Analytical Solution of Thermal Effects in Thin Disk Laser

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Abstract
In this work, the analytical solution of thermal effects in thin disk laser was derived using an integral transform method. Good agreements were found between the result of this work and other solutions for the same problem. The temperature distribution and its effect on laser crystal were obtained. It was found that increasing cooling of the thin disk leads to decrease in temperature difference across the crystal, which decreases the generated stress and strain and subsequently enhance beam quality (i.e. reduce optical path difference) also it was found that as cooling is increased the allowable pumping power (before fracture could occur) is increased too. It was found also that, reduce disk thickness has the same effect of increasing cooling of the disk due to the fact that as thickness decrease the heat could dissipate more efficiently.

1. Introduction

Thin disk laser is a diode-pumped laser, which could produce high output power with high efficiency and good beam quality [1-3]. More than 10 KW output power was achieved after its invention in 1993 and since then it attracts much attention due to the aforementioned specifications and compatibleness [4-7]. It is a composite of a crystal disk with a thickness of 100-200 \(\mu\)m, which is much, less than its diameter. Its faces are high reflectivity (HR) and the un-pumped face is fixed in perfect contact with a heat sink, coated for both the laser and the pump wavelength, see Fig. 1.

Many works devoted to studying temp., stress and fracture stress of this type of laser. Shang J. et al [8] used a finite element method to obtain temperature, stress, strain, optical path difference and thermal lensing in the thin crystal. Vahid S. et al [9] used Monte Carlo ray-tracing code along with the ANSYS package to predict the optical and structural behavior in end-pumped CW Yb: YAG disk lasers and the dependence of optical phase distortion on convection heat transfer. Guangzhi et al [10] derived an analytical model of the thermal effect and optical path difference (OPD) of a thin-disk laser to obtain the temperature distribution, stress, and strain using a super-Gaussian form pumping spot. In addition, Guangzhi et al [11] used a plane wave model with non-uniform temperature distribution in the thin-disk crystal to study the dynamic behavior of an end-pumped Yb: YAG thin-disk laser.

In this work, an analytical solution of a thin disk laser was derived using an integral transform method. The result of this work was compared with other works that used numerical and analytical solutions and good agreement was found. The temp. distribution and its effect on laser crystal and beam quality (such as optical path difference) were obtained. It was found that increasing cooling leads to decrease temperature difference across the crystal, which decreases the generated stress and subsequently, could increase the allowable pumping power before fracture could occur and enhance beam quality. It was also found that decreasing crystal thickness could extract heat more efficiently which reduces the thermal effect in the crystal.
2. Theory

The schematic diagram of the thin disk laser is shown in fig 1. The heat equation of the axis-symmetry form could simulate the physical domain of the micro-thickness crystal that has a cylindrical shape. Regarding constant thermal conductivity then the transient axis-symmetry form of heat transfer can be written as [12];

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{\alpha c \partial T}{\partial t}
\]

(1)

Where \( \rho \), T, c, \( \dot{q} \), k is the density in \((\text{kg/m}^3)\), temperature in \((\degree\text{K})\), the specific heat in \((\text{J/kg} \degree\text{K})\), the heat generation in \((\text{in W/m}^3)\) and the thermal conductivity in \((\text{W/m} \degree\text{K})\), respectively, also \( r \) and \( z \) are radial and the longitudinal coordinates in \((\text{m})\). Figure 1, shows YP: YAG thin disk laser with a back-surface cooling of 200 \( \mu \text{m} \) thickness and 4 mm in radius. The thin disk laser was pumped by uniform pumping power at the right end face, the pump light is reflected and propagates back to the opposite direction by a high reflection film.

Figure 1. Schematic diagram of thin disk laser

This design is suitable for quasi-three-level systems such as Yb: YAG due to its low working temperature and its necessarily high intense pump power for efficient operation. Therefore, it was chosen as the preferred active medium for a very thin disk. The heat generation density \( \dot{q} \) can be obtained by dividing the generated heat by the volume at which it was generated, assuming the power attenuation in omitted when the heat loading deposition is uniform[13], then

\[
\dot{q} = \frac{H_F \rho_{\text{abs}}}{\pi r^2 t}
\]

(2)

Where \( H_F \) the fraction of absorbing power that transforms to heat due to stokes shift and absorption of pumping and laser \( \approx 14.6\% \) for YP:YAG [14]. Fig 2 summarized boundary conditions, which can be written as:

Fig.2. Schematic diagram of thin disk laser crystal
\[ k \frac{\partial T(r,z,\varphi)}{\partial z} \bigg|_{z=0} = h_s(T_\infty - T) \quad \text{on } S_1 \quad \text{(3a)} \]
\[ k \frac{\partial T(r,z,\varphi)}{\partial z} \bigg|_{z=t} = h_s(T_\infty - T) \quad \text{on } S_2 \quad \text{(3b)} \]
\[ k \frac{\partial T(r,z,\varphi)}{\partial r} \bigg|_{r=D/2} = 0 \quad \text{on } S_3 \quad \text{(3c)} \]

where \( h_s \) is the right face natural convection heat transfer coefficient = 27.5 W/m²K, the radius of the crystal \( r_1 = D/2 \) is 4 mm, the radius of top hat pumping \( (r_p) \) is 3 mm. The back-face of the crystal is assumed to be subjected to very high convection heat transfer \( h_e = 10^5 \) W/m²K. The pumping power is assumed to equal to 1000 W.

Even YP: YAG was used in this work, many works devoted to using other laser material as thin-disk such as Nd: YAG, Nd: YVO4, Tm:YAG and Ti:sapphire [15-18].

The solution of the heat equation in laser crystal permits the prediction of the temperature distribution, which is the first step in determining the thermal effects in the thin disk laser. The transient temperature distribution through the laser rod can be determined by solving the axis-symmetry heat equation as shown below.

2.1 Analytical model

Figure 1 illustrates a YP: YAG thin disk laser with a back-surface cooling. The thin disk crystal was pumped from the centre of the right end face by a diode laser with a top-hat pumping profile. At the left end face, the pump light is reflected and propagates back to the opposite direction by a high reflection film and so on. \( P_{ab} \) is the absorbed pumped power in the laser crystal (W). Then using an integral transform method the solution of eq. (1) can be expressed as [19]:

\[ \theta(r, z) = T - T_\infty = \sum_{\beta_m} \sum_{\eta_p} \int_{0}^{\infty} \frac{J_0(\beta_m \eta_p r z)}{N_\beta N_\eta} e^{-a(\beta \eta_p + \eta_p) t} \left\{ \frac{\int_{0}^{\infty} e^{a(\beta \eta_p + \eta_p) t} g \, dt}{k} \right\} \frac{1}{\beta_m \eta_p} \right] e^{-a(\beta \eta_p + \eta_p) t} g \, dt \right\} \quad \text{(4)} \]

Where \( J_0 \) and \( J_1 \) are the Bessel functions. \( \beta_m \) and \( \eta_p \) are the roots of equations, which can be obtained by using the boundary conditions mentioned above [19]:

\[ \beta_m J_0(\beta_m r_2) - \frac{h_e}{k} J_1(\beta_m r_2) = 0 \quad \text{(5a)} \]
\[ \eta_p \tan \eta_p L + \frac{h_e}{k} = 0 \quad \text{(5b)} \]

Sequential values of the roots (i.e. \( \beta_m \) and \( \eta_p \) where \( m \) and \( p \) vary from 1 to infinity) were obtained by using the Newton Raphson method. It is to be noted that thirty roots were taken in the solution to ensure high accuracy.

\( H \) is equal to \( h_e/k \), and \( h_e \) is the back-face heat transfer coefficient and is assumed to be \( 10^5 \) W/m²K.

The Norms of the differential equation can be expressed as [19]:

\[ \frac{1}{N(\beta_m)} = \frac{1}{N_\beta} = \frac{2k^2 \beta_m^4}{\pi^2 J_0(\beta_m r_2)(h_e^2 + k^2 \beta_m^2)} \quad \text{(6)} \]

and:

\[ \text{IOP Publishing} \]

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\[ \frac{1}{N(\eta_p)} = \frac{1}{N_p} = \frac{2(\eta_p^2 + H^2)}{I(\eta_p^2 + H^2)} \]  

(7)

Then the solution can be written as:

\[ \theta(r, z) = T - T_\infty = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{J_0(\beta_m r) \cos(\eta_p (1-z))}{k N_m N_p (\beta_m + \eta_p^2) \beta_m} \frac{e^{-a(\beta_m + \eta_p^2)t}}{g} \left[ e^{a(\beta_m + \eta_p^2)t} \right] \]  

(8)

For a top hat pumping profile, the function \( g \) in the above equation can be expressed as[19]:

\[ g = \int_0^r r J_0(\beta_m r) \cos(\eta_p (1-z)) q \, dr \, dz \]  

(10)

Then carrying the integration lead to

\[ g = \frac{H f_p abs}{\pi r^2 i} \rho_s \frac{\sin(\eta_p l)}{\beta_m \eta_p} \frac{1}{\beta_m \eta_p} J_1(\beta_m r_0) \sin(\eta_p l) \]  

(11)

Then for a steady state where \( t=\infty \), the main solution for eq. (9) can be expressed as:

\[ \theta(r, z) = T - T_\infty = \frac{H f_p abs}{\pi r^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{J_0(\beta_m r) \cos(\eta_p (1-z))}{k N_m N_p (\beta_m + \eta_p^2) \beta_m \eta_p^2 i^2} \]  

(12)

Then the axial average temperature can be obtained based on the rule of finding average mathematically, which can be written as:

\[ \langle \theta(r) \rangle = \int_0^r \theta(r, z) \, dz \]  

(13)

\[ \langle \theta(r) \rangle = \frac{1}{r} \int_0^r \theta(r, z) \, dr \]  

(14)

This equation is helpful in obtaining radial (\( \sigma_r \)) and tangent (hoop) stress(\( \sigma_\theta \)) assume the axial stress is assumed zero due to the fact that the thickness of the disk crystal is much smaller than its radius then [20]:

\[ \langle \sigma_r(r) \rangle = aE \left( \frac{1}{r^4} \int_0^r \langle \theta(r) \rangle \, r \, dr - \frac{1}{r^2} \int_0^r \langle \theta(r) \rangle \, r \, dr \right) \]  

(15)

\[ \langle \sigma_\theta(r) \rangle = aE \left( \frac{1}{r^4} \int_0^r \langle \theta(r) \rangle \, r \, dr + \frac{1}{r^2} \int_0^r \langle \theta(r) \rangle \, r \, dr - \langle \theta(r) \rangle \right) \]  

(16)

Thus for average radial and hoop stress, eqs. (15,16) can be expressed as[20]:

\[ \langle \sigma_r(r) \rangle = aE \frac{H f_p abs}{\pi r^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin^2(\eta_p l) J_1(\beta_m r_0)}{k N_m N_p (\beta_m + \eta_p^2) \beta_m \eta_p^2 i^2} \left[ J_1(\beta_m r) \frac{\beta_m}{\beta_m^2} - J_1(\beta_m r_0) \frac{\beta_m}{\beta_m^2} \right] \]  

(17)

\[ \text{but } J_1(\beta_m r) = \frac{J_0(\beta_m r)}{2} + \frac{J_2(\beta_m r)}{2} \]  

then

\[ \langle \sigma_r(r) \rangle = aE \frac{H f_p abs}{\pi r^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin^2(\eta_p l) J_1(\beta_m r_0)}{k N_m N_p (\beta_m + \eta_p^2) \beta_m \eta_p^2 i^2} \left[ J_1(\beta_m r) \frac{\beta_m}{\beta_m^2} - J_1(\beta_m r_0) \frac{\beta_m}{\beta_m^2} + J_2(\beta_m r_0) \frac{1}{2} - J_2(\beta_m r_0) \frac{1}{2} \right] \]  

(18)

\[ \langle \sigma_\theta(r) \rangle = aE \frac{H f_p abs}{\pi r^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin^2(\eta_p l) J_1(\beta_m r_0)}{k N_m N_p (\beta_m + \eta_p^2) \beta_m \eta_p^2 i^2} \left[ J_1(\beta_m r) \frac{\beta_m}{\beta_m^2} + J_1(\beta_m r_0) \frac{1}{2} - J_0(\beta_m r_0) \right] \]  

(19)
\[
\langle \sigma_\theta (r) \rangle = aE \frac{H_f p_{abs}}{\pi r_o} \sum_p=1^n \sum_{m=1}^{m_0} \frac{\sin^2(\eta_p)}{2} \sum_{l=1}^{l_0} \frac{1}{kN_{m\mu}(\beta_{m\mu})^2} \left[ \frac{I_{e}(\beta_{m\mu}r^2)}{\rho m^2} + \frac{I_{o}(\beta_{m\mu}r^2)+I_{i}(\beta_{m\mu}r^2)}{2} \right] \]

By using plain strain approximation the strain can be written as;
\[
\langle \varepsilon_r \rangle = -\frac{1}{E} \left[ \langle \sigma_r \rangle - \nu \langle \sigma_\theta \rangle \right] + \alpha(\theta) \quad (21)
\]
\[
\langle \varepsilon_\theta \rangle = \frac{1}{E} \left[ \langle \sigma_\theta \rangle - \nu \langle \sigma_r \rangle \right] + \alpha(\theta) \quad (22)
\]
\[
\langle \varepsilon_z \rangle = -\frac{1}{E} \left[ \langle \sigma_z \rangle - \nu \langle \sigma_\theta \rangle \right] + \alpha(\theta) \quad (23)
\]

It is assumed that in the material , all shear strain is zero and the allowable tensile strength of YP:YAG is 175 MPa [22]. The maximum shear stress is used to predict failure, which is known as Tresca failure criterion then for plain strain approximation the Tresca stress is [23];
\[
\sigma_T = |\sigma_\theta - \sigma_r| \quad (24)
\]

2.2. Optical Path Difference in the Thin Disk Crystal

The temp. distribution, stress, strain and the bending deformation of the thin disk lead to (OPD), which acts as a curved mirror to the beam propagating. The effect of the thermal lens mainly depends on the thermal expansion and the variation of the refraction index due to thermal gradient and thermally induced birefringence and crystal deformation. These factors can be written as [24]:
\[
OPD(r) = 2 \int_0^1 \frac{\partial n}{\partial T} [T(r,z) - T_o] \cdot [1 + \varepsilon_z(r,z)] dz + \int_0^1 \Delta n_x(r,z) \cdot [1 + \varepsilon_x(r,z)] dz \\
+ \int_0^1 (n_o - 1) \cdot \varepsilon_x(r,z) dz - z_o(r) \quad (25)
\]

Where \( \Delta n_x(r,z) = \Delta n_r + \Delta n_\theta \) is the change of refractive index due to strain which can be obtained according to Neumann–Curie theorem, see reference [8,25]. It is assumed that the back-side laser crystal is soldered with a water-cooled heat sink, which is helpful in avoiding bending deformations [26]. This is especially true for Yb:YAG where the heat sink has enough deformation resistance and can be regarded as an ideal rigid body then the crystal bending can be omitted, so the last term in eq. (24) can be ignored[8,26].

3. Result and discussion

The axis-symmetry heat transfer equation was solved using the integral transfer method. A computer program was written in VB6 program to obtain the temp. distribution in thin disk crystal using eq.(12). Firstly, the program is designed to obtain thirty roots of equations (5a,5b) using the boundary conditions then calculating the Norms as written in eqs.(6,7), incorporating these values in eq.(12) then the temp. distribution thorough the thin disk laser could be obtained. The average temp. through laser thickness could be obtained using eq.(14) also the average stresses and strains thought the crystal can be obtained using eqs (17-23). Based on the above information, OPD can be calculated. The thin disk is attached to the heat sink where the heat transfer coefficient can be taken as large as \(10^7\) W/m².K. A natural convection is assumed at the front face of the disk. Due to small disk thickness and moderate temperature at the edge, an insulated circumferential boundary is assumed and because of the thinner disk and uniform pumping power, the heat generation inside crystal was assumed uniform, which simplified the solution. The obtained temp. distributions through the crystal were drawn using the Tecplot program as shown in fig 2. The hoop stress is also shown in fig 3.
These results are in very good agreement to the temp. distribution, stress and strain obtained by references [8, 10] where the maximum difference is not exceeding 4%.

An experiment measurement for OPD was carried in reference [8] for pumping Yb:Yag thin disk using the interference of the collimated He–Ne laser beam. For the pumping power of about 1000W power, pumping radius of 3 mm, the outside radius is 5 mm, the OPD of the disk crystal was found to be about 316.4 nm. Using the model of this work, the OPD was found to be 315 nm (i.e an accuracy of 0.4%).

![Figure 2](image-url)

**Figure 2.** Temperature distribution through laser crystal at pumping power of 1000 W, left face convection heat transfer coefficient $10^5$ W/m$^2$·K and 0.2 mm thickness.
Figure 3. Hoop stress distribution through laser crystal at the pumping power of 1000 W, the left face heat transfer coefficient is $10^5$ W/m$^2$.K and disk thickness of 0.2 mm.

The average temperature is obtained as the convection heat transfer coefficient is varied from $10^3$ to $10^5$ W/m$^2$.K which is shown in fig 4. It is clear that as the heat transfer coefficient reduced from $10^5$ W/m$^2$.K to $10^3$ W/m$^2$.K, the average temperature is increased. This is explainable since as the heat transfer coefficient is reduced then more heat is stored in the crystal rather than dissipated by the heat sink, which results in an increase in the temperature distribution as convection heat transfer is decreased.
The resulting Tresca stress is shown in fig 5. As mentioned earlier this stress indicates the stress at which failure could occur. In fig 5, it is clear that increasing the heat transfer coefficient results in decreasing in Tresca stress, which means that as the convection heat transfer coefficient increased more pumping power, could be pumped before failure could occur. The most effective parameters in calculating OPD is the axial strain. The variation of axial strain with a heat transfer coefficient is shown in fig 6. High convection heat transfer coefficient means more reduction in temperature distribution and its average value, which means less stress and subsequently less strain. So increasing the heat transfer coefficient results in decreasing axial strain and subsequently decrease in OPD, see fig 6, 7.

The effect of crystal thickness on temperature, strain and OPD is studied and it was found that decreasing length could decrease the average temperature through the laser crystal. Figure 8 shows the temperature profiles for three different thicknesses of Yb: YAG crystal. Increasing the crystal thickness naturally increases the temperature, stress and strain distribution that intensify thermal effects (see fig 8-11). This is because the heat is extracted more efficiently through thinner disks as thickness decreased.
Figure 5. Variation of average Tresca stress through thin laser crystal with different right face heat transfer coefficient, at pumping power of 1000 W and disk thickness of 0.2 mm.

Figure 6. Variation of average axial strain through thin laser crystal with different right face heat transfer coefficient at pumping power of 1000 W and disk thickness of 0.2 mm.
Figure 7. OPD through thin laser crystal with different left face convection heat transfer, the pumping power is 1000 W and a disk thickness of 0.2 mm.

Figure 8. The effect of increasing crystal thickness on crystal average temperature at the pumping power of 1000 W and a disk thickness of 0.2 mm, the heat transfer coefficient is $10^5$ W/m².K
Figure 9. Variation of Tresca stress with different thicknesses. at the pumping power of 1000 W, the heat transfer coefficient is $10^5$ W/m².K

Figure 10. Variation of average axial strain through thin laser crystal with different crystal thickness at the pumping power of 1000 W, the heat transfer coefficient is $10^5$ W/m².K
Figure 11. Variation of OPD through thin laser crystal with different crystal thickness at the pumping power of 1000 W, the heat transfer coefficient is $10^5$ W/m².K

It is noteworthy to mention that the program was tested to obtain the value of pumping power at which failure occurs under the above-mentioned condition. (i.e right face convection heat transfer coefficient $10^5$ W/m².K and 0.2 mm thickness). It was found that at absorbing power of 17.7 kW the Tresca stress reaches 175 MPa, which is known to be the failure stress. Decreasing the convection heat transfer coefficient to $10^3$ W/m².K could decrease the allowable power to about 16.3 kW. It is also to be noted that a test has been made to run the program at a thickness of 0.1 mm and it is found that the absorb power could be increase to about 37.6 kW before fracture stress could be reached.

4. Conclusions

An analytical solution for the thermal effects in thin disk crystal was derived using an integral transform method. The result of this work was compared with other works that used other methods and good agreement was found. By analyzing the result of this work, it was found that increasing convection heat transfer from the back-side of the crystal could, reduce the temperature distribution through the thin disk, which results in a decrease in the resulting strain, and subsequently reduce OPD. It is also found that increasing the heat transfer coefficient leads to increase in the possible allowable pumping power and enhances the beam quality by reducing OPD.

The effect of increasing the thickness of the disk was studied and it was found that increasing the disk thickness leads to increase in the temperature distribution and subsequently increase the resulting stress, strain and OPD which means that reducing disk thickness leads to increase in the possible allowable pumping power and can also enhance the beam quality by reducing OPD.

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