EQUATION OF STATE OF QUARK-NUCLEAR MATTER

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Quark-nuclear matter (QNM) is a many-body system containing hadrons and deconfined quarks. Starting from a microscopic quark-meson coupling (QMC) Hamiltonian with a density dependent quark-quark interaction, an effective quark-hadron Hamiltonian is constructed via a mapping procedure. The mapping is implemented with a unitary operator such that composites are redescribed by elementary-particle field operators that satisfy canonical commutation relations in an extended Fock space. Application of the unitary operator to the microscopic Hamiltonian leads to effective, hermitian operators that have a clear physical interpretation. At sufficiently high densities, the effective Hamiltonian contains interactions that lead to quark deconfinement. The equation of state of QNM is obtained using standard many-body techniques with the effective quark-hadron Hamiltonian. At low densities, the model is equivalent to a QMC model with confined quarks. Beyond a critical density, when quarks start to deconfine, the equation of state predicted for QNM is softer than the QMC equation of state with confined quarks.

1. Introduction

One of the most exciting open questions in the study of high density hadronic matter is the identification of the appropriate degrees of freedom to describe the different matter phases. For systems with matter densities several orders of magnitude larger than the nuclear saturation density, one expects a phase of deconfined matter composed of quarks and gluons whose properties very likely can be described by perturbative QCD. For ground state nuclei, there is a large body of experimental evidence that their gross properties can be described more economically employing hadronic degrees of freedom, rather than quarks and gluons. On the other hand, for matter at densities not asymptotically higher than the saturation density, like the ones in dense stars and produced in high-energy nuclear collisions, the situation seems to be very complicated, since hadrons and deconfined quarks and gluons can be simultaneously present in the system. Presently, it is not possible to employ QCD directly to study such systems and the use of effective, tractable models are essential for making
progress in the field.

One attractive model to study the different phases of hadronic matter in terms of explicit quark-gluon degrees of freedom is the quark-meson coupling (QMC) model, originally proposed by Guichon and subsequently improved by Saito and Thomas. For a list of references on further improvements of the model and recent work, see Ref. 2. In the QMC model, matter at low density is described as a system of nonoverlapping MIT bags interacting through effective scalar- and vector-meson degrees of freedom. The effective mesonic degrees of freedom couple directly to the quarks in the interior of the baryons. At very high density and/or temperature, when one expects that baryons and mesons dissolve, the entire system of quarks and gluons becomes confined within a single, big MIT bag.

In a regime of very high density, the description of hadronic matter in terms of nonoverlapping bags should of course break down, since once the relative distance between two bags becomes much smaller than the diameter of a bag, the individual bags loose their identity. The density for which this starts to happen is presently unknown within QCD.

In the present communication we introduce a generalization of the QMC model that allows to include quark deconfinement at high density. Our starting point is a relativistic quark potential model. From the model quark Hamiltonian, we construct a unitarily equivalent Hamiltonian that contains quark and hadron degrees of freedom. Starting from the Fock-space representation of single-hadron states, a unitary transformation is constructed such that the composite-hadron field operators are redescribed in terms of elementary-particle field operators in an extended Fock space. When the unitary transformation is applied to the quark Hamiltonian, effective, hermitian Hamiltonians with a clear physical interpretation are obtained. In particular, one of such effective Hamiltonians describes the deconfinement of quarks from the interior of the hadrons. The equation of state of QNM can be calculated using standard many-body techniques with the quark-hadron Hamiltonian. We will show that at low densities, the model is equivalent to the QMC model and, beyond a critical density, when quarks start to deconfine, the equation of state predicted for QNM is softer than the QMC equation of state.

2. QMC model with confined quarks

The nucleons are bound states of three constituent quarks. Constant scalar ($\sigma_0$) and vector ($\omega_0$) meson fields couple to the constituent quarks in the interior of the nucleons. Each constituent quark satisfies a Dirac equation of the
form
\[
\left[ -i \vec{\alpha} \cdot \vec{\nabla} + \beta^0 m_q^* + 1/(2(1 + \beta^0) V(r) \right] \psi(r) = E_q^* \psi(r),
\]
(1)
where
\[
m_q^* = m_q - g_q^2 \sigma_0, \quad E_q^* = \epsilon^* - g_q^2 \omega_0, \quad V(r) = \sigma r.
\]
(2)

The only difference with the model of Toki et al.\(^3\) is the form of the potential, while theirs is a harmonic oscillator, ours is a linearly rising one. For a linearly rising potential, the Dirac equation cannot be solved analytically. We use the saddle point variational principle (SPVP)\(^5\) to obtain an approximated solution. Since the Dirac Hamiltonian does not have a lower bound for the energy, the traditional nonrelativistic variational method cannot be employed. The SPVP amounts to minimizing (maximizing) the energy expectation value with respect to the variational parameters corresponding to the upper (lower) component of the Dirac wave function. We use as ansatz for the Dirac wave function\(^5\)
\[
\psi(r) = \left( \begin{array}{c} u(r) \\ i \vec{\sigma} \cdot \hat{r} v(r) \end{array} \right) \chi_s,
\]
(3)
with
\[
u(r) = i \gamma/\lambda \vec{\sigma} \cdot \vec{n} u(r),
\]
(4)
where \(N\) is a normalization constant, and \(\lambda\) and \(\gamma\) are the variational parameters. The parameters are found by minimizing the energy eigenvalue \(\epsilon\) with respect to \(\lambda\) and maximizing it with respect to \(\gamma\). We note here that for a harmonic oscillator potential, the SPVP with this ansatz leads to the exact solution.

Following the traditional path in the QMC model, we initially fix the parameters of the model in vacuum. The nucleon mass in vacuum (\(\sigma_0 = 0 = \omega_0\)) is given by
\[
M_N = 3 \epsilon - \epsilon_0 \lambda,
\]
(5)
where the last term above is used to take into account the c.m. energy of the three-quark state and other short-distance effects not taken into account by the confining potential, such as gluon exchange. \(\epsilon_0\) is fitted to obtain \(M_N = 939\) MeV. The value of the string tension is taken to be \(\sigma = 0.203\) GeV\(^2\) and \(m_q = 313\) MeV. With these parameters, the SPVP leads to \(\lambda = 2.38\) fm\(^{-1}\) and \(\gamma = 0.346\). The value required for \(\epsilon_0\) to fit the nucleon mass is 4.67 MeV fm.

Next, we proceed to obtain the energy of nuclear matter. Nuclear matter in the QMC model is modeled as a system of nucleons treated in the mean field approximation, in which the quarks remain confined within the nucleons.
The nucleon mass is now obtained as above, but now including the mean fields coupled to the quarks. The energy density of symmetrical nuclear matter is given by the traditional expression in the QMC model

\[
\frac{E}{V} = 4 \int_0^{k_F} \frac{d^3p}{(2\pi)^3} E_N^*(p) + 3 g_0^q \omega_0 \rho_B + \frac{1}{2} m_\sigma^2 \sigma_0^2 - \frac{1}{2} m_\omega^2 \omega_0^2,
\]

where \(E_N^* = \sqrt{p^2 + M_N^*} \), \(\rho_B\) is the baryon density and \(m_\sigma\) and \(m_\omega\) are the masses of the mesonic excitations. The next step consists in determining the mean fields. The vector mean field, from its equation of motion, is simply given in terms of \(\rho_B\), and the scalar field is obtained by minimizing \(E\) with respect to \(\sigma_0\), as usual. The coupling constants are then obtained by fitting \(E\) to the binding energy of nuclear matter at the saturation density, i.e. \(E/B - M_N = -15.7\) MeV at \(\rho_B = \rho_0 = 0.17\) fm\(^{-3}\) (or \(k_F = 1.36\) fm\(^{-1}\)), where \(B\) is the baryon number - in this case \(B\) is equal do the number of nucleons. Note that for each value of \(\rho_B\), one has to use the SPVP to obtain the in-medium values of \(\lambda\) and \(\gamma\). The values obtained for the coupling constants are \(g_0^q = 0.1355\) and \(3g_0^q = 6.285\). The incompressibility is found to be \(K = 248\) MeV.

In the next section we generalize the model to allow the deconfinement of quarks.

3. QMC model with quark deconfinement

Here we construct an effective Hamiltonian that contains hadron and quark degrees of freedom. The starting point is the quark model discussed in the previous section. In this model, the one-nucleon state can be written in a second-quantized notation as

\[
|\alpha\rangle = B_\alpha^\dagger |0\rangle, \quad B_\alpha^\dagger = \frac{1}{\sqrt{3!}} \Psi_{\alpha}^{\mu_1 \mu_2 \mu_3} q_{\mu_1}^\dagger q_{\mu_2}^\dagger q_{\mu_3}^\dagger,
\]

where the \(q_{\mu}^\dagger\)'s are constituent-quark creation operators and \(\Psi_{\alpha}^{\mu_1 \mu_2 \mu_3}\) is the Fock-space nucleon amplitude - for independent quarks, this is simply the product of three single-quark wave functions. The convention of summing over repeated indices is used throughly. The quark creation and annihilation operators satisfy the usual canonical anticommutation relations

\[
\{q_\mu, q_\nu^\dagger\} = \delta_{\mu\nu}, \quad \{q_\mu, q_\nu\} = \{q_\mu^\dagger, q_\nu^\dagger\} = 0.
\]

The index \(\alpha\) denotes the spatial and internal quantum numbers, such as internal and c.m. energies and the spin-isospin quantum numbers of the nucleon. Similarly, the quark indices \(\mu\) identify the spatial and internal quantum numbers as momentum, spin, flavor and color. The amplitude \(\Psi_{\alpha}^{\mu_1 \mu_2 \mu_3}\) is taken to be orthonormalized:

\[
\langle \alpha | \beta \rangle = \Psi_{\alpha}^{* \mu_1 \mu_2 \mu_3} \Psi_{\beta}^{\mu_1 \mu_2 \mu_3} = \delta_{\alpha \beta}.
\]
In the abbreviated notation we are using, the Hamiltonian corresponding to Eq. (1) can be written as

\[ H_q = T_q + V_{qq} = T(\mu) q_\mu^\dagger q_\mu + \frac{1}{2} V_{qq}(\mu\nu;\sigma\rho) q_\mu^\dagger q_\rho q_\nu q_\sigma, \]  

where \( V_{qq} \) is the confining potential.

Using the quark anticommutation relations of Eq. (8) and the normalization condition of Eq. (9), one can shown that the nucleon operators, \( B_\alpha \) and \( B_\alpha^\dagger \), satisfy the following anticommutation relations

\[ \{B_\alpha, B_\beta^\dagger\} = \delta_{\alpha\beta} - \Delta_{\alpha\beta}, \quad \{B_\alpha, B_\beta\} = 0, \]  

where

\[ \Delta_{\alpha\beta} = 3 \Psi^*_{\alpha\mu_1\mu_2\mu_3} \Psi_{\beta\nu_1\nu_2\nu_3} q_{\nu_1}^\dagger q_{\nu_2} q_{\nu_3}^\dagger - \frac{3}{2} \Psi^*_{\alpha\mu_1\mu_2\mu_3} \Psi^*_{\beta\nu_1\nu_2\nu_3} q_{\nu_1}^\dagger q_{\nu_2}^\dagger q_{\nu_3}^\dagger q_{\mu_1}, \]  

In addition, one has

\[ \{q_\mu, B_\alpha^\dagger\} = \sqrt{3} \frac{3}{2} \Psi^*_{\alpha\mu_2\mu_3} q_{\mu_2}^\dagger q_{\mu_3}, \quad \{q_\mu, B_\alpha\} = 0. \]  

The term \( \Delta_{\alpha\beta} \) is responsible for the noncanonical nature of the baryon anticommutator. This term and the nonzero value of Eq. (13) are manifestations of the composite nature of the baryons and the kinematical dependence of the quark operator and nucleon operators. This fact complicates enormously the mathematical treatment of many-body systems in which deconfined quarks and nucleons are simultaneously present. The mapping formalism of Ref. 4, known as the Fock-Tani (FT) representation 6, is a way to circumvent such complications. We will shortly review this formalism in the context of the present model. For further details, and applications for more general models, the reader is referred to Ref. 4.

The basic idea is to extend the original Fock space by introducing fictitious, or ideal nucleons that satisfy canonical anticommutation relations. The unitary operator is constructed in the extended Fock space such that

\[ |\alpha\rangle = B_\alpha^\dagger |0\rangle \rightarrow U^{-1}|\alpha\rangle \equiv |\alpha\rangle = b_\alpha^\dagger |0\rangle, \]  

where ideal baryon operators \( b_\alpha^\dagger \) and \( b_\alpha \) satisfy, by definition, canonical anticommutation relations

\[ \{b_\alpha, b_\beta^\dagger\} = \delta_{\alpha\beta}, \quad \{b_\alpha, b_\beta\} = 0. \]  

The state \( |0\rangle \) is the vacuum of both \( q \) and \( b \) degrees of freedom in the new representation. In addition, in the new representation, the quark operators \( q^\dagger \) and \( q \) are kinematically independent of the \( b_\alpha^\dagger \) and \( b_\alpha \)

\[ \{q_\mu, b_\alpha\} = \{q_\mu, b_\alpha^\dagger\} = 0. \]
The unitary operator $U$ can be constructed as a power series in the bound state amplitude $\Psi$. The rational for this is clear: in situations that the quarks remain confined in the interior of the nucleons, the term $\Delta_{\alpha\beta}$ plays no role, can be taken to be zero and the unitary operator becomes trivial. This is the situation for low densities, when the internal structures of the nucleons do not overlap significantly in the system. As the density of the system increases, the quark structures of different nucleons start to overlap. An expansion in powers of $\Psi$ offers a power counting procedure to construct the unitary operator.

The effective Hamiltonian is constructed by applying the unitary operator to the microscopic quark Hamiltonian of Eq. (10), $H_{\text{eff}} = U^{-1}H_{\text{q}}U$. The zeroth-order $U$ is trivial and not interesting. The first-order $U$ brings interesting effects. At this order, we denote the effective Hamiltonian by $H_{\text{eff}}^{(1)}$, where the superscript (1) means that $U$ has been evaluated up to the first order in $\Psi$. $H_{\text{eff}}^{(1)}$ can be written as

$$H_{\text{eff}}^{(1)} = T_q + H_b + \tilde{V}_{qq} + V_{qb} + \cdots. \quad (17)$$

The $\cdots$ refer to terms not relevant for our discussion here. $H_b = T_b + V_{bb}$, where $T_b$ is a single-nucleon energy and $V_{bb}$ is an effective nucleon-nucleon interaction without quark exchange. This term leads to the normal QMC model, in which the many-body system is described by nonoverlapping nucleons - no quark-exchange. In particular, it can describe Fock terms in the QMC model. The term $\tilde{V}_{qq}$ contains two-quark and three-quark interactions. It can be shown that if $\Psi$ is a bound-state eigenstate of the original quark Hamiltonian, $\tilde{V}_{qq}$ collapses to

$$V_{qq} = \frac{1}{2} V_{qq}(\mu\nu;\sigma\rho) q_{\mu}^\dagger q_{\nu}^\dagger q_{\rho} q_{\sigma} - E_{\alpha} B_{\alpha}^\dagger B_{\alpha}. \quad (18)$$

It is not difficult to show that this interaction leads to a quark Hamiltonian that has a positive semidefinite spectrum. That is, after the transformation, the resulting Hamiltonian involving only quark operators is unable to bind three quarks to form a nucleon, it describes only states in the continuum. The term $V_{qb}$ is given by

$$V_{qb} = \frac{1}{\sqrt{6}} \left[ H(\mu_1\mu_2;\sigma\rho) \Psi_{\beta}^{\mu_1\mu_2\sigma\rho} - H(\mu\nu;\sigma\rho) \Psi_{\beta}^{\mu\nu\sigma\rho} \Delta(\mu_1\mu_2\mu_3;\mu\nu\tau) \right] q_{\mu_1}^\dagger q_{\mu_2}^\dagger q_{\mu_3}^\dagger b_{\beta} + \text{h.c.}, \quad (19)$$

where h.c. denotes hermitian conjugation and $\Delta(\mu\nu\tau;\sigma\rho\lambda) = \sum_\alpha \Psi_\alpha^{\mu\nu\tau} \Psi_\alpha^{\sigma\rho\lambda}$ is known as the bound-state kernel. If $\Psi$ is a stationary state of the microscopic quark Hamiltonian, one obtains

$$V_{qb} = 0, \quad (20)$$
since $\Delta(\mu \nu \tau; \sigma \rho \lambda) \Psi^{\sigma \rho \lambda}_\alpha = \Psi^{\mu \nu \tau}_\alpha$. This result reflects the stability of the baryon bound state to spontaneous decay in the absence of external perturbations. This is clearly the case for a nucleon in vacuum. Also, in the QMC model, when one constructs the unitary transformation $U$ with a $\Psi$ that is an eigenstate of the microscopic quark Hamiltonian with the mean fields $\sigma_0$ and $\omega_0$ in it, the term $V_{bq}$ continues to be zero, and explicit quark degrees of freedom will not be present in the system at this order of $\Psi$.

Now, in a many-body system, the confining quark-quark interaction will become modified due to a variety of effects. Some of such effects, as self-energy corrections from quark loops, can be calculated within the model using standard many-body techniques. However, in a high density system there are other QCD effects that are not captured by the model, such as pair creation and gluonic interactions, that eventually will lead to quark deconfinement.

The formalism we just described allows to include in an effective way deconfinement in the QMC model. One generates an effective quark-hadron Hamiltonian as above using $\Psi$’s that are eigenstates of the QMC Hamiltonian, Eq. (1). Now, if $V(r)$ is modified such as that it does not lead to absolute confinement, the term $V_{bq}$, given by Eq. (19), is not zero. In a mean field perspective, the Hamiltonian of Eq. (17) leads to two Fermi seas, one for baryons and one for quarks. The crucial, and difficult point here is to obtain the relative abundances of baryons and quarks in the system. This can be evaluated in an approximated way as follows.

Let $Z$ be the fraction of baryons in the system,

$$\sum_\alpha \langle b_\alpha^\dagger b_\alpha \rangle = Z B, \quad \sum_\mu \langle q_\mu^\dagger q_\mu \rangle = (1 - Z) B, \quad (21)$$

where $B$ is as in the previous section the total baryon number. In the mean field approximation - or independent-particle approximation - and for sufficiently small $V_{bq}$, $Z$ can be estimated by the perturbative formula

$$Z^{-1} = 1 + (b|V_{bq}^\dagger |b \rangle \frac{P}{H_0 - E_0} V_{bq} |b \rangle), \quad (22)$$

where $H_0$ is $T_q + T_b$, and $P = 1 - |b\rangle \langle b|$ is a projection operator.

In order to evaluate Eq. (22), we postulate a density dependence for the confining interaction of the form \(^8\)

$$V(r) = \sigma r e^{-\mu^2 r^2}, \quad (23)$$

where $\mu$ is a prescribed function of $\rho_B$. We use a simple formula for $\mu$, such that it is zero for baryon densities below three times the normal nuclear matter density $\rho_0$, and for higher densities it increases linearly with the density as
Figure 1. The confining potential in vacuum (solid line) and in matter for two different baryon densities.

\[ \mu = \rho B/3\rho_0 - 1. \]  

In Fig. (1) we show the potential of Eq. (23) for zero density, and 5 and 10 times the saturation density of normal nuclear matter.

The energy density of the system can be written as

\[
\frac{E}{V} = 4 \int_0^{k_{bF}} \frac{d^3p}{(2\pi)^3} E_N^*(p) + 3 g_0^2 \omega_0 \rho_B + \frac{1}{2} m_\sigma^2 \sigma_0^2 - \frac{1}{2} m_\omega^2 \omega_0^2 
\]

\[ + 12 \int_0^{k_{qF}} \frac{d^3k}{(2\pi)^3} E_q^*(k), \tag{24} \]

where \( E_q^*(k) = \sqrt{k^2 + m_q^2} \) and the Fermi momenta \( k_{bF} \) and \( k_{qF} \) are related to the nucleon density and quark density as

\[ \rho_b = Z \rho_B = 2(k_{bF})^2/3\pi^2; \quad \rho_q = (1 - Z) \rho_B = 2(k_{qF})^2/\pi^2, \tag{25} \]

At this point, it is important to notice that since \( \mu \) only starts to operate for densities larger than three times the normal density, the coupling constants \( g_\sigma^2 \) and \( g_\omega^2 \) are the same as before. Of course, for higher densities, there is a somewhat complicated self-consistency problem to be solved, since \( Z \) is density dependent. Therefore, in the process of obtaining \( \sigma \), the iterative problem becomes more complicated.

In order to proceed, we need \( Z \) as a function of \( \rho_B \). It can be calculated numerically with our ansatz wave function given above. The calculation, however, involves multidimensional integrals that must be done using a Monte Carlo method.
Figure 2. Equation of state of quark nuclear matter. The solid line is for matter composed of nucleons only and the dashed line is for matter composed by nucleons and quarks.

Carlo integrator. For our purposes here, in this initial investigation we make some approximations. Initially we neglect the lower component of the Dirac spinor. This does not seem to be a too drastic approximation, since $\gamma$ in Eq. (4) is a small quantity. In this approximation, one obtains

$$Z^{-1} = 1 + \int d^3k_1 d^3k_2 d^3k_3 |\Phi_p(k_1, k_2, k_3)|^2 \frac{|F(k_1 - k_2)|^2}{\Delta E(p, k_1, k_2, k_3)},$$

(26)

with $F(\vec{q})$ given by

$$F(\vec{q}) = \int d^3k \Delta V(\vec{k}) e^{-k^2/\lambda^2} \left( e^{\vec{k} \cdot \vec{q}/\lambda^2} - 1 \right),$$

(27)

where $\Phi_p(k_1, k_2, k_3)$ is the three-quark wave function of the nucleon with c.m. momentum $\vec{p}$ (see Ref. 4), $\Delta E(p, k_1, k_2, k_3)$ is the difference between of the energies of the three unbound quarks and of the three quarks bound in the potential, and $\Delta V(\vec{k})$ is the Fourier transform of $\Delta V(r)$, where

$$\Delta V(r) = \sigma r \left( e^{-\mu^2 r^2} - 1 \right).$$

(28)

This clearly shows that once $\mu = 0$, i.e. the potential is density independent, one regains the original QMC model.

Now, Eqs. (26) and (27) still require a lot of numerical work. We simplify them further by making two additional approximations. The first one consists in neglecting the momentum dependence of the energy denominator and the
second one is to use an average value for $q^2$ in $F(q^2)$. Both approximations taken together seem not to be a bad approximation, since the energy denominator under the integral is dominated by low momenta. Now the problem consists in a single one dimensional integral that can easily be performed with a Gauss integration.

In Fig. (2) we present the results for the energy per baryon number, $E/B$ as a function of the ratio $\rho_B/\rho_0$. The solid line in this figure is the result for the QMC model of the previous section. The dashed line shows that the deconfining of quarks leads to a softening of the equation of state. It would be interesting to investigate the consequences of this softening for neutron-star phenomenology. Soft equations of state seem to be required to explain recent observational data of compact stellar objects.

4. Conclusions
We have generalized the QMC model to include quark deconfinement in matter. The model is based on an effective quark-hadron Hamiltonian obtained via a mapping procedure from a relativistic microscopic quark Hamiltonian with a density dependent quark-quark interaction. The equation of state of QNM was obtained using the effective quark-hadron Hamiltonian. It was found that beyond a critical density, when quarks start to deconfine, the equation of state predicted for QNM is softer than the usual QMC equation of state. Implications of this equation of state for the phenomenology of compact stellar objects were pointed out.

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