Confronting Einstein Yang Mills Higgs Dark Energy in light of observations

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Abstract

We study the observational aspects of Einstein Yang Mills Higgs Dark energy model and constrain the parameter space from the latest observational data from type Ia supernovae, observational Hubble data, baryon acoustic oscillation data and cosmic microwave background radiation shift parameter data. It is found from the analysis of data that the Higgs field in presence of gauge fields can successfully describe the present accelerated expansion of the universe consistent with the astrophysical observations.

1 Introduction

Cosmic acceleration discovered more than two decades ago by supernova projects [1,2] is perhaps the most important and fascinating phenomenon that still remains in mystery. This cosmic acceleration can be accounted for by invoking the presence of some exotic fluid dubbed dark energy with negative pressure to overcome the gravitational collapse and thereby resulting in the accelerated expansion of the universe [3–6]. From the observations it is evident that it constitute about 68% of the total energy density in the universe [7]. The cosmological constant \( \Lambda \) is the best fitted model so far to explain this recent accelerated expansion of the universe. However it suffers from two major theoretical problems known as fine tuning problem [8] and cosmic coincidence problem [9]. Despite being consistent with the observations, these problems of cosmological constant make the cosmologists search for alternatives.

A simplest alternative is the canonical scalar field model known as “quintessence” [9]. The potential of the canonical scalar field is so chosen that the field rolls very slowly at the present epoch resulting in the negative pressure of the field which leads to cosmic acceleration. This essentially requires the potential to be very flat with respect to the field \( \phi \) resulting in the mass of the field around \( 10^{-33} \) eV. The tracker behaviour of the scalar field model [10] helps to alleviate the problem of cosmic coincidence in the dark energy scenario. Explaining the late time cosmic acceleration is also possible from the modification of gravity at the large scales known as infrared modification of gravity. It is found that higher order curvature invariants play an important role in modification of gravity at large scales thereby leading to accelerated expansion of the universe at the present epoch [11–13]. Moreover higher dimensional models of gravity induces modification of Einstein’s gravity in the 3+1 dimensional effective theory at large scales leading to the accelerated expansion of the universe [14–15]. A large number of modified gravity models is tested to be free from ghost or tachyon instabilities and they also do not conflict with the solar system constraints (see [16] and references therein for a review on cosmology driven by modified gravity models). Of late the detections of GW170817 and GRB 170817A revealed the fact that the speed of gravitational waves differs from the speed...
of light by one part in $10^{15}$ [17, 21]. This discovery have severely constrained these modified
gravity models as well as other dark energy models [22, 23].

From the standard model of particle physics this is well known that all the particle in
the universe gets mass due to their interaction with the Higgs field [29, 30]. The dynamics of
FRW universe was studied in presence of non-abelian gauge fields invariant under SO(3) and
SU(2) gauge group or an arbitrary gauge group SO(N) [31, 33]. In the context of inflation
Einstein Yang-Mills Higgs action was first introduced [34, 35] to study the effect of gauge field
on inflation. Recently in [36], the dynamics of this non-abelian Higgs field coupled to gravity
was studied in the context of late time cosmic acceleration. In the work [37], considering the
interaction in SU(2) representation for Higgs field the authors have studied the dynamics of
cosmology in Einstein Yang-Mills Higgs to explain the recent accelerated expansion of the
Universe. What is not yet known is that the viability of this model in respect of cosmological
observations.

In the present work we study the viability of the Einstein Yang-Mills Higgs dark energy in
the context of observational data. We constrain the model from type Ia supernova data (SNe Ia),
observational Hubble data (OHD), baryon acoustic oscillation data (BAO) and cosmic
microwave background shift parameter data (CMB) and show that the model parameter
space is consistent with the cosmological observations thus making this a viable model for
dark energy to explain the current accelerated expansion of the universe. This paper is
organised as follows. In Sec. 2 we review the Einstein Yang-Mills Higgs action coupled to
gravity and the equations of motion in FRW background to study the dynamical system.
Construction of autonomous system and dynamics of cosmology is studied in Sec. 3. In Sec.
4 we discuss the various observational data and the formalism for analyses of those data. In
this section we also confront this Higgs dark energy model with the observational data and
present the results of our data analysis. Eventually we conclude in Sec. 5.

2 Einstein Yang-Mills Higgs action

In what follows we describe the Higgs dark energy in presence of gauge field in background
of Einstein’s gravitation. The Einstein Yang-Mills Higgs action is given by [36, 37],

$$S = \int d^4x \sqrt{-g} \left( \frac{M_{Pl}^2}{2} R - \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a - (D^\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi) + L_r + L_m \right),$$

(1)

where $M_{Pl}$ is the reduced Planck mass given by $M_{Pl} = 1/\sqrt{8\pi G}$, $g$ is the determinant
of spacetime metric, $\Phi$ is the complex Higgs doublet invariant under SU(2) gauge symmetry,
$L_r$ is the lagrangian for radiation and $L_m$ is the matter lagrangian. Here $F_{\mu\nu}^a$ is the rank-2
tensor that represents the non-Abelian gauge field and is given by

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \beta \epsilon^{abc}_{\mu\nu} A_\mu^b A_\nu^c,$$

(2)

where $A_\mu^a$ is the gauge field, $\beta$ is the coupling of SU(2) group and $\epsilon^{abc}_{\mu\nu}$ is the rank 3
Levi-Civita symbol. $D_\mu$ is the gauge covariant derivative given by

$$D_\mu = \nabla_\mu - i\beta \sigma^a_\mu A_\mu^a,$$

(3)
where $\nabla_\mu$ is the spacetime covariant derivative and $\sigma_a$ are the Pauli matrices. The complex Higgs doublet and its potential are respectively given by,

$$\Phi = \begin{pmatrix} \phi_1 + i\chi_1 \\ \phi_2 + i\chi_2 \end{pmatrix},$$

(4)

where $\phi_1, \phi_2, \chi_1, \chi_2$ are real scalar fields and

$$V(\Phi) = \frac{\lambda}{4} (\Phi\dagger\Phi - v^2)^2,$$

(5)

where $v$ is the vacuum expectation value (VEV) of Higgs field.

It is evident from the observations [7] that our universe is homogeneous and isotropic on large scales. Hence the background spacetime of the universe is described by the Friedmann Lemaître Robertson Walker (FRLW) metric and is given by in spherically symmetric coordinates,

$$ds^2 = -dt^2 + a^2(t) \left( dr^2 + r^2 d\Omega^2 \right),$$

(6)

where $t$ is the cosmological time, $a(t)$ is scale factor for expanding universe and $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$. The energy momentum tensor for the action in Eq. 1 is given by,

$$T_{\mu\nu} = -F_{\mu\nu}^a F_{\nu\alpha}^a - (D_\mu \Phi)\dagger(D_\nu \Phi) - (D_\nu \Phi)\dagger(D_\mu \Phi) + 2 \frac{\partial}{\partial g_{\mu\nu}} (\mathcal{L}_m + \mathcal{L}_r)$$

$$- g_{\mu\nu} \left[ -\frac{1}{4} F_{\mu\nu}^a F_{\alpha\beta}^a - (D^\mu \Phi)\dagger(D^\alpha \Phi) - V(\Phi) + \mathcal{L}_r + \mathcal{L}_m \right].$$

(7)

The Einstein tensor $G_{\mu\nu}$ is diagonal for FRLW background spacetime and hence the off diagonal terms of energy momentum tensor should vanish. This condition makes the gauge field become $A_\mu^a = \delta_\mu^a f(t)$ [36] where $f(t)$ is the only degree of freedom in the gauge sector as allowed from the FRLW spacetime of the universe, $a$ is the gauge index and $i$ is the spatial index. As discussed in [37], this condition is not sufficient to avoid the non-zero contribution to the momentum density arising from the interaction between the Yang-Mills field and the Higgs field. Hence another additional condition which is required to establish the isotropy in energy momentum tensor is to fix the gauge so that

$$\Phi(t) = \begin{pmatrix} \phi(t) \\ 0 \end{pmatrix},$$

(8)

where $\phi(t)$ is a real scalar field. With these choices of gauge field and gauge fixation for the Higgs field, we obtain

$$H^2 = \frac{1}{3M_{Pl}^2} \left[ \frac{3}{2} \frac{\dot{f}(t)^2}{a(t)^2} + \frac{\dot{\phi}(t)^2}{a(t)^2} + \frac{3}{2} \frac{\beta^2 f(t)^4}{a(t)^4} + \frac{3}{4} \frac{\beta^2 \phi(t)^2 f(t)^2}{a(t)^2} + V(\phi) + \rho_m + \rho_r \right],$$

(9)

$$\dot{H} = -\frac{1}{2M_{Pl}^2} \left[ \frac{3}{2} \frac{\dot{f}(t)^2}{a(t)^2} + 2 \frac{\dot{f}(t)^2}{a(t)^2} + 2 \frac{\beta^2 f(t)^4}{a(t)^4} + \frac{\beta^2 \phi(t)^2 f(t)^2}{2a(t)^2} + \rho_m + \frac{4}{3} \rho_r \right].$$

(10)
where $H$ is Hubble parameter given by $H = \dot{a}(t)/a(t)$ and $\rho_m, \rho_r$ are the matter and radiation density respectively. The equation of motions of the gauge and Higgs fields are respectively given by,

\begin{align}
\ddot{f}(t) + H\dot{f}(t) + \beta^2 \left[ 2\frac{f(t)^3}{a(t)^2} + \frac{f(t)\phi(t)^2}{2} \right] &= 0, \quad (11) \\
\ddot{\phi}(t) + 3H\dot{\phi}(t) + \frac{3\beta^2 f(t)^2 \phi(t)}{4a(t)^2} + \frac{dV(\phi)}{d\phi} &= 0, \quad (12)
\end{align}

where $V(\phi)$ is given by $V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)$. This is worth mentioning here that from Eq. (11) it is evident that there arises an effective potential for the gauge field with vanishing vacuum expectation value due to the interaction between the gauge field and the Higgs field. Moreover the same interaction leads to the effective potential of the Higgs field also as shown in the Eq. (12).

### 3 Dynamics of cosmology

To analyse the cosmological dynamics the following dimensionless variables are introduced here.

\begin{align}
x_1 &= \frac{\dot{f}}{\sqrt{2}aM_{Pl}H}, \quad y_1 = \frac{\beta f^2}{\sqrt{2}a^2M_{Pl}H}, \\
z_1 &= \frac{\beta f\phi}{2aM_{Pl}H}, \quad x_2 = \frac{\dot{\phi}}{\sqrt{3}M_{Pl}H}, \\
y_2 &= \sqrt{\frac{V(\phi)}{3M_{Pl}^2H^2}}, \quad r = \sqrt{\frac{\rho_r}{3M_{Pl}^2H^2}}, \\
m &= \sqrt{\frac{\rho_m}{3M_{Pl}^2H^2}}, \quad w_1 = \sqrt{\frac{2aM_{Pl}}{f}}, \quad (13)
\end{align}

The subscripts 1 and 2 refers to the dimensionless variables corresponding to gauge field and the Higgs field. With these choices the total energy density in the universe takes the form (from Eq. (9)),

\begin{equation}
x_1^2 + y_1^2 + z_1^2 + x_2^2 + y_2^2 + r^2 + m^2 = 1. \quad (14)
\end{equation}
The evolution equations of the autonomous system are given by

\[
\begin{align*}
x_1' &= x_1(q - 1) - w_1(2y_1^2 + z_1^2), \\
y_1' &= y_1(2x_1w_1 + q - 1), \\
z_1' &= z_1(x_1w_1 + q) + \frac{\sqrt{3}}{2}w_1y_1x_2, \\
x_2' &= x_2(q - 2) - z_1w_1(2\alpha y_2 + \frac{\sqrt{3}}{2}y_1), \\
y_2' &= y_2(q + 1) + \alpha w_1z_1x_2, \\
r' &= r(q - 1), \\
m' &= m\left(q - \frac{1}{2}\right), \\
w_1' &= w_1(1 - w_1x_1),
\end{align*}
\]

(15)

where the symbol prime denotes a derivative with respect to \( N = \ln a \), \( a \) being the scale factor of the universe and \( q \) is the deceleration parameter defined as \( q(t) = -\frac{\ddot{a}(t)a(t)}{a(t)^2} \). In terms of the dimensionless variables defined above the deceleration parameter takes the form

\[
q = \frac{1}{2}(1 + x_1^2 + y_1^2 - z_1^2 + 3x_2^2 - 3y_2^2 + r^2).
\]

(16)

Here \( \alpha \) is a dimensionless constant given by \( \alpha = \sqrt{\frac{\lambda}{2\beta^2}} \).

We solve the autonomous system for the initial conditions given by \( x_1 = 10^{-18}, y_1 = 10^{-18}, z_1 = 10^{-18}, x_2 = 10^{-18}, y_2 = 0.831, w_1 = 10^2 \) and \( r = 10^{-2} \) at \( z = 0 \) [36,37]. In the Fig. 2 we show the variation of density of radiation, matter and dark energy with the number of e-foldinds \( N \) and variation of the density parameters are shown in Fig. 1 for \( \Omega_m(0) = 0.31 \) and \( H_0 = 69 \text{Kmsec}^{-1}\text{Mpc}^{-1} \) and \( \alpha = 1 \). From these two figures this is evident that the dark energy dominates very recently. Moreover it is evident from Fig. 2 that the Higgs dark energy though varies initially but starts mimicking the cosmological constant around \( N = -12 \) i.e., well in the radiation dominated era. The plot of the deceleration parameter \( q \) and the effective equation of state \( \omega_{\text{eff}} \) of the universe for all the components i.e., radiation, matter and the dark energy are shown in Fig. 3. The acceleration of the universe corresponds to \( q < 0 \) and \( \omega_{\text{eff}} < -1/3 \).

4 Observational Constraints

In this era of precision cosmology models of dark energy are highly constrained. Passing the test of observational data only makes the model acceptable despite their theoretical viability. In this section we describe the observational data that are used to constrain the model parameter in this Einstein Yang-Mills Higgs dark energy and the formalism for the data analysis as well.

Supernovae Type Ia are accepted as the standard candles in astrophysical observations. Incidentally it happened to be the first probe for the discovery of the late time cosmic acceleration [1,2]. We consider here 279 Supernovae Type Ia (SNe Ia) observational data
Figure 1: Plot of the density parameters in the Universe with the number of e-foldinds $N$ given by $N = - \ln(1 + z)$ where $z$ is the corresponding redshift.

Figure 2: Plot of the density in the Universe number of e-foldinds $N$ given by $N = - \ln(1 + z)$ where $z$ is the corresponding redshift.
Figure 3: Plot of deceleration parameter $q$ and effective equation of state of the universe $\omega_{\text{eff}}$ with the number of e-foldings $N$ given by $N = -\ln(1 + z)$ where $z$ is the corresponding redshift.

from Pan-STARRS1 Medium Deep Survey in the redshift range $0.03 < z < 0.68$ along with the other SNe Ia data from Sloan Digital Sky Survey (SDSS) [38-40], SNLS [41,42], and ESSENCE [43-45] and SCP [46]. The combined data set known as Pantheon Sample [47] consists of 1048 SNe Ia data points in the redshift range $0.01 < z < 2.3$. The distance modulus for type Ia supernova as a function of the redshift is given by

$$\mu(z) = 5 \log_{10}(D_L(z)) + \mu_0,$$

where $D_L(z) = H_0 d_L(z)/c$ ($c$ is speed of light in free space) and $\mu_0 = 42.38 - 5 \log_{10} h$ for $H_0 = 100h$ Km Sec$^{-1}$ Mpc$^{-1}$. The chi-square for supernovae data is defined as

$$\chi^2_{\text{SN}}(p_s) = \sum_i \left[ \frac{\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i, p_s, \mu_0)}{\sigma_i} \right]^2,$$

where $p_s$ are the model parameters and $z_i$ are the redshifts of the observational supernovae type Ia data. $\mu_{\text{obs}}$ and $\mu_{\text{th}}$ are the observational and theoretical distance modulus respectively. The chi-square is marginalised over the nuisance parameter $\mu_0$ [48] and the marginalised chi-square is given by

$$\chi^2_{\text{SN}} = A(p_s) - \frac{B(p_s)^2}{C},$$
where $A$, $B$ and $C$ are given by

$$
A(p_s) = \sum_i \left[ \frac{\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i, p_s, \mu_0)}{\sigma_i} \right]^2, \tag{20}
$$

$$
B(p_s) = \sum_i \frac{\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i, p_s, \mu_0)}{\sigma_i^2}, \tag{21}
$$

$$
C = \sum_i \frac{1}{\sigma_i^2}, \tag{22}
$$

Cosmic microwave background shift parameter $R$ is a model independent parameter that can also be used to constrain the models of dark energy. It is obtained from the first peak of temperature anisotropy plot of the cosmic microwave background radiation. The CMBR shift parameter is defined as

$$
R(z_*) = (\Omega_m H_0^2)^{1/2} \int_{z_*}^{z_*} \frac{dz}{H(z)}, \tag{23}
$$

where $z_*$ corresponds to the redshift of the radiation matter decoupling epoch. The chi-square for CMBR shift parameter is defined as

$$
\chi^2_{\text{CMB}} = \left[ \frac{R_{\text{th}}(z_*, p_s) - R_{\text{obs}}(z_*)}{\sigma_R} \right]^2. \tag{24}
$$

Needless to mention that $p_s$ are the model parameters. We use the CMBR shift parameter from latest Planck observations $R = 1.7499 \pm 0.0088$ at the redshift of decoupling era $z_* = 1091.41 \ [49]$.

Observational Hubble data is a direct measurement of expansion rate of universe with the redshifts. It is another tool to constrain the dark energy models. The chi-square for observational Hubble data is given by

$$
\chi^2_{\text{OHD}} = \sum_i \left[ \frac{H_{\text{obs}}(z_i) - H_{\text{th}}(z_i, p_s)}{\sigma_i^2} \right]^2, \tag{25}
$$

We use the 31 data points of $H(z)$ for the purpose of $\chi^2_{\text{OHD}}$ analysis. The measurements of observational Hubble data are summarized in Tab. 1 [57].

Before the recombination epoch the baryons were tightly coupled to the photons and as a result of this tight coupling the acoustic oscillations created small density fluctuations in baryon photon plasma. In the expanding universe, this density fluctuations left an imprint in the large scale structures which provides a standard ruler in cosmology. Baryon acoustic oscillation is the powerful tool for constraining dark energy models. The sound horizon at a redshift $z_d$ for drag epoch is given by

$$
r_d = \frac{c}{\sqrt{3} \int_{z_d}^{\infty} \frac{dz}{\sqrt{1 + \frac{3\Omega_b^{(0)}}{4\Omega_r^{(0)} (1+z)} H(z)}}}, \tag{26}
$$
| $z$   | $H(z)$ [Km Sec$^{-1}$ Mpc$^{-1}$] | $\sigma_{H(z)}$ [Km Sec$^{-1}$ Mpc$^{-1}$] |
|-------|---------------------------------|----------------------------------|
| 0.07  | 69.0                            | 19.6                             |
| 0.09  | 69.0                            | 12.0                             |
| 0.12  | 68.6                            | 26.2                             |
| 0.17  | 83.0                            | 8.0                              |
| 0.179 | 75.0                            | 4.0                              |
| 0.199 | 75.0                            | 5.0                              |
| 0.2   | 72.9                            | 29.6                             |
| 0.27  | 77.0                            | 14.0                             |
| 0.28  | 88.8                            | 36.6                             |
| 0.352 | 83.0                            | 14.0                             |
| 0.3802| 83.0                            | 13.5                             |
| 0.4   | 95.0                            | 17.0                             |
| 0.4004| 77.0                            | 10.2                             |
| 0.4247| 87.1                            | 11.2                             |
| 0.4497| 92.8                            | 12.9                             |
| 0.47  | 89.0                            | 49.6                             |
| 0.4783| 80.9                            | 9.0                              |
| 0.48  | 97.0                            | 62.0                             |
| 0.593 | 104.0                           | 13.0                             |
| 0.68  | 92.0                            | 8.0                              |
| 0.781 | 105.0                           | 12.0                             |
| 0.875 | 125.0                           | 17.0                             |
| 0.88  | 90.0                            | 40.0                             |
| 0.9   | 117.0                           | 23.0                             |
| 1.037 | 154.0                           | 20.0                             |
| 1.3   | 168.0                           | 17.0                             |
| 1.363 | 160.0                           | 33.6                             |
| 1.43  | 177.0                           | 18.0                             |
| 1.53  | 140.0                           | 14.0                             |
| 1.75  | 202.0                           | 40.0                             |
| 1.965 | 186.5                           | 50.4                             |

Table 1: The 31 $H(z)$ data points [57].
where the drag redshift $z_d$ is given by

$$z_d = \frac{1291 \left( \Omega_m^{(0)} h^2 \right)^{0.251}}{1 + 0.659 \left( \Omega_m^{(0)} h^2 \right)^{0.282}} \left[ 1 + b_1 \left( \Omega_b^{(0)} h^2 \right)^{b_2} \right],$$

with

$$b_1 = 0.313 \left( \Omega_b^{(0)} h^2 \right)^{-0.410} \left[ 1 + 0.607 \left( \Omega_m^{(0)} h^2 \right)^{0.674} \right],$$
$$b_2 = 0.238 \left( \Omega_m^{(0)} h^2 \right)^{0.223},$$

and $\Omega_b^{(0)} h^2 = 0.02236$, $\Omega_m^{(0)} h^2 = 2.469 \times 10^{-5}$ [7]. In a spatially flat universe the angular diameter distance $D_A(z)$, the Hubble distance $D_H(z)$ and the effective distance $D_V(z)$ are respectively given by,

$$D_A(z) = \frac{c}{(1 + z)} \int_0^z \frac{dz}{H(z)},$$

$$D_H(z) = \frac{c}{H(z)},$$

$$D_V(z) = \left[ \left( \frac{d_L(z)}{1 + z} \right)^2 \frac{cz}{H(z)} \right]^{1/3},$$

where $c$ the speed of light in vacuum. Here we use both the isotropic and anisotropic BAO data that are tabulated in Tabs. 2 and 3 [58]. The covariance matrix $C$ associated with
the anisotropic BAO measurements is given by

\[
C = \begin{pmatrix}
0.0150 & -0.0357 & 0.0071 & -0.0100 & 0.0032 & -0.0036 & 0 & 0 \\
-0.0357 & 0.5304 & -0.0160 & 0.1766 & -0.0083 & 0.0616 & 0 & 0 \\
0.0071 & -0.0160 & 0.182 & -0.0323 & 0.0097 & -0.0131 & 0 & 0 \\
-0.0100 & 0.1766 & -0.0323 & 0.3267 & -0.0167 & 0.1450 & 0 & 0 \\
0.0032 & -0.0083 & 0.0097 & -0.0167 & 0.0243 & -0.0352 & 0 & 0 \\
-0.0036 & 0.0616 & -0.0131 & 0.1450 & -0.0352 & 0.2684 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.1358 & -0.0296 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.0296 & 0.0492
\end{pmatrix}.
\]

The total chi-square for isotropic and anisotropic BAO data is given by

\[
\chi_{\text{BAO}}^2 = \chi_{\text{iso}}^2 + \chi_{\text{aniso}}^2,
\]

where

\[
\chi_{\text{iso}}^2 = \sum_i \left[ \frac{D_V(z_i)/r_d - D_V(z_i,p_s)/r_d}{\sigma_i} \right]^2,
\]

\[
\chi_{\text{aniso}}^2 = X_{\text{aniso}}^T C^{-1} X_{\text{aniso}}
\]

where \( X_{\text{aniso}} \) is column matrix given by,

\[
X_{\text{aniso}} = \begin{pmatrix}
\frac{D_A(0.38)}{r_d} & -7.42 \\
\frac{D_H(0.38)}{r_d} & -24.97 \\
\frac{D_A(0.51)}{r_d} & -8.85 \\
\frac{D_H(0.51)}{r_d} & -22.31 \\
\frac{D_A(0.61)}{r_d} & -9.69 \\
\frac{D_H(0.61)}{r_d} & -20.49 \\
\frac{D_A(2.4)}{r_d} & -10.76 \\
\frac{D_H(2.4)}{r_d} & -8.94
\end{pmatrix}.
\]

The total combined chi-square for all the aforesaid data sets i.e., SNe Ia, CMB shift parameter, OHD, BAO is given by,

\[
\chi_{\text{tot}}^2 = \chi_{\text{SN}}^2 + \chi_{\text{OHD}}^2 + \chi_{\text{BAO}}^2 + \chi_{\text{CMB}}^2.
\]

We use this total chi-square defined in Eq. (37) for the data analysis purpose of the Yang-Mills Higgs dark energy model and constrain the parameters space.

In what follows, we describe the model parameters and the results of the chi-square analysis of the observational data. In this Higgs dark energy model we consider four parameters namely \( \alpha \), \( \Omega_m(0) \), \( H_0 \) and \( H_0 r_d/c \) to fit the chi-square with the latest observational data from SNe Ia, OHD, BAO and CMB. In the Fig. 4 we present the 68.3%, 90% and 99% confidence level plot for the parameters \( \Omega_m(0) \) and \( H_0 \) with the contour shading by the light blue, dark blue and cyan colours respectively. The total chi-square turns out to have a minima at
Figure 4: Observational constraints on the parameters space \((\Omega_m^{(0)} - H_0)\) at the 68.3% (light blue), 90% (dark blue) and 99% (cyan) confidence levels.

Figure 5: Observational constraints on the parameters space \((\Omega_m^{(0)} - H_0 r_d / c)\) at the 68.3% (light blue), 90% (dark blue) and 99% (cyan) confidence levels.
Figure 6: Observational constraints on the parameters space \((H_0 r_d/c - H_0)\) at the 68.3% (light blue), 90% (dark blue) and 99% (cyan) confidence levels.

\[ \Omega_m^{(0)} \simeq 0.315 \text{ and } H_0 \simeq 68.6 \text{KmSec}^{-1}\text{Mpc}^{-1} \text{ and } H_0 r_d/c \simeq 0.0335 \text{ which with the best-fit value of } H_0 \text{ and speed of light in vacuum gives } r_d \simeq 146.5 \text{ Mpc}. \]

\[ \text{Fig. 5 and 6 shows the observationally allowed parameters space in } \Omega_m^{(0)} - H_0 r_d/c \text{ and } H_0 r_d/c - H_0 \text{ at 68.3\%, 90\% and 99\% confidence levels with the same colours mentioned above. It is worth mentioning here that all this confidence contours corresponds to value of } \alpha = 1 \text{ [64]. The parameter } \alpha \text{ cannot be constrained from the present observational data we have considered here. A confidence contour is shown in Fig. 7 in the } \alpha - \Omega_m^{(0)} \text{ parameters space from where it is evident that the present data is unable to put any bound on the parameter } \alpha. \text{ With a total of 1091 data points from SNe Ia, OHD, BAO and CMB, we find from our data analysis a chi-square per degrees of freedom to be around 0.983 i.e., very close to 1 which in turn reflects the fact that the fitment of the model parameters are in good agreement with the observational data sets [65].} \]

5 Conclusion

In this work, we study Higgs dark energy model in presence of gauge field in light of observational data from supernovae type Ia, baryon acoustic oscillation, observational Hubble data and cosmic microwave background shift parameter data. In performing the data analysis, we considered the initial conditions at the present epoch for dynamical evolution of the autonomous system. The choice of initial condition for Higgs field is in consideration with the vacuum expectation value of Higgs \(v \sim 246 \text{ GeV} \) that leads to initial values of \(x_1 = 10^{-18}, y_1 = 10^{-18}, z_1 = 10^{-18}, x_2 = 10^{-18}, y_2 = 0.831, w_1 = 10^2 \text{ and } r = 10^{-2} \text{ at } z = 0 \text{ i.e., present epoch [36, 37]. These choice of initial conditions lead to correct cosmological dynamics for the observational universe as evident from Fig [2] and the cosmic acceleration is a recent phenomenon. Moreover from the same figure it appears that the Higgs dark} \]
energy starts mimicking cosmological constant well in the radiation dominated era. The
chi-square analysis of the observational data significantly constrains the model parameters
(\(\alpha\), \(\Omega_m^{(0)}\), \(H_0\), \(H_0r_d/c\)). The minimum combined chi-square for all the data sets is obtained
at the parameter values \((\Omega_m^{(0)}, H_0, H_0r_d/c) \sim (0.315, 68.6, 0.0335)\). Hence the sound horizon
at the redshift of drag epoch turns out to be around 146.5 Mpc which is in remarkably good
agreement with the Planck 2018 results [7]. Also this is worth mentioning here that the
chi-square per degrees of freedom is slightly greater than 0.98 which is the indication of a
good fitting of the model with the observational data [65]. However data is still unable to
provide any constraint on the model parameter \(\alpha\) as is evident from Fig. 7. Needless to
mention that the Higgs or the gauge field in the theory being minimally coupled to gravity
does not conflict with the observational evidences of gravitational wave detection [17,18,21].
Thus the Higgs field in presence of gauge field turns out to be a viable candidate for dark
energy so far as the observational data are concerned.

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