Production of doubly charmed baryons in $B$ decays

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Abstract

We study the doubly charmed baryonic $B$ decays $B \rightarrow \Xi_c \Lambda_c$ and $B \rightarrow \Lambda_c^+ \Lambda_c^- K$, recently observed by BELLE. We find that the unexpected large branching ratios (BRs) could be ascribed to the final state interactions (FSIs), which are dictated by $B \rightarrow DD_s^+ \rightarrow \Xi_c \Lambda_c^+$ and $B \rightarrow \bar{D}^0 D^0 K \rightarrow \Lambda_c^+ \Lambda_c^- K$. By utilizing the same mechanism, we predict that the BRs for $B^+ \rightarrow \bar{\Xi}_0^0 \Sigma_c^+$ and $B^+ \rightarrow \Sigma_c^\mp \Lambda_c^\pm K^+$ decays could be as large as $BR(B^+ \rightarrow \Sigma_c^\pm \Lambda_c^\pm)$ and $10^{-4}$, respectively. In addition, extending the FSIs to the processes associated with the creation of the $ss$ pair, $BR(B^+ \rightarrow \bar{\Omega}_c \Xi_c^\pm)$ at percent level is achievable.

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There have been many baryonic $B$ decays studied at $B$ factories. They are the charmless decays $B \to p(\Lambda)\bar{p}(\bar{\Lambda})K$, $B \to p\Lambda\pi$, and $B \to p(\Lambda)\bar{p}(\bar{\Lambda})$, in which the three-body baryonic decays have branching ratios (BRs) in magnitude of $O(10^{-6})$ while the BRs of two-body decays are limited to be less than $6.9 \times 10^{-7}$. In addition, the single charmed baryonic decay $B^0 \to \Sigma_c^{-}p\pi^+$ is measured to be $(2.38^{+0.63}_{-0.55} \pm 0.41 \pm 0.62) \times 10^{-4}$ while $\bar{B}^0 \to \Lambda_c^+\bar{p}$ and $B^- \to \Sigma_c(2455)^0\bar{p}$ are $(2.19^{+0.56}_{-0.49} \pm 0.32 \pm 0.57) \times 10^{-5}$ and $(3.67^{+0.74}_{-0.66} \pm 0.36 \pm 0.95) \times 10^{-5}$, respectively. Phenomenologically, the BRs of three-body baryonic $B$ decays are roughly one order of magnitude larger than those of corresponding two-body decays. The preference could be understood by the threshold enhancements occurring in the near threshold of baryon-pair invariant mass $\Sigma^0$. Moreover, one can easily find that in both three-body and two-body decays, the BRs for single charmed baryonic processes are two orders of magnitude larger than those for charmless decays. The differences could be attributed to the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, where single charmed (charmless) processes are associated with $V_{cb}(V_{ub})$. A more detailed review could be referred to Ref. [8].

However, when BELLE observes the two charmed baryons in the final state, the characters appearing in the single charmed and charmless decays subsequently are changed. According to BELLE’s results, the BRs $BR(B^+ \to \Lambda_c^+\Lambda_c^-K^+) = (6.5^{+1.0}_{-0.9} \pm 1.1 \pm 3.4) \times 10^{-4}$ and $BR(B^0 \to \Lambda_c^+\Lambda_c^-K^0) = (7.9^{+2.9}_{-2.3} \pm 1.2 \pm 4.1) \times 10^{-4}$ are measured with statistical significance of 16.4$\sigma$ and 6.6$\sigma$, and the products of BRs for two-body decays are $BR(B^+ \to \Xi_c^0\Lambda_c^+) \times BR(\bar{\Xi}_c^0 \to \bar{\Xi}_c^+\pi^-) = (4.8^{+1.0}_{-0.9} \pm 1.1 \pm 1.2) \times 10^{-5}$ and $BR(B^0 \to \bar{\Xi}_c^-\Lambda_c^+) \times BR(\Xi_c^- \to \Xi_c^+\pi^-\pi^-) = (9.3^{+3.7}_{-2.8} \pm 1.9 \pm 2.4) \times 10^{-5}$ with 8.7$\sigma$ and 3.8$\sigma$ significance, respectively. If we use the theoretical calculations $BR(\Xi_c^0 \to \Xi^-\pi^+) = 1.3\%$ and $BR(\Xi_c^+ \to \Xi^0\pi^+) = 3.9\%$ and the data $BR(\Xi_c^+ \to \Xi^0\pi^+)/BR(\Xi_c^+ \to \Xi^-\pi^+\pi^+) = 0.55 \pm 0.16$ [12], we can get $B^+ \to \Xi_c^0\Lambda_c^+ \approx 4.8 \times 10^{-3}$ and $B^0 \to \bar{\Xi}_c^-\Lambda_c^+ \approx 1.2 \times 10^{-3}$ [11]. Based on these results, one can find that there definitely appear two puzzles: (I) in terms of the BR of $B^0 \to \Sigma_c^{-}p\pi^+$, one can immediately find that due to phase space suppression, the BR of $B^+ \to \Lambda_c^+\Lambda_c^-K^+$ is $O(10^{-6})$ which is two orders of magnitude smaller than the observation; (II) It is expected that $BR(B \to \Xi_c\Lambda_c^+) \sim BR(B^0 \to \bar{\Lambda}_c^-p)$ in theoretical calculations [13], but the reality chooses $BR(B \to \Xi_c\Lambda_c^+) \gg BR(B^0 \to \bar{\Lambda}_c^-p)$. In this paper, we are going to investigate the possible mechanism to solve the unexpected large BRs in doubly charmed baryonic $B$ decays.

It has been noticed that final state interactions (FSIs) may play an important role on the
BRs, CP asymmetries (CPAs) and polarizations of vector mesons in two-body charmless and charmed mesonic B decays [14]. According to our previous discussions, the inconsistency in two charmed baryons production results from ordinary estimations and QCD calculations. That is, it should exist a new mechanism to enhance the BRs for the decays $B \to \bar{\Xi}_c \Lambda^+$ and $B \to \Lambda^+_c \Lambda^- K$. In Ref. [6], two different mechanisms have been introduced, in which the two-body and three-body decays are governed by the $\sigma$, $\pi^0$ and $\pi^\pm$ meson exchanges and charmonium-like resonance $X_{cc}$, respectively. However, according to the results of BELLE [9], there is no evidence for the existence of charmonium-like resonance. In addition, since the doubly charmed baryonic B decays are found in two-body and three-body decays simultaneously, we speculate that they are induced by the similar mechanisms. Hence, we consider that there exist other mechanisms which dictate the production of doubly charmed baryons. We speculate that FSIs play an essential role on the suffering puzzles. Furthermore, we propose that the FSIs are arisen from the inelastic scatterings $\bar{D}^0 D^+_s \to \bar{\Xi}_c \Lambda^+_c$ and $\bar{D}^0 D^0 \to \Lambda^+_c \Lambda^-_c$ and lead to $B \to \bar{D} D^+_s \to \bar{\Xi}_c \Lambda^+_c$ and $B \to \bar{D}^0 D^0 K \to \Lambda^+_c \Lambda^-_c K$. The illustrated flavor diagrams are displayed in Fig. 1. For convenience, we will use $B \to \bar{\Xi}_c \Lambda^+_c[\bar{D} D^+_s]$

![Flavor Diagrams](image)

**FIG. 1:** The flavor diagrams for (a) $B^+ \to \bar{\Xi}_c^0 \Lambda^+_c$ and (b) $B^+ \to \Lambda^+_c \Lambda^-_c K^+$ decays, in which the former is arisen from $B^+ \to \bar{D}^0 D^+_s$ while the latter is via $B^+ \to \bar{D}^0 D^0 K^+$. The dotted symbols represent the weak interactions.

and $B \to \Lambda^+_c \Lambda^-_c[\bar{D}^0 D^0] K$ instead of the decaying chains. Since charged B decays have better significance, in the following analysis we will concentrate on charged modes.

To be more clear for the calculations and the effective interactions induced by one-loop effects, we replace Fig. 1 of flavor diagrams with Fig. 2 of effective diagrams, where (a) and (b) are for two-body and three-body decays, respectively, and the squared (dotted) symbol stands for the strong (weak) interactions. Thus, according to Fig. 2(a), the decay amplitude
FIG. 2: The effective diagrams for (a) two-body and (b) three-body doubly charmed baryonic decays, where double-line denotes the $B$ meson and squared (dotted) symbol stands for the strong (weak) interactions.

for $B \to \Xi_c \Lambda_c^+$ could be expressed as

$$
\mathcal{M}(B \to \Xi_c \Lambda_c^+) = \bar{u}_{\Lambda_c} \int \frac{d^4q}{(2\pi)^4} \frac{g_{DD, B_c B_c}(m_B^2)}{q^2 - m_D^2 + i\varepsilon} \frac{g_{DD, s}(m_B^2)}{(p_B - q)^2 - m_D^2 + i\varepsilon} v_{\Xi_c}, \tag{1}
$$

where $u(v)$ expresses the Dirac spinor field of baryon (antibaryon), $\varepsilon$ is used for removing singularities, $m_{D(s)}$ is the mass of $D(s)$. $g_{DD, s}(m_B^2)$ and $g_{DD, B_c B_c}(m_B^2)$ denote the effective weak coupling for $B - D - D_s$ interaction and effective strong coupling for $D - D_s - B_c - B_c$ interaction with $B_c$ being a charmed baryon, respectively. Since $D$ and $D_s$ are the same in the $SU(3)$ flavor symmetry except the carrying isospins are different, we will assume that the effective strong interactions are the same for $D$ and $D_s$ except the effects from the isospin of system. Hence, we will use $g_{DD, B_c B_c}$ instead of $g_{DD, B_c B_c}$. Similarly, the same assumption is also applied to charmed baryons. We note that for two-body decays, because the rest frame of charmed baryon-pair is just the rest frame of $B$ meson, we have set the invariant mass of charmed baryon-pair, denoted by $\sqrt{k^2}$, to be $m_B$ the mass of $B$ meson. Therefore, the values of effective couplings are taken at the $m_B$ scale. However, for three-body decays, the scale should be chosen at $\sqrt{k^2}$ and is a variable. Although the momentum variable in the loop integration in principle has no limit, the main contribution will arise when the on-shell condition for the intermediate states $D$ and $D_s$ is satisfied, i.e. we can regard the production of doubly charmed baryons as the process $B \to \Xi_c \Lambda_c^{+} [DD_s^+]$, as illustrated by Fig. 1(a). Hence, in terms of narrow width approximation, $1/[(s - M^2)^2 + M^2 \Gamma_M^2] \approx \pi \delta(s - M^2)/M \Gamma_M$, the integration of Eq. (1) could be simplified as

$$
I_{D_s}(p_B^2) = \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_D^2} \frac{1}{(p_B - q)^2 - m_D^2} \approx -\frac{\langle \bar{p}_{D_s} \rangle}{8\pi \sqrt{p_B}}, \tag{2}
$$

with $\bar{p}_{D_s}$ being the spatial momentum of $D_s$ meson. Since $g_{BD, s}$ is associated with the
decays $B \to DD^+_s$, we can easily find its relationship to the BRs of $B \to DD^+_s$ by

$$BR(B \to DD^+_s) = \tau_B \frac{\bar{p}_{D_s}}{8\pi m_B^2} |g_{BDD_s}(m^2_B)|^2.$$  \hspace{1cm} (3)

By combining Eqs. (1), (2) and (3), we find that the BRs for $B \to \Xi_c \Lambda_c^+$ decays are related to those for $B \to DD^+_s$ by

$$BR(B \to \Xi_c \Lambda_c^+) = \frac{|\bar{p}_{\Lambda_c}| |\bar{p}_{D_s}|}{32\pi^2} \left(1 - \frac{(m_{\Xi_c} + m_{\Lambda_c})^2}{m_B^2}\right) |g_{DDB_cB_c}(m^2_B)|^2 BR(B \to DD^+_s), \hspace{1cm} (4)$$

where $m_{\Lambda_c(\Xi_c)}$ is the mass of $\Lambda_c(\Xi_c)$ baryon and $\bar{p}_{\Lambda_c}$ is the spatial momentum of $\Lambda_c$.

From Eq. (4), we clearly see that $BR(B \to \Xi_c \Lambda_c^+)$ can give a constraint on the parameter $g_{DDB_cB_c}(m^2_B)$. However, since the constraint is only suitable at the $m_B$ scale, it will not help us to understand three-body decays such as $B \to \Lambda_c^+ \Lambda_c^- K$ in which the involving invariant mass of the $\Lambda_c^+ \Lambda_c^-$ system is below the $m_B$ scale and the value varies between $2m_{\Lambda_c}$ and $m_B - m_K$. Moreover, the effects of FSIs should be strongly related to the momenta of final state particles, so that the contributions of energetic light baryons are much less important than those of slow heavy baryons. In sum, for giving a suitable effective strong coupling for $g_{DDB_cB_c}(k^2)$ which could be applied to various values of invariant mass, we will model the effective coupling in Lorentz covariant form. In the following, we show our way to determine the form of effective coupling. If we regard the scattering $DD \to B_cB_c$ as a t-channel process in which the intermediate particle is a doubly-charmed baryon, the scattering amplitude could be approximately described by

$$\frac{\bar{g}^2(k^2)}{m_B^2} \bar{u}_{B_c} \gamma_5 \bar{p}_D \frac{1}{\bar{p}_D - \bar{p}_{B_c} - m_{B_{cc}}} \gamma_5 \bar{p}_D v_{B_c} \approx \frac{\bar{g}^2(k^2)}{m_{B_{cc}}} \bar{u}_{B_c} v_{B_c},$$

where the fields of D meson have been neglected, $\bar{g}(k^2)$ expresses the coupling $D - B_c - B_{cc}$, $m_{B_{cc}}$ denotes the mass of exchanged doubly-charmed baryon and the small contributions from $p_D - p_{B_c}$ are neglected. Comparing to charmless baryonic decays, we speculate that the dominance of FSIs is due to the particles in the doubly charmed baryonic decays carrying lower spatial momenta. In order to display the momentum-dependent strong coupling, we further parametrize the coupling to be $\bar{g}^2(k^2) = g_c^2 m_{B_{cc}}^3 / (|p_{B_{c1}} - p_{B_{c2}}|^4 + \tilde{m}^4)$ where $p_{B_{c1(2)}}$ denote the four momenta of charmed baryons, $g_c$ is a dimensionless coupling constant and $\tilde{m}$ is the effective mass, which is used to remove the singularity when $p_{B_{c1}} = p_{B_{c2}}$. Accordingly, we model the effective coupling for the inelastic scattering $DD \to B_cB_c$ to be

$$g_{DDB_cB_c}(k^2) = g_c^2 \frac{m_{B_{cc}}^3}{|p_{B_{c1}} - p_{B_{c2}}|^4 + \tilde{m}^4}. \hspace{1cm} (5)$$
Since the $m_{B_c}$ could be estimated by the masses of constituent quarks, the undetermined parameters actually are $g_c$ and $\tilde{m}$.

Similar to the decays $B \to \bar{\Xi}_c \Lambda^+_c [D_{D^0 \bar{D}^0}]$, we find that the production of two charmed baryons in three-body $B$ decays could be arisen from the the inelastic scattering such as $B \to \Lambda^+_c \Lambda^-_c [D^0 \bar{D}^0] K$. According to the Fig. 2(b), the decay amplitude for $B \to \Lambda^+_c \Lambda^-_c K$ could be written as

$$M(B \to \Lambda^+_c \Lambda^-_c K) = \bar{u}_{\Lambda_c} \int \frac{d^4q}{(2\pi)^4} \frac{g_{DDB_c}(k^2) g_{BKDDD}(k^2, \cos \theta)}{2m_D^2 + i\varepsilon (p_B - q)^2 - m_D^2 + i\varepsilon} v_{\Lambda_c},$$

$$= C_I g_{DDB_c}(k^2) g_{BKDDD}(k^2, \cos \theta) I_D(k^2) \bar{u}_{\Lambda_c} v_{\Lambda_c},$$

(6)

where $C_I$ denotes the factor from the isospin wave function of system, taking to be $1/\sqrt{2}$ here, and $g_{BKDDD}(k^2, \cos \theta)$ represents the decay amplitude for $B \to \bar{D}^0 D^0 K$ decays. As known that the decay amplitude is Lorentz invariant, for convenience, we will set the working frame in the rest frame of charmed baryon-pair. Hence, $k^2$ stands for the invariant mass of $\Lambda^+_c \Lambda^-_c$ and is a variable. Angle $\theta$ is the polar angle of $\Lambda^+_c$ with respect to the momentum direction of $K$ meson. Since $B \to \bar{D}^0 D^0 K$ are color-allowed processes, it is a good approximation to assume that the processes are dominated by factorizable effects. Thus, by employing the factorization assumption, the decay amplitude for $B \to \bar{D}^0 D^0 K$ could be written as

$$g_{BKDDD}(k^2, \cos \theta) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs} a_1^{\text{eff}} \langle DK | \bar{c} \gamma^\mu s | 0 \rangle \langle \bar{D} | \bar{b} \gamma_\mu c | B \rangle$$

(7)

where $a_1^{\text{eff}}$ is the effective Wilson coefficient and the transition matrix elements are defined as

$$\langle \bar{D}(p_D) | \bar{b} \gamma_\mu c | B(p_B) \rangle = \left( p_B + p_D \right)_\mu - \frac{m_B^2 - m_D^2}{q^2} q_\mu F_1(q^2) + \frac{m_B^2 - m_D^2}{q^2} q_\mu F_0(q^2),$$

$$\langle D(p_D) K(p_K) | \bar{c} \gamma_\mu s | 0 \rangle = \left( p_D - p_K \right)_\mu - \frac{m_D^2 - m_K^2}{Q^2} Q_\mu F_1^{DK}(Q^2)$$

$$+ \frac{m_D^2 - m_K^2}{Q^2} Q_\mu F_0^{DK}(Q^2)$$

(8)

with $q = p_B - p_D$ and $Q = p_D + p_K$. In order to obtain the information of $g_{BKDDD}$ on $k^2$ and $\theta$, we have to analyze the decays $B \to \bar{D}^0 D^0 K$ themselves. Since the production of doubly charmed mesons is similar to that of doubly charmed baryons, in the following, $k^2$ and $\theta$ will be regarded as the invariant mass of $\bar{D}^0 D^0$ and $\Lambda^+_c \Lambda^-_c$ systems and the relative angle between $p_{D^0 \Lambda^+_c}$ and $p_K$, respectively. For deriving the decay rates as a function of invariant mass $k^2$ and angle $\theta$, the coordinates in the $k^2$ rest frame are chosen as follows:
\[ p_{B(K)} = (E_{B(K)}, 0, 0, |\vec{p}|) \] with \( E_{B(K)} = (m_B^2 + k^2 - m_K^2)/2\sqrt{k^2} \) and \(|\vec{p}| = \sqrt{E_K^2 - m_K^2}\) and \( p_{X(\bar{X})} = (\sqrt{k^2/2}, \pm |\vec{p}_X| \sin \theta, 0, \pm |\vec{p}_D| \cos \theta)\) with \(|\vec{p}_X| = \sqrt{K^2/2 \cdot \sqrt{1 - (2m_X)^2/k^2}}\) in which \( X \) could be \( D^0 \) meson or \( \Lambda^+_c \) baryon. Based on these coordinates, it is obvious that \( q^2 \) and \( Q^2 \) are functions of \( k^2 \) and \( \cos \theta \). This is the reason why we set the coupling \( g_{BKDD} \) as a function of \( k^2 \) and \( \cos \theta \). Hence, by including the phase space for three-body decay and using the chosen coordinates, the differential decay rate for \( B \to D^0D^0K \) is expressed by

\[
\frac{d\Gamma(B \to \bar{D}^0D^0K)}{dk^2d\cos \theta} = \frac{|\vec{p}_K'|}{2^8\pi^3m_B^2}|g_{BKDD}(k^2, \cos \theta)|^2 \sqrt{1 - \frac{(2m_D)^2}{k^2}} \tag{9}
\]

with \(|\vec{p}_K'| = \sqrt{E_K^2 - m_K^2}\) and \( E_K' = (m_B^2 - k^2 + m_K^2)/2m_B\). In terms of Eq. (9), similarly, we can also obtain the differential decay rate for \( B \to \Lambda^+_c\Lambda^-_c K \) decays as

\[
\frac{d\Gamma(B \to \Lambda^+_c\Lambda^-_c K)}{dk^2d\cos \theta} = \frac{|\vec{p}_K'|}{2^7\pi^3m_B^2}|I_D(k^2)|^2 \left(1 - \frac{(2m_{\Lambda_c})^2}{k^2}\right)^{3/2} \times |g_{DDB\Lambda_c}(k^2, \cos \theta)|^2 |C_I g_{DDB\Lambda_c}(k^2)|^2 . \tag{10}
\]

From Eqs. (7), (9) and (10), we immediately know that if we can determine \( g_{BKDD}(k^2, \cos \theta) \) from \( B \to \bar{D}^0D^0K \) and \( g_{DDB\Lambda_c}(k^2) \) from \( B \to \bar{\Xi}_c\Lambda^+_c \), then in principle we can make predictions on the BRs of \( B \to \Lambda^+_c\Lambda^-_c K \).

In order to understand whether we can predict the BRs for three-body doubly charmed baryonic decays, we have to know how many unknown parameters can be controlled by theoretical calculations and parametrizations and constrained by experimental measurements. At first, we discuss the unknowns of Eq. (5) for \( g_{DDB\Lambda_c} \). As mentioned before, \( m_{\Lambda_c} \) could be estimated by the masses of constitute quarks. If we chose \( m_c = 1.6 \text{ GeV} \) and the light quark \( m_q = 0.3 \text{ GeV} \), then we get \( m_{\Lambda_c} = 3.5 \text{ GeV} \). The estimated value is close to the particle \( \Xi^+_c(3520) \), observed recently by SELEX at FNAL [15]. Therefore, there remain two uncertain parameters \( g_c \) and \( \tilde{m} \) for effective coupling \( g_{DDB\Lambda_c} \). However, the observation of \( B^+ \to \bar{\Xi}_c^0\Lambda^+_c \) only can constrain their ratio. Thus, without further assumption, one unknown will be left. Nevertheless, we still can set the value of \( \tilde{m} \) by a proper approach. As stated early, the usage of \( \tilde{m} \) is to avoid the suffering problem when the spatial momentum of charmed baryon in the \( k^2 \) rest frame is approaching vanishment. For two-body decays, we can easily know that the spatial momentum of \( \Lambda^+_c \) is \(|\vec{p}_{\Lambda^+_c}| = 1.15 \text{ GeV} \). Therefore, the results are insensitive to \( \tilde{m} \) if its value is less than 1.5 GeV. However, for three-body decays, the available range of spatial momentum of \( \Lambda^+_c \) is \(|\vec{p}_{\Lambda^+_c}| = [0, 0.71] \text{ GeV} \). Needless to say, the
value choice of \( \tilde{m} \) will influence on the BRs of \( B \to \Lambda_c^+ \Lambda_c^- K \) significantly. To get a balance in both kinds of decays, the \( \tilde{m} \) can be chosen so that its value will not affect the two-body decays and still can protect three-body from overestimation. Since \(|p_{B,c} - p_{B,c}| \approx 2|p_{B,c}|\) in both two-body and three-body decays, we find the better value for \( \tilde{m} \) is around 1.0 GeV which is similar to the magnitude of momentum in two-body decays. Hence, if we accept this value as input, \( B^+ \to \Xi_0^0 \Lambda_c^+ \) can directly give a bound on \( g_c \).

Next, we concentrate on the effective coupling of Eq. (7). From the equation, we see that the main unknowns are from the form factors \( F_{1(0)} \) and \( F_{DK}^{1(0)} \). We can use a proper QCD approach, such as perturbative QCD \([16]\), relativistic quark model \([17]\) and light-front quark model \([18]\) etc, to calculate the transition form factors \( F_{1(0)} \). According to the results of light-front quark model, the explicit expressions can be given by \([18]\)

\[
F_1(q^2) = \frac{0.67}{(1 - 1.25q^2/m_B^2 + 0.39(q^2/m_B^2)^2)}, \quad F_0(q^2) = \frac{0.67}{(1 - 0.65q^2/m_B^2)}. \tag{11}
\]

As for the form factors \( F_{DK}^{1(0)} \), since there exists no good method to estimate the time-like form factors, what we can do is as usual to parametrize them to be

\[
F_{DK}^{1}(Q^2) = \frac{F_{DK}^{1(0)}(Q^2)}{1 + a_{DK} Q^2/m_X^2}, \quad F_{DK}^{0}(Q^2) = \frac{F_{DK}^{0(0)}(Q^2)}{1 + Q^2/m_X^2}, \tag{12}
\]

where we set \( m_X = m_B - m_D \) as the maximum invariant mass of \( DK \) system. We note that the parametrizations in Eq. (12) do not display the behavior predicated by PQCD at the large \( Q^2 \), i.e. \( F_{DK} \to 1/(Q^2 \ln(Q^2/L^2)) \) \([20]\); nevertheless it could be a good approximation since the dominant contributions for \( B \to D^0 \bar{D}^0 K \) are close to the threshold region of the invariant mass in the \( DK \) system. In terms of the parametrizations, we see that two parameters are needed to be determined. However, so far only the BRs of \( B \to \bar{D}^0 \bar{D}^0 K \) are measured. Hence, we only can determine the allowed ratio of \( a_{DK} \) and \( F_{DK}^{1(0)} \) by the data of \( B \to \bar{D}^0 \bar{D}^0 K \). If experiments can provide the information on angular distribution, we can further fix the remaining unknown. Nevertheless, we can still obtain some information on \( F_{DK}^{1(0)} \) by considering the two-body decay \( B^0 \to D_s^- K^+ \). It is known that the decay is governed by annihilation topology \([21]\) and the BR is measured to be \((3.8 \pm 1.3) \times 10^{-5} \) \([12]\). In terms of factorization assumption, the decay amplitude for \( B^0 \to D_s^- K^+ \) can be expressed by

\[
M(B^0 \to D_s^- K^+) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^{*} a_{D_s} f_B \langle D_s^- K^+ | \bar{s} p_B c | 0 \rangle \\
= \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^{*} a_{D_s} f_B (m_{D_s}^2 - m_K^2) F_{DK}^{D_s K}(m_B^2)
\]
where $f_B$ is the decay constant of $B$ meson, $a_2^{\text{eff}}$ is the color suppressed effective Wilson coefficient and nonfactorizable effects have been included, and the time-like form factors defined in Eq. (8) are used. If we neglect the differences between $D_s$ and $D$ mesons, we can get the value of form factor $F_{0}^{DK}$ at $m_B$ scale. By taking $V_{cb} = 0.041$, $f_B = 0.2$ GeV, $m_{D_s(K)} = 1.969(0.493)$ GeV, $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$, $\tau_{B^0} = 1.55 \times 10^{-12}$ s [12], $a_2^{\text{eff}} = 0.275$ [22], and $BR(B^0 \rightarrow D_s^- K^+) = 3.5 \times 10^{-5}$, we get $F_{0}^{DK}(m_B^2) \approx F_{0}^{DK}(m_B^2) = 1.0$. Relating the extracted value to the $F_{0}^{DK}(Q^2)$ parametrized in Eq. [12], we immediately obtain $F_{0}^{DK}(0) = 3.41$. If we adopt this value as input, the BRs of $B \rightarrow \bar{D}^0 D^0 K$ can fix the parameter $a_1^{DK}$. Consequently, by including proper measurements, we can determine the introduced parameters with errors from experiments.

In order to calculate the numerical values, besides the values taken before, we set other inputs as follows: $m_{Z_0} = 2.471$ GeV, $m_{\Lambda_c} = 2.285$ GeV, $m_{D^0} = 1.865$ GeV, $\tau_{B^+} = 1.67 \times 10^{-12}$ s, $BR(B^+ \rightarrow \bar{D}^0 D^+) = (1.3 \pm 0.4)\%$, $BR(B^+ \rightarrow \bar{D}^0 D^0 K) = (1.9 \pm 0.4) \times 10^{-3}$ [12] and $a_1^{\text{eff}} = 0.986$ [22]. Although in general it is not necessary to regard $m_{B_{cs}} = 3.5$ GeV, $\bar{m} = 1.0$ GeV and $F_{DK}(0) = 3.41$ as the inputs, however, it will be interesting to see whether by using these values, the predicted BRs of $B \rightarrow \Lambda_c^+ \Lambda_c^- K$ are consistent with the observations. In addition, we find that actually the BRs of $B \rightarrow \bar{D}^0 D^0 K$ are insensitive to the value of $a_1^{DK}$. That is, in our parametrizations, it is a good approximation to take $a_1^{DK} = 1.0$. Accordingly, we have $BR(B^+ \rightarrow \bar{D}^0 D^0 K) = 2.1 \times 10^{-3}$ and the differential BR as a function of invariant mass $M(\bar{D}^0 D^0)$ for $B^+ \rightarrow \bar{D}^0 D^0 K$ is presented in Fig. 3(a). To demonstrate the correlation between $g_c$ and $BR(B^+ \rightarrow \Lambda_c^+ \Lambda_c^- K)$, we further set $g_c = (3.1, 3.3, 3.5)$ so that the $BR(B^+ \rightarrow \Xi_c^0 \Lambda_c^+) = (3.4, 4.4, 5.6) \times 10^{-3}$ and $BR(B^+ \rightarrow \Lambda_c^+ \Lambda_c^- K) = (6.6, 8.5, 10.9) \times 10^{-4}$; then, we display the corresponding differential BR for $B^+ \rightarrow \Lambda_c^+ \Lambda_c^- K$ as a function of invariant mass $M(\Lambda_c^+ \Lambda_c^-)$ with dash-dotted, solid and dashed lines, respectively, in Fig. 3(b). From these results, we see clearly that the observations of $B \rightarrow \Xi_c \Lambda_c^+$ and $B \rightarrow \Lambda_c^+ \Lambda_c^- K$ could be understood by the similar FSIs. Furthermore, to illustrate the influence of the introduced parameters $F_{DK}(0)$, $a_{1}^{DK}$, $\bar{m}$ and $g_c$, we free these parameters and let the data decide the allowed ranges. The results are shown in Fig. 4. By Fig. 4(a), we confirm that the BRs of doubly charmed mesonic decays are not sensitive to $a_1^{DK}$. Taking $1\sigma$ error on $BR(B^+ \rightarrow \bar{D}^0 D^0 K)$, the range for the value of $F_{DK}(0)$ is $(3, 3.9)$. With the bound $(2.4, 5.3) \times 10^{-3}$ on $BR(B^+ \rightarrow \Xi_c^0 \Lambda_c^+)$ [10], from Fig. 4 we find that $BR(B^+ \rightarrow \Xi_c^0 \Lambda_c^+)$ is sensitive to $g_c$ but insensitive to $\bar{m}$ and we also see that
FIG. 3: The differential BRs for (a) $B^+ \to \bar{D}^0 D^0 K^+$ and (b) $B^+ \to \Lambda_c^+ \Lambda_c^- K^+$ as a function of invariant mass $M(D^0 \bar{D}^0)$ and $M(\Lambda_c^+ \Lambda_c^-)$, respectively, where we have set $F^{DK}(0) = 3.41$, $a^{DK} = 1.0$, $m_{B_{cc}} = 3.5$ GeV and $\tilde{m} = 1.0$ GeV. The dash-dotted, sold and dashed lines represent the contributions when $g_c = (3.1, 3.3, 3.5)$ and their corresponding BRs for $B^+ \to \Lambda_c^+ \Lambda_c^- K^+$ are $(6.6, 8.5, 10.9) \times 10^{-4}$, respectively.

FIG. 4: The contour plots for the correlations (a) between $(F^{DK}(0), a^{DK})$ and $BR(B^+ \to \bar{D}^0 D^0 K^+)$ and (b) between $(g_c, \tilde{m})$ and $BR(B^+ \to \Xi_c^0 \Lambda_c^+)$. The values (in units of $10^{-3}$) in the figures (a) and (b) denote the low and upper bounds of current data on BRs. (c) the possible BR for $B^+ \to \Lambda_c^+ \Lambda_c^- K^+$ when the involving parameters are satisfied with the bounds of figures (a) and (b), where the values (in units of $10^{-4}$) shown in the figure come from the data within $1\sigma$ statistical and systematic errors.

A wide range of $BR(B^+ \to \Lambda_c^+ \Lambda_c^- K^+)$ is allowed, including the data shown. We note that the values of BR (in units of $10^{-4}$) shown in the figure (c) only contain $1\sigma$ statistical and systemic errors. By the figure, we also find that the three-body doubly charmed baryonic decays are much more sensitive to $g_c$ and $\tilde{m}$, in particular $\tilde{m}$. Clearly, based on our modeling
of Eq. (5), the parametrizations of Eq. (12) and the limited information on data, in general we cannot make concrete predictions on the decays $B \rightarrow \Lambda_c^+ \Lambda_c^- K$.

Although at present we cannot give precise predictions on doubly charmed baryonic decays, however, in terms of the proposed mechanisms we can have some implications on other unobserved processes such as $B \rightarrow \Xi_c^0 \Sigma_c$ and $B \rightarrow \Lambda_c^+ \Sigma_c^- K$ etc. Since $\Sigma_c^+$ and $\Lambda_c^+$ have the same quark contents except the former is isospin $I = 1$ while the latter is $I = 0$, by utilizing Eqs. (1) and (5) and introducing a corrected factor $C_I = \sqrt{2/3}$ for isospin, we can easily obtain the BRs for $B \rightarrow \Xi_c^0 \Sigma_c$.

With the values of $g_c$ and $\tilde{m}$ which are satisfied with the measurement of $B^+ \rightarrow \Xi_c^0 \Lambda_c^+$, the estimated $BR(B^+ \rightarrow \Xi_c^0 \Lambda_c^+)$ is presented in Fig. 5(a) in which we have set $m_{\Sigma_c} = 2.452$ GeV and the values in the figure denote the available BR in units of $10^{-3}$. It is clear that although the phase space of the $\Xi_c^0 \Sigma_c^+$ mode is less than that of the $\Xi_c^0 \Lambda_c^+$ mode, by the modeling of Eq. (5), we predict that $B^+ \rightarrow \Xi_c^0 \Sigma_c^+$ and $B^+ \rightarrow \Xi_c^0 \Lambda_c^+$ have similar BRs, even the former could be larger than the latter. Similarly, in terms of Eq. (10) with the values of the parameters satisfied with the observation of $B^+ \rightarrow \Lambda_c^+ \Lambda_c^- K^+$ within $1\sigma$ error, the estimated $BR(B^+ \rightarrow \Lambda_c^+ \Sigma_c^- K^+)$ is displayed in Fig. 5(b) where the values in the figure are the available BR in units of $10^{-4}$. From the figure, we see that the BR of $B^+ \rightarrow \Lambda_c^+ \Sigma_c^- K^+$ could be as large as $10^{-4}$. If we extend the FSIs to the processes associated with the creation of the strange quark-antiquark pair, i.e. $s\bar{s}$ pair instead of $d\bar{d}$ pair of Fig. 1(a), and assume that the relevant strong couplings are the same, interestingly we obtain $BR(B^+ \rightarrow \Omega_c \Xi_c^+) \approx 5.5\%$. Although $\Omega_c \Xi_c^+$ has much less allowed phase space,
due to the enhancement of Eq. (5), we see that the BR for $B^+ \to \bar{\Omega}_c \Xi_c^+$ is not suppressed. However, since we have assumed that the effective couplings of FSIs for the production of doubly charmed baryons are the same, one cannot take the result of $B^+ \to \bar{\Omega}_c \Xi_c^+$ seriously. After all, the FSIs for real situations could be different in different processes. What we have displayed is that our FSIs could enhance the BR of process which the corresponding phase space is suppressed.

Finally, we make some remarks on other possible FSIs for three-body modes, such as $B \to \bar{\Xi}_c \Lambda_c \to \Lambda_c^+ \Lambda_c^- K$ and $B \to \bar{D} \bar{D}_s^+ \to \Lambda_c^+ \Lambda_c^- K$. As known that the $BR(B^+ \to \Lambda_c^+ \Lambda_c^- K^+)$ is only few factors smaller than $BR(B^+ \to \bar{\Xi}_c \Lambda_c^+)$, if we regard that the FSIs are inelastic scattering, by the loop and phase space suppressions, we expect that the contributions through $B \to \bar{\Xi}_c \Lambda_c \to \Lambda_c^+ \Lambda_c^- K$ should be small. As for the decay $B^+ \to \bar{D} \bar{D}_s^+ \to \Lambda_c^+ \Lambda_c^- K^+$ shown in Fig. 6(a), if its contribution is significant, one can speculate that the same FSIs could also arise from the doubly charmed mesonic decays such as $B^+ \to \bar{D} \bar{D} K^+$, shown in Fig. 6(b). From Fig. 6(a) and (b), we conjecture that the former should be smaller than the latter due to one more quark pair being produced. Since we have assumed that the dominant effects for $B^+ \to \bar{D} \bar{D} K^+$ are factorizable contributions, described by Eq. (7), if the decay chain $B^+ \to \bar{D} \bar{D}_s^+ \to \Lambda_c^+ \Lambda_c^- K^+$ is not negligible, in our analysis it could be regarded as a subleading effect.

**FIG. 6:** Flavor diagrams (a) for $B^+ \to \bar{D} \bar{D}_s^+ \to \Lambda_c^+ \Lambda_c^- K^+$ decay and (b) for $B^+ \to \bar{D} \bar{D}_s^+ \to \bar{D} \bar{D} K^+$ decay.

In summary, we have studied two-body and three-body doubly charmed baryonic $B$ decays. We find that the observed processes could be produced by FSIs $B^+ \to \bar{\Xi}_c \Lambda_c^+ [\bar{D} \bar{D}_s^+]$ and $B \to \Lambda_c^+ \Lambda_c^- [\bar{D} \bar{D}] K$. In terms of proper modeling for the effective strong coupling and factorization assumption, we find that with the constrained values of the parameters, the estimated results could be compatible with the current data. In addition, we also show the
implications of FSIs on $B^+ \to \Xi^0_c \Sigma^+_c$ and $B^+ \to \Lambda^+_c \Sigma^+_c K^+$ and get that their BRs could be as large as $BR(B^+ \to \Xi^0_c \Lambda^+_c)$ and $10^{-4}$, respectively. In addition, extending the FSIs to the processes associated with the creation of $s\bar{s}$ pair, $BR(B^+ \to \bar{\Omega}_c \Xi^+_c)$ at percent level is achievable.

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