Quantized large-bias current in the anomalous Floquet-Anderson insulator

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The AFAI carries a quantized current. Here we show that, in a two-terminal transport setup, the chiral edge mode carries a quantized current when completely filled. The AFAI’s chiral edge states exist at all quasienergies; each such chiral edge state is empty. Importantly, while quantized pumping in the Thouless pump is found in the adiabatic limit, the edge state is fully occupied, while the left-moving chiral edge state is empty. The current saturates to the quantized value $I = 1/T$ for $V \gtrsim 2\Omega$, where $\Omega = 2\pi/T$ is the driving frequency.

We study two-terminal transport through two-dimensional periodically driven systems in which all bulk Floquet eigenstates are localized by disorder. We focus on the Anomalous Floquet-Anderson Insulator (AFAI) phase, a topologically-nontrivial phase within this class, which hosts topologically protected chiral edge modes coexisting with its fully localized bulk. We show that the unique properties of the AFAI yield remarkable far-from-equilibrium transport signatures: for a large bias between leads, a quantized amount of charge is transported through the system each driving period. Upon increasing the bias, the chiral Floquet edge mode connecting source to drain becomes fully occupied and the current rapidly approaches its quantized value.

Topological phenomena, such as the quantum Hall effect [1] and Thouless’ adiabatic pump [2], are characterized by the precise quantization of certain transport properties. Recently, periodic driving has emerged as a versatile tool to control the topological characteristics of quantum systems [3,20]. Such “Floquet” systems can be realized in a wide variety of physical settings, including cold atomic, optical, and electronic systems [21-24]. The extent to which Floquet systems may host quantized transport is an important direction of investigation.

Interestingly, periodically-driven quantum systems host unique topological phases which cannot be realized by their static counterparts [5,25-40]. The richer topological classification of these systems is due to their discrete (rather than continuous) time translation symmetry, which is manifested as a periodicity of the quasienergy — the energy-like variable that characterizes the Floquet spectrum. Crucially, this structure provides the basis for wholly new types of quantized transport phenomena, also without analogues in static systems.

The first example of a quantized transport phenomenon unique to periodically-driven systems was uncovered in Ref. [2]. There, Thouless showed that the charge transmitted through an insulating one-dimensional system is quantized as an integer multiple of the fundamental charge when the system is adiabatically driven through a cycle in parameter space.

More recently, in Ref. [30] it was shown that two-dimensional, disordered, periodically driven systems host a topological phase (the AFAI phase) for which all bulk Floquet eigenstates are localized, while chiral edge states run along system boundaries. The AFAI’s chiral edge states exist at all quasienergies; each such chiral edge mode carries a quantized current when completely filled. Here we show that, in a two-terminal transport setup, the AFAI carries a quantized current $I = W_{2D}/T$ in the limit of large source-drain bias (see Fig. 1a). Here $W_{2D}$ is the winding number invariant that characterizes 2D periodically driven systems [23,30,41]. Associated with the quantized current, we find an inhomogeneous density profile in which the AFAI’s right-moving chiral edge state is fully occupied, while the left-moving chiral edge state is empty. Importantly, while quantized pumping in the Thouless pump is found in the adiabatic limit, the large-bias quantized current carried by the AFAI occurs for intermediate driving frequencies (comparable to the system’s natural bandwidth).

The AFAI phase occurs in two-dimensional systems, whose dynamics are governed by a time-periodic Hamiltonian $H_S(t) = H_S(t+T)$. The periodic driving gives rise to a unitary evolution $U_S(t) = T e^{-i \int_0^T dt H_S(t)}$, where $T$ denotes time ordering. The spectrum of the Floquet operator $U_S(T)$, given by $U_S(T) | \psi_n(0) \rangle = e^{-i \varepsilon_n T} | \psi_n(0) \rangle$, defines the Floquet states $| \psi_n(t) \rangle$ and their quasienergies $\{ \varepsilon_n \}$.

To study quantized transport in the AFAI phase, we consider a finite region of AFAI connected to two wide-bandwidth (non-driven) leads, as shown in Fig. 1b. The leads are indexed by $\lambda = \{ L, R \}$, standing for the left and right leads, respectively. Dynamics of the combined...
sublattices, with $n$ steps of equal length, $T/5$. In each step, the highlighted bonds are active with strength $J_{ij} = 5\pi/(2T)$, Eq. (2), while all others are set to 0. (b) Quasienergy spectrum of system-lead setup are described by the Hamiltonian $H(t) = H_S(t) + \sum_{\lambda=L,R} H_\lambda + \sum_{\lambda=L,R} H_{S\lambda}$, (1)

where $H_S(t) = H_S(t + T)$ is the time-periodic Hamiltonian of the AFAI system, $H_\lambda$ is the Hamiltonian describing lead $\lambda$, and $H_{S\lambda}$ describes the coupling between the system and lead $\lambda$. We treat each lead as an ideal Fermi reservoir, with filling characterized by an equilibrium Fermi-Dirac distribution with chemical potential $\mu_\lambda$ in lead $\lambda$. Specific forms for the Hamiltonian terms above will be given below. Throughout this paper, we use $e, \hbar = 1$.

For concreteness, we now give a specific model which realizes the AFAI phase. The square lattice tight-binding model, introduced in Ref. [30], is described by the Hamiltonian

$$H_S^{\text{clean}}(t) = \sum_{\langle ij \rangle} J_{ij}(t) c_i^\dagger c_j + \sum_i D c_i^\dagger c_i,$$

where $\{J_{ij}(t)\}$ are time-dependent nearest-neighbor hopping amplitudes. The piecewise-constant amplitudes $J_{ij}(t)$ are modulated according to the five step cycle depicted in Fig. 2a, where each step has length $T/5$. Within each step, all nonzero hopping amplitudes (bold bonds) have strength $J = \frac{2\pi}{3T}$; in the fifth interval, all $J_{ij} = 0$. The parameter $D$ is a staggered potential on the A and B sublattices, with $n_i = +1 (-1)$ for the A (B) sub-lattice.

Within the AFAI phase, realized for nonzero $w$ below a critical value [30], the system in an open geometry exhibits chiral edge states in coexistence with a fully-localized bulk. These chiral edge states are illustrated in the example spectra for the clean system ($w = 0$) in an infinite-strip geometry, shown in Fig. 2b.

We now study the steady-state current transported through the system when it is coupled to leads. To this end, we consider the Heisenberg equations of motion for the operators $c_j(t) = U(t) c_j(t_0) U^\dagger(t)$ and $a_\lambda^\dagger(t) = U(t)a_\lambda(t_0)U^\dagger(t)$, where $a_\lambda^\dagger$ is the fermionic annihilation operator on site $j$ of lead $\lambda$, and $U(t) = Te^{-i \int_0^t dt' H(t')}$ is the evolution operator for the full Hamiltonian, Eq. (1).

To simplify notation we introduce the operator vectors $a_\lambda = (\cdots a_{\lambda i} \cdots)^T$ and $c = (\cdots c_i \cdots)^T$, and express the system, lead, and system-lead coupling Hamiltonians in Eq. (1) as $H_S(t) = c^\dagger H_S^c(t) c$, $H_\lambda = a_\lambda^\dagger H_{\lambda c} a_\lambda$ and $H_{S\lambda} = c^\dagger H_{S\lambda c} a_\lambda + \text{h.c.}$, respectively. We leave the specific forms of the matrices $H_S^c$ and $H_{S\lambda c}$ unspecified for now.

The macroscopic leads are assumed to be attached in the very long past, such that the system operators $c(t)$ are completely determined by the distribution in the leads; i.e., there is no memory of any initial occupations in the system. We then write a formal solution for the Heisenberg equation of motion (we set $\hbar = 1$), $i\dot{c} = H_S^c c + \sum_\lambda H_{S\lambda c} a_\lambda$:

$$c(t) = \int dt' G(t,t') \left[ \sum_\lambda H_{S\lambda c} g_\lambda(t' - t_0)a_\lambda(t_0) \right],$$

where $g_\lambda(t) = -i \exp(-i H_{\lambda c} t) \theta(t)$ is the retarded propagator for lead $\lambda$ and $G(t,t')$ is the full Green’s function within the system. For the calculations below, it is convenient to furthermore define the Fourier-transformed Floquet Green’s function,

$$G^{(k)}(\mathcal{E}) = \frac{1}{T} \int_0^T dt \int_{-\infty}^\infty ds \ G(t, t-s) e^{i\xi_\mathcal{E} s} e^{i k t},$$

and

$$\xi_\mathcal{E}(\mathcal{E}) = H_{S\lambda c} \rho_\lambda(\mathcal{E}) H_{S\lambda c}^\dagger,$$

where $\rho_\lambda(\mathcal{E}) = \sum_n \delta(\mathcal{E} - E_{\lambda n}) |\lambda n\rangle \langle \lambda n|$ captures the density of states of lead $\lambda$, with $H_{\lambda c} |\lambda n\rangle = E_{\lambda n} |\lambda n\rangle$ [32].

The net current flowing into the right lead, averaged over one period, is given by

$$I = \frac{1}{T} \int_0^T dt i \langle [H(t), N_R(t)] \rangle,$$

where $N_R(t) = a_R^\dagger(t)a_R(t)$ is the number operator for the right lead. Through Eq. (3) we express the system operators $c(t)$ as linear combinations of the lead operators $a_\lambda(t_0)$ in the distant past (we take $t_0 \to -\infty$). Similarly, the lead operators $a_\lambda(t)$ can be written in terms of $a_\lambda(t_0)$. We assume that the state in each lead $\lambda$ is given by a Fermi distribution $f_\lambda$ with chemical potential $\mu_\lambda$ and
The absorption of \( E \) 
Here \( \alpha \) temperature \( T_\lambda \): \( \langle a_\lambda^\dagger(t_0)a_\lambda(t_0) \rangle = \delta_{n,m}f_\lambda(\epsilon_{\lambda n}) \), where \( a_\lambda^\dagger \) creates an electron in eigenstate \( \langle \lambda n \rangle \) in lead \( \lambda \) (see above). Using Eqs. (3)-(5) and the Fermi distributions for the leads, a standard calculation gives [43]:

\[
I = 2\pi \int_{-\infty}^{\infty} d\epsilon \sum_k \left\{ T_{RL}^{(k)}(\epsilon)T_{L}(\epsilon) - T_{RL}^{(k)}(\epsilon)T_{R}(\epsilon) \right\}.
\]

Here \( T_{RL}^{(k)}(\epsilon) \) is the probability for an electron at energy \( \epsilon \) to be transmitted from lead \( \lambda' \) to lead \( \lambda \) along with the absorption of \( k \) photons from the driving field.

As we now show, the steady-state time-averaged current carried by the AFAI, Eq. (7), is quantized in the limit of large bias, \( V \to \infty \), with \( \mu_L = V/2, \mu_R = -V/2 \). In this limit we may set \( f_L(\epsilon) = 1 \) and \( f_R(\epsilon) = 0 \), yielding

\[
I = \int_{-\infty}^{\infty} d\epsilon \sigma(\epsilon), \quad \sigma(\epsilon) = 2\pi \sum_k T_{RL}^{(k)}(\epsilon).
\]

In the following, we show the quantization of the current by relating \( \sigma(\epsilon) \) to the differential conductance of an associated static system. For illustration, we first consider the dominant processes contributing to \( \sigma(\epsilon) \), see Fig. 3. In each process a particle in the left lead with energy \( \epsilon \) scatters into a Floquet state of the system with quasienergy \( \epsilon \approx \epsilon + n\Omega \) [44]. The integer \( n \) is determined by our convention for Floquet states, \( \langle \psi_\epsilon(t) \rangle = e^{-i\eta t} \sum_m \phi_\epsilon^{(m)} e^{-i\eta mt} \), with \(-\Omega/2 \leq \epsilon < \Omega/2\).

\[
T_{RL}^{(k)}(\epsilon) = \text{Tr} \left[ G^{(k)}(\epsilon)\xi_\lambda(\epsilon) + k\Omega G^{(k)}(\epsilon)\xi_\lambda(\epsilon) \right].
\]

The operator \( H^{EZ} \) acts in enlarged Hilbert space, which is a tensor product of the original Hilbert space and a \((2M + 1)\)-dimensional auxiliary space, which we call the harmonic space.

As in Eq. (1), we write \( H^{EZ} = H^{EZ}_{S} + H^{EZ}_{\Lambda} + H^{EZ}_{\Omega} \). An eigenstate of \( H^{EZ}_{S} \) with energy \( \epsilon \) can be expanded as \( \langle \psi_\epsilon^{EZ} \rangle = \sum_m \phi_\epsilon^{(m)} \otimes |n\rangle \). The eigenvalues of \( H^{EZ}_{S} \) in the range \(-\Omega/2 \leq \epsilon < \Omega/2 \) approximate the quasienergy spectrum of \( U(\epsilon) = \mathcal{T} e^{-i\epsilon T} dh_T(t) \), becoming exact for \( M \to \infty \). Importantly, in this limit, for each \( \langle \psi_\epsilon^{EZ} \rangle \) there is a corresponding partner Floquet state with quasienergy \( \epsilon = \epsilon + m\Omega \) (with \( |\epsilon| \leq \Omega/2 \)) in the original driven problem: \( \langle \psi_\epsilon(t) \rangle = e^{-i\eta t} \sum_n \phi_\epsilon^{(n)} e^{-i\eta nt} \), with \( \langle \phi_\epsilon^{(n-m)} \rangle = \langle \phi_\epsilon^{(n)} \rangle \).

We now relate the relevant transport processes in the static “extended zone” (EZ) and Floquet pictures (see Fig. 3). Consider the differential conductance, \( \sigma_{EZ}(\mu) \), of the EZ system described by \( H^{EZ} \). Since the lead is not driven, the spectrum of \( H^{EZ}_{\Lambda} \) consists of \( 2M + 1 \) copies of that of \( H_{\Lambda} \), shifted by integer multiples of \( \Omega \); it can thus be viewed as a lead with many channels, labeled by the harmonic index. We define \( \sigma_{EZ}(\mu) \) by taking the Fermi level of the left and the right EZ leads to be \( \mu + \delta\mu \) and \( \mu - \delta\mu \), and take \(-\Omega/2 \leq \mu < \Omega/2 \) throughout [45].

Consider now the dominant processes contributing to \( \sigma_{EZ}(\mu) \). The system-lead coupling \( H^{EZ}_{\Omega} \) conserves the harmonic index. Therefore, a lead state with energy \( \epsilon \) and harmonic index \( n \) (which corresponds to a state of the physical lead with energy \( \epsilon + n\Omega \)) is coupled to the state \( \langle \psi_\epsilon^{EZ} \rangle \) through the component \( \langle \phi_\epsilon^{(n)} \rangle \). To obtain \( \sigma_{EZ}(\mu) \), we sum the contributions of states with energies close to \( \mu \) from all harmonic-index channels in both leads. Using the correspondence between \( \{\langle \psi_\epsilon^{EZ} \rangle\} \) and \( \{\langle \psi(t) \rangle\} \), for \( M \gg 1 \), we thus obtain (see App. A for detailed derivation):

\[
\sum_n \sigma(\mu - n\Omega) = \sigma_{EZ}(\mu).
\]
The leads are taken to have widths $W \gg \Omega$. The sample has dimensions $L \times W = 40 \times 20$ sites. The leads are taken to have widths $W_0 = W/2$. 

**Inset:** Bulk contribution to the steady state current, computed using a cylindrical geometry with contacts on opposite edges of the cylinder, for $w = 4.5/T$. Exponential decay of the bulk contribution with increasing $L$ indicates that the system is in the localized regime.

Numerical simulations.—To support the arguments above, we now numerically study the steady state current. We simulate the model described above, Eq. (1), for a range of system sizes and disorder strengths $w$, see Fig. 4. We take $D = \pi/(2T)$, and the leads to have constant density of states, $\rho_{0\lambda} = 1/J$. The lead-system coupling $H_{LS}$ is taken to yield $\xi_{\lambda}(E) = \sum_{\mathbf{r} \in W_0} \rho_{0\lambda} |\mathbf{r}/\mathbf{r}|$, where the sum runs over $W_0$ system sites directly adjacent to lead $\lambda$ (see Fig. 3b).

In the presence of disorder, all bulk states are localized and the current through the bulk vanishes exponentially with the distance between the leads. To probe this, we computed the current in a cylindrical geometry, with leads attached at opposite ends of the cylinder such that there were no edge states connecting the source and drain (shown in the inset of Fig. 4). As shown in Fig. 4 and the main panel of Fig. 4, for the Hall bar geometry of Fig. 4, the total current through the system saturates to the quantized value $I = 1/T$ in the insulating regime, for large (finite) bias.

As explained above, a quantized current is expected to flow in the AFAI when the edge-states exiting from the left lead are completely filled, while those exiting the right lead are empty. We confirm this picture (for a typical disorder realization) by mapping out the steady state time-averaged local density, $n_{\lambda} = \frac{1}{T} \int_0^T dt \langle c^\dagger_{\lambda}(t) c_{\lambda}(t) \rangle$, in Fig. 5. This situation is realized for “good” contacts, with appropriately strong couplings $\xi_{\lambda}$ and large enough contact width $W_0$ (see Fig. 5 and Appendix B).

To further investigate the spatial distribution of the current, we map out the period-averaged bond current density, $j_{ij} = \frac{1}{T} \int_0^T dt \text{Im}[J_{ij}(t) c^\dagger_{i}(t) c_{j}(t)]$, see Eq. (2). As shown in Fig. 5, the current density is concentrated at the boundary of the filled and empty regions. This result may at first seem counterintuitive since a) all states in the bulk are localized and b) we expect the quantized current to be carried by the chiral edge states. However, it is crucial to remember that the local current density $j_{ij}(t)$ includes contributions of both transport currents and magnetization current [11, 47]. The quantized transport current is indeed carried by the chiral edge state.

Summary.—In this work we demonstrated theoretically a new topological quantized transport phenomenon, occurring in disordered two-dimensional periodically
driven systems. In contrast to the equilibrium quantized Hall conductivity, in the AFAI we find a quantized current in the limit of large bias. Looking ahead, disorder-induced localization may provide a route for stabilizing interacting Floquet phases of matter by suppressing energy absorption from the periodic drive. Recently, several works proposed interacting analogues of the AFAI [48–52]. Determining whether quantized transport and other response functions can be used to probe these interacting phases will be crucial for further progress in the field.

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Appendix A: Sum rule

In this Appendix, we derive the sum rule appearing in Eq. (10). Consider $|\phi^{(k)}_\alpha^{\pm}(t)\rangle$, the $k$th Fourier component of the time-periodic Floquet state $|\phi_\alpha^{\pm}(t)\rangle$, which satisfies

$$
(H_S(t) \mp i\Gamma - i\frac{d}{dt}) |\phi_\alpha^{\mp}(t)\rangle = (\epsilon_\alpha \mp i\gamma_\alpha) |\phi_\alpha^{\mp}(t)\rangle. \tag{A1}
$$

In the above $\Gamma = \sum_\lambda \xi_\lambda$, where we have assumed a constant density of states in each lead. The $\mp$ symbol in $|\phi_\alpha^{\pm}(t)\rangle$ labels the right and left eigenstates of the non-Hermitian operator $(H_S(t) - i\Gamma - i\frac{d}{dt})$, respectively. The Floquet states satisfy the completeness and orthogonality relations,

$$
\sum_\alpha |\phi_\alpha^{\mp}(t)\rangle \langle \phi_\alpha^{\pm}(t)\rangle = \mathbb{I}, \quad \int_0^T dt \langle \phi_\alpha^{\mp}(t)\rangle \langle \phi_\beta^{\pm}(t)\rangle = \delta_{\alpha\beta}. \tag{A2}
$$

The Green’s function satisfies

$$
\left(i\frac{d}{dt} + \mathcal{E} - H_S(t) + i\Gamma\right) G(t, \mathcal{E}) = \mathbb{I}, \tag{A3}
$$

with $G(t, \mathcal{E}) = \sum_k e^{-ik\Omega t} G^{(k)}(\mathcal{E})$. The Floquet Green’s function can thus be written as:

$$
G^{(k)}(\mathcal{E}) = \sum_{n \in \mathbb{Z}} \frac{|\phi^{(n+k)-}_\alpha\rangle \langle \phi^{(n)+}_\alpha|}{\mathcal{E} - \epsilon_\alpha - n\Omega + i\gamma_\alpha}. \tag{A4}
$$

As written in Eq. (8) of the main text, the current is given by:

$$
I = 2\pi \sum_k \int_{-\infty}^{\infty} d\mathcal{E} \text{Tr} \left[ G^{(k)}(\mathcal{E}) \xi_R G^{(k)}(\mathcal{E}) \xi_L \right], \tag{A5}
$$

and we define

$$
\sigma(\mathcal{E}) = 2\pi \sum_k \text{Tr} \left[ G^{(k)}(\mathcal{E}) \xi_R G^{(k)}(\mathcal{E}) \xi_L \right]. \tag{A6}
$$

With this definition, the current is $I = \int_{-\infty}^{\infty} d\mathcal{E} \sigma(\mathcal{E})$, or equivalently:

$$
I = \int_{-\Omega/2}^{\Omega/2} d\epsilon \frac{dI}{d\epsilon}, \quad \frac{dI}{d\epsilon} = \sum_{n \in \mathbb{Z}} \sigma(\epsilon + n\Omega). \tag{A7}
$$

From Eq. (A3), we obtain:

$$
dI/d\epsilon = 2\pi \sum_{n,k} \text{Tr} \left[ G^{(k)}(\epsilon + n\Omega) \xi_R G^{(k)}(\epsilon + n\Omega) \xi_L \right]
= 2\pi \sum_{n,k} \frac{|\phi^{(m)+}_\alpha\rangle \langle \xi_L | \phi^{(q)+}_\beta\rangle}{\mathcal{E} - \epsilon_\alpha + (n-m)\Omega + i\gamma_\alpha} \frac{|\phi^{(q+k)-}_\beta\rangle \langle \xi_R | \phi^{(m+k)-}_\alpha\rangle}{\mathcal{E} - \epsilon_\beta + (n-q)\Omega + i\gamma_\beta}. \tag{A8}
$$
Recall that our goal is to relate $dI/d\varepsilon$ to the differential conductance of the static extended-zone (EZ) system, and thereby derive Eq. (10). The EZ Hamiltonian, $H^\text{EZ}$, is defined in Eq. (9). Considering that the system-lead coupling is time independent, and assuming a constant density of states, i.e., $(H^\text{S\Lambda}_m)_{nn} = \delta_{mn}H^\text{S\Lambda}$, the EZ eigenstates satisfy:

\[
(H \equiv i\Gamma^\text{EZ} - i \frac{d}{dt}) |\Phi_{\alpha,p}^-\rangle = (E_{\alpha,p} + i\gamma_{\alpha,p}) |\Phi_{\alpha,p}^+\rangle,
\]

(A8)

with $\Gamma^\text{EZ}_{mm} = \delta_{mn}\Gamma$, and where $-M \leq p \leq M$. Together, $\alpha$ and $\tilde{\Psi}$ provide a complete labeling of the EZ eigenstates. The left and right eigenstates form a complete set, with the relations: $\sum_{\alpha,p} |\Phi_{\alpha,p}^-\rangle |\Phi_{\alpha,p}^+\rangle = 1$, $(\Phi_{\alpha,p}^-|\Phi_{\beta,p'}^+\rangle = \delta_{\alpha\beta}\delta_{pp'}$. Correspondingly, the Green’s function of the EZ system satisfies:

\[
(E - H^\text{EZ} + i\Gamma^\text{EZ}) G^\text{EZ}(E) = 1.
\]

(A9)

We represent $G^\text{EZ}(E)$ in the basis of eigenstates above as

\[
G^\text{EZ}(E) = \sum_{\alpha,p} |\Phi_{\alpha,p}^-\rangle |\Phi_{\alpha,p}^+\rangle (E - E_{\alpha,p} - i\gamma_{\alpha,p})^{-1}.
\]

(A10)

The differential conductance of the EZ system at energy $\mu$ is given by

\[
\sigma^\text{EZ}(\mu) = \frac{dI^\text{EZ}(\mu)}{d\mu} = 2\pi \text{Tr} \left[ G^\text{EZ\uparrow}(\mu)\xi_L^\text{EZ} G^\text{EZ\downarrow}(\mu)\xi_R^\text{EZ} \right],
\]

(A11)

where $(\xi^\text{EZ}_\lambda)_{mn} = \delta_{mn}\xi_\lambda$. Substituting Eq. (A10) into Eq. (A11), and writing $|\Phi_{\alpha,p}^\pm\rangle = \sum_n |\Phi_{\alpha,p}^{(n)\pm}\rangle \otimes |n\rangle$, gives

\[
\sigma^\text{EZ}(\mu) = 2\pi \sum_{\alpha,p,\beta,p'} \frac{\langle m|\xi_L^\text{EZ}|m\rangle \langle m|\xi_R^\text{EZ}|-m-p\rangle \langle m|\xi_L^\text{EZ}|-m-p\rangle \langle m|\xi_R^\text{EZ}|-m-p\rangle}{\mu - E_{\alpha,p} - i\gamma_{\alpha,p} - E_{\beta,p'} + i\gamma_{\beta,p'}}.
\]

(A12)

We focus on values of $\mu$ between $-\omega/2$ and $\omega/2$. Crucially, for $\mu$ in this range, the main contribution to $\sigma^\text{EZ}(\mu)$ comes from states with $|E_{\alpha,p} - \mu| \lesssim \gamma_{\alpha,p}$. For $M$ [the parameter controlling the truncation of the EZ Hamiltonian, see text around Eq. (9)] much greater than 1, we have $\lim_{M \to \infty} \langle m|\Phi_{\alpha,p}^{(m)\pm}\rangle = |\phi_{\alpha}^{(m+p)\pm}\rangle$; the corresponding eigenvalues also satisfy $\lim_{M \to \infty} E_{\alpha,p} = \epsilon_{\alpha} + p\Omega$ and $\lim_{M \to \infty} \gamma_{\alpha,p} = \gamma_{\alpha}$. Thus,

\[
\sigma^\text{EZ}(\mu) = 2\pi \sum_{\alpha,\beta} \frac{\langle \phi_{\alpha}^{(m+p)}|\xi_L^\text{EZ}|-m-p\rangle \langle \phi_{\alpha}^{(m+p)}|\xi_R^\text{EZ}|-m-p\rangle \langle \phi_{\beta}^{(n+p)}|\xi_L^\text{EZ}|-n-p\rangle \langle \phi_{\beta}^{(n+p)}|\xi_R^\text{EZ}|-n-p\rangle}{\mu - \epsilon_{\alpha} - p\Omega + i\gamma_{\alpha} - \mu - \epsilon_{\beta} - p\Omega - i\gamma_{\beta}}.
\]

(A13)

After a relabeling of indices, the expression above matches that in Eq. (A7). Thus, comparing with Eq. (A6), we have shown $\sum_n \sigma(\mu + n\Omega) = \sigma^\text{EZ}(\mu)$.

Appendix B: Dependence on contact width

In order to achieve quantized two-terminal transport, it is necessary that the chiral edge states fully equilibrate with the leads in the contact region. For example, consider the current impinging on the drain contact. It is essential that each particle reaching the drain is absorbed into the drain lead with unit probability, to avoid it returning back to the source and diminishing the net transmitted current. The probability for the particle to be absorbed into the lead approaches one, exponentially with the width $W_0$ of the contact; the lengthscales associated with this exponential is controlled by the matrix elements $J_{S\Lambda}$ governing hopping between the system and the lead.

In Fig. 8 we demonstrate the convergence of the transmitted current as a function of $W_0$ and $J_{S\Lambda}$. We take $J_{S\Lambda}$ to be constant for all sites within the contact region. Here we focus on the weak coupling regime, where $J_{S\Lambda}$ is small compared with the hopping matrix elements within the system, $J$. As expected in this limit, the absorption length decreases with increasing $J_{S\Lambda}/J$. 


FIG. 6. Convergence of current to the quantized value as a function of contact width $W_0$, for three values of the system-lead coupling $J_{\text{SL}}$. Here we used $L \times W = 40 \times 20$ sites, disorder strength $w \times T = 1.5$, and constant density of states of the leads, $\rho_\lambda = 1/J$. 