Critical Current Enhancement due to an Electric Field in a Granular $d$-Wave Superconductor

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We study the effects of an electric-field in the transport properties of bulk granular superconductors with different kinds of disorder. We find that for a $d$-wave granular superconductor with random \pi-junctions the critical current always increases after applying a strong electric field, regardless of the polarity of the field. This result plus a change in the voltage as a function of the electric field are in good agreement with experimental results in ceramic high $T_c$ superconductors.

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Recent experiments in ceramic high-$T_c$ superconductors (HTCS) have found an enhancement in the critical current when applying an electric field $E$ through an insulating layer [1-4]. Previous studies of the electric field effects (EFE) in superconducting films have attributed the changes in the critical current to variations in the charge density or to a redistribution of carriers, which appear at the surface layer with depths of the order of the electrostatic screening length $d_E$ (in HTCS, $d_E \approx 5 \, \text{Å}$) [3]. Several experiments in ultrathin YBa$_2$Cu$_3$O$_{7-x}$ films ($\delta - 10 \, \text{nm}$ thick) have found that $E$ can affect $T_c$ and the IV characteristics [3], in good agreement with this picture [3]. In this case, there is either an enhancement or a depletion of the critical current depending on the polarity of $E$ [3]. The surprising observation in [1-4] of a strong EFE in bulk ceramic HTCS (1.5 mm thick), however, can not be explained by a surface effect. Moreover, for high enough electric fields, the critical current always increases regardless of the polarity of the field [3].

Rakhmanov and Rozhkov [4] have shown that an electric field can induce a change in the critical currents of the Josephson junctions present in granular samples. However, in their model the critical current either increases or decreases depending on the sign of $E$. Recently, Sergueenkov and José [3] have proposed that an electric field applied to a granular superconductor can produce a magneto-electric like effect, which could be indirectly related to the behavior of the critical current in [1-4], but no comparison with the experimental results was given.

Granular superconductors are usually described as a random network of superconducting grains coupled by Josephson weak links [1-10]. In the HTCS ceramics several experimental groups have found a paramagnetic Meissner effect (PME) at low magnetic fields [11]. Sigrist and Rice [12] proposed that this effect could be a consequence of the intrinsic unconventional pairing symmetry of the HTCS of $d_{x^2-y^2}$ type [12]. Depending on the relative orientation of the superconducting grains, it is possible to have weak links with negative Josephson coupling ($\pi$-junctions) [12,13] which, according to [11,12], give rise to the PME [14]. In this paper we will show that the presence of $\pi$-junctions in ceramic samples also explains the unusual electric field effects observed in [1-4].

We consider a 3-D cubic network of superconducting grains [1,2] at $\mathbf{n} = (n_x, n_y, n_z)$ and with unit vectors $\mathbf{\hat{y}} = x, \hat{y}, \hat{z}$. The current $I_{\mu}(\mathbf{n})$ between two grains $\mathbf{n}$ and $\mathbf{n} + \mathbf{\hat{y}}$ is given by the sum of the Josephson supercurrent plus a dissipative Ohmic current:

$$I_{\mu}(\mathbf{n}) = I_{\mu,0}^0 \sin \theta_{\mu}(\mathbf{n}) + \frac{\Phi_0}{c2\pi R} d\theta_{\mu}(\mathbf{n})$$

(1)

Here $\theta_{\mu}(\mathbf{n}) = \theta(\mathbf{n} + \mathbf{\hat{y}}) - \theta(\mathbf{n}) - A_{\mu}(\mathbf{n}, t)$ is the gauge invariant phase difference, with $\theta(\mathbf{n})$ the superconducting phase in each grain, and $A_{\mu}(\mathbf{n}, t) = \frac{\pi}{\Phi_0} \int_0^{n_{\mathbf{n}+\mathbf{\hat{y}}}} \mathbf{A} \cdot d\ell$ ($\Phi_0 = \hbar/2e$). The critical current of each junction is $I_{\mu,0}^0$, and $R$ is the normal state tunneling resistance between grains. Together with the conditions of current conservation, $\sum_{\mu} [I_{\mu}(\mathbf{n}) - I_{\mu}(\mathbf{n} - \mathbf{\hat{y}})] = 0$, this determines the dynamical equations for the Josephson network. We consider periodic boundary conditions (PBC) in a network with $N \times N \times N$ grains.

When an electric field $\mathbf{E}$ is applied in the $\hat{z}$ direction, the $z$ component of the vector potential is given by:

$$A_z(\mathbf{n}, t) = A_z(\mathbf{n}, 0) - \frac{2\pi cd}{\Phi_0} Et,$$

(2)

with $d$ the intergrain distance or junction thickness. This results in a high-frequency alternating supercurrent in the $z$-direction due to the ac Josephson effect [15]. In addition, we consider that the sample is driven by an external current density $I_{ext}$ along the $\hat{y}$ direction. Therefore, the vector potential term is $A_{\mu}(\mathbf{n}, t) = -\delta_{\mu,\hat{z}} \omega Et - \delta_{\mu,\hat{y}} \alpha_y(t)$, where the electric field frequency.
is \( \omega_E = 2\pi cE d/\Phi_0 \), and we consider that no external magnetic field is applied. (The experiments had zero magnetic field [4]). The dynamics of the external current density with PBC determines the dynamics of \( \alpha_y(t) \) as

\[
I_{\text{ext}} = \frac{1}{N^2} \sum_n I^n_{\alpha y} \sin \theta_y(n) + \frac{\Phi_0}{c^2 \pi R} \frac{d \alpha_y}{dt}.
\]  

(3)

The average voltage per junction induced by the driving current is then obtained by \( V = \frac{2\pi \alpha_y}{2\pi R} \frac{d \alpha_y}{dt} \).

Furthermore, we consider that the applied electric field is screened inside the grains and acting only in the insulating intergrain region of the junctions, of typical thickness \( d \approx 10 - 20 \mu \text{m} \). We also neglect the effects of intergrain capacitance \( C_1 \) and intragrain capacitance \( C_g \). In general, the effect of the capacitances is to screen the applied field \( E_{\text{ext}} \) inside the sample within an overall screening length \( \lambda_E \sim (C_1/C_g)^{1/2} \). Since \( C_g \ll C_1, \lambda_E \) is very large and we can consider that the internal field acting in the intergrain junctions is \( E_{\text{in}} = E \approx \alpha_y E_{\text{ext}} \) with the polarizability \( \alpha_y \ll 1 \). We also neglect the possible \( E \) dependence of the critical currents of the junctions. As shown in [4] this assumption may result in an increase or decrease of \( I^n_{\alpha y} \) depending on the sign of \( E_{\text{in}} \). We also neglect screening current effects (i.e. finite self-inductances). The self-induced magnetic fields are very important for the description of the critical state [16] at large magnetic fields, and for the PME at low magnetic fields [17]. The experiments of [14,15] were done at zero magnetic field and finite electric fields. Thus the most important approximation we make is to neglect the capacitances (i.e. the screening of \( E \)). After making all these physical approximations, we will now focus on the collective aspects of the ac Josephson effect induced by the electric field. In this model, the electric field scale is given by \( E_0 = R I_0(T = 0)/d = \pi \Delta_s(0)/2ed \), with \( \Delta_s(0) \) the superconducting energy gap; for the YBaCuO ceramics we have \( \Delta_s(0) \approx 20 \mu \text{eV} \), which gives \( E_0 \approx 30 \mu \text{eV/m} \), i.e. in the same range as the fields used in [4].

We assume that the effect of disorder in a granular superconductor at zero magnetic field mainly modifies the magnitude of the critical currents. In \( s \)-wave superconductors the sign of the Josephson coupling is always positive. In \( d \)-wave ceramics the sign of \( I^n_{\alpha y} \) is expected to vary randomly depending on the relative spatial orientation of the grains. Therefore, we consider two models of disorder: (i) Granular \( s \)-wave superconductor (GsS): We consider \( I^n_{\alpha y} \) a random variable with a uniform distribution in the interval \([I_0(1 - \Delta_c), I_0(1 + \Delta_c)]\) with \( \langle I^n_{\alpha y} \rangle = I_0 \) and \( \Delta_c < 1 \). (ii) Granular \( d \)-wave superconductors (GdS): We consider that there is a random concentration \( c \) of \( \pi \)-junctions with \( \langle I^n_{\alpha y} \rangle = -I_0 \), and \( I^n_{\alpha y} = I_0 \) with concentration \( 1 - c \) [17,19]. The latter model was used to explain the history-dependent paramagnetic effect [17], the anomalous microwave absorption [18] and the non-linear susceptibility and glassy behavior [19] observed experimentally in ceramic HTCS [11].

We have performed numerical simulations of systems of sizes \( N = 8,16 \) with a numerical integration of the dynamical equations with time step \( \Delta t = 0.1 \tau_j \), with \( \tau_j = \Phi_0/2\pi c R I_0 \), and we took \( 5 \times 10^4 \) integration steps. We calculated the current-voltage characteristics (IV) for different amounts of disorder and electric fields, averaging over \( 20 \) realizations of disorder in each case.

In the absence of disorder, the IV curves are unaffected by the electric field. In a perfect cubic network, a finite electric field induces an ac supercurrent \( I_0 \sin(\omega_E t) \) along the \( z \) axis only, and therefore the IV curves in the \( xy \) plane are not affected by the value of \( E \). When the disorder is small, the amplitude of the ac supercurrents in the \( z \) direction is random and, due to current conservation in each node of the network, this induces small ac-currents in the \( xy \) planes with random amplitudes \( \sim \Delta_c I_0 \sin(\omega_E t) \). Adding a small ac current to a Josephson junction reduces its effective dc critical current [5]. In fact, this is the effect we observe in the GsS model. In Fig. 1(a) we show the IV curve for \( \Delta_c = 0.6 \). There we see that after applying an electric field \( E \) the whole IV curve shifts to lower values of the current, the apparent critical current decreases, and therefore the voltage change \( \Delta V = V(E) - V(0) \) is positive for a given current \( I > I_c \). However, this is in the opposite direction of the experimental results of [4], where an increase in the critical current was observed.

![FIG. 1. Current-voltage characteristics before and after applying an electric field \( E \). (a) For a granular \( s \)-wave superconductor (GsS), with \( \Delta_c = 0.6 \). (b) For a granular \( d \)-wave superconductor (GdS) with concentration \( c = 0.5 \) of \( \pi \)-junctions. Voltages are normalized by \( N R I_0 \) and currents by \( N^2 I_0 \) and lattice sizes are \( 16 \times 16 \times 16 \).](image-url)
note that after applying the electric field the IV curve is shifted towards higher currents, i.e. the “apparent” critical current increases. If we change $E \rightarrow -E$ the IV curves overlap showing that the effect is independent of the polarity of the electric field. A similar effect was seen in the experiments of [3]: the IV curves shift upwards for large $E$ and almost overlap when changing the polarity of the field [20]. This result is surprising since in the usual electric field effect observed in films a change in the polarity of the field drastically changes the sign of the shift of the IV curves [3]. We also see in Fig.1(b) that the EFE is stronger near the critical current, and for large currents the change in voltage $\Delta V$ is smaller, which was also seen in experiment [11, 12]. If the amount of disorder in the GsS model is such that $\Delta c_1 > 1$, then a fraction $(\Delta c_1 - 1)/2\Delta c$ of the critical currents will be negative. This case is unrealistic for s-wave superconductors, but corresponds to a d-wave granular superconductor with a small concentration of $\pi$-junctions. In fact, we also find an increase of the critical current for $\Delta c_1 > 1$.

In Fig. 2 we study the relative change in voltage $\Delta V/V_0$ for a fixed bias current (above the critical current) as a function of $E$. We see that for the GsS model $\Delta V$ is positive, and only when negative critical currents are allowed ($\Delta c_1 > 1$), there is a decrease in the voltage, i.e. $\Delta V < 0$. The effect is stronger in the GdS model with $c = 0.5$ concentration of $\pi$-junctions. An effect of the same order, i.e. near a 50% decrease in voltage, and in the same scale of electric fields was observed in experiment. In Fig. 3 we show the dependence of the EFE with different amounts of disorder. For a given bias current we calculate $\Delta V/V_0$ after applying a large electric field of $E = 0.8E_0$. We see in Fig. 3(a) the voltage change in the GsS model as a function of $\Delta c$. It is clear that only when $\Delta c_1 > 1$ the $\Delta V$ can be negative. In Fig. 3(b) we study the same situation for the GdS model as function of the concentration of the $\pi$-junctions. We obtain that a small amount of $\pi$-junctions is enough to show a strong decrease in voltage at large electric fields. We see that there is a very rapid change in $\Delta V/V_0$ close to $c = 0.1$.

![Graph](image.png)

**FIG. 2.** Changes in voltages as a function of electric field for different kinds of disorder. $\Delta V/V_0 = (V(E) - V(0))/V(0)$. The results for the GsS model are for $I = 0.9I_0$ and for the GdS model, $I = 0.5I_0$. Electric fields are normalized by $E_0$.

It is interesting to note that there is another type of disorder that can give a negative $\Delta V/V_0$. In the presence of external magnetic fields, the vector potential has a component $A_{\mu}^H$ (n). For strong magnetic fields, $A_{\mu}^H$ (n) can be taken as a random variable in the interval $[0, 2\pi\Delta H]$, with the case $\Delta H = 1$ corresponding to the gauge glass model [21]. We have found that only when $\Delta H > 0.5$ we can obtain a decrease in the relative voltage, $\Delta V/V_0 < 0$, similar to the results obtained with the GdS model. The case $\Delta H > 0.5$ is when the effective couplings can take negative values. This case, even when interesting in itself, is unrealistic for this problem since it corresponds to ceramics at very large magnetic fields, while the experiments had zero magnetic field.

There are other interesting aspects of the experimental results. It was found that for low electric fields the critical current decreases and only for large fields it increases. A typical measurement shows that, when applying a current near the critical current, $\Delta V/V_0$ increases as a function of $E$ and, after reaching a maximum, it
falls and then becomes negative for large enough fields. Our random d-wave model for a granular superconductor also reproduces this effect in good agreement with experiment as we show in Fig. 4. We note that the value of $\Delta V/V_0$ at the maximum decreases when increasing the bias current, and only for large enough currents, the voltage change $\Delta V/V_0$ is always negative and decreasing with $E$ as previously shown in Fig. 2. The same dependence with the bias current has been observed in experiment (see for example Figs.3 and Fig.4 of [1] and Fig.3 of [2]). These results can be understood by looking at the shape of the IV curves at low electric fields. As we show in the inset of Fig.4, for low $E$ there is a crossing of the IV curves, which explains the behavior observed in $\Delta V/V_0$.

FIG. 4. Change in voltage as a function of $E$ for different driving currents for the GdS model with $c = 0.5$. Inset: Crossing of IV curves for different electric fields.

The experiments on the electric field effects in ceramic HTSC were surprising for two reasons: (i) a large electric field effect in a bulk ceramic sample was unexpected [1], (ii) an increase in the apparent critical current as a function of $E$ was found to be independent of the polarity of $E$ [2]. Here we have qualitatively explained all these experimental features with a simplified model of a granular superconductor, which has two basic ingredients: the ac Josephson effect induced by the electric field and the collective frustration effects due to $\pi$-junctions. There are many interesting open questions such as: the coexistence of this effect with the paramagnetic Meissner effect [1][2][17], the time dependent glassy dynamics as a function of electric and magnetic fields [3][4], the effects in a chiral glass state [19], and the effect of finite inductances and capacitances in a more realistic model. We expect that the results presented here will motivate further experimental and theoretical studies of this problem.

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could be easily explained by considering the dependence with $E$ of the Josephson critical currents $I_0(E)$. 

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