On Computation of Combined IVS EOP Series

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Abstract

Three topics related to computation of the combined IVS EOP series are discussed. The first one is the consistency of the VLBI EOP series with the IERS reference frames ITRF and ICRF. Not all IVS analysis centers use ITRF/ICRF as reference system for EOP solutions. Some of them realize global solution for simultaneous determination of CRF, TRF and EOP with no-net-rotation constrains w.r.t. ITRF/ICRF. Analysis shows that such a method can hardly provide consistency of computed EOP series with the IERS reference frames with sufficient accuracy. Direct use of ITRF and ICRF for computation of EOP series submitted to the IERS seems preferable. Second problem is the long-time stability of the IVS EOP series. Analysis of yearly biases w.r.t. C04 and NEOS is presented. A possible ways are proposed to save long time stability of the combined IVS EOP series. At last, various strategies of computation of weighted mean value are considered. It’s shown that usual methods used for this purpose do not provide satisfactory result for the error of the mean. A new method is proposed.

1 Introduction

The IVS combined EOP series computed at the IVS Analysis Coordinator Office located at the Geodetic Institute of the University of Bonn is available beginning from the end of 2000. Analysis of this series routinely provided by the IERS EOP Product Center at the Paris Observatory shows that its accuracy is better than accuracy of individual solutions provided by the IVS Analysis Centers. However, some topics related to the quality of the IVS combined EOP series seems to be investigated more carefully. This paper is intended to consider the following points:

• Consistency of the VLBI EOP series with the IERS reference frames ITRF and ICRF.
• Systematic stability of the VLBI EOP series.
• Computation of weighted mean values.

2 Consistency of IVS EOP series with IERS reference systems

According to the IVS Terms of Reference, IVS serves as the VLBI Technique Center for IERS. In turn, the IERS Terms of Reference said that one of the IERS primary objectives is providing Earth orientation parameters (EOP) required to transform between ICRF and ITRF. It is supposed that after completion of new IERS structure the IERS EOP product will be computed combining several EOP series delivered by the IERS Technique Centers one of which is the IVS. So, the evident goal of the IVS is computation of the combined IVS EOP series providing the transformation parameters between ITRS and ICRS.

However, not all IVS analysis centers use ITRF/ICRF as reference system for EOP solutions. Some of them realize global solution for simultaneous determination of CRF, TRF and EOP. To tie a global solution to IERS reference frames no-net-rotation constrains w.r.t. ITRF and ICRF are usually applied. The question is can such a method provide the consistency of VLBI EOP series with ITRF/ICRF with required accuracy?

Usually global VLBI solution is made using all available sessions and application of no-net-rotation provides zero translation and rotation of full set of stations and radio sources w.r.t. ITRF and ICRF.

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However, commonly speaking, it is not the case for the subset of stations participating in a particular session. Therefore EOP estimate obtained from processing of a session observations may systematically differ from ITRF/ICRF.

Besides, number of observations for stations and sources differ very much. Table 1 shows statistics of observations for stations and sources for all sessions and for NEOS-A ones. One can see that in fact the NEOS-A EOP series used in the IVS combined solution is practically defined by subset of 8 stations and 66 radio sources.

| Stations | Sources | Stations | Sources |
|----------|---------|----------|---------|
| Nsta    | Nobs    | Nsta    | Nobs    |
| 5        | 100–200 | 15       | 100–700 |
| 3        | 50–100  | 32       | 10–100  |
| 7        | 1–6     | 52       | 1–10    |
| 6        | < 1     | 50       | < 1     |
|          | 177     |          | 341     |

Let us see how close is the tie between different subsets of station and source catalogs with ITRF/ICRF. We use results of the USNO9903 global solution as most fresh publicly available one. Tables 3 and 2 present transformation parameters between USNO solution and ITRF(ICRF) for all common stations (sources), for all stations (sources) participating in the NEOS-A program, and for most frequently observing stations (observed sources). These data show that the transformation parameters including ones defining EOP system are not equal to zero and differ for various subsets of stations and sources. This mean that EOP system is not correspond to ITRF/ICRF and differ for various observational programs.

Therefore, CRF and TRF realization obtained from a global VLBI solution can hardly provide consistency of computed EOP series with ITRF/ICRF with sufficient accuracy. Direct use of the ITRF and ICRF for computation of EOP series submitted to the IERS seems preferable.

This does not mean indeed that the IVS Analysis centers should not compute global solutions. The reasonable strategy may be using individual CRF and TRF realizations for improving the IERS reference frames, and further using ICRF and ITRF for regular EOP computation. This strategy provides consistency between VLBI EOP series and the IERS reference frames and makes individual VLBI EOP series more homogeneous that allows to simplify combination procedure and improve quality of the IVS combined product.

Now the IERS and main space geodesy services are in the process of moving from ITRF97 to ITRF2000. What systematic changes in the VLBI EOP series can we expect? Table 4 shows the results of comparison between ITRF97 and ITRF2000 for different subsets of VLBI stations.

We also compared the EOP series computed at the IAA with ITRF97 and ITRF2000. The result is shown in Table 4. The computation was made with three radio source catalogues ICRF-Ext.1, RSC(IAA)99R02 and RSC(IAA)01R02, and no meaningful systematic differences between EOP series computed with ITRF97 and ITRF2000 was found.

Although ITRF2000 was constructed in such a way that no rotation w.r.t. ITRF97 was introduced, substantial value of rotation angle R2 (which corresponds to Y pole coordinate) is found both in direct comparison of two coordinate systems and in the result of EOP computation.

In conclusion, it’s important to mention that errors of inconsistency between EOP series and terrestrial and celestial reference frames are systematic ones and even their relatively small values can be substantial.

### 3 Long-time stability of IVS EOP series

Obviously, one of the main goal of maintenance of the IVS combined products is to provide systematically stable IVS EOP series. It is especially important now because the new IERS organization envisages computation of the IERS combined EOP series practically of the three series VLBI, GPS and SLR, and importance
Table 2: Transformation parameters between the TRF realizations USNO9903 and ITRF97 at epoch 1997.0 for different number of stations.

| N sta | 85  | 20  | 8  |
|---|---|---|---|
| T1, mm | -0.1 | -0.9 | -2.2 |
| σ  | 0.8 | 1.6 | 1.9 |
| T2, mm | -1.6 | -1.4 | -1.3 |
| σ  | 0.8 | 1.7 | 2.0 |
| T3, mm | -0.6 | -0.4 | -0.3 |
| σ  | 0.8 | 1.6 | 1.8 |
| D, ppb | 1.7 | 1.5 | 0.8 |
| σ  | 0.1 | 0.2 | 0.2 |
| R1, mas | -0.06 | -0.05 | -0.03 |
| σ  | 0.03 | 0.07 | 0.08 |
| R2, mas | 0.01 | 0.02 | 0.08 |
| σ  | 0.03 | 0.05 | 0.06 |
| R3, mas | 0.00 | 0.03 | 0.00 |
| σ  | 0.03 | 0.05 | 0.06 |

Table 3: Transformation parameters between the CRF realizations USNO9903 and ICRF-Ext.1 for different number of sources.

| N sou | 626 | 303 | 66 |
|---|---|---|---|
| A1, mas | 0.029 | 0.022 | 0.013 |
| σ  | 9  | 9  | 7  |
| A2, mas | 0.027 | 0.018 | 0.013 |
| σ  | 9  | 8  | 7  |
| A3, mas | -0.018 | -0.016 | -0.013 |
| σ  | 9  | 9  | 12 |
| Dα, mas | -0.001 | -0.001 | 0  |
| σ  | 0  | 0  | 0  |
| Dδ, mas | 0  | 0  | -0.001 |
| σ  | 0  | 0  | 0  |
| Bδ, mas | 0.054 | 0.042 | 0.086 |
| σ  | 9  | 10 | 12 |

Table 4: Transformation parameters between ITRF97 and ITRF2000 at epoch 1997.0 for different number of VLBI stations (98, 20, 8), nominal transformation parameters defined by the IERS (P0) and systematic differences between EOP series computed with ITRF97 and ITRF2000 with the OCCAM package.

| N sta | 98  | 20  | 8  | P0  | OCCAM |
|---|---|---|---|---|---|
| T1, mm | 6.9 | 7.8 | 7.8 | 6.7 |
| σ  | 0.5 | 0.9 | 1.4 |
| T2, mm | 3.9 | 3.9 | 3.6 | 6.1 |
| σ  | 0.5 | 1.0 | 1.6 |
| T3, mm | -20.2 | -20.1 | -21.3 | -18.5 |
| σ  | 0.5 | 0.9 | 1.4 |
| D, ppb | 1.5 | 1.3 | 0.9 | 1.55 |
| σ  | 0.1 | 0.1 | 0.2 |
| R1, mas | -0.17 | -0.19 | -0.20 | 0.0 | -0.15 |
| σ  | 0.02 | 0.04 | 0.06 |
| R2, mas | 0.01 | -0.01 | -0.01 | 0.0 | 0.09 |
| σ  | 0.02 | 0.03 | 0.05 |
| R3, mas | -0.01 | -0.03 | -0.03 | 0.0 | -0.07 |
| σ  | 0.02 | 0.02 | 0.04 |
| R3, mas/y | | | | -0.02 | -0.03 |
| σ  | | | | 0.02 |
of each of them is very large. Several factors make this task difficult and in the first place it is instability of individual series. The main reason for that are:

- Using individual periodically updated TRF and CRF realizations. As shown in the previous section these realization are not tied to the unique (IERS) reference frames with sufficient accuracy and, in fact, every VLBI session yields EOP estimates in its own system.

- Change in systematic errors of EOP series after modification of models, algorithms and software.

- Change of set of contributed VLBI Analysis Centers. Besides a difference in used reference systems, each EOP series has its own systematic peculiarities.

- Change of network configuration. This is well established fact, and it is not quite clear how to handle it properly. For instance, we can mention the problem of joining 9-year IRIS-A and 8-year NEOS-A programs to avoid EOP jump directly affected results of determination of 18.6-year nutation term.

For listed above and other reasons the VLBI EOP series show long-time instability. To investigated this effect we use five VLBI EOP series BKG00001, GSF2000A, IAAO9907, SPU00001, USN99003 over a 7-year interval from May 1993 till April 2000 (NEOS-A data only). The whole 7-year interval was split in 7 one-year ones and each series was compared with combined C04 and NEOS series at these one-year intervals. During computation six parameters of systematic differences between VLBI and combined series were estimated for every year. These are: bias, rate, amplitude of sine and cosine of annual term and semiannual terms. In such a way we obtained seven values for each of six parameters of model of systematic errors for each VLBI series. The final step of this analysis was the computation of RMS values from seven epochs. Such a approach to investigation of long-time stability is analogous to a method used at the Paris observatory during computation of the IERS combined products. Result of analysis of yearly biases is presented in Table 5.

### Table 5: Long-time stability of IVS EOP series (NEOS-A): statistics of yearly bias relative to the IERS C04 and NEOS combined series (7 years 1993.3–2000.3): bias, rate - result of approximation of yearly bias series by linear trend, rms - rms of residuals after removing trend.

| EOP | Bias (mas) | Rate (mas/yr) | RMS (mas) | Bias (mas) | Rate (mas/yr) | RMS (mas) |
|-----|------------|---------------|-----------|------------|---------------|-----------|
| C04 | BKG        | GSF           | IAA       | SPU        | USN           |
|     |            |               |           |            |               |           |
|     | 0.064      | -0.088        | -0.126    | -0.074     | -0.096        |
|     | 0.011      | 0.015         | -0.002    | 0.002      | 0.009         |
|     | 0.025      | 0.026         | 0.025     | 0.027      | 0.023         |
| Y   | bias       | rate          | rms       | bias       | rate          | rms       |
| mas | -0.249     | 0.015         | -0.065    | -0.030     | -0.034        |
|     | 0.064      | -0.003        | 0.052     | 0.049      | 0.043         |
|     | 0.040      | 0.311         | 0.049     | 0.043      | 0.044         |
| UT1 | bias       | rate          | rms       | bias       | rate          | rms       |
| 0.1 ms | 0.101    | -0.020        | -0.037    | -0.232     | -0.041        |
|     | 0.018      | -0.056        | -0.014    | 0.018      | -0.003        |
|     | 0.026      | 0.020         | 0.022     | 0.018      | 0.019         |
| dPsi | bias      | rate          | rms       | bias       | rate          | rms       |
| mas | -0.066     | -0.049        | -0.061    | 0.030      | 0.080         |
|     | -0.028     | -0.014        | 0.013     | 0.002      | -0.025        |
|     | 0.087      | 0.083         | 0.031     | 0.054      | 0.060         |
| dEps | bias     | rate          | rms       | bias       | rate          | rms       |
| mas | 0.001      | -0.008        | 0.047     | -0.021     | -0.043        |
|     | 0.006      | -0.005        | 0.007     | -0.012     | -0.009        |
|     | 0.020      | 0.009         | 0.031     | 0.031      | 0.016         |
| NEOS | BKG        | GSF           | IAA       | SPU        | USN           |
|     |            |               |           |            |               |           |
|     | 0.080      | -0.071        | -0.111    | -0.058     | -0.081        |
|     | 0.033      | 0.037         | 0.021     | 0.024      | 0.032         |
|     | 0.034      | 0.028         | 0.041     | 0.034      | 0.036         |
|     | -0.269     | -0.004        | -0.078    | -0.043     | -0.048        |
|     | 0.064      | -0.002        | 0.056     | 0.052      | 0.046         |
|     | 0.052      | 0.041         | 0.054     | 0.050      | 0.051         |
|     | 0.163      | 0.038         | 0.025     | -0.170     | 0.024         |
|     | 0.010      | -0.062        | -0.016    | 0.014      | -0.008        |
|     | 0.044      | 0.048         | 0.055     | 0.062      | 0.050         |
|     | 0.066      | 0.087         | 0.072     | 0.165      | 0.212         |
|     | -0.009     | 0.006         | 0.027     | 0.016      | -0.009        |
|     | 0.056      | 0.040         | 0.049     | 0.055      | 0.031         |
|     | 0.037      | 0.027         | 0.085     | 0.021      | -0.002        |
|     | 0.013      | 0.002         | 0.015     | -0.003     | -0.001        |
|     | 0.027      | 0.016         | 0.039     | 0.042      | 0.022         |

Obviously, this analysis cannot be fully objective because it depends on details of combination procedure (systematic corrections, weights, etc.) used during computation of C04 and NEOS series. One can see that
differences between the left and the right parts of Table 5 is sometimes quite large, especially for UT1-UTC. Maybe using IVS combined EOP series for such analysis would be preferable when it will have sufficient time span.

The results of analysis presented here and in the previous section confirm well known fact that each EOP series has own systematic errors and these errors are not stable at the required level of accuracy. Therefore it seems very important to develop appropriate strategy for computation of the IVS combined product to provide make its systematic stability. We would like to propose for discussion a possible strategy to keep long-time systematic stability of the IVS EOP combined series. This strategy includes the following steps.

1. Computation of the “reference” EOP series $EOP_0$ as the mean of existing long-time NEOS-A series fixed at epoch of computation with weights depending on long-time stability. Input series should be transformed to uniform TRF/CRF (preferably the IERS ones) as accurate as possible.

2. Using systematic corrections to individual series

$$dEOP_i = EOP_0 - EOP_i$$

derived from comparison with the reference series in further computations.

3. When an AC$_i$ updates EOP series new systematic correction can be computed as

$$(EOP_{i,old} - EOP_{i,new}) + dEOP_i.$$ 

4. When a new EOP series of a new AC is to be included in the IVS combination systematic correction to that series will be

$$dEOP_j = EOP_0 - EOP_j.$$ 

5. Periodical update of the reference series, e.g. when new ITRF or ICRF realization is accepted. Evidently, in such a case, appropriate care of careful tie between the new and the old reference series must be taken.

A separate problem is the transformation of EOP obtained on different networks to the reference series. However, hopefully improvement of ITRF and models of VLBI observations will eliminate this problem in the future.

4 Computation of weighted mean

Computation of the weighted mean of several estimates is usually the final step in each EOP (and all others) combining procedure. Let we have $n$ values $x_i$ with associated errors $s_i$, $i = 1 \ldots n$. Then we have a well known statistics [1, 2]

$$p_i = \frac{1}{s_i^2}, \quad p = \sum_{i=1}^{n} p_i, \quad x = \frac{\sum_{i=1}^{n} p_i x_i}{p},$$

$$H = \sum_{i=1}^{n} p_i (x_i - x)^2 = \sum_{i=1}^{n} \left[ \frac{(x_i - x)}{s_i} \right]^2, \quad \chi^2/dof = \frac{H}{n-1},$$

where $x$ is a estimate of the mean value. The question is how to estimate error $\sigma$ of the mean? Two classical approaches are:

Maximum likelihood approach if $\sigma_i$ are considered as absolute magnitudes of errors in $x_i$:

$$\sigma_1 = \frac{1}{\sqrt{p}}.$$
Least squares approach if $\sigma_i$ are considered as relative values of errors in $x_i$ and error of unit weight must be estimated from data itself:

$$\sigma_2 = \sqrt{\frac{\sum_{i=1}^{n} p_i (x_i - \bar{x})^2}{p(n-1)}} = \sqrt{\frac{H}{p(n-1)}} = \sigma_1 \sqrt{\frac{H}{n-1}}.$$

It is easy to see that $\sigma_1$ depends only on a priori errors in averaged values $x_i$ and $\sigma_1$ depends only on the scatter of $x_i$. Theoretically, solution of problem of choice between $\sigma_1$ and $\sigma_2$ depend on whether the scatter of $x_i$ is a result of random errors or there are systematic differences between estimates $x_i$. Obviously, both effect are present in most of practical applications.

That is a known problem in data processing and no rigorous solution is proposed. However some practical ways to handle it were considered in literature. Evidently, the most statistically substantial approach was made in [1,3]. According to this approach chi-square criteria is used to decide if the scatter of $x_i$ is result of random errors, and error of the mean $x$ is computed as

$$\sigma_3 = \begin{cases} 
\sigma_1, & \text{if } H \leq \chi^2(Q, n-1), \\
\sigma_2, & \text{if } H > \chi^2(Q, n-1),
\end{cases}$$

where $Q$ is a fiducial probability. Some other practical algorithms of choice between $\sigma_1$ and $\sigma_2$ were proposed too.

However, in practice, values of $\sigma_1$ and $\sigma_2$ may differ by several times. It leads to instability of $\sigma$ estimate. Table 6 shows some numerical examples of computation of weighted mean of two data points and its error (to compute $\sigma_3$ we use $Q=99\%$ which corresponds $\chi^2(0.99,1)=6.63$). One can see that no one value of $\sigma_1$, $\sigma_2$, $\sigma_3$ provides a satisfactory estimate of $\sigma$. Moreover, value of $\sigma_3$ depends not only on data sample $\{x_i, s_i\}$ but also on subjective choice of $Q$.

After many experiments with test data we decided in favor of simple formula

$$\sigma_4 = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{\frac{1}{p} \left( 1 + \frac{H}{n-1} \right)} ,$$

which can be called “combined” approach. The last column of Table 6 shows that such a approach can provide stable and realistic estimate of error of the mean.

More detailed consideration of this topic is given in [4].

5 Conclusions

Results of this study allow to make the following conclusions:

- Procedure of computation of the IVS combined EOP series must be “absolute”, i.e. independent on any reference, e.g. IERS, series. Otherwise details of combination procedure used during computation of “external” reference series (systematic corrections, weights) will affect the results of analysis.

- It seems preferable to use ITRF and ICRF by the all IVS Analysis Centers for computation of VLBI EOP series submitted to the IVS and IERS. Using individual TRF/CRF lead to difficulties in interpretation of results. Usual procedure of determination of systematic differences between EOP series provides correction only for “global” orientation between TRF/CRF. But, as it was shown above transformation parameters between individual TRF (CRF) realizations depend on sub-set of stations (sources) used for comparison. This means that, commonly speaking, every session produce EOP in its own system, which makes it difficult to transform an individual EOP series to ITRF/ICRF with sufficient accuracy.

- A reference EOP series based on IVS combined solution for fixed set of individual solutions can be used to save the long-time stability. Also, it is important to develop appropriate strategy to include new or updated solutions in the IVS combination, e.g. using strategy proposed in this paper.
Table 6: Numerical examples of computation of weighted mean (see explanation in text).

| No | $x_1$ | $x_2$ | $s_{1,2}$ | $x$ | $H$ | $\sigma_1$ | $\sigma_2$ | $\sigma_3$ | $\sigma_4$ |
|----|-------|-------|-----------|-----|-----|------------|------------|------------|------------|
| 1  | 1.0   | 1.0   | 0.5       | 1.0 | 0.00| 0.354      | 0.000      | 0.354      | 0.354      |
| 2  | 1.0   | 2.0   | 0.1       | 1.5 | 50.0| 0.071      | 0.500      | 0.500      | 0.505      |
| 3  | 0.2   |       |           | 12.5| 0.141| 0.500      | 0.500      | 0.500      | 0.520      |
| 4  | 0.3   |       |           | 5.56| 0.212| 0.500      | 0.212      | 0.500      | 0.543      |
| 5  | 0.5   |       |           | 2.00| 0.354| 0.500      | 0.354      | 0.354      | 0.612      |
| 6  | 1.0   |       |           | 0.50| 0.707| 0.500      | 0.707      | 0.707      | 0.866      |
| 7  | 2.0   |       |           | 0.12| 1.414| 0.500      | 1.414      | 1.414      | 1.500      |
| 8  | 10.0  | 20.0  | 0.1       | 15.0| 5000.0| 0.071     | 5.000      | 5.000      | 5.000      |
| 9  | 0.5   |       |           | 200.0| 0.354| 5.000      | 5.000      | 5.000      | 5.012      |
| 10 | 1.0   |       |           | 50.00| 0.707| 5.000      | 5.000      | 5.000      | 5.050      |
| 11 | 2.0   |       |           | 12.50| 1.414| 5.000      | 5.000      | 5.000      | 5.196      |
| 12 | 3.0   |       |           | 5.56 | 2.121| 5.000      | 2.121      | 2.121      | 5.431      |
| 13 | 5.0   |       |           | 2.00 | 3.536| 5.000      | 3.536      | 3.536      | 6.124      |
| 14 | 10.0  |       |           | 0.50 | 7.071| 5.000      | 7.071      | 7.071      | 8.660      |
| 15 | 20.0  |       |           | 0.12 |14.142| 5.000      |14.142      |14.142      |15.000      |
| 16 | 10.0  | 10.0  | 1.0       | 10.0| 0.00 | 0.707      | 0.000      | 0.707      | 0.707      |
| 17 | 10.0  | 11.0  | 10.5      | 11.0| 0.50 | 0.707      | 0.500      | 0.707      | 0.866      |
| 18 | 10.0  | 12.0  | 11.0      | 11.0| 2.00 | 0.707      | 1.000      | 0.707      | 1.225      |
| 19 | 10.0  | 13.0  | 11.5      | 11.5| 4.50 | 0.707      | 1.500      | 0.707      | 1.658      |
| 20 | 10.0  | 14.0  | 12.0      | 12.0| 8.00 | 0.707      | 2.000      | 2.000      | 2.121      |
| 21 | 10.0  | 15.0  | 12.5      | 12.5| 12.50| 0.707      | 2.500      | 2.500      | 2.598      |
| 22 | 10.0  | 16.0  | 13.0      | 13.0| 18.00| 0.707      | 3.000      | 3.000      | 3.082      |
| 23 | 10.0  | 17.0  | 13.5      | 24.50| 0.707| 3.500      | 3.500      | 3.500      | 3.571      |

- Weighting of individual series depending on their long-time stability seems useful for improvement of long-time stability of the IVS combined EOP series.
- Proposed method of computation of weighted mean EOP can be used to account for both formal error and scatter.

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