Overconfidence in Photometric Redshift Estimation

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ABSTRACT
We describe a new test of photometric redshift performance given a spectroscopic redshift sample. This test complements the traditional comparison of redshift differences by testing whether the probability density functions $p(z)$ have the correct width. We test two photometric redshift codes, BPZ and EAZY, on each of two data sets and find that BPZ is consistently overconfident (the $p(z)$ are too narrow) while EAZY produces approximately the correct level of confidence. We show that this is because EAZY models the uncertainty in its spectral energy distribution templates, and that post-hoc smoothing of the BPZ $p(z)$ provides a reasonable substitute for detailed modeling of template uncertainties. Either remedy still leaves a small surplus of galaxies with spectroscopic redshift very far from the peaks. Thus, better modeling of low-probability tails will be needed for high-precision work such as dark energy constraints with the Large Synoptic Survey Telescope and other large surveys.

Key words: surveys—galaxies: photometry—methods: statistical

1 INTRODUCTION
Photometric redshifts are of key importance to current and future galaxy surveys. A variety of methods have been demonstrated, falling broadly into two categories: empirical and template-based. Empirical methods predict redshifts from photometry by directly using the known spectroscopic redshifts of a subsample spanning the color and magnitude range of the main photometric sample. Template methods use models of galaxy spectral energy distributions (SEDs), which enable prediction of redshifts beyond the magnitude limit of the spectroscopic sample. See Hildebrandt et al. (2010) and Dahlen et al. (2013) for overviews and performance comparisons.

Until recently, photometric redshift performance comparisons (Hogg et al. 1998; Hildebrandt et al. 2008, 2010) have been based on casting the photometric redshift of a galaxy as a single number, but this glosses over some of the complexity inherent in these predictions. For example, a deep survey with a small number of filters is bound to encounter degeneracies in which both low- and high-redshift models are acceptable for some galaxies. Forcing a photometric redshift algorithm to choose only the single most likely model thus generates some wildly inaccurate redshift estimates, which are called “catastrophic outliers.” Capturing all the photometric redshift information in a probability density function $p(z)$ greatly reduces or eliminates these outliers (Fernández-Soto et al. 2002). Even if a particular $p(z)$ is not multiply peaked, it may be asymmetric, so that using the full $p(z)$ rather than a point estimate reduces bias (Mandelbaum et al. 2008) and thus reduces systematic errors on downstream science such as dark energy parameter estimation (Wittman 2009).

The works cited above established that the $p(z)$ paradigm offers better performance than point estimates, but point estimates are still more easily checked against spectroscopic redshift, by tabulating the mean and scatter in the quantity $z_p - z_s$ (spectroscopic redshift minus photometric redshift). The $p(z)$ paradigm offers no obvious generalization of this procedure. Indeed, codes that work internally with $p(z)$ often default to outputting a single error estimate for each galaxy. Hildebrandt et al. (2008) found that these error estimates are not predictive of the real errors, but this may simply reflect the underlying complexity of $p(z)$. The extensive performance comparison of Dahlen et al. (2013) did use $p(z)$ to derive 68% and 95% confidence intervals, and found that most codes are overconfident—their confidence intervals are too narrow. In this paper we present a tool for systematically testing overconfidence, and we show why the $p(z)$ output by template codes can be substantially overconfident. One limitation of our test is that spectroscopic subsamples may not be representative of the full photometric sample. However, this limitation also affects verification of point estimates, and is therefore separable from the question of how to assess the quality of $p(z)$—which are often broad, asymmetric, and/or multimodal functions—against the delta functions represented by spectroscopic redshifts.
2 MEASURING OVERCONFIDENCE

2.1 Conceptual explanation

By its nature, \( p(z) \) cannot be verified on a galaxy-by-galaxy basis, just as a single coin toss cannot determine whether a coin is fair. A large sample, accordingly, does support \( p(z) \) verification. A sample of 1000 galaxies, for example, should contain of order 10 galaxies whose spectroscopic redshift is in tension with the photometric redshift at the 99% level. If too many galaxies exhibit this much tension, the photometric redshifts collectively can be deemed overconfident, as they predict the spectroscopic redshifts with more precision than is supported by evidence. Similarly, if too few galaxies in the sample exhibit this much tension, the photometric redshifts collectively can be deemed underconfident.

In practice, overconfidence is far more common than underconfidence, when estimating almost anything. This may be due to the nature of error budgets: humans use judgment to identify the most salient sources of uncertainty worthy of quantification, whereas “subdominant” sources of uncertainty have little effect when added in quadrature and therefore do not merit quantification. Sources of uncertainty that initially do not seem salient may never be folded into the budget even if their true contribution is substantial. For example, most photometry codes base their uncertainties on photon noise and neglect sky modeling uncertainties. Neglecting this source of noise is justified in many cases, but the general pattern is to underrepresent some sources of noise without any compensating overrepresentation of other sources, so the final result is often overconfident.

We can check for overconfidence by asking whether 50% of galaxies have their spectroscopic redshift within their 50% credible interval (CI), 90% have spectroscopic redshift within their 90% CI, etc. Such checks do appear in the literature (e.g., Schmidt & Thorman 2013; Dahlen et al. 2013), but are usually implemented without a key feature that greatly assists with the interpretation. This key feature was evident already in the pioneering work of Fernández-Soto et al. (2002); here we explain it in more detail, use it to implement a systematic confidence test, and show how this test can lead to insight about the photometric redshift algorithms themselves.

For a confidence test, it is crucial that we choose the highest probability density (HPD) CI for any given credibility level. To see why, consider Figure 1, which shows a hypothetical posterior \( p(z) \). We could define a 20% CI by, say, starting at \( z = 0 \) and integrating the area under the posterior curve until we reach 20% of the total area under the curve, and indeed 20% of galaxies should have spectroscopic redshift within the 20% CI as defined this way. However, testing CIs defined this way would not test overconfidence—the tendency for \( p(z) \) to be too sharply peaked. We therefore define the 20% CI by lowering a threshold from the peak downward until the area under the parts of the curve intersected by the threshold equals 20% of the total area; this is the HPD 20% CI.

The same process leads to the 40%, 60%, and 80% CI in Figure 1 covering multiple separated redshift intervals. This is the only way to maintain the highest probability density and thereby probe for overconfidence. To illustrate, imagine this galaxy has \( z_s = 1.2 \), a result that we should judge to be very unlikely given this \( p(z) \). This indeed falls outside the HPD 99% CI, but falls inside the 50% CI if we define the CI by integrating only in a single contiguous region around the highest peak. The latter definition of CI gives us a false sense that the photometric redshift prediction was borne out. An example with two equal peaks is admittedly extreme, but the same principle is at work with unequal peaks or even a single asymmetric peak. Most confidence checks in the literature to date have not used the HPD CI, but authors should begin doing so; Section 4 shows that results can differ substantially when not using the HPD CI. For the remainder of the paper, references to CI should be understood as HPD CI unless otherwise stated.

2.2 Implementation

We want to know how many galaxies in a data set have \( z_s \) within their 1% CI, how many within their 2% CI, etc. Computationally, this implies a loop over credibility levels, with each iteration containing a loop over galaxies to check whether each galaxy meets the criterion. However, it is computationally more efficient to perform a one-time calculation for each galaxy, to find the CI that just barely includes the spectroscopic redshift. Referring again to Figure 1, imagine that we have not yet calculated any CI but we know the value of \( z_s \). We simply draw a horizontal line through \( p(z_s) \) to identify the relevant redshift intervals and compute the area under the curve in those intervals to find the credible level that just includes \( z_s \). Recording that this threshold credibility is, say, 32% is a highly efficient way of recording

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1 Bayesian statisticians use this term when speaking of Bayesian posteriors, and reserve the term confidence interval for the likelihood. This distinction does not often appear in the astronomy literature.
that this galaxy does not have $z_s$ within its 1% CI, nor its 2% CI, nor its 3% CI, etc, but does have $z_s$ within its 32% CI, and its 33% CI, and its 34% CI, etc.

The implementation is thus quite simple. We compute the threshold credibility $c_i$ for the $i$th galaxy with:

$$c_i = \sum_{z \in p_i(z) \geq p_i(z_{s,i})} p_i(z)$$

where $p_i(z)$ is the posterior for the $i$th galaxy, assumed to be normalized. The requirement that 1% of galaxies have $z_s$ within their 1% CI, etc, then translates into a requirement that $c$ be uniformly distributed from 0 to 1 (we drop the $i$ subscript when referring to collective properties of the $c_i$).

We test for this by computing the empirical cumulative distribution function $\hat{F}(c)$, which should equal $c$. Graphically, plotting $\hat{F}(c)$ resembles a q-q plot in which $\hat{F}$ is expected to match $c$, i.e., fall on a line through the origin with a slope of one. Overconfidence corresponds to $\hat{F}(c)$ falling below this line (too few galaxies have $z_s$ within a given CI). The statistical significance of such a departure can be measured with a Kolmogorov-Smirnov (KS) test. Of course, it is also possible for this test to reveal underconfidence. In either case, the method detects inaccurate error budgets.

3 APPLYING THE TEST

We tested the $p(z)$ estimated by two template-based photometric redshift codes, BPZ (Benítez 2000) and EAZY (Brammer et al. 2008). We are primarily interested in testing template methods because empirical methods should yield calibrated $p(z)$ by design. Because template methods pertain to yield redshifts beyond the magnitude limit at which $p(z)$ can be directly constructed from the photometric and spectroscopic data, they are the methods for which an independent test of $p(z)$ is most desirable. The test results will be data-dependent and must be interpreted accordingly. For example, any overconfidence in the underlying photometry will contribute to overconfidence in $p(z)$, and this contribution will be magnitude- and redshift-dependent. We therefore run each code on the same data, the Hubble Deep Field North (HDFN) seven-band photometry with 127 spectroscopic redshifts (Fernández-Soto et al. 1999) that ships with EAZY and shipped with earlier versions of BPZ. We run each code with the default templates and priors that are shipped with the code.

3.1 BPZ

We used BPZ version 1.99.3 with default templates and priors and the INTERP value set to 2 on the command line as recommended in the documentation. The resulting $\hat{F}(c)$ plot (Figure 2, middle curve) shows substantial overconfidence in HDFN data using the BPZ code shows substantial overconfidence overall (solid curve, $p < 10^{-15}$), with some magnitude dependence (dashed curves).

Figure 2. The $\hat{F}(c)$ plot for HDFN data using the BPZ code shows substantial overconfidence overall (solid curve, $p < 10^{-15}$), with some magnitude dependence (dashed curves).

more overconfidence in the brighter galaxies. A KS test indicates that the difference between the bright and faint distributions is not significant ($p = 0.44$). However, magnitude dependence will be a recurring issue so a few points should be clarified now. First, magnitude-dependent overconfidence does not automatically imply a problem in the photometric redshift algorithm, because uncertainties in the underlying photometry are already magnitude-dependent. Second, our method probes how well the uncertainty has been assessed rather than the uncertainties themselves, so one should not assume that faint galaxies will be more problematic. Indeed, Figure 2 shows that faint galaxies perform better. Photometry provides a useful analogy: faint galaxies have larger photometric uncertainties than bright galaxies, but the faint-galaxy uncertainty budget is probably more accurate because it is dominated by well-modeled photon noise rather than poorly modeled uncertainties in background subtraction, calibration, and color terms. In fact, because photometry uncertainties propagate into photometric redshift uncertainties, it is tempting to offer this as an explanation for the overconfidence pattern seen in Figure 2. However, the HDFN is a carefully calibrated and well-tested catalog, and we develop a more compelling explanation below.

Third, magnitude is correlated with redshift so disentangling the two variables may be difficult. If the magnitude trend here is really a redshift trend in disguise, splitting by redshift should reveal a greater divergence between subsamples, and low-redshift (bright) galaxies should have the most overconfidence. In fact, splitting by redshift shows the opposite in both respects, leading us to believe that magnitude is indeed the explanatory variable. An important conceptual point here is that use of a Bayesian prior on redshift prevents us from expecting all subsamples split by redshift to follow the $\hat{F}(c) = c$ relation. In a Bayesian framework, priors
can and should degrade the performance of some subsets in order to improve overall performance. As an everyday example, consider the batting averages of baseball players one month into the season. With each player having few at-bats, their averages vary widely, and applying a prior on batting average greatly improves our estimate of their “true” batting averages. But if, at the end of the season, we find the players with the best “true” batting averages and look back at our Bayesian estimate one month into the season, we will find that the prior biased their averages low; this was unavoidable if we were to improve the one-month estimates overall. Similarly, high-redshift subsamples must not be tested in isolation, and in this paper we do not plot $F_i(z)$ for subsamples split by redshift, even if we do a redshift split to check whether a magnitude trend could be a redshift trend in disguise.

We also tested BPZ using the test data it ships with, a catalog of 57 galaxies with spectroscopic redshift and seven-band photometry from the Hubble Ultra Deep Field (HUDF) catalog produced by Coe et al. (2006). We found trends similar to those illustrated here for the HDFN data, suggesting that overconfidence is a general feature of the $p_i(z)$ output by BPZ. A plausible mechanism for this is that BPZ, like most template codes, propagates uncertainty from the photometry only, and not from the templates. This would also explain why overconfidence is greater for bright galaxies; their smaller photometric uncertainties imply that template uncertainties are a larger share of the uncertainty budget. Further supporting this picture, we found that limiting the template set by turning off interpolation between templates (setting the INTERP parameter to 0) exacerbated the overconfidence. Increasing the INTERP parameter beyond 2 had little effect, presumably because SEDs vary in ways that cannot be captured with interpolation between the default templates.

The issue of template uncertainty was recognized by Fernández-Soto et al. (2002), who explored an empirical fix of convolving $p_i(z)$ with a Gaussian smoothing kernel. Although they cautioned that more sophisticated noise modeling would be required as data sets expanded in terms of both redshift and raw numbers, this approach performed well when they applied it to HDFN data. They assumed that the kernel width should scale as $(1 + z)$ and then optimized the prefactor by maximizing $p_i(z)$ for bright galaxies. This yielded a kernel with $\sigma = 0.065(1 + z)$; this kernel is “optimal” in the sense that it mimics the effect of galaxy SEDs varying from the template set used in their analysis, at the wavelengths used in their analysis, better than other kernels in its family. Smoothing with this kernel broadens $p_i(z)$ for each galaxy, but more so for bright galaxies because faint galaxies already have broad $p_i(z)$ due to their photometric uncertainties. Fernández-Soto et al. (2002) tested the performance of this procedure with a version of the $F_i(z)$ test, verifying (in our notation) $F(0.683), F(0.954)$, and $F(0.997)$. BPZ does have two parameters that nearly serve this function. The CONVOLVE_P parameter, if set, smooths $p_i(z)$ with a Gaussian of fixed width $\sigma = 0.03$. According to comments in the code the purpose of this feature is to combine multiple close peaks; for our purposes we can consider it as injecting a bit of template noise, but Figure 2 already includes this bit of smoothing because CONVOLVE_P is turned on by default. Therefore $\sigma = 0.03$ is too little smoothing to prevent overconfidence, at least for the data sets presented here. The other potentially relevant BPZ parameter is MIN_RMS, which according to the comments represents “intrinsic photo-z rms” (presumably due to the true SEDs of galaxies varying from the templates). In version 1.99.3 MIN_RMS is set to 0.05 (0.067 if the older “CWWSB” template set is used) but it does not affect the $p_i(z)$ written out to disk. It is used only to determine a few quantities derived from $p_i(z)$, such as upper and lower redshift limits and the fraction of the area under $p_i(z)$ that is near the highest peak.

We therefore use a post-processing step to smooth the $p_i(z)$ produced by BPZ in order to test the efficacy of the approach suggested by Fernández-Soto et al. (2002). In the absence of strong evidence that a redshift-dependent kernel is necessary, we tested redshift-independent kernels of various widths. We found that a Gaussian kernel with $\sigma = 0.11$ was optimal in the sense of balancing overconfidence with underconfidence, as seen in Figure 3. The resulting $F_i(z)$ is marginally consistent with uniformity for the overall sample and the faint subsample ($p = 0.07$ and 0.04 respectively), and entirely consistent for the bright subsample ($p = 0.89$). Given the median redshift (0.75) of the HUDF spectroscopic sample, this agrees well with the $\sigma = 0.065(1 + z)$ derived by Fernández-Soto et al. (2002) for the HDFN sample. We performed the same tests on the $p_i(z)$ output by BPZ for the HUDF sample and found the same result. Thus, smoothing $p_i(z)$ with a standard kernel may be an adequate substitute for modeling template noise in many situations.

Figure 3. The $F_i(z)$ plot for HDFN data using the BPZ code plus post-hoc $p_i(z)$ smoothing with a Gaussian kernel with $\sigma = 0.11$. All the resulting distributions are at least marginally consistent with uniformity, suggesting that smoothing $p_i(z)$ is a reasonable substitute for modeling template noise in BPZ.
3.2 EAZY

The EAZY code models template uncertainties as a function of rest-frame wavelength. This approach, called the template error function, has several virtues. First, there is strong physical motivation for assuming that template variance is a function of rest-frame wavelength (Hildebrandt et al. 2010). Second, any effect that depends on rest-frame wavelength propagates into $p_i(z)$ in a filter- and redshift-dependent way that cannot be fully mimicked by simply broadening $p_i(z)$. For example, consider a bimodal $p_i(z)$ with two similar well-separated peaks. The smoothing approach will blindly broaden both peaks, but the template error function may effectively broaden one peak much more than the other, according to the relevant rest-frame template uncertainties. Brammer et al. (2008) calibrate their template error function using a bright galaxy subsample with known spectroscopic redshifts, and provide the tools to recalibrate the error function if desired.

EAZY ships with the HDFN data, and Figure 4 shows the resulting $\hat{F}(c)$. For the entire sample (solid curve), there is only a modest amount of overconfidence, with a maximum deviation of 0.167 from the identity relation (78% of galaxies are within their 94% CI). According to the KS test, this meets standard criteria for statistical significance, but it is not overwhelming ($p = 0.0029$). Splitting into roughly equal subsamples by magnitude (dashed lines) reveals a substantial magnitude dependence, with overconfidence on the faint subsample and a bit of underconfidence on the bright subsample. To check whether this magnitude dependence could really be a redshift dependence, we split by redshift and find an even larger difference, in the sense of even more underconfidence for low-redshift galaxies than for bright galaxies. This implies that redshift could be the driving variable here; and as explained above, redshift variations in these tests could simply reflect the workings of the Bayesian prior on redshift. Thus, the magnitude dependence may be a feature rather than a flaw.

The relatively good performance of the overall sample and the lack of overconfidence in the bright subsample indicate that the template error function is generally serving its purpose in EAZY. Some overconfidence remains in the overall sample, but the good performance of the bright subsample suggests that this is not due to overconfidence in the templates. The remaining overconfidence may stem from other aspects of the algorithm or from the photometry; for example, unmodeled uncertainties in deblending or local background variations may affect faint galaxies more than bright galaxies and thereby fit this pattern. Investigating this hypothesis is beyond the scope of this paper, but we offer some recommendations in Section 5.

4 APPLICATION TO DEEP LENS SURVEY

The Deep Lens Survey (DLS; Wittman et al. 2002, 2006) is a ground-based 20 deg$^2$ BVRz survey that provides a counterpart to the HDFN and HUDF samples in terms of photometric uncertainties (larger from the ground) and filter set (DLS uses a minimal filter set to maximize the area covered). Schmidt & Thorman (2013, hereafter ST13) describe the DLS photometric redshifts and verify them using $\sim 10^4$ spectroscopic redshifts in a 1 deg$^2$ overlap region with the Prism Multi-object Survey (PRIMUS, Coil et al. 2010). ST13 tested the fraction of galaxies with spectroscopic redshift within six different CI and did not find systematic under- or overconfidence. However, our $\hat{F}(c)$ test on these data (Figure 5) reveals substantial overconfidence. The maximum departure from the desired line is nearly 0.30—only 53% of the galaxies are within their 82% CI—which a KS test deems overwhelmingly significant ($p < 10^{-300}$). Because of the large sample size—8719 galaxies after applying all the cuts applied by ST13—all departures visible in the DLS plots in this paper are significant.

The difference between the ST13 results and ours lies in the definition of CI. ST13 integrated around the highest peak and integrated symmetrically (in terms of area under $p_i(z)$) around that peak. Therefore, their CI are not HPD CI and should not be used to probe for overconfidence, as explained in Section 2.1.

Next, we probe for trends by splitting the data into subsamples. We found no visible difference in subsamples split by spectroscopic redshift and only small differences in subsamples split by galaxy spectral type (as determined by BPZ). We did find a modest trend with magnitude, with brighter galaxies showing more overconfidence (Figure 5, dashed curves), the same trend exhibited by BPZ on the HDFN and HUDF data. The sign of the magnitude trend, along with the lack of a redshift trend, again points to template noise. We therefore tried the $p_i(z)$ smoothing approach with a series of kernel widths, and in Figure 6 we plot the results using $\sigma = 0.055$ to illustrate the difficulty of defining an “optimal” kernel. The $\sigma = 0.055$ kernel shown here provides a quick way to remove most of the overconfidence, but leaves a discrepancy in the tails: 92.7% of galaxies are...
The $\hat{F}(F)$ plot for BPZ applied to DLS photometry of the PRIMUS spectroscopic sample shows overconfidence ($p < 10^{-300}$), preferentially in the brighter galaxies. This suggests the need to model template uncertainties within their 97.2% CI ($p \approx 10^{-15}$). A broader kernel would reduce the discrepancy in the tails, but would also introduce underconfidence elsewhere in the plot. Choosing the kernel by the sole criterion of minimizing the deviation between $\hat{F}(c)$ and $c$ may not be wise here because the bright subsample is already generally underconfident after smoothing, except in the tails. This suggests that template noise has been “modeled” about as well as it can be with a Gaussian smoothing kernel, and that an optimal kernel would include heavier tails. Note also the remaining overconfidence in the faint galaxies: this suggests that some photometric uncertainties remain unmodeled.

We also ran EAZY on the DLS data. The $\hat{F}(c)$ curve (Figure 7) is much better than for BPZ on the same data, but still departs significantly from the desired distribution in places (93.2% of galaxies are within their 96.8% CI, $p \approx 10^{-10}$). There is also some underconfidence where the empirical curves pass above the diagonal line in Figure 7. Although the EAZY and smoothed BPZ results have many dissimilarities (e.g., opposite magnitude trends), they both have a kink in the curve at $c \approx 0.8$: spectroscopic redshifts too frequently land very far from the $p(z)$ peak. This suggests non-Gaussian wings in the photometric uncertainties (from, e.g., deblending), the template uncertainties, or both. As discussed at the end of Section 3.2, these two sources of uncertainty can be decoupled by modeling the photometric uncertainties—including heavy tails—via simulations and repeat visits. Photometric redshift outliers can be substantially reduced simply by folding this heavetailed photometric model into a standard code such as BPZ (Wittman et al. 2007). Outliers remaining after this process are likely due to non-Gaussian template uncertainties.

Finally, we note that the DLS results presented so far...
could reflect differences between EAZY and BPZ other than the template error function:

- EAZY fits for a linear combination of templates with nonnegative coefficients, while BPZ considers templates separately (but interpolates between successive templates).
- BPZ has a type-dependent prior while EAZY does not. We ran EAZY on the DLS data using a prior as similar as possible to the ST13 BPZ prior, but due to this restriction the priors cannot be the same.
- We also used different templates. For EAZY we used the EAZY 1.0 templates 1–6, whereas the BPZ $p_i(z)$ we downloaded from the DLS data release website resulted from the more highly tuned ST13 templates.

We therefore tested whether the template error function is the primary factor responsible for reduced overconfidence in EAZY by running EAZY on the DLS data with the template error function turned off. We obtained an $F(c)$ distribution remarkably similar to the BPZ distribution shown in Figure 5, indicating that the template error function is indeed the primary cause of reduced overconfidence. Even with the template error function on, overconfidence rose when we restricted or eliminated EAZY’s linear combination feature. This suggests that two conditions must be met for appropriate confidence. First, the available templates must be able to match the gross features of the observations, either by combining many generic templates as EAZY does by default, or by tweaking a smaller number of templates as ST13 did before applying BPZ to the DLS. Second, spectral energy variations on finer wavelength scales can be modeled via the template error function.

5 SUMMARY AND DISCUSSION

Applications of photometric redshifts increasingly use each galaxy’s probability density function $p_i(z)$ rather than a single point estimate, and rightly so. However, tests of photometric redshift accuracy generally compare only the highest $p_i(z)$ peak to the spectroscopic redshift. This can lead to the false conclusion that a galaxy is a “catastrophic outlier” when in fact it is entirely predictable that a spectroscopic redshift sometimes falls on the second-highest peak, or in even lower-probability regions. We have defined a way to test all parts of $p_i(z)$ using the empirical cumulative distribution function $F$. This test determines the collective consistency of the $p(z)$ with the spectroscopic redshifts in way that probes specifically for the known failure mode of overconfidence. The test complements rather than replaces traditional tests because the latter are still necessary for measuring differences in terms of redshift.

We find that the $p(z)$ produced by BPZ (and presumably most other template methods) suffer from substantial overconfidence (e.g. only 32% of galaxies have true redshift within their 92% CI) because they do not account for variation in galaxy SEDs, so-called template noise. One code that does model template noise, EAZY, produced $p(z)$ with substantially less overconfidence, on each of two data sets that differ widely in terms of filter set and depth. Multiple independent arguments suggest that the improved performance is due to template uncertainty modeling rather than other differences between the codes. First, the most marked difference between BPZ and EAZY is with bright galaxies, for which template noise is a larger fraction of the uncertainty budget and for which priors should be relatively unimportant. Second, smoothing the BPZ $p_i(z)$—a crude model of the effect of template uncertainty—greatly reduces the BPZ overconfidence. Third, we turned the template uncertainty modeling off in EAZY and found overconfidence similar to BPZ.

The practical impact of this overconfidence is not immediately apparent from statements about the percentage of galaxies within a given CI. On an individual galaxy level, the practical impact is that the true redshift is not as well constrained as the $p_i(z)$ would indicate, and we can quantify this by specifying the amount of $p_i(z)$ smoothing required to eliminate the overconfidence (to the extent possible with smoothing rather than with more sophisticated modeling). We found that smoothing the BPZ, $p_i(z)$ with $\sigma = 0.09$ is adequate for the HUDF data set, and $\sigma \approx 0.06$ is adequate for the DLS; both numbers are consistent with the suggestion of $\sigma = 0.065(1 + z)$ by Fernández-Soto et al. (2002). The “optimal” kernel width may depend on filter set and other data details, as it reflects how much the SEDs vary from the templates at the rest wavelengths most heavily probed by the data, but $\sigma = 0.065(1 + z)$ seems to work with a variety of deep optical surveys. A fixed kernel does have the flexibility to work with varying numbers of filters, because it has appropriately less impact on the already-broad $p_i(z)$ produced by surveys with few filters.

For sets of galaxies such as a $z_p$ bin used in cosmic shear, the practical effect of overconfidence is that the true redshift distribution of the set is likely to be broader than the summed $p_i(z)$. For a hypothetical distribution of galaxies centered at $z = 1$ and with $\sigma_z = 0.2$, to first order the effect of smoothing with a $\sigma = 0.065(1 + z)$ kernel will be to broaden the bin to $\sigma_z = 0.24$. For weak lensing tomography with a next-generation imaging survey like LSST (Ivezic et al. 2008), Ma et al. (2006) report that the width of the redshift bins must be known to better than 0.01 to avoid substantial degradation of dark energy parameter constraints (where substantial degradation is defined as parameter constraints 1.5 times looser than in the case with perfect knowledge of the bin width). Accurate modeling of template uncertainty is therefore likely to be important in achieving the full potential of such surveys. In doing so, we must avoid the traditional anti-overconfidence tactic of multiplying the error bars by some factor in order to adopt a “conservative” estimate. Broadening $p(z)$ too much (underconfidence) results in overestimating the width of a redshift bin, which is equally harmful to much of the downstream science.

Although we have focused on smoothing the $p_i(z)$ as a convenient fix, template variance at a given rest wavelength propagates into different observed filters at different redshifts. Physical modeling of this process is therefore better than smoothing in principle. In either case, correcting this source of overconfidence helps expose other issues with the overall uncertainty budget. This is best illustrated by the comparison between Figures 5 and 6: an excess of spectroscopic redshifts very far from the $p(z)$ peaks is clearly visible.
in the upper right corner of Figure 6 but is masked by the much larger overconfidence trend in Figure 6. This pattern of outliers appears across a variety of codes and data sets, and points to the need to model heavy tails in probability distributions, including that of the underlying photometry. For large data sets and surveys we recommend decoupling photometric uncertainty from other issues by conducting targeted simulations and repeat observations to carefully calibrate the photometric uncertainty model. Better photometric uncertainty modeling will yield better photometric redshifts (Wittman et al. 2007), and in turn will enable photometric redshift confidence calibration to focus on physical modeling components such as templates. Because the photometric noise contribution is strongly magnitude-dependent, both types of modeling will be necessary to understand a survey over its full magnitude range.

The probability tails are potentially important for downstream science, because a small leakage of high-redshift galaxies into a low-redshift bin could add substantially to the naturally low lensing signal in that bin, while a small leakage in the other direction can substantially change the inferred luminosity function at high redshifts. Tracking these details requires tools other than the overconfidence test; for example, the leakage can be mapped with a \(z_s\) vs. \(z_p\) plot in which \(z_p\) is rendered as a cloud corresponding to \(p(z)\). In the end, the true redshift distribution of a photometric redshift bin may best be constrained by methods that are independent of any photometric redshift algorithm (e.g. the cross-correlation method, Newman 2008).

The overall uncertainty budget is strongly magnitude-dependent, so tests performed with bright spectroscopic samples should be interpreted carefully. Obtaining a truly representative spectroscopic subsample is difficult; for example, the PRIMUS spectroscopy in the DLS field has a 50% redshift success rate at \(R \approx 21.5\) (Cool et al. 2013) while the DLS photometry goes much deeper than that. Our conclusions regarding template uncertainty are robust, however, because the spectroscopic sample is most complete for bright galaxies, where template variance is the largest fraction of the uncertainty budget.

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REFERENCES

Benítez N., 2000, ApJ, 536, 571
Brammer G. B., van Dokkum P. G., Coppi P., 2008, ApJ, 686, 1503
Coe D., Benítez N., Sánchez S. F., Jee M., Bouwens R., Ford H., 2006, AJ, 132, 926
Cool A. L., et al., 2010, preprint, (arXiv:1011.4307)
Cool R. J., et al., 2013, ApJ, 767, 118
Dahlen T., et al., 2013, ApJ, 775, 93
Fernández-Soto A., Lanzetta K. M., Yahil A., 1999, ApJ, 513, 34
Fernández-Soto A., Lanzetta K. M., Chen H.-W., Levine B., Yahata N., 2002, MNRAS, 330, 889
Hildebrandt H., Wolf C., Benítez N., 2008, A&A, 480, 703
Hildebrandt H., et al., 2010, A&A, 523, AJ1
Hogg D. W., et al., 1998, AJ, 115, 1418
Ivezić Z., et al., 2008, preprint, (arXiv:0805.2366)
Ma Z., Hu W., Huterer D., 2006, ApJ, 636, 21
Mandelbaum R., et al., 2008, MNRAS, 386, 781
Newman J. A., 2008, ApJ, 684, 88
Schmidt S. J., Thorman P., 2013, MNRAS, 431, 2766
Wittman D., 2009, ApJ, 700, L174
Wittman D. M., et al., 2002, in Tyson J. A., Wolff S., eds, Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series Vol. 4836, Survey and Other Telescope Technologies and Discoveries. pp 73–82 (arXiv:astro-ph/0210118), doi:10.1117/12.457348
Wittman D., Dell’Antonio I. P., Hughes J. P., Margoniner V. E., Tyson J. A., Cohen J. G., Norman D., 2006, ApJ, 643, 128
Wittman D., Riechers P., Margoniner V. E., 2007, ApJ, 671, L109

MNRAS 000, 000–000 (0000)