The finite element method in the analysis of the stress and strain distribution in polyethylene elements of hip and knee joints endoprostheses

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Abstract. The paper presents the numerical analysis of stress and strain occurring in the most wearable parts of hip and knee joints endoprostheses. The complexity of the processes taking place in both, natural and artificial joints, makes it necessary to conduct the analysis on the 3D model based on already existing mathematical models. Most of the mechanical failures in alloplasty are caused by material fatigue. To cut down the risk of it, we can either increase the fatigue resistance of the material or decrease the load strain. It is extremely important to indicate the areas where damage or premature wear may occur. The Finite Elements Method makes it possible to calculate the stress and strain in particular elements of the tested models. All presented numerical calculations define quality conclusions concerning the influence of some parameters of endoprostheses on the values of stress and strain that are formed in polyethylene parts of endoprostheses of hip and knee joints. The obtained results help to reveal “weak points” in examined models and thus, counteract the subsequent effects resulting from premature wear of endoprosthesis elements. The numerical analysis was performed basing on the finite elements method using Autodesk Simulation Mechanical 2017 software and the ADINA 7.5.1.

Keywords: FEM, endoprosthesis, stress, strain

1 Introduction

The solutions used currently in endoprostheses have been focused on individual choice of the implant to suit the needs of patients. The simplest solution is to use a modular endoprosthesis. Apart from numerous opportunities for individualization, this solution allows for limitation of the extent of future revision joint surgeries.

At the initial stage of the endoprosthesis design, one should use analytical solutions aimed to indicate the areas where damages or premature wear of the components are most likely to occur \([1, 2, 3]\). Most of mechanical damages in hip joint replacement are due to material

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fatigue. All the presented numerical computations allow for drawing valuable conclusions concerning the effect of selected functional parameters on the level of stresses and strains in individual components of the endoprosthesis [4, 5].

The analyses performed in the study revealed that the polyethylene insert represents the weakest component of the endoprosthesis [6, 7]. The presented numerical analysis is the most efficient and clearest method to define distribution of stresses and strains present in components of hip and knee joint endoprostheses.

2 Numerical analysis of the load to the modular hip joint endoprosthesis

The geometrical model of the endoprosthesis was developed using the Autodesk Inventor 2017 software. It represents a modification of real endoprosthesis manufactured by Zimmer [8]. The model is composed of seven basic parts. Each component was designed separately as an object using the ipt standard. Next, individual components were collected to form a system with consideration for specific connections and ties between each other. The endoprosthesis is composed of the stem (distal and proximal) fixed by means of a lock screw, titanium or ceramic head, acetabulum, and the acetabulum housing. Furthermore, the acetabulum housing has holes for self-tapping screws that allow for fixation to the pelvis bone. Figure 1 illustrates a geometrical model of the adopted solution.

Fig. 1. Geometrical model

The materials used for individual parts were chosen based on the literature data [9, 10] and generally adopted solutions used for implantation of endoprostheses:
- stem (distal and proximal) made of Ti6Al4V,
- endoprosthesis head: Ti6Al4V or ZrO₂,
- internal acetabulum: UHMWPE,
- acetabulum housing: Ti6Al4V,
- self-tapping bone screws: Ti6Al4V,
- lock screw: Ti6Al4V.
Properties of the materials used are presented in Tab. 1.
Table 1. Parameters of materials used in simulations.

| Material           | Young's modulus E [MPa] | Coefficient Poisson's ratio (ν) | Yield point, [MPa] | Tensile strength, [MPa] |
|--------------------|-------------------------|---------------------------------|--------------------|------------------------|
| ZrO₂ ceramics      | 200                     | 0.3                             | 300                | 455                    |
| Ti6Al4V            | 1.08 x 10⁵              | 0.36                            | 900                | 900                    |
| UHMWPE             | 1.0 x 10³               | 0.4                             | 20                 | 30                     |

Numerical analysis was performed using the finite element method by means of Autodesk Simulation Mechanical 2017 software. The system of restraints and loads was based on the Będziński's active model, modified and simplified to facilitate simulation. The values of loads were adopted based on the literature data [5, 9, 11]. The model was restrained at all external surfaces of the stem, without the neck, and it simulated stabilization in the bone tissue. The loads with the values of 600 N and 750 N, respectively, were applied to the external surface of the endoprosthesis acetabulum housing. The stabilizing bone screws were also partially restrained, simulating the primary stabilization of the acetabulum housing in the pelvic bone. Fig. 2 illustrates the locations of restraints and surfaces to which the load was applied.

Fig. 2. Locations of adopted restraints and surfaces to which the loads were applied

In the area of tribological interaction of the system, the contact was modelled with the option of the head motion with relation to the acetabulum at adopted coefficient of friction $\mu = 0.2$. However, the presented static analysis does not concern the solutions in this area.
The figures below illustrate the example distributions of stress and strain (cross-section of the analysed system) for the pair of the Ti6Al4V head and UHMWPE acetabulum.

Fig. 3. Distribution of stress in the analysed system at the load of 600 N Endoprosthesis head made of Ti6Al4V alloy

Fig. 4. Distribution of strain in the analysed system at the load of 600 N Endoprosthesis head made of Ti6Al4V alloy

The highest values of stress at the load of 600 N and 750 N were observed in the endoprosthesis head just at the stem collar, reaching the values of 11 MPa for the loading force of 600 N and 14 MPa for the force of 750 N. The location at which maximal values of stress were observed in all cases is illustrated in Fig. 5.
The change in material pairs, consisting in the use of the endoprosthesis head made of ZrO$_2$, did not lead to noticeable differences in the results obtained. Maximal stresses for the analysed cases are illustrated in Fig. 6.

As expected, maximal stresses were observed in the acetabulum made of UHMWPE. However, they reached a very low levels and it can be concluded that they do not have an effect on stability of the system. Maximal strains in the analysed contact pairs are illustrated in Fig. 7.
The analysis performed in the study revealed that the values of stresses and strains in the analysed system at the adopted load similar to that in natural conditions are low. They do not cause formation of locations with accumulated stresses. Polyethylene components are not exposed to substantial strains, which can be a good predictor of a longer life of these components. The findings of the study lead to the conclusion that the adopted geometrical solutions are optimal and allow for long and failure-free use.

3 Stress analysis in the pair sled – flat insert for endoprosthesis

The paper’s objective was to analyze the state of stress occurring on the contact surfaces of metal sleds cooperating with flat polyethylene insert in knee joint endoprosthesis. It is to select the most suitable material which sleds should be made of in order to achieve an optimum stress distribution on the insert’s surface. The most requested parameters of the materials are:
- biotolerance,
- corrosion resistance,
- wear resistance,
- high tenacity values.

The materials for implants should be of low Young’s modulus (as close to the bone elasticity as possible), and low weight density. Titanium and its alloys with niobium [14, 16] present those features. When Young’s modulus value is close to the one of the bone being implanted (tibia or femoral) it very positively influences on bio-compatibility of the alloplastics. The relation between bone and implant is proper as far as stiffness is concerned. It excludes negative bone structure remodeling and stress shielding in the implanted area. When human body is implanted with a material of totally different elasticity than its own natural one, that leads to a significant backstay of the bone – implant structure. If Young’s modulus of the material is too high, like in the case of CoCrMo alloy, it causes decrease of the bone load or its total decay and the implant gets loose, because growths and restructuring the bone may only be conducted in proper load conditions.
It is critical to look for materials with elasticity features as close to the natural bones’ tissue as possible. There are currently being tested new titanium alloys with niobium and zirconium with much lower elasticity modulus of 55-60 GPa. Titanium alloys like Ti6Al4V and new ones like TiNbZr and TiNbZrTa, present very good mechanical features, however lower value of elasticity modulus of the alloys with niobium proves that the future of alloplastics belongs to them [12, 13, 14].

Low value of Young’s modulus in new titanium alloys, may cause decrease of the value of stress generated in polyethylene inserts, and assure more stable implementation of the endoprosthesis in the bone. Besides, the mentioned alloys do not consist of harmful for human body aluminum (Al).

Under all those restrictions, it is extremely difficult to find a proper material to build a knee joint endoprosthesis of. Applying FEM in calculations makes it possible to create a map of stress distribution for each alternative.

The calculations were conducted for partial endoprosthesis with flat polyethylene insert. They assigned reduced and contact stresses occurring in polyethylene inserts cooperating with metal sleds made of various titanium alloys and CoCrMo. Polyethylene UH MWPE is the weakest point of the endoprosthesis, that is why it is important to present the reduced stress distribution in the inserts.

To analyze the achieved results, the following formulas have been accepted: Formula (1) presents physical equations connecting stress and strain tensors’ values in 3D, in isotropic, linear – elastic [17].

\[
\sigma_x = \frac{E}{1 + \nu} \left[ \varepsilon_x + \frac{\nu}{1 - 2\nu} (\varepsilon_x + \varepsilon_y + \varepsilon_z) \right] \tau_{xy} = G\gamma_{xy} \\
\sigma_y = \frac{E}{1 + \nu} \left[ \varepsilon_y + \frac{\nu}{1 - 2\nu} (\varepsilon_x + \varepsilon_y + \varepsilon_z) \right] \tau_{yz} = G\gamma_{yz} \\
\sigma_z = \frac{E}{1 + \nu} \left[ \varepsilon_z + \frac{\nu}{1 - 2\nu} (\varepsilon_x + \varepsilon_y + \varepsilon_z) \right] \tau_{zx} = G\gamma_{zx}
\]  

(1)

Material constans:
- \(E\) – Young’s modulus (elasticity),
- \(G\) – Kirchoff’s modulus (non-dilatational strain),
- \(\nu\) – Poisson’s coefficient when solving the equation, we can define components of the stress as a function of the strain.

### 3.1 Reduces stress defined for polyethylene inserts of knee joint

The durability of endoprosthesis depends on mechanical and tribological features of its weakest element, which is polyethylene insert. Reduces stresses are usually normal substitute stress, compared to material strength during one-axis extension. The reduced stresses depend on all stress tensor’s components, and are presented by formula (2) [17]:

\[
\sigma_{\text{red}} = f(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \tau_{xy}, \tau_{xz}, \tau_{zy})
\]

or

\[
\sigma_{\text{red}} = f(\sigma_{11}, \sigma_{22}, \sigma_{33}),
\]

where:
\[ \sigma_{11}, \sigma_{22}, \sigma_{33} \text{ – main stress} \]
\[ \sigma_{\text{red}} \text{ – Huber-Mises’s reduced stress}. \]

Huber-Mises’s strength hypothesis, presents the formula (3) [7].

\[
\sigma_{\text{red}} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_{11} - \sigma_{22})^2 + (\sigma_{33} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2} 
\]

or

\[
\sigma_{\text{red}} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{zz} - \sigma_{yy})^2 + (\sigma_{xx} - \sigma_{zz})^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)}
\]

### 3.2 Knee joint endoprosthesis

The figure 8 presents the Oxford partial knee joint endoprosthesis by Zimmer Biomet, with spherical polyethylene insert. The femoral part copies the shape of condyles of femur bones. The femoral part is precisely fixed into the bone.

![Knee joint endoprosthesis Oxford by Zimmer Biomet](image)

**Fig. 8.** Knee joint endoprosthesis Oxford by Zimmer Biomet, 1 – metal sled, 2 – polyethylene insert, 3 – tibia part [18]

The materials used in the above endoprostheses are:
- metal alloy – femoral and tibia part (1 and 3),
- polyethylene UHMWPE – the insert (2).

That is the most commonly used pair of materials for knee joint endoprostheses. In some cases the titanium alloy is used as well, due to its high mechanical features and because it is lighter than CoCrMo alloy.

### 3.3 Partial knee joint endoprosthesis Oxford by Zimmer Biomet

When designing a simplified numerical model of endoprosthesis, it is important to draw a geometry of the endoprosthesis as true and similar to the real one knee joint as possible, not only as far as the shape is concerned but also considering the anatomical range of movements.

### 3.4 Criteria taken for numerical calculations

The contact stress occurs between two elements pressed to each other with force. They take place in certain areas and can reach quite high values even at respectively low value clamp.
Theoretical criteria for contact stress according to Hertz’s theory have been applied in the form as follows:
- contacting elements are made of homogeneous isotropic materials and Hooke’s law wise,
- the surfaces are fixed in the contact area of the element with smooth and regularly curved surfaces,
- when subjected to load there are only slight strains in the contact area,
- the contact area is relatively small when compared with the surfaces of the contacting elements,
- on the contact area there only occur normal strains.

3.5 Numerical model of knee joint endoprostheses

Numerical model is a simplified version of the original endoprosthesis, though the sleds’ geometry has been maintained. That enables us to keep the general shape of endoprosthesis and to quite closely imagine the strain distribution on the insert’s surface. The figure 9 presents the simplified sled model.

![Simplified sled model used for calculations](image)

Fig. 9. Simplified sled model used for calculations

The main purpose of the calculations was to define stress distribution on the surface of the polyethylene insert and right underneath it, where the sleds cooperate. There were three heights of polyethylene inserts analyzed: 8, 13 and 22 mm, and two sleds of cross section radiuses of 17 and 27 mm, respectively. The analyzed sleds were made of CoCrMo, Ti6Al4V, Ti13Nb13Zr, Ti12Mo6Zr2Fe, Ti NbZrTa. Figure 10 presents sleds’ geometry of the analyzed knee joint endoprosthesis.

Sleds’ geometric value, accepted as a specific parameter, defined cross-section radius of a sled. Constant geometric diameters are:
- Sled of geometry: $R = 28$ mm; $r_1 = 15$ mm; $r_2 = 27$ mm; $L = 46$ mm; $b = 17.5$ mm,
- Sled of geometry: $R = 26$ mm; $r_1 = 16$ mm; $r_2 = 17$ mm; $L = 45$ mm; $b = 16$ mm.

The most important dimension in the analysis is the height of the insert defined as $G$: $G_1 = 8$ mm, $G_2 = 13$ mm, $G_3 = 22$ mm.
Fig. 10. Knee joint endoprosthesis sled’s geometry – side and front view

There were conducted 30 numerical analysis for three various thicknesses of polyethylene inserts cooperating with two geometrically different sleds made of five different alloys. Each pair was subjected to load $F = 1500$ N. Simulations of the cases were conducted with the following, accepted physical features of the materials presented in the table 2.

Table 2. Mechanical features and weight density of the materials used for endoprostheses [15]

| Material         | Young’s modulus E [Gpa] | Poisson’s coefficient $\nu$ | Weight density $\rho$ [kg/m$^3$] |
|------------------|-------------------------|-----------------------------|----------------------------------|
| CoCrMo           | 210                     | 0.29                        | 8300                             |
| Ti6Al4V          | 110                     | 0.3                         | 4500                             |
| Ti13Nb13Zr       | 80                      | 0.3                         | 4510                             |
| Ti12Mo6Zr2Fe     | 73                      | 0.3                         | 4510                             |
| Ti NbZrTa        | 53                      | 0.3                         | 4490                             |
| UHMWPE           | 0.01                    | 0.4                         | 960                              |

3.6 The results of numerical analysis conducted with the use of Finite Elements Method and ADINA System 8.6.

The calculations prove that stress in endoprosthesis is concentrated in the polyethylene insert, right underneath the contact area of both elements, and highest stress is located right underneath the insert’s surface. Some examples of the calculations present figures 11, 12 and 13.

Fig. 11. Contact stress distribution occurring in a spherical polyethylene insert. Spherical insert 8 mm thick cooperates with a sled of radius 17 mm. Load 1500 N
Fig. 12. Contact stress distribution occurring in spherical polyethylene insert. Spherical insert 13 mm thick, cooperates with sled of radius 27 mm. Load 1500 N.

Fig. 13. Strain distribution occurring in spherical polyethylene insert. Spherical insert 8 mm thick, cooperates with sled of radius 27 mm. Load 1500 N.

3.7 Remarks on the results of the calculations

The lowest reduced stress was achieved for the model where the sled’s cross-section radius is 27 mm, and the sled is made of TiNbZrTa alloy and valued 8.88 MPa. The highest stress occurred in the model where the sled was made of CoCrMo alloy, and valued 32, 25 MPa, and the sled’s cross section radius was smallest and valued 17 mm. Figure 14 presents the influence of the cross section radius of the sled, thickness of the insert and kind of material which the sled is made of, on the value of the stress generated in the flat polyethylene insert.
The influence of the sled’s cross section radius and insert’s thickness on the value of the stress generated in the flat UHMWPE insert

4 Conclusions

1. The conducted numerical calculations and analysis undaubtedly prove that the future of knee joint alloplastics belongs to a group of new materials including titanium alloys, which when appropriately selected and combined as far as mechanical features are concerned (low Young’s modulus value), may significantly decrease the value of stress generated in polyethylene elements of endoprostheses.
2. Another important element influencing durability of endoprostheses is optimizing of the geometry of the implants both in the friction node and fixing area in the bone.
3. The analysis performed in the study revealed that the values of stresses and strains in the analysed system at the adopted load similar to that in natural conditions are low. They do not cause formation of locations with accumulated stresses. The analysis performed in the study revealed that the values of stresses and strains in the analysed system at the adopted load similar to that in natural conditions are low. They do not cause formation of locations with accumulated stresses.

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