$Nd \rightarrow ^3H\gamma$ with Effective Field Theory

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Abstract

The cross section of neutron-deuteron radiative capture $nd \rightarrow ^3H\gamma$ is calculated at energies relevant to Big-Bang nucleosynthesis ($20 \leq E \leq 200$ KeV) with pionless Effective Field Theory. At these energies, magnetic transition M1 gives the dominant contribution. The M1 amplitude is calculated up to next-to-next-to leading order (N$^2$LO) with insertion of three-body force. Results are in good agreement within few percent theoretical uncertainty in comparison with available calculated data below $E=200$ KeV.

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1 Introduction

Very low-energy radiative capture and weak capture reactions involving few-nucleon systems have considerable astrophysical relevance for studies of stellar structure evolution and big-bang nucleosynthesis. At these energies pionless Effective field theory (EFT) is an important tool for computing physical quantities.

Much of the strength of EFT lies in the fact that it can be applied without off shell ambiguities to systems with more nucleons. In past years, nuclear EFT has been applied to two-, three-, and four-nucleon systems [1-9] and recently developed pionless EFT is particularly suited to high-order and precision calculation. An example of a precise calculation is the reaction $np \rightarrow \gamma d$, which is relevant to big-bang nucleosynthesis (BBN), and the cross section for this process was computed to 1% error for center of mass energies $E \lesssim 1 \text{ MeV}$ [10].

On the other hand, The three-body EFT calculations have been so far confined to nucleon-deuteron system. For example nucleon-deuteron scattering in all channels except the $S_{1/2}$-wave can be calculated to high-orders using only two-nucleon input and the triton binding energy is found to be $B_3^{(EFT)} = 8.35 \text{MeV}$ in next to leading order close to the experimental $B_3^{(exp)} = 8.5 \text{MeV}$ [11]. Pionless EFT has been recently applied to the four-body system with contact interactions and large scattering length at leading order by Platter [12]. The binding energies of the $^4\text{He}$ tetramer and the alpha-particle have been calculated and it is in a good agreement with experimental value.

The radiative capture of neutrons and protons by deuterons and the inverse reactions, the photodisintegration of $^3\text{H}$ and $^3\text{He}$, have been investigated experimentally and theoretically over the last decades with some interest. In an experiment performed in recent years at TUNL [13, 14] the total cross section and vector and tensor analyzing power of the $pd \rightarrow ^3\text{He} \gamma$ process were measured at the center of mass energies below 55 KeV.

The theory of the $nd \rightarrow ^3\text{H} \gamma$ capture reaction has a long history. The $nd$ doublet or quartet state M1 transition calculated in Impulse Approximation, and explanation of smallness of its cross section when compared to the $np \rightarrow d \gamma$ reaction, $\sigma_T = 334.5 \pm 0.5 \text{mb}$, were pointed out by Schiff [15]. Later, Phillips [16] emphasized the importance of initial state interactions and two-body currents to capture reaction in the three-body model calculation, by considering a central, separable interaction. In recent years, a series of calculations of increasing sophistication with regard to the description of both the initial and final state wave functions and two-body current model were carried out [17]. These efforts culminated in the 1990 Friar et al. [18] calculation of the $nd \rightarrow ^3\text{H} \gamma$ total cross section, quartet capture fraction, and photon polarization, based on converged bound and continuum state Faddeev wave functions, corresponding to a variety of realistic Hamiltonian models with two- and three-nucleon interactions, and a nuclear electromagnetic current operator, including the long-range two-body components associated with pion exchange and the virtual excitation of intermediate $\Delta$ resonances.

For very low energy the $p-d$ and $n-d$ radiative capture, a magnetic dipole (M1) transition is a dominant contribution, which was studied in plane wave (Born) approximation by Friar et al. [18]. In these investigations the authors employed their configuration-space Faddeev calculations of the helium wave function, with inclusion of three-body forces and pion exchange
currents. Various trends, e.g., the correlation between cross sections and helium binding energies, and their potential dependence were pointed out. More recently a rather detailed investigation of such processes has been performed by Viviani et al. [19]. Their calculations employed the quite accurate three-nucleon bound- and continuum states obtained in the variational pair-correlated hyperspherical method, developed, tested and applied over years by this group.

We calculate very low energy cross section of radiative capture of neutrons by deuterons \( nd \rightarrow \gamma^3H \) at energies \( 20 \leq E \leq 200 \) KeV, relevant to Big-Bang nucleosynthesis, with pionless EFT. At these energies, magnetic transition M1 gives the dominant contribution. The M1 amplitude is calculated up to next-to-next-to leading order (N\(^2\)LO) with insertion of three-body force. Results show good agreement in comparison to ENDF [20] below \( E=200 \) KeV.

The organization of the paper is as follows: We first describe the relevant Lagrangian and scattering in doublet S-wave channel in Section 2. This section essentially introduced to define various parameters that enter in the expression for the cross section. The calculation of the total cross section is presented in sections 3. We tabulate the calculated cross sections for some energies relevant for BBN, discuss the theoretical errors, and compare our results with the corresponding values from the on-line ENDF/B-VI database [20] in Section 4. Summary and conclusions follow in Section 5.

2 \( ^2S_{1/2} \) neutron-deuteron scattering (triton channel)

The \( ^2S_{1/2} \) channel to which \(^3\)He and \(^3\)H belong is qualitatively different from the other three-nucleon channels. Consequently, \( ^2S_{1/2} \) describes the preferred mode for \( nd \rightarrow ^3H \gamma \) and \( pd \rightarrow ^3He \gamma \) processes. This difference can be traced back to the effect of the exclusion principle and the angular momentum repulsion barrier. In all the other channels, it is either the Pauli principle or an angular momentum barrier (or both) which forbids the three-particle to occupy the same point in space. As a consequence, the kernel describing the interaction among the three-nucleon, unlike in the bosonic case, is repulsive in these channels. The zero mode of the bosonic case, describes a bound state since it is a solution of the homogeneous version of the Faddeev equation. As such, it is not expected to appear in the case of repulsive kernels and, in fact, it does not. For the \(^3\)He or \(^3\)H channel however, the kernel is attractive and, as we will see below, closely related to the one in the bosonic case [11].

Let us first discuss the integral equation describing nucleon-deuteron scattering. We start with the three-nucleon lagrangian which is given by [11]:

\[
\mathcal{L} = N^\dagger \left( i \partial_0 + \frac{\nabla^2}{2M} \right) N + d_s^A d_s^{A\dagger} \left( -i \partial_0 - \frac{\nabla^2}{4M} + \Delta_s \right) d_s^A + d_t^\dagger \left( -i \partial_0 - \frac{\nabla^2}{4M} + \Delta_t \right) d_t^\dagger \\
+ t^\dagger \left( i \partial_0 + \frac{\nabla^2}{6M} + \frac{\gamma^2}{M} + \Omega \right) t - g_s \left( d_s^A (N^T P^A N) + \text{H.c.} \right) - g_t \left( d_t^\dagger (N^T P^t N) + \text{H.c.} \right) \\
- \omega_s \left( t^\dagger (\sigma^A N) d_s^A + \text{H.c.} \right) - \omega_t \left( t^\dagger (\sigma^t N) d_t^\dagger + \text{H.c.} \right) + \ldots ,
\]

(2.1)
where $N$ is the nucleon iso-doublet and the auxiliary fields $t$, $d^A_s$ and $d^i_t$ carry the quantum numbers of the $^3$He-$^3$H spin and isospin doublet, $^1S_0$ di-nucleon and the deuteron, respectively. The projectors $P^i$ and $P^A$ are defined by:

$$
P^i = \frac{1}{\sqrt{8}} \tau_2 \sigma_2 \sigma^i,
$$

$$
P^A = \frac{1}{\sqrt{8}} \sigma_2 \tau_2 \tau^A,
$$

where $A = 1, 2, 3$ and $i = 1, 2, 3$ are iso-triplet and vector indices and $\tau^A (\sigma^i)$ are isospin (spin) Pauli matrices.

One can write the Faddeev integral equation in the kinematics defined by the two cluster-configurations exist in the three-nucleon system. We follow the notation suggested by Grießhammer in [21]. The $Nd_t$-cluster with total spin $S = \frac{3}{2}$ or $S = \frac{1}{2}$, depending on whether the deuteron and nucleon spins are parallel or anti-parallel; and the $Nd_s$-cluster which has total spin $S = \frac{1}{2}$, as $d^A_s$ is a scalar. The leading-order three-particle amplitude is $\mathcal{O}(Q^{-2})$ (before wave-function renormalisation) and includes all diagrams built out of the leading two-body interactions. The resultant Faddeev integral equation is represented in Fig. 1. As the Lagrangian up to N$^2$LO does not mix partial waves or flip the spin of the

Figure 1: The Faddeev equation for $Nd$-scattering up to N$^2$LO. Thick solid line: propagator of the two intermediate auxiliary fields $d_s$ and $d_t$, denoted by $D$, see (2.5); $K$: propagator of the exchanged nucleon; $H$: three-body force and the cross denotes insertion of deuteron kinetic energy operator.

auxiliary fields, angular momentum is conserved in the quartet and doublet channels.

In the doublet channel, the Faddeev equation is two-dimensional in cluster-configuration.
space as both $Nd_t$- and $Nd_s$-configurations contribute \cite{21}:

$$\vec{t}_d(E; k, p) = 2\pi \left[ \mathcal{K}(E; k, p) \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix} + \mathcal{H}(E; \Lambda) \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right]$$

$$- \frac{1}{\pi} \int_0^\infty dq q^2 \left[ \mathcal{K}(E; q, p) \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} + \mathcal{H}(E; \Lambda) \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right]$$

$$\times D(E - \frac{q^2}{2M}, q) \vec{t}_d(E; k, q).$$

The vector

$$\vec{t}_d := \begin{pmatrix} t_{d,tt} \\ t_{d,ts} \end{pmatrix},$$

is built out of the two amplitudes which get mixed: $t_{d,tt}$ for the $Nd_t \rightarrow Nd_t$-process, and $t_{d,ts}$ for the $Nd_t \rightarrow Nd_s$-process. Furthermore,

$$D(p_0, p) := \begin{pmatrix} D_t(p_0, p) & 0 \\ 0 & D_s(p_0, p) \end{pmatrix},$$

is the propagator of the two intermediate auxiliary fields.

One included the specific three-body force term in the doublet-channel Faddeev equation \cite{23} at a given order $n$ of expansion as suggested by Bedaque et al. \cite{4, 5, 11, 21} if and only if that term is needed to cancel cut-off dependences in the observables which are stronger than cut-off dependence from the suppressed terms of order $(QR)^{n+1}$ where $Q$ is a typical external momentum and $R$ is the short distance scale in position space beyond which the EFT breaks down. This argument suggests that from the point of view of the EFT, cutoff dependences cancel order by order in the expansion. One can implement this idea by expanding the kernel of the integral equation perturbatively, and then iterate it by inserting it into integral equation. This partial re-summation arbitrarily includes higher order diagrams and consequently dose not improve the precision of calculation. The only necessary re-summation is the one present at LO. The partial re-summation of range effects is made only for convenience \cite{4}.

The integral equation is solved numerically by imposing a cut-off $\Lambda$. In that case, a unique solution exists in the $^2S_{1/2}$-channel for each $\Lambda$ and $\mathcal{H} = 0$, but no unique limit as $\Lambda \rightarrow \infty$. As long-distance phenomena must however be insensitive to details of the short-distance physics (and in particular of the regulator chosen), Bedaque et al. \cite{4, 5, 11, 21} showed that the system must be stabilized by a three-body force

$$\mathcal{H}(E; \Lambda) = \frac{2}{\Lambda^2} \sum_{n=0}^\infty H_{2n}(\Lambda) \left( \frac{ME + \gamma_1^2}{\Lambda^2} \right)^n = \frac{2H_0(\Lambda)}{\Lambda^2} + \frac{2H_2(\Lambda)}{\Lambda^4} (ME + \gamma_1^2) + \ldots.$$

which absorbs all dependence on the cut-off as $\Lambda \rightarrow \infty$. It is analytical in $E$ and can be obtained from a three-body Lagrangian, employing a three-nucleon auxiliary field analogous to the treatment of the two-nucleon channels \cite{11}.
$H_2$ is dimension-less but depends on the cut-off $\Lambda$ in a non-trivial way, as a renormalisation group analysis reveals: Instead of approaching a fixed-point as $\Lambda \to \infty$, it shows an oscillatory behavior known as “limit cycle” [4].

As one needs a three-body force at LO, $H_0 \sim Q^{-2}$, all three-body forces obtained by expanding $\mathcal{H}$ in powers of $E$ are also enhanced, with the interactions proportional to $H_2$ entering at N$^2$LO [11, 21]. The power-counting for the three-body forces is hence

$$H_0(\Lambda) \sim Q^{-2}, \ H_2(\Lambda) \sim Q^{-2}, \ H_{2n}(\Lambda) \sim Q^{-2}.$$  \hspace{1cm} (2.7)

The scattering phase-shift of the S-wave in the quartet and doublet channel is related to the renormalised on-shell amplitudes by

$$T_q = Z_t t_q = \frac{3\pi}{M} \frac{1}{k \cot \delta_q - ik}, \ T_{d,xy} = \frac{3\pi}{M} \frac{1}{k \cot \delta_{d,xy} - ik},$$ \hspace{1cm} (2.8)

where $x, y = s, t$ label the matrix entries in cluster-configuration space, and

$$\bar{T}_d = Z \bar{t}_d \quad \text{with} \quad Z := \begin{pmatrix} Z_t & 0 \\ 0 & \sqrt{Z_t Z_s} \end{pmatrix},$$ \hspace{1cm} (2.9)

is the renormalised doublet-amplitude and its wave-function renormalisation. In the doublet channel, the only observable process is nucleon-deuteron scattering, $Nd_t \to Nd_t$, i.e. $x = y = t$.

Nucleon-deuteron scattering is to N$^2$LO thus completely determined by four simple observables of $NN$-scattering: the deuteron binding energy, residue (or effective range), the scattering length and effective range of the $^1S_0$-channel. Only the $^2S_{1/2}$-channel has further unknowns, namely the strength of the three-body interaction $H_0$ at LO and NLO, and in addition of $H_2$ at N$^2$LO. They are determined by its measured scattering length $a_d$ [23] and the triton binding energy $B_d$, respectively:

$$a_d = (0.65 \pm 0.04) \ \text{fm}, \quad B_d = 8.48 \ \text{MeV}.$$ \hspace{1cm} (2.10)

3 Neutron-deuteron radiative capture process

In the KeV energy region, the processes $nd \to ^3H \gamma$ and $pd \to ^3He \gamma$ play an important role in nuclear astrophysics and in nuclear physics. In the standard big-bang nucleosynthesis theory the corresponding reaction rates are necessary to estimate the $^3$He-yield as well as the abundances of other light elements. The spin structure of the matrix elements of neutron radiative capture by deuteron is complicated but in very low energy for this reaction we can introduced three multipole transition that can be allowed by p-parity and angular momentum conservation i.e. $J^p = \frac{1}{2}^+ \to M1$ and $J^p = \frac{3}{2}^+ \to M1, E2$. The parameterization of the corresponding contribution to the matrix element to be build by the following contributions:

$$i(t^\dagger N)(\vec{D} \cdot \vec{e} \times \vec{k}),$$
\[
(t^\dagger \sigma_a N)(\vec{D} \times [\vec{e}^\ast \times \vec{k}])_a,
\]
\[
t^\dagger (\vec{\sigma} \cdot \vec{e}^\ast \vec{D} \cdot \vec{k} + \vec{\sigma} \cdot \vec{k} \vec{D} \cdot \vec{e}^\ast)N,
\]

where \( N, t, \vec{e}, \vec{D} \) and \( \vec{k} \) are the 2-component spinors of initial nucleon field, final \( ^3He \) (or \( ^3H \)) field, the 3-vector polarization of the produced photon, the 3-vector polarization of deuteron and the unit vector along the 3-momentum of the photons, respectively. The two structures in Eq. (3.1) correspond to the M1 transition. At thermal energies the reaction proceeds through S-wave capture predominantly via magnetic dipole transition, \( M_{1LSJ} \), where \( L=0, S=1/2,3/2 \) and \( i=1 \). To obtain the spin structure, which corresponds to a definite value of \( J \) for the entrance channel, it is necessary to build special linear combinations of products \( \vec{D}N \) and \( \vec{\sigma} \times \vec{D}N \), with \( J^P = \frac{1}{2} \) or \( J^P = \frac{3}{2} \):

\[
\phi_{1/2} = (i\vec{D} + \vec{\sigma} \times \vec{D})N \text{ and } (2i\vec{D} - \vec{\sigma} \times \vec{D})N.
\]

For both possible magnetic dipole transitions with \( J^P = \frac{1}{2} \) (amplitude \( g_1 \)) and \( J^P = \frac{3}{2} \) (amplitude \( g_3 \)) we can write:

\[
g_1 : t^\dagger (i\vec{D} \cdot \vec{e}^\ast \times \vec{k} + \vec{\sigma} \times \vec{D} \cdot \vec{e}^\ast \times \vec{k})N,
\]
\[
g_3 : t^\dagger (i\vec{D} \cdot \vec{e}^\ast \times \vec{k} + \vec{\sigma} \times \vec{D} \cdot \vec{e}^\ast \times \vec{k})N.
\]

The electric transition \( E_{1LSJ} \) for energies of less than 60 KeV does not contribute to the total cross section. Therefore \( E_{20(3/2)(3/2)} \) transition will not be considered in energies relevant to BBN calculation. The M1 amplitude receives contributions from the magnetic moments of the nucleon and dibaryon operators coupling to the magnetic field, which are described by the lagrange density involving fields:

\[
\mathcal{L}_B = \frac{e}{2M_N} N^\dagger (k_0 + k_1 \tau^3)\sigma.B + e \frac{L_1}{M_N \sqrt{\tau^{(1s_0)_1}(1s_1)}} d_{1j^3} d_{s3} B_j + H.C.
\]

where the unknown coefficient \( L_1 \), which contributes at order \( Q \) must either be predicted from QCD or determined experimentally in order to have model-independent predictive power.

The radiative capture cross section \( nd \rightarrow ^3H \gamma \) at very low energy is given by

\[
\sigma = \frac{2}{9} \frac{\alpha}{v_{rel}} \frac{p^3}{4M_N^2} \sum_{iLSJ} |\tilde{\chi}_{iLSJ}^L|^2,
\]

where

\[
\tilde{\chi}_{iLSJ}^L = \frac{\sqrt{6\pi}}{p\mu_N} \sqrt{4\pi} \chi_{iLSJ}^L,
\]
Figure 2: The Faddeev equation for $Nd$-radiative capture up to LO. Boxes indicate insertion of $Nd$-scattering amplitude up to LO from Fig. 1 (only up to first line of perturbative expansion of the Faddeev equation). Cross shows wave function renormalization in each step when triton is made and three-body interactions are shown with strength $H_0(\Lambda)$. Wavy line shows photon and small circles show magnetic photon interaction. The photon is minimally coupled. Remaining notation as in Fig. 1.
Figure 3: The NLO contribution can then be obtained by perturbing around the LO solution with the one deuteron kinetic energy operator insertion and last lines show NLO order of photon interaction with the lagrangian Eq. (3.3) with the $L_1$ vertices. Boxes indicate insertion of $Nd$-scattering amplitude for NLO from Fig. 1 (only second line of perturbative expansion of the Faddeev equation). Remaining notation as in Fig. 2.

with $\chi$ stands for either the electric or magnetic transition and $\mu_N$ is in nuclear magneton and $p$ is momentum of the incident neutron in the center of mass. There is an infinite number of diagrams contributing at leading order for M1 amplitude of the radiative capture cross section $nd \to {}^3H\gamma$, as shown in Fig. 2. We follow the same procedure as in Fig. 1: First, expand the kernel of the integral equation perturbatively in the same way of Fig. 1 and then iterate it by inserting it into the integral equation. We use the deuteron propagator in the integral equation, which is then solved numerically. The re-summation is necessary since according to the power counting all these diagrams contribute equally to the final amplitude. The
Figure 4: The $N^2$LO contribution can then be obtained by perturbing around the LO solution with the twice deuteron kinetic energy operator insertion. Boxes indicate insertion of $Nd$-scattering amplitude to $N^2$LO from Fig. 1. Remaining notation as in Fig. 2 and Fig. 3.

Figure 5: The two different contribution of $N^2$LO with twice deuteron kinetic energy operator insertion.

NLO and $N^2$LO contributions can then be obtained by perturbing around the LO solution with the one and twice deuteron kinetic energy operator insertion, respectively, as shown diagrammatically in Fig. 3 and Fig. 4. For $N^2$LO contribution due to very cumbersome
numerical calculation, one must substitute diagram of Fig. 5(a) by Fig. 5(b) in diagrams of Fig. 4, particularly when one needs to compute the full off-shell LO amplitude before inserting the deuteron kinetic energy terms.

We now turn to the amplitude to be used in the M1 calculation. The amplitude \( \vec{t}_d \) after properly iterated will be folded to electromagnetic interaction \( \mathcal{K}_{EM} \) order by order and integrated on momentum in each order. Finally the wave function renormalization in each order will be done when triton is made. We introduce for \( \mathcal{K}_{EM} \):

\[
\mathcal{K}_{EM} = \frac{1}{\sqrt{1 - \gamma r^{(3S_1)}}} \left[ \frac{1}{a^{(1S_0)}} + \frac{1}{2} r^{(1S_0)} |q|^2 - i |q| \right] \left[ \kappa_1 \frac{\gamma^2}{|q|^2 + \gamma^2} \left( \gamma - \frac{1}{a^{(1S_0)}} + \frac{1}{2} r^{(1S_0)} |q|^2 \right) + L_1 \frac{\gamma^2}{2} \right].
\] (3.6)

We have not considered the two following possible diagrams in our calculation. Photo-interaction directly with exchanged nucleon is shown in Fig. 6(a). For this interaction, we have \( p.A \) and in very low energy relevant to BBN, \( p \leq 200 \text{ KeV} \), this particular contribution (Fig. 6(a)) which could be appeared in Figures (2,3,4) has been neglected, because we can estimate it’s size from a naive power counting. In the integration of photon coupling to exchanged-nucleon vertices, we use \( q \sim Q \) and \( q \sim p \), where we have the \( Q/M^2 \) the wave function renormalization, \( M/Q^2 \) nucleon propagator and \( 1/My^2Q \) dibaryon propagator. Consequently, it’s size of contribution has been estimated to be < 1%. The other direct interaction is with three-body vertices \( \mathcal{H} \), up to order of our calculation only \( H_0, H_2 \) are considered. The direct interaction in order of \( H_0 \), for zero energy range, will be dealt with elsewhere. This diagram is shown in Fig. 6(b). Contribution of this diagram for the energy range of our calculation is ignored, because in very low energy for \( p \sim q \sim Q \) error for neglecting the contribution of this term is less than 1%. Contribution of diagrams with \( H_2 \) vertices generally appears at \( N^2 \)LO. At \( N^2 \)LO one can insert photon also to \( H_2 \) vertices. From (2.4), this contribution is \( (p^2/\Lambda^4)H_2 \) and is of the order of \( (k/\Lambda)^2 \) or \( (Q/\Lambda)^2 \) by power counting expansion. In numerical calculation we define \( H_2 \) such that it does not contribute at zero momentum, naturally for the energy range near zero momentum, insertion of photon to \( H_2 \) vertices for momentum \( p \sim 200 \text{ KeV} \), could be neglected and the error is estimated to be smaller than the estimation of error for insertion of photon to \( H_0 \) vertices.

The solution \( \vec{t}_d \) depends on \( \Lambda \) for arbitrary \( H_0, H_2 \) which in general are cutoff dependent, too. Since the low-energy amplitude can not depend on the cutoff, any explicit dependence on the cutoff has to be canceled by the cutoff dependence implicit in \( H_0, H_2 \), etc. We determine the cutoff dependence of the three-body forces by imposing this condition order by order in \( 1/\Lambda \) in the same way of [11]. In this process, we also determine which three-body force appears at every order of the expansion.

In order to have cutoff independent amplitude one must determine the three-body forces by demanding order by order in \( Q/\Lambda \)

\[
\mathcal{H}(E, \Lambda) + \frac{2}{\pi} \int_0^\Lambda dq \, q^2 \mathcal{D}(E - \frac{q^2}{2M}, q) [K(q, p) + \mathcal{H}(E, \Lambda)] t_\Lambda(q) = \text{const.}, \quad (3.7)
\]
Figure 6: Diagram (a) shows photon interaction with exchanged nucleon and diagram (b) shows interaction of photon with $H_0$ vertex.

This (possibly energy dependent) constant can without loss of generality be set to zero by absorbing it into a re-defined three body force

$$\mathcal{H}(E, \Lambda) \to \mathcal{H}(E, \Lambda) - \frac{\text{const.}}{1 + \frac{2}{\pi} \int dq \frac{q^2}{2M} D\left(E - \frac{q^2}{2M}, q\right) t_\Lambda(q)}.$$  \hspace{1cm} (3.8)

Figure 7: Semi-logarithm curve of the cutoff variation of phase shifts is shown between $\Lambda = 150$ MeV and $\Lambda = 500$ MeV as function of the center-of-mass momentum. The short dashed, long dashed and solid line correspond to LO, NLO and $N^2$LO, respectively.

Imposing (3.7) with const. = 0 order by order in the low energy expansion, we can determine which three-body forces are required at any given order, and how they depend
on the cutoff. As we are interested only in the UV behavior, i.e. in the asymptotics, and no IR divergences occur, we can safely neglect the IR limit of the integral. All calculations were performed with the cutoff at 400 MeV. At N^2LO, where we saw that \( H_2 \) is required the cutoff is varied between 150 and 500 MeV and we ignore other sources of error at low cutoffs, this is a reasonable estimate of the errors. For energy range of our calculation, below the break up reaction, we show cutoff variation of phase shifts between \( \Lambda = 150 \) MeV and \( \Lambda = 500 \) MeV as function of the center-of-mass momentum in Fig. 7. We can see very smooth slope and does however change significantly from order to order because the dominant correction is \( (\gamma/\Lambda)^n \) and for these energy range at every order these variations are nearly independent of variation of momentum. We confirm that in our calculation also in very low energy range the cutoff variation decreases steadily as we increase the order of the calculation and it is of the order of \( (k/\Lambda)^n, (\gamma/\Lambda)^n \), where \( n \) is the order of the calculation and \( \Lambda = 150 \) MeV is the smallest cutoff used. It is worth mentioning that the errors due to increasing momentum, as one would expect due to \( (k/\Lambda)^n \) also appear in our calculation in very low energy but these errors decrease when the order of calculation is increased up to N^2LO.

4 Numerical results for neutron-deuteron radiative capture

We numerically solved the Faddeev integral equation up to N^2LO. We used \( \hbar c = 197.327 \) MeV fm, a nucleon mass of \( M = 938.918 \) MeV, for the \( NN \) triplet channel a deuteron binding energy (momentum) of \( B = 2.225 \) MeV \( (\gamma_d = 45.7066 \) MeV), a residue of \( Z_d = 1.690(3) \), effective range \( r_0 \) = 2.73 fm, for the \( NN \) singlet channel an \( ^1S_0 \) scattering length of \( a_t = -23.714 \) fm and \( \mu_N = 5.050 \times 10^{-27} \) JT\(^{-1} \). We determine the two-nucleon parameters from the deuteron binding energy, triplet effective range (defined by an expansion around the deuteron pole, not at zero momentum), the singlet scattering length, effective range (defined by expanding at zero momentum), and two body capture process (obtained with comparison between experimental data and theoretical results for \( nd \rightarrow d\gamma \) process at zero energy [10]). We fix the three-body parameters as follows: because we defined \( H_2 \) such that it does not contribute at zero momentum scattering, one can first determine \( H_0 \) from the \( ^2S_1 \) scattering length \( a_3 = (0.65 \pm 0.04) \) fm [23].

At LO and NLO, this is the only three-body force entering, but at N^2LO, where we saw that \( H_2 \) is required, it is determined by the triton binding energy \( B_3 = 8.48 \) MeV. We solve integral equation by expansion in order of \( Q \) and properly iterating the kernel. Then, the resulted \( \vec{t}_d \) will be folded to electromagnetic interaction order by order and properly integrated on the involving momentum.

For our calculation source of error due to low cutoffs and low momentum for very low energy calculation is neglected. The cutoff variation decreases steadily as we increase the order of the calculation and is of the order of \( (k/\Lambda)^n, (\gamma/\Lambda)^n \), where \( n \) is the order of the calculation. We used \( \Lambda = 150 \) MeV, for the smallest cutoff.

The cross section calculation for neutron radiative capture by deuteron as function of
Figure 8: The cross section for neutron radiative capture by deuteron as function of the center-of-mass kinetic energy $E$ in Mev. The short dashed, long dashed and solid line correspond to the contribution from M1 capture up to LO, NLO and N$^2$LO, respectively.

Table 1: Neutron radiative capture by deuteron in micro barn and estimated error in percentage in comparison with the last column at every order up to N$^2$LO. Last column shows ENDF results for cross section [20].

| Energy (KeV) | $\sigma(\mu b)$ LO | error(%) | $\sigma(\mu b)$ NLO | error(%) | $\sigma(\mu b)$ N$^2$LO | error(%) | $\sigma(\mu b)$ ENDF | $\sigma(\mu b)$ ENDF |
|-------------|-----------------|----------|-----------------|----------|-----------------|----------|---------|---------|
| 40          | 1.64(6)         | 30       | 1.31(5)         | 4        | 1.25(0)         | 1.5      | 1.27(0) |         |
| 50          | 1.72(8)         | 24       | 1.45(8)         | 4        | 1.38(3)         | 0.7      | 1.39(0) |         |
| 60          | 1.90(5)         | 27       | 1.58(5)         | 5        | 1.50(1)         | 0.1      | 1.50(0) |         |
| 70          | 2.02(6)         | 25       | 1.72(0)         | 7        | 1.61(6)         | 0.1      | 1.61(0) |         |
| 80          | 2.11(9)         | 22       | 1.82(2)         | 6        | 1.73(3)         | 0.5      | 1.72(0) |         |
| 100         | 2.42(6)         | 25       | 2.04(4)         | 5        | 1.93(3)         | 0.5      | 1.94(0) |         |
| 140         | 2.88(9)         | 30       | 2.41(8)         | 9        | 2.30(3)         | 0.1      | 2.22(9) |         |

the center-of-mass energy at LO, NLO and N$^2$LO is shown in Fig. 8.

Table 1 shows numerical results for the EFT $nd \rightarrow \gamma$ cross section for various nucleon center of mass energies $E$ up to N$^2$LO and errors estimate at every order in comparison with the last column. The corresponding values for the cross section from the online evaluated nuclear data file ENDF/B-VI [20] are shown in the last column. The EFT results for this
cross section are presented up to only two significant digits (the third digit is shown for better comparison with ENDF data). In Fig. 9 we show the error due to available evaluated data ENDF [20] in percentage versus nucleon center-of-mass kinetic energy $E$ in KeV.

5 Conclusion

We have calculated the cross section of radiative capture process $nd \rightarrow ^3H\gamma$. We applied pionless EFT to find numerical results for the M1 contributions for this capture process for incident neutron energies relevant for BBN, $0.02 \leq E \leq 0.2$ MeV. At these energy our calculation is dominated by S-wave state and magnetic transition M1 contribution.

The error estimate in the cross section in comparison with evaluated nuclear data file ENDF [20] is shown in Fig. 9 and Table 1. Errors estimate are 20-30 percent at leading order, below 10 percent up to NLO and by insertion of three-body force at N$^2$LO, this error is reduced to below 1% percent. Specially, our calculation shows minimum error for energy 60-70 KeV up to at N$^2$LO. Comparison of the LO with the NLO and N$^2$LO results demonstrate convergence of the effective field theory. Finally, three-body forces will enter at higher orders of the EFT approach and reduce the theoretical uncertainty.

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