Can the $\Sigma^{-}nn$ System be Bound?

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Abstract

Motivated by the $\Sigma$-hypernuclear states reported in ($K^-$, $\pi^\pm$) experiments, we have explored the possibility that there exists a particle-stable $\Sigma^{-}nn$ bound state. For the Jülich $\tilde{A}$ hyperon-nucleon, realistic-force model, our calculations yield little reason to expect a positive-parity bound state in either the $J=\frac{1}{2}$ or the $J=\frac{3}{2}$ channels.

The question of the existence of $\Sigma$ hypernuclei bound states — narrow structure in hypernuclear spectra near the threshold for $\Sigma$ production in ($K^-$, $\pi$), etc. reactions — has intrigued physicists for more than a decade [1]. The widths of such states were estimated to be rather broad ($\sim 20$ MeV) due to strong $\Sigma N \rightarrow \Lambda N$ conversion [2], except in special cases. Particularly interesting special cases are the maximum isospin few-body systems such as $\Sigma^{-}nn$, which cannot decay via $\Sigma N \rightarrow \Lambda N$ conversion because of charge conservation. However, the analysis by Dover and Gal [3] of such maximum isospin states indicates that they are not expected to be the most bound. They concluded, based upon the strong spin-isospin dependence of the $\Sigma N$ interaction, that the $T=0, J=\frac{1}{2}$ $\Sigma NN$ state should lie lowest in energy — lower than the two $T=1, J=\frac{1}{2}$ states or the $T=1, J=\frac{3}{2}$ and the $T=2, J=\frac{1}{2}$ configurations. Unfortunately, the intrinsic width of the $T=0, J=\frac{1}{2}$ state was predicted to be much larger than the others. Thus, it was not anticipated that narrow $\Sigma$-hypernuclear few-body states would be observed.

The interest in $\Sigma NN$ states was recently rekindled by the report of Hayano et al [4] that narrow structure was observed below the $\Sigma$ threshold in the stopping kaon reaction $^4\text{He}(K^-, \pi^-)$. The structure in these data was confirmed by later in-flight
measurements [5] and is supported by earlier bubble chamber data [6] for the exclusive $K^- \, ^4\mathrm{He} \rightarrow \pi^- \Lambda p d$ reaction, which were recently reanalyzed [7]. This was surprising in view of the Dover and Gal analysis, in which the $T = \frac{1}{2}$ states were predicted to lie lower in energy but the $T = \frac{3}{2}$ states were predicted to have the narrower intrinsic widths. Narrow structure was actually observed in the $^4\mathrm{He}(K^-, \pi^-)$ reaction below the threshold for $\Sigma$ production, whereas no evidence for an enhancement in that region was observed in the $^4\mathrm{He}(K^-, \pi^+)$ spectra. The $(K^-, \pi^-)$ reaction leads to both $T = \frac{1}{2}$ and $T = \frac{3}{2}$ channels, while the $(K^-, \pi^+)$ reaction leads only to the $T = \frac{3}{2}$ channel. Thus, the observed structure was interpreted as a bound $^4\Sigma$ hypernucleus with quantum numbers $T = \frac{1}{2}, J = 0$. [The $(K^-, \pi)$ spin-flip amplitude is small.]

In fact, Harada et al. [8, 9] had predicted such an $A = 4$ bound state, based upon a central force approximation to the Nijmegen model D [10] hyperon-nucleon ($YN$) potential. Therefore, in spite of the theoretical analysis of Dover and Gal that suggests formation of a bound $\Sigma^-nn$ state is unlikely, one is led to ask whether state-of-the-art calculations based upon contemporary $YN$ potential models might indicate a possibility that the $T = 2, J = \frac{1}{2}$ or $J = \frac{3}{2}$ states could be observed experimentally, either as a bound state in the continuum or as a three-body resonance. {Garcilazo [11] argued on the basis of rank-one separable potentials that such a system is unbound.} One would prefer to explore all $\Sigma NN$ states, because $^3\mathrm{He}(K^-, \pi^\pm)$ experiments [12] can excite only $T_z = \pm 1$ states. [Target complications make the $^3\mathrm{H}(K^-, \pi^\pm)$ reaction to the $T_z = -2$ state more difficult.] However, including the $\Sigma N - \Lambda N$ coupling required by the $T = 1$ states leads to the technically difficult requirement that one must solve the three-body equations for the continuum. This has been accomplished for separable potentials [13], but not for local potential calculations. For that reason we have confined our investigation to the possible existence of a $T = 2$ bound state.

The Faddeev equations for the $\Sigma^-nn$ system were solved in momentum space using the technical apparatus described in Ref. [14]. The complication beyond standard triton calculations is that the $\Sigma$ can be distinguished from the two neutrons, which leads to a coupled pair of three-body equations instead of only the single equation that one finds for the comparable three-identical-particle problem. A more detailed presentation of $YNNN$ three-body bound-state equations can be found in Ref. [15].

The baryon-baryon interactions are assumed to act in all partial waves with $j \leq 1$ and with positive parity. This restriction yields a reasonable approximation to the converged binding energy in the three-nucleon system and can be expected to be sufficient for the purpose of determining whether a $\Sigma^-nn$ bound state might exist. The effect of higher partial waves is certainly smaller than the variations induced by the use of different baryon-baryon interaction models.
Because we work in momentum space, we considered the Jülich [16] hyperon-nucleon interaction models. In particular, we used the Jülich model A, an energy-independent one-boson-exchange approximation to the energy-dependent model A interaction. The s-wave effective range parameters for $\Sigma^+p$ scattering in these models are given in Table 1; we assumed equivalence for the $\Sigma^-n$ interaction for the purpose of this exercise. We would point out, however, that the Jülich model differs qualitatively from the Nijmegen models [10, 17, 18]. The Jülich models are attractive for both spins, whereas the Nijmegen models exhibit a repulsive spin-triplet interaction. Thus, we have chosen the realistic $\Sigma N$ potential model that is most likely to support a $\Sigma^-nn$ bound state. For the $nn$ interaction we employed the Nijmegen one-boson-exchange potential of Ref. [19].

### Table 1. The $\Sigma^+p$ scattering lengths and effective ranges in fm for the Jülich potential models listed

| Model     | Ref. | $a^*$ | $r_o^*$ | $a^t$ | $r_o^t$ |
|-----------|------|-------|---------|-------|---------|
| Jülich A  | [16] | -2.28 | 4.96    | -0.76 | 2.50    |
| Jülich $\tilde{A}$ | [16] | -2.26 | 5.22    | -0.76 | 0.78    |

Our search for a bound $\Sigma^-nn$ system with $J^\pi = \frac{1}{2}^+$ proved negative. In retrospect this is not surprising in view of the fact that the spin-singlet $\Sigma N$ interaction is the stronger, whereas the spin-triplet potential dominates: the average interaction is $\frac{3}{4} V^s + \frac{1}{4} V^t$. {Lack of binding was also found for the hypertriton using the $T = \frac{1}{2} \Lambda N - \Sigma N$ potentials of this same Jülich $\tilde{A}$ model [15].} To understand how far away a resonance might lie, we have multiplied the total interaction by a variable factor, increasing that factor until binding was achieved. A plot of the strength factor versus the binding energy obtained is shown in Fig. 1. Because the factor needed to produce binding is greater than 1.7, we do not expect any low-lying resonance in the $\Sigma^-nn$ system. In the $J^\pi = \frac{3}{2}^+$ case, a spectator $\Sigma^-$ must be at least in a p-wave relative to the $nn$ pair in order to reach spin-3/2, because s-wave neutrons will necessarily be paired to spin-0. Therefore, it was expected that the $J = \frac{3}{2}$ state will be unbound in view of the finding that there is no $J = \frac{1}{2}$ bound state. Indeed, that was the case.

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References

[1] See, for example, the review by Th. Walcher, Nucl. Phys. A479, 63c (1988) and the related papers on Σ-hypernuclei in that volume.

[2] A. Gal, Nukleonika 25, 447 (1980).

[3] C. B. Dover and A. Gal, Phys. Lett. B 110, 433 (1982).

[4] R. S. Hayano, T. Ishikawa, M. Iwasaki, H. Outa, E. Takada, H. Tamura, A. Sakaguchi, M. Aoki, and T. Yamazaki, Phys. Lett. B 231, 355 (1989); Nuovo Cimento A 102, 437 (1989).

[5] R. S. Hayano, Nucl. Phys. A547, 151c (1992).

[6] P. A. Katz et al. Phys. Rev. D, 1, 1267 (1970).

[7] R. H. Dalitz, D. H. Davis, and A. Deloff, Phys. Lett. B 236, 76 (1990).

[8] T. Harada, S. Shinmura, Y. Akaishi, and H. Tanaka, Soryusiron-Kenkyu 76, 25 (1987); Nuovo Cimento A 102, 76 (1990); Nucl. Phys. A507, 473 (1990).

[9] T. Harada and Y. Akaishi, Phys. Lett. B 262, 200 (1991).

[10] M. M. Nagels, T. A. Rijken, and J. J. de Swart, Phys. Rev. D 15, 2547 (1977).

[11] H. Garcilazo, J. Phys. G 13, L63 (1987).

[12] M. Barakat and E. V. Hungerford, Nucl. Phys. A547, 157c (1992).

[13] I. R. Afnan and B. F. Gibson, Phys. Rev. C 47, 1000 (1993).

[14] A. Stadler, W. Glöckle and P. U. Sauer, Phys. Rev. C 44, 2319 (1991).

[15] K. Miyagawa and W. Glöckle, Phys. Rev. C 48, 2576 (1993).

[16] A. Reuber, K. Holinde, and J. Speth, Czech. J. Phys. 42, 1115 (1992); K. Holinde, Nucl. Phys. A547, 255c (1992); B. Holzenkamp, K. Holinde, and J. Speth, Nucl. Phys. A500, 485 (1989).
[17] M. M. Nagels, T. A. Rijken, and J. J. de Swart, Phys. Rev. D 20, 1633 (1979).
[18] P. M. Maessen, T. A. Rijken, and J. J. de Swart, Phys. Rev. C 40, 2226 (1989).
[19] M. M. Nagels, T. A. Rijken, and J. J. de Swart, Phys. Rev. D 17, 768 (1978); T. A. Rijken, R. A. M. Klomp, and J. J. de Swart, Nijmegen preprint THEF-NYM-91.05.
Fig. 1 Strength factor by which the total $\Sigma N$ interaction is multiplied versus the $\Sigma^- nn$ binding energy. The circles represent the actual calculations, the solid line is drawn to guide the eye.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9402005v1
Fig. 1: A. Stadler and B.F. Gibson, Can the $\Sigma$nn System be Bound?