Potentials and superpotentials in the effective $N = 1$ supergravities from higher dimensions

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Abstract

We consider $N = 1$ superpotentials corresponding to gaugings of an underlying extended supergravity for a chiral multiplet in the $SU(1, 1)/U(1)$ manifold of curvature $2/3$. We analyze the resulting $D = 4$ scalar potentials, and show that they can describe different $N = 1$ phases of higher-dimensional supergravities, with broken or unbroken supersymmetry, flat or curved backgrounds, sliding or stabilized radius. As an application, we discuss the $D = 4$ effective theory of the detuned supersymmetric Randall-Sundrum model in two different approximation schemes.

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1. Introduction and summary

Compactifications of higher-dimensional supergravity and superstring theories preserving $N = 1$ supersymmetry in $D = 4$ dimensions, in its exact or spontaneously broken phase, have great phenomenological interest. Their low-energy effective theories \[1\] typically include a chiral multiplet whose spin-zero component has two real degrees of freedom: one parameterizes the volume of the internal space, the other one some internal gauge degree of freedom of the higher-dimensional theory. If we neglect the dynamics of all other fields, apart from the gravitational multiplet, this complex scalar field parameterizes the special Kähler manifold $SU(1,1)/U(1)$, with scalar curvature $2/3$. In a suitable field basis, we can decompose it as

$$T = t + i \tau, \quad (t \text{ and } \tau \text{ real}),$$

and write for its Kähler potential, in the standard notation of $N = 1$, $D = 4$ supergravity \[2\] and in $D = 4$ Planck mass units:

$$K(T, \overline{T}) = -3 \log(T + \overline{T}). \quad (2)$$

This is the case, for example, for the known compactifications of minimal $D = 5$ supergravity \[3, 4\] on the orbifold $S^1/Z_2$, both in the flat \[5\] and in the warped \[6, 7\] case. This is also true for compactifications of the same theory on the circle $S^1$ \[4\], which give $N = 2$, $D = 4$ supergravity coupled to a single vector multiplet. The same result is obtained in those higher-dimensional supergravity and superstring compactifications with branes, orientifolds and fluxes, where some of the moduli are fixed, but not the overall volume modulus \[8\].

In agreement with the special geometry of $N = 2$, $D = 4$ supergravity coupled to vector multiplets \[9, 10\], the Kähler potential \[2\] derives from the prepotential:

$$F = \frac{(X^1)^3}{X^0}. \quad (3)$$

Indeed \[10, 11\] :

$$K = -\log Y, \quad Y = i \left( \overline{X^I} F_I - X^I \overline{F_I} \right), \quad (4)$$

where a sum over the index $I = 0, 1$ is understood, $F_I \equiv \partial F / \partial X^I$, and $X^0$ is the compensating vector multiplet of the $N = 2$ superconformal theory. Introducing the unconstrained field $T = iX^1/X^0$, and choosing the gauge $X^0 = 1$, we obtain $Y = (T + \overline{T})^3$, which gives precisely the Kähler potential \[2\].
Given the Kähler potential (2), the superpotential \( w(T) \) encodes the information on the underlying higher-dimensional model. Schematically, considering only the \( T \) field and a generic \( w(T) \) amounts to reduce supersymmetry to four supercharges. The structure of \( w(T) \), however, should keep track of its higher-dimensional origin. With sixteen supercharges, as required minimally by the ten-dimensional supersymmetry algebra, a four-dimensional supergravity is completely specified by the real structure constants defining the gauging. Truncating this \( N = 4 \) theory \(^{12} \) to \( N = 1 \) leads to seven moduli (three associated with the complex structure, \( T_1, T_2, T_3 \), three associated with the Kähler structure, \( U_1, U_2, U_3 \), and the four-dimensional dilaton) or more. The \( N = 1 \) superpotential can then be obtained directly from the field-dependent gravitino mass term\(^1\)

\[
m_{3/2} = w e^{K/2} = (S + \overline{S})^{-1/2} \left( f_{i_1 i_2 i_3} \Phi^{i_1} \Phi^{i_2} \Phi^{i_3} + S \tilde{f}_{i_1 i_2 i_3} \Phi^{i_1} \Phi^{i_2} \Phi^{i_3} \right),
\]

where \( S \) is the complex dilaton parameterizing the \( SU(1,1)/U(1) \) manifold. The \( \Phi^{i_A} \) collectively denote all the other scalar fields of the \( N = 1 \) truncation, with \( A = 1, 2, 3 \) labeling the threefold degeneracy of the spectrum (the three \( T_A \) and \( U_A \) moduli for instance)\(^2\), and \((f_{i_1 i_2 i_3}, \tilde{f}_{i_1 i_2 i_3})\) are the real structure constants defining the gauging of the underlying \( N = 4 \) theory. These structure constants also induce, in general, a scalar potential. Notice that the expression (5) contains both perturbative and non-perturbative terms (with respect to \( S \)), as dictated by the \( SU(1,1) \) duality of \( N = 4 \) supergravity. The procedure outlined above is general enough to describe the dynamics of a variety of phases of the higher-dimensional theory in a duality-invariant way. Examples include effective Lagrangians for \( N = 4 \) strings \(^{14} \), finite-temperature phases of five-dimensional superstrings \(^{13, 15} \) and the effective description of partial (non-)perturbative supersymmetry breaking in string theory \(^{16} \). Freezing all the scalar fields \( \Phi^{i_A} \) apart from \( T \) will certainly lead to a polynomial superpotential \( w(T) \) of third order in \( T \). Its coefficients will be defined by the values assigned to the frozen fields, which are in general complex. Actually, it has been known for long \(^{17} \) that non-zero coefficients can be related to background values of antisymmetric tensors, \textit{aka} fluxes. We will therefore consider a superpotential of the form

\[
w(T) = m_0 - i m_1 T + 3 n^1 T^2 + i n^0 T^3, \tag{6}
\]

\(^1\)The procedure outlined here is described in detail in ref. \(^{13} \), section 4.

\(^2\)The exact relation between \( \Phi^{i_A} \) and, for instance, \( T_A \) and \( U_A \) follows from the solution of the \( N = 4 \) Poincaré constraints.
where \((m_0, m_1, n^0, n^1)\) are four arbitrary complex coefficients. Notice also that, since

\[
w = m_1 X^I - n^I F_I,
\]

a generic \(N = 2\) gauging of the theory \cite{18} with one vector multiplet only would correspond to the additional condition of taking the coefficients \((m_0, m_1, n^0, n^1)\) to be real, apart from an invisible overall phase in \(w\). Known examples of \(N = 2\) gaugings with one vector multiplet correspond to supersymmetric \(AdS_5\) backgrounds \cite{4}, and to the \(D = 4\) effective theories of supersymmetry-breaking compactifications on a flat \(D = 5\) background \cite{19, 20}, induced by twisted periodicity conditions à la Scherk-Schwarz \cite{21}. Some orientifold compactifications provide instead examples of superpotentials of the form \((6)\), but with complex coefficients \cite{8}. We will discuss in this paper another instructive example of this sort: the effective theory of the supersymmetric Randall-Sundrum (SUSY-RS) model with two branes \cite{22} and arbitrary tensions \cite{23, 24, 25, 7}.

Our goal in this paper is to study the general features of the \(T\) gaugings, in the generalized sense defined by eq. \((6)\) with complex coefficients. After giving a complete discussion of the ‘true’ \(N = 2\) gaugings, we confront the superpotential \((6)\) with another requirement, motivated by \(N = 1\) compactifications of higher-dimensional supergravities with negligible warping: the independence of the scalar potential from \(\tau\), which in this context is proportional to the internal component of an abelian gauge field. We then perform a similar analysis in a field basis that is more appropriate to discuss warped compactifications with non-negligible warping. We conclude the paper by discussing, as an illustration of our formalism, the effective theory of the detuned SUSY-RS model \cite{6, 7} in two different approximations. Some useful formulae for the backgrounds \cite{24} of the SUSY-RS model are collected in the appendix. The generalization of the results of this paper to the case of more chiral multiplets, relevant for the effective theories of superstring compactifications with fluxes, branes and orientifolds, is left for future work.

2. ‘True’ \(T\)-gaugings

We recall first that the scalar potential of a model with the Kähler potential of eq. \((2)\) and a generic superpotential \(w(T)\) is

\[
V = \frac{1}{(T + \bar{T})^2} \left[ \frac{|w_T|^2 (T + \bar{T})}{3} - (w_T \bar{w} + \bar{w}_T \bar{w}) \right],
\]

(8)
and also that the Kähler potential of eq. (2) has an $SU(1, 1)$ Kähler invariance, identified as a continuous $T$–duality:

$$T \rightarrow \frac{aT - ib}{icT + d}, \quad a, b, c, d \text{ real}, \quad ad - bc = 1.$$  \hspace{1cm} (9)

Invariance of the theory requires the following transformation of the superpotential:

$$w(T) \rightarrow (icT + d)^3 w \left( \frac{aT - ib}{icT + d} \right),$$  \hspace{1cm} (10)

which turns a cubic polynomial into another one, with transformed parameters. $T$–duality can then be used to eliminate some of them, as discussed below. Notice that both $SU(1, 1)$ duality and this form-invariance of the superpotential find their origin in the underlying $N = 4$ duality symmetry.

We concentrate in this section on ‘true’ $N = 2$ gaugings where the superpotential parameters $(m_0, m_1, n^0, n^1)$ in eq. (6) are real numbers. With the help of eq. (10), we can show that $SU(1, 1)$ can always be used to eliminate the cubic and linear term in $w(T)$, i.e. to set $n^0 = m_1 = 0$ (notice that, even if we start from real parameters and we allow for duality transformations, the general form of the superpotential includes at least an “electric” and a “magnetic” term). The proof is more easily given using the generators

$$\mathcal{I} : \quad T \rightarrow 1/T, \quad (a = d = 0, \ b = -c = 1),$$
$$\mathcal{S} : \quad T \rightarrow T - i, \quad (a = b = d = 1, \ c = 0).$$  \hspace{1cm} (11)

Under inversion $\mathcal{I}$, the superpotential parameters transform as

$$(m_0, m_1, 3n^1, n^0) \rightarrow (-n^0, -3n^1, m_1, m_0).$$  \hspace{1cm} (12)

A generic element of $SU(1, 1)$ is then of the form $S^C I S^B I S^A$, $(A, B, C$ real). First, by performing an appropriate duality transformation, we can always eliminate the cubic term, i.e. set $n^0 = 0$: it is sufficient to consider a shift of $T$ to eliminate the constant term, followed by an inversion. If the resulting superpotential is a quadratic polynomial, a shift of $T$ eliminates then the linear term. If it is linear in $T$, a shift of $T$ is used to eliminate the constant term and an inversion leads then to a single quadratic term. However, the discussion will be simple enough if we only eliminate the $T^3$ term, $n^0 = 0$, leaving $m_0, m_1$ and $n^1$ free.

With $n^0 = 0$, the scalar potential has the simple form

$$V = -\frac{1}{3t} \left[ m_1^2 + 9 m_0 n^1 - 3 m_1 n^1 \tau + 9 (n^1)^2 (t^2 + \tau^2) \right].$$  \hspace{1cm} (13)

\textsuperscript{3}Since $b = -c$, $\mathcal{I}$ squares to $-1$ when acting on the superpotential.
With $n^0 \neq 0$, the potential depends on the same powers of $t$ and $\tau$, but with more complicated numerical coefficients. Relaxing the requirement of real coefficients in the superpotential does not allow to remove the cubic term by duality transformations, and leads to additional powers of $t$ and $\tau$ in the scalar potential.

To discuss the phases of the theory, we need the expression of $f^T \equiv (2t)^{-1/2} [2 w_T t/3 - w]$, the auxiliary field which controls supersymmetry breaking, as well as the field-dependent gravitino mass:

$$m_{3/2}^2 = |w|^2 e^K = -\frac{V}{3} + (2t)^{-2} |f^T|^2. \quad (14)$$

The relation $V = -3 m_{3/2}^2$ holds then for a supersymmetric $AdS_4$ phase.

Taking into account that the allowed field configurations correspond to $t > 0$ and arbitrary $\tau$, we can now study the properties of eqs. (13)–(14) for different values of the gauging parameters and the corresponding allowed phases of the theory.

(I): $n^1 = m_1 = 0$, $m_0 \neq 0$.

This case corresponds to the original no-scale model \cite{26} with $V \equiv 0$, and spontaneously broken $D = 4$ supersymmetry in Minkowski space-time, with $m_{3/2}^2 = (2t)^{-3} m_0^2$. This includes the effective theory of Scherk-Schwarz compactifications \cite{21} of pure ungauged $D = 5$ supergravity on the orbifold $S^1/Z_2$, with a flat and constant bosonic background \cite{19,5}. At the full $N = 2$ level, this case would correspond to the $D = 4$ effective theory of a Scherk-Schwarz compactification on the circle $S^1$, of which the $N = 1$ theory is a consistent $Z_2$ truncation. Notice finally that a generic duality transformation (10) leads to the equivalent class of superpotentials:

$$w(T) = m_0 (icT + d)^3, \quad (15)$$

representing the most general no-scale model in the $T$ field basis.

(II): $n^1 = m_0 = 0$, $m_1 \neq 0$.

In this case the potential is negative definite, does not depend on $\tau$ and does not have any stationary point with respect to $t$: $V = -m_1^2/(3t)$. The educated reader will immediately notice that, since $m_{3/2}^2 = m_1^2 (t^2 + \tau^2)/(8t^3)$, for $\tau = 0$ we get the $AdS_5$ relation $V = -(8/3) m_{3/2}^2$. This is because, in the context of pure $D = 5$ supergravity, $m_1$ is proportional to the gauging parameter of the graviphoton. This gauging leads to $AdS_5$ supergravity \cite{4}. The relation with the present case can be established, for instance, by writing the $D = 5$
gravitino variation on a background with $D = 4$ Minkowski symmetry, as in the bulk of the SUSY-RS model. The result is identical to the gravitino variation with a superpotential linear in $T$. This is nothing more than a formal manipulation unless a boundary is applied to $AdS_5$, as in the $S^1/Z_2$ orbifold: in this case, however, boundary contributions do modify the effective $D = 4$ superpotential. The equivalent case

(III): $m_0, m_1 \neq 0, n^1 = 0$.

is generated by the shift $T \rightarrow T + im_0/m_1$.

(IV): $m_0 = m_1 = 0, n^1 \neq 0$.

This is the case of a purely “magnetic” gauging. It is equivalent to the previous case since it is connected to it by an inversion $I$: the potential depends then only on the real part of $(1/T)$.

(V): $m_0, n^1 \neq 0$.

In the case where both electric and magnetic terms are present, it is not restrictive to set $m_1 = 0$, since it can always be reached by a suitable duality transformation. As a function of $\tau$, the potential has a local maximum at $\tau = 0$. It then has a stationary point, at

$$\tau = 0$$

if and only if $m_0 n^1 > 0$. At this point,

$$V = -6(n^1)^2 t = -3m_{3/2}^2, \quad f^T = 0,$$

and one has a stable supersymmetric $AdS_4$ phase [27]. If, however, $m_0 n^1 < 0$, the field $t$ is not stabilized.

To summarize, ‘true’ $N = 2$ $T$-gaugings offer the following possibilities: broken supersymmetry in flat space, with a complex flat direction; unstable potential with one or no axionic flat direction, identified with $\tau$ up to a duality transformation; unbroken supersymmetry in $AdS_4$ with stabilized complex $T$. As will be discussed in detail later, none of these solutions is appropriate to describe the detuned SUSY-RS model.
3. Generalized $T$–gaugings

In the general case of the superpotential \([\mathcal{W}]\) with complex coefficients, we are interested in finding all cases where the scalar potential is independent of the axion $\tau$. This is suggested by the fact that, in compactifications with negligible warping, $\tau$ is proportional to the internal component of an abelian gauge field. The general solution to the above problem allows for a unique possibility, besides those corresponding to ‘true’ $N = 2$ gaugings discussed in section 2. Absorbing as usual an overall phase:

\[
(VI) : w(T) = \rho_0 + \rho_1 e^{i \varphi} T, \quad (\rho_0, \rho_1, \varphi \text{ real}, \rho_0, \rho_1 \geq 0).
\]

This case corresponds to $m_0 = \rho_0, \; m_1 = \rho_1 e^{i(\varphi+\pi/2)}, \; n^0 = n^1 = 0$, and is a generalization of cases (I)-(III) discussed in section 2. We then concentrate on the novel possibility represented by $\rho_0, \rho_1, \cos \varphi \neq 0$: as we will see in section 5, this corresponds to the effective theory of the detuned SUSY-RS model in the limit of small warping, when the compactification radius is small with respect to the $AdS_5$ radius. The scalar potential is:

\[
V = -\frac{\rho_1}{6 t^2} (3 \rho_0 \cos \varphi + 2 \rho_1 t),
\]

and has a minimum for $t = -3 \cos \varphi \rho_0/\rho_1$, which falls within the allowed field configurations if $\cos \varphi < 0$. At the minimum $V = \rho_1^3/(18 \rho_0 \cos \varphi) < 0$, corresponding to an $AdS_4$ background. Supersymmetry is unbroken for $\tau = \rho_0 \sin \varphi/\rho_1$, otherwise it is spontaneously broken. This is an example of $t$ stabilization in $AdS_4$.

To conclude this section, we notice that the decomposition of eq. \([\mathcal{W}]\) is stable under imaginary shifts, but unstable under the inversion of $T$, which mixes its real and imaginary part. Therefore, the requirement of a $\tau$-independent potential translates into the existence of a real flat direction when acting with the full $SU(1,1)$ duality group. Similar considerations apply to general analytic field redefinitions.

4. Potentials and superpotentials in the SUSY-RS basis

Another parameterization of the $SU(1,1)/U(1)$ Kähler manifold of curvature $2/3$, equivalent to the one considered in the previous sections, is

\[
K(U, \overline{U}) = -\log Y, \quad Y = \pm [e^{\lambda \pi (U + \overline{U})} - 1]^3.
\]

This particular parameterization has been used for writing the effective theory of the SUSY-RS model \([6, 7]\), in the limit of small $D = 4$ cosmological constant. Indeed, our
notation in eq. (20) has been chosen to fit such an interpretation, even if the present
discussion has a more general validity. As explained in section 5 and in the appendix, we
can identify $\lambda$ with the mass scale of the $D=5$ cosmological constant, in units of the
$D=5$ Planck mass. Moreover, we can set

$$U = r + i b,$$

where the ‘radion’ $r(x)$ describes the $D=4$ scalar fluctuation of the $D=5$ metric, and
the ‘axion’ $b(x)$ is proportional to the zero mode of the internal component $B_5$ of the
graviphoton. Because of the abelian $D=5$ gauge invariance, the scalar potential cannot
depend on $b(x)$.

We can now introduce, for notational convenience, the auxiliary variables

$$X = e^{\lambda \pi U} = e^{\lambda \pi (r + ib)}, \quad \overline{X} = e^{\lambda \pi \overline{U}} = e^{\lambda \pi (r - ib)}.$$  (22)

The relation between the parameterizations of eqs. (20) and (2) is then given by the
analytic field redefinition

$$T(X) = \pm \frac{X - 1}{X + 1}, \quad X(T) = \frac{1 \pm T}{1 \mp T},$$  (23)

which induces, neglecting as usual an overall phase, the following transformation of the
superpotential:

$$w(X) = w[T(X)] \frac{(1 + X)^3}{2 \sqrt{2}}, \quad w(T) = w[X(T)] \frac{(1 \mp T)^3}{2 \sqrt{2}}.$$  (24)

To reach the conventions of refs. [6, 7], we would need an additional Kähler transformation:

$$w(X) \rightarrow w(X) X^{-3}, \quad (X \overline{X} - 1) \rightarrow (X \overline{X} - 1)(X \overline{X})^{-1} = [1 - (X \overline{X})^{-1}].$$  (25)

The superpotential $w(X)$, corresponding to the superpotential $w(T)$ in (6) via (24), is
again a cubic polynomial. As in the previous section, we drop here the requirement of real
coefficients.

For a generic superpotential $w[X(U)]$, the corresponding scalar potential reads

$$V(U, \overline{U}) = \pm \frac{|w_U|^2[1 - e^{-\lambda \pi (U + \overline{U})}] - 3\lambda \pi (w_U \overline{w} + \overline{w} w) + (3\lambda \pi)^2|w|^2}{3\lambda^2 \pi^2[e^{\lambda \pi (U + \overline{U})} - 1]^2},$$  (26)

or, equivalently,

$$V(X, \overline{X}) = \pm \frac{|w_X|^2(X \overline{X} - 1) - 3(X w_X \overline{w} + \overline{X} w \overline{w}) + 9|w|^2}{3(X \overline{X} - 1)^2},$$  (27)
where the sign ambiguity comes from the definition of the $Y$ function, eq. (20). If we now ask for a non-trivial superpotential $w \neq 0$ such that the $D = 4$ scalar potential does not depend on the imaginary part of $U$ (the phase of $X$), as required by $D = 5$ gauge invariance, and we factor out for convenience an arbitrary phase, we obtain as general solution the four possibilities listed below.

(i) : $w(X) = \rho (1 + e^{i\varphi} X)^3, \quad (\rho, \varphi \text{ real, } \rho > 0)$. \hspace{1cm} (28)

This leads to a no-scale model with identically vanishing scalar potential, $V \equiv 0$. Both signs in eq. (20) are allowed, if we restrict the allowed field configurations to $|X| > 1$ and $|X| < 1$, respectively. Supersymmetry is spontaneously broken for all allowed values of $X$.

(ii) : $w(X) = \rho_0 + \rho_3 e^{i\varphi} X^3, \quad (\rho_0, \rho_3, \varphi \text{ real, } \rho_0, \rho_3 \geq 0)$. \hspace{1cm} (29)

This leads to the scalar potential $V = \pm 3 \left[ \rho_0^2 - \rho_3^2 (X\overline{X})^2 \right] / (X\overline{X} - 1)^2$. \hspace{1cm} (30)

This case includes the effective theory of the detuned Randall-Sundrum model in the limit of small $D = 4$ cosmological constant, as derived in [7] and discussed in section 5. The potential has stationary points for $X = 0$ and, if $\rho_3 > 0$, for $|X| = \rho_0 / \rho_3$. The case $X = 0$ is acceptable only if we choose the minus sign in eqs. (20) and (30): it corresponds to a stable vacuum with unbroken supersymmetry, $m_{3/2}^2 = - \langle V \rangle / 3 = \rho_0^2$; for $\rho_0 = 0$ it is Minkowski, and there is a classically massless complex scalar, even if with $\rho_3 > 0$ the potential has a quartic term with positive coefficient for the radion; for $\rho_0 > 0$ it is $AdS_4$.

To discuss the other stationary point we must consider $|X| < 1$ and the minus sign in eqs. (20) and (30) if $\rho_0 < \rho_3$, $|X| > 1$ and the plus sign if $\rho_0 > \rho_3$ (for $\rho_0 = \rho_3$ there is a singularity). Both cases give the same physics. At the minimum the potential is negative, $\langle V \rangle = -3 \rho_0^2 \rho_3^2 / |\rho_3^2 - \rho_0^2| < 0$, but its second derivative with respect to $|X|$ is positive, so we have a stable $AdS_4$ vacuum. Supersymmetry is spontaneously broken unless $3 \lambda \pi b + \varphi = \pm \pi$.

(iii) : $w(X) = \rho X^2, \quad (0 < \rho \text{ real})$. \hspace{1cm} (31)

The scalar potential reads

$V = \pm \frac{\rho^2 (X\overline{X})(X\overline{X} - 4)}{3(X\overline{X} - 1)^2}$. \hspace{1cm} (32)
Its only stationary point is for $X = 0$, so we must choose the minus sign in eqs. (20) and (32), which allows for the field configurations with $|X| < 1$. This leads to unbroken supersymmetry in a flat background, with a stable minimum at $\langle X \rangle = 0$. If this is interpreted as the effective theory coming from the compactification of a higher-dimensional supergravity, it amounts to the simplest example of moduli stabilization with unbroken supersymmetry in flat space. Notice that a similar result could be obtained in a much larger class of theories, characterized by: a Kähler potential $K(|X|^2)$, with arbitrary functional form as long as it does not depend on the phase of $X$ and it admits $X = 0$ among the allowed field configurations; a monomial superpotential $w(X) \propto X^n$, with $n \geq 2$.

(iv) : $w(X) = \rho X$, \hspace{1cm} (0 < \rho \text{ real}) .

The scalar potential reads

$$V = \pm \frac{\rho^2 (4X \bar{X} - 1)}{3(X \bar{X} - 1)^2} .$$

(34)

Its only stationary point is for $X = 0$, so we must choose the minus sign in eqs. (20) and (32), which allows for the field configurations with $|X| < 1$. However, it can be easily checked that in this case $X = 0$ is an unstable $dS_4$ maximum.

5. The effective theory of the detuned SUSY-RS model

As an illustration of the formalism described in the previous sections, we conclude by discussing the effective theory of the SUSY-RS model, in its detuned version. We shall see that such an effective theory can be formulated both in the $T$ basis and in the SUSY-RS basis, in two different approximations. In both cases, it corresponds to a specific example of the generalized gaugings classified in sections 3 and 4.

The effective $N = 1$, $D = 4$ theory of the SUSY-RS model is already known, both in the tuned [6] and in the detuned [7] case. We will present here an alternative, technically simpler derivation of its Kähler potential and superpotential, both in the SUSY-RS field basis of section 4 and in the $T$ field basis of section 3. For the reader’s convenience, our notation and some useful results on the backgrounds of the SUSY-RS model are spelled out in the appendix. With a slight variation with respect to [6, 7], we define the radion field $r(x)$ in such a way that its VEV coincides with $r_c$, the compactification radius in units of the $D = 5$ Planck mass $M_5$. This will allow us to derive the effective theory for the radion by simply replacing $r_c$ with $r(x)$ in the background ansatz:

$$ds^2 = a^2(r_c, y) \hat{g}_{\mu\nu}(x)dx^\mu dx^\nu + r_c^2 dy^2 ,$$

(35)
where the explicit form of $a^2(r_c, y)$ and the details of the notation are given in the appendix. This replacement should not be done, of course, in the equations that relate $r_c$ with the input parameters $(\lambda, \lambda_0, \lambda_\pi)$. Because of supersymmetry we can consider only the bosonic part of the $D = 5$ supergravity Lagrangian. Moreover, since we know from the tadpole analysis of [6, 7] the correct complexification of the radion and axion degrees of freedom, we can consider only the gravitational part $\mathcal{L}_g$ of the $D = 5$ bosonic Lagrangian, whose explicit form is given in the appendix, and neglect the part containing the graviphoton. After the replacements

$$r_c \rightarrow r(x), \quad a(r_c, y) \rightarrow a[r(x), y],$$

we obtain, neglecting as usual total derivatives:

$$\mathcal{L}_g = -\frac{1}{2} \Phi \hat{e}_4 \hat{R} + \frac{3}{4} \Phi \hat{e}_4 \hat{g}^{\mu \nu} (\partial_\mu \log \Phi)(\partial_\nu \log \Phi) - \frac{3}{4} \Phi \hat{e}_4 \hat{g}^{\mu \nu} (\partial_\mu \log r)(\partial_\nu \log r) + 6 \hat{e}_4 \left[ \frac{(a')^2 a^2}{r} + \lambda^2 r a^4 - \lambda_0 a^4 \delta(y) + \lambda_\pi a^4 \delta(y - \pi) \right].$$

(37)

Notice that the $D = 4$ Einstein term has an $x$-and-$y$-dependent $D = 5$ dilaton prefactor:

$$\Phi[r(x), y] = r(x) a^2[r(x), y].$$

We begin by observing that the radion kinetic term in eq. (37) is compatible with a $y$-dependent Kähler potential

$$K = -3 \log(U + \overline{U}), \quad \text{Re} U = r(x).$$

(39)

This is confirmed by the axion kinetic term, which arises from the $D = 5$ Maxwell term for the graviphoton $B_M$. Putting $B_5 = \sqrt{3/2} b(x)$, we get:

$$-\frac{1}{4} B_{MN} B^{MN} = -\frac{3}{4} \hat{e}_4 \Phi G^{55} \hat{g}^{\mu \nu} (\partial_\mu b)(\partial_\nu b) + \ldots = -\frac{3}{4} \hat{e}_4 \Phi r^{-2} \hat{g}^{\mu \nu} (\partial_\mu b)(\partial_\nu b) + \ldots .$$

(40)

The appropriate identification is then $\text{Im} U = b$. Before integration over $y$, the structure of the $D = 4$ kinetic terms in the $y$-dependent $D = 5$ bosonic Lagrangian is identical to the case of a compactification on $S^1/Z_2$ without warping.

To derive the Kähler potential and superpotential of the effective $D = 4$ supergravity, however, we should integrate over $y$ eqs. (37) and (40), and compare the result with the bosonic Lagrangian of $N = 1, D = 4$ Poincaré supergravity coupled to a chiral supermultiplet $U$ in an arbitrary frame:

$$\mathcal{L}_4 = -\frac{1}{2} \hat{e}_4 \Phi \hat{e}_4 \hat{R} + \frac{3}{4} \hat{e}_4 \Phi \hat{g}^{\mu \nu} (\partial_\mu \log \Phi_4)(\partial_\nu \log \Phi_4) + \hat{e}_4 \Phi_4 (K_{UU})^{-1} \hat{g}^{\mu \nu} (\partial_\mu U)(\partial_\nu \overline{U}) - \Phi_4^2 \hat{V}_4 .$$

(41)
where $\hat{V}_4$ is now the scalar potential in units of the $D = 4$ field-dependent Planck mass $M_4^2[r(x)] = \Phi_4[r(x)]$. If desired, the dilaton prefactor $\Phi_4$ can be eliminated by a field-dependent rescaling of the $D = 4$ metric $\hat{g}_{\mu\nu}(x)$. However, we do not need to integrate eq. (37) exactly. Both the original $D = 5$ supergravity and the effective $D = 4$ supergravity are being considered for small values of their respective cosmological constants, $\lambda$ and $\lambda_4$, with respect to their respective Planck masses, $M_5$ and $M_4$. We will then consider in the following two possible expansions, either in $\lambda_4$ or in $\lambda \pi r$, and derive the effective $D = 4$ supergravity in these two limits.

### 5.1: The $\lambda_4$ expansion

To perform an expansion in $\lambda_4$, we assume that $\lambda_4/\lambda \ll 1$ and take

$$ a[r(x), y] = \alpha e^{-\lambda r |y|} + \beta e^{+\lambda r |y|}, $$

choosing for simplicity

$$ \beta \simeq 1 - \frac{\lambda_4^2}{4 \lambda^2}, \quad \alpha \simeq \frac{\lambda_4^2}{4 \lambda^2}, $$

which is consistent with the exact expressions for $\alpha$ and $\beta$, given in eq. (63) of the appendix, at leading order in the expansion parameter. So doing, we assume to be close to the fine-tuned case $\lambda_0 = \lambda_\pi = -\lambda$. Since the fine-tuned case, corresponding to $\lambda_4 = 0$, has vanishing superpotential, the superpotential of the detuned case must be $O(\lambda_4)$. Therefore, in a consistent leading-order approximation, we must evaluate the Kähler potential in the limit $\lambda_4 = 0$, and the scalar potential $\hat{V}_4$ up to $O(\lambda_4^2)$.

Setting $\alpha = 0$ and $\beta = 1$, the $D = 4$ dilaton prefactor reads:

$$ \Phi_4(r) = \frac{1}{\lambda} \left( e^{2 \lambda \pi r} - 1 \right). $$

With the complexification $U(x) = r(x) + i b(x)$ as before, it corresponds to the Kähler potential of eqs. (20) and (22), with a plus sign in the present conventions. This result was found in [6]: the proof that the integrated axion kinetic term is compatible with the Kähler potential (20) is considerably more subtle.

We can also take the potential $\hat{V}_4(r)$ from eq. (67) of the appendix, and expand it up to order $\lambda_4^2/\lambda^2$. Doing so, we obtain the result of eq. (30), with the plus sign and:

$$ \rho_0 = \lambda_4 \left( 1 - e^{-2 \lambda \pi r_c} \right)^{1/2}, \quad \rho_3 = \lambda_4 \left( e^{2 \lambda \pi r_c} - 1 \right)^{1/2}. $$

This potential is minimized, as expected, for

$$ e^{2 \lambda \pi r} = \frac{\rho_0^2}{\rho_3^2} = \frac{(\lambda + \lambda_0) (\lambda - \lambda_\pi)}{(\lambda - \lambda_0) (\lambda + \lambda_\pi)}. $$
and its value at the minimum is, as expected,

$$\langle \hat{V}_4 \rangle = -\frac{3 \rho_0^2 \rho_3^2}{\rho_0^2 - \rho_3^2} = -3 \lambda_4^2. \quad (47)$$

We then derive, knowing that there is no potential for the axion $b(x)$, and using the results of section 4, the effective superpotential of eq. (29), in agreement with the results of [7].

In the $T$ field basis, this superpotential reads

$$w_{RS}(T) = \frac{1}{2 \sqrt{2}} \left[ \rho_0 (1 - T)^3 + \rho_3 e^{i \varphi} (1 + T)^3 \right], \quad (48)$$

and corresponds to one of the generalized $T$ gaugings discussed in section 3.

### 5.2: The $\lambda \pi r$ expansion

If we take the expressions for $\Phi_4(r_c)$ and $\hat{V}_4(r_c)$ given by eqs. (66)–(69) of the appendix, perform everywhere the replacement (36), and expand all the exponentials up to $O(\lambda \pi r)$, we obtain the following results. The $D = 4$ dilaton prefactor,

$$\Phi_4 = (\alpha + \beta)^2 2 \pi r(x), \quad (49)$$

gives a Kähler potential of the form (2), where we should now call $T \equiv r(x) + i b(x)$ the complex field that was previously called $U$.

As for the scalar potential $\hat{V}_4$, if we take $\alpha$ and $\beta$ to be both of order one we get, at the first non trivial order in the expansion parameter:

$$\hat{V}_4 = \frac{6 \lambda^2 \alpha \beta}{\pi (\alpha + \beta)^2} \frac{r_c - 2 r(x)}{r^2(x)}. \quad (50)$$

This is of the general form of eq. (19), thus we know from section 3 that it is generated by a superpotential of the form of eq. (18). As expected, the potential in eq. (50) is minimized for

$$\langle r(x) \rangle = r_c, \quad (51)$$

and at the minimum

$$\langle \hat{V}_4 \rangle = \frac{-3 \lambda^2}{2 \pi r_c (\alpha + \beta)^2} = -3 \frac{\lambda_4^2}{M_4^2}. \quad (52)$$

As already discussed in section 3, we have a stable $AdS_4$ background, with broken or unbroken supersymmetry according to the choice of $\langle b(x) \rangle$ along its flat direction.
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Appendix: backgrounds of the detuned SUSY-RS model

We collect in this appendix some results on the backgrounds [22, 24] of the detuned [24, 25] SUSY-RS model [22, 23] that can be useful to understand the derivation of its effective theory [6, 7] as presented in the text. For the purposes of the present paper it is sufficient to consider only the bosonic gravity sector of the theory, neglecting the graviphoton and the fermions.

The relevant part of the $D = 5$ Lagrangian reads, in units where the $D = 5$ Planck mass $M_5$ is set equal to one:

$$L_g = -\frac{1}{2} e_5 R_5 - e_5 \Lambda_5 - 6 e_4 \left[ \lambda_0 \delta(y) - \lambda_\pi \delta(y - \pi) \right].$$ (53)

In our notation, $E^A_M$ is the fünfbein, $M = [(\mu = 0, 1, 2, 3), 5]$ are curved space-time indices, $y = x^5$, $A = [(a = 0, 1, 2, 3), 5]$ are flat tangent-space indices, $e_5 = \det E^A_M$, $e_4 = \det E^a_\mu$, $\Lambda_5 \equiv -6 \lambda^2 < 0$ is the $D = 5$ cosmological constant in units of $M_5$, $R_5$ is the $D = 5$ scalar curvature in the conventions of [28], the $D = 5$ Minkowski metric is $\eta_{AB} = \text{diag}(-1, +1, +1, +1, +1)$, and the delta functions are normalized according to $\int_{-\pi}^{\pi} dy \, \delta(y) = 1$. On the right-hand side of eq. (53) we understand an overall factor of $M_5^3$, so that the input parameters $(\lambda, \lambda_0, \lambda_\pi)$ have all the dimension of a mass. The extrema of integration over $y$ are indeed $\pm \pi/M_5$, if we want the coordinate $y$ to have the dimension of a length. Supersymmetry requires $\lambda_0^2, \lambda_\pi^2 \leq \lambda^2$: in the following, it will not be restrictive to remove a twofold ambiguity and assume that $\lambda > 0$ and $-\lambda \leq \lambda_\pi \leq \lambda_0 \leq +\lambda$.

We are interested in backgrounds of the form

$$ds^2 = a^2(r_c, y) \tilde{g}_{\mu\nu}(x) dx^\mu dx^n + r_c^2 dy^2,$$ (54)
where \( \hat{g}_{\mu\nu} \) is a maximally symmetric \( D = 4 \) metric such that

\[
\hat{R}_{\mu\nu}(\hat{g}) = -\Lambda_4 \hat{g}_{\mu\nu} \equiv 3 \lambda_4^2 \hat{g}_{\mu\nu}, \tag{55}
\]

so that \( \Lambda_4 = -3 \lambda_4^2 \) can be interpreted as the \( D = 4 \) cosmological constant in units of the \( D = 4 \) Planck mass \( M_4 \), and the constant \( r_c > 0 \) can be interpreted, if we identify \( y \) with \( y + 2\pi \), as the compactification radius in units of \( M_5 \). Notice that the parameterization of eq. (54) is redundant, since we can rescale \( a^2(r_c, y) \) by an arbitrary constant factor and \( \hat{g}_{\mu\nu}(x) \) by its inverse: to fix this ambiguity we may require, for example, that \( a^2(r_c, 0) = 1 \), so that the \( D = 4 \) metric \( \hat{g}_{\mu\nu}(x) \) is identified with the \( D = 5 \) metric on the brane at \( y = 0 \).

However, we can also leave the normalization factor in \( a(r_c, y) \) undetermined, since it can always be reabsorbed in the definition of the \( D = 4 \) metric and of the \( D = 4 \) Planck mass: it will be fixed only when we want to give a definite \( D = 5 \) geometrical interpretation to the \( D = 4 \) metric. As appropriate for the detuned case \([24]\), we make the ansatz:

\[
a(r_c, y) = \alpha e^{-\lambda r_c|y|} + \beta e^{\lambda r_c|y|}, \tag{56}
\]

where \( \alpha \) and \( \beta \) are real dimensionless constants, and we understand periodicity for the function \( |y| \). The normalization condition \( a^2(r_c, 0) = 1 \) translates into

\[
\alpha + \beta = \pm 1. \tag{57}
\]

Depending on the problem under consideration, it may be convenient to make use of eq. (57) or to leave the overall normalization factor in \( \alpha \) and \( \beta \) undetermined. The bulk equations of motion require that

\[
\lambda_4^2 = 4 \lambda_2^2 \alpha \beta. \tag{58}
\]

The equations of motion at the fixed points (in the ‘upstairs’ picture) require

\[
\frac{\lambda_0}{\lambda} = \frac{\alpha - \beta}{\alpha + \beta}, \tag{59}
\]

and

\[
\frac{\lambda_\pi}{\lambda} = \frac{\alpha e^{-\lambda r_c \pi} - \beta e^{\lambda r_c \pi}}{\alpha e^{-\lambda r_c \pi} + \beta e^{\lambda r_c \pi}}. \tag{60}
\]

The last two conditions can be fulfilled for

\[
\beta (\lambda + \lambda_0) = \alpha (\lambda - \lambda_0), \tag{61}
\]

and

\[
e^{2\lambda r_c \pi} (\lambda - \lambda_0) (\lambda + \lambda_\pi) = (\lambda + \lambda_0) (\lambda - \lambda_\pi). \tag{62}
\]
Therefore, we can solve $D = 5$ Einstein’s equations everywhere, including the fixed points, with constant radius $r_c > 0$: in the fully detuned case, $-\lambda < \lambda_\pi < \lambda_0 < \lambda$, we have an $AdS_4$ background ($\lambda^2_4 > 0$), and the radius $r_c$ is uniquely determined by eq. (62); in the fully tuned case, $\lambda_0 = \lambda_\pi = \pm \lambda$, we have a $Minkowski_4$ ($\lambda^2_4 = 0$) background, and $r_c$ is undetermined. Partially tuned choices of $\lambda_0$ and $\lambda_\pi$ compatible with supersymmetry lead either to $r_c = 0$ ($\lambda_0 = \lambda_\pi \neq \pm \lambda$) or to $r_c = +\infty$ ($\lambda_0 = \lambda$ and/or $\lambda_\pi = -\lambda$). Notice that in the tuned case the background $a(r_c, y)$ is a single exponential, whereas in the fully detuned case it is always a double exponential. If we also assume the normalization condition in eq. (57), we can go further and find explicitly:

$$\alpha = \pm \frac{\lambda + \lambda_0}{2 \lambda}, \quad \beta = \pm \frac{\lambda - \lambda_0}{2 \lambda}, \quad \lambda_4^2 = \lambda^2 - \lambda_0^2. \quad (63)$$

With the line element of eq. (54), the $D = 5$ bosonic Lagrangian of eq. (63) reads

$$\mathcal{L}_g = -\frac{1}{2} \Phi \hat{e}_4 \hat{R}_4 + 6 \hat{e}_4 \left[ \frac{(a')^2 a^2}{r_c} + \lambda^2 r_c a^4 - \lambda_0 a^4 \delta(y) + \lambda_\pi a^4 \delta(y - \pi) \right], \quad (64)$$

where $\hat{e}_4 = |\det \hat{g}_{\mu\nu}|^{1/2}$, $\hat{R}_4$ is the 4D curvature scalar for the metric $\hat{g}_{\mu\nu}$, and we have safely neglected total derivatives. Notice that the $D = 4$ Einstein term is not canonically normalized, but has a $D = 5$ dilaton prefactor

$$\Phi(r_c, y) = r_c a^2(r_c, y). \quad (65)$$

The rest of eq. (64) contains $y$-dependent contributions to the scalar potential. After we integrate over $y$, the Einstein term remains non-canonical, with a $D = 4$ dilaton prefactor

$$\Phi_4(r_c) \equiv \int_{-\pi}^{\pi} dy \Phi(r_c, y) = \frac{1}{\lambda} \left[ \alpha^2 \left( 1 - e^{-2\lambda \pi r_c} \right) + \beta^2 \left( e^{2\lambda \pi r_c} - 1 \right) + 4\alpha \beta r_c \right], \quad (66)$$

which can be eliminated by a rescaling of the $D = 4$ metric $\hat{g}_{\mu\nu}$ or absorbed in the definition of the $D = 4$ $r_c$-dependent Planck mass, $M_4^2(r_c) = \Phi_4(r_c)$. Recalling that we deal with a non-canonical $D = 4$ Einstein term, we can now compute the $D = 4$ potential for the background field $r_c$, by integrating over $y$ the part of eq. (64) within square brackets. It is convenient to normalize the potential in units of $M_4^2(r_c)$. If we do so, we obtain:

$$\hat{V}_4(r_c) \equiv \frac{V_4(r_c)}{M_4^4(r_c)} = -6 \Phi_4^{-2} \hat{e}_4 \left[ I_{22}(r_c) + I_4(r_c) - \lambda_0 a^4(r_c, 0) + \lambda_\pi a^4(r_c, \pi) \right], \quad (67)$$

where

$$I_{22}(r_c) \equiv \int_{-\pi}^{\pi} dy \frac{(a')^2 a^2}{r_c} = \frac{\lambda}{2} \left[ \alpha^4 \left( 1 - e^{-4\lambda \pi r_c} \right) + \beta^4 \left( e^{4\lambda \pi r_c} - 1 \right) - 8\alpha^2 \beta^2 \lambda \pi r_c \right], \quad (68)$$
and
\[ I_4(r_c) \equiv \int_{-\pi}^{\pi} dy a^4 \lambda^2 r_c = \frac{\lambda}{2} \left[ \alpha^4 (1 - e^{-4\lambda \pi r_c}) + 8\alpha^3 \beta(1 - e^{-2\lambda \pi r_c}) + 8\alpha\beta^3(e^{2\lambda \pi r_c} - 1) + \beta^4(e^{4\lambda \pi r_c} - 1) + 24\alpha^2\beta^2\lambda \pi r_c \right]. \] (69)

The minimization of \( \hat{V}_4(r_c) \) is tedious, but of course reproduces all the results for \( r_c \) and \( \lambda_4 \) obtained from the \( D = 5 \) equations of motion, as functions of the input parameters \( (\lambda, \lambda_0, \lambda_\pi) \).
References

[1] E. Witten, Phys. Lett. B 155 (1985) 151;  
J. P. Derendinger, L. E. Ibáñez and H. P. Nilles, Nucl. Phys. B 267 (1986) 365;  
S. Ferrara, C. Kounnas and M. Porrati, Phys. Lett. B 181 (1986) 263.

[2] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Phys. Lett. B 116 (1982) 231 and Nucl. Phys. B 212 (1983) 413.

[3] E. Cremmer, in Superspace and supergravity, S. W. Hawking and M. Rocek eds., Cambridge University Press, 1981, pp. 267-282;  
A. H. Chamseddine and H. Nicolai, Phys. Lett. B 96 (1980) 89;  
R. D’Auria, E. Maina, T. Regge and P. Fre, Annals Phys. 135 (1981) 237;  
M. Gunaydin, G. Sierra and P. K. Townsend, Nucl. Phys. B 242 (1984) 244.

[4] M. Gunaydin, G. Sierra and P. K. Townsend, Nucl. Phys. B 253 (1985) 573.

[5] J. Bagger, F. Feruglio and F. Zwirner, JHEP 0202 (2002) 010 [arXiv:hep-th/0108010].

[6] M. A. Luty and R. Sundrum, Phys. Rev. D 64 (2001) 065012 [arXiv:hep-th/0012158];  
J. Bagger, D. Nemeschansky and R. J. Zhang, JHEP 0108 (2001) 057 [arXiv:hep-th/0012163].

[7] J. Bagger and M. Redi, arXiv:hep-th/0310086 and arXiv:hep-th/0312220.

[8] S. B. Giddings, S. Kachru and J. Polchinski, Phys. Rev. D 66 (2002) 106006 [arXiv:hep-th/0105097];  
S. Kachru, M. B. Schulz and S. Trivedi, JHEP 0310 (2003) 007 [arXiv:hep-th/0201028];  
R. D’Auria, S. Ferrara and S. Vaula, New J. Phys. 4 (2002) 71 [arXiv:hep-th/0206241];  
S. Ferrara and M. Porrati, Phys. Lett. B 545 (2002) 411 [arXiv:hep-th/0207135];  
S. Kachru, M. B. Schulz, P. K. Tripathy and S. P. Trivedi, JHEP 0303 (2003) 061 [arXiv:hep-th/0211182];  
L. Andrianopoli, R. D’Auria, S. Ferrara and M. A. Lledo, JHEP 0303 (2003) 044 [arXiv:hep-th/0302174];  
C. Angelantonj, S. Ferrara and M. Trigiante, JHEP 0310 (2003) 015 [arXiv:hep-th/0306185].
[9] B. de Wit and A. Van Proeyen, Nucl. Phys. B 245 (1984) 89.

[10] E. Cremmer, C. Kounnas, A. Van Proeyen, J. P. Derendinger, S. Ferrara, B. de Wit and L. Girardello, Nucl. Phys. B 250 (1985) 385.

[11] J. P. Derendinger, S. Ferrara, A. Masiero and A. Van Proeyen, Phys. Lett. B 140 (1984) 307.

[12] M. de Roo, Nucl. Phys. B 255 (1985) 515 and Phys. Lett. B 156 (1985) 331;  
E. Bergshoeff, I. G. Koh and E. Sezgin, Phys. Lett. B 155 (1985) 71;  
M. de Roo and P. Wagemans, Nucl. Phys. B 262 (1985) 644.

[13] I. Antoniadis, J. P. Derendinger and C. Kounnas, Nucl. Phys. B 551 (1999) 41  
arXiv:hep-th/9902032.

[14] A. Giveon and M. Porrati, Phys. Lett. B 246 (1990) 54 and Nucl. Phys. B 355 (1991) 422.

[15] I. Antoniadis and C. Kounnas, Phys. Lett. B 261 (1991) 369.

[16] E. Kiritsis and C. Kounnas, Nucl. Phys. B 503 (1997) 117  
arXiv:hep-th/9703059.

[17] J. P. Derendinger, L. E. Ibáñez and H. P. Nilles, Phys. Lett. B 155 (1985) 65;  
M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. B 156 (1985) 55.

[18] S. Ferrara, C. Kounnas, D. Lüst and F. Zwirner, Nucl. Phys. B 365 (1991) 431.

[19] S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner, Nucl. Phys. B 318 (1989) 75.

[20] M. Porrati and F. Zwirner, Nucl. Phys. B 326 (1989) 162.

[21] J. Scherk and J. H. Schwarz, Phys. Lett. B 82 (1979) 60 and Nucl. Phys. B 153 (1979) 61.

[22] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370  
arXiv:hep-ph/9905221.

[23] R. Altendorfer, J. Bagger and D. Nemeschansky, Phys. Rev. D 63 (2001) 125025  
arXiv:hep-th/0003117;
T. Gherghetta and A. Pomarol, Nucl. Phys. B 586 (2000) 141 [arXiv:hep-ph/0003129];
A. Falkowski, Z. Lalak and S. Pokorski, Phys. Lett. B 491 (2000) 172 [arXiv:hep-th/0004093] and Nucl. Phys. B 613 (2001) 189 [arXiv:hep-th/0102145];
E. Bergshoeff, R. Kallosh and A. Van Proeyen, JHEP 0010 (2000) 033 [arXiv:hep-th/0007044].

[24] N. Kaloper, Phys. Rev. D 60 (1999) 123506 [arXiv:hep-th/9905210].

[25] P. Brax, A. Falkowski and Z. Lalak, Phys. Lett. B 521 (2001) 105 [arXiv:hep-th/0107257];
J. Bagger and D. V. Belyaev, Phys. Rev. D 67 (2003) 025004 [arXiv:hep-th/0206024] and JHEP 0306 (2003) 013 [arXiv:hep-th/0306063];
Z. Lalak and R. Matyszkiewicz, Nucl. Phys. B 649 (2003) 389 [arXiv:hep-th/0210053];
Phys. Lett. B 562 (2003) 347 [arXiv:hep-th/0303227] and [arXiv:hep-th/0310269];
P. Brax and N. Chatillon, JHEP 0312 (2003) 026 [arXiv:hep-th/0309117].

[26] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, Phys. Lett. B 133 (1983) 61;
J. R. Ellis, C. Kounnas and D. V. Nanopoulos, Nucl. Phys. B 241 (1984) 406 and
Nucl. Phys. B 247 (1984) 373;
J. R. Ellis, A. B. Lahanas, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B 134 (1984) 429.

[27] P. Breitenlohner and D. Z. Freedman, Phys. Lett. B 115 (1982) 197 and Annals Phys. 144 (1982) 249;
P. K. Townsend, Phys. Lett. B 148 (1984) 55.

[28] S. Weinberg, *Gravitation and Cosmology*, John Wiley and Sons, 1972.