BARYOGENESIS VIA LEPTOGENESIS IN SO(10) MODELS

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We discuss the baryogenesis via leptogenesis mechanism within the supersymmetric and nonsupersymmetric SO(10) models. We find that the nonsupersymmetric model, endowed with an intermediate scale, is generally favoured, unless some fine tuning occurs in the supersymmetric case.
I. INTRODUCTION

The origin of the baryon asymmetry in the universe (baryogenesis) is a much discussed topic [1]. A popular mechanism is the baryogenesis via leptogenesis [2], where the out-of-equilibrium decays of heavy right-handed Majorana neutrinos generate a lepton asymmetry which is partially transformed into a baryon asymmetry by electroweak sphaleron processes [3]. A minimal framework required is the standard model with heavy right-handed neutrinos, but the mechanism is active also within unified theories [4], and in particular the \( SO(10) \) model [5], which naturally contains heavy right-handed neutrinos. The light left-handed Majorana neutrinos are obtained by means of the seesaw mechanism [6].

The baryogenesis via leptogenesis has been studied in many papers [7–12]. In this letter we address a specific issue, not explicitly considered before, namely the possibility to generate the baryon asymmetry within the nonsupersymmetric \( SO(10) \) model, characterized by the presence of an intermediate mass scale where both lepton number conservation and quark-lepton symmetry are broken. We match the result to the supersymmetric case, where the two effects occur at the unification scale, and no intermediate scale is present.

In section II we summarize the relevant formulas of the baryogenesis via leptogenesis mechanism. In section III we calculate the baryon asymmetry within the \( SO(10) \) model by using two distinct forms for the mass matrices of the right-handed neutrino, which correspond to the supersymmetric and nonsupersymmetric cases, respectively. In section IV we give our conclusions.

II. THE BARYOGENESIS VIA LEPTOGENESIS MECHANISM

As proposed in Ref. [2], a baryon asymmetry can be generated from a lepton asymmetry. We define the baryon asymmetry as [13]

\[
Y_B = \frac{n_B - n_{\bar{B}}}{s} = \frac{n_B - n_{\bar{B}}}{7n_\gamma} = \frac{\eta}{7}, \tag{1}
\]
where \( n_{B,\gamma} \) are number densities, \( s \) is the entropy density and \( \eta \) is the baryon-to-photon ratio. The range of \( Y_B \) required for a successful description of nucleosynthesis is \( Y_B = 10^{-11} - 10^{-10} \), see for example Ref. [14]. In the baryogenesis via leptogenesis framework, the baryon asymmetry is related to the lepton asymmetry [15],

\[
Y_B = \frac{a}{a - 1} Y_L, \quad a = \frac{8N_f + 4N_H}{22N_f + 13N_H},
\]

where \( N_f \) is the number of families and \( N_H \) the number of light Higgs doublets. For \( N_f = 3 \) and \( N_H = 1 \) or 2 (standard or supersymmetric case), it is \( a \approx 1/3 \) and \( Y_B \approx -Y_L/2 \).

The lepton asymmetry is written as [10]

\[
Y_L = d \frac{\epsilon_1}{g^*}
\]

where \( \epsilon_1 \) is a CP-violating asymmetry produced by the decay of the lightest heavy neutrino, \( d \) is a dilution factor which takes into account the washout effect of inverse decay and lepton number violating scattering, and \( g^* = 106.75 \) in the standard case or 228.75 in the supersymmetric case is the number of light degrees of freedom in the theory.

In the standard case \( \epsilon_1 \) is given by [17]

\[
\epsilon_1 = \frac{1}{8\pi v^2 (M_D^\dagger M_D)_{11}} \sum_{j=2,3} \text{Im}[(M_D^\dagger M_D)_{j1}]^2 f\left(\frac{M_j^2}{M_1^2}\right),
\]

where \( M_D \) is the Dirac neutrino mass matrix when the Majorana neutrino mass matrix \( M_R \) is diagonalized with eigenvalues \( M_i \) \((i = 1, 2, 3)\), \( v = 175 \) GeV is the VEV of the standard model Higgs doublet, and

\[
f(x) = \sqrt{x} \left[ 1 - (1 + x) \ln \frac{1 + x}{x} - \frac{1}{x - 1} \right].
\]

In the supersymmetric case \( v \to v \sin \beta \),

\[
f(x) = -\sqrt{x} \left[ \ln \frac{1 + x}{x} + \frac{2}{x - 1} \right],
\]

and a factor 4 is included in \( \epsilon_1 \), due to more decay channels. For a hierarchical spectrum of heavy neutrinos we have \( f \sim M_1/M_j \). The formula (4) is obtained by
calculating the interference between the tree level and one loop decay amplitudes of the lightest heavy neutrino, and includes vertex \[2\] and self-energy \[18\] corrections. The latter are dominant if \(M_1\) and \(M_j\) are nearly equal, in which case an enhancement of the asymmetry may occur.

The dilution factor should be obtained by solving the Boltzmann equations. We use an approximate solution \[19,4,20,10\]:

\[
d = \begin{cases} 
    0.24 \frac{1}{(\ln k)^{0.6}} & \text{for } k \gtrsim 10, \\
    \frac{1}{2k}, & \text{d} = 1 \end{cases}
\]

for \(k \gtrsim 10\), and

\[
d = \frac{1}{2k}, \quad d = 1
\]

for \(1 \lesssim k \lesssim 10\), \(0 \lesssim k \lesssim 1\), respectively, where the parameter \(k\) is

\[
k = \frac{M_P}{1.7 \times 32 \pi \sqrt{g^*}} \frac{(M_D M_D)^{11}}{M_1},
\]

and \(M_P\) is the Planck mass.

The baryon asymmetry depends on both the Dirac and the Majorana mass matrices of neutrinos. In the following section we adopt general approximate forms for these matrices and study the implications for leptogenesis.

### III. LEPTOGENESIS IN SO(10) MODELS

In unified \(SO(10)\) models, \(M_R\) is generated from the Yukawa coupling of right-handed neutrinos with the Higgs field that breaks the unification or the intermediate symmetry down to the standard model, see for example Ref. \[21\]. When such a Higgs field takes a VEV, the right-handed neutrinos get a Majorana mass. This happens because lepton number is broken at that scale. Therefore, in the supersymmetric case the mass scale of the right-handed neutrino is similar to the unification scale, \(M_R \sim M_U \sim 10^{16} \text{ GeV}\), while in the nonsupersymmetric case the scale of \(M_R\) is about the intermediate scale, \(M_R \sim M_I \sim 10^{11} \text{ GeV}\) \[23\]. On the other hand, \(M_U\) or \(M_I\) are also the scale of the quark-lepton symmetry, that is the gauge subgroup
$SU(4) \times SU(2)_L \times SU(2)_R$, where the $SU(4)$ component includes the lepton number as fourth color $[23]$. This framework gives $M_d \sim M_e$ and $M_u \sim M_{\nu}$, where $M_{d,u}$ are quark mass matrices, $M_e$ is the charged lepton mass matrix and $M_{\nu}$ is the Dirac neutrino mass matrix. The light (effective) neutrino mass matrix $M_L$ is obtained by means of the seesaw formula

$$M_L = -M_{\nu} M_R^{-1} M_{\nu}. \quad (8)$$

Since quark mixing is small, that is quark mass matrices are nearly diagonal, from quark-lepton symmetry we get nearly diagonal $M_e$ and $M_{\nu}$.

By inverting formula (8) with respect to $M_R$, in Ref. $[24]$ the approximate structures leading to the unification and intermediate scales were identified. The matrix $M_R$ should be nearly diagonal or nearly offdiagonal, respectively, with a strong hierarchy in the former case and a more moderate hierarchy in the latter. These two situations are similar to those discussed in Ref. $[25]$ in order to get a seesaw enhancement of lepton mixing. The condition $M_{R33} \simeq 0$ has been further discussed in Ref. $[26]$ (see also Ref. $[27]$). We assume large mixing of solar and atmospheric neutrinos. For the Dirac neutrino mass matrix we take

$$M_{\nu} = \frac{m_\tau}{m_b} \text{diag}(m_u, m_c, m_t), \quad (9)$$

where the ratio $m_\tau/m_b$ takes into account the running of quark masses with respect to lepton masses. For the mass matrix of right-handed neutrinos we take in the supersymmetric case $[24]$

$$M_R \simeq \left(\frac{m_\tau}{m_b}\right)^2 \left(\begin{array}{ccc}
m_u^2 & -\frac{m_u m_t}{\sqrt{2}} & \frac{m_u m_t}{\sqrt{2}} \\
-\frac{m_u m_t}{\sqrt{2}} & m_c^2 & -\frac{m_c m_t}{2} \\
\frac{m_u m_t}{\sqrt{2}} & -\frac{m_c m_t}{2} & m_t^2
\end{array}\right) \frac{1}{2m_1}, \quad (10)$$

where $m_1 \sim 10^{-3}$ eV is the mass of the lightest effective neutrino, and in the nonsupersymmetric case $[24, 26]$.
\[ M_R \simeq \left( \frac{m_\tau}{m_b} \right)^2 \begin{pmatrix} \frac{m_u^2}{\sqrt{2}} & -\frac{m_u m_c}{\sqrt{2}} & \frac{m_u m_t}{\sqrt{2}} \\ -\frac{m_u m_c}{\sqrt{2}} & 2U_{e3} m_e^2 & x \\ \frac{m_u m_t}{\sqrt{2}} & x & 0 \end{pmatrix} \begin{pmatrix} 1 \\ m_1 \end{pmatrix}, \]  

(11)

where \( U_{e3} \simeq 0.1 \) is the element 1-3 in the lepton mixing matrix and \( M_{R23} \) takes values from \( 10^{10} \) to \( 10^{12} \) GeV. Note that \( M_{R13} \sim 10^{11} \) GeV. We will discuss the different implications for leptogenesis of matrices (10) and (11), which correspond to distinct models. Since our main interest is the general result, especially the difference between the supersymmetric and nonsupersymmetric cases, we do not include phases and drop the imaginary part in Eqn.(4). We diagonalize \( M_R \) by the rotation \( U_R^T M_R U_R \), so that \( M_D = M_\nu U_R \), and insert both \( M_i \) and \( M_D \) is Eqn.(4). In this way the baryon asymmetry can be determined.

In the supersymmetric model we find \( Y_B \sim 10^{-15} - 10^{-14} \), where the range corresponds to moderate changes in \( M_R \). We use \( \sin \beta \simeq 1 \) for quark-lepton Yukawa unification of the third generation \[28\], although the baryon asymmetry depends very weakly on this parameter. The value of \( k \) is slightly larger than 1. This case is similar to the one studied in Ref. \[10\], where a sufficient amount of asymmetry is obtained only by fine tuning of some neutrino parameters.

In the nonsupersymmetric model, the baryon asymmetry is around the required order of magnitude \( Y_B \sim 10^{-11} \). In Fig. 1 we plot the result as a function of \( \log_{10} M_{R23} \). Here we have \( k \sim 10 - 10^3 \). For lower values of \( M_{R23} \) the baryon asymmetry undergoes a moderate increase and for higher values it drops towards the supersymmetric result.
IV. CONCLUSION

The main result of the present paper is that the nonsupersymmetric SO(10) model is favoured for leptogenesis with respect to the supersymmetric model. In fact, in the latter case a sufficient amount of baryon asymmetry can be obtained only by means of fine tuning, while the nonsupersymmetric model gives a baryon asymmetry of the same order as required.

By matching the present result with previous work [10,11], we realize that the supersymmetric model with full quark-lepton symmetry generally gives a too small asymmetry [10]. This can be avoided within the SU(5) model, where $M_\nu$ is no more related to $M_u$, by taking a moderate hierarchy in $M_\nu$ [11], or in the nonsupersymmetric model by means of a roughly offdiagonal $M_R$, corresponding to a moderate hierarchy of its eigenvalues.
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