An extended hot plate method for measurement of thermal conductivity variation with temperature of building materials

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Abstract: A new exploitation of the Hot Plate in Steady state HPS on the determination of thermal conductivity variation with temperature is presented. The mean temperature changement with temperature of the tested sample is ensured with varying the value of the voltage applied to the hot plate resistance, which enables to measure the thermal conductivity in different temperatures. Calculated thermal conductivity values obtained using a gypsum plaster sample, which specific heat capacity variation with temperature is accurately measured, validate the technic. Results show that the mean calculated thermal conductivity of the studied gypsum plaster is 0,143 W/m.k while the measured one is 0,148 W/m.k with a mean deviation of 3,81%. Moreover, the two trend lines of measured values and calculated ones have the same slope with a small intercept difference of 0,004.

1. Introduction
At lower temperatures, two classifications of experimental methods of the thermal conductivity determination exist: steady state and transient state methods. In the first category the sample thermal conductivity is directly calculated measuring the flow density and the temperature gradient once the system falls into equilibrium [1, 2]. In the second category the thermal diffusivity is measured instead, which takes into account the temperature fluctuations inside the sample, the thermal conductivity is then estimated from the thermal diffusivity knowing the specific heat and the density of the sample [3]. The hot plate method [4] is considered to be a first category technic for thermal conductivity measurements of small samples (0,01m²). In steady state, its straightforward elements incorporation made its use simple and easy, while keeping an acceptable accuracy of measurements. In the present study this technic is extended to the determination of the thermal conductivity variation with temperature of construction materials. A gypsum plaster sample was used to present and explain the experimental and numerical procedures. The specific heat variation with temperature of gypsum plaster was measured using an accurate differential scanning calorimeter in order to compare calculated and measured values of the thermal conductivity assuming a constant thermal diffusivity and density.

2. Description of used material
The sample was made in a circular mold of 10cm in diameter and 2cm in thickness mixing 190g of gypsum plaster with 118.75g of water respecting a water ratio of 0.625 as indicated in NF P 15-201-1, it was left on an ambient temperature of 25 °C and a RH of 80% for a sufficient time to consistency, subsequently introduced into an oven and dried at a temperature of 70 °C + -3 °C according to NF P 75-101. Once the sample mass becomes constant it is immediately coated on cellophane in order to keep its dry mass constant until measures.

3. Experimental approach:

3.1. Description of the stationary hot plate method

Figure 1 describes the experiment of the Hot Plate in Steady state HPS, the heating element is composed of a plane heating resistance of dimensions given in Table 1 inserted between the sample and the polyethylene having thermal conductivity of $\lambda_p = 0.043 \text{ W.m}^{-1}\text{K}^{-1}$. Most of the heat dissipated by the heating element of a resistance $R_h = 101\Omega$ passes through its upper part, the different temperatures are measured by a thermocouple type K with a resolution of 0.001K. The set is then placed between two aluminium blocks in order to ensure a constant temperature on the unheated faces of the sample and the polyethylene foam in one hand, in the other hand to underwrite a faster establishment of the steady state, the three presented thermocouples measure the temperatures $T_0, T_1, T_2$ which correspond to temperatures measured; in the centre of the lower face of the heating element, on the unheated face of the sample and on the unheated side of the polyethylene, respectively.

Admitting the explained configuration one can write in steady state and supposing that the transfer by conduction is on one-dimensional:

$$\phi_0 = \phi_1 + \phi_2 \quad (1)$$

$\phi_0$, $\phi_1$ and $\phi_2$ are expressed as:

$$\phi_0 = \frac{U.I}{S} \quad (2)$$

$$\phi_1 = \frac{\lambda}{e_{plaster}} (T_0 - T_1) \quad (3)$$

$$\phi_2 = \frac{\lambda}{e_p} (T_0 - T_2) \quad (4)$$

The voltage $U$ applied through the heating element is estimated at an accuracy of 0.01 V.

By combining the previous equations we can write that:

$$\lambda_{plaster} = \frac{e_{plaster}}{T_0 - T_1} \left[ \frac{U.I}{S} - \frac{\lambda}{e_p} (T_0 - T_2) \right] \quad (5)$$

| Table 1. Dimensions of the elements of the circular HPS |
|-----------------------------------------|------|------|------|
| Element                                 | Heating element | polyethylene | Aluminium block |
| Dimensions (Diameter, thickness)        | 100x002 mm      | 100x10 mm    | 100x55 mm      |
3.2. Using the HPS to determine the thermal conductivity variation
In this study the HPS elements are having a circular form; it is inserted in an insulated box in order to ensure a temperature stabilisation during the experimentation. In order to guarantee an acceptable accuracy the circular HPS (CHPS) is relied to a voltmeter and an ammeter to measure respectively the tension and the current as indicated in Figure 1. From equation (5) we noticed that the thermal conductivity, which corresponds to a mean temperature $T_m = (T_0 + T_1)/2$ of the sample, is proportional to the electric tension applied to the heating resistance. In order to evaluate the evolution of the thermal conductivity versus temperature the tension $U$ is changed in the generator respecting a voltage step equal to 1V. For high mean temperatures $T_m$ and for each value of $U$ the thermal conductivity is measured once the steady state is established. However for low $T_m$ four ice accumulators are disposed around the circular HPS as illustrated in Figure 2, which enables to measure thermal conductivity in low temperatures.

4. Results and discussion
4.1. Determination of thermal conductivity variation with temperature
Table 2 presents the measured thermal conductivity of plaster in the temperature range [19°C; 48°C], it is clear that the thermal conductivity slightly increases from 0.146 W/m·K for 19.1 °C to 0.154°C for 47.7 °C.

### Table 2. Measured data using the circular HPS.

| Circular HPS Disposition | \( U \) (V) | \( I \) (A) | \( T_0 \) (°C) | \( T_1 \) (°C) | \( T_2 \) (°C) | \( T_m \) (°C) | \( \lambda_{\text{plaster}} \) (W/m·K) |
|--------------------------|-------------|-------------|----------------|----------------|----------------|----------------|----------------|
| With ice accumulators    | 12.08       | 0.119       | 26.148         | 11.584         | 12.058         | 19.103         | 0.146          |
|                          | 13.08       | 0.129       | 29.225         | 12.049         | 12.569         | 20.897         | 0.145          |
|                          | 14.09       | 0.139       | 32.678         | 12.780         | 13.224         | 22.951         | 0.144          |
|                          | 15.08       | 0.149       | 36.292         | 13.513         | 13.853         | 25.072         | 0.144          |
|                          | 16.08       | 0.159       | 39.749         | 14.111         | 14.267         | 27.008         | 0.146          |
|                          | 17.08       | 0.169       | 44.004         | 15.242         | 15.174         | 29.589         | 0.147          |
| Without ice accumulators | 10.08       | 0.100       | 36.316         | 26.213         | 26.082         | 31.199         | 0.145          |
|                          | 11.08       | 0.109       | 38.847         | 26.782         | 26.616         | 32.732         | 0.145          |
|                          | 12.08       | 0.119       | 41.895         | 27.628         | 27.462         | 34.679         | 0.147          |
|                          | 13.08       | 0.129       | 44.687         | 28.185         | 27.978         | 36.333         | 0.150          |
|                          | 14.09       | 0.139       | 47.986         | 28.907         | 28.692         | 38.339         | 0.151          |
|                          | 15.08       | 0.149       | 51.700         | 29.732         | 29.642         | 40.671         | 0.151          |
|                          | 16.08       | 0.159       | 55.324         | 30.366         | 30.332         | 42.828         | 0.152          |
|                          | 17.08       | 0.169       | 59.63          | 31.578         | 31.062         | 45.346         | 0.151          |
|                          | 19.08       | 0.189       | 65.133         | 30.205         |                | 47.669         | 0.154          |

4.2. Validation of the thermal conductivity variation

In order to validate the measured thermal conductivities values in the previous sub-section, other values were calculated knowing:

- The density: Calculated knowing the sample mass and apparent dimensions at dry state. The density is equal to 1002.909 kg/m³.
- The thermal diffusivity: measured using the flash technic [5] in a mean temperature of the sample 28°C and calculated adopting the global minimization procedures detailed in [6]. The calculated thermal diffusivity is 1.767 \( 10^{-7} \) m²/s.
- The specific thermal capacity: measured using the differential scanning calorimeter \( \mu \text{DSC evo7} \) designed for the study of samples in isothermal modes over a wide temperature range [-45°C; 120°C], in this study the specific heat is measured in the temperature range [5°C; 50°C], the values of \( C_p \) corresponding to the studied temperatures in CHPS were extracted from the measures in this temperature range and collected in Table 3.

### Table 3. Measured specific thermal capacity of gypsum plaster using the \( \mu \text{DSC evo7} \)

| \( T_m \) (°C) | \( C_p \) (J·kg⁻¹·K⁻¹) | \( T_m \) (°C) | \( C_p \) (J·kg⁻¹·K⁻¹) | \( T_m \) (°C) | \( C_p \) (J·kg⁻¹·K⁻¹) |
|----------------|------------------------|----------------|------------------------|----------------|------------------------|
| 19.103         | 782.287                | 29.589         | 798.178                | 38.339         | 812.942                |
| 20.897         | 785.679                | 31.199         | 801.329                | 40.671         | 816.865                |
| 22.951         | 788.561                | 32.7315        | 803.463                | 42.828         | 820.828                |
| 25.0725        | 791.691                | 34.6785        | 806.943                | 45.346         | 824.533                |
| 27.008         | 794.014                | 36.3325        | 809.904                | 47.669         | 827.57                 |
The thermal conductivity variation is then calculated using equation (6) and assuming that the thermal diffusivity and density remain constant versus temperature.

\[ \lambda(T) = \rho \times a \times c(T) \]  

(6)

Figure 2 presumes the thermal conductivity distribution of both measured and calculated values of thermal conductivity and the deviation percentage between them. It is shown that both values are having a positive trend with the same line slope $3.10^{-4}$ and a small difference of the intercept of 0.004. The deviation, measured using equation (7), goes from 1.83% to 5.29% with a mean value of 3.81%. These results validate the current exploitation of the HPS.

\[ \text{Deviation(\%)} = \left| \frac{\lambda_{\text{calculated}} - \lambda_{\text{measured}}}{\lambda_{\text{calculated}}} \right| \times 100 \]  

(7)

Figure 3: calculated and measured values of Gypsum plaster thermal conductivity versus temperature.

5. Conclusion
A straightforward exploitation of the hot plate method allowing the measurement of thermal conductivity variation with temperature of construction materials has been presented. The main interest of this study stays in its capability to perform a series of measurements in steady state ensuring a fast establishment of the thermal conductivity distribution versus temperature while conserving the same state of the sample. An experiment has been conducted on a sample of gypsum plaster enabling to compare its values with calculated ones; adopting experimental values of the specific heat variation with temperature and assuming a constant thermal diffusivity and density. We have given in this study the resulting deviations between calculated and measured values of thermal conductivity variation in the temperature range of [19°C; 48°C], this range is mostly located in interior temperatures of Moroccan buildings. A larger range could be studied using a heat source inside the box which contains the CHPS.

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