Influence of Motion of Environment on Thermo-, Photo- and Diffusiophoresis of a Solid Aerosol Particle of the Spheroidal Form

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Abstract  In Stokes approach, the theoretical description of stationary motion of a large solid aerosol particle of the spheroidal form in external fields of temperature and concentration of gradients, on which powerful electromagnetic radiation in a binary gas mixture falls down, is carried out. At motion consideration it was supposed, that the average temperature of a surface of a particle slightly differs from the temperature of the gaseous environment surrounding it. In the course of the gasdynamics equations solution analytical expressions for force and speed of thermo-, photo-, and diffusion-phoresis taking into account influence of movement of environment are obtained.

Keywords  spheroid; thermophoresis; photophoresis; diffusiophoresis

1 Introduction

In a modern science and the technique, in areas of chemical technologies, hydrometeorology, preservations of the environment etc. multiphase mixes are widely applied. The disperse mixtures, consisting of two phases one of which is a particle and the other one is the viscous environment (gas or a liquid), hold the greatest interest today. Gas (liquid), with the particles suspended in it is called aerosol and the particles are called aerosol particles. Hydro- and aerosol particles can make considerable impact on a course of physical and physical-chemical processes of a various types in disperse systems (for example, processes of mass and heat exchange).

The size of particles of a disperse phase lies in a very wide limits: from macroscopic ($\sim 500\, \text{mkm}$) to molecular ($\sim 10\, \text{nm}$) values; a concentration of particles varies accordingly - from one particle to highly concentrated systems ($> 10^{10}$ $-3$). Nowadays, with development of nano-technologies and nano-materials, the essential prospect is represented by application of ultradisperse (nano-) particles, for example, in nano-electronics and nano-mechanics, etc.

The forces of the various nature can effect on the particles of disperse systems causing their ordered movement concerning the centre of inertia of the viscous environment. For example, the sedimentation occurs in the field of the gravitational force. In gaseous environments with non-uniform distribution of temperature there can be ordered movement of the particles which is caused, for example, by externally set gradients of temperature and concentration that is called thermophoresis and diffusiophoresis [1]. If movement is caused at the expense of internal sources of heat non-uniformly distributed in particle volume such movement is called photophoretic [2-4].

Average distance between aerosol particles of a considerable part of aerodisperse systems meeting in practice is much greater than a characteristic size of particles. In such systems the account of influence of an aerosol on development of physical process can be carried out, basing on knowledge of laws of dynamics of movement and heat - and mass exchange with infinite environment of separate aerosol particles. Mathematical modeling of evolution of aerosol systems and the solution of such important question as purposeful influence on aerosols is impossible without knowledge of laws of this type of behavior.

Many particles that can be found in industrial devices and nature, have the form of a surface different from spherical, for example, spheroidal (ellipsoid of rotation). Therefore studying of laws of motion of separate particles in gaseous (liquid) both homogeneous, and non-homogeneous environments is the important actual problem representing considerable theoretical and practical interest.

2 Materials and Methods

2.1. Problem of formulation. The large solid particle of the spheroidal form suspended in a binary gas mixture with temperature $T_\infty$, density $\rho_g$ and viscosity $\mu_g$ is considered. Let in this binary gas mixture by

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Within the limits of the formulated assumptions the distribution of speed $U_g$, pressure $P_g$, temperatures $T_g$ and $T_p$, and a relative concentration of the first component of the binary gas mixture $C_1$ are described by the following system of the equations [7]:

$$\nabla P_g = \mu_g \triangle U_g, \quad \text{div} U_g = 0,$$

$$\rho_g c_p \rho_p (U_g) \nabla T_g = \lambda_g \triangle T_g, \quad \triangle T_p = -\frac{q_p}{\lambda_p},$$

$$\left(U_g \nabla\right) C_1 = D_{12} \triangle C_1.$$  (3)

The system of equations (3) was solved with the following boundary conditions [5]:

$$\varepsilon = \varepsilon_0 : \quad U_\varepsilon = -\frac{cU \cosh \varepsilon}{H_\varepsilon} \cos \eta,$$

$$U_g = \frac{cU \sinh \varepsilon}{H_\varepsilon} \sin \eta - K_{TS} \nu_g (T_\nabla \cdot e_\eta) - K_{DS} D_{12} (\nabla \nabla C_1 \cdot e_\eta),$$

$$T_g = T_p, \quad \lambda_g \frac{\partial T_g}{\partial \varepsilon} = \lambda_p \frac{\partial T_p}{\partial \varepsilon}, \quad \frac{\partial C_1}{\partial \varepsilon} = 0.$$  (4)

$$\varepsilon \to \infty : \quad T_g \to T_\infty + |\nabla T_g| \cdot \sin \varepsilon \cos \eta,$$

$$C_1 \to C_\infty + |\nabla C_1| \cdot \sin \varepsilon \cos \eta,$$

$$P_g \to P_\infty; \quad U_g \to 0.$$  (5)

$$\varepsilon \to 0 \quad T_p \neq \infty$$  (6)

Here $e_\eta$, $e_\varepsilon$ are unit vectors of spheroidal system of co-ordinates; $U_\varepsilon$, $U_\eta$ - components of the mass speed of the gas $U_g$, $U = |U|$ - characteristic speed of the particle movement; $\lambda_g, \lambda_p$ - coefficients of the heat conductivity of the gas and the particle respectively; $\nu_g, \nu_p$ - kinematic and dynamic viscosity; $H_\varepsilon = c_\varepsilon \cosh^2 \varepsilon - \sin^2 \eta$ - Lamé coefficient; $K_{TS}$ and $K_{DS}$ - coefficients of thermal and diffusion slips which are determined by methods of the kinetic theory of gases. For example, at accommodation coefficients of a tangential impulse and the energy, equal to unity, the gaskinetic coefficient (in case of a spherical particle) $K_{TS} \approx 1.152$, $K_{DS} \approx 0.3$ [1, 8]. $\varepsilon = \varepsilon_0$ - the co-ordinate surface corresponding to a surface of the particle.

2.2. Distribution of temperature and relative concentration out of and in the particle. Let us make equations (3) and boundary conditions (4) - (6) dimensionless, having entered dimensionless temperature and speed as follows: $\tau_k = T_k / T_\infty$, $V_g = U_g / U, (k = g, p)$.

In the experiment except Reynolds’s and the Peclet dimensionless numbers there are two more controllable small parameters $\xi_1 = a |\nabla T_g| / T_\infty \ll 1$, $\xi_2 = a |\nabla C_1| \ll 1$, characterizing relative temperature drop and concentration over the size of a particle. For pure thermsophoresis the characteristic speed $U$ is of the order of magnitude $U \sim (\mu_g / \rho_g T_\nabla) |\nabla T_g|$, and for pure diffusion phoresis - $U \sim D_{12} |\nabla C_1|$. If we take this into account, the solution of the boundary problem (3) - (6) will be found in the form of expansion of corresponding physical sizes over powers of $\xi_1$. The small parameter $\xi_2$ is expressed through $\xi_1$, and also the Reynolds number.
\[ (\xi = Re = \rho g U a/\mu g \ll 1) \] calculated by the characteristic speed of pure thermophoresis coincides with \( \xi_t \), which proves the expansion over the given small parameter.

\[ V_g = V_{g0} + \xi_1 V_{g1} + \ldots, \quad \eta_g = \eta_{g0} + \xi_1 \eta_{g1} + \ldots \]  
(7)

Searching the force and the speed of thermo-, photo- and diffusiophoresis, we will be limited by the corrections of the first order smallness over \( \xi_t \). In order to find them it is necessary to know the distribution of speed, pressure, temperatures and concentration in the spheroid vicinity. Substituting (7) in (3), leaving terms of the order \( \xi_1 \), solving the obtained systems of the equations by the method of separation of variables, finally we obtain for zero and the first approximations

\[ t_{g0}(\lambda) = 1 + \gamma \lambda_0 \text{arccctg} \lambda, \quad C_{10} = C_1 = \infty \]  
(8)

\[ t_{g1}(\lambda) = 1 + (1 - \delta) \gamma \lambda_0 \text{arccctg} \lambda_0 + \delta \gamma \lambda_0 \text{arccctg} \lambda + \int_{\lambda_0}^{\lambda} f_0 \text{arccctg} \lambda d\lambda - \text{arccctg} \lambda \int_{\lambda_0}^{\lambda} f_0 d\lambda \]  
(9)

\[ t_{g1} = \cos \xi \frac{\lambda^3}{\lambda} + \frac{1}{\lambda} (\text{arccctg} \lambda - \frac{\lambda}{2} \text{arcctg}^2 \lambda) \]  
(10)

\[ \int_{\lambda_0}^{\lambda} f_1 (\text{arccctg} \lambda - 1) d\lambda + (\text{arccctg} \lambda - 1) \int_{\lambda_0}^{\lambda} f_1 d\lambda \]  
(11)

\[ C_{11} = \cos \xi \frac{\lambda^3}{\lambda} - \frac{\text{arccctg} \lambda - 1)}{\lambda_0} \frac{\lambda^3}{\lambda_0} (\text{arccctg} \lambda - 1) \]  
(12)

where \( \lambda = \sin \varphi, \lambda_0 = \sin \varphi_0, \delta = \lambda / \lambda_p, \gamma = \delta \) - dimensionless parameter characterizing heating of the spheroid; \( T_s = T_s / T_\infty, T_\infty - average temperature of the spheroid surface defined by the formula

\[ \frac{T_s}{T_\infty} = 1 + \frac{1}{4 \pi c \lambda_0 T_\infty} \int V q_p dV \]  
(13)

Integration in the formula (14) is carried out over the whole particle’s volume.

\[ f_n = \frac{2^{2n + 1}}{2 \lambda_p T_\infty} \int_{-1}^{1} c^2 q_p (\lambda^2 + x^2) P_n(x) dx, \quad x = \cos \varphi, P_n(x) - Legendre polynomials [9]. \]

The constants entering into expressions for fields of temperatures out of and in the particle (11), (12) are determined from corresponding boundary conditions on the spheroid surface. Considering, that later the expression for the coefficient is required, let us represent its explicit form

\[ \Gamma = -\frac{c(1 - \delta)}{a \Delta} + \frac{3}{4 \pi c^2 \lambda_p \lambda_0 T_\infty (1 + \lambda_0^2) \Delta} \int V q_p dV + \ldots + \frac{Pr \gamma \lambda_0}{ac} \{ A_2 (\text{arccctg} \lambda_0 - \frac{1 - \delta}{2 \Delta} \text{arccctg}^2 \lambda_0 - \frac{\delta}{(1 + \lambda_0^2) \Delta}) + A_1 (\text{arccctg} \lambda_0 + \frac{\delta \lambda_0 \text{arccctg} \lambda_0 - 1}{(1 + \lambda_0^2) \Delta}) \} \]  
(14)

\[ \Delta = (1 - \delta) \text{arccctg} \lambda_0 + \frac{\delta \lambda_0}{1 + \lambda_0^2} = \frac{1}{\lambda_0} \]  

### 2.3. Determination of the force and the speed of thermo-, photo-, and diffusiophoresis.

A general solution of hydrodynamic equations in the spheroid coordinate system has a form [5]:

\[ U_\varepsilon(\varepsilon, \eta) = \frac{U}{c \cos \varphi_\varepsilon} \cos \xi \{\lambda A_2 + [\lambda - (1 + \lambda_0^2) \text{arccctg} \lambda] A_1 + c^2 (1 + \lambda^2) \}, \]

\[ U_\eta(\varepsilon, \eta) = -\frac{U}{c \varphi_\varepsilon} \sin \xi \left\{ \frac{A_2}{\lambda} + [1 - \lambda \text{arccctg} \lambda] A_1 + c^2 \lambda \right\}, \]

\[ P_g(\varepsilon, \eta) = P_\infty + \frac{\mu_g U}{H_s} [(x^2 + \lambda^2) A_2]. \]

Constants of integration \( A_1, A_2 \), are determined from the boundary conditions of the spheroid surface, in particular,

\[ A_2 = \frac{2 c^2}{\lambda_0 + (1 - \lambda_0^2) \text{arccctg} \lambda_0 - \frac{2 K_T S}{U_s} \frac{c^2 \nu_g \sqrt{|\nabla T_s|} \lambda_0 - (1 + \lambda_0^2) \text{arccctg} \lambda_0}{T_\infty} \lambda_0 + (1 - \lambda_0^2) \text{arccctg} \lambda_0} \times \left( 1 + \lambda_0^2 \right) \Delta \left( 1 + \lambda_0^2 \right) \frac{3 a (1 - \lambda_0^2) \text{arccctg} \lambda_0}{4 \pi c \lambda_0 T_\infty} \int_V q_p dV - \frac{Pr \gamma \lambda_0}{2 \lambda_0 + (1 - \lambda_0^2) \text{arccctg} \lambda_0} [2 \text{arccctg}^2 \lambda_0 + 4 (\lambda_0 \text{arccctg} \lambda_0 - 1) + (1 - \lambda_0^2) (\lambda_0 \text{arccctg} \lambda_0 - 1)^2] \left( 1 + \lambda_0^2 \right) \frac{c^2 \langle C \rangle}{\lambda_0} \]  
(16)

Main force acting on the spheroid is determined by integration of the stress tensor over the surface of the aerosol particle [5, 6] and has a form:

\[ F_z = 4 \pi \mu_g U c A_2 \]  
(17)

Granting the explicit form of the coefficient \( A_2 \), we obtain the general expression for the force acting on the spheroidal particle, which additively consists of the force of the environment viscous resistance \( F_\mu \), thermophoretic force \( F_{th} \), photophoretic force \( F_{ph} \), proportional to the dipole moment of thermal sources density, non-uniformly distributed in the volume of the particle, force considering the influence of the environment movement, and the diffusion photoretic force \( F_{ph} \)

\[ F_z = F_\mu + F_{th} + F_{ph} + F_{th}, \]

\[ F_\mu = -8 \pi \mu_g U c \lambda_0 + (1 - \lambda_0^2) \text{arccctg} \lambda_0 \]  

\[ F_{th} = -8 K_T S \mu_g \nu_g \sqrt{|\nabla T_s|} \lambda_0 - (1 + \lambda_0^2) \text{arccctg} \lambda_0 \times \]

\[ T_\infty \lambda_0 + (1 - \lambda_0^2) \text{arccctg} \lambda_0 \]  

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\[
F_{ph} = -8\pi K_{TS} \xi \mu \nu_0 g \lambda_0 - (1 + \lambda_0^2) \arctgent g \lambda_0 \times \left( \frac{\varepsilon d}{(1 + \lambda_0^2)\Delta} (1 - \frac{Pr_{\infty} \gamma \lambda_0}{2(\lambda_0 + (1 - \lambda_0^2)\arctgent g \lambda_0)} \right) \times \left[ \frac{\varepsilon d}{(1 + \lambda_0^2)\Delta} \right] \int V q_p z dV - \frac{Pr_{\infty} \gamma \lambda_0}{2(\lambda_0 + (1 - \lambda_0^2)\arctgent g \lambda_0)} \left[ (\lambda_0\arctgent g \lambda_0 - 1)^2 \right] - 4(\lambda_0\arctgent g \lambda_0 - 1) + (\lambda_0\arctgent g \lambda_0 - 1)^2] + K_{DS} D_{12} |\nabla C_1|.
\]

In order to obtain an expression for the speed of the ordered movement of the spheroidal particle in the external set field of the gradient of temperature and concentration

\[
U = \frac{b}{a} K_{TS} \nu_0 g \left[ \frac{\nabla T_g}{T_\infty} \right] \left( 1 - \frac{1}{\lambda_0} \right) \arctgent g \lambda_0 \times \left( 1 - \frac{Pr_{\infty} \gamma \lambda_0}{2(\lambda_0 + (1 - \lambda_0^2)\arctgent g \lambda_0)} \right) \left[ \frac{\varepsilon d}{(1 + \lambda_0^2)\Delta} \right] \int V q_p z dV - \frac{Pr_{\infty} \gamma \lambda_0}{2(\lambda_0 + (1 - \lambda_0^2)\arctgent g \lambda_0)} \left[ (\lambda_0\arctgent g \lambda_0 - 1)^2 \right] - 4(\lambda_0\arctgent g \lambda_0 - 1) + (\lambda_0\arctgent g \lambda_0 - 1)^2] - K_{DS} D_{12} |\nabla C_1|.
\]

coinciding with the formulae (9) of the work [5].

In case of a sphere the formula (19) transforms in expression for pure thermophoresis speed of the spheroidal particle.

\[
U = \frac{b}{a} K_{TS} \nu_0 g \left[ \frac{\nabla T_g}{T_\infty} \right] \left( 1 - \frac{1}{\lambda_0} \right) \arctgent g \lambda_0 \times \left( 1 - \frac{Pr_{\infty} \gamma \lambda_0}{2(\lambda_0 + (1 - \lambda_0^2)\arctgent g \lambda_0)} \right) \left[ \frac{\varepsilon d}{(1 + \lambda_0^2)\Delta} \right] \int V q_p z dV - \frac{Pr_{\infty} \gamma \lambda_0}{2(\lambda_0 + (1 - \lambda_0^2)\arctgent g \lambda_0)} \left[ (\lambda_0\arctgent g \lambda_0 - 1)^2 \right] - 4(\lambda_0\arctgent g \lambda_0 - 1) + (\lambda_0\arctgent g \lambda_0 - 1)^2)] - K_{DS} D_{12} |\nabla C_1|.
\]

Without influence of the environment movement and internal sources of heat we have the classical formula for the thermophoresis speed of a large spherical particle [10, 11].

\[
U(a = b = R) = -K_{TS} \nu_0 g \left[ \frac{\nabla T_g}{T_\infty} \right] \left( 1 + \frac{2\delta}{1 + 2\delta} \right) \left( 1 - \frac{Pr_{\infty} \gamma}{3} \right) - K_{DS} D_{12} \left| \nabla C_1 \right|.
\]

To estimate, what influence renders the environment movement on the speed of thermo-and photophoresis of a spheroidal particle, it is necessary to concretize the nature of thermal sources non-uniformly distributed in its volume. As an example we will consider the most simple case when the particle absorbs radiation as a black body, i.e. the radiation absorption occurs in a thin layer in the thickness \( \delta \varepsilon \ll \varepsilon_0 \), adjoining to a heated up part of the particle surface. At that the density of thermal sources in a layer in the thickness \( \delta \varepsilon \) is equal [12, 13] to

\[
q_p(\varepsilon, \eta) = \begin{cases} 
-\frac{\cosh \varepsilon \cos \eta}{\varepsilon (\cosh^2 \varepsilon - \sin^2 \eta)\delta \varepsilon} I_0, & \eta \geq \frac{\pi}{2} \\
\varepsilon - \delta \varepsilon \leq \varepsilon \leq \varepsilon_0, & 0 < \eta \leq \frac{\pi}{2} \\
0, & \varepsilon_0 - \delta \varepsilon \leq \varepsilon \leq \varepsilon_0.
\end{cases}
\]

where \( I_0 \) is the intensity of falling radiation.
The expression for the speed of thermo-and photophoresis includes integrals $\int q_p \, dV$, $\int q_p \, z \, dV$. Substituting (23) in these integrals and accounting that $\delta \varkappa \ll \varkappa_0$ after integration one obtains:

$$\int q_p \, z \, dV = -\frac{2}{3} \pi \varkappa^3 \varkappa_0^2 (1 + \frac{1}{\varkappa_0^2})$$  \hspace{1cm} (24)

$$\int q_p \, dV = \pi \varkappa^2 \varkappa_0^2 (1 + \frac{1}{\varkappa_0^2})$$  \hspace{1cm} (25)

With accounting of (24), (25) the expression (19) has a form:

$$U = K_{TS} \nu_0 \delta (f_{th}^* + f_{ph}^* \xi) - K_{DS} D_{12} \nu \nabla C_1$$  \hspace{1cm} (26)

$$f_{th}^* = -\frac{b}{a} \frac{1}{t_s} \frac{\lambda}{\lambda_0} \arctan \lambda_0$$

$$\times (1 - P_{\infty}) \frac{\lambda_0}{2 \lambda_0 T_{\infty}} \frac{\lambda_0}{(\lambda_0 + 1) \lambda_0} \cdot (\lambda_0 \arctan \lambda_0 - 1)$$

$$f_{ph}^* = -\frac{b}{a} \frac{1}{t_s} \frac{\lambda}{\lambda_0} \arctan \lambda_0$$

$$\times (1 - P_{\infty}) \frac{\lambda_0}{2 \lambda_0 T_{\infty}} \frac{\lambda_0}{(\lambda_0 + 1) \lambda_0} \cdot (\lambda_0 \arctan \lambda_0 - 1)$$

$$f_{th}^* = -\frac{b}{a} \frac{1}{t_s} \frac{\lambda}{\lambda_0} \arctan \lambda_0$$

In case of the sphere the expression (26) has a form:

$$U(a = b = R) = -K_{TS} \frac{\nu_0}{t_s} \frac{\lambda}{\lambda_0} \frac{\lambda_0}{2 \lambda_0 T_{\infty}} \frac{2 \delta}{1 + 2 \delta} (1 - P_{\infty}) \frac{I_0 R}{12 \lambda_0 T_{\infty}}$$

$$-K_{TS} \frac{\nu_0}{t_s} \frac{\lambda}{\lambda_0} \frac{\lambda_0}{2 \lambda_0 T_{\infty}} \frac{3 P_{\infty}}{16} - 1$$

To illustrate the contribution of the form-factor (the relation of semiaxes of the spheroid), the influence of the environment movement and an internal thermal release for the speed thermo-and photophoresis (26), in the picture curves connecting values $f = f_{th}^* / f_{ph}^*$ at $\varkappa_0 = 300K$ with the intensity of falling radiation for particles of boride graphite ($\varkappa_p = 55W/(mK)$) with spheroidal (a curve 1 - taking into account the environment movement, a curve 2 - without movement) and spherical (a curve 3) forms of the surfaces suspended in air at $T_{\infty} = 300K$ and $P_g = 10^5 Pa$, for various relations of semiaxes of a spheroid $a$.

The numerical analysis has shown, that at the fix relation of semi-axes with increase in intensity of the falling radiation the total contribution of the motion of environment and of internal thermal release leads to monotonous reduction of the thermo- and photophoresis speed, and this reduction essentially depends on equatorial radius of a spheroid $a$.

Quantitative research of the discussed phenomenon for solid heated particles represents quite real experimental problem.
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