Abstracts of Seminars given at the Workshop on
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This pre-print contains the abstracts of seminars (including key references) presented at the ESI workshop on mathematical problems in quantum gravity held during July and August of 1996. Contributors include A. Ashtekar, J. Baez, F. Barbero, A. Barvinsky, F. Embacher, R. Gambini, D. Giulini, J. Halliwell, T. Jacobson, R. Loll, D. Marolf, K. Meissner, R. Myers, J. Pullin, M. Reisenberger, C. Rovelli, T. Strobl and T. Thiemann. While these contributions cover most of the talks given during the workshop, there were also a few additional speakers whose contributions were not received in time.
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Quantum theory of geometry
Abhay Ashtekar

This was primarily a review talk, based largely on joint work with Jerzy Lewandowski. Over the last three years, a new functional calculus has been developed on the quantum configuration space of general relativity without any reference to a background geometrical structure in space-time (such as a metric). The purpose of this talk was to indicate how this machinery can be applied to systematically construct a quantum theory of geometry. The kinematical Hilbert space of quantum gravity was presented. States represent polymer-like, 1-dimensional excitations of geometry. Regulated Operators corresponding to areas of 2-surfaces were introduced on the kinematical Hilbert space of quantum gravity and shown to be self-adjoint. Their full spectrum was presented. It is purely discrete and contains some physically interesting information. First, the “area gap”, i.e., the value of the smallest non-zero excitation, contains information about the global topology of the surface. Second, in the large eigenvalue limit, the eigenvalues become closer and closer to each other such that $|a_{n+1} - a_n| \leq [(l_P/2\sqrt{\alpha_n}) + O(l_P^2/a_n)]l_P^2$, where $a_n$ and $a_{n+1}$ are the consecutive eigenvalues and $l_P$ the Planck length. This shows why the continuum limit is such an excellent approximation. It also has a more interesting implication for the Hawking effect. Because we do not have an equal level spacing, the types of potential problems pointed out by Bekenstein and Mukhanov do not arise and the semi-classical approximation used by Hawking in his calculation of the black-body spectrum turns out to be excellent.

These and numerous other results are providing more and more intuition for the nature of quantum geometry. This framework is to quantum gravity what familiar differential geometry is to classical general relativity. Like differential geometry, it has no dynamical content; specific field equations are not involved. Just as the formulae for lengths, areas and volumes of differential geometry are valid in all theories of gravity (involving a space-time metric), the formulae, identities and theorems involving our quantum states and operators would hold in all dynamical theories of gravity (which include n-beins which are canonically conjugate to connections).

A. Ashtekar and J. Lewandowski, [gr-qc/9602046]; Class. & Quantum Grav. (in press).

Unforeseen non-commutativity between geometric operators
Abhay Ashtekar

This was a report on some joint work with Alejandro Corichi, Jerzy Lewandowski and Jose Antonio Zapata.
One of the implications of the work reported in the first talk is that area operators associated with different operators do not always commute. This is at first surprising because the classical formula for areas involves only triads, without any reference to connections and from the basic Poisson bracket relations one expects the triads to commute among themselves. It turns out, however, that the naive expectation is incorrect. The reason is that the formula for areas involves triads which are smeared only on 2-surfaces rather than in 3 dimensions and the Poisson brackets between such objects are, strictly speaking, singular. (Furthermore, in our framework based on holonomies, triad operators which are smeared in 3 dimensions are not likely to be well-defined!)

To analyze this issue in detail, we examine the Poisson algebra between the following phase space functions: cylindrical functions of (smooth) connections and triads smeared on 2-surfaces. (Incidentally, since the triads have density weight one, they are in fact 2-forms and it is thus geometrically natural to smear them on 2-surfaces.) If we assume naively that the smeared triads commute, we run into a problem with the Jacobi identity; the naive Poisson algebra is not a Lie algebra and is therefore incorrect. One can regulate the naive algebra carefully to obtain a Lie algebra. Then, one finds that the (2-dimensionally) smeared triads fail to Poisson commute. The commutators between the quantum triad operators just mirror this correct Poisson algebra and this is why the area operators fail to commute.

A. Ashtekar, A. Corichi, J. Lewandowski and J.A. Zapata, CGPG pre-print.

**Geometry of quantum mechanics**

Abhay Ashtekar

This talk summarized joint work with Troy Schilling which constitutes his 1996 Ph.D. thesis at Penn State.

In the way we normally formulate these theories, classical mechanics has deep roots in (symplectic) geometry while quantum mechanics is essentially algebraic. However, one can recast quantum mechanics in a geometric language which brings out the similarities and differences between the two theories. The idea is to pass from the Hilbert space to the space of rays, i.e. to the “true” space of states of quantum mechanics. The space of rays—or the projective Hilbert space, is in particular, a symplectic manifold, which happens to be equipped with a further Kähler structure. Regarding it as a symplectic manifold, one can repeat the familiar constructions from classical mechanics. For example, given any function, one can construct its Hamiltonian vector field. If one uses the expectation value of the Hamiltonian operator as the function, it turns out that the resulting “classical” symplectic evolution is precisely the (projection of the) Schrödinger evolution on the Hilbert space. Roughly, properties of quantum mechanics which it “shares” with classical mechanics use only the symplectic structure on the projective Hilbert space. The
“genuinely” quantum properties such as uncertainties and probabilities refer to the Kähler metric. Thus, purely in mathematical physics terms, one can regard quantum mechanics as a special case of classical mechanics, one in which the phase space happens to have a Kähler structure (which then enables one to do more.) This geometrical formulation of quantum mechanics sheds considerable light on the second quantization procedure and on semi-classical states and dynamics.

After the work was completed, we found that many of our results were discovered independently by a number of authors, most notably L. Hughstone and by R. Cirelli, A. Maniá and L. Pizzochero.

A. Ashtekar and T. Schilling; in: The Proceedings of the First Canadian-Mexican-American Physical Societies’ Conference, edited by A. Zapeda (American Institute of Physics, NY 1995.)

T. Schilling; *Geometry of Quantum Mechanics*, Ph.D. Thesis, Penn State, 1996.

**Probing quantum gravity through exactly soluble midi-superspaces**

*Abhay Ashtekar*

This talk summarized joint work with Monica Pierri which constituted part of her Ph.D. thesis.

The idea was to consider midi-superspaces which are simple enough to be exactly soluble both classically and quantum mechanically and use the solution to probe various nagging issues of quantum gravity such as the issue of time and the nature of the vacuum. The specific example presented was the midi-superspace of Einstein-Rosen (i.e., cylindrical) gravitational waves. This model was analyzed by Karel Kuchár already in the early seventies and by Michel Allen in the mid-eighties. However certain issues concerning boundary conditions, surface terms and functional analytic subtleties could not be discussed then. Using results on asymptotics and techniques of regularization that have been developed since then, their discussion can be completed to construct a complete quantum theory. We used this solution to construct a regulated, quantum space-time metric operator and address issues such as “light cone fluctuations”. That this is possible within the canonical framework is noteworthy since concerns are often expressed that the canonical quantization procedure may not be able to handle such “space-time” issues. The model has a well-defined, non-trivial Hamiltonian operator and a stable vacuum state (the eigenstate of the Hamiltonian with zero eigenvalue.) On general states, one can write the Schrödinger equation. However, the mathematical parameter in this equation has the physical interpretation of time only on semi-classical states. Finally, this solution can also be used to probe a key issue in our non-perturbative quantum gravity program: existence of operators
corresponding to traces of holonomies around closed loops. These operators do exist in spite of the fact that they involve smearing of the connection only in one dimension.

More recently, this model has been used to show the existence of certain unforeseen quantum gravity effects which can be large even when the space-time curvature is small.

A. Ashtekar and M. Pierri, gr-qc/9606085, J. Math. Phys., 37, 6250-70, (1996).
A. Ashtekar, gr-qc/9610008, Phys. Rev. Lett. 77, 4864-67 (1996).

Topological Quantum Field Theory
John Baez

The simplest sort of topological quantum field theory is $BF$ theory, where the Lagrangian is of the form $\text{tr}(BF)$, with $F$ being the curvature of a connection and $B$ being a Lie-algebra valued $(n-2)$-form in $n$ dimensions. When $n$ is 3 or 4 one can also add a "cosmological constant term" of form $\text{tr}(BBB)$ or $\text{tr}(BB)$, respectively. In this talk, I summarized what is known about $BF$ theory in dimensions 2, 3, and 4, as well as the equivalent state sum models. In particular, I described how state sum models of $BF$-like theories in 2 dimensions arise from certain monoids, while in 3 dimensions they arise from certain monoidal categories and in 4 dimensions from certain monoidal 2-categories (most notably the category of representations of a quantum group, which may be seen as a monoidal 2-category with one object). I also sketched how 4-dimensional $BF$ theory underlies Chern-Simons theory in 3 dimensions.

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John Baez and James Dolan, Higher-dimensional algebra and topological quantum field theory, Jour. Math. Phys. 36 (1995), 6073-6105.

John Baez, Four-dimensional $BF$ theory as a topological quantum field theory, to appear in Lett. Math. Phys., preprint available as q-alg/9507003.
The Entropy of 2-Part Systems

John Baez

I sketched the mathematical relationships between three constructions which might at first glance seem unrelated: Everett’s relative state formalism, the Gelfand-Naimark-Segal construction, and the construction of nontrivial spaces of states on ‘half’ of a 3-manifold from the single state of $BF$ theory with cosmological constant on the whole 3-manifold — the so-called ‘Chern-Simons state’. In all these constructions a single state gives rise to a Hilbert space of states. The last one gives a way of understanding the mathematical content of Smolin’s argument for the Bekenstein bound.

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John Baez, Quantum gravity and the algebra of tangles, Jour. Class. Quantum Grav. 10 (1993), 673-694.

Lee Smolin, Linking topological quantum field theory and non-perturbative quantum gravity, Jour. Math. Phys. 36 (1995), 6417-6455.

From Euclidean to Lorentzian Gravity: The Real Way

Fernando Barbero

The complex character of the Ashtekar variables has been one of the major issues to be understood in order to succeed in the quantization of general relativity by using the loop variables approach. The possibility of describing Lorentzian GR with a real Ashtekar connection can be realized by restricting oneself only to real canonical transformations in the transit from the SO(3)-ADM phase space to the SO(3)-Yang Mills phase space of the Ashtekar formalism [1]. The resulting Hamiltonian constraint is more complicated that the usual one but it is still written in terms of Ashtekar variables and, hence, loop variables can still be used to quantize it. From a Lagrangian point of view it is interesting to see if one can use local internal symmetries, instead of non-local ones, to write the action for Lorentzian general relativity. This can be achieved [2] by writing the Einstein-Hilbert action for a two parameter family of metrics whose signature can be adjusted at will by a suitable choice of these parameters.

References:

J. Fernando Barbero G. Phys.Rev.D51:5507-5510, (1995).

J. Fernando Barbero G. Phys.Rev.D54:1492-1499, (1996).
Semiclassical methods in the theory of constrained dynamics

A.O. Barvinsky

Operator realization of quantum constraints, the lowest-order structure functions and physical observables is found in the one-loop (linear in $\hbar$) approximation of the Dirac quantization for the general theory subject to first-class constraints. The general semiclassical solution of the quantum Dirac constraints is found. Semiclassical unitary equivalence of the Dirac and reduced phase-space quantization methods is established in terms of the conserved physical inner product in the space of physical states. The conservation of this inner product and its independence of the choice of gauge conditions is based on the Stokes theorem for a special closed form integrated over the physical subspace of the configuration space of the theory (superspace). Geometrical covariance properties of the quantum Dirac constraints with respect to contact canonical transformations and transformations of the basis of constraints is studied. Applications of these general methods of quantum constrained dynamics are considered in quantum cosmology of the early inflationary Universe.

References:

A.O. Barvinsky, The general semiclassical solution of the Wheeler-DeWitt equations and the issue of unitarity in quantum cosmology, Phys. Lett. B241 (1990) 201.

A.O. Barvinsky and V. Krykhtin, Dirac and BFV quantization methods in the 1-loop approximation: closure of the quantum constraint algebra and the conserved inner product, Class. Quantum Grav. 10 (1993) 1957.

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A.O.Barvinsky, Unitarity approach to quantum cosmology, Phys.Reports 230 (1993) 237.

A.O.Barvinsky, Geometry of the Dirac quantization of constrained systems, preprint ESI (1996).
Quantum Origin of the Early Universe and the Energy Scale of Inflation

A.O.Barvinsky

Quantum origin of the early inflationary Universe from the no-boundary and tunnelling quantum states is considered in the one-loop approximation of quantum cosmology. A universal effective action algorithm for the distribution function of chaotic inflationary cosmologies is derived for both of these states. The energy scale of inflation is calculated by finding a sharp probability peak in this distribution function for a tunnelling model driven by the inflaton field with large negative constant $\xi$ of nonminimal interaction. The sub-Planckian parameters of this peak (the mean value of the corresponding Hubble constant $H \approx 10^{-5} m_P$, its quantum width $\Delta H/H \approx 10^{-5}$ and the number of inflationary e-foldings $N \approx 60$) are found to be in good correspondence with the observational status of inflation theory, provided the coupling constants of the theory are constrained by a condition which is likely to be enforced by the (quasi) supersymmetric nature of the sub-Planckian particle physics model.

References:

A.O.Barvinsky and A.Yu.Kamenshchik, One-loop quantum cosmology: the normalizability of the Hartle-Hawking wave function and the probability of inflation, Class. Quantum Grav. 7 (1990) L181.

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A.O.Barvinsky and A.Yu.Kamenshchik, Tunnelling geometries: analyticity, unitarity and instantons in quantum cosmology, Phys.Rev. D50 (1994) 5093.

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Mode decomposition and unitarity in quantum cosmology
Franz Embacher

It is common folklore that the space of wave functions of quantum cosmology may not be decomposed into positive and negative frequency modes when the background structure (DeWitt metric and potential) does not admit symmetries. However, there are still perspectives for defining a generalized notion of preferred mode decomposition, starting from the space of solutions of the wave equation and the indefinite Klein-Gordon scalar product

\[ Q(\psi_1, \psi_2) = -\frac{i}{2} \int_{\Sigma} d\Sigma^\alpha (\psi_1^* \nabla_\alpha \psi_2 - \psi_2^* \nabla_\alpha \psi_1). \]

In the case of a positive potential \( U \) we outline a strategy in doing so.

The technical tool for analyzing the wave equation is the selection of a solution \( S \) of the Hamilton-Jacobi equation, which generates a congruence of classical trajectories, and a weight function \( D \). The pair \((S, D)\) is called WKB-branch. Within any WKB-branch, an operator \( H \) —satisfying a particular differential equation— may be chosen such that any solution of \( i\partial_t \chi = H \chi \) (with \( \partial_t \) the derivative along the trajectories) gives rise to a solution of the original wave equation \((-\nabla^2 + U)\psi = 0\). The wave functions constructed in this way define the space \( \mathcal{H}^+ \), generalizing the notion of positive frequency with respect to a WKB-branch.

The crucial step in the general strategy is to perform a natural choice of the operator \( H \) with respect to any WKB-branch, such that for any two infinitesimally close branches the respective decompositions coincide. It is achieved by iteratively solving the differential equation for \( H \), thus ending up with a formal expression (whose existence at least in simple cases can be inferred explicitly). Since arbitrarily high derivatives of the ingredients of the model (the DeWitt metric and the potential) appear, the proper mathematical existence of the preferred decomposition is likely to be related to the global structure of the model (and possibly to analyticity issues). Unitarity (i.e. a Schrödinger type evolution equation for wave functions) shows up only in terms of WKB-branches, much like components of a tensor show up only in a coordinate system. Details of this approach are presented in the two references cited below, along with speculations on a possible relation to the refined algebraic quantization program.

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F. Embacher, Decomposition and unitarity in quantum cosmology, preprint UWThPh-1996-64, gr-qc/9611006
F. Embacher, Mode decomposition and unitarity in quantum cosmology, Talk given at the Second Meeting on Constrained Dynamics and Quantum gravity, Santa Margherita Ligure, September 17–21, 1996, to appear in the Proceedings.
Gauge Invariance in the Extended Loop Representation

Rodolfo Gambini

The Gauss constraint in the extended loop representation is studied. It is shown that there is a sector of the state space that is gauge invariant. We determine necessary and sufficient conditions for states belonging to this sector. This conditions are satisfied by the extended Vassiliev invariants.

References:
C. Di Bartolo, R. Gambini, J. Griego, J. Pullin J Math. Phys. 36 6510, 1995.
C. Di Bartolo, ”The Gauss Constraint in the Extended Loop Representation” preprint gr-qc 9607014, 1996.

Views on Super-selection Rules in QT and QFT

Domenico Giulini

Many derivations of super-selection rules are purely formal in nature and hence do not make sufficiently clear the actual physical input that leads to them. In the context of quantum mechanics we critically review standard derivations of the super-selection rules for univalence and overall mass (so-called Bargmann super-selection rule.) The strong dependence on the required symmetry group is emphasized [1]. In particular, it is pointed out that in order for mass to define a super-selection rule it should be considered as a dynamical variable. We present a minimal extension of the dynamics of \( n \) point particles interacting via some Galilei invariant potential in which mass it also treated as dynamical variable. Here the classical symmetry group turns out to be given by an \( R \)-extension of the Galilei group. In contrast to the Galilei group its extension does have an action on the space of states including non-trivial superpositions of mass eigenstates. Hence no super-selection rules appear in this model [2]. Finally, we discuss the super-selection rule of electric charge (see chap. 6 of [1]). We emphasize the necessity of a consistent variational principle including charged configurations in its domain of differentiability. We present such a principle and show the unavoidable existence of additional degrees of freedom (“surface variables”) which measure the overall multipole moments of the charge distribution. Here super-selection rules result only if the surface variables are not in the algebra of observables, as it would be the case if one restricts to (quasi-) local observables. However, it is tempting to think of such a restriction as being conditioned dynamically and hence in principle only of approximate validity [3].
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Diffeomorphism Invariant Subspaces in Witten’s 2+1-Dimensional Quantum Gravity on $T^2 \times R$

*Domenico Giulini*

We address the rôle of large diffeomorphisms in Witten’s formulation of gravity as an ISO(2,1) gauge theory on a space-time $T^2 \times R$. In a “space-like sector” the classical phase space is $P = T^*(Q)$ with $Q = R^2 - \{0\}$ as configuration space. Using the vertical polarization the quantum state space becomes $H = L^2(Q, \mu_L)$, $\mu_L$ being the standard Lebesgue measure. By large diffeomorphisms we mean the orientation preserving mapping class group of $T^2$, given by $SL(2, Z)$. It acts by its defining representation on $Q$ and by its canonical lift on $P$. The action on the former is ‘wild’ in the sense that the isomorphism class of stabilizer subgroups is nowhere locally constant. The configuration space quotient is hence nowhere locally a manifold. In contrast, the lifted action on $P$ is free on the open and dense subset where the two vectors of coordinates and momenta are not perpendicular. The action of $SL(2, Z)$ on $H$ is given by composition in the argument of the function representing the element in $H$. We show that the action is fully reducible in terms of a direct integral over $R$ of vector spaces isomorphic to $L^2(S^2, d\varphi)$, which carry $SL(2, Z)$-irreducible sub-representations of certain irreducible representations of $SL(2, R)$ from the continuous series [1]. Hence for each measurable set $\Delta \subset R$ one finds a closed subspace $H_\Delta \subset H$ which is invariant under large diffeomorphisms. Now, if $SL(2, Z)$ is considered as gauge group, the observables should be contained in its commutant, implying in particular that no vector in $H$ defines a pure state for this algebra. $H$ is not the physical Hilbert space nor does it contain it. We conclude that the reduction of large diffeomorphisms should a priori not be regarded as a problem simpler than the reduction of diffeomorphisms generated by the constraints (identity component).

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D. Giulini, J. Louko: “Diffeomorphism Invariant Subspaces in Witten’s 2+1 Quantum Gravity on $R \times T^2$”, Class. Quant. Grav. 12, 2735-2745 (1995).
Mapping-Class Groups of General 3-Manifolds

Domenico Giulini

Let $\Sigma$ be a closed orientable 3-manifold, $\infty \in \Sigma$ a distinguished point, and $DF(\Sigma)$ the space of diffeomorphisms that fix the frames at $\infty$. We are interested in the mapping class groups $S(\Sigma) := DF(\Sigma)/D^0_F(\Sigma)$ for general $\Sigma$, where $D^0_F(\Sigma)$ denotes the identity component of $DF(\Sigma)$ [1]. As is well known, $\Sigma$ is diffeomorphic to a connected sum of finitely many (say $n$) and uniquely determined prime 3-manifolds $P_i$. [NB: $P$ is prime $\iff \pi_2(P) = 0$ or $P = S^2 \times S^1$ (the ‘handle’).] Using this, we think of $\Sigma$ as a configuration of $n$ elementary ‘objects’ attached to a common base along mutually disjoint connecting spheres [2]. An obvious subgroup of $S(\Sigma)$ is generated by the semi-direct product of permutations of diffeomorphic objects with their internal symmetry groups $S(P_i)$. This subgroup is in fact also a factor iff $P_i \neq S^2 \times S^1$ [3]. It is explained how a full presentation of $S(\Sigma)$ may be constructed using the Fuks-Rabinovich presentation for the automorphism group of free products [3]. For this one considers the natural map $h : S(\Sigma) \to Aut(\pi_1(\Sigma, \infty))$. If all $P_i$ satisfy that homotopic diffeomorphisms are also isotopic (no prime violating this is known), the kernel, $\ker(h)$, is known to be of the form $\mathbb{Z}_m^2$, $m \le n$, generated by certain rotations parallel to imbedded 2-spheres. The image, $\text{Im}(h)$, can be explicitly presented. In many cases it exhausts all of $Aut(\pi_1(\Sigma))$. Finally, one specifies the correct action of $\text{Im}(h)$ on $\ker(h)$, which results in a semi-direct product of these two groups. All this can be nicely exemplified explicitly for the connected sum of $n$ handles or $n$ PR$^3$’s [2][3].

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D. Giulini: “On the Configuration Space Topology in General Relativity”, Helv. Phys. Acta 68, 86-111 (1995).

D. Giulini: “3-Manifolds for Relativists”, Int.Jour. Theor. Phys. 33, 913-930 (1994).

D. Giulini: “The Group of Large Diffeomorphisms in General Relativity. Banach Center Publications, in press.
On the Probability of Entering a Spacetime Region in Non-Relativistic Quantum Mechanics

J.J. Halliwell and E. Zafiris

What is the probability of a particle entering a given region of space at any time between \( t_1 \) and \( t_2 \)? Standard quantum theory assigns probabilities to alternatives at a fixed moment of time and is not immediately suited to questions of this type. We use the decoherent histories approach to quantum theory to compute the probability of a non-relativistic particle entering a spacetime region. Aside from being of general formal interest, this question is relevant to the problems of arrival times and tunneling times. It may also be relevant to relativistic systems and in particular, to quantum gravity, where a variable playing the role of time may not exist. For a system consisting of a single non-relativistic particle, histories coarse–grained according to whether or not they pass through spacetime regions are generally not decoherent, except for very special initial states, and thus probabilities cannot be assigned. Decoherence may, however, be achieved by coupling the particle to an environment consisting of a set of harmonic oscillators in a thermal bath. Probabilities for spacetime coarse grainings are thus calculated by considering restricted density operator propagators of the quantum Brownian motion model, and we find approximate methods for calculating these. Another method of achieving decoherence, which we explore, is to consider a system consisting of a large number \( N \) of identical, non-interacting, free particles, and consider histories in which an imprecisely defined proportion of the particles cross the spacetime region. We find that there is decoherence, essentially due to statistics for large \( N \). We thus obtain general expressions for the probabilities for a variety of spacetime problems for a particle starting in an arbitrary initial state.

Issues in Black Hole Thermodynamics

T. Jacobson

This talk provided an overview of black hole thermodynamics and discussed recent progress and open questions. The issues discussed included: the generalized second law, entanglement entropy, the “holographic hypothesis”, and the statistical meaning of black hole entropy. In particular, the relation between matter field contributions to the entropy and the renormalization of Newton’s constant, the nature of the “bare” entropy, and Carlip’s counting of black hole states in 2+1-dimensional quantum gravity were discussed.
Origin of the Outgoing Black Hole Modes

T. Jacobson

The origin of the outgoing black hole modes is puzzling if no transplanckian reservoir at the horizon is available. In this talk I explained this puzzle and discussed models with high frequency dispersion, motivated by condensed matter analogies, which can resolve the puzzle. These models arose originally from Unruh’s sonic analog of a black hole, which is an inhomogeneous fluid flow with a “sonic horizon” where the flow speed exceeds the speed of sound. I explained how high frequency dispersion in the wave equation satisfied by the sound field (or its analog) leads to a process of “mode conversion”, whereby ingoing short wavelength modes are converted into long wavelength outgoing ones. Results of a calculation of the Hawking spectrum in one such model were described.

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Unruh, W.G., “Sonic analog of black holes and the effects of high frequencies on black hole evaporation”, Phys. Rev. D 51 (1995) 2827.

Corley, S. and Jacobson, T., “Hawking spectrum and high frequency dispersion”, Phys. Rev. D 54 (1996) 1568-1586.

Jacobson, T., “On the origin of the outgoing black hole modes”, Phys. Rev. D 53 (1996) 7082-7088.
Ashtekar’s formulation of general relativity admits an extension to degenerate metrics, and it appears that such metrics may play an important role in the quantization of the theory. Recently Matschull showed how this degenerate extension of GR can be described in a fully spacetime covariant manner, and he showed that the degenerate “geometries” allowed by the Ashtekar extension always possess a local “causal cone”, though the cone is collapsed in one or more dimensions. In this talk, the rank-1 sector of the classical theory was discussed, motivated by the degeneracy of the triad along the Wilson lines in quantum loop states. I showed that the classical lines behave like (1+1)-dimensional spacetimes with a pair of massless Dirac fields propagating along them (“connection waves”). Matschull’s causal structure is precisely the light cone for these waves. Further, if the lines form a congruence of closed loops, the holonomy must be the same on all the loops. Results for inclusion of matter and supergravity were also obtained in this work.

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Matschull, H.J., “Causal structure and diffeomorphisms in Ashtekar’s gravity”, Class. Quantum Grav. 13 (1996) 765-782.

Jacobson, T., “1+1 sector of 3+1 gravity”, Class. Quantum Grav. 13 (1996) L111-L116.

At the Vienna workshop, I reported about some new results which I obtained in the quantization of Hamiltonian lattice gravity. Because of the close structural resemblance of the calculations, these are also of relevance to the continuum loop quantization.

I am working with (a discretized version of) real connection variables and their canonically conjugate momenta \((A,E)\) on a cubic \(N^3\)-lattice. The natural scalar product at the kinematical level is therefore identical with that of the usual lattice gauge theory with a compact gauge group \((SO(3)\) or \(SU(2)\)), and the basic link operators (the link holonomy \(\hat{U}(l)\) and link momenta \(\hat{p}_i(l)\)) are self-adjoint. The Hamiltonian constraint is non-polynomial, but this can be handled along the lines described in [1].

Like in the continuum, one may define self-adjoint operators measuring volumes and areas, and the gauge-invariant states diagonalizing these operators are simple linear combinations of so-called spin network states. The one-dimensional Wilson loop states underlying this construction are obtained by simply multiplying together the one-dimensional
basic link holonomies (that form part of the single, fixed lattice), whereas in the continuum they are somewhat less natural composite objects depending on the connections $A$ and on arbitrary embedded loops in the three-dimensional spatial manifold $\Sigma$. Related to this is the fact that in the continuum theory – unlike on the lattice – there still exist natural Diff $\Sigma$-actions.

The first interesting property I found is the following: suppose one wanted to lattice-quantize the classical phase space function corresponding to the “area of a surface perpendicular to the 3-direction”, $\int d^2 x \sqrt{E^3 E^3}$. To reproduce the spectrum found by Ashtekar and Lewandowski in the continuum, one substitutes the continuum momenta $E^b(x)$ by symmetrized lattice momenta $\frac{1}{2}(p^+(n, b) + p^-(n, b))$, where $n$ is now a lattice vertex and the symmetrization is taken over the link momenta in positive and negative $b$-direction, both based at (i.e. transforming non-trivially at) $n$. Doing this, I realized that this operator is not diagonal in terms of certain volume eigenstates I had constructed previously on the lattice, that is, volume and area operators in general do not commute! (The commutator may still vanish on certain “simple” loop states.) I was at first greatly worried by this result, since the corresponding classical phase space functions do of course commute. Moreover, the calculations I did can be directly translated to the continuum, which means that also there areas and volumes do not commute. Details of this can be found in my forthcoming paper [2]. On the lattice there is an easy way around this problem: choose a different discretization for the term under the square root, namely, $\frac{1}{4}(p^+(n, b)^2 - p^- (n, b)^2)$ instead of $\frac{1}{4}(p^+(n, b) + p^- (n, b))^2$. This differs from the latter by terms of higher order in the lattice spacing $a$, as $a \to 0$ in the continuum limit, and is therefore an equally good operator from the point of view of the lattice theory. Being a sum of two laplacians, it commutes strongly with any other lattice operator.

I came across another curious feature while looking for simultaneous eigenvectors of the volume and the area operators on the lattice. Since their operator expressions reduce to sums over vertex contributions, one can diagonalize them separately at each intersection. The dual unit cubes around individual vertices may therefore be regarded as smallest building blocks of geometry. I looked at a particular family of states at some fixed vertex $n$, namely those where the six links meeting at $n$ (the lattice is cubic) have the same occupation number (or spin) $j = 1, 2, 3, \ldots$, and only differ by how the flux lines are contracted gauge-invariantly at $n$. One may then extract local length scales by computing $\sqrt{a_0}$ and $\sqrt[3]{v_0}$, with $a_0$ and $v_0$ the eigenvalues of the area (which for these states are the same in all three directions) and the volume. One finds that at least for the first few $j$ the length scale one obtains from the area is larger than that calculated from the volume, even if one always picks the state of highest volume from the entire set. For example, for $j = 1$, one finds $\sqrt{a} \sim 0.866$, $\sqrt[3]{v_{\text{max}}} \sim 0.821$, and for $j = 2$, $\sqrt{a} \sim 1.189$, $\sqrt[3]{v_{\text{max}}} \sim 1.077$, in suitable units. This behavior seems to persist also for higher $j$ [3]. It remains to be understood to what extent this is a general feature of local lattice states. If it were, it would be difficult to understand how one could construct states representing flat space, say, from those smallest building blocks.
D-branes and Black Hole entropy

Donald Marolf

A review was presented of recent calculations of black hole entropy in string theory, giving the general picture of how calculations of D-brane bound states are related to black hole entropy. The original calculation by Strominger and Vafa (Phys. Lett. B379, 99 (1996); hep-th/9601029) was outlined and some extensions were mentioned. A discussion along these lines and additional references can be found in the review by Horowitz, gr-qc/9604051.

Duality Symmetries in String and Field Theory

Krzysztof A. Meissner

The talk given at the ESI Workshop on Quantum Gravity in Vienna described duality symmetries both in string and in field theories. The subject is now being intensively studied for at least three reasons. The first one is that one aspect of duality (strong vs. weak coupling) gives us hope to probe normally inaccessible region of strongly coupled quantum field theories by showing their equivalence with (usually different) weakly coupled theories where the perturbation expansion can be trusted. This kind of equivalence was shown up to now in a limited number of theories (like N=2 supersymmetric Yang-Mills theories) but even these few examples are extremely helpful in understanding strongly interacting quantum field theories. The second reason is that they relate seemingly different string theories (like heterotic and IIB) which is seen upon compactifying them on two different manifolds and comparing the resulting effective actions. The number of dualities of that type is rapidly increasing and it led to speculations that all string theories are interconnected and in fact they all descend from one theory (called “M-theory” if 11-dimensional or “F-theory” if 12 dimensional) but compactified with different boundary conditions. The third reason that dualities are intensively studied is that they are “solution generating” symmetries i.e. starting with a solution to the equations of motion of a given theory and acting on it with elements of the duality group, we get a whole class of new solutions. The resulting solutions can be very complicated and rather impossible to get
by solving the equations of motion. Such global symmetries have also a conserved current which is very helpful in classifying the solutions. One particular example is the so called string cosmology with gravity coupled to a scalar (dilaton) and antisymmetric tensor where there is an $O(d, d)$ global symmetry where $d$ is the number of space dimensions. This symmetry allows for many generalizations and is always present in the gravitational sector of “string-inspired” effective actions.

**Black Hole Entropy from Strings and D-Branes**

*Robert Myers*

Finding a statistical mechanical interpretation of black hole entropy is an outstanding problem which has eluded physicists for over 20 years. Recently, progress into this question has been made using new insights from string theory. This progress is a spin-off from the work on string dualities, and the realization of the important role of extended objects beyond just strings. In particular, a class of extended objects known as D-branes [1] have proven very valuable from a calculational standpoint. It was found that different kinds of D-branes can be combined to produce black holes in a certain strong coupling limit. On the other hand in weak coupling, these systems are amenable to statistical mechanical analysis within string theory. These calculations were first carried out for a class of extremal black holes in five-dimensions [2], but then rapidly extended to a variety of other configurations [3-7]. Even though these calculations still apply to a relatively restricted class of black holes, they represent a breakthrough in our understanding of black hole entropy, since for the first time, we have some insight into the underlying microscopic degrees of freedom for a black hole.

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Quantum Gravi-dynamics as skein relations in knot space

Jorge Pullin

In the loop representation, states that are solutions of the diffeomorphism constraint are knot invariants. Typically, one tries to find a realization of the Hamiltonian constraint that acts on such states in order to find quantum states of the gravitational field. Such an action can never yield an operator in the space of knots, since the Hamiltonian is a non diffeomorphism invariant function of a point, and therefore cannot be realized in a space of diffeomorphism invariant states. We argue, however, that many proposals for regularized actions of the Hamiltonian can be decomposed into a non diffeomorphism invariant pre-factor times a topological operator. The latter can be realized in a space of diffeomorphism invariant wave functions. We analyze in particular a recently proposed lattice regularization [1]. In terms of it, the topological operator can be interpreted as a skein relation between intersecting knots [2]. This relation determines partially the knot polynomial that is the general solution of the Einstein equations. The indeterminacy is related to the fact that the theory does not have a single solution since it has local degrees of freedom. We show that certain knot invariants, which in the continuum [3] and extended [4] loop representations were found to solve formally the Hamiltonian constraint satisfy the skein relations found to represent the dynamics of general relativity, providing additional confirmation that they could be states of gravity. The calculations are carried out for a particular type of triple intersections; further studies will be needed to elucidate in a more general way if the states are actually compatible with the skein relations for general
intersections. The idea of viewing the constraint as partial skein relations is not confined to this particular approach and holds promise in the context of the recently proposed Hamiltonian for real Lorentzian gravity in terms of spin network states.

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A left-handed simplicial action for euclidean general relativity

Michael P. Reisenberger

An action for simplicial euclidean general relativity involving only left-handed fields is presented. The simplicial theory is shown to converge to continuum general relativity in the Plebanski formulation as the simplicial complex is refined.

An entirely analogous hyper-cubic lattice theory, which approximates Plebanski’s form of general relativity is also presented.
A four dimensional path integral formulation of simplicial euclidean general relativity (GR) corresponding to canonical GR in Ashtekar’s connection representation is presented. By integrating out the spacetime connection the path integral is turned into a sum over spins, analogous to the Ponzano-Regge model for 2+1 GR, corresponding to canonical GR in the spin network representation. In this latter case the path integral may be interpreted as a) a sum over world sheets of spin networks, and b) a sum over discrete spacetime geometries analogous to the discrete spatial geometries found in loop quantized canonical GR in the continuum. The discreteness of the 4-geometry should persist in a continuum limit of the simplicial model because it results from the discreteness of the spins of $SU(2)$ representations, not from the discreteness of the simplicial complex modeling spacetime. However, the existence of a continuum limit has not been established.

The path integral model is derived from a new classical simplicial action which in the classical continuum limit converges to the Plebanski action for GR.

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M. Reisenberger. A left-handed simplicial action for euclidean general relativity. gr-qc 9609002, 1996. (presents the classical simplicial theory.)

Black Hole Entropy from Loop Quantum Gravity

Carlo Rovelli

I have discussed recent ideas on the possibility of deriving the Bekenstein-Hawking formula, which states that the Entropy of a (non rotating) black hole is proportional to its Area, from Loop Quantum Gravity.
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Lessons from 1+1 Gravity

T. Strobl

Subject of our investigations is the wide class of generalized 2d dilaton gravity Lagrangians

\[ L[g, \Phi] = \int d^2 x \sqrt{|\det g|} \left[ U(\Phi) R + V(\Phi) + W(\Phi) \partial_\mu \Phi \partial^\mu \Phi \right], \]

where \( U, V, W \) are functions of the dilaton \( \Phi \). I briefly reviewed what is known about these (mini-superspace) models on the classical level [1]. In particular, for Lorentzian signature of the metric \( g \) all classical, diffeomorphism inequivalent solutions have been found. For not too specific choices of \( U, V \), and \( W \) these include perfectly smooth solutions on any ('reasonable') non-compact two-surface as well as various multi black hole configurations.

I then came to discuss the Dirac quantization of (1). Here the reformulation of this action in terms of so-called Poisson \( \sigma \)-models [2], comparable to the formulation of 2+1 gravity as Chern-Simons theory, is essential. In the context of (1) the target space of the \( \sigma \)-model is an \( R^3 \), equipped with a Poisson bracket induced by the choice of \( U, V \), and \( W \). Correspondingly this auxiliary \( R^3 \) foliates (stratifies) into (generically) two-dimensional symplectic sub-manifolds, characterized by a target space coordinate \( M \). On-shell the latter may be identified with the (generalized) mass of the spacetime, a Dirac observable of the theory. The general framework of Poisson \( \sigma \)-models allows to determine the spectrum of \( M \) by means of a simple analysis of the above foliation; e.g., \( \text{Spec}(M) \) is discrete, iff the respective symplectic leaves have non-trivial second homotopy. For some choices (but not for all) of \( U, V, W \) in (1) \( \text{Spec}(M) \) becomes discrete for Euclidean signature of the theory (i.e. of the metric \( g \)), but continuous for Lorentzian signature. If a similar relation holds for some (any) Dirac observable \( O \) of 4d Einstein gravity, the recently proposed generalized Wick transformation \( O_{\text{Lor}} = T O_{\text{Eucl}} T^{-1} \) cannot hold, at least in a strict sense.
In his treatment of the 2+1 black hole Carlip assigns the black hole entropy to states of a WZW boundary action located at the (stretched) horizon. In the 1+1 dimensional context of (1) such a boundary action will describe purely mechanical degrees of freedom. The following nice picture evolves [3]: The phase space of these edge degrees of freedom on a black hole spacetime of mass $M$ coincides precisely with the respective symplectic leaf in the above mentioned $R^3$; thus the fictitious point particles obtained above become 'alive' and physical. Despite this appealing picture, the entropy obtained in this way does not seem to agree with the one obtained by other, quite reliable semiclassical approaches.

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Quantum Spin Dynamics
Thomas Thiemann

An anomaly-free operator corresponding to the Wheeler-DeWitt constraint of Lorentzian, four-dimensional, canonical, non-perturbative vacuum gravity is constructed in the continuum. This operator is entirely free of factor ordering singularities and can be defined in symmetric and non-symmetric form.

We work in the real connection representation and obtain a well-defined quantum theory. We compute the complete solution to the Quantum Einstein Equations for the non-symmetric version of the operator and a physical inner product thereon.

The action of the Wheeler-DeWitt constraint on spin-network states is by annihilating, creating and rerouting the quanta of angular momentum associated with the edges of the underlying graph while the ADM-energy is essentially diagonalized by the spin-network states. We argue that the spin-network representation is the “non-linear Fock representation” of quantum gravity, thus justifying the term “Quantum Spin Dynamics (QSD)”.

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A Length Operator for Canonical Quantum Gravity

Thomas Thiemann

We construct an operator that measures the length of a curve in four-dimensional Lorentzian vacuum quantum gravity.

We work in a representation in which a $SU(2)$ connection is diagonal and it is therefore surprising that the operator obtained after regularization is densely defined, does not suffer from factor ordering singularities and does not require any renormalization.

We show that the length operator admits self-adjoint extensions and compute part of its spectrum which like its companions, the volume and area operators already constructed in the literature, is purely discrete and roughly is quantized in units of the Planck length.

The length operator contains full and direct information about all the components of the metric tensor which facilitates the construction of so-called weave states which approximate a given classical 3-geometry.

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