Class-based Rough Approximation with Dominance Principle

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Abstract

Dominance-based Rough Set Approach (DRSA), as the extension of Pawlak’s Rough Set theory, is effective and fundamentally important in Multiple Criteria Decision Analysis (MCDA). In previous DRSA models, the definitions of the upper and lower approximations are preserving the class unions rather than the singleton class. In this paper, we propose a new Class-based Rough Approximation with respect to a series of previous DRSA models, including Classical DRSA model, VC-DRSA model and VP-DRSA model. In addition, the new class-based reducts are investigated.

1. Introduction

Multiple Criteria Decision Analysis (MCDA) aims at providing the decision maker (DM) a knowledge recommendation while considering the finite objects evaluated from multiple viewpoints (known as criteria). Roy [9] considered four problems in MCDA, including criteria analysis, choice, ranking, sorting. The first one is the essential procedure for optimization of decision information and the latter three ones can produce specific decision outcomes.

Apart from several valid and classical MCDA approaches (see the state-of-the-art survey in [3]), the non-classical methods and techniques (like [1][2]) are significant since it attempts to address the risk and uncertainty of MCDA catering to the real world. Classical Rough Set Approach (CRSA for short) initially proposed by Pawlak (see [8]) is an effective mathematical tool for decision analysis. But, it fails to deal with the preference-ordered data in MCDA. In this reason, Dominance-based Rough Set Approach (DRSA for short) was generated by Greco and his colleagues [5][10]. Unlike the CRSA which makes use of the indiscernibility relations for construction of knowledge granular, DRSA considers the dominance relations of these preference-ordered data in given decision table.

The target by using DRSA is to induce the decision rule as classifier for providing the suitable assignment of both learning objects (from given decision table) and new objects. Recently, the classical DRSA model had been extended to VC-DRSA [4], VP-DRSA [6], etc.

In all previous DRSA models, the upper and lower approximations are defined in consideration of the union of decision class (i.e. upward union $C^U$ and downward union $C^D$). We call them as union-based rough approximation. In this paper, we attempt to investigate the issue: whether one singleton decision class can be used to define the upper and lower approximation in a series of DRSA models. To this end, we firstly analyze the partition of objects preserving one particular decision class, and provide a new Three Region Model (TRM). Then, we develop the so-called class-based rough approximation in a series of previous DRSA models, including the classical DRSA model, VC-DRSA model and VP-DRSA model. Finally, inspired by Inuiguchi’s initial works [6][7], the class-based criteria reduction is also studied.

This paper is organized as follows: The next section briefly reviews the basic principles of DRSA theory. Section 3 studies the class-based rough approximation in a series of DRSA models. Section 4 investigates the class-based criteria reduction. Finally, we draw the conclusion in section 5.

2. Background

In this section, we concisely revisit the basic theory of DRSA. Despite the various problem domains regarding MCDA, three elementary factors are usually involved, including objects, criteria and DM(s). These factors can generally be organized as decision table with columns of criteria and rows of objects. Formally, a decision table is the 4-tuple $S=(U,Q,V,f)$, which includes (1) a finite set of objects denoted by $U$, $x \in U = \{x_1,...,x_n\}$; (2) a finite set of criteria is denoted by
\( Q = C \cup D \), where condition criteria set \( C \neq \emptyset \), decision criteria set \( D \neq \emptyset \) (usually the singleton set \( D=\{d\} \)), and \( q \in Q = \{q_1, \ldots, q_n\} \); (3) the domain of criterion \( q \) denoted by \( V_q \), where \( V = \bigcup_{q \in Q} V_q \); (4) information function denoted by \( f_q(x) : U \times Q \to V \), where \( f_q(x) \in V_q \) for each \( q \in Q \), \( x \in U \).

The objective sets of rough approximations are the upward or downward unions of predefined decision classes (we call union-based rough approximations). Suppose the decision criterion \( \{d\} \) makes a partition of \( U \) into a finite number of classes \( CL=\{C_l, t=1, \ldots, l\} \). We assume that \( C_{t, u} \) is superior to \( C_{t, v} \) according to DM’s preference. Each object \( x \in U \) belongs to one and only one class \( C_l \in CL \). The upward and downward unions of classes are represented as:

\[
C_{l}\uparrow = \bigcup_{C_l} C_l \quad \text{and} \quad C_{l}\downarrow = \bigcup_{C_l} C_l \quad \text{where} \quad t=1, \ldots, l.
\]

Then, the following operational laws are valid:

\[
C_{l}\uparrow = C_l; \quad C_{l}\uparrow \cap C_{l}\uparrow = C_l; \quad C_{l}\uparrow = U - C_{l}\downarrow; \quad C_{l}\downarrow = U - C_{l}\uparrow; \quad C_{l}\downarrow = C_{l}\downarrow \cup \emptyset;
\]

The knowledge granules in DRSA theory are dominance cones. If two decision values are with the dominance relation like \( f_q(x) \geq f_q(y) \) for every considered criterion \( q \in P \subseteq C \), we say \( x \) dominates \( y \), denoted by \( _d x \cdot y \). The dominance relation is reflexive and transitive. With this in mind, the dominance cone of object \( x \) can be represented by:

\[
D_x^P = \{y \in U : _d x \cdot y\} \quad \text{and} \quad D_y^P = \{x \in U : _d x \cdot y\}.
\]

The key concept in DRSA theory is the Dominance Principle: if the decision value of object \( x \) is no worse than that of object \( y \), and \( x \) is getting assigned to a decision class \( C_l \) no worse than that of object \( y \), which is \( y \) is dominating \( x \) on \( D \). Objects satisfying the dominance principle are called consistent, and also, objects violating the dominance principle are called inconsistent. A decision table which contains inconsistent object is called inconsistency table. According to such dominance principle, the definition of rough approximations is given as follows.

- P-lower approximation of class union \( C_{l}\uparrow \) and \( C_{l}\downarrow \), denoted by \( P(C_{l}\uparrow) \) and \( P(C_{l}\downarrow) \) respectively, are represented as:

\[
P(C_{l}\uparrow) = \{x \in U : D_x^P \subseteq C_{l}\uparrow\}; \quad P(C_{l}\downarrow) = \{x \in U : D_x^P \subseteq C_{l}\downarrow\}.
\]

- P-upper approximation of class union \( C_{l}\uparrow \) and \( C_{l}\downarrow \), denoted by \( P(C_{l}\uparrow) \) and \( P(C_{l}\downarrow) \) respectively, are represented as:

\[
C_{l}\uparrow \cap C_{l}\downarrow = \emptyset; \quad P(C_{l}\downarrow) = \{x \in U : D_x^P \cap C_{l}\downarrow \neq \emptyset\}.
\]

Rough boundary region of class union \( C_{l}\uparrow \) and \( C_{l}\downarrow \), denoted by \( B_{n, l}(C_{l}\uparrow) \) and \( B_{n, l}(C_{l}\downarrow) \) respectively, are represented as:

\[
B_{n, l}(C_{l}\uparrow) = P(C_{l}\uparrow) \setminus P(C_{l}\downarrow); \quad B_{n, l}(C_{l}\downarrow) = P(C_{l}\downarrow) \setminus P(C_{l}\uparrow).
\]

Obviously, we have the properties:

\[
B_{n, l}(C_{l}\uparrow) = B_{n, l}(C_{l}\downarrow) = P(C_{l}\uparrow) \cap P(C_{l}\downarrow).
\]

In addition, the following properties are valid:

\[
P(C_{l}\uparrow) \subseteq C_{l}\uparrow \subseteq P(C_{l}\downarrow); \quad P(C_{l}\downarrow) \subseteq C_{l}\downarrow \subseteq P(C_{l}\uparrow); \quad P(C_{l}\downarrow) = U - P(C_{l}\uparrow); \quad P(C_{l}\uparrow) = U - P(C_{l}\downarrow);
\]

\[
P(C_{l}\downarrow) = U - P(C_{l}\uparrow); \quad P(C_{l}\uparrow) = U - P(C_{l}\downarrow).
\]

The definitions of the classical DRSA model are based on the strict dominance principle (as shown in above). Inspired by the Variable Precision Rough Set [11], which is the extension of CRSA via relaxation of strict indiscernibility relation, Greco et al. [10] provided the VC-DRSA model. This model accepts a limited number of inconsistency objects controlled by a predefined threshold called consistency level.

The lower approximations of VC-DRSA model can be represented as follows. For any \( P \subseteq C \), we have:

\[
P(C_{l}\uparrow)^P = \{x \in C_{l}\uparrow \mid \frac{|D_x^P \cap C_{l}\uparrow|}{|D_x^P|} \geq l\} \quad \text{and} \quad P(C_{l}\downarrow)^P = \{x \in C_{l}\downarrow \mid \frac{|D_x^P \cap C_{l}\downarrow|}{|D_x^P|} \geq l\}
\]

where \( l \) is called consistency level, which means that object \( x \) belongs to \( C_{l}\uparrow \) (or \( C_{l}\downarrow \)) with no ambiguity at level \( l \in (0,1] \).

Based on the definitions of lower approximation, we can further obtain the definitions of the upper approximations and the rough boundary regions as:

\[
P(C_{l}\uparrow) = U - P(C_{l}\downarrow)^P; \quad B_{n, l}(C_{l}\uparrow) = P(C_{l}\uparrow) \setminus P(C_{l}\downarrow)^P; \quad P(C_{l}\downarrow) = U - P(C_{l}\uparrow)^P; \quad B_{n, l}(C_{l}\downarrow) = P(C_{l}\downarrow) \setminus P(C_{l}\uparrow)^P.
\]

3. Class-based rough approximation

3.1. Class-based classical DRSA model

Classical DRSA model can be regarded as a special case of VC-DRSA model with the consistency level fulfilling \( l_1 = l_2 = 1 \) (the strict dominance principle), while,

\[
\frac{|D_x^P \cap C_{l}\uparrow|}{|D_x^P|} \geq l_1 \quad \text{and} \quad \frac{|D_x^P \cap C_{l}\downarrow|}{|D_x^P|} \geq l_1.
\]
considering the decision class $C_l$ and its two adjacent classes $C_{l-1}$ and $C_{l+1}$, the constraint conditions are given in Table 1. Each consistency level $l_i$ (or $l_j$) divides the entire objects into two regions: Low region and High region. These regions are constrained by different conditions. For the class $C_l$, the constraint conditions are (A), (B), (C), (D). For its adjacent classes $C_{l-1}$ and $C_{l+1}$, the constraint conditions are $(A')$, $(B')$, $(C')$, $(D')$.

Considering the class $C_l$, object $x \in C_l$ must be assigned to one of the four regions, as Region I, II, III, IV. Each region is constrained by two conditions which are defined by the dominance cores: $D^*_j(x)$ and $D^*_k(x)$, respectively. The details are shown in Table 2. And, Fig. 1 illustrates the partition of $U$ at consistency levels $l_i$ and $l_j$ in all decision classes $C_L = \{C_l | t = 1, ..., T\}$.

Based on such observations, we consider three regions in class-based classical DRSA model with respect to the predefined $C_l, (t = 2, ..., T - 1)$:

- **Low boundary region**, denoted by $P_l(C_l)$:
  $P_l(C_l) = \{x \in C_l : D^*_l(x) \subseteq C^*_l, \cap C^*_l \neq \emptyset\}$

- **Precise classification region**, denoted by $P^*(C_l)$:
  $P^*(C_l) = \{x \in C_l : D^*_l(x) \subseteq C^*_l \cap \cap C^*_l\}$

- **High boundary region**, denoted by $P_h(C_l)$:
  $P_h(C_l) = \{x \in C_l : D^*_l(x) \subseteq C^*_l \cup C^*_l \neq \emptyset\}$

Particularly, there are just $P_l(C_l)$ and $P^*(C_l)$ for class $C_l$, and just $P_l(C_l)$ and $P(C_l)$ for class $C_{l-1}$.

Corresponding to the Fig. 1, we have the following assertions:

- Region $P_l(C_l)$ consists of Region I and IV.
- Region $P(C_l)$ consists of Region II.

![Figure 1. The decision class $C_l$ as the partition of $U$ in DRSA models](image-url)
• Region $P^c_i(Cl_t)$ consists of Region III and IV.

We call the above definitions as Three Region Model (TRM).

Furthermore, if there are only two decision classes in a given decision table (i.e., pairwise comparison table, profit or non-profit, right or wrong), the TRM can be represented as follows:

**Definition (TRM in two-grade decision table)**

According to the given decision table, the decision criteria $d_{i}$ makes a partition of $U$ into two classes $S$ and $S'$ (suppose $S$ is superior to $S'$ in DM's preference).

And each $x \in U$ belongs to one and only one of such two predefined classes. The two-grade class-based rough approximations are represented as follows.

- Precise classification region of class $S$:
  $\text{P}(S) = \{x \in S : d_{i}(x) \subseteq S\}$

- Low boundary region of class $S$:
  $P_{l}(S) = \{x \in S : d_{i}(x) \subseteq S\}$

- High boundary region of class $S'$:
  $P^{h}(S') = \{x \in S' : d_{i}(x) \subseteq S\}$

- Precise classification region of class $S'$:
  $\text{P}(S') = \{x \in S' : d_{i}(x) \subseteq S\}$

And also, we can obtain following properties, which can be easily proved:

$P_{l}(S) = S - \text{P}(S)$; $P^{h}(S') = S' - \text{P}(S')$.

Next, we investigate the relationship between the definitions of the union-based and the class-based rough approximations. Considering the decision class $Cl_{t} \in Cl$ ($t = 2, \ldots, l-1$) and its adjacent classes $Cl_{u}$ and $Cl_{v}$, the following properties are valid:

$P(Cl_{t}) \cap P(Cl_{u}) = \{x \in Cl_{t} \cap P(Cl_{u}), d_{i}(x) \subseteq Cl_{u}\}$ and $D_{v}(x) \subseteq Cl_{v}$ = $\text{P}(Cl_{t})$

$P(Cl_{t}) \cap P(Cl_{v}) = \{x \in Cl_{t} \cap P(Cl_{v}), d_{i}(x) \subseteq Cl_{v}\}$ = $P^{h}(Cl_{t})$

$P(Cl_{t}) \cap Cl_{t} = \{x \in Cl_{t} : d_{i}(x) \subseteq Cl_{t}\}$ = $P_{l}(Cl_{t})$

$B_{u}(Cl_{t}) \cap Cl_{t} = \{x \in Cl_{t} : d_{i}(x) \subseteq Cl_{t}\}$ = $P_{l}(Cl_{t})$

$B_{v}(Cl_{t}) \cap Cl_{t} = \{x \in Cl_{t} : d_{i}(x) \subseteq Cl_{t}\}$ = $P^{h}(Cl_{t})$

$B_{u}(Cl_{v}) \cap Cl_{v} = B_{u}(Cl_{t}) \cap Cl_{v} = P_{l}(Cl_{v})$

$B_{v}(Cl_{v}) \cap Cl_{v} = B_{v}(Cl_{t}) \cap Cl_{v} = P^{h}(Cl_{v})$

$\bigcup_{t=v}^{\infty} (Cl_{t} - P_{l}(Cl_{t})) \subseteq P^{h}(Cl_{t})$

$\bigcap_{t=v}^{\infty} (Cl_{t} - P^{h}(Cl_{t})) \subseteq P_{l}(Cl_{t})$

### 3.2. Class-based VC-DRSA model

In this section, we investigate the TRM in VC-DRSA model. For any $P \subseteq U$ we say that $x \in U$ belongs to $Cl_{i}$ at consistency level $l_i \in (0,1)$, and $x \in U$ belongs to $Cl_{i}$ at consistency level $l_i \in (0,1)$. The concept of lower approximations at some consistency levels $l_i$ and $l_i$ are formally presented as:

$P^{l}(Cl_{i}) = \{x \in Cl_{i} : \frac{|D_{i}(x) \cap Cl_{i}|}{|D_{i}(x)|} \geq l_i\}$, $t = 1, \ldots, l$

$P^{h}(Cl_{i}) = \{x \in Cl_{i} : \frac{|D_{i}(x) \cap Cl_{i}|}{|D_{i}(x)|} \geq l_i\}$, $t = 1, \ldots, l$

Then, the TRM preserving the predefined class $Cl_{i}$ ($t = 2, \ldots, l-1$) can be presented as:

- **Low boundary region $P^{l}(Cl_{i})$**:
  $P^{l}(Cl_{i}) = B_{u}(Cl_{i}) \cap Cl_{i} = \{x \in Cl_{i} : Cl_{i} \subseteq Cl_{i}, \frac{D_{i}(x) \cap Cl_{i}}{|D_{i}(x)|} \leq l_i\}$

- **Precision classification region $P^{h}(Cl_{i})$**:
  $P^{h}(Cl_{i}) = \{x \in Cl_{i} : Cl_{i} \subseteq Cl_{i}, \frac{D_{i}(x) \cap Cl_{i}}{|D_{i}(x)|} \leq l_i\}$

- **High boundary region $P^{h}(Cl_{i})$**:
  $P^{h}(Cl_{i}) = \{x \in Cl_{i} : Cl_{i} \subseteq Cl_{i}, \frac{D_{i}(x) \cap Cl_{i}}{|D_{i}(x)|} \leq l_i\}$

Particularly, there are just $P^{l}(Cl_{i})$ and $P^{h}(Cl_{i})$ for class $Cl_{i}$ and just $P^{l}(Cl_{i})$ and $P^{h}(Cl_{i})$ for class $Cl_{i}$.

### 3.3. Class-based VP-DRSA model

Inuiguchi et al. [6] introduce the VP-DRSA model defined as follows: For any $P \subseteq C$, we say that $x \in U$ belongs to $Cl_{i}$ at precision level $l_i \in (0,1)$, and $x \in U$ belongs to $Cl_{i}$ at precision level $l_i \in (0,1)$. The concept of lower approximations at some precision levels $l_i$ and $l_i$ are formally presented as:

$P^{l}(Cl_{i}) = \{x \in Cl_{i} : \frac{|D_{i}(x) \cap Cl_{i}|}{|D_{i}(x)| + |D_{i}(x) \cap Cl_{i}|} \geq l_i\}$, $t = 1, \ldots, l$

$P^{h}(Cl_{i}) = \{x \in Cl_{i} : \frac{|D_{i}(x) \cap Cl_{i}|}{|D_{i}(x)| + |D_{i}(x) \cap Cl_{i}|} \geq l_i\}$, $t = 1, \ldots, l$

Particularly, when $D_{i}(x) \subseteq Cl_{i}$, we have $D_{i}(x) \cap Cl_{i} = \emptyset$, and $l_i = 1$. Accordingly, $P^{l}(Cl_{i})$ becomes DRSA lower approximation $P(Cl_{i})$. The same situation is happened in $P^{h}(Cl_{i})$.

The TRM with respect to the predefined class $Cl_{i}$ ($t = 2, \ldots, l-1$) can then be presented as follows:

- **Low boundary region $P^{l}(Cl_{i})$**:
  $P^{l}(Cl_{i}) = \{x \in Cl_{i} : Cl_{i} \subseteq Cl_{i}, \frac{D_{i}(x) \cap Cl_{i}}{|D_{i}(x)| + |D_{i}(x) \cap Cl_{i}|} \leq l_i\}$

- **Precision classification region $P^{h}(Cl_{i})$**:

\[
\begin{align*}
\mathcal{P}^{\alpha}(Cl_l) &= \{x \in Cl_l : \frac{|D_x^l(x) \cap Cl_l|}{|D_x^l(x) \cap Cl_l| + |D_x^l(x) \cap Cl_{l'}|} \geq l_l, \frac{|D_x^l(x) \cap Cl_{l'}|}{|D_x^l(x) \cap Cl_l| + |D_x^l(x) \cap Cl_{l'}|} \leq l_{l'} \} \\
\text{High boundary region } &\mathcal{P}^{\beta}(Cl_l) : \\
\mathcal{P}^{\beta}(Cl_l) &= \{x \in Cl_l : \frac{|D_x^l(x) \cap Cl_l|}{|D_x^l(x) \cap Cl_l| + |D_x^l(x) \cap Cl_{l'}|} < l_l \}
\end{align*}
\]

- High boundary region \( \mathcal{P}^{\beta}(Cl_l) \):

\[
\mathcal{P}^{\beta}(Cl_l) = \{x \in Cl_l : \frac{|D_x^l(x) \cap Cl_l|}{|D_x^l(x) \cap Cl_l| + |D_x^l(x) \cap Cl_{l'}|} < l_l \}
\]

Particularly, there are just \( \mathcal{P}^{\beta}(Cl_l) \) and \( \mathcal{P}^{\alpha}(Cl_l) \) for class \( Cl_l \) and just \( \mathcal{P}^{\beta}(Cl_l) \) for class \( Cl_{l'} \).

### 3.4. A discussion

Let us remark the two extensions of classical DRSA model: VC-DRSA model and VP-DRSA model. We firstly take the consistency and precision in class union \( Cl_l \) as example. In VC-DRSA model, consistency \( \alpha \) can be defined by:

\[
\alpha = \frac{|D_x^l(x) \cap Cl_l|}{|D_x^l(x) \cap Cl_l| + |D_x^l(x) \cap Cl_{l'}|}.
\]

If \( \alpha = 1 \) is satisfied, we have \( |D_x^l(x) \cap Cl_l| = 0 \). Then, we obtain \( D_x^l(x) \subseteq Cl_l \), which abides by the strict dominance principle of classical DRSA model. In VP-DRSA model, precision \( \beta \) can be defined by:

\[
\beta = \frac{|D_x^l(x) \cap Cl_l|}{|D_x^l(x) \cap Cl_l| + |D_x^l(x) \cap Cl_{l'}|}.
\]

Similarly, if \( \beta = 1 \) is satisfied, we have \( |D_x^l(x) \cap Cl_l| = 0 \). Then, we obtain \( D_x^l(x) \subseteq Cl_l \), which abides by the strict dominance principle of classical DRSA model.

Comparing the definition of consistency \( \alpha \) with that of precision \( \beta \), the only difference is shown as followings:

1. \( \alpha \) is related to dominance cone \( D_x^l(x) \);
2. \( \beta \) is related to dominance cone \( D_x^l(x) \) and \( D_x^l(x) \).

From the viewpoint of class-based rough approximation, we remark that the VP-DRSA model is focused on the Low and High boundary regions of TRM. More specifically, regarding the class \( Cl_l \), precision degree \( l_l \) is based on the investigation of objects \( x \in \mathcal{P}^{\alpha}(Cl_l) \). Similarly, precision degree \( l_{l'} \) is derived from the exploitation of assignment information of boundary region: \( x \in \mathcal{P}^{\beta}(Cl_l) \). Therefore, for VP-DRSA, we have the following assertions:

1. For each object \( x \in Cl_l \), the real value \( \beta \) represents to what extent, object \( x \) belongs to the High boundary region: \( \mathcal{P}^{\beta}(Cl_l) \), where,
   \[
   \beta = \frac{|D_x^l(x) \cap Cl_l|}{|D_x^l(x) \cap Cl_l| + |D_x^l(x) \cap Cl_{l'}|}.
   \]

2. For each object \( x \in Cl_l \), the real value \( \beta \) represents to what extent, object \( x \) belongs to the Low boundary region: \( \mathcal{P}^{\beta}(Cl_l) \), where,
   \[
   \beta = \frac{|D_x^l(x) \cap Cl_l|}{|D_x^l(x) \cap Cl_l| + |D_x^l(x) \cap Cl_{l'}|}.
   \]

As such, the predefined levels \( l_l \) and \( l_{l'} \) are used to control the precision degrees \( \beta_l \) and \( \beta_{l'} \) in definitions of lower approximation, respectively.

**Figure 2.** The illustration of constraint conditions preserving decision class \( Cl_l \).

### 4. Class-based criteria reduction

Kusunoki and Inuiguchi [7] studied the definitions of class-based rough approximations and also provided the new concepts of class-based reducts. The definition is given as follows:

**Definition** For \( P \subseteq C \) and \( t \in T \), lower and upper approximations of decision class \( Cl_l \) are defined by:
\( P(C_i) = \overline{P(C_i)} \cap P(C_i) ; \overline{T(C_i)} = \overline{T(C_i)} \cap \overline{T(C_i)} \). 

In this definition, \( P(C_i) \) is constrained by both conditions (B) and (C), which is also the precision classification region of TRM in classical DRSA model. And, \( T(C_i) \) is constrained by both conditions (D') and (A'). Please refer to the illustration of Fig. 2.

With this in mind, the following assertions are valid:

\( B_{\beta_i}(C_i) = T(C_i) - P(C_i) \)

\( P(C_i) = \{ x \in C_i \ ; \ D'_i(x) \subseteq C_i \} \) and \( D'_i(x) \subseteq C_i \}; \overline{T(C_i)} = U - P(C_i) - \overline{T(C_i)} \)

\( B_{\beta_i}(C_i) = U - P(C_i) - \overline{T(C_i)} \)

\( B_{\beta_i}(C_i) = U - P(C_i) - \overline{T(C_i)} \)

For \( P \subseteq C \ , \ P(C_i) \subseteq C(C_i) ; \overline{T(C_i)} \supseteq \overline{T(C_i)} \).

And also, the following assertion presented in [7] is actually not invalid:

\( P(C_i) = C_i - B_{\beta_i}(C_i) \).

It can be revised as the following assertion for describing the relations among \( C_i \ , \ P(C_i) \) and \( B_{\beta_i}(C_i) \):

\( B_{\beta_i}(C_i) \cap C_i = C_i - P(C_i) \)

According to the proposed TRM, the class-based reducts can be defined as follows:

**Definition (L-reduct):**

If a minimal subset \( P \subseteq C \) fulfills \( P(C_i) = C_i(C_i) \) for \( t = 1, \ldots, l \), this criteria subset is a Lower approximation reduct, denoted by L-reduct.

**Definition (L \( \beta \)-reduct):**

If a minimal subset \( P \subseteq C \) fulfills \( P(C_i) = C_i(C_i) \) for \( t = 1, \ldots, l \), this criteria subset is a Low boundary reduct, denoted by L\( \beta \)-reduct.

**Definition (H \( \beta \)-reduct):**

If a minimal subset \( P \subseteq C \) fulfills \( P(C_i) = C_i(C_i) \) for \( t = 1, \ldots, l \), this criteria subset is an High boundary reduct, denoted by H\( \beta \)-reduct.

**Proposition:**

If a criteria subset \( P \subseteq C \) is the H\( \beta \)-reduct as well as the L\( \beta \)-reduct, we assert this subset \( P \) is also the L-reduct.

**Proof:** It can be easily proved by using our proposed TRM of class-based rough approximation.

\[ \square \]

5. Conclusion

Unlike the union-based definitions in previous DRSA models, this paper attempts to develop the class-based definitions of rough approximation. Based on the analysis of the partition in one singleton decision class, a new Three Region Model is proposed. In addition, we study the relationship of definitions between union-based rough approximations and class-based rough approximations. Several consequential properties are provided in this paper. Finally, we provided the new class-based reducts with assistance of the Three Region Model.

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