Subwavelength waveguiding and imaging with a one-dimensional array of metallic H-fractals

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Abstract. We demonstrate, both experimentally and theoretically, subwavelength waveguiding and imaging through a one-dimensional (1D) array of 3D metallic H-fractals. The waveguide formed by the fractal array is subwavelength in all cross-sectional dimensions, thereby allowing compact designs for guided propagation of long-wavelength EM waves. The underlying physics is governed by the fractal metallic wire structure that allows subwavelength resonances. The measured results indicate that such waveguides can provide low-loss propagation of EM waves with a decay length in the range 10–40 m, achieved through coupling of resonant dipoles/multipoles between neighboring H-fractals along the wave propagation direction. By treating the waveguide as a 1D photonic crystal, the calculated bandstructure shows that in the neighborhood of peak transmission frequencies the dispersion relations are relatively flat as a function of the wave vector, implying low group velocities. This is experimentally verified. Owing to the increased spatial information carried at such transmission frequencies, imaging becomes possible and this was observed by placing an obstacle at the inlet and scanning the local field intensity at the other end of the waveguide.

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1. Introduction

Resonances can enable the transmission of electromagnetic (EM) and acoustic waves through subwavelength apertures [1]–[7]. A natural extension of this phenomenon is the subwavelength waveguiding through coupled arrays of resonant structures. This was indeed demonstrated recently by M Quinten et al. [8] and M L Brongersma et al. [9] for the efficient transport of EM energy through a chain of nanometer-size metallic particles [10], via their plasmon excitations [11]–[19]. Subwavelength waveguides can be especially important for microwaves and radio-frequency waves, owing to their importance in the radio-on-fiber (rof) communication technologies. But the large transverse dimension(s), necessary for such waves in order to overcome the cutoff threshold, means that the conventional waveguides are not practical in these cases. Thus a significant reduction in the transverse dimensions of the waveguides, through the use of resonant structures, may lead to useful applications in the low-loss transport of microwaves and radio-frequency waves [20].

In this work, we show that by using coupled local resonances of three-dimensional (3D) H-fractals, one can achieve subwavelength waveguiding in the microwave regime with reasonably high transmission efficiency. Due to the relatively flat dispersion relations (and hence slow EM wave group velocity) at the transmission peak frequencies, imaging through the waveguide becomes possible, and this is experimentally demonstrated. In what follows, we briefly introduce the 3D H-fractal in section 2. This is followed by a description of the transmission measurements and the associated H-fractal resonances in section 3. Finite-difference-time-domain (FDTD) simulations and comparison with phase measurements are presented in section 4, with the purpose of clarifying the details of the coupling between local resonances of the neighboring 3D H-fractals. In order to understand the transmission characteristics, we have carried out bandstructure calculations as well as measured the transmission loss. These results are presented in section 5. From the bandstructure it becomes clear that the peak transmission frequencies are associated with flat dispersions, thus implying slow group velocity and also the possibility of imaging. This is because a flat dispersion relation can mean a very small $\omega/k = v_g$, and physically it implies that there can be a significant amount of spatial information contained in a narrow frequency range, necessary for (single frequency) imaging. Both predictions—slow group velocity and imaging—are experimentally
Figure 1. (a)Iterative construction of the 3D H-fractals (from left to right: 3, 6 and 9 levels). (b) One of the projections of the 9-level 3D H-fractal is exactly a 6-level planar H-fractal.

verified. Details of the experiments and their results are presented in section 6. Concluding remarks are given in section 7.

2. Three-dimensional (3D) H-fractal

The starting point of a 3D H-fractal is a line of length $a$, defined as the first level of the fractal, parallel to the $z$-axis (see figure 1). The $(k + 1)$th level of the H-fractal, containing $2^k$ lines, is constructed along the direction orthogonal to the plane formed by the previous two levels’ lines, with the midpoint of each line connected to the ends of the $k$th level lines. The length of the $(k + 1)$th level lines is scaled as $a/2^{\text{quo}(k, 3)}$, where the notation $\text{quo}(m, n)$ denotes the quotient of integer $m$ divided by integer $n$. The construction process is illustrated in figure 1(a). This line structure therefore fills a $2a \times 2a \times 2a$ cube as the number of levels, $N$, approaches infinity. It is interesting to note that the projections of the 3D fractal along the three orthogonal directions, $x$, $y$ and $z$, are the planar H-fractals. One such projection is shown in figure 1(b).

The sample used in the experiment was a 6-level H-fractal with the length of the first level line equal to 26 mm, and thus the total dimensions of a single unit are $39 \times 39 \times 39$ mm$^3$. To construct the waveguide, 39 H-fractals were used to form a 1D array with a separation of 20 mm between the neighbors. The microwave power was fed into the center of the first-level line of a 3D H-fractal, located at the inlet of the 1D array. We denote the first H-fractal as an ‘antenna’, owing to its role as the power inlet.

3. Transmission measurements

Because of the anisotropic characteristics intrinsic to the H-fractal, there can be three different propagation directions in accordance with the different orientations of the 3D H-fractal. We denote by 1st $p$ the case when the first-level line of (all) 3D H-fractals is parallel to the propagating direction, and by 2nd $p$ and 3rd $p$ the respective cases when the second- and third-level lines of 3D H-fractals are parallel to the propagating directions. To measure the transmission spectrum, a separate H-fractal antenna was placed 20 mm away from the end of the waveguide to serve as the signal receiver, and both the antenna and the receiver were connected to an S-parameter network analyzer (Agilent 8720ES). All three propagation cases were measured and normalized to the transmission magnitude without the waveguide between the antenna and receiver, and the results are shown in figure 2. It is seen that the waveguide enhances the transmission by roughly 60 times at some frequencies.
Figure 2. The upper panel shows the transmission, normalized to the amplitude without the waveguide, for the 1st case (defined in the text) and its corresponding current distributions at the two peak frequencies. The middle panel shows the results for the 2nd case (defined in the text). The lower panel shows the results for the 3rd case (defined in the text).

We used FDTD simulations (Concerto 6.5) to delineate the field/current distribution in a single metallic fractal at the peak transmission frequencies. The absorption boundary condition was used in the simulations, with one H-fractal placed at the center of the simulation box. The incident plane wave was set at the peak transmission frequencies. A tiny loss was given to the metallic wires of the 3D H-fractals so as to facilitate the mapping of the current distribution, since the power dissipation \( P_d \) is directly related to the current in accordance with the relation \( P_d = I^2_s/Z_s \), where \( Z_s \) is the surface impedance for a good conductor. The results are shown in the insets to figures 2(a)–(c). It is noted that two peaks, one at 5.2 GHz and another at 9.7 GHz (both off the scale in figure 2), were not observed experimentally [21]. This is due to the difficulty in exciting the higher levels’ resonances when the microwave power is centrally fed to the first level.

4. Dipole and multipole couplings

By measuring the transmission along the three orientations, three different transmission characteristics with different peak transmission frequencies were identified. They correspond to

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4 Simulations were performed using the software CONCERTO 6.5 (Vector Fields Limited, England, 2006).
Figure 3. (a) The configuration of two neighboring H-fractals in the 1st \( p \) case, with the two planes (labeled 1 and 2) over which FDTD simulations were performed. Phase measurement was carried out over the red dashed line, located at the intersection between the two planes. (b) Simulated \( E_x \) field distribution on plane 1. \( E_x \) is the component of electric field along the propagation direction.) (c) Simulated \( E_x \) field distribution on plane 2. (d) Measured phase (black solid squares) across the red dashed line as indicated in (a), together with the simulation prediction (red open circles).

different resonant current distributions in each of the 3D H-fractals. Transmission requires the coupling of the resonances between the neighboring H-fractals. To understand these couplings, both FDTD simulations and phase measurements were carried out. Since the higher frequency modes were difficult to excite, only the three lowest resonant modes (0.7, 1.06 and 1.75 GHz, respectively, for the 1st \( p \), 2nd \( p \) and 3rd \( p \) cases) were studied. For the simulations, five H-fractals were placed at the center of the simulation box. Microwave power at three different resonant frequencies was center-fed at the first fractal of the waveguide. To verify the simulation results, local field scanning was performed along the red dashed lines shown in figures 3–5 with a step increment of 2 mm.

Simulated field distribution and phase measurement results for the 1st \( p \) case are shown in figure 3. Here, the waveguide configuration is shown in figure 3(a); the propagation direction is parallel to the first level of the fractals. The regions denoted by I and III are occupied by fractals, whereas region II is the interval between the two. The amplitude distribution of the electric field component (\( E_x \), along the propagation direction) is shown in figure 3(b) for plane 1 (the horizontal plane), and the distribution of \( E_x \) on plane 2 (the vertical plane) is shown in figure 3(c). From the simulated results, the field distribution is seen to have high symmetry. To verify the results in figures 3(b) and (c), we have measured the phase variation of \( E_x \) along the red dashed line shown in figure 3(a). The result is shown in figure 3(d) together with the
Figure 4. (a) The configuration of two neighboring H-fractals in the 2nd case, with the two planes (labeled 1 and 2) over which FDTD simulations were performed. Phase measurement was carried out over the red dashed line, located at the intersection between the two planes. (b) Simulated $E_x$ field distribution on plane 1. (c) Simulated $E_x$ field distribution on plane 2. (d) Measured phase (black solid squares) across the red dashed line as indicated in (a), together with the simulation prediction (red open circles).

Simulation predictions. Good agreement is obtained. From the field simulation and the phase measurement for the 1st case, we conclude that the waveguiding mechanism at this frequency (0.7 GHz) is due to the coupling between resonant dipoles generated in the neighboring H-fractals.

In figure 4, the simulated field distributions and phase measurement results at 1.06 GHz (the 2nd case) are shown. The configuration of the waveguide and the locations and orientations of planes 1 and 2 are delineated in figure 4(a). The relevant (simulated) field distributions of $E_x$ are shown in figure 4(b) for plane 1 and in figure 4(c) for plane 2. It is seen that there are more amplitude oscillations than those shown in figures 3(b) and (c). Phase measurement results along the red dashed line are plotted in figure 4(d) together with the simulation results. Good agreement between theory and experiment is again obtained. In this case, the wave is guided via the coupling of resonant quadrupoles between neighboring H-fractals [22].

The simulated distributions and phase measurement of $E_x$, for the 3rd case at the peak frequency 1.75 GHz, is shown in figure 5. The simulated field distributions for planes 1 and 2 are, respectively, shown in figures 5(b) and (c); the phase measurement results and the predictions from simulations (along the red dashed line shown in figure 5(a)) are plotted in figure 5(d). Here, the wave is guided via coupling of resonant higher-order multipoles between neighboring H-fractals.
Figure 5. (a) The configuration of two neighboring H-fractals in the 3rd \( p \) case, with the two planes (labeled 1 and 2) over which FDTD simulations were performed. Phase measurement was carried out over the red dashed line, located at the intersection between the two planes. (b) Simulated \( E_x \) field distribution on plane 1. (c) Simulated \( E_y \) field distribution on plane 2. (d) Measured phase (black solid squares) across the red dashed line as indicated in (a), together with the simulation prediction (red open circles).

5. Bandstructure and loss

Our waveguide may be viewed as a 1D photonic crystal comprising resonant H-fractal units. Coupling of the local resonances between neighboring H-fractals necessarily implies that there exist transmission bands. We have carried out bandstructure calculations for the three different polarizations. Since the wavelengths at the relevant transmission frequencies are much larger than the size of a single H-fractal, we treat each H-fractal as a homogeneous cube characterized by a frequency-dependent effective dielectric constant [21]:

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\begin{align*}
\varepsilon_x &= 0.40 + \frac{0.4}{1.15^2 - f^2} + \frac{28.0}{4.60^2 - f^2}, \\
\varepsilon_y &= 0.40 + \frac{1.0}{1.80^2 - f^2} + \frac{80.10}{9.52^2 - f^2}, \\
\varepsilon_z &= 0.80 + \frac{0.25}{0.72^2 - f^2} + \frac{3.10}{2.75^2 - f^2},
\end{align*}
\]

where the three directions \( x, y \) and \( z \) are those indicated in figure 1.
Figure 6. Bandstructures for the waveguide comprising a 1D array of H-fractals: (a) for the 1st $p$ case, (b) for the 2nd $p$ case and (c) for the 3rd $p$ case. The bandgap frequency regions are indicated by shading. The guided wave frequencies are delineated by dashed lines. They are noted to be close to the band edges and have relatively flat dispersions.

The equation for the field amplitude $u$, the component of the electric field along the propagation direction, is given by [23]

$$\nabla^2[e^{ikx}u(x)] + k_0^2 \varepsilon_z e^{ikx}u(x) = 0. \tag{2}$$

The periodic boundary condition is applied to $u$:

$$u(x + d) = u(x), \tag{3}$$

with $k$ being a 1D Bloch wavevector. Here, $k_0 = \omega/c$, $\omega$ being the angular frequency, $c$ denotes the light velocity, $\varepsilon_z$ is $\varepsilon_z$ in the 1st $p$ case, $\varepsilon_x$ in the 2nd $p$ case and $\varepsilon_y$ in the 3rd $p$ case, and $d$ is the lattice constant.

The commercial software COMSOL was employed [19] to solve equation (2) with the boundary condition given by equation (3). The bandstructures for the three cases are shown in figure 6, where (a), (b) and (c) correspond to the cases of the 1st $p$, 2nd $p$ and 3rd $p$, respectively. The bandgap frequency regions are indicated by light blue shading. It is seen that the peak transmission frequencies are close to the edges of the passing bands.

Transmission loss is an important characterizing parameter for the waveguide. We have experimentally measured the intensity of the guided waves along a 3D H-fractal array by using a scanning dipole antenna. The measurement schematics are shown in figure 7(a) where a dipole antenna, controlled by the Labview program, was used to detect the magnitude and phase at selected measurement locations. The measured electric field component is along the propagation direction. The magnitudes at six different locations for the 2nd $p$ case are shown in figure 7(b). It is seen that it follows an exponential behavior with a decay length of $\sim 15.7$ m. For the other cases, the decay lengths were similarly measured to yield 10.3 and 39.5 m for the 1st $p$ and 3rd $p$ cases, respectively.
Figure 7. (a) Schematic illustration showing the sample geometry with the scanning dipole and the measurement configuration. (b) Field intensity attenuation along the propagation direction. The solid line is the best exponential fit to the data.

6. Imaging with the H-fractal waveguide

From the calculated bandstructures shown in figure 6, the best waveguiding frequencies are found to be located close to the band edges, with a fairly flat dispersion. This is not surprising, since the waveguiding mechanism arises from the local resonances of the H-fractals. As a result, the group velocities at those frequencies should be much smaller than those of light in vacuum [24]–[26]. The flat dispersion also implies a high density of $(\mathbf{k})$ states at those frequencies; hence more spatial information may be transmitted. To test these predictions, experiments were performed for the 2nd $p$ case.

To measure the group velocity at the waveguiding frequency of 1.06 GHz and to verify the imaging capability, we formed a waveguide with 39 H-fractals oriented such that the second-level lines are parallel to the propagation direction. The whole array is 228 cm in length, or about eight wavelengths at the frequency of 1.06 GHz. Here the cross-sectional dimension is about 1/7 the wavelength. The EM power was fed in at the center of the first H-fractal, serving as the antenna, and another H-fractal antenna was put 20 mm away from the last fractal as a receiver. Both antennas were linked with an $S$-parameter network analyzer (Agilent 8720ES). The experimental result is shown in figure 8(a) as a function of frequency. The velocity at 1.06 GHz (the first guiding frequency of the 2nd $p$ case) can be evaluated by

$$|V_g| = \frac{\Delta \omega}{\Delta k},$$

From our measurements, $\Delta \omega = 2\pi \times 3.75 \times 10^6$ Hz and $\Delta k = \pi/4.4$ m$^{-1}$, so the group velocity is $3.3 \times 10^7$ m s$^{-1}$, about one-tenth of the value in vacuum.

To observe the image transported through the H-fractal waveguide, a paper plate with a piece of metal pasted on it was put between the source H-fractal antenna and the first H-fractal of the waveguide. The paper plate is a square with a side length of 40 cm, and the metal
Figure 8. (a) Measured phase plotted as a function of frequency. The shaded region indicates the neighborhood of the first peak frequency of the 2nd \( p \) case, from which we obtained the group velocity. (b) The schematic configuration of field scanning conducted in the near field of the last H-fractal of the waveguide. (c) Difference in the scanned intensities between the case with the obstacle and the case without the obstacle. The red rectangle encircles the corresponding position of the metal piece at the input plane.

pasted on the paper plate is a 9 cm × 16 cm rectangle. A local field scan of \( E_x \) (the component along the propagation direction) at a plane perpendicular to the propagation direction, 5 mm away from the last H-fractal, was performed. The same local field scan was performed again without the paper plate at the inlet. Figure 8(c) shows the difference between the two scanned results (the case without the plate minus that with the plate) after averaging over eight different frequencies—1.03, 1.035, 1.04, 1.045, 1.05, 1.055, 1.06 and 1.065 GHz—in the neighborhood of the guiding frequency (1.06 GHz). Because the metallic rectangle shields the EM wave immediately behind it, the (scanned) transmitted intensity clearly shows a less-than-average intensity at the same position of the output plane. Hence a rough image of the metal piece was observed.

7. Concluding remarks

We have demonstrated EM power transmission through a waveguide comprising a 1D array of subwavelength H-fractals. Due to the anisotropy of the 3D H-fractal, the transmission peak frequencies are dependent on the H-fractal orientation, thus providing multi-band characteristics. The waveguiding mechanism is due to the coupling between neighboring dipoles and multipoles arising from the H-fractal resonances. By regarding the waveguide as a 1D photonic crystal, the bandstructure was calculated for all three different orientations. It was found that all the peak frequencies are close to the edge of the passing band, with relatively flat dispersions. Measurements of transmission loss show decay lengths ranging from \( \sim 10 \) to 40 m. Imaging was demonstrated to be possible.
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