Vertical-plate free convection boundary layer disturbed by a row of circular cylinders: RANS-based numerical simulation

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Abstract. This paper presents the results of RANS-based numerical simulation of 3D flow dynamics and heat transfer in the turbulent free-convection boundary layer disturbed by a transverse periodic row of finite-height cylindrical obstacles. The disturbing row elements are mounted on a vertical isothermally heated plate along which the free-convection flow develops starting from a 2D state prescribed at the computational domain inlet. The height-to-diameter ratio of the cylindrical obstacles is varied from 1 to 3, and their surface is treated as adiabatic or isothermal. Vortex flow structures and local heat transfer patterns in the front and in the rear of the introduced macroroughness are analyzed.

1. Introduction

There are various devices where a free-convective boundary layer develops along a vertical heated surface. Studies of heat transfer intensification are of great importance for further improvements of such devices. The major efforts in this direction have been made for the case of low and moderate Grashof numbers, when the free convection flow can be treated as laminar. Typically, to enhance free-convection heat transfer from vertical surfaces, a system of vertical fins is installed on a heated plate. Last time, a special attention is paid to employment of V-shaped fins, which provide a more positive effect [1-3]. There are few studies dealing with problems of flow control and heat transfer intensification in the turbulent free convection boundary layer arising on a tall, vertical heated plate. With experimental facilities, some issues of heat transfer augmentation by fins of various heights were considered in Refs. [4, 5]. The authors of Ref. [6] employed heat transfer promoters in the form of a long flat plate and a row of short flat plates aligned at fixed steps in the boundary layer spanwise direction. The experiments performed have shown that in case of using a cross row of short plates one can achieve a rather considerably increase in the local heat transfer coefficients downstream of the promoter. In Refs [7, 8], the problem of heat transfer intensification in the front of a semi-infinite cylinder piercing the vertical-plate turbulent boundary layer was studied with 3D RANS-based computations, focusing at effects caused by horseshoe vortex structures. Results of an extended RANS-based study for the case of a finite-height cylinder disturbing the boundary layer were reported in Ref. [9], where the effects of the height-to-diameter ratio and the cylinder surface thermal conditions on 3D flow and heat transfer in the front and in the rear of the obstacle were analyzed.

The present paper covers results of RANS modeling of the 3D fluid dynamics and heat transfer phenomena in the vertical-plate turbulent free-convection boundary layer, which is disturbed by a row of finite-height cylindrical elements. The simulation was carried out using the k-ω SST turbulence...
model. With the ANSYS Fluent 16.0 CFD package, grid-independent steady-state numerical solutions were obtained varying the height-to-diameter ratio of the cylindrical obstacles and the type of thermal boundary conditions on their surface.

2. Problem formulation and computational setting

2.1. Description of the case
A scheme to the case considered, including the computational domain used, is shown in figure 1. On an isothermally heated plate (1), a cross row of cylinders (2) is installed. Each cylinder has diameter $d$ and height $h_c$. Its surface is assumed adiabatic or isothermal, kept at temperature equal to the plate temperature, $T_w$. The cylinders disturb the (originally 2D) turbulent free-convection boundary layer developing on the plate. Physical properties of the fluid are assumed constant.

Position of the row is characterized by a target value, \( Gr^{(0)}_\delta \), of the local Grashof number \( Gr^{\delta}_\delta = \frac{g \beta_T (T_w - T_a) \delta^3}{\nu^2} \) evaluated with the integral thickness of the layer. Here \( (T_w - T_a) \) is the difference between the plate temperature and the ambient temperature \( T_a \), \( g \) is gravity acceleration, \( \beta_T \) is volumetric expansion coefficient, \( \nu \) is kinematic viscosity. Superscript \(^{(0)}\) designates hereinafter a variable/parameter value at the obstacle position in case of its absence. The local boundary layer thickness \( \delta \) is calculated via integration of the normalized streamwise velocity \( \frac{v}{v_{\text{max}}} \) across the layer (over \( z \)-direction) from the plate to the point \( z = \delta_T \), where \( \delta_T \) is the thermal boundary layer thickness evaluated through prescribing the normalized-temperature 1\% deviation from the ambient temperature.

![Picture 1](image.png)

Picture 1. (a) Scheme to the case considered, (b) computational mesh on solid surfaces.

Following [9], the present simulation was carried out setting \( Gr^{(0)}_\delta = 1.4 \times 10^6 \). The Prandtl number was taken as 0.7. The ratio of the boundary layer thickness to the cylinder diameter was fixed, \( \delta/d = 3 \). The span pitch of the row was set to \( 2d \). Various computational cases are characterized by the cylinder height-to-diameter ratio, \( \Gamma = \frac{h_c}{d} \), and the type of thermal boundary conditions on the cylinder surface. Below, the computational cases are indicated by the following letter-digit combination: A# means adiabatic surface, B# is isothermal surface, # = 1, 2, 3 corresponds to \( \Gamma = 1, 2, 3 \).

2.2. Mathematical model and numerics
The Reynolds-averaged Navier-Stokes equations written with the Boussinesq approximation for the buoyancy force term and added by the energy equation are used for getting steady-state numerical solutions. The results presented below have been obtained with the k-\( \omega \) SST turbulence model, setting the turbulent Prandtl number to 0.85.

It is accepted that the flow past each cylinder in the row is symmetrical with respect to two vertical planes: one of them (central plane) passes through the cylinder axis, and the second one is the mid plane between neighboring cylinders. Consequently, the computational domain used for the present computations covered one half of a cylinder surface (figure 1a), and its width was set to half of the span period. The distances from the front edge and the back side of the cylinder to the inlet (3) and outlet (4) boundaries respectively were set to 10\( d \). The external boundary (5) parallel to the plate was placed at a distance of 5\( \delta \) from the wall.
No-slip condition was imposed on the plate and the cylinder surface (the thermal conditions on the walls are defined in Section 2.1). At the inlet section (3), the profiles of velocity, temperature and turbulence characteristics corresponding to the 2D turbulent free-convective boundary developed on a vertical heated plate were prescribed. These profiles were provided by preliminary 2D computations with the same turbulence model (details can be found in [9]). At the outlet section (4) and at the external boundary (5) parallel to the plate, the "pressure-outlet" condition, as implemented in the ANSYS Fluent, was set.

Quasi-structured multiblock grids were used, with a considerable refinement near the walls (surface-averages values of $y^+$ were about 0.1). A surface grid fragment is illustrated in figure 1b. The grids consisted of 1.5-2 million cells, depending on the height-to-diameter ratio of the cylinder. The choice of the grid topology and dimension was based on results of the grid-dependence study performed previously for the case of a single cylindrical obstacle [9].

3. Results

In all the cases computed, a complicated three-dimensional flow covering various types of vortex structures develops in the turbulent free-convective boundary layer interacting with the cross row of the obstacles. It is illustrated, in particular, by central-plane streamline patterns and maps of the velocity magnitude given in figure 2 for Cases B1-B3 (the velocity value is scaled by the buoyancy velocity $u_b=[g\beta(T_w-T_a)\nu^{1/3}]$). In each case, a horseshoe vortex structure is formed in front of the cylinder after the boundary layer separation. Note that in case of $\delta/d=3$, the maximum velocity in the approaching boundary layer is positioned at a distance of about $0.4d$ from the wall. A high velocity zone occurs near the cylinder top due to local flow acceleration. It is especially well pronounced in Cases B1 and B2. A large part of the cylinder top is occupied by the separation zone. The vortex flow pattern behind the cylinder is similar in Cases B1 and B2, but differs considerably in the highest cylinder case. For the first two cases, an extended recirculation zone with lower velocities forms in the near-wake of the cylinder that is typical of the flows past bluff bodies. Contrary to that, no large-scale recirculation zone occurs in Case B3. Instead of that, the fluid moves to the plate along the back-side of the obstacle, except a small vortex region adjacent to the cylinder top.

![Figure 2. Velocity magnitude maps and streamlines in the central vertical plane, Cases B1-B3.](image)

For each case of the B-family, figure 3 presents the plate-surface shear stress distribution with superimposed limiting surface streamlines (left part of a corresponding sub-plot), as well as the flow pattern at the plane positioned at a distance of $h_c/2$ from the plate (right part). The flow pattern is composed from a map of “tangential” velocity defined as $(u^2+v^2)^{1/2}$ and a set of streamlines in the plane. The legends given to the Case B1 sub-plot are applicable to other cases as well. Note, hereinafter the spatial coordinates used are normalized by the cylinder diameter $d$. In all the cases, flow dynamics is characterized by flow acceleration in the narrowed space between neighboring obstacles and high values of the plate surface friction in a horseshoe-like region going round the cylinder foot. The accelerated flow region is longer in cases of higher cylinders. Figure 3 shows again that in the highest obstacle case, significant differences in the structure of the flow are observed, as
compared with two other cases. Here, the vortex flow “footprints” on the plate are considerably displaced downstream, and a zone with concentrated normal-to-plate vorticity does not occur.

Distributions of the dimensionless temperature \( \theta = (T - T_a)/(T_w - T_a) \) over the central plane are presented in figure 4. A dramatic thinning of the thermal boundary layer is observed in the vicinity of the cylinder leading edge for all the cases. This phenomenon is due to action of the horseshoe vortex [7-9]. In the isothermal obstacle cases (B#), thermal layers form on the cylinder surface, and the near-wake region becomes warmer significantly. Notable also, that in Case B3, due to absence of pronounced recirculation in the rear of the cylinder, the thermal boundary layer on the plate is much thicker in this zone, as compared with Cases B1 and B2.

Patterns of limiting plate-surface streamlines obtained for the shortest and the longest cylinder cases are compared in figure 5. The left sub-plot (Case B1) contains both a focus-type point and a saddle point in the near-wake of the obstacle. In Case B3, these singular points are shifted considerably downstream, and only the focus point is seen at the surface fragment shown.

For the same cases, figure 6 presents a comparison of distributions of heat transfer coefficient, \( h \) (HTC), over the plate surface. The HTC contours in the front of the cylinder are similar, and in both cases there is a spot of highly intensified heat transfer. The HTC distributions in the rear of the obstacle differ considerably due to discussed-above specifics of flow dynamics in Cases B1 and B3.
Integral characteristics of the flow and heat transfer for all the cases considered are summarized in Table 1. These are (i) distance from the boundary layer separation point to the cylinder leading edge, \( \lambda_S \), (ii) distance from the back-side edge of the cylinder to the reattachment point, \( \lambda_R \), (iii-v) maximum (max) and average (< >) normalized values of the wall shear stress and HTC on the plate surface (PS) and the cylinder surface (CS), as well as (vi) values of the integral heat transfer intensification factor \( Q/Q^{(0)} \), the evaluation method of which is explained below.

The table data show in particular that the height of the cylinder affects moderately the dimensionless coordinate of the upstream separation of the boundary layer: the changes are within 9% over all the cases. It should be noted here that the ratio of the boundary layer thickness to the cylinder height, \( \delta/h_c \), varies together with variations of parameter \( I = h_c/d \), so that \( \delta/h_c = 1.0, 1.5, 3.0 \) when \( I = 1, 2, 3 \) at the fixed value of \( \delta/d = 3 \). An increase in the cylinder height leads, as expected, to a shift of the reattachment point downstream but disproportionately: \( \lambda_R \) increases less than two times with changing parameter \( I \) from 1 to 3. Maximum shear stress on the plate, as well as the average value on the cylinder surface, varies from one case to another within 5-10%. Changing of thermal boundary conditions on the cylinder surface has a significant impact on the peak HTC values that are always observed upstream of the row: in the isothermal cylinder cases the peak HTC is 17-18% lower than in the adiabatic cylinder cases.

| Case | A1   | A2   | A3   | B1   | B2   | B3   |
|------|------|------|------|------|------|------|
| \( \lambda_S \) | 0.760 | 0.821 | 0.828 | 0.756 | 0.810 | 0.823 |
| \( \lambda_R \) | 1.866 | 2.341 | 3.096 | 1.794 | 2.162 | 3.302 |
| max(\( \tau_w/\tau_w^{(0)} \))<PS> | 5.806 | 5.924 | 5.770 | 5.885 | 6.049 | 5.832 |
| max(\( h/h^{(0)} \))<PS> | 6.239 | 5.967 | 5.892 | 5.119 | 4.956 | 4.874 |
| \(<\tau_w/\tau_w^{(0)}\>)<CS> | 2.500 | 2.649 | 2.605 | 2.544 | 2.710 | 2.749 |
| \( Q/Q^{(0)} (L=10) \) | 1.738 | 1.841 | 1.487 | 1.773 | 2.090 | 2.028 |

For Case B2, figure 7 presents 3D views of the wall friction and wall heat flux distributions added also by a pattern of limiting surface streamlines (here, the fields calculated are multiplied according to symmetry and periodicity of the flow considered). Remarkably, that for the chosen span pitch of the row one does not observe any significant heat transfer augmentation in the vicinity of the mid lines passing between neighboring cylinders on the plate (figure 7b), despite figure 7a shows a considerable increase of the wall friction on these lines when the flow passes between the cylinders. So, it is one more example when the Reynolds analogy is not applicable.

Let us determine the integral heat transfer intensification factor as \( Q/Q^{(0)} = \int h dS / \int h^{(0)} dS \), where the HTC is integrated over a rectangle that covers the entire width of the computational domain and has a varied dimensionless length \( L \) in the streamwise direction. The lower boundary of the integration area is always located upstream of the cylinder, at \( y = -1.5d \). For Cases B1-B3, the cylinder
surface is included in the integration area as well. Intensification factor $Q/Q^0$ versus the integration area length is shown in figure 8. As seen, the highest rate of heat transfer augmentation is achieved in Cases B2, B3.

![Figure 7.](image.png)

**Figure 7.** (a) 3D view of wall friction distribution and pattern of limiting surface streamlines, (b) distribution of wall heat flux, Case B2.

![Figure 8.](image.png)

**Figure 8.** Integral heat transfer intensification factor: (a) adiabatic and (b) isothermal obstacles.

4. Conclusions

The RANS/k-ω-SST numerical simulation of three-dimensional flow dynamics and heat transfer in the turbulent vertical-plate free-convection boundary layer disturbed by a cross row of obstacles in the form of finite-height circular cylinders has been carried out. Influence of the obstacle height-to-diameter ratio, varied from 1 to 3, and the type of the cylinder-surface thermal boundary conditions on steady-state vortex flow structures and local heat transfer patterns in the front and in the rear of the introduced macroroughness has been analyzed. It has been revealed that, among the six cases considered, the highest rate of integral heat transfer augmentation is achieved when the cylindrical obstacles heated to same temperature as the vertical plate have the height-to-diameter ratio of 2…3.

Acknowledgments

The study is supported by the Russian Science Foundation under grants no. 18-19-00082.

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