Research Article

Dynamic Analysis and Robust Control of a Chaotic System with Hidden Attractor

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1. Introduction

As Dr Passival says, “Chaos science is like a river, made up of many tributaries, it comes from all science” [1]. In 1963, Lorenz first constructed a 3D quadratic polynomial ODEs system, and he found the first chaotic attractor in this system [2]. In 1971, Ruelle and Takens first used the chaotic theory in dynamic systems and explained the nature of turbulence and discovered the existence of “strange attractors” [3]. In the following decades, many chaotic attractors were discovered, such as Logistic mapping [4], Rössler attractor [5], Chen’s attractor [6], and Lü attractor [7]. For many years, chaotic systems and hyperchaotic systems with self-excited attractors have been the focus of investigation [8], and these systems have unstable equilibria on the boundary of their basins of attraction. In 2011, Leonov pointed out that there are hidden attractors in addition to self-excited attractors in the dynamical systems [9]. The attraction basins of hidden attractors cannot intersect the neighborhood of any equilibria, which is a very important feature for chaotic systems with hidden attractors. The Shilnikov condition cannot verify the chaotic characteristics of the chaotic systems with hidden attractors because of the absence of homologous or heterozygous orbits. Sprott gave some simple chaotic systems with no or one equilibrium by computer numerical search [10–13]. Then, researchers proposed chaotic systems with hidden attractors with only one stable equilibrium [14–18], no equilibrium [19–21], any number equilibria [22, 23], and a line equilibrium [24, 25]. Nowadays, chaotic systems with hidden attractors play an important role in nonlinear circuits [26], Van der Pol-Duffing oscillators [27], multi-level DC/DC converters [28], and relay systems with hysteresis [29]. In these dynamical systems, the existence of hidden oscillations can lead to the phenomenon of unstable states in life and industrial production. Hence, it is of great significance to understand the local and global dynamic behaviors of chaotic systems with hidden attractors.

In general, the appearance of hidden attractors is not hoped when a system operates normally. If it happens, this will lead to sudden and large chaotic oscillations and the system may collapse by the perturbation of hidden attractors. Actually, like two sides of a coin, there have been potential application values in the fields of chaos-based secure communication [30], image encryption [31], robust embedded biometric authentication system based on chaos.
2.2. Equilibria and Stability. Let $\dot{x}_1 = 0$, $\dot{x}_2 = 0$, and $\dot{x}_3 = 0$. At this time, the 3D jerk system (1) has only one equilibrium $O(0,0,0)$. Its Jacobian matrix is

$$J = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -b & -c \end{pmatrix}. \quad (3)$$

The characteristic polynomial equation is $f(\lambda) = \lambda^3 + c \lambda^2 + (b-c) \lambda + a$, according to Routh-Hurwitz criterion: When $c > 0$, $b > 0$, and $a > 0$, there are three negative real parts roots, and the 3D jerk system (1) has a stable node or stable node-foci. When $c > 0$, $a > 0$, and $bc - a < 0$, the 3D jerk system (1) has only one saddle-focus.

2.3. Phase Trajectories and Time Series of the System. The phase orbit of the system with different parameters can directly reflect its states and motion behavior. When the parameters of the 3D jerk system (1) are set as $a = 3.4, b = 1$, and $c = 4$ and the initial states $(x_1, x_2, x_3) = (-2, 0, 2.4)$, the 3D jerk system (1) is in chaotic state. The 3D phase space attractor diagram and the projection of attractors of 3D jerk system (1) on three coordinate planes are shown in Figure 1.

As shown in Figure 1, attractors cannot be found by means of equilibria. Therefore, attractors of 3D jerk system (1) are hidden attractors. The time series diagram of $x_1$, $x_2$, and $x_3$ can be obtained by integrating the 3D jerk system (1), as shown in Figure 2.

Figure 2 depicts the aperiodic characteristics of the system, and the oscillation curve is different from completely random noise; and each variable presents aperiodic changes in a continuous bounded range.

2.4. The Effect of Parameters on the System. The nonlinear behavior of the system is mainly influenced by the system parameters. In order to better study the complex dynamic behavior of the 3D jerk system (1), the numerical method is used to analyze the dynamic behavior of the system under different parameters, as shown in Table 1.
2.4.1. Phase Space Orbit. The parameters are fixed as \( b = 1 \) and \( c = 4 \). When the parameter \( a \) is selected as 3.31, 3.34, and 3.36, respectively, the phase diagrams of attractors with periodic 1, periodic 2, and periodic 4 can be obtained, as shown in Figure 3.

2.4.2. Poincare Section. A plane is cut out in the multidimensional phase space, and a pair of conjugate variables \( x_1 \) and \( x_2 \) are fixed values in this section, which is the Poincare section. Like Section 2.4.1, the parameters are fixed as \( b = 1 \) and \( c = 4 \). When the parameter \( a \) is selected as 3.31, 3.34, 3.36, and 3.40, respectively, the Poincare section diagram in the state of periodic 1, periodic 2, and periodic 4 periods and chaos can be obtained, as shown in Figure 4.

From Figure 4, when the parameter \( a \) is selected as 3.31, 3.34, and 3.36, the Poincare section has only one moving point or a few discrete points, indicating that the behavior of the system is periodic, but when the parameters are selected as \( a = 3.40 \), the Poincare section is a continuously dense point, and it has the characteristic of fractal structure, which indicates that chaotic behavior will occur.

2.4.3. Lyapunov Exponent. Initial value sensitivity of chaotic systems means that when given a very close initial value, the phase space will diverge with an exponential rate. Lyapunov exponent is to identify whether the system is chaotic or not, according to the characteristics of whether the phase orbit has diffusion [46]. In addition, according to Lyapunov dimension,

\[
D_L = j + \frac{1}{|\lambda_{j+1}|} \sum_{i=1}^{j} \lambda_i. \tag{4}
\]

Through Table 1, when \( \lambda_1 = 0.072, \ \lambda_2 = 0, \) and \( \lambda_3 = -0.069 \), substituting them into equation (4), the following equation can be obtained:

\[
D_L = j + \frac{1}{|\lambda_{j+1}|} \sum_{i=1}^{j} \lambda_i = 2 + \frac{\lambda_1 + \lambda_2}{|\lambda_3|} = 2.018. \tag{5}
\]

From equation (5), the Lyapunov dimension \( D_L = 2.018 \) for \( a = 3.4, b = 1, \) and \( c = 4 \), so the 3D jerk system (1) has fractal Lyapunov dimension, which verifies that system (1) is in a chaotic state when \( a = 3.4 \). In order to determine

![Figure 1: Chaotic attractor of the 3D jerk system (1) and projection in \( x_1 - x_2 \) plane, \( x_2 - x_3 \) plane, and \( x_3 - x_1 \) plane. (a) \( x_1 - x_2 - x_3 \). (b) \( x_1 - x_2 \). (c) \( x_3 - x_3 \). (d) \( x_3 - x_1 \).](image)

![Figure 2: The time series of 3D jerk system (1) in \( x_1, x_2, \) and \( x_3 \) plane.](image)

| PVs        | Dynamics     | LES                | PPs          | Ps          |
|------------|--------------|--------------------|--------------|-------------|
| \( a = 3.31, b = 1, c = 4 \) | Periodic 1   | \([0, -0.073, -3.927]\) | Figure 3(a) | Figure 4(a) |
| \( a = 3.34, b = 1, c = 4 \) | Periodic 2   | \([0, -0.056, -3.943]\) | Figure 3(b) | Figure 4(b) |
| \( a = 3.36, b = 1, c = 4 \) | Periodic 4   | \([0, -0.038, -3.964]\) | Figure 3(c) | Figure 4(c) |
| \( a = 3.40, b = 1, c = 4 \) | Chaos        | \([0.072, 0, -4.069]\) | Figure 1     | Figure 4(d) |
whether system (1) is chaotic or not, we should calculate the Lyapunov exponents spectrum of system (1) for fixed parameters $b, c$ and let $a$ vary. When $a$ varies in the interval $[3.2, 3.4]$, the Lyapunov exponents spectrum of the 3D jerk system (1) is shown in Figure 5, and the largest Lyapunov exponent of the 3D jerk system (1) is shown in Figure 6.

According to Figure 6, when $a \in [3.36, 3.4]$, the largest Lyapunov exponent $\lambda_1 > 0$ can be observed. This observation is also verified by Table 1.

2.4.4 Bifurcation. The bifurcation diagram of a system can also be used to analyze the state of a system within the parameter range. It can describe the bifurcation state that the state variable changes with the parameter change. Fixed parameters $b = 1$ and $c = 4$. When parameter $a \in [3.3, 3.4]$, the bifurcation diagram of the 3D jerk system (1) is shown in Figure 7.

Through the bifurcation state in Figure 7, when $a \in [3.31, 3.36]$, the 3D jerk system (1) exhibits periodic behaviors, and it presents typical states of periodic 1, periodic 2, and periodic 22; however, when $a = 3.40$, chaotic behavior occurs. Therefore, the 3D jerk system (1) evolves from periodic to chaotic. Then, combined with Lyapunov exponential spectrum in Figure 5, it is clear that the state of system (1) changes when the parameters change, which implies that chaotic behavior can only occur within a certain parameter range.

### 3. Design of Robust Feedback Controller

3.1 Provision of Robust Controller. Following the 3D jerk system (1), a class of master system with hidden attractors and uncertain parameters is given as

$$
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= x_3, \\
\dot{x}_3 &= -(a + \Delta_1)x_1 - (b + \Delta_2)x_2 - (c + \Delta_3)x_3 + (b + \Delta_2)x_1x_2,
\end{align*}
$$

where $x = (x_1, x_2, x_3)^T$ is the state variable of the master system, and the slave system can be described as follows:

$$
\begin{align*}
\dot{y}_1 &= y_2 + u_1(t), \\
\dot{y}_2 &= y_3 + u_2(t), \\
\dot{y}_3 &= -(a + \Delta_4)y_1 - (b + \Delta_2)y_2 - (c + \Delta_3)y_3 + y_2^2 + (b + \Delta_2)y_1y_2 + u_3(t),
\end{align*}
$$

where $y = (y_1, y_2, y_3)^T$ is the state variable of the slave system; $u_i(t)$ $(i = 1, 2, 3)$ is the external input control; $\Delta_i$ $(i = 1, 2, 3, 4)$ is the parameter uncertain term of master and slave chaotic systems. The synchronization error is defined as $e_i = y_i - x_i$ $(i = 1, 2, 3)$. Therefore, its error system can be expressed as
Before designing a finite-time robust feedback controller for the synchronization error system (1), the following Lemma 3.1, which plays an important role in the subsequent analysis, is recalled here for convenience.

**Lemma 1 (see [47]).** Provided that $V(t)$ is a differentiable and nonnegative scalar function and $V(t)$ satisfies the differential inequality $\dot{V}(t) \leq -\epsilon V^a(t)$, where $\epsilon$ and $\alpha$ are positive constants, $\epsilon > 1$ and $0 < \alpha < 1$, and then we have

\[
\begin{align*}
\dot{e}_1 &= e_2 + u_1(t), \\
\dot{e}_2 &= e_3 + u_2(t), \\
\dot{e}_3 &= -ae_1 - be_2 - ce_3 + y_1y_2(b + \Delta_3) - x_1x_2(b + \Delta_2) + y_2^2 - x_2^2 - \Delta_4y_1 - \Delta_5y_2 - \Delta_6y_3 + v_1x_1 + \Delta_2x_2 + \Delta_3x_3 + u_3(t).
\end{align*}
\]

(8)

where the finite time $t_M \leq V^{1-a}(0)/(\epsilon(1-a))$. $V(0)$ is the initial value and $V(0)$ is bounded.

Based on the above Lemma 3.1, to drive the synchronization error in error system (1) to zero in the finite time, a robust feedback controller is proposed as follows:

\[
\begin{align*}
u_1(t) &= -\epsilon_1^{y_1} + ae_1 - e_2, \\
u_2(t) &= -\epsilon_2^{y_2} + be_2 - e_3, \\
u_3(t) &= -\epsilon_3^{y_3} + ce_3 - y_1y_2b - \delta_3|y_1y_2|\text{sign}(e_3) + x_1x_2b + \delta_2|x_1x_2|\text{sign}(e_3) \\
&- y_2^2 + x_2^2 + \delta_4|x_1|\text{sign}(e_3) + \delta_5|y_2|\text{sign}(e_3) + \delta_6|x_3|\text{sign}(e_3) - \delta_1|x_1|\text{sign}(e_3) - \delta_2|x_2|\text{sign}(e_3) - \delta_3|x_3|\text{sign}(e_3)
\end{align*}
\]

(10)

where $0 < y_i < 1$ ($i = 1, 2, 3$) and remark $\gamma = \min_{1 \leq i \leq 3}\gamma_i$. 

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**Figure 4:** Periodic 1, 2, and 4 and chaotic Poincare section of 3D jerk system (1) with different $a$. (a) $a = 3 : 31$, $b = 1$, $c = 4$. (b) $a = 3 : 34$, $b = 1$, $c = 4$. (c) $a = 3 : 36$, $b = 1$, $c = 4$. (d) $a = 3 : 40$, $b = 1$, $c = 4$.

**Figure 5:** Lyapunov exponents spectrum of 3D jerk system (1) for $b = 1$ and $c = 4$. 

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**Complexity**
The parameter uncertainties as (8) into (10), the following proof is given.

Assumption 1. Assumes that positive $\delta_i$ $(i = 1, 2, 3 \ldots)$ makes the parameter uncertainties as $|\Delta_i| \leq \delta_i$ $(i = 1, 2, 3 \ldots)$.

**Theorem 1.** Under the action of robust feedback controller (10), the master system (6) and slave system (7) can achieve synchronization in finite time $T = 2^{-\sigma} V'(0)/\sigma$, where $\sigma = (1 - \gamma)/2$.

**Proof.** Construct the Lyapunov function $V(t) = 1/2 \sum_{i=1}^{3} e_i^2$. Differentiating $V(t)$ gives $\dot{V}(t) = 3 \sum_{i=1}^{3} e_i \dot{e}_i$. Substituting (8) into $\dot{V}(t)$, we can get:

$$\dot{V}(t) = e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3$$

$$= e_1 (e_2 + u_1 (t)) + e_2 (e_3 + u_2 (t))$$

$$+ e_3 (-a e_1 - b e_3 - c e_3 + y_1 y_2 (b + \Delta_3) - x_1 x_3 (b + \Delta_3))$$

$$+ y_1^2 - \Delta_4 y_1 - \Delta_5 y_2 - \Delta_6 y_3 + \Delta_1 x_1 + \Delta_2 x_2 + \Delta_3 x_3 + u_3 (t).$$

(11)

Then, substituting the controller in (10) into equation (11), we have

$$\dot{V}(t) = e_1 (e_2 + a e_3 - e_2 - e_1^2) + e_2 (e_3 + b e_3 - e_3 - e_2^2)$$

$$+ e_3 (-a e_1 - b e_3 - c e_3 + y_1 y_2 (b + \Delta_3) - x_1 x_3 (b + \Delta_3))$$

$$+ y_1^2 - \Delta_4 y_1 - \Delta_5 y_2 - \Delta_6 y_3 + \Delta_1 x_1 + \Delta_2 x_2 + \Delta_3 x_3 + u_3 (t).$$

(12)

Furthermore, simplifying equation (12) can be obtained as follows:

$$\dot{V}(t) = -e_1^2 + e_2^2 - e_3^2 + e_1 e_2 + e_1 e_3 - e_2 e_3$$

$$- e_3 y_1 y_2 \Delta_3 - \delta_1 \|y_1\| e_3$$

$$- e_3 y_1 y_3 - \Delta_4 y_1 + \delta_2 \|y_2\| e_3$$

$$+ e_3 x_1 \Delta_1 - \delta_3 \|x_1\| e_3$$

$$- e_3 x_3 \Delta_3 - \delta_4 \|x_3\| e_3.$$

(13)

where, based on Assumption 1, we can get

$$\dot{V}(t) \leq -e_1^{y_1+1} - e_2^{y_2+1} - e_3^{y_3+1}$$

(14)

and then we can know that

$$-e_1^{y_1+1} - e_2^{y_2+1} - e_3^{y_3+1} = -2^{y_{1/2}} \left[ \left( \frac{e_1^2}{2} \right)^{y_{1/2}} + \left( \frac{e_1^2}{2} \right)^{y_{1/2}} + \left( \frac{e_1^2}{2} \right)^{y_{1/2}} \right]$$

$$\leq -2^{y_{1/2}} \left( \frac{e_1^2}{2} + \frac{e_1^2}{2} + \frac{e_1^2}{2} \right)^{y_{1/2}} = -2^{y_{1/2}} V^{y_{1/2}}(t).$$

(15)
Thus, we have
\[ \dot{V}(t) \leq -2^{r+1/2}V^{r+1/2}(t). \] (16)

According to Lemma 1, we have
\[ T = \frac{V^{1-r+1/2}(0)}{2^{r+1/2}(1-(1+1/2))} = \frac{2^{1-r}V^{1-r/2}(0)}{1-y}. \] (17)

Let \( \sigma = 1 - y/2; \) there are \( T = 2^{\sigma-1}V^\sigma(0)/\sigma; \) when \( t \geq T, \)
e \rightarrow 0 (i = 1, 2, 3).

### 4. Circuit Implementation and Numerical Simulation

In this section, in order to verify the correctness of the 3D jerk system (1), the analog circuit is designed, and to illustrate the control effect of the robust feedback controller, the numerical simulation is presented.

#### 4.1. Circuit Implementation

We use LM741 operational amplifiers, AD633 analog multipliers, resistors, and capacitors to design analog circuit, where the gain of multiplier AD633 is 0.1, and the power voltage of operational amplifier LM741 is \( E = \pm 15V, \) and its output saturated voltages \( V_{\text{sat}} = \pm 13.5V. \) According to the 3D jerk system (1), a chaotic circuit with hidden attractor is designed, as shown in Figure 8. Due to the limitation of LM741 and AD633 working voltages, the output voltage of the system is reduced to 1/10 of the original. Compress them according to 10:1, and carry on time-scale transformation; let \( \tau = \tau_0 t, \) where \( \tau_0 = 100. \) Then the 3D jerk system (1) can be expressed as

\[
\begin{aligned}
&x_1 = -100(-x_2), \\
x_2 = -100(-x_3), \\
x_3 = -340x_1 - 100x_2 - 400x_3 - 100(-x_1)x_2 - 100(-x_2)x_2.
\end{aligned}
\] (18)

Applying Kirchhoff’s law and from Figure 8, the corresponding circuit equations are written as

\[
\begin{align*}
\frac{dx_1}{dt} &= -\frac{1}{R_1C_1}(-x_2), \\
\frac{dx_2}{dt} &= -\frac{1}{R_4C_2}(-x_3), \\
\frac{dx_3}{dt} &= -\frac{1}{R_3C_3}x_1 - \frac{1}{R_9C_3}x_2 - \frac{1}{R_{10}C_3}x_3 - \frac{1}{10R_6C_3}(-x_1)x_2 - \frac{1}{10R_{11}C_3}(-x_2)x_2.
\end{align*}
\] (19)

where \( x_1, x_2, \) and \( x_3 \) are related to the voltages on capacitators \( C_1, C_2, \) and \( C_3, \) respectively. The capacitors \( C_1 = C_2 = C_3 = 100 \text{nF}. \) Comparing equation (18) with equation (19) and making the corresponding coefficients be equal, one can obtain the values of resistances as \( R_1 = R_4 = R_7 = R_9 = R_{11} = 100k\Omega, R_8 = 29.41k\Omega, \) and \( R_{10} = 25k\Omega. \) The other resistances are \( R_2 = R_3 = R_5 = R_6 = R_{12} = R_{13} = 10k\Omega. \) Under the circuit parameters mentioned above, one can get a phase diagram on the digital oscilloscope as shown in Figure 9.

#### 4.2. Numerical Simulation of Robust Feedback Controller

For the master system (6) and slave system (7), the parameters are set as \( a = 3.4, b = 1, \) and \( c = 4, \) and when \( \Delta_1 = 0(i = 1, 2, 3, 4, 5, 6), \) the master-slave systems (6) and (7) are in chaotic state. The parameter uncertainties are taken as \( \Delta_1 = 0.01 \sin(x_3), \Delta_2 = 0.02 \cos(x_3), \Delta_3 = 0.01 \sin(y_3), \Delta_4 = 0.02 \sin(y_3), \Delta_5 = 0.01 \sin(y_3), \) and \( \Delta_6 = 0.01 \sin(y_3). \) So \( \delta_1 = 0.01, \delta_2 = 0.02, \delta_3 = 0.01, \delta_4 = 0.02, \delta_5 = 0.01, \) and \( \delta_6 = 0.01. \) The control parameters are chosen as \( y_1 = 0.6, y_2 = 0.8, \) and \( y_3 = 0.9. \) The system initial values are chosen as \( x_1 = -2, x_2 = 0, x_3 = 2.4 \) and \( y_1 = 7, y_2 = -3, y_3 = 7.4. \) Under the robust feedback controller (10), the synchronization result is shown in Figure 10.

In Figure 10, the states of master system \( x_i (i = 1, 2, 3) \) and slave system \( y_i (i = 1, 2, 3) \) tend to converge, and synchronization is realized in finite time, and the states \( e_1, e_2, \) and \( e_3, \) of the error system (8) are shown in Figure 11. From the above simulation results that the error system (8) realizes synchronization in finite time, the phase orbit of the synchronization error system gradually converges to the origin, the reliability of the synchronization method is verified, and the design of the robust feedback controller (10) meets the requirements.
Figure 8: The circuit principle diagram of the 3D jerk system (1).

Figure 9: The screenshots of the digital oscilloscope with the 3D jerk system (1) in (a) $x_1 - x_2$ plane, (b) $x_2 - x_3$ plane, and (c) $x_1 - x_3$ plane.
5. Conclusion

In this work, the dynamical behaviors of the 3D jerk system with hidden attractor are analyzed by numerical calculation. Furthermore, an analog circuit is designed for implementing and testing our system model. A very good qualitative consistency is shown between the circuit experimental results and the simulations of the theoretical model. For 3D jerk system synchronization and control, a finite-time robust feedback controller was proposed, and the synchronization of 3D jerk chaotic system with hidden attractor was realized in finite time. Finally, mathematical simulation result demonstrated that the performances of the proposed controller are excellent. However, there are still complex dynamics and the topological structure of this system should be exploited, and better synchronization control methods should be found. In addition, the synchronization and control of the master-slave system can be realized through circuit implementation. These will be provided in future works.

Data Availability

The data that support the findings of this study are available within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

All authors contributed equally to this work.

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