Vibrations of multilayer composite viscoelastic curved pipe under internal pressure

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Abstract. The paper presents a mathematical statement and the methods to solve the problem of natural vibrations of thin-walled multilayer composite viscoelastic curved pipes under internal pressure. A brief overview of well-known publications devoted to this problem is given. The stress state and flexibility of the pipes are investigated for two types of boundary conditions, taking into account internal pressure and viscoelastic properties of the material and geometrical and physical parameters of the pipe. The paper provides the results of numerical analysis of the spectra of the lower complex eigenfrequencies of the pipe depending on internal pressure, geometrical, structural parameters, rheological properties and boundary conditions. Ten lower eigenfrequencies were found and the corresponding vibration modes constructed. The paper presents a comparison of solution results with known solutions and experimental results.

1. Introduction

The problem of vibration is a relevant one in the oil-and-gas and aircraft industries. Pulsating flows and the intense vibrations initiated by them occur in power plants too [1]. An analysis of the pipelines behavior in nuclear power plants showed [2, 3], that the frequencies of vibrations vary from 0.5 to 1000 Hz, and the vibration amplitudes are 3-5 mm. The phenomenon of pulsations in the coolant circulation circuit [4] and equipment vibrations are the most acute. Cases of the loss-of-piping integrity are known. The fracture pattern indicates the fatigue nature of breaking. The origin and development of cracks is associated with elastic vibrations and hydraulic shock. Thus, the problem of pipelines is gaining important economic importance. When the frequencies of the excitation spectrum coincide with the eigenfrequencies of the pipeline, resonance develops in the system. One way to reduce the amplitudes of vibrations is to detune the dynamic system from resonances at the design stage [5, 6]. The eigenfrequencies of the pipeline should not coincide with the fundamental frequencies of the excitation spectrum. For this, a thorough calculation of the
hydrodynamic loading parameters and analysis of the dynamic stability of a pipeline using classical mechanics methods are needed [7, 8]. At present, regulatory documents [9–11] and reference books [12, 13] contain methods for calculating the static and dynamic parameters of pipelines made of homogeneous and isotropic materials. In these cases, as a rule, simplified rod models were used. To calculate the dynamic characteristics of curved pipes, the linear theory of rod vibrations[14, 15] and the formulas for circular arches[16] [17] were used. The rod forms of vibration are taken into account. Obviously, this approach is justified for thick-walled structures only. Generally, for complex structures, a plane or spatial design scheme of the theory of elasticity were used [27-29], the solution to the problem of natural vibrations for such structures becomes greatly complicated. As is well-known, the development and creation of composite structures and methods to assess their dynamic characteristics is a more complex problem. At that, the subject of design is the material itself, or rather its structure. The new material is designed taking into account technological capabilities for a given structure design and a given load. Only in this unity can the potentialities embodied in the composite be realized. However, the method for calculating the dynamic parameters of multilayer pipes, considering the layered fiber structure and anisotropy of the material, is currently practically absent. The effect of inhomogeneity in the material structure and the initial ovality and non-uniform thickness of the cross section on the stress state and dynamic properties of composite pipe, has not been adequately studied.

In this regard, the task of studying the dynamic properties of multilayer pipes and pipelines made of composite materials is of great practical importance. The need to solve an important scientific and technical problem related to the development of methods for calculating and designing multilayer composite pipes and pipelines in order to improve their dynamic properties and increase reliability determines the relevance of this study.

2. Statement of the problem
Consider a viscoelastic composite pipe whose centerline is an arc of a circle of radius \( R \) and length \( L \). The pipe has a cross-section with a nominal average radius \( r \) and wall thickness \( h \). Let us restrict ourselves to thin-walled long pipes of small curvature: \( h/r \leq 1/20, L/r \geq 4, r/R \leq 1/5 \). The pipe is considered as an element of a pipeline that conveys fluid. Consider the internal flow as a homogeneous one, the fluid - as a single-phase, ideal and incompressible one. Rigid weightless flanges that rest on fixed hinged supports close the end sections of the pipe.

The boundary conditions at \( s = 0 \) and \( s = L \) at points with coordinates \( (0 \leq s \leq L) \) \( \phi_0 = 90^\circ \) and \( \phi_0 = 270^\circ \) are zero, i.e. the displacement components are \( u = \theta = w = 0 \). To describe the viscoelastic properties of the body material, we accept the linear hereditary Boltzmann–Volterra theory; physical relations for the \( m \)-th viscoelastic element of the system are defined as [18]

\[
\sigma_{mk}^n(t) = \tilde{\lambda}_n \Theta^n(t) \delta_{mk} + 2 \tilde{\mu}_n \varepsilon_{mk}^n(t), \tag{1}
\]

where \( \tilde{\lambda}_n, \tilde{\mu}_n \) are the Volterra integral operators. Poisson’s \( \nu_n \) ratio in the proposed statement of the problem is assumed constant. This means that for a structurally homogeneous viscoelastic system, the modes of natural vibrations are equal to the eigenvectors of the corresponding elastic problem [19, 20]. Expressing integral operators from the well-known formulas in terms of \( \tilde{E}_n, \tilde{\nu}_n \), and considering that \( \tilde{v}_n = \nu_n = \text{const} \), instead of (1) we get

\[
\sigma_{mk}^n(t) = \frac{\tilde{E}_n}{1+\nu_n} \left[ \frac{\nu_n}{1-2\nu_n} \Theta^n(t) \delta_{mk} + \varepsilon_{mk}^n(t) \right], \tag{2}
\]

where \( \tilde{E}_n \) is the Volterra operator of the following form

\[
\tilde{E}_n \phi(t) = E_{on} \left[ \phi(t) - \int_0^t R_{En}(t-\tau) \phi(\tau) d\tau \right]. \tag{3}
\]

Here \( E_{on} \) is the instantaneous modulus of elasticity, and \( R_{En} \) is the relaxation kernel.
Given (1), the time function in equality (3) is \( \phi(t) = \exp(-i\omega t) \) with slowly varying amplitude. Assuming the smallness of integral term \( \int_0^\infty R(\tau) d\tau \), and using the freezing method [21], we replace relation (3) with an approximate one:

\[
\bar{E}_n(t) \approx E_{0j}[1 - \bar{E}_n^c(\omega_R) - i\bar{E}_n^s(\omega_R)]\phi(t) \equiv \bar{E}_n(t),
\]

where \( \bar{E}_n^c(\omega_R) = \int_0^\infty R_E(\tau) \cos \omega_R \tau d\tau \), \( \bar{E}_n^s(\omega_R) = \int_0^\infty R_E(\tau) \sin \omega_R \tau d\tau \) are the cosine and sine of the Fourier images of the relaxation kernel of the material, respectively, \( \omega_R \) is the real value. A three parametric relaxation kernel [30] \( R_E(t) = Ae^{-\beta \omega t} / t^{1-\alpha} \) is taken as a sample of a viscoelastic material. Present the following forms of motion under given boundary conditions [27,28]:

\[
w(s, \phi, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \cos n \phi \sin \frac{m\pi s}{L},
\]

\[
\vartheta(s, \phi, t) = -\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{n} w_{mn} \sin
\]

\[
u(s, \phi, t) = \frac{\pi R}{L} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m n \pi s}{L} w_{mn} \cos n \phi \cos \frac{m\pi s}{L}.
\]

Here \( u, \vartheta \) and \( w \) are the displacement components of the points of the shell middle surface in the axial, circumferential, and radial directions. The wavenumbers \( m \) and \( n \) characterize the vibration mode: \( m \) is the number of half-waves in the axial direction, \( n \) is the number of waves in the circumferential direction.

The following equation determines the kinetic energy of the pipe motion:

\[
K = \frac{1}{2} \rho_T r h m \int_0^L \int_0^{2\pi} \left[ \dot{u}^2 + \dot{\vartheta}^2 + \dot{w}^2 \right] h(\phi) ds d\phi = \frac{1}{8} (m_T) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[ \frac{n^2 + 1}{n^2} + \frac{m^2 \pi^2 r^2}{n^4 l^2} \right] \dot{w}_{mn}^2
\]

The viscoelastic potential, constructed based on the semi-momentless theory of multilayer thin shells and approximations (5), has the form:

\[
\Pi = \frac{1}{2} r \int_0^L \int_0^{2\pi} \left( B_{1m} \dot{\vartheta}^2 + D_{2m} \dot{w}_{mn}^2 \right) ds d\phi = \frac{\pi B_{1m} r L}{4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[ -\left( \frac{m\pi}{L} \right)^2 \frac{r}{n^2} w_{mn} + \frac{n - 2 w_{mn-1}}{n - 1} \frac{w_{mn}}{D} + \frac{n + 2 w_{mn+1}}{n + 1} \frac{w_{mn}}{D} \right] + \frac{\pi D_{2m} r^2}{4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( n^2 - 1 \right)^2 \dot{w}_{mn}^2.
\]

The axial deformation of the wall middle surface has the following form:

\[
\varepsilon_1 = \frac{1}{R} (w \cos \phi - \vartheta \sin \phi) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[ -\left( \frac{mn}{L} \right)^2 \frac{r}{n^2} w_{mn} + \frac{n - 2 w_{mn-1}}{n - 1} \frac{w_{mn}}{D} + \frac{n + 2 w_{mn+1}}{n + 1} \frac{w_{mn}}{D} \right] \cos n \phi \sin \frac{m\pi s}{L}.
\]

The change in wall curvature in the circumferential direction is

\[
k_2 = \frac{1}{r^2} \left( \frac{\partial^2 w}{\partial \phi^2} - \frac{\partial \vartheta}{\partial \phi} \right) = -\frac{1}{r^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( n^2 - 1 \right) w_{mn} \cos n \phi \sin \frac{m\pi s}{L}.
\]

The potential of external forces is

\[
W = \frac{1}{2} p_m \Delta V = \frac{1}{4} \pi p_m L \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( n^2 - 1 \right) w_{mn}^2.
\]

Here \( \Delta V \) is the change in internal cavity volume due to the pipe wall bending. Under pipe vibrations, the pressure \( p_m \) resists the wall bending, \( D = 2R \).

Using the Lagrange equations [22]:

\[
\frac{d}{dt} \left( \frac{\partial K}{\partial \dot{w}_{mn}} \right) + \frac{\partial E_{0j}}{\partial w_{mn}} - \frac{\partial (K - \Pi - W)}{\partial w_{mn}} = Q_{mn},
\]
and dependences (2) - (10), we obtain a coupled system of homogeneous differential equations with complex coefficients of the form
\[ [A][\ddot{w}] + D_{2m}(1 - \Gamma_n^R(\omega_R))[C][w] = 0, \quad (11) \]
where \( \Gamma_n^R(\omega_R) = \Gamma_n^C(\omega_R) + i\Gamma_n^S(\omega_R) \), and \( D_{2m} = \frac{h_m}{1 - \nu_{12}^2}E_{02} \), \( E_{02} \) is the instantaneous elastic modulus, \([A]\) is the diagonal matrix, the elements of which are determined by recurrence formulas:
\[
a_{nn} = \frac{n^2 + 1}{n^2} + \frac{m^2 n^2 r^2}{n^4 l^2}, \quad [C] \text{ is the square matrix, the elements of which are determined by the following recurrence formulas:}
\]
\[
c_{nm} = \frac{n^2 + 1}{n^2} + \frac{2\beta_m}{n^2} + \frac{1}{6}\zeta \lambda^2 (n^2 - 1)(n^2 - 1 + 3p'_m),
\]
\[
c_{nn+1} = -2\beta_m \frac{n^2 + n + 1}{n^2 + n + 1},
\]
\[
c_{11} = 2\beta_m^2, \quad c_{nn+2} = -2\beta_m \frac{n^2 + 2n - 3}{2(n^2 + 2n)}, \quad (12)\]
Here \( \beta_m = \left(\frac{mn}{L}\right)^2 rR \) and \( p'_m = \frac{pm^3}{3D_{2m}} \) are the dimensionless parameters, \( D_{2m} = \frac{h_m}{1 - \nu_{12}^2}E_{02} \).

Now the problem of natural vibrations of a viscoelastic curved pipe made of composite materials can be formulated as follows: it is required to find a nontrivial solution, i.e. a nonlinear complex parameter \( \omega^2 \) and a function \( w \) satisfying the homogeneous system of differential equation (11) with complex coefficients.

3. Solution method
To solve equation (11), note that the analysis of the structure of the matrix \([C]\) shows that it corresponds to generalized coordinates related to each other.

The interaction of the generalized coordinates is due to elastic constraints, the intensity of which is characterized by off-diagonal elements of the matrix \([C]\) and depends on the pipe length \( L = \theta_0 R \), where \( \theta_0 \) is the angle at center. The shorter the pipe, the larger the number of half-waves \( m \) in section \( L \) and the greater the curvature parameter \( r/R \), the stronger their interaction. As the radius of curvature \( R \) increases, the interaction of the generalized coordinates \( w_m \) becomes weaker. At \( 1/R \to 0 \), a passage to the limit of a thin-walled cylindrical shell occurs. In this case, a system of independent equations is obtained from equations (11):
\[
\dot{\omega}_n + \Omega_n^2 w_n = 0 \quad (n=2, 3, 4,...),
\]
\[
\Omega_n^2 = \omega_n^2 (1 - \Gamma_n^R(\omega_R)), \quad \omega_n^2 = \frac{2\pi D_{2m} n^2 (n^2 - 1)(n^2 - 1 + 3p'_m)}{m^2 r^2 (n^2 + 1)}, \quad (13)
\]
where \( \Omega_n^2 \) is the complex circular eigenfrequency.

At \( w(t) = 0 \), \( n=2, 3, 4,... \) we get a system of equations of bending vibrations of a hinged-supported straight rod:
\[
\ddot{w}_m + \Omega_m^2 w_m = 0 \quad (m=1, 2, 3,..),
\]
where
\[
\Omega_m^2 = \gamma^2 \frac{E_{02}(1 - \Gamma_m^R(\omega_R))}{m^2 r^2 (1 - \nu_{12}^2 v_{21}^2)}, \quad (14)
\]
At \( \nu_{12} = 0 \) or \( \nu_{21} = 0 \) \( \Gamma_m^R(\omega_R) = 0 \), the frequency \( \Omega_m \) expression coincides with the exact solution [23]. Varying (7) and (10) with respect to the generalized coordinates \( w_{m2} \), from the minimum condition for the total potential energy of the system we find:
\[
w_{m2} = \frac{12rR\lambda^2 m_0}{10 + 12\zeta_0(1 - \Gamma_m^R(\omega_R))(1 + p'_m) + \beta_m^2} w_{m1},
\]
where \( m_0 = \frac{m^2 r^2}{L^2}, \lambda = \frac{r^2}{R}, \zeta_0 = \frac{E_{02}}{E_{01}}, \)

At \( \lambda = 0 \), the coefficient of a composite curved pipe flexibility increase is determined by the formula [24]
\[
k = \frac{m_0}{m_0} = \frac{10 + 12\zeta_0(1 - \Gamma_m^R(\omega_R))(1 + p'_m)\lambda^{-2} + \beta_m^2}{1 + 12\zeta_0(1 - \Gamma_m^R(\omega_R))(1 + p'_m)\lambda^{-2} + \beta_m^2}, \quad (15)
\]
Here \( m'_0 \) and \( m_0 \) are the changes in the pipe axis curvature with / without considering the Karman effect, respectively.

4. Solution results and discussion

4.1 Flexibility assessment of viscoelastic composite curved pipes.

Consider the flexibility of viscoelastic composite pipe samples depending on boundary conditions and geometrical parameters. Table 1 presents geometrical characteristics of samples.

| Sample number | \( R, \text{mm} \) | \( r, \text{mm} \) | \( h, \text{mm} \) | \( \varphi_0 \) | \( r/R \) | \( h/r \) | \( \lambda \) |
|---------------|----------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 1             | 835            | 80          | 4.2         | 180°        | 1/10        | 1/20        | 0.50        |
| 2             | 1250           | 80          | 4.2         | 180°        | 1/15        | 1/20        | 0.70        |

Structure samples considered were made of Kevlar 49/PR-286 organic plastic with physical characteristics \( E_\alpha = 64,106\, \text{Pa}, E_\beta = 5,38\, \text{GPa}, \) \( G_{\alpha\beta} = 2,07\, \text{GPa}, \) \( v_{\alpha\beta} = 0.35 \) and the relaxation kernel parameters \( A = 0.048; \) \( \beta_1 = 0.05; \) \( \alpha_1 = 0.1. \) The number of monolayers is six. Effective elastic constants with a wall, as a multilayer orthotropic body, depending on reinforcement, are given in [25]. Table 2 presents the calculated values of the flexibility characteristics of a composite pipe with free edges; the values were calculated according to the formula (15). The results obtained showed that the greater the initial curvature of the pipe, the stronger the Karman effect.

| Sample number | Coefficients of flexibility increase \( k \) depending on reinforcement angles \( \pm \varphi_m \) |
|---------------|-----------------------------------------------|
|               | 50°              | 60°              | 70°              | 80°              | 90°              |
| 1             | 2.5              | 1.67             | 1.35             | 1.26             | 3.25             |
| 2             | 1.74             | 1.31             | 1.16             | 1.12             | 2.6              |

4.2 Analysis of complex eigenfrequencies of vibrations

Consider the spectra of lower complex frequencies and the corresponding eigenmodes of hinged-supported multilayer viscoelastic curved pipes with parameters: \( r = 80 \, \text{mm}, \) \( h / r = 1/40, \) and relaxation kernel parameters: \( A = 0.048; \) \( \beta_1 = 0.05; \) \( \alpha_1 = 0.10, \) at reinforcement angle of \( \phi_m = \pm 80^\circ, \) depending on initial curvature \( r/R = 1/40, 1/20, 1/15. \) Pipes have the same length \( L = 2.5\, \text{m}, \) but different bending angles: \( \theta_0 = 45^\circ, 90^\circ, 135^\circ, 180^\circ. \) The material is organic plastic Kevlar 49/PR-286. The number of monolayers is six. The solution of homogeneous differential equations (11) is sought in the form

\[
\{w\} = \{w_{mn}\} e^{-i\omega t},
\] (16)
where $w_{mn}$ are the amplitudes of the generalized displacements, $\omega = \omega_R + i\omega_I$ is the complex frequency.

Substituting (16) into (11), we obtain the following homogeneous algebraic equations

$$(-\omega^2[A] + D_{1m}(1 - \Gamma_n^*(\omega_R))[C])\{w_{mn}\} = 0.$$  \hspace{1cm} (16)

The frequency equation of the eigenvalue problem is written as:

$$[-\omega^2[A] + D_{1m}(1 - \Gamma_n^*(\omega_R))[C]] = 0.$$  \hspace{1cm} (17)

The roots of characteristic equation (17) are sought by the Mueller method [26].

**Table 3.** Change in real part of the complex eigenfrequency depending on initial curvature $r/R$

| $r/R$  | Real parts of complex eigenfrequencies $\omega_{Rm1}$, Hz |
|--------|------------------------------------------------------|
| 1/10   | - 62.41 153.5 515.1                                  |
| 1/15   | - 83.69 170.2 486.6                                  |
| 1/20   | - 94.54 190.9 455.2                                  |
| 1/40   | - 104.6 216.0 433.1                                  |
| Straight section of a pipe | 27.6 110.3 248.3 441.4 |

**Table 4.** Change in real part of the complex eigenfrequency depending on angle $\theta_0$

| $\theta_0$ | Real parts of complex eigenfrequencies, $\omega_{Rm2}$, Hz |
|------------|------------------------------------------------------|
| 45°        | 247.6 251.6 281.7 269.6                               |
| 90°        | 275.1 281.9 321.7 257.6                               |
| 135°       | 315.6 324.1 366.4 242.8                               |
| 180°       | 364.6 373.7 415.0 228.1                               |
| Straight section of a pipe | 237.8 243.3 258.6 289.8 |

Tables 3 and 4 show the change in real part of complex eigenfrequencies of multi-layered viscoelastic curved pipes depending on initial curvature $r/R$ and angle $\theta_0$, respectively. As seen from Table 4, at a decrease in bending angle $\theta_0$ the real parts of the lower complex frequency ($\omega_{Rm1}$), corresponding to $n = 1$ and $m = 2,3,4$ modes, increase, while the real parts of the higher frequency $\omega_{Rm2}$ and $\omega_{Rm3}$, corresponding to $n = 2,3$ and $m = 1,2,3,4$ modes, on the contrary, decrease. At the limit, they approach the eigenfrequencies of the vibrations of a straight section of composite elastic pipe.
5. Conclusions
1. The paper provided a mathematical problem and a method to assess the natural vibrations of thin-walled composite viscoelastic curved pipes using the theoretical preconditions of the approximate energy method.
2. A homogeneous differential equation was derived to estimate the natural vibrations of a cylindrical shell and a hinged-supported straight rod, taking into account the layered-fibrous structure and the anisotropy of the material.
3. The lower complex eigenfrequencies and vibration modes of hinged-supported multilayer composite curved pipes were obtained.
4. Based on the analysis, it was stated that at a decrease in initial curvature of the pipe, the lower frequencies increase, and the higher ones, on the contrary, decrease, and, at the limit, they approach the eigenfrequencies of a straight section of a pipe.

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