Electric dipole polarizability in neutron-rich Sn isotopes as a probe of nuclear isovector properties

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Abstract

The determination of nuclear symmetry energy, and in particular, its density dependence, is a long-standing problem for nuclear physics community. Previous studies have found that the product of electric dipole polarizability $a_D$ and symmetry energy $\Delta S$ at saturation density $\rho_0$ has a strong linear correlation with $L$, the slope parameter of symmetry energy. However, current uncertainty of $J$ hinders the precise constraint on $L$. We investigate the correlations between electric dipole polarizability $a_D$ (or times symmetry energy at saturation density $\rho_0$) in Sn isotopes and the slope parameter of symmetry energy $L$ using the quasiparticle random-phase approximation based on Skyrme Hartree-Fock-Bogoliubov. A strong and model-independent linear correlation between $a_D$ and $L$ is found in neutron-rich Sn isotopes where pygmy dipole resonance (PDR) gives a considerable contribution to $\Delta S$, attributed to the pairing correlations playing important roles through PDR. This newly discovered linear correlation would help one to constrain $L$ and neutron-skin thickness $\Delta R_{np}$ stably if $a_D$ is measured with high resolution in neutron-rich nuclei. Besides, a linear correlation between $a_D J$ in a nucleus around $\beta$-stability line and $a_D$ in a neutron-rich nucleus can be used to assess $a_D$ in neutron-rich nuclei.

Keywords: Electric dipole polarizability, Slope parameter of symmetry energy, Neutron-skin thickness

1. Introduction

The determination of nuclear equation of state (EoS) at high density is a challenge for both experimental and theoretical nuclear physics [1, 2], which is crucial for constraining current theoretical models [3, 4] and understanding many phenomena in astrophysics [5, 6]. The biggest uncertainty of EoS comes from its isovector parts, which are governed by the nuclear symmetry energy $S(\rho)$. The symmetry energy can be expanded as a function of $\varepsilon = (\rho - \rho_0)/3\rho_0$ by

$$ S(\rho) = J + L \varepsilon + \frac{1}{2} K_{sym} \varepsilon^2 + ... \quad (1) $$

where $J = S(\rho_0)$ is the symmetry energy at saturation density $\rho_0$, while $L = 3\rho_0 \left( \frac{\partial^2 S}{\partial \rho^2} \right)_{\rho_0}$ and $K_{sym} = 9\rho_0^2 \left( \frac{\partial^3 S}{\partial \rho^3} \right)_{\rho_0}$ correspond to the slope and curvature parameters at saturation density, respectively.

The slope parameter of symmetry energy $L$ determines the behavior of symmetry energy at high density, however, it varies a lot in different nuclear models. Constraints on $L$ can be obtained from heavy-ion collisions [1, 7], properties of neutron stars [5, 8], and nuclear properties of ground state and excited states of finite nuclei [9]. For example, it is revealed that $L$ is proportional to the neutron-skin thickness $\Delta R_{np}$ by droplet model [10, 11], which is further conformed by many microscopic models [12, 13]. However, the obstacle in the measurements of neutron radius hinders the access to high-resolution neutron skin data. As an alternative, charge radii difference $\Delta R_{p}$ between mirror nuclei is proposed as another possible way to constrain $L$ [14–16], which also faces difficulties in the measurements of charge radius in proton-rich nucleus.

The electric dipole ($E1$) excitation in nucleus is mainly composed of the giant dipole resonance (GDR), which is formed by the relative dipole oscillation between neutrons and protons, thus reflecting asymmetry information in nuclear EoS. The electric dipole polarizability $a_D$, being proportional to the inverse energy-weighted sum rule of $E1$ excitation, can be served as a possible probe for nuclear isovector properties. Theoretically, (quasiparticle) random-phase approximation [(Q)RPA] approach is widely used to describe small oscillations of nucleus, such as $E1$ excitations. The self-consistent (Q)RPA models have been developed based on Skyrme density functionals [17–19], Gogny density functionals [20, 21], and relativistic density functionals [22–25]. Global properties of GDR, such as centroid energies and electric dipole polarizabilities, can be well described within this approximation.

Based on these self-consistent (Q)RPA models, the correlations between electric dipole polarizability $a_D$ and other nuclear isovector properties have been investigated in recent years. Calculations performed by RPA model based on Skyrme density functionals SV-min series [26] and relativistic density functionals RMF-$\delta$-t series in $^{208}$Pb suggested a strong linear correlation between $a_D$ and neutron-skin thickness $\Delta R_{np}$ [27]. However, when one combines the results from a host of different nuclear density functionals, this linear correlation is not universal anymore [28]. Starting from droplet model, and further supported by RPA calculations based on many different Skyrme and rel-
ativistic density functionals in $^{208}$Pb, the product of dipole polarizability and symmetry energy at saturation density $\alpha_D J$ was suggested to be much better correlated with neutron-skin thickness and symmetry energy slope parameter $L$ than $\alpha_D$ alone is [29]. Based on this correlation, $L = 43 \pm (6)_{\text{exp}} \pm (8)_{\text{theor}} \pm (12)_{\text{est}}$ MeV was given by using the experimental $\alpha_D$ value in $^{208}$Pb [29], and the intervals $J = 30 \sim 35$ MeV and $L = 20 \sim 66$ MeV were further obtained by combining the measured polarizabilities in $^{68}$Ni, $^{120}$Sn and $^{208}$Pb [30]. Below saturation density, $\alpha_D$ in $^{208}$Pb was also found to be sensitive to both the symmetry energy $S(\rho_c)$ and slope parameter $L(\rho_c)$ at the subsaturation cross density $\rho_c = 0.11 fm^{-3}$ [31]. Since $S(\rho_c)$ is well constrained, $L(\rho_c)$ can be strongly constrained from experimental $\alpha_D$ in $^{208}$Pb [31]. At $\rho_c = \rho_0/3$, another linear correlation was built between $\alpha_D^{1/3}$ and $S(\rho_c)$ [32]. Besides, $\alpha_D$ between two different nuclei [33], as well as $\alpha_D J$ between two different nuclei [30], were also shown to have good linear correlations.

In recent years, the electric dipole polarizabilities $\alpha_D$ in $^{208}$Pb [34], $^{48}$Ca [35], and stable Sn isotopes [33, 36, 37] were measured with high resolution via polarized proton inelastic scattering at extreme forward angles [38]. For unstable nucleus $^{68}$Ni, $\alpha_D$ was also extracted by Coulomb excitation in inverse kinematics [39]. However, there are problems when one uses these high-resolution dipole polarizability data to constrain isovector properties: the constraints on $L$ or $\Delta R_{np}$ is either with big uncertainties due to the uncertainty of $J$ or in model-dependent ways. One way to solve the problem and constrain $L$ stably is to find a direct and model-independent correlation between $\alpha_D$ and $L$. Although the previous studies have shown that the model-independent linear correlation only exists between $\alpha_D J$ and $L$, it was only limited to stable nuclei or nuclei near $\beta$-stability line. It is well known that exotic phenomena will present when approaching to nuclei far from $\beta$-stability line, such as novel shell structures [40–44], new types of excitations [23, 45, 46], and so on. For $E1$ excitations, the pygmy dipole resonance (PDR) appears in neutron-rich nuclei [23, 45, 46], which would cause different characteristics of $E1$ excitations compared to the ones around $\beta$-stability line, and further affect $\alpha_D$. So an interesting question is if the linear correlation between $\alpha_D J$ and $L$ observed in stable nuclei still holds and new correlations would appear in neutron-rich nuclei.

Therefore, in our study we will explore the correlations between $\alpha_D$ and nuclear isovector properties such as slope parameter $L$ and neutron-skin thickness $\Delta R_{np}$, in even-even Sn isotopes from neutron-deficient $^{100}$Sn to neutron-rich $^{164}$Sn. The calculations are performed by QRPA based on Skyrme Hartree-Fock-Bogoliubov (HFB) model, in which the spherical symmetries are imposed. The linear correlations are evaluated by a least-square regression analysis. Based on the newly discovered correlations, constraints on $L$ and neutron-skin thickness will be discussed.

2. Theoretical Framework

We carry out a self-consistent HFB+QRPA calculation of $E1$ strength using 19 Skyrme functionals: SIII, STV, SV, SVI [47], SLy230a, SLy230b, SLy4, SLy5, SLy8 [52, 53], SAMi [54], SAM-J30, SAMi-J31, SAMi-J32, SAMi-J33 [55], SGi, SGII [48], SkM* [50], SkA [51]. The detailed formulas of QRPA on top of HFB can be found in Ref. [18]. The density-dependent zero-range surface pairing force is implemented in the particle-particle channel,

$$V_{pp}(r_1, r_2) = V_0 \left[ 1 - \frac{\rho(r)}{\rho_0} \right] \delta(r_1 - r_2)$$

where $\rho = (r_1 + r_2)/2$, and $\rho_0 = 0.16 fm^{-3}$ is the nuclear saturation density, while $V_0$ is adjusted by fitting neutron pairing gaps of $^{116-130}$Sn according to the five-point formula [56]. The electric dipole polarizability $\alpha_D$ is given by

$$\alpha_D = \frac{8\pi e^2}{9} m_{-1} m_{-1} = \sum \left| \langle \psi_e | F_{IV}^{(1)} | \psi_0 \rangle \right|^2 / E_v$$

where $\psi_e$ and $E_v$ are the eigenstates and eigenvalues of QRPA equations, and $\psi_0$ is the ground state. $m_{-1}$ is the inverse energy-weighted sum rule (EWSR), which is calculated using the isovector dipole operator

$$F_{IV}^{(1)} = \frac{N}{A} \sum_{p=1}^{Z} r_p Y_{1p} - \frac{Z}{A} \sum_{n=1}^{N} r_n Y_{1n}$$

where $A, N, Z$ denote mass number, neutron number, proton number, and $Y_{1p}$ are the spherical harmonics. In our calculations, the quasiparticle energy cutoff $E_{cut}$ is set as 90 MeV and the total angular momentum cutoff of quasiparticle $J_{max}$ is set as 21/2 to ensure the convergence of numerical results.

3. Results and Discussions

3.1. Correlations between $\alpha_D$ and nuclear isovector properties

First of all, we study if the previously discovered linear correlation between $\alpha_D J$ and $L$ holds in the whole tin isotopes from neutron-deficient ones to neutron-rich ones. So in Tab. 1, Pearson correlation coefficients (or Pearson’s coefficients) $r$ between $\alpha_D J$ and $L$ in even-even Sn isotopes from $^{100}$Sn to $^{160}$Sn, as well as the corresponding slopes $k$ of the regression lines are shown based on the HFB+QRPA calculations using 19 Skyrme density functionals. Pearson’s coefficient $r$ is a statistic that measures linear correlation between two variables, which is defined by the covariance of two variables divided by the product of their standard deviations. A value of $|r| = 1$ means that the two observables are fully linearly correlated while $r = 0$ are

| Nucleus | $^{100}$Sn | $^{110}$Sn | $^{120}$Sn | $^{130}$Sn | $^{140}$Sn | $^{150}$Sn | $^{160}$Sn |
|--------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $r$    | 0.965     | 0.966     | 0.974     | 0.961     | 0.966     | 0.940     | 0.937     |
| $k$ ($fm^3$) | 0.844 | 1.066 | 1.383 | 1.543 | 2.272 | 2.880 | 3.541 |
ing strong linear correlations between $\alpha$ only in stable nuclei as revealed in previous studies \cite{29}. The larger $k$ value totally uncorrelated. From Tab. 1, one can see the Pearson’s coefficients $r$ in the whole Sn isotopes are all above 0.9, showing strong linear correlations between $\alpha_D J$ and $L$. So it further proofs this linear correlation is a universal one which exists not only in stable nuclei as revealed in previous studies \cite{29} but also in neutron-deficient and neutron-rich nuclei. The corresponding slope $k$ of the regression line shows a clear increase trend with the increase of neutron number. The larger $k$ value means a more rapid increase of $\alpha_D J$ as a function of $L$, which gives a smaller range of $L$ under the same uncertainty of $\alpha_D J$. So the slope $k$ of the regression line is an important quantity to select good candidate nuclei as probes of nuclear isovector properties, which will be discussed in details in Sec. 3.2.

Although the above correlation is universal, it cannot provide a stiff constraint on the slope parameter of symmetry energy $L$ due to the uncertainty in the symmetry energy $J$. For example, by adopting $J = 31 \pm 2$ MeV, Roca-Maza et al. obtained $L = 43 \pm (6)_{\text{exp}} \pm (8)_{\text{theor}} \pm (12)_{\text{ext}}$ MeV, where the uncertainty $12$ MeV comes from the uncertainty of $J$ \cite{29}. So it would be better to find a direct correlation between $\alpha_D$ and $L$. Previous studies have shown that $L$ and $\alpha_D$ have a good linear correlation within some specific parameter family \cite{27}, however, by including different parameter families, this correlation becomes bad, for example, in $^{208}$Pb the Pearson’s coefficient $r$ was given as $r = 0.62$ \cite{29} and $r = 0.77$ \cite{28}. Here we recheck the correlation between the dipole polarizability $\alpha_D$ and the slope parameter $L$ of symmetry energy for the whole tin isotopes from neutron-deficient ones to neutron-rich ones, as shown in Fig. 1, to see if the previous conclusions still hold. In stable nucleus $^{120}$Sn, for some specific Skyrme parameter family, such as SAMi (green diamonds) or SIII-SVI (up blue triangles), one can observe a good linear correlation, in agreement with Ref. \cite{27}. However, when one includes more different Skyrme parameter sets, the linear correlation becomes poor, and the Pearson coefficient $r$ is around 0.8, again in agreement with the case in $^{208}$Pb \cite{28, 29}. Similar situations still exist in nuclei not far from the stability line such as $^{100,110,130}$Sn.

However, the cases become totally different in the neutron-rich nuclei. The coefficients are above 0.9 for the isotopes with mass number $A \geq 140$, which present strong correlations between $\alpha_D$ and $L$ in the neutron-rich Sn isotopes. After $A \geq 146$, the correlation between $\alpha_D$ and $L$ is even better than the one between $\alpha_D J$ and $L$. We stress here the assessments are carried out by different Skyrme functional families. For the neutron-rich nuclei of $A \geq 140$ with a clear linear correlation, we fur-
ther give the slopes $k$ of the regression lines. It is seen that $k$
becomes larger with the increase of neutron number, which
implies that the more neutron rich the nucleus is, the better probe
it can be served as for nuclear isovector properties, seeing de-
tailed discussions in Sec. 3.2.

To understand the above strong linear correlations in neutron-
rich Sn isotopes, we first investigate the role of pairing cor-
relations. So in Fig. 2 the correlations between $\alpha_D$ and $L$
in $^{120,140,150,160}$Sn are studied without considering pairing effec-
ts. For stable nucleus $^{120}$Sn, the correlation between $\alpha_D$ and $L$
is similar as the case with pairing correlations, where the Pea-
son’s coefficient is only slightly reduced without the inclusion
of pairing correlations. However, for these three neutron-rich
nuclei $^{140,150,160}$Sn, the linear correlations become much worse,
where the Pearson’s coefficients are largely reduced to the val-
ues 0.817, 0.873 and 0.867, respectively, being all below 0.9. It
shows that the pairing correlations play important roles in keep-
ing strong linear correlations between $\alpha_D$ and $L$ in neutron-rich
Sn isotopes.

On the other hand, for neutron-rich nuclei, the PDR appears
in the low-energy part of $E1$ transition strength distribution,
which would give big contributions to the dipole polarizabil-
ity. Since PDR represents an oscillation between neutron skin
and nearly isospin-saturated core, the correlations between its
strengths and symmetry energy were also explored \[27, 57–59\],
although it is still an open question. Inspired by this, we extract
the contributions of PDR to $\alpha_D$ in Sn isotopes in Fig. 3, where
the total dipole polarizabilities and contributions from PDR as
functions of mass number $A$ in even-even Sn isotopes calculated
by QRPA and RPA using Skyrme functional SLy4 are plotted.
According to the dipole strength distributions and the transition
densities, different energies are selected as the upper boundaries
of PDR for different Skyrme functionals, which are 9.0 MeV
for SVI, 10.0 MeV for SIII, SLy family, SkM, SkM\*, SGI, 10.5 MeV
for Ska, SAMi family, 11.0 MeV for SGI, 12.0 MeV

![Figure 3: (Color online) The dipole polarizabilities as functions of mass number $A$ in even-even Sn isotopes calculated by QRPA (square line) and RPA (circle line) using Skyrme functional SLy4. The total dipole polarizabilities (red) and the contributions from PDR (blue) are shown respectively.](image1)

![Figure 4: (Color online) Plots for dipole polarizability contributed by PDR against slope parameter of symmetry energy in $^{134,140,150,160}$Sn isotopes calculated by QRPA based on HFB with 19 Skyrme density functionals: SIII, SIV, SV, SIII (blue up triangles); SLy230a, SLy230b, SLy4, SLy5, SLy8 (red circles); SAMi, SAMi-J30, SAMi-J31, SAMi-J32, SAMi-J33 (green diamonds); SGI, SGIi, SkM, SkM\*, Ska (black squares). A linear fit is done for each nucleus (red solid line) with a corresponding Pearson’s coefficient $r$.](image2)

![Figure 5: (Color online) Plots for neutron-skin thickness $\Delta R_{np}$ against dipole polarizability $\alpha_D$ in $^{150,160}$Sn calculated by QRPA based on HFB with 19 Skyrme density functionals: SIII, SIV, SV, SIII (blue up triangles); SLy230a, SLy230b, SLy4, SLy5, SLy8 (red circles); SAMi, SAMi-J30, SAMi-J31, SAMi-J32, SAMi-J33 (green diamonds); SGI, SGIi, SkM, SkM\*, Ska (black squares). A regression line (red solid line) is obtained by a least-square linear fit of the calculated $\Delta R_{np}$ as a function of $\alpha_D$. $r$ is Pearson’s coefficient and $k$ (fm$^{-2}$) is the slope of regression line.](image3)
In Fig. 4 we further study the correlation between dipole polarizabilities $\alpha_D$ contributed by PDR and the slope parameter $L$ of symmetry energy in $^{134}$Sn, $^{140}$Sn, $^{150}$Sn, $^{160}$Sn isotopes. It shows that polarizability $\alpha_D$ of PDR has a good correlation with the slope parameter $L$ in general, which enhances the linear correlations between the total $\alpha_D$ and symmetry energy slope parameter $L$.

Apart from the correlation between $\alpha_D$ and $L$, the correlation between $\alpha_D$ and another important isovector property, i.e., neutron-skin thickness, is also investigated, and the plots for neutron-skin thickness against dipole polarizability in $^{150,160}$Sn are shown in Fig. 5. Not surprisingly, the linear correlations between $\Delta R_{np}$ and $\alpha_D$ in $^{150}$Sn and $^{160}$Sn are strong with $r = 0.939$ and $r = 0.947$ respectively, since the neutron-skin thickness $\Delta R_{np}$ and $L$ are reported to have a good linear correlation when $|N-Z|$ is large [15]. The slopes $k$ of regression lines, fitted by $\Delta R_{np}$ as a function of $\alpha_D$, are generally small in these neutron-rich nuclei, suggesting that $\alpha_D$ in neutron-rich nuclei can provide an effective constraint on neutron-skin thickness of the corresponding nuclei.

### 3.2. $\alpha_D$ as a probe of nuclear isovector properties

In Sec. 3.1, the correlations between $\alpha_D$ (or $\alpha_D J$) and nuclear isovector properties, e.g., $L$, $\Delta R_{np}$, are investigated for the whole tin isotopes, so in the following, we will analyse what information we can obtain from these correlations, and which nucleus could be treated as a proper probe of nuclear isovector properties in terms of dipole polarizabilities.

Experimentally, the dipole polarizabilities of $^{208}$Pb [34], $^{68}$Ni [39], $^{48}$Ca [35], and stable Sn isotopes [36] were measured with high resolution. The correlations between $\alpha_D J$ and $L$ are always strong for both stable nuclei and nuclei far from stability line from previous studies and our results in Sec. 3.1. So in Tab. 2, we show the constraints on the slope parameter of symmetry energy $L$ from experimental dipole polarizability values $\alpha_D^{exp}$ using correlation between $\alpha_D J$ and $L$ in these experimentally measured nuclei. The correlations between $\alpha_D J$ and $L$ are obtained by QRPA calculations using 19 Skyrme density functionals. The Pearson’s coefficient $r$ and the slope $k$ of the regression line $\beta$ = $\alpha_D J$ as a function of $L$ are also given. $J = 31.7 \pm 3.2$ MeV is adopted [2]. $\Delta L_{min}$ denotes the uncertainty coming from the uncertainty of $J$.

| Nucleus | $\alpha_D^{exp}$ (fm$^3$) | $r$ | $k$ (fm$^3$) | $L$ (MeV) | $\Delta L_{min}$ (MeV) |
|---------|----------------|-----|-------|----------|------------------|
| $^{208}$Pb | 19.6 ± 0.60 | 0.97 | 2.68 | 39.45 ± 34.15 | ±23.44 |
| $^{68}$Ni | 3.88 ± 0.31 | 0.96 | 0.56 | 33.25 ± 40.75 | ±22.12 |
| $^{48}$Ca | 2.07 ± 0.22 | 0.96 | 0.33 | 14.75 ± 44.25 | ±19.97 |
| $^{112}$Sn | 7.19 ± 0.50 | 0.97 | 1.10 | 12.80 ± 34.80 | ±20.87 |
| $^{114}$Sn | 7.29 ± 0.58 | 0.97 | 1.15 | 10.50 ± 36.00 | ±20.22 |
| $^{116}$Sn | 7.52 ± 0.51 | 0.97 | 1.23 | 12.25 ± 32.75 | ±19.52 |
| $^{118}$Sn | 7.91 ± 0.87 | 0.97 | 1.32 | 18.75 ± 40.75 | ±19.24 |
| $^{120}$Sn | 8.08 ± 0.60 | 0.97 | 1.38 | 17.90 ± 33.10 | ±18.70 |
| $^{122}$Sn | 7.99 ± 0.56 | 0.98 | 1.47 | 8.50 ± 31.50 | ±17.42 |

Figure 6: (Color online) The dipole polarizability $\alpha_D$ (a) in $^{208}$Pb and (b) in $^{160}$Sn as a function of the dipole polarizability in $^{150}$Sn. The dipole polarizability $\alpha_D$ (c) in $^{208}$Pb and (d) in $^{160}$Sn times the symmetry energy at saturation density $J$ as a function of the dipole polarizability in $^{150}$Sn. Calculations are done by QRPA based on HFB with 19 Skyrme density functionals: SIIV, SIV, SV, SVI (blue up triangles); SLY230a, SLY230b, SLY4, SLY5, SLY8 (red circles); SAMi, SAMi-J30, SAMi-J31, SAMi-J32, SAMi-J33 (green diamonds); SLG, SGII, SkM, SkM*, Skq (black squares). $r$ is the Pearson’s coefficient. Utilizing the experimental values of $\alpha_D$ in $^{208}$Pb [30, 34] and in $^{120}$Sn [36], and assuming $J = 31.7 \pm 3.2$ MeV [2], the dipole polarizability of $^{150}$Sn is predicted to be between 14.13 and 16.25 fm$^3$.
Table 3: Predictions of the dipole polarizabilities in neutron-rich Sn isotopes from experimental dipole polarizabilities of $^{208}$Pb [30, 34] and $^{124}$Sn [36, 37] using the correlations shown in Fig. 6 (c) and (d). The constrained values of slope parameter of symmetry energy L and neutron-skin thickness of neutron-rich Sn isotopes are also given from the correlations shown in Figs. 1 and Figs. 5. The Pearson’s coefficients r and slopes of regression line k fitted by dipole polarizability $\alpha_D$ as a function of L, as well as by neutron-skin thickness $\Delta R_{np}$ as a function of $\alpha_D$ are also shown respectively.

| Nuclei | $\alpha_D$ ($fm^3$) | $\alpha_D$ as a function of $L$ ($r$ k ($fm^3$/MeV)) | $\Delta R_{np}$ as a function of $\alpha_D$ ($\Delta$) | $\Delta R_{np}$ ($fm$) |
|--------|---------------------|----------------------------------------------------|----------------------------------------------------|---------------------|
| $^{110}$Sn | 11.97 ± 0.91 | 0.91 0.050 | 18.5 ± 0.89 0.32 | 0.295 ± 0.029 |
| $^{124}$Sn | 12.60 ± 0.96 | 0.93 0.054 | 19.4 ± 1.77 0.91 | 0.316 ± 0.031 |
| $^{124}$Sn | 13.25 ± 0.99 | 0.94 0.057 | 20.3 ± 1.73 0.91 | 0.338 ± 0.033 |
| $^{134}$Sn | 13.91 ± 1.02 | 0.96 0.060 | 21.1 ± 1.69 0.92 | 0.358 ± 0.034 |
| $^{146}$Sn | 16.25 ± 1.04 | 0.97 0.063 | 23.3 ± 1.65 0.93 | 0.377 ± 0.035 |
| $^{146}$Sn | 15.19 ± 0.98 | 0.98 0.065 | 22.3 ± 1.60 0.94 | 0.396 ± 0.036 |
| $^{150}$Sn | 15.79 ± 0.99 | 0.98 0.068 | 22.7 ± 1.60 0.94 | 0.414 ± 0.037 |
| $^{150}$Sn | 16.35 ± 1.12 | 0.99 0.071 | 23.1 ± 1.57 0.95 | 0.431 ± 0.038 |
| $^{156}$Sn | 16.84 ± 1.16 | 0.99 0.075 | 23.5 ± 1.55 0.94 | 0.447 ± 0.038 |
| $^{160}$Sn | 17.37 ± 1.21 | 0.99 0.078 | 23.5 ± 1.55 0.95 | 0.461 ± 0.039 |
| $^{170}$Sn | 17.81 ± 1.27 | 0.99 0.082 | 23.5 ± 1.55 0.95 | 0.474 ± 0.040 |

The correlations between electric dipole polarizability $\alpha_D$ (or times symmetry energy at saturation density $J$) and slope parameter of symmetry energy $L$ are studied in Sn isotopes preformed by QRPA based on Skyrme HFB theory. The previously found correlation between $\alpha_D J$ and $L$ is confirmed in the whole Sn isotopes from neutron-deficient ones to neutron-rich ones. The linear correlation between $\alpha_D$ and $L$ is not strong in stable tin isotopes and their surroundings, however, it becomes better for mass number $A > 132$, and strong correlations are found when $A \geq 140$ with the correlation coefficients $r > 0.9$, where PDR gives a considerable contribution to $\alpha_D$. The enhancement of this correlation between $\alpha_D$ and $L$ is attributed to the pairing correlations, which play important roles through PDR.

With the available high-resolution data of $\alpha_D$, the constraints on $L$ are obtained from the correlation between $\alpha_D J$ and $L$. Large uncertainties of $L$ are found, where more than half are contributed by the uncertainty from symmetry energy $\Delta J = \pm 3.2$ MeV. A proper candidate nucleus for constraining $L$ is the one with a small $\alpha_D/k$ value, where $k$ is the slope of regression line fitted by $\alpha_D J$ as a function of $L$. In stable Sn isotopes, the $\alpha_D/k$ becomes smaller towards neutron-rich side.

With the strong correlation between $\alpha_D$ and $L$ in neutron-rich Sn isotopes, $L$ can be constrained directly and more stiffer if experimental data of $\alpha_D$ with high resolution in these nuclei are known. At the moment, $\alpha_D$ in neutron-rich nuclei are predicted using the linear correlation between $\alpha_D J$ in a stable nucleus with experimental data and $\alpha_D$ in a neutron-rich nucleus. The measurements of electric dipole polarizability towards neutron-rich nuclei are called for. 

4. Summary

The correlations between electric dipole polarizability $\alpha_D$ of $^{140}$Sn, which is about the present accuracy for experimental measurement in dipole polarizability, could constrain $L$ within ±10 MeV, while with the same uncertainty of $\alpha_D$ in $^{150}$Sn, $L$ can be constrained within ±6 MeV. However, for these neutron-rich nuclei, the experimental data for dipole polarizabilities is still unavailable, so we first need to make predictions on $\alpha_D$ in neutron-rich nuclei.

In Fig. 6(a), we study the correlations of $\alpha_D$ between $^{208}$Pb and $^{150}$Sn. Although it was found that $\alpha_D$ between two stable nuclei, e.g., between $^{208}$Pb and $^{120}$Sn, have a good linear correlation [30, 33], this correlation is no longer well kept when it is extended to $\alpha_D$ between one stable nucleus and one neutron-rich nucleus, e.g., between $^{208}$Pb and $^{150}$Sn, as seen in Fig. 6(a).

The correlation between two neutron-rich nuclei, e.g., between $^{160}$Sn and $^{150}$Sn, is further checked in Fig. 6(b), and it becomes strong again. So one fails to predict $\alpha_D$ of neutron-rich nuclei from $\alpha_D$ of stable nuclei directly. Since both $\alpha_D J$ in stable nuclei and $\alpha_D$ in neutron-rich nuclei linearly correlate with $\alpha_D J$ in stable nuclei should also linearly correlate with $\alpha_D$ in neutron-rich nuclei. This is checked by our calculations in Fig. 6, where $\alpha_D J$ in $^{208}$Pb (c) and in $^{124}$Sn (d) as a function of $\alpha_D$ in $^{150}$Sn are plotted. Good linear correlations with $r = 0.950$ and 0.964 are found respectively, which can be used for the predictions of $\alpha_D$ in $^{150}$Sn as well as other neutron-rich nuclei. Utilizing the experimental $\alpha_D$ in $^{208}$Pb and $^{124}$Sn, shown in Tab. 2, and adopting $J = 31.7 \pm 3.2$ [2], $\alpha \in [12.26, 16.25]$ $fm^3$ and $\alpha_D \in [14.13, 18.29]$ $fm^3$ are obtained for $^{150}$Sn. The overlap $\alpha_D \in [14.13, 16.25]$ $fm^3$ is finally taken as the predicted value for $^{150}$Sn.

The same process can be done for other neutron-rich nuclei. The predicted $\alpha_D$ from $^{140}$Sn to $^{160}$Sn are given in Tab. 3, with which the corresponding constraints on $L$ and neutron-skin thickness $\Delta R_{np}$ are deduced and presented in Tab. 3 from the correlations between $\alpha_D$ and $L$, as well as between $\Delta R_{np}$ and $\alpha_D$. The corresponding Pearson’s coefficients $r$ of both correlations are shown in the table, and it can be seen that the linear correlations are very well kept for all these neutron-rich nuclei. Here since the $L$ values are constrained from the linear correlation between $\alpha_D$ and $L$ directly, the uncertainties become much smaller compared to those shown in Tab. 2. With the increase of neutron number, the slope of regression line fitted by $\alpha_D$ as a function of $L$ becomes larger, and as a result, the uncertainty of $L$ also becomes smaller until $^{150}$Sn even with an increasing uncertainty in the predicted $\alpha_D$. For the neutron-skin thickness, the slope of regression line fitted by $\Delta R_{np}$ as a function of $\alpha_D$ keeps almost a constant with increasing neutron numbers, yet the uncertainties of constrained neutron-skin thickness are becoming larger caused by the increasing uncertainties in $\alpha_D$. Due to the lack of experimental data of $\alpha_D$ in neutron-rich nuclei, the present constraints on $L$ shown in Tab. 3 in fact don’t give new information compared to the $L$ values obtained from the correlation between $L$ and $\alpha_D J$ in $^{208}$Pb and in $^{124}$Sn. However, the direct correlation between $\alpha_D$ and $L$ would show its special importance and effectiveness in constraining nuclear isovector properties when the experimental data of $\alpha_D$ in neutron-rich tin isotopes are available, so the measurements of dipole polarizability towards neutron-rich nuclei are strongly called for.
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