Population Synthesis of Ultraluminous X-ray Sources with Magnetized Neutron Stars

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Abstract—A model of a population of ultraluminous X-ray sources with magnetized neutron stars (NULXs) in a spiral galaxy with a star formation history as in the thin disk of the Milky Way is constructed by the hybrid population synthesis method. First, based on analytical approximations (BSE code), we compute a set of close binary systems (CBSs), potential precursors of NULXs, and, then, the evolution with mass accretion onto magnetized neutron stars (NSs) is computed by the MESA evolutionary code. The accretion rate onto NSs and the X-ray luminosity of sources are calculated for the models of sub- or supercritical disks and disks with advection. During accretion onto magnetized NSs, super-Eddington luminosities $L_X > 10^{38}$ erg s$^{-1}$ are reached already at the subcritical stage, when the energy release at the inner boundary of the disk defined by the NS magnetosphere is sub-Eddington one. Our calculations show that the standard evolution of CBSs, given the peculiarities of accretion onto magnetized NSs, allows the observed properties of NULXs (X-ray luminosities, NS spin periods, CBS orbital periods, and optical component masses) to be quantitatively explained without requiring additional model assumptions about the collimation of X-ray emission from NSs with a high observed super-Eddington luminosity. In a model galaxy with a star formation rate $\sim 5$ $M_\odot$ yr$^{-1}$ there can be several NULXs. The detection of a powerful wind from NULXs with $L_X \sim 10^{41}$ erg s$^{-1}$ may suggest supercritical accretion onto magnetized NSs.

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INTRODUCTION

Ultraluminous X-ray sources (hereafter ULXs) are point-like X-ray sources with an equivalent isotropic luminosity in the range $0.3-10$ keV exceeding $10^{39}$ erg s$^{-1}$. As a rule, ULXs are observed in external galaxies. Only one transient source, Swift J0243.6+6124 (Kennea et al. 2017), is known in the Milky Way (see Table 1). ULXs were discovered by the Einstein observatory in the early 1980s (Long and van Speybroeck 1983). ULXs are observed both in spiral and irregular galaxies and in ellipticals. The most luminous ULXs are encountered in star-forming galaxies. These are rare objects: in the local Universe ($< 40$ Mpc) there are about two ULX candidates per galaxy (irrespective of its type) (Walton et al. 2011; Earnshow et al. 2019; Kovlakas et al. 2020), which is most likely a consequence of the superposition of star formation, stellar evolution, and observational selection effects. The statistical properties of ULXs and their relation to the star formation rate in galaxies are summarized in Sazonov and Khabibullin (2017).

For a long time, it was assumed that the accretors in ULXs are intermediate-mass black holes accreting at sub-Eddington rates or stellar-mass black holes ($M \gtrsim 10$ $M_\odot$) accreting at super-Eddington rates (Mushotzky et al. 2004). Bachetti et al. (2014) found the first pulsating ULX, which allowed its accretor to be identified with a neutron star (NS). Furthermore, Brightman et al. (2018) identified the accretor in the source M 51 ULX-8 with a NS owing to the detection of a cyclotron resonance line typical of X-ray pulsars, although no pulsations have been observed (as yet). An analysis of the X-ray spectra for 18 ULXs (Koliopanos et al. 2017) showed that most of the ULXs could be accreting NSs with magnetic fields $\gtrsim 10^{12}$ G.

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The luminosity of ULXs is variable. ULXs can be divided quite arbitrarily into persistent ones, where $L_X$ changed by several-fold throughout the observations, and transients, where a single X-ray outburst during which $L_X$ exceeded $10^{39} \text{ erg s}^{-1}$ was observed (as a rule, these are X-ray binaries with a Be star). As of this writing, six persistent sources and four transients were known (see Table 1), for which such parameters as the X-ray luminosity $L_X$, orbital period $P_{\text{orb}}$, NS spin period $P^*$, and optical star mass $M_2$ were estimated. These parameters allow a detailed comparison with the population synthesis results to be made.

The models of a population of sources with NS accretors (hereafter NULXs) are being actively studied and have been previously published in Shao and Li (2015), Wiktorowicz et al. (2017, 2019), Marchant et al. (2017), and Misra et al. (2020). It was shown in these papers that within certain model assumptions about the pattern of supercritical flow and the characteristics of radiation during accretion onto compact objects in close binary systems (CBSs) models of a population of NULXs with properties close to the observed ones could be constructed. It was assumed that, by analogy with supercritical accretion disks around black holes, a geometrical focusing of the radiation from the inner regions of NULX accretion disks occurs during accretion onto NSs. As a result, for an observer inside the emission cone the equivalent isotropic luminosity estimate for the source can exceed its true luminosity (for a discussion, see, e.g., Wiktorowicz et al. 2017). The influence of NS magnetic fields on the generation of the source emission was not considered.

In this paper we model a population of NULXs in CBSs with magnetized NSs in the regimes of sub- and supercritical disk accretion. As physical models we consider the “standard” accretion disk model (Shakura and Sunyaev 1973), which was considered in more detail by Chashkina et al. (2017) as applied to accretion onto magnetized NSs, and the model of a supercritical disk with advection around a magnetized NS (Chashkina et al. 2019). The NULX population synthesis is accomplished in two steps. First, we use a modified BSE code (Hurley et al. 2002) to identify the range of parameters of CBSs with NSs that in the course of their subsequent evolution can potentially reach the stages of sub- and supercritical accretion. Thereafter, we compute the mass transfer onto the NS based on the evolutionary models preconstructed with the MESA code (Paxton et al. 2011). The star formation history in the Galactic disk and the normalization to the total mass of the stars in the disk are used in calculating the size of the population of sources in galaxies like the Milky Way.

The results of our computations show that within the adopted physical models of accretion, magnetized

### Table 1. Parameters of ultraluminous X-ray sources with NSs (NULXs). The asterisks in the first column mark the transient sources.

| No. | Source          | $L_{\text{min}}$ ($10^{39} \text{ erg s}^{-1}$) | $L_{\text{max}}$ ($10^{39} \text{ erg s}^{-1}$) | $P^*$ (s) | $P_{\text{orb}}$ (day) | $M_2$ ($M_\odot$) | References |
|-----|-----------------|-----------------------------------------------|-----------------------------------------------|----------|-------------------------|-------------------|------------|
| 1   | M 82 ULX-2      | 6.6 ± 0.3                                      | 18                                            | 1.37     | 2.53                    | >5.2              | [1]        |
| 2   | NGC 7793 P13    | 2.1                                            | 5 ± 0.5                                       | 0.415    | 63.9 ± 0.6              | 18–23             | [2, 3]     |
| 3   | NGC 5907 ULX-1  | 6.4                                            | 220 ± 30                                      | 1.43–1.14| 5.3 ± 0.9              | 2–6               | [10]       |
| 4   | M 51 ULX-7      | ≤0.3                                           | 10                                            | 2.8      | 2                       | 8–13              | [14, 15]   |
| 5   | M 51 ULX-8      | 2                                              | 20                                            |          | 8–400                   |                   | [16, 17]   |
| 6   | NGC 1313 X-2    | 14.4                                           | 19.9                                          | 1.5      | ≤3                      | ≤12               | [11, 12]   |
| 7*  | NGC 300 ULX-1   | 0.6                                            | 4.7                                           | 31.6     | 0.8–2.1 yr              | 8–10              | [4, 5]     |
| 8*  | Swift J0243     | 0                                              | 1.2–2.6                                       | 9.86     | 27.59                   |                   | [13]       |
| 9*  | SMC X-3         | 0.2                                            | 2.5                                           | 7.8      | 44.86                   | ≥3.7              | [6–8]      |
| 10* | NGC 2403 ULX-1  | ≤0.001                                         | 1.2                                           | 17.57    | 60–100                  |                   | [9]        |

[1]—Bachetti et al. (2014), [2]—Israel et al. (2017a), [3]—Motch et al. (2014), [4]—Carpano et al. (2018), [5]—Heida et al. (2019), [6]—Tsygankov et al. (2017), [7]—Cowley and Schmidtke (2004), [8]—Corbet et al. (2003), [9]—Trudolyubov et. al. (2007), [10]—Israel et al. (2017b), [11]—Grise et al. (2008), [12]—Sathyaprakash et al. (2019), [13]—Zhang et al. (2019), [14]—Rodríguez Castillo et al. (2020), [15]—Vasilopoulos et al. (2020), [16]—Brightman et al. (2020), [17]—Middleton et al. (2019).
NS formation parameters, and the pattern of mass transfer when the optical star fills its Roche lobe at the stages of sub- and supercritical accretion, it is possible to successfully explain both the positions and the expected number (per galaxy) of observed NULx on the “NS spin period—X-ray luminosity” \((P^* - L_X)\), “luminosity—orbital period” \((L_X - P_{orb})\), and \(P^* - P_{orb}\) diagrams.

ACCRETION ONTO MAGNETIZED NEUTRON STARS

It is well known that the main difference of the disk accretion onto magnetized NSs in CBSs with optical components overlying their Roche lobes from the classical picture of accretion onto black holes (Shakura and Sunyaev 1973) is due to the presence of an NS magnetosphere, at the boundary of which the accretion flow pattern changes (this phenomenon was considered in detail by Lipunov 1987). At sufficiently slow or moderate accretion rates in the disk all of the matter passing through the disk, after its interaction with the magnetosphere, falls on the NS surface, near which the main energy release observed in X rays occurs. When the accretion rate exceeds a certain value, the accretion can become supercritical. Supercritical accretion onto the NS occurs when the local energy release at the inner disk radius bounded by the NS magnetosphere exceeds the Eddington luminosity \(L_{Edd} \approx 1.5 \times 10^{38} (M_{NS}/M_\odot)\) erg s\(^{-1}\). An accretion rate \(M_{Edd} \approx 1.5 \times 10^{18}\) g s\(^{-1}\) corresponds to this luminosity.

In this case, the basic idea of supercritical accretion put forward in the pioneering paper by Shakura and Sunyaev (1973) about the outflow of matter from the disk within the radius at which the local energy release in the disk begins to exceed the Eddington limit (so-called spherization radius \(R_s\)) is modified (Lipunov 1982; King et al. 2017; Grebenev 2017). The Eddington energy release in the disk at the boundary of the NS magnetosphere changes the standard expression for the magnetospheric radius (Alföven radius), \(R_A \approx (\mu^2/M)^{2/7}\) (where \(\mu\) is the NS magnetic moment, \(M\) is the accretion rate). To a first approximation, the magnetospheric radius ceases to depend on the accretion rate, but is determined only by the NS magnetic field, \(R_A \approx \mu^{4/9}\).

The critical accretion rate \(M_{cr}\) at which the luminosity at the magnetospheric radius reaches the Eddington limit is determined from the condition for the spherization radius being equal to the magnetospheric radius, \(R_s = R_A\). For a typical NS mass of 1.4 \(M_\odot\), which will be used everywhere in our numerical estimates,

\[
M_{cr} \approx 3 \times 10^{10} \text{[g s}^{-1}] \mu_{30}^{4/9} \approx 30 M_{-8} \mu_{30}^{4/9}. \tag{1}
\]

Here and below, the NS magnetic moment and the accretion rate are expressed in units of \(\mu_{30} = \mu/(10^{30} \text{[G cm}^3]\)) and \(M_{-8} = M/(10^{-8} [M_\odot \text{ yr}^{-1}])\), respectively.

The magnetospheric radius during supercritical disk accretion can be written as

\[
R_A = \left(\frac{\mu^2}{M\sqrt{2GM}}\right)^{2/7} \approx 1.6 \times 10^8 \text{[cm]} \mu_{30}^{4/7} M_{-8}^{-2/7}, \quad \dot{M} < \dot{M}_{cr}, \tag{2}
\]

\[
R_A = \left(\frac{\mu^4GM}{2L_{Edd}^2}\right)^{1/9} \approx 6.5 \times 10^7 \text{[cm]} \mu_{30}^{4/9}, \quad \dot{M} > \dot{M}_{cr}. \tag{3}
\]

The NS spin period \(P^*\) will be assumed to be close to the equilibrium period \(P_{eq}\) determined from the condition for the Alfvén radius being equal to the corotation radius \(R_{co} = (GM^2/4\pi^2)^{1/3}\):

\[
P_{eq} \approx 0.9 [\text{s}] \mu_{30}^{4/9} M_{-8}^{-3/7}, \quad \dot{M} < \dot{M}_{cr}, \tag{4}
\]

at the subcritical stage and

\[
P_{eq} \approx 0.2 \text{[s]} \mu_{30}^{2/3}, \quad \dot{M} > \dot{M}_{cr}, \tag{5}
\]

at the supercritical stage. This approximation is justified at high accretion rates onto the NS, \(M \gtrsim M_{cr} \approx 3 \times 10^{-7} M_\odot\) yr\(^{-1}\), because at the subcritical stage the time it takes for equilibrium rotation to be established at a magnetospheric radius \(\sim 100\) NS radii, \(\tau_{su} = \omega/\dot{\omega} \sim 100 \text{[yr]} (M/M_{cr})^{-1} (P/1 \text{s})^{-1} \times \left(R_A/10^7 \text{[cm]}\right)^{-1/2} < \tau_{M}, \tag{6}
\]

where \(\tau_{M} = M/(dM/dt)\) is the timescale of the decrease in equilibrium period \(\tau_{eq} = 7/3 \tau_{M}\), where \(\tau_{M} = M/(dM/dt)\) is the timescale of the evolutionary changes in accretion rate (see examples of evolutionary tracks in Fig. 3). At the supercritical stage the magnetospheric radius is virtually constant, the NS periodically reaches the propeller stage interspersed with accretion episodes, which is apparently observed in M 82 X-2 (Tsygankov et al. 2016). In this case, the NS spin period remains near the equilibrium value and does not change significantly.

The NS accretion luminosity is determined mainly by the accretion rate onto its surface. At subcritical accretion rates \((\dot{M} < \dot{M}_{cr})\) it is equal to the accretion rate in the disk, while in the supercritical regime \((\dot{M} > \dot{M}_{cr})\) it is limited by the magnetospheric transmission,\(^1\) i.e.,

\[
L_X = 0.1 \dot{M} c^2 \approx L_{Edd} M_{-8}, \quad \dot{M} < \dot{M}_{cr}. \tag{6}
\]

\(^1\) It may well be that at very high accretion rates onto the NS part of the energy can be carried away by neutrinos (Basko and Sunyaev 1976; Mushtukov et al. 2018).
The behavior of the main parameters, namely the magnetospheric radius $R_A$, the NS equilibrium spin period $P_{eq}$, and the X-ray luminosity $L_X$, as a function of the accretion rate $\dot{M}$ onto a magnetized NS is schematically shown in Fig. 1 (see also Postnov et al. 2019).

The supercritical disk accretion onto a magnetized NS was analyzed in more detail by Chashkina et al. (2019). The NS magnetosphere is restructured at the outer magnetospheric radius $R_H$, which is equivalent to an increase in the critical accretion rate corresponding to the condition $\dot{M} > \dot{M}_{cr}$. This critical accretion rate (Eq. (66) in Chashkina et al. (2019)) can increase to

$$\dot{M}_{cr,\text{ad}} \sim 5 \times 10^{20} [\text{g s}^{-1}] \mu_{30}^{4/9}. \tag{8}$$

In our calculations we will use two models: with a conservative value of the critical accretion rate (1) for standard accretion disks and with a value higher by more than an order of magnitude (8) for advection disks. In the calculations we will ignore the weak dependence of the supercritical Alfvén radius on the accretion rate for advection disks (less than a factor of 2 as the accretion rate changes by three orders of magnitude, see Fig. 12 in Chashkina et al. (2019)).

It is important to note that Eq. (1) and especially Eq. (8) show that during accretion onto magnetized NSs, high X-ray luminosities $L_X > 10^{39}$ erg s$^{-1}$ typical of ULXs can be reached already at the subcritical accretion stage, i.e., when the local energy release at the inner disk radius has not reached the Eddington limit and the disk has not yet been pulled up by the radiation pressure to $H/R_A \sim 1$. In this case, the X-ray luminosity in the NS accretion column may be significantly super-Eddington (see, e.g., Mushtukov et al. 2015).

The possible beaming of X-ray emission from supercritical accretion disks around magnetized NSs should be discussed separately. The beaming factor $b$ is defined as the ratio of the observed X-ray luminosity $L_X$ to the luminosity of an isotropic source calculated from the observed flux $F_x$, $L_{iso} = 4\pi d^2 F_x$ ($d$ is the distance to the source), i.e., $b = L_X/L_{iso} \leq 1$. In the case of a source with radiation into oppositely directed cones with a half-angle $\theta$, we have $b = (1 - \cos \theta)$. This factor is commonly taken into account in the construction of NULX populations (see, e.g., Wiktorowicz et al. 2017). However, it is based on the extrapolation of the beaming of X-ray emission from supercritical disks around black holes, where $b \sim \dot{M}^{-2}$ (King 2009). In the case of magnetized NSs, as noted above, the contribution of the accretion energy release from the disk to the observed X-ray luminosity will be significantly lower than the contribution from the accretion column radiation near the NS surface (see Eq. (7)). There remains only the geometrical beaming of X-ray emission by a thick disk with a half-thickness $H/R_A \sim 1$. For the NS radiation shielded by a thick inner disk this factor $b = (1 - H/\sqrt{H^2 + R_A^2}) = (1 - 1/\sqrt{2})$ was included in our computations for both accretion and supercritical advection disks. We neglect the intrinsic beaming of X-ray emission from the accretion column, which is averaged by the NS rotation.

**Fig. 1.** Schematic behavior of the magnetospheric radius $R_A$, spherization radius $R_s$, NS equilibrium spin period $P_{eq}$, and X-ray luminosity $L_X$ as a function of the accretion rate $\dot{M}$ onto a NS with a magnetic moment $\mu$. 

$$L_X = L_{Edd} \left( \frac{R_A}{R_{NS}} \right) \simeq 65 L_{Edd} \mu_{30}^{4/9}, \tag{7}$$

$$\dot{M} > \dot{M}_{cr}. \tag{1}$$
THE METHOD OF CALCULATIONS

As mentioned above, for our computations we use a hybrid two-step method for the population synthesis of NULXs in CBSs that combines a rapid simplified computation before the Roche lobe overflow by the optical star in pair with the NS by the BSE code with a subsequent detailed computation of the stage with mass transfer by the MESA evolutionary code. Such a hybrid population synthesis method was successfully used, in particular, for modeling cataclysmic variables and SN Ia precursors (see, e.g., Chen et al. 2014; Goliash and Nelson 2015) as well as ULXs (Shao and Li 2015).

In the first step, using a modified BSE code based on analytical approximations for the description of the evolution of single and close binary stars, for $10^7$ initial massive binaries we model a population of NSs in pairs with nondegenerate stars (optical components) that can potentially become NULXs in the course of their evolution. The initial mass function of the primary components is assumed to be a power law (a Salpeter law, $dN/dM \sim M^{-2.35}$); the distribution in component mass ratio $q = M_2/M_1 \leq 1$ is taken to be uniform in the range from 0.1 to 1. The orbital eccentricity distribution is flat in the interval [0, 1]. The stellar binarity ratio is assumed to be 50% (2/3 of the stars are binary components). The initial distribution of the binaries follows the law $f(\log P_{\text{orb}}) \propto \log P_{\text{orb}}^{-0.55}$ (Sana et al. 2015). The stellar wind from massive stars and helium stars is described by the formulas from Vink et al. (2001) and Vink (2017), respectively. The orbital evolution of a binary with a common envelope was computed using the formalism of Webbink (1984) and de Kool (1990) with a parameter $\alpha = 1$. The parameter $\lambda$ characterizing the stellar envelope—core binding energy was taken from the calculations by Loveridge et al. (2011).

NSs with a fixed mass of 1.4 $M_\odot$ were assumed to be formed during the iron core collapse of massive stars. We used the approximations of the criteria for NS formation during the core collapse of massive stars from Giacobbo and Mapelli (2018). The nascent NSs get a kick that obeys a Maxwellian distribution with a dispersion $\sigma = 265$ km s$^{-1}$ (Hobbs et al. 2005). A kick of 30 km s$^{-1}$ (the arbitrariness of this poorly known quantity does not affect the results) was assigned to the few NSs formed as a result of the collapses triggered by the electron captures by the degenerate O–Ne–Mg core of a star with an initial mass in the range 8.5–8.8 $M_\odot$ (Siess and Lebreuilly 2018) that previously experienced a mass transfer in a CBS (the so-called e-capture SN, Miyaji et al. 1980). The NS magnetic moments obey a log-normal distribution (Faucher-Giguère and Kaspi 2006):

$$ f(\mu) \sim \exp \left[ -\frac{(\log \mu - \log \mu_0)^2}{\sigma^2} \right]. \quad (9) $$

In our computations we adopted $\log \mu_0 [\text{G cm}^3] = 30.6$ or $\log \mu_0 [\text{G cm}^3] = 31.6$ and $\log \sigma^2 [\text{G cm}^3] = 0.55$. The NS magnetic field decay during accretion was ignored, because the timescale of the mass transfer stage in the massive CBSs under consideration is much shorter than the possible timescale of the NS magnetic field decay. However, the NS magnetic field decay during accretion may turn out to be a factor reducing the duration and X-ray luminosity of the NULX stage in binaries with low-mass components $M_2 \lesssim 3 M_\odot$ (see an example of an evolutionary track in the Appendix).

Our computations were performed for stars with a solar metallicity ($Z = 0.02$). Given that all of the known NULXs have been discovered in the nearest spiral galaxies, this assumption also affects insignificantly the results. To calculate the number of NULXs, we used the formula for the star formation rate in the Galactic disk (Yu and Jeffery 2010):

$$ \frac{\text{SFR}(t)}{M_\odot \text{ yr}^{-1}} = \begin{cases} 11 e^{\frac{t-t_0}{0.1}} + 0.12(t-t_0), & t \geq t_0 \\ 0, & t < t_0 \end{cases} \quad (10) $$

where $t$ is the time (Gyr), $t_0 = 4$ Gyr is the time of the onset of star formation (Gyr), and the parameter $\tau = 9$ Gyr. In this model the mass of the Galactic thin disk at the present epoch (14 Gyr) is estimated as $M_G = 7.2 \times 10^{10} M_\odot$. This value is used below to estimate the total number of NULXs in a model galaxy.

In the second step of our modeling, the evolution of semidetached CBSs with NS components was computed in detail using the MESA code ( Paxton et al. 2011, version r12778). A grid of tracks with initial donor masses from 0.75 to 50 $M_\odot$ was constructed. The mass step was 0.25 and 2 $M_\odot$ in the ranges (0.75–10) $M_\odot$ and (10–50) $M_\odot$, respectively. The initial orbital semimajor axes were in the range $0.9 < \log(a_{\text{ini}}/R_\odot) < 3.5$ with a step $\log(a_{\text{ini}}/R_\odot) = 0.1$. Next, for each pair of parameters $[X, Y]$ (where $X, Y \equiv \{L_X, P_{\text{orb}}, P^*, M_2\}$) the duration of the NULX stage in a time interval $\Delta t_k$ in a cell $[X_i, Y_j]$ was calculated and convolved with

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2 In the original paper by Faucher-Giguère and Kaspi $\log \mu_0 = 30.35$ [G cm$^3$] for a NS radius of 10 km. We adopt $R_{\text{NS}} = 12$ km.
the formation probability of a given system (per unit mass) computed by the BSE code:

\[
\frac{\Delta N(t_k,t_k + \Delta t_k)}{\Delta X_i \Delta Y_j \langle m_{BSE} \rangle} = \frac{\sum_{l=1}^{N_{BSE,i,j}} \Delta t_{NULX}^{k,l}}{N_{MESA,i,j} \Delta t_k} \times \frac{N_{BSE,i,j}}{N_{BSE}} \times \frac{1}{\langle m_{BSE} \rangle}.
\]  

Here, \(\Delta t_{NULX}^{k,l}\) is the duration of the NULX stage for system \(l\), \(N_{MESA,i,j}\) and \(N_{BSE,i,j}\) are the numbers of systems in the cell \([X_i, Y_j]\) according to the MESA and BSE grids, respectively, \(N_{BSE} = 10^7\) is the number of computed BSE systems, and \(\langle m_{BSE} \rangle \approx 0.44 M_{\odot}\) is the average mass of the binary, given the initial mass function of the primary components and the component mass ratio distribution. The derived distribution at time \(t_k\) is convolved with the star formation history in the model galaxy (10) and is presented in the results of our computations for a time of 14 Gyr.

To describe the orbital evolution of CBSs at the mass loss stage, we used the formalism of Soberman et al. (1997), according to which the equation relating the angular momentum loss by the binary and the mass loss by the donor through the outer Lagrange point \(L_2\) is

\[
\dot{J}_{mt} = [(\dot{M}_{2,\alpha} + \alpha_{mt}(dM/dt)_{L1})M_1^2 \left(1 + (\dot{M}_{1,\alpha} + \beta_{mt}(dM/dt)_{L1})M_2^2\right) \times \frac{2a}{(M_1 + M_2)^2 P_{\text{orb}}} + \gamma_{mt} \delta_{mt}(dM/dt)_{L1}\sqrt{G(M_1 + M_2)a}.
\]

The dimensionless parameters \(\alpha_{mt}, \beta_{mt}\), and \(\delta_{mt}\) denote the fractions of matter being transferred through the inner Lagrange point \(L_1\) and lost by the CBS from the donor’s neighborhoods, the accretor, and the coplanar circumbinary torus with radius \(\gamma_{mt}^2 a\) (through the outer Lagrange point \(L_2\), respectively. Thus, without allowance for the stellar wind from the optical component (i.e., neglecting \(\dot{M}_{2,\alpha}\)), the mass transfer efficiency from the donor to the accretor is defined as \(f_{mt} = \min[1 - \alpha_{mt} - \beta_{mt} - \delta_{mt}, \frac{M_{mt}}{(dM/dt)_{L1}}]\).

Here, \(\dot{M}_{Edd}\) is the accretion rate onto the NS at which the Eddington luminosity is reached and the outflow of matter caused by the radiation pressure begins.

Observations of SS433 point to a mass loss from the binary system both in the form of a quasi-spherical stellar wind from a supercritical accretion disk and through the point \(L_2\) (for a review, see Cherepashchuk et al. 2020). The position of the point \(L_2\) depends on the binary component mass ratio and is characterized by \(\gamma_{mt}^2 \approx 1.2\), so that \(\gamma_{mt} \approx 1.1\) may be taken as the minimum possible value for the mass loss through \(L_2\) (see Fig. 1 in Cherepashchuk et al. (2018)). Recent observations of mass flows around SS433 with the VLTI GRAVITY optical interferometer (Weisberg et al. 2019) suggest a much more efficient angular momentum loss from the binary via a circumbinary disk corresponding to \(\gamma_{mt} \approx 5\) (Cherepashchuk et al. 2019). In our computations \(\gamma_{mt}\) was taken to be 3.0 as a compromise between its value at \(L_2 = 1.15\) and \(\gamma_{mt} \approx 5.0\) found for SS433 by Cherepashchuk et al. (2019).

In our computations we specified the following dimensionless parameters characterizing the mass transfer efficiency between the CBS components when the optical star \(M_2\) overflows its Roche lobe: \(\alpha_{mt} = 0.0\), \(\beta_{mt} = 0.0\) at the subcritical accretion stage and \(\beta_{mt} = ((dM/dt)_{L1} - \dot{M}_{Edd})/(dM/dt)_{L1}\) at the supercritical accretion stage. In the MESA code \(M_{Edd} \approx 1.5 \times 10^{-8} (M/M_{\odot}) M_{\odot} \text{yr}^{-1}\) is specified for compact objects of mass \(M\) without a magnetic field. In our case, in this formula \(M_{\text{cr}}\) should have been used instead of \(\dot{M}_{Edd}\), i.e., the orbital evolution of a CBS with mass transfer should have been computed separately for each case, which is very time-consuming. For our population synthesis computations we preferred to use a value fixed in the MESA code, i.e., we used the maximum possible value of \(\beta_{mt}\). Note that the orbital evolution is more sensitive to the non-conservativeness parameters \(\delta_{mt}\) and \(\gamma_{mt}\) than to the parameter \(\beta_{mt}\), which describes the Jeans mode of mass loss by the binary during supercritical accretion (see the Appendix for more details).

The non-conservativeness parameter due to the mass loss from the binary through the outer Lagrange point \(L_2\) was chosen to be \(\delta_{mt} = 0.1\). This value is motivated, in particular, by the observations of a change in the orbital periods of semidetached binaries with a mass transfer (see, e.g., Erdem and Öztürk 2014).

To compute the magnetorotational evolution of NSs, we used the formulas described in detail, for example, by Lipunov et al. (2009).

After the Roche lobe overflow by the donor, the mass transfer rate \((dM/dt)_{L1}\) through the inner Lagrange point, as a rule, increases very rapidly and begins to exceed the critical accretion rate onto the NS \(M_{\text{cr}}\). Two paths of subsequent evolution are possible in this case. Despite the high mass transfer rate \((dM/dt)_{L1}\), which reaches \(\sim 10^{-2} - 10^{-1} M_{\odot} \text{yr}^{-1}\) in several cases, the MESA evolutionary code finds the solutions of the system of stellar structure equations for which the star remains within the Roche lobe and
ends its evolution with the formation of a white dwarf or a helium star (depending on the initial mass).\textsuperscript{3} If, alternatively, no solution is found and the computations terminate, then this implies that the donor radius continues to increase and the binary is immersed in a common envelope. In both cases, we assume that the binary is a NULX as long as the accretion (X-ray) luminosity of the source exceeds $10^{39}$ erg s\textsuperscript{-1}.

RESULTS OF COMPUTATIONS

Examples of MESA Evolutionary Tracks with NULXs

The results of our computations of the duration of the disk accretion stage with a rate over $10^{-7} M_\odot \text{yr}^{-1}$ (the lower limit for potential NULXs harboring NSs with low magnetic fields) for a grid of models with initial semimajor axes $0.9 < \log(a_{ini}/R_\odot) < 3.5$ and donor masses $0.75–50 M_\odot$ in $M_{2,ini} - P_{\text{orb,ini}}$ coordinates are presented in Fig. 2 in $M_2 - P_{\text{orb,ini}}$ coordinates. The squares in this figure mark the binaries in which the NULX stage ends with the formation of a common envelope; two tracks with parameters close to those of observed sources are shown in Fig. 3 and are described in detail below. The circles in Fig. 2 indicate the binaries that experience a stable mass transfer over the entire NULX stage. The masses of the optical components in such binaries are $\lesssim 5 M_\odot$, while the known NULXs have masses $\gtrsim 5 M_\odot$. An example of an evolutionary track for binaries with a stable mass transfer is given in the Appendix (Fig. 11).

In the example on the left in Fig. 3 the initial donor mass is $3.5 M_\odot$, the orbital period at the Roche lobe overflow is 5 days, and the NS magnetic moment is $\mu_0 = 4 \times 10^{31} \text{ G cm}^3$. The upper panel shows the relation between the mass loss rate by the donor ($dM/dt$) and the binary orbital period. The color on the line (right scale) represents the donor mass.

\textsuperscript{3}A binary system with a Wolf–Rayet optical component is known (Qiu et al. 2019), but the nature of the accretor has not yet been determined.
The time dependences of the NS Alfvén radius \( R_A \), corotation radius \( R_{\text{c}} \), and spherization radius \( R_s \) are shown on the middle panel. The lower panel shows the time evolution of the NS spin period \( P^* \), its X-ray luminosity \( L_X \), and the mass loss rate by the donor and the accretion rate onto the NS. The subcritical accretion stages are bounded by the shaded regions. The quantity \( (dM/dt)_{\text{NS}} \) defines the magnetospheric transmission at the supercritical stage and the NS accretion luminosity \( L_X = 0.1(dM/dt)_{\text{NS}} c^2 \). The critical accretion rate \( M_{\text{cr}} \) was computed using the accretion disk model (Eq. (1)).

Since the donor mass exceeds significantly the accretor mass, the mass transfer rate through the inner Lagrange point \( (dM/dt)_{L1} \) very rapidly begins...
Fig. 4. Positions of the optical components $M_2$ on the Hertzsprung–Russell diagram at the NULX stage. On the left panel, the distribution of NULX over stage duration is shown in the color scale. On the right panel, the color of the points reflects the mass of the optical component. The black lines mark the tracks for stars with masses of 3.5 and $21.4 M_\odot$, the examples of whose evolution are presented in Fig. 3, at the Roche lobe overflow stage. The solid line segments mark the stages at which the CBSs are NULXs.

Fig. 5. “Optical star mass $M_2$–orbital period $P_{\text{orb}}$” model diagram. (a) The distribution of the number of CBSs with NSs at the onset of Roche lobe overflow by the donors obtained by computing $10^7$ tracks of binaries with the BSE code by taking into account the distributions in initial binary parameters (see the text). The color gradations represent the number of binaries normalized to the solar mass. (b) The distribution of the number of NULXs normalized to the solar mass in a model galaxy obtained by convolving the formation frequency of their precursors (the BSE code, upper panel) with the duration of the NULX stage (the MESA code, see Fig. 2) and the star formation history (Eq. (10)) 14 Gyr after the onset of star formation in the model galaxy. The NS magnetic field strength is determined from Eq. (9) with the mean $\mu_0 = 10^{11.6}$ G cm$^3$. The circles represent the persistent sources from Table 1. The black curves indicate the segments of the model tracks from Fig. 3 at the NULX stage.
to exceed the critical accretion rate onto the NS $M_{\text{cr}}$. The middle and bottom left panels in Fig. 3 clearly illustrate that the binary can formally ($L_X > 10^{39}$ erg s$^{-1}$) manifest itself as a NULX already at the short stage of subcritical accretion onto the NS. The mass loss by the donor and the NULX stage end with the formation of a common envelope. In this case, the NULX stage lasts for $\approx 40,000$ yr.

An example of the track of an optical star with an initial mass $M_2 = 21.4 M_\odot$ and an orbital period at the Roche lobe overflow $P_{\text{orb}} = 564$ days is presented on the right in Fig. 3. The NS magnetic field is the same as that in the example on the left in Fig. 3. For this binary the NULX stage with an X-ray luminosity over $10^{39}$ erg s$^{-1}$ is short (less than 1000 yr, see the middle and lower panels on the right) and also ends with the formation of a common envelope. Note that, in contrast to the example on the left, the NS spin period $P^*$ at the accretion stage (the upper curve on the lower right panel) initially increases to its equilibrium value $P_{\text{eq}}$ at a binary age of 7.534 Myr and then the NS spins up according to the law $P^* = P_{\text{eq}} \sim \dot{M}^{-6/7}$. In this example this is due to the NS transition from the ejector stage to the accretor one, bypassing the propeller stage.

Figure 4 shows the positions of the optical components of binaries at the NULX stage on the Hertzsprung–Russell diagram. Obviously, the overwhelming majority of them, despite the long duration of this stage, cannot be detected by present-day instruments due to their low luminosity. The known NULXs have masses $\geq 5 M_\odot$ (Table 1). The luminosity of stars with such a mass is close to $500 L_\odot$. Thus, the stars in the most populated part of the Hertzsprung–Russell diagram are “cut off.” At the same time, Fig. 4 shows that a significant fraction of the NULX donors must be red (super)giants. For instance, the optical component of NGC 30 ULX-1 is a red giant (Heida et al. 2019).

$$M_2 - P_{\text{orb}} \text{ Model Diagram}$$

The distribution of the number of binaries that are potential precursors of NULXs at the Roche lobe overflow by the donor on the “optical component mass–orbital period” ($M_2 - P_{\text{orb}}$) diagram computed by the BSE code for $10^7$ binaries is shown in Fig. 5a. A deficit of binaries with masses (12–14) $M_\odot$ and $P[\text{day}] \gtrsim 1.6$ is related to the transition from main-sequence donors to red-giant donors; previously, a similar deficit, but for slightly less massive binaries, was noted by Fragos et al. (2015).

The convolution of the formation probability of a CBS with a star of mass $M_2$ on the verge of Roche lobe overflow computed per solar mass by the BSE code (Fig. 5a) with the duration of the NULX stage computed with a grid of MESA models and the star formation history (Eq. (10)) gives the expected number of NULXs per solar mass in a model galaxy (Fig. 5b). Two groups of sources are distinguished in Fig. 5b: with optical component masses of several solar masses and orbital periods up to ten days and with masses $\sim 20$ solar masses and orbital periods from several tens to $\sim 100$ days. Examples of the evolutionary tracks falling into these regions of the diagram were described above and are presented in Fig. 3. The NULX stages for these sources are marked by the thick black lines. The positions of the observed sources 2 and 3 from Table 1 are seen to be close to the model tracks shown on the left and right panels of Fig. 3, respectively.

$L_X - P^*$ Diagram for Accreting NSs

The “X-ray luminosity $L_X$–NS spin period $P^* = P_{\text{eq}}$” model diagrams for NSs at the sub- and supercritical accretion stages are shown in Fig. 6 (the left column of panels). The color gradations in the figure indicate the number of binaries per solar mass. The results of our computations are presented for three models: disk accretion onto a NS with a log-normal magnetic field distribution with the mean magnetic moment $\log \mu_0 = 30.6$ G cm$^3$ corresponding to a surface magnetic field of a NS with a radius of 12 km $B_0 \approx 5 \times 10^{12}$ G (upper panel), the same model, but for an order of magnitude higher magnetic field ($\log \mu_0 = 31.6$ G cm$^3$, $B_0 \approx 5 \times 10^{13}$ G, middle panel), and the model of advection disks around NSs with the mean magnetic moment $\log \mu_0 = 31.6$ G cm$^3$ (lower panel). The dashed lines marked by the NS magnetic moments correspond to the dependence (4) of the NS equilibrium spin period $P_{\text{eq}}$ for subcritical disk accretion on the accretion rate (X-ray luminosity). We assume that under a slow evolutionary change in the accretion rate the NS spin has time to adjust itself to the equilibrium value and $P^* = P_{\text{eq}}(L_X)$, so that the NS spin period evolves along these lines. The dashed straight line ($dM/dt)_{\text{cr}}$ corresponds to the critical accretion rate $M_{\text{cr}}$ (the NS X-ray luminosity $L_X$ from Eq. (7)), so that the sources along this line and to the right of it are at the supercritical accretion stage. The circles and stars marked by the numbers as in Table 1 indicate the positions of the observed persistent and transient NULXs. The red square marks the position of a supercritical NULX with initial donor mass and orbital period as those in the example on the left in Fig. 3. The right panels show the results of our MESA computations for this source (the NS spin period, the X-ray luminosity, the mass transfer rate...
through \( L_1 \), and the accretion rate onto the NS) for NS magnetic moments \( \mu = 10^{30.6} \) and \( 10^{31.6} \) G cm\(^3\) and the models of accretion (upper and middle panels) and advection (lower panel) disks. For most of the time of the Roche lobe overflow by the donor such sources are at the supercritical accretion stage.

**Distributions of the Parameters of Model NULXs**

In our computations NULXs constitute a subset of NSs at the stage of disk accretion in a CBS with a luminosity \( L_X > 10^{39} \) erg s\(^{-1}\). Their distributions in observed parameters, i.e., optical donor star mass \( M_2 \), NS spin period \( P^* \), X-ray luminosity \( L_X \), and CBS orbital period \( P_{\text{orb}} \), are presented in the form of two-dimensional color diagrams in Figs. 7–9 for the same three models as those in the Fig. 6. The color gradations represent the number of binaries per solar mass in a model galaxy with the star formation history (10) and an age of 14 Gyr. As in Fig. 6, the circles and stars indicate the positions of the observed persistent and transient NULXs. The X-ray luminosity functions of NULXs (differential \( dN/d \log L_X \) and cumulative \( N(>L_X) \)) are constructed on the lower right panel. An analysis of X-ray observations shows (Mineo et al. 2012; Sazonov and Khabibullin 2017) that the number of high-mass X-ray binaries (HMXBs) in galaxies is proportional to the star formation rate (SFR). Therefore, the normalization of our computations to the mean SFR in a model galaxy (10) with an age of 13–14 Gyr, \( \langle \text{SFR} \rangle = 5 \, M_\odot \text{yr}^{-1} \), is presented on the right scale. Here, the stellar disk mass of \( 10^{11} \, M_\odot \) corresponds to \( \text{SFR} \approx 7 \, M_\odot \text{yr}^{-1} \).

It can be seen that even under conservative assumptions about the pattern of disk accretion and for a standard mean NS magnetic field (Fig. 7), the model reproduces the observed positions of the persistent NULXs (the circles in the figure). Note that most of the NULXs are explained by disk accretion onto a NS with a magnetic field in the range \( 10^{12} < B < 10^{14} \) G (see the \( L_X - P^* \) diagram on the middle right panel). Only one source (NGC 5907 ULX1) falls into the region of supercritical disk accretion onto a NS with a magnetic field \( \sim 10^{13} \) G. Note also that the transient NULXs (stars), whose luminosities reach super-Eddington values only during their outbursts, in quiescence with a low X-ray luminosity fall into the most densely populated region in Fig. 6.

An increase in the mean NS magnetic field to \( \sim 10^{13} \) G (Fig. 8) increases the NS equilibrium spin periods (Eqs. (4), (5)) and the limiting X-ray luminosities (7), extending the X-ray luminosity function for the brightest sources beyond \( 10^{41} \) erg s\(^{-1}\). Supercritical advection disks with a high accretion rate (8) increase the maximum possible accretion luminosity of NULXs above \( \sim 10^{41} \) erg s\(^{-1}\). However, the X-ray luminosity of such sources can be below this value (for a discussion of the maximum possible luminosities in model accretion columns, see Mushtukov et al. 2017). Note, however, that the presumed structure of the accretion flows affects weakly the positions of the model sources on the “optical star mass–orbital period” diagram.

**DISCUSSIONS AND CONCLUSIONS**

After the discovery of the first pulsating ULXs (Bachetti et al. 2014), it has become clear that accreting NSs can account for a significant fraction of the ULX population (see, e.g., Koliopanos et al. 2017; Walton et al. 2018). Population synthesis computations confirm this assumption, but they were carried out without allowance for the NS magnetic field. As shown in this paper, accretion onto magnetized NSs naturally reproduces the observed characteristics of NULXs without invoking additional hypotheses about the beaming of X-ray radiation from accreting NSs (cf. Wiktorowicz et al. 2017).

For our computations we used the hybrid population synthesis method in which the potential precursors of the objects of interest (NULXs in our case) are found by computing a large number (tens of millions) of binaries with given initial parameter distributions by the fast BSE analytical population synthesis code (Hurley et al. 2002), while the stages with a mass loss are computed with a grid of models constructed by the MESA evolutionary code (Paxton et al. 2011) with a careful allowance for the evolution of stars and the physical mass transfer processes in CBSs. The novelty of our computations is a detailed consideration of the possible X-ray luminosity during disk accretion onto magnetized NSs. At high accretion rates super-Eddington luminosities \( L_X > 10^{38} \) erg s\(^{-1}\) are reached even at the stage of subcritical accretion in the disk, while the supercritical stage begins only after the attainment of the local Eddington luminosity at the inner boundary of the disk bounded by the NS magnetosphere (\( R_A > R_{NS} \)). It is not surprising that for a NS with a magnetic field \( 10^{12}–10^{14} \) G in the standard CBS evolution model subcritical X-ray luminosities up to \( 10^{40}–10^{41} \) erg s\(^{-1}\) are naturally realized (see Fig. 6).

A significant factor that determines the characteristics of the accretion stage onto NSs (its duration, accretion rate, the possibility of common-envelope formation) is the non-conservativeness parameter \( \gamma_{mt} \) (the dimensionless angular momentum carried away from the CBs by the matter through the outer Lagrange point \( L_2 \)). We assumed a moderate fraction of the mass loss in this way (\( \delta_{mt} = 0.1 \)). However, our computations with the minimum possible value of
\( \gamma_{\text{mt}} \) = 1.15 do not reproduce the orbital periods of the observed NULXs. In contrast, the parameter \( \gamma_{\text{mt}} \) = 3 adopted by us (which is almost half the value deduced from the observations of SS433, see Cherepashchuk et al. (2019)) leads to satisfactory agreement of the results of our computations with the positions of the observed NULXs on the \( P^* - L_X \), \( P^* - P_{\text{orb}} \), \( P_{\text{orb}} - L_X \), \( M_2 - P_{\text{orb}} \), and \( M_2 - L_X \) diagrams (Figs. 7–9) for var-

**Fig. 6.** Distributions of NSs at the sub- and supercritical disk accretion stages in spin period and X-ray luminosity. Top left: the distributions for the model with accretion disk self-regulation at the supercritical accretion stage. The magnetic field strength is determined from Eq. (9) with a mean \( \mu_0 = 10^{30.6} \) G cm\(^3\). Middle left: the same for \( \mu_0 = 10^{31.6} \) G cm\(^3\). Bottom left: the same for the advection model of a supercritical disk and \( \mu_0 = 10^{31.6} \) G cm\(^3\). The observed NULXs are indicated by the open circles and stars and are marked by the numbers as in Table 1. The open square represents a model supercritical source with initial donor mass and orbital period as those in the example on the left panel of Fig. 3, but with different magnetic fields or in different models. Right: the characteristics of the model tracks for the same NULX, but for different NS magnetic fields and disk models (the accretion one at the top and in the middle, the advection one at the bottom.)
ious accretion disk models and mean NS magnetic fields.

Although, for completeness, we marked the positions of transient NULXs on the $P^*-L_X$ diagram by the stars, their evolution, of course, differs from the evolution of persistent sources: during their outbursts such sources rapidly move horizontally on this diagram, their spin period reflects the NS equilibrium spin period in quiescence, in which they stay for most of the time. Depending on the outburst amplitude, transient NULXs can move, for instance, away from the densely populated region of persistent NSs accreting matter from the disk or even from a quasi-spherical stellar wind (see the computations in Postnov et al. (2019)).

Apart from standard accretion disks, we used

Fig. 7. Model distributions of NULXs—magnetized NSs in CBSs at the sub- and supercritical disk accretion stages (leftward and rightward of the line $(dM/dt)_{cr}$, respectively, on the middle right panel) on the following diagrams (the panels from top to bottom and from left to right: $M_2- \log P_{orb}$, $\log P^* - \log P_{orb}$, $\log L_X - \log P_{orb}$, $\log M_2 - \log L_X$, $\log P^* - \log L_X$. The lower right panel shows the cumulative ($N > L_X$) and differential ($dN/(d \log L_X)$) X-ray luminosity functions of NULXs. The normalization is made to a star formation rate of $5 M_\odot$ yr$^{-1}$ at the present (14 Gyr) epoch, the scale on the right. The color gradations (top scale) indicate the number of sources per cell per solar mass. The mean NS magnetic moment is $\log \mu_0 = 30.6$ G cm$^3$. 
the model of supercritical advection disks (Lipunova 1999, Chashkina et al. 2019) that admits an even higher critical accretion rate onto NSs (Eq. (8) and the lower panel in Fig. 6), although X-ray luminosities over $10^{41}$ erg s$^{-1}$ from accretion columns are theoretically debatable (Mushtukov et al. 2017). Further observations of NULXs with ultrahigh luminosities would be important for choosing the models of possible accretion flows before and after their penetration into the NS magnetosphere.

In addition to the explanation of the observed X-ray luminosities and NS spin periods, the CBS models computed by us also explain the positions of NULXs with known orbital periods and mass estimates of the optical components (Figs. 7–9). As the luminosity function presented on the lower right panels of these figures shows, in a model galaxy with the chosen star formation history (10) and a mass of the thin disk of the order of its mass in the Milky Way or, equivalently, with a current star formation rate of $3–5$ $M_{\odot}$ yr$^{-1}$, there can be several NULXs with a luminosity up to $10^{40}$ erg s$^{-1}$. Note that for the population of NULXs in the model spiral galaxy, the distributions of NSs with a high mean magnetic

\[ N (\text{per cell per solar mass}) \]

\[ \mu_0 \text{ [G cm}^3 \text{]} = 31.6 \]

\[ \text{Fig. 8. Same as Fig. 7 for the mean NS magnetic moment } \log \mu_0 = 31.6 \text{ G cm}^3 \]. The black lines mark the model track of a CBS with initial optical star mass $M_2$ and orbital period $P_{\text{orb}}$ as those in the example on the left in Fig. 3.
field $\sim 5 \times 10^{13}$ G are more preferable than those following from an analysis of the population of radio pulsars (Faucher-Giguère and Kaspi 2006) (Figs. 8 and 9), although, of course, it is too bold to reach any conclusions about NS magnetic fields based on population synthesis models.

CONCLUSION

Our general conclusion is as follows. A consideration of the standard evolution of binary stars with a detailed treatment of the mass loss stage in CBSs with a NS component based on the MESA code allows the observed populations of NULXs in spiral galaxies to be quantitatively explained as magnetized NSs at the disk accretion stage without assuming any strong X-ray beaming. Sources with a high X-ray luminosity at the supercritical accretion stage can be most interesting from a physical point of view. Powerful outflows from supercritical accretion disks forming envelopes and nebulae around the binaries must be observed in such sources. The detection of optically thick outflows from Swift J0243 (Tao et al.

Fig. 9. Same as in Fig. 7 in the model of an advection disk around magnetized NSs (Chashkina et al. 2019) for the mean NS magnetic moment $\log \mu_0 = 31.6$ G cm$^3$. 

$N$ per cell per solar mass

$M_1$

$log P^* [s]$

$log P_{orb} [day]$

$log L_X$

$log (N > log L_X)$

$dN/dlog L_X$
and an expanding nebula around NGC 5907 (Belioire et al. 2020) can be a confirmation of this.

**APPENDIX**

INFLUENCE OF THE NON-CONSERVATIVE MASS TRANSFER PARAMETERS

Figure 10 presents the results of our evolution computations based on the MESA code for the binary from the example in Fig. 3 for various assumptions about the pattern of mass transfer when the optical star overflows its Roche lobe. The columns on the right show examples of the evolution without any mass loss through the outer Lagrange point $L_2$ (parameter $\delta_{mt} = 0$). The columns on the left show our computations with the assumptions made in this paper about the fraction of the mass loss from the binary through $L_2$, $\delta_{mt} = 0.1$, and the dimensionless specific angular momentum in the circumbinary disk, $\gamma_{mt} = 3$. The upper panels show our computations for the standard critical accretion rate adopted in MESA $\dot{M}_{cr} = \dot{M}_{Edd}$ in exceeding which an isotropic wind from the compact object begins (red curve). The lower panels show our computations for a critical accretion rate for the beginning of an isotropic wind in MESA set equal to $\dot{M}_{cr} = 10^{-6} M_\odot \text{yr}^{-1}$. It can be seen that the non-conservativeness parameters $\delta_{mt}$ and $\gamma_{mt}$ are much more significant than the parameter $\beta_{mt} = ((\dot{dM}/dt)_{L1} - \dot{M}_{cr})/(\dot{dM}/dt)_{L1}$ dependent on the adopted $\dot{M}_{cr}$. A non-conservative outflow of matter through the circumbinary disk gives rise to a common envelope and reduces the duration of the possible NULX stage.

EXAMPLE OF A TRACK WITH A STABLE MASS TRANSFER AT THE NULX STAGE

Figure 11 presents the evolutionary track of a binary with an optical star mass $M_2 = 2.5 M_\odot$ and an orbital period at the Roche lobe overflow $P_{\text{orb}} = 2.75$ days. The same parameters as those in Fig. 3 are shown. At the stage of a stable mass transfer the
NS is mainly at the accretion stage. No common-envelope stage occurs, the orbital period of the binary reaches its minimum and then increases. The evolution ends with the formation of a “NS + helium white dwarf” binary. The NULX stage lasts for more than 2 Myr. This time is sufficient for the NS magnetic field to decay due to the accretion of matter. The right panels in Fig. 11 present the same evolutionary track with the following model magnetic field decay (Osłowski et al. 2011):

$$\mu(t) = (\mu_0 - \mu_{\text{min}}) \times \exp \left[ -\frac{\Delta M}{0.025 \, M_\odot} \right] + \mu_{\text{min}},$$

where $\Delta M$ is the accreted mass and $\mu_{\text{min}} = 10^{26}$ G cm$^3$ is the minimum NS magnetic moment. It can be seen that the magnetic field decay leads to a decrease in the magnetospheric radius and a corresponding decrease in $\dot{M}_{\text{ct}} \sim R_A \sim \mu^{1/7}$ and X-ray...
luminosity. The duration of the NULX stage also decreases by half (Fig. 11, lower right panel). Note also that a NS with a decaying magnetic field decreases by half (Fig. 11, lower right panel). Note

luminosity. The duration of the NULX stage also mainly at the supercritical accretion stage. Of course, due to the great uncertainty in the quantitative NS magnetic field decay parameters, this example should be perceived as evidence of the tendency for the luminosity and the number of NULXs in binaries with low-mass components $M_2 \lesssim 3 M_\odot$ to decrease.

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