ARE THERE UNSTABLE PLANETARY SYSTEMS AROUND WHITE DWARFS?

JOHN H. DEBES AND STEINN SIGURDSSON

Department of Astronomy and Astrophysics, Pennsylvania State University, University Park, PA 16802

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ABSTRACT

The presence of planets around solar-type stars suggests that many white dwarfs should have relic planetary systems. While planets closer than ~5 AU will most likely not survive the post–main-sequence lifetime of their parent star, any planet with semimajor axis greater than 5 AU will survive, and its semimajor axis will increase as the central star loses mass. Since the stability of adjacent orbits to mutual planet-planet perturbations depends on the ratio of the planet mass to the central star’s mass, some planets in previously stable orbits around a star undergoing mass loss will become unstable. We show that when mass loss is slow, systems of two planets that are marginally stable can become unstable to close encounters, while for three planets the timescale for close approaches decreases significantly with increasing mass ratio. These processes could explain the presence of anomalous IR excesses around white dwarfs that cannot be explained by close companions, such as G29-38, and may also be an important factor in explaining the existence of DAZ white dwarfs. The onset of instability through changing mass ratios will also be a significant effect for planetary embryos gaining mass in protoplanetary disks.

Subject headings: planetary systems: formation — stars: evolution — stars: mass loss — white dwarfs

1. INTRODUCTION

The discovery of ~80 planets, and counting, around solar-type stars suggests that successful planet formation is quite common. The wealth of systems so unlike the solar system leads one to conclude that many aspects of planetary system formation and dynamical evolution have yet to be fully explored. One particularly interesting area is the long-term evolution of planetary systems specifically in the presence of post–main-sequence evolution of the central star. Observations of planets around post–main-sequence stars may provide additional information about the formation and evolution of planetary systems around main-sequence stars and can inform us about the long-term future of the solar system.

While planets at distances similar to those of the outer planets in the solar system will persist through post–main-sequence evolution (Duncan & Lissauer 1998), it is unlikely that close Jovian companions to such stars survive. As the star evolves it expands, engulfing anything up to ~1 AU (Sackmann, Boothroyd, & Kraemer 1993; Siess & Livio 1999a, 1999b). Outward of 1 AU, up to ~5 AU any planet’s orbit will decay through tidal transfer of angular momentum and be consumed within the envelope of the star (Rasio et al. 1996). Anything with less mass than a brown dwarf will not survive in the stellar envelope (Livio & Soker 1984; Soker, Harpaz, & Livio 1984).

Planets may still be observed in close orbits around white dwarfs if their orbits are significantly changed by some process that occurs after the asymptotic giant branch (AGB) phase. If the planets become unstable to close approaches with each other, their interaction would result in a planet close to the central star, a scenario similar to those proposed for the formation of close Jovian planets around main-sequence stars (Rasio & Ford 1996; Weidenschilling & Marzari 1996; Lin & Ida 1997). The results of close encounters between two Jovian planets have been studied in detail and show three possible outcomes: the two planets collide, leaving a large planet; one planet is ejected; or both planets remain in a new stable configuration (Ford, Havlícíková, & Rasio 2001). For planets starting out with semimajor axes greater than 5 AU, ~8% of unstable pairs will collide; the remaining ~92% will not. Of the systems that avoid collision, roughly 40% will settle into a configuration with a planet in a significantly closer and more eccentric orbit than in the initial system. Thus, the onset of instability after post–main-sequence mass loss may create white dwarf systems with planets at orbital radii scoured clear of the original inner planets during the star’s giant phase.

Duncan & Lissauer (1997), simulating the Uranian satellite system, found that Hill-stable systems can become unstable with an increasing mass ratio for satellites orbiting a central massive object. This important work led to one of the few systematic studies of the post–main-sequence evolution of planetary systems dynamically similar to the solar system (Duncan & Lissauer 1998). In that paper, Duncan & Lissauer found that the time to unstable close approaches for the planets followed a power-law relationship with the ratio of planetary mass to stellar mass, as an increasing fraction of mass was lost from the central star. At the level of individual planetary orbits, resonances may also play an important role in an adiabatically changing system, enhancing stability or creating instability. In this paper we generalize the specific results of Duncan & Lissauer (1998) to a range of system parameters applicable to a wider range of situations, such as those in the multiplanet extrasolar systems recently discovered.

Most likely, extrasolar planetary systems also possess Oort cloud analogues, as a direct result of planet formation (Oort 1950; Weissman 1999). For comets in the outer Oort cloud, orbital timescales are comparable to the more rapid late stages of post–main-sequence evolution, and the mass loss of a star is not adiabatic in the context of AGB evolution. If the mass loss is fairly symmetric, many of these comets survive the evolution and can later provide a steady flow of comets that impact the white dwarf directly or break up because of tidal strain and populate the inner system with dust, causing photospheric metal contamination (Stern,
Shull, & Brandt 1990; Alcock, Fristrom, & Siegelman 1986; Parriott & Alcock 1998). However, if the planet systems become unstable to close approaches after the AGB phase, the entire system becomes dynamically young, and new collisions and encounters can occur between surviving comets and planets. Many scenarios lead to a period of enhanced “late bombardment,” as cometary orbits are perturbed and the flow of comets into the inner system is enhanced. In this paper we investigate whether this mechanism can explain the flow of comets into the inner system is enhanced. In this paper we investigate whether this mechanism can explain the flow of comets into the inner system.

The stability of two planets against close approaches depends primarily on the masses of the planets relative to that of the central star and the separation of the two orbits. This separation is measured as \( \Delta = (a_2 - a_1)/a_1 \), where \( a_1 \) and \( a_2 \) are the inner and outer semimajor axes, respectively. A critical Hill separation, \( \Delta_c \), is then the minimum separation between two planets that ensures a lack of close approaches over all time (Hill 1886). A full treatment of the Hill stability of two planets in the case of static masses can be found in Gladman (1993). Several approximations can be made that simplify the full treatment, such as equal planetary masses and small eccentricities. The criterion is then given by

\[
\Delta_c \approx \sqrt{\frac{\mu}{\frac{1}{3} (e_1^2 + e_2^2) + 9 \mu^{2/3}}},
\]

where \( \mu \) is the ratio of the planets’ mass to the central star’s mass, \( e_1 \) and \( e_2 \) are the eccentricities of planets 1 and 2, and \( \Delta_c \) is in units of the inner planet’s semimajor axis \( a_1 \).

If either the mass of the planets or the mass of the star changes, the critical Hill radius will change as well. An increase in planet mass or a decrease in stellar mass will cause \( \mu \) to become larger, increasing the width of the zone in which orbits are unstable to close approaches. During post-main-sequence mass loss, the orbits of planets will widen as the central star loses mass. As long as this process is adiabatic, the planets will simply conserve their angular momentum and widen their orbits proportionally to the mass lost:

\[
a_{\text{new}} = a_{\text{old}} (M_1/M_f).
\]

However, since the orbits widen together by the same factor, \( \Delta \) remains the same. Thus, while the critical separation at which the two planets will become unstable widens, their relative separation remains unchanged. Orbits that are initially marginally stable, or close to being unstable, will become unstable to close planet-planet approaches as a consequence of the mass loss from the central star. In the case of planetary mass accretion in a protoplanetary disk, the orbits of the two planets will move to the same while \( \Delta \) increases, creating the same effect as if the star were losing mass.

The opposite case of stellar mass accretion, or planetary mass loss, works to make previously unstable regions more stable. However, since close approaches generally happen within a few tens of orbits, objects likely would be cleared out of an unstable region more quickly than the region could shrink.

### 2.2. Multiple-Planet Systems

We expect that multiple-planet systems should be common, e.g., the solar system, PSR 1257+12, and \( \nu \) Andromedae (Wolszczan 1994; Butler et al. 1999), and thus it is useful to develop an idea of how these systems remain stable. Chambers et al. (1996) found a relation between the separation of a system of planets and the time it would take for the system to suffer a close encounter:

\[
\log t = b \delta + c,
\]

where \( b \) and \( c \) are constants derived through numerical simulations. The symbol \( \delta \) is related to \( \Delta \) but is defined in a slightly different way. Here, \( \delta \) is the separation between two planets, \( (a_{i+1} - a_i) \), in units of mutual Hill radii (\( R_i \)), defined as

\[
R_i = \left( \frac{1}{3 \mu} \right)^{1/3} \frac{(a_{i+1} + a_i)}{2},
\]

where \( i \) can be from 1 to \( N_{\text{planets}} - 1 \) and we assume that the planets have equal masses and initially circular orbits. If the parameter \( \delta \) is the same for each pair of adjacent planets, the separations in units of AU will be different. For example, if we took three Jovian-mass planets (\( \mu \sim 10^{-3} \)) with \( \delta = 6.5 \) and the innermost Jovian at 5.2 AU from the central star, the next two planets would be at 9.4 and 16.7 AU. Compare to actual orbital radii of 9.6 and 19.1 AU for Saturn and Uranus, respectively, in our presumably stable-for-several-billion-years solar system. We add the obvious caveat that Saturn and Uranus are significantly less massive than Jupiter and have correspondingly weaker mutual interactions.

Adiabatic mass evolution will have the effect of shortening the time it takes for orbits to suffer close approaches. The knowledge of this has long been used to speed up numerical calculations (Duncan & Lissauer 1997 and references therein). However, this fact also leads to the hypothesis that planetary systems on the edge of stability for \( 10^{10} \) yr will be affected by mass loss. In general, the new time to close approaches for an initial \( \delta \) with a change in mass is given by

\[
\log \frac{t_f}{t_i} = (b' - b) \delta + (c' - c).
\]

We would expect \( c \) to have little or no change with a change of mass, since it represents the timescale for two planets at \( \delta \sim 0 \) to suffer a close approach. Mass loss will increase the
mutual Hill radii of the planets, which in turn will change $b$ to a new value we define as $b'$,

$$b' = \left( \frac{\mu_i}{\mu_j} \right)^{1/3} b,$$

(5)

where $\mu_i$ and $\mu_j$ are the final and initial mass ratios, respectively. Such behavior suggests that bodies that are stable over the lifetime of a planetary system will become unstable over a timescale several orders of magnitude smaller than their original timescale for instability, when the central star becomes a white dwarf, assuming that the relation of equation (2) holds for large $\delta$. It has been found that for the case of two planets with $\mu = 10^{-7}$, the parameters $b \approx 1.176$ and $c \approx -1.663$ (Chambers et al. 1996). If the three planets are each separated from their neighbors by $\delta = 6$, they will experience close encounters after $\sim 10^5$ or $10^6$ of the inner planet. For comparison, three planets with the same mass ratio separated by $\delta = 8$ will experience close encounters after $6 \times 10^7$ yr. Assuming that the central star loses half its mass, the timescale to close encounters will shorten by an order of magnitude for the first case and 2 orders of magnitude for the second.

2.3. When is Mass Evolution Adiabatic?

The question of whether mass evolution is adiabatic needs to be addressed. In the case of mass loss for solar-mass stars, roughly half the central star’s mass will be lost on the order of $10^8$ yr. A majority of the mass is lost at the tip of the AGB branch during a period of $\sim 10^9$ yr. Even the quickest rate of mass loss is much longer than one orbital period of a planet inward of 100 AU, the general region where planets are believed to have formed. Stars heavier than a solar mass probably have superwinds, which will cause significant mass loss on the order of a few hundred or thousand years (Vassiliadis & Wood 1993; Schröder, Winters, & Sedlmayr 1999). Whether this is important or not will be the subject of further study. Objects very far from the central star, such as Oort cloud object analogues, have orbital timescales comparable to the mass-loss timescale and will also not follow the adiabatic case. It should be noted that for Kuiper and Oort cloud distances, the mass loss by the star would become adiabatic if the asymptotic wind velocity were orders of magnitude smaller than the escape velocity at the surface of the star, since the crossing time of the wind would then be larger than the orbital timescale of the comets.

Mass gain of stars and planets is much slower than the orbital timescale of a planet. Accretion rates for protostars are on the order of $10^{-6} M_\odot$ yr$^{-1}$ (Shu, Adams, & Lizano 1987). The formation of giant planets through runaway gas accretion takes $\sim 10^7$ yr, the rough lifetime of gaseous protoplanetary disks (Pollack et al. 1996). If some giant planets are formed more quickly by more efficient runaway accretion, gravitational collapse (Boss 2000), or seeding through the formation of other planets (Armitage & Hansen 1999), they would not be described by the adiabatic case.

3. NUMERICAL METHODS

In order to test the hypothesis that adiabatic mass evolution should change the stability of planetary systems, we ran several numerical simulations of two-planet and multiplanet systems in circular orbits around a central star losing mass. The equations of motion were integrated using a Bulirsch-Stoer routine (Stoer & Bulirsch 1980; Press et al. 1992). Since the cases of mass loss by the central star and mass gain by the planet are the same, mass loss can be modeled in two ways. Either the star’s mass can be decreased or the planets’ masses can be increased. If the planets’ masses are increased, the time coordinate must be scaled to reflect the fact that the orbits are widening. To keep our investigations scalable, we chose the units of time to be orbits of the inner planet. We chose to increase the mass of the planets over a period of 1000 orbits. In the absence of mass evolution, energy and angular momentum were conserved to better than 1 part in $10^6$ for $10^5$ orbits. Since changing mass makes this a nonconservative system, energy and angular momentum could not be used as a test of accuracy. However, since the simulations were integrated until a close approach and then terminated, any error is similar to the case of no mass evolution. Several simulations without mass evolution were run, with stable results. A close approach was defined as an encounter separated by a radius of less than $2\mu^{2/5}$ (Gladman 1993 and references therein). This radius was chosen because at separations smaller than this the planet–planet system is dominant and the star becomes a perturbation. Other authors have chosen different criteria (Chambers et al. 1996), but the results are insensitive to the exact choice.

In the two-planet case, we started simulations at the critical separation predicted by equation (1), assuming no mass loss, and increased the separation between the two planets at regular intervals in $\Delta$. We integrated the equations of motion until a close approach, or for $10^5$ orbits. We increased $\Delta$ until it was 25% greater than what would be predicted in the presence of mass loss. These simulations were run an order of magnitude longer than Gladman (1993), and in the no-mass-loss case were consistent with what he found. The two planets were initially started with true anomalies separated by 180°. Our separations are, then, lower limits for the critical separation and thus truly reflect the minimum possible separation between orbits that remain stable. For multiple planets, $\delta$ was started at 2.2 and raised until several consecutive separations did not experience close encounters for $10^5$ orbits. Here random phases in the orbits were chosen, with the restriction that adjacent orbits be separated by at least 40°. Three separate runs with different random initial phases were performed to improve the statistics for each mass, as there is significant scatter in the actual time to a close approach for each separation.

4. RESULTS

4.1. Two Planets

We looked at a wide range of planetary masses for a solar-mass star, from a subterrestrial-sized planet ($\mu = 10^{-7}$) to a Jovian planet ($\mu = 10^{-3}$). Figure 1 shows the border for onset of instability in two-planet systems after mass loss. The dashed line represents the initial critical Hill radius for no mass loss. The solid line that goes through the points is the critical Hill radius for $\mu$ equal to twice that of the initial system, corresponding to the planets, doubling in mass or the central star’s losing half its mass. Several of the higher mass points are greater than predicted by the solid curve, an indication of higher order $\mu$ terms becoming important. It should be noted that these results are general
to any combination of planet and stellar mass that have these ratios.

In a few cases, separations predicted to become unstable after mass loss by the Hill criterion were stable for the length of our simulations. Particularly in the $\mu = 10^{-3}$ case, there was a large region in which the two planets suffered no close encounters (see Fig. 2). These orbits corresponded to a range of $\Delta$ from 0.32 to 0.37, which were predicted to be unstable under mass evolution from the simple scaling of the equation for $\Delta$. It is interesting to note that all of these orbits are close to the 3:2 resonance (see Fig. 3). For the same reason that the Hill radius will not change, these orbits will retain the ratio of their periods. The stability around the 3:2 resonance may be due to those separations being near but not in a region of resonance overlap (Wisdom 1980; Murray & Holman 2001); clearly, this conjecture needs to be confirmed, but that goes beyond the scope of this particular paper.

### 4.2. Multiple Planets

Figures 4, 5, and 6 show the results for three different runs, looking at three-planet systems in circular orbits. We looked at the mass ratios $\mu = 10^{-7}$, $10^{-5}$, and $10^{-3}$. The results are compared with simulations without mass loss, and the difference between the two is quite noticeable for the whole range of mass. It is important to note that for separations whose time to close approach is comparable to the mass-loss timescale, there is little change in behavior between the two cases. This is because the change in the time to close approach is smaller than the scatter in the simulations. Least-squares fitting of the static and mass-loss cases was performed to get the coefficients $b$, $c$, and $b'$. To test our assumption of $c$ not changing under mass evolution, we also measured $c'$, the intercept for the mass-loss case. Planets with initial separations in $\delta$ that were less than $2\sqrt{3}$, the two-planet stability criterion in units of $R_*$, were discarded. For the mass-loss case, data points for which the timescale of close approaches was comparable to the mass-loss timescale were also discarded. Once the coefficients were determined, they were compared with what was predicted from equation (2). Similarly, $b$ and $c$ from the $\mu = 10^{-7}$ case without mass loss were compared with the results of Chambers et al. (1996). Table 1 shows that within the uncertainties, $c$ indeed does not change with mass evolution and the slopes are consistent with predictions. In addition, our results for the static case with $\mu = 10^{-7}$ are consistent with the Chambers et al. (1996) values for $b$ and $c$. 
As mass increases, the presence of strong resonances becomes more important. This is a result of our choice of equal separations and equal masses; many of these resonances would disappear with small variations in mass, eccentricity, and inclination (Chambers et al. 1996), aspects that will be tested with further study. The presence of resonances is most easily seen in Figure 6, where $\mu = 10^{-3}$. In the range of $\delta = 4.4-5.2$, the points greatly depart from the predicted curve. The spike at $\delta = 5.2$ corresponds to the $2:1$ resonance. This particular example shows that the basic dynamics of a system undergoing adiabatic mass evolution favors stability near strong resonances. Such a process potentially could augment current ideas about how resonant extrasolar planets, such as those around GJ 876, formed (Snellgrove, Papaloizou, & Nelson 2001; Armitage et al. 2001; Murray, Paskowitz, & Holman 2001; Rivera & Lissauer 2000).

4.3. Observational Implications

These simulations have several observational implications, which can be broadly separated into two categories: the character and the signature of planetary systems around white dwarfs.

Surviving planets that are marginally stable will suffer close approaches soon after the star evolves into a white dwarf.

### Table 1

| $\mu$         | $b$             | $c$       | $b'$       | $c'$       |
|---------------|-----------------|-----------|------------|------------|
| $10^{-7}$     | $1.16 \pm 0.04$ | $-1.6 \pm 0.2$ | $0.87 \pm 0.05$ | $-1.4 \pm 0.3$ |
| $10^{-5}$     | $1.46 \pm 0.12$ | $-2.4 \pm 0.6$ | $1.14 \pm 0.05$ | $-2.5 \pm 0.3$ |
| $10^{-3}$     | $2.5 \pm 0.5$   | $-6 \pm 2$ | $1.2 \pm 0.5$ | $-3 \pm 2$ |

**Notes.** Coefficients are derived through numerical simulations of three planets in circular orbits undergoing both mass loss (primed coefficients) and no mass loss (unprimed coefficients). Errors quoted are 1 $\sigma$. The $\mu = 10^{-7}$ case can be compared with the results from Chambers et al. 1996, who determined that $b = 1.176 \pm 0.051$ and $c = -1.663 \pm 0.274$. 

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**Fig. 4.**—Comparison of the timescale to the first close approach for a system of three $\mu = 10^{-7}$ planets with and without mass loss. Asterisks represent static masses, and diamonds represent the presence of mass loss. The top line is given by least-squares fitting a line of slope $b$ and intercept $c$ for no mass loss. The bottom line is given by eq. (5).

**Fig. 5.**—Same as Fig. 4, but for the $\mu = 10^{-5}$ case. The slope and intercept of the top line were derived by fitting the numerical simulations without mass loss. The presence of strong resonances is particularly noticeable as enhanced stability around $\delta = 5.2$ for the mass-loss case, which corresponds to the $2:1$ resonance.

**Fig. 6.**—Same as Fig. 4, but for the $\mu = 10^{-3}$ case. Arrows indicate separations at which our simulations remained stable for $10^7$ orbits. The slope and intercept of the top line were derived by fitting the numerical simulations without mass loss. The presence of strong resonances is particularly noticeable as enhanced stability around $\delta = 5.2$ for the mass-loss case, which corresponds to the $2:1$ resonance.
dwarf, or possibly as early as the AGB phase. There are three possible end states for planets that suffer close approaches: ejection, collision, and a settling into a different and more stable configuration for all planets. The case of two planets has been studied carefully, and for two Jovian-mass planets with one planet starting at 5 AU, the probability of collision is roughly 8%, of ejection 35%, and of re-arrangement 57% (Ford et al. 2001). We naively assume that these results hold similarly for multiple planets as well, since collisions have been shown to hold for multi-planet systems (Lin & Ida 1997), while ejections and re-arrangements should have similar probability (certainly to within a factor of 2 or so). Ejections will leave planets that are closer to the white dwarf, while often a re-arrangement will leave one or two planets with larger semimajor axes (up to ~10^3 times greater) and one with a smaller semimajor axis (as close as 0.1 times smaller). Collisions are potentially more exciting, because as the two planets merge they essentially restart their cooling clock and thus will be anomalously luminous by 2 orders of magnitude for 10^6 yr (Burrows et al. 1997).

To estimate how many white dwarfs might have planets that collided ($F_c$), we can take the fraction of white dwarfs that have marginally stable planets and multiply them by the fraction of marginally stable planets that have collisions,

$$F_c = f_{pl} f_{ms} f_c,$$

where $f_{pl}$ is the fraction of white dwarfs with planets, $f_{ms}$ is the fraction of marginally stable planet systems, and $f_c$ is the fraction of marginally stable systems that suffer a collision. We can estimate the number of Jovian-sized planets around white dwarfs by looking at the number of young stars that still have significant disks after 1 Myr, the approximate time to form a Jovian planet. This has been found to be about 50% of young stars in nearby clusters (e.g., Haisch, Lada, & Lada 2001). Several numerical simulations (Barnes & Quinn 2001; Laughlin & Adams 1999; Quinlan 1992; Rivera & Lissauer 2000; Stepiński, Malhotra, & Black 2000) point to a high frequency of marginally stable systems around stars, as does the discovery of the marginally stable planetary systems around GJ 876 and HD 82943 (Marcy et al. 2001). However, factors such as multiple planets with widely different mass ratios could greatly change the effects of stability and need to be studied further. We estimate this fraction to be about 50% as well, although a large uncertainty is associated with this estimate. Taking the results above, we estimate, then, that ~2(f_{ms}/0.5)% of young white dwarfs should have the product of a recent planet-planet collision in orbit. Thus, we predict that observations of young ($\tau < 10^6$ yr) white dwarfs should reveal that ~2% have luminous planet-mass companions, some in orbits with semimajor axes smaller than the minimum 5 AU expected to survive the AGB phase. These planets would be detectable through their significant IR excess and should be distinguishable from brown dwarf companions.

A natural by-product of the formation of Jovian planets is the existence of a large cloud of comets at large heliocentric distances (Oort 1950; Weissman 1999). The survival of such a cloud through post–main-sequence evolution has been closely studied in the context of accounting for observed water emission in AGB stars and the presence of metals in DA white dwarfs (Stern et al. 1990; Alcock et al. 1986; Parriott & Alcock 1998). The general result to date is that comets at semimajor axes greater than a few hundred AU survive the AGB phase. After the AGB phase, massive comets are predicted to strike the central white dwarf at a rate of ~$10^{-4}$ yr^{-1}, depositing fresh metals in the white dwarf photosphere. Such a cometary influx can account for the DAZ phenomenon, but has difficulty explaining some of the strongest metal line systems, because of the short predicted residence times of metals in the photosphere of these white dwarfs. An alternate explanation for the origin of metals in white dwarfs is ISM accretion, whereby a steady drizzle of metal-rich dust is spherically accreted from the ambient ISM. Both scenarios have difficulty explaining the frequency of DAZ white dwarfs and accounting for the systems with strongest metal lines and shortest residence times, because of the high mass accretion rate required to sustain those systems.

Recent observations of the DAZ phenomenon do not seem to be consistent with either scenario (Zuckerman & Reid 1998). In one DAZ, G238-44, the diffusion time for metals is 3 days, which means that neither previous scenario can explain the high observed metal abundances or the stability of the metal lines (Holberg, Barstow, & Green 1997). Another white dwarf, G29-38, has a high abundance, as well as an infrared excess, possibly from a dust disk at small orbital radius (Zuckerman & Becklin 1987; Koester, Provencal, & Shipman 1997).

The evolution of a planetary system after post–main-sequence mass loss, coupled with the presence of an Oort cloud, may provide an alternative explanation for the DAZ phenomenon and, in particular, the peculiarities of G29-38 and G238-44.

4.3.1. Cometary Dynamics

The mass loss during post–main sequence is near impulsive for Oort cloud comets. Previous work has shown that a significant fraction of any Oort cloud-like objects will survive the mass-loss phase, even in the presence of mildly asymmetric mass loss (Alcock et al. 1986; Parriott & Alcock 1998). The immediate result of the mass-loss phase is to leave the remaining bound objects on orbits biased toward high eccentricity, but with similar initialperiastrons. Orbital timescales are on the order of 10^6 yr.

The number and typical size of Oort cloud objects is poorly constrained; canonical estimates scale to 1 km-sized comets, mostly composed of low-density ices and silicates, with masses of ~$10^{16}$ g each, with of order 10^{12} objects per star. Clearly there is a range of masses, and it is possible that the true numbers and masses of Oort cloud comets vary by several orders of magnitude from star to star. Dynamical effects also lead to a secular change in the amount of mass in any given Oort cloud.

External perturbations ensure a statistically steady flux of comets from the outer Oort cloud into the inner system, where interactions with Jovian planets lead to tidal disruption of comets (and direct collisions), scattering onto tightly bound orbits restricted to the inner system, ejection from the system, and injection into central star–encountering orbits. For the solar system, the flux of comets into orbits leading to collision with the Sun is of the order 10^{-2} yr^{-1}; of these a significant fraction undergo breakup before colliding with the Sun, with individual fragments colliding with the Sun over many orbital periods (Kreutz sungrazers). SOHO detects ~$10^2$ such objects per year in the solar system, or
one every 3 days or so on average. A single 1 km comet can fragment into $\sim 10^4$ fragments with sizes of order 50 m, consistent with those observed by SOHO and consistent with the collision rates estimated for both the parent comets and the fragments. Each fragment then deposits about $10^{12}$ g into the solar photosphere. Note that if the typical comet were 20 km rather than 1 km, the deposition rate would be about $10^{16}$ g every 3 days.

A white dwarf has a radius about 0.01 of the solar radius. Because of gravitational focusing, the cross section for collision for comets scattered into random orbits in the inner system is linear in radius, so the collision rate expected for a white dwarf with a solar-like Oort cloud is $10^{-4}$ yr$^{-1}$. However, the perturbation of the outer orbits due to AGB mass loss, combined with the expansion of the outer planet orbits, will drastically change this rate, leading to a new, late, “heavy bombardment” phase with significantly higher rates of comet influx into the inner system. If one of the outer (Jovian-mass) planets is scattered into a large ($a_j > a_i$) eccentric orbit after the onset of instability, as we expect to happen in about $\frac{1}{3}$ of the cases, then there will be strong periodic perturbations to the outer Kuiper belt and inner Oort cloud. About 10% of those systems will lead to the outermost bound planet being placed on very wide ($a_j > 10^3 a_i$), highly eccentric orbits, with orbital timescales comparable to the cometary orbital timescales. Perturbations of the Oort cloud from these planets lead to a persistent high flux of comets to the inner system, until the Oort cloud is depleted of comets.

The net effect of the dynamical rearrangement of the post-main-sequence planetary system is a greatly enhanced rate of cometary influx into the inner system, starting $10^7$–$10^8$ yr after the mass-loss phase, tapering off gradually with time on timescales of $10^9$–$10^9$ yr, leading to enhanced metal deposition to the white dwarf photosphere and, for some white dwarfs, increased dust formation in the inner system, depending on the final configuration of the outer planets.

Several processes affect the comet bombardment rate.

1. A fraction of the previously stably orbiting outer Kuiper belt objects that survived the AGB phase are injected into the inner system by newly established dynamical resonances with the outer planets over $\sim 10^8$ yr.

2. Planets ejected to the outer Oort cloud by planet-planet perturbations will randomize the orbits of a small (\[4(m/M)^2\]) fraction of the Oort cloud comets; some of these will enter the inner system, providing an enhanced flux of the normal Oort comet infall over $\sim 10^9$ yr.

3. Surviving inner planets, scattered to the smaller orbital radii, will trap the comets injected into the inner system, providing both direct tidal disruption at a few AU and a much higher influx of comets to very small radii, where they are tidally disrupted by the white dwarf (or, in rare cases, collide directly with it).

4. Dust from tidally disrupted cometary debris will be driven to the white dwarf surface by Poynting-Robertson drag, while larger debris will be dragged in through the Yarkovsky effect, both on a timescale shorter than the white dwarf cooling time.

We expect the Kuiper belt to be severely depleted by the post-main-sequence phase (Stern et al. 1990; Melnick et al. 2001); however, a substantial population of volatile depleted rocky bodies may survive the AGB phase. A substantial fraction of these burnt-out comets will become vulnerable to resonant perturbations by the surviving outer planets, now in new, wider orbits. The outer-belt objects have orbital periods ($\sim 10^4$–$10^5$ yr) that are becoming comparable to the shortest AGB mass-loss timescales, and therefore they will not generally expand adiabatically in proportion to the expansion of the planetary orbits. The solar Kuiper belt is inferred to have $\sim 10^5$ objects with size above 100 km, assuming a mass function with approximately equal mass per decade of mass, characteristic of such populations; we infer a population of $\sim 10^{11}$ Kuiper belt objects with size of about 1 km, at an orbital radius of order $10^3$ AU. Of order 1% of those will be vulnerable to the new dynamical resonances after the AGB phase, allowing for evaporative destruction and ejection; we estimate $\sim 10^8$ Kuiper belt objects will enter the inner system in the $10^7$–$10^8$ yr after the AGB phase. The rate will peak at $\sim 10^9$ yr and then decline as the reservoir of cometary bodies in orbits vulnerable to the new planetary resonances declines.

If there are multiple surviving Jovian planets, then the post-AGB planet-planet interactions will typically leave the inner planets on eccentric orbits, leading to broader resonances and a larger fraction of perturbed Kuiper belt objects. We expect that, in $\sim \frac{1}{3}$ of the cases for which there were multiple, initially marginally stable Jovian planets in the outer system, the final configuration will have an eccentric outer planet and a more tightly bound inner planet.

Some of the comets injected into the inner system will be tidally disrupted by the surviving Jovian planets, some will be ejected, and some will be injected into the inner system and be tidally disrupted by the white dwarf (and about 1% of those will directly impact the white dwarf). Dynamical timescales in the inner system are $\sim 10^2$ yr, and the probability of ejection or disruption per crossing time is of the order of $10^{-2}$ per crossing time, assuming that there is an inner planet scattered inward of 5 AU, matching the outer planet scattered to a wider orbital radius. So at any one time, $\geq 10^5$ Kuiper belt objects are in the inner system. The rate for tidal disruption by the surviving innermost Jovian planets is $\sim 10^{-6}$ yr$^{-1}$ per comet. Tidal disruption rates due to close approaches to the white dwarf may be as high as $\geq 10^{-4}$ yr$^{-1}$ per comet; the rates are uncertain because of the possibility of nongravitational processes breaking up the comet and deflecting debris. With each comet massing about $\geq 10^{16}$ g, by hypothesis, we get a flux of disrupted cometary material from the Kuiper belt remnant, assuming solar system–like populations, of $10^{14}$–$10^{16}$ g yr$^{-1}$. This is volatile depleted material, by hypothesis. Given our assumed mass function, disruption of rarer, more massive comets can sustain mass accretion rates an order of magnitude higher still for timescales comparable to the inner system dynamical timescales, in a small fraction of systems.

We can now compare our mechanism with the accretion rates needed to explain the constant, detected metal lines in G29-38 and G238-44, two DAZ white dwarfs with the highest measured abundances of Ca. An accretion rate has already been quoted in the literature for G238-44, where $\sim 3 \times 10^{17}$ g yr$^{-1}$ of solar-abundance ISM would need to be accreted (Holberg et al. 1997). In a volatile depleted case, only metals would be present, converting to only $\sim 6 \times 10^{15}$ g yr$^{-1}$ for cometary material. G29-38 also has a roughly estimated value of $\sim 1 \times 10^{19}$ g yr$^{-1}$, corresponding to
Both of these estimates were based on calculations made by Dupuis, Fontaine, & Wesemael (1993b) and Dupuis et al. (1992, 1993a), who use the ML 3 version of mixing-length (ML) theory. In fact, there are several other methods that can be used to model the convective layer of white dwarfs, including using other efficiencies of the ML theory and the CGM model of convection (Canuto, Goldman, & Mazzitelli 1996; see Althaus & Benvenuto 1998). The calculations differ by up to 4 orders of magnitude on the mass fraction \( q \) of the convection layer’s base in white dwarfs with \( T_{\text{eff}} \) similar to that of G29-38. Taking values of \( q \) for the base of the convection layer from Figures 4 and 6 of Althaus & Benvenuto (1998) and getting values for the diffusion timescale from Tables 5 and 6 of Paquette et al. (1986), one can estimate what steady state accretion rate \( G29-38 \) requires for the different models. The smallest rate came from ML 1 theory and the largest from ML 3 theory, with CGM having an intermediate value, giving a range of \( \sim 2 \times 10^{13} \) to \( \sim 4.4 \times 10^{17} \) g yr\(^{-1}\). We favor the CGM value of \( \sim 10^{15} \) g yr\(^{-1}\), which is consistent with the estimate based on observations conducted by Graham et al. (1990). The rate for G238-44 may be more robust, as a result of the fact that convective models converge for hotter white dwarfs.

Both rates are consistent with our scenario if either white dwarf has two Jovian-mass planets, one in a \( \lesssim 10 \) AU, eccentric orbit and another in a \( \lesssim 5 \) AU orbit, after rearrangement. Alternatively, their progenitors had a Kuiper belt population an order of magnitude richer than that inferred for the solar system. With a post–AGB age of \( \sim 6 \times 10^{8} \) yr and a mass of \( \sim 0.7 M_{\odot} \), the original main-sequence star of G29-38 was most likely more massive than \( 1 M_{\odot} \), and a more massive planetary and cometary system is not implausible. G238-44 is almost \( 10^{8} \) yr old and would represent an object close to the peak of predicted comet activity.

Our scenario may provide a consistent picture for the presence of DAZ white dwarfs and their anomalous properties (Zuckerman & Reid 1998). We don’t expect all white dwarfs to have metal lines. Only about \( 1/3 \) of those that possessed marginally stable planetary systems containing two or more Jovian planets at orbital radii greater than \( \sim 5 \) AU will be able to generate significant late cometary bombardment from the outer Kuiper belt and inner Oort cloud. Following an estimate similar to that in equation (6), we predict that about 14% of white dwarfs will be DAZs. Further, the rate will peak after \( \sim 10^{8} \) yr, after the planet-planet perturbations have had time to act, and then decline as the reservoir of perturbable comets is depleted. The convective layer of the white dwarf will also increase by several orders of magnitude over time, which would create a sharp drop in high-abundance DAZs with decreasing \( T_{\text{eff}} \). The drop would be greatest between 12,000 and 10,000 K, where the convective layer has its steepest increase (see Fig. 4 of Althaus & Benvenuto 1998), Zuckerman & Reid (1998), in conducting their survey of DAZ white dwarfs, estimated that \( \sim 20\% \) of white dwarfs were DAZs and that metal abundance dropped sharply with \( T_{\text{eff}} \) between 12,000 and 8000 K.

We expect DAZ white dwarfs to have (generally) multiple potentially detectable outer Jovian planets, whose orbits will show dynamical signatures of past planet-planet interaction, namely, an outer eccentric planet and an inner planet inside the radius scoured clean by the AGB phase.

\( \sim 2 \times 10^{17} \) g yr\(^{-1}\) in the volatile depleted case (Koester et al. 1997).

\[ \frac{d\Delta_{c}}{dt} = \mu^{-2/3} \frac{d\mu}{dt}. \] (7)

In the general case, \( d\mu/dt \) depends on two factors, the change in mass of the central star and the change in mass of the planets, given by

\[ \frac{d\mu}{dt} = \mu \left( \frac{d\ln M_{pl}}{dt} - \frac{d\ln M_{st}}{dt} \right). \] (8)

For the critical separation to widen, \( \mu \) must increase with time. Putting the two equations together gives the rate of change in \( \Delta_{c} \):

\[ \frac{d\Delta_{c}}{dt} = \mu^{1/3} \left( \frac{d\ln M_{pl}}{dt} - \frac{d\ln M_{st}}{dt} \right). \] (9)

The results of our multiplanet simulations are scalable to many situations, but for planet systems surviving around white dwarfs we are interested in timescales of \( \sim 10^{10} \) yr for solar-type stars to \( \sim 10^{8} \) yr for higher mass stars. The highest \( \delta \) we studied for \( \mu = 10^{-3} \) was roughly 5.2, which by equa-

5. DISCUSSION

Using the above results, we can compare the greatest fractional change in stability for two Jovian planets around different stars greater than 1 M\(_{\odot}\) that produce white dwarfs. We took the initial-final mass relation of Weidemann (2000) and calculated the critical separation without mass loss and with mass loss (\( \Delta_{c} \) and \( \Delta_{c}' \), respectively). As can be seen in Figure 7, the higher the initial mass of the star, the greater the fractional change. This is expected, since higher mass stars lose more mass to become white dwarfs. The best candidates for unstable planetary systems would be higher mass white dwarfs, if planet formation is equally efficient for the mass range considered here. The scheme conjectured in this paper provides a method for identifying and observing the remnant planetary systems of intermediate-mass stars, which might otherwise be hard to observe during their main-sequence lifetime.

One can predict the change in the critical separation at which two planets will remain stable, based on the change in \( \mu \) over time, simply by differentiating \( \Delta_{c} \) with time:

![Fig. 7.—Comparison of \( \Delta_{c} \) with and without mass loss, as a function of the central star’s original mass. The right panel shows the fractional change of \( \Delta_{c} \) when mass loss occurs. For both panels the fractional change in mass is calculated using the \( M_{f}-M_{i} \) relation of Weidemann (2000).](image-url)
tion (2) corresponds to a timescale to close approaches of $10^7$ orbits of the inner planet. After the central star loses half its mass, the timescale shortens to $\sim 2000$ orbits. For a planetary system with $\delta = 5.2$ to be stable over the main-sequence lifetime of the star, the minimum semimajor axis of the innermost planet for a higher mass star (say, $4 M_\odot$) would be 8.2 AU and for a solar-type star 100 AU. Longer integrations need to be performed to investigate the behavior of systems with larger values of $\delta$. We expect our results for timescales for onset of instability to scale to larger $\delta$; the initial computational effort we made here limited the exploration of slowly evolving systems with large $\delta$ in exchange for a broader exploration of the other initial condition parameters. It will also be instructive to model systems with unequal-mass planets, to explore the probability of ejection and hierarchical rearrangement as a function of planetary mass ratio.

By performing numerical integrations of two-planet and multiple-planet systems, we have shown that the stability of a system changes with mass evolution. In the specific case of mass loss as the central star of a planetary system becomes a white dwarf, we have found that previously marginally stable orbits can become unstable fairly rapidly after the mass-loss process. Coupled with our knowledge of the survival of material exterior to outer planets, such as Kuiper belt and Oort cloud analogues (Stern et al. 1990), a picture of the evolution of circumstellar material over the latter stages of a star’s lifetime becomes clear.

As a star reaches the red giant branch and AGB phases, inner planets are engulfed both by the expanding envelope of the star and through tidal dissipation. The surviving planets move slowly outward, conserving their angular momentum as the star loses its mass over several orbital periods of the planets. The planets may scrupt the resulting wind of the giant star (Soker 2001), and if they are on the very edge of stability undergo chaotic episodes during the AGB phase, creating some of the more exotic morphologies in the resulting planetary nebula. When the star becomes a white dwarf, two-planet systems that are marginally stable will become unstable and suffer close approaches, while for three or more planets the timescale to close approaches shortens by orders of magnitude. There are three possible outcomes once the planets start suffering close approaches: the planets collide, one planet is ejected, or the two planets remain but are in highly eccentric orbits (Ford et al. 2001). One major open question is how many marginally stable systems there are, but there are indications that many, if not most, general planetary systems should be close to instability (Barnes & Quinn 2001; Laughlin & Adams 1999; Quinnlan 1992; Rivera & Lissauer 2000; Stepinski et al. 2000).

Rocky material in the inner edge of the Kuiper belt, which is defined by the last stable orbits with respect to the planetary system, will follow the same fate as marginally stable planets, suffering close approaches with the planetary system and becoming scattered into the inner system, which increases the rate of close encounters with the planets or the central white dwarf. The surviving material at outer Kuiper belt and Oort cloud distances will have orbital periods comparable to the timescale of the central star’s mass loss. These objects have their eccentricity pumped up by the effectively instantaneous change in the central star’s mass, and then through interactions with planets they create a new dust disk around the white dwarf and contaminate the white dwarf photosphere to an observable extent.

The sensitivity of stability to changes in mass has implications for planet formation as well. Further research on the migration of Hill-stable regions while the planet/star mass ratio evolves may illuminate further the general issue of how Jovian planets in the process of formation become unstable to close encounters and gross changes in orbital parameters (Ford et al. 2001). One possibility is that the mass accretion of the planets occurs at a rate fast enough that $dM/dt > 0$. Other factors would need to be considered, in particular the interplay between the onset of rapid mass accretion by the planet and the accretion rate from the protoplanetary disk onto the central protostar. Gas drag and stellar mass accretion could work to stabilize orbits if the planets are embedded in a circumstellar disk, while orbital migration would change the relative separations of protoplanets. Since the stability of multiplanet systems is also sensitive to changes in mass ratio, this could help solve problems of isolation for planetary embryos and speed up the timescale for the production of giant planet cores.

The dependence of stability on both the mass of the planet and the mass of the central star suggests that stars of different masses may be more efficient at producing a certain size planet. This is exemplified by the fact that $\mu$ for a Jovian planet can change by an order of magnitude in either direction over the mass range of stars that might have planetary companions. For larger mass stars, planets can be more tightly spaced and still be mutually dynamically stable, which suggests that when planets are forming it is easier for them to become dynamically isolated in disks around more massive protostars. For lower mass stars, there is a wider annulus in which material is unstable to planetary gravitational perturbations, and so forming planets would have a larger reservoir of material to draw from. Other factors that need further research, such as a star’s temperature and radiation pressure, would play into this result as well and may dominate over this scenario. However, such effects will tend to reinforce the conclusion that less massive stars should be more efficient at creating more massive planets, while higher mass stars will produce more, lighter planets, if they are capable of forming planets at all. This prediction will be testable, as many space- and ground-based programs are devoting a great deal of effort to looking for planetary companions to stars.

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