THICK BRANE WORLD MODEL
FROM PERFECT FLUID

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Abstract

A (1 + d)-dimensional thick "brane world" model with varying Λ-term is considered. The model is generalized to the case of a chain of Ricci-flat internal spaces when the matter source is an anisotropic perfect fluid. The "horizontal" part of potential is obtained in the Newtonian approximation. In the multitemporal case (with a Λ-term) a set of equations for potentials is presented.

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1 Introduction

An interest in the so-called “brane world” models (see [1]-[22] and refs. therein) has recently greatly increased due to Refs. [8, 9, 10]. The idea of a brane world is rather simple. It is supposed that we are living on a (1 + 3)-dimensional thin (or thick) layer (3-brane) in multidimensional space-time and there exists a potential preventing us from leaving this layer, i.e. gauge and matter fields are localized on branes whereas gravity "lives" in the multidimensional bulk. Randall and Sundrum [10] suggested a construction for a confining potential (for an attractive potential see also [18]), using two symmetric copies of part of 5-dimensional anti-deSitter space, so that "our" 4-dimensional space-time is a "wedge of the edge". The confining potential has a $|y|$-type shape, where $y$ is the coordinate of the extra non-compact fifth dimension.

It should be noted that the main stream of the brane world studies is related to thin brane model, although it looks more natural for this "brane" to be thick rather than thin.

A simple 5-dimensional thick brane model was suggested in [14]. In this paper we start with a generalization of this model to $(d + 1)$-dimensional case, i.e. a multidimensional thick brane model with a $y$-dependent cosmological term is presented, where $y$ is an extra coordinate (Sec. 2). (For a review of models with the cosmological term see [23].) Then, we show that there exist a lot of alternative thick brane models that may be added to a "brane world collection" after certain investigations. As an example we consider a brane world model with a chain of Ricci-flat internal spaces (e.g., compact ones) and an anisotropic perfect fluid (see [25, 26] and references therein) as a matter source (Sec. 3). Instead of an $AdS_5$ solution and its thick brane extension, we start with the Euclidean version of the inflationary solution with perfect fluid from [28] defined on a product of Ricci-flat spaces: The metric of the solution is defined on the manifold $M = \mathbb{R} \times M_1 \times \ldots \times M_n$ and has the form

$$g = dy \otimes dy + \sum_{i=1}^{n} \exp[2\phi^i(y)]g^i,$$

where $(M_i, g^i)$ are Ricci-flat manifolds, $(i = 1, \ldots, n; n \geq 1)$ and $(M_1, g^1)$ is "our space-time". We obtain linearized equations for the potential in the Newtonian approximation and present their ("Newton-Yukawa-type") solutions (Sec. 3). In the "$\Lambda$-term" case, a multitemporal analogue of the set of equations on the potentials is presented (Sec. 2). (For brane-world models with extra timelike dimensions see, e.g., [12, 21] and references therein.)

2 Thick brane with varying cosmological term

2.1 The model

We first consider a model governed by the action

$$S = \int d^Dz \sqrt{|g|[R[g] − 2\Lambda]} + \int d^Dz \sqrt{|g_B|}\sigma,$$

(2.1)
where \( g = g_{MN}(z)dz^M \otimes dz^N \) is a metric, defined on the \((D = 1+d)\)-dimensional manifold

\[
M^D = \mathbb{R} \times M^d, \tag{2.2}
\]

\(z = (z^M) = (x^\mu, y)\), \(x^\mu\) are coordinates on \(M^d\), \(y\) is a coordinate on \(\mathbb{R}\), and

\[
g_B(y) = g_{\mu\nu}(x, y)dx^\mu \otimes dx^\nu \tag{2.3}
\]

is a \(y\)-parametrized family of brane metrics defined on sections \(\{y\} \times M^d\) isomorphic to \(M^d\), \(\mu, \nu = 0, \ldots, d-1\). We put \(|g_B| \equiv |\det(g_{\mu\nu}(x, y))| \neq 0\).

The cosmological term \(\Lambda\) and the brane tension \(\sigma\) are in general (smooth) functions on \(M^D\). We suppose that these functions only depend on the "extra" dimension, i.e.

\[
\Lambda = \Lambda(y), \quad \sigma = \sigma(y). \tag{2.4}
\]

We start with the simplest solutions to the equations of motion corresponding to the action (2.1) defined on the manifold (2.2) with \(\Lambda\) and \(\sigma\) of the form (2.4). The solution reads

\[
g = e^{2\phi(y)}g^1 + dy \otimes dy, \tag{2.5}
\]

where \(g^1\) is a Ricci-flat metric on \(M^d\) \((R_{\mu\nu}[g^1] = 0)\) and the extra-dimensional ("vertical") potential \(\phi = \phi(y)\) satisfies the following relations:

\[
d(d - 1)(\phi')^2 = -2\Lambda, \quad 2(d - 1)\phi'' = \sigma. \tag{2.6}
\]

Here and below \(\phi' = d\phi / dy = \phi_y\).

Indeed, the Hilbert-Einstein equations read

\[
R_{MN} - \frac{1}{2}g_{MN}R = T^B_{MN} - \Lambda g_{MN}, \tag{2.7}
\]

where the brane energy-momentum tensor for the block-diagonal metric \(g_{\mu y} = 0\) with \(g_{yy} = 0\) reads (see Appendix A)

\[
(T^B_{MN}) = \frac{\sigma}{2} \left( \begin{array}{cc} g_{\mu\nu} & 0 \\ 0 & 0 \end{array} \right). \tag{2.8}
\]

Using the relations for the Ricci-tensor components of \(g\) and the scalar curvature from Appendix B we get the "fine-tuning" relations (2.6).

Let us now consider the first example: flat pseudo-Euclidean metric \(g^1 = \eta = \eta_{\mu\nu}dx^\mu \otimes dx^\nu\), where \((\eta_{\mu\nu}) = \text{diag}(-1, +1, \ldots, +1)\). Then for \(\sigma = 0\), \(\phi(y) = ky\) with constant \(k\) satisfying

\[
\Lambda = -d(d - 1)k^2 / 2 \tag{2.9}
\]

we get the \(D\)-dimensional anti-de Sitter metric in (2.5).

For another choice

\[
\phi(y) = k|y|, \quad \sigma = 4(d - 1)k\delta(y) \tag{2.10}
\]

and \(\Lambda\) from (2.9) we get a thin \(d\)-dimensional brane, attractive for \(k > 0\) (for \(d = 4\) see [18]) and repulsive for \(k < 0\) (for \(d = 4\) see [10]).
2.2 Newtonian approximation

Consider small perturbations of the flat \( d \)-metric in the "Newtonian" approximation, i.e., we put

\[
g^1 = -\exp(2v(x, y))dt \otimes dt + (dx \otimes dx)_{d-1},
\]

(2.11)

where \( v(x, y) \) is a small enough "horizontal" part of the gravitational potential, \( t = x^0 \) is a time variable. Here and in what follows \( (dx \otimes dx) = \sum_{i=1}^{k} dx_i \otimes dx_i \).

We rewrite Eqs. (2.7) in the equivalent form

\[
R_{MN} = T_{BM}^{B} + \frac{g_{MN}}{D-2}(2\Lambda - T),
\]

(2.12)

\( T = T_{MN}^{B}g^{MN} \). Using the relation for the \( tt \)-component of the Ricci tensor of the metric (2.5) with \( g^1 \) from (2.11) (see Appendix B)

\[
R_{tt}[g] = e^{2(\phi + v)}[\phi'' + 2\phi'\phi' + \phi' + O(v^2)] + e^{-2\phi}\Delta v + \phi'' + (d+1)v'\phi' + O(v^2),
\]

(2.13)

we get from (2.12) and (2.6) a linearized equation for small \( v \)

\[
\Delta v + e^{2\phi}[v'' + (d+1)\phi'\rho' + m^2e^{-2\phi}\rho] = 0,
\]

(2.14)

where \( \Delta \) is the Laplace operator on \( \mathbb{R}^{d-1} \).

Eq. (2.14) can be easily solved by separation of variables, i.e. by seeking solutions as superposition of monoms: \( v = v_1(x)v_2(y) \). We are interested in spherically symmetric solutions with a certain behaviour at infinity \( |x| \to \infty \). The solution reads

\[
v = \frac{GM}{r^d} + \int_0^{+\infty} dm \rho(m, y) \frac{1}{r^d} \exp(-mr),
\]

(2.15)

where \( d = d-2 \) and

\[
\rho'' + (d+1)\rho' + m^2e^{-2\phi}\rho = 0.
\]

(2.16)

\( G \) is the \((d\text{-dimensional)}) gravitational constant, \( M \) is the mass and \( m \) is a spectral parameter. We restrict ourselves to the "Yukawa-type" part of the general solution. We thus consider a superposition of the Newtonian potential and the generalized Yukawa-type one.

2.3 Multitemporal generalization

Consider a multitemporal generalization of the metric (2.11), i.e.

\[
g^1 = -\sum_{i=1}^{n} \exp(2v_i(x, y))dt_i \otimes dt_i + (dx \otimes dx)_{d-n},
\]

(2.17)

where \( v_i(x, y) \) is a small enough part of the gravitational potential corresponding to \( t_i, i = 1, \ldots, n, n > 1 \). Using the relations for the Ricci-tensor components (see Appendix B)

\[
R_{tt_i}[g] = e^{2(\phi + v_i)}[\phi'' + 2\phi'\phi' + v_i'' + \phi' dv_i' + \sum_{j=1}^{n} v_j'] + O(v^2),
\]

(2.18)
we get from Eqs. (2.12) and (2.6) the following relations:

\[ \Delta v_i + e^{2\phi}[v_i'' + \phi'(dv_i' + \sum_{j=1}^{n} v_j')] = 0, \quad (2.19) \]

\( i = 1, \ldots, n \). Here \( \Delta \) is the Laplace operator on \( \mathbb{R}^{d-n} \). For \( n = 1 \) we get Eq. (2.14). Eqs. (2.13) can be easily solved in the symmetric case, when \( v_i = v \). In this case we get the relations (2.15) with \( \bar{d} = d - 1 - n \) and (2.16) with \( d \) replaced by \( d + n \). We note that in the vacuum case \( \phi = 0 \) the multitemporal analogs of the Schwarzschild and Tangherlini solutions from [24] give us exact solutions to the field equations.

### 3 Thick brane with perfect fluid on a product of \( n+1 \) spaces

Consider a generalization of the thick brane solution from the previous section to the case of \( n - 1 \) Ricci-flat spaces and perfect fluid as a matter source (see [25, 26] and references therein).

We take the metric

\[ g = dy \otimes dy + \sum_{i=1}^{n} \exp[2\phi^i(y)]g^i, \quad (3.1) \]

defined on the manifold

\[ M = \mathbb{R} \times M_1 \times \ldots \times M_n, \quad (3.2) \]

where the manifolds \( M_i \) with the metrics \( g^i \) are Ricci-flat spaces of dimensions \( d_i, \quad i = 1, \ldots, n; \quad n \geq 1 \). One of the spaces, say \((M_1, g^1)\), is by convention ”our space-time” and other spaces are ”internal”. Consider the Einstein equations

\[ R^M_N - \frac{1}{2} \delta^M_N R = \kappa^2 T^M_N, \quad (3.3) \]

where \( \kappa^2 \) is the gravitational constant and the energy-momentum tensor is adopted in the form

\[ (T^M_N) = \text{diag}(p_y, p_1 \delta^m_{k_1}, \ldots, p_n \delta^m_{k_n}). \quad (3.4) \]

Here \( p_y = p_y(y) \) is the pressure in the 1-dimensional \( y \)-space \( \mathbb{R} \) and \( p_i = p_i(y) \) is the pressure in \( M_i, \quad i = 1, \ldots, n \).

Using relations for Einstein tensor components \( E^M_N = R^M_N - \frac{1}{2} \delta^M_N R \) for the metric (3.1) (see Appendix C), we obtain from Einstein Eqs. (3.3)

\[ p_y = -\frac{1}{2\kappa^2}G_{ij}\phi^i_y \phi^j_y, \quad (3.5) \]

\[ p_i = -p_y - \frac{1}{\kappa^2 d_i}G_{ij}(\phi^j_{yy} + \phi^j_y d_k \phi^k_y), \quad (3.6) \]

\( i = 1, \ldots, n \). Here

\[ G_{ij} = d_i \delta_{ij} - d_i d_j, \quad (3.7) \]
are components of the minisuperspace metric, \( i, j = 1, \ldots, n \) \cite{28}. The minisuperscopic metric has a pseudo-Euclidean signature.

The pressures may be decomposed into "bulk" and "brane" parts

\[
p_i = p_i^{\text{bulk}} + p_i^{\text{br}} ,
\]

where by definition

\[
p_i^{\text{bulk}} = -p_y - \frac{1}{\kappa^2 d_i} G_{ij} \partial_j \phi \partial_k \phi_k, \quad i = 1, \ldots, n.
\]

(3.9)

\[
p_i^{\text{br}} = -\frac{1}{\kappa^2 d_i} G_{ij} \partial_j \phi \partial_{yy}, \quad i = 1, \ldots, n.
\]

(3.10)

Defining \( p_y^{\text{bulk}} = p_y, \ p_y^{\text{br}} = 0 \), we decompose the stress-energy tensor into a sum of two components: \( T^M_N = T_{\text{bulk}, M}^N + T_{\text{br}, M}^N \) where

\[
(T_{\text{bulk}, M}^N) = \text{diag}(p_y, p_i^{\text{bulk}} \delta_{k_1}, \ldots, p_n^{\text{bulk}} \delta_{k_n}), \quad (T_{\text{br}, M}^N) = \text{diag}(0, p_i^{\text{br}} \delta_{k_1}, \ldots, p_n^{\text{br}} \delta_{k_n}).
\]

As we shall see below these decompositions will be justified by the thin brane limit.

**One-function dependence.** Consider the following ansatz

\[
\phi^j(y) = -\kappa \frac{u^i}{\sqrt{-<u, u>}} f(y),
\]

(3.11)

where \( f(y) \) is a smooth function, \( u^i = G^{ij} u_j, \ u = (u_i) \in \mathbb{R}^n \),

\[
<u, v> = G^{ij} u_i v_j
\]

(3.12)

is a scalar product on \( \mathbb{R}^n \), \( u, v \in \mathbb{R}^n \), and

\[
G^{ij} = \frac{\delta^{ij}}{d_i} + \frac{1}{2 - D}
\]

(3.13)

are components of the matrix inverse to \( (G_{ij}) \) \( (D = 1 + \sum_{i=1}^n d_i) \) \cite{28}. Here we suppose that

\[
<u, u> > 0.
\]

(3.14)

As we shall see below, this condition is satisfied for isotropic case when all pressures are equal: \( p_i = p, \ i = 1, \ldots, n \).

For the pressures (3.9) and (3.10) we obtain in the special case (3.11)

\[
p_i^{\text{bulk}} = \left( \frac{<u, v>}{<u, u> d_i} - 1 \right) p_y, \quad (f'(y))^2 = 2p_y(y),
\]

(3.15)

\[
p_i^{\text{br}} = \frac{u_i}{\kappa d_i \sqrt{-<u, u>}} f''.
\]

(3.16)

Here the vector

\[
u_i^\Lambda = 2d_i,
\]

(3.17)

\( i = 1, \ldots, n \), corresponds to the cosmological constant case.
Since the relations for the metric and pressures are invariant under the replacement $u \mapsto \lambda u$ ($\lambda > 0$) we may normalize $u = (u_i)$ by the condition

$$< u^\Lambda, u > = < u, u > .$$

(3.18)

In this case Eq. (3.15) reads

$$p_{i}^{bulk} = \left( \frac{u_i}{d_i} - 1 \right) p_y,$$

(3.19)

$i = 1, \ldots, n$.

We note that for $f(y) = \sqrt{2p_y}$ the solution is nothing else but a Euclidean version of the (exponential) inflationary solutions from [26] (for $u = u^\Lambda$ see also [27]). For $n = 1$ one get $u = u^\Lambda$ due to the normalization condition (3.18), thus the perfect fluid generalizations are non-trivial only for $n > 1$.

**Isotropic case.** For $u_i = h d_i$, we get from (3.18) $h = 2$, i.e. we are led to cosmological constant case: $u^\Lambda = u$ with

$$u^i = \frac{2}{2 - D}, \quad < u, u > = -4 \frac{D - 1}{D - 2} < 0,$$

(3.20)

$i = 1, \ldots, n$. Thus the restriction (3.14) is satisfied identically. In this case we get

$$\phi^i(y) = \frac{\kappa}{\sqrt{(D - 1)(D - 2)}} f(y),$$

(3.21)

and

$$p_{i}^{br} = \frac{\sqrt{D - 2}}{\kappa \sqrt{D - 1}} f'' \quad \text{and} \quad p_{i}^{bulk} = p_{y} = (f'(y))^2/2,$$

(3.22)

$i = 1, \ldots, n$. Since all scale factors are equal, one can reduce the isotropic case to the 1-space case making the redefinition $g^1 + \ldots. g^n = \bar{g}^1$. The varying cosmological constant reads: $\Lambda(y) = -\kappa^2 p_y(y)$. For $n = 1$ we get the relations (2.6) with $2p_{1}^{br} = \sigma$.

**Thin brane.** Consider a special solution with

$$\phi^i(y) = \sqrt{2p_y} |y|,$$

(3.23)

$p_y > 0$. In this case we get a ”thin brane” with the pressures

$$p_{i}^{br} = \frac{u_i \sqrt{2p_y}}{\kappa d_i \sqrt{- < u, u >} 2\delta(y)},$$

(3.24)

$i = 1, \ldots, n$. We see that the thin-brane tension is proportional to a square root of the pressure in the $y$-dimension.

**Newtonian approximation.** Using a relation for $R_{tt}[g]$ from Appendix B in the multispace case with $g^1$ from (2.11), we get a modification of the Eq. (2.14) to the perfect-fluid case with $(n - 1)$ internal Ricci-flat spaces

$$\Delta v + e^{2\phi} \left\{ v'' + \sum_{i=1}^{n} d_i (\phi^i)' v' \right\} = 0,$$

(3.25)
where $\Delta$ is the Laplace operator on $\mathbb{R}^{d_1-1}$. Here all information about the perfect fluid and the internal spaces is hidden in the behavior of functions $\phi^i(u)$. The formal solution (3.25) coincides with that of (2.15) ($d = d_1$) but the equation for the density (2.16) should be modified as follows

$$\rho'' + \left( (\phi^1)' + \sum_{i=1}^n d_i (\phi^i)'' \right) \rho' + m^2 e^{-2\phi^1} \rho = 0.$$  

(3.26)

### 4 Conclusions and discussion

Here we have considered a generalization of the 5-dimensional thick brane model with a $y$-dependent cosmological term from [14] to the multidimensional case (Section 2) and to a multifactor product-space case, when the anisotropic perfect fluid (Sec. 3) is adopted as a matter source. These models have thin brane limits. In both cases we have obtained linearized equations for small "horizontal" potentials and their Newton-Yukawa-type solutions. In the "$\Lambda$-term" case we have also considered a multitemporal generalization and a set of linearized equations for potentials is written (it is solved in the symmetric case). In the perfect fluid case we have suggested two brane world models: (i) a general one, with more or less arbitrary $y$-dependent scale factors and pressures (decomposed into "perfect" and "brane" parts); (ii) a special model governed by one function $f(y)$ and the anisotropy parameters $u_i$. In the thin brane limit the latter coincides for $y > 0$ with the Wick-rotated inflationary solution from [26] (and symmetrized with respect to reflection in $y$).

A further consideration needs a globally consistent treatment of linearized gravity and exact solutions. Some special smeared (or regularized) thin brane solutions may be also considered (e.g., with a cosmological type metric $g^1$).

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**Appendix**

A. Energy-momentum tensor for thick brane

Consider the thick brane part of the action

$$S_B[g] = \int d^Dz \sqrt{|g_B|} \sigma(z)$$  

(A.1)

where $|g_B| = |\det g_{\mu\nu}| \neq 0$ ($\mu, \nu = 0, \ldots, d - 1$) and the metric $g = g_{MN}(z)dz^M \otimes dz^N$ is defined on the $(D = 1 + d)$-dimensional manifold (2.2).
We take the following representation of the metric

\[(g_{MN}) = \begin{pmatrix} g^B_{\mu\nu} & n_\mu \\ n_\nu & n_\rho n^\rho + b \end{pmatrix}, \tag{A.2} \]

where \(b = \det(g_{MN})/\det(g_{\mu\nu}) \neq 0\). For the inverse matrix we get

\[(g^{MN}) = \begin{pmatrix} g^B_{\mu\nu} + b^{-1}n^\mu n^\nu & -n_\mu b^{-1} \\ -n_\nu b^{-1} & b^{-1} \end{pmatrix}, \tag{A.3} \]

where \((g^B_{\mu\nu}) = (g_{\mu\nu})^{-1}\) and \(n_\mu = g^B_{\mu\nu} n_\nu\).

Variation of the action \(\delta S_B[g]\)

\[\int d^Dz \sqrt{|g^B|} \sigma \left[ g_{\mu\nu} \delta g^{\mu\nu} + 2n_\mu \delta g^{\mu\nu} + n_\rho n^\rho \delta g^{\mu\nu} \right] \tag{A.4}\]

implies the energy-momentum tensor for the thick brane

\[(T^B_{MN}) = \frac{\sigma^2}{2} \sqrt{\frac{|g^B|}{|g|}} \left( g_{\mu\nu} \frac{n_\mu}{n_\nu} n_\rho n^\rho \right). \tag{A.5}\]

We note that \(\det(T^B_{MN}) = 0\).

**B. Ricci-tensor components**

**(1 + d)-dimensional case.** Nonzero Ricci tensor components for the metric (2.5) are

\[R_{\mu\nu}[g] = -g_{\mu\nu}[\phi'' + d(\phi')^2], \tag{B.1}\]

\[R_{yy}[g] = -d[\phi'' + (\phi')^2]. \tag{B.2}\]

The scalar curvature for (2.5) is

\[R[g] = -2d\phi'' - d(d + 1)(\phi')^2. \tag{B.3}\]

For the metric (2.5) with \(g^1\) from (2.11) we get

\[R_{tt}[g] = e^{2(\phi + v)} \left\{ e^{-2\phi}[\Delta v + (\nabla v)^2] + \phi'' + v'' + (\phi' + v')(d\phi' + v') \right\}. \tag{B.4}\]

**Multitemporal case.** For the metric (2.5) with \(g^1\) from (2.17) we obtain

\[R_{tt}[g] = e^{2(\phi + v_i)} \left\{ e^{-2\phi} \left[ \nabla v_i + \sum_{j=1}^{n} \nabla v_i \nabla v_j \right] + \phi'' + v'' + (d-n)\phi'(\phi' + v_i') + (\phi' + v_i') \sum_{j=1}^{n} (\phi' + v_j') \right\}, \tag{B.5}\]

\(i = 1, \ldots, n\). Here \(\Delta\) is the Laplace operator on \(\mathbb{R}^{d-n}\).

**Product of \(n\) spaces.** Consider the product space metric (3.1) with \(g^1\) from (2.11) and \(d_1 = d\). Calculations give us

\[R_{tt}[g] = e^{2(\phi + v_i)} \left\{ e^{-2\phi} \left[ \Delta v + (\nabla v)^2 \right] + (\phi')'' + v'' + [(\phi')' + v']v' + \sum_{i=1}^{n} d_i(\phi')' \right\}, \tag{B.6}\]

\(\Delta\) being the Laplace operator on \(\mathbb{R}^{d_1-1}\).
C. Einstein tensor

Let us present expressions for the Einstein tensor $E_{MN} = R_{MN} - \frac{1}{2} g_{MN} R$ corresponding to the metric (3.1) (see Appendix in [29])

$$E_{uu} = -L \exp(-\gamma_0),$$  \hspace{1cm} (C.1)

$$E_{mni} = -\frac{1}{d_i} g_{mni} \left( \frac{d}{du} \frac{\partial L}{\partial (\phi^i)} - \frac{\partial L}{\partial \phi^i} \right) \exp(2\phi^i - \gamma_0)$$  \hspace{1cm} (C.2)

$i = 1, \ldots, n$, where

$$L = \frac{1}{2} \exp(\gamma_0) G_{ij}(\phi^j)'(\phi^j)'$$  \hspace{1cm} (C.3)

is Lagrangian and $\gamma_0 = \sum_{i=1}^{n} d_i \phi^i$.

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