Entropy-Statistical Analysis of the Air Conditioning System of a Passenger Airplane

Igor Valerievich Tishchenko 1, a),
Sergey Aleksandrovich Abalakin 1, b),
Vladislav Ivanovich Merkulov 1, c),
Anton Andreevich Zharov 1, d).

1 Bauman Moscow State Technical University, Moscow, Russian Federation
a) iv.tischenko@bmstu.ru
b) sserviuass@gmail.com
c) merkulov@bmstu.ru
d) zharov_a@bmstu.ru

ABSTRACT
This article discusses a static mathematical model of the air conditioning system of a passenger aircraft with moisture control. The model was developed to calculate various operation modes of this system, for one of which the entropy-statistical analysis was performed. In the analysis, the main nodes of the circuit are considered, an assessment of their influence on the overall efficiency of the air conditioning system is carried out, as well as the advantages of the given entropy-statistical analysis method for such systems are evaluated. The mathematical model was developed in the Matlab Simulink software package.

Keywords: Entropy-Statistical Analysis, Mathematical Model, Air Conditioning System, ACS, Aircraft.

INTRODUCTION
In modern aviation, it is impossible to imagine passenger aircraft without a modern air conditioning system (ACS). These systems solve various problems in the field of life support and comfortable existence of a person during an aircraft flight. Environmental parameters during aircraft operation vary over a wide range. The ACS is designed to regulate such air parameters as temperature, pressure, flow rate, moisture content, etc. At various flight modes, it can operate both in cooling and heating modes. ACS designing brings needs to comply with the following requirements for them: weight and dimensions, reliability, efficiency. These parameters significantly affect the total cost of ACS, the cost of its maintenance, operation, etc.

There are many options for ACS schemes. This article analyzes a high-pressure loop-type moisture separation scheme. Moisture has a great influence on the efficiency of ACS operation (Shustrov Yu. M. et al).
But besides this, it also affects the design features and options for the ACS operating modes. The presence of moisture during the ACS operation at negative temperatures leads to a deterioration in the efficiency of heat exchange units, which forces to use additional units to solve this problem, and also limits the range of operating modes. The ACS scheme with a loop is one of the options for combating the negative influence of moisture on ACS. This article will describe the mathematical model of a similar scheme for a passenger aircraft. Mathematical models are often used for refrigeration and cryogenic systems (Strizhenov E.M. et al 2019; Navasardyan E.S et al. 2016). This model allows carrying out a static calculation of the circuit in all ACS operating modes. On the basis of this model, the entropy-statistical analysis of the system was carried out for the selected flight mode of the aircraft. This method was developed at the Bauman Moscow State Technical University, an example of its description can be found in the study (Arkharov A.M et al. 2016). Based on this method, various cryogenic systems are actively analyzed (Arkharov A.M et al. 2016; Gareeva D.T and Lavrov N.A 2016). The entropy-statistical method of analyzing the system makes it possible to determine the effect of the individual ACS units efficiency.

**DESCRIPTION OF THE SYSTEM OPERATION**

The passenger aircraft ACS is a system designed to regulate the temperature, flow rate and moisture content of the air supplied to the aircraft. This article discusses the operation of an ACS with a three-wheel turbo-cooling unit and a high-pressure loop-type moisture separation. The ACS scheme analyzed in the article is shown in Fig. 1.

![ACS scheme](image)

**Figure 1. ACS scheme**
The source of compressed air in the ACS is the aircraft engine. Through the working path, air enters the primary heat exchanger (HE1), where it is cooled by outside air. Having passed it, the working air enters the compressor stage of the turbo-refrigerator (C), where it is further compressed to improve the separation of moisture later. After that, the air enters the secondary heat exchanger (HE2), where heat exchange with the outside air also takes place. The purging of the primary and secondary heat exchangers (HE1 and HE2) is provided by the operation of the fan stage (F) of the turbo-cooling unit or by the incoming air flow. After that, the air enters the so-called «loop», consisting of two heat exchangers (HE3 and HE4), an expander stage of a turbo-refrigeration unit (T) and a moisture separator (MS). In this «loop», the working air is cooled along the hot path due to the same air flow after it has been cooled. Working air enters the superheater (HE3) and then the condenser (HE4). In these heat exchangers, abundant moisture condensation occurs, which is then separated in a moisture separator (MS). In order to avoid the ingress of the remaining moisture into the expander stage, as well as to prevent premature condensation of the latter, the working air enters the evaporator through the cold path, where it is heated, providing heat removal from the hot path, and enters the expander stage (T) of the turbo-refrigeration unit. In the expander stage (T), the working air flow expands and cools, and the work performed by it is transferred along the shaft of the turbo-refrigeration unit to the compressor stage (C) and the fan stage (F). After that, the main part of the air flow enters the superheater (HE3) through the cold path, thereby providing heat removal from the hot path. The removed moisture in the moisture separator (MS) goes for injection into the purge path of the secondary heat exchanger of the ACS, thereby increasing the cooling efficiency in the primary and secondary heat exchangers (HE1 and HE2). The air conditioning system can operate in two modes: cooling and heating. In the heating mode, the outlet air temperature is regulated by removing the hot flow through the adjustment device (AD) and feeding it for mixing with the flow leaving the expander stage (T) of the turbo-refrigeration unit. The advantages of this scheme include small number of units required to create «loops», simplicity, reliability, high manufacturability of «loop» units, as well as use as a refrigerant in the working air loop.
DESCRIPTION OF THE MATHEMATICAL MODEL

To analyze the operation of the ACS, a mathematical model of the system was developed in the Matlab Simulink environment. The model performs a thermodynamic calculation of the system. Similar calculations of systems and units were carried out in (Merkulov V.I. et al. 2019; A. A. Zharov et al. 2019; I. V. Tishchenko et al. 2019). The model calculates the parameters of the ACS nodal points and some parameters of its nodes in particular. The use of this environment was due to the possibility of easy replacement of the components of the ACS units and thereby unification of the model for similar schemes. This scheme uses mathematical models of devices, considering the performance characteristics of their real counterparts. The model uses the method of successive iterations to calculate the entire circuit as a whole. This model considers the effect of the phase change of moisture on the temperature of the working air (in particular, it considers the effect of the phase change during the injection of moisture into the purge loop of the secondary heat exchanger, in other heat exchangers in the working path and the expander stage). The operation of the model is divided into two units. Unit №1 calculates the operation of the primary and secondary heat exchangers, the compressor stage, as well as the purge circuit with the fan stage. Unit №2 calculates the operation of the system «loop», which includes a superheater, condenser, moisture separator and turbine stage. Each unit has its own iteration parameters. Unit №1 sequentially calculates the parameters of the main nodal points and selects the purge air flow rate. Unit №2 determines the parameters of the nodal points, the amount of moisture removed and the power removed from the expander stage of the turbo-refrigerator.

The coordination of these two units operation also goes through the selection of parameters (inlet pressure, rotational speed of the turbo-refrigeration unit shaft, air bypass adjustment if the ACS operates in heating mode). Thus, the calculation of the entire system in any mode is carried out due to many successive iterations. As a result of the calculation, the mathematical model gives out the parameters at each of its points, which will later be used to analyze the operation of the entire system as a whole. An example of a design scheme is shown in Fig. 2.
SYSTEM OPERATION ANALYSIS

When developing and manufacturing an ACS, it is important to have a detailed understanding of the operation of such a system. For each ACS, it is necessary to carry out its own thermodynamic calculation and analysis of the operation of such a system. In most cases, the efficiency of the ACS is analyzed by its energy consumption (or rather, mainly by the consumption of hot air). The method of entropy-statistical analysis is actively used at the Bauman Moscow State Technical University, with the help of which it is possible to analyze in detail the operation of the ACS in cooling modes. This method allows you to determine the nodes, the losses in which have the greatest impact on the efficiency of ACS, and also allows you to assess the prospects for improving these nodes. In this paper, this method is applied to analyze the land cooling mode of the aircraft, as the most loaded for ACS. The calculation method is taken on the basis of the study (Arkharov A.M. 2014). The results of calculating the land mode of ACS using the mathematical model are presented in Table 1.
| №  | Pressure, kPa | Temp, K | Flow rate, kg/h | Moisture content in the form of steam, g/kg of dry air | Moisture content if the form of drops, g/kg of dry air | Enthalpy, kJ/kg | Entropy, kJ/kg·K |
|----|-------------|--------|----------------|------------------------------------------------------|-------------------------------------------------------|----------------|-----------------|
| A  | 101,301     | 310    | 1185           | 18                                                   | -                                                     | 310,303        | 6,898           |
| 0  | 304,65      | 473    | 1185           | 18                                                   | 0                                                     | 475,463        | 7,010           |
| 1  | 302,425     | 473    | 1185           | 18                                                   | 0                                                     | 475,465        | 7,012           |
| 2  | 280,602     | 378,994| 1185           | 18                                                   | 0                                                     | 379,681        | 6,808           |
| 4  | 375,503     | 424,192| 1183,8         | 18                                                   | 0                                                     | 425,504        | 6,838           |
| 6  | 348,652     | 314,583| 1183,8         | 14,764                                               | 3,236                                                 | 314,413        | 6,557           |
| 12 | 338,705     | 311,004| 1183,8         | 12,522                                               | 5,478                                                 | 310,818        | 6,553           |
| 13 | 329,388     | 298,716| 1183,8         | 6,351                                                | 11,658                                                | 298,425        | 6,521           |
| 16 | 316,235     | 299,136| 1163,8         | 6,931                                                | 0,816                                                 | 298,88         | 6,534           |
| 17 | 315,514     | 305,103| 1163,8         | 7,681                                                | 0,066                                                 | 304,907        | 6,554           |
| 19 | 101,301     | 275,423| 1144,1         | 4,516                                                | 3,232                                                 | 275,513        | 6,779           |
| 20 | 101,301     | 275,423| 1144,1         | 4,516                                                | 3,232                                                 | 275,513        | 6,779           |

| 7  | 101,301     | 299,561| 2788           | 22,45                                                | 0                                                     | 299,796        | 6,864           |
| 8  | 99,639      | 348,887| 2788           | 22,45                                                | 0                                                     | 349,498        | 7,022           |
| 9  | 98,791      | 389,104| 2788           | 22,45                                                | 0                                                     | 390,154        | 7,135           |
| 10 | 101,3       | 396,283| 2788           | 22,45                                                | 0                                                     | 397,425        | 7,146           |

**Table 1.** Estimated parameters of ACS

The calculation of cycle indicators along the working path was carried out according to the following formulas:

Specific cooling capacity of the cycle:

\[ q_c = (i_4 - i_2) \cdot \frac{G_4}{G_0} + (i_2 - i_6) \cdot \frac{G_6}{G_0} + (i_17 - i_19) \cdot \frac{G_{19}}{G_0} = 42,189 \frac{kJ}{kg} \]

Minimum specific work of the cycle:

\[ l_{min} = q_c \cdot \left( \frac{T_A}{T_A - T_{20}} \cdot \ln \left( \frac{T_A - T_{20}}{T_{19}} - 1 \right) + q_c \cdot \frac{T_A}{T_{20} - T_{19}} \cdot \ln \left( \frac{T_{20}}{T_{19}} - 1 \right) \right) \]

\[ = 9,512 \frac{kJ}{kg} \]
Real specific work of the engine:

\[ l_{real} = \frac{k}{k-1} \cdot R_{air} \cdot T_A \cdot \frac{(\frac{P_0}{P_A})^{\frac{k-1}{k}} - 1}{\eta_{eng}} = 160,881 \frac{kJ}{kg} \]

Actual value of the coefficient of performance:

\[ \varepsilon = \frac{q_c}{l_{real}} = 0.262 \]

**Specific work required to compensate for the production of entropy:**

For the engine:

\[ \Delta l_{eng} = \frac{k}{k-1} \cdot R_{air} \cdot T_A \cdot \left[ \left( \frac{P_0}{P_A} \right)^{\frac{k-1}{k}} - 1 \right] \cdot \left( \frac{1}{\eta_{eng}} - 1 \right) = 45,419 \frac{kJ}{kg} \]

For the primary heat exchanger:

\[ \Delta l_{he1} = T_A \cdot \left( (S_9 - S_8) \cdot \frac{G_9}{G_0} - (S_1 - S_2) \cdot \frac{G_2}{G_0} \right) = 18,923 \frac{kJ}{kg} \]

For the secondary heat exchanger:

\[ \Delta l_{he2} = T_A \cdot \left( (S_8 - S_7) \cdot \frac{G_8}{G_0} - (S_4 - S_6) \cdot \frac{G_6}{G_0} \right) = 28,268 \frac{kJ}{kg} \]

For the compressor:

\[ \Delta l_{comp} = \frac{k}{k-1} \cdot R_{air} \cdot T_2 \cdot \left[ \left( \frac{P_0}{P_2} \right)^{\frac{k-1}{k}} - 1 \right] \cdot \left( \frac{1}{\eta_{comp}} - 1 \right) = 12,306 \frac{kJ}{kg} \]

For the moisture separator:

\[ \Delta l_{ms} = T_A \cdot (S_{16} - S_{13}) \cdot \frac{G_{16}}{G_0} = 4,021 \frac{kJ}{kg} \]

For considering heat exchange with the environment:

\[ \Delta l_{im} = [T_A \cdot (S_A - S_{20}) - (i_a - i_{20})] \cdot \frac{G_{20}}{G_0} = 2,025 \frac{kJ}{kg} \]
For the heater:

\[ \Delta l_{he3} = T_A \cdot \left[ (S_6 - S_{12}) \cdot \frac{G_{12}}{G_0} - (S_{16} - S_{17}) \cdot \frac{G_{17}}{G_0} \right] = 7,261 \frac{kJ}{kg} \]

For the condenser:

\[ \Delta l_{he4} = T_A \cdot \left[ (S_{20} - S_{19}) \cdot \frac{G_{19}}{G_0} - (S_{12} - S_{13}) \cdot \frac{G_{13}}{G_0} \right] = 12,341 \frac{kJ}{kg} \]

For the expander:

\[ \Delta l_{exp} = \frac{k}{k - 1} \cdot R_{atm} \cdot T_{17} \cdot [1 - \left( \frac{P_{19}}{P_{17}} \right)^{\frac{k-1}{k}}] \cdot (1 - \eta_{turb}) = 17,659 \frac{kJ}{kg} \]

For considering hydraulic losses in the pipeline in front of the primary heat exchanger:

\[ \Delta l_{hydr} = T_A \cdot \left( S_1 - S_0 \right) \cdot \frac{G_1}{G_0} = 0,654 \frac{kJ}{kg} \]

Total cycle work:

\[ \Delta l_{total} = l_{min} + \Delta l_{he1} + \Delta l_{he2} + \Delta l_{comp} + \Delta l_{ms} + \Delta l_{im} + \Delta l_{exp} + \Delta l_{he3} + \Delta l_{he4} + \Delta l_{hydr} \]

\[ \Delta l_{total} = 158,388 \frac{kJ}{kg} \]

Calculation error:

\[ \frac{l_{real} - l_{total}}{l_{real}} = 1,25\% \]

The work cycle is shown in the T-S diagram in Fig. 3, and the ratio of work to compensate for the increase in entropy in Fig. 4.
CONCLUSION

As a result of this work, a static mathematical model of the ACS with a three-wheel turbo-refrigeration unit and a high-pressure «loop»-type moisture separation system was developed. This model makes it possible to quickly carry out system calculations in various flight modes, which simplifies the development of such a system. According to the developed mathematical model, an entropy-statistical analysis of the land mode was carried out, as the most loaded operation mode of the ACS, and on its basis, an assessment was made of the influence of various units on the efficiency of the system as a whole.
## LEGEND

| Symbol | Description |
|--------|-------------|
| $P$    | pressure (kPa) |
| $T$    | temperature (K) |
| $i$    | enthalpy (J×kg⁻¹) |
| $G$    | air flow rate (kg×s⁻¹) |
| $R_{air}$ | gas constant for air (287 J×kg⁻¹×K⁻¹) |
| $k$    | adiabatic exponent for air (1.41) |
| $S$    | entropy (J×kg⁻¹×K⁻¹) |
| $\eta$ | unit efficiency |

## REFERENCES

1. A.M. Arkharov, V.Y. Semenov, S.D. Krasnonosova (2016). An Entropy-Statistical Analysis of a Natural Gas Liquefaction Plant with External Nitrogen Cooling Cycle. Chemical and Petroleum Engineering, 51 (11-12) 725-735
2. A.M. Arkharov, V.Y. Semenov, S.D. Krasnonosova (2016). Methodology of Statistical Entropy Analysis of Small-Scale Natural Gas Liquefiers. Chemical and Petroleum Engineering, 51 (9) 665-673
3. D.T. Gareeva, N.A. Lavrov (2016) Entropy Production and Energy Loss During Mixing of Gases. Chemical and Petroleum Engineering, 51 (11-12) 759-764
4. E.M. Strizhenov, S.S. Chugaev, A.A. Zherdev (2019) Mathematical Model of The Process of Circuit Charging of an Adsorption Methane Storage System, Chemical and Petroleum Engineering 54 (9-10) 760-770.
5. E.S. Navasardyan, E.A. Antonov, I.A. Arkharov, A.A. Alexandrov (2016) Modeling of Processes in Microcryogenic Gas Cooler. Chemical and Petroleum Engineering 51 (9) 649-655.
6. I. V. Tishchenko, A. A. Zharov, V. S. Nikolaev, E. N. Pavlyuk Mathematical modeling of plate-fin heat exchanger in aircraft environmental control system (2019) AIP Conference Proceedings 2195, 020074
7. A. A. Zharov, V. P. Leonov, I. V. Tishchenko, L. B. Volokitin, I. A. Mikerov Designing a turbomachine of environmental control system for missile-carrier aircraft of air launch systems (2019) AIP Conference Proceedings 2171, 030012
8. A.M. Arkharov, 2014. Osnovy kriologii. Entropiyno-statisticheskiy analiz nizkotemperaturnymykh sistem [Fundamentals of cryology. Entropy-statistical analysis of low-temperature systems]. Bauman Moscow State Technical University Publishing House.
9. V.I. Merkulov, & I.V. Tishchenko, & S.A. Abalakin, (2019). The use of electrocompressors in helicopter air conditioning systems. Izvestiya MGTU MAMI №2(40) 59-66. DOI:10.31992/2074-0530-2019-40-2-59-66.