The measurements, parameters and construction of Web proxy cache

Dmitry Dolgikh

Samara State Aerospace University, Moscovskoe sh. 34a, Samara, 443086, Russia

Andrei Sukhov

Laboratory of Network Technologies, Samara Academy of Transport Engineering, 1 Bezymyanny per., 18, Samara, 443066, Russia

Abstract

The aim of this paper is an experimental study of cache systems in order to optimize proxy cache systems and to modernize construction principles. Our investigations lead to the criteria for the optimal use of storage capacity and allow the description of the basic effects of the ratio between construction parts, steady-state performance, optimal size, etc. We want to outline that the results obtained and the plan of the experiment follow from the theoretical model. Special consideration is given to the modification of the key formulas supposed by Wolman at al. [11].

Key words: Cache system, Zipf-like distribution, document lifetime, elements of cache construction, renewal of Web documents

1 Introduction

This paper considers approaches to the problem of optimization of proxy cache construction based on theoretical and experimental study. Until recently, caching was an optional service for users who voluntary configured their browsers to redirect request through a proxy. The Internet Service Providers interpose the caching systems in the strategic places at the organization boundaries.

1 Corresponding author

E-mail addresses: sukhov@ssau.ru (Andrei M. Sukhov), ddolgikh@ssau.ru (Dmitry G. Dolgikh)
In experimental research of the cache systems and construction of the mathematical models should lead to the growth of the caching effectiveness with minimal financial expenditures. A better algorithm that increases hit ratios by only several percent would be equivalent to a multiple growth in cache size. In order to find the optimal cache size Kelly and Reeves [8] are guided on economical methods like monetary cost of memory and bandwidth.

Our analysis is based on the model presented by Breslau et al. [2] and extended by Wolman et al. [11] to incorporate the steady-state behavior and documents’ rate of change. One difficulty in the study of Web caching is that there are many cache replacement policies and many factors affecting their performance. Wolman et al. parameterized the model using population size, population request rate, document rate of change, size of object universe, and popularity distribution of objects. Several researchers [7] include additional factors like object size, miss penalty, temporal locality, and long-term access frequency. Our research allows the calculation of the lower limit of the cache size corresponding to aggregated bandwidth of external links when performance mount to the effective level equal to 35%.

Such an approach can be easily generalized to describe any applications based on Zipf-like distribution such as Content Distribution Networks (CDN) [6], peer-to-peer systems [9,10], Internet search engines, etc.

The paper investigates how the different parameters of proxy cache influence on its performance. The methods of the measurements including the treatment of experimental data and analytical formulas for calculations are discussed. The correlations between the significant parameters are investigated and the corresponding Figures and Tables are constructed. On the basis of the new experimental data the analytical model is modernized and the addition to the cache construction and to replacement algorithm is proposed.

Special consideration is given to the modification of the key formulas introduced by Wolman at al. [11]. In order to describe the renewal effect of Web documents in the global network the alternative model is developed. The document rate of change $\mu(i)$ is supposed to depend on popularity index $i$ so the Zipf-like distribution with the new exponent $\alpha_R$ describes the mentioned effects of the Wolman at al. model. As a result of the special experiment the values of exponents $\alpha$, $\alpha_R$ and of the document rates of change $\mu_p$, $\mu_u$ are calculated.
2 Plan of the measurements

The scheme of the Web caching could be presented in the following way: there are users which send requests to the global network and receive information through a cache system as it is shown on the Fig. 1. Some documents are requested repeatedly and therefore they should be held in the cache system.

The relative frequency \( \theta_i \) of requests to Web pages follows Zipf-like distribution \([12]\). This distribution states that the relative probability of a request for the \( i \)’th most popular page is

\[
\theta_i = \frac{A}{i^\alpha},
\]

where \( A = \theta_1 \) is the probability of the most popular item and \( \alpha \) is a positive exponential value less then unity.

We proposed the following test: the size of proxy cache \( S_{eff} \) was varying. These values correspond to incoming traffic for one, two, three, and six days. All types of documents both cacheable and uncacheable are taken into account for calculating the ratio \( S_{eff}/\nu_{int} \).

The network of Samara State Aerospace University has been chosen as an experimental field. The proxy cache of SSAU is a two-processor Linux server with SQUID proxy installed. All hierarchical links were disconnected before the experiment began. The statistics of requests for the long time \( T_{st} \geq month \) were collected for each point.

In order to modify the model proposed by Wolman at al. we collected the requests during the time \( T_{st} \lesssim t_u \). The \( t_u \) is the mean lifetime of those documents, who’s popularity is \( \vartheta_i = 1 \) (\( \vartheta_i = \theta_i k \)).
### Table 1

**Primary results**

| $\frac{S_{int}}{Kbyte}$ | Rpd | $\nu_{int}^{\times 10^3}$ | $\nu_{out}^{\times 10^3}$ | H | $\nu_{out}^{Kbps}$ | $\nu_{int}^{Kbps}$ | $H^B$ | $E(C)$ | $E(S)$ | $T_{st}$ |
|---------------------------|-----|------------------------|------------------------|---|-----------------|-----------------|------|---------|--------|---------|
| 1.0                       | 56.5 | 42.5                   | 24.49                  | 42.6 | 38.6            | 9.13            | 8.13 | 10.5    | 31     |
| 2.15                      | 53.4 | 39.5                   | 28.08                  | 44.1 | 40.3            | 10.33           | 8.9  | 12.5    | 28     |
| 3.15                      | 69.8 | 47.3                   | 32.19                  | 46.8 | 41.5            | 11.17           | 7.25 | 13.7    | 31     |
| 5.96                      | 69.8 | 42.3                   | 36.75                  | 56.3 | 50.2            | 10.78           | 8.71 | 13.8    | 61     |

3 The original results and their processing

The primary results of experiments are summarized in the Table 1, where the following notations and abridgments are used:

- The variables $\nu_{int}$ and $\nu_{out}$ are the incoming and the outgoing request’s streams of cache proxy as it is shown on the Fig. 1. They are measured as the number of users requests per day (Rpd - request per day). It should be noted that the variable $\nu_{out} = \lambda N$ describes the request stream from a collective user that is a significant parameter of the Wolman at al. model.
- $H$ is the performance of the cache system or, in other words, hit ratio.
- The same variables with upper index B describe the system in the units of transmitting traffic ($Kbps$ - Kbit per second).
- $E(S)$ is the mean size of documents received from the global network directly.
- $E(C)$ is the mean size of documents from the cache.
- Finally, the time $T_{st}$ corresponds to the quantity of days when statistics are collected.

The statistics collected were processed by scripts specially written for the task. Originally, so-called cacheable documents that can be stored in a proxy are selected from the general list. Later the corresponding Zipf-like distribution was constructed where the documents were placed in the order of reducing of popularity index $\vartheta_i$. The fragment of this list is shown below:

```
112 http://www.ixbt.com/images/empty.gif (line 457), i.e. $\vartheta_{457} = 112$
111 http://cacheserver.myecom.net/main/images/adlogo.jpg (line 458)

2 http://zzz.net.ru/images/spacer.gif (line 78166 = M)

1 http://www.muz-tv.ru/chat/chat-top.html (line 200045)
```

The number of unique cacheable documents or the quantity of the lines in the list mentioned above is $p$. The line number of the last document, which was requested twice ($\vartheta_M = 2$), is $M$. 

4
In order to find the general number of cacheable documents $k$ the following sum was calculated

$$k = \sum_{i=1}^{p} \vartheta_i$$ \hspace{1cm} (2)$$

where $\vartheta_i$ is the number of cache requests for the $i$'th most popular document. The portion of cacheable documents $p_c$ is defined as

$$k = p_c \nu_{out} T_{st},$$ \hspace{1cm} (3)$$

where $K = \nu_{out} T_{st}$ is the general number of all documents received from the global network including both cacheable and uncacheable ones.

The value of $\alpha$’s was calculated using the equation

$$\alpha = 1 - 2M/\sum_{i=1}^{M} \vartheta_i = 1 - 2M/(k - p + M)$$ \hspace{1cm} (4)$$

Finally, analyzing the log files mean lifetime $t_u$ was calculated for those documents, with a popularity of $\vartheta_i = 1$. Such statistics also determine the lifetime $T_{eff}$ of cache objects with the citing index $\vartheta_i = 2$, i.e. those items, which have been stored in proxy cache, requested one time from a proxy, and deleted subsequently (see Tab. 2).

### 4 Basic correlation

The first family of the curves, which should be analyzed, is the dependence of $t_u$ and $T_{eff}$ on cache size $S_{eff}/\nu_{int}$. As it was discovered in the Ref. [5], these curves define the ratios between elements of cache construction:

- A kernel $S_k$ that contains popular documents with $\vartheta_i \geq 2$.
- An accessory part $S_u$ that keeps unpopular documents requested from the Internet once, i.e. $\vartheta_i = 1$. 

\begin{table}[h]
\centering
\caption{Parameters of proxy cache}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
$S_{eff}$ & $S_{int}$ & $\alpha$ & $t_u$ & $T_{eff}$ & $p_c$ & $M$ & $p$ & $k$
\hline
\hline
1.0 & 0.42 & 0.76 & 2.2 $\pm$ 0.9 & 3.8 $\pm$ 1.9 & 0.59 & 0.99 & 3.10 & 10.4
\hline
2.15 & 0.86 & 0.77 & 6.8 $\pm$ 1.9 & 9.1 $\pm$ 4.6 & 0.58 & 0.78 & 2.48 & 8.7
\hline
3.15 & 1.49 & 0.74 & 8.8 $\pm$ 2.9 & 8.5 $\pm$ 3.6 & 0.56 & 1.22 & 3.68 & 12.0
\hline
5.96 & 2.53 & 0.81 & 20.4 $\pm$ 2.4 & 18.9 $\pm$ 7.0 & 0.59 & 2.01 & 6.07 & 25.0
\hline
\end{tabular}
\end{table}
• A managing part $S_m$ that contains statistics of requests and rules for replacement of cache objects.

Earlier [4,5] we have got the following Eqs.

$$S_k = \frac{(1 - \alpha)H}{2} \nu_{out} T_{eff}$$  \hspace{1cm} (5)

$$\frac{S_k}{S_u} = \frac{T_{eff}}{(2^{1/\alpha} - 1)t_u} = \frac{M}{p - M} \frac{T_{eff}}{t_u}$$  \hspace{1cm} (6)

Analysis of the experimental data shown in the Tab. 2 and on the Fig. 2 leads to the facts that variables $t_u$ and $T_{eff}$ are directly proportional to the cache size $S_{eff}/\nu_{int}$ and can be considered as coincided values:

$$T_{eff} \simeq t_u$$  \hspace{1cm} (7)

The only deflection indicated for effectiveness of replacement algorithms is revealed at small cache size when performance is far from the optimal value.

In other words the kernel and accessory parts are approximately correlated as 1:2 or less then 40% of storage capacity has been used for the basic goal to store the repeatedly requested documents. It is remarkable that the number of documents $M$, which must be stored in the cache system for one and two months traffic, is less then three and six-days incoming stream correspondingly.

The second family of curves intended for study is the dependence of the system performance $H$ and $H^B$ on its relative size $S_{eff}/\nu_{int}$, see Fig 3.

The hit rate $H$ of the web cache is considered to grow in a log-like fashion as a function of cache size [1,2,3]. From the expression for cache performance

$$H = p_c \int_{1}^{S_k} \frac{A}{x^{\alpha}} dx,$$  \hspace{1cm} (8)
the following dependence appears

\[ \frac{H_1}{H_2} = \left( \frac{S_1}{S_2} \right)^{1-\alpha}, \]  

that allows us to talk about power fashion.

The Fig 3 illustrates the fact that the dependence between \( H \) and \( S_{eff}/\nu_{int} \) is successfully described by Eq. (9) with \( \alpha = 0.77 \) and the curve \( H^B \) is not predictable. This effect needs additional study especially because the Tab. 1 shows a positive difference between the mean size of cacheable documents \( E(S) \) and the mean size of all items \( E(C) \).

5 Renewal of Web documents

Rapid development of computer technology at the end of the last millennium lead to the appearance of a virtual world with its own laws. Unfortunately, during this period little attention was given to studying of fundamental principles of virtual life.

Nowadays we can afford some respite and some resources should be transferred to studying of the fundamental laws and to optimization of the vital systems on the basis of recent knowledge. Frankly speaking, any research area may be considered as a scientific field if and only if its basic principles are enveloped in mathematical form and new rules can be predicted on the basis of confirmed facts.

Our analysis is based on the model presented by Breslay et al [2] and extended by Wolman et al. [11] to incorporate document rate of change. The Wolman model yields formulas to predict steady-state properties of Web caching systems parameterized by population size \( N \), population request rates \( \lambda \), document rate of change \( \mu \), size of object universe \( n \) and a popularity distribution
The key principles of their theoretical study has been developed in our work [5] to describe the basic effects as a ratio between construction parts, steady-state performance, optimal size, etc. in an alternative way. We want to outline that the results received and the plan of the experiment follow from the theoretical model [5].

The key formulas from Wolman et al.

\[
C_N = \int_1^n \frac{1}{C x^\alpha} \left( \frac{1}{1 + \frac{\mu C x^\alpha}{\lambda N}} \right) dx \tag{10}
\]

\[
C = \int_1^n \frac{1}{x^\alpha} dx \tag{11}
\]

yield \( C_N \), the aggregate object hit ratio considering only cacheable objects. Document rates of change \( \mu \) were considered to take two different values, one for popular documents \( \mu_p \) and another for unpopular documents \( \mu_u \). An additional multiplier from Eq. (10) throws off one effective query to any document from cache during time \( T_{ch} = 1/\mu \) between its changes. Such an updating request must be redirected to the global network to get the renewed Web page.

In this paper we assume that the document rate of change \( \mu(i) \) depends on its popularity \( i \). The mathematical equivalent of this assertion is that the steady-state process is again described by Zipf-like distribution with a small \( \alpha_R \) as it is shown of the Fig. 4.

The difference between an ideal performance of a cache system and its real
value has been conditioned by the renewal of documents in the global network

\[ \Delta H = \left( \sum_{i=1}^{M} \vartheta_i - HK \right) / K \quad (12) \]

and can be investigated in an experimental way. Here

\[ \Delta k = \sum_{i=1}^{M} \vartheta_i - HK = k - HK + M - p, \quad (13) \]

\( \Delta k \) is the number of updating requests conditioned by renewal of Web documents and

\[ k_R = HK + p - M \quad (14) \]

Therefore we have to explore the log file collected for the time \( T_{st} \ll T_{eff} - \Delta T \) using the data from the Tab. 2. Then the condition \( M(T_{st}) \ll S_k \) has been fulfilled and all repeated requests are explained by the renewal effect. The ideal Zipf-like distribution corresponds to the upper line on the Fig. 4 with exponent \( \alpha \) from Eq. (4), but the real hit ratio \( H \) determines \( \alpha_R \) as

\[ \alpha_R = 1 - 2M/HK \quad (15) \]

Now it is easy to find \( \mu_i \)

\[ \mu(i) = \frac{\vartheta_i^{id} - \vartheta_i^{R}}{T_{st}} = \frac{(p/i)^\alpha - (p/i)^{\alpha_R}}{T_{st}}, \quad (16) \]

or

\[ \mu(i) = \frac{1}{T_{st}} \frac{1 - (i/p)^{\Delta \alpha}}{(i/p)^\alpha}. \quad (17) \]

We can consider \( \mu_u = \mu(p/4) \) and \( \mu_p = \mu(p/100) \), then the results can be summarized in the Table 3.

The key difference between our analytical model and presented by Wolman at al. model are:
The document rate of change $\mu(i)$ is described by continuous variables that depend on popularity $i$. Wolman et al. assume that the document rate of change $\mu$ takes two different values, one for popular documents $\mu_p$ and another for unpopular documents $\mu_u$.

The renewal of Web documents is incorporated in a steady-state process as a Zipf-like distribution with a small $\alpha_R$.

We assume also that size of cache system is restricted.

A considerable part of cacheable documents $p - M$ are requested from the global network only one time accordingly to Zipf-like distribution. The more accurate expression for an ideal hit ratio $H_i$ has been shown in Ref. [5]

$$H_i \leq 2^{(\alpha-1)/\alpha},$$

(18)

It has to be modified to taking into account the renewal effect:

$$H_i \leq 2^{(\alpha-1)/\alpha}(1 - \alpha)/(1 - \alpha_R),$$

(19)

Two types of correlations pretend to a role of fundamental laws that describe cache systems [4]:

- A Zipf-like distribution, see Eq. (1)
- Normalizing conditions or a sum of the probability to request the universe of $1 \leq n \leq k$ objects.

The above mentioned laws could be applied to the special points of Zipf distribution and two of them, $M$ and $p$, are used for the construction of the theory. Then the Zipf-like distribution leads to

$$\frac{AkR}{M^{\alpha_R}} = 2,$$

(20)

$$\frac{AkR}{p^{\alpha_R}} = 1.$$

(21)

Normalizing conditions for the first $M$ and $p$ documents from cache give

$$\int_1^M A x^{\alpha_R} dx = H_i$$

(22)

$$\int_1^p A x^{\alpha_R} dx = 1$$

(23)

Here $H_i$ is an ideal (steady-state) performance for cacheable documents. For
a real system the Eq. (22) has been transformed to

\[ H = p_c \int_1^{S_k} \frac{A}{x^{\alpha_R}} dx, \]  

(24)

where \( S_k \) is the number of cache objects in the kernel.

It is important that Zipf exponent \( \alpha \) grows as the experiment time \( T_{st} \) increases.

\[ \Delta \alpha = \alpha(T_{st}^2) - \alpha(T_{st}^1) = C \ln(T_{st}^2/T_{st}^1) \]  

(25)

6 Summary and future work

The aim of this paper is a study of a Web cache system in order to optimize proxy cache systems and to modernize construction principles. Our investigations lead to the criteria for the optimal usage of storage capacity and to allow to description of the basic effects as the ratio between the construction parts, steady-state performance, optimal size, etc. We want to outline that the results received and the plan of the experiment follow from the theoretical model.

Special consideration is given to the modification of the key formulas supposed by Wolman at al. [11]. The document rate of change \( \mu(i) \) is supposed to depend on popularity index \( i \) so the Zipf-like distribution with the new exponent \( \alpha_R \) describes the effects of the renewal of Web documents.

The main result of any research of cache system is finding a way for increasing hit ratio. We can conclude that with growth of \( S_k/S_u \) the hit rate \( H \) increases. A general feature of the current algorithm is the only documents requested two times and more during the time \( t_u \) are included in the kernel \( S_k \). It is a fact that \( M \) less then the cache size \( S_{eff} \), i.e. all necessary items could be stored in a cache system at the second, third and forth experimental points on the Tab. 2. Therefore we can make the conclusion that a caching algorithm based on a rigid tie of the mean parameters to the time \( t_u \) is ineffective.

One method of resolving the current situation is reconstructing of cache systems. Such a construction suggested in the Sec. 4 must be implemented to provide a rigid ratio between its elements. The requests’ statistics must be kept even for those cacheable documents that have been deleted from the cache. This statistics should exist for long time. This is one of the key differences between our construction and the existing ones that usually operate
only with the current set of documents in cache. An inseparable part of the new construction is the replacement algorithm based on Zipf law. Now we have prepared the corresponding application for an international patent.

An additional question, which will demanded a new experimental search, is the dependence that describes the renewal of Web documents. Probably, the ratio $\Delta \alpha / \alpha$ could be considered as a universal constant. The next direction for our development plans is an investigation of a model of cache interaction in hierarchical caching system.

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