EFFECT OF TENON DEPTH ON FLEXIBILITY OF MORTISE AND TENON JOINT

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ABSTRACT
This paper presents numerical analysis of the effects of tenon depth on flexibility of mortise and tenon joints. Numerical calculations are carried out with a linear elastic model for orthotropic material. Mathematical model is solved by finite element method. The results of the calculation indicate that a mortise and tenon joint becomes stiffer as tenon depth is increased. A satisfactory agreement was found between the experimental data taken from literature and obtained results, thus confirming the conclusions that were made.

Keywords: wood, mortise and tenon joint, finite element method, stiffness

1. INTRODUCTION
Frame structure represents the most widely used type of furniture constructions. Common construction manufactured by connecting wood elements with shape-adhesive joints. The stiffness and strength of frame structure depend on the properties of the employed joints. In order to be able to carry out analysis and optimization of construction it is necessary to know the joints properties. Attempts are being made to analysis and finding solutions that would improve joint properties [1,2,3].

2. RESEARCH OBJECTIVE AND METHODOLOGY
In the common structural analysis of a furniture frame, joints are modelled considering some idealizations. The joints are assumed to be ideally rigid or pinned. In fact, most joints in real wood structures are more or less flexible or semi-rigid. This study undertook to employ numerical method for the analysis of mortise and tenon joint stiffness. The objective was to determine the effects of tenon depth on flexibility of mortise and tenon T-type joints. Physical model (configuration, measurements and material, except glue type) of joint and loading diagram used in this study are taken from literature [3]. Results of rotational stiffness obtained in this study are compared with experimental values.

2.1. Mathematical model
The equation of momentum balance, expressed in the Cartesian tensor notation [4]

\[
\int_{S} \sigma_{ij} n_{j} dS + \int_{V} f_{i} dV = 0
\]

and of the constitutive relation for the elastic material

\[
\sigma_{ij} = C_{ijkl} \varepsilon_{kl} = \frac{1}{2} C_{ijkl} \left( \frac{\partial u_{i}}{\partial x_{j}} + \frac{\partial u_{j}}{\partial x_{i}} \right)
\]
describe the stress and strain of a loaded solid body in the static equilibrium. In the equations above, \( x_i \) are Cartesian spatial coordinates, \( V \) is the volume of solution domain bounded by the surface \( S \), \( \sigma_{ij} \) is the stress tensor, \( n_i \) is the outward unit normal to the surface \( S \), \( f_i \) the volume force, \( C_{ijkl} \) the elastic constant tensor components, \( \varepsilon_{kl} \) the strain tensor, and \( u_k \) the point displacement.

In order to complete the mathematical model, the boundary conditions have to be specified. The surface traction \( f_S \) and/or the displacement \( u_S \) at the domain boundaries are known, i.e.

\[
\sigma_{ij} n_j = f_{i_S} \quad \text{and} \quad u_i = u_{S_i} \quad \ldots \quad (3)
\]

Governing equations (1) combined with the constitutive relations (2) are solved by the numerical method based on the finite element. Calculations in this study are performed by employing the software package CATIA.

2.2. Physical model

Rectangular mortise and tenon joint was selected for examination. Geometries and measurements of the T-type end to side grain joints are shown in Fig. 1. and Tab. 1. All variables except tenon depth were held constant. Tenon depth varied from 12.7 mm to 76.2 mm.

| Tenon, mm | Mortise, mm |
|-----------|-------------|
| h_t       | h_m         |
| d_t       | d_m         |
| l_t+3,87  | l_m         |
| 12.7      | 12.9        |
| 25.4      | 25.6        |
| 38.1      | 38.3        |
| 50.8      | 51          |
| 63.5      | 63.7        |
| 76.2      | 76.4        |

A gap 0.1 mm tick was placed between the tenon and the mortise in which a glue bond was formed. The shoulder of the tenon (rail) was contact connected to the wall of the mortise (post). Calculation was carried out for maple wood (Acer saccharum Marsh.). Its elastic properties for wood density...
\( \rho = 0.57 \text{ g/cm}^3 \) and moisture content of 12% are presented in Tab. 2 [5]. Another material component is polyvinyl acetate (PVAC) glue. Selected elastic properties of the glue are \( E = 465.74 \text{ MPa} \) and \( \nu = 0.29 \) [2]. Loading diagram of the mortise and tenon joints is shown in Fig. 2. The post of joint was fixed at upper and lower end. Translation of the post lateral surface points was fixed in restrained direction \( x \). Loaded end of the rail was at a distance of 304.8 mm from the face of the post. Appropriate value (1/3 of ultimate load [3]) of bending moment was transformed into shear stress on rail cross section surface. Values of the bending moment acting on the joint and appropriate shear stress are presented in Tab. 3.

3. RESULTS

The results of the numerical calculation revealed that the tenon underwent rotation and deflection. Points at the upper tenon surface slipped outside the mortise and are moved downwards, while points at the bottom tenon surface slipped inside the mortise and moved downwards, Fig 3. The effect of the interaction between the rail and post is results both from the bending of the tenon and torsion of the glue line.

![Figure 3. Joint deformations - tenon depth: 12.7 mm (up), 76.2 mm (down; symmetry plane); scale 20:1](image)

![Figure 4. Position of the selected points](image)

The effect of the point position in the upper and bottom edge shoulder of tenon on the change in the point displacement was assessed. Position of the selected points is shown in Fig. 4. The rotation that occurs in semi-rigid joints is described by rotational stiffness \( R \). Angle change of selected joints was evaluated through the use of displacement results and rail width, Fig. 4. Values of the rotational stiffness were determined by means of the expression:

\[
R = \frac{M}{\phi} = \frac{H}{u_{up} + u_{bp}} \cdot M
\]

where: \( M \) - the bending moment acting on the joint (Nm), \( \phi \) - the angle change resulting from the joint behaviour, \( u_{up} \) and \( u_{bp} \) - the absolute values of the upper and bottom point \( u \) displacement (mm) and \( H \) - the rail with (mm).

The effect of the tenon depth on the change in rotational stiffness \( R \) was assessed. Values of rotational stiffness \( R \) for the six tenon depth are given in Tab. 4 and shown in Fig. 5. Description of test and detailed experimental results can be found in [3].

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These results indicate that the joint become increasingly stiffer as tenon depth is increased. For tenon depth 12.7 mm, the joint reached 9.7% (according to experimental results), 21.2% (symmetry plane) and 25.5% (outer plane) of the rotational stiffness it would have if the tenon depth was 76.2 mm. Differences between rotational stiffness obtained experimentally and numerical calculation are wide ranged.

Table 4. Rotational stiffness $R$ for the six tenon depth

| Tenon depth, mm | Exp. results* | Numerical results | Differ., % |
|-----------------|---------------|-------------------|------------|
| 12.7            | 5250.93       | 12310.70          | 134.45     |
| 25.4            | 8547.66       | 16193.77          | 89.45      |
| 38.1            | 1587.52       | 21311.57          |            |
| 50.8            | 26151.35      | 28347.76          | 8.40       |
| 63.5            | 28826.53      | 36701.97          | 27.32      |
| 76.2            | 54066.99      |                   |            |

* Experimental results are taken from [3]; ** Fault

4. CONCLUSION

The performed investigations revealed that the numerical procedure used in the study provide a convenient method of obtaining the information needed for determining rotational stiffness for shape-adhesive wood joints. The research model could be used for optimization of joints in wood constructions. Many variables affecting the stiffness of joints could be evaluated individually or it could be analysed the interaction of variables on the joint properties. The knowledge of joint properties brings product engineers to simulate real behaviour wood structure under service loads.

On the basis of the performed numerical calculation, the following conclusions were drawn. It is evident from the analysis of rotational stiffness that the joints become stiffer as tenon depth is increased. Similarity between experimental and numerical results of rotational stiffness allows conclusion that the research model was designed correctly.

5. REFERENCES

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