Photopion Production in Black-Hole Jets and Flat-Spectrum Radio Quasars as PeV Neutrino Sources

Charles D. Dermer, Kohta Murase, & Yoshiyuki Inoue

1 Space Science Division, Code 7653, Naval Research Laboratory, Washington, DC 20375-5352; charles.dermer@nrl.navy.mil
2 Institute for Advanced Study, 1 Einstein Dr., Princeton, NJ 08540, USA; murase@ias.edu
3 Institute of Space and Astronautical Science, Japan AeroSpace Exploration Agency, 3-1-1 Yoshinodai, Chuo-ku, Sagamihara, Kanagawa, 252-5210, Japan

Abstract
The IceCube collaboration has reported neutrinos with energies between ~ 30 TeV and a few PeV that are significantly enhanced over the cosmic-ray induced atmospheric background. Viable high-energy neutrino sources must contain very high-energy and ultra-high-energy cosmic rays while efficiently making PeV neutrinos. Gamma-ray Bursts (GRBs) and blazars have been considered as candidate cosmic-ray accelerators. GRBs, including low-luminosity GRBs, can be efficient PeV neutrino emitters for low bulk Lorentz factor outflows, although the photopion production efficiency needs to be tuned to simultaneously explain ultra-high-energy cosmic rays. Photopion production efficiency of cosmic-rays accelerated in the inner jets of flat spectrum radio quasars (FSRQs) is ~ 1 – 10% due to interactions with photons of the broad-line region (BLR), whereas BL Lac objects are not effective PeV neutrino sources due to the lack of external radiation fields. Photopion threshold effects with BLR photons suppress neutrino production below ~ 1 PeV, which imply that neutrinos from other sources would dominate over the diffuse neutrino intensity at sub-PeV energies. Reduction of the \( \gg \) PeV neutrino flux can be expected when curving cosmic-ray proton distributions are employed. Considering a log-parabolic function to describe the cosmic-ray distribution, we discuss possible implications for particle acceleration in black-hole jets. Our results encourage a search for IceCube PeV neutrino events associated with \( \gamma \)-ray loud FSRQs using Fermi-LAT data. In our model, as found in our previous work, the neutrino flux is suppressed below 1 PeV, which can be tested with increased IceCube exposure.

Keywords: neutrinos, \( \gamma \) rays, blazars, \( \gamma \)-ray bursts, IceCube, ultra-high energy cosmic rays

1. Introduction
The IceCube Collaboration has reported evidence for extragalactic neutrinos (1), which opens up an important multi-messenger connection between photons, neutrinos, and high-energy cosmic rays. Using two years of IceCube data in its 79 and 86 string configuration, 28 fully contained events were identified between 30 and 1200 TeV, of which 21 were shower-like, and the remainder track-like. Using three years of data, 37 events are reported, with 28 shower-like and 9 track-like events (2). All of the highest energy neutrinos, with energies of 1040, 2044 TeV, and energy uncertainties of \( \approx 12\% \), are shower-like. The ratio of showers and tracks is a consequence of the larger effective area of IceCube for neutrino interactions (3), different neutrino-flavor opacities through the Earth, and the analysis requirement that the events are fully contained.

The combined significance of the data is \( \approx 5.7 \sigma \) over predicted background, systematic uncertainties and uncertain charm contribution, with the significance of separate low-energy (LE, \( \approx 25 – 500 \) TeV) and high-energy (HE, \( \gtrsim 0.5 \) PeV) neutrino enhancements less significant. Indeed, the available evidence for a suppression of neutrino production near ~ 0.5 PeV is not statistically significant, either in the two-year or three-year data sets. At higher energies, between \( \approx 2 \) and 10 PeV, 3 – 6 neutrinos were predicted for a proton spectrum with index \( -2 \), whereas none were reported (1,2). Thus the existence of a high-energy cutoff above the energies of the two ~ 1 PeV events and the recently announced ~ 2 PeV event (2, 4, 5), is statistically favored though not definitely established. The neutrino flux is adequately fit with a ~2 neutrino spectrum down to low energies (1), and is inconsistent with a diffuse cosmogenic origin of neutrinos from UHECRs in the intergalactic medium (2, 3, 8).

Here we consider whether neutrino production with photons of the broad-line region (BLR) of flat spectrum radio quasars (FSRQs), can account for the features of the PeV neutrinos detected by IceCube. This process was first considered in detail by Atoyan & Dermer (9), though suggestions of neutrino production from FSRQs were made earlier (10, 11). If atomic-line radiation in the BLR dominates neutrino production through photopion processes, suppression of the neutrino flux from FSRQs at energies \( \lesssim 1 \) PeV is expected. These cutoffs are easily understood by noting that for threshold pion production, \( \gamma_p \epsilon_{\pi} \gtrsim m_{\pi}/m_\nu \approx 300 \). For neutrinos formed with \( \approx 5\% \) of the incident proton energy, then a cutoff is expected at \( E_\nu \approx 0.05 m_\nu c^2 (m_{\pi}/m_\nu) \approx 1 \) PeV, taking \( \epsilon_{\pi} \approx 2 \times 10^{-5} \) for the Lyman \( \alpha \) photon energy.

In this paper, we study the emission of HE neutrinos produced by photopion processes in extragalactic black-hole jet sources, focusing in particular on FSRQs. Inefficient Fermi ac-
celeration competing with strong photodisintegration energy losses due to atomic-line photons in the BLR of FSRQs is shown to give proton distributions with cutoffs at ≈ 10^{16} eV. In related work [16], we calculate the diffuse neutrino background from the superposition of distant blazars, where we also find a suppression of the neutrino flux below 1 PeV.

2. Photopion efficiency with internal target electron synchrotron photons

Photopion production of high-energy (≈ 10^{14} – 10^{17} eV) cosmic rays in the intense BLR and internal radiation fields of blazars is more energetically efficient than secondary nuclear production in proton-ion collisions, provided the threshold for pion production is achieved [12]. To calculate the proton energy-loss timescale through photopion losses, we use the approximation $K_{\gamma \pi^-}(\varepsilon) \approx \theta H(\varepsilon - \varepsilon_{thr})$ for the product of the inelasticity and photopion production cross section, where $\varepsilon$ is the invariant photon energy in the particle rest frame. Here $\theta = 70 \mu B$, and $\varepsilon_{thr} \approx 400$. The Heaviside function $H(x)$ is 1 for $x \geq 0$ and $H(x) = 0$ otherwise.

The timescale $t_{\gamma \pi^-}^{-1}(\varepsilon'_p)$ for a proton of energy $m_p c^2 \varepsilon'_p$ to lose energy through photopion production is given by

$$t_{\gamma \pi^-}^{-1}(\varepsilon'_p) = \frac{c}{\varepsilon'_p} \int_{\varepsilon_{thr}/2\varepsilon_p}^{\infty} d\varepsilon' n_{pk}'(\varepsilon') \left[ 1 - \frac{\varepsilon_{thr}}{4\varepsilon_p \varepsilon'} \right]^2$$

with primes referring to comoving fluid-quantum frames. The term $n_{pk}'(\varepsilon')$ is the comoving synchrotron photon number density, $\varepsilon'$ is the comoving dimensionless photon energy, and the pitch-angle diffusion timescale of the particles is assumed to be rapid enough to isotropize the particle distribution in the fluid frame.

We adopt expressions for the nonthermal synchrotron luminosity radiated by an isotropic comoving electron synchrotron distribution $\gamma' \delta(N(\gamma'))$ described by a log-parabola function, where $\gamma'_e$ is the electron Lorentz factor in the comoving frame [14]. In this approximation, the synchrotron luminosity spectrum

$$\epsilon L_{syn}(\epsilon) = \nu x^{1-\delta_{var}}$$

where $\nu = 10^{-1/4_b}(\nu L_{\nu, pk}^{syn})$, $b = \hat{b} \ln 10$ is the log-parabola width parameter of the electron distribution, $x = \sqrt{\varepsilon'/\varepsilon_p}$, $\epsilon' = \epsilon/\delta_D$, $\varepsilon'_b = \varepsilon_p/\delta_D$, and $\delta_D$ is the Doppler factor. The peak synchrotron luminosity $\nu L_{\nu, pk}^{syn}$ at peak synchrotron frequency $\nu_i = 10^{17} \varepsilon_p/8.1 \times 10^{-7} \nu_{pk, 14}$ is derived directly from the data for a source at redshift $z$. In the blob formulation,

$$n_{pk}'(\varepsilon') = \frac{e^\epsilon' d(\epsilon')}{m_e c^2 \epsilon'^2} = \frac{\epsilon L_{syn}(\epsilon)}{4 \pi m_e c^3 \epsilon^2 \nu_{pk}^{1/2} \hat{b}_0 f_0}$$

where $f_0 \approx 1/3$. In the blast-wave formulation, $f_0 \equiv 1$, $\Gamma \equiv \delta_D$, and $n_{pk}'(\varepsilon' = \epsilon L_{syn}(\epsilon)/4 \pi m_e c^3 \epsilon^2 \nu_{pk}^{1/2}$, with $r \approx \epsilon T_{max}^{1/2}$, leading to effectively equivalent results [13][16]. For blazar calculations using the blob formulation, $\delta_D \equiv \Gamma$ is assumed. Synchrotron self-absorption is not important for PeV neutrino production in blazars and GRBs (see Appendix A), and internal and source

![Fig. 1](image) Integral $I_\gamma(\bar{x})$, from eq. (5). Thin curves show the high-energy approximation given in the text.

$\gamma$ opacity is less important for the neutrino spectrum than the $\gamma$-ray spectral energy distribution (SED) [4][14].

The photopion radiative efficiency of ultra-high-energy cosmic ray (UHECR; energies $\gtrsim 10^{17}$ eV) protons with Lorentz factor $\gamma_p \approx \delta_D \gamma'_p$ is defined by the expression $\eta_{\gamma \pi^-} \equiv t_{\gamma \pi^-}^{-1}/t_{\gamma \pi^-}(\gamma_p)$, where $t_{\gamma \pi^-}^{-1}$ is defined as $t_{\gamma \pi^-}^{-1} \equiv \delta_D \Gamma_{var} \equiv \Gamma_{var}$ is the comoving dynamical timescale. Eqs. (1) – (3) imply that the efficiency of UHECR protons to lose energy through photopion production with internal synchrotron photons is

$$\eta_{\gamma \pi^-} = \frac{I_{s}(\bar{x})}{I_{\gamma}(\bar{x})},$$

where

$$I_{s}(\bar{x}) \equiv \int_0^\infty dx x^{2-\hat{b}} \ln \left[ 1 - \frac{\nu_i}{\bar{x}} \right]$$

with $\bar{x} \equiv \sqrt{\varepsilon_{thr}/2\nu_c \varepsilon_p}$,

$$\eta_s = \frac{\hat{b} 10^{1/4_b}(\nu L_{\nu, pk}^{syn})}{2 \pi m_e c^3 T_{max}^{1/2} \delta D_0 f_0 \epsilon_0} \approx 1.5 \times 10^5 \frac{10^{3/4_b} L_{48}}{T_{max}(s) \delta D_0 f_0 \epsilon_0}$$

and $L_{48} \equiv \nu L_{\nu, pk}^{syn}/(10^{48}$ erg s$^{-1}$). This expression likewise applies to a spherical blast-wave geometry, letting $\hat{b} \rightarrow 1$ and taking $f_0 \approx 1$. Note that $I_{s}(\bar{x}) \rightarrow 10^{3/4_b} \sqrt{\bar{x}} \ln \bar{x}$ in the limit $\bar{x} \ll 1 \approx 1$, and $I_{s}(\bar{x}) \approx 4 \bar{x}^{1-k}/[(k-1)(k+3)]$ in the limit $\bar{x} \gg 1$, where $k \equiv 2 + \hat{b} \ln \bar{x}$. Fig. 1 shows a numerical integration of $I_{s}(\bar{x})$, eq. (5), for different values of $b$, compared to the $\bar{x} \gg 1$ asymptotes. When $\bar{x} \ll 1$, corresponding to large $\gamma_p$, the production efficiency, eq. (4), approaches a constant value.

3. Internal synchrotron photopion production efficiency of black-hole jet sources

Fig. 2 shows calculations of photopion production efficiencies using eqs. (4) – (7) for different classes of black-hole jet sources, using characteristic parameter values given in Table 1. Here we take $b = 1$. A value of $b$ near unity is implied for
the electron distribution by fitting the nonthermal synchrotron emission of 3C 279 (14), and we assume it also applies for the proton distribution. The Doppler factors are similar to values implied by equipartition leptonic models (14). Only the synchrotron radiation field is assumed to be important for photopion production in the sources considered in Fig. 2; the higher-energy synchrotron self-Compton radiation fields have too few photons to be effective targets for photopion production. What is most notable is the extreme sensitivity of the efficiency to $\delta$ or $\delta_D$, with $\eta_{\phi\pi} \propto \Gamma^{-3}$ at large proton energies. For LGRBs and SGRBs, low ($\Gamma \sim 100$) outflows are potentially much more neutrino luminous than for high ($\Gamma \sim 1000$) bursts. Fermi-LAT results suggest that the most powerful GRBs are those with the largest bulk Lorentz factor outflows (13), but to optimize neutrino luminosity, a smaller value of $\Gamma$ is required (Appendix B). This suggests examining neutrino production from GRBs that can be shown to have small $\Gamma$ factors, e.g., GRB 090926A whose Fermi-LAT spectrum shows a cutoff that suggests that $\Gamma \sim 200 - 700$ (13).

The photomeson efficiency of LLGRBs is poorly known due to the large uncertainty in determining $\Gamma$ and $t_{\text{var}}$. For a hydrodynamic jet to penetrate the star, $\Gamma \sim 5$ is suggested (13). The synchrotron self-absorption interpretation of the low-energy spectrum also indicates that $\Gamma \sim 5$ and dissipation radiates the photosphere (20). Values of $\Gamma \sim 5-20$ are considered in (21); see also (22, 23). Related to the LLGRBs are shock-breakout GRBs, where the dissipation is caused by transrelativistic ejecta with $\Gamma \sim 1$, and GRBs where neutrino production takes place in the star (24, 25). We consider a broad range of $\Gamma$ between 2 and 30, and take $t_{\text{var}} = 100$ s.

Photopion production efficiency $\eta_{\phi\pi}$ of high-energy protons with internal synchrotron luminosity is largest for small $\delta_D$, because the photon density is largest, so all injected power is reprocessed into neutrinos, $\gamma$ rays and neutrons. For internal processes, the produced radiation can be assumed to be isotropically emitted in the comoving fluid frame, so the neutrino luminosity $L_\nu \propto \delta^2 \gamma \delta' \Gamma' \propto \delta^2 L_\gamma$, with the Jet Doppler opening angle decreasing $\propto \delta^{-1}$. This leads to a characteristic Doppler factor $\delta_D$ when $\eta_{\phi\pi} \approx 1$ that optimizes neutrino luminosity; see Appendix B.

4. Photopion production efficiency in black-hole jet sources with external radiation fields.

We now treat the case of black-hole jet sources with strong external radiation fields, most notably FSRQs, though LSP and ISP BL Lac objects with peak synchrotron frequencies $\lesssim 10^{15}$ Hz can also have external radiation fields with significant energy densities. In contrast, HSP BL Lac objects have radiatively inefficient accretion flows and generally lack evidence for optically thick accretion disks or luminous BLDs, so external radiation fields are usually neglected. As we have seen for equipartition values, and as has been shown earlier by detailed Monte Carlo simulations (27), blazars without external radiation fields radiate the bulk of the neutrinos’ energy at $\eta_{\phi\pi} \approx 10^{17}$ eV, and would have difficulty explaining the IceCube PeV neutrinos unless the Doppler factor was unusually low. As shown in Appendix B, $\delta_D \lesssim 4$ is required for lower-energy neutrino production in HSP BL objects, and would represent a system far from equipartition with large $\gamma$-ray opacity that would produce absorption features that have not been observed in the SEDs of BL Lac objects (13).

External radiation fields arise from accretion-disk radiation absorbed by and reradiated from the molecular torus and BLR clouds, and scattered by electrons (for recent reviews of AGN and blazar physics, see (28, 29)). The highly anisotropic direct accretion-disk radiation field is shown in Appendix C to be unimportant for the production of PeV neutrinos.

External radiation fields from the accretion-disk radiation reprocessed by BLR clouds and the IR torus is assumed to have an isotropic distribution in the black-hole frame. Studies of anisotropies in the scattered radiation field (30, 31) show that locations within and close to the inner edge of the scattered radiation field have approximately isotropic external fields. An increasing fraction of tail-on photons develop as the jet becomes closer to the outer edge of the scattering zone. Calculations of $\gamma$-ray and neutrino SEDs entail a reaction-rate factor $(1 - \beta_\mu)$ that reduces the importance of tail-on photons. The assumption of isotropy is a good first approximation well within the radiation reprocessing region, but should be relaxed in further studies.

The transformation of an isotropic monochromatic external radiation field with energy density $\eta_0$ and photon energy $E_0$ to the fluid frame is easily performed using the transformation law

| #  | Source Class | $\nu_{\text{ph}}^{\text{psyn}}$ (10$^{48}$ erg s$^{-1}$) | $t_{\text{var}}$ (s) | $\delta_D \approx \Gamma$ | $\nu_{\text{ph,14}}$ (10$^{15}$ Hz) |
|----|--------------|-----------------|-----------------|------------------|-----------------|
| 1a,b | LGRB$^a$ | 1000 | 0.1 | 100, 1000 | 2 $\times 10^5$ |
| 2a,b | SGRB$^b$ | 1000 | 10$^{-3}$ | 100, 1000 | 10$^6$ |
| 3a,b | LLGRBs$^c$ | 0.1 | 100 | 2, 30 | 10$^4$ |
| 4a | BL Lac$^d$ | 0.001 | 10$^5$ | 5 | 10$^5$ |
| 4b | BL Lac$^d$ | 0.003 | 100 | 100 | 10$^3$ |
| 5a | FSRQ$^e$ | 0.03 | 10$^6$ | 10 | 0.1 |
| 5b | FSRQ | 0.1 | 10$^4$ | 30 | 0.1 |

$^a$ Long Duration GRB

$^b$ Short Duration GRB

$^c$ Low-luminosity GRBs; (21)

$^d$ High-synchrotron peaked BL Lac object

$^e$ Flat Spectrum Radio Quasars

---

1 By “equipartition” we mean equality of the energy densities of the magnetic-field and nonthermal electrons, that is, $u'_B = \zeta u'_e$, with $\zeta \approx 1$. If instead the all-particle energy density $u'_\text{par} = u'_B + u'_\text{par}'$ is related to the magnetic field according to the equipartition relation $u'_\text{par} = \zeta u'_B$, where $u'_\text{par}'$ is the energy density of hadrons in the blazar jet, this modifies the equipartition expressions by replacing $\zeta$ with $\zeta(1 + \beta_\mu)$ in the relation $u'_B = \zeta u'_\gamma$, where the hadron-electron loading factor $\eta_{\text{he}} \equiv u'_\text{par}/u'_e$. For large hadronic loading, when $\eta_{\text{he}} \gg 1$, the power requirements increase with accompanying spectral effects on the Compton component due to smaller values of $\delta_D$ and larger values of $\Gamma'$.
the target radiation field of extent $m$ and multiplying by angle. Substituting this expression into eq. (1), noting eq. (3) in jet plasma moving with relativistic energy density loss rate for protons with escaping energy $E_p$. For the parameters from Table 1 for long soft GRBs (LLGRBs), short monochromatic radiation, production with internal synchrotron photons. $u'(\epsilon', \Omega') = u(\epsilon, \Omega)/[\Gamma(1 + \beta \epsilon')]^3$ for the specific spectral energy density $u(\epsilon, \Omega)$ (see eq. (5.24) in (15)). For a highly relativistic ($\Gamma \gg 1$) flow, one obtains the spectral energy density $u' \approx (u_0/2\Gamma\epsilon(\epsilon_0)^3)H(\epsilon'; 0, 2\Gamma\epsilon_0)$, after integrating over angle. Substituting this expression into eq. (1), noting eq. (3) and multiplying by $t'_{\gamma\pi}$, gives the efficiency

$$\eta_{\gamma\pi}^{\text{ext}} = \eta_0[1 - \left(1 + \frac{\ln y_u}{y_u}\right)H(y_u - 1)], \quad \eta_0 \equiv \frac{\delta u_0 R}{m_c c^2 \epsilon_0}, \quad (8)$$

where $y_u = (4\epsilon_0 \gamma_p/\epsilon_0)^2$ and the pathlength $R \lesssim R_{\text{ext}}$ through the target radiation field of extent $R_{\text{ext}}$. The comoving energy-loss rate for protons with escaping energy $E_p = m_p c^2 \gamma_p \approx m_p c^2 \gamma_p$ that lose energy through photopion processes with photons of a locally isotropic external radiation fields is therefore given by

$$-\gamma_b' R(\gamma_p) = \frac{c^2 \gamma_p}{m_c c^2} \int_{\ln(y_u/4\gamma_p)}^{\infty} \frac{du}{u} \left[1 - \left(1 + \frac{\ln y_u}{y_u}\right)H(y_u - 1)\right], \quad (9)$$

where $\gamma_b \equiv (4\epsilon_0 \gamma_p/\epsilon_0)^2$. In comparison with a proton bound in jet plasma moving with $\Gamma \gg 1$, the corresponding efficiency of a neutron or proton traveling rectilinearly is $\eta_{\gamma\pi}^{\text{ext}} = \eta_0 (1 - 4/y_u)H(y_u - 4)$. We consider radiation fields associated with (1) the BLR, (2) the infrared-emitting dust torus, and (3) scattered accretion-disk photons, all of which provide target photons for photopion production with cosmic rays coming from the jet. In the first case, the Ly $\alpha$ radiation field dominates. For external isotropic monochromatic radiation, $\epsilon_{\text{in}}(\epsilon) = \epsilon_{\text{in}}(\epsilon - \epsilon_0)$. In the specific case of Ly $\alpha$ photons, $\epsilon_0 = 2 \times 10^{-5}$ is the Ly $\alpha$ photon energy in $m_c c^2$ units. A spectrum of BLR lines has at most a small effect on the photon spectrum of Compton-scattered radiation and similarly has a small effect on the neutrino spectrum except near the spectral cutoffs. Nevertheless, we superpose a spectrum of lines in our subsequent neutrino production spectrum calculations.

For quasi-thermal infrared radiation from a dusty torus surrounding the black hole, $\epsilon_{\text{IR}}(\epsilon) = 15\epsilon_{\text{IR}}(\epsilon/\Theta)^4/\Gamma(\epsilon^2 \exp(-\epsilon/\Theta)) - 1]$, where the effective IR temperature $T_{\text{IR}} = m_c c^2 \Theta/k_B$, and $u_{\text{IR}}$ is the energy density of the torus field, restricted by the blackbody limit to $u_{\text{IR}} < u_{\text{BB}}(T) \approx 0.008(T/1000 \text{K})^4 \text{ erg cm}^{-3}$. The third case involving scattered accretion-disk radiation is approximated by $\epsilon_{\text{disk}}(\epsilon) = u_{\text{disk}}(\epsilon/\epsilon_{\text{max}})^{2/3} \exp(-\epsilon/\epsilon_{\text{max}})$, where $u_{\text{disk}} = L_{\text{disk}}(\Theta/s_c)\Gamma(\alpha)4\pi R_{\text{sc}}^2 c$, $L_{\text{disk}}$ is the accretion-disk luminosity, and $s_c$ is the Thomson depth through the scattering volume of radius $R_{\text{sc}}$. For a Shakura-Sunyaev spectrum, $\alpha = 4/3$, $\Gamma(4/3) = 0.893 \ldots$, and $\epsilon_{\text{max}}$ corresponds to the dimensionless temperature of the accretion disk near the innermost stable orbit, which must be $\gtrsim 2 \times 10^{-5}$ in order to make strong Ly $\alpha$ radiation. In the calculations, we take $m_c c^2 \epsilon_{\text{max}} = 20 \text{ eV}$.

Fig. 3 shows a calculation of the photopion production efficiency using typical parameters for $\gamma$-ray loud FSRQs. Com-
Table 2. BLR Emission Lines Included in the Modeling of Neutrino Production.\textsuperscript{a}

| Line                | E (eV) |
|---------------------|--------|
| H Ly \(\alpha\)     | 100    |
| C IV                | 10.2   |
| He Ly \(\alpha\)    | 22.0   |
| Broad feature\textsuperscript{b} | 10.00  |
| Mg II               | 52.0   |
| N V                 | 8.00   |
| O VI + Ly \(\beta\) | 13.2   |
| C III + Si III      | 6.53   |

\textsuperscript{a} Line strengths are expressed as a ratio of the line flux to the H Ly \(\alpha\) flux; see Teller et al. \textsuperscript{[30]}; Cerruti et al. \textsuperscript{[34]}.

\textsuperscript{b} Broad feature at \(\sim 1600\AA\) has equivalent width of \(\approx 38.5\AA\) and is treated as a monochromatic line.

pared to the sources in Fig. 2, the presence of the external radiation field of the BLR, as well as the scattered accretion-disk radiation field, is extremely important for neutrino production in FSRQs \textsuperscript{[3]}. In this calculation, we take the energy density of the BLR radiation field \(\epsilon_{BLR} = 0.026f_{BLR}/0.1\) erg cm\(^{-3}\) \textsuperscript{[35]}, where \(f_{BLR}\) is the covering factor for atomic-line production. The BLR radiation is dominated by Ly \(\alpha\), but we also consider a range of lines with strengths given by analyses of AGN spectra \textsuperscript{[34] \textsuperscript{[36]}}, as given in Table 2. Furthermore, we assume that He Ly \(\alpha\) lines are present with an energy density of one-half the Ly \(\alpha\) energy density \textsuperscript{[16 \textsuperscript{[37]}]}. For the IR radiation field of the dust torus, we set \(\epsilon_{IR} = 10^{-3}\) erg cm\(^{-3}\) and assume it has an effective temperature of 1200 K \textsuperscript{[38]}.

In addition, an electron column with effective Thomson scattering depth of \(\tau_{esc} = 0.01\) in a region of extent \(R_{esc} = 0.1\) pc is used in Fig. 3 to define the scattered accretion-disk radiation, which is approximated by a Shakura-Sunyaev spectrum with temperature of 20 eV and \(L_{disk} = 10^{46}\) erg s\(^{-1}\). The direct accretion-disk radiation field provides another external photon target \textsuperscript{[29]}, but is unimportant for the production of PeV neutrinos (Appendix C), and is important for Compton scattering only if the emission region is within \(\approx 10^{15}\) cm of the accretion disk \textsuperscript{[40]}.

In the calculations of photopion efficiency, \(R\) is equated with \(cT^2t_{var}\) for interactions with the internal radiation fields. For external radiation processes, where photopion production can occur only as long as the jet remains within the target radiation field, the only requirement is that \(R \lesssim R_{ext}\). For a BLR with \(R_{ext} \sim 0.1\) pc, a photopion production efficiency \(\eta_{proc} \approx 0.03\) can be expected for \(\gtrsim 10^{16}\) eV protons in both the quiescent and flaring phases of FSRQs.

5. Neutrino production spectrum from photopion processes in black-hole jet sources

Following \textsuperscript{[15 \textsuperscript{[41]}]}, we derive the neutrino luminosity spectrum for neutrinos made with energy of \(m_c c^2\epsilon_n\) by using a formalism where the photopion production cross section is divided into separate step functions. Here we consider a three-step function model, corresponding to single-, double-, and multi-pion production that approximates the cross section and results from Monte Carlo simulations \textsuperscript{[42 \textsuperscript{[43]}]}. Photopion interactions taking place with the invariant dimensionless photon energy \(\bar{\epsilon}_i\), in the range \(\epsilon_i \lesssim \bar{\epsilon}_i < \epsilon_{sd}, \bar{\epsilon}_i = 1, 2, 3\), have cross section \(\sigma_l\), neutrino multiplicity \(\xi\), and fractional energy \(\chi_i\) of the neutrino secondary (compared to the incident photon energy). The parameters for the model are given in Table 3, including \(\beta\)-decay neutrinos from the decay of neutrinos formed in photopion processes. Here we assume that neutrinos are produced one-half of the time in photohadronic processes for single and double \(\pi\) production, and one-third of the time for multi-\(\pi\) production. This gives a rough approximation to the SOPHIA 2.0 event-generator neutron conversion efficiency (see Fig. 11 in \textsuperscript{[43]}).

For neutrino production from proton interactions with the internal synchrotron radiation field, the synchrotron emission is assumed to be radiated by a distribution of electrons that are isotropically distributed in the comoving jet frame and described by a log-parabola function \textsuperscript{[14]}.

The synchrotron luminosity spectrum is given by eq. \textsuperscript{[2]}, and the synchrotron photon spectrum coming from relativistic electrons is given by eq. \textsuperscript{[3]}. The photohadronic production cross section for secondary neutrinos is approximated by

\[
\frac{d\sigma(\bar{\epsilon}_i)}{d\bar{\epsilon}_i d\Omega_i} = \sum_{i=1}^{3} \xi_i \sigma_l(H(\bar{\epsilon}_i; \epsilon_i, \epsilon_{sd})\delta(\Omega_i - \Omega_{i1})\delta(\epsilon_i - \epsilon_{i}) + \frac{\chi_i m_p c^2}{m_e}) ,
\]

making the co-directional approximation that the secondaries travel in the same direction as the primary ultra-relativistic proton, and that the secondary energy is a fixed fraction \(\chi\) of the primary energy. Here \(\bar{\epsilon}_i = \gamma_i^\prime \epsilon_i (1 - \mu_i)\) is the invariant collision energy, and \(H(x; a_1, b_1) = 1\) if \(a_1 \leq x \leq b_1\), and \(H(x; a_1, b_1) = 0\) otherwise.

For the description of the proton spectrum in the blob, we also adopt the log-parabola function, and assume for simplicity

\begin{table}[h]
\centering
\caption{Parameters for secondary neutrinos formed in photomeson production and neutron \(\beta\)-decay.\textsuperscript{a}}
\begin{tabular}{|c|c|c|c|}
\hline
Parameter & Single \(\pi\) & Double \(\pi\) & Multi \(\pi\) \\
\hline
\(\sigma_l(\mu\text{b})\) & 340 & 180 & 120 \\
\(\epsilon_i\) & 390 & 980 & 3200 \\
\(\epsilon_{sd}\) & 980 & 3200 & \text{\textsuperscript{\infty}} \\
\(\xi\) & 3/2 & 4 & 6 \\
\(\chi_i\) & 0.05 & 0.05 & 0.05 \\
\(\xi_{\beta}\) & 1/2 & 1/2 & 1/3 \\
\(\chi_{\beta}\) & \(10^{-3}\) & \(10^{-3}\) & \(10^{-3}\) \\
\hline
\end{tabular}
\textsuperscript{a} For neutron \(\beta\)-decay neutrinos, the same parameters as for photopion neutrinos are used except for multiplicity \(\xi_{\beta}\) and mean fractional energy \(\chi_{\beta}\).
\end{table}
that the log-parabola width parameter $b$ is the same for protons as electrons (differing from the treatment in Ref. [10]). The spectrum of protons with Lorentz factor $\gamma_p = \delta_D \gamma_p^\prime$ is therefore given by

$$\gamma_p^2 N_p(\gamma_p^\prime) = K x_p^{-b \ln x_p},$$

(11)

where $K \equiv E_p/m_p c^2 I_1(b), x_p = \gamma_p/\gamma_{pk} = \gamma_p^\prime/\gamma_{pk}^\prime, E_p^\prime$ is the total comoving energy of the nonthermal protons, and $I_1(b) = \sqrt{\pi \ln 10/b}$ [14]. Because $\epsilon_i L(\epsilon_i, \Omega_i) = \delta_{i2} \delta_{i2}^\prime L^\prime(\epsilon'_i, \Omega'_i)$, one obtains

$$4\pi\epsilon_i L^\prime m(\epsilon_i, \Omega_i) = \frac{3}{16 \pi f_0 \chi m_p c^2 \sigma_{var}} \sum_{i=1}^3 K \epsilon_i m_i c \epsilon_i^2 \omega_i \chi^{4-b \ln \chi}$$

$$\times \int_0^\infty d\epsilon' \frac{\epsilon'^{1-b \ln \epsilon'}}{\epsilon'^4} [\min(\epsilon_{a,i}, 2\gamma_p^\prime \epsilon')]^2 - \epsilon_i^2 \right).$$

(12)

for the neutrino production spectrum from photodisruptive interactions with synchrotron photons. Here $\chi = \gamma_p/\gamma_{pk}, \gamma_{pk} = m_e/\chi m_p,$ and $\gamma_{pk}^\prime = \gamma_p/\delta_D$.

We follow the technique of Ref. [44] to derive the production spectrum of neutrinos formed when protons interact with photons of an external isotropic radiation field, by transforming the particle distribution to the source frame directly (see also [44]). The result is

$$4\pi\epsilon_i L_{\epsilon\gamma} m(\epsilon_i, \Omega_i) = \frac{3}{16 \pi f_0 \chi m_p c^2 \sigma_{var}} \sum_{i=1}^3 K \epsilon_i m_i c \epsilon_i^2 \omega_i \chi^{4-b \ln \chi}$$

$$\times \int_0^\infty d\epsilon' \frac{\epsilon'^{1-b \ln \epsilon'}}{\epsilon'^4} [\min(\epsilon_{a,i}, 2\gamma_p^\prime \epsilon')]^2 - \epsilon_i^2 \right).$$

(13)

Note the $\delta_D^2$ dependence [44]. The $\delta$-function approximation to the neutrino production spectrum does not give a good representation to the low-energy cutoff of the neutrino spectrum, which follows a number spectral index of $-1$ (43). For pion-decay neutrinos formed with target synchrotron, BLR, scattered accretion-disk and IR photons, we improve the approximation by correcting the neutrino spectrum by adding a low-energy extension with $\nu F_\nu$ equal index to $+1$ if the $\nu F_\nu$ spectrum calculated in the $\delta$-function approximation to the mean neutrino energy becomes harder than $+1$. No correction is made for the spectrum of $\beta$-decay neutrinos in the $\delta$-function approximation for average neutrino energy. For detailed numerical calculations, see, e.g., Ref. [46].

Fig. 4 shows a calculation of the luminosity spectrum of neutrinos of all flavors from an FSRQ with $\delta_D = \Gamma = 30$, using parameters of a flaring blazar given in Table 1. The radiation fields are assumed isotropic with energy densities $u_{\text{BLR}} = 0.026 \text{ erg cm}^{-3}$ for the BLR field, $u_{\text{IR}} = 0.001 \text{ erg cm}^{-3}$ for the graybody IR field. For the scattered accretion-disk field, $\tau_{sc} = 0.01$ is assumed. The proton spectrum is described by a log-parabola function with log-parabola width $b = 1$ and principal Lorentz factor $\gamma_{pk} = \Gamma \gamma_{pk} = 10^{1.5}$. Separate single-, double- and multi-pion components comprising the neutrino luminosity spectrum produced by the BLR field are shown by the light dotted curves for the photodisruptive and $\beta$-decay neutrinos. Separate components of the neutrino spectra from photodisruptive interactions with the synchrotron, BLR, IR, and scattered accretion-disk radiation are labeled.

Fig. 4 shows a calculation of the luminosity spectrum of neutrinos of all flavors produced by a curving distribution of protons in a flaring FSRQ like 3C 279 with a peak synchrotron frequency of $10^{13} \text{ Hz}$ and peak synchrotron luminosity of $10^{24} \text{ erg s}^{-1}$ (parameters of Table 1). The log-parabola width parameter $b = 1$ is assumed for both the electron and proton distributions. Here and below, we take $E_p = 10^{53} / \Gamma \text{ erg}$, which implies sub-Eddington jet powers for jet emissions occurring no more frequently than once every $10^3 M_\odot \text{s}$, where $M_\odot$ is the black-hole mass in units of $10^9 M_\odot$ (we take $M_\odot = 1$). The separate components for single-pion, double-pion, and multi-pion production from interactions with the BLR radiation are shown for both the pion-decay and neutron $\beta$-decay neutrinos. In this calculation, the neutron principal Lorentz factor $\gamma_{nk} = 10^{1.5}$, corresponding to source-frame principal proton energies of $E_p \approx 3 \times 10^{16} \text{ eV}$. Because the efficiency for synchrotron interactions in low-synchrotron peaked blazars is low until $E_p \gtrsim 10^{20} \text{ eV}$, as seen in Fig. 3, neutrino production from the synchrotron component is consequently very small. Interactions with the blazar BLR radiation is most important, resulting for this value of $\gamma_{pk}$ in a neutrino luminosity spectrum peaked at a few PeV, and with a cutoff below $\approx 1 \text{ PeV}$. Comparisons between luminosity spectra of neutrinos of all flavors for parameters corresponding to the quiescent phase of blazars, and for different values of $\gamma_{pk}$ and $b$, as labeled, are shown in Fig. 5. As can be seen, the low-energy hardening in the neutrino spectrum below $\approx 1 \text{ PeV}$ is insensitive to the assumed values of $\gamma_{pk}$ and $b$.

6. Discussion

We have calculated the efficiency of neutrinos produced by photodisruptive interactions of protons with internal and external target photons in black-hole jet sources. Neutrino spectra were calculated semi-analytically for the chosen parameters. After
summarizing (1) data from IceCube motivating this study, we discuss (2) the UHECR/neutrino connection, (3) particle acceleration in jets, and (4) the contributions of FSRQs and blazars to the diffuse neutrino background.

6.1. Extragalactic Neutrinos with IceCube

The IceCube Collaboration has reported compelling evidence for the first detection of high-energy neutrinos from extragalactic sources. The sources of the neutrinos remain unknown. Candidate astrophysical sources include powerful γ-ray sources such as blazars, GRBs, and young pulsars or magnetars. Other possibilities, e.g., structure formation shocks and star-forming galaxies, are not excluded. Here we have argued that FSRQs are ≥1 PeV neutrino sources.

IceCube searches have not, however, found statistically compelling counterparts by correlating neutrino arrival directions and times with pre-selected lists of candidate neutrino point sources, including FSRQs. An early search using 22-string data over 276 days live time found no significant excess other than 1 event associated with PKS 1622-297. Upper limits for an $E^{-2}$ neutrino spectrum from candidate γ-ray emitting AGNs were at the level of $\approx 1.6 \times 10^{-12} \Phi_{90} \text{ erg cm}^{-2} \text{ s}^{-1}$, $15 \lesssim \Phi_{90} \lesssim 600$, for neutrinos with energies $E_{\nu}$ from $\approx 100$ TeV to $\approx 10$ PeV. The upper limit for 3C 279 was a factor $\gtrsim 30$ above model predictions (43, 49).

Improved point-source searches in 22-string and 40-string configurations during 2007–2009 were reported for both flaring and persistent sources in Ref. (49). Recent 86-string data taken over 1373 days live time give IceCube limits of $\approx 10^{-12}$ erg cm$^{-2}$ s$^{-1}$ for $1 \text{ TeV} \lesssim E_{\nu} \lesssim 1 \text{ PeV}$ in the northern sky, and $\approx 10^{-11}$ erg cm$^{-2}$ s$^{-1}$ for $100 \text{ TeV} \lesssim E_{\nu} \lesssim 100 \text{ PeV}$ in the southern sky (50).

Source γ-ray fluxes provide an upper limit to the neutrino flux because of the decay of $n^0$ and $\pi^\pm$ formed in photopion process will produce secondaries that initiate γ-ray cascades that cannot overproduce the measured γ-ray fluxes. The brightest γ-ray blazars, namely 3C 279, 3C 273, and 3C 454.3, have average $>100 \text{ MeV}$ fluxes at the level of $\approx \text{ few } \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}$ (51). These limits rule out a hypothetical blazar model where the γ rays are entirely associated with photohadronic processes, but the success of leptonic models for blazar γ radiation (29) means that only a small fraction of the high-energy radiation from blazars can be hadronically induced. Particular interest for neutrino counterpart association attaches to unusual very-high-energy (VHE; $\gtrsim 100 \text{ GeV}$) flaring episodes in FSRQs, such as 3C 279 (52) and PKS 1222+216 (53). Furthermore, analysis of associations between GeV-TeV sources and IceCube neutrino arrival directions finds counterpart TeV BL Lac objects and pulsar wind nebulae (54). In principle, two-zone models for these objects could achieve the required flux (55) by adjusting the cosmic-ray spectral index and cutoff energy to appropriate values, but one has to take into account contributions from FSRQs for a detailed comparison.

6.2. UHECR/High-Energy Neutrino Connection

High-energy neutrino sources are obvious UHECR source candidates, though production of PeV neutrinos requires protons with energies of “only” $E_p \approx 10^{16} - 10^{17}$ eV. The close connection between neutrino and UHECR production implies the well-known Waxman-Bahcall (WB) bound on the diffuse neutrino intensity at the level of $\approx 3 \times 10^{-8} \text{ GeV cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$ (56), and the similarity of the IceCube PeV neutrino flux with the WB bound has been noted (57). Nevertheless, our results show that the relationship between the diffuse neutrino and UHECR intensities leading to the WB bound depends on fine-tuning the neutrino production and escape probability of UHECRs. The situation is even worse if the UHECRs are ions rather than protons, because the photo-dissintegration cross section for ions is larger than the photodisintegration cross section and neutrino production is less efficient (58, 59).

For GRBs and HSP BL Lac objects, the most significant radiation field for photopion production is the internal synchrotron field. Appendix B gives the Doppler factor for optimal neutrino production, $\delta_D$, and the typical energy $E_\nu$ of the produced neutrinos in terms of the apparent isotropic synchrotron luminosity, the peak synchrotron frequency, and the minimum variability time. As a consequence of the low value of the peak synchrotron frequency, FSRQs formally require $\delta_D \approx 70$ to make $\approx 10^{20}$ eV neutrinos, but would have to accelerate protons to $\gtrsim 10^{21}$ eV. GRBs and BL Lac objects with small Doppler factors can effectively make $\approx 100$ PeV neutrinos from photopion losses on internal synchrotron photons. The SEDs in one-zone models of such low-Doppler factor BL Lac objects would, however, be strongly distorted by internal γγ absorption. Furthermore, provided that $t_{\text{var}}$ and $\Gamma$ (or $\delta_D$) are sufficiently small so that the internal target photon density is large (eq. (4)), efficient
photopion and neutrino production can take place in GRBs, including LLGRBs. GRBs are also extremely powerful, so can accelerate protons to \( \gtrsim 10^{20} \) eV from simple arguments regarding Fermi acceleration (e.g., \( 63 \)). Except under special conditions that \( \eta_{\text{gr}} \sim 1 \) is realized for typical \( \Gamma \), however, GRBs would be weak neutrino and strong UHECR sources when \( \Gamma \) is large, and strong neutrino sources with quenched UHECR production when \( \Gamma \) is small (Fig. 2).

This difficulty also exists for blazars. HSP BL Lac objects are always inefficient PeV neutrino producers for the assumed parameters, as seen in Fig. 2, and when they are efficient PeV neutrino emitters, \( \gamma \)-ray opacity is large, contrary to the appearance of the \( \gamma \)-ray SEDs of these objects. Because their SEDs are well described by nonthermal synchrotron self-Compton models, values of Doppler factor and fluid magnetic field can be determined, which are similar to values found in equipartition modeling (14). Using such values from spectral modeling, along with the Hillas (61) condition to define the maximum possible particle energy, Ref. (62) found that HSP BL Lac are not capable of accelerating protons to \( E_p \gtrsim 10^{19} \) eV. If BL Lac objects are the sources of the UHECRs, then a transition from light to heavy composition would be required, as indicated in Auger (63) (though not HiRes; (63)) analyses of UHECRs. Indeed, BL Lac objects and their off-axis counterparts (i.e., FRI radio galaxies) may be favored as sources of UHECRs because they are found within the GZK radius, and their \( \gamma \)-ray emissivity greatly exceeds the UHECR emissivity (64). If the UHECR source spectrum has a log-parabolic type behavior, then the second-knee and ankle structures in the cosmic-ray spectrum, as well as compositional changes, could result from a superposition of UHECR injection spectra modified by transport and energy losses, just as it is for power-law injection. Fits to the UHECR spectrum from blazar sources is, however, beyond the scope of the present work.

Escaping UHECRs from the jets of BL Lac objects can explain various peculiarities in blazar physics, including the hardening of the deabsorbed TeV spectrum for most models of the extragalactic background light (EBL), and the existence of an unusual, weakly variable class of TeV blazars (63, 66, 67). Indeed, production of neutrinos formed by high energy cosmic-ray protons from a blazar source in transit through the intergalactic medium has been proposed as an explanation for the PeV events (68) based on calculations made prior to the detections (69). The model as proposed cannot, however, explain PeV neutrinos and UHECRs simultaneously. This is because the maximum cosmic-ray energy has to be tuned to \( \lesssim 10^{17.5} \) eV in order not to overproduce multi-PeV neutrinos. In addition, high EBL models, which are challenged by GRB observations (70), are needed. Moreover, the neutrino spectrum must harden below \( \approx 1 \) PeV because the EBL is cutoff above \( \approx 13.6 \) eV. This model therefore needs other components such as star-forming galaxies and galaxy clusters to explain sub-PeV neutrino events.

Neutrino production from proton interactions in the inner jets of FSRQs differs significantly from the preceding types of sources by virtue of the strong external radiation fields that are required when modeling their \( \gamma \)-ray SEDs. Indeed, FSRQs are defined by the strength of their broad lines. Though leptonic models appear adequate to explain the broadband SEDs of FSRQs, a hadronic component can explain observations of VHE \( \gamma \) rays in FSRQs (71). The calculations presented here show that if high-energy cosmic-ray protons are accelerated in the inner jets of FSRQs, photopion losses with \( \approx 1 \) – 10% efficiency is found for both FSRQs in their quiescent and flaring states, but that the proton spectrum must soften at \( E_p \gtrsim 10^{16} \) eV due to the assumed log-parabolic function, and can therefore not be significant UHECR sources. The dominant radiation field is the BLR radiation, though scattered accretion-disk radiation and, at higher proton energies, IR radiation, can also result in efficient photopion losses.

Figs. 4 and 5 show that a distinct low-energy hardening in the neutrino spectrum below 1 PeV is formed, as explained in the Introduction. Compared to the sharp cutoff found for a single monochromatic external radiation field, some smoothing is formed by a distribution of target photons from atomic lines, a stronger scattered accretion-disk radiation field, and a low-energy extension of the neutrino number spectrum \( \propto E^{-1}_\nu \). Even the inclusion of a distribution in redshift \( \z \) of FSRQs in the calculation of the diffuse neutrino flux from blazars (see Figs. 13 – 16 in (19)), which range broadly from \( \z \approx 0.5 \) to \( \z \approx 2 \) (72), is not sufficient to remove this low-energy hardening, which appears to be a robust feature of FSRQs. If the hardening is not found in IceCube data, then other sources must be considered to fill in the gap and explain LE neutrinos.

If the spectra of cosmic rays in star-forming galaxies are like our Galaxy’s cosmic-ray spectrum, with the cosmic-ray proton spectrum softening at the knee (\( \approx 3 \) PeV), then the neutrino-production spectrum through secondary nuclear processes should soften at \( \approx 0.05 \times 3 \) PeV, or at \( \approx 150 \) TeV. The LE neutrinos could then be due to superposition of neutrino emissions from star-forming/starburst galaxies or galaxy clusters and groups. A higher energy cutoff in the proton spectrum of star-forming galaxies or galaxy clusters and groups may consistently explain the PeV neutrinos if the cosmic-ray number index is \( \lesssim 2.1 \) – 2.2, in order to avoid overproducing the extragalactic \( \gamma \)-ray background (73). Such sources of \( \approx 0.03 – 2 \) PeV neutrinos require cosmic rays of \( \approx 0.6 – 80 \) PeV energy (for a typical redshift \( \z \approx 1 \)). Furthermore, neutrino emission from nuclear collisions in the Fermi bubbles might explain some though not all of the LE neutrinos (74, 75).

If scattered accretion-disk radiation is the dominant radiation field for photopion production, then a cutoff below \( \approx 10^{14} \) eV can result if the accretion-disk radiation has an effective temperature of \( \approx 35 \) eV (8) (the mean photon energy is \( \approx 3 \times \) the temperature). This way to make LE neutrinos is, however, problematic by requiring unusually large (\( \gtrsim 10 \) eV) effective temperatures for the accretion disks in FSRQs and large scattering depths \( \tau_{sc} \sim 1 \), leading to an associated \( \gamma \gamma \) opacity that strongly attenuates the blazar \( \gamma \)-ray spectrum down to a few GeV. Neutrino production from the cores of AGNs, without distinguishing radio-loud and radio-quiet sub-categories, was proposed in Ref. (76). As originally formulated, this model over-produces the IceCube neutrino intensity by a large factor, but could have the observed flux level by renormalizing the injected

\[ \eta_{\text{gr}} \approx 1 \]
cosmic-ray flux (77).

To summarize, given large baryon loading (16), FSRQs represent viable sources of the IceCube PeV neutrinos, but make a hardening below ~1 PeV, so that the neutrinos with \( E_\nu \ll 1 \) PeV have to be made by another neutrino source class. Furthermore, the cosmic-ray energy distributions in FSRQs must have a high-energy softening, which we have modeled with a log-parabola spectrum, meaning that FSRQs cannot be the sources of the UHECRs. For this, HSP BL Lac objects are favored in terms of emissivity and existence of sources within the GZK radius, provided that their particle distribution extends, unlike in FSRQs, to ultra-high energies.

6.3. Cosmic-ray acceleration in black-hole jets

A log-parabola function has been assumed for the proton spectrum, which is a departure from power-law particle spectra that are usually assumed. The mechanisms accelerating particles in blazars and the prompt-phase emissions of GRBs are highly uncertain. A curving log-parabola function can often give a better fit to the blazar SED, with fewer parameters, than electron spectra formed by the injection of power-laws followed by adiabatic losses and radiative cooling (34, 14). In both blazars and prompt emissions of GRBs, the synchrotron radiation spectrum never reaches its maximum energy of \( \approx 100 \) MeV, so that a slower, second-order acceleration scenario that results in curving particle distributions may be favored. (The delayed onset of \( \gamma \)-ray emissions at GeV energies could, however, be synchrotron radiation from first-order Fermi acceleration of electrons at the external blast-wave shock (78, 79).) A curving proton distribution, or a soft power-law distribution, is consistent with the lack of a large flux of multi-PeV neutrinos. Nonlinear effects in first-order acceleration make concave particle spectra, opposite to the behavior required to explain the IceCube data. On the other hand, a long acceleration timescale compared to escape could cause a cutoff at high energies in the particle spectra formed in first-order Fermi acceleration.

The simplest characterization of the maximum particle energy is to suppose that the relevant mechanism is Fermi acceleration, which operates on timescales longer than the Larmor time \( t'_q = E'/(QB'c) \), where \( E = \Gamma E' \) is the escaping particle energy and \( Q = Ze \) is its charge. For first-order Fermi acceleration, this implies a characteristic timescale \( t_{F1} = f_1 t'_q \), with \( f_1 \gtrsim 1 \). If the dynamical timescale \( t'_{\text{dyn}} \) during which the accelerator is active is determined by the measured variability timescale \( t_{\text{var}} \), then \( t'_{\text{dyn}} \approx \Gamma t_{\text{var}} \approx \beta_{\text{rel}} t_{\text{var}} \), and

\[
\frac{t'_{F1}}{t'_{\text{dyn}}} \approx f_1 \left( \frac{E}{Q B' c \Gamma^2 t_{\text{var}}} \right) .
\]

The condition \( t'_{F1}/t'_{\text{dyn}} \approx 1 \), with \( f_1 \approx 1 \), is a restatement of the Hillas (61) condition that gives the maximum energy \( E \) of a particle with charge \( Q \).

Using simple forms for particle acceleration derived in Ref. (81) for gyroresonant acceleration of protons with Alfvénic turbulence, the corresponding relation for second-order Fermi ac-

![Fig. 6](image-url)  
**Fig. 6** Ratios of proton acceleration and escape timescales to the dynamical timescale in the fluid frame are plotted as a function of escaping proton energy \( E_p \) for FSRQ parameters given in the legend. Acceleration efficiency reaches a maximum for both simplified descriptions of first-order (F1) and second-order (F2) Fermi acceleration, shown by heavy solid and dotted lines, respectively, with progressively lighter lines corresponding to a reduction in the acceleration efficiency by an order-of-magnitude. Ratios of maximum escape times to the dynamical time through diffusive gyrosoront joint-angle scattering and Bohm diffusion are shown by the labels “F2, esc” and “Bohm, esc,” respectively. Calculations for second-order Fermi acceleration assume \( q = 5/3 \). For the chosen parameters, protons cannot be accelerated to energies found in the cross-hatched region according to the Hillas criterion.

acceleration is

\[
\frac{t'_{F2}}{t'_{\text{dyn}}} \approx f_2 \left( \frac{E}{Q B' c \Gamma^2 t_{\text{var}}} \right)^{2-q} .
\]

Here \( q \) is the index of turbulence, with \( q = 5/3 \) for Kolmogorov turbulence and \( q = 3/2 \) for Kraichnan turbulence, and the term \( f_2 \approx 2q/[(\pi q - 1)\beta_3^3 \zeta] \), where \( \zeta \) is the fraction of magnetic-field energy density in Alfvénic turbulence, and \( \beta_A \) is the Alfvén speed.

In Fig. 6, we plot the ratios of the comoving particle acceleration, energy-loss, and escape timescales to the dynamical timescale \( t'_{\text{dyn}} \approx \Gamma t_{\text{var}} \). Parameters are appropriate to a model flaring blazar, and are given in the figure legend. For second-order processes, a Kolmogorov turbulence spectrum (\( q = 5/3 \)) is assumed. The characteristic times to accelerate protons to energies \( E_p \), divided by \( t'_{\text{dyn}} \), are plotted for first-order acceleration (F1), from eq. (14), and second-order acceleration (F2), from eq. (15), by solid and dotted lines, respectively. The progressively lighter lines take \( f_1, f_2 = 1, 10, 10^2, 10^3 \), respectively. The assumed parameters permit acceleration of protons to \( \approx 8 \times 10^{19} \) eV with maximum efficiency. Protons with energies greater than this energy, shown by the cross-hatched region, are not allowed by the Hillas condition.
Fig. 7 shows the timescale for photohadronic energy losses with proton acceleration to and is shown in Fig. 6 by the dot-dashed line denoted “Bohm, esc.”

represents a scattering shell of Thomson depth not escape on timescales shorter than (80). This ratio cannot be less than unity because particles can scatter with Alfvénic turbulence, using the expression

\[ \frac{t'_{\text{F2,esc}}}{t'_d} \approx \max[1, \frac{\pi}{8}(q-1)(2-q)(4-q)c^2] \left( \frac{E}{QB'c^2t_{\text{var}}} \right)^{q-2} \]  

(16)

This ratio cannot be less than unity because particles cannot escape on timescales shorter than \( t'_d \). We set the coefficient \( \frac{\pi}{8}(q-1)(2-q)(4-q)c^2 \) equal to unity in order to give the longest possible escape timescale through gyroresonant diffusion in the dashed line denoted “F2,esc” in Fig. 6. (A second dashed line assumes a factor of 10 more rapid escape, corresponding to an order-of-magnitude reduction in the plasma turbulence energy density.) But note that Eq. (16) assumes a picture where there are open magnetic field lines along which the particles diffuse and escape from the jet plasma. A more realistic picture for blazars might be Bohm diffusion in a randomly oriented magnetic field. The ratio of the Bohm diffusion timescale to the dynamical timescale takes the simple form

\[ \frac{t'_{\text{Bohm,esc}}}{t'_d} \approx \max[1, \left( \frac{E}{QBc^2t_{\text{var}}} \right)^{-1}] \]  

(17)

and is shown in Fig. 6 by the dot-dashed line denoted “Bohm,esc.”

Maximum particle energy is also limited by radiative losses. Fig. 7 shows the timescale for hadronic energy losses with scattered accretion disk, BLR, and IR torus photons, for parameters of a flaring FSRQ. In this case, photopion losses will limit proton acceleration to \( \lesssim 10^{17} \) eV in first-order acceleration if \( f_t \approx 10^3 \) (i.e., an acceleration efficiency of 0.001%), assuming Bohm diffusion. In comparison, a value of \( f_t \gtrsim 1000 \) for second-order acceleration limits proton acceleration to \( \approx 10^{16} \) eV, assuming Bohm diffusion. The acceleration efficiency is difficult to estimate in either case, and depends on the uncertain level of turbulence and the Alfvén speed. But it is important to note that in both first- and second-order Fermi acceleration, a characteristic maximum proton energy of \( \approx 10^{16} \) eV, i.e., \( \gamma_{pk} \approx 10^7 \), is a consequence of energy losses off the BLR radiation when the acceleration efficiency is sufficiently small and photohadronic losses with the BLR are sufficiently large. This feature of particle acceleration could explain the apparent cutoff in multi-PeV neutrino events observed with IceCube.

Second-order Fermi acceleration gives a curving accelerated particle distribution resulting from diffusive acceleration, but inefficient first-order acceleration will also produce a curving spectrum with a spectral cutoff due to photohadronic losses with BLR photons. As noted previously, we chose a value of the log-parabola parameter \( b = 1 \) based on fits to nonthermal electron synchrotron spectra in blazars [14]. In principle, \( b \) can be derived by comparing the proton distribution formed as a consequence of particle acceleration, energy-loss and escape, or by directly using theoretical particle spectra resulting from scenarios involving Fermi acceleration [51, 82, 83, 84, 85].

6.4. Diffuse neutrino intensity

The large directional uncertainty makes association of IceCube neutrinos with point sources difficult, particularly for shower events. Although a truly diffuse cosmogenic origin of the IceCube neutrinos is ruled out, as noted in the Introduction, the PeV neutrinos may be associated with large fluence FSRQs. No convincing associations have been made (see Section 6.1), and we can speak of the PeV neutrinos as “diffuse,” even though they may be due to the superposition of blazars that are not individually resolved by IceCube but resolved by Fermi.

PeV neutrinos originating from FSRQs will unavoidably be accompanied by \( \gamma \)-rays, so that \( \gamma \)-ray fluxes provide perhaps the best index to search for high-energy neutrino sources. FSRQs make a significant contribution to the \( \gamma \)-ray background [86, 87]. A calculation of the “diffuse” neutrino intensity based on the blazar sequence is presented in Ref. [16]. Besides searches for neutrinos from known point-source blazar directions, another method to test this model is to compare the probability for high-\( \gamma \)-ray fluence FSRQs to be found in PeV neutrino directional error ellipses with sources described by the same fluence distribution that are distributed randomly on the sky. With only three > 1 PeV neutrinos so far reported with IceCube, such tests are as yet statistically challenging (e.g., [88], but will become more promising as exposure, both with IceCube and Fermi, grows with time.

Regarding all 28 [1] and now 37 [2] excess neutrino events, the implied intensity of the excess IceCube neutrino flux is at the level of \( \sim 3 \times 10^{-8} \) GeV/cm²-s-sr [1, 57]. The PeV neutrinos alone contribute more than 50% of this intensity. This can be compared with the integrated \( \gamma \)-ray intensity of FSRQs measured with Fermi-LAT between 100 MeV and 100 GeV [72]. The cumulative energy-flux (\( \Phi \)) distribution measured in the
range of $10^{-11} \leq \Phi$ (erg/cm$^2$-s) $\leq 10^{-9}$ implies an FSRQ $\gamma$-ray intensity $\approx 5 \times 10^{-7}$ GeV/cm$^2$-s-sr, which is a lower limit given that FSRQs with $\Phi$ outside this range are not counted. Because neutrino production will unavoidably produce $\gamma$ rays with comparable intensity that, though generated at very high energy, can cascade into the LAT energy range, this estimate indicates that $\approx 10\%$ of the FSRQ $\gamma$-ray emission could be produced by hadronic jet processes. Diffuse 100 TeV – PeV neutrinos from one-zone models of BL Lac objects require low Doppler factors that would imprint strong $\gamma\gamma$ opacity features on the SED, unlike the observed SEDs of BL Lac objects.

7. Conclusions

In order to avoid overproduction of $\approx$ PeV neutrinos by cosmic-ray protons in FSRQs, a typical FSRQ proton spectrum (reflecting an average over many sources) that softens at energies $\gtrsim$ 100 PeV is required. The proton distribution could be in the form of a broken power law or an exponentially cutoff power law, but here we consider a log-parabola function. The lack of high-energy neutrinos can then be explained if $b \approx 1$ and the principal Lorentz factor $\gamma_{pk} \lesssim 10^9$. The corresponding proton energies, $\lesssim 10^{17}$ eV, are well below energies needed to explain the UHECRs.

Indeed, FSRQs and their off-axis counterparts cannot be the principal sources of UHECRs extending to $\approx 10^{20}$ eV, as they are not found within the GZK radius, and their $\gamma$-ray energy production rate per unit volume, if comparable to the required UHECR emissivity, is inadequate in the local universe (unlike the case of BL Lac objects). The presence of strong external radiation fields in the inner jets of FSRQs may inhibit acceleration of protons and ions to ultra-high energies, in the same way that the mean electron Lorentz factors in FSRQs are much less than those in BL Lac objects [89, 90] due, apparently, to radiative cooling. This behavior of the electron distribution, which helps explain the blazar sequence relation, is also peak luminosity with the frequency of the synchrotron peak, would have analogously behavior for hadrons, in accord with the hypothesis that BL Lac objects are the sources of the UHECRs. Intermediate and high-synchrotron peaked BL Lac objects are then predicted to be candidates for detection of EeV neutrino point sources by the Askaryan Radio Array [91]. A separate study is required to demonstrate whether a superposition of curved UHECR injection spectra from blazars at various redshifts reproduces the UHECR spectrum.

In related work [16], we considered the latest blazar gamma-ray luminosity function in order to derive the diffuse intensity of neutrinos made in blazar jets, which is dominated by production in FSRQs due to their strong external radiation fields. There we find a similar result [16] using a power-law cosmic-ray spectrum to explain UHECRs. Our calculations of neutrino spectra from FSRQs show a hardening below $\approx$ 1 PeV from the spectrum of decaying pions. Superposition of emission from blazars at various redshifts is not sufficient to conceal this low-energy cutoff [16]. Evidence for a suppression of neutrinos below $\approx$ 1 PeV would support this model, but the existence of a gap in the neutrino spectrum at these energies is not statistically significant [1, 2]. As IceCube exposure grows, our model will be tested by measuring the $\gtrsim$ 100 TeV neutrino spectrum. If FSRQs are the sources of the IceCube PeV neutrinos, a second component is unavoidably required to explain the $\lesssim$ 300 TeV neutrino events, for which star-forming galaxies, galaxy clusters and groups, or a higher prompt atmospheric neutrino background provide plausible explanations. Lack of evidence for a gap between a few hundred TeV and $\approx$ 1 PeV in the IceCube neutrino spectrum would instead provide evidence for a single-source model of neutrino production, for example, a nuclear production model where neutrinos originate from cosmic-ray reservoirs (such as galaxies and galaxy assemblies) with typical cosmic-ray spectra described by a power-law with index harder than $\approx$ 2 and a break or cutoff near 100 PeV [73, 92]. By comparison, in the model studied here where PeV neutrinos are produced by photopion processes in the inner jets of FSRQs, a suppression of the neutrino flux below $\approx$ 1 PeV is predicted, and can furthermore be tested by identifying high $\gamma$-ray fluence FSRQs in PeV neutrino error boxes.

Acknowledgments
We wish to thank J. Becerra, E. Blaufuss, J. Finke, A. Kusenko, B. Lacki, B. Lott, A. Reimer, and K. Schatto for discussions and correspondence. We would like to acknowledge the very useful report of the referee, which helped clarify the issues surrounding this model. The work of C.D.D. is supported by the Office of Naval Research. K.M. is supported by NASA through Hubble Fellowship, Grant No. 51310.01 awarded by the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Inc., for NASA, under Contract No. NAS 5-26555.

Appendix A. Synchrotron Self-Absorption (SSA) Frequency in Blazars

Synchrotron spectra of blazars may be self-absorbed at low radio frequencies. Following Ref. [14], the SSA optical depth of magnetoeactive plasma with average magnetic field $B'$ and an electron Lorentz-factor $\gamma'$ distribution described by the log-parabola function $\gamma'^2N(\gamma') = K\gamma'^{-b-\log\gamma'}$, where $\gamma \equiv \gamma'/\gamma_{pk}$, is given in the $\delta$-function approximation for photons with comoving dimensionless energy $\epsilon'$ by

$$\tau_{\epsilon'} = \frac{8\pi}{9} \frac{r_0}{m_e c^2 I_1(b)} \frac{u_{\epsilon'} u_{\epsilon''}}{u_{\epsilon''}} \frac{\mathcal{F}(\hat{\gamma})}{\epsilon' \epsilon''} \left( \gamma'^{-b-\log\gamma'} \right).$$

(A.1)

Here $u_{\epsilon'}$ is the nonthermal electron energy density, $u_{\epsilon''} = B'^2/8\pi$, $u_{\epsilon''} = B'^2/8\pi$ is the critical field energy density, $B_{cr} = 4.41 \times 10^{13}$ G, $r_0 \equiv c \delta \tau_{\epsilon''}$, $I_1(b) = \sqrt{\pi} \ln 10/b$, $\Lambda_c = h/\epsilon m_e c$ is the Compton wavelength, and

$$\mathcal{F}(\hat{\gamma}) \equiv (1 + \frac{b}{2} \log \hat{\gamma}) \hat{\gamma}^{-b-\log \hat{\gamma}},$$

(A.2)

with

$$\hat{\gamma} \equiv \sqrt{\epsilon_{SSA}/2B^2 \delta \tau_{\epsilon''}} \gamma'^{-b-\log \gamma'}/\gamma_{pk}.$$

(A.3)
Note that $\mathcal{F}(\gamma = 1) = 1$.

Defining the SSA frequency by $\tau_{\text{SSA}}$ gives

$$\epsilon_{\text{SSA}} = \delta_0 \nu_{\text{SSA}} = \delta_D^{4/3} \left[ \frac{8\pi mc^2 \epsilon_{\text{tot}}}{9mc^2 I_1(b)} \left( \frac{\xi_u \mu_p}{u_{\text{cr}}} \right) \mathcal{F}(\gamma) \right]^{1/3} \quad \text{(A.4)}$$

Relating $\mu' = \xi_u \mu_{\text{cr}}$ through the parameter $\xi_u$, one can show that the equipartition condition $\xi_u = 1$ is close to the minimum jet power condition $\delta_D$. Note that eq. (A.4) is transcendental through the dependence of $\gamma$ on $\epsilon_{\text{SSA}}$.

Eq. (A.4) gives the SSA frequency

$$\nu_{\text{SSA}} \approx 140 \text{ GHz} \left( \frac{\xi_{\text{f}}}{I_1(b)} \right)^{1/3} B^{4/3}(G) \left( \frac{0.6}{10} \right)^{4/3}, \quad \text{(A.5)}$$

dropping the slowly varying factor $\mathcal{F}^{1/3}(\gamma)$ and defining $b = b_0 / c_{\text{th}}$.

For a blast-wave geometry, the SSA frequency of nonthermal proton production is found when

$$\frac{\delta_D}{\delta_{\text{BL}}} \approx 4.0 \left( \frac{L_{48}}{t_{4} - 3} \right)^{1/4}, \quad E_{\nu,\text{BL}} \approx 150 \sqrt{\frac{L_{48}}{t_{4} - 3}} \text{ TeV} \quad \text{(B.4)}$$

With such low Doppler factors, GeV – TeV $\gamma$ rays would be strongly attenuated by $\gamma\gamma$ pair production $\mathcal{P}_{\gamma\gamma}$, whereas BL Lac objects show no indication of internal $\gamma\gamma$ absorption. Furthermore, such low Doppler factors correspond to systems far from equipartition $\delta_D$, which are disfavored energetically.

For the low synchrotron-peaked FSRQs, with $\epsilon_{\text{pk}} \approx 10^{13} \text{ Hz}$, or $\epsilon_{\text{pk}} \approx 10^{-7} e_7$,

$$\delta_{\text{FS}} \approx 71 \left( \frac{L_{48}}{t_{4} - 7} \right)^{1/4}, \quad E_{\nu,\text{FS}} \approx 4.7 \times 10^{20} \sqrt{\frac{L_{48}}{t_{4} - 7}} \text{ eV} \quad \text{(B.5)}$$

The class of intermediate synchrotron-peaked blazars with $10^{-6} \lesssim \epsilon_{\text{pk}} \lesssim 10^{-5}$ would be favored to make EeV neutrinos by this logic. Provided that a broadband spectrum of protons is accelerated with a number index of $\approx 2$, specific values of Doppler factor for GRBs and HSP BL Lac objects optimize $\sim 100 \text{ TeV}$ neutrino production.

### Appendix B. Photoproduction Efficiency and Neutrino Luminosity

We optimize neutrino luminosity for neutrinos $E_{\nu} = \chi m_p \delta_0 E_{\gamma} \mu_p$ formed from protons with Lorentz factors $\gamma_{\mu} = \delta_D \gamma_{\mu}' = \gamma_{\mu}/\chi m_p, \chi \approx 0.05$ (see also [93]). Assume that a fraction $k_p$ of the jet power $L_{\gamma\gamma}$ is converted into nonthermal proton power $L_{p,\gamma} \sim k_p L_{\gamma\gamma}$. Because $dt' = dt/\Gamma, E_{\gamma}' = E_{\gamma}/\Gamma, L_{p,\gamma}' = L_p \approx L_{\gamma\gamma}/\Gamma^2$ (for a blast wave).

Assuming that the target photons are isotropically distributed in the fluid frame, $L_\nu = \delta_l L'_\nu$ (for a blob). Further, $L'_\nu = \Gamma \chi L_p \mu_p \mu_\nu$, where $\mu_\nu = \mu_\nu(\tilde{x}, \chi)$, and $\eta_\nu \equiv K_{\nu} / \delta_0$ is given by eq. (7), where $\tilde{x} = \delta_0 \sqrt{\tilde{x}_0 \mu_\nu / (2E_{\gamma}(\tilde{x}) \mu_p)}$.

From inspection, optimal neutrino production from a black-hole jet source occurs for $\tilde{x} \approx 1$, so at $\tilde{x} \approx 1, I_\nu(\tilde{x}) \approx 1$, and the optimal Doppler factor in terms of neutrino production is defined by

$$\delta_0 \equiv \frac{11L_{48}}{t_{4}^{1/4} (s) f_0^{1/3} 10^{16} e_7^{1/4} \mu_p} \quad \text{B.1}$$

For a blast-wave geometry, $f_0 = 1$, and $f_0 = 1/3$ for a blob [14]. Defining $L_\nu = L/10^4 \text{ erg s}^{-1}$ and $\epsilon_\nu = \epsilon_{\text{pk}} / 10^4$,

$$\delta_{\text{GRB}} \approx 170 \left( \frac{L_{\nu 2}}{t_{\text{var}}(0.1 s) \epsilon_\nu} \right)^{1/4} \quad \text{(B.2)},$$

and the condition $\tilde{x} \approx 1$ implies that the efficiency is maximized for neutrinos formed at energy $E_{\nu} \approx \delta_0 \delta_{\text{GRB}} \chi m_p / 2\epsilon_\nu$, which for GRBs, implies

$$E_{\nu,\text{GRB}} \approx 270 \sqrt{L_{\nu 2} / t_{\text{var}}(0.1 s) \epsilon_\nu^3} \text{ TeV} \quad \text{(B.3)}$$

For an HSP BL Lac object, the most luminous neutrino fluxes from internal processes are found when

$$\delta_{BL} \approx 4.0 \left( \frac{L_{48}}{t_{4} - 3} \right)^{1/4}, \quad E_{\nu,BL} \approx 150 \sqrt{\frac{L_{48}}{t_{4} - 3}} \text{ TeV} \quad \text{(B.4)}$$

### Appendix C. Threshold Lorentz Factor for Photoproduction from Direct Accretion-Disk Radiation

Consider accretion-disk photons passing through a plasma jet moving outward with bulk Lorentz factor $\Gamma = 1 / \sqrt{1 - \beta^2}$ along the axis of the accretion disk. In the jet fluid frame, the threshold condition for photoproduction by ultra-relativistic protons is simply $\gamma_{\mu}' (1 - \mu') > \epsilon_{\text{thr}}$, using notation of the Section 2. Here $c_{\text{th}} = \Gamma c (1 - \beta \mu)$, and $\mu = (\mu - \beta \mu) / (1 - \beta \mu)$. The term $\rho = r / \sqrt{r^2 + R^2}$ is the cosine angle of the photon emitted by the accretion disk at radius $R$ from the nucleus that intercepts the jet at distance $r$ along the polar axis of the accretion disk.

The mean photon energy radiated from an optically thick Shakura-Sunyaev accretion disk is $m_e c^2 \epsilon(R) = 77 q R^{-3/4} \text{ eV}$, where $q = (\ell_{\text{EDd}} / M_\odot)^{3/4}, 10^3 M_\odot$, $M_\odot$ is the black hole mass, $\ell_{\text{EDd}}$ is the ratio of the radiative luminosity to the Eddington luminosity, and $\eta \approx 0.1$ is the efficiency of the accretion disk for converting accretion power into luminosity $L_{\text{edd}}$. The tildes refer to quantities measured in units of the gravitational radius $r_g = GM/c^2$. Writing $E(R) \approx 1.5 \times 10^{-5} q R^{-3/4}$ gives the threshold condition

$$\gamma_{\mu,\text{thr}} \equiv \Gamma \gamma_{\mu}' \approx \frac{A_T^{3/4}}{1 - \beta / \sqrt{1 + x^2}}, \quad \text{(C.1)}$$

for photoproduction by a proton with Lorentz factor $\gamma_{\mu}'$. Here $A \equiv \epsilon_{\text{thr}} / 1.5 \times 10^{-4} q$. Differentiating gives the minimum value of $\gamma_{\mu}'$ which, for $R \gg 1$, is at $\gamma_{\mu,\text{thr}} \approx 8 \times 10^{3.2/4} / q$. This can be rewritten to give the minimum energy $E_{\mu,\text{thr}} \approx 2.4 \times 10^{-7} q^{-1/4} (r/100 R) q^{3/4} \text{ eV}$, which is independent of $\Gamma \gg 1$. A value of $q \approx 0.1$ gives typical temperatures of the optically-thick accretion disk in FSRQs, which leads to even higher values of $E_{\mu,\text{thr}}$. Unless we consider extreme inner jet models with
$r < 100 r_g$, we can therefore neglect photopion production from the direct accretion-disk radiation field for the production of PeV neutrinos.

References

[1] IceCube Collaboration, Aartsen, M. G., Abbasi, R., et al. 2013, Science, 342, 1242885. [arXiv:1311.5238]
[2] IceCube Collaboration, Aartsen, M. G., Ackermann, M., Adams, J., et al. 2014, Phys. Rev. Lett., accepted. [arXiv:1405.5303]
[3] IceCube Collaboration, Aartsen, M. G., et al. 2013, Phys. Rev. Lett. 111, 021103
[4] Spencer R. Klein for the IceCube Collaboration 2013, 2013 Intl. Cosmic Ray Conf. [arXiv:1311.6519]
[5] Anchordoqui, L. A., Barger, V., Cholis, I., et al. 2014, Journal of High Energy Astrophysics, 1, 1
[6] Aartsen, M. G., Abbasi, R., Ackermann, M., et al. 2013, Phys. Rev. D, 88, 112008
[7] Roulet, E., Sigl, G., van Viet, A., & Mollerach, S. 2013, JCAP, 1, 28
[8] Laha, R., Beacom, J. F., Dasgupta, B., Horiuchi, S., & Murase, K. 2013, Phys. Rev. D, 88, 043009
[9] Atoyan, A., & Dermer, C. D. 2001, Physical Review Letters, 87, 221102;
 [10] Mannheim, K., Stanek, T., & Biermann, P. L. 1992, Astron. Astrophys., 260, L1
[11] Rachen, J. P., & Mészáros, P. 1998, Phys. Rev. D, 58, 123005
[12] Atoyan, A. M., & Dermer, C. D. 2003, Astrophys. J., 586, 79
[13] Stecker, F. W. 1969, Physical Review, 180, 1264
[14] Dermer, C. D., Cerruti, M., Lott, B., Boisson, C., & Zech, A. 2014, Astrophys. J., 782, 82
[15] Dermer, C. D., & Menon, G. 2009, High Energy Radiation from Black Holes: Gamma Rays, Cosmic Rays, and Neutrinos (Princeton University Press)
[16] Murase, K., Inoue, Y., & Dermer, C. D., 2014, Phys. Rev. D, 90, 023007
[17] Cenko, S. B., Friel, D., Harrison, F. A., et al. 2011, Astrophys. J., 749, 63
[18] Murase, K., Ioka, K., Nagataki, S., & Nakamura, T. 2008, Phys. Rev. D, 78, 023005
[19] Waxman, E. 2013, arXiv:1312.0558
[20] Wang, X.-Y., Razzaque, S., & Mészáros, P. 2008, Astrophys. J., 677, 432
[21] Murase, K., Ioka, K., Nagataki, S., & Nakamura, T. 2008, Phys. Rev. D, 78, 023005
[22] Waxman, E. 1995, Physical Review Letters, 75, 386
[23] Hillas, A. M. 1984, Ann. Rev. Astron. Astrophys., 22, 425
[24] Abraham, J., Abreu, P., Aglietta, M., et al. 2010, Physical Review Letters, 104, 091101
[25] Dermer, C. D., Cerruti, M., Lott, B., Boisson, C., & Zech, A. 2013, Astrophys. J., 755, 147
[26] Takami, H., Murase, K., Nagataki, S., & Sato, K. 2009, Astroparticle Phys., 31, 201
[27] Ahlers, M., & Murase, K. 2014, Phys. Rev. D, 90, 023016
[28] Murase, K., Ahlers, M., & Lacki, B. C. 2013, Phys. Rev. D, 88, 121301
[29] Murase, K., Ioka, K., Nagataki, S., & Nakamura, T. 2008, Phys. Rev. D, 78, 023005
[30] Waxman, E., & Bahcall, J. 1999, Phys. Rev. D, 59, 023002
[31] Ahlers, M., & Murase, K. 2012, Astrophys. J. Letters, 751, L11
[32] Kalashev, O. E., Kusenko, A., & Essey, W. 2013, Physical Review Letters, 111, 041103
[33] Murase, K., & Ioka, K. 2012, Astroparticle Phys., 33, 81
[34] Waxman, E., & Bahcall, J. 2001, Astroparticle Physics, 15, 121
[35] Kalashev, O. E., Kusenko, A., & Essey, W. 2013, Physical Review Letters, 77, 160
[36] Ahlers, M., & Murase, K. 2012, Astrophys. J. Letters, 751, L11
[37] Essey, W., & Kusenko, A. 2010, Astroparticle Physics, 33, 81
[38] Dermer, C. D., Murase, K., & Takami, H. 2012, Astrophys. J., 749, 63
[39] Takami, H., Murase, K., Nagataki, S., & Sato, K. 2009, Astroparticle Physics, 31, 201
[40] Murase, K., Ioka, K., Nagataki, S., & Nakamura, T. 2008, Phys. Rev. D, 78, 023005
[41] Ahlers, M., & Murase, K. 2012, Astroparticle Phys., 33, 81
[86] Inoue, Y., & Totani, T. 2009, Astrophys. J., 702, 523
[87] Ajello, M., Shaw, M. S., Romani, R. W., et al. 2012, Astrophys. J., 751, 108
[88] Krauß, F., Kadler, M., Mannheim, K., et al. 2014, Astron. Astrophys., 566, L7
[89] Fossati, G., Maraschi, L., Celotti, A., Comastri, A., & Ghisellini, G. 1998, MNRAS, 299, 433
[90] Ghisellini, G., Celotti, A., Fossati, G., Maraschi, L., & Comastri, A. 1998, MNRAS, 301, 451
[91] ARA Collaboration, Allison, P., Auffenberg, J., et al. 2012, Astroparticle Physics, 35, 457
[92] Tamborra, I., Ando, S., & Murase, K. 2014, arXiv:1404.1189
[93] Dermer, C. D., Ramirez-Ruiz, E., & Le, T. 2007, Astrophys. J. Letters, 664, L67