The black hole information paradox
and highly squeezed interior quantum fluctuations

Naritaka Oshita

Department of Physics, Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan
Research Center for the Early Universe (RESCEU), Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan

E-mail: naritaka@resceu.s.u-tokyo.ac.jp

Received 24 April 2017, revised 3 August 2017
Accepted for publication 8 August 2017
Published 4 September 2017

Abstract
Almheiri, Marolf, Polchinski, and Sully argued that, for a consistent black hole evaporation process, the horizon of a sufficiently old black hole should be replaced by a ‘firewall’ at which an infalling observer burns up, which obviously leads to the violation of the equivalence principle. We propose that once the infalling partner of an outgoing Hawking particle approaches a black hole singularity, it experiences decoherence and the loss of its entanglement with the outgoing Hawking particle. This implies we would no longer need firewalls to avoid the black hole information paradox.

Keywords: black hole information paradox, quantum entanglement, decoherence

(Some figures may appear in colour only in the online journal)

1. Introduction

The black hole information paradox [1] is one of the most profound problems in physics, which might lead to a deeper understanding of the relation between general relativity and quantum theory. If radiation from a black hole is thermal, the final state of black hole should be a mixed state even for a black hole originating from a gravitationally collapsing pure state. This process is forbidden in unitarity of quantum mechanics. Therefore, it is expected that the radiation from a black hole would be non-thermal.

Susskind, Thorlacius and Ugrum proposed the black hole complementarity principle [2–4], which gives a phenomenological picture for the evaporation of a black hole that explains how the non-thermal radiation could be emitted from a black hole. This proposal is consistent with
three postulates, which are briefly given as follows: (postulate 1) Hawking radiation is in a 
pure state, (postulate 2) outside the region near the horizon of a massive black hole, physics 
can be described by an effective field theory of general relativity plus quantum field theory, 
and (postulate 3) a black hole is regarded as a quantum system with discrete energy levels 
whose number is the exponential of the Bekenstein entropy \[5\] of a black hole.

In 2012, however, Almheiri, Marolf, Polchinski and Sully (AMPS) pointed out in \[6\] 
that postulate 1, postulate 2, and the equivalence principle are mutually inconsistent for an 
old black hole \[7–9\] an idea that we briefly review here. Let us consider an old black hole 
with early Hawking radiation A, late Hawking radiation B and infalling quanta behind the 
horizon C. A and B have to be fully entangled so that the final state of the black hole is a pure 
state (postulate 1). On the other hand, according to quantum field theory in curved spacetime, 
B and C, pair-created particles, are also fully entangled (postulate 2). That is, according to 
postulate 1 and 2, B should be fully entangled simultaneously with both A and C. This con-
tradicts the \textit{monogamy of entanglement} that forbids any quantum system being entangled 
with two independent systems fully and simultaneously. AMPS then proposed ‘\textit{firewalls}’, 
high-energy quanta at horizons energetic enough to break the entanglement of Hawking pairs, 
which would get rid of the inconsistency between postulate 1 and 2. However, the existence 
of firewalls implies that the free falling observer going across the horizon has a dramatic 
experience: the observer burns up at the horizon. That is, firewalls amounts to abandoning the 
equivalence principle.

In this paper a sufficient reason for rejecting the AMPS firewall concept as a solution to the 
black hole information paradox is presented. It was previously pointed out that the black hole 
information paradox only manifests limitations of semiclassical theory, rather than presents a 
conflict between any fundamental principles \[10\]. It was proved that firewalls are excluded by 
Einstein’s field equations for black holes of mass exceeding the Planck mass \[11\], and demon-
strated that the AMPS argument is based on an over-counting of internal black hole states 
including those that are singular in the past \[12\]. Here we show that an infalling mode inside 
a black hole C is infinitely squeezed due to the gravitational effect of a black hole, which 
makes the infalling mode highly sensitive to decoherence\[1\] and leads to the loss of its entan-
glement with the outgoing mode B (figure 1). This means that there would be no violation of 
monogamy of entanglement around a black hole and the black hole complementarity principle 
can be consistent with the equivalence principle.

The plan of this paper is as follows. In section 2 we will introduce a quantum state around 
a black hole formed from gravitational collapse and will describe how we calculate the time 
evolution of the quantum state. The resolution to the firewall paradox is described in section 3. 
We will show that the quantum state of a Hawking pair, which is initially an entangled state, 
would become a separable state due to environment-induced decoherence (see e.g. \[23\] for the 
review of decoherence). In section 4 we will confirm the consistency between our proposal, 
explaining how the Hawking pair evolves to a separable state from an initially entangled state, 
and the previous works that investigated how the purity of the Hawking radiation will be real-
ized. Section 5 is dedicated to conclusions.

\section{2. Formalism}

The Unruh vacuum state \[24\] is the quantum state on an eternal black hole spacetime which 
models the late time properties of the \textit{in vacuum} of a collapsing star, which is denoted by

\footnote{This mechanism is closely related to the quantum-to-classical transition of quantum fluctuations in a de Sitter 
spacetime that has been well investigated by \[13–23\].}
in⟩, that contains no Hawking particle at the past infinity. The Unruh vacuum is associated with the infalling modes and the outgoing modes that have positive frequency with respect to the Killing vector $\partial_t$ and $\partial_T$, respectively, where $t$ is the Schwarzschild time and $T$ is the Kruskal time. Introducing the vacuum state $|0\rangle_c$ for the infalling modes and $|0\rangle_b$ for the outgoing modes, the Unruh vacuum state can be expressed as $|U\rangle = |0\rangle_c |0\rangle_b$, and the relation between the in vacuum state $|\text{in}\rangle$ and the Unruh vacuum state $|U\rangle$ has the form \[ |\text{in}\rangle \propto \frac{1}{\sqrt{Z_\omega}} \sum_{n=0}^{\infty} e^{-\pi \omega n(\omega)/\kappa} (b^\dagger_\omega)^n (c^\dagger_\omega)^n / n! |0\rangle_c |0\rangle_b, \] (1)

where $b^\dagger_\omega$ and $c^\dagger_\omega$ are creation operators for the states $|0\rangle_b$ and $|0\rangle_c$, respectively, $n(\omega)$ is the number of particles with mode $\omega$, $\kappa \equiv (4GM)^{-1}$ is the surface gravity, and $Z_\omega \equiv (1 - e^{-\pi \omega/\kappa})^{-1}$.

In the following, we will use the Unruh vacuum state as a quantum state around a black hole although modeling the quantum state around the collapsing star with the (outgoing) Kruskal mode has not been fully successful and may demand us to take into account the technical issues, e.g. the backscattering effect in the definition of $|0\rangle_c$ and $|0\rangle_b$.\footnote{See \cite{26} for more details. They have discussed the definition of the standard quantum states around a black hole, including the Unruh vacuum state, focussing on the differences between fermionic and bosonic quantum field.}

The relation (1) implies that the infalling modes are fully entangled with the outgoing modes, which is the problematic entanglement and should be broken for the purity of the Hawking radiation as is pointed out by AMPS \cite{6}. In the following, we will neglect multi-pair creations because the states of $n$-particules are suppressed by the exponential factor $e^{-\pi \omega n/\kappa}$ and their cumulative contribution to the entanglement entropy (EE) between the infalling and outgoing mode is negligibly small.\footnote{The EE is given by $-\sum_{n=0}^{\infty} p_n \ln p_n$, where $p_n$ is the probability for $n$-pair creation. Using equation (1) with the typical energy of a Hawking particle $\omega = \kappa$, we can find that the cumulative contribution of the multi-pair creations to the EE is less than 0.1%.}

For simplicity and to grasp the essence, we consider here a generically entangled state
\[ |\text{in}\rangle \to \sqrt{1 - p^2} |0\rangle_c |0\rangle_b + p |1\rangle_c |1\rangle_b. \] (2)

Figure 1. The infalling mode near the horizon, C on the hyper surface $\Sigma$, can hold coherence, whereas the infalling mode in the vicinity of the singularity, C on the hyper surface $\Sigma'$, exits the particle horizon (dashed line) and loses causal contact as a whole, which leads to the decoherence of the infalling mode. As a result, the entanglement of the Hawking pairs disappears and its state becomes separable.
\[ |1\rangle_c = \int_0^\infty d\omega \varphi_c(\omega) |1, \omega\rangle_c, \quad (3) \]

\[ |1\rangle_b = \int_0^\infty d\omega \varphi_b(\omega) |1, \omega\rangle_b, \quad (4) \]

where \( |1, \omega\rangle_c \equiv c^+_0 |0\rangle_c, \quad |1, \omega\rangle_b \equiv b^+_0 |0\rangle_b \), \( p \) is a real number satisfying \( 0 < |p| < 1/\sqrt{2} \), and \( \varphi_c(\omega) (\varphi_b(\omega)) \) is a function satisfying \( \int d\omega |\varphi_c(\omega)|^2 = 1 \) (\( \int d\omega |\varphi_b(\omega)|^2 = 1 \)), which ensures that \(|1\rangle_c \) (\( |1\rangle_b \)) is a one-particle state of an infalling (outgoing) localized wave packet. In the latter part of this paper, we will show that this entanglement is broken by the existence of the singularity, which is caused by the decoherence of an infalling mode. An infalling mode inside a black hole is redshifted as \( \lambda = \lambda_0 \sqrt{2GM/r - 1} \), where \( \lambda_0 \) is the initial wavelength, and it diverges in the limit of \( r \to 0 \). Therefore, the infalling mode exits the particle horizon near the singularity and loses causal contact as a whole (figure 1), which is responsible for the squeezing (EPR-like correlation) of the infalling mode, that has the role to retain its coherent structure [14], and decoherence as is discussed in section 3.

We consider a massless scalar field \( \phi \) on the Schwarzschild spacetime with a mass \( M \) whose metric is given as \( ds^2 = f(r) dr^2 - f^{-1}(r) d\rho^2 - r^2 d\Omega^2 \) with \( f(r) \equiv 1 - 2GM/r \), where \( d\Omega^2 \) denotes the line element of a two-sphere \( d\Omega^2 \equiv d\theta^2 + \sin^2 \theta d\phi^2 \). Using the tortoise coordinate \( r^* = r + 2GM \ln |1 - r/(2GM)| \), we can rewrite it as \( ds^2 = g_{\mu\nu} dx^\mu dx^\nu \equiv f(r) \left[d\rho^2 - dr^* \right] - r^2 d\Omega^2 \). In order to describe the infinite squeezing of an infalling mode, let us investigate the dynamics of the vacuum \( |0angle_c \) inside the black hole \( r < 2GM \). The action \( S \) is given as

\[ S = \int d^4s \mathcal{L} = -\frac{1}{2} \int d^4s \sqrt{-G} \partial^\mu \varphi \partial_\mu \varphi = -\frac{1}{2} \int d^4s \sum_{lm} \left[ \chi'''_{lm} - 2\chi''_{lm} \omega^2 \mathcal{G} + \mathcal{G}^2 \chi''_{lm} \right] \equiv \frac{1}{2} \int d^4s \sum_{lm} \left[ \frac{\partial^2}{\partial r^* \partial \omega^2} - f(r) \left( \frac{2GM}{r^3} + \frac{l(l+1)}{r^2} \right) \right] \chi_{lm} = 0. \quad (5) \]

where we decompose the field \( \phi \) into partial waves with an angular momentum \( l \) as \( \phi \equiv \sum_{lm} \chi_{lm} Y_{lm}/r \), a prime and a dot denote differentiation with respect to \( r^* \) and \( t \) respectively, and \( \mathcal{G} \equiv r'/r \). From the action (5), the Euler–Lagrange equation can be derived as

\[ \left[ \frac{\partial^2}{\partial r^* \partial \omega^2} - f(r) \left( \frac{2GM}{r^3} + \frac{l(l+1)}{r^2} \right) \right] \chi_{lm} = 0. \quad (6) \]

We find that the mode functions satisfying (6) are almost independent of the angular momentum \( l \) in the vicinity of the singularity because \( l(l+1)/r^2 \) in (6) can be ignored for \( r \ll 2GM \). We are interested in the behavior of an infalling mode near the singularity, and therefore, we set \( l = 0 \) and omit the suffixes \( (l,m) \) in the following. The time like coordinate inside the black hole is \( r^* \), therefore, the conjugate momentum \( \pi \) of the field \( \chi \) is given as [30]

\[ \pi \equiv \partial \mathcal{L} / \partial \dot{\chi} = \chi' - \mathcal{G} \dot{\chi} \quad (7) \]

and then the Hamiltonian is

\[ H = \int dt \frac{1}{2} \left[ \pi^2 + \dot{\chi}^2 + 2\mathcal{G} \pi \chi \right]. \quad (8) \]

\(^4\)For example, taking the functions \( \varphi_c(\omega) \) and \( \varphi_b(\omega) \) to be \( (\Delta E)^{-1/2} \) for \( \omega_h < \omega < \omega_h + \Delta E \) and zero elsewhere, we can reproduce the wave packet introduced in [27, 28] where \( \omega_h \) is the typical energy of a Hawking particle and \( \Delta E \) gives the dispersion scale of the wave packet with \( \sim 1/\Delta E \).

\(^5\)For a review of gravitational redshift, see, e.g. [29].
We can decompose the field $\chi$ and its conjugate momentum $\pi$ as

$$
\chi \equiv \int_{-\infty}^{+\infty} \frac{d\omega}{\sqrt{2\pi}} \tilde{\chi}_\omega(r^*) e^{-i\omega t} + \text{(O.M.)} \equiv \int_{-\infty}^{+\infty} \frac{d\omega}{\sqrt{2\pi}} \left[ c_{\omega} \tilde{\chi}_\omega(r^*) e^{-i\omega t} + c_{\omega}^* \tilde{\chi}_\omega(r^*) e^{+i\omega t} \right] \theta(\omega) + \text{(O.M.)},
$$

(9)

$$
\pi \equiv \int_{-\infty}^{+\infty} \frac{d\omega}{\sqrt{2\pi}} \tilde{\pi}_\omega(r^*) e^{-i\omega t} + \text{(O.M.)} \equiv -i \int_{-\infty}^{+\infty} \frac{d\omega}{\sqrt{2\pi}} \left[ c_{\omega} \tilde{\pi}_\omega(r^*) e^{-i\omega t} - c_{\omega}^* \tilde{\pi}_\omega(r^*) e^{+i\omega t} \right] \theta(\omega) + \text{(O.M.)},
$$

(10)

where (O.M.) denotes the outgoing modes and $\theta(\omega)$ is a step function: $\theta(\omega) = 1$ for $\omega > 0$ and $\theta(\omega) = 0$ for $\omega < 0$. The canonical commutation relation is $[\chi_\omega, \pi^*_{\omega'}] = i\delta(\omega - \omega')$. In the following, we will omit the suffix $\omega$ for simplicity. From (7) and the canonical commutation relation, we obtain the Wronskian condition as $(\tilde{\chi}^* \tilde{\chi} - \chi^* \chi^*) = i$.

The third term in (8) is responsible for the squeezing of infalling modes [13–17], which becomes stronger as $r^* \to 0$ as is shown later. To investigate the dynamics of the states $|0\rangle_c$ and $|1, \omega\rangle_c$, we first derive the wave functions for them, $\Psi_0[\chi]$ and $\Psi_1[\chi]$, that satisfy $c|0\rangle_c = 0$ and $|1, \omega\rangle_c = c^\dagger|0\rangle_c$, respectively. From (9) and (10), we can rewrite the former in the Schrödinger representation as $[\tilde{\chi} + iy^{-1}(r^*, \omega)\tilde{\pi}] |0\rangle_c = 0$, where $\gamma(r^*, \omega) \equiv \tilde{\pi}^* \tilde{\chi}^*$. Replacing the conjugate momentum $\tilde{\pi}$ by $-i\partial/\partial \chi^*$, we obtain the wave function $\Psi_0[\chi]$ of the state $|0\rangle_c$ as

$$
\Psi_0[\chi] = \sqrt{\frac{2\gamma}{\pi}} \exp \left[ -\gamma(r^*, \omega) \tilde{\chi}^* \chi \right],
$$

(11)

where $\gamma_R \equiv \Re[\gamma(r^*, \omega)]$. On the other hand, $|1, \omega\rangle_c$ satisfies $|1, \omega\rangle_c = c^\dagger|0\rangle_c$, and hence we obtain $\Psi_1[\chi] \propto (\tilde{\chi} - \gamma^{-1}(r^*)\partial/\partial \chi^*) \Psi_0[\chi]$, which leads to

$$
\Psi_1[\chi] = \sqrt{\frac{2\gamma}{\pi}} \tilde{\chi} \exp \left[ -\gamma(r^*, \omega) \chi \tilde{\chi}^* \right].
$$

(12)

The function $\gamma$ can be calculated numerically from (6).

### 3. Decoherence near a black hole singularity

In the following we show that the density matrix $\rho_{co}$ of the quantum state (2) is reduced to a separable density matrix $\rho_{de}$ due to the decoherence once the infalling mode reaches the vicinity of the singularity, namely, $\rho_{co} \to \rho_{de}$ for $r^* \to 0$. To this end, we first show that the infalling mode becomes highly squeezed as the mode approaches the singularity, and secondly, that the squeezed state is highly sensitive to decoherence. The density matrix $\rho_{co}$ can be written as

$$
\rho_{co} \equiv (1 - p^2)|0\rangle_c \langle 0|_c \otimes |0\rangle_b \langle 0|_b + p^2 |1\rangle_c \langle 1|_c \otimes |1\rangle_b \langle 1|_b
$$

$$
+ p \sqrt{1-p^2} (|1\rangle_c \langle 0|_c \otimes |1\rangle_b \langle 0|_b + |1\rangle_c \langle 1|_c \otimes |0\rangle_b \langle 1|_b),
$$

(13)

and as shown later, the separable density matrix $\rho_{de}$ is

$$
\rho_{de} \equiv (1 - p^2)|0\rangle_c \langle 0|_c \otimes |0\rangle_b \langle 0|_b + p^2 |1\rangle_c \langle 1|_c \otimes |1\rangle_b \langle 1|_b.
$$

Hence, we will show that the third and fourth terms in (13) disappear, that is, $\rho_{co} \to \rho_{de}$, as the infalling mode approaches the vicinity of the singularity.

---

6 When a density matrix $\rho$ can be rewritten as $\rho = \sum_i \rho_u^{(i)} \otimes \rho_{b}^{(i)}$ with $\sum_i \rho_i = 1$, the density matrix $\rho$ is said to be ‘separable’, and this means that there is no entanglement.
Let us consider the time evolution of the non-diagonal terms of $\rho_{\text{co}}$. Using (3), $|0,c\rangle\langle 1,c|$, and $|1,c\rangle\langle 0,c|$ in the non-diagonal terms can be decomposed as

$$
|0,c\rangle\langle 1,c| = \int d\omega \varphi^*_c(\omega)|0,c\rangle\langle 1,\omega|_c,
$$
$$
|1,c\rangle\langle 0,c| = \int d\omega \varphi_c(\omega)|1,\omega\rangle\langle 0|_c
$$

(15)

respectively, and we will show the decay of $|0,c\rangle\langle 1,c|$ and $|1,c\rangle\langle 0,c|$ by calculating the time evolution of $|0,c\rangle\langle 1,\omega|_c$ and $|1,\omega\rangle\langle 0|_c$, $|0,c\rangle\langle 1,\omega|_c$ and $|1,\omega\rangle\langle 0|_c$, component of the Wigner function of $\rho_{\text{co}}$, $W^{(c)}_{01}$ and $W^{(c)}_{10}$, are given as

$$
W^{(c)}_{01} = \int \int d\omega dx d\eta e^{-i(\omega x + \eta \chi)} \langle \chi - \frac{x}{2}|0,c\rangle\langle 1,\omega|_c |\chi + \frac{x}{2}\rangle
$$

(16)

where we used (11) and (12) and the suffixes $R$ and $I$ represent the real and imaginary part, respectively. We numerically confirmed that they are infinitely squeezed in the limit of $r^* \to 0$ with $2GM\omega = 0.5$ (figures 2(a)–(c)) and the ratio $\gamma_I/\gamma_R \propto \sinh 2s$ diverges in the vicinity of the singularity, $\gamma_I/\gamma_R \to -\infty$, where $s$ is the squeezing parameter. This means that $s$ also diverges, $|s| \to \infty$, as $r^* \to 0$ (see e.g. [13]).

Secondly, we will show that an infinitely squeezed state with an environment is highly fragile against decoherence, in which the environment plays an important role. For instance, let us consider a double-slit experiment with electrons in which they create an interference pattern (non-diagonal density matrix). If they are exposed to thermal noise (environment), the pattern will be coarse-grained and will disappear (decoherence). This is the intuitive interpretation for the role of environment in decoherence. We here take into account the environment as follows. The field $\chi$ can be separated into two parts, the long-wavelength part as the system (an infalling Hawking particle) and the short-wavelength part as the environment (vacuum fluctuations). We here regard only the modes with wavelengths much shorter than the gravitational curvature radius of black hole as the short-wavelength part, as in the stochastic inflation scheme [21, 31, 32]. Therefore, the environment can be regarded as a coherent state with a good approximation and we can consider the decoherence by tracing out the coherent environment. It is shown that tracing out the coherent environment is corresponding to convolving (coarse-graining) the system’s Wigner function (16) with that of a coherent state $W_E$ [33] (see also [34, 35]),

$$
W_E \equiv \pi^{-2} \exp \left( -|\chi|^2 - |\bar{\eta}|^2 \right).
$$

(17)

Taking the convolution of (16) and (17), the non-diagonal term of the coarse-grained Wigner function $W^{(c)}_{01} = W^{(c)}_{01}*$ is obtained as

$$
W^{(c)}_{01} = (W^{(c)}_{01} * W_E) = \frac{Q^2}{\pi^2} (\chi - i\bar{\eta}) \exp \left( -|Q|^2 \left( |\chi|^2 + |\bar{\eta}|^2 \right) + 2\gamma_R (|\chi|^2 + |\bar{\eta}|^2) + (\gamma_I/\gamma_R) \bar{\eta} \chi^3 \right)
$$

(18)

where $Q \equiv \sqrt{2\gamma_R}/(1 + \gamma_I)$. In the limit of $r^* \to 0$, the real and imaginary parts of the function $\gamma(r^*, \omega)$ diverge and hence $Q$ asymptotically approaches zero. Therefore, the non-diagonal term $W^{(c)}_{01}$ is decaying as approaching the singularity (figures 2(a)–(c)), which means that the Hawking pair will experience decoherence as the infalling mode approaches singularity
since the effect of decoherence on a density matrix is essentially the decay of its non-diagonal terms, see e.g. [23]. Although general relativity and quantum field theory are, of course, no longer valid near the singularity at \( r \lesssim r_{Pl} = \frac{2GM}{(M_{Pl}/M)^{2/3}} \), where \( M_{Pl} \) is the Planck mass, the decoherence is almost completed at \( r \gg r_{Pl} \) in the case of interest, namely a massive black hole \( M \gg M_{Pl} \) (remember postulate 2). That is, the above estimates suggest that the squeezing becomes so strong that the decoherence can take place well before the modes reach \( r \sim r_{Pl} \), and therefore using a (semi)classical spacetime picture of the mode evolution should still be reliable.

As is shown above, the intense squeezing leads to the decay of the non-diagonal terms. Therefore, the third and fourth terms in (13), containing the non-diagonal components \( \{|1, \omega \rangle_c \langle 0| \) and \( \{|0, \omega \rangle_c \langle 1| \) (see (15)), decay due to decoherence and this leads to the transition of the state \( \rho_{co} \rightarrow \rho_{de} = (1 - p^2)\{|0\rangle_c \langle 0| \otimes \{|0\rangle_b \langle 0| + p^2\{|1\rangle_c \langle 1| \otimes \{|1\rangle_b \langle 1| \). This implies that the entanglement of Hawking pairs decays as the infalling mode approaches singularity.

4. Microscopic picture of information recovery

We can apply the loss of the entanglement between a Hawking pair to the black hole information paradox. According to our proposal, the entanglement between B and C is broken when C approaches singularity. Therefore, the timescale on which the entanglement is broken is of the order of the free fall timescale, \( t_F \sim 2GM \), measured by a freely falling observer\(^8\). In other\(^7\) the gravitational curvature is of the order of \( M_{Pl}^2 \) at \( r \sim r_{Pl} \), see e.g. [29].

\(^7\)The gravitational curvature is of the order of \( M_{Pl}^2 \) at \( r \sim r_{Pl} \), see e.g. [29].

\(^8\)The local velocity is given by \( v_{loc} = (dr/dt)/(1 - 2GM/r) \). We can obtain \( v_{loc} = -1 \) and \( v_{loc} = -(2GM/r)^{1/2} \) for a light ray and for a massive particle falling from rest at infinity respectively [36]. Integrating the inverse of local velocity from \( r = 2GM \) to \( r = 0 \), we find out that the free fall time scale \( t_F \) is of the order of \( 2GM \) in both cases.
In words, we cannot avoid the entanglement between B and C only during the moment of the free fall $\sim t_F$. Therefore, we have to discuss how the scenario proposed here is consistent with the monogamy of entanglement and the previous works [37, 38], in which the timescale of information recovery is carefully discussed at the microscopic level.

In [37], the radiation around a gravitationally collapsing shell was analytically investigated and it was shown that the correlations between the Hawking particles (between A and B) are initially zero but grow on the timescale of $t_F$ for an observer far from the black hole. [38] also pointed out that the microscopic timescale of information recovery may be of the order of $t_F$ by considering the interaction between a collapsing shell and the Hawking radiation. For these reasons, we can conclude that the entanglement between A and B would be initially zero and gradually appears on the timescale of $t_F$, and B can be allowed to be entangled with C only for the short time $\sim t_F$, which is quite consistent with our scenario. This implies that B would not be fully entangled with A and C simultaneously (figure 3), and therefore there is no violation of the monogamy of entanglement.

5. Conclusions

We showed that a Hawking pair becomes a separable state from an entangled state by pointing out that high squeezing and decoherence occur inside a black hole. The analysis was made with a simplified state (2) and the environment interacting with the infalling Hawking modes whose Wigner function is given by (17). The interaction with the environment can be effectively taken into account by smearing out the Wigner function of the infalling mode with that of the environment (18). As a result, we showed that the non-diagonal terms of the density matrix for the Hawking pair would decay quickly compared to the black hole evaporation timescale, which implies that the decoherence would be caused by the interior gravitational effect and that the entanglement between Hawking pairs will be broken. It should be emphasized that although general relativity and quantum field theory would break down near the singularity, our proposal is valid as long as the mass of black hole is much larger than the Planck mass, $M \gg M_{Pl}$ [2, 6]. According to our proposal, we would no longer need firewalls. We believe that our work can be important for understanding how the states of Hawking pairs of particles become separable, and how the black hole information paradox can be solved.
Acknowledgments

The author thanks T Nakama, Y Tada, D Yamauchi, and J Yokoyama for helpful comments. This work was partially supported by Grant-in-Aid for JSPS Fellow No.16J01780.

References

[1] Hawking S W 1976 Phys. Rev. D 14 2460
[2] Susskind L, Thorlacius L and Uglum J 1993 Phys. Rev. D 48 3743
[3] ’t Hooft G 1990 Nucl. Phys. B 335 138
[4] Susskind L and Thorlacius L 1994 Phys. Rev. D 49 966
[5] Bekenstein J D 1974 Phys. Rev. D 9 3292
[6] Almheiri A, Marolf D, Polchinski J and Sully J 2013 J. High Energy Phys. JHEP02(2013)062
[7] Page D N 1993 Phys. Rev. Lett. 71 1291
[8] Page D N 1993 Phys. Rev. Lett. 71 3743
[9] Page D N 2013 J. Cosmol. Astropart. Phys. JCAP09(2013)028
[10] Nomura Y and Salzetta N 2016 Phys. Lett. B 761 62
[11] Abramowicz M A, Klu W and Lasota J-P 2014 Phys. Rev. Lett. 112 091301
[12] Page D N 2014 J. Cosmol. Astropart. Phys. JCAP06(2014)051
[13] Polarski D and Starobinsky A A 1996 Class. Quantum Grav. 13 377
[14] Kiefer C, Lothar I, Polarski D and Starobinsky A A 2007 Class. Quantum Grav. 24 1699
[15] Lesgourgues J, Polarski D and Starobinsky A A 1997 Nucl. Phys. B 497 479
[16] Kiefer C and Polarski D 2009 Adv. Sci. Lett. 2 164
[17] Kiefer C, Polarski D and Starobinsky A A 1998 Int. J. Mod. Phys. D 7 455
[18] Campo D and Parentani R 2005 Phys. Rev. D 72 045015
[19] Lyth D H and Seery D 2008 Phys. Lett. B 662 309
[20] Kiefer C, Polarski D and Starobinsky A A 2000 Phys. Rev. D 62 043518
[21] Burgess C P, Holman R and Hoover D 2008 Phys. Rev. D 77 063534
[22] Kiefer C and Polarski D 1998 Ann. Phys. 7 137
[23] Giulini D, Joos E, Kiefer C, Kupsch J and Stamatescu I-O 2003 Decoherence and the Appearance of a Classical World in Quantum Theory (Berlin: Springer)
[24] Unruh W G 1976 Phys. Rev. D 14 870
  Boulware D G 1975 Phys. Rev. D 11 1404
  Hartle J B and Hawking S W 1976 Phys. Rev. D 13 2188
[25] Brout R, Massar S, Parentani R and Spindel P 1995 Phys. Rep. 269 329
[26] Casals M, Dolan S R, Nolan B C, Oterweill A and Winstanley E 2013 Phys. Rev. D 87 064027
[27] Hawking S W 1975 Commun. Math. Phys. 43 199
  Hawking S W 1976 Commun. Math. Phys. 46 206
[28] Audretsch J and Muller R 1992 Phys. Rev. D 45 613
[29] Misner C W, Thorne K S and Wheeler J A 1973 Gravitation (San Francisco, CA: W H Freeman)
[30] Yajnik U A and Narayan K 1998 Class. Quantum Grav. 15 1315
[31] Starobinsky A A and Yokoyama J 1994 Phys. Rev. D 50 6357
[32] Calzetta E A and Hu B B 2008 Nonequilibrium Quantum Field Theory (Cambridge: Cambridge University Press)
[33] Kanada-En’yo Y 2015 Prog. Theor. Exp. Phys. 2015 05D101
[34] Husimi K 1940 Proc. Phys. Math. Soc. Japan 22 264
[35] Takahashi K 1986 J. Phys. Soc. Japan 55 762
[36] Raine D and Thomas E 2009 Black Holes: an Introduction 2nd edn (London: Imperial College Press)
[37] Saini A and Stojkovic D 2015 Phys. Rev. Lett. 114 111301
[38] Kawai H and Yokokura Y 2016 Phys. Rev. D 93 044011