Solving a problem in the quantum way

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Abstract
A general quantum algorithm for solving a problem is discussed. The number of steps required to solve a problem using this method is independent of the number of cases that has to be considered classically. Hence, it is more efficient than existing classical algorithms or quantum algorithm, which requires $O(\sqrt{N})$ steps.

1 Introduction
Quantum computation [1,2,3] offers a unique class of algorithms based on quantum parallelism. It has been shown that certain problems like factorization [4] and database [5] search can be done more efficiently in a quantum computer. However designing new quantum algorithms is not an easy task since our intuitions gained from the world around cannot guide us in the quantum realm. In this letter, I propose a general quantum algorithm, which can be applied to all problems where we can classify the numbers in the initial superposition as solutions non-solutions to the given problem.

2 The Problem.
The problem is to find an $x$ such that :

$$C_n(x) = 0$$

$C_n(x)$ can be a mathematical expression or a set of logical statements. It is known that there are $n$ values of $x$, which satisfies this condition, and they may
be any number between 0 and $2^k - 1$. For example, $Cn(x)$ can be an $n^{th}$ order polynomial equation, which has $n$ solutions.

3 Solving in the quantum way

The model of the quantum computer I have in mind consists of registers $X$ and $Y$, a quantum processor that is made up of a quantum network whose action on the registers $X$ and $Y$ is represented by the operator $Uc$. In addition, a third register $Z$ if needed. A $k$-bit quantum register can be in an equally weighted superposition of 0 to $2^k - 1$ numbers. We prepare such a superposition in the $X$ register. We can classify the numbers present in the superposition into two groups. We also define a new variable $y$ as follows.

**Group 1:** The values of $x$ for which $Cn(x) = 0$

**Group 2:** The values of $x$ for which $Cn(x) \neq 0$

A new variable $y$ is defined as follows

$y(x) = 0$ if $Cn(x) = 0$

$y(x) = 1$ if $Cn(x) \neq 0$

Our quantum processor can compute $y(x)$ for all values of $x$. An equally weighted superposition of the form

$2^{-k/2} \sum_{j=0}^{2^k-1} |j\rangle$

is prepared in the $X$ register and the $Y$ register is initially kept in the state $|0\rangle_y$. Perform the operation

$Uc |x\rangle_x |0\rangle_y \rightarrow |x\rangle_x |y\rangle_y$

The state of the system $|Q\rangle$ after the operation can be represented as

$|Q\rangle = a \sum |x_s\rangle_x |1\rangle_y + b \sum |x_{ns}\rangle_x |0\rangle_y$

such that $|a|^2 + |b|^2 = 1$

where $x_s$ are the values which are solutions and $x_{ns}$ are not solutions to the condition $Cn(x) = 0$
Now measure the $Y$ register. If $Y$ register projects to the state $|1\rangle_y$, then due to the entanglement between the $X$ and $Y$ registers, the $X$ register will contain a superposition of numbers that are solutions. Measuring the $X$ register will yield a solution. Due to the fact that the number of solutions in the initial superposition is less compared to the number of non solutions, $|a|^2 << |b|^2$. Therefore probability that the $Y$ register projects to the state $|1\rangle_y$ is very less due to the fact that out of the $2^k$ entries, only $n$ of them are the solutions to the condition. Thus the probability to get $|1\rangle_y$ while measuring the $Y$ register is only $n/2^k$. This means that most of the times the $Y$ register is projected to the state $|0\rangle_y$. If this is the case, the $X$ register will now contain a superposition of numbers that are not solutions to the condition $Cn(x) = 0$. We denote this register by $X^\sim$.

With this $X^\sim$ register, how can we obtain a superposition of solutions? For this purpose we prepare a superposition of all numbers between 0 and $2^k - 1$ in a new register $Z$, and by interfering the $X^\sim$ register and the $Z$ register common modes of the registers get vanished and now $Z$ register will contain a superposition of solutions and measuring $Z$ register, we get one of the solutions.

3.1 The algorithm can be written as

Step1:
Prepare an equally weighted superposition of all numbers between 0 and $2^k - 1$ in the $X$ register.

Step2:
Perform the operation $Uc |x\rangle_x |0\rangle_y \rightarrow |x\rangle_x |y\rangle_y$

Step3:
Measure $Y$ register; If one is obtained measure the $X$ register. If zero is obtained, perform the interference operation and measure the $Z$ register.

4 Conclusion

This General method can be applied to all cases where we can classify the numbers in the initial superposition as ‘solution’ and ‘not solution’ to the given condition. For a classical computer, to solve this problem, it has to check all the numbers between $0$ and $2^k - 1$, which
requires $2^{k-1}$ steps on the average. A Quantum computer working on Grover’s search algorithm will take around $2^{k/2}$ steps. The method that we have discussed above, requires only a finite number of steps, which is independent of the number of cases that has to be considered classically. Hence, for utilizing the advantages of the quantum parallelism in full, this method or something similar to this has to be developed. The proposed interference step needs further investigation. I hope that an efficient method for performing this operation can be found through further research.

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