CD and PMD Effect on Cyclostationarity-Based Timing Recovery for Optical Coherent Receivers

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Abstract—Timing recovery is critical for synchronizing the clocks at the transmitting and receiving ends of a digital coherent communication system. The core of timing recovery is to determine reliably the current sampling error of the local digitizer so that the timing circuit may lock to a stable operation point. Conventional timing phase detectors need to adapt to the optical fiber channel so that the common effects of this channel, such as chromatic dispersion (CD) and polarization mode dispersion (PMD), on the timing phase extraction must be understood. Here we exploit the cyclostationarity of the optical signal and derive a model for studying the CD and PMD effect. We prove that the CD-adjusted cyclic correlation matrix contains full information about timing and PMD, and the determinant of the matrix is a timing phase detector immune to both CD and PMD. We also obtain other results such as a completely PMD-independent CD estimator, etc. Our analysis is supported by both simulations and experiments over a field implemented optical cable.

Index Terms—Timing recovery, polarization mode dispersion, signal processing.

I. INTRODUCTION

 Timing recovery plays a critical role in the digital signal processing (DSP) chain of optical coherent receivers for synchronizing the local clock to the incoming data stream [1], [2], [3]. The core of timing recovery is a timing phase detector (TPD), which identifies the current sampling phase of the receiver analog-to-digital converter (ADC) [4], often equivalent to detecting the phase of a generated clock tone at the frequency equal to the signal baudrate. The core can also be a timing error detector (TED), which produces error signals indicating the deviation of current sampling instance from the optimal sampling point. While finding a proper TPD or TED is a classic problem in digital communication systems, and there are numerous methods for solving it [5], [6], [7], one should pay attention to various fiber channel effects on the existing timing recovery solutions in optical communications [8]. Such effects include fiber chromatic dispersion (CD), state of polarization (SOP) rotation, polarization mode dispersion (PMD), and fiber nonlinearity, etc. Considerable efforts have been made to understand the CD and PMD effect on the timing recovery [9], [10]. It appears however that there are still more to discover especially concerning the spectral correlation involving the two data-carrying polarizations. Both CD and PMD effects can be accounted for and simple TED algorithms can be derived from the so-called cyclic correlation matrix. In particular, we prove that the determinant of the proper matrix is a valid TPD regardless of the CD and PMD conditions of optical fiber and hence can be performed before CD and PMD equalization in the DSP chain. More analysis on the spectral correlation properties of the common timing recovery methods can also found in [11], [12].

The effect of CD on timing recovery is static and hence it is a common practice to extract timing phase after CD equalization. Because the clock tone is sensitive to CD, it also becomes a popular way to estimate the CD based on the magnitude of clock tone [13]. In contrast, the PMD effect is dynamic and the half symbol first-order differential group delay (DGD) rotated by 45-degree is well identified as the worst case for timing recovery. In fact, based on our analysis in this paper, the worst cases correspond to the slow principal state of polarization (PSP) of optical fiber lying in the $s_2$-$s_3$ plane of Stokes space (Section III-A). It then follows that one could rotate the signal polarization away from the worst case scenario prior to the timing recovery. Previous studies include adaptive polarization rotation to maximize the time-averaged error signals of Gardner TED [10], adaptive phase and/or polarization rotation to maximize cost functions similar as the clock-tone strength [14], [15], SOP rotation to minimize the correlation between two polarizations [16]. Other timing phase extraction points such as the output of channel equalizer and carrier recovery are discussed in [17].

It has been understood that the PMD effect is related to the polarization correlation. However, we haven’t seen a complete analysis on what exactly the relation is and its impact on the timing recovery. In this paper, we prove that the spectral correlation and the cyclic correlation functions are powerful tools for analyzing the channel effects on the timing recovery. We analyze in...
detail the CD and PMD effect on the class of timing error detectors based on the second-order cyclostationarity of data-carrying signals. We have proposed several algorithms in that class including the a completely PMD-insensitive CD estimator, a PMD matrix estimator, the DGD and PSP vector estimation, and the TED for negligible DGD, for DGD smaller than half unit interval, and for all channel conditions. We corroborate our findings with numerical simulations of an optical coherent communication system exploiting the polarization division multiplexed 16-ary quadrature amplitude modulation (PDM-16QAM) and field tests with a 153 km subterranean optical cable.

II. PRINCIPLE OF TIMING RECOVERY

Consider the general linear modulation carried by one of two orthogonal polarizations (x polarization) of light

\[ x(t) = \sum_{n=-\infty}^{\infty} a_n g(t - \tau_g - nT_0) \]  

where \( \{a_n\} \) are the complex-valued symbols randomly selected from a signal constellation, \( T_0 \) is one symbol duration, \( g(t) \) is the real-valued pulse shaping function, and \( \tau_g \) is the group delay causing the varying timing phase. The signal \( g(t) \) at the \( y \) polarization is defined similarly but with symbols \( \{b_n\} \) that are uncorrelated with \( \{a_n\} \). The cyclic autocorrelation function (CAF) of \( x(t) \) is given by

\[ R_x^\alpha(\tau) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} E \left\{ x(t + \frac{\tau}{2}) x^*(t - \frac{\tau}{2}) e^{-j2\pi\alpha t} \right\} dt \]

\[ = \frac{1}{T_0^2} \int_{-\infty}^{\infty} g(t - \tau_g + \frac{\tau}{2}) g(t - \tau_g - \frac{\tau}{2}) e^{-j2\pi\alpha t} dt \]

where \( j = \sqrt{-1} \), \( E\{\cdot\} \) is the expectation operator and the cyclic frequency \( \alpha \) is an integer multiple of the signal baudrate. The spectral correlation function (SCF) of \( x(t) \) can be found as the Fourier transform of the CAF

\[ S_x^\alpha(f) = \int_{-\infty}^{\infty} R_x^\alpha e^{-j2\pi f \tau} d\tau \]

\[ = \frac{1}{T_0^2} G\left( f + \frac{\alpha}{2} \right) G^*\left( f - \frac{\alpha}{2} \right) e^{-j2\pi\alpha\tau_g} \],

which can be viewed as the generalization of ordinary power spectral density (a special case of SCF with \( \alpha = 0 \)). When \( G(f) \) is bandlimited with a nonzero excess bandwidth, the SCF is nonzero only for a few cyclic frequencies. Define the cyclic periodogram as

\[ P_x^\alpha(f) = \frac{1}{W} X\left( f + \frac{\alpha}{2} \right) X^*\left( f - \frac{\alpha}{2} \right) \]

where \( X(f) \) is the Fourier transform of a finite segment of \( x(t) \) that has time duration \( W \). Like the ordinary periodogram, the cyclic periodogram serves as a practical estimate of true SCF. It follows that the CAF can be calculated practically by taking the inverse Fourier transform of the cyclic periodogram. Fig. 1 shows simulated SCF and CAF curves for a common modulation format used in coherent optical systems. The fiber CD caused the shift of CAF peak, which is explained in more detail in Section IV. Moreover, it is easy to verify the spectral correlation is perfect for common digital modulations in the sense that the correlation coefficient \( S_x^\alpha(f) / \sqrt{S_x(f + \alpha/2)S_x(f - \alpha/2)} \) has modulus one for \( \alpha \neq 0 \). For \( x(t) \) at the transmitter, the spectral components separated by \( \alpha \) are effectively carrying the same information. However, channel impairments such as CD and PMD will reduce the level of spectral correlation, which is analyzed in Section III and IV.

In contrast to the real-valued power spectral density, the SCF is in general complex-valued due to the presence of phase term involving \( \tau_g \) for \( \alpha \neq 0 \) and also the complex-valued \( G(f) \). Note that (5) is a general expression of SCF for linear modulations with an arbitrary pulse shaping function \( g(t) \), real or complex. However, when the pulse shaping \( g(t) \) is an even and real function such as a raised cosine function, its Fourier transform \( G(f) \) is also even and real. This is the reason that enables us to retrieve the timing phase term by averaging the SCF with \( \alpha = 1/T_0 \), the baudrate, over a certain range of frequency. That is \( \int S_x^\alpha df = A \cdot e^{-j2\pi\alpha\tau_g} \) according to (5), where \( A \) is a real-valued coefficient when \( G(f) \) is real.\(^1\) The integration range is usually small around the zero frequency such that we compute effectively the correlation between upper and lower bands of the signal around \( f = \pm 1/(2T_0) \). It is obvious that the real and the imaginary part follows

\[ \text{Re} \int S_x^\alpha df = A \cdot \cos(2\pi\alpha\tau_g) \]

\[ \text{Im} \int S_x^\alpha df = -A \cdot \sin(2\pi\alpha\tau_g) \]

\(^1\)We restrict our discussion to real and even \( g(t) \) and ignore CD and other nonlinear phase variations across the channel frequency response. Or, one may consider those effects negligible in the integration range of SCF.
where the imaginary part forms the well-known timing error indicator (or detector) of Godard’s method [5]. By using (4), we obtain easily the timing error detector

\[ e_T = -\text{Im} \int S_x^u d\tau = -\text{Im} R_x^0(0) \]  
(9)

where we have replaced the expectation with time average over a large \( T \). The last line is recognized as the square timing error detector [7] which computes the complex coefficient of the spectral line of \( |x|^2 \) at frequency \( \alpha = 1/T_0 \). It can be shown [18] that in order to maximize the SNR of the spectral line, \( x(t) \) needs to be the matched filtered signal at the receiver. The TED is usually embedded in a feedback loop which adjusts the ADC sampling phase constantly to lock on the optimal timing instances.

### III. PMD EFFECT

The formulation (2−5) works equally well for the \( y \) polarization. Also, the cyclic cross-correlation function (CCF) and the SCF between the two orthogonal polarizations can be similarly developed. They can be written as matrices of form

\[ C(\tau) = \begin{bmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{bmatrix}, \quad S(f) = \begin{bmatrix} S_{xx} & S_{xy} \\ S_{yx} & S_{yy} \end{bmatrix} \]  
(11)

where \( R_{xx} \) and \( R_{yy} \) are given respectively by replacing in (2) the first and the second \( x(t) \) by \( y(t) \). In ideal cases (i.e., no CD, no PMD, and stationary noise), the \( S \) matrix is (up to a common phase term) an identity matrix, \( S = e^{-j2\pi\alpha\tau_s}I \), at a given frequency because the data carried by \( x \) and \( y \) polarization are uncorrelated. Hence, according to (9), the CAF matrix

\[ C(0) = \int S(f) d\tau = A e^{-j2\pi\alpha\tau_s}I, \]  
(12)

where \( A \) is a real-valued coefficient.

When considering only the first-order PMD of optical fiber without CD and other nonlinear phase variations across the channel bandwidth, we describe the change of output polarization state \( E \) given the same input state over angular frequency \( \omega \) by the first-order differential equation [19 Eq. (5.2)]

\[ \frac{dE}{d\omega} = -j(\tau_g + H)E, \]  
(13)

where \( \tau_g \) is a constant, and the matrix \( H \) is Hermitian when the polarization dependent loss (PDL) is neglected. It represents the rotation of output Stokes vector about the principal state of polarization (PSP) when frequency changes. The rotation rate is precisely the differential group delay (DGD). The general solution of (13) is expressed as

\[ E(\omega) = e^{-j(\tau_g + H)\Delta\omega} E(\omega_0), \]  
(14)

where \( \Delta\omega = \omega - \omega_0 \) and \( \omega_0 \) is an initial frequency. Since the input polarization states are the same at the two frequencies \( f \pm \alpha/2 \) due to the complete spectral correlation, we chose \( \omega_0 = f - \alpha/2 \) and hence \( \Delta\omega = 2\pi\alpha \). It follows \( U = e^{-j2\pi\alpha H} \) is a constant unitary matrix. Therefore, when neglecting PDL, the spectral components are related by the PMD matrix \( U \)

\[ \begin{bmatrix} X(f + \alpha/2) \\ Y(f + \alpha/2) \end{bmatrix} = e^{-j\phi_0}U \begin{bmatrix} X(f - \alpha/2) \\ Y(f - \alpha/2) \end{bmatrix}, \]  
(15)

where \( \phi_0 = 2\pi\alpha\tau_g \) is the common phase induced by the group delay, \( E_{1,2}' \) are Jones vectors, \( X, Y \) are respectively the Fourier transform of time truncated \( x(t) \) and \( y(t) \) at the receiver. The PMD matrix can be written in the form

\[ U = \begin{bmatrix} a & -b^* \\ b & a^* \end{bmatrix} \text{ with } aa^* + bb^* = 1, \]  
(16)

and admits the expansion [19]

\[ U = \cos(\pi\alpha\tau_{\text{DGD}})I - j\sin(\pi\alpha\tau_{\text{DGD}})[\vec{\rho} \cdot \vec{\sigma}], \]  
(17)

where \( \vec{\rho} = [p_1, p_2, p_3] \) is the slow PSP Stokes vector associated with the eigenvalue \( \rho = e^{-j\pi\alpha\tau_{\text{DGD}}} \) of \( U \), \( \vec{\sigma} = [\sigma_1, \sigma_2, \sigma_3] \) are the three Pauli matrices

\[ \sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix}, \]  
(18)

and \( \vec{\rho} \cdot \vec{\sigma} = p_1\sigma_1 + p_2\sigma_2 + p_3\sigma_3 \).

#### A. Cyclic Periodogram

Let \( E_{1,2}' \) be the two output Jones vectors at frequencies \( f \pm \alpha/2 \) when this is no PMD and polarization rotation. Namely, they should coincide with their input states except for a common phase change. Since the PMD is absent, we set \( U \) to be the identity matrix in (15) and hence \( E_1 = e^{-j\phi_0}E_2 \). When considering in addition a global polarization rotation denoted by a unitary matrix \( V \), we simply multiply both vectors by \( V \), i.e., \( E_{1,2}' \rightarrow VE_{1,2}' \). Furthermore, when considering the PMD effect, we chose \( E_{2}' \rightarrow V E_2 \) as the referenced polarization and \( E_{1}' \) follows (15). The relation described above is summarized in Fig. 2. Using these relations, we obtain the matrix of cyclic periodogram

\[ P' = E_1' (E_2')^H UV (E_1 (E_2)^H)V^H = UV PV^H, \]  
(19)

where \( P = E_1 (E_2)^H \) is the matrix of cyclic periodogram without polarization rotation and PMD. In analogy to the ordinary periodogram, the SCF matrix \( S \) can be estimated by taking the expectation of, or more practically by time averaging, both sides of (19).
We have seen in Section II that the frequency integrated SCF forms a TED. Hence, by integrating both sides of (19) over all frequencies and denoting \( \hat{P}' = \int P'df \) and \( \hat{P} = \int P'df \), we see that

\[
\hat{C}(0) = \hat{P}' = UV \hat{P}V^H \approx e^{-j\phi_0}U
\]  

(20)
because \( \hat{P} \approx e^{-j\phi_0}I \) and the first equality is due to the Fourier relation in (12). We have chosen \( A = 1 \) assuming proper normalization and \( \hat{C} \) is the estimate of \( C \) in (11). Namely, the frequency integrated cyclic periodogram matrix \( \hat{P}' \) or the estimated CAF matrix \( \hat{C} \) at \( \tau = 0 \) is in fact an estimate of the PMD matrix \( U \), with a common phase related to the group delay. Matrices \( \hat{P}' \) and \( \hat{P} \) also have four elements

\[
\hat{P}' = \begin{bmatrix}
    P_{xx}' & P_{xy}' \\
    P_{yx}' & P_{yy}'
\end{bmatrix}, \quad \hat{P} = \begin{bmatrix}
    P_{xx} & P_{xy} \\
    P_{yx} & P_{yy}
\end{bmatrix}.
\]  

(21)

For \( U \) given by (17), it is easily seen that \( \hat{P}' = \pm e^{-j\phi_0} [\vec{P} \cdot \vec{\sigma}] \) whenever \( \tau_{\text{DGD}} = 0.5\alpha^{-1} \mod(\alpha^{-1}) \), commonly known as the half symbol DGD, and furthermore \( P_{xx}' = P_{yy} = 0 \) whenever \( p_1 = 0 \), i.e., the Stokes vector of fiber slow PSP has zero \( \alpha_1 \) component. This defines the worst cases for exploiting any linear combination of \( P_{xx} \) and \( P_{yy} \) as TED.

### B. Case of Small DGD

When the DGD is negligible such as in short reach fiber communications, the PMD matrix \( U \) is approaching an identity matrix such that

\[
\text{trace}(\hat{P}') = \text{trace}(V \hat{P}V^H) \approx 2e^{-j\phi_0},
\]  

(22)

the imaginary part of which, \( -\text{Im}\{\text{trace}(\hat{P}')\} \), is a valid timing error detector completely independent of the polarization rotation due to the trace operation. Note that since \( \hat{P} \) is basically a noisy identity matrix, the first element of \( \hat{P}' \), the \( P_{xx}' \) in (21), should also be a timing detector. However, it can be shown that the SNR of \( P_{xx}' \) is lower than that of \( \text{trace}(\hat{P}') = P_{xx} + P_{yy} \), leading to a worse jitter performance. This is because \( P_{xx}' \) in fact involves all four elements of \( \hat{P} \) and does not cancel completely the effect of \( V \).

### C. Case of Large DGD

When the DGD is large, we first note that \( \text{trace}(\hat{P}') \) has a clear dependence on the PMD [cf. (20)]. But the quantity

\[
\text{trace}(\hat{P}'U^H) = \text{trace}(UV \hat{P}V^HU^H) \approx 2e^{-j\phi_0}
\]  

(23)
is an exact timing phase detector independent of PMD, provided we know \( U \). However, there is no obvious way to estimate the true \( U \) without being affected by the timing phase [cf. (20)]. On the other hand, it is remarkable that despite the potentially time-varying phase term \( e^{-j\phi_0} \), both DGD and PSP can be estimated from the matrix \( \hat{P}' \). Since \( U \) has two eigenvalues \( \rho = e^{\pm j\pi\alpha} \), the DGD can be estimated from the two eigenvalues of \( \hat{P}' \), the estimate of \( U \), via

\[
\hat{\tau}_{DGD} = \left| \arg\{\rho_1\rho_2^*\}/(2\pi\alpha) \right|
\]  

(24)

with a wrapped range from zero to half symbol duration due to the multi-valued argument function \( \arg(\cdot) \) and the PSP vector can be estimated via

\[
\hat{p}_i = k \cdot \text{Im} \left[ \text{trace}(\sigma_i \hat{P}') \cdot \text{trace}(\hat{P}'\hat{P}'^H) \right]
\]  

(25)

where \( i = 1, 2, 3 \) and \( k \) is a constant such that the estimated vector is normalized. It is easy to verify that \( \hat{p}_i \) has a dependence \( \sin(2\pi\alpha\tau_{DGD}) \) on the true DGD \( \tau_{DGD} \). We can combine (24) and (25) to form an estimate of \( U \) that is independent of timing phase, i.e.,

\[
\hat{U} = \cos(\pi\alpha\tau_{DGD})I - j\sin(\pi\alpha\tau_{DGD})[\vec{P} \cdot \vec{\sigma}].
\]  

(26)

Note that the sign of \( \hat{U} \) depends on both \( \tau_{DGD} \) and \( \tau_{DGD} \) and cannot be determined based on \( \tau_{DGD} \) alone. This sign ambiguity prohibits the use of \( \text{trace}(\hat{P}'U^H) \) as a timing phase detector for an arbitrary DGD. It does work when the true DGD is strictly less than half of the symbol duration. The comparison of typical TED curves of (22) and (23) are shown in Fig. 3. Clearly, \( \text{trace}(\hat{P}') \) is better than a simple \( P_{xx}' \) for a small DGD, whereas \( \text{trace}(\hat{P}'U^H) \) is better than \( \text{trace}(\hat{P}') \) for a large DGD.

### D. All DGD Cases

On the other hand, we see that the determinant of \( \hat{P}' \)

\[
\det(\hat{P}') = \det(UV \hat{P}V^H) = e^{-2j\phi_0}
\]  

(27)
is PMD independent [20]. Its imaginary part, \( -\text{Im}\{\det(\hat{P}')\} \), is a timing error detector with characteristic curves shown in Fig. 4(d). The TED curve is stable in the presence of random polarization rotation whereas all other compared TED algorithms become unstable. It has two positive zero-crossings and hence two locking points. Depending on the initial state, the timing recovery loop may lock to one of the two points. For instance, when the initial timing phase is \( >0.25 \) or \( \sim-0.25 \) unit interval (i.e., one symbol), it locks to the symbol edges. This property combined with the varying polarization mixing due to PMD would not be a prominent problem if fractional-spaced channel
equalizer is used after the timing unit. Otherwise, it might affect the DSP performance when the symbol-spaced equalizer is used afterwards. This is a common problem for all TEDs considering the fact that the concept of optimal sampling phase is complicated by the combined effect of locking point and PMD.

Note that the TED based on (27) is completely transparent to the first-order PMD and polarization rotation, and due to the relation of SCF with CAF [cf. (10) and (20)], it is also clear that all entries of matrix \(C\) (and hence its determinant) are insensitive to the carrier phase. In addition, TED formed based on (22) and (27) can be considered as generalized Godard algorithm, since they all use spectral correlation as underlying principle, and (27) is slightly more complex than the simple trace operation of (22). They rely on taking the fast Fourier transform (FFT) of signals and hence is more suitable to be implemented with shared FFT with other DSP modules for reducing hardware complexity. One may argue that in the presence of nonlinear frequency-domain phase variation other than CD, the TED \(\text{det}(P')\) has to be implemented after the channel equalizer, in which case the extra FFT may be a complexity concern compared to the time-domain algorithm such as Gardner’s TED.

In light of (10) and (27), we could also extract directly a clock tone, independent of PMD, at the baudrate from the spectrum

\[
F = F_{xx} F_{yy} - F_{xy} F_{yx},
\]

where \(F_{xx}\) is the fast Fourier transform (FFT) of \(xx^*\), \(F_{xy}\) is the FFT of \(xy^*\), etc., and \(x, y\) are signals from two polarizations after CD compensation.

### E. Adaptive Method

First introduced in [9] and furthered studied in [14], the linear combination of the elements of matrix \(\tilde{P}'\) seems a feasible way to deal with the PMD effect. Consider the quantity

\[
q = P'_{xx} + e^{j\phi_{xy}} P'_{xy} + e^{j\phi_{yx}} P'_{yx} + e^{j\phi_{yy}} P'_{yy},
\]

the imaginary part of which, \(-\text{Im}\{q\}\), can be used as the timing error detector if the phase vector \([\phi_1, \phi_2, \phi_3]\) is adapted jointly with the timing loop such that the above quantity converges to a purely real-valued number when the joint loop is locked. The update at time instance \(k\) is written as

\[
\phi^k = \phi^{k-1} - \mu \text{Im}(e^{j\phi_{xx} - 1} P'_{xx}), \quad l = xy, yx, yy
\]

It effectively treats the four entries of the SCF matrix (19) as independent timing phase detectors. Note that unlike previous timing error detectors, the timing loop does not lock to the true phase \(\phi_0\) via the adaptive method but to the phase of \(P'_{xx}\), which is \(\phi_0\) plus the phase of the first element of \(U\). This adds noise to the timing loop considering \(U\) is an erratic effect in practice.

### IV. CD Effect

It is well-known that the chromatic dispersion of optical fiber incurs a quadratic phase change over the entire signal spectrum. The frequency response of CD is given by

\[
H = \exp(jKf^2)
\]

with \(K = \pi\lambda^2 DL/c\), where \(\lambda\) is the wavelength, \(D\) dispersion coefficient, \(L\) fiber length, and \(c\) the speed of light. According to (5), the CD effect results in a linear phase change in the SCF with the change rate over frequency equal to \(2K\alpha\). The linear phase suppresses the timing error upon integration in (9). The solution to restore the timing sensitivity is to estimate the CD value from the slope of phase in (21) and then remove the linear phase from SCF during integration [14]. On the other hand, the linear phase in SCF is equivalent to a constant time shift in CAF due to their Fourier relation. The CAF is a pulse-like function and hence the time shift can be identified by peak searching of \(|R^n_\tau|\). Namely,

\[
\tau_{CD} = \arg \max |R^n_\tau(\tau)|^2.
\]

Note that the CAF can be computed efficiently by taking the inverse Fourier transform of SCF. The CAF peak location can also distinguish the sign of CD, provided the time shift due to CD is no larger than half of the CAF duration. Moreover, it is easy to verify that the length of CAF determines the maximum range of CD estimate not the resolution. A higher resolution of CD estimation can only be obtained by a higher sampling rate of the signal. After obtaining the delay of CAF peak \(\tau_{CD}\), the timing error can be detected simply by using \(\text{Im}R^n_\tau(\tau_{CD})\) [12]. Assuming little dependence on polarization, the CD effect is modeled as a common term in both CAF and SCF. In particular, it causes a time shift in the CAF matrix and a linear phase change in the SCF matrix in (11). The development in the last section is similar in the presence of CD, except that the matrix relation in (12) should be modified as

\[
C(\tau_{CD}) = \int S(f)e^{j2\pi f \tau_{CD}} df = Ae^{-j2\pi\tau_0}\tau_I
\]

and in the presence of first-order PMD, \(\tilde{C}(\tau_{CD}) \approx e^{-j\phi_0}U\) is an PMD matrix estimate insensitive to both CD and polarization rotation. Note that the form of \(U\) given in (16) clearly suggests that the search of CAF peak is affected by the PMD. Namely,
using $|R_{xx}^\alpha| |R_{yx}^\alpha|^2$ or any linear combination of them will loss sensitivity at $\tau_{CD}$ when $a$, the first element of $U$, is approaching zero. However, the PMD effect is irrelevant if we use $|R_{xx}^\alpha| + |R_{yx}^\alpha|$, because $|a|^2 + |b|^2 = 1$. Namely,

$$\tau_{CD} = \arg \max \{ |R_{xx}^\alpha(\tau)|^2 + |R_{yx}^\alpha(\tau)|^2 \}.$$  

(34)

Therefore, we find a way to estimate the CD value completely independent of polarization rotation, PMD and timing phase. After that, the matrix $\hat{C}(\tau_{CD})$ can be worked with to detect timing error according to the methods introduced in the last section.

Fig. 5 shows the CD estimation results when (32) and (34) are used respectively. A half-symbol DGD and all possible PSP states are considered in the simulation. The results confirm that (34) is insensitive to the PMD. The previous results suggest that the timing recovery could be performed without actual CD and/or PMD compensation, which could be useful in certain practical use cases. The results based on second-order cyclostationarity are summarized as follows. For $\alpha = 1/T_0$, the estimator of CD is given by

$$\widehat{DL} = \frac{cT_0}{\lambda^2} \cdot \tau_{CD}$$  

(35)

where $\tau_{CD} = \arg \max \{ |R_{xx}^\alpha(\tau)|^2 + |R_{yx}^\alpha(\tau)|^2 \}$.

The estimator of PMD matrix $U$ is given by

$$\widehat{U}_T = \hat{C}(\tau_{CD}) \approx e^{-j2\pi\alpha\tau_g}U,$$  

(36)

where $\tau_g$ is the group delay.

The DGD estimator is given by

$$\widehat{\tau}_{DGD} = \arg \{ \rho_1\rho_2^* \} / (2\pi\alpha),$$  

(37)

where $\rho_1$ and $\rho_2$ are the two eigenvalues of $\widehat{U}_T$.

The PSP estimator is given by $\hat{\beta} = [\hat{p}_1, \hat{p}_2, \hat{p}_3]$, with

$$\hat{p}_i = k \cdot \text{Im} \left[ \text{trace} \left( \sigma_i \widehat{U}_T \right) \cdot \text{trace}(\widehat{U}_T)^H \right],$$  

(38)

where $\sigma_i$ is one of the three Pauli matrices.
The CD estimator is insensitive to polarization rotation, PMD and timing error. The PMD matrix estimator is immune to polarization rotation but affected by timing error. The DGD and PSP estimators are not affected by polarization rotation and timing error, but are valid only when the true DGD is less than $T_0/2$.

The timing error detector for the case when the DGD is negligible is given by

$$e_T = \text{Im} \text{trace}(U_T^H),$$

and for the case when DGD is smaller than $T_0/2$, is

$$e_T = \text{Im} \text{trace}(U_T U_T^H),$$

where $U = \cos(\pi \alpha_{\text{DGD}})I - j \sin(\pi \alpha_{\text{DGD}})[\hat{p} \cdot \sigma]$, and for larger DGD to use the adaptive combination of $P_{xx}$, $P_{yy}$, and $P_{xy}$, and finally for all channel conditions

$$e_T = \det(U_T).$$

V. HIGHER-ORDER CYCLOSTATIONARITY

The studies presented in previous sections are based on second-order cyclostationarity of common signals, which is however not suitable for the analysis of severely filtered signal with no excess bandwidth. Fortunately, the statistics of higher-order cyclostationarity can be exploited for proper timing phase detection in such cases (to be published). Although not well recognized by the research community, most of the proposed timing phase detectors for such signals, such as those listed in Table I, are based on the fourth-order cyclostationarity. However, the CD and PMD effect on this class of timing phase detector remains unknown. It is not a straightforward task of extending the analysis in this paper to higher order statistics. It is however clear that efforts are required to study the spectral properties of higher-order correlation, which certainly is an interesting direction for future research.

VI. SIMULATION AND FIELD TEST

We perform numerical simulations of a 32 GBaud PDM-16QAM system with 100 km standard single mode fiber (SSMF) transmission. The laser linewidth is 100 kHz. The laser frequency offset is considered negligible or has been compensated prior to the timing recovery. The fiber SOP rotation speed is 50,000 rd/s. The jitter effect of ADC is simulated as a sinusoidal model with a 30 kHz frequency and $0.6T_0$ peak deviation of sampling instance. The fiber DGD also varies from 6 ps to 16 ps in a sinusoidal manner with a frequency of 260 kHz. It provides a critical situation for testing the performance of various TED algorithms. The DSP chain includes in sequence CD estimation, CD compensation, timing recovery, least mean squares (LMS) dual-polarization channel equalization with embedded phase lock loop (PLL) for carrier phase recovery. When the LMS equalizer works in the symbol spaced mode, the performance of timing recovery becomes prominent. The simulation results are shown in Fig. 6. Note that the LMS equalizer has 7 and 13 taps respectively for the symbol-spaced and fractional-spaced mode for fair comparison. The TED gives the best performance for all cases but the differences are less significant when fractional-spaced equalizer is utilized.

We exploit the field optical fiber cable illustrated in Fig. 8 with coherent detection and offline DSP for testing the TED algorithms in a realistic scenario. It has several SMFs which can be interconnected at the cable ends so that the signals circulate inside the fiber loop. The total length of the fiber loop is measured to be 153 km consisting 9 interconnected SMFs. An optical amplifier with 20 dB gain is used at the input to the 3rd, 6th, and 8th SMF, respectively. A 32 GBaud PDM-16QAM signal carried by a 1550 nm laser light is launched with 0 dBm power into the fiber loop and coherently detected at the receiver end. We use an 80 GSa/s oscilloscope to sample the detected signal and run signal processing offline. Fig. 9 shows the long term CD estimation results when (32) and (34) are used.
It confirms the stability of (34) by comparing Fig. 9(a) and (c) where polarization scrambling is enabled at the fiber input. The maximal SOP speed is limited to 2000 rd/s by the device in our lab. Note that without scrambling, the natural condition of the field cable is fairly stable as shown in Fig. 9(b). A 1-hour BER test is performed and the results are shown in Fig. 7. It is again confirmed that the TED $-\text{Im}\{\det(P')\}$ gives the best performance for all cases.

VII. CONCLUSION

We have studied in detail the CD and PMD effect on the second-order cyclostationarity-based timing recovery by modeling the channel effect based on the spectral correlation and cyclic autocorrelation matrices. Several timing error detector algorithms are derived from those matrices for various application scenarios. Their features and performances are analyzed and compared, and the one based on the determinant of spectral correlation matrix is found to be independent of PMD and polarization rotation. In addition, we have proposed robust CD and PMD estimators based on the same spectral correlation model. Both numerical simulations and field tests are conducted to verify the theory and proposed algorithms.

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