y^N M_N^{-1} \lambda_\nu, \quad (21)
y_d = \lambda_q M_Q^{-1} Y^D M_D^{-1} \lambda_d, \quad y_e = \lambda_l M_L^{-1} Y^E M_E^{-1} \lambda_e. \quad (22)
\]

In this picture, hierarchies of the SM Yukawa interactions are generated by hierarchies of \(\lambda f, M_F, \) and \(Y^F\). The terms with \(\tilde{Y}\) do not contribute SM Yukawa matrices in the first order of \(\lambda/M\). However, the leading approximation cannot be valid for the third generation, and we should keep in mind it is just a “thumb counting”.

4 Partially Composite Pati–Salam Unification

In order to explore the unified origin of flavor structure, it is reasonable to consider Grand Unified Theory (GUT) \([39]\). The lopsided texture can be embedded to SO(10) \([24, 28, 29]\), \(E_6\) \([25, 27]\), and originally \(E_7\) \([23]\). These GUTs are well researched because they predict proton decay. A latest bound of the proton decay is \(\tau / B(p \to e^+ \pi^0) > 1.6 \times 10^{34}\) years at 90% confidence level \([10]\). In contrast, the Pati–Salam (PS) unified model \([11]\) has no proton decay, unless the model has no \(\bar{f}_c\) \([y\phi_{ij} + \tilde{y}\epsilon_{ijkl}\phi_{kl}] f_j\) type coupling with 6 representation Higgs \(\phi_{ij}\) under \(SU(4)_c\) \([56]\). Since the proton decay have not been observed for a long time, it is somewhat reasonable to consider a GUT model with no proton decay.

The lopsided texture in Pati–Salam unification is realized by FN mechanism \([26]\). They predicted FN charges \(n_{i1} = 2 \sim 3, n_{\nu_1} = 4\) that are little bit larger than the bound \(n_{i1} + n_{\nu_1} \lesssim 5.5\).

In this paper, we will consider a lopsided flavor texture compatible with thermal leptogenesis in partially composite Pati–Salam unification. GUT with composite Higgs is discussed in some literatures \([54, 58]\).

Usually, the symmetry breaking of the PS unification is achieved by the following two Higgs fields \(\Sigma(15, 1, 1), \Delta_R(10, 1, 3)\) under the group \(G_{PS} \equiv SU(4)_c \times SU(2)_L \times SU(2)_R\):

\[
\langle \Sigma \rangle = V \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}, \quad \langle \Delta_R \rangle = V' \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \quad (23)
\]

However, it is difficult to induce different flavor structure between quarks and leptons, unless an exponential coupling such as \(e^{-y \Sigma / \Lambda}\) is introduced.

Alternatively, this paper utilize a bi-fundamental representation

\[
H_R(4, 1, 2) = (u_{RH}, d_{RH}, \nu_{RH}, e_{RH}), \quad (24)
\]

\(^2\)We can obtain the same results from integrating out the heavy (composite) fields by solving the equations \(\partial L / \partial Q = \partial L / \partial U = \cdots = 0\) for the whole Lagrangian.
under $G_{PS}$\cite{26}. This Higgs corresponds to the 16 representation of SO(10) and truly minimal Higgs\cite{29} in the left-right symmetric model. $H_R$ breaks the group $G_{PS}$ to SM by obtaining a VEV in the “right-handed neutrino” direction:

$$\langle H_R \rangle = \langle \nu_{RH} \rangle \sim 2 \times 10^{16} \text{GeV},$$

and then the breaking scale is determined uniquely.

The indices of $H_R$ can be contracted in two ways, that correspond to the breaking of subgroup:

$$SU(4) : H_R^{\alpha a} H_R^{\beta a} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & V^2 \end{pmatrix}, \quad SU(2)_R : H_R^{\alpha a} H_R^{\alpha b} = \begin{pmatrix} V^2 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

for $\alpha, \beta = 1 - 4$ and $a, b = 1, 2$. The key observation is that if these VEVs\cite{28} induces the linear mixing $\lambda_f$, the difference of quarks and leptons can be generated.

### 4.1 The Model with $n_{\nu i} = (n + 1, n, n)$: Small - Large $\tan \beta$ Case

Here we construct the Pati–Salam model with partial compositeness, compatible with thermal leptogenesis. The FN charges of right-handed neutrinos are arbitrary as long as $n_{\nu 1} \lesssim 4.5$ holds. From the viewpoint of unification, it is rational to consider the case $n_{\nu i} \simeq n_{q i} = (3, 2, 0)$ or $n_{\nu i} \simeq n_{l i} = (n + 1, n, n)$. In both cases, the field contents of the model is in Table 3.

| $SU(4)_c$ | $SU(2)_L$ | $SU(2)_R$ |
|-----------|-----------|-----------|
| $\tilde{f}_{L i}$ | 4 | 2 | 1 |
| $\tilde{f}_{R i}$ | 4 | 1 | 2 |
| $\tilde{F}_{(L,R)i}$ | 4 | 2 | 1 |
| $\tilde{F}'_{(L,R)i}$ | 4 | 1 | 2 |
| $\Phi$ | 1 | 2 | 2 |
| $H_R$ | 4 | 1 | 2 |

Table 3: The charge assignments of the fermions and Higgs fields under the gauge symmetries. The index $i = 1 - 3$ represents flavor of the fermion.

First let us check the case $n_{\nu i} = (n + 1, n, n)$, where $M_{\nu R}$ has mild-hierarchy like $m_{\nu}$. If the bound $5.5 \gtrsim n_{\nu 1} + n_{11}$ holds, the FN charge is restricted to $1.7 \gtrsim n$. However, the charges are not determined because the Davidson–Ibarra bound does not holds for mild-hierarchical $\nu_{R i}$\cite{15,19}.

The relevant part of the Lagrangian in PS GUT is given by

$$\mathcal{L}_{\text{composite}} = \bar{F}_i (i \partial - M_{F_i}) F_i + \bar{F}'_i (i \partial - M_{F'_i}) F'_i + Y^F_{ij} \bar{F}_{Li} \Phi F_{Rj} + \text{h.c.},$$

$$\mathcal{L}_{\text{elementary}} = i \bar{f}_{Li} \partial f_{Li} + i \bar{f}_{Ri} \partial f_{Ri},$$

$$\mathcal{L}_{\text{mixing}} = \lambda^f_{ij} \bar{f}_{Li} F_{Rj} + \lambda^{f'}_{ij} \bar{F}'_{Li} f_{Rj} + \text{h.c.},$$

where $\lambda^f_{ij}$ and $\lambda^{f'}_{ij}$ are Yukawa couplings.
where $F = (Q, L)$ and $F' = (U, D, N, E)$. The Lagrangian corresponds to the special case of Eqs. (15)-(18) with the relations
\[ \lambda^q = \lambda^l = \lambda^{f_L}, \quad M_Q = M_L = M_F, \quad Y^U = Y^D = Y^N = Y^E, \quad (30) \]
\[ \lambda^u = \lambda^d = \lambda^e = \lambda^{f_R}, \quad M_U = M_D = M_N = M_E = M_{F'}. \quad (31) \]

If the gauge symmetry is not broken, it leads to the unrealistic GUT relation
\[ y_{SM}^f = \lambda^{f_L} M_{F^{-1}} Y^F M_{F'}^{-1} \lambda^{f_R}. \quad (32) \]

Conversely, the GUT relation can be broken if the combination of VEV \[ \langle \phi \rangle \] contributes the mass parameters.

Here we assume following non-renormalizable couplings, that is often utilized in contexts of GUT \[ \text{[26]} \]:
\[ \mathcal{L}'_{\text{mixing}} = \frac{c_{ij}^{f_L}}{\Lambda} f_{Li} \bar{f}_{Li} (H_R H_R^\dagger)^{\alpha \beta} F_{Rj}^{\alpha \beta} + \frac{c_{ij}^{f_R}}{\Lambda} f_{Li} \bar{f}_{Li} [(H_R H_R^\dagger)^{\alpha \beta} \delta_{ab} - (H_R^\dagger H_R)^{\alpha \beta} \delta_{ab}] f_{Rj}^{\beta \dagger} + \text{h.c.}, \quad (33) \]

where $H_R^{\alpha a} \equiv i(\sigma^2)^{ab} H_R^{\alpha b}$ is the conjugate field which transforms $(4^*, 1, 2)$ under $G_{PS}$. $\Lambda$ is the cutoff scale and $c_{ij}^{f_L, f_R}$ are dimensionless couplings.

After the spontaneous symmetry breaking (SSB) \[ \text{[26]} \], we obtain new linear mixing terms:
\[ \mathcal{L}'_{\text{mixing}} = \frac{c_{ij}^{f_L}}{\Lambda} f_{Li} \bar{f}_{Li} |N_{Li}|^2 R_{ij} + \frac{c_{ij}^{f_R}}{\Lambda} f_{Li} \bar{f}_{Li} (|N_{Li}|^2 R_{ij} - |D_{Li}|^2 R_{ij} - |E_{Li}|^2 R_{ij}) + \text{h.c.} \]
\[ = \frac{c_{ij}^{f_L}}{\Lambda} f_{Li} \bar{f}_{Li} |N_{Li}|^2 R_{ij} + \frac{c_{ij}^{f_R}}{\Lambda} f_{Li} \bar{f}_{Li} (|N_{Li}|^2 R_{ij} - |D_{Li}|^2 R_{ij}) + \text{h.c.}, \]
\[ \text{where } \zeta_{ij}^{f_L,R} \equiv c_{ij}^{f_L,R} V^2 / \Lambda. \text{ Then the linear mixing terms are changed as} \]
\[ \lambda^q = \lambda^{f_L}, \lambda^l = \lambda^{f_L} + \zeta^{f_L}, \quad (36) \]
\[ \lambda^u = \lambda^{f_R}, \lambda^d = \lambda^{f_R} - \zeta^{f_R}, \lambda^e = \lambda^{f_R} + \zeta^{f_R}, \lambda^e = \lambda^{f_R}. \quad (37) \]

Since $Y^E \sim O(1)$ is assumed in the partially composite models, the flavor structure is induced from $\lambda^f, \zeta^f$ and $M_F$. Tentatively we assume large $\tan \beta$ case $(n + 1, n, n) = (1, 0, 0)$. The new linear mixing terms should have the following form
\[ \zeta_{ij}^{f_L} M_{F_{ij}}^{-1} \lesssim \begin{pmatrix} \lambda^1 & \lambda^1 & \lambda^1 \\ 0 & \lambda^0 & \lambda^0 \\ 0 & \lambda^0 & \lambda^0 \end{pmatrix}, \quad M_{F_{ij}}^{-1} \zeta_{ij}^{f_R} \lesssim \begin{pmatrix} \lambda^1 & \lambda^0 & \lambda^0 \\ \lambda^1 & \lambda^0 & \lambda^0 \\ \lambda^1 & \lambda^0 & \lambda^0 \end{pmatrix}, \quad (38) \]
\[ \lambda_{ij}^{f_L} M_{F_{ij}}^{-1} \lesssim \begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ \lambda^0 & \lambda^0 & \lambda^0 \end{pmatrix}, \quad M_{F_{ij}}^{-1} \lambda_{ij}^{f_R} \lesssim \begin{pmatrix} \lambda^3 & \lambda^2 & \lambda^0 \\ \lambda^3 & \lambda^2 & \lambda^0 \\ \lambda^3 & \lambda^2 & \lambda^0 \end{pmatrix}, \quad (38) \]

to reconstruct the lopsided texture. The symbol $\lesssim$ represents some components of matrices can be smaller than the right-hand side, as far as the texture is realized. By these assumptions, $\zeta^f \gg \lambda^l$ holds for lighter generations.
Substituting Eq. (38) to Eq. (32), FN charge of $\nu_R$ for $n = 0$ in Table 2 is realized. The effect of each GUT breaking can be written schematically as

$$\begin{bmatrix}
    u_L(3,2,0) & u_R(3,2,0) \\
    d_L(3,2,0) & d_R(1,0,0) \\
    \nu_L(1,0,0) & \nu_R(1,0,0) \\
    e_L(1,0,0) & e_R(3,2,0)
\end{bmatrix}
= \begin{bmatrix}
    (3,2,0) & (3,2,0) \\
    (3,2,0) & (3,2,0) \\
    (3,2,0) & (3,2,0) \\
    (3,2,0) & (3,2,0)
\end{bmatrix}
+ \begin{bmatrix}
    (1,0,0) & (1,0,0) \\
    (1,0,0) & (1,0,0) \\
    (1,0,0) & (1,0,0) \\
    (1,0,0)
\end{bmatrix}
- \begin{bmatrix}
    (1,0,0)
\end{bmatrix}$$

lopsided texture = $G_{PS}$ invariant + $SU(4)_c$ - $SU(2)_R$.

Georgi–Jarlskog factor [61] does not realized in this construction. However, the difference between $\lambda^l$ and $\lambda^d$ in Eqs. (30), (37) can induce desirable effects.

For small tan $\beta$ case ($n + 1,n,n) = (3,2,2)$, lopsided texture is realized by a similar treatment. In this case quarks should receive GUT breaking flavor structure. The situation is realized by the following interactions

$$\mathcal{L'}_{\text{mixing}} = \frac{c_{ij}^L H_{Li}}{\Lambda} \left[ \text{tr}(H_R H_R^\dagger) \delta^{\alpha\beta} - (H_R H_R^\dagger)^{\alpha\beta} \right] F_{Rj}^{\beta a}$$

$$+ \frac{c_{ij}^R}{\Lambda} \left[ \text{tr}(H_R H_R^\dagger) \delta^{\alpha\beta} - (H_R H_R^\dagger)^{\alpha\beta} \right] F_{Rj}^{\beta b} + \text{h.c.}, \quad (39)$$

where $\text{tr}(H_R H_R^\dagger) = H_R^{\gamma c} H_R^{\gamma c}$. After the SSB, it leads to

$$\mathcal{L'}_{\text{mixing}} = \frac{c_{ij}^L}{\Lambda} \left[ \bar{q}_{Li} Q_{Rj} + \bar{D}_{Li} d_{Rj} + \bar{D}_{Li} d_{Rj} + \bar{E}_{Li} e_{Rj} \right] + \text{h.c.}, \quad (40)$$

$$= \frac{c_{ij}^L}{\Lambda} \left[ \bar{q}_{Li} Q_{Rj} + \bar{D}_{Li} u_{Rj} - \bar{E}_{Li} e_{Rj} \right] + \text{h.c.} \quad (41)$$

If the parameters $\zeta_{ij}^L, M_{F,c}^{-1}$ and $\lambda_{ij}^L, M_{F,F}^{-1}$ are assumed to have textures with FN charges (3,2,0) and (3,2,2), in the same way to Eq. (38), the lopsided texture with small tan $\beta$ is realized.

The situation can be written schematically as

$$\begin{bmatrix}
    u_L(3,2,0) & u_R(3,2,0) \\
    d_L(3,2,0) & d_R(3,2,0) \\
    \nu_L(3,2,2) & \nu_R(3,2,2) \\
    e_L(3,2,2) & e_R(3,2,0)
\end{bmatrix}
= \begin{bmatrix}
    (3,2,2) & (3,2,2) \\
    (3,2,2) & (3,2,2) \\
    (3,2,2) & (3,2,2) \\
    (3,2,2) & (3,2,2)
\end{bmatrix}
+ \begin{bmatrix}
    (3,2,0) & (3,2,0) \\
    (3,2,0) & (3,2,0) \\
    (3,2,0) & (3,2,0) \\
    (3,2,0)
\end{bmatrix}
- \begin{bmatrix}
    (3,2,0)
\end{bmatrix}$$

lopsided texture = $G_{PS}$ invariant + $SU(4)_c$ - $SU(2)_R$.

However, the interactions of the form $[\text{tr}(H_R H_R^\dagger) \delta^{\alpha\beta} - (H_R H_R^\dagger)^{\alpha\beta}]$ seem to be unnatural from the viewpoint of GUT. Therefore, the large tan $\beta$ case $n_{\nu_i} = (1,0,0)$ in which leptons receive mass term seems to be natural. If the VEV of GUT breaking Higgs mediates some flavor structure, they contribute to some mass term. Then, this statement can be hold for even in other Pati–Salam model, that does not assume the partial compositeness.

4.2 Majorana Neutrino Mass

The lepton number violation with the partial compositeness is discussed in literature [62–64]. In Refs. [62, 63] the majorana mass is introduced by an interaction between
heavy (composite) lepton and SM Higgs doublet $\tilde{L}_L^c L_L H H/\Lambda$. In Ref. [34], the majorana mass is introduced to elemental right-handed neutrino, $M_{Ri} \nu_R^c$. In models with type-I seesaw, the large mixing of light neutrino can not be realized unless the hierarchy of $Y_{\nu}$ and $M_R$ is compensated. Then two matrices $Y_{\nu}$ and $M_R$ should be induced from same mechanism. For this purpose, we consider the other case, in which the heavy (composite) lepton has heavy majorana mass (by GUT Higgs):

$$\mathcal{L}_{\text{majorana}} = - \frac{c_{ij}^{NL}}{2\Lambda} F_{L'i}^c a a H_R^{1\alpha a} H_R^{1\beta b} F_{Lj}^c + \text{c.c.} - \frac{c_{ij}^{NR}}{2\Lambda} F_{R'i}^c a a H_R^{1\alpha a} H_R^{1\beta b} F_{Rj}^c + \text{h.c.}.$$  (43)

The Lagrangian relevant to the majorana mass term is found to be

$$\mathcal{L} \ni \zeta_{i j}^{f_R} \tilde{N}_{L_i} \nu_{R_j} - \tilde{N}_{L_i} M_{N_i} N_{R_i} - \frac{1}{2} m_{Lij} \tilde{N}_{L_i}^c N_{L_j} - \frac{1}{2} m_{Rij} \tilde{N}_{R_i}^c N_{R_j} + \text{h.c.},$$  (44)

where $m_{(L,R)ij} = c_{ij}^{N(L,R)} V^2 / 2\Lambda, M_{Ni} = M_{F'i}$.

In order to obtain the mass of the right-handed neutrinos $\nu_{Ri}$, heavy fields $N_{L,R}$ should be integrated out using their equation of motions.

$$\frac{\partial \mathcal{L}}{\partial \tilde{N}_{Li}} = -m_{Lij} N_{L_j} + \zeta_{i j}^{f_R} \nu_{R_j} - M_{N_i} N_{R_i} + \text{h.c.},$$  (45)

$$\frac{\partial \mathcal{L}}{\partial \tilde{N}_{Ri}^c} = -m_{Rij} N_{R_j} - M_{N_i} N_{L_i}^c + \text{h.c.}.$$  (46)

Therefore,

$$\frac{\partial \mathcal{L}}{\partial N_{Li}} = 0 \Rightarrow \zeta_{i j}^{f_R} \nu_{R_j} = -M_{N_i}^{-1} m_{Rij} \tilde{N}_{Rj} + M_{N_i} N_{R_i},$$  (47)

$$\frac{\partial \mathcal{L}}{\partial N_{Ri}^c} = 0 \Rightarrow -M_{N_i}^{-1} m_{Rij} N_{R_j} = N_{Li}^c.$$  (48)

The first term in Eq. (17) can be neglected than the second one because the scales $\Lambda, M_N$ is larger than the GUT scale $V$. Substituting these equations to the Lagrangian (44), We obtain

$$\mathcal{L} \ni -\frac{1}{2} M_{\nu Rij} \nu_R^c \nu_{Rj} + \text{h.c.}, \quad M_{\nu Rij} = (\zeta_{i j}^{f_R} M_{N_i}^{-1} m_{Rj} M_{N_i}^{-1} \zeta_{i j}^{f_R})_{ij}. $$ (49)

This formula is similar to Yukawa interactions Eq. (22). The term contains $m_{Lij}$ does not contribute the majorana mass at first order.

Moreover, the light neutrino mass can be obtained from the seesaw formula by integrated out the neutrino $\nu_R$

$$m_{\nu} = \frac{v^2}{2} y_{\nu} M^{-1}_{\nu R} y_{\nu}^T$$

$$= \frac{v^2}{2} (\zeta_{i j}^{f_L} M_{F_i}^{-1} Y F_{F'} \zeta_{i j}^{f_R})(\zeta_{i j}^{f_R} M_{N_i}^{-1} m_{Rj} M_{N_i}^{-1} \zeta_{i j}^{f_R})^{-1} (\zeta_{i j}^{f_L} M_{F_i}^{-1} Y F_{F'} \zeta_{i j}^{f_R})^T$$

$$= \frac{v^2}{2} (\zeta_{i j}^{f_L} M_{F_i}^{-1} Y F_{F'} m_{Rj}^{-1} Y F_{F'} M_{F'}^{-1} \zeta_{i j}^{f_L})^T.$$  (52)
In order to reproduce the lopsided texture, the flavor structure should be generated from $\zeta^L M^{-1}_F$. In other words, $Y^F m^1_R Y^{FT}$ and $Y^F$ should have (almost) same flavor texture and then $m^1_R Y^{FT} \simeq 1_3$. Since the LNV mass has the form $m_R \propto c^{NR}_{ij}$, it is found that heavy (composite) sector should have (almost) same flavor structure (c.f, $Y^F_{ij} \propto c^{NR}_{ij}$) to reproduce the large mixing of neutrinos by the seesaw formula.

### 4.3 The Model with $n_{\nu i} = (3, 2, 0)$: Medium - Large $\tan \beta$ Case

Next we will consider the case $n_{\nu i} = (3, 2, 0)$, where $M_{\nu R}$ has strong hierarchy like $Y_u$. From the bound (13), $5 \gtrsim n_{\nu 1} + n_{l1}$, the FN charge of the lightest lepton doublet is bounded as $2.5 \gtrsim n_{l1}$. Then, small $\tan \beta$ is forbidden in this case. Actually, a $E_6$ model with $n_{\nu i} = (3, 2, 0)$ and small $\tan \beta$ naively fails thermal leptogenesis [46].

Similar to the previous case, we introduce new linear mixing terms that break the GUT relation (32). For example,

$$\mathcal{L}^\prime_{\text{mixing}} = \frac{c^f_{ij}}{\Lambda} \bar{l} \bar{i} \alpha a (H_R H_R^\dagger)^{\alpha \beta} F_{Rj}^{\beta a} + \frac{c^f_{ij}}{\Lambda} \bar{l} \bar{i} \alpha a [-(\tilde{H}_R^\dagger \tilde{H}_R)^{ab}]^{\alpha \beta} \tilde{H}_R^{iaa} \tilde{H}_R^{ibb} \tilde{F}_{Rj}^{\beta b} + \text{h.c.}.$$ (53)

By the SSB (26), we obtain

$$\mathcal{L}^\prime_{\text{mixing}} = \zeta^L_{ij} \bar{l} \bar{i} \bar{l} \bar{l} \bar{l}_{Rj} + \zeta^R_{ij} \bar{l} \bar{i} \bar{l} \bar{l} \bar{l}_{Rj} + \text{h.c.},$$ (54)

$$= \zeta^L_{ij} \bar{l} \bar{i} \bar{l} \bar{l} \bar{l}_{Rj} - \zeta^R_{ij} \bar{l} \bar{i} \bar{l} \bar{l} \bar{l}_{Rj} + \text{h.c.},$$ (55)

where $\zeta^L_{ij} \equiv c^L_{ij} \nu^2 / \Lambda$.

The later discussion, including the majorana mass, is done in the same way to the previous case. However, this case seems to be little artificial because the GUT breaking can not be factorized to $SU(4)_c$ and $SU(2)_R$.

### 5 Conclusions and Discussion

In this paper, we consider a lopsided flavor texture compatible with thermal leptogenesis in partially composite Pati–Salam unification. The Davidson–Ibarra bound for the successful thermal leptogenesis can be recast to the Froggatt–Nielsen (FN) charge of the lopsided texture. We found the FN charge $n_{\nu 1}$ of the lightest right-handed neutrino $\nu_{R1}$ can not be larger than an upper bound, $n_{\nu 1} \lesssim 4.5$.

In order to explore the unified origin of flavor structure, it is reasonable to consider Grand Unified Theory (GUT). Since the proton decay have not been observed for a long time, it is somewhat reasonable to consider the Pati–Salam unification, a GUT model with no proton decay. From the viewpoint of unification, it is rational to consider the case $n_{\nu i} \simeq n_{qi} = (3, 2, 0)$ or $n_{\nu i} \simeq n_{li} = (n + 1, n, n)$.

To realize these FN charges, we utilize the partial compositeness. In this picture, the hierarchies of the Yukawa matrices is a consequence of mixing between massless chiral fermions $f$ and heavy vector fermions $F$. That is induced by the linear mixing terms $\zeta^{L,R}_{ij}$. 

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\( \lambda \bar{f}_L f_R \) and \( \lambda' \bar{F}_L f'_R \). If the GUT breaking Higgs contributes these linear mixing terms, the resulting Yukawa interactions can be different between quarks and leptons.

For this purpose, we use the bi-fundamental Higgs \( H_R(4,1,2) \) under the PS group \( G_{PS} = SU(4) \times SU(2)_L \times SU(2)_R \). A particular set of non-renormalizable couplings between this GUT Higgs and fermions generates GUT breaking linear mixing.

As a result, the large \( \tan \beta \) case \( n_{\nu_i} = (1,0,0) \) in which leptons receive mass term seems to be natural. Moreover, it is found that heavy (composite) neutrino sector should have (almost) same flavor structure to reproduce the large mixing of neutrinos by the seesaw formula. If the VEV of GUT breaking Higgs mediates some flavor structure, they contribute to some mass term. Then, this statement can be hold for even in other Pati–Salam model, that does not assume the partial compositeness.

In the construction of this model, several points have not been discussed enough: UV completion of the non-renormalizable couplings, precise values of the flavor structures and its origin, and so on. We leave it for our future work.

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