Determination of necking time in tensile test specimens, under high-temperature creep conditions, subjected to distribution of stresses over the cross-section

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Abstract. The work describes an experimental research of creep of cylindrical tensile test specimens made of aluminum alloy D16T at a constant temperature of 400\textdegree C. The issue to be examined was the necking at different values of initial tensile stresses. The use of a developed noncontacting measuring system allowed us to see variations in the specimen shape and to estimate the true stress in various times. Based on the obtained experimental data, several criteria were proposed for describing the point of time at which the necking occurs (necking point). Calculations were carried out at various values of the parameters in these criteria. The relative interval of deformation time in which the test specimen is uniformly stretched was also determined.

1. Introduction
It is known that the gradual fracture of tensile test specimens of constant cross-section occurs with instability such as necking, which results from inhomogeneity of the material microstructure. Upon necking, the material loses its capacity to resist a given load, or its load-bearing capacity; therefore, it is of critical importance to be able to estimate this point of time.

In [1], it is stated that, in 1885, Consid\`ere suggested that when an elastoplastic material is examined, the maximum force criteria should be used as a necking condition. According to this condition, the thinning occurs as soon as the stress attains its maximum value in the specimen exposed to the uniaxial tension (beginning in the area, where the dependence of the tensile force $P$ on the deformation $\varepsilon$ decreases). In the following, this “maximum force approach” [1] was improved (Swift [2], Hill [3], and Hora [4]).

A more refined criterion for determining the necking time was obtained by Michel [5]. To study the creep, one of the classical methods for determining the value of necking time $\tau$ is the time approach according to which it is assumed that $\tau \approx (0.9 \div 0.99) \cdot t^*$, where $t^*$ is the time prior to the fracture. To all appearance, this approach is true for high-strength metals, e.g., such as Cr-Mo-V of steel [6]. Other criteria also reveal the necking shortly before fracture, such as those published in [7] and [8].
Table 1. Chemical composition of the alloy under study.

| Element | Value          |
|---------|----------------|
| Fe      | < 0.5          |
| Si      | < 0.5          |
| Mn      | 0.3–0.9        |
| Cr      | < 0.1          |
| Ti      | < 0.15         |
| Cu      | 3.8–4.9        |
| Mg      | 1.2–1.8        |
| Zn      | < 0.25         |
| other   | 0.15           |

A. Malygin [9] proposed a deformation approach in which the homogeneous deformation \( \varepsilon_d \) is limited by the value \( 1/n (\varepsilon_d < 1/n) \), where \( n \) is the exponent in an exponential law of the creep formation.

A specific feature of all known criteria is that a real shape of the specimen is not taken into account. The creep tests are usually carried out in a closed oven, and therefore the only characteristic of the deformed state to be measured in the course of real experiments is the dependence of the specimen elongation \( l \) on the time \( t \). In the room temperature tests, it is possible to use an approach, where the tests are periodically interrupted for measurements of the specimen shape [10]. But the use of this approach is not acceptable for studying the high-temperature creep. The development of known and generalized criteria [1–10] is stipulated exactly by this. The method of noncontacting measurements, developed by the author of [11], enables us to measure the real shape of specimens exposed to tensile stresses at a high temperature and, thereby, to register the necking occurrence.

To know the point of time at which homogeneous deformation of solids becomes inhomogeneous is of utmost interest for the correct description of material resistance to a given load. In the paper, various criteria are proposed for objective determining this point of time on the basis of real experiments carried out.

2. Setup and performance of real experiments

To carry out experiments, specimens of a circular cross-section were selected, made of D16T (Al-Cu-Mg) alloy whose chemical composition is shown in table 1. The specimen shape was selected according to the international standard ISO 783-99. The initial length of the test portion was \( l_0 = 23–31 \text{ mm} \), and the initial diameter was \( d_0 = 4 \text{ or 5 mm} \). All specimens were cut out from the same bar. For each value of the initial tensile stress \( \sigma_0 \), the tests were carried out with different amounts of specimens; \( k_3, k_4, k_5 \) were given constant values. Table 2 presents all details. In the bottom line, the values of \( \bar{\tau}_m (\%) \) are given calculated as average values of \( \bar{\tau} \) except for one maximum and one minimum value in each column.

The experiments were carried out at the Institute of Mechanics of Lomonosov Moscow State University, using the “IMEX-5” test facility. First, the specimen fixed on rods was heated up to an operation temperature, then the camera set to shoot the specimen elongation at given intervals was switched on. The specimen was quickly loaded up to a given level of initial stress \( \sigma_{0,i} \) (\( i = 1, \ldots, 4 \)) using a weight \( P_{0,i} \) in the lower part of the test facility. The further deformation was caused under creep conditions with a constant tensile force up to the fracture.

For experiments, one used a noncontacting system to measure the geometrical parameters of the specimen. The system was developed by the author of [11]. The specimen geometry was measured remotely, with pictures taken during the experiment.

The detailed data on experimental research, used in the present paper, are given in [12].

2.1. Accuracy of measurements

In addition to the optical system of measurements [11], a contact inductive sensor is fixed to measure the elongation of the specimen between the grips of the test facility. The comparison of curves corresponding to experiments 1 and 4 is illustrated in figure 1. The maximum deviation
Figure 1. Elongation $\Delta l(t)$ of specimens 1 and 4 measured by a contact method (solid line) and an optical method (dashed line).

Table 2. Necking times $\tau$ in specimens of a circular cross-section with various parameters $k_3$, $k_4$, and $k_5$ for various criteria.

| No. | $\sigma_0$ [MPa] | $t^*$ [sec] | $l_0^*$ [mm] | $\tau_3$ [sec] $\bar{\tau}_3$ [%] | $\tau_3$ [sec] $\bar{\tau}_3$ [%] | $\tau_4$ [sec] $\bar{\tau}_4$ [%] | $\tau_5$ [sec] $\bar{\tau}_5$ [%] | $\tau_5$ [sec] $\bar{\tau}_5$ [%] |
|-----|------------------|-------------|-------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| 1   | 24.5             | 692         | 35.5        | 540 (78)             | 610 (88)             | 530 (77)             | 580 (84)             | 500 (72)             | 530 (77)             |
| 2   | 24.5             | 843         | 29          | —                    | 350 (46)             | 200 (26)             | 440 (57)             | 80 (10)              | 220 (29)             |
| 3   | 24.5             | 965         | 32.5        | 750 (78)             | 850 (88)             | 770 (80)             | 820 (85)             | 760 (79)             | 820 (85)             |
| 4   | 19.6             | 1733        | 42.5        | 1600 (92)            | 1660 (95)            | 1500 (86)            | 1720 (99)            | 1500 (86)            | 1720 (99)            |
| 5   | 19.6             | 1174        | 33.4        | —                    | 820 (72)             | 800 (70)             | 990 (86)             | 630 (55)             | 850 (74)             |
| 6   | 19.6             | 1575        | 32.9        | 1310 (82)            | 1400 (88)            | 1260 (79)            | 1430 (90)            | 1100 (69)            | 1420 (89)            |
| 7   | 19.6             | 1129        | 36.4        | 900 (80)             | 900 (80)             | 610 (54)             | 915 (81)             | 610 (54)             | 885 (78)             |
| 8   | 19.6             | 1407        | 40.8        | 850 (59)             | 1400 (98)            | 780 (55)             | 1400 (98)            | —                    | 1380 (96)            |
| 9   | 19.6             | 1707        | 37.2        | 1250 (73)            | 1250 (73)            | 1500 (87)            | 1500 (87)            | 1100 (64)            | 1400 (82)            |
| 10  | 17.6             | 2865        | 33.6        | 2520 (88)            | 2700 (94)            | 700 (24)             | 2640 (92)            | 700 (24)             | 2600 (91)            |
| 11  | 17.6             | 1772        | 28.3        | 1580 (90)            | 1650 (94)            | —                    | 1450 (83)            | —                    | 1400 (80)            |

$\bar{\tau}_m$ [%] 81.3 87.3 64.2 87.4 59.7 83.5

between the methods for measuring the elongation, which are shown by the solid blue and dashed red lines, correspondingly, was 0.3 mm.

2.2. Tangent circle radius
To consider the stress distribution over the specimen cross-section according to [13], it is required to know the values of radii of longitudinal tangent circles $R$ on the smallest cross-section of the neck at an arbitrary time. The radii of tangent circles were calculated by the least-square method for small ($\sim 1–3$ mm of length) longitudinal parts of the specimen contours, measured
Figure 2. Time-dependence of the tangent circle radius $R$ on the smallest cross-section. Specimens 1, 2, 6, and 7.

on the smallest cross-section, in each picture taken by the camera. The standard dependencies of $R(t)$ are presented in figure 2. From the necking occurrence and up to the point of fracture, the radius value decreases to 5–10 mm, while almost linear fracture occurs (figure 2).

3. Determination of the necking time

3.1. General points

To determine the necking time, one uses the Heaviside function

$$\Psi(t) = H[\varphi(t, l(t), d(t), k_i, \ldots)],$$

(1)

where $\varphi$ is a function of parameters of mechanical tests, $l$ is the length of the specimen, and $d$ is its diameter. The $\Psi(t)$ functions take the values 0 or 1; the value 0 describes no deformation, and the value 1 characterizes the necking in the specimen. At $t = \tau$, stepwise changes in the function $\Psi(t)$ take place, from 0 to 1. Further, the $\Psi(t)$ functions with various types of dependence of $\varphi$ are called criteria for the necking occurrence. Additionally, we introduce the relative time $\bar{t} = t/t^*$, and from here, the relative necking time $\bar{\tau} = \tau/t^*$.

Let us consider a specimen under homogenous deformation, i.e., a virtual specimen of diameter $d^0(t)$ subjected to deformation on the basis of the same pattern of changes $l(t)$ (to be observed in a “living” experiment), and this homogenous deformation of the whole test section lasts throughout the tension time. Based on the assumption that the volume of the cylindrical specimen is constant, it follows that

$$d^0(t) = d_0 \sqrt{\frac{l_0}{l(t)}}.$$  

(2)

As for the virtual specimen, its tensile stress is constant over the cross-section and the longitudinal coordinate $y$, because there is no distorting effect of the neck. Then, for each longitudinal cross-section, we obtain

$$\sigma^0(t) = \sigma_0 \sqrt{\frac{l_0}{l(t)}}.$$  

(3)

With no initial defects of the specimen structure, the real specimen deformation proceeds in the same way as that of the virtual one, up to the necking time. At a moment $t = \tau$, the deformation of the real specimen begins to differ from that of the virtual one.
3.2. Geometrical criterion
The experiments show that the deformation processes in all the experiments are axially symmetric. At an arbitrary point of time $t$, the circular specimen is characterized by the length $l(t)$ and the distribution of diameters $d(t, y)$. Let us assume that the necking criterion has the form

$$\Psi_3(t) = H[d^0(t) - \min_y \{d(t, y)\} - k_3].$$ (4)

When the tests results were processed for each experiment according to (4), the time $\tau$ was calculated at which the value of $\Psi_3$ varies stepwise from 0 to 1. The dependence $\Psi_3(t)$ is illustrated in figure 3 for specimens 3 and 9. If the dependence $\Psi_3(t)$ fluctuates (i.e., is unstable), then the time $\tau$ is selected as an average value. The time values of necking according to (4) in all the experiments are shown in table 2. Their statistical processing allows us to concluded the following: the mathematical expectation of the necking time $\bar{\tau}_3$ for the $\Psi_3$ criterion is 81% for $k_3 = 0.1$ mm and 87% for $k_3 = 0.2$ mm.

3.3. Kinematic criteria
The necking is related to a local increase in the tensile stress in a specimen. Therefore, to according to (3), during the necking, the maximum active tension $\sigma_{\text{max}}(t)$ in the real specimen and the stress $\sigma^0(t)$ in the virtual one start to differ. A similar kinematic approach was described in [14]. If we assume that the stress is constant over the cross-section of the specimen, then $\sigma = \sigma_{m1}(t, y) = P_0/S(t, y) = 4P_0[\pi d^2(t, y)]^{-1}$, and then $\Psi_4$ becomes

$$\sigma_{m1}(t, y_{\text{min}}) = \max_y (\sigma_{m1}(t, y)),$$

$$\Psi_4(t) = H[\sigma_{m1}(t, y_{\text{min}}) - \sigma^0(t) - k_4].$$ (5)

Table 2 shows calculations of the necking time for the $\Psi_4$ criterion. The average value of the necking time $\bar{\tau}_4$, on the basis of $\Psi_4$, is 64% for $k_4 = 0.3$ MPa and 87% for $k_4 = 1.4$ MPa.

On the basis of [13, 15], we conclude that, on the occurrence of a state of inhomogeneous deformation, the distribution of axial stress over the cross-section $y_{\text{min}}$ of the minimum area is

**Figure 3.** Dependence $\Psi_3(t)$ for $k_3 = 0.1$ mm (solid line) and $k_3 = 0.2$ mm (dashed line) for cylindrically shaped specimen (experiments 3 ($a$) and 9 ($b$)).
Figure 4. Dependence $\sigma_0(t)$ (solid line), $\sigma_{m1}(t)$ (dashed line), and $\sigma_{m2}(t)$ (dotted line) for the cross-section $y_{\min}$ and $\Psi_4(t)$ and $\Psi_5(t)$ criteria for $k_4 = k_5 = 0.3$ MPa (solid line) and $k_4 = k_5 = 1.4$ MPa (dotted line) (experiments 1 (a) and 5 (b)).

not constant and can be obtained from the parabolic expression

$$\sigma(t, y_{\min}, r) = \sigma_{m1}(t, y_{\min}) \left[ 1 + \frac{0.25d^2(t, y_{\min}) - r^2}{Rd(t, y_{\min})} \right],$$

where $R$ is the radius of neck curvature on the center line, $r$ is the distance from an arbitrary point of the minimum cross-section to the specimen axis. This implies that the maximum stress value over the smallest cross-section, with the inhomogeneity taken into account, can be obtained at $r = 0$ as

$$\max_r(\sigma(t, y_{\min}, r)) = \sigma_{m2}(t, y_{\min}) = \sigma_{m1}(t, y_{\min}) \left[ 1 + \frac{d(t, y_{\min})}{4R} \right].$$

Therefore, replacing the criterion of constant stress over the specimen cross-section in the $\Psi_4$ criterion with an approach with a correcting expression according to [7] and taking $d_{\min}(t) = d(t, y_{\min})$, we obtain

$$\Psi_5(t) = H \left[ \sigma_{m1}(t, y_{\min}) \left( 1 + \frac{d_{\min}}{4R} \right) - \sigma^0(t) - k_5 \right].$$

Therefore, $\Psi_5$ and $\Psi_4$ differ in the additional additive component $P_0/[\pi Rd_{\min}(t)]$, which describes the increase in the stress at the specimen center relative to the average value due to the necking effect.

Figure 4 illustrates Experiments 1 and 5, where the curves of smoothed values of $\sigma_0(t)$, $\sigma_{m1}(t)$ and $\sigma_{m2}(t)$ are given for the cross-section $y_{\min}$, as well as values of $\Psi_4$ and $\Psi_5$ are given for the parameters $k_4 = k_5 = 0.3$ MPa and $k_4 = k_5 = 1.4$ MPa. The deviations of the $\Psi_4$ and $\Psi_5$ curves can be explained by the corresponding fluctuations of the initial (chaotic) dependence $\sigma(t)$, which is not presented in the graph.

According to $\Psi_5$, the average necking time $\bar{\tau}_5$ is 60% for $k_5 = 0.3$ MPa and 83% for $k_5 = 1.4$ MPa. Using the visual method for determining the time, we obtain the necking time equal to 61% (average value) of the time prior to fracture (curve $\sigma_{m1}(t)$) and 50.6% (curve $\sigma_{m2}(t)$). The $\Psi_4$ and $\Psi_5$ criteria, based on the experimental data, must be considered as basic ones to be used to determine the necking point.
Conclusions
The use of complex criteria (5) and (6) based on the introduction of a homogenous (virtual) specimen, allows one to conclude that the uniform deformation may take only 60% of the specimen life cycle. This must be taken into account in calculations of the load bearing capacity. As for geometrical criterion $\Psi_3$ (the difference between the diameter of a homogenous specimen and the minimum diameter of the specimen from the experiment) with $k_3 = 0.1$ mm, the necking time is 81%, and this is 87% for $k_3 = 0.2$ mm. Based on the $\Psi_4$ kinematic criteria (the difference between the stresses within a homogenous specimen and a real one) and based on the assumption that the stress value is constant over the cross-section, the necking time is 64% for $k_4 = 0.3$ MPa and 87% for $k_4 = 1.4$ MPa. For the kinematic criterion $\Psi_5$ (the difference between the stresses within a homogenous specimen and a real one) with the nonuniform character of stresses over the cross-section taken into account, the necking time is 60% for $k_5 = 0.3$ MPa and 83% for $k_5 = 1.4$ MPa. To estimate the necking time, it is most expedient to use the force criterion $\Psi_5$.

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