A Method for Determining CP Violating Phase $\gamma^*$

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Abstract

A new way of determining the phases of weak amplitudes in charged $B$ decays based on SU(3) symmetry is proposed. The CP violating phase $\gamma$ can now be determined without the previous difficulty associated with electroweak penguins.

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Detection of CP violation and verification of the unitarity triangle of the CKM matrix is a major goal of B factories \[1\]. Decisive information about the origin of CP violation will be obtained if the three phases \[\alpha = \arg(-V_{td}V_{tb}^*/V_{ub}^*V_{ud}), \beta = \arg(-V_{cd}V_{cb}^*/V_{tb}^*V_{td})\] and \[\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}^*V_{cd})\] can be independently measured experimentally \[2\]. The sum of these three phases must be equal to 180° if the Standard Model with three generations is the model for CP violation. There have been many studies to measure these phases.

The phase \(\beta\) can be determined unambiguously by measuring time variation asymmetry in \(\bar{B}^0(B^0) \rightarrow \psi K_S\) decay rates \[3,4\]. The phase \(\alpha\) can be measured in \(B^- \rightarrow \pi\pi\) and \(B \rightarrow \rho\pi\) decays \[3,4\]. In these decays, there are contributions from the tree and the penguin (both strong and electroweak) amplitudes. The methods proposed in Refs. \[3,4\] are valid even if the strong penguin contributions are included. The inclusion of the electroweak penguin contributions may contaminate the result. However, because the electroweak penguin effects are small in this case, the error in \(\alpha\) determination are small.

Several methods using \(\Delta S = 1\) \(B\) decays to measure the phase \(\gamma\) had been proposed \[5,6\]. Most had assumed that the effects from electroweak penguin could be neglected. It has been recently shown by us \[7\] that this assumption is badly violated for top quark mass of order 170 GeV. For \(\Delta S = 0\) hadronic \(B\) decays, the strong penguin effects are much smaller than the leading tree contributions, and thus electroweak penguin effects which are even smaller can be safely ignored. In \(\Delta S = 1\) decays, because of the large enhancement factor \(|V_{tb}V_{ts}^*/V_{ub}^*V_{us}| \approx 55\), the strong penguins dominate and the electroweak penguin effects are comparable to the tree contributions. This invalidates methods proposed in Refs. \[5,6\]. In this letter we give further consideration to measuring \(\gamma\) using \(\Delta S = 1\) decays, although other methods have been discussed in the literature \[2,8\].

An interesting method has recently been proposed by Hernandez, Gronau, London and Rosner \[9\] using SU(3) relations between the decay amplitudes for \(B^- \rightarrow \pi^0K^-, \pi^-\bar{K}^0, \pi^0\bar{K}^0, \pi^+K^-, B^- \rightarrow \pi^-\pi^0\), and \(B_s \rightarrow \eta\pi^0\). This method requires the reconstruction of the quadrangle from \(B^- \rightarrow \pi^0K^-, \pi^-\bar{K}^0, \pi^0\bar{K}^0, \pi^+K^-\) decays. In order to do so, one not only needs to measure all the four \(B \rightarrow \pi K\) decay amplitude but also needs to measure the
rare decay amplitude $B_s \to \eta \pi^0$. It has been shown that this last decay is a pure $\Delta I = 1$ transition, with the dominant contribution from the electroweak penguin. However, the branching ratio is extremely small $O(10^{-7})$ \cite{10}. In this letter we propose a new method to measure $\gamma$ using $\Delta S = 1$ decays $B^- \to \pi^- K^0, \pi^0 K^-, \eta K^-$, and the $\Delta S = 0$ decay $B^- \to \pi^- \pi^0$, which relies on SU(3) symmetry. This method is free from the electroweak penguin contamination problem, and further, all decays involved have relatively large ($O(10^{-5})$) branching ratios. More importantly these measurements can in principle be carried out at present facilities like CLEO or CDF/D0.

In the SM the most general effective Hamiltonian for hadronic $B$ decays can be written as follows:

$$H_{eff}^q = \frac{G_F}{\sqrt{2}} V_{ub} V_{uq}^* (c_1 O_1^q + c_2 O_2^q) - \sum_{i=3}^{10} V_{ib} V_{iq}^* c_i O_i^q + H.C.,$$  

(1)

where the $O_i^q$ are defined as

$$O_1^q = \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) u_\beta \bar{u}_\beta \gamma^\mu (1 - \gamma_5) b_\alpha, \quad O_2^q = \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) u_\beta \bar{u}_\alpha \gamma^\mu (1 - \gamma_5) b_\beta,$$

$$O_{3,5}^q = \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \bar{q}_\beta \gamma_\mu (1 \mp \gamma_5) q', \quad O_{4,6}^q = \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \bar{q}_\beta \gamma_\mu (1 \mp \gamma_5) q'_\alpha,$$

$$O_{7,9}^q = \frac{3}{2} \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \bar{u}_q \gamma^\mu (1 \pm \gamma_5) q', \quad O_{8,10}^q = \frac{3}{2} \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \bar{u}_q \gamma^\mu (1 \pm \gamma_5) q'_\alpha.$$

Here $q'$ is summed over u, d, and s. For $\Delta S = 0$ processes, $q = d$, and for $\Delta S = 1$ processes, $q = s$. $O_2, O_1$ are the tree level and QCD corrected operators. $O_{3-6}$ are the strong gluon induced penguin operators, and operators $O_{7-10}$ are due to $\gamma$ and Z exchange, and the “box” diagrams at one loop level (electroweak penguin). The Wilson coefficients $c_i$ are defined at the scale of $\mu \approx m_b$ which have been evaluated to the next-to-leading order in QCD \cite{11}. In the above we have neglected small contributions from u and c quark loop contributions proportional to $V_{ub} V_{uq}^*$.

We can parametrise the decay amplitude of $B$ as

$$A = <\text{final state}|H_{eff}^q|B> = V_{ub} V_{uq}^* T(q) + V_{ib} V_{iq}^* P(q),$$  

(3)

where $T(q)$ contains the tree contribution, while $P(q)$ contains penguin contributions. SU(3) symmetry will lead to specific relations among $B$ decay amplitudes.
SU(3) relations for B decays have been studied by several authors [12,13]. We will follow the notation used in Ref. [13]. The operators $Q_{1,2}, O_{3-6},$ and $O_{7-10}$ transform under SU(3) symmetry as $\bar{3}_a + \bar{3}_b + 6 + \mathbf{15}, \bar{3},$ and $\bar{3}_a + \bar{3}_b + 6 + \mathbf{15},$ respectively. In general, we can write the SU(3) invariant amplitude for $B$ to two octet pseudoscalar mesons in the following form

$$T = A_T^{(3)} B_i H(3)^i (M^k_l M^l_k) + C_T^{(3)} B_i M^k_l M^l_k H(3)^i + A_T^{(6)} B_i H(6)^{ij} M^l_j M^l_i + C_T^{(6)} B_i M^l_j H(6)^{ik} M^l_k + A_T^{(15)} B_i H(15)^{ij} M^l_j M^l_i + C_T^{(15)} B_i M^l_j H(15)^{ik} M^l_k,$$

(4)

where $B_i = (B^-, \bar{B}_0, \bar{B}_s^0)$ is a SU(3) triplet, $M^l_i$ is the SU(3) pseudoscalar octet, and the matrices $H$ represent the transformation properties of the operators $O_{1-10}$. $H(6)$ is a traceless tensor that is antisymmetric on its upper indices, and $H(15)$ is also a traceless tensor but is symmetric on its upper indices. For $q = d$, the non-zero entries of the $H$ matrices are given by

$$H(\bar{3})^2 = 1, \quad H(6)^{12}_1 = H(6)^{23}_3 = 1, \quad H(6)^{21}_1 = H(6)^{32}_3 = -1,$$

(5)

$$H(\mathbf{15})^{12}_1 = H(\mathbf{15})^{21}_1 = 3, \quad H(\mathbf{15})^{22}_2 = -2, \quad H(\mathbf{15})^{32}_3 = H(\mathbf{15})^{23}_3 = -1.$$

For $q = s$, the non-zero entries are

$$H(\bar{3})^3 = 1, \quad H(6)^{13}_1 = H(6)^{32}_2 = 1, \quad H(6)^{31}_1 = H(6)^{23}_2 = -1,$$

$$H(\mathbf{15})^{13}_1 = H(\mathbf{15})^{31}_1 = 3, \quad H(\mathbf{15})^{32}_3 = -2, \quad H(\mathbf{15})^{32}_2 = H(\mathbf{15})^{23}_2 = -1.$$

(6)

In terms of the SU(3) invariant amplitudes, the decay amplitudes $T(\pi\pi)$, $T(\pi K)$ for $\bar{B}^0 \to \pi\pi$, $\bar{B}^0 \to \pi K$ are given by

$$T(\pi^- K^0) = C_T^{(3)} + A_T^{(6)} - C_T^{(6)} + 3A_T^{(15)} - C_T^{(15)} ,$$

$$T(\pi^0 K^-) = \frac{1}{\sqrt{2}}(C_T^{(3)} + A_T^{(6)} - C_T^{(6)} + 3A_T^{(15)} + 7C_T^{(15)}),$$

$$T(\eta s K^-) = \frac{1}{\sqrt{6}}(-C_T^{(3)} - A_T^{(6)} + C_T^{(6)} - 3A_T^{(15)} + 9C_T^{(15)}),$$

$$T(\pi^0 \pi^-) = \frac{8}{\sqrt{2}}C_T^{(15)},$$

(7)
We also have similar relations for the amplitude $P(q)$. The corresponding SU(3) invariant amplitudes will be denoted by $A_i^P$ and $C_i^P$. It is easy to obtain the following triangle relation from above:

$$\sqrt{2}A(\pi^0 K^-) - 2A(\pi^- \bar{K}^0) = \sqrt{6}A(\eta_8 K^-). \quad (8)$$

For the moment if we ignore $\eta - \eta'$ mixing, it is clear that we can construct this triangle from the experimentally measured rates for the various $B^-$ decays. A similar triangle can also be constructed for the modes of $B^+$ decay:

$$\sqrt{2}\bar{A}(\pi^0 K^+) - 2\bar{A}(\pi^+ K^0) = \sqrt{6}\bar{A}(\eta_8 K^+). \quad (9)$$

We shall now use a hypothesis that the tree contribution to the mode $B^- \rightarrow \pi^- \bar{K}^0$ is negligible [14]. This is verified in factorization approximation and had been assumed by Ref. [6,9]. Further, if we work in Wolfenstein parametrization of the CKM matrix, the amplitude $A(\pi^- \bar{K}^0)$ contains no weak phase. Then

$$A(\pi^- \bar{K}^0) = \bar{A}(\pi^+ K^0). \quad (10)$$

We can now obtain the magnitude and relative phases of $A(\pi^0(\eta_s)K^-)$ and $\bar{A}(\pi^0(\eta_s)K^+)$ subject to two fold ambiguities related to whether the triangles for the $B^-$ and $B^+$ decays are on the same side (solution a) or opposite side (solution b) of $A(\pi^- \bar{K}^0)$ as shown in Figure 1.

Now we construct two complex quantities (shown in Figure 1)

$$B = \sqrt{2}A(\pi^0 K^-) - A(\pi^- \bar{K}^0) = 8(|V_{ub}V_{us}^*|e^{-i\gamma}C_{F15}^T + |V_{tb}V_{ts}^*|C_{P15}^T). \quad (11)$$

and

$$\bar{B} = \sqrt{2}\bar{A}(\pi^0 K^+) - \bar{A}(\pi^+ K^0) = 8(|V_{ub}V_{us}^*|e^{i\gamma}C_{F15}^T + |V_{tb}V_{ts}^*|C_{P15}^T). \quad (12)$$

Then

$$B - \bar{B} = -i16|V_{ub}V_{us}^*|C_{F15}^T \sin \gamma. \quad (13)$$
To determine $\sin \gamma$, we need a way of measuring $C^T_{T_{15}}$. We achieve this by relating $C^T_{T_{15}}$ to the amplitude $A(\pi^-\pi^0)$ for $B^- \to \pi^-\pi^0$. In the SU(3) limit this decay amplitude is given by

$$A(\pi^-\pi^0) = \frac{8}{\sqrt{2}} (|V_{ub}V_{ud}^*|e^{-i\gamma}C^T_{T_{15}} + |V_{tb}V_{ts}^*|C^P_{T_{15}}). \quad (14)$$

Here the penguin contribution $C^P_{T_{15}}$ arises from the electroweak penguin only, and contributes less than 4\% \cite{note1}. We therefore obtain

$$B - \bar{B} = -i2\sqrt{2}e^{i\delta_{T_{15}}} |V_{us}| |A(\pi^-\pi^0)| \sin \gamma, \quad (15)$$

where $\delta_{T_{15}}$ is the strong final state rescattering phase of $C^T_{T_{15}}$. Thus the magnitude of $B - \bar{B}$ can be used to determine $\sin \gamma$. The phase of $B - \bar{B}$ gives us information of strong phase $\delta_{T_{15}}$ relative to the strong phase of $A(\pi^-\bar{K}^0)$. The two solutions for $B - \bar{B}$ in Figure 1 corresponding to a smaller value (solution $(B - \bar{B})_a$) and a larger value (solution $(B - \bar{B})_b$) for $\sin \gamma$. The larger value is likely to be ruled out because the resulting $\sin \gamma$ may very well exceed unity.

Similarly, one can use the combination

$$\tilde{B} = \sqrt{2}A(\pi^0K^-) + \sqrt{6}A(\eta_8K^-) = 16(|V_{ub}V_{us}^*|e^{-i\gamma}C^T_{T_{15}} + |V_{tb}V_{ts}^*|C^P_{T_{15}}); \quad (16)$$

$$\tilde{\bar{B}} = \sqrt{2}\overline{A}(\pi^0K^+) + \sqrt{6}\overline{A}(\eta_8K^+) = 16(|V_{ub}V_{us}^*|e^{i\gamma}C^T_{T_{15}} + |V_{tb}V_{ts}^*|C^P_{T_{15}}),$$

to determine $\gamma$. In this case we have

$$\tilde{B} - \tilde{\bar{B}} = -i4\sqrt{2}e^{i\delta_{T_{15}}} |V_{us}| |A(\pi^-\pi^0)| \sin \gamma, \quad (17)$$

The results obtained hold in the exact SU(3) limit. SU(3) breaking effects in several ways will affect the above relations. These include $\eta - \eta'$ mixing effect, the breaking effects in form factors and mass differences. Due to the $\eta - \eta'$ mixing effect to determine $A(\eta_8K^-)$, we need to determine the decay amplitudes $A(\eta K^-)$ and $A(\eta' K^-)$. These amplitudes can be obtained from experiments. We can then construct

$$A(\eta_8K^-) = \cos \theta A(\eta K^-) + \sin \theta A(\eta' K^-), \quad (18)$$
where $\theta \approx 20^0$ is the $\eta - \eta'$ mixing angle. In principle we need to know the relative phase of $A(\eta'K^-)$ and $A(\eta K^-)$. Since $\sin\theta$ is small, this phase is important only if $A(\eta'K^-)$ is much larger than $A(\eta K^-)$. There are no reliable methods to evaluate the form factors at present. A factorization approximation calculation indicates that the large part of the effect is due to different decay constants and can be corrected by changing $A(\pi^-\pi^0)$ to $(f_K/f_\pi)A(\pi^0\pi^-)$ in eqs.(15,16,17), and $A(\eta_8K^-)$ to $(f_K/f_\eta)A(\eta_8K^-)$ in eqs. (8,9,16).

A similar analysis can be carried out for $B^- \rightarrow \rho^0K^{*-}, \rho^-K^{*0}, \omega K^*-,$ and $B^- \rightarrow \rho^0\rho^-$. where SU(3) relations are expected to hold also. In this case, $\omega - \phi$ mixing is “ideal” mixing with $\phi$ a pure $s\bar{s}$ state.

One might think the same analysis can be identically applied to $B^- \rightarrow \rho^-K^0, \rho^0K^-, \omega K^-$ and $B^- \rightarrow \rho^-\pi^0$, or $B^- \rightarrow \pi^-K^{*0}, \pi^0K^{*-}, \eta K^{*-}$ and $B^- \rightarrow \pi^-\rho^0$, separately. In each of the above two cases, there is similar triangle relations analogous to eq.(8) among the first three decay amplitudes. However, similar relations to eq.(15,17) are no longer valid. This is because in this case there is no bose statistics. There are two ways of writing SU(3) invariant relations, for example, for $H_3$, we can write

$$H_{eff} = C^V_i B_i V_{ij} M^j_k H^k_3 + C^M_i B_i M_{ij} V^{j}_k H^k_3,$$

where $V^j_i$ is the octet-vector meson. If $C^V = C^M$ one would have the desired relations. However, there is no reasons for $C^V$ and $C^M$ to be equal. Similarly for other SU(3) invariant amplitudes. Because of lack of this equality, similar relations to eqs. (15,17) do not exist.

In conclusion we have proposed a new method to determine the phase $\gamma$ which is free from contamination by electroweak penguin contributions. All decays involved have branching ratios of order $10^{-5}$, and are in principle measurable at existing or future $B$ facilities.
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FIG. 1. The triangle relations and the two solutions a and b for $B - \bar{B}$. Lines A, 1, 2, 3, 4 are the amplitudes $A(\pi^-K^0)$, $\sqrt{2}A(\pi^0K^-)$, $\sqrt{6}A(\eta_8K^-)$, $B$, and $\bar{B}$, respectively. The dashed lines are for the corresponding anti-B decay amplitudes.