MULTI-OBJECTIVE OPTIMAL PLACEMENT OF PMUS CONSIDERING CHANNEL LIMITATIONS AND VARIABLE PMU COSTS USING NSGA-II

B. Vedik\textsuperscript{1}, Chandan Kumar Shiva\textsuperscript{2}

\textsuperscript{1,2}Department of Electrical and Electronics Engineering, S R Engineering College, Warangal, Telangana State, Pin: 506371

\textsuperscript{1}b.vedik@gmail.com, \textsuperscript{2}chandankumarshiva@gmail.com

Corresponding Author: Chandan Kumar Shiva

https://doi.org/10.26782/jmcms.2020.07.00012

Abstract

In wide area monitoring system, phasor measurement units (PMUs) plays a vital role in providing synchronized measurements with the help of Global Positioning System (GPS). In conventional optimal PMU placement methodology these PMUs are placed optimally across the power system network ensuring completely observable. It is found in literature, that most of them neglect the PMU channel limitations, variable PMU costs, and measurement redundancy improvement. To address this problem, in the present paper an optimal PMU problem is addressed by optimizing the two objective functions that are conflicting in nature, namely, minimization of PMU installation cost and maximization of measurement redundancy at the same time. In order to allocate PMUs, both channel limitation and variable cost of PMUs has been considered. A non-dominated sorting genetic algorithm-II (NSGA-II) based methodology is proposed to solve the combinatorial optimization problem. The Pareto optimal solution obtained using the concept of crowding distance and non-dominated sorting. A multi-criteria decision making technique based on VIKOR method is utilized for finding the best compromise solution from the set of Pareto-optimal solution obtained through NSGA-II. To verify the effectiveness and reliability, the proposed approach is tested on IEEE 14-bus, 30-bus, and 57-bus systems.

Keywords: PMU placement, Power System, NSGA-II, VIKOR method.

I. Introduction

With the rapid increase in power demands, utilizing the transmission lines to its limits to transfer power, restructuring of the power system, and other factors have made the power system to operate close to their stability margin. Therefore, to ensure the stable operation of the power system, the state of the power system need to be monitored. This is traditionally performed using supervisory control and data...
acquisition (SCADA) system with the help of conventional measurements. However, these measurements are not synchronized thus limiting the SCADA system to deliver real-time operating of the electric power system [I]. To obtain the real-time control and monitoring of the power system, the deployment of phasor measurement units (PMUs) in the power system network is becoming ineluctable [XVIII]. These PMUs provide synchronized measurements of voltage phasor of the installed bus and current phasors in the branches connected to this bus with the help of the time signal provided by the Global Position System (GPS). Further, a PMU installed at a bus can make neighbouring buses observable by utilizing Ohm’s law and Kirchhoff’s laws. Therefore, the power system network can be made completely observable with a reduced number of PMUs than the number of buses [XIV]. Thus, the main goal of PMU placement problem is to minimize the total number of PMUs expected to be deployed such that the system is completely observable. To tackle this problem, various techniques have been proposed in the literature. These techniques are broadly divided into numerical, topological, and evolutionary optimization algorithms. The numerical techniques are based on the information obtained from the rank of the Jacobian or gain matrix. The system is said to be observable if the Jacobian matrix is of full rank. Topological techniques are based on the concept of minimum spanning tree. Unlike numerical and topological methods, evolutionary techniques do not require to state a set of constraints to attain the required objective. Further, the special attribute of these techniques is that the parameters of the methods are tuned adaptably to attain an optimum solution. Furthermore, these techniques consider unobservable buses as part of the objective function. Therefore, the observability of the system is attained by minimizing this objective function [I].

In [III], a non-dominant sorting genetic algorithm (NSGA) has been proposed to solve two objective functions that are conflicting in nature, namely, minimization of PMU installation cost and maximization of measurement redundancy at the same time. It has been observed in [III] that rather than single optimum solution better tradeoff solutions known as Pareto optimal solutions are obtained between two conflicting objectives. However, this technique suffers from slow operational speed, premature convergence, and consists of many tuning of algorithm parameters, which limits its application [V]. In order to overcome these limitations, a non-dominating sorting differential evolution technique and multi-objective biogeography based optimization techniques are suggested in [V], [XIII] and fuzzy set theory concept is utilized to choose the best compromised solutions among the Pareto optimal solutions. In [I], integer programming method is proposed to solve PMU placement problem by converting two objectives, i.e. minimization of PMUs installation cost and measurement redundancy maximization into single objective function using weighting factor. Further, a realistic methodology in finding optimal solution is suggested by taking PMU channel limitations into account. A similar approach to solve multi-objective PMU placement problem is suggested in [XVI] using cellular learning automata evolutionary technique. The main contribution in [XVI] is that the existence of conventional measurements is considered in obtaining the optimal PMU number. A binary gravitational search algorithm is suggested in [XVIII] to solve multi-objective PMU placement problem by giving equal weightage to both the...
objectives. However, the main disadvantages of this weighting factor approach [I], [XVI] are that the optimum solution highly depends on weighted values and it is difficult to assign appropriate weight value. Consequently, the solution lacks the diversity and extends to its major limitations [XII]. Further, this approach leads to single optimal solution where there exist various compromised solutions. In [VI], a weighted goal programming framework is proposed to solve PMU placement problem by considering installation cost of PMUs, gross error detection, and observability of the system. Therefore, a new index known as vulnerability index is utilized to measure the vulnerability of each branch in the power system for gross error detection. A two-step optimal PMU placement problem is suggested in [II]. In the first step, minimal number of PMUs required to make the system observable is identified by proposing a new rule in considering zero injection buses. Then making use of PMU locations obtained in first step measurement redundancy is maximized in the next step. Further, various contingency conditions such as single line and single PMU failure are considered. In [XIV], sequential quadratic programming technique is utilized to find the optimal number of PMUs and their locations. The proposed approach is executed multiple number of times with dissimilar starting points to obtain different optimal PMU locations. After obtaining multiple solutions, PMU placement location with maximum redundancy is selected based on system observability redundancy index. Further, existing of traditional measurements, PMU loss, and limited channel capacity of PMU are integrated into the PMU placement formulation. In [XX] suggests non-dominated sorting genetic algorithm-II (NSGA-II) to solve two conflicting objectives simultaneously by considering existence of traditional measurements in the electric power system network. However, this approach does not consider the PMU channel limitations and corresponding variable cost. In [VIII], multi-objective PMU placement problem is solved using two approaches, namely, augmented epsilon-constraint and weighting approach. Consequently, to compare these two methods, two new indices are suggested. It is identified using these indices that augmented epsilon-constraint provides better competence. A new integer linear programming formulation is proposed in [IX] to optimize two conflicting objectives using multi-objective mathematical programming. Further, a new index known as minimum distance to utopia point, is employed to choice the best desired locations among the available Pareto optimal fronts. In [X], apart from minimization of number of PMUs, a new conflicting objective function known as maximization of voltage stability level has been considered to solve PMU placement problem. This multi-objective optimization problem is solved using fuzzified binary artificial bee colony technique. In [VII], the authors not only considered minimizing the number of PMUs to be deployed across the network and maximization of voltage stability levels of the system but also a new objective function minimization of cost of communication cost. This tri-objective PMU placement problem is solved using binary ant-lion optimization technique. Similarly, a tri-objective PMU placement problem is solved using hybrid particle swarm-krill herd method in [XIX]. The main contribution in [XIX] is by introducing two new fitness functions namely, minimization of cost of IEDs and minimization of state estimation error to solve optimal PMU placement problem. A mixed integer linear programming framework is considered to solve PMU placement problem using
two-phase branch-and-bound method in [XV]. In [XV], the authors neglected the radial buses by pre-assigning a PMU on a bus connected to the radial buses such that measurement redundancy is increased. It is observed from the above literature that the influence of channel limitation on PMU placement locations and corresponding installation cost of PMUs has been not considered in solving multi-objective PMU placement problem. Therefore, in the present work a non-dominated sorting genetic algorithm-II (NSGA-II) has been employed to solved both the objective functions, namely, minimization of PMU installation cost and measurement redundancy maximization at the same time by considering the effect of channel limitation and corresponding variable cost on number of PMUs to be deployed across the power system network.

The rest of the paper is organized in the following way. Section II introduces the problem formulation and multi-objective optimization model. Solution methodology is presented in Section III. Results and discussion for various case studies are presented in Section IV. Finally, Section V concludes the paper.

II. PMU Placement Problem Formulation

The main aim of multi-objective optimal PMU placement problem is to determine the PMU placement locations such that the installation cost of PMUs is minimized and measurement redundancy is maximized simultaneously such that electric power system is completely observability.

II.i. Minimization of Installation Cost of PMUS

To determine the placement locations of PMUs, system connectivity matrix (A) which shows the interconnection of transmission lines between the buses is required [I]. This connectivity matrix (A) for an N bus system is computed as follows.

$$A = \begin{cases} 1, & \text{if } i = j \text{ or if buses } i \text{ and } j \text{ are connected} \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

To minimize the installation cost of PMUs, the objective function is formulated as given in (2) without sacrificing the system observability of the system.

$$\min F_i = W'X$$

such that $$[A][X] \geq b$$

where, $$W_i$$ denotes the cost of PMU installed at bus $$i$$, $$b$$ is a unit vector, and $$X$$ is PMU placement vector.

$$X = \begin{cases} 1, & \text{if a PMU installed at bus } i \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

In (1), the cost of PMU installed at a bus depends on various factors such as CT and PT connections, number of channels, GPS receiver, power and ground connections. It can be observed that the installation cost of PMU at a bus is mainly influenced by number of channels of PMUs, i.e. PMU installation cost increases with...
increase in number of channels. In the present work, a PMU with one channel is conceived as 1 p.u. For each added channel, PMU cost is incremented by a factor of 0.1 p.u [I].

II.ii. Maximization of Redundancy (R)

Another main objective of the PMU placement problem is to enhance the reliability of the system by increasing the measurement redundancy. Measurement redundancy (r) of the system refers to the number of times a bus $i$ is said to be observable directly or indirectly by installing a PMU at a bus $i$ or neighbouring bus $j$. The primary advantage of enhancing the redundancy is that the system remains observable even in the case of a single line or PMU outage condition [IX].

In the present work, the measurement redundancy (r) is computed based on the number of buses that are observable even in case of line or PMU outage at the same instant of time. To calculate measurement redundancy $r$, the number of buses that are observable even by removing all the PMUs or all the lines one by one is computed [IX]. Therefore, the system operator should ensure maximum measurement redundancy (r) value to enhance reliability. In other words, maximization of redundancy (r) refers to minimization of $N - r$. Hence, maximization of redundancy is formulated as follows.

$$\min F_z = N - r$$

such that $\text{rank}(H) = N$  

(4)

where, $H$ denotes the Jacobian matrix.

Therefore, in the present work, multi-objective PMU problem is formulated as

$$\min W'X, N - r$$

such that $[A][X] \geq b, \text{rank}(H) = N$  

(5)

III. Solution Methodology

III.i. Non-Dominated Sorting Genetic Algorithm-II

NSGA-II is proposed by Deb et al. [XI] in the year 2002 to solve multiple-objectives at the same time. This algorithm is based upon three main concepts, namely, non-dominate sorting, fast crowded distance calculation, and crowded-comparison procedure. These concepts are explained as follows:

a) Non-dominated sorting: Similar to the other algorithm, initially the population is initialized. The chromosomes in the population represent the set of solutions to the given problem. Now, each chromosome in the population is evaluated with respect to all the objective functions. Then, based upon the non-domination the chromosomes in the population are sorted and allocated to different fronts. A chromosome $i$ in the population is said to non-dominant if it dominates all the remaining chromosomes with respect to at least one objective function value. Initially, these non-dominated chromosomes are identified and allocated to the first non-dominated front. This first non-dominated front is assigned with the highest
weight. Similarly, second non-dominated front with remaining chromosomes (excluding the chromosomes assigned to first non-dominated front) is obtained. This second non-dominated front with high weight, however, less than the weight assigned to the first non-dominated front. This process is repeated until each chromosome in the population is included in one of the fronts [XI], [XIII], [XX].

b) Crowded distance computation: The main objective for performing crowded distance operation is to enhance the diversity of chromosomes in the population. This operator measures how close a chromosome to its fellow chromosomes. This operation is performed on each front. In order to assess the density of chromosomes in a front, the average distance on either side of this point for each objective function is computed. This value assists as an approximation of the boundary of the cuboid obtained using the nearest neighbours as the vertices. This is termed as crowding distance. Now, based upon the crowding distance the chromosomes are sorted in ascending order for every fitness function. Then, the chromosomes with the highest and lowest crowding distance value are assigned as infinite value. Further, for all the intermediate chromosomes the absolute normalized difference between two adjacent chromosomes is computed. This process is repeated for each objective function. Now, the complete crowding distance is computed by adding the individual distance values conforming to every objective function. Each objective function is normalized before calculating the crowding distance [XI], [XIII], [XX].

c) Crowded-comparison: This operator helps in selecting chromosomes at different stages of the method such that non-dominated solutions are uniformly distributed. So as to do that this operator utilizes two attributes of every chromosome, namely, the weight of the non-domination front and crowding distance. Therefore, when selecting between two chromosomes, a chromosome with the highest non-dominant front weight is selected. However, if both the chromosomes belong to same non-dominant front then chromosome with the highest crowding distance is selected [XI], [XIII], [XX].

The simulation procedure of NSGA-II is depicted in Figure 1 and comprehensive elucidation can be found in [XI], [XIII], [XX].

III.ii. VIKOR Method

After obtaining the results from NSGA-II, it is required to select a solution from Pareto-optimal set to apply for any specific application based on supplementary knowledge of the application. In the present work, VIKOR technique is utilized to select the pre-eminent compromised solution among different objective functions. To select the best solution VIKOR provides the compromised ranking to each solution in the Pareto-optimal set. The ranking is provided by VIKOR method is based on the measure of chumminess of the present solution with respect to the ideal solution of the corresponding objective function [IV], [XVII]. The compromise-ranking VIKOR algorithm has the following steps.
Fig. 1: Flow chart of NSGA-II [XX].

Step 1: In the first step, the solutions from Pareto-optimal set acquired using NSGA-II are represented as \(\{x_1, x_2, \ldots, x_m\}\). The merit of the \(i^{th}\) objective function for \(j^{th}\) Pareto-optimal solution is represented as \(f_{ij}\). Here, \(m\) represents the number of Pareto-optimal solutions and \(n\) denotes the number of objective functions [IV], [XVII].

Step 2: Identify the ideal \(f_{i}^{\ast}\) and poorest \(f_{i}^{\ast}\) values of each fitness functions.

\[
\begin{align*}
    f_{i}^{\ast} &= \max_{j} f_{ij}, f_{i}^{\ast} = \min_{j} f_{ij}, \text{ if the } i^{th} \text{ objective function represents a gain}; \\
    f_{i}^{\ast} &= \min_{j} f_{ij}, f_{i}^{\ast} = \max_{j} f_{ij}, \text{ if the } i^{th} \text{ objective function represents a cost}. 
\end{align*}
\] (6)

Step 3: Now calculate the \(S_i\) and \(R_i\) values using the following equations.
\[ S_j = \sum_{i=1}^{n} w_i \left( f_i^* - f_i^j \right) / \left( f_i^* - f_i^r \right), \]
\[ R_j = \max_{i} \left[ w_i \left( f_i^* - f_i^j \right) / \left( f_i^* - f_i^r \right) \right], \]

where, \( w_i \) denotes the objective function respective weights.

**Step 4:** Next, the rank \( Q_j \) of each Pareto-optimal solution is calculated using the following equations

\[ Q_j = v(S_j - S^*) / (S^* - S^r) + (1-v)(R_j - R^*) / (R^* - R^r) \]

Where, \( S^* = \min_{j} S_j, S^r = \max_{j} S_j, R^* = \min_{j} R_j, R^r = \max_{j} R_j \); and weight for the scheme of maximum group utility is represented as \( v \) and \( 1-v \) is the weight of the individual regret.

**Step 5:** Now, the value of \( Q_j \) with less value is selected as the best compromised alternative solution among various objective functions [IV], [XVII].

**IV. Results and Discussions**

In the present work, the multi-objective PMU placement problem has been tackled by considering three test systems, namely, IEEE 14-bus, IEEE 30-bus, and IEEE 57-bus systems without considering the presence of zero injection buses. To consider the influence of channel limitation on the installation cost of PMUs, optimal PMU placement problem is performed by considering three case studies, namely, PMUs available with only two-channel capacity, PMUs available with only three channel capacity, and PMUs with variable channel capacity with variable cost. Further, the proposed NSGA-II algorithm control parameters considered in the paper are population size of 100, the maximum number of iterations of 75, mutation operator of 0.1, and a crossover rate of 0.8.

**Case Study 1: PMUs Available With Only Two Channel Capacity**

In this case study, PMUs with two-channel capacity has been considered. Further, the installation cost of these PMUs is considered to be equal to 1.0 p.u. The results obtained using the proposed method is tabulated in Tables 1 to 3 for IEEE 14-bus, IEEE 30-bus, and IEEE 57-bus systems respectively. It consists of five columns. The first column denotes the number of PMUs installed in the system. The second column represents the number of buses to become unobservable for a given number of PMUs in the first column in the event of loss of single PMU or line outage. Measurement redundancy (\( r \)) for a given number of PMUs is given in column three. The compromised ranking computed using the VIKOR method for a given number of PMUs and measurement redundancy obtained using NSGA-II is given in column four. Optimal PMU placement obtained for conflicting objectives is given in column five. It can be observed from Tables 1 to 3 that for given channel capacity, the number of PMUs necessary to install across system increases with an increase in redundancy. It can be perceived that a minimum of five PMUs, 11 PMUs, and 19 PMUs were obtained.
PMUs are required to make system observable for IEEE 14-bus, IEEE 30-bus, and IEEE 57-bus systems respectively with minimum redundancy. Further, a maximum of 11 PMUs, 23 PMUs, and 52 PMUs are required to make system observable for IEEE 14-bus, IEEE 30-bus, and IEEE 57-bus systems respectively with maximum redundancy. The best-compromised solution between two conflicting objectives is selected based upon the compromised ranking factor Q computed using the VIKOR method. It can be observed that the value of Q decreases with rise in number of PMUs to a certain stage and then increases. The best-compromised solution is designated based upon the value of Q. Smaller the value of Q better is the solution because it provides a maximum group utility of the majority and a minimum individual regret of the opponent, the obtained compromise solution is acceptable by decision-makers[IV], [XVII].

After comparison of contribution factor Q values for Pareto-optimal solutions depicted in Tables 1 to 3, the best-compromised solution for optimal PMU placement for considered test systems are shown in bold. It can be observed from Tables 1 to 3, that best-compromised solution obtained for considered test systems with a number of PMUs of 8 with measurement redundancy of 10, number of PMUs of 16 with measurement redundancy of 18, and number of PMUs of 31 with measurement redundancy of 36 respectively.

### Table 1: Multi-objective OPP Scheme of Two Channel IEEE 14-bus test system.

| \(N_{PMUs}\) | \(N-r\) | \(r\) | \(Q\) | Optimal Placement |
|---|---|---|---|---|
| 5 | 13 | 1 | 1 | 2, 6, 7, 9, 12 |
| 6 | 10 | 4 | 0.6026 | 2, 6, 6, 7, 7, 9 |
| 7 | 7 | 7 | 0.2051 | 2, 2, 6, 6, 7, 9, 10 |
| 8 | 4 | 10 | 0 | 2, 2, 6, 6, 7, 7, 9, 12 |
| 9 | 2 | 12 | 0.2 | 2, 3, 5, 6, 6, 7, 7, 9, 10 |
| 10 | 1 | 13 | 0.6 | 2, 4, 5, 6, 6, 7, 9, 9, 10, 12 |
| 10 | 0 | 14 | 1 | 2, 2, 3, 6, 6, 7, 7, 9, 10, 13 |
| 11 | 0 | 14 | 1 | 2, 4, 5, 6, 6, 7, 7, 9, 9, 10, 12 |
Table 2: Multi-objective OPP Scheme of Two Channel IEEE 30-bus system.

| N_{PMUs} | N-r | r   | Q    | Optimal Placement          |
|----------|-----|-----|------|-----------------------------|
| 11       | 27  | 3   | 1    | 1, 7, 8, 9, 10, 12, 14, 19, 22, 24, 27 |
| 12       | 24  | 6   | 0.8  | 1, 2, 7, 8, 9, 10, 12, 14, 19, 22, 24, 27 |
| 13       | 21  | 9   | 0.6  | 1, 7, 8, 9, 10, 12, 14, 19, 22, 23, 24, 27, 27 |
| 14       | 18  | 12  | 0.4  | 1, 2, 7, 8, 9, 10, 12, 14, 16, 19, 22, 24, 25, 27 |
| 15       | 15  | 15  | 0.2  | 1, 2, 7, 8, 9, 10, 12, 14, 16, 19, 22, 24, 25, 27, 29 |
| 16       | 12  | 18  | 0    | 1, 2, 7, 8, 9, 10, 12, 14, 16, 18, 19, 22, 24, 25, 27, 29 |
| 17       | 10  | 20  | 0.0833 | 1, 2, 6, 7, 8, 9, 10, 12, 14, 15, 16, 19, 22, 24, 25, 27, 29 |
| 18       | 8   | 22  | 0.1917 | 1, 2, 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 19, 22, 24, 25, 27, 29 |
| 19       | 6   | 24  | 0.3   | 1, 2, 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 19, 22, 24, 25, 27, 29 |
| 20       | 4   | 26  | 0.4083 | 1, 2, 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 19, 22, 24, 25, 27, 29 |
| 21       | 3   | 27  | 0.65  | 1, 2, 4, 7, 8, 9, 10, 10, 11, 12, 14, 16, 18, 19, 22, 22, 24, 25, 27, 29 |
| 22       | 1   | 29  | 0.7583 | 1, 2, 4, 6, 7, 8, 9, 10, 10, 11, 12, 14, 16, 18, 19, 22, 22, 24, 25, 27, 29 |
| 23       | 0   | 30  | 1     | 1, 2, 4, 6, 7, 8, 9, 10, 10, 11, 12, 14, 16, 18, 19, 22, 22, 24, 25, 27, 29 |

Table 3: Multi-objective OPP Scheme of Two Channel IEEE 57-bus system.

| N_{PMUs} | N- \( r \) | Q    | Optimal Placement          |
|----------|------------|------|-----------------------------|
| 19       | 57         | 0    | 1, 4, 6, 11, 12, 19, 22, 26, 29, 30, 32, 36, 38, 39, 42, 45, 46, 51, 54, |
| 20       | 54         | 3    | 0.9206 1, 4, 6, 9, 11, 12, 19, 22, 26, 29, 30, 32, 36, 38, 39, 42, 45, 46, 51, 54, |
| 21       | 51         | 6    | 0.8412 1, 4, 5, 6, 11, 12, 19, 22, 26, 29, 30, 32, 36, 38, 39, 42, 45, 46, 51, 54, |
| 22       | 48         | 9    | 0.7617 1, 4, 5, 6, 11, 12, 14, 19, 22, 26, 29, 30, 32, 36, 38, 39, 42, 45, 46, 51, 53, |
| 23       | 45         | 12   | 0.6825 1, 4, 5, 6, 11, 12, 12, 19, 22, 26, 29, 30, 32, 36, 38, 39, 42, 45, 46, 51, 53, |
| 24       | 42         | 15   | 0.6029 1, 4, 5, 6, 11, 12, 14, 19, 20, 22, 26, 27, 29, 30, 32, 36, 37, 38, 39, 42, 45, |
| 25       | 39         | 18   | 0.5235 1, 4, 5, 6, 11, 12, 14, 19, 20, 22, 26, 27, 29, 30, 32, 35, 36, 37, 38, 39, 42, |
| 26       | 36         | 21   | 0.4441 1, 4, 5, 6, 11, 12, 14, 19, 20, 22, 24, 26, 27, 29, 30, 32, 35, 36, 37, 38, 39, |
| 27       | 33         | 24   | 0.3647 1, 4, 5, 6, 11, 12, 19, 22, 26, 28, 29, 30, 32, 32, 35, 36, 37, 38, 39, 42, |
| 28       | 30         | 27   | 0.2852 1, 3, 4, 5, 6, 11, 12, 16, 19, 22, 26, 28, 29, 30, 32, 35, 36, 37, 38, 39, |
| 29       | 27         | 30   | 0.2058 1, 4, 5, 6, 11, 12, 12, 14, 19, 20, 22, 26, 28, 29, 30, 32, 32, 35, 36, 38, 38, |
| 30       | 24         | 33   | 0.1264 1, 4, 5, 6, 9, 11, 12, 12, 14, 19, 20, 22, 26, 28, 29, 30, 32, 32, 35, 36, 38, 38, |

Copyright reserved © J. Mech. Cont.& Math. Sci.
B. Vedił et al
Case Study 2: PMUs Available With Only Three Channel Capacity

Similar to case study 1, here the PMUs available with three-channel capacity is considered to solve multi-objective PMU placement problem. The results obtained using NSGA-II for IEEE 14-bus, IEEE 30-bus, and IEEE 57-bus system is tabulated in Tables 4 to 6. It can be observed from Tables 4 to 6 that for given channel capacity, the number of PMUs necessary to install across system increases with an increase in redundancy. It can be observed that a minimum of four PMUs, 10 PMUs, and 17 PMUs are required to make system observable for IEEE 14-bus, IEEE 30-bus, and IEEE 57-bus systems respectively with minimum redundancy. Further, a maximum of 9 PMUs, 22 PMUs, and 38 PMUs are required to make system observable for IEEE 14-bus, IEEE 30-bus, and IEEE 57-bus systems respectively with maximum redundancy. The best-compromised solution between two conflicting objectives is selected based upon the compromised ranking factor Q computed using the VIKOR method. After comparison of contribution factor Q values for Pareto-optimal solutions depicted in Tables 4 to 6, the best-compromised solution for optimal PMU placement for considered test systems are shown in bold. It can be observed from Tables 4 to 6, that best-compromised solution obtained for considered test systems with a number of PMUs of 6 with measurement redundancy of 10,
number of PMUs of 15 with measurement redundancy of 22, and number of PMUs of 26 with measurement redundancy of 36 respectively.

Table 4: Multi-objective OPP Scheme of Three Channel IEEE 14-bus test system.

| N_PMUs | N-r | r   | Q     | Optimal Placement |
|--------|-----|-----|-------|-------------------|
| 4      | 12  | 2   | 1     | 2, 6, 7, 9        |
| 5      | 8   | 6   | 0.4722| 2, 6, 7, 9, 13    |
| 6      | 4   | 10  | 0     | 2, 2, 6, 7, 9, 13 |
| 7      | 2   | 12  | 0.2292| 2, 2, 6, 7, 9, 13 |
| 8      | 1   | 13  | 0.6146| 2, 2, 6, 7, 8, 9, 10, 13 |
| 9      | 0   | 14  | 1     | 2, 2, 5, 6, 7, 8, 9, 10, 13 |

Table 5: Multi-objective OPP Scheme of Three Channel IEEE 30-bus system.

| N_PMUs | N-r | r   | Q     | Optimal Placement |
|--------|-----|-----|-------|-------------------|
| 10     | 24  | 6   | 1     | 1, 5, 9, 10, 12, 15, 19, 25, 27, 28 |
| 11     | 20  | 10  | 0.6905| 1, 5, 9, 10, 12, 15, 19, 25, 27, 28 |
| 12     | 17  | 13  | 0.5   | 1, 5, 9, 10, 10, 12, 15, 19, 25, 27, 28 |
| 13     | 14  | 16  | 0.3095| 1, 5, 9, 10, 10, 12, 15, 19, 25, 27, 28 |
| 14     | 11  | 19  | 0.119 | 1, 3, 5, 9, 10, 10, 12, 15, 19, 25, 27, 28 |
| 15     | 8   | 22  | 0     | 1, 3, 5, 9, 10, 10, 12, 15, 19, 22, 25, 27, 28 |
| 16     | 6   | 24  | 0.0714| 1, 3, 5, 7, 9, 10, 10, 12, 15, 19, 22, 25, 27, 28 |
| 17     | 4   | 26  | 0.1429| 1, 3, 5, 7, 9, 10, 10, 12, 12, 15, 15, 19, 22, 25, 27, 28 |
| 18     | 3   | 27  | 0.2976| 1, 3, 5, 7, 9, 10, 10, 12, 12, 15, 15, 19, 22, 25, 26, 27, 28 |
| 20     | 2   | 28  | 0.6905| 1, 3, 5, 7, 8, 9, 10, 10, 12, 12, 15, 15, 19, 22, 25, 26, 27, 28 |
| 21     | 1   | 29  | 0.8452| 1, 3, 5, 7, 8, 9, 10, 10, 10, 12, 12, 15, 15, 19, 20, 22, 25, 26, 27, 28 |
| 22     | 0   | 30  | 1     | 1, 3, 5, 7, 8, 9, 10, 10, 11, 12, 12, 13, 15, 15, 19, 20, 22, 25, 27, 28 |
Table 6: Multi-objective OPP Scheme of Three Channel IEEE 57-bus system.

| N_PMUs | N- | r  | Q     | Optimal Placement |
|--------|----|----|-------|-------------------|
| 17     | 53 | 4  | 1     | 3, 6, 12, 15, 19, 22, 26, 29, 30, 32, 36, 38, 41, 46, 51, 54, 57 |
| 18     | 49 | 8  | 0.8651| 3, 6, 12, 15, 19, 22, 26, 29, 30, 32, 36, 38, 41, 46, 51, 54, 57 |
| 19     | 45 | 12 | 0.7301| 1, 3, 6, 12, 15, 19, 22, 26, 29, 30, 32, 36, 38, 41, 46, 51, 54, 57 |
| 20     | 41 | 16 | 0.5952| 3, 4, 6, 12, 15, 19, 22, 26, 29, 30, 32, 36, 38, 41, 46, 51, 54, 56, 57 |
| 21     | 27 | 20 | 0.4603| 3, 4, 6, 9, 12, 15, 19, 22, 26, 29, 30, 32, 36, 38, 41, 46, 51, 54, 57 |
| 22     | 33 | 24 | 0.3254| 1, 3, 6, 12, 13, 15, 19, 22, 24, 26, 29, 30, 32, 36, 38, 41, 46, 51, 54, 57 |
| 23     | 20 | 27 | 0.1819| 1, 3, 6, 9, 12, 15, 19, 22, 26, 29, 30, 32, 36, 38, 41, 46, 47, 51, 54, 57 |
| 24     | 25 | 33 | 0.1101| 1, 3, 6, 9, 12, 15, 19, 20, 22, 24, 26, 29, 30, 32, 36, 41, 46, 47, 51, 54, 57 |
| 25     | 21 | 36 | 0.0667| 1, 3, 6, 9, 12, 15, 19, 20, 22, 24, 26, 28, 29, 30, 32, 36, 41, 46, 51, 54, 57 |
| 26     | 18 | 39 | 0.0861| 1, 3, 6, 9, 12, 15, 19, 20, 22, 24, 26, 29, 30, 32, 34, 36, 38, 41, 46, 51, 54, 57 |
| 27     | 15 | 42 | 0.1056| 1, 3, 6, 9, 12, 15, 19, 20, 22, 24, 26, 28, 29, 30, 32, 34, 36, 38, 41, 46, 51, 54, 57 |
| 28     | 12 | 45 | 0.125 | 1, 3, 6, 9, 12, 15, 19, 20, 22, 24, 26, 28, 29, 30, 32, 34, 36, 38, 41, 46, 51, 54, 57 |
| 29     | 10 | 47 | 0.1911| 1, 3, 4, 6, 9, 12, 15, 19, 20, 22, 24, 26, 28, 29, 30, 32, 34, 36, 38, 41, 46, 47, 51, 54, 57 |
| 30     | 8  | 49 | 0.2572| 1, 3, 4, 6, 9, 12, 13, 15, 19, 20, 22, 24, 26, 28, 29, 30, 32, 34, 36, 38, 41, 46, 47, 50, 51, 53, 54, 56, 57 |
| 31     | 2  | 51 | 0.3233| 1, 3, 4, 6, 9, 12, 13, 15, 19, 20, 22, 24, 26, 28, 29, 30, 32, 34, 36, 37, 38, 38, 41, 46, 51, 54, 55, 57 |
| 32     | 1  | 53 | 0.5022| 1, 3, 4, 6, 6, 9, 12, 13, 15, 19, 20, 22, 24, 26, 28, 29, 30, 32, 34, 35, 37, 38, 38, 41, 46, 47, 50, 51, 53, 54, 56, 57 |
| 33     | 4  | 53 | 0.3894| 1, 3, 4, 6, 9, 12, 13, 15, 19, 20, 22, 24, 26, 28, 29, 30, 32, 34, 36, 37, 38, 38, 41, 46, 47, 47, 50, 51, 53, 54, 56, 57 |
| 34     | 3  | 54 | 0.5022| 1, 3, 4, 6, 6, 9, 12, 13, 15, 19, 20, 22, 24, 26, 28, 29, 30, 32, 34, 36, 37, 38, 38, 41, 46, 47, 50, 51, 53, 54, 56, 57 |
| 35     | 2  | 55 | 0.615 | 1, 3, 4, 6, 8, 9, 12, 13, 15, 19, 20, 22, 24, 26, 28, 29, 30, 32, 33, 34, 36, 37, 38, 38, 41, 46, 47, 50, 51, 53, 54, 56, 57 |
| 36     | 1  | 56 | 0.7287| 1, 3, 4, 6, 8, 9, 12, 13, 15, 19, 20, 22, 24, 26, 28, 29, 30, 32, 33, 34, 36, 37, 38, 38, 41, 46, 47, 47, 50, 51, 53, 56, 54, 57 |

Case Study 3: PMUS Available With Variable Channel Capacity

In this case study, unlike case studies 1 and 2, PMUs with variable channel capacity having variable cost has been considered to solve multi-objective PMU...
placement problem. If a PMU to be placed at a bus connected to four other buses, a four-channel capacity PMU is considered. Similarly, if a PMU to be placed at a bus connected to three other buses, a three-channel capacity PMU is considered. Thus, the installation cost of PMU at four and three bus system is 1.3 p.u. and 1.2 p.u. respectively. The results obtained using NSGA-II for IEEE 14-bus, IEEE 30-bus, and IEEE 57-bus system is tabulated in Tables 7 to 9. It can be observed from Tables 7 to 9 that for given channel capacity, the number of PMUs necessary to install across system increases with an increase in redundancy. It can be observed that a minimum of four PMUs, 10 PMUs, and 17 PMUs are required to make system observable for IEEE 14-bus, IEEE 30-bus, and IEEE 57-bus systems respectively with minimum redundancy. Further, a maximum of 9 PMUs, 21 PMUs, and 37 PMUs are required to make system observable for IEEE 14-bus, IEEE 30-bus, and IEEE 57-bus systems respectively with maximum redundancy. The best-compromised solution between two conflicting objectives is selected based upon the compromised ranking factor Q computed using the VIKOR method. After comparison of contribution factor Q values for Pareto-optimal solutions depicted in Tables 7 to 9, the best-compromised solution for optimal PMU placement for considered test systems are shown in bold. It can be observed from Tables 7 to 9, that best-compromised solution obtained for considered test systems with a number of PMUs of 5 with measurement redundancy of 8, number of PMUs of 15 with measurement redundancy of 24, and number of PMUs of 25 with measurement redundancy of 41 respectively.

Table 7: Multi-objective OPP of IEEE 14-bus test system considering the unequal cost.

| N_{PMUs} | N-r | PMU cost (p.u.) | r | Q     | Optimal Placement |
|----------|-----|-----------------|---|-------|-------------------|
| 4        | 14  | 4.6             | 0 | 1.0000| 2, 8, 10, 13      |
| 4        | 12  | 4.8             | 2 | 0.6875| 2, 7, 10, 13      |
| 5        | 11  | 5.7             | 3 | 0.7626| 1, 2, 8, 10, 13   |
| 5        | 10  | 5.8             | 4 | 0.6063| 2, 7, 8, 10, 13   |
| 5        | 9   | 5.9             | 5 | 0.4501| 2, 8, 9, 10, 13   |
| 5        | 7   | 6.1             | 7 | 0.1375| 2, 6, 8, 9, 13    |
| 5        | 6   | 6.3             | 8 | 0.0102| 2, 6, 7, 9, 13    |
| 6        | 5   | 7.1             | 9 | 0.1074| 2, 6, 7, 8, 10, 13|
| 6        | 4   | 7.2             | 10| 0.0267| 2, 6, 8, 9, 10, 13|
| 7        | 3   | 8.1             | 11| 0.2916| 1, 2, 8, 9, 10, 12, 13|
| 7        | 2   | 8.3             | 12| 0.2541| 1, 2, 6, 8, 9, 10, 13|
| 8        | 1   | 9.4             | 13| 0.6054| 1, 2, 3, 6, 8, 9, 10, 13|
| 9        | 0   | 10.6            | 14| 1.0000| 1, 2, 3, 6, 7, 8, 9, 10, 13|
Table 8: Multi-objective OPP of IEEE 30-bus test system considering the unequal cost.

| N_PMU | N-r | PMU cost | r   | Q         | Optimal Placement  |
|-------|-----|----------|-----|-----------|--------------------|
| 10    | 17  | 12.6     | 13  | 1.0000    | 3,6,7,10,11,12,15,19,25,27 |
| 11    | 14  | 13.7     | 16  | 0.6563    | 1,3,6,7,10,11,12,15,19,25,27 |
| 12    | 13  | 14.4     | 17  | 0.6074    | 1,3,7,8,10,11,12,15,19,22,25,27 |
| 12    | 12  | 14.8     | 18  | 0.4994    | 1,3,6,7,10,11,12,15,19,21,25,27 |
| 12    | 11  | 14.9     | 19  | 0.3323    | 1,3,6,7,10,11,12,15,19,24,25,27 |
| 13    | 10  | 15.5     | 20  | 0.2637    | 1,3,5,7,8,10,11,12,15,19,22,25,27 |
| 13    | 9   | 16       | 21  | 0.1754    | 1,3,6,7,10,11,12,15,19,22,25,27,29 |
| 14    | 8   | 16.6     | 22  | 0.1068    | 1,3,5,7,8,10,11,12,15,17,19,22,25,27 |
| 14    | 7   | 17.1     | 23  | 0.0186    | 1,3,6,7,10,11,12,15,17,19,24,25,27,29 |
| 15    | 6   | 17.7     | 24  | 0.0174    | 1,3,5,7,8,10,11,12,15,17,19,22,25,27,29 |
| 16    | 5   | 18.7     | 25  | 0.1497    | 1,3,5,7,8,10,11,12,15,17,19,22,25,26,27,29 |
| 17    | 4   | 19.7     | 26  | 0.2821    | 1,3,5,7,8,10,11,12,13,15,17,19,22,25,26,27,2 |
| 18    | 3   | 20.8     | 27  | 0.4414    | 1,3,5,7,8,10,11,12,13,15,17,19,20,22,25,26,2 |
| 19    | 2   | 22       | 28  | 0.6276    | 1,3,5,7,8,10,11,12,13,15,17,19,20,22,25,26,2,7 |
| 20    | 1   | 23.2     | 29  | 0.8138    | 1,3,5,7,8,9,10,11,12,13,15,17,19,20,22,24,25, |
| 21    | 0   | 24.4     | 30  | 1.0000    | 1,3,5,7,8,9,10,11,12,13,15,17,19,20,22,24,25, |

Table 9: Multi-objective OPP of IEEE 57-bus test system considering the unequal cost.

| N_PMU | N-r | PMU cost | r   | Q         | Optimal Placement  |
|-------|-----|----------|-----|-----------|--------------------|
| 17    | 48  | 20.3     | 9   | 1.0000    | 1,4,6,13,20,22,25,27,29,32,36,39,41,45,47,51,54 |
| 18    | 45  | 21.4     | 12  | 0.9225    | 1,4,6,13,20,22,25,27,29,30,32,36,39,41,45,47,51,54 |
| 18    | 44  | 21.5     | 13  | 0.8773    | 1,4,6,13,20,22,24,25,27,29,32,36,39,41,45,47,51,54 |
| 18    | 43  | 21.6     | 14  | 0.8321    | 1,4,6,13,20,22,25,27,29,32,36,39,41,45,47,51,54,56 |
| 18    | 42  | 21.7     | 15  | 0.7870    | 1,4,6,12,13,20,22,25,27,29,32,36,39,41,45,47,51,54 |
| 19    | 41  | 22.6     | 16  | 0.7998    | 1,4,6,13,20,22,25,27,29,30,32,36,39,41,45,47,48,51,54 |
| 19    | 40  | 22.7     | 17  | 0.7546    | 1,4,6,13,20,22,25,27,29,30,32,36,39,41,45,47,51,54,56 |
| 19    | 39  | 22.8     | 18  | 0.7095    | 1,4,6,12,13,20,22,25,27,29,31,32,36,39,41,45,47,51,54 |
| 19    | 37  | 23       | 20  | 0.6191    | 1,4,6,12,13,20,22,25,27,29,32,36,39,41,45,47,51,54,56 |

Copyright reserved © J. Mech. Cont. & Math. Sci.
B. Vedik et al
| 20 | 36 | 23.9 | 21 | 0.6319 | 1,4,6,13,20,22,25,27,29,30,32,36,39,41,45,47,48,51,54,56 |
| 20 | 35 | 24   | 22 | 0.5868 | 1,4,6,12,13,20,22,24,25,27,29,31,32,36,39,41,45,47,51,54 |
| 20 | 34 | 24.1 | 23 | 0.5416 | 1,4,6,12,13,20,22,25,27,29,31,32,36,39,41,45,47,51,54 |
| 20 | 33 | 24.2 | 24 | 0.4964 | 1,4,6,12,13,20,22,25,27,29,32,36,39,41,45,47,51,54 |
| 20 | 32 | 24.4 | 25 | 0.4585 | 1,4,6,12,13,20,22,25,27,29,32,36,39,41,45,47,51,54 |
| 21 | 31 | 25.2 | 26 | 0.4641 | 1,4,6,12,13,20,22,25,27,29,32,36,39,41,45,46,47,50,51,54 |
| 21 | 30 | 25.3 | 27 | 0.4189 | 1,4,6,12,13,20,22,24,25,27,29,32,36,39,41,45,47,51,54 |
| 21 | 29 | 25.4 | 28 | 0.3737 | 1,4,6,12,13,20,22,24,25,27,29,32,36,39,41,45,47,51,54 |
| 21 | 28 | 25.6 | 29 | 0.3358 | 1,4,6,12,13,20,22,24,25,27,29,32,36,38,39,41,45,47,51,54 |
| 22 | 27 | 26.4 | 30 | 0.3414 | 1,4,6,12,13,19,20,22,24,25,27,29,32,36,39,41,45,46,47,51,54,56 |
| 22 | 26 | 26.5 | 31 | 0.2962 | 1,4,6,12,13,20,22,24,25,27,29,32,36,39,41,45,47,48,49,51,54 |
| 22 | 25 | 26.7 | 32 | 0.2583 | 1,2,4,6,12,13,20,22,24,25,27,29,32,36,38,39,41,45,47,48,49,51,54 |
| 23 | 24 | 27.5 | 33 | 0.2638 | 1,2,4,6,12,13,19,20,22,24,25,27,29,31,32,36,39,41,45,47,48,49,51,54 |
| 23 | 23 | 27.6 | 34 | 0.2187 | 1,2,4,6,12,13,19,20,22,24,25,27,29,32,36,39,41,45,47,48,49,51,54 |
| 23 | 22 | 27.8 | 35 | 0.1808 | 1,2,4,6,12,13,20,22,24,25,27,29,31,32,36,39,41,45,47,48,49,51,54,56 |
| 24 | 21 | 28.6 | 36 | 0.1863 | 1,2,4,6,12,13,19,20,22,24,25,27,29,31,32,36,39,41,45,47,48,49,51,54 |
| 24 | 20 | 28.7 | 37 | 0.1411 | 1,2,4,6,12,13,19,20,22,24,25,27,29,32,36,39,41,45,47,48,49,51,54 |
| 24 | 19 | 28.9 | 38 | 0.1032 | 1,2,4,6,12,13,19,20,22,24,25,27,29,31,32,36,39,41,45,47,48,49,51,54 |
| 25 | 18 | 29.7 | 39 | 0.1323 | 1,2,4,6,12,13,19,20,22,24,25,27,29,31,32,36,39,41,45,47,48,49,51,54 |
| 25 | 17 | 29.9 | 40 | 0.1187 | 1,2,4,6,12,13,19,20,22,24,25,27,29,30,32,36,38,39,41,45,47,48,49,51,54 |
| 25 | 16 | 30   | 41 | 0.0944 | 1,2,4,6,12,13,19,20,22,24,25,27,29,31,32,36,38,39,41,45,47,48,49,51,54 |
| 26 | 15 | 30.8 | 42 | 0.1456 | 1,2,4,6,12,13,19,20,22,24,25,27,29,31,32,36,39,41,45,47,48,49,51,54 |
| 26 | 14 | 31   | 43 | 0.1320 | 1,2,4,6,12,13,19,20,22,24,25,27,29,30,32,34,36,38,39,41,44,45,47,48,49,51,54 |
| 26 | 13 | 31.1 | 44 | 0.1076 | 1,2,4,6,12,13,19,20,22,24,25,27,29,31,32,36,38,39,41,44,45,47,48,49,51,54 |
| 27 | 12 | 31.9 | 45 | 0.1588 | 1,2,4,6,12,13,19,20,22,24,25,27,29,32,34,36,39,41,44,45,47,48,49,51,54 |
| 27 | 10 | 32.2 | 47 | 0.1209 | 1,2,4,6,9,12,19,20,22,24,25,27,29,31,32,36,38,39,41,45,47,50,51,52,54,56 |
| 28 | 7  | 33.3 | 50 | 0.1342 | 1,2,4,6,9,12,19,20,22,24,25,27,29,31,32,35,36,38,39,41,45,46,47,50,51,52,54,56,57 |
| 30 | 6  | 35.5 | 51 | 0.3365 | 1,2,4,6,9,12,19,20,22,24,25,27,29,30,32,34,35,36,38,39,41,45,46,47,50,51,52,54,56,57 |
Comparison of Results Obtained Using Three Case Studies

The cost incurred in installing multi-objective PMU placement problem for all the three case studies are compared and shown in Table 10. Table 10 consists of four columns that represent test system, number of PMUs, measurement redundancy respectively. It can be observed from this table that the cost acquired by conceiving variable channel capacity is less when compared to a fixed number of channel capacity during the installation of PMUs on a bus. For example, consider the IEEE 57-bus system the installation cost of PMU with the variable cost is less i.e. 30 when compared to 34.1 and 31.2 for two-channel and three-channel capacity respectively. Further, the number of PMUs required for installation is less i.e. 25 when compared to 31 and 26 for two-channel and three-channel capacity respectively. Furthermore, measurement redundancy by conceiving variable cost is high when compared to the other two case studies.

Table 10: Comparison of three case studies

| Test System     | N_PMU | r   | Installation cost (p.u.) |
|-----------------|-------|-----|--------------------------|
| **IEEE 14-bus system** |       |     |                          |
| Two-Channel Capacity | 8     | 10  | 8.8                      |
| Three Channel Capacity | 6     | 10  | 7.2                      |
| Variable Cost     | 5     | 8   | 6.3                      |
| **IEEE 30-bus system** |       |     |                          |
| Two-Channel Capacity | 16    | 18  | 17.6                     |
| Three Channel Capacity | 15    | 22  | 17                       |
| Variable Cost     | 15    | 24  | 17.7                     |
| **IEEE 57-bus system** |       |     |                          |
| Two-Channel Capacity | 31    | 36  | 34.1                     |
| Three Channel Capacity | 26    | 36  | 31.2                     |
| Variable Cost     | 25    | 41  | 30                       |
V. Conclusions

In the present work, a multi-objective PMU placement problem to enhance the reliability of the system and minimize the installation cost of PMUs simultaneously has been proposed using NSGA-II. The optimal placement of PMUs has been determined by considering channel limitations and variable PMUs cost. The Pareto-optimal solutions for these two conflicting objectives have been acquired by employing non-dominated sorting and crowded mechanism of NSGA-II. Then, VIKOR technique has been utilized to find the best-compromised solution among the Pareto-optimal solutions. It has been identified that the installation cost incurred in placing PMUs with high redundancy is achieved by considering variable PMU costs. The effectiveness and reliability of the proposed method is verified by applying to various standard IEEE test systems. This work can be extended by considering more objective functions like minimizing the cost of communication infrastructure and maximizing the voltage stability.

References

I. A. Enshaee, R. A. Hooshmand, and F. H. Fesharaki, “A new method for optimal placement of phasor measurement units to maintain full network observability under various contingencies,” Electr. Power Syst. Res., vol. 89, pp. 1–10, 2012.

II. A. Mahari and H. Seyedi, “Optimal PMU placement for power system observability using BICA, considering measurement redundancy,” Electr. Power Syst. Res., vol. 103, pp. 78–85, 2013.

III. B. Milošević and M. Begović, “Nondominated sorting genetic algorithm for optimal phasor measurement placement,” IEEE Trans. Power Syst., vol. 18, no. 1, pp. 69–75, 2003.

IV. C. Chang, “A modified VIKOR method for multiple criteria analysis,” Environ. Monit. Assess., vol. 168, pp. 339–344, Sep. 2010.

V. C. Peng, H. Sun, and J. Guo, “Multi-objective optimal PMU placement using a non-dominated sorting differential evolution algorithm,” Int. J. Electr. Power Energy Syst., vol. 32, no. 8, pp. 886–892, 2010.

VI. C. Ruben, S. C. Dhulipala, A. S. Bretas, Y. Guan, and N. G. Bretas, “Multi-objective MILP model for PMU allocation considering enhanced gross error detection: A weighted goal programming framework,” Electr. Power Syst. Res., vol. 182, p. 106235, 2020.
VII. H. Manoharan, S. Srikrishna, G. Sivarajan, and A. Manoharan, “Economical placement of PMUs considering observability and voltage stability using binary coded ant lion optimization,” Int. Trans. Electr. Energy Syst., vol. 28, no. 9, pp. 1–19, 2018.

VIII. J. Aghaei, A. Baharvandi, A. Rabiee, and M. A. Akbari, “Probabilistic PMU Placement in Electric Power Networks: An MILP-Based Multiobjective Model,” IEEE Trans. Ind. Informatics, vol. 11, no. 2, pp. 332–341, 2015.

IX. J. Aghaei, A. Baharvandi, M. A. Akbari, K. M. Muttaqi, M. R. Asban, and A. Heidari, “Multi-objective Phasor Measurement Unit Placement in Electric Power Networks: Integer Linear Programming Formulation,” Electr. Power Components Syst., vol. 43, no. 17, pp. 1902–1911, 2015.

X. K. Arul jeyaraj, V. Rajasekaran, S. K. Nandha Kumar, and K. Chandrasekaran, “A multi-objective placement of phasor measurement units using fuzzified artificial bee colony algorithm, considering system observability and voltage stability,” J. Exp. Theor. Artif. Intell., vol. 28, no. 1–2, pp. 113–136, Mar. 2016.

XI. K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, “A fast and elitist multiobjective genetic algorithm: NSGA-II,” IEEE Trans. Evol. Comput., vol. 6, no. 2, pp. 182–197, Apr. 2002.

XII. K. Jamuna and K. S. Swarup, “Multi-objective biogeography based optimization for optimal PMU placement,” Appl. Soft Comput. J., vol. 12, no. 5, pp. 1503–1510, 2012.

XIII. M. Basu, “Dynamic economic emission dispatch using non dominated sorting genetic algorithm-II,” Int. J. Electr. Power Energy Syst., vol. 30, no. 2, pp. 140–149, 2008.

XIV. N. P. Theodorakatos, N. M. Manousakis, and G. N. Korres, “A sequential quadratic programming method for contingency-constrained phasor measurement unit placement,” Int. Trans. Electr. Energy Syst., vol. 25, no. 12, pp. 3185–3211, Dec. 2015.

XV. R. Babu and B. Bhattacharyya, “Strategic placements of PMUs for power network observability considering redundancy measurement,” Meas. J. Int. Meas. Confed., vol. 134, pp. 606–623, 2019.

XVI. S. M. Mazhari, H. Monsef, H. Lesani, and A. Fereidunian, “A multi-objective PMU placement method considering measurement redundancy and observability value under contingencies,” IEEE Trans. Power Syst., vol. 28, no. 3, pp. 2136–2146, 2013.

XVII. S. Opricovic and G.-H. Tzeng, “Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS,” Eur. J. Oper. Res., vol. 156, no. 2, pp. 445–455, Jul. 2004.

Copyright reserved © J. Mech. Cont.& Math. Sci.
B. Vedik et al
XVIII. S. P. Singh and S. P. Singh, “A Multi-objective PMU Placement Method in Power System via Binary Gravitational Search Algorithm,” Electr. Power Components Syst., vol. 45, no. 16, pp. 1832–1845, 2017.

XIX. S. Prasad and D. M. V. Kumar, “Robust meter placement for active distribution state estimation using a new multi-objective optimization model,” IET Sci. Meas. Technol., vol. 12, no. 8, pp. 1047–1057, 2018.

XX. V. Basetti and C. Ashwani Kumar, “Optimal Multi-Objective Hybrid Measurement Placement Using NSGA-II,” i-manager’s J. Power Syst. Eng., vol. 2, no. 3, p. 28, 2014.