Simple Solution to the Strangeness Horn Description Puzzle

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We propose to use the thermal model with the multi-component hard-core radii to describe
the hadron yield ratios from the low AGS to the highest RHIC energies. It is demonstrated
that the variation of the hard-core radii of pions and kaons enable us to drastically improve
the fit quality of the measured mid-rapidity data and for the first time to completely describe
the Strangeness Horn behavior as the function of the energy of collision without spoiling the
fit quality of other ratios. The best global fit is found for the vanishing hard-core radius of
pions and for the hard-core radius of kaons being equal to 0.35 fm, whereas the hard-core
radius of all other mesons is fixed to 0.3 fm and that one of baryons is fixed to 0.5 fm.
It is argued that the multi-component hadron resonance gas model opens us a principal
possibility to determine the second virial coefficients of hadron-hadron interaction.

I. INTRODUCTION

The hadron resonance gas model 1 [1, 2] is the only theoretical tool allowing us to extract information
about the chemical freeze-out (FO) stage of the relativistic heavy ion collisions. Although its systematic
application to the experimental data description began about fifteen years ago [3], many features of this
model are not well studied [4, 5]. Thus, very recently in a critical analysis of the hadron resonance gas
model [5] it was shown that for the description of the hadron multiplicities the baryon charge conservation
and the isospin conservation, used in one of the most successful versions of this model [1], should be
essentially modified, whereas for the description of the hadron yield ratios these conservation laws are not
necessary at all. Although the discussion about the reliable chemical FO criterion has a long history [1, 6],
only recently it was demonstrated that none of the previously suggested chemical FO criteria, including
the most popular one of constant energy per particle $E/N \simeq 1.1$ GeV [6], is robust [5], if the realistic
particle table with the hadron masses up to 2.5 GeV is used. At the same time in [5] it was shown that
despite an essential difference with the approach used in [1], the both versions of the hadron resonance gas

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We apologize for not quoting even the major works on this model which are well known, but the list is so long that we have
to choose just the papers strictly related to our discussion.
model demonstrate almost the same value 7.18 for the entropy per particle at chemical FO. Thus, it turns out that the criterion of the constant entropy per particle at chemical FO is, indeed, a reliable one. It is interesting that the constant entropy per particle at chemical FO was also found in [7], but the way of hard core repulsion used in this work is too different from the traditional one used in the hadron resonance gas model [1, 2, 5] and, hence, in contrast to the results of [1, 2, 5], the model used in [7] leads to a simultaneous fulfillment of a few chemical FO criteria.

One of the traditional difficulties of the hadron resonance gas model is related to the Strangeness Horn description which up to now is far from being satisfactory, although very different formulations of the thermal model are used for this purpose [1, 5, 7, 8]. Note that the principal importance to improve the Strangeness Horn description can be easily understood from the fact that just the non-monotonic behavior of the $K^+/\pi^+$ ratio as the function of the center of mass energy of collision is often claimed to be one of a few existing signals of the onset on deconfinement [9–11]. The previous attempts [1, 7, 8] to describe the Strangeness Horn behavior without spoiling the quality of other particle ratios fit and the thorough analysis performed in [5] led us to a conclusion that further improvement of the hadron resonance gas model can be achieved, if we consider the pion and kaon hard-core radii, as an independent fitting parameters. Evidently, this would allow us to have two additional fitting parameters to improve the fit quality of the Strangeness Horn without spoiling the other hadron yield ratios. The physical idea behind such an approach is that the hadronic hard-core radii are the effective parameters which include the contributions of the repulsion and attraction. Since the parameters of hadron-hadron interaction are, generally speaking, individual for each kind of hadrons, then each kind of hadrons can have its own hard-core radius.

The work is organized as follows. Section II contains the main equations of the model. The results are discussed in Section III, while the last Section contains our conclusions.

II. MULTI-COMPONENT HADRON GAS AND HARD-CORE RADII

The hadron resonance gas model is a successful compromise between the right choice of the physically relevant degrees of freedom and the simple parameterization of their interaction. Its theoretical justification is based on a simple fact found rather long ago [12] that for temperatures below 170 MeV the interacting mixture of stable hadrons and their resonances behaves as the mixture of nearly ideal gases of stable particles which in this case include both the hadrons and the resonances taken with their averaged masses. The reason for such a behavior is nearly a complete cancellation between the attraction and repulsion contributions. The resulting deviation from the ideal gas (a weak repulsion) is usually attributed to the second virial coefficients $b_{ij}$ defined for the hadrons of $i$-th and $j$-th kinds. Since the equations of state of the hadron resonance gas have the Van der Waals type repulsion, the coefficients $b_{ij}$ are called as excluded volumes. Up to now the hadron resonance gas model employed only two basic parameters related to hadron-hadron repulsion: the model of [3] had one excluded volume for pions and another for all other
hadrons, whereas the model [13] suggested to consider one common hard-core radius for mesons $R_m$ and another hard-core radius for baryons $R_b$. However, none of the models developed in [3, 13] were correct [4], since they did not include the second virial coefficient of the crossed type $b_{ij}$ with $i \neq j$, i.e. between the hadrons of different kinds. On the one hand the realization of this fact led to a systematic description of the data with a single hard-core radius for all hadrons [1], and on the other hand it also led to the development of the multi-component Van der Waals gas models [4, 14, 15].

The success of the hadron resonance gas model, the one [1] or two component [4, 5], in the data description may look surprising at the first glance, but this is not just a single example of a simple statistical model that is able to efficiently account for the complex features of interaction between the constituents. One should remember, although the interaction between the clusters of many molecules in the real gases or interaction between the nuclear fragments is no less, but more complex than interaction of hadrons, the successful statistical models for such systems are well known [16–20], and, nevertheless, these models are able not only to describe the low density states of real gases [16, 18] or that ones of nuclear fragments [19, 20], but they are able to successfully model the condensation of these gases into the corresponding liquids at rather high densities. And one should also remember that these models employ a few statistical parameters only and use rather simple, but physically adequate (!) parameterization for the many-body effects. Thus, the whole point is that in all these successful examples [16–19] the employed parameters which characterize the interaction are effective from the very beginning and only at very low densities the models recover the virial expansion up to the second virial coefficients. Therefore, one should not be surprised that the second virial coefficients $b_{ij}$ of the hadron resonance gas are some effective parameters which account for (to large extent) a cancellation of the attractive and repulsive contributions, and which, in principle, could be individual characteristics for each hadronic pair. However, the overall success of the hadron resonance gas model evidences that the number of independent parameters should be essentially smaller then the number of hadron types. Thus, below we demonstrate that the available data favor just the set of the excluded volumes defined as $b_{ij} \equiv \frac{2\pi}{3}(R_i + R_j)^3$ via the hard-core radii of pions $R_\pi$, kaons $R_K$, baryons $R_b$ and the radius for all other mesons $R_m$. In what follows we give the main equations of the multi-component formulation referring to [4, 5] for a detailed derivation.

Consider the Boltzmann gas of $N$ hadron species in a volume $V$ that has the temperature $T$, the baryonic chemical potential $\mu_B$, the strange chemical potential $\mu_S$ and the chemical potential of the isospin third component $\mu_{I3}$. The system pressure $p$ and the $K$-th charge density $n^K_i$ ($K \in \{B, S, I3\}$) of the i-th hadron sort are given by the expressions ($\mathcal{B}$ denotes a symmetric matrix of the second virial coefficients
with the elements $b_{ij}$)

$$p = T \sum_{i=1}^{N} \xi_{i} , \quad n_{i}^{K} = Q_{i}^{K} \xi_{i} \left[ 1 + \frac{\xi_{i}^{T} B_{\xi}}{\sum_{j=1}^{N} \xi_{j}} \right]^{-1} , \quad \xi = \begin{pmatrix} \xi_{1} \\ \xi_{2} \\ \vdots \\ \xi_{s} \end{pmatrix} ,$$

(1)

where the variables $\xi_{i}$ are the solution of the following system

$$\xi_{i} = \phi_{i}(T) \exp \left( \frac{\mu_{i}}{T} - \sum_{j=1}^{N} 2 \xi_{j} b_{ij} + \frac{\xi_{i}^{T} B_{\xi}}{\sum_{j=1}^{N} \xi_{j}} \right) , \quad \phi_{i}(T) = \frac{g_{i}}{(2\pi)^{3}} \int \exp \left( -\sqrt{\frac{k^{2} + m_{i}^{2}}{T}} \right) d^{3}k .$$

(2)

Here the full chemical potential of the $i$-th hadron sort $\mu_{i} \equiv Q_{i}^{B} \mu_{B} + Q_{i}^{S} \mu_{S} + Q_{i}^{I3} \mu_{I3}$ is expressed in terms of the corresponding charges $Q_{i}^{K}$ and their chemical potentials, $\phi_{i}(T)$ denotes the thermal particle density of the $i$-th hadron sort of mass $m_{i}$ and degeneracy $g_{i}$, and $\xi_{i}^{T}$ denotes the row of variables $\xi_{i}$.

In a special case when all the elements of the second virial coefficients matrix are equal $b_{ij} = v_{0}$ Eqs. (1)–(2), evidently, reproduce the one component model with the pressure

$$p = T \sum_{i=1}^{N} \phi_{i}(T) \exp \left( \frac{\mu_{i} - p v_{0}}{T} \right) ,$$

(3)

which defines the particle density of $i$-th kind of hadron as $n_{i} = \frac{\phi_{i}(T)}{T + p v_{0}/T} \exp \left[ \frac{\mu_{i} - p v_{0}}{T} \right]$. The latter shows that the ratios of two particle densities defined by (3) match that ones of the mixture of the corresponding ideal gases for an arbitrary value of $v_{0}$, while the particle densities themselves may essentially differ from the particle densities of the ideal gas.

It is known that the resonance width is important at low temperatures [1]. Similarly to [1], the width $\Gamma_{i}$ of the resonance of mean mass $m_{i}$ is modeled by replacing the Boltzmann distribution function in the particle thermal density (2) by its average over the Breit-Wigner mass distribution as

$$\int \exp \left( -\sqrt{\frac{k^{2} + m_{i}^{2}}{T}} \right) d^{3}k \rightarrow \int_{M_{0}}^{\infty} \frac{dx}{(x-m_{i})^{2} + \Gamma_{i}^{2}/4} \int \exp \left( -\sqrt{\frac{k^{2} + x^{2}}{T}} \right) d^{3}k ,$$

(4)

where $M_{0}$ is the dominant decay channel mass. Such a substitution provides a simple, but reliable approximation to account for the resonance width.

The contribution of the resonance decays is accounted for as usual: the total density of hadron $X$ consists of the thermal part $n_{X}^{th}$ and the decay ones:

$$n_{X}^{tot} = n_{X}^{th} + n_{X}^{decay} = n_{X}^{th} + \sum_{Y} n_{Y}^{th} Br(Y \rightarrow X) ,$$

(5)

where $Br(Y \rightarrow X)$ is the decay branching of the $Y$-th hadron into the hadron $X$. The masses, the widths and the strong decay branchings of all hadrons were taken from the particle tables used by the thermodynamic code THERMUS [21].
The strange charge conservation completes the list of equations used. Since in strong decays the strangeness is conserved, then it is sufficient to impose the vanishing of the total strangeness for thermal densities at chemical FO, i.e. to determine the strange chemical potential $\mu_S$ from the equation\[ \sum_{i=1}^{N} n_i^S = 0. \] As it was shown recently [5] the baryonic charge and isospin conservation laws should not be imposed to fit the hadron multiplicity since they lead to unphysically huge FO volumes. Therefore, in this work for the data at given energy of collision we use the following fitting parameters: temperature $T$, baryonic chemical potential $\mu_B$ and the chemical potential of the third projection of isospin $\mu_{I3}$. Note that such a procedure is completely consistent with fitting the hadron multiplicities instead of hadron yield ratios [5], the main difference is only that to fit the hadron multiplicities one has to use the chemical FO volume $V$ as an additional parameter. As it was explained earlier the global fitting parameters are the hard-core radii of pions $R_\pi$, kaons $R_K$, baryons $R_b$ and that one for all other mesons $R_m$. In addition, to demonstrate the pure effect of the radii variation we do not include any strangeness suppression factor into simulations. Then one should expect some minor problems with the description of multi-strange baryons.

III. RESULTS

The recent comprehensive analysis [5] performed for different hard-core radii of all mesons $R_m$ and all baryons $R_b$ clearly showed us that the good description of the data can be achieved for many pairs of these radii. However, the ratios are more stable during the fitting, if $R_m = 0.3$ fm and $R_b = 0.5$ fm. Right these values of hard-core radii were fixed and then we fitted the data by the $\chi^2/dof$-criterion for different values of the pion $R_\pi$ and kaon $R_K$ hard-core radii taken below 0.5 fm each. The minimal value of $\chi^2/dof \approx 1.018$ for the energies in the range $\sqrt{s_{NN}} = 2.7, 3.3, 3.8, 4.3, 4.9, 6.3, 7.6, 8.8, 12, 17, 130, 200$ GeV (for details see below) was obtained for $R_\pi = 0$ fm and $R_K = 0.35$ fm.

Since in the present approach there is no principle difference between fitting the absolute hadron yields at mid-rapidity or their ratios, we prefer to fit ratios in order to reduce the volume of numerical efforts. In our choice of the data sets we basically followed Ref. [1]. Thus, at the AGS energy range of collisions ($\sqrt{s_{NN}} = 2.7 - 4.9$ GeV) the data are available for the kinetic beam energies from 2 to 10.7 AGeV. For the beam energies 2, 4, 6 and 8 AGeV there are only a few data points available: the yields for pions [22, 23], for protons [24, 25], for kaons [23] (except for 2 AGeV), for $\Lambda$ hyperons the integrated over 4$\pi$ data are available [26]. For the beam energy 6 AGeV there exist the $\Xi^-$ hyperon data integrated over 4$\pi$ geometry [27]. However, the data for the $\Lambda$ and $\Xi^-$ hyperons have to be corrected [1], and instead of the raw experimental data we used their corrected values of Ref. [1]. For the highest AGS center of mass energy $\sqrt{s_{NN}} = 4.9$ GeV (or the beam energy 10.7 AGeV) in addition to the mentioned data for pions, (anti)protons and kaons there exist data for $\phi$ meson [28], for $\Lambda$ hyperon [29] and $\bar{\Lambda}$ hyperon [30].

As one can see from the left panel of Fig. [1] the quality of the fit achieved for $\sqrt{s_{NN}} = 4.9$ GeV is extremely good even for $\bar{\Lambda}/\Lambda$ and $\Lambda/\pi^-$ ratios, i.e. for the most problematic ratios of [1]. This is related to
FIG. 1: The particle yield ratios described by the present multi-component hadron gas model. The best fit for $\sqrt{s_{NN}} = 4.9 \text{ GeV}$ is obtained for $T \simeq 131 \text{ MeV}$, $\mu_B \simeq 539 \text{ MeV}$, $\mu_{I3} \simeq -16 \text{ MeV}$ (left panel), whereas for $\sqrt{s_{NN}} = 17 \text{ GeV}$ (right panel) it is obtained for $T \simeq 147.6 \text{ MeV}$, $\mu_B \simeq 218 \text{ MeV}$, $\mu_{I3} \simeq -2.1 \text{ MeV}$. A yield ratio of two particles is denoted by the ratio of their respective symbols.

an essential improvement of the kaons and their ratios in the present model. Thus, the $\bar{\Lambda}$ anomaly [1, 30] is not seen at this energy.

In the SPS energy range we used only the NA49 mid-rapidity data for all ratios. There are two main reasons for such a selection. First, the NA49 are self-consistent and have relatively small error bars for all energies. Second, it is well known that right these data are traditionally the most difficult ones to be described within the thermal model [1, 7, 8, 31, 32]. Therefore, in order to demonstrate the new possibilities of the multi-component hadron resonance model we concentrate on the NA49 data fitting. In contrast to [1], we included into the fit procedure $\Omega/\pi^-$ and $\Xi/\pi^-$ ratios, but excluded from it the dependent $\Xi/\Lambda$ and $\Omega/\Xi$ ratios for hyperons. The results for the highest SPS energy $\sqrt{s_{NN}} = 17.3 \text{ GeV}$ are shown in the right panel of Fig. 1. These results are compared to the NA49 mid-rapidity data for pions, kaons and (anti)protons [33, 34], for the set of strange (anti)hyperons [35–37] and for $\phi$ meson [38].

The variation of the $R_\pi$ and $R_K$ radii immediately allowed us to notably improve the description of $K^+/\pi^+$ ratio and all the ratios involving the strange hyperons and pions (for instance, look at $\Xi^-/\pi^-$ and $\Omega/\pi^-$). This also led to a slight improvement of $K^-/K^+$ ratio. However, a slight change for the total strangeness of kaons means a larger change of the strange hyperons densities. Although the most problematic ratios at this energy, namely $\bar{\Lambda}/\Lambda$, $\bar{\Xi}^-/\Xi^-$ are $\bar{\Omega}/\Omega$, are improved only marginally compared to [1], but as one can see from the right panel of Fig. 1 the crossed ratios of $\Xi/\Lambda$ and $\Omega/\Xi$, which were not fitted, are automatically reproduced well. The obtained results for the chemical FO temperature $T$ and baryonic chemical potential $\mu_B$ almost coincide with the values $T \simeq 152 \text{ MeV}$, $\mu_B \simeq 226 \text{ MeV}$ found in [1] for this energy for the fitting the NA49 data alone. However, the resulting quality of our fit at this energy of collision is essentially better: $\chi^2/dof \simeq 1.57$ determined in this work against $\chi^2/dof \simeq 2.78$ found in
The same trend is seen for all the SPS energies: a small variation of kaon hard-core radius and a vanishing pionic hard-core radius systematically improve the fit quality of the NA49 data. After such a fitting of the most ‘hard’ data it is clear that a high quality description of other data sets (or of all data sets) is possible, if the experimental data of different collaborations are reanalyzed and become consistent with each other.

Since the RHIC high energy data of different collaborations agree with each other, we just analyzed the STAR results for \( \sqrt{s_{NN}} = 130 \text{ GeV} \) [39–42] and 200 GeV [42–44]. For the main subject of the present work the exotic ratios are not of a great importance and, hence, for the RHIC energies we fitted the same set of hadronic ratios as for the highest SPS energy (see the right panel of Fig. 1). The variation of the pionic and kaonic hard-core radii practically does not affects the fit quality at the RHIC energies. This is clearly seen from the comparison of the best fit results for \( \sqrt{s_{NN}} = 130 \text{ GeV} \) found here \( T \simeq 163.1 \text{ MeV}, \mu_B \simeq 27.3 \text{ MeV} \) and the values \( T \simeq 162.5 \pm 5.5 \text{ MeV}, \mu_B \simeq 35 \pm 11 \text{ MeV} \) found in [1] for a combined fit of PHENIX and STAR data assuming that the pion data do not contain any contribution from weak decays. The better agreement is seen for the chemical FO parameters at \( \sqrt{s_{NN}} = 200 \text{ GeV} \) obtained here \( T \simeq 162.2 \text{ MeV}, \mu_B \simeq 17.6 \text{ MeV} \) and the values \( T \simeq 160.5 \pm 2 \text{ MeV}, \mu_B \simeq 20 \pm 4 \text{ MeV} \) found in [1] for a combined fit of PHENIX and STAR data excluding \( \bar{p}/\pi^- \) and \( \phi/K^- \) ratios.

The above results are not surprising, since from Fig. 2 it is seen that the present multi-component fit almost reproduces the values of the chemical FO temperature and baryonic chemical potential obtained in the model with a single hard-core radius \( R = 0.3 \text{ fm} \) [5] (squares) are also shown.

FIG. 2: \( \sqrt{s_{NN}} \) dependence of the chemical FO temperature (left panel) and baryonic chemical potential (right panel) found here within the multi-component model (circles). For a comparison the corresponding quantities for a model with a single hard-core radius \( R = 0.3 \text{ fm} \) [5] (squares) are also shown.
best description of the Strangeness Horn, i.e. $K^+ / \pi^+$ ratio, alone with $\Lambda / \pi^-$ ratio, is demonstrated in Fig. 3. These two ratios were thoroughly studied within the one component hadron resonance gas model in [1] and then their analysis (together with other ratios) was continued in [8]. As we mentioned earlier for the one component model the fit of ratios is not affected by the excluded volume correction. If, however, one imposes some additional constraints like the artificial baryon charge conservation criticized in [5], then some small correction (below 5%) can appear. This is the reason why the one component model fit of $K^+ / \pi^+$ and $\Lambda / \pi^-$ ratios found in [1, 8] for a single hard-core radius $R = 0.3$ fm is essentially worse than the one component model fit with the same hard-core radius shown in Fig. 2. Evidently, in Refs. [1, 8] the fit of $\Lambda / \pi^-$ ratio indicates a problem with too steep rise as a function of $\sqrt{s_{NN}}$ compared to the data, while both of these ratios show too slow decrease compared to the data. Note that such a behavior of $K^+ / \pi^+$ and $\Lambda / \pi^-$ ratios is typical for almost all statistical models (see Figs. 7 and 8 and the corresponding discussion in [7]). Evidently, the too steep rise in $\Lambda / \pi^-$ behavior is a consequence of the $\bar{\Lambda}$ anomaly [1, 30] discussed above. The one component fit of the present approach does not indicate such difficulties for $\Lambda / \pi^-$ ratio, while the slow decrease in $K^+ / \pi^+$ ratio still is there. However, the multi-component approach really removes such a defect in $K^+ / \pi^+$ without spoiling the other ratios including $\Lambda / \pi^-$ one as it is seen from Fig. 2. The results for other ratios are available and will be shown in a longer work.

Actually the best fit for $K^+ / \pi^+$ ratio found here practically coincides with the dashed curve drawn in Fig. 4 of [8] which was obtained assuming an existence of the Hagedorn mass spectrum of hadrons [15]. However, we do not share this hope of the authors of Ref. [8]. Also we do not agree with such an estimate and, hence, we cannot accept it as a real solution of the puzzling problem. Our skepticism is based on the following facts. First of all, we note that inclusion of the hypothetical states with the masses up to 3 GeV should essentially modify not only $K^+ / \pi^+$ results, but all other ratios in uncontrollable way. Then
the authors of [8] would spoil their own results reported in many nice works. Second, it is unclear to us why in this case one should stop at hadron mass of 3 GeV and not to increase it till 10 GeV or even to 100 GeV? It is clear that already in the former case the one component excluded volume description would lead to a whole complex of problems that are typical to the Hagedorn mass spectrum and in order to get rid of them one will have to unavoidably introduce the excluded volume which is proportional to a mass of a heavy resonance. Such models are well known [46, 47], but then in the framework of such models the heavy hadronic resonances should be regarded as quark-gluon bags and the whole treatment should be completely changed.

Furthermore, in [8] the estimates based on the Hagedorn mass spectrum inclusion were done without accounting for the large resonance width and without knowing the branching ratios of these hypothetical resonances. Although the mass dependence of width of heavy resonances was found within the finite width model of quark gluon bags [46, 47] and recently it was successfully verified on the Regge trajectories of heavy mesons [48], the possible channels of their decays and the corresponding branching ratios are completely unknown yet. However, the more serious issue is that the finite width model [46, 47] explains that the ‘missing’ hadrons with the masses above 2.5 GeV and with the large width are the quark gluon bags, which are extremely suppressed (by fifteen-sixteen orders of magnitude) compared to the stable hadrons up to the temperatures of about half of the Hagedorn temperature. Therefore, we again come to a conclusion that the ‘missing’ hadrons should not be included into the hadron resonance gas model spectrum, but they should be attributed to the spectrum of quark gluon bags. The practical consequence out of these facts is as follows: due to the short life time $\tau \simeq \sqrt{M/M_0}$ of 0.5 fm/c of the bag of mass $M \geq M_0 \simeq 2.5$ GeV, by the time of chemical FO such bags should have been, probably, completely decayed into the stable hadrons and light hadron resonances. Since up to the moment of chemical FO the chemical equilibrium is assumed to exist, then all thermodynamic quantities of pions (or other particles appeared from these bags) should (locally) have their equilibrium values in accordance with the hadron resonance gas model spectrum, i.e. at chemical FO the result of the bag decays should not be seen.

IV. CONCLUSIONS

In the present work we considered the hadron resonance gas model with multi-component hard-core radii, i.e. we treat the pion $R_\pi$ and kaon $R_K$ hard-core radii as independent fitting parameters compared to the hard-core radius of baryons $R_b = 0.5$ fm and that one of all other mesons $R_m = 0.3$ fm. Such an approach allows us to essentially improve the quality of the global fit of hadron yield ratios derived from the mid-rapidity data measured at the AGS, SPS and two highest RHIC energies. Thus, for $R_\pi = 0$ fm and $R_K = 0.35$ fm we found $\chi^2/dof \simeq 1.018$ which is the best value for the global fit compared to other analyses. It is necessary to stress that at SPS energies the present fit included only the NA49 data,
which are usually hard to be fitted by the hadron resonance gas model. The suggested approach allows us to drastically improve the quality of the data description and, as a consequence, it is able to completely describe the Strangenes Horn irregular behavior for the first time. Thus, the developed approach gives a simple solution to the puzzle of the Strangenes Horn description without the need to use the hypothetical hadron resonances of masses up to 3 GeV which, so far, are not observed in the experiments.

The found small values of hard-core radii are consistent with the results of analysis done in [12] that up to the temperatures of about 170 MeV the hadron-hadron repulsive and attractive interaction contributions into the system pressure practically compensate each other. In fact, we determined the second virial coefficients of hadrons using the statistical model with the multi-component hard-core repulsion. Evidently, such an approach can be used to further improve the description of other particle ratios. Thus, the suggested multi-component model provides us with a practical way to extract the second virial coefficients for all hadrons and tabulate them as a function of temperature as this is done for usual gases. The main problem, however, is related to the poor quality of existing experimental data. The present analysis clearly shows that to accurately determine the hadronic second virial coefficients and, thus, to provide the community with the data allowing in principle to extract the statistical measure of interaction for any pair of hadrons we need much better data up to $\sqrt{s_{NN}} \simeq 20$ GeV. We hope that the Dubna Nuclotron and the future colliders NICA and FAIR will successfully resolve at least the half of this task.

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