Nonlinear Aeroelastic Modeling of a Folding Wing Structure

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Abstract. The nonlinear aeroelastic response of a folding wing with the hinge free-play nonlinearity is investigated. A nonlinear structural dynamic model is established based on the modal synthesis of free-substructures by modification of the connected relationships, and is unnecessary to re-establish it if the stiffness changes. The nonlinear aeroelastic equation is obtained by rational function approximation of the unsteady aerodynamic force, and solved by Runge-Kutta method to predict the aeroelastic response of the folding wing. The results show that the free-play at hinges changes the aeroelastic characteristic of the folding wing, leading to the occurrence of the limit cycle oscillation within a certain velocity range. Moreover, the sensitivity to the free-play is different for the inboard and outboard hinges. If the free-play is controlled within a certain value, the motion of nonlinear aeroelastic system of the folding wing can be considered linear approximately.

1. Introduction
With the development of aircraft design technologies and for the purpose of improving the performance of the aircraft in different flight conditions, some aircraft designers turned their attention to the morphing aircraft from traditional fixed-wing aircraft [1], because morphing aircraft can change shape and size in-flight to enable a single vehicle to perform multiple mission roles. The majority of shape change occurs in the wing, resulting in a wide variety of changes in aerodynamic and structural features [2].
Many factors, such as large deformation, free-play nonlinearity, etc., can contribute to the nonlinear aeroelastic response of a folding wing [3]. Among them, the hinge free-play is often the main source of nonlinearity. Lee et al [4] analyzed the nonlinear aeroelastic response of a folding wing considering hinge free-play nonlinearity using ZAERO and MSC.NASTRAN and obtained the condition for the limit cycle oscillation (LCO), in their research works, fictitious mass method [5] was employed to improve the accuracy and efficiency of modal-based structural analysis. Bae et al [6,7] and Kim et al [8,9] studied the nonlinear aeroelastic response of an aircraft wing with control surface using the fictitious mass modal approach and the description function approach. Their numerical results showed the nonlinearities changed the flutter characteristics of the wing and LCO occurred within different velocities ranges. Yang et al [10] developed a free-interface component mode synthesis method which could be used to establish governing equations and to analyze the characteristics of nonlinear aeroelastic systems, and the results of the numerical simulations and the wind tunnel tests indicated the same trends and critical velocities.
Two methods have been developed for structural nonlinear aeroelastic analysis. The first one is the fictitious mass mode synthesis method. In this method, a fictitious mass is added to the mass matrix...
corresponding to the degree of freedom where structural changes will occur to improve local structural characteristics. However there is not a definite standard to quantify the fictitious mass. It is possible that different values could be used for different structures, which limits to the applicability of this method in nonlinear aeroelastic modeling. Furthermore, it may cause numerical difficulties if the value of fictitious mass becomes too large. Another kind of method is Craig-Bampton component mode synthesis. In this method, the dynamics of a structure is described by selected sets of normal modes of individual component structures, plus a set of static vectors that account for the coupling at each interface where component structures are connected. Since the degrees of freedom at the interfaces still remain, which results to the equation with high order and the computational cost of the general aerodynamic influence coefficient matrix.

In the present work, the aeroelastic equation for a folding wing with structural nonlinearities is established and the nonlinear aeroelastic analysis is carried out. The free-play structural dynamic equation is derived by the free interface component mode synthesis, in which the continuity at interfaces is modified and the nonlinear internal force is taken into consideration. The unsteady aerodynamic force is introduced into the structural dynamic equation by the virtual work principle. Finally, the aeroelastic equation for a folding wing with multi-degree of freedom structural nonlinearities is achieved and the aeroelastic responses are obtained.

2. Nonlinear structural dynamic model of a folding wing based on the free interface component mode synthesis

The folding wing consists of three components: the fuselage, the inboard wing and the outboard wing, denoted as A, B, C, respectively. The material has a Young’s modulus of $7.1 \times 10^{10}$ Pa, Poisson’s ratio of 0.33 and density of $2.7 \times 10^3$ kg/m$^3$, representing a typical aluminum alloy. All components are modeled by plates and discretized using the CQUAD4 elements with the thickness of 2 mm in MSC.NASTRAN. Each node of this element has six degrees of freedom: three translations, $u_x, u_y, u_z$, and three rotations, $\theta_x, \theta_y, \theta_z$. A hinge between component A and component B is model by a set of torsional spring and the multi-point constraint (MPC). The rotational degree of freedom $\theta_x$ is coupled using torsional springs with the stiffness of $K_x$, while the other five degrees of freedom are defined via MPC. Similarly, hinge between components B and C is modeled by a set of torsional spring with the stiffness of $K_y$ and the MPC. Figure 1 shows the finite element model for the folding wing.

![Figure 1. Finite element model for a folding wing (X represent hinges position).](image)

2.1. Structural Dynamic Equation

The detailed finite element model of a folding wing typically has hundreds of thousands degrees of freedom, which makes the component mode synthesis method a natural choice for structural dynamic analysis. It enables the dynamic analysis to be performed on highly reduced matrices to save computational time. It also makes modifications possible to reduce the number of design iterations [11]. Hou first introduced the free interface component mode synthesis method. But this method was...
found to result in unacceptable errors in certain cases. Later Rubin [12] and Craig and Chang [13] introduced modifications to overcome this problem and their modified free interface component mode synthesis method has been widely used since. However, the elastic connection and the nonlinear internal force are not taken into consideration in the traditional free interface component mode synthesis. In the present work, modifications are made in the continuity at the interface so that this method can be used to establish the nonlinear structural dynamic equation. Figure 2 illustrates the procedure for obtaining the nonlinear structural dynamic equation.

First, the structural dynamic equation of each undamped substructure can be written as:

\[ M \ddot{u} + K u = f \]  

\[ \lambda = A, B, C \]  

(1)

where \( M \) is the mass matrix, \( K \) is the stiffness matrix, \( u \) is the displacement vector, \( f \) is the external force, \( f_I \) is the internal force applied to the degrees of freedom at interfaces, and \( B \) is the Boolean matrix.

The nodal displacement vector for each substructure includes displacements of the internal nodes and the displacements of the interfacial nodes. In preparation for the component mode synthesis, one can write the structural dynamic equation of each substructure in a rearranged and partitioned form. In present model, the hinges are modeled using the torsional spring and the MPC. Therefore the degrees of freedom at the interfaces are divided into two parts, displacements given by the MPC and displacements due to the torsional springs. Here, substructure A is taken as an example and its dynamic equation is as follows:

\[
\begin{bmatrix}
M_{ii}^A & M_{im}^A & M_{ms}^A \\
M_{mi}^A & M_{mm}^A & M_{ms}^A \\
M_{mi}^{sym} & M_{mm}^{sym} & M_{ms}^{sym}
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_{i}^A \\
\ddot{u}_{m}^A \\
\ddot{u}_{s}^A
\end{bmatrix}
+ \begin{bmatrix}
K_{ii}^A & K_{im}^A & K_{ms}^A \\
K_{mi}^A & K_{mm}^A & K_{ms}^A \\
K_{mi}^{sym} & K_{mm}^{sym} & K_{ms}^{sym}
\end{bmatrix}
\begin{bmatrix}
u_{i}^A \\
u_{m}^A \\
u_{s}^A
\end{bmatrix}
= \begin{bmatrix}
f_{i}^A \\
f_{m}^A \\
f_{s}^A
\end{bmatrix}
\]  

(2)

In the above equation, subscript \( i \) represents the terms refers to the internal nodal, subscript \( m \) represents the terms refers to the MPC, and subscript \( s \) represents the terms refers to the torsional spring. When the degrees of freedom at the interfaces are set free, the modal analysis of the substructure A is carried out as follows:

\[
u^A = \begin{bmatrix}
\nu_i^A \\
\nu_m^A \\
\nu_s^A
\end{bmatrix} = \Phi_i^A \Psi_i^A + \Psi_s^A f_s^A = [\Phi_i^A \quad \Psi_s^A] \begin{bmatrix}
p_i^A \\
f_s^A
\end{bmatrix}
\]  

(3)
where $\mathbf{p}_i^c$ is the modal coordinates, $\Phi_i^c$ is the kept modes, $\Psi_i^c = (\mathbf{K}^c)^{-1}\mathbf{B}^{st} - \Phi_i(A_i^c)^{-1}\Phi_i^c\mathbf{B}^{st}$ is the residual modes.

Substituting Eq. (3) into Eq. (2) leads to:

$$\ddot{\mathbf{M}}\ddot{\mathbf{p}}^c + \ddot{\mathbf{K}}\ddot{\mathbf{p}}^c = \ddot{\mathbf{f}}^c$$

where $\ddot{\mathbf{M}} = [\Phi_i^c \quad \psi_i^c]^T\ddot{\mathbf{M}}[\Phi_i^c \quad \psi_i^c] = \begin{bmatrix} \dddot{\mathbf{M}}^c_{ii} & 0 \\ 0 & \dddot{\mathbf{M}}^c_{ii} \end{bmatrix}$, $\dddot{\mathbf{K}} = [\Phi_i^c \quad \psi_i^c]^T\dddot{\mathbf{K}}[\Phi_i^c \quad \psi_i^c] = \begin{bmatrix} K_i^c & 0 \\ 0 & K_i^c \end{bmatrix}$.

$\ddot{\mathbf{p}}^c = [p_i^{stT} \quad f_i^{stT}]$, $\ddot{\mathbf{f}}^c = [\Phi_i^c \quad \psi_i^c]^T\mathbf{B}^{st}\dddot{\mathbf{f}}^c$.

Similarly, the above procedures can be applied to substructures B and C. Therefore the uncoupling structural dynamic equation can be expressed as:

$$\ddot{\mathbf{M}}\ddot{\mathbf{p}} + \ddot{\mathbf{K}}\ddot{\mathbf{p}} = \ddot{\mathbf{f}}$$

where $\ddot{\mathbf{M}} = \text{diag}(\dddot{\mathbf{M}}^A, \dddot{\mathbf{M}}^B, \dddot{\mathbf{M}}^C)$, $\ddot{\mathbf{K}} = \text{diag}(\dddot{\mathbf{K}}^A, \dddot{\mathbf{K}}^B, \dddot{\mathbf{K}}^C)$, $\dddot{\mathbf{f}} = [\dddot{\mathbf{f}}^A \quad \dddot{\mathbf{f}}^B \quad \dddot{\mathbf{f}}^C]^T$.

$\ddot{\mathbf{p}} = [\dddot{\mathbf{p}}^A \quad \dddot{\mathbf{p}}^B \quad \dddot{\mathbf{p}}^C]^T$, $\ddot{\mathbf{f}} = [\dddot{\mathbf{f}}^A \quad \dddot{\mathbf{f}}^B \quad \dddot{\mathbf{f}}^C]^T$.

The continuity condition at the interface includes the displacement continuity and the force/moment continuity. The displacement continuity can be expressed as:

$$\begin{cases} \mathbf{u}_m^A = \mathbf{u}_m^B, & \mathbf{u}_m^B = \mathbf{u}_m^C \\ \mathbf{u}_s^A = \mathbf{u}_s^B + \delta_1, & \mathbf{u}_s^B = \mathbf{u}_s^C + \delta_2 \end{cases}$$

The force/moment continuity can be expressed as:

$$\begin{cases} f_m^A + f_m^B = 0, & f_m^B = f_m^C = 0 \\ f_s^A = K_1\delta_1, & f_s^B = -K_1\delta_1, \quad f_s^A + f_s^B = 0 \\ f_s^B = K_2\delta_2, & f_s^C = -K_2\delta_2, \quad f_s^B + f_s^C = 0 \end{cases}$$

Because of the torsional springs at interfaces, the force and moment are equal at both sides but the corresponding displacements are not equal, there should be a relative shift $\delta$. By modal coordinate transformation for each substructure, the displacement condition at interfaces can be expressed as:

$$\begin{align*}
B_m^A(\Phi_i^m \psi_{im}^m + \psi_{im}^m \Phi_i^m) &= B_m^B(\Phi_i^m \psi_{im}^m + \psi_{im}^m \Phi_i^m) \\
B_m^B(\Phi_i^m \psi_{im}^m + \psi_{im}^m \Phi_i^m) &= B_m^C(\Phi_i^m \psi_{im}^m + \psi_{im}^m \Phi_i^m) \\
B_s^A(\Phi_i^m \psi_{im}^m + \psi_{im}^m \Phi_i^m) &= B_s^B(\Phi_i^m \psi_{im}^m + \psi_{im}^m \Phi_i^m) + \delta_1 \\
B_s^B(\Phi_i^m \psi_{im}^m + \psi_{im}^m \Phi_i^m) &= B_s^C(\Phi_i^m \psi_{im}^m + \psi_{im}^m \Phi_i^m) + \delta_2
\end{align*}$$

where $B_m^A$ and $B_s^A$ represent the projection matrix related to MPC and torsional spring respectively, and $\psi_{im}^m$ and $\psi_{im}^m$ represent the residual modes related to MPC and torsional spring respectively. By considering the force/moment continuity condition, one can obtain:

$$\begin{align*}
f_m^A &= -f_m^B = (B_m^m \psi_{im}^m + B_m^m \psi_{im}^m)^{-1}(-B_m^m \Phi_i \psi_{im}^m + B_m^m \Phi_i \psi_{im}^m) \\
f_s^A &= -f_s^B = (B_s^m \psi_{im}^m + B_s^m \psi_{im}^m)^{-1}(-B_s^m \Phi_i \psi_{im}^m + B_s^m \Phi_i \psi_{im}^m - \delta_1) \\
f_m^C &= -f_m^B = (B_m^m \psi_{im}^m + B_m^m \psi_{im}^m)^{-1}(B_m^m \Phi_i \psi_{im}^m - B_m^m \Phi_i \psi_{im}^m - \delta_1) \\
f_s^C &= -f_s^B = (B_s^m \psi_{im}^m + B_s^m \psi_{im}^m)^{-1}(B_s^m \Phi_i \psi_{im}^m + B_s^m \Phi_i \psi_{im}^m + \delta_2)
\end{align*}$$
where $A_{11} = B^4 \Phi_{n}^4 + B^n \Phi_{m}^n$, $G_{11} = B^4 \Phi_{n}^4$, $A_{22} = B^4 \Phi_{n}^4 + B^n \Phi_{m}^n$, $G_{22} = B^4 \Phi_{n}^4$, $A_{33} = B^4 \Phi_{n}^4 + B^n \Phi_{m}^n$, $G_{33} = B^4 \Phi_{n}^4$, $A_{44} = B^4 \Phi_{n}^4 + B^n \Phi_{m}^n$, $G_{44} = B^4 \Phi_{n}^4$, $A = B^4 \Phi_{n}^4$, the above equation can be expressed:

$$
\begin{align*}
\mathbf{p} = f^{\text{BF}} = \mathbf{S}^q = \begin{bmatrix}
\mathbf{p}_1^q \\
\mathbf{t}_1^q \\
\mathbf{t}_2^q \\
\vdots \\
\mathbf{t}_n^q \\
\mathbf{f}_1^q \\
\mathbf{f}_2^q \\
\vdots \\
\mathbf{f}_m^q \\
\end{bmatrix}
\end{align*}
$$

Substituting Eq. (10) into Eq. (5) and pre-multiplying $S'$, one can obtain:

$$
\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f}
$$

where $\mathbf{M} = S'^{T}\mathbf{M}\mathbf{S}, \mathbf{K} = S'^{T}\mathbf{K}\mathbf{S}, \mathbf{K} = S'^{T}\mathbf{K}\mathbf{S}$. In general, the mass and stiffness matrices are coupled. But since the size of the matrix is reduced, the computational efficiency is improved.

Now consider the right hand term $S'^{T}$. The general forces corresponding to the modal coordinates $\mathbf{b}_i^s, \mathbf{p}_i^s, \mathbf{p}_i^c$ are zero, which is consistent with the result of the traditional free interface component mode synthesis. The following discusses the general forces corresponding to $\delta_1^s, \delta_2^s$.

$$
- (A_{22}^{-1})^T \psi_{d}^{BT} B_{s}^{BT} \mathbf{q}_s + (A_{22}^{-1})^T \psi_{d}^{BT} B_{f}^{BT} \mathbf{f}_s
$$

$$
- (A_{22}^{-1})^T (\psi_{d}^{BT} B_{s}^{BT} + \psi_{d}^{BT} B_{f}^{BT}) \mathbf{f}_s
$$

$$
- (\psi_{d}^{BT} B_{s}^{BT} + \psi_{d}^{BT} B_{f}^{BT})^{-1} (\psi_{d}^{BT} B_{s}^{BT} + \psi_{d}^{BT} B_{f}^{BT}) \mathbf{f}_s
$$

As can be seen from (12) and (13), the generalized force of the whole folding wing structure contains the torque by the torsional spring at the interfaces. The final generalized force can be expressed as:

$$
\tilde{\mathbf{f}} = \begin{bmatrix}
0^T \\
0^T \\
0^T \\
\mathbf{K}_s \delta_1^s \\
\mathbf{K}_s \delta_2^s
\end{bmatrix}
$$

In this case, the generalized force depends on $K_s, K_f$ and $\delta_1^s, \delta_2^s$. Obviously, the nonlinear internal force can also be explicitly expressed in the structural dynamics equation.

### 2.2. Aeroelastic equation in state space

In this section, the aerodynamic force for a folding wing is obtained by the doublet lattice method. By modal coordinate transformation of the aerodynamic force, the aeroelastic equation for the folding wing can be written as

$$
\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f} + q_s S'^{T} \mathbf{Q}\mathbf{s}
$$

Note that the modal matrix of each substructure is not affected as $\mathbf{f}$ changes, and it is not necessary to recalculate $\mathbf{Q}$. The nonzero elements in $\mathbf{f}$ due to the nonlinear internal forces caused by the torsional spring can be expressed as nonlinear functions of the generalized coordinates. The free-play
nonlinearity is generally modeled by expressing the internal force as a piecewise nonlinear function of the generalized displacement. Let $\theta$ be the generalized displacement at the hinge, figure 3 shows the free-play nonlinearity given in reference 3. In this model the torque is expressed as a piecewise nonlinear function of $\theta$.

$$
\begin{cases}
K_\delta(\theta - \delta), (\theta > \delta) \\
0, (-\delta \leq \theta \leq \delta) \\
K_\delta(\theta + \delta), (\theta < -\delta)
\end{cases}
$$

where $\delta$ is the amount of clearance. When $\delta$ is set to zero, the moment is a linear function of $\theta$. Generally, the unsteady aerodynamic influence coefficient matrix is calculated for discrete reduced frequency rather than as a continuous function of the circular frequency. Thus, the aerodynamic influence coefficient matrices can be approximated by rational functions. There are many methods of rational function approximation. The minimum state method is used here for simplicity and fast computation time. Introducing the aerodynamic states $q_\alpha$, one can obtain the aeroelastic equation in the state space:

$$
\begin{bmatrix}
q \\
\dot{q}
\end{bmatrix} =
\begin{bmatrix}
0 & I & 0 \\
-M_\alpha K & -M_\alpha D & q_\alpha M D_s \\
0 & E_s & (V/R_s) q_\alpha
\end{bmatrix}
\begin{bmatrix}
q \\
\dot{q}
\end{bmatrix} + M_\alpha f
$$

where $M = M - q_\alpha (b/V)^2 S A S$, $D = -q_\alpha (b/V) S A S$, $K = K - q_\alpha S A S$. Here, $A_0$, $A_1$, $A_2$, $D_s$, and $E_s$ are unknown matrices in the minimum state approximation, which can be calculated from the least square fit, $R_s$ is the aerodynamic lag matrix, and $b$ is the reference length. Integrating Eq. (17) gives the time response for a nonlinear system. Here, the Runge-Kutta algorithm is used to solve the response for the folding wing.

3. Analysis and discussion

The DMAP language is used to extract the mass and stiffness matrices of each component in MSC.NASTRAN. The unsteady aerodynamic forces are achieved using Doublet Lattice Method (DLM). Finally, the aeroelastic equation of motion in state space form is expressed as the minimum state approximation. The six kept modes of each substructure are used to establish the nonlinear structural dynamic equation for the folding wing based on the free interface component mode synthesis. At the interface, ten degrees of freedom are coupled by the MPC and torsional springs. Adding the four aerodynamic states to the dynamic equation, the order of the final aeroelastic equation in state space is 48. A MATLAB program is written based on the Runge-Kutta algorithm to solve the response for the folding wing.

3.1. Model validation

In order to verify the present model, which introduces generalized displacements at the inner and outer hinges, the natural frequency and the flutter boundary for the linear system are compared with those by MSC.NASTRAN, here $K_s = K_n = 1 \text{Nm/rad}$. The corresponding natural frequency of the folding wing is given in table 1. Since the substructures are connected by a hinge and the remaining nodes at the
interface are free, a lower first-order frequency occurs. The free interface mode synthesis introduces the residual mode, which makes up the error caused by the truncation of the high order modes. The results show a good agreement with MSC.NASTRAN. The corresponding mode shapes are showed in figure 4.

Table 1. Comparative result of natural frequencies.

| Mode order | Present (Hz) | MSC.NASTRAN (Hz) | Deviation |
|------------|--------------|-------------------|-----------|
| 1          | 0.3545       | 0.3915            | -9.45%    |
| 2          | 3.5245       | 3.6686            | -3.93%    |
| 3          | 10.5703      | 10.5182           | 0.50%     |
| 4          | 20.2140      | 19.0352           | 6.19%     |
| 5          | 26.6760      | 25.3796           | 5.11%     |
| 6          | 28.3108      | 27.0206           | 4.77%     |

Figure 4. Mode shapes comparison (in each group, left: present; right: MSC.NASTRAN)  
(a) The first mode shape; (b) The second mode shape; (c) The third mode shape; (d) The fourth mode shape; (e) The fifth mode shape; (f) The sixth mode shape.

The first mode is dominated by the first bending mode of component C. The second mode is dominated by the second bending mode of component C. The third mode is dominated by the first bending mode of component B and the first torsion of component C. As for the last three modes, it is not easy to tell the dominated shape.

Table 2 compares the flutter boundary and shows that the result of the present method is in good agreement with MSC.NASTRAN.

Table 2. Comparison of flutter boundary.

| Present(m/s) | MSC.NASTRAN(m/s) | Deviation |
|--------------|------------------|-----------|
| 31.25        | 33.03            | -5.39%    |

3.2. Free play nonlinear aeroelastic analysis

In order to analyze the effect of the free play on the aeroelastic responses for the folding wing, three different clearances ($\delta = 0.02^\circ, 0.2^\circ, 1.0^\circ$) are taken into consideration. The initial condition is a vertical displacement of 0.01mm. For convenience, the tip is selected for motion characteristics. ($V_f = 31.25m/s$ is the linear flutter velocity by the present method).

3.2.1 Inner hinge with free play nonlinearity. Some simulations show that there are three different kinds of motion, the convergent motion, limit cycle oscillation and divergent oscillation, when the inner hinge has the free-play nonlinearity. The motion map is shown in figure 5.

When the clearance $\delta$ equals to $0.02^\circ$, the limit cycle oscillation occurs within a much smaller range of the velocity. The response is a divergent oscillation occurs when the velocity is greater than the linear flutter velocity. If the velocity is less than the flutter velocity, the response is damped out as time proceeds because the aerodynamic damping exists in the nonlinear aeroelastic system. The response becomes complex as the velocity increases and the limit cycle oscillation and divergent motion take
place. The range of the velocity corresponding to the damped motion decreases as the clearance increases.

3.2.2 Outer hinge with free play nonlinearity. Similarly, three kinds of motion occur when the outer hinge has free-play nonlinearity.

The motion map of folding wing with free play at the outboard hinge is shown in figure 6. As the clearance increases, the range of the velocity corresponding to the damped motion greatly decreases when the free-play exists in the outer hinge. As for $\delta = 0.2^\circ$ and $\delta = 1.0^\circ$, the ranges of the velocity corresponding to the limit cycle oscillation and divergent motion both increase. When the clearance $\delta$ equals to $0.02^\circ$, a harmonic oscillation occurs at about the linear flutter velocity. The response is divergent when the velocity is greater than a critical value. Obviously, there exists this characteristic in the linear aeroelastic system. A typical limit cycle oscillation occurs near the linear flutter velocity in the case of $\delta = 0.02^\circ$. At the same time, the response is divergent at the velocity greater than the linear flutter velocity, which is the characteristic of an approximately linear or weakly nonlinear aeroelastic system. As the clearance increases, the velocity at which the response becomes divergent decreases.

![Figure 5. Motion map of folding wing with free play at the inboard hinge.](image1)

![Figure 6. Motion map of folding wing with free play at the outboard hinge.](image2)

4. Conclusions

In this study, the aeroelastic equation for a folding wing with structural nonlinearities is established and the dynamic response is obtained. The following conclusions can be drawn from the present study:

1) The nonlinear structural dynamic equation is established based on the free interface component mode synthesis, in which the nonlinear internal force can be explicitly expressed in the structural dynamics equation. When the stiffness changes, it is not necessary to reestablish the structural dynamic model. Moreover, the order of the final aeroelastic equation is equal to the summation of kept modal degree of freedom and the generalized coordinates, which is relatively low. While comparing the frequencies of each mode order and flutter boundary obtained by the aeroelastic equation and MSC.NASTRAN, all the relative errors are no more than 10%, hence this method attractive for engineering applications.

2) The aeroelastic equation is derived from the integration of the nonlinear structural equation and the unsteady aerodynamic force. The aeroelastic response analysis is simplified, which does not require to recalculate the generalized aerodynamic influence matrix when the structural stiffness changes.

3) When the free-play exists in a folding wing and the clearance is within a certain value, the system can be regarded as an approximately linear system. The limit cycle oscillation will occur at some clearance values.
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