Search for a bound H-dibaryon using local six-quark interpolating operators

Jeremy Green
Anthony Francis   Parikshit Junnarkar   Chuan Miao
Thomas Rae   Hartmut Wittig

Institut für Kernphysik,
Helmholtz-Institut Mainz,
and PRISMA Cluster of Excellence,
Johannes Gutenberg-Universität Mainz

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Perhaps a Stable Dihyperon*

R. L. Jaffe†
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305, and Department of Physics and Laboratory of Nuclear Science,‡ Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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In the quark bag model, the same gluon-exchange forces which make the proton lighter than the Δ(1236) bind six quarks to form a stable, flavor-singlet (with strangeness of -2) \( J^P = 0^+ \) dihyeron (H) at 2150 MeV. Another isosinglet dihyeron (H*) with \( J^P = 1^+ \) at 2335 MeV should appear as a bump in \( \Lambda \Lambda \) invariant-mass plots. Production and decay systematics of the H are discussed.

TABLE I. Quantum numbers and masses of S-wave dibaryons.

| SU(6)_{CS} representation | \( C_6 \) | \( J \) | SU(3)_{I} representation | Mass in the limit \( m_s = 0 \) (MeV) |
|---------------------------|--------|------|----------------------|------------------|
| 490                       | 144    | 0    | \( \bar{1} \)        | 1760             |
| 896                       | 120    | 1, 2 | \( \bar{8} \)        | 1986             |
| 280                       | 96     | 1    | \( \bar{10} \)       | 2165             |
| 175                       | 96     | 1    | \( \bar{10}^* \)     | 2165             |
| 189                       | 80     | 0, 2 | \( \bar{27} \)       | 2242             |
| 35                        | 48     | 1    | \( \bar{35} \)       | 2507             |
| 1                         | 0      | 0    | \( \bar{28} \)       | 2799             |

Proposed dibaryon with \( I = 0, S = -2, J^P = 0^+ \).
The strongest constraint comes from the “Nagara” event from E373 at KEK, which found a $^6\Lambda\Lambda$He double-hypernucleus with binding energy

$$B_{\Lambda\Lambda} = 6.91 \pm 0.16 \text{ MeV}.$$ 

The absence of a strong decay $^6\Lambda\Lambda\text{He} \to ^4\text{He} + H$ implies

$$m_H > 2m_\Lambda - B_{\Lambda\Lambda}.$$
Calculations have found a bound H-dibaryon using $m_\pi > m_\pi^{\text{phys}}$.

| collab. | method       | $N_f$ | action     | $N_{\text{vol}}$ | $m_\pi$ (MeV) | $B_H$ (MeV) |
|---------|--------------|-------|------------|------------------|---------------|-------------|
| NPLQCD  | 2pt          | 3     | clover     | 3                | 806           | 74.6(3.3)(3.4) |
|         | 2+1 aniso    | 4     | clover     | 4                | 390           | 13.2(1.8)(4.0) |
|         | -clover      | 1     |            |                  | 230           | -0.6(8.9)(10.3) |
| HALQCD  | B-B potentials | 3     | clover     | 1                | 1171          | 48(4)        |
|         |              | 3     |            |                  | 1015          | 32.9(4.5)(6.6) |
|         |              | 1     |            |                  | 837           | 37.4(4.4)(7.3) |
|         |              | 1     |            |                  | 672           | 35.6(7.4)(4.0) |
|         |              | 1     |            |                  | 469           | 26(4)        |
CLS “E” ensembles:

- $N_f = 2$, $O(a)$-improved Wilson fermions.
- $a = 0.063$ fm, $64 \times 32^3$.
- Two pion masses: 451 MeV (E5) and 1 GeV (E1).
- Quenched strange quark.

$k_s$ tuned such that

$$R_3 \equiv \frac{m_K^2 - \frac{1}{2} m_{\pi}^2}{m_{\Omega}^2}$$

has its physical value.

We found $k_s$ close to $k_{ud}$ for E1, so we set it to be equal.
Six-quark interpolating operators

Forming the product of six positive-parity-projected (two-component) quark fields,

\[ [abcdef] = \varepsilon^{ijk} \varepsilon^{lmn} (b_i^T C \gamma_5 P + c_j)(e_l^T C \gamma_5 P + f_m)(a_k^T C \gamma_5 P + d_n), \]

where \( P_+ = (1 + \gamma_4)/2 \), there are two local interpolating operators in the H-dibaryon channel:

\[ H^1 = \frac{1}{48} ([sudsud] - [udusds] - [dudsus]), \]
\[ H^{27} = \frac{1}{48 \sqrt{3}} (3[sudsud] + [udusds] + [dudsus]), \]

which belong to the singlet and 27-plet irreps of \( SU(3)_f \).

To further expand the set of operators, we also vary the level of quark-field smearing.
Timeslice-normalized smearing

Standard smearing is a polynomial in hopping term $H$:

$$\tilde{q}(\vec{x}, t) = \sum_{\vec{y}} S(\vec{x}, \vec{y}; t) q(\vec{y}, t) = (1 + \alpha H)^n q.$$ 

This introduces noise, of which broadest part can be reduced by normalizing:

$$\tilde{q}_{N1}(\vec{x}, t) = \frac{1}{N(\vec{x}, t)} \tilde{q}(\vec{x}, t), \quad N(\vec{x}, t) = \sqrt{\sum_{\vec{y}, a, b} |S_{ab}(\vec{x}, \vec{y}; t)|^2},$$

but this is difficult to do at the sink. Instead, we compute the timeslice-summed normalization, using stochastic estimation:

$$N(t)^2 = \sum_{\vec{x}, \vec{y}, a, b} |S_{ab}(\vec{x}, \vec{y}; t)|^2 \approx \frac{1}{n_{\text{noise}}} \sum_{\vec{x}, \vec{y}, a, b, i} |S_{ab}(\vec{x}, \vec{y}; t) \eta_b^{(i)}(\vec{y}, t)|^2,$$

so that the smeared quark fields are defined as $\tilde{q}_N(\vec{x}, t) = \frac{1}{N(t)} \tilde{q}(\vec{x}, t)$.

This procedure can be applied after a production run, to reduce the noise from smearing. For a dibaryon operator, $C_N(t_f, t_i) = (\frac{1}{N(t_i)N(t_f)})^6 C(t_f, t_i)$. 

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Lattice 2014 8 / 24
Smearing normalization

One configuration from E1; $\alpha \approx 0.75$, $n = 280$, $N_{\text{noise}} = 160 + \text{color-dilution}$
We compute the matrix of two-point functions,

\[
C_{ij}(t) = \sum_{\vec{x}} \langle O_i(t_0 + t, \vec{x}) O_j^\dagger(t_0, \vec{x}_0) \rangle,
\]

and find effective masses from both its diagonal elements,

\[
m_{\text{eff},i}(t) = \frac{1}{\Delta t} \log \frac{C_{ii}(t)}{C_{ii}(t + \Delta t)},
\]

and from solving the generalized eigenvalue problem (GEVP),

\[
C_{ij}(t + \Delta t)v_j(t) = \lambda(t)C_{ij}(t)v_j(t); \quad m_{\text{eff}}(t) = \frac{-\log \lambda(t)}{\Delta t}.
\]

(We use $\Delta t = 3a$.)

These will approach the ground-state mass from above, with exponentially-decaying excite-state contamination.
All-mode averaging

Reduce costs by computing most samples using low-precision propagator solves; correct the bias with the difference between a high-precision and low-precision sample, evaluated at the same source:

\[ O = O_{x_0} - O_{x_0}^{(apx)} + \frac{1}{N_{\Delta x}} \sum_{\Delta x} O_{x_0 + \Delta x}^{(apx)}. \]

The resulting variance is reduced by a factor of

\[ 2(1 - \frac{1}{N_{\Delta x}})(1 - r) + R^{corr} + \frac{1}{N_{\Delta x}}, \]

where

- \( r \) is the correlation between \( O_{x_0} \) and \( O_{x_0}^{(apx)} \),
- \( R^{corr} \) is the average correlation between \( O_{x_0 + \Delta x}^{(apx)} \) and \( O_{x_0 + \Delta x'}^{(apx)} \).

We get a \( \approx 2 \times \) speed-up for the propagators on E5; larger improvements will be expected as we go to lighter pion masses.

→ see poster by Eigo Shintani
Blocking algorithm for contractions

Pre-contract three propagators into a color-singlet at the source, e.g.,

\[ \mathcal{B}[sll]_{\alpha' \beta' \gamma', \alpha}^{a'b'c'} = \epsilon^{abc} (C_{\gamma 5} P_+)_{\beta' \gamma} (S_s)_{\alpha' \alpha}^{a'a} (S_l)_{\beta' \beta}^{b'b} (S_l)_{\gamma' \gamma}^{c'c}, \]

then sum over permutations when contracting at the sink, e.g.,

\[ [sudsud] = (C_{\gamma 5} P_+)_{\alpha \beta} (C_{\gamma 5} P_+)_{\alpha_1' \alpha_2'} \epsilon^{a_1'b_1'c_1'} \epsilon^{a_2'b_2'c_2'} (C_{\gamma 5} P_+)_{\beta_1' \gamma_1'} (C_{\gamma 5} P_+)_{\beta_2' \gamma_2'} \]

\[ \sum_{\sigma_s, \sigma_u, \sigma_d} (-1)^{\sigma} \mathcal{B}[sll]_{\alpha' \sigma_s(1) \beta' \sigma_u(1) \gamma' \sigma_d(1)}^{a'_s \sigma_s(1) b'_s \sigma_u(1) c'_s \sigma_d(1)} \mathcal{B}[sll]_{\alpha' \sigma_s(2) \beta' \sigma_u(2) \gamma' \sigma_d(2)}^{a'_s \sigma_s(2) b'_s \sigma_u(2) c'_s \sigma_d(2)}, \]

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E5 ensemble

- $m_\pi = 451 \text{ MeV}$
- $m_\pi L = 4.6$
- 1881 gauge configurations.
- One source point with high- and low-precision solves.
- Sixteen source points with low-precision solves.
- Use both $P_+$ and $P_-$ projectors for forward/backward-propagating states. This corresponds to

\[ 1881 \times 16 \times 2 = 60192 \text{ samples}. \]

- Both point and smeared ($n = 140$) quark fields. Combined with $H^1$ and $H^{27}$, this gives four interpolating operators.
E5: two-point functions, smeared

Cross-term is suppressed by 2–3 orders of magnitude.

\[ \langle H^1(t)H^{1\dagger}(0) \rangle \]
\[ \langle H^{27}(t)H^{27\dagger}(0) \rangle \]
\[ -\langle H^{27}(t)H^{1\dagger}(0) \rangle \]
E5: effective masses, diagonal correlators

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Lattice 2014 15 / 24
Improvement over smeared $H^1$ is small; no bound H-dibaryon seen.
E1 ensemble

- $m_\pi = 1 \text{ GeV}$
- $m_\pi L = 10$
- 168 gauge configurations.
- One source point with high- and low-precision solves.
- 128 source points with low-precision solves.
- Use both $P_+$ and $P_-$ projectors for forward/backward-propagating states. This corresponds to 
  \[168 \times 128 \times 2 = 43008 \text{ samples} \]
- $\kappa_s = \kappa_{ud}$, thus no mixing between $SU(3)_f$ singlet and 27-plet irreps.
- Three quark-field smearing: wide ($n = 280$), medium ($n = 140$), and narrow ($n = 70$). This gives three interpolating operators.
E1: average correlation between sources, medium $H^1$

$$R_{corr} \equiv \frac{1}{N_{x_0}^2} \sum_{x_0 \neq y_0} \frac{\text{cov}(O_{x_0}, O_{y_0})}{\sigma(O_{x_0}) \sigma(O_{y_0})}; \sigma_{AMA}^2 \propto \frac{1}{N_{x_0}} + R_{corr} + \ldots$$
E1: effect of timeslice-normalized smearing

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Wide smearing is too noisy to be useful in GEVP.
Improvement over medium-smeread $H^1$ is small; no bound H-dibaryon seen.
E1: comparison against other calculations

If the plateau for $t/a \in [11, 14]$ the ground state, then there is a discrepancy.
Possible sources of discrepancy with NPLQCD

- Insufficient statistics; real plateau possibly not yet reached in our calculation.
- Different size of overlap with the ground state: the two calculations use different kinds of interpolating operators,

\[
C_{\text{Mainz}}(t) = \sum_{\vec{x}} \langle (qqqqq)(\vec{x}, t)(\bar{q}\bar{q}\bar{q}\bar{q}\bar{q})(0, 0) \rangle,
\]
\[
C_{\text{NPLQCD}}(t) = \sum_{\vec{x}, \vec{y}} \langle (qq)(\vec{x}, t)(qq)(\vec{y}, t)(\bar{q}\bar{q}\bar{q}\bar{q}\bar{q})(0, 0) \rangle.
\]

- Different analysis of two-point functions:
  - We use a symmetric set-up and solve the GEVP; up to statistical fluctuations, the ground-state mass will be approached from above.
  - NPLQCD uses asymmetric correlators and the matrix-Prony method; plateaus may be approached from below.
- We use a quenched strange quark.
- Our calculations lack a \( L \to \infty \) extrapolation.
Calculation done on two ensembles with $m_\pi = 451$ MeV and 1 GeV.

All-mode-averaging and timeslice-normalized smearing help to reduce noise.

Present data do not show a bound H-dibaryon.

Future plans:
- Increase statistics.
- Explore adding two-baryon operators to the basis of interpolators.