Experimental study and performance comparison on various types of rectangular piano key side weirs at a 120° section of a 180° curved channel

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Abstract
Application of side weirs with high effective length is necessary to discharge excessive flows, to control the flow in water conveyance systems, and irrigation and drainage systems. Most of the studies on the side weirs have been conducted on the straight channels and linear weirs. The flow pattern on the outer arc of the curved channels and its suitability for side weir can be used and combined with the piano key weirs. So far, no comparison has been made on rectangular piano key side weirs (RPKSW) at a 120° Section of a 180° Curved Channel. In this study, an experimental study was performed on A-, B-, C-, and D-type RPKSW at a bend angle of 120 degrees. The results showed that the specific energy at two ends of the RPKSWs was the same, with a slight difference of 3.4% for A-Type, 1.3% for B-Type, 1.1% for C-Type, and 1.8% for D-Type weirs. The discharge coefficients of the studied weirs were also investigated, and it was concluded that B-Type weir has better performance than other weirs. On average, the discharge coefficient of B-Type weir was 9.9%, 21.2%, and 24.1% higher than that of A-Type, C-Type, and D-Type weir, respectively. It was shown that the ratio of P/h1 is the main parameter affecting the weir discharge coefficient. Finally, an empirical equation was proposed for each weir. The proposed equation has MAE = 0.028 for A-Type weir, MAE = 0.041 for B-Type weir, MAE = 0.049 for C-Type weir, and MAE = 0.053 for D-Type weir.

Keywords Piano key side weir (RPKSW) · Discharge coefficient · Bend channel · Specific energy · Free flow condition

Introduction
Weirs are typically designed and constructed in two general ways: perpendicular (inline) or parallel (lateral) to the flow direction. The weir constructed parallel to the flow direction along with the main channel on the channel wall are referred to side weir (Izadinia and Heidarpour 2016). The main tasks of the lateral weirs are to discharge the excess water from the main channel in the municipal sewer systems, distribution channels, and to control the river flooding and regulate the water level (Zaji and Bonakdari 2017). Side weirs can exhibit different behaviors depending on the position and type of weir (Oertel 2015). To increase the discharge capacity of side weirs, the weir length should be increased; however, these conditions are not easy to provide (in some cases, it is impossible). For this purpose, the conditions must be created to increase the weir length in a small width (Haghiabi et al. 2018). It also makes it easier to use the outer arch of the bend channel to increase the discharge capacity. According to previous studies, the flow in the curved channels and river meanders is inclined toward the outer arch. In some cases, the side weirs in the straight channels are not capable of discharging the required flow rate. The flow in the curved channel is much more complex than the flow in other channels so that the centrifugal force is the effective force on the arc and the rivers that are usually curved (Agaccioglu et al. 2012).
Piano key weir (PKW) is nonlinear weirs that have better benefits than the other weirs due to position, discharge capacity, and the upstream head of the weir (Li et al. 2020). These weirs usually are located perpendicular to the flow direction. Rectangular piano key weirs (RPKWs) in the straight channels as inline weir were first investigated by Blanch and Lempérière (Erpicum et al. 2013). Afterwards, many experimental studies have been conducted on PKWs (Machiels et al. 2011; Ribeiro et al. 2012). Anderson and Tullis (2012) investigated experimentally inline RPKW and labyrinth weir. They indicated that the RPKW discharge coefficient at a straight channel (inline) was greater than that of the labyrinth weirs. Erpicum et al. (2014) showed that the ratio of weir height to width in RPKW was more efficient than the other parameters. Akbari et al. (2019) used different models of artificial intelligence methods to predict the discharge coefficient in gated piano key weirs (GPKW) and concluded that the accuracy of the GPR model is higher than the other models. Kumar et al. (2020) studied the trapezoidal piano key weir (TPKW) and evaluated the discharge equation of these weirs. They finally developed an equation using the M5 intelligent method for determining the discharge coefficient.

Numerous experiments on side weirs with different shapes in various types of canals have been investigated in order to understand appropriately the behavior of these weirs and to evaluate the discharge coefficient and efficiency of these weirs. Many experimental studies were conducted on the discharge capacity side weirs in a straight channel. Side weir in subcritical flow was investigated in the rectangular straight channels by many researchers (Ramamurthy et al. 1995; Borghet et al. 1999; Huagao et al. 2007; Mohammed and Mohammed 2011; Vatankhah 2012). In addition, many experimental investigations have been conducted on the labyrinth weir in the straight channels. Emiroglu et al. (2010) investigated the hydraulic performance of the labyrinth side weir in a straight channel and determined the discharge coefficient of this weir. It was found that the labyrinth weir with an angle of 45 degrees has the highest discharge coefficient among the weirs. Parvaneh and Borghet (2009) used oblique weir to increase the discharge coefficient in the straight channel and examined the specific energy of the first and second edges of the weir. They also concluded that these weirs have 35% higher efficiency than rectangular weirs. Nezami et al. (2015) examined the labyrinth weir discharge coefficient. They proposed some empirical relationships to calculate the discharge coefficient of these weirs. Emiroglu et al. (2014) used one-cycle and two-cycle trapezoidal labyrinth side weirs and examined the discharge coefficient of these weirs.

Various experiments have been conducted on the curved channel with different shapes of weir. Fares and Herbertson (1993) investigated the side weir at a bend channel with an angle of 60° and analyzed the flow pattern under these conditions. Agaccioglu and Yüksel (1998) analyzed the rectangular side weir in a curved channel. They showed that the discharge coefficient of the rectangular side weir at a 120°C Section is the highest discharge coefficient for $L/b < 1$. Coşar and Agaccioglu (2004) used triangular weir as a lateral weir in a curved channel. They found that the triangular weir with an angle of 120°C degrees has the highest discharge coefficient among the weirs. Ağacıçıoğlu et al. (2007) investigated the effect of side weir at a curved channel with an angle of 30° on scouring the sedimentary bed. They concluded that the scour occurs in the outer arch. Bilhan et al. (2011) investigated the discharge coefficient for labyrinth side weir using the intelligence model to predict in the curved channels and concluded that the accuracy of artificial neural networks (ANN) is acceptable. Agaccioglu et al. (2012) used a rectangular sharp-crest side weir at different arc angles. They concluded that the equation developed for the weir with one-cycle is not suitable for weir with two-cycle. The development of secondary currents after the weir in critical flow conditions was also observed in this study. They concluded that a stagnation point is created after the side weir. Few experiments have been performed on the different types of PKWs in straight and curved channel. Experimental investigations on piano key weirs as side weir are limited. Karimi et al. (2018) investigated C-Type RPKSW in a straight channel. They showed that C-Type RPKSWs and rectangular labyrinth side weirs are more effective than related linear side weirs in terms of discharge capacity. Saghari et al. (2019) examined type of A trapezoidal piano key side weirs (TPKSWs) in a curved channel. They presented an empirical equation to calculate the discharge coefficient of a A-type TPKSW. Mehri et al. (2018), in an experimental study, compared the discharge coefficient of C-type RPKSWs at two angles of 30 and 120 degrees and concluded that the weir discharge coefficient at angle of 120 degrees was greater than that of for the angle of 30 degrees. Also, Mehri et al. (2019) developed an optimal model by combining Kolmogorov–Gabor equations and neural networks for predicting the coefficient of discharge of C-Type RPKSW in a curved channel and showed that this developed group method data handling (DGMDH) model has an appropriate accuracy. The difference between RPKW and RPKSW is shown in Fig. 1.

Many studies have shown the angle of 120°C is proper for side weirs with $L/b < 1$ (Agaccioglu and Yüksel 1998; Coşar and Agaccioglu 2004; Mehri et al. 2018). So far, no comparison has been made on A, B, C, and D type of RPKSW at a 120° Section of a 180° Curved Channel. The main purpose of this study was to evaluate, compare and select the best side weir based on the hydraulic performance of the A-, B-, C-, and D-types RPKSWs as an alternative to a nonlinear side weir in 120°C Section of a 180°C rigid curved channels. According to the features and superiority
of this type of nonlinear weirs and based on the optimal hydraulic performance, a superior RPKSW is recommended, especially, for water intake or flow diversion in the regions where there are space restrictions.

**Materials and methods**

As mentioned before, the purpose of this study was to investigate, compare and select a superior side weir based on the hydraulic performance of a variety of RPKSW (A-, B-, C-, and D-type) as alternatives to nonlinear side weir in rigid curved channels with clear-water flow conditions.

**Experimental work**

This study was conducted at the hydraulic laboratory of Soil Conservation and Watershed Management Institute (Iran). The experiments were conducted in a curved flume with a diameter of 4 m, length of 17 m, depth of 0.5 m, and width of 0.5 m, with 0.001-bed slope (Fig. 2). The experimental model consisted, type of A, B, C, and D, RPKSW (Figs. 3, 4 and 5). The characteristics of the weirs and channel used in the present study are presented in Tables 1 and 2. The RPKSWs and channel were made of plexiglass. The lateral channel, set at 120° to the main channel. A sluice gate was set at the end of the main channel for flow depth control. The overflow rate was measured by two calibrated sharp-crested weirs downstream of the main channel and the collection channel for RPKSW. A triangular weir was installed downstream of the main channel to measure the flow rate. In addition, the outflow from side weir was measured by a rectangular weir. PV09 profile indicator (Delft Hydraulics) was used to measure water surface profile.

**Fundamentals of side weir theory**

Given that the flow in the side weir is spatially varied flow (SVF), the flow conditions in the main channel are SVF with decreasing discharge. The differential equation for SVF is written as follows:

$$\frac{dy}{dx} = \frac{S_0 - S_f - \left( \frac{Q}{gA^2} \right) \left( \frac{dQ}{dx} \right)}{1 - \left( \frac{Q^2}{gA^3} \right)}$$

where, \(y\) is water depth in main channel, \(x\) is the direction of flow, \(S_0\) is bed slope, \(S_f\) is the energy slope, \(Q\) is the main channel discharge, \(dQ/dx\) is the variation of discharge per unit width of side weir. In these conditions, the discharge per unit weir width is calculated as follows:

$$q = \frac{-dQ}{dx} = C_d \frac{2}{3} \sqrt{2gh} \left( h - p \right)^{3/2}$$

Therefore, the discharge equation of side weir is:
where \( Q_1 \) is the discharge at the upstream in the main channel, \( Q_2 \) is the discharge at the downstream in the main channel, \( Q_w \) is the discharge over side weir, \( C_d \) is the discharge coefficient (DeMarchi coefficient), \( g \) is the gravitational acceleration, and \( h \) is the water depth in the channel and \( P \) is the weir height. Mehri et al. (2019) proposed Eq. (4):

\[
C_d = \frac{F_1}{\sqrt[3]{\frac{V_1^2}{g} \cdot \frac{L}{h_1} \cdot \frac{P}{R_c} \cdot \frac{P_d}{B}}}
\]

where, \( C_d \) is discharge coefficient, \( F_1 \) is the Froude number, \( V_1 \) is the mean flow velocity of the main channel (first edge), \( g \) is the gravitational acceleration, \( L \) is the length of the side weir, \( b \) is the width of the main channel, \( P \) is the weir height, \( h_1 \) is the flow depth at the upstream end of the side weir, \( R_c \) is the radius of the main channel center, \( P_d \) is the height of the foundation, and \( B \) is the weir length along with the flow direction. Also, to analyze the changes in the specific energy (\( E \)) was calculated using Eqs. 5 and 6:

\[
E_1 = h_1 + \frac{V_1^2}{2g}
\]

\[
E_2 = h_2 + \frac{V_2^2}{2g}
\]

where, \( E \) is specific energy (m), \( h_1 \) = Upstream flow depth (m), \( h_2 \) = Downstream flow depth (m), \( V_1 \) is Upstream mean velocity (m/s) and \( V_2 \) is Downstream mean velocity (m/s).

Mean velocity (\( V \)) is calculated using Eq. 7

\[
V = \frac{Q}{A}
\]

To evaluate the accuracy of the calculated equation, the mean absolute error (MAE) was used.

\[
MAE = \frac{1}{N} \sum_{i=1}^{N} |C_d(\text{observed}) - C_d(\text{calculated})|
\]

**Results and discussion**

The flow over the RPKSWs consists of three parts. Inflow from the input key of the side weir, outflow from the output keys downstream of the RPKSW, and the flow through the parallel walls to the flow direction. According to the experiments, as the flow through the main axis of the curved channel approaches the side weir, it is re-directed near the weir. According to the experiments, the water surface profile at the beginning of the weir is not linear and follows the flow pattern in the side weirs. The flow is deflected at the first edge of the weir and tends to the second edge of the weir. In addition to the inlet and outlet keys, the weir side edges are effective in flow
discharge and a large part discharge is passed through these side walls. Furthermore, the sloped part makes the weir to be aerated and prevents the submergence of the weir.

**Specific energy**

Given the arc-shaped of the channel and the use of RPKSW, the DeMarchi principle, which is the constant energy at the first and second edges of the weir in the straight channel and rectangular sharp-crest weir, is for complex current conditions (the arc-shaped channel and the nonlinear weir). The specific energy the first and second edges of the weir ($E_1$ and $E_2$) was calculated (Fig. 6). To investigate the variations of the specific energy, the average percentage of changes in specific energy ($E$) was calculated. This value is 3.4% for A-Type p RPKSW, 1.3% for B-Type RPKSW, 1.1% for C-Type RPKSW, and 1.8% for D-Type RPKSW. It can be assumed that the specific energy under these conditions is equal to the first and second edges of the weir.
Fig. 4 A schematic view of RPKSW

Fig. 5 Weirs used in the laboratory
Variations of discharge coefficient in A-, B-, C-, and D-type RPKSWs

Laboratory study of the discharge coefficient in RPKSWs and their differences with each other is very important, indicating the performance of these weirs. The discharge coefficient of the studied weirs in the curved channel is determined using the experimental evaluation of the discharge coefficients for four types of RPKSWs. Figures 7 and 8 show the discharge coefficients in the weirs compared to the dimensionless ratio of $P/h_1$. As can be observed, the discharge coefficient of these weirs has a significant relationship with the ratio $P/h_1$, so that the coefficient of discharge of RPKSW increases with increasing the dimensionless ratio of $P/h_1$. This is due to the shortening of the separation zone, which increases the effective weir length and, therefore, the discharge coefficient. In contrast, by decreasing this ratio, separation length and the submergence rate in input keys of the RPKSW increase, reducing the discharge coefficient. It can be revealed that B-type RPKSW has better hydraulic performance than other RPKSWs, which is consistent with previous studies in the straight channel when the weir was perpendicular to the flow direction. On average, the discharge coefficient of B-type weir is 9.9%, 21.2%, and 24.1% higher than that of A-, C-, and D-type weirs, respectively. Figures 9 and 10 show the variations of the discharge coefficient against the Froude number in three weirs with different lengths and widths. In this figure, the discharge coefficient of B-type RPKSW against the Froude number is the highest among the studied weirs. However, the dependence of this coefficient on the Froude number is much less than that of the parameter $P/h_1$. The arrangement of the keys and inlet openings are of the important issues causing the change in the discharge coefficient in the RPKSWs. As can be seen, B-type weir has enough space for the flow to pass under the inlet key and to reach the edges, whereas this is slightly obvious in A-type weir and do not observe in C- and D-types. The most important difference among the weirs is the additional space to be added under the inlet in B-type weir.

Figure 11 shows the discharge coefficients in the weirs compared to the dimensionless ratio of $P_d/B$, $L/R_c$, and $L/b$. Our investigation reveals that the $P_d/B$ parameter was an important parameter on the discharge coefficient while the

### Table 1  Characteristics of the used weirs

| Weir type | Weir crest length (l(m)) | Height of weir crest (P(m)) Width of the inlet and outlet keys (w_c, w_o(m)) | Width of the inlet key (P_i (m)) | Width of side weir (L (m)) | Height of the foundation (P_d (m)) |
|-----------|--------------------------|---------------------------------------------------------------------------|-----------------------------|-----------------|-------------------------------|
| A1        | 1.25                     | 0.11, 0.14, 0.18, 0.21 | 0.0605                      | 0.11             | 0.25                          | 0, 0.03, 0.07, 0.10 |
| B1        | 1.25                     | 0.11, 0.14, 0.18, 0.21 | 0.0605                      | 0.11             | 0.25                          | 0, 0.03, 0.07, 0.10 |
| C1        | 1.25                     | 0.11, 0.14, 0.18, 0.21 | 0.0605                      | 0.11             | 0.25                          | 0, 0.03, 0.07, 0.10 |
| D1        | 1.25                     | 0.11, 0.14, 0.18, 0.21 | 0.0605                      | 0.11             | 0.25                          | 0, 0.03, 0.07, 0.10 |
| A2        | 1.75                     | 0.15, 0.18              | 0.0845                      | 0.15             | 0.35                          | 0, 0.03             |
| B2        | 1.75                     | 0.15, 0.18              | 0.0845                      | 0.15             | 0.35                          | 0, 0.03             |
| C2        | 1.75                     | 0.15, 0.18              | 0.0845                      | 0.15             | 0.35                          | 0, 0.03             |
| D2        | 1.75                     | 0.15, 0.18              | 0.0845                      | 0.15             | 0.35                          | 0, 0.03             |

### Table 2  The variables related to the flume

| Variable | Signal | Unit | Value |
|----------|--------|------|-------|
| Flume long | – | M | 17 |
| Flume width | b | M | 0.5 |
| Inner radius of flume | $R_1$ | M | 2 |
| Radius of main channel centerline | $R_c$ | M | 2.25 |
| Outer radius of flume | $R_1$ | M | 2.5 |
| Channel depth | – | M | 0.5 |
| Channel slope | S | – | 0.001 |
| Discharge at the upstream | $Q_1$ | Lit/s | 6.2–41 |
| Discharge over side weir | $Q_w$ | Lit/s | 3–30 |
| Length (width) of side weir (m) | L | M | 0.35 |
| Angle | α | – | 120 |
effect of this parameter in the design can be ignored. Moreover, the $L/b$ parameter is one of the parameters that should be considered in the design. Figure 11 shows an increase in discharge coefficient by raising this dimensionless parameter. The $L/R_c$ parameter affects the discharge coefficient, and it has directly related to this parameter. However, the effect of these parameters on the discharge coefficient is low and has a lower correlation.

A comparison is made between previous research on a sharp rectangular and sharp-crested weir in a curved channel and the present study with piano key weir. Figure 12 shows that there is a great difference between the discharge coefficients of a rectangular sharp-crested weir with a piano key weir. The greater effective length as well as the design of this weir is the main reason for this performance.

**Propose an equation for calculating the weir discharge coefficient**

In this study, an equation was presented for each A, B, C, and D-type RPKSW. Table 3 presents the proposed equations for calculating the discharge coefficient. As can be seen, the proposed equations have $\text{MAE} = 0.028$ and $R^2 = 0.9984$ for A-Type weir, $\text{MAE} = 0.041$ and $R^2 = 0.9942$ for B-Type weir, $\text{MAE} = 0.049$ and $R^2 = 0.89$ for C-Type weir, and $\text{MAE} = 0.053$ and $R^2 = 0.93$ for D-Type weir. As can be seen in Fig. 13, the weir discharge coefficient in the experimental conditions has been acceptably calculated.
Fig. 7 Variations of discharge coefficient of RPKSW vs. the ration of $P/h_1$ with $L = 25$ cm.

![Graph](image1)

Fig. 8 Variations of discharge coefficient of RPKSW vs. the ration of $P/h_1$ with $L = 35$ cm.

![Graph](image2)
Conclusions

In this study, the hydraulic performance of A, B, C, and D-type RPKSWs at an arc angle of 120 degrees was experimentally investigated. The results of this study can be summarized as follows: Specific energy for the curved channel is also stable at an angle of 120 degrees with a nonlinear side weir, and the difference in $E_1$ and $E_2$ is negligible in this study. The flow passed through the RPKSW has three main parts, including flow from input keys, output keys, and lateral edges of the weir. When these weirs are used as side weir, the flow is inclined to the second edge of the weir and flow discharge is greater than the keys near the second edge. Due to the high discharge coefficient of the RPKSW, these side weirs may solve considerably the major problems encountering in mountainous regions, meanders and bend channels with limitations in design, cost, and layout of the weirs, where the increase in the width of the lateral channel
is not possible. Finally, among the studied weirs, the discharge coefficient measured in the B-type RPKSW is greater than that of the A-, C-, and D-type RPKSWs at 120°C. The authors of this paper recommend that the effect of sediment on the discharge coefficient of RPKSW in a curved channel should be investigated.

**Table 3** The proposed equations for RPKSWs

| Weir   | Equation                                      | MAE  | $R^2$ |
|--------|-----------------------------------------------|------|------|
| A-Type | $C_d = 1.38 \left( \frac{L}{h_1} \right)^2 - 2.22 \left( \frac{L_d}{w} \right) (Fr) + 0.56$ | 0.028 | 0.96 |
| B-Type | $C_d = 11.04 \left( \frac{L}{h_1} \right)^2 - 1.66 \left( \frac{w}{h} \right) \left( \frac{L}{h_1} \right) - 10.04 \left( \frac{L}{h_1} \right) + 3.25$ | 0.041 | 0.95 |
| C-Type | $C_d = 5.87 \left( \frac{L}{h_1} \right)^2 - 0.98 \left( \frac{w}{h} \right) \left( \frac{L}{h_1} \right) - 5.45 \left( \frac{L}{h_1} \right) + 2.16$ | 0.049 | 0.89 |
| D-Type | $C_d = 7.06 \left( \frac{L}{h_1} \right)^2 + 2.31 \left( \frac{L}{L_d} \right) \left( \frac{L}{h_1} \right) - 8.43 \left( \frac{L}{h_1} \right) + 2.68$ | 0.053 | 0.93 |

**Fig. 13** Evaluation of the accuracy of the proposed equations
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