SIMULATION OF THICK ACCRETION DISKS
WITH STANDING SHOCKS
BY SMOOTHED PARTICLE HYDRODYNAMICS

Diego Molteni
University of Palermo, 90100 Palermo, Italy

Giuseppe Lanzafame
Osservatorio Astronomico, University of Catania, Catania, Italy

Sandip K. Chakrabarti*
Astronomy and Astrophysics
The University of Chicago, 5640 S. Ellis Av., Chicago, Il, 60637

and

Aspen Center for Physics, Aspen, Colorado 81611
Abstract

We present results of numerical simulation of inviscid thick accretion disks and wind flows around black holes. We use Smoothed Particle Hydrodynamics (SPH) technique for this purpose. Formation of thick disks are found to be preceded by shock waves travelling away from the centrifugal barrier. For a large range of the parameter space, the travelling shock settles at a distance close to the location obtained by a one-and-a-half dimensional model of inviscid accretion disks. Occasionally, it is observed that accretion processes are aided by the formation of oblique shock waves, particularly in the initial transient phase. The post-shock region (where infall velocity suddenly becomes very small) resembles that of the usual model of thick accretion disk discussed in the literature, though they have considerable turbulence. The flow subsequently becomes supersonic before falling into the black hole. In a large number of cases which we simulate, we find the formation of strong winds which are hot and subsonic when originated from the disk surface very close to the black hole but become supersonic within a few tens of the Schwarzschild radius of the blackhole. In the case of accretion of high angular momentum flow, very little amount of matter is accreted directly onto the black hole. Most of the matter is, however, first squeezed to a small volume close to the black hole, and subsequently expands and is expelled as a strong wind. It is quite possible that this expulsion of matter and the formation of cosmic radio jets is aided by the shock heating in the inner parts of the accretion disks.

Subject Headings: accretion, accretion disks - black hole physics - hydrodynamics - shock waves

* On temporary leave of absence from Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay-400005, INDIA (Permanent Address)
1. INTRODUCTION

In the literature, some results of the numerical simulation of thick accretion disks using finite difference methods are present (Wilson, 1978; Hawley, Smarr & Wilson, 1984, 1985). These works show that the thick accretion disks could form during the accretion of matter which has a significant angular momentum. It is found that shocks are formed which travel outward in the disks. This work also points out that 'hollow' jet like features are produced which propagate roughly along the funnel wall. Subsequently, Eggum, Coroniti and Katz (hereafter ECK, 1987, 1988) simulated Keplerian disks with viscosity and radiation transport. These solutions do not produce shock waves, but strong winds are produced. In paper I, (Chakrabarti & Molteni, 1993) we presented basic motivations of studying disk models which include shock waves as well as some results of one-dimensional simulations showing that in an inviscid thin disk standing shocks should be quite common.

In the present paper, we simulate the formation of thick disks, keeping in mind that not only can travelling shocks form as noted by earlier workers, but standing shocks can form as well. In view of the recent advances in understanding of shock formation in the ‘thick’ disk (see, Chakrabarti 1989, 1990a), we have a priori knowledge of the parameter space (spanned by, say, the energy $E$ and angular momentum $\lambda$ of the flow) for which shocks may or may not form in the disk. Guided by the input from this analytical work (which is, at best, valid for one-and-a-half dimensional inviscid disks, where the vertical motion is neglected compared to the other velocity components), we have been able to understand in a fairly complete manner the status of shock formation in thick accretion disks as well as the manner in which winds may originate. Our principal conclusions are: (a) for a significant range of energy and angular momentum of the flow, the thick disks may contain standing shock waves; (b) models which ignore the vertical motion
inside the disk are reasonably good, particularly in the preshock region of the flow; (c) shock locations predicted by the one-and-a-half dimensional models are at lower radial distances than the shocks locations actually obtained in the simulations – this discrepancy is removed when the effects of the turbulent pressure in the post-shock region and the extra compression due to the vertical motion are included; (d) of the two shock locations predicted by the analytical work, the shock located farther away from the black hole is stable – this conclusion is similar to that obtained for one dimensional thin accretion flows [paper I]; (e) there could be considerable mixing in the matter in the postshock region near a black hole; and finally, (f) a strong supersonic wind can originate from regions close to the black hole, and the shock heating in this region is probably an essential ingredient for such behavior.

A major difference in the manner in which shocks form in a thin disk (Paper I) and a thick disk (of the present work) is that whereas in the thin (one dimensional) disk, some perturbation must be introduced in order to have a shock, in the thick disk, perturbations are always present in the form of turbulence. Thus, we do not obtain any solution which does not contain shocks, stationary or non-stationary. Our work also brings about a major departure from the standard models of disks in astrophysics. From the results of paper I and the present paper, we claim that shock waves are more common in accretion disks than hitherto realised, and their effects must be included in interpreting observed results from AGNs and stellar disks. This statement is particularly true when the disk is radiation pressure driven rather than viscous driven, i.e., when the infall timescale due to pressure gradient force is shorter compared to the timescale of the transport of angular momentum by viscosity.

The plan of the present paper is the following: In the next Section, we briefly present
analytical models of thick accretion disks which contain significant radial motions (unlike the canonical thick disk models of, say, Paczyński and Wiita, 1980, where motions other than that in the azimuthal direction are ignored). These models can be found in detail in Chakrabarti (1989, 1990a). However we now qualitatively include the effects of the turbulent pressure in the post-shock region and the extra compression due to significant vertical velocity component. In §3, we produce results of a few numerical simulations. Finally, in §4, we summarize our results and make concluding remarks.

2. SHOCK LOCATIONS IN A 1.5D FLOW WITH TURBULENCE

We assume a rotating, axisymmetric, adiabatic, accretion flow in vertical equilibrium, near a black hole. We take Newtonian models for the non-rotating central compact object as given in terms of the Paczyński & Wiita (1980) potential. We also assume a polytropic equation of state for the accreting (or, outflowing) matter, \( P = K \rho^\gamma \), where, \( P \) and \( \rho \) are the isotropic pressure and the matter density respectively, \( \gamma \) is the adiabatic index (assumed in this paper to be constant throughout the flow, and is related to the polytropic index \( n \) by \( \gamma = 1 + 1/n \)) and \( K \) is related to the specific entropy of the flow \( s \): \( s = \text{const} \) implies \( K = \text{const} \). We assume that the flow is non-dissipative, so that the specific angular momentum \( \lambda \) is constant everywhere. This also implies that the entropy density, and thus \( K \) can vary only at the shock. A complete solution of the stationary model requires the equations of energy, angular momentum and mass conservation supplied by transonic conditions at the critical points and the Rankine-Hugoniot conditions at the shock. The general procedure followed is the same as is presented in Chakrabarti (1989, 1990a). Presently, however, we include the qualitative effects of the turbulent pressure in the post-shock flow in order to obtain shock locations in the thick, turbulent disk. When the vertical component of velocity is significant, its infall toward the equatorial
plane causes an extra compression of the flow. Effects of this has also been qualitatively included. Inclusion of these effects show that the parameter space in which shocks can form may be vastly enhanced than what is described in Chakrabarti (1989, 1990a).

In what follows, we use the mass of the black hole $M$, the velocity of light $c$ and the Schwarzschild radius $R_g = 2GM/c^2$ as the units of mass, velocity and distance respectively. The dimensionless energy conservation law can be written as,

$$E = \frac{\vartheta^2}{2} + \frac{a^2}{\gamma - 1} + \frac{\lambda^2}{2x^2} - \frac{1}{2(x - 1)}$$

(1a)

Here, $\vartheta$ and $a$ are the non-dimensional radial velocity and sound speed, $x$ is the non-dimensional radial distance. Apart from an unimportant geometric factor, the mass conservation equation is given by,

$$\dot{M} = \vartheta \rho x h$$

(1b)

where $h$ is the constant half-thickness of the flow which, assuming hydrostatic equilibrium in vertical direction, is given by $h = ax^{1/2}(x-1)$. It is useful to write the mass conservation equation in terms of $\vartheta$ and $a$ in the following way,

$$\dot{\mathcal{M}} = \vartheta a^{2n+1}x^{1/2}(x - 1)$$

(2)

We shall use the phrase ‘accretion rate’ for this quantity, keeping in mind, however, that $\dot{\mathcal{M}} \sim \dot{M} K^n$ does not remain constant at the shock because of the generation of entropy. The shock conditions which we employ here are the following (subscripts ‘-’ and ‘+’ refer to quantities before and after the shock): The energy conservation equation,

$$E_+ = E_-,$$

(3a)

the pressure balance condition,

$$W_+ + W_T + \Sigma_+\vartheta_+^2 = W_- + \Sigma_-\vartheta_-^2$$

(3b)
and the baryon number conservation equation,

\[ \dot{M}_+ = \dot{M}_- \quad (3c) \]

Here, \( \Sigma \) and \( W \) are the density and pressure respectively which are integrated in the vertical direction (Chakrabarti, 1989). In Equation 3b, we include turbulent pressure \( W_T \sim \alpha \Sigma a^2 \), \( \alpha \) is assumed to be a constant \((<1)\) measuring the effects of turbulence. A part of \( W_T \) is contributed by an extra compression of the flow due to the infalling matter from the vertical direction, as the vertical velocity could be significant. In order to have a shock, the radial flow must be supersonic, i.e., the stationary flow must pass through a sonic point. The sonic point conditions are derived in Chakrabarti (1989) and will not be discussed here in detail. However, we derive here the Mach number relation, which enables one to obtain the shocks locations very easily.

From Equations 3(a-c) and (2), at the shock location \( x = x_s \), we obtain,

\[
\frac{1}{2} M_+^2 a_+^2 + \frac{a_+^2}{\gamma - 1} = \frac{1}{2} M_-^2 a_-^2 + \frac{a_-^2}{\gamma - 1} \quad (4a)
\]

\[
\dot{M}_+ = M_+ a_+^{2n+2} x_s^{1/2} (x_s - 1) \quad (4b)
\]

\[
\dot{M}_- = M_- a_-^{2n+2} x_s^{1/2} (x_s - 1) \quad (4c)
\]

\[
\frac{a_+^{2n+3}}{M_+} \left[ \frac{2}{3\gamma - 1} + (\alpha + M_+^2) \right] = \frac{a_-^{2n+3}}{M_-} \left[ \frac{2}{3\gamma - 1} + (\alpha + M_-^2) \right] \quad (4d)
\]

After elimination of some variables, we obtain the relationship between the pre-shock and the post-shock mach numbers, as given by,

\[
M_+ M_- = \frac{2 + \alpha(3\gamma - 1)}{(3\gamma - 1)^2 - C(\gamma - 1)} \quad (5)
\]

in terms of the Mach invariant function, \( C = C(M) \) at the shock,

\[
C = \frac{[2 + (3\gamma - 1)(\alpha + M^2)]^2}{M^2[2 + (\gamma - 1)M^2]} \quad (6)
\]
Figure 1 shows the parameter space spanned by the specific energy $\mathcal{E}$ and specific angular momentum $\lambda$ of the flow for which standing shocks may be formed in the disk when $\alpha = 0$ is chosen (Chakrabarti 1989, 1990a). This region, bounded by three curves, is labelled with $\Sigma$. Numerically, however, standing shocks were found even outside this region, because in general, some turbulence is always present in the thick disk. In Figure 2, we show the effects of turbulence on the location of the shocks ($X_{s3}$ of Chakrabarti 1989) for $\lambda = 1.65$ for a range of $\alpha$ and specific energy $\mathcal{E}$. It is clear that shocks form farther from the hole, as the turbulent pressure as well as the extra compression due to vertical motion goes up. Another important point to note is that the range of energy for which shocks form with $\alpha \neq 0.0$ is much higher than the range obtained with $\alpha = 0.0$. This shows that the chance of shock formation is much higher in a turbulent disk, particularly, for low enough energy.

3. RESULTS OF SIMULATION OF THICK ACCRETION DISKS AND WINDS

The Smoothed Particle Hydrodynamics (SPH) method that we use has been primarily developed to deal with fluid dynamics in astrophysical context (Lucy 1977; Gingold & Monaghan 1977; for a recent review, see, Monaghan, 1992). In Paper I, we have presented the procedure for the implementation of the code in axisymmetric cylindrical co-ordinates. The results described in this Section are obtained with a range of initial conditions which may prevail in realistic circumstances. We study the following diverse cases: (A) the injection rate is uniform with height at the outer edge and (B) the injection rate is such that at the outer edge, the disk vertical structure is isothermal. We simulate (B) with low as well as high angular momentum. Since in this case, the flow is in equilibrium at the injection radius, the infall is less ‘violent’ than in case (A), and it takes longer time to
reach a steady state solution. Due to limitations in computing time, we had to stop the simulation in case (B) before a *complete* steady state is reached, though the simulations were carried out to times much longer than the infall timescale. In all the cases, we inject matter only in one quadrant and use reflection boundary condition to obtain the solutions in other quadrants.

We note here that $\dot{M}$ (but not $\dot{\dot{M}}$) is an eigenvalue of our problem and is fixed by a choice of the input parameters $\mathcal{E}, \lambda$. We therefore chose the density of matter at the outer edge to be unity and the rate of matter injection is automatically adjusted to achieve a steady state flow. The reference density is obtained by the actual accretion rate $\dot{M}$. For an accretion rate of $\dot{M} = \dot{m}\dot{M}_{Edd}$, the reference density is

$$\rho_{\text{ref}} = \frac{\dot{m}\dot{M}_{Edd}}{4\pi c X_{\text{ref}}^2}$$

(7a)

where, $X_{\text{ref}} = \frac{2GM}{c^2}$, and the reference temperature is,

$$T_{\text{ref}} = \left[3c^2\gamma - 1\right]^{1/4}$$

(7b)

with $a$ as the radiation constant.

Figures 3(a-d) show results of a Case (A) simulation. Particles with angular momentum $\lambda = 1.65$ and energy $\mathcal{E} = 0.006$ are injected at the outer edge of the disk at $x = 30.0$. The result shown is at $T = 700$ when the flow has achieved steady state. The total number of particles is 60,000. A shock is clearly formed at $x \sim 16.6$. Using the 1.5D model without any turbulent pressure ($\alpha = 0.0$) the shock location for these parameters is at $X_{s3} = 11.2$, which is much closer to the black hole than the location we observe here. The post-shock flow clearly contains by a strong vertical motion (ignored in the 1.5D model) as well as turbulence. Assuming the shifted location of the shock is entirely due to the turbulent pressure, we obtain $\alpha = 0.14$, which is very realistic (see, Fig. 2).
A weaker oblique shock is also produced in this example. Later we will show a stronger case of oblique shock formation (see, Fig. 5b below).

In Figure 3a, X-Z locations of the ‘pseudo’-particles are presented. In Figure 3b, we zoom a region closer to the hole, in order to show detailed behavior of the flow. We plot arrows at one in every five particles for clarity. The length of an arrow is proportional to the Mach number of the flow at the location where the arrow is originated ($M = 1$ corresponds to 0.2 in length). A few observations could be made in this context: (a) The shock is very sharp (width of about $0.2 – 0.4$ Schwarzschild radius), (b) The subsonic flow in the post-shock region becomes highly supersonic before plunging into the black hole, (c) The postshock flow is first diverted away from the equatorial plane due to the presence of a very strong turbulence between $r = 7.0$ and $r = 14.0$ and then enter into the hole at an angle. (d) A part of the flow is diverted altogether from the black hole in the form of winds. This latter behavior is due to the heating and subsequent expansion in the postshock region. The overall behavior of the Mach number is clear in Figure 3c, where the contours of constant Mach number are plotted. The contours are drawn in the linear scale with minimum, maximum and interval given by: $M_{\text{min}} = 0.03$, $M_{\text{max}} = 2.0$ and $\Delta M = 0.2$. Note in particular that the wind becomes supersonic at distances as close as $x \sim 22$. In Figure 3d, we show the contours (in linear scale) of constant temperature (dimensionless) of the flow with $T_{\text{min}} = 0.14$, $T_{\text{max}} = 1.2$ and $\Delta T = 0.06$. Note that the contours of constant temperature resemble those of canonical thick disk models (cf. Paczyński and Wiita, 1980) in the post shock region. This is due to the fact that in the immediate vicinity of the postshock region, the radial velocity is very small, and our simulation reproduces the original thick disk solution. Our result indicates that the postshock region of the flow could be considered as the thick accretion disk.
In the above example, we showed only the final result. At initial stage of the simulation the shock is formed very close to the hole, which subsequently travelled outward to reach its stable location. This behavior is very similar to what was observed in Paper I. Similarly, by reducing the vertical height of the disk, we recover the result that the shock at \( x \sim 16.6 \) actually corresponds to \( X_{s3} \), the outer shock of the analytical solution of Chakrabarti (1989). Thus, as in Paper I, only the shock at \( X_{s3} \) is found to be stable.

In Case (B) simulations, matter is assumed to be in hydrostatic equilibrium in the vertical direction at the outer edge. Figure 4(a-d) shows the result of a simulation for \( \lambda = 1.625, \mathcal{E} = 0.007 \) where isothermal vertical structure at the outer edge is used. This case is so chosen that the parameters lie at the boundary between shock and no-shock solutions in analytical 1.5D model (with \( \alpha = 0 \)). At a little higher energy a weak shock should form at around \( X_{s3} = 6.9 \). The observed shock in the simulation at \( \sim 14.0 \) corresponds to \( \alpha = 0.2 \) in our present model. The parameter space for which shocks may form is vastly bigger in presence of turbulence and vertical motion in the disk. This behavior is found to be particularly significant when low angular momentum flow is considered. This is because the turbulent pressure becomes comparable or more than the centrifugal pressure of matter close to the black hole.

In the next example, we study the formation of winds, as well as the interaction of winds with the accreting matter. We choose the injection radius at \( x = 30 \) and the wind formed is allowed to go much beyond this radius. The thickness of the disk at the outer edge (\( x = 32 \)) is chosen to be such that the density falls off to a value of about 10 percent of the equatorial density. Figure 4a shows the flow behavior. Length of the arrows are proportional to the local Mach numbers. Note that a shock is formed at about \( x = 14 \) where the flow becomes subsonic, and another shock is formed in the region where
the wind is interacting with the injected matter. The interaction heats up the gas and matter is turned around to join the strong wind that is developed above the disk. Figure 4b shows contours of constant Mach number in linear scale. The minimum, maximum and the interval of Mach number are: $M_{\text{min}} = 0.04$, $M_{\text{max}} = 4.0$, $\Delta M = 0.2$. Notice that the shock strength (namely, Mach number jump at the shock) at the interaction region is much higher compared to the shock on the equatorial plane. Also note that the wind formed certainly becomes supersonic by $x \sim 30$. In Figure 4c, we show the contours of constant temperature in linear scale with $T_{\text{min}} = 0.076$, $T_{\text{max}} = 0.8$ and $\Delta T = 0.02$. As in Figure 3d, the temperature distribution in the post shock flow also resemble as that in the thick accretion disks of Paczyński and Wiita (1980). In Figure 4d, we show the ratio of the outflow rate to the injection rate as a function of time. The outflow commences at $T \sim 400$ and steadily rises to about 10 percent. The rapid fluctuation throughout is mostly due to noise, typical of SPH technique. We stop the simulation at about $T = 2000$, at which time the outflow ratio was slowly increasing.

We present another Case (B) simulation where we use a high angular momentum ($\lambda = 1.80$). The objective was to simulate disks which are much larger as well as thicker. We inject matter at $x = 100$ and the thickness chosen to be $x = 70$ at this edge. The disk is vertically cut off at $x = 70$ where the density is 10 percent of the equatorial density. Because of limitations on computer processing time, we stop the simulations at $T = 1780$ when there are 63,345 particles in the region of integration. The number was still increasing very slowly, indicating that the steady state was not reached yet. Because of high angular momentum, shocks are produced at a very large radius ($X_{s3} = 50.0$ in the 1.5D model with $\alpha = 0$). There is a considerable backflow of the matter (bounced off the centrifugal barrier close to the hole) which diverted the flow off the equatorial plane. Interaction with the diverted flow with the infalling matter from a very large vertical
height causes oblique shocks to develop. Figure 5a provides the Mach number distribution in this case, showing a vertical strong shock forming at about $x = 40$. Oblique shocks are also seen. In Figure 5b, we zoom the region with triple shocks which are remarkably resolved in our simulation. The flow coming closer to the equatorial plane is diverted away by the first shock, but a second oblique shock is seen to refract matter toward the black hole. A weak backflow near the equatorial plane is also seen. Such backflows were also present in the initial transient phase of the Case (A) simulation (Figure 3). However, it developed into small scale turbulence when otherwise steady state was reached. We expect that similar behavior will prevail in the present case also.

5. SUMMARY AND CONCLUDING REMARKS

In this paper, we have studied the nature of the Rankine-Hugoniot shocks in thick accretion disks using Smoothed Particle Hydrodynamics method. Our numerical simulations indicate that shocks are essential ingredients in the formation of a thick accretion disk, and that they could be present for a large range of initial parameters. Unlike simulations by earlier workers, we find disks with stationary shocks. The initial injection condition at the outer edge was chosen from the 1.5D analytical models and location of the shock appears to be roughly in agreement with the prediction from 1.5D model, though, inclusion of the effects of turbulence in the post shock region as well as the extra compression (both of which we model qualitatively here) renders a better agreement between the numerical and the theoretical results. In Paper I, we noted that the outer shock at $X_{s3}$ (Chakrabarti, 1989) was chosen by the flow and not the one located at $X_{s2}$. We find similar result in our present 2D simulation also. An important difference, however, is that, whereas, in the one dimensional simulations (Paper I), one had to use ‘perturbed’ initial condition in order to produce shocks, in the two dimensional simula-
tions, the turbulence present is sufficient to ‘induce’ shock formation. Thus shocks are always present in our simulations even when the parameter space is so chosen that they are non-stationary.

Our observations regarding shock formation inside the disk are particularly true when the viscosity of the disk is very low and the flow is mostly pressure driven rather than viscous driven. Chakrabarti (1990b), in his study of standing shocks in viscous isothermal disks, found that the shock located at $X_{s3}$ becomes weaker (and ultimately disappear) as viscosity is increased. This result, together with the present simulation, indicates why in the standard models of the accretion disks where the flow is viscous driven (Shakura & Sunyaev 1973) one need not be concerned with shock formation. Indeed, the simulation of viscous, initially Keplerian, axisymmetric accretion disks (as opposed to our constant angular momentum disk) by ECK (1987, 1988) does not show any shock waves. In this case, viscosity reduces the angular momentum significantly to sub-Keplerian values throughout the disk [see, Fig. 10 of ECK (1988)] so that the flow faces a very weak centrifugal barrier. The flow does not strongly ‘bounce back’ at the barrier and as a result, the winds are found to have much weaker kinematic flux. Winds in these simulations carry a fraction of a percent of the infalling matter (as opposed to our simulation where a sometimes almost ten percent of matter is blown away) and the specific angular momentum carried by the wind is smaller as well. These simulations show strong, non-steady equatorial outflows, the physical origin of which are not immediately obvious.

A major problem in accretion disk theory is to provide a suitable prescription for viscosity mechanisms which may be operating in the disk. Recently, a significant progress has been made in the literature. It appears that non-axisymmetric spiral waves (Spruit, 1987; Hanawa, 1988; Chakrabarti, 1990c) or internal waves (Vishniac & Diamond, 1992
and references therein) or violent instabilities developing in presence of small vertical component of magnetic fields (Balbus & Hawley, 1992 and references therein) could play a significant role in angular momentum transport. In future, we plan to incorporate non-axisymmetry as well as explicit viscosity to verify some of these assertions.

One of the most significant results of our simulation is that, for the first time, it is realised that the winds (a precursor of radio jets?) with larger kinematic flux are produced in those cases where strong shocks are also present, indicating that the shock heating may be an important ingredient in the ejection of matter from the disk surface. Our simulation also indicates that a very thorough mixing of matter (not obvious in the figures we presented here) may be taking place in the post-shock region before a part of it is expelled from the disk as winds. A corollary of this process is that, in the event matter undergoes a significant amount of nucleosynthesis (Chakrabarti, Jin and Arnett, 1987) in the post-shock region of the disk (particularly valid in low mass X-ray binaries), the wind may be rich in metallicity and contribute significantly to the metallicity of the galaxy which is observed. This is clearly an important problem, and we plan to pursue this in the near future.
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FIGURE CAPTIONS

Fig. 1: Parameter space (bounded by three curves and is labeled as $\Sigma$) spanned by the specific energy $\mathcal{E}$ and specific angular momentum $\lambda$ of the flow for which standing shocks may be formed in the analytical 1.5D disk models.

Fig. 2: Effects of turbulence is seen on the location of the stable shock for a range of $\alpha$ (as labeled) and specific energy $\mathcal{E}$. Specific angular momentum $\lambda$ (in units of $2GM/c$) is chosen to be 1.65. With the increase in $\alpha$ the shock seems to be pushed away from the black hole and the range of energy for which shocks may be formed seems to go up.

Fig. 3(a-d): Results of a simulation of thick accretion disks with shocks (seen here at $x \sim 16.6$) in which matter is injected with angular momentum $\lambda = 1.65$ and energy $\mathcal{E} = 0.006$. Total number of particles are 60,000. (a) X-Z coordinates of the particles, (b) Mach number field of the flow close to the shock, (c) contours of constant Mach number, and (d) contours of constant non-dimensional temperature. See text for details.

Fig. 4(a-d): Result of a simulation of thick accretion disks with shocks (seen here at $x = 14$) for $\lambda = 1.625$ and $\mathcal{E} = 0.007$. (a) Mach number field of the flow, (b) contours of constant Mach number, (c) contours of constant temperature, and (d) ratio of the outflow rate and the injection rate of matter as a function of time. See text for details.

Fig. 5(a-b) Simulations results of a thick disk with a high angular momentum ($\lambda = 1.80$). (a) Contours of constant Mach number showing a vertical shock at $x = 40$ and (b) Mach number field showing the formation of oblique shocks which divert flows towards the black hole.