On $e^+e^- \rightarrow W^+W^-$ at LEP2*

D. Schildknecht

ABSTRACT

We briefly discuss the first direct experimental evidence from LEP2 on non-vanishing trilinear couplings among the weak vector bosons. Subsequently we review the improved Born approximation to include radiative corrections. Provided the appropriate high-energy scale is used for the SU(2) gauge-coupling, the improved Born approximation is sufficiently accurate for all practical purposes at LEP2.

* Supported by EU-Project CHRX-CT 94-0579. To appear in the Proceedings of the EU-network meeting, Ouranoupolis, Greece, April 1997, edited by A. Nicolaidis
The process of W-pair production in $e^+e^-$ annihilation is presently studied experimentally at LEP2. In the future, it will be one of the outstanding processes at a linear collider in the TeV energy range. The process of W-pair production yields direct experimental information on the non-Abelian couplings characteristic for the $SU(2)\times U(1)$ electroweak theory and allows to put bounds on potential non-standard $Z_0W^+W^-$ and $\gamma W^+W^-$ couplings [1,2]. Indeed, even after a fairly short time of running, LEP2 results from the L3 collaboration have provided direct experimental evidence [3] for the existence of a genuine trilinear coupling between the members of the $W^\pm, W^0$ triplet characteristic for the non-Abelian structure of the basic Lagrangian. A vanishing genuine trilinear coupling in the basic electroweak Lagrangian, $\hat{g} = 0$, is excluded at 95 % C.L. in a two-parameter, $\hat{g}, \Delta \kappa_\gamma \equiv \kappa_\gamma - 1$, analysis of the experimental data, where $\kappa_\gamma$ denotes the electromagnetic dipole coupling of the $W^\pm$. This is shown in fig. 1 taken from ref. [3]. The analysis of the experimental data, without much loss of generality, is based on custodial SU(2) symmetry for the couplings among the vector bosons, i.e. on imposing restoration of SU(2) for vanishing electromagnetic coupling, $e = 0$ [4,2]. Under this constraint, deviations from the standard value, of the $Z_\mu\nu$ dipole coupling, $\kappa_Z = 1$, are proportional to $\Delta \kappa_\gamma$, thus reducing the number of independent dim.4 couplings in the phenomenological vector-boson Lagrangian from three to two, the $Z^0W^+W^-$ coupling, $g_{ZW^+W^-}$, and the electromagnetic dipole coupling $\kappa_\gamma$ of the $W^\pm$. In fig. 1, the experimental bounds on the deviations of these couplings from their Standard Model values, $\delta_Z \equiv g_{ZW^+W^-} \cot \theta_W$ and $\Delta \kappa_\gamma \equiv \kappa_\gamma - 1$, are shown.

In what follows, I will concentrate on briefly describing a simple approximate treatment of radiative correction to $e^+e^- \rightarrow W^+W^-$ relevant at LEP2 energies. It is based on a recent paper in collaboration with Masaaki Kuroda and Ingolf Kuss [5], which in turn rests heavily on previous work by Böhm, Denner and Dittmaier [6]. Essentially, I wish to stress that an accuracy of the order of 0.5 % can be reached for the radiatively corrected differential cross section of $e^+e^- \rightarrow W^+W^-$ at LEP2 energies by supplementing the Born approximation by Coulomb corrections and initial-state-radiation. An appropriate choice of the scale, in particular for the SU(2) gauge coupling, and also for the electromagnetic coupling in the Born approximation is a prerequisite for obtaining this accuracy of approximately 0.5 %.

The Born amplitude is most transparently derived [5] directly from the basic Lagrangian of the $SU(2)\times U(1)$ theory by treating $BW_3$ mixing perturbatively (to all orders of the mixing). Compare the Feynman diagrams in fig. 2. Upon replacing $g^2$ by $e^2$ via $g^2 \equiv e^2/(1-e/g^2_W)$, from the Feynman diagrams of fig. 2, one obtains the Born amplitude directly in terms of the SU(2) coupling, $g_W$, and the electromagnetic coupling, $e$,

$$\mathcal{M}_{Born}(\kappa, \lambda_+, \lambda_-, s, t) = g_W^2 \frac{1}{2} \delta_{\kappa_\gamma} \mathcal{M}_I + e^2 \mathcal{M}_Q,$$  \hspace{1cm} (1)
\[ \delta Z = g_{ZWW} \cot \theta_W \]

\[ \Delta \kappa_\gamma = \kappa_\gamma - 1 \]

Figure 1: Contour curves of 68 % and 95 % confidence level in the \((\delta Z, \Delta \kappa_\gamma)\) plane are shown as solid and dashed lines. Expectations due to vanishing ZWW and weak couplings, \(g_{ZWW} = 0\) and \(g = 0\), are indicated by the dotted and dashed-dotted lines. (From [3]).
Figure 2: Evaluating the electroweak Born approximation in the $BW_3$ basis.
with the weak isospin amplitude (modified by the replacement of $M_{W^\pm}^2 \rightarrow M_Z^2 = (1 + g^2/g_{W^\pm}^2)M_W^2$ in the propagator via $BW^3$ mixing) being given by

$$\mathcal{M}_I = \frac{1}{s-M_Z^2} \mathcal{M}_s + \frac{1}{t} \mathcal{M}_t,$$

(2)

while the electromagnetic amplitude has the double-pole structure

$$\mathcal{M}_Q = -\frac{M_Z^2}{s(s-M_Z^2)} \mathcal{M}_s.$$

(3)

For the explicit form of the s-channel and t-channel quantities, $\mathcal{M}_s$ and $\mathcal{M}_t$ in terms of the kinematic variables $s$ and $t$ and the particle helicities we refer to ref. [7].

The calculation of the cross section for $e^+e^- \rightarrow W^+W^-$ from (1) requires a specification of the scale at which the SU(2) gauge coupling, $g_{W^\pm}$, and the electromagnetic coupling, $e$, in (1) are to be defined. As $W$-pairs are produced at energies of $2M_{W^\pm} \lesssim \sqrt{s} \sim 200$ GeV at LEP2, it is natural to choose a high-energy scale, such as $\sqrt{s}$. We expect it to be sufficiently accurate to use the scale $M_W \approx M_Z$ instead of $\sqrt{s}$. Accordingly, we define $g_{W^\pm}(M_W^2)$ by the leptonic branching ratio of the $W^\pm$ boson, $\Gamma^{W}_l$ [8],

$$g_{W^\pm}^2(M_W^2) = 48\pi \frac{\Gamma^{W}_l}{M_{W^\pm}}.$$  

(4)

The $W$ branching ratio not being experimentally known with sufficient accuracy, it must be expressed in terms of the Fermi coupling, $G_{\mu}$. The SU(2) gauge coupling (4) then becomes [8]

$$g_{W^\pm}^2(M_W^2) = \frac{4\sqrt{2}G_{\mu}M_{W^\pm}^2}{1 + \Delta y^{SC}},$$

(5)

where the one-loop correction, $\Delta y^{SC}$, derives from the change of scale from $\mu^\pm$ decay, where $G_{\mu}$ is defined, to $W^\pm$ decay. It (obviously) contains a sum of two contributions,

$$\Delta y^{SC} = \Delta y^{SC}_{ferm} + \Delta y^{SC}_{bos},$$

(6)

The fermion contribution, essentially due to contributions of the light leptons and quarks to the $W$ propagator, leads to an increase of $g_{W^\pm}$, when taken by itself, as

$$\Delta y^{SC}_{ferm}|_{m_t=180 GeV} = -7.79 \times 10^{-3}.1$$

(7)

The bosonic contribution, on the other hand, with

$$\Delta y^{SC}_{bos} = 11.1 \times 10^{-3},$$

(8)

1The dependence on the mass of the top quark, $m_t$, is practically negligible, as $\Delta y^{SC}_{ferm}|_{m_t \rightarrow \infty} = -\frac{3\alpha(M_Z^2)}{4\pi \alpha_s} \approx -8.01 \times 10^{-3}$. 

5
leads to a decrease of $g_{W\pm}$. The overall correction,

$$\Delta y^{SC} = 3.3 \times 10^{-3}, \quad (9)$$

which is practically independent of the top quark and Higgs boson masses, thus results in a decrease of the coupling $g_{W\pm}(M^2_{W\pm})$ in (9) relative to the low-energy value, $g^2_{W\pm}(0) \equiv 4\sqrt{2}G_\mu M^2_{W\pm}$. Finally, we have to specify the electromagnetic coupling, $e$, in (9), which is given by [9]

$$\left(\frac{e^2}{4\pi}\right)^{-1} = \alpha^{-1}(M^2_Z) = 128.89 \pm 0.09. \quad (10)$$

Improving the Born approximation (1) by Coulomb corrections and initial state radiation [6], the differential cross section for $W^{\pm}$ pair production in $e^+e^-$ annihilation becomes [5]

$$\left(\frac{d\sigma}{d\Omega}\right)_{IBA} = \frac{\beta}{64\pi^2 s} \left[ \frac{2\sqrt{2}G_\mu M^2_{W\pm}}{1 + \Delta y^{SC} \mathcal{M}_I^2 \delta \kappa - 4\pi\alpha(M^2_Z) \mathcal{M}_Q^2} + \left(\frac{d\sigma}{d\Omega}\right)_{Coul} (1 - \beta^2)^2 + \left(\frac{d\sigma}{d\Omega}\right)_{ISR} \right]. \quad (11)$$

We note that the weak interaction term in (11) is dominant relative to the electromagnetic one. Accordingly, the effect of choosing the appropriate high-energy scale in $g_{W\pm}$ is most important and may easily be estimated. Neglecting $\mathcal{M}_Q$, the decrease of the normalized cross section due to $\Delta y^{SC} \neq 0$,

$$\delta \Delta_{IBA} = \left(\frac{d\sigma}{d\Omega}\right)_{IBA}(\Delta y^{SC} \neq 0) - \left(\frac{d\sigma}{d\Omega}\right)_{IBA}(\Delta y^{SC} = 0), \quad (12)$$

becomes

$$\delta \Delta_{IBA} \simeq -2\Delta y^{SC} = -0.66\%. \quad (13)$$

The quality of the improved Born approximation (11) is obtained by adding $\delta \Delta_{IBA}$ to the quantity $\Delta_{IBA}$, which results from comparing (11) for $\Delta y^{SC} = 0$ with the full one-loop results [6,7,10]. The detailed numerical results results in Table 1 [5] indeed show that the improved Born approximation with inclusion of $\Delta y^{SC} \neq 0$ yields an accuracy below 1 %, which is sufficiently accurate for all practical purposes.

We finally comment on the significance of the appropriate choice of the high-energy scale in the weak coupling, $g^2_{W}(M^2_W)$, with respect to recent one-loop calculations [11] which incorporate the decay of the $W^{\pm}$ into 4 fermions in a gauge-invariant formulation. These calculations take into account fermion-loops only. While interesting as a first step towards a full one-loop evaluation of $e^+e^- \to 4$ fermions, the numerical results of a calculation including fermion loops only can easily be estimated within the present framework of stable $W^{\pm}$ to enlarge the cross section appreciably. In fact, taking into account fermion loops only, the estimate (13) changes sign and becomes

$$\delta \Delta_{IBA}|_{ferm} \simeq -2\Delta y^{SC}_{ferm, m_t=180GeV} \simeq +1.56\%, \quad (14)$$
| angle   | unpolarized |                      | left-handed |                      |
|---------|-------------|----------------------|-------------|----------------------|
|         | $\Delta_{IBA}$ | $\delta\Delta_{IBA}$ | $\Delta_{IBA} + \delta\Delta_{IBA}$ | $\Delta_{IBA}$ | $\delta\Delta_{IBA}$ | $\Delta_{IBA} + \delta\Delta_{IBA}$ |
|         |              |                      |             |                      |
| $\sqrt{s} = 161$ GeV |              |                      |             |                      |
| total   | 1.45        | -0.72                | 0.73        | 1.45                 | -0.72                | 0.73                     |
| 10      | 1.63        | -0.73                | 0.90        | 1.63                 | -0.73                | 0.90                     |
| 90      | 1.44        | -0.72                | 0.72        | 1.44                 | -0.72                | 0.72                     |
| 170     | 1.26        | -0.70                | 0.56        | 1.26                 | -0.70                | 0.56                     |
| $\sqrt{s} = 165$ GeV |              |                      |             |                      |
| total   | 1.27        | -0.71                | 0.56        | 1.28                 | -0.71                | 0.57                     |
| 10      | 1.67        | -0.74                | 0.93        | 1.67                 | -0.74                | 0.93                     |
| 90      | 1.17        | -0.71                | 0.46        | 1.18                 | -0.71                | 0.47                     |
| 170     | 0.75        | -0.67                | 0.08        | 0.77                 | -0.67                | 0.10                     |
| $\sqrt{s} = 175$ GeV |              |                      |             |                      |
| total   | 1.26        | -0.71                | 0.55        | 1.28                 | -0.71                | 0.57                     |
| 10      | 1.71        | -0.75                | 0.96        | 1.71                 | -0.75                | 0.96                     |
| 90      | 1.03        | -0.69                | 0.34        | 1.06                 | -0.70                | 0.36                     |
| 170     | 0.59        | -0.62                | -0.03       | 0.69                 | -0.63                | 0.06                     |
| $\sqrt{s} = 184$ GeV |              |                      |             |                      |
| total   | 1.02        | -0.70                | 0.32        | 1.06                 | -0.71                | 0.35                     |
| 10      | 1.57        | -0.75                | 0.82        | 1.57                 | -0.75                | 0.82                     |
| 90      | 0.67        | -0.68                | -0.01       | 0.72                 | -0.69                | 0.03                     |
| 170     | 0.10        | -0.58                | -0.48       | 0.32                 | -0.64                | -0.32                    |
| $\sqrt{s} = 190$ GeV |              |                      |             |                      |
| total   | 1.24        | -0.70                | 0.54        | 1.28                 | -0.71                | 0.57                     |
| 10      | 1.67        | -0.74                | 0.93        | 1.67                 | -0.75                | 0.92                     |
| 90      | 0.95        | -0.68                | 0.27        | 1.01                 | -0.69                | 0.32                     |
| 170     | 0.58        | -0.57                | 0.01        | 0.83                 | -0.59                | 0.24                     |
| $\sqrt{s} = 205$ GeV |              |                      |             |                      |
| total   | 1.60        | -0.70                | 0.90        | 1.65                 | -0.71                | 0.94                     |
| 10      | 1.77        | -0.74                | 1.03        | 1.77                 | -0.74                | 1.03                     |
| 90      | 1.55        | -0.66                | 0.89        | 1.64                 | -0.68                | 0.96                     |
| 170     | 1.61        | -0.53                | 1.08        | 1.94                 | -0.56                | 1.38                     |

Table 1: The Table shows the quality of the improved Born approximation (IBA) for the total (defined by integrating over $10^0 \lesssim \vartheta \lesssim 170^0$) and the differential cross section (for $W^-$-production angles $\vartheta$ of $10^0$, $90^0$ and $170^0$) for $e^+e^- \rightarrow W^+W^-$ at various energies for unpolarized and left-handed electrons. The final percentage deviation, $\Delta_{IBA} + \delta\Delta_{IBA}$, of the IBA from the full one-loop result is obtained by adding the correction $\delta\Delta_{IBA}$ resulting from using the appropriate high energy scale in the SU(2) coupling, to the percentage deviation, $\Delta_{IBA}$, based on using the low-energy scale in the SU(2) coupling, i.e. $\Delta g^{SC} = 0$. (From [5])
and the total deviation from the full one-loop results (using $\Delta_{IBA} \simeq 1.2\%$ from Table 1) rises to values of

$$\Delta_{IBA} + \delta \Delta_{IBA}|_{\text{ferm}} \simeq 2.8\%.$$  \hspace{1cm} (15)

Accordingly, results from fermion-loop calculations including the decay of the $W^\pm$ are expected to overestimate the cross section by almost 3 \% relative to the (so far unknown) outcome of a calculation of $e^+e^- \rightarrow 4$ fermions including bosonic loops as well. It is gratifying, that a simple procedure immediately suggests itself for improving the large discrepancy (15). One simply has to approximate the bosonic loop corrections by using the substitution

$$G_\mu \rightarrow G_\mu/(1 + \Delta y_{bos}^{SC})$$  \hspace{1cm} (16)

with $\Delta y_{bos}^{SC} = 11.1 \times 10^{-3}$ in the four-fermion production amplitudes. Substitution (16) practically amounts to using $g_{W^\pm}(M_W^2)$ in four-fermion production as well. With substitution (16), it is indeed to be expected that the deviation of four-fermion production in the fermion-loop scheme will be diminished from the above estimated value of $\simeq 2.8\%$ to a value below 1 \%.

In conclusion, evaluating the Born approximation for $e^+e^- \rightarrow W^+W^-$ with the SU(2) gauge coupling and the electromagnetic coupling at the appropriate high-energy scale, and supplementing with Coulomb corrections and initial state radiation, yields differential cross sections which are sufficiently accurate for all practical purposes at LEP2.

Calculations of $e^+e^- \rightarrow 4$ fermions in the fermion-loop scheme overestimate the true cross section by a non-negligible amount, unless bosonic loops are globally included by the appropriate choice of the SU(2) gauge coupling. And last not least, first experimental results from LEP2 gave direct experimental evidence for a non-vanishing trilinear coupling among the vector bosons characteristic for the non-Abelian structure of the $SU(2) \times U(1)$ electroweak theory.

**Acknowledgement**

It is a pleasure to thank Masaaki Kuroda and Ingolf Kuss for collaboration on the subject matter of the present paper and Stefan Dittmaier for useful discussions. The stimulating atmosphere and the magnificent hospitality in Ouranoupolis, Greece, at the occasion of the European network meeting organized by Argyris Nicolaidis, is gratefully acknowledged.
References

1. M. Bilenky, J.L. Kneur, F.M. Renard and D. Schildknecht, Nucl. Phys. B409 (1993) 22; Nucl. Phys. B419 (1994) 240.

2. I. Kuss and D. Schildknecht, Phys. Lett. B383 (1996) 470.

3. The L3 Collaboration, CERN-PPE/97/98 (July 1997), submitted to Phys. Lett. B

4. J. Maalampi, D. Schildknecht and K.H. Schwarz, Phys. Lett. B166 (1986) 361; M. Kuroda, J. Maalampi, K.H. Schwarz and D. Schildknecht, Nucl. Phys. B284 (1987) 271; M. Kuroda, J. Maalampi, D. Schildknecht and K.H. Schwarz, Phys. Lett B190 (1987) 217.

5. M. Kuroda, I. Kuss and D. Schildknecht, BI-TP 97/15, hep-ph/9705294, to appear in Phys. Lett. B

6. M. Böhm, A. Denner and S. Dittmaier, Nucl. Phys. B376 (1992) 443; err. ibid. B391 (1993) 483.

7. W. Beenakker et al., in Physics at LEP2, eds. G. Altarelli, T. Sjöstrand, F. Zwirner, CERN 96-01 Vol. 1, p. 79, hep-ph/9602351.

8. S. Dittmaier, D. Schildknecht and G. Weiglein, Nucl. Phys. B465 (1996) 3.

9. H. Burkhardt and B. Pietrzyk, Phys. Lett. B356 (1995) 398; S. Eidelman and F. Jegerlehner, Z. Phys. C67 (1995) 585.

10. S. Dittmaier, Talk at the 3rd International Symposium on Radiative Corrections Cracow, Poland, August 1996, hep-ph/9610522, Acta Physica Polonica B28 (1997) 619.

11. W. Beenakker et al., hep-ph/9612260, NIKHEF 96-031, PSI-PR-96-41.