Anderson, A.; Reznikov, A.
Minimal Riesz energy on balanced fractal sets. (English) J. Math. Anal. Appl. 529, No. 1, Article ID 127663, 14 p. (2024)

Summary: We investigate the asymptotic behavior of minimal $N$-point Riesz $s$-energy on fractal sets of non-integer dimension, with algebraically dependent contraction ratios. For $s$ bigger than the dimension of the set $A$, we prove the asymptotic behavior of the minimal $N$-point Riesz $s$-energy of $A$ along explicit subsequences, but we show that the general asymptotic behavior does not exist.

MSC:
28Axx Classical measure theory
52Cxx Discrete geometry
52Axx General convexity

Keywords:
best-packing points; Cantor sets; equilibrium configurations; minimal discrete energy; renewal theory; Riesz potentials

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References:
[1] Alishahi, K.; Zamani, M., The spherical ensemble and uniform distribution of points on the sphere, Electron. J. Probab., 20 (2015) · Zbl 1327.60022
[2] Beltrán, C.; Marzo, J.; Ortega-Cerdà, J., Energy and discrepancy of rotationally invariant determinantal point processes in high dimensional spheres, J. Complex., 37, 76-109 (2016) · Zbl 1366.60079
[3] Borodachov, S., Asymptotics for the minimum Riesz energy and best-packing on sets of finite packing premeasure, Publ. Mat., 56, 1, 225-254 (2012) · Zbl 1245.28005
[4] Borodachov, S. V.; Hardin, D. P.; Saff, E. B., Asymptotics of best-packing on rectifiable sets, Proc. Am. Math. Soc., 135, 68, 2369-2381 (2007) · Zbl 1124.28004
[5] Borodachov, S. V.; Hardin, D. P.; Saff, E. B., Discrete Energy on Rectifiable Sets, Springer Monographs in Mathematics (2019), Springer: Springer New York · Zbl 1437.41002
[6] Brauchart, J. S., About the second term of the asymptotics for optimal Riesz energy on the sphere in the potential-theoretical case, Integral Transforms Spec. Funct., 17, 5, 321-328 (2006) · Zbl 1100.41018
[7] Brauchart, J. S.; Reznikov, A. B.; Saff, E. B.; Sloan, I. H.; Wang, Y. G.; Womersley, R. S., Random point sets on the sphere—hole radii, covering, and separation, Exp. Math., 27, 1, 62-81 (2018) · Zbl 1386.52019
[8] Cohn, H.; Kumar, A.; Miller, S. D.; Radchenko, D.; Viazovska, M., The sphere packing problem in dimension 24, Ann. Math. (2), 185, 3, 1017-1033 (2017) · Zbl 1370.52037
[9] Cohn, H.; Kumar, A.; Miller, S. D.; Radchenko, D.; Viazovska, M., Universal optimality of the \( e^{\sqrt{8}n} \) leech lattices and interpolation formulas (2019), arXiv preprint
[10] Damelin, S. B.; Maymeskul, V., On point energies, separation radius and mesh norm for s-extremal configurations on compact sets in \( \mathbb{R}^{1} \), J. Complex., 21, 6, 845-863 (2005) · Zbl 1084.52521
[11] Erdélyi, T.; Saff, E. B., Riesz polarization inequalities in higher dimensions, J. Approx. Theory, 171, 128-147 (2013) · Zbl 1282.31003
[12] Falconer, K. J., The Geometry of Fractal Sets, Cambridge Tracts in Mathematics, vol. 85 (1986), Cambridge University Press: Cambridge University Press Cambridge · Zbl 0587.28003
[13] Feller, W., An Introduction to Probability Theory and Its Applications. Vol. I (1957), John Wiley and Sons, Inc.: John Wiley and Sons, Inc. New York, Chapman and Hall, Ltd., London · Zbl 0039.13201
[14] Feller, W., An Introduction to Probability Theory and Its Applications. Vol. II (1966), John Wiley & Sons, Inc. New York-London-Sydney · Zbl 0138.10207
[15] Hardin, D.; Saff, E., Discretizing manifolds via minimum energy points, Not. Am. Math. Soc., 10, 1186-1194 (2004) · Zbl 1058.31003
[16] Hardin, D. P.; Saff, E. B., Minimal Riesz energy point configurations for rectifiable d-dimensional manifolds, Adv. Math., 193, 1, 174-204 (2003) · Zbl 1192.49048
[17] Hutchinson, J., Fractals and self-similarity, Indiana Univ. Math. J., 30, 73-74 (1981) · Zbl 0598.28011
Lalley, S., Renewal theory, available at · Zbl 0551.60094

Lalley, S. P., The packing and covering functions of some self-similar fractals, Indiana Univ. Math. J., 37, 3, 699-709 (1988) · Zbl 0665.28005

Lalley, S. P., Renewal theorems in symbolic dynamics, with applications to geodesic flows, non-Euclidean tessellations and their fractal limits, Acta Math., 163, 1-2, 1-55 (1989) · Zbl 0701.58021

Mattila, P., Geometry of Sets and Measures in Euclidean Spaces: Fractals and Rectifiability, Cambridge Studies in Advanced Mathematics, vol. 44 (1995), Cambridge University Press: Cambridge University Press Cambridge · Zbl 0819.28004

Petrache, M.; Serfaty, S., Crystallization for Coulomb and Riesz interactions as a consequence of the Cohn-Kumar conjecture, Proc. Am. Math. Soc., 148, 7, 3047-3057 (2020) · Zbl 1439.52025

Reznikov, A.; Vlasiuk, O., Riesz energy on self-similar sets, Proc. Am. Math. Soc. (2018), in press, arXiv preprint

Viazovska, M., Sharp sphere packings, (Proceedings of the International Congress of Mathematicians—Rio de Janeiro 2018. Vol. II. Invited Lectures (2018), World Sci. Publ.: World Sci. Publ. Hackensack, NJ), 455-466 · Zbl 1447.52021

Viazovska, M. S., The sphere packing problem in dimension 8, Ann. Math. (2), 185, 3, 991-1015 (2017) · Zbl 1373.52025

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