The Aharonov-Casher effect for spin-1 particles in non-commutative quantum mechanics

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Abstract. By using a generalized Bopp’s shift formulation, instead of star product method, we investigate the Aharonov-Casher(AC) effect for a spin-1 neutral particle in non-commutative(NC) quantum mechanics. After solving the Kemmer equations both on a non-commutative space and a non-commutative phase space, we obtain the corrections to the topological phase of the AC effect for a spin-1 neutral particle both on a NC space and a NC phase space.

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1 Introduction

Recently, there has been an increasing interest in the study of physics on a non-commutative space. Apart from non-commutative field theories, there are many papers devoted to the study of various aspects of quantum mechanics on a NC space with usual (commutative) time coordinate [1]-[8]. For example, the Aharonov-Bohm phase on a NC space has been investigated in Refs.[1]-[8]. For example, the Aharonov-Bohm phase on a NC space has been investigated in Refs.[1]-[8]. The Aharonov-Casher phase for a spin-1 neutral particle on a NC space and a NC phase space has been studied in Refs.[1]-[8]. For example, the Aharonov-Bohm phase on a NC space has been investigated in Refs.[1]-[8].

It is interesting to obtain the corrections to the topological phase of the AC effect for a spin-1 neutral particle both on a NC space and a NC phase space by using the new method in Ref.[3].

This article is organized as follows: in section 2 by using the Lagrangian formulation, we discuss the AC effect on a commutative space. In section 3 we study the AC effect on a NC space and a NC phase space by using the new method in Ref.[3].

2 AC effect for spin-1 particles on a commutative space time

In this section by following Ref.[9] we review briefly the Aharonov-Casher effect of a spin-1 particle on a commutative space time. The Lagrangian for a free spin-1 particle of mass m is

\[ L = \frac{1}{2m} (\beta^\nu \partial_\nu - m) \phi, \]

where the 10×10 matrices \( \beta^\nu \) are generalization of the 4×4 Dirac gamma matrices, and it can be chosen as follows[9]-[12]

\[ \beta^0 = \begin{pmatrix} \hat{O} & \hat{O} & I & o^\dagger \\ \hat{O} & \hat{O} & \hat{O} & o^\dagger \\ I & \hat{O} & \hat{O} & o^\dagger \\ o & o & o & 0 \end{pmatrix}, \]

\[ \beta^j = \begin{pmatrix} \hat{O} & \hat{O} & \hat{O} & o^\dagger \\ \hat{O} & \hat{O} & S^j & o^\dagger \\ I & -S^j & \hat{O} & o^\dagger \\ -iK^j & o & o & 0 \end{pmatrix}, \]

with \( j = 1, 2, 3 \). The elements of the 10×10 matrices \( \beta^\nu \) are given by the matrices

\[ \hat{O} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \]

\[ S^1 = i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, S^2 = i \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, S^3 = i \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \]

\[ o = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}, \quad K^1 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, \]

\[ K^2 = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}, \quad K^3 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}. \]

The above \( \beta \) matrices satisfy the following relation

\[ \beta_\nu \beta_\lambda \beta_\rho + \beta_\rho \beta_\lambda \beta_\nu = \beta_\nu g_{\lambda \rho} + \beta_\rho g_{\nu \lambda}. \]

Other algebraic properties of the Kemmer \( \beta \)-matrices were given in Ref. [10]; the metric tensor is \( g_{\lambda \rho} = \text{diag}(1, -1, -1, -1) \). The Kemmer equation of motion is

\[ (i \beta^\nu \partial_\nu - m) \phi = 0. \]
The Lagrangian for a spin-1 neutral particle with a magnetic dipole moment \( \mu_m \) interacting with the electromagnetic field has the form

\[
L = \bar{\psi}(i\gamma^\nu \partial_\nu + \frac{1}{2}\mu_m S_{\lambda\rho} F^{\lambda\rho} - m)\psi,
\]

where \( F^{\lambda\rho} \) is the field strength of the electromagnetic field; \( S_{\lambda\rho} \) is the Dirac \( \sigma_{\lambda\rho} \) like spin operator, which can be defined as

\[
S_{\lambda\rho} = \frac{1}{2} (\beta_\lambda \beta_\rho - \beta_\rho \beta_\lambda).
\]

It follows that in the presence of an electromagnetic field, the Kemmer equation of motion of a spin-1 neutral particle with a magnetic moment \( \mu_m \) is

\[
(i\beta^\nu \partial_\nu + \frac{1}{2}\mu_m S_{\lambda\rho} F^{\lambda\rho} - m)\psi = 0.
\]

The aim is to find a solution of the above equation, which can be written in the following form

\[
\phi = e^{-i\xi_3 \int A' \cdot dr} \phi_0,
\]

where \( \phi_0 \) is a solution of (3); the spin-1 pseudo-vector operator \( \xi_\nu \) in (7) is defined as

\[
\xi_\nu = \frac{1}{2} i \epsilon_{\nu\lambda\rho\sigma} \beta^\lambda \beta^\rho \beta^\sigma,
\]

where \( \epsilon_{\nu\lambda\rho\sigma} \) is the Levi-Civita symbol in four dimensions. Now we need to find the explicit form of the vector \( A' \) in (7). To do this, first we write Eq. (3) for \( 0 \)

\[
(i\beta^\nu \partial_\nu - m) e^{-i\xi_3 \int A' \cdot dr} \phi = 0.
\]

Then the equivalence of (3) and (4) can be obtained by imposing the following two conditions

\[
e^{-i\xi_3 \int A' \cdot dr} \beta^\nu e^{i\xi_3 \int A' \cdot dr} = \beta^\nu,
\]

and

\[
- \beta^\nu \xi_3 A'_\nu \phi = \frac{1}{2} \mu_m S_{\lambda\rho} F^{\lambda\rho} \phi = \mu_m S_{\lambda\rho} F^{\lambda\rho} \phi
\]

By comparing Eq. (10) with the Baker-Housdorff formula

\[
e^{-i\xi_3 \nu \beta^\nu} e^{i\xi_3 \beta^\nu} = \beta^\nu + \nu (-i \xi_3) \beta^\nu + \frac{1}{2}\nu (-i \xi_3)^2 \beta^\nu + \ldots,
\]

one obtains, \( [\xi_3, \beta^\nu] = 0 \); \( \nu \) in (12) stands for the path ordering of the integral in the phase. If \( \nu \neq 3 \) this commutation relation is automatically satisfied. For \( \nu = 3 \), by using (2) and (8), one finds that the commutator does not vanish. Therefore in order to fulfill the first condition the particle is restricted to move in \( x - y \) plane, that is, \( p_z = 0 \). In particular \( \partial_3 \phi = 0 \) and \( A_3' = 0 \). From second condition (11), by using (2), (4) and (8), one obtains

\[
A'_1 = -2\mu_m E_2,
\]

\[
A'_2 = 2\mu_m E_1.
\]

Thus the AC phase for a neutral spin-1 particle moving in a 2+1 space time under the influence of a pure electric field produced by a uniformly charged infinitely long filament perpendicular to the plane is

\[
\phi_{AC} = \xi_3 \int A' \cdot dr = 2\mu_m \xi_3 \int (E_1 dx_2 - E_2 dx_1)
\]

\[
= 2\mu_m \xi_3 e^{i k} \int E_i dx_k.
\]

The above equation can also be written as in Ref. [9]

\[
\phi_{AC} = \xi_3 \int A' \cdot dr = \xi_3 \int (\nabla \times A') \cdot dS
\]

\[
= 2\mu_m \xi_3 \int (\nabla \cdot E) dS = 2\mu_m \xi_3 \lambda_e,
\]

where \( \lambda_e \) is the charge density of the filament. This spin-1 AC phase is a purely quantum mechanical effect and has no classical interpretation. One may note that the AC phase for spin-1 particles is exactly the same as in the case of spin-\( \frac{1}{2} \), except that the spin and spinor have changed. The factor of two shows that the phase is twice that accumulated by a spin-\( \frac{1}{2} \) particle with the same magnetic dipole moment coupling constant, in the same electric field.

3 AC effect for spin-1 particles on a non-commutative space

On a NC space the coordinate and momentum operators satisfy the following commutation relations (we take \( \hbar = 1 \) unit)

\[
[\hat{x}_i, \hat{x}_j] = i \Theta_{ij}, \quad [\hat{p}_i, \hat{p}_j] = 0, \quad [\hat{x}_i, \hat{p}_j] = i \delta_{ij},
\]

where \( \Theta_{ij} \) is an element of an antisymmetric matrix, it is related to the energy scale and it represents the non-commutativity of the NC space; \( \hat{x}_i \) and \( \hat{p}_i \) are the coordinate and momentum operators on a NC space.

By replacing the usual product in (3) with a star product (Moyal-Weyl product), the Kemmer equation for a spin-1 neutral particle with a magnetic dipole moment \( \mu_m \), on the NC space, can be written as

\[
(i\beta^\nu \partial_\nu + \frac{1}{2}\mu_m S_{\lambda\rho} F^{\lambda\rho} - m)\phi = 0.
\]

The star product between two functions is defined by

\[
(f \ast g)(x) = e^{i \Theta_{ij} \partial_i \hat{a}_j} f(x_i) g(x_j)
\]

\[
= f(x) g(x) + i \Theta_{ij} \partial_i f \partial_j g |_{x_i = x_j} + O(\Theta^2).
\]

(18)

here \( f(x) \) and \( g(x) \) are two arbitrary functions.

On a NC space the star product can be changed into an ordinary product by a Bopp’s shift, that is, by shifting coordinates \( x_\nu \) with

\[
x_\nu \rightarrow \hat{x}_\nu = x_\nu - \frac{1}{2} \Theta_{\nu\lambda} p^\lambda.
\]

(19)
Now, let us consider the non-commutative Kemmer equation (17). To replace the star product in (17) with an ordinary product, the $F_{\nu \lambda}$ must, up to the first order of the NC parameter $\Theta$, be shifted as

$$F_{\nu \lambda} \rightarrow \hat{F}_{\nu \lambda} = F_{\nu \lambda} + \frac{1}{2} \Theta^{\sigma \rho} p_{\sigma} \partial_{\rho} F_{\nu \lambda}.$$  

which is equivalent to the Bopp’s shift (13). Then the Kemmer equation on a NC space has the form

$$(i \beta^\nu \partial_\nu + \frac{1}{2} \mu m S_{\lambda \rho} \hat{F}^{\lambda \rho} - m) \phi = 0.$$  

In a similar way as the commuting space, the solution of the above equation can also be written as

$$\phi = e^{-i \xi_3 \int \hat{A}' \cdot dr} \phi_0.$$  

To determine $\hat{A}'$ we write the free Kemmer equation as

$$(i \beta^\nu \partial_\nu - m) e^{i \xi_3 \int \hat{A}' \cdot dr} \phi = 0.$$  

The equivalence of (21) and (23) gives the following two conditions

$$e^{-i \xi_3 \int \hat{A}' \cdot dr} \beta^\nu e^{i \xi_3 \int \hat{A}' \cdot dr} = \beta^\nu$$

and

$$- \beta^\nu \xi_3 A'_\nu \phi = \frac{1}{2} \mu m S_{\lambda \rho} \hat{F}^{\lambda \rho} \phi = \mu m S_{0 \nu} \hat{F}^{\nu 0} \phi.$$  

By using the Baker-Housdorf formula (12), the first condition (24) implies that, $[\xi_3, \beta^\nu] = 0$. If $\nu \neq 3$ then this commutation relation is automatically satisfied. For $\nu = 3$, by using (2) and (8), one finds that the commutator does not vanish. Therefore in order to fulfill the first condition we restrict ourselves to 2 + 1 space-time. In particular $\partial_3 \phi = 0$ and $A'_3 = 0$. From second condition (25) by using (2), (5) and (8), one obtains

$$\hat{A}'_1 = -2 \mu m \hat{F}^{02} = -2 \mu m F^{02} - \frac{1}{2} \Theta^{ij} p_i \partial_j F^{02},$$
$$\hat{A}'_2 = 2 \mu m \hat{F}^{01} = 2 \mu m F^{01} + \frac{1}{2} \Theta^{ij} p_i \partial_j F^{01},$$

with $\Theta^{ij} = \Theta^{ji}, \Theta^{00} = \Theta^{0 \nu} = 0; \epsilon^{ij} = -\epsilon^{ji}, \epsilon^{12} = +1$. Thus the AC phase for a neutral spin-1 particle moving in a 2 + 1 non-commutative space under the influence of a pure electric field produced by a uniformly charged infinitely long filament perpendicular to the plane is

$$\hat{\phi}_{AC} = \xi_3 \int A' \cdot dr = 2 \mu m \xi_3 \varepsilon^{lk} \int E_l d x_k + \mu m \xi_3 \Theta^{ij} \varepsilon^{lk} \int p_i \partial_j E_l d x_k.$$  

In a similar way as in spin-$\frac{1}{2}$ [4, 5], the momentum on a NC space for a spin-1 neutral particle can also be written as

$$p_i = m v_i + (E \times \mu)_i + O(\theta),$$  

where $\mu = 2 \mu m S$, and $S$ is the spin operator of the spin-1. By inserting (26) into (27), we have

$$\hat{\phi}_{AC} = \phi_{AC} + \delta \phi_{NCS},$$  

where $\phi_{AC}$ is the AC phase in (14) on a commuting space; the additional phase $\delta \phi_{NCS}$, related to the non-commutativity of space, is given by

$$\delta \phi_{NCS} = \mu m \xi_3 \varepsilon^{lk} \int [k_i - (\mu \times E)]_i \partial_j E_l d x_k.$$  

4 AC effect for spin-1 particles on a non-commutative phase space

In section 3 we have investigated the AC effect for a neutral spin-1 particle on a NC space, where space-momentum, and space-space are non-commuting, but momentum-momentum are commuting. The Bose-Einstein statistics in non-commutative quantum mechanics requires both space-space and momentum-momentum non-commuting. The NC space with non-commuting momentum-momentum is called NC phase space. In this section we study the AC phase on a NC phase space. On a NC phase space, the commutation relation in (16) should be replaced by

$$[\hat{p}_i, \hat{p}_j] = i \tilde{\Theta}_{ij},$$  

where $\tilde{\Theta}$ is the antisymmetric matrix, its elements represent the non-commutative property of the momenta. Then the Kemmer equation for AC problem on a NC phase space has the form

$$(- \beta^\nu p_\nu + \frac{1}{2} \mu m S_{\lambda \rho} \hat{F}^{\lambda \rho} - m) \phi = 0.$$  

The star product in (32) on a NC phase space can be replaced by the usual product in two steps, first we need to replace $x_i$ and $p_i$ by a generalized Bopp’s shift as

$$x_i \rightarrow \hat{x}_i = \alpha x_i - \frac{1}{2 \alpha} \Theta_{i \lambda} p^\lambda,$$
$$p_i \rightarrow \hat{p}_i = \alpha p_i + \frac{1}{2 \alpha} \tilde{\Theta}_{i \lambda} x^\lambda,$$

where $\alpha$ is the scaling parameter, and it is related to the non-commutativity of the phase space via $\Theta_{i \lambda} = 4 \alpha \tau (\alpha^2 -$...
1) \( 1 \cdot I \), here \( I \) is a unit matrix. Then we also need to rewrite the shift in (20) as

\[
F_{\nu \lambda} \rightarrow \tilde{F}_{\nu \lambda} = \alpha F_{\nu \lambda} + \frac{1}{2\alpha} \Theta^{\mu \nu} p_\mu \partial_\nu F_{\nu \lambda}.
\]

Thus the Kemmer equation for AC problem on a NC phase space has the form

\[
(\beta'' p_\nu + \frac{1}{2} \mu_\nu S_{\lambda \rho} \tilde{F}^{\lambda \rho} - m) \phi = 0.
\]

Since \( \alpha \neq 0 \), the above equation can be written as

\[
\left( -\beta'' p_\nu - \frac{1}{2\alpha} \beta'' \tilde{\Theta}^\nu_\rho x^\rho + \frac{1}{2} \mu_\nu S_{\lambda \rho} (\tilde{F}^{\lambda \rho} + \frac{1}{2\alpha} \Theta^{\mu \nu} p_\mu \partial_\nu \tilde{F}^{\lambda \rho}) - m' \right) \phi = 0.
\]

where \( m' = m/\alpha \). We write the above equation in the following form

\[
(\beta'' p_\nu - m') e^{-i\xi_3} \int A' \cdot d\phi = 0.
\]

To have the equivalence of (36) and (37), we impose the following two conditions

\[
e^{-i\xi_3} \int A' \cdot d\phi = \beta''
\]

and

\[
-\beta'' \xi_3 \hat{A}_i' \phi = \frac{1}{2\alpha} \mu_\nu S_{\lambda \rho} \tilde{F}^{\lambda \rho} \phi = \frac{\mu_\nu}{\alpha} \Theta^\nu \mu S_{\nu \lambda} \tilde{F}_i^0 01 \phi.
\]

In an analogous way as in NC space, from (38) and (39), one may obtain

\[
\hat{\mathcal{A}}_1' = -2 \mu_\mu \frac{\Theta^0}{\alpha} F^{02} = -2 \mu_\mu F^{02} - 2 \mu_\mu \frac{1}{2\alpha^2} \Theta^{ij} p_i p_j F^{02}
\]

\[
= -2 \mu_\mu E_2 - \frac{\mu_\mu \theta}{\alpha^2} \epsilon^{ij} p_i \partial_j E_2,
\]

\[
\hat{\mathcal{A}}_2' = 2 \mu_\mu \frac{\Theta^0}{\alpha} F^{01} = 2 \mu_\mu F^{01} + 2 \mu_\mu \frac{1}{2\alpha^2} \Theta^{ij} p_i \partial_j F^{01}
\]

\[
= 2 \mu_\mu E_1 + \frac{\mu_\mu \theta}{\alpha^2} \epsilon^{ij} p_i \partial_j E_1,
\]

\[
\hat{\mathcal{A}}_3' = 0.
\]

Thus the AC phase for a neutral spin-1 particle moving in a \( 2 + 1 \) non-commutative phase space under the influence of a pure electric field produced by a uniformly charged infinitely long filament perpendicular to the plane is given by

\[
\hat{\phi}_{AC} = \frac{1}{2\alpha^2} \int \Theta^\nu_\lambda x^\nu dx^\nu + \xi_3 \int A' \cdot dx
\]

\[
= \frac{\theta}{\alpha^2} E \int x_j dx_i + 2 \mu_\nu \xi_3 \epsilon^{jk} \int E_i dx_k
\]

\[
+ \mu_\nu \xi_3 \frac{\theta}{\alpha^2} \epsilon^{ij} \epsilon^{jk} \int p_i \partial_j E_0 dx_k.
\]

By \( p_i = k_i' + (E \times \mu)_i + O(\theta) \), and \( k_i' = m' v_i, \mu = 2 \mu_\nu S \), one obtains

\[
\hat{\phi}_{AC} = \phi_{AC} + \delta \phi_{NCS} + \delta \phi_{NCPS},
\]

where \( \phi_{AC} \) is the AC phase in (14) on a commuting space; \( \delta \phi_{NCS} \) is the space-space non-commuting contribution to the AC phase in (14), and its explicit form is given in (30); the last term \( \delta \phi_{NCPS} \) is the momentum-momentum non-commuting contribution to the AC phase in (14), and it has the form

\[
\delta \phi_{NCPS} = \frac{\theta}{2\alpha^2} \int \epsilon^{ij} x_j dx_i
\]

\[
+ \left( \frac{1}{\alpha^2} - 1 \right) \mu_\nu \xi_3 \epsilon^{ij} \epsilon^{jk} \int \left[ k_i' - (E)_i \right] \partial_j E_i dx_k.
\]

which represents the non-commutativity of the momenta. The first term in (33) comes from the momentum-momentum non-commutativity; the second term is a velocity dependent correction and does not have the topological properties of the commutative AC effect and could modify the phase shift; the third term is a correction to the vortex and does not contribute to the line spectrum. In two dimensional non-commutative plane, \( \Theta_{ij} = \theta \epsilon_{ij} \), and the two NC parameters \( \theta \) and \( \theta \) are related by \( \theta = 4\alpha^2 (1 - \alpha^2/\theta) \). When \( \alpha = 1 \), which leads to \( \theta = 0 \), the AC phase on a NC phase space case reduces to the AC phase on a NC space case, i.e., \( \delta \phi_{NCPS} = 0 \) and equation (24) changes into equation (29).

5 Conclusions

In this paper in order to study the AC effect both on a non-commutative space and on a non-commutative phase space, we use the shift method, instead of star product formulation. Our shift method is equivalent to the star product method, i.e., the Kemmer equation with star product can be replaced by Bopp’s shift together with the shift which we defined in (20) for a NC space and in (31) for a NC phase space. The additional AC phase in (29) on a NC space is the same as the result of Ref. (1), where the star product method has been used. Furthermore, by considering the momentum-momentum non-commutativity we obtained the NC phase space corrections to the topological phase of the AC effect for a spin-1 neutral particle. We note that the corrections (30) and (33) to the topological phase (14) or (15) of the AC effect for a spin-1 neutral particle both on a NC space and a NC phase space can be obtained from spin-\( \frac{1}{2} \) corrections through the replacement \( \gamma^0 \theta_{12} \rightarrow \xi_3 \). One may conclude that, apart from the spin operators, the AC phase for a higher spin neutral particle is the same as in the case of spin-\( \frac{1}{2} \) and spin-1 in non-commutative quantum mechanics.

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