Neutrino Mixings and Fermion Masses in Supersymmetric SU(5)

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Abstract

We consider neutrino mixings in supersymmetric SU(5) supplemented by a $U(1)$ flavor symmetry. In particular, we show how bi-maximal neutrino mixings can be realized. Two scenarios for implementing the small mixing angle MSW solution, with one involving a sterile state $\nu_s$ and maximal $\nu_\mu - \nu_\tau$ mixing to resolve the atmospheric anomaly, are also discussed. Finally, a new mechanism for eliminating the asymptotic relations $m_\mu^{(0)} = m_s^{(0)}$ and $m_e^{(0)} = m_d^{(0)}$, while retaining $m_\tau^{(0)} = m_b^{(0)}$, is presented. It employs ‘matter’ multiplets in the $15 + 1\bar{5}$ of SU(5), and is consistent with the various oscillation scenarios, as well as with the unification at $M_{GUT}$ of the three gauge couplings.

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The recent atmospheric neutrino results [1] from Superkamiokande (SK) have greatly enhanced the case for new physics beyond the standard model (SM). The data is found to be consistent with a two-flavor model involving $\nu_\mu - \nu_x$ oscillations, where $\nu_x$ could either be $\nu_\tau$ or a (new) sterile neutrino $\nu_s$. The mixing angle is estimated to be large, $\sin^2 2\theta \sim 0.8$, with $\Delta m^2 \sim 10^{-2} - 10^{-3}$eV$^2$. It is difficult to see how neutrino masses of order $10^{-1}$eV (or larger, in case some masses are degenerate) can arise from the SM or its minimal supersymmetric extension (MSSM), without involving new physics at some ‘intermediate’ mass scale. Non-zero neutrino masses from Planck scale ($M_P = 2.4 \times 10^{18}$ GeV) suppressed dimension five operators are expected to be $\sim 10^{-5}$eV (or so).

The solar neutrino data offers independent evidence for neutrino oscillations [2]. The data is consistent with either vacuum oscillations in which $\sin^2 2\theta \sim 0.7 - 1.0$ and $\Delta m^2 \sim 5 \times 10^{-11} - 10^{-10}$eV$^2$, or the small mixing angle MSW solution with $\sin^2 2\theta \sim \text{few} \cdot 10^{-3}$ and $\Delta m^2 \sim \text{few} \cdot 10^{-6}$eV$^2$. It is found that the data favors a vacuum solution involving only active neutrinos, while the MSW solution can involve a sterile neutrino ($\nu_e \rightarrow \nu_s$).

In a recent paper [3], we presented a systematic approach, based on $\nu_{\text{MSSM}}$ (MSSM augmented with the seesaw mechanism) and its extension for incorporating the various two-flavor neutrino oscillations allowed by the atmospheric and solar neutrino data. It utilizes in an essential way a flavor $U(1)$ symmetry [4], whose spontaneous breaking by (the scalar component of) a MSSM singlet field $X$ yields an important ‘expansion’ parameter $\epsilon \equiv \langle X \rangle / M_P (\simeq 0.2)$. The $U(1)$ symmetry together with $\epsilon$, also plays an important role in understanding the quark and charged lepton mass hierarchies, as well as the magnitudes of the CKM matrix elements. The approach we followed was especially guided by the desire to realize maximal mixing ($\sin^2 2\theta = 1$) to explain the atmospheric anomaly, which seems favored by the SK data. A general mechanism for achieving this was presented in ref. [3], and it was noted that except for $\nu_{\text{MSSM}}$ and its SU(5) extension, this typically required the existence of a new (sterile) neutrino in order to simultaneously account for the solar and atmospheric neutrino puzzles. For instance, maximal $\nu_\mu - \nu_\tau$ oscillations can be accompanied by the small mixing angle MSW solution involving $\nu_e$ and $\nu_s$. It was pointed out that a $U(1) - R$ symmetry can be particularly effective at keeping a sterile neutrino light.

The purpose of this letter is twofold. First, we would like to demonstrate how a variety of two-flavor neutrino oscillations scenarios, that are compatible with the current atmospheric and solar neutrino data, can be realized within a $SU(5)$ setting, supplemented by singlet (right handed) superfields and a $U(1)$ flavor symmetry. In particular, we show how bi-maximal mixings [5] among the active neutrinos, which is consistent with the latest atmospheric and solar neutrino experiments [6], can be realized. To do this, we first utilize a version of the seesaw mechanism discussed in [7] to obtain large (including maximal) mixing in the $\nu_\mu - \nu_\tau$ sector, with only one neutrino acquiring a non-zero mass. We then invoke the mechanism described in [3] to implement maximal $\nu_e - \nu_{\mu,\tau}$ mixing...
to resolve the solar neutrino puzzle. This is followed by a discussion of how the small mixing angle MSW solution can be realized in two distinct ways, with one (non-minimal) approach yielding a light sterile state and gives possibility of existence of the neutrino hot dark matter. Our second goal here is to provide a new mechanism for eliminating the unacceptable asymptotic relations predicted in $SU(5)$ with minimal higgs, to wit, $m_{\mu}^{(0)} = m_{\tau}^{(0)}$ and $m_{e}^{(0)} = m_{d}^{(0)}$, retaining in the process the desirable relation $m_{\tau}^{(0)} = m_{b}^{(0)}$. This we achieve by introducing a pair of $15 + \overline{15}$ ‘matter’ multiplets (rather than a 45-plet of higgs [8] as is commonly done). It should be stressed that this mechanism is consistent with the various oscillations scenarios, and it also preserves the unification at $M_{GUT}$ of the three gauge couplings.

The minimal $SU(5)$ model contains the following ‘matter’ multiplets (we assume a $Z_{2}$ ‘matter’ parity which distinguishes the ‘matter’ and ‘higgs’ supermultiplets):

$$10_{\alpha} = (q, u^{c})_{\alpha}, \quad 5_{\alpha} = (d^{c}, l)_{\alpha}, \quad (1)$$

where $\alpha = 1, 2, 3$ denotes the flavor index. The transformation properties under $SU(3)_{c} \times SU(2)_{L}$ of the various superfields are:

$$q(3, 2), \quad u^{c}(\overline{3}, 1), \quad e^{c}(1, 1), \quad d^{c}(\overline{3}, 1), \quad l(1, 2). \quad (2)$$

The higgs multiplets consist of $\Sigma(24)$, $H(5)$, $\overline{H}(\overline{5})$, where the 24-plet breaks $SU(5)$ to $SU(3)_{c} \times SU(2)_{L} \times U(1)_{Y}$, and $H(5)$ ($\overline{H}(\overline{5})$) contains the electroweak doublet $h_{u}$ ($h_{d}$) which provides masses for the up quarks (down quarks and charged leptons).

We now introduce a $U(1)$ flavor symmetry with a judicious choice of flavor charges which yield the observed quark and charged lepton mass hierarchies, as well as the magnitudes of the CKM matrix elements. For the asymptotic Yukawa couplings we will assume the following ratios:

$$\lambda_{u} : \lambda_{c} : \lambda_{t} \sim \epsilon^{6} : \epsilon^{4} : 1,$$

$$\lambda_{d} : \lambda_{s} : \lambda_{b} \sim \epsilon^{5} : \epsilon^{2} : 1,$$

$$\lambda_{e} : \lambda_{\mu} : \lambda_{\tau} \sim \epsilon^{5} : \epsilon^{2} : 1. \quad (3)$$

For the CKM matrix elements we take

$$V_{us} \sim \epsilon, \quad V_{cb} \sim \epsilon^{2}, \quad V_{ub} \sim \epsilon^{3}. \quad (4)$$

One of our goals is to achieve large (including maximal) mixing in the $\nu_{\mu} - \nu_{\tau}$ sector [9].

These considerations determine the following Yukawa matrix structure for the down quarks and charged leptons (the matrix elements are determined only up to factors of order unity; for simplicity, we will ignore CP violation):

2
\[
Y_{d,e} \sim \begin{pmatrix}
\tilde{5}_1 & 10_1 & 10_2 & 10_3 \\
\tilde{5}_2 & e^5 & e^4 & e^2 \\
\tilde{5}_3 & e^3 & e^2 & 1 \\
\end{pmatrix} e^a,
\]

where \(a = 0, 1, 2\) determines the value of the MSSM parameter \(\tan\beta(\sim \frac{m_t}{m_b}e^a)\). The texture in (5) is realized for the following prescription of the \(U(1)\) charges of the various supermultiplets:

\[
\begin{align*}
Q_{5_3} &= Q_{5_2} = q_1, & Q_{5_1} &= q_1 - 2, \\
Q_{10_3} &= q_2, & Q_{10_2} &= q_2 - 2, & Q_{10_1} &= q_2 - 3, \\
Q_H &= -a - q_1 - q_2, & Q_H &= -2q_2.
\end{align*}
\]

Here and below the \(U(1)\) charges of the supermultiplets are presented in units of the charge of the \(X\) superfield (\(Q_X = 1\)). The charges \(q_1\) and \(q_2\) are arbitrary for the time being. Note that \(\tilde{5}_2\) and \(\tilde{5}_3\) carry the same \(U(1)\) charge in order to accommodate large mixing in the \(\nu_{\mu} - \nu_\tau\) sector [9].

Similarly, taking into account (6), the up quark Yukawa matrix has the following texture

\[
Y_u \sim \begin{pmatrix}
10_1 & 10_2 & 10_3 \\
10_2 & e^5 & e^4 & e^2 \\
10_3 & e^3 & e^2 & 1 \\
\end{pmatrix} e^a.
\]

We see that large (hopefully even maximal) mixing can arise in the \(\nu_{\mu} - \nu_\tau\) sector in a rather straightforward manner [8]. In order to achieve the desired mass splitting \(\Delta m^2 \sim 10^{-2} - 10^{-3} \text{ eV}^2\), let us choose \(q_1 - 2q_2 = 0\), and introduce a SU(5) singlet superfield \(N\), with \(Q_N = 0\). Consider the superpotential couplings:

\[
W_N = M_N N^2 + (a e^{2\tilde{5}_1} + b \tilde{5}_2 + c \tilde{5}_3) H N,
\]

where the coefficients \(a, b, c\) are all of order unity. Through the seesaw mechanism we obtain the following mass for the ‘light’ mass eigenstate

\[
m_{\nu_3} \sim \frac{h^2}{M_N} \sim \sqrt{\Delta m^2_{\nu_{\mu\tau}}}.
\]

Clearly, the two remaining states are still massless.
In order to achieve large (especially maximal) mixing, say between $\nu_e - \nu_\mu$, to resolve the solar neutrino puzzle, we could try assigning a zero $U(1)$ charge to $\bar{5}_1$. However, coupled with the charge assignment $10_1(-5)$, this implies $V_{us} \sim \epsilon^3$ which is unacceptable! (In $\nu$MSSM this strategy may work because the quarks and leptons belong to separate multiplets). We are thus led to exploit the maximal mixing scenarios discussed in ref \cite{3,10}. We introduce two new SU(5) singlet states $N_1, N_2$ with charges $Q_{N_1} = -2$ and $Q_{N_2} = 2$, and consider the following two (Dirac and Majorana) matrices:

\[
\begin{pmatrix}
\bar{5}_1 & N_2 \\
\bar{5}_2 & e^2 \\
\bar{5}_3 & e^2 \\
\end{pmatrix} \kappa H, \quad \begin{pmatrix}
N_1 & N_2 \\
N_1 & e^4 \\
N_2 & 1 \\
\end{pmatrix} M_N, \quad (10)
\]

where $\kappa$ is some dimensionless coefficient, and $M_N$ is an appropriate heavy scale. In the basis in which the couplings (8) (responsible for generation of the charged lepton masses) are diagonal, the first matrix in (10) will be modified, and the appropriate ‘Dirac’ and ‘Majorana’ couplings will have the forms:

\[
M_D = \begin{pmatrix}
\epsilon^4 & 1 \\
\epsilon^2 & e^2 \\
\epsilon^2 & e^2 \\
\end{pmatrix} \kappa h_u, \quad M_R = \begin{pmatrix}
\epsilon^4 & 1 \\
1 & 0 \\
\end{pmatrix} M_N. \quad (11)
\]

Note that the hierarchical structure of the couplings in (8) is unchanged. Working in this basis, the matrix which diagonalizes the ‘light’ neutrino mass matrix will coincide with the physical lepton mixing matrix.

Together with (8) and (10), the $U(1)$ symmetry also permits the coupling $M' \epsilon^2 N_1 N$. Assuming

\[
M_N \gg M' \epsilon^2, \quad M_N \approx M'^2 / M_N, \quad (12)
\]

after integrating out the $N$ and $N_{1,2}$ states, the seesaw mechanism yields:

\[
\hat{m}_\nu = \hat{A} m + B m', \quad (13)
\]

where

\[
m \equiv \frac{h_u^2}{M_N}, \quad m' \equiv \frac{\kappa^2 \epsilon^2 h_u^2}{M_N}, \quad (14)
\]

and
A = \begin{pmatrix} a^2 \epsilon^4 & ab \epsilon^2 & ace \epsilon^2 \\ ab \epsilon^2 & b^2 & bc \\ ace \epsilon^2 & bc & \epsilon^2 \end{pmatrix} m ,

\hat{B} = \begin{pmatrix} \epsilon^2 & 1 & 1 \\ 1 & \epsilon^2 & \epsilon^2 \\ 1 & \epsilon^2 & \epsilon^2 \end{pmatrix} m' . \tag{15}

Note that \( \text{Det} \hat{B} = 0 \) since \( \hat{B} \) is obtained through the exchange of the two heavy \( N_{1,2} \) states. The matrix \( \hat{A} \) has only one nonzero eigenvalue.

For \( \kappa \sim 1, M_N \sim 4 \cdot 10^{16} \text{ GeV} \) and \( M_N \sim 10^{15} \text{ GeV} \) (the conditions in (12) are satisfied for \( M' < 10^{15} \text{ GeV} \)) one obtains

\[ m_{\nu_3} \simeq m(b^2 + c^2 + a^2 \epsilon^4) \sim 3 \cdot 10^{-2} \text{ eV} , \]
\[ m_{\nu_1} \simeq m_{\nu_2} \simeq m' \sim 3 \cdot 10^{-5} \text{ eV} . \tag{16} \]

Ignoring CP violation the neutrino mass matrix (13) can be diagonalized by the orthogonal transformations \( \nu_{\alpha} = U_{\alpha i}^\nu \nu_i \), where \( \alpha = e, \mu, \tau \) denotes flavor indices, and \( i = 1, 2, 3 \) the mass eigenstates. One finds

\[ U_{\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} e^{i \phi} & s_1 \\ -\frac{1}{\sqrt{2}} e^{-i \phi} & \frac{1}{\sqrt{2}} e^{i \phi} & s_\theta \\ \frac{1}{\sqrt{2}} e^{-i \phi} & -\frac{1}{\sqrt{2}} e^{i \phi} & c_\theta \end{pmatrix} + \mathcal{O}(\epsilon^2) , \tag{17} \]

with

\[ \tan \theta = \frac{b}{c} , \quad s_1 = \frac{a e^2}{\sqrt{b^2 + c^2}} , \quad s_\theta \equiv \sin \theta , \quad c_\theta \equiv \cos \theta . \tag{18} \]

For the solar and atmospheric neutrino oscillation parameters we find

\[ \Delta m_{21}^2 \sim 2 m^2 e^2 \simeq 10^{-10} \text{ eV}^2 , \]
\[ \mathcal{A}(\nu_e \to \nu_{\mu,\tau}) = 1 - \mathcal{O}(\epsilon^4) , \tag{19} \]

and

\[ \Delta m_{32}^2 \simeq m_{\nu_3}^2 \sim 10^{-3} \text{ eV}^2 , \]
\[ A(\nu_\mu \to \nu_\tau) = \frac{4b^2c^2}{(b^2 + c^2)^2} - O(\epsilon^4) . \] (20)

Here \( A \) determines the transition amplitude. We have therefore realized maximal \( \nu_e - \nu_\mu,\tau \) mixing as promised! In the \( \nu_\mu - \nu_\tau \) system large mixing can be obtained quite naturally for \( b \sim a \), while maximal mixing holds for \( b \simeq c \). In this case the \( \nu_e \) oscillations are 50\% into \( \nu_\mu \) and 50\% into \( \nu_\tau \).

Next let us briefly discuss other possible neutrino oscillation scenarios which can be realized in the framework of \( SU(5) \) GUT. Without applying any particular mechanism, the charged sector contribution to \( \nu_e - \nu_\mu,\tau \) mixing is expected to be \( \theta_{e\mu} \sim \theta_{e\tau} \sim \epsilon^2 \) (see (3)), which has the right magnitude for the small angle MSW solution of the solar neutrino puzzle. By introducing only the \( N \) right handed neutrino, from (8), (13), the neutrino mass matrix will be \( mA \), providing still large (or even maximal) \( \nu_\mu - \nu_\tau \) oscillations, and massless \( \nu_1, \nu_2 \) states. To obtain the relevant mass scale for solar neutrino oscillations, one can introduce a state \( N' \) with \( \mathcal{U}(1) \) charge \( Q_{N'} = 0 \) (we still work with the combination \( q_1 - 2q_2 = 0 \)). With couplings

\[ W_{N'} = M_{N'}N'^2 + (\epsilon^2\bar{5}_1 + \bar{5}_2 + \bar{5}_3)HN', \] (21)

and a suppressed term \( M_1NN' \) (\( M_1 \ll M_N \)), after integrating out the \( N' \) state, and taking \( M_{N'} \sim 10^{16} \) GeV, we get \( m_{\nu_2} \sim \frac{h_u^2}{M_{N'}} \sim 10^{-3} \) eV, the desired value for the solar neutrino oscillation parameter \( (\Delta m^2_{21} \sim 10^{-6} \text{eV}^2) \).

This scenario, as well as the previously discussed case, can naturally provide large \( \nu_\mu - \nu_\tau \) mixings. For obtaining maximal mixing to 1\% accuracy, the condition \( b^2 = \epsilon^2 \) should hold to 10\% accuracy. The mechanism of [3] (which we applied above for obtaining maximal \( \nu_e - \nu_\mu,\tau \) mixings) does not work for \( \nu_\mu - \nu_\tau \) system, since these flavors carry the same \( \mathcal{U}(1) \) charge. However, another scenario involving a sterile neutrino state, suggesting maximal \( \nu_\mu - \nu_s \) mixing for atmospheric neutrino deficit, can still be realized. The solar neutrino puzzle can be resolved in this case through the small angle MSW oscillations.

To see this, consider the prescription of the \( \mathcal{U}(1) \) charges (6), with \( q_1 - 2q_2 = 11/2 \). Introducing a right handed neutrino \( N'' \), and a light sterile state \( \nu_s \), with charges \( Q_{N''} = -11/2 \), \( Q_{\nu_s} = -43/2 \), the relevant couplings are

\[ W_{N''\nu_s} = \kappa' \left( \epsilon^2\bar{5}_1 + \bar{5}_2 + \bar{5}_3 \right) HN'' + \epsilon^{11}M_PN''^2 \]

\[ \epsilon^{16} \left( \epsilon^2\bar{5}_1 + \bar{5}_2 + \bar{5}_3 \right) H\nu_s + \epsilon^{43}M_P\nu_s^2 , \] (22)

where \( \kappa' \) is a dimensionless coupling. Integrating out \( N'' \), and using the notations

\[ m = \epsilon^{16}h_u \ , \ m' = \frac{\kappa'^2h_u^2}{M_P\epsilon^{11}} \ , \ m_{\nu_s} = M_P\epsilon^{43} , \] (23)
the mass matrix for the light neutrinos is given by

\[
m_
u = \begin{pmatrix}
\nu_e & \nu_\mu & \nu_\tau & \nu_s \\
\nu_e & m_l' c^4 & m_l' c^2 & m_e c^2 \\
\nu_\mu & m_l' c^2 & m_l' & m \\
\nu_\tau & m_l' c^2 & m_l' & m \\
\nu_s & m c^2 & m & m
\end{pmatrix}.
\]

(24)

From (23), taking \( \kappa' \sim 10^{-3}, \epsilon \simeq 0.2 \), one has

\[
m \simeq 1 \text{eV} , \quad m' \simeq 10^{-3} \text{eV} , \quad m_{\nu_s} \simeq 2 \cdot 10^{-3} \text{eV} .
\]

(25)

From (24) and (23),

\[
\Delta m^2_{\odot} \simeq m'^2 \simeq 10^{-6} \text{eV}^2 ,
\]

\[
\mathcal{A}(\nu_e \to \nu_{\mu,\tau}) = \sin^2 2\theta_{e\tau} \sim 4\epsilon^4 \simeq 6 \cdot 10^{-3} ,
\]

(26)

\[
\Delta m^2_{\odot} \simeq 2m m_{\nu_s} \simeq 3 \cdot 10^{-3} \text{eV}^2 ,
\]

\[
\mathcal{A}(\nu_\mu \to \nu_s) = 1 - \mathcal{O}\left(\frac{m^2_{\nu_e}}{m^2}\right) .
\]

(27)

We see that maximal \( \nu_\mu - \nu_s \) mixing is realized by a proper choice of the \( U(1) \) charges of the appropriate fields. The sterile neutrino is kept light with the help of the \( U(1) \) symmetry [11, 10]. Having a lower value for \( m_{\nu_s} \sim 10^{-3} \text{eV} \), we can still obtain the same value for \( \Delta m^2_{\odot} \) for \( m \sim 3 \text{eV} \) (this value for \( m \) in (25) is obtained for \( \epsilon \simeq 0.21 \). In this case one of the active neutrinos has mass \( m_{\nu_2} \sim 3 \text{eV} \), and therefore contributes roughly 15% to the critical energy density of the universe. Models of structure formation with cold and hot dark matter [12] are in good agreement with the observations.

In summary, we have shown how different scenarios for large (even maximal) \( \nu_\mu - \nu_x \) neutrino mixings are realized in supersymmetric \( SU(5) \) in order to accommodate the atmospheric neutrino data. The solar neutrino puzzle can be explained by either maximal angle vacuum oscillations (bi-maximal scenario), or through the small mixing angle MSW solution.

We now move on to the problematic asymptotic mass relations \( m_{\mu}^{(0)} = m_s^{(0)} \) and \( m_e^{(0)} = m_d^{(0)} \). These arise from the coupling \( 10 \cdot 5 \cdot \bar{H} \) which contains the mass generating terms \( q_d h_d \) and \( l e^c h_d \). In order to break this mass degeneracy (we would like to retain \( m_{\tau}^{(0)} = m_{\nu_s}^{(0)} \)), let us introduce a pair of ‘matter’ superfields belonging to the \( 15 + \overline{15} \) of \( SU(5) \) where, under \( SU(3)_c \times SU(2)_L \),

\[
15 = (3, 2) + (6, 1) + (1, 3) .
\]

(28)
It is clear that with $\Sigma 10 \Gamma 15$ type couplings, only $q$ states from 10 plets will mix with the corresponding states in 15. This only affects the down and up quark mass matrices, but not the charged lepton Yukawa couplings. This mixing enables us to break the down quark-charged lepton mass degeneracy. Let us note that this mechanism differs from those suggested previously [8].

With the following prescription of the $U(1)$ charges of $(15 + \Gamma 15)_{1,2}$ and $\Sigma$ multiplets

$$
Q_{15_1} = q_2 - 6, \quad Q_{15_2} = q_2 - 5, \quad Q_\Sigma = 0,
Q_{\Gamma 15_1} = 1 - q_2, \quad Q_{\Gamma 15_2} = 2 - q_2,
$$

the relevant couplings are

$$
\begin{pmatrix}
10_1 & 10_2 & 10_3 \\
\epsilon^2 & \epsilon & 0 \\
\epsilon & 1 & 0
\end{pmatrix} \Sigma,
\begin{pmatrix}
15_1 & 15_2 \\
\epsilon^5 & \epsilon^4 & \epsilon^3
\end{pmatrix} M_P.
$$

(30)

Through (29) choice and (30) couplings, $q_2$ is undetermined and therefore, charged fermion sector is consistent with various oscillation scenarios discussed above.

Choosing a basis in which the couplings (3) and the mass matrix for 15-plets (in (30)) are diagonal, the mass matrix relevant for down quarks is given by

$$
\hat{M}_d = \frac{d^c}{\bar{q}_{10}} \begin{pmatrix}
q_{10} & q_{15} \\
Y^D_h d & \hat{M}_{15}
\end{pmatrix},
$$

(31)

where

$$
\hat{C} = \begin{pmatrix}
c_{11} \epsilon^2 & c_{12} \epsilon & c_{13} \epsilon^3 \\
c_{21} \epsilon & c_{22} & c_{23} \epsilon^2
\end{pmatrix} M_P \epsilon_G,
\hat{M}_{15} = \begin{pmatrix}
\epsilon^5 & 0 \\
0 & \epsilon^3
\end{pmatrix} M_P,
$$

(32)

and

$$
Y^D_e = \text{Diag} \left( a_1 \epsilon^5, a_2 \epsilon^2, a_3 \right) \epsilon^a
$$

($\epsilon_G \equiv \langle \Sigma \rangle / M_P$). For $c_{12} \lesssim 1/5 (\simeq \epsilon)$ and other couplings of order unity, integration in (31) of the heavy states leads to the down quark mass matrix:

$$
\hat{m}_d = \frac{d^c}{\bar{q}_{10}} \begin{pmatrix}
q'_1 & \lambda' \epsilon^5 & q'_2 \\
\lambda'' \epsilon^5 & \lambda_1 \epsilon^3 & \epsilon^4 \\
\epsilon^6 & \epsilon^4 & a_3
\end{pmatrix} e^a h_d,
$$

(34)

where $\lambda_{1,2}, \lambda'$ and $\lambda''$ are some coupling constants. From (33) and (34) we have the desired relation
\[ \lambda_b = \lambda_r = a_3 e^a, \]  

while the unwanted mass degeneracy between the down quarks and charged leptons of the two ‘light’ families are avoided. Note that the desired ratios (see (3)) between the down quark Yukawa couplings still hold. Analyzing the couplings in (30), one can verify that the ‘light’ left handed quarks \( q'_{1,2} \) reside in 10\(_{1,2} \) and 15\(_{1,2} \) states (respectively), weighted in comparable order. This means that the hierarchical structure of the couplings (7) will not be altered, and the magnitudes of the up type quark Yukawa constants will still be given by (3).

Finally, let us note that by choosing basis (32), (33), and using (7), (34) the desired magnitudes of the CKM matrix elements (see (4)) are obtained. Furthermore, the unification of the three gauge coupling constants at \( M_{GUT} \) is not affected, since all the additional fragments form complete \( SU(5) \) multiplets, and decouple without significant splitting.

In conclusion, we have shown how a variety of neutrino oscillation scenarios, that are consistent with the experimental data from solar and atmospheric neutrino experiments, can be realized within a \( SU(5) \) scheme supplemented by a \( U(1) \) flavor symmetry and some gauge singlet (right handed) neutrinos. We showed, in particular, how bi-maximal mixings can be achieved without fine tuning the parameters. Two scenarios for realizing the small mixing angle MSW solution, with one involving a sterile state and giving possibility of existence of the neutrino hot dark matter, are also displayed. Note that in this scheme neutrinoless double \( \beta \) decay and other flavor changing processes are strongly suppressed.

Finally, consistent with these scenarios, a new mechanism for resolving the unacceptable mass relations involving the two light families (of down quarks and charged leptons) is presented. This was achieved by introducing the 15-plets of \( SU(5) \) [13]. The status of nucleon decay in this modified scheme is essentially the same as for minimal \( SU(5) \).

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