Magnetic Field Effect in a Two-dimensional Array of Short Josephson Junctions

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July 31, 2021

PACS number(s) 74.50+r, 74.60+Ge, 74.20+De, 05.40+j

Abstract
We study analytically the effect of a constant magnetic field on the dynamics of a two dimensional Josephson array. The magnetic field induces spatially dependent states and coupling between rows, even in the absence of an external load. Numerical simulations support these conclusions.

1 Introduction

Arrays of Josephson junctions have attracted increasing attention in recent years. One reason for this interest is the possibility of using Josephson arrays as millimeter-wave oscillators and amplifiers. While single junctions might in principle be used for such purposes, in practice they deliver very little power when coupled to an external load \cite{1}. Tilley proposed \cite{2} using 1D series arrays working coherently to match typical load impedances. Unfortunately the mechanism of internal coupling of series arrays has proven to be weak, thus requiring stringent conditions on the fabrication tolerance \cite{3}.
Two dimensional arrays have been investigated in the hope that some internal mechanism might prove to be effective in coherently phase locking the junctions. Experimentally, encouraging results have been reported on the emitted power \[4\] and linewidth \[5\]. A heuristic explanation for the success of 2D arrays is that the presence of superconductive loops in the system provides a further coupling mechanism among the junctions that is absent in 1D series arrays; indeed evidence that fluxons (a $2\pi$ wrap of the superconductor phase trapped in a superconductive loop) do play a role in 2D arrays has been reported with the LTSEM technique \[6\]. On the other hand, theoretical analysis of bare 2D arrays (i.e. arrays not coupled to an external load) in the absence of any magnetic field shows that the uniform in-phase solution has similar neutral stability features as bare 1D series arrays \[7, 8\]. Indeed, recent simulations on one class of disordered 2D arrays suggest that the external load is responsible (in large part or entirely) for the coherent behavior observed there \[9\]. These results point out the need for a deeper fundamental understanding of the dynamics of 2D arrays. The inclusion of magnetic field effects has perhaps been slowed by the fact that appropriate models of 2D arrays in presence of magnetic field are in general rather complicated \[10, 11\], so that a direct theoretical attack on the problem is formidable.

The purpose of this paper is to build up some analytic insight into the dynamics of fluxon states in two dimensional arrays, especially the role played by the magnetic field. Rather than study directly the general case of $M \times N$ arrays (see Figure 1a), we focus on two simpler configurations. First, we consider the case of a single row of plaquettes (i.e. a $2 \times N$ array, see Fig. 1b) biased by a constant transverse current. This is similar to the problem of a 1D parallel array studied elsewhere \[12, 13\], except that we include junctions in the
horizontal branches. For bias currents not too close to the critical current, we construct fluxon solutions whose spatial structure depends on the presence of the magnetic field. While this is somewhat enlightening, the $2 \times N$ array is too simple to help us understand certain important aspects of the two dimensional problem. Thus, we turn next to the case of a double row of plaquettes (i.e. a $3 \times N$ array, see Fig. 1c.) This is perhaps the simplest arrangement which fully captures the essential features of a 2D array, insofar as it allows us to study the effect of weak interactions between the rows. We find that a fluxon state in the top row induces a fluxon state in the bottom row; moreover, the resulting dynamical state has a definite phase relation between the fluxons in the two rows, in qualitative agreement with numerical simulations. Remarkably, this relative phase becomes undetermined in the zero-field limit. We conclude that the magnetic field breaks the symmetry responsible for the neutral stability found in bare 2D arrays [8].

2 The model

A two dimensional array of $N \times M$ short Josephson junctions can be represented by the equivalent circuit depicted in Fig. 1a. The most important effect we want to study is the interaction between rows. To this end we consider a simplified model which includes only the self inductances of each loop but ignores mutual inductances between loops. With this simplification, following the usual analysis for superconductive loops [14], the equation of motion for this system (in normalized units and for junctions of negligible capacitance, see Fig. 1 for notation) are[15, 16]

\[
\dot{V}_{l,j} = - \sin V_{l,j} + \gamma +
\]
\[
\frac{1}{\beta_l}[V_{l+1,j} - 2V_{l,j} + V_{l-1,j} + H_{l,j} - H_{l,j+1} + H_{l-1,j+1} - H_{l-1,j}] \\
\dot{H}_{l,j} = -\sin H_{l,j} + \\
\frac{1}{\beta_l}[H_{l,j+1} - 2H_{l,j} + H_{l,j-1} + V_{l,j} - V_{l+1,j} + V_{l+1,j-1} - V_{l,j-1}]
\]

where \( l = 2, \ldots, M - 1 \) and \( j = 2, \ldots, N - 1 \). Here, \( V_{l,j} \) and \( H_{l,j} \) are the Josephson phase differences of the vertical and horizontal junctions, respectively, \( \gamma = I_B/I_0 \) is the normalized bias current, \( \beta_l = \frac{2\pi L_l I_0}{\Phi_0} \) is the usual SQUID parameter, \( \eta = \frac{\Phi^e}{L_l I_0} \) is the normalized external flux per elementary cell (\( \Phi^e \)), \( R \) and \( I_0 \) are the normal resistance and the critical current of the junctions, respectively, \( L \) is the self inductance of the superconducting loop, and the unit of time is \( \frac{\hbar}{2eRI_0} \).

The boundary conditions are (\( N \) denotes the total number of vertical junctions and \( M \) the total number of horizontal junctions):

\[
\dot{V}_{1,j} = -\sin V_{1,j} + \gamma - \eta + \\
\frac{1}{\beta_l}[V_{2,j} - V_{1,j} + H_{1,j} - H_{1,j+1}] \\
\dot{V}_{N,j} = -\sin V_{N,j} + \gamma + \eta + \\
\frac{1}{\beta_l}[V_{N-1,j} - V_{N,j} - H_{N-1,j+1} - H_{N-1,j}] \\
\dot{H}_{l,1} = -\sin H_{l,1} + \eta + \\
\frac{1}{\beta_l}[H_{l,2} - H_{l,1} + V_{l,1} - V_{l+1,1}] \\
\dot{H}_{l,M} = -\sin H_{l,M} - \eta + \\
\frac{1}{\beta_l}[H_{l,M-1} - H_{l,M} - V_{l,M-1} + V_{l+1,M-1}]
\]

In this section and the next, we consider the case of a single row of plaquettes which
is current biased in the transverse direction (Fig. 1b). This system is similar to the 1D parallel array of short Josephson junctions studied elsewhere\[12, 13\], but for the presence of junctions in the horizontal branches. There are two horizontal Josephson junctions for each vertical junction; however, we can find solutions where the dynamics of the upper junction and the lower junction are not independent, but rather satisfy

\[ H_{l,1}(t) = -H_{l,2}(t) = H_l(t) \] (8)

where \( H_{l,2} \) is the Josephson phase across the \( l^{th} \) horizontal junction in the upper branch and \( H_{l,1} \) is the Josephson phase difference across the \( l^{th} \) lower branch, an identity that allows us to introduce the simplified notation \( H_l \), as indicated.

To see that such dynamical states exist, add Eq.(5) to Eq.(6) with \( M = 2 \) to get

\[ \dot{H}_{l,1} + \sin H_{l,1} = -\dot{H}_{l,2} - \sin H_{l,2}. \] (9)

This is satisfied by \( H_{l,1} = -H_{l,2} \) provided either (i) the initial conditions are the same for the two junctions or (ii) this is an attracting state. (Physically, we can arrange for identical initial conditions if, before applying a driving current, we allow the system to relax to the steady state \( H_{l,1} = \dot{H}_{l,1} = 0 \).

We emphasize that our analysis relies on our neglecting the off diagonal terms of the inductance matrix \[10\]: the mutual inductances make the problem much more complicated.

Summarizing, our equations for the 1D row of plaquettes are:

\[
\dot{V}_l = -\sin V_l + \gamma + \frac{1}{\beta_l} [2(H_l - H_{l-1}) + V_{l-1} - 2V_l + V_{l+1}]; \quad l = 2, \ldots, N - 1
\] (10)

\[
\dot{H}_l = -\sin H_l + \frac{1}{\beta_l} [V_l - V_{l+1} - 2H_l] + \eta; \quad l = 1, \ldots, N - 1
\] (11)
where \( V_l \) is the Josephson phase difference across the \( l^{th} \) vertical junction. The equations for the vertical junctions at the left and right ends are:

\[
\dot{V}_1 = - \sin V_1 - \eta + \gamma + \frac{1}{\beta_l} [2H_1 - V_1 + V_2] \tag{12}
\]

\[
\dot{V}_N = - \sin V_N + \eta + \gamma + \frac{1}{\beta_l} [V_{N-1} - V_N - 2H_{N-1}] \tag{13}
\]

### 3 Approximate solution for the isolated row

To find an approximate solution for the isolated row of plaquettes we apply a scheme of successive approximations. To first approximation, we assume that the horizontal junctions are completely inactive \((H_l \simeq 0)\), as suggested by numerical simulations of Eqs.(9-12) (see Fig. 2). Under this hypothesis the analysis is similar to that carried out for a continuous (long) Josephson junction in Ref. [17]. For sufficiently large bias currents \( \gamma \gg 1 \), we can approximate the solution as the sum of a linear term and a small oscillating term:

\[
V_l(t) = \theta_{l,0} + vt + X_l(t) \tag{14}
\]

where the \( \theta_{l,0} \) and \( v \) are constants to be determined self-consistently. Linearizing Eq.(9) yields

\[
\dot{X}_l(t) = - v + X_l(t) + \sin (\theta_{l,0} + vt) + \cos (\theta_{l,0} + vt)X_l(t) + \frac{1}{\beta_l} [\theta_{l-1,0} - 2\theta_{l,0} + \theta_{l+1,0} + X_{l-1}(t) - 2X_l(t) + X_{l+1}(t)] \tag{15}
\]

After the balancing of the constants and assuming a stationary wave profile \( X_l(t) = A(t)e^{i(kl-\omega t)} \) we obtain the following equation for the wave amplitude:

\[
\dot{A}(t) = [i\omega + \cos (\theta_{l,0} + vt) + \frac{2}{\beta_l} (\cos k - 1)]A(t) + \sin (\theta_{l,0} + vt)e^{-i(kl-\omega t)} \tag{16}
\]
The solution of the associated homogeneous equation decays exponentially with time, so except for transient behavior it is sufficient to seek a particular solution of Eq. (16). We find

\[ V_l(t) = \theta_{l,0} + vt + A_v \sin (\theta_{l,0} + vt) + B_v \cos (\theta_{l,0} + vt) \]  \hspace{1cm} (17)

\[ \theta_{l,0} = l \eta \beta_l \] \hspace{1cm} (18)

where \( A_v \) and \( B_v \) are constants, and Eq. (17) is necessary to satisfy the boundary conditions. Notice that the solution does not contain the wave number \( k \); this parameter has canceled out. This solution can be viewed as a travelling wave in the sense that the equation of motion in two adjacent cells is the same after a fixed time delay \( \Delta t = \beta l \eta / v \). The propagation velocity of the wave is given by the physical distance between two cells divided by this time. It is important to note that no signal is in fact propagating across the system: in fact the time delay between two junctions is zero if the magnetic field is zero. In other words, this is the phase velocity of the wave rather than the group velocity. A similar estimate for this velocity was derived in Ref. [11].

The three parameters that appear in Eq. (16), namely \( v, A_v \) and \( B_v \), are fixed by separately balancing the constant, sine, and cosine terms in Eq. (9):

\[ B_v = 2(\gamma - v) \] \hspace{1cm} (19)

\[ \frac{2A_v}{\beta_l}(\cos \eta \beta_l - 1) + vB_v = 1 \] \hspace{1cm} (20)

\[ vA_v - \frac{2B_v}{\beta_l}(\cos \eta \beta_l - 1) = 0 \] \hspace{1cm} (21)

In the limit of very large \( \gamma \) the solution is

\[ v \approx \gamma, \quad A_v \approx 0 \quad B_v \approx \frac{1}{\gamma} \] \hspace{1cm} (22)
In this limit the time delay is simply $\Delta t = \beta \eta / \gamma$. Although derived for bias currents $\gamma >> 1$, this formal restriction is not required in practice. For example, Fig. 3 compares the formula for the time delay and typical results from numerical simulations with $\gamma = 3/2$. The agreement is quite good. This is because the key approximation is that the oscillations are nearly sinusoidal, which is valid as long as the bias current is not too close to the critical current; of course the agreement improves with increasing $\gamma$. From Fig. 3 it is evident that the formula systematically underestimates the actual value. This is reasonable because the estimate is based on the approximation $v = \gamma$, but this overestimates the velocity (a better approximation is $v = (\gamma^2 - 1)^{1/2}$).

The next step is to obtain an approximate solution for the dynamics of the horizontal junctions. We proceed using the same approximations as before, inserting into Eq. (10) the approximate solution for the vertical junctions, Eqs. (16,17). We write the solution as a constant plus an oscillating term:

$$H_l(t) = H_0 + Y_l(t) \quad (23)$$

and assume that the oscillating part is small ($Y_l << 1$).

Physically, the absence of a term which grows linearly in the time (compare Eq.(13)) means that there is no d.c. voltage across the horizontal junctions ($\langle \dot{H}_l \rangle = 0$). This is a reasonable assumption since the horizontal branches are unbiased; it is also what we find in the numerical simulations.

Proceeding as for the vertical junctions we obtain for $H_i(t)$ a solution of the form

$$H_i(t) = H_0 + A_H \sin (\theta_{i,0} + vt) + B_H \cos (\theta_{i,0} + vt) \quad (24)$$

where $H_0$, $A_H$ and $B_H$ are again to be determined using harmonic balance.
constants fixes $H_0$ to be

$$\sin H_0 = \frac{-2H_0}{\beta_l}. \tag{25}$$

The physical meaning of this constant is analogous to the classical SQUID phase shift induced by a magnetic field trapped in the loop [14], the factor 2 takes into account the fact that in this case there 4 rather than 2 junctions in the elementary loop. In the limit of high bias current ($\gamma > \gamma > 1$) and no trapped magnetic field ($H_0 = 0$) we find

$$A_H = \frac{1}{\gamma^2(\beta_l + 2)^2} \left[ -\sin \eta \beta_l + \frac{1}{\beta_l} \frac{(1 - \cos \eta \beta_l)}{\gamma (\beta_l + 2)} \right] - \frac{\eta \beta_l}{\gamma (\beta_l + 2)} \tag{26}$$

$$B_H = \frac{1}{\beta_l \gamma^3(\beta_l + 2)} \left[ -\sin \eta \beta_l + \frac{1}{\beta_l} \frac{(1 - \cos \eta \beta_l)}{\gamma (\beta_l + 2)} \right] \tag{27}$$

in agreement with our numerical simulations. (For example, for the same parameters as in figure 2, we find agreement to better than 20%.) Note that when the applied magnetic field vanishes ($\eta = 0$), Eqs.(25,26) give $A_H = B_H = 0$ so the horizontal junctions are inactive. When the magnetic field is present this is no longer true; nevertheless, the amplitude of the oscillations for the horizontal junctions are much smaller than those of the vertical junctions.

At this stage one could carry the analysis further, inserting the solution (23) back into Eq. (9), and repeating the harmonic balance procedure to get a more accurate expression for the phases $V_l(t)$ and the velocity $v$, then iterating the scheme for the horizontal junctions, and so on. However, for our purposes the estimates Eqs. (18-20) and Eqs. (24-26) are adequate. Let us summarize the main results of this section:

1) For $\gamma > > 1$ the solution of the 1D array can be approximated analytically by retaining only the first Fourier component. This approximation is a common one which has been used in previous studies of a single junction.
2) In this limit there is a clear difference between horizontal and vertical junctions, which results from the anisotropic bias current: the vertical junction phases increase without limit (on average linearly in time), while the horizontal junction phases oscillate about a fixed value.

3) The magnetic field is responsible for the spatial non-uniformity of the dynamics: if the applied magnetic field is zero then the vertical junctions oscillate synchronously \( [\theta_{l,0} = 0 \text{ for all } l, \text{ compare Eq. (10)}] \) and the horizontal junctions are inactive.

4. Coupling between two rows

We now extend the analysis to the case of two rows of plaquettes. Our main interest is to study the interactions between rows. To do this, we proceed as follows: the solution for the first row is assumed to coincide with the solution for the isolated row, and with this imposed we solve the equation for the second row. In other words we seek the solution of one row driven by the unperturbed solution of the other row. Although this scheme is "undemocratic", it has the virtue that the dynamical equations are tractable using the same approximations as in the last section. The equation for the second row reads:

\[
\dot{V}_{l,2} = -\sin V_{l,2} + \frac{1}{\beta_l} [V_{l+1,2} - 2V_{l+1,2} + V_{l-1,2} + (A_H - A_H \cos (\eta \beta_l) + B_H \sin (\eta \beta_l)) \sin (V_{l,0} + vt)
+ (B_H - A_H \sin (\eta \beta_l) - B_H \cos (\eta \beta_l)) \cos (V_{l,0} + vt)] + \gamma \]  

(28)

whose asymptotic solution, in the same sense discussed for the single row, is:

\[
V_{l,2} = \theta_{l,0} + \delta + vt + A_v \sin (\theta_{l,0} + \delta + vt) + B_v \cos (\theta_{l,0} + \delta + vt) .
\]  

(29)
Again, the parameters $\delta, \overline{A_v}$, and $\overline{B_v}$ are determined by a set of algebraic equations which result from harmonic balance, namely

$$\overline{B_v} = 2(\gamma + v)$$

$$\sin \delta \left\{ -A_v v + \frac{1}{\beta_l} \left[ 2B_v (\cos(\eta \beta_l) - 1) \right] \right\} = \cos \delta \left\{ \overline{B_v} v - 1 + \frac{2A_v}{\beta_l} \left[ \cos(\eta \beta_l) - 1 \right] \right\} +$$

$$\frac{1}{\beta_l} \left[ A_H (1 - \cos(\eta \beta_l)) + B_H \sin(\eta \beta_l) \right]$$

$$\sin \delta \left\{ -\overline{B_v} v + 1 - \frac{2A_v}{\beta_l} \left[ \cos(\eta \beta_l) - 1 \right] \right\} = \cos \delta \left\{ -A_v v + \frac{1}{\beta_l} \left[ 2\overline{B_v} v (\cos(\eta \beta_l) - 1) \right] \right\} +$$

$$\frac{1}{\beta_l} \left[ B_H (1 - \cos(\eta \beta_l)) + A_H \sin(\eta \beta_l) \right].$$

The most striking feature of these equations is that if either $\eta = 0$ (no magnetic field) or $\beta_l \rightarrow 0$ (uncoupled limit) then $\delta$ is undetermined, i.e. the phase shift between the two rows is arbitrary. In other words the observed value of $\delta$ can be anything, and depends on the initial conditions. On the contrary, even a tiny magnetic field leads to the selection of a specific value of $\delta$. To check that this conclusion is not an artifact of our approximation scheme we have performed numerical simulations of the full dynamical equations for a two-row array, and we have indeed found (see Fig. 4) that in the presence of a magnetic field the final asymptotic value of the phase shift $\delta$ is zero, regardless of the initial conditions and also regardless of the values of the parameters $\gamma$ and $\beta_l$. Thus, while the simulations qualitatively support our analysis, quantitatively they do not: Eqs. (29-31) predict an asymptotic value of $\delta$ that is parameter dependent and not, in general, equal to zero (see Fig. 4). We suspect that this disagreement in the value of $\delta$ is an artifact of our separation of the system into a ”slave” row and a ”master” row, and that an analysis which treats the two rows on an equal footing would lead to a more accurate value of the phase shift.
5 Discussion and Conclusion

Our analysis has shown that it is possible to induce a spatially dependent solution in 2D arrays by the application of a magnetic field. As a consequence the magnetic field produces a coupling between rows, even in the absence of an external load. In the limit of zero magnetic field the phase shift between rows (for the simplest case of two rows) becomes arbitrary, which signals that the dynamics is only neutrally stable.

This finding may have practical importance for the application of Josephson junction arrays as local oscillators. In the absence of an external load, it is known that (in the context of lump circuit equations) the inphase state of 2D arrays is neutrally stable\[7, 8\]. Neutrally stable dynamical states (other than the inphase state) also occur in a variety of 1D series arrays both with and without external loads \[18, 19, 20, 21, 22, 8, 23\]. One drawback to neutrally stable dynamics is their intrinsic sensitivity to noise. As a result, it is desirable to modify these arrays in a way which will stabilize the dynamics (i.e. make the target dynamical state a bona fide attractor). One way to do this is to couple the array to an appropriate external load, but this can have the disadvantage of limiting the frequency range over which the array can operate\[8\]. An alternative possibility suggested by the present work is to induce coupling via the application of a magnetic field. Of course, this also makes the dynamics spatially non-uniform, which may itself be a drawback for applications.

We reiterate that our conclusions are based on a number of assumptions: we have i) included only self-inductances; ii) assumed that the junction parameters are identical; iii) neglected the effects of any external (parasitic) load; and iv) ignored higher harmonics in the junction oscillations. The virtue of these assumptions is that, within the context of the
idealized model, we have achieved some level of analytic understanding of a notoriously complex nonlinear system. One direction for future theoretical work is to extend our analysis to include these other effects. Of these, the presence of a parasitic load and higher harmonics could be handled straightforwardly within the same framework. In contrast, the presence of mutual inductance and disorder (i.e. variations among the junction parameters) is more difficult; nevertheless, we can make an educated guess as to how they might modify the dynamics.

Consider first the role of mutual inductances. Physically, mutual inductance provides a mechanism for coupling non-neighboring junctions in a similar manner as does the self-inductance of a single loop. This is reflected in the governing dynamical equations: self-inductance introduces a next-nearest-neighbor coupling; mutual inductance will introduce further couplings whose strength, however, diminishes with distance. Thus, we expect the inclusion of mutual inductance to increase the net coupling strength between junctions, thereby providing additional interactions between rows which are responsible for breaking the inherent neutral stability. However, we expect nothing fundamentally new in the observed dynamical behavior.

Turning next to the effects of having non-identical junctions in the array, we can learn from some recent theoretical studies on bare two-dimensional arrays in the absence of a magnetic field. These show that disorder spontaneously induces shunt currents (transverse to the direction of the imposed bias current) which tend to compensate for the mismatch between junctions within a row; however, the inter-row dynamics remains neutrally stable. These findings are consistent with those of Kautz on disordered 2D arrays with an external load, who also found that disorder apparently plays an unimportant
dynamical role in coupling rows. Consequently, we expect that small amounts of disorder will not greatly change the dynamical behavior we have described.

Of course, pulling together these various effects within a single theoretical framework is a challenging task. On the other hand, within the context of the idealized model studied here, we have achieved some level of analytic understanding of these complex nonlinear dynamical systems.

6 Acknowledgement

We wish to thank Y. Braiman, T. Doderer, R.P. Huebener, S.G. Lachenmann, and B. Larsen and especially S. Benz and R.L. Kautz for useful comments and discussions. The work was partially supported a grant from the U.S. Office of Naval Research under contract number N00014-J-91-1257. GF wishes to thank Georgia Tech for their hospitality during the preparation of this work and the EU for financial support through the Human Capital and Mobility program (Contract No ERBCHRXCT920068).

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**Figure Captions**

Fig. 1 Schematic circuit model for a) a two-dimensional array; b) a single row of plaquettes; c) two rows.

Fig. 2 $\dot{V}_i(t)$ and $\dot{H}_i(t)$ for a 1D row. Parameters of the simulations are: $N = 10$, $\beta_l = 4$, $\eta = \pi/4$, $\gamma = 1.5$.

Fig. 3 Time delay of the voltage peak between two adjacent cells compared with theoretical estimate. Parameters of the simulations are: $N = 10$, $\beta_l = 1$, $\gamma = 1.5$.

Fig. 4 Time evolution of two vertical junctions in the same column for a) $\eta = 0$ and b) $\eta = \pi/4$. Parameters of the simulations are: $M = 3$, $N = 10$, $\beta_l = 1$, $\gamma = 1.5$.  

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