Relativistic verifiable delegation of quantum computation

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Abstract

The importance of being able to verify quantum computation delegated to remote servers increases with recent development of quantum technologies. In some of the proposed protocols for this task, a client delegates her quantum computation to non-communicating servers. The fact that the servers do not communicate is not physically justified and it is essential for the proof of security of such protocols. For the best of our knowledge, we present in this work the first verifiable delegation scheme where a classical client delegates her quantum computation to two entangled servers that are allowed to communicate, but respecting the plausible assumption that information cannot be propagated faster than speed of light. We achieve this result by proposing the first one-round two-prover game for the Local Hamiltonian problem where provers only need polynomial time quantum computation and access to copies of the groundstate of the Hamiltonian.

1 Introduction

With the recent progress in the development of quantum technologies, large-scale quantum computers may be available in a not-so-distant future. Their costs and infrastructure requirements make it impractical for them to be ubiquitous, however clients could send their quantum computation to be performed remotely by a quantum server in the cloud [Cas17], broadening the use of quantum advantage to solve computational problems (see Ref. [Mon16] for such examples). For the clients, it is a major concern whether the quantum servers are performing the correct computation and quantum speedup is really being experienced.

In order to solve this problem, we aim a protocol for verifiable delegation of quantum computation where the client exchanges messages with the server, and, at the end of the protocol, either the client holds the output of her computation, or she detects that the server is defective. Ideally, the client is a classical computer and an honest server only needs polynomial-time quantum computation to answer correctly. Protocols of this form could also be used for validating devices that claim to have quantum computational power, but in this work we focus on the point of view of verifiable delegation of computation.

There are efficient protocols that can perform this task if the model is relaxed, for instance giving limited quantum power and quantum communication to the client [FK12, ABOEM17, Bro15, Mor14, MF16]. In this work, we focus on a second line of protocols, where a classical client delegates her computation to two non-communicating quantum servers. Although the servers are supposed to share and maintain entangled states, which is feasible in principle but technologically challenging, these protocols are “plug-and-play” in the sense that the client only needs classical communication with the quantum servers.

Following standard notation in these protocols, we start calling the client and servers by verifier and provers, respectively. The security of such protocols relies on the so called self-testing of non-local games. We consider games where a verifier interacts with non-communicating provers by exchanging one round

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of classical communication and, based on the correlation of the provers’ answers, the verifier decides to accept or reject. The goal of the provers is to maximize the acceptance probability in the game and they can share a common strategy before the game starts. A game is non-local [Bel64] whenever there exists a quantum strategy for the provers that achieves acceptance probability strictly higher than any classical strategy, allowing the verifier to certify that the provers share some entanglement, if the classical bound is surpassed. Self-testing [MY04] goes one step further, proving that if the correlation of the provers’ answers is close to the optimal quantum value, the provers’ strategy is close to the honest strategy.

Reichardt, Unger and Vazirani [RUV13] used the ideas of self-testing to propose a verifiable delegation scheme where the verifier interleaves questions of non-local games and instructions for the computation, and from the point of view of the provers, these two types of questions are indistinguishable. In this case, the correctness of the quantum computation is inherited by the guarantees achieved in self-testing. Follow-up works [McK16, GKW15, HPDF15, FH15, NV17, CGJV17] have used the same approach in order to propose more efficient protocols. However, in all of these protocols, the fact that the provers do not communicate is unjustified and enforced by the model.

For the best of our knowledge, in this work we present the first protocol for verifiable delegation of quantum computation to multiple entangled provers where the provers are allowed to communicate respecting the assumption that information cannot be transmitted faster than speed of light. We achieve such protocol by showing a non-local game for Local Hamiltonian problem, where the verifier plays against two provers in one round of classical communication. In this game, honest provers perform polynomial time quantum computation on copies of the groundstate of the Hamiltonian. This non-local game is of independent interest since it was an open question if a one-round game for Local Hamiltonian problem could be achieved with only two efficient provers. We then convert this non-local game into a delegation protocol using the circuit-to-hamiltonian construction, and show how to drop the non-communication assumption.

1.1 Our contributions

Non-local games for Local Hamiltonians The first technical contribution of this work is presenting one-round two-prover game for the Local Hamiltonian problem, where honest provers only need access to quantum polynomial time computation, copies of the groundstate of the Hamiltonian and shared EPR pairs. More concretely, we show how to construct a game $G(H)$ based on a Hamiltonian $H$ acting on $n$ qubits, which is a sum of tensor product of Paulis $\sigma_X, \sigma_Z$ and $\sigma_I$. We show that the upper and lower bound on the maximum acceptance probability in $G(H)$ is tightly related to the groundstate energy of $H$. Then, based on $G(H)$, we devise a game $\tilde{G}(H)$ such that if the groundstate energy of $H$ is low, then the maximum acceptance probability in $\tilde{G}(H)$ is at least $\frac{1}{2} + \Delta$, while if the groundstate energy is high, the acceptance probability in the game is at most $\frac{1}{2} - \Delta$. We describe now the main ideas of $G(H)$.

The game is composed by two tests: the Pauli Braiding Test (PBT) [NV17], where the verifier checks if the provers share the expected state and perform the indicated Pauli measurements, and the Energy Test (ET), where the verifier estimates the groundstate energy of $H$.

The same structure was used in a different way in the non-local game for LH proposed by Natarajan and Vidick [NV17] (and implicitly in Ji [Ji16]). In their game, 7 provers are expected to share the encoding of the groundstate of $H$ under a stabilizer quantum error correcting code. In ET, the provers estimate the groundstate energy by jointly performing the measurements on the state, while PBT checks if the provers share a correct encoding of some state and if they perform the indicated measurements. The provers receive questions consisting in a Pauli tensor product observable and they answer with the one-bit outcome of the measurement on their share of the state. The need of 7 provers comes from the fact that the verifier must
test if the provers are committed to an encoded state and use it in all of their measurements. It is an open problem if the number of provers can be decreased in this setup.

In this work, we are able to reduce the number of provers to 2 by making them asymmetric. In ET, one of the provers holds the groundstate of $H$ and teleports it to the second prover, who is responsible for measuring it. In our case, PBT checks if the provers share EPR pairs and if the second prover’s measurements are correct. We remark that no test is needed for the state, since the chosen measurement is not known by the first prover. A drawback of this approach is that the size of the answers is polynomial in $n$: in order to estimate the energy of the groundstate, the verifier must correct the output of several one-qubit measurements due to Pauli errors from quantum teleportation, instead of considering it as a tensor product measurement as in NV. We leave as an open problem devising a one-round two-prover game for Local Hamiltonians with constant-size answers.

We state now the key ideas to upper bound the maximum acceptance probability of $G(H)$. The behavior of the second prover in ET can be verified thanks to PBT, since the two tests are indistinguishable to him. On the other hand, the first prover can perfectly distinguish PBT and ET, but he has no information about the measurement being performed. We show that his optimal strategy is to teleport the groundstate of $H$, but in this case the acceptance probability is low if the groundstate energy is high.

**Relativistic delegation of quantum computation** Non-local games for Local Hamiltonians are easily converted into protocols for verifiably delegate quantum computation through the circuit-to-hamiltonian construction [FH15 NV17]. This construction provides a reduction from a quantum circuit $Q$ to an instance $H_Q$ of LH, such that $H_Q$ has low groundstate energy iff $Q$ accepts with high probability. Consequently, non-local games for $H_Q$ correspond to a delegation scheme for circuit $Q$. Using our non-local game, we have a verifiable delegation scheme for $Q$ where the verifier interacts with two non-communicating entangled provers in one-round of classical communication.

As argued before, the non-communication restriction is unrealistic and we show how to replace it in our case by the No Superluminal Signaling (NSS) principle. The NSS states that information cannot propagate faster than the speed of light, which is one of the foundations of Theory of Special Relativity [Ein05]. For this reason, protocols whose correctness relies on NSS are known as relativistic protocols.

The first relativistic protocol is due to Kent [Ken99], who showed the existence of relativistic information-theoretical secure bit commitment, which is impossible in the general case [May97 LC97]. Since then, several other relativistic protocols were proposed for bit commitment [Ken11 Ken12 LKB+13 LKB+15 CCL15], verification of space-time position of agents [CGMO09 KMS11 Unr14 CL15 LL11 BCF+11], oblivious transfer [PG16] and zero-knowledge proof systems [CL17].

Relativistic protocols can be achieved by fixing the position of the provers and the verifier in such a way that the information transmitted between the provers takes much longer than an upper-bound of the duration of the honest protocol. The verifier could abort whenever the answers arrive too late, since the provers could have communicated and the security of the protocol is compromised. The space-time diagram of such interactions is depicted in Figure 1. We show also how to prevent more sophisticated attacks where malicious provers move closer to the verifier in order to receive the message earlier, being able to cheat in the previous protocol.

The fact that the provers communicate after the verifier receives their responses is not harmful since this cannot change the output of the protocol. The provers even do not learn any additional private information, given that our protocol is not blind, i.e. they know the input $x$, due to the circuit-to-hamiltonian construction. We leave as an open question if it is possible to create a relativistic and blind verifiable delegation scheme for quantum computation, or proving that this is improbable, in the lines of Ref. [ACGK17].
The circuit-to-hamiltonian construction also causes an overhead on the resources needed by honest provers. Namely, in our protocol the provers need $\tilde{O}(ng^2)$ EPR pairs for delegating the computation of a quantum circuit acting on $n$ qubits and composed by $g$ gates, while some non-relativistic protocols need only $O(g)$ EPR pairs [CGJV17]. We leave as an open problem finding more efficient relativistic protocol for delegating quantum computation.

We remark that our protocol can be seen as a special case of the delegation scheme of Coladangelo et al. [CGJV17] for circuits that consist of a probabilistic distribution of Pauli measurements, avoiding the complexity due to T-gates.

**Non-local games for QMA.** In Complexity Theory, the connection between the PCP theorem [AS98, ALM+98, Din07] and multi-prover games [BOGKW88] has had a lot of fruitful consequences, such as tighter inapproximability results [Raz98]. It is still an open question if a quantum version of the PCP theorem holds [AAV13], and it is not known a multi-prover version of it. Recently, there has been some effort in proposing multi-prover games for QMA [FV15, Ji16, NV17, GKP16, CGJV17], pursuing a better understanding of the quantum PCP conjecture. Since XZ Local Hamiltonian problem is QMA-complete, our result directly implies a one-round two-prover game for QMA.

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Organization

The remainder of this paper is organized as follows. In Section 2 we give the necessary preliminaries. In Section 3 we present the non-local game for Local Hamiltonian problem. In Section 4 we present the relativistic protocol for verifiable delegated quantum computation.

2 Preliminaries

2.1 Notation

We denote $[n]$ as the set $\{1, ..., n\}$. For a finite set $S$, we denote $x \in S$ as $x$ being an uniformly random element from $S$. For a complex number $x = a + ib$, $a, b \in \mathbb{R}$, we define its norm $|x|$ by $\sqrt{a^2 + b^2}$. We assume that all Hilbert spaces are finite-dimensional. For a Hilbert space $H$, $L(H)$ is the set of linear operators on $H$ and for a linear operator $M \in L(H)$, we denote $\lambda_0(M)$ as its smallest eigenvalue and $\|M\|$ as its the maximum singular value.

We review now the notation for some standard Quantum Computation concepts used in this work. A pure quantum state with $k$ qubits is a unit vector in the Hilbert space $\mathbb{C}^{2^k}$. We denote $\{|i\rangle\}_{i \in \{0,1\}^k}$ as the canonical basis for $\mathbb{C}^{2^k}$. For Hilbert spaces $H_1$ and $H_2$, we shorthand the state $|\psi_1\rangle \otimes |\psi_2\rangle \in H_1 \otimes H_2$ by $|\psi_1\rangle|\psi_2\rangle$. For a vector $|\psi\rangle = \sum_{i \in \{0,1\}^k} a_i|i\rangle$, its norm is defined as $\|v\| := (\sum_{1 \leq i \leq d} |a_i|^2)^{1/2}$.

A mixed state is a probabilistic distribution of pure quantum states $\{(|\psi_1\rangle, ..., |\psi_m\rangle)\}$, and they are represented by its density matrix $\rho = \sum_i p_i |\psi_i\rangle \langle \psi_i|$, which is a positive-definite matrix in $L(\mathbb{C}^{2^k})$ with trace 1. We denote by $D(H)$ the set of all density operators in the Hilbert space $H$. A bipartite state $\rho_{AB} \in D(H_A \otimes H_B)$ is a quantum state shared by the parties $A$ and $B$, such that $A$ holds the state $\rho_A = \text{Tr}_B(\rho_{AB}) \in H_A$ and $B$ holds $\rho_B = \text{Tr}_A(\rho_{AB}) \in H_B$.

A $n$-qubit binary observable $O$ is a Hermitian matrix that squares to the identity. $O$ has eigenvalues $\pm 1$ and we can write $O = O^+ - O^-$, where $O^+$ and $O^-$ are projectors and $O^+ + O^- = I$. A measurement of a state $\rho$ with respect to $O$ outputs $+1$ with probability $\text{Tr}(O^+ \rho)$ and $-1$ with probability $\text{Tr}(O^- \rho)$, and the expectation of the output of $\rho$ with respect to $O$ is $\text{Tr}(O \rho)$. We denote $\text{Obs}(H)$ as the set of binary observables on the Hilbert space $H$.

We will use the letters $X$, $Z$ and $I$ to denote questions in multi-prover games, the letters in the sans-serif font $\mathcal{X}$, $\mathcal{Z}$ and $\mathcal{I}$ to denote unitaries and $\sigma_X$, $\sigma_Z$ and $\sigma_I$ to denote observables such that

$$I = \sigma_I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad X = \sigma_X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad Z = \sigma_Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$  

We denote the Bell basis for $\mathbb{C}^4$ as $\{|\Phi_{ab}\rangle\}_{a,b \in \{0,1\}}$ for

$$|\Phi_{ab}\rangle = (X^a Z^b \otimes I) \left( \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \right),$$

and the state $|\Phi_{00}\rangle$ is called an EPR pair.

2.2 Complexity classes

We start by defining BQP, the complexity class that contains the problems that can be solved efficiently by quantum computers.  

\footnote{We refer to Ref. [NC00] for a detailed introduction of these notions.}
Definition 1 (BQP). A promise problem $A = (A_{\text{yes}}, A_{\text{no}})$ is in BQP if and only if there exist a polynomial $p$ and a polynomial-time uniform family of quantum circuits $\{Q_n\}$ such that the following holds. $Q_n$ takes as input a string $x \in \Sigma^*$ with $|x| = n$, and $p(n)$ ancilla qubits in state $|0\rangle^{\otimes p(n)}$ and then outputs $|1\rangle$ (accepts) or $|0\rangle$ (rejects). The acceptance probability of $x$ is such that

Completeness. If $x \in A_{\text{yes}}$, then $Q_n$ accepts $x$ with probability at least $1 - \exp(-n)$.

Soundness. If $x \in A_{\text{no}}$, then $Q_n$ accepts $x$ with probability at most $\exp(-n)$.

We define now QMA, the quantum analog of NP, which contains problems whose solution can be verified efficiently with quantum proofs.

Definition 2 (QMA). A promise problem $A = (A_{\text{yes}}, A_{\text{no}})$ is in QMA if and only if there exist polynomials $p$, $q$ and a polynomial-time uniform family of quantum circuits $\{Q_n\}$, where $Q_n$ takes as input a string $x \in \Sigma^*$ with $|x| = n$, a $p(n)$-qubit quantum state, and $q(n)$ ancilla qubits in state $|0\rangle^{\otimes q(n)}$, such that:

Completeness. If $x \in A_{\text{yes}}$, then there exists a $p(n)$-qubit quantum state $|\psi\rangle$ such that $Q_n$ accepts $(x, |\psi\rangle)$ with probability at least $1 - \exp(-n)$.

Soundness. If $x \in A_{\text{no}}$, then for any $p(n)$-qubit quantum state $|\psi\rangle$, $Q_n$ accepts $(x, |\psi\rangle)$ with probability at most $\exp(-n)$.

These two complexity classes are usually defined with constant completeness and soundness errors, but using standard techniques [MW05, NWZ11], this error can be reduced to be exponentially small in $n$.

For a circuit $Q_{|x|}$ on input $x$, we denote $Q_x$ as the circuit $Q_{|x|}$ with its input hardcoded to $x$ and acting only on ancilla qubits (and witness state when it is the case).

2.3 Non-local games, Self-testing and the Pauli Braiding Test

We consider games where a verifier plays against two provers in the following way. The verifier sends questions to the provers according to a publicly known distribution and the provers answer back to the verifier. Based on the correlation of the answers, the verifier decides to accept or reject according to a rule that is also publicly known. The provers share a common strategy before the game starts in order to maximize the acceptance probability in the game, but they do not communicate afterwards.

For a game $G$, its classical value $\omega(G)$ is the maximum acceptance probability in the game if the provers share classical randomness, while the quantum value $\omega^*(G)$ is the maximum acceptance probability if they are allowed to follow a quantum strategy, i.e. share a quantum state and apply measurements on it. Non-local games (or Bell tests) [Bel64] are such games where $\omega^*(G) > \omega(G)$ and they have played a major role in Quantum Information Theory, since they allow the verifier to certify that there exists some quantumness in the strategy of the provers, if the classical bound is surpassed.

Self-testing (also known as device-independent certification or rigidity theorems) of a non-local game $G$ allows us to achieve stronger conclusions by showing that if the acceptance probability on $G$ is close to $\omega^*(G)$, then the strategy of the provers is close to the ideal one, up to local isometries.

Pauli Braiding Test. We present now the Pauli Braiding Test (PBT), a non-local game proposed by Natarajan and Vidick [NV17]. PBT allows the verifier to certify that two provers share $t$ EPR pairs\(^2\) and perform the indicated measurements, which consist of tensors of Pauli observables.

\(^2\)In the original result, NV have proved a more general result where an encoding of a specified stabilizer quantum error correcting code can be certified.
In PBT, each prover receives questions in the form \( W \in \{X, Z, I\}^t \), and each one is answered with some \( b \in \{-1, +1\} \). For \( W \in \{X, Z\} \) and \( a \in \{0, 1\} \), we have \( W(a) \in \{X, Z, I\}^t \) where \( W(a)_i = W_i \) if \( a_i = 1 \) and \( W(a)_i = I \) otherwise. In the honest strategy, the provers share \( t \) EPR pairs and measure them with respect to the observable \( \sigma_W = \bigotimes_{i \in [t]} \sigma_{W_i} \) on question \( W \).

However, the provers could deviate and perform an arbitrary strategy, by sharing an entangled state \( |\psi\rangle_{AB} \in \mathcal{H}_A \otimes \mathcal{H}_B \) and performing projective measurements \( \tau^A_W \) and \( \tau^B_W \) for each possible question \( W \). NV showed that if the provers pass the PBT test with probability \( 1 - \varepsilon \), their strategy is, up to local isometries, \( O(\sqrt{\varepsilon}) \)-close to sharing \( t \) EPR pairs and measuring \( \sigma_W \) on question \( W \).

PBT is divided in three different tests, which are performed equal probability. The first one, the Consistency Test, checks if the measurement performed by both provers on question \( W \) are equivalent, i.e. \( \tau^A_W \otimes I_B |\psi\rangle_{AB} \) is close to \( I_A \otimes \tau^B_W |\psi\rangle_{AB} \). In the Linearity Test, the verifier checks if the measurement performed by the provers are linear, i.e. \( \tau^A_{W(d)} \tau^A_{W(d')} \otimes I_B |\psi\rangle_{AB} \) is close to \( \tau^A_{W(d+d')} \otimes I_B |\psi\rangle_{AB} \). Finally, in the Anti-commutation Test, the verifier checks if the provers’ measurements follow commutation/anti-commutation rules consistent with the honest measurements, namely \( \tau^A_{W(a)} \tau^A_{W(a')} \otimes I_B |\psi\rangle_{AB} \) is close to \( (-1)^{|\{W_i \neq W'_i \text{ and } a_i = a'_i = 1\}|} \tau^A_{W(a')} \tau^A_{W(a)} \otimes I_B |\psi\rangle_{AB} \).

The Consistency Test and Linearity Test are very simple and are described in Figure 2. For the Anti-commutation Test, we can use non-local games that allow the verifier to check that the provers share a constant number of EPR pairs and perform Pauli measurements on them. In this work we use the Magic Square game since there is a perfect quantum strategy for it, and we present it in Appendix A.

The verifier performs the following steps, with probability \( \frac{1}{3} \) each:

(A) Consistency test

1. The verifier picks \( W \in_R \{X, Z\} \) and \( a \in \{0, 1\} \).
2. The verifier sends \( W(a) \) to both provers.
3. The verifier accepts if the provers’ answers are equal.

(B) Linearity test

1. The verifier picks \( W \in_R \{X, Z\} \) and \( a, a' \in \{0, 1\} \).
2. The verifier sends \( \langle W(a), W(a') \rangle \) to \( P_1 \) and \( W' \in_R \{W(a), W(a')\} \) to \( P_2 \).
3. The verifier receives \( b, b' \in \{\pm 1\} \) from \( P_1 \) and \( c \in \{\pm 1\} \) from \( P_2 \).
4. The verifier accepts if \( b = c \) when \( W' = W(a) \) or \( b' = c \) when \( W' = W(a') \).

(C) Anti-commutation test

1. The verifier makes the provers play Magic Square games in parallel with the \( t \) EPR pairs (see Appendix A).

Figure 2: Pauli Braiding Test

\footnote{We allow for simplicity only strategies where the shared state is pure and the measurements are projective, but this assumption is without loss of generality.}
Theorem 3 (Theorem 14 of [NV17]). Suppose $|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$ and $W(a) \in \text{Obs}(\mathcal{H}_A)$, for $W \in \{X, Z\}$, and $a \in \{0, 1\}$. Suppose a strategy for the players that has success probability at least $1 - \varepsilon$ in the Pauli Braiding Test described in Figure 2. Then there exist isometries $V_D : \mathcal{H}_D \rightarrow ((\mathbb{C}^2)^\otimes t)_D \otimes \mathcal{H}_D$ for $D \in \{A, B\}$, such that
\[
\|(V_A \otimes V_B)|\psi\rangle_\mathrm{AB} - |\Phi_{00}\rangle_{\mathrm{AB}}^{\otimes t}\|_{\mathrm{AUX}}^2 = O(\sqrt{\varepsilon}),
\]
and on expectation over $W \in \{X, Z\}$,
\[
\mathbb{E}_{a \in \{0,1\}} \|(W(a) - V_A^t(\sigma_W(a) \otimes I)V_A) \otimes I_B)|\psi\rangle\|^2 = O(\sqrt{\varepsilon}).
\]
Moreover, if the provers share $|\Phi_{00}\rangle_{\mathrm{AB}}^{\otimes t}$, and measure with the observables $\otimes \sigma_{W_i}$ on question $W$, they pass the test with probability 1.

We remark that PBT allows the verifier to test for EPR pairs and Pauli measurements in such a way that robustness is independent of the number of EPR pairs.

2.4 Local Hamiltonian problem

In quantum mechanics, the evolution of quantum systems is described by Hermitian operators called Hamiltonians. Inspired by nature, where particles that are far apart tend not to interact, an input for the $k$-Local Hamiltonian problem ($k$-LH) consists in $m$ terms $H_1, \ldots, H_m$, where each one describes the evolution of at most $k$ qubits. For some parameters $\alpha, \beta \in \mathbb{R}$, $\alpha < \beta$, the question is if there is a global state such that its energy in respect of $H = \frac{1}{m} \sum_{i \in [m]} H_i$ is at most $\alpha$ or all states have energy at least $\beta$. The area studying the above problem is called quantum Hamiltonian complexity [Osb12, GHLS15], a topic that lies in the intersection of physics, computer science and mathematics.

Kitaev first proved that for some $\beta - \alpha \geq \frac{1}{\text{poly}(n)}$, 5-LH is complete for the class QMA [KSV02]. Kitaev’s result has subsequently been improved, reducing the locality of the Hamiltonians [KR03, KKR06] and restricting their structure [OT10, CMT14, HNN13, Ji16, BJSW16, BC16]. In this work, we are particularly interested in the version of LH where all the terms are the tensor product of $\sigma_X$, $\sigma_Z$ and $\sigma_I$.

Definition 4 (XZ Local Hamiltonian). The XZ $k$-Local Hamiltonian problem, for $k \in \mathbb{Z}^+$ and parameters $\alpha, \beta \in [0, 1]$ with $\alpha < \beta$, is the following promise problem. Let $n$ be the number of qubits of a quantum system. The input is a sequence of $m(n)$ values $\gamma_1, \ldots, \gamma_{m(n)} \in [-1, 1]$ and $m(n)$ Hamiltonians $H_1, \ldots, H_{m(n)}$ where $m$ is a polynomial in $n$, and for each $i \in [m(n)]$, $H_i$ is of the form $\otimes_{j \in [n]} \sigma_{W_j} \in \{\sigma_X, \sigma_Z, \sigma_I\}^\otimes n$ with $|\{j \in [n] \text{ and } \sigma_{W_j} \neq \sigma_I\}| \leq k$. For $H \overset{\text{def}}{=} \frac{1}{m(n)} \sum_{i=1}^{m(n)} \gamma_i H_i$, one of the following two conditions hold.

Yes. There exists a state $|\psi\rangle \in \mathbb{C}^{2^n}$ such that $\langle \psi | H | \psi \rangle \leq \alpha(n)$

No. For all states $|\psi\rangle \in \mathbb{C}^{2^n}$ it holds that $\langle \psi | H | \psi \rangle \geq \beta(n)$.

Whenever the value of $n$ is clear from the context, we call $\alpha(n)$, $\beta(n)$ and $m(n)$ by $\alpha$, $\beta$ and $m$. The XZ $k$-LH problem was proved QMA-complete by Ji [Ji16], for $k = 5$, and Cubitt and Montanaro [CM14], for $k = 2$.

Lemma 5 (Lemma 22 of [Ji16], Lemma 22 of [CM14]). There exist $\alpha, \beta \in \mathbb{R}$, such that $\beta - \alpha \geq \frac{1}{\text{poly}(n)}$ such that XZ $k$-Local Hamiltonian is QMA-complete, for some constant $k$. 

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It is an open question if $k$-LH is QMA-complete for $\beta - \alpha = O(1)$ while maintaining $k$ constant [AAV13]. However, it is possible to achieve this gap at the cost of increasing the locality of the Hamiltonian [NV17].

**Lemma 6** (Lemma 26 of [NV17]). Let $H$ be an $n$-qubit Hamiltonian with minimum energy $\lambda_0(H) \geq 0$ and such that $\|H\| \leq 1$. Let $\alpha, \beta \geq \frac{1}{\text{poly}(n)}$ and $\alpha < \beta$ for all $n$. Let $H'$ be the following Hamiltonian on $(\beta - \alpha)^{-1}n$ qubits

$$H' = \sigma_1^{\otimes na} - (\sigma_1^{\otimes n} - (H - a^{-1}\sigma_1^{\otimes n}))^{\otimes a},$$

where $a = (\beta - \alpha)^{-1}$.

It follows that if $\lambda_0(H) \leq \alpha$ then $\lambda_0(H') \leq \frac{1}{2}$, while if $\lambda_0(H) \geq \beta$ then $\lambda_0(H') \geq 1$. Moreover if $H$ is a XZ Local Hamiltonian, then so is $H'$.

Finally, we define now non-local games for Local Hamiltonian problems.

**Definition 7** (Non-local games for Hamiltonians). A non-local game for the Local Hamiltonian problem consists in a reduction from a Hamiltonian $H$ acting on $n$ qubits to a non-local game $G(H)$ where a verifier plays against $r$ provers, and for some parameters $\alpha, \beta, c, s \in [0, 1]$, for $\alpha < \beta$ and $c > s$, the following holds.

**Completeness.** If $\lambda_0(H) \leq \alpha$, then $\omega^*(G(H)) \geq c$

**Soundness.** If $\lambda_0(H) \geq \beta$, then $\omega^*(G(H)) \leq s$.

### 3 Non-local game for Local Hamiltonian

In this section, we define our non-local game for Local Hamiltonian problem. We start with a XZ Hamiltonian $H = \frac{1}{m} \sum_{i=1}^{m} \gamma_i H_i$ acting on $n$ qubits and $\alpha, \beta \in (0, 1)$ with $\alpha < \beta$. We propose then the Hamiltonian Test $G(H)$, a non-local game based on $H$, whose maximum acceptance probability upper and lower bounds are tightly related to $\lambda_0(H)$. Based on $G(H)$, we show how to construct another non-local game $\tilde{G}(H)$ such that there exists some universal constant $\Delta > 0$ such that if $\lambda_0(H) \leq \alpha$, then $\omega^*(\tilde{G}(H)) \geq \frac{1}{2} + \Delta$, whereas if $\lambda_0(H) \geq \beta$, then $\omega^*(\tilde{G}(H)) \leq \frac{1}{2} - \Delta$. The techniques used to devise $G(H)$ and $\tilde{G}(H)$ are based on Ref. [Ji16] [NV17].

We describe now the Hamiltonian Test, which is composed by the Pauli Braiding Test (PBT) from Section 2.3 and the Energy Test (ET), which allows the verifier estimate $\lambda_0(H)$. The provers are expected to share $t$ EPR pairs and the first prover holds a copy of the groundstate of $H$. In ET, the verifier picks $l \in [m], W \in \{X, Z\}^l$ and $e \in \{0, 1\}^l$, and chooses $T_1, ..., T_n \in [i]$ such that $W(e) T_i$ matches the $i$-th Pauli observable of $H_i$. By setting $t = O(n \log n)$, it is possible to choose such positions for a random $W(e)$ with overwhelming probability. The verifier sends $T_1, ..., T_n$ to the first prover, who is supposed to teleport the groundstate of $H$ through the EPR pairs in these positions. As in PBT, the verifier sends $W(e)$ to the second prover, who is supposed to measure his EPR halves with the corresponding observables. The values of $T_1, ..., T_n$ were chosen in a way that the first prover teleports the groundstate of $H$ in the exact positions of the measurement according to $H_i$. With the outcomes of the teleportation measurements, the verifier can correct the output of the measurement of the second prover and estimate $\lambda_0(H)$. The full description of the game is presented in fig.

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The verifier performs each of the following steps with probability $1 - p$ and $p$, respectively:

(A) Pauli Braiding Test

(B) Energy Test

1. The verifier picks $W \in \{X, Z\}^l$, $e \in \{0, 1\}^l$ and $l \in [m]$
2. The verifier picks positions $T_1, \ldots, T_n$ such that $H_l = \bigotimes \sigma_{W(e)T_i}$
3. The verifier sends $T_1, \ldots, T_n$ to the first prover and $W(e)$ to the second prover.
4. The first prover answers with $a, b \in \{0, 1\}^n$ and the second prover with $c \in \{+1, -1\}^l$.
5. Let $d \in \{-1, +1\}^n$ such that $d_i = (-1)^a_i c_{T_i}$ if $W_{T_i} = X$ and $d_i = (-1)^b_i c_{T_i}$ if $W_{T_i} = Z$.
6. If $\prod_{i \in [n]} d_i \neq \text{sign}(\gamma_l)$, the verifier accepts.
7. Otherwise, the verifier rejects with probability $|\gamma_l|$.

Figure 3: Hamiltonian Test $G(H)$ for a XZ Hamiltonian $H$.

We show now that if the provers follow the honest strategy, then the acceptance probability is

$$\omega_h(H) \overset{\text{def}}{=} 1 - p \left( \frac{1}{2m} \sum_{l \in [m]} |\gamma_l| - \frac{1}{2} \lambda_0(H) \right).$$

**Lemma 8.** Let $H = \sum_{l \in [m]} \gamma_l H_l$ be a XZ Hamiltonian and let $G(H)$ be the Hamiltonian-self test for $H$, described in Figure 3. If the provers use the honest strategy in PBT, the maximum acceptance probability in $G(H)$ is $\omega_h(H)$. Moreover, this probability is achieved if the first prover behaves honestly in ET.

**Proof.** Since PBT and ET are indistinguishable to the second prover, he also follows the honest strategy in ET and the acceptance probability in $G(H)$ depends uniquely in the strategy of the first prover in ET.

Let $a, b \in \{0, 1\}^n$ be the answers of the first prover in ET and $\tau$ be the reduced state held by the second prover on the positions $T_1, \ldots, T_n$ of his EPR halves, after the teleportation.

For a fixed $H_l$, the verifier rejects with probability

$$\frac{|\gamma_l| + \gamma_l \mathbb{E} [\prod_{i \in [n]} d_i]}{2}. \quad (1)$$

We notice that measuring a qubit $|\phi\rangle$ in the Z-basis with outcome $f \in \{\pm 1\}$ is equivalent of considering the outcome $(-1)^f f$ when measuring $X^f Z^b |\phi\rangle$ in the same basis. An analog argument follows also for the X-basis. Therefore, by fixing the answers of the first prover, instead of considering that the second prover measured $\tau$ in respect of $H_l$ with outcome $c$, we consider that he measured $\rho = Z^b X^a \tau X^a Z^b$ with respect to $H_l$ with outcome $d$. In this case, by taking $\prod_{i \in [n]} d_i$ as the outcome of the measurement of $H_l$ on $\rho$, and averaging over all $l \in [m]$, it follows from eq. (1) that the verifier rejects in ET with probability

$$\frac{1}{m} \sum_{l \in [m]} \frac{|\gamma_l| + \gamma_l \text{Tr}(\rho H_l)}{2} = \frac{1}{2m} \sum_{l \in [m]} |\gamma_l| + \frac{1}{2} \text{Tr}(\rho H),$$

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and this value is minimized when \( \rho \) is the groundstate of \( H \). In this case the overall acceptance probability in \( G(H) \) is at most

\[
1 - p \left( \frac{1}{2m} \sum_{i \in [m]} |\gamma_i| - \frac{1}{2} \lambda_0(H) \right) = \omega_h(H).
\]

Finally, this acceptance probability is achieved if the first prover teleports the groundstate \( |\psi\rangle \) of \( H \) and report the honest outcomes from the teleportation, since \( \tau = X^aZ^b|\psi\rangle\langle\psi|Z^bX^a \) and \( \rho = |\psi\rangle\langle\psi| \). \( \Box \)

We show now that for every \( \eta > 0 \), if we choose the probability of running ET \( p = O(\sqrt{\eta}) \), then \( \omega^*(G(H)) \leq \omega_h(H) + \eta \). To prove this, we use the self-testing of PBT to certify the measurements of the second prover in ET. In this way, we can bound the acceptance probability in \( G(H) \) with Lemma 8.

**Lemma 9.** Let \( H \) and \( G(H) \) be defined as Lemma 8. For every \( \eta > 0 \), there is some value of \( p = O(\sqrt{\eta}) \) such that \( \omega^*(G(H)) \leq \omega_h(H) + \eta \).

**Proof.** Let \( S \) be the strategy of the provers, which results in acceptance probabilities \( 1 - \epsilon \) in PBT and \( 1 - \frac{1}{2m} \sum_{i \in [m]} |\gamma_i| - \frac{1}{2} \lambda_0(H) + \delta \) in ET, for some \( \epsilon \) and \( \delta \).

By Lemma 3 their strategy in PBT is \( O(\sqrt{\epsilon}) \)-close to the honest strategy, up to the local isometries \( V_A \) and \( V_B \). Let \( S_h \) be the strategy where the provers follow the honest strategy in PBT and, for ET, the first prover performs the same operations of \( S \), but considering the isometry \( V_A \) from Theorem 3. Since the measurements performed by the provers in \( S \) and \( S_h \) are \( O(\sqrt{\epsilon}) \)-close to each other, considering the isometries, the distributions of the corresponding transcripts have statistical distance at most \( O(\sqrt{\epsilon}) \). Therefore, the provers following strategy \( S_h \) are accepted in ET with probability at least

\[
1 - \frac{1}{2m} \sum_{i \in [m]} |\gamma_i| - \frac{1}{2} \lambda_0(H) + \delta - O(\sqrt{\epsilon}).
\]

Since in \( S_h \) the provers perform the honest strategy in PBT, it follows from Lemma 8 that

\[
1 - \frac{1}{2m} \sum_{i \in [m]} |\gamma_i| - \frac{1}{2} \lambda_0(H) + \delta - O(\sqrt{\epsilon}) \leq 1 - \frac{1}{2m} \sum_{i \in [m]} |\gamma_i| - \frac{1}{2} \lambda_0(H),
\]

which implies that \( \delta \leq C\sqrt{\epsilon} \), for some constant \( C \).

The original strategy \( S \) leads to acceptance probability at most

\[
(1 - p)(1 - \epsilon) + p \left( 1 - \frac{1}{2m} \sum_{i \in [m]} |\gamma_i| - \frac{\lambda_0(H)}{2} + C\sqrt{\epsilon} \right) = \omega_h(H) - (1 - p)\epsilon + pC\sqrt{\epsilon}.
\]

For any \( \eta \), we can pick \( p = \min\left\{ \frac{\sqrt{\epsilon}}{D}, 1 \right\} \), for \( D \geq 2C \), and it follows that

\[
pC\sqrt{\epsilon} - (1 - p)\epsilon \leq \frac{2C}{D} \sqrt{\epsilon} - \epsilon \leq \sqrt{\epsilon} - \epsilon \leq \eta
\]

and therefore the maximum acceptance probability is at most \( \omega_h(H) + \eta \). \( \Box \)

Based on Lemmas 8 and 9 we propose a game \( \tilde{G}(H) \) such that for a universal constant \( \Delta \), we can choose a value of \( p \) such that \( \omega^*(G(H)) \) is at least \( \frac{1}{2} + \Delta \) or at most \( \frac{1}{2} + \Delta \), depending if \( \lambda_0(H) \leq \alpha \) or \( \lambda_0(H) \geq \beta \), respectively.
**Theorem 10.** There exists a universal constant $\Delta$ such that the following holds. Let $H = \sum_{l \in m} \gamma_l H_l$ be XZ $k$-Local Hamiltonian acting on $n$ qubits with parameters $\alpha, \beta \in (0, 1)$, for $\beta > \alpha$. There exists a classical verifier one-round two-prover non-local game such that

- if $\lambda_0(H) \leq \alpha$, then the verifier accepts with probability at least $\frac{1}{2} + \Delta$; and
- if $\lambda_0(H) \geq \beta$, then the verifier accepts with probability at most $\frac{1}{2} - \Delta$.

Moreover, each message is $\tilde{O}(n(\beta - \alpha)^{-1})$-bit long.

**Proof.** Lemma 6 states that from $H$ we can construct a Hamiltonian $H'$ such that

$$\lambda_0(H) \leq \alpha \Rightarrow \lambda_0(H') \leq \frac{1}{2} \quad \text{and} \quad \lambda_0(H) \geq \beta \Rightarrow \lambda_0(H') \geq 1,$$

and $H' = \sum_{l \in m'} \gamma'_l H'_l$ is an instance of XZ Local Hamiltonian problem.

We now bound the maximum acceptance probability of the Hamiltonian Test on $H'$, relating it to the groundstate energy of $H$. From Lemma 8 it follows that

$$\lambda_0(H) \leq \alpha \Rightarrow \omega^*(G(H')) \geq 1 - p \left( \frac{1}{2m} \sum_{l \in [m]} |\gamma'_l| - \frac{1}{4} \right) \overset{\text{def}}{=} c,$$

while from Lemma 9 for any $\eta > 0$ and some $p \leq C\sqrt{\eta}$, we have that

$$\lambda_0(H) \geq \beta \Rightarrow \omega^*(G(H')) \leq 1 - p \left( \frac{1}{2m} \sum_{l \in [m]} |\gamma'_l| - \frac{1}{2} \right) + \eta = c - \frac{C\sqrt{\eta}}{4} + \eta.$$

By choosing $\eta$ to be a constant such that $\eta' \overset{\text{def}}{=} \frac{C\sqrt{\eta}}{4} - \eta > 0$, it follows that

$$\lambda_0(H) \leq \alpha \Rightarrow \omega^*(G(H')) \geq c \quad \text{and} \quad \lambda_0(H) \geq \beta \Rightarrow \omega^*(G(H')) \leq c - \eta'.$$

We describe now the game $\tilde{G}(H)$ that achieves the completeness and soundness properties stated in the theorem. In this game, the verifier accepts with probability $\frac{1}{2} - \frac{2c - \eta'}{4}$, rejects with probability $\frac{2c - \eta'}{4}$ or play $G(H')$ with probability $\frac{1}{2}$. Within this new game, if $\lambda_0(H) \leq \alpha$ then $\omega^*(\tilde{G}(H')) \leq \frac{1}{2} + \frac{\eta'}{4}$, whereas when $\lambda_0(H) \geq \beta$, we have that $\omega^*(\tilde{G}(H')) \leq \frac{1}{2} - \frac{\eta'}{4}$. 

It follows from Lemma 8 that XZ $k$-Local Hamiltonian is QMA-complete, and in this case, Theorem 10 implies directly a one-round two-prover game for QMA, where the provers perform polynomial time quantum computation on copies of the QMA witness.

**Corollary 11.** There exists an one-round classical verifier two-prover game for QMA, where honest provers perform quantum polynomial time computation on copies of the QMA witness. The verifier and provers send $O(\text{poly}(n))$-bit messages.
4 Relativistic delegation of quantum computation

In this section we present our protocol for relativistic delegation of quantum computation. We start by showing that every one-round protocol with two non-communicating provers can be converted into a relativistic one, using standard techniques in relativistic cryptography. Finally we show how to define a delegation protocol based in a non-local game for the Local Hamiltonian problem.

In a first attempt for making the relativistic protocol, the provers and the verifier could be placed in such a way that the time it takes for the provers to communicate is longer than the time of the honest protocol. In this case the verifier can abort whenever the provers’ message take too long to arrive and this protocol is secure as long as it is possible to rely on the position of the provers during the protocol. However, if the provers could move in order to receive the message earlier, then security is compromised. Using techniques from relativistic cryptography, we show how to prevent this type of attacks by placing two trusted agents at the expected position of the provers.

Lemma 12. Every classical one-round protocol with two non-communicating provers can be converted into a protocol where the provers are allowed to communicate at most as fast as speed of light.

Proof. Let us assume that the provers can be forced to stay at an arbitrary position in space. We use the unit system, where the speed of light is $c = 1$, in order to simplify the equations. The provers and the verifier are placed in a line, with the verifier at position 0, the first prover at position $-t_0$ and the second prover at position $t_0$. The value $t_0$ is chosen to be much larger than an upper-bound of the time complexity of the provers in the honest protocol, denoted by $t_1$.

The message from the verifier to the provers arrive at time $t_0$. The expected time for the provers to perform the computation and answer back is $t_1 + t_0$, whereas the time it takes for the provers to send a message to each other is $2t_0 \gg t_1 + t_0$. Therefore, the verifier aborts whenever the provers’ answers arrive after time $3t_0$ since the security of the protocol is compromised. We depict this protocol in fig. 1.

The previous argument works if we can rely on the position of the provers, but in some settings we require the protocol to be robust against malicious provers that may move in order to receive the verifier’s messages earlier, being able to collude and break the security of the protocol. This attack is depicted in fig. 4a. We can prevent this type of attacks by adding two trusted agents at the expected position of the provers. The verifier sends the message to the agents through a secure channel and the agents transmit the information to the provers. The provers perform their computation and then report their answers to the agents, who transmit the messages to the verifier. The secure channel can be implemented with the verifier and the agents sharing one-time pad keys, and then messages can be exchanged in a perfectly secure way. This protocol is depicted in fig. 4b.

We finally describe the relativistic protocol for delegation of quantum computation.

Corollary 13. There exists a universal constant $\Delta$ such that the following holds. There exists a protocol for a promise problem $A = (A_{\text{yes}}, A_{\text{no}})$ in BQP where a classical verifier exchanges one-round of communication with two entangled provers such that if $x \in A_{\text{yes}}$ then the verifier accepts with probability at least $\frac{1}{2} + \Delta$, whereas if $x \in A_{\text{no}}$, then the verifier accepts with probability at most $\frac{1}{2} - \Delta$. Moreover, the honest provers only need polynomial-time quantum computation and the provers are allowed to communicate respecting NSS.

Proof. From the quantum circuit $Q_x$ for deciding $A$, the circuit-to-hamiltonian construction (see Appendix B) allows us to create an instance $H_{Q_x}$ of XZ 5-Local Hamiltonian problem, such that if $x \in A_{\text{yes}}$, then $\lambda_0(H_{Q_x}) \leq \exp(-n)$, whereas if $x \in A_{\text{no}}$, then $\lambda_0(H_{Q_x}) \geq \frac{1}{\text{poly}(n)}$. 

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Using the non-local game of Theorem 10 with $H_{Q_x}$, we have a delegation protocol for $Q_x$ with the desired completeness/soundness gap. The history state of $H_{Q_x}$ can be constructed efficiently by the provers, as well as the performed measurements. Finally, this protocol can be made relativistic using the construction of Lemma 12.

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A Magic Square game

The Magic Square or Mermin-Peres game [Mer90, Per90], is a two-prover non-local game where one of the provers is asked a row \( r \in \{1, 2, 3\} \) and the second prover is asked with a column \( c \in \{1, 2, 3\} \). The first and second prover answer with \( a_1, a_2 \in \{\pm 1\} \) and \( b_1, b_2 \in \{\pm 1\} \), respectively. By setting \( a_3 = a_1 \oplus a_2 \) and \( b_3 = b_1 \oplus b_2 \), the provers win the game if \( a_c = b_r \).

If the provers follow a classical strategy, their maximum winning probability in this game is \( \frac{8}{9} \), while we describe now a quantum strategy that makes them win with probability 1. The provers share two EPR pairs and, on question \( r \) (resp. \( c \)), the prover performs the measurements indicated in the first two columns (resp. rows) of row \( r \) (resp. column \( c \)) of the following table

| \( IZ \) | \( ZI \) | \( ZZ \) |
| \( XI \) | \( IX \) | \( XX \) |
| \( XZ \) | \( ZX \) | \( YY \) |
and answer with the outcomes of the measurements. The values $a_3$ and $b_3$ should correspond to the measurement of the EPR pairs according to the third column and row, respectively.

The self-testing theorem proved by Wu, Bancal and Scarani [WBMS16] states that if the provers win the Magic Square game with probability close to $1$, they share two EPR pairs and the measurements performed are close to the honest Pauli measurements, up to local isometries.

**Lemma 14.** Suppose a strategy for the provers, using state $|\psi\rangle$ and observables $W$, succeeds with probability at least $1 - \varepsilon$ in the Magic Square game. Then there exist isometries $V_D : \mathcal{H}_D \rightarrow (\mathbb{C}^2 \otimes \mathbb{C}^2)_{D'} \otimes \mathcal{H}_{D'}$ for $D \in \{A, B\}$ and a state $|\text{AUX}\rangle_{AB} \in \mathcal{H}_A \otimes \mathcal{H}_B$ such that

$$\| (V_A \otimes V_B) |\psi\rangle_{AB} - |\Phi_{00}\rangle_{A'B'} |\text{AUX}\rangle_{AB} \|^2 = O(\sqrt{\varepsilon}),$$

and for $W \in \{I, X, Z\}$,

$$\| (W - V_A^\dagger \sigma_W V_A) \otimes I_B |\psi\rangle \|^2 = O(\sqrt{\varepsilon}).$$

**B Circuit-to-hamiltonian construction**

Feynman [Fey86], in his pioneering work where he suggests the use of the quantum structure of matter as a computational resource, has shown the construction of a time-independent Hamiltonian that is able to simulate the evolution of a quantum circuit. This construction is now known as the circuit-to-hamiltonian construction and it is a central point in proving QMA-completeness of Local Hamiltonian problems [KSV02, BT14] and the universality of adiabatic quantum computation [AvDK+04]. More related to this work, it has also been used in the context of delegation of quantum computation [FH15, NV17]. We describe now such construction.

Let $Q = U_T \ldots U_1$ be a quantum circuit on $n$ qubits, decomposed on $T$ 2-qubit gates. Let us assume for simplicity that the circuit $Q$ is applied on the initial state $|\psi\rangle$. The Hamiltonian $H_Q$ acts on $n$ working qubits, as the circuit $Q$, and an extra clock-register of $c$ qubits to count the operations steps from 0 to $T$. The number of bits in the clock register depends on the representation of the time steps: if we represent it in binary, we take $c = \log T$; for some applications, it is better to represent it in unary, where time $t$ will be encoded as $T - t$ “0”s followed by $t$ “1”s. For the remainder of the section we abstract the representation of the clock register and we write $|t\rangle_{\text{clock}}$ for the correct encoding of time $t$. We will construct $H_Q$ such that its groundstate is

$$\sum_{t=0}^{T} |t\rangle_{\text{clock}} \otimes U_t \ldots U_1 |\psi\rangle,$$

which is known as the history state of $Q$. As noticed by Fitzsimons and Hajdušek [FH15], the history state of $H_Q$ can be computed in quantum polynomial time if the initial state $|\psi\rangle$ is provided.

**Lemma 15.** Provided the initial state $|\psi\rangle$ of $Q$, the history state $\sum_{t=0}^{T} |t\rangle_{\text{clock}} \otimes U_t \ldots U_1 |\psi\rangle$ can be prepared in time polynomial in $T$.

The Hamiltonian $H_Q$ is decomposed in three parts: the initialization terms, the propagation terms and clock terms. As we see later, output terms are also needed for some applications.

The initialization terms check if the groundstate is a computation that start in a valid initial state $|\psi\rangle$. For instance, if $|\psi\rangle = |0\rangle^\otimes n$, then for each $i \in [n]$, the following term will be added to $H_Q$

$$|0\rangle \langle 0|_{\text{clock}} \otimes |1\rangle \langle 1|_i.$$
The interpretation of these terms is that they add some “penalty” for states where the computation does not start with a $|0\rangle^\otimes n$.

The propagation terms check if all the intermediate steps $U_0, \ldots, U_T$ are simulated in the Hamiltonian. For each step $t \in [T]$, the following Hamiltonian is added to $H_Q$

$$\frac{1}{2} \left( -|t\rangle\langle t - 1|_{\text{clock}} \otimes U_t - |t\rangle\langle t - 1|_{\text{clock}} \otimes U_t^+ + |t\rangle\langle t|_{\text{clock}} \otimes I + |t - 1\rangle\langle t - 1|_{\text{clock}} \otimes I \right),$$

where the second part of the tensor product acts on the same qubits of $U_t$.

The clock terms are added in order to check if the clock register contains only correct encodings of time. For instance, if time is encoded in unary, the clock terms check if there is no $1$ followed by a $0$ in the clock register. More concretely, in the unary representation, for every $i \in [T]$, the following term is added to $H_Q$

$$|10\rangle\langle 10|_{i,i+1},$$

and it acts on qubits $i$ and $i + 1$ of the clock register.

One can easily see by inspection that the state in eq. (2) is the only state that has energy 0 according the previous terms.

In some applications, we need also to check some properties of the output of $Q$. For instance in delegation protocols, we are interested in the probability that $Q$ outputs $|1\rangle$ (we usually say in this case that $Q$ accepts). In these cases, we want to construct $H_Q$ such that its frustration is related to the acceptance probability of the circuit: if $Q$ outputs $|1\rangle$ with probability at least $c$, then $\lambda_0(H_Q) \leq \alpha$, while if $Q$ outputs $|1\rangle$ with probability at most $s$, then $\lambda_0(H_Q) \geq \beta$. For this task, we add the following term to the $H_Q$ that acts on the clock register and on the output qubit

$$|T\rangle\langle T|_{\text{clock}} \otimes |0\rangle\langle 0|_{\text{output}}.$$

The following theorem was then proved by Kitaev [KSV02].

**Theorem 16** (Sections 14.4.3 and 14.4.4 of [KSV02]). Let $Q$ be a quantum circuit composed by $T$ gates that computes on some initial state $|\phi\rangle$ and then decides to accept or reject. Let $H_Q$ be the 5-Local Hamiltonian created with the circuit-to-hamiltonian with unary clock on $Q$.

**Completeness.** If the acceptance probability is at least $1 - \varepsilon$, then $\lambda_0(H_Q) \leq \frac{\varepsilon}{T + 1}$.

**Soundness.** If the acceptance probability is at most $\varepsilon$, then $\lambda_0(H_Q) \geq c\frac{1 - \sqrt{\varepsilon}}{T^2}$, for some constant $c$.

Ji [Ji16] has proved that Kitaev’s construction can be converted into an XZ Local Hamiltonian, by choosing a suitable gate-set for the circuit $Q$.

**Theorem 17** (Lemma 22 of [Ji16]). Let $Q$ be a quantum circuit composed of gates in the following universal gate-set $\{\text{CNOT}, X, \cos\left(\frac{\pi}{8}\right)X + \sin\left(\frac{\pi}{8}\right)Z\}$. Then $H_Q$ from theorem 16 can be written as a XZ 5-Local Hamiltonian.