ACCRETION DISK WARPING BY RESONANT RELAXATION: THE CASE OF MASER DISK NGC 4258

MICHAL BREGMAN AND TAL ALEXANDER
Faculty of Physics, Weizmann Institute of Science, P.O. Box 26, Rehovot 76100, Israel
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ABSTRACT

The maser disk around the massive black hole (MBH) in active galaxy NGC 4258 exhibits an $O(10^6)$ warp on the $O(0.1 \text{ pc})$ scale. The physics driving the warp is still debated. Suggested mechanisms include torquing by relativistic frame dragging or by radiation pressure. We propose here a new warping mechanism: resonant torquing of the disk by stars in the dense cusp around the MBH. We show that resonant torquing can induce such a warp over a wide range of observed and deduced physical parameters of the maser disk.

Key words: accretion, accretion disks – galaxies: individual (NGC 4258) – galaxies: nuclei – masers – stellar dynamics

1. INTRODUCTION

There is compelling evidence that massive black holes (MBHs) of mass $10^6 M_\odot \lesssim M_* \lesssim \text{ few } \times 10^9 M_\odot$ exist in the centers of most galaxies (Ferrarese & Merritt 2000; Gebhardt et al. 2003; Shields et al. 2003). The MBHs acquired most of their mass by efficient luminous accretion (Soltan 1982; Yu & Tremaine 2002), which implies accretion from an optically thick, geometrically thin accretion disk (Shakura & Sunyaev 1973). On larger spatial scales, such disks may fragment and form stars on disk-like orbits (Kolygalov & Syunyaev 1980; Shlosman & Begelman 1989; Goodman 2003), as observed in the Galactic center (Beloborodov & Levin 2004; Paumard et al. 2006) and elsewhere (Pizzella et al. 2002). Gas disks, often marginally stable and with properties similar to star disk progenitors (Milošavljević & Loeb 2004), are detected in the radio by H$_2$O maser emission (for a list of known circumnuclear maser disks, see Maloney 2002; for more candidates, see Braatz & Gugliucci 2008).

The interest in processes that affect accretion disks stems from the possible implications for MBH growth, for the luminosity evolution of active galactic nuclei (AGNs) and for nuclear star formation. Conversely, the disk properties may provide information about dynamical processes that operate very close to the MBH, and may be relevant for other phenomena there, such as tidal interactions with the MBH or the emission of gravitational waves (e.g., Alexander 2005, 2007).

Maser-emitting nuclear accretion disks are unique, clean probes of the near environment of MBHs. The first discovered and best-studied maser disk, NGC 4258, is also the thinnest and most Keplerian (to better than 1%; Maloney 2002) nuclear disk yet discovered. Radio H$_2$O maser observations of the disk morphology, velocities, and accelerations allow accurate measurements of the MBH mass and the distance to the host galaxy (Herrnstein et al. 1996). Together with optical and X-ray observations, these provide estimates of the mass accretion rate through the disk (Neufeld & Maloney 1995). The NGC 4258 maser disk shows a clear $O(10^6)$ warp on the few $\times 0.1$ scale (Herrnstein et al. 1996), whose origin is still a matter of debate. Warps are possibly also observed in the maser disks of Circinus (Greenhill et al. 2003) and NGC 3393 (Kondratko et al. 2008).

Several mechanisms were proposed to explain the warped disk in NGC 4258. The Bardeen & Petterson (1975) (BP) effect (torquing of a viscous disk by the general relativistic frame dragging) can warp a disk that is initially misaligned with the MBH spin by dragging its inner regions to the MBH equatorial plane (lodato & Pringle 2006; Caproni et al. 2007; Martin 2008). Instability of a thin disk to radiation pressure from the central source can result in the amplification of a small pre-existing warp (Pringle 1996; Maloney et al. 1996). The disk’s self-gravity can support equilibrium-warped disk configurations (Ulubay-Siddiki et al. 2007), if these are excited by other means. One feature shared by these mechanisms is that they require some initial asymmetry in the disk or disk/MBH alignment for the warp to develop.

Here we consider the implications of the dense stellar cusp around the gas disk. Various dynamical scenarios predict the formation of steep stellar density cusps $n(r) \propto (r/r_h)^{-\gamma}$ ($0.5 \lesssim \gamma \lesssim 2.5$) within the MBH radius of influence $r_h$ (e.g., Bahcall & Wolf 1977). The empirical $M_*/\sigma$ correlation implies that the cusp density scales as $M_*^{-1/2}$, so that the relatively low-mass MBHs, where maser disks are found, are surrounded by a very dense stellar cusp (Alexander 2007). Strong mass segregation (Alexander & Hopman 2009), if present, further concentrates stellar-mass black holes (SBHs) to the center. The stars in the cusp move in a nearly spherical potential dominated by the MBH, and therefore orbit on nearly fixed planes. This symmetry leads to the rapid torquing of any test particle by the process of resonant relaxation (RR; Rauch & Tremaine 1996 (RT96); Hopman & Alexander 2006). In particular, gas streams on circular orbits undergo “vector RR,” which changes the orbital plane, but not their eccentricity. RR thus induces warps by exchanging angular momentum between the stars and the disk.

This Letter is organized as follows. RR dynamics are summarized in Section 2. An analytical model for RR torquing of a disk is described in Section 3, and applied to the NGC 4258 maser in Section 4. Our results are presented in Section 5 and summarized in Section 6.

2. RESONANT RELAXATION DYNAMICS

RR is a rapid relaxation mechanism of the angular momentum $\mathbf{L}$, which operates in near-symmetric potentials that suppress the evolution of the stellar orbits (e.g., fixed ellipses in a Kepler potential, or planar rosettes in a spherical potential). In such systems, the residual torque on a test mass by the orbit-averaged mass distributions of $N_\star$ stars of mass $M_\star$ within distance $r$ from the center, $|\mathbf{T}| \sim N_\star^{1/2} G M_\star/r$, remains constant on timescales shorter than the coherence time $t_0$, as long as perturbations due to
deviations from the perfect symmetry remain small. The change in \( L \) then grows coherently as \( t \) on timescales \( P \ll t \ll t_0 \), where \( P = 2\pi \sqrt{r^3}/G(M_*+N,M_*) \) is the circular orbital period. When \( t \gg t_0 \), the large accumulated change over the coherence time, \( \langle \Delta L \rangle = |T|t_0 \), becomes the step size of a rapid random walk, \( \langle \Delta L/L \rangle(t) = |\langle \Delta L \rangle/L_0 \rangle/|\sqrt{t/t_0} = \theta/|T/T_{RR} | \), where \( L_0 = \sqrt{G(M_*+N,M_*)r} \) is the circular specific angular momentum for the test mass’ orbital energy, and \( T_{RR} \) is thereby defined as the RR timescale, when \( |\Delta L/L_0| = 1 \). The slower the loss of coherence (“quenching”), the more efficient is RR (shorter \( T_{RR} \). RR can be orders of magnitude faster than non-coherent two-body relaxation.

In a near-Keplerian potential the orbits are nearly fixed ellipses, and the coherence time is set by the faster of general relativistic (GR) precession or precession by the potential of the enclosed stellar mass. In this case, RR can change both the direction and magnitude of \( L \) (“scalar RR”). However, the torques along \( L \) fall to zero for a test mass on a circular orbit (Gürkan & Hopman 2007), and so circular orbits, such as those of gas streams in an accretion disk, remain circular \((\Delta T_\perp = 0)\), but their orientation evolves rapidly \((\Delta T_\perp \neq 0)\) until RR itself randomizes the orbital planes to a degree where coherence is lost (“self-quenching”). Vector RR is “vector RR” until RR itself randomizes the orbital planes to a degree where coherence is lost (“self-quenching”). Vector RR is not quenched by precession \((\Delta T_\perp \neq 0)\) between orbit-averaged rosettes, and so can it operate both very near the MBH, where GR precession is fast, and far from the MBH where the potential is spherical but no longer Keplerian. The coherence time for the GR precession is fast, and far from the MBH where the potential \( L_{\perp}/L_0 \) by Equation (1), \( f_w = w_2/w_1 = (R_2/R_1)^{-\gamma/2} \) \((f_w = 0.6\) for \( \gamma = 1.5\)). In the limit of small warp inclination angles, as is the case here, \( f_w \approx \omega_0/\gamma_1 \) and \( \omega \simeq \omega_1/\gamma_1 \), \((\frac{1}{2} - 2f_w \cos(\varphi_1 - \varphi_2) + f_w^2) \). The maser disk in NGC 4258 displays an \( \omega = 8^\circ \) relative warp between the limits of the masing region at \( R_1 = 0.14, R_2 = 0.28 \) (Herrnstein et al. 1996), which after averaging over uncorrelated \( \varphi_1 \) and \( \varphi_2 \), implies that locally, \( \delta_1 \approx 10^\circ \) and \( \delta_2 \approx 6^\circ \).

3. ACCRETION DISK WARping BY RESonANT RELAXATION

The gas in a thin accretion disk flows slowly into the MBH on nearly circular orbits. Over time, the orbit associated with a ring element of the disk shrinks, and the torques on it change. RR can significantly affect the ring only if the typical RR timescale, \( T_{RR} \), is shorter than the radial inflow timescale, \( t_r \sim R^2/v \), which is set by the disk’s tangential in-plane viscosity coefficient \( \nu \), where \( R \) is the radius along the disk’s mid-plane. The presence of a warp also induces a vertical shear viscosity \( \nu_{zz} \), which resists the external torques by diffusing the warp on a timescale \( t_w = R^2/\nu \). Effective warping thus also requires that \( T_{RR} \) is shorter than the warp diffusion timescale. In addition to these timescale constraints, the torquing stars must carry enough residual angular momentum to exchange with the disk and warp it. Therefore, both a timescale condition (Section 3.1) and an angular momentum condition (Section 3.2) must be satisfied for RR warping to occur.

3.1. RR Timescale Condition

The vector RR timescale (Equation (2)) estimated above for stars enclosed within radius \( r \) and acting on a mass orbiting at the same typical radius \( R \), can be easily generalized to a mass (disk ring element) at a different radius \( R \), by noting that \( T_{RR} \) scales with the torque as \( |T|^{-2} \), and that the typical torque on the ring element is decreased by a factor \( r/R \) when \( R > r \) and by a factor \( R/r \) when \( R < r \).

\[
T_{RR}(r, R) = T_{RR}(r/R^2) \propto r^{2(\theta+\nu)/2},
\]

where \( \Theta = \text{sgn}(r-R) \), and the last proportionality holds in the Keplerian limit. In order to efficiently warp the disk by a factor \( w \), \( T_{RR} \) must be shorter by at least some factor \( \epsilon_t \leq 1 \) than the faster of the two viscous processes, \( t_{w} = \text{min}(t_r, t_w) \).

\[
\epsilon_t t_w(R) \geq wT_{RR}(r, R).
\]

Equality in Equation (4) defines an upper limit \( r^{(+)}_t \) for a given \( R < r \) and a lower limit \( r^{(-)}_t \) for \( R > r \) (assuming \( 0 < \gamma < 4 \); recall that \( 0.5 < \gamma < 2.5 \) for realistic cusp models)

\[
r^{(\theta)}(R) = \left[ \frac{\sqrt{N_{iso} M_*/M_0}}{P_{iso}} \frac{R}{t_{iso}} \right]^{(1/2(\theta+\nu)/2)},
\]

where \( N_{iso} \) is the formally extrapolated number of stars within the last stable circular orbit \( t_{iso} \), and \( P_{iso} \) is the formal Keplerian orbital period there. When there exists a solution with \( r^{(-)}_t < r^{(+)}_t \) then stars in the volume \( r^{(-)}_t \leq r \leq r^{(+)}_t \) induce fast enough vector RR on the disk at \( R \) to warp it by a factor \( w \).

Note that the timescale condition implies that the RR torque dominates over the viscous torques, since \( |T_{RR}| \sim L_c/T_{RR} > L_c/t_w \sim \max(|T_c|, |T_w|) \).

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3.2. RR Angular Momentum Condition

Efficient torquing of the disk mass in a ring \((R, R + \Delta R)\) by stars in a spherical shell \((r, r + \Delta r)\) requires that the disk’s angular momentum be smaller by at least some factor \(\epsilon_L \leq 1\) than the random residual angular momentum carried by the stars, assumed to be isotropically distributed (this is a conservative criterion, since it neglects the possibility of warping by the transfer of angular momentum from the disk to the stars). The total angular momentum in both the disk and the stellar cusp is dominated by the large scales (e.g., a disk with surface density \(\Sigma \propto R^{\gamma}\) (Section 4.2) and temperature that falls with \(R\) has \(s > -3/2\) (Equation (9)) and \(L_d \Delta M_d \propto R^{3/2 + n} \Delta R\), where \(M_d\) is the disk mass enclosed within \(R\); a stellar cusp with \(\gamma < 2\) (Section 4.1) has \(L_c \Delta (\sqrt{\Sigma} M_c) \propto R^{1-\gamma/2} \Delta R\). We therefore approximate the angular momentum criterion by requiring that the angular momentum in the disk within \(R\) be smaller than the residual angular momentum in the stars within \(r\),

\[
w M_d(R) L_c(R) \leq \epsilon_L \sqrt{\Sigma_c(r)} M_c L_c(r) \propto (r/r_\text{iso})^{2-\gamma/2}.
\]

Equation in Equation (6) defines a lower limit \(r_L\) (assuming \(\gamma < 4\))

\[
\frac{r_L}{r_\text{iso}} = \frac{1}{\epsilon_L \sqrt{\Sigma_c} M_c} \left( \frac{R}{r_\text{iso}} \right)^{1/2(2-\gamma/2)}.
\]

When there exists a solution such that \(r(\gamma) = \max (r_\gamma, r_L) < r^{(\gamma)}\), then the stars in the volume \(r^{(\gamma)} < r < r^{(+\gamma)}\) carry enough residual angular momentum to significantly torque the disk enclosed within \(R\), on a short enough timescale.

Note that the angular momentum condition implies that the RR torque dominates over the disk’s self-gravity, since the self-gravity torques \(|T_d| \sim GM_d/r \ll G \sqrt{\Sigma} M_c/r \sim |T_{RR}|\).

4. NGC 4258 MODEL

In order to apply the RR torquing mechanism to the maser disk in NGC 4258, it is necessary to specify the stellar cusp density profile, the disk’s viscous timescale, \(t_{visc}(R)\) (Equation (4)), and its enclosed mass \(M_d(R)\) (Equation (6)). We adopt here simple, observationally motivated models of the cusp and disk.

4.1. Galactic Nucleus Model

NGC 4258 is a spiral galaxy at a distance of \(7.2 \pm 0.3\) Mpc with an \(M_\bullet \sim 3.7 \times 10^7 M_\odot\) central MBH (Herrnstein et al. 1999). The empirical \(M_\bullet/\sigma\) relation (Ferrarese & Merritt 2000; Gebhardt et al. 2003; Shields et al. 2003) implies that \(\sigma \simeq 150 \text{ km s}^{-1}\) and the MBH’s radius of dynamical influence is \(r_h = GM_\bullet/\sigma^2 \simeq 7\) pc, where the enclosed stellar mass is \(\mu_n M_\bullet\), with \(\mu_n \sim O(1)\). On the distances spanned by the maser spots, \(R_1 = 0.14\) to \(R_2 = 0.28\) pc (Herrnstein et al. 1996), main-sequence stars dominate the population with a relatively flat \(\sim 1.5\) power-law density profile, typical of the low-mass component in a mass-segregated population (Bahcall & Wolf 1977; Alexander & Hopman 2009). We model the central cusp of NGC 4258 as a power-law cusp, \(N_c(< r) = \mu_n M_\bullet (M_c/r_h)^{3-\gamma}\), with \(\gamma = 1.5\), \(r_h = 7\) pc, \(\mu_n = 2\) (the formal value for an MBH-less singular isothermal distribution), and \(M_\bullet = 1 M_\odot\) stars. The corresponding vector RR timescale is \(T_{RR} \sim \text{few} \times 10^7\) yr across the disk.

4.2. Maser Disk Model

Following Caproni et al. (2007), we assume a power-law surface density profile for the disk, \(\Sigma(R) = \Sigma_1 (R/R_1)^\alpha\). We assume the structure equations of a stationary, geosymmetrically thin, optically thick Keplerian disk around an MBH of mass \(M_\bullet\) (e.g., Frank et al. 2002). These can be expressed in terms of the mass accretion rate \(\dot{M}\), the dimensionless viscosity parameter \(\alpha \equiv v/c_s H\), the surface density power-law index \(s\), and \(\rho(R_1)\), the mid-plane mass density of H2 at \(R_1\) (the disk mass is assumed to be mostly in H2). The normalization \(\Sigma_1\) at \(R_1\) is fixed by

\[
\Sigma^3 = M \rho^2/\alpha \Omega,
\]

where \(\Omega = \sqrt{GM_\bullet/R^3}\) is the Keplerian frequency, \(H\) is the disk scale height, and \(c_s\) is the isothermal sound speed,

\[
H^2 = \dot{M}/2 \pi \alpha c_s, \quad c_s^2 = \dot{M} \Omega/(\pi \alpha \Sigma).
\]
For an ideal gas, \( c_s = \sqrt{kT/\mu} \) with \( T \) the gas temperature and \( \mu \) the mean molecular mass (\( \mu = 2m_p \) assumed). The radial inflow and the warp diffusion viscous timescales are then

\[
\tau_r \sim R^2/\alpha c_s H, \quad \tau_w \sim R^2/\alpha^2 c_s H, \tag{10}
\]

where \( \alpha = \max\{f(\alpha), \alpha_{\text{max}}\} \) with \( f(\alpha) \simeq (1 + 7\alpha^2)/(4(1 + \alpha^2)) \) (Ogilvie 1999) and \( \alpha_{\text{max}} \sim 3-4 \), with some uncertainty (Lodato & Pringle 2007)

Caproni et al. (2007) combine observational and theoretical constraints and deduce that the physical parameters of the NGC 4258 maser disk lie in the range \( 10^{-6} \lesssim \epsilon M \lesssim 10^{-4} \msun \ yr^{-1} \), where \( \epsilon \) is the radiative accretion efficiency (but see higher estimate \( M \gtrsim 10^{-3} \msun \ yr^{-1} \) for the ADAF model; Lasota et al. 1996), and 0.03 \( \lesssim \alpha \lesssim 0.2 \). They consider a range of possible disk profiles, \( -2 \leq s \leq 0 \). Efficient production of H2O maser emission constrains the disk’s physical parameters: the \( \H2 \) density, \( 10^7 \text{ cm}^{-3} < n_{\H2} < 10^{11} \text{ cm}^{-3} \), the gas temperature, \( 400 \text{ K} < T < 1000 \text{ K} \), and the gas pressure, \( 10^{10} \text{ K cm}^{-3} < p/k < 10^{13} \text{ K cm}^{-3} \) (Maloney 2002). In addition, the disk aspect ratio is constrained by observations to \( H/R \lesssim 0.002 \) (Moran 2008). The disk is gravitationally stable only if \( M_\star/M_\bullet < H/R \).

5. RESULTS

Figure 1 shows the region in the disk’s \((M, \alpha, s, n_{\H2})\) parameter space where masing is possible with the aspect ratio and disk mass constraints (Section 4.2), and where both the timescale (Equation (4) with \( \epsilon_1 = 1, \alpha_{\text{max}} = 3 \)) and angular momentum conditions (Equation (6) with \( \epsilon_2 = 1 \)) are satisfied for \( 0 \lesssim \alpha \lesssim 1 \) between 0.14 and 0.28 pc, so that RR torquing can warp the disk by at least 8\( \circ \), on average. We find that such an RR-induced warp is possible over a wide range of the observed and deduced physical parameters of the maser disk. RR warping is less efficient for disks with a very steep density profile, high density, or a high mass accretion rate. We conclude that the RR mechanism can affect the accretion rate and direction, and thereby MBH mass and spin evolution.

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