Induction Motor Flux Estimation using Nonlinear Sliding Observers

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Abstract: A nonlinear sliding flux was proposed for an induction motor. Its dynamic observation errors converge asymptotically to zero, independently from the inputs. The aim of this work was to study the robustness of this observer with respect to the variation of the rotor resistance known to be a crucial parameter for the control. The dynamic performance of this sliding observer was compared to that of Verghese observer via a simulation of an IM driven by U/F control in open loop.

Key words: Nonlinear sliding observer, induction motor, rotor flux estimation

INTRODUCTION

DC motors have been used extensively in the industry because of the simple control strategies required to achieve good performance, in variable speed applications. However, in comparison with their counterparts, IM drives, DC drives result more expensive and less robust devices, not to mention the maintenance they require due to the commutator. Because they are highly nonlinear, thus requiring much more control complex algorithms, IM drives were rarely used in control applications in the past.

Nowadays, as a consequence of the important progress realized in nonlinear control theory and power electronics, the AC drives, by using new control techniques, have proved to outperform the DC ones. Among these techniques, both field oriented control (FOC) and nonlinear input-output decoupling have emerged as powerful tools for high performance control of induction machines[1,2]. The main drawback of both algorithms is the need of flux sensors, which are to be inserted in the air gap and involve a redesign of the machine, which reduces reliability and implies both additional costs and technological difficulties. For this reason, flux observers have been widely investigated[3,4]: they are rather sensitive with respect to rotor resistance variations.

Starting with[5], rotor resistance estimators have been studied[6,7] but most contributions rely on simplifying assumptions and definite results are still not completely available since, no method applies when the motor is in low-speed regime.

Class of sliding mode observers: Consider the $n^{th}$ order nonlinear system:

$$\dot{x} = f(x,u), \quad x \in R^n; \quad u \in R^d$$

(1)

and for convenience, consider a vector of measurements:

$$y = Cx, \quad y \in R^r$$

(2)

The system is assumed to be observable and the observer is defined with the following structure:

$$\dot{\hat{x}} = \hat{f}(\hat{x}, y, u) + KI_y$$

(3)
where $\hat{x} \in \mathbb{R}^n$, $\hat{f}$ is our model of $f$, $K$ is $n \times r$ gain matrix to be specified and
\[
I_s = [\text{sgn}(s_1), \text{sgn}(s_2), \ldots, \text{sgn}(s_r)]^T
\] (4)
where
\[
[s_1, s_2, \ldots, s_r]^T = S = \Gamma(y-C\hat{x})
\] (5)
and $\Gamma$ is $r \times q$ matrix to be specified. Defining the error vector $\tilde{y} = C\hat{x}$ and $\tilde{x} = (x - \hat{x})$, one has
\[
\dot{\tilde{x}} = \Delta f - K\tilde{x}
\] (6)

where $\Delta f = f(x, u) - \hat{f}(\tilde{x}, y, u)$

The $r \times r$ matrix $\Gamma CK$ is invisible with an appropriate choice for $\Gamma$ and $K$. Thus, from (6) and (7) the dynamics on the reduced order manifold is given by:
\[
\dot{x} = (I - K(\Gamma CK)^{-1}\Gamma C)\Delta f
\] (8)
with $\Gamma C\tilde{x} = 0$. The structure of $\Delta f$ must be known before any further analysis can be done.

**Verghese observer:** The dynamic behaviour of an induction motor working under no saturation of its magnetic circuits can be described in a fixed stator reference $(a-b)$ frame\(^{[3]}\) by:
\[
\begin{align*}
\frac{d\phi_{a,r}}{dt} &= -b\phi_{a,r} + ai_{a,r} - \omega p\phi_{b,r}, \\
\frac{d\phi_{b,r}}{dt} &= -b\phi_{b,r} + ai_{b,r} + \omega p\phi_{a,r}, \\
\frac{di_{a,r}}{dt} &= \gamma_1 V_{a,r} - \gamma_1 i_{a,r} + \gamma_2 \phi_{b,r} + \gamma_3 \phi_{a,r} + \gamma_5 \phi_{b,r}, \\
\frac{di_{b,r}}{dt} &= \gamma_1 V_{b,r} - \gamma_1 i_{b,r} + \gamma_2 \phi_{a,r} - \gamma_3 \phi_{b,r} + \gamma_5 \phi_{a,r}, \\
\frac{d\omega}{dt} &= p(M/L_s)(i_{a,r}\phi_{b,r} - i_{b,r}\phi_{a,r}) - T_L + f_s\omega
\end{align*}
\] (9)
with $\alpha_s = pM/J_L$, $b = R_s/L_s = 1/T_r$, $\sigma = 1 - (M_s^2/L_s L_r)$, $a = R_s M/L_r$, $\gamma_1 = (R_r/\sigma L_r) + (R_r M_s^2/\sigma L_s L_r)$, $\gamma_2 = M R_r/\sigma L_s L_r$, $\gamma_3 = M p/\sigma L_s L_r$, $\gamma_4 = 1/\sigma L_s$

where: $(\phi_{a,r}, \phi_{b,r}), (i_{a,r}, i_{b,r}), T_L, J, \sigma, f_s, R_s, L_s, R_r, L_r, M, T_r, \omega$ and $p$ are respectively the rotor fluxes, the stator currents, the torque load, the moment of inertia, the leakage and sticky friction coefficients, the rotor and stator windings resistances and inductances, the mutual inductance, the rotor time constant, the mechanical speed and the number of pole pairs. Therefore, setting $x = (\alpha_s, \phi_{a,r}, \phi_{b,r}, i_{a,r}, i_{b,r})^T$, (9) is written in the form:
\[
\begin{align*}
\dot{x}_1 &= \alpha_s(x_5 - x_4 - x_3) - \alpha_2 T_L - \alpha_3 x_2, \\
\dot{x}_2 &= \alpha_2 x_3 - bx_5 - px_3 x_4, \\
\dot{x}_3 &= ax_4 - bx_6 + px_5 x_4, \\
\dot{x}_4 &= -\gamma_1 x_4 + \gamma_2 x_3 + \gamma_3 x_1 + \gamma_4 v_1, \\
\dot{x}_5 &= -\gamma_1 x_3 + \gamma_2 x_2 - \gamma_3 x_5 + \gamma_4 v_2.
\end{align*}
\] (10)

The Verghese observer model which is a copy of the first four equations of (9) where added a corrective term due to a prediction error, is written in compact form as\(^{[3]}\):
\[
\begin{align*}
\begin{bmatrix}
\dot{i}_s \\
\dot{\phi}_r
\end{bmatrix} &= 
\begin{bmatrix}
-\gamma_1 I \\
(M/c T_r) I
\end{bmatrix}
\begin{bmatrix}
1
0
\end{bmatrix}
+ 
\begin{bmatrix}
\dot{i}_s \\
\dot{\phi}_r
\end{bmatrix} \\

\begin{bmatrix}
1 & 0
\end{bmatrix}
\begin{bmatrix}
I_p/c & 0
\end{bmatrix}
+ 
\begin{bmatrix}
k_1 I + k_2 w_p J \\
k_3 I + k_4 w_p J
\end{bmatrix}
(\dot{i}_s - i_s)
\end{align*}
\] where
\[
\begin{align*}
\phi &= [\phi_{\alpha,r}, \phi_{\beta,r}]^T, \\
i &= [i_{a,r}, i_{b,r}]^T, \quad \nu_r = [V_{a,r}, V_{b,r}]^T, \\
L &= \begin{bmatrix}1 & 0 \\0 & 1 \end{bmatrix}, \quad J = \begin{bmatrix}0 & -1 \\1 & 0 \end{bmatrix}, \quad c = \sigma L_s L_r,
\end{align*}
\]

$(\dot{i}_s, \dot{\phi}_r)$ are the derivatives of $(i_s, \phi_r)$, $k_1$ are scalars and $\omega_s = p\omega$ is the electrical speed of the rotor. The dynamics of the observation error $e = \hat{x} - x$ is given by:
\[
\begin{align*}
e &= \begin{bmatrix}
(k_1 - \gamma_1) I \\
(M/c T_r) I
\end{bmatrix}
\begin{bmatrix}
I_k + \nu_r & 0
\end{bmatrix}
+ 
\begin{bmatrix}
\dot{i}_s \\
\dot{\phi}_r
\end{bmatrix} \\

\begin{bmatrix}
k_3 + (M/c T_r) I \\
(M/c T_r) I
\end{bmatrix}
\begin{bmatrix}
1 & -1
\end{bmatrix}
\begin{bmatrix}
I_p/c & 0
\end{bmatrix}
+ 
\begin{bmatrix}
k_2 J & -M/c J \\
k_4 J & J
\end{bmatrix}
\begin{bmatrix}
1 & 0
\end{bmatrix}
\begin{bmatrix}
k_1 I + k_2 w_p J \\
k_3 I + k_4 w_p J
\end{bmatrix}
\begin{bmatrix}
1 & 0
\end{bmatrix}
\begin{bmatrix}
1 & 0
\end{bmatrix}
e.
\end{align*}
\] (11)

If $k_1$ and $k_3$ are selected such that: $k_1 - \gamma_1 = -k_2/T_r$ and $k_3 + (M/c T_r) = -k_4/T_r$ the error dynamics becomes: $\dot{e} = AQe$

where: $A = \begin{bmatrix}k_1 I & -(M/c) I \\k_1 I & I \end{bmatrix}$

and: $Q = \begin{bmatrix}(1/T_r) J + \omega_s J & 0 \\0 & (1/T_r) J + \omega_s J \end{bmatrix}$.

The freedom that one has in choosing $k_2$ and $k_4$ is used to place the eigenvalues of $A$ in pairs at arbitrary locations, as is verified by noting that the characteristic polynomial of $A$ is:
\[
\begin{bmatrix}p^2 - (1 + k_2)p + k_2 + k_4 (M/c) \end{bmatrix}^2
\]
If the eigenvalues of A are \(p_1\) (twice) and \(p_2\) (twice), then the eigenvalues of the matrix product in (11) can be shown to be:

\[
\begin{bmatrix}
-1/T_z + j\omega_s \\
-1/T_z + j\omega_s 
\end{bmatrix} \begin{bmatrix} p_1 \\
p_2 \end{bmatrix}
\]

Hence, if the speed is (nearly) constant, the error dynamics is approximately governed by these eigenvalues. If it is time-varying, we will attempt a Lyapunov analysis.

Flux sliding mode observer: The proposed type of sliding mode based observer of (9) can be written as:

\[
\begin{align*}
\dot{x}_1 &= \alpha_1(x_1, \dot{x}_1, \dot{x}_2, x_2) - \alpha_2, T_z - \alpha_3, x_1 + K_i, I + \theta_1(x_1 - \dot{x}_1) \\
\dot{x}_2 &= \dot{x}_2 - b\dot{x}_2 - px_1, x_1 + K_i, I \\
\dot{x}_3 &= -\gamma_1, x_1 + \gamma_2, x_2 + y_1, x_1, \dot{x}_2 + y_1, x_4 + K_s, I \\
\dot{x}_4 &= -\gamma_2, x_1 + \gamma_3, x_2 + y_2, x_3 + y_3, x_4 + K_s, I \\
\dot{x}_5 &= -\gamma_3, x_1 + \gamma_4, x_2 + y_4, x_3 + y_4, x_5 + K_s, I
\end{align*}
\]

where \(K_i, I_s = \dot{\lambda}_i, \text{sign}(s_1) + \dot{\lambda}_i, \text{sign}(s_2)\). for \(i \in \{1, \ldots, 5\}\), \(K_i, q_1\) are the observers gains. The sliding surface \(S\) is given by:

\[
S = M \begin{bmatrix} x_1 - \dot{x}_1 \\
\dot{x}_2 - \dot{x}_2 \\
\dot{x}_3 - \dot{x}_3 \\
\dot{x}_4 - \dot{x}_4 \\
\dot{x}_5 - \dot{x}_5 \end{bmatrix} = \begin{bmatrix} s_1 \\
s_2 \\
s_3 \\
s_4 \\
s_5 \end{bmatrix} = 0
\]

The stability analysis consists of determining \(K_i, K_s\) such that the surface \(S = 0\) is the attractive. Then \(K_1, K_2, K_3, q_1\) are determined such that the reduced order system obtained when \(S = 0\) is locally stable to 0 in the attractive domain defined as follows:

Let us consider the Lyapunov function \(V = \frac{S^T S}{2}\) such that \((S = 0 \rightarrow e_4 = e_5 = 0)\) with M as a regular matrix. The attractive of sliding surface \(S = 0\) is given by:

\[
\begin{aligned}
\dot{V} &< 0 \\
\dot{V} &< 0
\end{aligned}
\]

where:

\[
\begin{align*}
W &:= M^{-1} \begin{bmatrix} e_2 \\
e_3 \end{bmatrix} \begin{bmatrix} \lambda_{11} & \lambda_{12} \\
\lambda_{21} & \lambda_{22} \end{bmatrix} \begin{bmatrix} e_2 \\
e_3 \end{bmatrix} + \begin{bmatrix} K_4 \\
K_5 \end{bmatrix} = M^{-1} \Delta \\
\Delta &:= \begin{bmatrix} \delta_1 & 0 \\
0 & \delta_2 \end{bmatrix}, \delta_1, \delta_2 > 0.
\end{align*}
\]

From the singular perturbation theory, the dynamics of \(\Omega\) is supposed to be a slow variable with respect to the currents and the flux dynamics. The conditions of attraction between \(s_1, e_2\) and \(s_2, e_3\) are decoupled. So (12) is obtained within the set defined by the following inequalities:

\[
\begin{align*}
\text{if } s_1 > 0 & \text{ then } e_2 < \delta_1 \\
\text{if } s_1 < 0 & \text{ then } e_2 > -\delta_1 \\
\text{if } s_2 > 0 & \text{ then } e_3 < \delta_2 \\
\text{if } s_2 < 0 & \text{ then } e_3 > -\delta_2
\end{align*}
\]

On the sliding surface, \(S = 0\) which is invariant, the vector \(I_s = I_s\) is given by:

\[
\begin{bmatrix} 0 \\
0 \\
0 \\
0 \\
0 \end{bmatrix} = M^{-1} \begin{bmatrix} e_2 \\
e_3 \end{bmatrix} \begin{bmatrix} \delta_1 & 0 \\
0 & \delta_2 \end{bmatrix} + I_s
\]

From the definition of equivalent vector, one obtains the dynamics error after a finite time \(t_0\) which is reduced to:

\[
\begin{align*}
\dot{e}_2 &= \dot{H}_1 \begin{bmatrix} e_2 \\
e_3 \end{bmatrix} \begin{bmatrix} \delta_1 & 0 \\
0 & \delta_2 \end{bmatrix} \begin{bmatrix} e_2 \\
e_3 \end{bmatrix} + I_s \\
\end{align*}
\]

Thus:

\[
\begin{bmatrix} \lambda_{11} & \lambda_{12} \\
\lambda_{21} & \lambda_{22} \end{bmatrix} = \begin{bmatrix} \alpha_1 x_5 & \alpha_1 x_4 \end{bmatrix}
\]

The reduced system of the observer errors can be written as:

\[
\begin{align*}
\dot{e}_1 &= -\alpha_1(x_5, e_2 - e_4) - \lambda_1, \delta_1 + \alpha_2, \lambda_2, \delta_2 \\
\dot{e}_3 &= -\lambda_2, \delta_2 - px_1, e_2 - K_s, I \\
\dot{e}_4 &= \gamma_2, e_1 + \gamma_2, x_1, e_2 + \gamma_2, x_2 + \gamma_2, x_3 + \gamma_2, x_4 + K_s, I \\
\dot{e}_5 &= \gamma_3, e_1 + \gamma_3, x_1, e_2 + \gamma_3, x_2 + \gamma_3, x_3 + \gamma_3, x_4 + K_s, I
\end{align*}
\]

Digital simulation: A combination of the two observers presented in Sections 3 and 4, is simulated in Matlab/Simulink for the simultaneous estimation of rotor fluxes. The combined estimation is depicted in the block diagram of Fig. 1.
A digital simulation of the proposed combined scheme illustrates its behaviour when the motor is operating under U/F control in open loop with full load capability (The IM data in simulation are given in the Appendix).

Figure 2a shows the reference speed of the motor with the measured one and Fig. 2b shows the first component of the rotor flux.

From Fig. 2c, we verify that observation errors converge to zero in steady-state operations but, as stated previously, they tend to their maximum value, in low-speed regime, particularly at zero-crossing. Figure 2d illustrates the well-known insensitivity property of sliding modes with respect to disturbances.

Figure 2e and 2f show the robustness of the two observers in case of an instantaneous rotor resistance increase of 50% in the IM drive.

CONCLUSION

In this study, a novel combined scheme for concurrent estimation of the rotor fluxes of an IM was presented. The proposed method is based on two nonlinear observers. The simulation results show a good performance of the sliding mode estimation scheme.

It has been shown that sliding mode observer design methods based on the prescribed form of the Lyapunov function candidate can be successfully applied. The simulation results are suggesting that design can be implemented based on the mechanical motion measurement only, thus avoiding flux variable measurement. In addition, the simplicity of the algorithm makes it suitable for an on-line implementation.

In further work, the authors intend to study high-order sliding modes, in both control and observation for an induction machine and a pneumatic robot, to remove the chattering effect which is known to be the main drawback of the standard sliding modes.

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Appendix

| Machines parameters |
|----------------------|
| Total Stator Inductance | 0.142 H |
| Total Rotor Inductance | 0.076 H |
| Mutual Inductance     | 0.099 H |
| Stator Resistance     | 1.633Ω  |
| Rotor Resistance      | 0.93 Ω  |
| Rotor Inertia (IM + load) | 0.029 Kgm² |
| Number of Pole pairs  | 2       |

| Rated magnitudes      |
|-----------------------|
| Direct voltage        | 450 V   |
| Load Torque           | +7 & -7Nm |
| Speed                 | 1430 rpm |
| Stator Flux           | 0.59 Wb |
| Power                 | 1.5 Kw  |
| Coefficient of sticky friction | 0.0038 Nms/rd |

Fig. 1: Flux observers for induction motor

Fig. 2: Simulation results
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