Assessment of technological resources for the production of composite products based on mathematical methods

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Abstracts. This article is devoted to the selection and justification of the use of the corresponding mathematical methods in the formation of a composite product production program and the assessment of the importance of provided technological resources. The choice of marginal profit as criteria of the optimality of the production program is proved. The formalized mathematical formulations of integer and dual tasks of linear programming are presented. The tasks of analyzing the annual release plan for a range of six products is solved. It is concluded that the usage of mathematical methods of linear programming and the theory of dual is successful decision in the field of development of composite production programs.

Key words: composite products, technological production resources, economic and mathematical methods.

1. Introduction

The current level of technological development necessitates the usage of materials that have special properties that are not inherent in traditional structural materials [10]. In this regard, in various areas of technology, composite materials are being actively introduced. [5]. Which leads to breakthrough growth of the composite industry. Thus, according to the estimates of the reputable statistical portal Statista, the global market for composite materials will grow in 2017 to 2024 from 76.71 to 130.83 billion US dollars [9].

High-intensity development of the industry requires effective management of production [4]. This task can be attributed to the field of engineering. One of the important sections of engineering is the development of recommendations to achieve the best result from the spent investments in the organization of the technological process of production [3]. The specificity of the manufacturing process for the production of composite products suggests the possibility of using versatile capital assets for the production of a wide range of products applicable in various engineering industries [1]. This raises the problem: what types of products are the most profitable for the company to produce taking into account the available technological resources and in the conditions of a limited available stock of working hours of capital assets?

The purpose of this study is the selection and justification of the use of the corresponding mathematical methods for the formation of the optimal production program for the composite industry enterprise.
2. Research methods

The task of finding the best distribution of limited technological resources between manufactured products is the task of integer linear programming. In order to build a mathematical model of this task, it is necessary to choose criteria of optimality [8]. It is appropriate to use marginal profit as criteria of optimality, since its specific value does not depend on the volume of output and allows to bring out an objective function that is linear in volume of output [7]. At the same time, the maximum of margin profit and the maximum of operating profit - a key indicator of the effectiveness of the main production activities of the enterprise [6], are achieved at the same point in the space of acceptable release plans [2].

The limitations are, on the one hand, the available working time funds for each type of main technical equipment (capital assets) and production plans for each type of product on the other. The total system of conditions is as follows:

\[
\begin{align*}
\sum_{j=1}^{n} X_j \cdot a_{ij} & \leq b_i, \quad \forall i \in \{1; m]\ \\
X_j & \geq B_j, \quad \forall j \in J^1 \\
X_j & = B_j, \quad \forall j \in J^2 \\
X_j & \in \mathbb{Z} \\
\sum_{j=1}^{n} X_j \cdot p_j & \rightarrow \max
\end{align*}
\]

Where \( j \) – sequence number of product; \( i \) – sequence number of technical equipment; \( a_{ij} \) – time spent on processing the \( j \)-product on \( i \)-type of technical equipment; \( b_i \) – Fund of working hours of the \( i \)-type of technical equipment; \( p_j \) – marginal profit from the sale of the \( j \)-product; \( B_j \) – Production plan for type \( j \) product; \( J^1 \) – set of products for which it is permissible to overfulfill the plan, for the reason that they can be sold on the open market; \( J^2 \) – set of products for which exact fulfillment of the plan is required, that is, they cannot be sold on the open market and are available only on pre-order; \( X_j \) – the number of manufactured products of type \( j \).

The solution to this problem is the vector of optimal volume with fixed current restrictions [12]. To further improvement of the proposed solution, it is necessary to consider the effect of restrictions on the objective function. It is required to determine the deficiency of resources and the criticality of product release plans. A quantitative assessment of these parameters will make it possible to draw conclusions about possible ways to increase profits by revising obligations to supply certain types of products or by attracting additional equipment working time.

This analysis is performed by solving the dual linear programming task, in the following statement:

\[
\begin{align*}
\sum_{i=1}^{m} y_i \cdot a_{ij} - V_j & \geq p_j, \quad \forall j \in \{1; n\} \\
y_i & \geq 0, \quad \forall i \in \{1; m\} \\
V_j & \geq 0, \quad \forall j \in J^1 \\
\sum_{i=1}^{m} y_i \cdot b_i - \sum_{j=1}^{n} V_j \cdot B_j & \rightarrow \min
\end{align*}
\]

Where: \( y_i \) – dual resource rating; \( V_j \) – dual product rating.

An economically dual resource rating can be interpreted as the rental cost of an hour of operation of this type of technical equipment. Thus, if an enterprise has the opportunity, in one way or another, to increase up to some point the available fund of working time of technical equipment with unit costs less than the value of the dual rating of the hours of operation of this technical equipment, then you it is feasible to take this opportunity. The result of making the described management decision will be an increase in the total marginal profit by

\[
\sum_{i=1}^{m} (y_i - r_i) \cdot \Delta b_i
\]

Where \( r_i \) – the rental cost of an hour of operation of \( i \)-type of technical equipment; \( \Delta b_i \) – increase in available working time of \( i \)-type of technical equipment.

The positive dual rating of a product can be interpreted as the amount of a penalty for failure to fulfill a delivery plan per one unit of a product. If the enterprise has the opportunity to refuse to release a certain number of products included in the plan and also having a positive dual rating, while suffering specific losses associated with the payment of the penalty, lower than the dual rating, then this opportunity should
be taken. The negative dual rating of a product can be interpreted as the amount of discounts provided in conditions of price discrimination that a manufacturer can offer to increase demand for this type of product. If it is possible to sell an additional unit of products that have a negative dual rating, lowering the price by an amount less than the absolute value of the dual rating, then this move will bring additional profit [11].

The result of making the described management decisions will be an increase in the total marginal profit by:

$$\sum_{j=1}^{n} P_j$$ (4)

Where $P_j$ – profit margin increase due to changes in the plan for the j product type, calculating as follows:

$$P_j = \begin{cases} (V_j-l_j)\Delta B_j, & V_j>0 \\ -(V_j+s_j)\Delta B_j, & V_j<0 \\ 0, & V_j=0 \end{cases}$$ (5)

Where $l_j$ – the amount of the penalty paid for the refusal to supply one unit of the j product type; $\Delta B_j$ – change of the planned production volume for the j product type; $s_j$ – discount provided for an additional unit of the j product type.

For a relatively small nomenclature of production, the numerical solution of the two tasks formulated above can be implemented in the MS Excel using the Solver Add-in. Also, this tool provides an opportunity to assess the boundaries in which the criticality of resources does not change and the linearity of profits by changing restrictions is keeping the same. Described ratings can be extracted from the Sensitivity Report. [13].

3. Results

In this study, we analyzed the annual plan for the production of a product range consisting of six positions: a helicopter blade, a covering for the main beam of a helicopter tail rotor, a tactical helmet, and a fender of the car, hood of the car and body panel of a car, taking into consideration that the company operates in one shift. In view of the specifics of the products, the products «Helicopter blade» a «Covering for the main beam of a helicopter tail rotor» are made on pre-order, and therefore the volume of products is strictly equal to the production plan. Other types of products are produced in the amount not less than planned. The following solution is obtained for the direct integer linear programming task.

| Technical equipment type | Products | Fund of working hours of technical equipment (hours/year) | Total technical equipment runtime |
|--------------------------|----------|----------------------------------------------------------|----------------------------------|
| Vacuum pump              |          |                                                          |                                  |
| Oven                     |          |                                                          |                                  |
| Autoclave                |          |                                                          |                                  |
| Milling machine          |          |                                                          |                                  |
| Vertical drilling machine|          |                                                          |                                  |
| Grinding machine         |          |                                                          |                                  |
| Spray booth              |          |                                                          |                                  |

Table 1. Solution direct integer linear programming task
The optimal production plan with fixed restrictions is presented in the line «Production volume». The total marginal profit provided by this release plan is presented in the last cell of the «Total marginal profit» line.

To further improve the proposed production plan, it is necessary to consider the solution of the dual task, reflecting the influence of restrictions on the objective function.

### Table 2. Solution of dual task

| Technical equipment type | Helicopter blade | Covering for the main beam of a helicopter tail rotor | Tactical helmet | Fender of the car | Hood of the car | Body panel of the car | Fund of working hours of technical equipment (hours/year) | Shadow price |
|--------------------------|-----------------|---------------------------------|-----------------|------------------|---------------|----------------------|----------------------------------------------------------|-------------|
| Vacuum pump              | 0.5             | 4                               | 0.5             | 3                | 4             | 2                    | 1682.15                                                  | 20 000      |
| Oven                     | 0               | 2.5                             | 2.5             | 2.5              | 2.5           | 0                    | 1781.10                                                  | 0           |
| Autoclave                | 3               | 0                               | 0               | 0                | 2.5           | 0                    | 1781.10                                                  | 16 000      |
| Milling machine          | 1.2             | 1.2                             | 0.3             | 0.5              | 0.7           | 1.5                  | 1721.73                                                  | 0           |
| Vertical drilling machine| 0.3             | 0.5                             | 0.2             | 0.1              | 0.3           | 0.3                  | 1840.47                                                   | 0           |
| Grinding machine         | 1               | 0.2                             | 0.5             | 0                | 0             | 2                    | 1840.47                                                   | 0           |
| Spray booth              | 2               | 0.5                             | 1               | 1                | 0             | 1840.47              | 0                                                        |             |
| Infrared lamp 1          | 1               | 1                               | 1               | 1                | 1             | 1880.05              | 0                                                        |             |
| Infrared lamp 2          | 1               | 1                               | 0               | 0                | 0             | 1880.05              | 0                                                        |             |
| Infrared lamp 3          | 1               | 1                               | 0               | 0                | 0             | 1880.05              | 0                                                        |             |
| Infrared lamp 4          | 1               | 1                               | 0               | 0                | 0             | 1880.05              | 0                                                        |             |
| Release plan             | 280             | 70                              | 100             | 70               | 55            | 70                   | 61 274 600                                               |             |
| Reduced Cost             | -19 400         | 25 820                          | 0               | 40 000           | 30 000        | 0                    | 61 315 200                                               |             |
| Marginal profit per unit | 77 400          | 54 180                          | 10 000          | 20 000           | 50 000        | 80 000               | 61 315 200                                               |             |
| Total marginal profit    | 21 672 000      | 3792 600                        | 158 000         | 1400 000         | 275 000       | 30 080 000           | 61 274 600                                               |             |
| Total marginal profit    | 21 672 000      | 3792 600                        | 158 000         | 1400 000         | 275 000       | 30 080 000           | 61 274 600                                               |             |
According to the calculation results, it was determined that the «Vacuum Pump» and the «Autoclave» are lacking resources, the dual ratings of which are greater than 0. Each additional hour of operation of these types of technical equipment will bring an additional 20,000 and 16,000 rubles into the objective function. Products with dual ratings <0 ("Helicopter Blade") are profitable for production, but are produced strictly according to plan. An increase in the output plan for these products would increase the value of the objective function by the absolute value of the dual valuation of 19,400 rubles.

The types of products which have received dual ratings = 0 («tactical helmet», «body panel») are produced in accordance with the optimal plan; Products with dual ratings> 0 («Covering for the main beam of a helicopter tail rotor», «fender», «hood of the car») are not profitable for production, thus reducing the plan per unit of this type of product will lead to an increase in the value of the objective function by the absolute value of the corresponding dual assessment (25,820 rubles for the «Covering for the main beam of a helicopter tail rotor»; 40,000 rubles for the «fender» and 30,000 rubles for the «hood of the car»)

The boundaries within which the calculated dual ratings are valid can be extracted from the Sensitivity report.

### Table 3. The Sensitivity Report

| Products dual ratings | Cell  | Name                                           | Final Value | Reduced Cost \( -V_j \) | Objective Coefficient | Allowable Increase | Allowable Decrease |
|-----------------------|-------|------------------------------------------------|-------------|-------------------------|---------------------|--------------------|--------------------|
| $B$15                 | Helicopter blade | 280 | 19400 | 77400 | 19400 | $\infty$          |
| $C$15                 | Covering for the main beam of a helicopter tail rotor | 70 | -25820 | 54180 | 25820 | $\infty$          |
| $D$15                 | Tactical helmet | 158.54 | 0 | 10000 | 5105.26 | 3227.50        |
| $E$15                 | Fender of the car | 70 | -40000 | 20000 | 40000 | $\infty$          |
| $F$15                 | Hood of the car | 55 | -30000 | 50000 | 30000 | $\infty$          |
| $G$15                 | Body panel of the car | 376.44 | 0 | 80000 | $\infty$ | 16166.67        |

| Technical equipment dual ratings | Cell | Name             | Final Value | Shadow price \( y_i \) | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|----------------------------------|------|------------------|-------------|--------------------------|---------------------|--------------------|--------------------|
| $I$3                              | Vacuum pump | 1682.15 | 20000 | 1682.15 | 179.45 | 29.27           |
| $I$4                              | Oven | 883.85 | 0 | 1781.10 | $\infty$ | 897.25        |
| $I$5                              | Autoclave | 1781.1 | 16000 | 1781.10 | 36.59 | 224.31        |
| $I$6                              | Milling machine | 1105.722 | 0 | 1721.73 | $\infty$ | 616.01        |
| $I$7                              | Vertical drilling machine | 287.14 | 0 | 1840.47 | $\infty$ | 1553.33        |
| $I$8                              | Grinding machine | 1126.15 | 0 | 1840.47 | $\infty$ | 714.32        |
| $I$9                              | Spray booth | 904.27 | 0 | 1840.47 | $\infty$ | 936.20        |
| $I$10                             | Infrared lamp 1 | 633.54 | 0 | 1880.05 | $\infty$ | 1246.51        |
| $I$11                             | Infrared lamp 2 | 350.00 | 0 | 1880.05 | $\infty$ | 1530.05        |
| $I$12                             | Infrared lamp 3 | 350.00 | 0 | 1880.05 | $\infty$ | 1530.05        |
| $I$13                             | Infrared lamp 4 | 350.00 | 0 | 1880.05 | $\infty$ | 1530.05        |

The boundaries in which dual ratings are keeping the same are contained in the columns «Allowable Increase» and «Allowable Decrease».

### 4. Conclusion

According to the results of the study, it is clear that the mathematical methods of integer linear programming and duality theory can be effectively used in solving engineering issues during the process of the formation of a composite production program and assessing the significance of available technological resources.
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