Parameter identification for active mass damper controlled systems

C C Chang¹, J F Wang², and C C Lin³

¹ Assistant Researcher, National Center for Research on Earthquake Engineering, National Applied Research Laboratories, 200, Sec. 3, Xinhai Rd., Taipei 10668, Taiwan, R.O.C. ccchang@ncree.narl.org.tw

² Assistant Research Fellow, 921 Earthquake Museum of Taiwan, National Museum of Natural Science, Taichung 41364, Taiwan, R.O.C.

³ Distinguished Professor, Department of Civil Engineering, National Chung Hsing University, Taichung 40227, Taiwan, R.O.C.

Abstract. Active control systems have already been installed in real structures and are able to decrease the wind- and earthquake-induced responses, while the active mass damper (AMD) is one of the most popular types of such systems. In practice, an AMD is generally assembled in-situ along with the construction of a building. In such a case, the AMD and the building is coupled as an entire system. After the construction is completed, the dynamic properties of the AMD subsystem and the primary building itself are unknown and cannot be identified individually to verify their design demands. For this purpose, a methodology is developed to obtain the feedback gain of the AMD controller and the dynamic properties of the primary building based on the complex eigen-parameters of the coupled building-AMD system. By means of the theoretical derivation in state-space, the non-classical damping feature of the system is characterized. This methodology can be combined with any state-space based system identification technique as a procedure to achieve the goal on the basis of the acceleration measurements of the building-AMD system. Results from numerical verifications show that the procedure is capable of extracting parameters and is applicable for AMD implementation practices.

1. Introduction
In recent years, remarkable progress has been made in the field of active control of civil engineering structures subjected to environmental loadings such as winds and earthquakes [1, 2]. Various control methods were proposed with new control devices to different civil engineering structures. Among them, the active control method has attracted intensive theoretical and experimental attention[3-7]. Since the application of active control in civil engineering field in 1970 [8], several control approaches have been investigated, such as LQ [7,9], LQR [6,10], and $H_\infty$ control [11-13]. Active control systems have already been installed in real structures and are able to decrease the wind- and earthquake-induced responses, while the active mass damper (AMD) is one of the most popular types of such systems. In practical, an AMD is generally assembled in-situ along with the construction of a building.
However, there are still many issues to be solved for active control techniques towards real implementation, such as system instability due to inappropriate selection of control parameters and control force execution time delay, which may significantly deteriorate the control performance. In such a case, the AMD and the building is coupled as an entire system. After the construction is completed, the dynamic properties of the AMD subsystem and the primary building itself are unknown and cannot be identified individually to verify their design demands. For this purpose, a methodology is developed to obtain the feedback gain of the AMD controller and the dynamic properties of the primary building based on the complex eigen-parameters of the coupled building-AMD system. By means of the theoretical derivation in state-space, the non-classical damping feature of the system is characterized. This methodology can be combined with any state-space based system identification technique as a procedure to achieve the goal on the basis of the acceleration measurements of the building-AMD system. Results from numerical verifications show that the procedure is capable of extracting parameters and is applicable for AMD implementation practices.

2. AMD-controlled systems

2.1. Equations of motion of AMD-controlled system

For a \( n \)-DOF discrete-parameter structure with a AMD attached on the \( l \)th DOF subject to dynamic loadings and active control forces, as shown in figure 1. The equations of motion take the form as

\[
M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = E_1w(t) + B_1u(t)
\]  

where \( M \), \( C \) and \( K \) are the \( N \times N \) (where \( N = n + 1 \)) system mass, damping and stiffness matrices, respectively, and can be expressed in detail as

\[
M = \begin{bmatrix}
m_{p_1} & & & \\
& m_{p_2} & & \\
& & \ddots & \\
& & & m_{p_n}
\end{bmatrix}_{N \times N}, \quad C = \begin{bmatrix}
c_{p_1} & c_{p_2} & -c_{p_2} & 0 & \cdots & \cdots & 0 \\
-c_{p_2} & c_{p_2} + c_{s_2} & -c_{s_2} & 0 & \cdots & \cdots & \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{bmatrix}_{N \times N},
\]

\[
K = \begin{bmatrix}
k_{p_1} + k_{s_1} & -k_{s_1} & 0 & \cdots & \cdots & \cdots & 0 \\
-k_{s_1} & k_{s_1} + k_{p_2} & -k_{p_2} & 0 & \cdots & \cdots & \\
0 & \cdots & \ddots & \ddots & \ddots & \ddots & \\
0 & \cdots & \ddots & \ddots & \ddots & \ddots & \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & k_{s_n}
\end{bmatrix}_{N \times N}
\]

where \( m_{p_i} \) and \( m_s \) represent the masses of the \( i \)th DOF and AMD, respectively; \( c_{p_i} \) and \( c_s \) represent the damping coefficients of the \( i \)th DOF and AMD, respectively; \( k_{p_i} \) and \( k_s \) represent the stiffness coefficients of the \( i \)th DOF and AMD, respectively. In addition, \( x(t) \) is the \( N \)-dimensional displacement vector, and can be expressed in detail as

\[
x = \begin{bmatrix}
x_{p_1} \\
x_{p_2} \\
\vdots \\
x_{p_n} \\
x_s
\end{bmatrix}_{N+1}
\]

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where $x_{p_i}$ and $x_s$ represent the displacement of the $i$th DOF and AMD relative to the ground, respectively. The notation $w(t)$ is the $r$-dimensional external excitation vector and $u(t)$ is the $q$-dimensional control force vector. The $n \times q$ matrix $B_i$ and $n \times r$ matrix $E_i$ respectively define the locations of control forces and excitations. In the situation of ground acceleration, $\ddot{x}_g(t)$, the $B_i$ and $E_i$ can be expressed as

$$B_i = \{0 \ 0 \ \ldots \ -1^{(i)} \ \ldots \ 0\}_N^T$$  \hspace{1cm} \hspace{1cm} (2e)$$

$$E_i = \{-m_{p_1} \ -m_{p_2} \ \ldots \ -m_{p_i} \ \ldots \ -m_{p_n} \ -m_{s}\}_N^T$$  \hspace{1cm} \hspace{1cm} (2f)$$

Represented in state-space form, equation (1) can be rewritten as

$$\dot{X}(t) = AX(t) + E\ddot{x}_g(t) + Bu(t)$$  \hspace{1cm} \hspace{1cm} (3)$$

where

$$X = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}, \quad A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -M^{-1}B \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ -M^{-1}E_i \end{bmatrix}$$  \hspace{1cm} \hspace{1cm} (4)$$

are $2N \times 1$ state vector, $2N \times 2N$ system matrix, $2N \times 1$ controller location matrix and $2N \times 1$ external excitation location matrix, respectively. In this paper, active control forces are determined using the $H_{\infty}$ direct output feedback control theory developed by Lin et al. [11, 12].

Figure 1. System model of a multistory building structure with AMD
2.2. $H_\infty$ direct output feedback control theory

Define a $p \times 1$ control output vector $Z(t)$ and an $s \times 1$ output measurement vector $y(t)$ as

$$Z(t) = C_1 X(t) + Du(t)$$

$$y(t) = C_2 X(t)$$

where $C_1$, $D$ and $C_2$ are $p \times 2N$, $p \times q$ and $s \times 2N$ matrices. The direct output feedback control force is calculated by

$$u(t) = Gy(t) = GC_2 X(t)$$

where $G$ is a $q \times s$ time-invariant feedback gain matrix. According to $H_\infty$ control algorithm, the $H_\infty$ norm of transfer function matrix $T_{zw}(j\omega)$ of control output $Z(j\omega)$ with respect to external excitation $w(j\omega)$ takes the form

$$\|T_{zw}(j\omega)\|_\infty = \sup_{\|w(j\omega)\|_2} \frac{\|Z(j\omega)\|_2}{\|w(j\omega)\|_2}$$

where $j = \sqrt{-1}$, and sup is defined as the supremum over all $w(t)$. According to the optimal $H_\infty$ control algorithm (Yaesh and Shaked 1997), the $H_\infty$ norm of transfer function matrix $T_{zw}(j\omega)$ satisfies the following constraint

$$\|T_{zw}(j\omega)\|_\infty = \sup_{\|w(j\omega)\|_2} \frac{\|Z(j\omega)\|_2}{\|w(j\omega)\|_2} < \gamma$$

where $\gamma$ is a positive attenuation constant which denotes a measure of control performance. Adopting a smaller value of $\gamma$ means that more stringent performance of control system is required. It has been proved [12] that an optimal $H_\infty$ control system is asymptotically stable if there exists a matrix $P \geq 0$ that satisfies the following Riccati equation

$$(A + BGC_1)^T P + P(A + BGC_2) + \frac{1}{\gamma^2} PEE^T P + (C_1 + DGC_2)^T (C_1 + DGC_2) = 0$$

(10)

One way to design the optimal $H_\infty$ output feedback gain is to solve Eq. (10) with minimizing the Entropy of $T_{zw}(j\omega)$ which takes the form [11]

$$E_{\infty}(T_{zw}, \gamma) = \text{tr}\{EE^T P\}$$

(11)

where $\text{tr}\{\cdot\}$ denotes the trace of a square matrix. Then, the optimization problem to obtain gain matrix $G$ is converted to minimize the Entropy in Eq. (11) subject to the constraint of Eq. (10). The Lagrangian $L$ can be introduced as

$$L(G, P, \lambda) = \text{tr}\{EE^T P + \lambda[(A + BGC_1)^T P + P(A + BGC_2) + \frac{1}{\gamma^2} PEE^T P + (C_1 + DGC_2)^T (C_1 + DGC_2)]\}$$

(12)

where $\lambda$ is a $8 \times 8$ Lagrangian multiplier matrix. For simplicity and without loss of generality, the necessary and sufficient conditions for minimization of $L(G, P, \lambda)$ are derived and expressed by

$$\frac{\partial L}{\partial \lambda} = (A + BGC_1)^T P + P(A + BGC_2) + \frac{1}{\gamma^2} PEE^T P + (C_1 + DGC_2)^T (C_1 + DGC_2) = 0$$

(13a)

$$\frac{\partial L}{\partial P} = (A + BGC_1 + \frac{1}{\gamma^2} EE^T P)\lambda + (A + BGC_2 + \frac{1}{\gamma^2} EE^T P)^T + EE^T = 0$$

(13b)
\[
\frac{\partial L}{\partial G} = B^T P \lambda \mathbf{C}_2^T + (\mathbf{C}_2^T G^T \mathbf{D}^T \mathbf{D})^T G \mathbf{C}_2 \mathbf{C}_2^T = 0
\]  
(13c)

The gain can be obtained by solving equations (13a)-(13c) for the direct output feedback.

3. Identification theory for subsystem and primary structure of AMD-controlled systems

3.1. AMD-controlled system matrix

For an \( n \)-story shear building with an AMD attached on the \( l \)th floor, the equation of the combined system in state-space form can be written as equation (3). Substituting equation (8) into equation (3) gives

\[
\dot{\mathbf{X}}(t) = (\mathbf{A} + \mathbf{B} \mathbf{G} \mathbf{C}_2) \mathbf{X}(t) + \mathbf{E} \ddot{x}_g(t) = \mathbf{\tilde{A}} \mathbf{X}(t) + \mathbf{E} \ddot{x}_g(t)
\]  
(14)

where \( \mathbf{\tilde{A}} \) represents the AMD-controlled system matrix, and \( \mathbf{\tilde{A}} = \mathbf{A} + \mathbf{B} \mathbf{G} \mathbf{C}_2 \).

In the situation that the velocity measured on the \( l \)th floor in which AMD is located related to the ground, \( \dot{x}_{nl} \), is used as direct output feedback response, the gain matrix \( \mathbf{G} \) and \( \mathbf{C}_2 \) can be expressed as

\[
\mathbf{G} = [g_r]; \quad \mathbf{C}_2 = \{0 \ldots 0 \ldots 0 \mid 0 \ldots 1 \ldots 0\}_{1 \times 2N}
\]  
(15)

where \( g_r \) is velocity gain of the \( l \)th floor in which AMD is located. From equation (8), the control force is calculated by

\[
u(t) = g_r \dot{x}_{nl}
\]  
(16)

And AMD-controlled system matrix can be expressed in detail as

\[
\mathbf{\tilde{A}} = \mathbf{A} + \mathbf{B} \mathbf{G} \mathbf{C}_2 = \begin{bmatrix}
0 & 1 \\
-M^{-1} \mathbf{K} & -M^{-1}(\mathbf{C} + \mathbf{B} g_r)
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-M^{-1} \mathbf{K} & -M^{-1} \mathbf{C}
\end{bmatrix}
\]  
(17)

where

\[
\mathbf{G} \mathbf{C}_2 = \{0 \ldots 0 \ldots 0 \mid 0 \ldots g_r \ldots 0\}_{1 \times 2N} = \{0 \ G_r\}
\]  
(18)

3.2. Identification theory

The terms \( \lambda_j \) and \( \Psi_j \) denote the \( j \)th eigenvalue and its corresponding \( 2N \times 1 \) eigenvector of the system matrix \( \mathbf{\tilde{A}} \). The physical-domain system matrix is related to the \( j \)th eigenparameters by using the state-space characteristic equation

\[
(\mathbf{\tilde{A}} - \lambda_j I) \Psi_j = 0
\]  
(19)

Substituting equation (17) into equation (19) gives

\[
\begin{bmatrix}
0 & 1 \\
-M^{-1} \mathbf{K} & -M^{-1} \mathbf{C}
\end{bmatrix} - \lambda_j \begin{bmatrix}
0 & 1 \\
0 & 1
\end{bmatrix} \Psi_j = 0
\]  
(20)

Furthermore, the expression of the lower half of equation (20), where contains information regarding the dynamic properties of the system, gives

\[
[\mathbf{L} \quad \mathbf{R}] \Psi_j = 0
\]  
(21)

where
3.2.1. Identification of velocity gain matrix $v_g$ of AMD

From the last row of equation (21),

$$ k_s (\psi_{j,i} - \psi_{N,i}) + c_s (\psi_{N+1,i} - \psi_{2N,i}) - g_v \psi_{N+1,i} = m_s \lambda_j \psi_{2N,i} $$

(23)

As $c_s = k_s = 0$, the velocity gain matrix $g_v$ of AMD can be obtained and expressed as

$$ g_v = \frac{-m_s \lambda_j \psi_{2N,i}}{\psi_{N+1,i}} (j = 1, 2, \ldots, or 2N) $$

(24)

As some components with stiffness and damping are installed in the AMD ($c_s \neq 0$ and $k_s \neq 0$), $k_s$, $c_s$, and $g_v$ can be derived on the basis of three different complex mode shape and then obtained by
3.2.2. Identification of the primary building

(1) Damping and stiffness coefficients of the /th story in which the AMD is located

To obtain the dynamic parameters of primary building, the derivation begins at the /th row of equation (20), and then the damping coefficient of the /th story of the primary building can be derived and expressed as

\[
\begin{align*}
\mathbf{c}_p &= \mathbf{c}_{p,1} \left( \begin{array}{cc}
\psi_{i,j} - \psi_{N,i,j} & \psi_{N+1,i,j} - \psi_{2N,i,j} & \cdots & \psi_{N+l-1,i,j} \\
\psi_{N+1,i,j} - \psi_{N+2,i,j} & \cdots & \psi_{N+l-2,i,j} & \psi_{N+l-1,i,j} \\
\psi_{N+l-1,i,j} - \psi_{N+l,i,j} & \cdots & \psi_{N+1+2N-l,j} & \psi_{2N+1+2N-l,j} \\
\cdots & \cdots & \cdots & \cdots
\end{array} \right) \mathbf{m}_i \lambda_i \psi_{N+1,i,j} \\
\mathbf{g}_p &= \mathbf{g}_{p,1} \left( \begin{array}{cc}
\psi_{i,j} - \psi_{N,i,j} & \psi_{N+1,i,j} - \psi_{2N,i,j} & \cdots & \psi_{N+l-1,i,j} \\
\psi_{N+1,i,j} - \psi_{N+2,i,j} & \cdots & \psi_{N+l-2,i,j} & \psi_{N+l-1,i,j} \\
\psi_{N+l-1,i,j} - \psi_{N+l,i,j} & \cdots & \psi_{N+1+2N-l,j} & \psi_{2N+1+2N-l,j} \\
\cdots & \cdots & \cdots & \cdots
\end{array} \right) \mathbf{m}_i \lambda_i \psi_{N+1,i,j}
\end{align*}
\]  

(25)

where \( \lambda_i \) and \( \phi_i \) are the eigenvalues and eigenvectors of \( \mathbf{M} \)

Next, any complex modes can be selected (e.g. /th and /th modes), and the stiffness coefficient of the /th story of the primary structure can be derived on the basis of equation (26) as

\[
\begin{align*}
k_p &= \left[ \frac{\psi_{l-1,k} - \psi_{l,k} - \psi_{l-2,k} - \psi_{l,k}}{\psi_{N+l,k} - \psi_{N+l,k}} \right]^{-1} \\
&= \frac{\hat{C}_{l,k} - \hat{C}_{l,k} - \hat{C}_{l,k}}{\psi_{N+l,k} - \psi_{N+l,k}} + \frac{\hat{K}_{l,k} - \hat{K}_{l,k}}{\psi_{N+l,k} - \psi_{N+l,k}} \\
&= \frac{\hat{C}_{l,k} - \hat{C}_{l,k}}{\psi_{N+l,k} - \psi_{N+l,k}} + \frac{\hat{K}_{l,k} - \hat{K}_{l,k}}{\psi_{N+l,k} - \psi_{N+l,k}} \\
&= \frac{\hat{C}_{l,k} - \hat{C}_{l,k}}{\psi_{N+l,k} - \psi_{N+l,k}} + \frac{\hat{K}_{l,k} - \hat{K}_{l,k}}{\psi_{N+l,k} - \psi_{N+l,k}} + \frac{\hat{K}_{l,k} - \hat{K}_{l,k}}{\psi_{N+l,k} - \psi_{N+l,k}}
\end{align*}
\]

(27)

where \( \hat{K}_{l,k} = k_{p,i} (\psi_{l,k} - \psi_{l,k}) \) , \( \hat{C}_{l,k} = c_{p,i} (\psi_{N+l,k} - \psi_{N+l,k}) \) , \( \hat{K}_{l,j} = k_{p,i} (\psi_{l,j} - \psi_{l,j}) \) , and \( \hat{C}_{l,j} = c_{p,i} (\psi_{N+l,j} - \psi_{N+l,j}) - g_{p,i} (\psi_{N+l,j} - \psi_{N+l,j}) \) . In equation (27), parameters \( k_p \) , \( c_p \) and \( g_p \) can be solved from equation (24) or equation (25) which determine \( \hat{K}_{l,k} \) , \( \hat{C}_{l,k} \) , \( \hat{K}_{l,j} \) , and \( \hat{C}_{l,j} \) . In addition, the information related to the \((l+1)\) story (i.e., \( \hat{K}_{l+1,k} \) , \( \hat{C}_{l+1,k} \) , \( \hat{K}_{l+1,j} \) and \( \hat{C}_{l+1,j} \)) can be solved in advance (as shown in the next section). The floor masses of the primary building and AMD are assumed to be known, and equations (26) and (27) are then solved sequentially.

(2) Damping and stiffness coefficient of the top story (at which the AMD is not installed)

For the top story, equation (27) can be applied by setting \( \hat{K}_{l+1,k} = \hat{K}_{l+1,k} = \hat{C}_{l+1,j} = \hat{C}_{l+1,j} = 0 \) . In addition, assigning \( l=n \) in equation (27) lead to
Furthermore, setting \( c_{p\neq l} = k_{p\neq l} = 0 \) and \( l = n \) in equation (26) gives

\[
k_{p_n} = \begin{bmatrix}
\psi_{n-1,k} - \psi_{n,k} & \psi_{n-1,j} - \psi_{n,j} \\
\psi_{n,k} - \psi_{n-1,k} & \psi_{n,j} - \psi_{n-1,j}
\end{bmatrix}^{-1}
\cdot
\begin{bmatrix}
\lambda_{j} \psi_{n-1,j} m_{p_n} - \lambda_{j} \psi_{n,j} m_{p_n} \\
\psi_{n,k} - \psi_{n-1,k} & \psi_{n,j} - \psi_{n-1,j}
\end{bmatrix}
+ \hat{C}_{n_k} \begin{bmatrix}
\psi_{n,k} - \psi_{n-1,k} \\
\psi_{n,j} - \psi_{n-1,j}
\end{bmatrix} + \hat{K}_{n_k} \begin{bmatrix}
\psi_{n,k} - \psi_{n-1,k} \\
\psi_{n,j} - \psi_{n-1,j}
\end{bmatrix}
\quad (28)
\]

Equation (28) can be used to calculate the damping and stiffness coefficient of the top story. In a situation in which the AMD is not installed at the \( n \)th story, then \( k\text{,}_n\text{,} c\text{,}_n\text{,} \hat{C}_{n_k}\text{,} \hat{K}_{n_k}\text{,} \) are all equal to zero. The Damping and stiffness coefficient of the top story can be calculated by

\[
k_{p_n} = \begin{bmatrix}
\psi_{n-1,k} - \psi_{n,k} & \psi_{n-1,j} - \psi_{n,j} \\
\psi_{n,k} - \psi_{n-1,k} & \psi_{n,j} - \psi_{n-1,j}
\end{bmatrix}^{-1}
\cdot
\begin{bmatrix}
\lambda_{j} \psi_{n-1,j} m_{p_n} - \lambda_{j} \psi_{n,j} m_{p_n} \\
\psi_{n,k} - \psi_{n-1,k} & \psi_{n,j} - \psi_{n-1,j}
\end{bmatrix}
+ \hat{C}_{n_k} \begin{bmatrix}
\psi_{n,k} - \psi_{n-1,k} \\
\psi_{n,j} - \psi_{n-1,j}
\end{bmatrix} + \hat{K}_{n_k} \begin{bmatrix}
\psi_{n,k} - \psi_{n-1,k} \\
\psi_{n,j} - \psi_{n-1,j}
\end{bmatrix}
\quad (29)
\]

In a situation in which the AMD is not installed at the \( n \)th story, then \( k\text{,}_n\text{,} c\text{,}_n\text{,} \hat{C}_{n_k}\text{,} \hat{K}_{n_k}\text{,} \) are all equal to zero. The Damping and stiffness coefficient of the top story can be calculated by

\[
k_{p_n} = \begin{bmatrix}
\psi_{n-1,k} - \psi_{n,k} & \psi_{n-1,j} - \psi_{n,j} \\
\psi_{n,k} - \psi_{n-1,k} & \psi_{n,j} - \psi_{n-1,j}
\end{bmatrix}^{-1}
\cdot
\begin{bmatrix}
\lambda_{j} \psi_{n-1,j} m_{p_n} - \lambda_{j} \psi_{n,j} m_{p_n} \\
\psi_{n,k} - \psi_{n-1,k} & \psi_{n,j} - \psi_{n-1,j}
\end{bmatrix}
+ \hat{C}_{n_k} \begin{bmatrix}
\psi_{n,k} - \psi_{n-1,k} \\
\psi_{n,j} - \psi_{n-1,j}
\end{bmatrix} + \hat{K}_{n_k} \begin{bmatrix}
\psi_{n,k} - \psi_{n-1,k} \\
\psi_{n,j} - \psi_{n-1,j}
\end{bmatrix}
\quad (30)
\]

Equation (30) can be used to calculate the damping and stiffness coefficient of the other inner story (at which the AMD is not installed) at the \( i \)th story, by setting \( k\text{,}_c\text{,} c\text{,}_c\text{,} \hat{C}_{c_k}\text{,} \hat{K}_{c_k}\text{,} \) are all equal to zero. The following equation can be derived

\[
k_{p_i} = \begin{bmatrix}
\psi_{i-1,k} - \psi_{i,k} & \psi_{i-1,j} - \psi_{i,j} \\
\psi_{i,k} - \psi_{i-1,k} & \psi_{i,j} - \psi_{i-1,j}
\end{bmatrix}^{-1}
\cdot
\begin{bmatrix}
\lambda_{j} \psi_{i-1,j} m_{p_i} - \lambda_{j} \psi_{i,j} m_{p_i} \\
\psi_{i,k} - \psi_{i-1,k} & \psi_{i,j} - \psi_{i-1,j}
\end{bmatrix}
+ \hat{C}_{i_k} \begin{bmatrix}
\psi_{i,k} - \psi_{i-1,k} \\
\psi_{i,j} - \psi_{i-1,j}
\end{bmatrix} + \hat{K}_{i_k} \begin{bmatrix}
\psi_{i,k} - \psi_{i-1,k} \\
\psi_{i,j} - \psi_{i-1,j}
\end{bmatrix}
\quad (31)
\]

Equations (26) and (27) can be applied to calculate the stiffness and damping coefficient of the other inner story (at which the AMD is not installed) at the \( i \)th story, by setting \( k\text{,}_c\text{,} c\text{,}_c\text{,} \hat{C}_{c_k}\text{,} \hat{K}_{c_k}\text{,} \) are all equal to zero. The following equation can be derived

\[
k_{p_i} = \begin{bmatrix}
\psi_{i-1,k} - \psi_{i,k} & \psi_{i-1,j} - \psi_{i,j} \\
\psi_{i,k} - \psi_{i-1,k} & \psi_{i,j} - \psi_{i-1,j}
\end{bmatrix}^{-1}
\cdot
\begin{bmatrix}
\lambda_{j} \psi_{i-1,j} m_{p_i} - \lambda_{j} \psi_{i,j} m_{p_i} \\
\psi_{i,k} - \psi_{i-1,k} & \psi_{i,j} - \psi_{i-1,j}
\end{bmatrix}
+ \hat{C}_{i_k} \begin{bmatrix}
\psi_{i,k} - \psi_{i-1,k} \\
\psi_{i,j} - \psi_{i-1,j}
\end{bmatrix} + \hat{K}_{i_k} \begin{bmatrix}
\psi_{i,k} - \psi_{i-1,k} \\
\psi_{i,j} - \psi_{i-1,j}
\end{bmatrix}
\quad (32)
\]

For the first story (i.e., when \( l = 1 \)), \( \psi_{i-1,j} \text{,} \psi_{i-1,k} \text{,} \psi_{i,j} \text{,} \psi_{i,k} \text{,} \psi_{i+1,j} \text{,} \psi_{i+1,k} \) are eliminated and are all equal to zero in equations (26) and (27). The following equation can be derived

\[
k_{p_1} = \begin{bmatrix}
\psi_{1,k} - \psi_{1,j} \\
\psi_{1,j} - \psi_{1,k}
\end{bmatrix}^{-1}
\cdot
\begin{bmatrix}
\lambda_{j} \psi_{1,j} m_{p_1} - \lambda_{j} \psi_{1,j} m_{p_1} \\
\psi_{1,k} - \psi_{1,j}
\end{bmatrix}
+ \hat{C}_{1_k} \begin{bmatrix}
\psi_{1,k} - \psi_{1,j} \\
\psi_{1,j} - \psi_{1,k}
\end{bmatrix} + \hat{K}_{1_k} \begin{bmatrix}
\psi_{1,k} - \psi_{1,j} \\
\psi_{1,j} - \psi_{1,k}
\end{bmatrix}
\quad (33)
\]

For the first story (i.e., when \( l = 1 \)), \( \psi_{i-1,j} \text{,} \psi_{i-1,k} \text{,} \psi_{i,j} \text{,} \psi_{i,k} \text{,} \psi_{i+1,j} \text{,} \psi_{i+1,k} \) are eliminated and are all equal to zero in equations (26) and (27). The following equation can be derived

\[
k_{p_1} = \begin{bmatrix}
\psi_{1,k} - \psi_{1,j} \\
\psi_{1,j} - \psi_{1,k}
\end{bmatrix}^{-1}
\cdot
\begin{bmatrix}
\lambda_{j} \psi_{1,j} m_{p_1} - \lambda_{j} \psi_{1,j} m_{p_1} \\
\psi_{1,k} - \psi_{1,j}
\end{bmatrix}
+ \hat{C}_{1_k} \begin{bmatrix}
\psi_{1,k} - \psi_{1,j} \\
\psi_{1,j} - \psi_{1,k}
\end{bmatrix} + \hat{K}_{1_k} \begin{bmatrix}
\psi_{1,k} - \psi_{1,j} \\
\psi_{1,j} - \psi_{1,k}
\end{bmatrix}
\quad (34)
3.3. Identification Procedure

The following analysis procedure is used for extracting the feedback gain of the AMD controller and the dynamic properties of the primary building for displacement and velocity feedback: 1) The masses of each floor and AMD are known; 2) The acceleration response of each floor and AMD are needed to be measured for AMD-controlled system; 3) To identify the eigenvalues and eigenvectors from the acceleration measurements of the building-AMD system, the System Realization Using Information Matrix (SRIM) identification technique [14-17] is employed based on the selected system input and output measurements for the input–output situation. Regarding the output-only situation, the stochastic subspace identification (SSI)[17, 18] method is employed on the basis of the output response of the building–AMD system. Transforming discrete time into continuous time may obtain the optimal realization of the state–space system matrix $\tilde{A}$. The continuous-time eigenvalue $\lambda_j$ and eigenvector $\Psi_j$ of system matrix $\tilde{A}$ are computed; 4) In the situation of $c_s = k_s = 0$, the velocity feedback gain of AMD can be obtained from equation (24) based on selecting one set of eigenparameters within the second mode or the higher-mode. Whereas some components with stiffness and damping are installed in the AMD ($c_s \neq 0$ and $k_s \neq 0$), $k_s$, $c_s$, and $g$ can be obtained from equation (25) by selecting three sets of eigenparameters. 5) According to the equations in Section 3.2.2, the stiffness and damping coefficient of each floor can be obtained by selecting any two sets of eigenparameters complex modes.

4. Numerical verifications

To verify the accuracy of the aforementioned methodology, numerical studies for two three-story building were conducted. The primary structure of the first case (CASE-1) is a shear-type building, whereas the structure of the second case (CASE-2) is a real benchmark building, for which the damping and stiffness matrices are full matrices. The structural parameters of CASE-2 were synthesized from a modal testing performed on an actual 3-story steel frame erected in the earthquake simulation laboratory of the National Center for Research on Earthquake Engineering (NCREE) in Taipei, Taiwan. The physical and normal-modal parameters of these two buildings are presented in Tables 1 and 2. The accuracy of the proposed parameter-extraction procedure was investigated using different building types to examine its applicability to evaluating AMD performance in real building-AMD structures.

An AMD with a mass of 360 kg was assumed. The AMD was assumed to be installed on the roof (third floor) of the building. The stiffness and damping coefficient of AMD are 1 kN/m and 0.1 kN-sec/m, respectively. According to optimal $H_\infty$ control algorithm in Section 2, the velocity gains of the AMD on the roof floor were designed as $g = -4389$ for CASE-1 and $g = -4756$ for CASE-2, respectively. For input-output situation, the ground acceleration recorded at National Chung Hsing University (NCHU) in the east-west direction during the 1999 Taiwan Chi-Chi earthquake was used as the base excitation. Figures 2 and 3 show the acceleration responses of the roof floor under the ground motion of the 1999 Taiwan Chi-Chi earthquake with and without AMD.

Table 3 presents the obtained parameters of the AMD and primary building by using the proposed method. As shown in Table 3, $\{\hat{c}_{01}, \hat{c}_{02}, \hat{c}_{03}\}$ and $\{\hat{\xi}_{01}, \hat{\xi}_{02}, \hat{\xi}_{03}\}$ were the identified real modal frequencies and damping ratios of the reconstructed shear building based on $\{\hat{c}_{p1}, \hat{c}_{p2}, \hat{c}_{p3}\}$ and $\{\hat{k}_{p1}, \hat{k}_{p2}, \hat{k}_{p3}\}$. In theory, the proposed method can be implemented by selecting any two sets of eigenvalues and eigenvectors, namely $(\lambda_j, \Psi_j)$ and $(\hat{\lambda}_k, \hat{\Psi}_k)$. The results of CASE-1 showed that the
identified parameters of the AMD and primary building were the same as the original parameters (Table 2), regardless of the selection of $(j, k)$ set. For CASE-2, the parameters of the AMD and primary building were also accurately identified regardless of the selection of $(j, k)$ set. The results showed that the identified “pseudoshear” building was related to the selection of $(j, k)$; it was difficult to assess which set of the identified physical parameters could optimally represent the CASE-2 building. However, although the modal parameters of the reconstructed shear building could not fully represent the original building, selecting the $(j, k)$ pairs resulted in the optimal estimation of the corresponding building mode. Specifically, selecting $(j, k) = (1, 2)$ or $(3, 4)$ resulted in the optimal $\omega_1$ and $\xi_1$, selecting $(j, k) = (5, 6)$ resulted in the optimal $\omega_2$ and $\xi_2$, and continued in a trend of optimally selected pairs and resulting estimations. These results indicate that it is always possible to obtain modal parameters of a building that are more accurate by selecting various $(j, k)$ pairs.

Table 1. Physical parameters of two 3-story buildings in the numerical study.

| Physical parameters | CASE-1: Ideal shear building | CASE-2: NCREE benchmark building |
|---------------------|------------------------------|---------------------------------|
| Mass matrix (kg)    | $\begin{bmatrix} 6000 & 0 & 0 \\ 0 & 6000 & 0 \\ 0 & 0 & 6000 \end{bmatrix}$ | $\begin{bmatrix} 6000 & 0 & 0 \\ 0 & 6000 & 0 \\ 0 & 0 & 6000 \end{bmatrix}$ |
| Damping matrix (kN-sec/m) | $\begin{bmatrix} 13.6 & -7.90 & 0 \\ -7.90 & 13.3 & -5.40 \\ 0 & -5.40 & 5.40 \end{bmatrix}$ | $\begin{bmatrix} 13.28 & -8.701 & 1.463 \\ -8.701 & 13.557 & -5.774 \\ 1.463 & -5.774 & 5.657 \end{bmatrix}$ |
| Stiffness matrix (kN/m) | $\begin{bmatrix} 3300 & -1600 & 0 \\ -1600 & 3100 & -1500 \\ 0 & -1500 & 1500 \end{bmatrix}$ | $\begin{bmatrix} 3406 & -1814 & 138.2 \\ -1814 & 3329 & -1672 \\ 138.2 & -1672 & 1509 \end{bmatrix}$ |

Table 2. Real modal parameters of two 3-story buildings in the numerical study.

| Modal parameters | CASE-1: Ideal shear building | CASE-2: NCREE benchmark building |
|-----------------|-------------------------------|---------------------------------|
| Frequency, (Hz) | $\{\omega_0, \omega_1, \omega_2\}$ | $\{\omega_0, \omega_1, \omega_2\}$ |
|                 | $\{1.17, 3.22, 4.65\}$       | $\{1.04, 3.17, 4.86\}$         |
| Damping ratio, (%) | $\{\xi_0, \xi_1, \xi_2\}$ | $\{\xi_0, \xi_1, \xi_2\}$ |
|                 | $\{1.45, 3.65, 6.32\}$       | $\{1.99, 3.07, 6.43\}$         |
| Mode shape,     | $\{\varphi_0, \varphi_1, \varphi_2\}$ | $\{\varphi_0, \varphi_1, \varphi_2\}$ |
|                 | $\{0.421, -1.194, 2.124\}$   | $\{0.408, -1.247, 1.954\}$   |
|                 | $\{0.783, -0.634, -2.420\}$  | $\{0.784, -0.627, -2.291\}$  |
|                 | $\{1.000, 1.000, 1.000\}$    | $\{1.000, 1.000, 1.000\}$    |
Table 3. Identified parameters of the AMD and primary buildings in the numerical study

| Identified Parameters | \((j_1,j_2,j_3)\) or \((j,k)\) | CASE-1 | CASE-2 |
|-----------------------|---------------------------------|--------|--------|
| **AMD**               |                                 |        |        |
| \(\hat{g}_v\)         | \((1,2,3), (4,5,6)\)            | -4387  | -4756  |
| \(\hat{k}_j (kN/m)\)  | \((1,2), (3,4)\) \((5,6), (7,8)\) | 1      | 1      |
| \(\hat{c}_j (kN-sec/m)\) | \((5.7, 7.9, 5.4)\)             |        |        |
| **Primary building (physical)** |                         |        |        |
| \(\hat{c}_{p_1}, \hat{c}_{p_2}, \hat{c}_{p_3}\) \((kN-sec/m)\) | \((1,2)\) \((3,4)\) \((5,6)\) \((7,8)\) | \(\{8.55, 7.86, 6.71\}\) | \(\{8.28, 8.12, 5.31\}\) |
| \(\hat{k}_{p_1}, \hat{k}_{p_2}, \hat{k}_{p_3}\) \((kN/m)\) | \((3,4)\) \((5,6)\) \((7,8)\) | \[1326, 1187, 1301\] | \[1367, 1206, 1179\] |
| **Primary building (modal)** |                         |        |        |
| \(\hat{\omega}_{b_1}, \hat{\omega}_{b_2}, \hat{\omega}_{b_3}\) \((Hz)\) | \((1,2)\) \((3,4)\) \((5,6)\) \((7,8)\) | \[2.07, 5.32, 7.84\] | \[2.07, 5.32, 7.84\] |
| \(\hat{\xi}_{b_1}, \hat{\xi}_{b_2}, \hat{\xi}_{b_3}\) \((\%)\) | \((3,4)\) \((5,6)\) \((7,8)\) | \[1.45, 3.65, 6.32\] | \[1.45, 3.65, 6.32\] |

5. Conclusions
In this paper, an analysis procedure was developed for extracting the dynamic parameters of an AMD and primary building based on acceleration measurements of a combined building-AMD system. This procedure was first performed by identifying the eigenvalues and eigenvectors from the acceleration measurements of the building-AMD system. These complex eigenparameters were then used to calculate the respective parameters of the AMD and primary building by using the derived analytical formulas. Although the developed theory is based on the shear building assumption, the numerical-simulation results show that the extracted AMD parameters are influenced little by the building model, whereas the building parameters of different modes can be accurately extracted by selecting appropriately complex modes. It is concluded that the proposed analysis procedure can be appropriately...
applied to the health monitoring of a combined building-AMD system and used for ensuring the performance of the AMD control system.

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