Testing the theory of QGP-induced energy loss at RHIC and the LHC

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Abstract

We compare an analytic model of jet quenching, based on the GLV non-Abelian energy loss formalism, to numerical results for the centrality dependent suppression of hadron cross sections in Au + Au and Cu + Cu collisions at RHIC. Simulations of neutral pion quenching versus the size of the colliding nuclear system are presented to high transverse momentum, \( p_T \). At low and moderate \( p_T \), we study the contribution of medium-induced gluon bremsstrahlung to single inclusive hadron production. In Pb + Pb collisions at the LHC, the redistribution of the lost energy is shown to play a critical role in yielding nuclear suppression that does not violate the participant scaling limit.

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1. Introduction

Strong suppression of single inclusive pions and charged hadrons at large transverse momentum, as large as \( p_T = 20 \text{ GeV} \) [1,2], is arguably one of the most fascinating phenomena from the heavy ion program at the Relativistic Heavy Ion Collider (RHIC). It signifies the transition from soft, collective, and strongly model-dependent physics to the high-\( p_T \) or high-\( E_T \) production of particles and jets that is well understood in terms of the perturbative quantum chromodynamics (pQCD) factorization approach. Calibrated hard probes can thus be used to sample the properties of the medium created in collisions of heavy nuclei [3]. With capabilities to detect \( p_T \sim 50 \text{ GeV} \) pions and \( E_T \sim 500 \text{ GeV} \) jets with good statistics, experiments at the Large Hadron Collider (LHC) will be able to critically test such perturbative calculations of hadron and jet modification in the quark–gluon plasma (QGP) at a new energy frontier.

Following the discovery [4] of jet quenching in Au + Au collisions and its verification through \( d + Au \) measurements [5] at RHIC, heavy ion theory has emphasized the need for a systematic study of the energy and system size dependence of leading-particle attenuation [3,6]. Previous measurements have been limited to transverse momenta \( p_T \lesssim 10 \text{ GeV} \) and Au + Au collisions at RHIC [7]. More recent results up to \( p_T \sim 20 \text{ GeV} \) [1,2] in both Au + Au and Cu + Cu reactions were shown to be compatible with several theoretical estimates, emphasizing the final-state QGP-induced suppression of jets [6,8,9]. In this Letter, we compare for the first time analytic [3] and numerical [6] models of jet absorption in order to establish the role of sub-leading effects, such as the running of the strong coupling constant \( \alpha_s \) with the Debye screening scale \( \mu \sim gT \) in the plasma. We present our predictions for \( \pi^0 \) quenching versus centrality, \( p_T \) and \( \sqrt{s} \), based on the Gyulassy–Levai–Vitev (GLV) approach to the medium-induced non-Abelian energy loss [10]. We note that redistribution of the lost energy in small- and moderate-\( p_T \) hadrons plays a significant role in the transition from high-\( p_T \) suppression to low-\( p_T \) enhancement of the away-side two particle correlations [11]. In this Letter we give results for single inclusive pions and identify the range of transverse momenta at RHIC and the LHC where the gluon feedback is important.

In Section 2 we present a simple analytic model for the system size, \( N_{\text{part}} \), dependence of jet quenching in nucleus–nucleus collisions. Section 3 contains select intermediate results from the GLV theory, including the evaluation of the mean gluon number \( \langle N_g \rangle \) and fractional energy loss \( \langle \Delta E / E \rangle \). The derivation of the radiative gluon contribution to small- and moderate-
hadron production in the region of 5 GeV at RHIC. In a finite size dependence of jet quenching, we use the approximate GLV formalism. The first one associates the hard parton production cross section with the kinematic modification of the momentum fraction $z = p_h^\perp / p_T^\perp$ in the fragmentation function $D_{h/c}(z)$, leaving the hard parton production cross section $d\sigma^c / dy d^2 p_T$ unmodified. The second approach reduces the jet cross section in the presence of the medium but leaves $D_{h/c}(z)$ unaltered. It can conveniently be implemented in an analytic model of QGP-induced leading-hadron suppression. We take the underlying parton production cross section to be of power law type,

$$d\sigma^c / dy d^2 p_T = \frac{A}{(p_0 + p_T^\perp)^n} \approx \frac{A}{p_T^\perp}, \quad \text{if } p_T^\perp \gg p_0,$$

where $n = n(y, p_T^\perp, \sqrt{s})$ and $p_0 \sim 2.5$ GeV at RHIC. In a finite $p_T^\perp$ range, fixed rapidity $y$ and center of mass energy $\sqrt{s}$, a constant $n = n(y, p_T^\perp, \sqrt{s})$ is a good approximation. At RHIC, in the region of $5 < p_T^\perp < 10$ GeV, the spectra scale roughly as $n_q = 7$, $n_g = 8.4$. Fragmentation functions are convoluted with the partonic cross section as follows:

$$d\sigma^h / dy d^2 p_T = \sum_c \int_{z_{\min}}^1 dz \frac{d\sigma^c(p_T^\perp / z)}{dy d^2 p_T^\perp} \frac{1}{z^2} D_{h/c}(z)$$

$$\approx \sum_c \frac{1}{p_T^\perp} \int_{z_{\min}}^1 dz \frac{d\sigma^c(p_T^\perp / z)}{dy d^2 p_T^\perp} \frac{1}{z^2} D_{h/c}(z)$$

In Eq. (2), $z_{\min} = p_T^\perp / p_T^\perp_{\max}$ and the subsequent approximation is most reliable when $z_{\min} \ll 1$. It should be noted that $z$ will also depend on the partonic and hadronic species.

The second input to the analytic model comes from the radiative energy loss formalism [10]. To understand the system size dependence of jet quenching, we use the approximate GLV formula which relates $\Delta E$ to the size and the soft parton rapidity density of the medium. For $(1 + 1)$D Bjorken expansion, in the limit of large parton energy $2E / \mu^2 L \gg 1$, we find [12]

$$\Delta E \approx \frac{9}{4} C_R \pi a_0^3 A_\perp \frac{dN^g}{dy} \frac{2E}{\mu^2 L} + \ldots$$

In Eq. (3), $L$ is the jet path length in the medium and $A_\perp$ is the transverse area. $C_R = 4/3$ (3) for quarks (gluons), respectively, is the quadratic Casimir in the fundamental (adjoint) representation of SU(3). Numerical simulations of $\Delta E / E$ clearly indicate a weaker dependence of the fractional energy loss on the jet energy than given in Eq. (3).

The key to understanding the dependence of jet quenching on the heavy ion species and centrality is the effective atomic mass number, $A_{\text{eff}}$, or the number of participants, $N_{\text{part}}$, dependence of the characteristic plasma parameters in Eq. (3) [3],

$$dN^g / dy \propto dN^h / dy \propto A_{\text{eff}} \propto N_{\text{part}}^1,$$  

Therefore, the fractional energy loss scales approximately as

$$\epsilon = \Delta E / E \propto A_{\text{eff}}^2 / N_{\text{part}}^3,$$  

up to logarithmic corrections from Eq. (3). If a parton loses this momentum fraction $\epsilon$ during its propagation in the medium to escape with momentum $p_T^\text{quench}$, immediately after the hard collision $p_T^\text{pp} = p_T^\text{quench} / (1 - \epsilon)$. Noting the additional Jacobian $|d^2 p_T^\text{quench} / d^2 p_T^\perp| = (1 - \epsilon)^2$, we find for the quenched hadronic spectrum per elementary $NN$ collision

$$dN^h / dy d^2 p_T = \sum_c \frac{d\sigma^c(p_T^\perp / (1 - \epsilon)(z))}{dy d^2 p_T^\perp} \frac{1}{(1 - \epsilon)^2(z)} D_{h/c}(z)$$

$$\approx (1 - \epsilon_{\text{eff}})^{-n} \sum \frac{A}{p_T^\perp} \frac{D_{h/c}(z)_{(n-2)}}{z_{n-2}^2}.$$  

In Eq. (7), $\epsilon_{\text{eff}}$ is the average over all parton species and accounts for the color charge, geometry and multi-gluon fluctuations. From this result we can easily derive the system size dependence of the nuclear modification factor:

$$R_{AA} = \frac{\langle dN_{AA}^h / dy d^2 p_T \rangle_{\text{exp.}}}{\langle dN_{AA}^h / dy d^2 p_T \rangle_{\text{th.}}},$$

$$k = (n / n_{\text{part}}^2) k_{\text{exp}} / (n / n_{\text{part}}^2) k_{\text{th.}}$$

In Eq. (8) $k / (n - 2)$ is the proportionality coefficient in Eq. (6) which depends on the microscopic properties of the medium but not on its size. With $n \gg 1$ and experimentally measured and theoretically calculated suppression, $\sim 5$ fold in central Au + Au collisions at RHIC, $k_{\text{exp}} = N_{\text{part}}^2 / k_{\text{th.}}$ is small. We thus predict that the logarithm of nuclear suppression, Eq. (8), has simple power law dependence on the system size,

$$\ln R_{AA} = -k N_{\text{part}}^2 / (2n - 2).$$  

where the leading correction goes as $k^2 N_{\text{part}}^2 / (2n - 2).$
We focus on the most central nuclear collisions and use impact parameters $b = 1 - 3$ fm depending on the atomic mass $A$. An optical Glauber model calculation is used to evaluate $N_{\text{part}}^{2/3}$ with $\sigma_{pp}^{0} = 42$ mb and a Wood–Saxon nuclear density with results given in Table 1. The analytic prediction for the system size dependence of jet quenching is shown in Fig. 1. It is fixed by the magnitude of the suppression established in central Au + Au collisions [7] and consistent with existing simulations [12]. From this analysis, we expect a factor $\sim 2$ suppression in central Cu + Cu collisions. Comparison to the preliminary PHENIX $p_T > 7$ GeV data [1] and STAR $p_T > 6$ GeV data [2] is also shown in the bottom panel of Fig. 1 with good agreement within the experimental uncertainties. Dashed and solid lines illustrate the difference between Eqs. (8) and (9) when normalized to the same suppression in central Au + Au collisions.

The main advantage of the GLV analytic model is the ability to provide guidance on the magnitude of the QGP-induced jet quenching versus centrality and address a large body of experimental data. Its limitations include a fixed coupling constant $\alpha_s$, implementation of only the mean, though suitably reduced to reflect the effects of multi-gluon fluctuations, energy loss $\Delta E$ and the inability to incorporate additional nuclear effects, such as the Cronin multiple scattering [5] and nuclear shadowing [15]. It also relies on a reference numerical calculation in central $A + A$ collisions in the same $p_T$ and $y$ range as well as $\sqrt{s}$ [12]. The deviation of $dN^S/dy$ from the exact participant scaling in Eq. (4) may lead to less quenching and improved agreement with the data in peripheral reactions, but is here neglected.

### 3. Numerical evaluation of the QGP-induced energy loss

The solution for the differential in energy, $\omega$, and transverse momentum, $k$, spectrum of medium induced gluon radiation has been obtained order by order in the correlations between the multiple scattering centers in nuclear matter using the reaction operator approach [10]:

$$\omega \frac{dN_g}{d\omega d^2k} \approx \sum_{n=1}^{\infty} C_Rg_{\omega} \prod_{i=1}^{n-1} \int_{0}^{\lambda g(i)} d\Delta z_i \left[ \frac{1}{\lambda g(i)} \frac{d2q_i}{d^2q_i} \right]$$

$$\times \left[ \left( \frac{2C_{1,\ldots,n}}{\sum_{m=1}^{n} B_{m+1,\ldots,n}(m,\ldots,n)} \right) \left( \frac{\sum_{k=2}^{m} \cos \left( \sum_{k=1}^{m} \alpha_{i,k} \Delta z_k \right) - \cos \left( \sum_{k=1}^{m} \alpha_{i,k} \Delta z_k \right) \right) \right].$$

(10)

Here $q_i$ are the momentum transfers from the medium, distributed according to a normalized elastic differential cross section $\sigma_{el}(i)^{-1} d\sigma_{el}(i)/d^2q_i$, and $\Delta z_k = z_k - z_{k-1}$ are the separations of the subsequent scattering centers. In Eq. (10), the color current propagators and inverse formation times are denoted by

$$C_{m,\ldots,n} = \frac{1}{2} \nabla_k \ln (k - q_m - \cdots - q_n)^2,$$

$$B_{m+1,\ldots,n}(m,\ldots,n) = C_{(m+1,\ldots,n)} - C_{(m,\ldots,n)}.$$

$$\omega_{i,m,\ldots,n}(k) = \frac{(k - q_m - \cdots - q_n)^2}{2\omega}.$$  

(11)

### Table 1

Summary of the relevant quenching parameter $N_{\text{part}}^{2/3}$ (rounded) at a fixed impact parameter $b$ for select heavy ion species

| Species | $^9\text{Be}$ | $^{16}\text{O}$ | $^{28}\text{Si}$ | $^{32}\text{S}$ | $^{56}\text{Fe}$ | $^{64}\text{Cu}$ | $^{197}\text{Au}$ | $^{208}\text{Pb}$ | $^{238}\text{U}$ |
|---------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| $b$ [fm] | 1 | 1 | 1.5 | 1.5 | 2 | 2 | 3 | 3 | 3 |
| $N_{\text{part}}^{2/3}$ | 5 | 8 | 12 | 14 | 20 | 22 | 48 | 50 | 55 |
Numerical results have been obtained to third order in the opacity, \((L/\lambda_g)\), with all correlations of up to four scattering centers, including the initial hard interaction. To speed up the evaluation of the squared amplitudes, the oscillating quantum coherence phases between the points of interaction were converted to Lorentzians using an exponentially falling geometry with no sharp edges. The requirement that the mean locations \(z_k, k = 1, \ldots, n\) of \(n\) scatters are the same for an uniform soft parton distribution fixes the parameter \(L_e\) of such geometry [10]:

\[
\langle z_0 - z_k \rangle = k \frac{L}{n + 1} \quad \Rightarrow \quad L_e = \frac{L}{n + 1}.
\] (12)

Bjorken \((1 + 1)\)D expansion of the plasma is accounted for as follows:

\[
\rho(z_k) = \rho(z_0)\frac{z_0}{z_k},
\mu(z_k) = \mu(z_0)\left(\frac{z_0}{z_k}\right)^{1/3},
\lambda_g(z_k) = \lambda_g(z_0)\left(\frac{z_0}{z_k}\right)^{1/3},
\] (13)

and the kinematic constraints,

\[
\mu/Q \leq \kappa^+ / E^+ \approx \omega / E \leq 1,
\mu \leq |\kappa| \leq \sqrt{Q^2 \min(x, 1 - x)},
\] (14)

have been incorporated for consistency with our previous work [12]. It was recently shown that for physical on-shell final-state gluons the medium induced radiation is infrared and collinear safe [11]. This allows relaxation of the \(\mu/Q \leq x\), \(\mu \leq |\kappa|\) constraints in the future, though it should be noted that the Debye screening scale still controls the small-\(k\) and small-\(\omega\) cancellations between the single- and double-Born diagrams in the opacity expansion [10].

To evaluate the effect of multiple gluon emission and arrive at a probabilistic distribution \(\rho(\epsilon)\) for the fractional energy loss \(\epsilon = \Delta E / E = \sum_{i=1}^{\epsilon} \epsilon_i, \epsilon_i = \omega_i / E\), we are motivated by an independent Poisson gluon emission ansatz, but incorporate kinematic constraints [13] in contrast to alternative approaches [14]:

\[
P(\epsilon) = \sum_{n=0}^{\infty} P_n(\epsilon), \quad P_0(\epsilon) = e^{-(N_g)} \delta(\epsilon),
\]

\[
P_n(\epsilon) = \frac{1}{n!} \int_{0}^{\epsilon} d\epsilon' P_{n-1}(\epsilon - \epsilon') \frac{dN_g}{d\epsilon'} (\epsilon' = \omega / E).
\] (15)

We normalize this probability density to unity and Eq. (15) ensures that at every step energy is conserved. As a consequence, for small jet energies and large \(\Delta E / E\) the gluon distribution is distinctly non-Poisson. We evaluate the mean energy loss as follows:

\[
\int_{0}^{1} d\epsilon P(\epsilon) = 1, \quad \int_{0}^{1} d\epsilon \epsilon P(\epsilon) = \left\langle \frac{\Delta E}{E} \right\rangle.
\] (16)

In Ref. [12], we considered jet production following the binary collision density \(T_{AA}(b)\) in central Au + Au reactions at \(\sqrt{s} = 200\) GeV. In an elementary hard interaction inclusive jets are distributed uniformly in azimuth relative to the reaction plane. We calculated \(\langle \Delta E \rangle_{\text{geom}}\), using the line integral, Eq. (10), through the \((1 + 1)D\) Bjorken expanding medium density by correctly weighing the amount of lost energy with the jet production rate. Cylindrical geometry with radius \(L = 6\) fm that gives the same mean energy loss \(\langle \Delta E \rangle\) for uniform initial soft parton rapidity density was then constrained. Using Eqs. (4) and (5) we can determine the effective length \(L\), transverse area \(A_L = \pi L^2\), gluon rapidity density \(dN_g/dy\) and effective atomic mass \(A_{\text{eff}}\) for shadowing applications [15] in interacting heavy ion systems of different size. Table 2 summarizes the parameters used in our calculation of central, mid-central and peripheral Au + Au and Cu + Cu collisions at \(\sqrt{s} = 200\) GeV at RHIC. For central collisions with \(\sqrt{s} = 5.5\) TeV at the LHC, we use \(dN_g/dy = 2000, 3000\) and \(4000\) to test the sensitivity of jet quenching to the QGP properties.

The calculated fractional energy loss and mean gluon number for quark and gluon jets versus their energy for the centralities and densities discussed above are shown in the left and right panels of Fig. 2. Only in the limit \(E_{\text{jet}} \rightarrow \infty, \Delta E / E \rightarrow 0\) does the energy loss for quarks and gluons approach the naive ratio \(\Delta E^q / \Delta E^g = C_A / C_F = 9/4\). For large fractional energy losses this ratio is determined by the \(\Delta E \leq E\) constraint. In this regime, no simple scaling arguments related to the properties of the dense nuclear matter and the color charge are applicable. It should be noted that at high \(E_{\text{jet}}\) the fractional energy loss is not large even at the LHC.

In our calculation, the strong coupling constant \(\alpha_s\) is not used as a free parameter but evaluated at the typical scale \(\mu\) in the elastic scattering cross section \(\sigma_{el}(\epsilon)\) and the gluon mean free path \(\lambda_g(\epsilon) = 1/\sigma_{el}(\epsilon)\). At the radiation vertex, \(\alpha_s(k^2)\) is also sensitive to the increase of the temperature or density of the medium, \(\rho \propto T^3\), with the increase of \(dN_g/dy\) at a fixed transverse area \(A_L\). The effects described here lead to sublinear dependence of the energy loss on \(dN_g/dy\). For the LHC example, given in Fig. 2, this can be a 50% correction for large \(E_{\text{jet}}\) and even more significant at low \(E_{\text{jet}}\) when compared to Eq. (3). Conversely, at low parton densities in peripheral collisions the energy loss will be larger than naively expected.

| Centrality     | 0–10%  | 20–30% | 60–80% |
|----------------|--------|--------|--------|
| \(N_{\text{part}}\) | 328    | 167    | 21     |
| \(dN_g/dy\)     | 800–1175 | 410–600 | 50–75  |
| \(L [\text{fm}]\) | 6      | 4.8    | 2.4    |
| \(A_{\text{eff}}\) | 197    | 99     | 12     |

Estimated \(dN_g/dy\), \(L\) and \(A_{\text{eff}}\) versus \(N_{\text{part}}\) for central, semi-central and peripheral Au + Au collisions (top table) and Cu + Cu collisions (bottom table) at RHIC.
Another important point, seen in the right panel of Fig. 2, is that except for very low jet energies the mean gluon number \(\langle N_g \rangle\) is not small \((\ll 1)\) and the probability of not radiating gluons, \(P_0 = \exp(-\langle N_g \rangle)\), is never large. Full results for \(P(\epsilon)\) were shown in [8,13]. We find that the probability density ansatz yields \(\epsilon\) much more uniformly distributed in the domain \([0, 1]\) and our calculations retain sensitivity to the local slope and the parton species contribution to the differential inclusive hadron production cross section, see Eq. (8).

We finally note that multi-gluon fluctuations, given by Eq. (15), reduce the jet quenching effect relative to the application of the mean \((\Delta E/E)\) shown in Fig. 2 [13]. This can be seen by comparing \(\epsilon_{\text{eff}} \approx 0.2\) in central Au + Au reactions, obtained from Eq. (8), to the fractional energy loss for 10 GeV quark jets, \(\epsilon_{\text{eff}} \ll \langle \Delta E/E \rangle\). To investigate the scaling of energy loss with \(N_{\text{part}}\) we select quark jets of \(E_{\text{jet}} = 10, 20\) and 30 GeV at midrapidity at RHIC and \(N_{\text{part}} = 9, 21, 55, 103, 167\) and 328. These cover fractional energy losses \(0 < \langle \Delta E/E \rangle < 0.4\) with numerical results shown in Fig. 3. Power law fits, also shown, give exponents \(n = 0.60 - 0.63\) that are not very different from the naive \(n = 2/3\) expectation from Eq. (6), which was used in Fig. 1. We conclude that the deviation between the calculated energy loss with kinematic constraints and running strong coupling constants and its asymptotic fixed \(\alpha_s\) analytic behavior is smaller when the variation of the energy loss is associated with a change in the system size rather than a large change in the density of the medium alone.

4. Nuclear effects on inclusive hadron production

The lowest order perturbative QCD cross section for single inclusive hadrons in nucleon–nucleon (NN) reactions, including non-vanishing transverse momentum \(k_{a,b}\) distributions of the incoming partons, is given by

\[
\frac{d\sigma_{\text{NN}}}{dy \, d^2 p_T} = K \sum_{abcd} \int_{x_{a\min} \, y_{b\min}}^{1} \int_{x_{a\, \min}}^{1} \, dx_a \, dx_b \int d^2 k_{a} \, d^2 k_b \\
\times f(k_a) f(k_b) \phi_{a/N}(x_a, \mu_f) \phi_{b/N}(x_b, \mu_f) \\
\times \frac{1}{\pi z_c} \frac{d\sigma_{ab\rightarrow cd}}{dt} \, D_{h/c}(z_c, \mu_f).
\]

Here, \(z_c = p_T / p_T^c\), \(x_{a,b} = p_{a,b}^+/p_{a,b}^\perp\). In our notation, \(\phi_{a,b/N}(x_{a,b}, \mu_f)\) are the parton distribution functions (PDFs), \(D_{h/c}(z_c, \mu_f)\) are the fragmentation functions (FFs) and we have chosen the factorization/fragmentation, and renormalization scales \(\mu_f = \mu_f = p_T^c\). The distribution of non-zero transverse momenta of the incoming partons is parametrized as follows

\[
f(k_{a,b}) = \frac{1}{\pi (k_{a,b}^2)} \exp\left(-k_{a,b}^2 / (k_{a,b}^2)\right).
\]

The physical requirement for hard partonic scattering is ensured by \(k_{a,b} < x_{a,b} \sqrt{s}\) and \(K = 1.5\) is a phenomenological K-factor at RHIC. For further details see [6,8]. Comparison to the PHENIX measurement of \(\pi^\pm\) production in \(\sqrt{s} = 200\) GeV \(p + p\) collisions at RHIC is shown in the insert of Fig. 4.

Nuclear effects can be incorporated in the pQCD formalism, Eq. (17), and fall in two categories: medium-induced kinematic modifications to the perturbative formulas and possibly universal modifications to the PDFs and FFs. An example of the latter are parameterizations of nuclear shadowing [15],...
production. One possibility is that the fraction of the hadrons from imperceptible to study this effect for single inclusive particle production in central Au + Au collisions at $\sqrt{s} = 200$ GeV for medium density $dN_g/dy = 800–1175$ and $T_{AAu} = 23$ mb$^{-1}$. The same calculation for Cu + Cu collisions for medium density $dN_g/dy = 255–370$ and $T_{CuCu} = 4.5$ mb$^{-1}$. The insert shows the cross section for $\pi^0$ production in p + p collisions to LO pQCD. Data is from PHENIX [7].

$$\frac{1}{A} S_{q,g/A}(x, \mu_f) = \left( \frac{Z}{A} S_{q,g/A}(x, \mu_f) \phi_{q,g/p}(x, \mu_f) + \frac{N}{A} S_{q,g/A}(x, \mu_f) \phi_{q,g/n}(x, \mu_f) \right).$$

(19)

In future work, we will combine dynamical calculations of coherent nuclear enhanced power corrections with other elastic and inelastic effects in nuclear matter [15]. Transverse momentum broadening of the incoming partons, leads to enhancement of the differential particle distributions at $p_T \sim$ few GeV [5] and can be accounted for in Eqs. (17), (18) as follows:

$$\langle k^2_{a,b} \rangle = \langle k^2_{a,b} \rangle_{\text{vac}} + \langle k^2_{a,b} \rangle_{\text{med}}.$$  

(20)

Here, $\langle k^2_{a,b} \rangle_{\text{med}} = (2\mu^2 L_{a,b}/\lambda_{a,b})\xi$ and the typical momentum transfers squared, $\mu^2,$ mean free paths, $\lambda_{a,b},$ and parton propagation lengths, $L_{a,b},$ refer to cold nuclear matter [5,6].

In the QGP, final-state energy loss is the dominant effect that alters the single and double inclusive hadron production cross sections [5,6]. Application to the attenuation of leading hadrons as a kinematic modification of the momentum fraction, $z,$ in the FFs $D_{h/c}(z)$ is considered standard [6,8,9]. The redistribution of the lost energy in soft and moderate $p_T$ hadrons was only recently derived in the pQCD approach, first for away-side two particle correlations [11]. The established dramatic transition from the high-$p_T$ factor of four suppression ($R_{AA} \sim 0.25$) to the low-$p_T$ factor of two enhancement ($R_{AA} \sim 2$) makes it imperative to study this effect for single inclusive particle production. One possibility is that the fraction of the hadrons from the bremsstrahlung gluons is negligible or small over the full accessible $p_T$ range. At the other extreme, a very large fraction may compromise the current jet quenching phenomenology, leading to $R_{AA} \sim 1$ at moderate transverse momenta even in dense matter.

With this motivation, we first give results for the modification of inclusive hadron production from final-state radiative energy loss. It can be represented as

$$D_{h/c}(z) \Rightarrow \int_0^{1-\epsilon} d\epsilon P(\epsilon) \frac{1}{1-\epsilon} D_{h/c} \left( \frac{z}{1-\epsilon} \right) + \int_0^1 d\epsilon \frac{dN_g}{d\epsilon}(\epsilon) \frac{1}{\epsilon} D_{h/g} \left( \frac{z}{\epsilon} \right).$$

(21)

Here, $P(\epsilon)$ is calculated from Eqs. (10), (15) and $dN_g/d\epsilon$ is the distribution of the average gluons versus $\epsilon = \omega/E$ so that

$$\int_0^1 d\epsilon \frac{dN_g}{d\epsilon}(\epsilon) = \langle N_g \rangle,$$

$$\int_0^1 d\epsilon \frac{dN_g}{d\epsilon}(\epsilon) \left( \frac{\Delta E}{E} \right) = \langle \Delta E \rangle$$

(22)

It is easy to verify the momentum sum rule for all hadronic fragments from the attenuated jet and the radiative gluons. With appropriate changes of variables:

$$\sum_k \int_0^1 dz \frac{dN_k}{d\epsilon} D_{h/c}(z) \Rightarrow \int_0^1 d\epsilon (1-\epsilon) P(\epsilon) \sum_k \int_0^{1-\epsilon} \frac{dz}{1-\epsilon} \frac{z}{1-\epsilon} D_{h/c} \left( \frac{z}{1-\epsilon} \right)$$

$$+ \int_0^1 d\epsilon \frac{dN_g}{d\epsilon}(\epsilon) \sum_k \int_0^\epsilon \frac{dz}{\epsilon} \frac{z}{\epsilon} D_{h/g} \left( \frac{z}{\epsilon} \right) = 1 - \langle \epsilon \rangle + \langle \epsilon \rangle = 1.$$  

(23)

Fig. 4 shows the invariant $\pi^0$ multiplicity in central Au + Au and Cu + Cu reactions. Data is from PHENIX [7]. At high $p_T$, comparisons been the jet quenching theory and the experimental measurement can (and should) also be made for the differential cross sections. At low $p_T$, a deviation is present due to the fixed order baseline pQCD calculation and QGP effects are more accurately studied via $R_{AA}(p_T)$.

5. Quenching of inclusive pions at RHIC and the LHC

Having evaluated the energy loss of quark and gluon jets in the QGP media specified in Table 2, we calculate the quenched pion spectra using Eqs. (17), (19), (20) and (21). Fig. 5 shows $R_{AA}(p_T)$ for central, semi-central and peripheral collisions. The predictions in Cu + Cu reactions are for a constant suppression factor, as in Au + Au, at high $p_T$. Preliminary PHENIX data, first compared to this theory in Ref. [1], are also included.
It should be noted that even in peripheral reactions there can be noticeable particle attenuation. This is larger than the analytic estimates due to sub-leading effects of the medium density on the parton energy loss, discussed in the previous Section, see Fig. 5. Whether such effects are observable or compensated by the non-uniform QGP density in the transverse plane is subject to experimental verification. Finally, we make the important observation that for similar densities and system sizes, for example $dN^g/dy = 410$ in mid-central Au + Au and $dN^g/dy = 370$ in central Cu + Cu, the magnitude of the predicted pion suppression is similar.

$\text{Pb + Pb collisions at the LHC represent the future energy frontier of QGP studies in heavy ion reactions. We have explored the sensitivity of } R_{AA}(p_T) \text{ to the parton rapidity density in central nuclear reactions with } dN^g/dy \approx 2000, 3000 \text{ and } 4000. \text{ In [9], a seven-fold increase of the medium density in going form RHIC to the LHC was assumed. In this work we adhere to a more modest two- to four-fold increase of the soft hadron rapidity density and emphasize that future measurements of jet quenching must be correlated to } dN^g/dy \approx 3(2)dN^{ch}/dy \text{ [6,8,12] to verify the consistency of the phenomenological results.}$

From Figs. 5–7, we conclude that at low and moderate $p_T$, jet suppression at the LHC is larger than at RHIC. However, at high $p_T > 30–50$ GeV this ordering is reversed. The physics reason for our result is that the fractional energy loss $(\Delta E/E)$ of 100 GeV quark and gluon jets at the LHC is not very different from that of 25 GeV jets at RHIC, see Fig. 2. In addition, the exponent in the power law dependence of the hadronic and the underlying partonic spectrum $n(y, p_T, \sqrt{s})$ changes (decreases) from $\sqrt{s} = 200$ GeV to $\sqrt{s} = 5.5$ TeV. This is shown in the insert of Fig. 6 and affects the magnitude of calculated nuclear suppression, Eq. (8). LHC will soon provide an extended $p_T$ range to test the correlation of $R_{AA}(p_T)$ with the stiffness of the differential particle spectra.

To assess the importance of the gluon feedback term in Eq. (21), we extend, in Fig. 7, the calculation of $R_{AA}(p_T)$ for central Au + Au and Pb + Pb collisions to low and moderate transverse momenta. At RHIC the redistribution of the lost energy leads to small, $\sim 25\%$, modification of the neutral pion cross section in the $p_T \sim 1–2$ GeV range and modest improvement in the theoretical description of that data. In contrast, at the LHC the fragmentation of medium-induced gluons is a much more significant $\geq 100\%$ correction to the low- and moderate-$p_T$ $\pi^0$ production rate.

The need for the more consistent treatment of jet energy loss, Eq. (21), is also illustrated by comparing $R_{AA}(p_T)$ in Fig. 7 to the participant scaling ratio: $(N_{\text{part}}/2)/N_{\text{coll}}$. At high $p_T$ there is no lower limit on the quenching of jets. At low $p_T$ the total available energy of the collision, $\sim (N_{\text{part}}/2)/\sqrt{s}$, suggests participant scaling of bulk particle production confirmed by hydrodynamic calculations [16]. Previous estimates of leading-particle suppression at low and moderate $p_T$ have violated this limit [6,9]. Fig. 7 shows that the gluon feedback can ensure numerically $R_{AA}(p_T) \geq (N_{\text{part}}/2)/N_{\text{coll}}$ at mid-rapidity at the LHC, for the densities considered here, and is important everywhere in the region $p_T \leq 15$ GeV.

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1 With $\sigma_{pp} = 42 \text{ mb (65 mb)}$ we obtain $(N_{\text{part}}/2)/N_{\text{coll}} = 0.17(0.11)$ at RHIC (LHC), respectively.
6. Conclusions

In this Letter we presented predictions for the nuclear modification of inclusive neutral pion production in Au + Au and Cu + Cu collisions at $\sqrt{s} = 200$ GeV versus centrality and transverse momentum. Pb + Pb reactions at $\sqrt{s} = 5.5$ TeV at the LHC were also discussed and our calculations at mid-rapidity accounted for Cronin multiple scattering [5], nuclear shadowing [15] and final-state radiative energy loss in the quark–gluon plasma [10]. Elastic energy loss was not considered here, since its effects are still under debate [17] relative to the attenuation of jets via gluon bremsstrahlung. We compared our numerical results to a simplified analytic model for centrality dependent jet quenching [6] and showed that there are non-negligible corrections in the evaluation and implementation of radiative energy loss related to the temperature or $\mu$ dependence of the strong coupling constant. While the dependence of the observable hadron suppression on $dN/dy$ was shown to be sub-linear, this calculation retains sensitivity to both the properties of the medium and the underlying perturbative baseline cross sections.

At low and moderate transverse momenta we derived the contribution to single inclusive pion production from the fragmentation of medium-induced gluons. At RHIC we found this effect to be a modest, $\lesssim 25\%$, correction. Our result should be contrasted with the case of back-to-back dihadron correlations where the redistribution of the lost energy controls the QGP-induced transition from suppression to enhancement of large-angle inclusive two particle production [11]. At the LHC, however, even in inclusive one pion calculations, gluon feedback is shown to alter the $p_T \lesssim 15$ GeV cross section in central Pb + Pb reactions by as much as a factor of two. In summary, results reported in this Letter not only provide a more consistent theoretical framework to treat the effects of medium-induced gluon bremsstrahlung but also rectify the over-quenching of jets at low $p_T$ in the limit of large fractional energy loss [6,9].

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