The study of circumsphere and insphere of a regular polyhedron

D Noviyanti and H P Lestari
Department of Mathematics, Universitas Negeri Yogyakarta, Yogyakarta, Indonesia
E-mail: dian.noviyanti2015@student.uny.ac.id

Abstract. A regular polygon is a polygon that is both equilateral and equiangular. One of the properties of the regular polygon is having a circumcircle and an incircle. The analogy of a regular polygon on plane is a regular polyhedron in space, while the analogy of a circle on plane is a sphere in space. Relationship between regular polygon and circle on plane can be studied in space. The purpose of this paper is to show the existence and the characteristics of a circumsphere and an insphere of the regular polyhedron. The result show that each regular polyhedron has a circumsphere and an insphere. The center of the circumsphere of a regular polyhedron is intersection of the perpendicular bisector planes of the regular polyhedron and the center of the insphere of a regular polyhedron is intersection of the angle bisector planes of the regular polyhedron. The characteristics of a circumsphere and an insphere of a regular polyhedron are: 1) The center of a circumsphere and an insphere of a regular polyhedron is coincide; 2) The length of radius of a circumsphere and an insphere of a regular polyhedron meet the equation

\[ R = \frac{s/2 \sin \left( \frac{180\degree}{2e} \right)}{\sin \left( \frac{180\degree}{2e} \right) \cos \left( \frac{\theta}{2} \right)}; \]

\[ r = \frac{s/2 \cos \left( \frac{180\degree}{2e} \right) \tan \left( \frac{\theta}{2} \right)}{\tan \left( \frac{\theta}{2} \right)}. \]

1. Introduction
Analogy is a similarity, two objects of different types but each has one characteristic, both the same position or characteristic. In mathematics, analogies can occur for objects on plane and objects in space. One example is a regular polygon on plane is a regular polyhedron in space[1].

We define a regular polygon in the plane to be a polygon that is both equilateral and equiangular[2]. A figures bounded by regular polygon is a regular polyhedron. All of its faces meeting at each vertex and all dihedral angles equal[2].

This similarity is the basic of the analogy between a regular polygon on plane is a regular polyhedron in space. Based on its number and types of sides, there are only five figures, tetrahedron, cube, octahedron, icosahedron, and dodecahedron[1]. Analogy can also be used on circle and sphere. Both are sets of points that are the same distance to a certain point. However, the circle is on the plane, while the sphere is in space[4].

The similarity between a regular polygon and a regular polyhedron not only based on their sides and angles. One of the properties of regular polygon is having circumcircle and incircle. In regular polygon there is a point which is the same distance to the vertices and its side which are called of the center of regular polygon[3]. The point is the center of circumcircle who through its vertices and the center of incircle who tangent its sides. Therefore, it must be shown if the characteristic can also apply in space. A regular polyhedron is an analogy of a regular polygon, a sphere is an analogy of a circle, it will show the existence of the circumsphere and insphere of a regular polyhedron and how its characteristic.
Pradana had previously shown the existence of the circumsphere and the insphere of a regular tetrahedron. He analogous lines in space, its perpendicular bisector line and angle bisector plane. The line and planes will intersect at a point which is the same distance to the vertices and the sides, this point is called the center of circumsphere and insphere of the regular tetrahedron. The same method can be used to indicate the center of the circumsphere and insphere of cube, octahedron, icosahedron, and dodecahedron.

After show the existance of the circumsphere and insphere of a regular polyhedron, it must be shown the characteristic of the circumsphere and insphere. The characteristics studied are the center point and the radius of the circumsphere and insphere. The purpose of this paper is to show the existance and the characteristics of a circumsphere and an insphere of the regular polyhedron.

2. Discussion and result
A polyhedron is a figures bounded by polygon. When two polygons meet, they must have an entire edge in common. These plane polygons are the faces of the polyhedron. Their edges are the edges of the polyhedron, and their vertices are the vertices of the polyhedron. Where two faces meet along an edge, we have a dihedral angle. At a vertex, the angle in any face passing through that vertex is called a face angle. The collection of all the face at a vertex makes a solid angle[2]. If the faces of polyhedron is a regular polygon and each solid angle have same number of face angle then the polyhedron named a regular polyhedron[3]

In a regular polyhedron, the only possible configurations at a single verte are 3, 4, or 5 triangles, 3 square, or 3 pentagons[2]. This possible prove that there is 5 types of a regular polyhedron, tetrahedron, cube, octahedron, icosahedron, and dodecahedron.

A regular polyhedron have some characteristics, all the dihedral angle equal and for any two vertices, there is a rigid motion of the figure onto itself sending the first vertex to the second[2].

| Table 1. Properties of a regular polyhedron |
|---------------------------------------------|
| Regular Polyhedron | \( v \) | \( e \) | \( n \) | \( \theta \) |
|---------------------|-------|-------|-------|---------|
| Tetrahedron         | 4     | 6     | 4     | 70°31’44’’ |
| Cube                | 8     | 12    | 6     | 90°     |
| Octahedron          | 6     | 12    | 8     | 109°28’16’’ |
| Dodecahedron        | 20    | 30    | 12    | 116°33’54’’ |
| Icosahedron         | 12    | 30    | 20    | 138°11’23’’ |

In the Table 1. \( v \) is the total number of vertice, \( e \) is the total number of edge, \( n \) is the total number of face, and \( \theta \) is the dihedral angle at each edge[7].

2.1. Circumsphere and insphere of a regular Ppolyhedron
In a regular polyhedron, if all the vertices of polyhedron lie on sphere, we will say that the polyhedron is inscribed in the sphere or a regular polyhedron have a circumsphere[2]. The existence of circumsphere can be proved by show that in regular polyhedron there is a point which is the same distance to each vertex, the point is the center of circumsphere of a regular polyhedron.

Pradana has show that each of tetrahedron has a circumsphere. He show that the center of the circumsphere can be obtained from intersection of the perpendicular bisector lines. However, this research will use the perpendicular bisectors plane.

For example, there are tetrahedron \( D.ABC \), it will be shown that there is a point \( O \) such that \( OA = OB = OC = OD \). To show the circumsphere of a regular polyhedron, use the following method.

1) The point that is the same distance to the endpoint of the line segment.

In regular polygon, the center of the circumsphere can be obtained from the intersection of the perpendicular bisectors line[3]. Each point that is lie on the perpendicular bisectors line will be the same distance to the endpoint of its side. Adopting the statement, the perpendicular bisectors line will be analogous to space in the form of a plane dividing a line segment at the midpoint and perpendicular to the line segment. This field is then called the perpendicular bisector plane.
Theorem 1. A point lies on the perpendicular bisector plane of a line segment, if and only if a point is the same distance from the endpoints of the line segment. [6].

Proof. There is a plane that divides perpendicularly to the line segment \( AB \) at the midpoint \( M \). Suppose the plane \( \alpha \) is the plane that is perpendicular to the line segment \( AB \) at point \( M \). This means that the distance between the line segments \( AM \) and \( BM \) is equal. Next, a point \( P \) is chosen that is located in the plane \( \alpha \), so that there is an \( AMP \) triangle and a \( BMP \) triangle.

![Figure 1. \( \triangle AMP \) and \( \triangle BMP \)](image)

Triangle \( \triangle AMP \) and \( \triangle BMP \) are congruent triangles. The congruences between \( \triangle AMP \) and \( \triangle BMP \) obtained because of the S-A-S criteria[5]. Point \( M \) is the midpoint of the line segment \( AB \), then \( AM \cong BM \). The angle \( AMP \) and \( BMP \) are right angles so \( \angle AMP \cong \angle BMP \). The last, the line segment \( MP \) which is the side of \( \triangle AMP \) and \( \triangle BMP \). The congruences causes \( AP \) and \( BP \) also congruent. Therefore, the distance between point \( P \) to point \( A \) is the same as the distance from point \( P \) to point \( B \). Point \( P \) is a point located in the plane \( \alpha \), so that for each point in the plane \( \alpha \) is the same is the same distance to points \( A \) and \( B \).

Next, show the plane \( \beta \), this is the perpendicular bisector of line segment \( BC \) and the plane \( \gamma \) which is the perpendicular bisector of line segment \( BD \).

2) The center of the circumsphere of a regular polyhedron

Triangles \( \triangle ABD \) and \( \triangle BCD \) are planes that contain the tetrahedron \( DABC \) and intersect the triangle \( ABC \), so the are not parallel, so the \( a \) and \( \beta \) are also not parallel. The two planes that are not parallel will intersect, the intersection of two planes is a line. The intersection between the \( a \) and \( \beta \) is line \( l \).

Plane \( \gamma \) and line \( l \) are also not parallel. A plane and a line which are not parallel will intersect and the intersection is a point called point \( O \). Point \( O \) is lie on line \( l \), while line \( l \) is the intersection between plane \( \alpha \) and \( \beta \). Therefore the distance between points \( O \) to points \( A \), \( B \) and \( C \) is the same, then \( OA = OB = OC \).

Point \( O \) is the intersection between plane \( \gamma \) and line \( l \), so point \( O \) lies on the plane \( \gamma \). The plane \( \gamma \) is a plane that is the same distance from points \( B \) and \( D \), then \( OB = OD \), thus \( OA = OB = OC = OD \). It can be concluded that point \( O \) is the same distance as \( A \), \( B \), \( C \), and \( D \). This construction show that the point \( O \) is equidistant from all the vertices.

![Figure 2. A point \( O \)](image)
Next, the same method can be used to show that there is a point which is the same distance to all vertex of a regular polyhedron. In regular polyhedron, each edge has a perpendicular bisector plane. Since all the dihedral angles are equal, the construction could have been started anywhere. Thus there are rotations sending any vertex to an adjacent vertex, and a succession of these will be a rigid motion sending any vertex to any other vertex[2]. Therefore, to show the center points of a regular polyhedron, only three perpendicular bisectors plane are needed that are not parallel, so that the intersection can be appoint.

Figure 3

In figure 3 show that each regular polyhedron has a circumsphere. The center of the circumsphere of a regular polyhedron is an intersection of the perpendicular bisector planes of the regular polyhedron.

In a regular polyhedron, if all the faces of polyhedron tangent to a sphere, we will say that the polyhedron is circumscribed about the sphere or a regular polyhedron has an insphere[2]. The existence of insphere can be proved by shows that in regular polyhedron there is a point which is the same distance to each side, the point is the center of insphere of a regular polyhedron.

Pradana has shown that each of tetrahedron has an insphere. He shows that the center of the insphere can be obtained from the intersection of the angle bisector lines. The same method can be used to show that a regular polyhedron has an insphere. To show the circumsphere of a regular polyhedron, use the following method.

1) The point that is the same distance to the endpoint of two planes.

To show the circumsphere of a regular polyhedron, he uses a plane that divides the dihedral angle into equal. The property of this plane is that every point which lies on this planes will be the same distance to the two plane of the dihedral angle. The angle bisector plane is an analogy from the angle bisector on a plane.

For example, there are cube $ABCD.EFGH$, it will be shown that there is a point $P$ such that $PA = PB = PC = PD = PE = PF = PG = PH$. Described a plane $\delta$ that divides the angle between the $ABCD$ and $ADHE$ equally. Based on the properties of the angle bisector plane, each point which lies on $\delta$ plane will be the same distance from the $ABCD$ and $ADHE$. Next, show the plane $\varepsilon$, this is the angle bisector plane between the $ABCD$ and $ABFE$ and the plane $\eta$ which is the angle bisector plane between the $ABCD$ and $BCGF$.

2) The center of the insphere of a regular polyhedron

Square $ADHE$ and $ABFE$ are planes that contain the cube $ABCD.EFGH$ and intersect the square $ABCD$, so the two are not parallel, so the $\delta$ and $\varepsilon$ are also not parallel. The two planes that are not parallel will intersect, the intersection of two planes is a line. The intersection between the $\delta$ and $\varepsilon$ is line $y$.

Plane $\eta$ and line $y$ are also not parallel. A plane and a line that are not parallel will intersect and the intersection is a point called point $P$. Point $P$ is lying on line $y$, while line $y$ is the intersection between plane $\delta$ and $\varepsilon$. Therefore the distance between points $P$ to planes $ABCD$, $ABFE$, $ADHE$, and $BCGF$ is the same.

Planes $\delta$, $\varepsilon$ and $\eta$ is intersected at point $P$ and has the same distance to $ABCD$, $ABFE$, $ADHE$, and $BCGF$. Because the dihedral angles are all equal, this construction propagates around the whole surface to show that the point $P$ is equidistant from all the faces. Hence the cube is circumscribed in a sphere with center $P$. 

---

Figure 3. Circumsphere of a regular polyhedron
Figure 4. A point $P$

Next, the same method can be used to show the existence of an insphere of octahedron, icosahedron, and dodecahedron. In regular polyhedron, each dihedral angle has an angle bisector plane. Since all the dihedral angles are equal, only three angle bisectors plane is needed that are not parallel, so that the intersection can be a point.

Figure 5. Insphere of a regular polyhedron

In figure 5 show that each regular polyhedron has an insphere. The center of the insphere of a regular polyhedron is an intersection of the angle bisector planes of the regular polyhedron.

2.2. The characteristics of circumsphere and insphere of a regular polyhedron

The existence of circumsphere and insphere of a regular polyhedron shows that the analogy of the circumcircle and incircle of a regular polygon also applies to regular polyhedron. The center point of the circumcircle and incircle of a regular polygon is coincided. The analogy of the center point of the regular polygon will be proven whether it also applies to the circumsphere and insphere of a regular polyhedron.

Pradana shows that the center of the circumsphere in a tetrahedron coincides with the center of insphere. The same method can be used to show that the center of the circumsphere and insphere of the cube, octahedron, icosahedron, and dodecahedron coincide.

Suppose there is a cube $ABCD.EFGH$. There is a point $P$, where the point $P$ is the center point of the circumsphere, then the distance of the point $P$ to the vertex is the same.

Figure 6. A cube with center point $P$

Points $I$ and $J$ are determined so that $PI \perp ABCD$ and $PJ \perp BCGF$. Points $A$, $P$, $B$, and $I$ can form two congruent triangles, $\triangle API$ and $\triangle BPI$. The congruences obtained because $PA \cong PB$.  

Points $I$ and $J$ are determined so that $PI \perp ABCD$ and $PJ \perp BCGF$. Points $A$, $P$, $B$, and $I$ can form two congruent triangles, $\triangle API$ and $\triangle BPI$. The congruences obtained because $PA \cong PB$. 

Points $I$ and $J$ are determined so that $PI \perp ABCD$ and $PJ \perp BCGF$. Points $A$, $P$, $B$, and $I$ can form two congruent triangles, $\triangle API$ and $\triangle BPI$. The congruences obtained because $PA \cong PB$. 

Points $I$ and $J$ are determined so that $PI \perp ABCD$ and $PJ \perp BCGF$. Points $A$, $P$, $B$, and $I$ can form two congruent triangles, $\triangle API$ and $\triangle BPI$. The congruences obtained because $PA \cong PB$. 

Points $I$ and $J$ are determined so that $PI \perp ABCD$ and $PJ \perp BCGF$. Points $A$, $P$, $B$, and $I$ can form two congruent triangles, $\triangle API$ and $\triangle BPI$. The congruences obtained because $PA \cong PB$. 

Points $I$ and $J$ are determined so that $PI \perp ABCD$ and $PJ \perp BCGF$. Points $A$, $P$, $B$, and $I$ can form two congruent triangles, $\triangle API$ and $\triangle BPI$. The congruences obtained because $PA \cong PB$. 

Points $I$ and $J$ are determined so that $PI \perp ABCD$ and $PJ \perp BCGF$. Points $A$, $P$, $B$, and $I$ can form two congruent triangles, $\triangle API$ and $\triangle BPI$. The congruences obtained because $PA \cong PB$. 

Points $I$ and $J$ are determined so that $PI \perp ABCD$ and $PJ \perp BCGF$. Points $A$, $P$, $B$, and $I$ can form two congruent triangles, $\triangle API$ and $\triangle BPI$. The congruences obtained because $PA \cong PB$. 

Points $I$ and $J$ are determined so that $PI \perp ABCD$ and $PJ \perp BCGF$. Points $A$, $P$, $B$, and $I$ can form two congruent triangles, $\triangle API$ and $\triangle BPI$. The congruences obtained because $PA \cong PB$. 

Points $I$ and $J$ are determined so that $PI \perp ABCD$ and $PJ \perp BCGF$. Points $A$, $P$, $B$, and $I$ can form two congruent triangles, $\triangle API$ and $\triangle BPI$. The congruences obtained because $PA \cong PB$. 

Points $I$ and $J$ are determined so that $PI \perp ABCD$ and $PJ \perp BCGF$. Points $A$, $P$, $B$, and $I$ can form two congruent triangles, $\triangle API$ and $\triangle BPI$. The congruences obtained because $PA \cong PB$.
\( \angle AIP \cong \angle BIP \) and line segment \( \overline{PI} \) which is the side of \( \triangle API \) and \( \triangle BPI \), so \( \triangle API \) and \( \triangle BPI \) is congruent.

Points \( F, P, B, \) and \( J \) also can form two congruent triangles, \( \triangle BPJ \) and \( \triangle FPJ \). The congruences obtained because \( \overline{PB} \cong \overline{PF} \), \( \angle BPJ \cong \angle FJP \), and line segment \( \overline{PJ} \) which is the side of \( \triangle BPJ \) and \( \triangle FPJ \), so \( \triangle BPJ \) and \( \triangle FPJ \) is congruent.

Describe \( \triangle A'BP \) with \( I \) on \( \overline{AB} \) as well as \( \overline{A'I \perp A'B} \). Note that \( \triangle A'PI \cong \triangle API \). Also describe \( \triangle BF'P \) with \( J \) on \( \overline{BF} \) as well as \( \overline{F'P} \parallel \overline{A'B} \).

![Figure 7. Triangle \( \triangle A'BP \) and \( \triangle BF'P \)](image)

According to Murdanu, for example points \( A, B, C \) lie on line \( g \) and points \( D, E, F \) lies on line \( h \). If \( \overline{AB} \cong \overline{DE} \) and \( \overline{BC} \cong \overline{EF} \), so \( \overline{AC} \cong \overline{DF} \), then on \( \triangle A'BP \) and \( \triangle BF'P \), This is because of \( \overline{A'I} \cong \overline{BI} \cong \overline{BJ} \cong \overline{F'J} \). The line segments \( \overline{A'P}, \overline{BP}, \overline{F'P} \) is congruent, all three are congruent because they are the same length as the radius of circumsphere. The triangle \( BF'P \) is congruent with \( \triangle A'BP \) because of the S-S-S criteria[5].

Triangle \( BF'P \) and \( \triangle A'BP \) is congruent, then \( \angle PA'B \cong \angle BPA' \cong \angle PBF' \cong \angle PF'B \). Angle \( \angle PBA' \) and \( \angle PBF' \) are congruent, then \( \angle PBI \cong \angle PBJ \) because \( I \) lie on \( \overline{A'B} \) and \( J \) lie on \( \overline{BF} \). The congruences between \( \angle PBI \) and \( \angle PBJ \) show that \( \angle PBI \) and \( \angle PBJ \) are congruent.

Triangle \( \triangle PBI \) dan \( \triangle PBJ \) is congruent, then \( \angle PBI \cong \angle PBJ \), \( \angle PIB \cong \angle PJB \), and \( \overline{PI} \cong \overline{PJ} \), so the distance between point \( P \) to \( ABCD \) and \( BCGF \) is the same.

A similar method can be used to show that the point \( P \) is equal to the six planes that bounded the cube. It is proved that the center point of the circumsphere and insphere is coincided.

![Figure 8. Circumsphere and insphere of a regular polyhedron](image)

In figure 8 show that the center point of the circumsphere and insphere is coincided. The circumcircle and incircle of a regular polygon have the same center point, so it can be concluded that the analogy of the center points of the regular polygon also applies to the circumsphere and insphere of the regular polyhedron.

The radius of the circumcircle of a regular polygon is a measure of the distance from the center of the circumcircle to the vertex of the regular polygon, while the incircle is a measure of the distance from the center of the incircle to the side of the regular polygon.

The analogy of the regular polyhedron in space is a regular polygon on plane, so the radius of the circumsphere is also a measure of the distance from the center point of circumsphere to the vertex of a regular polyhedron and can be called by \( R \). the radius of the insphere is a measure of the distance from the center point of circumsphere to the side of a regular polyhedron and can be called by \( r \).
There is a regular tetrahedron with center point $O$, then the radius of circumsphere is $R$, the radius of insphere is $r$, and the length of an edge is $s$.

![Figure 9. Triangle $\triangle OEB$](image1)

In figure 9, line segment $\overline{OE}$ is the radius of insphere and called by $r$. Line segment $\overline{OB}$ the radius of circumsphere and called by $R$. The value of $R$ and $r$ can be found using the Pythagorean Theorem, which is $R^2 = EB^2 + r^2$ [5]. The value of $\overline{EB}$ can be found using the sine rule and obtained $\overline{EB} = \frac{s\sqrt{3}}{3}$ [5]. Before looking for an $R$-value, it is necessary to find the value of $r$. The line segment $\overline{DE}$ is the height of the tetrahedron, where $\overline{DE} = R + r$, then $R = \overline{DE} - r$. The value of $\overline{DE}$ can be found from the Pythagorean Theorem on $\triangle BDE$ and obtained $\overline{DE} = \frac{s\sqrt{6}}{3}$. Next, the value of $\overline{DE}$ and $\overline{EB}$ can be substituted in the equation $(\overline{DE} - r)^2 = EB^2 + r^2$ and obtained $r = \frac{s\sqrt{6}}{12}$. The value of $r$ can be substituted in the equation $R = \overline{DE} - r$ and obtained the value of $R = \frac{s\sqrt{6}}{4}$.

Next, to determine the values of $R$ and $r$ of the cube, you can also use the sine rules and the Pythagorean Theorem.

![Figure 10. Triangle $\triangle PIB$](image2)

In figure 10, the line segment $\overline{PI}$ is the radius of insphere and called by $r$. Line segment $\overline{PB}$ the radius of circumsphere and called by $R$. The value of $R$ and $r$ can be found using the Pythagorean Theorem, which is $R^2 = IB^2 + r^2$ [5]. The value of $\overline{IB}$ can be found using the sine rule and obtained $\overline{IB} = \frac{s\sqrt{2}}{2}$ [5]. Before looking for an $R$-value, it is necessary to find the value of $r$. The radius of insphere is the half of height of the cube, then $r = \frac{s}{2}$. The value of $r$ can be substituted in the equation $R^2 = IB^2 + r^2$ and obtained the value of $R = \frac{s\sqrt{3}}{2}$.

Next, to determine the values of $R$ and $r$ of the octahedron, you can also use the sine rules and the Pythagorean Theorem.
In figure 11, line segment $OG$ is the radius of insphere and called by $r$. Line segment $PO$ the radius of circumcircle and called by $R$. The value of $R$ and $r$ can be found using the Pythagorean Theorem, which is $R^2 = GB^2 + r^2$ [5]. The value of $GB$ can be found using the sine rule and obtained $GB = \frac{s\sqrt{3}}{3}$ [5]. Before looking for an $r$-value, it is necessary to find the value of $R$. The value of $R$ can be found using the sine rule and obtained $R = \frac{s\sqrt{3}}{2}$. Next, the value of $R$ and $GB$ can be substituted in the equation $R^2 = GB^2 + r^2$ and obtained the value of $r = \frac{s\sqrt{6}}{6}$.

Next, the same method can also be used to find the values of $R$ and $r$ of the icosahedron and dodecahedron. In dodecahedron, the value of $R = 1.4012s$ and $r = 1.1134s$, while in icosahedron the value of $R = 0.951s$ and $r = 0.7557s$.

After obtaining the values of $R$ and $r$ of the regular polyhedron, a general formula is obtained for finding the values of $R$ and $r$ and is written in the following theorem.

**Theorem 2.** There is a regular polyhedron, $n$ the total number of face, $v$ the total number of vertice, $e$ the total number of edge, $s$ is the length of an edge, $R$ the radius of the circumsphere, $r$ the radius of the inscribed sphere, $\theta$ the dihedral angle at each edge. The radius of the circumsphere and insphere meet the equation:

$$R = \frac{s}{2} \sin \left(\frac{180^\circ}{2e/n}\right) \cos \left(\frac{\theta}{2}\right)$$

and

$$r = \frac{s}{2} \cot \left(\frac{180^\circ}{2e/n}\right) \tan \left(\frac{\theta}{2}\right)$$

### 3. Conclusion

According to the discussion we can conclude that each regular polyhedron has a circumsphere and an insphere. The center of the circumsphere of a regular polyhedron is an intersection of the perpendicular bisector planes of the regular polyhedron and the center of the insphere of a regular polyhedron is an intersection of the angle bisector planes of the regular polyhedron. The characteristics of a circumsphere and an insphere of a regular polyhedron are: 1) The center of a circumsphere and an insphere of a regular polyhedron is coincide; 2) The length of radius of a circumsphere and an insphere of a regular polyhedron meet the equation:

$$R = \frac{s}{2} \sin \left(\frac{180^\circ}{2e/n}\right) \cos \left(\frac{\theta}{2}\right)$$

$$r = \frac{s}{2} \cot \left(\frac{180^\circ}{2e/n}\right) \tan \left(\frac{\theta}{2}\right)$$

### References

[1] Budhi W S & Kartasasmita B G 2015 *Berpikir Matematis Matematika untuk Semua* (Jakarta: Penerbit Erlangga) 73-76

[2] Hartshorne R 2000 *Geometry: Euclid and Beyond* (New York: Springer-Verlag New York, Inc) 435-442

[3] Keedy M L 1967 *Exploring Geometry* (New York: Holt, Rinehart and Winston, Inc) 289 449

[4] Pradana P & Lestari H P 2017 *Kajian Bola-Luar dan Bola-Dalam pada Bidang-Empat*. Jurnal Matematika Vol. 6 No. 1 56-65
[5] Venema G A 2012 *Foundations of Geometry 2nd Edition* (Boston: Pearson) 64-65 94 114
[6] Murdanu 2003 *Geometri Euclides Secara Deduktif-Aksiomatik* (Yogyakarta) 6
[7] Zwilinger D 2018 CRC Standard Mathematical Tables and Formulas 33rd Edition (New York: Taylor & Francis Group, LLC) 262