A comparative study of sub-grid scale (SGS) models in the large eddy simulation (LES) of liquid sloshing

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Abstract. A comparative study is carried out in which the effectiveness of general-purpose isotropic subgrid-scale (SGS) models in the Large Eddy Simulation (LES) of the liquid sloshing problem is investigated. It has been observed that for large sloshing amplitudes Standard Smagorinsky Model smoothed out the results excessively and was incapable of estimating the peaks in pressure values on the container walls correctly. On the other hand, with the cost of extra complexity added to the algorithm, the dynamic procedure was more successful in calculating the pressure peaks on the container walls.

1. Introduction

When a liquid that is partially filling a container is considered, a two-phase problem is faced in which the second phase is the gas that is occupying the remaining space. The density ratio between the phases is so high that under normal conditions the heavier liquid and the lighter gas distinguish from each other by an interface called the free-surface of the liquid.

This free-surface of the liquid is very sensitive to perturbations. Even a very little amount of external disturbance applied directly into the liquid body or to its container will lead to small distortions, movements or periodic fluctuations on the free-surface which will last until all of this extra energy is dissipated. This physical phenomenon is called the liquid sloshing.

A wide range of intensities for the liquid sloshing from the smallest fluctuation on a calm clean free-surface to linear, non-linear, breaking or overturning surface waves or even run-ups to the container walls or to the container ceiling can be defined. The extreme point of the sloshing intensity would be the chaotic mixing case when the major free-surface is totally destroyed and broken into randomly splitting/merging liquid/gas interfaces between tiny droplets of any size, chunks of larger liquid masses, gas bubbles and larger gas spaces.

The large liquid body movements may create highly localized impact pressures on the container walls which may in turn cause structural damage and may even create sufficient moment to effect the stability of the vehicle which carries the container (Celebi & Akyildiz, 2001). Therefore, correct estimation of peak pressures on the container walls has a critical importance. The effects of excitation frequency and amplitudes, baffle configurations and liquid fill depths have been extensively investigated in the previous works (Celebi & Akyildiz, 2001, 2002a,b). But these works have been limited by the numerical unstability problem which arose for higher slosh intensities due to occurrence of turbulence, the transition from homogeneous...
There are recent studies where the turbulence in the sloshing liquid body is taken into account. Rhee (2005) used the standard $k - \epsilon$ turbulence model to simulate liquid tank sloshing at low filling level conditions. He compared the static pressure histories at a probe computed with and without using the turbulence model and observed small differences between the two curves. Rhee (2005) also noted some small unphysical fluctuations, irregularities and instabilities in the laminar solution and pockets of non-negligible turbulence levels in both air and water regions on the root mean square of the turbulent velocity fluctuation results. Godderidge et al. (2009) also used the standard $k - \epsilon$ turbulence model but they solved the transport equations separately for each phase in an inhomogeneous multiphase model. Their results showed that an inhomogeneous approach was more appropriate in violent-sloshing simulations. There are also few reported studies that follow the Large Eddy Simulation (LES) approach for liquid sloshing problem (Hirosi et al., 2005; Liu & Lin, 2008). In both studies the standard Smagorinsky sub-grid scale (SGS) model was used.

The use of those general-purpose turbulence models which had been originally developed for isotropic homogeneous turbulence of the single-phase flows is a possible and an already available option for this problem. However, sloshing is actually an anisotropic inhomogeneous two-phase problem with its special physics. Therefore, a detailed investigation about the suitability of the available subgrid-scale (SGS) models for sloshing problem is needed. Within the scope of this study, the effectiveness various SGS Models have been compared to each other in a set of sloshing problem cases from small to large intensities.

2. Governing Equations

In the LES approach, the velocity field is separated into large scales and small scales by a spatial filtering operation. The filtered governing continuity and the momentum equations are:

$$\nabla \cdot \mathbf{U} = 0. \quad (1)$$

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot (\mathbf{U} \mathbf{U}) = -\frac{1}{\rho} \nabla p + \nabla \cdot (\tau - \tau_{sgs}) + \mathbf{G} \quad (2)$$

where, $\mathbf{U}$ is the filtered velocity field, $p$ is the filtered pressure field, $\mathbf{G}$ is the gravity and non-inertial body acceleration, $\rho$ is the density, $t$ is time, $\tau$ is the filtered viscous stress tensor and $\tau_{sgs} = \mathbf{UU} - \mathbf{U} \mathbf{U}$ is the SGS stress tensor which arises after the filtering process and modeled by a SGS Model. The detailed formulations of common SGS Models can be found in Lesieur & Métais (1996).

3. Definition of the Problem

A hexahedron tank (92x46x62 cm) which was used in the experimental study on sloshing Akyildiz & Unal (2005) is used as the prototype in this mathematical model. The physical experiment of Akyildiz & Unal (2005) included application of different roll motion amplitude and frequencies to this tank which was filled with three different liquid depths. The pressure variation data at various pressure transducer locations has been recorded at various pressure transducer locations which are shown in Fig. 1.

In the scope of this study, the tank is filled 25% with water, the amplitude of roll motion is 4 degrees and the pressure variation data is recorded at probe 1. Six different roll motion frequencies have been applied which covers linear and nonlinear zones as the intensity is increased. The applied frequencies are 0.88 rad/s, 1.185 rad/s, 1.63 rad/s, 2.0 rad/s, 3 rad/s and
4.025 rad/s. Here, 4.025 rad/s is the first natural frequency of the liquid for the given container geometry and liquid fill depth.

The results of the Laminar Model, Smagosinsky Model, Mixed Smagorinsky Model and Localized Dynamic Smagorinsky are compared.

4. Results

For the frequencies 0.88 rad/s, 1.185 rad/s, 1.63 rad/s the simulations were carried out using 46x23x31 uniform grids and the results successfully matched the already available experimental data. The SGS stresses are insignificant and the results are insensitive to the SGS Model that is used (Fig. 2).

For the frequencies 2.0 rad/s, 3 rad/s and 4.025 rad/s the simulations were carried out using three different mesh resolutions (coarse with 46x23x31 grids, fine with 92x46x62 grids and very fine with 184x92x124 grids) and the results of the fine mesh and very fine mesh coincided. In this region, the SGS Stresses became significant and the results were sensitive to the SGS Model used.

For 2.0 rad/s, the results of the Laminar Model at very fine mesh resolution is considered as the exact solution. It has been observed that the results of the Localized Dynamic Smagorinsky Model were identical to the exact solution, while the results of the Smagorinsky Model and the Mixed Smagorinsky Model deviated from the exact solution with a very small amount (Fig. 3).
For 3 rad/s, it has been observed that Smagorinsky and Mixed Smagorinsky Models successfully filtered out the small scale fluctuations and returned the large scale results in an acceptable range. The results of the Localized Dynamic Smagorinsky Model contained more details but those small details are useless from the practical point of view (Fig. 4).

For 4.025 rad/s, the Smagorinsky Model filters out the small scale fluctuations and returns the large scale results however, it fails to predict the peaks in the pressure values. From the practical point of view, in order to estimate the structure loads on the walls correctly, the peaks in pressure variations has to be precisely calculated. On the other hand, Localized Dynamic Smagorinsky Model did not miss the peak pressure values giving safer results (Fig. 5).

5. Discussions

Sloshing is a unique problem with its special physical characteristics. In general, the source of all the kinetic energy in the problem is the resultant acceleration vector of the 6-degree-of-freedom (6-DOF) motion of the container and earth’s gravity. These external factors lead to body forces which vary in time, in direction and in magnitude within the sloshing liquid and appear as the source term in the governing Navier-Stokes Equations. It should be emphasized that there is no necessary dependence in how the container will move at different time instants. So, the source term that appear in Navier-Stokes Equations varies by an external function in time. Therefore, large variations in the source term (due to large variations in container acceleration) has drastic effects like re-defining the problem again at every discrete time steps.
Since sloshing is inherently a dynamic problem, there may be cases where the standard Smagorinsky Model which uses a predefined constant Smagorinsky coefficient throughout the whole transient simulation returns acceptable results but there may be other occasions where the results are overly smoothed out.

Dynamic procedures in SGS Models re-calculate the Smagorinsky coefficient for every discrete time step, but the procedures usually depend on averaging in homogeneous directions but it is not easy to define a homogeneous direction in this problem. Localized versions of the dynamic procedures are more suitable but they have the extra cost of re-calculating and storing the Smagorinsky coefficient as a field variable instead of a single scalar value.

Another unique physical aspect in this problem is the high ratio of inertia among the phases due to the high ratio of density. This effects how momentum and the kinetic energy is transfered between the phases through the interface at grid and sub-grid scales and more importantly what happens as the turbulence in the liquid body approaches to and reaches to the free-surface. In their study on turbulent shear flows, Shen & Yue (2001) showed that the interaction between the turbulence and the free-surface is \textit{unisotropic} and \textit{inhomogeneous}. It is unisotropic because it is different in direction normal to and tangential to the free-surface and it is inhomogeneous because parameters such as the Smagorinsky coefficient vary by the distance to the free-surface.

For future work, a new SGS Model which can handle the unisotrophy near the free surface may produce more accurate results and therefore it may be possible to use coarser meshes and less number of iterations for convergence at discrete time steps and eventually this may relieve the extra load of computational complexity that come with the inevitable requirement of localized dynamic approach which is mentioned above.

6. Conclusions

The effectiveness of various SGS Models (Smagorinsky, Mixed Smagorinsky and Localized Dynamic Smagorinsky) have been compared in a set of sloshing scenarios. For frequencies of 0.88 rad/s, 1.185 rad/s, 1.63 rad/s the results were not sensitive to the SGS Model that is used. For 2.0 rad/s, the Localized Dynamic Smagorinsky Model returned exact results while Smagorinsky, Mixed Smagorinsky Models slightly deviated from the exact results. For 3.0 rad/s, Smagorinsky and Mixed Smagorinsky Models successfully resolved the large-scales, while filtering out the small-scale details. On the other hand, the Localized Dynamic Smagorinsky Model returned unnecessary large amount of details in solution, as an addition to its cost of re-calculation and storage of the Smagorinsky coefficient at every time step and as a field variable. For 4.025 rad/s, while the Localized Dynamic Smagorinsky Model successfully estimated the peak values in the pressure variations, the Smagorinsky Model failed since it overly smoothed out the solution.
References

AKYILDIZ, H. & UNAL, E. 2005 Experimental investigation of pressure distribution on a rectangular tank due to the liquid sloshing. Ocean Eng. 32, 1503–1516.

CELEBI, M. S. & AKYILDIZ, H. 2001 Numerical computation of pressure in a rigid rectangular tank due to large amplitude liquid sloshing. Turkish Journal of Engineering and Environmental Sciences 25, 659–674.

CELEBI, M. S. & AKYILDIZ, H. 2002a Nonlinear modelling of liquid sloshing in a moving rectangular tank. Ocean Eng. 29, 1527–1553.

CELEBI, M. S. & AKYILDIZ, H. 2002b Numerical computation of hydrodynamic loads on walls of a rigid rectangular tank due to large amplitude liquid sloshing. Turkish Journal of Engineering and Environmental Sciences 26, 429–446.

GODDERIDGE, B., TURNOCK, S., TAN, M. & EARL, C. 2009 An investigation of multiphase cfd modelling of a lateral sloshing tank. Computers and Fluids 38, 183–193.

HIROSHI, N., EIJI, T. & SHIGEKI, N. 2005 Reproduction of sloshing phenomena in rectangular tank by les. Journal of the Visualization Society of Japan 25, 17–18.

LESIEUR, M. & MÉTAIS, O. 1996 New trends in large-eddy simulation of turbulence. Ann. Rev. Fluid Mech. 28, 45–82.

LIU, D. & LIN, P. 2008 A numerical study of three-dimensional liquid sloshing in tanks. J. Comput. Phys. 227, 3921–3939.

RHEE, S. H. 2005 Unstructured grid based reynolds-averaged navier-stokes method for liquid tank sloshing. J. Fluids Eng. 127, 572–582.

SHEN, L. & YUE, D. K. P. 2001 Large-eddy simulation of free-surface turbulence. J. Fluid Mech. 440, 75–116.