Glueball decay in the Fock-Tani formalism

Mario L. L. da Silva*, Daniel T. da Silva*, Dimiter Hadjimichef†,* and Cesar A. Z. Vasconcellos*

*Instituto de Física, Universidade Federal do Rio Grande do Sul, Caixa Postal 15051, CEP 91501-970, Porto Alegre, RS, Brazil.
†Departamento de Física, Instituto de Física e Matemática, Universidade Federal de Pelotas, Campus Universitário, CEP 96010-900, Pelotas, RS, Brazil.

Abstract. We investigate the two-meson decay modes for $f_0(1500)$. In this calculation we consider this resonance as a glueball. The Fock-Tani formalism is introduced to calculate the decay width.

Keywords: Glueball decay; Fock-Tani Formalism

PACS: 12.39.Mk; 12.39.Pn

INTRODUCTION

The gluon self-coupling in QCD opens the possibility of existing bound states of pure gauge fields known as glueballs. Even though theoretically acceptable, the question still remains unanswered: do bound states of gluons actually exist? Glueballs are predicted by many models and by lattice calculations. In experiments glueballs are supposed to be produced in gluon-rich environments. The most important reactions to study gluonic degrees of freedom are radiative $J/\psi$ decays, central productions processes and antiproton-proton annihilation.

Numerous technical difficulties have so far been present in our understanding of their properties in experiments, largely because glueball states can mix strongly with nearby $q\bar{q}$ resonances [1],[2]. However recent experimental and lattice studies of $0^{++}$, $2^{++}$ and $0^{-+}$ glueballs seem to be convergent.

In the present we shall apply the Fock-Tani formalism [3] to glueball decay. In particular the resonance $f_0(1500)$ shall be considered. On theoretical grounds, a simple potential model with massive constituent gluons, namely the model of Cornwall and Soni [4],[5] has received attention recently [6],[7] for spectroscopic calculations. The results obtained are consistent with lattice and experiment.

THE FOCK-TANI FORMALISM

Now let us to apply the Fock-Tani formalism in the microscopic Hamiltonian to obtain an effective Hamiltonian. In the Fock-Tani formalism we can write the glueball and the meson creation operators in the following form

$$G_\alpha^\dagger = \frac{1}{\sqrt{2}} \Phi^{\mu\nu}_\alpha a^\dagger_\mu a^\dagger_\nu; \quad M^\dagger_\alpha = \Psi^{\mu\nu}_\alpha q^\dagger_\mu q^\dagger_\nu.$$
The gluon creation \( a_\mu^\dagger \) and annihilation \( a_\mu \) operators obey the following commutation relations \( [a_\mu,a_\nu] = 0 \) and \( [a_\mu,a_\nu^\dagger] = \delta_{\mu\nu} \). While the quark creation \( q_\nu^\dagger \), annihilation \( q_\mu \), the antiquark creation \( \bar{q}_\nu^\dagger \) and annihilation \( \bar{q}_\mu \) operators obey the following anticommutation relations \( \{q_\mu,q_\nu\} = \{\bar{q}_\mu,\bar{q}_\nu\} = \{q_\mu,q_\nu^\dagger\} = \{\bar{q}_\mu,\bar{q}_\nu^\dagger\} = \delta_{\mu\nu} \). In (1) \( \Phi_{\alpha}^{\mu\nu} \) and \( \Psi_{\alpha}^{\mu\nu} \) are the bound-state wave-functions for two-gluons and two-quarks respectively. The composite glueball and meson operators satisfy non-canonical commutation relations

\[
\begin{align*}
[G_\alpha,G_\beta] &= 0 ; 
[G_\alpha,G_\beta^\dagger] = \delta_{\alpha\beta} + \Delta_{\alpha\beta} \\
[M_\alpha,M_\beta] &= 0 ; 
[M_\alpha,M_\beta^\dagger] = \delta_{\alpha\beta} + \Delta_{\alpha\beta}
\end{align*}
\]

(2)

The “ideal particles” which obey canonical relations

\[
\begin{align*}
[g_\alpha,g_\beta] &= 0 ; 
[g_\alpha,g_\beta^\dagger] = \delta_{\alpha\beta} \\
[m_\alpha,m_\beta] &= 0 ; 
[m_\alpha,m_\beta^\dagger] = \delta_{\alpha\beta}
\end{align*}
\]

(3)

This way one can transform the composite state \( |\alpha\rangle \) into an ideal state \( |\alpha\rangle \), in the glueball case for example we have

\[
|\alpha\rangle = U^{-1} \left( -\frac{\pi}{2} \right) G_\alpha^\dagger |0\rangle = g_\alpha^\dagger |0\rangle
\]

where \( U = \exp(tF) \) and \( F \) is the generator of the glueball transformation given by

\[
F = \sum_\alpha g_\alpha^\dagger \tilde{G}_\alpha - \tilde{G}_\alpha g_\alpha
\]

(4)

with

\[
\tilde{G}_\alpha = G_\alpha - \frac{1}{2} \Delta_{\alpha\beta} G_\beta - \frac{1}{2} G_\beta^\dagger [\Delta_{\beta\gamma},G_\alpha] G_\gamma.
\]

In order to obtain the effective potential one has to use (4) in a set of Heisenberg-like equations for the basic operators \( g,\tilde{G},a \)

\[
\frac{dg_\alpha(t)}{dt} = [g_\alpha,F] = \tilde{G}_\alpha ; 
\frac{d\tilde{G}_\alpha(t)}{dt} = [\tilde{G}_\alpha(t),F] = -g_\alpha.
\]

The simplicity of these equations are not present in the equations for \( a \)

\[
\frac{da_\mu(t)}{dt} = [a_\mu,F] = -\sqrt{2} \phi^{\mu\nu}_{\beta} a_\nu^\dagger g_\beta + \sqrt{2} \phi^{\mu\nu}_{\beta} a_\nu^\dagger \Delta_\alpha a_\beta g_\beta + \phi^{\mu\nu}_{\beta} (a_\beta^\dagger g_\nu - a_\nu^\dagger g_\beta G_\beta) + \sqrt{2} (\phi^{\mu\rho}_{\alpha} \phi^{\rho\gamma}_{\beta} \phi^{\gamma\rho}_{\gamma} + \phi^{\mu\rho}_{\alpha} \phi^{\rho\gamma}_{\rho} \phi^{\gamma\rho}_{\gamma}) G^{\dagger}_{\gamma} a_\mu^\dagger G_\beta g_\beta.
\]
The solution for these equations can be found order by order in the wave functions. For zero order one has 
\[ a_\mu^{(0)} = a_\mu, \quad g_\alpha^{(0)}(t) = G_\alpha \sin t + g_\alpha \cos t \]
and \[ G_\alpha^{(0)}(t) = G_\beta \cos t - g_\beta \sin t. \]
In the first order \[ g_\alpha^{(1)} = 0, \quad G_\beta^{(1)} = 0 \]
and \[ a_\mu^{(1)}(t) = \sqrt{2} \Phi^\mu_\nu \Phi^\nu_\beta. \]
If we repeat a similar calculation for mesons let us to obtain the following equations solution:
\[ q_\mu^{(0)} = q_\mu, \]
\[ \bar{q}_\mu^{(0)} = \bar{q}_\mu, \]
\[ q_\mu^{(1)}(t) = \Psi^\mu_\nu \bar{q}_\nu m_\beta \]
and \[ \bar{q}_\mu^{(1)}(t) = -\Psi^\mu_\nu \bar{q}_\nu m_\beta. \]

**THE MICROSCOPIC MODEL**

The microscopic model adopted here must contain explicit quark and gluon degrees of freedom, so we obtain a microscopic Hamiltonian of the following form
\[ H = g^2 \int d^3x d^3y \Psi^\dagger(\vec{x}) \gamma^0 \gamma^i A_i^\mu(\vec{x}) \frac{\lambda^\mu_\nu}{2} \Psi(\vec{x}) \Psi^\dagger(\vec{y}) \gamma^0 \gamma^j A_j^\nu(\vec{y}) \frac{\lambda^\nu_\rho}{2} \Psi(\vec{y}) \]
(5)

Where the quark and the gluon fields are respectively \[ \Psi(\vec{x}) = \sum_s \int \frac{d^3k}{(2\pi)^3} [u(k,s)q(k,s) + v(-k,s)\bar{q}^\dagger(-k,s)]e^{i\vec{k} \cdot \vec{x}} \]
(6)
and
\[ A_i^\mu(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2a_k}} [a_i^\dagger(\vec{k}) + a_i^\dagger(-\vec{k})]e^{i\vec{k} \cdot \vec{x}} \]
(7)

We choose this Hamiltonian due to its form that allow to obtain a operators structure of this type \[ q^\dagger_q^\dagger q^\dagger_q^\dagger a a \]

**THE FOCK-TANI FORMALISM APPLICATION**

Now we are going to apply the Fock-Tani formalism to the microscopic Hamiltonian
\[ H_{FT} = U^{-1}HU \]
(8)
which gives rise to an effective interaction \( H_{FT} \). To find this Hamiltonian we have to calculate the transformed operators for quarks and gluons by a technique known as the equation of motion technique. The resulting \( H_{FT} \) for the glueball decay \( G \to mm \) is represented by two diagrams which appear in Fig. (1).

Analyzing these diagrams, of Fig. (1), it is clear that in the first one there is no color conservation. The glueball’s wave-function \( \Phi \) is written as a product
\[ \Phi^\mu_\nu = \chi^{s_p s_v}_{\alpha} C^{c\mu c\nu} \Phi^\mu_\nu \]
(9)
\[ \chi^{s_p s_v}_{\alpha} \] is the spin contribution, with \( A_\alpha \equiv \{ S_\alpha, S^3_\alpha \} \), where \( S_\alpha \) is the glueball’s total spin index and \( S^3_\alpha \) the index of the spin’s third component; \( C^{c\mu c\nu} \) is the color component.
given by $\frac{1}{\sqrt{8}} \delta^{\mu \nu}$ and the spatial wave-function is

$$\Phi_{\vec{P} \alpha} = \delta^{(3)}(\vec{P} - \vec{p}_\mu - \vec{p}_\nu) \left( \frac{1}{\pi b^2} \right)^{3/2} e^{-\frac{1}{8b^2}(\vec{p}_\mu - \vec{p}_\nu)^2}. \quad (10)$$

The expectation value of $r^2$ gives a relation between the rms radius $r_0$ and $\beta$ of the form $\beta = \sqrt{1.5/r_0}$. The meson wave function $\Psi$ is similar with parameter $b$ replacing $\beta$. To determine the decay rate, we evaluate the matrix element between the states $|i\rangle = g_\alpha |0\rangle$ and $|f\rangle = m_\beta^* m_\tau^* |0\rangle$ which is of the form $\langle f | H_{FT} | i \rangle = \delta(\vec{p}_\alpha - \vec{p}_\beta - \vec{p}_\gamma) \delta_{hf}$. The $h_{fi}$ decay amplitude can be combined with a relativistic phase space to give the differential decay rate

$$\frac{d\Gamma_{\alpha \rightarrow \beta \gamma}}{d\Omega} = 2\pi \frac{P E_\beta E_\gamma}{M_\alpha} |h_{fi}|^2. \quad (11)$$

After several manipulations we obtain the following result

$$\langle f | H_{FT} | i \rangle = \frac{8\alpha_s}{3\pi} \left( \frac{1}{\pi b^2} \right)^{3/4} \int dq \frac{q^2}{\sqrt{q^2 + m_q^2}} \left( 1 - \frac{q^2}{4m_q^2} - \frac{q^2}{4m_s^2} \right) e^{-\left(\frac{1}{2b^2} + \frac{1}{4b^2}\right)q^2} \quad (12)$$

Finally one can write the decay amplitude for the $f_0$ into two mesons

$$\Gamma_{f_0 \rightarrow M_1 M_2} = \frac{512\alpha_s^2}{9} \frac{P E_{M_1} E_{M_2}}{M_{f_0}} \left( \frac{1}{\pi b^2} \right)^{3/2} \mathcal{J}^2 \quad (13)$$

where

$$\mathcal{J} = \int dq \frac{q^2}{\sqrt{q^2 + m_q^2}} \left( 1 - \frac{q^2}{4m_q^2} - \frac{q^2}{4m_s^2} \right) e^{-\left(\frac{1}{2b^2} + \frac{1}{4b^2}\right)q^2} \quad (14)$$

with $m_q$ the $u$ and $d$ quark mass and $m_s$ the mass of the $s$ quark. The decays that are studied are for the following processes $f \rightarrow \pi \pi$, $f \rightarrow K\bar{K}$ and $f \rightarrow \eta \eta$. The parameters used are $b = 0.34$ GeV, $m_q = 0.33$, $m_q/m_s = 0.6$, $\alpha_s = 0.6$. 

FIGURE 1. Diagrams for glueball decay

\[ \text{FIGURE 1. Diagrams for glueball decay} \]
CONCLUSIONS

The Fock-Tani formalism is proven appropriate not only for hadron scattering but for decay. The example decay process $f_0(1500) \rightarrow \pi\pi; K\bar{K}$ and $\eta\eta$ in the Fock-Tani formalism is studied. The same procedure can be used for other $f_0(M)$ and for heavier scalar mesons and compared with similar calculations which include mixtures.

ACKNOWLEDGMENTS

The authors acknowledges support from the Fundação de Amparo à Pesquisa do Estado do Rio Grande do Sul - FAPERGS. M.L.L.S. acknowledges support from the Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq. D.T.S. acknowledges support from the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - CAPES.

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