Goujad, Elise; Möller, Martin

Counting Feynman-like graphs: quasimodularity and Siegel-Veech weight. (English)
J. Eur. Math. Soc. (JEMS) 22, No. 2, 365-412 (2020).

Summary: We prove the quasimodularity of generating functions for counting torus covers, with and without Siegel-Veech weight. Our proof is based on analyzing decompositions of flat surfaces into horizontal cylinders. The quasimodularity arises from representing the generating series as a contour integral of quasi-elliptic functions. This provides an alternative proof of the quasimodularity results of S. Bloch and A. Okounkov [Adv. Math. 149, No. 1, 1–60 (2000; Zbl 0978.17016)], A. Eskin and A. Okounkov [Invent. Math. 145, No. 1, 59–103 (2001; Zbl 1019.32014)] and D. Chen et al. [J. Am. Math. Soc. 31, No. 4, 1059–1163 (2018; Zbl 1404.32025)], and generalizes the results of J. Böhm et al. [J. Reine Angew. Math. 732, 211–246 (2017; Zbl 1390.14191)] for simple ramification covers.

MSC:
05C30  Enumeration in graph theory
05A15  Exact enumeration problems, generating functions
11F11  Holomorphic modular forms of integral weight
32G15  Moduli of Riemann surfaces, Teichmüller theory (complex-analytic aspects in several variables)
14H30  Coverings of curves, fundamental group

Keywords:
quasimodular forms; Feynman graphs; Siegel-Veech constants; tropical covers; correspondence theorem

Full Text: DOI  arXiv

References:
[1] Lando, S. K., Zvonkin, A. K.: Graphs on Surfaces and Their Applications. Encyclopaedia Math. Sci. 141, Springer, Berlin (2004)Zbl 1040.05001 MR 2036721 · Zbl 1040.05001
[2] Lassalle, M.: An explicit formula for the characters of the symmetric group. Math. Ann.340, 383-405 (2008)Zbl 1182.20011 MR 2368985 · Zbl 1182.20011
[3] Li, S.: Feynman graph integrals and almost modular forms. Comm. Number Theory Phys.6, 129-157 (2012)Zbl 1270.81142 MR 2955933 · Zbl 1270.81142
[4] Milas, A.: Formal differential operators, vertex operator algebras and zeta-values. II. J. Pure Appl. Algebra183, 191-244 (2003)Zbl 1036.17019 MR 1992047 · Zbl 1036.17019
[5] Okounkov, A., Olshanski, G.: Shifted Schur functions. Algebra i Analiz9, no. 2, 73–146 (1997) (in Russian)Zbl 0894.05053 MR 1468548
[6] Okounkov, A., Pandharipande, R.: Gromov-Witten theory, Hurwitz theory, and completed cycles. Ann. of Math. (2)163, 517-560 (2006)Zbl 1105.14076 MR 2199225 · Zbl 1105.14076
[7] Shadrin, S., Spitz, L., Zvonkine, D.: On double Hurwitz numbers with completed cycles. J. London Math. Soc. (2)86, 407-432 (2012)Zbl 1252.05218 MR 2980918 · Zbl 1252.05218
[8] Viviani, F.: Tropicalizing vs. compactifying the Torelli morphism. In: Tropical and Non-Archimedean Geometry, Contemp. Math. 605, Amer. Math. Soc., Providence, RI, 181-210 (2013)Zbl 1320.14018 MR 3204272 · Zbl 1320.14018
[9] Zagier, D.: Periods of modular forms and Jacobi theta functions. Invent. Math.104, 449-465 (1991)Zbl 0742.11029 MR 1106744 · Zbl 0742.11029
[10] Zagier, D.: Partitions, quasimodular forms, and the Bloch-Okounkov theorem. Ramamunjan J.41, 345-368 (2016)Zbl 1352.05021 MR 3574638 · Zbl 1352.05021
[11] Zorich, A.

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.