Acceleration of particles in Einstein-Maxwell-Dilaton black hole

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Abstract

It has been recently pointed out that, under certain conditions, the energy of particles accelerated by black holes in the center-of-mass frame can become arbitrarily high. In this Letter, we study the collision of two particles around the four-dimensional Kaluza-Klein black hole in Einstein-Maxwell-Dilaton theory. We find that the center-of-mass energy for a pair of colliding particles is unlimited at the horizon of charged nonrotating and extremal rotating Kaluza-Klein black hole.

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I. INTRODUCTION

The Planck scale defines the meeting point of gravity and quantum mechanics. The probe of the Planck-scale physics also contribute to the discovery of extra dimensions of space-time and the Grand Unification Theory. However, comparing with the Planck energy $10^{16}$ TeV, the largest terrestrial accelerator, the Large Hadron Collider, which could detect physics at collision energy $10^1$ TeV, is too low to probe the Planck-scale physics. There is a very, very long distance between the Planck scale and our current experiment technique. So, other new physical mechanisms should be proposed for probing the Planck-scale physics. The particles collision around a black hole may provide such a possible approach.

Bañados, Silk and West (BSW) \cite{1} recently investigated the maximum center-of-mass energy of particles collision around a Kerr black holes. Their result showed that the maximum energy grows with $a$ which is the unit angular momentum of the black hole. Furthermore, as the black hole becomes extremal, they found a fascinating and important property of the extremal Kerr black hole that two particles freely falling from rest at spatial infinity can collide at the horizon with arbitrarily high center-of-mass (CM) energy, which could play as a role of particle accelerators and provide a visible probe of Planck-scale physics. However, the back hole must have a maximally angular momentum and one of the colliding particles should have orbital angular momentum per unit rest mass $l = 2$. Subsequently, in \cite{2, 3}, the authors argued that the CM energy is in fact limited, because there always exists a small deviation of the spin of the astrophysical black hole from its maximal value.

According to the work of Thorne \cite{23}, the dimensionless spin of astrophysical black holes should not exceed $a = 0.998$. In terms of the small parameter $\epsilon = 1 - a$, Jacobson and Sotiriou got the maximal CM energy $E_{CM} \sim 4.06\epsilon^{-1/4} + \mathcal{O}(\epsilon^{1/4})$ in \cite{3}. Taking $a = 0.998$ as a limit, one can obtain the maximal CM energy per unit mass 19.20, which is a finite value. Meanwhile, Lake showed that the CM energy of the collision at the inner horizon of the black hole is generically divergent \cite{4} when the colliding particles take any angular momentum per unit rest mass that could fall into the black hole. But soon he claimed that the collision at the inner horizon actually could not take place, which leads to no divergence of the CM energy \cite{5}. On the other hand, Grib and Pavlov suggested that the CM energy can be unlimited in the case of multiple scattering \cite{6, 9}. The universal property of acceleration of particles for black holes was investigated in \cite{10, 12}. In \cite{13, 14}, the property of
the CM energy for two colliding particles in the background of charged spinning black hole was discussed. One of the important results of [13, 14] is that the CM energy can still be unlimited despite the deviation of the spin from its maximal value. The BSW’s approach was then applied to the collision of a particle plunging from the innermost stable circular orbit and last stable orbit near the horizon in [15, 16]. There are also some investigate focus on naked singularities [17–20] and (anti-) de Sitter Backgrounds [21].

In the present paper, we will investigate the property of the collision of particles in the background of Einstein-Maxwell-Dilaton gravity. Firstly we calculate equations of motion in Kaluza-Klein black hole in Einstein-Maxwell-Dilaton theory. Then, we study the CM energy for the collision taking place at the horizon of three cases of Kaluza-Klein black hole. We find the CM energy can still be unlimited for a pair of point particles colliding at the horizon with some fine tunings for the charged onorotating case and the extremal rotating case. And the result of the near-extremal case shows that the CM energy is in fact limited for the critical angular momentum is too large for the geodesics of particle to reach the horizon. Then, for a near-extremal Kaluza-Klein black hole, in terms of the small parameter \( \epsilon = 1 - a \), because of the difficulty in finding the exact result, we obtain the numerical result of the maximal CM energy per unit mass for some different value of \( \epsilon \).

This paper is organized as follows. In Section 2, we give a brief review of four-dimensional Kaluza-Klein black hole and obtain the equations of motion in the background of Kaluza-Klein black hole spacetime. In Section 3, we study the CM energy for the collision taking place at the horizon for three different cases of Kaluza-Klein black hole. The last section is devoted to conclusion and discussion.

II. EQUATIONS OF MOTION IN KALUZA-KLEIN BLACK HOLE

We begin with a briefly review of the Kaluza-Klein black hole. Kaluza-Klein black hole solution is derived by a dimensional reduction of the boosted five-dimensional Kerr solution to four dimensions. It is also an exact solution of Einstein-Maxwell-Dilaton action. The metric is explicitly given by [22]

\[
ds^2 = \frac{1 - Z}{B} dt^2 - \frac{2aZ \sin^2 \theta}{B \sqrt{1 - \nu^2}} dtd\phi + \frac{B\Sigma}{\Delta} dr^2 + B\Sigma d\theta^2 + \left[ B(v^2 + a^2) + a^2 \sin^2 \theta \frac{Z}{B} \right] \sin^2 \theta d\phi^2 ,
\]

(1)
where

\[
Z = \frac{2\mu r}{\Sigma}, \\
B = \sqrt{1 + \frac{\nu^2 Z}{1 - \nu^2}}, \\
\Sigma = r^2 + a^2 \cos^2 \theta, \\
\Delta = r^2 - 2\mu r + a^2. 
\] (2)

The gauge potential is given by

\[
A = \frac{\nu}{2(1 - \nu^2)} \left( \frac{Z}{B^2} \right) dt - a\nu \sin^2 \theta \left( \frac{Z}{B^2} \right) d\varphi. 
\] (3)

The physical mass \( M \), the charge \( Q \), and the angular momentum \( J \) are expressed with the boost parameter \( \nu \), mass parameter \( \mu \), and specific angular momentum \( a \) as

\[
M = \mu \left[ 1 + \frac{\nu^2}{2(1 - \nu^2)} \right], \\
Q = \frac{\mu \nu}{1 - \nu^2}, \\
J = \frac{\mu a}{\sqrt{1 - \nu^2}}. 
\] (4)

The outer and inner horizons are respectively given by

\[
r_{\pm} = \mu \pm \sqrt{\mu^2 - a^2}. 
\] (5)

Thus, \( \mu = a \) corresponds to the extremal black hole with one degenerate horizon. The components of the inverse metric are

\[
g^{tt} = -\frac{B(r^2 + a^2) + a^2 \sin^2 \theta \frac{Z}{B}}{B\Sigma}, \\
g^{rr} = \frac{\Delta}{B\Sigma}, g^{\theta\theta} = \frac{1}{B\Sigma}, \\
g^{\varphi\varphi} = \frac{1 - Z}{B\Delta \sin^2 \theta}, g^{t\varphi} = -\frac{aZ}{B\Delta \sqrt{1 - \nu^2}}. 
\] (6)

Because there are two killing vectors \( \left( \frac{\partial}{\partial t} \right)^\mu \) and \( \left( \frac{\partial}{\partial \varphi} \right)^\mu \), we have the conserved quantities along a geodesic motion for a test particle with charge \( e \) as follow

\[
E = -g_{\mu\sigma}(\frac{\partial}{\partial t})^\mu [u^\sigma + eA^\sigma] = -(g_{tt}\dot{t} + g_{t\varphi}\dot{\varphi} + eA_t), 
\] (7)

\[
L = g_{\mu\sigma}(\frac{\partial}{\partial \varphi})^\mu [u^\sigma + eA^\sigma] = g_{t\varphi}\dot{t} + g_{\varphi\varphi}\dot{\varphi} - eA_\varphi, 
\] (8)

4
where $E$ and $L$ correspond the constants energy and angular momentum along a geodesic motion, respectively. It is easy to solve the above for $\dot{t}$ and $\dot{\varphi}$

$$
\dot{t} = \frac{B(r^2 + a^2) + a^2 \frac{Z \sin^2 \theta}{\Delta} (E - eA_t)}{\Delta} - \frac{aZ}{B \Delta \sqrt{1 - \nu^2}} (L + eA_\varphi),
$$

$$
\dot{\varphi} = \frac{aZ}{B \Delta \sqrt{1 - \nu^2}} (E - eA_t) + \frac{1 - Z}{B \Delta \sin^2 \theta} (L + eA_\varphi).
$$

(9)

Substituting these into the normalization condition $g_{\mu\nu}u^\mu u^\nu = -1$ on the equatorial plane, $\theta = \frac{\pi}{2}$ and $\dot{\theta} = 0$, one will arrive

$$
\dot{r} = \frac{\Delta}{B \Sigma} R(r),
$$

(10)

where

$$
\frac{\Delta}{B \Sigma} R^2(r) = \frac{B(r^2 + a^2) + a^2 \frac{Z}{B} (E - eA_t)^2 - 2 \frac{aZ}{B \Delta \sqrt{1 - \nu^2}} (E - eA_t)(L + eA_\varphi) - \frac{1 - Z}{B \Delta} (L + eA_\varphi)^2 - 1.}
$$

(11)

Now we have obtained the equations of motion on the equatorial plane in Kaluza-Klein black hole. In the next section, we will turn to the CM energy for particles collision in the background of Kaluza-Klein black hole.

III. CENTER-OF-MASS ENERGY FOR COLLISION IN KALUZA-KLEIN BLACK HOLE

The energy in the center-of-mass frame for a pair of point particles colliding should be computed by the formula [1]

$$
E_{CM} = \sqrt{2m_0 \sqrt{1 - g_{\mu\nu}u_{1}^{\mu}u_{2}^{\nu}}},
$$

(12)

where $u_{1}^{\mu}$ and $u_{2}^{\nu}$ are the 4-velocities of the two particles. For the case that the particles begin at rest at infinity and the collision energy comes solely from gravitational acceleration, it is obvious to see that particles follow geodesics with the energy $E \geq 1$. Consider two particles coming from infinity with $E_1 = E_2 = 1$ and approaching the black hole with different angular momenta $L_1$ and $L_2$. Taking into account the metric of the Kaluza-Klein black hole (15) on the equatorial plane, we obtain the CM energy for collision with the help of (9) (10) and (12)
\[ \frac{E_{\text{CM}}^2}{2m_0^2} = 1 + \frac{K}{B\Delta} , \]  

(13)

where

\[ K = \left[ B^2(r^2 + a^2) + a^2 Z \right] (1 - e_1 A_t)(1 - e_2 A_t) + (Z - 1)(L_1 + e_1 A_\varphi)(L_2 + e_2 A_\varphi) \]

- \[ \frac{aZ}{\sqrt{1 - \nu^2}} \left[ (1 - e_1 A_t)(L_2 + e_2 A_\varphi) + (1 - e_2 A_t)(L_1 + e_1 A_\varphi) \right] \]

- \( B^2(r^2 + a^2) + a^2 Z \left[ (1 - e_1 A_t)^2 + (Z - 1)(L_1 + e_1 A_\varphi)^2 \right] \)

- \( \frac{2aZ}{\sqrt{1 - \nu^2}} \left[ (1 - e_1 A_t)(L_1 + e_1 A_\varphi) - B\Delta \right] ^{\frac{1}{2}} \]

\[ \times \left\{ B^2(r^2 + a^2) + a^2 Z \left[ (1 - e_2 A_t)^2 + (Z - 1)(L_2 + e_2 A_\varphi)^2 \right] \right. \]

- \( \frac{2aZ}{\sqrt{1 - \nu^2}} \left[ (1 - e_2 A_t)(L_2 + e_2 A_\varphi) - B\Delta \right] ^{\frac{1}{2}} \]  

(14)

We have obtained the CM energy of two colliding particles in general Kaluza-Klein spacetime. Then we will investigate the CM energy in different cases of the Kaluza-Klein black hole.

A. the charged nonrotating Kaluza-Klein black hole

The acceleration of particles by charged nonrotating black holes has been discussed in [11]. However in [11], only the Reissner-Nordström black hole was mentioned. It should be pointed that the charged nonrotating Kaluza-Klein spacetime is very different from the Reissner-Nordström case for there is only one event horizon which lead to the absent of the extremal nonrotating Kaluza-Klein black hole. The metric of charged nonrotating Kaluza-Klein spacetime is

\[ ds^2 = -\frac{\Delta}{r^2 B}dt^2 + \frac{r^2 B}{\Delta}dr^2 + Br^2d\theta^2 + Br^2 \sin^2 \theta d\varphi^2 , \]  

(15)

where

\[ B = \sqrt{1 + \frac{\nu^2 Z}{1 - \nu^2}} , \]

\[ Z = \frac{2\mu}{r} , \]

\[ \Delta = r^2 - 2\mu r . \]  

(16)
The gauge potential is given by
\[ A = \frac{\nu}{2(1 - \nu^2)} \frac{Z}{B^2} dt. \] (17)

The horizon lies at \( r_h = 2\mu \). According to Eq. (13) and Eq. (14), we can read the CM energy of two radial motion particles colliding in charged nonrotating Kaluza-Klein spacetime as
\[ \frac{E_{CM}^2}{2m_0^2} = 1 + \frac{K_1}{B\Delta}, \] (18)

where
\[ K_1 = B^2r^2(1 - e_1A_t)(1 - e_2A_t) - \sqrt{B^2r^2(1 - e_1A_t)^2 - B\Delta}[B^2r^2(1 - e_2A_t)^2 - B\Delta]. \] (19)

It appears that \( E_{CM}^2 \) diverges at \( r = r_h \), but this is not true because, although not totally evident, the numerator vanishes at that point as well. After some calculations, the CM energy reads as
\[ \frac{E_{CM}^2}{2m_0^2} = 1 + \frac{1}{2} \left( \frac{1 - e_2\nu}{1 - e_1\nu} + \frac{1 - e_1\nu}{1 - e_2\nu} \right). \] (20)

If one of the particles participating in the collision has the critical charge \( e = \frac{2}{\nu} \), the CM energy will blow up at the horizon. Thus we have shown that non-extremal black hole could also play as a role of particle accelerators and provide a visible probe of Planck-scale physics.

**B. the extremal charged rotating Kaluza-Klein black hole**

In the case \( a = \mu \), which corresponds to the extremal Kaluza-Klein black hole, we obtain the form of the CM energy of two uncharged particles colliding at the degenerate horizon after some tedious calculations
\[ E_{CM}^{KK}(r \rightarrow r_+) = \sqrt{2}m_0 \left[ 1 + \frac{(1 + \nu^2)(L_1 - L_2)^2}{2\sqrt{1 - \nu^4}(L_1 - \frac{2\mu}{\sqrt{1 - \nu^2}})(L_2 - \frac{2\mu}{\sqrt{1 - \nu^2}})} \right. \\
+ \frac{1}{2} \left( \frac{L_1 - \frac{2\mu}{\sqrt{1 - \nu^2}}}{L_2 - \frac{2\mu}{\sqrt{1 - \nu^2}}} + \frac{L_2 - \frac{2\mu}{\sqrt{1 - \nu^2}}}{L_1 - \frac{2\mu}{\sqrt{1 - \nu^2}}} \right)^{\frac{1}{2}}, \] (21)

Clearly, when \( L_1 \) or \( L_2 \) takes the critical angular momentum \( L_c = \frac{2\mu}{\sqrt{1 - \nu^2}} \), the CM energy \( E_{CM}^{KK} \) will be unlimited, which means that the particles can collide with arbitrarily high CM energy at the horizon. We expect that, in the case \( \nu = 0 \), the CM energy \( (21) \) in the background of a Kaluza-Klein black hole should reduce to the one in the background of a
FIG. 1: For an extremal Kaluza-Klein Black hole with $J = \sqrt{3}/4$ and $M = 1$ (a) The variation of $s = \dot{r}^2$ with radius for three different values of angular momentum. (b) The variation of $E_{CM}^{KK}$ with radius for three combinations of $L_1$ and $L_2$. For $L_1 = L_c$, $E_{CM}^{KK}$ blows up at the horizon.

FIG. 2: For an extremal Kaluza-Klein Black hole with $J = \sqrt{5}/9$ and $M = 1$ (a) The variation of $s = \dot{r}^2$ with radius for three different values of angular momentum. (b) The variation of $E = E_{CM}^{KK}$ with radius for three combinations of $L_1$ and $L_2$. For $L_1 = L_c$, $E_{CM}^{KK}$ blows up at the horizon.

Kerr black hole. After some calculations, we obtain the CM energy is exactly consistent with that of [1] in the case $\nu = 0$.

We plot $\dot{r}^2$ and $E_{CM}^{KK}$ in Fig. 1 and Fig. 2, from which we can see that there exists a critical angular momentum $L_c = \frac{2\mu}{\sqrt{1-\nu^2}}$ for the geodesics of particle to reach the horizon. If $L > L_c$,
the geodesics never reach the horizon. On the other hand, if the angular momentum is too small, the particle will fall into the black hole and the CM energy for the collision is limited. However, when \( L_1 \) or \( L_2 \) takes the angular momentum \( L = \frac{2\mu}{\sqrt{1-\nu^2}} \), the CM energy is unlimited with no restrictions on the angular momentum per unit mass \( \frac{J}{m} \) of the black hole. As a result, it may provide a unique probe to the Planck-scale physics.

### C. the near-extremal charged rotating Kaluza-Klein black hole

For the near-extremal case, we also obtain the CM energy at the outer horizon

\[
E_{CM}^{KK}(r \to r_+) = \sqrt{2}m_0 \left[ 1 + \frac{\left(\frac{r_+^2}{\mu^2} + \nu^2\right)(L_1 - L_2)^2}{2B(r_+)(1 - \nu^2)(L_1 - \frac{r_+^2 + a^2}{a\sqrt{1-\nu^2}})(L_2 - \frac{r_+^2 + a^2}{a\sqrt{1-\nu^2}})} + \frac{1}{2} \left( \frac{L_1 - \frac{r_+^2 + a^2}{a\sqrt{1-\nu^2}}}{L_2 - \frac{r_+^2 + a^2}{a\sqrt{1-\nu^2}}} + \frac{L_2 - \frac{r_+^2 + a^2}{a\sqrt{1-\nu^2}}}{L_1 - \frac{r_+^2 + a^2}{a\sqrt{1-\nu^2}}} \right) \right]^{\frac{1}{2}}.
\]  

(22)

**TABLE I**: The CM energy per unit rest mass \( \frac{E_{CM}}{m_0} \) for a KK black hole with spin \( a = 1 - \epsilon \) and \( L_1 = L_{\text{max}}, L_2 = 0 \).

| \( \nu \) | \( \epsilon=0.1 \) | \( \epsilon=0.01 \) | \( \epsilon=0.001 \) | \( \epsilon=0.0001 \) | \( \epsilon=0.00001 \) |
|---|---|---|---|---|---|
| 0 | 4.21767 | 7.10481 | 12.43999 | 22.01962 | 39.10101 |
| 0.1 | 4.21931 | 7.10426 | 12.43576 | 22.00991 | 39.08243 |
| 0.2 | 4.22482 | 7.10430 | 12.42667 | 21.98763 | 39.03917 |
| 0.3 | 4.23623 | 7.11010 | 12.42368 | 21.97348 | 39.00878 |
| 0.4 | 4.25740 | 7.13106 | 12.44599 | 22.00338 | 39.05618 |
| 0.5 | 4.29529 | 7.18270 | 12.52408 | 22.13337 | 39.28214 |
| 0.6 | 4.36278 | 7.29163 | 12.70810 | 22.45441 | 39.84939 |
| 0.7 | 4.48641 | 7.50928 | 13.09206 | 23.13507 | 41.05854 |
| 0.8 | 4.73260 | 7.95838 | 13.89599 | 24.56740 | 43.60709 |
| 0.9 | 5.34624 | 9.07637 | 15.89588 | 28.12961 | 49.94483 |

Though, when \( L_1 \) or \( L_2 \) takes the critical angular momentum \( L_c = \frac{r_+^2 + a^2}{a\sqrt{1-\nu^2}} \), the CM energy \( E_{CM}^{KK} \) will be unlimited. Unfortunately, the critical angular momentum \( L_c \) is too large for the geodesics of particle to reach the horizon. When we take \( \mu = 1 \) and one of the particles
falling without orbital angular momentum, for different small parameter $\epsilon = 1 - a$ and different $\nu$, we list the numerical result of the maximal CM energy per unit mass in table \[1\].

The result shows that the CM energy is in fact limited and the maximal CM energy grows very slowly as the black hole spin approaches its maximal value.

IV. CONCLUSION AND DISCUSSION

In this paper, we have investigated the property of CM energy for two colliding particles in Kaluza-Klein black hole. Like the Kerr-Newman black hole, Kaluza-Klein black hole is another exact solutions in the Einstein-Maxwell-dilaton theory in four-dimensional spacetime. With the vanished charge $Q$, it just describes the Kerr black hole. So, it gives a restriction that our result should not be in contradiction with that of Kerr black hole when $Q = 0$ which we have proved with the calculation. Our results show that the extremal Kaluza-Klein black holes could serve as particle accelerators to arbitrarily high CM energy when one of the colliding particles have the fine angular momentum $L = \frac{2\mu}{\sqrt{1 - \nu^2}}$. For the near-extremal case. In terms of the small parameter $\epsilon = 1 - a$, we also obtain the numerical result of the maximal CM energy per unit mass for some different value of $\epsilon$ and $\nu$. Our near-extremal result shows that the CM energy will not be so high, even in the very near-extremal black hole. On the other hand, with the vanished angular momentum $J$, the Kaluza-Klein black hole does not describe the Reissner-Nordström black hole. Our result in the nonrotating case shows that the CM energy of two charged colliding particles could also blow up, thus the non-extremal black hole could also play as a role of particle accelerators and provide a visible probe of Planck-scale physics.

Lastly, we want to give some comments on the naked singularity. If we consider a near-extremal Kaluza-Klein naked singularity, we find it is possible to collide between an ingoing and another outgoing particle just like the case in \[19\]. Then the CM energy for collision in that case reads

$$\frac{E_{\text{CM}}^2}{2m_0^2} = 1 + \frac{\mathcal{K}}{B\Delta},$$

(23)
where

\[
\mathcal{K} = [B^2(r^2 + a^2) + a^2Z](1 - e_1A_t)(1 - e_2A_t) + (Z - 1)(L_1 + e_1A_\phi)(L_2 + e_2A_\phi)
- \frac{aZ}{\sqrt{1 - \nu^2}}[(1 - e_1A_t)(L_2 + e_2A_\phi) + (1 - e_2A_t)(L_1 + e_1A_\phi)]
+ \left\{B^2(r^2 + a^2) + a^2Z\right\}(1 - e_1A_t)^2 + (Z - 1)(L_1 + e_1A_\phi)^2
- \frac{2aZ}{\sqrt{1 - \nu^2}}(1 - e_1A_t)(L_1 + e_1A_\phi) - B\Delta \right\}^{\frac{1}{2}}
\times \left\{B^2(r^2 + a^2) + a^2Z\right\}(1 - e_2A_t)^2 + (Z - 1)(L_2 + e_2A_\phi)^2
- \frac{2aZ}{\sqrt{1 - \nu^2}}(1 - e_2A_t)(L_2 + e_2A_\phi) - B\Delta \right\}^{\frac{1}{2}}. \tag{24}
\]

Thus, in the limit where the deviation of a extremal Kaluza-Klein naked singularity is very small ($\Delta \rightarrow 0$), we find that the numerator in Eq.(23) will not vanish which means the center of mass energy of collision between two particles is arbitrarily large. And this process showed in [19] could also extend to any other naked singularity.

However, our calculations is performed without considering the back reaction effect of accelerated particles pair to the background geometry of Kaluza-Klein black hole. It is shown that particles can be accelerated to arbitrarily high CM energy. So the background geometry may be destroyed and the back reaction effect should not be ignored. On the other hand, high energy assembled in small scale will lead to gravitational collapse. So the Planck-scale physics induced by the collision of particles pair with arbitrarily high CM energy is protected by the event horizon formed due to gravitational collapse and can not be observed by the external observations. Maybe, to explore the field theory interpretation of this classical effect is also an interesting topic in the future.

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