Determining Exoplanetary Oblateness Using Transit Depth Variations

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Abstract

The measurement of an exoplanet’s oblateness and obliquity provides insights into the planet’s internal structure and formation history. Previous work using small differences in the shape of the transit light curve has been moderately successful, but was hampered by the small signal and extreme photometric precision required. The measurement of changes in transit depth, caused by the spin precession of an oblate planet, was proposed as an alternative method. Here, we present the first attempt to measure these changes. Using \textit{Kepler} photometry, we examined the brown dwarf Kepler-39b and the warm Saturn Kepler-427b. We could not reliably constrain the oblateness of Kepler-39b. We find transit depth variations for Kepler-427b at 90.1% significance (1.65\sigma) consistent with a precession period of $P_{\text{prec}} = 5.45_{-0.37}^{+0.46}$ years and an oblateness of $f = 0.19_{-0.16}^{+0.32}$. This oblateness is comparable to solar system gas giants and would raise questions about the dynamics and tidal synchronization of Kepler-427b.

\textit{Key words:} planetary systems – techniques: photometric

1. Introduction

The determination of the oblateness ($f$) of an exoplanet would shed light on that planet’s internal structure, dynamics, and formation history. Specifically, it may be possible to empirically constrain the rotation rate, $P_{\text{rot}}$, obliquity, $\theta$, and gravitational zonal quadrupole moment, $J_2$, of an exoplanet.

Two methods for determining the oblateness of an exoplanet from transit photometry have been proposed. As a result of the planet being slightly aspherical, an oblate exoplanet’s transit light curve differs slightly from that of a perfectly spherical planet with the same cross-sectional area; this effect occurs primarily during the ingress and egress phases of the transit (Hui & Seager 2002; Seager & Hui 2002; Barnes & Fortney 2003). Efforts to observe this effect have met with mixed results. Carter & Winn (2010a) used \textit{Spitzer Space Telescope} photometry to constrain the oblateness of HD 189733b to be less than that of Saturn, the most oblate solar system planet. A similar search through \textit{Kepler} candidates conducted by Zhu et al. (2014) yielded a tentative detection of oblateness for the $\sim 20 M_{\oplus}$ object Kepler-39b (KOI 423.01) and constraints on the oblateness of three other \textit{Kepler} candidates. These efforts were hampered by the short duration of the expected signal and its relatively small amplitude, $\sim 200$ ppm for Saturn-like oblateness and $\sim 2$ ppm for a hot Jupiter (Carter & Winn 2010b).

A second signal of planetary oblateness was identified by Carter & Winn (2010a). If the oblate planet has non-zero obliquity, its spin-axis will precess, and the projected area of the planet, and hence the observed transit depth, will change over the period of that precession. For planets as oblate as Jupiter or Saturn, the transit depth may change by $\sim 1\%$, or $\sim 100$ ppm for a Jupiter-like planet around a Sun-like star (Carter & Winn 2010b). Carter & Winn suggest that this is a more feasible observable, within the precision likely to be achieved by \textit{Kepler}.

In this paper, we present an attempt to observe this signal in \textit{Kepler} photometry. We begin with a summary of the expected signal in Section 2, discuss our transit depth measurement method in Section 3, describe our oblateness detection technique in Section 4, present results in Section 5, and discuss our findings in Section 6.

2. Planetary Oblateness and the Transit Signal

Following Carter & Winn (2010b), we consider the planet to be an oblate spheroid. The oblateness, or flatness, is

$$f = \frac{R_{\text{eq}} - R_{\text{pol}}}{R_{\text{eq}}} ,$$

where $R_{\text{eq}}$ and $R_{\text{pol}}$ are the equatorial and polar radii, respectively. For rotationally induced oblateness,

$$f \approx \frac{3}{2} J_2 + \frac{1}{2} \frac{R_{\text{eq}}^3}{G M_p} \left( \frac{2 \pi}{P_{\text{rot}}} \right)^2 ,$$

where $M_p$ is the planet mass (Murray & Dermott 1999). The obliquity, $\theta$, is the angle between the planet’s orbit-normal and the spin-axis of the planet. We define $\phi(t)$ as the azimuth of the spin-axis projected onto the orbit-plane. For the case of uniform precession,

$$\phi(t) = \frac{2 \pi (t - t_0)}{P_{\text{prec}}} ,$$

where $P_{\text{prec}}$ is the precession period and $t_0$ is a phase offset. This precession causes the projected area of the exoplanet to change, changing the planet-to-star area ratio, $\delta(t)$. Carter & Winn (2010b) derive

$$\delta(t) = k^2 \sqrt{1 - \epsilon^2} \left[ 1 - \sin \theta \cos \phi \sin i + \cos \theta \cos i \right]$$

for the areal ratio of the planet to the star, where $k = R_{\text{eq}}/R_\star$ is the planet-to-star radius ratio, $i$ is the transit inclination, and $\epsilon$ is the ellipticity,

$$\epsilon = \sqrt{1 - (1 - f)^2} .$$
From Equation (4), the authors show that the amplitude of the transit depth variations is determined by a combination of oblateness and obliquity:

$$\left(\frac{\delta_{\text{max}}}{\delta_{\text{min}}}\right)^2 - 1 \approx 2f \sin^2 \theta,$$

(6)

where the approximation is valid when $f$ is small and $i \approx 90^\circ$. While the amplitude of depth variations can be readily determined from the photometric data, there is a degeneracy between $f$ and $\theta$. Since the obliquity determines the extent that the aspect angle of the planet changes over the course of its precession, even very high oblateness can be compensated by low obliquity, producing low amplitude variations. This degeneracy cannot be broken without additional constraints (Carter & Winn 2010b).

The timescale of the change in transit depth is determined by $P_{\text{prec}}$, which, for uniform precession in a fixed orbit, is $2 \pi/ (\alpha \cos \theta)$, where

$$\alpha = \frac{3}{2} \left(\frac{2 \pi}{P_{\text{orb}}}ight)^2 \left(\frac{P_{\text{rot}}}{2 \pi}\right) J_2 \lambda$$

(7)

is the precession constant (Ward 1975; Ward & Hamilton 2004) and $\lambda = 1/(M_p R_p^3)$ is the normalized moment of inertia of the planet. Using $\lambda/J_2 = 13.5$, estimated for Saturn (Ward & Hamilton 2004), we obtain the scaling relation

$$P_{\text{prec}} = 13.3 \text{ years} \times \left(\frac{P_{\text{orb}}}{15 \text{ days}}\right)^2 \left(\frac{10 \text{ hr}}{P_{\text{rot}}}\right) \left(\frac{\lambda/J_2}{13.5}\right) \left(\frac{1}{\cos \theta}\right).$$

(8)

As this relation shows, the stronger torques from the host star on planets with short periods reduce the precession period. This enables easier detection. But this effect must be balanced against the tidal synchronization of a planet by its host star. If the planet is too close, tidal interaction with the star will slow its rotation period to its orbital period, greatly diminishing any rotationally induced oblateness. The rate at which a planet’s spin, $\omega$, is diminished is

$$\frac{d\omega}{dt} = -\frac{9}{4} \frac{GM_p^2}{M_* Q_p} \frac{R_p^3}{a^6 \lambda},$$

(9)

where $Q_p$ is the planet’s tidal dissipation factor, $M_*$ is the stellar mass, $G$ is the gravitational constant, and $a$ is the semimajor axis of the planet’s orbit (Goldreich & Soter 1966). Integrating this equation with an initial rotation period of $P_{\text{rot},i}$ yields a spin-down time of

$$\tau_{\text{spin}} = 1.22 \text{ Gyr} \times \left(\frac{M_p}{M_{\text{Jup}}}\right) \left(\frac{Q_p}{10^{6.5}}\right) \left(\frac{\lambda}{0.25}\right) \left(\frac{P_{\text{orb}}}{15 \text{ days}}\right)^4 \left(\frac{R_p}{R_{\text{eq}}}\right)^3 \left(\frac{10 \text{ hr}}{P_{\text{rot},i}}\right) - \left(\frac{10 \text{ hr}}{P_{\text{rot},i}}\right).$$

(10)

Both the tidal synchronization timescale and precession period are strongly dependent on the orbital period.

2.1. Candidate Selection

Based on the above properties of the expected signal, we selected a handful of candidates to scrutinize for evidence of transit depth variations. We began with an expansion of the “sweet spot” identified by Carter & Winn (2010b). Carter & Winn suggested that candidates with $P_{\text{orb}} \approx 15$–30 days were likely to have both precession periods observable over Kepler’s planned six-year mission and spin-down timescales of ~1 Gyr. Because of the shorter actual duration of the primary Kepler mission, and the considerable uncertainty in spin-down time estimates, we included planets within a period range of 10–30 days in our search.

To select gas giants, we required $R_p > 6 R_{\text{Jup}}$. We then restricted our search to confirmed planets only. This was primarily motivated by an estimated false-positive rate for Kepler giant planet candidates with $P < 400$ days of 54.6 ± 6.5% (Santerne et al. 2016). It also allowed for independent determination of stellar and planetary parameters, particularly the mass of the planet.

At the time of the study, only nine planets matched these criteria. To avoid complication from overlapping transits and transit timing variations, we also eliminated the five confirmed multiplanet systems, leaving four candidates. From these four, we selected Kepler-39b (KOI 423.01) and Kepler-427b (KOI 192.01), detailed in Table 1.

### Table 1. Candidate Systems

| Kepler-39 | Kepler-427 |
|-----------|------------|
| Star mass $M_*$ [$M_\odot$] | 1.26±0.07 | 0.96±0.06 |
| Star radius $R_*$ [$R_\odot$] | 1.25±0.03 | 1.35±0.20 |
| Orbital period $P_{\text{orb}}$ [day] | 21.087 | 10.291 |
| Semimajor axis $a$ [au] | 0.162±0.003 | 0.091±0.010 |
| Planet mass $M_p$ [$M_{\text{Jup}}$] | 19.1±1.0 | 0.29±0.09 |
| Planet radius $R_p$ [$R_{\text{Jup}}$] | 1.11±0.03 | 1.23±0.21 |

**Note.** Values for Kepler-39 from the circular orbit model in Bonomo et al. (2015). Kepler-427 parameters from Hebrard et al. (2014).
et al. 2013). Despite this, some quarters (or months, in the case of short cadence photometry) still exhibited an overall slope, especially in data with pronounced “ramp up” events. Assuming this long-term trend to be artificial, we de-trended each quarter (or month) using a degree 2 polynomial.

3.2. Light Curve Normalization with Starspots

Cool starspots on the disk of the star produce quasi-periodic variations in stellar flux that can complicate the interpretation of transit light curves (see Czesla et al. 2009; Carter et al. 2011; Sanchis-Ojeda et al. 2013). As illustrated in Figure 1, starspots outside the transit chord reduce the total flux from the star. But since the flux blocked by the planet is unchanged, the relative transit depth is increased. Czesla et al. (2009) found that correcting for this can change the transit depth by ~1%—comparable to the expected signal from oblateness.

We follow the method outlined by Czesla et al., normalizing the difference between the measured flux and estimated stellar flux by the estimated “unspotted” flux:

\[ F_{\text{norm},i} = 1 + \frac{F_i - F_{\text{star},i}}{F_{\text{unspotted}}} \]

where \( F_i \) is the measured flux value at a time \( t_i \), \( F_{\text{star},i} \) is the estimated stellar flux at that time, \( F_{\text{unspotted}} \) is the estimated flux from the star showing a “clean” photospheric surface, and \( F_{\text{norm},i} \) is the resulting normalized flux at \( t_i \). We modeled \( F_{\text{star},i} \), the stellar flux local to the transit, by fitting a low-order (typically degree 2) polynomial to out-of-transit (OOT) data on either side of the transit. Accurate determination of \( F_{\text{unspotted}} \) was more challenging; it is difficult to determine when, if ever, we observe a clean disk. We took the maximum observed flux over each quarter (or month for short cadence photometry) as \( F_{\text{unspotted}} \) for that interval. To avoid an estimate biased by transient brightening events, the maximum was taken from a running average of the observed flux. This approach assumes that any variation in stellar brightness over the quarter is caused by starspots and not by an overall change in luminosity.

An example of the fit of a local stellar flux model, subsequent normalization of the transit, fit to a transit model, and resulting residuals is shown in Figure 2.

3.3. Changes in Crowding

Contamination of the photometric aperture by light from other stars also complicates accurate measurement of the transit depth. The PDC-MAP pipeline corrects this “crowding,” but time-varying errors in this procedure can produce changes in the measured transit depth. The process is analogous to starspots; the flux blocked by the planet is constant, but the estimated total flux from the star changes, yielding a change in the normalized transit depth. Van Eylen et al. (2013) found that the transit depth of the hot Jupiter HAT-P-7b varied by ~1% from season to season, orders of magnitude higher than the observed precision of transit depth measurements within a season. The authors suggested several possible causes, including incorrect crowding correction. Subsequently, Gandolfi et al. (2015) found significant seasonal changes in transit depths measured from the PDC-MAP photometry of Kepler-423 (KOI 183). Since the changes in transit depths were highly correlated \( p = 0.15\% \) with the quarterly crowding metric and were absent in the uncorrected SAP photometry, these authors attributed the changes entirely to the PDC-MAP crowding correction.

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3 The Kepler Space Telescope performed an attitude adjustment four times per year to keep its solar panels pointed sunward. This causes each target to fall on a different CCD each “season.”

4 Each change in Kepler season marks a new quarter, so data from Q1 and Q5 are from the same season, one year apart.
3.4. Measuring Transit Depth

Transit models were then fit to the normalized light curves in a two-step process. We first phase-folded ~20 transits and fit a typical quadratic limb-darkening transit model (Mandel & Agol 2002) using \( \chi^2 \) minimization. The initial values were taken from the NASA Exoplanet Archive.\(^5\) Then, to extract the transit depths, we fixed the limb-darkening model and orbital parameters, and we fit between one and a few transits with only the radius ratio, \( p \), and transit mid-time, \( t_0 \), as free parameters. The estimated 1\( \sigma \) error in each depth measurement was determined from propagation of the PDC pipeline estimated photometric error. The estimated transit depth errors varied from star to star, with a typical value of ~1\%. Transits were rejected if the data coverage over the transit was too sparse or large data gaps prevented an accurate fit to the out-of-transit flux.

\(^5\) http://exoplanetarchive.ipac.caltech.edu

4. Determination of Oblateness

We examined the time series of measured transit depths for each candidate for signs of transit depth variations attributable to the spin precession of an oblate planet. To begin, we converted the series of measured transit depths into a time series of fractional transit depth variation, \( TTV \), from the mean observed depth.\(^6\) Because transit depths may vary systematically from season to season in Kepler photometry (Van Eylen et al. 2013), we normalized the depths from each season by the mean of the observed depths from that season:

\[
TTV = \frac{\delta_{\text{obs,s}} - \delta_{\text{obs,s}}}{\delta_{\text{obs,s}}},
\]

where the index \( s \) indicates the season.

We constructed a corresponding model of the relative transit depth variation, \( TTV_m \), using Equation (4) in Section 2, in which we substituted for \( \phi \) and \( \epsilon \) using (3) and (5), respectively. As can be seen from these equations, this model has the precession period, \( P_{\text{pre}} \); oblateness, \( f \); obliquity, \( \theta \); transit inclination, \( i \); and a phase offset, \( t_0 \), as free parameters. For a given choice of these parameters, we used Equation (4) to calculate the modeled transit depth, \( \delta_m \), at the observed transit times. We then normalized the modeled depths from each season by the mean of the modeled depths from that season as in Equation (12). Because we expect the planet-to-star radius ratio, \( k = R_\text{pl}/R_\star \), and the stellar limb-darkening model to remain constant over the observation period, this normalization by \( \delta \) makes both the observed and modeled \( TTV \) series independent of these parameters.

4.1. Fitting the \( TTV \) Model

Fits of this model to transit depth time series are generally not unique. High oblateness, for example, can be compensated by very low obliquity or very slow precession. We explored this parameter space using emcee, a Markov Chain Monte Carlo Ensemble sampler written by Foreman-Mackey et al. (2013). We used uniform priors over the appropriate physical range (e.g., \( f \in [0, 1] \)) for every model parameter but the transit inclination. The inclinations of our candidates were constrained by a normal prior based on the results of previous studies. For Kepler-39b we used \( \mathcal{N}(89.23, 0.014) \) (Bonomo et al. 2015), while for Kepler-427b our prior was \( \mathcal{N}(89.50, 0.25) \) (Hébrard et al. 2014).

We also restricted the precession period. First, we imposed a lower bound of 2 years. This is lower than expected for a Saturn-like planet on the inner edge of our period range with modest rotational oblateness, \( P_{\text{rot}} = 18 \text{ hr} \Rightarrow f \approx 0.05 \), and reduced the likelihood of overfitting to sparse data or detecting a spurious signal associated with Kepler’s ~1 year orbit. Second, because of the finite duration of the observations, a planet model with significant oblateness can match even unvarying transit depth measurements if compensated by a long enough precession period. To avoid this degeneracy, we required that the precession period be short enough that we could have observed a rise and fall in the measured transit depths. Given the ~4 year duration of the Kepler mission, and that the transit depth variations peak twice per precession period, our final requirement was \( P_{\text{pre}} \in [2, 16] \) years.

\(^6\) This definition is slightly different from that introduced in Carter & Winn (2010), who normalized by the minimum transit depth.
To test the robustness and reliability of our MCMC approach, for each candidate we performed two injection and recovery tests. The first test was a spherical planet. Because oblateness is a positive-definite quantity, our marginalized posterior distributions of \( f \) will have a positive bias (see, for example, Zakamska et al. 2011). The spherical planet test gauged the extent of the bias and probed for other possible false-positive signals. The second injected planet was Saturn-like. It had an oblateness \( f = 0.1 \), an obliquity \( \theta = 30^\circ \), the same transit inclination as the real candidate, and a precession period determined by Equation (8), assuming the same orbital period as the candidate, a 10 hr rotation period and \( \lambda/2 = 13.5 \). In both cases, the planets were scaled so that the transit depth matched that measured for the real candidate.

We ran our MCMC analysis with 200 parallel chains taking 15,000 steps. We discarded the first \( \sim 10,000 \) steps as the burn-in period. We assessed convergence by inspection of the walker trajectories and by splitting the remaining part of the chain in half and comparing the resulting posteriors. The resulting MCMC model was determined by taking the median of the marginalized posterior distribution of each parameter with the 16th and 84th percentiles taken as approximate \( 1\sigma \) bounds.

4.2. Statistical Significance

The likelihood function was calculated using the errors estimated for each transit as described in Section 3. These estimates, however, may underestimate the true error. We performed a series of injection and recovery tests to assess our measurement accuracy and gauge the resulting false-positive rate.

4.2.1. Simulated Transit Light Curves

When Carter & Winn (2010b) investigated the detectability of transit depth variations in simulated Kepler photometry, they used a Gaussian white-noise model with a simulated transit light curve. They found that a single transit’s depth could be recovered to within \( \approx 0.6\% \) for a star with a Kepler magnitude of 13. With the full 4 years of data in hand, we attempted to improve this estimate by injecting transits into real Kepler photometry.

For each candidate, we constructed a series of simulated Kepler data sets using the photometry for that star. First, we removed all known transits from the data. We considered the remaining photometric data to consist of the true stellar flux plus some noise-induced offset: \( F_{\text{meas}} = F_{\text{true}} + F_{\text{offset}} \). We calculated the running average of the photometry and took this as our model for the stellar flux, \( F_{\text{true}} \). We then added artificial Mandel & Agol (2002) transit models to the stellar flux: \( F_{\text{true}} \rightarrow F_{\text{true}} \). The simulated data were then generated by applying the same offset that was present in the real data: \( F_{\text{sim}} = F_{\text{true}} + F_{\text{offset}} \).

After generating the artificial Kepler observations, we then fed the data into our analysis pipeline. We tested our analysis using both a constant and variable planet-to-star radius ratio to generate the injected transit models. For each star we generated \( \sim 1000 \) simulated transits and compared our pipeline’s estimated error, derived from the Kepler pipeline’s reported photometric uncertainty, to the actual accuracy obtained from comparing the recovered transit parameters to the injected ones. We found that propagation of the photometric precision tends to underestimate the transit depth measurement error. An example of the error distribution is shown in Figure 3 for Kepler-39 (KOI 423).

4.2.2. Simulated Transit Depth Series

Next, we investigated the false-positive rate for oblateness detection using synthetic \( T \) vs \( V \) data. We generated a \( T \) vs \( V \) time series for a spherical planet in two ways, by sampling from the error distributions described above and by bootstrapping from the observed \( T \) vs \( V \) series. To account for the possibility of seasonally correlated data, we used block bootstrapping: synthetic data for each season were resampled from that season’s observations. For each time series, we then calculated the \( \Delta \chi^2 = \chi^2_{\text{obs}} - \chi^2_{\text{true}} \) value for the minimum-\( \chi^2 \) oblate planet model and a spherical planet model. These values were used to create both a simulated and bootstrapped distribution of \( \Delta \chi^2 \) values for that star. Given an oblate model fit to Kepler data and two \( \Delta \chi^2 \) distributions from simulated \( T \) vs \( V \) series, we calculate \( p \)-values from the fraction of simulated \( \Delta \chi^2 \) values higher than the one obtained by the model and report the more conservative estimate.

5. Results

Of the nine candidates matching our period criteria and having \( R_p > 6 R_J \), we selected two, Kepler-39b (KOI 423.01) and Kepler-427b (KOI 192.01), for close evaluation based on their large transit depths (\( \sim 1\% \)) and independently measured masses. The results are presented below and summarized in Table 2.

5.1. Kepler-39b (KOI 423.01)

Zhu et al. (2014) conducted a search for oblate planets using Kepler short cadence photometry to identify deviations from the light curve of a perfectly spherical transiting planet. The \( \sim 20 M_J \) Kepler-39b (KOI 423.01) was the only object found with likely non-zero oblateness. Though the measured projected oblateness was high, \( f_{\perp} = 0.22 \pm 0.11 \), the authors
cautioned that the finding might not be robust due to inconsistency in the best-fit models when different subsets of short cadence transits were examined.

With this tentative detection, and an orbital period of approximately 21 days, Kepler-39b presented a promising target for the T8V method. We fit normalized transit models to 46 of the 69 transits spanning 3.93 years of long cadence Kepler photometry. Kepler-39 is a 1.29 M$_\odot$ star with a Kepler band magnitude of 14.33 (Bonomo et al. 2015). Due to the relative faintness of the star, the long cadence data has a photometric precision of ~200 ppm. The standard deviation of errors in our T8V signal, as determined by our injection and recovery test, was $\sigma = 173$ ppm (see Figure 3). We found no correlation between the measured transit depth and the Kepler quarterly crowding metric.

The measured relative T8V and the best-fit model are plotted in Figure 4. For visual clarity, we omitted the separate seasonal normalizations and show the variations relative to the mean of all the observed depths. The MCMC analysis yielded modest oblateness, $f = 0.13^{+0.04}_{-0.03}$, with a rapid precession period of $P_{\text{prec}} = 2.04^{+0.02}_{-0.02}$ years. The full model is detailed in Table 2, and the posterior distributions are shown in Figure 5. The precession period is tightly bound at the lower edge of the range of allowed periods. The joint $f$-$\theta$ distribution shows the expected degenerate relationship (Equation (6)), with zero oblateness excluded and low oblateness allowed only at very high obliquities.

The cumulative distribution of $\Delta \chi^2$ values for the simulated ensemble of spherical planet models is shown in Figure 11. The recovered oblate model scored 61.54, corresponding to $p = 0.002$, or significant at the $3.09\sigma$ level.

We created similar $\Delta \chi^2$ distributions based on the simulated data we generated for injected spherical and Saturn-like planets. The $p$-values obtained from these distributions are also shown in Figure 11. The recovered model from the injected spherical planet had $p = 0.37$, or $0.90\sigma$, indicating an oblate model provides no statistical advantage over a spherical planet model. The absence of a robust detection is corroborated by the posterior distributions shown in Figure 6. For $f$ and $f_0$, we recovered our Gaussian and uniform priors, respectively. The posteriors for $f$ and $\theta$ display the expected degeneracy, while the joint distribution shows that the data support models with near-zero oblateness even for modest obliquity. The $P_{\text{prec}}$ posterior spans the allowed range but has a broad peak in probability density centered near 6 years.

The recovered model for the Saturn-like planet was also not statistically significant, with $p = 0.75$, corresponding to $0.32\sigma$. As before, the posterior distributions (Figure 7) reflect the absence of a detection, while in this case the $P_{\text{prec}}$ posterior shows unexpected narrow peaks at periods of 2 and 3 years. The failure to detect the injected Saturn-like planet was not surprising, given that the injected period, $P_{\text{prec}} = 30.35$ years, exceeds the upper limit we imposed on the precession period, but the high probability density at low periods was unexpected. This anomaly in the precession period posterior for the Saturn-like case occurs at roughly the same period as the signal in the real photometry, suggesting that detection may be spurious.

5.2. Kepler-427b (KOI 192.01)

Kepler-427 (KOI 192), is a 0.96 $\pm$ 0.06 M$_\odot$ star hosting a 0.29 $\pm$ 0.09 M$_J$ planet on a 10.3 day orbit (Hébrard et al. 2014). Like Kepler-39, Kepler-427 is a relatively dim star with

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### Table 2

| Planet          | $P_{\text{prec}}$ (year) | $f$    | $\theta$ ($^\circ$) | $i$ ($^\circ$) | $f_0$ (year) |
|-----------------|---------------------------|--------|---------------------|---------------|--------------|
| Kepler-39b      | 2.04$^{+0.02}_{-0.02}$    | 0.13$^{+0.04}_{-0.03}$ | 50.65$^{+25.86}_{-21.01}$ | 89.24$^{+0.12}_{-0.12}$ | $-0.12^{+5.44}_{-5.38}$ |
| Kepler-427b     | 5.45$^{+0.17}_{-0.16}$    | 0.19$^{+0.04}_{-0.03}$ | 13.62$^{+7.95}_{-5.5}$   | 89.44$^{+0.46}_{-0.46}$ | 0.31$^{+8.21}_{-5.33}$ |

**Kepler-39 (KOI 423.01) Simulated Planets**

| Spherical (Injected) | ... | 0.0 | 0 | 89.23 | ... |
|---------------------|-----|-----|---|------|-----|
| Spherical (Recovered)| 6.74$^{+4.14}_{-4.2}$ | 0.15$^{+0.03}_{-0.11}$ | 17.60$^{+31.14}_{-12.21}$ | 89.23$^{+0.12}_{-0.12}$ | $-0.21^{+5.66}_{-5.38}$ |
| Saturn-like (Injected) | 30.35 | 0.10 | 30.0 | 89.23 | 13.71 |
| Saturn-like (Recovered) | 7.46$^{+4.99}_{-4.14}$ | 0.14$^{+0.03}_{-0.11}$ | 11.69$^{+27.60}_{-8.85}$ | 89.23$^{+0.12}_{-0.12}$ | 0.02$^{+5.44}_{-4.41}$ |

**Kepler-427 (KOI 192.01) Simulated Planets**

| Spherical (Injected) | ... | 0.0 | 0 | 89.50 | ... |
|---------------------|-----|-----|---|------|-----|
| Spherical (Recovered)| 5.60$^{+3.38}_{-3.2}$ | 0.12$^{+0.03}_{-0.10}$ | 9.64$^{+24.61}_{-7.53}$ | 89.5$^{+0.44}_{-0.44}$ | 0.00$^{+5.36}_{-4.46}$ |
| Saturn-like (Injected) | 7.23 | 0.10 | 30.0 | 89.50 | 1.26 |
| Saturn-like (Recovered) | 8.11$^{+3.78}_{-3.2}$ | 0.20$^{+0.03}_{-0.10}$ | 19.68$^{+10.87}_{-8.09}$ | 89.35$^{+0.51}_{-0.50}$ | 0.43$^{+5.05}_{-4.48}$ |

**Note.** The reported values are the medians along with the 16th and 84th percentiles.

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![Figure 4](image-url)  
**Figure 4.** Transit depth variation relative to mean of all observed depths for Kepler-39b. The data are plotted in blue with estimated 1$\sigma$ uncertainty. The red solid line is the best-fit oblate planet model.
a Kepler band magnitude of 14.22 (Hébrard et al. 2014), resulting in photometric precision of approximately 185 ppm. From recovery of simulated transits we obtained a standard deviation of errors of $\sigma = 140$ ppm (see Figure 12). Again, we find no significant correlation between the Kepler quarterly crowding metric and the observed transit depths.

We fit 104 long cadence transits spanning nearly 4 years of Kepler photometry, with the resulting relative TTV measurements and the best-fit model plotted in Figure 13. The oblate planet model had $\Delta \chi^2 = 35.80$, which, in comparison to the distribution shown in Figure 14, yields $p = 0.099$, or 1.65$\sigma$. From the MCMC posterior distributions (Figure 8) we obtained $f = 0.19^{+0.32}_{-0.16}$ with a precession period of $P_{\text{prec}} = 5.45^{+0.46}_{-0.37}$ years.

The model recovered for the simulated spherical planet matches the expectation for unvarying transit depths. The posterior distributions (Figure 9) largely recover the input priors, with the familiar degenerate structure for $f$ and $\theta$. The posterior distribution of precession period shows a short-period peak analogous to that recorded in the Saturn-like simulation for Kepler-39b. The oblate model fit is not significant, scoring $p = 0.818$, or 0.23$\sigma$, indicating no improvement on a spherical model.

In the case of the simulated Saturn-like planet, the oblate model is a highly significant improvement on a spherical planet fit, achieving $p < 0.002$, or >3.09$\sigma$. In addition, with the exception of the precession period, all the input parameters

Figure 5. Posterior probability distributions from MCMC analysis of Kepler-39b transit depths. Contours are drawn at 0.5$\sigma$ intervals from 0.5–2.0$\sigma$ and the plots have been smoothed to remove noisy features.
were recovered in the estimated 68% confidence interval (see Figure 10). The precession period was only narrowly outside the interval. The poorly resolved period is likely due to the 4 year duration of TV data compared to the 7.23 year injected period. We recovered $f \sin^2 \theta$ to within 12.5% of the input value, but, in part due to the $f-\theta$ degeneracy (Equation (6)), we only recovered $f$ to within 50% and $\theta$ to within 35%.

The recovery of an injected candidate and the absence of a false-positive detection in the null simulation indicate that the 1.65σ confidence level of the recovered signal in the real Kepler data is likely valid.

To confirm that the observed change in transit depth was not a byproduct of our transit normalization technique, we repeated our analysis without the starspot correction (Section 3.2). Kepler-427 only showed starspot-like brightness fluctuations in a few quarters. As a consequence, the average change in recovered transit depths was only $\sim 8$ ppm, less than 0.1% of the transit depth. The change in measured depths affected the model recovered from our MCMC analysis, but for each parameter, the recovered values with or without the starspot correction are mutually consistent—the median value obtained with one method is within the estimated 1σ uncertainty of the median value obtained with the other normalization. In particular, the measured precession period changed by $\sim 0.5\%$, while $f \sin^2 \theta$ changed by $\sim 7\%$. Finally, removing the starspot correction marginally diminishes the statistical

Figure 6. Posterior probability distributions from MCMC analysis of transit depths from spherical planet transits injected into Kepler-39 photometry. Contours are drawn at 0.5σ intervals from 0.5–2.0σ and the plots have been smoothed to remove noisy features.
significance of the detection, yielding \( p = 0.112 \), or \( 1.59\sigma \). Based on these findings, we conclude that the observed TdV signal is not an artifact of our normalization procedure.

6. Discussion

In this paper, we present the first attempt to detect the rotational oblateness of an exoplanet through long-term changes in transit depth. We examined confirmed Kepler planets with periods between 10 and 30 days and radii greater than \( 6 R_\oplus \) for signs of transit depth variations consistent with the spin-axis precession of an oblate planet. Of the nine planets matching our criteria, we selected two for close analysis, Kepler-39b (KOI 423.01) and Kepler-427b (KOI 192.01). We searched for an oblate signature using a Markov Chain Monte Carlo approach and assessed the significance of the recovered oblate planet fits.

6.1. Kepler-39b (KOI 423.01)

Although we detected oblateness \( (f = 0.13^{+0.12}_{-0.04}) \) with high significance in Kepler-39b, based on the additional injection and recovery tests, this detection is likely a false positive. The posterior distributions of precession period recovered from the true and Saturn-like simulated Kepler-39 photometry

![Figure 7. Posterior probability distributions from MCMC analysis of transit depths from Saturn-like planet transits injected into Kepler-39 photometry. Contours are drawn at 0.5\( \sigma \) intervals from 0.5–2.0\( \sigma \) and the plots have been smoothed to remove noisy features.](image-url)
Figures 5 and 7 both show excess probability mass at low periods. The recovered 2.04 year period is exactly twice the 372.5 day *Kepler* orbit and suggests an explanation. For $i \approx 90^\circ$, the TV peaks have comparable amplitude, creating a quasi-periodic signal matching the *Kepler* year. While we have corrected for sharp changes in transit depth at quarter boundaries caused by the spacecraft’s seasonal rolls (see Section 3.3), other systematics have been observed. In a study of M giant variability, Bányai et al. (2013) found one-quarter of their targets exhibited smooth flux variations with 372.5 day periods. The high statistical significance ($p = 0.002, 3.09\sigma$) of the observed transit depth variations may reflect detection of this effect or another similar systematic.

The physical plausibility of such rapid precession casts additional doubt on our detection. Assuming the detected oblateness to be rotationally induced, we can calculate the required rotation rate and estimated precession period using Equations (2) and (8). Bonomo et al. (2015) recently updated the parameters for the Kepler-39 system. They find that the previously measured eccentricity may be spurious, so we adopt the values from the circular model for simplicity: $M_p = 19.1 \pm 1 M_J$ and a mean radius $R_m = \sqrt{R_{eq} R_{pol}} = 1.11 \pm 0.03 R_J$. For

![Figure 8](image-url)
planets in hydrostatic equilibrium, we can apply the Darwin–Radau approximation (Murray & Dermott 1999):

\[ \frac{J_2}{f} = -\frac{3}{10} + \frac{5}{2} \lambda - \frac{15}{8} \lambda^2. \]  

(13)

Assuming, conservatively, that \( \lambda = 0.4 \), corresponding to a uniform density sphere, then \( J_2 = 0.052 \) and the required rotation period to produce the measured oblateness is \( P_{rot} = 2.7 \) hr. The resulting precession period is \( P_{prec} = 87 \) years. Under the more common assumption that \( \lambda \approx 0.23 \) for giant planets, \( P_{prec} \approx 150 \) years. Relaxing the Darwin–Radau constraint, assuming the observed oblateness can be supported at \( P_{rot} \sim 5 \) hr, and that \( \cos \theta \sim 1 \), the observed precession period requires \( \lambda/J_2 \sim 0.5 \), considerably less than the value of 13.5 for Saturn (Ward & Hamilton 2004).

The presence of exomoons around Kepler-39b could alter the effective value of \( \lambda/J_2 \) and drive faster precession; Saturn’s satellites reduce its calculated precession period by a factor of four (Carter & Winn 2010b). The effective value of \( \lambda/J_2 \) is given by \( (\lambda + l)/(J_2 + f) \), where

\[ l = \sum_i \frac{m_i}{M_P} \left( \frac{a_i}{R_{ext}} \right)^2 \frac{P_{rot}}{P_{orb,i}}, \]  

(14)

Figure 9. Posterior probability distributions from MCMC analysis of transit depths from spherical planet transits injected into Kepler-427 photometry. Contours are drawn at 0.5\( \sigma \) intervals from 0.5–2.0\( \sigma \) and the plots have been smoothed to remove noisy features.
$j = \frac{1}{2} \sum_i \frac{m_i}{M_p} \left( \frac{a_i}{R_{eq}} \right)^2 \frac{\sin(\theta - I_i)}{\sin \theta}$.

Figure 10. Posterior probability distributions from MCMC analysis of transit depths from Saturn-like planet transits injected into Kepler-427 photometry. Contours are drawn at 0.5σ intervals from 0.5–2.0σ and the plots have been smoothed to remove noisy features.

where $m_i$, $a_i$, $P_{\text{orb},i}$, and $I_i$ are the satellite’s mass, semimajor axis, orbital period, and inclination relative to the planet’s equator (Ward & Hamilton 2004). Prograde satellites on circular orbits are only stable with a semimajor axis $a \lesssim 0.4895 R_{\text{Hill}}$ (Domingos et al. 2006; Schlichting & Sari 2008). Under the generous (and inconsistent) assumptions that $\lambda = 0.1334$ (corresponding to $J_2 \approx 0$ in the Darwin–Radau relationship), $J_2 = 0.052$, and that the satellite has $a = 0.4895 R_{\text{Hill}}$, then the

required reduction in $\lambda/J_2$ is achieved at $m \approx 5 M_\oplus$. By comparison, Jupiter and Saturn’s largest satellites, Ganymede and Titan, have masses of 0.025 $M_\oplus$ and 0.022 $M_\oplus$, respectively (Showman & Malhotra 1999; Jacobson et al. 2006), and each orbit at $\sim 0.02$ times the Hill radius of their respective hosts. Modeling satellite formation around gas giants, Canup & Ward (2006) find that competing processes naturally limit satellite systems to a total mass of $\sim 10^{-4} M_p$, or $\sim 0.6 M_\oplus$ for Kepler-39b. A massive satellite of Kepler-39b, therefore, seems to be an unlikely explanation for the observed precession period.

If our detection is indeed a false positive, these results are consistent with the findings of Zhu et al. (2014). Using their
assumptions and measured values of $f_0 = 0.22 \pm 0.11$ and $\theta_0 = -40^\circ$, we obtain an expected precession period of $\sim 100$ years, much longer than could be detected from 4 years of Kepler observations. Updated measurements of the Kepler-39 system (Bonomo et al. 2015) have reduced the age estimate from $5.1 \pm 1.5$ Gyr to $1.0^{+0.7}_{-0.5}$ Gyr (or $2.1^{+0.8}_{-0.5}$ Gyr if the observed eccentricity is real), greatly decreasing the time available to de-spin the planet and bolstering the claim that the rapid rotation could be primordial.

6.2. Kepler-427b (KOI 192.01)

We detected variation in the transit depths of Kepler-427b consistent with moderate oblateness ($f = 0.19^{+0.22}_{-0.16}$) at a significance level of $1.65\sigma$. In contrast to Kepler-39b, we do not believe this to be a data artifact. While the simulated null case does show an excess of probability density at short precession periods, there is no indication of this signal in the recovered distributions for the Saturn-like case or the real data. Furthermore, the solutions found for the real data and Saturn-like planet are not located in the same region of parameter space as the signal found for the null case.

If the observed signal is caused by spin precession, then armed with the measured oblateness and precession period, and assuming the Darwin–Radau approximation, we can infer a range of values for $\lambda$, $J_2$, and $P_{\text{rot}}$. Adopting $M_\text{p} = 0.29 \pm 0.09$ $M_\oplus$ and $R_\text{p} = 1.23 \pm 0.21$ $R_\oplus$ from Hébrard et al. (2014), we find $\lambda \simeq 0.16^{+0.08}_{-0.02}$, $J_2 \simeq 9000^{+3000}_{-3000} \times 10^{-6}$, and $P_{\text{rot}} \simeq 15^{+22}_{-12}$ hr. By comparison, Jupiter has $J_2 = 14695.62 \pm 0.29 \times 10^{-6}$ and
a rotation rate of 10 hr.\(^7\) The detailed interior structure of Jupiter is still uncertain, but common model-derived values for the normalized moment of inertia are \(\lambda = 0.26387 - 0.26394\) (Hubbard & Militzer 2016). Saturn, with nearly the same mass as Kepler-427b, has \(J_2 = 16290.71 \pm 0.27 \times 10^{-6}\) (Jacobson et al. 2006) and a rotation period of 10.7 hr. Hubbard & Marley (1989) present an interior model for Saturn with \(\lambda = 0.22037\), while Ward & Hamilton (2004) propose a dynamical origin for Saturn’s obliquity that requires \(0.2233 < \lambda < 0.2452\). Most of the range of our inferred values is outside that expected from solar system gas giants, but there is some overlap.

The rotation period we measure is significantly shorter than the orbital period of the planet. Using Equation (10), assuming an initial rotation period of 9 hr, near the breakup rotation rate, and an optimistic tidal dissipation factor, \(Q = 10^{6.5}\), we calculate that only \(\sim 10\) Myr are required to spin down Kepler-427b to the rotation periods inferred by our measurement. This is much shorter than the estimated system age of \(7 \pm 4\) Gyr, an apparent contradiction. Equation (10) is highly approximate and is derived assuming a circular orbit and a planet with its spin-axis aligned to the orbit-normal. Planets on eccentric orbits do not directly synchronize, instead entering a planet with its spin-axis aligned to the orbit-normal. Planets on eccentric orbits do not directly synchronize, instead entering a quasi-synchronous spin state while they retain significant eccentricity (Dobbs-Dixon et al. 2004). Hébrard et al. (2014) were not able to significantly measure the eccentricity of Kepler-427b, providing only a 99% upper bound of \(e < 0.57\). Kepler-427b lies in the “period-valley”—the sparsely populated region between hot Jupiters and more distant gas giant planets (\(10 \lesssim P \lesssim 85\) days)—making tidal circularization a precarious assumption. Hébrard et al. suggest that follow-up measurements could more precisely constrain the orbital eccentricity, stellar age and radius, and the planetary radius, potentially resolving the spin-down time problem.

6.3. Other Causes of Transit Variations

Changes in the transit geometry can also alter the measured transit depth. To confirm our 90.1% confidence level we examined several possible sources of false positives.

6.3.1. Nodal Precession

If Kepler-427b is undergoing sufficiently rapid nodal precession—the precession of the orbital plane—the impact parameter of the transit would change during our observations. In conjunction with limb darkening, this would change the measured transit depth over time. The nodal precession rate is given by

\[
\omega_{\text{nodal}} = -\frac{3}{2} \frac{R_p^2}{a} \frac{J_{2,\ast}}{(1 - e^2)^3} \frac{2\pi}{P_{\text{orb}}} \cos \psi, \tag{16}
\]

where \(\psi\) is the angle between the orbit-normal and the spin-axis of the star and \(J_{2,\ast}\) is the star’s zonal quadrupole moment (Murray & Dermott 1999; Barnes et al. 2013). For the Kepler-427 system, \(R_* = 1.35R_J\), and \(a = 0.091\) au (Hébrard et al. 2014). For the stellar quadrupole moment, we adopted the Sun-like value \(J_{2,\ast} \sim 10^{-7}\) (Ulrich & Hawkins 1981; Mecheri et al. 2004). Assuming, conservatively, that \(\cos \psi \approx 1\) and that the eccentricity is near the upper bound, \(e = 0.57\), we obtain \(|\omega_{\text{nodal}}| \sim 10^{-15}\) rad s\(^{-1}\). This is a nodal precession period of \(\sim 40\) Myr, too slow to account for the observed changes in transit depth.

6.3.2. Apsidal Precession

If Kepler-427b is on an eccentric orbit, then apsidal precession, or periastron precession, would change the star–planet distance during the transit over time. If the transit is not observed edge-on, this would cause the transit impact parameter, and observed transit depth, to change over time. Ragozzine & Wolf (2009) calculated the expected apsidal precession rate for a hot Jupiter. Following their formulation, we find the dominant term is precession driven by the planet’s rotational bulge:

\[
\omega_{\text{rot,p}} = \frac{k_{2,p}}{2} \frac{R_p^3}{a} \left(\frac{2\pi}{P_{\text{rot}}}\right)^2 \times \frac{a^3}{GM_\star(1 - e^2)^2} \left(\frac{2\pi}{P_{\text{orb}}}\right)^2, \tag{17}
\]

where \(k_{2,p}\) is the planet’s Love number. Assuming \(e = 0.57\), \(P_{\text{rot}} = 15\) hr, and a Saturn-like Love number, \(k_{2,p} = 0.4\) (Lainey et al. 2017), we calculate \(\omega_{\text{rot,p}} = 3.2 \times 10^{-11}\) s\(^{-1}\). To include the remaining terms from Ragozzine & Wolf, we assumed a stellar rotation period of \(\sim 10\) days, appropriate for Sun-like stars with ages of \(\sim 1\) Gyr (do Nascimento et al. 2014), and consistent with \(v\sin i_k = 3 \pm 1\) km s\(^{-1}\), measured by Hébrard et al. (2014). Following Ragozzine & Wolf, we adopted a stellar Love number \(k_{2,\ast} = 0.03\). Under these assumptions, the total precession induced is \(\omega_{\text{apspidal}} = 4.5 \times 10^{-11}\) rad s\(^{-1}\). This includes, in order of diminishing importance, the effects of the rotational bulge of the planet, the tidal bulge on the planet, general relativity, the rotational bulge of the star, and the tidal bulge on the star. The corresponding apsidal precession period of 4.4 kyr is too long to explain the observed change in transit depth.

6.3.3. Three-body Interactions

The preceding calculations only considered a two-body system. Dynamical interactions with another planet in the Kepler-427 system could drive faster precession or create secular variation in the eccentricity or inclination of Kepler-427b’s orbit. While we cannot definitively exclude this possibility, we found no evidence of timing variations in the transits of Kepler-427b, consistent with the TTV survey of the entire Kepler data set undertaken by Holczer et al. (2016). Additionally, no evidence of another companion is reported in the HARPS-N spectrograph observations conducted by Hébrard et al. (2014).

Given the expected long precession periods and lack of evidence for strong interactions with another body, we conclude that changes in orbital geometry are not the cause of the observed transit depth variations and that the 90.1% confidence estimate is reliable.

6.4. Conclusion

We have presented the first attempt at detecting the oblateness of an exoplanet through changes in transit depth caused by spin precession. We examined two planets in detail.
While we were unable to detect the oblateness of Kepler-39b, this is broadly consistent with the findings of Zhu et al. (2014), given the expected long precession period. We find transit depth changes consistent with an oblateness comparable to solar system gas giants for Kepler-427b, but with a significance of only 90.1% (1.65σ). Kepler-427b is a warm Saturn in the period-valley $P \approx 10–85$ days, a class of planets with an unclear formation mechanism (Santerne et al. 2016). Confirming and improving this oblateness detection and further constraining the bulk properties of Kepler-427b would illuminate the planet’s internal structure, possibly providing insight into period-valley giant planet formation, making the Kepler-427 system an attractive target for follow-up observations.

Current and near-future missions, such as $K2$ (Howell et al. 2014) using Kepler and the Transiting Exoplanet Survey Satellite (Ricker et al. 2015), probably do not have the long time baseline required to detect these transit depth variations. TESS, however, is likely to provide a wealth of more easily characterizable targets in the period range where these effects are measurable, allowing for long-term follow-up using other instruments. Additionally, TESS’s high-quality short cadence photometry will provide a rich data set for the method attempted by Carter & Winn (2010a) and Zhu et al. (2014). Efforts combining both methods promise to greatly expand our understanding of gas giant structure and formation.

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Software: emcee (Foreman-Mackey et al. 2013).

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