Neuro-Fuzzy Approach for Estimating Energy Dissipation in Skimming Flow over Stepped Spillways

Farzin Salmasi · Mehmet Özger

Abstract In a stepped spillway, the spillway face is provided with a series of steps from near the crest to the toe. The energy dissipation caused by the steps reduces the size of the energy dissipator, generally provided at the toe of the spillway. The hydraulics of stepped spillways is investigated by carrying out laboratory experiments, building models to explain the data, and testing the robustness of the models developed here using a neuro-fuzzy approach. The experiments consist of twenty different stepped spillways tested in a horizontal laboratory flume, a wide range of discharge values, three weir slopes of 15°, 25°, and 45° and different step numbers from 3 to 50 on the ogee surface. The main objective of this paper was to investigate the applicability and accuracy of the neuro-fuzzy approach in estimating the proper values of energy dissipation of skimming flow regime over stepped spillways because of the imprecise, insufficient, ambiguous and uncertain data available. A neuro-fuzzy approach was developed to relate the input and output (energy dissipation) variables. Multiple regression equations based on dimensional analysis theory were developed for computing energy dissipation over stepped spillways. The determination coefficients for the suggested neuro-fuzzy model in training and testing process are 0.974 and 0.966, respectively. It was found that the neuro-fuzzy approach formulation of the problem of solving for the energy dissipation over stepped spillways is more successful than that by regression equations.

Keywords Energy dissipation · Stepped spillway · Neuro-fuzzy · Skimming regime

List of symbols

- \( b \) Spillway width
- \( E_1 \) Energy downstream of spillway before hydraulic jump
- \( E_0 \) Total energy upstream of spillway
- \( \Delta E \) The difference between energy upstream and downstream of the spillway \( (\Delta E = E_0 - E_1) \)
- \( F_r \) Supercritical Froude number \( = \frac{V_1}{\sqrt{gy_1}} \)
- \( g \) Acceleration due to gravity
1 Introduction

Stepped spillways have been built in the past, but there is some interest in them because of significant cost savings. Stepped spillways have many applications in dam structures, river engineering, and soil conservation works. Their stilling basins can be constructed at lower costs as energy dissipation in these spillways is high. Stepped spillways have been used for centuries. Recently, new construction materials such as roller compacted concrete (RCC) and design techniques have increased the interest in stepped spillways [1]. The steps produce considerable energy dissipation along the spillway and reduce the size of the required downstream energy dissipation basin. The various flow regimes for stepped spillways are as follows: (i) nappe or jet flow regime: the water flows as a succession of free-falling nappes at small discharges [2]; (ii) skimming flow regime: Most prototype spillways operate at large discharges per unit width for which the water skims as a coherent stream over the pseudo-bottom (i.e., not solid bottom) formed by step edges, which is characterized by significant form losses and momentum transfer from the main stream to the recirculation zones (e.g., [3,4]); and (iii) transition flow regime: For an intermediate range of flow rates, a transition flow regime is observed between the above regimes [5].

Figure 1 shows the schematic classification of nappe, transition and skimming flow regimes.

The hydraulics of flow over stepped spillways is difficult due to the complexity of flow regimes, physical characteristics, and various hydraulic effects such as turbulence. Their implementations in the form of stepped chutes/spillways
have become a popular method for handling flood releases. Arguably, a better insight into their hydraulics can enhance their popularity and this is the focus of this paper. Many studies have been carried out to investigate different aspects of overflow in stepped spillways but the hydraulics of stepped spillways by neuro-fuzzy approach has not been fully investigated due to the complexity of flow patterns and resistance.

The primary focus of this research is to investigate the accuracy of a fuzzy rule system approach for estimating the energy dissipation of the skimming flow regime over stepped spillway because of the imprecise, insufficient, ambiguous and uncertain data available. The application of the proposed approach was performed using the measured data for energy dissipation available from the experimental analyses [6]; hence, its performance was tested using some parameters for error estimation.

In recent years, artificial intelligence (AI) techniques such as neuro-fuzzy, artificial neural network (ANN), and genetic programming (GP) models have attracted researchers in many disciplines of science and engineering, since they are capable of correlating large and complex data sets without any prior knowledge of the relationships among them. Notably, ANNs have been applied to other hydraulic problems, e.g., Yuhong and Wenxin [7] to predict the friction factor of open channel flow, Kisi [8] to predict the mean monthly stream flow, Rakhshandehroo et al. [9] for forecasting groundwater level in Shiraz plain and Eslamian et al. [10] to estimate Penman–Monteith reference evapotranspiration. The authors are not aware of the application of neuro-fuzzy to stepped spillways.

Ozger and Yildirim [11] investigated the accuracy of a fuzzy rule system approach to determine the relationship between pipe roughness, Reynolds number, and friction factor. A neuro-fuzzy approach was developed to relate the input (pipe roughness and Reynolds number) and output (friction coefficient) variables. The application of the proposed approach was performed for the data derived from the Moody’s diagram. The performance of the proposed model was compared with respect to conventional procedures using some statistical parameters for error estimation. The comparison test results reveal that through fuzzy rules and membership functions, the friction factor can be identified precisely.

Yildirim and Ozger [12] applied neuro-fuzzy approach in estimating Hazen–Williams friction coefficient (C_{HW}) for small-diameter polyethylene pipes. The examination results indicated that through fuzzy rules and membership functions, the proposed model can be successfully used to identify the proper values of the C_{HW} coefficient; hence, accurately estimate friction losses through smooth PE pipes.

2 Theoretical Bases and Model Implementation

2.1 Experimental Setup

Experiments on stepped spillways test runs were carried out at the Hydraulic Laboratory of Water Engineering Department, Shahid Chamran University (SCU) in Ahvaz city and Iran. The test runs were installed in two flumes, (i) ~0.5 m wide, 8 m in length, and 1.6 m in height; (ii) ~0.25 m wide, 10 m in length, and 0.60 m in height. Tables 1 and 2 show some geometrical characteristics of the physical models of stepped spillways with 50 and 25 cm width, respectively.

The flow through the flume was controlled at the end of the laboratory flume by a gate to form a hydraulic jump at the weir toe to enable flow measurements. Thus, discharge values were measured by a calibrated sharp triangle weir (of 53° angle) installed at the downstream of the flume. Discharge water was supplied by a pump (maximum value 50 l/s). Discharge values ranged from 7 to 50 l/s with an accuracy level of ±0.9 l/s. Upstream water levels were measured using a point gauge within ±0.1 mm accuracy. All measurements were taken along the centreline of the flume.

| Table 2 Geometrical characteristics of physical models of stepped spillways of 25 cm width and average height of 32 cm |
|---|---|---|---|
| Steps number (N) | Steps height h (cm) | Steps length l (cm) | Spillway slope (S) |
| 3 | 10.5 | 10.5 | 45 |
| 5 | 3.9 | 3.9 | 45 |
| 10 | 1.96 | 1.96 | 45 |
| 15 | 1.3 | 1.3 | 45 |
| 35 | 0.8 | 0.8 | 45 |
| 5 | 5.6 | 12 | 25 |
| 10 | 2.8 | 6 | 25 |
| 20 | 5.6 | 12 | 25 |
| 10 | 2.8 | 6 | 25 |
| 15 | 1.9 | 4.07 | 25 |
| 5 | 6.1 | 22.77 | 15 |
| 10 | 3 | 11.2 | 15 |
| 15 | 2 | 7.46 | 15 |
| 30 | 1 | 3.73 | 15 |

Table 1 Geometrical characteristics of physical models of stepped spillways of 50 cm width and average height of 100 cm

| Steps number (N) | Steps height h (cm) | Steps length l (cm) | Spillway slope (S) |
|---|---|---|---|
| 5 | 17.22 | 17.22 | 45 |
| 10 | 8.5 | 8.5 | 45 |
| 15 | 5.5 | 5.5 | 45 |
| 20 | 4.3 | 4.3 | 45 |
| 35 | 2.46 | 2.46 | 45 |
| 50 | 1.72 | 1.72 | 45 |
In each test run, water depth was measured 0.60 m upstream of the chute, \( y_0 \), and after the hydraulic jump, \( y_2 \). The thin flow and air entrainment at the spillway toe made it difficult to measure the flow depth (\( y_1 \)) accurately. Pegram et al. [13] calculated energy dissipation using the conjugate water depths of the hydraulic jump (\( y_2 \)). In the present study, \( y_2 \) was measured with an accuracy level of \( \pm 2 \) mm, where there were few bubbles and less undulation in the tail water.

2.2 Experimental Data

A total of 154 test runs were carried out in the skimming flow regime with three different slopes (\( S = 15^\circ, 25^\circ \) and \( 45^\circ \)), seven different step numbers (Tables 1, 2), and varying discharge rates and the measurements for each test run comprised discharge values and two values of water depth (Table 4 in “Appendix”). It should be mentioned that the total number of experiments was 250 including the nappe, transition and skimming flow regimes, but in this study, only skimming data were used for simulation of energy dissipation. Selection of step numbers was based on laboratory flume facilities.

A decision was made to use 68% of these data points (104 data points) for training and 32% of the total data points (50 point data) for testing the predictions of the model.

The procedure for selecting training and prediction data was based on plotting relative energy dissipation, drop number, Froude number, slope, number of steps versus discharge and selecting representative data points from high, medium and low ranges. The measurement data points are given in “Appendix”

2.3 Dimensional Analysis

Upstream energy head (\( E_0 \)), downstream energy head (\( E_1 \)) and relative energy dissipation (\( \Delta E/E_0 \)) are calculated as follows (Fig. 2):

\[
E_0 = H_w + y_0 + \frac{V_0^2}{2g} = H_w + y_0 + \frac{q^2}{2g(H_w + y_0)^2} \quad (1)
\]

\[
E_1 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{q^2}{2g\gamma_1^2} \quad (2)
\]

\[
\frac{\Delta E}{E_0} = \frac{E_0 - E_1}{E_0} = 1 - \frac{E_1}{E_0} \quad (3)
\]

where \( g \) is acceleration due to gravity, \( H_w \) is total spillway height measured with a point gauge after the installation of the spillway at the flume, \( y_0 \) is the depth of the flow at a set distance upstream of the spillway and above the spillway crest, \( q \) is discharge per unit width, and \( V_0 \) is the approach velocity. The depth, \( y_1 \), was calculated using the conjugate depth (\( y_2 \)) expressed as:

\[
y_1 = \frac{y_2}{2} \left( \sqrt{1 + 8F_{r2}^2} - 1 \right) \quad (4)
\]

where \( F_{r2} \) is Froude number (\( F_{r2} = V_2/\sqrt{g\gamma_2} \)), \( V_2 \) and \( y_2 \) are the velocity and water depth at Sect. 2 (after hydraulic jump and the re-establishment of subcritical flow), respectively.

Generally, energy dissipation depends on hydraulic and geometric variables expressed as:

\[
\frac{\Delta E}{E_0} = f(q, l, h, H_w, g, N) \quad (5)
\]

where \( l \) is step length, \( h \) is step height, and \( N \) is number of steps. In all tests, discharge was regulated in a way to form hydraulic jump at spillway toe, so that supercritical flow at the downstream of the spillway toe may occur (Froude number \( > 1 \)). Although both depth values of \( y_1 \) and \( y_2 \) were measured, only \( y_2 \) values were used to calculate the energy dissipation by means of Eqs. (1)-(4).

The fundamental variables that are important in the hydraulics stepped spillways are geometrical parameters such as total spillway height (\( H_w \)), step length (\( l \)), step height (\( h \)), spillway slope (\( S \)), number of steps (\( N \)); and hydraulic parameters such as discharge per unit width of canal (\( q \)), energy upstream of weir (\( E_0 \)) and energy downstream of weir (\( E_1 \)) defined in Eqs. (1) and (2) respectively. Using the Buckingham \( \Pi \)-theorem, relative energy dissipation can be expressed as:

\[
\frac{\Delta E}{E_0} = f(q^2/gH_w^3, h/l, N, \gamma_c/h, F_{r1}) \quad (6)
\]

Equation (6) can be rewritten as Eq. (7):

\[
\frac{\Delta E}{E_0} = f(DN, S, N, \gamma_c/h, F_{r1}) \quad (7)
\]
Fig. 3 The model structure of the proposed FIS

FIS: 5 inputs, 1 outputs, 32 rules

where $DN = q^2/gH_w^3$, $y_c$ is critical depth ($y_c = (q^2/g)^{1/3}$) and $S = h/l$ is the spillway slope. The parameter $DN$ is termed drop number, similar to drop number presented first by Rand [14]. Froude number at Sect. 1 is defined by: $Fr_1 = V_1/\sqrt{gy_1}$. The average flow velocity at any section ($V = q/y$) was calculated as the measured flow rate per unit width ($q = Q/b$) where $Q$ is total discharge, $b$ is the spillway width, and $y$ is depth of water at the appropriate section.

2.4 Implementation of Adaptive Neuro-Fuzzy Inference System (ANFIS)

Adaptive neuro-fuzzy inference system (ANFIS), first introduced by Jang [15], is a universal approximator. The neuro-fuzzy model used in this study implements the Takagi–Sugeno (TS) fuzzy approach [16] to obtain the value of the output variable from input variables. Here, the fuzzy inference system (FIS) has five inputs: drop number ($DN$), spillway slope ($S$), number of steps ($N$), critical depth to step height ($y_c/h$) and Froude number ($Fr_1$); and one output relative energy dissipation ($\Delta E/E_0$).

In this study, ANFIS is selected to train the proposed model. ANFIS Editor GUI of MATLAB is used for implementation. The data (154) is divided into two parts which are the training (104) and testing (50) parts. The testing data is selected randomly, and it is independent from training data. An initial FIS model structure should be specified prior to FIS training. The FIS model consists of grid partitioning techniques which apply grid partition on the data. To train the FIS, backpropagation gradient descent method is employed. The number of training epochs is 1,000, and the training error tolerance is set to 0.01. After the FIS is trained, the model is validated using the testing data that is independent from the data used to train the FIS.

A schematic of the model structure is given in Fig. 3. The FIS model consists of 3 membership functions (MFs) for each input. Gaussian type MF is chosen for input and constant type for output membership function. The MFs for the trained FIS are given in Fig. 4.

2.5 Performance Criteria

The two error measures are used to compare the performance of the various models: determination coefficient ($r^2$) and root mean square error (RMSE). The study also uses relative error (RE) that is defined as follows:

$$RE = \frac{|T_{True} - T_{Estimated}|}{T_{True}}$$

where $T_{True}$ is from experimental tests and $T_{Estimated}$ is calculated.

3 Results

Figure 5 presents: $\Delta E/E_0$ versus $DN$ and shows that by increasing $DN$, the relative energy dissipation is reduced. Energy dissipation is reduced with a high gradient until $DN$ reaches a value approximately equal to 0.01 but above this value ($DN > 0.01$), energy dissipation becomes independent of $DN$, i.e., it becomes less efficient.

Figure 6 presents: $\Delta E/E_0$ versus $y_c/h$ and shows that by increasing $y_c/h$, the relative energy dissipation is reduced. But discrepancies among data points are high, and it seems that $y_c/h$ does not affect the variation of $\Delta E/E_0$ solely.

Figure 7 presents: $\Delta E/E_0$ versus $Fr_1$ and discrepancies among data points are high, and it seems that $Fr_1$ does not reflect the variation of $\Delta E/E_0$ solely.
Scatter plots of energy dissipation versus to $N$ and $S$ indicate that the correlation is lower than that of the $Fr_1$, $y_c/h$ and DN, so their scatter plots are not presented in this study.

The performance of the ANFIS model for calculating energy dissipation of flow over stepped spillways was investigated by plotting a scatter diagram, as shown in Figs. 8 and 9 for training and testing, respectively. The determination coefficients of the proposed neuro-fuzzy model in training and testing process are 0.974 and 0.966, respectively. This suggests high accuracy of implementation of neuro-fuzzy model for estimating the energy dissipation of flows over stepped spillways.

3.1 Regression Analyses

Multiple regression analyses were performed with different combinations of the dimensionless parameters that appear in Eq. (7). Several linear and nonlinear multiple regressions were conducted using the Statistical Package for Social Science (SPSS) software version 17.

The fitted equations for physical models are given by Eqs. (9)–(12):

$$
\Delta E/E_0 = -1.063Fr_1^{2.119} + 350.365(y_c/h)^{-0.02}
+ 473.031(DN)^{-0.014} + 61.544N^{0.052}
- 858.114S^{0.007}, \quad r^2 = 0.928
$$

(9)
\[
\frac{\Delta E}{E_0} = 26.991(F_{r1})^{0.075}(y_c/h)^{-0.298}(DN)^{-0.048} \\
\times (N)^{0.255}(S)^{-0.063}, \quad r^2 = 0.865 \quad (10)
\]
\[
\frac{\Delta E}{E_0} = -172464.174(F_{r1})^{1.791\times10^{-5}} + 172471.549 \\
\times (DN)^{-4.747\times10^{-5}}, \quad r^2 = 0.903 \quad (11)
\]

An overview of Eqs. (9)–(12) indicates that the correlation of \(y_c/h\), \(N\) and \(S\) with energy dissipation is lower (compare Eqs. 10 and 11) than that of the \(F_{r1}\) and \(DN\).

In practice, Eq. (11) can be helpful because it includes simple correlations among \(\Delta E/E_0\), \(F_{r1}\) and \(DN\) with \(r^2 = 0.903\). With more accuracy, Eq. (9) with \(r^2 = 0.928\) can be used.

The performance of regression analysis using Eq. (11) versus the observed data points is shown in Fig. 10, according to
Table 3 Error measurements in regression equations and ANFIS for testing data

| Determination coefficient ($r^2$) | RE (average) | RE (minimum) | RE (maximum) | Equation |
|----------------------------------|--------------|--------------|--------------|----------|
| 0.908                            | 0.0962       | 0.0022       | 1.0158       | 9        |
| 0.892                            | 0.1133       | 0.00097      | 1.1879       | 11       |
| 0.966                            | 0.0545       | 0.00056      | 0.3349       | ANFIS    |

which regression Eq. (11) is capable of predicting the values of energy dissipation.

A comparison among Eqs. 9, 11 and the ANFIS approximation approach in Table 3 demonstrates that ANFIS approach is more accurate than regression analysis. From Table 3, $\text{RE (Average)} = 0.054$ and $r^2 = 0.966$ for the ANFIS approach.

4 Conclusions

The paper applies neuro-fuzzy technique for predicting the energy dissipation in stepped spillways. The data is obtained from laboratory experimental tests on physical stepped spillways. In the proposed fuzzy rule system approach, drop number (DN), Froude number ($F_{r1}$), slope (S), number of steps (N) and ($y_c/h$) are the input variables, whereas the relative energy dissipation is employed as the output variable.

Preliminary test runs identified the optimum ANFIS model. The trained network of ANFIS model was able to predict the response with $r^2$ and RE equal to 0.966 and 0.054, respectively (Table 3). The application of the proposed approach was performed using the measured data for the relative energy dissipation available from the experimental analysis.

The performance of the proposed model was tested using some parameters for error estimation. The results indicate that through fuzzy rules and membership functions, the proposed model can be used with the highest degree of accuracy to identify the proper values of energy dissipation; hence, accurately estimate relative energy dissipation.

Energy dissipation with regression equations had some edge over the fuzzy rules model, both visually and quantitatively. Although the performance of the regression equations in terms of $r^2$ and RE was good, these values were slightly lower than those predicted by means of the neuro-fuzzy approach, i.e., the latter was found to be more accurate than regression analysis.

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Appendix: Experimental Results

Note-Total data points are 154. A decision was made to use 104 data points for training and 50 point data for testing the model.
See Table 4.

Table 4 Experimental results of stepped spillway

| Row | $\Delta E/E_0$ | $F_{r1}$ | $y_c/h$ | $q^2/gH_w^3$ | N | S | Row | $\Delta E/E_0$ | $F_{r1}$ | $y_c/h$ | $q^2/gH_w^3$ | N | S |
|-----|---------------|---------|---------|---------------|---|---|-----|---------------|---------|---------|---------------|---|---|
| 1   | 61.523        | 4.706   | 0.608   | 0.001105      | 5 | 45| 78  | 21.720        | 3.719   | 2.880   | 0.050072      | 5 | 45|
| 2   | 60.530        | 4.877   | 0.596   | 0.001044      | 5 | 45| 79  | 32.943        | 3.440   | 2.582   | 0.036092      | 5 | 45|
| 3   | 63.390        | 4.758   | 0.565   | 0.000890      | 5 | 45| 80  | 18.065        | 5.320   | 1.710   | 0.010492      | 5 | 45|
| 4   | 65.931        | 4.734   | 0.524   | 0.000710      | 5 | 45| 81  | 74.946        | 3.549   | 0.709   | 0.000747      | 5 | 45|
| 5   | 70.308        | 4.658   | 0.459   | 0.000476      | 5 | 45| 82  | 86.357        | 4.415   | 0.281   | 0.000047      | 5 | 45|
| 6   | 78.157        | 3.931   | 0.404   | 0.000325      | 5 | 45| 83  | 24.643        | 3.232   | 6.885   | 0.084616      | 10| 45|
| 7   | 70.203        | 6.223   | 0.314   | 0.000153      | 5 | 45| 84  | 16.689        | 4.086   | 5.428   | 0.041449      | 10| 45|
| 8   | 84.571        | 4.534   | 0.233   | 0.000063      | 5 | 45| 85  | 19.022        | 4.039   | 5.300   | 0.038602      | 10| 45|
| 9   | 60.486        | 4.807   | 1.231   | 0.001111      | 10| 45| 86  | 39.911        | 4.382   | 3.083   | 0.007594      | 10| 45|
| 10  | 61.312        | 4.979   | 1.145   | 0.000896      | 10| 45| 87  | 54.504        | 6.423   | 1.280   | 0.000543      | 10| 45|
| 11  | 67.193        | 5.283   | 0.878   | 0.000404      | 10| 45| 88  | 93.836        | 1.455   | 0.640   | 0.000068      | 10| 45|
| 12  | 80.723        | 3.642   | 0.780   | 0.000283      | 10| 45| 89  | 22.915        | 3.249   | 10.603  | 0.092298      | 15| 45|
| 13  | 65.084        | 7.361   | 0.601   | 0.000130      | 10| 45| 90  | 23.783        | 3.791   | 8.040   | 0.040247      | 15| 45|
| 14  | 84.384        | 3.650   | 0.619   | 0.000141      | 10| 45| 91  | 31.232        | 4.566   | 5.172   | 0.010711      | 15| 45|
| 15  | 81.226        | 5.346   | 0.473   | 0.000063      | 10| 45| 92  | 42.199        | 9.339   | 1.475   | 0.000249      | 15| 45|
| 16  | 86.788        | 4.085   | 0.454   | 0.000056      | 10| 45| 93  | 23.388        | 3.692   | 6.333   | 0.047045      | 15| 25|
| Row | $\Delta E/E_0$ | $F_{r_1}$ | $\gamma_c/h$ | $q^2/gH_w^2$ | N   | S   | Row | $\Delta E/E_0$ | $F_{r_1}$ | $\gamma_c/h$ | $q^2/gH_w^2$ | N   | S   |
|-----|----------------|----------|-----------|--------------|-----|-----|-----|----------------|----------|-----------|--------------|-----|-----|
| 17  | 87.066         | 4.101    | 0.442     | 0.000051     | 10  | 45  | 18  | 84.818         | 5.505    | 0.364     | 0.000029     | 10  | 45  |
| 19  | 53.397         | 5.346    | 1.956     | 0.0000271    | 15  | 45  | 20  | 60.260         | 4.820    | 1.875     | 0.0000121    | 15  | 45  |
| 21  | 84.818         | 5.505    | 0.364     | 0.000029     | 10  | 45  | 22  | 53.397         | 5.346    | 1.956     | 0.0000271    | 15  | 45  |
| 23  | 60.260         | 4.820    | 1.875     | 0.0000121    | 15  | 45  | 24  | 64.960         | 6.336    | 1.124     | 0.0000241    | 15  | 45  |
| 25  | 71.521         | 5.883    | 0.991     | 0.0000165    | 15  | 45  | 26  | 83.994         | 4.279    | 0.800     | 0.0000087    | 15  | 45  |
| 27  | 88.529         | 3.764    | 0.654     | 0.000047     | 15  | 45  | 28  | 92.395         | 3.750    | 0.427     | 0.000013     | 15  | 45  |
| 29  | 56.414         | 5.158    | 2.548     | 0.0001249    | 20  | 45  | 30  | 54.585         | 5.391    | 2.513     | 0.0000122    | 20  | 45  |
| 31  | 57.646         | 5.155    | 2.467     | 0.0001061    | 20  | 45  | 32  | 57.767         | 5.249    | 2.399     | 0.0000976    | 20  | 45  |
| 33  | 54.751         | 5.615    | 2.365     | 0.0000935    | 20  | 45  | 34  | 59.966         | 5.199    | 2.286     | 0.000844     | 20  | 45  |
| 35  | 56.414         | 5.158    | 2.548     | 0.0001249    | 20  | 45  | 36  | 60.962         | 5.239    | 2.198     | 0.0000750    | 20  | 45  |
| 37  | 71.836         | 5.084    | 1.589     | 0.0000284    | 20  | 45  | 38  | 77.825         | 4.573    | 1.402     | 0.0000195    | 20  | 45  |
| 39  | 90.285         | 2.673    | 1.055     | 0.000083     | 20  | 45  | 40  | 91.466         | 3.424    | 0.714     | 0.000026     | 20  | 45  |
| 41  | 95.710         | 3.243    | 0.372     | 0.0000004    | 20  | 45  | 42  | 42.443         | 6.275    | 4.484     | 0.001343     | 35  | 45  |
| 43  | 46.099         | 6.045    | 4.383     | 0.001253     | 35  | 45  | 44  | 45.403         | 6.271    | 4.222     | 0.001120     | 35  | 45  |
| 45  | 47.449         | 6.526    | 4.043     | 0.000984     | 35  | 45  | 46  | 51.015         | 6.087    | 3.887     | 0.000874     | 35  | 45  |
| 47  | 53.859         | 6.078    | 3.639     | 0.000718     | 35  | 45  | 48  | 58.269         | 5.884    | 3.398     | 0.000584     | 35  | 45  |
| 49  | 63.573         | 5.571    | 3.144     | 0.000463     | 35  | 45  | 50  | 68.903         | 5.805    | 2.499     | 0.000232     | 35  | 45  |
| 51  | 83.291         | 4.175    | 1.943     | 0.000109     | 35  | 45  | 52  | 91.516         | 2.925    | 1.406     | 0.000041     | 35  | 45  |
| 53  | 94.435         | 2.919    | 0.908     | 0.000011     | 35  | 45  | 54  | 46.559         | 6.271    | 6.038     | 0.001040     | 50  | 45  |
| 55  | 68.387         | 5.267    | 4.245     | 0.000361     | 50  | 45  | 56  | 67.240         | 5.425    | 4.245     | 0.000361     | 50  | 45  |
| 57  | 55.561         | 7.034    | 4.174     | 0.000344     | 50  | 45  | 58  | 58.334         | 6.770    | 4.098     | 0.000325     | 50  | 45  |
| 59  | 63.225         | 6.202    | 4.027     | 0.000309     | 50  | 45  | 60  | 58.139         | 6.883    | 4.027     | 0.000309     | 50  | 45  |
| 61  | 69.063         | 5.410    | 4.000     | 0.000302     | 50  | 45  | 62  | 77.825         | 4.573    | 1.402     | 0.0000195    | 20  | 45  |

Table 4 continued
Table 4 continued

| Row | $\Delta E/E_0$ | $F_{r1}$ | $\gamma_c/h$ | $q^2/gH_w^3$ | N | S | Row | $\Delta E/E_0$ | $F_{r1}$ | $\gamma_c/h$ | $q^2/gH_w^3$ | N | S |
|-----|----------------|--------|-------------|--------------|---|---|-----|----------------|--------|-------------|--------------|---|---|
| 64  | 68.069         | 6.013  | 3.596       | 0.000220     | 50| 45| 141 | 68.842         | 3.952  | 0.584       | 0.000135     | 5 | 15|
| 65  | 79.533         | 4.282  | 3.437       | 0.000192     | 50| 45| 142 | 68.882         | 4.275  | 0.490       | 0.000779     | 5 | 15|
| 66  | 76.888         | 4.910  | 3.301       | 0.000170     | 50| 45| 143 | 83.096         | 4.530  | 0.233       | 0.000804     | 5 | 15|
| 67  | 71.326         | 6.503  | 2.884       | 0.000113     | 50| 45| 144 | 20.522         | 3.244  | 5.008       | 0.108685     | 10| 15|
| 68  | 85.997         | 3.985  | 2.493       | 0.000073     | 50| 45| 145 | 27.132         | 3.277  | 4.263       | 0.067043     | 10| 15|
| 69  | 90.110         | 3.264  | 2.162       | 0.000048     | 50| 45| 146 | 32.461         | 3.349  | 3.655       | 0.042257     | 10| 15|
| 70  | 95.096         | 1.669  | 1.796       | 0.000027     | 50| 45| 147 | 34.180         | 3.740  | 2.964       | 0.022532     | 10| 15|
| 71  | 92.397         | 3.082  | 1.742       | 0.000205     | 50| 45| 148 | 44.370         | 3.742  | 2.349       | 0.011221     | 10| 15|
| 72  | 94.628         | 2.715  | 1.376       | 0.000012     | 50| 45| 149 | 44.105         | 4.156  | 2.030       | 0.000743     | 10| 15|
| 73  | 96.580         | 2.165  | 1.047       | 0.000005     | 50| 45| 150 | 51.193         | 4.420  | 1.568       | 0.003338     | 10| 15|
| 74  | 20.735         | 3.628  | 1.179       | 0.006239     | 3 | 45| 151 | 56.819         | 4.907  | 1.172       | 0.001392     | 10| 15|
| 75  | 28.297         | 3.900  | 0.899       | 0.026757     | 3 | 45| 152 | 80.929         | 3.253  | 0.776       | 0.000404     | 10| 15|
| 76  | 47.728         | 3.832  | 0.599       | 0.007919     | 3 | 45| 153 | 95.475         | 0.598  | 0.298       | 0.000023     | 10| 15|
| 77  | 62.845         | 4.358  | 0.331       | 0.001332     | 3 | 45| 154 | 96.217         | 0.686  | 0.259       | 0.000015     | 10| 15|

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