Optical solitons in a trinal-channel inverted nonlinear photonic crystal

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Inverted nonlinear photonic crystals are the crystals featuring competition between linear and nonlinear lattices, with minima of the linear potential coinciding with maxima of the nonlinear pseudopotential, and vice versa. In this paper, a new type of inverted nonlinear photonic crystal is constructed by juxtaposing three kinds of channels into a period. These three channels are a purely linear channel, a saturable self-focusing nonlinear channel, and a saturable self-defocusing nonlinear channel. This optical device is assumed to be fabricated by means of SU-8 polymer material periodically doped with two types of active dyes. The nonlinear propagation of a light field inside this device (passing along the channel) can be described by a nonlinear Schrödinger equation. Stable multi-peak fundamental and dipole solitons are found in the first gap of the system. These solitons sufficiently exhibit some interesting digital properties, which may have potential in optical communications.

Keywords: active materials; trinal-channel; linear and nonlinear potentials; inverted nonlinear photonic crystal; multi-peak soliton

1. Introduction

The dynamics of discrete systems is one of research hotspots in optics. Arrays or lattices of evanescently coupled waveguides are prime examples of discrete systems in which discrete optical dynamics can be observed and investigated. Optical fields propagating in such discrete systems exhibit novel phenomena. However, traditional coupled arrays or lattices, such as AlGaAs waveguide arrays, periodically poled lithium niobate waveguides, photorefractive crystals, and liquid crystals via optical induction, are almost composed of by passive materials. It is well known that active materials (AMs) have plenty of optical characteristics, such as the strong dispersion, the complex dielectric constant, and the strong variation of the dispersion relation near the resonance. Therefore, waveguide arrays made by the active materials show advantages of freedom to manipulate, and offer low threshold, in comparison with their passive counterparts. These properties qualify the active waveguides to have many potential optical applications, such as soliton or bullet generation, optical switching, optical...
tical storage, PT-symmetry formation, and nonlinear optical frequency conversion.

Recently, a 2D imaginary part photonic crystal (IPPhC) (or resonantly absorbing waveguide arrays, RAWA) has been realized by the technique of multi-beam interference holography lithography and back-filling, which allows the active materials to be doped into a homogeneous background with spatial structure. The active material of this IPPhC is chosen to be the Rhodamine B (RhB, a dye featuring saturable absorption), which can be doped in a homogeneous SU-8 background to form the IPPhC. In the low power region, where the nonlinearity of the active material is not excited, the IPPhC shows a significant efficiency on color-separation, which may have a potential to construct a new kind of color filter matrix in display technology and industry. In the high power region, where the nonlinearity of the active material cannot be neglected, the nonlinear localized modes, i.e. solitons, show some interesting power-dependent properties. The centers of the solitons can switch between the linear and nonlinear stripe as the power is increasing.

In this paper, we introduce a kind of one-dimensional discrete optical system, which is developed from the color filter matrix in Ref. and consider it within the high power region. This system is assumed to be built on the basis of the SU-8 doped with two kinds of active materials (alias, two kinds of dyes) featuring saturable absorption peak in different wavelength respectively. In Section II, a description to this model is presented in detail, and a nonlinear Schrödinger equation, which describe the nonlinear propagation of the light field though the system, is drawn in the paraxial approximation. In section III, a numerical simulation is carried out. Multi-peak fundamental and dipole solitons, which show many interesting digital properties, are found in the first band gap of the system. And the paper is concluded in Section IV.

Fig. 1. The diagram of 1-D optical device. The green and red areas are doped with AMs, and the blue ones are the background media. $R_1(x)$ and $R_2(x)$ are the structure functions of the two AMs. $d_1 \rightarrow d_3$ are the widths of the channels A-C, respectively.
2. The model

The structure of our device is displayed in Fig. 1. The propagating light field through the device can be described (in the paraxial approximation) with the underlying equation:

\[ i \frac{\partial E}{\partial z} = -\frac{1}{2k} \frac{\partial^2 E}{\partial x^2} - \sum_{j=1}^{2} \left( k_0 \Delta n_j + \frac{i \alpha_j}{2} \right) R_j(x) E \]

(2.1)

In Eq. (2.1), \( k = k_0 n \), where \( n \) is the refractive index of the background, \( k_0 \) is the vacuum vector. \( \alpha_j \) (\( j = 1, 2 \)) are the absorption coefficients of the two AMs, respectively. \( \Delta n_j \) (\( j = 1, 2 \)) are the corresponding refractive index difference (RID) induced by the two absorption coefficients, respectively. The relation between the absorption coefficient and the correspondent RID is assumed to be satisfied by the Kramers-Kronig (K-K) relationship. \( R_j(x) \) (\( j = 1, 2 \)) are the two structure functions of the AMs. \( R_j(x) = 1 \) describes the areas doped with AM \( j \). In Fig. 1, AM1 is doped into the green stripes while AM2 is doped into the red ones, and blue stripes in the figure represent the background (alias, pure SU-8). These three channels are juxtaposed into one period.

We assume that the absorption coefficients of the two AMs have a Lorentzian shape, which are shown in FIG. 2(a) (We assume the absorption centers of them are \( \lambda = 480 \text{ nm} \) and \( 560 \text{ nm} \), respectively). According to the K-K relation, their RIDs are figured out and shown in FIG. 2(b).

![Fig. 2. (Color online) (a) The absorption curves of the two AMs. (b) The refractive index difference curves of two AMs. The arrow is the wavelength of the light we chosen for incidence the device.](image)

In theory, there may exist such a possibility: at the region between the two absorption peaks of the AMs (See in Fig. 2(b) i.e. \( \lambda = 520 \text{ nm} \)), we obtain the RID of the two AMs higher and lower the background, respectively. By neglecting the absorption coefficients at this wavelength, Eq. (2.2) can be simplified as:
iE_z = -\frac{1}{2k} E_{xx} + \sum_{j=1}^{2} \frac{K_j(x)}{1 + |E|^2} E \tag{2.2}

where \( K_j(x) = -k_0 \Delta n_j(\lambda) R_j(x) \) \( (j = 1, 2) \).

Figure 2(b) shows that, for AM1, \( \Delta n_1(\lambda) > 0 \), while for AM2, \( \Delta n_2(\lambda) < 0 \). For convenience, if we further assume \( |\Delta n_2(\lambda)| = |\Delta n_2(\lambda)| = \kappa \), Eq. (2.2) can be simplified as the following dimensionless nonlinear Schrödinger equation:

\[ iU_z = -\frac{1}{2} U_{xx} - \frac{V_1(x)}{1 + |U|^2} U + \frac{V_2(x)}{1 + |U|^2} U \tag{2.3} \]

where \( U \) is a dimensionless light field; \( V_j(x) = V_0 R_j(x) \) \( (j = 1, 2) \) with \( V_0 = k_0 \kappa \); \( k \) is removed by rescaling of \( x \). From Fig. 1 we know that the light field will experience saturable self-focusing (SF) in channel C (Red color stripe) and saturable self-defocusing (SDF) in channel B (Green color stripe), while in channel A (Blue color stripe), the field is linear propagation.

![Fig. 3. Total linear potential of Eq. (2.3)](image)

If we take off the saturable nonlinearity from Eq. (2.3), the total linear potential \( V(x) = V_1(x) - V_2(x) \) converts into a one-dimensional periodical potential with a fine structure. By considering the nonlinearity, the modulation to the nonlinear potential is exactly equal to \(-V(x)\), which forms a combination of competing \( \pi \)-out-of-phase juxtaposed linear potential and nonlinear potential. The maxima of the refractive index coincides with the maxima of the local strength of the self-defocusing nonlinearity (the bottom of the linear potential coincide with the top of the nonlinear potential), while the minima of the refractive index coincides with the maxima of the self-focusing nonlinearity (the top of the linear potential coincide with the bottom of the nonlinear potential). The medium of this type is naturally called an inverted nonlinear photonic crystals (INPCs). However, previous INPCs mainly contain one type of nonlinearity (self-focusing or defocusing); in our model, the mixture of two types of nonlinearity is considered.

In the following section, we will carry out the numerical simulations on Eq. (2.3) and find that this system can afford the existence of a rich variety of multi-peak solitons. The numbers of the peaks in the soliton solutions show interesting digital properties, which may have potential in developing new types of optical devices.
3. Numerical Results

We assume that the stationary solutions of Eq. (2.3) can be described with
\[ U(x, z) = u(x)e^{-i\mu z}, \]
where \(-\mu\) is the propagation constant of the soliton. The stability of the stationary solitons can be numerically identified by the computation of eigenvalues for small perturbations or direct simulation. For the eigenvalue problem, the perturbed solution is given as
\[ U = (u + we^{i\lambda z} + ve^{-i\lambda^* z})e^{-i\mu z}. \]
Substitution of this ansatz into Eq. (2.3) and linearization can lead to the eigenvalue problem
\[
\begin{pmatrix}
\frac{1}{2} \frac{d^2}{dx^2} + \mu - \sum_i \frac{V_i(x)}{F}\left(1 - \frac{u^2}{F}\right) \\
\sum_i \frac{V_i(x)}{F}u^2
\end{pmatrix} \begin{pmatrix} w \\ v \end{pmatrix} = \lambda \begin{pmatrix} w \\ v \end{pmatrix}
\]
(3.1)
where \( F = 1 + u^2 \) and \( i = 1, 2 \). The solution \( u \) is stable if all the eigenvalues of Eq. (3.1) are real.

We select the Newton-Jocobi method to figure out the soliton solutions. In the numerical simulation, we choose the modulation depths of \( V_1(x) \) and \( V_2(x) \) to be \( V_0 = 0.3 \), and the widths of the three stripes to be \( d_1 = d_2 = d_3 = 2.25 \). We find that valid soliton solutions seem mainly converge in the vicinity of zero of the propagation constant \(-\mu\). Some multi-peak fundamental and dipole solitons can exist stably.

The fundamental solitons with the number of peaks equal to 1, 2, 3, 4, 5 and 7 are shown in Fig. 4.

The stabilities of these soliton solutions are identified by the computation of eigenvalues for small perturbations and direct simulation. The numerical results show that all of them are stable. Typical example (5 peaks case) of such analysis is shown in Fig. 5.

Multi-peak dipole solitons can also be found. Figure 4 displays some multi-peak dipole solitons with 1+1 peaks, 1+2 peaks, 1+3 peaks, 1+6 peaks, 2+2 peaks and 3+5 peaks. Typical example of the stability analysis is shown in Fig. 6.

All of above solitons appear in the region of \(-0.06 < \mu < 0.01\), which is very close to the zero of the propagation constant. According to Fig. 8, this region belongs to the first bank gap of the periodical structure. For the solution of multi-peak fundamental solitons and the multi-peak dipole solitons with an anti-symmetric profile, it has been report that they can be found in the nonlinear photonic crystals with self-defocusing nonlinearity. And the multi-peak dipole solitons with an asymmetry profile only appear in the surface of the lattice. However, the asymmetry dipole solitons (ie. Fig 6(b), 6(d) and 6(f) inside the lattice are observed for the first time though our model. Moreover, the peaks number of the solitons features interesting digital property, which may have potential in optical communication.
Fig. 4. (Color online) The amplitudes of some multi-peak fundamental solitons. (a) 1-peak with $\mu = 0.005, P = 3.0453$. (b) 2-peaks with $\mu = 0.00123, P = 8.702$. (c) 3-peaks with $\mu = 0.00535, P = 20.6291$. (d) 4-peaks with $\mu = -0.0085, P = 15.3901$. (e) 5-peaks with $\mu = 0.00238, P = 35.6062$. (f) 7-peaks with $\mu = 0.00239, P = 55.1836$.

4. CONCLUSION

Photonic waves propagating under the combination of linear and nonlinear lattice potentials exhibit a number of dynamics. In this paper, a new type of optical system is built, which juxtaposes three channels (Linear, self-focusing and -defocusing nonlinear) into a period and features the inverted modulation of the linear and nonlinear potentials. The structural inspiration of the medium of this type comes from the recent experimental work in color filter matrix, which use the SU-8 polymer material periodically doped with two types active dyes, which
can feature saturable nonlinearity. This model can be described by a nonlinear schrödinger equation with two types of saturable nonlinear terms. The numerical simulation finds that some multi-peak fundamental and dipole solitons can stable exist in the first band gap of the lattice. These solitons show some interesting digital properties, which may have potential in optical communication. Furthermore, using properly designed holographic patterns onto the SU-8 polymer, various complex spatial structures of the distribution of the dopant concentration can be photoinduced in the 2D geometry, such as quasicrystals, honeycomb lattices, defect lattices, and ring lattices etc. Such 2D structures may have their own spectra of potential applications.

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Fig. 6. (Color online) The amplitudes of some multi-peak dipole solitons. (a) $1+1$ peaks with $\mu = 0.00133$, $P = 5.7490$. (b) $1+2$ peaks with $\mu = -0.0137$, $P = 7.1972$. (c) $1+3$ peaks with $\mu = 0.00228$, $P = 20.2986$. (d) $1+6$ peaks with $\mu = 0.00256$, $P = 49.4150$. (e) $2+2$ peaks with $\mu = 0.00232$, $P = 18.6655$. (f) $3+5$ peaks with $\mu = 0.0053388$, $P = 116.8560$.

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Fig. 7. (Color online) The stability of the soliton in Fig. 6(b). (a) The evolution of the soliton perturbed by 1% noises. (b) The growing rate of the soliton.

Fig. 8. Band structure of the device.

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