Spectator effects in semileptonic decay of charmed baryons

M.B. Voloshin
Theoretical Physics Institute, University of Minnesota, Minneapolis, MN 55455
and
Institute of Theoretical and Experimental Physics, Moscow, 117259

Abstract

It is shown that the absolute rate of the semileptonic decays of the \( \Xi_c \) baryons can be as much as two times higher than the semileptonic decay rate of \( \Lambda_c \) due to the Pauli interference of the strange quark. This interference effect is still larger in the semileptonic decay rate of \( \Omega_c \). An experimental measurement of the difference of these rates would thus provide an important piece of information on the spectator effects in decays of hadrons containing a heavy quark.
1 Introduction

The differences of the lifetimes of hadrons containing a heavy quark are attracting considerable interest ever since the early experimental evidence for unequal lifetimes of the charmed $D^\pm$ and $D^0$ mesons. Presently measured lifetimes of weakly decaying charmed hadrons span more than one order of magnitude: $\tau(D^\pm)/\tau(\Omega_c) \sim 20$. For the hadrons, containing the $b$ quark, the relative differences in lifetimes are much smaller and the experimental situation with their precise values is still in flux. These lifetime differences reflect the effects of spectator light quarks in heavy hadrons and become to a certain degree tractable theoretically in the limit of large mass of the heavy decaying quark $Q$ in terms of an expansion in the inverse powers of its mass $m_Q$. The leading term in this expansion is the ‘parton’ decay rate $\Gamma_{\text{part}} \propto m_Q^5$ of a free heavy quark, which sets the overall scale for the decay rates of the heavy hadrons containing $Q$ and which, naturally, does not depend on the spectator quarks or antiquarks. The spectator-flavor dependence arises through two mechanisms:\cite{2-5}: the weak scattering (WS) and the Pauli interference (PI). The weak scattering corresponds to a cross-channel of the parton decay, generically $Q \to q_1 q_2 \bar{q}_3$, where either a spectator light quark in a baryon scatters off the heavy quark into two light ones: $q_3 Q \to q_1 q_2$, or the spectator antiquark in a meson, say $\bar{q}_1$, annihilates with the $Q$ into $q_2 q_3$ via the $W$ boson. The Pauli interference effect arises when one of the final quarks in the decay of $Q$ is identical to a spectator light quark in the initial hadron. The contribution of both the WS and the PI to the decay rate of a hadron is suppressed with respect to the parton decay rate by the factor $m_Q^{-3}$. However, they come with a large numerical coefficient, typically $16\pi^2/3$, reflecting mainly the difference of numerical factors in the two-body final phase space vs the three-body one, which makes these effects hundred percent significant for the charmed hadrons, as is confirmed by the experimental data.

A quantitative description of the spectator-flavor dependent effects in inclusive decay rates of charmed hadrons is problematic for two reasons: poor knowledge of light quark matrix elements over the hadrons (light quark wave functions in a naive constituent quark model) and the fact that for charmed hadrons the spectator effects are comparable in magnitude to the leading ‘parton’ term. The latter obstacle in the way of a quantitative theoretical picture disappears for the $b$ hadrons, while the former one, the lack of understanding the hadronic matrix elements, stays. It can be noticed however, that an application of the calculated in the order $m_Q^{-3}$ WS and PI effects to lifetime differences of the charmed hadrons is possible at a semi-quantitative level. In particular it has allowed to predict\cite{5} the hier-
archy of the lifetimes of the charmed mesons and baryons in agreement with the present data, while then existing data were either missing (for $\Xi^0_c$), or were contradicting to the presently established pattern of the hierarchy (for $D_s$ and for $\Omega_c$). Therefore the data on the inclusive decay rates of charmed hadrons may provide a better information on the hadronic matrix elements, describing the spectator effects, than the very model-dependent theoretical evaluation\cite{5}, thus allowing for better estimates of the differences of inclusive decay rates of $b$ hadrons by a rescaling of the heavy quark mass.

The extrapolation to the $b$ hadrons is especially interesting in view of the recent data\cite{6,7}, indicating a large difference of the lifetimes of $\Lambda_b$ and the $B$ mesons: $\tau(B)/\tau(\Lambda_B) = 1.30 \pm 0.07$, while typical theoretical estimates for the deviation of this ratio from unity range from few percent\cite{5} to at most 0.1\cite{8}.

The purpose of the present paper is to point out that an additional information on the spectator quark matrix elements in the $SU(3)_{FL}$ triplet of heavy baryons consisting of $\Lambda_Q$ and two $\Xi_Q$ hyperons can be obtained by measuring the difference of the inclusive semileptonic decay rates of $\Lambda_c$ and the $\Xi_c$ hyperons. Moreover, this difference should be large: the rate of these decays for the $\Xi_c$ baryons (equal between $\Xi^+_c$ and $\Xi^0_c$ up to the CKM suppressed contributions) may exceed that for the $\Lambda_c$ by a factor of two. Besides testing this prediction, following from a simple extension of the analysis\cite{5} of non-leptonic decays, an experimental measurement of these semileptonic decay rates would allow to separate the relevant matrix elements for each of the spectator light quarks: WS for the $d$ quark and PI for the $u$ and $s$ quarks.

Indeed, the lifetime difference between the baryons and the mesons is contributed not only by the WS and the PI but also by a flavor-independent effect of order $m_Q^{-2}$ related to the motion and the chromo-magnetic interaction of the heavy quark inside hadron\cite{9} and the same mechanism contributes also to the difference of decay rates of baryons with different spin arrangement. (For a recent review see Ref. [8].) Thus for the triplet ($\Lambda_c, \Xi_c$) the differences of the total rates provide only two inputs for three matrix elements relevant for WS and PI. The difference of the semileptonic decay rates in this triplet is determined only by the PI for the $s$ quark, which thus would provide the third experimental input.

Even stronger effect of the positive PI for the $s$ quark on semileptonic decay rate should be expected for the hyperon $\Omega_c$. The $\Omega_c$ is known to have the fastest total decay rate in agreement with the theoretical prediction\cite{5}. Comparing the PI effects in the nonleptonic and semileptonic decays, leads to the conclusion that the semileptonic decay of $\Omega_c$ is enhanced approximately by the same amount as its nonleptonic decay rate. Thus one may expect a
quite sizeable value of the branching ratio $B(\Omega_c \to e^+ \nu X)$, up to approximately 10 - 15%.

2 Spectator effects in nonleptonic decays

A systematic description of the inclusive decay rates of heavy hadrons is performed\[2,4,5] by applying the operator product expansion in powers of $m_Q^{-1}$ to the ‘effective Lagrangian’ arising in the second order in the weak-interaction Lagrangian $L_W$:

$$L_{\text{eff}} = 2 \text{Im} \left[ i \int d^4x e^{iqx} T \{ L_W(x), L_W(0) \} \right],$$

in terms of which effective Lagrangian the inclusive decay rate of a heavy hadron $X_Q$ is given as\[6]

$$\Gamma_X = \langle X_Q | L_{\text{eff}} | X_Q \rangle.$$

The leading term in $L_{\text{eff}}$ gives the ‘parton’ decay rate. For the nonleptonic decay of the charmed quark one uses in eq.(1) the nonleptonic part of the weak Lagrangian and finds for the CKM unsuppressed part

$$L_{\text{eff, nl}}^{(0)} = \frac{G_F^2 m_c^5}{64 \pi^3} \eta_{nl} \langle \bar{c} c \rangle,$$

where the factor $\eta_{nl}$ describes the QCD corrections to the decay of free heavy quark and the CKM mixing parameters $V_{ud}$ and $V_{cs}$ are approximated by one. For the semileptonic decay rate the analogous expression is

$$L_{\text{eff, sl}}^{(0)} = \frac{G_F^2 m_c^5}{192 \pi^3} \eta_{sl} \langle \bar{c} c \rangle.$$

The standard ‘parton’ prediction for the decay rate of charmed hadrons is obtained by applying the equation (2) and taking into account that $\langle X_c | (\bar{c} c) | X_c \rangle \approx 1$ up to corrections of order $m_c^{-2}$. The full set of corrections of order $m_c^{-2}$ arise from the corrections to the latter matrix element and also from the contribution to the $L_{\text{eff}}$ of the dimension 5 operator $(\bar{c} \sigma_{\mu\nu} G_{\mu\nu} c)$. These corrections are studied in detail\[9,8] for both nonleptonic and semileptonic decays. These terms split the decay rates of heavy baryons from those of heavy mesons and also between heavy baryons of different spin arrangement. However they do not depend on the light spectator quark flavor, and do not split the decay rates within an $SU(3)_F$ multiplet.

\[1\] The nonrelativistic normalization of heavy quark states is systematically used throughout this paper, so that e.g. $\langle Q | Q' | Q \rangle = 1$. 

3
i.e., say, between the Λc and the Ξc hyperons. In this paper we are mainly concerned with the latter splittings, thus in what follows we leave the $m_c^{-2}$ terms aside.

The flavor-dependent terms in the $L_{eff}$ describing the WS and the PI mechanisms arise in the order $m_Q^{-3}$ in the form of dimension 6 four-quark operators. For the purpose of further discussion we reproduce here the complete expression\(^{[3]}\) for these terms in the effective Lagrangian for nonleptonic decays of charmed hadrons\(^{[3]}\) adjusted for a different convention about the sign of $\gamma_5$ and for a different normalization of the heavy quark states:

$$L_{eff,\,nl}^{(3)} = \frac{G_{F}^2 \, m_c^2}{2\pi} \left\{ \frac{1}{2} \left[ C_+^2 + C_-^2 + \frac{1}{3} \left( 1 - \kappa^{1/2} \right) \left( C_+^2 - C_-^2 \right) \right] \left( \overline{\tau} \Gamma_\mu c \right) \left( \overline{d} \Gamma_\mu d \right) + \right.$$

$$\frac{1}{2} \left( C_+^2 - C_-^2 \right) \kappa^{1/2} \left( \overline{\tau} \Gamma_\mu d \right) \left( \overline{\tau} \Gamma_\mu c \right) + \frac{1}{3} \left( C_+^2 - C_-^2 \right) \kappa^{1/2} \left( \kappa^{-2/9} - 1 \right) \left( \overline{\tau} \Gamma_\mu t^a c \right) j_{\mu}^a - \left(5\right)$$

$$\frac{1}{8} \left[ (C_+ + C_-)^2 + \frac{3}{3} \left( 1 - \kappa^{-1/2} \right) \left( 5 C_+^2 + C_-^2 - 6 C_+ C_- \right) \right] \left( \overline{\tau} \Gamma_\mu c \right) + \frac{2}{3} \overline{\tau} \gamma_\mu \gamma_5 c \left( \overline{\tau} \Gamma_\mu u \right) -$$

$$\frac{1}{8} \kappa^{1/2} \left( 5 C_+^2 + C_-^2 - 6 C_+ C_- \right) \left( \overline{\tau} \Gamma_\mu c \right) + \frac{2}{3} \overline{\tau} \gamma_\mu \gamma_5 c \left( \overline{\tau} \Gamma_\mu s \right) -$$

$$\frac{1}{8} \kappa^{1/2} \left( 5 C_+^2 + C_-^2 - 6 C_+ C_- \right) \left( \overline{\tau} \Gamma_\mu c \right) + \frac{2}{3} \overline{\tau} \gamma_\mu \gamma_5 c \left( \overline{\tau} \Gamma_\mu s \right) -$$

$$\frac{1}{6} \kappa^{1/2} \left( \kappa^{-2/9} - 1 \right) \left( 5 C_+^2 + C_-^2 \right) \left( \overline{\tau} \Gamma_\mu t^a c \right) + \frac{2}{3} \overline{\tau} \gamma_\mu \gamma_5 t^a c \left( j_{\mu}^a \right) \right\} .$$

In this formula $\Gamma_\mu = \gamma_\mu \left( 1 - \gamma_5 \right)$, $C_+$ and $C_-$ are the standard coefficients in the RG renormalization of the nonleptonic weak interaction from $m_W$ down to the charmed quark mass $m_c$: $C_- = C_+^{-2} = (\alpha_s(m_c)/\alpha_s(m_W))^{4/6}$, the powers of the parameter $\kappa = (\alpha_s(\mu)/\alpha_s(m_c))$ describe the ‘hybrid’\(^{[3]}\) renormalization of the operators from the scale $m_c$ down to a low normalization scale $\mu$, and $j_{\mu}^a = \overline{\tau} \gamma_\mu t^a u + \overline{d} \gamma_\mu t^a d + \overline{s} \gamma_\mu t^a s$ is the color current of the light quarks ($t^a = \lambda^a/2$ being the color generators).

The terms in eq.\(^{[3]}\) with the $SU(3)_F$ singlet operator $j_{\mu}^a$ do not produce differences of the decay rates within the same $SU(3)_F$ multiplet, while the rest terms containing the $d$, $u$, and $s$ quarks with different coefficients give rise to those differences. For the charmed hyperons the term with the $d$ quark describes the WS\(^{[3]}\), while those with the $u$ and $s$ describe the interference between the quark produced in the $c$ quark decay with that in the initial hyperon. The color antisymmetry of the baryon wave function relates the matrix elements\(^{[3]}\)

\(^{[2]}\)This reiterating of the old result may also be helpful in view of the recently renewed interest to the problem\(^{[10]}\)\(^{[11]}\).

\(^{[3]}\)In mesons this term describes the interference of the $\overline{d}$ quark and is responsible for the difference of the lifetime of $D^\pm$ from those of $D^0$ and $D_s$. 

4
over baryons of the operators with a crossed color arrangement: \( \langle X_c | (\bar{c}_T T_\mu c_k) (\bar{q}_k \Gamma_\mu q_i) | X_c \rangle = - \langle X_c | (\bar{c}_T T_\mu c_k) (\bar{q} \Gamma_\mu q) | X_c \rangle \) for any spinor structure \( T_\mu \). Furthermore, in the leading order in \( m_c^{-1} \) the axial current of the charmed quark is proportional to its spin, which is decoupled from that of the light quark pair in the spin \( \frac{1}{2} \) hyperons \( \Lambda_c, \Xi_c \). Thus for these baryons only the vector current of the charmed quarks contributes and the matrix elements of the effective Lagrangian in eq.(3) reduce to those of the operators \( O_q = (\bar{c} \gamma_\mu c) (\bar{q} \gamma_\mu q) \) with \( q = u, d, \) or \( s \). Clearly, these matrix elements are unknown, which makes a further analysis model dependent. Within an additive constituent quark model applied to operators at a low normalization point \( \mu \) the shift of the decay rates of the baryons due to the effective Lagrangian (6) can be written as:

\[
\Delta \Gamma_{nl}(\Lambda_c) = c_d \langle O_d \rangle_{\Lambda_c} + c_u \langle O_u \rangle_{\Lambda_c} \\
\Delta \Gamma_{nl}(\Xi_c^+) = c_s \langle O_s \rangle_{\Xi_c^+} + c_u \langle O_u \rangle_{\Xi_c^+} \\
\Delta \Gamma_{nl}(\Xi_c^0) = c_d \langle O_d \rangle_{\Xi_c^0} + c_s \langle O_s \rangle_{\Xi_c^0} 
\]

with \( \langle O_q \rangle_{X_c} = \langle X_c | O_q | X_c \rangle \) and the coefficients \( c_q(\mu) \) read off eq.(6):

\[
c_d = \frac{G_F^2 m_c^2}{4\pi} \left[ C_+^2 + C_-^2 + \frac{1}{3} (4\kappa_{1/2}^2 - 1) (C_+^2 - C_-^2) \right] \\
c_u = -\frac{G_F^2 m_c^2}{16\pi} \left[ (C_+ + C_-)^2 + \frac{1}{3} (1 - 4\kappa_{1/2}^2) (5C_+^2 + C_-^2 - 6C_+ C_-) \right] \\
c_s = -\frac{G_F^2 m_c^2}{16\pi} \left[ (C_+ - C_-)^2 + \frac{1}{3} (1 - 4\kappa_{1/2}^2) (5C_+^2 + C_-^2 + 6C_+ C_-) \right].
\]

The numerical values of these coefficients depend on the choice of the normalization point \( \mu \) at which the constituent quark model may be applied and, to a lesser extent, on the value of \( \alpha_s(m_c) \). Varying \( \mu \) in the limits such that \( \alpha_s(\mu) \) varies between 0.5 and 1, one finds that the coefficients \( c_d \) and \( c_s \) are positive and \( c_u \) is negative and that \( c_d \approx -3c_u \) with \( c_s \) being approximately equal, or slightly smaller than \( c_d \). If one further assumes that the matrix elements of each of the operators \( O_q \) over the corresponding constituent quark are the same, one finds the hierarchy of the decay rates within the triplet of \( \Lambda_c \) and \( \Xi_c \) hyperons in a qualitative agreement with the presently observed.

The absolute value of the splitting of the rates is even more difficult to predict, since this requires knowledge of the absolute magnitude of the matrix elements \( \langle O_q \rangle_{X_c} \). A very approximate estimate in a naive nonrelativistic model indicates that the effect of these operators on the decay rates should be comparable to the ‘parton’ decay rate.
3 Spectator effects in semileptonic decays

An additional effect, which has been disregarded in the previous studies, but whose experimental observation can be quite helpful in understanding the spectator effects in inclusive decay rates of heavy hadrons, is a similar PI of the $s$ quark produced in the underlying charmed quark decay $c \rightarrow s l^+ \nu$. The effective Lagrangian describing this interference effect can be readily deduced from the nonleptonic one in eq.(6). This amounts to taking into account the overall color factor, setting the coefficients $C_+ \text{ and } C_-$ equal to one, and keeping only the contribution of the $\overline{\sigma} \Gamma_\mu s$ operator (operators with $u$ and $d$ quarks still appear, through a ‘penguin’ type renormalization, but only in terms with $j^a_{\mu}$). In this way one finds an analog of eq.(6) for the spectator effects in semileptonic decays:

$$L^{(3)}_{\text{eff, } sl} = \frac{G_F^2 m_c^2}{12\pi} \left[ (\kappa^{1/2} - 1) \left( \overline{c} \Gamma_{\mu} c + \frac{2}{3} \overline{c} \gamma_{\mu} \gamma_5 c \right) (\overline{s} \Gamma_\mu s) \right] - 3 \kappa^{1/2} \left( \overline{c} \Gamma_{\mu} c_k + \frac{2}{3} \overline{c} \gamma_{\mu} \gamma_5 c_k \right) (\overline{s} \Gamma_{\mu} s_i) - \kappa^{1/2} (\kappa^{-2/9} - 1) (\overline{c} \Gamma_{\mu} t^a c + \frac{2}{3} \overline{c} \gamma_{\mu} \gamma_5 t^a c) j^a_{\mu} \right].$$

Proceeding in the same way as for the nonleptonic decays, i.e. leaving out the $SU(3)_F$ symmetric operator containing $j^a_{\mu}$ and also using the color antisymmetry of the baryon wave function, one finds the shift of the semileptonic decay rates of the $\Xi_c$ baryons due to PI of the $s$ quark:

$$\Delta \Gamma(\Xi_c \rightarrow e^+ \nu X) = c_{sl} \langle O_s \rangle_{\Xi_c}$$

with the same operator $O_s$ as in eqs.(7) and the coefficient $c_{sl}$ given by

$$c_{sl} = \frac{G_F^2 m_c^2}{12\pi} \left( 4 \kappa^{1/2} - 1 \right).$$

Clearly, the semileptonic decay rates of $\Xi^+_c$ and $\Xi^0_c$ should be equal (up to CKM suppressed contributions) due to the isotopic symmetry.

Allowing for the variation of $\alpha_s(\mu)$ at the low normalization point between $\alpha_s(\mu) = 0.5$ and $\alpha_s(\mu) = 1$, one can readily see that with a better than 10% accuracy the following relation holds $c_{sl} \approx c_s/3$, where the factor of 1/3 is clearly due to the overall color suppression of the semileptonic decay. In other words, the relative shift of the decay rate due to the PI of the $s$ quark is approximately the same for semileptonic decays as for the nonleptonic ones.

From the observed differences of lifetimes of the charmed baryons and using the additive quark model relations (7) one can conservatively estimate that the effect of the constructive PI of the $s$ quark in these baryons enhances the nonleptonic decay rate by at least 1 ps$^{-1}$ in absolute units. Therefore according to the above estimate of the relative magnitude of the
coefficients $c_{sl}$ and $c_s$ the effect of the PI in the semileptonic decay rates should amount to at least $0.3 \text{ ps}^{-1}$, which is somewhat larger or approximately equal to the measured semileptonic decay rate of $\Lambda_c$: $\Gamma(\Lambda_c \to e^+ \nu X) = 0.23 \pm 0.9 \text{ ps}^{-1}$. Therefore we come to the conclusion that the semileptonic decay rate of the $\Xi_c$ baryons can exceed that of the $\Lambda_c$ by a factor of two, or more.

The effects of the strange quark interference are especially strong in the weakly decaying baryon $\Omega_c$ and are considered\cite{5} to be primarily responsible for the fact that this hyperon has the shortest lifetime among weakly decaying charmed hadrons. Since these effects are approximately proportional for the semileptonic and the nonleptonic decay, one should expect that the semileptonic decay rate of the $\Omega_c$ is enhanced proportionally to the total decay rate. Thus the semileptonic branching ratio of $\Omega_c$ should essentially be determined by the usual overall color factor and the relative perturbative enhancement (by the coefficients $C_+$ and $C_-$) of the nonleptonic decay. Thus one should expect the branching ratio $B(\Omega_c \to e^+ \nu X)$ close to 10-15%.

4 Summary

The differences of the inclusive weak decay rates of charmed and beauty particles still present an interesting problem for a theoretical and experimental study. The theoretical description of these differences in terms of expansion in the inverse heavy quark mass runs into difficulty of poor knowledge of the hadronic matrix elements of the relevant operators. This uncertainty can be somewhat reduced by measurements for the charmed hadrons, where the relative differences of the rates are large. In this respect the measurement of the differences of semileptonic decay rates of charmed baryon would provide an important information on the magnitude of the enhancement of the decay rates due to the constructive Pauli interference of the $s$ quarks. According to the estimates presented in this paper, the differences of the absolute semileptonic decay rates can be large: $\Gamma(\Xi_c \to e^+ \nu X) \gtrsim 2 \Gamma(\Lambda_c \to e^+ \nu X)$. This interference effect should be further enhanced in the semileptonic decays of the $\Omega_c$, due to the presence of two strange quarks in the baryon. The estimates of the present paper suggest that the enhancement of the semileptonic decays of $\Omega_c$ should be approximately proportional to the overall enhancement of its total decay rate. Thus the semileptonic branching ratio of $\Omega_c$ can be as large as 10-15%.

This work is supported, in part, by the DOE grant DE-AC02-83ER40105.
References

[1] W. Bacino et.al. (DELCO Coll.), Phys. Rev. Lett. 45 (1980) 329.

[2] M.A. Shifman and M.B. Voloshin, (1981) unpublished, presented in the review V.A.Khoze and M.A. Shifman, Sov. Phys. Usp. 26 (1983) 387.

[3] N. Bilic, B. Guberina and J. Trampetic, Nucl. Phys. B248 (1984) 261.

[4] M.A. Shifman and M.B. Voloshin, Sov. J. Nucl. Phys. 41 (1985) 120.

[5] M.A. Shifman and M.B. Voloshin, Sov. Phys. JEPT 64 (1986) 698.

[6] I.J. Kroll, Masses and Lifetimes of B hadrons, talk presented at the 17th Int. Symp. on Lepton and Photon Interactions, Beijing, August 10-15 1995.

[7] G. Apollinari (CDF collaboration), presented at the Aspen Winter Conference on Particle Physics, Aspen, Co., January 1996.

[8] I.I. Bigi, Univ. Notre Dame report UND-HEP-95-BIG02, June 1995; [hep-ph/9508408].

[9] I.I. Bigi, N.G. Uraltsev, and A.I. Vainshtein, Phys. Lett. B293 (1992) 30(1992), erratum – ibid. B297, 477 (1993).

[10] M. Neubert and C.T. Sachraida, report CERN-TH/96-19, SHEP 96-03, March 1996; [hep-ph/9603202].

[11] G. Altarelli, G. Martinelli, S. Petrarca and F. Rapuano, report CERN-TH/96-77, ROME1 prep. 1143/96, April 1996; [hep-ph/9604202].

[12] M.A. Shifman and M.B. Voloshin, Sov. J. Nucl. Phys. 45 (1987) 292.