Sweep-Soil Interaction: a Mathematical Model

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Abstract. Cutting-edge tillage machinery design involves the evolving theory of implement-soil interaction. This paper proposes a mathematical model that represents the performance indicators of a sweep (soil deformation, draught, contact pressure) as a function of design and operating factors (note: this paper covers low-crown sweeps). Analysis of these equations and the results of field experiments show that increasing the nose angle will reduce the drag and require less force to cut the stubbles. As the cutting angle increases, more force is required to cut through; the penetration ability is directly proportional to \( \cos \alpha \).

1. Introduction
Research into the mechanics behind implement-soil interaction is becoming ever more important for tillage mechanics. Such research is critical for designing novel tillage machinery and for improving tillage techniques.

Mathematical modeling of implement-soil interaction, which affects the operating parameters of implements, has a certain role to play in theoretical studies. How accurately the operating parameters of agricultural machinery could be optimized depends on the rational choice of mathematical models describing soil deformation in a broad range of density and moisture values.

Scientifically and practically, mathematical models of sweep-soil interaction are both interesting and needed. The model is essentially intended to optimize the geometry of the sweep so as to minimize the draught.

2. Theory
Sweep-soil interaction modeling has been covered by many researchers for various regions, soils, and climates [1,2,3,4]. Tillage modeling should adequately reflect the rupture resistance and shear strength of soil [6,7]. Let us introduce a Cartesian coordinate system bound to the sweep, see Figure 1:
Let the straight lines OA (the central line), OC (the left horizontal line) and OB (the right horizontal line) be the cutting edges of the sweep. The figure shows only the front of the sweep (around the tip), where $AD \perp BC$, $AC \perp OC$, $AB' \perp OB$, $AD' \perp OD$, $AD' \perp OA$.

The key geometric parameters of a sweep are: $\alpha$, the pitch; $2\gamma$, the nose angle; $h$, the height to which the sweep raises a slice of soil (the lift), cm; $2l$ is the sweep width, cm; $H$ is the tillage depth, cm.

These geometric parameters of a sweep are bound by the following ratios:

$$
\sin \alpha = \frac{\sin \gamma \cdot \sin x}{\sqrt{\cos^2 x + \sin^2 \gamma \cdot \sin^2 x}}
$$

$$
\cos \alpha = \frac{\cos x}{\sqrt{\cos^2 x + \sin^2 \gamma \cdot \sin^2 x}}
$$

$$
tg \alpha = \sin \gamma \cdot tgx
$$

$$
\sin \beta = \sqrt{1 - \cos^2 \gamma \cdot \sin^2 x}
$$

$$
\cos \beta = \cos \gamma \cdot \sin x
$$

$$
tg \beta = \frac{\sqrt{1 - \cos^2 \gamma \cdot \sin^2 x}}{\cos \gamma \cdot \sin x}
$$

where $\beta$ is the angle between the normal to the plowshare and the OZ axis; $x$ is the cutting angle.

As the sweep moves through soil, soil particles interact with the lateral surfaces of the sweep, which is why the perpendicular to the cutting surface is important for describing such interaction:

The interacting forces are: $F_{yx}$ the inertia forces, $F_{sp}$ the friction, $F_{gr}$ the gravity, $F_{p}$ the ground reaction forces, $F_{yp}$ the elastic forces.

Let us analyze the inertia forces induced by the dispersal of soil particles. Let the sweep be submerged in soil and moving along the OX axis in the negative direction. Sweep-soil interaction takes place as follows: as the sweep travels forward by a distance $\epsilon$, soil accumulates elastic energy due to elastic soil crumbling. Normal and tangential stresses in soil are rising within this timeframe. As the latter reach a critical value, soil layers under the cutting edge (the plowshare blade) are destroyed, a process that manifests itself as a fracture that propagates along the surface approaching the soil surface. The elastic energy released by such fracturing is spent to make the fracture and to disperse loose soil. Fracture surface consists of two planes intersecting along a straight line tilted at an angle $\phi$ to the soil surface. The middle line, where these planes intersect, and the soil surface make
the angle $2\gamma$.

Consider the soil layer in the shaded area, see Figure 2. Let the apex of the bottom angle in this area have the coordinates $(x', y', z')=(0,0,0)$ and the velocity $\vec{v}=(v,0,0)$ before the motion begins. Once the soil is fractured, the withdrawn loose soil layer will slide, on one side, along the cutting surface of the sweep (the section OA); its other side will slide along the fracture surface (the section OB).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{soil_fracturing_chart.png}
\caption{Soil fracturing chart.}
\end{figure}

Let the bottom angle move to $(x'', y'', z'')=\delta \cos \theta (\cos \phi, \sin \phi, \tan \theta)$ after some relatively short time $\Delta t$, where $\delta, \theta$ are unknown parameters.

Since the bottom angle BAD has been sliding along the fracture surface all the time, the vector $(x'', y'', z'')$ lies in the fracture surface plane and is orthogonal to the vector $\vec{n}=(1,\tan \phi, -\tan \gamma)$. Besides, the vector $(\epsilon, 0, 0) + (x'', y'', z'')$ is the trajectory of the bottom angle moving along the sweep surface.

Therefore, the vector $(\epsilon, 0, 0) + (x'', y'', z'')$ is orthogonal to the vector $\vec{n}$. Thus, we have two constraints resulting from the assumption that the withdrawn loose soil layer is sliding along the sweep surface and the fracture surface alike:

1. $\cos \phi + \tan \phi \sin \phi - \tan \gamma \tan \theta = 0$,
2. $-\cos \phi + \tan \alpha \sin \phi - \tan \gamma \tan \theta = \frac{\epsilon}{\delta \cos \theta}$.

Sweep-soil interaction makes the soil block reach the following velocity:

\[ v' = \lim_{\Delta t \to 0} \frac{(x'', y'', z'') - (x', y', z')}{\Delta t} = \frac{v}{(\tan \alpha + \tan \phi) \sin \phi} (\cos \phi, \sin \phi, \tan \theta) = \frac{v}{(\tan \alpha + \tan \phi) \sin \phi} (\cos \phi, \sin \phi, \tan (\cos \phi + \tan \phi \sin \phi)) = \frac{v}{(\tan \alpha + \tan \phi) (\tan \phi, 1, \tan \gamma (\cos \phi + \tan \phi))} \]

The vector of soil particle velocity is changing over some time $\Delta t$ i.e. while the soil is subject to elastic crumbling and until the implement penetrates the soil to a distance $\epsilon = v \cdot \Delta t$, one can evaluate the force needed to attain such change in velocity. Assuming there is a constant acceleration $\ddot{a}$, obtain:
\[
\alpha = \frac{\gamma (\text{ctg } \phi, 1, \text{tg } (\text{ctg } \phi + \text{ctg } \phi))}{\Delta (\text{ctg } \alpha + \text{ctg } \phi)}.
\]

Given that the desired force is concentrated along the cutting edge of the implement, obtain:

Let us further describe the forces withdrawn soil is exposed to.

The soil layer of the ABOD section in Figure 2 is moving at the velocity \(v\) along the AB surface and at the velocity \(v' = \bar{v} + \bar{v}'\) along the surface AD, i.e. these surfaces carry friction forces whose direction is opposite to the soil layer motion direction.

This soil block moves along the sweep surface; thus, the sweep-soil contact surface AD carries a friction force directed oppositely to the vector \(-\bar{v}' = - (\bar{v} + \bar{v}')\).

The friction force is proportional to the normal soil-on-sweep pressure and the soil-steel friction coefficient \(f\).

Since soil layers (Area III) under the sliding surface DC lie on lower layers, they press the surface DC due to gravity; this pressure is proportional to the height of the soil layer above the corresponding sliding surface point: \(F_{\text{CD}}^DC(s) = F_{\text{DC}}^DC(s) \rho \bar{g}\), where \(\bar{g} = (0, -1, 0)\).

Since Areas 2 and 3 are moving against each other, there is a friction force between them that is proportional to normal pressure and to the soil-soil friction coefficient \(-f\).

For space consideration, this paper only presents the final equation of forces the surface CD is exposed to:

\[
F_{\text{CD}}(s) = F_{\text{CD}}^C(s) \left( \frac{\text{ctg } \phi}{1 + \text{ctg}^2 \phi + \text{ctg}^2 \gamma} \right) \left( 1 - \frac{1}{1 + \text{ctg}^2 \phi + \text{ctg}^2 \gamma} \times \right.
\]

\[
\left. \left( \text{ctg } \phi, -1 - \text{ctg}^2 \gamma, \text{tg } \phi \cdot \text{tg } \gamma \right) - \frac{f \cdot \text{ctg } \phi}{1 + \text{ctg}^2 \phi + \text{ctg}^2 \gamma \cdot (\text{ctg } \phi + \text{ctg } \phi)^2} \times \right.
\]

\[
\left. \left( \text{ctg } \phi, 1, \text{tg } (\text{ctg } \phi + \text{ctg } \phi) \right) \right) \right),
\]

where \(F_{\text{CD}}(s)\) is the height of the soil layer above the point (s) in the sliding surface CD.

The resultant force can be found by finding the volume of soil lying on the surface CD. For the first approximation, assume

\[
\int F_{\text{CD}}(s) ds = \frac{1}{2} H^2 \text{ctg } \phi.
\]

Let normal pressure forces acting upon the sliding surfaces AD and AB be unknowns to be found from the constraint: the soil layer ABCD is moving evenly at the velocity:

\[
\bar{v}' = \frac{\gamma (\text{ctg } \phi, 1, \text{tg } (\text{ctg } \phi + \text{ctg } \phi))}{\text{ctg } \alpha + \text{ctg } \phi}.
\]
Let \( F_{AD}(s) \) be the normal pressure force acting on the sweep surface. Given the friction force, find that the forces the surface \( AD \) is exposed to can be written as follows:

\[
\begin{align*}
\mathbf{F}_{AD}(s) &= \frac{F_{AD}(s)(-1, \cot \alpha, \cot \gamma)}{1 + \cot^2 \alpha + \cot^2 \gamma} - \frac{f^* \cdot F_{AD}(s) \cdot (\cot \alpha + \cot \varphi + \cot \phi, 1, \tan \gamma(\cot \varphi + \cot \phi))}{1 + \tan^2 \gamma(\cot \varphi + \cot \phi)^2 (\cot \alpha + \cot \varphi + \cot \phi)^2} \\
\mathbf{F}_{AB}(s) &= \frac{F_{AB}(s)(1, \cot \phi, -\cot \gamma)}{1 + \cot^2 \phi + \cot^2 \gamma} - \frac{f^* \cdot F_{AB}(s) \cdot (\cot \phi + \cot \phi, 1, \tan \gamma(\cot \phi + \cot \phi))}{1 + \tan^2 \gamma(\cot \phi + \cot \phi)^2 (\cot \phi + \cot \phi + \cot \phi)^2}
\end{align*}
\]

For the surface \( AB \), similarly find:

\[
\begin{align*}
\mathbf{F}_{AD}(s) &= \frac{F_{AD}(s)(-1, \cot \alpha, \cot \gamma)}{1 + \cot^2 \alpha + \cot^2 \gamma} - \frac{f^* \cdot F_{AD}(s) \cdot (\cot \alpha + \cot \varphi + \cot \phi, 1, \tan \gamma(\cot \varphi + \cot \phi))}{1 + \tan^2 \gamma(\cot \varphi + \cot \phi)^2 (\cot \alpha + \cot \varphi + \cot \phi)^2} \\
\mathbf{F}_{AB}(s) &= \frac{F_{AB}(s)(1, \cot \phi, -\cot \gamma)}{1 + \cot^2 \phi + \cot^2 \gamma} - \frac{f^* \cdot F_{AB}(s) \cdot (\cot \phi + \cot \phi, 1, \tan \gamma(\cot \phi + \cot \phi))}{1 + \tan^2 \gamma(\cot \phi + \cot \phi)^2 (\cot \phi + \cot \phi + \cot \phi)^2}
\end{align*}
\]

The soil volume filling Area II is also exposed to gravity:

\[
\mathbf{F}_{grav} = \rho \cdot \ddot{g} \cdot H \cdot h \cdot l(\cot \alpha + \cot \phi), \text{ where } \ddot{g} = g \cdot (0, -1, 0).
\]

Describe the resultant force acting on the soil block in the area ABCD:

\[
\mathbf{F} = \mathbf{F}_{AD} + \mathbf{F}_{AB} + \mathbf{F}_{CD} + \mathbf{F}_{grav}, \text{ where }
\]

\[
\begin{align*}
\mathbf{F}_{CD} &= \frac{\rho \cdot H^2}{2} \cdot \cot \phi \left( \ddot{g} + \frac{f^*((\ddot{g}, \dddot{n}, \dddot{v}))}{\sqrt{(\dddot{n}, \dddot{n}, \dddot{v}, \dddot{v})}} \right) \\
\mathbf{F}_{AD} &= \frac{F_{AD}}{\sqrt{(\dddot{n}, \dddot{n})}} + f^* \cdot F_{AD} \cdot \frac{\dddot{v}}{\sqrt{(\dddot{v}, \dddot{v})}} \\
\mathbf{F}_{AB} &= \frac{F_{AB}}{\sqrt{(\dddot{n}, \dddot{n})}} + f^* \cdot F_{AB} \cdot \frac{\dddot{v}}{\sqrt{(\dddot{v}, \dddot{v})}} \\
\mathbf{F}_{grav} &= \rho \cdot \ddot{g} \cdot H \cdot h \cdot l(\cot \alpha + \cot \phi)
\end{align*}
\]

where \( \rho \) is the density of loose soil; \( H \) is the depth of tillage; \( h \) is the sweep lift; \( F_{AD} \) is the soil pressure on the cutting surface of the sweep; \( F_{AB} \) is the soil-on-soil pressure in the surface before the sweep; \( F_{grav} \) is the gravity of the soil block removed.

Given that the soil block under consideration is moving at a constant velocity, the following ratio must hold:

\[
\mathbf{R} = \mathbf{F}_{AD} + \mathbf{F}_{AB} + \mathbf{F}_{CD} + \mathbf{F}_{grav} = 0
\]

from which we can now find these values:

\[
X = \cot \varphi, Z = F_{AB}, Y = F_{AD}/
\]

Introduce the notation:

\[
X = \cot \varphi, A = \cot \gamma, B = \tan \gamma, C = \cot \phi, D = \cot \alpha, U = X + E, W = B(X + C),
\]

\[
R_{1} = C_{p}gH^{2}/2, T_{1} = E_{p}gHhl, F_{1} = f^*, F_{2} = f^*, N_{1} = \sqrt{(n, n, n)}, N_{2} = \sqrt{(n, n, n)}, V_{1} = \sqrt{(v, v, v)}, V_{2} = \sqrt{(v, v, v)}, H_{1} = \frac{R_{1} \cdot F_{1} \cdot C \cdot X}{N_{1} \cdot V_{1}}, H_{2} = \frac{V_{2} + F_{2} \cdot U \cdot N_{2}}{N_{2} \cdot V_{2}}.
\]

\[
J_{1} = (V_{1} \cdot F_{1} \cdot X \cdot N_{1} / (N_{1} \cdot V_{1})).
\]

Break the constraint down into components: \( \mathbf{R} = (R_{1}, R_{2}, R_{3}) \)

\[
\begin{align*}
R_{1} &= Y_{1} \cdot (1/N_{2} \cdot F_{2} \cdot U / N_{2}) + Z_{1} / (N_{1} \cdot N_{1} \cdot V_{1} \cdot X) \cdot R_{1} \cdot F_{1} \cdot C \cdot X \cdot (N_{1} \cdot V_{1}) = 0, \\
R_{2} &= -T_{1} \cdot R(1 + F_{1} \cdot C / (N_{1} \cdot V_{1})) + Y_{1} / (N_{2} \cdot F_{2} \cdot V_{2}) + Z_{1} / (C_{1} \cdot N_{1} \cdot V_{1} \cdot F_{1} \cdot V_{1}) = 0, \\
R_{3} &= -R_{1} \cdot F_{1} \cdot C \cdot W / (N_{1} \cdot N_{1} \cdot V_{1} \cdot X) + Y(A / N_{2} \cdot F_{2} \cdot W / N_{2}) - Z / (A / N_{1} \cdot F_{1} \cdot W / N_{1}) = 0.
\end{align*}
\]

Given \( R_{3} = 0 \): \( Y = A_{1} / (B_{1} \cdot Z \cdot C_{1}) \), \( A_{1} = N_{2} \cdot V_{2} / (D_{2} \cdot V_{2} \cdot F_{2} \cdot N_{2}), B_{1} = T + R(1 + F_{1} \cdot C / (N_{1} \cdot V_{1})), C_{1} = (C_{1} \cdot V_{1} / F_{1} \cdot V_{1}) / (N_{1} \cdot V_{1}). \)

Substitute this expression in the constraint \( R_{3} = 0 \) to find:

\[
Z = -(D_{1} + A_{1} \cdot B_{1} \cdot E_{1}) / (G_{1} + A_{1} \cdot C_{1} \cdot E_{1}), \text{ where } D_{1} = W \cdot F_{1} \cdot C / (N_{1} \cdot N_{1} \cdot V_{1}).
\]
\[ E1 = \frac{(A \cdot V_2 - W \cdot F_2 \cdot N_2)}{(N_2 \cdot V_2)}, \]
\[ G1 = \frac{(A1 \cdot V_1 + W \cdot F_1 \cdot N_1)}{(N_1 \cdot V_1)}. \]

Substitute the expression for \( Y \) and \( Z \) in the constraint \( R_1 = 0 \) to obtain a nonlinear equation for the unknown variable \( X \); by solving it by any iterative method (e.g. by halving), find: \( X = \cot \phi, \ Y = F_{AB}, \ Z = F_{AD}. \)

Sweep draught can be found by the formula:
\[ F_{drgt} = v^2 s X + Y II, \text{ where } s = \rho HI/E. \]

Calculations run on an PC show that a greater nose angle increases the drag and results in a lesser force needed to cut the stubbles. A greater cutting angle is associated with a greater cutting force, whereas the penetration ability is directly proportional to \( \cos \alpha \). Soil backpressure for tillage is sufficient at the nose angle \( 2\gamma = 75^\circ \) [1,5, 8].

3. References
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