The Effect of Noise on the Dirac Phase of Electron in The Presence of Screw Dislocation

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Abstract

The effect of noise on the Dirac phase of electron in the presence of screw dislocation is studied. An uncorrelated noise, which coincides with the nature of thermal fluctuations, is adopted. Results indicate that the Dirac phase is robust against existing noise in the system.

PACS numbers: 03.65.Vf, 61.72.Lk, 05.40.-a, 02.40.Ky

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I. INTRODUCTION

Some of solids have a crystalline structure which is not an ideal structure and has defects. These defects have strong effect on physical properties of the medium [1–13]. Therefore, study them is important in solid state physics.

Elastic stresses in solids due to the defects mathematically corresponds to a non-Euclidean space. This is the Katanaev-Volovich approach to the theory of defects in solids which is analogous to three dimensional gravity [14, 15]. Dealing with a non-Euclidean metric is easier than using complicated boundary conditions. In the continuum limit, which is valid at distance much larger than the lattice spacing, the solid is described by a Riemann-Cartan manifold with curvature and torsion. Topological defects such as disclination and dislocation are described by curvature and torsion, respectively.

In 1931, Dirac [16] showed that when a particle transports in an external electromagnetic field, its wave function acquires a phase term in addition to usual dynamic phase factor. The change of the phase under the transfer along a close counter is proportional to the field flux through the counter. This additional phase is known as Dirac phase and is a non-integrable phase factor [17]. Non-integrable phase factor appears in many different area of physics [18].

Furtado and coworkers [19] investigated the Dirac phase of electron in the presence of screw dislocation using the Katanaev-Volovich approach and showed that it is analogous to Aharonov-Bohm effect [20]. In this case the Burger vector of dislocation plays the role of electromagnetic flux.

In this paper, we study the effect of noise, which usually presents in physical systems, on the dirac phase of electrons in media with screw dislocation. Theoretical results indicate that the dirac phase factor of screw dislocation is robust against fluctuations.

The paper is organized as follow. In section II, we review the Dirac phase of electrons in the presence of screw dislocation. In section III, The effect of noise on the mentioned Dirac phase factor is studied. The selected model of noise coincides with the nature of thermal fluctuations. At the end, the conclusion is presented.
II. DIRAC PHASE IN THE PRESENCE OF SCREW DISLOCATION

In a screw dislocation the Burger vector is parallel to the dislocation line. This type of defect, which corresponds to a singular torsion \[21\] along the defect line, is described by the following metric \[19, 22\]

\[
ds^2 = g_{ij} dx^i dx^j = (dz + \beta d\phi)^2 + d\rho^2 + \rho^2 d\phi^2,\]

(1)

where \(\beta\) is a parameter related to the Burger vector \(\vec{b}\) by \(\beta = \frac{b}{2\pi}\). This metric carries torsion but no curvature.

The schrödinger equation for an electron in the presence of defect is given by

\[
-\frac{\hbar^2}{2m} \nabla^2 \psi(\rho, \varphi, z, t) = i\hbar \frac{\partial}{\partial t} \psi(\rho, \varphi, z, t),
\]

(2)

where the Laplace-Beltrami operator is given by \(\nabla^2 = \frac{1}{\sqrt{g}} \partial_i (g^{ij} \sqrt{g} \partial_j)\). \(g\) is the determinant of the metric tensor \(g_{ij}\). The metric tensor \(g_{ij}\) and its inverse \(g^{ij} = (g_{ij})^{-1}\) are as follow

\[
g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \beta^2 + \rho^2 & \beta \\ 0 & \beta & 1 \end{pmatrix}, \quad g^{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\rho^2} & -\frac{\beta}{\rho^2} \\ 0 & -\frac{\beta}{\rho^2} & \frac{\beta^2 + \rho^2}{\rho^4} \end{pmatrix}.
\]

(3)

Using the metric tensor (3) for screw dislocation, the schrödinger equation (2) takes the form

\[
-\frac{\hbar^2}{2m} \left\{ \partial_z^2 + \frac{1}{\rho} \partial_\rho (\rho \partial_\rho) + \frac{1}{\rho^2} (\partial_\varphi - \beta \partial_z)^2 \right\} \psi(\rho, \varphi, z, t) = i\hbar \frac{\partial}{\partial t} \psi(\rho, \varphi, z, t),
\]

(4)

whose solution can be written as

\[
\psi(\rho, \varphi, z, t) = e^{-iE\frac{t}{\hbar}} e^{ikz} \psi(\rho, \varphi).
\]

In this way the schrödinger equation (4) is given by

\[
-\frac{\hbar^2}{2m} \left\{ -k^2 + \frac{1}{\rho} \partial_\rho (\rho \partial_\rho) + \frac{1}{\rho^2} (\partial_\varphi - i\beta k)^2 \right\} \psi(\rho, \varphi) = E \psi(\rho, \varphi),
\]

(5)

Equation (5) implies that \(\partial_\varphi \rightarrow \partial_\varphi - ik\beta\) with respect to the defect free case \((\beta = 0)\) in which the Laplacian operator is given in a flat space. In the other words, the angular momentum changes according to \(l \rightarrow l - k\beta\). The angular momentum of the electron is modified by the presence of the defect which is due to the torque exerted by the strain field.
of the dislocation. Thus, an electron in the presence of screw dislocation behaves like an electron in the presence of a magnetic flux. The corresponding vector gauge potential is

\[ A = \frac{k\beta}{\rho} \hat{e}_\varphi. \]  

(6)

According to this correspondence, the Dirac phase factor method \[16, 23\] can be used. Therefore, the solution of the Schrödinger equation (5) can be written as

\[ \psi(\rho, \varphi) = \exp \left\{ i \int A \cdot d\mathbf{r} \right\} \psi_0(\rho, \varphi), \]

(7)

where \( \psi_0(\rho, \varphi) \) is the solution of the defect free case. Substituting (6) in (7) leads to

\[ \psi(\rho, \varphi) = e^{i \int^{\varphi}_0 k\beta d\varphi} \psi_0(\rho, \varphi). \]

This means that \( \psi(\rho, \varphi) \) differs from \( \psi_0(\rho, \varphi) \) just in a factor \( e^{i\gamma} \), where

\[ \gamma = \int^{\varphi}_0 k\beta d\varphi, \]

(8)

and is known as Dirac phase factor. This phase depends on electron trajectory.

### III. THE EFFECT OF NOISE ON THE DIRAC PHASE

The presence of noise is an unavoidable subject in physical systems. The noise for example, can be due to ubiquitous thermal fluctuations in the system. In this section we study the effect of noise on the Dirac phase factor. The noise does not affect the the magnitude of wave vector, \( k \). The only effect of noise is to change Burger vector according to

\[ \mathbf{b}(t) = \mathbf{b}_0 + N(t), \]

(9)

where the index "0" indicates the absence of noise and \( N(t) \) is the noise term. The noise is a random process with zero average and small amplitude compared to \( \mathbf{b}_0 \). So, the effect of noise on the metric is negligible due to the smallness of the noise amplitude. Naturally, the metric can be considered like (1) just by having beta as a function of time, \( \beta(t) = \frac{b(t)}{2\pi} \). Consequently, by following the same procedure, \( \gamma \) leads to the result (8) while beta is now time dependent and the electron trajectory is a fluctuating one. It fluctuates about the noiseless trajectory of the electron (assumed cyclic with period \( T \)). Using (9) the resulting change in \( \gamma \) during time \( T \) will be

\[ \Delta \gamma = \frac{k}{T} \int_0^T (b - b_0) dt. \]
In deriving above we used \( d\varphi = \frac{2\pi}{T} dt \). \( \mathbf{b} \) does not return to its original direction because of the random noise and a non-cyclic contribution also appears. According to the definition of non-integrable phase for non-cyclic evolution \[24\], this term can be removed and the above result still holds \[25\].

Expanding \( b = |\mathbf{b}_0 + \mathbf{N}| \) in terms of \( \mathbf{b}_0 \) to the first order in the noise, yields

\[
\Delta \gamma = \frac{k}{T} \int_0^T \mathbf{b}_0 \cdot \frac{\mathbf{N}}{\mathbf{b}_0} dt.
\]  

(10)

The noise has zero average, thus \( < \Delta \gamma > = 0 \). It means that the average value of \( \gamma \) coincides with its noiseless value. In order to compute the probability distribution of \( \Delta \gamma \), a definite model for the noise is needed. We want to compute the second moment of \( \Delta \gamma \) which is related to the width of the probability distribution function.

A suitable model for noise with respect to the physical nature of thermal fluctuations is the uncorrelated noise defined by

\[
< N_i(t) > = 0, \quad < N_i(t) N_j(\hat{t}) > = 2D\delta(t - \hat{t})\delta_{ij},
\]

where angular bracket denotes ensemble average. Using this definition for noise, the second moment of \( \Delta \gamma \) which is defined in (10) leads to

\[
< \Delta \gamma^2(T) > = \frac{k^2}{T}.
\]

It increases with second power of wave vector and means that the probability distribution function for the change of \( \gamma \) phase, which reflects the effect of fluctuations, decreases by choosing low energy electrons. Also it slows down in time by the factor \( \frac{1}{T} \) so \( \gamma \) coincides with its noiseless value in the adiabatic limit \( T \to \infty \). As a result, in spite of dynamic phase, the Dirac phase is robust against existing noise in the system.

IV. CONCLUSION

The effect of noise, which is presented in physical systems, on the Dirac phase of electrons in media with screw dislocation is studied. The selected noise is an uncorrelated random noise which is coincided with the nature of thermal fluctuations. The second moment of the change in the Dirac phase, which is related to the width of the probability distribution
function, slows down in time by the factor $\frac{1}{T}$. It indicates that Dirac phase is robust against fluctuations.

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