Probing early-time correlations in heavy ion collisions

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Abstract. We investigate the common influence of correlations arising from initial state parton density fluctuations on measured multiplicity and momentum fluctuations as well as flow fluctuations. We calculate both a universal correlation scale in an initial stage Glasma flux tube picture and the modification to these correlations from later stage hydrodynamic flow. We find quantitative agreement with measurements over a range of collision systems and energies.

1. Introduction

In the first instants of every heavy ion collision event a unique system is produced. This happens because of the random orientation of nucleons before the collision and the finite probability for nucleon-nucleon or partonic interaction at the instant of the collision. The irregular nature of these first moments leads to inhomogeneous or “lumpy” energy/parton density deposits in the collision volume and the distribution of these lumps defines a distinct event geometry. Assuming that the subsequent evolution of the system progresses from the initial shape in a deterministic way, fluctuations in the initial geometry could explain the origin of odd harmonic flow coefficient measurements such as $v_3$. Just as measurements of harmonic flow coefficients of all orders, $v_n$, can provide insight into the initial geometrical distribution of lumps, fluctuations of $v_n$ can provide information about the lumps themselves.

We argue that partons emerging from the same source or lump are more likely to share the same phase space throughout the collision lifetime and this correlation simultaneously gives rise to observed multiplicity and transverse momentum fluctuations as well as fluctuations of harmonic flow coefficients [1, 2]. In Sec. 2, we define these observables and demonstrate that they are related by a common model independent momentum correlation function. In Sec. 3, we consider contributions to this correlation function from the flow modification of initial state spatial correlations of partons originating from common density lumps. In this formulation parton density lumps emerge from the dissolution of Color Glass Condensate (CGC) -Glasma flux tubes. We find the strength of this correlation is determined by the size, number, and parton multiplicity of the lumps and sets a common scale that governs most of the centrality and center of mass collision energy dependence [3, 4]. Finally, we summarize in Sec. 4.

2. Correlations and Fluctuations

Fluctuation observables are often measured utilizing two-particle correlation methods. The pair distribution of hadrons with momenta $p_1$ and $p_2$ is $\rho_2(p_1, p_2) = dN/dp_1 dp_2$ and in the absence
of correlations, it factorizes into the square of the singles distribution, \( \rho_2(p_1, p_2) \rightarrow \rho_1(p_1)\rho_1(p_2) \) where \( \rho_1(p) = dN/dp \). In this case, fluctuation measurements would depend only on event by event differences in \( \rho_1(p) \). Alternatively, if correlations do exist, then fluctuation observables fundamentally depend on their behavior. In this work, we consider contributions to pairwise correlations only from the existence of parton density lumps, specifically, the correlation induced between particles that originate at the same transverse position. We remark, however, that the formulae in this section apply in general to any two-particle correlation in the system.

In the presence of correlations, we can write the pair distribution as \( \rho_2(p_1, p_2) = \rho_1(p_1)\rho_1(p_2) + r(p_1, p_2) \), where \( r(p_1, p_2) \) represents the genuine two-particle correlation function [1, 2, 5, 6]. We can isolate these correlations by writing

\[
r(p_1, p_2) = \rho_2(p_1, p_2) - \rho_1(p_1)\rho_1(p_2).
\]

Integrating over all momenta yields the “correlation strength” defined as

\[
R \equiv \frac{1}{(N)^2} \int r(p_1, p_2) dp_1 dp_2 = \frac{\langle (N(N-1)) - (N)^2 \rangle}{(N)^2} = \frac{\text{Var}(N) - \langle N \rangle}{(N)^2}.
\]

This quantity distinguishes multiplicity fluctuations as non-Poissonian particle production since Poissonian distributions have the property that \( \text{Var}(N) = \langle N \rangle \). Consequently, non-zero values of \( R \) indicate a correlation in the production of hadrons. As suggested by the name, the correlation strength, \( R \), sets the scale of all fluctuation measurements related to two-particle correlations.

Dynamic fluctuations of transverse momentum are defined using the covariance

\[
\langle \delta p_{1i} \delta p_{2j} \rangle = \frac{\sum_{i \neq j} \delta p_{1i} \delta p_{2j}}{(N(N-1))} = \int dp_1 dp_2 r(p_1, p_2) \langle N(N-1) \rangle \delta p_{1i} \delta p_{2j},
\]

where \( \delta p_{1i} = p_{1i} - \langle p_i \rangle \) and the average transverse momentum is \( \langle p_i \rangle = \langle p_1 \rangle / (N) \) for \( p_1 = \sum p_{1i} \) the total transverse momentum in an event [7, 8, 9, 10]. This quantity vanishes when particles \( i \) and \( j \) are uncorrelated. Equation (3) sums the product of differences in particle momenta from the average for each hadron pair. Pairs in which both particles have higher than average momentum add to \( \langle \delta p_{1i} \delta p_{2j} \rangle \). Lower-than-average pairs also add to the covariance, while high/low pairs subtract from it. In global equilibrium \( \langle \delta p_{1i} \delta p_{2j} \rangle \equiv 0 \). Positive measurements of \( \langle \delta p_{1i} \delta p_{2j} \rangle \) could be explained if, for example, particle production is dominated by jets, or a collectively expanding medium is comprised of spots of locally equilibrated matter. In the next section we investigate the latter case and take the locally equilibrated spots of fluid to be the results of initial state parton density lumps.

The correlation strength, \( R \), sets the scale of (3) in the sense that the same (correlated) pairs are counted in (2), only here they are weighted by their momenta. Significantly, both observables count correlated pairs independent of their direction and therefore do not depend on the asymmetry of the system. However, because of the momentum weights, (3) is sensitive to the average expansion and temperature.

The relative angles of correlated pairs is quantified by harmonic flow correlation and fluctuation measurements. These measurements reflect both global correlations based on the shape of the collision as well as correlations represented by (1). The global anisotropy of the collision can be characterized by the moments \( v_n = \langle \cos n(\phi - \phi_{RP}) \rangle \), where \( \langle ... \rangle \) represents an average over particles and events and \( \phi_{RP} \) is the reaction plane angle defined by the plane spanned by the impact parameter \( b \) and the beam direction. Measurements utilizing two-particle correlations do not require knowledge of the reaction plane and are defined as

\[
v_n(2)^2 = \frac{\langle \sum_{i \neq j} \cos n(\phi_i - \phi_j) \rangle}{\langle N(N-1) \rangle} = \int \frac{\rho_2(p_1, p_2)}{(N(N-1))} \cos n(\phi_1 - \phi_2) dp_1 dp_2
\]
Cast in terms of the pair distribution, we use (1), to write (4) as

\[ v_n(2)^2 = \langle v_n \rangle^2 + 2\sigma_n^2, \]

where the flow coefficient relative to the reaction plane is \( \langle v_n \rangle = \int \rho_1(p) \cos(n(\phi - \Psi_{pp})d\mathbf{p} \) comes from the single particle distribution and the factor of two in the fluctuation term \( \sigma \) is conventional [14]. Assuming the four-particle harmonic flow correlation measurement is approximately \( v_n\{4\} \approx \langle v_n \rangle \), as in Ref. [14], we have

\[ c_n^2 = \frac{v_n\{2\}^2 - v_n\{4\}^2}{2} = \int d\mathbf{p}_1 d\mathbf{p}_2 \frac{r(p_1, p_2)}{2\langle N(N - 1) \rangle} \cos n\Delta\phi, \]

where \( \Delta\phi = \phi_1 - \phi_2 \) is the relative azimuthal angle between the pair. This observable quantifies the contribution of genuine two-particle correlations to the harmonic flow coefficients (4). Although this contribution is often regarded as “non-flow”, anisotropic flow modifies the relative angles of correlated pairs emerging from common density sources. Given the assumption that a density lump produces a locally thermalized fluid element, the push on this element from flow completely determines the \( \Delta\phi \) distribution of partons in that cell. We discuss this idea further in the next section.

3. Glasma Correlations

We examine correlations arising only from particles emerging from a common source and their modifications from the transverse expansion of the system. We take the sources to be initial state density lumps which are consequences of the dissipation of CGC-Glasma flux tubes. In general one can describe two-particle spatial correlations with \( c(x_1, x_2) = n_2(x_1, x_2) - n_1(x_1)n_1(x_2) \). Here \( x_1 \) and \( x_2 \) are transverse spatial coordinates. If the pair distribution, \( n_2(x_1, x_2) \), contains no correlations, it factorizes into the square of the singles distribution, \( n_1(x_1) \) and \( c(x_1, x_2) \) vanishes. In our flux tube lump picture we write

\[ c(x_1, x_2) = \langle N \rangle^2 R \delta(r_1) \rho_{FT}(R_t). \]

The delta function in relative transverse elliptical coordinates \( r_t = r_{t1} - r_{t2} \) enforces the condition that correlated partons originate from the same flux tube source, assuming the flux tube transverse size is small compared to that of the collision area. The flux tube probability distribution, written in average coordinates \( R_t = \langle r_{t1} + r_{t2} \rangle / 2 \), is \( \rho_{FT}(R_t) \approx (2\pi R_A^2) (1 - R_t^2/R_A^2) \). Here the shape of \( \rho_{FT} \) resembles the nuclear thickness function and the area of overlap region. The distribution \( \rho_{FT} \) represents an average over all possible shapes, which we have taken to be a simple ellipse.

The correlation strength, \( R \), governs the magnitude of correlations in (6). To compute its value we implement a CGC-Glasma framework in which the initial moments of a collision event is comprised of a system of longitudinally long color flux tubes each with transverse size \( \sim Q_s^{-2} \). Here \( Q_s \) is the scale of momentum transfer in collisions of dense partonic systems that signals the onset of gluon saturation. For recent review see [15]. Each event produces a random number of \( K \) flux tubes that is proportional to the transverse area \( R_A^2 \) divided by the area per flux tube, \( \sim Q_s^{-2} \) [16]. \( K \) fluctuates from event to event with average \( \langle K \rangle \), and we calculate in Ref. [3] that \( R \propto \langle K \rangle^{-1} \). Each Glasma flux tube yields an average multiplicity of \( \sim \alpha_s^{-1}(Q_s) \) gluons and as in Ref. [16], the number of gluons in a rapidity interval \( \Delta y \) is then \( \langle N \rangle = (dN/dy) \Delta y \sim \alpha_s^{-1}(Q_s) \langle K \rangle \). Combining these two results we find the Glasma correlation scale, \( R \langle dN/dy \rangle = \alpha_s^{-1}(Q_s^2) \), which is dimensionless and depends only on the saturation scale, \( Q_s^2 \) [1, 17]. Measurements of the ridge at various beam energies, target masses, and centralities fix the dimensionless coefficient \( \alpha \) and are in accord with the leading-order dependence [3, 4]. As a comparison, the correlation strength is equivalent to the inverse of the width parameter of negative binomial multiplicity distributions [18] as measured by the PHENIX collaboration [19]. The comparisons are shown in Fig. 1 for various beam energies.
Correlations (6) are modified by the transverse expansion of the system. Depending on its initial position a parton receives a transverse boost from flow; partons originating near the center get a small push while those near the periphery get a larger push. Additionally, the geometry of the collision influences this boost since, depending on the shape, some directions have faster expansions than others. The key is that, on average, partons originating at the same position experience the same boost. Their transverse momenta is therefore enhanced in a common way; pairs experiencing a larger transverse boost end up with more columnated final transverse momenta. In this manner, correlations in coordinate space (6) result in correlations in momentum space (1). In this spirit we describe (1) as

$$r(p_1, p_2) = \int c(x_1, x_2) f(x_1, p_1) f(x_2, p_2) d\Gamma_1 d\Gamma_2. \tag{7}$$

Here (6) links blast-wave descriptions of the Boltzmann phase-space densities $f(x, p)$ for two particles with Cooper-Frye freeze out surfaces $d\Gamma [1, 2]$.

We are now in a position to calculate (3) and (5) using (7). In Fig. 2 we compare $dN/d\eta(\delta p_{t1}\delta p_{t2})$ to data from the STAR and ALICE collaborations [10, 20]. In Fig. 3 we compare flow fluctuations in terms of the observable $\sigma_{\nu_n}/\langle \nu_n \rangle = ((\nu_n{2})^2 - \nu_n{4})/((\nu_n{2})^2 + \nu_n{4})^{1/2}$ which resembles the so-called “coefficient of variation” defined as the standard deviation divided by the mean. Care should be taken here since the definition of $\sigma_{\nu_n}^2$, Eq.(5), is not strictly the variance. In both cases the closest agreement with data occurs in central collisions, the region where the Glasma description is most valid. If flow fluctuations in this region arise only from fluctuations in the shape of the collision area, then their governing correlation function would predict $\langle \delta p_{t1}\delta p_{t2} \rangle = 0$ and have almost no dependence on collision energy. The energy dependence of $R$ is dominated by that of the saturation scale $Q_s$ and we note that the most significant result here is the simultaneous agreement of all mentioned observables with changes in collision system and center of mass energy.

Lastly in Fig. 4 we show $v_3$ fluctuations from Glasma. We further emphasize that the energy dependence in Fig. 4 is in good accord with data, again supporting the Glasma scaling with $Q_s^2$. To use (5) to calculate $v_3\{2\}$, we must come to grips with the fact that our symmetric blast wave parametrization assumes $v_3\{4\} = 0$. While this seems to be the case for the STAR measurements, ALICE has measured a non-zero $v_3\{4\}$ for Pb+Pb collisions at 2.76 TeV. Therefore, we prefer to report only our calculation of $\sigma_3^2$.

4. Summary
In this work we study the connection between correlations and fluctuations of multiplicity, momentum, and harmonic flow coefficients. We argue in Sec. 2 that given the existence of
genuine two-particle correlations in the hadron pair distribution, (1), multiplicity fluctuations, momentum fluctuations, and flow fluctuations are all governed by a common scale $R$. This is the first calculation to link these observables. We then consider contributions to the genuine correlations due to fluctuating or lumpy initial parton distributions where correlated partons emerge from the same source/lump. We take the origin of initial state parton density lumps from Glasma flux tubes and present our model calculations in Sec. 3. The reasonable agreement of our calculations with the collision energy dependence of the measurements supports saturation based descriptions of the initial state. Further improvements of this method will come with implementation of event-by-event hydrodynamic calculations which inherently embody the dynamics of collisions that our current expansion model only approximates.

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