Theory of radiative and rare $B$ decays

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Abstract. We present a concise theoretical overview of radiative and rare $B$ decays mediated by flavour-changing neutral-current transitions of the type $b \rightarrow s(d)\gamma$ and $b \rightarrow s(d)\ell\ell$.

INTRODUCTION

Thanks to the efforts of $B$ factories, the exploration of the mechanism of quark-flavour mixing is now entering a new interesting era. The precise measurements of mixing-induced CP violation and tree-level allowed semileptonic transition have provided an important consistency check of the SM, and a precise determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The next goal is to understand if there is still room for new sources of flavour symmetry breaking close to the electroweak scale. From this perspective, radiative and rare $B$ decays mediated by flavour-changing neutral current (FCNC) amplitudes represent a fundamental tool (see e.g. Ref. [1]).

Beside the experimental sensitivity, the conditions which allow to perform significant NP searches in rare decays can be summarized as follows: i) decay amplitude dominated by electroweak dynamics, and thus enhanced sensitivity to non-standard contributions; ii) small theoretical error within the SM, or good control of both perturbative and non-perturbative corrections. In the rest of this talk we shall analyze at which level these conditions are satisfied in various decay modes.

INCLUSIVE FCNC $B$ DECAYS

Inclusive rare $B$ decays such as $B \rightarrow X_s\gamma$, $B \rightarrow X_s\ell^+\ell^-$ and $B \rightarrow X_s\nu\bar{\nu}$ are the natural framework for high-precision studies of FCNCs in the $\Delta B = 1$ sector [2]. Perturbative QCD and heavy-quark expansion form a solid theoretical framework to describe these processes: inclusive hadronic rates are related to those of free $b$ quarks, calculable in perturbation theory, by means of a systematic expansion in inverse powers of the $b$-quark mass.

The starting point of the perturbative partonic calculation is the determination of a low-energy effective Hamiltonian, renormalized at a scale $\mu = O(m_b)$, obtained by integrating out the heavy degrees of freedom of the theory. For $b \rightarrow s$ transitions –within the SM– this can be written as

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}}V_{ts}^*V_{tb} \sum_{i=1}^{10} C_i(\mu)Q_i + \text{h.c.} \quad (1)$$

where $Q_{1...6}$ are four-quark operators, $Q_8$ is the chromomagnetic operator and

$$Q_7 = \frac{e^2}{4\pi^2} \bar{s}_L\gamma^\mu m_b b_R F^\mu\nu$$

$$Q_9 = \frac{e^2}{4\pi^2} \bar{s}_L\gamma^\mu b_L \bar{\ell}\gamma^\mu \ell$$

$$Q_{10} = \frac{e^2}{4\pi^2} \bar{s}_L\gamma^\mu b_L \bar{\ell}\gamma^\mu \gamma_5 \ell$$

$$Q_{\nu} = \frac{e^2}{4\pi^2 s_w} \bar{s}_L\gamma^\mu b_L \bar{\nu}_L \gamma_\nu \nu_L$$

are the leading FCNC electroweak operators. Within the SM, the coefficients of all the operators in Eq. (2) receive a large non-decoupling contribution from top-quark loops at the electroweak scale. But the $m_t$ dependence is not the same for the four operators, reflecting a different $SU(2)_L$-breaking structure, which can be affected in a rather different way by new-physics contributions [3,4].

The calculation of the rare decay rates then involves three distinct steps: i) the determination of the initial conditions of the Wilson coefficients at the electroweak scale; ii) the evolution by means of renormalization-group equations (RGEs) of the $C_i$ down to $\mu = O(m_b)$; iii) the evaluation of the hadronic matrix elements of the effective operators at $\mu = O(m_b)$, including both perturbative and non-perturbative QCD corrections. Each of the three steps must be taken to matching orders of accuracy in powers of the strong coupling constants $\alpha_s$ and of the large logs generated by the RGE running. The interesting short-distance (electroweak) dynamics that we
would like to test enters only in the first step; the following two steps are fundamental ingredients to reduce and control the theoretical error.

The first two steps (initial conditions and RGEs) are process independent and are common also to exclusive modes. Nonetheless, the organization of the leading-log (LL) series is not the same for the three underlying partonic processes, or the four operators in Eq. \[Q_7\]:

\[
b \rightarrow s \gamma.\]

Here only \(Q_7\) has a non-vanishing matrix element at the tree level. The large logarithms generated by mixing of four-quark operators into \(Q_7\) (see Fig. 1) play a very important role and enhance the partonic rate by a factor of almost three. Since this mixing vanishes at the one-loop level, a full treatment of QCD corrections beyond lowest order is a rather non-trivial task. This has been achieved already a few years ago, thanks to the joint effort of many authors (see e.g. Ref. and references therein), and is nowadays a rather mature subject. All the ingredients of the partonic calculation have been cross-checked by more than one group. In particular, very recently an independent confirmation of the three-loop mixing of \(Q_7\) and \(Q_{1…6}\) – till few months ago the only piece of the calculation performed by one group only – has been presented.

\[
b \rightarrow s \ell^+ \ell^- .\]

The three operators with non-vanishing tree-level matrix elements are \(Q_7, Q_9\) and \(Q_{10}\). Similarly to \(Q_7\), QCD corrections are very important also for \(Q_9\). Since \(Q_9\) mixes with four-quark operators already at the one-loop level (see Fig. 1), the organization of the LL series for \(b \rightarrow s \ell^+ \ell^-\) is different than in \(b \rightarrow s \gamma\); the NLL level is much simpler (no three-loop mixing involved), but less precise. An accuracy below the 10% level on the decay rate (a precision similar to the NLL level in \(b \rightarrow s \gamma\)) is reached here only with a NNLL calculation. All the missing ingredients to reach this goal has finally become available. In particular, NNLL initial conditions and anomalous dimension matrix can be found in Ref. and respectively.

It is worth to stress that the impact of QCD corrections is very limited in the axial-current operator \(Q_{10}\). This operator does not mix with four-quark operators and is completely dominated by short-distance contributions. Together with \(Q_7, Q_{10}\) belongs to the theoretically clean \(\mathcal{O}(G_F^2)\) hard-GIM-protected part of the effective Hamiltonian. Thus observables more sensitive to \(Q_{10}\), such as the forward-backward (FB) lepton asymmetry in \(b \rightarrow s \ell^+ \ell^-\), have a reduced QCD uncertainty and a stronger sensitivity to possible non-standard phenomena.

\[
b \rightarrow s \ell^- \ell^+ .\]

In this case only \(Q_7\) is involved. Similarly to \(Q_{10}\), QCD corrections play a very minor role since there is no mixing with four-quark operators. As a result, the only non-trivial step of the perturbative calculation for \(b \rightarrow s \ell^- \ell^+\) decays is the determination of the initial condition of \(C_V\) at the electroweak scale: this is known with a precision around 1% within the SM.

These two processes-independent steps of the calculation, namely the determination of the effective Hamiltonian renormalized at a low scale \(\mu = \mathcal{O}(m_b)\), can easily be transferred from the \(b \rightarrow s\) case to the \(b \rightarrow d\) one. The only difference is the richer CKM structure of the \(b \rightarrow d\) Hamiltonian, with two independent non-negligible terms \(V_{td}V_{ub}^*\) and \(V_{td}V_{ub}\).

The situation is very different for the last step of the calculation, namely the evaluation of the hadronic matrix elements. The latter strongly depend on the specific process and the specific observable we are interested in (e.g. fully inclusive rate or differential distribution). In the following we shall review the results of this step (and thus the final numerical predictions) for some of the most interesting \(b \rightarrow s\) observables.

### B → Xsγ [the most effective “NP killer”]

The inclusive \(B \rightarrow X_s \gamma\) rate is the most precise and clean short-distance information that we have, at present, on \(\Delta B = 1\) FCNCs. Combining the precise measurements by ALEPH, BaBar, Belle and CLEO, the world average reads

\[
\mathcal{B}(B \rightarrow X_s \gamma)^{\text{exp}} = (3.34 \pm 0.38) \times 10^{-4} \tag{3}
\]

On the theory side, the NLL partonic calculation of the matrix elements, performed first in Ref. for the leading terms, has recently been cross-checked and completed in Ref. Perturbative corrections due to higher-order electroweak effects have also been analyzed (see Ref. and references therein).

Non-perturbative \(1/m_b\) corrections are well under control in the total rate. In particular, \(\mathcal{O}(1/m_b)\) corrections vanish in the ratio \(\Gamma(B \rightarrow X_s \gamma)/\Gamma(B \rightarrow X_s \ell\nu)\), and the \(\mathcal{O}(1/m_b^2)\) ones are known and amount to few per cent. Also non-perturbative effects associated to charm-quark loops have been estimated and found to be very small. The most serious problem of non-perturbative origin is related to the (unavoidable) experimental cut in the photon energy spectrum that prevents

\[
(\text{left})\quad \text{and}\quad (\text{right})\]

\[
\begin{align*}
\text{FIGURE 1.} & \quad \text{Representative diagrams for the mixing of four-quark operators into } Q_7 \text{ (left) and } Q_9 \text{ (right).}
\end{align*}
\]
the measurement from being fully inclusive [17, 19]. With the present cut by CLEO $E_T > 2.0$ GeV [20], this uncertainty is smaller but non-negligible with respect to the error of the perturbative calculation. The latter is around 10% and its main source is the uncertainty in the error of the perturbative calculation. The latter is certainly useful to further constrain this observable. A critical discussion about the error in $\Delta \Gamma_{\text{CP}}(B \to X_s \gamma)$ do not exceed the 10%–30% level [4]. Improved measurements of $\Delta \Gamma_{\text{CP}}(B \to X_s \gamma)$ are certainly useful to further constrain this possibility. However, since the experimental error has reached the level of the theoretical one, it will be very difficult to clearly identify possible deviations from the SM, if any, in this observable.

Hopes to detect new-physics signals are still open through the CP-violating asymmetry

$$\Delta \Gamma_{\text{CP}}(B \to X_s \gamma) = \frac{\Gamma(B \to X_s \gamma) - \Gamma(\bar{B} \to X_s \gamma)}{\Gamma(B \to X_s \gamma) + \Gamma(\bar{B} \to X_s \gamma)}, \quad (5)$$

This is expected to be below 1% within the SM [22, 24], but could easily reach $\mathcal{O}(10\%)$ values beyond the SM, even in the absence of large effects in the total $B \to X_s \gamma$ rate. This is indeed one of the main expectations in models with sizable NP effects in $\Delta \Gamma_{\text{CP}}(B \to \phi K_S)$. The present measurement of $\Delta \Gamma_{\text{CP}}(B \to X_s \gamma)$ is consistent with zero [11], but the sensitivity is still one order of magnitude above the SM level.

$B \to X_s \ell^+ \ell^-$ [the present frontier]

Both Belle [25] and BaBar [26] have recently announced a clear evidence ($\approx 5\sigma$) of the $B \to X_s \ell^+ \ell^-$ decay. The two results are compatible and are both based on a semi-inclusive analysis (the hadronic system is reconstructed from a kaon plus 0 to 4 pions, with at most one $\pi^0$). Their combination [11]

$$\mathcal{B}(B \to X_s \ell^+ \ell^-)_{\text{exp}} = (6.2 \pm 1.1^{+1.6}_{-1.3}) \times 10^{-6}. \quad (6)$$

represents a very useful new piece of information about $\Delta B = 1$ FCNCs, with considerable margin of improvement in the near future.

In principle, these decays offer a phenomenology richer than $B \to X_s \gamma$, with more than one interesting observable. The joint effort of several groups has recently allowed to evaluate at the NNLL level all the matrix element necessary for the two main kinematical distributions: the dilepton spectrum [27, 28, 29] and the lepton FB asymmetry [20, 31].

In addition to the non-perturbative corrections due to the finite $b$ quark mass, $B \to X_s \ell^+ \ell^-$ transitions suffer from specific non-perturbative effects due to long-lived $c\bar{c}$ intermediate states ($B \to X_s c\bar{c} \to X_s \ell^+ \ell^-$). The heavy-quark expansion, which allow to evaluate the $\Lambda_{\text{QCD}}/m_b$ terms, is rapidly convergent and leads to small corrections for sufficiently inclusive observables [32, 33]. A consistent treatment of the second type of effects requires kinematical cuts in order to avoid the large non-perturbative background of the narrow $c\bar{c}$ resonances (see Fig. 2). These two requirements are somehow in conflict [28, 33]; nonetheless, we can identify two perturbative windows, defined by:

$$q^2 \equiv M_{\ell^+ \ell^-}^2 \in \begin{cases} 1 \text{ GeV}^2, 6 \text{ GeV}^2 \quad \text{(low)}, \\ \text{high:} \quad q^2 > 14.4 \text{ GeV}^2 \end{cases}$$

where reliable predictions can be performed [28]. It is worth to emphasize that the two regions have complementary virtues and disadvantages:

- **Virtues of the low-$q^2$ region**: reliable $q^2$ spectrum; small $1/m_b$ corrections; sensitivity to the interference of $C_9$ and $C_9'$; high rate.
- **Disadvantages of the low-$q^2$ region**: difficult to perform a fully inclusive measurement (severe cuts on the dilepton energy and/or the hadronic invariant mass); long-distance effects due to processes of the type $B \to \Psi X_s \to X_s + X^\prime \ell^+ \ell^-$ not fully under control; non-negligible scale and $m_b$ dependence.
- **Virtues of the high-$q^2$ region**: negligible scale and $m_b$ dependence due to the strong sensitivity to $|C_{10}|^2$; easier to perform a fully inclusive measurement (small hadronic invariant mass); negligible long-distance effects of the type $B \to \Psi X_s \to X_s + X^\prime \ell^+ \ell^-$. 


• Disadvantages of the high-$q^2$ region: $q^2$ spectrum not reliable (only the integrated rate can be predicted); sizeable $1/m_b$ corrections (effective expansion in $1/(m_b - \sqrt{s_{min}})$ [23]); low rate.

Given this situation, we believe that future experiments should try to measure the branching ratios in both regions and report separately the two results. These two measurements are indeed affected by different systematic uncertainties (of theoretical nature) and provide a different short-distance information. The NNLL SM predictions for the two clean windows [23],

\[
\mathcal{B}(B \to X_s \ell^+ \ell^-)_{\text{low}}^{\text{SM}} = (1.63 \pm 0.20) \times 10^{-6}, \\
\mathcal{B}(B \to X_s \ell^+ \ell^-)_{\text{high}}^{\text{SM}} = (4.04 \pm 0.78) \times 10^{-7}, \quad (7)
\]

are still affected by a considerable error; however, in both cases the uncertainty is mainly of parametric nature and could be substantially improved in the future. In particular, the large error in the high-$q^2$ region is mainly due to the uncertainty in the relation between the physical $q^2$ interval and the corresponding interval for the partonic calculation (i.e. the uncertainty in the relation between $m_b$ and the physical hadron mass), which can be improved with better data on charged-current semileptonic modes. According to the recent analysis of Ref. [29], both results in (7) should be decreased by $\approx 4\%$ to take into account the leading electroweak corrections.

The two results in (7) cannot be directly confronted with [23], which includes an extrapolation to the full $q^2$ spectrum. The updated SM expectation for this extrapolated branching ratio is $(4.6 \pm 0.8) \times 10^{-6}$ [23] (in agreement with the previous estimate of Ref. [34]), and it is consistent with the present experimental world average.

We stress that this prediction is already saturated by irreducible theoretical errors and, contrary to the results in [7], is very difficult to improve it further.

As anticipated, some of the most interesting short-distance tests in $\mathcal{B}(B \to X_s \ell^+ \ell^-)$ decays can be performed by means of the FF asymmetry of the dilepton distribution:

\[
\mathcal{A}_{\text{FB}}(q^2) = \frac{\left(\frac{1}{d\mathcal{B}(B \to X_s \ell^+ \ell^-)/dq^2} \int_{-1}^{1} d\cos\theta_\ell \right)_{\text{high}}}{\left(\frac{1}{d\mathcal{B}(B \to X_s \ell^+ \ell^-)/dq^2} \int_{-1}^{1} d\cos\theta_\ell \right)_{\text{low}}} \text{sgn}(\cos\theta_\ell), \quad (8)
\]

where $\theta_\ell$ is the angle between $\ell^+$ and $B$ momenta in the dilepton centre-of-mass frame. Here the SM predict a zero for $q_0 = q_0^0/m_B^2 = 0.162 \pm 0.008$ [34, 51]: a very precise prediction which could easily be modified beyond the SM, even in absence of significant non-standard effects on the total rate.

**EXCLUSIVE MODES**

On general grounds, theoretical predictions for exclusive FCNC decays are more difficult. The simplest cases are processes with at most one hadron in the final state. Here there has been a substantial progress in the last few years, both by means of analytic approaches [35] and by means of Lattice QCD [36], but still the overall theoretical uncertainty is around 20% at the amplitude level. The largest source of uncertainty is typically the normalization of the hadronic form factors, an error that can be substantially reduced in appropriate ratios or differential distributions. These type of observables become particularly interesting in channels where, because of irreducible experimental problems, the short-distance amplitude cannot be extracted from corresponding inclusive modes. Two of such examples are the ratio

\[
R_\gamma(\rho/K^*) = \frac{\mathcal{B}(B \to \rho \gamma)}{\mathcal{B}(B \to K^* \gamma)}, \quad (9)
\]

and the normalized FF asymmetry in $B \to K^* \ell^+ \ell^-$. The ratio $R_\gamma(\rho/K^*)$ is one of the most promising tool to extract short-distance properties about the $b \to s \gamma$ amplitude. On the experimental side, the combination of the bounds on charged and neutral channels, in the isospin limit, leads to $R_\gamma(\rho/K^*) < 0.047$ at 90% C.L. [37]. On the theory side, the $B \to V \gamma$ amplitudes which determine this ratio have been analyzed beyond naive factorization by several authors [38, 39, 40]. Within the SM one can write

\[
R_\gamma(\rho/K^*) = \left| \frac{V_{td}}{V_{ts}} \right|^2 \left( \frac{m_B^2 - m_{\rho^0}^2}{m_B^2 - m_{K^*}^2} \right)^3 \xi^2 (1 - \Delta R), \quad (10)
\]
isospin) breaking due to 1

short-distance information: 1) The position of the zero
tensor form factors [42]. There are three main features
indicate a larger value,
the SM expectation. However, preliminary Lattice results
light-cone sum rule estimate
ζ [39].

of the relative sign of
CP-violating phase.

The largest source of uncertainty is

FIGURE 4. Zero of the forward-backward asymmetry in
B \rightarrow K^+\ell^+\ell^- at LO and NLO. The band reflects all theo-
retical uncertainties from parameters and scale dependence
combined [44].

B_{s,d} \rightarrow \ell^+\ell^- [the future frontier]

The purely leptonic decays constitute a special case
among exclusive transitions. Within the SM only the
axial-current operator, Q_{10}, induces a non-vanishing
contribution to these decays. As a result, the short-distance
contribution is not diluted by the mixing with four-quark
operators. Moreover, the hadronic matrix element involved
is the simplest we can consider, namely the B-
meson decay constant
\langle 0|\bar{q}\gamma_\mu p_b|B_q(p)\rangle = ip_{\mu} f_{B_q}
(11)

Reliable estimates of f_{B_d} and f_{B_s} are obtained at present
from lattice calculations and in the future it will be pos-
sible to cross-check these results by means of the B \rightarrow
\ell^+\ell^- rate. Modulo the determination of f_{B_s}, the theo-
retical cleanness of B_{s,d} \rightarrow \ell^+\ell^- decays is comparable to
that of the golden modes K_L \rightarrow \pi^0\nu\bar{\nu} and B \rightarrow X_{s,d}\nu\bar{\nu}.

The price to pay for this theoretically-clean amplitude
is a strong helicity suppression for \ell = \mu (and \ell = e),
or the channels with the best experimental signature. Emplo-
ying the full NLO expression of C_{10} [10], we can write

\mathcal{B}(B_s \rightarrow \mu^+\mu^-)^{\text{SM}} = 3.1 \times 10^{-9} \left( \frac{|V_{td}|}{0.04} \right)^2 
\times \left( \frac{f_{B_s}}{0.21 \text{ GeV}} \right)^2 \left( \frac{\tau_{B_s}}{1.6 \text{ ps}} \right) \left( \frac{m_1/m_2}{166 \text{ GeV}} \right)^{3.12} 
\mathcal{B}(B_s \rightarrow \tau^+\tau^-)^{\text{SM}} = 215 \mathcal{B}(B_s \rightarrow \mu^+\mu^-)^{\text{SM}}

The corresponding B_d modes are both suppressed by an addi-
tional factor [V_{td}/V_{ts}]^2 = (4.0 \pm 0.8) \times 10^{-2}. The pre-
sent experimental bound closest to SM expectations is the one obtained by CDF on B_s \rightarrow \mu^+\mu^- [14,44]:

\mathcal{B}(B_s \rightarrow \mu^+\mu^-) < 9.5 \times 10^{-7} (95\% \text{ CL}) ,
which is still very far from the SM level. The latter will certainly not be reached before the LHC era.

As emphasized in the recent literature [45, 46, 47], the purely leptonic decays of \( B_s \) and \( B_d \) mesons are excellent probes of several new-physics models and, particularly, of scalar FCNCs. Scalar FCNC operators, such as \( b_{\mu\mu}b\bar{b}H_L \), are present within the SM but are absolutely negligible because of the smallness of down-type Yukawa couplings. On the other hand, these amplitudes could be non-negligible in models with an extended Higgs sector. In particular, within the MSSM, where two Higgs doublets are coupled separately to up- and down-type quarks, a strong enhancement of scalar FCNCs can occur at large tan\( \beta = v_u/v_d \) [45]. This effect is very small in non-helicity-suppressed \( B \) decays and in \( K \) decays (because of the small Yukawa couplings), but could enhance \( B \to \ell^+\ell^- \) rates by orders of magnitude. As pointed out in Ref. [48], \( \mathcal{O}(10\%) \) enhancements in \( \mathcal{B}(B \to \ell^+\ell^-) \) correspond to \( \mathcal{O}(10\%) \) breaking of universality in \( \mathcal{B}(B \to K\mu^+\mu^-) \) vs. \( \mathcal{B}(B \to K\ell^+\ell^-) \). Therefore, the present search for \( B \to \ell^+\ell^- \) at CDF is already quite interesting, even if the sensitivity is well above the SM level. In a long-term perspective, the discovery of such processes is definitely one of the most interesting items in the \( B \)-physics program of hadron colliders.

**CONCLUSIONS**

Rare FCNC decays of \( B \) mesons provide a unique opportunity to perform high-precision studies of quark-flavour mixing. The \( B \to X_c \gamma \) rate, where both experimental and theoretical errors have reached a comparable level around 10\%, represents the highest peak in our present knowledge of FCNCs. The lack of deviations from SM expectations in \( \Gamma(B \to X_c \gamma) \) should not discourage the measurement of other clean and independent FCNC observables, such as the forward–backward asymmetry in \( B \to X_c \ell^+\ell^- \) or the \( B \to \ell^+\ell^- \) rates. Even if new physics will first be discovered elsewhere, the experimental study of these theoretically-clean observables would still be very useful to investigate the flavour structure of any new-physics scenario.

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**REFERENCES**

1. A. J. Buras, [hep-ph/0301203]; Y. Nir, *Nucl. Phys. Proc. Suppl.*, **117**, 111 (2003) [hep-ph/0208080]; G. Isidori, *Int. J. Mod. Phys. A*, **17**, 3078 (2002) [hep-ph/0110255].
2. For a recent review see T. Hurth, *Rev. Mod. Phys.*, **75**, 1159 (2003) [hep-ph/0212364].
3. S. Bertolini, F. Borzumati and A. Masiero, *Phys. Lett. B*, **192**, 437 (1987); A. Ali, G. F. Giudice and T. Mannel, *Z. Phys. C*, **67**, 417 (1995) [hep-ph/9408213].
4. G. D’Ambrosio et al., *Nucl. Phys. B*, **645**, 155 (2002) [hep-ph/0207036]; A. J. Buras et al., *Phys. Lett. B*, **500**, 161 (2001) [hep-ph/0007085]; A. J. Buras, *Acta Phys. Polon. B*, **34**, 5615 (2003) [hep-ph/0310208].
5. S. Bertolini, F. Borzumati, A. Masiero, *Phys. Rev. Lett.*, **59**, 180 (1987); N. G. Deshpande et al., *Phys. Rev. Lett.*, **59**, 183 (1987).
6. A. J. Buras and M. Misiak, *Acta Phys. Polon. B*, **33**, 2597 (2002) [hep-ph/0207151].
7. K. Chetyrkin, M. Misiak and M. Munz, *Phys. Lett. B*, **400**, 206 (1997) [425, 414 (1997)] (E) [hep-ph/9612313].
8. P. Gambino, M. Gorbahn and U. Haisch, *Nucl. Phys. B*, **673**, 238 (2003) [hep-ph/0306079].
9. C. Bobeth, M. Misiak and J. Urban, *Nucl. Phys. B*, **574**, 291 (2000) [hep-ph/9910220].
10. G. Buchalla and A. J. Buras, *Nucl. Phys. B*, **548**, 309 (1999) [hep-ph/9901288]; M. Misiak and J. Urban, *Phys. Lett. B*, **451**, 161 (1999) [hep-ph/9901728].
11. M. Nakao, [hep-ex/0312041].
12. C. Greub, T. Hurth and D. Wyler, *Phys. Rev. D*, **54**, 3350 (1996) [hep-ph/9603404].
13. A. J. Buras, A. Czarnecki, M. Misiak and J. Urban, *Nucl. Phys. B*, **631**, 219 (2002) [hep-ph/0203135]; 611, 488 (2001) [hep-ph/0105160].
14. P. Gambino and U. Haisch, *JHEP*, **0110**, 020 (2001) [hep-ph/0109058].
15. A. F. Falk, M. Luke and M. J. Savage, *Phys. Rev. D*, **49**, 3367 (1994) [hep-ph/9308288].
16. M. B. Voloshin, *Phys. Lett. B*, **397**, 275 (1997) [hep-ph/9612483]; A. Grant, A. Morgan, S. Nussinov and R. Peccei, *Phys. Rev. D*, **56**, 3511 (1997) [hep-ph/9702380].
17. Z. Ligeti, L. J. Randall and M. B. Wise, *Phys. Lett. B*, **402**, 178 (1997) [hep-ph/9702212].
18. G. Buchalla, G. Isidori and S. J. Key, *Nucl. Phys. B*, **511**, 594 (1998) [hep-ph/9705233].
19. A. L. Kagan and M. Neubert, *Eur. Phys. J. C*, **7**, 5 (1999) [hep-ph/9805303].
20. D. Cassel, these proceedings; S. Chen et al. [CLEO Collab.], [hep-ex/0108032].
21. P. Gambino and M. Misiak, *Nucl. Phys. B*, **611**, 338 (2001) [hep-ph/0104034].
22. T. Hurth, E. Lunghi and W. Porod, [hep-ph/0312260].
23. M. Ciuchini et al., eConf C0304052 (2003) WG307 [hep-ph/0308013].
24. A. L. Kagan and M. Neubert, *Phys. Rev. D*, **58**, 094012 (1998) [hep-ph/9803368].
25. J. Kaneko et al. [Belle Collaboration], *Phys. Rev. Lett.*, **90**, 021801 (2003) [hep-ex/0208029].
26. B. Aubert et al. [BaBar Collaboration], [hep-ex/0308016].
27. H. H. Asatryan, H. M. Asatryan, C. Greub and M. Walker, *Phys. Lett. B*, **507**, 162 (2001) [hep-ph/0103087]; *Phys. Rev. D*, **65**, 074004 (2002) [hep-ph/0109140].
28. A. Ghinculov, T. Hurth, G. Isidori and Y. P. Yao, \textit{Nucl. Phys. Proc. Suppl.}, 116, 284 (2003) [hep-ph/0211197].

29. C. Bobeth, P. Gambino, M. Gorbahn and U. Haisch, \textit{hep-ph/0312090}.

30. A. Ghinculov, T. Hurth, G. Isidori and Y. P. Yao, \textit{Nucl. Phys. B}, 648, 254 (2003) [hep-ph/0211197].

31. H. M. Asatrian, K. Bieri, C. Greub and A. Hovhannisyan, \textit{Phys. Rev. D}, 66, 094013 (2002) [hep-ph/0209006]; H. M. Asatrian, H. H. Asatryan, A. Hovhannisyan and V. Poghosyan, \textit{hep-ph/0311187}.

32. A. Ali, G. Hiller, L. T. Handoko and T. Morozumi, \textit{Phys. Rev. D}, 55, 4105 (1997) [hep-ph/9609449].

33. G. Buchalla and G. Isidori, \textit{Nucl. Phys. B}, 525, 333 (1998) [hep-ph/9801456].

34. A. Ali, E. Lunghi, C. Greub and G. Hiller, \textit{Phys. Rev. D}, 66, 034002 (2002) [hep-ph/0112300].

35. I. Stewart, these proceedings.

36. A. El-Khadra, these proceedings.

37. B. Aubert \textit{et al.} [BaBar Collaboration], \textit{hep-ex/0207073}.

38. B. Grinstein and D. Pirjol, \textit{Phys. Rev. D}, 62, 093002 (2000) [hep-ph/0002216]; A. Ali and A. Y. Parkhomenko, \textit{Eur. Phys. J. C}, 23, 89 (2002) [hep-ph/0105302]; S. W. Bosch and G. Buchalla, \textit{Nucl. Phys. B}, 621, 459 (2002) [hep-ph/0106081].

39. A. Ali and E. Lunghi, \textit{Eur. Phys. J. C}, 26, 195 (2002) [hep-ph/0206242]; T. Hurth and E. Lunghi, eConf C0304052 (2003) WG206 [hep-ph/0307142].

40. M. Beneke, T. Feldmann and D. Seidel, \textit{Nucl. Phys. B}, 612, 25 (2001) [hep-ph/0106067].

41. D. Becirevic and J. Reyes, hep-lat/0309131.

42. G. Burdman, \textit{Phys. Rev. D}, 57, 4254 (1998) [hep-ph/9710550].

43. G. Buchalla, G. Hiller and G. Isidori, \textit{Phys. Rev. D}, 63, 014015 (2001) [hep-ph/0006136].

44. C.-J. Lin, these proceedings.

45. C. Hamzaoui, M. Pospelov and M. Toharia, \textit{Phys. Rev. D}, 59, 095005 (1999) [hep-ph/9807350]; K. S. Babu and C. Kolda, \textit{Phys. Rev. Lett.}, 84, 228 (2000) [hep-ph/9909476]; G. Isidori and A. Retico, \textit{JHEP}, 0111, 001 (2001) [hep-ph/0110121]; A. J. Buras \textit{et al.}, \textit{Nucl. Phys. B}, 659, 3 (2003) [hep-ph/0210145]; A. Dedes and A. Pilafitis, \textit{Phys. Rev. D}, 67, 013012 (2003) [hep-ph/0209306]; A. Dedes, \textit{Mod. Phys. Lett.} A, 18, 2627 (2003) [hep-ph/0309233].

46. C. S. Huang \textit{et al.}, \textit{Phys. Rev. D}, 59, 011701 (1999) [hep-ph/9803460]; \textit{Phys. Rev. D}, 63, 114021 (2001) [hep-ph/0006250]; S. R. Choudhury and N. Gaur, \textit{Phys. Lett. B}, 451, 86 (1999) [hep-ph/9810307]; P.H. Chankowski and L. Slawianowska, \textit{Phys. Rev. D}, 63, 054012 (2001) [hep-ph/0008046]; C. Bobeth \textit{et al.}, \textit{Phys. Rev. D}, 64, 074014 (2001) [hep-ph/0104284]; \textit{Phys. Rev. D}, 66, 074021 (2002) [hep-ph/0204225]; Z. Xiong and J. M. Yang, \textit{Nucl. Phys. B}, 628, 193 (2002) [hep-ph/0105260]; A. Dedes \textit{et al.}, \textit{Phys. Rev. Lett.}, 87, 251804 (2001) [hep-ph/0108037]; S. w. Baek, P. Ko and W. Y. Song, \textit{Phys. Rev. Lett.}, 89, 271801 (2002) [hep-ph/0205259]; \textit{JHEP}, 0303, 054 (2003) [hep-ph/0208112]; J. K. Mizukoshi, X. Tata and Y. Wang, \textit{Phys. Rev. D}, 66, 115003 (2002) [hep-ph/0208078]; C. S. Huang and X. H. Wu, \textit{Nucl. Phys. B}, 657, 304 (2003) [hep-ph/0212220]; G. L. Kane, C. Kolda and J. E. Lennon, \textit{hep-ph/0310042}.

47. G. Isidori and A. Retico, \textit{JHEP}, 0209, 063 (2002) [hep-ph/0208159]; R. Fleischer, G. Isidori and J. Matias, \textit{JHEP}, 0305, 053 (2003) [hep-ph/0302229]; A. J. Buras, \textit{Phys. Lett. B}, 566, 115 (2003) [hep-ph/0303060].