Abstract

It is shown that the manner of introducing the interaction between a spin 1 particle and external classical gravitational field can be successfully unified with the approach that occurred with regard to a spin 1/2 particle and was first developed by Tetrode, Weyl, Fock, Ivanenko. On that way a generally relativistical Duffin-Kemmer equation is constructed. So, the manner of extending the flat space Dirac equation to general relativity case indicates clearly that the Lorentz group underlies equally both these theories. In other words, the Lorentz group retains its importance and significance at changing the Minkowski space model to an arbitrary curved space-time. In contrast to this, at generalizing the Proca formulation, we automatically destroy any relations to the Lorentz group, although the definition itself for a spin 1 particle as an elementary object was based on just this group. Such a gravity’s sensitiveness to the fermion-boson division might appear rather strange and unattractive asymmetry, being subjected to the criticism. Moreover, just this feature has brought about a plenty of speculation about this matter. In any case, this peculiarity of particle-gravity field interaction is recorded almost in every handbook.

In the paper, on the base of the Duffin-Kemmer formalism developed, the problem of a vector particle in the Abelian monopole potential is considered. The content is: 1. On Duffin-Kemmer formalism in the Riemannian space-time; 2. On wave functions of a spin 1 particle in the monopole field; 3. On connection with the Proca approach; 4. Discret symmetry.
1. On Duffin-Kemmer formalism in the Riemannian space-time

A generally acceptable point of view is that description of interaction between a quantum mechanical particle and an external classical gravitational field looks basically different in accordance as whether fermion or boson is meant. So, the starting flat space (Dirac) equation

\[(i \gamma^a \partial_a - m) \Psi(x) = 0\]

as well known, we have to generalize through the use of the tetrad formalism according to the Tetrode-Weyl-Fock-Ivanenko (TWFI) procedure [1-9]. With regard to a vector bosons [10-33], a totally different approach is generally used: it consists in ordinary formal changing all involved tensors and usual derivative \(\partial_a\) into general relativity ones. For example, in case of a vector (spin 1) particle, the flat space Proca equations

\[(\partial_a \Psi_b - \partial_b \Psi_a) = m \Psi_{ab}, \quad \partial^b \Psi_{ab} = m \Psi_a\]

being subjected to the formal change

\[\partial_a \rightarrow \nabla_a, \quad \Psi_a \rightarrow \Psi_a, \quad \Psi_{ab} \rightarrow \Psi_{a\beta}\]

results in

\[(\nabla_a \Psi_\beta - \nabla_\beta \Psi_a) = m \Psi_{a\beta}, \quad \nabla^\beta \Psi_{a\beta} = m \Psi_a\]

However, it is known already for a long time that all particles of the theory, irrespective of whether bosons or fermions are meant, obey in a curved background space-time a unique TWFI approach (see, for example, in [8,9]). But admittedly, in the common literature, they do not use consistently this universal formalism. Although the widely spread method of light tetrad or Newman-Penrose formalism [34,35]) is certainly a renewed and modified variant of the TWFI above mentioned approach, the Newman-Penrose method was developed in accordance with its own special intrinsic requirements and with no clearly visible relations to the conventional TWFI approach (such a correlation is potentially implied rather than observed really).

As a matter of fact, a potentially existing (general relativity) Duffin-Kemmmmer \((D-K)\) equation for a spin 1 particle, apparently, is not widely adopted. But, as evidenced by many examples, sometimes it is desirable if not necessary, to depart from constructions of common use in order to arrive at a simpler or more suitable one for a particular situation. Bellow, we develop some aspects of this generalized \(D-K\) theory, that are essential to real practical calculations (I adhere an unpublished work of the three authors [...]). This method will be successfully applied further in Sec.2 to a spin 1 particle-monopole problem.

So, let us take up considering this matter in more detail. We start from a flat space equation in its matrix (Duffin-Kemmer) form [10]:

\[(i \beta^a \partial_a - m) \Phi(x) = 0\]

where \(\Phi(x)\) is a ten component column-function; \(\beta^a\) is \((10 \times 10)\) -matrices; in the Cartesian representation they are

\[\Phi = (\Phi_0 \Phi_1 \Phi_2 \Phi_3; \Phi_{01} \Phi_{02} \Phi_{03}; \Phi_{23} \Phi_{31} \Phi_{12})\],

\[\beta^a = \begin{pmatrix} 0 & \kappa^a \\ \lambda^a & 0 \end{pmatrix} = (\kappa^a \oplus \lambda^a), \quad (\kappa^a)_{[mn]} = -i(\delta^m_j g^{na} - \delta^a_j g^{ma})\].
\begin{align*}
\lambda_{[mn]}^i &= -i(g^{ia} \delta^j_n - \delta^a_i \delta^j_m) = -i \delta_{mn}^{ij} \\
(\lambda^a)_{[mn]}^i &= i[\delta_{mn}^{ij} g^{aj} - \delta_{mn}^{ij} g^{ai}],
\end{align*}

(1.2b)

\((g^{\alpha\beta}) = \text{diag}(+1, -1, -1, -1)\) is the Minkowski metric tensor; the sectional matrix structure introduced here will be used below. By using this representation (5.2b), we can easily verify the major properties of \(\beta^a\):

\[\beta^c \beta^a \beta^b = \begin{pmatrix} 0 & \kappa^a \kappa^c \lambda^b & 0 \\
\kappa^c \kappa^a \lambda^b & 0 & \kappa^c \kappa^a \lambda^b \\
0 & \kappa^c \kappa^a \lambda^b & 0 \end{pmatrix}, \quad (\lambda^c \kappa^a \lambda^b)_{[mn]}^i = i[\delta_{mn}^{ij} g^{aj} - \delta_{mn}^{ij} g^{ai}],\]

and then

\[(\kappa^c \lambda^a \kappa^b)_{[mn]}^i = i [\delta_{ij} (g^{cm} g^{bn} - g^{cn} g^{bm}) + g^{ac} (\delta_{ij} g^{mb} - \delta_{ij} g^{nb})] \]

To follow the TWFI procedure, the equation (1.2a) must be extended to a Riemannian space-time (with a metric \(g_{\alpha\beta}(x)\) and its concomitant tetrad \(e^a_{(\alpha)}(x)\)) according to

\[\left[ i \beta^a(x) (\partial_\alpha + B_a(x)) - m \right] \Phi(x) = 0 \quad (1.3)\]

where

\[\beta^a(x) = \beta^a e^a_{(\alpha)}(x), \quad B_a(x) = \frac{1}{2} j^{ab} e^a_{(\alpha)} \nabla_\alpha (e_{(\beta)}), \quad j^{ab} = (\beta^a \beta^b - \beta^b \beta^a).\]

This equation contains the tetrad \(e^a_{(\alpha)}(x)\) explicitly. Therefore, there must exist a possibility to demonstrate the equivalence of any variants of this equation associated with various tetrads:

\[e^a_{(\alpha)}(x) \text{ and } e^a_{(\beta)}(x) = L^b_a(x) e^a_{(\alpha)}(x) \quad (1.4a)\]

\((L(x)\) is an arbitrary local Lorentz transformation). We will show that two such equations

\[\left[ i \beta^a(x) (\partial_\alpha + B_a(x)) - m \right] \Phi(x) = 0, \quad \left[ i \beta^a(x) (\partial_\alpha + B'_a(x)) - m \right] \Phi'(x) = 0 \quad (1.4b)\]

generating in tetrads \(e^a_{(\alpha)}(x)\) and \(e^a_{(\beta)}(x)\), respectively, can be converted into each other through the transformation \(\Phi(x) = S(x) \Phi'(x)\):

\[\begin{pmatrix} \phi'_a(x) \\ \phi'_{[ab]}(x) \end{pmatrix} = \begin{pmatrix} L^i_a & 0 \\ 0 & L^m_a L^n_b \end{pmatrix} \begin{pmatrix} \phi_i(x) \\ \phi_{[mn]}(x) \end{pmatrix} \quad (1.4c)\]

here the \(L(x)\) is the same as in the relation (1.4a). So, starting from the first equation in (1.4b), let us obtain an equation for \(\Phi'(x)\). Allowing for \(\Phi(x) = S(x) \Phi(x)\), we get

\[\left[ i S \beta^a S^{-1} (\partial_\alpha + S B^a S^{-1} + S \partial_\alpha S^{-1}) - m \right] \Phi'(x) = 0 \]

A task that faces us now is of verifying the relationships

\[S(x) \beta^a(x) S^{-1}(x) = \beta'^a(x), \quad (1.5a)\]

\[[S(x) B(x) S^{-1}(x) + S(x) \partial_\alpha S^{-1}(x)] = 0. \quad (1.5b)\]
The first one can be rewritten as
\[ S(x) \beta^a e^\alpha_{(a)}(x) S^{-1}(x) = \beta^b e^\alpha_{(b)}(x) \]
from where, taking into account the relation (1.4a) between tetrads, we come to
\[ S(x) \beta^a S^{-1}(x) = \beta^b L^a_b(x). \tag{1.5c} \]
The latter condition is of great familiarity in \( D - K \) theory; one can verify it through the use of the sectional structure of \( \beta^a \), which provides two sub-relations:
\[ L(x) \kappa^a \left[ L^{-1}(x) \otimes L(x)^{-1} \right] = \kappa^b L^a_b(x), \quad \left[ L(x) \otimes L(x) \right] \lambda^a L(x)^{-1} = \lambda^b L^a_b(x). \tag{1.5d} \]
Those latter will be satisfied identically, after we take explicit form of \( \kappa^a \) and \( \lambda^a \) into account and also allow for the \( L^a_b \) being pseudo orthogonal: \( g^{al} (L^{-1})_l^k(x) = g^{kb} L^a_b(x) \).
Now, let us pass to the proof of the relationship (1.5b). By using the determining relation for \( D - K \) connection
\[ B^a_\alpha(x) = \frac{1}{2} j^{ab} e^\beta_{(a)} \nabla_\alpha (e_{(b)}^\beta), \quad j^{ab} = (\beta^a \beta^b - \beta^b \beta^a) \]
and also the formula (1.5c), we get
\[ S(x) \partial_\alpha S^{-1}(x) = [B'_\alpha(x) - \frac{1}{2} j^{mn} L_m^n(x) g_{ab} \partial_\alpha L_n^b(x)]. \]
In a sequence, the (1.5b) results in
\[ S(x) \partial_\alpha S^{-1}(x) = \frac{1}{2} L_m^a(x) g_{ab} (\partial_\alpha L_n^b(x)). \]
The latter condition is an identity: this is readily verified through the use of sectional structure of all involved matrices.

Thus, the equations from (1.4b) are translated into each other; thereby, they manifest a gauge symmetry under local Lorentz transformations (in a complete analogy with more familiar Dirac particle case [1-9]). In the same time, the wave function from this equation represents scalar quantity relative to general coordinate transformations: that is, if \( x^\alpha \rightarrow x'^\alpha = f^\alpha(x) \), then \( \Phi'(x) = \Phi(x) \).

It remains to demonstrate that this \( D - K \) formulation can be inverted into the Proca formalism in terms of general relativity tensors. To this end, as a first step, let us allow for the sectional structure of \( \beta^a, J^{ab} \) and \( \Phi(x) \) in the \( D - K \) equation; then instead of (1.3) we get
\[ i \left[ \lambda^c e^\alpha_{(c)} \right] (\partial_\alpha + \kappa^b \lambda^b e^\beta_{(b)} \nabla_\alpha e_{(b)}^\beta) \right]_{[mn]} \Phi_I = m \Phi_{[mn]}, \]
\[ i \left[ \kappa^c e^\alpha_{(c)} \right] \left( \partial_\alpha + \lambda^a \kappa^b e^\beta_{(a)} \nabla_\alpha e_{(b)}^\beta \right) \right]_{[m]} \Phi_{[mn]} = m \Phi_I \tag{1.6a} \]
which, after taking into account the explicit form of \((\lambda^c, \lambda^c \kappa^a, \lambda^b, \kappa^c, \kappa^c \lambda^c, \kappa^b)\), lead to
\[ \left[ (e^\alpha_{(a)} \partial_\alpha \Phi_b - e^\alpha_{(b)} \partial_\alpha \Phi_a) + (\gamma^c_{ab} - \gamma^c_{ba}) \Phi_c \right] = m \Phi_{ab}, \]
\[ \left[ e^{(b)\alpha} \partial_\alpha \Phi_{ab} + \gamma^{nb}_{n} \Phi_{ab} + \gamma_{a}^{mn} \Phi_{mn} \right] = m \Phi_a. \tag{1.6b} \]
In turn, these will represent just the Proca equations (1.1c) after they are rewritten in terms of tetrad components according to

\[
\Phi_a(x) = e_{(a)}^\alpha(x) \Phi_\alpha(x), \quad \Phi_{ab}(x) = e_{(a)}^\alpha(x) e_{(b)}^\beta(x) \Phi_{\alpha\beta}(x)
\]  

(1.7)

the symbol \( \gamma_{abc}(x) \) is used to designate a rotational Ricci coefficients:

\[
\gamma_{abc}(x) = - (\nabla_\beta e_{(a)}^\alpha(x) e_{(b)}^\beta(x) e_{(c)}^\gamma).
\]

So, as evidenced by the above, the manner of introducing the interaction between a spin 1 particle and external classical gravitational field can be successfully unified with the approach that occurred with regard to a spin 1/2 particle and was first developed by Tetrode, Weyl, Fock, Ivanenko. One should attach great significance to that possibility of unification. Moreover, its absence would be a very strange fact indeed because it touches concepts of great physical significance. Let us discuss this matter in more detail.

The manner of extending the flat space Dirac equation to general relativity case indicates clearly that the Lorentz group underlies equally both these theories. In other words, the Lorentz group retains its importance and significance at changing the Minkowski space model to an arbitrary curved space-time. In contrast to this, at generalizing the Proca formulation, we automatically destroy any relations to the Lorentz group, although the definition itself for a spin 1 particle as an elementary object was based on just this group. Such a gravity sensitiveness to the fermion-boson division might appear rather strange and unattractive asymmetry, being subjected to the criticism. Moreover, just this feature has brought about a plenty of speculation about this matter. In any case, this peculiarity of particle-gravity field interaction is recorded almost in every handbook. By my mind, the possibility itself of rewriting the tetrad-based Duffin-Kemmer equation in terms of general relativity tensors looks very surprising indeed.

2. On wave functions of a spin 1 particle in the monopole field

Now, on the base of Duffin-Kemmer (D-K) formalism, let us consider the problem of a vector particle in the Abelian monopole potential. The starting D-K equation in the spherical tetrad takes the form

\[
\left[ i \beta^0 \partial_t + i (\beta^3 \partial_r + \frac{1}{r} (\beta^1 j^{31} + \beta^2 j^{32})) + \frac{1}{r} \Sigma^{\kappa}_{\theta,\phi} - \frac{mc}{\hbar} \right] \Phi(x) = 0
\]  

(2.1a)

where

\[
\Sigma^{\kappa}_{\theta,\phi} = \left[ i \beta^1 \partial_\theta + \beta^2 \frac{i \partial + (\beta j^{12} - \kappa) \cos \theta}{\sin \theta} \right]
\]

(2.1b)

These relations are very close to analogous ones used in the electronic case [36] ; variations concern only the explicit expressions for matrices: \( \gamma^a, \sigma^{ab} \) are to be changed into \( \beta^a, J^{ab} \).

Below, we will use the cyclic basis for Duffin-Kemmer matrices:
$$\beta^0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\beta^3 = \begin{pmatrix} 0 & 0 & 0 & 0 & i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & i & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\beta^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & -i & 0 & +i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 \\ -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\beta^2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +i & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +i & 0 & +i & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -i & 0 & +i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$
correspondingly, the matrix $ij^{12}$ has a diagonal structure

$$ij^{12} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & t_3 & 0 & 0 \\ 0 & 0 & t_3 & 0 \\ 0 & 0 & 0 & t_3 \end{pmatrix}, \quad t_3 = \begin{pmatrix} +1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$  

In the given tetrad representation, three components of the total conserved momentum are (compare with [37,38])

$$j_i^\sigma = \left[ l_1 + \frac{\cos \phi}{\sin \theta} (ij^{12} - \kappa) \right], \quad j_2^\sigma = \left[ l_2 + \frac{\sin \phi}{\sin \theta} (ij^{12} - \kappa) \right], \quad j_3^\sigma = l_3. \quad (2.2a)$$

Correspondingly, according to the general procedure [36], the particle’s wave functions with fixed quantum number $(\epsilon, j, m)$ are to be constructed as follows:

$$\Phi_{ijm}(x) = e^{-i\epsilon t} [f_1(r) D_\kappa, f_2(r) D_{\kappa-1}, f_3(r) D_\kappa, f_4(r) D_{\kappa+1}, f_5(r) D_{\kappa-1}, f_6(r) D_\kappa, f_7(r) D_{\kappa+1}, f_8(r) D_{\kappa-1}, f_9(r) D_\kappa, f_{10}(r) D_{\kappa+1}] \quad (2.2b)$$

here, $D_\sigma \equiv D_{-m_\sigma} (\phi, \theta, 0)$.

At finding 10 radial equations for $f_1, \ldots, f_{10}$, we are to use the six recursive relations [39]

\[
\begin{align*}
\partial_{\theta} D_{\kappa-1} &= (a D_{\kappa-2} - c D_\kappa), \\
&= \frac{-m - (\kappa - 1) \cos \theta}{\sin \theta} D_{\kappa-1} = (-a D_{\kappa-2} - c D_\kappa), \\
\partial_{\theta} D_\kappa &= (c D_{\kappa-1} - d D_{\kappa+1}), \\
&= \frac{-m - \kappa \cos \theta}{\sin \theta} D_\kappa = (-c D_{\kappa-1} - d D_{\kappa+1}), \\
\partial_{\theta} D_{\kappa+1} &= (d D_\kappa - b D_{\kappa+2}), \\
&= \frac{-m - (\kappa + 1) \cos \theta}{\sin \theta} D_{\kappa+1} = (-d D_\kappa - b D_{\kappa+2})
\end{align*}
\]

where

\[
\begin{align*}
a &= \frac{1}{2} \sqrt{(j + \kappa - 1)(j - \kappa + 2)}, \\
b &= \frac{1}{2} \sqrt{(j - \kappa - 1)(j + \kappa + 2)}, \\
c &= \frac{1}{2} \sqrt{(j + \kappa)(j - \kappa + 1)}, \\
d &= \frac{1}{2} \sqrt{(j - \kappa)(j + \kappa + 1)}.
\end{align*}
\]

Allowing for the following intermediate results

$$\Sigma_{\theta, \phi}^\epsilon \Phi = \exp -i\epsilon t \sqrt{2} \begin{pmatrix} ( -c f_5 - d f_7 ) D_\kappa \\
- i c f_9 D_{\kappa-1} \\
( -i c f_8 + i d f_{10} ) D_\kappa \\
- i d f_9 D_{\kappa+1} \\
c f_1 D_{\kappa-1} \\
0 \\
d f_1 D_{\kappa+1} \\
- i c f_3 D_{\kappa-1} \\
( + i c f_2 - i d f_4 ) D_\kappa \\
+ i d f_3 D_{\kappa+1} \end{pmatrix}; \quad (2.3a)$$
from (2.1a) we produce

\[
\begin{pmatrix}
0 \\
-i f_5 D_{\kappa-1} \\
i f_6 D_{\kappa} \\
i f_7 D_{\kappa+1} \\
-i f_2 D_{\kappa-1} \\
i f_3 D_{\kappa} \\
i f_4 D_{\kappa+1} \\
0 \\
0
\end{pmatrix}
\]

\[
i \beta^0 \partial_t \Phi = \epsilon \exp(-i \epsilon t) \begin{pmatrix}
0 \\
-i f_5 D_{\kappa-1} \\
i f_6 D_{\kappa} \\
i f_7 D_{\kappa+1} \\
-i f_2 D_{\kappa-1} \\
i f_3 D_{\kappa} \\
i f_4 D_{\kappa+1} \\
0 \\
0
\end{pmatrix};
\]

\[
i (\beta^3 \partial_r + \frac{1}{r} (\beta^1 \beta^{31} + \beta^2 \beta^{32})) \Phi_{\epsilon m} = \exp(-i \epsilon t)
\]

\[
\begin{pmatrix}
0 \\
(\ -d/dr - 2/r \ ) f_6 D_{\kappa} \\
i (d/dr + i/r) f_8 D_{\kappa-1} \\
0 \\
0 \\
0 \\
(\ -i d/dr - i/r \ ) f_2 D_{\kappa-1} \\
(\ i d/dr + i/r \ ) f_4 D_{\kappa+1}
\end{pmatrix}
\]

from (2.1a) we produce

\[-(\frac{d}{dr} + \frac{2}{r}) f_6 - \frac{\sqrt{2}}{r} (c f_5 + d f_7) - m f_1 = 0,
\]

\[i \epsilon f_5 + i(\frac{d}{dr} + \frac{1}{r}) f_8 + i \frac{\sqrt{2} c}{r} f_9 - m f_2 = 0,
\]

\[i \epsilon f_6 + \frac{2i}{r} (-c f_8 + d f_10) - m f_3 = 0,
\]

\[i \epsilon f_7 - i(\frac{d}{dr} + \frac{1}{r}) f_10 - i \frac{\sqrt{2} d}{r} f_9 - m f_4 = 0,
\]

\[i \epsilon f_2 + \frac{\sqrt{2} c}{r} f_1 - m f_5 = 0, \quad -i \epsilon f_3 - \frac{d}{dr} f_1 - m f_6 = 0,
\]

\[-i \epsilon f_4 + \frac{\sqrt{2} d}{r} f_1 - m f_7 = 0, \quad -i(\frac{d}{dr} + \frac{1}{r}) f_2 - i \frac{\sqrt{2} c}{r} f_3 - m f_8 = 0
\]

\[i \frac{\sqrt{2}}{r} (c f_2 - d f_4) - m f_9 = 0, \quad i(\frac{d}{dr} + \frac{1}{r}) f_4 + \frac{i \sqrt{2} d}{r} f_3 - m f_{10} = 0.
\]

Parametres \( j \) are allowed to take values (we have to draw distinction between \( \kappa = \pm 1/2 \) and all remaining \( \kappa \)):

\[
\text{if } \kappa = \pm 1/2, \quad \text{then } \quad j = | \kappa |, | \kappa | + 1, \ldots;
\]

\[
\text{if } \kappa = \pm 1, \pm 3/2, \ldots \quad \text{then } \quad j = | \kappa | - 1, | \kappa |, | \kappa | + 1, \ldots
\]

In both cases, the states of minimal \( j \) (respectively \( j_{\text{min.}} = | \kappa | \) and \( j_{\text{min.}} = | \kappa | - 1 \)) are to be considered separately: the radial system (2.4) is not valid for those states.
Let us consider the state with $j_{\text{min}} = |\kappa| - 1$. First, one ought to investigate the $j_{\text{min}} = 0$ situation arisen at $\kappa = \pm 1$; the relevant wave function does not depend on the $\theta, \phi$ variables at all. Let $\kappa = +1$ and $j_{\text{min}} = 0$, then we start with the substitution

$$\Phi^0(t, r) = \exp -i\epsilon t [0, f_2, 0, 0, f_5, 0, 0; f_8, 0, 0]$$

(2.6a)

It is readily verified that the $\Sigma_{\theta, \phi}$ operator acts on $\Phi_0$ as a null operator: $\Sigma_{\theta, \phi} \Phi_0 = 0$; because the identity $(i j^{12} - \kappa) \Phi_0 \equiv 0$ holds. As a result, we produce only three non-trivial (as one should expect) equations:

$$i \epsilon f_5 + i \left( \frac{d}{dr} + \frac{1}{r} \right) f_8 - m f_2 = 0$$

$$- i f_2 - m f_5 = 0, \quad - i \left( \frac{d}{dr} + \frac{1}{r} \right) f_2 - m f_8 = 0$$

(2.6b)

From here, it follows

$$f_5 = - i \frac{\epsilon}{m} f_2, \quad f_8 = - i \frac{\epsilon}{m} \left( \frac{d}{dr} + \frac{1}{r} \right) f_2$$

and the function $f_2 (F_2 = \frac{1}{r} f_2)$ satisfies the equation

$$\left( \frac{d^2}{dr^2} + \epsilon^2 - m^2 \right) F_2 = 0$$

(2.6c)

The latter provides us with an exponential solution of the same kind as in the electronic case, that is a candidate for a possible bound state. The situation with $j_{\text{min}} = 0$ and $\kappa = -1$ looks completely analogous:

$$\Phi^0(t, r) = \exp -i\epsilon t [0, 0, 0, f_4, 0, 0, f_7, 0, 0, f_10]$$

(2.7a)

and the radial equations

$$i \epsilon f_7 - i \left( \frac{d}{dr} + \frac{1}{r} \right) f_{10} - m f_4 = 0,$$

$$- i f_4 - m f_7 = 0, \quad - i \left( \frac{d}{dr} + \frac{1}{r} \right) f_4 - m f_{10} = 0$$

(2.7b)

and eventually we get

$$f_7 = - i \frac{\epsilon}{m} f_4, \quad f_{10} = i \frac{\epsilon}{m} \left( \frac{d}{dr} + \frac{1}{r} \right) f_2,$$

$$\left( \frac{d^2}{dr^2} + \epsilon^2 - m^2 \right) F_4 = 0, \quad \left( F_4 = \frac{1}{r} f_4 \right).$$

(2.7c)

Now, we pass on the case of minimal $j_{\text{min}} = |\kappa| - 1$ with higher values of $\kappa$: $\kappa = \pm 3/2, \pm 2, \ldots$ First, let $\kappa$ be positive, then we have start with a substitution

$$\kappa \geq 3/2 : \quad \Phi^0 = \exp -i\epsilon t [0, f_2 D_{\kappa - 1}, 0, 0; f_5 D_{\kappa - 1}, 0, 0; f_8 D_{\kappa - 1}, 0, 0]$$

(2.8a)

Using the recursive relations

$$\partial_\theta D_{\kappa - 1} = \sqrt{\frac{\kappa - 1}{2}} D_{\kappa - 2}, \quad \frac{-m - (\kappa - 1) \cos \theta}{\sin \theta} D_{\kappa - 1} = - \sqrt{\frac{\kappa - 1}{2}} D_{\kappa - 2}$$
we find

\[ i\beta^1 \Phi^0 = \exp -iet i \left( \frac{\kappa - 1}{2} \right) \]

\[ \begin{pmatrix} -if_5 D_{\kappa-2} \\ 0 \\ +f_8 D_{\kappa-2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -f_2 D_{\kappa-2} \\ 0 \end{pmatrix} \]

\[ \beta^2 i \frac{\partial \phi + (ij^{12} - \kappa) \cos \theta}{\sin \theta} \Phi^0 = e^{-iet \left( \frac{\kappa - 1}{2} \right)} \begin{pmatrix} -f_5 D_{\kappa-2} \\ 0 \\ -if_8 D_{\kappa-2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ +if_2 D_{\kappa-2} \\ 0 \end{pmatrix} \]

and further we produce \( \Sigma_{\theta,\phi} \Phi^0 = 0 \). Therefore, the radial functions \( f_2, f_5, f_8 \) satisfy again the same system (2.6b). The case of \( j_{\min.} = |\kappa| - 1 \) with negative \( \kappa \) looks completely similar to the above:

\[ \kappa \leq -3/2 : \quad \Phi^0 = e^{-iet \left[ 0, 0, 0, f_4 D_{\kappa+1}, 0, 0, f_7 D_{\kappa+1}, 0, 0, f_9 D_{\kappa+1} \right]} \] (2.9)

the identity \( \Sigma_{\theta,\phi} \Phi^0 \equiv 0 \) also holds and a radial system coincides with (2.7b). So, the description of \( j_{\min.} = |\kappa| - 1 \) states has been completed; all of them provide us with solutions of a special exponential kind which potentially might be related to a bound state and therefore these solutions are of special physical interest. In the same time, unfortunately, it is a unique case that we have managed to solve entirely up to their radial equations.

Now, let us pass on the states with \( j = |\kappa| \) that which are to be regarded whether as \( j_{\min.} = |\kappa| \) states at \( \kappa = \pm1/2 \) or non-minimal \( j \) states at all other values of \( \kappa \). Let \( j = |\kappa| \) and \( \kappa \) be positive (\( \kappa \geq +1/2 \)), then we have to begin with a substitution (the radial functions at all \( D_{j=m,\kappa+1} \) in \( \Phi(x) \) are equated to zero)

\[ \kappa \geq +1/2 : \]

\[ \Phi_{ejm}(x) = \exp -iet \left[ f_1(r) D_{\kappa}; f_2(r) D_{\kappa-1}, f_3(r) D_{\kappa}, 0; f_5(r) D_{\kappa-1}, f_6(r) D_{\kappa}, 0; f_8(r) D_{\kappa-1}, f_9(r) D_{\kappa}, 0 \right] \] (2.10a)

For \( \Sigma_{\theta,\phi} \Phi \) we get
\[
\Sigma_{\theta,\phi} \Phi = \exp -i \epsilon t \sqrt{\kappa} \left( \begin{array}{c}
-f_5 D_{\kappa} \\
+if_9 D_{\kappa-1} \\
-if_8 D_{\kappa} \\
0 \\
-f_1 D_{\kappa-1} \\
0 \\
0 \\
-if_3 D_{\kappa-1} \\
+if_2 D_{\kappa} \\
0
\end{array} \right) \]

and further we produce the radial system

\[\begin{align*}
-(d \frac{d}{dr} + \frac{2}{r}) f_6 - \sqrt{-\kappa} \frac{f_5}{r} - m f_1 &= 0, \\
i \epsilon f_5 &+ i(\frac{d}{dr} + \frac{1}{r}) f_8 + i \sqrt{-\kappa} f_9 - m f_2 = 0,
\end{align*}\]

\[\begin{align*}
i \epsilon f_6 - i \sqrt{-\kappa} f_8 - m f_3 &= 0, \\
0 &= 0, \\
i \epsilon f_2 &+ \sqrt{-\kappa} f_1 - m f_5 = 0,
\end{align*}\]

\[\begin{align*}
-i \epsilon f_3 - \frac{d}{dr} f_1 - m f_6 &= 0, \\
0 &= 0, \\
i \epsilon f_2 &+ \sqrt{-\kappa} f_3 - m f_8 = 0,
\end{align*}\]

\[i \sqrt{-\kappa} \frac{f_2}{r} - m f_9 = 0, \\
0 &= 0 .
\] (2.10b)

In an analogous way one can consider the \( j = |\kappa| \) states at negative \( \kappa \): \( \kappa \leq -1/2 \):

\[\Psi = \exp -i \epsilon t \left[ f_1 D_{\kappa}, 0, f_3 D_{\kappa}, f_4 D_{\kappa+1}, 0, f_6 D_{\kappa}, f_7 D_{\kappa+1}, 0, f_9 D_{\kappa}, f_{10} D_{\kappa+1} \right] ;\] (2.11a)

\[\begin{align*}
\left( \frac{d}{dr} + \frac{2}{r} \right) f_6 + \sqrt{-\kappa} \frac{f_7}{r} + m f_1 &= 0, \\
0 &= 0, \\
i \epsilon f_6 &- i \sqrt{-\kappa} f_{10} - m f_3 = 0 ,
\end{align*}\]

\[\begin{align*}
i \epsilon f_7 - i \sqrt{-\kappa} f_9 &- i(\frac{d}{dr} + \frac{1}{r}) f_{10} - m f_4 = 0 , \\
0 &= 0 ,
\end{align*}\]

\[\begin{align*}
i \epsilon f_5 + \frac{d}{dr} f_1 + m f_6 &= 0, \\
-i \epsilon f_4 &+ \sqrt{-\kappa} f_1 + m f_7 = 0, \\
0 &= 0 ,
\end{align*}\]

\[\begin{align*}
i \sqrt{-\kappa} \frac{f_4}{r} + m f_9 &= 0, \\
i \epsilon f_2 &+ \sqrt{-\kappa} f_3 - m f_{10} = 0 .
\] (2.11b)

Thus, the task of finding radial equations has been completely solved. All those systems look rather involved, so we are reasons to question its easy analysis in terms of any standard special functions. It can be noted that the ten equations established above fall naturally into 4 plus 6 sub-groups: those six give us a possibility to express the functions \( f_5, \ldots, f_{10} \) in terms of \( f_1, \ldots, f_4 \). Thereby, we can reduce the first order system of 10 equations to a second order system of 4 ones. Evidently, those four relation will represent a still complicated system.
3. On connection with the Proca approach

At analyzing the above radial system, any additional information can be useful. In particularly, as well known, there must exist a first order differential condition on the vector constituent of 10-dimensional wave function, namely, the so-called generalized Lorentz relation. Let us work out it explicitly in this monopole situation. To this end, instead of D-K formalism it will be more convenient to use the Proca formalism (see Sec.2):

\[ D_\alpha \Psi_\beta - D_\beta \Psi_\alpha = \frac{mc}{\hbar} \Psi_{\alpha\beta}, \quad D^\alpha \Psi_{\alpha\beta} = \frac{mc}{\hbar} \Psi_\beta \]  
(3.1a)

where \( D_\alpha = (\nabla_\alpha + i \frac{e}{\hbar c} A_\alpha) \); \( A_\alpha \) is an electromagnetic potential (here, it is presented by Schwinger monopole potential \( A_\phi = g \cos \phi \)). After the operator \( D_\alpha \) acts on the second equation in (3.1a), we will get

\[ \frac{mc}{\hbar} (\nabla_\alpha + i \frac{e}{\hbar c} A_\alpha) \Psi^\alpha = i \frac{e}{2\hbar c} F_{\alpha\beta} \Psi^{\alpha\beta} \]  
(3.1b)

where, \( F_{\alpha\beta} = (\partial_\alpha A_\beta - \partial_\beta A_\alpha) \). When \( A_\alpha = 0 \), this relations provides us with the usual Lorentz condition \( \nabla_\alpha \Psi^\alpha = 0 \).

Now, we face to translate this relationship (3.1b) from Proca representation into the Duffin-Kemmer’s. All above, instead of \( \Psi^\alpha \) and \( \Psi^{\alpha\beta} \) we have to introduce their tetrad components: \( \Psi^\alpha = e^{(a)\alpha} \Psi_a \), \( \Psi^{\alpha\beta} = e^{(a)\alpha} e^{(b)\beta} \Psi_{ab} \). Correspondingly, the (3.1b) will take on the form

\[ \frac{mc}{\hbar} \left[ e^{(a)\alpha} \Psi_a + e^{(a)\alpha} \partial_\alpha \right] \Psi_a + i \frac{e}{\hbar c} A^\alpha \Psi_a = i \frac{e}{2\hbar c} F^{ab} \Psi_{ab} \]  
(3.1c)

The coordinate representatives of the monopole \( A_\phi = g \cos \theta \), \( F_{\theta\phi} = -g \sin \theta \) have the following tetrad description

\[ A^2 = e^{(2)\phi} A_\phi = -\frac{g \cos \theta}{r \sin \theta}, \quad F^{12} = e^{(1)\theta} e^{(2)\phi} F_{\theta\phi} = -\frac{g}{r^2} \]  
(3.1d)

In addition, on simple straightforward computation, we find

\[ e^{(0)\alpha} = 0, \quad e^{(1)\alpha} = -\frac{\cos \theta}{r \sin \theta}, \quad e^{(2)\alpha} = 0, \quad e^{(3)A\alpha} = -\frac{2}{r} \]  
(3.1e)

The functions \( \Psi_a \) and \( \Psi_{ab} \) involved in (3.1c), relate to the 10 constituents of \( D-K \) column \( \Phi \) as follows (this represents translating from cyclic basis into Cartesian one; \( W \equiv -1/\sqrt{2} \))

\[
\begin{pmatrix}
\Phi_0 \\
\Phi_1 \\
\Phi_2 \\
\Phi_3 \\
\Phi_{01} \\
\Phi_{02} \\
\Phi_{03} \\
\Phi_{23} \\
\Phi_{31} \\
\Phi_{12}
\end{pmatrix}
= \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -W & 0 & +W & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -iW & 0 & -iW & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -W & 0 & +W & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -iW & 0 & -iW & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
= \begin{pmatrix}
f_1 D_\kappa \\
f_2 D_{\kappa-1} \\
f_3 D_\kappa \\
f_4 D_{\kappa+1} \\
f_5 D_{\kappa-1} \\
f_6 D_\kappa \\
f_7 D_{\kappa+1} \\
f_8 D_{\kappa-1} \\
f_9 D_\kappa \\
f_{10} D_{\kappa+1}
\end{pmatrix}
\]  
(3.2a)
In the following we need only the components $\Psi_0, \Psi_1, \Psi_2, \Psi_3, \Psi_{12}$:

$\Psi_0 = e^{-i\epsilon t} f_1 D_\kappa, \quad \Psi_3 = e^{-i\epsilon t} f_3 D_\kappa, \quad \Psi_1 = e^{-i\epsilon t} \frac{1}{\sqrt{2}} (-f_2 D_{\kappa-1} + f_4 D_{\kappa+1}),$

$\Psi_2 = e^{-i\epsilon t} \frac{i}{\sqrt{2}} (-f_2 D_{\kappa-1} - f_4 D_{\kappa+1}), \quad \Psi_{12} = e^{-i\epsilon t} f_9 D_\kappa \quad (3.2b)$

Allowing for (3.2b) and (3.1d,e), the condition (3.1c) has taken the form:

$$\frac{mc}{\hbar} \left[ \frac{1}{\sqrt{2}} r f_2 \left( \partial_\theta D_{\kappa-1} - \frac{m + (\kappa - 1) \cos \theta}{\sin \theta} D_{\kappa-1} \right) - \frac{1}{\sqrt{2}} r f_4 \left( \partial_\theta D_{\kappa+1} + \frac{m + (\kappa + 1) \cos \theta}{\sin \theta} D_{\kappa+1} \right) \right. + D_\kappa \left( -\frac{2}{r} f_3 - i \frac{\epsilon}{\hbar c} f_1 - \frac{d}{dr} f_3 \right) = -i \frac{\kappa}{r^2} f_9 D_\kappa \quad (3.3a)$$

After having used the recursive relations

$$\partial_\theta D_{\kappa-1} - \frac{m + (\kappa - 1) \cos \theta}{\sin \theta} D_{\kappa-1} = -\sqrt{(j - \kappa + 1)(j + \kappa)} D_\kappa,$$

$$\partial_\theta D_{\kappa+1} - \frac{m + (\kappa + 1) \cos \theta}{\sin \theta} D_{\kappa+1} = -\sqrt{(j + \kappa + 1)(j - \kappa)} D_\kappa$$

(which are easily derived from the used above) we eventually arrive at

$$\frac{mc}{\hbar} - i \frac{\epsilon}{\hbar c} f_1 + \left( \frac{d}{dr} + \frac{2}{r} \right) f_3 - \frac{1}{\sqrt{2}} (c f_2 + d f_4) = -i \frac{\kappa}{r^2} f_9 \quad (3.3b)$$

If $j = |\kappa|, \kappa \geq +1/2$, one gets

$$\frac{mc}{\hbar} \left[ -i \frac{\epsilon}{\hbar c} f_1 + \left( \frac{d}{dr} + \frac{2}{r} \right) f_3 - \frac{\sqrt{\kappa}}{\sqrt{r}} f_2 \right] = -i \frac{\kappa}{r^2} f_9 \quad (3.3c)$$

if $j = |\kappa|, \kappa \leq -1/2$, one gets

$$\frac{mc}{\hbar} \left[ -i \frac{\epsilon}{\hbar c} f_1 + \left( \frac{d}{dr} + \frac{2}{r} \right) f_3 - \frac{\sqrt{-\kappa}}{\sqrt{r}} f_2 \right] = -i \frac{\kappa}{r^2} f_9 \quad (3.3d)$$

4. Discret symmetry.

Now, let us take up else one question, namely, concerning a problem of discrete symmetry at the vector particle - monopole case. As was shown in [36], at the electron-monopole case there exists some composite operator $\hat{N} = [\hat{\pi} \otimes \hat{P}_{\text{monopole}} \otimes \hat{P}]$. It would seems that the same possibility is realized also in case of vector particle. Indeed, a direct extension of the above to a new situation: $\hat{N}_{\text{vect.}} = [\hat{\pi} \otimes \hat{P}_{\text{vect.}} \otimes \hat{P}]$ affords formally an operator with analogous commuting properties, that is,

$$[\hat{N}_{\text{vect.}}, \hat{H}_{\text{vect.}}] = 0, \quad [\hat{N}_{\text{vect.}}, \hat{J}_{\text{vect.}}] = 0.$$

However, as it will be verified bellow, such an operator cannot be diagonalized on Duffin-Kemmer wave functions found above. This matter is worth considering in more detail.
The vector ordinary $P$-reflection operator in Cartesian tetrad, is

$$
\hat{P}_{\text{Cart.}} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -I & 0 & 0 \\
0 & 0 & -I & 0 \\
0 & 0 & 0 & +I
\end{pmatrix}
$$

(4.1)

where a symbol "I" denotes a unit $3 \times 3$ matrix. After translating this $\hat{P}_{\text{Cart.}}$-operator into the spherical tetrad’s basis according to $\hat{P}_{\text{sph.}} = O(\theta, \phi) \hat{P}_{\text{Cart.}} O^{-1}(\theta, \phi)$, where $(O(\theta, \phi)$ is a 10-dimension rotational matrix associated with the spinor gauge transformation used in case of electronic field, it takes on the form (the standart cyclic basis in the vector space is used)

$$
\hat{P}_{\text{sph.}}^{\text{cycl.}} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & +E & 0 & 0 \\
0 & 0 & +E & 0 \\
0 & 0 & 0 & -E
\end{pmatrix}, \quad \text{where} \quad E \equiv \begin{pmatrix}
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0
\end{pmatrix}.
$$

(4.2)

From the equation on proper values

$$
[\hat{\pi} \otimes \hat{P}_{\text{sph.}}^{\text{cycl.}} \otimes \hat{P}] \Phi_{jm}^{eg} = N \Phi_{jm}^{eg}
$$

it follows

$$
N = (-1)^{j+1} : \quad f_1 = f_3 = f_6 = 0, \quad f_4 = -f_2, \quad f_7 = -f_5, \quad f_{10} = +f_8; \quad (4.3b)
$$

$$
N = (-1)^j : \quad f_9 = 0, \quad f_4 = + f_2, \quad f_7 = + f_5, \quad f_{10} = - f_8 \quad (4.3c)
$$

these relations are exactly the same which had arisen from diagonalizing the ordinary $P$-reflection operator in case of a free vector field: $[\hat{P}_{\text{sph.}}^{\text{cycl.}} \otimes \hat{P}] \Phi^0 = P \Phi^0$.

Let us try imposing these additional relations (4.3b) or (4.3c) on radial functions $f_1(r), \ldots, f_{10}(r)$ obeying the system (2.4). On direct verification, one concludes that a system so achieved is not self-consistent. This means that the $\hat{N}$ operator, though commuting with the vector $eg$-Hamiltonian, cannot be regarded as an observable quantity measured simultaneously with vector particle-monopole’s Hamiltonian. For example, in case (4.3b), one has

$$
-(\frac{d}{dr} + \frac{2}{r}) 0 - \frac{\sqrt{2}}{r} (c - d) f_5 - m 0 = 0, \quad \text{that is} \quad f_5 \equiv 0;
$$

$$
i \varepsilon 0 + i(\frac{d}{dr} + \frac{1}{r}) f_8 + i \frac{\sqrt{2}c}{r} f_9 - m f_2 = 0;$$

$$
i \varepsilon 0 + \frac{2i}{r} (c + d) f_8 - m 0 = 0, \quad \text{that is} \quad f_8 \equiv 0;
$$

$$
i \varepsilon 0 - i(\frac{d}{dr} + \frac{1}{r}) 0 - \frac{\sqrt{2}d}{r} f_9 - m f_2 = 0;$$

$$
i \varepsilon f_2 + \frac{\sqrt{2}c}{r} 0 - m 0 = 0, \quad \text{that is} \quad f_2 \equiv 0, \quad f_9 \equiv 0;$$

that is
\[-i\epsilon 0 - \frac{d}{dr} 0 - m 0 = 0 \quad , \quad -i\epsilon 0 + \frac{\sqrt{2}d}{r} 0 - m , \]
\[0 = 0 \quad , \quad -i(\frac{d}{dr} + \frac{1}{r}) 0 - i\frac{\sqrt{2}c}{r} 0 - m 0 = 0 . \]

So, all the \( f_i(r) \) turn out to be equal to zero; but such a solution is not of interest because of its triviality.

Here one gives some added comment on extending the vector particle-monopole formalism constructed above to an arbitrary background space-time with spherical symmetry. The relevant Duffin-Kemmer \( eg \)-equation is taken in the form

\[\left[ i\beta^0 \left( e^{-\nu/2} \partial_t + \frac{1}{2} \frac{\partial\nu}{\partial r} e^{-\nu/2} j^{03} \right) + +i\beta^3 \left( e^{-\mu/2} \partial_r + \frac{1}{2} \frac{\partial\mu}{\partial t} e^{-\nu/2} j^{03} \right) + \right. \]
\[\left. - \frac{i}{r} e^{-\mu/2} \left( \beta^1 \beta^{12} + \beta^2 \beta^{23} \right) + \frac{1}{r} \sum_{\theta,\phi} - \frac{mc}{\hbar} \right] \Phi(x) = 0 . \]

Therefore, almost all done above for the flat space model will be easily taken into a curved space model with only several evident changes.

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