Robust Consensus of Higher-Order Multi-Agent Systems With Attrition and Inclusion of Agents and Switching Topologies

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Abstract—Some of the issues associated with the practical applications of consensus of multi-agent systems (MAS) include switching topologies, attrition and inclusion of agents from an existing network, and model uncertainties of agents. In this paper, a single distributed dynamic state-feedback protocol referred to as the Robust Attrition-Inclusion Distributed Dynamic (RAIDD) consensus protocol, is synthesized for achieving the consensus of MAS with attrition and inclusion of linear time-invariant higher-order uncertain homogeneous agents and switching topologies. A state consensus problem termed as the Robust Attrition-Inclusion (RAI) consensus problem is formulated to find this RAIDD consensus protocol. To solve this RAI consensus problem, first, the sufficient condition for the existence of the RAIDD protocol is obtained using the \( \nu \)-gap metric-based simultaneous stabilization approach. Next, the RAIDD consensus protocol is attained using the Glover-McFarlane robust stabilization method if the sufficient condition is satisfied. The performance of this RAIDD protocol is validated by numerical simulations.

Index Terms—Attrition, consensus, inclusion, multi-agent systems, simultaneous stabilization, switching topologies, \( \nu \)-gap metric

I. INTRODUCTION

A multi-agent system (MAS) is made up of multiple independently operated autonomous agents that can work together as a group through communication. The state consensus problem is a fundamental issue in the cooperative control of MASs. This problem is concerned with the synthesis of a distributed consensus protocol that drives the desired states of all the agents to a common value. Now, to reach consensus with the distributed protocol, each agent must be able to access the states of its neighboring agents via a communication network or sensing devices. This type of MAS with communicating agents is modeled using the model of the dynamics of each agent, a communication protocol that describes the interaction among the agents, and a graph that represents the interconnection topologies between agents. The potential applications of consensus are in flocking, formation control, oscillation synchronization, firefighting, multi-agent rendezvous, and satellite reconfiguration. It is worth noting that some of the practical applications of consensus have issues. For example, in the consensus of mobile agents the communication topologies between agents need to switch between several fixed topologies from time to time due to finite communication radius as well as due to the presence of an obstacle between two agents. Furthermore, the number of agents in some applications, such as firefighting, can change over time as some agents are relieved or new agents join the existing network depending upon the workload. In this paper, the removal and addition of agents from an existing network of agents are referred to as attrition and inclusion of agents, respectively. Moreover, the model of each agent is of higher-order and could possess parametric as well as unmodeled dynamics uncertainties. Considering all the aforementioned issues, some practical applications require a single distributed robust consensus protocol that can handle a varying number of higher-order agents, switching topologies, and model uncertainties.

Concerning the cooperative control of MAS with attrition of agents, a cooperative relay tracking strategy is developed in [1] to ensure successful tracking even when a second-order agent quits tracking due to malfunction. In the case of the multiagent tracking systems that are subjected to agent failure followed by the agent replacement, a modified nonsingular terminal sliding mode control scheme and event-triggered coordination strategies were proposed in [2]. In [3]-[5], consensus recovery methods are proposed to compensate for the undesirable effects caused by the removal of agents while retaining the consensus property. The consensus problem for high-order MASs with switching topologies and time-varying delays is studied in [6]. Here, the consensus problem is converted into an \( L_2-L_\infty \) control problem employing the tree-type transformation approach. Also, the consensus with the prescribed \( L_2-L_\infty \) performance is ensured through sufficient conditions that are derived using linear matrix inequalities (LMIs). In [7], the consensus problem of MAS with switch-
ing topologies is transformed into an $H_{\infty}$ control problem. The sufficient condition is derived in terms of LMIs to ensure consensus of the MAS. Following this, a distributed dynamic output feedback protocol is developed where the system matrix of the protocol is designed by solving two LMIs. Moreover, a distributed algorithm is developed in [8] using an iterative learning rule for the consensus tracking control of MASs with switching topologies and disturbances. The LMI-based necessary and sufficient conditions for the convergence of the consensus tracking objectives are also presented. Further solutions to the consensus problem with switching topologies employing LMIs can be found in [9]-[12]. The consensus problems of MAS with fixed/switching topologies and time-delays is discussed in [13]. In this paper, a Lyapunov function-based disagreement function is used to study the convergence characteristics of consensus protocols. In [14], the sufficient conditions to design a distributed protocol for the consensus of identical linear time-invariant (LTI) MASs subjected to bounded external disturbance, switching topologies, and directed communication network graph are proposed. These conditions are based on $L_2$ gain and RMS bounded disturbances. The Lyapunov stability theory is then used to investigate the stability characteristics of the proposed controllers. Maria Elena Valcher et al. [15] describes each agent of the MAS using a single-input stabilizable state-space model and then investigate the consensus problem under arbitrary switching for identical MAS with switching communication topology. Also, the consensusability of this MAS is illustrated by constructing a common quadratic positive definite Lyapunov function describing the evolution of the disagreement vector for the switched system. Guanghui Wen et al. [16] discusses the distributed $H_{\infty}$ consensus problem of MASs with higher-order linear dynamics and switching directed topologies. It is demonstrated here that if the protocol’s feedback gain matrix is properly designed and the coupling strength among neighboring agents is greater than a derived positive value, then distributed $H_{\infty}$ consensus the problem can be solved. The exponential state consensus problem for hierarchical multi-agent dynamical systems with switching topology and inter-layer communication delay is addressed in [17]. In this paper, the stability theory of switched systems and graph theory of hierarchical network topology are utilized to derive sufficient conditions for accomplishing the exponential hierarchical average consensus. The robust consensus of linear MASs with agents subject to heterogeneous additive stable perturbations is addressed in [18]. To design dynamic output-feedback protocols, two methods based on an algebraic Riccati equation and some scalar/matrix inequalities are proposed. Moreover, in [19], the sufficient condition in terms of LMIs is derived for the robust $H_{\infty}$ consensus control of MASs with model parameter uncertainties and external disturbances. Further, the traditional $H_{\infty}$ controller design is utilized in [20] to design a consensus protocol for the MAS with second-order dynamics that is subjected to parameter uncertainties and external disturbances. Also, the asymptotical convergence of agents along with desired $H_{\infty}$ performance is assured through suitable sufficient conditions. For a class of second-order multi-agent dynamic systems with disturbances and unmodeled agent dynamics, continuous distributed consensus protocols that enable global asymptotic consensus tracking are designed in [21]. These protocols are developed with the help of an identifier that estimates unknown disturbances and unmodeled agent dynamics.

The determination of a single controller that stabilizes a finite number of systems is referred to as the simultaneous stabilization (SS) problem [23]-[24]. The SS problem of more than two systems has no closed-form solution due to its NP-hardness [25]-[26]. Hence, iterative algorithms are utilized to solve SS problem. For example, an LMI-based iterative algorithm is developed in [27] and a bi-level optimization-based decomposition strategy is utilized in [28] to solve the SS problem. The SS problem is solved in [29]-[31] by first determining the sufficiency condition for the existence of the simultaneously stabilizing controller. Then, this condition is solved using a robust stabilization controller that is synthesized around the central plant (system). In [29], the central plant is obtained by solving a 2-block optimization problem. However, the central plant is identified in [30]-[31] using the maximum $\nu$-gap metric of the systems that requires SS. The definition of this $\nu$-gap metric-based central plant and the maximum $\nu$-gap metric of the systems is given in Section II. It is shown in [32]-[33] that the state consensus problem of MAS with $N$ identical LTI agents can be expressed as the SS problem of $N-1$ independent systems. In these papers, the consensus problem of MAS has been studied using an LMI-based SS approach. One needs to notice that the methods discussed in the preceding articles do not generate a single distributed consensus protocol that achieves consensus of MAS with attrition and inclusion of higher-order uncertain agents and switching topologies.

In this paper, the existing MAS is supposed to have $N$ agents. From this MAS, either attrition of $P$ agents or inclusion of $M$ agents at a time is considered. Subsequently, the problem of finding a single robust distributed dynamic state-feedback consensus protocol that achieves consensus of MAS with attrition and inclusion of LTI higher-order uncertain homogeneous agents and switching topologies is stated as the Robust Attrition-Inclusion (RAI) consensus problem. Also, the protocol that solves the RAI consensus problem is referred to as the Robust Attrition-Inclusion Distributed Dynamic (RAIDD) consensus protocol. In addition, the actual dynamics of every agent considered here is uncertain. The nominal linear dynamics of each agent is identical. Moreover, the uncertainty in the actual linear dynamics of each agent is also assumed to be homogeneous. This uncertainty is represented by bounded perturbations in the system and input matrices of the state-space model of the nominal linear dynamics. In this article, the RAIDD consensus protocol is synthesized in two steps. In the first step, the sufficient condition for the existence of this protocol is obtained using the $\nu$-gap metric-based SS method. Next, the RAIDD consensus protocol is attained using the Glover-McFarlane robust stabilisation method presented in [38] if the sufficient condition is satisfied. The main contributions of this article are the following.

1) To the best of the author’s knowledge, this is the first paper to propose a method for generating a distributed consensus control that accomplishes consensus of MAS.
with the varying number of higher-order agents, switching topologies, and model uncertainties.

2) The sufficient condition for the existence of the RAIDD consensus protocol, which depends on the Hankel norm of right coprime factors and the $\nu$-gap metric-based central plant’s maximum $\nu$-gap metric, is developed using the $\nu$-gap metric-based SS method.

3) A RAIDD consensus protocol is developed for the RAI consensus of four unmanned underwater vehicles (UUVs), with attrition and inclusion of one UUV. The performance of this protocol is validated by numerical simulations.

The rest of this paper is organized as follows. The preliminaries and notation are given in Section II. In Section III, the problem statement is presented. The synthesis of the RAIDD consensus protocol is described in Section IV. The simulation results are discussed in Section V. In Section VI, conclusions are summarized.

II. PRELIMINARIES AND NOTATION

This section introduces various definitions and notations, as well as some of the fundamental concepts of graph theory and $\nu$-gap metric-based simultaneous stabilization.

A. Notation

In this paper, $\mathbb{R}$, $\mathbb{R}_{\geq 0}$, $\mathbb{R}^n$, and $\mathbb{R}^{n \times m}$ denote the set of real numbers, the set of non-negative real numbers, the set of $n$-column real vectors, and the set of all real matrices of dimension $n \times m$, respectively. $\mathbb{N}$ is the set of natural numbers, $\mathbb{N}^+$ is the set of positive real numbers without zero, and $\mathbb{N}_0^a$ is the set of natural numbers from $a$ to $b$ ($\mathbb{N}_0^b = \{a, \ldots, b\}$, $a, b \in \mathbb{N}, a < b$). $n_{C_2}$ denotes $n(n - 1)/2$. $A \otimes B$ indicates the Kronecker product of the $A$ and $B$ matrices. $C_2$ symbolizes the open left-hand side of complex plane. The superscript $'T'$ denotes matrix/vector transpose. $A = (a_{ij}) \in \mathbb{R}^{n \times m}$ denotes a real matrix with $n$ rows and $m$ columns. Here, $a_{ij}$ is the element at the $i$th row and $j$th column of the matrix. The zero matrix with $n$ rows and $m$ columns is represented using $0_{n,m}$. The $\max\{}$ and $\min\{}$ symbolize the maximum and the minimum element of the set, respectively. The $\lambda_{\text{max}}[A]$ represents the largest eigenvalue of the matrix. $R$ and $\mathbb{R}^{\infty}$ are the set of all proper and stable rational transfer function matrices. Likewise, $\mathcal{R}$ is the set of all proper rational transfer function matrices. For a transfer function matrix, $P(s)$, its $H_\infty$ norm, determinant, Hankel norm, and winding number are denoted by $\|P(s)\|_\infty$, $\det(P(s))$, $\|P(s)\|_H$, and $\text{wno}(P(s))$ respectively. Besides these, $P(s) := (A, B, C, D)$ denotes the short form of $P(s) := C(sI - A)^{-1}B + D$. Also, $P(s)^*$ represents $P^T(-s)$.

B. Definitions

Definition 2.1: Generalized stability margin : Consider a closed-loop (CL) system, $[P(s), K]$, with the controller, $K$. Then, the generalized stability margin, $b_{P, K} \in [0, 1]$, is defined as $[40]

\[ b_{P, K} = \begin{cases} \|\mathcal{Y}\|_1^{-1} & \text{if } [P(s), K] \text{ is internally stable} \\ 0 & \text{otherwise} \end{cases} \]

where $\mathcal{Y} = \begin{bmatrix} P(s) \\ I \end{bmatrix}(I - K P(s))^{-1}[I - K]$.\]

Definition 2.2: $\nu$-gap metric : Consider two systems, $P_1(s)$ and $P_2(s)$. Let $[N_1(s) \in \mathbb{R}^{\infty}, M_1(s) \in \mathbb{R}^{\infty}]$ and $[N_1(s) \in \mathbb{R}^{\infty}, M_1(s) \in \mathbb{R}^{\infty}]$ be the right and left coprime factors of $P_1(s)$, respectively. Likewise, $[N_2(s) \in \mathbb{R}^{\infty}, M_2(s) \in \mathbb{R}^{\infty}]$ and $[N_2(s) \in \mathbb{R}^{\infty}, M_2(s) \in \mathbb{R}^{\infty}]$ be the right and left coprime factors of $P_2(s)$, respectively. Then, $\nu$-gap metric, $\delta_\nu(P_1(j\omega), P_2(j\omega)) \in [0, 1]$, of $P_1(s)$ and $P_2(s)$ is defined as $[41]

\[ \delta_\nu(P_1(j\omega), P_2(j\omega)) = \begin{cases} \|\Phi(P_1(j\omega), P_2(j\omega))\|_\infty & \text{if } \det(\Theta(j\omega)) \neq 0 \text{ and } \text{wno}(\Theta(j\omega)) \\ 1 & \text{otherwise} \end{cases} \]

where $\Phi(P_1(j\omega), P_2(j\omega)) = -N_2(s)M_1(s) + M_2(s)N_1(s)$ and $\Theta(j\omega) = N_2^\ast(s)N_1(s) + M_2^\ast(s)M_1(s)$. The $\nu$-gap metric measures the distance between systems, and if this distance is closer to zero, then any controller that works well with one system will also work well with the other.

Definition 2.3: Maximum $\nu$-gap metric of the plant : Let $Q = \{P_1(s), \ldots, P_f(s), \ldots, P_\varepsilon(s)\}$ be a finite set of systems. Then, the maximum $\nu$-gap metric of $P_i(s) \in Q$, $\epsilon_i$, is defined as $[31]

\[ \epsilon_i = \max \{ \delta_\nu(P_i(j\omega), P_f(j\omega)) \mid P_i(s), P_f(s) \in Q \land f \in \mathbb{N}^\varepsilon_0 \}. \]

Definition 2.4: $\nu$-gap metric-based central plant of $Q$ : The $\nu$-gap metric-based central plant of $Q$ is defined as the system whose maximum $\nu$-gap metric is the smallest among the maximum $\nu$-gap metrics of all the plants of $Q$ $[31]$. This definition implies that the $\nu$-gap metric-based central plant is the system that is closest to all other systems belonging to $Q$ in terms of $\nu$-gap metric. In this paper, the $\nu$-gap metric-based central plant of a set and any parameters associated with it are denoted by the subscript ‘cp’.

Definition 2.5: Hankel norm : Consider a stable system, $P(s)$. Then, the Hankel norm of $P(s)$ is given by

\[ \|P(s)\|_H = \sqrt{\lambda_{\text{max}}[W_c W_o]} \]

where $W_c$, $W_o$, and $\lambda_{\text{max}}[W_c W_o]$ are respectively the controllability Gramian and the observability Gramian.

C. Graph Theory for Formulating RAI Consensus Problem

The primary focus of this paper is the synthesis of a RAIDD consensus protocol that achieves consensus of MAS with $N \in \mathbb{N}$, $(N - P) \in \mathbb{N}$, and $(N + M) \in \mathbb{N}$ agents as well as switching topologies. The communication network topologies among these agents are represented using Undirected Simple (US) graphs. These graphs do not have any loops or parallel edges. The number of US graphs that can be formed with $N$, $N - P$, and $N + M$ nodes is $2^Nc_2 < \infty$, $2^{(N - P)c_2} < \infty$, and $2^{(N + M)c_2} < \infty$, respectively. The graphs of MAS need to be connected for achieving consensus. Following this, let the number of Connected Undirected Simple (CUS) graphs of the
The graphs associated with these matrices are undirected and \( L \) and \( \hat{L} \) eigenvector, respectively. \( \hat{\lambda}_0 \) be the maximum \( \nu \)-gap metric of \( P_{cp}(s) \). Now, the concept of the \( \nu \)-gap metric-based SS method is stated as the following.

- A single controller can provide similar CL characteristics to all the plants in \( Q \) if that controller is synthesized around \( P_{cp}(s) \). This is because \( P_{cp}(s) \) is closest to all other plants belonging to \( Q \) as \( \epsilon_{cp} \) is the smallest among the maximum \( \nu \)-gap metrics of all other plants of \( Q \).

The possibility of providing similar CL characteristics to all the plants in \( Q \) with a stabilizing controller of \( P_{cp}(s) \) increases when \( \epsilon_{cp} \) approaches zero.

In the \( \nu \)-gap metric-based SS method, the simultaneous stabilizing controller is generated by solving the sufficient condition that depends on \( P_{cp}(s) \) and \( \epsilon_{cp} \). This condition is given as [31]

\[
b_{P_{cp},K} > \epsilon_{cp}
\]

where \( b_{P_{cp},K} \) and \( \epsilon_{cp} \) are given by (11) and (3), respectively. The identification of \( P_{cp}(s) \) and \( \epsilon_{cp} \) is required for solving (11). This is accomplished by following the steps given below.

**Step 1:** Find the maximum \( \nu \)-gap metrics of all the plants of \( Q \) using (4).

**Step 2:** Identify the smallest value among the maximum \( \nu \)-gap metrics. This gives \( \epsilon_{cp} \) and the plant associated with this value is \( P_{cp}(s) \).

For more details about the \( \nu \)-gap metric-based SS method, one needs to refer to the supporting material of [30].

### III. Problem Statement

The RAI consensus problem is formulated in this section. For this purpose, consider a MAS of identical but uncertain higher-order agents. The uncertainty in the actual dynamics of these agents is also identical and parametric in nature. The nominal and actual dynamics of these agents are described by LTI systems. Following this, let \( P_i(s) \) be the transfer function of the nominal dynamics of the \( i \)-th agent of the MAS. Also, assume that all the states of \( P_i(s) \) are measured. The state-space form of \( P_i(s) \) is then given as

\[
P_i(s) : \begin{bmatrix} x_i = A x_i + B u_i \\ y_i = C x_i \end{bmatrix}
\]

where \( x_f \in \mathbb{R}^n \), \( u_f \in \mathbb{R}^m \), \( y_f \in \mathbb{R}^n \), \( A \in \mathbb{R}^{n \times n} \), \( B \in \mathbb{R}^{n \times m} \), and \( C = I \in \mathbb{R}^{n \times n} \) represent state vector, input vector, output vector, system matrix, input matrix, and output matrix of the \( i \)-th agent, respectively. Moreover, \( n \) and \( m \) are the number of states and inputs of the \( i \)-th agent, respectively. Furthermore, \( A \) is not Hurwitz and \((A, B)\) is stabilizable. Let \( \hat{P}_i(s) \) be the transfer function of actual (perturbed system) dynamics of \( i \)-th agent. The state-space form of \( \hat{P}_i(s) \) is given as

\[
\hat{P}_i(s) : \begin{bmatrix} \hat{x}_i = \hat{A} x_i + \hat{B} u_i \\ \hat{y}_i = C x_i \end{bmatrix}
\]
where $\tilde{A} \in \mathbb{R}^{n \times n} = A + \Delta A$ and $\tilde{B} \in \mathbb{R}^{n \times m} = B + \Delta B$ are the system and input matrices, respectively. Here, the $\Delta A \in \mathbb{R}^{n \times n}$ and $\Delta B \in \mathbb{R}^{n \times m}$ are the perturbations of $A$ and $B$ matrices of the nominal dynamics, respectively, due to the parametric uncertainties in the system dynamics. When $\Delta A = 0_{n,n}$ and $\Delta B = 0_{n,m}$, then (13) represents nominal dynamics of the $i$th agent.

In this paper, we consider three operational scenarios for the consensus of MAS with attrition and inclusion of agents. These scenarios are the following.

1) **scenario 1**: There is no attrition and inclusion of agents in this scenario. The MAS operates with a fixed number of agents, say $N \in \mathbb{N}_2$, in the whole operating time.

2) **scenario 2**: The MAS begins its operation with $N$ agents and $P \in \mathbb{N}^+$ agents are later removed at a given time instant. Consequently, the number of agents is not constant and varies between $N$ and $N - P$ over the course of the operation. Also, note that the maximum value of $P$ is $N - 2$.

3) **scenario 3**: The MAS commences its operations with $N$ agents. Afterward, at a given time instant, $M \in \mathbb{N}^+ < \infty$ agents were included to the MAS. In this case, the number of agents varies between $N$ and $N + M$. Note that the dynamics of $M$ agents is identical to those of $N$ existing agents.

Moreover, the communication network topologies between agents of the MAS are allowed to switch from time to time so that the network graph remains connected. Also, we assume that the graphs of the communication network topologies formed after the attrition or inclusion of agents from the MAS remain connected. Consequently, when the MAS with $\\tilde{A}$ operates over a time interval, then the CUS graphs and corresponding Laplacian matrices belong to the sets, $\{\tilde{G}_1, \ldots, \tilde{G}_p, \ldots, \tilde{G}_k\}$ and $\{\tilde{L}_1, \ldots, \tilde{L}_h, \ldots, \tilde{L}_r\}$, respectively, switch at desired time instants.

2) **scenario 2**: The MAS operates over a time interval, then the CUS graphs and corresponding Laplacian matrices belong to the sets, $\{\tilde{G}_1, \ldots, \tilde{G}_p, \ldots, \tilde{G}_r\}$ and $\{\tilde{L}_1, \ldots, \tilde{L}_h, \ldots, \tilde{L}_s\}$, respectively, switch at desired time instants.

3) **scenario 3**: The MAS operates over a time interval, then the CUS graphs and corresponding Laplacian matrices belong to the sets, $\{\hat{G}_1, \ldots, \hat{G}_h, \ldots, \hat{G}_r\}$ and $\{\hat{L}_1, \ldots, \hat{L}_h, \ldots, \hat{L}_s\}$, respectively, switch at desired time instants.

In view of the aforementioned three scenarios and switching topologies, the RAIIID consensus protocol needs to achieve consensus of the MAS with $N$, $N - P$, and $N + M$ uncertain agents that are connected using the communication network topologies whose graphs are $\tilde{G}_h \forall h \in \mathbb{N}_2$, $\hat{G}_h \forall h \in \mathbb{N}_3$, and $\hat{G}_h \forall h \in \mathbb{N}_3$, respectively. Now, consider the distributed dynamic protocol, $K_i(s)$, whose state-space form is given by

$$K_i(s) : \begin{cases} \dot{v}_i = K_A v_i + K_B \delta_i \\ u_i = K_C v_i + K_D \delta_i \end{cases}$$  \hspace{1cm} (14)
IV. SYNTHESIS OF ROBUST ATTITUION-INCLUSION DISTRIBUTED CONSENSUS PROTOCOL

The sufficient condition for the existence of $K_A$, $K_B$, $K_C$, and $K_D$, as well as the design procedure for the RAIDD consensus protocol, are proposed in this section. To develop this sufficient condition, we define the following.

1) Let $P$ be a finite set that contains all the eigenvalues of $\hat{L}_h \forall h \in N_0^p$, $\hat{L}_h \forall h \in N_0^n$, and $\hat{L}_h \forall h \in N_0^t$, except their first eigenvalue, zero. The th element of $P$ is denoted by $\lambda_i$. Also, the cardinality of $P$ is $n(P) = \xi$ where $\xi = k(N - 1) + r(N - P - 1) + p(N + M - 1)$.

Now, $\hat{P}_i(s) \forall i \in N_1^n$ are defined as

$$\hat{P}_i(s) : \begin{cases} \hat{x}_i = A\hat{x}_i + \lambda_i B\hat{u}_i \\ \tilde{y}_i = C\hat{x}_i; \forall i \in N_1^n \end{cases}$$

(22)

where $\hat{x}_i \in R^n$, $\hat{u}_i \in R^m$, and $\tilde{y}_i \in R^n$ are the state, input, and output vectors, respectively. Also, $(A, \lambda_iB)$ is stabilizable.

2) Define $Q$ as $Q = \{\hat{P}_i(s) \mid \hat{P}_i(s) =: (A, \lambda_iB, C, D = 0_{n,m}) \forall i \in N_1^n\}$. The maximum $\nu$-gap metric of the th system belongs to $Q$ is given by

$$\epsilon_i = \max \{\delta_\nu(\hat{P}_i(j\omega), \hat{P}_f(j\omega)) \mid \hat{P}_i(j\omega), \hat{P}_f(j\omega) \in Q, \forall f \in N_1^n\} \in [0, 1]$$

(23)

Also, the central plant of $Q$ and its maximum $\nu$-gap metric be $\hat{P}_{cp}(s) \in Q$ and $\epsilon_{cp}$, respectively. Furthermore, the normalized right coprime factors of $\hat{P}_{cp}(s)$ are $\hat{N}_{cp}(s) \in RH_{\infty}$ and $\hat{M}_{cp}(s) \in RH_{\infty}$ with $\det(M_{cp}(s) \neq 0)$.

3) Let $\hat{P}_i(s) \forall i \in N_1^n$ be the perturbed systems of $\hat{P}_i(s) \in Q \forall i \in N_1^n$, respectively. These systems arise due to the perturbation of $A$ and $B$ matrices mentioned in (22) to form $\hat{A} = A + \Delta A$ and $\hat{B} = B + \Delta B$. Subsequently, the function, $\Psi : R^{nxn} \times R_{nxm} \rightarrow [0, 1]$ is defined for $\Psi = \max \{\delta_\nu(\hat{P}_i(j\omega), \hat{P}_f(j\omega)) \mid \hat{P}_i(j\omega), \hat{P}_f(j\omega) \in Q, \forall f \in N_1^n\}$

$$\hat{P}_i(s) =: (\hat{A}, \lambda_iB, C, D) \forall i \in N_1^n$$

(24)

Also, note that (23), (24), and $dom(\Psi)$ indicate that $\Psi = \epsilon_{cp}$ when $\Delta A = 0_{n,n}$ and $\Delta B = 0_{n,m}$.

In the following theorem, the sufficient condition for the existence of $K_A$, $K_B$, $K_C$, and $K_D$ is proposed.

Theorem 4.1: The $K_A$, $K_B$, $K_C$, and $K_D$ exist such that the closed-loop systems given in [16], [17], and [18] satisfy [19], [20], and [21], respectively, if all the systems belong to $Q$ can be simultaneously stabilized using a full state feedback controller, $\hat{K}(s)$, which is given as

$$\hat{K}(s) : \begin{cases} \hat{v} = K_A\hat{v} + K_B\hat{x}_i \\ \hat{u}_i = K_C\hat{v} + K_D\hat{x}_i \end{cases}$$

(26)

where $\hat{v}$ is the state vector with appropriate dimension. Consequently, the consensus problem of MAS with $N$, $N - P$, and $N + M$ agents with their nominal dynamics (given in [12]) is equivalent to the following SS problem. Determine a full state feedback controller of the form given in (26) that simultaneously stabilizes all the systems belonging to $Q$. Let us assume $\hat{P}_{cp}$ and $\epsilon_{cp}$ are identified for developing the existence condition for the RAIDD consensus protocol based on the concept of $\nu$-gap metric-based SS method proposed in [30]- [31]. Then, the sufficient condition for the SS all the systems belong to $Q$ is given as

$$b_{\hat{P}_{cp}} > \epsilon_{cp}$$

(27)

Equation (11) implies that a controller needs to be obtained in the first place to solve (27). Hence, (27) does not indicate the existence of a simultaneous stabilizing controller without synthesizing a controller. To derive the controller independent sufficient condition for the SS of all the systems belonging to $Q$, let $\hat{b}_{\hat{P}_{cp}}$ be the maximum generalized stability margin of $\hat{P}_{cp}(s)$. This margin indicates the largest infinity norm of left/right coprime factor perturbations for which $\hat{P}_{cp}(s), \hat{K}(s)$ remains stable. From [39], $b_{\hat{P}_{cp}}$ is given by

$$b_{\hat{P}_{cp}} = \sqrt{1 - \| [\hat{N}_{cp}(s) \hat{M}_{cp}(s)]^T \|_{\infty}^2}$$

(28)

Further, assume $Q$ as an uncertainty set with $\hat{P}_{cp}(s) \in Q$ as its nominal system and the systems belonging to $Q \setminus \hat{P}_{cp}(s)$ as the perturbed systems of $\hat{P}_{cp}(s)$. Let $\hat{P}_f(s) \in Q \setminus \hat{P}_{cp}(s)$ be a perturbed system of $\hat{P}_{cp}(s)$. Also, $\epsilon_{cp}$ be the least upper bound on the normalized right coprime factor perturbations of $\hat{P}_{cp}(s)$ to form $\hat{P}_f(s)$. Additionally, $\Delta_{N_{cp}}(s) \in RH_{\infty}$ and $\Delta_{M_{cp}}(s) \in RH_{\infty}$ be the normalized right coprime factor perturbations of $\hat{N}_{cp}(s)$ and $\hat{M}_{cp}(s)$, respectively. These perturbations satisfy the condition given by

$$\| [\Delta_{N_{cp}}(s) \Delta_{M_{cp}}(s)]^T \|_{\infty} \leq \epsilon_{cp}.$$

(29)

Subsequently, $\hat{P}_{cp}(s)$ and $\hat{P}_f(s)$ are defined as

$$\hat{P}_{cp}(s) = \hat{N}_{cp}(s)\hat{M}_{cp}^{-1}(s)$$

(30)

$$\hat{P}_f(s) = (\hat{N}_{cp}(s) + \Delta_{N_{cp}}(s))(\hat{M}_{cp}(s) + \Delta_{M_{cp}}(s))^{-1}$$

(31)

Now, there always exists a full state feedback controller of the form given in (26) that stabilizes both $\hat{P}_{cp}(s)$ and $\hat{P}_f(s)$ when the condition given by

$$b_{\hat{P}_{cp}} \geq \sqrt{1 - \| [\hat{N}_{cp}(s) \hat{M}_{cp}(s)]^T \|_{\infty}^2} > \epsilon_{cp}$$

(32)

holds [38]. Even though $Q$ is considered as a uncertainty set, the systems belong to it are known. Therefore, the condition given in (29) becomes

$$\epsilon_{cp} = \| [\Delta_{N_{cp}}(s) \Delta_{M_{cp}}(s)]^T \|_{\infty}.$$

(33)
Also, the relation between \( \delta_v(\hat{P}_c(p), \hat{P}_f(p)) \) and their normalized right coprime factor perturbations is given as [43]
\[
\delta_v(\hat{P}_c(p), \hat{P}_f(p)) = ||| \Delta_{R_c}(s) \hat{M}_{L_c}(s) |||_{\infty}^{P_c}
\]
(34)
Let the largest infinity norm of the right coprime factor perturbations between \( \hat{P}_c(p) \) and the systems belong to \( Q \setminus \{ \hat{P}_c(p) \} \) be \( e_{\infty}^{max} \). Following [29] and (34), \( e_{\infty}^{max} \) can be written as [31]
\[
e_{\infty}^{max} = \max\{\delta_v(\hat{P}_c(p), \hat{P}_f(p)) \mid \hat{P}_c(p), \hat{P}_f(p) \in Q, \forall f \in \mathbb{R}^1\}.
\]
(35)
Using the definition of maximum \( \nu \)-gap metric, we can write (35) as
\[
e_{\infty}^{max} = e_{\infty}.
\]
(36)
Substituting (36) in (32) results in
\[
b_{\infty}^{max} = \sqrt{1 - ||[\mathbf{N}_{cp}(s) \hat{M}_{cp}(s)]^T ||_H^2} > e_{\infty}
\]
(37)
When (37) holds, then there always exists a full state feedback controller of the form given in (26) that simultaneously stabilizes all the plants of \( Q \) as \( e_{\infty} \geq e_{\infty}, \forall f \in \mathbb{R}^n \). Hence, there exists \( K_A, K_B, K_C, \) and \( K_D \) such that the CL systems given in (16), (17), and (18) with \( A = 0_{n,n} \) and \( B = 0_{n,m} \) satisfy (19), (20), and (21), respectively. Moreover, it is important to note that validating (37) does not require any controller.

**Case b:** Here, let \( \Delta A \neq 0_{n,n} \) and \( \Delta B \neq 0_{n,m} \). Furthermore, consider two ball of systems, \( B(\hat{P}_c(p), b_{\infty}^{max}, \mathbf{P}_c, R) = \{ \mathbf{G}(s) \mid \delta_v(\hat{P}_c(p), \mathbf{G}(s)) < b_{\infty}^{max} \} \) and \( B(\hat{P}_c(p), e_{\infty}) = \{ \hat{\mathbf{G}}(s) \mid \delta_v(\hat{P}_c(p), \hat{\mathbf{G}}(s)) \leq e_{\infty} \} \). The \( \nu \)-gap metrics between \( \hat{P}_c(p) \) and all the systems belong to \( B(\hat{P}_c(p), e_{\infty}) \) is less than or equal to \( e_{\infty} \) which is less than
\[
\sqrt{1 - ||[\mathbf{N}_{cp}(s) \hat{M}_{cp}(s)]^T ||_H^2}.
\]
Hence, if (37) holds, then there exists a full state feedback controller that simultaneously stabilizes all the systems belonging to \( B(\hat{P}_c(p), e_{\infty}) \). Subsequently, for the existence of a full state feedback controller that simultaneously stabilizes all the perturbed systems, \( \hat{P}_c(p), \forall i \in \mathbb{R}^n \), necessitates
\[
\hat{P}_c(p), \forall i \in \mathbb{R}^n
\]
(38)
For (38) to hold, the \( \nu \)-gap metrics between \( \hat{P}_c(p) \) and \( \hat{P}_c(p), \forall i \in \mathbb{R}^n \) need to be less than
\[
\sqrt{1 - ||[\mathbf{N}_{cp}(s) \hat{M}_{cp}(s)]^T ||_H^2}, \ i.e., \ | \delta_v(\hat{P}_c(p), \hat{P}_c(p), \forall i \in \mathbb{R}^n) | \leq e_{\infty}.
\]
The condition given in (25) needs to be true for this to happen. Note that the condition given in (25) and the condition given in (37) will be the same when \( \Delta A = 0_{n,n}, \Delta B = 0_{n,m} \) as \( \Delta A = 0_{n,n}, \Delta B = 0_{n,m} \) is true. Hence, there exists a full state feedback controller that simultaneously stabilizes all the systems, \( \hat{P}_c(p), \forall i \in \mathbb{R}^n \), and its perturbed systems, \( \hat{P}_c(p), \forall i \in \mathbb{R}^n \), if the condition given in (25) holds true. Therefore, \( K_A, K_B, K_C, \) and \( K_D \) exist such that the closed-loop systems given in (16), (17), and (18) satisfy (19), (20), and (21), respectively, if the condition given in (25) holds. This establishes the proof.

To realize sufficient condition given in (25) requires \( \hat{P}_c(p), e_{\infty}, \Delta A, \) and \( \Delta B \). Here, \( \hat{P}_c(p), e_{\infty} \) is identified by following the steps given in Section III.D. Once \( \hat{P}_c(p), e_{\infty} \) is identified, then \( \Delta_{R_c}(s) \) and \( \Delta_{L_c}(s) \) are obtained and thereafter
\[
\sqrt{1 - ||[\mathbf{N}_{cp}(s) \hat{M}_{cp}(s)]^T ||_H^2}
\]
(39)
where \( a_{ij} \in \mathbb{R}_{>0}, b_{ij} \in \mathbb{R}_{>0}, c_{iu} \in \mathbb{R}_{>0} \), and \( d_{iu} \in \mathbb{R}_{>0} \). It is possible to compute the value of \( \Psi \) associated with each element of \( \Xi \) and see if these values satisfy the condition given in (25). Those values of \( \Psi \) that satisfy the condition given in (25) yield upper and lower bounds on the elements of \( \Delta A \) and \( \Delta B \) for which consensus of \( N, N - P, \) and \( N + M \) uncertain agents can be achieved. Consequently, the sufficient condition stated by (25) is tractable as \( \sqrt{1 - ||[\mathbf{N}_{cp}(s) \hat{M}_{cp}(s)]^T ||_H^2} \) and \( \Psi \) are determinable. Now, \( \hat{K}(s) \) is obtained utilizing the Glover-McFarlane method proposed in (38) using Matlab function, ncfsys, when the condition given in (25) holds for the desired set of \( (\Delta A, \Delta B) \). ncfsys finds a \( \hat{K}(s) \) that achieves the maximum generalized stability margin that is equal to
\[
\sqrt{1 - ||[\mathbf{N}_{cp}(s) \hat{M}_{cp}(s)]^T ||_H^2}.
\]
Hence, \( \hat{K}(s) \) stabilizes all the systems belonging to \( B(\hat{P}_c(p), b_{\infty}^{max}, \mathbf{P}_c, R) \). Once \( \hat{K}(s) \) is attained, \( \hat{K}(s) \) is established using the state-space matrices of \( \hat{K}(s) \). Eventually, the RAIDD consensus protocol given in (14) can be synthesized by following the steps given below.

**Step 1:** Find the values of the maximum \( \nu \)-gap metrics of all the plants using (22).

**Step 2:** Identify the smallest value of the maximum \( \nu \)-gap metrics. This gives \( e_{\infty} \) and the system associated with this value is the central plant.

**Step 3:** If \( \sqrt{1 - ||[\mathbf{N}_{cp}(s) \hat{M}_{cp}(s)]^T ||_H^2} > e_{\infty} \), then
a) Obtain the set of \( \Delta A \) and \( \Delta B \) by varying their elements within lower bound and upper bound. For this set, compute corresponding set of \( \Psi \).

b) If \( \sqrt{1 - ||[\mathbf{N}_{cp}(s) \hat{M}_{cp}(s)]^T ||_H^2} > \Psi \), then Synthesize \( \hat{K}(s) \) using ncfsys and thereafter establish \( \hat{K}(s) \) using the state-space matrices of \( \hat{K}(s) \). Else stop.

**Step 4:** Else stop.

**V. Simulation Results**

In this section, a RAIDD consensus protocol is synthesized for the consensus of MAS with \( N = 4, N - P = 3, \) and \( N + M = 5 \) unmanned underwater vehicles (UUVs) whose
nominal dynamics [14] in state-space form is described by \( \text{(12)} \) where

\[
A = \begin{bmatrix} -0.7 & -0.3 & 0 \\ 1 & 0 & 0 \\ 0 & -v_0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.035 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{40}
\]

The pitch angular velocity, pitch angle, the depth, and the deflection of the control surface from the stern plane of the UUV are symbolized by \( q_i, \theta_i, d_i \), and \( u_i \), respectively. Then, the state vector, \( x_i \), and the input vector, \( u_i \), of \( \text{(12)} \) are defined as \( x_i = [q_i, \theta_i, d_i]^T \) and \( u_i = |u_i| \), respectively. The uncertain parameter of the UUV’s state-space model is the surge velocity \( v \). The nominal value of \( v \) is represented by \( v_0 \) and its value is 0.3 m/s. The lower and upper bound of \( v \) are 0.225 m/s and 0.375 m/s, respectively. Hence, the \( \Delta A \) in its interval matrix form is given by

\[
\Delta A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \Delta v \\ 0 & \Delta v & 0 \end{bmatrix}; \quad \Delta v \in [-0.075, +0.075]. \tag{41}
\]

The input matrix has no uncertainty and hence \( \Delta B = 0_{n,m} \). Also, the number of CUS graphs considered for the synthesis of RAIDD protocol is \( k = 3, p = 4 \), and \( r = 3 \). These graphs are shown in Figs. 2(a)-8(a). The number of agents thereafter used to constitute \( Q \). Now following Step 1 and Step 2 described in Section IV, \( \epsilon_{cp} \) is identified as 0.4293 and \( \hat{P}_{cp}(s) \in \mathcal{Q} \) as

\[
\hat{P}_{cp}(s) = \begin{cases} 
\dot{x}_i = Ax_i + 2Bu_i \\
\gamma = Cx_i.
\end{cases} \tag{42}
\]

Using this \( \hat{P}_{cp}(s) \), \( \hat{N}_{cp}(s) \) and \( \hat{M}_{cp}(s) \) are obtained, and \( \sqrt{(1 - \|N_{cp}(s) \hat{M}_{cp}(s)\|^2_2)} \) is computed to be 0.6539, which is greater than 0.4293. Hence, the condition given by \( \text{(37)} \) holds. Subsequently, \( \Xi \) is formed using \( \Delta A \) given in \( \text{(41)} \) and \( \Delta B = 0_{n,m} \). For this \( \Xi, \Psi \) is determined and its values are less than 0.6539 as indicated by Fig. 4. Then, the state-space matrices of \( \hat{K}(s) \) are computed using Glover-McFarlane method proposed in [39] and are given as

\[
K_A = \begin{bmatrix} -0.3227 & -0.3283 \\ 0.658 & -0.5469 \end{bmatrix}, \tag{43}
\]

\[
K_B = \begin{bmatrix} 0.01976 & -0.05098 & 0.4598 \\ -0.01496 & 0.1107 & -0.4072 \end{bmatrix}, \tag{44}
\]

\[
K_C = \begin{bmatrix} -0.2959 & 0.09703 \end{bmatrix}, \quad K_D = \begin{bmatrix} -0.003565 & -0.2504 & 1.13 \end{bmatrix}. \tag{45}
\]

All the closed-loop systems, \( [\hat{P}_i(s), \hat{K}(s)] \forall i \in \mathbb{N}^2_1 \), are stable because the trajectories of all their eigenvalues belong to \( \mathcal{C}_- \) as shown in Fig. 5 even when \( \Delta v \) is varied from \(-0.075 \) to 0.075. Hence, \( \hat{K}(s) \) simultaneously stabilizes \( P_i(s) \forall i \in \mathbb{N}^2_1 \) and its perturbed systems. Consequently, \( K(s) \) is formed and the values of \( k, p, \) and \( r \) suggest that \( \xi \) is 29. \( \mathcal{P} \) is then formed using the eigenvalues of \( \hat{L}_h \forall h \in \mathbb{N}^2_1 \), \( \hat{L}_h \forall h \in \mathbb{N}^2_1 \), and \( \hat{L}_h \forall h \in \mathbb{N}^2_1 \). Subsequently, the state-space forms of \( \hat{P}_i(s) \forall i \in \mathbb{N}^2_1 \) are determined using the matrices given in \( \text{(40)} \) and the eigenvalues belonging to \( \mathcal{P} \). These systems are using the state-space matrices of \( \hat{K}(s) \) given in \( \text{(45)} \). The effectiveness of this \( K(s) \) is evaluated using the four cases
of simulations listed below.

**Case 1:** The nominal dynamics of the UUVs is used in this case. The MAS begins its operations with four UUVs. The fifth UUV is then added at the 200th s. Following that, two UUVs are removed at the 500th s.

**Case 2:** Here also nominal dynamics of the UUVs is used. Initially, the MAS begins its operation with 4 UUVs. Later, one UUV is removed at 200th s. Thereafter, at 500th s, 4th and 5th UUVs are added.

**Case 3** and **Case 4:** In these cases, **Case 1** and **Case 2** are repeated with uncertain dynamics in which \( v \) varied from 0.3750 m/s to 0.2550 m/s.

In all the four cases, desired communication network topologies are switched at 1 s. The simulation results of all the four cases are shown in Figs. 6–19. The state trajectories shown in these figures indicate that the consensus of MAS with 4, 3, and 5 UUVs is accomplished even with parametric uncertainties and switching topologies.
Fig. 8. Response of the CL MAS that begins its operation with four UUVs. Later, the fifth UUV is added to the CL MAS at the 200th s and two UUVs are removed from the CL MAS at the 500th s. Also, $v_o=0.3750$ m/s and the communication network topologies of the CL MAS are switched at 1 s.

Fig. 9. Response of the CL MAS that begins its operation with four UUVs. Later, the fifth UUV is added to the CL MAS at the 200th s and two UUVs are removed from the CL MAS at the 500th s. Also, $v_o=0.3450$ m/s and the communication network topologies of the CL MAS are switched at 1 s.

Fig. 10. Response of the CL MAS that begins its operation with four UUVs. Later, the fifth UUV is added to the CL MAS at the 200th s and two UUVs are removed from the CL MAS at the 500th s. Also, $v_o=0.3150$ m/s and the communication network topologies of the CL MAS are switched at 1 s.
Fig. 11. Response of the CL MAS that begins its operation with four UUVs. Later, the fifth UUV is added to the CL MAS at the 200th s and two UUVs are removed from the CL MAS at the 500th s. Also, \( v_0 = 0.2850 \) m/s and the communication network topologies of the CL MAS are switched at 1 s.

Fig. 12. Response of the CL MAS that begins its operation with four UUVs. Later, the fifth UUV is added to the CL MAS at the 200th s and two UUVs are removed from the CL MAS at the 500th s. Also, \( v_0 = 0.2550 \) m/s and the communication network topologies of the CL MAS are switched at 1 s.
Fig. 13. Response of the CL MAS that begins its operation with four UUVs. Later, the fifth UUV is added to the CL MAS at the 200th s and two UUVs are removed from the CL MAS at the 500th s. Also, $v_0=0.2250$ m/s and the communication network topologies of the CL MAS are switched at 1 s.

Fig. 14. Response of the CL MAS that begins its operation with four UUVs. Later, one UUV is removed from the CL MAS at the 200th s and two UUVs are added to the CL MAS at the 500th s. Also, $v_0=0.3450$ m/s and the communication network topologies of the CL MAS are switched at 1 s.
(a) case 4: pitch angular velocity response

(b) case 4: pitch angular velocity response

(c) case 4: pitch angle response

Fig. 16. Response of the CL MAS that begins its operation with four UUVs. Later, one UUV is removed from the CL MAS at the 200th s and two UUVs are added to the CL MAS at the 500th s. Also, \( v_0 = 0.3150 \) m/s and the communication network topologies of the CL MAS are switched at 1 s.

Fig. 17. Response of the CL MAS that begins its operation with four UUVs. Later, one UUV is removed from the CL MAS at the 200th s and two UUVs are added to the CL MAS at the 500th s. Also, \( v_0 = 0.2850 \) m/s and the communication network topologies of the CL MAS are switched at 1 s.

Fig. 18. Response of the CL MAS that begins its operation with four UUVs. Later, one UUV is removed from the CL MAS at the 200th s and two UUVs are added to the CL MAS at the 500th s. Also, \( v_0 = 0.2550 \) m/s and the communication network topologies of the CL MAS are switched at 1 s.
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VI. CONCLUSION

Based on the gap metric-based simultaneous stabilization method, a RAIDD consensus protocol is developed for the consensus of MAS with attrition and inclusion of LTI higher-order uncertain homogeneous agents and switching topologies. This protocol is based on the gap metric-based simultaneous stabilization method for the consensus of MAS with attrition and inclusion of LTI higher-order uncertain homogeneous agents and switching topologies. The sufficient conditions for the existence of the RAIDD consensus protocol are given. The effectiveness of the RAIDD consensus protocol is validated through numerical simulations of the closed-loop system comprising the RAIDD protocol and the MAS with 4, 3, and 5 UUVs. The state trajectories of agents indicate that the consensus of MAS with 4, 3, and 5 UUVs is achieved even with model uncertainties and switching topologies.

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