Influence of initial nozzle opening on the maximum pressure head at the impulse turbine inlet and set of needle closing law

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Abstract. The maximum pressure head at the impulse turbine inlet is one of the most important parameters for operation of the impulse turbine. In order to determine the most critical operating condition for the maximum pressure head rise at the impulse turbine inlet, the formula which demonstrates the relation between relative pressure head and the initial opening is obtained through theoretical derivation. It shows a good agreement with the numerical results by the method of characteristics. A parabolic shaped flow characteristic of the nozzle is utilized in the derivation. Besides, the fact that the maximum pressure often occurs in the moment of the first wave reflection for the hydropower plants with impulse turbines is considered. The formula indicates that the maximum of pressure head rise increases monotonously with the initial opening decreasing. Therefore, the most critical condition for the maximum pressure head at the impulse turbine inlet is the condition of load rejection from small opening rather than the load rejection condition from full opening in the rated or maximum head. When adopting linear or two-speed closing law, the derived formula can be used to calculate the closing speed and the opening of break point, realizing the optimization of closure.

1. Introduction

When designing the hydraulic conveyance system of hydropower station with impulse turbines, the maximum pressure head during operation conditions is concluded by hydraulic transients calculation. It’s used to guide the design of units and pipeline. The maximum pressure head at the impulse turbine inlet is generally considered to occur in the operating conditions of load rejection from full opening in the rated or maximum head [1]. However, when computing some hydropower stations with impulse turbines by numerical simulation, such as Delsi-Tanisagua in Ecuador, Nam Phay and Shenshu hydropower station, it’s found that the maximum pressure head occurs in the load rejection condition from small or middle opening. And the pressure head is much larger than that of full opening. The traditional calculation of water hammer assumes the nozzle opening linear change. But in practice, it’s the needle that operates under linear closure. As a result, the relation of nozzle opening to needle stroke is non-linear. The relation between nozzle discharge and needle stroke is deduced after analyzing the characteristics of the impulse turbine nozzle component [2]. This paper derives the parabolic equation for relative opening with respect to needle stroke on this basis. With the help of the parabolic equation, a new water hammer calculation formula is derived on the base of water hammer interlocking equation. The influence of initial opening on maximum pressure head is analyzed and
verified by numerical simulation of actual hydropower stations. According to the given relative pressure head rise, the derived formula can be used to set linear or two-speed closing law. Lastly, the method is introduced.

2. Relation between initial nozzle opening and maximum relative pressure head rise

2.1. Flow characteristic of impulse turbine nozzles
The impulse turbine is equipped with cigar-shaped needles. The needle and the injector are on the same axis. When the needle moves along the axis, the nozzle discharge is changed smoothly. The injector design is shown in figure 1 in which $d_{\text{max}}$ is nozzle outlet diameter, $\alpha$ needle angle, $\beta$ nozzle outlet angle and $S_{\text{max}}$ maximum needle stroke.

![Figure 1. Injector design](image)

The equation for nozzle discharge is derived as follows [2, 3]:

$$Q = \pi \left[ \tan \frac{\alpha}{2} d_{\text{max}} \Delta S - \tan^2 \frac{\alpha}{2} \Delta S^2 \right] \varphi \sqrt{2gH_0 \cos \left( \frac{\alpha + \beta}{4} \right)}$$

where $\Delta S$ is needle stroke, $\varphi$ is nozzle discharge coefficient and $H_0$ is the net head.

In the injector, the water flows in the annulus and the projected nozzle area $A_j$ of the annulus perpendicular to the flow direction may be written as:

$$A_j = \pi \left[ \tan \frac{\alpha}{2} d_{\text{max}} \Delta S - \tan^2 \frac{\alpha}{2} \Delta S^2 \right] \cos \left( \frac{\alpha + \beta}{4} \right)$$

Let $n = \tan \frac{\alpha}{2} S_{\text{r}}^2 / (d_{\text{max}} S_{\text{r}} - \tan \frac{\alpha}{2} S_{\text{r}}^2)$ and the relative nozzle opening may be written as:

$$\tau = A_j / A_r = (1 + n)x - nx^2$$

in which $x$ is the relative needle stroke, $x \in [0, 1]$, $S_{\text{r}}$ is the rated needle stroke, $A_j$ is the nozzle area and $A_r$ is the nozzle area of rated needle stroke.

Note: * The nozzle opening $\tau$, needle stroke $x$ and pressure head rise $\xi$ mentioned below refer to relative values. $\xi = \Delta H / H_0$.

When the needle adopts linear closure, as is shown in figure 2, the nozzle opening is parabolic correlated to needle stroke. Referring to equation (1), an equation for discharge when the needle adopts linear closure may be written as:

$$Q = (1 + n)x - nx^2 A_r \varphi \sqrt{2gH_0}$$

Which $Z$ is the number of jets. When $n = 0$, the equation (4) comes to be the traditional equation[3] in orifice flow model.
2.2 Water hammer calculation when needle adopting linear closure

The wave reflection in the injector leads to water hammer. The maximum pressure head may happen in the moment of reflection time of $2L/c$ or at the end of needle closure period [4, 5, 6].

1. If the needle closing time from initial stroke $x_0$ satisfies $t = x_0T_S \leq 2L/c$, the maximum pressure head may occur at the beginning of needle closing, which can be referred to as direct hammer. Therefore, when the initial nozzle opening satisfies the equation

$$\tau_0 \leq (1 + n)\frac{2L}{cT_S} - n\left(\frac{2L}{cT_S}\right)^2 = \frac{1 + n}{p} - \frac{n}{p^2},$$

the equation for calculating pressure head of direct hammer may be written as:

$$\xi_d = \frac{cV_{max}}{gH_0} \tau_0 = 2\rho \tau_0$$

eq.(5)

in which $\rho$ is the water hammer constant, $p = \frac{cV_{max}}{2gH_0}$; $L$ is the pipe length; $c$ is the wave speed; $g$ is the gravitational acceleration; $V_{max}$ is the maximum flow velocity; $T_S$ is the total closing time at rated condition; $p = cT_S/(2L) = T_S/t_r$, $t_r$ is the water hammer reflection time, $t_r = 2L/c$, $p$ is not limited to an integer.

2. When the criteria $\rho\tau_0 < 1$ is satisfied, the maximum pressure occurs $2L/c$ seconds after the needle starts closing. The variation of needle stroke is $\Delta x = t_r/T_s = 2L/(cT_s)$ during the reflection time $T_s$ while the variation of nozzle opening is $\Delta \tau = \tau_0 - \tau_1 = n\Delta x^2 - 2nx_0\Delta x + (1 + n)\Delta x$. Let $\rho\Delta x = \sigma$ and then $\sigma = \frac{LV_{max}}{gH_0T_s}$. The pressure head rise is usually less than 50% of the static head. Hence, $\sqrt{1 + \xi_1}$ can be substituted by $1 + \xi_1/2$. Simplifying the equation $\tau_1\sqrt{1 + \xi_1} = \tau_0 - \frac{1}{2\rho} \xi_1$, which is used to calculate the maximum pressure head occurring $2L/c$ seconds, we obtain:

$$\xi_1 = \frac{2\sigma^2 + \sqrt{(1 + n)^2 - 4n\tau_0\rho\sigma}}{\rho^2\tau_0 - \sqrt{(1 + n)^2 - 4n\tau_0\rho\sigma} - n\sigma^2 + \rho}$$

eq.(6)

Let $n = 0$, then the equation (6) may be written as $\xi_1 = \frac{2\sigma}{1 + \rho\tau_0 - \sigma}$, which is in accordance with the traditional calculation equation.

Therefore, when the initial opening is in the interval of $[\frac{1 + n}{p} - \frac{n}{p^2}, \frac{1}{p}]$, the maximum pressure occurs $2L/c$ seconds and the derivative of $\xi_1$ is less than zero. The pressure head increases as the initial opening decreases. By substituting $\tau_{0c} = \frac{1 + n}{p} - \frac{n}{p^2}$ into equation (6), we obtain:

$$\xi_1 = \frac{2\rho^2}{p^2}[(1 + n)p - n] = 2\sigma[1 + (1 - \frac{1}{p})n]$$

eq.(7)

3. When $\rho\tau_0 > 1$, the maximum value of $\xi_1$ occurs at the end of needle closing which is referred to as final water hammer[7]. The water hammer equation is written as:

$$(\tau_{n+1} - \tau_n)\sqrt{1 + \xi_m} = -\Delta \tau \sqrt{1 + \xi_m} = -\frac{1}{\rho} \xi_m$$

eq.(8)
Solving for $\xi_m$ yields the following equation:

$$\xi_m = \frac{2(\rho \Delta \tau)}{\sqrt{(\rho \Delta \tau)^2 + 2^2 - (\rho \Delta \tau)}}$$  \hspace{1cm} \text{eq.(9)}$$

in which $\Delta \tau = n \Delta x^2 - 2nx_n \Delta x + (1 + n) \Delta x$, $\rho \Delta x = \sigma$, and $\rho \tau_n = 1$, $\tau_n = (1 + n)x_n - nx_n^2$.

Substituting these into equation (9) yields the following equation:

$$\xi_m = \frac{2(\sigma \sqrt{\left[ (1 + n)^2 - \frac{4n}{\rho} + \frac{n}{p} \right]})}{\sigma \sqrt{\left[ (1 + n)^2 - \frac{4n}{\rho} + \frac{n}{p} \right]} + 2^2 - \sigma \frac{2}{\sqrt{\left[ (1 + n)^2 - \frac{4n}{\rho} + \frac{n}{p} \right]}}}$$  \hspace{1cm} \text{eq.(10)}$$

When $n = 0$, we obtain $\xi_m = \frac{\sigma}{2}(\sigma + \sqrt{\sigma^2 + 4})$, which is in accordance with the traditional equation[3]. Substitute $\sqrt{1 + \xi_i}$ for $1 + \xi_i/2$ and equation (10) is simplified as:

$$\xi_m = \frac{2 \rho \Delta \tau}{2 - \rho \Delta \tau} = \frac{2 \sigma \sqrt{\left[ (1 + n)^2 - \frac{4n}{\rho} + \frac{n}{p} \right]}}{2 - \sigma \frac{2}{\sqrt{\left[ (1 + n)^2 - \frac{4n}{\rho} + \frac{n}{p} \right]}}}$$  \hspace{1cm} \text{eq.(11)}$$

2.3. Relation between initial nozzle opening and maximum relative pressure head rise

The turbine units may reject load from any initial opening in emergency shut-down case. From the above analysis, it is clear that $\tau_{\theta_{\rho}} = \frac{1 + n}{\rho} - \frac{n}{p^2}$ and $\tau_{\theta} = \frac{1}{\rho}$ are the criteria for different water hammer phenomena. As the head for hydropower plants with impulse turbines is high, the water hammer constant $\rho$ is small and the phenomenon as shown in figure 3 and figure 4 often occurs. The relative pressure head rise $\xi$ with respect to different initial opening $\tau_{\theta}$ is analysed, as shown below.

Figure 3. Relative pressure head rise for different initial opening (1)

1. As shown in figure 3, when $1/\rho > 1$, the direct hammer phenomenon will occur if the initial opening is in the interval of $[0, \tau_{\theta_{\rho}}]$. While the maximum pressure occurs $2L/c$ seconds if the initial opening is in the interval of $[\tau_{\theta_{\rho}}, 1]$. In this case, the most critical condition is that the units reject load.
from small initial opening $\tau_{\theta r}$ and the responding maximum pressure head rise is

$$\xi_{\text{max}} = 2\sigma[1 + (1 - \frac{1}{p})n].$$

2. As shown in figure 4, when $\tau_{\theta r} < 1/ \rho < 1$, the direct hammer phenomenon will occur if the initial opening is in the interval of $[0, \tau_{\theta r}]$. The maximum pressure occurs $2L/c$ seconds if the initial opening is in the interval of $[\tau_{\theta r}, 1/ \rho]$. While the maximum pressure occurs at the end of needle closing if the initial opening is in the interval of $[1/ \rho, 1]$. In this case, the most critical condition is still that the units reject load from small initial opening $\tau_{\theta r}$ and the responding maximum pressure head rise is

$$\xi_{\text{max}} = 2\sigma[1 + (1 - \frac{1}{p})n].$$

Based on the above analysis, we obtain that the most critical condition is that the units reject load from initial opening $\tau_{\theta r}$ and the responding pressure head is the maximum value. The condition should be paid more attention when computing transient pressure of hydropower plants with impulse turbines.

3. Set Of needle closing law

Selecting the appropriate needle closing law is one of the most effective and economical measures to ensure the safe operation in hydropower plants. The unit discharge remains nearly constant at the same opening in the Hill charts of the impulse turbine. Hence, the linear closure or two-speed closure whose speed in the first closing phase is faster than the second one is often adopted [8]. According to the given relative pressure head rise, the linear or two-speed closure can be set utilizing derived equation in this paper.

3.1. Setting linear closing law

For hydropower plants with impulse turbines, rejecting load from initial opening $\tau_{\theta r}$ is the most critical condition. Substituting $\xi_c$ into equation (7) yields the following equation:

$$T_s = \frac{4L V_{\text{max}} n}{c V_{\text{max}} (1+n) - \sqrt{[c V_{\text{max}} (1+n)]^2 - 4cgH_0 V_{\text{max}} n \xi_c}}$$

eq.(12)

Let $n = 0$, a simplified equation can be obtained: $T_s = \frac{2L V_{\text{max}}}{gH_0 \xi_c}$. Figure 6 shows the maximum relative pressure head rise with respect to different initial opening when the needle closes linearly.

**Figure 6.** Relative pressure head rise for different initial opening in linear closure

**Figure 7.** Needle two-speed closure
Note that \( x_0 \) is the needle stroke corresponding to the most critical opening \( \tau_{0r} \).

### 3.2. Setting two-speed closing law

As the most critical opening \( \tau_{0r} \) is usually small, the break point is often set in the interval of \([\tau_{0r},1]\) when setting two-speed closing law.

1. Assume the opening of break point is \( x_\tau \). When the phenomenon shown in figure 3 occurs in hydropower station, \( x_\tau \) will be the most critical opening in the interval of \([,1]x_\tau\) in linear closing law. While \( \tau_{0r} \) is the most critical opening in the interval of \([0,]x_\tau\). The most two critical openings should be considered to be the control point and the \( \xi_c \) to be the maximum relative pressure head rise when setting two-speed closing law. Then the needle closing speed can be figured out. This method can guarantee that the pressure rise is less than the required value \( \xi_c \) when units rejecting load from any initial opening. As shown in figure 7, the opening changes from 1 to break point \( \tau_\times \) in the first closing phase and changes from \( x_\tau \) to 0 in the second closing phase. \( T_s \) is total closing time corresponding to different closing speed. Details of the analysis follow.

The first closing phase: Utilizing \( \xi_1 = \frac{2(\zeta_0 + \sqrt{(1+n)^2 - 4n\tau_\times \rho \sigma})}{\rho^2 \tau_\times - (1+n)^2 - 4n\tau_\times \rho \sigma - n\sigma^2 + \rho} = \xi_c \), we obtain:

\[
T_{s1} = \frac{(2 + \xi_1)\sqrt{(1+n)^2 - 4n\tau_\times} + \sqrt{M_2 - 8n(2 + \xi_1)\tau_\times}}{c\xi_c \tau_\times + M_1} + M_1
\]

in which, \( M_1 = \frac{2gH_0 \xi_c}{LV_{max}} \), \( M_2 = (2 + \xi_1)^2 (1+n)^2 + \frac{4gH_0 \xi_c (2\xi_c n + 4n)}{cV_{max}} \).

The second closing phase: \( \tau_{0r} \) is the most critical opening and the total closing time can be obtained according to the analysis of linear closing law.

\[
T_{s2} = \frac{4LV_{max} n}{cV_{max} (1+n) - \sqrt{(cV_{max} (1+n))^2 - 4cgH_0 V_{max} n\xi_c}}
\]

The opening \( \tau_\times \) and stroke \( x_\tau \) satisfy equation (3). The total needle closing time of two-speed closure may be written:

\[
T_s = t_1 + t_2 = (1 - x_\tau)T_{s1} + x_\tau T_{s2}
\]

When \( n = 0 \), the simplified equations for \( T_{s1}, T_{s2} \) can be obtained. As it is hard to obtain the extreme value for equation (15), we substitute the simplified equations into equation (15). The total closing time \( T_s \) obtains the minimum value when \( \tau_\times = \left[\left(\frac{(2 + \xi_c)(\rho + 1)}{2}\right) - 1\right] / \rho \).

Therefore, the opening of break point is \( \tau_\times = \left[\left(\frac{(2 + \xi_c)(\rho + 1)}{2}\right) - 1\right] / \rho \) if the phenomenon in figure 3 occurs in hydropower station and the two-speed closure is adopted. The closing speed of the first phase is \( k_1 = 1/T_{s1} \) and the second is \( k_2 = 1/T_{s2} \) as shown in figure 7.

2. When the phenomenon shown in figure 4 occurs in hydropower station, the opening of the two control points are still \( \tau_\times \) and \( \tau_{0r} \), respectively.

The first closing phase: The closing speed is still \( 1/T_{s1} \) if \( \tau_\times \) is in the interval of \([\tau_{0r},1/\rho]\). If
is in the interval of \([1/\rho, 1]\), utilizing \(\xi_m = \frac{2\sigma[(1+n)^2 - \frac{4n}{\rho} + \frac{n}{p}]}{2 - \sigma[(1+n)^2 - \frac{4n}{\rho} + \frac{n}{p}]} = \xi_e\) and solving for \(T_3\), one obtains:

\[
T_{33} = \frac{2j}{k^2 + 8j\xi_e - k}
\]
eq (16)

in which \(j = \frac{2L^2V_{\text{max}}n(2+\xi_e)}{cgH_0}\), \(k = LV_{\text{max}}(2+\xi_e)\sqrt{(1+n)^2 - \frac{4n}{\rho}} / (gH_0)\).

The second closing phase: \(\tau_{0r}\) is still the control point and the corresponding closing speed is \(1/T_{52}\).

If \(\tau_x\) is in the interval of \([1/\rho, 1]\), the maximum pressure head is a constant while the opening is changing. Therefore, the total closing time \(T_S\) obtains the minimum value when \(\tau_x = 1/\rho\) and the closing rates are \(1/T_{53}\) and \(1/T_{52}\), respectively.

Hence, when \(\left(\frac{(2+\xi_e)(1+\rho)}{2} - 1\right)/\rho < 1/\rho\), the opening of break point should be \(\tau_x = \left(\frac{(2+\xi_e)(1+\rho)}{2} - 1\right)/\rho\) and the closing speed of the first phase is \(k_1 = 1/T_{53}\) while the second is \(k_2 = 1/T_{52}\). When \(\left(\frac{(2+\xi_e)(1+\rho)}{2} - 1\right)/\rho \geq 1/\rho\), the opening of break point should be \(\tau_x = 1/\rho\) and the closing speed of the first phase is \(k_3 = 1/T_{53}\) while the second is still \(k_2 = 1/T_{52}\).

Considering that the flow rate will decline as the slope of closing law decreases if the initial opening is at the first stroke \([\tau_z, 1]\), the relative pressure head rise will be less than \(\xi_e\). Therefore, the closing speed of the first phase could be increased appropriately to optimize the closing law.

4. Mathematical model validation

In numerical simulation, the method of characteristics incorporating the boundary conditions, such as for upstream and impulse turbine units, are used to develop simplified mathematical model of actual hydropower plants [5, 7, 9]. The grid spacing satisfies the Courant stability condition and the basic time step is \(\Delta t = 0.01s\).

Model A: The upstream reservoir level is 1105m and the installation elevation is 405m. The tunnel length is 9030m and the diameter of the tunnel is 1.5m. The wave speed is 1000 m/s and the friction losses are neglected. The reflection time is \(t_r = 18.06s\). The turbine model is CJC601-L-180/4×14.5 with its rated head and rated speed being 700m and 600r/min, respectively. The rated discharge is 7.082m³/s and rated output is 44.5MW.

(1) According to the data of the injector structure, it can be obtained \(n = 0.3\). Therefore, the equation can be expressed as: \(Q = [(1+0.3)x - 0.3x^2]S_eZ\varphi\sqrt{2gH_0}\). The comparison of field data and results of derived equation and traditional equation is shown in figure 8.
(2) Comparison of numerical and calculated results

1) Set of closing law

The pressure head rise at the impulse turbine inlet is usually required to be less than 20% of the rated head. Substitute $\xi_{c} = 0.2$ into derived equations and traditional equations and compare the calculated results with the numerical results, as presented in table 1:

Table 1. Optimization of two-speed closure for different calculation methods

| Calculation method         | $T_{S}$ for linear closure | Two-speed closure |          |          |          |          |
|---------------------------|----------------------------|-------------------|----------|----------|----------|----------|
|                           |                            | Break point $\tau_{z}$ | closing speed for first phase $k_{1}$ | closing speed for second phase $k_{2}$ | Total closing time $T_{S}$ |
| Numerical simulation      | 64                         | 0.69              | 0.0200   | 0.0156   | 58.68    |
| Derived equation          | 63.8                       | 0.66              | 0.0196   | 0.0157   | 58.40    |
| Traditional equation      | 52.5                       | 0.66              | 0.0206   | 0.0190   | 51.11    |

The closing speed of the first stroke can be increased to $k_{1} = 0.0258$ and then the total closing time is optimized to 55.47s.

2) The maximum relative pressure head rise

With the total linear closing time of 64s, the maximum relative pressure head rise of calculation and corresponding numerical simulation is presented in table 2.

Table 2. Maximum relative pressure head rise by different methods

| Calculation method         | Most critical opening $\tau_{0r}$ | maximum relative pressure head rise $\xi_{max}$ | Relative error $(100\%)$ |
|---------------------------|-----------------------------------|-----------------------------------------------|--------------------------|
| Numerical simulation      | 0.3430                            | 0.1995 (18.06s)                               | /                        |
| Derived equation          | 0.3429                            | 0.199 (18.06s)                                | 0.25                     |
| Traditional equation      | 0.2820                            | 0.164 (18.06s)                                | 17.79                    |

a The relative error is referred to the percent between calculated and numerical results, the same below.

b The figures in the brackets refers to the time when maximum occurs, the same below.

3) Analysis

As can be seen from validation of model A: ① The equation for nozzle discharge and needle stroke obtained by the data of injector structure reflects the flow characteristic of impulse turbine nozzles better than the traditional equation, as shown in figure 8; ② In optimization of linear closure, the calculated closing time by derived equation and computed result agree closely for about 64s, which is larger than the calculated one by traditional equation. In optimization of two-speed closure, the break points calculated by the derived and traditional equation, $\tau_{z} = 0.66$, are the same, which is slightly less than numerical result 0.69. The closing time 58.40s calculated by derived equation is close to the numerical result 58.68s and larger than result 51.11s calculated by traditional equation. ③
The most critical opening calculated by derived equation and computed result agree closely for about 0.343, which is larger than the calculated one for 0.282 by traditional equation. The pressure head rise respectively obtained by the derived equation and numerical simulation agree closely with the relative error being only 0.25%. While the relative error between the numerical result and calculated result by traditional equation is 17.79%.

Model B: The upstream reservoir level is 1451.3m and the installation elevation is 956.3m. The tunnel length between the surge tank and the units is 1340m and the diameter of the tunnel is 1.45m. The wave speed is 1000 m/s and the friction losses are neglected. The reflection time is $t_r = 2.233s$.

The turbine model is CJ (Blizzard) -L-203.8/6×17.4 with its rated head and rated speed being 495m and 450r/min, respectively. The rated discharge is 14.10m$^3$/s and rated output is 62.30MW. The flow equation is obtained:

$$Q = [(1 + 0.622)x - 0.622x^2]S_gZ\varphi\sqrt{2gH_0}.$$ The unit adopts linear closure with the total closing time being 54.36s.

| Initial opening $\tau_{0r}$ | Numerical simulation | Derived equation | Relative error (100%) | Traditional equation | Relative error (100%) |
|-----------------------------|----------------------|------------------|------------------------|----------------------|------------------------|
| 1.000                       | 24.6 (37.8s)         | 16.92            | 31.27                  | 13.18                | 46.47                  |
| 0.2515                      | 28.5 (2.30s)         | 26.08            | 8.7                    | 17.27                | 39.55                  |
| 0.0876                      | 42.6 (2.23s)         | 38.86            | 8.9                    | 22.18                | 48.00                  |

It is clear from Table 3 that the results calculated by equation (5) and equation (6) show a good agreement with the numerical results. But accuracy of equation (10) is low. The calculation precision of derived equations is still higher than the traditional equations.

### 5. Discussion

For high-head hydropower station with impulse turbines, the maximum pressure often occurs $2L/c$ seconds when units rejecting load from an initial opening. A parabola relation exists between nozzle opening and needle stroke when the needle adopts linear closure, as shown in figure 9. With the initial opening decreasing, the discharge gradient $dQ/dx$ increases and the discharge variation $\Delta Q$ increases in the reflection time of $2L/c$. Therefore, the relative pressure head rise $\xi$ increases as the initial opening (initial needle stroke) decreases. The most critical initial opening is $\tau_{0r} = \frac{1+n}{p} - \frac{n}{p^2}$ and the responding maximum value is $\xi_{max} = 2\sigma[1 + (1 - \frac{1}{p})n]$.

In the process of needle closing, the discharge change for derived equation (curve B) and traditional equation (line A) is shown in figure 9. The slopes of line A and curve B are the same when $x = 0.5$. When calculating the direct hammer, the opening calculated by derived equation is larger than that calculated by traditional equation at the same stroke. So the pressure calculated by the former is larger than that calculated by the latter. If the maximum pressure occurs $2L/c$ seconds, the pressure obtained by derived equation is larger than the result obtained by traditional equation. This is because the discharge variation $\Delta Q_1$ in curve B is larger than discharge variation $\Delta Q_2$ in line A in the reflection time of $2L/c$. When calculating the final hammer, the equation may be written as

$$\xi = \frac{2\rho\Delta x}{2 - \rho\Delta t}.$$ If $\rho\tau = 1$ occurs in the range where the slope of curve B is smaller than that of line A, the discharge variation in curve B is smaller than discharge variation in line A. Then the pressure calculated by derived equation will be less than the result by traditional equation. Conversely, the pressure calculated by derived equation is larger than the result by traditional equation. Head loss is
one of the factors that leads to the error because the method of characteristics considers the head loss while the theoretical analysis does not. The error for final hammer is large as the closing time is long. From the above analysis, it is clear that the results calculated by equation (5) and equation (6) are close to numerical results. The calculation precision of derived equations is higher than the traditional equations. But the equation used to calculate the final hammer should be further studied.

![Diagram](image)

**Figure 9.** Discharge change

6. **Conclusion**

1. The parabolic equation of opening and needle stroke obtained by nozzle structure of impulse turbine can accurately reflect the nozzle flow characteristic. On this basis, the water hammer equations are derived when needle is closed linearly. The agreement between the calculated and numerical results is good except for the final hammer. The equation for final hammer can be further studied to improve the accuracy.

2. For high-head hydropower stations with impulse turbines, rejecting load from small opening is more critical than from full opening. The most critical opening and the corresponding maximum pressure can be calculated utilizing the derived equations. The mathematical models, which are developed by method of characteristics, have verified the accuracy of derived equations.

3. According to the given relative pressure head rise, the linear or two-speed closing law can be set and further optimized.

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8. **References**

[1] You Q S, Lai X, Li T T, Zhong Q S and Liang B 2013 *J. Hydroelectric engineering*. 32(6) 267–271
[2] Zhong Q S, Lai X and You Q S 2012 *J. Water resource and power*. 30(9) 130–132
[3] Zhou W T and Zhou X Q 2007 *Pelton Turbine Basic Theory and Design* (Beijing: China Water & Power Press)
[4] Liu Q Z 1997 *Hydropower Station* (Beijing: China Water & Power Press)
[5] M.Hanif C 2014 *Applied hydraulic transients* (New York: Springer)
[6] Zhang Y L, Miao M F and Ma J M 2010 *J. Water science and engineering*. 3(2) 174–189
[7] E.B. W and V.L. S 1982 *Fluid Transients* (Ann Arbor, Mich: FEB Press)
[8] A.V and B. F 2009 *J. Mechanical Engineering*. B 16(3) 222–228
[9] Uros K, Anton B and Petar V 2009 *J. Mechanical Engineering*. 55 369–380