Semi-Active Control Performance Index for magneto-rheological dampers considering s-structure interaction

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Abstract. In this study, an instantaneous optimal control performance index for structures with magneto-rheological (MR) dampers is analytically defined. Absolute acceleration and absolute velocity feedback are implemented to this simple performance index. The performance index is another application of instantaneous optimal control algorithms. A three story structure incorporating an MR damper is presented as a numerical example. Modified Bouc Wen damper force model is used for obtaining the MR damper force. The electrical current need for the operation of the MR damper is taken into account by using a piece wise invertible MR damper model. Near fault earthquakes are used in the dynamic simulations. Seismic energy-based evaluations are carried out. Uncontrolled structural response reduction performance and power requirement of the MR damper is investigated. The analysis shows that proposed performance index is effective in reducing uncontrolled structural seismic responses.

1. Introduction and Previous Studies
Semi-active control systems are very good alternatives to active control systems in earthquake engineering applications as their power requirements are relatively small in comparison to active control systems. Magneto-rheological (MR) dampers are one of the most promising semi-active control devices. There have been some interesting efforts on control algorithms of MR dampers during the last decade.

A semi active control method for MR dampers was analytically proposed to suppress earthquake structural vibration in Zizouni et al. [1]. Their controller design was based on Linear Quadratic Regulator (LQR) with a robust feedback law to control the input energy of MR damper. Their numerical example was a three story structure with an MR damper installed in the first floor [1]. A six-story benchmark problem with semi-active controller based on artificial neural network (ANN) was studied numerically in Vadtala et al. [2]. Linear quadratic regulator (LQR) was used to generate optimal control forces. Their MR damper was modeled based on parallel plate Bouc-Wen model [2]. The effectiveness of optimal semi-active dampers for reaching the optimum gains of the response of the adjacent buildings connected by magneto-rheological (MR) dampers subjected to seismic motion was examined analytically in Uz [3]. For his numerical simulations a SIMULINK block in MATLAB program was developed to compute the desired control forces at each floor level and to the obtain number of dampers [3]. A new self-powered and sensing semi-active control system based on MR damper was presented in Wang et al. [4]. Numerical simulations for seismic protection of elevated bridges equipped with their system excited by two historical earthquakes were conducted [4]. Synthesis of full-state and limited state flexible mode shape (FMS)-based controllers for suppression of free and forced vibration of a cantilever beam fully and partially treated with the (MR) fluid was given in Rajamohan et al. [5]. Response attenuation of seismically excited adjacent buildings connected by a MR damper was studied using semi-active LQR controller design in Mohebbi and Bagherkhani [6]. Numerical results for a 5-story and a 3-story interconnected buildings were obtained in terms of peak and root mean square responses [6]. Another
analytical study on MR dampers is on optimal static output feedback (OSOF) control to obtain the control force desired from a MR damper fitted between ground and first story of a three-story numerical building model Chandiramani and Purohit [7]. They used the modified Bouc–Wen model for obtaining damper response. Two semi-active control methods for seismic protection of structures using MR dampers were presented in Bitaraf et al. [8]. They used a simple shear frame structure incorporating two MR dampers for numerical simulations under two far field and two near field earthquakes [8]. A direct semi-active control method introduced to mitigate the seismic responses of structures equipped with magneto-rheological (MR) dampers was presented in Mohajer et al. [9]. Their algorithm was applied to control seismic vibrations of a three-story and a eleven-story sample shear building that were equipped with the MR damper control system [9].

An instantaneous optimal control performance index for MR dampers is presented in this paper. Simple soil-structure interaction is considered in the derivation of the formulation. The mathematical formulation of the system and performance index are given below.

2. System Model Considering Soil-Structure Interaction (SSI)
A shear structure incorporating an MR damper with fixed base and simple SSI effect is shown in figure 1. MR damper is attached between the ground and first story. SSI effect is simply taken into account by swaying displacement $x_0$ and rocking displacement $\theta_0$ of the foundation. $x_0$ is the earthquake displacement and $x_i$’s ($i=1$ to $n$) are the relative displacements of the stories. Where $n$ is the total number of the stories. The dynamic equation of the motion of this structure under the effect of an earthquake can be written as

$$
M_{SSI} \ddot{X}(t) + C_{SSI} \dot{X}(t) + K_{SSI} X(t) = -M_{soil} \ddot{x}_g(t) + L F(\dot{x}, i, t)
$$

(1)

here $M_{SSI}$, $C_{SSI}$, and $K_{SSI}$ are mass, stiffness, and damping matrices of the $n$ story structure considering SSI respectively. $M_{soil}$ is the soil related mass matrix defined below Bekdas and Nigdeli [10]. A single MR damper is implemented to this structure. $X(t)$, $\dot{X}(t)$, and $\ddot{X}(t)$ are the $(n+2)$ dimensional time dependent relative acceleration, relative velocity and relative displacement vectors, respectively. $L$ is the $(n+2) \times r$ dimensional location matrix of MR damper forces, where $r$ is the number of MR dampers in the system. For the specific example used in this study $r$ is 1. $m_i$, $c_i$, $k_i$ and $I_i$ in Fig. 1 are the mass, damping, stiffness and mass moment of inertia of each story. $F(\dot{x}, i, t)$ is the velocity and time dependent MR damper force and $i$ represents the electrical current.

![Figure 1. Building model considering SSI](image)

Structural parameters considering SSI can be defined as [10-11].
In the expressions given above $c_s$ and $k_s$ are the swaying damping and stiffness, while $c_r$ and $k_r$ are the rocking damping and stiffness of the foundation. Swaying damping and stiffness are related with the lateral vibration of the foundation, while rocking stiffness and damping are related with the rotation of the foundation.

$m_b$ is the mass and $I_b$ is the moment of inertia of the foundation. $z_i$ ($i=1$ to $n$) is the length between the base and the corresponding story. $I_i$ is the moment of inertia of each story. $\mathbf{M}$ is the conventional ($n$ by $n$) diagonal mass matrix of the superstructure, which is Diag($m_1, m_2, m_3, \ldots, m_n$). The other sub matrices are damping and stiffness matrices of the superstructure $\mathbf{C}$, $\mathbf{K}$ ($n$ by $n$ dimensional). More information on the formulation given above can be obtained in [10-11].

The semi-active MR damper model used in this paper is piece-wise input-invertible and the detailed formulation is not shown here because of space constraints. Piece-wise input-invertible MR damper formulation is related to the electrical power requirement of the device. The formulation in details can be read from [12-13]. The modified Bouc-Wen model can be defined with the following formulas. The MR damper force $F(\dot{x}, i, t)$ in Eq. (1) can be calculated as

$$F(\dot{x}, i, t) = c_i \dot{y} + k_i (x - x_0)$$ (3)

here $\dot{x}$ and $x$ are the damper velocity and displacement. MR damper initial displacement is $x_0$. The other parameters in Eq. (3) can be defined as

$$\dot{y} = \frac{1}{(c_0 + c_i)} \left[ \alpha z + c_0 \dot{x} + k_0 (x - y) \right]$$ (4)
$$\dot{u} = - \eta (u - i)$$ (5)
$$\alpha = \alpha(u) = \alpha_a + \alpha_s u \quad , \quad c_i = c_i(u) = c_{ia} + c_{is} u \quad , \quad c_0(u) = c_{0a} + c_{0s} u$$ (6)
$$\dot{\gamma} = - \gamma [\dot{x} - \dot{\gamma}] [\dot{x}]^{q-1} - \beta (\dot{x} - \dot{y}) [\dot{x}]^q + A(\dot{x} - \dot{y})$$ (7)
γ, β and α are adjustable parameters for controlling the linearity and behavior of the MR damper before and after yielding. More information on formulation of the MR damper force approach, can be found in [14,16]. In the next section formulation of the proposed performance index is given briefly.

3. Instantaneous Optimal Control Performance Index

An instantaneous optimal control performance index (Ins-Abs) for MR dampers considering simple SSI is analytically defined in this section. This is another version of instantaneous optimal control algorithms which were defined in the pioneering work of Yang et al. [15]. Before defining the performance index, absolute accelerations should be explained. The absolute accelerations for the given system can be written by using the parameters defined in Eq.(1) as

\[ \ddot{X}_{abs}(t) = \ddot{X}(t) + v \dot{x}_g(t) \]

(8)

Here \( v \) is the \((n+2)\) dimensional location vector of earthquake excitation. Similarly, absolute velocities can be presented as

\[ \dot{X}_{abs}(t) = \dot{X}(t) + v \dot{x}_g(t) \]

(9)

where \( \dot{x}_g(t) \) is the velocity of the earthquake. The state vector considering absolute displacement and absolute velocity can be defined by

\[ Z_{SSI-abs}(t) = [X(t) \dot{X}(t)]^T + [v \dot{x}_g(t) \dot{x}_g(t)]^T \]

(10)

here \( x_g(t) \) is the displacement of the earthquake. The first term on the right hand side of the above equation is the conventional state vector. After defining the absolute state vector, the time dependent performance index can be stated as

\[ J_{abs}(t) = \int_0^{\infty} Z_{SSI-abs}^T(t) Q Z_{SSI-abs}(t) + F_d^T(t) R F_d(t) \]

(11)

Here \( Q \) and \( R \) denote the control weighting matrices. To obtain the control force the expression given above should be minimized. By necessary mathematical operations, which are not shown here, the resulting proposed control force can be obtained as

\[ F_d(t) = -R^{-1}B^T Q \dot{Z}_{SSI-abs}(t) \]

(12)

It should be noted here that the formulation of resulting control force and instantaneous optimal control performance are given very briefly by omitting many details here. More information on the analytical formulation and detailed derivation of this performance index can be obtained from the unpublished work of the author by request (which is submitted for possible publication and currently under review). Numerical example is shown in the next section.

4. Numerical Example

In all numerical calculations, the simulations created in MATLAB Simulink are used and the 4th order Runge Kutta method is chosen as the numerical method. During the dynamic analysis, it is observed that the difference of the numerical method in the use of Simulink did not affect the results much. However, the commonly used Runge Kutta method is preferred over Euler, Dormand-Prince, Heun and extrapolation methods. A simplified three-story scaled test structure is used as a numerical example. The test structure is a shear building with single translational degree of freedom per story. The height of each story is taken as 0.5 m. Three different cases of soil for the foundation of the building are considered in the numerical example to discuss the soil-structure interaction effect. These soil cases are fixed base,
dense soil and soft soil. The properties of dense and soft soil are given in Table 1. Table 1 includes the values of swaying damping ($c_s$) and stiffness ($k_s$) and the rocking damping ($c_r$) and stiffness ($k_r$) of the foundation. All these parameters are scaled as well. In the fixed base case SSI effects are not taken into account. The mass of the base is 200 kg, while the mass of each story is identical and equal to 100 kg. The stiffness of each story is identical and equal to $6.84 \times 10^3$ N/cm. The damping of each story is also identical and equal to 50 Ns/m. The mass moment of inertia of the foundation is taken as 19,600 kgm$^2$ and mass moment of inertia of each story is 13,100 kgm$^2$. The MR damper used in the numerical model has a 3 kN capacity. MR damper model parameters are shown in table 2. These parameters are taken from [16,17].

Table 1. Properties of the soil types

| Soil Type   | $k_s$(N/m)  | $k_r$(N/m)  | $c_s$(Ns/m)  | $c_r$(Ns/m)  |
|-------------|-------------|-------------|--------------|--------------|
| Soft Soil   | $1.91 \times 10^5$ | $7.53\times10^7$ | $2.19\times10^4$ | $2.26\times10^6$ |
| Dense Soil  | $5.75 \times 10^6$ | $1.91\times10^9$ | $1.32 \times 10^5$ | $1.15\times10^7$ |

Table 2. Damper parameters for modified Bouc Wen model.

| Damper parameter | Value       | Damper parameter | Value       |
|------------------|-------------|------------------|-------------|
| $c_{0a}$         | 21.0 N.s/cm | $\alpha_a$      | 140 N/cm    |
| $c_{0b}$         | 3.50 N.s/cm | $\alpha_b$      | 695 N/cm.v  |
| $k_0$            | 46.9 N/cm   | $\gamma$        | 363 cm$^2$  |
| $c_{1a}$         | 283.0 N.s/cm| $\beta$         | 363 cm$^2$  |
| $c_{1b}$         | 2.95 N.s/cm | A                | 301         |
| $k_1$            | 5.00 N/cm   | $\eta$          | 2           |
| $x_0$            | 14.3 cm     |                 | 190 s$^{-1}$|

Northridge (1989) and Loma Prieta (1984) earthquakes are used in this study. The peak ground accelerations of these earthquakes are; 0.63 g for Northridge and 0.44 g for Loma Prieta. The dynamic simulations are performed for 20 s. It should be stated here that as the example structure is scaled, the earthquake excitations used in this study are scaled as well. A scaling factor of 1/10 is used on the earthquakes. Weighting matrices $Q$ and $R$ that were defined in Eq. 11 are selected as follows.

$$Q = \begin{bmatrix} K_{SSI} & C_{SSI} \\ C_{SSI} & M_{SSI} \end{bmatrix} ; \quad R = 10^{-3}$$

(13)

In this section abbreviations are defined as NC: uncontrolled case, fx : fixed base, ds : dense soil and ss : soft soil case. For the scaled structure, the maximum displacement responses obtained from dynamic analysis for all different control cases and soil types are shown in table 3.
Table 3. Maximum Displacement Responses of the Scaled Structure

| Soil and Control Case | Northridge (cm) | Loma Prieta (cm) |
|-----------------------|-----------------|-----------------|
| Story No              | 1               | 2               | 3               | 1               | 2               | 3               |
| NC-fx                 | 0.40            | 0.62            | 0.81            | 0.40            | 0.52            | 0.72            |
| NC-ds                 | 0.94            | 1.06            | 1.08            | 0.42            | 0.78            | 1.16            |
| NC-ss                 | 0.56            | 0.77            | 0.95            | 0.43            | 0.78            | 1.16            |
| Ins-Abs-fx            | 0.37            | 0.50            | 0.62            | 0.17            | 0.30            | 0.37            |
| Ins-Abs-ds            | 0.60            | 0.80            | 1.02            | 0.32            | 0.49            | 0.86            |
| Ins-Abs-ss            | 0.20            | 0.25            | 0.30            | 0.33            | 0.46            | 0.82            |

It can be obtained from table 3 that the maximum uncontrolled response reduction percentages for Ins-Abs control algorithms range between 7% to 23% for the Northridge earthquake. Table 3 also shows that SSI effects increased structural displacements. For the Northridge earthquake, the maximum uncontrolled story displacement reduction percentage for dense soil case is to the extent of 36% for Ins-Abs. For the Loma Prieta earthquake and dense soil case the maximum uncontrolled response reduction percentages range from 23% to 58% for Ins-Abs. MR damper hysteresis force velocity curves are presented in figure 2 a & b for fixed base case and both earthquakes. The velocity is the relative velocity of first story with respect to the ground, as the MR damper is assumed to be attached between first and second story. Ins-Abs control produces small control forces. Changes in the absolute accelerations are investigated in this paper for different soil types. The maximum absolute acceleration of each story for the uncontrolled cases are presented in figure 3 for both earthquakes. It can be indicated from figure 3 that consideration of SSI significantly increased the absolute accelerations in comparison with the fixed base case. The dense soil case produced higher absolute accelerations than the soft-soil case. It should be stated here that, this comparison is given for absolute accelerations. If relative accelerations were investigated, the expected result was that soft soil case should have produced the maximum relative accelerations.

Figure 2. MR Damper Force Velocity Curves for Fixed Base Structure (a: Northridge earthquake, b: Loma Prieta earthquake)
5. Seismic Energy Based Comparison

Before giving the numerical comparison about seismic energies, the energy distribution equation of a structure is briefly defined. The total seismic energy input of a structure with SSI can be obtained by multiplying Eq. 1 with the transpose of velocity matrix $\mathbf{X}(t)$ and by taking the integrals in each side of the equation from 0 to $t$. This operation yield to the energy equilibrium of the system as

$$
\int_0^t \mathbf{X}(t) \mathbf{M}_{SSI} \dot{\mathbf{X}}(t) dt + \int_0^t \mathbf{X}(t) \mathbf{C}_{SSI} \ddot{\mathbf{X}}(t) dt + \int_0^t \mathbf{X}(t) \mathbf{K}_{SSI} \mathbf{X}(t) dt =
$$

$$
-\int_0^t \mathbf{X}(t) \mathbf{M}_{soil} \ddot{\mathbf{x}}_g(t) dt + \int_0^t \mathbf{X}(t) \mathbf{L} \mathbf{F} (\ddot{\mathbf{x}}, \dot{\mathbf{x}}, t) dt
$$

(14)

The first integral term in Eq. 14 is the kinetic energy, the second term is the damping energy, and the third term is the strain energy. The first term on the right hand side of Eq. 14 is the total seismic energy input of the structure and the second term on the right hand side is the semi-active control energy. The change in the total maximum seismic energy (SE) is investigated for different soil types. For Loma Prieta earthquake, change in the maximum seismic energy for different soil and control types is shown in table 4. For the Loma Prieta earthquake and fixed base case, uncontrolled structure seismic energies are reduced by 5% for Ins-Abs control. For the soft soil case, it is found that Ins-Abs decreased the uncontrolled seismic energy by 10%. For dense soil and Loma Prieta earthquake Ins Abs case increased the seismic energy to the extent of 24%. The conclusions obtained from this study are given in the next section.

![Figure 3. Changes in the Absolute Accelerations for Different Soil Types](image)
Table 4. Maximum Seismic Energies

| Control Case↓ | Loma Prieta (Joule) |
|---------------|----------------------|
| Soil Case→    | fx  ss  ds           |
| NC            | 302  554  1782       |
| Ins-Abs       | 287  501  2229       |

6. Conclusion
An instantaneous optimal control performance index for magneto-rheological dampers is proposed in this study. SSI is included in the derivation of the performance index and in the numerical simulations. The conclusions obtained from this study are presented below.

- It is obtained that the proposed performance index reduces both uncontrolled structural displacements and absolute accelerations.
- Seismic energy based evaluations showed that SSI effects increase the seismic energy of the fixed base structure.
- Considering SSI effects increases the absolute accelerations as well as the structural displacements.

Finally, it should be stated here that all the results given in this paper are preliminary and obtained from numerical simulations of a scaled test structure. Future studies will include the dynamic analysis of the performance indices considering different soil flexibility parameters. Shear wave velocity, density and hysteretic damping of the soil will be considered in future studies.

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