Chapter 1

The Formation of Primordial Luminous Objects

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1.1 Introduction

The scientific belief that the universe evolves in time is one of the legacies of the theory of the Big Bang.

The concept that the universe has an history started to attract the interest of cosmologists soon after the first formulation of the theory: already Gamow (1948; 1949) investigated how and when galaxies could have been formed in the context of the expanding Universe.

However, the specific topic of the formation (and of the fate) of the first objects dates to two decades later, when no objects with metallicities as low as those predicted by primordial nucleosynthesis ($Z < 10^{-10} \sim 10^{-8} Z_{\odot}$) were found. Such concerns were addressed in two seminal papers by Peebles & Dicke (1968; hereafter PD68) and by Doroshkevich, Zel’Dovich & Novikov (1967; hereafter DZN67)$^1$, introducing the idea that some objects could have formed before the stars we presently observe.

$^1$ This paper is in Russian and we base our comments on indirect knowledge (e.g. from the Novikov & Zel’Dovich 1967 review).
Introduction

(i) Both PD68 and DZN67 suggest a mass of $\sim 10^5 M_\odot$ for the first generation of bound systems, based on the considerations on the cosmological Jeans length (Gamow 1948; Peebles 1965) and the possible shape of the power spectrum.

(ii) They point out the role of thermal instabilities in the formation of the proto-galactic bound object, and of the cooling of the gas inside it; in particular, PD68 introduces H$_2$ cooling and chemistry in the calculations about the contraction of the gas.

(iii) Even if they do not specifically address the occurrence of fragmentation, these papers make two very different assumptions: PD68 assumes that the gas will fragment into “normal” stars to form globular clusters, while DZN67 assumes that fragmentation does not occur, and that a single “super-star” forms.

(iv) Finally, some feedback effects as considered (e.g. Peebles & Dicke considered the effects of supernovae).

Today most of the research focuses on the issues when fragmentation may occur, what objects are formed and how they influence subsequent structure formation.

In these notes we will leave the discussion of feedback to lecture notes by Ferrara & Salvaterra and by Madau & Haardt in this same book and focus only on the aspects of the formation of the first objects.

The advent of cosmological numerical hydrodynamics in particular allow a fresh new look at these questions. Hence, these notes will touch on aspects of theoretical cosmology to chemistry, computer science, hydrodynamics and atomic physics. For further reading and more references on the subject we refer the reader to other relevant reviews such as Barkana & Loeb 2001, and more recently Ciardi & Ferrara 2004, Glover 2004 and Bromm & Larson 2004.

In this notes, we try to give a brief introduction to only the most relevant aspects. We will start with a brief overview of the relevant cosmological concepts in section 2, followed by a discussion of the properties of primordial material (with particular emphasis to its cooling and its chemistry) in section 3. We will then review the technique and the results of numerical simulations in sections 4 and 5: the former will deal with detailed 3D simulations of the formation of gaseous clouds which are likely to transform into luminous objects, while the latter will examine results (mostly from 1D codes) about the modalities of such transformation. Finally, in section 6 we will critically discuss the results of the previous sections, examining their consequences and comparing them to our present knowledge of the universe.
1.2 Physical cosmology

In the following of this notes we will adopt the modern physical description of the universe (dating back to at least 1920), based upon the “cosmological principle”, which affirms that on cosmological scales the distributions of matter and energy should be homogeneous and isotropic, whose metric is the Robertson-Walker metric. Although derived from mainly philosophical arguments, the cosmological principle is also supported by observations such as the isotropy of the CMB (e.g. Wu et al. 1999).

We will also make some additional, general assumptions which are quite common in present-day cosmology, and which are believed to be the best explanations for a series of observations. That is:

(i) The cosmological structures we observe at present (galaxies, clusters of galaxies etc.) formed because of gravitational instability of pre-existing, much shallower fluctuations;

(ii) Most of the matter in the universe is in the form of “Cold Dark Matter”, that is, of some kind of elementary particle (or particles) that has not been discovered at present. Cold Dark Matter particles are assumed to have a negligibly small cross section for electromagnetic interactions (i.e. to be dark), and to move at non-relativistic speeds (i.e. to be cold).

1.2.1 Fluctuations in the Early Universe

1.2.1.1 Inflation

Inflation is a mechanism which was first proposed by Guth (1981) and (in a different way) by Starobinsky (1979, 1980), and has since been included in a number of different “inflationary theories” (see Guth 2004 for a review upon which we base this paragraph).

The basic assumption of inflationary models is the existence of states with negative pressure; a typical explanation is that some unidentified kind of scalar field (commonly referred to as inflaton) temporarily keeps part of the universe in a “false vacuum” state, in which the energy density must be approximately constant (that is, it can not “diluted” by expansion) at some value $\rho_f c^2$, implying a negative pressure $p_f = -\rho_f c^2$. Inserting these two expressions in the first Friedmann cosmological equation

$$\ddot{a}(t) = -\frac{4\pi}{3} G \left( \rho + \frac{3 P}{c^2} \right) a(t)$$

(1.1)

(where $a(t)$ is the scale factor at cosmic time $t$) it is easy to derive that the considered region expands exponentially: $a(t) \propto e^{t/t_f}$ with $t_f = (8\pi G \rho_f/3)^{-1/2}$; the epoch in which the universe undergoes such exponential expansion is called inflation.
Figure 1.1. Effect of universe geometry on the observed angular size of fluctuations in the CMBR. If the universe is closed (left panel) “hot spots” appear larger than actual size; if the universe is flat (middle panel), “hot spots” appear with their actual size; if the universe is open, “hot spots” appear smaller than their actual size.

If $\rho_f$ is taken to be at about the grand unified theory scale, we get $t_f \sim 10^{-38}$ s, corresponding to an Hubble length of $c t_f \sim 10^{-28}$ cm; if the inflationary phase is long enough (a lower limit for this is about 65 e-folding times, corresponding to an expansion by a factor $\sim 10^{28}$), it smooths the metric, so that the expanding region approaches a de Sitter flat space, regardless of initial conditions. Furthermore, when inflation ends, the energy stored in the inflaton field is finally released, thermalizes and leads to the hot mixture of particles assumed by the standard big bang picture.

Inflation helps explaining several fundamental observations, which were just assumed as initial conditions in non inflationary models:

(i) The Hubble expansion: repulsive gravity associated with false vacuum is exactly the kind of force needed to set up a motion pattern in which every two particles are moving apart with a velocity proportional to their distance.

(ii) The homogeneity and isotropy: in “classical” cosmology the remarkable uniformity of the Cosmic Microwave Background Radiation (CMBR) cannot be “explained”, because different regions of the present CMBR sky never were causally connected. Instead, in inflationary models the whole CMBR sky was causally connected before inflation started, and uniformity can be established at that time.
(iii) The flatness problem: a flat Friedman-Robertson-Walker model universe (i.e. with \( \Omega(t) = \rho_{\text{tot}}(t)/\rho_{\text{crit}}(t) = 1 \), where \( \rho_{\text{tot}}(t) \) is the cosmological mean density, including the “dark energy” term, and \( \rho_{\text{crit}}(t) = 3H(t)^2/8\pi G \), \( H(t) \equiv \dot{a}(t)/a(t) \) being the Hubble parameter) always remains flat, but if at an early time \( \Omega \) was just slightly different from 1, the difference should have grown as fast as \( (\Omega - 1) \propto t^{2/3} \). All the observational results (e.g. the Bennet et al. 2003 WMAP result \( \Omega_0 = 1.02 \pm 0.02 \)) show that at present \( \Omega \) is quite close to 1, implying that at the Planck time (\( t \approx 10^{-43} \) s) the closeness was really amazing. Inflation removes the need for this incredibly accurate fine tuning, since during an inflationary phase \( \Omega \) is driven towards 1 as \( (\Omega - 1) \propto e^{-2Ht} \).

(iv) The absence of magnetic mono-poles: grand unified theories, combined with classical cosmology, predict that the universe should be largely dominated by magnetic mono-poles, which instead have never been observed. Inflation provides an explanation, since it can dilute the mono-pole density to completely negligible levels.

(v) the anisotropy properties of the CMBR radiation: inflation provides a prediction for the power spectrum of fluctuations, which should be generated by quantum fluctuations and nearly scale-invariant, a prediction in close agreement with the WMAP results (see the next subsection).

A peculiar property of inflation is that most inflationary models predict that inflation does not stop everywhere at the same time, but just in localized “patches” in a succession which continues eternally; since each “patch” (such as the one we would be living in) can be considered a whole universe, it can be said that inflation produces an infinite number of universes.

### 1.2.1.2 Primordial fluctuation evolution - Dark Matter

Inflation predicts the statistical properties of the density perturbation field, defined as

\[
\delta(x) = \frac{\rho(r) - \bar{\rho}}{\bar{\rho}}
\]

(1.2)

where \( r \) is the proper coordinate, \( x = r/a \) is the comoving coordinate, and \( \bar{\rho} \) is the mean matter density.

In fact, if we look at the Fourier components in terms of the comoving wave-vectors \( k \)

\[
\delta(k) = \int \delta(x) \ e^{-ikx} \ d^3x
\]

(1.3)

the inflationary prediction is that the perturbation field is a Gaussian random field, that the various \( k \) modes are independent, and that the power
Physical cosmology

Figure 1.2. Schematic shape of the DM power spectrum after accounting for the effects of free streaming and of the processing that takes place for fluctuations entering the horizon before $t_{\text{eq}}$.

spectrum $P(k)$ (where $k \equiv |\mathbf{k}|$) is close to scale-invariant, i.e., it is given by a power law

$$P(k) \equiv \langle |\delta_k|^2 \rangle \propto k^{n_s}, \quad \text{with } n_s \simeq 1. \quad (1.4)$$

This prediction applies to the primordial power spectrum; in order to make comparisons with observations, we need to include effects which subsequently modified its shape:

(i) Free streaming: the dark matter particles are in motion, and since they are believed to interact only through gravity, they freely propagate from overdense to underdense regions, wiping out the density perturbations. However, at any given cosmic time $t$, this will affect only wavelengths smaller than the free streaming length, i.e. the proper distance $l_{\text{FS}}(t)$ which a dark matter particle can have travelled in time $t$. It can be shown (see e.g. Padmanabhan 1993, §4.6) that this scale depends primarily on the epoch $t_{\text{nr}}$ when the dark matter particles become non-relativistic: before that epoch they move at the speed of light, and can cover a proper distance $l_{\text{FS}}(t_{\text{nr}}) \sim 2ct_{\text{nr}}$, corresponding to a comoving distance $\lambda_{\text{FS}} = (a_0/a_{\text{nr}})2ct_{\text{nr}}$; after becoming non-relativistic, particle motions become much slower than cosmological expansion, and the evolution after $t_{\text{nr}}$ only increases $\lambda_{\text{FS}}$ by a relatively small factor. In turn, $t_{\text{nr}}$ depends primarily on the mass $m_{\text{DM}}$
of dark matter particles, so that
\[ \lambda_{FS} \sim 5 \times 10^{-3} (\Omega_{DM} h^2)^{1/3} \left( \frac{m_{DM}}{1 \text{ GeV}} \right)^{-4/3} \text{pc} \]  
(1.5)
corresponding to a mass scale
\[ M_{FS} \sim 6 \times 10^{-15} (\Omega_{DM} h^2) \left( \frac{m_{DM}}{1 \text{ GeV}} \right)^{-4} M_\odot. \]  
(1.6)

Since the most favoured candidates for the DM particles are Weakly Interacting Massive Particles (WIMPs) with mass between 0.1 and 100 GeV, we probably have \( M_{FS} \lesssim 10^{-8} M_\odot \). Some super-symmetric theories (e.g. Schwartz et al. 2001, and more recently Green, Hohmann & Schwarz 2004) instead point towards \( M_{FS} \sim 10^{-6} M_\odot \), and Diemand, Moore & Stadel (2005) used this result in order to argue that the first structure which formed in the early Universe were Earth-mass dark-matter haloes (but see also Zhao et al. 2005 for criticism about this result);

(ii) Growth rate changes: in general, perturbations grow because of gravity; however, the details of the process change with time, leaving an imprint on the final processed spectrum. The time \( t_{eq} \) when the matter density becomes larger than the radiation density is particularly important: before \( t_{eq} \), perturbations with \( \lambda \) larger than the Hubble radius \( c/H(t) \) grow as \( \delta \propto a^2 \), while the growth of smaller perturbations is almost completely suppressed; after \( t_{eq} \) both large and small fluctuations grow as \( \delta \propto a \). Because of these differences, the size of the Hubble radius at \( t_{eq} \), \( \lambda_{eq} \approx 13 (\Omega h^2)^{-1} \text{Mpc} \) (in terms of mass, \( M_{eq} \approx 3.2 \times 10^{14} (\Omega h^2)^{-2} M_\odot \)) separates two different spectral regimes. In the wavelength range \( \lambda_{FS} \leq \lambda \leq \lambda_{eq} \) the growth of fluctuations pauses between the time they enter the Hubble radius and \( t_{eq} \). As a result the slope of the processed spectrum is changed, and \( P(k) \propto k^{-4} \approx k^{-3} \). Instead, at scales \( \lambda > \lambda_{eq} \) all the fluctuations keep growing at the same rate at all times, and the shape of the power spectrum remains similar to the primordial one, \( P(k) \propto k^{n_s} \approx k^1 \).

WMAP (Bennett et al. 2003) measured the spectral index, obtaining \( n_s = 0.99 \pm 0.04 \), and did not detect deviations from gaussianity, both results in agreement with inflationary predictions.

This kind of spectrum, in which fluctuations are typically larger on small scales, leads naturally to hierarchical structure formation, since small-scale fluctuations are the first to become non-linear (i.e. to reach \( \delta \sim 1 \)), collapse and form some kind of astronomical object. It is also worth remarking that the very first objects, coming from the highest peaks of \( \delta(x) \), are typically located where modes \( \delta(k) \) of different wavelength make some kind of “constructive interference”: the very first objects are likely to
be on top of larger ones, and they are likely to be clustered together, rather than uniformly distributed. For this reason, it is also very likely that the halos where these objects formed have long since been incorporated inside larger objects, such as the bulges of $M_*$ galaxies or the cD galaxy at the centre of galaxy clusters (see e.g. White & Springel 1999).

### 1.2.1.3 Fluctuation evolution - Baryons

Before the equivalence epoch $t_{eq}$ the baryons evolve in the same way as dark matter. Instead, in the matter dominated era they behave differently: we mentioned that all the dark matter fluctuations which were not erased by free streaming grew as $\delta \propto a \propto (1 + z)^{-1}$, but this does not apply to baryons. In fact, baryons decouple from radiation only at $t_{dec}$, significantly later than $t_{eq}$ (we remind that $1 + z_{eq} \sim 10^4$, while $1 + z_{dec} \simeq 10^3$). The persistence of the coupling with radiation prevents the growth of baryonic fluctuations on all scales; even worse, on relatively small scales all the fluctuations in the baryonic component are erased through a mechanism similar to free streaming. Such effect takes place below the so-called Silk scale (Silk 1968), which is given by the average distance that the photons (and the baryons coupled with them) can diffuse before $t = t_{dec}$; this translates into a comoving distance

$$\lambda_S \simeq 3.5 \left( \frac{\Omega}{\Omega_b} \right)^{1/2} (\Omega h^2)^{-3/4} \text{ Mpc}$$

(1.7)
Figure 1.4. Angular CMB power spectrum of temperature (top panel) and temperature-polarization (bottom panel) as obtained by the WMAP satellite (from Bennett et al. 2003). The line shows the best-fit with a $\Lambda$CDM model, and grey areas represent cosmic variance.

(where $\Omega_b$ is the baryonic contribution to $\Omega$) and encloses a mass

$$M_S \simeq 6.2 \times 10^{12} \left( \frac{\Omega}{\Omega_b} \right)^{3/2} (\Omega h^2)^{-5/4} M_\odot. \quad (1.8)$$

This result was a major problem for cosmology before the existence
of dark matter started to be assumed: it implies that in a purely baryonic universe there should be no structures on a scale smaller than that of galaxy clusters (if $\Omega = \Omega_b \simeq 0.1$). Furthermore, even fluctuations which were not erased can grow only by a factor $1 + z_{\text{dec}}$ between decoupling and present time, and this is not enough to take a typical CMBR fluctuation (of amplitude $\delta \sim 10^{-5}$) into the non-linear regime. The introduction of Cold Dark Matter solved this problem, since after recombination the baryons are finally free to fall inside dark matter potential wells, whose growth was unaffected by the photon drag.

It can be found that after decoupling from radiation, the baryonic fluctuations quickly “reach” the levels of dark matter fluctuations, evolving as

$$\delta_b = \delta_{\text{DM}} \left(1 - \frac{1 + z}{1 + z_{\text{dec}}} \right),$$

so that the existing dark matter potential wells “regenerate” baryonic fluctuations, including the ones below the Silk scale.

This result is correct as long as pressure does not play a role, that is for objects with a mass larger than the cosmological Jeans mass $M_J \propto T^{3/2}/\rho^{1/2}$. Such mass behaves differently at high and low redshift. Before a redshift $z_{\text{Compton}} \simeq 130$ we have that the temperature of the baryons is still coupled to that of the CMB because of Compton scattering of radiation on the residual free electrons; for this reason, $T_b(z) \simeq T_{\text{CMB}}(z) \propto (1 + z)$, and as $\rho(z) \propto (1 + z)^3$ the value of $M_J$ is constant:

$$M_J(z) \simeq 1.35 \times 10^5 \left(\frac{\Omega_m h^2}{0.15}\right)^{-1/2} M_\odot \quad (\text{for } z_{\text{dec}} \lesssim z \lesssim z_{\text{Compton}})$$

(1.10)

where $\Omega_m = \Omega_b + \Omega_{\text{DM}}$ is the total matter density (baryons plus dark matter). At lower redshifts the Jeans mass evolves as

$$M_J(z) \simeq 5.7 \times 10^3 \left(\frac{\Omega_m h^2}{0.15}\right)^{-1/2} \left(\frac{\Omega_b h^2}{0.022}\right)^{-3/5} \left(\frac{1 + z}{10}\right)^{3/2} M_\odot \quad (\text{for } z \lesssim z_{\text{Compton}}).$$

(1.11)

1.2.2 From fluctuations to cosmological structures

1.2.2.1 Non-linear evolution

When the density contrast $\delta$ becomes comparable to unity, the evolution becomes non-linear, and the Fourier modes no more evolve independently. The easiest way to study this phase is through the “real space” $\delta(x)$ (rather
than its Fourier components $\delta(k)$, considering the idealized case of the collapse of a spherically symmetric overdensity, and in particular the collapse of a top-hat fluctuation. It is well known that, through some simple further assumption (such as that “spherical shells do not cross”), the time evolution of the radius $R$ of a top-hat perturbation (see e.g. Padmabhan 1993, §8.2 for the treatment of a slightly more general case) can be written down as

$$R(t) = \frac{R_i}{2} \frac{1 + \delta_i}{\delta_i - (\Omega_i^{-1} - 1)} [1 - \cos(\theta(t))]$$

(1.12)

where $R_i = R(t_i)$, $\delta_i = \delta(t_i)$ and $\Omega_i = \Omega(t_i)$ are the “initial conditions” for the fluctuation evolution, and $\theta$ is defined by the equation

$$\frac{\theta(t) - \sin(\theta(t))}{2H_i \Omega_i^{1/2} \delta_i - (\Omega_i^{-1} - 1)^{3/2}} \approx t$$

(1.13)

where again $H_i$ is the Hubble parameter at $t_i$, and the last approximate equality is valid only as long as $\delta_i \ll 1$ (that is, a sufficiently early $t_i$ must be chosen). The fluctuation radius $R$ reaches a maximum $R_{ta}$ at the so-called turn-around epoch (when $\theta = \pi$) when the overdense region finally detaches itself from the Hubble flow and starts contracting. However, while eq. (1.12) suggests an infinite contraction to $R = 0$ (when $\theta = 2\pi$), the vi-
olent relaxation process (Lynden-Bell 1967) prevents this from happening, leading to a configuration in virial equilibrium at $R_{\text{vir}} \approx R_{\text{ta}}/2$.

Here, we summarise some well known, useful findings of this model. First of all, combining the evolution of the background density evolution and of eq. \ref{eq:1.12} it is possible to estimate the density contrast evolution

$$\delta(t) = \frac{9}{2} \left( \frac{\theta - \sin \theta}{1 - \cos \theta} \right)^3 - 1$$  \hspace{1cm} (1.14)

which leads to some noteworthy observation, such as that the density contrast at turn-around is $\delta_{\text{ta}} = (9\pi^2/16) - 1 \approx 4.6$, which at virialization becomes $\delta_{\text{vir}} = \Delta_c$, where it can be usually assumed that $\Delta_c \approx 18\pi^2$ but sometimes higher order approximations are necessary, such as the one in Bryan & Norman 1998,

$$\Delta_c = 18\pi^2 + 82(1 - \Omega_m^z) - 39(1 - \Omega_m^z)^2$$  \hspace{1cm} (1.15)

with

$$\Omega_m^z = \Omega_m(1 + z)^3 \Omega_\Lambda + \Omega_k(1 + z)^2$$  \hspace{1cm} (1.16)

where $\Omega_\Lambda$ is the dark energy density, and $\Omega_k = 1 - \Omega_m - \Omega_\Lambda$ is the curvature.

From this, it is possible to estimate the virial radius

$$R_{\text{vir}} \approx 0.784 \left( \frac{M}{10^8 M_\odot} \right)^{1/3} \left( \frac{\Omega_m}{\Omega_m^z} \right)^{-1/3} \left( \frac{\Delta_c}{18\pi^2} \right)^{-1/3} \left( \frac{1 + z}{10} \right)^{1/2} h^{-1} \text{kpc}$$  \hspace{1cm} (1.17)

the circular velocity for such an halo

$$V_{\text{circ}} \approx 23.4 \left( \frac{M}{10^8 M_\odot} \right)^{1/3} \left( \frac{\Omega_m}{\Omega_m^z} \right)^{-1/6} \left( \frac{\Delta_c}{18\pi^2} \right)^{1/2} \left( \frac{1 + z}{10} \right)^{1/2} \text{km s}^{-1}$$  \hspace{1cm} (1.18)

and the virial temperature

$$T_{\text{vir}} \approx 19800 \left( \frac{M}{10^8 M_\odot} \right)^{2/3} \left( \frac{\Omega_m}{\Omega_m^z} \right)^{1/3} \left( \frac{\Delta_c}{18\pi^2} \right) \left( \frac{1 + z}{10} \right) \left( \frac{\mu}{0.6} \right) \text{K}$$  \hspace{1cm} (1.19)

1.2.2.2 The Press-Schechter formalism

The simple top-hat model is at the core of the so-called Press-Schechter formalism (Press & Schechter 1974, but see also the contribution by Sommerville in this same book), which predicts the density of virialized halos of a given mass at a given redshift. This model assumes that the distribution of the smoothed density field $\delta_M$ (where $M$ is the mass scale of the smoothing) at a certain redshift $z_0$ is Gaussian with a variance $\sigma_M$, so that the probability of having $\delta_M$ larger than a given $\delta_{\text{crit}}$ is

$$P(\delta_M > \delta_{\text{crit}}) = \int_{\delta_{\text{crit}}}^{\infty} \frac{1}{(2\pi)^{1/2} \sigma_M} e^{-\frac{x^2}{2\sigma_M^2}} dx;$$  \hspace{1cm} (1.20)
Figure 1.6. Characteristic mass of $1\sigma$ (bottom solid curve), $2\sigma$ (middle solid curve) and $3\sigma$ (top solid curve) collapsing halos as a function of redshift. These were obtained from the Eisenstein & Hu (1999) power spectrum, assuming an $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$ cosmology. The dashed curves show the minimum mass which is required for the baryons to be able to cool and collapse (see next section) in case of pure atomic cooling (upper curve) and of molecular cooling (lower curve). (from Barkana & Loeb 2001).

a common choice is $z_0 = 0$, requiring $\delta_M$ to be estimated through a purely linear evolution of the primordial power spectrum.

The Press-Schechter model then chooses $\delta_{\text{crit}} = \delta_{\text{crit}}(z)$ (but it is also possible to assume a constant $\delta_{\text{crit}}$ and make $\sigma_M$ a function of redshift; see e.g. Viana & Liddle 1996) and assumes that this probability (multiplied by a factor of 2 - see Bond et al. 1991 for an explanation of this extra factor) also gives the fraction mass which at a redshift $z$ is inside virialized halos of mass $M$ or larger. This can be differentiated over $M$ in order to get the
Halo mass functions at several redshifts (from bottom left to bottom right, $z = 30$, $z = 20$, $z = 10$, $z = 5$ and $z = 0$, respectively). The assumed power spectrum and cosmology are the same as in fig. 1.6 (from Barkana & Loeb 2001).

mass distribution at each given redshift

$$
\frac{dn}{dM} = \frac{2}{(2\pi)^{1/2}} \frac{\rho_m}{M} \frac{d\ln(1/\sigma_M)}{dM} \frac{\delta_{\text{crit}}(z)}{\sigma_M} e^{-\frac{\delta_{\text{crit}}(z)^2}{2\sigma_M^2}} \quad (1.21)
$$

In this way, the abundance of halos is completely determined through the two functions $\delta_{\text{crit}}(z)$ and $\sigma_M$. The first one is commonly written as $\delta_{\text{crit}}(z) = \delta_0 / D(z)$, where $D(z)$ is the growth factor ($D(z) \simeq (1 + z)^{-1}$ for Einstein-de Sitter models; see Peebles 1993 for a more general expression), coming from cosmology; instead $\delta_0$ is usually taken to be 1.686, since the top-hat model predicts that an object virializes at the time when the linear theory estimates its overdensity at $\delta = 1.686$. Instead $\sigma_M$ depends from the power spectrum; for example, figures 1.6 and 1.7 are based on the Eisenstein & Hu (1999) results$^2$.

$^2$ The authors of this paper also provide some very useful codes for dealing
1.3 Primordial gas properties

1.3.1 Cooling

The typical densities reached by the gas after virialization (of the order of 
\[ n_B \equiv \rho_B/m_H \sim 0.01 \Omega_b[(1 + z_{\text{vir}})]/10^3 \text{cm}^{-3} \] are far too low for the gas to condense and form an object like a star. The only way to proceed further in the collapse and in the formation of luminous objects is to remove the gas thermal energy through radiative cooling.

For this reason, cooling processes are important in determining where, when and how the first luminous objects will form.

In Fig. 1.8 it is possible to see that the cooling of primordial (i.e. metal-free) atomic gas at temperatures below \( \sim 10^4 \text{ K} \) is dramatically inefficient, because in that temperature range the gap between the fundamental and the lowest excited levels (\( \simeq 10.2 \text{ eV for H atoms} \)) is so much larger than the thermal energy \( \sim k_B T < \sim 1 \text{ eV} \) that very few atoms get collisionally excited, and very few photons are emitted through the corresponding de-excitations.

This is important because in all hierarchical scenarios the first objects to virialize are the smallest ones, and such halos have the lowest virial temperatures. If the primordial gas were completely atomic, the first luminous objects would probably form relatively late (\( z < \sim 10 - 15 \)), in moderately massive halos (\( M \sim 10^8 M_\odot \)) with \( T_{\text{vir}} > \sim 10^4 \text{ K} \).

However, it is also possible to see from Fig. 1.8 that the presence of molecules in small amounts \( f_{\text{H}_2} \equiv 2 n_{\text{H}_2}/(n_{\text{H}} + 2 n_{\text{H}_2}) > 5 \times 10^{-4} \); the dashed curve in Fig. 1.8 was obtained assuming \( f_{\text{H}_2} = 10^{-3} \) can dramatically affect the cooling properties of primordial gas at low temperatures, making low mass halos virializing at high redshift (\( z > \sim 20 \)) the most likely sites for the formation of the first luminous objects.

1.3.2 Molecular cooling

In the current scenario for the formation of primordial objects, the most relevant molecule is \( \text{H}_2 \). The only reason for this is the high abundance of \( \text{H}_2 \) when compared with all other molecules. In fact, the radiating properties of an \( \text{H}_2 \) molecule are very poor: because of the absence of a dipole moment, radiation is emitted only through weak quadrupole transitions. In addition, the energy difference between the \( \text{H}_2 \) ground state and the lowest \( \text{H}_2 \) excited roto-vibrational levels is relatively large (\( \Delta E_{01}/k_B \gtrsim 200 \text{ K} \)), between the fundamental and the first excited level; however, such transition is prohibited by quadrupole selection rules, and the lowest energy gap for a quadrupole transition is \( \Delta E_{02}/k_B \approx 510 \text{ K} \)), further reducing the cooling efficiency at low temperatures.

with the power spectrum and the Press-Schechter formalism at the web page
http://background.uchicago.edu/~whu/transfer/transferpage.html
Figure 1.8. Cooling rate per atomic mass unit of metal-free gas, as a function of temperature. The solid line shows assumes the gas to be completely atomic (the two peaks correspond to H and He excitations, while the high temperature tail is dominated by free-free processes) and drops to about zero below $T \sim 10^4$ K; the dashed line shows the contribution of a small ($f_{H_2} = 10^{-3}$) fraction of molecular hydrogen, which contributes extra-cooling in the range $100 \text{ K} \lesssim T \lesssim 10^4$ K (from Barkana & Loeb 2001).

Apart from H$_2$ the most relevant molecular coolants are HD and LiH. In the following, we will briefly list the cooling rates (mainly taken from Galli & Palla 1998, hereafter GP98; see also Hollenbach & McKee 1979, Lepp & Shull 1983, 1984 and Martin et al. 1996, Le Bourlot et al. 1999, Flower et al. 2000) of H$_2$ and of the other two possibly relevant species$^3$.

We note that Bromm et al. 1999 included HD cooling in some of their

$^3$ In Fig. 1.10 the cooling rate for H$_2^+$ is shown, too. But while it is still marginally possible that HD or LiH cooling can play some kind of role in the primordial universe, this is much more unlikely for H$_2^+$, mainly because its under-abundance with respect to H$_2$ is always much larger than the difference in the cooling rates; for this reasons we choose to omit a detailed discussion of H$_2^+$ cooling.
1.3.2.1 \( \text{H}_2 \) cooling rate

The \( \text{H}_2 \) cooling rate per molecule \( \Lambda_{\text{H}_2}(\rho, T) \) can be conveniently expressed in the form:

\[
\Lambda_{\text{H}_2}(\rho, T) = \frac{\Lambda_{\text{H}_2, \text{LTE}}(T)}{1 + \frac{\Lambda_{\text{H}_2, \text{LTE}}(T)}{n_{\text{H}} \Lambda_{\text{H}_2, \rho \rightarrow 0}(T)}}
\]

(1.22)

where \( \Lambda_{\text{H}_2, \text{LTE}}(T) \) and \( n_{\text{H}} \Lambda_{\text{H}_2, \rho \rightarrow 0} \) are the high and low density limits of the cooling rate (which apply at \( n \gtrsim 10^4 \text{ cm}^{-3} \) and at \( n \lesssim 10^2 \text{ cm}^{-3} \), respectively).
The high density (or LTE, from the Local Thermal Equilibrium assumption which holds in these conditions) limit of the cooling rate per H$_2$ molecule is given by Hollenbach & McKee (1979):

$$\Lambda_{H_2, \text{LTE}}(T) = \frac{9.5 \times 10^{-22} T_3^{3.76}}{1 + 0.12 T_3} e^{-(4.13 T_3)^3} + 3 \times 10^{-24} e^{-\frac{0.55}{T_3}} + 6.7 \times 10^{-19} e^{-\frac{5.86}{T_3}} + 1.6 \times 10^{-18} e^{-\frac{4.87}{T_3}} \text{ erg s}^{-1} \ (1.23)$$

where $T_3 \equiv T/(1000 \text{ K})$. Note that the first row in the formula accounts for rotational cooling, while the second row accounts for the first two vibrational terms.

For the low density limit, GP98 found that in the relevant temperature range ($10 \text{ K} \leq T \leq 10^4 \text{ K}$) the cooling rate $\Lambda_{H_2, \rho \rightarrow 0}$ is independent from density, and is well approximated by

$$\log \Lambda_{H_2, \rho \rightarrow 0}(T) \simeq -103.0 + 97.59 \log T - 48.05(\log T)^2 + 10.80(\log T)^3 - 0.9032(\log T)^4 \ (1.24)$$

where $T$ and $\Lambda_{H_2, \rho \rightarrow 0}$ are expressed in $K$ and $\text{erg cm}^{-3}\text{s}^{-1}$, respectively.

Note that even if both $\Lambda_{H_2, \text{LTE}}$ and $\Lambda_{H_2, \rho \rightarrow 0}$ do not depend on density, $\Lambda_{H_2}(\rho, T)$ is independent of $\rho$ only in the high density limit.

1.3.2.2 HD and LiH cooling rates

The cooling rates of HD and of LiH are more complicated (see Flower et al. 2000 for HD and Bogleux & Galli 1997 for LiH), but in the low density limit (and in the temperature range $10 \text{ K} \leq T \leq 1000 \text{ K}$) it is possible to use the relatively simple expressions given by GP98.

For HD, we have that the low density limit of the cooling rate per molecule, $\Lambda_{\text{HD, \rho \rightarrow 0}}$, is:

$$\Lambda_{\text{HD, \rho \rightarrow 0}}(T) \simeq 2 \gamma_{10} E_{10} e^{-\frac{E_{10}}{k_B T}} + (5/3) \gamma_{21} E_{21} e^{-\frac{E_{21}}{k_B T}} \ (1.25)$$

where $E_{10}$ and $E_{21}$ are the energy gaps between HD levels 1 and 0 and levels 2 and 1, respectively; they are usually expressed as $E_{10} = k_B T_{10}$ and $E_{21} = k_B T_{21}$, with $T_{10} \simeq 128 \text{ K}$ and $T_{21} \simeq 255 \text{ K}$.

$\gamma_{10}$ and $\gamma_{21}$ are the approximate collisional de-excitation rates for the 1-0 and 2-1 transitions, and are given by

$$\gamma_{10} \simeq 4.4 \times 10^{-12} + 3.6 \times 10^{-13} T^{0.77} \ (1.26)$$

$$\gamma_{21} \simeq 4.1 \times 10^{-12} + 2.1 \times 10^{-13} T^{0.92} \ (1.27)$$

where we use the numerical value of $T$ (in Kelvin) and the rates are expressed in $\text{cm}^3\text{s}^{-1}$.
Figure 1.10. Comparison of the low density \((n \lesssim 10 \text{ cm}^{-3})\) cooling rates per molecule of several molecular species, in particular \(\text{H}_2\), HD and LiH (from GP98). Note that at \(T \sim 100\text{ K}\) both HD and LiH molecules are more than \(10^3\) times more efficient coolants than \(\text{H}_2\) molecules, but this difference is believed to be compensated by the much higher \(\text{H}_2\) abundance (see \textit{e.g.}, Bromm \textit{et al.} 2002). The plot also shows the cooling due to \(\text{H}-\text{H}_2^+\) and \(e^-\text{-H}_2^+\) collisions, but these contributions are never important because of the very low \(\text{H}_2^+\) abundance.

For LiH instead we have that the same density limit of the cooling rate per molecule, \(\Lambda_{\text{LiH}, \rho \rightarrow 0}\) can be fitted by:

\[
\log_{10}(\Lambda_{\text{LiH}, \rho \rightarrow 0}) = c_0 + c_1 \log_{10} T + c_2(\log_{10} T)^2 + c_3(\log_{10} T)^3 + c_4(\log_{10} T)^4
\]

(1.28)

where \(c_0 = -31.47\), \(c_1 = 8.817\), \(c_2 = 4.144\), \(c_3 = 0.8292\) and \(c_4 = -0.04996\), assuming that \(T\) is expressed in Kelvin, and that \(\Lambda_{\text{LiH}, \rho \rightarrow 0}\) is expressed in \(\text{erg cm}^3\text{s}^{-1}\).
1.3.2.3 Cooling at high densities: Collision Induced Emission

During the formation of a protostar an important role is played by the so called Collision-Induced Emission (CIE; very often known as Collision-Induced Absorption, or CIA), a process which requires pretty high densities \( n = \rho / m_H \gtrsim 10^{13} - 10^{14} \text{ cm}^{-3} \) to become important (see Lenzuni et al. 1991, Frommold 1993, Ripamonti & Abel 2004). In fact, Collision-Induced Emission takes place when a collision between two \( \text{H}_2 \) molecules (or \( \text{H}_2 \) and \( \text{H} \), or \( \text{H}_2 \) and \( \text{He} \), or even \( \text{H} \) and \( \text{He} \)) takes place: during the collision, the interacting pair briefly acts as a “super-molecule” with a nonzero electric dipole, and a much higher probability of emitting (CIE) or absorbing (CIA) a photon than an unperturbed \( \text{H}_2 \) molecule (whose dipole
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is 0). Because of the very short durations of the collisions, \(( \lesssim 10^{-12} \text{ s})\), such mechanism can compete with “normal” emission only in high density environments. Furthermore, because of the short durations of the interactions, collision-induced lines become very broad and merge into a continuum: the \(\text{H}_2\) CIE spectrum only shows broad peaks corresponding to vibrational bands, rather than a large number of narrow roto-vibrational lines. This is also important because self-absorption is much less relevant than for line emission, and in primordial proto-stars simulations CIE cooling can be treated with the optically thin approximation up to \(n \sim 10^{16} \text{ cm}^{-3}\) (for \(\text{H}_2\) lines, the optically thin approximation breaks at about \(n \sim 10^{10} \text{ cm}^{-3}\)). In figure 1.11 we show the CIE cooling rate for a gas with \(n = 10^{14} \text{ cm}^{-3}\), as a function of temperature and for different chemical compositions; at temperatures between 400 and 7000 K. For \(\text{H}_2\) abundances \(f_{\text{H}_2} \equiv \frac{2n_{\text{H}_2}}{(n_{\text{H}} + n_{\text{H}} + 2n_{\text{H}_2})} \gtrsim 0.5\) the total CIE cooling rate can be approximated by the simple expression (see Ripamonti & Abel 2004)

\[
L_{\text{CIE}}(\rho, T, X, f_{\text{H}_2}) \simeq 0.072 \rho T^4 X f_{\text{H}_2} \text{ erg g}^{-1} \text{ s}^{-1} \tag{1.29}
\]

where the density is in \(\text{g cm}^{-3}\), the temperature is in K, and \(X \simeq 0.75\) is the hydrogen fraction (by mass).

1.3.3 Chemistry

Since molecules play such an important role in the formation of the first luminous objects, it is important to include a proper treatment of their abundance evolution. However, a full treatment should keep into account \(\sim 20\) different species and \(\sim 100\) different reactions. For instance, GP98 give a chemical network of 87 reactions, which includes \(e^{-}, \text{H}, \text{H}^{+}, \text{H}^{-}, \text{D}, \text{D}^{+}, \text{He}, \text{He}^{+}, \text{He}^{++}, \text{Li}, \text{Li}^{+}, \text{Li}^{-}, \text{H}_2, \text{H}_2^{+}, \text{HD}, \text{HD}^{+}, \text{HeH}^{+}, \text{LiH}, \text{LiH}^{+}, \text{H}_2^{3+}\) and \(\text{H}_2\text{D}^{+}\); even their minimal model is too complicated to be described here, so we will just describe the most basic processes involved in \(\text{H}_2\) formation. However, we note that the papers by Abel et al. (1997)\(^4\) and by GP98 provide a much more accurate description of primordial chemistry.

1.3.3.1 Atomic Hydrogen and free electrons

Apart from molecule formation (see below), the main reactions involving Hydrogen (and Helium, to which we can apply all the arguments below) are ionizations and recombinations.

\(^4\) The collisional rate coefficients given in the Abel et al. (1997) paper can be readily obtained through a FORTRAN code available on the web page [http://www.astro.psu.edu/users/tabel/PGas/LCA-CM.html](http://www.astro.psu.edu/users/tabel/PGas/LCA-CM.html); also note that on Tom Abel’s web site [http://www.tomabel.com/](http://www.tomabel.com/) it is possible to find several useful informations about the primordial universe in general.
Ionizations can be produced both by radiation (H + hν → H+ + e−) and by collisions, mainly with free electrons (H + e− → H+ + 2e−) but also with other H atoms (H + H → H+e− + H). Photoionizations dominate over collisions as long as UV photons with energy above the H ionization threshold are present (see e.g. Osterbrock 1989), but this is not always the case before the formation of the first luminous objects, when only the CMB radiation is present; even after some sources of radiation have appeared, it is likely that their influence will be mainly local, at least until reionization. However, in the primordial universe the electron temperature is low, and collisional ionizations are relatively rare.

Recombinations (H+ + e− → H + hν) have much higher specific rates, since they do not have a high energy threshold. They probably dominate

Table 1.1. Reaction rates for some of the most important reactions in primordial gas, plus the main reactions involved in the formation of HD. In the formulae, T∗ is the temperature of the radiation field (for our purposes, the temperature of the CMB radiation) and temperatures need to be expressed in Kelvin; j(νLW) is the radiation flux (in erg s−1 cm−2) at the central frequency νLW of the Lyman-Werner bands, hνLW = 12.87 eV. The rates come from compilations given by Tegmark et al. 1997 (reactions 1-7), Palla et al. 1983 (reactions 8-13), Abel et al. 1997 (reaction 14) and Bromm et al. 2002 (reactions 15-19).

| Reaction | Rate |
|----------|------|
| H+ + e− → H + hν | k1 = 1.88 × 10−10 T.0.644 cm3 s−1 |
| H + e− → H+ + hν | k2 = 1.83 × 10−18 T−0.88 cm3 s−1 |
| H− + H → H2 + e− | k3 = 1.3 × 10−9 cm3 s−1 |
| H− + hν → H + e− | k4 = 0.114 T2.13 e−8650/T∗ cm3 s−1 |
| H+ + H → H2 + hν | k5 = 1.85 × 10−23 T1.8 cm3 s−1 |
| H2 + H → H2 + +H+ | k6 = 6.4 × 10−10 cm3 s−1 |
| H2 + hν → H+ + H | k7 = 6.36 × 10−7 cm3 s−1 |
| H + H + H → H2 + H | k8 = 5.5 × 10−29 T−1 cm6 s−1 |
| H2 + H → H + H + H | k9 = 6.5 × 10−7 T−1/2 e−52000/T × \(1−e−60000/T) cm6 s−1 |
| H + H + H2 → H2 + H2 | k10 = k8/3 |
| H2 + H2 → H + H + H2 | k11 = k9/3 |
| H + e− → H+ + e− + e− | k12 = 5.8 × 10−11 T1/2 e−158600/T cm3 s−1 |
| H + H → H+ + e− + H | k13 = 1.7 × 10−4 \(\frac{k}{k_{12}}\) |
| H2 + hν → H+ + H | k14 = 1.1 × 10^6 j(νLW) s−1 |
| D+ + e− → D + hν | k15 = 8.4 × 10−11 T−0.5 \(\frac{\nu}{\nu_{LW}}\)−0.2 [1 + \(\frac{\nu}{\nu_{LW}}\)−0.7] cm3 s−1 |
| D + H+ → D+ + H | k16 = 3.7 × 10−10 T0.28 e−43/T cm3 s−1 |
| D+ + H → D + H+ | k17 = 3.7 × 10−10 T0.28 cm3 s−1 |
| D+ + H2 → H+ + HD | k18 = 2.1 × 10−9 cm3 s−1 |
| HD + H+ → H2 + D | k19 = 1.0 × 10−9 e−464/T cm3 s−1 |
the evolution of free electrons, which can be relatively abundant as residuals from the recombination era (e.g. Peebles 1993), and are important for H\(_2\) formation (see below).

Simple approximations for the reaction rates of collisional ionizations and recombinations are given in Table 1.1.

### 1.3.3.2 H\(_2\) formation and disruption

At present, H\(_2\) is commonly believed to form mainly through reactions taking place on the surface of dust grains (but see e.g. Cazaux & Spaans 2004). Such a mechanism cannot work in the primordial universe, when the metal (and dust) abundance was negligible. The first two mechanisms leading to formation of H\(_2\) in a primordial environment was described by McDowell (1961) and by Saslaw & Zipoy (1967); soon after the publication of this latter paper, H\(_2\) started to be included in theories of structure formation (such as the PD68 paper about globular cluster formation through H\(_2\) cooling). Both these mechanisms consist of two stages, involving either e\(^{-}\) or H\(^+\) as catalyzers. The first (and usually most important) goes through the reactions

\[
\begin{align*}
H + e^- &\rightarrow H^- + h\nu \quad (1.30) \\
H^- + H &\rightarrow H_2 - + e^- \quad (1.31)
\end{align*}
\]

while the second proceeds as

\[
\begin{align*}
H^+ + H &\rightarrow H_2^+ + h\nu \quad (1.32) \\
H_2^+ + H &\rightarrow H_2 + + H^+ \quad (1.33)
\end{align*}
\]

In both cases, H\(_2\) production can fail at the intermediate stage if a photodissociation occurs (H\(^-\) + h\(\nu\) \rightarrow H + e\(^-\), or H\(_2^+\) + h\(\nu\) \rightarrow H\(^+\) + H). The rates of all these reactions are listed in Table 1.1.

By combining the reaction rates of these reactions, it is possible (see Tegmark et al. 1997; hereafter T97) to obtain an approximate evolution for the ionization fraction \(x \equiv n_{H^+}/n\) and the H\(_2\) fraction \(f_{H_2} \equiv 2n_{H_2}/n\) (here \(n\) is the total density of protons, \(n \simeq n_H + n_{H^+} + 2n_{H_2}\)):

\[
x(t) \simeq x_0 \frac{1}{1 + x_0 nk_1 t} \quad (1.34)
\]

\[
f_{H_2}(t) \simeq f_0 + 2 \frac{k_m}{k_1} \ln(1 + x_0 nk_1 t) \quad \text{with} \quad (1.35)
\]

\[
k_m = \frac{k_2 k_3}{k_3 + k_4/[n(1-x)]} + \frac{k_5 k_6}{k_6 + k_7/[n(1-x)]} \quad (1.36)
\]

where the various \(k_i\) are the reaction rates given in Table 1.1 and \(x_0\) and \(f_0\) are the initial fractions of ionized atoms and of H\(_2\) molecules.
Another mechanism for H\(_2\) formation, which becomes important at (cosmologically) very high densities \(n_\text{H} \gtrsim 10^8\) cm\(^{-3}\) are the so-called 3-body reactions described by Palla \textit{et al.} (1983). While the previous mechanisms are limited by the small abundance of free electrons (or of H\(^+\)), reactions such as

\[
\text{H} + \text{H} + \text{H} \rightarrow \text{H}_2 + \text{H}
\]

(1.37)
can rapidly convert all the hydrogen into molecular form, provided the density is high enough. For this reason, they are likely play an important role during the formation of a primordial protostar.

Finally, H\(_2\) can be dissociated through collisional processes such as reactions 9 in table 1.1 but probably the most important process is its photo-dissociation by photons in the Lyman-Werner bands (11.26-13.6 eV; but Abel \textit{et al.} 1997 found that the most important range is between 12.24 and 13.51 eV). These photons are below the H ionization threshold, therefore they can diffuse to large distances from their source. So, any primordial source emitting a relevant number of UV photons (\textit{e.g.} a \(\sim 100M_\odot\) star) is likely to have a major feedback effect, since it can strongly reduce the amount of H\(_2\) in the halo where it formed (and also in neighbouring halos), inhibiting further star formation.

1.3.3.3 \textit{Approximate predictions}

The above information can be used for making approximate predictions about the properties of the halos hosting the first luminous objects. Such kind of predictions was started by Couchman \& Rees (1986), and especially by T97 and their example was followed and improved by several authors (see below).

The basic idea is to have a simplified model for the evolution of the H\(_2\) fraction and the cooling rate inside spherical top-hat fluctuations, in order to check whether after virialization the baryons are able to cool (and keep collapsing), or they just “settle down” at about the virial radius. Such approximate models are fast to compute, and it is easy to explore the gas behaviour on a large range of virialization redshifts and fluctuation masses (or, equivalently, virial temperatures). In this way, for each virialization redshift it is possible to obtain the minimum molecular fraction which is needed for the collapse to proceed, and the minimum size of halos where such abundance is achieved. The results interestingly point out that the molecular fraction threshold separating collapsing and non-collapsing objects has an almost redshift-independent value of \(f_{\text{H}_2} \sim 5 \times 10^{-4}\) (see fig. 1.12). Instead, the minimum halo mass actually evolves with redshift (see fig. 1.13).

Predictions about the ability of the baryons inside each kind of halo to keep collapsing after virialization can then be combined with Press-Schechter predictions about the actual abundances of halos. For instance,
Figure 1.12. Comparison of the H\textsubscript{2} fraction needed for an halo to collapse and H\textsubscript{2} fraction which can be formed inside the halo in an Hubble time (from T97). This is shown as a function of halo virial temperature and for three different virialization redshifts (z = 100: solid; z = 50: short dashes; z = 25: long dashes). The three dots mark the minimum H\textsubscript{2} abundance which is needed for collapse at the three considered redshift, and it can be seen that they all are at \( f_{H_2} \sim 5 \times 10^{-4} \).

in fig. the solid black (almost vertical) line shows where the masses of 3\( \sigma \) fluctuations lie, as a function of redshift. So, if we decide to neglect the rare fluctuations at more than 3\( \sigma \) from the average, that figure tells us that the first luminous objects can start forming only at \( z \lesssim 30 \), in objects with a total mass \( \gtrsim 2 \times 10^6 M_\odot \).

Such result is subject to a number of uncertainties, both about the “details” of the model and about the processes it neglects; here are some of the more interesting developments:

(i) Abel et al. (1998) found that the minimum mass is strongly affected by the uncertainties in the adopted H\textsubscript{2} cooling function, with differences that could reach a factor \( \sim 10 \) in the minimum mass estimate

(ii) Fuller & Couchman (2000) used numerical simulations of single, high-\( \sigma \) density peaks in order to improve the spherical collapse approximation
Figure 1.13. Evolution with virialization redshift of the minimum mass which is able to cool, collapse and possibly form stars. Only the halos whose \((z_{\text{vir}}, M)\) fall outside the filled region are able to cool fast enough. The filled region is made of a red part, where CMB radiation prevents the cooling, and a yellow part where the cooling is slow because of a dearth of \(H_2\) molecules. The two parallel dashed lines correspond to virial temperatures of \(10^4\) K (the highest one) and \(10^3\) K (the lowest one), while the almost vertical line in the middle of the plot corresponds to \(3\sigma\) peaks in a SCDM \((\Omega_m = 1, \Omega_{\Lambda} = 0)\) cosmology.

(iii) Machacek et al. (2001) and Kitayama et al. (2001) investigated the influence of background radiation

(iv) Yoshida et al. (2003) used larger-scale numerical simulations and found that also the merging history of an halo could play a role, since frequent mergings heat the gas and prevent or delay the collapse of \(\sim 30\%\) of the halos.

As a result of the improved modeling (and of a different set of cosmological parameters as ΛCDM has substituted SCDM), in the most recent papers the value of the minimum halo mass for the formation of the first luminous objects is somewhat reduced to the range \(0.5–1 \times 10^6\ M_\odot\), with a weak dependence on redshift and a stronger dependence on other param-
1.4 Numerical cosmological hydrodynamics

Numerical simulations are an important tool for a large range of astrophysical problems. This is especially true for the study of primordial stars, given the absence of direct observational data about these stars, and the relatively scarce indirect evidence we have.

1.4.1 Adaptive refinement codes (ENZO)

The two problems which immediately emerge when setting up a simulation of primordial star formation are the dynamical range and the required accuracy in solving the hydrodynamical equations.

When studying the formation of objects in a cosmological context, we need both to simulate a large enough volume (with a box size of at least 100 comoving kpc), and to resolve objects of the size of a star ($\sim 10^{11} \text{ cm}$ in the case of the Sun), about 11 orders of magnitude smaller.

This huge difference in the relevant scales of the problem is obviously a problem. It can be attenuated by the use of Smoothed Particle Hydrodynamics (SPH; see e.g. Monaghan 1992), whose Lagrangian nature has some benign effects, as the simulated particles are likely to concentrate (i.e., provide resolution) in the regions where the maximum resolution is needed. However, even if this kind of method can actually be employed (see Bromm et al. 1999, 2002), it has at least two important drawbacks. First of all, the positive effects mentioned above cannot bridge in a completely satisfactory way the extreme dynamical range we just mentioned, since the mass resolution is normally fixed once and for all at the beginning of the simulation. Second, SPH is known to have poor shock resolution properties, which casts doubts on the results when the hydrodynamics becomes important.

The best presently available solution for satisfying both requirements is the combination of an Eulerian approach (which is good for the hydrodynamical part) and an adaptive technique (which can extend the dynamical range). This is known as Adaptive Mesh Refinement (AMR; see e.g. Berger & Colella 1989; Norman 2004), and basically consists in having the simulation volume represented by a hierarchy of nested grids (meshes) which are created according to resolution needs.

In particular, the simulations we are going to describe in the following paragraphs were made with the code ENZO\(^5\) (see O’Shea et al. 2004 and references therein for a full description). Briefly, ENZO includes the treatment of gravitational physics through N-body techniques, and the treat-

\(^5\) The ENZO code can be retrieved at the web site [http://cosmos.ucsd.edu/enzo/](http://cosmos.ucsd.edu/enzo/)
Figure 1.14. Overview of the evolution leading to the formation of a primordial star. The top row shows the gas density, centered at the pre-galactic object within which the star is formed. The four projections are labelled with their redshifts. Pre-galactic objects form from very small density fluctuations and continuously merge to form larger objects. The middle and bottom rows show thin slices through the gas density and temperature at the final simulation stage. The four pairs of slices are labelled with the scale on which they were taken, starting from 6 (proper) kpc (the size of the simulated volume) and zooming in down to 0.06 pc (12,000 AU). In the left panels, the larger scale structures of filaments and sheets are seen. At their intersections, a pre-galactic object of $\sim 10^6 M_\odot$ is formed. The temperature slice (second panel, bottom row) shows how the gas shock heats as it falls into the pre-galactic object. After passing the accretion shock, the material forms hydrogen molecules and starts to cool. The cooling material accumulates at the centre of the object and forms the high-redshift analog to a molecular cloud (third panel from the right), which is dense and cold ($T \sim 200K$). Deep within the molecular cloud, a core of $\sim 100 M_\odot$, a few hundred K warmer, is formed (right panel) within which a $\sim 1 M_\odot$ fully molecular object is formed (yellow region in the right panel of the middle row). (from ABN02).
ment of hydrodynamics through the piecewise parabolic method (PPM) of Woodward & Colella 1984 (as modified by Bryan et al. 1995 in order to adapt to cosmological simulations). ENZO can optionally include the treatment of gas cooling (in particular H\textsubscript{2} cooling, as described in the preceding sections), of primordial non-equilibrium chemistry (see the preceding sections) and of UV background models (e.g. the ones by Haardt & Madau 1996) and heuristic prescriptions (see Cen & Ostriker 1992) for star formation in larger-scale simulations.

1.4.2 Formation of the first star

The use of AMR codes for cosmological simulations of the formation of the first objects in the universe was pioneered by Abel et al. (2000) and further refined by Abel et al. (2002; hereafter ABN02), where the dynamic range covered by the simulations was larger than 5 orders of magnitude in scale length, i.e. the wide range between an almost cosmological (\(\gtrsim 100\) comoving kpc, i.e. \(\gtrsim 5\) proper kpc) and an almost stellar scale (\(\lesssim 1000\) AU \(\sim 10^{-2}\) pc). In the whole process, the AMR code kept introducing new (finer resolution) meshes whenever the density exceeded some thresholds, or the Jeans length was resolved by less than 64 grid cells. The simulations were started at a redshift \(z = 100\) from cosmologically consistent initial conditions\(^6\), and the code also followed the non-equilibrium chemistry of the primordial gas, and included an optically thin treatment of radiative losses from atomic and molecular lines, and from Compton cooling.

The main limitation of these simulations was the assumption that the cooling proceeds in the optically thin limit; such assumption breaks down when the optical depth inside H\textsubscript{2} lines reaches unity (corresponding to a Jeans length \(\sim 10^3\) AU \(\sim 0.01\) pc). However, the simulations were halted only when the optical depth at line centres becomes larger than 10 (Jeans length of about \(\sim 10\) AU (\(\sim 10^{-4}\) pc), since it was unclear whether Doppler shifts could delay the transition to the optically thick regime.

1.4.2.1 Summary of the evolution: radial profiles

In Figures 1.14 and 1.15 we show the evolution of gas properties both in pictures and in plots of spherically averaged quantities, as presented in Abel et al. (2002). From these figures, and in particular from the local minima in the infall velocity (Fig. 1.15e) it is possible to identify four characteristic mass scales:

\(^6\) Such conditions were taken from an SCDM model with \(\Omega_\Lambda = 0, \Omega_m = 1, \Omega_b = 0.06, H_0 = 50\) km/s Mpc\(^{-1}\) which is quite different from the “concordance” \(\Lambda\)CDM model. However, the final results are believed to be only marginally affected by differences in these cosmological parameters.
Figure 1.15. Radial mass-weighted averages of physical quantities at seven different simulation times. (A) Particle number density in cm$^{-3}$ as a function of radius; the bottom line corresponds to $z = 19$, and moving upwards the “steps” from one line to the next are of $9 \times 10^6$ yr, $3 \times 10^5$ yr, $3 \times 10^4$ yr, 3000 yr, 1500 yr, and 200 yr, respectively; the uppermost line shows the simulation final state, at $z = 18.181164$. The two lines between 0.01 and 200 pc give the DM mass density (in GeV cm$^{-3}$) at $z = 19$ and the final time, respectively. (B) Enclosed gas mass. (C) Mass fractions of H and H$_2$. (D) Temperature. (E) Radial velocity of the baryons; the bottom line in (E) shows the negative value of the local speed of sound at the final time. In all panels the same output times correspond to the same line styles. (from ABN02).
(i) The mass scale of the pre-galactic halo, of $\sim 7 \times 10^5 M_\odot$ in total, consistent with the approximate predictions discussed in the previous sections.

(ii) The mass scale of a “primordial molecular cloud”, $\sim 4000 M_\odot$: the molecular fraction in this region is actually very low ($\lesssim 10^{-3}$), but it is enough to reduce the gas temperature from the virial value ($\gtrsim 10^3$ K) to $\sim 200$ K.

(iii) The mass scale of a “fragment”, $\sim 100 M_\odot$, which is determined by the change in the H$_2$ cooling properties at a density $n \sim 10^4$ cm$^{-3}$ (for a complete discussion, see Bromm et al. 1999, 2002), when Local Thermal Equilibrium is reached and the cooling rate dependence on density flattens to $\Lambda \propto n$ (from $\Lambda \propto n^2$); this is also the first mass scale where the gas mass exceeds the Bonnor-Ebert mass (Ebert 1955, Bonnor 1956) $M_{BE}(T, n, \mu) \simeq 61 M_\odot T^{3/2} n^{-1/2} \mu^{-2}$ (with $T$ in Kelvin and $n$ in cm$^{-3}$), indicating an unstable collapse.

(iv) The mass scale of the “molecular core”, $\sim 1 M_\odot$ is determined by the onset of 3-body reactions at densities in the range $n \sim 10^8 - 10^{10}$ cm$^{-3}$, which leads to the complete conversion of hydrogen into molecular form; at this stage, the infall flow becomes supersonic (which is the precondition for the appearance of an accretion shock and of a central hydrostatic flow); the increase in the H$_2$ abundance due to the formation of this molecular core also leads to the transition to the optically thick regime.

1.4.2.2 Angular momentum

In Fig. 1.16 we show the evolution of the radial distribution of average specific angular momentum (and related quantities). It is remarkable that in ABN02 simulations rotational support does not halt the collapse at any stage, even if this could be a natural expectation.

There are two reasons for such (apparently odd) fact:

(i) As can be seen in panel A of Fig. 1.16 the collapse starts with the central gas having much less specific angular momentum than the average (this is typical of halos produced by gravitational collapse; see e.g. Quinn & Zurek 1988). That is, the gas in the central regions starts with little angular momentum to lose in the first place.

(ii) Second, some form of angular momentum transport is clearly active, as is demonstrated by the decrease in the central specific angular momentum. Turbulence is likely to be the explanation: at any radius, there will be both low and high angular momentum material, and redistribution will happen because of pressure forces or shock waves: lower angular momentum material will selectively sink inwards, displacing higher angular momentum gas. It is notable that this kind of transport will be suppressed in situations in which the collapse occurs
Figure 1.16. Radial mass weighted averages of angular momentum-related quantities at different times (the same as in fig. 1.15). (A) Specific angular momentum $L$. (B) Rotational speed in units of Keplerian velocity $v_{\text{Kepl}} \equiv (GM_\odot/r)^{1/2}$. (C) Rotational speed ($L/r$). (D) Rotational speed in units of the sound speed $c_s$ (from ABN02).

on the dynamical time scale, rather than the longer cooling time scale: this is the likely reason why this kind of mechanism has not been observed in simulations of present day star formation (see e.g. Burkert & Bodenheimer 1993).

1.4.3 SPH results

Bromm et al. (1999, 2002) have performed simulations of primordial star formation using an SPH code which included essentially the same physical processes as the simulations we just described. Apart from the numerical technique, the main differences were that they included deuterium in some of their models, and that their initial conditions were not fully cosmological
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(e.g., since they choose to study isolated halos, the angular momenta are assigned as initial conditions, rather than generated by tidal interactions with neighbouring halo).

In Fig. 1.17 and 1.18 we show the results of one of their simulations, which give results in essential agreement with those we have previously discussed.

An interesting extra-result of this kind of simulation is that the authors are able to assess the mass evolution of the various gaseous clumps in the hypothesis that feedback is unimportant (which could be the case if the clumps directly form intermediate mass black holes without emitting much radiation, or if fragmentation to a quite small stellar mass scale happens). They find that a significant fraction (∼0.5) of the halo gas should end up inside one of the gas clumps, although it is not clear at all whether this gas will form stars (or some other kind of object). Furthermore, they find that clumps are likely to increase their mass on a timescale of about 10^{7} yr (roughly corresponding to the initial time scale of the simulated halo), both because of gas accretion and of mergers, and they could easily reach masses >∼10^{4} M_{\odot}. Obviously, this result is heavily dependent on the not very realistic assumed lack of feedback.

1.5 Protostar formation and accretion

Full three-dimensional simulations (such as the ones of ABN02) are not able to reach the stage when a star is really formed. They are usually stopped at densities (n ∼ 10^{10} − 10^{11} cm^{-3}) which are much lower than typical stellar densities (ρ ∼ 1 g cm^{-3}, n ∼ 10^{24} cm^{-3}). In fact, at low densities the gas is optically thin at all frequencies, and it is not necessary to include radiative transfer in order to estimate the gas cooling. Instead, at densities n ∼ 10^{10} cm^{-3} some of the dominant H_2 roto-vibrational lines become optically thick and require the treatment of radiative transfer.

For this kind of problem, this is a prohibitive computational burden, and at present the actual formation of a protostar can not be fully investigated through self-consistent 3-D simulations. In order to proceed further, it is necessary to introduce some kind of simplification in the problem.

1.5.1 Analytical results

Historically, there were several studies based on analytical arguments (i.e. stability analysis) or single-zone models, leading to different conclusions about the properties of the final object.

An early example is the paper by Yoneyama (1972), in which it was argued that fragmentation takes place until the opacity limit is reached. However, Yoneyama looked at the opacity limit of the entire “cloud” (roughly corresponding to one of the 10^{5}−10^{6} M_{\odot} mini-halos we consider at present;
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Figure 1.17. Typical result of the SPH simulations by Bromm et al. 2002. The figure shows the morphology of a simulated $2 \times 10^6 M_\odot$ halo virializing at $z \sim 30$ just after the formation of the first clump of mass $1400 M_\odot$ (which is likely to produce stars). The two top row panels show the distribution of the dark matter (which is undergoing violent relaxation). The two bottom panels show the distribution of gas, which has developed a lumpy morphology and settled at the centre of the potential well.

originally, this was mass the scale proposed by PD68) rather than the putative fragments and arrived at the conclusion that fragmentation stopped for masses $\lesssim 60 M_\odot$.

More recently, Palla et al. 1983 pointed to the increase of the cooling rate at $n \gtrsim 10^8 \text{ cm}^{-3}$ (due to H$_2$ fast formation through 3-body reactions) as a possible trigger for instability, leading to fragmentation on mass scales...
Figure 1.18. Gas properties in the SPH simulations by Bromm et al. 2002. The figure shows the properties of the particles shown in Fig. Panel (a) shows the free electron abundance $x_e$ as a fraction of H number density. Panel (b) shows the H$_2$ abundance $f_{H_2}$ (note that even the highest density particles have $n_H \lesssim 10^8$ cm$^{-3}$, so 3-body reactions are unimportant and $f_{H_2} \lesssim 10^{-3}$). Panel (c) shows the gas temperature (low density gas gets to high temperatures because H$_2$ cooling is inefficient). Panel (d) shows the value of the Jeans mass which can be obtained by using each particle temperature and density. All the panels have the hydrogen number density $n_{H}$ on the X axis.

of $\sim 0.1$ $M_\odot$; such instability has actually been observed in the simulations of Abel et al. (2003), but it does not lead to fragmentation because its growth is too slow (see also Sabano & Yoshii 1977, Silk 1983, Omukai & Yoshii 2003 and Ripamonti & Abel 2004 for analytical fragmentation criteria and their application to this case).
Figure 1.19. Proto-stellar collapse, formation of an hydrostatic core and start of the proto-stellar accretion phase as can be found with 1-D simulations. The four panel show the profiles of density (top left; a), infall velocity (top right; b), temperature (bottom left;d) and H$_2$ abundance (bottom right; d) as a function of enclosed mass at 10 different evolutionary stages (0=initial conditions; 9=final stage of the computation). The most relevant phases are the rapid formation of H$_2$ (2-3) and the formation of a shock on the surface of the hydrostatic core (5-6-7), followed by the onset by accretion. Also note the almost perfect power-law behaviour of the density profile before core formation. (from Ripamonti et al. 2002).
Figure 1.20. Comparison of the accretion and the Kelvin-Helmoltz time scales for a primordial protostar, as a function of protostellar mass. The Kelvin-Helmoltz contraction time (obtained using the ZAMS luminosity as given by Schaerer 2002) is shown as the solid black line, while the dashed line and the solid line with circles show the time which is needed to accrete each mass of gas; they are based on the results of ABN02 and differ slightly in the way in which they were obtained. The dotted lines mark three constant accretion rates of $10^{-2}$ (bottom line), $5 \times 10^{-3}$ (middle line) and $10^{-3} \, M_\odot/\text{yr}$ (top line).

1.5.2 Mono-dimensional models

1.5.2.1 The formation of the protostellar core

A less simplified approach relies on 1D studies, such as those of Omukai & Nishi (1998) and Ripamonti et al. (2002): in such studies the heavy price of assuming that the collapsing object always remains spherical (which makes impossible to investigate the effects of angular momentum, and prevents fragmentation) is compensated by the ability to properly treat all the other physical processes which are believed to be important: first of all, the detailed radiative transfer of radiation emitted in the numerous $\text{H}_2$ resonance lines, then the transition to continuum cooling (at first from molecular Collision-Induced Emission, and later from atomic processes) and finally
Figure 1.21. Evolution of the protostellar radius as a function of protostellar mass in the models of Omukai & Palla (2003). The models differ in the assumed accretion rates. The dotted curves show the results of models where $\dot{M}_{\text{core}}$ is assumed to be constant at values of $1.1 \times 10^{-3} \ M_\odot$/yr (dotted curve starting at $R_{\text{core}} \simeq 30 \ R_\odot$), $2.2 \times 10^{-3} \ M_\odot$/yr (starting at $R_{\text{core}} \simeq 40 \ R_\odot$), $4.4 \times 10^{-3} \ M_\odot$/yr (“fiducial” model; starting at $R_{\text{core}} \simeq 50 \ R_\odot$) and $8.8 \times 10^{-3} \ M_\odot$/yr (starting at $R_{\text{core}} \simeq 65 \ R_\odot$). The solid thick curve shows the evolution of a model in which the accretion rate was obtained from the extrapolation of the ABN02 data.

the non-ideal equation of state which becomes necessary at high densities, when the gas becomes increasingly hot and largely ionized.

Such studies find that the collapse initially proceeds in a self-similar fashion (in good agreement with the solution found by Larson 1969 and Penston 1969); at the start of this phase (stages 1 and 2 in fig. 1.19) to be compared with the profiles at corresponding densities in fig. 1.15, their results can be compared with those of 3-D simulations, and the agreement
is good despite the difference in the assumed initial conditions: this is likely due to the self-similar properties. At later stages the comparison is not as good, but this is presumably due to the differences in the cooling rate (the 3D models at $n > 10^{10}$ cm$^{-3}$ only include the optically thin cooling rate).

It is noticeable that the transition to optically thick cooling does not stop the self-similar phase, contrary to theoretical expectations (see e.g. Rees 1976). The reason is that the reduction in the cooling rate is smooth, and is further delayed by the onset of CIE cooling and by the start of H$_2$ dissociation, which acts as an heat well (each H$_2$ dissociation absorbs 4.48 eV of thermal energy) and provides an alternative “disposal” mechanism for the excess thermal energy.

The self-similar phase proceeds until the central density is $n > 10^{21}$ cm$^{-3}$ $\sim 10^{-3}$ g cm$^{-3}$, when a small ($M_{\text{core},0} \sim 3 \times 10^{-3} M_\odot$) hydrostatic core eventually forms. Such core starts accreting the surrounding gas at a very high rate ($\dot{M}_{\text{core}} \sim 0.01 - 0.1 M_\odot$ yr$^{-1}$), comparable to the theoretical predictions of Stahler et al. 1980

$$\dot{M}_{\text{core}} \sim \frac{c_s^3}{G} \simeq 4 \times 10^{-3} \left( \frac{T}{1000 \text{ K}} \right)^{3/2} \left( \frac{\mu}{1.27} \right)^{-3/2} M_\odot \text{ yr}^{-1}$$

where $c_s = [k_B T/(\mu m_H)]^{1/2}$ is the isothermal sound speed of a gas with mean molecular weight $\mu$.

Although the Omukai & Nishi (1998) and the Ripamonti et al. (2002) studies predict that such a high accretion rate will decline (approximately as $\dot{M}_{\text{core}} \propto t^{-0.3}$), it is clear that, if the accretion proceeds unimpeded, it could lead to the formation of a star with a mass comparable to the whole proto-stellar cloud where the proto-star is born, that is, about $100 M_\odot$; such a process would take a quite short time of about $10^4 - 10^5$ yr.

This scenario could be deeply affected by feedback effects: even not considering the radiation coming from the interior of the proto-star, the energy released by the shocked material accreting onto the surface can reach very high values, comparable with the Eddington luminosity, and is likely to affect the accretion flow.

Figure 1.20 compares the accretion time scale (which can be extrapolated from the data at the end of the ABN02 simulations) to the Kelvin-Helmoltz timescale,

$$t_{KH} = \frac{GM^2}{RL}$$

where $L$ is the luminosity of the protostar. This plot tells us that the accretion timescale is so fast that the stellar interior has very little space for a “re-adjustment” (which could possibly stop the accretion) before reaching a mass of $\sim 10 - 100 M_\odot$.

However, it can be argued that such a readjustment is not necessary, since in the first stages most of the protostellar luminosity comes from the
accretion process itself:

\[ L_{\text{acc}} \simeq \frac{GM_{\text{core}} \dot{M}_{\text{core}}}{R_{\text{core}}} \]

\[ \simeq 8.5 \times 10^{37} \left( \frac{M_{\text{core}}}{M_\odot} \right) \left( \frac{\dot{M}_{\text{core}}}{0.01 M_\odot/\text{yr}} \right) \left( \frac{R_{\text{core}}}{10^{12} \text{ cm}} \right)^{-1} \text{ erg s}^{-1} \] (1.40)

where we have inserted realistic values for \( M_{\text{core}} \), \( \dot{M}_{\text{core}} \) and \( R_{\text{core}} \). This luminosity is quite close to the Eddington luminosity \( (L_{\text{Edd}} \simeq 1.3 \times 10^{38} (M/M_\odot) \text{ erg s}^{-1}) \), and radiation pressure could have a major effect.

Ripamonti et al. (2002) found that this luminosity is not able to stop the accretion, but they do not properly trace the internal structure of the core, so their results cannot be trusted except in the very initial stages of accretion (say, when \( M_{\text{core}} \lesssim 0.1 M_\odot \)).

A better suited approach was used in studies by Stahler et al. (1986) and, more recently, by Omukai & Palla (2001, 2003), who assumed that the accretion can be described as a sequence of steady-state accretion flows onto a growing core. The core is modelled hydrostatically, as a normal stellar interior (including the treatment of nuclear burning), while the accreting envelope is modelled through the steady state assumption, in conjunction with the condition that outside the “photosphere” (the region where the optical depth for a photon to escape the cloud is \( \gtrsim 1 \)) the accreting material is in free-fall. As shown in Fig. 1.21, Omukai & Palla (2003) find that even if feedback luminosity deeply affects the structure of the accreting envelope, it is never able to stop the accretion before the proto-stellar mass reaches \( 60 - 100 M_\odot \) as a minimum; after that, the final mass of the protostar depends on the assumed mass accretion rate \( \dot{M}_{\text{core}} \): for \( \dot{M}_{\text{core}} \lesssim 4 \times 10^{-3} M_\odot \text{ yr}^{-1} \), the accretion can proceed unimpeded until the gas is completely depleted (or the star explodes as a supernova); otherwise, the radiation pressure is finally able to revert the accretion; with high accretion rates this happens sooner, and the final stellar mass is relatively low, while for accretion rates only slightly above the critical value of \( \sim 4 \times 10^{-3} M_\odot \text{ yr}^{-1} \) the stellar mass can reach about 300 \( M_\odot \).

If such predictions are correct, the primordial Initial Mass Function is likely to be much different from the present one, reflecting mainly the mass spectrum of the gas fragments from which the stars originate, and a mass of \( \sim 100 M_\odot \) could be typical for a primordial star. However, we note that this important results could change as a result of better modeling. For example, deep modifications of the envelope structure are quite possible if a frequency-dependent opacity (rather than the mean opacity used in the cited studies) is included.
1.6 Discussion

The previous sections show that, although not certain at all (because of the big uncertainty about feedback effects), numerical simulations tend to favour the hypothesis that primordial stars had a larger typical mass than present-day stars.

If this is true, it could indeed solve some observational puzzle, such as why we have never observed a single zero-metallicity star (answer: they have exploded as supernovae and/or transformed into compact objects at a very high redshift), and maybe help explaining the relatively high metallicities \( Z \gtrsim 10^{-4} Z_\odot \) even in low column density systems) measured in the Lyman \( \alpha \) forest (see e.g. Cowie & Songaila 1998), or the high (\( \sim \) solar) metallicities we observe in the spectra of some quasars already at redshift 6 (Fan et al. 2000, 2001, 2003, 2004; Walter et al. 2003).

However, a top-heavy primordial IMF also runs into a series of problems, which we will discuss in the remaining of this section.

1.6.1 UV Radiation feedback

First of all, if a moderately massive star (even \( M \gtrsim 20-30 M_\odot \) is likely to be enough) actually forms in the centre of an halo in the way shown by ABN02, it will emit copious amounts of UV radiation, which will produce important feedback effects. Indeed, the scarcity of metals in stellar atmospheres (see Schaerer 2002, and also Tumlinson & Shull 2000, Bromm Kudritzki & Loeb 2001) results in UV fluxes which are significantly larger than for stars of the same mass but with “normal” metallicity. Furthermore, this same scarcity of metals is likely to result in a negligibly small density of dust particles, further advancing the UV fluxes at large distances from the sources when compared to the present-day situation.

Massive primordial objects are also likely to end up into black holes (either directly or after having formed a star\(^7\)), which could be the seeds of present day super-massive black holes (see e.g. Volonteri, Haardt & Madau 2003). If accretion can take place onto these black holes (also known as mini-quasars), they are likely to emit an important amount of radiation in the UV and the X-rays (this could also be important in explaining the WMAP result about Thomson scattering optical depth; see e.g. Madau et al. 2003).

UV photons can have a series of different effects. First of all, we already mentioned in the section about chemistry that Lyman-Werner photons (\( 11.26 \text{ eV} \leq h\nu \leq 13.6 \text{ eV} \)) can dissociate \( \text{H}_2 \) molecules, preventing

\(^7\) Here we think of a star as an object where quasi-hydrostatic equilibrium is reached because of stable nuclear burning; according to this definition, a black hole can be formed without passing through a truly stellar phase, even if it is very likely to emit significant amounts of radiation in the process.
Figure 1.22. Average flux (in units of erg cm$^{-2}$ s$^{-1}$ sr$^{-1}$ Hz$^{-1}$) at an observation redshift $z_{\text{obs}} = 25$ coming from sources turning on at $z = 35$. The top panel shows the effects of absorption of neutral hydrogen and helium, strongly reducing the flux between $\sim 13.6$ eV and $\sim 1$ keV (the solid and dashed lines show the absorbed and unabsorbed flux, respectively). The bottom panel shows the same quantities in a much smaller energy range, in order to illustrate the sawtooth modulation due to line absorption below 13.6 eV. (from Haiman, Rees & Loeb 1997).

Further star formation. Such photons are effective even at large distances from their sources, and even in a neutral universe, since their energy is below the H ionization threshold; the only obstacle they find on their way are some of the Lyman transitions of H, which can scatter these photons or remove them from the band; this results in a “sawtooth” modulation (see fig. 1.22) of the spectrum. Haiman, Rees & Loeb (1997) argue that this negative feedback could conceivably bring primordial star formation to an
Figure 1.23. Dynamical evolution of the HII region produced by a 200 $M_\odot$ star. Panels refer to ionization fraction profiles (top left), temperature distributions (top right), densities (bottom left) and velocities (bottom right). The dashed line in the density panel is for the Strömgren density (the density required to form a Strömgren sphere and therefore initially bind the ionization front) at a given radius. Profiles are given at times (from top to bottom in the density panel; from left to right in the others) 0 (density panel), 63 kyr (all panels), 82 kyr (ionization and temperature panels), 95 kyr (ionization panel) 126.9 kyr (all panels), 317 kyr (all panels), 1.1 Myr (temperature, density and velocity panels) and 2.2 Myr (all panels), which is the approximate main sequence lifetime of a 200 $M_\odot$ star. (From Whalen, Abel & Norman 2004).

early stop; however, other authors found that this is not necessarily the case (Ciardi et al. 2000; Omukai 2001; Glover & Brand 2001), or that the feedback effect on H$_2$ abundance can be moderated by the effects of X-ray photons coming from mini-quasars (Haiman Abel and Rees 2000; Glover & Brand 2003).

A second obvious effect from UV photons is to ionize the interstellar and the intergalactic medium; it is unclear whether these kind of objects are important in the reionization history of the universe, but they will definitely form HII regions in their immediate neighbourhoods. In fig. 1.23 we show
the results of Whalen, Abel & Norman 2004 about the evolution of an HII region produced by a 200$M_\odot$ star inside a halo with the same density profile as found in the ABN02 paper: in the beginning the ionization front is D-type (that is, it is “trapped” because of the density) and expands because a shock forms in front of it; later (after about 70 kyr from the start of the UV emission) the ionization front reaches regions of lower densities and becomes R-type (radiation driven), expanding much faster than the shock. However, the shock keeps moving at speeds of the order of 30 km s$^{-1}$ even after the source of ionizing photons is turned off. Such a speed is much larger than the rotational velocities of the mini-halos where primordial star formation is supposed to be taking place ($\lesssim$ 10 km s$^{-1}$), so this shocks are likely to expel a very large fraction of the original gas content of the mini-halo. UV emission could even lead to the photo-evaporation of neighbouring mini-halos, similar to what Barkana & Loeb 1999 (see also Shapiro, Iliev & Raga 2004) found in the slightly different context to cosmological reionization.

Since the dynamical timescale of a mini-halo is $\gtrsim 10^7$ yr and is longer than the timescale for this kind of phenomena (definitely shorter than the $\sim 1−10$ Myr main sequence lifespan of massive stars, and very likely to be of the order of $\sim 10^4−10^5$ yr), the star formation in one mini-halo is likely to stop almost immediately after the first (massive) object forms. This means that, unless two or more stars form exactly at the same time (within $\sim 1\%$ of the mini-halo dynamical time), each mini-halo will form exactly one massive star.

1.6.2 Supernovae feedback and metallicities

After producing plenty of UV photons during their life, primordial massive stars are likely to explode as supernovae. This must be true for some fraction of them, otherwise it would be impossible to pollute the IGM with metals, as mass loss from zero-metal stars is believed to be unimportant (this applies to stars with $M \lesssim 500 M_\odot$; see Baraffe, Heger & Woosley 2001, and also Heger et al. 2002); however, it is clearly possible that some fraction of primordial stars directly evolve into black holes.

In figure [1.2] we show the results about the fate of zero metallicity stars, as obtained by Heger & Woosley 2002 (but see also Heger et al. 2003), indirectly confirming this picture and suggesting that pair-instability supernovae could play a major role if the primordial IMF is really skewed towards masses $\gtrsim 100 M_\odot$.

This has a series of consequences. First of all, supernova explosion are one of the very few phenomena directly involving primordial stars which we can realistically hope to observe. Wise & Abel (2004; see also Marri & Ferrara 1998 and Marri, Ferrara & Pozzetti 2000 for the effects of gravitational lensing) investigated the expected number of such supernovae by means of
Figure 1.24. The “final mass function” of non-rotating metal-free stars as a function of their initial mass, as given by from Heger & Woosley 2002. The thick black curve gives the final mass of the collapsed remnant, and the thick gray curve and the mass of the star at the start of the event that produces that remnant (mass loss, SN etc.); for zero-metal stars this happens to be mostly coincident with the dotted line, corresponding to no mass loss. Below initial masses of \(5 \times 10 M_\odot\) white dwarfs are formed; above that, initial masses of up to \(\sim 25 M_\odot\) lead to neutron stars. At larger masses, black holes can form in different ways (through fall-back of SN ejecta or directly). Pair instability starts to appear above \(\sim 100 M_\odot\), and becomes very important at \(\sim 140 M_\odot\): stars with initial masses in the \(140 - 260 M_\odot\) range are believed to completely disrupt the star, leaving no remnant; instead, above this range pair instability is believed to lead to a complete collapse of the star into a black hole.
a semi-analytic model combining Press-Schechter formalism, an evolving minimum mass for star forming halos and negative feedbacks, finding the rates shown in Fig. 1.25 if such objects are pair-instability supernovae with masses $\sim 175 \, M_\odot$; they should be detectable by future missions such as JWST$^8$; if some of them are associated to Gamma Ray Bursts, the recently launched Swift$^9$ satellite has a slim chance of observing them at a redshifts $\sim 30$ (see Gou et al. 2004).

Supernova explosions obviously have hydrodynamical effects on the gas of the mini-halo where they presumably formed, especially in the case of the particularly violent pair-instability supernovae (see e.g. Ober, El Eid & Fricke 1983), and they are likely to expel it even if it was not removed by the effects of UV radiation (see e.g. MacLow & Ferrara 1999 and Ferrara & Tolstoy 2000), further reducing the probability of having more than 1 star per mini-halo. The similarity with UV effects extends to the influence on neighbouring halos which can be wiped out by SN explosions (Sigward, Ferrara & Scannapieco 2004).

Finally, supernovae provide a mechanism for spreading metals into the IGM, as discussed e.g. by Madau, Ferrara & Rees 2001. In turn, these metals will modify the cooling properties of the gas, and lead to star formation with a normal (Salpeter-like) IMF. If this is true, and if pair-instability supernovae dominate the yields from primordial stars, we should be able to distinguish the peculiar pair-supernova abundance pattern (described in Heger & Woosley 2001) as a “signature” of primordial origin.

Searches for low-metallicity stars have a long history (see e.g. Beers 1999, 2000), and their inability to find stars with metallicities $Z \leq 10^{-4} \, Z_\odot$ led to speculation that this metallicity marked the transition to a normal IMF (see e.g. Bromm et al. 2001, Schneider et al. 2002). However, Christlieb et al. 2002 finally reported the discovery of an extremely metal poor star (HE0107-5240) with $[Fe/H] = -5.3^{10}$, a level compatible with a “second generation” star (i.e., a star formed from material enriched by very few supernovae; for comparison, Wise & Abel 2004 find that primordial SNe could enrich the IGM to $[Fe/H] \sim -4.1$, although this is probably an upper limit). The abundance patterns in this star are quite strange (for example, the carbon abundance is slightly less than 1/10 solar), but a much better fit is provided by supernova yields of moderately massive stars ($15 \sim 40 \, M_\odot$) rather than from yields predicted for pair-instability supernovae coming from very massive stars. However, at the moment no model can satisfactorily fit all the observed abundances (see Bessel, Christlieb & Gustafsson 2004).

Even if the primordial nature of this star must still be established, it represents a cautionary tale about the currently favoured predictions of a
large number of massive or very massive primordial stars, and a reminder that better theoretical modeling is still needed, with particular regard to feedback effects during the stellar accretion phase.

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Figure 1.25. Primordial supernova properties as reported by Wise & Abel 2004. The right panels show the differential rate of primordial supernovae per unit redshift (top) and the cumulative rate (bottom); both are per year of observation and per square degree. The left panels show the critical halo mass (in $M_\odot$) for primordial star formation (top), the comoving number density (in $Mpc^{-3}$) of halos above the critical mass (middle) and the predicted specific intensity of the soft UV background in the Lyman-Werner band (in erg s$^{-1}$ cm$^{-2}$ Hz$^{-1}$ sr$^{-1}$, bottom). The three lines in each panel refer to fixed primordial stellar masses of $100 M_\odot$ (solid), $200 M_\odot$ (dotted) and $500 M_\odot$ (dashed).
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