Hierarchical Summarization of Metric Changes

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ABSTRACT
We study changes in metrics that are defined on a cartesian product of trees. Such metrics occur naturally in many practical applications, where a global metric (such as revenue) can be broken down along several hierarchical dimensions (such as location, gender, etc).

Given a change in a metric, our goal is to identify a small set of non-overlapping data segments that account for the change. An organization interested in improving the metric can then focus their attention on these data segments.

Our key contribution is an algorithm that mimics the operation of a hierarchical organization of analysts. The algorithm has been successfully applied, for example within Google Adwords to help advertisers triage the performance of their advertising campaigns.

We show that the algorithm is optimal for two dimensions, and has an approximation ratio $\log^{d-2}(n+1)$ for $d \geq 3$ dimensions, where $n$ is the number of input data segments. For the Adwords application, we can show that our algorithm is in fact a 2-approximation.

Mathematically, we identify a certain data pattern called a conflict that both guides the design of the algorithm, and plays a central role in the hardness results. We use these conflicts to both derive a lower bound of $1.144^{d-2}$ (again $d \geq 3$) for our algorithm, and to show that the problem is NP-hard, justifying the focus on approximation.

1. MOTIVATION
Organizations use metrics to track, analyze, and improve the performance of their businesses. The organization might be a company, a government, an advertiser or a website developer. And the metric might be the revenue of the company, the level of unemployment in a country, or the number of clicks for an online advertising campaign. Indeed, our interest in this problem stems from creating tools that help analysts reason about Google’s revenue, and to help Google’s advertisers reason about the performance of their advertising campaigns.

Metrics vary because of changes in the business environment. One common task of a data scientist is to determine what drives changes in a metric over time. In particular, they want to identify the segments of the business where the change is most pronounced. This helps decision-makers within the organization to counter these changes if they are negative, or amplify the changes if they are positive.

EXAMPLE 1. Consider a government data scientist analyzing an increase in unemployment. She does this by comparing the current month’s employment data to the previous month’s data to figure out what caused the increase.

The domain of this data - the employment market - can be sliced along many dimensions, such as geography, industry sector, demographics, etc. into a very large collection of granular submarkets, each of which has its own variation in employment. Naturally, this analysis proceeds in two steps: 1) The summarization step: Identify a small set of submarkets which account for a majority of the variation in overall unemployment. 2) Design fixes for negative trends. This second step is most often manually intensive and case specific. Therefore, one hopes that the first step narrows focus meaningfully.

It is commonly observed that hierarchical data lends itself naturally to summarization (cf. OLAP [13]). For instance, the geography dimension in the above example has a natural hierarchy: metros, states, countries, and so on. If all the metros in a state have similar unemployment trends, it is more concise to report the state slice as an output of the summary rather than each metro separately. Industry sectors and demographics also have similar hierarchical representations. Organizations are similarly structured hierarchically. The hierarchies aid the allocation, coordination and supervision of tasks that are intended to improve organizational metrics (cf. Organizational Theory [23]). Just as hierarchies in the data inform step 1 from Example 1, hierarchies in the organization help effective delegation required to accomplish step 2. For instance, many companies have separate hierarchies for Sales, Finance and Product functions. Data scientists can help organize performance metrics and assign responsibilities for fixes to the right substructure of the right functional hierarchy.

When determining the submarkets that drive a metric change in step 1, it is important to avoid “double-counting”, which can happen if the resulting submarkets are not independent. This is a problem, since it is possible that the same drop in employment is visible in several overlapping slices of data. For instance, a regional slice (e.g. California), and...
an industry sector (e.g., construction), may both account for the same underlying change in unemployment. Double-counting causes several conceptual problems. It might prevent responsibilities for a fix from being clearly assigned, it might lead to duplicated efforts from two different parts of an organization, or it might lead to the illusion of greater progress than was actually made. For instance, if the construction division and the California division both work on the same issue, or both take credit for a fix, then this credit assignment does not add up. We will therefore insist that the list of submarkets from step 1 are non-overlapping.

**Informal problem statement:** Identify a small list of non-overlapping sub-segments that account for the majority of a variation in a key metric, where the space of candidate sub-segments is determined by a few, hierarchical dimensions.

As we discuss in Section 7 in greater detail, there are several formulations of the “drill-down” problem. Most of these formulations attempt to summarize patterns in the data. They use a combination of information-theoretic models and input from a human analyst to do so. In contrast, we seek to summarize the source of a high-level change in a metric. As we will show, this problem is more amenable to nearly complete automation. Our model is inspired directly by the excellent models of Fagin et al [14] and Sarawagi [26]. Unfortunately, these papers used lattices to model the hierarchical organization of analysts. As we discuss in Section 6, we have applied this algorithm within large decision support products. One of these products helps advertisers reason about their advertising spend, and the other helps website developers reason about their traffic.

## 2. PROBLEM STATEMENT

With the motivation from the previous section, we are ready to state our model and problem formally.

### 2.1 Definitions

We consider multi-dimensional datasets where each dimension can be modeled as a rooted tree. In a single (rooted) tree $T$, we say that two tree-nodes $p$ and $q$ overlap if either $p = q$ or they share an ancestor-descendant relationship, otherwise they are called non-overlapping.

We extend this definition to a cartesian product of trees. For a product of trees $P = T_1 \times T_2 \times \cdots \times T_d$, we say that two nodes $p = (p_1, \ldots, p_d)$ and $q = (q_1, \ldots, q_d)$ overlap if for every dimension $i$, the tree-nodes $p_i$ and $q_i$ overlap. Consequently, if a pair of nodes does not overlap, then there exists a (possibly non-unique) dimension $i$ such that the tree-nodes $p_i$ and $q_i$ do not overlap, and we say that nodes $p$ and $q$ do not overlap along dimension $i$.

A node $p = (p_1, \ldots, p_d)$ is in the subspace of a node $q = (q_1, \ldots, q_d)$ if for every $i$, $p_i$ is either a descendant of $q_i$ in the tree $T_i$, or $p_i = q_i$. We define $Sub(v)$ to be the set of nodes that are in the subspace of $v$. We denote the root of the product of the trees as $r = (r_1, \ldots, r_d)$, where each $r_i$ is the root of $T_i$.

Finally, a set $S$ of nodes is overlap-free if no two nodes in $S$ overlap.

**Example 2.** Figure 1 depicts the cartesian product of two trees $T_1$ and $T_2$, each of depth 2. Each node is a pair of tree nodes. The solid lines are edges along $T_2$ and the dotted lines are edges along $T_1$.

![Figure 1: The cartesian product of two trees $T_1$ and $T_2$, each of depth 2. Each node is a pair of tree nodes. The solid lines are edges along $T_2$ and the dotted lines are edges along $T_1$.](image)

### 2.2 Formal Problem Statement

With these definitions we can now formally state our problem.

**SUMMARIZE**

**Input:** Trees $T_1, T_2, \ldots, T_d$ with the set of tree-nodes $V := T_1 \times T_2 \times \cdots \times T_d$, a non-negative weight function $w : V \rightarrow \mathbb{R}^+$, and a maximum output size $k$.

**Output:** Subset $S \subseteq V$, such that $|S| \leq k$, $S$ is overlap-free, and $w(S) := \sum_{s \in S} w(s)$ is maximal under these restrictions.

The main parameters of this problem are the number of dimensions $d$, and $n := |V|$, the size of the input set. We express hardness and runtime results in terms of these two variables.

### 2.3 Modeling Metric Changes

Let us briefly discuss how our problem definition relates to the summarization of metric changes. Assume that we are comparing metrics for two points in time, and that the value of the metric is defined for every leaf node in Figure 1. Via aggregation, this defines the metric values for every internal node as well.

Let us define the weight function $w$ to be the absolute value of the difference in the metric values for the two time points. (We discuss refinements to this weight function in Section 6.1.)

Why does this weight function result in effective summarization? Consider two patterns of metric changes.

1. Two children of some node change their metrics in the same direction. Then the node’s weight is the sum of children’s weights.
2. Two children of some node change their metrics in different directions. Then the node’s weight is smaller in magnitude than one of the children, possibly both.

In the first case, it is better to pick the node rather than the children. In the second case, it is better to pick one or both children rather than the parent. Further, notice that nodes may have multiple parents in different dimensions. So a given input may exhibit both patterns simultaneously, and then it is advantageous to summarize along certain dimensions.

To make this concrete, consider the topology in Figure 1. Suppose that \( k = 2 \) in the above definition. First let us suppose that \( (a_1, a_2) \) and \( (b_1, a_2) \) both go up by some quantity \( x \), while \( (a_1, b_2) \) and \( (b_1, b_2) \) fall by the same quantity \( x \). In essence, the change is along dimension 2.

Notice that the pair \( (a_1, a_2) \) and the pair \( (a_1, b_2) \) and \( (b_1, b_2) \) follow pattern 2. Computing the weights shows us that the optimal solution is the overlap-free set consisting of the nodes \( (r_1, a_2) \) and \( (r_1, b_2) \), reflecting the change along dimension 2. Each of these nodes has a weight of \( 2x \). The other two internal nodes and the root all have a weight of 0.

3. THE CASCADING ANALYSTS ALGORITHM

As we will discuss in Section 3, Summarize cannot be solved optimally in polynomial time unless P=NP. Therefore we will now attempt to identify a good approximation algorithm for Summarize. In this section, we describe the “Cascading Analysts” algorithm that achieves that goal.

3.1 Conflicts

Our algorithm will achieve an optimal solution for a more restricted version of Summarize, namely where the solution is additionally required to be conflict-free.

The presence of a conflict prevents a set of nodes from being recursively subdivided one dimension at a time, even though the set of nodes is possibly overlap-free. This definition and the example that follows elaborate.

**Definition 3.** A conflict is a set \( C \subseteq V \) such that for every dimension \( i \) there is a \( (c_1, \ldots, c_k) \in C \) such that for all \( (x_1, \ldots, x_d) \in C \), \( x_i \) is a descendant of \( c_i \), or \( x_i = c_i \).

A conflict can be overlap-free. Here is the simplest example of a conflict that is also overlap-free.

**Example 4.** Consider three trees \( T_1, T_2, T_3 \), each of height two, and each with two leaves. Tree \( T_1 \) has root \( r_1 \), left and right children \( a_1 \) and \( b_1 \) respectively. The conflict is defined by the set of nodes consisting of \( (r_1, a_2, b_2) \), \( (a_1, b_2, b_3) \) and \( (b_1, a_2, b_3) \).

Conflicts play a central role in both the positive and the negative results in our paper. For instance, our algorithm will find the optimal conflict-free solution, i.e., no subset of the nodes output by our algorithm contain a conflict. But it will only be approximate for Summarize: given the inputs in Example 4, our algorithm will only output two of the three nodes, even though the three nodes do not overlap. But we will show that the Summarize problem is NP-hard even over problem instances of three trees of height two like the one in Example 4 except with many more leaves. On the other hand, the optimal conflict-free solution can be found by recursively subdividing the product space of trees as we will show next. Each substep of our algorithm resembles a standard drill-down that an analyst would perform.

The more visually inclined may benefit from a geometric view of a conflict. Figure 2 depicts the conflict in Example 4. The three trees in Example 4 correspond to the three dimensions in the figure. The three nodes correspond to the three cuboids in the figure. Note that for every dimension, there is a cuboid that spans it completely, i.e., every other cuboid “overlaps” with it in this dimension, or in the words of the definition of a conflict, is a descendant of this cuboid in this dimension. This overlap prevents us from passing an axis aligned cutting plane through the space that would separate the cuboids without cutting at least one of them. This is the source of the approximation in our algorithm. Our algorithm will recursively perform axis aligned cuts and lose some of the nodes (cuboids) in the process. But as our hardness results in Section 3 show, these conflicts are exactly what make the problem inherently hard, so some of this loss is inevitable.

3.2 Algorithm

For each \( v \in P \), our algorithm computes subsets \( S(v, 0) \), \( S(v, 1) \), ..., \( S(v, k) \) such that for all \( j \in \{0, \ldots, k\} \): \( S(v, j) \subseteq \text{Sub}(v) \), \( |S(v, j)| \leq j \), \( S(v, j) \) is a conflict-free set, and \( S(v, j) \) is a maximum weight set satisfying these constraints. The set \( S(r, k) \) is the output of our algorithm, where we let \( r := (r_1, r_2, \ldots, r_d) \) is the root of the product space. The sets \( S(v, j) \) are computed bottom-up on the tree product space \( P := T_1 \times T_2 \times \cdots \times T_d \) via dynamic programming:

1. **Base case:** If \( v \) is a leaf, i.e. \( \text{Sub}(v) = \{v\} \), then we assign the sets in the “obvious” way: \( S(v, 0) := \emptyset \), and \( S(v, j) := \{v\} \) for \( j \geq 1 \).

2. **Recursive step:** If \( \text{Sub}(v) \neq \{v\} \), we proceed as follows. We let \( S(v, 0) := \emptyset \). For \( j \in \{1, \ldots, k\} \), we set \( S(v, j) \) as the maximum weight solution among these possibilities:

\[ S(v, j) := \max \left( \sum_{w \in S(v, j-1)} \sum_{v \in \text{Sub}(w)} S(v, j-1) \right) \]

\[ \text{subject to } \sum_{w \in S(v, j-1)} \sum_{v \in \text{Sub}(w)} S(v, j-1) = \text{weight}(v) \]

\[ \text{for all } v \in S(v, j-1) \]

\[ \text{and } S(v, j) \subseteq \text{Sub}(v) \]

\[ \text{and } |S(v, j)| \leq j \]

\[ \text{and } S(v, j) \text{ is conflict-free} \]

\[ \text{and } S(v, j) \text{ is a maximum weight set satisfying these constraints} \]

**This dynamic program can either be implemented on a single machine, or distributed using MapReduce.**
• The singleton set \{v\}.
• The \(d\) or fewer solutions \(S_i(v, j)\) that stem from repeating steps 2a and 2b below for those dimensions \(i\) along which \(v\) has children, i.e., \(v_i\) is not a leaf of \(T_i\).

2a. Breakdown along a dimension \(i\): Let \(C_i(v)\) be the set of children of \(v\) in dimension \(i\). So if \(v = (v_1, v_2, \ldots, v_d)\), then:

\[
C_i(v) = \{ (v_1, \ldots, v_{i-1}, c, v_{i+1}, \ldots, v_d) \mid c \text{ is child of } v_i \text{ in } T_i \}
\]

(This is the typical breakdown of the space that an “analyst” interested in dimension \(i\) would consider. The algorithm performs a recursive sequence of these, hence the name “Cascading Analysts”.)

We let the maximal solution \(S_i(v, j)\) along dimension \(i\) be the largest weight union of sets of its children \(S(c_i, j)\) where \(c_i \in C_i(v)\) (all \(c_i\) distinct) and \(\sum_{j} j \leq j\). Note that the number of sets \(S(c_i, j)\) can be anything from 0 to \(j\). This step can be accomplished using a simple dynamic program (not to be confused by the dynamic program over the tree structure) over the children. Here are the details:

2b. Combining child solutions: This simple dynamic program orders the children in \(C_i(v)\) in a sequence. Let \(C_i''(v)\) be the first \(m\) nodes in \(C_i(v)\) and \(c_m\) be the \(m\)-th child in this sequence. Mirroring the definition of \(S(v, j)\), let \(S(C_i''(v), j)\) be the optimal conflict-free solution of cardinality at most \(j\) in the subspace of the first \(m\) children of node \(v\). The base case is when \(m = 1\); here we set \(S(C_i''(v), j) = S(c_1, j)\). The recursive step is that we compute \(S(C_i''(v), j)\) by selecting the best union of \(S(C_i''(v), p)\) and \(S(c_m, q)\), where \(p + q \leq j\). Then, the optimal solution along dimension \(i\) is defined by \(S(C_i(v), j)\), where \(\ell = |C_i(v)|\).

Lemma 5. The Cascading Analysts algorithm will output a conflict-free set.

Proof. We prove this claim bottom-up, mirroring the structure of the algorithm. For the base case, when \(Sub(v) = \{v\}\), \(S(v, j)\) is either an empty set or a singleton set; both are conflict-free. When \(Sub(v) \neq \{v\}\), if \(S(v, j) = \{v\}\) then it is conflict-free by itself. Otherwise, there is a dimension \(i\) such that \(S(v, j)\) is the union of sets each of which is contained within the subspace of a child \(c \in C_i(v)\).

Suppose that there is a conflict \(Q\) in the union. Clearly \(Q\) cannot be contained entirely with the subspace of any child, because inductively, these sets are each conflict-free. So \(Q\) must span the subspace of at least two distinct children. But then \(Q\) cannot be conflict-free because there is no node \(q \in Q\) that is an ancestor along dimension \(i\) to nodes in the subspace of both these children, violating the condition from the definition of conflicts. So we have a contradiction.

Lemma 6. The Cascading Analysts algorithm will output a maximum weight overlap-free, conflict-free set.

Proof. We first show that the \(S(v, j)\) are overlap-free. This can be proved inductively. It is obviously true for leaves of the product space (where \(Sub(v) = \{v\}\)). When combining \(S(c_i, j)\) from different children \(c_i\) note that for elements of \(Sub(c_i)\) and \(Sub(c_j)\) have empty intersection in the dimension that we split on (\(i\) in the above description). Thus, their union is overlap-free.

Let \(S_C\) be a maximum weight, overlap-free, conflict-free solution for an instance of SUMMARIZE. We will show that the weight of the output of the Cascading Analysts algorithm is at least \(w(S_C)\). Since the cascading analysts algorithm outputs a conflict-free set (Lemma 5), this proves the lemma.

We show the following by induction: for all \(v, j\), the weight of \(S(v, j)\) is at least the weight of the maximum weight conflict-free subset of size \(j\) of \(Sub(v)\). Clearly this is true if \(v\) is a leaf, so we just need to consider the induction step.

Suppose \(w(S(v, j)) < w(C)\), where \(|C| = j\) and \(C \subseteq Sub(v)\) is a conflict-free set. Since \(C\) is conflict-free, there has to be a dimension \(d'\) such that \(\forall w \in C : w_{d'} \neq v_{d'}\). Let \(D\) be the children of \(v\) in dimension \(d'\). Then there are children \(c_1, \ldots, c_r \in D\) such that \(C = C_1 \cup \cdots \cup C_r\), where \(C_i \subseteq Sub(c_i)\).

We know that \(w(S(C_i, |C_i|)) \geq w(C_i)\) by induction hypothesis. The combination of these sets will be considered by the algorithm, thus \(w(S(v, j)) \geq \sum w(S(C_i, |C_i|)) \geq w(C_1) = w(C)\), contradicting our assumption that the lemma did not hold.

3.3 Running Time Analysis

There are \(n\) choices of \(v\) and \(k\) choices of \(j\) for which we need to compute the \(S(v, j)\). With standard techniques such as memoization, each of these needs to be touched only once. For a fixed \(v\), and fixed \(j\), we can combine the child solutions to form \(S(v, j)\) by a linear pass over the children (as in step 2a of the algorithm in Section 3.2). Each step in this pass incorporates an additional child and takes time \(O(j)\). Since a child can have at most \(d\) parents, the total cost of this step for a single node is \(O(dj)\). Noting that \(j \leq k\), this gives us a total runtime of \(O(ndk^2)\).

Remark 7. Note that the size of the input grows multiplicatively in the number of trees. For example, if each tree has size 10, then \(|V| = 10^d\). Even reading the input becomes impractical for \(d > 10\). Fortunately for us, there are compelling practical applications where \(d\) is fairly small, i.e., \(d \leq 5\). This is true of our applications in Section 4.

4. PERFORMANCE GUARANTEE

We first show that our algorithm is optimal for two dimensions, followed by an approximation guarantee of \(\left(\log_2(n+1)\right)^d - 2\) for the case of three or more dimensions.

Theorem 8. The Cascading Analysts algorithm solves SUMMARIZE optimally when \(d = 2\).

Proof. We show that when \(d = 2\), every overlap-free solution is also conflict-free. With Lemma 6 this concludes the proof.

All we have to argue is that if a set of nodes \(C\) constitutes a conflict, then it also contains an overlap. If \(C\) is a conflict, then there are two nodes \(x, y \in C, x = (x_1, x_2), y = (y_1, y_2)\), such that for all \((c_1, c_2) \in C\), \(x_1\) is an ancestor of \(c_1\) and \(y_2\) is an ancestor of \(c_2\). If \(x = y\) then this node overlaps with all other nodes in \(C\), completing the proof. If \(x \neq y\), then (as just stated) \(x_1\) is an ancestor of \(y_1\) and \(y_2\) is an ancestor of \(x_2\). Therefore \(x\) and \(y\) overlap, completing the proof.

Theorem 9. For \(d \geq 2\), let \(d\) trees \(T_1, \ldots, T_d\) have sizes \(n_1, \ldots, n_d\) respectively. The Cascading Analysts algorithm is a \(\left(\log_2(n+1)\right)^d - 2\)-approximation algorithm for such an instance of the SUMMARIZE problem, where \(m = \max n_i\).

Alternatively, The Cascading Analysts algorithm is a \(\left(\log_2(n+1)\right)^d - 2\)-approximation algorithm.
Proof. The second theorem statement is easily implied by the first because \( m \leq n \), so we now prove the first statement. Our proof is by induction over \( d \). The base case for \( d = 2 \) follows from Theorem 5, so for the following assume \( d > 2 \).

Given a tree, let an (ordered) path be a sequence of nodes \( v_1, \ldots, v_t \) in the tree where \( v_{i+1} \) is a child of \( v_i \) for each \( i \). We say that two paths \( p_1, p_2 \) overlap if some node \( v_1 \in p_1 \) overlaps with some node \( v_2 \in p_2 \). The following combinatorial lemma is fundamental to our proof.

Lemma 10. For every rooted tree with \( \ell \) leaves, there exists a partition of its nodes into \([\log_2(\ell + 1)]\) groups, such that each group is a set of paths, and no two paths in a group overlap.

We defer the proof of Lemma 10 and first use it to finish the proof of Theorem 11.

For a **Summarize** instance \( P \), let \( \text{Opt}(P) \) denote its optimal solution weight. Let \( \beta := [\log_2(m + 1)] \). Using Lemma 10, we can decompose \( T_1 \) into disjoint groups \( T = G_1 \cup \cdots \cup G_m \), where \( q \leq [\log_2(n + 1)] \leq \beta \).

Let \( P_{G_i} \) denote the **Summarize** problem restricted to \( G_i \times T_2 \times \cdots \times T_d \). Then we have \( \sum_{i \in I} \text{Opt}(P_{G_i}) \geq \text{Opt}(P) \).

Wlog assume that \( \text{Opt}(P_{G_i}) \) has the largest weight among the subproblems, and therefore \( \text{Opt}(P_{G_i}) \geq \text{Opt}(P)/g \geq \text{Opt}(P)/\beta \).

Recall that \( G_i \) is a set of non-overlapping paths. For each such path \( p \) in \( G_i \), consider the **Summarize** problem \( P_p \) over \( p \times T_2 \times \cdots \times T_d \). Then we have \( \sum_{p \in G_i} \text{Opt}(P_p) \geq \text{Opt}(P_{G_i}) \).

We remove the first dimension from \( P_p \) to form a problem \( P_p' \) over \( T_2 \times \cdots \times T_d \), by setting \( w'(t_2, \ldots, t_d) = \max_{v \in p} w(v, t_2, \ldots, t_d) \) for all \((t_2, \ldots, t_d) \in T_2 \times \cdots \times T_d \). Note that \( \text{Opt}(P_p') = \text{Opt}(P_p) \), and a conflict-free solution for \( P_p' \) can be mapped back to a conflict-free solution for \( P_p \) with the same weight, by replacing each \((t_2, \ldots, t_d)\) by \((v, t_2, \ldots, t_d)\) where \( v = \text{argmax}_{v \in p} w(v, t_2, \ldots, t_d) \).

Note that \( P_p' \) has \( d - 1 \) dimensions. By inductive hypothesis, it has a conflict-free solution \( S_p' \) with \( w'(S_p') \geq \text{Opt}(P_p')/\beta^{d-3} \). The corresponding conflict-free solution \( S_p \) for \( P_p \) also satisfies \( w(S_p) \geq \text{Opt}(P_p)/\beta^{d-3} \). Since the \( p \) in \( G_i \) are non-overlapping, the union of solutions \( S := \cup_{p \in G_i} S_p \) must be a conflict-free solution for \( P_{G_i} \).

Combining our insights leads to \( w(S) = \sum_{p \in G_i} w(S_p) \geq \sum_{p \in G_i} \text{Opt}(P_p)/\beta^{d-3} \geq \text{Opt}(P_{G_i})/\beta^{d-3} \geq \text{Opt}(P)/\beta^{d-3} = \text{Opt}(P)/\beta^{d-3} \).

It remains to show Lemma 10. We will prove the following slightly stronger generalization to forests instead. Forests are sets of rooted trees, and nodes in different trees in a forest are considered to be not overlapping.

Lemma 11. For every forest with \( \ell \) leaves, there exists a partition of its nodes into exactly \([\log_2(\ell + 1)]\) groups, such that each group consists of a set of paths, and no two paths in a group overlap.

Proof. We prove Lemma 11 by induction on \( \ell \). Given a forest, let \( v_1, \ldots, v_ℓ \) be a preorder traversal ordering of its leaves. Let \( m := \lfloor \ell/2 \rfloor \) be the index of the middle leaf, and let \( p \) be the path from \( v_m \) all the way to its root. If \( p \) contains all nodes in the forest, we are done. Otherwise consider the forest over \( v_1, \ldots, v_{m-1} \), and the forest over \( v_{m+1}, \ldots, v_\ell \) respectively. The two forests are separated by the path \( p \). In particular, no node in the first forest overlaps with any node in the second forest. We inductively apply the lemma to the two (smaller) forests respectively, and obtain \([\log_2(m-1+1)]\) groups for the first forest, and \([\log_2(\ell-m+1)]\) groups for the second forest, both of which have size at most \([\log_2(\ell+1)] - 1\). No group from the first forest overlaps with any group from the second forest. Hence we can set \( G_i \) to be the union of the \( i \)-th group for the first forest with the \( i \)-th group for the second forest, for \( i = 1, \ldots, [\log_2(\ell+1)] - 1 \), and no two paths in \( G_i \) overlap. Finally, we finish our construction by setting \( G_i([\log_2(\ell+1)]) := \{p\} \).

Remark 12. An alternative construction for Lemma 11 but with a looser bound is by an induction over the height of the forest, where at each step, we take one root-to-leaf path from each root of the forest, to form a group, and then proceed with the rest of the forest. Each such step reduces the height of the forest by 1, and we end up having as many groups as the height of the forest. In practical applications, the depths of the hierarchies are usually bounded by a small constant such as 3, so that this construction gives a better approximation bound.

Remark 13. In our applications, the worst-case approximation ratio is usually a small constant (2 – 4). These applications have at most two large dimensions, which do not contribute to the approximation ratio via the proof of Theorem 3. Notice that the proof allows to leave out any two dimensions from the bound, and we may as well leave out the two dimensions that correspond to the largest trees. Further, the other dimensions all have height at most 2. So by Remark 13 we get an approximation factor that is \( 2^{d-2} \), where \( d \) is the number of dimensions. For instance, in the AdWords use-case, we get an approximation ratio of 2. In practice, the approximation could be even better due to the absence of conflicts. We discuss this in Remark 16.

5. HARDNESS AND LOWER BOUNDS

5.1 Hardness results

We have seen that our algorithm solves **Summarize** exactly for \( d \leq 2 \), and provided approximation guarantees for \( d \geq 3 \). The following theorem shows that an exact solution even for \( d = 3 \) is likely infeasible.

Theorem 14. **Summarize** is NP-hard for \( d = 3 \).

Proof. We show this by reduction from (directed) **Maximum Independent Set** (MIS). Given a directed graph \((V, E)\) (an instance of MIS), we construct an instance of...
Summarize with \( d = 3 \) as follows. Let us call the three trees \( A, B, C \) with roots \( a, b, c \), respectively. All the trees have height 2. In trees \( A \) and \( B \) we have one child per vertex \( v \in V \), called \( a_v \) and \( b_v \), respectively. In the third tree, we have one child per edge \( (v, w) \in E \), called \( c_{v, w} \) (see Figure 3).

The weight function \( w \) has non-zero weight on the following nodes:

- node \( N_v := (a_v, b_v, c) \) has weight 1 for every \( v \in V \), and
- nodes \( N_{v, w} := (a_v, b_v, c_{v, w}) \) and \( N'_{v, w} := (a_v, b_v, c_{v, w}) \) have weight \( \beta := 1 + \varepsilon \) for every \( (v, w) \in E \).

We set \( k = \infty \), so that any overlap-free set \( S \) is a valid solution. Note that the reduction is polynomial-time; the number of nodes in the Summarize instance is \( O(|E| \cdot |V|^2) \).

We claim that there is a solution to

\[
\sum_{(v, w) \in E} w(S) \geq m + \beta|E|
\]

This implies that Summarize is NP-hard.

\( \Rightarrow \) Let \( V' \subseteq V \) be an independent set with \( |V'| = m \). Let \( S \) be the union of the sets \( S_1 := \{N_v \mid v \in V' \}, S_2 := \{N_{v, w} \mid v \notin V' \}, S_3 := \{N'_{v, w} \mid v \in V' \wedge w \notin V' \} \).

Since for all edges \( (v, w) \) either \( v \) or \( w \) is not in \( V' \), we have that either \( N_v \in S_2 \) or \( N_{v, w} \in S_3 \), and thus \( |S_2 \cup S_3| = |E| \). Therefore, we have \( \sum_{(v, w) \in E} w(S) = m + \beta|E| \). It remains to show that \( S \) is overlap-free. There are no overlaps in \( S_2 \cup S_3 \); since no two elements overlap in the third dimension \( \{c_{v, w}\} \), neither are there overlaps in \( S_1 \cup S_2 \), since elements differ in the first dimension, and \( S_1 \cup S_3 \), since elements differ in the second dimension.

\( \Leftarrow \) Given a solution \( S \) to the Summarize problem with \( w(S) \geq m + \beta|E| \), we need to construct an independent set of size at least \( m \). For each edge \( (v, w) \), \( S \) can contain at most one of \( N_{v, w} \) and \( N'_{v, w} \), since the two nodes overlap. However, wlog, we can assume that \( S \) contains exactly one of them: It is not hard to see that if \( S \) contains neither, and adding one of them would create an overlap, then \( S \) has to contain either \( N_v \) or \( N_{v, w} \). However replacing e.g. \( N_v \) by \( N_{v, w} \) will increase the weight of \( S \) by \( \beta - 1 = \varepsilon \). Note that this also implies that for any edge \( (v, w) \), \( N_v \) and \( N_w \) cannot both be in \( S \).

So wlog, \( S \) contains exactly one of \( N_{v, w} \) and \( N'_{v, w} \) for each edge \( (v, w) \). Since \( w(S) \geq m + \beta|E| \), at least \( m \) of the weight comes from nodes \( N_v \). Thus, the set \( V' := \{v \mid N_v \in S\} \) satisfies \( |V'| \geq m \). As we observed before, for each edge \( (v, w) \), not both \( v \) and \( w \) can be in \( V' \), thus it is an independent set of the desired size.

### 5.2 Lower bounds for the algorithm

We now construct “hard” input instances for Summarize for which our algorithm outputs a solution that has \((2/3)^{d/3}\) of the weight of the optimal solution, when \( d \) is a multiple of 3. It follows that our algorithm is at best a \((2/3)^{d/3} > 1.144^{d/2}\) approximation algorithm. Our strategy will be to construct an instance with lots of conflicts.

**Theorem 15.** For every integer \( m \geq 1 \), there is an instance of Summarize with \( d = 3m \) dimensions that has an overlap-free solution with weight \( 3^m \), while the optimal conflict-free solution has weight \( 2^m \).

**Proof.** Note that for \( m = 1 \), such a problem instance is given by the conflict in Example 4. We obtain the general case by raising this example to the \("m-th power", as follows.

Our instance of Summarize has \( d = 3m \) dimensions, where every dimension has a tree with root \( r_i \) and two children \( a_i \) and \( b_i \). We group the dimensions into sets of size three, and define for \( i \in \{1, \ldots, m\} \):

\[
S_i := \{(a_{3i-2}, b_{3i-1}, r_i), (b_{3i-2}, r_{3i-1}, a_i), (r_{3i-2}, a_{3i-1}, b_i)\}
\]

Note that these are copies of Example 4 restricted to dimensions \( 3i - 2, 3i - 1, 3i \). The weight function \( w \) is 1 for all nodes in \( S := S_1 \times S_2 \times \cdots \times S_m \); all other nodes have weight zero. We set \( k = \infty \).

By construction, \( S \) has \( 3^m \) elements, and therefore a total weight of \( 3^m \). For the first claim, we now show that \( S \) is overlap-free. Consider two different elements of \( S \). Clearly, they must differ on some factor \( S_i \). But by definition of \( S \), that means that they are disjoint.

For the second claim, we now inductively prove that every conflict-free solution of this problem instance has a weight of at most \( 2^m \). For \( m = 1 \), the claim follows from Example 4. For \( m \geq 2 \), let \( T \subseteq S \) be a conflict-free solution. We can assume \(|T| > 1\), since otherwise \( T \) clearly is of size less than \( 2^m \).

Since \( T \) is conflict-free, there must be a dimension \( i \) such that \( x_i \in \{a_i, b_i\} \) for all \( x \in T \) (as per the definition of a conflict). By symmetry of our construction, we can wlog assume \( i = d \). Then \( T \) can be decomposed as a disjoint union \( T = T_a \cup T_b \), where \( T_a = T \cap \{x | x_d = a_d\} \) and \( T_b = T \cap \{x | x_d = b_d\} \). By construction of \( S \), the nodes in \( T_a \) have the same values in factor \( S_m \). Removing the last three dimensions from \( T_a \), we obtain a set of nodes in \( S_1 \times \cdots \times S_{m-1} \) with the same cardinality. This set is also conflict-free (otherwise \( T_a \) would contain a conflict), and forms a solution to the instance of size \( m - 1 \). By induction, \( T_a \) can have size at most \( 2^{m-1} \). Similarly, \(|T_b| \leq 2^{m-1} \), and therefore \(|T| = |T_a| + |T_b| \leq 2^m \).

**Remark 16 (Role of Conflicts).** Note the fundamental role played by conflicts in the proofs of Theorem 14 and in Theorem 1. The simple conflict in Example 4 underlies the constructions in both proofs. As stated by Lemma 3 in the absence of conflicts, Summarize can be solved optimally. Therefore an interesting open question is to ask how frequently large-weight, non-overlapping conflict structures arise in practice. In the context of summarizing metric changes, it is likely that these are fairly rare because for Example 4 to manifest, there have to be three fluctuations, each from three separate causes, but the causes are such that they don't overlap with each other. It would be worthwhile to test this conjecture in practice.

**Remark 17 (Dense input versus sparse input).** There is a significant difference between our work and Multi-structural Databases [14] in how the input is provided. In [12], the input consists only of the subset \( V' \) of nodes that have non-zero weight. Let us call this a sparse input. In contrast, we assume that the weights are specified for every node \( v \in V \). That is, we assume that the input is dense. We chose this modeling assumption because in practice (i.e., for the applications in Section 2), we found that almost all nodes in \( V \) had non-zero weight. Even though our algorithm is described for the dense case, it is straightforward to apply it to the sparse case as well.
Sparness plays a critical role in the hardness results of \ref{thm:hardness}. They perform a reduction from independent set (a well-known NP-hard problem). Their reduction can be done in polynomial time only if the input is sparse. In fact, in their reduction, the number of nodes is equal to the number of trees.

These results therefore do not imply that the dense case is also NP-hard. In principle, it is possible that NP-hardness disappears because we ‘only’ need to be polynomial in the product of the sizes of the trees. Theorem \ref{thm:hardness} shows that the problem is NP-hard even with dense input. So the hardness is not due to the density of input, but due to the presence of conflicts.

Remark 18 (Comparison to Rectangle Packing). It is instructive to compare \textsc{Summarize} to the problem of the max-weight packing of axis-aligned hyper-rectangles in a space so that no two rectangles spatially overlap. Nodes in our problem correspond to hyper-rectangles in that setting (cf. \ref{thm:hardness,thm:hardness2,thm:hardness3}). That problem is optimally solvable for one dimension and NP-Hard for two more more dimensions. It can be shown that every instance of our problem is an instance of that problem, but not vice versa. This is because a tree cannot model the case where two hyper-rectangles intersect along a certain dimension, but neither contains the other. What Theorems \ref{thm:hardness} and \ref{thm:hardness2} together show is that the restriction to hierarchical hyper-rectangles now allows positive results for two dimensions and “postpones” the NP-Hardness to three or more dimensions. We borrow some proof ideas for Theorem \ref{thm:hardness2} from \[6\].

6. APPLICATIONS OF THE ALGORITHM

The Cascading Analysts algorithm is fairly general. The key choices when applying the algorithm to a specific context are to pick the metrics and dimensions to apply them over, and a sensible weight function. We have applied the Cascading Analysts algorithm to helping advertisers debug their advertising campaigns via a report called the “top movers report” \[5\], and to helping websites analyze their traffic within Google Analytics \[2\].

6.1 Interesting Weight Functions

In Section \[2\] we discussed a very simple weight function used to analyze metric changes. For each node \(v\), \(w(v)\) was set to the absolute value of the difference in the metric values for the node between two time periods \(|t_v - l_v|\), where \(l_v\) is the metric value for the current time period, and \(t_v\) is the metric value for a past time period (pre-period). In this section, we present other alternatives that result in different types of summarization.

6.1.1 Modeling Composition Change

If the data displays a generally increasing trend (or a generally decreasing trend), it is possible that almost all the slices are data are generally increasing. So the weight function \(w(v) = |t_v - l_v|\) essentially becomes \(w(v) = t_v - l_v\), and the root node is a degenerate, optimal solution, because it has as least as much weight as any set of non-overlapping nodes. In practice we may still want to separate low growth slices from high growth ones, because the former could still be improved. A simple option is to compare the mix or the composition of the metric rather than magnitude of the metric, that is

\[
w(v) := \left| \frac{t_v}{\sum_{v \in V} t_v} - \frac{l_v}{\sum_{v \in V} l_v} \right|
\]  

(1)

This way, the output of \textsc{Summarize} consists of nodes that were a large fraction of the market in one of the two time periods, and a relatively small fraction in the other.

This technique is also useful in performing Benchmarking \[1\]. In Benchmarking the goal is to compare the metric for a business against similar businesses. For instance, comparing the traffic of one messaging app against the traffic of another app across dimensions such as location, user age, etc. Here, \(t_v\)’s correspond to amount of traffic for the protagonist’s company/app and \(l_v\)’s represent traffic for a benchmark app. It is usually the case that one of the businesses/apps is much larger, and therefore it makes more sense to compare the composition of their markets as in Equation \[1\].

6.1.2 Modeling Absolute Change v/s Relative Change

A slightly different issue is that a large relative change in a metric (say from $500 to $1000) may be more interesting than a small relative change in metric (say from $1000 to $11000), because the latter is probably due to usual fluctuations (noise), whereas the former is a real event worth responding to. However, focusing entirely on relative change could produce tiny segments as output (say the metric changes from 1 to 100). In practice, it makes sense to weigh relative changes to some extent. To model this, we apply a standard technique called a Box-Cox transform \[10\] by setting the weight \(w(v) := \frac{(t_v - m) - (l_v - m)}{1 - m}\). For \(m = 0\), this reduces to the absolute value of the difference \(|t_v - l_v|\). When \(m \rightarrow 1\), this approaches \(\log(t_v) - \log(l_v)\), which models a relative difference. In practice we found it useful to set \(m\) in the range \([0.1, 0.3]\).

6.2 Analyzing the Performance of Ad Campaigns

Google’s Adwords platform supports advertisers that run pay-per-click advertising campaigns. There are many factors that affect the performance of a campaign \[3\]. The advertiser’s own actions, an advertiser’s competitor’s actions, a change in user behavior, seasonal effects, etc. Given that large amounts of money are often at stake, most advertisers monitor and react to changes in their spend very carefully.

A routine task for advertisers is therefore to (1) periodically identify the “top moving” data-segments, and (2) react to these changes sensibly. We seek to automate (1). Adapting the cascading analysts algorithm involves carefully choosing the dimensions and the weight function.

We use three hierarchical dimensions to partition campaign performance. The first is the campaign-adgroup-keyword hierarchy. Each campaign can have multiple adgroups, and an adgroup can have multiple keywords, forming a tree structure. For example, a gift shop can have one campaign for flowers and one campaign for cards. Within the flower campaign, it can have one adgroup for each zip code that the shop serves. A second hierarchical dimension partitions the user by the kind of device they use. The top level split can be Mobile, Desktop, and Tablet, and each of these can further be split by the make and model of the device. And the third
dimension is the platform on which the ads were shown, for instance, Google search, on the Web, or on Youtube.

We briefly describe how the weight function is modeled. The metrics of interest are the spend of the advertiser, the number of clicks received by the ads, and the number of views (impressions) received by the ads. We usually compare two time-slices. The weight function is modeled as the BoxCox transformation applied to the values of the metric in the two time-periods (see Section 6.1.2).

6.3 Understanding Website Traffic

Google Analytics helps website and app developers understand their users and identify opportunities to improve their website (and similarly for phone apps) [4]. There are many factors that affect the traffic to a website. Changes in user interests, buzz about the website in social media, advertising campaigns that direct traffic to the website, changes to the website that make it more or less engaging, etc. The “Automated Insights” feature of Google Analytics [2] analyzes the traffic data and identifies opportunities or room for improvement. The Cascading analysts algorithm is used within a section of this feature that identifies focus-worthy segments of traffic. The feature involves other algorithms and curation to make the insights actionable.

We now discuss the metrics and the dimensions. Google Analytics is used by a variety of apps and websites, for example by content providers like large newspapers, ecommerce platforms, personal websites or blogs, mobile apps, games, etc. Different dimensions and metrics are important for different businesses. Consequently, Google Analytics has a very large set of dimensions and metrics that it supports. Some metrics include visits to the website, number of users, number of sessions, and a metric called goals whose semantics are user-defined. Some examples of dimensions include the source of the traffic to the website (search engines, social network sites, blogs, direct navigation), medium (was the visit due to an ad, an email, a referral), geographic dimensions (continent, country, city), device related dimensions (as in our AdWords example above) etc. The Cascading analysts is applied to several coherent groupings of dimensions and metrics. For instance, we may run the algorithm to compare the composition (see Section 6.1.1) of visits in one month versus another, with three dimensions like source, geography and device. (Here we compare compositions rather than the raw metric magnitudes because large seasonal effects could make all the data trend up or down.) This produces several candidate segments that are then turned into insights reports.

7. RELATED WORK

7.1 OLAP/Drill-Down

There is a large body of literature on OLAP [13]. As discussed in the introduction, there is justified interest in automating data analysis for it. There is work on automating or helping the automation of drill-downs [29, 27, 30, 16, 21]. These attempts to summarize patterns in the data use information-theoretic approaches rather than explain the difference in aggregate metrics. Candan et al [12] propose an extension to OLAP drill-down that takes visualization real estate into account, by clustering attribute values. Again, this is not targeted to a specific task like explaining the change in aggregate metrics.

The database community has devoted a lot of attention to the problem of processing taxonomical data. For example [8, 25, 24, 22] consider the same or related models. Broadly, they concentrate on the design issues such as query languages, whereas we focus on computational issues and optimization.

There is recent work by Joglekar, Garcia-Molina, and Parameswaran [18], which we call Smart Drill-Down, that attempts to “cover” the data. They trade off dual
recognizing that the problem is hard to solve over a lattice, both papers extrinsically convert the lattice into a tree by sequentially composing dimensions (pasting one dimension below another). This precludes certain data-cubes from being candidate solutions.

**Example 19.** Consider US employment data in a two dimensional space: Location $\times$ Gender. If we compose the dimensions strictly as location before gender, then the following pair of non-overlapping nodes will never be considered in the same solution: (North-east, Male) and (New York, Female). We can only reach both of these nodes by first splitting the space on gender, and then splitting each of subspaces independently on location.

One may try all possible sequential compositions of the dimension trees - arguably an efficient operation when the number of dimensions is a constant - and pick the best one. The following example proves that even with this improvement, we can only expect a $\Omega(n^{1/4})$-approximate solution. By Theorem 5, our algorithm is optimal for two dimensions.

So, we have a significant improvement even for two dimensions.

**Example 20.** Consider an instance of Summarize with two dimensions, each with the same topology: a star with $\sqrt{m}$ strands (paths), each with $\sqrt{m}$ nodes, and a root. Suppose that the strands are labeled $1, \ldots, \sqrt{m}$ from left to right, and the tree-nodes within a strand are labeled $1, \ldots, \sqrt{m}$, then we can label each tree-node by a pair (strand index, within-strand index). Notice that $n = (m + 1)^2$.

Suppose further that the weights are in $\{0, 1\}$. The only lattice nodes with a weight of 1 are the $m$ nodes indexed by the quadruple $(i, j), (j, i)$ for $i \in \{1, \ldots, \sqrt{m}\}$ and $j \in \{1, \ldots, \sqrt{m}\}$. The optimal solution has value $m$ because none of the non-zero weight nodes overlap—if two nodes overlap in one dimension, then they belong to different strands in the other dimension.

Now, by Observation 13, for a sequential composition $T_1$ followed by $T_2$, we cannot pick a pair of lattice nodes that belong to the same strand in dimension $T_1$, yielding a solution of size at most $\sqrt{m}$. The argument for the other sequential composition is symmetric, and we have at best a $\sqrt{m}$ approximation.

**8. CONCLUSION**

We study the problem of summarizing metric changes across hierarchical dimensions. Our main contribution is a new algorithm called Cascading Analysts that is a significant improvement over prior algorithms for this problem. We apply this algorithm to two widely used business-intelligence tools (Google Adwords and Google Analytics) that help advertisers and website/app developers analyze their core metrics.

Studying concrete applications gave us an interesting lens on the computational hardness of the summarization problem. We identified a practically relevant restriction of the previously studied model of hierarchical data to a product space of trees – prior work studied lattices. Without this restriction, the problem on lattices is computationally hard, although, this hardness is relatively uninteresting, i.e., it stems purely from the “dimensionality” of data. In practice, we note that the summarization problem is useful to
solve even when there are a few (say less than five) dimensions as in our applications. Further investigation reveals a more interesting source of hardness—the presence of structures called conflicts that occur in data with three or more dimensions. Fortunately, this source of hardness does not preclude approximation.

One direction of future work is to better understand the prevalence of conflicts in an “average case” sense. Our belief (see Remark 16) is that large weight conflicts ought to be rare in practice. It would be interesting to formalize this in a beyond-worst-case-analysis sense.

9. REFERENCES

[1] Benchmarking. https://en.wikipedia.org/wiki/Benchmarking.

[2] Explore important insights from your data automatically. https://analytics.googleblog.com/2016/09/explore-important-insights-from-your.html.

[3] Google adwords. https://en.wikipedia.org/wiki/AdWords.

[4] Google analytics. https://en.wikipedia.org/wiki/Google_Analytics.

[5] Understanding the top movers report in adwords. https://support.google.com/adwords/answer/2985776?hl=en.

[6] P. K. Agarwal, M. van Kreveld, and S. Suri. Label placement by maximum independent set in rectangles. *Computational Geometry*, 11(3):209 – 218, 1998.

[7] M.-F. Balcan, B. Manthey, H. Röglin, and T. Roughgarden. Analysis of Algorithms Beyond the Worst Case (Dagstuhl Seminar 14372). *Dagstuhl Reports*, 4(9):30–49, 2015.

[8] O. Ben-Yitzhak, N. Golbandi, N. Har’El, R. Lempel, A. Neumann, S. Ofek-Koifman, D. Sheinwald, E. Shekita, B. Szajder, and S. Yogev. Beyond basic faceted search. In *Proceedings of the international conference on Web search and web data mining*, WSDM ’08, pages 33–44, New York, NY, USA, 2008. ACM.

[9] P. Berman, B. Dasgupta, S. Muthukrishnan, and S. Ramaswami. Improved approximation algorithms for rectangle tiling and packing (extended abstract). In *Proc. 12th ACM-SIAM Symp. on Disc. Alg*, pages 427–436, 2001.

[10] G. E. Box and D. Cox. An analysis of transformations, 1964.

[11] S. Bu, L. V. S. Lakshmanan, and R. T. Ng. Mdl summarization with holes. In *VLDB*, pages 433–444, 2005.

[12] K. S. Candan, H. Cao, Y. Qi, and M. L. Sapino. Alphasmum: size-constrained table summarization using value lattices. In *EDBT*, pages 96–107, 2009.

[13] E. F. Codd, S. B. Codd, and C. T. Salley. Providing OLAP (On-Line Analytical Processing) to User-Analysis: An IT Mandate, 1993.

[14] R. Fagin, R. Guha, R. Kumar, J. Novak, D. Sivakumar, and A. Tomkins. Multi-structural databases. In *Proceedings of the twenty-fourth ACM SIGMOD-SIGACT-SIGART symposium on Principles of database systems*, PODS ’05, pages 184–195, New York, NY, USA, 2005. ACM.

[15] R. Fagin, P. Kolaitis, R. Kumar, J. Novak, D. Sivakumar, and A. Tomkins. Efficient implementation of large-scale multi-structural databases. In *Proceedings of the 31st International Conference on Very Large Data Bases*, VLDB ’05, pages 958–969. VLDB Endowment, 2005.

[16] K. E. Gebaly, P. Agrawal, L. Golab, F. Korn, and D. Srivastava. Interpretable and informative explanations of outcomes. *PVldb*, pages 61–72, 2014.

[17] F. Geerts, B. Goethals, and T. Mielikinen. Tiling databases. In *Discovery Science*, pages 278–289, 2004.

[18] M. Joglekar, H. Garcia-Molina, and A. Parameswaran. Smart drill-down: A new data exploration operator. *Proc. VLDB Endow.*, 8(12):1928–1931, Aug. 2015.

[19] S. Khanna, S. Muthukrishnan, and M. Paterson. On approximating rectangle tiling and packing. In *Proceedings of the Ninth Annual ACM-SIAM Symposium on Discrete Algorithms*, SODA ’98, pages 384–393, Philadelphia, PA, USA, 1998. Society for Industrial and Applied Mathematics.

[20] L. V. S. Lakshmanan, R. T. Ng, C. X. Wang, X. Zhou, and T. J. Johnson. The generalized mdl approach for summarization. In *VLDB*, pages 766–777, 2002.

[21] M. Mampaey, N. Tatti, and J. Vreeken. Tell me what i need to know: Succinctly summarizing data with itemsets. In *KDD*, pages 573–581, 2011.

[22] D. Martinenghi and R. Torlone. Querying databases with taxonomies. In J. Parsons, M. Saeki, P. Shoval, C. Woo, and Y. Wand, editors, *Conceptual Modeling ER 2010*, volume 6412 of *Lecture Notes in Computer Science*, pages 377–390. Springer Berlin / Heidelberg, 2010. 10.1007/978-3-642-16373-9:27.

[23] D. Pugh. *Organization Theory Edited by D.S. Pugh: Selected Readings*. Penguin modern management readings. Penguin, 1971.

[24] Y. Qi, K. S. Candan, J. Tatemura, S. Chen, and F. Liao. Supporting olap operations over imperfectly integrated taxonomies. In *Proceedings of the 2008 ACM SIGMOD international conference on Management of data*, SIGMOD ’08, pages 875–888, New York, NY, USA, 2008. ACM.

[25] R. Ramakrishnan and B.-C. Chen. Exploratory mining in cube space. *Data Mining and Knowledge Discovery*, 15:29–54, 2007. 10.1007/s10618-007-0063-0.

[26] S. Sarawagi. Explaining differences in multidimensional aggregates. In *Proceedings of the 25th International Conference on Very Large Data Bases*, VLDB ’99, pages 42–53, San Francisco, CA, USA, 1999. Morgan Kaufmann Publishers Inc.

[27] S. Sarawagi. User-adaptive exploration of multidimensional data. In *VLDB*, pages 307–316, 2000.

[28] S. Sarawagi. User-Adaptive Exploration of Multidimensional Data. In *VLDB*, pages 307–316. Morgan Kaufmann, 2000.

[29] S. Sarawagi. User-cognizant multidimensional analysis. *The VLDB Journal*, pages 224–239, 2001.

[30] S. Sarawagi, R. Agrawal, and N. Megiddo. Discovery-driven exploration of olap data cubes. In *EDBT*, pages 168–182, 1998.

[31] S. Sarawagi and G. Sathe. I3: Intelligent, interactive
investigation of olap data cubes. In *Proceedings of the 2000 ACM SIGMOD International Conference on Management of Data*, SIGMOD ’00, pages 589–, New York, NY, USA, 2000. ACM.

[32] T. Sellam and M. L. Kersten. Meet charles, big data query advisor. In *CIDR’13*, pages –1–1, 2013.

[33] Y. Xiang, R. Jin, D. Fuhr, and F. F. Dragan. Succinct summarization of transactional databases: an overlapped hyperrectangle scheme. In *KDD*, pages 758–766, 2008.