Improvement of student’s mathematical communication ability using M-APOS approach

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Abstract. Calculus course is a much needed material in the development of science and technology. Calculus is the important for learning the next many topic. Students have difficulties in understanding the calculus. The lecturers often explain subject matter by giving examples of problems and exercises, so the concepts are less meaningful. Students only learn to solve problems mechanically. Considering those problems, one way that can be used to overcome the problem is to design a learning tools. The designed teaching materials should be able to summarize the existing concepts so that students are easily understood the concept. Lecturer can use M-APOS modification approach M-APOS (Modification-action - process - object – schema). The purpose of this study was to produce M-APOS based calculus lecturing tool that valid, practical and effective to improve students' mathematical communication ability. This research is a development research. The result of this research show that the learning tools of calculus based on M-APOS approach are valid both in terms of content and construct. The developed M-APOS based calculus learning tools have met the practical criteria both from the aspect of implementation, ease and required time. Yet, the developed M-APOS based calculus learning tools have not been effective in improving students' mathematical communication ability.

1. Introduction
Calculus courses are important to science and technology [1], [2]. Because Calculus is the basic for learning other topics. Many science applications require calculus. For that all students studying science and technology require calculus in their studies (Douglas: in [2]).

From the experience of the researcher in teaching calculus, there many students have difficulties in understanding the material. Whereas they have learned most of the calculus material in high school. Low level of understanding of students, can be seen from various factors, such as in terms of pedagogy, reading sources, content, and prerequisites material.

In pedagogic perspective, Glass [3] argues that calculus teaching traditionally emphasize in simplification by using certain formulas or tricks to solve problems. This is good enough in helping students to solve routine problems or exam questions, but not enough to improve student problem solving ability. For less-experienced lecturers, teaching calculus are widely focused on text-based and teacher-centered lectures (Jordan, [5]). Students are given the practice of counting problems in various way, but do not understand what they are doing essentially. Thus, students have difficulties in understanding a given calculus concept.
Glass [3] also revealed that most textbooks emphasize simplicity in the presentation of matter. Calculus books often describe a topic using graphical representation, while many students experience the relationship between graphs and equations. Much research has shown that students have problems about the limits, derivatives, and integrals because of a lack of understanding about prerequisite materials such as continuous function and formal mathematical definitions [4].

In terms of prerequisite materials, it was found that many students with problems related to functions for example have not been able to determine the domain, codomain, and range of a function and draw a function’ graph. In fact, the concept of function is central to modern mathematics including calculus [6], [4]. Another difficulty is in doing algebraic manipulation [7].

As a result of the above difficulties, there is a misconception on the students. For derivative topics, Ubuz [8] reported that in general, misconception on derivative experienced by students are as follows: (a) derivatives at a point produce a function of a derivative, (b) the equation of the tangent line is a derived function, (c) the derivative in a point is the equation of the tangent line, and (d) the derivative at a point is the value of the tangent equation at that point. This is due to (a) lack of differentiation of concepts occurring in the same context or the confusion of a concept with another concept (b) incompatibility of exploration from specific to general case, and (c) lack of understanding of graphical representation.

The cause might be that students accept only a concept in the final form not a process. Lecturers explain the subject matter are less meaningful. They are only trained to solve problems mechanically. Rarely students are engaged in abstraction of the facts that exist in taking a conclusion from a concept.

The role of lecturers in learning is not giving the final answer to the students' questions, but directs them to construct mathematical knowledge to obtain the correct mathematical structure. In order to mathematical knowledge and structure can be achieved well, students must relate it into their own experience. Thus, teaching materials that will help students in understanding the concept is required. As experienced by the researcher, calculus material are given too much so that students are less focused to follow the lecture. Learning tools have to be designed to summarize the existing concept so that students can understand easily. An approach that allows students to construct their knowledge must be use in instructional design.

One of the mental construction theories that can be used to assist students in constructing their knowledge is the M-APOS (APOS modification) approach. APOS (action - process - object - schema) is a theory developed by Dubinsky and his colleagues [9]. APOS theory embraces the social constructivism developed by Vygotsky. The students' mathematical knowledge and understanding is the result of their construction and interaction with others in understanding mathematics [10].

Action is the transformation of mental objects to obtain other mental objects. Transformation is done by taking action against external instructions, which provides details on which steps should be taken. A person is said to experience an action if he focuses his mental process on trying to understand the concept given [11], [5]. At this stage students are given a set of facts and mathematical concepts that they already know. Using these facts and concepts, students are asked to perform various mathematical activities. When they do this activity then the students are doing the process.

When an action is repeated and the individual reflects on the action, the action is inter-processed into a process, ie an internal construct created by performing the same action. Individuals who have constructed the concept process can describe the steps of the transformation without actually doing it [12].

When student reflect on operations that are applied to a particular process, being aware of the process as a totality, realizing that the transformation (both action and process) can be done, and can actually construct that transformation, the student will interpret the process as an object. The process has been encapsulated into an object. Someone is said to have a mathematical concept if he has been able to treat the concept as a cognitive object. Then students can also break down the object back to the process because it comes from when the properties of the object will be used.
From the above process then formed a scheme in the student. Scheme is a comprehensive set of actions, processes, objects, and other schema, which are connected in a structured form in one's mind. This schema is reliable in dealing with math problems. APOS theories can be used to improve the ability of high order thinking and as a tool to assess students' understanding of the more basic mathematical concepts [13]. There have been many studies using APOS theories such as [12], [13].

Learning strategies in APOS theory use the ADL learning cycle, namely: activities, class discussions, and exercises. Activities carried out in the computer laboratory in doing programming tasks in groups of 3-5 people to develop students' mental construction. The programming tasks are not limited to time (it can be done outside of the official laboratory work schedule). The main purpose of this activity is to give students basic experience to provide correct answers.

Discussion activities are conducted in the classroom. Activities are conducted with tasks that are still associated with activities in computer labs or work on structured worksheets. Student involvement in the discussions aims to give students the opportunity to reflect on what they have done in the laboratory and on the task they are working on. In this class discussion, students also provide definitions, explanations, and reviews to relate them to what students have been thinking about.

In the exercise activities, students are given some problems to be solved in groups. This activity is expected to be done outside of classroom and laboratory activities and can be either homework on laboratory tasks or the task of structured worksheets. The purpose of these exercises is to reinforce mathematical concepts that have been constructed, apply learned concepts, and start thinking about what will be learned in the next activity.

The purpose of this research is to see the improvement of students' mathematical communication ability after using M-APOS approach. Mathematical communication is: a) students' ability to explain unique ways to solve problems, b) students' ability to construct and explain presentations of real-world phenomena graphically, phrases or sentences, equations, tables and physical presentations, and c) students' ability to give guesses about geometric images.

Greene and Schulman [14] stated that mathematical communication is: (a) central force for students in formulating concepts and strategies; (b) the student's success asset of approaches and solutions in mathematical exploration and investigation; (c) a facility for students to communicate with their friends for sharing thoughts and discoveries, brainstorming, assessing and sharpening ideas to convince others.

Meanwhile, in [15] was stated that the standard of mathematical communication is the emphasis of teaching mathematics on the students' ability in terms of: (a) organizing and consolidating their thinking through communication; (b) communicate their thinking coherently (in a logical order) and clear to their friends, teachers and others; (c) analyze and evaluate thinking and strategies employed by others; and (d) using mathematical language to express mathematical ideas correctly.

Mathematical communication can be developed through various ways, including through group discussion. Nurlaelah [14] suggests that communication ability are important when students discuss. In the discussion students are expected to be able to express, explain, describe, hear, ask and work together to bring students to a deep understanding of mathematics. According to Cobb [14] by communicating their knowledge, renegotiation can occur among students and the role of teachers is expected only as a guide in the learning process.

2. Research Methods
This research was a development research (Plomp model) which is purposed to develop a valid, practical, and effective lecturing tool of calculus based on M-APOS approach. The development process consists of 3 phases. Development or prototyping phase produced Product 1, Product 2, and Product 3. Product 1 was designing and doing self check and expert reviews so that it was valid. The product is valid then proceed to product 2. In product 2, it is done by asking three students about the product. After the product was revised then a field tests are carried out in conditions similar to actual conditions. Field tests were carried out to see the level of tools practicability that had been designed.
Practicality was viewed through observation, interviews and questionnaires. The effectiveness of M-APOS based calculus lecturing tools can be known from the results of test.

3. Results and Discussion

In this paper will be discussed the result of students’ ability before and after the lecture using the product with the M-APOS approach. The data was obtained from the research data on preliminary tests and final tests on student mathematical communication. The results of the test as in follow.

| Test                          | N  | \( \bar{X} \) | S   | \( X_{\text{max}} \) | \( X_{\text{min}} \) |
|-------------------------------|----|-------------|-----|---------------------|---------------------|
| Before learning process      | 56 | 11.75       | 4.64| 20                  | 3                   |
| After learning process       | 56 | 12.84       | 4.46| 20                  | 1                   |

From Table 1 the mean of students’ ability after M-APOS approach is higher than before using M-APOS approach. Standard deviation before treatment is higher than after treatment. From the mean value and standard deviation of the data is seen that the result after treatment is better.

To ascertain whether the increase is significant, the mean comparison test was performed. The test used was paired sample t-test. The pre-requisite analysis of this test are normality test and homogeneity of variance of data. Normality test of data distribution of student communication ability before and after treatment used Kolmogorov-Smirnov test. Testing criteria: data is normal distributed if the value of Sig. greater than the real level (= 0.05) and not normally distributed otherwise. The result of normality test as in Table 2.

| Test                          | Nilai Sig. | Criteria |
|-------------------------------|------------|----------|
| Before learning process      | 0.450      | Normal   |
| After learning process       | 0.728      | Normal   |

Based on the normality test, the data of students’ ability normally distributed so that t test was performed. The result of this t test as in Table 3.

| Data                          | Mean | SD  | T   | df  | Sig.  |
|-------------------------------|------|-----|-----|-----|-------|
| Mathematical Communication Ability | -1.089 | 4.31 | 1.890 | 55  | 0.064 |

Based on Table 3, the data of mathematical communication ability has Sig value. greater than the real level (= 0.05), thus we accept \( H_0 \). This means that students’ mathematical communication abilities after and before using M-APOS-based learning tools did not differ significantly. Nevertheless, there had been a slight increase, it is predicted that if this learning is implemented, after a long period it can improve students’ ability.

4. Conclusions

This research was obtained the following conclusions : a) The developed M-APOS-based learning calculus tools are valid in both content and constructs criteria, b) The developed M-APOS-based learning calculus tools already meet the practical criteria in terms of implementation, ease and required time, c) The developed M-APOS-based learning calculus tool has not been effective yet.

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