PARTON DISTRIBUTION WITHIN VIRTUAL PHOTON AND DIFRACTIVE PHOTOPRODUCTION IN DIS

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Abstract

A simple relativistic model is suggested that elucidates qualitatively the quark-antiquark distribution within virtual photon. The diffractive hadroproduction in DIS initiated by highly virtual photon $\gamma^*(Q^2)$ is discussed in more detail. The main result is that the contribution of large transverse scale $q\bar{q}$-fluctuations of the photon is just sufficient to produce the cross section of its hadron-like strong interaction (in particular, its diffraction into hadrons) of the same $Q^2$-dependence as the total cross section of $\gamma^*p$-interaction. It is why observed in DIS fraction of photon diffractive hadroproduction in $\gamma^*p$ interaction is quite large and does not vary with $Q^2$.

The unambiguous experimental evidence has been obtained [1] in DIS that the cross section of photon diffraction into hadrons in the process $\gamma^*(Q^2) + p \rightarrow hadrons$ is unexpectedly large even at $Q^2 \gg 1 \text{ GeV}^2$, and that its fraction in the total cross section of $\gamma^*p$ interaction is almost independent of $Q^2$. The data obtained at HERA, using the H1 detector, as well as their theoretical interpretation in terms of the proton diffractive structure function were summarized in ref. [2]. Meanwhile, being of essentially non-perturbative origination, these results unavoidably imply the significant role which play fluctuations of highly virtual space-like photon into hadronic states of sufficiently large transverse size. That is why the analysis in terms of the photon structure function seems to be more adequate. The account of rare fluctuations of such a type was shown would like to present a very simplified two-particle model which demonstrates that the mean transverse size of highly virtual space-like photon treated as a function of two variables,
$Q^2$ and $x$ ($x$ being the familiar Feynman variable - energy fraction carried by (anti)quark), is really expected to peak sharply at sufficiently small $x \sim Q^{-2}$, having irrespectively of $Q^2$ its maximum value about $1 \text{ GeV}^{-1}$. No doubt, these rare large scale parton configurations of the photon $\gamma^*(Q^2)$ should produce the diffraction pattern similar to that observed in the scattering of the real photon (vector mesons). Since the width of the peak is shown to be proportional to $Q^{-2}$, one can easily understand that their contribution to the cross section of $\gamma^*p$ interaction exhibits nearly the same $Q^2$-dependence as the bulk of short-range ("point-like") $\gamma^*p$ collisions. Being the striking qualitative effect, this feature of virtual photon interaction is expected to manifest itself also within the frameworks of more realistic approaches.

In the present communication, the $q\bar{q}$-distribution within space-like photon is linked to the quark structure of time-like hadronic states in the relevant channel. No unreliable perturbation theory calculations are invoked, however the nonperturbatively motivated treatment is paid by implication of some specific model assumptions.

The space-like virtual photon state is assumed to be related to the time-like $q\bar{q}$-states in the conventional quantum mechanical manner: the contribution of a given state is proportional to the (lightcone) time $\Delta t$ which this state "spends", having the energy of the virtual photon $\gamma^*(Q^2)$ (or vice versa), in the accordance with the energy-time uncertainty relation. The states to be allowed for incorporate the low laying resonances ($\rho_0, \omega, \text{etc.}$) as well as the continuum background which contributes predominantly to the parton distribution at large $Q^2$. It is easy to estimate that for the state with the eigenmass $M \gg 1 \text{ GeV}$ and momentum $\vec{P}$ ($|\vec{P}|/M \equiv P_z/M = \gamma \to \infty$)

$$\Delta t \sim \frac{1}{\sqrt{\vec{P}^2 + M^2} - \sqrt{\vec{P}^2 - Q^2}} \simeq \frac{2|\vec{P}|}{M^2 + Q^2}$$

Hence, if a certain wave function can be attributed to space-like $q\bar{q}$-fluctuation of four momentum squared equal to $-Q^2$, then the coefficients in its decomposition into the series (integral) over the eigenfunctions of time-like states are reasonably expected to be proportional to $(M^2 + Q^2)^{-1}$, and therefore, the parton distribution in the time-like state of mass $M$ contributes to that in the above space-like fluctuation with the weight
proportional to \((M^2 + Q^2)^{-2}\).

As a very simple realization of this guess, one can consider the two-particle model of spinless quark and antiquark of mass \(m_q\), interacting via the potential \(U\) of the simplest form: \(U = 0\) within the relativistically contracted (along the axis \(z\)) ellipsoid of transverse radius \(R\) about some few GeV\(^{-1}\) and \(U = \infty\) outside it. Since the highly excited states, \(4m_q^2 \ll M^2 \sim Q^2\) are of primary significance at large \(Q^2\), one can easily anticipate that what follows should be almost independent of the precise shape of potential within this ellipsoid, provided that the effectively non-transparent potential wall near its surface exists: being far above, these states do not "feel" the peculiarities of quark interaction therein. Of course, being associated with peculiarities of the potential \(U\), quantization of the mass \(M\) is irrelevant too.

The corresponding relativistic Shrödinger equation

\[
\left\{ \sqrt{p_1^2 + m_q^2} + \sqrt{p_2^2 + m_q^2} + U - \gamma M \right\} \psi(p_1, p_2) = 0
\]  

(2)

should be supplemented by the condition

\[
p_1 + p_2 = \vec{P}
\]  

(3)

The solution (more precisely, its absolute value) of Eq.(2) is obviously peaked near its classical limit:

\[
\sqrt{p_1^2 + m_q^2} + \sqrt{p_2^2 + m_q^2} - \gamma M = 0
\]

For the highly excited states, \(M \gg 2m_q\), which are just relevant, the quasiclassical approximation should be valid, i.e., the width of quantum distribution of particle momenta around this strict constraint is relatively small:

\[
\left| \frac{\sqrt{p_1^2 + m_q^2} + \sqrt{p_2^2 + m_q^2}}{\gamma M} - 1 \right| \approx \frac{\pi}{RM} \ll 1
\]

That is why in the context of the following discussion, the precise solution of Eq.(2) can be reasonably replaced by

\[
\psi(p_1, p_2) \approx \delta\left( \sqrt{p_1^2 + m_q^2} + \sqrt{p_2^2 + m_q^2} - \gamma M \right)
\]  

(4)

\(^1\)The many particle (evolution) aspect of the problem is briefly discussed below.
\(^2\)The exactly solvable example in support of the following estimate.
At $\vec{P}^2 \gg M^2$ this solution describes the eigenstates of the angular momentum $\hat{J}$ (in particular, $J = 1$) as well (for the moment, the normalization is put aside) because in the above limit the approximate relations $|\vec{p}_1| \simeq p_{1z}$, $|\vec{p}_2| \simeq p_{2z}$ and $J \simeq J_z$ are maintained, and therefore, operator $\hat{J}$ is essentially commutative with what is enclosed in the curly brackets of Eq.(2). Nevertheless, the selection of states with $\hat{J} = 1$ crucially affects the subject: since the radial excitations are in life only, the states of different masses $M$ contribute to the integral over $M$ with the same (independent of $M$) weight. Just because of this restriction, no factor of the type $\sum_{J=0}^{M/\alpha_0} (2J + 1)$ appears in the integrand in the middle of Eq.(6).

Making use the well known relativistic kinematics, one can replace the variables in Eq.(4) by the more convenient ones: Feynman variable $x \simeq p_{1z}/(p_{1z} + p_{2z}) \equiv p_{1z}/|\vec{P}|$ and, so-called, transverse mass $m_T = \sqrt{p_T^2 + m_q^2}$ where $p_T$ is the absolute value of quark transverse momentum. Being expressed in terms of new variables, Eq.(4) reads (as $\gamma \rightarrow \infty$)

$$\psi \sim \delta\left(\frac{m_T}{\sqrt{x(1-x)}} - M\right)$$

Thus, for the quark distribution $dW$ in a virtual state with $Q^2 \gg 1 \text{ GeV}^2$ one gets

$$\int \frac{dW}{d^3\vec{p}_1d^3\vec{p}_2}\delta^3(\vec{p}_1 + \vec{p}_2 - \vec{P})d^3(\vec{p}_1 + \vec{p}_2) \sim$$

$$\left| \int_{2m_q}^\infty \frac{\delta(m_T/\sqrt{x(1-x)} - M)dM}{M^2 + Q^2} \right|^2 = \frac{1}{[Q^2 + m_T^2/x(1-x)]^2}$$

The standard relativistic kinematics gives (as $\gamma \rightarrow \infty$)

$$d(p_{1z} - p_{2z}) \simeq \frac{\gamma m_T dx}{2x^{3/2}(1-x)^{3/2}}$$

and, finally, the normalized (and integrated over the azimuthal angle) quark distribution is obtained in the form:

$$\frac{dW}{dm_Tdx} \simeq \frac{4Q}{\pi x^{3/2}(1-x)^{3/2}} \frac{m_T^2}{[Q^2 + m_T^2/x(1-x)]^2}$$
This distribution contains much more information on the object, than the familiar inclusive structure function itself (the latter one is obtained by integration of Eq. (7) over \( m_T \) which results obviously in the loss of some important details: one can easily check that it is almost perfectly plateau-like except of very narrow intervals (\( \sim m_q^2/Q^2 \) near \( x = 0 \) and \( x = 1 \) where it tends to zero). The most essential feature of this distribution shows up, if one estimates the mean transverse size of the virtual \( q\bar{q} \)-fluctuation as the function of variables \( x \) and \( Q^2 \)

\[
< r(x, Q^2) > \simeq \frac{1}{m_T} \simeq \int_{m_q}^{\infty} \frac{dW}{m_T} \frac{dW}{dm_T dx} \simeq \frac{1}{\pi m_q} \frac{2y}{1 + y^2} \quad (8)
\]

where

\[
y = \frac{Q\sqrt{x(1 - x)}}{m_q}
\]

Thus, the mean transverse size of \( q\bar{q} \)-state \( < r(x, Q^2) > \) is peaked sharply at

\[
x(1 - x) = \frac{m_q^2}{Q^2}
\]

i.e., very close to \( x = 0 \), see Fig.1 (in the framework of the oversimplified two-particle model under consideration the peak near \( x = 1 \) is nothing else, than again the above one only associated with the second parton). The main point is that its maximum value is equal to

\[
r_{\text{max}} \simeq (\pi m_q)^{-1} \geq 1 \text{ GeV}^{-1}, \quad (9)
\]

irrespectively of \( Q^2 \), being about \( Q/4m_q \) times larger, than its typical "point-like" expectation (i.e., than its value at \( x \approx 0.5 \)). One can estimate the value of \( r_{\text{max}} \) under the well known assumption that the light quark masses are proportional to the distance between them, \( m_q \approx Kr \), where \( K \approx \frac{1}{2\pi} \text{ GeV}^{-2} \). Then, the typical scale of large-size \( q\bar{q} \)-fluctuations is \( r_{\text{max}} \simeq \sqrt{2} \text{ GeV}^{-1} \) (it is worthy to note that this scale is just about correlation radius for perturbative gluons motivated by the lattice studies \([3]\)), and corresponding quark mass \( m_q \simeq Kr_{\text{max}} \simeq 220 \text{ MeV} \). Since it is only slightly less, than the mass of constituent quark (valon), the effective radius of quarks in large-size \( q\bar{q} \)-fluctuations is just of the order of (or slightly less, than) that of valons which was estimated \([4]\) to
be about (1.0 - 1.8) GeV$^{-1}$. Thus, the large size $q\bar{q}$-fluctuations just make quark and antiquark spatially resolvable from each other, both color and electric dipoles becoming recognized and, hence, strong and electromagnetic interactions between $q$ and $\bar{q}$ coming in life.

The considered $q\bar{q}$-fluctuations can be reasonably related to the virtual photon $\gamma^*(Q^2)$ and its interaction with photon as it is shown in Fig. 2. The account of small size fluctuations ($q\bar{q}$, $e^+e^-$, etc.), $r \sim Q^{-1}$, is nothing else, than the dual ("microscopic") way of description of the "ordinary point-like" photon (in a sense, one can even say that they are not fluctuations at all), and perturbative treatment of these fluctuations is, at least, doubtful (the equivalence of both representations is emphasizes in Fig. 2(a) by matching them with an effective coupling constant about 1), while the large size (true) fluctuations, $r \sim r_{\text{max}}$, are coupled to the photon by the electromagnetic constant $\alpha$ but show up for it the hadron-like strong interaction with proton (an effective coupling constant about 1 in Fig. 2(b)). That is why in the process $\gamma^*p \to \text{hadrons}$ the virtual photon can be essentially thought of as an object described by the above $q\bar{q}$-distribution and coupled to the proton by the constant $\alpha$.

The large size $q\bar{q}$-fluctuations should show up the hadron-like strong interaction with the cross section which is caused by the degree of color descreening, i.e., is of the order of $\pi B(r) < r^2(x, Q^2) \simeq 0.6 B(r)/m_q^2$ where $B(r)$ is some unknown phenomenological non-transparency (blackness) factor, $0 = B(0) < B(r_{\text{max}}) \simeq 1$. Alongside with this, the probability of such fluctuations decreases from 1 to $\alpha$ due to the electrical charge descreening (see above). To get an estimate one can take $B(r) \equiv 0$ and the fluctuation probability equal to 1 at $r < r(0)$ (the electromagnetic "point-like" $\gamma^*p$ interaction shows up only) and $B(r) \simeq 1$ within the range $r_0 \leq r \leq r_{\text{max}}$, the $q\bar{q}$-fluctuation probability being proportional to $2\pi\alpha$ (strong $\gamma^*p$ interaction is dominated). Then, being integrated over $x$, the above cross section of hadron-like $\gamma^*p$ interaction is obviously proportional to $Q^{-2}$ (the width of the dashed domain in Fig.1), and therefore, its ratio to the cross section of the "point-like" $\gamma^*p$ interaction should be nearly independent of $Q^2$. It is easy

3This distribution resembles, to some extent, perturbatively motivated distributions although it is far from being identical with them.
to estimate that these two contributions are nearly equal at quite reasonable value of $r_0$, $r_0 \simeq 0.9\ r_{\text{max}} \simeq 1.25\ GeV^{-1}$. At the same time, the part of the virtual photon interaction associated with its hadron-like fluctuations should obviously show up the features of the real photon one. In this case the fraction of virtual photon diffractive hadroproduction observed in DIS could be easily understood, if one adopts that it is about 20% of the total cross section caused by its hadron-like interaction (just like for the real photon).

Of course, for being more realistic, the considered model needs some essential improvements. Nevertheless, its striking qualitative features can, most probably, stand against the necessary modifications which are of two kinds. First, even in the framework of the two-particle approximation one has to allow for the direct quark-antiquark interaction via gluonic exchange that should result in formation of an effective (linear in the "distance"
\[ \sqrt{4R^2 + \gamma^2(z_1 - z_2)^2} \] confining potential. As it was already mentioned, the relevant corrections do not affect noticeably the above results. Second, the many particle aspect of the problem is to be taken into account. In the spirit of the suggested approach it should be equivalent to the replacement of the above artificial non-transparent potential wall and corresponding infinite set of excited two-quark (time-like) states by the real many particle (quark and gluon) state continuum. One can reasonably anticipate that energy dissipation over the many degrees of freedom (at the same $M$) is only favorable for appearing the low $x$ partons and, consequently, for realization of the large size states. In terms of perturbation theory one can say that evolution and gluon bremsstrahlung (especially important is the latter one produced by the low $x$ quark or antiquark) enlarge the population of low energy partons and could lead, most probably, to an enhancement of the large scale effects. Anyway, the distributions (7) and (8) can be used as quite reasonable starting points for the calculation of more realistic parton distribution within highly virtual space-like photon. In particular, they can be chosen as the initial conditions which are well known to influence significantly the result of solution of the evolution equation. At the same time, the above discussions provides some insight to the essential complications that are to be faced in the program of usage of DIS as an effective tool for investigation

\footnote{The interference between corresponding amplitudes is undoubtedly unessential because of qualitatively different mechanism of the final state generation.}
of the short-range structure of hadrons.

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