Clutter Suppression for Wideband Radar STAP

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Abstract—Traditional space-time (ST) adaptive processing (STAP) theory is based on the assumption of narrowband or “zero-bandwidth,” where the decorrelation within the ST snapshot is ignored. However, with radar bandwidths increasing, this assumption becomes invalid due to the deteriorated decorrelation of the received signals within the ST snapshot. The decorrelation directly causes the dispersion of the received signals in both spatial and temporal domains, leading to the spreading of the clutter spectrum in the 2-D frequency (Doppler-spatial frequency) domain. With the spreading of the clutter spectrum, the clutter suppression notch in the traditional STAP filters is widened, resulting in a relative poor ability to detect slow-moving targets. In this article, we focus on the clutter suppression for wideband radar STAP. A generalized signal model of the ground clutter is first established for the wideband array radar. Using this outcome, we analyze the influence of bandwidth on the characteristics of the ground clutter and quantitatively describe the 2-D spreading of the ground clutter on the Doppler-spatial frequency plane. Moreover, the model of clutter covariance matrix for wideband STAP (W-STAP) is established. Finally, a 2-D keystone transform (KT) algorithm, referred to as ST KT (ST-KT), is proposed to eliminate the spreading of the ground clutter in the 2-D frequency domain caused by increasing bandwidths. Simulation results are employed to validate the theoretical analysis and verify the overperformance of the ST-KT based W-STAP method in terms of the output signal-to-clutter-plus-noise ratio (SCNR) of moving targets.

Index Terms—Ground moving target indication (GMTI), keystone transform (KT), space-time (ST) adaptive processing (STAP), wideband radar.

I. INTRODUCTION

SPACE-TIME (ST) adaptive processing (STAP) is known as a 2-D adaptive filtering technique for airborne surveillance radar to detect ground moving targets within severe and dynamic clutter and jamming environments [1], [2]. It is generally recognized that STAP can be seen as an extension of the 1-D spatial-only adaptive array processing, and was first introduced by Brennan and Reed to the radar community in 1973 [3]. Since then, STAP has been vigorously researched, with a number of theories and methods being proposed [4]–[6].

The early research work of STAP focused mainly on two challenges. The first one is known as computational complexity required by an optimum filter, which approximately reaches the order of $O\{M^3N^3\}$ [2], [4]–[6], where $M$ denotes the degree of freedom (DoF) in the temporal domain [the number of the pulses in a coherent processing interval (CPI)] and $N$ stands for the DoF in the spatial domain (the number of the array elements), primarily due to the covariance matrix’s inversion operation. The second is known as slow convergence associated with fully STAP [6], [7]. Usually, it is suggested that at least $2MN$ independent and identically distributed (i.i.d) training samples should be required to achieve an average output signal-to-interference-plus-noise ratio (SINR) loss of 3 dB between the fully adaptive and optimal filters [7]. However, this cannot be achieved in a dynamic interference environment. Slow convergence rate coupled with heavy computational burden significantly limits the fully STAP architecture in practical implementations. To address these problems, a diverse set of reduced-dimension STAP [4]–[6], [8], [9] and reduced-rank STAP approaches have been proposed [6], [10]–[12]. These approaches, adopting suboptimal adaptive filters instead of the optimal adaptive filter, significantly reduce the computational burden as well as sample requirements, making STAP feasible in practice.

At the end of the last century, by analyzing some measured datasets, such as the Mountaintop dataset collected by Defense Advanced Research Projects Agency (DARPA) [13] and the multichannel airborne radar measurement (MCARM) data collected by the Air Force Research Laboratory at Rome [14], researchers began to pay attention to another challenging problem, i.e., heterogeneous clutter environments, that limits the practical use of STAP [15]. Before this practice, it is usually assumed that the training samples employed to generate 2-D weights are i.i.d with the cell under test (CUT). Such training samples are characterized as being homogeneous. However, unfortunately, such sample set is hard to acquire in real world, due to rapidly changing clutter environments. Moreover, a variety of outliers may also be contained in the sample set, making the situation even worse. Such training samples are characterized as being heterogeneous [15], which causes significant degradation of the established STAP methods. To remedy this, a number of STAP algorithms have been proposed to deal with heterogeneous clutter environments and to reduce the deleterious effects of outliers [15]–[18]. At the beginning of this century, the knowledge-aided sensor signal processing and expert reasoning (KASSPER) program was
developed by DARPA, which promotes the development of a new research branch of STAP, i.e., knowledge-aided STAP (KA-STAP) [19]. This kind of method incorporates a variety of prior knowledge into the traditional training schemes, resulting in promising performance improvement in real-world environments [20]–[23].

To date, STAP is recognized as an advanced ground moving target detection technique for airborne radar. However, most of the existing STAP approaches suffer from significant performance loss when the instantaneous bandwidth of radar is relatively large. As we have known, traditional STAP methods are based on the assumption of narrowband or “zero-bandwidth,” where no decorrelation exists between the signals received on different array elements and at different pulses [2]–[6]. Nevertheless, when the radar bandwidth increases, the decorrelation of the received signals in either the temporal or the spatial domain can no longer be neglected.

In fact, the influence of bandwidths on adaptive processing was first recognized and investigated in array signal processing [24], [25]. The broadened bandwidth causes serious dispersion of interferences across array, leading to deleterious effects on interference-cancellation performance. To handle this problem, a number of wideband (broadband) beamforming and interference cancellation methods were proposed [24]–[29]. In general, the existing wideband beamforming methods can be categorized into three classes [26], i.e., subband processing methods [27], tapped delay line techniques [24], [28], and frequency invariant beamforming methods based on 2-D or 3-D arrays [25], [29].

As we know, STAP is originally seen as an extension of the 1-D adaptive array signal processing. Hence, with increasing bandwidth, the performance loss for the traditional STAP methods is inevitable [30]. The situation is even worse than that for 1-D array processing, as the major undesired signal for STAP, i.e., the echoes of the ground clutter, is correlated both in spatial and temporal domains [31]. Thus, in case of wideband, the decorrelation within the ST snapshot makes the echo from each ground clutter source disperse across array elements and between pulses. As a result, the 2-D spectrum in the Doppler-spatial frequency domain of the ground clutter spreads evidently in both Doppler and spatial frequencies, and thereby degrades the slow-moving target detection performance of the current STAP methods. Moreover, increasing bandwidth also induces increasing mismatches between the actual target steering vector and the ideal steering vector used to calculate the adaptive weight in the traditional STAP architecture, resulting in additional performance loss for the moving targets with different velocities.

It is no doubt that a wideband radar system provides more advantages, such as fine range resolution that benefits the identification and classification of targets. With the development of hardware, an increasing number of new radar systems adopt wideband waveforms instead of narrowband waveforms [1]. Thus, the performance of the so-called wideband STAP (W-STAP) techniques becomes a key factor that affects the moving target detection ability of current and next-generation airborne radar systems.

Generally speaking, current W-STAP approaches can be classified into two classes, i.e., subband STAP [32]–[34] and 3-D-STAP [35]–[37]. The subband methods decompose the received data into a parallel bank of subbands with the bandwidths narrow enough so that the narrowband assumption is valid. Afterwards, a traditional STAP filter is applied on each of the subband data to suppress the interference and the outputs from all the subbands are recombined to reconstruct the high-resolution data. It is no doubt that the motivation of using the subband methods, i.e., reducing the bandwidth of data to meet the narrowband assumption, is correct and the improved performance is also verified by simulations. However, it still has obvious drawbacks that greatly limit its practical applications. First, for a bank of say $K$ subbands, a single STAP processing thread is repeated $K$ times, which increases the complexity of the system as well as the computational burden. Second, since each channel of the subband data undergoes adaptive processing independently, inaccuracy would be introduced more or less when reconstructing the final wideband data, leading to the distortion of waveforms for the moving targets. Finally, the subband processing is somewhat like a compromise solution to the problem of wideband clutter, which cannot deal with the clutter spreading thoroughly.

The second type of W-STAP is 3-D-STAP, where “3-D” refers to spatial, slow-time and fast-time (or equivalently range frequency) domains. In this method, the dimensionality of the adaptive processing problem is increased from $NM$ to $NML$, where $L$ is the number of the fast-time DoF. The order of the computational complexity increases from $O\{M^3N^3\}$ to $O\{M^3N^3L^3\}$, and the demand of i.i.d sample support is increased $L$ times, which are hardly to achieve in practice.

In this article, we focus our research on clutter suppression for W-STAP. Instead of using the signal model adopted in the traditional narrowband STAP, where the echo from each single clutter source is modeled as a 2-D single-frequency signal, we present here a generalized signal model of the single ground clutter sources for wideband airborne array radar. Based on this model, we first give a detailed analysis on how the bandwidth affects the ST characteristics of the ground clutter, with analytical expressions derived to quantitatively describe the 2-D spreading of each single ground clutter source. Afterward, the model of the clutter covariance matrix in case of wideband is established to characterize the echo of the ground clutter received by wideband array radar. Then, a 2-D ST keystone transform (ST-KT) algorithm is proposed to eliminate the 2-D spreading of the ground clutter, and thereby to improve the performance of W-STAP. As verified by the simulation results later, the proposed 2-D ST-KT algorithm can thoroughly cope with the spreading of clutter caused by the increasing bandwidth. Moreover, it also gains the ability to re-focus the moving targets in the 3-D datacube, resulting in the further improvement in terms of the output signal-to-clutter-plus-noise ratio (SCNR). Part of this work was presented in [38].

Overall, the main contributions of this article are presented as follows.
1) A deep analysis on how the ground clutter spreads on the 2-D frequency (Doppler-spatial frequency) domain when the bandwidth increases is provided, with closed-form expressions to quantitatively describe the direction and extension of the spreading presented.

2) A model of the clutter covariance matrix for W-STAP is established, thus providing a feasible way to directly generate the ideal clutter-plus-noise covariance matrix in case of wideband. As we know, the clutter covariance matrix is a key factor used to both describe the statistical characteristics of the received ground clutter and evaluate the performance of different STAP filters. Hence, we expect that this theoretical work (and also the theoretical work in the first contribution) may have reference value for the future research of W-STAP.

3) Motivated by the performance of the traditional KT method in the synthetic aperture radar (SAR) community, we propose a so-called 2-D SK-KT algorithm to eliminate the spreading of clutter in the 2-D frequency domain, which is proved to be a promising way to improve the performance of W-STAP.

The rest of this article is organized as follows. In Section II, a generalized signal model of single ground clutter sources is established for wideband airborne array radar. In Section III, the influence of bandwidth onto the ST characteristics of single clutter sources is analyzed both in the fast-time and fast-time frequency domains. In Section IV, the statistical characteristics of the ground clutter is analyzed in case of wideband, with the model of wideband clutter covariance matrix established. In Section V, a 2-D ST-KT algorithm is proposed to address the problem of 2-D clutter spreading, and hence improve the performance of W-STAP. Simulated data are employed to validate the proposed ST-KT algorithm. Finally, Section VI concludes this article.

II. SIGNAL MODEL OF SINGLE CLUTTER SOURCES FOR WIDEBAND AIRBORNE ARRAY RADAR

As mentioned in [2]–[5], the echoes of the ground clutter can be modeled as the superposition of a large number of independent and discrete clutter sources evenly distributed in the azimuth and elevation angles. In this section, we first establish the signal model of single clutter sources for airborne pulse-Doppler radar with an array antenna, which is then employed as a basis to investigate the influence of bandwidth on the ST characteristics of the ground clutter. Note that, we do not hold any assumption on the bandwidth of radar, and here derive a generalized form to describe the echo signal of single ground clutter sources, which is suitable for different bandwidths.

A 3-D geometry of data collection (Cartesian coordinate system) and its 2-D top view are shown in Fig. 1(a) and (b), respectively. The array system under consideration is a uniform linear array (ULA) that is horizontally oriented and parallel to X-axis. The airborne platform is at an altitude denoted by $H$ and moving at a constant velocity denoted by $v_a$. The crab angle, defined to be the angle between the flight direction and the ULA axis, is represented by $\beta$.

Let continuous variable $t$ denotes the slow-time, continuous variable $\eta$ refers to the along-array position, i.e., the distance between the element under consideration and the reference element along the ULA axis (the element in the central position on the ULA is defined as the reference element in this article), and assume that the coordinate of the reference element in the Cartesian coordinate system is $(0,0,H)$ when $t=0$. Thus, the instantaneous coordinate of each array element can then be represented as $(v_a \cos \beta t + \eta, v_a \sin \beta t, H)$.

Now, consider that a stationary scatterer (single clutter source), denoted by $P$, is on the ground in Fig. 1(a). Let $R_0$ represent the distance of $P$ to the reference element, $\alpha$ and $\theta$ denote the azimuth and elevation angles of $P$ with respect to the reference element, respectively, when $t=0$. The coordinate of $P$ in Fig. 1(a) is then given by $(R_0 \cos \theta \sin \alpha, R_0 \cos \theta \cos \alpha, 0)$. Therefore, the instantaneous range from this clutter source to each array element is obtained from [see (1), as shown at the bottom of the next page].

By expanding the above binary function in a Taylor series [39] about $(t, \eta) = (0,0)$, we have

$$R(t, \eta) = R(0, 0) + R_t'(0, 0)t + R_\eta'(0, 0) \eta + o(t, \eta)$$

where $R_t'(0, 0)$ and $R_\eta'(0, 0)$ are calculated from

$$R_t'(0, 0) = \frac{\partial R(t, \eta)}{\partial t} \bigg|_{(t, \eta) = (0,0)}$$

and

$$R_\eta'(0, 0) = \frac{\partial R(t, \eta)}{\partial \eta} \bigg|_{(t, \eta) = (0,0)}$$
respectively, with \((\partial R(t, \eta))/\partial t\) and \((\partial R(t, \eta))/\partial \eta\) denoting the first partial derivatives of \(R(t, \eta)\) with respect to \(t\) and \(\eta\), which are expressed as

\[
\frac{\partial R(t, \eta)}{\partial t} = \frac{(v_a \cos \beta t + \eta - R_0 \cos \theta \sin \alpha) v_a \cos \beta}{R(t, \eta)} + \frac{(v_a \sin \beta t - R_0 \cos \theta \cos \alpha) v_a \sin \beta}{R(t, \eta)}
\]

and

\[
\frac{\partial R(t, \eta)}{\partial \eta} = \frac{(v_a \cos \beta t + \eta - R_0 \cos \theta \sin \alpha)}{R(t, \eta)}
\]

respectively. \(\phi(t, \eta)\) stands for the quadratic and higher order terms of the expansion. Substituting \((t, \eta) = (0, 0)\) into (5) and (6), and using (2), we have

\[
R(t, \eta) = R_0 - [v_a \cos \theta \sin(\alpha + \beta)] t - [\cos \theta \sin \alpha] \eta + \phi(t, \eta)
\]

where \(R(0, 0)\) is replaced by \(R_0\) for simplification. Note that, in general STAP implementations, the number of the pulses contained in one CPI is relatively small, and the size of the array mounted on a moving platform is also limited, suggesting that both the distance travelled by the platform in one CPI and the length of ULA are far smaller than the distance between the clutter source and the radar. As a result, the quadratic and higher order terms in (7) are very small and hence can be ignored in the following discussion. Thus, the range equation in (7) can be substituted by the approximate expression given by

\[
R(t, \eta) \approx R_0 - [v_a \cos \theta \sin(\alpha + \beta)] t - [\cos \theta \sin \alpha] \eta.
\]

We denote the waveform of the transmitted signal after pulse compression (point spread function), with the variable \(\tau\) denoting the fast-time, \(f_c\) is the carrier frequency of the radar, \(A_1\) is a constant depending on the radar cross section (RCS) of the scatterer, and \(c\) is the speed of light. By defining

\[
\phi(t, \eta) = \frac{-2\pi f_c R_{TW}(t, \eta)}{c}
\]

we can rewrite (10) in a simplified form as follows:

\[
s_R(\tau, t, \eta) = s_c \left[ \tau - \frac{R_{TW}(t, \eta)}{c} \right] e^{j\phi(t, \eta)}.
\]

Thus, the echo signal of the clutter source is actually comprised of two terms, which are represented by

\[
s_1(\tau, t, \eta) = s_c \left[ \tau - \frac{R_{TW}(t, \eta)}{c} \right]
\]

and

\[
s_2(\tau, t, \eta) = e^{j\phi(t, \eta)}
\]

respectively. As we can see, the first term, i.e., the envelop induced by \(A_1\), can be ignored in the following discussion. Thus, the range equation in (7) can be substituted by the approximate expression given by

\[
R(t, \eta) \approx R_0 - [v_a \cos \theta \sin(\alpha + \beta)] t - [\cos \theta \sin \alpha] \eta.
\]
the ULA. The discrete form of compression, window (taper) function used in the procedure of the pulse respectively, \( M \) and elements, respectively, single-frequency signal in the ST domain. 

\[ \text{echo signal from each clutter source is recognized as a 2-D single-frequency signal in the ST domain.} \]

Let \( m \) and \( n \) denote the indexes of pulses and array elements, respectively, \( f_{\text{PRF}} \) represent the pulse repetition frequency (PRF), and \( d \) denote the interelement distance of the ULA. The discrete form of \( s_2(t, \eta) \) is then given by

\[ s_2(n, m) = e^{j2\pi \tilde{F}_d(\alpha, \theta)} e^{j2\pi \tilde{F}_s(\alpha, \theta)} \]

where

\[ \tilde{F}_d(\alpha, \theta) = \frac{F_d(\alpha, \theta)}{f_{\text{PRF}}} \]

(20)

and

\[ \tilde{F}_s(\alpha, \theta) = F_s(\alpha, \theta) d \]

(21)

are referred to as the normalized Doppler and normalized spatial frequencies in the literature [4]–[6]. By introducing the notation of the matrix, (19) can also be rewritten in a vector form referred to as ST steering vector with respect to clutter path \((\alpha, \theta)\) in the literature [2]–[6], which is given by

\[ s_2(\alpha, \theta) = s_T(\alpha, \theta) \otimes s_S(\alpha, \theta) \]

(22)

where

\[ s_T(\alpha, \theta) = \left[ e^{j2\pi \tilde{F}_d(\alpha, \theta)}, e^{j4\pi \tilde{F}_d(\alpha, \theta)}, \ldots, e^{j2\pi M \tilde{F}_d(\alpha, \theta)} \right]^T \]

(23)

and

\[ s_S(\alpha, \theta) = \left[ e^{j2\pi \tilde{F}_s(\alpha, \theta)}, e^{j4\pi \tilde{F}_s(\alpha, \theta)}, \ldots, e^{j2\pi N \tilde{F}_s(\alpha, \theta)} \right]^T \]

(24)

are the steering vectors in the temporal and spatial domains, respectively. \( \otimes \) stands for the Kronecker product, and superscript \( T \) denotes the operation of transposition.

We now turn our attention to the RM term. Without loss of generality, we assume the transmitted signal is a linear frequency modulation (LFM) signal (which is widely employed in airborne pulse-Doppler radar) given by

\[ s_{\text{LFM}}(t) = \text{rect} \left[ \frac{t}{T_p} \right] e^{-j \frac{B}{2} \pi^2 t^2} \]

(25)

where \( T_p \) and \( B \) are the pulse length and bandwidth of the LFM signal, respectively. Assuming that there is no additional window (taper) function used in the procedure of the pulse compression, \( s_1(t, t, \eta) \) can be denoted by the following sinc function:

\[ s_1(t, t, \eta) = A_2 \text{sinc} \left( B \left( \frac{\tau - T_{\text{TW}}(t, \eta)}{c} \right) \right) \]

(26)

where \( A_2 \) is a constant that depends on different methods used for pulse compression. Substituting (9), (16), and (17) into (26) and omitting constant \( A_2 \), we obtain

\[ s_1(t, t, \eta) = \text{sinc} \left( B \left( \frac{\tau - R_0}{c} \right) + \frac{B}{f_c} [F_d(\alpha, \theta)t + F_s(\alpha, \theta)\eta] \right). \]

(27)

By introducing the discrete forms of variable \( t \) and \( \eta \), (27) can also be expressed as a column vector given by

\[ s_1(\tau, \alpha, \theta) = B \left( \frac{\tau - 2R_0 c}{c} \right) + \frac{B}{f_c} [F_d(\alpha, \theta)t + F_s(\alpha, \theta)\eta] \]

Thus, the vector form of the echo signal (the ST steering vector) from the clutter source localized at \((\alpha, \theta)\) can be expressed as the Hadamard product of (28) and (22), which is given by

\[ s_R(\tau, \alpha, \theta) = s_1(\tau, \alpha, \theta) \otimes s_2(\alpha, \theta) \]

(29)

where \( \otimes \) stands for the Hadamard product. Now, substituting (27) and (18) into (12), the continuous form of the echo signal is obtained [see (30), as shown at the bottom of the next page.]

From the above formulas, it is clear that the difference between the signal model used in the traditional narrowband STAP and the generalized model given by (29) and (30) mainly lies in the RM term. In the traditional narrowband STAP theory, the RM term is not considered because it varies slightly between pulses and across the array. However, in the case of wideband, the increasing bandwidth enlarges the RM evidently, leading to the decorrelation in both temporal and spatial domains.

Based on the signal model provided by (29) as well as its continuous form in (30), we will present thorough discussion on the influence of bandwidth onto the ST characteristics of single clutter sources in Section III.

### III. Influence of Bandwidth on ST Characteristics of Single Clutter Sources

In this section, the influence of bandwidth on the ST characteristics of single clutter sources is investigated in detail. As discussed in Section II, the major difference between the signal model presented in this article and that in the traditional STAP theory is the RM term. So it can be deduced that the RM within the snapshot is a key factor affecting the ST characteristics of the ground clutter. Consequently, we now turn our attention to the RM term and investigate how it affects the 2-D spectrum of the single clutter sources in case of wideband.

In Sections III-A and III-B, the characteristics of the RM term is analyzed in the fast-time and fast-time frequency (range frequency) domains, respectively, and quantitative results to evaluate the 2-D spectrum spreading of single clutter sources are also presented.
A. Analysis in the Fast-Time Domain

Using the Fourier property, the multiplication of \( s_1(t, \tau, \eta) \) and \( s_2(t, \eta) \) in (12) is equivalent to the 2-D convolution in the 2-D frequency (Doppler-spatial frequency) domain given by

\[
S_R(\tau, f_d, f_s) = S_1(\tau, f_d, f_s) \ast S_2(f_d, f_s) \tag{31}
\]

where \( S_1(\tau, f_d, f_s) \) and \( S_2(f_d, f_s) \) are the 2D spectra of \( s_1(\tau, t, \eta) \) and \( s_2(t, \eta) \), i.e., the 2-D Fourier transforms (2-D-FT) to \( s_1(\tau, t, \eta) \) and \( s_2(t, \eta) \) with respect to \( (t, \eta) \), \( S_R(\tau, f_d, f_s) \) is the 2-D spectrum of the clutter source, variables \( f_d \) and \( f_s \) are used to identify the Doppler and spatial frequencies, respectively, and \( \ast \) stands for the operation of 2-D convolution. Considering that \( s_2(t, \eta) \) is a single-frequency signal in the ST domain, the 2-D-FT of \( s_2(t, \eta) \) is given by

\[
S_2(f_d, f_s) = \delta_{2D}[f_d - F_d(\alpha, \theta), f_s - F_s(\alpha, \theta)] \tag{32}
\]

where \( \delta_{2D}[f_d, f_s] \) is the 2-D impulse function in 2-D frequency domain. Substituting (32) into (31), the 2-D spectrum of the clutter source localized at \((\alpha, \theta)\) is obtained from

\[
S_R(\tau, f_d, f_s) = S_1[\tau, f_d - F_d(\alpha, \theta), f_s - F_s(\alpha, \theta)]. \tag{33}
\]

As we can see, the 2-D spectrum of the clutter source is equivalent to the displayed spectrum of the RM term. Applying a 2-D-FT to (27) with respect to \((\tau, \eta)\), we obtain the 2-D spectrum of \( s_1(\tau, t, \eta) \) [see (34), as shown at the bottom of this page], where \( \delta() \) is the 1-D impulse function, and \( \text{rect}(\cdot) \) denotes the rectangular function. Considering that the constant term, i.e., \( \left( \left( f_d F_d(\alpha, \theta) \right) / |B| \right) \), and the exponential term, i.e., \( e^{j2\pi \frac{2\theta_0}{c} f_s} \), will not affect the shape of \( S_1(\tau, f_d, f_s) \), we omit these terms for simplification and rewrite (34) as follows:

\[
S_1(f_d, f_s) \approx \text{rect} \left[ \frac{f_d}{BF_d(\alpha, \theta)} \right] \delta \left[ f_s - \frac{F_s(\alpha, \theta)}{F_d(\alpha, \theta)} f_d \right]. \tag{35}
\]

The above formula indicating the shape of \( S_1(f_d, f_s) \) allows us to quantitatively describe the spectrum spreading of the clutter source. The impulse function, with its peaks (\( \delta(0) \)) localized along the line denoted by

\[
f_s = \frac{F_s(\alpha, \theta)}{F_d(\alpha, \theta)} f_d \tag{36}
\]

on the 2-D \((f_d - f_s)\) plane, indicates the direction of spectrum spreading, and the rectangular function defines the range of spreading, which is given by

\[
-\frac{BF_d(\alpha, \theta)}{2f_c} < f_d < \frac{BF_d(\alpha, \theta)}{2f_c}. \tag{37}
\]

From (35) to (37), we observe that the spectrum spreading of the clutter source can be exactly described by a line segment on the \( f_d - f_s \) plane, where the slope is determined by the ratio of the spatial frequency to the Doppler frequency of the clutter source, and the length is proportional to the radar bandwidth. In other words, these equations accurately answer the question how the ground clutter shall spread in the 2-D frequency domain under wideband conditions.

We now show examples to support the above discussions and to compare the spectrum spreading in cases of narrowband and wideband. The major parameters used for this simulation are listed in Table I, and two different bandwidths, i.e., 10 and 240 MHz, are chosen for the cases of narrowband and wideband, respectively.

In Fig. 2, the characteristics of the RM term in both the ST and 2-D frequency domains are demonstrated. We first consider the case of narrowband and set the bandwidth to be 10 MHz. The amplitude of the RM term in the ST domain is shown in Fig. 2(a). As can be seen, for relatively small bandwidth, the RM varies slightly within the snapshot (see the values on the color-bar), implying that the data are strongly correlated across pulses and also across elements. Applying a 2-D fast Fourier transform (FFT) to the RM term shown in Fig. 2(a), the 2-D spectrum is obtained and displaced in Fig. 2(b), where no evident spectrum spreading is recognized. The bandwidth is then set to be 240 MHz to simulate the case of wideband, and the relevant results are displaced in Fig. 2(c) and (d). As we can see from Fig. 2(c), the amplitude of the RM term varies significantly as compared with that of Fig. 2(a), leading to considerable decorrelation within the snapshot. As a result, the 2-D spectrum of the RM term spreads in the 2-D frequency domain, which is clearly seen from

\[
s_R(\tau, t, \eta) = \text{sinc} \left\{ B \left( \tau - \frac{2R_0}{c} \right) + \frac{B}{f_c} [F_d(\alpha, \theta)t + F_s(\alpha, \theta)\eta] \right\} e^{j2\pi F_d(\alpha, \theta) t} e^{j2\pi F_s(\alpha, \theta) \eta} \tag{30}
\]

\[
S_1(\tau, f_d, f_s) = \frac{f_d F_d(\alpha, \theta)}{B} \text{rect} \left[ \frac{f_d}{BF_d(\alpha, \theta)} \right] \delta \left( f_s - \frac{F_s(\alpha, \theta)}{F_d(\alpha, \theta)} f_d \right) e^{j2\pi \frac{2\theta_0}{c f_c} f_s} \tag{34}
\]
Fig. 2. Demonstrations of RM in the ST and 2-D frequency domains with different bandwidths. (a) RM in ST domain when the bandwidths is equal to 10 MHz. (b) RM in the 2-D frequency domain when the bandwidths is equal to 10 MHz. (c) RM in ST domain when the bandwidths is equal to 240 MHz. (d) RM in the 2-D frequency domain when the bandwidths is equal to 10 MHz.

Fig. 2(d). To validate the quantitative analysis on the spectrum spreading of the RM term, the line denoted by (36) and the upper and lower bounds denoted by (37) are also plotted in Fig. 2(d). The white dash line stands for (36) and the pair of red dash lines shows the bounds in (37). As we can see, positions of the lines and the locus of the 2-D spectrum are exactly corresponding, which directly validates the theoretical analysis.

The final 2-D spectra of the clutter source in cases of narrowband and wideband are displayed in Fig. 3(a) and (b), respectively. As we can see, the spectrum spreading in case of wideband is more evident as compared with the case of narrowband. We also note that the two subfigures are actually the displaced spectra of the RM term shown in Fig. 2(b) and (d). Hence, it is clear that the spectrum spreading of the clutter source is fully determined by the RM term.

B. Analysis in the Fast-Time Frequency Domain

In this section, the characteristics of the RM term as well as the spectrum spreading of single clutter sources are analyzed in the fast-time frequency (range frequency) domain. Applying a FT to (27) with respect to $\tau$, the RM term in the fast-time frequency domain is obtained from

$$S_1(f_r, t, \eta) = A_3 \text{rect} \left( \frac{f_r}{B} \right) e^{j2\pi f_r \eta} e^{j2\pi \left( F_d(\alpha, \theta)t + F_s(\alpha, \theta)\eta \right)}$$

(38)

where variable $f_r$ is used to identify the fast-time frequency, and $A_3$ is also a constant. Then, we define

$$\Delta F_d(f_r, \alpha, \theta) = \frac{f_r}{f_c} F_d(\alpha, \theta)$$

(39)
and
\[ \Delta F_d(f_r, \alpha, \theta) = \frac{f_r}{f_c} F_d(\alpha, \theta) \] (40)
to be the fast-time frequency dependent changes of Doppler and spatial frequencies. Substituting (39) and (40) into (38), and omitting constant \( A_2 \) and the exponential term \( e^{(i2\pi f_0 f_r/c)} \) denoting the constant time-delay, we obtain
\[ S_t(f_r, t, \eta) = \text{rect} \left( \frac{f_r}{B} \right) e^{i2\pi \left[ \Delta F_d(f_r, \alpha, \theta) + \Delta F_f(f_r, \alpha, \theta) \right] \eta} \] . (41)

Multiplying (41) by (18), the received signal in the fast-time frequency domain is expressed as
\[ S_R(f_r, t, \eta) = \text{rect} \left( \frac{f_r}{B} \right) e^{i2\pi \left[ F_d(\alpha, \theta) + \Delta F_d(f_r, \alpha, \theta) \right] t} e^{i2\pi \left[ F_f(\alpha, \theta) + \Delta F_f(f_r, \alpha, \theta) \right] \eta} . \] (42)

Comparing (42) with (18), we find that the RM term introduces additional Doppler and spatial frequencies to the traditional 2-D single-frequency signal, and the additional frequencies depend on the fast-time frequency. Consequently, when transferred back to the fast-time domain, the 2D -spectrum of the echo signal spreads. Dividing (39) by (40), the coupling relationship between the changes of Doppler and spatial frequencies is obtained from
\[ \Delta F_f(f_r, \alpha, \theta) = \frac{F_f(\alpha, \theta)}{F_d(\alpha, \theta)} \Delta F_d(f_r, \alpha, \theta) \] . (43)

Considering the rectangular function in (41), the range of \( f_r \) is given by
\[ \frac{B}{2} < f_r < \frac{B}{2} . \] (44)

Substituting (39) in to (44) yields the range of \( \Delta F_d(f_r, \alpha, \theta) \)
\[ \frac{BF_d(\alpha, \theta)}{2f_c} < \Delta F_d(f_r, \alpha, \theta) < \frac{BF_d(\alpha, \theta)}{2f_c} . \] (45)

From (43) and (45), we find that the RM induced additional Doppler and spatial frequencies have the coupled relationship that can be described by a line segment on the \( f_d-f_r \) plane. We also note that the slope in (43) and the boundary given by (45) are exactly identical with those shown in (36) and (37), which again verifies the quantitative analysis on the 2-D spectrum spreading of the single clutter sources.

Another similar simulation is employed to validate the above discussion in the fast-time frequency domain, with all the related parameters the same as those listed in Table I. In Fig. 4, the centers of the 2-D spectrum of the clutter source on different fast-time frequencies are marked by circles with different colors. As we can see, the center of the clutter spectrum varies with the fast-time frequency, indicating the spreading of the clutter in the 2-D frequency domain. We also note that Fig. 4 is in accordance with the 2-D spectrum of the clutter source shown in Fig. 3(b).

From the above discussions on the 2-D spectrum of single clutter sources, we witness that the RM within the snapshot, which is assumed to be negligible in the traditional STAP theory, is the major aspect that induces the 2-D spectrum spreading of the ground clutter, when the radar bandwidth increases.

IV. ST CHARACTERISTICS OF GROUND CLUTTER FOR WIDEBAND RADAR

For airborne surveillance radar working on the air-to-ground mode, the ground surface is the major source of clutter. Of all the components of interference, ground clutter is one of the most complicated factors, which is distributed in both Doppler and spatial frequencies domains. As it has been mentioned in the literature [30], [31], obvious spectrum spreading in the 2-D frequency domain for the ground clutter is observed in case of wideband. The spectrum spreading of the ground clutter makes the clutter suppressing notch of the adaptive filter broader than the corresponding narrowband case, and thereby degrades the slow-moving target detection ability of current STAP methods.

In this section, based on the signal model presented in Section II, a generalized model is developed for ground clutter in the ST snapshot for a given range gate, when a wideband radar is considered. Based on this analysis, the model of the clutter covariance matrix in case of wideband is established, which enables us to investigate the influences of bandwidth on the ST characteristics of the ground clutter. Note that, since we only focus on the influences of bandwidth onto STAP in this article, the clutter model presented here is a generalized model, where other factors, such as intrinsic clutter motion, range ambiguity, and imbalance of receiving channels are not considered herein. In addition, we also assume that the ground is flat, which is acceptable for airborne radar [2], [4]–[6].

In the Cartesian coordinate system defined in Fig. 1, the position of any discrete clutter source on the ground is described by its azimuth and elevation angles, i.e., \( \alpha \) and \( \theta \). Since ground clutter is distributed in both the two directions on the ground, theoretically, clutter sources localized at all the azimuth and elevation angles will contribute to the received echo signals. The final clutter component consists of the superposition of echoes from all the azimuths and elevations. Consequently, the clutter vector in the ST snapshot at the range gate with respect to fast-time \( \tau \) can be denoted by the following twofold integral:
\[ \chi_r(\tau) = \int_0^\infty \int_{-\pi/2}^{\pi/2} c(\alpha, \theta) s(\tau, \alpha, \theta) d\theta d\alpha \] (46)
where \( c(\alpha, \theta) \) is the amplitude of the clutter source localized at \( (\alpha, \theta) \). For convenience, an approximation to the continuous
field of clutter shown in (46) is constructed, in which the clutter return in each snapshot is modeled as the superposition of a large number of discrete independent clutter sources evenly distributed in both azimuth and elevation angles. Letting $i$ and $k$ denote the indexes of the discrete azimuth and elevation locations, the azimuth and elevation locations of the $(i,k)$th clutter source are described by $\alpha_i$ and $\theta_k$, and the amplitude is denoted by $c_{i,k}$. Thus, we obtain the discrete form expression of (46), which is given by

$$\chi_c(\tau) = \sum_{i}^{N_c} \sum_{k}^{N_e} c_{i,k} s(\tau, \alpha_i, \theta_k)$$  \hspace{1cm} (47)$$

where $N_c$ and $N_e$ denote the total numbers of clutter sources in azimuth and elevation, respectively.

Assuming that the echoes from different clutter sources are uncorrelated (a common assumption in the field of STAP [2]–[6]), the ST covariance matrix of $\chi_c(\tau)$ is then given by

$$\mathbf{R}_c(\tau) = \mathcal{E}[\chi_c(\tau)\chi_c^H(\tau)]$$

$$= \sum_{i}^{N_c} \sum_{k}^{N_e} \sigma_{c,i,k}^2 s(\tau, \alpha_i, \theta_k)s^H(\tau, \alpha_i, \theta_k)$$  \hspace{1cm} (48)$$

where

$$\sigma_{c,i,k}^2 = \mathcal{E}[c_{i,k}c_{i,k}^*]$$  \hspace{1cm} (49)$$

is the power of the $(i,k)$th clutter source, with $\mathcal{E}[\cdot]$ denoting the expectation operator. Introducing (29) into (48), we obtain the final clutter covariance matrix as follows:

$$\mathbf{R}_c(\tau) = \sum_{i}^{N_c} \sum_{k}^{N_e} \sigma_{c,i,k}^2 \left\{ [s_1(\tau, \alpha_i, \theta_k)s_1^H(\tau, \alpha_i, \theta_k)] \right\}$$

$$\circ [s_2(\alpha_i, \theta_k)s_2^H(\alpha_i, \theta_k)] \}$$  \hspace{1cm} (50)$$

Equation (50) establishes a model of the clutter covariance matrix for wideband airborne array radar, and also provides a numerical method to calculate the ideal clutter covariance matrix in the research of W-STAP.

By comparing (50) to the classical model of the clutter covariance matrix used in the narrowband STAP [4], it is clear that the difference lies in the additional matrix generated by the self-exterior product of the RM vector for each clutter source, i.e., $s_1(\tau, \alpha_i, \theta_k)s_1^H(\tau, \alpha_i, \theta_k)$. As discussed in Section III, the RM term leads to additional spreading for each single clutter source along the line defined by (36) on the $f_d - f_s$ plane. Thus, each clutter source will spread along different directions in the 2-D frequency domain, leading to the spreading of the entire clutter spectrum.

In Fig. 5, the spectrum spreading of a group of clutter sources are shown. All the clutter sources are localized at an identical elevation angle ($30^\circ$), but distributed in different azimuth positions. The total number of the clutter sources is 17, and the azimuth positions of them are set to be $[-80^\circ, -70^\circ, \ldots, 70^\circ, 80^\circ]$. All the other relevance parameters are the same as those given in Table I. As we can see, since the clutter sources localized at different azimuth positions have different central Doppler and spatial frequencies, the spreading of them in the 2-D frequency domain differs from each other.

In Fig. 6, the 2-D minimum variance distortionless response (MVDR) spectra of the ground clutter in cases of wideband and narrowband are provided, where the 2-D spreading of clutter spectrum caused by increasing bandwidth is clearly seen. The parameters employed here are the same as those listed in Table I. To clearly show the entire spectra of the ground clutter from all the azimuth directions, a cosine-shape antenna pattern is assumed, but the back lobe of the antenna is ignored. By comparing the two MVDR spectra, it is obvious that the spectrum of the ground clutter spreads, in both Doppler and spatial frequencies, when the bandwidth of radar increases. Moreover, the shape of the spectrum shown in
Fig. 7. Eigenspectra of the ground clutter with different bandwidths.

In Fig. 7, the ranks of the ground clutter in cases of wideband and narrowband are tested via the eigenspectra of the ground clutter. As we can see, instead of the “cliff-like” eigenspectrum in case of narrowband (10 MHz), the spread clutter in case of wideband (240 MHz) generates a smoother “slope-like” eigenspectrum. That means the rank of the ground clutter is enlarged when the bandwidth increases, indicating additional performance degradation of reduced-rank and reduced-dimension STAP algorithms [8]–[12], such as joint domain localized (JDL) STAP [9], eigen-canceller [10], and the like.

Fig. 8 compares the performance of the traditional STAP methods in cases of wideband and narrowband by the calculated SCNR loss. The SCNR loss of a STAP algorithm is defined as the output SCNR relative to the output signal-to-noise ratio (SNR) of the matched filter in an interference-free environment, which is given by [2], [4]

\[
L_{SCNR}(f_d, f_s) = \frac{1}{MN} \left| w^H(f_d, f_s)v(f_d, f_s) \right|^2
\]

(51)

where \( v(f_d, f_s) \) is the ST steering vector for the moving target with Doppler and spatial frequencies equal to \( f_d \) and \( f_s \), respectively, \( R \) is the clutter-plus-noise covariance matrix, and \( w(f_d, f_s) \) is the adaptive weight corresponding to Doppler frequency \( f_d \) and spatial frequency \( f_s \). In this article, all the SCNR loss curves are corresponding to the boresight direction of the beam (\( f_s = 0 \)). In Fig. 8(a), the performance of the optimum STAP filter is tested, while in Fig. 8(b), the performance of a classical reduced-dimension STAP algorithm, i.e., the 3×3 JDL-STAP algorithm [10], is evaluated. As we can see from these curves, the clutter suppressing notch in case of wideband for either of the fully and partial optimum STAP algorithm is much broader than that of the corresponding narrowband case, implying that the current STAP algorithms have a worse slow-moving target detection ability when the radar bandwidth increases.

V. 2-D ST-KT FOR W-STAP

Compared with narrowband radar, the slow-moving target detection performance for wideband array radar degrades when using the traditional STAP methods due to the spreading clutter spectrum in both Doppler and spatial frequencies domains. As discussed in Sections II-IV, the difference between the clutter model used in traditional narrowband STAP and that proposed in this article is the RM term. We also deduced that the RM term is the major reason for the 2-D spreading of the ground clutter in case of wideband. Therefore, how to eliminate the RM term in the echo signals or to greatly reduce its influences becomes a key issue for W-STAP.

As we have mentioned in Section I, most of the current W-STAP methods use the subband processing or 3-D adaptive processing approaches to reduce the impact induced by increasing bandwidth. However, both of them have certain drawbacks such as heavy computational burden and performance loss caused by mismatches between subbands, limiting their applications in practice. In this section, we present a 2-D ST-KT algorithm, which is expected to thoroughly eliminate the RM term within each snapshot, and make the narrowband assumption revalidated.

A. 2-D ST-KT

The keystone transform (KT) is well known as a classical method in the field of SAR imaging, especially for SAR imaging of moving targets [41], [42]. It is widely used because its ability to compensate arbitrary linear range mitigations for moving targets or ground stationary scatterers, without the a priori information of target motion. The key step in the initial version of KT is a 1-D interpolation to rescale the slow-time axis for each fast-time frequency, which eliminates the range-Doppler (RD) coupling effect and hence removes the linear RM in the received signals.
With the capability of KT in mind, and based on our analysis on the RM in the range frequency domain (in Section III-B), we now extend the traditional 1-D KT into the 2-D ST domain and propose here a so-called ST-KT method for W-STAP.

As we have discussed in Section III, the RM in the received datacube induces the coupling effect between the fast-time and ST (slow-time and along-array position) domains, resulting in the 2-D spreading of the clutter. Hence, by decoupling the fast-time and ST domains, the 2-D spreading of the clutter is expected to be eliminated. However, the decoupling operation is hard to achieve in the fast-time domain. Thus, we first apply a FFT with respect to the fast-time (range) on the datacube, and then try to decouple the echo in the range frequency-ST domain. Since some related analysis has been provided in Section III-B, the signal model of single clutter source in the range frequency domain in (38) is employed here directly.

Now, substituting (39) and (40) into (38) and omitting the constant $A_3$, we rewrite the expression of the received data in the range frequency domain as

$$S_R(f_r, t, \eta) = \text{rect} \left[ \frac{f_r}{B} e^{-j 2 \pi \eta \frac{c}{c}} e^{j 2 \pi \frac{c}{c} F_d(\alpha, \theta) t} e^{j 2 \pi \frac{c}{c} F_s(\alpha, \theta) \eta} \right].$$

(52)

As we can see from the last two exponential terms, i.e.,

$$e^{j 2 \pi \eta \frac{c}{c} F_d(\alpha, \theta) t} e^{j 2 \pi \frac{c}{c} F_s(\alpha, \theta) \eta},$$

because of the RM term, the Doppler and spatial frequencies of the clutter source, i.e.,

$$e^{j 2 \pi \eta \frac{c}{c} F_d(\alpha, \theta) t} e^{j 2 \pi \frac{c}{c} F_s(\alpha, \theta) \eta},$$

vary with the range frequency $f_r$, implying the coupling effect between the range frequency, slow-time, and along-array position. Specifically, the first exponential term represents the coupling between the range frequency and slow-time, while the second exponential term denotes the coupling between the range frequency and along-array position.

To eliminate this coupling effect, we propose to rescale the slow-time axis and the along-array position axis for each range frequency by the following 2-D transform:

$$\begin{align*}
t &= \frac{f_r}{f_c + f_r} \tilde{t} \\
\eta &= \frac{f_c}{f_r + f_c} \tilde{\eta}
\end{align*}$$

(53)

where $\tilde{t}$ and $\tilde{\eta}$ are new variables of the rescaled slow-time and the along-array position, respectively. Substituting (53) into (52), we obtain

$$S_R(f_r, \tilde{t}, \tilde{\eta}) = \text{rect} \left[ \frac{f_r}{B} e^{-j 2 \pi \eta \frac{c}{c} \tilde{t}} e^{j 2 \pi \frac{c}{c} F_d(\alpha, \theta) \tilde{t} + F_s(\alpha, \theta) \tilde{\eta}} \right].$$

(54)

It is clear that the 2-D transform in (53) decouples the range frequency and slow-time, and also the range frequency and along-array position, making the Doppler and spatial frequencies, i.e., $F_d(\alpha, \theta)$ and $F_s(\alpha, \theta)$, independent on the range frequency in the new ST coordinate system ($\tilde{t}, \tilde{\eta}$).

Transforming (54) back to the range domain, the new datacube is expressed as

$$s_R(\tau, \tilde{t}, \tilde{\eta}) = \text{sinc} \left[ B \left( \tau - \frac{2 R_0}{c} \right) \right] e^{j 2 \pi \left[ F_d(\alpha, \theta) \tilde{t} + F_s(\alpha, \theta) \tilde{\eta} \right]}.$$

(55)

Now, as we can see, the received data of the single clutter source become single-frequency in the ST domain, which is identical to the signal model adopted in the traditional STAP. Consequently, the 2-D spreading of the clutter caused by the RM term is effectively eliminated.

Note that, like the classical 1-D KT method, the transform in (53) also cannot eliminate the quadratic and higher order RM terms, but as mentioned before, the contributions of these terms are negligible for airborne array radar (small size) and short CPI. In practice, the proposed 2-D transform in (53) can be also achieved by a 2-D interpolation in the ST domain for each range frequency [41]. Considering that there is actually no coupling between the slow-time and the along-array position in (52), the 2-D transform can be decomposed into two 1-D interpolations, i.e., the interpolations in the slow-time and spatial domains for each range frequency, which can reduce the computational burden.

We refer to the proposed algorithm as ST-KT, and provide the signal processing flowchart in Fig. 9. In this algorithm, a FFT is first applied with respect to the fast-time to transform the received datacube to the range frequency-ST domain, followed by the two 1-D interpolations to rescale the slow-time and along-array position, respectively. Afterwards, the data are transformed back into the fast-time domain via an inverse FFT (IFFT) with respect to the range frequency to generate the new datacube for the following STAP operations. It is worth mentioning that, the order of the two 1-D interpolations in the ST-KT can be exchanged (we can either apply the interpolation in slow-time or along-array position first). In Fig. 10, a demonstration of the ST-KT (the two interpolations) to the 3-D datacube is provided, where, as we can see, the proposed SK-KT looks like a 3-D keystone in the range frequency-ST domain.

In Table II, a brief comparison of the major additional computational load between the W-STAP algorithm based on ST-KT and two existing W-STAP methods, i.e., the subband STAP [32]–[34] and 3-D-STAP [35]–[37], is provided. Note that the additional computational load mentioned here only refers to the computational load induced by special operations when adopting different W-STAP methods, which means the common computation load, i.e., the computations of STAP.
filtering \(O(M^3N^3)\) for each of the range CUT when the fully adaptive processing is considered) for different methods are not included. In this table, \(L\) denotes the number of the additional fast-time DoF introduced by the 3-D-STAP method, \(L_{\text{cut}}\) is the number of all the range CUT, \(K_{\text{sub}}\) is the number of subbands used in the subband STAP method, \(K_{\text{inp}}\) is the number of the neighbor points employed to calculate the value on the query point in the 1-D interpolations of ST-KT (the basic sinc-interpolation approach is considered here).

As we can see, the 3-D-STAP method carries the highest computational burden, since it introduces additional DoF in the adaptive processing. Considering the number of the neighbor points in the two 1-D interpolations is usually small as compared with the \(M\) or \(N\), the proposed ST-KT algorithm is superior to the subband STAP method in terms of computational efficiency.

Moreover, some of the computational efficient methods for traditional KT, such as the chirp scaling based KT [42], and the chirp-Z based KT [43], can also be easily integrated into the proposed ST-KT algorithm, which can further improve the computational efficiency in practical exercises.

### B. Simulation Results

In this section, simulated echo data of the ground clutter are generated to evaluate the capability of the proposed ST-KT algorithm to eliminate the 2-D spectrum spreading. To simulate the echoes of the ground clutter more realistically, millions of discrete clutter sources with random backscattering coefficients are set on the ground, each of which generates an echo signal independently for an airborne array radar. The distance between each pair of the adjacent clutter sources is set to be less than either of the range and azimuth resolutions, and the amplitudes of the clutter sources are zero-mean complex Gaussian distributed random variables with identical variance and independent with each other. The echo signals from all the clutter sources are finally summed up and compressed in range to generate a 3-D datacube, which is then used as samples to test the performance of the STAP algorithms. The major parameters for this simulation are listed in Table III.

Fig. 11 illustrates the feasibility of the proposed ST-KT algorithm by the estimated MVDR spectra when KTs is applied on the datacube or not. All the spectra are estimated from the simulated datacube using 512 range samples. In Fig. 11(a), as we can see, the MVDR spectrum spreads obviously in the 2-D frequency domain, like that shown in Fig. 6(a). To compare the performance of the proposed ST-KT to that of the traditional 1-D KT algorithm, we first, respectively, rescale the slow-time and along-array position on the datacube (apply the 1-D KT interpolation on the slow-time and spatial domains, respectively), and show the corresponding MVDR spectra in Fig. 11(b) and (c). As we can see, neither of the 1-D interpolations can fully solve the spectrum spreading problem, although they have the ability to eliminate part of the spreading. Take Fig. 11(c) for example, applying a

| Parameter                      | Value          |
|-------------------------------|----------------|
| Carrier frequency             | 1GHz           |
| Bandwidth                     | 240MHz         |
| Platform speed                | 75m/s          |
| Crab angle                    | 20°            |
| PRF                           | 1000Hz         |
| Number of array elements      | 32             |
| Number of coherent processing pulses | 32          |
| Inter-element distance of the ULA | Half-wavelength |
| Elevation of the scene center | 35°            |
| Antenna pattern               | Cosine type without backlobe |
| Number of clutter sources on the ground | 4000(range)* |
| Distance between the scene center and radar | 5000(azimuth) |
| Input clutter-to-noise ratio per element per pulse | 15km |

\[20\text{dB}\]
cross-array KT interpolation can only eliminate the spectrum spreading in the area where the Doppler frequency is close to 0. In Fig. 11(d), the MVDR spectrum is estimated after applying ST-KT to the datacube. As we can see, the spreading of the clutter in the 2-D frequency domain is totally eliminated, similar to the MVDR spectrum in case of narrowband.

Figs. 12 and 13 evaluate the performance of the proposed ST-KT algorithm by the estimated SCNR loss when the ST-KT is applied on the datacube or not. All the curves are calculated via (51), with \( R \) replaced by its maximum likelihood estimate [2], [7]. To provide a deep investigation on the improvements brought by ST-KT, we first assume that the bandwidth just affects the echo of the ground clutter, but has no influence on the moving target signals. That means the target steering vector in (51), i.e., \( \mathbf{v}(f_d, f_s) \), is assumed to be single-frequency in the ST domain, as that used in the traditional narrowband STAP, when calculating the SCNR loss curves. The related results are shown in Fig. 12. As we can see, ST-KT narrows the clutter suppressing notches for both the fully STAP and JD L STAP algorithms, implying improved slow-moving target detection performance in case of wideband. Moreover, by comparing the two subfigures, we find the improvement brought by ST-KT in terms of the width of the clutter suppressing notch is more evident for the reduce-dimension STAP method. This behavior is really desirable, since in most practical applications, reduce-dimension STAP methods are used instead of fully STAP methods due to computational efficiency as well as fast convergence.

Note that, although the SCNR loss curves shown in Fig. 12 have indicated the improvement brought by ST-KT, there is
still a key factor not considered when generating the curves. That is, the influence of bandwidth on the echoes of the moving targets. When the bandwidth increases, the RM within the snapshot induced by platform motion as well as self-motion for moving target is not negligible, and should also be considered when evaluating the performance of the STAP methods. Therefore, we now consider that the bandwidth affects both the echo signals of the ground clutter and moving targets. The target steering vector is generated via the signal model proposed in Section II when calculating the SCNR loss curves, and the results are shown in Fig. 13. As we can see, without ST-KT, the performance of the fully STAP and JDL STAP methods in terms of the SCNR is even poorer, as compared with Fig. 12. The output SCNR drops significantly not only for slow-moving targets but also for fast-moving targets. This is because in the traditional STAP algorithms, the target steering vector used to generate the adaptive weight differs from the real target steering vector in case of wideband. This mismatch will then attenuate the power of moving target when it passes the so-called matched filter in the architecture of the STAP filter. With the relative velocity between the platform and the moving target increasing, the RM within the snapshot becomes more evident, leading to the increasing mismatch between the two steering vectors.

It also can be seen from the two figures that the curves estimated from the datacube with ST-KT change slightly as compared with the counterparts shown in Fig. 12. To explain that, we first remember that the traditional 1-D KT is originally designed for moving target imaging. It has the ability to compensate arbitrary linear RM of the target to be imaged, including that induced by platform motion, self-motion, or both of them. When it is extended to the ST domain, this desirable ability remains. Therefore, after ST-KT, the moving target is “refocused” in the 3-D datacube and the mismatch between the target steering vector used to generate the adaptive weight and the real steering vector is eliminated.

By comparing the SCNR curves in Fig. 13, it is seen that the proposed ST-KT algorithm significantly improves the performance of W-STAP, in terms of the output SCNR for both slow-moving and fast-moving targets.

After the testing of the SCNR curves, we now use simulated moving targets to compare different STAP methods in case of wideband (the radar bandwidth is set to be 240 MHz). Two simulated moving targets, referred to as target 1 and target 2, are added in the simulated clutter-plus-noise background. Both of the two targets are localized at the boresight direction of the beam (with normalized spatial frequency equal to 0). Target 1 is used to denote a slow-moving target (with self-motion induced normalized Doppler display equal to 0.11), while target 2 is used to simulate a fast-moving target (with self-motion induced normalized Doppler display equal to 0.42). The input SNR (per element per pulse) of the two targets are set to be 5 and 0 dB, respectively. In Fig. 14(a), the RD map of the simulated data (including targets, clutter, and noise) is shown, where the positions of the two targets are marked by the red squares. In Fig. 14(b), the RD map of the simulated data with respect to the two moving targets (where the clutter and noise is absent) is shown.

To test the ability of ST-KT to refocus moving targets, the RD maps of the two moving targets before and after ST-KT processing are provided in Figs. 15 and 16 (contour figures are employed to demonstrate the changes more clearly). As is clearly seen from these figures, the ST-KT refocuses both of the two moving targets, and thereby reduces the SCNR loss caused by motion-induced defocusing. Moreover, by comparing Figs. 15 and 16, we find that the improvement of SCNR for the fast-moving targets is more significant than that for the slow-moving targets. This is because the defocusing induced by self-motion of fast-moving targets is even worse than that of slow-moving targets, and ST-KT can refocus moving targets with arbitrary linear range mitigations. Hence, the enhancement of signal power brought by ST-KT is more significant for the fast-moving targets.

Now, the improvement of the moving target detection ability brought by ST-KT for W-STAP is verified by the output residue (output power after adaptive processing) with respect to the two moving targets. Three STAP approaches i.e., the traditional JDL algorithm, the JDL algorithm with subband processing, and the JDL algorithm with ST-KT, are utilized to process the simulated data separately for comparison. The number of the training samples for all the three methods is set to be 40, and the number of subband is set to be 8 (the bandwidth of each subband is 30 MHz) when applying the subband STAP. Fig. 17 shows the range profiles...
Fig. 14. RD maps of the simulated data. (a) RD map of the entire simulated data (including moving targets, clutter, and noise). (b) RD map of the interference-free data.

Fig. 15. RD maps of Target 1 before and after ST-KT. (a) RD map before ST-KT. (b) RD map after ST-KT.

Fig. 16. RD maps of Target 2 before and after ST-KT. (a) RD map before ST-KT. (b) RD map after ST-KT.

Fig. 17. Range profiles of the output residue with respect to the two moving targets. (a) Range profile of the output residue for Target 1. (b) Range profile of the output residue for Target 2.

of the output residue for the Doppler bins where the two moving targets are present. In both of the two subfigures, the moving targets are localized in the center. As we can see, both the subband processing method and ST-KT based
Fig. 18. Estimated MVDR spectra of the simulated ground clutter and jamming with and without KT interpolations. (a) Spectrum is generated without KT. (b) Spectrum is generated with slow-time domain KT interpolation. (c) Spectrum is generated with spatial domain KT interpolation. (d) Spectrum is generated with 2-D ST-KT.

STAP method overperform the traditional STAP method in terms of the output SCNR of moving targets. However, the improvement brought by subband processing is limited, as it cannot eliminate the 2-D spectrum spreading thoroughly. Among all the three approaches, the ST-KT method gains the best performance in terms of the output SCNR for both the fast-moving and slow-moving moving targets.

Considering that STAP is designed not only to mitigate the ground clutter but also to suppress the jamming, we now add a jamming source into the simulation and evaluate the performance of the proposed ST-KT in the environment where both clutter and jamming are present. A land-based barrage noise jamming is considered here as an example. Both the azimuth and elevation angles of the jamming source are set to be $30^\circ$, and the jamming-to-noise ratio (JNR) is 35 dB. Then, the echo signals from the simulated ground clutter as well as this jamming are summed up and compressed in range to generate the datacube.

In Fig. 18, the estimated MVDR spectra from the datacube with and without KT interpolations are provided to validate the ST-KT algorithm in the clutter-plus-jamming environments. As mentioned in the literature [2]–[6], jamming is assumed to be uncorrelated between pulses but correlated between array elements. That is to say, the contribution of jamming in the temporal domain is identical to that of noise, while the contribution in the spatial domain is the same as that of the point-target. Thus, in case of narrowband, the echo of jamming is recognized as the signal-frequency signal in the spatial domain, but occupies all the Doppler band. However, with increasing bandwidth, dispersion of jamming is deteriorated across the array, leading to the spectrum spreading in the spatial frequency domain, which is clearly shown in Fig. 18(a). Since the spreading is in the spatial frequency domain, the traditional slow-time domain KT fails to eliminate it, as shown in Fig. 18(b). The spreading spectrum of jamming causes a broadened jamming suppression notch, thereby degrades the performance of the traditional STAP methods. From our discussion made in Section III, it can be deduced that the dispersion of jamming in the spatial domain is induced by the RM in the spatial domain. Hence, it is expected to address this problem via cross-array KT interpolation. In Fig. 18(c), the estimated spectrum from the datacube with spatial domain KT interpolation is shown. As compared with Fig. 18(a), the spreading of jamming in the spatial frequency is well eliminated, although the spreading of clutter is not totally solved, which is the same as that shown in Fig. 11(c). Fig. 18(d) shows the MVDR spectrum estimated from the datacube when ST-KT is applied. Both the spectrum spreading with respect to the clutter and jamming are eliminated, demonstrating that our proposed algorithm is also promising in the environment where both clutter and jamming are present.

VI. CONCLUSION

With increasing bandwidths, the narrowband assumption employed in the traditional STAP theory becomes invalid, and the performance of the corresponding STAP methods drops significantly. In this article, we first present a detailed analysis on the influence of the increasing bandwidths on the ST characteristics of the ground clutter. Especially, analytical expressions are derived to quantitatively describe the direction
and extension of the spreading of single clutter source on the Doppler-spatial frequency plane, which accurately answers the question how the ground clutter shall spread in the 2-D frequency domain under wideband conditions. Moreover, the contribution of the ground clutter to ST snapshots in case of wideband is investigated, with the model of the clutter covariance matrix for W-STAP established.

Finally, to remedy the degraded performance of the current STAP methods in wideband cases, a 2-D ST-KT algorithm is proposed to eliminate the decorrelation within the snapshot. As validated by simulation results, by applying ST-KT to the datacube prior to adaptive processing, both traditional fully and partial STAP methods gain significant improvement in terms of the output SCNR for both slow-moving and fast-moving targets, as compared with the case when ST-KT is not applied, which proves the feasibility of this proposed algorithm for W-STAP.

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