Topologically Protected Magnetic Helix for All-Spin-Based Applications

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The recent years have witnessed an emergence of the field of all-spin-based devices without any flow of charge. An ultimate goal of this scientific direction is the realization of full spectrum of spin-based networks like in modern electronics. The concepts of energy storing elements, indispensable for those networks, are so far lacking. Analyzing analytically the size dependent properties of magnetic chains that are coupled via either exchange or long-range dipolar or Ruderman-Kittel-Kasuya-Yosida interactions, we discover a particularly simple law: magnetic configurations corresponding to helices with integer number of twists, that are commensurate with the chain’s length, are energetically stable. This finding, supported by simulations and an experimentally benchmarked model, agrees with the study [J. Appl. Phys. 111, 07E116 (2012)] showing that boundaries can topologically stabilize structures that are not stable otherwise. On that basis an energy storing element is proposed.

A challenge for the solid state physics at the nanoscale is to develop energetically efficient information and communication technologies. While spintronics, spin caloritronics and magnonics focus on the interaction of spins with charges, heat currents, or external fields, the most recent strategy is to create devices that do not require the spin to charge conversion, but use the spin degree of freedom only to store and process information. This idea is very appealing, because of the variety of degrees of freedom only to store and process information. The proposed device can be realized in structured multilayers and superlattices, colloids, and molecular systems. In order to transmit and process information without electric currents or external fields intrinsic interactions are needed. The most ubiquitous in different kinds of magnetic systems interactions are the direct and indirect (e.g. Ruderman-Kittel-Kasuya-Yosida (RKKY)) exchange, and the dipolar coupling. Short, antiferromagnetically coupled via RKKY or dipolar interaction chains (spin leads), have been already utilized to transmit the information of the state of an “input” ferromagnetic dot to the ”gate” dot. It has been also realized that detailed structure of those antiferromagnetic (AF) states depend on the chain length. For such nanosized systems, however, there is up to now no clear understanding how the competition between the finite length of those chains and the order of interactions manifest itself in time-dependent magnetic order. Especially intriguing is the aspect elaborated in, showing that boundary conditions might induce topologically protected excitations. Clearly, this lack of knowledge hampers further development of the all-spin-based information technology.

In this paper, studying theoretically the size-dependence of magnetic order in chains with exchange, RKKY, and dipolar interactions, we discover a particularly simple law: magnetic states corresponding to the modulated helices with integer number of twists, that are commensurate with the chain’s length, are energetically stable and correspond to local energy minima separated by energy maxima. The AF state, used so far for the spin leads, is a limiting, low energy case of the double helix with an infinite periodicity. With increasing number of twists the energy of the double helix increases but, once achieved, remains stable at small temperatures. The higher energy levels can be reached by rotating one of the chain ends like the winding up of spring driven clocks. In contrast to the clocks, a magnetic system may be “clicked into place” for integer number of twists.

We confirm these results using Monte-Carlo (MC) and Landau-Lifshitz-Gilbert Spin Dynamics (SD) simulations and a magneto-mechanical model. We also demonstrate that a chain can be forced towards one of the topologically stable states by rotation of end-spins. By further rotation, the helix can be winded up towards higher energy levels and pinned in this stable state to store energy. Unpinning this spin at a later time leads to release of the stored energy, which can be used to perform work or transfer information. The proposed device can be realized in structured multilayers, chains of nanoparticles or molecules, nanoarrays, and atomic chains.

The starting point of our calculations is a linear (along x-axis) chain of N dipoles coupled by either exchange or RKKY or dipolar interaction and possessing a uniaxial (easy axis or easy plane) anisotropy K arising due to the magnetocrystalline anisotropy, particle shape, or higher order multipolar contributions. Here we describe results for the most complicated dipolar coupling. However, they can be straightforwardly adopted to the RKKY or exchange interactions.

\[ H = D \sum_{ij} (S_i \cdot S_j - 3(S_i \cdot r_{ij})(S_j \cdot r_{ij})) - K \sum_i (S_i \cdot r_{ij})^2 \]  

(1)

where \( S_i \) is a three-dimensional unit vector and \( r_{ij} \) denotes the distance-vector between moments \( i \) and \( j \). All
K = 2 respectively, while the cyan line shows
E of modulated helices in a dipolar chain consisting of ten
δ
modulated helices with

FIG. 1: (a,b) One- and two-dimensional representation of the
energy of modulated helices in a dipolar chain consisting of ten
levels. The thick lines in (c) show the envelope lines of double-
sublattices corresponding to the magnetic moments (arrows) in
two sublattices. The dashed parabola in (a) corresponds to
the energy of the harmonic spiral S(x) = S(sin δx, 0, cos δx);
the black, red, and blue curves correspond to E(δ) of mod-
ulated helices with K = 0 and q = 0, q = π/N, and q = 2π/N
respectively, while the cyan line shows E(δ) for q = π/N and
K = 2.5D. The energy scale in (b) goes linearly from −0.8D
(blue) to +0.8D (yellow); (d) Three-dimensional representa-
tion of an intermediate helix state found in the SD simulations
for N=81 (video SI2) for K \approx −1.23\mu_0 M_0^2 at T < 1K; (e)
End-configuration of SD-simulations for K = 0 and identical
to (d) starting configuration.

These metastable states look like.

To answer this question analytically we utilize the
method used for construction of spin-density waves. The
magnetic structure is regarded as a superposition of spirals in the 2D Brillouin zone under the requirement of the constant magnetic moment at all sites. The
energy of the constructed structure is then analyzed. The
spin-spirals can be described via vectors δ and q in the
form (i) S(r) = S[\sin(\delta r), \cos(\delta r), \sin(\delta r)\sin(\delta r), \cos(\delta r)]
or (ii) S(r) = S[\sin(\delta r), \cos(\delta r)\sin(\delta r), \cos(\delta r)\cos(\delta r)].
For q = 0 and r \parallel Ox, for example, (i) gives a spin-
spiral S(x) = S[\sin(\delta x), \cos(\delta x)] in the xz-plane. The
energy of this spin configuration as a function of δ is plotted as dashed line in Fig. 1(a). It is seen that the
energy is minimal \(E_{\text{min1}}\) at \(\delta = \pi\); i.e., corresponds
to the antiferromagnetic (AF) alignment of neighboring
spins and is known as the ground state of a chain with
easy xx-plane (see Fig. 1(c)). There is only one min-
umin in this case and the envelope lines of AF structure
Fig. 1(c) are straight. If, however, (δ, q) differ from
these special values the energy spectrum changes dra-
matically: it adopts many local energy minima and max-
ima in good agreement with our analytical analysis. The
contour plot of the energy \(E(\delta, q)\) for a chain of \(N = 10\)
dipoles is shown in Fig. 1(b). Several cross-sections of
this two-dimensional energy surface for different K are
exemplified in Fig. 1(a). Fascinatingly, the minima oc-
cur for all \(\{q, \delta\} = \{\pm \frac{m\pi}{N}, \pm \frac{m\pi}{N}\}\) for \(K < -2.3D\) and
\(\{q, \delta\} = \{\pm \frac{m\pi}{N}, \pi + \frac{m\pi}{N}\}\) for \(K > 2.3D\) with in-
teger \(m \in [0, N]\) and \(n \in [0, N]\). In other words, the
energy spectrum becomes discrete and the energy gaps
become size dependent; i.e., we observe all components
of quantum confinement. The deepest minima; i.e., sta-
blest energy levels correspond to magnetic helices (MH)
with integer number of turns along the chain as shown in
Fig. 1(c) and can be uniquely described using quantum
numbers \(m\) and \(n\) defining the two characteristic,
commensurate to the chain length wave vectors δ and q.
The number of helix-turns is given by the smaller wave-
vector. For example if \(\delta \geq \pi\) then \(q = \pi/N\) corresponds
to one helix-turn of the entire chain.

To explore complete configurational space beyond the
two-wave-vectors approximation MC and SD simula-
tions have been performed. Additionally, a magnetome-
chanical model presented in Fig. 2 and described in
has been constructed. While the MC procedure has
been developed to describe the equilibrium properties of
many-body systems, the SD gives an exact dynamical
path from one configuration to another. The details of
the numerical procedure are described in
. According
to the MC analysis the stable equilibrium configura-
tion of a dipolar chain of length \(N\) for \(kT > 0.1D\) and
\(-1.2D < K < 2D\) indeed corresponds to \(E_{\text{min2}}\) in
Fig. 1(c); i.e., to \(\{q, \delta\} = (\pi/N, \pi \pm \frac{\pi}{N}\) \).
Only at lower temperatures or higher anisotropy the ground AF state
can be achieved. Our SD simulations demonstrate that

information about the saturation magnetization \(M_0\) of
a particle is as usual hidden in the interaction constant
\(D = \mu_0 M_0^2/2\). First, we look for the exact total num-
ber of critical points of Hamiltonian \(H\) solving the set
of equations \(\nabla \sum_{i,j} H = 0\) as explained in SI19. We re-
veal \(3 \cdot 2^N\) stationary points (minima, maxima, or saddle
points) related to non-collinear solutions and the ques-
tion is how the magnetic configurations corresponding to
Starting with the passes through several energy MH states predicted above. The exact dynamical path towards the global energy minimum strongly depends on the starting configuration and the exact dynamical path towards the global energy minimum. Thus, the starting configuration and the exact dynamical path towards the global energy minimum. Next, we want to use this property to achieve different \((\delta, q)\) states artificially. To do so we propose to rotate magnetization of one of the terminal dipoles introducing energy into the chain. The consequences of this procedure are documented in MC simulations (Fig. 3), SD simulations (videos SI3,4,6), and magnetomechanical model (Video SI5).

The Monte-Carlo simulations have been performed for chains of length \(N = 70a\). The equilibrium winding up process is shown in Fig. 3(a). Initially, the slow annealing procedure has been applied. At the end of relaxation, at \(kT = 0.05D\), the chain has adopted the MH state with \(q = \pi/N\); i.e., the entire chain acquired a half of a modulation period. This state was taken as the start configuration for the winding up process. Next, the first spin has been rotated with the velocity \(\pi/2\cdot n/MCS\), because our analysis showed that the period of \(2\cdot 10^5\) MCS was long enough to achieve a new equilibrium state. After nine subsequent rotations, which are illustrated Fig. 3(a), the chain arrived at the stable \(q = 5\pi/2N\) helix. The other end of the chain remained free. In Fig. 3(b) the time dependence of the \(z\)-component of magnetization for the first and the last spins are monitored. One sees that the rotation of the last spin is somewhat delayed comparably to the rotation of the first spin because of the system’s retarding in the stable energy levels. Another interesting observation concerns the propagation velocity of a knot in the spiral. The velocity of propagation decreases with increasing \(q\) as shown by dashed red line and can be described by a function of the form \(v = at^2\) with the negative acceleration \(a\). The backward process is depicted in Fig. 3(c),(d). The first spin was fixed now and the chain’s relaxation was monitored. The process was mirrored and the spiral was wound down performing work. The rotation was again decelerated as shown in Fig. 3(d). The whole process, however, took larger time as the spiral has been released at the local energy minimum.

The MC results are in very good agreement with SD simulations shown in video material SI3,4 for dipolar coupling and SI6 for ferromagnetic exchange interaction. The time scale depends on the strength of magnetic interactions and damping parameters. For a chain made out of \(35\times35\times2\) nm nanoparticles, interparticle distance \(20\) nm, and Gilbert damping of 0.01, the entire winding up process of video SI3,4 corresponds to milliseconds. Additionally, SD reveals new interesting aspects of the winding process. The first, evident conclusion is that the velocity of the entire process depends on the velocity of forced rotation of the first spin. The unexpected finding is that formation of each new turn of the magnetic helix is accompanied by an abrupt change in the magnetization of several dipoles in case of dipolar inter-
The probably most important one concerns the new way of energy storage. One of the oldest but still actively utilized in a number of applications methods is the spring windup technique. An example familiar to everyone is a clockwork device mechanically powered by a mainspring. In this method one end of a mechanical spring is fixed, while the other is rotated until the spring is wound up. Then the latter spring terminal is released and the stored potential energy transforms into the kinetic energy of a clock arrow, a motor, a pump etc.

Similar procedure can be applied to the MH’s. By rotation of one chain’s end via local fields or spin-polarized currents the helix can be forced towards smaller periodicities and, thus, higher energy. The system can be left in this new stable configuration to store introduced energy. The stored energy can be then transformed into its mechanical or magnetic counterparts to make work as visualized in videos\textsuperscript{19} and Fig. 3.

In conclusion, the results described above demonstrate that the finite magnetic chains coupled by a exchange, RKKY or dipolar interactions possess a quantized energy spectrum depending on the chain geometry, material of elements, and the shape of particles. This unique energy spectrum results in topologically protected configurations in form of magnetic helices with integer number of revolutions, explains recent experiments\textsuperscript{13} and opens broad perspectives for future investigations concerning the dynamics of this nontrivial system and quantum effects on a length scale of a few Å.

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\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig3.png}
\caption{MC simulations of a linear chain of 70 dipoles with $K \approx -1.23 \mu_B M_s^2$ at $kT < 1 K$. (a) Winding up of the spiral. Each snapshot of $S_z(r)$ corresponds to a relaxed state. The very left moment (red) has been rotated with velocity of $(\pi/2)/2 \cdot 10^5$ MCS; (b) Time dependence of the very left (black line) and the very right (red line) moment; (c) Release of the stored energy: the left moment is fixed downwards; (d) Non-linear time dependence of the chain-averaged $\langle S_z \rangle$. A typical angle distribution in a relaxed helix is shown as inset.}
\end{figure}

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