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A robust just-in-time flow shop scheduling problem with outsourcing option on subcontractors

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ABSTRACT
Scheduling is known as a great part of production planning in manufacturing systems. Flow Shop Scheduling (FSS) problem deals with the determination of the optimal sequence of jobs processing on machines in a fixed order. This paper addresses a novel robust FSS problem with outsourcing option where jobs can be either scheduled for inside or outsourced to one of the available subcontractors. Capacity limitation for inside resource, just-in-time delivery policy and uncertain processing time are the key assumptions of the proposed model. The objective is to minimize the total-weighted time required to complete all jobs and the total cost of outsourcing. So, a Robust Mixed-Integer Linear Programming (RMILP) model is proposed to accommodate the problem with the real-world conditions. Finally, the obtained results show the effects of the robustness in optimizing the model under uncertainty condition. Moreover, the comparison analysis demonstrated the superiority of our proposed model against the previous Non-Linear Programming (NLP) model in the literature.

1. Introduction
Flow shop is a production system where all machines are organized based on operational jobs. Flow Shop Scheduling (FSS) problem involves determining an optimal schedule for jobs processing on machines and has been a research benchmark for many years. Optimization algorithms for two and three-machine flow shop problems have been developed in conjunction with different targets. As the majority of FSS problems are NP-hard; i.e., problems with non-polynomial run time, all of the exact, heuristic and meta-heuristic solution methods seek to minimize total completion time (makespan) (Chung, Flynn, & Kirca, 2006).

There are three types of main objective functions for scheduling problems in flow shop system:

(A) Based on jobs completion time,
(B) Based on delivery time,
(C) Based on the inventory and implementation cost of facilities/equipment.
There are various categories regarding completion time of jobs including all jobs completion time, average completion time, weighted completion time, and weighted average completion time (Khalili & Tavakkoli-Moghaddam, 2012).

Other objective functions include total flow time, average flow time, weighted total flow time, weighted average flow time, maximum delay, total delay, average delay, maximum earliness, total earliness, average earliness, weighted average delay and completion time variance which have been examined in the recent literature (Xu & Xiao, 2011).

One of the major issues in production planning is the optimal utilization of production resources; some of these resources are production machinery that should be exploited efficiently and effectively. On the other hand, there should always exist a significant volume of products and parts outside the production system as outsourcing is a necessary way to improve scheduling performance in various companies (Mokhtari & Abadi, 2013; Qi, 2011). Moreover, it will cause manufacturers applying higher concentration on more important and more valuable issues in the workplace, this research focused on this issue. Nowadays many companies are outsourcing their jobs rather than implementing direct management process (Behnamian, Fatemi Ghomi, & Zandieh, 2010). Most of the manufacturers are faced with the conditions that demands exceed the capacity of production (Lee, Jeong, & Moon, 2002; Mokhtari & Abadi, 2013). In this situation, they will outsource all or some of their jobs to a subcontractor or several subcontractors rather than design large amounts of productions or pay enormous costs of warehousing and inventory holding. Indeed, proper planning for outsourcing can reduce product delivery time and total costs as well as making the manufacturer plant more competitive. To achieve such benefits, managers have to make the decision about the amount of products to be produced and the amount of products to be outsourced (Mokhtari & Abadi, 2013).

One of the main aspects of this research is providing a solution for manufacturing companies that have a delay in delivering orders. The suggestion of this research is to divide jobs between themselves and their other subcontractors. This makes many orders executed in parallel between different production units. The main feature of this research is that it provides an optimal decision for between manufacturing firms to increase productivity as well as reduce the delay in the delivery of orders and increasing customers’ satisfaction.

Therefore, this research presents an efficient mathematical model to minimize completion time and outsourcing costs by scheduling the in-house production and considering outsourcing option in addition to maintaining the integration of the problem. Furthermore, the model is developed using Robust Optimization (RO) method to cover the uncertain nature of the processing time parameter.

The rest of the paper is organized as follows. Section 2 reviews the relevant works in the literature. Section 3 introduces the proposed RO formulation. The problem, mathematical model and its robust counterpart formulation are discussed in Section 4, and then Section 5 presents the computational results including model validation, comparison results, and sensitivity analysis. Section 6 provides a discussion of the obtained results of the research. Finally, conclusions and outlook of the research are described in Section 7.

2. Literature review

Chung et al. (2006) applied a Branch-and-Bound algorithm for solving FSS problems of $m$-machine aiming at total tardiness minimization. They could demonstrate the
efficiency of the proposed algorithm by conducting extensive computational experiments on test problems for less than 20 jobs. Naderi, Zandieh, Balagh, & Roshanaei (2009) presented an improved Simulated Annealing (SA) algorithm for hybrid flow shops with sequence-dependent setup and transportation times to minimize total tardiness and maximum completion time. The proposed algorithm applies a Taguchi method to adjust algorithm parameters to reduce the frequency of testing. Khalili and Tavakkoli-Moghaddam (2012) proposed a bi-objective FSS problem with the aim of minimizing the maximum completion time and total weighted delay. In their study, it was assumed that there is no need for everything to be processed on all machines. Furthermore, the transportation time between stations is considered in the model. They developed a Multi-Objective Electromagnetic Algorithm (MOEM) to solve the problem. This algorithm applies the electromagnetic theory absorption and disposal mechanisms.

Behnamian et al. (2010) introduced a hybrid meta-heuristic algorithm to solve a hybrid FSS problem with sequence-dependent preparation time that aims to minimize total earliness and delays. The algorithm includes three parts: 1) the first part generates a primary population using Ant Colony Optimization (ACO) algorithm, 2) the second part applies SA algorithm, and 3) the third part implements the local search to improve the solutions. (Liao & Huang, 2010) introduced three linear programming models for the non-permutation FSS problem with the objective function of minimizing all delays. They developed a Tabu Search (Bertsimas & Sim, 2003) algorithm to solve the problem.

Some important research has been done in parallel machine environment considering different assumptions such as resource constraints (Hou, 2013), sequence-dependent setup times (Hou, 2013), machine eligibility constraint in automotive gear industry (Gokhale & Mathirajan, 2012), single batch delivery of production by vehicles in a supply chain (Cakici, Mason, Geismar, & Fowler, 2014), and acceptance or rejection possibility of jobs in order to minimize total penalty cost from rejections and make-span of accepted jobs (Zhang, Lu, & Yuan, 2009). Mokhtari and Abadi (Mokhtari & Abadi, 2013) designed a Non-Linear Programming (NLP) model to solve a single-stage scheduling problem with outsourcing option with the aim of total cost minimization. It was assumed that subcontractors are capable to undertake all the outsourced jobs. They developed a heuristic algorithm to decompose the problem and solve them in a short run time.

In uncertainty condition, Rahmani and Heydari (Rahmani & Heydari, 2014) developed a study of FSS problem to minimize the make-span, while maintaining stability and robustness, and incorporating uncertain processing times and the unexpected arrival of new jobs. Other production scheduling studies have included random processing times (Kamburowski, 2000). Nagasawa, Ikeda, and Irohara (2015) presented a robust FSS problem with random processing time for reduction of peak power consumption. They used a scenario-based RO approach to implement randomness nature of the parameters.

Recently, Fu, Ding, Wang, & Wang (2018) developed a mathematical model for a flowshop scheduling problem by considering multiple objectives, time-dependent processing time and uncertainty. They implemented a fireworks algorithm with some local search strategies to solve the problem. Brum and Ritt (Zhang et al., 2009) developed an algorithm with automatic configuration to solve the permutation FSS problem for minimizing total completion time. They investigated how algorithmic components can be combined to form a full heuristic search method. They compared their proposed methodology with several local search procedures in the literature to show its superiority.
After reviewing the most contributed research, it is clarified that the previous works didn’t focus on the application of RO in the field of FSS with outsourcing option and just-in-time policy. Therefore, it can be concluded that studying robust just-in-time FSS with outsourcing option and considering due dates, resource constraints, and uncertain processing time on different machines is the main contribution of this research. As a consequence, this research focuses on several available suppliers to be selected for subcontracting and considers the scheduling of outsourced jobs as well as in-house in a flow shop system. Moreover, an RO approach based on Bertsimas and Sim (2003) is applied to the model to cope with the real world uncertain condition of the problem.

3. Robust optimization approach

Among existing approaches for dealing with uncertainty, RO is an efficient and reliable approach, which has been expanded to deal with uncertainty in recent years (Tirkolaee, Mahdavi, & Esfahani, 2018a). In this approach, we look for solutions close to optimal which has a high possibility to be feasible. In other words, we can guarantee the feasibility of solutions while the optimality of the solutions is ignored. The aim of RO is to find a solution to show the least modifications against uncertainty.

Generally, the robust models are divided into two groups (Xu & Xiao, 2011):

1. The first group looks for a solution to optimize the objective function for the worst scenario.
2. The second group imposes some conditions on solutions. In this way, a solution which can handle these imposing conditions is considered as a robust solution.

In fact, RO approach is a suitable method to create a primary plan so that the uncertainty of data leads to the least possible modifications. Mulvey, Vanderbei, and Zenios (1995) introduced an approach which combined the formulations of ideal programming with the scenarios of data. In the early 1970s, Soyster (1973) presented a linear optimization model that gives the best feasible solution for all input data so that each input data can get any value from an interval. This approach tends to find solutions, which are over-conservative. It means that for assuring the robustness of solutions in this approach, we get far from the optimization of the problem. Ben-Tal and Nemirovski (1999) presented an efficient algorithm for solving convex optimization problems under uncertainty of data assuming that data are uncertain in elliptic sets. While regarding the obtained robust formulations from the conic quadratic problem, these approaches cannot be implemented for discrete optimization problems. Bertsimas and Sim (2003) proposed a different technique to control the level of conservatism. This approach has the advantage of presenting a linear optimization model. Thus, it applies to discrete optimization models.

Hence, for formulating the linear RO problems, we can apply several different approaches, which have been developed. Three main RO models and methods include Soyster (1973), Ben-Tal and Nemirovski (1999) and Bertsimas and Sim (2003). In this research, we implement Bertsimas’ and Sim’s model for formulating the problem due to its advantages (Tirkolaee, Goli, Bakhshi, & Mahdavi, 2017).

The procedure of the robust model development is described in the following.
3.1. Robust counterpart formulation

In this section, the proposed robust-mixed-integer linear programming (RMILP) model is explained such that uncertain parameters are used in the objective functions and constraints. The steps of the robust model development are discussed as follows.

First, consider the following optimization problem:

\[
\begin{align*}
\text{minimize} & \quad c^T x \\
\text{subject to} & \quad Ax \leq b, l \leq x \leq u
\end{align*}
\] (1)

The intervals of uncertainty are defined as follows (Tirkolaee et al., 2017).

Each coefficient of \(a_{ij}\) is as an independent random variable which turns into \(\tilde{a}_{ij}\) by the symmetrical distribution that takes value in \([\hat{a}_{ij} - \bar{a}_{ij}, \hat{a}_{ij} + \bar{a}_{ij}]\), where \(\hat{a}_{ij}\) shows the derivation from the nominal value of \(\tilde{a}_{ij}\) which is \(\hat{a}_{ij}\). Each coefficient of the objective function (\(c_j\)) takes value in the interval of \([\hat{c}_j - d_j, \hat{c}_j + d_j]\) where \(d_j\) shows the derivation of the nominal value of \(\hat{c}_j\) which is defined as \(\hat{c}_j\). It has to be mentioned that the aim of the robust model is to obtain the maximum regret and only one side of the interval is considered; i.e., it is assumed that \(c_j\) takes value in \([\hat{c}_j, \hat{c}_j + d_j]\).

As the critical conducting parameter, \(\Gamma_i\) is defined as the conservatism level of robustness. For \(i^{th}\) constraint of the problem as \(a_{it}^T x \leq b_i; J_i\) is a set of uncertainty coefficients in \(i^{th}\) row. For each \(i^{th}\) row, \(\Gamma_i\) is defined which may not be an integer number so that \(\Gamma_i \in [0, |J_i|]\). The role of \(\Gamma_i\) is to set the robustness degree of the problem as the level of solution conservation. Bertsimas and Sim (2003) demonstrated that with a low possibility all the coefficients might lead to uncertainty. Thus, it is assumed that up to \(\Gamma_i\) of these coefficients are allowed to change, and just one coefficient of \(a_{it}\) can change up to \((\Gamma_i - \Gamma_i) \hat{a}_{ij}\). In other words, it is assumed that only one subset of the coefficients is allowed to affect the solution procedure undesirably (Bertsimas & Sim, 2003). With this assumption, it is guaranteed that if this happens naturally, our robust optimal solution will be strongly feasible. Moreover, regarding the symmetric distribution of variables, even if the number of changed coefficients exceeds \(\Gamma_i\), optimal will remain feasible with a huge probability. Thus, \(\Gamma_i\) is called the conservatism level for \(i^{th}\) constraint which ensures the robustness adjustment against the protection level of the solution (Tirkolaee et al., 2017). On the other hand, the parameter \(\Gamma_0\) controls the robustness conservatism level in the objective function. Therefore, the goal is to find the value of the optimal solution so that \(\Gamma_0\) coefficients of the objective function will change and leave the most effect on the solution.

Generally, the higher amounts of \(\Gamma_0\) increase the conservatism level against the more costs added to the objective function. Here, \(\Gamma_0\) has to be an integer (Bertsimas & Sim, 2003). Based on this, the proposed RMILP is constructed step-by-step as follows:

\[
\begin{align*}
\text{minimize} & \quad c^T x + \max_{\{s_\delta | s_\delta \leq b_\delta, |s_\delta| \leq \Gamma_0\}} \left\{ \sum_{j \in s_\delta} d_j x_j \right\} \\
\text{subject to} & \quad Ax \leq b, l \leq x \leq u
\end{align*}
\] (2)

subject to
\[
\sum_j a_{ij} x_j + \max_{\{s \cup \{i\}|s \subseteq H, |s| \leq |\Gamma_i \cap J_i| \backslash s\}} \left\{ \sum_j \hat{a}_{ij} |x_j| + (\Gamma_i - \Gamma_i)\hat{a}_{it_i}|x_t| \right\} \leq b_i \forall i \in I,
\]

\[l \leq x \leq u.\] (4)

Equations. (2)-(4) propose an NLP model so that the optimization is not guaranteed. To linearize this model, Theorem 1 is defined (Naderi et al., 2009).

(5) **Theorem 1**: The conservation function of \textit{ith} constraint is calculated for the given vector of \(x^*\):

\[
\beta_i(x^*, \Gamma_i) = \max_{\{s \cup \{i\}|s \subseteq H, |s| \leq |\Gamma_i \cap J_i| \backslash s\}} \left\{ \sum_j \hat{a}_{ij} |x_j^*| + (\Gamma_i - \Gamma_i)\hat{a}_{it}|x^*_{t}| \right\},
\]

where it is equal to the optimal value of objective function (6) of the following problem which is linear (Rahmani & Heydari, 2014).

\[
\beta_i(x^*, \Gamma_i) = \max \sum_j \hat{a}_{ij} |x_j^*| z_{ij}
\] (6)

subject to

\[
\sum_{j \in I_i} z_{ij} \leq \Gamma_i \quad \forall i \in I,
\] (7)

\[0 \leq z_{ij} \leq 1 \quad \forall i, j \in I_i.
\] (8)

Now, the dual mode of the above problem; i.e., Equation. (6)-(8) is:

\[
\min c^T x + z_0 \Gamma_0 + \sum_{j \in J_0} r_{0j}
\] (9)

subject to

\[
\sum_j a_{ij} x_j + z_i \Gamma_i + \sum_{j \in J_i} r_{ij} \leq b_i \forall i,
\] (10)

\[z_0 + r_{0j} \geq e_{ij} y_j \quad \forall j \in J_0,
\] (11)

\[z_i + r_{ij} \geq \hat{a}_{ij} y_j \quad \forall i \neq 0, j \in I_i,
\] (12)

\[r_{ij} \geq 0 \quad \forall i, j \in I_i,
\] (13)

\[y_j \geq 0 \quad \forall j,
\] (14)

\[z_i \geq 0 \quad \forall i,
\] (15)
\[ -y_j \leq x_j \leq y_j \quad \forall j, \]  \hspace{1cm} (16)

\[ u_j \leq x_j \leq l_j \quad \forall j. \]  \hspace{1cm} (17)

The variables of \((z_0, z_i, r_{ij}, y_j)\) are used to apply the protection levels to the model and adjust the robustness of the solutions (Nagasawa et al., 2015).

4. Problem description and mathematical model

Problem description and the subsequent mathematical model development are the basis of the optimizations as we can survey in different types of optimization problems (Alinaghian, Amanipour, & Tirkolaee, 2014; Babaee Tirkolaee, Abbasian, Soltani, & Ghaffarian, 2019; Goli, Aazami, & Jabbarzadeh, 2018; Goli & Davoodi, 2018; Goli, Tirkolaee, Malmir, Bian, & Sangaiah, 2019; Hosseinabadi & Tirkolaee, 2018; Sangaiah, Tirkolaee, Goli, & Dehnavi-Arani, 2019; Tirkolaee, Alinaghian, Hosseinabadi, Sasi, & Sangaiah, 2018; Tirkolaee, Goli, Hematian, Sangaiah, & Han, 2019; Tirkolaee, Hosseinabadi, Soltani, Sangaiah, & Wang, 2018; Tirkolaee, Mahdavi, & Esfahani, 2018b). The basic model of our proposed problem is a nonlinear programming model which has been presented in Mokhtari and Abadi (2013). Our proposed model develops the main model by considering job due date, availability of resources and uncertain processing time in addition to linearizing the model so that a novel RMILP model is designed.

Each job \(j \in N\) is processed by machine \(k \in M\) with the processing time of \(p_{jk}\) and cost of \(y^k_j\). It is considered that some of these machines are stationed inside, and some of them are stationed outside of the manufacturer site as subcontractors. In this case, the inside jobs are considered with zero cost and the cost of the outsourced jobs is interpreted as the payments to the subcontractors. The other point is the consideration of completion time for the predetermined jobs. Considering the constraint of available resources inside the manufacturer plant may lead to an interaction between outsourcing and production in manufacturer plant in addition to cost and completion time factors.

The model assumptions can be summarized as follows:

1. The processing time of each job on each machine is assumed to be uncertain,
2. Each machine can process different jobs,
3. Split processing of jobs is not allowed (a job processing on a machine should not be divided into different operational time intervals),
4. All jobs are independent and available for processing at time zero,
5. Jobs setup time on machines are independent in sequences and has a given processing time,
6. The processing cost of each job is a certain value,
7. Some of the manufacturing machines are assumed to be outside of the plant,
8. Outsourced jobs have a certain additional processing time, but they can be processed parallel with other inside jobs,
9. The optimal scheduling should be non-delay scheduling as a just-in-time delivery policy.

Sets, indices, parameters, and variables of the problem are defined as below:
According to the explanations mentioned above, the problem $P$ is modeled as follows:

$$(Problem \ P) \ \text{minimize} \ \sum_{j \in N} \sum_{k \in M} \left( W_j C_{jk} + y_j^k \sum_{i \in N \cup 0} x_{ij}^k \right)$$

(subject to)

$$\sum_{k \in M} \sum_{i \in N \cup 0} x_{ij}^k = 1 \ \forall j \in N,$$
The objective function (18) represents the sum of total weighted completion time and total processing cost. Equation (19) guarantees that each job is processed only once and Equation (20) demonstrates that there is only one starting job to be processed on each machine. Equation (21) plays the same role as flow conservation condition in network flow problems ensures that the jobs are correctly sequenced within a partial schedule on machine \( k \in M \) and also ensures that at most one job has to be the last job on the machine. Equation (22) calculates the completion time \( C_{jk} \) by considering the relationships between two adjacent jobs. Equation (23) expresses that the completion time of job \( j \) should not exceed its due date. Equation (24) indicates the capacity limitation of available resources to complete the jobs inside the manufacturer plant. Equation (25) describes types of variables.

In the following, it is attempted to schedule each machine using Smith’s rule (Goli et al., 2019). This rule states that in an optimal solution of \( R \mid \| \sum w_j C_j \) problem, all the applied schedules on the machines have Shortest Weighted Processing Time (WSPT). As a result, it is needed to investigate such schedules that comply with Smith’s rule for all machines. \( A^k_j \) denotes a group of jobs \( i \) following WSPT on machine \( k \) that job \( j \) is scheduled before job \( i \) and \( B^k_j \) is regarded as a group of jobs \( i \) following WSPT on machine \( k \) that job \( j \) is scheduled after job \( i \). By applying the WSPT rule, a new kind of \( P \) problem is generated, which is shown by \( P' \):

\[
\text{Problem} \ P' \ \text{minimize} \quad \sum_{j \in N} \sum_{k \in M} \left( w_j C_{jk} + y^k_j \sum_{i \in B^k_j \cup 0} x^k_{ij} \right)
\]

subject to

\[
\sum_{k \in M_1} \sum_{i \in B^k_j \cup 0} x^k_{ij} = 1 \quad \forall j \in N,
\]

\[
\sum_{j \in N} x^k_{0j} \leq 1 \quad \forall k \in M.
\]

Since the feasibility space of solutions decreased in problem \( P' \) (based on Smith rule), less computational time is needed to solve it. The above problem is a special kind of unrelated parallel machines with \( n \) jobs and \( m_1 + m_2 \) machines. The structure of its objective function cannot be found in any scheduling problem of unrelated machines.
\[
\sum_{i \in B^k} x^k_{ij} = \sum_{i \in A^k \cup n + 1} x^k_{ij} \quad \forall j \in N, k \in M, \tag{29}
\]

\[
C_{jk} = p_{jk} x^k_{ij} + \sum_{i \in B^k_j} (C_{ik} + p_{jk}) x^k_{ij} \quad \forall j \in N, k \in M, \tag{30}
\]

\[
C_{jk} \leq d_j \quad \forall j \in N, k \in M, \tag{31}
\]

\[
\sum_{k \in M} \sum_{i \in B^k_j} \sum_{j \in N} \text{dem}_{ij} x^k_{ij} \leq \text{Av}_r \quad \forall r \in R, \tag{32}
\]

\[
x^k_{ij} \in \{0, 1\} \quad \forall i, j \in N, k \in M. \tag{33}
\]

### 4.1. Linearization of the model

As it is clear, Equation (30) makes the model non-linear due to the multiplication of \(x^k_{ij}\) (a binary variable) by \(C_{ik}\) (a positive continuous variable) which occurs similarly to Equation (22). However, it can be linearized easily by defining a new variable of \(h^k_{ij}\) and applying Equations (34) to (38) to the model instead of Equation (30). This would result in a mixed-integer linear programming (MILP) model.

\[
h^k_{ij} \leq C_{ik} \quad \forall i, j \in N, \forall k \in M, \tag{34}
\]

\[
h^k_{ij} \leq \text{MM} x^k_{ij} \quad \forall i, j \in N, \forall k \in M, \tag{35}
\]

\[
h^k_{ij} \geq C_{ik} - \text{MM} \left(1 - x^k_{ij}\right) \quad \forall i, j \in N, \forall k \in M, \tag{36}
\]

\[
C_{jk} = p_{jk} x^k_{ij} + \sum_{i \in B^k_j} (h^k_{ij} + p_{jk} x^k_{ij}) \quad \forall i, j \in N, \forall k \in M, \tag{37}
\]

\[
h^k_{ij} \geq 0 \forall i, j \in N, \forall k \in M \tag{38}
\]

### 4.2. Robust counterpart formulation of the problem

Robust counterpart of the proposed model or RMILP model is formulated based on Bertsimas and Sim (2003). The uncertain parameter is \(p_{jk}\) which changes in \([\hat{p}_{jk} - \bar{p}_{jk}, \bar{p}_{jk} + \bar{p}_{jk}]\). This parameter causes uncertainty both in the objective function and constraint (Equation (37)). As defined before, for calculating each equation of completion time of job \(j\) on machine \(k\), parameter \(\Gamma_{jk}\) is defined so that \(\Gamma_{jk} \in [0, \max\{J_{jk}\}]\) is defined as a member of uncertainty coefficients set. Now, we have:
\[
C_{jk} = p_{jk} \left( x_{ij}^k + \sum_{i \in B_j^k} x_i^k \right) + \sum_{i \in B_j^k} h_{ij}^k \quad \forall j \in N, \forall k \in M,
\]

\[
C_{jk} = \sum_{i \in B_j^k} p_{jk} x_i^k + \sum_{i \in B_j^k} h_{ij}^k \quad \forall j \in N, \forall k \in M,
\]

Then,

\[
C_{jk} = p_{jk} \left( x_{ij}^k + \sum_{i \in B_j^k} x_i^k \right) + \sum_{i \in B_j^k} h_{ij}^k +
\]

\[
\max \left\{ s_jk \cup \{ t_jk \} \mid s_jk \subseteq J_k, s_jk \leq T_{jk}, t_jk \in J_k \setminus s_jk \right\} \left\{ \sum_{i \in s_jk} \hat{p}_{jk} x_{ij}^{k*} + (\Gamma_{jk} - \Gamma_{jk}) \hat{p}_{jk} x_{ij}^{k*} \right\}
\]

\[
\forall k \in M, \forall j \in N,
\]

\[
\beta_{jk}(x^*, \Gamma_{jk}) = \max \left\{ s_jk \cup \{ t_jk \} \mid s_jk \subseteq J_k, s_jk \leq T_{jk}, t_jk \in J_k \setminus s_jk \right\} \left\{ \sum_{i \in s_jk} \hat{p}_{jk} x_{ij}^{k*} + (\Gamma_{jk} - \Gamma_{jk}) \hat{p}_{jk} x_{ij}^{k*} \right\}
\]

\[
\forall k \in M, \forall j \in N.
\]

where \( \Gamma_{jk} \) can take any integer or non-integer value, the main point is that, if \( \Gamma_{jk} = 0 \), then \( \beta_{jk}(x^*, \Gamma_{jk}) = 0 \) and the robust problem turns into the nominal problem. Similarly, if \( \Gamma_{jk} = |J_k| \), Bertsimas’ and Sim’s method would be equal to Soyster’s method. Then, by changing \( \Gamma_{jk} \) in \([0,|J_k|]\) interval, the robustness is guaranteed. Here, \( t_{jk} \) is a subset of \( J_k \) such that \( t_{jk} \in J_k \setminus s_{jk} \), and \( s_{jk} \) is a subset of \( J_k \) that is equal such that the number of its members is at most \( \Gamma_{jk} \).

Now we have the previous problem as:

\[
\beta_{jk}(x^*, \Gamma_{jk}) = \min \sum_{i \in s_{jk}} \hat{p}_{jk} x_{ij}^{k*}
\]

subject to

\[
\sum_{i \in s_{jk}} z_{ij}^k \leq \Gamma_{jk} \quad \forall k \in M, \forall j \in N,
\]

\[
0 \leq z_{ij}^k \leq 1 \quad \forall k \in M, \forall i \in s_{jk}, \forall j \in N.
\]

Now, the dual mode of the problem is:

\[
\beta_{jk}(x^*, \Gamma_{jk}) = \max \sum_{i \in s_{jk}} r_{ij}^k + \Gamma_{jk} z_{jk}
\]
subject to
\[ z_{jk} + r_{ij}^k \geq \hat{p}_{jk} x_{ij}^k \quad \forall k \in M, \forall i \in s_{jk}, \forall j \in N, \quad (47) \]
\[ r_{ij}^k \geq 0, z_{jk} \geq 0 \quad \forall k \in M, \forall i \in s_{jk}, \forall j \in N. \quad (48) \]

where \( r_{ij}^k \) and \( z_{jk} \) are used to control the protection level against the possible fluctuations of the uncertain processing time parameter within the defined interval. Now the final linear robust form of the constraint is as follows:
\[ C_{jk} = \hat{p}_{jk} x_{ij}^k + \sum_{i \in s_{jk}} (h_{ij}^k + \hat{p}_{jk} x_{ij}^k) + z_{jk} \Gamma_{jk} + \sum_{i \in s_{jk}} r_{ij}^k \quad \forall i, j \in N, \forall k \in M, \quad (49) \]

subject to
\[ z_{jk} + r_{ij}^k \geq E_{jk} \hat{p}_{jk} \quad \forall i \in s_{jk}, \forall j \in N, \forall k \in M, \quad (50) \]
\[ -E_{jk} \leq x_{ij}^k \leq E_{jk} \quad \forall i \in s_{jk}, \forall j \in N, \forall k \in M, \quad (51) \]
\[ r_{ij}^k \geq 0, z_{jk} \geq 0, h_{ij}^k \geq 0, E_{jk} \geq 0 \quad \forall i \in s_{jk}, \forall j \in N, \forall k \in M. \quad (52) \]

Now the final RMILP model is:
\[
\text{minimize} \sum_{j \in N} \sum_{k \in M} \left( w_j C_{jk} + y_j^k \sum_{i \in B_{ik}^j} x_{ij}^k \right) \quad (53)
\]

subject to
\[ \sum_{k \in M} \sum_{i \in B_{ik}^j} x_{ij}^k = 1 \quad \forall j \in N, \quad (54) \]
\[ \sum_{j \in N} x_{ij}^k \leq 1 \quad \forall k \in M, \quad (55) \]
\[ \sum_{i \in B_{ik}^j \cup \emptyset} x_{ij}^k = \sum_{i \in A_{ik}^j \cup \emptyset} x_{ji}^k \quad \forall j \in N, k \in M, \quad (56) \]
\[ C_{jk} = p_{jk} x_{ij}^k + \sum_{i \in s_{jk}} (h_{ij}^k + p_{jk} x_{ij}^k) + z_{jk} \Gamma_{jk} + \sum_{i \in s_{jk}} r_{ij}^k \quad \forall i, j \in N, \forall k \in M, \quad (57) \]
\[ h_{ij}^k \leq C_{ik} \quad \forall i, j \in N, \forall k \in M \quad (58) \]
\[ h_{ij}^k \leq M x_{ij}^k \quad \forall i, j \in N, \forall k \in M, \quad (59) \]
\[ h_{ij}^k \geq C_{ik} - MM(1 - x_{ij}^k) \quad \forall i, j \in N, \forall k \in M, \] (60)

\[ z_{jk} + r_{ij}^k \geq E_{jk} \hat{p}_{jk} \quad \forall i \in s_{jk}, \forall j \in N, \forall k \in M, \] (61)

\[-E_{jk} \leq x_{ij}^k \leq E_{jk} \quad \forall i \in s_{jk}, \forall j \in N, \forall k \in M, \] (62)

\[ C_{jk} \leq d_j \quad \forall j \in N, k \in M, \] (63)

\[ \sum_{k \in M1} \sum_{i \in B_j^k \cup \emptyset} \sum_{j \in N} \text{dem}_{jkx_{ij}^k} \leq Av_r \quad \forall r \in R, \] (64)

\[ x_{ij}^k \in \{0, 1\}, h_{ij}^k \geq 0, \quad \forall i \in B_j^k \cup \emptyset, \forall j \in N, \forall k \in M, \] (65)

\[ r_{ij}^k \geq 0, z_{jk} \geq 0, E_{jk} \geq 0 \quad \forall i \in s_{jk}, \forall j \in N, \forall k \in M. \] (66)

5. Computational results

In this section, the applied steps for solving the proposed RMILP are explained.

**Step 1:** Based on Bertsimas and Sim approach, \( \Gamma \ (0 \leq \Gamma \leq |j| \times |k|) \) is determined for each \( j \) and each \( k \) in Equation. (45). So it is considered to be \( (|j| \times |k|)/2 \) as a golden mean.

**Step 2:** After choosing the value of \( \Gamma \), the deviation value of the processing time is defined as \( \hat{p} = \alpha \times p \) (\( \alpha \) is the predetermined parameter, namely, deviation value).

In determining the amount of changed processing time, the below cases should be considered:

1. If \( \Gamma \) is an integer, \( \Gamma \) jobs should be chosen and the processing time should be increased.
2. Otherwise, if \( \Gamma \) is not an integer, \( \Gamma \) jobs are chosen for increasing the processing time. For \( \Gamma - 1 \) jobs, we act like case 1 and for the last job, the non-integer part is multiplied by the deviation value of \( \hat{p} \), that is \( \hat{p}_{\text{lastjob}} = \alpha \times (\Gamma - 1) \times p \).

**Step 3:** We do this for all the processing jobs by machines in any solution to get the new matrix of processing time.

**Step 4:** After applying Steps 1 to 3, the robust solution is achieved.

5.1. Design of experiments

In this section, some instance problems are generated randomly to study the efficiency and verification of the proposed model and also to evaluate the effects of robustness. Fifteen scenarios including random instance problems are designed in different sizes. The information of the problems is given in Table 1. The input parameters are generated uniformly. Parameters \( w_j \sim U(3, 5), \ p_{jk} \sim U(4, 7), \ y_j^k \sim U(3, 5), \ d_j \sim U(12, 25), \ \text{dem}_{jk} \sim U(2, 10), \) and \( Av_r \) are considered to be 80% of the total demand of jobs in different problems.
It should be noted that problem 6 is regarded as a real case of Teram Chap Company which is one of the pioneers of printing and packaging industry in Iran, especially in box manufacturing (Teram Industrial Group, 2018). The most demanded products include luxury, chocolate boxes, beverages and food boxes, cosmetics, and pharmaceutical packaging boxes. The problem is studied for processing 20 chosen jobs on 3 in-house machines and 3 possible subcontractors to produce chocolate and food boxes. The values of the input parameters have been assigned based on the experts of the company.

Initially, both deterministic and robust models are solved using data given in Table 1. CPLEX solver of GAMS software version 23.6 is used to solve the model by a laptop of Intel Core i7, (8 GB RAM). Furthermore, a runtime limitation of 3600 is implemented to report the obtained solution. The computational results are shown in Table 2. The results of solving the robust counterpart problem are represented for the deviation levels of \( \hat{p} = 0.10p \) and \( \hat{p} = 0.2p \).

Figure 1 shows the differences between objective function values of deterministic and robust problems. As we can see, these differences are more tangible in large-sized problems. In fact, it has been clear that the value of the processing time plays an important role in making this difference. Managers should pay careful attention to these different deviation levels of processing times that have a direct impact on the objective and make it worse when increase.

Figure 2 depicts the reported run times up to 3600 seconds. As it is obvious, run time value in the robust problems is higher than deterministic problems. It demonstrates that the robust problem adds more complexity to the problem. Moreover, we can conclude that robust problems have a worse objective function due to its effort to generate a feasible solution in different uncertain conditions that will be done by imposing additional costs.

5.2. Sensitivity analysis

In this section, a sensitivity analysis is performed on \( dem_{jrk} \); i.e., the demand of job \( j \) for resource \( r \) for being processed on machine \( k \). Note that the parameter of processing time is the most effective which was investigated by RO approach. Here, three different changes ranges are considered to be analyzed. Parameter different values are specified in \((0.8 \ dem_{jrk}, dem_{jrk}, 1.2 \ dem_{jrk})\), which expresses a 20% decrease in the first range and a 20% increase in the last range. Table 3 shows the obtained results. As an example of

| Problem No. | Number of jobs | Number of in-house machines | Number of outsourcing machine |
|-------------|----------------|----------------------------|-------------------------------|
| 1           | 3              | 1                          | 1                             |
| 2           | 5              | 2                          | 1                             |
| 3           | 6              | 1                          | 2                             |
| 4           | 8              | 3                          | 2                             |
| 5           | 10             | 3                          | 3                             |
| 6           | 12             | 3                          | 3                             |
| 7           | 15             | 4                          | 3                             |
| 8           | 18             | 4                          | 4                             |
| 9           | 20             | 4                          | 4                             |
| 10          | 22             | 5                          | 4                             |
| 11          | 23             | 5                          | 5                             |
| 12          | 24             | 5                          | 5                             |
| 13          | 25             | 6                          | 5                             |
| 14          | 28             | 6                          | 6                             |
| 15          | 30             | 7                          | 6                             |
a real case, the behavior of the objective function against the changes of $dem_{jk}$ is depicted in Figure 3 for problem 6.

According to Figure 3, we can find that objective function value will be reduced relatively by decreasing the demand and similarly increases against demand reduction with different rates of change. We can conclude that the objective value has approximately a linear relation with the demand parameter. Thus, we can achieve an optimal strategy concerning the demand level control and by considering a tradeoff between in-house production and outsourcing based on different conditions.

5.3. Results comparison with Other solution techniques

Since the proposed model is an extended model of the previous one in the literature which was introduced by Mokhtari and Abadi (2013), a comparison is conducted to evaluate the superiority of our model. To this end, the applied solutions techniques for the proposed NLP model in the previous research including LINGO and Lagrangian method are considered to investigate the advantages and disadvantages of our deterministic MILP model. Six instances problems extracted from Mokhtari and Abadi (2013) are studied and the comparison results are represented in Table 4. It should be noted that our suggested MILP model is solved without capacity limitation of available resources (which was not studied in Mokhtari and Abadi (2013)) and without the 3600 runtime limitation. The run time limitation is set to 12 h for the comparison of CPLEX with LINGO according to Mokhtari & Abadi (2013).

As can be seen in Table 4, our proposed model and solver could yield better results compared to the NLP model of Mokhtari and Abadi (2013) in terms of Q-gap and runtime. In fact, our MILP model is solved exactly within remarkable lower runtimes by CPLEX. However, a Lagrangian heuristic could find good solutions within lower runtimes than CPLEX and LINGO. Figure 4 depicts the run time comparisons.

It should be noted that the considered gap is calculated as follows:

| Problem No. | Objective Value | Run time (second) | Objective Value | Run time (second) | Objective Value | Run time (second) |
|-------------|-----------------|-------------------|-----------------|-------------------|-----------------|-------------------|
| 1           | 57.51           | 0.73              | 57.51           | 1.34              | 57.51           | 1.48              |
| 2           | 109.66          | 1.23              | 110.89          | 1.78              | 112.12          | 1.8               |
| 3           | 146.04          | 2.52              | 148.89          | 6.06              | 151.74          | 7.19              |
| 4           | 181.89          | 4.56              | 179.84          | 8.23              | 182.55          | 9.84              |
| 5           | 234.92          | 24.11             | 239.65          | 6.23              | 242.65          | 82.21             |
| 6           | 304.86          | 67.1              | 311.2           | 394.54            | 316.71          | 640.2             |
| 7           | 443.75          | 354.6             | 454.23          | 1982.87           | 465.11          | 1972.4            |
| 8           | 531.24          | 1590.1            | 548.92          | 3420.4            | 561.37          | 3600              |
| 9           | 598.23          | 2019.04           | 616.87          | 3600              | 629.45          | 3600              |
| 10          | 654.08          | 3180.7            | 678.19          | 3600              | 693.3           | 3600              |
| 11          | 698.26          | 3600              | 721.08          | 3600              | 729.58          | 3600              |
| 12          | 769.23          | 3600              | 785.74          | 3600              | 798.65          | 3600              |
| 13          | 819.81          | 3600              | 836.02          | 3600              | 852.83          | 3600              |
| 14          | 903.24          | 3600              | 931.64          | 3600              | 954.29          | 3600              |
| 15          | 937.43          | 3600              | 977.63          | 3600              | 994.06          | 3600              |
| Ave.        | 3726.7          | 310.26            | 3960.9          | 1540.68           | 4079.1          | 1660.76           |
Figure 1. Objective functions comparison.
Figure 2. Run time comparison.
\[ \text{Gap}(\%) = 100 \times \frac{\text{(Lagrangian solution value} - \text{LINGO solution value})}{\text{LINGO solution value}} \]  

\[(67)\]

6. Discussion

The numerical results showed that the uncertainty yields a situation with more complexity and higher total cost. Furthermore, it made the model more complicated to be solved in comparison with the deterministic model. It has been found that the objective function values

Table 3. Computational results of sensitivity analysis.

| Problem No. | Deterministic | Robust (\( \hat{p} = 0.1p \)) | Robust (\( \hat{p} = 0.2p \)) |
|-------------|---------------|-------------------------------|-------------------------------|
|             | \((0.8 \text{dem}_{xk})\) | \((\text{dem}_{xk})\) | \((1.2 \text{dem}_{xk})\) | \((0.8 \text{dem}_{xk})\) | \((\text{dem}_{xk})\) | \((1.2 \text{dem}_{xk})\) | \((0.8 \text{dem}_{xk})\) | \((\text{dem}_{xk})\) | \((1.2 \text{dem}_{xk})\) |
| 1           | 56.07         | 57.51                         | 58.95                         | 56.07         | 57.51                         | 58.95                         | 56.07         | 57.51                         | 58.95                         |
| 2           | 106.48        | 109.66                        | 112.83                        | 107.72        | 110.89                        | 114.06                        | 108.95        | 112.12                        | 115.29                        |
| 3           | 148.24        | 146.04                        | 155.74                        | 145.02        | 148.89                        | 152.77                        | 147.87        | 151.74                        | 155.62                        |
| 4           | 176.16        | 181.89                        | 187.35                        | 174.13        | 179.84                        | 185.56                        | 176.83        | 182.55                        | 188.26                        |
| 5           | 228.47        | 234.92                        | 241.48                        | 235.9         | 239.65                        | 247.65                        | 235.9         | 242.65                        | 249.4                         |
| 6           | 296.33        | 304.86                        | 312.82                        | 301.47        | 311.63                        | 319.99                        | 307.61        | 316.71                        | 323.27                        |
| 7           | 434.32        | 443.75                        | 452.2                         | 442.5         | 454.23                        | 464.13                        | 455.48        | 465.11                        | 476.56                        |
| 8           | 522.82        | 531.24                        | 541.43                        | 537.3         | 548.92                        | 551.81                        | 549.95        | 561.37                        | 574.62                        |
| 9           | 587.6         | 598.23                        | 610.47                        | 605.04        | 616.87                        | 629.94                        | 615.82        | 629.45                        | 643.71                        |
| 10          | 637.63        | 654.08                        | 669.31                        | 659.44        | 678.19                        | 694.87                        | 675.05        | 693.5                         | 712.7                         |
| 11          | 674.67        | 698.26                        | 714.2                         | 706.58        | 721.08                        | 742.18                        | 710.83        | 729.58                        | 750.67                        |
| 12          | 745.49        | 769.23                        | 786.61                        | 769.8         | 785.74                        | 803.32                        | 778.53        | 798.65                        | 823.67                        |
| 13          | 794.16        | 819.81                        | 841.25                        | 821.05        | 836.02                        | 854.56                        | 834.93        | 852.83                        | 875.98                        |
| 14          | 882.69        | 903.24                        | 922.79                        | 914.6         | 931.64                        | 953.94                        | 933.2         | 954.29                        | 973.04                        |
| 15          | 917.03        | 937.43                        | 958.99                        | 959.39        | 977.63                        | 1003.23                       | 969.04        | 994.06                        | 1012.36                       |
| Ave.        | 480.54        | 492.67                        | 504.42                        | 495.73        | 506.55                        | 776.68                        | 503.73        | 516.14                        | 528.94                        |

Figure 3. Sensitivity analysis performed on the deterministic and robust model.
and the run time values are bigger in the robust problems of 1–15. In the real world condition, managers are faced with uncertainty and need to provide an overview of decision-making processes in these different uncertain conditions. For this purpose, as a real case example in Teram Chap Company, the proposed methodology provided comparing results for both deterministic and robust problem to show the behavior of the objective value. On the other hand, a sensitivity analysis was performed on the parameter of demand for in-house production resources to evaluate the objective behavior alternatively. According to this proposed methodology, it was revealed that the robust problem with the uncertainty level of 0.1 makes a significant increase in total cost in comparison with the deterministic problem. However, the next increase in the uncertainty level of 0.2 is not as large as it is and these differences become more remarkable with the increase of the problem size. This happens also in sensitivity analysis result so that the objective value changes from the robust problem with the uncertainty level of 0.2 to the robust problem with the uncertainty level of 0.1 is smaller than the objective value changes from the robust problem with the uncertainty level of 0.1 to the deterministic problem.

Moreover, the final comparison analysis between our MILP model and the proposed NLP model by Mokhtari and Abadi (2013) demonstrated that CPLEX can generate

| Problem No. | \( |N| \) | \( |M_1| \) | \( |M_2| \) | NLP model of Mokhtari and Abadi (Mokhtari & Abadi, 2013) | Our MILP model | Lagrangian heuristic | LINGO | CPLEX for MILP model |
|-------------|---------|---------|---------|---------------------------------|----------------|---------------------|-------|---------------------|
| 1           | 25      | 6       | 5       | 0.026                           | 0              | 1.13                 | 5853  | 4102                 |
| 2           | 30      | 6       | 5       | 0.042                           | 0              | 2.31                 | 10,854| 4620                 |
| 3           | 45      | 6       | 5       | 0.093                           | 0              | 6.08                 | 24,243| 7826                 |
| 4           | 70      | 8       | 5       | 0.016                           | 0              | 18.91               | >43,200| 12,166               |
| 5           | 90      | 8       | 5       | 0.014                           | 0              | 41.61               | >43,200| 19,607               |
| 6           | 120     | 10      | 5       | 0.019                           | 0              | 59.74               | >43,200| 32,904               |

Figure 4. Run time comparisons of CPLEX, Lagrangian method and LINGO.
high-quality solutions in comparison with LINGO and Lagrangian method, however, the Lagrangian method is able to find near-optimal solutions within lower run times. As a great achievement, the results of this research may be considered as an applicable managerial tool to help them in the decision-making process under uncertainty.

7. Conclusion and outlook

Scheduling problem is one of the most well-known issues in production system studies. Due to the increasing importance of on-time delivery of jobs to the customers, this paper suggests a single-stage just-in-time production-scheduling model with an option of outsourcing. The goal of the problem is to find the optimal sequence of jobs for each machine. To analyze more real word assumptions, the due date of jobs and resource capacity constraint are studied in the model. Moreover, the uncertain nature of processing time is evaluated by the robust optimization approach. The proposed methodology is validated by generating and solving 15 different-sized instance problems. The results show that uncertain jobs processing time lead to having different solutions compared to the deterministic problem, and these differences mostly increase machines and outsourcing to the subcontractors. Then, in the sensitivity analysis, the importance of the processing cost parameter on objective function values in deterministic and robust problems is studied. Finally, the superiority of our proposed MILP was tested compared to the other solution techniques in the literature.

For future works, the following suggestions may be considered:

- Adding a planning horizon into the model such as a weekly periodic scheduling model,
- Studying other objective functions such as reliability maximization of scheduling,
- Applying other methods to deal with uncertainty such as stochastic programming,
- Developing efficient meta-heuristics to solve the large-sized problems in a reasonable run time.

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