Mathematical model of bisubject qualimetric arbitrary objects evaluation

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Abstract. An analytical basis and the process of formalization of arbitrary objects bisubject qualimetric evaluation mathematical model information spaces are developed. The model is applicable in solving problems of control over both technical and socio-economic systems for objects evaluation using systems of parameters generated by different subjects taking into account their performance and priorities of decision-making.

1. Statement of the research problem
From the standpoint of system analysis, an object of any nature with a certain system of characteristics can be analysed from the perspective of different subjects, each of which generates its own system of parameters by which the analysed object can be described [1, 2, 4]. If such systems of parameters are of numerical nature, there is a problem of inconsistency of the systems parameters that characterize the object from the perspective of various subjects and the lack of evaluation methods and means [6, 7].

In this study, we consider the problem of forming a mathematical model to convert the levels of the object quality measuring in the system of one subject parameters to the levels of its measurement in the system of another subject parameters.

2. Analytical basis of the mathematical model
The mathematical model of arbitrary objects bisubject qualimetric evaluation is based on the following axiomatic basis ensuring the integrity of the subsequent multivariate analysis of data characterizing these objects.

1. There is some \( n \)-dimensional \( (n \to \infty) \) information space \( N \) determining a system of parameters describing a multi-parameter object from the perspective of the subject \( \Omega \), \( m \)-dimensional \( (m \to \infty) \) information space \( M \) determining a set of parametric clusters that describe superpositionally synergistically generated characteristics of the object from the position of the subject \( \Omega \), and \( l \)-dimensional \( (l \to \infty) \) information space \( L \) determining a system of parameters describing a multi-parameter object from the perspective of the subject \( \Xi \).

2. The \( m \)-dimensional \( (m \to \infty) \) information space \( M \) and the \( l \)-dimensional \( (l \to \infty) \) information space \( L \) are subsets of the \( n \)-dimensional \( (n \to \infty) \) information space \( N \). The elements of information spaces \( N, M \) and \( L \) are information parameters that describe the studied object. The information parameters are the parameters containing numeric information about the object.
3. There are sets $D, A$ and $B$, the elements of which are, respectively, parameters $D_i, i=1:n$ describing the multivariable object from the position of the subject $\Omega$, parametric clusters $A_j, j=1:m$ describing superpositional synergistically generated characteristics of the object from the position of the subject $\Omega$, and parameters $b_j, k=1:l$ describing the multivariable object from the position of the subject $\Xi$. 

4. Each $i$ th, $i=1:n$ axis $O_N N_i$ ($[O_N N_i]$) ray) of the space $N$ maps a set of numerical values of the parameter $D_i, i=1:n$. On the $[O_N N_i]$ ray, a vector $e_{N_i}$ with starting point $O_N$ is marked. If the point is $D_{pi} \in [O_N N_i]$ the vector is $\overline{O_N D_{pi}} = q_{pi} \cdot e_{N_i}$, where $q_{pi}$ is a coordinate of the vector $\overline{O_N D_{pi}}$ on the axis $O_N N_i$ interpreted as a numeric value of the parameter $D_i, i=1:n$, which characterizes the $p$ th object from the position of the subject $\Omega$.

5. Each $j$ th, $j=1:m$ axis $O_M M_j$ ($[O_M M_j]$ ray) of the space $M$ maps a set of numerical values of the parametric cluster $A_j, j=1:m$. On the ray $[O_M M_j]$ there is a vector $e_{M_j}$ with the starting point $O_M$. If the point is $A_{pj} \in [O_M M_j]$, the vector is $\overline{O_M A_{pj}} = s_{pj} \cdot e_{M_j}$, where $s_{pj}$ is a coordinate of the vector $\overline{O_M A_{pj}}$ on the axis $O_M M_j$, interpreted as a numeric value of the parametric cluster $A_j, j=1:m$, which characterizes the $p$ th object from the position of the subject $\Omega$.

6. Each $k$ th, $k=1:l$ axis $O_L L_k$ ($[O_L L_k]$ ray) of the space $L$ maps a set of numerical values of the parameter $B_k, k=1:l$. On the ray $[O_L L_k]$ there is a vector $e_{L_k}$ with the starting point $O_L$. If the point is $B_{pk} \in [O_L L_k]$, the vector is $\overline{O_L B_{pk}} = h_{pk} \cdot e_{L_k}$, where $h_{pk}$ is a coordinate of the vector $\overline{O_L B_{pk}}$ on the axis $O_L L_k$, interpreted as a numeric value of the parameter $B_k, k=1:l$, which characterizes the $p$ th object from the position of the subject $\Xi$.

3. The process of the mathematical model information spaces formalizing

Formalization of the arbitrary objects bisubject qualimetric evaluation mathematical model information spaces is performed on its axiomatic basis and is as follows.

1. The set of parameters describing the $p$ th object from the position of the subject $\Omega$ is displayed in the $n$-dimensional space $N$ with a point $N_p (q_{p1}; q_{p2}; \ldots q_{pr}; \ldots q_{pn})$, where $q_{pi}$ is a projection of the point $N_p$ on the axis $O_N N_i, r=1:n$.

2. Each $i$ th, $i=1:n$ axis $O_N N_i$ has a quantitative non-negative scale of values, i.e. each point $q_{i}$ on the $i$ th axis corresponds to a certain numerical value of the parameter $D_i$ that describes the object from the position of the subject $\Omega$. In particular, the value $q_{pr} = 0$ indicates that the parameter $D_r, r=1:n$, is not used in the description of the $p$ th object, and the value $q_{pr} \neq 0$ means that the parameter $D_r$ is used in the description of the $p$ th object.

For the further analysis of values $q_{i}, i=1:n$, it is advisable to establish qualimetric levels (numerical values) of the parameter $D_i$: threshold $Q^\min_i$, advanced $Q^i$ and high $Q^w_i$. The value $q_{pi}$ at which $q_{pi} \geq Q^w_i$, implies that the $p$ th object has a high level value of the parameter $D_i$ that describes it from the position of the subject $\Omega$. 

A set of characteristics $X_p$ describing the $p$ th object from the position of the subject $\Omega$ is functionally dependent on the numerical values of all the coordinates of the point $N_i$, $i = 1: n$ of the information space $N$:

$$X_p = F_1(q_{p1}; q_{p2}, \ldots; q_{pj} \ldots q_{pm})$$ (1)

In the case where the parameter $D_r$, $r = 1: n$, is not used in the description the $p$ th object, the expression (1) becomes:

$$X_p = F_1(q_{p1}; q_{p2}, \ldots; q_{pr-1}; \Omega, q_{pr+1} \ldots q_{pm})$$ (2)

3. Each $j$ th, $j = 1: m$ axis $O_M M_j$ has a quantitative non-negative scale of values, i.e. each point $s_j$ on the $j$ th axis corresponds to a certain numerical value of the parameter $A_j$ that describes the object from the position of the subject $\Omega$. In particular, the value $s_{pw} = 0$ indicates that the parametric cluster $A_w$, $w = 1: m$, is not used in the description of the $p$ th object, and the value $s_{pw} \neq 0$ means that the parametric cluster $A_w$ is used in the description of the $p$ th object.

Characterization of the $p$ th object by all parametric clusters $A_w$, $w = 1: m$, is reflected in the $m$-dimensional space $M$ with the point $M_p(s_{p1}; s_{p2}; \ldots; s_{pw}; \ldots; s_{pm})$, where $s_{pw}$ is a projection of the point $M_p$ on the axis $O_M M_w$, $w = 1: m$.

For the further analysis of the values $s_j$, $j = 1: m$, it is advisable to establish the qualimetric levels (numerical values) of the parametric cluster $A_j$: threshold $s_{\text{max}}$, advanced $S^w_j$ and high $S^w_j$. The value $s_{pj}$, at which $s_{pj} \geq S^w_j$, implies that the $p$ th object has a high level value of the parametric cluster $A_j$ that describes it from the position of the subject $\Omega$.

Superposition characteristics $Y_p$ describing the $p$ th object from the position of the subject $\Omega$ functionally depends on the numerical values of all the coordinates of the point $M_j$, $j = 1: m$ of the information space $M$:

$$Y_p = F_2(s_{p1}; s_{p2}; \ldots; s_{pw}; \ldots; s_{pm})$$ (3)

In the case where the parametric cluster $A_w$, $w = 1: m$, is not used in the description of the $p$ th object, the expression (3) becomes:

$$Y_p = F_2(s_{p1}; s_{p2}; \ldots; s_{pw-1}; \Omega, s_{pw+1} \ldots s_{pm})$$ (4)

Consequently, the value $s_j$ of the parametric cluster $A_j$, $j = 1: m$, is functionally dependent on the values of all coordinates $q_i$, $i = 1: n$, of the point $N_i$ of the information space $N$ corresponding to the numerical values of the parameters $D_i$, $i = 1: n$, included in the analyzed parametric cluster $A_j$ in the form:

$$s_j = f(q_1; q_2; \ldots; q_i; \ldots q_n)$$ (5)

In the case where the parameter $D_r$, $r = 1: n$, is not included in the parametric cluster $A_j$, $j = 1: m$ the expression (5) becomes:

$$s_j = f(q_1; q_2; \ldots; q_{r-1}; \Omega, q_{r+1} \ldots q_n)$$ (6)

In addition, the superposition characteristic $Y_p$ of all parametric clusters describing the $p$ th object from the position of the subject $\Omega$, is also determined by the values of the characteristics set $X_p$. 


describing the $p$ th object with a system of parameters $D_i, \; i = \overline{1:n}$, from the position of this subject, i.e.:

$$Y_p = \Psi(X_p). \tag{7}$$

4. Each $k$ th, $k = \overline{1:l}$ axis $O_kL_k$ has a quantitative non-negative scale of values, i.e. each point $h_k$ on the $k$ th axis corresponds to a certain numerical value of the parameter $B_k$ that describes the object from the position of the subject $\Xi$. In particular, the value $h_{pv} = 0$ indicates that the parameter $B_v, \; v = \overline{1:l}$, is not used in the description of the $p$ th object, and the value $h_{pv} \neq 0$ means that this parameter is used in the description of the $p$ th object.

Regarding the subsequent analysis of values $h_k, \; k = \overline{1:l}$, it is advisable to establish the qualimetric levels (numerical values) of the parameter $B_k$: threshold $H_{min}$, advanced $H^{+}$, and high $H^{**}$. For example, the value $h_{pk}$, at which $h_{pk} \geq H^{**}$, implies that the $p$ th object has a high level value of the parameter $B_k$ that describes this object from the position of the subject $\Xi$.

The value $h_k$ of the parameter $B_k, \; k = \overline{1:l}$, is functionally dependent on the values $q_i$ of the parameters system $D_i, \; i = \overline{1:n}$, as the coordinates of the point $N_i$ of the information space $N$:

$$h_k = g(q_1; q_2; \ldots; q_i; \ldots q_n). \tag{8}$$

In the case where the parameter $D_r, \; r = \overline{1:n}$, describing the $p$ th object from the position of the subject $\Omega$ is not used as a component of the parameters set $B_k, \; k = \overline{1:l}$, describing the $p$ th object from the position of the subject $\Xi$, the expression (8) becomes:

$$h_k = g(q_1; q_2; \ldots; q_{r-1}; 0; q_{r+1}; \ldots q_n). \tag{9}$$

5. The system of parameters possessed by the $p$ th object from the position of the subject $\Xi$ is displayed in the $l$-dimensional space $L$ by the point $L_p(h_{p1}; h_{p2}; \ldots; h_{pv}; \ldots; h_{pl})$, where $h_{pv}$ is a projection of the point $L_p$ on the axis $O_kL_k, \; v = \overline{1:l}$.

Characteristic $Z_p$ that describes the $p$ th object from the position of the subject $\Xi$ is functionally dependent on the values of all coordinates of the point $L_k, \; k = \overline{1:l}$, of the information space $L$:

$$Z_p = F_k(h_{p1}; h_{p2}; \ldots; h_{pv}; \ldots; h_{pl}). \tag{10}$$

In the case where the parameter $B_k, \; k = \overline{1:l}$, is not used in the description of the $p$ th object, the expression (10) becomes:

$$Z_p = F_k(h_{p1}; h_{p2}; \ldots; h_{pk-1}; 0; h_{pk+1}; \ldots; h_{pl}). \tag{11}$$

In addition, the value of the characteristics set $Z_p$ that describes the $p$ th object from the position of the subject $\Xi$ is functionally determined by the value of the coordinates set $X_p$ describing the $p$ th object from the position of the subject $\Omega$ by means of the parameters system $D_i, \; i = \overline{1:n}$, i.e.:

$$Z_p = G(X_p). \tag{12}$$

Consequently, there is a functional relationship between the value of the characteristics set $Z_p$ describing the $p$ th object from the position of the subject $\Xi$ and the total synergetic superposition characteristic $Y_p$ of all parametric clusters describing the $p$ th object from the position of the subject $\Omega$, which can be represented as:

$$Z_p = \Phi(Y_p). \tag{13}$$
4. Conclusion
The developed axiomatic basis of the arbitrary objects bisubject qualimetric evaluation mathematical model is based on the formalization of the two information spaces, one of which is determined by the system of parameters describing the $p$th object from the position of the subject $\Omega$, and the other one – by the system of parameters describing the same object from the position of the subject $\Xi$.

In the process of the formed axiomatic basis practical use, subsets of information spaces are determined, which are directly involved in the description of the object from the position of the specific subjects, the system of their parameters descriptions, as well as units of measure. The problem of specific relationship (12) and (13) identification is solved using the actual description of the same object from the positions of different subjects.

This mathematical model is applicable to both technical and social objects [3-5]. For example, the quality of any type of equipment can be described by its technical and technological characteristics, which have a great degree of importance from the standpoint of the manufacturer, and the performance, which is a priority of the consumer. A similar approach can be used when searching for solutions to the following problem: how, using the values of the graduates’ educational achievements levels reflected in their high school diploma, to determine the degree of their professional competencies formation, which is essential for employers while filling the vacant posts of technical officers.

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