Mathematical modeling of the polarization characteristics of radio waves in the Earth's equatorial ionosphere

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Abstract. Using the bicharacteristics method, mathematical modeling of the polarization parameters of P-band radio waves in the Earth’s ionosphere in the vicinity of the equatorial anomaly along the meridian path was carried out. The influence of the ionosphere on the phase deviation and the angle of the Faraday rotation during the passage of the spacecraft is studied taking into account changes in the magnitude and orientation of the Earth’s magnetic field.

1. Introduction. Formulation of the problem

This work is devoted to the mathematical modeling of the polarization characteristics of radio signals in the Earth's ionosphere. In [1–6], high-latitude and mid-latitude models of the ionosphere were considered. A low-latitude ionosphere model was selected (figure 1), which includes the equatorial anomaly. The model is based on radio tomography data [7, 8, 9], and therefore, more adequately than the average empirical models, reflects the real structure of the ionospheric plasma.

Figure 1 shows the distribution of the electron concentration along the 121° east longitude meridian, with the latitude (Lat) increasing with increasing coordinate x:

\[ x = (\rho + R_z) \sin(Lat - Lat0), \]

where \( \rho \) is the shortest distance from the surface of the earth to the observation point, and \( R_z \) – Earth radius. The value \( x=0 \) corresponds to \( Lat0=24^\circ N \). Data refer to September 3, 1994, time 06:20UT. As can be seen from figure 1, the work uses a local coordinate system, with the y axis directed perpendicular to the plane of the picture from the reader, that is, to the west.

The calculations were performed for an operating frequency of 430 MHz (~0.7 m) corresponding to the P-band. The relevance of the study of the radio wave propagation through the Earth’s ionosphere of a given frequency band is determined by the planning of space experiments with the aim of remote sensing of the Earth’s surface from space [10]. The meter and decimeter bands are used in problems of radio location and radiotomography when reconstructing the profile of the electron concentration of the ionospheric plasma [8, 9].

In addition to the distribution of electron concentration, an important role in the study of the polarization characteristics of radio waves the magnitude and orientation of the Earth's magnetic field are played. The orientation of the magnetic field relative to the local coordinate system is defined by two angles \( \gamma \) and \( \varphi \):

\[ H_{0x} = H_0 \cos \gamma \cos \varphi, \quad H_{0y} = H_0 \cos \gamma \sin \varphi, \quad H_{0z} = H_0 \sin \gamma, \]

\( (2) \)
which vary with latitude. The angle $\phi$ is opposite to the angle of magnetic declination $D$, and the angle $\gamma$ is opposite to the angle of magnetic inclination $I$.

![Figure 1. Distribution of electron concentration (gray color) in the ionosphere. Green indicates the surface of the earth.](image)

In figure 2 the dependence of the angle $\phi = -D$ on latitude along the surface of the earth is shown, and figure 3 the dependence for the angle $\gamma = -I$ is shown [11]. Analyzing figure 2, we see that along the selected path (that is, along the meridian), the magnetic declination, as one would expect, does not change significantly, therefore, it has little effect on the simulation results.

![Figure 2. Dependence of the angle $\phi$ on latitude.](image)

On the contrary, magnetic inclination, as can be seen from figure 3, varies significantly from 16 to 45 degrees ($\gamma = -I$). Therefore, accounting for such a change when performing calculations is desirable.
Figure 3. Dependence of the angle $\gamma$ on latitude.

Figure 4 shows the dependence on the latitude of the magnitude of the magnetic field along the earth's surface. It can be seen that with latitude, the field strength increases, although the real values do not change very significantly.

Figure 4. Dependence of the magnetic field strengths on latitude.

Note that the dependence on the latitude of the angles of magnetic declination and inclination can be approximated with sufficient accuracy by a parabola, while for reliable approximation of the dependence of the magnetic field strengths in figure 4, the cubic term must be taken into account too.

When performing numerical modeling, the change in the values of the angles $\varphi$, $\gamma$ and the magnitude of the magnetic field intensity $H_0$ from latitude was taken into account. The altitude dependence of the magnetic field is taken into account too.

2. Ray paths

In figure 5 ray paths in the propagation plane $(x,z)$ are plotted in different colors in accordance with the spectrum of the rainbow. It is assumed that at an altitude of 400 km from the surface of the Earth a moving point source of radiation (spacecraft – SC) is located. As shown in the figure, the receiver is located on the surface of the Earth at the origin. When the spacecraft moves from left to
right, the angle of inclination of the rays varies from 150 to 30 degrees relative to the positive direction of the axis $x$. The figure shows the trajectories arriving at the observation point. To calculate the parameters of the ray paths, a system of bi-characteristics:

$$\frac{d\hat{r}}{d\tau} = -\frac{\partial \Gamma}{\partial \hat{k}} \left/ \frac{\partial \Gamma}{\partial \omega} \right. ,$$

$$\frac{d\hat{k}}{d\tau} = \frac{\partial \Gamma}{\partial \hat{r}} \left/ \frac{\partial \Gamma}{\partial \omega} \right.$$

with the Hamiltonian

$$\Gamma = k_x^2 + k_y^2 + k_z^2 - \frac{\omega^2}{c^2} e(\hat{r}, \hat{k}, \omega) ,$$

is applied [12-15].

In the formulas (3), (4) $\hat{r} = (x, y, z)$ – coordinates of the observation point, $\hat{k}$ – wave vector, $\omega = 2\pi f$ – circular radiation frequency, $f$ – working frequency, $\tau$ – group time and simultaneously a parameter along the ray path, and $e(\hat{r}, \hat{k}, \omega)$ – effective permittivity of the propagation medium [16-17].

![Figure 5](image)

**Figure 5.** Ray paths against the background of the distribution of the electron concentration of the ionosphere.

Since the radiation frequency of the source in this problem is much higher than the plasma frequency $\omega_p$, one can reasonably assume that

$$e(\hat{r}, \hat{k}, \omega) = 1 - \frac{\omega_p^2}{\omega^2} ,$$

$$\omega_p^2 = \frac{4\pi e^2 N(\hat{r})}{m_e} ,$$

where $m_e$ – the electron mass, $e$ – the electron charge, $N(\hat{r})$ – the electron concentration. The initial wave vector $\hat{k}(0)$ is determined by the ray exit angle $\omega_0$.

In figure 6 the dependence of group time on the coordinate $x$ is shown. Dependence is quasi-parabolic. At this frequency, the influence of the ionosphere on the change in group time (group delays) is extremely small. Therefore, the shape of the curve is determined solely by the trajectory of the spacecraft.
3. Phase deviation

In figure 7 the dependences of the rates of phase changes (derived phase with respect to group time) on height are shown:

\[
\Phi'(t) = \frac{dx}{dt} k_x(t) + \frac{dy}{dt} k_y(t) + \frac{dz}{dt} k_z(t) - \omega = \Phi'(t) - \omega. \tag{6}
\]

The color palette corresponds to figure 5.

As shown in [1–6], the minima of the curves correspond to the positions of the maxima of the ionospheric layers, and the local maxima correspond to interlayer valleys. In figure 7 we see a deep minimum at a height corresponding to the maximum value of the electron concentration in the equatorial anomaly, as well as the minima and maxima in the lower part of the figure, different for the rays crossing the ionospheric layer at different angles. This corresponds to the structure of the ionosphere shown in figure 1 and 5.

Knowing the phase derivative (6), it is easy to determine the dependence of the phase deviation on group time at the signal receiving point (figure 8). Having calculated by the formula:

\[
\Phi(t) = \int_0^\eta \Phi'(\eta) \, d\eta, \tag{7}
\]

the phase deviation \(\Delta\Phi\) could be found

\[
\Delta\Phi = \Phi - \omega R / c, \tag{8}
\]

where \(\omega R / c\) – ray phase along the trajectory in the absence of an ionospheric layer.

It can be seen that the curve in figure 8 has two branches. The upper branch corresponds to the right side of figure 5, and the lower branch – the left. The leftmost value corresponds to the ray arriving at the receiver with the SC located strictly above the source. For this case, the signal propagation time is minimal. The irregularity of both the upper and lower branches should be noted, which is explained by the significant influence of ionospheric layers on phase deviation.

In figure 9 the dependence of the phase deviation on the horizontal coordinate \(x\) is shown. It can be seen that the absolute value of the deviation for the left side of the figure is greater than for the right, which is explained by the higher density of electron concentration on the left (see figure 1). Figure 9 is completely consistent with figure 8.
Figure 7. Dependence of the rate of the phase change on height.

Figure 8. The dependence of the phase deviation $\Delta \Phi$ on the group time $t$.

4. Faraday rotation
In the P-band, we can assume that the ordinary and extraordinary waves propagate along the same ray path. Then we can talk about the rotation effect of the plane of polarization, and the angle of the Faraday rotation can be calculated by integration along the ray path [1-3, 18-20]:  

\[ \frac{d \Phi}{dt} \times 10^{-3} \]
\[ \Omega(t) = \frac{1}{2} \frac{\omega^2}{c} \int \Delta \mu \left( (dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2 \right)^{1/2} dt. \]  

(9)

**Figure 9.** The dependence of the phase deviation \( \Delta \phi \) on the horizontal distance \( x \).

In the formula (9) as \( \Delta \mu \):

\[ \Delta \mu = \frac{1}{2} \sqrt{u^2 \sin^4 \theta + 4(1-v)^2 u \cos^2 \theta} \]

(10)

the difference in the refractive indices of the ordinary and extraordinary waves is indicated. In formula (10), the parameter \( u \) is the ratio of the square of the gyrofrequency to the square of the circular frequency:

\[ u = \frac{\omega_H^2}{m^2 c^2 \omega^2}, \]

(11)

and the angle \( \theta \) is the angle between the wave vector \( \vec{k} \) and the Earth's magnetic field vector \( \vec{H}_0 \).

Since the differences \( \Delta \mu \) of the refractive indices of the ordinary and extraordinary waves are crucial for estimating the angle of the Faraday rotation, in figure 10 the dependences of \( \Delta \mu \) on height are shown. The color palette corresponds to figures 5 and 7. Curve shapes correlate with electron concentration profiles along trajectories. It can be seen that for the rays corresponding to the right side of the figure, the values at the upper maximum are significantly larger than for the rays corresponding to the left side of the figure. As for the bottom of figure 10, in this region various rays pass through the ionosphere with a significantly different electron concentration (figure 1), which is reflected in figure 10.

The dependence of the Faraday rotation angle \( \Omega \) on group time at the signal receiving point, calculated by formula (9), is shown in figure 11. As in figure 8, the upper branch of the curve corresponds to the left side of figure 5, where the ionosphere is denser and the lower branch of the curve corresponds to the right side. The vertical ray corresponds to the left edge of figure 11. In this case, the Faraday rotation angle is about 50°.

Figure 12 shows the dependence of the angle of the Faraday rotation on the coordinate \( x \). It is necessary to pay attention to the break point of the graph at which the nature of the behavior of the function changes significantly. At this point, the magnetic field vector is orthogonal to the wave.
vector, and the angle of the Faraday rotation is close to zero. The maximum value of the angle of Faraday rotation on this spacecraft trajectory exceeds $100^\circ$.

Figure 10. Dependence of the difference in the refractive indices of an ordinary and extraordinary wave on the height along the rays.

Figure 11. The dependence of the angle of the Faraday rotation $\Omega$ on the group time $t$. 
Figure 12. Dependence of the angle of the Faraday rotation $\Omega$ on the horizontal distance $x$.

5. Conclusion

Thus, using the bicharacteristics method [12,14,21], mathematical modeling of the polarization parameters of the $P$-band radio waves in the Earth’s ionosphere in the vicinity of the equatorial anomaly was performed. The influence of the ionosphere on the angle of the Faraday rotation and phase deviation during the spacecraft flight in the meridian direction was studied taking into account changes in the magnitude and orientation of the magnetic field.

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