Investigation of the dynamics of a small spacecraft elastic element temperature change under a temperature shock considering a penumbral section

A V Sedelnikov¹, D I Orlov¹, Yu D Leskova¹

¹ Samara National Research University, 34, Moskovskoye shosse, Samara, 443086, Russia
E-mail: axe_backdraft@inbox.ru

Abstract. In this paper, a study of the temperature shock of small technological spacecraft large elastic elements, considering the penumbral and shadow areas of its orbit, is carried out. A comparison of the dynamics changes in the temperature field of an elastic element with and without consideration of the penumbral section is presented. Conclusions are made about the significance of the penumbral section for modeling the dynamics of the small spacecraft under a temperature shock orbital motion in order to create favorable conditions for micro-accelerations for the successful implementation of gravitationally sensitive technological processes. The results of the work can be used in the small technological spacecraft design and the development of algorithms that reduce the negative impact of temperature shock on the micro-acceleration field of the small spacecraft inner environment.

1. Introduction
When designing small technological spacecraft, it is necessary to take into account the impact of the large elastic elements temperature shock on the micro-acceleration field of the inner environment. For successful implementation of gravity-sensitive processes, it is necessary to significantly limit the level of micro-accelerations in the zone of technological equipment placement [1-3]. In [5], the significance of the impact of temperature shock on the micro-acceleration field was studied without taking into account the penumbral section of the orbit. A simplified model of the Earth's shadow without the penumbral section is also studied in the works of other authors [6-9]. Considering the penumbral section of the orbit, the dynamics of large elastic elements temperature changes in solving the initial boundary value problem given in [5] will not change, if we assume that the accepted initial field of temperature distribution corresponds to the end of the penumbral section and the beginning of the light section of the small spacecraft orbit. However, in this case, it is more correct to change the initial conditions by assuming a uniform temperature field at the end of the shadow and the beginning of the penumbral sections.

This can be explained by the fact that the boundary conditions for all external surfaces of a large elastic element are the same inside the shadow section. Therefore, in the presence of a long shadow section, the temperature of large elastic elements can come to an equilibrium state, representing a uniform field. In the penumbral section, one of the surfaces will be illuminated by an increasing flow of solar radiation, so it is difficult to imagine a uniform temperature distribution.
Thus, the dynamics of the small spacecraft large elastic elements temperature change will differ from the evaluation made when solving the initial boundary value problem [5].

2. Models and methods of research

When evaluating the temperature dynamics of the small spacecraft large elastic elements, we will use a cone-shaped model of the Earth's shadow (figure 1).

![Figure 1. Cone-shaped model of the Earth's shadow](image)

In figure 1, the dotted line indicates the orbit of a small spacecraft. The OO\textsubscript{1} line connects the centers of the Sun and Earth. AB and A’B’ are the shadow section of the orbit for the cone-shaped and cylindrical models of the Earth's shadow, respectively. CA and DB are penumbral sections of the orbit for a cone-shaped model of the Earth's shadow.

Let’s consider a solid central body with one elastic element rigidly attached to it as a model of a small spacecraft (figure 2).

![Figure 2. The model of a small spacecraft with a large elastic element](image)

Let’s consider the temperature shock when the small spacecraft exits the Earth's shadow. This corresponds to the movement of the small spacecraft in its orbit along the BD section (figure 1). To do this, we set the initial boundary value problem of thermal conductivity. As in [5], we will use a one-dimensional model of thermal conductivity:
\[
T(x, y, z, 0) = 200 \ K , (2)
\]

where the moment of time \( t_0 = 0 \) corresponds to the arrival of the small spacecraft at point B on the border of the shadow and penumbral sections of its orbit (figure 1). In [5], a cylindrical shadow model was used, so this initial condition corresponded to the arrival of the small spacecraft at a point on the border of the shadow and light sections of its orbit (figure 1).

The boundary conditions for different parts of the orbit are the following:

a) shadow section AB:
\[
\lambda \frac{\partial T}{\partial n} + \varepsilon \sigma (T^4 - T_s^4) = 0 \ (x, y, z) \in \Omega , (3)
\]

where \( \varepsilon \) – degree of blackness, \( \sigma \) – Boltzmann constant, \( T \) – temperature, \( T_s \) – ambient temperature, \( n \) – a normal to the surface of the solar panel power frame, \( \Omega \) – boundary surface of the frame (\( S_1 \)-\( S_6 \)).

b) penumbra section BD:
\[
\begin{align*}
\lambda \frac{\partial T}{\partial n} + &\varepsilon \sigma (T^4 - T_s^4) = Q_1(t) (x, y, z) \in \Omega_1, \\
\lambda \frac{\partial T}{\partial n} + &\varepsilon \sigma (T^4 - T_s^4) = 0 \ (x, y, z) \in \Omega_2
\end{align*}
\]

Where \( Q_1(t) \)-variable solar radiation flux, varying from 0 to \( Q=1400 \ \text{W/m}^2 \), \( \Omega_1 \) – boundary surface of the frame (\( S_1 \)), \( \Omega_2 \) – boundary surface of the frame (\( S_2 \)-\( S_6 \)).

c) light section DC:
\[
\begin{align*}
\lambda \frac{\partial T}{\partial n} + &\varepsilon \sigma (T^4 - T_s^4) = Q \ (x, y, z) \in \Omega_1, \\
\lambda \frac{\partial T}{\partial n} + &\varepsilon \sigma (T^4 - T_s^4) = 0 \ (x, y, z) \in \Omega_2
\end{align*}
\]

Thus, the initial boundary value problem using a cone–shaped shadow model (1) - (5) is set for estimating the dynamics of small spacecraft large elastic element temperature change when it leaves the Earth’s shadow.

### 3. Numerical modeling

To solve the initial boundary value problem, it is necessary to approximate the dependence \( Q_1(t) \) included in equation (4). Let’s use the simplest linear approximation:
\[
Q_1(t) = a t , (6)
\]

where \( a \) is an unknown constant coefficient.

Since at point B \( Q_1(t)=0 \), the linear dependence (6) must not have a free term, we need to define the specific orbit of a small spacecraft and determine the duration of the penumbral section BD to estimate the coefficient \( a \). In this work, we will consider the circular orbit of the small spacecraft with a height of 600 km. Such an orbit generally corresponds to the orbits of the implemented projects of technological and biomedical "Foton" [10, 11] and "Bion" medium-class spacecraft types [12, 13].

Simple calculations of angles \( \alpha_1 \) and \( \alpha_2 \) considering the known radiuses of the Earth and the Sun, as well as the distance between them (figure 1) show that \( \alpha_1 \approx 0,268^\circ \) and \( \alpha_2 \approx 0,263^\circ \). The length of the BD arc will be equal to (figure 3):
where $r_o$ is the radius of the Earth, $h$ is the height of the small spacecraft orbit.

**Figure 3.** Calculating the length of the penumbral section of the small spacecraft orbit

By the law of sines from a triangle $O, DE$:

$$\beta_1 = \arcsin \left[ \frac{r_o}{r_o + h} \sin (90 + \alpha_1) \right] = 66,053^\circ \cdot (8)$$

Then:

$$\beta_1 = 90 - \alpha_1 - \beta_1 = 23,679^\circ \cdot (9)$$

By the law of sines from a triangle $O, BE$:

$$\beta_2 = \arcsin \left[ \frac{r_o}{r_o + h} \sin (90 - \alpha_2) \right] = 66,053^\circ \cdot (10)$$

Then:

$$\beta_2 = 90 + \alpha_2 - \beta_2 = 24,210^\circ \cdot (11)$$

Substituting the values obtained in (8)– (11) into expression (7):

$$s_{BD} = 64,605 \text{ km}. \cdot (12)$$

The orbital speed of the small spacecraft is determined by the dependence:

$$v = \frac{G M}{r_0 + h} = 7,577 \text{ km/s}. \cdot (13)$$

Assuming the orbital velocity (13) to be constant, let’s estimate the transit time of the penumbral section of the orbit BD:

$$t_{BD} = \frac{s_{BD}}{v} \approx 8,526 \text{ s}. \cdot (14)$$

Thus, the boundary conditions are distributed over time as follows:

- $t = 0$ initial condition (2) and boundary conditions (3);
- $0 < t \leq 8,526 \text{ s}$ boundary conditions (4);
- $t > 8,526 \text{ s}$ boundary conditions (5).
Then the approximation of the solar radiation flux in the penumbra of the Earth (6), considering (14), will be the following:

\[ Q_{tt} = \frac{t}{8.526} Q, \quad (15) \]

Numerical simulation was performed in the ANSYS package. The characteristics of the elastic element used in numerical modeling are shown in table 1 (quoted in [5]).

The parameter values used in the numerical simulation of solving the heat conduction problem

| Parameter                              | Designation | Value          | Dimension          |
|----------------------------------------|-------------|----------------|--------------------|
| Solar panel frame material             | –           | MA2            | –                  |
| Coefficient of thermal conductivity    | \( \lambda \) | 96.3           | \( W/(m \cdot K) \) |
| Stefan-Boltzmann constant             | \( \sigma \) | \( 5.67 \times 10^{-8} \) | \( W/(m^2 \cdot K^\circ) \) |
| External heat flux                     | \( Q \)     | 1400           | \( W/m^2 \)        |
| Vacuum temperature                     | \( T_c \)   | 3              | \( K \)             |
| Initial temperature of the solar panel frame | \( T(x, y, z, 0) \) | 200 | \( K \) |
| Specific heat                          | \( c \)     | 1130.4         | \( J/(kg \cdot K) \) |
| Density                                | \( \rho \)  | 1780           | \( kg/m^3 \)        |
| Time step                              | \( \Delta t \) | 0.04          | \( s \)             |
| Layer thickness                        | \( \Delta z \) | 1.5           | \( mm \)            |
| Solar panel frame thickness            | \( h \)     | 6              | \( mm \)            |
| Solar panel length                     | \( l \)     | 0.5            | \( m \)             |

As a result of numerical simulation, the dynamics of the elastic element temperature field for a temperature shock when a small spacecraft exits the Earth's shadow is obtained. It is shown in figure 4. During the simulation, the entire plate was divided into four layers of thickness.

![Figure 4](image)

**Figure 4.** The temperature change dynamics of the inner layers (1–4) of the small spacecraft elastic element

In figure 4, a solid line shows the temperature dynamics in the case of using a cylindrical model of the Earth's shadow (quoted in [5]). The dotted line shows the temperature dynamics when using a cone-shaped model of the Earth's shadow. The penumbral section is indicated in figure 4 as Zone 1. Zone 2 is the light section of the small spacecraft orbit. The results obtained differ somewhat from the...
results presented in [5]. The heating of the elastic element, considering the penumbral section of the orbit, occurs more smoothly than in [5].

4. Conclusion and summary

The cone-shaped model of the Earth's shadow gives more adequate results of warming up the elastic element of the small spacecraft under a temperature shock than the cylindrical model of the shadow. In this case, the temperature shock itself does not occur at point B (figure 1) on the border of the shadow and penumbral sections of the orbit, but is shifted to the penumbral section. When considering the cone-shaped model of the Earth's shadow, we should talk about a section of the orbit for which the time derivative of temperature is sufficiently large, and not about a specific point at which a temperature shock occurs. By setting specific parameters of the small spacecraft, we can create a criterion for selecting this section based on restrictions, for example, on the level of micro-accelerations [3].

A number of conclusions can be drawn from the results of these studies.
– For the considered circular orbit with a height of 600 km, the penumbral section is significant. It substantially affects the dynamics of large elastic elements temperature changes under temperature shock. This orbit was chosen on the basis of actual projects of technological and biomedical spacecraft (“Bion-M” # 1 – 575 km, “Foton-M” # 4 – 525 km).
– Taking into account the penumbral section allows us to use the initial conditions in the form (2) more correctly than when using the cylindrical model of the Earth's shadow. On the one hand, the gradual solar radiation power growth in penumbral orbit section is taken into account, on the other hand, the initial condition of uniform temperature field at the moment, when the small spacecraft exits the shadow section, is retained.
– Micro-accelerations resulting from a temperature shock are somewhat reduced due to the «smearing» of the temperature shock time. If the temperature shock is a point in the cylindrical shadow model (the point in figure 1), then for the cone-shaped shadow model it is implemented at a certain section of the orbit with the time derivative of the temperature maximum values. In case of a point temperature shock, the maximum value of this derivative obviously corresponds to the small spacecraft exit point from the Earth's shadow (the point in figure 1). However, this disadvantage of the cylindrical Earth shadow model can be turned into an advantage. The overstatement of the micro-acceleration estimation gives it a certain margin of safety compared to the estimation of the cone-shaped shadow model. Therefore, from the point of view of estimating micro-accelerations, the use of different models of the Earth's shadow is under discussion - it depends on the specific task being solved.

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