Two-dimensional (2D) numerical modelling of rainfall induced overland flow, infiltration and soil erosion: comparison with laboratory rainfall-runoff simulations on a two-directional slope soil flume

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Abstract: This paper presents a two-dimensional (2D) numerical model of soil erosion and sediment transport resulting from rainfall induced overland flow. It is a spatial and temporal dynamic model combining physical and empirical laws and comprises: i) An overland flow module that solves the two-dimensional unsteady water flow equations on an infiltrating surface; ii) A soil infiltration module that uses a combined Horton-SCS scheme; and iii) A soil erosion and sediment transport module that solves the two-dimensional sediment transport equation, distinguishing between rill erosion, interrill erosion and sediment deposition.

The performance of the model was evaluated by comparing its results with observed data from laboratory rainfall-runoff experiments on a two-directional 2.00 × 2.00 m² soil flume set at 1% and 10% slopes in the x- and y-directions, respectively. The x-direction produced remarkably lower runoff and transported sediments than the y-direction. The numerical model significantly underestimated x-direction lower values of runoff and transported sediments. However, in the y-direction the model presented very good performance. Overall, in total terms (x- plus y-direction), the numerically simulated graphs of runoff and sediment transport were in very good agreement with corresponding experimental measurements, demonstrating the laboratory proof-of-concept of the model.

Keywords: Two-dimensional modelling; Overland flow; Soil erosion; Horton-SCS infiltration; Two-directional laboratory soil flume; Rainfall simulation.

INTRODUCTION

Soil erosion has been widely recognized as a serious environmental degradation problem throughout the history (Montgomery, 2007). It can reduce soil fertility and productivity (Boardman et al., 2009) and increase the transport of sediment and pollutants to freshwater bodies (Rickson, 2014). Accurate prediction of soil erosion is therefore essential in land and water management (Panagos and Katsoyiannis, 2019).

Mathematical models are cost-effective tools for improving our understanding of erosion processes and predict its effects on soil and water quality (Panagos and Katsoyiannis, 2019). A robust mathematical model can provide a cost-effective tool by which many scenarios can be simulated and compared in order to find the best alternative of addressing a particular problem (Batista et al., 2019; Singh and Woolhiser, 2002). Consequently, a wide spectrum of soil erosion models, ranging from simple empirical formulas, such as the Universal Soil Loss Equation (USLE, Wischmeier and Smith, 1978) or its revised version (RUSLE, Renard et al., 1997), to comprehensive physically based distributed descriptions, such as the KINematic runoff and EROsion (KINEROS, Woolhiser et al., 1990), the Water Erosion Prediction Project (WEPP, Flanagan and Nearing, 1995), or the EUROpean Soil Erosion Model (EUROSEM, Morgan et al., 1998), have been proposed for the prediction of soil erosion and sediment transport (Batista et al., 2019; Singh and Woolhiser, 2002).

Physically based models are generally the most scientifically robust and flexible models and provide an understanding of the fundamental and non-stationary processes involved in the detachment, transport and deposition of sediments and provide an access to their spatial and temporal variation (Nearing, 2000). However, due to the complexity of such processes, fully physically based models have not yet become a practical tool (Stroosnijder, 2005). Their parametrisation is complex, and they are data intensive. Also, such data always carry a level of uncertainty, are expensive and time consuming and, therefore, most of the times the amount of data needed is not readily available (Stroosnijder, 2005).

Reduced-scale laboratory experiments using soil flumes and rainfall simulators allow observing the fundamental mechanisms in the complex hydrologic processes (i.e. overland flow generation, infiltration, erosion) under controlled conditions (Abrantes et al., 2018; de Lima et al., 2013, 2019; Montenegro et al., 2013; Prats et al., 2017). They allow to investigate effect of specific factors such as slope geometry, soil conditions and
rainfall characteristics and allow for a quicker and easier way to access good-quality data, such as peak discharges, sediment concentrations and time to peaks. The availability of such good-quality data provides a good chance to evaluate the performance of numerical models and improve them (Abrantes et al., 2015, 2019; Cuomo et al., 2016; Deng et al., 2005; Isidoro and de Lima, 2013; Silveira et al., 2016; Singh and de Lima, 2018).

This paper presents a two-dimensional (2D) numerical model of soil erosion and sediment transport resulting from rainfall induced overland flow. The main objective of this study is to present a proof of concept of the model performance in simulating soil erosion and sediment transport produced by single rainfall events in a two-directional (i.e. slope in x- and y-directions) laboratory soil flume on a steady flat surface. It is expected that, even on a flat soil surface, a two-dimensional model will perform better than a one-dimensional model in the case of two-directional flow and spatially distributed rainfall. The performance of the model was evaluated for the x- and y-directions individually. To the best of our knowledge, the essential novelty of this study resides in the originality of the soil flume experiments that allows to evaluate surface runoff in x- and y-directions individually.

GOVERNING EQUATIONS

The model is a spatial and temporal dynamic model combining physical and empirical laws and comprises three main modules: i) An overland flow module that solves the two-dimensional unsteady water flow equations on an infiltrating surface; ii) A soil infiltration module that uses a modified version of the empirical Horton’s infiltration equation; and iii) A soil erosion and sediment transport module that solves the two-dimensional sediment transport equation.

Overland flow and soil erosion

Overland flow was described by the two-dimensional shallow-flow equations commonly referred to as the Saint-Venant equations, which include the equation of continuity and two equations of motion for the coordinate directions x and y (Zhang and Cundy, 1989):

\[
\frac{\partial h}{\partial t} + \frac{\partial v_x h}{\partial x} + \frac{\partial v_y h}{\partial y} = p - i 
\]

(1)

\[
\frac{\partial v_x h}{\partial t} + \frac{\partial}{\partial x}\left(v_x^2 + \frac{1}{2}gh^2\right) + \frac{\partial v_y v_x h}{\partial y} = gh \left(-\frac{\partial Z}{\partial x} - S_{fx}\right)
\]

(2)

\[
\frac{\partial v_y h}{\partial t} + \frac{\partial v_x v_y h}{\partial x} + \frac{\partial}{\partial y}\left(v_y^2 + \frac{1}{2}gh^2\right) = gh \left(-\frac{\partial Z}{\partial y} - S_{fy}\right)
\]

(3)

where \(h\) is the water depth, \(v_x\) and \(v_y\) are the depth-averaged flow velocity components in the x and y directions, respectively, \(p\) is the rainfall intensity, \(i\) is the infiltration rate, \(g\) the gravitational constant, \(Z\) is the bed elevation, \(t\) is the time and \(S_{fx}\) and \(S_{fy}\) are the friction slopes in the x and y directions, respectively and were approximated by the Manning’s formula and expressed in terms of conservation variables, as:

\[
S_{fx} = \frac{n^2 v_x \left(v_x^2 + v_y^2\right)^{3/2}}{h^{5/3}}, \quad S_{fy} = \frac{n^2 v_y \left(v_x^2 + v_y^2\right)^{3/2}}{h^{5/3}}
\]

(4)

where \(n\) is the Manning’s roughness coefficient. Assumptions and derivation of Eqs. (1–3) can be found in detail in Zhang and Cundy (1989).

Equations used for soil erosion and sediment transport resulting from rainfall induced overland flow vary significantly due to different understanding and treatments of the sediment detachment, transport and deposition mechanisms (Flanagan and Nearing, 1995; Morgan et al., 1998; Woolhiser et al., 1990). Raindrop impact and/or overland flow can detach sediments from the soil surface. A critical force needs to be exerted by either a raindrop or a flow before detachment occur. Transport of detached material can occur as the result of raindrops and flow acting singly or together. Sediment deposition occurs when the flow can no longer support the suspended sediments, usually as result of a decrease in the flow transport capacity (Kinnell, 2005).

According to these principles, soil erosion was divided into three main mechanisms: i) Interrill erosion, that reflects the detachment and transport of sediments by the action of raindrops; ii) Rill erosion, that reflects the detachment and transport of sediments by the action of overland flow; and iii) Sediment deposition, that reflects the settling down of sediments. These mechanisms were described by the following two-dimensional sediment transport equation:

\[
\frac{\partial h c}{\partial t} + \frac{\partial v_x h c}{\partial x} + \frac{\partial v_y h c}{\partial y} = e_i + e_r - d
\]

(5)

where \(c\) is the overland flow mass sediment concentration and \(e_i\) and \(e_r\) are the volumetric interrill and rill erosion, respectively, and \(d\) is the sediment deposition.

The right side of Eq. (5) represents the constant exchange of sediment particles in the vertical between the soil surface and the flow, and its terms can be expressed as follows (Cao et al., 2002; Deng et al., 2008):

\[
e_i = \rho_s \frac{P^2}{\partial h} \exp\left(-\eta h\right)
\]

(6)

\[
e_r = \begin{cases} 
\rho_s \xi \left(\theta - \theta_c\right) \frac{1}{\eta h d_i^{-1/2}} \sqrt{v_x^2 + v_y^2} & \text{if } \theta > \theta_c \\
0 & \text{if } \theta \leq \theta_c
\end{cases}
\]

(7)

\[
d = \rho_s \omega c \left(1 - \alpha c\right)^2
\]

(8)

where \(d_i\) is the mean sediment particle diameter, \(\theta_c\) is the dimensionless critical Shields parameter for initiation of sediment movement and \(\theta\) is the dimensionless flow shear stress and can be expressed as (Liu and Beljadid, 2017):

\[
\theta = \frac{n^2}{\left(\rho_s / \rho - 1\right) \eta h^{5/3} \left(v_x^2 + v_y^2\right)}
\]

(9)

where \(\rho\) and \(\rho_s\) are the density of clear water and sediment particles, respectively, \(\eta\) is the settling velocity of a single sediment particle in tranquil water (Cheng, 1997):
of the overland flow and α describes the difference between the bed surface sediment concentration and the overland flow sediment concentration and can be approximated as:

\[ \alpha = \min \left( 2, \frac{1 - p_s}{c} \right) \]  

where \( p_s \) is the soil porosity.

The numerical methods used to solve the governing equations and to address the initial and boundary conditions are presented in "APPENDIX A". The governing equations (Eqs. (1–3) and (5)) were solved using the explicit finite-difference method based on the MacCormack operator-splitting scheme. In addition, specific numerical procedures were developed to handle the wet/dry front, in such a way that numerical simulations can start on an initially dry surface, allowing a more realistic prediction of the interaction between rainfall, infiltration and overland flow.

### Infiltration

Infiltration was computed using a modified version of the Horton’s infiltration equation (Horton, 1933) with a calibration methodology of its parameters based on formal analogies with the SCS-CN method (USDA, 2004). The result was presented in Gabellani et al. (2008) and is a general relation between SCS-CN and modified parameters of Horton’s method.

Horton (1933) proposed an exponential decay equation to describe the variation in time of the infiltration capacity of the soil during a rainfall event as:

\[ f(t) = f_\infty + (f_0 - f_\infty) e^{-kt} \]  

where \( f(t) \) is the infiltration rate at time \( t \) from the beginning of the rainfall event, \( f_\infty \) and \( f_0 \) are the final (minimum) and initial (maximum) infiltration rates, respectively, and \( k \) is the exponential time decay coefficient.

The main restrictions to the application of Horton’s equation in its original form are the difficulty of considering rainfall with intensities lower than \( f_0 \), the impossibility to describe the effect of dry periods inside the rainfall event, and the difficulty of obtaining reliable estimates for its parameters. This modified version of the Horton’s infiltration accounts for a relation between the infiltration capacity and soil moisture conditions for a more successful parameter calibration and accounts for intermittent and low-intensity rainfall events, namely lower than \( f_0 \).

According to this methodology, the upper soil layer is modelled as a linear reservoir with a water volume \( V(t) \) varying in time between 0 for dry soil condition and \( V_{\text{max}} \) for saturated soil condition. \( V(t) \) varies with the infiltration in the upper soil layer \( i(t) \) and the percolation to deeper soil layers \( i_p(t) \), according to the following mass-balance equation:

\[ \frac{\partial V}{\partial t} = i(t) - i_p(t) \]  

\( i(t) \) is simultaneously regulated by the rainfall intensity \( p(t) \) and the infiltration capacity \( f(t) \), as follows:

\[ i(t) = \begin{cases} p(t) & \text{if } p(t) \leq f(t) \\ f(t) & \text{if } p(t) > f(t) \end{cases} \]  

Both \( f(t) \) and \( i_p(t) \) vary linearly with \( V(t) \), as follows:

\[ f(t) = f_0 - (f_0 - f_\infty) \frac{V(t)}{V_{\text{max}}} \]  

\[ i_p(t) = f_\infty - \frac{V(t)}{V_{\text{max}}} \]  

For dry soil condition, i.e. \( V(t) = 0 \), \( f_0 \) and \( i_p(t) \) assume the values of \( f_0 \) and 0, respectively. For saturated soil condition, i.e. \( V(t) = V_{\text{max}} \), both are equal to \( f_\infty \).

Substituting Eqs. (14–16) in Eq. (13) and integrating it in \( \Delta t \), results in:

\[ V(t_{i+1}) = \begin{cases} p(t_i) V_{\text{max}} + c f_\infty V_{\text{max}} (1 - e^{-k\Delta t}) & \text{if } p(t_i) \leq f(t_i) \\ V_{\text{max}} & \text{if } p(t_i) > f(t_i) \end{cases} \]  

where \( V(t_{i+1}) = V(t) \) at \( t = t_{i+1} \) and \( p(t) \) and \( f(t) \) are the values of \( p(t) \) and \( f(t) \) at \( t = t_i \) and are assumed to be constant between \( t_i \) and \( t_{i+1} \).

The calibration of the parameters \( f_0, f_\infty \) and \( V_{\text{max}} \) was performed based on an analytical derivation of the Horton’s equation and combination with the SCS-CN method (USDA, 2004), resulting in:

\[ f_0 = \frac{V_{\text{max}}}{0.87 T_p} \ln \frac{V_{\text{max}}}{P + 0.87 V_{\text{max}}} \]  

\[ f_\infty = c_f f_0 \]  

where \( T_p \) is the duration of the rainfall event, \( P \) is the cumulated rainfall of the event and \( c_f \) is a calibration parameter varying from 0 to 1. \( V_{\text{max}} \) can be compared the maximum retention capacity \( S \) of the SCS-CN method, calculated as:

\[ V_{\text{max}} = S = 25.4 \frac{10000}{(10000 \left( \frac{10}{CN} \right) - 10)} \]  

where \( CN \) is the dimensionless curve number parameter.

It should be noted that this methodology is only valid from the moment that the cumulative rainfall \( P \) exceeds the initial abstraction \( I_a \), i.e. \( P > I_a = 0.2S \). Till that moment, all rainfall is considered to infiltrate into the upper soil layer, therefore becoming water input to the water reservoir, i.e. \( V(t) \).

### METHODOLOGY

#### Experimental tests

To evaluate the proposed two-dimensional numerical model, data from laboratory rainfall-runoff experiments on a two-directional free-drainage square soil flume were used. The experiments were performed on the laboratory setup schematised
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Fig. 1. Experimental tests: a) Sketch of the laboratory set-up with the square soil flume and rainfall simulator comprising the water reservoir and pump, hydraulic circuit and nozzles (adapted from Deng et al., 2008); b) Photograph of the 2.00 × 2.00 m² soil flume with adjustable slope in x- and y-directions (represented by the arrows), with indication of downslope gutters; and c) Top-view scheme of the downslope gutters and outlets to where runoff is conveyed and collected.

in Fig. 1. The flume has a perforated bottom sheet covered with a geotextile blanket to allow for free drainage of percolated water underneath the soil layer. The 2.00 × 2.00 m² soil flume has adjustable slope in two directions, i.e. slope in x- and y-directions. The flume was adjusted to a slope of 1% in the x-direction and 10% in the y-direction (Fig. 1b). Two gutters, each one placed along the downslope end of each slope direction, convey the surface runoff into two individual outlets (Fig. 1c).

The soil used in the experiments, characterised as sandy-loam (USDA, 1993), was comprised of 11.5% clay, 9.8% silt, 78.7% sand. Prior to the experiments, the soil was air-dried, sieved through a 5 mm mesh screen and well mixed to ensure uniformity. The soil was uniformly spread in the flume over a geotextile to allow for free drainage of water. The topsoil was flattened and tapped to attain a smooth surface, i.e. without rough elements such as microtopographic protuberances. The soil layer presented an uniform bulk density of 1565 kg m⁻³ with a uniform thickness of 0.1 m. Laboratory permeability tests yielded a saturated hydraulic conductivity of 5.70 × 10⁻⁵ m s⁻¹ and a saturated soil water content of 39%.

The rainfall simulator was comprised of three downward-oriented full-cone nozzles, a support structure, in which the nozzles were installed, and a hydraulic circuit connected to a water pump, water reservoir and tap water supply system from the public network. The nozzles (Fig. 1a) were positioned in a straight line with its direction parallel to the direction of the higher slope (i.e. y-direction), with an equal spacing of 0.95 m between them at an average height of 2.50 m from the geometric centre of the flume soil surface. The working pressure on the nozzles was kept approximately constant at 50 kPa, producing rainfall at an average intensity of 211 mm h⁻¹ at the soil flume surface, with a uniformity coefficient of 64.6%, calculated according to Christiansen (1942). Such extreme intensity was used previously in, e.g. Deng et al. (2008), to evaluate the performance of a soil erosion model. Spatial distribution of the rainfall intensity at the soil surface is shown in Fig. 2.

Fig. 2. Rainfall intensity spatial distribution at the soil surface level. Major isohyets (black lines) are in m s⁻¹. Interval between minor isohyets (grey lines) is 0.50 × 10⁻⁵ m s⁻¹. The arrows represent the slope in x- and y-directions.
The experiments consisted on four rainfall simulations each lasting 5 min, with a 48 h interval of no rain between them. The first rainfall was simulated on an initially dry soil condition with a low soil water content of 0.1%. Initial soil moisture for the following events was the resulting from the previous rainfall simulations and the 48 h drying period. After the end of an event, the amount of transported sediments was replaced for the next rainfall event. This was done by removing the entire 10 mm of topsoil layer, mixing it with the replacing amount of soil and spreading it uniformly in the flume over the remaining 90 mm of soil layer. The new topsoil was flattened and tapped to attain a new smooth soil surface, similar to the one in the beginning of the experiments.

Samples of surface runoff of the x- and y-directions were collected separately at the two outlets located at the downslope end of the soil flume, using metal containers. Sediments transported by surface runoff were estimated by drying of samples at a temperature of 110 ºC for 24–48 h, in a low temperature oven. Free water percolation below the flume was not measured.

**Model parameterisation**

The following parameters were estimated using the data collected from the laboratory experiments: i) Clear water density (\(\rho\)) and kinematic viscosity (\(\eta\)), for water at a temperature of 20°C, were fixed in 998.2 kg m\(^{-3}\) and 1.003 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}, respectively, according to Vennard and Street (1975); ii) Mean sediment particle diameter (\(d_s\)) was estimated from the soil granulometric analysis and was fixed to 4.00 \times 10^{-4} \text{ m}; iii) Soil porosity (\(\psi\)) was estimated as equivalent to the saturated water content and was fixed to 0.39; and iv) Sediment particles density (\(\rho_s\)) was estimated using the measured soil bulk density of 1565 kg m\(^{-3}\) and estimated porosity of 0.39 and was fixed on 2566 kg m\(^{-3}\).

The following parameters were estimated using the data from similar experimental work using the same laboratory set-up (i.e. soil, soil flume, rainfall simulator) and similar numerical work using similar numerical equations to express soil erosion (Deng et al., 2008): i) Manning’s roughness coefficient (\(n\)) was fixed to 0.0265 s m\(^{-1/3}\); and ii) Interrill and rill erosion control parameters (\(s\) and \(c\)) were fixed in 1.38 m\(^{-1}\) and 8.20 \times 10^{-3} \text{ m}, respectively.

The dimensionless critical Shields parameter (\(\theta_s\)) was estimated from Petit (1994) and was fixed on 0.047.

The parameters involved in the numerical simulation of infiltration (\(CN\) and \(c\)) were obtained after calibration till achievement of the best model performance. Such parameters are listed in Table 1.

Spatial discretization in both x- and y-directions (\(Ax\) and \(Ay\)) was fixed to 0.01 m.

**Table 1. Infiltration parameters used in the proposed model, for each of the four rainfall-runoff events.**

| Infiltration parameter | Rainfall event | 1\(^{st}\) | 2\(^{nd}\) | 3\(^{rd}\) | 4\(^{th}\) |
|------------------------|----------------|---------|---------|---------|---------|
| \(CN\) (-)             |                | 95.0    | 99.0    | 99.5    | 99.5    |
| \(c\) (-)              |                | 0.200   | 0.050   | 0.050   | 0.025   |

**Model evaluation**

Different criteria were used to evaluate the performance of the numerical model. Firstly, the accuracy of estimated runoff peak (\(Q_p\)) and volume (\(V\)) and transported sediments peak (\(Q_{sp}\)) and total mass (\(M_s\)) and was quantified using the relative error (\(E_r\)):

\[
E_r = \frac{\text{Obs} - \text{Mod}}{\text{Obs}} \times 100
\]

where Obs and Mod represent the observed and modelled data, respectively.

Secondly, the goodness of fit of the shape of the estimated runoff hydrographs and graphs of transported sediments was quantified using the Coefficient of determination (\(r^2\)) and the Nash-Sutcliffe model efficiency coefficient (\(NS\)) according to Nash and Sutcliffe (1970):

\[
r^2 = \left( 1 - \frac{\sum_{i=1}^{n} ((\text{Obs}_i - \text{Obs}) \times (\text{Mod}_i - \text{Mod}))^2}{\sum_{i=1}^{n} (\text{Obs}_i - \text{Obs})^2} \right) \left( \frac{\sum_{i=1}^{n} (\text{Mod}_i - \text{Mod})^2}{\sum_{i=1}^{n} (\text{Obs}_i - \text{Obs})^2} \right)
\]

\[
NS = 1 - \frac{\sum_{i=1}^{n} ((\text{Obs}_i - \text{Mod}_i)^2)}{\sum_{i=1}^{n} (\text{Obs}_i - \text{Obs})^2}
\]

where Obs and Mod, represent the observed and modelled data at point \(i\), respectively, \(\text{Obs}\) and \(\text{Mod}\) represent the average observed and modelled data and \(n\) is the total number of data points.

**RESULTS AND DISCUSSION**

Graphs of experimentally observed (symbols) and numerically modelled (solid curves) runoff (left) and transported sediments (right) for the four simulated rainfall events, are shown in Fig. 3. Results of experimentally observed (Obs) and numerically modelled (Mod) runoff peak (\(Q_p\)) and runoff volume (\(V\)) and transported sediments peak (\(Q_{sp}\)) and total mass (\(M_s\)), for the four rainfall-runoff events, are shown in Tables 2 and 3; the Relative error (\(E_r\)), the Coefficient of determination (\(r^2\)) and the Nash-Sutcliffe model efficiency coefficient (\(NS\)) comparing observed to modelled results are also shown.

When compared to the higher slope of 10% in the y-direction, the lower slope of 1% in the x-direction produced remarkably lower runoff and transported sediments (Fig. 3). Overall, this situation was more pronounced in the numerical model than in the experimental tests (see \(E_r\) in Tables 2 and 3). The numerical model significantly underestimated x-direction runoff and transported sediments, both in terms of peak and total amounts. In terms of numerical modelling goodness of fit, this situation translated in a good performance in the y-direction as opposed to a poor performance in the x-direction (see \(r^2\) and \(NS\) in Tables 2 and 3). For the x-direction, \(NS\) was always negative for both runoff and transported sediments. For the y-direction, \(NS\) was always higher than 0.75 meaning a very good performance. The poor performance observed for the x-direction may be related to the numerical treatment considered to address the wetting/drying front. The numerical treatment considered in this study rectifies the negative depth, ensuring mass continuity. However, as observed in Martins et al. (2017), the momentum is not corrected which may result in spurious oscillations of the modelled data and poorer model performance. Despite this results, \(r^2\) values were always higher than 0.75, meaning a good correlation between observed and modelled data, even for the x-direction. Only runoff results in...
the x-direction in the first rainfall event showed a different behaviour from the other results. Here, x-direction runoff was overestimated and $r^2$ value was close to 0.

As expected, due to the initial dry soil condition, observed time to runoff was significantly higher in the first rainfall event (60 s) when compared to the following events (20, 15 and 14 s). Also, the hydrograph and sediment transport graph in the first event presented significantly less steep rising limbs and lower peaks. The last three rainfall events presented similar results due to the similar initial wet condition of the soil (i.e. 24 h dry period between rainfall events). In numerical terms, since infiltration parameters were calibrated according to each rainfall event, the initial soil moisture condition did not have a visible impact in the performance model.

Fig. 3. Graphs of observed (symbols) and modelled (solid curves) runoff (left) and transported sediments (right) in the x-direction (primary axis) and y-direction (secondary axis, respectively) and for each of the four rainfall-runoff events: a) 1st rainfall event; b) 2nd rainfall event; c) 3rd rainfall event; and d) 4th rainfall event.
Table 2. Observed (Obs) and modelled (Mod) results of runoff peak (Qp) and runoff volume (V) for the four rainfall-runoff events. Relative error (Er), Coefficient of determination (r²) and Nash-Sutcliffe model efficiency coefficient (NS) are shown.

| Rainfall event | Direction | Data | Runoff |
|----------------|-----------|------|--------|
|                |           |      | Qp     | V      | r²   | NS    |
|                |           |      | ml s⁻¹ | %      |      |       |
| 1              | x         | Obs  | 22.94  | 79.57  | 2.39 | 69.47 | 0.02  | –1.16 |
|                |           | Mod  | 4.69   |        | 0.73 |       |       |
|                | y         | Obs  | 149.06 | –2.38  | 24.18| 25.77 | –6.56 | 0.93  | 0.90  |
|                |           | Mod  | 152.61 |        |      |       |       |
|                | Total     | Obs  | 157.67 | 0.23   | 26.58| 26.50 | 0.29  | 0.96  | 0.95  |
|                |           | Mod  | 157.30 |        |      |       |       |
| 2              | x         | Obs  | 5.34   | –50.39 | 1.09 | 1.97  | –80.33| 0.51  | –2.61 |
|                |           | Mod  | 4.69   |        |      |       |       |
|                | y         | Obs  | 228.73 | 4.03   | 55.70| 56.09 | –0.72 | 0.99  | 0.99  |
|                |           | Mod  | 219.52 |        |      |       |       |
|                | Total     | Obs  | 230.43 | 1.25   | 56.79| 58.06 | –2.25 | 0.99  | 0.98  |
|                |           | Mod  | 227.55 |        |      |       |       |
| 3              | x         | Obs  | 4.58   | –84.05 | 1.25 | 2.28  | –82.28| 0.80  | –12.99|
|                |           | Mod  | 8.03   |        |      |       |       |
|                | y         | Obs  | 232.15 | 2.41   | 64.11| 62.73 | 2.15  | 0.96  | 0.92  |
|                |           | Mod  | 226.54 |        |      |       |       |
|                | Total     | Obs  | 236.66 | 0.71   | 65.36| 65.01 | 0.54  | 0.96  | 0.92  |
|                |           | Mod  | 234.97 |        |      |       |       |
| 4              | x         | Obs  | 5.96   | –42.31 | 1.27 | 2.30  | –81.39| 0.83  | –3.01 |
|                |           | Mod  | 8.43   |        |      |       |       |
|                | y         | Obs  | 237.82 | 4.15   | 63.68| 63.35 | 0.51  | 0.99  | 0.98  |
|                |           | Mod  | 227.95 |        |      |       |       |
|                | Total     | Obs  | 241.80 | 2.22   | 64.95| 65.66 | –1.09 | 0.99  | 0.98  |
|                |           | Mod  | 234.97 |        |      |       |       |

Table 3. Observed (Obs) and modelled (Mod) results of transported sediment peak (Qsp) and total mass (Ms) for the four rainfall-runoff events. Relative error (Er), Coefficient of determination (r²) and Nash-Sutcliffe model efficiency coefficient (NS) are shown.

| Rainfall event | Direction | Data | Transported sediments |
|----------------|-----------|------|-----------------------|
|                |           |      | Qsp × 10²  E_r (%) | Ms  E_r (%) | r²  | NS    |
|                |           |      | g m⁻² s⁻¹ | (%)     | g    | (%)     |       |
| 1              | x         | Obs  | 0.50     | –13.70  | 2.31 | –2.83  | 0.79  | –0.79 |
|                |           | Mod  | 0.57     |         | 2.38 |         |       |
|                | y         | Obs  | 34.07    | –10.89  | 188.97| –7.95  | 0.85  | 0.80  |
|                |           | Mod  | 37.78    |         | 204.09| –9.51  | 0.85  | 0.80  |
|                | Total     | Obs  | 34.57    | –10.93  | 191.28| –7.89  | 0.85  | 0.80  |
|                |           | Mod  | 38.35    |         | 206.38| –7.89  | 0.85  | 0.80  |
| 2              | x         | Obs  | 4.96     | 70.03   | 17.26| 13.07  | 24.28 | 0.82  | –0.08 |
|                |           | Mod  | 1.49     |         | 1.25 | 2.28   | –81.39| 0.83  | –3.01 |
|                | y         | Obs  | 74.28    | 72.44   | 634.54| 658.74 | –3.81 | 0.85  | 0.80  |
|                |           | Mod  | 74.90    |         | 634.54| 658.74 | –3.81 | 0.85  | 0.80  |
|                | Total     | Obs  | 75.61    | 6.90    | 651.80| 671.81 | –3.07 | 0.85  | 0.79  |
|                |           | Mod  | 70.39    |         | 651.80| 671.81 | –3.07 | 0.85  | 0.79  |
| 3              | x         | Obs  | 0.85     | –88.47  | 8.68 | 16.44  | –139.52| 0.75  | –8.97 |
|                |           | Mod  | 1.61     |         | 16.44| 6.86   | –139.52| 0.75  | –8.97 |
|                | y         | Obs  | 69.39    | –4.46   | 690.12| 744.73 | –12.26| 0.91  | 0.79  |
|                |           | Mod  | 72.49    |         | 744.73| 690.12 | –12.26| 0.91  | 0.79  |
|                | Total     | Obs  | 69.99    | –5.87   | 696.33| 791.17 | –13.62| 0.91  | 0.79  |
|                |           | Mod  | 74.10    |         | 791.17| 696.33 | –13.62| 0.91  | 0.79  |
| 4              | x         | Obs  | 1.17     | –39.22  | 9.28 | 16.78  | –80.77| 0.87  | –2.02 |
|                |           | Mod  | 1.63     |         | 16.78| 9.28   | –80.77| 0.87  | –2.02 |
|                | y         | Obs  | 74.05    | 1.11    | 683.33| 787.41 | –14.39| 0.88  | 0.76  |
|                |           | Mod  | 73.23    |         | 683.33| 787.41 | –14.39| 0.88  | 0.76  |
|                | Total     | Obs  | 74.69    | –0.22   | 697.61| 804.19 | –15.28| 0.88  | 0.76  |
|                |           | Mod  | 74.86    |         | 804.19| 697.61 | –15.28| 0.88  | 0.76  |

Since runoff and transported sediments in the x-direction were almost meaningless, the numerical performance of the model in total terms (i.e. x- plus y-direction) was considered to be very good. Overall, runoff was slightly better modelled than sediment transport. However, as stated before, the infiltration parameters were calibrated according to each rainfall event due to changes in initial soil moisture condition with the consecutive rainfall events; i.e. the initial soil moisture of an event was the resulting from the previous rainfall simulation. On the contrary, sediment transport parameters were not calibrated; they were estimated using data collected from laboratory experiments and were fixed for all rainfall events. Even so, the performance of the model in simulating the total sediment transport was very good.
This study started from the premise that a two-dimensional model will perform better than a one-dimensional model in the case of spatially distributed rainfall. It is acknowledged that rainfall spatial distribution is often heterogeneous and complex, and its adequate representation is crucial to accurate runoff prediction (Beven et al., 2012). In this sense, only two-dimensional distributed models, as the one in developed in this study, fully represent and explore the heterogeneity and complexity of the rainfall spatial distribution. However, only a comparison with results from a one-dimensional model can ascertain this premise.

The developed model presented in this study considered steady flat soil surface and therefore did not considered the spatiotemporal evolution of the soil surface microrelief due to erosion processes, such as the formation of rills and depressions. The same simplification is common in many soil erosion models, including USLE (Wischmeier and Smith, 1978), RUSLE (Renard et al., 1997), KINEROS (Woolhiser et al., 1990), WEPP (Flanagan and Nearing, 1995) and EUROSEM (Morgan et al., 1998). The not consideration of the complex changes of the soil surface microrelief may result in large errors in soil erosion modelling Wu et al. (2020). However, in the present laboratory experiments with a short duration rainfall and small scale soil flume, no significant relief features, such as rills, have formed on the soil surface. Therefore, despite the possible errors due to this simplification, it is not expected that they have been significant.

CONCLUSION

A two-dimensional mathematical model was developed for simulating soil erosion and sediment transport resulting from rainfall induced overland flow. The model comprised: i) Two-dimensional unsteady water flow equations on an infiltrating surface; ii) Combined Horton-SCS infiltration scheme; and iii) Two-dimensional sediment transport equation with three distinct soil erosion processes (interrill erosion, rill erosion and sediment deposition).

Overall, if the sum of the x- and y-direction results is considered, the numerically simulated graphs of runoff and sediment transport were in very good agreement with corresponding experimental measurements, demonstrating the laboratory proof-of-concept of the model. However, if x- and y-directions results are analysed separately, the numerical model was only able to properly simulate the runoff and sediment transport observed in the y-direction with the higher slope of 10%. A poor agreement was observed for the remarkably lower values of runoff and sediment transport observed in the x-direction with the lower slope of 1%. Overall, since the infiltration parameters were calibrated, the initial moisture condition of the soil did not had impact on the performance of the numerical model.

Naturally, the results from this study limits the model to the descriptions of the known experimental conditions and extrapolations beyond those limits should not be made, e.g. the artificial soil sample with disturbed/sieved soil, the flat surface does not represent natural soil conditions and the steady state of the surface does not represent the spatiotemporal evolution of the soil surface microrelief. Therefore, future tests should be conducted to evaluate the performance of the model in more complex and natural field conditions and other temporal and spatial scales, such as longer rainfall events at hillslope scale.

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APPENDIX A
MacCormack scheme

In this paper, the governing equations were solved using the explicit finite-difference method based on the MacCormack operator-splitting scheme (Garcia and Kahawita, 1986; MacCormack, 1971; Simões, 2006). For simplification, Equations (1–3) and (5) were further rewritten in the following vector format:

\[
\frac{\partial U}{\partial t} + \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = gh \left( \frac{\partial Z_x}{\partial x} + \frac{\partial Z_y}{\partial y} \right) + G \tag{A1}
\]

in which:

\[
U = \begin{bmatrix} h \\ \nu_x h \\ \nu_y h \\ h_c \end{bmatrix}, \quad E_x = \begin{bmatrix} v_x h \\ v_x \nu_y h \\ v_x h_c \end{bmatrix}, \quad E_y = \begin{bmatrix} v_y h \\ v_y \nu_x h \\ v_y h_c \end{bmatrix},
\]

\[
Z_x = \begin{bmatrix} 0 \\ -Z \\ 0 \\ 0 \end{bmatrix}, \quad Z_y = \begin{bmatrix} 0 \\ 0 \\ -Z \\ 0 \end{bmatrix}, \quad G = \begin{bmatrix} p - i \\ -gh \nu_{x} \nu_{x}h \\ -gh \nu_{y} \nu_{y}h \\ e - d \end{bmatrix} \tag{A2}
\]

Eq. (A1) was divided into two separate one-dimensional problems by the operator-splitting technique as:

\[
\frac{\partial U}{\partial t} + \frac{\partial E_x}{\partial x} = gh \frac{\partial Z_x}{\partial x} + G, \quad \frac{\partial U}{\partial t} + \frac{\partial E_y}{\partial y} = gh \frac{\partial Z_y}{\partial y} + G \tag{A3}
\]

where the solution of \(U\) at time \((n+1)\Delta t\), for the computational point \((i,j)\), i.e. \(U(i\Delta t,j\Delta t,(n+1)\Delta t)\), can be obtained as follows:

\[
U_{i,j}^{n+1} = L_x^1 \frac{\Delta t}{2} L_y^1 \frac{\Delta t}{2} L_x^2 \frac{\Delta t}{2} L_y^2 \frac{\Delta t}{2} U_{i,j}^n \tag{A4}
\]

where \(L_x^1\) and \(L_y^1\) are one-dimensional finite-difference operators, each one composed of a predictor-corrector computational sequence.

In each of the \(L_x^1\) and \(L_y^1\) operators, the solution is advanced by a time step \(\Delta t/2\) as if the derivatives in the other direction were absent. Therefore, each operator is computed twice to gain the solution at the next step.

Taking \(L_x^1\) and \(L_y^1\) operators as example, their solution can be written as:

\[
L_x^1 \text{ predictor sequence (backward differences)}:
\]

\[
U_{i,j}^p = U_{i,j}^o - \frac{\Delta t}{2\Delta x} \left( E_{x,i,j}^p - E_{x,i-1,j}^o \right) + g \left( \frac{h_{i,j}^o + h_{i-1,j}^o}{2} \right) \frac{\Delta t}{2\Delta x} \left( Z_{x,i,j}^p - Z_{x,i-1,j}^o \right) + \frac{\Delta t}{2} \left( \frac{G_{i,j}^o + G_{i-1,j}^o}{2} \right) \tag{A5}
\]

\[
L_x^1 \text{ corrector sequence (forward differences)}:
\]

\[
U_{i,j}^c = \frac{1}{2} \left[ U_{i,j}^o + U_{i,j}^p - \frac{\Delta t}{2\Delta x} \left( E_{x,i+1,j}^p - E_{x,i,j}^o \right) + g \left( \frac{h_{i+1,j}^p + h_{i,j}^p}{2} \right) \frac{\Delta t}{2\Delta x} \left( Z_{x,i+1,j}^p - Z_{x,i,j}^p \right) + \frac{\Delta t}{2} \left( \frac{G_{i+1,j}^o + G_{i,j}^o}{2} \right) \right] \tag{A6}
\]

\[
L_y^1 \text{ predictor sequence (forward differences)}:
\]

\[
U_{i,j}^p = U_{i,j}^o - \frac{\Delta t}{2\Delta y} \left( E_{y,i,j}^p - E_{y,i,j-1}^o \right) + g \left( \frac{h_{i,j}^p + h_{i,j-1}^p}{2} \right) \frac{\Delta t}{2\Delta y} \left( Z_{y,i,j}^p - Z_{y,i,j-1}^p \right) + \frac{\Delta t}{2} \left( \frac{G_{i,j}^o + G_{i,j-1}^o}{2} \right) \tag{A7}
\]

\[
L_y^1 \text{ corrector sequence (backward differences)}:
\]

\[
U_{i,j}^c = \frac{1}{2} \left[ U_{i,j}^o + U_{i,j}^p - \frac{\Delta t}{2\Delta y} \left( E_{y,i+1,j}^p - E_{y,i,j}^p \right) + g \left( \frac{h_{i+1,j}^p + h_{i,j}^p}{2} \right) \frac{\Delta t}{2\Delta y} \left( Z_{y,i+1,j}^p - Z_{y,i,j}^p \right) + \frac{\Delta t}{2} \left( \frac{G_{i+1,j}^o + G_{i,j}^o}{2} \right) \right] \tag{A8}
\]
where the superscript $o$ indicates that results from the previous operator (or time step in case of $Lx_1$) should be used and superscript $p$ indicates that predicted quantities are used to obtain the corrected quantities denoted by the superscript $c$. $Lx_1$ and $Ly_1$ are like $Lx$ and $Ly$ except that a forward difference is used in the predictor step and a backward difference is used in the corrector step.

Although derivatives are discretized to first-order accuracy, the operator-splitting technique achieve second-order accuracy in space and time. The stability of the scheme can be determined by the Courant-Friedrichs-Lewy condition, which for the two-dimensional case is:

$$
\Delta t \leq \frac{CFL}{\sqrt{\Delta x^2 + \Delta y^2}} \left( \sqrt{v_x^2 + v_y^2} + \sqrt{g h} \right)_{\text{max}}
$$

(A9)

where $CFL$ is the Courant-Friedrichs-Lewy number which can take values up to 1.

**Initial conditions**

The numerical model considers rain falling on initially dry soil bed. In terms of initial conditions this translates into flow depths, flow velocities and sediment concentrations all set equal to zero for all computational points. Also, it is possible that, after the rainfall event, the soil surface dries up again. This originates numerical complications that need to be solved using specific procedures. One is the surging of very shallow water depths, in which case velocities need a special treatment (Esteves et al., 2000). The other is the wetting/drying front that can originate computational negative water depths that need to be corrected for the next time step (Martins et al., 2017).

In the case of very shallow water depths, velocities need a special treatment because the motion of a very shallow flow is not correctly described by Eqs. (1–3). Also, unrealistically large values of the friction slope are computed when such water depths occur. Therefore, in this model, for water depths lower than 0.1 mm, velocities were computed considering the kinematic approximation, as follows:

$$
v_x = \frac{1}{n} \frac{\partial Z}{\partial x}, \quad v_y = \frac{1}{n} \frac{\partial Z}{\partial y}
$$

(A10)

In the case of negative depths, the following procedure was implemented. Firstly, after each time step, the computational points were differentiated between having negative depths ($\eta_{ij} = 0$) and having null or positive depths ($\eta_{ij} = 1$). Secondly, whenever a point with a negative depth is identified ($\eta_{ij} = 0$), the sum of the positive depths of the four closer adjacent neighbours is calculated ($h_{\text{sum}_{ij}}$).

$$
h_{\text{sum}_{ij}} = (\eta h)_{i+1,j} + (\eta h)_{i-1,j} + (\eta h)_{i,j+1} + (\eta h)_{i,j-1} \quad \text{if} \quad \eta_{ij} = 0
$$

(A11)

Finally, if $h_{\text{sum}_{ij}}$ is greater than the absolute value of the negative depth of the respective point ($|h_{\text{neg}_{ij}}|$), the negative depth is set to zero and the positive depths of the four neighbours that contributed to $h_{\text{sum}_{ij}}$ are reduced a fraction, as exemplified for the neighbour point $h_{i+1,j}$:

$$
h_{i+1,j} = h_{i+1,j} - \frac{h_{\text{neg}_{ij}}}{h_{\text{sum}_{ij}}} h_{i+1,j} \quad \text{if} \quad \eta_{ij} = 0 \quad \text{and} \quad \eta_{i+1,j} \quad \text{and} \quad h_{\text{sum}_{ij}} > |h_{\text{neg}_{ij}}|
$$

(A12)

If $h_{\text{sum}_{ij}} < |h_{\text{neg}_{ij}}|$ the positive depths of the next four closer adjacent neighbours (e.g. diagonal neighbours) are added to the sum and the process is repeated.

**Boundary conditions**

In the numerical model, the physical domain was represented as a square divided in a uniform grid with longitudinal and transversal slope in $x$ and $y$ directions, respectively. Each direction is represented with a closed boundary (i.e. solid wall limiting the flow) at the upper end and an open boundary (i.e. outlet) at the lower end. Geometry and boundary characteristics of the physical domain are better explained in the “METHODOLOGY” section.

Apart from the common elimination of the normal velocity component at the upper closed boundaries, an additional condition of zero tangential velocity at the wall has shown to improve the numerical solution (Garcia and Kahawita, 1986; Simões, 2006):

$$
v_{x,UB,j} = 0 = v_{y,UB,j} \quad v_{x,UB,i} = 0 = v_{y,UB,i}
$$

(A13)

where $(UB,j)$ and $(i,UB)$ represent the computational points at the upper closed boundaries in the $x$ and $y$ direction, respectively.

Since the flow was always supercritical, the lower open boundaries were considered free and no special treatment was necessary (Garcia and Kahawita, 1986; Simões, 2006).