Reactive Power Pricing Model Considering the Randomness of Wind Power Output

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Abstract. With the increase of wind power capacity integrated into grid, the influence of the randomness of wind power output on the reactive power distribution of grid is gradually highlighted. Meanwhile, the power market reform puts forward higher requirements for reasonable pricing of reactive power service. Based on it, the article combined the optimal power flow model considering wind power randomness with integrated cost allocation method to price reactive power. Meanwhile, considering the advantages and disadvantages of the present cost allocation method and marginal cost pricing, an integrated cost allocation method based on optimal power flow tracing is proposed. The model realized the optimal power flow distribution of reactive power with the minimal integrated cost and wind power integration, under the premise of guaranteeing the balance of reactive power pricing. Finally, through the analysis of multi-scenario calculation examples and the stochastic simulation of wind power outputs, the article compared the results of the model pricing and the marginal cost pricing, which proved that the model is accurate and effective.

Keywords. Reactive power pricing; wind power output model; optimal power flow calculation; integrated cost allocation method CLC number: TM315 Document code: A

1. Introduction
With the continuous development of renewable energy, wind power plays an important role in the global power generation capacity depending on its proven technique and commercialized development prospect. The value of reactive power service is recovered mainly by means of user power factor or one-part tariff. Economic signals cannot be transferred based on the demand of reactive power, so the reactive power value cannot be reflected. Therefore, in the future open electricity market of wind power integration with high permeability, how to comprehensively consider the characteristics of wind power generators, embody the feature and value of reactive power service and reasonably price reactive power is of great significance in the development of the future reactive power auxiliary market in our country.

The integrated cost allocation method based on optimal power flow tracing is proposed in the article after the advantages and disadvantages of the two methods above are integrated, and the reasonable allocation of reactive power cost is realized in accordance with the principle of power flow tracing on the premise of guaranteeing the minimum total cost of reactive power supply of the system and the
balance of payments. Meanwhile, due to mutually independent studies on wind power integration and reactive power pricing by the predecessors, the optimal power flow calculation model considering wind power output randomness is proposed in the article. The model takes the total cost of reactive power supply as the objective function, wind driven generator as PQ node joining constraint condition, wind speed as the variable of wind power output to calculate optimal power flow based on repeatedly stochastic simulation of wind power output, eventually pricing reactive power in combination with the integrated cost allocation method.

2. Establishment of Wind Power Model

According to the tested wind data in our country, the probability distribution of annual average wind speed in the most regions of our country basically conforms to Weibull distribution:

\[ f(v) = \frac{k}{c} \left( \frac{v}{c} \right)^{k-1} \exp \left(-\left( \frac{v}{c} \right)^k \right) \]

When the electrical loss and the wake effect of the wind power plant are ignored, the active power of the wind driven generator is mainly determined by the wind speed of fan wheel hub height, and the output active power and wind speed approximately accord with piecewise linear function.

\[
P_w = \begin{cases} 
0 & v \leq v_{ci} \\
kv + k_2 & v_{ci} < v < v_r \\
P_c & v_r \leq v < v_{co} \\
0 & v \geq v_{co} 
\end{cases}
\]  

(1)

\[
k_i = \frac{P_c}{v_r - v_{ci}}
\]

(2)

\[
k_2 = -k_1
\]

(3)

Where: \( P_c \) - rated power of wind driven generator;

\( v_{ci}, v_r, v_{co} \) - cut-in wind speed, rated wind speed, and cut-out wind speed.

When the reactive power of wind power is calculated, considering that asynchronous generator absorbs reactive power from the system to establish a magnetic field and the absorbed reactive power size is closely related to the output active power and terminal voltage, the expression of output reactive power is derived from the structure of the asynchronous generator.

![Figure 1. Simplified equivalent circuit diagram of asynchronous generator](image-url)
As output active power $P_e$ is generally known, output reactive power can be acquired by bringing the formula above into Formula (6):

$$Q_e = -\frac{U^2}{x_m} P_e x_k$$

Acquired from Formulas (4)-(6):

$$Z = jx_m//\left(\frac{r_2 + jx_i}{s}\right) = \frac{s r_2 x_m^2 + j\left(\frac{r_2^2 x_m + s^2 x_k^2 x_m + s^4 x_k x_m^2}{r_2^2 + s^2 (x_k + x_m)^2}\right)}{r_2^2 + s^2 (x_k + x_m)^2}$$

$$P_e = -\frac{U^2}{(r_2/s)^2 + x_k^2}$$

$$Q_e = \frac{U^2}{x_m} \frac{P_e x_k}{r_2/s}$$

$$= \frac{-r_2^2 + s^2 x_k x_m + s^2 x_k x_m^2}{s r_2 x_m}$$

$$s = \frac{U^2 r_2^2 - \sqrt{U^4 r_2^4 - 4 P_e^2 x_k^2 r_2^2}}{2 P_e x_k^2}$$

When the optimal power flow for a certain time is calculated, it can be approximately thought that wind power output is determined by the wind speed at the moment. Meanwhile, to simplify the calculation model of optimal power flow, it is assumed in the article that the unified type and capacity of fan are adopted in the wind power plant, and the electrical loss and wake effect of the plant are ignored.

3. Calculation Model of Optimal Power Flow Considering Wind Power Randomness

The problems such as minimum active power cost and electricity purchasing cost, etc. are mainly considered in the optimal power flow distribution in the references of the predecessors. The article takes the minimum total cost of reactive power supply as the target to reflect the importance of reactive power service in the system. On account of wind power generation consuming natural wind only rather than fossil energy, all wind power shall be first used in dispatching to reduce the use of fossil energy. Therefore, the consumption cost of the conventional power units and the minimum use cost of reactive compensation equipment are only considered in the calculated objective function of optimal power flow instead of the cost of wind power generator. Wind power randomness is reflected in constraint in the form of PQ node.

1. Objective function:

$$\min F = \sum_{i=1}^{N} \left[ C_{vgi}(P_{gi}) + C_{vgi}(Q_{gi}) + C_{vgi}(Q_{i}) \right]$$
In the formula above, \( C_{gpi}(P_i), C_{gqi}(Q_i), C_{cqi}(Q_{ci}) \) are respectively active cost function, reactive cost function of generator connected by Node i and cost function of reactive compensation equipment connected by Node i, as shown in Formula (8).

\[
\begin{align*}
C_{gpi} P_i &= a_i P_i^2 + b_i P_i + c_i \\
C_{gqi} Q_i &= k \left[ C_{gpi} S_{Gi, \max} - C_{gqi} \sqrt{S_{Gi, \max}^2 - Q_{Gi}^2} \right] \\
C_{cqi} Q_{ci} &= \frac{e \times Q_{ci}}{y \times 365 \times 24 \times f_i}
\end{align*}
\]  

(8)

Where, \( a_i, b_i, c_i \) are constants; \( k \) is generally taken as 5%-10%; \( y \) is generally taken as 10-15 years, \( \varepsilon \) is approximately taken as \( 2/3 \);

2. Constraint condition:

\[
P_{Gi} - P_{Li} + P_{WIND_i} \sum_{j=1}^{N} U_j (G_j \cos \theta_j + B_j \sin \theta_j) = 0
\]

\[
Q_{Gi} - Q_{Li} - Q_{WIND_i} \sum_{j=1}^{N} U_j (G_j \sin \theta_j - B_j \cos \theta_j) = 0
\]

\[
P_{WIND_i} \leq P_{WIND_i} \leq P_{WIND_i}
\]

\[
Q_{WIND_i} \leq Q_{WIND_i} \leq Q_{WIND_i}
\]

\[
\begin{align*}
P_{Gi} & \leq P_{Gi} \leq P_{Gi} \\
Q_{Gi} & \leq Q_{Gi} \leq Q_{Gi} \\
Q_{Ci} & \leq Q_{Ci} \leq Q_{Ci} \\
U_i & \leq U_i \leq U_i \\
\theta_j & \leq \theta_j \leq \theta_j
\end{align*}
\]  

(11)

\[
P_j \leq |P_j| \leq P_j
\]

\[
Q_j \leq |Q_j| \leq Q_j
\]

(12)

Where: \( P_{Gi}, Q_{Gi} \) - active power and reactive power of generator connected by Node i, respectively;
\( P_{WIND_i}, Q_{WIND_i} \) - active power and reactive power of wind driven generator connected by Node i, respectively;
\( P_{Li}, Q_{Li} \) - active power and reactive power of load connected by Node i, respectively;
\( U_i \) - Voltage amplitude value of Node i;
\( \theta_{ij}, P_{ij}, Q_{ij} \) - phase angle difference between heads and ends of circuit \( ij \), transmitted active and reactive power on the circuit;

\[ \underline{\mathbf{\xi}}, \underline{\mathbf{\eta}} \] - upper limit value, lower limit value of corresponding variable;

Formula (10) is active and reactive power equilibrium constraint, Formula (210) is active and reactive peak value constraint of wind power output, Formula (311) is active and reactive output peak value constraint of generator; voltage amplitude value and phase angle constraint of generation price and compensation constraint of reactive compensation equipment; Formula (412) is transmitted active and reactive constraint.

Due to wind power output having randomness, the article simulates wind power output at random based on Monte Carlo method, and makes different optimal power flow calculations for various wind power outputs, thereby acquiring the systematic power flow distribution of minimum reactive power cost.

4. Integrated cost Allocation Method Based on Optimal Power Flow Tracing

The calculation of optimal power flow ensures the minimum integrated cost of reactive power in the system. We can obtain the source of reactive power in every node based on optimal power flow tracing, consequently achieving reasonable cost allocation in accordance with the power share used by loads.

4.1. Power Flow Tracing Theory

The article traces reactive power and determines cost allocation coefficient with the downstream tracing method based on the principle of proportional distribution.

The downstream power matrix \( \mathbf{S}^s \) is defined:

\[
\mathbf{S}^s = \begin{cases} 0 & i \neq j, S_{ij} \geq 0 \\ S_{ij} & i \neq j, S_{ij} < 0 \\ S_i & i = j \end{cases}
\]  

\( (13) \)

Where:

\( S_{ij} \) - power of Node \( i \) on the circuit branch \( ij \) flowing to Node \( j \) after the calculation of power flow;

\( S_i \) - total injection power of Node \( i \);

As the node power is equal to the sum of branch power of every inflow node (or outflow node), \( \mathbf{S}^s \) is the main diagonally-dominant matrix.

The column vector \( b_{j}^{\text{max}} \) of proportionality factor of Node \( i \) and the column vector \( S_{j}^{\text{max}} \) of source power are defined.

\[
b_{j}^{\text{max}} = \left[ b_{1j}^{\text{max}}, b_{2j}^{\text{max}}, \ldots, b_{ij}^{\text{max}}, \ldots, b_{n}^{\text{max}} \right]^T
\]

\( (14) \)

\[
B_{i}^{\text{max}} = \left[ b_{1i}^{\text{max}}, b_{2i}^{\text{max}}, \ldots, b_{ji}^{\text{max}}, \ldots, b_{ni}^{\text{max}} \right]^T
\]

\( (15) \)

\[
S_{j}^{\text{max}} = \left[ 0, \ldots, S_{Gj}, \ldots, 0 \right]^T
\]

\( (16) \)

\[
S^{\text{max}} = \text{diag} \left[ S_{G1}, S_{G2}, \ldots, S_{Gn} \right]
\]

\( (17) \)

Where:

\( n \) - Number of nodes in the system;
Specific gravity of power generated by power supply on Node i in the load power of Node j;
- $S_{i,j}$ - Sum of power generated by power supply on Node i and the power of injected node i in every branch, i.e. injection power of Node i;
- $S_{\text{man}}$ - $n$ order diagonal matrix, diagonal element is $S_{Gi}$.

For single power supply i, the proportionality coefficient $b_{i,\text{man}}$ of the generated power allocated to every node load is:

$$b_{i,\text{man}} = \left(S^S\right)^{-1} S_{\text{man}}$$  \hspace{1cm} (18)

When there are $n$ power supplies in the system, the coefficient matrix of the generated power allocated to every node is:

$$B_{i,\text{man}} = \left(S^S\right)^{-1} S_{\text{man}}$$  \hspace{1cm} (19)

After the power flow distribution is determined, the power flow and power supply output of every branch are also determined, thereby acquiring $S^S$ and $S_{\text{man}}$ matrix. The allocation matrix (cost allocation coefficient matrix) $B_{i,\text{man}}$ of every power supply output on the $n$ nodes can be acquired from the formula above.

### 4.2. Reactive Power Pricing Process Based on Cost Allocation Coefficient Matrix

After the calculation of optimal power flow considering wind power, the reactive power price of every reactive power load node can be obtained according to the reactive power cost function and power flow distribution of every reactive power supply. The steps are as follows:

1. Calculation of optimal power flow considering wind power;
2. Downstream power matrix is $S^S$ written in Formula (13) according to the power value of every circuit;
3. According to the output of generator (including wind driven generator) and reactive compensation equipment connected by every node, source power matrix $S_{\text{man}}$ is written;
4. Cost allocation coefficient matrix $B_{i,\text{man}}$ is obtained according to $B_{i,\text{man}} = \left(S^S\right)^{-1} S_{\text{man}}$;
5. According to the cost function of reactive power, the total reactive costs $W = [W_1, W_2, \ldots, W_c]^T$ of generator and reactive compensation equipment connected by every node are calculated respectively, in which $W_i$ is the sum of reactive power cost of generator and reactive compensation equipment connected on Node $i$;
6. Column vector of reactive power cost undertaken by reactive load of every node obtained based on $W_i = B_{i,\text{man}}W = [W_{i1}, W_{i2}, \ldots, W_{ic}]^T$, in which $W_{li}$ is reactive power cost undertaken by reactive power load of Node $i$;
7. Average reactive power price of Node $i$ obtained according to $Q_{\text{price}} = \frac{W_{li}}{Q_i}$, in which $Q_i$ is reactive power load of Node $i$.

### 5. Conclusion

The reactive power pricing method of randomness considering fan output is studied in the article through the combination of wind power grid-connected model with reactive power pricing based on power flow tracing. Firstly, the calculation of optimal power flow is made to the access to asynchronous generator. The minimum reactive power cost is taken as a target, and the randomness of wind power output is
added in the constraint calculation, improving the authenticity and accuracy of optimal power flow calculation model. Secondly, the integrated cost allocation method based on power flow tracing principle is adopted to price reactive power, thereby avoiding the disadvantage of payment imbalance of real-time power pricing.

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