Fast-to-Alfvén Mode Conversion Mediated by Hall Current. II. Application to the Solar Atmosphere

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Abstract

Coupling between fast magnetoacoustic and Alfvén waves can be observed in fully ionized plasmas mediated by stratification and 3D geometrical effects. In Paper I, Cally & Khomenko have shown that in a weakly ionized plasma, such as the solar photosphere and chromosphere, the Hall current introduces a new coupling mechanism. The present study extends the results from Paper I to the case of warm plasma. We report on numerical experiments where mode transformation is studied using quasi-realistic stratification in thermodynamic parameters resembling the solar atmosphere. This redresses the limitation of the cold plasma approximation assumed in Paper I, in particular allowing the complete process of coupling between fast and slow magnetoacoustic modes and subsequent coupling of the fast mode to the Alfvén mode through the Hall current. Our results confirm the efficacy of the mechanism proposed in Paper I for the solar case. We observe that the efficiency of the transformation is a sensitive function of the angle between the wave propagation direction and the magnetic field, and of the wave frequency. The efficiency increases when the field direction and the wave direction are aligned for increasing wave frequencies. After scaling our results to typical solar values, the maximum amplitude of the transformed Alfvén waves, for a frequency of 1 Hz, corresponds to an energy flux (measured above the height of peak Hall coupling) of ~10^3 W m^-2, based on an amplitude of 500 m s^-1 at \( \beta = 1 \), which is sufficient to play a major role in both quiet and active region coronal heating.

Key words: magnetohydrodynamics (MHD) – Sun: chromosphere – Sun: magnetic fields – Sun: oscillations – waves

1. Introduction

Alfvén waves (Alfvén 1942) are plasma perturbations whose restoring force is magnetic tension instead of gas or magnetic pressure. They carry energy along the magnetic field lines but the motion of the charged particles, which provide the inertia, and the magnetic field perturbation are transverse to the field.

In the solar atmosphere, the presence of gradients and strong vertical stratification allow for the process of mode transformation. Fast/slow magnetoacoustic coupling takes place where the acoustic and Alfvén speeds match \( (c_s = v_A) \). In the solar atmosphere the stratification-induced mode transformation usually occurs somewhere in the upper photosphere and low chromosphere, depending on the strength of the magnetic structures. In sunspot umbra it is typically subsurface. The mathematical formalism of mode conversion was developed by Schunker & Cally (2006) and Cally (2006, 2007) based on the generalized ray theory of Tracy et al. (2003). In the solar literature, the equipartition layer is either defined as where the plasma \( \beta \) or the ratio of acoustic and Alfvén speeds squared, \( c_s^2/v_A^2 \), reach unity. These measures differ by the factor \( \gamma/2 \). The wave speed equipartition is the more physically relevant criterion, but in practice there is little difference.

Ideal-magnetohydrodynamic (MHD) fast-to-Alfvén transformation happens around where the fast wave reflects the density stratification. This is typically well above the equipartition level, but depends on wave frequency and horizontal wavenumber. In a low \( \beta \) plasma, the reflection point is near where the horizontal phase speed matches the Alfvén speed, and so is higher for high frequency and for near-vertical propagation. This process could be an important mechanism to provide energy to the solar corona. This is so because unlike fast waves, which reflect in the transition region (TR) or chromosphere due to the rapidly increasing Alfvén speed, Alfvén waves generated through mode transformation close to the TR are more able to overcome this barrier, and to reach the corona. The efficiency of geometrical mode conversion depends on the local relative inclination between the wave vector and the background magnetic field (Cally & Goossens 2008; Cally 2009; Cally & Hansen 2011; Khomenko & Cally 2011, 2012; Felipe 2012). Observations realized by De Pontieu et al. (2007), Tomczyk et al. (2007), or Jess et al. (2009) have shown that Alfvén waves are everywhere in the corona. Later on, observations from McIntosh et al. (2011) and Jess et al. (2012) have shown that these “Alfvénic” waves are of sufficient amplitude to heat some regions and contribute to the acceleration of the solar wind. Recently, Srivastava et al. (2017) have reported the first observation of high-frequency torsional Alfvén waves (~12–42 mHz) in the solar chromosphere.

In general, astrophysical plasmas are formed by electrons, ions, neutrals, and dust grains and all these particles interact with the magnetic field, either directly or via collisional coupling between charged particles and neutrals. When differing inertia and collisional interactions produce a drift between electrons and ions, ideal MHD theory has to be modified to include the Hall effect. This introduces a new Hall electric field proportional to the cross product of the current density and the magnetic field, which thereby contributes to a generalized Ohm’s law. In order to treat the Hall effect, one has to apply so-called Hall-MHD theory.
In the atmosphere of the Sun or solar-like stars, the plasma can be weakly ionized, reaching, for example, an ionization fraction as low as \( f \sim 10^{-4} \) around the Sun’s temperature minimum. Under these conditions, Cheung & Cameron (2012) investigated the effects of ambipolar diffusion and Hall currents on the formation of structures in photospheric magnetoconvection, showing how the Hall current can couple magnetocoustic and Alfvén waves. Later on, Cally & Khomenko (2015) provided a corrected theory of Hall-current mediated coupling between the fast and Alfvén waves in cold (i.e., high beta) plasmas. They demonstrated that coupling efficiency is proportional to the dimensionless Hall parameter \( \epsilon_{\text{Hall}} = \omega / \Omega_i f \), where \( \Omega_i \) is the mean ion gyro-frequency and \( \omega \) is the wave frequency. Due to the smallness of the ionization fraction \( f \), coupling can be produced even for relatively low-frequency waves. They show that the Hall effect produces an oscillation between the Alfvén and magneto-sonic states and the precession would be the beating between those modes. They found that this coupling occurs in places where the wave vector is nearly aligned with the magnetic field vector. It is also more efficient for relatively low magnetic field strengths, and for higher frequency waves. Cally & Khomenko (2015) speculated that such processes as reconnection, where high-frequency waves are produced, can be affected.

Unlike the geometrical mode conversion studied by Cally & Goossens (2008), Khomenko & Cally (2012), or Felipe (2012) among others, Hall current induced mode conversion can happen even if initially the wave vector and the magnetic field vector are in the same two-dimensional plane. Moreover, once the transformation happens, the waves keep traveling long distance nearly aligned with the magnetic field so they could transfer energy to the surrounding plasma during the precession process.

The purpose of the current paper is to advance the initial work by Cally & Khomenko (2015), relaxing the approximation of cold plasmas. That approximation excludes acoustic modes. Nevertheless, the current schematic picture of mode generation, propagation, and transformation in a stratified solar atmosphere suggests that acoustic waves play an important role in the chain of wave energy transmission to the corona. Acoustic \( p \)-modes propagating below the surface (and being fast modes there because \( c_s > v_A \)) reach the equipartition layer where they are partially transformed into fast magnetic waves. These subsequently refract and reflect back to the solar surface. As mentioned above, Alfvén waves are produced through a secondary transformation while the fast waves refract. By considering cold plasma, the first process (fast acoustic to fast magnetic transformation) is not considered, and, therefore, the efficiency of the production of Alfvén waves through the Hall current mechanism cannot be evaluated correctly for the solar case.

Here we perform simulations of the complete process beginning with imposed acoustic wave generation below the surface, and all subsequent transformations, including the Hall effect, are addressed on the way to the corona. We take account of realistic plasma stratification for the Sun, and realistic values of the magnetic field. In Section 2 we show the set of equations corresponding to a single-fluid description with a generalized Ohm’s law including the Hall term. In Section 3 we present our numerical setup. In Section 4 we show the results of our numerical experiments, while in Section 5 a brief summary is presented.

2. Equations

For this work we adopt the Hall-MHD formulation for a partially ionized solar plasma. We neglect all other nonideal effects (such as ambipolar diffusion and the battery effect) and consider only the nonideal Hall term under the single-fluid approach (Khomenko et al. 2014; Ballester et al. 2018). In this approximation, the equations to be solved are the continuity equation,

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,
\]

the momentum conservation equation,

\[
\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot \left( \rho \mathbf{v} \mathbf{v} + \left( p + \frac{B^2}{2\mu_0} \right) \mathbf{I} - \frac{\mathbf{B} \mathbf{B}}{\mu_0} \right) = \rho \mathbf{g},
\]

the induction equation,

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[ (\mathbf{v} \times \mathbf{B}) - \eta_H \frac{\mu_0}{|B|} (\mathbf{J} \times \mathbf{B}) \right],
\]

where we retained the convective and Hall terms on the right-hand side. Here \( \eta_H \) is the Hall coefficient and is written in units of a diffusivity coefficient \((l^2 s^{-1})\), i.e., \(m^2 s^{-1}\) in SI,

\[
\eta_H = \frac{|B|}{en_s \mu_0}. \tag{4}
\]

The total energy conservation equation,

\[
\frac{\partial e_{\text{tot}}}{\partial t} + \nabla \cdot \left( \frac{\rho \mathbf{v}^2}{2} + \frac{p}{\gamma - 1} \mathbf{v} + \frac{1}{\mu_0} \nabla \cdot [\mathbf{B} \times (\mathbf{v} \times \mathbf{B})] \right) = \rho \mathbf{v} \cdot \mathbf{g}, \tag{5}
\]

is written in terms of the total energy density per volume unit \( e_{\text{tot}} \), which is the sum of the kinetic, internal, and magnetic energies,

\[
e_{\text{tot}} = \frac{1}{2} \rho \mathbf{v}^2 + \frac{p}{\gamma - 1} + \frac{B^2}{2\mu_0}. \tag{6}
\]

The electric current density \( \mathbf{J} \) is defined through

\[
\mu_0 \mathbf{J} = \nabla \times \mathbf{B}, \tag{7}
\]

and Gauss’s law for magnetism is

\[
\nabla \cdot \mathbf{B} = 0. \tag{8}
\]

To close the system, the equation of state for an ideal gas is used. The equations above assume charge neutrality \( n_e = n_i \) and negligible electron mass \( (m_e \ll m_i) \).

3. Numerical Setup

The numerical experiments described in this work are produced by the MANCHA3D code (Khomenko & Collados 2006), which solves the nonlinear nonideal 3D single-fluid MHD equations for the perturbations written in conservative form (Felipe et al. 2010). Our recently implemented numerical treatment of the Hall term is described in Gonzalez-Morales et al. (2018).

The modeled solar stratification is built using a 2.5D approximation, that is, vectors are three-dimensional objects.
but the derivatives are applied only in two directions, one horizontal and one vertical, taking those to be $x$ and $z$. The model is horizontally homogeneous.

The vertical stratification is built by merging the standard solar Model-S of Christensen-Dalsgaard et al. (1996) with the chromospheric model VAL-C of Vernazza et al. (1981) smoothly at height $z = 140$ km. Because the upper layers of the solar convection zone are super-adiabatic and unstable against convection, we modify the stratification in thermodynamic parameters to make it convectively stable and avoid the generation of modes that grow exponentially. To do this, we apply the procedure described by Schunker et al. (2011) solving the system iteratively and considering the hydrostatic equilibrium condition to the stratification with a constant value for the gravity $g_0 = 273.98 \, \text{m s}^{-2}$ and a constant adiabatic coefficient $\gamma = 5/3$. The resulting stratification is illustrated in Figure 1. The numerical box is $L_x = 1.5 \, \text{Mm}$ tall, extending from $z = -0.5 \, \text{Mm}$ to $z = 1 \, \text{Mm}$.

Finally, a constant and uniform magnetic field $B_0$ with adjustable inclination $\theta$ with respect to the vertical direction is included. The strength of the magnetic field is chosen based on the arguments explained below in this section.

To excite the waves in the simulation domain we impose an acoustic-gravity wave with a given frequency and horizontal wavenumber as a bottom boundary condition. The analytic solution at the bottom boundary is calculated according to Mihalas & Mihalas (1984), and provides a self-consistent perturbation in pressure, density, temperature, and velocity. The temperature gradient and magnetic field are neglected in this analytical solution. The neglect of magnetic field is justified because the bottom boundary is located in the region where $v_A \ll c_s$, and we are interested in exciting acoustic fast waves. The analytical solutions applied are identical to Khomenko & Cally (2012) and are provided in Appendix A.

For the top boundary we use a perfectly matched layer (Berenger 1994, 1996; Hu 1996, 2001; Hesthaven 1998; Parchevsky & Kosovichev 2009) but slightly modified. We apply the absorption coefficients over all the split variables to obtain a better attenuation and stability for certain magnetic field angles. The horizontal boundaries are set to periodic. The horizontal size of the domain, $L_x$, is determined by the value of $k_x$. Table 1 provides the details of the simulation runs. Our goal is to study how the efficiency of the transformation depends on the wave frequency. Therefore, the wave frequency is chosen to be a free parameter varying from simulation to simulation. We used frequencies from 0.01 to 1 Hz.

To initiate the simulation one has to specify the horizontal wavenumber, $k_x$, for the given frequency. Because we are interested in keeping the wave vector in the same direction for different source frequencies we calculate the components $(k_x, k_z)$ for a given angle $\alpha$ by writing the dispersion relation in terms of the wave vector modulus:

$$k = \frac{\omega}{c_s} \sqrt{\frac{\omega_c^2 - \omega^2}{\omega_s^2 \sin^2 \alpha - \omega^2}},$$

where $\omega_c = \gamma g/2 c_s$ is the acoustic cutoff frequency and $\omega_s = 2 \omega_k \sqrt{\gamma - 1/\gamma}$ is the Brunt–Väisälä frequency. The wave vector components are then calculated as $k_x = |k| \sin \alpha$ and $k_z = |k| \cos \alpha$, where $\alpha$ is the angle between the vertical and the wavenumber vector, see Figure 4(a). We choose $\alpha = 10^\circ$.

Figure 2 shows the behavior of $k$ according to Equation (9) and the wavenumber vector components $k_x, k_z$ as a function of the source frequency for a fixed angle $\alpha = 10^\circ$.

In order to select the background magnetic field strength $B_0$ for our experiments we consider the behavior of the Hall

![Figure 1](image1.png)

**Figure 1.** Pressure (blue), density (green), and temperature (red) as a function of vertical coordinate in the model atmosphere assumed in this study. The dotted–dashed vertical line indicates the location of the $\beta = 1$ layer. The dashed line corresponds to the location of the maximum of the Hall parameter ($\epsilon_{\text{max}}$). The blue rectangle indicates the location of our numerical box.

![Figure 2](image2.png)

**Figure 2.** Modulus of the wavenumber $k$ as a function of the wave frequency at the bottom boundary of the numerical box, calculated according to Equation (9). The blue vertical dotted–dashed lines are the selected frequencies for the experiments. The red dotted–dashed vertical line is acoustic cutoff frequency $\omega_{ac}$ and the green one $\omega_{bc}$ both calculated at the bottom of the numerical domain.

| Table 1 | Selected Source Frequency and Relevant Parameters for the Numerical Setups |
|---------|-----------------------------|
| $\nu$ (Hz) | $k_x$ (Mm$^{-1}$) | $k_z$ (Mm$^{-1}$) | $dx$ (km) | $dz$ (km) | $n_x$ | $n_z$ |
| 0.01 | 0.85 | 4.84 | 147.21 | 10 | 50 | 172 |
| 0.02 | 1.77 | 10.02 | 71.13 | 10 | 50 | 172 |
| 0.05 | 4.46 | 25.28 | 28.19 | 5 | 50 | 332 |
| 0.1 | 8.92 | 50.62 | 14.08 | 1 | 50 | 1612 |
| 1 | 89.29 | 506.40 | 1.41 | 0.05 | 50 | 32012 |
parameter $\epsilon_{\text{Hall}}$, defined according to:

$$
\epsilon_{\text{Hall}}(\nu, B) = \frac{\omega}{f\Omega_i} = \frac{2\pi p_0 \nu}{q_i n_e B_0},
$$

(10)

where $p_0$ is the background density, $q_i$ is the electron charge, $n_e$ is the electron number density, $\nu$ is the wave source frequency, and $B_0$ is the background magnetic field. Figure 3 shows the Hall parameter, written in units of the background magnetic field, calculated for our stratification of thermodynamic parameters, as a function of height, for several wave frequencies. Independently of the frequency and the value of $B_0$, $\epsilon_{\text{Hall}}$ has a maximum around $z \approx 390$ km. Therefore, we choose $B_0$ to place the equipartition layer $c_i = v_A$ below the height where $\epsilon_{\text{Hall}}$ is maximum. This is because we wish to study the process of conversion between fast magnetic and Alfvén waves, and the fast magnetic waves are to be produced first through the primarily geometrical transformation at the $c_i = v_A$ layer. To affect this compromise, we choose a magnetic field $B_0 = 500$ G, which locates the equipartition layer at $z \approx 228$ km, just below the maximum of the Hall parameter and inside the region where mode transformation can take place.

4. Alfvén Wave Production in the Warm Plasma

First we confirm that Hall-current mediated transformation is indeed taking place in our model. For that we separate the contribution of the three wave modes. This can be done relatively easily in our numerical experiment, given the 2.5D setup and the knowledge of the direction of the wave vector and the magnetic field vector direction. We calculated the projection of the velocity vector following Cally & Goossens (2008). This decomposition has been successfully used previously to identify the three wave modes propagating into a magnetized medium in similar models (e.g., Felipe et al. 2010; Khomenko & Cally 2011, 2012; Felipe 2012). Figure 4(b) shows a schematic diagram for the decomposition. The longitudinal component of the velocity given by $v_{\text{long}}$, the one parallel to the magnetic field, selects the slow magnetoacoustic wave in a low-$\beta$ plasma. The other two components are perpendicular to the magnetic field. The component perpendicular to both $B$ and $\nabla p$ (direction of the background pressure gradient, coinciding with the direction of gravitational stratification), given by $v_{\text{perp}}$, selects the Alfvén wave. Finally, the component perpendicular to the previous two, $v_{\text{tran}}$, selects the fast magnetoacoustic wave in the low-$\beta$ plasma. Mathematically, this new basis can be written as:

$$
e_{\text{long}} = (\cos \theta \sin \phi, \sin \phi \sin \theta, \cos \phi),$$  

$$e_{\text{perp}} = (-\cos \phi \sin^2 \theta \sin \phi, 1 - \sin^2 \phi \sin^2 \theta, -\cos \theta \sin \theta \sin \phi),$$  

$$e_{\text{tran}} = (-\cos \theta, 0, \cos \phi \sin \theta),$$

(11)

where $\theta$ is the magnetic field angle with the vertical $z$ axis, and the $\phi$ the azimuth angle measured from the $XZ$-plane.

In a 2.5D case, we set the azimuth $\phi = 0$, so the expressions for the components simplifies to:

$$e_{\text{long}} = (\sin \theta, 0, \cos \theta),$$  

$$e_{\text{perp}} = (0, 1, 0),$$  

$$e_{\text{tran}} = (-\cos \theta, 0, \sin \theta),$$

(12)

this means that $v_{\text{perp}} = v_i$, so the $v_i$ component of the velocity field is the one that selects Alfvén waves.

A note of caution must be taken regarding the above decomposition. Following the book of Goossens (2003), chapter 5, the component $e_{\text{perp}}$ chooses the asymptotic polarization direction for the Alfvén mode in a plasma with any value of $\beta$. However, two other directions, $e_{\text{long}}$ and $e_{\text{tran}}$ generally provide a mixture of the fast and slow magnetoacoustic modes. The particular contribution of each of the modes into $e_{\text{long}}$ and $e_{\text{tran}}$ depends on plasma $\beta$, see Appendix B. In the limit of low-$\beta$, as mentioned above, most contribution to $e_{\text{long}}$ comes from the longitudinal slow magnetoacoustic mode, while the direction $e_{\text{tran}}$ selects the fast magnetoacoustic mode in this case.

As mentioned in Section 3, our source produces acoustic-gravity waves at the bottom of the numerical box. Since the plasma $\beta$ is large there, the waves generated by the source are fast, essentially acoustic, waves. These waves propagate upward and suffer a first mode transformation at the equipartition layer where the acoustic and Alfvén speeds coincide, splitting into the slow (essentially acoustic) wave component and fast (essentially magnetic) wave component in $v_A > c_s$. Due to the geometry of our numerical experiment
setup, the wave vector $k$ and the magnetic field vector lie in the same vertical plane (the $x$-$z$ plane in Figure 4(b)). Therefore, in ideal MHD with the Hall term switched off, only fast and slow MHD waves can exist in our system, and no transformation to the Alfvén wave can take place. Mathematically, the velocity component $v_\parallel$ is exactly zero, because there is no coupling out of the $x$-$z$ plane.

However, when the Hall term is switched on, a secondary mode transformation can take place. This transformation happens when the fast magnetic mode, generated by the primary mode transformation at $v_A = c_s$, enters into the region where the Hall parameter becomes important, see Figure 3. This way, Alfvén waves are produced from fast magnetic waves. This double mode transformation can be seen in Figure 5.$^5$

In the first example, we see in the right panel of Figure 5 how the fast (acoustic) wave propagates with the acoustic speed upward and how the fast and slow waves are generated after the primary mode transformation around 200 km height in the photosphere. The existence of both fast and slow components can be appreciated from different inclination of ridges in the middle and right panels. The fast (magnetic) mode ridges are much steeper above 200 km, indicating faster propagation speed. The fast magnetic mode can also be observed reflecting back to the subphotosphere, which is seen as downward-inclined ridges and from the interference pattern below 200 km. The ridges become vertical above the reflection height, where the wave is evanescent. Such behavior is well known and has been observed before in many simulations (Khomenko & Collados 2009; Khomenko & Cally 2011; Felipe 2012; Khomenko & Cally 2012; Santamaria et al. 2015). The new feature one can observe in Figure 5 is the generation of the Alfvén mode, seen as a nonzero $v_\parallel$ component in the left panel. This component starts to appear at heights around 400 km in the photosphere, coinciding with heights where the Hall parameter is maximum; see the left panel of Figure 5. Nevertheless, the Hall-mediated transformation occurs throughout the height range. The amplitude of the Alfvén waves produced by this mechanism is highest immediately above the height where the Hall parameter is maximum.

Although the height of the numerical box is insufficient to completely encompass the region where the fast wave reflection takes place, part of the fast mode that is reflected downward again travels through the region with high Hall parameter values. This way, downward propagating Alfvén waves are produced (ridges below $\epsilon_{\text{max}}$ in the left panel of Figure 5). A similar behavior was observed by Khomenko & Cally (2011, 2012) and Felipe (2012). However, in their simulations the Alfvén waves were produced through geometrical mode transformation, and not through Hall-mediated transformation.

One may notice that the amplitude of the Alfvén waves produced through the Hall-mediated transformation in Figure 5 is rather small, being about four orders of magnitude smaller than the amplitude of the slow acoustic mode at the equipartition layer. Nevertheless, the results of the theoretical investigation in cold plasma (Cally & Khomenko 2015) suggest that the amplitude of the Alfvén waves is a sensitive function of their frequency and of the inclination between the magnetic field and the wave vector. In order to study these dependencies in warm plasmas, we have repeated the simulations of Figure 5, but with different inclinations of magnetic field (from 0° to 90°) and different wave frequencies (as indicated in Table 1).

Figure 6 shows the results of these experiments. It presents the time–height diagrams for the perpendicular component of the projected velocity ($v_{\text{verp}} = v_\parallel$) for three of the selected frequencies, $\nu = 0.01, 0.1,$ and 1 Hz (columns) and five different inclination angles of the magnetic field, $\theta = 0°$, $20°$, $40°$, $60°$, and $80°$ (rows). All simulations share the same numerical setup including number of grid points per wavelength in the horizontal direction and the magnitude of the numerical diffusivity (necessary for the stability of the simulations). Nevertheless, it is unavoidable that higher frequency waves are affected more by numerical effects, and therefore their amplitude is inevitably lower than it should be. In order to account for these numerical effects and to standardize the experiments on the same scale, the velocities for a given frequency are scaled by setting the amplitude of the longitudinal component of the velocity $v_{\text{long}}$ at the equipartition layer height in the simulation with $\theta = 10°$ to 500 m s$^{-1}$. The rest of the amplitudes and perturbed quantities for this frequency are then scaled according to this factor.

We observe in Figure 6 that the amplitudes of both upward and downward propagating Alfvén waves depend on the inclination and frequency. In particular, the amplitude of the down-going wave becomes progressively smaller for larger magnetic field inclination angles. Similarly, simulations with

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$^5$ Fast-Alfvén coupling is also produced by 3D geometric effects, though these are absent in the 2.5D system.
Figure 6. Time–height diagrams of the $v_{\text{perp}} = v_y$ (Alfvén wave) velocity component in simulations with varying frequency (panels from left to right) and varying inclination angle of the magnetic field (panels from top to bottom). The Alfvén waves are generated through Hall mediated mode transformation. The values of the frequencies and inclination angles are indicated in each panel. The square box marks the region used to calculate the wave amplitudes in the stationary regime in the simulations summarized in Figure 8.
higher frequencies show reduced down-going Alfvén waves, as measured by the amplitude. In all cases, the amplitudes of the down-going waves are smaller than of the up-going waves. The amplitudes of the up-going waves significantly increase with increasing frequency. The region where these amplitudes are maximal are located at or immediately above the height with maximum Hall parameter.

In order to confirm the visual impression about the presence of the Alfvén waves and their direction of propagation, and also in order to quantify our results in terms of energy supply to the upper layers, we computed the magnetic Poynting flux carried by waves:

$$F_{\text{mag}} = (B_1 \times (v_1 \times B_0/\mu_0)),$$  \quad (13)

and the acoustic flux

$$F_{\text{acu}} = \langle p_1 v_1 \rangle,$$  \quad (14)

where the subscript “1” denotes a small perturbation in velocity ($v_1$), magnetic field ($B_1$), and pressure ($p_1$). The angled brackets denote phase averages. Using Equations (11) together with Equation (13) we obtain the longitudinal component of the magnetic wave energy flux. Considering only the perpendicular component of the velocity, we obtain a longitudinal magnetic flux associated exclusively with Alfvén waves:

$$F_{\text{Alfvén}} = -B_{1,\perp} v_{1,\perp} B_{0,\parallel}/\mu_0.$$  \quad (15)

Figure 7 shows the Alfvénic flux in simulations with varying magnetic field inclination angle and frequency in the same format as Figure 6. The amplitudes of the velocity and magnetic field oscillations were normalized for each frequency setting $V_{\text{long}} = 500 \text{ m s}^{-1}$ at $\beta = 1$ height, in the simulation with $\theta = 10^\circ$, as before. The results for the flux confirm that, indeed the ridges with the opposite inclination in Figure 6 correspond to downward magnetic flux (blue color). They indeed vanish (in comparison to the upward fluxes) when the inclination angle and wave frequency are increased, though notice that the magnitude of the fluxes is orders of magnitude higher at 1 mHz than at the lower frequencies. Also, we observe how this flux increases up to a maximum value at a certain inclination angle, and then starts to decrease; see the column for $\nu = 100 \text{ mHz}$ for example. The magnetic flux for a given inclination angle increases with increasing frequency.

Finally, gathering together the results of all simulations, we have computed the amplitude of the perpendicular velocity once the system reaches a stationary regime. Figure 8 shows the results of this calculation as a function of inclination angle for all considered frequencies. This figure nicely summarizes the behavior mentioned above. First, we see that the amplitude of Alfvén waves increases with increasing wave frequency. The maximum amplitude reached for waves of 1 Hz constitutes about 30% of the amplitude of the longitudinal wave component, which is significant. Next, for a given frequency, there is magnetic field inclination where the amplitude of the generated Alfvén waves reaches a maximum value. For low-frequency waves this maximum falls at large inclination angles. But for progressively larger frequencies, the inclination with maximum amplitude approaches asymptotically the inclination of $10^\circ$, i.e., $\theta = \alpha$, the inclination of the wave vector. In Figure 9 we can see similar behavior in the perpendicular projection of the perturbed magnetic field.

As mentioned previously, Hall-mediated transformation acts everywhere in the numerical domain and its efficiency increases with wave frequency. Because our numerical scheme contains numerical noise, it is important to carefully choose parameters such as the filtering cadence and the artificial diffusivity to keep this noise as low as possible to avoid its growth and the artificial Hall-mediated transformation that can be introduced into the simulations. Our chosen setup parameters made the simulations computationally very costly at progressively higher frequencies, so we had to truncate our numerical analysis at 1 Hz. Nevertheless, Figure 8 clearly suggests that for higher frequencies, the amplitudes would increase further. In order to quantify this increase we performed fitting to the maximum values of velocity versus angle, using the following power law:

$$\max (|v_{\parallel}|) = A(\theta - \theta_0)^m.$$  \quad (16)

The best fit is for $A = 4475.88 \text{ deg m s}^{-1}$, $m = -2.28$, $\theta_0 = 15^\circ.5$. According to these parameters, an asymptotic angle with maximum velocity lies around $15^\circ.5$. Nevertheless, this particular number should only be taken as an indication. Possibly, simulations with better sampling in $\theta$ would allow us to make a more precise fit. In addition, the amplitude of waves in numerical simulations is affected by numerical effects (such as numerical diffusivity). Although we kept the numerical parameters as similar as possible between all simulations, it is still possible that the numerical diffusivity affects the higher frequency waves (especially those at 1 Hz) more severely. Nevertheless, given all the uncertainties, we conclude that the amount of energy transferred from fast to Alfvén modes can be considerable. For a reference, Figure 9 provides the corresponding amplitudes of the magnetic field perturbation. For all the frequencies except 1 Hz, the amplitudes of $B_1$ do not reach above 1 G. This means that detecting such perturbations in observations would only be possible with the highest sensitivity polarimeters on the largest-aperture telescopes such as DKIST or future EST.

In Figure 10, we collect the results for the average Alfvénic flux for all the inclinations and frequencies. This flux is calculated at $z = 450 \text{ km}$, above the height with the maximum Hall parameter. This figure allows us to quantify the energy input by Alfvén waves into the higher layers. The Alfvénic flux reaches a maximum around $10^3 \text{ W m}^{-2}$ at $15^\circ$ for the 1 Hz wave. The spatio-temporal root mean square (rms) of the Alfvén flux, calculated for this frequency inside the green box marked in Figure 8, is about $780 \text{ W m}^{-2}$. Making a similar fit to the Alfvénic flux as before, the asymptotic angle is $\theta_0 = 10^\circ.36$, very close to the inclination angle of $k$ with respect to the background magnetic field.

The values of the magnetic flux of Alfvén waves are to be compared to the available acoustic flux at some reference layer. We take for the reference the equipartition layer as the layer where the acoustic waves start to transform. Figure 11 shows the mean value of the longitudinal acoustic flux at the equipartition level. We observe that, in general, the acoustic flux pumped into the corona and then available to be transformed into Alfvénic flux slightly increases with frequency. The behavior of the curve for 1 Hz is slightly different from the others, probably due to numerical effects on this simulation. The values of the acoustic flux vary in the range of $10^2$--$10^4 \text{ W m}^{-2}$ for all the frequencies. One can conclude that
only for the highest frequency of 1 Hz does the Alfvén flux shown in Figure 10 contribute a significant fraction of the available acoustic flux.

On the other hand, in the full nonlinear regime, which is not explored here, acoustic waves are much more subject to attenuation (by shocking for example) in the chromosphere...
Our results might therefore be expected to overestimate the acoustic wave flux reaching higher levels.

5. Discussion and Conclusion

In this paper simulations are performed of Hall current mediated mode conversion to Alfvén waves for plasma parameters approximating the solar atmosphere. The numerical solution allows us to relax the cold plasma approximation assumed in the initial analytic study of Paper I. We consider an acoustic-gravity wave with various frequencies propagating upward from the lower boundary of our simulation domain located in the high plasma $\beta$ region below the photosphere. Our simulations indeed show the presence of Alfvén waves when the Hall effect is acting. Therefore, we confirm that this effect can be responsible for coupling fast magnetoacoustic and Alfvén waves even when there is no cross-field wave propagation (the usual coupling mechanism).

In Paper I, Cally & Khomenko (2015) concluded that the transformation is more efficient for vertical fields and wave propagation aligned with the field. The efficiency is also a sensitive function of the Hall parameter, and therefore it increases for increasing wave frequency $\nu$ and decreasing ionization fraction $f$. Our numerical experiments in warm plasma have partially confirmed this picture, but also shown a more nuanced behavior. We conclude that the efficiency of the transformation for low-frequency waves is maximal for strongly inclined fields (50°–70°). However, for waves at higher frequencies, the maximum becomes progressively aligned with the field. The asymptotic fit for the perpendicular velocity shows that the alignment between the directions of $k$ and $B$ is within 5°.5 for waves with frequencies above 1 Hz. A
similar fit to the Alfvénic flux curves results in a difference between \( k \) and \( B \) of just 0\(^{\circ}36\).

A further conclusion concerns the absolute value of the effect. As discussed in the Introduction, in warm plasmas the transformation is a two-step process. First acoustic fast waves are partially transformed into magnetic fast waves at the \( v_A = c_s \) equipartition layer, and then the latter are transformed into Alfvén waves progressively where Hall coupling operates. For that to happen there should be a specific relation between the location of the equipartition layer and the level with maximum Hall parameter \( \xi_{\text{Hall}} \). While the latter depends exclusively on the temperature structure of the atmosphere, the former is a function of the magnetic field. For the Hall-mediated transformation to be efficient one needs to simultaneously satisfy both conditions: (1) that the layer with \( v_A = c_s \) is located below the layer with maximum \( \xi_{\text{Hall}} \), therefore \( B_0 \) has to be sufficiently large to have \( v_A = c_s \) located deep enough; (2) that \( B \) is sufficiently small to maximize the value of \( \xi_{\text{Hall}} \). In our simulations we set \( B_0 = 500 \, \text{G} \) to satisfy both conditions. We reach the maximum amplitudes of Hall-excited Alfvén waves, about 30\% of \( v_{\text{long}} \), for waves of the highest studied frequency, i.e., 1 Hz.

In practice, the process of Hall-mediated transformation acts in addition to the geometrical mode transformation to Alfvén waves suggested by Cally & Goossens (2008). It does not need any particular relation between the wave vector and the orientation of the magnetic field, and, our simulations suggest that the maximum transformation can occur for a broad range of magnetic field inclinations, depending on the wave frequency. Also, the considerations above suggest that the process would be efficient for intermediate field strengths of the order of \( h \, \text{G} \), comparable to those existing in solar network and quiet areas. Therefore, this process could provide a constant energy supply be means of Alfvén waves to the solar corona.

The 3D geometric fast/Alfvén coupling mechanism occurs near the fast wave reflection height, where the horizontal phase speed \( \omega k_{h} \) equals the Alfvén speed (in a low \( \beta \) plasma), and \( k_{h} = (k_{2}^{2} + k_{3}^{2})^{1/2} \) is the horizontal wavenumber. At frequencies of a few mHz this may occur somewhere in the low-to-mid chromosphere, depending on magnetic field strength and \( k_{h} \). However, at the high frequencies considered above (1 Hz etc.), reflection may not occur until the TR is reached. In that case, the two coupling regions (Hall and geometric) are spatially separated and would operate independently: Hall coupling would operate in the weakly ionized low chromosphere, and geometric coupling would set in once the TR is reached.

In our modeling, for waves of 1 Hz, the average Alfvénic energy flux we obtain at 450 km height (above the height of peak Hall coupling) is about \( 10^{7} \, \text{W} \, \text{m}^{-2} \) with an rms of 780 W m\(^{-2} \). These numbers are bigger than what is required for heating the corona above quiet-Sun regions, which is about 100–300 W m\(^{-2} \), but are of the order of what is needed for heating the corona above active regions (Withbroe & Noyes 1977). The numbers we provide were obtained after normalizing the wave amplitude of \( v_{\text{long}} \) to be 500 m s\(^{-1} \) at \( \beta = 1 \) which, using Equation (14), corresponds to an average acoustic wave energy flux of \( \sim 3.8 \times 10^{5} \, \text{W} \, \text{m}^{-2} \) and a spatiotemporal rms value of about 368 W m\(^{-2} \). These values depend of the adopted base amplitude, which is probably overestimated at this frequency.

Theoretical estimates and measurements of the wave energy flux for such high-frequency waves are uncertain (Fossum & Carlsson 2006). These authors provide measurements of the acoustic energy flux from TRACE observations for frequencies up to a few hundred mHz, showing an exponential decrease of flux with frequency. The flux measured at the largest frequencies by these authors makes 0.1–1 W m\(^{-2} \). Simulations of acoustic wave generation by turbulence show a maximum wave energy at frequencies around 0.1 Hz with a strong decrease toward the higher frequencies (Musielak et al. 1994). The acoustic energy flux at 1 Hz reported by Musielak et al. (1994) makes \( 10^{3} \, \text{W} \, \text{m}^{-2} \). Assuming we can convert 1\% of this acoustic flux into Alfvén waves and bring it to the solar corona, a good fraction of the energy needed to compensate its losses would be provided by Hall coupling. Although this flux is small compared to the peak acoustic flux at the same height, the Alfvén flux is far more able to penetrate to the corona, and so is more relevant to coronal heating.

While fast modes refract and reflect in low-\( \beta \) regions and slow modes rapidly shock and damp, Alfvén waves can reach upper regions of the solar atmosphere due to their incompressible nature (\( \nabla \cdot v = 0 \)), making them an attractive mechanism to transport energy up to the corona. It is then a challenge to find mechanisms to dissipate Alfvén energy there.

Recently, simulations from Santamaria et al. (2017) have shown that slow magnetoacoustic shock waves coming from the chromosphere can trigger a jet-like structure of slow magnetoacoustic waves with frequencies up to 80 mHz around null points. These shock waves could be converted into fast modes around \( \beta = 1 \) regions with complex topology, and then they can be converted into Alfvén waves via the Hall term. Because the presence of null points is theoretically predicted almost everywhere, this double transformation could be an important source of production of Alfvén waves in complex magnetic topologies.

Possibly, Hall mediated conversion can be important for other astrophysical scenarios, e.g., in star formation regions or reconnection events as well as in other cooler stars. Further investigations of 3D effects via the azimuth (\( \phi \)) and stratification dependence, and taking into account other scenarios may also be interesting.

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Appendix A
Bottom Boundary Condition

For our experiments we impose a bottom boundary condition as a source. This source introduces an acoustic-gravity wave of a given frequency \( \omega \) and wavenumber \( k \). The analytical solution
for the perturbations in density, pressure, temperature, and velocity are calculated according to Mihalas & Mihalas (1984), providing a self-consistent solution. The temperature gradient and magnetic field are neglected in this formulation because the bottom boundary is located in the region where \( v_A \ll c_s \) and they are unimportant at this depth. The analytical solutions applied are identical to those used by Khomenko & Cally (2012):

\[
\delta V_c = V_0 \exp \left( \frac{z}{2H} + k_{\gamma z} \right) \sin(\omega t - k_{\gamma z} - k_{\gamma} x), \quad \text{(17a)}
\]

\[
\frac{\delta p}{\rho_0} = V_0 |P| \exp \left( \frac{z}{2H} + k_{\gamma z} \right) \sin(\omega t - k_{\gamma z} - k_{\gamma} x + \phi_p), \quad \text{(17b)}
\]

\[
\frac{\delta \rho}{\rho_0} = V_0 |R| \exp \left( \frac{z}{2H} + k_{\gamma z} \right) \sin(\omega t - k_{\gamma z} - k_{\gamma} x + \phi_r), \quad \text{(17c)}
\]

\[
\delta V_r = V_0 |U| \exp \left( \frac{z}{2H} + k_{\gamma z} \right) \sin(\omega t - k_{\gamma z} - k_{\gamma} x + \phi_r), \quad \text{(17d)}
\]

where with the \( \delta \) symbol we are indicating a perturbed quantity, \( \rho_0 \) and \( \rho_0 \) are the pressure and density at the bottom, \( V_0 \) is the initial amplitude for the imposed perturbation in velocity, \( H = c_s^2/2g \) is the pressure scale height, \( k_z \) is the horizontal wavenumber, \( k_{\gamma} \) is the vertical wavenumber made up of a real part \( k_{\gamma} \) and an imaginary part \( k_{\gamma} \), and \( \omega = 2\pi\nu \) is the angular frequency of the perturbation. \( |P|, |R|, \) and \( |U| \) are the amplitudes given by

\[
|P| = \frac{\gamma \omega}{\omega^2 - c_s^2 k_z^2} \left[ k_{\gamma}^2 + \left( k_{\gamma} + \gamma - \frac{2}{2H}\gamma \right)^2 \right], \quad \text{(18a)}
\]

\[
|R| = \frac{\gamma \omega}{\omega^2 - c_s^2 k_z^2} \left[ k_{\gamma}^2 + \left( k_{\gamma} + \frac{\gamma - 1}{\gamma H} \frac{c_s^2 k_z^2}{\omega^2} - \frac{1}{2H} \right)^2 \right], \quad \text{(18b)}
\]

\[
|U| = \frac{c_s^2 k_z}{\gamma \omega} |P|, \quad \text{(18c)}
\]

where \( \gamma \) is the adiabatic coefficient. The phases \( \phi_p, \phi_u, \) and \( \phi_r \) are given by

\[
\phi_p = \phi_u = \arctan \left( \frac{k_{\gamma}}{k_{\gamma} + \frac{1}{2Hk_{\gamma}}} \frac{\gamma - \frac{2}{2H}}{\gamma} \right), \quad \text{(19a)}
\]

\[
\phi_r = \arctan \left( \frac{k_{\gamma}}{k_{\gamma} + \frac{\gamma - 1}{\gamma Hk_{\gamma}} \frac{c_s^2 k_z^2}{\omega^2} - \frac{1}{2Hk_{\gamma}}} \right). \quad \text{(19b)}
\]

The vertical wavenumber is

\[
k_z = k_{\gamma} + ik_{\gamma} = \sqrt{\omega^2 - \omega_s^2} - \frac{\omega^2 - \omega_s^2}{c_s^2}, \quad \text{(20)}
\]

where the cutoff frequency is

\[
\omega_c = \frac{\gamma g}{2c_s}, \quad \text{(21)}
\]

and the Brunt–Väisälä frequency is

\[
\omega_B = \frac{2\omega_c}{\gamma} \sqrt{\gamma - 1}. \quad \text{(22)}
\]

The dispersion relation can be written in terms of the wavenumber modulus \( k \) and the propagation angle \( \alpha \) as

\[
k = \frac{\omega}{c_s} \sqrt{\frac{\omega_c^2 - \omega^2}{\omega^2 \sin^2 \alpha - \omega^2}}. \quad \text{(23)}
\]

**Appendix B**

A Note on the Velocity Projections

In a semiempirical solar stratification, as the one we had considered for our experiments, the identification of the different modes is a tough task because they are physically mixed. In this paper, in order to carry out the mode selection we have used the properties of MHD waves and made the change of base using the triad \((e_{\text{long}}, e_{\text{perp}}, e_{\text{tran}})\) as used by Cally \& Goossens (2008), also mention as the mixed field line/magnetic surface triad by Goedbloed et al. (2010),

\[
e_{\text{long}} = \cos \phi \sin \theta \, e_x + \sin \phi \sin \theta \, e_y + \cos \theta \, e_z, \quad \text{e_{perp} = -cos \phi \sin^2 \theta \, e_x + cos \phi \sin \theta \, e_y + cos \theta \, e_z}, \quad \text{e_{tran} = (1 - sin^2 \phi \sin \theta \, e_x - \cos \theta \sin \theta \, e_y + \cos \phi \sin \theta \, e_z).} \quad \text{(24)}
\]

This new basis allows us to select the Alfvén mode (for any plasma \( \beta \)). However, \( e_{\text{long}} \) and \( e_{\text{tran}} \) generally mix a selection of slow/fast magnetoacoustic modes depending on plasma \( \beta \).

To verify the behavior of the projections, we can use the ideal MHD formulation and consider a simple case with an infinite uniform plasma with a constant vertical magnetic field \( B = B_0 \, e_z \) and \( k_y = 0 \). In this case, the transformation between the bases is simplified to

\[
e_{\text{long}} = e_x, \quad e_{\text{perp}} = e_y, \quad e_{\text{tran}} = e_z. \quad \text{(25)}
\]

We can define the displacement associated to the Alfvén wave and the slow/fast magnetoacoustic waves as

\[
X_F = (\xi \cdot e_{\text{tran}}) e_{\text{tran}} = \xi_x e_x, \quad X_A = (\xi \cdot e_{\text{perp}}) e_{\text{perp}} = \xi_y e_y, \quad X_S = (\xi \cdot e_{\text{long}}) e_{\text{long}} = \xi_z e_z. \quad \text{(26)}
\]

The eigenfunctions for the incompressible Alfvén wave \( \xi_A \), and both the slow and fast magnetoacoustic waves, \( \xi_{(S,F)} \), in this case are given by

\[
\xi_A = \xi_x e_x, \quad \xi_{S,F} = \xi_x e_x + \frac{\omega^2 - k_z^2 \omega_s^2}{\omega_s^2} e_z. \quad \text{(27)}
\]

\[
\xi_{S,F} = \xi_x \left( e_x + \frac{\omega^2 - k_z^2 \omega_s^2}{\omega_s^2} e_z \right). \quad \text{(28)}
\]
\[ \xi_F = \xi_x \left( \frac{\omega_F^2}{\omega_F^2 - k^2 v_A^2} k_x e_x + e_z \right) \]
\[ = \xi_x \left( e_x + \frac{\omega_F^2 - k^2 v_A^2}{\omega_F^2} k_x e_z \right) . \quad (29) \]

where the slow/fast magnetoacoustic frequencies, \( \omega_{(S,F)} \), are
\[ \omega_{(S,F)} = \frac{k^2 (c_s^2 + v_C^2)}{2} \times \left\{ 1 \pm \left( 1 - \frac{4 \omega_C^2}{k^2 (c_s^2 + v_A^2)} \right)^{1/2} \right\} , \quad (30) \]

with \( \omega_C = k v_C \) being the cusp frequency, and \( v_C \) the cusp velocity \( v_C^2 = c_s^2 v_A^2 / (c_s^2 + v_A^2) \). It is clear from Equation (27) that \( \xi_A = X_A \) holds in general. While from Equation (28) and (29) it can be seen that the slow and fast mode displacements are formed by a linear combination of eigen displacements (26). In the limit of low plasma \( \beta \),
\[ \xi_S \approx X_S, \quad \xi_F \approx X_F. \quad (31) \]
while in the limit of high plasma \( \beta \),
\[ \xi_S \approx \xi_x (e_x - \frac{k_x}{k_z} e_z), \]
\[ \xi_F = \xi_x (e_x + \frac{k_x}{k_z} e_z). \quad (32) \]

So the above projection only allows us to separate fast and slow magnetoacoustic modes in the limit of low plasma \( \beta \). More details about this can be found in Goossens (2003), chapter 5.

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**References**

Alfvén, H. 1942, Natur, 150, 405

Ballester, J. L., Alexeev, I., Collados, M., et al. 2018, SSRv, 214, 58

Berenger, J.-P. 1994, JCoPh, 114, 185

Berenger, J.-P. 1996, JCoPh, 127, 363

Cally, P. S. 2006, RSPTA, 364, 333

Cally, P. S. 2007, AN, 328, 286

Cally, P. S. 2009, MNJAS, 395, 1309

Cally, P. S., & Goossens, M. 2008, SoPh, 251, 251

Cally, P. S., & Hansen, S. C. 2011, ApJ, 738, 119

Cally, P. S., & Khomenko, E. 2015, ApJ, 814, 106

Cheung, M. C. M., & Cameron, R. H. 2012, ApJ, 750, 6

Christensen-Dalsgaard, J., Dappen, W., Ajukov, S. V., et al. 1996, Sci, 272, 1286

De Pontieu, B., McIntosh, S. W., Carlsson, M., et al. 2007, Sci, 318, 1574

Felipe, T. 2012, ApJ, 758, 96

Felipe, T., Khomenko, E., & Collados, M. 2010, ApJ, 719, 357

Fossum, A., & Carlsson, M. 2006, ApJ, 646, 579

Goedbloed, J. P., Keppens, R., & Poedts, S. 2010, Advanced Magnetohydrodynamics, With Applications to Laboratory and Astrophysical Plasmas (Cambridge: Cambridge Univ. Press) (http://adsabs.harvard.edu/abs/2010adma.book.GG)

González-Morales, P. A., Khomenko, E., Downes, T. P., & de Vicente, A. 2018, A&A, 615, A67

Goossens, M. 2003, An Introduction to Plasma Astrophysics and Magnetohydrodynamics, Vol. 294 (Dordrecht: Kluwer)

Hesthaven, J. S. 1998, JCoPh, 142, 129

Hu, F. Q. 1996, JCoPh, 129, 201

Hu, F. Q. 2001, JCoPh, 173, 455

Jess, D. B., Mathioudakis, M., Erdélyi, R., et al. 2009, Sci, 323, 1582

Jess, D. B., Pascoe, D. J., Christian, D. J., et al. 2012, ApJL, 744, L5

Khomenko, E., & Cally, P. S. 2011, JPhCS, 271, 012042

Khomenko, E., & Cally, P. S. 2012, ApJ, 746, 68

Khomenko, E., & Collados, M. 2006, ApJ, 653, 739

Khomenko, E., & Collados, M. 2009, A&A, 506, L5

Khomenko, E., Collados, M., Díaz, A., & Vitats, N. 2014, PhPl, 21, 092901

McIntosh, S. W., De Pontieu, B., Carlsson, M., et al. 2011, Natur, 475, 477

Mihalas, D., & Mihalas, B. W. 1984, Foundations of Radiation Hydrodynamics (New York: Oxford Univ. Press)

Musielak, Z. E., Rosner, R., Stein, R. F., & Ulmschneider, P. 1994, ApJ, 423, 474

Parchevsky, K. V., & Kosovichev, A. G. 2009, ApJ, 694, 573

Santamaria, I. C., Khomenko, E., & Collados, M. 2015, A&A, 577, A70

Santamaria, I. C., Khomenko, E., Collados, M., & de Vicente, A. 2017, A&A, 602, A43

Schunker, H., & Cally, P. S. 2006, MNJAS, 372, 551

Schunker, H., Cameron, R. H., Gizon, L., & Moradi, H. 2011, SoPh, 271, 1

Srivastava, A. K., Shetye, J., Murawski, K., et al. 2017, NatSR, 7, 43147

Tomczyk, S., McIntosh, S. W., Keil, S. L., et al. 2007, Sci, 317, 1192

Tracy, E. R., Kaufman, A. N., & Brizard, A. J. 2003, PhPl, 10, 2147

Vernazza, J. E., Avrett, E. H., & Loeser, R. 1981, ApJS, 45, 635

Withbroe, G. L., & Noyes, R. W. 1977, ARA&A, 15, 363