ON THE ORBITAL EVOLUTION OF A GIANT PLANET PAIR EMBEDDED IN A GASEOUS DISK. II. A SATURN–JUPITER CONFIGURATION

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ABSTRACT

We carry out a series of high-resolution ($1024 \times 1024$) hydrodynamic simulations to investigate the orbital evolution of a Saturn–Jupiter pair embedded in a gaseous disk. This work extends the results of our previous work by exploring a different orbital configuration—Jupiter lies outside Saturn ($q < 1$, where $q \equiv M_i/M_o$ is the mass ratio of the inner planet and the outer one). We focus on the effects of different initial separations ($d$) between the two planets and the various surface density profiles of the disk, where $\sigma \propto r^{-\alpha}$. We also compare the results of different orbital configurations of the planet pair. Our results show that (1) when the initial separation is relatively large ($d > d_{\text{L},i}$, where $d_{\text{L},i}$ is the distance between Jupiter and its first inner Lindblad resonance), the two planets undergo divergent migration. However, the inward migration of Saturn could be halted when Jupiter compresses the inner disk in which Saturn is embedded. (2) Convergent migration occurs when the initial separation is smaller ($d < d_{\text{L},i}$) and the density slope of the disk is nearly flat ($\alpha < 1/2$). Saturn is then forced by Jupiter to migrate inward where the two planets are trapped into mean motion resonances (MMRs), and Saturn may get very close to the central star. (3) In the case of $q < 1$, the eccentricity of Saturn could be excited to a very high value ($e_{S_i} \sim 0.4–0.5$) by the MMRs and the system could maintain stability. These results explain the formation of MMRs in the exoplanet systems where the outer planet is more massive than the inner one. It also helps us to understand the origin of the “hot Jupiter/Saturn” with a highly eccentric orbit.

Key words: planet–disk interactions – protoplanetary disks

Online-only material: color figures

1. INTRODUCTION

The existence of more than 40 multiple planet systems have been affirmed so far. The observational facts show that almost one-fourth of them contain two or more planets locked in mean motion resonances (MMRs). This ratio keeps growing as new detection methods are adopted, e.g., the transit time variation method, which is particularly suitable for detecting low-mass planets (comparable to Jupiter) open a gap in the gaseous disk (Masset et al. 2006; Morbidelli et al. 2008). Otherwise, they could be captured by the high-order MMRs of inward migrating giants (Zhou et al. 2005; Fogg & Nelson 2007; Raymond et al. 2006). These processes explain well the formation of terrestrial planets in “hot Jupiter/Saturn” systems, but could not account for the low-order MMRs in systems whose inner planet is as massive as Saturn or even Jupiter, e.g., Gliese 876, HD 160691, and HD 128311.

So far, only a few works have considered the orbital configuration in which the two giant planets have a mass ratio of $q < 1$. Kley et al. (2005) studied the orbital evolution of Gliese 876, which has $q \approx 0.31$. They simulated convergent migration for the two planets and successfully represented their observed
focus on the effects of various surface density slopes (outer) pair embedded in a gaseous protostellar disk. We therefore, following Paper I, we continue to which is most suitable for studying the dynamics of this mass typical planet pair in this range is a Jupiter–Saturn pair, pairs trapped in MMRs have of these domain settings can be found in Paper I.

mechanism to bring about convergent migration is required in a cavity at the center of the disk, e.g., the rotation of the magnetic gas accretion onto the central star should be so vigorous that the inner part of the disk is depleted much earlier before the formation and migration of the giant planets. Other processes may also result in a cavity at the center of the disk, e.g., the rotation of the magnetic field. However, the relatively small radii of the cavity limit this assumption to specific cases. A more general and self-consistent mechanism to bring about convergent migration is required in a whole and regular disk. To construct a full-region disk instead of a ring, we adopt the Cartesian computational domain. Details of these domain settings can be found in Paper I.

As shown in Table 1, we note that most of the planet pairs trapped in MMRs have \( q \approx 0.3–0.6 \). A familiar and typical planet pair in this range is a Jupiter–Saturn pair, which is most suitable for studying the dynamics of this mass configuration. Therefore, following Paper I, we continue to investigate the orbital evolution of a Saturn (inner)–Jupiter (outer) pair embedded in a gaseous protostellar disk. We focus on the effects of various surface density slopes (\( \alpha = -d \ln(\sigma)/d \ln(r) \)) and various initial separations \( d \) between the two planets. We will show the following. (1) Although under divergent migration, when Jupiter migrates inward, it could halt the inward migration of Saturn by compressing the inner disk in which Saturn is embedded. (2) When the initial separation is smaller than the distance from Jupiter to its fattest inner Lindblad resonance \( d_{\text{IL}} \) and the density slope of the disk is nearly flat, the two planets may undergo convergent migration and then be trapped into MMRs. (3) The eccentricity excitation of MMRs overwhelm the damping of the gas disk. Saturn may get very close to the central star (\( a_s < 1 \text{ AU} \)), preserving a very high eccentricity. (4) We also compare the results between different orbital configurations—\( q > 1 \) and \( q < 1 \). These results help reveal the orbital architecture formation of some resonant exoplanet systems, e.g., HD 160691 and Gliese 876, as well as the formation of a “hot Jupiter/Saturn” with a highly eccentric orbit.

This paper is organized as follows. The model including the numerical methods and the initial settings is introduced in Section 2, our results and the analysis are presented in Section 3, some discussions are carried out in Section 4, and the conclusions are summarized in Section 5.

2. NUMERICAL SETUP

2.1. Physical Model

We simulate the full dynamical interaction of a system including a solar-type star, a Saturn mass giant (inner planet), a Jupiter mass giant (outer planet), and a two-dimensional (2D) gas disk. The star is fixed at the origin of the system with both the planets and the disk surrounding it; thus, the whole system is accelerated by the gravity of the planets and the disk. For efficiency, we ignore the self-gravitating effect of the gas. Therefore, the gravity exerted on the gas only comes from the central star, the two giant planets, and the acceleration of the origin.

For numerical convenience, the gravitational constant \( G \) is set to 1. The solar mass (\( M_\odot \)) and the initial semimajor axis of Saturn (\( R_\odot = 5.2 \text{ AU} \)) are set in units of mass and length, respectively. We locate Saturn initially inside the orbit of Jupiter; thus, the mass ratio of the two planets is \( q = 0.33 \). Our aim is to investigate the formation of MMRs and the consequential evolution of the system with this orbital configuration. Considering that Saturn and Jupiter would mostly undergo type II migration, we adopt a relatively large viscosity of \( \nu = 5 \times 10^{-5} \) to accelerate the evolution and run simulations up to 2500–5000 initial orbital periods of the inner planet.

We adopt a polytropic equation of state and set the disk aspect ratio to be \( H/r = 0.04 \). The evolution of the gas under the gravity of the star and the planets is solved by the 2D Godunov code Antares, which is based on the exact Riemann solution for isothermal or polytropic gas. While the dynamics of the two planets under the potential of the star and gas disk are integrated by an eighth-order Runge–Kutta integrator, the global time step is the minimum of the hydrodynamical part and the orbit integration part. Details of the numerical method as well as the computational configuration have been described well in our previous works (Paper I). A comparison with other well-studied codes has also been presented in Paper I.

2.2. Initial Condition

One of the issues that we expect to figure out is the effect of the surface density profile on the disk. In this paper, we try several typical density profiles, which are only functions of disk radii \( \sigma = \sigma_0 r^\alpha \). As shown in Table 2, the initial density distribution varies from flat to very steep: \( \sigma_0, \sigma_0 r^{-1/2}, \sigma_0 r^{-1}, \) and \( \sigma_0 r^{-3/2} \). \( \sigma_0 \) is set to be 0.0006 in our units, which corresponds to a height-integrated surface density of \( \sim 200 \text{ g cm}^{-2} \). The density slope on the gas disk results in a pressure gradient. To ensure that there is no radial flow at the beginning, we set the radial velocity \( u_{\phi} \) to be 0 and adjust the initial angular velocity of the gas to \( u_{\phi} = r \Omega_e \) to balance the pressure and central gravity, where \( \Omega \) depends on \( \alpha \).
Another issue is the role of the initial separation between Saturn and Jupiter. As shown in Table 2, we choose three separations: \(d \equiv a_{J0} - a_{S0} = 1, 0.5, \) and 0.25. For numerical convenience, we set the initial semi-major axis of the inner planet (Saturn) as the length unit (the initial location of Saturn is always \(a_{S0} = 1\)). Then we adjust the initial locations of the outer one (Jupiter) to obtain different separations. When \(d = 1\), Jupiter is initially located at \(a_{J0} = 2\). At such a large distance, the mutual interaction due to gravity is negligible, so the two planets could migrate independent of each other at the very beginning. When \(d = 0.5\), Jupiter is initially located at \(a_{J0} = 1.5\). The position of the \(P_J : P_S = 2:1\) MMR, where \(P_J\) and \(P_S\) are the orbital periods of Jupiter and Saturn, respectively, is now at \(r \approx 0.94\), which is a little bit inside the initial location of Saturn. If divergent migration of the two planets occurs, Saturn would pass through the position of the 2:1 resonance with Jupiter soon after the release moment. Furthermore, the first inner Lindblad resonance (at \(\Omega = \Omega_J + \kappa/m\), where \(\kappa\) is the epicycle frequency and \(m = -2\)) of Jupiter is located around \(\sim 0.88–0.94\) depending on the density slope \(\alpha\). Now the separation between the two planets is smaller than the distance between Jupiter and its first (also the furthest) inner Lindblad resonance, \(d \lesssim d_{ILr}\). Thus, the inner Lindblad torque of Jupiter should be reduced by the existence of Saturn at the beginning of the evolution. Finally, when \(d = 0.25\), Jupiter is initially located at \(a_{J0} = 1.25\). This location ensures that Saturn passes through the \(P_J : P_S = 3:2\) MMR of Jupiter, the location of which is at \(r = 0.95\). More importantly, this small separation \(d \lesssim 3r_{H}\) allows mutual scattering due to gravity between the two planets at the beginning of the evolution, where \(r_{nlH} = 0.085\) is the mutual Hill radius of Saturn and Jupiter at this configuration:

\[
r_{nlH} = \left(\frac{M_J + M_S}{3 M_\odot}\right)^{\frac{1}{3}} \left(\frac{a_J + a_S}{2}\right).
\]

The initial settings of the disk do not take into account the gravitational perturbation of the planets. Instead, we adopt the “quiet-start” prescription to set up a dynamical equilibrium to ensure that the streamlines of the gas are always closed when the planet is growing. Basically, we set the initial masses of the planets to be negligible, then we fix their orbits and increase their masses to Saturn and Jupiter masses adiabatically. At the end of their growth, we release them at the same moment and start the evolution. Details of this prescription have also been described in Zhang et al. (2008).

### 3. RESULTS

#### 3.1. Divergent Migration and Suppression of Inward Migration

Our main results are summarized in Table 2. There are two main variables: the density slope \(\alpha\) and the initial separation \(d\) between the two planets. To avoid confusion, we present the results mainly according to the sequence of the initial separations \(d\). And for each \(d\), we present the effects of the different density slopes first and then explain these effects together.

First, we start with a relatively large initial separation \((d = 1)\). As introduced in the previous section, the mutual interaction due to gravity between Saturn and Jupiter is now negligible \((d > 5r_{H})\). Our results show that, at large separation, Saturn and Jupiter will generally undergo divergent migration or achieve equilibrium, depending on the surface density profile.

1. When the disk is very steep \((\alpha > 1/2)\), Jupiter digs a clear gap and migrates outward after the release. In the meantime, Saturn migrates inward very fast until it clears its co-orbital region (type III migration). Then Saturn starts to follow the viscous evolution of the disk, and its fast inward migration is halted or even reversed. This result is consistent with our previous result that the massive planet follows the movement of the global disk and moves outward when \(\alpha > 1/2\). Figure 1 shows the orbital evolutions of the two giant planets embedded in different surface density slopes when \(d = 1\).

2. When the disk is nearly flat \((\alpha \lesssim 1/2)\), both planets are under inward migration. Since Saturn is still surrounded by gas at the moment of release, it migrates inward much faster than Jupiter during the first stage of evolution. As Saturn keeps clearing its vicinity, inward migration is then reduced gently. In fact, we observed an equilibrium state where Jupiter and Saturn both stop migrating inward and maintain their separation when \(\alpha = 1/2\); see panel (b) in Figure 1. This is also consistent with the results of Paper I, which showed that the direction of viscous movement changes sign around \(\alpha = 1/2\):

\[
r = \frac{1}{2\pi r\sigma} \frac{d\Gamma_v}{dr} = -3\nu \left(\frac{1}{2} - \alpha\right) r^{-1}.
\]

where \(\Gamma_v\) is the viscous torque,

\[
\Gamma_v = 2\pi r^2 \nu \sigma \frac{r d\Omega}{dr} \sim r^{1/2-a},
\]

which stays constant over different radii \(r\) when \(\alpha = 1/2\) by assuming a constant viscosity \(\nu\) across the disk.

According to Figure 1, one may find that both the inward and the outward type II migration of Saturn are suppressed. After several hundred orbit evolution, the whole disk has been separated well into two parts—an inner disk and an outer disk—by the gap opened by Jupiter. Saturn has also dug a gap in the inner disk. Although it is much weaker than that of Jupiter, this gap ensures that Saturn will undergo type II migration. Because of the large initial separation and divergent migration at the beginning stage, the gaps of the two planets will not overlap soon. Thus, the inner disk and Saturn could be treated as a sub-system that is shepherded by the tidal torque of Jupiter; see Figures 2 and 3. Then the surface density profile \(\alpha\) makes some difference.

### Table 2: Summary of Our Simulations

| Case | Configuration | Separation, \(d\) | \(\alpha\) | Relative migration | Resonance |
|------|---------------|------------------|----------|-------------------|-----------|
| 1    | S-J           | 1                | \(\sim r^0\) | Divergent         | \(2:1\)   |
| 2    | S-J           | 1                | \(\sim r^{-1/2}\) | Equilibrium       | \(\cdots\) |
| 3    | S-J           | 1                | \(\sim r^{-1}\) | Divergent         | \(\cdots\) |
| 4    | S-J           | 1                | \(\sim r^{-3/2}\) | Divergent         | \(\cdots\) |
| 5    | S-J           | 0.5              | \(\sim r^0\) | Convergent        | 2:1       |
| 6    | S-J           | 0.5              | \(\sim r^{-1/2}\) | Convergent        | 2:1       |
| 7    | S-J           | 0.5              | \(\sim r^{-1}\) | Divergent         | \(\cdots\) |
| 8    | S-J           | 0.5              | \(\sim r^{-3/2}\) | Divergent         | \(\cdots\) |
| 9    | S-J           | 0.25             | \(\sim r^0\) | Convergent        | 3:2       |
| 10   | S-J           | 0.25             | \(\sim r^{-1/2}\) | Convergent        | 2:1       |
| 11   | S-J           | 0.25             | \(\sim r^{-1}\) | Convergent        | 2:1       |
| 12   | S-J           | 0.25             | \(\sim r^{-3/2}\) | Convergent        | 2:1       |
Figure 1. Semimajor axis evolutions of Saturn and Jupiter embedded in a gas disk whose surface density slope varies from flat ($\sigma \propto r^0$) to very steep ($\sigma \propto r^{-3/2}$). The dash-dotted curve in panel (a) corresponds to the migration of a single Saturn, which is faster than that in the planet pair case. This indicates that the inward migration of Saturn is suppressed by the existence of Jupiter. Furthermore, the inward migration may be halted (panel (b)) or even reversed (panels (c) and (d)) as the surface density becomes steeper.

(A color version of this figure is available in the online journal.)

Figure 2. Density map of the gas disk at different evolution stages, where the initial density slope $\alpha = 0$ and the initial separation $d = 1$. Note that after about 500 $P_{S0}$ evolution from release, the inner disk is compressed and shepherded by Jupiter while Saturn is digging a gap in it. This ensures that Saturn moves with the inner disk.

(A color version of this figure is available in the online journal.)
If the disk was nearly flat ($\alpha \lesssim 1/2$), Jupiter migrates inward gently and pushes the gas of the inner disk toward the central star. Thus, the local surface density distribution of the inner disk is changed. As shown in Figure 4, the surface density profile of the inner disk is changed from flat ($\alpha \lesssim 1/2$) to relatively steep ($\alpha \gtrsim 1$). When the local disk mass exceeds the planet mass, $\pi a_p \sigma \gtrsim M_p$, the migration of the planet is then disk dominated. To exceed the mass of Saturn, the disk requires a minimum local density $\sigma_{\text{min}} \gtrsim 1.5 \times 10^{-4}$ inside the orbit of Saturn when Saturn is located at $a_S = 0.8$, which is much smaller than the present density of the inner disk ($\sigma > 6 \times 10^{-4}$). This ensures that the type II migration of Saturn is dominated by the disk evolution, which means Saturn follows the movement of the inner disk. On one hand, the gas which flows across the orbit of Saturn from outside to inside exerts an additional positive corotation torque on Saturn; see panels (b) and (d) in Figure 4. On the other hand, as the inner part becomes denser, the inner disk tends to spread outward. However, the outward diffusion of the inner disk is suppressed by Jupiter (as well as the disk outside Jupiter’s orbit); thus, this local density profile is maintained (panels (a) and (c) of Figure 4). As a result, the inward migration of Saturn is slowed down or even halted.

If the disk is relatively steep ($\alpha > 1/2$), Jupiter tends to migrate outward. The inner disk could now expand outward and Saturn follows the movement of the gas. However, the Hill radius of Jupiter increases as it moves outward and the expansion of the inner disk is limited by the increasing width of the gap. Thus, the outward migration of Saturn is also slower than that of Jupiter; see panels (c) and (d) in Figure 1.

The above results indicate that Saturn could migrate slower than Jupiter under some conditions when they are both undergoing type II migration. If the surface density of disk is steep, both Saturn and Jupiter will migrate outward and thus result in divergent migration, while in a nearly flat disk, the inward migration of Saturn is suppressed. Due to the large initial separation, the two planets would achieve an equilibrium state (panels (a) and (b) in Figure 1). How could they then approach each other further and get into MMRs? One possible way is reducing their initial separation to enlarge the interactions between them at the very beginning of the evolution.

### 3.2. Convergent Migration and MMR Captures

Next, we try a moderate separation $d = 0.5$ by setting Jupiter at $a_{J0} = 1.5$ and Saturn at $a_{S0} = 1$. At this distance, the initial mutual interaction due to the gravity of the two planets is still not important ($d > 4r_{\text{H}}$). However, the indirect interaction becomes significant since Saturn is initially located near the position of the 2:1 MMR with Jupiter, and furthermore, it is also close to the location of the farthest inner Lindblad resonance ($m = -2$) of Jupiter. Our results show that the surface density profile also plays a great role in the final results.

1. When $\alpha > 1/2$, we get divergent migrations which are similar to the cases with $d = 1$. The divergent rate is higher when a larger $\alpha$ is adopted. An important difference is that Jupiter migrates inward for a while right after the moment of release and turns back outward while Saturn moves far away inward; see panel (c) in Figure 5. This phenomenon
indicates that, at small separation, the total inner Lindblad torque exerted on Jupiter is weakened by Saturn (at the beginning of evolution).

2. When $\alpha \leq 1/2$, we find that Jupiter migrates inward much faster than it does in the $d = 1$ cases and the inward migration of Saturn is substantially slowed down. At this condition, we observe a gently convergent migration between Saturn and Jupiter. This slow convergent migration then results in the 2:1 MMR of the two planets and Saturn is forced to migrate further inward by Jupiter. Their eccentricities are greatly excited and maintained by the resonance, especially for that of Saturn ($e_S \sim 0.4–0.5$); see panels (a) and (b) in Figure 5.

These results can be understood as follows. At the beginning of the evolution, Saturn sweeps out the gas at the positions of the inner Lindblad resonances of Jupiter. Thus, the total inner Lindblad torque exerted on Jupiter is weakened. The outer Lindblad torque then dominates the migration of Jupiter for a while. When $\alpha > 1/2$, their orbits become divergent because Saturn migrates inward faster than Jupiter at the first stage of the evolution. When their separation becomes large enough, Saturn’s effect vanishes. Thus, the inner Lindblad torque overwhelms the outer one again and Jupiter starts to migrate outward. Panel (b) in Figure 6 shows the torques exerted on Jupiter versus time when $\alpha = 3/2$. It is clear that the inner torque decreases for a while around the moment of release. Then the inner torque recovers and overwhelms the outer one, while Saturn is quickly dragged inward by the corotation torque at the beginning (panel (c) of Figure 6). Then, as its co-orbital region is cleared, Saturn follows the expansion of the inner disk and moves outward. Figure 7 shows the semi-analysis of each $m$th Lindblad torque (inner and outer) exerted on Saturn (obtained by the same method in Paper I). One may find that the total outer Lindblad torque decreases a lot after the release while the inner one increases a little bit, and thus the net torque becomes positive.

The situation is different when $\alpha \leq 1/2$. Although the decrease rate of the inner Lindblad torque is slowed down when the separation between Saturn and Jupiter is increasing, the outer Lindblad torque still dominates the migration of Jupiter and pushes it inward slowly. In the meantime, the inward migration rate of Saturn decreases even more. This is mainly due to three reasons. First, as we have mentioned before, the inwardly migrating Jupiter compresses the inner disk and the steepened local surface density profile makes the net torque exerted on Saturn vanish there. Figure 8 shows each $m$th ($|m| = 2–80$) Lindblad torque exerted on Saturn at different time points. It is clear that during the first $300P_{\text{Saturn}}$ evolution, the total outer Lindblad torque is greatly reduced to the same value as the
Figure 5. Orbital evolutions of Saturn and Jupiter embedded in a gas disk with a surface density slope $\alpha$ equal to 0 (panel a), $1/2$ (panel b), and $3/2$ (panel c). The initial separation between the two planets is reduced to $d = 0.5$. Convergent migration happens when the disk is nearly flat, $\alpha \lesssim 1/2$.

(A color version of this figure is available in the online journal.)

Figure 6. Panel (a): the surface density evolution when $\alpha = 3/2$. Panel (b): the evolution of torques exerted on Jupiter. Note that the jump around $T = 250P_{S0}$ indicates the recovery of the inner Lindblad torque as Saturn migrates farther away. Although the outer Lindblad torque is initially larger, the inner Lindblad torque decreases slower than the outer one and makes the net torque positive soon after the release. Panel (c): the evolution of torques exerted on Saturn. Note that the corotation torque decreases to zero as Saturn digs a gap in the inner disk and the outer Lindblad torque decreases much faster than the inner one.

(A color version of this figure is available in the online journal.)
inner one. Second, the gas outside Saturn is pushed inward by the tidal torques of Jupiter. Parts of the gas flow across Saturn’s orbit and generate an additional positive corotation torque on Saturn, which drives Saturn to move outward (see Figure 9). Third, the disk between Saturn and Jupiter is heavily weakened by the gap created by Jupiter. As Jupiter approaches Saturn, more higher order \( |m| > 2 \) inner Lindblad torques of Jupiter and outer Lindblad torques of Saturn are cleared. Thus, both parts of the disk, outside Jupiter and inside Saturn, push the two planets toward each other further. Finally, when the disk between them is swept out gently, a common gap forms (see Figure 10).

Since this convergent migration is slow, the 2:1 MMR is then a robust outcome. When Jupiter captures Saturn into MMR, Saturn is forced to migrate inward to the vicinity of the central star, preserving a high eccentricity \( e_S \sim 0.4–0.5 \) (Figure 5). This result is consistent with observations, e.g., Gliese 876 and HD 128311. Faster convergent migration and higher MMR may be achieved by reducing the initial separation between the two planets further.

At last, we try a small separation \( d = 0.25 \) by setting Jupiter initially at \( a_J = 1.25 \) and Saturn initially at \( a_S = 1 \). Since \( d < 3r_{\text{inH}} \), the mutual interaction due to gravity between the giant planets becomes important at the beginning of the evolution. Our results show that, although the steep surface density prevents fast inward migration, it does not determine the direction of the migration as in the previous cases. Now, the two planets migrate inward even when \( \alpha = 3/2 \). The results are similar to the cases that have \( d = 0.5 \) and \( \alpha < 1/2 \), but the inward migration of the planet pair becomes much faster and more unstable. As a result of this fast convergent migration, Saturn is trapped into a 3:2 MMR with Jupiter when the disk is flat at \( \alpha = 0 \). For the other surface density slopes \( \alpha \geq 1/2 \), the 2:1 MMR is still the preferred outcome. The eccentricity of Saturn never gets higher than 0.3 in the 3:2 MMR; see Figure 11.

3.3. Comparison between \( q > 1 \) and \( q < 1 \)

As we have shown in the previous section and in Paper I, both \( q > 1 \) and \( q < 1 \) configurations could result in convergent migration of the two planets and the planets being trapped in MMRs. The observations show that more than half of the exoplanet pairs trapped in MMRs are of the \( q < 1 \) configuration. It should be interesting to compare the results of these two configurations.

First, the major difference between the cases of \( q > 1 \) and \( q < 1 \) is the direction of the common migration of the planet pair after they are trapped into MMRs. When \( q > 1 \), the common inward migration is halted and the planet pair starts to migrate outward. When \( q < 1 \), common inward migration is preferred; see Figures 5 and 11.

Since Jupiter is much more massive, Saturn in fact follows the migration of Jupiter when they are locked in MMRs. The migration of Jupiter is dominated by the torque balance between the gas disk inside and outside its orbit. Thus, the surface density slope of the gas disk is an important issue (presented in Paper I).
When the difference: when Saturn lies outside Jupiter ($q > 1$), the total outer Lindblad torque exerted on Jupiter is weakened and the inner Lindblad torque becomes relatively stronger. Thus, the inward migration of Jupiter will be slowed down, halted, or even reversed. When Saturn lies inside Jupiter ($q < 1$), the situation is also similar: the total inner Lindblad torque exerted on Jupiter is reduced when Saturn sweeps out the gas inside Jupiter and is also similar: the total inner Lindblad torque exerted on Jupiter is reduced when Saturn sweeps out the gas inside Jupiter and the net inner Lindblad torque exerted on Jupiter decreases. So, Jupiter will migrate inward and push Saturn inward as well.

Second, the eccentricity of Saturn could be excited to a higher value in the $q < 1$ configuration. When Saturn lies outside Jupiter ($q > 1$), its maximum eccentricity could be excited to $e_S \sim 0.2$ by the 2:1 MMR and to $e_S \sim 0.15$ by the 3:2 MMR. When $e_S > 0.15$, the 3:2 MMR breaks, and the eccentricity is damped by the gas disk quickly. Due to the damping effect of the gas, the eccentricity of Saturn never increases to $e_S > 0.2$ even in the 2:1 MMR. However, when Saturn lies inside Jupiter, its maximum eccentricity could be excited to $e_S \sim 0.4$–0.5 by the 2:1 MMR, and the system is still stable (Figure 5).

This difference could be understood by considering the different structures of the heavily perturbed disk. When Saturn is located outside Jupiter ($q > 1$), the gas disk would be separated into two parts after the two gaps overlap. As the eccentricity keeps growing, Saturn gets closer to the outer edge of the common gap or even cuts into the disk. If the resonance is strong enough, e.g., the 2:1 MMR, the effect of damping and excitation will allow the system to achieve equilibrium. Otherwise the MMR would probably break at high eccentricity, e.g., in the 3:2 MMR (see Paper I). The situation seems to be the same in the $q < 1$ configuration since Saturn will meet the inner edge of the common gap. However, since the two planets tend to migrate inward together after the common gap is formed, Saturn usually forms an inner cavity at the very center (Figure 10). With both Saturn and Jupiter located within the cavity, the damping effect of the gas is negligible. Thus, the eccentricity of Saturn could be estimated by mutual dynamical analysis. Denoting the eccentricity of Saturn by $e_{S0}$ before the $p + 1:p$ resonance and by $a_{J0}$, $a_J$ the semimajor axes of Jupiter before and after resonance, respectively, we can obtain the eccentricity of Saturn $e_S$ which is excited by the resonance through the following equation (Malhotra 1995):

$$e_S^2 = e_{S0}^2 + \frac{1}{p + 1} \ln \left( \frac{a_{J0}}{a_J} \right).$$

For the 2:1 MMR, when $e_{S0} = 0.01$, $a_{J0} = 1.05$ at $T \approx 1000P_{S0}$, and $a_J = 0.58$ at $T \approx 5000P_{S0}$, both the above equation and our result gives $e_S = 0.5$, which are in good agreement with each other (see Figure 5).

The third difference is the frequency and stability of the planet pair in MMRs. As shown here and in Paper I, the two planets undergo convergent migration more easily in the $q > 1$ configuration. Convergent migration and MMRs are robust outcomes for all surface density slopes when $q > 1$. However, when $q < 1$, the two planets are under divergent migration in almost half of the simulations.

It seems that most of the exoplanet pairs trapped in the MMRs should be of the $q > 1$ configuration (where the inner planet is more massive than the outer one). However, by considering the above-mentioned differences and the migration process of the two planets, we find that the MMRs in $q < 1$ are more stable than those in $q > 1$. In the case of $q > 1$, Saturn and Jupiter usually reverse their migration outward when they are locked in MMRs. As Jupiter moves outward, Saturn is pushed outward further and their orbits are in fact divergent. As the separation between them increases, the mutual interaction due to gravity becomes weaker and Saturn is easily scattered away by the corotation torque, especially at a high eccentricity (unless the gas has already depleted). For $q < 1$, there is no such problem since the two planets migrate inward and form an inner cavity at the center. Our long-time evolution also proves this stability...
Figure 10. Density map at different stages of evolution. The right panels show a close-up of the inner disk in which Saturn is embedded. (A color version of this figure is available in the online journal.)

\( T \geq 8000 P_{50} \); see Figure 12). This explains why the relatively high frequency of exoplanet systems that have the configuration of \( q < 1 \) contain MMRs; see Table 1. This relatively high frequency may also be a result of observational bias where the massive outermost planet is easier to detect by the radial velocity method.

4. DISCUSSIONS

In our simulations, the onset of convergent migration is much earlier than the emergence of the inner cavity. Therefore, in our results, the cavity should not account for the suppression of the inward migration of Saturn. We would like to indicate that there are two essentials that make convergent migration happen: the steepened density slope on the inner disk (compressed by the pre-formed giant planet) and the proper separation between Saturn and Jupiter.

The disk discussed here should be gas-full, otherwise it could not support the long-range type II migration of massive planets. This usually happens at the early stage of the evolution of a protostellar disk. When a massive planet is forming, say Jupiter, it will substantially change the structure of the gas disk by digging a clear gap. The gas interior to its orbit will be pushed inward and accumulate on the inner disk. If the timescale of gas accretion onto the central star (and the other processes leading to the loss of gas at the center) is longer than the timescale of this compression, the local density slope on the inner disk will steepen. The typical gas accretion rate of a T Tauri star is around \( \sim 10^{-7} M_{\odot} \ yr^{-1} \), while the gap opening process is only \( \sim 100 \) orbits if we drop a mature Jupiter in an unperturbed disk (in fact, we have already employed a “quiet start” method in our simulations to simulate the growth of planets and avoid the unreal initial impact to the gas. The planets all start with a mass of 0.1 \( M_{\oplus} \) Earth masses; see Section 2.2 and Paper I).

If we take into account the growth of the planet (core and gas accretion), the gap opening process will be prolonged. However, according to the core accretion model of giant planet formation, the core mass stays below 15 \( M_{\oplus} \) for most of the time. Since the lowest mass required to open a gap in a typical protostellar disk (\( H/r = 0.04, v = 10^{-5} \)) is around 30–50 \( M_{\oplus} \), the planet would not change the disk significantly during this stage. As soon as its core mass reaches the critical mass \( M_{\text{core}} = 15 M_{\oplus} \) and equals...
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Figure 11. Orbital evolutions of Saturn and Jupiter embedded in a gas disk with a surface density slope \( \alpha \) equal to 0 (panel (a)), \( 1/2 \) (panel (b)), and \( 3/2 \) (panel (c)). The initial separation between the two planets is reduced to \( d = 0.25 \). The convergent migration becomes faster at small initial separation and the 3:2 MMR is reached when \( \alpha = 0 \) (panel (a)).

(A color version of this figure is available in the online journal.)

Besides the surface density factor, a proper separation between the two giant planets is also required. If the separation is small, the gas disk between Saturn and Jupiter would be weakened substantially when the two planets are digging gaps on the disk. For Jupiter (Saturn), the torques that come from the gas inside (outside) its orbit are reduced and it would be pushed inward (outward) by the disk outside (inside) its orbit. And, as the two planets get closer, the torque unbalance of each planet becomes more serious and the two planets would get closer. Thus, the total effect is that the gas disk tends to keep pushing the two giant planets toward each other when the gas between their orbits becomes more and more tenuous. When the two planets get close enough (the gaps usually had already overlapped), the mutual gravitational interaction between the two planets would prevent them from further approaching, e.g., MMR.

Our results show that the maximum separation leading to convergent migration is approximately \( d \lesssim 5 r_{\text{MH}} \) (when \( \alpha = 0 \)). According to the core accretion model, the embryos of planets will accrete all the solid mass within their vicinities and achieve their isolation masses. This isolation separation between embryos is also \( d_{\text{iso}} \sim 5 r_{\text{MH}} \). If we take into account the migration of light planets (embryos) in the gas disk, the planets may get closer before they become massive enough to open gaps on the disk. Furthermore, the massive gas giants usually form in an area a little further outside the snow line, where the surface density of the disk increases significantly due to the icing of water, and the collision timescale of solid grains is not long enough to prevent the effective growth of the planet core. So, it is reasonable to expect that some giant planet cores emerge with proper separations. This means that the initial conditions leading to convergent migration should not be rare, despite \( q > 1 \) or \( q < 1 \) (\( q \) is the mass ratio of the inner planet and the outer planet).

To focus on the effects of the surface density slope of the disk and the initial separation between the two planets, we assume the two planets form simultaneously and the gas accretions onto planets (and the growth of planet) are not taken into account in this series of works (in both Papers I and II). However, this does not mean that the accretion process is negligible. In fact, the gas accretion should become more vigorous on Saturn, whose gap formation process is prolonged by Jupiter. Thus, we add some additional discussions about the accretion processes of the planets here. Many factors need to be considered when the growth of the planet is included, e.g., the formation sequence of the two planets, the time required to build a critical mass core, and the proper descriptions of accretion rate and range. We find that it becomes too complicated to concentrate on the dynamic evolution of the planet pair if all the factors are taken into account. To avoid the complexity, one could adopt a reasonable assumption that Jupiter forms first while Saturn is still a planet core undergoing gas accretion. Although in a multiple planet system the accretion process of the planet makes the orbital evolution more complicated by leading to mass of the planets growing and various migration rates, we believe that it may not change our main result—the convergent migration—qualitatively.
Figure 12. Long-time evolution of the $a = 0$ and $d = 0.5$ case. This system is stable to at least $8000P_{50}$ at a high eccentricity of Saturn ($e_s \lesssim 0.6$) when the 2:1 MMR is preserved. Saturn gets close to the central star at $a_s < 1\text{AU}$.

(A color version of this figure is available in the online journal.)

First of all, the accretion onto Jupiter could be neglected since the clear gap prevents the effective gas accretion process. Then we only need to consider the accretion onto Saturn. One could further assume the core of Saturn is already around $15\,M_\oplus$ and it undergoes fast type I migration initially. Since the direction of type I migration is always (or in most cases) inward, the convergent migration should still be a robust outcome when the planet core (Saturn) lies outside Jupiter ($q > 1$). Comparing to the cases we studied in this paper, the growing Saturn may get closer to Jupiter and be trapped in MMRs with higher $p$ ($p + 1 : p$ is the orbit ratio of the two planets). As the mass of the Saturn core keeps increasing, the high-$p$ MMRs would become unstable and lead to breaks or re-captures of MMRs (the resonances may overlap easily for massive planets at high $p$ and lead to instability; see Paper I).

When the planet core lies inside Jupiter ($q < 1$), the two planets would undergo divergent migration at the beginning. However, the accretion process is fully runaway when the core mass is above $15\,M_\oplus$. It will take only 200–400 orbits to achieve $1\,M_J$ for an accreting planet embedded in a disk as dense as the one we adopt here and its orbit decay is less than 20% during this process (d’Angelo & Lubow 2008). When the mass of the inner planet becomes massive enough to open a clear gap, its migration rate will decrease significantly. Furthermore, considering the accreting gas, the inner planet should become more massive (than Saturn) and dig a wider gap, which weakens the disk between the two planets further. Thus, the gas outside the planet pair (inside the inner planet and outside the outer planet) tends to push the two planets toward each other (the inner planet only gets angular momentum from the inner disk and moves outward; the outer planet only loses angular momentum to the outer disk and moves inward) and results in a more compact orbital configuration. So, it may also lead to convergent migration in the $q < 1$ configuration, when we consider the gas accretion onto the Saturn core.

A more self-consistent simulation should include two accreting and interacting planet cores as well as the thermal evolution of gas around planets, which would lead to much more complicated orbital evolution, and thus stepping into a new aspect of orbital evolution investigation. The relational results are under preparation now.

We also note that the characteristics of resonant systems with $q < 1$ are the relatively large eccentricity and the short orbit period of the inner giant planet, e.g., GJ 876, HD 128311, HD 45364, and HD 60532. These characteristic orbits are easier to detect by radial velocity methods. Thus, this observational bias also explains the relatively high frequency of the resonant system in the $q < 1$ configuration. The orbital eccentricities of planets in these real systems are all less than 0.3, which indicates that there may exist other mechanisms that restrain...
the planets’ eccentricities when gas damping is absent. One possible mechanism which needs to be addressed further is the interaction with the planetesimal disk after the depletion of gas.

The tidal dissipation that arises from the star will drive the resonant planet pair out of resonance when they are close to the center (D. N. C. Lin et al. 2010, in preparation). The eccentricity of the inner planet could be damped by this tidal dissipation as well. So, the “hot Jupiter/Saturn” would probably form in the $q < 1$ system as follows: at first, the fast inward migration of the inner planet is suppressed by the outer giant planet. Then the two planets are locked into MMRs and migrate inward together. They dig an inner cavity at the center of the gas disk and the eccentricity of the inner planet is excited to a high value by the resonance. As the inner planet gets closer to the star, the tidal dissipation gently drives it out of MMR with the outer one. After the depletion of gas, the inward migration of the outer planet stops, while the inner one falls toward the center due to tidal interaction with the star. Finally, the two planets undergo divergent migration again and the inner one approaches the central star more closely with a moderate eccentricity which had been damped by the tidal dissipation of the star. To ensure the validity of this process, we need to carry out further $N$-body integration associated with tidal damping by adopting the results of this work as the initial conditions.

5. CONCLUSIONS

Following Paper I, we continue the investigation of the orbital evolution of Saturn and Jupiter embedded in a protostellar disk by running a series of 2D high-resolution hydrodynamic simulations. The main aim of this work is to find out whether convergent migration also happens in a system with $q < 1$ (where the more massive giant planet is initially located outside the lighter one). To do so, we switch the initial positions of Saturn and Jupiter to achieve $q < 1$ and focus on the effects of various surface density profiles of the gas disk and different initial separations between the two planets. From our results and analysis, we summarize our conclusions as follows.

1. The type II migration of Saturn could be suppressed by Jupiter when $q < 1$. As Jupiter digs a deep gap, the gas disk is cut into two parts—an inner disk and an outer disk. Saturn also digs a gap in the inner disk and follows the viscous movement of gas. Being shepherded by the tidal torque of Jupiter, the expansion of the inner disk is suppressed. When the surface density slope is steep ($\alpha > 1/2$), Jupiter migrates outward and the inner disk expands. The width of the gap increases as Jupiter moves outward ($a_J$ increases) and the expansion of the inner disk is limited, thus the outward migration of Saturn is limited as well. When the surface density is nearly flat ($\alpha \leq 1/2$), Jupiter tends to migrate inward. Being compressed by Jupiter, the inner disk becomes denser and the local surface density slope $\alpha$ increases. Thus, the inner disk tends to spread outward and fight against the tidal compression of Jupiter. When the expansion and compression effects achieve equilibrium, the inward migration of Saturn is slowed down or even halted.

In a system of $q < 1$, this mechanism provides a way to halt the inward migration of the inner giant planet and does not require the overlapping of gaps. In fact, this suppression happens as early as the outer massive planet starts to compress the inner disk and makes the convergent migration possible. We also note that the inner planet should also be massive enough to open a gap in the disk, otherwise it will not follow the viscous evolution of the inner disk and this mechanism will not be valid. The lowest mass required should be $30–50 M_\oplus$, depending on the scale height and viscosity of the disk.

2. Convergent migration could also happen in the $q < 1$ configuration under some circumstances. The two main factors that account for the occurrence of the convergent migration are the nearly flat surface density profile and the relatively small separation between the two planets. On the one hand, as we have concluded above, when $\alpha < 1/2$, the inward migration of Saturn would be suppressed by the steepened local surface density slope on the inner disk. On the other hand, as the initial separation between the two planets is reduced, the total inner (outer) Lindblad torque exerted on Jupiter (Saturn) is weakened by the gap created by Saturn (Jupiter). Thus, the outer torque overwhelms the inner one and pushes Jupiter inward faster than the regular type II migration. In the meantime, the inward migration of Saturn is suppressed and it migrates more slowly than Jupiter. As a result, the net orbital movement of the two planets is convergent.

The 2:1 MMR is a usual outcome when Jupiter and Saturn approach each other adiabatically. If the initial separation becomes smaller, e.g., $d \leq 0.25 \sim 3r_{\text{mt}}$, the inward migration of Jupiter is much faster and the two planets may achieve the 3:2 MMR. However, because of the mutual scattering due to gravity, the migrations of both planets are unstable when the initial separation is too small.

3. In the case of $q < 1$, after Saturn and Jupiter have been locked into MMRs, they will migrate inward together instead of migrating outward, and the eccentricity of Saturn could be excited much higher than that of the $q > 1$ configuration. After the stage of convergent migration, the gaps of the two planets overlap. As the disk between the two planets is cleared, Saturn is pushed outward by the inner disk and Jupiter is pushed inward by the outer disk. When the two approaching planets are locked into MMR, the mutual gravity interaction prevents them from getting close too quickly, and the separation between them becomes steady. Since Jupiter is much more massive, Saturn is then pushed inward by Jupiter. As they move toward the center, the inner disk is swept out and forms an inner cavity.

This cavity plays a great role. First, without the damping effect of gas, the eccentricity of Saturn could be excited to $e_S \sim 0.5$ (in the 2:1 MMR), which is much higher than that of the $q > 1$ configuration and is consistent with the result of the analysis of two resonant planets. Second, without the scattering triggered by the massive gas within the co-orbital zone of the planet, Saturn is able to maintain high eccentricity and be pushed to the vicinity of the central star. Our long-time evolution shows that the inward migration of Saturn and Jupiter is stable at high eccentricity for at least $T \geq 8000 P_{50}$. Thus, this cavity in fact ensures the stability of a highly eccentric system, e.g., “hot Jupiter/Saturn.”

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