Relation between the theoretical and average observed curvatures of mining terrain

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Abstract. In the article have been shown the obtaining methods of average graph of terrain curvatures caused by underground mining exploitation and their theoretical graph calculated on the base of subsidence forecasted by the use of the geometric–integral theory of mining exploitation influences. The average graph of curvatures measured after the ending of excavation in the 338/2 and 358/1 hard coal seams, was designated using the smoothed splines and the least squares method. First it has been done an approximation of the subsidence measured values (with help of the R-project computer program) and then, it has been determined an average graph of the observed curvatures. The choice criterion of proper value of smoothing parameter of approximating function was applied for the subsidence. Right value of smoothing parameter occurs when the standard deviation between the average and predicted graphs of subsidence reaches the minimal value. Average values of measured curvatures were compared with the theoretical values of curvatures. They were calculated on the base of subsidence forecasted by the use of the EDN-OPN computer program and the Bialek’s formula with the values of its parameters designated from the results of geodesic measurements. It was determined a relation between the theoretical values of curvatures and their average measured values. This relationship allows to evaluate the mathematical model which was used to calculation of the curvatures theoretical values.

1. Introduction
Every mining plant, which strives for a concession to an extraction of the useful ore, has to prepare a forecast of mining exploitation influences on terrain surface, buildings and technical infrastructure objects.

These forecasts can be made by the use of the various theories of exploitation influences, e.g. the geometric-integral theories [1], [2], a cellular automata theory [3], [4], and with the different values of these theories parameters [5], [6].

Credibility of made predictions can be only checked when the excavation will be completed. Then, the proper geodesic surveys are being done and their results are comparing with the surveys results done before an exploitation has begun. On the base of this, there are determined the deformations indicators values, for example of subsidence, inclinations, curvatures, horizontal strains.

The forecasted graphs of deformations indicators shouldn’t be compared with the deformations indicators graphs directly received from the results of geodesic measurements (they are irregular), but with their average graphs obtained on the way of approximation. This irregularity is mainly caused...
by the measurements errors, a randomness of deformation process and the cracks of superficial layer of rock mass [7].

The average graphs of measured deformations indicators can be obtained by the use of the various, approximating functions, e.g. the orthogonal polynomials [8]. Unfortunately, these functions aren’t flexible (especially in case of curvatures) and require a division of the whole graph of deformation indicator into the sections.

Because of that, in the article were applied the smoothed splines functions to determination of an average graph of terrain curvatures observed as a result of underground mining exploitation conducted in two hard coal seams.

Then, there have been compared the theoretical graph of curvatures with the average graph of curvatures measured after the ending of hard coal extraction from the 338/2 and 358/1 seams.

The theoretical curvatures have been calculated from the subsidence forecasted by the use of a geometric-integral theory of mining exploitation influences which takes into account an existence of the operating margin [2], namely the Bialek’s formula with the values of its parameters calculated from the geodesic measurements results.

To evaluate the credibility of made forecast, it was determined the equation of linear regression between the theoretical and average measured curvatures [9].

2. Average graph of observed curvatures
In this section have been presented an example of operation of hard coal which was carried out by some mining plant in Poland, in two coal seams located at the large depth; a description of the geodesic measurements which have been taken on the observational points stabilized in the ground above the exploitation field; the obtainment ways of the observed and average graphs of curvatures.

2.1. Mining exploitation case
Some hard coal mine, which is located in the Upper Silesian Coal Basin in Poland, in the years 2001 – 2006 realized an extraction of hard coal from the 338/2 and 358/1 seams. Exploitation was conducted at the great depth and by the use of longwall system with cave-in of the roof rocks. Slope of coal layers was small and amounted to 6°. Hard coal deposit was inclined in the south-eastern direction.

Coal extraction from the 338/2 seam was conducted at the depth of 600 m and at the height of 3.0 m. Within five years (2001 – 2006), there have been exploited the following longwalls: B-1, B-2, B-3, B-4, C-1.

Hard coal seam 358/1 was exploited at the depth of 1000 m, in the years 2002 – 2006. Exploitation of seven longwalls (from the B-1 to the B-7) was running at the height of 2.9 m.

2.2. Geodesic observations
Shaping of terrain surface curvatures, under the exploitation influence of two hard coal seams, has been observed by the use of the geodesic measurements. These measurements were carried out at 53 points of the observational line N° 8. This line was established above the B-2 longwall in the 338/2 seam, perpendicularly to its run. The average length of line segments was amounted 35 m.

Surveys of the points heights have been taken by the use of precise leveller. Accuracy of measurements was amounted 0.1 mm. The heights of points were defined in the Kronstadt altitude system. It was used a geometric levelling from the middle with the double targeting.

The sections lengths have been measured by the use of the distance-meter device with an accuracy of 1 mm.

2.3. Graph of the observed curvatures
Curvatures observed on the neighbouring sections of the measuring line N° 8 have been obtained from the subsidence values measured on three neighbouring points and an average length of these sections. It presents the formula (1).
The curvatures, noted on the measuring sections of the observational line No 8 after the exploitation end in the 338/2 and 358/1 seams, have been presented at the Figure 1.

![Curvature graph](image)

Figure 1. The curvatures observed along the measuring line No 8 after an extraction ending in the 338/2 and 358/1 hard coal seams

Subsidence of measuring points have been calculated from the points heights differences. Heights of points have been measured before the beginning of exploitation and after the termination of exploitation of two hard coal seams. Subsidence were calculated from the formula:

\[ S_i^{meas} = H_i^A - H_i^B \]

where:
- \( H_i^A \) – absolute height of the \( i \) point after the exploitation termination [mm];
- \( H_i^B \) – absolute height of the \( i \) point before the exploitation beginning [mm];
- \( S_i^{meas} \) – measured subsidence of the \( i \) point [mm].

2.4. Average graph of the observed curvatures

Average values of the observed curvatures [10], [11] have been calculated from the approximated values of measured subsidence and the average length of two neighbouring segments, from the below formula [12]. This formula is analogous as the formula (1):

\[ C_{i-1,i+1}^{meas} = \frac{S_{i-1}^{meas} - 2S_i^{meas} + S_{i+1}^{meas}}{L_{i-1,i+1}^{meas}} \]
where:

- \( C_{i-1,i+1}^{\text{aver}} \) – average value of curvature observed on the \( i-1,i \); \( i,i+1 \) sections of measuring line \([10^{-6} \cdot 1/m]\);
- \( L_{i-1,i}^{\text{meas}} \) – measured length of the \( i-1,i \) section \([m]\);
- \( S_{i-1}^{\text{approx}} \) – approximated value of subsidence measured in the \( i-1 \) point \([mm]\);
- \( i \) – measuring point.

Figure 2 presents the average values of curvatures observed along the measuring line \( N^2 8 \) after the end of exploitation in the \( 338/2 \) and \( 358/1 \) coal seams.

**Figure 2.** The average curvatures observed on the measuring line \( N^2 8 \) after the termination of exploitation of the \( 338/2 \) and \( 358/1 \) coal seams

Approximation of the average values of measured subsidence has been conducted by the use of the smoothed splines functions [13], [14] and the least squares method. It has been applied the `smooth.spline` function from the R computer program. This function is based on the following arguments:

- \( df \) – number of the freedom degree assuming the values from 0 to \( n \), where \( n \) is number of measuring points;
- \( spar \) – smoothing parameter of approximating function assuming the values from 0 to 1, where 0 denotes a total lack of smoothing and approximating function assumes the form of approximated function;
- \( \lambda \) – penalized criterion;
- \( cv \) – cross validation, when \( cv = \text{truth} \) then we deal with the leave-one-out validation, if \( cv = \text{false} \) then we deal with the general cross validation (GCV).

To choose the right value of the smoothing parameter \( spar \) [15], it was used a residuals minimization criterion between the approximated and forecasted values of subsidence [10], [11].
Predicted values of subsidence have been calculated by the use of the Bialek’s formula with the values of its parameters obtained from the surveys results.

In the Table 1 have been presented the parameters values of the smooth.spline function and an approximation [16].

Table 1. The parameters values of subsidence approximation

| Parameter | Value             |
|-----------|-------------------|
| $D_f$     | 20.95             |
| $S_{par}$ | 0.41              |
| $A$       | $4.53 \cdot 10^6$|
| $GCV$     | 403.36            |
| $\sum(S_{fore} - S_{approx})^2$ [mm$^2$] | 858891.36 |
| $\sum(S_{meas} - S_{approx})^2$ [mm$^2$] | 6148.87 |

3. Theoretical graph of curvatures

Theoretical values of curvatures have been calculated on the base of the forecasted values of subsidence by the use of the Bialek’s formula with the parameters values specified from the results of geodesic measurements:

$$C_{i-1,i+1}^{theor} = 2 \frac{S_{i-1}^{fore} - 2S_{i}^{fore} + S_{i+1}^{fore}}{L_{i-1,i}^{meas} + L_{i,i+1}^{meas}},$$

where:
- $C_{i-1,i+1}^{theor}$ – theoretical value of curvature on the $i-1,i;i,i+1$ sections of measuring line [$10^{-6} \cdot 1/m$];
- $L_{i-1,i}^{meas}$ measured length of the $i-1,i$ section [m];
- $S_{i-1}^{fore}$ – forecasted value of subsidence by the use of the Bialek’s formula in the $i-1$ point [mm];
- $i$ – measuring point.

At the Figure 3 has been shown the theoretical graph of curvatures along the observational line $N^2 8$ after the exploitation ending in the 338/2 and 358/1 coal seams, obtained from the subsidence forecasted by the use of the Bialek’s formula.

![Theoretical curvatures](image-url)
Subsidence forecast has been made by the use of the *EDBJ1* computer program. In calculations was assumed the Bialek’s formula which is classified as the geometric-integral theory of mining exploitation influences. This formula takes into account: the far influences (the $\beta$-angle), an exploitation margin (the $A_1$ parameter), an old goaf reactivation, a desymmetrization of subsidence trough profile relative to an exploitation edge (the $r_1$, $r_2$-radius) and the exploitation influences coming from several coal seams (the $a$ parameter). The parameters values of this formula have been determined from the subsidence measured after the termination of exploitation in two hard coal seams, namely: $a = 0.890; \tan \beta = 2.616; A_1 = 0.227$.

$$S_f = (1 - a_2)s(r_1) + a_2s(r_2) - A_1 \left( \frac{2 + \frac{a_3}{2}}{A_3} \frac{s(r_1)r_1(r_2)}{r_1^2 + s(r_2)} \right) + [r_1y(r_2)]^2, \quad (5)$$

where:

- $\beta$ – range angle of main influences [°];
- $\gamma$ – simplified octahedral strain [mm/m];
- $A_1$ – dimensionless multiplier which takes into account an asymmetry of subsidence trough profile;
- $S_f$ – final subsidence [mm];
- $a_2$ – coefficient constant for each subsidence trough, it defines what part of final subsidence was calculated by the use of the $r_1$ radius ($a_2 = 0.4 \div 1.25A_1$);
- $h$ – exploitation depth [m];
- $r_1$ – radius of influences dispersion for each subsidence trough [m];
- $r_2 = 2r_1$;
- $s(r_1), s(r_2)$ – subsidence calculated using the Knothe’s formula with the use of two different radiuses of influences dispersion $r_1, r_2$ [mm].

4. Relation between the theoretical and average observed curvatures

Relationship the theoretical graph of curvatures with the average observed graph of curvatures has been shown by the use of a linear regression equation and statistics published in the Table 2. There have been calculated: the $P_{Cm}$ percentages of the extreme average values of observed curvatures which correspond the extreme theoretical values of curvatures, the $\sigma_C$ standard deviation between the average observed and theoretical values of curvatures, the $R_C$ correlation coefficient between these curvatures.

**Table 2. Relation between the average observed curvatures and the theoretical curvatures**

| Parameter                  | Value  |
|----------------------------|--------|
| $P_{Cm}$ [\%]             | 109.58 |
| $P_{Cm}$ [\%]             | 92.06  |
| $\sigma_C$ [10⁻⁶, 1/m]    | 14.04  |
| $R_C$                      | 0.9030 |

The equation of linear regression between the theoretical curvatures and the average observed curvatures is shown at the Figure 4.
5. Conclusions

Figures 1 and 2 indicate that the minimum values of measured and average curvatures occur in the same point number 831, which is located about 1418 meters from the first point of line and at the end of run of the B-2 longwall in the 338/2 seam. But the maximum value of average curvatures occurs in another point than the maximum value of observed curvatures, namely in point number 827 (about 1258 meters from the beginning of observational line) instead in point number 828 (about 1287 meters from the line beginning). The maximum curvatures occur nearby the southern run of the B-4 longwall in the 358/1 hard coal seam.

From the Figures 2 and 3 result that the minimal value of curvatures theoretical graph occur in the different place than the minimal value of curvatures average graph (in the 832 point, about 1491 m from the first measuring point and outside of exploitation field of the B-2 longwall in the 338/2 coal seam, instead in the 831 point). From these figures can be concluded that the maximal values of theoretical and average curvatures have been noted in the same point number 827.

Data juxtaposed in the Table 2 show that the theoretical graph of curvatures, obtained on the base of subsidence forecasted by the use of the Bialek’s formula with the determined values of its parameters, fits very well to the average graph of observed curvatures, obtained from the measured subsidence approximated by the use of the smoothed splines functions. It’s confirmed by the high values of the \( R^2 \) correlation coefficient (above 0.9), the \( P_C \) percentages of the extreme average values of observed curvatures which correspond the extreme theoretical values of curvatures (both close to 100 %) and rather by the small value of the \( \sigma_C \) standard deviation (about \( 14 \cdot 10^{-6} \) 1/m).

The equation of linear regression between the theoretical and average observed curvatures \( C_{\text{aver}} = 0.8118 C_{\text{theor}} \) indicates that the theoretical values of curvatures are over 23 % larger than the average values of observed curvatures.

Taking into account the correlation coefficient value, it can be seen that the relation between the theoretical and average observed curvatures is stronger than in case of this relation determined for an operation of three coal seams [9].

Due to the above arguments, it can be said that the forecasts of mining exploitation influences on terrain surface, made by the use of the Bialek’s formula with the values of its parameters \((a, \tan \beta, A_1)\) designated from the surveys results, are credible and fairly accurate.

Figure 4. Linear regression equation between the theoretical and average observed curvatures at the measuring line No. 8 after the extraction termination in the 338/2 and the 358/1 hard coal seams
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