Kaon HBT radii from perfect fluid dynamics using the Buda-Lund model

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In this paper we summarize the ellipsoidally symmetric Buda-Lund model’s results on HBT radii. We calculate the Bose-Einstein correlation function from the model and derive formulas for the transverse momentum dependence of the correlation radii in the Bertsch-Pratt system of out, side and longitudinal directions. We show a comparison to $\sqrt{s_{NN}} = 200\, GeV$ RHIC PHENIX two-pion correlation data and make prediction on the same observable for different particles.

1. Perfect fluid hydrodynamics

Perfect fluid hydrodynamics is based on local conservation of entropy and four-momentum. The fluid is perfect if the four-momentum tensor is diagonal in the local rest frame. The conservation equations are closed by the equation of state, which gives the relationship between energy density $\epsilon$, pressure $p$. Typically $\epsilon - B = \kappa(p + B)$, where $B$ stands for a bag constant ($B = 0$ in the hadronic phase, non-zero in a QGP phase), and $\kappa$ may be a constant, but can be an arbitrary temperature dependent function.

There are only a few exact solutions for these equations. One (and historically the first) is the famous Landau-Khalatnikov solution discovered more than 50 years ago [1,2,3]. This is a 1+1 dimensional solution, and has realistic properties: it describes a 1+1 dimensional expansion, does not lack acceleration and predicts an approximately Gaussian rapidity distribution.

Another renowned solution of relativistic hydrodynamics is the Hwa-Bjorken solution [4,5,6], which is a simple, explicit and exact, but accelerationless solution. This solution is boost-invariant in its original form, but
this approximation fails to describe the data [7, 8]. However, the solution allowed Bjorken to obtain a simple estimate of the initial energy density reached in high energy reactions from final state hadronic observables.

There are solutions which interpolate between the above two solutions [9, 10], are explicit and describe a relativistic acceleration.

2. The Buda-Lund model

We focus here on the analytic approach in exploring the consequences of the presence of such perfect fluids in high energy heavy ion experiments in Au+Au collisions at RHIC. Such exact analytic solutions were published recently in refs. [9, 10, 11, 12, 13]. A tool, that is based on the above listed exact, dynamical hydro solutions, is the Buda-Lund hydro model of refs. [14, 15].

The Buda-Lund hydro model successfully describes BRAHMS, PHENIX, PHOBOS and STAR data on identified single particle spectra and the transverse mass dependent Bose-Einstein or HBT radii as well as the pseudorapidity distribution of charged particles in central Au+Au collisions both at $\sqrt{s_{\text{NN}}} = 130$ GeV [16] and at $\sqrt{s_{\text{NN}}} = 200$ GeV [17] and in p+p collisions at $\sqrt{s} = 200$ GeV [18], as well as data from Pb+Pb collisions at CERN SPS [19] and h+p reactions at CERN SPS [20, 21]. The model is defined with the help of its emission function; to take into account the effects of long-lived resonances, it utilizes the core-halo model [22]. It describes an expanding fireball of ellipsoidal symmetry (with the time-dependent principal axes of the ellipsoid being $X$, $Y$ and $Z$).

3. HBT from the Buda-Lund model

Let us calculate the two-particle Bose-Einstein correlation function from the Buda-Lund source function of the Buda-Lund model as a function of $q = p_1 - p_2$, the four-momentum difference of the two particles. The result is

$$C(q) = 1 + \lambda e^{-q^2 \Delta \tau^2 - q^2 R^2_{\tau, x} - q^2 R^2_{\tau, y} - q^2 R^2_{\tau, z}},$$

(1)

with $\lambda$ being the intercept parameter (square of the ratio of particles emitted from the core versus from the halo [22]), and

$$\frac{1}{\Delta \tau^2} = \frac{1}{\Delta \tau^2} + \frac{m_t}{T_0} \frac{d^2}{d \tau^2},$$

(2)

$$R^2_{\tau, x} = X^2 \left(1 + m_t (a^2 + \dot{X}^2)/T_0\right)^{-1},$$

(3)

$$R^2_{\tau, y} = Y^2 \left(1 + m_t (a^2 + \dot{Y}^2)/T_0\right)^{-1},$$

(4)
$$R^2_{t,z} = Z^2 \left( 1 + m_t (a^2 + \dot{Z}^2) / T_0 \right)^{-1},$$  \tag{5}$$

with $X, \dot{Y}, \dot{Z}$ being the time-derivative of the principal axes, $m_t$ the average transverse mass of the pair. $T_0$ is the central temperature at the freeze-out, $\Delta \tau$ is the mean emission duration and $\tau_0$ is the freeze-out time. Furthermore, $a$ and $d$ are the spatial and time-like temperature gradients, defined as $a^2 = \langle \Delta T^2 \rangle_\perp$ and $d^2 = \langle \Delta T^2 \rangle_\tau$. From the mass-shell constraint one finds $q_0 = \beta_x q_x + \beta_y q_y + \beta_z q_z$, if expressed by the average velocity $\beta$. Thus we can rewrite eq. [1] with modified radii to

$$C(q) = 1 + \lambda_* \exp \left( - \sum_{i,j=x,y,z} R^2_{i,j} q_i q_j \right), \text{ where}  \tag{6}$$

$$R^2_{i,i} = R^2_{i,i} + \beta_i^2 \Delta \tau_i^2, \text{ and } R^2_{i,j} = \beta_i \beta_j \Delta \tau_i^2,  \tag{7}$$

From this, we can calculate the radii in the Bertsch-Pratt frame [22] of

out (o, pointing towards the average momentum of the actual pair, rotated from $x$ by an azimuthal angle $\varphi$), longitudinal (l, pointing towards the beam direction) directions and side (s, perpendicular to both l and o) directions. The detailed calculations are described in ref. [24]. These include azimuthally sensitive oscillating cross-terms. However, due to space limitations, the angle dependent radii are not shown here. If one averages on the azimuthal angle, and goes into the LCMS frame (where $\beta_l = \beta_s = 0$), the Bertsch-Pratt radii are:

$$R^2_o = (R^2_{x,x} + R^2_{y,y})^{-1} + \beta_i^2 \Delta \tau_i^2,  \tag{8}$$

$$R^2_s = (R^2_{x,x} + R^2_{y,y})^{-1},  \tag{9}$$

$$R^2_l = R^2_{z,z}.  \tag{10}$$

These can be fitted then to the data [23] as in ref. [26], see fig. [1]. This allows us to predict the transverse momentum dependence of the HBT radii of two-kaon correlations as well: if they are plotted versus $m_t$, the data of all particles fall on the same curve. This is also shown for kaons on fig. [1].

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Fig. 1. HBT radii from the axially Buda-Lund model from ref. [26], compared to data of ref [25]. We also show a prediction for kaon HBT radii on this plot: these overlap with that of pions if plotted versus transverse mass $m_t$.

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