Extra-Acceleration at Large Scale due to Merging at Small Scales

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(Dated: May 22, 2018)

Merging of cosmic objects at small scales is regarded as a possible source for explaining the extra-acceleration of the universe at large scale. Inspired by chemical association, merging of cosmic objects introduces a correction term in the perfect fluid equation of state in the form of \( P = w\rho + b\rho^2\). Alternative relations for the energy density and scale factor evolution in the FRLW framework are obtained that coincide with the conventional results in the perfect fluid limit. Invoking the observational constraint for the equation of state parameter \( w \) and the deceleration parameter, we show that the simultaneous merging of cosmic objects such as galaxies on one hand and voids on the other, will act as a source of extra-accelerating in the universe at large scale.

PACS numbers: 98.80.Jk, 95.36.+x, 98.80.Bp

I. INTRODUCTION

The inflationary models show acceleration in the early universe[1–3]. As of 1981 it is believed that in the far past time, the universe exponentially accelerated and then came out of this phase after a very short period. Before decoupling, the universe expansion was driven by radiation and particles, but after that driven only by matter. Thus, it was thought that the expansion was slowing down[4–7]. However, in 1998–99, contrary to the above picture, the accelerated expansion was confirmed by two groups from observations of supernova Ia explosion. WMAP and Planck recently confirmed the acceleration in the late universe[10, 11]. Naturally, a real model of cosmology requires a source of energy to drive acceleration in the present universe. To search a suitable source for dark energy it is important to know the nature of the effective cosmic fluid[4].

Modern cosmology strongly depends on particle physics, and the role of scalar field is very significant in it[3]. So, cosmologists proposed scalar fields as a source of dark energy in several models. Some of the most important of them are Quintessence[12], K-essence[13], tachyon scalar[14], Chaplygin gas[15, 16], phantom scalar[17, 18] and generalized Chaplygin gas[19], and so on. On the geometrical side, it has been proposed that the general relativity should be modified to account for dark energy, where some curvature functions are assumed to explain late acceleration. Some models in this regard include \( f(R) \), and \( f(T) \) gravity[20–23], where \( R \) and \( T \) are scalar curvature and scalar torsion, respectively. In this work, we try to explain the present accelerating universe by introducing some kind of real cosmic fluid (RCF).

The perfect cosmic fluid (PCF) for the whole universe with the equation of state (EOS)[4, 24],

\[
P = w\rho,
\]

is usually invoked in most cosmological models, where \( P, w \) and \( \rho \) are the pressure, EOS parameter and energy density of the cosmic fluid, respectively. However in the real universe, it can not accurately describe the physical properties of the cosmic fluid. One problem in the ideal gas law (Eq.1) is what happens to the regions with higher densities in the limit of higher densities. As the volume of the regions becomes so small, their densities become higher. In the ideal gas law it is assumed that the particles are point-like without any volumes and interactions, but in the real physical world is much better to be modeled by a more real gas system.

In fact, there exist some phenomena occurring between cosmic objects in the RCF. Merging of the cosmic objects is a common phenomenon at cosmological scale. Galaxies and galaxy clusters, and even voids are all always merging together to form bigger ones. We think that probably the merging may be responsible for the extra-acceleration of the universe at large scale.

The cosmic fluid, as we know is so dilute that, as a whole we can consider it as an ideal gas. However, because of the importance of merging phenomena we would like to include its effects into the behavior of the cosmic fluid. Since the cosmic fluid is very dilute, merging occurs between only two objects \( B \), that we can write this as a simple chemical association reaction; \( 2B = A \)[25]. Here cosmic object \( A \) represents the merged one formed from two initial objects \( B \). If \( g(= N/V) \) stands for the number

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density of the cosmic fluid then, based on the proposed merging reaction we have \( \varrho_B = \varrho - 2\varrho_A \), where subscribe \( B \) and \( A \) stand for the number densities of the cosmic fluid before and after the merging process, respectively. The equilibrium constant of the merging process is [25]

\[
K(T) = \frac{\varrho_A}{(\varrho - 2\varrho_A)^2} \approx \frac{\varrho_A}{\varrho^2},
\]

here \( \varrho_A \ll \varrho \). The whole number density of the cosmic fluid due to merging is changed from \( \varrho \) to \( (\varrho - 2\varrho_A) + \varrho_A = \varrho - \varrho_A \). So we can write \( PV = (\varrho - \varrho_A)RT \) or

\[
\frac{PV}{Nk_B T} = 1 - \frac{\varrho_A}{\varrho} = 1 - K(T)\varrho,
\]

here \( K(T) > 0 \). Although the volume of the merged regions are small fractions of the total cosmos volume, they are becoming more important in the smaller scales and their contributions will become larger and larger fraction as depicted in FIG.1. Taking into account the contribution and effect of the volumes and interactions of these merged regions also play an important role in the dynamics of the large scale of the cosmos. The main task of this article is to address this very common and important process.

The PCF (1) does not account the merging process and its effects. Let’s suppose that in addition to presence of conventional cosmic fluid with the EOS (1), there is another process in the small scale of the universe; their merging with each other. In the present universe, the galaxies and galaxy clusters in one hand, and vast voids in the other hand are the possible natural candidates for these merging objects [26]. We will thus know that they contribute some portions of the volume of the whole cosmic fluid. This assumption, based on the real gas theory (accounting the merging process) by using (3) and \( E = \rho V = (1/w)Nk_B T \), leads to the following form of EOS[24, 25],

\[
P = w\rho + b\rho^2,
\]

where,

\[
b = -K(T)w^2/k_B T,
\]

is proportional to the equilibrium constant \( K(T) \). \( k_B \) and \( T \) are the Boltzmann constant and temperature, respectively.

Note that the parameter \( b \), taking into account the merging process is temperature dependent and negative. It seems that at lower temperatures the absolute values of it increases meaning the merging of the cosmic objects come more into play at lower and lower temperatures as expected. Thus the real cosmic gas (4), is containing of two parts: The normal part \( w\rho \), occupy the larger contribution of the volume of the universe with low density. Another part of the cosmic fluid is the merged one, contains some cosmic objects such as galaxies and galaxy clusters, and even small voids . Their volumes are small in comparison to the total volume of the universe but their densities are higher than the density average of the whole universe.

![FIG. 1: Real Cosmic fluid includes some higher (lower)-dense merging regions[27].](image)

**II. THE MAIN COSMOLOGICAL RELATIONS**

**A. Energy Density in Terms of Scale Factor**

One of the most important properties of the Universe is its energy density. Energy density is defined as the energy within a region, divided by the volume of that region. Since the universe is homogeneous, it does not matter where the supposed regions are located. How can it be large? It depends on the epoch under consideration. In the early universe, before galaxy formation gets underway, such regions should contain many tiny particles. After the formation of galaxies the regions must contain many galaxies, and after the formation of galaxy clusters it must contain many galaxy clusters [27].

The Friedmann equations tells us the effect of gravity on the expansion rate. According to them, that effect is related to the scale factor itself, and the energy density [4–7]:

\[
\frac{\dot{a}^2(t)}{a^2(t)} + \frac{kc^2}{R^2 a^2(t)} = \frac{8\pi G N}{3} \rho
\]

and

\[
\frac{\dot{a}(t)}{a(t)} = -\frac{4\pi G N}{c^2} \left( p + \frac{1}{3} \rho c^2 \right)
\]

where \( a(t) \) represents the scale factor and \( k \) is the curvature signature. These equations can be combined to get [5, 6],

\[
\frac{d}{dt} (\rho c^2 a^3) = -p \frac{da^3}{dt}
\]
We shall study the scaling behavior of the energy density in terms of scale factor. For the perfect fluid Eq.(1), from (8) we have the standard result,

$$\rho = \rho_0 a^{-3(1+w)}.$$ \hspace{1cm} (9)

It turns out that the expansion rate depends on where the energy density comes from. The part of the energy density that comes form particles are being attracted towards each other by the force of gravity which is what one would expect. Strangely though, the part of the energy density that comes from the cosmological constant (as a candidate) has the opposite effect and it increases the expansion rate of the large scale universe\cite{27}. Instead of cosmological constant, as we have mentioned above, the merging of higher(lower)-dense cosmic objects are now introduced to account for extra-accelerations observed in our universe. To have a real cosmology with these two parts of the energy density, we invoke our proposed EOS Eq.(4). After some calculations the final relation for energy density obtain as (Appendix A),

$$\rho_r = \rho_0 a^{-3(1+w)} \left(1 - \frac{b\rho_0}{(1+w)} \left(1 - a^{-3(1+w)}\right)\right).$$ \hspace{1cm} (10)

Here, subscript r, represents the real cosmic fluid (RCF).

**B. Evolution of the Scale Factor**

We will proceed to solve equation (6) for the time evolution of the scale factor \(a(t)\). Here we assume that second term of the Eq.(10), as the first perturbation term for the energy density of the real universe.

In order to find the scale factor as a function of time, the equation (10) can now be combined with (6). This goal will be achieved by invoking perturbation theory by taking \(\lambda\) as the perturbation coupling parameter and assuming that

$$a_r = a + \lambda a_1,$$ \hspace{1cm} (11)

where \(a_1\) is the perturbative part of the scale factor to be determined. We thus find (with another rationale assumption that \(k = 0\)):

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G N}{3} \rho_0 a^{-3(1+w)}.$$ \hspace{1cm} (12)

For the reference part with known solution and the generalized one to be solved:

$$\left(\frac{\dot{a}_r}{a_r}\right)^2 = \frac{8\pi G N}{3} \rho_r,$$

$$= \frac{8\pi G N}{3} \rho_0 a^{-3(1+w)} \times \left(1 - \frac{\lambda b\rho_0}{(1+w)} \left(1 - a^{-3(1+w)}\right)\right).$$ \hspace{1cm} (13)

After some calculations (Appendix B), the final expression for the scale factor will be obtained as:

$$a_r(t) = \left(1 - \frac{3 b\rho_0 n^2}{4} \left(\frac{t}{t_0}\right)^n\right) + \frac{6 b\rho_0 n^2}{4} \left(\frac{t}{t_0}\right)^{n-1} - \frac{3 b\rho_0 n^2}{4} \left(\frac{t}{t_0}\right)^{n-2}.$$ \hspace{1cm} (14)

Where we have,

$$n = \frac{2}{3(1+w)}.$$ \hspace{1cm} (15)

**III. SOME RESULTS AND POSSIBLE INTERPRETATIONS**

Let us define \(\alpha = \frac{b\rho_0}{(1+w)}\) to recast relation (10) to

$$\rho_r = (1 - \alpha)\rho_0 a^{-3(1+w)} + \alpha\rho_0 a^{-6(1+w)}.$$ \hspace{1cm} (16)

Compared to the conventional relation (9), ours is divided into two different parts; the first part is essentially the same as standard one, except that now a certain fraction of it has been subtracted. The second part has exactly the same fraction coming from the second term of the equation of state (4) containing the equilibrium constant. We can state that the first term of (16) relates to regions with higher energy density if and only if the second term relates to regions with lower density or vice versa (see FIG. 2). As expected in the limit of \(\alpha = 0\), Eq.(16) returns to the conventional result (9). However, for the present universe containing RCF as interacting gas with \(\alpha \neq 0\), the correction term gets important and is applicable to the regions like of the galaxies, galaxy clusters, and voids.

Since, we will get the standard solution at the limit of \(\alpha = 0\), \(\alpha\) can be regarded as a correction factor to it (9),

$$0 \leq |\alpha| < 1.$$ \hspace{1cm} (17)

At the other limit \(\alpha \rightarrow 1\), the contribution of second term of Eq.(16), becomes dominant. **Note that the \(\alpha\) values are related to the \(K\), intensity of the merging phenomenon among cosmic objects. The gravitational merging observed in the galaxies, galaxy clusters, and even voids can be candidates for this phenomenon.**

As seen the \(\alpha\) values depend on three parameters \(\rho_0\), \(b\), and \(w\). \(\rho_0\) is known, hence the values of \(b\) and \(w\) should be adjusted to give the suitable \(\alpha\) values\cite{17}. From the definition of \(\alpha = \frac{b\rho_0}{(1+w)}\), in order to get acceptable values for it one should have \(b\rho_0 \approx 1 + w\). Since \(\rho_0 \sim 10^{-10}\) and \(w \approx -1\), one can infer that \(b\) should assume large values. In general, when \(w \rightarrow -1\), \(b \rightarrow 0\), and reversely, when \(w\) goes away from \(-1\), \(b\) assume much larger values.

The observational data show that parameter \(w\) has a very narrow range around \(w = -1\) with more likelihood
to the side of $w \lesssim -1$ [11]. So, for this range of $w$, we have $\alpha > 0$. The positive sign for the energy density in second term of Eq.(16), can mean the gravitational merging of cosmic objects, but with the negative sign in the first term of Eq.(16) that can mean as repulsive gravitational force (expansion). This additional repulsion force in the inter galactic voids causes extra-acceleration in the large scale universe. On the other hand, for the range $w \gtrsim -1$, we have $\alpha < 0$. The negative sign for the energy density in second term of Eq.(16), could mean as a very large gravitational repulsion due to merging of the vast voids (see FIG. 2). In figure 3 the energy density $\rho/\rho_0$ is plotted for the range $(-1.5 < w < -0.5)$ for the PCF model and RCF models with $\alpha = 0.3$ and $\alpha = -0.3$. As shown in the figure 3, except for the particular value $w = -1$, the energy density of the universe at large scale with the assumption of gravitational merging, is always lower than the energy density of cosmic gas with non-merging objects.

A. Discussions on the Deceleration Parameter

We can write equation (14) as,

$$a_r(t) = (1-\gamma)\left(\frac{t}{t_0}\right)^n + 2\gamma \left(\frac{t}{t_0}\right)^{n-1} - \gamma \left(\frac{t}{t_0}\right)^{n-2},$$

(18)

where we take $\gamma = \frac{3b\rho_0n^2}{4} = \frac{b\rho_0}{3(1+w)^2} = \frac{n\alpha}{2}$. As can be seen, we have in the limit of $t \to t_0$, $a \to a_0$. For the case of $\alpha = 0$, we have the conventional form of the scale factor, i.e. $a(t) = \left(\frac{t}{t_0}\right)^n$. Also, for $(t/t_0) \to 0$, we should have $a \to 0$.

The deceleration parameter $q$, in cosmology is defined as [4, 7],

$$q = -\frac{H}{aH^2} = -\frac{a\ddot{a}}{(\dot{a})^2}t=t_0$$

(19)

The larger value of $q$ with negative sign shows more rapid acceleration. It is a way to quantify the accelerated expanding of present universe at $t = t_0$. After simple calculations we will obtain the Hubble and deceleration parameters as,

$$H_r = H_p = n$$

$$q_r = -1 + \frac{1}{n} + \frac{\alpha}{n},$$

(20)

by considering $b < 0$, the sign of $\frac{\alpha}{n} = \frac{3b\rho_0}{2}$ is always negative. So, we can rewrite

$$q_r = q_p - \frac{\alpha}{n}$$

(21)

and consequently we have $q_r$ larger than $q_p$ with negative sign.

Let us compare the deceleration parameters of both PCF and RCF models in terms of parameter $w$. In figure 3 the deceleration parameters $q$ is plotted versus $w$ in the range of $(-1.05 < w < -0.95)$ for the PCF model and RCF models with $\alpha = 0.3$ and $\alpha = -0.3$. As seen, the three plots cross each other at a particular value $w = -1$. Since the observed values for parameter $w$ for type Ia supernovae, the equation of state of dark energy is constrained to $w = -1.006 \pm 0.045$ [11], by consideration $w = -1.006$, we obtain deceleration parameter for PCF model $q = -1.009$, and for RCF model, $q = -1.012$. The significant result is that the acceleration for the both phantom ($w \lesssim -1, \alpha > 0$) and non-phantom ($w \gtrsim -1, \alpha < 0$) phases of the cosmic fluid in
the RCF model is larger than PCF model (In figure 4 and 5, blue lines situated under red line). According to relation (16) and the results obtained for the deceleration parameter, it seems that the extra-acceleration at large scale of cosmos coming from the integration and merging of the cosmic objects in the small scales of the universe. These cosmic objects can include the galaxies, the galaxy clusters and the small voids on the one hand and the vast voids on the other. In figure 2, we implicitly showed that the shrinking in higher-dense regions causes expanding in the lower-dense parts of the cosmic gas, similar to what is found in soap bubbles [26, 36].

In this work we have presented the basic ideas and mathematical derivations of model. It seems the RCF model is able to explain dark energy and dark matter simultaneously. The work on these subjects based on this theory is underway. Also, later works will naturally be devoted to the observational and some more interesting consequences.

Acknowledgments

EY would like to acknowledge David H. Lyth, Hassan Firouzjahi, Shant Baghram, Saeed Tavasoli, Majid Mohsenzadeh and Mohammad. V. Takook for their help in improving the manuscript. EY would like to thank school of Astronomy at IPM for the material and spiritual support during preparing of this research. This work has been supported by the Islamic Azad University, Ayatollah Amoli Branch, Amol, Iran.

Appendix A: Derivation of the Energy Density

Combination of two Eqs. (8) and (4) gives:

\[ \frac{d\rho}{(1+w)\rho + b\rho^2} = -3\frac{da}{a}, \]  

(A1)

which can be integrated to

\[ \frac{\ln(\rho)}{(1+w)} - \frac{\ln(1+w+b\rho)}{(1+w)} = \ln(ca^{-3}) \]  

(A2)

or

\[ \frac{\rho}{1+w+b\rho} = Ca^{3(1+w)} \]  

(A3)

where \( c \) and \( C = \ln(c) \) are constants. The latter can be rearranged to get the following analytical formula:

\[ \rho = C(1+w)a^{-3(1+w)} \]  

(A4)

In order to find \( C \) we employ the usual conditions at which \( \rho = \rho_0 \) and \( a = 1 \) when \( t = t_0 \), where subscript zero indicate the corresponding value at the present time. This leads to

\[ C = \frac{\rho_0}{1+w+b\rho_0} \]  

(A5)

from which one obtains:

\[ \rho = \frac{\rho_0(1+w)a^{-3(1+w)}}{1+w+b\rho_0 - b\rho_0a^{-3(1+w)}} \]  

(A6)

Since \( \rho_0 \) is a so tiny quantity we can first write Eq.(A6) as

\[ \rho = \frac{\rho_0a^{-3(1+w)}}{1 + \frac{b\rho_0a^{-3(1+w)}}{(1+w)}(1-a^{-3(1+w)})}, \]  

(A7)

and then expand it up to the first order to obtain final relation (10).
Appendix B: Derivation of the Scale Factor

In combination with Eq.(11), Eq. (13) becomes:
\[
\left( \frac{\dot{a} + \lambda a_1}{a + \lambda a_1} \right)^2 = \frac{8 \pi G_N}{3} \frac{\rho_0 (a_0 + \lambda a_1)^{-3(1+w)}}{a_0} \times \left( 1 - \frac{\lambda b \rho_0}{1 + w} \right) \left( 1 - \frac{a + \lambda a_1}{a} \right)^{-3(1+w)} \left( 1 - \frac{a}{1 + w} \right) \left( 1 - \frac{a - 3(1+w)}{1 + w} \right)
\]

Or after taking out the reference system from both sides:
\[
\left( \frac{\dot{a}}{a} \right)^2 \left( 1 + \lambda a_1/a \right)^2 = \frac{8 \pi G_N}{3} \frac{\rho_0 a^{-3(1+w)}(1 + \lambda a_1/a)^{-3(1+w)}}{a_0} \times \left( 1 - \frac{\lambda b \rho_0}{1 + w} \right) \left( 1 - \frac{a^{-3(1+w)}(1 + \lambda a_1/a)^{-3(1+w)}}{a} \right) \left( 1 - \frac{a}{1 + w} \right) \left( 1 - \frac{a - 3(1+w)}{1 + w} \right)
\]

Removing the reference parts we are left with:
\[
\left( 1 + \lambda a_1/a \right)^2 = \left( 1 + \lambda a_1/a \right)^{-3(1+w)} \times \left( 1 - \frac{\lambda b \rho_0}{1 + w} \right) \left( 1 - \frac{a^{-3(1+w)}(1 + \lambda a_1/a)^{-3(1+w)}}{a} \right) \left( 1 - \frac{a}{1 + w} \right) \left( 1 - \frac{a - 3(1+w)}{1 + w} \right)
\]

In order to make this equation be solvable we will expand both sides in terms of \( \lambda \) up to first order:
\[
1 + 2 \left( \frac{\dot{a}}{a} - \frac{a_1}{a} \right) \lambda + ... = 1 + \left( -\frac{b \rho_0 (1 - a^{-3(1+w)})}{1 + w} - \frac{3(1 + w)a_1}{a} \right) \lambda + ...
\]

Fortunately, this will lead to a linear differential equation for finding \( a_1 \) in terms of known \( a \):
\[
2 \frac{a_1}{\dot{a}} - \frac{2a_1}{a} = 3 \frac{(1+w)a_1}{a} - \frac{b \rho_0 (1 - a^{-3(1+w)})}{1 + w},
\]

which may be cast in a simpler form:
\[
\dot{a}_1 + \left( \frac{3(1+w)}{2} - 1 \right) \left( \frac{\dot{a}}{a} \right) a_1 = -\frac{b \rho_0 (1 - a^{-3(1+w)})}{2(1 + w)}.
\]

Let’s invoke the reference system solution. It reads as
\[
a(t) = \left( \frac{t}{t_0} \right)^n, \text{ satisfying naturally the condition at } t = t_0,
\]

\[
a = 1. \text{ Also, putting it in Eq.(12) leads to (15), and } \frac{\dot{a}}{a} = \frac{n}{t} \text{ needed later. Plugging them into Eq.(B6) will transform it into:}
\]

\[
\dot{a}_1 + \left( 1 - \frac{n}{t} \right) a_1 = -\frac{3b \rho_0 n^2}{4t} \left( \frac{t}{t_0} \right)^n - \left( \frac{t}{t_0} \right)^{n-2}.
\]

This is a simple first order differential equation with the following solution:
\[
a_1 = -\frac{3b \rho_0 n^2}{4} \left( \frac{t}{t_0} \right)^n + \left( \frac{t}{t_0} \right)^{n-2} + \frac{6b \rho_0 n^2}{4} \left( \frac{t}{t_0} \right)^{n-1}.
\]

Where we have used the condition at \( t = t_0, a_1 = 0 \), and hence the final expression for the scale factor will be obtained as (14).

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