A Satisficing Game Theoretic Approach to Multi-Aircraft Collision Avoidance

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Abstract. We propose a distributed approach to multi-aircraft real-time conflict resolution based on satisficing game theory which considers reducing flight collisions and improving economic efficiency. In our conflict game model, aircraft as exclusively self-interested agent are assigned different statuses by a priority ranking mechanism proposed. According to the state of one's own priority, they take others’ preferences into consideration to choose the most satisficing conflict resolution strategy from policy set including speed and heading modification strategies. The simulation results on the random flight scenario of two concentric circles show that our method which adopts speed and heading adjustment to avoid conflicts is promising in terms of safety and system efficiency.

1. Introduction
Along with the development of air transportation industry, flight conflicts are becoming increasingly prominent as aircraft density in airspace increases dramatically. Collision avoidance plays a vital role in order to ensure the airspace security and improve the capacity of airspace. For conflict resolution, the objective is to provide suitable maneuvers to avoid a predicted conflict. Much research has been devoted over the last decade on conflict resolution tasks.

A review of research in this area of conflict resolution is presented in a study[1]. Short-term conflict resolution using genetic algorithm(GA)[2,3] and optimal control theory[4,5] has been paid attention extensively. These methods are large in computing capacity and can’t predict conflict within 5 minutes. Force fields approach[6] is explored to resolve real-time conflict problem. However, the kind of algorithm is not always guaranteed to generate flyable trajectories. Tomlin et al.[7,8] propose non-cooperative two-person zero-sum game theory to design controllers for safety specifications. Krozel et al.[9] introduce the distributed real-time conflict resolution algorithms. A myopic strategy selects the alternative that requires the smallest heading adjustment to resolve collisions in pairs. Archibald et al.[10] adopt a satisficing game theory to resolve real-time aircraft collisions. The maneuvering strategies of conflict resolution are only heading modification, and the flight speed is constant.

In our paper, we present a decentralized approach to multi-aircraft real-time conflict resolution based on the satisficing game theory. The approach adopts flight velocity vector adjustments as maneuvering strategies of collision avoidance focused on the two dimensional case. A resolution maneuver set is obtained for each player by selectability and rejectability. The proposed approach is illustrated in simulation of the random flight scenario of two concentric circles. It is showed that our approach could real-time resolve multi-aircraft conflicts efficiently and robustly.
The mathematic formulation of multi-aircraft conflicts and real-time resolution methodology is detailed in the second part. In the third part, we describe the numerical simulation and results. The paper is concluded in the last part.

2. Collision Avoidance Model

2.1. Conflict Prediction

We assume that each aircraft is aware of flight critical information pertaining to the others within a given detectable radius \( R_D \) and \( A \) is a set of all aircraft operating in the airspace where there are \( M \) aircraft, and any aircraft \( a_i \) satisfies \( a_i \in A \). In order to illustrate the flight collision, circular protected zone of aircraft, namely near miss radius \( R_{NM} \) and collision radius \( R_C \), satisfies \( R_{NM} \gg R_C \), as shown in figure 1. When the distance of current locations between \( a_i \) and \( a_j \) represented as \( d(i,j) \) is smaller than or equal to \( R_C \), it will lead to a collision. If \( R_C < d(i,j) \leq R_{NM} \), a near miss event will occur. Collisions are weighted more heavily than near misses.

![Figure 1. Example of near miss zone and collision zone. Yellow represents near miss zone and red represents collision zone, with \( R_{NM} \gg R_C \).](image1.png)

The conflict prediction method used in our study is based on searching for the closest point of approach (CPA) between two aircraft within look-forward time window \( T_w = 8 \text{ min} \), which can be mathematically obtained by linear predictions according to their current action, as shown in figure 2. We define \( d_{nm}(i,j) \) to represent the minimum distance at CPA between \( a_i \) and \( a_j \), and \( d_{CPA}(i,j) \) is the distance from \( a_i \)'s current position to \( a_i \)'s CPA between \( a_i \) and \( a_j \). If \( d_{nm}(i,j) \leq R_{NM} \) and \( d(i,j) < R_D \), there are potential collisions between \( a_i \) and \( a_j \) to be denoted as a conflict vector \( w_{ij} = (d_{nm}(i,j),d(i,j)) \in W^2 \), where \( W^2 \) is the conflict space of pair-wise aircraft. It is supposed that \( E_i \) is a set of all the potential conflict with \( a_i \) represented as \( E_i = \{w_{ij} | j = 1,\ldots,i-1,i+1,\ldots,M\} \).

![Figure 2. Representation of a conflict. It shows the position of CPA between \( a_i \) and \( a_j \), the minimum predicted distance \( d_{nm}(i,j) \) at CPA, and the distance \( d_{CPA}(i,j) \) from \( a_i \)'s current position to \( a_i \)'s CPA between \( a_i \) and \( a_j \).](image2.png)
In games, the playing pure strategy \( s_i^n = (n = 1, \ldots, N) \) of individual \( a_i (i = 1, \ldots, M) \) is from the discrete set of optional strategies \( S_i (s_i^n \in S_i) \), where \( N \) is the total number of \( a_i \)'s optional strategies, namely the dimensions of \( S_i \). \( a_i \) possesses a mapping from different strategy combinations \((s_i^n, s_i^n', \ldots, s_i^{n_M}, \ldots, s_i^{n_M})\) to rejectable and selectable payoffs. The mapping from strategies space to rejectable and selectable payoffs space can be mathematically constructed as \( H : S_i \times S_j \rightarrow W^2 \), namely the mapping from strategies space to conflicts space.

### 2.2. Priority Ranking Mechanism

A priority ranking mechanism is introduced to represent individuals’ different social status before games. The rules of defining ranking are as follows: firstly, those individuals within 5 nmi away from the destination possess higher ranks than all the others; secondly, in each subset, individuals with greater current flight delay cause higher ranks; thirdly, individuals in each subset are allocated ranks respectively by their current flight time, with longer flight time bringing higher ranks; finally, individuals in the same subset with the same delay and flight time are ranked according to the remaining planned flight time, with larger remaining time resulting in higher ranks. It is supposed that the priority ranking mechanism leads to a unique priority for each individual, and that rank orderings in different games are consistent from the perspective of all individuals.

In a round of games, individual \( a_i \) determines its neighbors of game according to the potential collisions set \( E_i \) and individuals’ priority ranking. The individuals with higher ranking will ignore potential conflicts with ones of lower priority and satisfy its own interests absolutely, while ones with lower ranking will preferentially consider potential conflicts and cost its own interests to benefit superior ranking individuals and the whole group. Therefore, the set of \( a_i \)'s game neighbors is all individuals with higher ranking than \( a_i \) in \( E_i \), and represented as \( E_i' \).

### 2.3. Conflict Resolution

In our study, each aircraft chooses its maneuvering strategies of conflict resolution according to its priority ranking, game neighbors, and selectable and rejectable payoff. Aircraft adopt flight velocity vector changes as the flight strategies to avoid collisions. At each time step, each aircraft selects one resolving strategy from five speed and five direction options as its behavior at the next time step. We assumed that heading and speed changes are instantaneous. The five direction modifications include straight flight, moderate turns left and right (2.5°), and sharp turns left or right (5°). The five speed modifications include flight according to planned speed, acceleration ±5% based on planned speed, and acceleration ±10% based on planned speed[11].

#### 2.3.1. Rejectability

The details of computing the rejectable payoff are as follows. Let \( s_i \) signify \( a_i \)'s current flight action. The rejectable payoff function is represented as

\[
P_r(s_i) = \frac{1}{1 + \sum_{s_j \neq s_i} F(s_i, s_j)}.
\] (1)

The function \( F(s_i, s_j) \) is defined as

\[
F(s_i, s_j) = \begin{cases} 
2\alpha, & \text{if } d_{\min}(i, j) \leq R_c \\
\alpha, & \text{if } R_c < d_{\min}(i, j) \leq R_{\text{cut}} \\
0, & \text{otherwise}
\end{cases}
\] (2)

where \( \alpha \) is defined as
Each aircraft selects a strategy according to the rules described in previous section and updates its heading, speed and position, the new information is then broadcasted to all the others within its detectable zone. For all aircraft in the airspace, it is supposed that the value of the detectable radius is same, set as $R_{detect} = 50nmi$. To maintain calculation accuracy, the frequency of updates of resolution strategy is once each second in our simulation.

$$\alpha = \begin{cases} (2 - \frac{d_{min}(i, j)}{R_{SM}})^{\beta} - \frac{1}{d_{CPA}(i, j)}, & \text{if } d_{CPA}(i, j) \leq 3R_{SM} \\ \frac{1}{d_{CPA}(i, j)^{\beta}}, & \text{otherwise} \end{cases}$$

(3)

where the parameter $\beta$ is experimentally tuned variable, set as $\beta = \frac{2}{3}$. If there are conflicts in every optional strategy, the maximum rejectable options will be the ones with the most distant conflicts. Those strategies leading to maximum rejectable payoff function $P_{ia}(s^{*}_i)$ are seen as safe strategy subset $\overline{S}_i = \{s \mid s = \arg \max_{s^{*}_i} P_{ia}(s^{*}_i), \overline{S}_i \subset S_i \}$. If there is only strategy in $\overline{S}_i$, aircraft $a_i$ will execute the only one as its behavior at the next time step. If the number of the elements in $\overline{S}_i$ is more than one or there is no conflict for all strategy option, aircraft’s action decisions will depend on selectability.

2.3.2. Selectability. Most flight missions require that the aircraft reaches the destination as timely as possible, so selectability that reflects goal achievement according to flight mission requirements is influenced by the required time of arrival. $a_i$ selects the strategy to satisfy the required time of arrival to the greatest extent possible and minimize the flight delay. We assume that $a_i$’s current location coordinate is represented as $L'_i = (x'_i, y'_i)$, and its planned location coordinate at the next time step is $L^{p}_i = (x'_i, y'^{p}_i)$, $a_i$’s possible location at the next time step is determined by selected strategy $s^{*}_i$ from the safety subset $\overline{S}_i$, represented as $L'_i + s^{v}_i$, where $s^{v}_i$ is the vector form of $s^{*}_i$ to denote flight speed and direction. Then the selectable payoff function is defined as

$$P_{ia}(s^{*}_i) = \exp(-|L'_i + s^{v}_i - L^{p}_i|),$$

(4)

where $s^{v}_i \in \overline{S}_i$. The planned location coordinate at the next time step is defined as

$$L^{p}_i = \begin{cases} L'_i + \frac{\overline{L}_iL^{p}_i}{T^{p}_i - T'_i}, & \text{if } T^{p}_i > T'_i \\ L'_i, & \text{otherwise} \end{cases}$$

(5)

where $L'_i = (x'_i, y'_i)$ is the coordinate of $a_i$’s destination, $T^{p}_i$ is its planned flight time, $T'_i$ is its current flight time, and $\overline{L}_iL^{p}_i$ is the vector from its current position $L'_i$ to its destination $L^{p}_i$. The strategy with maximum selectable payoff function $P_{ia}(s^{*}_i)$ will be executed by $a_i$ in the next time step, specifically,

$$s^{\text{next}}_i = \arg \max_{s^{*}_i \in \overline{S}_i} P_{ia}(s^{*}_i).$$

(6)

3. Simulation Results and Discussion

Our simulation environment is similar to the one used in some previous studies[9,10]. In the airspace, all aircraft cruise at the same altitude and at the same planned speed $v_p = 500mph$. Each aircraft selects a strategy according to the rules described in previous section and updates its heading, speed and position, the new information is then broadcasted to all the others within its detectable zone. For all aircraft in the airspace, it is supposed that the value of the detectable radius is same, set as $R_{detect} = 50nmi$. To maintain calculation accuracy, the frequency of updates of resolution strategy is once each second in our simulation.
3.1. Evaluation Metrics

In order to evaluate our approach to collision avoidance, appropriate metrics will be used to guarantee that both safety and efficiency objectives are met. Safety and system efficiency evaluation concerns are similar to the ones used in other studies[10]. The most important metric is flight safety for any method of conflict resolution. Two types of separation violations are reported: system collisions and system near misses. The collision event described in previous section is the case of $R_c \leq 900\text{ ft}$ between any two aircraft. System collisions is defined as

$$C = \sum_{m=1}^{T_s} C_m, \quad (7)$$

where $C_m$ is the number of collisions at mth time step and $T_s$ is total flight time of whole system. When $R_NM \leq 5\text{ nmi}$ between any two aircraft, the near miss event happens. System near misses is defined to be similar to system collisions,

$$NM = \sum_{m=1}^{T_s} NM_m, \quad (8)$$

where $NM_m$ is the number of near misses at mth time step.

Conflict resolution maneuvers would lead to deviating from aircraft’s projected paths, consuming more resources, and bring greater economic losses. System efficiency reflects the deviation to which the aircraft are able to follow their projected paths to destinations. A good method of collision avoidance should maintain the high system efficiency while ensuring safe flight. The fight efficiency of aircraft $a_i$ is defined as $EFF_i = \frac{T_{ip}}{T_i}$, where $T_{ip}$ is $a_i$’s projected flight time along its projected paths, and $T_i$ is $a_i$’s actual flight time to reach its destination. Then system efficiency is represented as

$$SE = \frac{1}{M} \sum_{i=1}^{M} EFF_i. \quad (9)$$

Additionally, the standard deviation of efficiency is used as an evaluation of fairness of flight delays, represented as

$$SDE = \frac{1}{\sqrt{M}} \left( \sum_{i=1}^{M} (EFF_i - SE)^2 \right)^{1/2}. \quad (10)$$

3.2. Simulation Results

In our study, the simulation process is divided into three types according to different maneuvering strategies: only speed changes, only heading changes, and velocity vector changes.

Figure 3. The scenario of random flights.
In order to identify the robust of our model, a random traffic patterns scenario[9] is used. In the random flight scenario, there are two concentric circles. The radii of the outer circle and the inner circle are 60nmi and 50nmi respectively. Each aircraft starts at a random point on the outer circle and arrives at a random point on the inner circle as its destination, and its projected path must pass through the inner circle, as shown in figure 3. In this scenario, conflicts only occur in the inner circle. The inner circle represents the test airspace and the 10nmi ring buffer guarantees that no possible collision exists in the inner circle at initial period of generating aircraft. Because of the randomness of the scenario, it is a great examination of collisions avoidance algorithms with diverse random geometries. When each simulation run starts, a new aircraft emerges with approximately 5s intervals until the upper limit of the number of aircraft is reached in the concentric circles. The upper limit of the number of aircraft denotes an associated traffic density $\mu$ in this scenario. When an aircraft arrives at their destinations, a new aircraft is generated to replace it with the same start and end. The simulation results that run for up to 60 minutes are collected and averaged over 20 runs at each group of data.

**Table 1.** The result for random flights scenario in the case of only speed changes

| $\mu$ | Near Misses | Collisions | SE | SDE |
|-------|-------------|------------|----|-----|
| 3     | 0           | 0          | 1  | 0   |
| 6     | 0.02409771  | 0          | 1  | 0   |
| 9     | 0.170067375 | 0.000222099| 1  | 0   |
| 12    | 0.3285664   | 0.000111049| 0.9983245 | 0.00085069 |
| 15    | 0.6291387   | 0          | 0.9983005 | 0.00135001 |
| 18    | 0.98722465  | 0.000166574| 0.9981165 | 0.0018207 |
| 20    | 1.37991535  | 0.000333149| 0.998004 | 0.00211189 |
| 22    | 1.3602859   | 0.000277624| 1   | 0   |
| 24    | 2.0086165   | 0.000402555| 0.997895 | 0.00246308 |
| 26    | 1.9560525   | 0.000277623| 0.997534 | 0.00349094 |
| 28    | 2.7481115   | 0.000499722| 0.9979945| 0.00228207 |
| 30    | 2.977541    | 0.000652416| 0.99711 | 0.00434006 |
| 32    | 3.6777205   | 0.001096613| 0.996751 | 0.00579429 |

**Table 2.** The result for random flights scenario in the case of only heading changes

| $\mu$ | Near Misses | Collisions | SE    | SDE    |
|-------|-------------|------------|-------|--------|
| 3     | 0           | 0          | 1     | 0      |
| 6     | 0.023806223 | 0          | 0.9999628 | 0.001636319 |
| 9     | 0.16461992  | 0          | 0.99222205 | 0.018389865 |
| 12    | 0.38903991  | 0          | 0.9951527 | 0.00894775 |
| 15    | 0.70884775  | 0          | 0.9830611 | 0.042024318 |
| 18    | 1.01013355  | 0.000083287| 0.98135935 | 0.043837987 |
| 20    | 1.22247345  | 0          | 0.97308325 | 0.053015962 |
| 22    | 1.7353      | 0.00059689 | 0.96891295 | 0.059064045 |
| 24    | 2.5912115   | 0.000624653| 0.9544303 | 0.084839645 |
| 26    | 2.6465705   | 0.001110492| 0.95361955 | 0.08323377 |
| 28    | 3.9120355   | 0.005496944| 0.9381217 | 0.0871449 |
| 30    | 4.968198    | 0.006538036| 0.91691315 | 0.110845485 |
| 32    | 6.4531805   | 0.012395889| 0.89199145 | 0.122369525 |
Table 1 shows the simulation results in the case of only speed adjustments as maneuvering strategies. The number of near misses and collision events increase and system efficiency doesn’t change much in the vicinity of \( SE = 1 \) as the traffic density \( \mu \) grows. In the case of only heading changes, the simulation results for collision avoidance are exhibited in table 2. When \( \mu \) is increased, the number of near misses and collisions grows and system efficiency lowers. Comparing the two methods, the strategy of alone heading changes can improve the performance of flight safety slightly, whereas its system efficiency is sacrificed as the cost. The simulation results for velocity vector adjustments as resolution strategies are exhibited in table 3. The near miss event increases and system efficiency decreases slightly with growing \( \mu \). It is noted that the strategy of speed and heading adjustments greatly improves the performance of flight safety in contrast to two others, and its system efficiency is slightly less than one of only speed changes as resolution strategies.

### Table 3. The result for random flights scenario in the case of velocity vector change

| \( \mu \)   | Evaluation Metrics |
|-----------|--------------------|
|           | Near Misses | Collisions | SE   | SDE   |
| 3         | 0          | 0          | 1    | 0     |
| 6         | 0          | 0          | 1    | 0     |
| 9         | 0.0843987  | 0          | 1    | 0     |
| 12        | 0.3658218  | 0          | 1    | 0     |
| 15        | 0.5918375  | 0          | 0.9997827 | 0.0054207 |
| 18        | 0.98057    | 0          | 0.9935034 | 0.022086  |
| 20        | 1.3594895  | 0          | 0.9862456 | 0.0384128 |
| 22        | 1.6042109  | 0          | 0.980139 | 0.0481082 |
| 24        | 1.7628547  | 0          | 0.9808618 | 0.0460986 |
| 26        | 2.0192789  | 0          | 0.977478778 | 0.0596417 |
| 28        | 2.728651   | 0          | 0.9596697 | 0.0815321 |
| 29        | 2.964632   | 0.0003054 | 0.961747 | 0.081553 |
| 30        | 3.145503   | 0          | 0.9624844 | 0.075867  |
| 32        | 3.339284   | 0.0009717 | 0.9538982 | 0.1006478 |

In order to prove the high efficiency of our algorithm, a contrast experiment is shown in figure 4. The two contrastive methods are Archibald’s full satisficing algorithm[10] and Krozel’s decentralized look-ahead approach[9]. In the scenario of random flights, the contrastive results confirm that our approach which uses speed and heading adjustments as game strategies offers significantly better performance for system efficiency in contrast to two others.

![Figure 4](image-url)  

**Figure 4.** A comparison of system efficiency among three algorithms including our method, Archibald’s full satisficing algorithm and Krozel’s decentralized look-ahead approach.
4. Conclusion

Further developments of collision avoidance are required for a better management of flight conflicts with the increasing air traffic densities of the next decade. In the study, we present a decentralized approach to multi-aircraft real-time conflict resolution based on the satisficing game theory. The approach adopts speed and heading modification as maneuvering strategies of collision avoidance. The simulation results prove our approach can offer good performance of safety and highly efficient flight. Most importantly, the reported collision events and system efficiency demonstrate that the approach of velocity vector adjustments is promising in the case of high density autonomous flight.

Acknowledgments

This work is supported by Beijing Natural Science Foundation(Grant No. 4174092).

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