A hybrid experimental validation of nonlinear energy-based control with noisy and biased measurements

Guanming Liang
Dalian Yuming Senior High School
*153205617@qq.com

Abstract. Current validation of feedback control law typically relies on simulations that may not capture the actual noise and bias characteristics or on the experiments that is more expensive and less manageable. This paper proposes a hybrid experimental strategy to validate the effectiveness of nonlinear feedback control law for attitude control of Unmanned Aircraft Vehicles (UAVs). We use the typical measurement device, Micro-Electro-Mechanic Systems (MEMS) gyroscope, with a turntable to capture the exact measurement noise and bias. We implement the gyroscope with different up-pointing axes and rotation speeds to imitate different bias. Then, these measured angular velocities are combined into the numerical simulation of a rigid body dynamics with feedback control law to examine the performance of the nonlinear feedback control law. Results show that trajectories of the closed-loop Euler angle are close to the designed reference trajectory and the estimated bias accurately reflecting the measured bias. This hybrid experiments validate the effectiveness of energy based nonlinear feedback control law with noisy and biased experimental measurements using a much less cost.

1. Introduction
The use of UAVs (Unmanned Aircraft Vehicles) in the military and civilian sectors have become increasingly widespread over the last few decades [1]. There are five main categories overall. UAVs are put to use in various areas including military defense, surveillance, and guidance. Depending on their sizes and payload, UAVs can be divided into five categories: full-scale, medium-scale, small-scale, mini-scale and micro-scale. UAVs with different sizes are put to use in different sectors [2].

Many researches have adopted the popular Kalman filter approach in positioning and navigating given its powerful estimation capabilities. What’s more, as Tereshkov [3] points out, with the Kalman filter, only a simple update procedure is needed to include various bias sources. However, other researches directly point out the inherent deficiencies and drawbacks of the Kalman-filtering method despite its advantages. There are better methods that are less complex than the Kalman filter, such as the moving average method proposed in Kirkko-Jaakkola et al. [4] and the direct complementary filter proposed in Tayebi and McGilvray [5]. In Tereshkov [3], the author specifies the reasons for its complexity. In details, the Kalman filter requires on-line computation of the error covariance matrix, which squares the number of necessary update operations at each time step. Secondly, the Jacobians of dynamics and measurement functions must be provided. Thirdly, the optimality and even the convergence of the filter are not guaranteed for nonlinear systems. Fourthly, the formal nature of the Kalman filter makes it non-intuitive and difficult to tune. Many researchers have been working on systems that reduce the complexity of the Kalman filter while enhancing its estimation capabilities [3].
Apart from the Kalman filter, other control laws have been proposed to stabilize the attitude of the UAVs with bias. The early use of Euler Angles for parametrization of the movement of the quadrotor in its fixed frame of reference comes with geometric singularities and nonlinearities of trigonometric function representations in 3×3 matrices. Therefore, many researchers start using 4-parameter quaternions to model the attitude of quadrotors. To further enhance its stability, a PD, LQR and back-stepping technique for handling complexity in four-parameter quaternions is proposed in Guerrero-Sánchez et al. [2] to decrease the bias and improve the performance of the attitude and position control laws.

Different methods had been proposed to handle such biases. In Thienel [6], researchers attempt to decrease the bias of gyroscope measurements by combining the results of three adaptive nonlinear observers for calibration of spacecraft gyros. Mahony et al. [7] also proposed the nonlinear complementary filter on the special orthogonal group to estimate the attitude and measurement bias. Past research had found various bias sources from the complex linear calculation method, sub-system failures and lack of failure accommodation and reconfiguration as well as biased measurements of position and attitude. Others also include inherent biases of the experiment itself, such as sensor bias (noise) and outer disturbances such as wind gusts in Pounds et al. [8] and effects of ambient temperature.

In order to re-balance these biases, researchers used MATLAB to initially simulate the various kinds of biases. Different calculation models for the biases were proposed according to different experimental data. Many researchers use the simpler nonlinear methods instead of the more traditional linearization method with more complex trigonometric functions. Bias and disturbances are also considered in Pounds et al. [8], where they proposed a combined non-linear control-estimator for Vertical Take-off and Landing (VTOLs) mini Aerial Vehicles (mAVs) based on previous filters by previous quaternion-based filters proposed by other authors. This improved filter help UAVs better characterize noises and maintain attitude stability despite of significant bias. However, Kirkko-Jaakkola et al. [4] points out its inaccuracy due to the constant change of components every time the gyroscope is powered up. Therefore, the article proposed calibrating the data at all times to predict future biases. They examined the bias instability and the 1/f noise process which is characterized by the constant variance of its bias. The co-authors use this property to predict the eventual bias. Other models suppose the bias to vary with respect to time, most prominently, the exponentially decaying model proposed in Thienel [6].

Theoretically, the stabilization of UAVs renders the energy as the Lyapunov function certifying stability of the system with control [5, 9, 10, 11, 12]. Similarly, Lizarralde and Wen [13] extended the passivity-based attitude (similar to energy-based) control strategies to the case in which there is no available angular velocity. This energy-based approach considers the nonlinearity into the consideration, but its effectiveness under practical environment with noisy and biased measurements typically requires experimental validation. However, there is currently not a low-cost approach to initially validate the feedback control law under experimental measurements, typically containing noise and bias.

In this paper, I propose a hybrid experimental approach to validate the effectiveness of the nonlinear feedback control law under noisy and biased measurements using much less cost. This approach obtains the noisy and biased measurements from the experiments but apply the feedback control law in the numerical simulation. I derived a feedback control law with attitude described by rotational matrices. This feedback control law selects the energy as the Lyapunov function certifying theoretically the asymptotic stability of the closed-loop system through Barbalat’s lemma. After simulations in section 3, I went on to collect data from the Micro-Electro-Mechanical Systems (MEMS) Inertial Measurement Unit (IMU) set on a turntable with different angular velocities and up-point axes. I thus displayed the similarity between the results in the closed-loop Euler Angle trajectory and the designed reference trajectory, thereby validating that the system under this specific control law can reach a state of stability despite noise and bias interference.

2. Basic theoretical derivation and algorithms

I follow the control laws proposed by Pounds et al. [8]. According to the equation of kinematics and dynamics of a rigid body:
\[
\dot{R} = R\Omega_x, \quad \text{and} \\
I\dot{\Omega} = \Omega_x I\Omega + \Gamma \tag{1}
\]

Then we denote \(R^d, \Omega^d, \tau^d\) as the desired (targeting) orientation, desired angular velocity, and the desire torque, respectively with the desired dynamics as:
\[
\dot{R}^d = R^d\Omega^d_x, \quad \text{and} \\
I\dot{\Omega}^d = \tau^d \tag{3}
\]

Furthermore, we denote the noise-free measurement of the angular velocity \(\tilde{\Omega}\) as the sum of the actual value and the bias; i.e., \(\tilde{\Omega} = \Omega + b\). With this notation, the orientation error \(\hat{R} = R^d\dot{R}\) evolves like:
\[
\dot{\hat{R}} = R^{dt}R + R^{dt}R\Omega_x \\
= (R^{d\Omega}_x)^T R + R^{dt}R\Omega_x \\
= -\Omega^d_x\dot{R} + \dot{\tilde{R}} \Omega_x \\
= \left[\tilde{R}, \Omega^d_x\right] + \tilde{R}(\Omega - \Omega^d)_x \tag{5}
\]

Thus, I can denote the estimation for \(b\) as \(\hat{b}\) and the estimation error as \(\hat{b} = b - \hat{b}\):
\[
\hat{R} = \left[\hat{R}, \Omega^d_x\right] + \tilde{R}(\Omega - \Omega^d - \hat{b})_x - \tilde{R}\hat{b}_x \tag{7}
\]

I will make further additions to \(\tilde{\Omega}\), for example, white noise. I define:
\[
\epsilon = \tilde{\Omega} - \Omega^d - \hat{b}, \\
I\hat{\Omega} = (I\Omega)_x + \Gamma, \text{ and} \\
I\hat{\Omega}^d = \tau^d. \tag{15}
\]

Through the substitution of values above, we have
\[
I\dot{\epsilon} = I(\tilde{\Omega} - \hat{\Omega}^d - \hat{b}) = (I\epsilon)_x \Omega - (I\hat{b})_x \Omega + (I\hat{\Omega}^d)_x \Omega + (I\Omega^d)_x \Omega + \Gamma - I\hat{b} - \tau^d \tag{16}
\]

Let
\[
\alpha = (I\hat{b})_x \epsilon, \\
\tilde{\Gamma} = \Gamma - \tau^d - (I\hat{b})_x \tilde{b} + (I\Omega^d)_x \tilde{b} - (I\tilde{\Omega}^d)_x \hat{b}, \\
I\dot{\epsilon} = -\left(\Omega + \tilde{b}\right)_x I\epsilon + \alpha + [\xi, \epsilon - (I\tilde{\Omega})_x] \tilde{b} + \tilde{\Gamma} - I\hat{b}, \text{ and} \\
\tilde{\Gamma} = -k_\epsilon \epsilon - k_R I^{-1} \text{vex} (Pa(\tilde{R})) - \tilde{a} + I\hat{b}. \tag{21}
\]

We define:
\[
\hat{\alpha} = k_\alpha \epsilon, \\
\hat{\tilde{b}} = -k_\xi k_R I^{-1} \text{vex} (Pa(\tilde{R})) + k_\phi ([\xi, \epsilon - (I\tilde{\Omega})_x] \tilde{b}, \tag{22}
\]

\[
V = \frac{k_R}{2} \text{tr} \left(I\epsilon - \tilde{R}\right) + \frac{1}{2} (I\epsilon)^T (I\epsilon). \tag{24}
\]

Using the property of the Lie Bracket \(\text{tr} [\tilde{R}, \Omega^d_x] = 0\) and that of skew-symmetric matrices \((I\epsilon)^T (\Omega + \tilde{b})_x (I\epsilon) = 0\), we have:
\[
\dot{V} = -\frac{k_\xi}{2} \text{tr} (R \epsilon_x - \tilde{R} b_x) + (I\epsilon)^T (\alpha + [\xi, \epsilon - (I\tilde{\Omega})_x] \tilde{b} - k_\epsilon \epsilon - k_R I^{-1} \text{vex} (Pa(A))), \tag{25}
\]

where \(Pa\) is an algorithm for the rotational matrices and \(\hat{\alpha} = \alpha - \hat{\alpha}\). Note that for any matrix \(A\) and any vector \(m\), we have
\[ tr(Am_x) = tr(Pa(A)m_x) = -2m^T vhx(Pa(A)). \] (27)

Thus, we get:
\[ \dot{\nu} = -k_R b^T vhx(Pa(\nu)) - k_e e^T I e + (Ie)^T (\hat{\alpha} + [\Omega_x I - (I\Omega_x)] b). \] (28)

According to the Lyapunov function:
\[ L = V + \frac{1}{2} b_k b_k^{-1} \nu + \frac{1}{2} \hat{\alpha}^T k_a^{-1} \hat{\alpha} \]
\[ \dot{L} = \dot{\nu} + \frac{1}{2} \dot{b}_k b_k^{-1} \nu + \dot{\hat{\alpha}} k_a^{-1} \hat{\alpha} = -k_e e^T I e \Rightarrow \dot{L} = 0 \] (29)
when \( \epsilon = 0 \). We use Barbalat’s lemma to ensure that the non-autonomous system is stable [14]. For \( \dot{L} = 0 \), it gives
\[ \hat{\alpha} = 0 \] (30)
\[ \dot{\nu} = k_a \nu \] (31)
\[ \hat{\nu} = \nu \nu \] (32)
\[ \dot{\hat{\nu}} = \frac{1}{2} \dot{b}_k b_k^{-1} \nu + \dot{\hat{\alpha}} k_a^{-1} \hat{\alpha} = -k_e e^T I e \Rightarrow \dot{L} = 0 \] (33)

According to \( L = constant \), we can easily get:
\[ \dot{b} = -k_\nu k_\nu \nu \nu (Pa(\nu)) \hat{\alpha} + [\Omega_x I - (I\Omega_x)] b + i^{-1} b_k^{-1} \nu = 0 \] (34)

The solutions for the differential equation are either \( b \to constant \) with \( \dot{\nu} = 0 \) or \( \nu \nu = 0 \) with \( \nu \to \infty \), under which \( L \) diverges that is impossible. Therefore, the asymptotic stability of the error of estimated bias \( \dot{\nu} \) and \( \hat{\alpha} \) are proved; i.e.,
\[ \dot{\nu} \to 0, \]
\[ \hat{\alpha} \to 0. \] (35)

3. Simulations

I simulated the changes in the quadrotor’s angular velocity and other related parameters in MATLAB with regard to the bias changing with respect to time. The data in the simulations come from a Wit-motion gyroscope. The time-invariant rotational inertia is:
\[ I = \begin{bmatrix} 0.0797 & 0 & 0 \\ 0 & 0.0797 & 0 \\ 0 & 0 & 0.149 \end{bmatrix} \] [8].

Let us denote the yaw as \( \psi \), the pitch as \( \theta \), and the roll as \( \phi \). The initial angle and angular velocity are \( \varphi = 10^5 \) and \( \theta = 0.1 rad/s \). The initial bias \( b \) is: \( [0.2 \ 0.1 \ -0.1]^T \). Using some of the critical algorithms and theoretical derivations in section 2, where bias related vectors
\[ \hat{\alpha} = k_a I e, \]
\[ \dot{\nu} = -k_\nu k_\nu \nu \nu (Pa(\nu)) + k_\nu ((I\Omega_x) - (I\Omega_x))Ie. \] (37)

We have the Euler’s equation of motion related to the torques imposed on the quadrotor to obtain a differential equation for angular velocity:
\[ \Gamma = \omega_0 \times L_0 \omega_0 + L_0 \omega_0, \] (39)
which can be separated into three components of the Euler angle–\( \varphi \), \( \theta \), and \( \psi \).

We also obtained the same \( \Gamma \) in a different form in the theoretical deduction in section 2:
\[ \Gamma = -k_\nu - k_\nu I^{-1} \nu \nu (Pa(\nu)) - \hat{\alpha} + I \dot{\hat{\nu}}, \] (40)
\[ \Gamma = \hat{\Gamma} + \tau_\nu + (I \dot{\hat{\nu}}) \nu - (I \Omega^d)_x \nu + (I \Omega^d)_x \nu. \] (41)

We can easily obtain the simultaneous solution for angular velocity based on these two equations. The central equation of the simulation is \( \Omega = \Omega + b + \nu \nu \), where \( b \) is the sensor bias and \( \mu \) is the noise. For bias estimation, I considered white noise as the sensor bias and changed the variance noise coefficient \( \sigma \) (this means variance of noise, initially set at 0.1) and \( k_a = k_\nu = k_\nu = 10 \) (initial value) to different values.
Based on this, I then used MATLAB to plot graphs of the angular velocity for the three Euler angles—yaw, pitch and roll—changing with respect to time, but nevertheless exponentially converging to the initial value of angular velocity given sufficient time. After changing the three scalar gains from 10 to 50, the graphs display a denser distribution with a sharp decrease in the amplitude of the decaying function around the eventual value. This signifies that the three variables respond more frequently and the bias of a larger scalar gain is thus smaller than before. This is similar to a simple harmonic oscillation where the period decreases as $k$ increases. On the whole, Figures 1-6 resembles damped oscillation where the amplitude gradually decreases to 0 and exponentially decays to a constant according to its mathematical form. When I changed the scalar gains $k_\varepsilon$ from 0.1 to 10, the graphs display a similar trend with a sharper decrease in its amplitude from the right beginning and shorter period.

4. Experiments and Data

The experiments were designed as following: I first let the gyroscope stay motionless and obtain its bias. Then I collected data from an Inertial Measurement Unit (IMU) placed on a turntable with three different speeds of 10, 15, and 30 seconds per round. Since the constant angular velocity of the turntable is known, I averaged the recorded angular velocity of the gyroscope and set the average of white noise $\mu$ at zero. Therefore, the average of the recorded angular velocity is the same as the real value (average) of $b$ ($b$ is a constant). After importing the data into MATLAB, I substituted $\hat{b} + \mu$ in the numerical relationship $\hat{\Omega} = \Omega + b + \mu$ in Section 2 (measured angular velocity) with the data I collected. Then I used MATLAB to plot graphs of the angular velocity. It is displayed as minor, rough and irregular edges on the graphs of angular velocity, $\hat{\Theta}$ and $\hat{\Phi}$ around the x, y and z axis. I then compared the graphs of the original simulations in section 3 with the graphs obtained from the collected data and discovered a close resemblance between the two, therefore verifying the validity of the algorithm proposed in section 2.

![Figure 1](image1.png)  
**Figure 1:** Estimated bias: Euler Angle, with (a), the gain in the feedback control law is set up as $k_a = k_b = k_R = 10, k_\varepsilon = 0.1$; with (b), the gain in the feedback control law is set up as $k_\varepsilon = 0.1$.  

![Figure 2](image2.png)  
**Figure 2:** Estimated bias: $\hat{\Theta}$, with (a), the gain in the feedback control law is set up as $k_a = k_b = k_R = 10, k_\varepsilon = 0.1$; Euler Angle, with (b), the gain in the feedback control law is set up as $k_\varepsilon = 0.1, k_a = k_b = k_R = 50$. 

4. Experiments and Data
As shown in the graphs, the previous theoretical simulations and actual results from the experiment display high similarity, specifically from the graphs of the Euler Angle and the desired Euler Angle with respect to the three Euler angles $\theta$, $\psi$, and $\phi$. The desired Euler Angle has smooth edges while the distribution of the data-based Euler Angle fluctuate in places where there are biases and noise. The fluctuation magnitude is positively correlated with the magnitude of the bias. Furthermore, the amplitude and period of the biases tend to be larger at the beginning and gradually converge to a constant.

Since I switched the turntable to three speed levels and collected experimental data for each up-pointing axis and speed, we can compare the magnitude of the biases for different speeds, which we easily perceive through the fluctuation rates. As shown in 5(b), 6(a) and 6(b), when the speed increases from 10 to 30, all with the same up-pointing axis, we get greater fluctuation amplitudes around the central line, whereas for common simulations, there is no fluctuations caused by any bias. The bias is also different when I compare different up-pointing axes rotating at the same speed. For example, when you compare the bias for all three up-pointing axes at the speed of 30, we can clearly observe that the magnitude of bias is the largest when revolving around the x axis, the second largest around the z axis and the smallest around the y axis.

There is a particular characteristic for $\hat{b}$. We can easily perceive the up-pointing axis through the figures of $\hat{b}$ because $\hat{b}$ has the largest absolute value for the component on the axis which faces upwards. The components with the largest absolute value for each case are always negative.

Here figure 15 shows the IMU (Inertial Measurement Unit) gyroscope and turntable that were used in the experiment.

![Figure 3: Estimated bias: $\hat{b}$, with (a), the gain in the feedback control law is set up as $k_\epsilon = 0.1$, $k_\alpha = k_\beta = k_R = 50$; $\hat{a}$, with (b), the gain in the feedback control law is set up as $k_\epsilon = 0.1$, $k_\alpha = k_\beta = k_R = 50$.](image)

![Figure 4: Euler angle with (a) and $\hat{b}$ with (b), the gain in the feedback control law is set up as $k_\alpha = k_\beta = k_R = k_\epsilon = 10$.](image)
Figure 5: (a) $\hat{a}$, the gain in the feedback control law is set up as $k_a = k_b = k_c = k_d = 10$; (b) Estimated bias Euler Angle with the x axis up-pointing and rotating speed $\pi/15 \text{ rad/s}$. The gain in the feedback control law is set up as $k_a = k_d = 10, k_c = 1, k_b = 30$.

Figure 6: Estimated bias Euler Angle with (a) the x axis up-pointing and rotating speed $2\pi/15 \text{ rad/s}$ and (b) the x axis up-pointing and rotating speed $\pi/5 \text{ rad/s}$ The gain in the feedback control law is set up as $k_a = k_d = 10, k_c = 1, k_b = 30$.

Figure 7: Estimated bias Euler Angle with (a) the y axis up-pointing and rotating speed $\pi/15 \text{ rad/s}$ and (b) the y axis up-pointing and rotating speed $2\pi/15 \text{ rad/s}$ The gain in the feedback control law is set up as $k_a = k_d = 10, k_c = 1, k_b = 30$. 


Figure 8: Estimated bias Euler Angle with (a) the y axis up-pointing and rotating speed $\pi/5 \text{ rad/s}$ and (b) the z axis up-pointing and rotating speed $\pi/15 \text{ rad/s}$. The gain in the feedback control law is set up as $k_R = k_\alpha = 10, k_\varepsilon = 1, k_b = 30$.

Figure 9: Estimated bias Euler Angle with (a) the z axis up-pointing and rotating speed $2\pi/5 \text{ rad/s}$ and (b) the z axis up-pointing and rotating speed $\pi/5 \text{ rad/s}$. The gain in the feedback control law is set up as $k_R = k_\alpha = 10, k_\varepsilon = 1, k_b = 30$.

Figure 10: Estimated bias $\vec{b}$ with (a) the x axis up-pointing and rotating speed $\pi/15 \text{ rad/s}$ and (b) the x axis up-pointing and rotating speed $2\pi/15 \text{ rad/s}$. The gain in the feedback control law is set up as $k_R = k_\alpha = 10, k_\varepsilon = 1, k_b = 30$. 
Figure 11: Estimated bias $\hat{b}$ with (a) the x axis up-pointing and rotating speed $\pi/5 \text{ rad/s}$ and (b) the y axis up-pointing and rotating speed $\pi/15 \text{ rad/s}$. The gain in the feedback control law is set up as $k_R = k_\alpha = 10, k_\epsilon = 1, k_b = 30$.

Figure 12: Estimated bias $\hat{b}$ with (a) the y axis up-pointing and rotating speed $2\pi/15 \text{ rad/s}$ and (b) the y axis up-pointing and rotating speed $10 \text{ seconds/round}$. The gain in the feedback control law is set up as $k_R = k_\alpha = 10, k_\epsilon = 1, k_b = 30$.

Figure 13: Estimated bias $\hat{b}$ with (a) the z axis up-pointing and rotating speed $\pi/15 \text{ rad/s}$ and (b) the z axis up-pointing and rotating speed $2\pi/15 \text{ rad/s}$. The gain in the feedback control law is set up as $k_R = k_\alpha = 10, k_\epsilon = 1, k_b = 30$. 
Figure 14: Estimated bias $\hat{\mathbf{b}}$ with the $z$ axis up-pointing and rotating speed $\pi/5 \text{ rad/s}$. The gain in the feedback control law is set up as $k_x = k_a = 10, k_e = 1, k_b = 30$.

(a) (b)

Figure 15: (a) Turntable; (b) Wit-Motion MEMS gyroscope and Bluetooth connector

5. Conclusion
In this paper, I propose a hybrid experiments strategy to explore the validity of the feedback control law derived through employing energy as a Lyapunov function. To demonstrate the stability of the quadrotor using this control law, I initially simulated the results using the feedback control law with different initial values, specifically, changing the scalar gains to study the influence of these gains on closed-loop system response. Then I collected data from the Wit-motion Micro-Electro-Mechanic Systems (MEMS) Inertial Measurement Unit (IMU) rotating on a turntable with different speeds and up-pointing rotating axes to imitate different bias in measurements. Although we can see that there are biases causing partial fluctuations, the general trend of the Euler angle is stable and gradually converging to the reference trajectory. The estimated bias $\hat{\mathbf{b}}$ is also reflecting the bias from gyroscope measurements. This is a form of hybrid experiment technique with both theoretical and experimental results indicating the validity of the feedback control law in maintaining the stability of the UAV throughout its flight.

References
[1] F. Kendoul. Survey of advances in guidance, navigation, and control of unmanned rotorcraft systems. Journal of Field Robotics, 29(2):315–378, 2012.
[2] M. E. Guerrero-Sánchez, H. Abaunza, P. Castillo, R. Lozano, and C. D. García-Beltrán. Quadrotor energy-based control laws: a unit-quaternion approach. Journal of Intelligent & Robotic Systems, 88(2-4):347–377, 2017.

[3] V. M. Tereshkov. An intuitive approach to inertial sensor bias estimation. International Journal of Navigation and Observation, 2013, 2013.

[4] M. Kirkko-Jaakkola, J. Collin, and J. Takala. Bias prediction for mems gyroscopes. IEEE Sensors Journal, 12(6):2157–2163, 2012.

[5] A. Tayebi and S. McGilvray. Attitude stabilization of a vtol quadrotor aircraft. IEEE Transactions on control systems technology, 14(3):562–571, 2006.

[6] J. K. Thienel. Nonlinear observer/controller designs for spacecraft attitude control systems with uncalibrated gyros. PhD thesis, 2004.

[7] R. Mahony, T. Hamel, and J.-M. Pflimlin. Nonlinear complementary filters on the special orthogonal group. IEEE Transactions on automatic control, 53(5):1203–1217, 2008.

[8] P. Pounds, T. Hamel, and R. Mahony. Attitude control of rigid body dynamics from biased imu measurements. In 2007 46th IEEE Conference on Decision and Control, pages 4620–4625. IEEE, 2007.

[9] J.-Y. Wen and K. Kreutz-Delgado. The attitude control problem. IEEE Transactions on Automatic control, 36(10):1148–1162, 1991.

[10] H. Liu, X. Wang, and Y. Zhong. Quaternion-based robust attitude control for uncertain robotic quadrotors. IEEE Transactions on Industrial Informatics, 11(2):406–415, 2015.

[11] C. Liu and L. Dong. Physics-based control education: energy, dissipation, and structure assignments. European Journal of Physics, 40(3):035006, 2019a.

[12] C. Liu and L. Dong. Stabilization of lagrange points in circular restricted three-body problem: A porthamiltonian approach. Physics Letters A, 383(16):1907–1914, 2019b.

[13] F. Lizarralde and J. T. Wen. Attitude control without angular velocity measurement: A passivity approach. IEEE transactions on Automatic Control, 41(3):468–472, 1996.

[14] H. K. Khalil. Nonlinear systems. Upper Saddle River, 2002.