Three-Flavor Analysis of Neutrino Mixing with and without Mass Hierarchy†

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Abstract

I summarize the results of barely model-dependent phenomenological analyses of the structure of the neutrino flavor mixing. The analyses are based on the three-flavor mixing framework without sterile neutrinos and utilize the hints from solar and atmospheric neutrino observations as well as that from mixed dark matter cosmology. It will be demonstrated that the features of the analysis is sharply distinguished by the two cases (I) with and (II) without dark matter mass scale, and by whether one (or two) mass is dominant (OMD) or the three states are almost degenerate (ADN). The global features of the neutrino mixing is illuminated for these different mass patterns.

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In this talk, I summarize the progress in our understanding of the structure of lepton mixing matrix based on the analyses of neutrino mixing hinted by various experiments. I will put special emphasis on the fact that very weakly model-dependent approach exists toward determining patterns of neutrino mixing matrix. Probably, now is the time for us to start to think about the lepton flavor mixing, a fascinating subject which may guide us beyond the standard model of particle physics.

Usually the standard electroweak theory is formulated by giving neutrinos zero mass. Do we have any experimental indication which is contradictory to the simplest possibility? I believe that the answer is yes. We do have three hints, whose two may be called as the “direct experimental evidence”, while whose last is an intriguing suggestion from cosmology. The first of the former category is the energy-dependent modulation of the solar neutrino energy spectrum observed by four different experiments, the chlorine, the Kamiokande II-III, GALLEX and SAGE [1]. It is now very difficult to believe that the data from these four experiments can be reconciled with some sensible modification of the standard solar model. It is due to the low $^{71}$Ga rate which is well below 100 SNU and the missing (or strongly suppressed) $^{7}$Be neutrino situation enforced by the chlorine and the Kamiokande experiments.

The second is the anomaly in the $\nu_\mu/\nu_e$ ratio in the atmospheric neutrinos observed by the Kamiokande, IMB and Soudan 2 experiments [2]. In spite of the fact that the anomaly is not observed by NUSEX and the Frejus detectors [3], the evidence in the Kamiokande experiment is so impressive that forces us to take it seriously. The existence (or not) of the anomaly will be checked by the well-operating Super-Kamiokande and the relevant region of the oscillation parameters will be probed by the planned long-baseline neutrino oscillation experiments. I note that the Japanese project, KEK $\rightarrow$ Super-Kamiokande, has already been funded and will run in early 1999.

The third hint comes from the mixed (=hot and cold) dark matter cosmology [4]. It is one of the rival models which can account for the structure formation in the universe in a way consistent with the COBE observation. The neutrinos with masses of the order of $\sim$
10 eV provides the best candidate for hot component of the dark matter. In fact, they are the only candidate particles that are known to exist in nature. We have to mention that the mixed dark matter cosmology itself cannot be regarded as compelling evidence for dark matter mass neutrinos.

The important point which I am going to stress in this talk is that the existence, or nonexistence, of neutrinos with dark matter mass scale sharply distinguishes the features of the analysis of the neutrino mixing. I hope that clarification of this point finally allows us to obtain unambiguous criteria of how the (non-) existence of the dark matter mass neutrinos can be signaled experimentally. It is the ultimate goal of our investigation that the complete classification of the neutrino mixing pattern constrained by various experiments eventually leads us to the understanding of the physics of lepton flavor mixing.

It is useful to classify neutrino mass patterns into I with and II without dark matter mass neutrinos. It is then important to classify the case I into two classes:

I-OMD: One-Mass Dominance in type I (1 or 2 species of dark matter mass neutrinos)

I-ADN: Almost Degenerate Neutrinos (all of which have dark matter mass)

Then the case II may be denoted as II-ADN because there is no natural reason, besides dark matter cosmology, for expecting either intermediate or larger neutrino masses. So now the three types of mass pattern, I-OMD, I-ADN, and II-ADN, are on your shopping list.

Let us define more precisely what we mean by I-OMD and I-II-ADN to make our subsequent analyses unambiguous. By I-OMD we imply the mass pattern (a) \( m_3^2 \gg m_1^2 \approx m_2^2 \) where \( \Delta m_{13}^2 \approx \Delta m_{23}^2 \sim 10 \text{eV}^2 \) and much larger than \( \Delta m_{12}^2 \), or the pattern (b) \( m_3^2 \ll m_1^2 \approx m_2^2 \) where \( |\Delta m_{13}^2| \approx |\Delta m_{23}^2| \sim 10 \text{eV}^2 \). In the latter case (b) the term OMD is not quite correct; it is the two-mass dominant case. But we employ this terminology because it is becoming popular. Without matter effect the neutrino oscillation phenomenon itself does not distinguish between (a) and (b). The smaller \( \Delta m_{12}^2 \) can be taken as either \( \Delta m_{12}^2 \sim 10^{-6} \text{eV}^2 \), or \( \Delta m_{12}^2 \sim 10^{-2} \text{eV}^2 \), corresponding respectively to the solar neutrino and the atmospheric neutrino anomalies. Similarly we assume that the degeneracy in I-II-ADN mass pattern is
due to the solar and the atmospheric neutrino mass scales. Thus, the mass scales involved in the mass patterns which will be used in our following analysis entirely comes from one of the hints for neutrino masses that we mentioned before.

We discuss in the following how these three neutrino mass patterns, I-OMD, I-ADN, and II-ADN, can be constrained by various experiments. We examine, one by one, (1) reactor and accelerator experiments, (2) solar neutrino experiments, (3) atmospheric neutrino experiments, and (4) double $\beta$ decay experiments. While detailed model building of the mixed dark matter cosmology may imply additional constraints, we rely on none of them quantitatively, except for the order of magnitude scale $\sim10\text{eV}$ of dark matter mass. We believe that it is a reasonable attitude because systematic errors of the cosmological observations are difficult to estimate in most cases.

We use the standard form of Cabibbo-Kobayashi-Maskawa quark mixing matrix

$$U = \begin{bmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{bmatrix}, \quad (1)$$

for the neutrino mixing matrix throughout the following analyses. Its use became rather standard also in the analysis of the neutrino mixing.

(1) Accelerator and reactor experiments

The feature of the constraints from the accelerator and the reactor experiments markedly differ between mass patterns I-OMD and I-II-ADN. By now it is well known that tremendous amount of constraints exist for the I-OMD case, but only a very mild one for I-II-ADN. The clear recognition of these points began with the works in [5,6], and the bounds on the mixing parameters are worked out quantitatively in detail by Fogli, Lisi and Scioscia [7] by using the explicit three-flavor angle definition as ours. The constraint for the I-OMD case is schematically drawn on $s_{23}^2 - s_{13}^2$ plane in Fig. 1.

(2) Solar neutrino experiments
The constraints from solar neutrino observation are again much milder for the mass pattern I-II-ADN. It is only quite recently that the constraints for this case with the MSW mechanism has been worked out [8] in a full three-flavor setting under the assumption of the $\Delta m^2$-hierarchy ($\neq$ mass hierarchy). They found, quite remarkably, that the small-$s_{12}$ and the large-$s_{12}$ solutions found in the two-flavor analyses are merely a part of the edge at $s_{13} = 0$ in much broader 90% CL allowed region on $s_{12}^2 - s_{13}^2$ plane. Namely, the two small-$s_{13}$ solutions fuse into a single one at relatively large value of $s_{13}^2$ at around $s_{13}^2 = 0.33$, and the solution survives up to $s_{13}^2 \simeq 0.6$. At the moment, the result obtained in [8] constitutes our best knowledge of the three-flavor analysis of the solar neutrino constraint for the case I-II-ADN.

In the mass pattern I-OMD one can place stronger constraints by using the solar neutrino observation. One can show quite generally that a severe bound exists for $s_{13}^2$, $s_{13}^2 \lesssim$ a few %. It can be seen in Fig. 1, though schematically, that with dark matter mass neutrinos of OMD type mass pattern the accelerator and the reactor data constrain $s_{13}^2$ either less than a few % level or very close to 1. Then, the useful formula [9]

\[ P^{(3)}(\nu_e \rightarrow \nu_e) = c_{13}^4 P^{(2)}(\nu_e \rightarrow \nu_e) + s_{13}^4 \]

(2)

which relates the electron neutrino survival probabilities in the three- and the two-flavor frameworks implies that the large-$s_{13}$ branch is not consistent with the solar neutrino deficit. Notice that (2) holds also for the vacuum neutrino oscillation and hence the conclusion prevails in the vacuum oscillation solution. Thus, the lower two domains in Fig. 1 are the allowed region by the terrestrial experiments and the solar neutrino observation for the case of I-OMD.

(3) Atmospheric neutrino experiments

The constraint from the atmospheric neutrino experiments gives rise to quite different allowed region on $s_{13}^2 - s_{23}^2$ plane for the I-OMD and I-II-ADN cases. In the case of I-II-ADN mass pattern the allowed region is a $\Gamma$ (gamma)-shaped region on $s_{13}^2 - s_{23}^2$ plane as drawn,
again schematically, in Fig. 2. Despite the uncertainty we mention in the footnote\footnote{Unlike the case of the solar neutrino analysis the theoretical analysis of the atmospheric neutrino anomaly seems to be plagued with great uncertainties. It is realized by several researchers who tried to reproduce the features of the Monte-Carlo simulation (without neutrino oscillation) by Kamiokande group that it is quite difficult to achieve the goal. It might be due to the fact that theorists neither know the detector performance, nor have a chance of looking into the details of the Kamiokande Monte-Carlo simulation (including its structure in energy-zenith angle plane) which cannot be read off from the published data. I must emphasize that unless one is able to reproduce the Kamiokande Monte-Carlo result with full use of the realistic neutrino flux, the reliability of the detailed shape of the obtained allowed region is subject to the great uncertainties.} this global feature seems to hold irrespective of the the details of the analysis. On the other hand, in the case of I-OMD the atmospheric neutrino anomaly must be taken care of by either almost pure $\nu_\mu \to \nu_\tau$ or $\nu_\mu \to \nu_e$ oscillations, if it is entirely due to the neutrino oscillation. It is because the constraints from accelerator and reactor experiments are so restrictive as to confine the allowed mixing parameters into the almost pure two-flavor channel. (We must bear in mind, however, that the full three-flavor analysis of the atmospheric neutrino data in this context has not been done so far.) By these simple considerations one can easily conclude that the relevant regions for accounting for the atmospheric neutrino anomaly in the I-OMD mass pattern is the upper horizontal region ($\nu_\mu \to \nu_\tau$) and the lower-left corner ($\nu_\mu \to \nu_e$) on $s_{13}^2 - s_{23}^2$ plane, as indicated in Fig. 1.

I note that the allowed regions for the cases of I-OMD and I-II-ADN are so different (they do not overlap!) is essentially due to the artifact of our definition of the mixing angle in the restricted framework of three-flavor mixing. If we work with an extended framework of I-OMD plus one sterile neutrino with the label 0 (zero) the mixing angles corresponding to $\theta_{13}$ and $\theta_{23}$ in the I-II-ADN scenario are “$\theta_{02}$” and “$\theta_{12}$”, respectively.

(4) Double $\beta$ decay experiments
So far we have discussed the so-called “$\Delta m^2$ physics” in which only the mass-squared differences, not their absolute values, are the relevant quantities. This is the reason why our foregoing discussions did not distinguish between the mass patterns I-ADN and II-ADN, and between I-OMD (a) ($m_3^2 \gg m_1^2 \approx m_2^2$) and I-OMD (b) ($m_3^2 \ll m_1^2 \approx m_2^2$). Let us now address the experimental quantity that can distinguish these two cases. It is the neutrinoless double $\beta$ decay experiments by which one can constrain the Majorana neutrinos.

Within the framework of I-ADN mass pattern $<m_{\nu e}>$, the observable in neutrinoless double $\beta$ decay, can be greatly simplified. To understand the point let me remind you the generic expression of $<m_{\nu e}>$,

$$<m_{\nu e}> = |c_{12}^2 c_{13}^2 m_1 e^{-i(\beta + \gamma)} + s_{12}^2 c_{13}^2 m_2 e^{i(\beta - \gamma)} + s_{13}^2 e^{2i(\gamma - \delta)},|$$

where $\beta$ and $\gamma$ are the extra CP-violating phases characteristic to Majorana neutrinos. In the I-ADN case one can ignore mass differences against the nearly degenerate neutrino masses themselves. In the CP-conserving case, as it is most obvious, the cancellation can take place depending upon the patterns of CP parities. Let us take the convention that $\eta_1 = +$ and denote them collectively as $(\eta_1, \eta_2, \eta_3) \equiv (1, e^{2i\beta}, e^{i(\beta + \gamma + 2\delta)}) = (+ + -)$ etc. Then,

$$r \equiv \frac{<m_{\nu e}>}{m} = \begin{cases} 1 & \text{for } (+ + +) \\ |1 - 2s_{13}^2| & \text{for } (+ + -) \\ |1 - 2s_{12}^2 c_{13}^2| & \text{for } (+ - +) \\ |1 - 2c_{12}^2 c_{13}^2| & \text{for } (+ - -) \end{cases}$$

Let us refer to the ratio $<m_{\nu e}> / m$ as $r$ hereafter.

An analysis based on the mixed dark matter scenario [10] gives degenerate dark matter neutrino mass in the range of $2.3 \text{ eV} \lesssim m_\nu \lesssim 4.5 \text{ eV}$, which can be translated to $0.15 \lesssim r \lesssim 0.29$ assuming the most stringent bound $<m_{\nu e}> \lesssim 0.68 \text{ eV}$ [11]. Then, we have a constraint which can be drawn on $s_{12}^2 - s_{13}^2$ plane, as indicated in Fig. 3. The figure is for general CP-noninvariant cases and is based on our analysis done in Ref. [12], to which we refer for details.
In Fig. 3 the darker shaded region is the allowed region with 90% CL for the three-flavor MSW solution to the solar neutrino problem obtained by Fogli et al. One can conclude that either the large-$s_{13}$ or the large-$s_{12}$ MSW solutions of the solar neutrino problem are preferred by the double $\beta$ decay constraint in the I-ADN scenario.

The double $\beta$ decay constraint matters also for the I-OMD mass pattern. I briefly discuss the point. Generally speaking, the double $\beta$ decay constraint is difficult to achieve in the I-OMD (a) mass pattern because there is no chance of cancellation between two dark-matter masses. The constraint is, however, easily met in the small-$s_{13}$ region because the largest mass $m_3$ is multiplied by $s_{13}^2$. On the other hand, in I-OMD (b) mass pattern the cancellation is possible. In fact the cancellation is automatic in the large-$s_{13}$ region. Thus, the double $\beta$ decay constraints act in a rather nontrivial way in the I-OMD cases. For detail I refer Ref. [5].

In conclusion we have presented the results of our analysis of the neutrino mixing with and without mass hierarchy in the framework of three-flavor neutrinos without steriles. In the OMD case the mixing pattern is tightly constrained and is determined to be essentially unique when the double $\beta$ constraint is imposed. In the ADN mass pattern with dark matter scale we also obtain interesting constraints which prefer either the large-$s_{13}$ or the large-$s_{12}$ MSW solutions of the solar neutrino problem.
Fig. 1

Fig. 2
Fig.3: The areas inside the solid and the dashed lines are the allowed regions by the neutrinoless double $\beta$ decay experiments in general CP non-conserving cases where the CP violating phases $\beta$, $\gamma$ and $\delta$ are treated as unconstrained. The solid and the dashed lines are for $r < 0.15$ which corresponds to $m=4.5$eV, and for $r < 0.29$ which corresponds to $m=2.3$ eV, respectively. The darker shaded areas bounded by the thinner lines are the allowed regions with 90% CL for the three-flavor MSW solution of the solar neutrino problem obtained by Fogli et al. [8].
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