Signal Reconstruction With Semidefinite Relaxation Optimization in Uplink Massive MIMO Systems

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Abstract—In this letter, we consider a single group massive MIMO (Multi-Input Multi-Output) system, which consists of one receiver deployed with large number of antennas and multiple users with each having single antenna. In the uplink transmissions from users to base station, we focus on the efficient signal reconstruction of all the users’ transmitted symbols. We build a statistic decision matrix based on the received signals, whose eigenvectors and eigenvalues constitute the key components of our signal reconstruction algorithm. Then, an optimization problem is formulated and we convert this problem to obtaining the optimal weights by solving Semidefinite Relaxation (SDR) optimization and obtaining the optimal rotation phases. Finally, an iterative algorithm framework is proposed for multi-user signal reconstruction. Numerical results are carried out to verify the bit-error-ratio (BER) performance compared with several coherent detection and noncoherent transmission schemes.

Index Terms—Massive MIMO system, signal reconstruction, semidefinite relaxation optimization, uplink transmission, BER performance.

I. INTRODUCTION

MASSIVE multiple-input multiple-output (MIMO) is a hot topic research in recent years and has been considered as one promising technology for the future wireless communication systems, since it can provide many benefits, such as great improvements in throughput, efficient radiated energy and simplification of the multiple access layer [1]. Due to the law of large numbers, the channel hardens so that frequency domain scheduling no longer pays off. Each terminal can be given the whole bandwidth, which renders most of the control signaling redundant.

In the uplink transmissions, the CSI (Channel State Information) estimations at the based station are obtained from the signals transmitted by the multiple users in TDD (Time-Division Duplex) mode. In FDD (Frequency-Division Duplex) mode, the CSI estimations are obtained with the CSI feedback from the multiple users by using the different transmission frequency. The one is that a large number of radio frequency (RF) chains increase the energy consumptions. The other is RF chains occupy an increasing amount of spectral resources. In addition, the transmission schemes without the CSI training or estimation become more necessary and practical. The most noncoherent transmission schemes focus on elaborately designing the transmitting signals. In massive MIMO systems, it is crucial to fully utilize the receiving freedom of the receiver’s large-scale antennas in order to design the noncoherent transmission strategy. The unitary space-time modulation [2] was proposed earlier for avoiding the need of CSI estimation at the receiver. However, this scheme was designed for the classical MIMO systems. An energy-detection-based noncoherent communication scheme for multiuser massive single-input multiple-output (SIMO) systems with multiple access channels was proposed in [3], where an Amplitude Shift Keying (ASK) modulation was described. A differential Phase Shift Keying (DPSK) aided large-scale MIMO system was designed in [4], which proposed a transmitting constellation design by allocating different power factors to each transmitter. In [5], the authors proposed new DPSK-aided non-coherent constellation types for the large-scale MIMO, which does not rely on strict power control and yet provides the considerable performance. To design well noncoherent transmitting signal structure, authors in [6] proposed a $2 \times 2$ noncoherent space-time signal block in point-to-point massive MIMO system. In addition, a different noncoherent transmission scheme based on the new concept of additively uniquely decomposable constellation pairs (AUDCPs) was proposed in [7] for the massive MIMO space-time block transmission. Then, QAM-Division uniquely-factorable multiuser space time modulation (UF-MUSTM) [8] was also proposed by the same research team. These recently proposed schemes show competitive performance. Similarly, uniquely factorable constellation (UFC) was also studied in cooperative cognitive radio systems [9]. In the extremely fast time varying channels, authors in [10] strengthened the importance of non-coherent schemes with OFDM signaling.

Motivations: In this letter, we alternatively turn our attention on an efficient signal processing and reconstruction at the receiver. We expect to reduce the cost of pilots or training sequences as much as possible in an approximately non-coherent transmission.

Contributions: (1) A statistic decision matrix is built based on the received signals, whose eigenvectors and eigenvalues constitute the key components of our signal reconstruction algorithm; (2) An optimization problem is refined to reconstruct one user’s transmitted signals. We convert this problem to two equivalent sub-problems, i.e., obtain the optimal weights by solving SDR optimization and obtain the optimal rotation phases; (3) Iterative algorithm framework is proposed for multi-user signal reconstruction.

II. SYSTEM AND TRANSMISSION MODEL

We consider $K$ single-antenna transmitters (users) and one receiver (base-station) deployed with $N$ antennas, $N \gg K$. This letter focuses on the uplink transmission where all the
users transmit their symbols simultaneously and independently. We assume no accurate channel state information (CSI) is available at either the users or the base station. A quasi-static channel fading with channel coherence time $T$ can be modeled as $h_k = [h_{k,1}, h_{k,2}, \ldots, h_{k,N}]^T$, where $h_{k,n}$ denotes the fading coefficient between the $k$-th user to the $n$-th antenna of the base station, $k = 1, 2, \ldots, K, n = 1, 2, \ldots, N$. Generally, the channel fading is modeled as $h_{k,n} = \sqrt{\beta} h_{k,n}$, where $\beta$ denotes the large-scale fading and $\bar{h}_{k,n}$ denotes the small-scale fading. The large-scale fading $\beta_k$ is related with the user’s physical position. Hence, $\beta_k$ changes negligibly for a relative long observation duration (usually in the order of second). The small-scale fading $h_{k,n}$ changes much faster than large-scale fading. Usually, the sampling duration is in the order of microsecond. The channel coherence time $T$ spans at least several hundreds of sampling intervals (in the order of milliseconds) in most mobile communication scenarios. Hence, we assume the small-scale fading $h_{k,n}$ is unchanged until the next coherence time block. We model $h_{k,n}$ during one coherence time $T$ as the random variable with independent and identical distributed (i.i.d.) complex Gaussian $CN(0,1)$.

We denote the vector $s_k = [s_{k,1}, s_{k,2}, \ldots, s_{k,L}]$ as the transmitted symbols from the $k$-th user during the $L$ time slots, $LT_s < T$, where $T_s$ denotes the duration between the adjacent transmit symbols. The symbol $s_{k,l}$ is selected from the same power-normalized constellation, $k = 1, 2, \ldots, K, l = 1, 2, \ldots, L$. Then, during the $l$-th time slot, the received signal at the base station can be expressed as

$$y_l = \sum_{k=1}^{K} \sqrt{P_k} h_{k,s_{k,l}} + \xi_l, \quad (1)$$

where $P_k$ denotes the transmitting power of the $k$-th user, $\xi_l$ denotes the noise vector with each element having i.i.d. complex Gaussian distribution with zero mean and noise power $\sigma^2$. After the $L$ observations at base station, the whole received signals can be compactly combined as

$$Y = [y_1, y_2, \ldots, y_L] = \mathbf{H} \mathbf{A}_p \mathbf{S} + \mathbf{X}, \quad (2)$$

where $\mathbf{H} = [h_1, h_2, \ldots, h_K]$ with dimension $N \times K$ denotes the equivalent channel matrix, $\mathbf{A}_p = \text{diag}(\sqrt{P_1}, \sqrt{P_2}, \ldots, \sqrt{P_K})$ denotes the diagonal power matrix, $\mathbf{S} = [s_{1}, s_{2}, \ldots, s_{L}]^T$ with dimension $L \times K$ denotes the combined transmitted symbol matrix of all the users, $\mathbf{X} = [\xi_1, \xi_2, \ldots, \xi_L]$ denotes the additive complex Gaussian noises.

Considering the large-scale fading $\beta_k$ and small fading $h_{k,n}$, we express the equivalent channel vector as $\mathbf{H} = \mathbf{H}_\beta \mathbf{H}_n$, where $\beta = \text{diag}(\sqrt{\beta_1}, \sqrt{\beta_2}, \ldots, \sqrt{\beta_K})$, $\mathbf{H}_\beta = [h_{1}, h_{2}, \ldots, h_{N}]$ and $\mathbf{H}_n = [h_{k,1}, h_{k,2}, \ldots, h_{k,N}]^T$. In order to overcome the influence of large-scale fading on received signal average energy, transmitting power is allocated as $P_k = c^2/\beta_k$, where $c$ is a constant. Then, we have $\mathbf{A}_p \beta = c^2 \mathbf{I}_K$. The transceiver model in (2) can be rewritten as $Y = c \mathbf{H} \mathbf{S} + \mathbf{X}$.

It is worth noting that the received signals in (2) has the dimension $N \times L$, where $N$ is largely greater than $K$ in our massive MIMO scenario. That is to say, it is possible for us to build a detector to extract the transmitted symbols directly from the received signals without extra operations for channel information acquisition.

III. SIGNAL RECONSTRUCTION WITH STATISTICAL EIGENVECTORS

Preliminarily, we first introduce several useful properties of very large antenna arrays. Assuming $\mathbf{p}$ and $\mathbf{q}$ to be two $N \times 1$ mutually independent vectors whose elements are i.i.d. zero-mean complex Gaussian random variables with variances of $\sigma_p^2$ and $\sigma_q^2$ respectively. According to the probability statistical theory [11], we have $\mathbf{p}^H \mathbf{p} \xrightarrow{a.s.} N \sigma_p^2$, $\mathbf{p}^H \mathbf{q} \xrightarrow{a.s.} 0$, when $\mathbf{p}$ and $\mathbf{q}$ denotes the almost sure convergence in distribution when $N$ approaches to the infinity. Therefore, in our transmission model (2), we have $\mathbf{H}^H \mathbf{H}/N \xrightarrow{a.s.} \mathbf{I}_K$, $\mathbf{X}^H \mathbf{X}/N \xrightarrow{a.s.} \sigma^2 \mathbf{I}_L$, $\mathbf{X}^H \mathbf{H}/N \xrightarrow{a.s.} 0_{L \times K}$, $\mathbf{H}^H \mathbf{X}/N \xrightarrow{a.s.} 0_{K \times L}$.

Since there is no accurate CSI known at base station, we just build a new $L \times L$ statistic decision matrix $\Sigma = \mathbf{Y}^H \mathbf{Y}/N$ as

$$\Sigma = c^2 \mathbf{H}^H \mathbf{H}/N + \sum_{c \mathbf{S}^H \mathbf{H} = \mathbf{Y}} \mathbf{H}^H \mathbf{X} + \mathbf{H}^H \mathbf{H} + \frac{\mathbf{X}^H \mathbf{X}}{N}.$$ \hspace{1cm} (3)

Combined with (2), the asymptotic expression of this statistic decision matrix, when $N$ is very large, can be expressed as

$$\Sigma \xrightarrow{N \rightarrow \infty} c^2 \mathbf{H}^H \mathbf{H} + \sigma^2 \mathbf{I}_L.$$ \hspace{1cm} (4)

Clearly we observe that $\Sigma \xrightarrow{N \rightarrow \infty}$ contains no channel fading gains but the only transmitted symbol matrix $\mathbf{S}$ and the statistical noise power $\sigma^2$. It means we can obtain the symbol estimations regardless of the channel coefficients.

In this letter, we expect to propose a signal reconstruction method based on the decision statistic matrix $\Sigma$. Before to do that, we consider the noise-free case as the exploratory investigation. Based on (4), we can easily obtain the noise-free statistic decision matrix as $\Sigma_N \xrightarrow{N \rightarrow \infty} c^2 (s_1, s_2, \ldots, s_K)$.

Due to $\mathbf{S} = [s_1^T, s_2^T, \ldots, s_L^T]^T$, the structure of $\Sigma_N$ suggests that it is possible to extract the individual symbols $s_k, k = 1, 2, \ldots, K$. Since $\Sigma_N \xrightarrow{N \rightarrow \infty}$ is a hermitian symmetric matrix, it can be decomposed as $\Sigma_N = \mathbf{U} \mathbf{A} \mathbf{U}^H$, where $\mathbf{U}$ denotes the $L \times L$ unitary matrix consisting of the eigenvectors, and the $\mathbf{A}$ is the corresponding diagonal matrix consisting of the eigenvalues. Furthermore, because the rank of $\Sigma_N$ is $K$, it has only $K$ nonzero eigenvalues. Therefore, the transmitted symbols $\mathbf{S}$ can be reconstructed from these eigenvalues and eigenvectors

$$\mathbf{S} = \Theta \mathbf{Q} \mathbf{A}^\frac{1}{2} \mathbf{U}^H,$$ \hspace{1cm} (5)

where $\mathbf{A} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_K)$ with $\lambda_i$ being the eigenvalue, $\mathbf{U} = [u_1, u_2, \ldots, u_K]$ with $u_i$ being the corresponding eigenvector, $\Theta = \text{diag}(e^{j\theta_1}, e^{j\theta_2}, \ldots, e^{j\theta_K})$, $\theta_k \in [0, 2\pi], k = 1, 2, \ldots, K$. Here, $\Theta$ reflects the linear relationship between the $K$ eigenvectors and the element $e^{j\theta_k}$ in $\Theta$ reflects the phase rotation. This reconstruction method is easy to verify by performing $(\Theta \mathbf{Q} \mathbf{A}^\frac{1}{2} \mathbf{U}^H) \Theta \mathbf{Q} \mathbf{A}^\frac{1}{2} \mathbf{U}^H = \mathbf{S}$, given the $L \times L$ statistic decision matrix in (3), we can refer to the noise-free and infinite $N$ case, i.e., $\Sigma_N = \mathbf{S}$, since the noise terms in (3) can not be ignored, the results and analyses will not be valid in (5). Due to the nonlinear nature of eigen decomposition, it is difficult to give precise analysis of the effect of noise terms on the eigen properties of the matrix. However, we can also perform...
the reconstruction processes with similar idea. We observe that the term $\mathbf{H}^H \mathbf{H} / N$ in (3) is a diagonal-dominant matrix, and the term $(\mathbf{S}^H \mathbf{H}^H \mathbf{S} + \mathbf{S}^H \mathbf{H} + \mathbf{S}^H \mathbf{S}) / N$ goes to very small values resulting from the independence between random variable $\mathbf{H}$ and $\mathbf{S}$. Here, we assume $L \geq K$. That is to say there must be $K$ nonzero eigenvalues clearly larger than the others among all the $L$ eigenvalues of $\mathbf{S}$ in (3). Therefore, we just focus on the dominant eigenvalues.

Based on the above analyses, the $k$-th user’s estimation can be reconstructed as

$$\hat{s}_k = e^{j\theta_k} \sum_{i=1}^{K} \alpha_{k,i} \sqrt{\lambda_i} u_i^H,$$

where $\lambda_i$ is the eigenvalue of the decision statistic matrix $\mathbf{S}$, $u_i$ denotes the eigenvector with $\mathbf{S} u_i = \lambda_i u_i$, $\theta_k \in [0, 2\pi)$. Certainly, the parameters $\alpha_{k,i}$ and $\theta_k$ are needed to be determined according to some metric. From (6), we know the $k$-th user’s estimation $\hat{s}_k$ is related with parameters $\alpha_{k,i}$ and $\theta_k$. Here, we could define a metric function $\varepsilon(\alpha_{k,1}, \alpha_{k,2}, \cdots, \alpha_{k,K}, \theta_k)$ with the average symbol energy based on the assumption of the normalized average power of the transmitted symbols

$$\varepsilon(\alpha_{k,1}, \alpha_{k,2}, \cdots, \alpha_{k,K}, \theta_k) = \sum_{l=1}^{L} \left| |\hat{s}_k[l]|^2 - \chi \right|^2,$$

where $\hat{s}_k[l]$ denotes the $l$-th element of vector $\hat{s}_k$, $\chi$ means the expected average power. The optimal values of parameters $\alpha_{k,i}$ and $\theta_k$ should make the reconstructed symbols $\hat{s}_k$ minimizing the metric function $\varepsilon(\alpha_{k,1}, \alpha_{k,2}, \cdots, \alpha_{k,K}, \theta_k)$. From (5), we have known the $(k,l)$-th element $\alpha_{k,i}$ of the unitary matrix $\mathbf{Q}$ should satisfy the condition $|\alpha_{k,1}|^2 + |\alpha_{k,2}|^2 + \cdots + |\alpha_{k,K}|^2 = 1$. Hence, the optimal parameters can be obtained with solving the following problem

(P1) \[ \min_{\alpha_{k,1}, \alpha_{k,2}, \cdots, \alpha_{k,K}, \theta_k} \varepsilon(\alpha_{k,1}, \alpha_{k,2}, \cdots, \alpha_{k,K}, \theta_k) \]

s.t. \[ \sum_{i=1}^{K} \alpha_{k,i} = 1, \quad k = k', \quad 0 < \theta_k < 2\pi. \]

We observe it’s very computationally complex to obtain the whole $K(K+1)$ optimal parameters according the above optimization problem (P1), especially when $K$ is large. More efficient algorithm will be designed in the next section.

IV. RECONSTRUCTION ALGORITHM WITH SDR OPTIMIZATION

Carefully, we observe that the value of $\theta_k$ in (6) and (7) can not take the parameter $\theta_k$. Let $\bar{s}_k = \sum_{i=1}^{K} \alpha_{k,i} \sqrt{\lambda_i} u_i^H = x_k^T \tilde{\Lambda} \tilde{U}^H$, where $x_k = [\alpha_{k,1}, \alpha_{k,2}, \cdots, \alpha_{k,K}]^T$, the diagonal matrix $\tilde{\Lambda} = \text{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_2}, \cdots, \sqrt{\lambda_K})$ with $\lambda_i$ being the first $K$ eigenvalues of the $L \times L$ decision statistic matrix $\mathbf{S}$, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_K$, $u_i$ is the eigenvector corresponding to $\lambda_i$. In order to give a convenient representation, we define $\mathbf{V} = \tilde{\Lambda} \tilde{U}^H$ and rewrite $\mathbf{V}$ with $L$ column vectors, i.e., $\mathbf{V} = \tilde{\Lambda} \tilde{U}^H = [v_1, v_2, \cdots, v_L].$

A. Reconstruction Using SDR Optimization

As for the optimization (P1), it is not easy to solve this problem directly. Alternatively, we find that a problem of minimizing a summation can be converted to a problem of minimizing the maximum of the summands. Hence, the problem (P1) can be reformulated as

\[
\begin{align*}
\text{(P2)} \quad & \min_{l=1,2,\ldots,L} \max_{\tilde{s}_k[l]} \left| |\tilde{s}_k[l]|^2 - \chi \right| \\
\text{s.t.} \quad & \mathbf{x}_k^H \mathbf{x}_k = 1.
\end{align*}
\]

Although (P2) takes away the summation of (P1), it still belongs to the class of NP-hard problems [12]. However, we can see $|\tilde{s}_k[l]|^2 = x_k^T v_l$ and $|\tilde{s}_k[l]|^2 = x_k^T v_l x_k^H = \text{tr} (v_l^H \mathbf{x}_k^H \mathbf{x}_k^H)$. Here, we notice $x_k^H \mathbf{x}_k$ is a $L \times L$ matrix and rank($x_k^H \mathbf{x}_k$) = 1. Clearly, this is a nonconvex Quadratically Constrained Quadratic Programs (QCQPs). According to [12], it can be efficiently solved by Semidefinite Relaxed (SDR). Therefore, let $\mathbf{x}_k = x_k^H \mathbf{x}_k^H$, the problem (P2) can be reformulated as

(P3) \[ \min_{t \in \mathbb{R}^P} \|x_k\|_{t} \]

s.t. \[ -t \leq \text{tr} (v_l^H \mathbf{x}_k) - \chi \leq t, \quad l = 1, 2, \ldots, L, \]

\[ \text{tr} (\mathbf{x}_k) = 1, \quad \mathbf{x}_k \succ 0. \]

This optimization problem can be efficiently solved by using the cvx toolbox to obtain the global optimal solution $\mathbf{x}_k^*$. The final procedure of using SDR is how to convert a globally optimal solution $\mathbf{x}_k^*$ into our expected solution. Generally speaking, the rank of this globally optimal solution $\mathbf{x}_k^*$ is larger than one, that is to say we must somehow extract a vector feasible for (9). There are two methods to do this extraction. The one is to apply a rank-one approximation on $\mathbf{x}_k^*$. Specially, let $r = \text{rank}(\mathbf{x}_k^*)$, and then $\mathbf{x}_k^* = \sum_{i=1}^{r} \lambda_i q_i q_i^H$, where $\lambda_1' \geq \lambda_2' \geq \cdots \geq \lambda_r' > 0$ and $q_1, q_2, \cdots, q_r$ are the respective eigenvectors. Here we know the best rank-one approximation of $\mathbf{x}_k^*$ can be given by $\lambda_1' q_1 q_1^H$. Therefore, the candidate solution can be defined as $\tilde{s}_k = \sqrt{\lambda_1} q_1$. The other method is to use randomization to extract an approximate QCQP solution from an SDR solution $\mathbf{x}_k^*$. Considering the $K \times 1$ complex random vector $\eta$ according to the Guassian distribution with zero mean and covariance $\mathbf{x}_k^*$, $\eta \sim \mathcal{CN}(0, \mathbf{x}_k^*)$. Then, with generated random vectors $\eta_m, m = 1, 2, \cdots, M_{\text{random}}$, we normalize $\eta_m$ as $x_m = \eta_m / \|\eta_m\|$. Finally, the best solution can be determined as $m^* = \arg\min_{m} \max_{l=1,2,\ldots,L} \text{tr} (v_l^H x_m^H x_m^H - \chi).$

We obtain the best approximated solution $\mathbf{x}_k^* = \tilde{s}_m^*$.

Here we obtain the temporary reconstructed vector with optimal weight $\mathbf{x}_k^*$ by using SDR optimization as

$$\tilde{s}_k = (\tilde{s}_k)^T \tilde{\Lambda} \tilde{U}^H.$$
B. Recognition With Embedded Training Sequences

It is natural to use a few structured sequences to address the phase-rotation and assignment ambiguity problems which are mentioned above. Since there are $K$ users in our communication scenario, it needs $K$ training sequences $c_1, c_2, \cdots, c_K$ for users with each sequence having length $L'$. To successfully determine the index of user, the orthogonality between every two sequences must be assured. A small piece of training sequences is embedded into the transmission frame structure for every user, which results in a slight reduction of the transmission rate by $L'/L$. Therefore, the decrease in throughput is negligible. In addition, it should be noted that it is not necessary to have the same start location of all the users’ training sequences. In other words, the training sequences in transmission frames could be asynchronous. Based on the estimation $\hat{s}_k^*$ in (11), these training sequences can help us to achieve the following three goals.

1) With the cross correlation processing between the estimation $\hat{s}_k^*$ and the possible local training sequences $c_1, c_2, \cdots, c_K$, we can determine the user index $k^*$ which corresponds to the maximum correlation peak. This is due to orthogonal and quasi-orthogonal properties between training sequences.

2) Meanwhile, we can get the location of the training sequence in the estimation $\hat{s}_k^*$ with searching the maximum correlation peak. We note the starting point of the training sequence $c_{k^*}$ in $\hat{s}_k^*$ as $l_{u_k}$.

3) Perform the phase estimation $\theta_{k^*}$ based on the above results

$$\hat{\theta}_{k^*} = \arg \left( \frac{1}{L'} \hat{s}_{k^*}^{H} \cdot |l_{u_k} \cdots l_{u_K} | \right),$$

where $\hat{s}_{k^*}^{H} \cdot |l_{u_k} \cdots l_{u_K} | = [\hat{s}_{k^*}^{H} |l_{u_k}], \hat{s}_{k^*}^{H} |l_{u_k} + 1], \cdots, \hat{s}_{k^*}^{H} |l_{u_k} + L' - 1]$, and here $\hat{s}_{k^*}^{H} |l_{u_k} + l]$ denotes the $(l_{u_k} + l)$-th element of $\hat{s}_k^*$.

Based on the result $\hat{s}_k^*$ solved by SDR in IV-A and the above procedures, we finally obtain the estimation of user $k^*$ as $\hat{s}_{k^*} = \hat{s}_k^* e^{-j\hat{\theta}_{k^*}}$.

C. Successive Extraction for All the Users

It is easy to see that the optimization (P3) in IV-A and the procedures in IV-B just give out the estimation $\hat{s}_{k^*}$ of the $k^*$-th user. Then, the final question is how to extract the estimations of the other users’ transmitted symbol vectors by using the similar method described in the previous subsections. Actually, we observe that the information from the other users’ estimations of the other users’ transmitted symbol vectors by the methods and analysis in IV-A and IV-B, we can iteratively use the SDR optimization and recognition with remaining embedded training sequences to successively obtain the all users’ estimation.

D. Complexity Analysis

The algorithm complexity of the proposed signal reconstruction with SDR is measured in terms of the number of required multiplications. The complexity of construct $\Sigma$ and decomposition $U \Lambda U^H$ is caused by the matrix computation.

The complexity of the SDR has been verified as a powerful, computationally efficient approximation technique for many nonconvex QCQPs. As for determining $k^*$ and $\theta_{k^*}$, the complexity is mainly caused by the sequence correlation operations, which is related with the length of training sequences $L'$.

TABLE I shows the computational complexity comparisons between our proposed scheme and the several existed literatures. For the uplink signal detection schemes with CSI, the linear detectors, such as zero-forcing (ZF) and minimum mean squared error (MMSE), involves inversion of high dimension matrices which has unacceptable computational complexity. The low-complexity algorithms based on approximate matrix inverse for near-optimal detection in massive MIMO systems are of great interest, e.g., [13]–[15]. For the uplink signal detection schemes without using CSI, several noncoherent detector have also been extensively researched [3], [4], [7], [8]. Although in our proposed scheme a piece of training sequences with length $L'$ are inserted in the transmitted symbols, they are not used to estimate the CSI at receiver. Therefore, we treat the scheme in this letter as the detection without CSI.

From the comparisons in TABLE I, we find the complexity of schemes with CSI is related with the number of users. This is mainly derived from matrix inverse operations, for example the MMSE detector $\hat{s} = (\hat{H}^H \hat{H} + \sigma^2 I_K)^{-1} \hat{H}^H y$. Comparably, the complexity of schemes without CSI varies greatly according to the different algorithm. Our proposed scheme is mainly related with the length of the received signals. According to the asymptotic property of the massive channels, the large $L$ will improve the performance of our proposed scheme. However, the complexity will also relatively increase. Hence, a trade-off should be made in the practical scenario between the channel coherent time, performance requirement and complexity limitations.
In addition, the parameter feasible solution can be basically solved in polynomial time.

Improve the performance because large slowly (logarithmically) with the computation precision. Observation period. Although the SDR complexity scales to ensure a relatively constant large-scale fading during the observation period. Although the SDR complexity scales slowly (logarithmically) with the computation precision $\epsilon$, its feasible solution can be basically solved in a polynomial time. In addition, the parameter $L'$ in Sec.IV-B is also much smaller than the number of antennas at receiver, which has nearly no impact on the detection performance.

V. NUMERICAL RESULTS

In this section, we carry out several numerical simulations to show how the proposed signal reconstruction scheme works and to examine its statistical performance of the bit error ratio (BER) at the receiver. We mainly investigate the performance comparisons with different antenna number and the channel quality signal-to-noise ratio (SNR). All the user’s transmitted symbols are randomly selected from the same given constellation. Power $P_k = \beta_k^2/\beta_k$ is allocated to the $k$-th user so as to mitigate the large-scale fading, where $c = 1$ in simulations. The SDR problem is solved by the CVX toolbox [16].

Fig.1 shows the BER performance with the increasing number of receive antennas. We consider the observation length $L = 200$ and channel SNR = 10dB, BPSK, QPSK and 16QAM constellations are considered for the multiple users. Comparisons are also performed with several typical noncoherent schemes in massive MIMO systems [3], [4], [7], [8] described in “Introduction”. We can observe that our proposed scheme shows competitive BER performance especially in the small $K$ and small constellations. When the large constellation is used, the large number of antennas is needed to achieve a moderate BER. In addition, under the same configuration conditions, the proposed scheme is slightly better than AAUDCPs-MaSTBC [7], while the schemes QAM-Division UF-MUSTM [8], Energy-based [3] and Diff-QPSK [4] require more antennas to achieve similar BER performance.

Fig.2 gives a performance comparisons between the proposed signal reconstruction algorithm and the detectors with using CSI, i.e., the optimal maximum likelihood (ML), minimum mean squared error (MMSE) detection, the complexity-reduced iterative sequential detection [13] and Gauss-Seidel soft-output signal detection [14]. There exist performance gap between our proposed scheme and the optimal ML detection. However, we can see the performance gap gradually decreases with the increasing $N$. In addition, We observe that the proposed algorithm is slightly better than the MMSE detection.

VI. CONCLUSION

In summary, the mathematical essence of this algorithm comes from the quasi-orthogonality property of any two independently random vectors when the length of vectors is large according to the probability statistical theory. Therefore, by building a statistical decision matrix with the observed received signals, this property could effectively alleviate the accurate CSI acquirement in the massive MIMO systems. Then, we formulate the original problem of reconstructing the every user’s transmitted symbols from the statistical decision matrix and we convert this problem to an SDR optimization problem. In terms of generality, in our proposed scheme, each user can essentially use different constellation. The extra recognitions of the possible constellation size and shape are necessary, which will be studied in future work.

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