Future deceleration due to cosmic backreaction in presence of the event horizon

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ABSTRACT
The present acceleration of the Universe leads to the formation of a cosmological future event horizon. We explore the effects of the event horizon on cosmological backreaction due to inhomogeneities in the universe. Beginning from the onset of the present accelerating era, we show that backreaction in presence of the event horizon causes acceleration to slow down in the subsequent evolution. Transition to another decelerating era could ensue eventually at a future epoch, ensuring avoidance of a big rip.

1 INTRODUCTION
There exists overwhelming observational evidence for the present acceleration of the Universe (Perlmutter 1998; Kowalski 2008; Hicken 2009; Seikel & Schwarz 2009). The accelerating universe leads to a future event horizon from beyond which it is not possible for any signal to reach us. On the other hand, observations also tell us that our Universe is inhomogeneous up to the scales of super clusters of galaxies. The idea that backreaction originating from density inhomogeneities could lead to modifications in evolution of the universe as described by the background Friedmann-Robertson-Walker (FRW) metric at large scales has gained popularity in recent years (Buchert 2000; Buchert & Carfora 2008; Wiegand & Buchert 2010; Zalaletdinov 1992; Buchert & Carfora 2003; Rasanen 2004, 2008, 2010; Kolb et al. 2003, 2008; Paranjape & Singh 2008; Singh 2011; Mattson & Mattson 2010; Wiltshire 2007a,b; Gasperini et al. 2009). Here we show that backreaction in the presence of the cosmological event horizon could have a remarkable consequence of ushering in another decelerated era beyond the present accelerating epoch.

In spite of numerous creative ideas proposed for the present acceleration (Saha 2001; Copeland et al. 2006), there is still a lack of convincing explanation of this phenomenon. The simplest possible explanation provided by a cosmological constant is endowed with conceptual problems (Weinberg 1988). Alternative mechanisms based on either modifications of the gravitational theory, or invoking extra fields suffer from the coincidence problem, as to why the era of acceleration begins around the same era when the Universe becomes structured. The ultimate fate of our Universe remains clouded in considerable mystery. Backreaction from inhomogeneities provides an interesting platform for investigating this issue without invoking additional physics, since the effects of backreaction gain strength as the inhomogeneities develop into structures around the present era.

Approaches have been developed to calculate the effect of inhomegeneous matter distribution on the evolution of the Universe (Buchert 2000; Zalaletdinov 1992; Kolb et al. 2009). Arguments in favour of the viability of backreaction seem rather compelling (Kolb et al. 2008), though there exists debate on the impact of inhomogeneities on observables of an overall homogeneous FRW model (Paranjape & Singh 2008; Ishibashi & Wald 2004; Singh 2011), and on the magnitude of backreaction modulated by the effect of shear between overdense and underdense regions (Mattson & Mattson 2010). Using the framework formulated by Buchert (Buchert 2000) it has been shown (Rasanen 2004, 2008, 2010) that backreaction could lead to an accelerated expansion during the present epoch. A notable application of the formalism has been developed by Wiltshire showing an apparent volume acceleration of the universe based on the different lapse of time between the underdense and overdense regions (Wiltshire 2007a,b). Further, gauge invariant averages in the Buchert framework have also been constructed recently (Gasperini et al. 2009).

While upcoming observations may ultimately decide whether backreaction from density inhomogeneities drives the present acceleration, the above studies (Buchert 2000; Buchert & Carfora 2008; Wiegand & Buchert 2010; Buchert & Carfora 2003; Rasanen 2004, 2008, 2010; Kolb et al. 2003, 2008; Wiltshire 2007a,b; Gasperini et al. 2009) have highlighted that backreaction could be an important ingredient of the evolution of our Universe. Here we explore this issue with a fresh perspective, viz., the impact of the event horizon on cosmological backreaction. The currently accelerating epoch dictates the existence of an event horizon since the transition from the previously matter dominated decelerating expansion. Since backreaction is evaluated from the global distribution of matter inhomogeneities, the event horizon demarcates the spatial regions which are causally connected to us and hence impact the evolution of our part of the Universe. In the present work we investigate the consequences of backreaction in presence of the horizon. Such an approach has remained unexplored in previous studies of backreaction. It may be noted that the for-
malism of back reaction (Buchert 2000, Buchert & Carfora 2008, Wiegand & Buchert 2010) has been criticized on the grounds that the average is taken on a space like hypersurface, while observations are made along and inside the past light cone (Ishibashi & Wald 2000). Our present analysis, by considering an effect due to the event horizon, introduces an element of light cone physics from a somewhat different perspective. We show that backreaction with the event horizon could lead to the possibility of transition to a decelerated future era.

2 THE BACKREACTION FRAMEWORK

In the framework developed by Buchert (Buchert 2000, Buchert & Carfora 2008, Wiegand & Buchert 2010) for the Universe filled with an irrotational fluid of dust the spacetime is foliated into flow-orthogonal hypersurfaces featuring the line-element $ds^2 = -dt^2 + g_{ij} dx^i dx^j$, where the proper time $t$ labels the hypersurfaces and $x^i$ are Gaussian normal coordinates (locating free-falling fluid elements or generalized fundamental observers) in the hypersurfaces, and $g^{ij}$ is the full inhomogeneous three metric of the hypersurfaces of constant proper time. For a compact spatial domain $D$ whose volume is given by $|D| = \int_D d\mu_g$, where $d\mu_g = \sqrt{\det g(t, X, X^2)} dX^1 dX^2 dX^3$, the scale factor $a_D(t) = \left[ \frac{|D|}{|D_0|} \right]^{1/3}$ encodes the average stretch of all directions of the domain. The Einstein equations then lead to Buchert (Buchert 2000, Buchert & Carfora 2008, Wiegand & Buchert 2010)

$$3 \frac{\ddot{a}_D}{a_D} = -4\pi G \langle \rho \rangle_D + Q_D + \Lambda$$
$$3H_D^2 = 8\pi G \langle \rho \rangle_D - \frac{1}{2} \langle R \rangle_D - \frac{1}{2} Q_D + \Lambda$$

where the average of the scalar quantities on the domain $D$ is defined as $(f)_D = \frac{1}{|D|} \int_D f d\mu_g = |D|^{-1} \int f d\mu_g$, and where $\rho$, $\rho$ and $H_D$ denote the local matter density, the global matter density, and the Hubble rate $H = \dot{a}_D/a_D$ respectively. The kinematical backreaction $Q_D$ is defined as $Q_D = \frac{3}{2} \left( \langle \theta^2 \rangle_D - \langle \theta \rangle_D^2 \right) - 2\sigma_D^2$, where $\theta$ is the local expansion rate and $\sigma_D^2 = 1/2 \sigma_{ij} \sigma^{ij}$ is the squared rate of shear. $Q_D$ encodes the departure from homogeneity.

The “global” domain $D$ is assumed to be separated into subregions $F \ell$ which themselves consist of elementary space entities $F \ell^{(\alpha)}$ that may be associated with some averaging length scale, i.e., $D = \cup \cup F \ell$, where $F \ell = \cup \cup F \ell^{(\alpha)}$ and $F \ell^{(\alpha)} \cap F \ell^{(\beta)} = \emptyset$ for all $\alpha \neq \beta$ and $\ell \neq m$. Analogous to the scale factor for the global domain, a scale factor $a_\ell$ for each of the subregions $F \ell$ can be defined such that $|D|_\ell = \int_{F \ell^{(\alpha)}} d\mu_g$, and hence $a_\ell^2 = \sum \lambda_\ell a_\ell^2$, where $\lambda_\ell = |F \ell^{(\alpha)}|/|D|_\ell$ is the initial volume fraction of the subregion $F \ell$. The average of the scalar valued function $f$ on the domain $D$, may then be split into the averages of $f$ on the subregions $F \ell$ in the form, $(f)_D = \sum \lambda_\ell (f)_{F \ell}$, where $\lambda_\ell = |F \ell^{(\alpha)}|/|D|_\ell$, is the volume fraction of the subregion $F \ell$. Due to the $(\theta^2)_D$ term, the expression for the backreaction $Q_D$ is given by

$$Q_D = \sum \lambda_\ell Q_\ell + 3 \sum \lambda_\ell \lambda_m (H_\ell - H_m)^2$$

where, $Q_\ell$ and $H_\ell$ are defined in $F \ell$ in the same way as $Q_D$ and $H_D$ are defined in $D$. The shear part $(\sigma^2)_F \ell$ is completely absorbed in $Q_D$ whereas the variance of the local expansion rates $(\theta^2)_D - (\theta)_{F \ell}^2$ is partly contained in $Q_D$ but also generates the extra term $3 \sum \lambda_\ell \lambda_m (H_\ell - H_m)^2$. This is because the part of the variance that is present in $Q_D$, namely $(\theta^2)_F \ell - (\theta)_{F \ell}^2$ only takes into account points inside $F \ell$. To restore the variance that comes from combining points of $F \ell$ with others in $F_m$, the extra term containing the averaged Hubble rate emerges. Note here that the above formulation of the backreaction holds in the case when there is no interaction between the overdense and the underdense subregions.

Now from Eq.$(\ref{eq:backreaction})$ one gets

$$\frac{\ddot{a}_D}{a_D} = \sum \lambda_\ell \frac{a_\ell}{a_D} \frac{\ddot{a}_\ell}{a_\ell} + \sum \lambda_\ell \lambda_m (H_\ell - H_m)^2$$

Following the simplifying assumption of Ref. Wiegand & Buchert (2010), (which captures the essential physics) we work with only two subregions. Clubbing those parts of $D$ which consist of initial overdensity as $M$ (called “wall”), and those with initial underdensity as $E$ (called “void”), such that $D = M \cup E$, one obtains $H_D = \lambda_M H_M + \lambda_E H_E$, with similar expressions for $(\rho)_D$ and $(\langle R \rangle)_D$, and

$$\frac{\ddot{\lambda}_M}{\lambda_M} = \lambda_M \frac{\ddot{a}_M}{a_M} + \lambda_E \frac{\ddot{a}_E}{a_E} + 2 \lambda_M \lambda_E (H_M - H_E)^2$$

Here $\lambda_M + \lambda_E = 1$, with $\lambda_M = |M|/|D|$ and $\lambda_E = |E|/|D|$. Since the global domain $D$ is large enough for a scale of homogeneity to be associated with it, one can write $|D|_\ell = \int_{F \ell} \sqrt{-g} d^4 X \approx f(r) a_\ell^{-1}(t)$, where $f(r)$ is a function of the FRW comoving radial coordinate $r$. It then follows that $a_D \approx \left( \frac{f(r)}{|D|_\ell} \right)^{1/3} a_F$, and hence, the volume average scale factor $a_D$ and the FRW scale factor $a_F$ are related by $a_D \approx c_F a_F$, where $c_F$ is constant in time. Thus, $H_F \approx H_D$, where $H_F$ is the FRW Hubble parameter associated with $D$. Though in general $H_D$ and $H_F$ could differ on even large scales Wiegand & Buchert (2010), the above approximation is valid for small metric perturbations.

3 EFFECT OF EVENT HORIZON

We now come to the central issue of the paper, as to what happens to the evolution of the universe once the present stage of acceleration sets in. Note henceforth, we do not need to necessarily assume that the acceleration is due to backreaction Wiegand & Buchert (2010, Rasanen 2004). For the purpose of our present analysis, it suffices to consider the observed accelerated phase of the universe Seikel & Schwarz (2000) that could occur due to any of a variety of mechanisms Sahni (2004, Copeland et al. 2006). Given that we are undergoing a stage of acceleration since transition from an era of structure formation, our aim here is to explore the subsequent evolution of the Universe due to the effects of backreaction in presence of the cosmic event horizon. Though
in general, spatial and light cone distances and corresponding accelerations could be different, as shown explicitly in the framework of LTB models (Bolečko & Andersson 2008), an approximation for the event horizon which forms at the onset of acceleration could be defined by

\[ r_h = a_D \int_{0}^{\infty} \frac{dt'}{a_D(t')} \] (5)

in the same spirit as \( a_D = c_D a_F \).

Following the Buchert framework (Buchert 2004, Wiegand & Buchert 2010), as discussed above, the global domain \( D \) is divided into a collection of overdense regions \( M = \bigcup_{j} M_j \), with total volume \( |M|_g = \sum_{j} |M_j|_g \), and underdense regions \( E = \bigcup_{i} E_i \) with corresponding volume \( |E|_g = \sum_{i} |E_i|_g \). Assuming that the scale factors of the regions \( E_i \) and \( M_j \) are respectively given by \( a_{E_i} = c_{E_i} t^\beta \) and \( a_{M_j} = c_{M_j} t^\beta \), where \( \alpha, \beta, c_E, \) and \( c_M \) are constants, one has

\[ a_E^2 = c_E^3 t^{3\alpha}; \quad a_M^3 = c_M^{3\beta} \] (6)

where \( c_E^3 = \sum_{i} c_{E_i}^3 |E_i|_g |E_i|_g \) is a constant, and similarly for \( c_M \). The volume fraction of the subdomain \( M \) is given by \( \lambda_M = |M|_g / |E|_g \) which can be rewritten in terms of the corresponding scale factors as \( \lambda_M = a_{M_g} / a_{E_g} \). Since an event horizon forms, only those regions of \( D \) that are within the event horizon are accessible to us. Hence, in this case an apparent volume fraction \( \lambda_{M_h} \) given by \( \lambda_{M_h} = a_{M_h} = \frac{c_M^4 |M|_g / 8\pi \kappa}{r_h^3} \) is introduced. From eq. (5) it follows that

\[ \lambda_{M_h} = \frac{c_M^4 |M|_g / 8\pi \kappa}{r_h^3} \] (7)

where \( c_M^4 = 3c_M^4 |M|_g / 4\pi \) is a constant. Normalizing the total accessible volume in the presence of the event horizon, we can write

\[ \lambda_{E_h} = 1 - \lambda_{M_h} \] (8)

where \( \lambda_{E_h} \) is the apparent volume fraction for the subdomain \( E \). It hence follows that the global acceleration equation (4) is now given by

\[ \ddot{a}_D = \frac{c_M^4 t^3 \beta (\beta - 1)}{r_h^3} + \left( 1 - \frac{c_M^4 t^3 \beta}{r_h^3} \right) \frac{\alpha (\alpha - 1)}{t^2} + 2 \frac{c_M^4 t^3 \beta}{r_h^3} \left( 1 - \frac{c_M^4 t^3 \beta}{r_h^3} \right) \left( \frac{\beta - \alpha}{t} \right)^2 \] (9)

In order to obtain the future evolution of the universe with backreaction in presence of the event horizon, one has to solve the above equation for the scale factor with the event horizon \( r_h \) given by Eq. (9). In what follows we will eventually obtain numerical solutions of the above integro-differential equations. However, it is first instructive to obtain some physical insight of the evolution by taking recourse to a simple approximation.

To this end, let us for the moment model the onset of the present acceleration of the Universe by an exponential expansion, keeping our analysis close to observations. Specifically, we set \( a_D \propto e^{H_D t} \) in Eq. (9). (We will see later that this rather crude approximation does indeed give rise to results that are qualitatively similar to the ones obtained through numerical analysis). Using \( H_D = H_{DE} \), where \( H_D \) is the FRW Hubble parameter associated with \( D \), it follows that \( r_h = H_{DE}^{-1} \), a constant which we substitute in Eq. (9). With this substitution, the global acceleration \( \ddot{a}_D \) vanishes at times given by

\[ t^\beta = \frac{r_h^3}{4 (\beta - \alpha) c_M^3} \left[ (3\beta - \alpha - 1) \pm \sqrt{(3\beta - \alpha - 1)^2 + 8\alpha (\alpha - 1)} \right] \] (10)

The scale factor of the “wall” grows as \( t^\beta \), where \( 1/2 < \beta < 2/3 \). Eq. (10) corresponds to real time solutions for \( \alpha > \frac{1}{2} \left( \frac{1}{2} + 2 \sqrt{2/3 (1 - \beta)} \right) \).

Now, let us consider the following two cases separately:

Case I: \( \alpha < 1 \) and \( \beta < 2/3 \). There exist two real solutions corresponding to two values of time when the global acceleration vanishes. In Fig.1 we plot a dimensionless global acceleration parameter \( \ddot{a}_D / \dot{a}_D H^2_0 \) with time using eq. (3). The curves (i) and (ii) correspond to this case showing that the Universe first enters the epoch of acceleration due to backreaction, which subsequently slows down and finally vanishes at the onset of another decelerating era in the future. Case II: \( \alpha > 1 \) and \( \beta < 2/3 \). From (10) it follows that there is only one real solution (minus sign for the square root). This case models the Universe which accelerates due to some other mechanism (not backreaction), but subsequently enters an epoch of deceleration due to backreaction of inhomogeneities in the presence of the event horizon (see curves (iii) and (iv) of Fig.1).

The plots in Fig.1 have been done taking the standard values of the parameters \( r_h = H_D^{-1} = 4.36 \times 10^{27} \) s while choosing the appropriate range for the parameters \( \alpha \) and \( \beta \), as given in the figure caption. Based on the N-body simulation values used in (Wiegand & Buchert 2010), we also take \( \lambda_{M,0} = 0.09 \). Using the relation \( z_T = \exp[H_D (t_0 - t_T)] - 1 \), where \( z_T \) corresponds to the transition time in the past, the red-shifts for the transition could be estimated. For example, for the data used in curve (i), the transition from deceleration to acceleration occurs at \( z_T \approx 0.844 \), and for curve (ii) we have \( z_T \approx 0.914 \) (which are close to the \( \Lambda CDM \)
The dimensionless global acceleration parameter \( \frac{\ddot{a}}{a^2 H_0^2} \) is plotted versus time \( t \) as obtained through numerical integration, with the ‘initial condition of \( q_0 = -0.7 \). The values for the various parameters used are chosen to be the same as in the corresponding plots of Fig.1.

Thus, the evolution of the scale factor is now governed by the set of coupled differential equations \( \ref{eqn:1} \) and \( \ref{eqn:4} \). We numerically integrate these equations by using as an “initial condition” the observational constraint \( q_0 \approx -0.7 \), where \( q_0 \) is the current value of the deceleration parameter, and using the solution for the scale factor plot the global acceleration versus time in Fig.2 (thus all the curves in Fig.2 are set to intersect at the point \( (t_0, q_0) \)). The values of the other parameters including \( \alpha \) and \( \beta \) are chosen to be the same as in the corresponding curves of the exponential case. As can be seen from Fig.2, the nature of the plots is quite similar to the ones that were obtained in the case assuming a constant event horizon, with the \( \alpha > 1 \) curves signifying only one transition between acceleration and deceleration in the future. The differences in the various slopes and also in the scale for the dimensionless global acceleration parameter in the two cases arise as a result of the approximation of constant horizon used in the former, as well as due to the choice of the condition \( q_0 \approx -0.7 \) used in the latter.

4 CONCLUSIONS

To summarize, in this work we have explored the effect of backreaction due to inhomogeneities on the evolution of the Universe undergoing present acceleration. We have shown that the presence of the cosmic event horizon causes the acceleration to slow down significantly with time. Our results indicate the fascinating possibility of backreaction being responsible not only for the present acceleration as shown in earlier works (Wiegand & Buchert 2010, Rasanen 2010), but also leading to a transition to another decelerated era in the future. Another possibility following from our analysis is of the Universe currently accelerating due to a different mechanism (Sahni 2004; Copeland et al. 2006), but with backreaction (Buchert 2000; Buchert & Carfora 2008; Wiegand & Buchert 2010) later causing acceleration to slow down. Our prediction of the future slowing down of acceleration seems to fit smoothly with the earlier era of structure formation and the transition to acceleration in the standard CDM model, as shown here (transition red-shift \( z_T \approx 0.8 \)).

Before concluding, in context of the formalism used in the present work it may be worthwhile to recapitulate some of the present debate in the literature regarding averaging on a space like hypersurface (Buchert 2000; Buchert & Carfora 2008; Wiegand & Buchert 2010) as compared to taking the average on the past light cone. The usefulness of the expansion rate averaged on any hypersurface is determined by relating it to observed quantities. It has been observed that the redshift and distance can be expressed in terms of the average geometry alone, provided that the contribution of the null shear is negligible (Rasanen 2009). Observationally, the shear is known to be indeed small (Munshi et al. 2008). Nonetheless, it has been claimed that neither averaging on a constant time hypersurface nor light cone averaging is easy to connect with the observations corresponding to parameters of the CDM model (Kolb & Lamb 2003). The task of developing a procedure for light cone averaging is an ambitious program and till date there is no standard formalism to do so. In a recent paper three different types of light cone averaging have been proposed (Gasperini et al. 2011), though much work remains to be done in order to apply their technique to the problem of cosmic acceleration. On the other hand, our present work introduces an element of light cone physics from another perspective by considering an effect due to the event horizon.

Finally, it may be noted that though the event horizon is observer dependent, it follows from the symmetry of the equations \( \ref{eqn:1} \) and \( \ref{eqn:4} \) that our analysis leads to similar conclusions for a “void” centric observer, as it does for a “wall” centric one. Note also that our analysis is valid while the event horizon exists. Hence, if the acceleration vanishes at some epoch in the future, one needs to consider backreaction without the event horizon beyond that epoch. The scales which crossed outside the horizon earlier, will begin re-entering with backreaction from their associated inhomogeneities impacting the evolution. Such a scenario is somewhat reminiscent of the horizon crossing of modes during inflation in the early universe, and their subsequent reentry with rich cosmological consequences. In the present context, the impact on the global evolution of the reentering scales needs to be studied further. Moreover, it would be worthwhile to investigate effects of the cosmic event horizon on other models of backreaction in order to make more generic predictions of observational interest.

References

Bolejko K., and Andersson L., 2008, JCAP 10, 003
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Buchert T., 2000, Gen. Rel. Grav. 32, 105
Buchert T., and Carfora M., 2003, Phys. Rev. Lett. 90, 031101
Buchert T., and Carfora M., 2008, Class. Quant. Grav. 25, 195001
Copeland E. J., Sami M., Tsujikawa S., 2006, Int. J. Mod. Phys. D 15, 1753
Gasperini M., Marozzi G., and Veneziano G., 2009, JCAP 0903, 011
Gasperini M., Marozzi G., Nugier F., and Veneziano G., 2011, JCAP 07, 008
Hicken M., et al., 2009, APJ 700, 1097
Ishibashi A., and Wald R. M, 2006, Class. Quant. Grav. 23, 235
Kolb E. W., Matarrese S., Notari A., and Riotto A., 2005, Phys. Rev. D 71, 023524
Kolb E. W., Marra S., Matarrese S., 2008, Phys. Rev. D 78, 103002
Kolb E. W., and Lamb C. R., 2009, arXiv: 0911.3852
Kowalski M., et al., 2008, APJ 686, 749
Mattsson M., and Mattsson T., 2010, JCAP 10, 021
Melchiorri A., Pagano L., and Pandolfi S., 2007, Phys. Rev. D 76, 041301(R)
Munshi D., et al., 2008, Phys. Rep. 462, 67
Paranjape A., and Singh T. P., 2008, Phys. Rev. D 78, 063522
Perlmutter S., et al., 1998, Nature 391, 51
Rasanen S., 2004, JCAP 0402, 003
Rasanen S., 2008, JCAP 0804, 026
Rasanen S., 2009, JCAP 02, 011
Rasanen S., 2010, Phys. Rev. D 81, 103512
Sahni V., 2004, Lect. Notes Phys. 653, 141
Seikel M., and Schwarz D. J., 2009, JCAP 02, 024
Singh T. P., 2011, preprint arXiv:1105.3450
Wiegand A., and Buchert T., 2010, Phys. Rev. D 82, 023523
Weinberg S., 1989, Rev. Mod. Phys. 61, 1
Wiltshire D. L., 2007a, New J. Phys. 9, 377
Wiltshire D. L., 2007b, Phys. Rev. Lett. 99, 251101
Zalaletdinov R., 1992, Gen. Rel. Grav. 24, 1015