Creep Behaviour of Reinforced Concrete Beam under Long-Term Bending

Jienda Xin\textsuperscript{1,2,3}, Yi Liu\textsuperscript{1,2,3}, Guoxin Zhang\textsuperscript{3}, Juan Wang\textsuperscript{3}, Zhenhong Wang\textsuperscript{1,2,3} and Wenqian Hou\textsuperscript{3}

\textsuperscript{1} State Key Laboratory of Simulation and Regulation of Water Cycle in River Basin, China Institute of Water Resources and Hydropower Research, Beijing 100038, China;
\textsuperscript{2} Key Laboratory of Construction and Safety of Hydraulic Engineering of Ministry of Water Resources, China Institute of Water Resources and Hydropower Research, Beijing 100038, China;
\textsuperscript{3} Department of Structure and Materials, China Institute of Water Resources and Hydropower Research, Beijing 100038, China

Email: xinjd@iwhr.com

Abstract. Concrete structure after casting will be subjected to external load, peripheral components and other constraints, the overall performance of the structure under a long-term loading state will change continuously. In practical engineering, reinforcement is usually calculated and configured based on external load, but the role of reinforcement in the continuous loading process of concrete structure is not fully considered. Based on the elastic creep theory, this paper deduces the concrete creep calculation formula considering the influence of reinforcement, and analyses the law of the influence of reinforcement ratio and loading age on the strain and stress of concrete. The results show that the reinforcement can effectively restrain the deformation of concrete and reduce the load level of concrete.

Keywords. Reinforced concrete; Creep; Age; Reinforcement ratio.

1. Introduction

Concrete as a kind of structural material, has a very rapid application development and has become an indispensable part of construction engineering. Concrete will have creep effect under continuous load \cite{1}. This effect will affect the service life and performance of concrete structures: on the one hand, creep can cause stress relaxation and reduce the risk of concrete cracking; On the other hand, creep will cause the loss of profit of prestressed concrete structure, and adversely affect the normal use of the structure. Therefore, it is particularly necessary to study the mechanical properties of concrete under the long-term loading.

In recent years, many scholars have carried out theoretical analysis and experimental research on concrete creep characteristics, and achieved a lot of results \cite{2-4}. Mu et al. \cite{5} conducted creep analysis on steel-reinforced concrete beam members and studied the variation rules of parameters such as strain and section bending moment. Xue et al. \cite{6} studied the creep of prestressed reinforced concrete beam members, taking into account the influence of concrete types and prestressed tendons. Pan et al. \cite{7} studied the influence of reinforcement on the shrinkage and creep properties of HPC. Koga et al. \cite{8} studied the beam deflection and crack width of prestressed reinforced concrete beams under long-term load. Mari. Antonio et al. \cite{9} analyzed the variation rule of deflection of cracked reinforced concrete
beams under long-term load in tension zone. Generally, the role of reinforcement is to limit crack width and improve bending deflection [10], but its "crack resistance" role is not obvious [11], so it is difficult to give full play to the advantages of reinforcement. In China's existing specifications [12], the configuration of steel bars is only to consider its "crack limit" function, with a certain number of temperature reinforcement. As a matter of fact, the influence of creep cannot be ignored because the concrete is in the state of loading for a long time after casting. If the influence of reinforcement is considered, the stress-strain state of concrete is more concerned.

Therefore, based on the basic theory of elastic creep, this paper deduces the calculation formula of concrete beam creep considering the influence of reinforcement, analyzes the variation law of strain and load of reinforcement and concrete under different loading, and provides a theoretical basis for the application of reinforcement in engineering.

2. Creep Theory of Concrete

The creep characteristics of concrete were discovered in the early 20th century, and since then, relevant computational theories have made great progress, including effective modulus method, aging theory and elastic creep theory, etc. [1]. Elastic creep theory is based on the assumption that there is a linear relationship between deformation and stress. When stress changes, the total creep deformation $\varepsilon_{total}(t)$ at age $t$ can be calculated according to the sum of the creep deformation caused by the corresponding stress increment, which is the so-called superposition principle

$$
\varepsilon_{total}(t) = \varepsilon(t_0)\left[\frac{1}{E(t_0)} + C(t_0)\right] + \int_{t_0}^{t} \left[\frac{1}{E(\tau)} + C(\tau, \tau)\right]d\sigma(\tau)
$$

where $\sigma(t_0)$ is the initial stress of concrete; $E(t_0)$ is the initial elastic modulus of concrete; $C(t, t_0)$ is the specific creep of concrete at age $t$ with a loading age of $t_0$.

3. Creep of Concrete Considering Effect of Reinforcement

3.1. Basic Assumption

The basic condition that reinforcement and concrete can form a composite structural material is that there is a reliable bond between them. For newly cast reinforced concrete structures, the deformation of reinforcement and concrete coordinated under the early age loading is an important factor to be considered. Mimura et al. [13] conducted pull tests on reinforced concrete specimens at different ages, and found that under low load levels, the strains of reinforcement and concrete were basically the same, and the bond force at the initial stage of pouring could ensure the two to work together. Therefore, the basic assumptions of formula derivation in this paper are as follows:

1. The section of the specimen conforms to the assumption of plane section;
2. There is no slip between reinforcement and concrete;
3. The creep properties of concrete are consistent;
4. Concrete does not crack.

3.2. Deformation Behavior of Reinforced Concrete

After the load is applied, the load will be transferred to the reinforcement with time due to the creep effect of concrete. By dividing the whole calculation time into small periods, the final strain value of concrete can be obtained through continuous cyclic calculation. The elastic strain increment and creep strain increment of concrete in each time period are [14]

$$
\Delta \varepsilon_n^c = \int_{t_n}^{t_{n+1}} \frac{1}{E(\tau)} \frac{d\sigma}{d\tau} d\tau = \frac{\Delta \sigma_{n-1}}{E(\tau_n)}
$$
\[ \Delta \varepsilon^c = \eta_n + \Delta \sigma_n C(t_n, \tau_n) \]  \hspace{1cm} (3)

\[ \Delta \varepsilon_n = \Delta \varepsilon^c + \Delta \varepsilon^c_n \]  \hspace{1cm} (4)

where

\[ \Delta \sigma_n = \bar{E}_n (\Delta \varepsilon_n - \eta_n) \]  \hspace{1cm} (5)

\[ \Delta \bar{E}_n = \frac{E(\tau_n)}{1 + E(\tau_n)C(t_n, \tau_n)} \]  \hspace{1cm} (6)

\[ \eta_n = \sum (1 - e^{-\tau_n \Delta t_n}) \omega_{sn} \]  \hspace{1cm} (7)

\[ \omega_{sn} = \omega_{s,n-1} e^{-\tau_n \Delta t_n} + \Delta \sigma_{n-1} \psi_s (\tau_{n-1}) e^{-0.5 \tau_n \Delta t_{n-1}} \]  \hspace{1cm} (8)

\[ \omega_{s1} = \Delta \sigma_0 \psi_s (\tau_0) \]  \hspace{1cm} (9)

\[ \psi_s = \sum f_i + g_i \tau_{p_i} \]  \hspace{1cm} (10)

**Figure 1.** Stress distribution of reinforced concrete beam.

Figure 1 shows the stress distribution of reinforced concrete beam. The moment \( M \) together undertaken by concrete and reinforcement can be expressed by

\[ M = \frac{bh^2}{6} - \frac{\pi D^4}{8h} - \frac{\pi D^2 (h - 2a_i)^2}{2h} ] \sigma_c + \frac{A'_i \sigma'_i (h - a_i)}{2} + A_i \sigma_i (h - a_i) \]  \hspace{1cm} (11)

where \( b \) is width of beam; \( h \) is height of beam; \( D \) is diameter of reinforcement; \( \sigma_c \) is the concrete stress at outermost edge of compression zone; \( \sigma'_i, \sigma_i, A'_i \) and \( A_i \) is stress and area of reinforcement at compression and tension zone, respectively. \( a_i \) is the distance between the gravity center of reinforcement and outermost edge of concrete.

The stress and strain of reinforcement at compression and tension zone is same due to symmetrical reinforcement

\[ \varepsilon'_s = \varepsilon_s \]  \hspace{1cm} (12)

\[ \sigma'_s = \sigma_s \]  \hspace{1cm} (13)

\[ \varepsilon'_c = \frac{h - 2a_s}{h} \varepsilon'_c \]  \hspace{1cm} (14)

Therefore, Equation (11) can be rewritten as

\[ M = \frac{bh^2}{6} - \frac{\pi D^4}{8h} - \frac{\pi D^2 (h - 2a_i)^2}{2h} ] \sigma_c + \frac{A'_i \sigma'_i (h - 2a_i)}{2} \]  \hspace{1cm} (15)

The increment of moment \( \Delta M \) is
The increment of reinforcement strain is

$$\Delta \varepsilon_s = \frac{h-2a_s}{h} \Delta \varepsilon_c$$

(17)

Combining Equations (3), (16) and (17), one can obtain

$$\Delta \sigma_c = \frac{\Delta M - A_s E_s \eta}{6 - \frac{\pi D^4}{8h} - \frac{\pi D^2 (h - 2a_s)^2}{2h}} \frac{h-2a_s}{h}$$

(18)

$$\Delta \sigma'_c = \frac{\Delta M + \frac{bh^2 - \pi D^4}{6 - \frac{\pi D^2 (h - 2a_s)^2}{2h} - \frac{E_n}{E_n} \eta}}{6 - \frac{\pi D^4}{8h} - \frac{\pi D^2 (h - 2a_s)^2}{2h}} \frac{E_s (h-2a_s)}{h}$$

(19)

Based on the above formula, the variation rules of the parameters such as the strain and stress of reinforcement and concrete at different times can be calculated by programming.

4. Case Analysis

Figure 2 shows the dimensions of reinforced concrete beam. $b=140\, \text{mm}$, $h=160\, \text{mm}$, $a_s=37\, \text{mm}$, $D=14\, \text{mm}$, and the reinforcement ratio is 2%. According to CEB-FIP [15], the evolution of elastic modulus $E_c = 26.51 \times 10^4 \, \text{GPa}$, the elastic modulus of reinforcement $E_s = 200 \, \text{GPa}$. Based on the design code for hydraulic concrete structures [12], the cracking force of reinforced concrete $N_{cr} = 10.5 \, \text{kN}$, so the loading force is $0.5 \, N_{cr}$, and the applied moment $M_{cr} = 1.136 \, \text{kN} \cdot \text{m}$. In the current case, the loading age is 28d, and the loading duration is 200d. The specific creep is [14]:

$$C(t, \tau) = \sum_{j=1}^{2} (f_i + g_i \tau^{-p_i}) [1 - e^{-r_i(t - \tau)}]$$

(20)

Parameters in Equation (20) take the specific value shown in table 1.

| Parameter | $f_i (10^{-4})$ | $g_i (10^{-4})$ | $p_i$ | $r_i$ |
|-----------|-----------------|-----------------|-------|-------|
| $i=1$     | 0.1045          | 0.9614          | 0.45  | 0.3   |
| $i=2$     | 0.2360          | 0.4012          | 0.45  | 0.005 |

![Figure 2. Dimensions of reinforced concrete beam.](image)
Based on the assumed conditions of the case, the strain and stress of concrete and reinforcement in the compression zone and tension zone of reinforced concrete specimen are equal, but the signs are opposite. Therefore, the compression zone of the specimen is taken as the analysis object below.

4.1. Strain Evolution

Figure 3 shows the strain curves of the outermost concrete and adjacent reinforcement in the compression zone. It can be seen that with the growth of age, the strain of reinforcement and concrete increases rapidly in the early stage and gradually slows down in the later stage, which is consistent with the creep change law of concrete. Component separation of concrete strain is shown in figure 4. Due to the creep of concrete, the elastic strain of concrete strain generated by initial load is 58με, which increases to 109με after 200d of loading duration.

![Figure 3](image)

**Figure 3.** Strain curves of the outermost concrete and adjacent reinforcement in the compression zone.

![Figure 4](image)

**Figure 4.** (a) Section of reinforced concrete beam, (b) Strain distribution after 200d loading, (c) Instantaneous elastic strain distribution, and (d) Creep strain distribution.

4.2. Stress Evolution

The creep characteristic of concrete causes stress redistribution of reinforced concrete specimens after loading. The stress of concrete decreases continuously, while the stress level of reinforcement increases continuously, as shown in figure 5. Similar to the variation curve of strain, the stress curve of reinforcement and concrete also shows the characteristics of rapid growth in the early stage and gradual growth in the late stage. The stress of concrete decreases from 1.54 to 1.17MPa, and the decrease ratio is about 23.7%. The stress of the reinforcement increases by nearly two times compared with that at the initial loading.
Figure 5. Stress curves of the outermost concrete and adjacent reinforcement in the compression zone.

4.3. Stress Transfer Ratio
Figure 6 shows the change curve of concrete stress transfer ratio with age. With the decrease of concrete stress level, the bending moment of concrete itself is gradually transferred to the reinforcement, and the stress transfer ratio tends to moderate with the increase of loading time. After 200 d loading duration, the transfer ratio of concrete stress is about 23.7%.

Figure 6. Concrete stress transfer ratio.

5. Conclusions
In this paper, formula for calculating the stress change of reinforced concrete beams considering creep is derived. The calculation results show that, due to creep characteristics of concrete, the stress transfer ratio of concrete increases with the loading duration, which increases rapidly in the early stage and tends to be gentle in the later stage. After 200 d loading duration, the transfer ratio of concrete stress is about 23.7%.

References
[1] Huang G X, Hui R Y, Wang X J. 2011 Creep and shrinkage of concrete. Beijing: China Electric Power Press.
[2] Ranaivomanana N, Multon S, Turatsinze A. 2013 Basic creep of concrete under compression, tension and bending. Construction and Building Materials 38 173–180.
[3] Rossi P, Tailhan J L, Maou F L, Gaillet L, Martin E. 2012 Basic creep of concretes investigation of the physical mechanisms by using acoustic emission. Cement and Concrete Research 42 61–73.
[4] Reinhardt H W, Rinder T. 2006 Tensile creep of high-strength concrete. Journal of Advanced Concrete Technology 4 (2) 277-283.
[5] Mu G B, Wang L G, Wang C. 2007 Creep effect analysis of steel reinforced concrete beam. Journal of Northeastern University (Natural Science) 28 (5) 733-736.

[6] Xue W C, Hu Y M, Wang W. 2008 Experiment on creep behaviors of prestressed concrete beams China Journal of Highway and Transport 21 (4) 61-66.

[7] Pan Z F, Lv Z T, Meng S P. 2009 Experimental study on the influence of steel on high-strength concrete creep and shrinkage China Civil Engineering Journal 42 (2) 11-16.

[8] Koga H, Watanabe H, Takeuchi Y, Aoyama H, Kitano Y. 2009 Experimental study on the time dependent flexural behavior of prestressed reinforced concrete beams. Creep, Shrinkage and Durability Mechanics of Concrete and Concrete Structures-Proceedings of the 8th Int. Conference on Creep, Shrinkage and Durability Mechanics of Concrete and Concrete Structures 1 781-786.

[9] Antonio R, Bairán J M, Duarte N. 2010 Long-term deflections in cracked reinforced concrete flexural members Engineering Structures 32 (3) 829-842.

[10] Piyasena R, Loo Y C, Fragomeni S. 2003 Determination of crack spacing and crack width in reinforced concrete beams. Structural Engineering and Mechanics 15 (2) 159-180.

[11] Song W, Yuan Y, Gong J. 2002 Experimental research on tensile performance of reinforced concrete Journal of Southeast University (Natural Science Edition) 32 Sup.:98-101.

[12] SL 191-2008. Design code for hydraulic concrete structures. Beijing: China Water & Power Press, 2009.

[13] Mimura Y, Yoshitake I, Zhang W. 2011 Uniaxial tension test of slender reinforced early age concrete members. Materials 4:1345-1359.

[14] Zhu B F. 2009 Thermal stresses and temperature control of mass concrete. Beijing: China Water & Power Press.

[15] CEB-FIP. International recommendations for the design and construction of concrete structures. London: Cement and Concrete Association, 1970.