Conference Paper

Solving a Real Problem in Plastic Industry: A Case in Trim-loss Problem

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Abstract

In this paper, a cutting plane model is presented for solving a problem in a cast polypropylene (CPP) plastic film manufacturer. The company produces plastic rolls from plastic pellets with widths ranging from 3050 mm to 3250 mm. The plastic rolls are trimmed according to customer’s orders. In prior to the trimming process, the production planning and inventory control (PPIC) department scheduled the machines and arranged the plastic trim compositions manually. In this work, the plastic trimming problem is solved by applying the trim loss model. Since trimmed loss problem is an NP-hard problem. In this case, the permutations are selected in advance so that the total length is feasible to the machine length. The computation is carried out using visual basic for application (VBA). The model outcomes are then used for optimizing the machine scheduling process. Modified earliest due date is proposed to schedule in which machines customer’s orders should be done. The machines scheduling represents the company conditions and the cutting production can be scheduled for daily basis.

Keywords: Cutting plane; cutting stock; earliest due date; machine scheduling; non polinomial-hard problem.

1. Introduction

In recent years, real-world trim loss (or cutting stock) problems have challenged researchers in optimization areas due to the diversity of the problems. There are many solutions that have been developed in practice. Delorme et al. [1], modeled the bin packing and cutting stock problem, Furini and Malaguti [2], use the mixed-integer model to solve the cutting-stock problem for multiple stock size, Kallrath et al. [3], solved the real-world cutting stock problem in the paper industry. Besides modeling in linear programming, many researchers also solved those problems heuristically. Cui et al. [4] developed a new model and proposed a two-phase heuristics algorithm to solve the cutting problem with usable leftovers. Tanir et al. [5], proposed heuristic dynamic programming to solve the cutting stock problem in steel industries. While Rietz and Dempe [6] used the linear relaxation called a gap to solve that problem.
In this work, a cutting plane is modeled for solving a problem in a cast polypropylene (CPP) plastic film manufacturer. The plastic is used as food packaging. The company produces plastic rolls from plastic pellets with widths ranging from 3 050 mm to 3 250 mm. The plastic rolls are trimmed according to customer’s orders. The waste produced from trimming large plastic rolls is recycled into second-grade plastics pellets. This second-grade plastics pallet will be sold to other company with a lower profit than the primary product of this company. In prior to the trimming process, the PPIC department scheduled the machines and arranged the plastic trim compositions manually. The modified earliest due date (EDD) algorithm is used to schedule the jobs so that the waste produced from trimming large plastics rolls is minimized.

2. Model

2.1. Trim-loss problem

Many researchers developed the trim-loss problem models, Belov [7] resumed those models in his work. One of the basic model is given in the Bazaraa et al. [8] as follow

\[ \text{Minimize } \sum_{j=1}^{n} X_j \]

subject to

\[ \sum_{j=1}^{n} a_{ij} X_j \geq D_i, \]

\[ x_i \geq 0 \quad i = 1, \ldots, n \]

\[ x_i \text{ integer, } j = 1, \ldots, n \quad j = 1, \ldots, n \]

It should be noted that the trim-loss problem is generally an NP-hard problem. In this case, the permutations are selected in advance so that the total length is feasible to the machine length. To meet the real condition in the company, the basic model has been extended as the proposed model as follows.

\[ \text{Min } \sum_{j=1}^{n} W_j X_j + \sum_{i=1}^{m} L_i \left( \left( \sum_{j=1}^{n} a_{ij} X_j \right) - D_i \right) \]  

such that.

\[ \sum_{j=1}^{n} a_{ij} X_j \geq D_i \quad i = 1, \ldots, m \]  

\[ W_j, a_{ij}, X_j, D_i \geq 0 \quad j = 1, \ldots, n \]  

\[ W_j, a_{ij}, X_j, D_i \text{ integer} \]
where,

- \( W_i \): waste produced if the large plastic roll is trimmed using permutation \( j \)
- \( L_i \): the length of product \( i \)
- \( a_{ij} \): number of product \( i \) if the large plastic roll is trimmed using permutation \( j \)
- \( X_j \): decision variable -- number of trim roll if the large plastic roll is cut follows the order of permutation \( j \)
- \( D_i \): demand for product \( i \)
- \( n \): number of feasible permutation
- \( m \): number of products

Here, the product is the small plastic roll, which is produced by trimming the large plastic roll. The objective function of this model, Equation 1, is to minimize the waste. The total number of products produced by trimming the large plastic roll should not exceed the demand in Equation 2. All of variables are non-negative (Equation 3) and integer (Equation 4).

### 2.2. Machine scheduling

In prior to make the schedule of the machine, it is necessary to calculate the time needed to process a job (\( p_j \)) and convert the total waste in mass (kg) of the total waste in correspond to Equation 5, Equation 6 and Equation 7, respectively.

\[
\text{Mass} = \text{Volume} \times \rho \quad (5)
\]

\[
\text{Volume} = \text{thickness} \times \text{Length} \times \sum \text{Width} \quad (6)
\]

The time needed is formulated as follows:

\[
\text{Time} = \frac{\text{Length}}{\text{Velocity}} \times N \quad (7)
\]

The velocity of the machine depends on the thickness of the plastics. The modified earliest due date (EDD) is used to solve this problem (see e.g. Pinedo [9]). Modified EDD is common for solving industrial problems, e.g. Demir et al. [10] solved the process planning using weighted EDD. In this case, the EDD is modified such that the waste produced by each job in a parallel system is minimum. In this case, the decision variables are due date, time for finishing a job in each machine, and weight of a job. The weight of the job is calculated by dividing due date to the priority of that job. The priority value is decided by the company. Due date in this scheduling problem is not a date, but only
a priority number since the customers do not give the exact due date. The larger the priority number refers to the short due date.

For every scheduled job, first waste differences are calculated if the jobs are produced by Egan 1 or Egan 3 (since both machines are identic) and Egan 2. Then, the weighted due date is defined as waste difference divided by weight. Afterward, the weighted due date is sorted descending (see Table 5 as an example) and finally the sorted job is achieved. Job 1 should be scheduled in advance since it can cause greater waste if it is scheduled on an unsuitable machine. The company also can give a certain priority to a job based on customer demand. The developed software for solving this problem has flexibility in determining the job priority. It can be calculated as waste differences divided by weight or directly determined by the company.

2.3. Numerical example

The company has three machines for rolling the plastic, Egan 1, Egan 2 and Egan 3. Egan 1 and Egan 3 can roll the plastic sheet with a width from 2 900 mm to 3 100 mm, while the Egan 2 can roll the plastic sheet with a width from 3 100 mm to 3 300 mm. There are two types of plastic, transparent and metallic. Metallic plastic is produced by coating the transparent one, and it is coated after the transparent plastic is trimmed according to its order. Additionally, the plastic thickness is also varying. It depends on the customer's order. However, one roll plastic will have the same thickness. Hence, the trim loss problem for transparent plastic which is already grouped based on its thickness is modeled.

In this model, first, the feasible permutation need to find first. For example, there is an order coded as OPUS 731 #25 25 12000. This code means, the plastic’s type is OPUS 731, with thickness 25 μm and length 12 000 m. The customer needs 20 rolls plastic with length 920 mm, 23 rolls with length 1 000 mm and 38 rolls with length 1 040 mm. Since the order has max length 1 040 mm, it is better to use Egan 2, which can roll plastic up to 3 300 mm. From the large roll, i.e. 3 300 mm, it can produce max three small rolls with the same length (see Table 1). All possible permutations from the number of small rolls are depicted in Figure 1.

Not all possible permutations have a feasible solution. For this purpose, the non-feasible solution is erased (see Table 2, permutation number 5 and number 7) before the optimum solution is calculated. Afterward, the permutation that has waste greater than or equal one of the small roll lengths is excluded (see Table 2, permutation no. 3).
TABLE 1: Number of small rolls with the same length can be produced from the large one

| Machine       | Length | Max | Number |
|---------------|--------|-----|--------|
| Egan 2 3 300 mm | 920 mm | 3   | 0,1,2,3 |
|               | 1000 mm| 3   | 0,1,2,3 |
|               | 1040 mm| 3   | 0,1,2,3 |

Figure 1: All possible permutation.

However, since machine Egan 2 can be adjusted from 3 100 mm to 3 300 mm, then the permutation should be adjusted so that the total length of trimming products is in between 3 100 mm to 3 300 mm. In the adjusted permutation algorithm, we check the feasibility solution and the total of trimming length. Both measurements should be in the range of the machine setting. The adjusted permutation of this case is given in Table 3. There are 10 permutations with a total length of 3 100 mm to 3 120 mm.
Using the adjusted permutation and excel solver for solving Equation 1 to Equation 4, the optimal result is obtained as it is listed in Table 4. The demand is fulfilled with the total waste 2 780 mm. The company should produce 27 rolls of OPUS 731 #25 25 12000, and after trimming the large rolls, it produces total waste 2 780 mm (in total width of small rolls).
2.4. Scheduling the machines

Before scheduling the machines, the needed time was calculated and convert the waste from length to mass (kg). To convert the waste length to mass, Equation 5 is used. In this case, since all unit measurements are in mm, then

\[
Mass(kg) = \frac{\text{Thickness} \times \text{Length} \times \sum \text{Width}}{10^6} \times 0.91
\]  

\[
Mass(kg) = \frac{25 \times 12000 \times 2780}{10^6} \times 0.91 = 758.94
\]  

The time needed is calculated using Equation 7. For example, suppose the plastic thickness is 25 µm and the machine velocity is 248 m min\(^{-1}\). The more thickness of the product, the slower the velocity of the production process of the machine is. In order to produce 27 large rolls of OPUS 731 #25 25 12000, the time required is as follows

\[
Time = \frac{12000}{248} \times 27 = 1 \ 306.45 = 21.77 \ h.
\]  

The objective of the machine scheduling is to minimize makespan. As it is stated, the company has three machines, Egan 1, Egan 2 and Egan 3. Machines Egan 1 and Egan 3 are identical. Those machines work in parallel and work based on the given jobs. There is no pre-emption in the process while the machines operate. In this study, due to the processing time needed in calculating the adjusted permutation, the scheduling is limited only for seven jobs. In every job, it has maximum seven jobs and seven types of small plastic roll width. Use \(Rm||L_{\text{max}}\) to notate this condition. For scheduling, the jobs, mass of the produced waste, time needed to process the jobs for every machine, and the priority of the jobs (weight) should be known in advance.

The modified EDD is applied to schedule the jobs. The modified EDD calculation for this small example is given in Table 5. The sorted job is given in the last column of Table 5, and the respective Gant chart is depicted in Figure 2. For this example, Job 1 is scheduled in machine Egan 1, or Egan 3; Job 3 can be done also in Egan 1 or Egan 3 since those two jobs have the smallest waste if they are produced in Egan 1 or Egan 3. Since the Job 1 in Egan 1 is already scheduled, then the Job 3 should be scheduled in Egan 3. Egan 1 and Egan 3 now are occupied, so Job 2 only can be scheduled in Egan 2; the last Job 4 also should be scheduled in Egan 2, since Egan 1 and Egan 3 are occupied. The total waste produced using this proposed schedule is 981.7 kg, and the time completion for the whole orders (makespan) is 21.77 h.
### Table 5: Scheduling Case Example

| Job | weight = DueDate / Priority | Waste E1 (kg) | Waste E2 (kg) | Waste E3 (kg) | Waste Diff (kg) | Job Priority = Waste Diff / weight | #Roll E1 (h) | #Roll E2 (h) | #Roll E3 (h) | Sorted Job |
|-----|-----------------------------|---------------|---------------|---------------|----------------|-----------------------------------|-------------|-------------|-------------|------------|
| 1   | weight=1/2 = 0.5            | 0.0           | 824.5         | 0.0           | 824.5          | 1.648.9                           | 21.77       | 21.77       | 21.77       | Job 1      |
| 2   | weight=1/1=1                | 160.5         | 370.2         | 160.5         | 209.7          | 209.7                             | 3.87        | 3.87        | 3.87        | Job 3      |
| 3   | weight=2/1 = 2              | 0.0           | 457.7         | 0.0           | 457.7          | 228.9                             | 11.29       | 11.29       | 11.29       | Job 2      |
| 4   | weight=2/1 = 2              | 384.1         | 611.5         | 384.1         | 227.4          | 113.7                             | 7.90        | 7.90        | 7.90        | Job 4      |

![Gantt Chart](image)

**Figure 2**: Scheduling result in a Gant chart.

### 3. Conclusion

This study attempted to solve the problem for a plastic company to minimize the plastics waste produced by trimming large plastics rolls. Additionally, the jobs are scheduled so that the plastics waste is minimized. This study performed a model using trim-loss problem and modified the earliest due date algorithm for scheduling the jobs. In the current state, the algorithm only works for daily basis scheduling. In the future, the weekly basis scheduling for minimizing waste would be proposed to meet the company's requirements.

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