ABC-LogitBoost for Multi-class Classification

Ping Li
Department of Statistical Science
Faculty of Computing and Information Science
Cornell University
Ithaca, NY 14853
pingli@cornell.edu

Abstract
We develop abc-logitboost, based on the prior work on abc-boost [10] and robust logitboost [11]. Our extensive experiments on a variety of datasets demonstrate the considerable improvement of abc-logitboost over logitboost and abc-mart.

1 Introduction
Boosting\textsuperscript{1} algorithms [14, 4, 5, 2, 15, 7, 13, 6] have become very successful in machine learning. This study revisits logitboost [7] under the framework of adaptive base class boost (abc-boost) in [10], for multi-class classification.

We denote a training dataset by \( \{y_i, x_i\}_{i=1}^{N} \), where \( N \) is the number of feature vectors (samples), \( x_i \) is the \( i \)th feature vector, and \( y_i \in \{0, 1, 2, ..., K-1\} \) is the \( i \)th class label, where \( K \geq 3 \) in multi-class classification.

Both logitboost [7] and mart (multiple additive regression trees) [6] algorithms can be viewed as generalizations to the logistic regression model, which assumes the class probabilities \( p_{i,k} \) to be

\[
p_{i,k} = \Pr(y_i = k | x_i) = \frac{e^{F_{i,k}(x_i)}}{\sum_{s=0}^{K-1} e^{F_{i,s}(x_i)}}, \quad i = 1, 2, ..., N, \tag{1}
\]

While traditional logistic regression assumes \( F_{i,k}(x_i) = \beta^T x_i \), logitboost and mart adopt the flexible “additive model,” which is a function of \( M \) terms:

\[
F^{(M)}(x) = \sum_{m=1}^{M} \rho_m h(x; a_m), \tag{2}
\]

where \( h(x; a_m) \), the base learner, is typically a regression tree. The parameters, \( \rho_m \) and \( a_m \), are learned from the data, by maximum likelihood, which is equivalent to minimizing the negative log-likelihood loss

\[
L = \sum_{i=1}^{N} L_i, \quad L_i = -\sum_{k=0}^{K-1} r_{i,k} \log p_{i,k} \tag{3}
\]

where \( r_{i,k} = 1 \) if \( y_i = k \) and \( r_{i,k} = 0 \) otherwise.

For identifiability, the “sum-to-zero” constraint, \( \sum_{k=0}^{K-1} F_{i,k} = 0 \), is usually adopted [7, 6, 17, 9, 16, 18].

1.1 Logitboost
As described in Alg. 1, [7] builds the additive model (2) by a greedy stage-wise procedure, using a second-order (diagonal) approximation, which requires knowing the first two derivatives of the loss function (3) with respective

\textsuperscript{1}The idea of abc-logitboost was included in an unfunded grant proposal submitted in early December 2008.
to the function values $F_{i,k}$ \cite{7} obtained:

$$\frac{\partial L_i}{\partial F_{i,k}} = -(r_{i,k} - p_{i,k}), \quad \frac{\partial^2 L_i}{\partial F_{i,k}^2} = p_{i,k} (1 - p_{i,k}).$$

Those derivatives can be derived by assuming no relations among $F_{i,k}$, $k = 0$ to $K - 1$. However, \cite{7} used the “sum-to-zero” constraint $\sum_{k=0}^{K-1} F_{i,k} = 0$ throughout the paper and they provided an alternative explanation. \cite{7} showed (4) by conditioning on a “base class” and noticed the resultant derivatives are independent of the particular choice of the base class.

Algorithm 1 LogitBoost\cite{7, Alg. 6}. $\nu$ is the shrinkage (e.g., $\nu = 0.1$).

0: $r_{i,k} = 1$, if $y_i = k$, $r_{i,k} = 0$ otherwise.
1: $F_{i,k} = 0$, $p_{i,k} = \frac{1}{K}$, $k = 0$ to $K - 1$, $i = 1$ to $N$
2: For $m = 1$ to $M$ Do
3: For $k = 0$ to $K - 1$, Do
4: Compute $w_{i,k} = p_{i,k} (1 - p_{i,k})$.
5: Compute $z_{i,k} = r_{i,k} - p_{i,k} p_{i,k} (1 - p_{i,k})$.
6: Fit the function $f_{i,k}$ by a weighted least-square of $z_{i,k}$ to $x_i$ with weights $w_{i,k}$.
7: $F_{i,k} = F_{i,k} + \nu \frac{K-1}{K} \left( f_{i,k} - \frac{1}{K} \sum_{k=0}^{K-1} f_{i,k} \right)$
8: End
9: $p_{i,k} = \exp(F_{i,k}) / \sum_{s=0}^{K-1} \exp(F_{i,s})$, $k = 0$ to $K - 1$, $i = 1$ to $N$
10: End

At each stage, logitboost fits an individual regression function separately for each class. This is analogous to the popular individualized regression approach in multinomial logistic regression, which is known \cite{3, 1} to result in loss of statistical efficiency, compared to the full (conditional) maximum likelihood approach.

On the other hand, in order to use trees as base learner, the diagonal approximation appears to be a must, at least from the practical perspective.

1.2 Adaptive Base Class Boost

\cite{10} derived the derivatives of (3) under the sum-to-zero constraint. Without loss of generality, we can assume that class 0 is the base class. For any $k \neq 0$,

$$\frac{\partial L_i}{\partial F_{i,k}} = (r_{i,0} - p_{i,0}) - (r_{i,k} - p_{i,k}), \quad \frac{\partial^2 L_i}{\partial F_{i,k}^2} = p_{i,0} (1 - p_{i,0}) + p_{i,k} (1 - p_{i,k}) + 2 p_{i,0} p_{i,k}.$$  \hspace{1cm} (5)

The base class must be identified at each boosting iteration during training. \cite{10} suggested an exhaustive procedure to adaptively find the best base class to minimize the training loss (3) at each iteration.

\cite{10} combined the idea of abc-boost with mart. The algorithm, abc-mart, achieved good performance in multi-class classification on the datasets used in \cite{10}.

1.3 Our Contributions

We propose abc-logitboost, by combining abc-boost with robust logitboost\cite{11}. Our extensive experiments will demonstrate that abc-logitboost can considerably improve logitboost and abc-mart on a variety of datasets.

2 Robust Logitboost

Our work is based on robust logitboost\cite{11}, which differs from the original logitboost algorithm. Thus, this section provides an introduction to robust logitboost.

\cite{6, 8} commented that logitboost (Alg. 1) can be numerically unstable. The original paper\cite{7} suggested some “crucial implementation protections” on page 17 of \cite{7}:
In Line 5 of Alg. 1, compute the response \( z_{i,k} \) by \( \frac{1}{p_{i,k}} \) (if \( r_{i,k} = 1 \)) or \( \frac{1}{1-p_{i,k}} \) (if \( r_{i,k} = 0 \)).

- Bound the response \( |z_{i,k}| \) by \( z_{\text{max}} \in [2, 4] \).

Note that the above operations are applied to each individual sample. The goal is to ensure that the response \( |z_{i,k}| \) is not too large (Note that \( |z_{i,k}| > 1 \) always). On the other hand, we should hope to use larger \( |z_{i,k}| \) to better capture the data variation. Therefore, the thresholding occurs very frequently and it is expected that some of the useful information is lost.

[11] demonstrated that, if implemented carefully, logitboost is almost identical to \( \text{mart} \). The only difference is the tree-splitting criterion.

### 2.1 The Tree-Splitting Criterion Using the Second-Order Information

Consider \( N \) weights \( w_{i} \), and \( N \) response values \( z_{i}, i = 1 \) to \( N \), which are assumed to be ordered according to the sorted order of the corresponding feature values. The tree-splitting procedure is to find the index \( s \), \( 1 \leq s < N \), such that the weighted mean square error (MSE) is reduced the most if split at \( s \). That is, we seek \( s \) to maximize

\[
\text{Gain}(s) = \text{MSE}_T - (\text{MSE}_L + \text{MSE}_R) = \sum_{i=1}^{N} (z_i - \bar{z})^2 w_i - \left( \sum_{s=1}^{s} (z_i - \bar{z}_L)^2 w_i + \sum_{i=s+1}^{N} (z_i - \bar{z}_R)^2 w_i \right)
\]

where \( \bar{z} = \frac{\sum_{i=1}^{N} z_i w_i}{\sum_{i=1}^{N} w_i} \), \( \bar{z}_L = \frac{\sum_{i=1}^{s} z_i w_i}{\sum_{i=1}^{s} w_i} \), and \( \bar{z}_R = \frac{\sum_{i=s+1}^{N} z_i w_i}{\sum_{i=s+1}^{N} w_i} \). After simplification, we obtain

\[
\text{Gain}(s) = \left[ \frac{\sum_{i=1}^{s} z_i w_i}{\sum_{i=1}^{s} w_i} \right]^2 + \left[ \frac{\sum_{i=s+1}^{N} z_i w_i}{\sum_{i=s+1}^{N} w_i} \right]^2 - \left[ \frac{\sum_{i=1}^{N} z_i w_i}{\sum_{i=1}^{N} w_i} \right]^2
\]

Plugging in \( w_{i} = p_{i,k}(1 - p_{i,k}) \), and \( z_{i} = \frac{r_{i,k} - p_{i,k}}{p_{i,k}(1 - p_{i,k})} \) as in Alg. 1, yields

\[
\text{Gain}(s) = \left[ \frac{\sum_{i=1}^{s} r_{i,k} - p_{i,k}}{\sum_{i=1}^{s} p_{i,k}(1 - p_{i,k})} \right]^2 + \left[ \frac{\sum_{i=s+1}^{N} r_{i,k} - p_{i,k}}{\sum_{i=s+1}^{N} p_{i,k}(1 - p_{i,k})} \right]^2 - \left[ \frac{\sum_{i=1}^{N} r_{i,k} - p_{i,k}}{\sum_{i=1}^{N} p_{i,k}(1 - p_{i,k})} \right]^2
\]

Because the computations involve \( \sum_{i=1}^{s} p_{i,k}(1 - p_{i,k}) \) as a group, this procedure is actually numerically stable.

In comparison, \( \text{mart} [6] \) only used the first order information to construct the trees, i.e.,

\[
\text{MARTGain}(s) = \left[ \sum_{i=1}^{s} r_{i,k} - p_{i,k} \right]^2 + \left[ \sum_{i=s+1}^{N} r_{i,k} - p_{i,k} \right]^2 - \left[ \sum_{i=1}^{N} r_{i,k} - p_{i,k} \right]^2
\]

### 2.2 The Robust Logitboost Algorithm

**Algorithm 2** Robust logitboost, which is very similar to \( \text{mart} \), except for Line 4.

1. \( F_{i,k} = 0, p_{i,k} = \frac{1}{K}, k = 0 \) to \( K - 1 \), \( i = 1 \) to \( N \)
2. For \( m = 1 \) to \( M \) Do
3. For \( k = 0 \) to \( K - 1 \) Do
4. \( \{ R_{j,k,m} \}_{j=1}^{J} = J \)-terminal node regression tree from \( \{ r_{i,k} - p_{i,k}, x_{i} \}_{i=1}^{N} \), with weights \( p_{i,k}(1 - p_{i,k}) \) as in Section 2.1.
5. \( \beta_{j,k,m} = \frac{K-1}{K} \frac{\sum_{x_{i} \in R_{j,k,m}} r_{i,k} - p_{i,k}}{\sum_{x_{i} \in \overline{R}_{j,k,m}} (1 - p_{i,k}) p_{i,k}} \)
6. \( F_{i,k} = F_{i,k} + \nu \sum_{j=1}^{J} \beta_{j,k,m} 1_{x_{i} \in R_{j,k,m}} \)
7. End
8. \( p_{i,k} = \exp(F_{i,k}) / \sum_{s=0}^{K-1} \exp(F_{i,s}), k = 0 \) to \( K - 1 \), \( i = 1 \) to \( N \)
9. End
Alg. 2 describes \textit{robust logitboost} using the tree-splitting criterion developed in Section 2.1. Note that after trees are constructed, the values of the terminal nodes are computed by

\[
\sum_{\text{node}} z_{i,k} w_{i,k} = \sum_{\text{node}} p_{i,k} - \frac{\sum_{\text{node}} r_{i,k}}{\sum_{\text{node}} p_{i,k} (1 - p_{i,k})},
\]

which explains Line 5 of Alg. 2.

### 2.2.1 Three Main Parameters: $J$, $\nu$, and $M$

Alg. 2 has three main parameters, to which the performance is not very sensitive, as long as they fall in some reasonable range. This is a very significant advantage in practice. The number of terminal nodes, $J$, determines the capacity of the base learner. [6] suggested $J = 6$. [7, 18] commented that $J > 10$ is unlikely. In our experience, for large datasets (or moderate datasets in high-dimensions), $J = 20$ is often a reasonable choice; also see [12].

The shrinkage, $\nu$, should be large enough to make sufficient progress at each step and small enough to avoid over-fitting. [6] suggested $\nu \leq 0.1$. Normally, $\nu = 0.1$ is used.

The number of iterations, $M$, is largely determined by the affordable computing time. A commonly-regarded merit of boosting is that over-fitting can be largely avoided for reasonable $J$ and $\nu$.

### 3 Adaptive Base Class Logitboost

**Algorithm 3 Abc-logitboost** using the exhaustive search strategy for the base class, as suggested in [10]. The vector $B$ stores the base class numbers.

1: $F_{i,k} = 0$, $p_{i,k} = \frac{1}{K}$, $k = 0$ to $K - 1$, $i = 1$ to $N$
2: For $m = 1$ to $M$ Do
3: For $b = 0$ to $K - 1$, Do
4: For $k = 0$ to $K - 1$, $k \neq b$, Do
5: \[ (R_{j,k,m})^j = \{ - (r_{i,k} - p_{i,k}) + (r_{i,k} - p_{i,k}), \quad \forall i \}_{i=1}^N \]
   : with weights $p_{i,b}(1 - p_{i,b}) + p_{i,k}(1 - p_{i,k}) + 2p_{i,k}p_{i,b}$, as in Section 2.1.
6: \[ \beta_{j,k,m} = \frac{\sum_{i,j,k,m} x_{i} r_{i,k,m} p_{i,b} (1 - p_{i,b}) + p_{i,k} (1 - p_{i,k}) + 2p_{i,k}p_{i,b}}{\sum_{i,j,k,m} x_{i} r_{i,k,m} p_{i,b} (1 - p_{i,b}) + p_{i,k} (1 - p_{i,k}) + 2p_{i,k}p_{i,b}} \]
7: \[ G_{i,b,k} = F_{i,k} + \nu \sum_{j=1}^{J} \beta_{j,k,m} x_{i} \in R_{j,k,m} \]
8: End
9: $G_{i,b,k} = - \sum_{k \neq b} G_{i,k,b}$
10: $q_{i,k} = \exp(G_{i,k,b})/\sum_{k=0}^{K-1} \exp(G_{i,k,b})$
11: $L^{(b)} = - \sum_{i=1}^{N} \sum_{k=0}^{K-1} r_{i,k} \log (q_{i,k})$
12: End
13: $B(m) = \arg\min_b L^{(b)}$
14: $F_{i,k} = G_{i,k,B(m)}$
15: $p_{i,k} = \exp(F_{i,k})/\sum_{k=0}^{K-1} \exp(F_{i,k})$
16: End

The recently proposed \textit{abc-boost} [10] algorithm consists of two key components:

1. Using the widely-used \textit{sum-to-zero} constraint [7, 6, 17, 9, 16, 18] on the loss function, one can formulate boosting algorithms only for $K - 1$ classes, by using one class as the base class.

2. At each boosting iteration, adaptively select the base class according to the training loss. [10] suggested an exhaustive search strategy.

[10] combined \textit{abc-boost} with \textit{mart} to develop \textit{abc-mart}. [10] demonstrated the good performance of \textit{abc-mart} compared to \textit{mart}. This study will illustrate that \textit{abc-logitboost}, the combination of \textit{abc-boost} with (robust) \textit{logitboost}, will further reduce the test errors, at least on a variety of datasets.
Alg. 3 presents *abc-logitboost*, using the derivatives in (5) and the same exhaustive search strategy as in *abc-mart*. Again, *abc-logitboost* differs from *abc-mart* only in the tree-splitting procedure (Line 5 in Alg. 3).

4 Experiments

Table 1 lists the datasets in our experiments, which include all the datasets used in [10], plus Mnist10k.

Table 1: For *Letter*, *Pendigits*, *Zipcode*, *Optdigits* and *Isolet*, we used the standard (default) training and test sets. For *Covertype*, we use the same split in [10]. For Mnist10k, we used the original 10000 test samples in the original Mnist dataset for training, and the original 60000 training samples for testing. Also, as explained in [10], *Letter2k* (*Letter4k*) used the last 2000 (4000) samples of *Letter* for training and the remaining 18000 (16000) for testing, from the original *Letter* dataset.

| dataset     | K | # training | # test  | # features |
|-------------|---|------------|---------|------------|
| Covertype   | 7 | 290506     | 290506  | 54         |
| Mnist10k    | 10| 10000      | 60000   | 784        |
| Letter2k    | 26| 2000       | 18000   | 16         |
| Letter4k    | 26| 4000       | 16000   | 16         |
| Letter      | 26| 16000      | 4000    | 16         |
| Pendigits   | 10| 7494       | 3498    | 16         |
| Zipcode     | 10| 7291       | 2007    | 256        |
| Optdigits   | 10| 3823       | 1797    | 64         |
| Isolet      | 26| 6218       | 1559    | 617        |

Note that *Zipcode*, *Optdigits*, and *Isolet* are very small datasets (especially the testing sets). They may not necessarily provide a reliable comparison of different algorithms. Since they are popular datasets, we nevertheless include them in our experiments.

Recall *logitboost* has three main parameters, $J$, $\nu$, and $M$. Since overfitting is largely avoided, we simply let $M = 10000$ ($M = 5000$ only for *Covertype*), unless the machine zero is reached. The performance is not sensitive to $\nu$ (as long as $\nu \leq 0.1$). The performance is also not too sensitive to $J$ in a good range.

Ideally, we would like to show that, for every reasonable combination of $J$ and $\nu$ (using $M$ as large as possible), *abc-logitboost* exhibits consistent improvement over (robust) *logitboost*. For most datasets, we experimented with every combination of $J \in \{4, 6, 8, 10, 12, 14, 16, 18, 20\}$ and $\nu \in \{0.04, 0.06, 0.08, 0.1\}$.

We provide a summary of the experiments after presenting the detailed results on Mnist10k.

4.1 Experiments on the Mnist10k Dataset

For this dataset, we experimented with every combination of $J \in \{4, 6, 8, 10, 12, 14, 16, 18, 20\}$ and $\nu \in \{0.04, 0.06, 0.08, 0.1\}$. We trained till the loss (3) reached the machine zero, to exhaust the capacity of the learner so that we could provide a reliable comparison, up to $M = 10000$ iterations.

Figures 1 and 2 present the mis-classification errors for every $\nu$, $J$, and $M$:

- Essentially no overfitting is observed, especially for *abc-logitboost*. This is why we simply report the smallest test error in Table 2.
- The performance is not sensitive to $\nu$.
- The performance is not very sensitive to $J$, for $J = 8$ to 20.

Interestingly, *abc-logitboost* sometimes needed more iterations to reach machine zero than *logitboost*. This can be explained in part by the fact that the “$\nu$” in *logitboost* is not precisely the same “$\nu$” in *abc-logitboost* [10]. This is also why we would like to experiment with a range of $\nu$ values.

---

2We also did limited experiments on the original Mnist dataset (i.e., 60000 training samples and 10000 testing samples), the test mis-classification error rate was about 1.3%.
Table 2 summarizes the smallest test mis-classification errors along with the relative improvements (denoted by \( R_{err} \)) of abc-logitboost over logitboost. For most \( J \) and \( \nu \), abc-logitboost exhibits about \( R_{err} = 12 \sim 15(\%) \) smaller test mis-classification errors than logitboost. The \( P \)-values range from \( 1.9 \times 10^{-10} \) to \( 3.9 \times 10^{-5} \), although they are not reported in Table 2.

Table 2: Mnist10k. The test mis-classification errors of logitboost and abc-logitboost, along with the relative improvement \( R_{err} (\%) \). For each \( J \) and \( \nu \), we report the smallest values in Figures 1 and 2. Each cell contains three numbers, which are logitboost error, abc-logitboost error, and relative improvement \( R_{err} (\%) \).

| \( \nu \) | \( J = 4 \) | \( J = 6 \) | \( J = 8 \) | \( J = 10 \) | \( J = 12 \) | \( J = 14 \) | \( J = 16 \) | \( J = 18 \) | \( J = 20 \) |
|-------|------|------|------|------|------|------|------|------|------|
| 0.04  | 2911 | 2623 | 2536 | 2486 | 2435 | 2399 | 2341 | 2384 | 2307 |
| 0.06  | 2658 | 2255 | 2541 | 2472 | 2435 | 2399 | 2397 | 2384 | 2307 |
| 0.08  | 2884 | 2244 | 2521 | 2447 | 2424 | 2407 | 2405 | 2404 | 2404 |
| 0.10  | 2876 | 2224 | 2517 | 2444 | 2424 | 2407 | 2405 | 2404 | 2404 |

The original abc-boost paper[10] did not include experiments on Mnist10k. Thus, in this study, Table 3 summarizes the smallest test mis-classification errors for mart and abc-mart. Again, we can see very consistent and considerable improvement of abc-mart over mart. Also, comparing Tables 2 and 3, we can see that abc-logitboost also significantly improves abc-mart.

Table 3: Mnist10k. The test mis-classification errors of mart and abc-mart, along with the relative improvement \( R_{err} (\%) \). For each \( J \) and \( \nu \), we report the smallest values in Figures 1 and 2. Each cell contains three numbers, which are mart error, abc-mart error, and relative improvement \( R_{err} (\%) \).

| \( \nu \) | \( J = 4 \) | \( J = 6 \) | \( J = 8 \) | \( J = 10 \) | \( J = 12 \) | \( J = 14 \) | \( J = 16 \) | \( J = 18 \) | \( J = 20 \) |
|-------|------|------|------|------|------|------|------|------|------|
| 0.04  | 3346 | 3054 | 3040 | 2979 | 2912 | 2885 | 2852 | 2831 |
| 0.06  | 3308 | 3009 | 3012 | 2941 | 2897 | 2879 | 2860 | 2833 |
| 0.08  | 3302 | 3009 | 3000 | 2957 | 2906 | 2874 | 2865 | 2833 |
| 0.10  | 3287 | 2927 | 2993 | 2947 | 2887 | 2864 | 2852 | 2813 |
Figure 1: **Mnist10k**. The test mis-classification errors, for logitboost and abc-logitboost. $J = 12$ to 20.
Figure 2: **Mnist10k.** The test mis-classification errors, for logitboost and abc-logitboost. $J = 4$ to 10.
4.2 Summary of Test Mis-Classification Errors

Table 4 summarizes the test errors, which are the overall best (smallest) test mis-classification errors. In the table, \( R_{err} (%) \) is the relative improvement of test performance. The \( P \)-values tested the statistical significance if \textit{abc-logitboost} achieved smaller error rates than \textit{logitboost}.

To compare \textit{abc-logitboost} with \textit{abc-mart}, Table 4 also includes the test errors for \textit{abc-mart} and the \( P \)-values (i.e., \( P \)-value (2)) for testing the statistical significance if \textit{abc-logitboost} achieved smaller error rates than \textit{abc-mart}. The comparisons indicate that there is a clear performance gap between \textit{abc-logitboost} and \textit{abc-mart}, especially on the large datasets.

Table 4: Summary of test mis-classification errors.

| Dataset  | logit | abc-logit | \( R_{err} \) (%) | \( P \)-value | abc-mart | \( P \)-value (2) |
|----------|-------|-----------|-------------------|--------------|----------|-------------------|
| Covertype| 10759 | 9693      | 9.9               | \( 1.6 \times 10^{-4} \) | 10375    | \( 4.8 \times 10^{-7} \) |
| Mnist10k | 2357  | 2048      | 13.1              | \( 1.0 \times 10^{-6} \) | 2425     | \( 4.6 \times 10^{-9} \) |
| Letter2k | 2257  | 1984      | 12.1              | \( 4.0 \times 10^{-6} \) | 2180     | \( 6.2 \times 10^{-4} \) |
| Letter4k | 1220  | 1031      | 15.5              | \( 1.8 \times 10^{-5} \) | 1126     | 0.017             |
| Letter   | 107   | 89        | 16.8              | \( 9.7 \times 10^{-3} \) | 99       | 0.23              |
| Pendigits| 109   | 90        | 17.4              | \( 8.6 \times 10^{-3} \) | 100      | 0.23              |
| Zipcode  | 103   | 92        | 10.7              | 0.21         | 100      | 0.28              |
| Optdigits| 49    | 38        | 22.5              | 0.11         | 43       | 0.29              |
| Isolet   | 62    | 55        | 11.3              | 0.25         | 64       | 0.20              |

4.3 Experiments on the \textit{Covertype} Dataset

Table 5 summarizes the smallest test mis-classification errors of \textit{logitboost} and \textit{abc-logitboost}, along with the relative improvements (\( R_{err} \)). Since this is a fairly large dataset, we only experimented with \( \nu = 0.1 \) and \( J = 10 \) and 20.

Table 5: \textit{Covertype}. We report the test mis-classification errors of \textit{logitboost} and \textit{abc-logitboost}, together with the relative improvements (\( R_{err} \), \%) in parentheses.

| \( \nu \) | \( M \) | \( J \) | logit | abc-logit |
|----------|-------|-------|-------|-----------|
| 0.1      | 1000  | 10    | 29865 | 23774 (20.4) |
| 0.1      | 1000  | 20    | 19443 | 14443 (25.7) |
| 0.1      | 2000  | 10    | 21620 | 16991 (21.4) |
| 0.1      | 2000  | 20    | 13914 | 11336 (18.5) |
| 0.1      | 3000  | 10    | 17805 | 14295 (19.7) |
| 0.1      | 3000  | 20    | 12076 | 10399 (13.9) |
| 0.1      | 5000  | 10    | 14698 | 12185 (17.1) |
| 0.1      | 5000  | 20    | 10759 | 9693 (9.9)   |

The results on \textit{Covertype} are reported differently from other datasets. \textit{Covertype} is fairly large. Building a very large model (e.g., \( M = 5000 \) boosting steps) would be expensive. Testing a very large model at run-time can be costly or infeasible for certain applications (e.g., search engines). Therefore, it is often important to examine the performance of the algorithm at much earlier boosting iterations. Table 5 shows that \textit{abc-logitboost} may improve \textit{logitboost} as much as \( R_{err} \approx 20\% \), as opposed to the reported \( R_{err} = 9.9\% \) in Table 4.
4.4 Experiments on the Letter2k Dataset

Table 6: *Letter2k*. The test mis-classification errors of *logitboost* and *abc-logitboost*, along with the relative improvement $R_{err}$ (%). Each cell contains three numbers, which are *logitboost error*, *abc-logitboost error*, and relative improvement $R_{err}$ (%).

| $\nu$ = 0.04 | $\nu$ = 0.06 | $\nu$ = 0.08 | $\nu$ = 0.1 |
|---------------|---------------|---------------|---------------|
| $J = 4$ | 2576 2317 10.1 | 2535 2294 9.5 | 2545 2252 11.5 | 2523 2224 11.9 |
| $J = 6$ | 2389 2133 10.7 | 2391 2111 11.7 | 2376 2070 12.9 |
| $J = 8$ | 2325 2074 10.8 | 2299 2046 11.0 | 2298 2033 11.5 | 2271 2025 10.8 |
| $J = 10$ | 2294 2041 11.0 | 2292 1995 13.0 | 2279 2018 11.5 | 2276 2000 12.1 |
| $J = 12$ | 2314 2010 13.1 | 2304 1990 13.6 | 2311 2010 13.0 | 2268 2018 11.0 |
| $J = 14$ | 2315 2015 13.0 | 2300 2003 12.9 | 2312 2003 13.4 | 2277 2024 11.1 |
| $J = 16$ | 2302 2022 12.2 | 2394 1996 13.0 | 2276 3002 12.0 | 2257 1997 11.5 |
| $J = 18$ | 2295 2041 11.1 | 2275 2021 11.2 | 2301 1984 13.8 | 2281 2020 11.4 |
| $J = 20$ | 2280 2047 10.2 | 2267 2020 10.9 | 2294 2020 11.9 | 2306 2031 11.9 |

4.5 Experiments on the Letter4k Dataset

Table 7: *Letter4k*. The test mis-classification errors of *logitboost* and *abc-logitboost*, along with the relative improvement $R_{err}$ (%).

| $\nu$ = 0.04 | $\nu$ = 0.06 | $\nu$ = 0.08 | $\nu$ = 0.1 |
|---------------|---------------|---------------|---------------|
| $J = 4$ | 1460 1295 11.3 | 1471 1232 16.2 | 1452 1199 17.4 | 1446 1204 16.7 |
| $J = 6$ | 1390 1135 18.3 | 1394 1116 20.0 | 1382 1088 21.3 | 1374 1070 22.1 |
| $J = 8$ | 1336 1078 19.3 | 1332 1074 19.4 | 1311 1062 19.0 | 1297 1042 20.0 |
| $J = 10$ | 1289 1051 18.5 | 1285 1065 17.1 | 1280 1031 19.5 | 1273 1046 17.8 |
| $J = 12$ | 1251 1055 15.7 | 1247 1065 14.6 | 1261 1044 17.2 | 1243 1051 15.4 |
| $J = 14$ | 1247 1060 15.0 | 1233 1050 14.8 | 1251 1037 17.1 | 1244 1060 14.8 |
| $J = 16$ | 1244 1070 14.0 | 1227 1064 13.3 | 1231 1044 15.2 | 1228 1038 15.5 |
| $J = 18$ | 1243 1057 15.0 | 1250 1037 17.0 | 1234 1049 15.0 | 1220 1055 13.5 |
| $J = 20$ | 1226 1078 12.0 | 1242 1069 13.9 | 1242 1054 15.1 | 1235 1051 14.9 |
4.6 Experiments on the Letter Dataset

Table 8: Letter. The test mis-classification errors of logitboost and abc-logitboost, along with the relative improvement $R_{err}$ (%).

| $\nu = 0.04$ | $\nu = 0.06$ | $\nu = 0.08$ | $\nu = 0.1$ |
|-------------|-------------|-------------|-------------|
| $J = 4$     | 149 125     | 151 121     | 148 122     | 149 119     |
| $J = 6$     | 130 112     | 132 107     | 133 101     | 129 102     |
| $J = 8$     | 129 104     | 125 102     | 131 93      | 113 95      |
| $J = 10$    | 114 101     | 115 100     | 123 96      | 117 93      |
| $J = 12$    | 112 96      | 115 100     | 107 95      | 112 95      |
| $J = 14$    | 110 96      | 113 98      | 113 94      | 110 89      |
| $J = 16$    | 111 97      | 113 94      | 109 93      | 109 95      |
| $J = 18$    | 114 95      | 112 92      | 111 96      | 117 93      |
| $J = 20$    | 113 95      | 113 97      | 115 93      | 113 89      |

4.7 Experiments on the Pendigits Dataset

Table 9: Pendigits. The test mis-classification errors of logitboost and abc-logitboost, along with the relative improvement $R_{err}$ (%).

| $\nu = 0.04$ | $\nu = 0.06$ | $\nu = 0.08$ | $\nu = 0.1$ |
|-------------|-------------|-------------|-------------|
| $J = 4$     | 119 92      | 120 93      | 118 90      | 119 92      |
| $J = 6$     | 111 98      | 111 97      | 111 96      | 110 93      |
| $J = 8$     | 116 97      | 117 94      | 115 95      | 114 93      |
| $J = 10$    | 116 100     | 115 98      | 116 97      | 111 97      |
| $J = 12$    | 117 98      | 113 98      | 113 98      | 114 98      |
| $J = 14$    | 113 100     | 115 101     | 112 99      | 114 98      |
| $J = 16$    | 112 100     | 118 97      | 112 98      | 111 96      |
| $J = 18$    | 114 102     | 112 97      | 109 99      | 112 97      |
| $J = 20$    | 112 106     | 116 102     | 113 100     | 107 100     |
4.8 Experiments on the Zipcode Dataset

Table 10: Zipcode. The test mis-classification errors of logitboost and abc-logitboost, along with the relative improvement $R_{err} (%)$.

| $\nu$ = 0.04 | $\nu$ = 0.06 | $\nu$ = 0.08 | $\nu$ = 0.1 |
|-------------|-------------|-------------|-------------|
| $J = 4$     | 114 111 2.6 | 117 108 7.6 | 111 114 -2.7 | 115 107 7.0 |
| $J = 6$     | 109 101 7.3 | 107 102 4.6 | 106 98 7.5  | 110 99 10.0 |
| $J = 8$     | 110 99 10.0 | 108 95 12.0 | 108 96 11.1 | 108 98 9.3  |
| $J = 10$    | 111 97 12.6 | 110 94 14.5 | 106 97 8.5  | 103 94 8.7  |
| $J = 12$    | 111 98 11.7 | 112 98 12.5 | 111 99 10.8 | 108 93 13.9 |
| $J = 14$    | 112 100 10.7| 108 99 8.3  | 110 97 11.8 | 114 92 19.3 |
| $J = 16$    | 111 98 11.7 | 114 95 16.7 | 110 99 10.0 | 111 98 11.7 |
| $J = 18$    | 112 96 14.2 | 114 98 14.0 | 109 101 7.3 | 113 98 13.3 |
| $J = 20$    | 114 97 14.9 | 108 96 11.1 | 109 100 8.3 | 116 96 17.2 |

4.9 Experiments on the Optdigits Dataset

Table 11: Optdigits. The test mis-classification errors of logitboost and abc-logitboost, along with the relative improvement $R_{err} (%)$.

| $\nu$ = 0.04 | $\nu$ = 0.06 | $\nu$ = 0.08 | $\nu$ = 0.1 |
|-------------|-------------|-------------|-------------|
| $J = 4$     | 52 41 21.2  | 50 42 16.0  | 50 40 20.0  | 49 41 16.3  |
| $J = 6$     | 52 43 17.3  | 52 45 13.5  | 53 44 17.0  | 52 38 26.9  |
| $J = 8$     | 55 44 20.0  | 55 44 20.0  | 53 45 15.1  | 54 45 16.7  |
| $J = 10$    | 57 50 12.3  | 56 50 10.7  | 54 46 14.8  | 55 42 23.6  |
| $J = 12$    | 52 50 3.8   | 55 48 12.7  | 54 47 13.0  | 54 46 14.8  |
| $J = 14$    | 58 48 17.2  | 55 46 16.4  | 56 51 8.9   | 53 48 9.4   |
| $J = 16$    | 61 54 11.5  | 57 51 10.5  | 58 49 15.5  | 56 46 17.9  |
| $J = 18$    | 65 54 16.9  | 64 55 14.0  | 60 53 11.7  | 66 51 22.7  |
| $J = 20$    | 63 61 3.2   | 61 56 8.2   | 64 55 14.1  | 64 55 14.1  |

12
4.10 Experiments on the Isolet Dataset

For this dataset, [10] only experimented with $\nu = 0.1$ for mart and abc-mart. We add the experiment results for $\nu = 0.06$.

Table 12: Isolet. The test mis-classification errors of logitboost and abc-logitboost, along with the relative improvement $R_{err}$ (%).

| $J$ | $\nu = 0.06$ | $\nu = 0.1$ |
|-----|-------------|-------------|
| 4   | 65          | 55 15.4     |
| 6   | 67          | 59 11.9     |
| 8   | 72          | 57 20.8     |
| 10  | 73          | 61 16.4     |
| 12  | 75          | 63 16.0     |
| 14  | 74          | 65 12.2     |
| 16  | 70          | 64 8.6      |
| 18  | 74          | 67 9.5      |
| 20  | 71          | 63 11.3     |

Table 13: Isolet. The test mis-classification errors of mart and abc-mart, along with the relative improvement $R_{err}$ (%).

| $J$ | $\nu = 0.06$ | $\nu = 0.1$ |
|-----|-------------|-------------|
| 4   | 81          | 68 16.1     |
| 6   | 86          | 71 17.4     |
| 8   | 86          | 72 16.3     |
| 10  | 87          | 74 14.9     |
| 12  | 93          | 73 21.5     |
| 14  | 92          | 73 20.7     |
| 16  | 91          | 73 19.8     |
| 18  | 86          | 75 12.8     |
| 20  | 95          | 79 16.8     |

5 Conclusion

Multi-class classification is a fundamental task in machine learning. This paper presents the abc-logitboost algorithm and demonstrates its considerable improvements over logitboost and abc-mart on a variety of datasets.

There is one interesting UCI dataset named Poker, with 25K training samples and 1 million testing samples. Our experiments showed that abc-boost could achieve an accuracy $> 90\%$ (i.e., the error rate $< 10\%$). Interestingly, using LibSVM, an accuracy of about 60% was obtained$^3$. We will report the results in a separate paper.

References

[1] Alan Agresti. *Categorical Data Analysis*. John Wiley & Sons, Inc., Hoboken, NJ, second edition, 2002.

[2] Peter Bartlett, Yoav Freund, Wee Sun Lee, and Robert E. Schapire. Boosting the margin: a new explanation for the effectiveness of voting methods. *The Annals of Statistics*, 26(5):1651–1686, 1998.

$^3$Chih-Jen Lin. Private communications in May 2009 and August 2009.
[3] Colin B. Begg and Robert Gray. Calculation of polychotomous logistic regression parameters using individualized regressions. *Biometrika*, 71(1):11–18, 1984.

[4] Yoav Freund. Boosting a weak learning algorithm by majority. *Inf. Comput.*, 121(2):256–285, 1995.

[5] Yoav Freund and Robert E. Schapire. A decision-theoretic generalization of on-line learning and an application to boosting. *J. Comput. Syst. Sci.*, 55(1):119–139, 1997.

[6] Jerome H. Friedman. Greedy function approximation: A gradient boosting machine. *The Annals of Statistics*, 29(5):1189–1232, 2001.

[7] Jerome H. Friedman, Trevor J. Hastie, and Robert Tibshirani. Additive logistic regression: a statistical view of boosting. *The Annals of Statistics*, 28(2):337–407, 2000.

[8] Jerome H. Friedman, Trevor J. Hastie, and Robert Tibshirani. Response to evidence contrary to the statistical view of boosting. *Journal of Machine Learning Research*, 9:175–180, 2008.

[9] Yoonkyung Lee, Yi Lin, and Grace Wahba. Multicategory support vector machines: Theory and application to the classification of microarray data and satellite radiance data. *Journal of the American Statistical Association*, 99(465):67–81, 2004.

[10] Ping Li. Abc-boost: Adaptive base class boost for multi-class classification. In *ICML*, Montreal, Canada, 2009.

[11] Ping Li. Robust logitboost. Technical report, Department of Statistical Science, Cornell University, 2009.

[12] Ping Li, Christopher J.C. Burges, and Qiang Wu. Mcrank: Learning to rank using classification and gradient boosting. In *NIPS*, Vancouver, BC, Canada, 2008.

[13] Liew Mason, Jonathan Baxter, Peter Bartlett, and Marcus Frean. Boosting algorithms as gradient descent. In *NIPS*, 2000.

[14] Robert Schapire. The strength of weak learnability. *Machine Learning*, 5(2):197–227, 1990.

[15] Robert E. Schapire and Yoram Singer. Improved boosting algorithms using confidence-rated predictions. *Machine Learning*, 37(3):297–336, 1999.

[16] Ambuj Tewari and Peter L. Bartlett. On the consistency of multiclass classification methods. *Journal of Machine Learning Research*, 8:1007–1025, 2007.

[17] Tong Zhang. Statistical analysis of some multi-category large margin classification methods. *Journal of Machine Learning Research*, 5:1225–1251, 2004.

[18] Hui Zou, Ji Zhu, and Trevor Hastie. New multicategory boosting algorithms based on multicategory fisher-consistent losses. *The Annals of Applied Statistics*, 2(4):1290–1306, 2008.