Minimizing Total Cost in Integrated Scheduling of Production and Distribution Using Ant Colony Optimization

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Abstract. The integrated scheduling is a scheduling that applied in more than one problem. In this paper, we discuss about integrated scheduling of distribution and production problem in manufacturer. In the case of production, jobs are processed in several machines. In the case of distribution, one vehicle is used to distribute the product to the several customers. The objective of this paper is minimizing total cost in the production and distribution process. Since these problems are included in NP-hard problem, we suggest ant colony optimization method to solve both parts of the problem. We generate random data to verify the performance of the proposed method. Our result show that the performance of the method is excellent.

1. Introduction

Now days, the integrated scheduling is a topic that has been considered by researchers. This topic is more complicated than the common scheduling. The difference is the integrated scheduling can resolve more than one problem at once. Those problems can be a combination from production scheduling, maintenance, distribution, and other part of supply chain management problems. The purpose of integrated scheduling is overall optimization.

The integrated scheduling concerns on all activities of supply chain management. For example, two integrated issues discussed in [1] are machine scheduling and transportation considerations. Integrating preventive maintenance planning and production scheduling [2]. A novel genetic algorithm was proposed by [3] to solve transportation and production scheduling problem. Integrating of scheduling and packing [4]. Supply chain scheduling and coordination [5]. Batch delivery and production coordination [6]. Immune clonal selection algorithm was proposed by [7] to maximize the profit in integrated of production scheduling and maintenance planning. Three integrated issues, those are production, inventory and location-allocation decisions, discussed in [8].

In this paper, we discuss about the integrated scheduling of distribution and production issues at manufacturer. In the case of production, the job is process at several machines and the production scheduling type is job shop. Job shop scheduling is a scheduling of n jobs that is processed exactly once on m machines [9]. When the machine is started, the operation process cannot be interrupted and one machine can process one job only at the certain time. The studies that discussed about job shop,
methods and techniques used for job shop scheduling [10], conventional and new solution techniques for solving job shop scheduling problem [11].

In the case of distribution, one vehicle is used to distribute the products to the customers. This problem is included on the travelling salesman problem. In a condition where a salesman goes from his own city, who wants to find the shortest path can be trough to visit all the customers in several different cities, then get back to his own city. Each of customers is visited exactly once [12]. Some of research that solved TSP are exact method for solving traveling salesman problem [13], using standard heuristics to solving traveling salesman problem of scheduling in AS/RS [14], and solving traveling salesman problem by evolutionary algorithm [15].

The objective of this paper is minimizing total cost in the distribution and production process. Since these problems are included in \( NP-hard \), we suggest ant colony optimization method to solve both parts of the problem. Ant colony is used as a method to find the optimal distribution and production schedule. In the case of production, the purpose is minimizing the production cost. The minimum cost of production is obtained from the minimum number of total production time. In the case of distribution, the purpose is minimizing cost of distribution. The minimum cost of distribution is obtained from the minimum of total distance in distribution.

2. Problem characteristics and assumption

In this paper, the production type that will be optimized is job shop scheduling problem. According to [16], the characteristics of job shop scheduling problem are listed below.

1. Job shop scheduling has some job that has to be finished, represented by \( J = \{ J_1, J_2, \ldots, J_n \} \).

2. Job shop scheduling has some machine that is used to finish every operation, represented by \( M = \{ M_1, M_2, \ldots, M_m \} \).

3. Every job has several operations that has to be finished in the time start from ready time to due time. A job \( i \) has several operation which are represented by \( O_i = \{ O_{i1}, O_{i2}, \ldots, O_{in} \} \).

Let a set job \( J = \{ Job_1, Job_2, \ldots, Job_n \} \), \( K_i \) is a completion time of \( Job_i \) and total production time is symbolized by \( R \), formulated as below.

\[
R = \max\{K_1, K_2, K_3, \ldots, K_n\} \quad (1)
\]

In the distribution scheduling, one vehicle is used to distribute the products to the customers. This problem is included on the travelling salesman problem. Generally, the characteristic of TSP is listed below [17].

1. The tour begins and ends at the same point, that is manufacturer.

2. All the points except manufacturer, which means customers, are visited exactly once.

3. The salesman cannot be back to the manufacturer before all customers are visited.

If the customer set that must be visited is symbolized by \( C_i = \{ C_1, C_2, C_3, \ldots, C_v \} \) and the distance from \( C_a \) to \( C_b \) is symbolized by \( d_{ab} \) is, so the total distance that can be reached at the distribution process is formulated as follows.

\[
D = \sum_{i=1}^{C_{i=v-1}} d_{piC(i)piC(i+1)} + d_{piC(v)piC(1)} \quad (2)
\]
$R$ and $D$ are the objective functions that will be optimized.

3. Ant colony optimization
We suggest ant colony optimization as a method to minimize total cost of the integrated distribution and production scheduling problem. In this paper, we use the algorithm adapted from [18].

1. Coding

$k$ number of ants is used to determine the optimal schedule. At every path $(i, j)$ which selected by ant, there is additions of pheromones at that path. At the path that is not passed by ants, the pheromone levels are decreased, because there is a rate of pheromone evaporation. In this paper, the rate of pheromone evaporation is constant.

| Parameters | Meaning |
|------------|---------|
| $k$        | number of ants |
| $\tau_{ij}$ | pheromone intensity at path $(i, j)$ |
| $\rho$     | rate of pheromone evaporation |
| $\alpha$   | constant controller of the pheromone intensity |
| $\beta$    | constant controller of visibility |

Table 1. Parameters in ant colony algorithm.

In the production scheduling, $\eta_{ab} = \frac{1}{t_{ab}}$ is a value of visibility obtained from reciprocal processing time of a job $a$ in the machine $b$, while in distribution scheduling, $\eta_{ab} = \frac{1}{d_{ab}}$ is the value of visibility obtained from reciprocal distance from the customer $a$ to $b$.

2. Status transmission

The rule of transmission status is showed by the following equation. This is a probability of the ant–$k$ that is lie at node $i$ and choose to go to node $j$.

$$P_{ij}^k(t) = \begin{cases} \frac{[\tau_{ij}(t)]^\alpha[\eta_{ij}]^\beta}{\sum_{i=1}^n[\tau_{ij}(t)]^\alpha[\eta_{ij}]^\beta}, & \text{if } (i, j) \in S_k \\ 0, & \text{if } (i, j) \notin S_k. \end{cases}$$ (3)

In the production scheduling, $S_k$ contains the operations list of every jobs must be completed at a machine. Every operation that has been done by a machine will not be included in tabu list. Meanwhile, in the distribution scheduling, $S_k$ contains the customers list must be visited. Every customer that has been visited will be removed from the tabu list.
3. Pheromone update locally

After every ant completes his tour, local update pheromone is carried out with this following formula.

\[ \tau_{ij}(t + 1) = \Delta \tau_{ij}(t) + (1 - \rho) \tau_{ij}(t) \]  

\[ \Delta \tau_{ij}(t) = \left\{ \begin{array}{ll} \frac{[\tau_{ij}(t)]^a \eta_{ij}^\beta}{\sum_{n=1}^{N} [\tau_{ij}(t)]^a \eta_{ij(t)}^\beta}, & \text{if } (i, j) \text{ is the path where ants pass} \\ 0, & \text{otherwise.} \end{array} \right. \]  

The parameter of pheromone evaporation level is bounded in range \( 0 \leq \rho \leq 1 \).

4. Pheromone update globally

After several \( k \) ants complete their own tour, then the global pheromone update is carried out. The formulas of global update of pheromone for each problem are shown below.

\[ \tau_{ij}(t + 1) = \Delta \tau_{ij}(t) + (1 - \rho) \tau_{ij}(t) \]  

In the production scheduling,

\[ \Delta \tau_{ij}^k(t) = \left\{ \begin{array}{ll} \frac{Q}{B_k}, & \text{if } (i, j) \text{ is the path where ants pass} \\ 0, & \text{otherwise.} \end{array} \right. \]  

In the distribution scheduling,

\[ \Delta \tau_{ij}^k(t) = \left\{ \begin{array}{ll} \frac{Q}{D_k}, & \text{if } (i, j) \text{ is the path where ants pass} \\ 0, & \text{otherwise.} \end{array} \right. \]  

Only the path included in the best tour will increase the level of pheromone.

4. Simulations

In the case of production, we use job shop scheduling problem with four jobs and four machines. In this problem, to test the performance of Ant Colony Optimization, we do the several changes of parameter value and take notes in every experiment.

The data of processing time has been generated randomly to represent job shop with four jobs and four machines. Data of processing time that we use in production part are shown in table below.

| Table 2. Processing time three jobs in three machines. |
|-------------------------------|--------|--------|--------|--------|
| J1 | M1 | M2 | M3 | M4 |
|----|----|----|----|----|
| J1 | 37 | 58 | 56 | 40 |
| J2 | 45 | 80 | 27 | 35 |
| J3 | 64 | 29 | 46 | 75 |
| J4 | 33 | 42 | 65 | 51 |
Several experiments have been tried based on the data above. The output obtained from each experiments are shown below.

**Figure 1.** Results of experiment with the value of parameters used are $k = 1$, $\rho = 0.1$, and the iteration is 5. (a) $\alpha = 1$, $\beta = 1$, the minimum processing time is 340 and order of job is J3-J4-J1-J2. (b) $\alpha = 0.3$, $\beta = 0.5$, the minimum processing time is 340 and order of job is J3-J4-J1-J2.

**Figure 2.** Results of experiment with value of parameters used are $\rho = \beta = \alpha = 0.5$, $k = 3$. (a) use 20 iterations, the minimum processing time is 340 and order of job is J3-J4-J1-J2. (b) use 50 iterations, the minimum processing time is 340 and order of job is J3-J4-J1-J2.

**Figure 3.** Results of experiment with value of parameters used are $\rho = \beta = \alpha = 0.5$, and the
iteration is 100 (a) the number of ant is 1, the minimum processing time is 340 and order of job is J3-J4-J1-J2. (b) the number of ant is 20, the minimum processing time is 340 and order of job is J3-J4-J1-J2.

Based on the experiments result, we obtain the minimum processing time is 340 and order of job is J3-J4-J1-J2. Let the constant cost of production process is $c_1$, so the minimum cost is $340(c_1)$.

We compare the total production time obtained by ant colony optimization with the manual calculation to validate the results. On job shop scheduling of four jobs on four machines, there are 24 possible order of job. All possibilities are listed below.

| Order of job     | Total processing time | Order of job     | Total processing time |
|------------------|-----------------------|------------------|-----------------------|
| J1 - J2 - J3 - J4| 376                   | J3 - J1 - J2 - J4| 387                   |
| J1 - J2 - J4 - J3| 408                   | J3 - J1 - J4 - J2| 366                   |
| J1 - J3 - J2 - J4| 384                   | J3 - J2 - J4 - J1| 392                   |
| J1 - J3 - J4 - J2| 358                   | J3 - J2 - J1 - J4| 419                   |
| J1 - J4 - J3 - J2| 377                   | J3 - J4 - J1 - J2| 340                   |
| J1 - J4 - J2 - J3| 377                   | J3 - J4 - J2 - J1| 376                   |
| J2 - J1 - J3 - J4| 411                   | J4 - J1 - J2 - J3| 363                   |
| J2 - J1 - J4 - J3| 430                   | J4 - J1 - J3 - J2| 352                   |
| J2 - J3 - J4 - J1| 366                   | J4 - J2 - J3 - J1| 348                   |
| J2 - J3 - J1 - J4| 384                   | J4 - J2 - J1 - J3| 393                   |
| J2 - J4 - J1 - J3| 409                   | J4 - J3 - J1 - J2| 341                   |
| J2 - J4 - J3 - J1| 398                   | J4 - J3 - J2 - J1| 376                   |

Total processing time from 24 order of job has been calculated with manual process. The minimum processing time from all possibilities is 340 with the optimal schedule is J3-J4-J2-J1. This comparison proves that the ant colony method works very well. Optimal result obtained from the heuristic approach is the same as exact solution. Even in the third experiment, by using more ants and iterations, the optimal solution is obtained since the first iteration. This method is really useful for solving bigger problems.

In the next discussion, distribution part, we use 40 data, one of them is represent the location of manufacturer and the others are represent the location of customers. The random data have been generated to represent the traveling salesman problem with 40 locations. Every location is represented by coordinate $(x, y)$. All the value of $x$ and $y$ are shown below.
Several experiments have been tried based on the data above. With the parameters $\rho = 0.5, \alpha = 1, \beta = 1$. The output obtained from each experiment is shown below.

| Coordinates of $x$ | Coordinates of $y$ | Coordinates of $x$ | Coordinates of $y$ |
|-------------------|-------------------|-------------------|-------------------|
| 20                | 64                | 130               | 15                |
| 89                | 37                | 122               | 90                |
| 22                | 55                | 45                | 70                |
| 29                | 18                | 39                | 100               |
| 43                | 49                | 25                | 120               |
| 99                | 40                | 93                | 87                |
| 72                | 84                | 80                | 96                |
| 53                | 19                | 75                | 110               |
| 31                | 57                | 90                | 23                |
| 50                | 20                | 81                | 52                |
| 110               | 33                | 77                | 92                |
| 70                | 28                | 31                | 105               |
| 48                | 60                | 18                | 115               |
| 55                | 47                | 127               | 12                |
| 69                | 79                | 130               | 125               |
| 15                | 30                | 45                | 108               |
| 125               | 25                | 62                | 122               |
| 30                | 10                | 13                | 127               |
| 10                | 45                | 57                | 130               |
| 100               | 82                | 120               | 30                |

**Figure 4.** The number of ants is 50 and iteration is 100, and the total distance obtained from this trial is 765.6428. (a) distribution route and (b) length of tour function.
Figure 5. The number of ant is 500 and iteration is 1000, and the total distance obtained from this trial is 677.8603. (a) distribution route and (b) length of tour function.

Figure 6. The number of ant is 500 and iteration is 5000, and the total distance obtained from this trial is 669.0953. (a) distribution route and (b) length of tour function.

From the experiments, we know that the number of ants and the iterations used greatly affect the results. The higher number of ants and the iterations used, the better the results obtained. Based on the experiments, the minimum value of total distance of distribution route is 669.0953. Let the constant cost of distribution process is \( c_2 \), so the minimum cost is \( 669.0953(c_2) \).
5. Conclusion

Focus of this paper is an approach of ant colony algorithm for solving integrated scheduling problem. The problem included distribution and production scheduling. From the experiments carried out, ant colony optimization can solve the problem with great satisfaction. In the case of production, the result of the approach is the same as the exact solution, as well as in the case of distribution, ant colony optimization is able to find the optimal solution in a bigger problem.

However, the result of this study shown that solutions that obtained depend on the number of ant or iteration. Less number of ants used causes a random change of route which obtain a different output in each iterations. Meanwhile a less number of iterations allows a less variety of the obtained output, so that, it is possible that the obtained result is not optimum. While Ant Colony Optimization is applied with higher number of ant and iteration in both of problem, we obtain the great result. The experimental showed that the method we proposed is better in convergence speed and even the ability to get the best solution in a bigger problem.

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