Highly-stable generation of vector beams through a common-path interferometer and a DMD

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Abstract
Complex vector modes of light, non-separable in their spatial and polarisation degrees of freedom, are revolutionising a wide variety of research fields. It is therefore not surprising that the generation techniques have evolved quite dramatically since their inception. At present it is common to use computer-controlled devices, among which digital micromirror devices have become popular. Some of the reason for this are their low-cost, their polarisation-insensitive and their high-refresh rates. As such, in this manuscript we put forward a novel technique characterised by its high stability, which is achieved through a common-path interferometer. We demonstrate the capabilities of this technique experimentally, first by generating arbitrary vector modes on a higher-order Poincaré sphere, secondly, by generating vector modes in different coordinates systems and finally, by generating various vector modes simultaneously. Our technique will find applications in fields such as optical manipulations, optical communications, optical metrology, among others.

Keywords: complex vector beams, digital micromirror devices, stokes polarimetry

(Some figures may appear in colour only in the online journal)

1. Introduction

The on-demand manipulation of the properties of light, such as, amplitude, phase, polarisation or frequency, is amongst the most popular topics of late [1, 2]. In particular, complex vector modes, featuring a non-homogeneous transverse polarisation distribution, are attracting the attention of a wide research community due to their potential applications in fields such as, optical manipulations, high-resolution microscopy, optical metrology, classical and quantum communications, amongst others [3–11]. Such vector modes are mathematically described as a non-separable superposition of the spatial and polarisation degrees of freedom (DoFs) in a similar way to entangled states, a feature that has given rise to a wide variety of quantum-like analogies [9, 12–18]. In regards to the generation techniques, they have evolved quite dramatically since their inception, specially after the invention of liquid crystal spatial light modulators (SLMs), which enabled for the first time the on-demand generation of vector beams with arbitrary spatial shapes and polarisation distributions [19–24]. More recently, digital micromirror devices (DMD) gain popularity due to their high-speed refresh rates, polarisation-insensitive properties, as well as their low cost [25–35]. Their potential
has been demonstrated in the generation of vector modes in different coordinate systems [36–39]. Since the first DMD generation techniques implemented without taking advantage of their polarization-insensitive properties (see for example [28]), techniques have evolved in various ways. This is the case of the technique reported in [34], where the polarization-insensitive property of DMDs was exploited for the first time.

Here, a DMD is impinged with two expanded beams at opposite angles, $2^\circ$ and $-2^\circ$ respectively, bearing orthogonal polarisation. The DMD is then addressed with a multiplexed hologram encoding the two holograms that constitute the vector beam, each with a unique spatial grating to deflect the first diffraction order of each beam along the same propagation axis, where the vector beam is generated. In this approach it is crucial to place the DMD exactly at the plane where the two input beams cross each other.

In this manuscript we propose a novel generation technique which takes full advantage of the properties of DMDs and the high-stability feature of a common-path interferometric array. Even though a similar technique with SLMs has been previously demonstrated, this is limited to a maximum refresh rate of 60 Hz and to an input horizontal polarisation due to the inherent properties of SLMs [40]. On the contrary, our present technique does not require a specific input polarisation, even though, diagonal polarisation might be preferred. In addition, the DMD screen is not split into two sections but rather it incorporates a multiplexing approach to generate both constituting beams from a single hologram, which allows an easier alignment. Crucially, this technique also allows the simultaneous generation of multiple vector beams with independent properties, as we also demonstrated experimentally [20].

2. Theoretical considerations

2.1. Tailoring light with a digital micromirror device

As it has been widely demonstrated, DMDs have the capability to modulate the amplitude $A(x,y)$ and phase $\phi(x,y)$ of a complex optical field $U(x,y) = A(x,y)e^{i\phi(x,y)}$ through a binary hologram [41]. The transmission function that allows to encode the amplitude and phase information, has the specific form [28]:

$$ T(x,y) = \frac{1}{2} + \frac{1}{2} \text{sgn} \{ \cos[p(x,y)] + \cos[q(x,y)] \}, $$

where the information of the amplitude and phase are encoded in the terms $q(x,y)$ and $p(x,y)$, respectively, and computed through the expressions:

$$ q(x,y) = \arcsin \{ A(x,y)/A_{\text{max}} \}, $$

$$ p(x,y) = \phi(x,y) + 2\pi(vx + \eta y). $$

Here, $\text{sgn} \{ \cdot \}$ is the sign function and $A_{\text{max}}$ represents the maximum value of the amplitude $A(x,y)$. Further, the second term in $p(x,y)$ is an additional linear phase grating that controls the separation of the multiple diffraction orders according to the frequency specified by the parameters $\nu$ and $\eta$. As an example, figure 1 shows the binary holograms that are displayed on a DMD to generate, from left to right, the modes $LG^0_{10}$, $IG^1_{01}$, $PG$ and $MG^0_0$.

![Figure 1](image)

**Figure 1.** Example of the binary holograms encoded on a DMD to generate, from left to right, the modes $LG^0_{10}$, $IG^1_{01}$, $PG$ and $MG^0_0$.

3. Generation of highly-stable arbitrary vector modes

3.1. Experimental setup

The novel experimental technique to generate vector modes is based on a polarisation-insensitive Digital Micromirror Device (DMD, DLCR4710EVM-G2 with pixel size 5.4 $\mu$m from Texas Instruments) and a common-path interferometer of the Sagnac type, as schematically illustrated in figure 2. Here, a continuous wave laser beam ($\lambda = 633$ nm) with horizontal polarisation, expanded and collimated by a microscope objective MO and lens $L_1$ ($f = 300$ mm) to approximate a flat wavefront, is sent through a half-wave plate oriented at 22.5° (HWP) and rotates the polarisation state to +45°. Lenses $L_2$ ($f = 300$ mm) and $L_3$ ($f = 200$ mm) form a telescope to image the flat wavefront onto the DMD. In the DMD screen, two multiplexed binary holograms, which contain the amplitude and phase information of the constituting orthogonal
3.2. Polarisation reconstruction and concurrence

To reconstruct the transverse polarisation distribution, the Stokes parameter are determined from four intensity measurements according to the equations [45]

\[
S_0 = I_0, \quad S_1 = 2I_H - S_0, \quad S_2 = 2I_D - S_0, \quad S_3 = 2I_R - S_0,
\]

(3)

where \(I_R, I_L, I_H, I_D\) represent the intensities associated to right and left circular polarisation, horizontal and diagonal polarisation, respectively, with the total intensity represented by \(I_0\). Such intensities are measured with the help of QWP, a HWP and a LP, as schematically shown in figure 2 and recorded with the CCD camera (Stokes measurement stage). More precisely, the intensities associated to horizontal and diagonal polarisation are measured using a HWP2 with its fast axis orientated at 0° and 22.5°, respectively, followed by a linear polariser orientated at 0°. The intensities associated to the right and left circular polarisation are measured using QWP2 with its fast axis at −45° and 45°, respectively, followed by a linear polariser orientated at 0°. An example of the experimental Stokes parameters recontructed from such intensity measurements, are shown in figure 3(a) (top), compared with a numerical simulation (bottom), for the specific case of a radially polarised vector vortex mode. In general, a cylindrical vector vortex beam can be mathematically expressed as:

\[
U_{\ell p}^{LG} (\rho, \varphi) = \cos \theta \ LG_{\ell p}^*(\rho, \varphi) \hat{e}_r + e^{2i\alpha} \sin \theta \ LG_{\ell p}^\ast (\rho, \varphi) \hat{e}_\ell,
\]

(4)

where \(LG_p^\ast (\rho, \varphi) \propto (\rho/w_0)^l e^{-\rho^2/w_0^2} L_p^{(l)}(2\rho^2/w_0^2) e^{i\ell \varphi}\) is the Laguerre-Gauss beam at \(z=0\) [48]. \((\rho, \varphi)\) are the polar coordinates, \(w_0\) is the beam waist, \(\ell\) is known as the topological charge, \(p\) is the radial index and \(L_p^{(l)}(\cdot)\) is the associated Laguerre polynomial. For our particular case, we used the parameters \(\theta = \frac{\pi}{4}\), \(\alpha = 0\), \(\ell = 1\) and \(p = 0\).

In each spatial point over the transverse plane, we reconstructed the corresponding polarisation ellipse via the Stokes parameters. To achieve that, we computed the following quantities [45]:

\[
A = \sqrt{\frac{1}{2} \left( S_0 + \sqrt{S_1^2 + S_2^2} \right)}, \quad \gamma = \frac{1}{2} \arctan(S_2/S_1),
\]

(6)

\[
B = \sqrt{\frac{1}{2} \left( S_0 - \sqrt{S_1^2 + S_2^2} \right)},
\]

(7)

where \(A\) is the semi–major axis, \(B\) is the semi–minor axis, \(\gamma\) is the angular orientuation of the ellipse, and the sign of \(S_1\) determines the handedness of the polarisation state. The reconstructed polarisation for each studied vector beam, is plotted on top of the corresponding intensity. For example figure 3(b) shows the polarisation distribution of the vector mode \(U^{LG}_{10}\), with \((\theta, \alpha) = (\pi/4, 0)\), experiment on the top, numerical simulation on the bottom.
An additional example of the Stokes parameters and the reconstructed transverse polarisation distribution of a helical Mathieu–Gauss vector mode is shown in figure 3(d), experiment on top and numerical simulation on the bottom. This mode was experimentally generated as detailed in [38], where the parameters \( a = 0.56, k = 16.5 \times 10^3 \, \text{m}^{-1}, e = 0.9 \) and \( m = 6, \theta = \frac{\pi}{3} \) and \( \alpha = 0 \) were used.

The quality of the generated vector beams, was quantified through the degree of concurrence \( C \) defined as [46]:

\[
C = \sqrt{1 - \left( \frac{S_1}{S_0} \right)^2 - \left( \frac{S_2}{S_0} \right)^2 - \left( \frac{S_3}{S_0} \right)^2},
\]

with \( C \in [0, 1] \) where 0 is assigned to pure scalar modes and 1 to pure vector modes. Here \( S_j = \int_{-\infty}^{\infty} S_j dA \) is the integral of the Stokes parameters (equation (3)) over the entire transverse plane.

### 3.3. Experimental results

To exemplify the capabilities of our technique, first, we present a comparison between experimental results and numerical simulations of cylindrical vector vortex beams. Our setup allows a delicate control over the degree of concurrence and polarisation distribution, by digitally adjusting the parameters \( \alpha \) and \( \theta \). In figure 4 we show the results for the specific case \( U_{LG}^{1,0} \). Figure 4(a) shows the corresponding Higher–Order Poincaré Sphere (HOPS) with the modes \( LG_0^{1,0} \) and \( LG_0^{1,1} \) represented on the North and South poles, respectively. In such representation, all vector modes with varying degrees of concurrence and inter-modal phases are represented as points \((2\theta, 2\alpha)\) on the surface of the HOPS. The numbers from 5 to 8 correspond to the four vector modes shown in figure 4(b). In a similar way, the numbers from 5 to 8 correspond to the four vector modes shown in figure 4(c) with coordinates \((\theta, \alpha) = (0, 0), (\frac{\pi}{2}, 0), (\frac{\pi}{2}, \frac{\pi}{2}), (\frac{\pi}{2}, 0)\). In this case, the modes follow the path along a geodesic connecting the North and South poles. Notice the high similarity between the experimental and numerically simulated results.

As a second example, we demonstrate that our experimental setup can generate vector modes from different paraxial families. The transverse polarisation distribution overlapped
with the intensity profile of a representative set of vector modes, generated in different coordinate systems is shown in figure 5. Figures 5(a) and (b) show two cylindrical vector modes $U_{LG}^{−1,0}$ and $U_{LG}^{2,1}$, respectively. In figures 5(c) and (d) we show two helical Ince–Gauss vector modes $U_{LG}^{p,m,ε}$, where the indices $\{p,m\} \in \mathbb{N}$ follow the relation $0 \leqslant m \leqslant p$ for even functions and $1 \leqslant m \leqslant p$ for odd functions, while $ε$ represents the eccentricity. In this case, the spatial mode of each component in equation (2), takes the form of the helical Ince–Gauss modes with opposite handedness, as described in [36]. The parameters $\{p,m,ε\}$ for this specific instance are $\{5,3,2\}$ and $\{8,8,2\}$, respectively, and the intermodal phase and weighting parameter are $\{α,θ\} = \{0,\frac{π}{4}\}$. In a similar way, in figures 5(e) and (f) we show two helical Mathieu–Gauss vector modes $U_{MG}^{m,e}$, generated as outlined in [38]. The parameters we used for the first case are $a = 0.56$, $k_t = 16.5 \times 10^3$ m$^{-1}$, $e = 0.9$ and $m = 6$, whereas for the second case they are $a = 0.2$, $k_t = 15.5 \times 10^3$ m$^{-1}$, $e = 0.9$ and $m = 4$. For this mode, $a$ is the semi–minor axis of the elliptical coordinates, $k_t$ is the transverse component of the wave vector, $e$ is the eccentricity of the elliptical coordinates related to the semi–focal distance $f$ as $e = f/a$, and $m$ is the order of the vector mode.

Finally, in figures 5(g) and (h) we show the results for two travelling parabolic–Gauss vector modes $U_{TPG}^{n}$. In the first case $\{α,θ\} = \{0,\frac{π}{4}\}$, and for the second case $\{α,θ\} = \{\frac{π}{2},\frac{π}{4}\}$, that where generated with the parameters $n = 3$ and $k_t = 1.9 \times 10^5$ m$^{-1}$, for more details please refer to [39]. Notice the high similarity between the experimental and numerical simulations, which demonstrates qualitatively the high accuracy of this device. In addition, we measured the value of concurrence $C$ for each mode. The latter quantifies the accuracy of our device. Finally and with the aim of testing the multiplexing capabilities of our setup, we generated multiple vector beams with independent polarisation distributions and/spatial shape, using a single hologram. In figure 6(a) we...
show a representative set of nine different cylindrical vector modes $U^m_{\ell p}$, whose values $\ell$ and $p$ are given as insets in the top-right corner of each mode. In addition, their specific values $(\alpha, \theta)$ are given below each mode. In figure 6(b) we show a set of nine Ince–Gauss vector modes $U^{p,m,\varepsilon}_{\ell p}$ which evolve from polar to Cartesian-Gauss symmetry, by taking different values of eccentricity $\varepsilon$. Their specific values $(\varepsilon, \alpha, \theta)$ are also given in the bottom of each mode. As a technical note, the generation of the nine simultaneous vector modes implies the use of different values for the linear phase grating by adjusting $\nu, \eta$ in equation (1), for each mode. The nature of the DMD and the binary hologram programmed on the modulator, generate a vast amount of diffraction orders. The components of the desired set of vector modes, propagate along a common optical path and are recombined using a Sagnac interferometer, as described in the experimental setup (figure 2). Crucially, the background noise of the generated modes depends on the chosen spatial mode basis. For instance, the noise is higher when using the HIG than for LG spatial basis. This is due to the spatial filtering, since the generated modes are difficult to isolate from each other, and the diffraction orders may be close to the desired vector modes. In addition, if the vector modes are separated to reduce the noise, the spatial intensity distribution is degraded.

4. Conclusions

In this manuscript we proposed a novel technique to generate arbitrary complex vector modes. This technique is based on a polarisation insensitive DMD which is becoming very popular in the generation of vector modes as it is a low-cost device, polarisation insensitive and with high refresh rates. It is precisely its polarisation-insensitive property that is fully exploited in this proposal, which in combination with a common-path interferometer, makes it highly stable. In this technique, two diagonally polarised, orthogonal modes propagating at different angles are generated simultaneously from a single hologram displayed on the DMD screen. Both modes enter a triangular Sagnac interferometer, which is formed by a PBS and two mirrors. Each of the two modes is split into its horizontal and vertical polarisation components by the PBS, and after a round trip exit through a contiguous port. The two mirrors are adjusted to ensure the overlap between two of the modes with orthogonal polarisation, which generate the vector beam. We demonstrated this technique experimentally, first by generating arbitrary vector modes on a Higher-order Poincaré sphere; later by generating vector modes with arbitrary transverse profiles, and finally by generating several vector modes from a single hologram. This technique might be of high relevance in optical trapping techniques, which require fast reconfiguration of the vector beams [10].

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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Conflict of interest

The authors declare that there are no conflicts of interest related to this article.

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