Emergence of $d \pm ip$-wave superconducting state at the edge of $d$-wave superconductors mediated by Andreev-bound-state-driven ferromagnetic fluctuations

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We propose a mechanism of spin-triplet superconductivity at the edge of $d$-wave superconductors. Recent theoretical research in $d$-wave superconductors predicted that strong ferromagnetic (FM) fluctuations are induced by large density of states due to edge Andreev bound states (ABS). Here, we construct the linearized gap equation for the edge-induced superconductivity, and perform a numerical study based on a large cluster Hubbard model with bulk $d$-wave superconducting (SC) gap. We find that ABS-induced strong FM fluctuations mediate the $d \pm ip$-wave SC state, in which the time-reversal symmetry is broken. The edge-induced $p$-wave transition temperature $T_{cp}$ is slightly lower than the bulk $d$-wave one $T_{cd}$, and the Majorana bound state may be created at the endpoint of the edge.

I. INTRODUCTION

In cuprate high-$T_c$ superconductors, spin fluctuations induce various kind of interesting phenomena. For example, $d$-wave superconductivity is mediated by the antiferromagnetic (AFM) fluctuations \[1–6\]. Non-Fermi-liquid transport phenomena such as $T$-linear resistivity, Curie-Weiss behavior of the Hall coefficient, and the modified Kohler rule between the magnetoresistance and Hall angle ($\Delta \rho / \rho_0 \propto (\sigma_{xy}/\sigma_{xx})^2$) are understood as the effects of strong AFM fluctuations on the Fermi liquid state \[7–13\]. Moreover, recently discovered axial and uniform charge density wave (CDW) \[14–17\] has been theoretically understood as the spin-fluctuation-driven CDW due to Aslamazov-Larkin vertex correction mechanism. \[18–23\].

In addition, by introducing real-space structures such as surfaces and impurities, interesting non-trivial critical phenomena emerge in correlated electron systems. In cuprate superconductors, non-magnetic impurities enhance the spin fluctuations around them \[24–32\]. In the two-dimensional Hubbard model with the (1, 1) edge, the ferromagnetic (FM) fluctuations develop along the edge \[33\]. These phenomena are caused by the Friedel oscillation in the local density of states (LDOS) since the large LDOS sites near the real-space structure drive the system toward the magnetic criticality.

In contrast, in the superconducting (SC) states, studies of the effects of real-space structures on the electron correlation were limited until recently. Recently, several interesting impurity-induced \[34, 35\] and surface-induced \[36\] critical phenomena have been analyzed theoretically. The key ingredient is the edge-induced Andreev bound states (ABS) in the $d$-wave superconductors \[37–42\], which is observed in the STM experiment as the zero-bias conductance peak \[43, 44\]. In a previous paper \[36\], the present authors revealed that the huge edge DOS due to the ABS triggers very strong FM fluctuations around the (1, 1) edge, by carrying out site-dependent random-phase approximation (RPA) and modified fluctuation-exchange (FLEX) approximation. In this case, the strong FM fluctuations may induce exotic phenomena such as the triplet superconductivity \[47–51\].

As well-known, the emergence of surface or interface induced SC state that is not realized in the bulk has been studied very actively. Near the (1, 1) edge of the $d_{x^2-y^2}$-wave superconductor, the $s$-wave superconductivity can emerge by using the ABS, and an $d \pm is$-wave SC state is realized \[52–54\]. In this case, time-reversal symmetry is broken and the edge current flows along the edge. This emergence of time-reversal breaking superconductivity at the domain wall is also discussed with regards to the polycrystalline YBa$_2$Cu$_3$O$_{7-x}$ (YBCO) \[55–57\] and twinned iron-based superconductor FeSe in the nematic phase \[58\]. However, the site-dependence of pairing interaction has not been taken into consideration, although FM fluctuations are strongly enhanced near the edge of the Hubbard model.

In this paper, we theoretically predict the emergence of the triplet superconductivity near the (1, 1) edge of the $d$-wave superconductors. The origin of the triplet gap is the strong FM fluctuations triggered by the ABS due to the sign-change in the $d$-wave SC gap. We first develop the linearized gap equation for the edge-supercconductivity, and apply it to a two-dimensional cluster Hubbard model with the (1, 1) edge in the bulk $d$-wave SC state. The site-dependent pairing interaction is obtained based on the microscopic calculation by the RPA or GV$^2$-FLEX \[30\]. We reveal that the phase difference between the edge triplet gap and the bulk $d$-wave gap is $\pi/2$ in the $k$-space. That is, exotic edge-induced $d \pm ip$-wave SC state is expected to be realized at $T = T_{cp}$, which is slightly lower than the bulk $d$-wave transition temperature $T_{cd}$. The present study may offer an interesting platform of realizing exotic SC states.

II. THEORETICAL METHOD OF TRIPLET GAP EQUATION

To study the edge-induced triplet superconductivity, we construct a two-dimensional square lattice Hubbard
To simplify the calculation, we actually use the square lattice showed in (c) instead of (b). (d) $T$-dependence of $\alpha_{S}$ in the RPA. The inset shows the $T$-dependence of the bulk $d$-wave gap in (a). We set the transition temperature of the $d$-wave superconductivity as $T_{cd} = 0.04$. At $T = T_{M}$, $\alpha_{S}$ reaches unity.

The $d$-wave SC state:

$$\mathcal{H} = \sum_{i,j,\sigma} t_{i,j} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$+ \sum_{i,j} \Delta_{i,j} \left( c_{i\uparrow}^\dagger c_{j\downarrow} + c_{i\downarrow}^\dagger c_{j\uparrow} \right),$$

(1)

where $t_{i,j}$ is the hopping integral between sites $i$ and $j$. We set the nearest, next nearest, and third-nearest hopping integrals as $(t, t', t'') = (-1, 1/6, -1/5)$, which correspond to the YBCO TB model. $c_{i\sigma}^\dagger$ and $c_{i\sigma}$ are creation and annihilation operators of an electron with spin $\sigma$, respectively. $U$ is the on-site Coulomb interaction, and $\Delta_{i,j} \equiv \Delta_{i,j}^{\uparrow\downarrow}$ is the bulk $d$-wave SC gap. Figure 1(a) shows the Fermi surface of the periodic tight-binding (TB) model at filling $n = 0.95$. Fig. 1(b) shows the square lattice with the (1, 1) edge. For convenience, we analyze the (1, 1) edge model with the one-site unit cell structure shown in Fig. 1(c).

The bulk $d$-wave SC gap in the $(k_x, y, y')$ representation $\Delta_{y,y'}^{\uparrow\downarrow}(k_x)$ satisfies

$$\Delta_{\uparrow\downarrow} = -\Delta_{\downarrow\uparrow},$$

(2)

$$\Delta_{y,y'}^{\uparrow\downarrow}(k_x) = -\Delta_{y',y}^{\downarrow\uparrow}(-k_x).$$

(3)

is the definition of the singlet superconductivity and comes from the anticommutation relation of the fermion. By using (2) and (3), we obtain

$$\Delta_{y,y'}(k_x) = \Delta_{y',y}(-k_x).$$

(4)

For numerical calculation, we set the bulk $d$-wave gap in the $(k_x, y, y')$ representation as

$$\Delta_{y,y'}(k_x, T) = \Delta(T) \left\{ \frac{e^{-ik_x} - 1}{2} \delta_{y,y'+1} + \frac{e^{ik_x} - 1}{2} \delta_{y,y'-1} \right\},$$

(5)

$$\Delta(T) = \Delta_0 \tanh \left( 1.74 \sqrt{\frac{T_{cd}}{T} - 1} \right),$$

(6)

where $\Delta(T)$ is the temperature-dependent $d$-wave gap and $\Delta_0 \equiv \Delta(T = 0)$. Note that $\Delta(k, T) = \Delta(T) (\cos k_x - \cos k_y)$ in a bulk $d$-wave superconductor. $T_{cd}$ is the transition temperature of the $d$-wave superconductivity. The $d$-wave gap is real in the $(x, y, y')$ representation. It is easy to verify the Fourier transformation of the $d$-wave gap in Eq. (5) is real. Therefore, the following relation holds:

$$\Delta_{y,y'}(k_x) = \Delta_{y,y'}(k_x).$$

(7)

Next, we calculate $N_y \times N_y$ Green functions in the $d$-wave SC state $\mathcal{G}$, $\mathcal{F}$, and $\mathcal{F}^{\dagger}$ as follows:

$$\left( \begin{array}{c} \mathcal{G}(k_x, \varepsilon_n) \\ \mathcal{F}(k_x, \varepsilon_n) \\ \mathcal{F}^{\dagger}(k_x, \varepsilon_n) \end{array} \right) = \left( \begin{array}{c} \varepsilon_n \mathbb{1} - \mathcal{H}(k_x) \\ -\Delta(k_x) \\ \varepsilon_n \mathbb{1} + \mathcal{H}(k_x) \end{array} \right)^{-1},$$

(8)

where $\varepsilon_n = (2n + 1) \pi iT$ is the fermion Matsubara frequency. Here, $(\mathcal{H}(k_x)y,y') = H^{0}_{y,y'}(k_x)$ is the TB hamiltonian. $\mathcal{F}$ and $\mathcal{F}^{\dagger}$ are anomalous Green functions, which are finite only in the bulk $d$-wave SC state. Note that the Green function $\mathcal{F}$ satisfies the relation

$$\mathcal{F}^{\dagger} = -\mathcal{F}^{\dagger},$$

(9)

In this model, we can obtain the enhancement in the FM fluctuations at the edge by the RPA or $GV^{I}.FLEX$ approximation [36]. Fig. 1(d) shows the $T$-dependence of the Stoner factor $\alpha_{S}$ in the RPA. The inset shows the $T$-dependence of the bulk $d$-wave gap given by (a). As $T$ decreases, $\alpha_{S}$ increases drastically in the $d$-wave SC state, and reaches unity at $T = T_{M}$.

Next, we analyze the edge-induced triplet superconductivity in the presence of the bulk $d$-wave SC gap. The triplet SC gap in the $(k_x, y, y')$ representation $\phi_{y,y'}^{\uparrow\downarrow}(k_x)$ satisfies the relations

$$\phi_{\uparrow\downarrow} = \phi^{\uparrow\downarrow},$$

(10)

$$\phi_{y,y'}^{\uparrow\downarrow}(k_x) = -\phi_{y',y}^{\downarrow\uparrow}(-k_x).$$

(11)

is the definition of the triplet superconductivity and comes from the anticommutation relation of the fermion. From (10) and (11), the triplet gap follows

$$\phi_{y,y'}(k_x) = -\phi_{y',y}(-k_x).$$

(12)
To decide the edge-induced SC state, we must obtain the phase difference between the bulk \(d\)-wave gap and the edge \(d\)-wave gap. Although we can use the Bogoliubov-de Gennes (BdG) equation, we have to perform heavy self-consistent calculation at various temperatures. To make the theoretical analysis much more efficient, we develop the linearized gap equation for the edge superconductivity, by linearizing the BdG equation only for \(\phi\) and \(\hat{\phi}\). In this method, by just performing the diagonalization, we can address the emergence of the triplet superconductivity by the temperature-dependence of the eigenvalue. We use the relation (9) and (10) in the derivation of the linearized gap equation, and it is given as

\[
\lambda\phi_{y,y'}(k_x) = -T \sum_{k_{x,\nu}',Y',n} V_{y,y'}(k_x - k_{x,\nu}',\varepsilon_n - \varepsilon_0) \\
\times \{ G_{y,y'}(k_{x,\nu}',\varepsilon_n) \phi_{Y,Y'}(k_{x,\nu}') G_{y',Y'}(-k_{x,\nu}',-\varepsilon_n) \\
- F_{y,y'}(k_{x,\nu}',\varepsilon_n) \hat{\phi}_{Y,Y'}(k_{x,\nu}') F_{y',Y'}(-k_{x,\nu}',-\varepsilon_n) \},
\]

\[
\lambda\phi^\dagger_{y,y'}(k_x) = -T \sum_{k_{x,\nu}',Y',n} V_{y,y'}(k_x - k_{x,\nu}',\varepsilon_n - \varepsilon_0) \\
\times \{ G_{y,y'}(-k_{x,\nu}',-\varepsilon_n) \phi^\dagger_{Y,Y'}(k_{x,\nu}') G_{y',Y'}(k_{x,\nu}',\varepsilon_n) \\
- F_{y,y'}(k_{x,\nu}',\varepsilon_n) \hat{\phi}^\dagger_{Y,Y'}(k_{x,\nu}') F_{y',Y'}(k_{x,\nu}',\varepsilon_n) \},
\]

\[
\hat{V}(q_x,\omega_l) = U^2 \left( -\frac{1}{2} \chi^s(q_x,0) - \frac{1}{2} \chi^c(q_x,0) \right) C(\omega_l,\omega_d),
\]

where \(\lambda\) is the eigenvalue of the gap equation. When \(\lambda \geq 1\), the triplet superconductivity emerges and coexists with the bulk \(d\)-wave superconductivity. \(\hat{V}(q_x,\omega_l)\) is the site-dependent pairing interaction for triplet superconductivity. \(\chi^s(c)(q_x,0)\) is the static spin (charge) susceptibility in the \(d\)-wave SC state obtained by the RPA or \(GV^{t}\)-FLEX approximation. Here, \(\omega_l = 2\pi i T\) is the boson Matsubara frequency. \(C(\omega_l,\omega_d) = \omega_d^2 / (|\omega_l|^2 + \omega_d^2)\) is a cut off function, where \(\omega_d\) is the cutoff energy, and we set \(\omega_d = 0.5\). We then solve the gap equation (13) under the restriction (12). Note that the first and second terms of the gap equation have different sign due to the relation (9). This fact greatly affects the phase difference between the bulk gap function and the edge one.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{(color online) Diagram of the linearized triplet SC gap equation in the presence of the bulk \(d\)-wave SC gap. The undulating lines are pairing interactions of the triplet superconductivity. The line with a single arrow represents the Green function \(G\) and the line with double arrows represents anomalous Green functions \(F\) and \(F^\dagger\).}
\end{figure}

\section{III. Numerical Result of Triplet Gap Equation}

In this section, we analyze the linearized triplet gap equation (13). \(k_x\)-mesh is \(N_x = 64\), site number along \(y\)-direction is \(N_y = 64\), the number of Matsubara frequencies is 1024. The transition temperature of the bulk \(d\)-wave superconductivity is \(T_{cd} = 0.04\). The Coulomb interaction is \(U = 2.25\) in the RPA, and \(U = 2.65\) in the \(GV^{t}\)-FLEX. Here, the unit of energy is \(|t|\), which corresponds to \(\sim 0.4\) eV in cuprate superconductors. In addition, we define \(\Delta_{\text{max}}\) as the maximum value of the \(d\)-wave gap on the Fermi surface. In the present model, \(\Delta_{\text{max}} = 1.76\Delta_0^{\text{cd}}\) for \(n = 0.95\). Experimentally, \(4 < 2\Delta_{\text{max}}/T_{cd} < 10\) in YBCO \cite{59,60}. Therefore, in the RPA, we set \(\Delta_0^{\text{cd}} = 0.06\) or 0.09, which corresponds to \(\Delta_{\text{max}} = 5.28\) or 7.92 for \(T_{cd} = 0.04\).

\subsection{A. \(d \pm ip\)-wave SC state}

First, we analyze the linearized triplet gap equation for the pairing interaction calculated by the RPA. Figure 3 shows \(k_x\)-dependence of the obtained triplet gap in the same layer \(y\). This is the \(p_x\)-wave gap with a node at \(k_x = 0\). It can emerge at the edge because there are finite LDOS and large triplet pairing interactions due to the ABS.

Next, we discuss the phase difference between the \(d\)- and \(p\)-wave gap. The triplet SC gap in the real space \(\phi_{x,y,y'}\) is represented by the Fourier transformation on
FIG. 3. (color online) $k_x$-dependence of obtained $p_x$-wave SC gap $\phi_{y,y'}(k_x)$ for $\Delta_0 = 0.09$ at $T = 0.0375$. The pairing interaction is calculated by the RPA. $y = 1$ and $y = 32$ correspond to the edge and bulk, respectively. We normalize the gap as $\max|\phi_{y,y'}(k_x)| = 1$.

The $x$-direction of $\phi_{y,y'}(k_x)$. By using (12), we obtain

$$\phi_{x,y,y'} = -\left\{ \sum_{k_x} \phi_{y,y'}^{\dagger}(k_x)e^{ik_xx}\right\}^*.$$  \hspace{1cm} (15)

The relation holds for the general triplet SC gap. On the other hand, the obtained $p$-wave gap satisfies

$$\phi_{y,y'}(k_x) = -\phi_{y,y'}^{\dagger}(k_x),$$  \hspace{1cm} (16)

in the present numerical study. Therefore, the obtained $p$-wave gap is a real function in real space $\phi_{x,y,y'} = \phi_{x,y,y'}^{\dagger}$. In this case, the phase difference is $\pm \pi/2$ in the $k$-space, and this is the $d \pm ip$-wave SC state. We find that the edge $d \pm ip$-wave SC state is stabilized by the coexistence of the bulk $d$-wave superconductivity and the edge-induced triplet superconductivity.

The reason of this phase difference $\pm \pi/2$ is understood by evaluating the contribution from the second term of (13a). Since the triplet pairing interaction $V_{y,y'}(k_x - k'_x, \varepsilon_n - \varepsilon_0)$ has large value only at the edge ($y = 1$), we can approximately evaluate the contribution to $\phi_{1,1}(k_x)$ from second term of (13a) by setting $Y = Y' = 1$,

$$\approx -T \sum_{k'_x, n} |V_{1,1}(k_x - k'_x, \varepsilon_n - \varepsilon_0)||F_{1,1}(k'_x, \varepsilon_n)|^2 \phi_{1,1}^{\dagger}(k'_x).$$  \hspace{1cm} (17)

Here, $V_{y,y'}(k_x - k'_x, \varepsilon_n - \varepsilon_0)$ has a large peak at $k_x = k'_x$. Therefore, the triplet superconductivity is stabilized when $\phi_{1,1}^{\dagger}(k_x) = \phi_{1,1}^{\dagger}(k_x) = -\phi_{1,1}(k_x)$, and it is actually confirmed by numerical calculation.

In the $d \pm ip$-wave SC state, the time-reversal (TR) symmetry is broken. To verify it, we apply the time-reversal operator $\Theta = -i\sigma^y K$ to the present gap functions.

$$\Delta_{y,y'}^{\dagger}(k_x) + \phi_{y,y'}^{\dagger}(k_x) \to -\Delta_{y,y'}^{\dagger}(k_x) - \phi_{y,y'}^{\dagger}(k_x).$$  \hspace{1cm} (18)

By using the conditions (2), (7), (10), and (16), we confirm that the $d + ip$-wave gap changes to the $d - ip$-wave gap. In Appendix A, we calculate the LDOS in the $d \pm ip$-wave SC state. The LDOS for up spin electrons and that for down spin electrons are separated since the time-reversal symmetry is broken in the $d \pm ip$-wave SC state.

B. Temperature-dependence of $\lambda$

Next, we examine the $T$-dependence of the eigenvalue of the edge $p$-wave superconductivity. We denote the eigenvalue in the $d$-wave superconductivity and normal state as $\lambda$ and $\lambda^{(n)}$, respectively. Figure 4 shows the $T$-dependence of the eigenvalue based on the RPA. $\lambda^{(n)}$ hardly increases and does not reach unity. On the other hand, $\lambda$ increases drastically as $T$ decreases and exceeds unity below $T_{cp} \lesssim T_{cd}$. At these temperatures, the $d \pm ip$-wave SC state is realized. Note that the edge ferromagnetic order is realized at $T_M \lesssim T_{cp}$. For $\Delta_0 = 0.09$ (2$\Delta_{\max}/T_{cd} = 7.92$), the increase in $\lambda$ is more drastic than that for $\Delta_0 = 0.06$ (2$\Delta_{\max}/T_{cd} = 5.28$) due to the stronger development of the FM fluctuations as shown in the Fig. 1(d).

FIG. 4. (color online) $T$-dependence of $\lambda$ for the pairing interaction by the RPA. The red and green line represent $\lambda$ for $\Delta_0 = 0.06$ and 0.09, respectively. The blue line shows $\lambda^{(n)}$ in the normal state ($\Delta_0 = 0$). Below $T_{cp}$, the $p$-wave superconductivity emerges. At $T = T_M$, $\alpha_S$ reaches unity in the RPA.

To examine the effect of the FM fluctuations on the increase in $\lambda$, we analyze two types of gap equations, (i) and (ii), from which the effect of the $d$-wave gap is partially subtracted. In (i), we use the pairing interaction in the normal state $\hat{V}$ in the $d$-wave SC state, and denote the eigenvalue as $\lambda'$. In (ii), we
replace the Green functions $\hat{G}$, $\hat{F}$ and $\hat{F}^\dagger$ with those in the normal state, $G^0$ and $F = F^\dagger = 0$. We denote the eigenvalue as $\lambda''$. Figure 5 shows the $T$-dependence of $\lambda'$ and $\lambda''$. We see that $\lambda'$ is strongly suppressed, and it does not reach unity. On the other hand, $\lambda''$ is almost equal to $\lambda$ and exceed unity at $T \lesssim T_{\text{cp}}$. Therefore, the drastic increase in $\lambda$ under $T_{\text{cd}}$ is mainly due to the ABS-driven FM fluctuations.

![FIG. 5. (color online) $T$-dependence of $\lambda'$ and $\lambda''$.](image)

We obtain $\Delta_0 = 0.09$ The red dotted line and blue solid line represent $\lambda$ and $\lambda''$ in the $d$-wave SC state and normal state, respectively.

C. Result of the GV$^I$-FLEX approximation

In this study, we analyze the linearized triplet gap equation for the pairing interaction calculated by the GV$^I$-FLEX approximation in the $(1,1)$ edge cluster model [33]. In the conventional FLEX, the negative feedback effect on spin susceptibility near an impurity is overestimated since the vertex corrections for the spin susceptibility is not considered [32]. In the modified FLEX, the cancellation between negative feedback and vertex corrections is assumed, and then reliable results are obtained for the single impurity problem [32].

$\Delta_0^{\ast}$ is the renormalized gap by the normal self-energy. We obtain $\Delta_0^{\ast} \approx 0.087$ and $\Delta_{\text{max}}^{\ast}/T_{\text{cd}} \approx 7.69$ for $\Delta_0 = 0.12$, and $\Delta_0^{\ast} \approx 0.058$ and $\Delta_{\text{max}}^{\ast}/T_{\text{cd}} \approx 5.11$ for $\Delta_0 = 0.08$. To simplify the analysis, the normal self-energy is not included in the Green functions in the gap equation.

Figure 6 shows the $T$-dependence of $\lambda$ based on the GV$^I$-FLEX. $\lambda$ increases as $T$ decreases also in the GV$^I$-FLEX. In the case of $\Delta_0 = 0.08$, $\lambda$ exceeds unity at $T \approx 0.02$. For $\Delta_0 = 0.12$, the increase in $\lambda$ is sharper than that for $\Delta_0 = 0.08$ because of the stronger development of the FM fluctuations. The increase in $\lambda$ becomes milder than that in the RPA due to the negative feedback effect of self-energy. However, we obtain the emergence of a $d \pm ip$-wave superconductivity even if the self-energy is considered. Note that the $T$-dependence of $\lambda$ based on the RPA and GV$^I$-FLEX is comparable when $(2\Delta_{\text{max}}^{\ast}/T_{\text{cd}})_{\text{RPA}} \approx (2\Delta_{\text{max}}^{\ast}/T_{\text{cd}})_{\text{FLEX}}$.

![FIG. 6. (color online) $T$-dependence of $\lambda$ for the pairing interaction by the GV$^I$-FLEX. $\Delta_0^{\ast}$ is renormalized gap by the self-energy. We obtain $\Delta_0^{\ast} = 0.058$ for $\Delta_0 = 0.08$ and $\Delta_0^{\ast} = 0.087$ for $\Delta_0 = 0.12$.](image)

D. Effect of finite $d$-wave coherence length on edge-induced triplet superconductivity

In this section, we discuss the emergence of the $p$-wave superconductivity when the $d$-wave gap is suppressed for the finite range $1 \leq y \leq \xi_d$, where $\xi_d$ is the coherence length of the $d$-wave superconductivity. We set the $y$-dependence of the $d$-wave gap as follows:

$$\Delta_y(y',k_x,T) \left( 1 - \exp \left( \frac{y + y' - 2}{2\xi_d} \right) \right). \tag{19}$$

We note that the SC FLEX approximation [4] is applied to the edge cluster model, the obtained $d$-wave gap for $y \lesssim \xi_d$ should be naturally suppressed. Instead, we set $\xi_d$ as a parameter to simplify the analysis. From the experimental results [61–64], we can estimate $\xi_d$ to be 3 sites for $T \ll T_{\text{cd}}$. For $T \lesssim T_{\text{cd}}$, $\xi_d \gg 3$ because of the relation $\xi_d \propto (1 - T/T_{\text{cd}})^{-1/2}$ in the GL theory. Thus, we set $\xi_d = 3$ and 10 in the present analysis.

Figure 7 (a) shows the site-dependence of the $d$-wave gap expressed by (19). Fig. 7 (b) is the obtained LDOS at the edge. Although the height of the peak becomes lower, the peak structure due to the ABS still exists for finite $\xi_d$. In our previous paper, we confirmed that $\alpha_S$ increases as $T$ decreases for finite $\xi_d$.

Then, we analyze the gap equation based on the RPA for finite $\xi_d$. Figure 8 shows the $T$-dependence of $\lambda$. For $\Delta_0 = 0.09$, $\lambda$ increases as the temperature decreases and exceeds unity even for finite $\xi_d$. On the other hand, the increase in $\lambda$ is mild for $\Delta_0 = 0.06$ and $\xi_d$, and $\lambda \approx 0.68$ even at $T = 0.03$. Therefore, the strong increase in $\lambda$ is realized under the conditions $2\Delta_{\text{max}}^{\ast}/T_{\text{cd}} \gtrsim 6$ and $\xi_d \ll 10$. These conditions are satisfied in real cuprate superconductors.
where $n$ is 1,2,3,4. This section, we calculate the super edge current in the $d$ wave for the $i$th site.

**FIG. 8.** (color online) $T$-dependence of $\lambda$ for (a) $\Delta_0 = 0.06$ or (b) $\Delta_0 = 0.09$ with finite $\xi_d$. The pairing interaction is calculated by the RPA for finite $\xi_d$.

**FIG. 9.** (color online) $y$-dependence of super edge current $\langle J_y^x \rangle$ in the $d + ip$- and $d + is$-wave SC state. We set $\Delta_0 = 0.09$ and $\phi_{y_{y'}} = 0.09$. We set the size of edge $s$-wave gap as $\Delta^* = 0.09$.

To explain why the spontaneous edge current cancels in the $d + ip$-wave SC state, we consider the Green function $G^y_{y', x} (k_x, \epsilon_n)$, which corresponds to the transfer process of up spin electron from site $y'$ to $y$. Here, we evaluate an example of its second order term in proportion to $\Delta \phi^d$:

$\delta G_{y, y'}^{x, \uparrow \uparrow} (k_x, \epsilon_n) = -G_{y, y'}^{0} \Delta_{y, y'} (k_x)$

$\times G_{y_{y'}, y}^{0} (k_x, \epsilon_n) G_{y', y}^{0} (k_x, \epsilon_n), \quad (24)$

where $G_{y, y'}^{0}(k_x, \epsilon_n)$ is the Green function in the normal state. Then, the inverse transfer process of $G_{y, y'}^{0}(k_x, \epsilon_n)$ is given by

$\delta G_{y', y}^{x, \uparrow \uparrow} (k_x, \epsilon_n) = -G_{y', y}^{0} (k_x, \epsilon_n) \phi^{\dagger}_{y, y'} (k_x)$

$\times G_{y, y'}^{0} (k_x, \epsilon_n) \phi^{\dagger}_{y', y} (k_x)$

$\times G_{y', y}^{0} (k_x, \epsilon_n) \phi^{\dagger}_{y, y'} (k_x)$

$\times G_{y, y'}^{0} (k_x, \epsilon_n) \phi^{\dagger}_{y', y} (k_x)$. 

Then, the total super current is given by $\langle J_y^x \rangle = \sum_y \langle J_y^x \rangle$.
\[ x \times G_{y2,y3}^{0}(k_x,-\varepsilon_n)\Delta_{y2,y3}^{\uparrow\downarrow}(-k_x)G_{y1,y}^{0}(-k_x,\varepsilon_n). \]  

Note that \( \tilde{G}^{0} \) satisfies \( G_{y,y}^{0}(k_x,\varepsilon_n) = G_{y,y}^{0}(-k_x,\varepsilon_n) \). In addition, by using (4), (7), (12), and (16), we obtain \( \delta G_{y,y}^{\uparrow\downarrow}(k_x,\varepsilon_n) = \delta G_{y,y}^{\uparrow\downarrow}(-k_x,\varepsilon_n) \). Therefore, \( n_{y,y}^{\sigma}(k_x) = n_{y,y}^{\sigma}(-k_x) \) holds and therefore the current does not flow.

V. SUMMARY

In this paper, we demonstrated that the \( d \pm ip \)-wave SC state is realized at the (1, 1) edge of the \( d \)-wave superconductors due to the ABS-induced strong FM fluctuations. We studied the two-dimensional cluster Hubbard model with the edge in the presence of the bulk \( d \)-wave SC gap. To analyze the edge-induced SC gap, we constructed a linearized triplet SC gap equation in the presence of the bulk \( d \)-wave SC gap. The site-dependent pairing interaction is calculated using the RPA or \( GV'F \) model. The obtained phase difference between the bulk \( d \)-wave gap and the edge \( p \)-wave gap is \( \pi/2 \) in the \( k \)-space, and it is the \( d \pm ip \)-wave SC state in which the time-reversal symmetry is broken. Next, we examined the dependence of the eigenvalue \( \lambda \) for the edge-induced SC state. Below the bulk \( d \)-wave transition temperature \( T_{cd} \), \( \lambda \) for the triplet state increases drastically as \( T \) decreases, and exceeds unity at \( T = T_{tp} \). Therefore, the \( d \pm ip \)-wave SC state is realized at \( T_{tp} \approx T_{cd} \). In the \( d \pm ip \)-wave SC state, the edge current does not flow irrespective of the time-reversal symmetry braking.

The result of the present study indicates the emergence of the Majorana bound state at the endpoint of the (1, 1) edge \[66\] since pure triplet \( p \)-wave superconductivity emerges at the edge layer. Thus, the present study of the edge-induced novel superconductivity induced by the ABS-driven strong correlation may offer an interesting platform of SC devises.

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Appendix A: LDOS in the \( d \pm ip \)-wave SC state

Here, we discuss the LDOS in the \( d \pm ip \)-wave SC state. We assume that the d-vector of the \( p \)-wave superconductivity is normal to \( xy \) plane. We use the \( p \)-wave gap obtained by the numerical analysis. The LDOS is given by

\[ D_{\downarrow}(\varepsilon) = \frac{1}{\pi} \sum_{k_x,\sigma} \text{Im} \delta G_{y,y}^{\downarrow\downarrow}(k_x,\varepsilon - i\delta). \]  

We set \( \delta = 0.03 \) in the numerical calculation. Figure 10 shows the obtained LDOS at the edge. The LDOS for up spin electrons and that for down spin electrons are separated since the time-reversal symmetry is broken in the \( d \pm ip \)-wave SC state.

FIG. 10. (color online) \( \varepsilon \)-dependence of the LDOS at the (1, 1) edge in the \( d \pm ip \)-wave SC state. We set \( \Delta_0 = 0.09 \) and \( \max|\delta_{i,j}| = 0.05 \). The red dashed line and blue dotted line \( ^{i,j} \) represent the LDOS for up and down spin, respectively. The green solid line is the sum of spins.

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