Three-dimensional electrodynamic code ELF

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Abstract. The paper is devoted to the description of the three-dimensional ELF code developed in RFNC-VNIITF. This code is intended to simulate the problems of electrodynamics, plasma physics, acceleration of electrons and ions on laser and electrophysical facilities. The code is based on solving the Vlasov kinetic equation and the Maxwell equations system. To solve the Vlasov equations, the method of particles in cells is used. An explicitly finite difference scheme of the second order of accuracy is used to solve the Maxwell equations. A set of analytical tests (propagation of plane electromagnetic wave in plasma, motion of an electron in electric and magnetic fields, etc.) is given and a good agreement of the numerical solution with them is shown. The order of convergence of the numerical algorithms also determined. The calculations of the accelerated electrons in plasma wakefield are presented, and the results of comparison with experiment for the parameters for the laser facility of RFNC-VNIITF are demonstrated.

1. Introduction
When electromagnetic pulses with femtosecond length of the laser interact with charged particles and plasma, a multiple nonlinear collective processes arises [1, 2]. These processes include generation of a plasma wave, stochastic and vacuum heating of electrons, betatron oscillations of electrons, acceleration of electrons in a plasma wave, and others. Due to the interaction of accelerated electrons with the laser pulse, the transparency coefficient of the plasma changes, which leads to self-focusing and filamentation of the laser pulse (LP) [1]. Simulation of such processes is performed using kinetic approaches based on the PIC method. Recently, three-dimensional PIC codes like VORPAL [3], OSIRIS [4], QuickPIC [5], VLPL [6]. Using the 3D3V PIC ELF (Electrons Laser Focus) code [7], laser acceleration of charged particles from a plasma of arbitrary composition and density in a collisionless limit is studied. The code is written in C++ (MPI). The mathematical model consists of the Vlasov kinetic equations for the distribution functions of various particle varieties and the system of Maxwell equations describing the dynamics of electromagnetic fields.

2. Equations of the relativistic electrodynamics
The collisionless relativistic plasma is described by the Vlasov kinetic equations for the distribution functions $f_\alpha(t, \vec{r}, \vec{p})$ of particles of different varieties [8, 9, 10]

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}(\vec{p}) \cdot \nabla_\vec{r} f_\alpha + \vec{F}_\alpha \cdot \nabla_\vec{p} f_\alpha = 0,$$

$$\vec{p} = m_\alpha \gamma \vec{v}, \quad \gamma = \sqrt{1 - v^2/c^2}, \quad \vec{F}_\alpha = q_\alpha \left( \vec{E} + \frac{1}{c} \left[ \vec{v} \times \vec{B} \right] \right),$$
where \( q_\alpha, m_\alpha \) – is the charge and mass of a particle of type \( \alpha \).

Maxwell’s equations

\[
\frac{\partial \vec{E}}{\partial t} = c \left[ \nabla \times \vec{B} \right] - 4\pi \vec{J}, \quad \nabla \cdot \vec{E} = 4\pi \rho, \tag{3}
\]

\[
\frac{\partial \vec{B}}{\partial t} = -c \left[ \nabla \times \vec{E} \right], \quad \nabla \cdot \vec{B} = 0. \tag{4}
\]

The space charge density and current density are defined as sums over all sorts of particles of zero and first moments of the distribution functions in the momentum space \( R^3(\vec{p}) \)

\[
\rho(t, \vec{r}) = \sum_\alpha q_\alpha \int f_\alpha(t, \vec{r}, \vec{p}) \, d\vec{p}, \quad \vec{J}(t, \vec{r}) = \sum_\alpha q_\alpha \int \vec{v} f_\alpha(t, \vec{r}, \vec{p}) \, d\vec{p}. \tag{5}
\]

The system (1) – (5) is closed.

3. The numerical model

3.1. Discretization of the distribution function

The discretization of the distribution function is performed by the PIC method. The distribution function of particles of type \( \alpha \) is approximated by a finite number of \( N_\alpha \) computational particles (or macroparticles), each one containing \( N_p \) of real particles. \( N_p \) can be variable, which allows calculating a wide range of plasma densities.

\[
f_\alpha(t, \vec{r}, \vec{p}) = \frac{N_\alpha}{\sum_{\alpha=1}^{N_\alpha}} q_\alpha R(\vec{r}, \vec{r}_\alpha) \delta (\vec{p} - \vec{p}_\alpha), \tag{6}
\]

where \( \vec{r}_\alpha \) and \( \vec{p}_\alpha \) – are the coordinate and momentum of a particle of type \( \alpha \) in phase space, \( R(\vec{r}, \vec{r}_\alpha) \) – is the function of the particle kernel, \( \delta (\vec{p}) \) is the three-dimensional Dirac function. The kernel function is positive definite one and satisfies the normalization condition which guarantees the conservation of charge. The kernel determines the shape, size and distribution of the charge inside the particle \( \int_{R^3(\vec{r})} R(\vec{r}, \vec{r}_\alpha) \, d\vec{r} = 1 \). The kernel function is positive definite one and satisfies the normalization condition which guarantees the conservation of charge.

The kernel determines the shape, size and distribution of the charge inside the particle. In [10] a set of computational kernels is provided that ensure the monotonicity of the solution. In practice, the spatial function of the kernel \( R(\vec{r}, \vec{r}_\alpha) \) is represented as a product of one-dimensional kernels \( R(\vec{r}, \vec{r}_\alpha) = R_{1D}(x - x_\alpha) \cdot R_{1D}(y - y_\alpha) \cdot R_{1D}(z - z_\alpha) \).

In the ELF code, a one-dimensional kernel is a generalized function for which polynomial forms of kernels up to the 5th order of smoothness are realized. The most common is the classical PIC kernel, where \( h \) – is the size of the grid cell:

\[
R_{1D}(x) = \begin{cases} \frac{1}{\pi \Delta}, & |x| \leq \Delta, \\ 0, & |x| > \Delta. \end{cases} \quad \Delta = h/2. \tag{7}
\]

The substitution of the approximation of the distribution function (6) into the Vlasov equation (1) and the integration of the latter with an arbitrary smooth compactly supported function \( \psi(\vec{p}) \) over the phase space leads to an identity under the conditions [10]

\[
\frac{d\vec{r}_\alpha}{dt} = \vec{v}_\alpha, \quad \vec{v}_\alpha = \frac{\vec{p}_\alpha}{m\gamma}, \tag{8}
\]
\[
\frac{d\vec{\beta}_\alpha}{dt} = q_\alpha \left( \vec{E}(t, \vec{r}_\alpha) + \frac{1}{c} [\vec{v}_\alpha \times \vec{B}(t, \vec{r}_\alpha)] \right),
\]

(9)

where the quantities \( \vec{E}(t, \vec{r}_\alpha) \) and \( \vec{B}(t, \vec{r}_\alpha) \) were obtained by weighting the spatial distribution of the field along the kernel of the particle,

\[
\vec{E}(t, \vec{r}_\alpha) = \int \vec{E}(t, \vec{r}) R(\vec{r}, \vec{r}_\alpha) \, d\vec{r}, \quad \vec{B}(t, \vec{r}_\alpha) = \int \vec{B}(t, \vec{r}) R(\vec{r}, \vec{r}_\alpha) \, d\vec{r}.
\]

(10)

As a result, the solution of the Vlasov equations (1) – (4) was reduced to the solution of the ODE system (8) – (9) for describing the motion of an individual particulate.

3.2. Integration of the equations of motion

To solve the equations of motion in the ELF code, a step-over scheme is implemented, in which the central-difference approximation of the equations of motion is used. This second-order time-accuracy scheme is known as the Boris scheme [9]. The scheme is reversible in time. Equations of motion of the macroparticle (8) – (9) in the difference form, are written without the index \( \alpha \),

\[
\frac{\vec{r}^{n+1} - \vec{r}^n}{\Delta t} = \vec{\nu}^{n+1/2},
\]

(11)

\[
\frac{\vec{u}^{n+1/2} - \vec{u}^{n-1/2}}{\Delta t} = \frac{q}{m} \left( \vec{E}^n + \frac{\vec{u}^{n+1/2} - \vec{u}^{n-1/2}}{2\gamma^n} \times \vec{B}^n \right),
\]

(12)

\[
\gamma^n = \sqrt{1 + \left( \frac{u^-}{c} \right)^2} = \sqrt{1 + \left( \frac{u^+}{c} \right)^2},
\]

(13)

where \( \vec{u} \equiv \gamma \vec{v}, n \) is the step number in time, \( n \pm 1/2 \) is the half step, \( \Delta t \) is the step size, \( u^+ \) and \( u^- \) are the auxiliary velocities defined below.

Since the electric field basically changes the modulus of the particle’s velocity, and the magnetic field changes the direction of its motion, the velocity at half-integer steps in time is represented in the form

\[
\vec{u}^{n-1/2} = \vec{u}^- - \frac{q\vec{E}^n \Delta t}{2m}, \quad \vec{u}^{n+1/2} = \vec{u}^+ + \frac{q\vec{E}^n \Delta t}{2m}.
\]

(14)

From (14) and (12) we calculate the change in the angle \( \theta \) between the vectors \( \vec{u} \) and \( \vec{B} \)

\[
\frac{\vec{u}^+ - \vec{u}^-}{\Delta t} = \frac{q}{2\gamma^m} (\vec{u}^+ + \vec{u}^-) \times \vec{B},
\]

(15)

\[
|\tan \frac{\theta}{2}| = \frac{|\vec{u}^+ - \vec{u}^-|}{|\vec{u}^+ + \vec{u}^-|} = \frac{q|\vec{B}|\Delta t}{2\gamma m}, \quad \theta = 2 \arctan \left( \frac{qB \Delta t}{2\gamma m} \right).
\]

(16)

The velocity components in equation (15) are perpendicular to the magnetic field. To calculate the rotational motion due to the magnetic field, an auxiliary vector \( \vec{u}' \), is introduced, which is perpendicular to the difference of the vectors \( (\vec{u}^+ - \vec{u}^-) \) and to the magnetic field vector \( \vec{B} \).

\[
\vec{B} \vec{u}' = \vec{u}^- + \vec{u}^- \times \vec{b}^n.
\]

The vector \( \vec{b} \) is determined to take into account that the angle between \( \vec{u}^- \) and \( \vec{u}^+ \) is \( \theta/2 \), \( \vec{b}^n = \frac{qB_n \Delta t}{2\gamma m} \). Equation (15) is written in a simplified form \( \vec{u}^+ - \vec{u}^- = (\vec{u}^+ + \vec{u}^-) \times \vec{t} \).

Taking into account that the difference \( \vec{u}^+ - \vec{u}^- \) is parallel to the vector product \( \vec{u}' \times \vec{B} \), the vector \( \vec{u}^+ \) calculated as \( \vec{u}^+ = \vec{u}^- + \vec{u}' \times \vec{s}^n \). The vector \( \vec{s} \) is parallel to \( \vec{B} \), its value is determined from the condition that the square of the velocity is constant when the particle rotates \( \vec{s}^n = (2\vec{b}^n)/(1 + (\vec{b}^2)^n) \).
3.3. Solution of the Maxwell equations
The finite-difference approximation of the system of equations (3) – (4) is performed according to the explicit conservative scheme proposed in [11] for modelling the propagation and scattering of electromagnetic waves.

The Yee’s scheme at each time step ensures the condition that the divergence of the magnetic field equal to zero if, for \( t = 0 \) this condition is fulfilled [12]. The scheme is stable under the Courant condition \( \Delta t < (c\sqrt{\Delta x}^{-2} + (\Delta y)^{-2} + (\Delta z)^{-2})^{-1} \).

3.4. Calculation of charge and current densities
The charge and current densities depend on the grid kernel \( R_{i,j,k} \) and can be calculated from the summation formulas for the contributions from each particle of the type \( \alpha \):

\[
\rho_{i,j,k} = \sum_{\alpha} \sum_{p=1}^{N_{\alpha}} q_{\alpha} R_{i,j,k}(x_{p\alpha}, y_{p\alpha}, z_{p\alpha}),
\]

\[
J_{i,j,k}^{(x)} = \sum_{\alpha} \sum_{p=1}^{N_{\alpha}} q_{\alpha} v_{x\alpha} R_{i,j,k}(x_{p\alpha}, y_{p\alpha}, z_{p\alpha}),
\]

\[
J_{i,j,k}^{(y)} = \sum_{\alpha} \sum_{p=1}^{N_{\alpha}} q_{\alpha} v_{y\alpha} R_{i,j,k}(x_{p\alpha}, y_{p\alpha}, z_{p\alpha}),
\]

\[
J_{i,j,k}^{(z)} = \sum_{\alpha} \sum_{p=1}^{N_{\alpha}} q_{\alpha} v_{z\alpha} R_{i,j,k}(x_{p\alpha}, y_{p\alpha}, z_{p\alpha}),
\]

where the grid kernel is represented in the form

\[
R_{i,j,k}(x_{p\alpha}, y_{p\alpha}, z_{p\alpha}) = R(x_{i} - x_{p\alpha}, y_{j} - y_{p\alpha}, z_{k} - z_{p\alpha}),
\]

\( x_{i}, y_{j}, z_{k} \) – coordinates of grid nodes. The mesh kernel satisfies the normalization condition

\[
\sum_{i,j,k} R_{i,j,k}(x_{p\alpha}, y_{p\alpha}, z_{p\alpha}) = 1.
\]

By virtue of the discrete representation of equations (18) – (20)

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0,
\]

the law of conservation of charge does not hold with absolute accuracy.

In this case, for the correction of the electric field, the Poisson equation (3) (right) is solved [8, 9]. Recently, various numerical methods have been developed for solving the system of equations of electrodynamics, which do not require additional field correction [12, 13, 14, 15]. These are the so-called ”charge conservation methods”, which are based on the calculation of flows satisfying the difference equation of continuity

\[
\frac{\rho_{i+\frac{1}{2},j,k}^{n+1} - \rho_{i+\frac{1}{2},j,k}^{n}}{\frac{\Delta t}{\Delta x}} + \frac{J_{i+\frac{1}{2},j,k}^{(x)n+\frac{1}{2}} - J_{i+\frac{1}{2},j,k}^{(x)n+\frac{1}{2}}}{\frac{\Delta t}{\Delta y}} + \frac{J_{i+\frac{1}{2},j,k}^{(y)n+\frac{1}{2}} - J_{i+\frac{1}{2},j,k}^{(y)n+\frac{1}{2}}}{\frac{\Delta t}{\Delta z}} = 0.
\]
In the ELF code the scheme from [12] is implemented. According to this scheme, the flow of a charge in the motion of one macroparticle is divided into a sum of twelve auxiliary particles, each of which moves only along one direction: $x$, $y$ or $z$. The sum of the flows is $q_{a_i}\vec{v}_a$, since the contribution of each flow is taken with the according weight.

The algorithm for calculating the flows is as follows. Taking into account the linearity of the continuity equation (24) for each individual particle with the kernel (21), the coordinates $x, y, z$ and the charge $q$ the vector $\vec{W}$ are defined:

$$J_{i+1,j+1/2,k+1/2}^{(x)} - J_{i,j+1/2,k+1/2}^{(x)} = -\frac{\Delta x}{\Delta t} W_{i,j+1/2,k+1/2}^{(x)},$$ (25)

$$J_{i+1,j+1,k+1/2}^{(y)} - J_{i,j+1/2,k+1/2}^{(y)} = -\frac{\Delta y}{\Delta t} W_{i,j+1/2,k+1/2}^{(y)},$$ (26)

$$J_{i+1,j+1/2,k+1} - J_{i+1/2,j+1/2,k+1} = -\frac{\Delta z}{\Delta t} W_{i+1/2,j+1/2,k}^{(z)}.$$ (27)

From equations (17), (24) and (25) the expression for the sum of the components of the vector $\vec{W}$ (upper and lower indices are omitted) is $W^{(x)} + W^{(y)} + W^{(z)} = R(x + \Delta x, y + \Delta y, z + \Delta z) - R(x, y, z)$.

The contribution to the current density from one particle is calculated by the equations (25). Summation over all particles of each form determines the components of the vector $\vec{J}$ that satisfy the law of conservation of charge.

### 3.5. Boundary conditions

Conditions on the boundaries of the computational domain are set separately for fields and particles. For particles, the conditions for absorption, reflection, injection, periodic, and free escape beyond the calculated region are determined. The periodic boundary conditions, the conditions of reflection, absorption, and entry through the boundary are determined for the electromagnetic field of the LP.

### 4. Test calculations

#### 4.1. Propagation of a plane electromagnetic wave in a vacuum

A plane linearly polarized electromagnetic wave with a wavelength $\lambda = 1$ mkm is given in the calculated region of size $5\lambda \times 5\lambda \times 5\lambda$ at the initial moment $t = 0$.

An initial distribution of electromagnetic field components:

$$E_x(x, y, z, 0) = 0, \quad E_y(x, y, z, 0) = E_0 \sin(kx), \quad E_z(x, y, z, 0) = 0,$$ (28)

$$B_x(x, y, z, 0) = 0, \quad B_y(x, y, z, 0) = 0, \quad B_z(x, y, z, 0) = B_0 \sin(kx).$$ (29)

The amplitudes of the electric and magnetic fields are $E_0 = 3.21 \cdot 10^{12}$ V/m, $B_0 = 1.07 \cdot 10^4$ T, the corresponding dimensionless quantity $\alpha_0 = \frac{eE_0}{m_0c} = 1$.

On all boundaries of the computational domain, periodic boundary conditions were given. The exact solution has the form

$$E_x(x, y, z, t) = 0, \quad E_y(x, y, z, t) = E_{x0} (\omega t - kx, y, z), \quad E_z(x, y, z, t) = 0,$$ (30)

$$B_x(x, y, z, t) = 0, \quad B_y(x, y, z, t) = 0, \quad B_z(x, y, z, t) = B_{z0} (\omega t - kx, y, z).$$ (31)

Calculations have been made up to the time $t = 10/\lambda c^{-1}$ on a sequence of difference grids with the step size in space $\Delta x = \Delta y = \Delta z = \lambda/16, \lambda/32, \lambda/64$ and $\lambda/128$. Figure 1 shows a comparison of numerical solutions with the exact one (30). It can be seen that when the grid is refined, the numerical solution approaches the exact one.
For determining of the order of convergence, the norms $C$ and $L_2$, shown in Figure 2, were calculated as $\delta_C = \max ||y - u_h||$ and $\delta_{L_2} = (\sum_k \Delta(y_k - u_k)^2)^{1/2}$, where $y$ is the numerical solution, and $u_h$ is the projection of the exact solution onto the grid. The order of convergence was defined as

$$\beta_C = \log \frac{\Delta_{i+1}}{\Delta_i} \frac{(\delta_C)_i}{(\delta_C)_{i+1}}, \quad \beta_{L_2} = \log \frac{\Delta_{i+1}}{\Delta_i} \frac{(\delta_{L_2})_i}{(\delta_{L_2})_{i+1}}. \quad (32)$$

The order of convergence was varied in the range $[1.98, 2.02]$, which corresponds to the order of the chosen scheme for solving the Maxwell equations.

4.2. Motion of an electron in a constant uniform electric field
A constant electric field $\vec{E} = (10^{13}; 0; 0)$ V/m is given in the computational domain of $3.2 \times 3.2$ mkm. At the initial moment of time in the location $x_0 = 3.075$, $y_0 = 0.1$ mkm there is an electron with momentum components $p_{0x} = 0$, $p_{0y} = 0.9m_e c$. The electron is accelerated by an electric field. The analytical solution of the problem is described by parametric equations [16]

$$x(t) = \frac{1}{cE} \sqrt{\varepsilon_0^2 + (ceEt)^2}, \quad y(t) = \frac{p_{0c}}{cE} \frac{Arsh(ceEt)}{\varepsilon_0} \quad (33)$$

or by the equation of a chain line

$$x = \frac{\varepsilon_0}{cE} \frac{eEy}{p_0 c}. \quad (34)$$

Here $\varepsilon_0 = \sqrt{m^2 c^4 + c^2 p_0^2}$ is the kinetic energy at $t = 0$, when $p_x = 0$, $p_y = p_0$.

A comparison with the exact solution was carried out for numerical solutions obtained on square calculated grids with the number of cells $N_c = 2^n$, $n = 4, \ldots, 10$. The calculation ended at time of 10 fs. Figure 3 shows a comparison of electron trajectories. It can be seen that when the grid is refined, the numerical solution approaches the exact one.

The values of the norms $C$ and $L_2$ are shown in Figure 4. Boris’s scheme, described in Section 3.2 for solving the equations of motion, has the order of convergence depending on the grid size in the range $[0.63, 0.81]$.

4.3. Motion of an electron in a homogeneous magnetic field
A constant magnetic field $\vec{H} = (0, 0, 100)$ T is given in the calculated area $20 \times 20 \times 20$ mkm. At the initial time point, at the point $\vec{r} = (x_0, y_0, z_0) = (R_0, 0, 0)$ there is an electron with
momentum components $\vec{p} = (0, 0.5, 0.066)mc$, $R_0 = 8.52$ mkm. The electron moves under the action of a constant magnetic field.

The analytical trajectory of an electron is a screw line (see Figure 6, left) with an axis along the magnetic field vector and the radius $r_0 = \frac{v_0}{\omega_{ce} E_0}$ [16]

$$x(t) = x_0 + r_0 \sin(\omega t + \alpha), \quad y(t) = y_0 + r_0 \cos(\omega t + \alpha), \quad z(t) = z_0 + v_{0z} t,$$

where $v_0 = \sqrt{\left(\frac{p_0 x c^2}{\varepsilon_0}\right)^2 + \left(\frac{p_0 y c^2}{\varepsilon_0}\right)^2}$ – is the electron velocity in the $(x0y)$ plane, $v_{0z}$ – is the initial velocity of the electron in the direction of the field, $\mathcal{E} = \mathcal{E}_0 = c\sqrt{m_e c^2 + p_{0x}^2 + p_{0y}^2 + p_{0z}^2}$ is the electron energy (the magnetic field does not change the electron energy), $\mathcal{E}_{oe} = mc^2$ – the electron rest energy, $\omega_{ce} = \frac{eH}{mc^2}$ – the cyclotron frequency, $\omega = \frac{E_0 E_{oe}}{\mathcal{E}_0}$ – the cyclic frequency of rotation of the electron in a plane perpendicular to the field.

A comparison with the exact solution was carried out for numerical solutions obtained on cubic grids with the number of cells $N_c = 2^n$, $n = 4, \ldots, 9$. End of the calculation time is 1 ps. Estimations give the values

$$\mathcal{E}_0 \approx 9.15 \times 10^{-7}[\text{erg}], \quad v_0 \approx 1.34 \times 10^{10}[\text{cm/s}], \quad r_0 \approx 8.521 \times 10^{-4}[\text{cm}] \approx R_0.$$
The electron trajectory and the dependence of the \( C \) and \( L_2 \) norms of the numerical solution error on the size of the mesh are shown in Figure 6. It is seen that when refining the grid, the numerical solution approaches the exact one with the order of convergence depending on the grid size in the range [0.98, 1.0].

4.4. Motion of a single electron in a plane electromagnetic wave

A plane linearly polarized electromagnetic wave whose components have the form (30) is given in the calculated region \( 2\lambda \times 2\lambda \times 2\lambda \) at the initial time, \( \lambda = 1 \text{ mkm} \) the field amplitude corresponds to \( a_0 = 1 \). At the point with coordinates \((0,0,0)\) here is an electron with zero momentum components \( p_x^0 = p_y^0 = p_z^0 = 0 \). On all boundaries of the computational domain, periodic boundary conditions are given. In this case, the electron trajectory is described by a system of equations [17]

\[
\begin{align*}
\dot{p}_x &= 0.5a_0^2 \sin^2 \tau, \\
\dot{p}_y &= a_0 \sin \tau, \\
\dot{p}_z &= 0, \\
\dot{\bar{x}} &= \bar{p}_x d\tau, \\
\dot{\bar{y}} &= \bar{p}_y d\tau, \\
\dot{\bar{z}} &= 0, \\
\tau &= \bar{t} - \bar{x}, \\
x(\tau) &= a_0^2 (\tau - \sin \tau \cos \tau)/(4k), \\
y(\tau) &= a_0 (1 - \cos \tau)/k, \\
z(\tau) &= 0.
\end{align*}
\]

Normalization: \( \bar{t} = \omega t, \bar{x} = \frac{\omega}{c} x, \bar{p}_x = \frac{p_x}{m_e c} \).

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig7.png}
\caption{Dependence of the momentum components on the time for a single electron in a linearly polarized plane wave with a constant amplitude.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig8.png}
\caption{Dependence of the error rates of the numerical solution on the quality of the difference grid.}
\end{figure}

A comparison with the exact solution was carried out for numerical solutions obtained on cubic computational grids with varying degrees of detail up to the time point \( t = 2\lambda c^{-1} \). The grid spacing is \( \Delta = \lambda/16, \lambda/32, \lambda/64, \lambda/128 \) and \( \lambda/256 \). Figure 7 shows the calculated and analytical dependence of the electron momentum on time. It can be seen that when the grid is refined, the numerical solution approaches the exact one. The values of the norms \( C \) and \( L_2 \) and the order of convergence in these norms were calculated for the quantity \( \gamma^2 = 1 + p_x^2 + p_y^2 + p_z^2 \). The comparison showed that Boris’s scheme for solving the equations of motion has the order of convergence in the range [0.99, 1.07] – see Figure 8. The difference with the declared 2nd order of the scheme is apparently due to the error of linear interpolation of the fields from grid nodes to the coordinate of the particle.
4.5. Absorption of electromagnetic waves in the skin layer of high-density plasma

A flat electromagnetic wave of amplitude $a_0 = 5$ enters the one-dimensional computational domain of size $2\lambda$ from the left boundary. At a distance $\lambda$ from the boundary, a hydrogen plasma with an electron density $n_e = 10n_c \approx 4 \cdot 10^{25} \text{cm}^{-3}$ is given, where $n_c = m_e\omega_p^2(4\pi e^2)^{-1}$ is the critical density [2]. The step of the calculated grid is calculated from the requirement of the resolution of the plasma wave $\lambda_p = 2\pi c\omega_p^{-1}$, $\omega_p = \sqrt{4\pi e^2 n_e m_e^{-1}}$. It is known that an electromagnetic wave can not propagate in a plasma with a density higher than the critical one. However, thermal electrons can penetrate into the overcritical regions to a depth of the skin layer $l_s = c\omega_p^{-1}$. An analytical solution is given in [18]

$$E(t, x) = a_0 \cos(\omega t - kx) \exp(-x/l_s).$$  \hspace{1cm} (39)

The numerical solution of the electric field distribution of an electromagnetic wave near the plasma-vacuum boundary is shown in Figure 9.

![Figure 9](image1.png)

**Figure 9.** Field distribution at time $t = 3.08$ fs, corresponding to the reflection of the first wave of the electromagnetic package. The dotted line denotes the initial boundary of the plasma.

![Figure 10](image2.png)

**Figure 10.** Field distribution at time $t = 3.08$ fs, corresponding to the reflection of the first wave of the electromagnetic package. The dotted line denotes the initial boundary of the plasma (back of the target).

It can be seen that on a detailed numerical grid the solution converges to the analytic one. Calculations show that the plasma wave must be described by at least 30 cells.

5. Conclusion

This paper presents a description of the 3D ELF code for electrodynamics, plasma physics, electron and ion acceleration in a laser pulse, and in an electrophysical installation.

The results of the testing confirmed the ability of the code to simulate the propagation processes of the electromagnetic pulse, the interaction of charged particles with the electromagnetic field, both for individual particles and in the case of large space charges and currents. The order of convergence of the numerical algorithms used is determined.

A code is a tool for the analysis and prediction of experimental data on laser and electrophysical installations of VNIITF. Further development of the ELF code involves improving the algorithms – increasing accuracy, efficiency of the calculation.

The development of ELF code continues taking into account the specific features of laser and electrophysical problems and for the analysis of the effect of magnetic fields in a plasma in a self-consistent approximation with kinetic approach within the framework of a hybrid PIC + MHD code. The latter is necessary for studying the effect of fast electrons on the compression of laser targets and the stage of maximum compression of the Z-pinch in the near-axis region.
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