Heavy baryon spectroscopy in QCD

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We perform a systematic study of the masses of charmed and bottom baryons in the framework of the QCD sum rule approach. Contributions of the operators up to dimension six are included in operator product expansion. The resulting heavy baryon masses from the calculations are well consistent with the experimental values, and predictions to the spectroscopy of the unobserved bottom baryons are also presented.

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I. INTRODUCTION

During the past several years there has been tremendous progress in the experimental investigations of the heavy baryon spectroscopy. With the precise measurement for the mass of Λ_b by the CDF collaboration [1], the D0 collaboration proclaimed the observation of Ξ_b[2], which was quickly confirmed by the CDF collaboration [3]. The first observations of Σ_b and Σ_b[2] have been reported by the CDF [4]. The BABAR collaboration announced the observation of Ω∗_c[5] and the production of Ω_c from B decays [6]. The Ξ_c as well as the excited states of Ξ_c were set forth by the Belle and the BABAR collaborations [7]. Therefore, a large amount of experimental data on charmed and bottom baryons has become available.

On the other hand, various theoretical models have been used to study heavy baryon masses, such as quark models [8, 9], mass formulas [10], and lattice QCD calculations [11]. From QCD sum rules [12], masses of the heavy baryons were primarily calculated in the heavy-quark limit [13], and subsequently in the heavy quark effective theory [14, 15, 16]. In Refs. [17], the calculations for the heavy baryons began with the full theory. Recently the masses of Ξ_Q and Ω_Q[18] were tested in QCD sum rules [18, 19]. Other renewed works were inspired by the current significant observations continually [20, 21, 22]. Proceeding from the motivation to evaluate the spectroscopy of the heavy baryons systematically in full QCD, we shall study mass sum rules for the heavy baryons, with the technique developed in [23, 24]. For the cases of Ξ_Q and Ω_Q[18, 19], the interpolating currents utilized in this work are not entirely the same as the ones used in Refs [18, 19].

The paper is organized as follows. In Sec II QCD sum rules for the heavy baryons are derived. Section III contains numerical analysis, a brief summary, and some discussions.

II. QCD SUM RULES FOR THE HEAVY BARYONS

It is of interest to apply the QCD sum rule to study the heavy baryons composed of a heavy quark (Q = c, b) and two light (u, d, or s) quarks. The basic point is to choose the suitable interpolating current. For the ground states, the currents are correlated with the spin-parity quantum numbers J^P_L = 0^+ and J^P_L = 1^+ for the light diquark system, along with the heavy quark forming the state with J^P = 1^+ and the pair of degenerate states with J^P = 1^− and J^P = 3^+, which may determine the choice of Γ_k and Γ'_k matrices in baryonic currents [13]. For the Γ'_k matrices for the excited baryons with I(J^P) = 0(1^−), it might be referred to the current of the heavy-light vector meson [25]. For the baryons with J = 3/2, the currents may be gained from those of baryons with J = 1/2 using SU(3) symmetry relations [26]. Thus, the following forms of currents are adopted [13, 17, 25, 26, 27]:

\[ j_{Λ_Q} = \varepsilon_{abc}(q_1^T T_k q_2) \Gamma'_k Q_c, \]
For each invariant function $\Pi$, logically, the correlator can be expressed as a dispersion integral over a physical spectral function.

Lorentz covariance implies that the two-point correlation function has the form given by Wilson coefficients, while long-distance confinement effects are attributed to power corrections.

\[
j_{\Lambda Q} = \varepsilon_{abc} \left[ \frac{2}{\sqrt{3}} (q_{1a}^T C T_k Q_b) \Gamma'_k q_c + \frac{1}{\sqrt{3}} (q_{1a}^T C T_k q_{2b}) \Gamma'_k Q_c \right],
\]

\[
j_{\Xi Q} = \varepsilon_{abc} (q_{1a}^T C T_k s_b) \Gamma'_k Q_c,
\]

\[
j_{\Xi'^2} = \varepsilon_{abc} (q_{1a}^T C T_k s_b) \Gamma'_k Q_c,
\]

\[
j_{\Xi'^3} = \varepsilon_{abc} \left[ \frac{2}{\sqrt{3}} (q_{1a}^T C T_k Q_b) \Gamma'_k s_c + \frac{1}{\sqrt{3}} (q_{1a}^T C T_k s_b) \Gamma'_k Q_c \right],
\]

\[
j_{\Xi'^2} = \varepsilon_{abc} (q_{1a}^T C T_k s_b) \Gamma'_k Q_c,
\]

\[
j_{\Xi'^3} = \varepsilon_{abc} \left[ \frac{2}{\sqrt{3}} (q_{1a}^T C T_k Q_b) \Gamma'_k s_c + \frac{1}{\sqrt{3}} (q_{1a}^T C T_k s_b) \Gamma'_k Q_c \right].
\]

Here the index $T$ means matrix transposition, $C$ is the charge conjugation matrix, $a, b, c$ are color indices, and $q_1, q_2$ are $u$ or $d$ depending on the concrete quark contents of the corresponding heavy baryons. The choice of $\Gamma_k$ and $\Gamma'_k$ matrices are shown in Table I.

**TABLE I**: The choice of $\Gamma_k$ and $\Gamma'_k$ matrices in baryonic currents

| State (quark content) | $J^P$ | $S_l$ | $L_l$ | $J^{P/2}$ | $\Gamma_k$ | $\Gamma'_k$ |
|-----------------------|-------|-------|-------|------------|-------------|-------------|
| $\Xi Q (qsQ)$         | $\frac{1}{2}$ | 0     | 0     | 0$^+$      | $\gamma_5$  | 1           |
| $\Omega Q (ssQ)$      | $\frac{3}{2}$ | 1     | 0     | 1$^+$      | $\gamma_{\mu}$ | $\gamma_{\mu} \gamma_5$ |
| $\Omega'^1 (ssQ)$     | $\frac{3}{2}$ | 1     | 0     | 1$^+$      | $\gamma_{\mu}$ | $\gamma_{\mu} \gamma_5$ |
| $\Lambda Q (qqQ)$     | $\frac{1}{2}$ | 0     | 1     | 1$^-$      | $\gamma_5$  | $\gamma_{\mu}$ |
| $\Lambda'^1 (qqQ)$    | $\frac{3}{2}$ | 0     | 1     | 1$^-$      | $\gamma_5$  | $\gamma_{\mu}$ |

The QCD sum rules for the heavy baryons are constructed from the two-point correlation function

\[
\Pi(q^2) = i \int d^4 x e^{i q \cdot x} \langle 0 | T[j(x)\overline{j}(0)] | 0 \rangle.
\]

Lorentz covariance implies that the two-point correlation function has the form

\[
\Pi(q^2) = \Pi_1(q^2) + \Phi \Pi_2(q^2).
\]

For each invariant function $\Pi_1$ and $\Pi_2$, a sum rule can be obtained, which is shown below. Phenomenologically, the correlator can be expressed as a dispersion integral over a physical spectral function

\[
\Pi(q^2) = \lambda_H \frac{\Phi + M_H}{M^2_H - q^2} + \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\text{Im} \Pi_{\text{phen}}(s)}{s - q^2} + \text{subtractions},
\]

where $M_H$ denotes the mass of the heavy baryon. In obtaining the above expression, the Dirac and Rarita-Schwinger spinor sums

\[
\sum_s N(q,s)\bar{N}(q,s) = \Phi + M_H,
\]

for spin-$\frac{1}{2}$ baryon, and

\[
\sum_s N_\mu(q,s)\bar{N}_\nu(q,s) = (\Phi + M_H)(g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{q_\mu \gamma_\nu - q_\nu \gamma_\mu}{3M_H} - \frac{2q_\mu q_\nu}{3M^2_H}),
\]

for spin-$\frac{3}{2}$ baryon have been used. In the operator product expansion (OPE) side, short-distance effects are given by Wilson coefficients, while long-distance confinement effects are attributed to power corrections.
and parameterized in terms of vacuum condensates. Hence

\[ \Pi_i(q^2) = \Pi_i^{\text{pert}}(q^2) + \Pi_i^{\text{cond}}(q^2), i = 1, 2. \tag{7} \]

We work at leading order in \( \alpha_s \) and consider condensates up to dimension six. The strange quark is dealt as a light one and the diagrams are considered up to order \( m_s \). To keep the heavy-quark mass finite, the momentum-space expression for the heavy-quark propagator is used. We follow Refs. [23, 24] and calculate the light-quark part of the correlation function in the coordinate space, which is then Fourier-transformed to the momentum space in \( D \) dimension. The resulting light-quark part is combined with the heavy-quark part before it is dimensionally regularized at \( D = 4 \). For the heavy-quark propagator with two and three gluons attached, the momentum-space expressions given in Ref. [25] are used. With Eq. [7], the correlation function in the OPE side in terms of a dispersion relation can be written as

\[ \Pi_i(q^2) = \int_{m_Q^2}^{\infty} ds \frac{\rho_i(s)}{s - q^2} + \Pi_i^{\text{cond}}(q^2), \tag{8} \]

where the spectral density is given by the imaginary part of the correlation function

\[ \rho_i(s) = \frac{1}{\pi} \text{Im} \Pi_i^{\text{OPE}}(s). \tag{9} \]

After equating the two expressions for \( \Pi(q^2) \), assuming quark-hadron duality, and making a Borel transform, the sum rules can be written as

\[ \lambda_H^2 M_H e^{-M_H^2/M^2} = \int_{m_Q^2}^{s_0} ds \rho_1(s) e^{-s/M^2} + \hat{B} \Pi_1^{\text{cond}}, \tag{10} \]

\[ \lambda_H^2 e^{-M_H^2/M^2} = \int_{m_Q^2}^{s_0} ds \rho_2(s) e^{-s/M^2} + \hat{B} \Pi_2^{\text{cond}}. \tag{11} \]

To eliminate the baryon coupling constant \( \lambda_H \) and extract the \( M_H \), first take the derivative of Eq. [10] with respect to \(-\frac{1}{M^2}\), divide the result by Eq. [10] itself, and similarly deal with Eq. [11] to yield

\[ M_H^2 = \left\{ \int_{m_Q^2}^{s_0} ds \rho_1(s) e^{-s/M^2} + d/d\left(-\frac{1}{M^2}\right) \hat{B} \Pi_1^{\text{cond}}(s) \right\}/\left\{ \int_{m_Q^2}^{s_0} ds \rho_1(s) e^{-s/M^2} + \hat{B} \Pi_1^{\text{cond}}(s) \right\}, \tag{12} \]

\[ M_H^2 = \left\{ \int_{m_Q^2}^{s_0} ds \rho_2(s) e^{-s/M^2} + d/d\left(-\frac{1}{M^2}\right) \hat{B} \Pi_2^{\text{cond}}(s) \right\}/\left\{ \int_{m_Q^2}^{s_0} ds \rho_2(s) e^{-s/M^2} + \hat{B} \Pi_2^{\text{cond}}(s) \right\}, \tag{13} \]

where

\[ \rho_i(s) = \rho_i^{\text{pert}}(s) + \rho_i^{(q\bar{q})}(s) + \rho_i^{(s\bar{s})}(s) + \rho_i^{(G^2)}(s), \tag{14} \]

with

\[ \rho_i^{\text{pert}}(s) = \frac{3}{2\pi^4} m_Q \int_0^1 d\alpha \frac{(1-\alpha)^2}{\alpha^2} (m_Q^2 - \alpha s)^2, \tag{15} \]

\[ \rho_i^{(G^2)}(s) = \frac{(g^2 G^2)}{2\pi^4} m_Q \int_0^1 d\alpha \frac{(1-\alpha)^2}{\alpha^2} [2], \tag{16} \]

\[ \hat{B} \Pi_1^{\text{cond}} = \frac{\langle g^2 G^2 \rangle}{3 \cdot 2^3 \pi^4} m_Q^3 \int_0^1 d\alpha \frac{(1-\alpha)^2}{\alpha^3} e^{-m_Q^2/(\alpha M^2)} \]

\[ + \frac{2 \langle q\bar{q} \rangle^2}{3} m_Q e^{-m_Q^2/M^2} \]
for $\Lambda_Q$ baryons,

$$\rho_1^{\text{pert}}(s) = \frac{1}{2^{1/2}} m_Q \int_0^1 \frac{d\alpha}{\alpha} \frac{(1-\alpha)^2}{\alpha^2} (m_Q^2 - sa)^2,$$

$$\rho_1^{(\bar{q}q)}(s) = \frac{1}{3} \int_0^1 \frac{d\alpha}{\alpha} (m_Q^2 - sa),$$

$$\rho_1^{(G^2)}(s) = \frac{g^2 G^2}{3 \cdot 2^{3/2} m_Q} \int_0^1 \frac{d\alpha}{\alpha^2} (1-\alpha)^2 e^{-m_Q^2/(\alpha M^2)}$$

$$+ \frac{5/2}{3} (\bar{q}q)^2 m_Q e^{-m_Q^2/M^2}$$

$$- \frac{g^2 G^3}{3 \cdot 2^{1/2}} m_Q \int_0^1 \frac{d\alpha}{\alpha^2} (1-\alpha)^2 (3 - \frac{m_Q^2}{\alpha M^2}) e^{-m_Q^2/(\alpha M^2)},$$

$$\rho_2^{\text{pert}}(s) = \frac{1}{2^{1/2}} m_Q \int_0^1 \frac{d\alpha}{\alpha} \frac{(1-\alpha)^2}{\alpha^2} (m_Q^2 - sa)^2,$$

$$\rho_2^{(\bar{q}q)}(s) = \frac{1}{3} \int_0^1 \frac{d\alpha}{\alpha} (m_Q^2 - sa),$$

$$\rho_2^{(G^2)}(s) = \frac{g^2 G^2}{3 \cdot 2^{3/2} m_Q} \int_0^1 \frac{d\alpha}{\alpha^2} (1-\alpha)^2 (1-3\alpha) (m_Q^2 - sa)^2,$$

$$\tilde{B}_{\Pi_2}^{\text{cond}} = \frac{g^2 G^2}{3 \cdot 2^{3/2}} m_Q \int_0^1 \frac{d\alpha}{\alpha^2} (1-\alpha)^2 (1-3\alpha) (m_Q^2 - sa)^2,$$

$$- \frac{2}{3} (\bar{q}q)^2 m_Q e^{-m_Q^2/M^2}$$

$$- \frac{g^2 G^3}{3 \cdot 2^{1/2}} m_Q \int_0^1 \frac{d\alpha}{\alpha^2} (1-\alpha)^2 [\alpha (1-4\alpha + 9\alpha^2) - 2(1-4\alpha + 5\alpha^2)] m_Q^2 e^{-m_Q^2/(\alpha M^2)},$$
\begin{align}
\rho_1^{(G^2)}(s) &= \frac{\langle q^2 G^2 \rangle}{3 \cdot 2^9 \pi^4} m_Q \int_0^1 da \left[ \frac{1 - \alpha}{\alpha} \right]^2 + 2, \\
\tilde{\Pi}_1^{\text{cond}} &= -\frac{\langle g^2 G^2 \rangle}{3 \cdot 2^9 \pi^4} m_Q \int_0^1 da \left( \frac{1 - \alpha}{\alpha} \right)^2 e^{-m_Q^2/(\alpha M^2)} \\
&\quad + \frac{\langle gqGq \rangle}{2^5 \pi^2} m_s m_Q e^{-m_Q^2/M^2} \\
&\quad + \frac{\langle gsGs \rangle}{3 \cdot 2^4 \pi^2} m_s m_Q e^{-m_Q^2/M^2} \\
&\quad + \frac{\langle \bar{q}q \rangle \langle \bar{s}s \rangle}{6} m_Q e^{-m_Q^2/M^2} \\
&\quad - \frac{\langle g^3 G^3 \rangle}{3 \cdot 2^{11} \pi^4} m_Q \int_0^1 da \left( \frac{1 - \alpha}{\alpha} \right)^2 (3 - \frac{m_Q^2}{\alpha M^2}) e^{-m_Q^2/(\alpha M^2)}, \\
\rho_2^{(\bar{q}q)}(s) &= \frac{3 \cdot 2^9 \pi^4}{2^5 \pi^2} m_s \left[ 1 - \left( \frac{m_Q^2}{s} \right)^2 \right], \\
\rho_2^{(\bar{s}s)}(s) &= \frac{\langle \bar{s}s \rangle}{2^5 \pi^2} m_s \left[ 1 - \left( \frac{m_Q^2}{s} \right)^2 \right], \\
\rho_2^{(G^2)}(s) &= \frac{\langle q^2 G^2 \rangle}{3 \cdot 2^9 \pi^4} \left[ 1 - \left( \frac{m_Q^2}{s} \right)^2 \right], \\
\tilde{\Pi}_2^{\text{cond}} &= -\frac{\langle g^2 G^2 \rangle}{3 \cdot 2^9 \pi^4} m_Q \int_0^1 da \left( \frac{1 - \alpha}{\alpha} \right)^2 e^{-m_Q^2/(\alpha M^2)} \\
&\quad + \frac{\langle gqGq \rangle}{2^5 \pi^2} m_s m_Q e^{-m_Q^2/M^2} \\
&\quad + \frac{\langle gsGs \rangle}{3 \cdot 2^4 \pi^2} m_s m_Q e^{-m_Q^2/M^2} \\
&\quad + \frac{\langle \bar{q}q \rangle \langle \bar{s}s \rangle}{6} e^{-m_Q^2/M^2} \\
&\quad - \frac{\langle g^3 G^3 \rangle}{3 \cdot 2^{11} \pi^4} m_Q \int_0^1 da \left( \frac{1 - \alpha}{\alpha} \right)^2 (1 - \frac{2m_Q^2}{\alpha M^2}) e^{-m_Q^2/(\alpha M^2)},
\end{align}

for \( \Xi_Q \) baryons,

\begin{align}
\rho_1^{\text{pert}}(s) &= \frac{3 \cdot 2^9 \pi^4}{2^5 \pi^2} m_Q \int_\Lambda^1 da \left( \frac{1 - \alpha}{\alpha} \right)^2 (m_Q^2 - sa)^2, \\
\rho_1^{(\bar{q}q)}(s) &= \frac{2 \langle \bar{q}q \rangle}{2^5 \pi^2} m_s m_Q \left( 1 - \frac{m_Q^2}{s} \right), \\
\rho_1^{(\bar{s}s)}(s) &= \frac{\langle \bar{s}s \rangle}{2^5 \pi^2} m_s m_Q \left( 1 - \frac{m_Q^2}{s} \right), \\
\rho_1^{(G^2)}(s) &= \frac{\langle q^2 G^2 \rangle}{2^6 \pi^4} m_Q \int_\Lambda da \left[ \frac{1 - \alpha}{\alpha} \right]^2 - 2, \\
\tilde{\Pi}_1^{\text{cond}} &= -\frac{\langle g^2 G^2 \rangle}{3 \cdot 2^6 \pi^4} m_Q \int_0^1 da \left( \frac{1 - \alpha}{\alpha} \right)^2 e^{-m_Q^2/(\alpha M^2)} \\
&\quad + \frac{\langle gqGq \rangle}{2^5 \pi^2} m_s m_Q e^{-m_Q^2/M^2} \\
&\quad - \frac{\langle gsGs \rangle}{3 \pi^2} m_s m_Q e^{-m_Q^2/M^2} \\
&\quad + \frac{8 \langle \bar{q}q \rangle \langle \bar{s}s \rangle}{3} m_Q e^{-m_Q^2/M^2} \\
&\quad - \frac{\langle g^3 G^3 \rangle}{3 \cdot 2^7 \pi^4} m_Q \int_0^1 da \left( \frac{1 - \alpha}{\alpha} \right)^2 (3 - \frac{m_Q^2}{\alpha M^2}) e^{-m_Q^2/(\alpha M^2)},
\end{align}
\[ \rho_2^{\text{pert}}(s) = \frac{3}{2^5\pi^4} \int_0^1 d\alpha \frac{(1 - \alpha)^2}{\alpha} (m_Q^2 - s\alpha)^2, \]  
\[ \rho_2^{(\bar{q}q)}(s) = -\frac{\langle \bar{q}q \rangle}{2\pi^2} m_s[1 - \frac{m_Q^2}{s}], \]  
\[ \rho_2^{(s\bar{s})}(s) = \frac{\langle s\bar{s} \rangle}{2\pi^2} m_s[1 - \frac{m_Q^2}{s}], \]  
\[ \rho_2^{(G^2)}(s) = -\frac{\langle g^2G^2 \rangle}{2\pi^4} [1 - \frac{m_Q^2}{s}], \]  
\[ \hat{\Pi}_2^{\text{cond}} = -\frac{\langle g^2G^2 \rangle}{3\cdot 2^9\pi^4} m_Q \int_0^1 d\alpha \frac{(1 - \alpha)^2}{\alpha} e^{-m_Q^2/(\alpha M^2)} \]  
\[ + \frac{\langle \bar{q}q \cdot Gq \rangle}{2\pi^2} m_s e^{-m_Q^2/M^2} \]  
\[ - \frac{\langle g\bar{s}\sigma \cdot Gs \rangle}{3\cdot 2\pi^2} m_s e^{-m_Q^2/M^2} \]  
\[ + \frac{4\langle \bar{q}q \rangle \langle \bar{s}s \rangle}{3} e^{-m_Q^2/M^2} \]  
\[ - \frac{\langle g^3G^3 \rangle}{3\cdot 2^9\pi^4} m_Q \int_0^1 d\alpha \frac{(1 - \alpha)^2}{\alpha} e^{-m_Q^2/(\alpha M^2)} \]  
\[ - \frac{2\langle \bar{q}q \rangle}{2\pi^2} m_Q [1 - \frac{m_Q^2}{s}]^2 - \frac{\langle \bar{q}q \rangle}{3\cdot 2\pi^2} m_s[1 - \frac{m_Q^2}{s}], \]  
\[ \rho_2^{\text{pert}}(s) = -\frac{1}{2^5\pi^4} \int_0^1 d\alpha \frac{(1 - \alpha)^2}{\alpha} (1 - 3\alpha)(m_Q^2 - s\alpha)^2, \]  
\[ \rho_2^{(\bar{q}q)}(s) = -\frac{2\langle \bar{q}q \rangle}{3\pi^2} m_Q[1 - \frac{m_Q^2}{s}]^2 - \frac{\langle \bar{q}q \rangle}{3\cdot 2\pi^2} m_s[1 - \frac{m_Q^2}{s}], \]  
\[ \rho_2^{(s\bar{s})}(s) = \frac{\langle s\bar{s} \rangle}{3\cdot 2^2\pi^2} m_s \int_0^1 d\alpha (3 - \alpha), \]  
for \( \Xi_Q \) baryons.
\[
\rho^{(G^2)}_2(s) = \frac{\langle g^2 G^2 \rangle}{3 \cdot 2^9 \pi^4 m_Q^3} \int_0^1 d\alpha (1 - \alpha)^2 \left[ \frac{2}{(m_Q^2 - s \alpha)^2} + \frac{1}{2} \right] d\alpha (\alpha - 1)(m_Q^2 - s \alpha), \tag{57}
\]

\[
\tilde{\Pi}^{\text{cond}}_2 = \frac{\langle g^2 G^2 \rangle}{3 \cdot 2^9 \pi^4 m_Q^3} \int_0^1 d\alpha (1 - \alpha)^2 (1 - 3\alpha) \frac{e^{-\tilde{m}_Q^2/(\alpha M^2)}}{\alpha^3}
+ \frac{\langle q\bar{q} \cdot G_q \rangle}{3 \pi^2} m_Q \int_0^1 d\alpha e^{-\tilde{m}_Q^2/(\alpha M^2)}
+ \frac{\langle q\bar{q} \cdot G_q \rangle}{3 \cdot 2^4 \pi^2} m_s e^{-\tilde{m}_Q^2/M^2}
- \frac{\langle q\bar{s} \cdot G_s \rangle}{3 \cdot 2^4 \pi^2} m_s e^{-\tilde{m}_Q^2/M^2}
- \frac{\langle q\bar{s} \cdot G_s \rangle}{3 \cdot 2^9 \pi^4} m_s e^{-\tilde{m}_Q^2/M^2}
+ 4 \langle \bar{q}q \rangle \langle s\bar{s} \rangle \pi^2 (1 - 2 m_s m_Q) e^{-\tilde{m}_Q^2/M^2}
+ \frac{\langle g^3 G^3 \rangle}{3 \cdot 2^9 \pi^4} \int_0^1 d\alpha \frac{1 - \alpha}{\alpha} \left[ \frac{1}{(m_Q^2 - s \alpha)^2} - \frac{1}{(m_Q^2 - s \alpha)^2} \right] e^{-\tilde{m}_Q^2/(\alpha M^2)}, \tag{58}
\]

for \( \Xi^*_Q \) baryons,

\[
\rho^{\text{pert}}_1(s) = \frac{3}{2^8 \pi^4} m_Q \int_0^1 d\alpha \left[ \frac{1 - \alpha}{\alpha} \right]^2 (m_Q^2 - s \alpha)^2, \tag{59}
\]

\[
\rho^{(\bar{q}q)}_1(s) = - \frac{\langle \bar{q}q \rangle}{2^4 \pi^2} m_s m_Q (1 - \frac{m_Q^2}{s}), \tag{60}
\]

\[
\rho^{(s\bar{s})}_1(s) = \frac{\langle s\bar{s} \rangle}{2^4 \pi^2} m_s m_Q (1 - \frac{m_Q^2}{s}), \tag{61}
\]

\[
\rho^{(G^2)}_1(s) = \frac{\langle g^2 G^2 \rangle}{2^7 \pi^4} m_Q \int_0^1 d\alpha \left[ \frac{(1 - \alpha)^2}{\alpha} \right] e^{-\tilde{m}_Q^2/(\alpha M^2)} \tag{62}
\]

\[
\tilde{\Pi}^{\text{cond}}_1 = - \frac{\langle g^3 G^3 \rangle}{3 \cdot 2^9 \pi^4} m_Q \int_0^1 d\alpha \frac{(1 - \alpha)^2}{\alpha^3} e^{-\tilde{m}_Q^2/(\alpha M^2)}
+ \frac{\langle q\bar{q} \cdot G_q \rangle}{2^4 \pi^2} m_s m_Q e^{-\tilde{m}_Q^2/M^2}
+ \frac{\langle q\bar{s} \cdot G_s \rangle}{3 \cdot 2^4 \pi^2} m_s m_Q e^{-\tilde{m}_Q^2/M^2}
+ \frac{2}{3} \langle \bar{q}q \rangle \langle s\bar{s} \rangle \pi^2 m_s e^{-\tilde{m}_Q^2/M^2}
- \frac{\langle g^3 G^3 \rangle}{3 \cdot 2^8 \pi^4} m_Q \int_0^1 d\alpha \frac{(1 - \alpha)^2}{\alpha^3} \left[ 3 - \frac{m_Q^2}{\alpha M^2} \right] e^{-\tilde{m}_Q^2/(\alpha M^2)}, \tag{63}
\]

\[
\rho^{\text{pert}}_2(s) = - \frac{3}{2^8 \pi^4} \int_0^1 d\alpha \left[ \frac{1 - \alpha}{\alpha} \right]^2 (m_Q^2 - s \alpha)^2, \tag{64}
\]

\[
\rho^{(\bar{q}q)}_2(s) = \frac{\langle \bar{q}q \rangle}{2^4 \pi^2} m_s \left[ 1 - \frac{(m_Q^2)^2}{s} \right], \tag{65}
\]

\[
\rho^{(s\bar{s})}_2(s) = - \frac{\langle s\bar{s} \rangle}{2^4 \pi^2} m_s \left[ 1 - \frac{(m_Q^2)^2}{s} \right], \tag{66}
\]

\[
\rho^{(G^2)}_2(s) = - \frac{\langle g^2 G^2 \rangle}{2^7 \pi^4} \left[ 1 - \frac{(m_Q^2)^2}{s} \right], \tag{67}
\]

\[
\tilde{\Pi}^{\text{cond}}_2 = \frac{\langle g^2 G^2 \rangle}{3 \cdot 2^9 \pi^4} m_Q \int_0^1 d\alpha \left[ \frac{(1 - \alpha)^2}{\alpha} \right] e^{-\tilde{m}_Q^2/(\alpha M^2)}
- \frac{\langle q\bar{q} \cdot G_q \rangle}{2^4 \pi^2} m_s e^{-\tilde{m}_Q^2/M^2}
- \frac{\langle q\bar{s} \cdot G_s \rangle}{3 \cdot 2^4 \pi^2} m_s e^{-\tilde{m}_Q^2/M^2}
- \frac{\langle g^3 G^3 \rangle}{3 \cdot 2^8 \pi^4} m_Q \int_0^1 d\alpha \frac{(1 - \alpha)^2}{\alpha^3} \left[ 3 - \frac{m_Q^2}{\alpha M^2} \right] e^{-\tilde{m}_Q^2/(\alpha M^2)}, \tag{68}
\]
\[
\frac{1}{3} \langle \bar{q}q \rangle \langle \bar{s}s \rangle e^{-m_Q^2/M^2} + \frac{(g^3 G^3)}{3 \cdot 2^9 \pi^4} \int_0^1 d\alpha \left( \frac{1 - \alpha}{\alpha^2} \right)^2 (1 - \frac{2m_Q^2}{\alpha M^2}) e^{-m_Q^2/(\alpha M^2)},
\]

for \( \Xi_{1Q} \) baryons,

\[
\rho_1^{\text{pert}}(s) = \frac{1}{3^2 \cdot 2^9 \pi^4} m_Q \int_0^1 d\alpha \left( \frac{1 - \alpha}{\alpha^2} \right)^2 (m_Q^2 - \alpha s)^2 \frac{1}{2^2 \pi^2} m_s \int_0^1 d\alpha \frac{1 - \alpha}{\alpha} (m_Q^2 - \alpha s)^2,
\]

\[
\rho_1^{\langle \bar{q}q \rangle}(s) = \frac{5\langle \bar{q}q \rangle}{3 \cdot 2^9 \pi^4} m_s m_Q (1 - \frac{m_Q^2}{s}),
\]

\[
\rho_1^{\langle \bar{s}s \rangle}(s) = \frac{2\langle \bar{s}s \rangle}{3^2 \pi^2} \int_0^1 d\alpha (m_Q^2 - \alpha s) + \frac{\langle \bar{s}s \rangle}{3 \cdot 2^9 \pi^4} m_s m_Q (1 - \frac{m_Q^2}{s}),
\]

\[
\rho_1^{\langle g^2 G^2 \rangle}(s) = \frac{\langle g^2 G^2 \rangle}{3 \cdot 2^9 \pi^4} m_Q \int_0^1 d\alpha \left( \frac{1 - \alpha}{\alpha^2} \right)^2 + \frac{\langle g^2 G^2 \rangle}{3 \cdot 2^9 \pi^4} m_s \int_0^1 d\alpha \frac{1 - \alpha}{\alpha} (m_Q^2 - \alpha s),
\]

\[
\bar{B}_1^{\text{cond}} = \frac{(g^2 G^2)}{3 \cdot 2^9 \pi^4} m_Q \int_0^1 d\alpha \left( \frac{1 - \alpha}{\alpha^2} \right)^2 e^{-m_Q^2/(\alpha M^2)}
\]

\[
+ \frac{\langle g\bar{q}G \rangle}{3 \cdot 2^9 \pi^4} m_s m_Q e^{-m_Q^2/M^2}
\]

\[
+ \frac{5\langle g\bar{q} \cdot Gq \rangle}{3 \cdot 2^9 \pi^4} m_s m_Q e^{-m_Q^2/M^2}
\]

\[
+ \frac{\langle g\bar{s} \cdot Gs \rangle}{3 \cdot 2^9 \pi^4} m_s m_Q e^{-m_Q^2/M^2}
\]

\[
+ \frac{5}{3^2} \frac{\langle g\bar{q} \rangle \langle \bar{s}s \rangle}{3 \cdot 2^9 \pi^4} m_Q e^{-m_Q^2/M^2}
\]

\[
+ \frac{(g^3 G^3)}{3 \cdot 2^9 \pi^4} m_Q \int_0^1 d\alpha \left( \frac{1 - \alpha}{\alpha^2} \right)^2 (m_Q^2 - \alpha s) \frac{1}{2^2 \pi^2} m_s \int_0^1 d\alpha \frac{1 - \alpha}{\alpha} (m_Q^2 - \alpha s) - \frac{2(1 - 2\alpha)m_Q^2}{\alpha M^2} e^{-m_Q^2/(\alpha M^2)},
\]

\[
\rho_2^{\text{pert}}(s) = \frac{1}{3^2 \cdot 2^9 \pi^4} \int_0^1 d\alpha \left( \frac{1 - \alpha}{\alpha} \right)^2 (1 - 3\alpha) (m_Q^2 - \alpha s)^2,
\]

\[
\rho_2^{\langle \bar{q}q \rangle}(s) = \frac{\langle \bar{q}q \rangle}{3 \cdot 2^9 \pi^4} m_Q (1 - \frac{m_Q^2}{s})^2 + \frac{\langle \bar{q}q \rangle}{3 \cdot 2^9 \pi^4} m_s [1 - \frac{m_Q^2}{s}],
\]

\[
\rho_2^{\langle \bar{s}s \rangle}(s) = \frac{\langle \bar{s}s \rangle}{3 \cdot 2^9 \pi^4} m_s \int_0^1 d\alpha (3 - \alpha),
\]

\[
\rho_2^{\langle g^2 G^2 \rangle}(s) = \frac{(g^2 G^2)}{3 \cdot 2^9 \pi^4} \left[ - \frac{1}{2} + \frac{m_Q^2}{s} \right] + \int_0^1 d\alpha \frac{1 - \alpha}{\alpha} (m_Q^2 - \alpha s),
\]

\[
\bar{B}_2^{\text{cond}} = -\frac{(g^2 G^2)}{3 \cdot 2^9 \pi^4} m_Q \int_0^1 d\alpha \left( \frac{1 - \alpha}{\alpha^2} \right)^2 (1 - 3\alpha) e^{-m_Q^2/(\alpha M^2)}
\]

\[
- \frac{\langle g\bar{q}G \rangle}{3 \cdot 2^9 \pi^4} m_Q \int_0^1 d\alpha e^{-m_Q^2/(\alpha M^2)}
\]

\[
- \frac{\langle g\bar{q} \cdot Gq \rangle}{3 \cdot 2^9 \pi^4} m_s e^{-m_Q^2/M^2}
\]

\[
+ \frac{\langle g\bar{s} \cdot Gs \rangle}{3 \cdot 2^9 \pi^4} m_s \int_0^1 d\alpha e^{-m_Q^2/(\alpha M^2)}
\]

\[
- \frac{\langle g\bar{s} \cdot Gs \rangle}{3 \cdot 2^9 \pi^4} m_s e^{-m_Q^2/M^2}
\]

\[
- \frac{\langle \bar{q}q \rangle \langle \bar{s}s \rangle}{3^2} (1 - \frac{2m_Q m_s}{M^2}) e^{-m_Q^2/M^2}
\]
for \( B \) baryons, and \( \Xi_Q \) baryons,

\[
\rho_1^{\text{pert}}(s) = \frac{3}{2^5 \pi^4} m_Q \int_0^1 \frac{da \, \left(1 - \frac{a}{\alpha}\right)^2 (m_Q^2 - sa)^2}{\alpha^4},
\]

\[
\rho_1^{(ss)}(s) = \frac{3}{2^4 \pi^2} m_s m_Q \left(1 - \frac{m_Q^2}{s}\right),
\]

\[
\rho_1^{(G^2)}(s) = \frac{1}{2^6 \pi^4} m_Q \int_0^1 \frac{da \, [(1 - \frac{a}{\alpha})^2 - 2]}{\alpha^3},
\]

\[
\mathcal{B}_{II_1}^{\text{cond}} = -\frac{\langle g^2 G^2 \rangle}{3 \cdot 2^6 \pi^4} m_Q^3 \int_0^1 \frac{da \, (1 - \frac{a}{\alpha})^2 e^{-\frac{1}{\alpha M^2} m_Q^2}}{\alpha^4} \Bigg[ \frac{g s \sigma - G s}{3 \pi^2} m_s m_Q e^{-\frac{m_Q^2}{\alpha M^2}} + \frac{8}{3} m_Q e^{-\frac{m_Q^2}{M^2}} 
\]

\[
- \frac{\langle g^3 G^3 \rangle}{3 \cdot 2^8 \pi^4} m_Q \int_0^1 \frac{da \, (1 - \frac{a}{\alpha})^2 e^{-\frac{1}{\alpha M^2} m_Q^2}}{\alpha^3} \left(3 - \frac{m_Q^2}{\alpha M^2}\right) e^{-\frac{m_Q^2}{\alpha M^2}},
\]

\[
\rho_2^{\text{pert}}(s) = \frac{3}{2^5 \pi^4} \int_0^1 \frac{da \, (1 - \frac{a}{\alpha})^2 (m_Q^2 - sa)^2}{\alpha^4},
\]

\[
\rho_2^{(ss)}(s) = \frac{3}{2^5 \pi^2} m_s \left(1 - \frac{m_Q^2}{s}\right),
\]

\[
\rho_2^{(G^2)}(s) = \frac{1}{2^5 \pi^4} \left[1 - \frac{m_Q^2}{s}\right],
\]

\[
\mathcal{B}_{II_2}^{\text{cond}} = -\frac{\langle g^2 G^2 \rangle}{3 \cdot 2^7 \pi^4} m_Q^3 \int_0^1 \frac{da \, (1 - \frac{a}{\alpha})^2 e^{-\frac{1}{\alpha M^2} m_Q^2}}{\alpha^4} \left[ \frac{g s \sigma - G s}{3 \pi^2} m_s e^{-\frac{m_Q^2}{M^2}} + \frac{4}{3} e^{-\frac{m_Q^2}{M^2}} 
\]

\[
- \frac{\langle g^3 G^3 \rangle}{3 \cdot 2^9 \pi^4} m_Q \int_0^1 \frac{da \, (1 - \frac{a}{\alpha})^2 (1 - \frac{2 m_Q^2}{\alpha M^2}) e^{-\frac{1}{\alpha M^2} m_Q^2}}{\alpha^4},
\]

for \( \Omega_Q \) baryons, and

\[
\rho_1^{\text{pert}}(s) = \frac{1}{2^4 \pi^4} m_Q \int_0^1 \frac{da \, \left(1 - \frac{a}{\alpha}\right)^2 (m_Q^2 - sa)^2}{\alpha^4} + \frac{1}{2^4 \pi^4} m_s \int_0^1 \frac{da \, \left(1 - \frac{a}{\alpha}\right)(m_Q^2 - sa)^2}{\alpha^4},
\]

\[
\rho_1^{(ss)}(s) = \frac{2^4 \langle \bar{s} s \rangle}{3 \pi^2} \int_0^1 \frac{da \, (m_Q^2 - sa)}{\alpha^2} - \frac{19 \langle \bar{s} s \rangle}{3 \pi^2} m_s m_Q \left(1 - \frac{m_Q^2}{s}\right),
\]

\[
\rho_1^{(G^2)}(s) = \frac{1}{2^5 \pi^4} \left[1 - \frac{m_Q^2}{s}\right],
\]

\[
\mathcal{B}_{II_1}^{\text{cond}} = \frac{\langle g^2 G^2 \rangle}{3^2 \cdot 2^6 \pi^4} m_Q^3 \int_0^1 \frac{da \, \left(1 - \frac{a}{\alpha}\right)^2 e^{-\frac{1}{\alpha M^2} m_Q^2}}{\alpha^4} \left[ \frac{g s \sigma - G s}{3 \pi^2} m_s m_Q e^{-\frac{m_Q^2}{M^2}} + \frac{5}{3} \left(\frac{\bar{s} s}{3}\right)^2 e^{-\frac{m_Q^2}{M^2}} 
\]

\[
- \frac{5}{3} \left(\frac{\bar{s} s}{3}\right)^2 \int_0^1 \frac{da \, \left(1 - \frac{a}{\alpha}\right)^2 e^{-\frac{1}{\alpha M^2} m_Q^2}}{\alpha^4} \left(1 + \frac{m_Q^2}{M^2}\right) e^{-\frac{m_Q^2}{M^2}}.
\]
which the stability of the Borel curves should not be sensitive. The Borel windows are fixed in this
with
continuum contributions from Eq. (12) for
should be larger than the condensate contributions while the upper one is got by the pole contribution
merely owing to Borel windows, not involving the ones rootin
g the quark masses and
II, we present our results for the masses of the heavy baryons and compare with experimental data and
have been averaged to decrease the systematic errors. It is worth noting that the uncertainty in the results are
been averaged to decrease the systematic errors. It is worth noting that the uncertainty in the results are
for
baryons. The lower limit of integration is given by

\[ \rho_2^{(ss)}(s) = \frac{\langle ss \rangle}{3 \cdot 2\pi^2 m_s} \int_0^1 d\alpha \left( 3 - 5\alpha \right) - \frac{2 \langle ss \rangle}{3\pi^2 m_Q(1 - \frac{m_Q^2}{s})^2}, \]

\[ \rho_2^{(2\pi)}(s) = \left( \frac{g^2 G^2}{3 \cdot 2\pi^2} \right) \int_0^1 d\alpha \left( 1 + 2\alpha \right) - \frac{2m_Q^2}{\alpha(1 - \frac{m_Q^2}{s})^2}, \]

\[ B \Pi_2^{cond} = \left( \frac{g^2 G^2}{3 \cdot 2\pi^2} \right) \int_0^1 d\alpha \left( 1 - \frac{m_Q^2}{\alpha^2} \right) e^{-m_Q^2/\alpha M^2} \]

\[ - \frac{\langle g\bar{s}\sigma \cdot Gs \rangle}{3\pi^2} m_Q \int_0^1 d\alpha e^{-m_Q^2/\alpha M^2} \]

\[ + \frac{4\langle ss \rangle^2}{3} \int_0^1 d\alpha e^{-m_Q^2/\alpha M^2} \]

\[ - \frac{\langle g\bar{q}\sigma \cdot Gq \rangle}{3 \cdot 2\pi^2} m_s e^{-m_s^2/M^2} \]

\[ + \left( \frac{g^3 G^3}{3 \cdot 2\pi^4} \right) \int_0^1 d\alpha \frac{1 - \alpha}{\alpha^3} \left[ (1 - 4\alpha - 3\alpha^2) - 2(1 - 4\alpha + \alpha^2 \frac{m_Q^2}{M^2}) \right] e^{-m_Q^2/\alpha M^2} \]

\[ - \frac{\langle g^3 G^3 \rangle}{3 \cdot 2\pi^4} m_s m_Q \int_0^1 d\alpha \left( 1 - \frac{m_Q^2}{\alpha^2 M^2} \right) e^{-m_Q^2/\alpha M^2} \]

\[ \text{for } \Omega_Q \text{ baryons. The lower limit of integration is given by } \Lambda = m_Q^2/s. \]

**III. NUMERICAL RESULTS AND DISCUSSIONS**

In the numerical analysis, the input values are taken as
with

\[ m_s = 0.13 \text{ GeV}, \langle qq \rangle = -(0.23)^3 \text{ GeV}^3, \langle ss \rangle = 0.8 \langle qq \rangle, \langle g\bar{q}\sigma \cdot Gq \rangle = m_Q^2 \langle qq \rangle, m_Q^2 = 0.8 \text{ GeV}^2, \langle g^2 G^2 \rangle = 0.5 \text{ GeV}^4, \text{ and } \langle g^3 G^3 \rangle = 0.045 \text{ GeV}^6. \]

The proper ranges of the thresholds can be determined, to
which the stability of the Borel curves should not be sensitive. The Borel windows are fixed in this
way: the lower limit constraint for
is obtained from the condition that the perturbative contribution
should be larger than the condensate contributions while the upper one is got by the pole contribution
larger than the continuum contribution. Giving an illustration, the comparison between pole and
continuum contributions from Eq. (12) for \( \sqrt{s_0} = 3.4 \text{ GeV} \) for \( \Omega_c \) is shown in Fig. 1(a), and its OPE
convergence by comparing the different contributions is shown in Fig. 1(b). The analysis for the others
has similarly been done, but the corresponding figures are not listed to keep the paper from being too
lengthy. Accordingly, the threshold are taken as those values exhibited in corresponding figures, and Borel
windows are \( M^2 = 1.5 \sim 3.0 \text{ GeV}^2 \) and \( 4.5 \sim 6.0 \text{ GeV}^2 \) for charmed and bottom baryons, respectively.

The Borel curves for the dependence on \( M^2 \) of the heavy baryon masses are shown in Figs. 2-6. In Table
we present our results for the masses of the heavy baryons and compare with experimental data and
other theoretical approaches. In the numerical evaluation, the results gained from the two sum rules have
been averaged to decrease the systematic errors. It is worth noting that the uncertainty in the results are
merely owing to Borel windows, not involving the ones rooting in the variation of the quark masses and
QCD parameters. In conclusion, we have employed the QCD sum rule approach to calculate the masses of charmed and bottom baryons including the contributions of the operators up to dimension six in OPE. The final results extracted from the sum rules are well compatible with the existing experimental data. Predictions to the spectroscopy of the unobserved bottom baryons are also presented. Although there has been enormous progress in experimental aspects and many theoretical works have been done for the heavy baryons, plenty of problems are desiderated to resolve. It is worth elucidating that most of the $J^P$ quantum numbers for the heavy baryons have not been determined experimentally, but are assigned by PDG on the basis of quark model predictions, which are waiting for experimental identification, especially for several higher excited states. More data on bottom baryons are earnestly expected by putting into operation the Large Hadron Collider, which may supply a gap of experimental data in the near future. From the theoretical point of view, it might be meaningful to reanalyze the QCD sum rules in full theory for the heavy baryons, taking into account the QCD $O(\alpha_s)$ corrections to improve the results, which are not involved in this work.

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![Graph](image1.png)

**FIG. 1:** In (a), the dashed line shows the relative pole contribution (the pole contribution divided by the total, pole plus continuum contribution) and the solid line shows the relative continuum contribution from Eq. (12) for $\sqrt{s_0} = 3.4$ GeV for $\Omega_c$. Its OPE convergence is shown in (b) by comparing the perturbative, quark condensate, two-gluon condensate, three-gluon condensate, and mixed condensate contributions.
FIG. 2: The dependence on $M^2$ for the masses of $\Lambda_{1c}$, $\Lambda_{1c}^*$, $\Lambda_{1b}$, and $\Lambda_{1b}^*$. The continuum thresholds are orderly taken as $\sqrt{s_0} = 3.1 \sim 3.3$ GeV, $\sqrt{s_0} = 3.2 \sim 3.4$ GeV, $\sqrt{s_0} = 6.5 \sim 6.7$ GeV, and $\sqrt{s_0} = 6.6 \sim 6.8$ GeV. (a) and (c) are from the sum rule [12], (b) and (d) from the sum rule [13].

FIG. 3: The dependence on $M^2$ for the masses of $\Xi_c$ and $\Xi_b$. The continuum thresholds are taken as $\sqrt{s_0} = 3.0 \sim 3.2$ GeV, and $\sqrt{s_0} = 6.3 \sim 6.5$ GeV, respectively. (a) are from the sum rule [12], (b) from the sum rule [13].

FIG. 4: The dependence on $M^2$ for the masses of $\Xi_c^*$, $\Xi_b^*$, $\Xi_c^*$, and $\Xi_b^*$. The continuum thresholds are orderly taken as $\sqrt{s_0} = 3.1 \sim 3.3$ GeV, $\sqrt{s_0} = 3.2 \sim 3.4$ GeV, $\sqrt{s_0} = 6.6 \sim 6.8$ GeV, and $\sqrt{s_0} = 6.6 \sim 6.8$ GeV. (a) and (c) are from the sum rule [12], (b) and (d) from the sum rule [13].
FIG. 5: The dependence on $M^2$ for the masses of $\Xi_{1c}$, $\Xi_{1c}^*$, $\Xi_{1b}$, and $\Xi_{1b}^*$. The continuum thresholds are orderly taken as $\sqrt{s_0} = 3.4 \sim 3.6$ GeV, $\sqrt{s_0} = 3.5 \sim 3.7$ GeV, $\sqrt{s_0} = 6.7 \sim 6.9$ GeV, and $\sqrt{s_0} = 6.8 \sim 7.0$ GeV. (a) and (c) are from the sum rule (12), (b) and (d) from the sum rule (13).

FIG. 6: The dependence on $M^2$ for the masses of $\Omega_c$, $\Omega_{c}^*$, $\Omega_{b}$, and $\Omega_{b}^*$. The continuum thresholds are orderly taken as $\sqrt{s_0} = 3.3 \sim 3.5$ GeV, $\sqrt{s_0} = 3.4 \sim 3.6$ GeV, $\sqrt{s_0} = 6.6 \sim 6.8$ GeV, and $\sqrt{s_0} = 6.7 \sim 6.9$ GeV. (a) and (c) are from the sum rule (12), (b) and (d) from the sum rule (13).

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TABLE II: The mass spectra of charmed and bottom baryons (mass in unit of MeV except for “Our works”)

| Baryon | J^P | S_L | J^L | Experiment | Our works (GeV) | Refs. | [8] | Ref. [10] | Ref. [11] | Ref. [21] |
|--------|-----|-----|-----|------------|-----------------|-------|-----|----------|----------|----------|
| Λ_b    | 1/2 | 0   | 0   | 5619.7 ± 1.2 | 5.69 ± 0.13     | 5622  | 5620 | 5672     | 5637±0.68 |
| Λ_b^+ | 1/2 | 0   | 0   | 2286.46 ± 0.14| 2.31 ± 0.19     | 2297  | 2285 | 2290     | 2271±0.57 |
| Σ_b    | 1/2 | 1   | 0   | 5807.8 ± 2.0 | 5.73 ± 0.21     | 5805  | 5820 | 5847     | 5809±0.62 |
| Σ_b^+ | 1/2 | 1   | 1   | 5815.2 ± 1.0 | 5.81 ± 0.19     | 5834  | 5850 | 5871     | 5835±0.62 |
| Σ_b^-  | 1/2 | 1   | 1   | 5836.4 ± 2.0 | 5.81 ± 0.19     | 5834  | 5850 | 5871     | 5835±0.62 |
| Σ_c(2455)^0 | 1/2 | 1   | 0   | 2453.76 ± 0.18 | 2.40 ± 0.31 | 2439  | 2453 | 2452     | 2411±0.93 |
| Σ_c(2520)^0 | 1/2 | 1   | 0   | 2518.0 ± 0.5  | 2.56 ± 0.24     | 2518  | 2520 | 2538     | 2534±0.96 |
| Λ_c(2593)^+ | 1/2 | 0   | 1   | 2595.4 ± 0.6  | 2.53 ± 0.22     | 2598  |      |          |          |
| Λ_c(2625)^+ | 1/2 | 0   | 1   | 2628.1 ± 0.6  | 2.58 ± 0.24     | 2628  |      |          |          |
| Ξ_b^+  | 1/2 | 1   | 1   | 2578.0 ± 2.9  | 2.50 ± 0.29     | 2578  | 2580 | 2599     | 2598±0.97 |
| Ξ_b(2645)^0 | 1/2 | 1   | 1   | 2641.6 ± 1.2  | 2.64 ± 0.22     | 2654  | 2650 | 2680     | 2634±0.94 |
| Ξ_c(2790)^0 | 1/2 | 0   | 1   | 2791.9 ± 3.3  | 2.65 ± 0.27     | 2801  |      |          |          |
| Ξ_c(2815)^0 | 1/2 | 0   | 1   | 2818.2 ± 2.1  | 2.69 ± 0.29     | 2820  |      |          |          |
| Ω_b^0  | 1/2 | 1   | 0   | 2697.5 ± 2.6  | 2.62 ± 0.29     | 2698  | 2710 | 2678     | 2664±0.102 |
| Ω_b(2768)^0 | 1/2 | 1   | 0   | 2768.3 ± 3.0  | 2.74 ± 0.23     | 2768  | 2770 | 2752     | 2790±0.105 |
| Σ_b    | 1/2 | 0   | 1   | 5.85 ± 0.15   | 5.90 ± 0.16     | 5947  |      |          |          |
| Σ_b^+  | 1/2 | 0   | 1   | 5.87 ± 0.20   | 5.94 ± 0.17     | 5963  | 5980 | 5959     | 5929±0.79 |
| Σ_b^-  | 1/2 | 0   | 1   | 5.95 ± 0.16   | 5.99 ± 0.17     | 6119  |      |          |          |
| Ξ_b^+  | 1/2 | 0   | 1   | 6.00 ± 0.16   | 6090  | 6090 | 6060     | 6063±0.81 |
| Ξ_b^-  | 1/2 | 0   | 1   | 6.00 ± 0.16   | 6090  | 6090 | 6060     | 6063±0.81 |

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