Dynamic Behaviors of Supersonic Granular Media under Vertical Vibration
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Abstract
We present experimental study of vibrofluidized granular materials by high speed photography. Statistical results present the averaged dynamic behaviors of granular materials in one cycle, including the variations of height, velocity and mechanical energy of the center of mass. Furthermore, time-space distribution of granular temperature which corresponds to the random kinetic energy shows that a temperature peak forms in the compression period and propagates upward with a steepened front. The Mach number in the steepened front is found to be greater than unity, indicating a shock propagating in the supersonic granular media.

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1 Introduction
Granular materials are conglomerations of macroscopic particles. They are so ubiquitous that they attract interest from people all over the world for a long time. It has been known that with different energy input, granular materials behave like solids, liquids or gases [1]. When fluidized, granular materials show many interesting phenomena such as surface pattern [2], convection [3], heap formation [4], localized excitation [5] and so on. To describe dynamical behaviors of fluidized granular materials, properties such as density, pressure and temperature have been defined and various theories have been introduced [6]. Correspondingly noninvasive methods for detecting these properties from statistical average of experimental data is created [7]. Moreover computer simulations [8] are performed to compare with experimental results. Recent simulation results [9] indicate that it is relatively easy for vertical vibrated granular materials to be supersonic. In this paper,
we use high speed photography to explore time-dependent behaviors of a two-dimensional vibrofluidized granular materials and give evidence of supersonic granular flow by experimental results of granular temperature waves.

2 Method

The experiment is conducted with a rectangular container mounted on the vibration exciter (Brüel & Kjær 4805), which is controlled by a function generator (type HP 3314A). We use steel spheres with diameter $d = 7\text{mm}$ and density $\rho = 7900\text{kg/m}^3$. The container is made up of two parallel $155\text{mm}(\text{length}) \times 280\text{mm}(\text{height})$ glass plates separated by $7.5\text{mm}$ vertically adhered in a Plexiglas bracket. Particle number $N$ changes from 60 to 210. The vibrator undergoes sinusoidal vibrations controlled by driving parameters frequency $f$ and nondimensional acceleration $\Gamma = 4\pi^2 f^2 A/g$, where $A$ is the driving amplitude, and $g$ the gravitational acceleration. High speed camera (Redlake MASD MotionScope PCI 2000sc) is employed to record the movements of all spheres. A frequency multiplication and phase lock circuit is used to generate external trigger signals for the camera. The acquisition rate is $N_p f$ with the multiple number $N_p = 25$.

Image processing technique [7] is used to track locations and velocities of particles. Every recorded image ($22d(\text{wide}) \times 30d(\text{height})$) is divided uniformly into $10(\text{wide}) \times 30(\text{height})$ cells. The cells are numbered by indices $i$ and $j$ which specify the positions of the cells along the horizontal and vertical directions. To obtain time-dependant behaviors of granular materials, average is performed at $N_p$ phase points over all the 326 cycles captured. Granular temperature is designated by $T(i, j, k) = \frac{1}{N_c} \sum_{n=1}^{N_c} \frac{1}{2} |\mathbf{v}_n - \mathbf{v}_b(i, j, k)|^2$, in which $N_c$ is total number of particles in cell $(i, j)$ at time $k$, $\mathbf{v}_n$ is velocity vector of the $n$th particle, and background velocity $\mathbf{v}_b(i, j, k)$ is the mean of the particle velocities in cell $(i, j)$ at the same time $k$ for all the cycles.

3 Results and Discussion

With the increase of $\Gamma$ from a critical value and other parameters ($f = 15\text{Hz}$, $N = 60$) fixed, granular materials become fluidized from upper to lower layers. The packing fraction decreases and particles change locations relative to their neighbors frequently. The squared amplitude spectrum of the vertical displacement of the center of mass (c.m.) is used as an abbreviation
for the center of mass hereafter) in laboratory reference frame with $\Gamma = 5$ is shown in Fig. 1. It is composed of a noisy background and two lines at the basic and second harmonic. The spectrum line at the basic harmonic is about 3 order of magnitude larger than that at the second harmonic. This indicates that granular materials as a whole move periodically with the vibrating frequency of the plate. Although spectrum lines at higher harmonic begin to surpass the background noise and become comparable with that of the second harmonic with the increase of $N$ from 60 to 150 and other parameters fixed, the periodicity still exists for the spectrum line at the basic frequency keeps one order larger than that at higher harmonic. The periodicity also exists with other driving parameters we used.

Fig. 2 shows time-dependant behaviors of the center of mass, which indicates that granular materials as a whole undergo three stages in one cycle. In the first stage (from $4\pi/25$ to $22\pi/25$), granular materials fly freely. The trajectory of the center of mass is parabolic, its velocity decreases almost linearly with time and the mechanical energy (defined as $E_{c.m.} = NmgY_{c.m.} + NmV_{c.m.}^2/2$, in which $m$ is the mass of one particle) fixes at about $27.5mJ$. In Stage 2 (from $22\pi/25$ to $38\pi/25$), granular materials collide with the plate and compress. Height of the center of mass continues to decrease but deviates from originally parabolic track. Particles at lower layers change their moving direction by colliding with the plate which leads to the increase of negative c.m. velocity. In this stage the mechanical energy
Figure 2: Height (a), velocity (b) and mechanical velocity (c) of the center of mass as a function of time with driving parameters $\Gamma = 5$ and $f = 15\, \text{Hz}$, time 0 corresponds to the time when the plate is at its maximum height.

of c.m. drastically decreases, which is due to the decrease of both the height and scalar velocity. The third stage is the restoration stage. It starts at $38\pi/25$ when granular materials reach the most compact state. After this time c.m. of granular materials changes its direction and moves upward and all of the $Y_{c.m.}$, $V_{c.m.}$ and $E_{c.m.}$ increase. This stage lasts until $4\pi/25$ at the next vibration cycle when granular layer begins to leave the plate and fly freely again. Then a new cycle begins. Experiments with different driving parameters ($f = 15\, \text{Hz}$ and $20\, \text{Hz}$, $\Gamma = 2$ to 8) and particle numbers ($N = 60$ to 210) are also performed and similar time-dependent behaviors of the center of mass are obtained except for some parameters with which the well of input energy exists [10].

Vibrofluidized granular materials are strongly dissipative for inelastic collisions between particles. The vibrating plate who acts as energy source to keep granular media fluidized are also the source of mechanical waves propagating in the media. Fig. 3 shows the propagation of a temperature wave in one vibration cycle. As granular materials fly freely and expands, the granular temperature as a whole decreases by the dissipation. In the second stage, as an obstacle the plate collide with granular materials. The velocities of particles at the lower layers drop to that of the plate while particles at higher
Figure 3: Granular temperature as a function of height from $\text{Time} = 1.52\pi$ to $\text{Time} = 2.96\pi$ of the next cycle with parameters the same as those in Fig. 2.

layers still fly downward freely. Thus at $\text{Time} = 1.52\pi$ an active area of collisions between particles as well as a temperature peak is formed. Between $\text{Time} = 1.52\pi$ and $1.76\pi$, the value of the temperature peak increases to a peak value $0.097\, mJ$ at about $1.76\pi$ and then slowly decreases as time elapses. After $\text{Time} = 1.76\pi$, the perturbation of granular temperature propagates upward with a uniform velocity $v_T = 1.51\, m/s$. At $\text{Time} = 2.96\pi$ of the next cycle, the wave transmit to height 11 as its amplitude decays. With the granular temperature and volume fraction obtained by experiments, we calculate the wave propagation velocity and the Mach number [9, 11]. The average phase velocity ($c = 0.438\, m/s$) is comparable with the granular flow velocity relative to the vibrating plate. The Mach number increases to be greater than unity at the steepened temperature front when granular layer collides with the plate. At the end time of the compression period $\text{Time} = 1.52\pi$ the Mach number reaches its maximum 1.71, which indicates that there exist supersonic granular flow as the collisions between particles and the plate occur.
4 Conclusions

In summary, we present experimental study of supersonic granular materials. Power spectrum of the height of the center of mass indicates that granular materials as a whole move periodically with the driving frequency. Statistical averages of the height, velocity and mechanical energy of the center of mass in one vibration cycle indicate that granular materials undergo three stages: free flight stage, compression stage and restoration stage. Time-space distribution of granular temperature shows that a temperature peak forms within the compression period of the granular materials and propagates upward with a steepened front when the particles collide with the plate and compress. The existence of this steepened temperature front and the maximum Mach number in this media demonstrate that it is easy for vertical vibrated granular flow to be supersonic. This is due to that the acoustic velocity in granular materials is less than that of granular flow relative to the vibrating plate.

Internal wave propagations of mechanical waves take the role of transporting energy from the plate to granular layer. Study of shock propagation in vibrofluidized granular materials and its relationship with surface instability will be the subject of the future work, which may provide useful information about the mechanism of surface waves and other interesting phenomena of vibrofluidized granular materials.

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