Particle Spectrum in the Non-Minimal Supersymmetric Standard Model with $\tan \beta \simeq m_t/m_b$

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Abstract

We present a detailed discussion of the particle spectrum of the Non-Minimal Supersymmetric Standard Model (NMSSM), containing two Higgs doublets and a singlet, in the limit $\tan \beta \simeq m_t/m_b$. This is compared with the corresponding particle spectrum of the Minimal Supersymmetric Standard Model (MSSM). In this limit the singlet vacuum expectation value is forced to be large, of the order of 10 TeV, and the singlet decouples from the lightest scalar Higgs boson and the neutralinos. With the exception of the lightest Higgs boson, the particle spectrum in the model turns out to be heavy. The radiatively corrected lightest Higgs boson mass is in the neighbourhood of $\sim 130$ GeV.

Keywords: Supersymmetry; Non-minimal and Minimal models; Renormalization Group analysis.

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1. Introduction.

The Non-Minimal Supersymmetric Standard Model (NMSSM) [1], where the particle content of the Minimal Supersymmetric Standard Model (MSSM) [2] is extended by the addition of a gauge singlet chiral superfield $S$, and one in which dimensionful couplings are eliminated through the introduction of a discrete $Z_3$ symmetry, offers an interesting and viable alternative to the MSSM. It has a significantly richer phenomenology and a typically larger parameter space. Furthermore, with the prospect of LHC running in the not so distant future, it is important to consider variations of the MSSM in order to test the stability of its predictions so that search strategies may be appropriately extended or modified in order to discover low energy supersymmetry.

The predictiveness of the MSSM is vastly enhanced when the crucial parameter $\tan \beta \equiv v_2/v_1$, the ratio of the vacuum expectation values of the two Higgs doublets $H_2$ and $H_1$, required to give masses to the up-type and the down-type (and charged leptons) quarks, respectively, is constrained via the requirement that the b-quark mass come out in the right range [3,4] in supersymmetric unification [5] (our normalization is $v \equiv \sqrt{v_1^2 + v_2^2} = 174$GeV, and the mass of the Z boson is defined such that $m_Z^2 = \frac{1}{2}(g^2 + g'^2)v^2$, where $g$ and $g'$ are the gauge couplings of $SU(2)$ and $U(1)$, respectively). One particularly predictive framework is based on the assumption that the heaviest generation fermions lie in a unique 16-dimensional representation of the unifying gauge group $SO(10)$ with the Higgs doublets in a 10-dimensional representation of the group[3]. This implies that the top-quark, b-quark and $\tau$ lepton Yukawa interactions arise from a $h.16.16.10$ term in the superpotential at the unification scale $M_X$ determined from gauge coupling unification. The coupled system of differential equations for the gauge couplings and Yukawa couplings are then evolved down to present energies from $M_X$, and $\tan \beta$ is determined from the accurately measured value of $m_\tau = 1.78$ GeV. When $h(=h_t = h_b = h_\tau)$ is chosen in such
a manner as to yield a value for $m_b(m_b)$ in its “observed” range of 4.25 $\pm$ 0.10 GeV [6], a rather good prediction for the top-quark mass parameter $m_t(m_t)$ is obtained, which with the present central value of $\alpha_S(M_Z) = 0.12$ lies in the range favoured by the experimental data [7]. Here $\tan \beta$ is found to saturate what is considered to be a theoretical upper bound on its value of $m_t/m_b$ and the Yukawa coupling $h$ is found to come out to be rather large $O(1 - 3)$ with a certain insensitivity to the exact value since it is near a fixed point of its evolution. In $SU(5)$ type unification where $\tan \beta$ is free, the region $\tan \beta \approx 1$ is also a region which is favoured for the unification of the b-quark and $\tau$-lepton masses from the observed data [4]. One crucial difference between the two extremes discussed above is that in the $SO(10)$ case the Yukawa couplings of the b-quark (and that of the $\tau$ lepton) always remain comparable to that of the top-quark, with the observed hierarchy in the masses of these quarks arising from the large value of $\tan \beta$, while in the $SU(5)$ case the Yukawa couplings of the b-quark and the $\tau$-lepton are negligible in comparison with that of the top-quark. Furthermore, with large $\tan \beta$ in the MSSM, the mass of the lightest higgs boson is expected to be no larger than 140 GeV [8].

The above discussion about unification does not involve in any great detail the remaining aspects of the embedding of the standard model into a supersymmetric grand unified framework. The minimal supersymmetric extension of the standard model requires, besides the superpartners, the introduction of an additional Higgs doublet, and indeed with this matter content and an additional symmetry known as matter parity [2], to forbid couplings that lead to rapid nucleon decay, it is possible to construct a self-consistent and highly successful framework of the MSSM. Despite its many successes it may be premature to confine our attention only to the MSSM, especially because of the presence of the dimensionful Higgs bilinear parameter $\mu$ in the superpotential. Recently, a systematic study of the simplest alternative to the MSSM, the NMSSM, in the limit of large $\tan \beta$
was undertaken [9], wherein the bilinear term $\mu H_1 H_2$ of the MSSM is replaced by

$$\lambda S H_1 H_2 + \frac{1}{3}k S^3,$$

(1)

with the effective "$\mu$" term generated by the vacuum expectation value $< S > (\equiv s) \neq 0$.

This model is particularly interesting since it does not affect the positive features of the MSSM including gauge coupling unification [10], and allows a test of the stability of the features of the MSSM such as the upper bound on the mass of the lightest Higgs boson with favourable results. In Ref. [9] it was shown that the lightest Higgs boson mass in NMSSM, in this limit, is $\lesssim 140$ GeV.

In this paper we study the particle spectrum of the NMSSM in the limit of large $\tan \beta$ and compare it with the corresponding spectrum obtained in the MSSM. We carry out a renormalization group analysis of this model with universal boundary conditions and analyze the renormalization group improved tree-level potential at the scale $Q_0$. The cut off scale for the renormalization group evolution is chosen to be the geometric mean of the scalar top quark masses which is roughly equal to that of the geometric mean of the scalar b-quark masses as well, since during the course of their evolution the Yukawa couplings of the $t$ and $b$-quarks are equal up to their hypercharges and the relatively minor contribution of the $\tau$-lepton. Whereas in the MSSM the parameters $\mu$ and $B$ (the soft susy parameter characterizing the bilinear term in the scalar potential) do not enter into the evolution of the other parameters of the model at one-loop level, the situation encountered here is drastically different with a systematic search in the parameter space having to be performed with all parameters coupled from the outset. Our analysis of the minimization conditions that ensure a vacuum gives rise to severe fine tuning problems, that are worse in NMSSM as compared to the ones that arise in MSSM. The problems are further compounded by having to satisfy the constraints of three minimization conditions, rather than two such conditions that occur in MSSM. In previous studies of the model
where $\tan \beta$ was free, the tuning of parameters was possible in order to meet all the requisite criteria, viz., minimization conditions, requirement that the vacuum preserve electric charge and colour, etc. However, in the present case where $\tan \beta$ is fixed and large, what we find is a highly correlated system.

2. The Model

We recall the basic features of the NMSSM in what follows. The model is characterised by the following couplings in the superpotential

$$ W = h_t Q \cdot H_2 t_R^c + h_t Q \cdot H_1 b_R^c + h_t L \cdot H_1 \tau_R^c + \lambda S H_1 H_2 + \frac{1}{3} k S^3, $$

where we have written only the interactions of the heaviest generation and the Higgs sector (doublet and singlet) of the theory. In addition, one has to add to the potential obtained from (2) the most general terms that break supersymmetry softly which, in the conventions of Ref. [9], are:

$$ (h_t A_t \tilde{Q} \cdot H_2 \tilde{t}_R^c + h_t \tilde{Q}.H_1 \tilde{b}_R^c + h_t A_r \tilde{L} \cdot H_1 \tilde{\tau}_R^c + \lambda A_\lambda H_1 \cdot H_2 S + \frac{1}{3} k A_k S^3) + \text{h.c.} $$

$$ + m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + m_S^2 |S|^2 + m_Q^2 |\tilde{Q}|^2 + m_{\tilde{t}_R}^2 |\tilde{t}_R|^2 + m_{\tilde{b}_R}^2 |\tilde{b}_R|^2 + m_{\tilde{\tau}_R}^2 |\tilde{\tau}_R|^2. $$

The conventions for the gaugino masses follow those of the MSSM [11]. The minimization conditions (evaluated at $Q_0$ after all the parameters are evolved via their one-loop renormalization group equations down to this scale) are:

$$ m_{H_1}^2 = -\lambda \frac{v_2}{v_1} s(A_\lambda + k s) - \lambda^2 (v_2^2 + s^2) + \frac{1}{4} (g^2 + g'^2) (v_2^2 - v_1^2), $$

$$ m_{H_2}^2 = -\lambda \frac{v_1}{v_2} s(A_\lambda + k s) - \lambda^2 (v_1^2 + s^2) + \frac{1}{4} (g^2 + g'^2) (v_1^2 - v_2^2), $$

$$ m_S^2 = -\lambda^2 (v_1^2 + v_2^2) - 2k^2 s^2 - 2\lambda s v_1 v_2 - k A_k s - \frac{\lambda A_\lambda v_1 v_2}{s}. $$

One may rewrite the first two minimization equations to obtain [9]

$$ \tan^2 \beta = \frac{m_{Z}/2 + m_{H_1}^2 + \lambda^2 s^2}{m_{Z}/2 + m_{H_2}^2 + \lambda^2 s^2}, $$

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\begin{align}
\sin 2\beta &= \frac{(-2\lambda s)(A_\lambda + ks)}{m_{H_1}^2 + m_{H_2}^2 + \lambda^2(2s^2 + v^2)}. \tag{7}
\end{align}

Note that eq. (6) guarantees that, as in the MSSM, \(\tan \beta\) must lie between 1 and \(m_t/m_b\) [9]. Eq. (6) also shows that, to achieve a large \(\tan \beta\), with the essential degeneracy of \(m_{H_1}^2\) and \(m_{H_2}^2\) enforced by the renormalization group equations, the denominator of the equation has to come out to be small at low scale, implying the fine-tuning condition

\[ m_{H_2}^2 + \lambda^2 s^2 \approx -m_Z^2/2. \tag{8}\]

Here one notes that the correspondence with the MSSM will occur in a certain well defined manner with the identification of \(\lambda s\) with \(\mu\). Similarly one has to identify \(A_\lambda + ks\) with \(B\). It has been shown [9] that in the large \(\tan \beta\) case this identification occurs in a novel way that is not generic to the model, say, in the limit of \(\tan \beta \approx 1\). Furthermore, eq. (7) implies

\[ A_\lambda \approx -ks. \tag{9}\]

This is similar to the condition in the MSSM that \(B \approx 0\). Here the situation is far worse since \(A_\lambda\) is not a parameter that is fixed at \(Q_0\) but is present from the outset. This is the first of the fine-tuning problems that we encounter.

A rearrangement of eq. (7) yields

\[ \lambda s(A_\lambda + ks) = \tan \beta(-m_{H_2}^2 - \lambda^2 s^2) + m_Z^2 \frac{(\tan \beta^2 - 1)}{2(\tan \beta^2 + 1)} - \frac{\lambda^2 v^2 \sin 2\beta}{2}. \tag{10}\]

In eq. (10) it is legitimate to discard the last term for the case of large \(\tan \beta\), and one sees here that with the identification of the appropriate parameters in terms of the MSSM parameters as described earlier, one recovers all the analogous MSSM relations for all values of the other parameters without having to go through a limiting procedure [12], as is the case when \(\tan \beta\) is arbitrary.
The next fine tuning condition we encounter is related to the third minimization condition which we rewrite as:

\[ m_S^2 = -\lambda^2 v^2 - 2A_\lambda^2 + \frac{\lambda v^2 A_\lambda \sin 2\beta}{k} + A_\lambda A_k + \frac{\lambda k v^2 \sin 2\beta}{2}. \] (11)

In order to satisfy eq. (11), viz., that \( m_S^2 \) come out positive (see e.g., Fig. 1) in order to have a physically acceptable ground state, one must have at least have large cancellations between the fourth and the first two terms since the terms proportional to \( \sin 2\beta \) are negligible. This requires that \( A_k \) and \( A_\lambda \) come out with the same sign and that the product be sufficiently large. It has been shown that this condition leads to problems with finding solutions with sufficiently small trilinear couplings in magnitude [9].

The first step in the study [9] is to estimate the scale \( M_X \) with the choice of the SUSY breaking scale \( Q_0 \sim 1 \, \text{TeV} \). For \( \alpha_S(m_Z) = 0.12, \, Q_0 = 1 \, \text{TeV} \) and \( \alpha = 1/128 \), we find upon integrating the one-loop beta functions, \( M_X = 1.9 \times 10^{16} \, \text{GeV} \) and the unified gauge coupling \( \alpha_G(M_X) = 1/25.6 \). Next we choose a value for the unified Yukawa coupling \( h \) of \( O(1) \). The free parameters of the model are \( (M_{1/2}, \, m_0, \, A, \, \lambda, \, k) \), which are the common gaugino mass, the common scalar mass, the common trilinear scalar coupling and the two additional Yukawa couplings, respectively. Note that our convention [9] requires us to choose \( \lambda > 0 \) and \( k < 0 \) in order to conserve CP in the Yukawa sector of the model[12].

Writing down the coupled system of renormalization group (RG) equations for the 24 parameters of the model [13,14] that are coupled to each other, including the contributions of \( h_b \) and \( h_\tau \) [9], we compare the numerical values of the mass parameters that enter the left hand sides of the minimization equations (3)-(5) with the combinations of the parameters that enter the right hand side of these equations as obtained from the RG evolution. It turns out that the first of the minimization conditions eq. (3) is the one which is most sensitive to the choice of intial conditions reflecting the fine tuning discussed earlier. Furthermore, in order to guarantee the absence of electric-charge breaking vacua,
we impose the constraint $|A| < 3m_0$ [15]. For the NMSSM at large $\tan \beta$ this choice may have to be strengthened further due to the presence of large yukawa couplings for the b-quark. The situation is considerably less restrictive when mild non-universality is allowed and, for instance, if strick Yukawa unification is relaxed. Given these uncertainties, we choose to work with the present constraint [9].

The parameter space of the model is scanned by taking values of the input parameters ($M_{1/2}, m_0, A, h, \lambda, k$) at the GUT scale $M_X$ which are then evolved down to present energies $Q_0$ through the RG evolution to obtain the values of the crucial parameters $\tan \beta, r(\equiv s/v), A_\lambda, A_k, ks$ in addition to the soft masses that appear on the LHS of eq. (3)-(5). The parameters in Ref. [9] were chosen so as to study, $r_1, r_2$ and $r_3$, the differences in the LHS and RHS of eqs. (3)-(5), divided by the right-hand side of each, and to minimize the magnitude of each of these and then study the change in sign that these suffer as the parameters are varied. In Ref. [9] the enormous difficulties faced in trying to achieve a simultaneous solution to $r_i = 0, i = 1, 2, 3$ were noted. In particular, it was shown that $r_3 = 0$ required the presence of values for $|A|/m_0$ of almost 3 or more. This requirement will have a profound impact on the particle spectrum under discussion as we will show below.

We also ensure that the value of $\Delta E$, the difference in the value of the scalar potential computed with the scalar fields attaining their vacuum expectation values, $(v_1, v_2, s)$ and its value computed at the symmetric minimum is negative making the $SU(2) \times U(1)$ breaking minimum energetically favourable. Furthermore, the squared mass of the charged Higgs boson [12] was also computed in order to single out only those points in the parameter space where it is positive in order to not break electric charge spontaneously. The intricate relationship between the various free parameters enforced by the rough scale invariance enjoyed by the evolution equations has been emphasized [9]. Finally, we note
that $r$, the ratio of the singlet to the doublet expectation value persistently remains large for the choice of parameters considered with large $\tan \beta$, corresponding to vacuum expectation value of the singlet $s$ being of the order of 10 TeV. This is substantially different from what happens in the case of $\tan \beta \simeq 1$ [12]. We note that the singlet expectation value is not constrained by the experimental data.

3. Results and Discussion

Having described the NMSSM with large $\tan \beta$ in some detail in the previous section [9], we now turn to obtaining the particle spectrum of the model.

In Fig. 1, we show a typical evolution of the three soft SUSY breaking mass parameters $m_{H_1}^2$, $m_{H_2}^2$, and $m_S^2$ from $M_X$ down to the low scale $Q_0$ with a choice of parameters such that all constraints are satisfied and we are in the neighbourhood of an $SU(2) \times U(1)$ breaking vacuum. We note that because of the possibility of large value of $m_t(m_t) = 181$ GeV, we have a large value for $h$ so that the Yukawa couplings dominate the evolution of these parameters over the gauge couplings. This in turn forces the mass parameters to remain large at large momentum scales compared to their values at smaller momentum scales.

In supersymmetric theories with R-parity conservation, the lightest supersymmetric particle is generally assumed to be the neutralino. In NMSSM, the neutralino mass matrix is a $5 \times 5$ matrix whose general properties are discussed in [16]. The parameters that determine the mass matrix are $\lambda, k, s, \tan \beta, M_1$ and $M_2$, where $M_1$ and $M_2$ are the SUSY breaking bino and wino masses, respectively. Choosing the input parameters at the GUT scale so that all the constraints are satisfied in the manner detailed in the previous section, we obtain the values of the parameters which enter the neutralino mass matrix at the weak scale. We then evaluate the neutralino mass matrix numerically. The chargino mass matrix, on the other hand, is the same as in the MSSM with $\mu$ replaced by $\lambda s$. The
chargino masses can, therefore, be obtained analytically.

One result of this procedure is shown in Fig. 2, where we plot the lightest neutralino masses for a specific choice of input parameters (see table 1), which are similar to ones detailed in Ref. [9], as a function of the top quark mass. From our scan of the parameter space, we have found that the lightest neutralino is almost a pure bino in the limit of large $\tan \beta$. Furthermore, all other neutralinos except the heaviest one have a negligible singlet component. This indicates that the singlet completely decouples from the lighter neutralino spectrum. The mass of the lightest neutralino in this case is determined by the simple mass relation for the bino

$$M_1 \simeq \left(\frac{\alpha_1(M_G)}{\alpha_G}\right) M_{1/2} \simeq 0.45 M_{1/2}. \quad (12)$$

The masses of the heavier neutralinos lie in the range of $0.5 - 1$ TeV. Following the same procedure, as in the case of the neutralinos, of obtaining the parameters entering the chargino mass matrix from the RG evolution of parameters at the GUT scale, we obtain the lightest chargino mass as a function of $m_t(m_t)$ also displayed in Fig. 2. The lightest chargino mass bears a relation to $M_{1/2}$ similar to the neutralino relation, with $\alpha_1$ in eq. (12) replaced by $\alpha_2$ reflecting that it is primarily a charged wino. The heavier chargino mass is found to be $\simeq 1$ TeV. The gluino mass is found to be 1.6 TeV for the choice of parameters of Fig. 2 and follows from a relation similar to eq. (12) with $\alpha_1$ replaced by $\alpha_S$.

We come now to the spectrum of CP-even Higgs bosons of the model. The mass matrix of these Higgs bosons in a $3 \times 3$ matrix which has been discussed extensively in the literature [17,18,19]. Nevertheless, in order to understand the quantitative features of the results we have obtained for the lightest CP-even higgs we need to go into some detail regarding the actual choices of parameters entering the computation and the correlations between the various elements of the spectrum. Such a discussion has been made available
[8] for the case of the MSSM and we will offer a comparison for the model at hand in the present case. In Fig. 3, we plot for typical and reasonable values of the input parameters in the region where the vacuum is expected to lie, the mass of the lightest CP-even Higgs boson as a function of the top-quark mass $m_t(m_t)$ in the range that is most favoured under these boundary conditions [3,8,20,21]. The choice of parameters here is closely related to the family of solutions studied extensively in Ref. [9] and would serve as a typical example of the numbers we have explored. Also, the features seen in this figure may be understood in terms of several of the entries appearing in Table 1 and in particular with those of the corresponding entries for the CP-odd neutral higgs masses. In the MSSM, for instance, it is well known that when the mass of its unique CP-odd boson $m_A >> m_Z$, the substantive part of the radiative correction is picked up by the lighter of the CP-even bosons, $h^0$. As $m_A$ approaches $m_Z$, the radiative corrections are now shifted to the heavier of the CP-even higgs bosons, $H^0$. Such a feature is observed here: for those choices of parameters in Table 1 that yield a somewhat smaller $m_{P_1}$, we find that the radiative corrections to the lighest CP-even higgs, $h_1$ are smaller. Due to the complexity of the system under investigation and the difficulty to control precisely the numerical confidence in the choice of the parameter $\lambda$ for a given $h$ with all other parameters held fixed, we do not know how precisely close the choice of parameters of Table 1 are to a genuine ground state. Furthermore, we note that the clarity with which the correlations have been observed in the MSSM between $m_A$ and $m_{h^0}$ does not have a simple parallel here due to the presence of a large number of physical states. A more precise, albeit prohibitively time consuming, determination could then ensure that the spurious wobble seen in Fig. 3 is eliminated and establish a more reliable correlation between increasing $h$ and the rise of the mass of $h_1$ and the correlations with $m_{P_1}$. Furthermore, a refinement of the choice of parameters when a more complete computation based on the minimization of the one-loop effective
potential could stabilize the figures presented here.

We note that the lightest Higgs bosons mass $\sim 130$ GeV for a wide range of parameters which nearly saturates the upper bound of $140$ GeV [9]. The mass of the lightest Higgs boson in the NMSSM lies in the same range as in the MSSM with large $\tan \beta$. This is a consequence of the largeness of $\tan \beta$: the contribution to the tree level mass which depends on the tri-linear couplings $\lambda$ is small, being proportional to $\sin^2 2\beta$, so that the upper bound on the lightest Higgs mass reduces to the corresponding upper bound in the MSSM when appropriate identification of parameters is made. We also note that the upper bound on the lightest Higgs mass depends only logarithmically on $r$, and hence on the singlet vacuum expectation value $s$, in the limit of large $r$, which, therefore decouples from the bound [22]. Furthermore, the lightest Higgs boson in almost a pure doublet Higgs $(\text{Re} \, H_2^0)$, with the singlet component being less than $1\%$ in the entire range of parameters considered. It is only the second heavier CP even Higgs boson $h_2$ that is predominantly a singlet. Its mass ranges between $740$ GeV and $2.3$ TeV. The heaviest CP even Higgs boson $h_3$ is again predominantly a doublet Higgs $(\text{Re} \, H_1^0)$ with its mass varying between $4 - 6$ TeV. This implies that all the CP-even Higgs bosons, except the lightest one, decouple from the spectrum. The results presented above, that the lightest Higgs boson is almost purely a doublet Higgs at large $\tan \beta$ is in contrast to the situation with low values of $\tan \beta$, where the lightest CP-even Higgs boson contains a large admixture of the gauge singlet field $S$ [12,23,24].

On the other hand, we note from Table 1 that the two CP-odd Higgs bosons, $P_1$ and $P_2$ in the model are heavy, their masses being in the range $2$ TeV and $6$ TeV, respectively. Also, the lightest CP-odd state is predominantly a Higgs singlet, thereby effectively decoupling from the rest of the spectrum. The charged Higgs boson mass $m_C$ lies, for most of the cases that we have studied, in the range $1 - 2$ TeV.
In order to discuss the feature of the remainder of the spectrum, viz., the sfermion spectrum let us first recall some features of the spectrum of the MSSM. In MSSM, it has been observed \[8\] that the presence of large Yukawa couplings for the b-quark as well as the \(\tau\) lepton and also the presence of large trilinear couplings could lead to the lighter of the scalar \(\tau\)'s tending to become lighter than the lightest neutralino, which is the most favored candidate in such models for the lightest supersymmetric particle. In particular, in order to overcome such cosmological constraint for given values of \(M_{1/2}\), lower bounds on \(m_0\) were found to emerge. In turn, increasing \(m_0\) implies ever decreasing \(m_A\) thus leading to further constraints on the parameter space of the MSSM \[8\]. The competing tendencies between the lighter scalar tau mass and \(m_A\) have been shown to play an extraordinary role in MSSM for large values of \(\tan \beta\) in establishing a lower bound \(\sim 450\) GeV on \(M_{1/2}\). Given the complexity of the system of equations, it has not been possible to to extract similar lower bounds on \(M_{1/2}\) in NMSSM. Nevertheless, in Ref. \[9\] the intimate link between the ground states of the two models has been established and a much more sophisticated and time consuming analysis of the present model is also likely to yield a lower bound that is unlikely to be very different from the one obtained in MSSM. As a result, in confining ourselves to numbers of this magnitude and higher, we find a heavy spectrum. More recently \[25\] further experimental constraints on MSSM have been taken into account resulting in an extension of the minimal assumptions at \(M_X\) by including non-universality for scalar masses. Indeed, in the present analysis similar problems have been encountered with some of the choice of parameters studied in Ref. \[9\], with \(m_{\tilde{\tau}_1}\) tending to lie below the mass of the lightest neutralino due to the persistent presence of large Yukawa couplings and more so due to that of the large trilinear couplings. Nevertheless, given the fact that the present work minimizes the tree-level potential, and that the violations of cosmological constraints are not serious in that minor adjustments
of $|A/m_0|$ solve this problem efficiently, we consider the regions of the parameter space we have studied to be reasonable ones. Furthermore, it could be that the extension of minimal boundary conditions along the lines of Ref. [25] could provide alternative and elegant solutions to this problem, while preserving the existence of relatively light scalar $\tau$'s as a prediction of the unification of Yukawa couplings in both the minimal and nonminimal supersymmetric standard models.

The heaviest sfermions in the spectrum of NMSSM as in MSSM are the scalar quarks. The scalar quarks here, as in the MSSM, tend to be much heavier, in the TeV range. The $SO(10)$ property that the scalar b-quarks are as massive as the scalar top-quarks is preserved in the NMSSM.

4. Conclusions

In the detailed analysis presented here, we have shown that, except for the lightest CP-even Higgs boson, all the particles implied by supersymmetry are heavy for large values of $\tan\beta$. The gauge singlet field $S$ decouples both from the lightest Higgs boson as well as the neutralinos. This is in contrast to the situation that one obtains for the model at low values of $\tan\beta$. The LSP of the model continues to be, as in MSSM, the lightest neutralino that is primarily a bino in composition, with the lighter scalar $\tau$ with a mass in the neighbourhood of the LSP mass. The remainder of the spectrum tends to be heavy, from 1 to a few TeV. We note that the NMSSM in the large $\tan\beta$ regime rests on a delicately hinged system of equations and constraints. Although it provides a good testing ground for the stability of the predictions of the MSSM, in practice it deserves great care in its treatment.
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Table Caption

Table 1. Sample of points in the parameter space and the computed values of different mass parameters [all masses in units of GeV].

Figure Captions

Fig. 1 The evolution of soft supersymmetry breaking mass parameters from the grand unified scale $M_X$ to $Q_0$ defined in the text. The input parameters are $M_{1/2} = -700$, $m_0 = 800$ and $A = 1600$ (all in GeV). The other parameters are $h = 1.5$, $\lambda = 0.40$ and $k = -0.10$. The associated value of the top quark mass is 181 GeV.

Fig. 2 The lightest neutralino and chargino masses as a function of $m_t(m_t)$. The input parameters are $M_{1/2} = -700$, $m_0 = 800$, $A = 2200$ (all in GeV), with the remaining parameters varied to guarantee a solution.

Fig. 3 The lightest CP-even Higgs boson mass as a function of $m_t$. The range of parameters is as in Fig. 2.
| $M_{1/2}$ | $m_0$ | $A$ | $h$ | $\lambda$ | $k$ | $m_t(m_t)$ | $m_{h_1}$ | $m_{h_2}$ | $m_{h_3}$ | $m_{P_1}$ | $m_{P_2}$ | $m_C$ |
|---------|-------|-----|-----|----------|----|------------|----------|----------|----------|----------|----------|-------|
| -700    | 800   | 2200| 0.75| 0.1      | -0.1 | 170.5      | 131.0    | 2314     | 6039     | 3048     | 6918     | 5077  |
| -700    | 800   | 2200| 1.00| 0.2      | -0.1 | 176.5      | 132.5    | 1326     | 4822     | 2479     | 6216     | 3031  |
| -700    | 800   | 2200| 1.25| 0.3      | -0.1 | 179.6      | 130.3    | 977      | 4342     | 2238     | 5990     | 1862  |
| -700    | 800   | 2200| 1.50| 0.4      | -0.1 | 181.4      | 119.0    | 774      | 4297     | 2099     | 6394     | 949   |
| -700    | 800   | 2200| 1.75| 0.4      | -0.1 | 182.5      | 133.0    | 1031     | 5116     | 2275     | 6800     | 2901  |
| -700    | 800   | 2200| 2.00| 0.5      | -0.1 | 183.3      | 128.0    | 858      | 4823     | 2153     | 6636     | 2227  |
| -700    | 800   | 2200| 2.25| 0.6      | -0.1 | 183.8      | 119.0    | 740      | 4644     | 2067     | 6557     | 1668  |

Table 1
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