A New Estimation of Road Safety Index in Transportation Systems with Fuzzy-DEA Method: A Case Study on Roads of East Azarbaijan Province in Iran

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ABSTRACT
Road safety problem is an important issue which is always considered as one of the main benchmarks to assess growth and development in many countries. Many factors influence in calculating the road safety index. In the past, for calculating the road safety index, all factors affecting road safety were considered as desirable indicators. In this paper, the road safety factors are considered in two categories: desirable and undesirable indicators. Moreover, the observed values of the factors affecting the road safety index in practice are sometimes uncertain. In this study, fuzzy data envelopment analysis method is applied to calculate and evaluate road safety index. A generalised fuzzy index-maximising model is proposed to obtain an optimal road safety index, in which the optimal index is obtained as the maximum value of sum of weighted desirable—undesirable factors. In addition, the problem of ranking of the roads is investigated with regard to the optimal composite index. In order to rank the roads, a new index is suggested that discriminates the efficient roads. Finally, the proposed method is applied to evaluate the safety problem for a case study on the roads of East Azarbaijan Province in Iran.

1. Introduction
Nowadays, the road transportation system has become an essential part of the modern world, and with the development of societies and consequently, the development of the transportation system, a new evolution has taken place in contemporary social and economic relations. The roads have a key role in the transportation system, as it serves as arteries of the economy in each country. On the other hand, over 1.3 million people die on the world’s roads and tens of millions are injured or disabled every year according to the report published by World Health Organization (WHO)\textsuperscript{[1]}. Thus, road accidents and resulting casualties are considered one of the main issues of public health which have incurred enormous economic costs. Moreover, road safety has always been a matter of concern for traffic authorities across the country. In this regard, road safety is one of the
principles in transportations and, it needs more attention and consideration with regard to the development of technology.

In developed countries, the issue of road safety has been addressed and necessary actions have been initiated to reduce accidents and their consequences. However, even more studies are needed in order to achieve a higher level of safety, in which identifying factors contributing to road accidents and evaluating the factors is essential.

For every organisation, performance evaluation is one of the important tasks of management team and it has always been an important criterion for organisational stakeholders and also for empowering human resources. Recently, data envelopment analysis (DEA) is considered as one of the main approaches that is used as a powerful tool in performance evaluation problems due to the capability and flexibility of this method. DEA models are useful techniques that are used in accident analysis and it is also found a very valuable tool in identifying and analyzing factors affecting the road safety.

Odeck [2] studied the road safety problem applying DEA methodology. He examined efficiency and productivity for the transport sector of the Norwegian Public Roads Administration. Hermans et al. [3] evaluated the road safety performance to construct a composite road safety indicator. They considered three methods, including factor analysis, analytic hierarchy process, and DEA, to obtain weights of indicators in the process of calculating the road safety performance index. Some other studies have been carried out to evaluate the road safety performance [4–8]. In the latest study, Teimourzadeh et al. [8] proposed a DEA-based method to estimate the road safety index. They considered risk factors on the roads in two classified groups as desirable and undesirable factors and suggested strategies for improving the performance of inefficient roads. However, the observed values of the dataset are sometimes uncertain for the factors affecting the road safety index in practice. Fuzzy approach is one of the suitable techniques to take into account the uncertainty of the data [9]. Some researchers have used the fuzzy set theory in the mathematical model literature [10–17].

In recent years, the road safety problem has been studied by some other researchers in fuzzy environment. Bao et al. [18] proposed a hierarchical fuzzy TOPSIS method to introduce a combined index of the multilayer safety performance factors with regard to experts’ opinions. The proposed hierarchical method was also applied to performance evaluation for a set of European countries. In the following, in 2012 Bao et al. [19] proposed an improved hierarchical fuzzy TOPSIS method to define a new composite index for evaluating safety performance of the considered European countries according to a hopeful intelligent decision support system. Shen et al. [20] provided a fuzzy DEA-based composite index model to create the composite indicator that applied the fuzzy ranking approach for solving the proposed fuzzy model. They constructed a composite index of alcohol performance using the proposed method and incorporated quantitative and qualitative factors into a performance index for a case study of the 28 European countries. Behnood et al. [21] proposed a DEA-based road safety method to evaluate efficiency and constructed a fuzzy decision-making system to transform DEA efficiency scores into a rule-based system. They studied 30 different regions in Iran as well as defined strategies and plans to analyze alternative strategies for policy makers. Amini et al. [22] suggested a DEA method to evaluate the performance of Iranian provinces with road safety efficiency scores for deterministic and non-deterministic situations. In fact, they considered uncertainty environment of data on the basis of credibility theory. Omrani et al. [23] proposed a DEA-based road
safety model in the evaluation system that is combined with group best-worst method and incorporated opinions of decision-makers’ preferences into the decision-making process.

In this paper, a new method, based on a composite index of roads, is presented for the evaluation of the index of road safety when desirable and undesirable indicators exist. Moreover, risk factors on the road safety are studied using experts’ opinions given in a fuzzy environment. A generalised fuzzy index-maximising model is proposed to obtain an optimal index which maximises the sum of weighted road safety indicators. The proposed generalised fuzzy index-maximising model is solved using \( \alpha \)-level-based approach that is transformed into a family of parametric programming problems. In this study, data information is considered in a triangular fuzzy number form. Moreover, a new definition is proposed for efficient roads from the road safety perspective. Subsequently, the ranking problem and classification of roads are studied according to the optimal road safety index. For this purpose, a new proxy is suggested that discriminates all of the efficient roads. Finally, the proposed method is applied to evaluate the safety problem for a case study on the roads of East Azarbaijan Province in Iran.

This paper is organised into five sections. In Section 2, road safety factors are provided. In Section 3, a new method is presented for evaluating the Road Safety Index with fuzzy DEA approach. In Section 3, all roads of East Azarbaijan province in Iran are evaluated and analyzed applying the proposed method. Conclusions of the proposed research are provided in Section 4.

2. Road Safety Factors

Many factors influence the road safety and some of these factors include traffic flow volume, geometric characteristics of the road, status of the bridges, underpasses and intersections, type of business on the roadsides, people’s driving cultures, the traffic rules and regulations, police surveillance, number of black spot points, installed warning signs, capability of the vehicles and elimination of technical shortcomings of the vehicles before moving, existence of the lights on the road and tunnels, development of dual carriageway, equipping the road with new technologies, monitoring the speed of the vehicles, and education of the drivers.

Researchers considered different safety factors for the road safety evaluation, based on their methods. Odeck [2] used three indicators for evaluating road safety for heavy vehicle. These indicators are the number of vehicles controlled, technical controls in halls and along the roads, and seat belt controls. Hermans et al. [3,4] used some strategies to obtain a composite road safety index. The chosen risk factors to obtain the index are alcohol and drugs, Speed, Protective systems, infrastructures, vehicle, trauma care, and daytime running lights. Shen et al. [5,6] proposed three models: DEA-based road safety model, the cross-efficiency model, and the categorical DEA model. They considered population, number of passengers, and number of passenger cars as input factors and number of road deaths as output. Moreover, Teimourzadeh et al. [8] suggested a DEA-based method for computing the road safety index. Their considered risk factors are included road infrastructure, trauma care, visibility, speed control, and traffic signs as desirable indicators as well as vehicle, black spot points, unauthorised speed, and protective systems as undesirable indicators.
In this paper, desirable and undesirable indicators are selected as follows:

**Desirable indicators**

(i) Road Infrastructure Indicator (RII): this indicator is considered for several factors, including equivalence of the traffic flow volume with the capacity of the road, safety status of the bridges and underpasses located on the roads, qualitative status of the shoulder of the roads, intersection, and exchange of cross-roads with the roads, and geometric design (direction, longitudinal and transverse slopes, horizontal radius of the horizontal and vertical arch, visibility distances)

(ii) Safety Equipment Indicator (SEI): status of road marking in accordance with the rules, compliance status of regulations with road signs (warning signs, law enforcement, and intelligence), and long-distance safety shields (Gard rill)

(iii) Visibility Indicator (VI): The status of road lighting, where by-laws are necessary on the basis of traffic at night

(iv) Speed Control Indicator (SCI): Qualitative and quantitative status of surveillance cameras and speed control cameras

(v) Trauma Care Indicator (TCI): Road emergency trauma care status in terms of the speed for the presence at the scene of accident as well as services delivery

**Undesirable outputs**

(i) Roadsides Business Indicator (RBI): Disruption of the roads due to roadside businesses, including shops, workshops, factories, etc.

(ii) Road Accidents Indicator (RAI): rate of road traffic accidents

3. New Estimation Model of Road Safety Index

In this section, a new method is proposed for computing road safety index with regard to the desirable and undesirable indicators. According to the classification of indicators, a fuzzy DEA model is applied in which the weighted sum of desirable indicators obtains the maximum value, and the weighted sum of undesirable indicators obtains the minimum value.

Suppose that there are \( n \) roads to estimate and then evaluate optimal road safety index. In this paper, \( \tilde{y}_{rij} \) is denoted for the fuzzy vector of the desirable indicator \( r \) (\( r = 1, 2, \ldots, 5 \)), and \( \tilde{y}_{tij} \) is also denoted for the fuzzy vector of undesirable indicator \( t \) (\( t = 1, 2 \)) for road \( j \) (\( j = 1, 2, \ldots, n \)). Besides, \( w_r \) and \( w_t \) are supposed for the vector of desirable indicator \( r \), and the vector of undesirable indicator \( t \). In this paper, a generalised fuzzy index-maximising model is proposed for estimating the optimal road safety index of road \( R_k \) which can be calculated by the following fuzzy programming problem.

\[
CI_k = \max \sum_{r=1}^{5} w_r \tilde{y}_{rjk} - \sum_{t=1}^{2} w_t^b \tilde{y}_{tjk}^b
\]

s.t. \( \sum_{r=1}^{5} w_r \tilde{y}_{rj} - \sum_{t=1}^{2} w_t^b \tilde{y}_{tj}^b \leq \tilde{1}; \quad \forall j \)
\[ w_r \geq 0; \quad \forall r \]
\[ w_t^b \geq 0; \quad \forall t \]

(1)

where notation ’∼’ shows that the data are as fuzzy numbers. The weighted composite index is constrained to be less than or equal to one for all the roads. Moreover, optimal objective function value \( C_{Ik} \) is defined as the optimal road safety index according to the maximum obtained value of the weighted component index of the desirable and undesirable factors. Furthermore, the generalised fuzzy index-maximising model (1) is equivalent to a fuzzy input-oriented undesirable DEA model with \( m \) fuzzy desirable outputs, \( s \) fuzzy undesirable outputs and one dummy input of 1 for all decision-making units under the assumption of constant returns to scale.

Model (1) is a fuzzy model and there are several methods to solve this model. These methods have been classified in the literature into four main categories: the fuzzy ranking approach, the possibility approach, the tolerance approach, and the \( \alpha \)-level-based approach [24–27]. The \( \alpha \)-level-based approach is one of the most popular techniques to address situations where the fuzzy models are considered. In this approach, the fuzzy model is transformed into a family of parametric programming problems based on the \( \alpha \)-cuts of the membership functions of the fuzzy numbers. In this paper, the \( \alpha \)-level-based approach is applied to solve the fuzzy DEA model (1). For this purpose, the presented method, by Saati et al [10], is used to transform the fuzzy model a family of crisp parametric linear models.

In this study, fuzzy data are considered as triangular fuzzy numbers. Let \( \tilde{y}_{ij} = (y_{ij}^L, y_{ij}^M, y_{ij}^U) \) and \( \tilde{y}_{bj} = (y_{bj}^L, y_{bj}^M, y_{bj}^U) \) be the triangular fuzzy desirable and the undesirable indicators, respectively. The \( \alpha \)-cut of fuzzy factors is defined as follows:

\[
(\tilde{y}_{ij})_\alpha = [(y_{ij})_i^L, (y_{ij})_i^U] = [\alpha y_{ij}^M + (1 - \alpha) y_{ij}^L, \alpha y_{ij}^M + (1 - \alpha) y_{ij}^U]; \forall r, j
\]
\[
(\tilde{y}_{bj})_\alpha = [(y_{bj})_j^L, (y_{bj})_j^U] = [\alpha y_{bj}^M + (1 - \alpha) y_{bj}^L, \alpha y_{bj}^M + (1 - \alpha) y_{bj}^U]; \forall t, j
\]

(2)

By applying the \( \alpha \)-cut of fuzzy data, an interval programming problem is obtained which can be solved as a crisp linear model with some variable substitutions. The substitute variables \( \hat{y}_{ij} \) and \( \hat{y}_{bj} \) are introduced for intervals of \( \alpha \)-cut of fuzzy desirable indicator and fuzzy undesirable indicator, respectively. Therefore, the proposed fuzzy model (1) is written as follows.

\[
C_{Ik} = \max \sum_{r=1}^{5} w_r \hat{y}_{rk} - \sum_{t=1}^{2} w_t^b \hat{y}_{tk}^b
\]

s.t. \[
\sum_{r=1}^{5} w_r \hat{y}_{ij} - \sum_{t=1}^{2} w_t^b \hat{y}_{ij}^b \leq 1; \quad \forall j
\]
\[
\alpha y_{ij}^M + (1 - \alpha) y_{ij}^L \leq \hat{y}_{ij} \leq \alpha y_{ij}^M + (1 - \alpha) y_{ij}^U; \quad \forall r, j
\]
\[
\alpha y_{ij}^M + (1 - \alpha) y_{ij}^L \leq \hat{y}_{ij} \leq \alpha y_{ij}^M + (1 - \alpha) y_{ij}^U; \quad \forall t, j
\]
\[
\begin{align*}
wr & \geq 0; \quad \forall r \\
\omega^b_t & \geq 0; \quad \forall t
\end{align*}
\]

Note that model (3) is a nonlinear programming problem, and will be required to convert into a linear programming problem. The substitutions \( Y_{rj} = \hat{w}_r \hat{y}_{tj} \); \( \forall r, j \) and \( Y^b_{tj} = \hat{w}^b_t \hat{y}^b_{tj} \); \( \forall t, j \) are used to linearise model (3). According to the substitute variables, model (3) is transformed into the following linear model:

\[
CI_k = \max \left( \sum_{r=1}^{5} Y_{rk} - \sum_{t=1}^{2} Y^b_{tk} \right)
\]

s.t. \[
\sum_{r=1}^{5} Y_{rj} - \sum_{t=1}^{2} Y^b_{tj} \leq 1; \quad \forall j
\]

\[
\hat{w}_r \left[ \alpha y^M_{tj} + (1 - \alpha) y^L_{tj} \right] \leq Y_{rj} \leq \hat{w}_r \left[ \alpha y^M_{tj} + (1 - \alpha) y^U_{tj} \right]; \quad \forall r, j
\]

\[
\hat{w}^b_t \left[ \alpha y^M_{tj} + (1 - \alpha) y^L_{tj} \right] \leq Y^b_{tj} \leq \hat{w}^b_t \left[ \alpha y^M_{tj} + (1 - \alpha) y^U_{tj} \right]; \quad \forall t, j
\]

\[
w_r \geq 0; \quad \forall r
\]

\[
\omega^b_t \geq 0; \quad \forall t
\]

Model (4) is a parametric linear programming model that parameter \( \alpha \) belongs to the interval \([0,1]\). The objective function of this model reflects the value of the optimal composite index for a given \( \alpha \), with regard to the maximum value of the sum of weighted desirable–undesirable factors. Moreover, the obtained model (4) can be solved for different parameters \( \alpha \) to inspect changes of the optimal composite index when the possibility level \( \alpha \) is different. It makes that the best and the worst value of the optimal composite index of under consideration road may be determined.

**Theorem 3.1:** The linear programming model (4) is feasible and its optimal value is bounded and the optimal objective function value is less than or equal to one.

**Proof:** To prove the feasibility of model (4), it is enough to introduce a feasible solution. The following particular solution is a feasible solution of the linear model (4):

\[
Y_{rj} = 0; \quad \forall r, j
\]

\[
Y^b_{tj} = 0; \quad \forall t, j
\]

\[
w_r = 0; \quad \forall r
\]

\[
\omega^b_t = 0; \quad \forall t
\]

Therefore, this model is always feasible. Moreover, in accordance with the first constraint \( \sum_{r=1}^{5} Y_{rj} - \sum_{t=1}^{2} Y^b_{tj} \leq 1; \quad \forall j \), as well as with regard to the suggested feasible solution (5), the optimal value of (4) is bounded. Therefore, the optimal composite index belongs to the interval \([0,1]\). This completes the proof.

\[\blacksquare\]
Definition 3.1: A road is defined as an efficient road from the road safety perspective, when the optimal composite index $CI_k$ is equal to one.

The problem of ranking roads is essential for decision-makers. In this study, the obtained optimal composite index $CI_k$ is defined as a benchmark index to the rank of roads. The aim of choosing this parametric DEA model is to consider the effect of ranking with different values of parameters $\alpha$. In the proposed method, however, it may be happening that a number of roads are evaluated and then introduced as an efficient road conforming to Definition 3.1. In this situation, all efficient roads will not be possible to be ranked. Hence, it is necessary to consider a proxy for identifying the efficient roads, whose that their composite index equals to one. For such a case a new composite index is suggested which can be determined for each efficient road through the following model:

$$CIP_k = \max \sum_{r=1}^{5} Y_{rk} - \sum_{t=1}^{2} Y_{btk}$$

s.t. $$\sum_{r=1}^{5} Y_{rj} - \sum_{t=1}^{2} Y_{b tj} \leq 1; \quad \forall j \neq k$$

$$w_r \left[ \alpha y_{ij}^M + (1 - \alpha) y_{ij}^L \right] \leq Y_{rij} \leq w_r \left[ \alpha y_{ij}^M + (1 - \alpha) y_{ij}^U \right]; \quad \forall r, j$$

$$w_t \left[ \alpha y_{ij}^M + (1 - \alpha) y_{ij}^L \right] \leq Y_{b tj} \leq w_t \left[ \alpha y_{ij}^M + (1 - \alpha) y_{ij}^U \right]; \quad \forall t, j$$

$$w_r \geq 0; \quad \forall r$$

$$w_t \geq 0; \quad \forall t$$

(6)

Model (6) is solved to obtain the composite index $CIP_k$ for a given parameter $\alpha$. Consequently, the proposed index is used for ranking efficient roads in light of the weighted desirable–undesirable factors. In fact, the suggested model (6) is introduced according to the frameworks of super-efficiency method proposed in Andersen and Petersen [28].

4. Case Study

In this section, 41 roads of East Azarbaijan Province in Iran are studied based on the road safety index that is constructed with some of the desirable and undesirable indicators. These roads are Tabriz-Sufian, Ilkhchi-Azarshahr, Tabriz-Islamic island, Tabriz-Hashtrood, Hashtrood-Mianeh, Bostanabad-Tabriz (Old Road), Tabriz-Ilkhchi, Kasai highway-tunnel Shiboli, Western Ring road of Tabriz-Sufian, Azarshahr-Ajabshir, Tabriz-Ahar, Ahar-Kaleybar, Ahar-Meshginshahr, Ahar-Hurand, Sarab-Nir, Hashtrood-Maragheh, Maragheh-Bonab, Jolfa-Marand, Marand-Koshksaray, Spacecraft-Three Way Ovagli, Jolfa-Nurduz, Jolfa-Poldasht, Mianeh-Zanjjan, The Three Way Of Sarcham-Khalkhal, Mianeh-Zanjjan (old road), Bonab-Ajabshir, Bonab-Malekan, Malekan-Miandoab, Heris-The Three Way of Heris, Three ways of Heris-Ahar, Sarab-Bostanabad, Bostanabad-Mianeh, highway of Tabriz-Bostanabad, Mianeh-Qarah Aghaj, Sufian-Shabestar, Sufian-Marand, Shabestar-Tasuj, Tasuj-Salmas, Kaleybar-Jananlu, Khajeh-Varzaqan, Three Way of Khajeh-Three Way of Heris and these roads are denoted as $R_1, R_2, \ldots, R_{41}$, respectively.

Three resources are being used for identifying factors affecting road safety including the WHO, the SARTRE project on social attitudes to road traffic risk in Europe, and
shown in Tables 1 and 2, respectively. Road safety index for data in Tables 1 and 2 is calculated using our proposed model (model 4). In this calculation, Gams with C++ software server are used.

Iran Road Maintenance and Transportation Organization [1,29]. All risk factors are classified as desirable and undesirable indicators, as they are listed in the sources. The selected desirable indicators are Road Infrastructure Indicator (RII), Safety Equipment Indicator (SEI), Visibility Indicator (VI), Speed Control Indicator (SCI), and Trauma Care Indicator (TCI). Moreover, the selected undesirable indicators are Roadsides Business Indicator (RBI) and Road Accidents Indicator (RAI). The dataset for the fuzzy desirable indicators and the fuzzy undesirable indicators for this application are listed in Tables 1 and 2, respectively.

Five fuzzy desirable indicators and two fuzzy undesirable indicators for each road are shown in Tables 1 and 2, respectively. Road safety index for data in Tables 1 and 2 is calculated using our proposed model (model 4). In this calculation, Gams with C++ software server are used.
Table 2. Fuzzy data of the undesirable indicators.

| Road | RBI | RAI |
|------|-----|-----|
| R1   | (12.144, 15.852, 26.444) | (15.024, 22.83, 39.924) |
| R2   | (11.832, 15.54, 26.132) | (15.112, 22.918, 40.012) |
| R3   | (3.88, 5.06, 8.324) | (4.36, 6.406, 11.66) |
| R4   | (3.535, 4.636, 7.671) | (3.94, 6.406, 11.66) |
| R5   | (4.66, 6.156, 10.336) | (5.44, 8.206, 14.94) |
| R6   | (9.76, 12.836, 21.596) | (13.96, 21.406, 37.76) |
| R7   | (11.23, 14.78, 24.914) | (13.648, 23.454, 40.548) |
| R8   | (9.07, 11.988, 20.29) | (11.2, 17.206, 30.6) |
| R9   | (3.1, 3.964, 6.312) | (3.28, 4.606, 8.38) |
| R10  | (9.415, 12.412, 20.943) | (11.62, 17.806, 31.57) |
| R11  | (12.768, 16.476, 27.068) | (15.648, 23.454, 40.548) |
| R12  | (8.98, 11.74, 19.584) | (10.96, 16.606, 29.26) |
| R13  | (6.13, 8.1, 13.654) | (7.36, 11.206, 20.16) |
| R14  | (4.57, 5.908, 9.63) | (5.2, 7.606, 13.6) |
| R15  | (11.52, 15.228, 25.82) | (14.8, 22.606, 39.7) |
| R16  | (12.456, 16.164, 26.756) | (15.336, 23.142, 40.236) |
| R17  | (4.225, 5.484, 8.977) | (4.78, 7.006, 12.63) |
| R18  | (10.105, 13.26, 22.249) | (12.46, 19.006, 33.51) |
| R19  | (10.795, 14.108, 23.555) | (13.3, 20.206, 35.45) |
| R20  | (8.635, 11.316, 18.931) | (10.54, 16.006, 28.29) |
| R21  | (8.29, 10.892, 18.278) | (10.12, 15.406, 27.32) |

The maximum value of the sum of weighted desirable and undesirable factors is obtained with parametric model (4) for different parameters $\alpha$. The optimal objective function score is considered as the value of the optimal road safety index for the given $\alpha$.

By applying the proposed model (4) for different $\alpha$ values, it is observed that changing the parameters $\alpha$ at the beginning of the interval $[0, 1]$ does not have much effect on the values of the optimal indices. However, the values of the optimal road safety index undergo significant changes when it gets close to the end of the interval $[0, 1]$. Accordingly, the values of the optimal road safety index are calculated on the basis of model (4) for $\alpha = 0, 0.25, 0.5, 0.7, 0.8, 0.9, 1$ which are shown in columns 2–8 of Table 3.

The optimal values for road safety index are crisp numbers belonging to the interval $[0, 1]$. For efficient roads, optimal road safety index equals to one, while for inefficient roads, the road safety index is smaller than one. The following points can be taken from the results in Table 3.

1. As the parameter $\alpha$ increases, the optimal road safety index becomes smaller.
2. If a road is evaluated as an inefficient road for parameter $\alpha = 0$, then it is always an inefficient road. The optimal road safety index of roads R1, R2, R7, R11, R15, R16, R31 are smaller than one, which means these roads are inefficient. Therefore, these roads are always inefficient and unsafe for all values of parameter $\alpha$. Moreover, it can be expressed that these roads are not in an appropriate safety level and more attention should be paid to these roads.
3. If a road is evaluated as an efficient road for parameter $\alpha = 1$, then this road is classified as an efficient road. The optimal road safety index of road R9 is equal to one for $\alpha = 1$ which means this road is efficient. As a result, R9 is always defined as an efficient road for each parameter $\alpha$ in $[0, 1]$. Therefore, it can be concluded that road R9 is the safest road.
Table 3. The optimal road safety index.

| Road | $\alpha = 0$ | $\alpha = 0.25$ | $\alpha = 0.5$ | $\alpha = 0.7$ | $\alpha = 0.8$ | $\alpha = 0.9$ | $\alpha = 1$ |
|------|-------------|----------------|----------------|----------------|----------------|----------------|----------------|
| R1   | 0.6055      | 0.3786         | 0.2665         | 0.1930         | 0.1596         | 0.1281         | 0.0985         |
| R2   | 0.6849      | 0.4346         | 0.3055         | 0.2269         | 0.1912         | 0.1576         | 0.1259         |
| R3   | 1           | 1              | 1              | 1              | 1              | 1              | 0.9517         |
| R4   | 1           | 1              | 1              | 1              | 1              | 0.9750         | 1              |
| R5   | 1           | 1              | 1              | 1              | 1              | 0.9155         | 0.8367         |
| R6   | 1           | 0.8787         | 0.6321         | 0.5133         | 0.4598         | 0.4096         | 0.3625         |
| R7   | 0.9287      | 0.6085         | 0.4289         | 0.3356         | 0.2934         | 0.2538         | 0.2164         |
| R8   | 1           | 0.9847         | 0.7051         | 0.5521         | 0.4958         | 0.4430         | 0.3935         |
| R9   | 1           | 1              | 1              | 1              | 1              | 0.9762         | 0.8937         |
| R10  | 1           | 0.9377         | 0.6744         | 0.5508         | 0.4951         | 0.4429         | 0.3940         |
| R11  | 0.5083      | 0.3213         | 0.2241         | 0.1574         | 0.1271         | 0.0988         | 0.0740         |
| R12  | 1           | 1              | 0.7810         | 0.6200         | 0.5594         | 0.5027         | 0.4495         |
| R13  | 1           | 1              | 1              | 0.9219         | 0.8314         | 0.7577         | 0.6887         |
| R14  | 1           | 1              | 1              | 1              | 0.9762         | 0.8937         |               |
| R15  | 0.7664      | 0.4925         | 0.3464         | 0.2627         | 0.2248         | 0.1891         | 0.1554         |
| R16  | 0.5898      | 0.3698         | 0.2650         | 0.1933         | 0.1607         | 0.1302         | 0.1032         |
| R17  | 1           | 1              | 1              | 1              | 1              | 1              | 0.9222         |
| R18  | 1           | 0.8460         | 0.5993         | 0.4778         | 0.4264         | 0.3782         | 0.3330         |
| R19  | 1           | 0.7270         | 0.5066         | 0.4019         | 0.3548         | 0.3106         | 0.2690         |
| R20  | 1           | 1              | 0.8268         | 0.6575         | 0.5947         | 0.5361         | 0.4811         |
| R21  | 1           | 1              | 0.8727         | 0.6950         | 0.6301         | 0.5694         | 0.5126         |
| R22  | 1           | 1              | 1              | 1              | 1              | 0.9762         | 0.8937         |
| R23  | 1           | 1              | 1              | 0.9935         | 0.8961         | 0.8183         | 0.7457         |
| R24  | 1           | 1              | 1              | 1              | 1              | 0.9635         |               |
| R25  | 1           | 0.6741         | 0.4700         | 0.3705         | 0.3254         | 0.2832         | 0.2435         |
| R26  | 1           | 1              | 1              | 0.8419         | 0.7631         | 0.6939         | 0.6291         |
| R27  | 1           | 1              | 0.9631         | 0.7647         | 0.6929         | 0.6277         | 0.5666         |
| R28  | 1           | 1              | 1              | 1              | 0.9184         | 0.8402         |               |
| R29  | 1           | 1              | 1              | 0.8031         | 0.7277         | 0.6605         | 0.5976         |
| R30  | 1           | 1              | 1              | 0.8807         | 0.7984         | 0.7272         | 0.6606         |
| R31  | 0.8466      | 0.5495         | 0.3866         | 0.2982         | 0.2581         | 0.2204         | 0.1849         |
| R32  | 1           | 1              | 1              | 1              | 0.9314         | 0.8517         | 0.7772         |
| R33  | 1           | 1              | 1              | 0.9547         | 0.8607         | 0.7850         | 0.7142         |
| R34  | 1           | 0.9160         | 0.7299         | 0.6629         | 0.6003         | 0.5411         |               |
| R35  | 1           | 0.7351         | 0.5826         | 0.5241         | 0.4694         | 0.4180         |               |
| R36  | 1           | 0.9233         | 0.7319         | 0.6636         | 0.6003         | 0.5411         |               |
| R37  | 1           | 1              | 0.9195         | 0.8337         | 0.7606         | 0.6921         |               |
| R38  | 1           | 1              | 1              | 1              | 0.9428         | 0.8622         |               |
| R39  | 1           | 0.7870         | 0.5534         | 0.4404         | 0.3911         | 0.3449         | 0.3015         |
| R40  | 1           | 0.6635         | 0.4672         | 0.3691         | 0.3248         | 0.2831         | 0.2439         |
| R41  | 1           | 1              | 1              | 1              | 0.9669         | 0.8850         | 0.8087         |

(4) Moreover, the optimal road safety index of six roads R3, R4, R9, R17, R22, and R24 is equal to one for parameters $0 \leq \alpha \leq 0.9$ which means these roads are efficient roads (the safest roads). In fact, these roads have a constant level of road safety for $0 \leq \alpha \leq 0.9$ meaning that by changing the parameter $\alpha$, their road safety indexes do not change.

From Table 3, it can be seen that as the length of the $\alpha$-cut interval of the fuzzy data becomes greater (or the value of parameter $\alpha$ reduces), the number of efficient roads and safe roads increases. It is clear when the length of the $\alpha$-cut interval becomes greater, these roads, from the safety point of the view, cannot be distinguished. On the other hand, as the length of the $\alpha$-cut interval of the fuzzy data gets smaller (or the value of $\alpha$ becomes close to one), the number of efficient roads decreases. Therefore, as data flexibility increases, these...
roads are barely distinguishable, while closeness of parameter \( \alpha \) to one makes that roads are completely distinguishable and in this case, there is only one road as an efficient road.

From Table 3 it can be concluded that there are many roads that can be introduced as efficient roads and it is hard to rank the roads, according to the optimal values of the road safety index. To approach this challenge, a new composite index is defined through model (6) which can be used for ranking the efficient road, as mentioned in section 3. The obtained results for the new composite index are shown in Table 4.

The values of the new optimal road safety index are listed in Table 4, in which values greater than one are for efficient roads (bolded scores) and for inefficient roads, the scores are smaller than one.

From Table 4 it can be observed that the rank of the roads does not change much for different values of parameters \( \alpha \). Also several points can be taken from Table 4 as follows.

**Table 4.** Ranking roads with a new optimal index.

| Rank | Road \( \alpha = 0 \) | Road \( \alpha = 0.25 \) | Road \( \alpha = 0.5 \) | Road \( \alpha = 0.7 \) | Road \( \alpha = 0.8 \) | Road \( \alpha = 0.9 \) | Road \( \alpha = 1 \) |
|------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| 1    | R22              | 3.0918           | 2.1454           | 1.6732           | 1.8123           | 4.0441           | 1.5411           |
| 2    | R24              | 3.0582           | 2.1205           | 1.6003           | 1.3019           | 1.1810           | 1.0742           |
| 3    | R4               | 3.0187           | 2.0926           | 1.5801           | 1.2842           | 1.1643           | 1.0617           |
| 4    | R3               | 2.9791           | 2.0617           | 1.5586           | 1.2659           | 1.1473           | 1.0582           |
| 5    | R17              | 2.9348           | 1.9209           | 1.5323           | 1.2423           | 1.1246           | 1.0379           |
| 6    | R14              | 2.8556           | 1.9730           | 1.5064           | 1.2197           | 1.1053           | 1.0065           |
| 7    | R38              | 2.7734           | 1.9603           | 1.4635           | 1.1839           | 1.0705           | R14 0.9762       |
| 8    | R5               | 2.6972           | 1.9141           | 1.4176           | 1.1451           | 1.0345           | R38 0.9428       |
| 9    | R28              | 2.6839           | 1.8611           | 1.3778           | 1.1123           | 1.0045           | R28 0.9184       |
| 10   | R41              | 2.6017           | 1.8535           | 1.3736           | 1.1099           | 1.0029           | R5 0.9155        |
| 11   | R32              | 2.5195           | 1.7945           | 1.3277           | 1.1071           | R41 0.9669       |
| 12   | R23              | 2.4374           | 1.7355           | 1.2818           | 1.0323           | R32 0.9314       |
| 13   | R9               | 2.4163           | 1.6765           | 1.2360           | 0.9935           | R23 0.8961       |
| 14   | R33              | 2.3552           | 1.6175           | 1.1901           | 0.9547           | R33 0.8607       |
| 15   | R13              | 2.2790           | 1.5646           | 1.1503           | 0.9219           | R13 0.8337       |
| 16   | R37              | 2.2656           | 1.5570           | 1.1460           | 0.9195           | R37 0.8314       |
| 17   | R30              | 2.1835           | 1.4980           | 1.1002           | 0.8807           | R30 0.7984       |
| 18   | R26              | 2.1013           | 1.4390           | 1.0543           | 0.8419           | R26 0.7631       |
| 19   | R29              | 2.0191           | 1.3800           | 1.0085           | 0.8031           | R29 0.7277       |
| 20   | R27              | 1.9374           | 1.3215           | 0.9631           | 0.7647           | R27 0.6929       |
| 21   | R36              | 1.8613           | 1.2686           | 0.9233           | 0.7319           | R36 0.6636       |
| 22   | R34              | 1.8449           | 1.2580           | 0.9160           | 0.7299           | R34 0.6629       |
| 23   | R21              | 1.7652           | 1.2015           | 0.8727           | 0.6950           | R21 0.6301       |
| 24   | R20              | 1.6830           | 1.1425           | 0.8268           | 0.6575           | R20 0.5947       |
| 25   | R12              | 1.6009           | 1.0835           | 0.7810           | 0.6200           | R12 0.5594       |
| 26   | R35              | 1.5187           | 1.0245           | 0.7351           | 0.5826           | R35 0.5241       |
| 27   | R8               | 1.4615           | 0.9847           | 0.7051           | 0.5521           | R8 0.4958       |
| 28   | R10              | 1.3913           | 0.9377           | 0.6744           | 0.5508           | R10 0.4951       |
| 29   | R6               | 1.3091           | 0.8787           | 0.6321           | 0.5133           | R6 0.4598       |
| 30   | R18              | 1.2648           | 0.8460           | 0.5993           | 0.4778           | R18 0.4264       |
| 31   | R39              | 1.1826           | 0.7870           | 0.5534           | 0.4404           | R39 0.3911       |
| 32   | R19              | 1.0995           | 0.7270           | 0.5066           | 0.4019           | R19 0.3548       |
| 33   | R25              | 1.0233           | 0.6741           | 0.4705           | 0.3705           | R25 0.3254       |
| 34   | R40              | 1.0069           | 0.6635           | 0.4672           | 0.3691           | R40 0.3248       |
| 35   | R7               | 0.9287           | 0.6085           | 0.4289           | 0.3356           | R7 0.2934       |
| 36   | R31              | 0.8466           | 0.5495           | 0.3866           | 0.2982           | R31 0.2581       |
| 37   | R15              | 0.7664           | 0.4925           | 0.3464           | 0.2627           | R15 0.2248       |
| 38   | R2               | 0.6849           | 0.4346           | 0.3055           | 0.2269           | R2 0.1912       |
| 39   | R1               | 0.6055           | 0.3786           | 0.2665           | 0.1933           | R1 0.1607       |
| 40   | R16              | 0.5898           | 0.3698           | 0.2650           | 0.1930           | R16 0.1596       |
| 41   | R11              | 0.5083           | 0.3213           | 0.2241           | 0.1574           | R11 0.1271       |

Note: The values were bold as their efficient roads values are equal to one to greater than one.
Table 5. Classifying roads.

| Class 1 | Class 2 | Class 3 | Class 4 | Class 5 | Class 6 | Class 7 | Class 8 Inefficient |
|---------|---------|---------|---------|---------|---------|---------|---------------------|
| Roads   | R9      | R22     | R14     | R41     | R23     | R27     | R8                  | R7 |
|         | R4      | R38     | R32     | R33     | R13     | R36     | R10                 | R31|
|         | R24     | R5      | R5      | R37     | R21     | R34     | R6                  | R15|
|         | R3      | R28     | R28     | R30     | R20     | R18     | R2                  | R1 |
|         | R17     |         |         | R26     | R12     | R19     | R16                 | R11|
|         |         |         |         | R29     | R35     | R25     | R40                 |     |

Figure 1. Pyramid diagram of the road classification.

1. The rank of 20 roads including R33, R30, R26, R29, R27, R36, R34, R21, R20, R12, R35, R6, R18, R39, R19, R7, R31, R15, R2, and R11 is always constant for all of parameters \( \alpha \).
2. The rank of 9 roads including R9, R22, R3, R17, R14, R38, R41, R32, and R23 is always constant for the values of parameter \( \alpha = 0.5, 0.7, 0.8, 0.9 \).
3. The roads R9 and R22 have the best performance in terms of road safety index and these roads have the highest level of safety and are the safest. R22 has the first rank for \( \alpha = 0 \) and 0.25, while this road is in the second rank for other values of \( \alpha \). On the other hand, R9 is in the 13th rank for \( \alpha = 0 \) and is in the 7th rank for \( \alpha = 0.25 \), while this road is in the first rank for other values of \( \alpha \).
4. When parameter \( \alpha \) approaches to 0, data flexibility increases, consequently minor changes happen in the rank of the roads.
5. The road R11 always has the worst performance in terms of road safety index and has the last rank for all parameters. Therefore, it can be concluded that road R9 has the lowest level of safety and is actually the most unsafe road.
It should be noted that given different values of $\alpha$, all efficient roads can be highlighted. Also the studied roads can be ranked with the optimal values of a new composite index.

Using the data in Table 4, the roads can be classified into 8 classes based on the optimal index, as shown in Table 5.

The following points can be taken from Table 5. The roads classified as class 1 are always efficient and safe. The roads located in class 2 are efficient for the values of parameters $0 \leq \alpha \leq 0.9$. The roads in class 3 are efficient for the values of parameters $0 \leq \alpha \leq 0.8$. The road in class 4, class 5, and class 6 also can be described for the different values of parameters for $\alpha$ in the same way. The roads in class 7 are efficient for the values of parameters $\alpha = 0$. Finally, the roads in class 8 are always inefficient for all values of parameters $\alpha$ that are shown in the last column of Table 5. These classifications are set up in a pyramid diagram, as shown in Figure 1.

5. Conclusion

Road safety index always has been a vital parameter from the road safety system perspective. This index classifies the road in terms of road user behavior, vehicle safety, and infrastructural factors. In the classic methods, all factors affecting road safety were considered as desirable indicators for estimating the road safety index. In this paper, the road safety factors were considered in two categories, desirable and undesirable indicators in which the risk factors are examined by combining experts’ opinions in a fuzzy environment. Moreover, a Fuzzy DEA model was applied for evaluating the safety performance of the roads. A generalised fuzzy index-maximising model was proposed for computing an optimal road safety index. The proposed fuzzy model was transformed into a family of parametric linear model, according to the $\alpha$-level-based approach. In this study, data information was considered in a triangular fuzzy number form. Moreover, a new definition was proposed for efficient roads from the road safety viewpoint. In addition, the problem of ranking and classification of roads was investigated with regard to the optimal road safety index. In this study, the proposed method was applied to evaluate the safety problem for a case study on the roads of East Azarbaijan Province in Iran. Furthermore, the full ranking of the roads was achieved in which all efficient roads are completely distinguished. Finally, the studied roads were listed in 8 classes based on the optimal road safety index.

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References

[1] World Health Organization. Global status report on alcohol and health 2018. Geneva: World Health Organization; 2019.
[2] Odeck J. Identifying traffic safety best practice: an application of DEA and Malmquist indices. Omega. 2006;34(1):28–40.
[3] Hermans E, Van den Bossche F, Wets G. Combining road safety information in a performance index. Accid Anal Prev. 2008;40(4):1337–1344.
[4] Hermans E, Van den Bossche F, Wets G. Uncertainty assessment of the road safety index. Reliab Eng Syst Saf. 2009;94(7):1220–1228.
[5] Shen Y, Hermans E, Ruan D, et al. A generalized multiple layer data envelopment analysis model for hierarchichal structure assessment: a case study in road safety performance evaluation. Expert Syst Appl. 2011;38(12):15262–15272.
[6] Shen Y, Hermans E, Brijs T, et al. Road safety risk evaluation and target setting using data envelopment analysis and its extensions. Accid Anal Prev. 2012;48:430–441.
[7] Shen Y, Hermans E, Bao Q, et al. Road safety development in Europe: a decade of changes (2001–2010). Accid Anal Prev. 2013;60:85–94.
[8] Teimourzadeh K, Pourmahmoud J, Kordrostami S. A novel approach to evaluate the road safety index: a case study in the roads of East Azerbaijan Province in Iran. Iran J Manage Stud. 2019;12(2):39–59.
[9] Zadeh LA. Fuzzy sets. Inform Contr. 1965;8:338–353.
[10] Saati SM, Memariani A, Jahanshahloo GR. Efficiency analysis and ranking of DMUs with fuzzy data. Fuzzy Optim Decis Mak. 2002;1(3):255–267.
[11] Nasseri SH, Ahmadi Khatir M. Fuzzy stochastic undesirable two-stage data envelopment analysis models with application to banking industry. J Intell Fuzzy Syst. 2019;37:7047–7057.
[12] Pourmahmoud J, Bafekr Sharak N. Measuring cost efficiency with new fuzzy DEA models. Int J Fuzzy Syst. 2018;20:155–162.
[13] Nasseri SH, Kiaei H. Ranking of efficient units on the basis of distance from virtual ideal and anti-ideal units. Int J Appl Decis Sci. 2019;12(4):361–374.
[14] Hatami-Marbini A, Tavana M, Emrouznejad A, et al. Efficiency measurement in fuzzy additive data envelopment analysis. Int J Ind Syst Eng. 2011;10(1):1–20.
[15] Nasseri SH, Taghi-Nezhad N, Ebrahimnejad A. A note on ranking fuzzy numbers with an area method using circumcenter of centroids. Fuzzy Inf Eng. 2017;9(2):259–268.
[16] Nasseri S, Bavandi S. Fuzzy stochastic linear fractional programming based on fuzzy mathematical programming. Fuzzy Inf Eng. 2018;10(3):324–338.
[17] Allahviranloo T, Nuraei R, Ghanbari M, et al. A new metric for L–R fuzzy numbers and its application in fuzzy linear systems. Soft Comput. 2012;16(10):1743–1754.
[18] Bao Q, Ruan D, Shen Y, et al. Creating a composite road safety performance index by a hierarchical fuzzy TOPSIS approach. In: 2010 IEEE International Conference on Intelligent Systems and Knowledge Engineering. IEEE; 2010. p. 458–463.
[19] Bao Q, Ruan D, Shen Y, et al. Improved hierarchical fuzzy TOPSIS for road safety performance evaluation. Knowl Based Syst. 2012;32:84–90.
[20] Shen Y, Hermans E, Brijs T, et al. Fuzzy data envelopment analysis in composite indicator construction. In: Performance measurement with fuzzy data envelopment analysis. Berlin: Springer; 2014. p. 89–100.
[21] Behnood HR, Ayati E, Brijs T, et al. A fuzzy decision-support system in road safety planning; 2017.
[22] Amini M, Dabbagh R, Omrani H. A fuzzy data envelopment analysis based on credibility theory for estimating road safety. Decision Sci Lett. 2019;8(3):275–284.
[23] Omrani H, Amini M, Alizadeh A. An integrated group best-worst method – Data envelopment analysis approach for evaluating road safety: a case of Iran. Measurement. 2020;152:107330.
[24] Karsak EE. Using data envelopment analysis for evaluating flexible manufacturing systems in the presence of imprecise data. The Int J Adv Manuf Technol. 2008;35(9–10):867–874.
[25] Lertworasirikul S, Fang S, Nuttle H, et al. Fuzzy data envelopment analysis. Proceedings of the 9th Bellman Continuum, Beijing; 2002. p. 342.
[26] Lertworasirikul S, Fang S-C, Joines JA, et al. Fuzzy data envelopment analysis (DEA): a possibility approach. Fuzzy Sets Syst. 2003;139(2):379–394.
[27] Lertworasirikul S, Fang S-C, Nuttle HL, et al. Fuzzy BCC model for data envelopment analysis. Fuzzy Optim Decis Mak. 2003;2(4):337–358.
[28] Andersen P, Petersen NC. A procedure for ranking efficient units in data envelopment analysis. Manage Sci. 1993;39(10):1261–1264.
[29] SafetyNet, W. State of the art report on road safety performance indicators. EU Integrated Project SafetyNet; 2005.