What are the causes for the spread of GLE parameters deduced from NM data?

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Abstract. Investigations have shown that the analysis results of ground level enhancements (GLEs) based on neutron monitor (NM) data for a selected event can differ considerably depending the procedure used. This may have significant consequences e.g. for the assessment of radiation doses at flight altitudes. The reasons for the spread of the GLE parameters deduced from NM data can be manifold and are at present unclear. They include differences in specific properties of the various analysis procedures (e.g. NM response functions, different ways in taking into account the dynamics of the Earth’s magnetospheric field), different characterisations of the solar particle flux near Earth as well as the specific selection of NM stations used for the analysis. In the present paper we quantitatively investigate this problem for a time interval during the maximum phase of the GLE on 13 December 2006. We present and discuss the changes in the resulting GLE parameters when using different NM response functions, different model representations of the Earth’s magnetospheric field as well as different assumptions for the solar particle spectrum and pitch angle distribution near Earth. The results of the study are expected to yield a basis for the reduction in the spread of the GLE parameters deduced from NM data.

1. Introduction

The worldwide network of neutron monitors (NMs) together with the geomagnetic field acts as a giant spectrometer and enables to determine the spectral variations of the galactic cosmic rays near Earth and the characteristics of sporadic solar cosmic ray (SCR) events in the energy range of \(\sim 500\) MeV to \(\sim 15\) GeV. Since the introduction of NMs in the 1950s about 70 SCR events were observed by the ground-based cosmic ray detectors. The analysis of these so-called ground level enhancements (GLEs) is essential to understand the particle acceleration mechanisms at or near the Sun and the transport of the SCRs in the interplanetary and near-Earth space. Beside this aspect in fundamental research, the investigation of GLEs is also of particular interest as e.g. the energetic solar cosmic rays may significantly increase the radiation dose rates at flight altitudes.

The results of GLE analysis based on NM data are used as input for e.g. the determination of the radiation doses along flight routes during a GLE. However, investigations in the past \cite{1,2} have shown that for a selected event the results of different GLE NM analysis procedures may differ considerably. The comparison of published GLE characteristics e.g. during the maximum phase of the GLE on 20 January 2005 shows differences between the highest and lowest published
solar proton intensity in the direction of maximum flux at 1 GV of almost two orders of magnitude [2]. The reasons for the spread of the GLE parameters deduced from NM data are at present unclear and may be manifold. They include differences in specific properties of the various analysis procedures, different characterisations of the solar particle flux near Earth as well as the specific selection of NM stations used for the analysis.

In this paper we describe recent work where we investigated the influence of the differences in specific properties of the various analysis procedures on the resulting GLE characteristics during the maximum phase of the GLE on 13 December 2006 (~0300 UT). This solar cosmic-ray event is ranked among the largest in solar cycle 23. The NM stations Oulu with 90% and Apatity with 78% measured the highest relative count rate increases.

2. Investigations

In the following we present the effects on the GLE results when using different NM response functions (section 2.1), different descriptions of the pitch angle distribution (section 2.2), different considerations of the dynamics of the Earth’s magnetospheric field (section 2.3), and different selection and number of used NM stations (section 2.4).

The response of a NM to relativistic solar protons can be expressed in the following simplified form:

$$\Delta N_i(t) = \sum_{P=P^i_c(t)}^\infty S_i(P) \cdot J_{||}(P,t) \cdot F(\delta_i(P,t),t) \cdot \Delta P$$

where

- $\Delta N_i(t)$ count rate increase at the NM station $i$ due to solar protons as function of time $t$,
- $P$ particle rigidity,
- $P^i_c(t)$ effective vertical geomagnetic cutoff rigidity at the location of the NM station $i$,
- $S_i(P)$ yield function of NM station $i$,
- $J_{||}(P,t)$ solar particle intensity near Earth in the direction of maximum flux (source direction, usually the field vector of the interplanetary magnetic field near Earth),
- $F(\delta,t)$ pitch angle distribution of solar particles,
- $\delta$ pitch angle, i.e. angular distance between the velocity direction of the particle and the magnetic field vector,
- $\delta_i(P,t)$ angular distance between the source direction (usually the field vector of the interplanetary magnetic field near Earth) and the rigidity dependent particle arrival direction outside the geomagnetosphere (asymptotic direction) for particles of vertical incidence at the top of the atmosphere above the location of the NM station $i$,
- $\Delta P$ rigidity interval.

The GLE characteristics are determined by minimizing the sum of squared differences between $\Delta N_{\text{calc}}$ and $\Delta N_{\text{obs}}$ for the selected set of NMs using a trial and error procedure.

For the investigations under sections 2.1, 2.3 and 2.4 an exponential form for the pitch angle distribution, $F(\delta,t)$, with two free parameters $A(t)$ and $B(t)$ was used as proposed by Bombardieri [3]:

$$F(\delta,t) = \exp \left( \frac{-0.5 \cdot (\delta - \sin(\delta) \cdot \cos(\delta))}{A(t) - 0.5 \cdot (A(t) - B(t)) \cdot (1 - \cos(\delta))} \right)$$

(2)
The trajectories of the solar cosmic ray protons (asymptotic directions, effective vertical cutoff rigidities) are computed with MAGNETOCOSMICS [4]. The inner geomagnetic field was described by the IGRF field (epoch 2005.0) [5] and the outer field by the Tsyganenko model 1989 [6] with the $K_p$-index on 13 December 2006 at 0300 UT ($K_p = 3$) for the sections 2.1, 2.2 and 2.4. We assumed a pure power law in rigidity for the differential rigidity spectrum of the solar cosmic ray protons. The data of the following 36 NM stations of the worldwide network were used for the GLE analysis presented in sections 2.1, 2.2 and 2.3: Alma Ata, Apatity 18NM64, Athens, Cape Schmidt, Fort Smith, Hermanus, Inuvik, Irkutsk, Irkutsk2, Irkutsk3, Jungfraujoch NM64, Jungfraujoch IGY, Kerguelen, Kiel, Kingston, Larc, Lomnický Štít, Magadan, Mawson, McMurdo, Moscow 24NM64, Moscow 6NM64, Nain, Norilsk, Novosibirsk, OLC, Oulu, Peawanuck, Potchefstroom, Rome, Sanae, Terre Adelie, Tixie Bay, Tsumeb, Thule, and Yakutsk.

2.1. Effect of different NM response functions

We considered the following three NM response functions: coupling function by Dorman and Yanke [7] (including modifications for rigidities < 2.78 GV according to Belov and Struminsky [8]), the specific yield function by Debrunner at al. [9] and the parameterized yield function by Flückiger et al. [10]. The coupling function derived by Dorman and Yanke [7] is based mainly on NM measurements during latitude surveys, whereas the yield functions by Debrunner at al. [9] and Flückiger et al. [10] are determined by Monte Carlo simulations of the transport of the particles in the atmosphere and of the detection of incident particles in the NM. The pitch angle distribution was described by the formula by Bombardieri [3]. The asymptotic directions as well as the effective vertical cutoff rigidities were computed with the IGRF combined with the Tsyganenko 1989 model with $K_p$-index equal to 3. All 36 NM stations listed in section 2 were used for the GLE analysis.

Figures 1 and 2 show the deduced differential rigidity solar proton spectra and the resulting pitch angle distributions for the time interval 0305-0310 UT during the GLE on 13 December 2006.

![Figure 1](image1.png)  
**Figure 1.** Deduced SCR spectra during the GLE on 13 December 2006 for the time interval 0305-0310 UT by using different NM response functions.

![Figure 2](image2.png)  
**Figure 2.** Deduced pitch angle distributions during the GLE on 13 December 2006 for the time interval 0305-0310 UT by using different NM response functions.

The spectrum obtained with the coupling function by Dorman and Yanke [7] is harder than
the ones derived with the two yield functions by Debrunner et al. [9] and by Flückiger et al. [10]. The relative difference between the solar proton flux in direction of maximum intensity $J_{\parallel}^{\text{Flückiger et al.}}$ and $J_{\parallel}^{\text{Dorman and Yanke}}$ is $\sim300\%$ at 1 GV, $\sim120\%$ at 2 GV and $\sim10\%$ at 5 GV, whereas it is only $\sim20\%$ at 1 GV, $\sim1\%$ at 2 GV, and $\sim10\%$ at 5 GV between $J_{\parallel}^{\text{Flückiger et al.}}$ and $J_{\parallel}^{\text{Debrunner et al.}}$. The additional radiation dose rate$^1$ produced by the SCR latitudes $55^\circ$–$90^\circ$ and at a typical cruise altitude of $\sim10.5$ km ($250$ g/cm$^2$) during the GLE maximum is up to $\sim30$ $\mu$Sv/h according to the results with the coupling function by Dorman and Yanke [7] and maximal $\sim65$ $\mu$Sv/h with the yield function by Debrunner et al. [9] and $\sim75$ $\mu$Sv/h with the yield function by Flückiger et al. [10].

### 2.2. Effect of different descriptions of the pitch angle distribution

For this part of the analysis we used the yield function by Debrunner et al. [9], the solar cosmic ray proton trajectories computed based on the geomagnetic field models IGRF combined with the Tsyganenko 1989 model with $K_p$-index = 3, and the data of all 36 NM stations listed in section 2.

According to the literature different formulas for the description of the pitch angle distribution $F(\delta,t)$ are adopted. We investigated the effect of the following five functional forms of the pitch angle distribution on the deduced GLE parameters:

- Bombardieri [3] used an exponential form with two free parameters, $A$ and $B$, see Formula 2.
- Vashenyuk et al. [11] utilized an exponential form with only one free parameter, $c$:
  \[
  F(\delta,t) = \exp\left(\frac{\delta^2}{c(t)}\right)
  \]  
  (3)

- Plainaki et al. [12] also used an exponential form with a single free parameter, $n_a$:
  \[
  F(\delta,t) = \exp\left(-n_a^2(t) \cdot \sin^2\left(\frac{\delta}{2}\right)\right)
  \]
  (4)

- The Bern group used a piece by piece linear function with typically five interpolation values, see the pink line in Figure 3.

- Matthiä [13] used a simple linear dependence:
  \[
  F(\delta,t) = \begin{cases} 
  1 - b(t) \cdot \delta & : \text{if } b(t) \cdot \delta < 1 \\
  0 & : \text{otherwise}
  \end{cases}
  \]
  (5)

The resulting GLE parameters show only small differences. Figure 3 shows the determined solar cosmic ray proton flux in the direction of maximum flux and Figure 4 the deduced pitch angle distributions. The difference in the solar proton intensity in direction of maximum flux is $\sim30\%$ at 1 GV, $\sim8\%$ at 2 GV, and $\sim2\%$ at 5 GV.

### 2.3. Effect of different consideration of dynamics of the Earth’s magnetic field

For this investigation we used the yield function by Flückiger et al. [10], the pitch angle distribution by Bombardieri [3], and the data of all 36 NM stations as listed in section 2.

The effect of different considerations of the dynamics of the Earth’s magnetic field on the GLE analysis was investigated in determining the asymptotic directions and the effective vertical cutoff rigidities by using the following three different models of the Earth’s magnetic field models:

\[\text{In this paper the term “radiation dose rate” is used for “ambient dose equivalent rate”}\]
Figure 3. Deduced SCR spectra during the GLE on 13 December 2006 for the time interval 0305-0310 UT by using different forms of pitch angle distributions.

(i) IGRF [5] + Tsyganenko model 1989 [6], $K_p = 3$
(ii) IGRF [5] + Tsyganenko model 1989 [6], $K_p = 0$
(iii) only IGRF [5]

Figure 4. Determined pitch angle distributions during the GLE on 13 December 2006 for the time interval 0305-0310 UT by using different forms of pitch angle distributions.

Figure 5 shows as an example the asymptotic geographic latitude and longitude, as function of the rigidity, for protons of vertical incidence arriving at the top of the atmosphere above the NM station Terre Adelie. The computations were made with MAGNETOCOSMICS [4] for the different assumptions of the geomagnetic field models.

Figure 5. Asymptotic latitude (left) and longitude (right) in geographic coordinates for protons of vertical incidence at the top of the atmosphere above the NM station Terre Adelie on 13 December 2006, 0300 UT and for different assumptions of the geomagnetic field models.

The deduced solar proton spectra and pitch angle distributions are plotted in Figures 6 and 7. The solar proton flux in the rigidity range 1–5 GV is 50-30% higher when only the IGRF
The geomagnetic field is used for the trajectory computations compared to the deduced values when using the IGRF model [5] combined with the model by Tsyganenko 89 [6] with $K_p=3$. The differences in the GLE parameters are marginal when using IGRF + Tsyganenko 89 model with $K_p=0$ and with $K_p=3$.

The resulting differences in the radiation dose rates at high geographic latitudes and at a typical flight altitude of 10.5 km are on average only small, $\sim 10\%-20\%$. Only on some selected locations the absolute deviations of additional SCR contribution are up $\sim 20 \mu$Sv/h.

2.4. Effect of the selection and the number of used NM stations

For this investigations the GLE analysis was executed by removing the data of individual or of a group of NM stations such as NMs at mountain altitudes. The following NM stations were considered as mountain stations: Alma Ata, Irkutsk2, Irkutsk3, Jungfraujoch NM64, Jungfraujoch IGY, Lomnický Štít. The yield function by Debrunner et al. [9] and the pitch angle distribution by Bombardieri [3] were used for the computations. The deduced results for the solar proton spectra and the pitch angle distributions are shown in Figures 8 and 9.

At an altitude of 10.5 km (atmospheric depth $\sim 250 \text{ g/cm}^2$) and at restricted locations the radiation dose rate caused by solar cosmic rays in the time interval considered is maximal $\sim 65 \mu$Sv/h, if all NM stations, as listed under section 2, are included in the GLE analysis and at most $\sim 130 \mu$Sv/h when the data of the NM stations Apatity and Oulu, i.e. the NM stations showing the largest count rate increase during this GLE, were not considered for the GLE analysis.

3. Summary and Conclusions

We have attempted to identify the reasons for the differences in the results of GLE analysis by the different GLE analysis procedures. The results of the investigations show that especially the
used response functions as well as the used geomagnetic field models with different considerations of the dynamics of the geomagnetic field for the trajectory computations have a relevant effect on the deduced GLE parameters. Also the selection of the used NM data may significantly affect the results of a GLE analysis. However, the outcome of this work does not conclusively reveal the reasons for the sometimes diverse results of GLE analysis published in the literature. In addition, other causes not addressed in this study like e.g. different trajectory tracing techniques in the determination of cutoff rigidities and asymptotic directions may also have an effect. Therefore, more detailed exchange of information on the different GLE analysis procedures between the specialists is encouraged and appreciated in view of a harmonization of the GLE analysis procedures. In particular there is a need to reduce the uncertainties in the response function of NMs mainly at the lower rigidity regime, i.e. $\lesssim 2$ GV. Reliable and undisputed results of GLE analysis are particularly important, as non cosmic ray specialists use these results in space weather applications, as e.g. the determination of radiation dose assessments at flight altitudes.

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