Stability of RS brane for tachyonic scalars

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Abstract

We study the stability of the RS brane embedded in the AdS$_5$ vacuum of 5d gauged supergravity, where many tachyonic scalars exist. We consider a model in which these scalars couple to the brane such that the BPS conditions are satisfied to preserve the bulk supersymmetric configuration. In this case, we find that these tachyons are not trapped on the brane and only the massless dilaton is localized. As a result, the braneworld is stable. Further, the effective action of trapped fields is studied by using the brane running method developed recently, and we find that the action is independent of the brane position.
1 Introduction

It is an interesting idea to consider our 4d world as a thin three-brane embedded in $AdS_5$ space-time as proposed by Randall-Sundrum (RS) [1, 2]. This bulk space is considered as an extended part of $AdS_5 \times S^5$, which is realized near the accumulated D3 branes in the type IIB superstring theory. In this context, the bulk theory can be supposed as the maximally symmetrized 5d gauged supergravity with massive Kaluza-Klein (KK) modes of $S^5$. In this theory, many tachyonic scalars are included, and they play an important role [7, 8, 9] in the context of AdS/CFT correspondence [10, 11, 12, 13]. Since their masses are within the Bleitenrhoner-Freedman bound [14], then the bulk is stable. While, it is known that the bulk tachyons can be trapped on the brane as 4d tachyons when the tachyons are free and the brane action is given by the tension only [15]. Then the braneworld is unstable in such a case.

In our present model, the brane is embedded in the AdS_5 in such a way to keep the BPS conditions [3, 4] and to preserve the bulk supersymmetric configurations. This is realized by a special form of the brane action or the brane-scalar coupling [5, 6], then the situation for the localization is non-trivially changed. So, it is important to make clear the localization problem in the present model based on the gauged supergravity. Our purpose is to examine the stability of the braneworld through the investigation of the localization for tachyons in our brane model. It would be necessary to make clear this point when we try to construct the braneworld based on the superstring theory compactified to $AdS_5 \times S^5$.

Another point to be studied here is the effective action of the brane. In the original RS model, the brane action is simply expressed by a tension parameter only. In the present case, however, it is given as a function of scalar fields as mentioned above. So we might need some principle what kind of brane action we choose. The most important factor would be the stability of the braneworld. Since the form of the brane action is deeply related to the localization of the bulk fields, then it also controls the stability of the braneworld.

However, in general, the action is not invariant for the shift of the brane position even if a form of the action is fixed at some point. Actually we need various kinds of terms in the brane action when the brane position is changed by keeping the background configuration and its fluctuation modes to be unchanged [16, 17, 18, 19, 20]. This procedure is called as brane running and it would provide a renormalization group flow of the brane action in the following sense. The fields on the brane could couple to the bulk modes, and interact with them. From the viewpoint of AdS/CFT, this interaction can be interpreted as the one with the cutoff CFT living on the brane. Through this interaction, the brane action would receive cutoff dependent corrections. As a result, the parameters of the brane action are running and they varies according to the flow equations obtained by the brane running method. In other words, the parameters of the brane action on a different position are related each other by these flow equations. Then a simple brane action of RS model is regarded as the one obtained under a special...
renormalization condition at an appropriate renormalization point. So it might be changed to a more complicated or an unstable form of action when the renormalization point is shifted even if the original form is simple and stable.

According to this idea, we examine the running behavior of the brane action for a scalar field, which is trapped on the brane, and derive the effective action. Through this action, we can see the effect on the brane from the bulk of gauged supergravity and also get an aspect for the stability of the system.

In Section 2, the model used here is set, and the localization of scalar fields and the stability of the braneworld are examined in Section 3. In Section 4, the effective action for the brane is obtained by the brane running method, and the effect of the bulk on the brane is discussed. Concluding remarks are given in the final section.

2 Setting of the model

As a bulk action, consider the bosonic part of a truncated 5d gauged supergravity

\[ S_g = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R - \frac{1}{2} \sum I (\partial \phi_I)^2 - V(\phi) \right\} + \frac{1}{2\kappa^2} 2 \int d^4x \sqrt{-g} K, \]

where \( K \) is the extrinsic curvature on the boundary, and the potential \( V \) of scalars \( \phi_I \) is written by their superpotential \( W(\phi_I) \) as

\[ V = \frac{v^2}{8} \sum I \left( \frac{\partial W}{\partial \phi_I} \right)^2 - \frac{v^2}{3} W^2. \]

The gauge coupling parameter \( v \) is fixed from the AdS\(_5\) vacuum \([21]\) by fixing the radius of AdS as a unit length. And we are considering in the Einstein frame \(^2\). The other ingredient is the brane action, which is given here as \([6]\)

\[ S_b = -v \int d^4x dy \sqrt{-g} W(\phi_I) \delta(y - y_h). \]

The reason for choosing of this form is made clear in the followings, and the brane position \( y_h \) is set as \( y_h = 0 \) hereafter for simplicity.

The background solutions are obtained under the ansatz, \( \phi_I = \phi_I(y) \) and

\[ ds^2 = A^2(y) \eta_{\mu \nu} dx^\mu dx^\nu + dy^2. \]

Here \( \eta_{\mu \nu} = \text{diag}(-1,1,1,1) \) and the coordinates parallel to the brane are denoted by \( x^\mu = (t, x^i) \), \( y \) being the coordinate transverse to the brane. In this set up, the four

\(^2\)Here we take the following definition, \( R^\mu_\nu\lambda\sigma = \partial_\lambda \Gamma^\mu_\nu\sigma - \cdots \), \( R_{\nu \sigma} = R^\rho_\nu\rho_\sigma \) and \( \eta_{AB} = \text{diag}(-1,1,1,1,1) \). Five dimensional suffices are denoted by capital Latin and four dimensional ones by Greek letters.
dimensional slice perpendicular to $y-$axis is the Minkowsky space-time, and the BPS solutions for the bulk configuration are obtained by solving the following first order equations \[3\],

$$
\phi'_I = \frac{v}{2} \frac{\partial W}{\partial \phi_I}, \quad \frac{A'}{A} = -\frac{v}{3} W,
$$

where $' = d/dy$. The solutions of \[4\] satisfy the equations of motion of $S_g$ as is well-known, and they preserve supersymmetry [3, 4] in the bulk. Further, the equations \[5\] at the brane position coincide with the boundary conditions in solving the equations of motion of the total action, $S = S_g + S_b$, iff $S_b$ was given by the form of \[3\]. This is the reason why the brane action is given as \[3\]. Then, in this case, the brane can be embedded at any point of $y$ since the boundary conditions are satisfied at any $y$ for the solutions of \[5\]. Another point to be noticed is that this system could be extended to a supersymmetric braneworld solutions by adding other necessary terms on the brane \[5\].

As for the scalar fields in the 5d gauged supergravity, there exist many tachyonic fields and they play an important role in the gauge/gravity correspondence since they couple to the relevant operators of $\mathcal{N} = 4$ SYM as deformations of the CFT on the boundary. When we consider a free tachyonic scalar which does not couple to the brane, this scalar is trapped on the brane as a 4d tachyonic scalar [15]. In this case, the braneworld destabilizes and the existence of such a tachyon in the bulk would be rejected. In the present case, however, the situation is different since the the scalars couple to the brane through $W(\phi)$ in a special form. We can see in the next section that this coupling changes the situation of the stability for braneworld.

### 3 Scalar localization

In order to make clear the issue, we consider here a simple AdS$_5$ solution, $A = e^{-|y|}$ and $\phi_1 = 0$. Here we set the parameters as $\kappa^2 = 2$ and $v = -2$ for simplicity. According to [15], we examine the localization of scalars in terms of the linearized equations. For simplicity, consider the case of one scalar, $\phi$, which is expanded as $\phi = \bar{\phi} + \chi$ around its classical solution $\bar{\phi}(= 0)$. So the equation of $\chi$ is obtained as

$$
\chi'' + \frac{4 A'}{A} \chi' + \frac{m^2}{A^2} \chi = \left( \frac{\partial^2 V(0)}{\partial \phi^2} - 2 \frac{\partial^2 W(0)}{\partial \phi^2} \delta(y) \right) \chi, \quad (6)
$$

where $m^2$ is the four dimensional mass square of $\chi$. Due to the second term of the right hand side in \[6\], the following boundary condition is needed,

$$
\chi'(0) = -\frac{\partial^2 W(0)}{\partial \phi^2} \chi(0). \quad (7)
$$

Hereafter we denote as $V^{(i)} = \frac{\partial^2 V(0)}{\partial \phi^2}$ and $W^{(i)} = \frac{\partial^2 W(0)}{\partial \phi^2}$, then the equation \[6\] can be rewritten into the following form,

$$
[-\partial_z^2 + V(z)]u(z) = m^2 u(z), \quad (8)
$$
\[ V(z) = \frac{9}{4}(\partial_y A)^2 + \frac{3}{2} A \partial_y^2 A + A^2 V^{(2)} - 2W^{(2)} \delta(y), \]  
where we introduced \( u(z) \) and \( z \) defined as \( u = A^{-3/2} \chi \) and \( \partial z/\partial y = \pm A^{-1} \).

For \( A = e^{-|y|} \), the bound state wave function is solved as \( u = x^{1/2} K_\nu(|m|) \) where \( x = |z| + 1 \) and \( \nu = \sqrt{4 + V^{(2)}} \). Since the bound state should be restricted to the region \( m^2 \leq 0 \), the solution is written in terms of the absolute value of negative \( m^2 = -|m|^2 \), and \( K_\nu(x) \) is the modified Bessel function. Then the boundary condition \( (7) \) is obtained as

\[ \left( 2 + \nu + W^{(2)} \right) K_\nu(|m|) = |m| K_{\nu+1}(|m|), \]

where the AdS\(_5\) radius is taken as unit and \( W^{(0)} = -3/2 \) since we set as \( \kappa^2 = 2 \) and \( v = -2 \). Further we demanded \( W^{(1)} = 0 \), which is needed for \( \bar{\phi} = 0 \) and satisfied for all the known superpotentials. And from \( (2) \),

\[ M^2 \equiv V^{(2)} = 4W^{(2)} + (W^{(2)})^2, \quad \nu = |2 + W^{(2)}|. \]

So the boundary condition \( (10) \) depends only on the parameter \( W^{(2)} \). For \( W^{(2)} > -2 \) \( (M^2 > -4) \), \( (10) \) is written as

\[ 2\nu = \frac{|m|K_{\nu+1}(|m|)}{K_\nu(|m|)}. \]

and this is satisfied only for \( m = 0 \). While for \( W^{(2)} \leq -2 \), we have \( \nu = -(2 + W^{(2)}) \), then \( 2 + \nu + W^{(2)} = 0 \). In this case, \( (10) \) is satisfied only for \( \nu = 0 \) and \( m = 0 \), then \( W^{(2)} = -2 \) and \( M^2 = -4 \). After all, we find that there is no tachyonic bound state and the trapped state might be seen in the case of \( M^2 \geq -4 \), within the BF bound, only for \( m = 0 \).

In order to see whether the zero mode, \( m = 0 \), is really localized or not, we must check over the normalizability of the mode with respect to \( y \)-integration. From \( (6) \), the \( y \)-dependent part of \( \chi \) for \( m = 0 \) is obtained by parametrizing as \( \chi = \chi_t(y)\chi_b(x) \),

\[ \chi_t(y) = e^{(2-\nu)y}. \]

From the normalizability of the kinetic term, we demand

\[ \int_0^\infty dy A^2(y)\chi_t(y)^2 = \int_0^\infty dy e^{2(1-\nu)y} < \infty, \]

then \( \nu < 1 \) or \( M^2 > -3 \).

On the other hand, we must also demand the normalizability of the potential \( V(\chi) \) simultaneously. In general, however, \( V(\chi) \) is expressed by infinite power series as \( V(\chi) = \sum \chi^i/i! \), so we must demand \( 2 - \nu \leq 0 \) for the normalizability of all these terms. Then the condition \( M^2 \geq 0 \) is needed. This means that the bulk mass square of the trapped state must be non-negative. Then we can conclude that the bulk tachyonic scalar \( (M^2 < 0) \) can not be trapped neither as a tachyon nor as a zero mode.
For the dilaton, the potential is zero, \( V(\phi) = 0 \), and \( W(\phi) = 0 \) \[22\]. The situation for the localization is the same with the case of a free scalar field which does not couple to the brane. We find that this field can be trapped as a massless scalar on the brane, and it does not affect the stability of the brane solution.

After all, the RS braneworld is stable against for any tachyonic scalar of the gauged supergravity when we take the brane action as \[3\]. In order to see the necessity of this form for its stability, consider a small modification of \[3\] without changing the background configuration. For example, consider the following brane action,

\[
S_b = -v \int d^4x dy \sqrt{-g} \left( W(\phi) + \frac{1}{2} \epsilon \phi^2 \right) \delta(y - y_b). \tag{15}
\]

with a small parameter \( \epsilon \). Here the bulk action is not changed. In this case, the bulk AdS_5 is preserved, but the boundary condition \[12\] is changed as

\[
2\nu + \epsilon = \frac{|m| K_{\nu+1}(|m|)}{K_{\nu}(|m|)}, \tag{16}
\]

and we find that the scalar is trapped as a tachyon for the cases of \( \{ \nu < 1, \epsilon < 0 \} \) and \( \{ \nu \geq 1, \epsilon > 0 \} \). In general, in the gauged supergravity, both kinds of scalars of \( \nu < 1 \) and \( \nu \geq 1 \) are included, then we could not get a stable brane solutions for a finite value of \( \epsilon \). Then the stable solution would be restricted to the case of \( \epsilon = 0 \). This implies that the background configuration should be changed when the BPS conditions are broken. In the next section, we study the effective action how these bulk scalars are observed on the brane.

### 4 Effective action and brane running

Here we estimate the effective brane action \( S_b^{\text{eff}} \) to study how we observe the bulk fields on the brane. From the viewpoint of path-integral formulation, it can be obtained as \[23\ [24\ [25\ [26\]

\[
S_b^{\text{eff}} = \frac{1}{2} S_b + \ln Z_5(g, \phi) \tag{17}
\]

\[
Z_5(g, \phi) = \int_{G(x,0),\phi(x,0)\text{fixed}} DG D\phi \exp \left( i \int_{y \geq 0} d^4x dy L_g \right), \tag{18}
\]

where \( g \) and \( \phi \) represent the boundary value of \( G(x, y) \) and \( \phi(x, y) \) respectively. And \( S_b \) and \( S_g \equiv \int d^4x dy L_g \) are defined in the previous section.

As for \( \ln Z_5(g, \phi) \), it is related to the conformal field theory (CFT) on the boundary (\( y \rightarrow -\infty \)) in the context of AdS/CFT correspondence. And it can be estimated by WKB approximation in terms of a classical solution as

\[
\ln Z_5(g) = S_{\text{CT}} + S_{\text{CFT}} \tag{19}
\]
where $S_{CT}$ represent the divergent term at $y \to -\infty$. The CFT generating functional $S_{CFT}$, which is finite at $y \to -\infty$, is obtained by subtracting the divergent $S_{CT}$ from $\ln Z_5(g)$. However, here, $y$ is taken at some point, $y_0$, off the boundary $y = -\infty$, then $S_{CFT}$ represents a "cut-off" CFT and it is written by the fields at $y = y_0$ not at $y = -\infty$. In this context, $\ln Z_5(g)$ has been estimated in terms of the asymptotic expansion of the equation of motion, and we find the counter term for a scalar of conformal dimension $\Delta$ as $^{27, 28}$

$$S_{CT}^{s} = \int_{y=y_0} d^4x \sqrt{-g} \left( \frac{4 - \Delta}{2} \phi^2 - \frac{1}{4(\Delta - 3)} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi + \cdots \right),$$

for $\Delta \neq 3$. While for $\Delta = 3$, the coefficient $-\frac{1}{4(\Delta - 3)}$ of the kinetic term is replaced by $+\frac{1}{4} \ln(\epsilon) \equiv y_0/2$, which corresponds to the conformal anomaly due to the scalar. This is seen also in the brane running method as shown below, and this kinetic term is canceled out in the effective action. So the kinetic term of this scalar field disappears. However the scalar of $\Delta = 3$ ($M^2 = -3$) is not trapped as shown in the previous section, so we should consider the above action is useful only for the classical solution as on-shell action in the case of $\Delta = 3$ scalar. Here we concentrate on the scalar whose zero mode is trapped. It is the dilaton of $\Delta = 4$ in the present case. We notice that the same result with (20) is obtained for this scalar simply by integrating the $y$-dependent part in the 5d action as

$$S_{CT}^{s} = \int_{y=y_0} d^4x \sqrt{-g} \left( -\frac{1}{4} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi + \cdots \right),$$

which is precisely equivalent to (20) for $\Delta = 4$.

In the next, we estimate the brane action $S_{b}$ at an arbitrary point of $y$ by using the brane running method to obtain the effective action for the trapped scalar field $\phi$. The scalar is expanded as $\phi = \bar{\phi} + \chi$. For $\bar{\phi} = 0$, the linearized equation for $\chi$ is obtained as

$$\chi'' + 4 \frac{A'}{A} \chi' + \frac{q^2}{A^2} \chi = \left( \frac{\partial^2 V(0)}{\partial \phi^2} - 2 \frac{\partial^2 W(0)}{\partial \phi^2} \delta(y) \right) \chi,$$

where $q^2$ is the four dimensional momentum square of $\chi$ and we take $y_0 = 0$ for simplicity. Then the boundary condition for $\chi$ is written as,

$$\frac{\chi'(0)}{\chi(0)} = -\frac{\partial^2 W(0)}{\partial \phi^2} \big|_{y=0}.$$

We extend this equation to the region, $y > y_0 (= 0)$, where the running brane arrived, by introducing the scalar part of running brane action as

$$S_{b}^{(s)} = -\int d^4x \sqrt{-g} \left\{ \sum_{i=0}^{\infty} \frac{\chi_i^i}{i!} s_{(i)}(y) + \frac{1}{2} \tau_k(y)(\partial \chi)^2 + \cdots \right\},$$

(24)
where dots represent other higher derivative and non-local terms. Then the extended boundary condition is given as

$$2 \chi'(y) \chi(y) = s^{(2)}(y) - \tau_k(y) \frac{-q^2}{A^2(y)} + O \left( \frac{q^2}{A^2(y)} \right)^2,$$

(25)

with the initial values, $s^{(2)}(0) = -2 \frac{\partial^2 W(0)}{\partial \phi^2} \equiv -2 W^{(2)}$ and $\tau_k(0) = 0$. By differentiating (25) with respect to $y$ and using (22), we obtain the following $\chi$-independent flow equations,

$$s^{(2)}' = -\frac{1}{2} s^{(2)^2} - 4 \frac{A'}{A} s^{(2)} + 2 V^{(2)},$$

(26)

$$\tau'_k = -s^{(2)} \tau_k - 2 \frac{A'}{A} \tau_k + 2,$$

(27)

where $V^{(2)} \equiv \frac{\partial^2 V(0)}{\partial \phi^2} = (W^{(2)})^2 - 4 W^{(2)}$. These equations are solved as

$$s^{(2)} = \text{const.} = -2 W^{(2)}, \quad \tau_k = -\frac{e^{2 \tilde{\mu} y}}{\tilde{\mu}},$$

(28)

where $\tilde{\mu} = 1 + W^{(2)}$. The above solution for $\tau_k$ is useful for $W^{(2)} \neq -1$. In the case of $W^{(2)} = -1$, we find $M^2 = -3 (= V^{(2)})$ and $\Delta = 3$. And we obtain

$$\tau_k = 2 y = \ln(\epsilon),$$

(29)

where $\epsilon$ is the cutoff used in [27, 28]. As mentioned in the previous section, this kinetic term cancels out in $\frac{1}{2} S_b + S_{CT}$. However, as shown in the previous section, the scalar of $M^2 < 0$ is not trapped, so we don’t consider the effective action for this scalar. However the above result [29] is very interesting since this reproduces the conformal anomaly which has been shown in the holographic approach [27, 28]. So we could see again the anomaly through the brane running method as shown in the case of pure gravity [20].

Using the above results, we can obtain the effective brane action at an arbitrary point of running position, or at an arbitrary mass scale of the 4d field theory. The only trapped scalar here is the dilaton, for which $M^2 = W^{(2)} = 0$, then

$$s^{(2)} = 0, \quad \tau_k = e^{2y} - 1.$$

(30)

After all, we arrive at the following effective brane action for the trapped scalar,

$$S_{s}^{s_{\text{eff}}} = \frac{1}{2} S_{b}^{s_{\text{eff}}} + S_{CT}^{s_{\text{eff}}} = \int d^4 x \sqrt{-\hat{g}} \left( -\frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \cdots \right),$$

(31)

where the metric $\hat{g}^{\mu\nu}(x)$ is defined as $ds^2 = A^2(y)\hat{g}_{\mu\nu}(x)dx^\mu dx^\nu + dy^2$. This result implies that the low energy action is independent of the brane position as shown for
the Einstein term in the gravitational case [20]. As a result, we have an effective brane action which includes the Einstein term and the dilaton as

$$S_{\text{eff}}^{b} = \int d^4x \sqrt{-\hat{g}} \left( \frac{1}{4\mu \kappa^2} \hat{R} - \frac{1}{4} \hat{g}^{\mu \nu} \partial_{\mu} \phi \partial_{\nu} \phi + \cdots \right) + S_{\text{CFT}},$$

(32)

Except for the higher derivative terms, the action is independent of the energy-scale $y$. In other words, we can see the same low energy theory at any mass scale. This would be the reflection of the conformal invariance of the AdS bulk. And all the tachyonic scalars are living in the bulk and they affect the fields on brane through $S_{\text{CFT}}$, but the above low energy part is not affected.

5 Concluding remarks

The stability of the RS braneworld is examined in the AdS$_5$ vacuum of gauged supergravity. Although there are several tachyonic scalars in this model, their masses are within the bound of Breitenlohner-Friedman. Then the bulk is stable. While, a tachyonic scalar in the bulk could be trapped on the brane as a four dimensional tachyon when there is no brane-scalar coupling, then the brane becomes unstable in this case. In the present case, we set a brane-scalar coupling in such a way that the supersymmetric bulk configuration is preserved. In other words, the BPS conditions are satisfied in the bulk and also on the brane. Just on the brane position, the BPS conditions are equivalent to the boundary conditions of the equations of motion. Then the brane, in our model, can be embedded at any point in the bulk when the solutions are BPS.

In this case the situation for the trapping of tachyons is non-trivial, and we find that the tachyons are not trapped on the brane in our model mentioned above. Among the scalars in the model, only the massless dilaton is trapped. Then our braneworld is stable against many tachyonic fields in the bulk. This stability is supported by the bulk supersymmetry and the BPS conditions satisfied up to the brane position. Actually, we can see that the braneworld becomes unstable for a small modification, which breaks the BPS conditions, of the brane action.

For the trapped dilaton field, the effective action is studied by the brane running method to see the effect from the bulk to the brane action. The action given at some point of the fifth coordinate $y$ may represent the one obtained at the cutoff of $\ln \epsilon = y$. The cutoff dependence of this action can be seen by shifting $y$ according to the brane running method. After performing this method, we could find that the action of relevant terms are independent of the brane position $y$. This result is similar to the case of the Einstein action for the trapped graviton. We can say that this $y$-independence is the reflection of the bulk supersymmetry or conformal symmetry of CFT on the brane. We find further the anomaly for the kinetic term of the scalar field of conformal dimension $\Delta = 3$. This coincides with the one found in the holographic
approach to obtain the on-shell action. Then we could assure that the brane running method would give a correct flow of the parameters in the brane action.

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