Scalar Mesons in a Relativistic Quark Model with Instanton-Induced Forces

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Abstract

In a relativistic quark model with linear confinement and an instanton-induced interaction which solves the $\eta-\eta'$ puzzle, scalar mesons are found as almost pure SU(3) flavor states. This suggests a new interpretation of the scalar nonet: We propose that the recently discovered $f_0(1500)$ is not a glueball but the scalar (mainly)–octet meson for which the $K\bar{K}$ decay mode is suppressed. The mainly–singlet state is tentatively identified with the $f_0(980)$. The isovector and isodoublet states correspond to the $a_0(1450)$ and $K^*(1430)$, respectively.
I. INTRODUCTION

The spectrum of scalar mesons is puzzling. The number of resonances found in the region from 1 to 2 GeV exceeds the number of states which can be accommodated in conventional quark models. Extra states are interpreted alternatively as $K\bar{K}$ molecules, glueballs, multiquark states or hybrids. In particular the $f_0(1500)$ is presently considered as prime glueball candidate. This state is seen in decays into $\pi\pi$, $\eta\eta$, $\eta\eta'$, and into 's' (where 's' stands for the scalar $\pi\pi$ interaction). It has been observed in $\bar{p}p$ annihilation at rest and in flight, in Pomeron–Pomeron interactions and in radiative $J/\psi$ decays.

All these processes are supposed to be 'glue–rich' and enhance the chance of observing glueballs. The mass of the $f_0(1500)$ compares very well with predictions of lattice gauge calculations. The main argument in favor of the glueball interpretation is, however, the peculiar decay pattern. It decays strongly into $\pi\pi$ but not into $K\bar{K}$. Assuming the $f_0(1300)$ to be one of the two isoscalar states, the second isoscalar state should decay preferentially into $K\bar{K}$. Hence the $f_0(1300)$ and the $f_0(1500)$ cannot be the two isoscalar states of one meson nonet.

These arguments lead naturally to the hypothesis that the $f_0(1500)$ is a glueball. This interpretation then requires the existence of a further scalar state which is mainly $s\bar{s}$ and should have a mass of about 1700 MeV, possibly the old $\Theta(1690)$. The scalar meson nonet would be nearly ideally mixed. The peculiar decay properties of the $f_0(1500)$ can be reproduced by tuning its mixing with the $f_0(1300)$ $n\bar{n}$ and the (predicted) $f_0(1700)$ $s\bar{s}$ state. The $f_0(1500)$ is hence interpreted as glueball state with strong mixing with close–by conventional scalar mesons.

In this paper we propose a radically different interpretation of the spectrum of light scalar mesons. The states are interpreted as conventional $q\bar{q}$ states, but with very small SU(3) mixing angle, governed dynamically by 't Hooft’s instanton-induced interaction. The comparison of the predicted mass spectrum with data gives a surprisingly good agreement; no further states are needed. The reduced $K\bar{K}$ partial decay width of the $f_0(1500)$ is qualitatively explained by its flavor structure.

II. EXPERIMENTAL SCALAR MESON MASS SPECTRUM

We start our discussion with a short review of the experimental situation. The Particle Data Group lists two scalar isodoublet states, the $K_0^*(1430)$ and the $K_0^{*+}(1950)$, and one isovector $a_0(980)$. A second isovector meson has been discovered recently, the $a_0(1450)$, with mass and width of $(M, \Gamma) = (1450 \pm 40$ MeV, $270 \pm 40$ MeV), respectively. There are 5 isoscalar states in the Review of Particle Properties, the $f_0(980)$, $f_0(1300)$, $f_0(1370)$, $f_0(1525)$, $f_0(1590)$ and 2 further possibly scalar states, the $f_J(1710)$ seen in radiative $J/\psi$ decays (the old $\Theta(1690)$) and an $\eta-\eta$ resonance $X(1740)$ produced in $p\bar{p}$ annihilation in flight and in charge–exchange. A rather narrow isoscalar state with $(M, \Gamma) = (1450 \pm 5$ MeV, $54 \pm 7$ MeV) has been reported in: it is produced in central collisions and seen in decays into $4\pi$. A recent reanalysis of data on $J/\psi \to \gamma 2\pi^+2\pi^-$ revealed the existence of three isoscalar
states at 1505 MeV, 1750 MeV and at 2104 MeV \[7\]. These are certainly more resonances than a quark model can accommodate. Hence we first try to identify those which we will interpret as \(q \bar{q}\) mesons.

The \(K_0^*(1430)\) is the only strange candidate with scalar quantum numbers; it gives a natural mass scale for the \(1^3P_0\) light–quark nonet. We use the \(K_0^*(1950)\) to estimate the excitation energy of scalar radial excitations to 520 MeV. This may be compared to the mass difference of 370 MeV between the \(\chi_{b0}(2P)\) and \(\chi_{b0}(1P)\) states.

It has been convincingly argued, that the narrow \(a_0(980)\), which has also been seen as a narrow structure in \(\eta\pi\)-scattering, can be generated by meson-meson dynamics alone \[10,11\]. This interpretation of the \(a_0(980)\) leaves the \(a_0(1450)\) as the \(1^3P_0\) quark-antiquark state. In analogy, it is mostly assumed that the \(f_0(980)\) is a \(K\bar{K}\) molecule. The mass degeneracy and their proximity to the \(K\bar{K}\) threshold seem to require that the nature of both states must be the same. On the other hand, the \(K\bar{K}\) interaction in isospin \(I=1\) and \(I=0\) are very different \[12\]. The extremely attractive \(I=0\) interaction may not support a loosely bound state. Instead, it may just define the pole position of the \(f_0(980)\) \(q\bar{q}\) resonance. Indeed, Morgan and Pennington find a \(f_0(980)\) pole structure characteristic for a genuine resonance of the constituents and not of a weakly bound system \[13,14\]. The \(I=1\) \(K\bar{K}\) interaction is weak and may generate a \(K\bar{K}\) molecule.

The next two states, the \(f_0(1300)\) and \(f_0(1370)\), deserve particular attention. We follow the arguments of \[13,14\] and assume that the \(\pi\pi\) interactions produce a very broad \(f_0(1000)\) state, and a comparatively narrow \(f_0(980)\) giving rise to the dip at 980 MeV in the squared \(\pi\pi\) scattering amplitude \(T_{11}\). In this scenario the \(f_0(1300)\) is interpreted as the high-mass part of the \(f_0(1000)\). In experiments the \(f_0(1000)\) shows up as a resonance at 1300 MeV because of the pronounced dip in \(|T_{11}|^2\) at 1 GeV. The \(f_0(1000)\) has an extremely large width; thus the resonance interpretation is questionable. It could be generated by t–channel exchanges instead of inter–quark forces. As for the second state, we do not consider the scalar \(4\pi\) resonance seen in \(NN\) annihilation into \(5\pi\) by Gaspero \[15\], by the Obelix \[16\] and the Crystal Barrel \[17\] collaborations as established. Likely, the mass of it is compatible with 1500 MeV \[18\].

Finally, we notice several claims for resonant structures close to 1500 MeV including the states \(f_0(1450), f_0(1500)\) and \(f_0(1590)\). Their masses, widths and decay branching ratios are incompatible within the errors quoted by the groups. Nevertheless, we do not consider it as plausible, that so many scalar isoscalar states exist in such a narrow mass gap. Instead we take the various states as manifestations of one object which we call \(f_0(1500)\).

These arguments lead us to the suggestion to identify the following states as members of the ground state scalar \(q\bar{q}\) nonet:

\[
a_0(1450), \quad K_0^*(1430), \quad f_0(980), \quad f_0(1500) \quad (1)\]

In view of the huge mass splitting a grouping of these states into the flavor nonet at first sight does not seem very obvious. In addition a quark model interpretation for the isoscalar resonance at 1500 MeV meets serious difficulties, because it is expected to be an \(s\bar{s}\) state and hence strongly decay into \(K\bar{K}\). This is not observed and consequently it is considered as
a serious candidate for a glueball. We will argue that both issues are in fact closely related. If the \( f_0(1500) \) is interpreted as a \( q\bar{q} \) state, it must have a special structure, which suppresses the decay into kaons. In \([3]\), it is interpreted as glueball mixing with the \( f_0(1300) \) and one further state at a mass of 1600 - 1800 MeV which is mostly \( s\bar{s} \). In our scenario the \( f_0(1500) \) has an \( s\bar{s} \) and an \( n\bar{n} \) component, with a negative sign suppressing \( K\bar{K} \) decays by destructive interference. We are in fact not unfamiliar with such a situation: it occurs already for the pseudoscalars in the form of the \( \eta \) and the \( \eta' \) splitting and mixing. Also the order of magnitude of the mass splitting is similar. However, for pseudoscalars the state with a negative sign between the strange and nonstrange components (i.e. the \( \eta \)-meson) is the lowest state. In the next section, we will present a quark model which relates scalar and pseudoscalar mesons in more detail and could offer in fact a consistent explanation of the scalar mesons without introducing additional yet unobserved states as in \([3]\).

### III. A RELATIVISTIC QUARK MODEL OF MESONS WITH INSTANTON-INDUCED FORCES

The model is formally based on the Bethe-Salpeter equation for quark-antiquark amplitudes, with free quark propagators with an effective constituent quark mass, an instantaneous interaction kernel that models confinement and a residual interaction based on instanton effects \([19,20]\). In such a framework the mass spectrum is obtained from the equation

\[
\Phi_P(p) = \int dp^0 S^F_1 \left( P/2 + p \right) \int \frac{d^3p'}{(2\pi)^4} \left[ -i V(p, p') \Phi_P(p') \right] S^F_2 \left( -P/2 + p \right) \tag{2}
\]

where in the CM-frame \( P = (M, \vec{0}) \) and \( S^F_i(p) = i/(p - m_i + i\epsilon) \) with a constituent-quark mass \( m_n = 306 \text{ MeV} \) and \( m_s = 503 \text{ MeV} \) for nonstrange and strange quarks, respectively. The interaction kernel \( V \) comprises a confinement part \( V_C \):

\[
[V_C(p, p') \Phi(p')] = V_C((p - p')^2) \frac{1}{2} \left[ \Phi(p') - \gamma^0 \Phi(p') \gamma^0 \right] \tag{3}
\]

where the scalar function \( V_C \) in coordinate space is given by a linearly rising potential \( V_C(r) = a_c + b_c r \), with \( a_c = -1751 \text{ MeV} \) and \( b_c = 2076 \text{ MeV} \). The special Dirac structure of \( V_C \) was chosen in order to obtain a maximal cancelation of unwanted spin-orbit splitting. The quark masses and confinement parameters have been fixed to reproduce the overall meson spectrum except for the mesons with spin zero. We find a very good description of the meson masses, especially for the Regge-behavior, which will be presented in a detailed study \([21]\).

In addition the kernel contains a residual interaction \( V_T \) in the form of ’t Hooft’s instanton-induced interaction:

\[
[V_T(p, p') \Phi(p')] = 4 G \left[ 1 \text{ tr } (\Phi(p')) + \gamma^5 \text{ tr } (\Phi(p') \gamma^5) \right] \tag{4}
\]
where \( G(g, g') \) is a flavor matrix containing the coupling constants \( g = 13.2 \text{MeV fm}^3 \) and \( g' = 11.8 \text{MeV fm}^3 \), which multiply the \( n\bar{n} \)- and \( n\bar{s} \)-interaction, respectively. Here summation over flavor and a regulating Gaussian function with a range \( \lambda = 0.3 \text{ fm} \) have been suppressed (see [22] for more details). The parameters of the 't Hooft interaction have been fixed in order to reproduce the spectrum of the pseudoscalar states, as this force has been suggested to describe the breaking of the \( U_A(1) \) symmetry.

As it stands, with this instantaneous interaction kernel the Bethe-Salpeter equation is reduced to the three dimensional Salpeter equation, which can be solved by standard techniques: all details can be found elsewhere [22,23], together with a discussion of the relativistic quark amplitudes and their behavior under Lorentz-boosts, which improves dramatically the values computed for form factors and electroweak decay values of deeply bound quark states in comparison to nonrelativistic quark models [24].

In fact we have used the same instanton-induced interaction before in a nonrelativistic reduction within the framework of the Schrödinger equation, both for mesons and baryons [25] and obtained a satisfactory description of the low lying hadronic mass spectrum. In particular 't Hooft’s interaction naturally explained (consistently with the major splitting in the baryon spectrum) the splitting and configuration mixing of the pseudoscalar nonet. This feature remains in the present relativistic treatment, see Fig. 1. However, in the nonrelativistic reduction, which is essentially an expansion in average quark momenta divided by the constituent quark mass (a quantity of the order one!), 't Hooft’s force vanishes for scalar mesons.

The calculated masses of the ground state scalar mesons are predicted to be

\[
a_0(1370), \quad K^*_0(1430), \quad f_0(980), \quad f_0(1470)
\]

which is in fair agreement with [24], especially in view of the fact, that the parameters were chosen to reproduce the spectrum in other meson sectors.

It is interesting to compare the full low–lying spectra of the scalar and pseudoscalar nonets in some more detail. Figs. 1,2 show that the mass splitting in the scalar octet is reduced. This is reasonable since the average relative momenta should be larger for orbital excitations, and thus the different quark masses breaking the \( SU(3) \) symmetry play a less important role. Also the flavor splitting between singlet and octet is much more pronounced than in the corresponding pseudoscalar case.

On the basis of the pure quark model it is hard to decide, whether to identify the low lying mainly singlet state with \( f_0(980) \) or with \( f_0(1000) \). Ultimately, this can be decided only from a genuine coupled channel approach, which includes both \( q\bar{q} \) and meson-meson states. For the time being, we tentatively prefer the identification with \( f_0(980) \), since at least one of the states of the nonet should couple to \( K\bar{K} \).

The decay properties of the scalar mesons certainly depend most sensitively on the ratio
of the $n\bar{n}$ and $s\bar{s}$ component of the wave function. Following the parameterization of Rosner [26], we define

$$|f_0⟩ = X_{f_0} |n\bar{n}⟩ + Y_{f_0} |s\bar{s}⟩$$

$$|f'_0⟩ = X'_{f_0} |n\bar{n}⟩ + Y'_{f_0} |s\bar{s}⟩$$

(6)

Although the radial wave functions of the nonstrange and strange component in our model are different, we have calculated the value of the coefficients $X$ and $Y$ from the relativistic norm [22] and the apparent relative sign of the amplitudes. The values are given in Table I and indicate, that due to the instanton-induced interaction the $f_0$ is almost a SU(3) flavor singlet and the $f'_0$ is almost an octet.

The scalar mesons decay electro-magnetically into two photons as well as into a photon and a vector meson. Emission of photons is certainly an important test–ground to decide on the glueball– or $q\bar{q}$–nature of the $f_0(1500)$ since gluons do not couple to photons. Corresponding $\gamma\gamma$ results in the tensor sector agree very favorably with data, which indicates the reliability of the calculations. The results will be published in a forthcoming paper [27].

The two–photon and the photon–vector-meson widths have been calculated in the framework of the Salpeter model following the relativistic procedure outlined in [23,24] including e.g. explicitly the negative energy components. The results on scalar mesons are shown in Table II, where we compare results with and without the instanton-induced interaction. The scarce experimental data are also listed. Again we will refrain from an ultimate experimental identification of the $f_0$ state, although the large calculated $\gamma\gamma$-width better corresponds to the result found for the $f_0(1300)$, which we quoted as $f_0(1000)$. The decrease of the $\gamma\gamma$ width of the lowest $f_0$ is merely due to the decrease of phase space, whereas the decrease of the $\gamma\gamma$ width of the $f'_0(1500)$ comes from the destructive interference between the $n\bar{n}$ and $s\bar{s}$ component. Unfortunately, the data are too poor and do not allow to distinguish the two models.

The radiative decays into a vector meson again are mainly determined by phase space except for the $f'_0(1500)$, where the results are very sensitive to the flavor structure of this meson. In particular the decays into $\rho\gamma$ and $\omega\gamma$ are sensitive to the $n\bar{n}$, the $\Phi$ to the $s\bar{s}$ component of the wave function.

An outstanding feature of the scalar mesons is their peculiar strong decay pattern. As a full calculation of hadronic decay amplitudes in the Salpeter formalism is tuff and still under investigation, we will merely quote the calculated mixing angle of the scalar states, which can be used to estimate the branching ratios, see [3]. The mixing of the singlet and octet $f_0$ is parameterized as

$$|f_0⟩ = \sin(\Theta_S) |f_{0,8}⟩ + \cos(\Theta_S) |f_{0,1}⟩$$

$$|f'_0⟩ = \cos(\Theta_S) |f_{0,8}⟩ - \sin(\Theta_S) |f_{0,1}⟩$$

(7)

and use the relative $n\bar{n}$ and $s\bar{s}$ amplitudes given in Table I to estimate $\Theta_S$. The admixture of the $n\bar{n}$ component leads to a mixing angle of approximately $\Theta_S = 6^\circ$ for $f'_0$, which already decreases the $K\bar{K}$ amplitude by a factor of 4 compared to a purely $s\bar{s}$ state ($\Theta_S = 35.3^\circ$), while increasing the $\pi\pi$-amplitude [3].
We therefore stress that the instanton interaction naturally leads to a mixing of the $n\bar{n}$ and $s\bar{s}$ component of the $f_0'(1500)$, with the tendency to suppress $K\bar{K}$ decays, although the effect quantitatively does not suffice to describe the unusual decay pattern of this meson. Therefore more theoretical effort is needed in order to arrive at a quantitative quark model prediction. We also encourage experiments, which quantify the partial width especially into $K\bar{K}$. In addition, a measurement the decay properties of the $a_0(1450)$ could decide, whether it is the isovector partner of the $f_0(1300)$ or the $f_0(1500)$, and whether our interpretation of the scalar meson nonet is correct.

**IV. CONCLUSION**

We have presented a relativistic model of mesons, which reproduces the meson mass spectrum and sheds new light on the structure of the scalar meson nonet. The $a_0(980)$ is interpreted as $K\bar{K}$ molecule or threshold effect; there are two isoscalar resonances, the narrow $f_0(980)$ and the broad $f_0(1000)$. One of is supposed to form the flavor–singlet state of the scalar nonet while the $f_0(1500)$ is considered as flavor octet state. We prefer to interpret the $f_0(980)$ as $q\bar{q}$ state, because of its strong coupling to $\bar{K}K$. The scalar mesons are governed dynamically by 't Hooft's instanton-induced interaction. This force, which solved naturally the $\eta-\eta'$ puzzle, thus also explains quantitatively the unusual pattern of the scalar mesons consisting of an almost SU(3) octet at about 1400 MeV and a low lying SU(3) singlet at 1000 MeV.

We presented results for the modification of the two photon widths coming from the flavor mixing due to the instanton force, which may serve for an experimental verification of our model. In addition, this concept has to be further explored in the description of the strong decay widths, for which work is in progress.

The identification of the calculated states with experimental data on the basis of the present model is not yet conclusive: Nevertheless, we do believe, that the scalar particles, very much like the pseudoscalars, exhibit large splitting and mixing properties reflecting instanton effects, and that this should be considered also when invoking other mechanisms such as mixing with glueballs and multi-meson states.

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FIG. 1. Schematic splitting of pseudoscalar flavor nonets with confinement interaction (left), with confinement and instanton-induced force (middle) compared to the compilation by the Particle Data Group [1] (right).

FIG. 2. Schematic splitting of pseudoscalar flavor nonets with confinement interaction (left), with confinement and instanton-induced force (middle) compared to the experimental spectrum interpreted as $q\bar{q}$ states [12] (right).
TABLE I. Calculated $\gamma\gamma$-widths in eV with and without instanton-induced interaction compared to experimental data [1]

| Process                  | Full Calculation | Confinement Only | Experimental [1] |
|--------------------------|------------------|------------------|------------------|
| $f_0(980) \rightarrow \gamma\gamma$ | 1810             | 3400             | $560 \pm 110 \ (f_0(980))$ |
|                          |                  |                  | $5400 \pm 2300 \ (f_0(1300))$ |
| $f_0(1500) \rightarrow \gamma\gamma$ | 155              | 280              |                  |
| $a_0(1450) \rightarrow \gamma\gamma$ | 1370             | 1230             |                  |
| $f_0(980) \rightarrow \rho\gamma$     | 14.9             | 190              |                  |
| $f_0(980) \rightarrow \omega\gamma$   | 1.57             | 20               |                  |
| $f'_0(1500) \rightarrow \rho\gamma$   | 160              | 0                |                  |
| $f'_0(1500) \rightarrow \omega\gamma$ | 17.4             | 0                |                  |
| $f'_0(1500) \rightarrow \Phi\gamma$   | 87               | 101              |                  |
| $a_0(1450) \rightarrow \rho\gamma$    | 34               | 21               |                  |
| $a_0(1450) \rightarrow \omega\gamma$  | 290              | 180              |                  |
| $K_0(1430) \rightarrow K^*\gamma$     | 190              | 124              |                  |

TABLE II. $f_0, f'_0$ mixing parameters from relativistic norm

| Meson    | X    | Y    |
|----------|------|------|
| $f_0(980)$ | 0.92 | 0.40 |
| $f'_0(1500)$ | 0.48 | -0.88 |