Effect of Ambient Temperature on Catenary System

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Abstract. This paper investigates the effect of the ambient temperature change on the overhead catenary system (OCS). Thermal expansion/contraction of the components of the overhead catenary system (OCS) are considered. Simulation results show that the temperature stress has a much greater influence on the longitudinal positioning of the contact wire than on the horizontal and vertical positioning. When ambient temperature reduces to -60°C or rises to 60°C from design temperature of 20°C, the maximum longitudinal displacement of the positioner can reach as high as 304 mm, while the maximum horizontal and vertical displacement of the positioner is within 4 mm. The decrease of temperature leads to the increase of cable tension, and the increase of temperature leads to the decrease of cable tension. When the temperature decreases from 20°C to -60°C or rises to 60°C, the tension change rate of contact wire is less than 1%, while that of messenger wire is not more than 0.01%.

1. Introduction
Pantograph catenary system is of great importance in an electrified railway line [1]. It is the key equipment of energy supply to high-speed train. Unfortunately, the catenary system exposes to the open air for a long time period during the train operation. So, the catenary is vulnerable to the influence of weather condition. With bigger geographical coverage of railway transportation, the operational environment of a high-speed train is becoming more complex and harsher. Since overhead catenary cables are quite long, so they are very sensitive to changes in the ambient temperature [2]. The change of tensile force and spatial position of catenary cables will inevitably affect the dynamic match between pantograph and catenary (PAC) during train operation.

The direct effect of temperature change on catenary is its spatial re-positioning. That is, due to the phenomenon of thermal expansion/contraction, the catenary will experience elongation or shortening accordingly [3]. The change of temperature will change the geometric position of the contact wire, which will affect the tensile stress in it [4]. Studying a geometrically nonlinear suspension structure shows that the rise of temperature results in the relaxation of contact wire and drop of temperature results in contraction leading to bigger tensile stress [5].

At present, there are not many studies on the ambient temperature of OCL system. In this paper, the influence of ambient temperature on the spatial position and tension of catenary is studied. This study
is not only helpful to guide the construction layout of catenary system, but also to study the influence of ambient temperature on current collection of pantograph catenary system.

2. Mathematical Model of Catenary with Thermal Effect

Simple stitched catenary system was established in this paper. The simple stitched catenary system is mainly composed of contact wire, messenger wire, dropper and support positioning devices. The relevant parameters of OCS are extracted from the reference [6]. The catenary is a continuous system. It can be modeled using the Finite Element Method (FEM) techniques [7].

The catenary system can be viewed as a series of cables fixed at both ends. The dynamic equation of the cable fixed at both ends without considering temperature is as follows [8]:

\[
m \frac{\partial^2 v}{\partial t^2} + c_v \frac{\partial v}{\partial t} - H \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial x^2} \right) \left[ \frac{dy}{dx} \frac{\partial v}{\partial x} + \frac{1}{2} \left( \frac{\partial v}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \right] dx = P_n(x,t) \tag{1}
\]

\[
m \frac{\partial^2 w}{\partial t^2} + c_w \frac{\partial w}{\partial t} - H \left( \frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial x^2} \right) \left[ \frac{dy}{dx} \frac{\partial v}{\partial x} + \frac{1}{2} \left( \frac{\partial v}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \right] dx = P_n(x,t) \tag{2}
\]

Where \( v(x,t) \) and \( w(x,t) \) are the vertical and lateral displacements of a point on the cable respectively; \( m \) is the cable mass per unit length. \( c_v \) and \( c_w \) are the viscous damping coefficients per unit length; \( E \) is Young modulus, \( A \) is the cross-sectional area of the cable; \( H \) is the initial tensile force under the reference temperature, \( H = \frac{mgL^2}{8b} \); \( b \) is the vertical displacement under the initial configuration of the cable; \( g \) is the gravitational acceleration and \( y(x) = \frac{4b(L-x)x}{L^2} \) is the cable profile.

Making Eq. (1) and Eq. (2) dimensionless, we can get the following results:

\[
\frac{\partial^2 v^*}{\partial t^2} + c^{*} \frac{\partial v^*}{\partial t} - \Theta \left( \frac{d^2 y^*}{dx^2} + \frac{d^2 v^*}{dx^2} \right) \left[ \frac{dy^*}{dx} \frac{\partial v^*}{\partial x} + \frac{1}{2} \left( \frac{\partial v^*}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial w^*}{\partial x} \right)^2 \right] dx = P^*_v(x,t) \tag{3}
\]

\[
\frac{\partial^2 w^*}{\partial t^2} + c^{*} \frac{\partial w^*}{\partial t} - \Theta \left( \frac{d^2 w^*}{dx^2} + \frac{d^2 w^*}{dx^2} \right) \left[ \frac{dy^*}{dx} \frac{\partial v^*}{\partial x} + \frac{1}{2} \left( \frac{\partial v^*}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial w^*}{\partial x} \right)^2 \right] dx = P^*_w(x,t) \tag{4}
\]

Where \( v^* = \frac{v}{L} \), \( w^* = \frac{w}{L} \), \( x^* = \frac{x}{L} \), \( y^* = \frac{y}{L} \), \( t^* = \frac{t}{\sqrt{\frac{8b}{g} \frac{c_{v^*}}{m}}}, \ c^*_{v^*} = \frac{8b}{g} \frac{c_{v^*}}{m}, \ p^* = \frac{p_{n^*}(x,t)L}{H} \), \( \Theta = \frac{E^*}{H} \).

Assuming that the temperature variation is uniform along the length and cross section of the cable, and the Young modulus, viscous damping coefficients and boundary conditions are all independent of the thermal effect. In order to consider the influence of temperature change on the suspended cable, the dimensionless cable tensile force variation factor \( \chi^* = \frac{H^*_{\Delta T}}{H} \) is introduced, where \( H_{\Delta T} \) is the tensile force under the temperature change \( \Delta T \). The equations of motion of the suspended cable considering the thermal effect can thus be obtained as follows:

\[
\frac{\partial^2 v^*}{\partial t^2} + c^{*} \frac{\partial v^*}{\partial t} - \chi^* \frac{d^2 v^*}{dx^2} + \Theta \left( \frac{1}{\chi^*} \frac{d^2 v^*}{dx^2} \right) \left[ \frac{1}{\chi^*^2} \frac{dy^*}{dx} \frac{\partial v^*}{\partial x} + \frac{1}{2} \left( \frac{\partial v^*}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial w^*}{\partial x} \right)^2 \right] dx = P^*_v(x,t) \tag{5}
\]
\[
\frac{\partial^2 \bar{W}}{\partial t^2} + c_v \frac{\partial \bar{W}}{\partial t} - \chi^2 \frac{\partial^2 \bar{W}}{\partial x^2} - \Theta \left( \frac{d^2 \bar{W}}{dx^2} \right) \int \frac{1}{\chi^2} \frac{\partial^2 \bar{v}}{\partial x^2} + \frac{1}{2} \left( \frac{\partial \bar{W}}{\partial x} \right)^2 \right] dx = P_1^* (x,t) \quad (6)
\]

Galerkin method is used to solve Eq. (5) and Eq. (6) numerically. The solution can be written as a summation of many terms, each term being variable separated as follows.

\[
\bar{v}(x,t) = \sum_{n=1}^{N} q_n(t) \varphi_n(x) \quad (7)
\]

Where \(N\) is the number of terms; \(q_n(t)\) is the generalized coordinate and \(\varphi_n(x)\) is the mode shape.

Substituting Eq. (7) into Eq. (5) and Eq. (6) and considering boundary conditions, the results of n-th symmetric mode shapes can be obtained as follows

\[
\varphi_n = \tilde{\xi} \left[ 1 - \cos \left( \frac{\omega_n}{\chi} x \right) - \tan \left( \frac{\omega_n}{2\chi} \right) \sin \left( \frac{\omega_n}{\chi} x \right) \right], \quad (n=1, 3, 5, \ldots) \quad (8)
\]

Where \(\tilde{\xi}\) is determined by the normalization conditions of the mode shapes. The symmetric mode frequencies \(\omega_n\) can be calculated by Eq. (9).

\[
\tan \left( \frac{\omega_n}{2\chi} \right) = \frac{\omega_n}{2\chi} - \frac{(\omega_n)^3}{2\chi^2} \quad (9)
\]

Where \(\chi^2\) is the Irvine parameter which can be expressed as \(\chi^2 = \frac{\left( \frac{mgL}{H} \right)^2}{EA} \).

The anti-symmetric mode shapes can be expressed as follows

\[
\varphi_n = \sqrt{2} \sin (n \pi x), \quad \omega_n = \chi n \pi \quad \text{ (n=2, 4, 6, \ldots)} \quad (10)
\]

Substituting Eq.(7) ~ Eq.(10) into Eq.(5) and Eq.(6), the cable dynamic equation considering temperature can be obtained by multiplying \(\varphi_n\) on both sides of the formula and integrating along the length of the cable

\[
M\ddot{q}_v + C\dot{q}_v + K_v = Q_v \quad (11)
\]

\[
M\ddot{q}_w + C\dot{q}_w + K_w = Q_w \quad (12)
\]

3. Influence of Temperature on Spatial Location of OCS

The temperature stress will change the spatial position and state of the OCS, and affect the dynamic match of pantograph and catenary when electrified train passes through the section. Fig. 1 depicts the OCS profile at two different temperature: the design profile at 20°C and the hot profile at 50°C.
The 9-span OCS model is established, and the central anchor is set in the middle of the fifth span. The number of positioner increases along the extension direction of catenary, and the number is 1~8. The design temperature of OCS is 20°C. It is assumed that the contact wire, messenger wire, auxiliary messenger wire, dropper, positioner, cantilever and other components of catenary are uniformly stretched or compressed with the change of temperature within the range of ambient temperature change.

3.1. Longitudinal Displacement of the Positioner

In order to ensure that the pantograph slide plate of the electric locomotive is in good contact with the contact wire during the operation, it is necessary to locate the contact wire according to the operation requirements of pantograph with the positioning device. Among them, the positioner is an important part to determine the spatial position of contact wire. The dynamic matching between pantograph and catenary is directly affected by the spatial position of positioner. The thermal stress caused by thermal expansion/contraction will change the position of the positioner of OCS.

Fig. 2 shows the variation trend of the longitudinal static influence of the positioner with the temperature. It shows that the longitudinal static response of the positioner on the contact wire is linearly related to the temperature, and the deformation is directly proportional to the distance from the point to the center anchor. When the temperature rises to 60°C, the head / end of the contact wire is pulled up about 152 mm along the contact wire. When the temperature is reduced to -60°C, the head / end of the contact wire is shortened by about 304 mm.

The longitudinal static response of overhead contact system is symmetrical with respect to the central anchor. The longitudinal static response direction of each positioner on the same side of the central anchor is the same, and the direction of the opposite side is opposite. This is because the linear expansion coefficient of cable material is considered as customized in the range of ambient temperature change.

![Figure 1. OCS Profiles at: (a) 20°C (Design), and (b) 50°C (Hot).](image1)

![Figure 2. Longitudinal displacement.](image2)

![Figure 3. Vertical displacement.](image3)
3.2. Vertical Displacement of the Positioner

Fig. 3 shows the change of vertical position at the positioner caused by temperature stress. The vertical static response of the central anchor knot varies linearly with the air temperature. When the temperature rises to 60°C, the vertical deformation displacement of the central anchor knot does not exceed 1.5mm. When the temperature drops to -60°C, the vertical deformation displacement of the central anchor knot does not exceed 2.5mm.

The vertical static response of No.1 to No.8 positioners changes linearly with air temperature. And the change trend is similar to that of the central anchor: when the temperature rises, the central anchor point drops vertically; when the temperature decreases, the central anchor point rises vertically. When the temperature is reduced to -60°C, the maximum vertical rise of positioner is 2.2mm. When the temperature rises to 60°C, the maximum vertical settlement of the positioner is 1.2mm.

The results show that the temperature change has little effect on the vertical static response of the positioner. The vertical static response of the positioner is within 2.5mm when the temperature rises from 20°C to 60°C or decreases to -60°C.

3.3. Lateral Displacement of the Positioner

Fig. 4 shows the change of lateral position at the positioner caused by temperature stress. It can be seen from the figure that the lateral static response of the central anchor varies linearly with the temperature. When the temperature rises to 60°C, the transverse deformation displacement does not exceed 2 mm; when the temperature decreases to -60°C, the transverse deformation displacement does not exceed 3.5 mm.

The lateral static response of No.1~8 positioners is basically linear with the temperature. And the change law is similar to that of the central anchor: when the air temperature rises, the positioner moves along the direction close to the central line of the catenary; when the temperature drops, the positioner moves along the direction far away from the central line of the catenary. When the temperature drops to -60°C, the lateral displacement of No.1 positioner is the largest, which is 3.43mm. When the temperature rises to 60°C, the lateral displacement of No.1 positioner is the largest, 1.71mm.

Therefore, the influence of temperature change on the lateral static response of the positioner is relatively small. When the temperature rises from 20°C to 60°C or decreases to -60°C, the lateral static response at the positioner is controlled within 3.5mm.

From the influence of temperature change on the spatial position of catenary, the influence of temperature change on the vertical and lateral spatial position of catenary is not obvious, but the impact on the longitudinal spatial position of catenary is more obvious. The change of temperature causes the conductor to expand and contract linearly along the cable longitudinal direction.
4. Tension of the Catenary

The spatial position of OCS is changed due to temperature change. In addition to the positioner positioning the contact wire in the horizontal direction perpendicular to the line, the catenary has to bear the tension along the line. When the spatial position of the catenary changes, it will inevitably lead to the change of cable tension in the catenary system.

4.1. Tension of Contact Wire

Fig.5 shows the distribution of contact wire tension along the catenary when the ambient temperature is -60°C, 20°C and 60°C respectively. When the ambient temperature is the design temperature of catenary (20°C), the relative variation of ambient temperature is 0 °C. At this time, the distribution of contact wire tension along the line is the design tension of contact wire (27kN), as shown in Fig. 5(b).

When the ambient temperature is -60°C, compared with the design temperature of catenary, the ambient temperature decreases, and the contact wire shrinks longitudinally under the action of temperature stress, as shown in Fig. 5(a). Because the central anchor is set in the middle of the catenary, the contact wire shrinks towards the central anchor after the temperature drops. The closer the contact wire is to the central anchor knot, the greater the contact wire tension is, and the smaller the contact wire tension is farther away from the central anchor. When the temperature drops to -60°C, the maximum contact wire tension appears in the fifth span, reaching 27.177kN; the minimum contact wire tension appears in the first span and the ninth span, reaching 26.999kN, which is close to the initial contact wire tension.

When the ambient temperature is 60 °C, relative to the design temperature of the catenary, the ambient temperature rises, and the contact wire elongates longitudinally under the action of temperature stress. Similarly, due to the role of the central anchor in the middle of the catenary, after the temperature rises, the contact wire is far away from the central anchor and elongates. The closer the contact wire is to the central anchor, the smaller the tension is, and the more far away from the central anchor is, the greater the tension is, as shown in Fig.5(c). When the temperature rises to 60°C, the maximum contact wire tension appears in the first span and the ninth span, which is 27.000kN; the minimum contact wire tension appears in the fifth span, which is 26.912kN.

Figure 5. Tension distribution of contact wire at different temperatures: (a) -60°C; (b) 20°C; (c) 60°C
Fig. 6 shows the change of contact wire tension after the change of ambient temperature. It can be seen from the figure that when the temperature decreases, the maximum tension of the contact wire increases continuously. When the temperature decreases from 20°C to -60°C, the maximum tension of the contact wire increases by 0.177kN, increasing by 0.66%; when the temperature increases, the maximum tension of the contact wire is basically maintained. When the temperature decreases, the minimum value of the contact wire tension basically remains unchanged; when the temperature increases, the minimum value of the contact wire tension decreases continuously. When the temperature increases from 20°C to 60°C, the minimum value of the contact wire tension decreases by 0.088kN, with a decrease of 0.33%.

4.2. Tension of Messenger Wire

Fig. 7 shows the distribution of messenger wire tension along the catenary when the ambient temperature is -60°C, 20°C and 60°C. When the ambient temperature is the design temperature of catenary (20°C), the distribution of catenary tension along the line is close to the design tension of catenary (21kN), as shown in Fig. 7(b).

When the ambient temperature is -60°C, compared with the design temperature of catenary, the ambient temperature decreases, and the catenary shrinks longitudinally under the action of temperature stress. When the temperature decreases to -60°C, the maximum and minimum tension of the messenger wire increase slightly.

When the ambient temperature is 60°C, compared with the design temperature of catenary, the ambient temperature rises, and the catenary is longitudinally extended under the action of temperature stress. When the temperature rises to 60°C, the maximum and minimum tension of the messenger wire decrease slightly.
Figure 7. Tension distribution of messenger wire at different temperatures: (a) -60°C; (b) 20°C; (c) 60°C

Figure 8. The influence of temperature on the tension of the messenger wire

Fig. 8 shows the change of messenger wire tension after the change of ambient temperature. It can be seen from the figure that tension and temperature are negatively related. The higher the temperature, the smaller the tension, and the lower the temperature, the greater the tension. When the temperature decreases from 20°C to -60°C, the maximum tension of the contact wire increases by 0.017kN, increasing by 0.008%; when the temperature increases from 20°C to 60°C, the minimum tension of the contact wire decreases by 0.004kN, increasing by 0.002%. When the temperature decreases from 20°C to -60°C, the minimum tension of the contact wire increases by 0.003kN, increasing by 0.001%; when the temperature increases from 20°C to 60°C, the minimum tension of the contact wire decreases by 0.007kN, increasing by 0.003%.

The change of temperature will cause the tension of contact wire and messenger wire to change, but the tension of contact wire is greatly affected by temperature, and the tension of messenger wire is also affected by temperature, but the influence is very small. The tension of the contact wire in each span of an anchor section is not consistent due to the change of temperature. The closer to the central anchor, the greater the influence of temperature on the tension of the contact wire.

5. Conclusion
This paper proposes an extended mathematical model, considering the ambient temperature effect on the catenary system. Comprehensive parametric studies are carried out. The influence of temperature stress on the location of positioners and tension of catenary are studied. Simulation results show that the influence of temperature stress on the longitudinal position of contact wire is significant. The vertical and lateral spatial positions of the contact wire are not significantly affected by the temperature. The
temperature decreases from 20°C to -60°C or rises to 60°C, and the change in the lateral and vertical positions of the positioner does not exceed 4mm. The tension of contact wire and messenger wire is affected by temperature. The tension of contact wire is greatly affected by the temperature change, while the tension of messenger wire is less affected by the temperature change. Specifically, when the ambient temperature is reduced from 20°C to -60°C, the change range of contact wire tension is not more than 1%, and that of messenger wire is not more than 0.01%. The closer to the center anchor, the greater the influence of temperature on the tension of contact wire.

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