D-effects in Toroidally Compactified Type II String Theory

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We review exact results obtained for $R^4$ couplings in maximally supersymmetric type II string theories. These couplings offer a privileged scene to understand the rules of semiclassical calculus in string theory. Upon expansion in weak string coupling, they reveal an infinite sum of non-perturbative $e^{-1/g}$ effects that can be imputed to euclidean D-branes wrapped on cycles of the compactification manifolds. They also shed light on the relation between Dp-branes and D-(p-2)branes, D-strings and $(p,q)$ strings, instanton sums and soliton loops. The latter interpretation takes over in $D \leq 6$ in order to account for the $e^{-1/g^2}$ effects, still mysterious from the point of view of instanton calculus.

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Abstract: We review exact results obtained for $R^4$ couplings in maximally supersymmetric type II string theories. These couplings offer a privileged scene to understand the rules of semiclassical calculus in string theory. Upon expansion in weak string coupling, they reveal an infinite sum of non-perturbative $e^{-1/g}$ effects that can be imputed to euclidean D-branes wrapped on cycles of the compactification manifolds. They also shed light on the relation between Dp-branes and D-(p-2)branes, D-strings and $(p,q)$ strings, instanton sums and soliton loops. The latter interpretation takes over in $D \leq 6$ in order to account for the $e^{-1/g^2}$ effects, still mysterious from the point of view of instanton calculus.

1 Introduction

Extending string theory beyond the perturbative regime has become an important challenge over the last two years. Whereas string theory was originally formulated as a perturbative genus expansion, string duality has allowed the determination of a number of exact non-perturbative results, starting with the vector multiplet geometry in $N = 2$ four-dimensional string theories. In that case, the metric is given exactly at tree-level on the type II side, and the $e^{-1/\alpha'}$ world-sheet instanton corrections translate into $e^{-1/g^2}$ space-time instantons on the Heterotic side. On the other hand, the large order behaviour of the string perturbation series allows much stronger $e^{-1/g}$ non-perturbative effects, a typical feature of the old matrix models [1]. While such instanton effects are absent from ordinary gauge theories, they are naturally accommodated in string theory as the D-branes of type II theories, or their Wick rotated analogues. Their contributions to various couplings may be determined by duality, and one may hope to learn the rules for including and weighting them from a close scrutiny of these results. Although less ambitious, this approach aims at extending string theory into the semi-classical regime. This is only meaningful if the perturbative corrections can be brought under control, an achievement of supersymmetry in many cases.

Due to their classical action $e^{-1/\sqrt{\text{Im}} S}$ being non-holomorphic in the chiral dilaton field $S$, D-effects cannot correct the vector multiplet geometry. This restriction however does not apply to the hypermultiplet geometry, and this is where this type of contribution was first investigated [2]. In the case of a single hypermultiplet, the symmetry restrictions on the hyperkähler metric are strong enough to determine it uniquely [3]. It then exhibits an infinite sum of non-perturbative effects of order $e^{-nA/g}$, where $A$ is the area of the three-cycle parametrized by the hypermultiplet, and $n$ the summation integer. These effects can be interpreted as euclidean D2-branes wrapped $n$ times around the three-cycle, and altogether resolve the the logarithmic conifold singularity of the metric. This is dual to the resolution of the conifold singularity on the vector moduli space, which occurs via loops of massless solitons [4].

Significant progress in the understanding of the rules of instanton calculus has been achieved by examining simpler settings with more supersymmetries, namely toroidal compactifications of type II strings. U-duality often allows to completely determine amplitudes, which due to their supersymmetric
structure can receive corrections from BPS saturated instanton configurations only. Upon expansion in the weak coupling regime, they reveal infinite sums of non-perturbative effects, corresponding to D-branes wrapped on various cycles of the compactification torus. After recalling in Section 2 some background on instanton corrections in string theory, we shall review in Section 3 the results obtained on the exact $R^4$ couplings in $N = 8$ vacua\footnote{SUSY charges are counted in four-dimensional units.} and their interpretation. The reader is referred to Ref.\[5\] for a detailed discussion on instanton corrections in Type II string theory compactified on a curved manifold $K_3$ corresponding to the second part of my talk at this workshop. Related work in the context of type I string theory appeared in Ref.\[6\].

2 Instanton corrections and BPS-saturated amplitudes.

Instantons of ordinary Quantum Field Theories are saddle points of the Euclidean fundamental action $S$. They usually come in continuous families, and the integration measure $\mu$ for their collective coordinates $\phi$ can be obtained by a change of variables from the canonical fields. Part of the collective coordinates are generated by the symmetries broken by the instanton background and which are restored upon integration over $\phi$. The Gaussian fluctuations around the instanton background give rise to the fluctuation determinant $\det Q$, and the correlation functions take the symbolic form

$$\langle \rangle = \sum_{\text{sectors}} \int d\mu \langle \rangle \frac{1}{\sqrt{\det Q}} e^{-S_I}$$

In the presence of supersymmetry, the bosonic and fermionic oscillators cancel in the determinant, leaving the zero-modes only. The SUSY generators broken by the instanton background generate fermionic zero-modes that have to be saturated by a sufficient number of vertex insertions. This restricts the terms in the effective action which can receive corrections. When supersymmetry is extended, instantons can break SUSY only partially, and accordingly, terms in the effective action which are related to fermionic terms with a low number of fermions, can receive corrections from a restricted set of instantons. These are the so called BPS saturated couplings. The $1/2$-BPS saturated couplings include the $R^4$ couplings in $N = 8$, $R^2, F^4$ couplings in $N = 4$, the kinetic terms of $N = 2$ (including the hypermultiplet metric), and the D-terms of the scalar potential in $N = 1$.

Unfortunately, string theory so far does not come with a fundamental action, and the rules of instanton calculus have to be inferred rather than derived. It is however still possible to use the supergravity low-energy description of string theory to determine the possible $1/2$-BPS instantons. These include the Neveu-Schwartz (NS) 5-brane, and the type II D-branes. In order to yield finite action saddle points, such objects have to be wrapped on cycles of the compactification manifold. This in particular means that $1/2$-BPS saturated couplings in ten-dimensional heterotic string, type IIA and type I are exact in perturbation theory, while the type IIB couplings can receive corrections from the D-instantons already in ten dimensions. This is the case we now turn to.

3 $R^4$ couplings in maximally supersymmetric theories

The $R^4$ coupling in ten-dimensional type IIB theory offers the most simple setting to investigate instanton corrections. It is given in the Einstein frame by a function $f_{10}^R(\tau, \bar{\tau})$ of the only scalar $\tau = \alpha + i e^{-\phi}$ (where $e^\phi = g$ is the string coupling and $\alpha$ the Ramond scalar):

$$S_{R^4} = \int d^{10}x \sqrt{g_E} f_{10}^R(\tau, \bar{\tau}) \left( t_8 t_8 + \frac{1}{8} \epsilon_{10} \epsilon_{10} \right) R^4$$

which can be evaluated at tree-level and one-loop order:

$$f_{10}^R(\tau, \bar{\tau}) = e^{\phi/2} \left( 2\zeta(3)e^{-2\phi} + \frac{2\pi^2}{3} \right)$$

\footnote{SUSY charges are counted in four-dimensional units.}
As it stands, \( f_{10}^B \) is not invariant under the type IIB \( SL(2, \mathbb{Z})_B \) symmetry, under which \( \tau \) transforms as a modular parameter. It was realized in Ref. \([7]\) that one could supplemented it by an infinite sum of non-perturbative effects which restore the \( SL(2, \mathbb{Z})_B \) invariance:

\[
f_{10}^B = 2\zeta(3)e^{-3\phi/2} + \frac{2\pi^2}{3}e^{\phi/2} + 4\pi e^{-\phi/2} \sum_{m \neq 0} \sum_{n \neq 0} \left| \frac{m}{n} \right| K_1 \left( 2\pi e^{-\phi} |mn| \right) e^{2\pi imnA} = \sum_{(m,n) \neq 0} \left( \frac{\tau_2}{|m + n\tau|^2} \right)^{3/2}
\]

(4)

The above equality follows by Poisson resummation on the integer \( m \), after separating the contributions \( (m \neq 0, n = 0) \). \( \zeta(3) \) is Apery’s transcendental number. Using the saddle point approximation \( K_1(z) = \sqrt{\frac{\pi}{2z}}e^{-z} \left( 1 + O(1/z) \right) \) of the Bessel function, the double sum approximates to

\[
2\pi \sum_{m \neq 0} \sum_{n \neq 0} \left| \frac{mn}{n^2} \right| e^{-2\pi |mn| (e^{-\phi} \pm i\alpha)} \left( 1 + O(e^\phi) \right)
\]

(5)

which has the right form to be interpreted as a sum of D-instantons effects with action \( S_{cl} = a + i e^{-\phi} \).

The product of integers \( N = mn \) can be interpreted as the charge of the D-instantons, or alternatively the number of elementary D-instantons put together, while the measure \( \mu(N) = \sum_{n|N} \frac{1}{n} \) should be computable from a Matrix model analysis \([3]\). \( O(e^{-\phi}) \) corrections correspond to perturbative corrections around the instanton background.

Although the conjecture in Eq. (1) has not been rigorously proved due to the lack of mathematical knowledge about Eisenstein series, strong evidence has already been given \([4, 5, 10]\). It has also been extended to other type IIB ten-dimensional 1/2-BPS couplings in Refs. \([11]\). We now want to extend this conjecture to type IIB theory compactified on a N-dimensional torus \( T^N \). That these lower-dimensional results have a sensible D-brane expansion will strengthen the evidence for Eq. (4). Under compactification on a circle, no further instantons appear in the type IIB theory (in agreement with the fact that the U-duality group is still \( SL(2, \mathbb{Z}) \)), so that the \( R^4 \) coupling is simply obtained from Eq. (4) by multiplying by the length of the circle. This is easily translated on the type IIA side by T-duality:

\[
f_{9}^A = 2\zeta(3)r_A e^{-2\phi} + \frac{2\pi^2}{3r_A} + 0 + 4\pi e^{-\phi} \sum_{m \neq 0} \sum_{n \neq 0} \left| \frac{m}{n} \right| K_1 \left( 2\pi r_A e^{-\phi} |mn| \right) e^{2\pi imnA}
\]

(6)

reproducing the tree-level and one-loop contributions together with the contributions of D0-brane instantons wrapped on the circle, with action \( S_A = A + i r_A e^{-\phi} \), where \( A \) is the Wilson line of the Ramond one-form on the circle \([6]\). Upon decompactification to ten dimensions, all non-perturbative corrections together with the (degenerate) one-loop term vanish, and it follows that the \( R^4 \) coupling is exact at one-loop in ten dimensions \([6]\).

We now would like to determine the contributions from higher branes to \( R^4 \) couplings. This can be achieved by arranging the moduli into a symmetric coset representative \( M \) transforming in the adjoint of the \( E_{11-D} \) U-duality group, and postulating that the result is given by the Eisenstein series

\[
f_D(M) = \sum_{m \neq 0} \left[ m^4 M m \right]^{-2}
\]

(7)

This was carried out in Ref. \([3]\) for compactifications on \( T^2 \) and \( T^3 \), and the Eisenstein series was shown to reproduce the expected \( (p, q) \) string instanton corrections together with the perturbative terms. It however becomes rather impractical for compactifications to lower dimensions, and we instead follow another route \([4]\). The tree-level, (degenerate) one-loop and D0-brane contributions can actually be

\[\text{Here and henceforth, curl letters denote Ramond fields.}\]

\[\text{There is also a one-loop } R^4 \text{ coupling but with a different kinematical structure. The latter would also subist in the M-theory decompactification limit.}\]
written for any toroidal compactification, by simply replacing the circle of radius \(nr_A\) by any cycle of radius \(\sqrt{n^i g_{ij} n^j}\), where \(g_{ij}\) denotes the metric on the \(N\)-torus with volume \(V\):

\[
f^A_{D0} = 2\zeta(3) V e^{-2\phi} + 2 \sum_{n^i \neq 0} \frac{1}{n^i g_{ij} n^j} + 4\pi V \tau_2 \sum_{m \neq 0, n^i \neq 0} \frac{|m|}{\sqrt{n^i g_{ij} n^j}} K_1 \left( 2\pi |m| e^{-\phi} \sqrt{n^i g_{ij} n^j} \right) e^{2\pi i m A_i}\]

This expansion can actually be obtained from a (formally divergent) one-loop computation in eleven-dimensional supergravity \(\cite{12}\).

\[
f^A_{D0} = 2\pi V_{11} \int_0^\infty dt t^{5/2} \sum_{n^i \neq 0} e^{-t \mathcal{M}^2} = V_{11} \sum_{n^i \neq 0} (\mathcal{M}^2)^{-3/2}, \quad \mathcal{M}^2 = n^i g_{ij} n^j,
\]

where \(g_{IJ}\) now denotes the volume \(V_{11}\) metric on the \(N+1\)-torus. The equality of Eqs. \(\ref{eq:10}\) and \(\ref{eq:11}\) follow by Poisson resummation on the integer \(n^{11}\). This is in agreement with the fact that the D0-branes are the Kaluza-Klein modes of the supergravity multiplet upon reduction to ten dimensions \(\cite{15}\).

Equation \(\ref{eq:10}\) offers a very good starting point for determining the contributions of higher branes to \(R^4\) couplings in toroidally compactified type II theories. Indeed, all \(Dp\)-branes can be reached from the D0-branes (or from the D-instantons, for that matter) by a sequence of T-dualities. Indeed, decomposing the metric on the torus as

\[
ds^2 = R^2(dx^1 + A_a dx^a)^2 + dx^a g_{ab} dx^b,
\]

where the indices \(a, b\) run from 1 to \(N - 1\), and applying a T-duality

\[
R \leftrightarrow 1/R, \quad A_a \leftrightarrow B_{1a}, \quad A_{-a} \leftrightarrow B_{-a}, \quad B_{ab} \leftrightarrow B_{ab} - A_a B_b + B_a A_b, \quad e^{-2\phi} R = \text{const.},
\]

the classical action of a D0-brane

\[
S_{D0} = e^{-\phi} \sqrt{n^i g_{ij} n^j + i n^i A_i}
\]

turns into

\[
S_{D1} = e^{-\phi} \sqrt{\left( n + \frac{1}{2} n^{ij} B_{ij} \right)^2 + \frac{1}{2} n^{ij} g_{ik} g_{jl} n^{kl} + i \left( n A + \frac{1}{2} n^{ij} \hat{B}_{ij} \right)},
\]

where we reinterpreted \(n^i\) as a singlet \(n\) and \(n^a\) as the component of a two-form \(n^{1a}\). The integers \(n^{a\beta}\) in Eq. \(\ref{eq:13}\) do not have a counterpart in Eq. \(\ref{eq:11}\), but their presence is inferred from the requirement of \(SL(N, \mathbb{Z})\) symmetry. \(\hat{B}\) coincides with the Ramond two-form up to a linear shift in \(B\) \(\cite{14}\). Equation \(\ref{eq:13}\) can be interpreted as the classical action of a D-string wrapped on a two cycle of the torus. Indeed, evaluating the Born-Infeld action

\[
S_{BI} = \int \frac{1}{g} \sqrt{\det(\hat{G} + \hat{B} + F)} + i e^{\hat{B} + F} \wedge \mathcal{R},
\]

on a configuration

\[
X^i(\sigma^\alpha) = N^i_a \sigma^a, \quad n^{ij} = e^{\alpha \beta} N^i_a N^j_b, \quad F_{\alpha \beta} = e_{\alpha \beta} n
\]

precisely reproduces Eq. \(\ref{eq:13}\). The action for \(n^{ij} = 0\) reduces to the action of \(n\) type IIB D-instantons. The integer \(n\), corresponding to the flux of the \(U(1)\) gauge field on the D-string, therefore describe the number of D-instantons bound to the string. The contribution of D-strings and D-instantons can therefore be summarized in the following series:

\[
f^B_{D1} = 4\pi V e^{-\phi} \sum_{m \neq 0, (n, n^i) \neq 0} \frac{|m|}{\sqrt{\det(\hat{G} + \hat{B} + F)}} K_1 \left( 2\pi |m| e^{-\phi} \right) \sqrt{\det(\hat{G} + \hat{B} + F)} e^{2\pi i m \int \hat{A}_+ + A_F}
\]

While this result is by construction T-duality invariant when \(D \geq 7\) (in lower dimensions, the D-3 brane has to be included), it is not guaranteed to be U-duality invariant. In order to check the invariance
under $SL(2, \mathbb{Z})_B$, which together with the T-duality $SO(N, N, \mathbb{Z})$ generates the full U-duality group, it is convenient to perform a Poisson resummation on the integer $n \to p$ (this amounts to going to the $\theta$ vacuum in the world-sheet gauge theory) and rename the integer $m \to q$. Under this operation, the Bessel function $K_1(\theta)$ turns into $K_{1/2}(\theta) = \sqrt{\frac{\pi}{2\theta}} e^{-x}$, and the above expression becomes

$$f_{D1}^B = 4\pi V \sum_{1 \neq 0} \sum_{r/s=1} \sum_{n^{ij}} e^{-2\pi i |p+q|} \sqrt{n^{ij}} g_{ij} g_{kl} n^{ij} + 2\pi i n^{ij} \left(qB_{ij} - pB_{ij}\right) \left(\frac{\pi}{nq}\right)$$

This sum can now be interpreted as the contribution of $(p, q)$ strings to the four-graviton amplitude. Indeed, the term with $(p, q) = (1, 0)$ corresponds to the one-loop world-sheet instantons on the fundamental string, and the other terms are obtained by replacing the string tension and the NS two-form by

$$\alpha' \to \frac{\alpha'}{|p + q|}, \quad B_{ij} \to pB_{ij} - qB_{ij}$$

as appropriate for a $(p, q)$ string. Note that as expected, only strings with $p$ and $q$ coprime contribute. This is a rather dramatic change of perspective, since what was previously interpreted as a sum of instanton effects now is now described as a soliton sum on the BPS excitations of the $(p, q)$ string.

This sequence of T-duality can be pursued in order to obtain the contributions of higher branes:

$$f_{D2}^A = e^{-\phi} \left[ (n^i + \frac{1}{2} n^{ijk} B_{jk}) g_{il} (n^l + \frac{1}{2} n^{mn} B_{mn}) + \frac{1}{6} n^{ijk} g_{il} g_{jm} g_{kn} n^{mn} \right]^{1/2} + i \left( n^i A_i + \frac{1}{6} n^{ijk} C_{ijk} \right)$$

$$f_{D3}^B = e^{-\phi} \left[ (n^i + \frac{1}{2} n^{ijkl} B_{ijkl})^2 + \frac{3}{2} (n^i + \frac{1}{2} n^{ijkl} B_{ijkl}) g_{im} g_{jn} \left( n^{mn} + \frac{1}{2} n^{mpq} B_{pq} \right) \right] + i \left( n^i \tilde{A}_i + \frac{1}{24} n^{ijkl} \tilde{B}_{ijkl} + \frac{1}{24} n^{ijkl} \tilde{C}_{ijkl} \right)$$

which can again be interpreted as the Born-Infeld action for D2- (resp. D3-) branes wrapped on three-cycles $n^{ijk}$ (resp. $n^{ijkl}$), with fluxes $n^i$ (resp. $n^{ij}$). The integer $n$ is now interpreted as the gauge instanton number $\frac{1}{2} e^{\alpha i \beta \delta} F_{\alpha \beta} F_{\gamma \delta}$ on the D3-brane world-volume. By construction, these actions are again T-duality invariant for $D \geq 6$ (resp. $D \geq 5$). In fact, one can see the expression under the square root of Eq. (19) and (20) as the norm of the two spinors of $SO(N, N)$ induced by the coset representative $g + B$ of $SO(N, N)/(SO(N) \times SO(N))$, after decomposition in terms of $Sl(N)$ irreducible representations [16]. However, U-duality is not guaranteed, and in fact not fulfilled. Indeed, going to the representation of Eq. (8), we find that the D2-brane sum translates into

$$M^2 = R_{11}^2 \left( n^{11} + A n^i + \frac{1}{6} n^{ijk} C_{ijk} \right)^2 + \left( n^i + \frac{1}{2} n^{ijkl} B_{ijkl} \right) g_{il} \left( n^l + \frac{1}{2} n^{mn} B_{mn} \right) + \frac{R_{11}^2}{6} n^{ijk} \frac{g_{ij} g_{km} g_{ln}}{R_{11}^2} n^{lmn}$$

so that $Sl(11 - D, \mathbb{Z})$ invariance with respect to the metric $ds_{11}^2 = R_{11}^2 (dx^1 + A_i dx^i)^2 + \frac{1}{R_{11}^2} dx^i g_{ij} dx^j$ can be only be recovered by extending the three-form $n^{ijk}$ into a four-form $n^{ijkl}$:

$$M^2 = \left( n^i + \frac{1}{6} n^{ijkl} C_{ijkl}^{(11)} \right) g_{IM} \left( n^M + \frac{1}{6} n^{MNPO} C_{NOPQ}^{(11)} \right) + \frac{R_{11}^2}{24} n^{ijkl} g_{IM} g_{JN} g_{KP} g_{LQ} n^{MNPO}$$

Whereas the D-brane charges $n^i, n^{ijk}$ contribute $O(1/g^2)$ to $M^2$ (and therefore $e^{-1/g}$ after Poisson resummation on $n^{11}$), the extra charge $n^{ijkl}$ starting to occur for $D \leq 6$, contributes $O(1/g^4)$ to $M^2$. It therefore belongs to another T-duality multiplet, and generates $e^{-1/g^2}$ non-perturbative effects in the corresponding Eisenstein series. Such contributions are usually imputed to the NS fivebrane, but the latter cannot generate instanton effects when $D > 4$. The situation is not any better on the type IIB
side, where $Sl(2, \mathbb{Z})_B$ invariance requires the introduction of an extra four-form $m^{ijkl}$ transforming as a doublet with the D3-brane wrapping number $n^{ijkl}$.

This puzzle can be resolved by noting that the mass formula (22) is nothing but the mass formula for BPS strings of M-theory [10], corresponding to membranes and fivebranes wrapped on a one-cycle or a four-cycle of the compactification torus $T^{N+1}$. These states are identified with the states of the momentum multiplet of M-theory in the Discrete Light-Cone Quantization [11], upon unwinding them from the light-cone circle. The $R^4$ coupling is therefore given by the sum of the one-loop amplitudes of all BPS strings obtained by reduction of the M-theory extended states on $T^{N+1}$. This is a rather plausible result, since after all our strategy was to covariantize the fundamental string contribution under the U-duality group.

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[18] Another way out may be that the $R^4$ couplings are not given by Eisenstein series for $D \leq 6$. An indication that this may be the case is the fact that the $D = 6$ $SO(5,5,\mathbb{Z})$ Eisenstein series has so far resisted to our many attempts to extract the perturbative contributions.

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