Chiral Effective Action With Heavy Quark Symmetry

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ABSTRACT

We derive an effective action combining chiral and heavy quark symmetry, using approximate bosonization techniques of QCD. We explicitly show that the heavy-quark limit is compatible with the large $N_c$ (number of color) limit in the meson sector, and derive specific couplings between the light and heavy mesons ($D$, $D^*$, ...) and their chiral partners. The relevance of this effective action to solitons with heavy quarks describing heavy baryons is discussed.
The constraints of chiral symmetry on low energy processes have led to a wealth of predictions ranging from strong to weak interactions. In general, chiral symmetry constrains the dynamics of pions and kaons by organizing the scattering amplitudes in powers of the light meson momenta. Recently, it was suggested that chiral symmetry put also constraints on the soft part of processes involving pions and heavy mesons such as $D$ and $B$ \[1\].

It was suggested by Shuryak \[2\] and more recently by Isgur and Wise \[3, 4\] that if the mass of one quark is taken to infinity, the dynamics of the heavy quark $Q$ is independent of its mass and spin. (We will refer to this limit as Isgur-Wise limit or IW limit in short.) As a result, a new spin symmetry develops in hadronic processes involving one heavy quark, leading to a degeneracy of, say, $D$ with $D^*$ and $B$ with $B^*$. Several relations \[3, 4\] have been recently derived showing that the excitation spectra and form factors are indeed independent of the mass and spin of the heavy quark, a result analogous to the hydrogen atom.

The purpose of this letter is to provide a short derivation of the effective action for QCD processes involving heavy and light but nonstrange mesons, using approximate bosonization schemes, a detailed discussion of which can be found in refs. \[5\]. In the presence of the light vector mesons our effective action differs from the one presently in use in the literature \[1\]. We show that the heavy-quark (IW) limit is compatible with the large $N_c$ (number of color) limit in the meson sector, and argue that heavy baryons \textit{may} be described as solitons, as previously suggested \[6\] in the context of the Callan-Klebanov model \[7\] and recently advocated by Manohar and collaborators \[8\]. While our method could be readily extended to strange mesons as well, here we shall restrict our discussion to two light flavors $q = (u, d)$ and a heavy flavor $(Q)$. The heavy mesons $(\pi Q, \eta Q, \ldots)$. 

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$\mathcal{O} Q$) transform as singlets under chiral symmetry.

If the mass of the heavy quark is infinitely large, then the heavy quark momentum is large and conserved $P_\mu = m_Q v_\mu$. In this limit, there is a velocity superselection rule \cite{12}. In the effective theory with heavy quarks, this translates to a different heavy quark (antiquark) field $Q^\pm_v(x)$ for each velocity $v$. The latter carries momenta of the order of the QCD scale $\Lambda$ and will be referred to as soft. To display this we follow Georgi \cite{12} and define

$$Q(x) = \frac{1 + i \not \! P}{2} e^{-i m_Q v \cdot x} Q^+_v(x) + \frac{1 - i \not \! P}{2} e^{i m_Q v \cdot x} Q^-_v(x).$$

As a result, the free QCD action reads

$$S = \sum_v \int d^4 x \left( \overline{q} \left( i \not \! D - m_q \right) q + Q_v \left( i \not \! v \cdot \partial \right) Q_v \right).$$

The action (2) is flavor $U(2)_L \times U(2)_R$ symmetric (for $m = 0$) and invariant under independent spin rotations of the quark and the antiquark (Isgur-Wise symmetry). The latter follows from the fact that the spin effects are down by powers of $1/m_Q$. Finally, the decomposition (1) is invariant under velocity shifts of the order of $\Lambda$ – a point recently stressed by Luke and Manohar \cite{13}. These conclusions are unaffected by the introduction of gluons to leading order.

Approximate bosonization schemes for QCD have been discussed extensively in the literature \cite{5}. We will apply them here to the heavy-light system. The idea consists of integrating out the short wavelength ($k >> \Lambda$) content of the light quarks generating massive constituent quarks with multiquark interactions (as in the instanton liquid model for instance) admixed with bare but soft heavy quarks. In the long wavelength limit, approximate bosonization schemes can be used to generate an effective action as a gradient expansion in the slowly varying fields that intermingles heavy-light dynamics.
Specifically, if we denote the heavy meson fields by
\[
\hat{H}_\pm = \frac{1 + \slashed{P}}{2} (\gamma^\mu \hat{P}_{\mu, \pm}^* + i\gamma_5 \hat{P}_\pm)\gamma_5^\pm + \text{h.c.}
\]  
where \(\hat{P}_+^a \sim \bar{Q}_R Q_v\), \(\hat{P}_{\mu, +}^a \sim \bar{Q}_R^* \gamma_\mu Q_v\) \((+ \rightarrow -\text{ corresponds to } R \rightarrow L)\) are the pseudoscalar and vector bare heavy mesons with specific light chirality, then standard arguments yield
\[
S = \sum_v \int \bar{\psi} \left( 1_2 i\gamma_\mu \hat{\psi} + 1_3 i\gamma_\mu v \cdot \partial + 1_2 (\hat{L} \gamma_5^+ + \hat{R} \gamma_5^-) - 1_2 (M \gamma_5^+ + M^\dagger \gamma_5^-) + \hat{H}_+ + \hat{H}_- \right) \psi
\]  
Here \(\hat{L}_\mu \sim \bar{q}_L \gamma_\mu q_L\) and \(\hat{R}_\mu \sim \bar{q}_R \gamma_\mu q_R\) are the bare light vector fields, valued in \(U(2)_L\) and \(U(2)_R\) respectively, \(1_2 = \text{diag}(1, 1, 0)\), \(1_3 = \text{diag}(0, 0, 1)\) are the projectors onto the light and heavy sectors respectively and we are using the short-hand notation \(\gamma_5^\pm \equiv \frac{1}{2} (1 \pm \gamma_5)\).

The quark field \(\psi\) in (4) stands for \(\psi = (q; Q_v)\). The \(\hat{P}\)’s are off diagonal in flavor space, so that \(H = H_a T^a\) with \(a = 1, 2\) and \(T^1 = (\lambda_4 - i\lambda_5)/2\), \(T^2 = (\lambda_6 - i\lambda_7)/2\).

The action (4) enjoys both heavy-quark spin symmetry denoted \(SU(2)_Q\) and rigid chiral symmetry, \(U(2)_L \times U(2)_R\). We have set the light-quark masses to zero and ignored the usual assumption-dependent part related to the effective potential in \(M^\dagger M\). Here we will just assume that chiral \(U(2)_L \times U(2)_R\) is spontaneously broken to \(SU(2)_V\) with the appearance of three Goldstone bosons (the chiral anomaly taking care of the \(U(1)_A\)) around a chiral symmetric condensate. With this in mind, we will decompose the \(2 \times 2\) complex matrix \(M\) as follows (a pertinent choice of gauge to avoid doubling of the Goldstone bosons will be specified below)
\[
M = \xi_L^\dagger \Sigma \xi_R
\]  
with the \(\xi\)’s as elements in the coset \(SU(2)_L \times SU(2)_R/SU(2)_V\). In the vacuum (saddle point) \(\Sigma\) is diagonal and constant along the light directions \((u, d)\). As a result, the “bosonized” QCD action is in addition invariant under local \(SU(2)_V\) symmetry : \(\xi_L \rightarrow h(x)\xi_L g_L^\dagger\), \(\xi_R \rightarrow h(x)\xi_R g_R^\dagger\) and \(\Sigma \rightarrow h(x)\Sigma h(x)^\dagger\).
The constituent (dressed) quark field $\chi$ relates to the bare quark field $\psi$, through

$$\chi_{L,R} = (\xi_{L,R} q_{L,R}; Q_v).$$

In terms of the constituent field, the “bosonized” action reads

$$S = \sum_v \int d^4x \chi \left( 1_2i\frac{\partial}{\partial \chi} + 1_3i\frac{\partial}{\partial \chi} v \cdot \partial + 1_2(\xi_{R} \phi \xi_{R}^\dagger \gamma_0^+ + \xi_{L} \phi \xi_{L}^\dagger \gamma_0^-) + 1_2(\xi_{L} \bar{\phi} \xi_{L}^\dagger \gamma_0^- + \xi_{R} \bar{\phi} \xi_{R}^\dagger \gamma_0^+) - 1_2(\Sigma \gamma_0^- + \Sigma^+ \gamma_0^-) + H + H + G + G \chi. \right)$$

(6)

It is invariant under local $SU(2)_V$ symmetry and global $SU(2)_Q$ symmetry. Now, let us define the dressed fields

$$L_\mu = \xi_L \hat{\xi}^\dagger_L \mu + i\xi_L \partial_\mu \xi_L^\dagger,$$

$$R_\mu = \xi_R \hat{\xi}^\dagger_R \mu + i\xi_R \partial_\mu \xi_R^\dagger,$$

(7)

with

$$H = \frac{1 + \gamma_5}{2}(\gamma^\mu P^\mu + i\gamma_5 P) = \frac{1 + \gamma_5}{2}(\gamma^\mu(P^\mu + \xi_{R}^\dagger + P_{\mu}^* - \xi_{L}^\dagger)),$$

$$G = \frac{1 + \gamma_5}{2}(\gamma^\mu\gamma_5 Q^\mu + Q) = \frac{1 + \gamma_5}{2}(\gamma^\mu\gamma_5(P^\mu + \xi_{R}^\dagger - P_{\mu}^* - \xi_{L}^\dagger)) \right)$$

(8)

where the new $H$ and $G$ fields refer to $(D,D^*)$ and their chiral partners $(\tilde{D},\tilde{D}^*)$ respectively. The heavy vector fields are transverse $v^\mu P^\mu = v^\mu Q^\mu = 0$. The “bosonized” QCD action in the dressed fields becomes

$$S = \sum_v \int d^4x \chi \left( 1_2(i\nabla_L - \Sigma)\gamma_5^- + 1_3(i\nabla_R - \Sigma)\gamma_5^+ + 1_3i\phi v \cdot \partial + H + H + G + G \chi \right)$$

(9)

where the covariant $L,R$ derivatives are : $\nabla_L = \partial - iL$ and $\nabla_R = \partial - iR$. $\bar{H}$ and $\bar{G}$ are related, respectively, to $H$ and $G$ through

$$\bar{H} = \gamma^0 H + \gamma^0,$$

$$\bar{G} = \gamma^0 G + \gamma^0.$$

(10)

The light-light and heavy-light quark dynamics follows from the dressed action (9) through a derivative expansion. The momenta are bounded from above by a number.
times $\Lambda$ and from below by a soft quark mass $\epsilon \sim \Lambda^2/m_Q$ acquired by the long wavelength components of the heavy-quark propagator, if we recall that the heavy-quark condensate vanishes as $\Lambda^4/m_Q$. This effect is dominant in the infrared regime for the soft part of the heavy-quark field $Q_v$. Our expansion of (9) will be understood in the sense of $m_Q/\Lambda \to \infty$.

To second order, the heavy-light induced action reads

$$S_H = N_c \text{Tr} \left( \mathbf{1}_2 \Delta_l H \mathbf{1}_3 \Delta_h \overline{H} \right) - N_c \text{Tr} \left( \mathbf{1}_2 \Delta_l \left( \mathbf{V} \Delta_l + \mathbf{A} \Delta_l \gamma_5 \right) H \mathbf{1}_3 \Delta_h \overline{H} \right) + \cdots$$

(11)

where the functional trace includes tracing over space, flavor and spin indices. We have denoted $\Delta_l = (i\partial - \Sigma)^{-1}$ and $\Delta_h = (i\partial_v \cdot \partial)^{-1}$, and

$$\gamma_5^+ R + \gamma_5^− L = \mathbf{V} + \gamma_5 A.$$  

(12)

The ellipsis in (11) stands for higher insertions of vectors and axials. A similar action is expected for the heavy chiral partners $G$'s. To the order considered, there are also cross terms generated by the axial current.

After carrying out the trace over space in (4) and renormalizing the heavy-quark fields, $H \to H/\sqrt{Z_H}$ and $G \to G/\sqrt{Z_G}$, we obtain to leading order in the gradient expansion for the $H$'s \((s^P_l = \frac{1}{2}^-)\)

$$\mathcal{L}^H_v = -\frac{i}{2} \text{Tr} \left( \tilde{H} v^\mu \partial_\mu H - v^\mu \partial_\mu \tilde{H} H \right)$$

$$+ \text{Tr} V_\mu \tilde{H} H v^\mu - g_H \text{Tr} A_\mu \gamma^\mu \gamma_5 \tilde{H} H + m_H \text{Tr} \tilde{H} H$$

(13)

and for the $G$'s \((s^P_l = \frac{1}{2}^+)\) \(\Box\)

$$\mathcal{L}^G_v = +\frac{i}{2} \text{Tr} \left( G v^\mu \partial_\mu G - v^\mu \partial_\mu \tilde{G} G \right)$$

$$- \text{Tr} V_\mu \tilde{G} G v^\mu - g_G \text{Tr} A_\mu \gamma^\mu \gamma_5 \tilde{G} G + m_G \text{Tr} \tilde{G} G$$

(14)

\(^1\)We label $H$ and $G$ by the total angular momentum $s_l$ and parity $P_l$ of the light degrees of freedom.
The parameters in (13,14) are given by \((P_l = \pm\) is the parity of the light part of \(H,G)\)

\[
Z_{P_l} = N_c \int_\varepsilon^\Lambda \frac{d^4Q}{(2\pi)^4} \frac{-1/(v\cdot Q) + P_l 2\Sigma/(Q^2 - \Sigma^2)}{(Q^2 - \Sigma^2)^2} \\
g_{P_l} = \frac{N_c}{Z_{P_l}} \int_\varepsilon^\Lambda \frac{d^4Q}{(2\pi)^4} \frac{1}{v\cdot Q} \frac{-Q^2/3 + \Sigma^2}{(Q^2 - \Sigma^2)^2} \rightarrow 1 \\
m_{P_l} = \frac{N_c}{Z_{P_l}} \int_\varepsilon^\Lambda \frac{d^4Q}{(2\pi)^4} \frac{1 + P_l \Sigma/(v\cdot Q)}{Q^2 - \Sigma^2} \rightarrow -P_l \Sigma
\]

(15)

with \(\epsilon \sim \Lambda^2/m_Q\) and the limits in (13) follow from \(m_Q/\Lambda \rightarrow \infty\). Note that \(m_H\) and \(m_G\) are of order \(m_0^Q\). The interaction between the \(H\)'s and the \(G\)'s is given by

\[
L_{vG}^{H} = -\sqrt{Z_H/Z_G} \text{Tr}(\gamma_5 G v^\mu A_\mu) - \sqrt{Z_G/Z_H} \text{Tr}(\gamma_5 H \gamma^\mu A_\mu) 
\]

(16)

Notice that \(Z_G/Z_H = g_H/g_G\) reduces to 1 in the heavy quark limit. We observe that the mass splittings between the \(H\)'s (\(D,D^*,...\)) and their chiral partners the \(G\)'s (\(\tilde{D},\tilde{D}^*,...\)) imply the following mass relations to order \(m_0^Q N_c^9\)

\[
m(\tilde{D}^*) - m(D^*) = m(\tilde{D}) - m(D) = \Sigma
\]

(17)

where again \(\Sigma\) is the constituent mass of the light quarks in the chiral limit. This is expected since the \(D, D^*\) are S-wave mesons, while the \(\tilde{D}, \tilde{D}^*\) are P-wave mesons (not yet observed). In the nonrelativistic quark model the difference is centrifugal and of order \(m_0^Q\). This point has been appreciated already by Shuryak in the context of bag models and QCD sum rules.

Since the decomposition (13) doubles the scalar degrees of freedom, a proper gauge fixing in \(\xi\)'s is required. We choose the “unitary gauge” \(\xi_{L}^\dagger = \xi_R \equiv \xi = e^{i\pi/2f^*}\). In a minimal model with only pions (model I), we have for the vector and axial currents

\[
V_\mu = +\frac{i}{2} \left( \xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi \right), \\
A_\mu = +\frac{i}{2} \left( \xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi \right).
\]

(18)

\footnote{The observed \(D_1(2400)\) and \(D_2^*(2460)\) are components of the \(s_l^P = \frac{3}{2}^+\) multiplet.}
If we were to introduce vector mesons (ω, ρ, a_1(1270)) (model II) then the vector and axial currents are *entirely* vectorial

\[
V_\mu = \omega_\mu + \rho_\mu \\
A_\mu = A_\mu - \frac{\alpha}{f_\pi} \partial_\mu \pi
\]  

(19)

where \( \alpha = (m_\rho/m_A)^2 \) follows after eliminating the πa_1 mixing at tree level in the light-light sector [1]. The relations (19) identify the vector fields with the constituent vector currents, e.g. \( \omega_\mu \sim \chi \gamma_\mu \chi \), as can be checked explicitly in (3). They also enforce vector dominance in the light-light sector. Clearly other constructions are also theoretically possible in which the amount of pion dressing is intermediate between model I and model II, using the relations (7). Of course, the issue of how the effective fields are physically defined (dressed) can only be resolved by comparing the various model predictions with experiment.

The renormalized effective action involving both \( H \) and \( G \) and their interactions is invariant under local \( SU(2)_V \) (or more precisely \( U(2)_V \) including the singlet) symmetry (h), in which \( V \) transforms as a gauge field, \( A \) transforms covariantly and \( H,G \to Hh^\dagger, Gh^\dagger \) and \( \overline{H}, \overline{G} \to \overline{h}H, \overline{h}G \). It is also invariant under heavy-quark symmetry \( SU(2)_Q \) (S), \( H,G \to SH, SG \) and \( \overline{H}, \overline{G} \to \overline{S}H, \overline{S}G^\dagger \).

Our renormalized effective action (13) with only pions (model I) is entirely consistent with the one suggested by Wise and others (11) (aside from the mass term). Our derivation suggests \( g_H = 1 \) with a specific *sign* assignment for the axial term. The magnitude of \( g_H \) is consistent with the constituent quark model. The rest of our effective action is also consistent with the effective action written down recently by [11].

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\(^3\)Note that in this case the soliton approach to heavy hyperons follows from binding \( Q, Q^* \) instead of \( P, P^* \) to Skyrmions.

\(^4\)In the convention of [10, 11] the P-wave heavy quark field \( H_\alpha \) relates to our \( G \) through \( H_\alpha =\)
The introduction of vector mesons along with pions (model II) which is a more realistic description of the light-light sector yields a totally new effective action for the heavy-light sector. The identification of the currents is entirely vectorial in this case. The pion coupling to the heavy particles occurs solely through the longitudinal component of the axial current. In this case the $\pi$-HH coupling is quenched in comparison with the $a_1$-HH coupling (1 in this case), $g_{\pi HH} = (m_\rho/m_A)^2 = .37$, improving the decay width $D^* \rightarrow D\pi$ of model I by about $1/2$, in the limit where the heavy quark mass is infinite.\footnote{The recent CLEO collaboration data seem to indicate a somewhat smaller empirical value $g_H \approx .58$ with a large error bar. See \cite{9} for an analysis.}

Our arguments suggest that in the heavy-quark limit and in the chiral limit, a similar effective action should involve the chiral partners of the heavy pseudoscalars and vectors, here denoted by $G$. Their role in the soliton scenario might be important for the description of opposite-parity heavy-baryon states. They also allow for a qualitative estimation for the coefficients involved, and provide a rationale for the systematic expansion in both $k_\pi/(\sqrt{N_c}\Lambda)$ (derivative or $1/N_c$ expansion) and $\Lambda/m_Q$ (heavy-quark expansion).

The scaling with $N_c$ of the overall heavy-light effective action before renormalization, suggests that the heavy-light system should be entrusted with the same weight as the light-light counterpart (which is well known and hence omitted in our discussion) in the large $N_c$ limit, implying that a soliton description for heavy baryons is perhaps justified \cite{1}. The large $N_c$ limit appears to be compatible with the heavy-quark limit in the meson sector. This point is a priori not obvious since terms of the form $m_Q/N_c\Lambda$ and others cannot be ruled out on general grounds. Thus our approximate bosonization version of QCD yields a long wavelength description that appears to be consistent with the one advocated recently by \cite{11}. This is to be contrasted with the “colored” mesonic description recently discussed $G(v_\alpha - \gamma_\alpha)/\sqrt{3}$.\footnote{\citel{9}}
by Ellis et al [15] where it was argued that in QCD the presence of a large current quark mass forces the bosonization to be $U(N_f N_c) \times U(N_f N_c)$ \textit{(i.e., colored bosons)} as opposed to the factorized bosonization scheme $(U(N_f) \times U(N_f)) \otimes (U(N_c) \times U(N_c))$ \textit{(colorless bosons)}. Here $N_f$ is the number of flavours. It would be interesting to see how the approximate bosonization scheme discussed here works in QCD in comparison to the exact solutions to the 't Hooft equations for heavy-light systems [16]. A similar comparison is also warranted for the solutions discussed by Ellis \textit{et al.}

We note that that in the meson sector the $HH\pi$ interaction is of order $1/\sqrt{N_c}$ as expected. In the soliton sector, however, the effect of this interaction is in \textit{principle} of order $N_c$, since we expect $H \sim \sqrt{N_c}$ and $\pi \sim \sqrt{N_c}$ \textit{(i.e., semiclassical field)}. However, in the heavy-quark limit, the heavy meson field is \textit{in general} localized over a range of the order of $1/(N_c m_Q)$ affecting the energy to order $N_c^0$. Hence, one would expect heavy baryons composed of pions and $P, P^*$ \textit{(or} $Q, Q^*)$ \textit{to emerge to order} $N_c^0$ or lower.\(^6\) The emergence of the heavy baryon spectrum depends crucially on whether the $P, P^*$ bind to the soliton to order $N_c^0$.

In the Callan-Klebanov scheme, the presence of the Wess-Zumino term causes P-wave kaons to bind to the soliton \textit{(here} $U_0 = \xi^2_0$) \textit{to order} $N_c^0$. The bound state carries good grand spin $K = I + J$ \textit{(here} $1/2^-$) \textit{and heavy-flavor quantum number}. However states with good isospin ($I$) and angular momentum ($J$) emerge after \textit{“cranking”} \textit{(or rotating)} the kaon-soliton bound state as a \textit{whole}. This means that heavy meson soliton states with good quantum numbers are expected only to order $1/N_c$. In the scheme recently advocated \(^6\)The sharp localisation of the heavy quark field $H$ in the heavy mass, large $N_c$ limit may be at odd with the soft character of $H$ and thus invalidate \textit{even further} the use of the derivative expansion in the soliton sector of heavy baryons than in that of light-quark baryons. \textit{(The derivative expansion for skyrmions in the light-quark sector is bad for the same reason that the standard chiral perturbation theory does not work with baryons and pions.) Also there might be problems with the commutativity of the heavy mass, large $N_c$ limit in the same sector. These points are presently under investigations.
by Manohar and collaborators\cite{8}, the heavy mesons are treated as free wavepackets of the size of the Compton wavelength of the heavy quark. In this case only the soliton is cranked leaving the heavy meson field unrotated and good quantum states composed of the soliton and the heavy meson appear already to order $N_c^0$. At this order, all the baryons of flavor $Q$ (i.e., $\Lambda$, $\Sigma$, $\Sigma^*$ etc.) are degenerate. The splitting $\Sigma - \Lambda$ occurs to order $1/N_c$, while $\Sigma^* - \Sigma$ occurs to order $1/m_Q N_c$. In this scheme, the Wess-Zumino term plays no role.

In both approaches, good quantum numbers and fine-structure (or $\Sigma - \Lambda$) splitting are obtained to order $1/N_c$. In the latter case, the Isgur-Wise symmetry is automatic to that order but in the former case, terms involving vector mesons have to be added to restore the IW symmetry.

An interesting question to ask is: Is it possible to interpolate the Callan-Klebanov model to the heavy-quark limit? We have no clear answer but we can think of two possibilities.

- As noted above, in the heavy-quark limit, the heavy mesons decouple from the Wess-Zumino term. Therefore, the primary source of binding in the conventional form of the Callan-Klebanov model would be lost. If there is no other source of binding, then the bound state would disappear into the continuum for a finite but heavy-quark mass (following the disappearance of the Wess-Zumino term), causing the decoupling of the heavy kaon from the soliton to order $N_c^0$. This would signal the breakdown of the Callan-Klebanov picture at some large quark mass and the only viable heavy skyrmion would be the one advocated in [8].

- Although the topological Wess-Zumino term does not survive heavy quark mass, the binding could still persist all the way to the heavy-quark limit. In fact our effective action with the inclusion of the omega yields naturally a coupling of the form
\( \text{Tr}(HH) \cdot \omega \) between the heavy meson and the omega (model II). If we recall that the light-light effective action induces a pion-omega coupling of the form \( N_c \omega^\mu B^\mu \), where \( B^\mu \) is the properly normalized baryon current, then by eliminating the \( \omega^0 \) part through its constraint equation, we generate a term in the Hamiltonian density of the form (heavy omega)

\[
\mathcal{H} \sim \frac{g^2}{2m^2_\omega} (N_c B^0 + v^0 \text{Tr}(HH))^2
\]  

(20)

Here \( g \sim 1/\sqrt{N_c} \) is the vector gauge coupling. With our conventions, the omega-three-pion coupling is \( g_\omega = N_c g \sim \sqrt{N_c} \). Clearly the choice \( \text{Tr}(HH)v^0 \sim -B^0 \), can lower the energy to order \( N_c^0 \). This argument is similar to the one advanced by [14].

We see no \textit{a priori} reason for suppressing the heavy meson couplings to the light vector mesons. In this case it would appear that the Callan-Klebanov description would go \textit{smoothly} over to the the IW limit, provided the appropriate vector mesons are introduced to restore the IW symmetry (see also footnote 6). Approaching the IW limit in this way is not perhaps very elegant but what is surprising is that it can be done at all.

Which of the two pictures is realized in Nature is an intriguing question which we hope to answer.

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