Stage analysis of delayed-choice and quantum eraser experiments

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Delayed-choice and quantum eraser experiments have attracted much interest recently, both theoretically and experimentally. In particular, they have prompted suggestions that quantum mechanics involves acausal effects. Using a recently developed approach which takes apparatus into account, we present a detailed analysis of various double-slit experiments to show that this is never the case. Instead, quantum experiments can be described in terms of a novel concept of time called stages. These can cut across the conventional linear time parameter as experienced in the laboratory and appear to violate causality.

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I. INTRODUCTION

Delayed-choice \[1, 2\], quantum eraser \[3\] and delayed-choice quantum eraser \[4\] experiments have led to suggestions that interference patterns formed by particles impacting on a screen may be influenced in some way by decisions made long after those particles had landed on that screen. Our objective in this paper is to show by a detailed analysis of various experiments that quantum principles do not support those suggestions.

In our analysis, we shall make out a case for the adoption of a perception of time in quantum observation different to that used in classical mechanics. To understand what we mean, it is important to keep a clear distinction between the concepts of systems under observation (SUOs), such as photons, and apparatus. Our formalism will reflect this difference consistently. Standard unitary Schrödinger evolution can be maintained for states of SUOs in between preparation and outcome, but apparatus appears to follow different rules.

Recent quantum experiments \[2, 3, 4, 5\] are consistent with and support the view that the passage of “detector time” is synonymous with quantum information acquisition occurring in a sequence of stages \[6\]. Stages have rules which are not precisely those of classical information acquisition, and it appears to be this which accounts for much of the well-known difficulty we have in explaining on a classical level various quantum mechanical experiments involving quantum interference.

These rules conform with known physics. For example, quantum information acquisition never violates the light-cone constraints of relativity: classical information cannot be acquired between spacelike intervals. Quantum correlations which appear to violate the principle of Einstein locality actually always require observations to be completed before those correlations can be defined, and this completion always takes place in a classically consistent matter.

Another rule is that quantum information in the form of SUO states can be shielded against the effects of decoherence and preserved in a state of stasis for arbitrarily long periods of laboratory time. This is most evident in the Heisenberg picture and is confirmed by the observation of light from distant stars and galaxies. It is also one way to understand particle decay experiments \[7\].

Yet another rule is that the observation of different components of entangled states is best discussed in terms of stages rather than linear laboratory time. This rule is responsible for the apparent acausality in the delayed-choice experiments we are interested in: observations involving separate detectors can be taken in apparently random order relative to laboratory time without affecting correlations. This was confirmed in the case of the double-slit quantum eraser experiment by Walborn et al. \[8\], who specifically looked at this issue.

Our approach uses a formalism that we have developed for the analysis of time-dependent quantum apparatus networks \[9\]. We have recently applied it to the Franson-Bell experiment \[10, 11\], an experiment that appears to involve acausal quantum interference.

In contrast to standard approaches which tend to focus on the quantum mechanics of systems under observation (SUOs) such as photons, our approach focuses on the detecting apparatus as well. This permits a stage-by-stage analysis of the processes involved in typical quantum optics experiments, starting from state preparation, through the various modules making up the apparatus and ending up with the final state detectors. The formalism is particularly good at giving coincidence rates, which are crucial to many recent quantum optics experiments such as the delayed-choice quantum eraser.

Our notation serves two purposes. First, it provides an efficient method for dealing with quantum networks of great complexity and can be readily encoded into computer algebra packages. Second, it distinguishes between the quantum states of apparatus detectors and those conventionally associated with SUOs such as photons. The formalism is completely consistent with all standard quantum principles.

We make a number of standard assumptions throughout our analysis. First, complete efficiency is assumed, but of course, no real experiment is like that. However, many experiments show precisely those important quan-
II. THE DOUBLE-SLIT EXPERIMENT

In this section we discuss the double-slit (DS) experiment. This experiment features prominently as a component of the delayed-choice quantum eraser experiment and Wheeler’s delayed-choice experiment discussed subsequently.

Figure 1 shows a schematic diagram representing the main features of the DS experiment. Our notation is as follows. In such a diagram, $A_n^i$ represents the labstate $\mathcal{A}_{i,n}^\dagger|0,n\rangle$ of the $i^{th}$ elementary signal detector located at the indicated place in the apparatus network at stage $\Omega_n$. Here $\mathcal{A}_{i,n}^\dagger$ is the associated signal creation operator and $|0,n\rangle$ is the void state of the apparatus $S$ at stage $\Omega_n$. A stage can often be identified with a particular moment or period of laboratory time (i.e., something approximating an instant of simultaneity in the laboratory rest frame), but this need not be the case at all. In particular, a given stage during a delayed-choice experiment may involve an enormous interval of laboratory time, far beyond any notion of simultaneity. What is crucial in the definition of a stage is that all the detectors associated with a given stage are effectively and mutually spacelike relative to each other. In other words, no information in any form is transmitted between them within that stage. Successive stages are defined in terms of either actual or potential transmission of information between them in either classical and quantum forms. Exactly what this means will be made clearer during our discussion of delayed-choice experiments.

In the DS experiment, the source of the photon beam impinging on the double-slit is denoted by $A_0^1$ in Figure 1, the subscript denoting that it occurs at stage $\Omega_0$ and the superscript denoting that there is a single photon source. The initial labstate is taken to be of the form

$$\Psi_0 = \Psi_0^1 s_0^1 A_0^1,$$

where $s_0^1$ is the normalized spin state of the photon concerned and $\Psi_0^1$ is a complex valued normalized factor related to the initial beam characteristics. In our notation, $\Psi_0\Psi_0 = |\Psi_0^1|^2$.

The initial labstate is not normalized to unity because the formalism actually determines relative probability rates for photon signal production during relevant photon coherence times and related times connected with the passage of wave-trains through the apparatus. Related to this is the requirement for specific contextual information about the apparatus to be taken into account. For instance, given a very large detecting screen, some of its detectors would signal photon detection much earlier on in a given run than other detectors further way from the slits. Wheeler’s delayed-choice experiment is an example where such contextual information is crucial to the discussion.

In the basic DS experiment, there are no issues with photon spin, so actually the $s_0^1$ term in (1) is redundant here. However, photon pair spin is a factor in the double-slit quantum eraser experiment discussed later on, so we include a photon spin term here to show how we deal with it in our formalism.

The next step is to compute the effective evolution operator $U_{1.0}$ which takes the initial labstate from $\Psi_0$ to $\Psi_1$ in the transition from stage $\Omega_0$ to $\Omega_1$. Referring to Figure 1, we write

$$U_{1.0} s_0^1 A_0^1 = \alpha^1 s_1^1 A_1^1 + \alpha^2 s_1^1 A_2^2,$$

where $\alpha^1$ and $\alpha^2$ are complex coefficients satisfying the rule $|\alpha^1|^2 + |\alpha^2|^2 = 1$. We do not require at this point to have a symmetrical double-slit device, so $\alpha^1$ and $\alpha^2$ are not assumed equal in magnitude. Moreover, each slit could induce a separate phase change in that part of the beam passing through it, so this is left open as well.

The transition from stage $\Omega_0$ to stage $\Omega_1$ involves a change in Hilbert space dimension from 4 to 8, according
to the counting rules of our formalism. However, the effective dimension involved with stage $\Omega_0$ is just 1 and we find

$$U_{1,0} \simeq s_1^{1} s_0^{1} \sum_{a=1}^{2} \alpha^a A_1^a \bar{A}_0^1,$$

(3)

where $s_1^{1}$ and $\bar{A}_1^{1}$ are the respective duals of $s_0^{1}$ and $A_0^{1}$ and $\simeq$ denotes “effective”.

The next step is to calculate $U_{2,1}$, the effective evolution operator from $\Omega_1$ to $\Omega_2$, at which point we have completed our dynamical account of the DS experiment. Referring again to Figure 1, we write

$$U_{2,0} s_1^{1} A_1^a = \sum_{i=1}^{S} V_i^{,a} s_2^{1} A_2^i, \quad a = 1, 2,$$

(4)

where we have modeled the detecting screen as consisting of a large number $S$ of photon detectors. This accords with actual experiments, which never actually involve a continuum of detectors. If necessary, we are free to take $S$ as large as required in order to model an observed probability pattern to the accuracy required. Note that the detectors in this collection need not be assumed to be coplanar: they could be distributed throughout a three-dimensional region of physical space. Our discussion is perfectly general.

What keeps a track of total probabilities are the semi-unitarity relations between the complex coefficients \{V_i^{,a}\}, which represent the transition amplitudes from emitters based in $\Omega_1$ to detectors in $\Omega_2$. These relations take the form

$$\sum_{i=1}^{S} \bar{V}_i^{,a} V_i^{,b} = \delta_{ab}, \quad a, b = 1, 2,$$

(5)

where $\bar{V}_i^{,a}$ denotes the complex conjugate of $V_i^{,a}$. Again, this is perfectly general. There is no need for our purposes to specify any particular form for the $V_i^{,a}$ coefficients, provided the basic semi-unitarity conditions [6] are satisfied. It is traditional in standard discussions of the double-slit experiment to solve the Schrödinger equation for a monochromatic beam impinging on a screen from two point sources, but this is not necessary in a discussion such as ours: a more general formulation reveals the important features of the experiment more clearly and precisely.

From (4) we find the effective transition operator $U_{2,1}$ from $\Omega_1$ to $\Omega_2$ to be given by

$$U_{2,1} \simeq s_1^{1} s_0^{1} \sum_{i=1}^{S} A_2^i \bar{V}_i^{,a} A_0^{1}.$$

(6)

The total effective evolution operator $U_{2,0}$ is given by the product $U_{2,1} U_{1,0}$ and is found to be

$$U_{2,0} = s_2^{1} s_0^{1} \sum_{i=1}^{S} \sum_{a=1}^{2} \alpha^a A_2^i \bar{V}_i^{,a} A_0^{1}.$$

(7)

The next step is to calculate the generalized Kraus operators $M_{2,0}^{i}$, defined by

$$M_{2,0}^{i} \equiv \bar{A}_2^i U_{2,0} = s_2^{1} s_0^{1} \sum_{a=1}^{2} \alpha^a V_i^{,a} A_0^{1}, \quad i = 1, 2, \ldots, S.$$

(8)

In principle, there are $2^S$ such operators, counting multiple coincidence cases, but in the case of the DS experiment, we know we are dealing with single photon interference. This means that only the $S$ one-signal Kraus operators are non-zero, as given by (8) and there is one of these for each of the screen detecting sites $A_2^i$.

Next, we construct the generalized POVM elements $E_{2,0}^{i}$ from the generalized Kraus operators. By definition, $E_{2,0}^{i} = M_{2,0}^{i} M_{2,0}^{i}$, where there is no sum over $i$. We find

$$E_{2,0}^{i} = s_0^{1} s_0^{1} \sum_{a,b=1}^{2} \delta^a \bar{\delta}^b V_i^{,a} V_i^{,b} A_0^{1} \bar{A}_0^{1}.$$

(9)

By construction these are all positive operators. As a check on the physical correctness of our formalism, we can use the semi-unitary relations [5] to show that

$$\sum_{i=1}^{S} E_{2,0}^{i} = s_0^{1} s_0^{1} A_0^{1} \bar{A}_0^{1} = I_0^{\text{EFF}},$$

(10)

where $I_0^{\text{EFF}}$ is the effective identity operator for stage $\Omega_0$.

The outcome signal rates $\Pr(A_2^{i} | \Psi_0)$ are defined by $\Pr(A_2^{i} | \Psi_0) = \Psi_0 E_{2,0}^{i} \Psi_0$ and are found to be

$$\Pr(A_2^{i} | \Psi_0) = |\Psi_0^{1}|^2 \sum_{a,b=1}^{2} \alpha^a \bar{\delta}^b V_i^{,a} V_i^{,b}.$$

(11)

These rates contain the quantum interference contributions expected from quantum principles. Using the semi-unitarity relations [5], these rates add up as expected to the beam production rate $|\Psi_0^{1}|^2$, as required from probability conservation.

This completes our analysis of the double slit experiment.

### III. DELAYED-CHOICE QUANTUM ERASER

We turn now to the delayed-choice quantum eraser. The basic experiment is shown in Figure 2 [5]. A photon source $A_1^{1}$ sends a beam onto a crystal which is arranged to produce a superposition of coherent photon pairs associated with sites $A$ and $B$ as shown. One component of each pair is collimated and passed onto a screen containing detectors $A_2^{1}, A_2^{2}, \ldots, A_2^{S}$, exactly as for the DS experiment discussed in the previous section. In effect, positions $A$ and $B$ act as a pair of slits for a DS experiment.
FIG. 2: The delayed-choice quantum eraser experiment

In this case, however, the situation is more complicated. For each pair, one component is passed into the DS screen whilst the other passes into a prism $P$ which deflects it onto a beam-splitter. Component $A^2_1$ is passed onto beam-splitter $BS_1$ whilst component $A^2_2$ is passed onto beam-splitter $BS_2$. Each of these beam-splitters acts on its own incident beam and splits it further into two. In each case, the reflected component is passed onto a photon detector, either at $A^1_3$ or at $A^1_4$, whilst the transmitted component is passed onto a third beam-splitter $BS_3$, where interference takes place, with subsequent detection at $A^2_3$ and $A^2_4$.

Various discussions of this arrangement suggest that choices made by the experimentalist at $BS_3$ can influence the interference patterns seen on the screen containing $A^1_2, \ldots, A^1_5$, even though the signals in that screen may have been captured much earlier.

We proceed with our stage analysis as follows. As with the DS experiment, we represent our initial source at stage $\Omega_0$ by the labstate $\Psi_0 = \Psi_0^1 A_0^1$.

The next stage $\Omega_1$ is defined by the production of two correlated photon pairs. These pairs are each assumed spinless. The formalism can readily deal with any situation where this is not the case. The first pair is generated at point $A$ on the crystal whilst the other pair is generated at point $B$. The transformation from $\Omega_0 \rightarrow \Omega_1$ is given by

$$U_{1,0} s_0^1 A_0^1 = \frac{1}{\sqrt{2}} \left[ s_1^L s_1^R A_0^1 A_1^1 + s_1^L s_1^R A_0^1 A_1^2 \right] + \frac{\beta}{\sqrt{2}} \left[ s_1^L s_1^R A_0^1 A_1^3 + s_1^L s_1^R A_0^1 A_1^4 \right]$$

(12)

where $|\alpha|^2 + |\beta|^2 = 1$ and $s_1^L R$ represents the spin state of a left-handed circularly polarized photon moving along direction $A_1^1$, and so on. Here terms such as $A_1^1 A_1^2$ represent the two-signal labstate $A_1^+ \Psi_0^1 A_0^1\{0, n\}$. Hence the effective transition operator is

$$U_{1,0} = s_1^1 s_0^1 \left[ \alpha A_1^1 A_1^1 + \beta A_1^2 A_1^2 \right]$$

(13)

where

$$s_n^1 = \frac{1}{\sqrt{2}} \left[ s_n^L s_n^2 R + s_n^L R s_n^2 L \right]$$

(14)

represents an entangled two photon state of total angular momentum zero at stage $\Omega_1$, $n = 1, 2, 3$. Although individual photon wave components get channelled into four possible directions as shown, the internal total angular moment state remains unaffected during this particular experiment.

The next stage change is from $\Omega_1$ to $\Omega_2$ and given by

$$U_{2,1} s_1^1 A_1^1 A_1^2 = s_1^1 s_2^1 \sum_{i=1}^S V_{i-A} A_2^i \left( t_1 A_2^S + i r_1 A_2^{S+1} \right)$$

$$U_{2,1} s_1^1 A_1^3 A_1^4 = s_1^1 s_2^1 \sum_{i=1}^S V_{i-B} A_2^i \left( t_2 A_2^{S+3} + i r_2 A_2^{S+4} \right).$$

Here, one component beam from each pair is focused on the detecting screen whilst the other component is channelled onto either beam-splitter $BS_1$ or $BS_2$, as shown. The $\{V_{i-A}\}$ represents the amplitudes for landing on the screen from the pair sourced from point $A$, and similarly for the pair sourced from point $B$. The coefficients $t_i, r_i$ are characteristic transmission and reflection parameters associated with $BS_i$. It is very useful not to set these parameters to the conventional value $1/\sqrt{2}$ but to keep them open and available to be changed. It is in these parameters that we shall encode the observer’s freedom of choice in this particular experiment.

From the above we find the effective transition operator

$$U_{2,1} = s_2^1 s_1^1 A_1^1 A_1^2 \left[ V_{i-A} \{t_1 A_2^S + i r_1 A_2^{S+1}\} A_1^2 + V_{i-B} \{t_2 A_2^{S+3} + i r_2 A_2^{S+4}\} A_1^4 \right].$$

(16)

The final transition is from stage $\Omega_2$ to $\Omega_3$ and involves four terms:

$$U_{3,2} s_2^1 A_2^1 A_2^2 S^{+1} = s_2^1 A_3^1 A_3^{S+1},$$

$$U_{3,2} s_2^1 A_2^3 A_2^4 S^{+2} = s_2^1 A_3^3 [A_3^1 A_3^{S+3} + i r_3 A_3^{S+2}]$$

$$U_{3,2} s_2^1 A_2^5 A_2^{S+3} = s_2^1 A_3^5 [A_3^1 A_3^{S+2} + i r_3 A_3^{S+3}]$$

$$U_{3,2} s_2^1 A_2^7 A_2^{S+4} = s_2^1 A_3^7 A_3^{S+4}.$$  

(17)

This gives

$$U_{3,2} = s_2^1 \sum_{i=1}^S A_i^1 A_i^2 [A_i^{S+1} A_i^{S+1} + \{t_3 A_i^{S+3} + i r_3 A_i^{S+2}\} A_i^{S+2}$$

$$+ \{t_3 A_i^{S+2} + i r_3 A_i^{S+3}\} A_i^{S+3} + A_i^{S+4} A_i^{S+4}]$$

(18)
The complete evolution operators is given by $U_{3.0} \equiv U_{3.2}U_{2.1}U_{1.0}$. Using the above results we find
\[
U_{3.0} = s_3^{1/2} \sum_{i=1}^S A_i \left\{ \frac{1}{2} \left[ \begin{array}{c} \alpha V^{i.A} r_1 A_3^{S+1} + \beta V^{i.B} r_2 A_3^{S+4} \\
+ |i_r V^{i.A} t_1 \alpha + t_3 V^{i.B} t_2 \beta| A_3^{S+2} \\
+ |t_3 V^{i.A} t_1 \alpha + i_r V^{i.B} t_2 \beta| A_3^{S+3} \end{array} \right] \right\} A_0.
\]

(19)

There are four Kraus operators associated with each detector on the screen, each of the form
\[
M_{3.0,0}^{S+k} = \bar{A}_3^{S+k} U_{3.0}, \quad i = 1, 2, \ldots, S, \quad k = 1, 2, 3, 4.
\]

(20)

We find
\[
M_{3.0,0}^{S+1} = s_3^{1/2} \bar{A}_3^{S+1},
M_{3.0,0}^{S+2} = s_3^{1/2} (r_3 V^{i.A} t_1 \alpha + t_3 V^{i.B} t_2 \beta) \bar{A}_3^1,
M_{3.0,0}^{S+3} = s_3^{1/2} (i_r V^{i.A} t_1 \alpha + i_r V^{i.B} t_2 \beta) \bar{A}_3^1,
M_{3.0,0}^{S+4} = s_3^{1/2} (i_r V^{i.B}) r_2 \bar{A}_3^2.
\]

These give four POVMs associated with each detector on the screen:
\[
E_{3.0}^{i.S+1} = M_{3.0}^{i.S+1} M_{3.0}^{i.S+1} = r_1^2 |\alpha|^2 V^{i.A} |s_0^3 s_0^0 A_3^1 \bar{A}_3|,
E_{3.0}^{i.S+2} = M_{3.0}^{i.S+2} M_{3.0}^{i.S+2} = |i_r V^{i.A} t_1 \alpha + t_3 V^{i.B} t_2 \beta|^2 s_0^0 s_0^0 A_3^1 \bar{A}_3|,
E_{3.0}^{i.S+3} = M_{3.0}^{i.S+3} M_{3.0}^{i.S+3} = |t_3 V^{i.A} t_1 \alpha + i_r V^{i.B} t_2 \beta|^2 s_0^0 s_0^0 A_3^1 \bar{A}_3|,
E_{3.0}^{i.S+4} = M_{3.0}^{i.S+4} M_{3.0}^{i.S+4} = r_2^2 |\beta|^2 V^{i.B} |s_0^3 s_0^0 A_3^1 \bar{A}_3|.
\]

It is straightforward to check that
\[
\sum_{i=1}^S \sum_{k=1}^4 E_{3.0}^{i.S+k} = s_3^{1/2} s_3^{1/2} \bar{A}_3^{S+k} A_3^1 = I_0^{\text{Eff}},
\]

the effective identity operator for the initial stage Hilbert space.

There are four coincidence rates $\Pr(A_3^1 A_3^{S+k} | \Psi_0)$ associated with each detector on the screen, involving one of the detectors $A_3^{S+k}$, $k = 1, 2, 3, 4$. These rates are defined by
\[
\Pr(A_3^1 A_3^{S+k} | \Psi_0) = \Psi_0 E_{3.0}^{i.S+k} \Psi_0.
\]

(24)

We find
\[
\Pr(A_3^1 A_3^{S+1} | \Psi_0) = |\Psi_0|^2 r_1^2 |\alpha|^2 |V^{i.A}|^2,
\Pr(A_3^1 A_3^{S+2} | \Psi_0) = |\Psi_0|^2 |i_r V^{i.A} t_1 \alpha + t_3 V^{i.B} t_2 \beta|^2,
\Pr(A_3^1 A_3^{S+3} | \Psi_0) = |\Psi_0|^2 |t_3 V^{i.A} t_1 \alpha + i_r V^{i.B} t_2 \beta|^2,
\Pr(A_3^1 A_3^{S+4} | \Psi_0) = |\Psi_0|^2 r_2^2 |\beta|^2 |V^{i.B}|^2.
\]

(25)

There are several observations to be made about these results.

1. The parameters $t_i, r_1$ for beam-splitter $B_S$ represent places in the apparatus where the experimentalist could make changes, either before or after signals have been registered on the screen during any given run of the experiment. In other words, choices can be made at $BS_1$, $BS_2$ and $BS_3$ which affect various incidence rates. The question is, does any change made by the experimentalist at any beam-splitter affect anything that has been measured before that change was made? In particular, can any change in $BS_3$ affect what has already happened on the screen?

By inspection of (20), we see that no change in $t_3$ or $r_3$, subject to $t_3^2 + r_3^2 = 1$, has any effect whatsoever on $\Pr(A_3^1 A_3^{S+1} | \Psi_0)$ or $\Pr(A_3^1 A_3^{S+4} | \Psi_0)$. These coincidence rates actually involve signal detection completed during earlier stages. The conclusion therefore is that any suggestion that delayed-choice can erase information acquired in the past is false and misleading.

2. It is true that changes in $t_3$ and $r_3$ affect $\Pr(A_3^1 A_3^{S+2} | \Psi_0)$ and $\Pr(A_3^1 A_3^{S+3} | \Psi_0)$. However no acausality is involved, because a coincidence rate is undefined until signals from both detectors involved have been counted. $\Pr(A_3^1 A_3^{S+2} | \Psi_0)$ and $\Pr(A_3^1 A_3^{S+3} | \Psi_0)$ cannot be measured until after the choice of $t_3$ and $r_3$.

Suggestion is that events in stage $\Omega_3$ could influence events in earlier stages do not take into account the crucial role of post-selection in such experiments. The proper way to understand what is happening is to view the role of the four detectors $A_3^{S+k}$ as a post-selection processing of data already accumulated on the screen.

3. If we look at the total counting rates at each of the four detectors $A_3^{S+k} \equiv \sum_{i=1}^S \Pr(A_3^i A_3^{S+1} | \Psi_0)$, $k = 1, 2, 3, 4$, we find
\[
\Pr(A_3^{S+1} | \Psi_0) = |\Psi_0|^2 r_1^2 |\alpha|^2,
\Pr(A_3^{S+2} | \Psi_0) = |\Psi_0|^2 (t_3^2 r_3^2 |\alpha|^2 + t_2^2 r_2^2 |\beta|^2),
\Pr(A_3^{S+3} | \Psi_0) = |\Psi_0|^2 (t_3^2 r_3^2 |\alpha|^2 + t_2^2 r_2^2 |\beta|^2),
\Pr(A_3^{S+4} | \Psi_0) = |\Psi_0|^2 r_2^2 |\beta|^2.
\]

(26)

Using the semi-unitarity of the $\{V^{i.A}\}$ and $\{V^{i.B}\}$ coefficients. Again, changes made at $BS_3$ would have no effect on $\Pr(A_3^{S+1} | \Psi_0)$ or $\Pr(A_3^{S+4} | \Psi_0)$.

4. If we look at the total count rate for a given detector on the screen, we find
\[
\Pr(A_3^i | \Psi_0) = \sum_{k=1}^4 \Pr(A_3^i A_3^{S+k} | \Psi_0)
= |\Psi_0|^2 (|\alpha|^2 |V^{i.A}|^2 + |\beta|^2 |V^{i.B}|^2),
\]

(27)

which shows that no changes at any of the beam-splitters affects the pattern observed on the screen.
5. Significantly, changes made at either B$S_1$ and/or B$S_2$ would have an effect on the counting rates at $A_3^{S+2}$ and $A_3^{S+3}$. That is physically possible because B$S_1$ and B$S_2$ are involved in stage $\Omega_2$, which is earlier than $\Omega_3$.

6. This particular experiment is a good one to illustrate the concept of *stage*. None of the detectors $A_i^n$ is assumed to have an enduring identity throughout time. In that sense, they do not represent the devices *per se* constructed in a laboratory, which usually persist as physical objects during many separate runs of an experiment. Rather, the $A_i^n$ represent a potential for information transfer between the observer and the apparatus in stage $\Omega_n$. As discussed in our account of the Fanson-Bell experiment [11], context in the form of which-path information can determine the dynamics of the $A_i^n$.

The observer, who is controlling the apparatus, has the freedom to decide whether or not to actually look at a given detector at any given time to see if a photon has been registered or not. In the quantum eraser experiment, $A_1^3, A_2^3, A_3^3$ and $A_4^3$ are all in the same stage, even though their individual actual laboratory times could be very different.

What is most remarkable about quantum processes is that if a signal is not observed at a given detector at a given time, that detector can act as a source for signals observed later on. Moreover, quantum rules tells us to add signal amplitudes whenever several such detectors are involved, as in the DS experiment and then take the square modulus in order to calculate relative probabilities.

**IV. WHICH-PATH MEASURE**

The double-slit and eraser experiments discussed above belong to an important class of experiment which, to use colloquial terminology, provide partial or complete information about which path a photon had taken in its journey from initial to final stages. Another important experiment which belongs to this category is the Fanson-Bell experiment, which we have recently discussed [11]. We shall discuss below another example, Wheeler’s delayed-choice experiment.

Each of these experiments carries with it contextual attributes arising from the experimental setup which determine the extent to which paths can be determined from the data or not. For example, the double-slit experiment with both slits open gives zero information about which slit a particular detected photon came from. On the other hand, the same setup with one of the slits blocked up gives us total information as to where any of the detected photons originated.

It is of interest therefore to find some measure or parameter $\Phi$ which is characteristic of any given experimental setup and which gives us an indication as to how much which-path information we could extract. In the absence of any deeper analysis, possibly based on entropic grounds, our choice is to define $\Phi$ as the total probability of determining for sure full path information from a single detected photon, i.e.,

$$\Phi \equiv \text{Prob}(\text{full path information|single photon anywhere})$$

(28)

In the case of the double-slit experiment discussed above we find $\Phi_{DS} = 0$. On the other hand, in the case of the delayed-choice quantum eraser discussed above, we find

$$\Phi = \frac{\text{Pr}(A_3^{S+1} | \Psi_0) + \text{Pr}(A_3^{S+4} | \Psi_0)}{|\Psi_0|2} = r_1^2 |\alpha|^2 + r_2^2 |\beta|^2.$$  

(29)

In the conventional symmetric situation when $r_1 = r_2 = 1/\sqrt{2}$, $\Phi = 1/2$ as we should expect. When $r_1 = r_2 = 0$, any single photon detected in an off-screen detector would occur only either in $A_3^{S+2}$ or else $A_3^{S+3}$ and no path information could normally be obtained. However, there is a pathology in this case, because $r_1 = r_2 = 0$ gives $\text{Pr}(A_3^{S+2} | \Psi_0) = |\Psi_0|^2 \{t_3^2 |\alpha|^2 + t_2^2 |\beta|^2\}$ and $\text{Pr}(A_3^{S+3} | \Psi_0) = |\Psi_0|^2 \{t_2^2 |\alpha|^2 + t_3^2 |\beta|^2\}$. If the experimentalist had set $t_3 = 0$ or $t_3 = 0$, then a single photon detected at $A_3^{S+2}$ or $A_3^{S+3}$ would now give information about which path had been taken. Of course, this is equivalent to having no beam-splitters and is therefore of limited value.

In the next section we shall discuss Wheeler’s delayed-choice experiment and determine the which-path parameter for it.

**V. WHEELER’S DELAYED-CHOICE EXPERIMENT**

Wheeler’s delayed-choice experiment can be regarded as a double-slit experiment with a modified screen. Some
of the detectors can receive quantum signals from both slits, whilst the others can receive a signal from only one of the slits. The interest this experiment generates comes from the possibility that the observer can decide in principle which detector receives which signal/s after light has left the two slits. A recent experiment which confirms quantum expectation was done with a Mach-Zehnder interferometer, such that the final beam-splitter could be removed whilst the light was on its way from the first beam-splitter [2].

An idealized version of this experiment is shown in Figure 3. The details are much the same as the DS experiment studied first, but with the difference that now there are three groups of detectors on the screen. \( A_2 \) to \( A_3 \) can each receive a quantum amplitude from \( A_1 \) only, \( A_2^{R+1} \) to \( A_2^{R+S} \) can each receive quantum amplitudes from \( A_1 \) and from \( A_2 \), and from \( A_2^{R+S+1} \) to \( A_2^{R+S+T} \) can each receive a quantum amplitude from \( A_2^2 \) only. We can imagine that the experimentalist can shuffle the values of \( R, S \) and \( T \) during any given run after they were sure that light had left the two slits and before any impact on the screen. Of course, any actual experiment would require a lot of analysis of the data, post-selecting signals corresponding to equivalent values of \( R, S \) and \( T \).

We can encode the dynamics by writing

\[
U_{2,1} s_1 A_1^a = s_2^1 \sum_{i=1}^{R+S+T} V^{i,a} A_2^i, \quad a = 1, 2, \quad (30)
\]

with the condition that

\[
V^{i,1} = 0, \quad i > R + S, \quad V^{i,2} = 0, \quad i \leq R. \quad (31)
\]

The semi-unitarity relations are then equivalent to

\[
\sum_{i=1}^{R+S} |V^{i,1}|^2 = 1, \quad \sum_{i=R+1}^{R+S} \bar{V}^{i,1} V^{i,2} = 0, \quad \sum_{i=R+S+1}^{R+S+T} |V^{i,2}|^2 = 1. \quad (32)
\]

Applying the results found for the DS experiment we find

\[
\begin{align*}
\Pr(A_2^i | \Psi_0) &= |\Psi_0^{1,2}|^2 \alpha^1 V^{i,1}, \quad 1 \leq i \leq R, \\
P(A_2^i | \Psi_0) &= |\Psi_0^{1,2}|^2 \sum_{a,b=1}^{2} \bar{a}^a a^b \bar{V}^{i,a} V^{i,b}, \quad R < i \leq R + S, \\
P(A_2^i | \Psi_0) &= |\Psi_0^{1,2}|^2 |\alpha^2 V^{i,2}|, \quad R + S < i \leq R + S + T. \quad (33)
\end{align*}
\]

From this we find the which-path parameter to be

\[
\Phi = |\alpha^1|^2 \sum_{i=1}^{R} |V^{i,1}|^2 + |\alpha^2|^2 \sum_{i=R+S+1}^{R+S+T} |V^{i,2}|. \quad (34)
\]

This reduces to unity when \( S = 0 \) as expected and zero when both \( R \) and \( T \) are zero.

The recent experiment of Jacques et al [2] is equivalent to the above scenario with \( R + S + T = 2 \); the configuration with the second beam-splitter removed corresponds to \( R = T = 1, S = 0 \), whilst that with the second beam-splitter in operation corresponds to \( R = T = 0, S = 2 \).

The above experiments have not involved photon spin significantly. The experiment we discuss next requires a careful analysis of spin.

Prior to the delayed-choice quantum eraser experiment of Jacques et al [2], the double-slit quantum eraser experiment of Walborn et al [3] had demonstrated the empirical validity of the stage concept in quantum mechanics. Their experiment is discussed in two parts.

A. No polarization control

The first part of the experiment is shown in Figure 4. A spinless photon pair is produced, with one photon \( p \) passed onto a double-slit and then onto a screen, whilst the other photon \( s \) is passed onto a detector. Coincidence measurements are taken involving fixed position screen impacts and \( p \) photon detection, with no polarization input involved.

With an initial state \( \Psi_0 = \Psi_0^{1} s_0^2 A_1^1 \), the evolution from \( \Omega_0 \rightarrow \Omega_1 \) is given by the operator

\[
U_{1,0} = \frac{1}{\sqrt{2}} \{ s_1^H s_1^p V + s_1^V s_1^p H \} s_0^1 A_1^1 \bar{A}_0^3, \quad (35)
\]

where \( H \) and \( V \) are the horizontal and vertical polarization degrees of freedom. The next step is \( \Omega_1 \rightarrow \Omega_2 \), with evolution operator

\[
U_{2,1} = \sum_{a=1}^{2} \alpha^a \{ s_2^H s_2^p V s_1^H s_1^p V + s_2^V s_2^p V s_1^H s_1^p V \} A_2^a \bar{A}_2^a \bar{A}_1^3 \bar{A}_1^1, \quad (36)
\]

where as with the basic double slit experiment discussed in §II, \( |\alpha^1|^2 + |\alpha^2|^2 = 1 \).

The final stage transition \( \Omega_2 \rightarrow \Omega_3 \) is described by

\[
\begin{align*}
\end{align*}
\]
have equal and opposite effects, given by

\[ \Psi = s^1H \rightarrow s^{1L}, \quad s^1V \rightarrow i s^{1R}, \]
\[ s^{2H} \rightarrow s^{2R}, \quad s^2V \rightarrow -i s^{2L}, \quad (42) \]

where \( H, V \) represent horizontal and vertical plane polarizations, whilst \( R, L \) represent right-handed and left-handed circularly polarized states. In addition, the observer can insert a plane polarizer in front of the photon detector. Our formalism deals with this as if this were a choice.

The details of the evolution are the same as for the unpolarized situation up to stage \( \Omega_2 \), i.e., we have

\[ U_{2,0} = \frac{1}{\sqrt{2}}\left( s^2H \ s^2V + s^2V \ s^2H \right) \]
\[ = \frac{1}{\sqrt{2}}\left( \{i s^+_3 + s^-_3\} A_3^1 \times \{ s^+_3 A_3^1 - s^-_3 A_3^2X \} \right), \quad (43) \]

In the next step from \( \Omega_2 \rightarrow \Omega_3 \) we first transform to circularly polarized states and then rewrite them in terms of the linear polarization vectors \([+] \) and \([-] \), defined by

\[ |R\rangle = \frac{(1-i)}{2}\{|+\rangle + |-\rangle\}, \quad |L\rangle = \frac{(1-i)}{2}\{|+\rangle - |-\rangle\}. \quad (44) \]

Our conventions are exactly those in [2]. Then we find

\[ U_{3,2} s^2H s^2V A_2^1 A_2^1 = s^2L s^2V A_3^1 Y \]
\[ = \frac{1}{\sqrt{2}} \left( \{i s^+_3 + s^-_3\} A_3^1 \times \{ s^+_3 A_3^1 - s^-_3 A_3^2X \} \right), \quad (45) \]

etc., where the label \( X \) indicates a choice. If \( X = 1 \), that corresponds to no polarizer placed in front of the detector of the \( p \) photon, whereas \( X = 2 \) corresponds to an ability to detect two possible polarizations at that detector. The result is

\[ U_{3,2} = \frac{(1-i)}{2\sqrt{2}} \left( \begin{array}{c}
\{ s^+_3 A_3^1 - s^-_3 A_3^2X \} \times \\
\{ i s^+_3 + s^-_3 \} A_3^1 A_2^2 + A_3^2 A_2^2
\end{array} \right) \times \\
\left( \begin{array}{c}
\{ s^+_3 A_3^2 - s^-_3 A_3^2X \} \times \\
\{ i s^+_3 - s^-_3 \} A_3^3 A_2^3 + A_3^3 A_2^3
\end{array} \right). \quad (46) \]

The final evolution operator \( U_{4,3} \) involves screen impacts and is given by

\[ U_{4,3} = \sum_{a=1}^{S} \sum_{i=1}^{S} V_{i,a} A_4^i \left( s^{1+} s^{3+} + s^{1-} s^{3-} \right) \times \\
\left[ \{ s^{1+} s^{3+} A_4^1 A_3^1 + s^{1-} s^{3-} A_4^2 A_3^2X \} \right] \quad (47) \]

Hence we find

\[ U_{4,0} = \frac{(1-i)}{2} \sum_{i=1}^{S} A_4^i \times \\
\left\{ \alpha^1 V_{i,1} \left[ i s^+_4 A_4^1 A_3^1 - s^-_4 s^-_4 A_4^2X \right] + \alpha^2 V_{i,2} \left[ s^+_4 A_4^1 A_3^2 + i s^+_4 s^-_4 A_4^2X \right] \right\} \quad (48) \]

These demonstrate double-slit interference, because detection of the \( p \) photon provides no which-way information.

**B. Polarization control**

The experiment is now repeated with some modifications, shown schematically in Figure 5. Two quarter-wavelength polarizers \( P_1 \) and \( P_2 \) are introduced, \( P_1 \) in front of slit 1 and \( P_2 \) in front of slit 2. These polarizers have equal and opposite effects, given by

\[ \text{FIG. 5: Double-slit quantum eraser with polarization control.} \]
We are now in a position to make a choice as to what happens at the \( p \) detector. First, we remove the \( p \) detector polarizer.

### C. Case I: no erasure

For this scenario, we take \( X = 1 \). Then the total evolution operator is

\[
U_{4,0} = \frac{(1-i)}{2} \sum_{i=1}^{S} A_i^1 A_4^p \times \left\{ \alpha^1 V_{i,1}[i s_4^+ s_4^+ p^+ - s_4^- s_4^- p^-] + \alpha^2 V_{i,2}[s_4^+ p^+ - is_4^- s_4^- p^+] \right\} s_0^1 A_0^i,
\]

This gives the Kraus operators

\[
M_{4,0}^{i,p} = \frac{(1-i)}{2} \left\{ \alpha^1 V_{i,1}[i s_4^+ s_4^+ p^+ - s_4^- s_4^- p^-] + \alpha^2 V_{i,2}[s_4^+ p^+ - is_4^- s_4^- p^+] \right\} s_0^1 A_0^i,
\]

from which we construct the POVMs

\[
E_{4,0}^{i,p} = \{ |\alpha^1 V_{i,1}|^2 + |\alpha^2 V_{i,2}|^2 \} s_0^1 A_0^i A_0^i \quad (51)
\]

Hence the coincidence rates involving screen site \( i \) and the \( p \) detector are

\[
\text{Pr}(A_i^1 A_p^p | \Psi_0) = |\Psi_0|^2 \{ |\alpha^1 V_{i,1}|^2 + |\alpha^2 V_{i,2}|^2 \}, \quad i = 1, 2, \ldots, S,
\]

which show no interference. This is precisely what was observed by Walborn et al\[\text{3}\]. Essentially, placing \( P_1 \) and \( P_2 \) in front of their respective slits destroys in principle the lack of which-way information so evident in the conventional unpolarized double-slit experiment discussed earlier. In the current scenario, the experimentalist could if so desired have determined the spin of every photon impacting on the screen and thereby determine from which slit it had come. It is the mere possibility of doing this that destroys the interference pattern.

### D. Case II: erasure: \( X = 2 \)

Now we consider the effect of inserting a polarizing filter in front of the \( p \) detector. In this case, the presence of a polarizing filter with variable angle at the \( p \) detector is equivalent to placing a Wollaston prism there with two output beams with mutually orthogonal polarizations. In our approach, this is described in terms of two detectors, \( A_4^1 \) and \( A_4^2 \), rather than one.

In this scenario, the total evolution operator is given by setting \( X = 2 \) in (48):

\[
U_{4,0} = \frac{(1-i)}{2} \sum_{i=1}^{S} A_i^1 \times \left\{ \alpha^1 V_{i,1}[i s_4^+ s_4^+ A_4^p - s_4^- s_4^- A_4^p] + \alpha^2 V_{i,2}[s_4^+ A_4^p - i s_4^- s_4^- A_4^p] \right\} s_0^1 A_0^i.
\]

From this we find the Kraus operators

\[
M_{4,0}^{i,0} = \frac{(1-i)}{2} s_4^+ s_4^+ \{ \alpha^1 V_{i,1} + \alpha^2 V_{i,2} \} s_0^1 A_0^i,
\]

\[
M_{4,0}^{i,p} = \frac{(1-i)}{2} s_4^- s_4^- \{ \alpha^1 V_{i,1} + i \alpha^2 V_{i,2} \} s_0^1 A_0^i.
\]

and then the POVMs

\[
E_{4,0}^{i,0} = \frac{1}{2} |\alpha^1 V_{i,1}| + \alpha^2 V_{i,2}| s_0^1 A_0^i A_0^i \quad (54)
\]

\[
E_{4,0}^{i,p} = \frac{1}{2} |\alpha^1 V_{i,1} + i \alpha^2 V_{i,2}| s_0^1 A_0^i A_0^i. \quad (55)
\]

Hence the two coincidence transition rate patterns are given by

\[
\text{Pr}(A_i^1 A_p^p | \Psi_0) = \frac{1}{2} |\alpha^1 V_{i,1} + \alpha^2 V_{i,2}|^2 |\Psi_0|^2 \quad (56)
\]

These now show interference, with one showing what would normally be described as a fringe pattern whilst the other showing an antifringer pattern. Essentially, the insertion of the polarizer in front of the \( p \) photon detector erases the which-path information which previously gave a non-interference pattern on the screen.

Most significantly, Walborn et al repeated the experiment with the screen and \( p \) photon detection order reversed with significant time differences and found no change in the results. This is strong evidence for the validity of the stages concept in such quantum process.

### VII. CONCLUDING REMARKS

Our analysis supports the notion that quantum mechanics never actually involves causality. We should be worried if it did, for then our entire view of what probability and information represent would need drastic revision. However, several factors would appear to conspire to make it look otherwise. It is the case that some quantum interference experiments do suggest causality to the unwary. We have in mind here not only the delayed-choice scenarios discussed here but also the Franson-Bell experiment \[\text{9, 10, 11}\]. In that experiment, the lack of which-path information involves non-locality in time as well as non-locality in space in a most spectacular fashion.

However, detailed analysis always reveals the basic fact that interference phenomena arise from a lack of information about quantum states and have nothing specifically to do with the properties of particles per se. It may be reasonable to talk about "photon self-interference" when we know we are dealing with one-signal experiments, but as the Franson-Bell experiment and more recent ones demonstrate \[\text{3}\], two-photon state interference occurs under circumstances when individual photons simply do not "overlap".
Conceptual problems arise when our classical conditioning is relied on too much. We would like to believe in photons as particles and we would like to believe that time runs continuously. Both concepts have their uses, but quantum mechanics requires a generalization of both. In the case of the former, experiments tell us that we have to deal with interference of states not particles. In the case of the latter, we cannot expect quantum processes to evolve strictly according to an integrable timetable, such as coordinate time, or even the physical time in a laboratory. What is important is whether or not quantum information has been extracted. If it has been placed “on hold”, as can be seen in our analysis of the delayed-choice eraser and the double-slit eraser, then it can remain in a stage which could in principle persist until the end of the universe. This is one way of understanding unstable particles: an undecayed unstable particle is one trapped in an information bubble.

We end with a remarkable quote from experimentalists who have done real experiments in this area:

“Once more, we find that Nature behaves in agreement with the predictions of Quantum Mechanics even in surprising situations where a tension with Relativity seems to appear.”

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