Cosmic rays and non-thermal emission in simulated galaxies. I. Electron and proton spectra compared to Voyager-1 data

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ABSTRACT

Current-day cosmic ray (CR) propagation studies use static Milky-Way models and fit parametrized source distributions to data. Instead, we use three-dimensional magnetohydrodynamical (MHD) simulations of isolated galaxies with the moving-mesh code Arepo that self-consistently accounts for hydrodynamic effects of CR protons. In post-processing, we calculate their steady-state spectra, taking into account all relevant loss processes. We show that this steady-state assumption is well justified in the disc and generally for regions that emit non-thermal radio and gamma rays. Additionally, we model the spectra of primary electrons, accelerated by supernova remnants, and secondary electrons and positrons produced in hadronic CR proton interactions with the gas. We find that proton spectra above 10 GeV only weakly depend on galactic radius, while they acquire a radial dependence at lower energies due to Coulomb interactions. Radiative losses steepen the spectra of primary CR electrons in the central galactic regions while diffusive losses dominate in the outskirts. Secondary electrons exhibit a steeper spectrum than primaries because they originate from the transported steeper CR proton spectra. Consistent with Voyager-1 and AMS-02 data, our models (i) show a turn-over of proton spectra below GeV energies due to Coulomb interactions so that electrons start to dominate the total particle spectra and (ii) match the shape of the positron fraction up to 10 GeV. We conclude that our steady-state CR modeling in MHD-CR galaxy simulations is sufficiently realistic to capture the dominant transport effects shaping their spectra, arguing for a full MHD treatment to accurately model CR transport in the future.

Key words: astroparticle physics – cosmic rays – local interstellar matter – methods: numerical – MHD.

1 INTRODUCTION

Relativistic particles (so called CRs) are extremely rare in the interstellar medium (ISM): only one in ~ 109 particles is a CR particle. The CR population is mostly composed of protons with a small admixture of heavier nuclei, electrons and positrons. Despite the rarity of these highly energetic particles in terms of number density, their energy density is comparable to the thermal, magnetic and kinetic counterpart (Boulares & Cox 1990; Zweibel 2013). Hence, they inevitably play a crucial role in galaxy formation and evolution, e.g., by driving galactic winds and regulating star formation. This has been suggested in several different settings, such as one-dimensional (1D) flux tube models (Breitschwerdt et al. 1991; Zirakashvili et al. 1996; Ptuskin et al. 1997; Everett et al. 2008; Samui et al. 2010; Recchia et al. 2016) or three-dimensional simulations of the ISM (Hanasz et al. 2013; Girichidis et al. 2016; Simpson et al. 2016; Farber et al. 2018). In addition, CR-hydrodynamic simulations of forming galaxies in isolation (Jubelgas et al. 2008; Uhlig et al. 2012; Booth et al. 2013; Salem & Bryan 2014; Pakmor et al. 2016c; Ruszkowski et al. 2017; Pfrommer et al. 2017b; Jacob et al. 2018; Dashyan & Dubois 2020) or in cosmological environments (Salem et al. 2014; Buck et al. 2020; Hopkins et al. 2020a,b) have shown the relevance of CRs in regulating the star formation rate (SFR) of galaxies and their ability to launch galactic winds. Up to PeV particle energies, CR protons are assumed to be mostly accelerated at the remnants of supernovae (SNe) and therefore of Galactic origin (e.g. Ginzburg & Syrovatskii 1964; Jokipii & Morfill 1985). The mechanism of diffusive shock acceleration leads to a distribution of CRs that can be expressed as a power law in momentum (Blandford & Eichler 1987). However, the locally observed spectrum is different from the freshly injected one, in terms of spectral distribution as well as composition, enabling theorists to infer CR propagation and interactions throughout the ISM. In particular, the abundance of secondary particle species that are created via CR interactions with the ISM provide insight into the propagation mechanisms (Strong et al. 2007; Grenier et al. 2015).

The strong connection between CRs and the physical properties of a galaxy can be deduced from observations of their non-thermal emission in radio (van der Kruit 1971; Condon 1992; Bell 2003) and gamma rays (Ackermann et al. 2012b; Rojas-Bravo & Araya 2016; Linden 2017) that tightly correlate with indicators of star formation. Since core-collapse SNe explode only ~ 5 – 30 Myr after star formation, a fresh population of CRs is closely connected to the existence of a young stellar population, whose ultraviolet (UV)
radiation typically gets absorbed by dust and re-emitted in the far-infrared (FIR). Thus, under the assumption of calorimetry, i.e. that CRs lose most of their energy due to emission before they can escape, the FIR luminosity of star forming galaxies is expected to be strongly connected to the non-thermal emission arising from their CR population (Pohl 1994; Völk 1989; Lisenfeld et al. 1996). Indeed, the radio synchrotron emission from CR electrons is found to be linearly correlated with the FIR luminosity of star forming galaxies. The correlation holds over many orders of magnitude from dwarf galaxies up to strongly starburst systems with a remarkably low scatter. This challenges our understanding of the processes in place, and needs fine-tuning of several parameters in order to explain the observations (Lacki et al. 2010). In this context, the relevance of primary versus secondary electrons has been discussed, the latter being claimed to play an important role especially in highly star-forming galaxies.

Similarly, the gamma-ray luminosities are found to strongly correlate with the FIR luminosities of star forming galaxies, but deviate from this relation at low SFRs (Ajello et al. 2020). This has been attributed to non-radiative losses of CRs coming into play in low-density galaxies, where the calorimetric assumption can not be fulfilled (Thompson et al. 2007; Strong et al. 2010; Lacki et al. 2011; Pfrommer et al. 2017b; Kornecki et al. 2020).

There are a number of numerical propagation models in the literature, e.g. GALPROP (Strong & Moskalenko 1998), DRAGON (Evoli et al. 2008), PICARD (Kissmann 2014) and Usine (Maurin 2020). They solve the CR propagation equation on a grid while parametrizing the distribution of supernova remnants (SNR) with various input parameters, while aiming to match all constraints given by the observations of CR nuclei, electrons and positrons, as well as the observed gamma-ray and synchrotron emission of our galaxy. However, these models combine different models for the source distributions and independent inferences of the density and magnetic field of the Milky Way. As such, they are not self-consistently emerging from an MHD simulation, which limits their predictive power.

In an extra-galactic context, it is common to use one-zone leaky-box models of star-forming galaxies in order to explain various aspects of their observed non-thermal emission quantities (e.g. Torres 2004; Lacki et al. 2010, 2011; Yoast-Hull et al. 2013). These models prescribe one galaxy with a characteristic scale height, a fixed magnetic field and gas density, motivated by observations. Additionally, Heesen et al. (2016) and Miskolczi et al. (2019) use 1D CR transport models in order to explain the observed radio emission in the halos of star-forming spiral galaxies. However, there is no contact made to CR propagation models in the Milky Way, that probe the same underlying physics, and these one-zone models require fitting a number of parameters that are observationally not constrained.

We aim to develop a complementary approach to previous models by performing three-dimensional MHD simulations of forming galaxies. These simulations include CR protons as a relativistic fluid that is dynamically coupled to the MHD equations in the advection-diffusion approximation. We inject CRs at SNe that trace the sites of active star formation in the simulations. The dynamical impact of CR protons is thus included in the evolution of these galaxies. The spectral details of the CR protons as well as the CR electron and positron physics are investigated in post-processing.

This paper is the first of a series of three papers. Here, we focus on studying the spectra of CR protons, primary and secondary electrons, that we assume to be in steady-state in each computational cell. Our approach is aimed to be predictive, since our source function is not tuned in order to exactly reproduce the observations, but results from modeling star-formation and CR physics self-consistently in our MHD simulations. Hence, we obtain observables that can be related to recent measurements of the Milky Way, such as the spectra of CR protons and electrons, as well as the fraction of positrons. The aspects of the resulting non-thermal emission will be studied in two following papers. Werhahn et al. (2021a, hereafter Paper II) provides insights into the gamma-ray emission from CRs in simulated galaxies, both in terms of total luminosities and spectral energy distributions and Werhahn et al. (2021b, hereafter Paper III) analyses the radio-synchrotron emission from the primary and secondary electron population. We present our simulations in Section 2. The modeling of CRs is explained in Section 3, where we detail how we obtain the spectra of CR protons, primary electrons and model the production of secondary electrons and positrons. Our resulting maps and spectra are discussed in Section 4, where we compare the latter to observations. Finally, we summarise our findings in Section 5. Appendix A provides more details on CR loss rates and the normalization of primary electron and proton spectra. Additionally, in Appendix B we describe the production of secondary electrons and positrons, show our parametrization of the cross section of pion production, and compare the electron source function to an analytical approximation.

2 SIMULATIONS

We perform MHD simulations of isolated galactic discs with the moving mesh code Arepo (Springel 2010; Pakmor et al. 2016a), which simulates magnetic fields with ideal MHD (Pakmor & Springel 2013). The simulations performed in this study are similar to those in Pfrommer et al. (2017b) and use the one-moment CR hydrodynamics algorithm (Pakmor et al. 2016b; Pfrommer et al. 2017a). In order to cover the entire mass spectrum of galaxies from dwarfs to Milky Way-like galaxies, we simulate dark matter halo masses ranging from \( M_{200} = 10^{10} \) to \( 10^{12} \) M\(_{\odot}\). The gas cloud is initially assumed to be in hydrostatic equilibrium with the halo. It contains \( 10^7 \) gas cells, each carrying a target mass of 155 M\(_{\odot}\) \times \( M_{200}/(10^{10} \text{M}_{\odot})\), embedded in a dark matter halo that follows an NFW profile (Navarro et al. 1997). This is characterised by a concentration \( c_{200} = r_{200}/r_S\), where \( r_s \) is the characteristic scale radius of the NFW profile and the radius \( r_{200} \) encloses a mean density that corresponds to 200 times the critical cosmic density. We assume a baryon mass fraction of \( \Omega_b/\Omega_m = 0.155 \) and assign initial angular momentum to the halo, which is parametrized by a spin parameter \( J = j/(\sqrt{2} R_{\text{vir}} V_{\text{vir}}) = 0.05 \), where \( j = J/M \) is the specific angular momentum of the halo and \( R_{\text{vir}} \) and \( V_{\text{vir}} \) denote the virial radius and velocity of the halo, respectively. Our choice of the radial distribution of \( J \) is in agreement with results from full cosmological simulations (Bullock et al. 2001). We switch on cooling at \( t = 0 \) which is fastest in the center. This causes loss of pressure support and infall of the gas while it conserves

\[^1\] We enforce that the gas mass of all Voronoi cells remains within a factor of two of the target mass by explicitly refining and de-refining the mesh cells.
its specific angular momentum. As a result, a galactic disc starts to form from the inside out. While this problem set-up is axisymmetric, the simulation result is not axisymmetric due to the probabilistic star formation model that we will explain in the following.

The simulations follow a simplified model of star formation and instantaneous core-collapse SN feedback (Springel & Hernquist 2003), in which regions above a critical threshold density are stochastically forming stars with an expectation value consistent with the observed Kennicutt (1998) law. CRs are instantaneously injected at the SNe, and obtain a fraction $\zeta_{\rm SN}$ of the kinetic energy of the SN explosion. This implies that the CR energy gain of a cell with SFR $\dot{m}_*$ is given by $\Delta E_{\rm CR} = \zeta_{\rm SN} \epsilon_{\rm SN} \dot{m}_* \Delta t$, where $\epsilon_{\rm SN} = 10^{49}$ erg $M_\odot^{-1}$ is the released specific energy.

We adopt an initial seed magnetic field before collapse of the gas cloud of $B_0 = 10^{-10}$ G and an injection efficiency of $\zeta_{\rm SN} = 0.05$ and 0.10. The injected CRs are advected with the gas, while adiabatic changes in the CR energy are taken into account. We also account for CR losses due to Coulomb interactions as well as hadronic losses as a consequence of inelastic collisions with the thermal ISM. Furthermore, we include anisotropic diffusion of CRs along the magnetic field, as described in Pakmor et al. (2016b) and adopt a parallel CR diffusion coefficient along the local magnetic field of $D = 10^{28}$ cm$^2$ s$^{-1}$. For halo masses of $10^{12} M_\odot$, we adopt different values of the concentration parameter $c_{200}$ for the dark matter halo.

Table 1 provides an overview of the different combinations of injection efficiency and concentration parameters that we use in our simulations of the $10^{12} M_\odot$ halo, which we analyse here. We focus on this Milky Way-sized galaxy in this paper, whereas the smaller galaxies will be analysed in Paper II and Paper III. Figure 1 depicts a snapshot of the simulation with $10^{12} M_\odot$ and $c_{200} = 7$ after $t = 1.1$ Gyr of evolution. The morphologies of the gas column density $\Sigma$ and the SFR density $\Sigma_{\rm SFR}$, which are manifested by spiral structures, self-consistently determine the CR source distribution via our star formation model and hence give rise to a similar structure of the CR energy density. The CR pressure gradient was successful in driving an asymmetric outflow from the center that shows the largest SFR, which has evacuated underdense channels above and below the disc (visible in the column density map).

Despite the axisymmetric problem setup, we have seen that the emergent galactic winds are not symmetric with respect to the disc. In order to understand the reason for this asymmetric outflow morphology, we study the dependence of the CR distribution on...
Figure 2. We show face-on views (top panels) and edge-on views (bottom panels) of slices of the CR energy density $e_{\text{cr}}$ through three different simulations with the same halo mass $M_{200} = 10^{12} M_\odot$ at 1 Gyr, but different concentration parameters $c_{200}$ and injection efficiencies $\zeta_{\text{SN}}$ as indicated in the panels. The central panel shows our fiducial galaxy that we will analyse in the following.

Figure 3. Time evolution of the SFR (left-hand panel) and the volume-averaged thermal and CR energy densities in a disk of radius 10 kpc and total height 1 kpc (right-hand panel) for our Milky Way-like galaxy with a halo mass of $10^{12} M_\odot$. We vary halo concentration parameter $c_{200} = \{7, 12\}$ and CR energy acceleration efficiency $\zeta = \{0.05, 0.1\}$. Note that our fiducial galaxy is characterised by the combination $c_{200} = 7$ and $\zeta = 0.05$ (shown in yellow).

various parameters in Fig. 2, which shows slices of the CR energy density. Prime among those parameters is i) the CR acceleration efficiency $\zeta_{\text{SN}}$ that determines the amount of injected CR energy at SNe and hence sets the CR gradient strength that is able to drive galactic winds and ii) the halo concentration $c_{200}$ that determines the potential depth of the halo. Increasing values of $c_{200}$ imply a larger density and hence a deeper dark matter potential. Indeed, Fig. 2 shows that larger values of $c_{200}$ imply more compact discs and weaker outflows while larger values of $\zeta_{\text{SN}}$ increase the outflow strengths. Hence, small changes in these parameters have large consequences on whether CRs can drive outflows in Milky Way-mass galaxies and shape the particular outflow morphologies. This is only possible because CR-driven winds are getting weaker towards the mass scale of Milky Way-mass galaxies (Uhlig et al. 2012; Jacob
et al. 2018) so that the onset of a CR-driven wind represents an unstable phenomenon and critically depends on the acceleration strength (i.e., the CR gradient) and the gravitational attraction of the density of dark matter and the amount of stars, which increases with time and furthermore deepens the central potential. Hence, the asymmetry arises as an emergent phenomenon that is the result of the outflow taking the path of least resistance away from the galaxy, which may be blocked or obscured in one hemisphere of the galaxy.

Note that despite the different galactic outflow appearances and strengths, the global properties such as formed stellar mass or average thermal or CR energy are rather similar among these Milky Way-like models studied here. This is exemplified in Fig. 3 by analysing the SFRs (left-hand side) and volume-averaged thermal and CR energy densities in a disk of fixed radius and height for our three galaxies (right-hand side). The peak SFR increases by a factor of three when the halo concentration is increased from $\zeta_{\text{SN}} = 7$ to 12 due to the different compression upon the initial collapse, but this leaves little impact on the average thermal and CR energies. In particular, the latter quantity differs by approximately a factor of two, which exactly resembles the difference in energy injection efficiency at SNRs.

The time evolution of our simulations is exemplified in Figs. 4 and 5 for a simulation with $M_{200} = 10^{12} M_\odot$, $\zeta_{\text{SN}} = 12$, $B_0 = 10^{-10}$ and $\zeta_{\text{SN}} = 0.05$. In particular, maps of the gas density in Fig. 4 show the formation of a rotationally supported disc at a few hundred Myrs after the initial gas cloud has started to collapse. Because CRs are injected at remnants of SN explosions, they reside in the star-forming disc before they are transported through advection or diffusion. For the parameters chosen here, the CR density in the disc is sufficiently high after 600 Myrs in order to bend and open up the toroidal magnetic field, enabling them to diffuse into the halo and drive an outflow. Due to the decreasing SFR and hence decreasing CR injection rate, the CR gradient weakens over time and the outflow eventually dissolves another 300 Myrs later.

The initial collapse of the gas cloud results in a short starburst, followed by an exponentially declining SFR. This is exemplified in Figs. 3 and 5. Additionally in Fig. 5, we show the variance of CR protons and secondary electrons, where $E$ denotes the total particle energy. It assumes that the injection of CRs, given by the source term $q(E) = dN/(dE \, dV \, dt)$, the CR production rate per unit volume and unit energy, is balanced by cooling, i.e., energy losses $b(E) = -dE/dt$, and escape from the system. The latter includes advection and diffusive losses, which are combined in an energy-dependent escape timescale $\tau_{\text{esc}} = (1/\tau_{\text{adv}} + 1/\tau_{\text{diff}})$. For each CR population, we solve the following equation

$$\frac{f(E)}{\tau_{\text{esc}}} - \frac{d}{dE} \left[ f(E) b(E) \right] = q(E),$$

which can be solved using the Green’s function

$$G(E, E') = \frac{1}{b(E)} \exp \left( - \frac{E'}{E} \int_0^E \frac{1}{\tau_{\text{esc}}(y) b(y)} \, dy \right).$$

We obtain the steady-state distribution $f(E)$ in each cell from an injected source function of CRs, $q(E')$, by integrating over the initial energy $E'$, i.e.,

$$f(E) = \int_E^\infty dE' q(E') G(E, E'),$$

where $q(E) = q\left[p(E)/dE\right]$ and the injection spectra of CR electrons and protons are given in terms of a power-law in momentum,

$$q_i(p_i) \, dp_i = C_i p_i^{-\alpha_{\text{inj}}} \exp[-(p_i/p_{\text{cut},i})^n] \, dp_i,$$

where the normalised particle momentum is given by

$$p_i = \frac{p_i}{m_i c} = \left( \frac{E_i}{m_i c^2} \right)^2 - 1,$$

the subscript $i = e, p$ denotes the CR species and $n = 1$ for protons and $n = 2$ for electrons. We assume that both protons and primary electrons share the same injection spectral index of $\alpha_{\text{inj}} = 2.2$ (Lacki & Thompson 2013) and assume cutoff momenta for protons, $p_{\text{cut},p} = 1 \text{ PeV} / m_p c^2$ (Gaisser 1990), and electrons, $p_{\text{cut},e} = 20 \text{ TeV} / m_e c^2$ (Vink 2012). We inject CR protons at SNRs and transport their energy density with our MHD code (Pfrommer et al. 2017a; see also Section 2) the source term for the electrons is unspecified in our formalism and may include all relevant sources (SNRs, pulsar-wind nebulae, gamma-ray binaries). In principle, we could implement different source spectra for leptonic and hadronic CRs but leave the investigation of this degree of freedom to future work.

In order to solve for the spectral energy distribution, in practice we discretise the spectrum in logarithmically equidistant momentum bins and integrate Eq. (2) for every momentum bin, before we perform the integration over the Green’s function in Eq. (3) by using the trapezoidal rule.

### 3.1.1 Energy losses and timescales

The energy losses are evaluated locally in every cell, using the present physical properties of the cell. In order to separately solve Eq. (1) for CR protons and electrons, we have to consider the corresponding loss processes $b(E) = \dot{E}$ for each species.

The simulations already account for the CR proton losses. But in order to obtain a representation of the spectral distribution of CRs, we assume a steady-state and solve the diffusion-loss equation for CR protons. This might give us a modified spectral index
Figure 4. Temporal evolution of slices of the gas density $\rho$ (top panels) and slices of the cosmic ray energy density $\varepsilon_{\text{cr}}$ (bottom panels) for a galaxy with a halo mass of $M_{200} = 10^{12} M_\odot$, concentration $c_{200} = 12$, initial magnetic field $B_0 = 10^{-10}$ G and CR acceleration efficiency at SNe $\zeta_{\text{SN}} = 0.05$. Each panel shows the face-on view on top of the edge-on view, respectively.
CRs and non-thermal emission in galaxies I.

Figure 5. Evolution of the mean spectral energy density $f$ of each CR species at 10 GeV, 100 GeV and 1 TeV over time for the simulation shown in Fig. 4. The spectra are averaged over the gas scale height and the radius which includes 99 per cent of the hadronic gamma-ray luminosity (see Paper II, for the calculation of the non-thermal emission processes). In addition, we show the SFR (grey line) as a function of time.

Protons. CR protons lose energy due to (i) hadronic interactions with the ambient medium, which produce pions (Eq. A1) and (ii) through Coulomb interactions (Eq. A3), which heat the ISM. In order to avoid double-counting of adiabatic loss and gain processes we only account for this effect in our MHD simulations and neglect the spectral changes associated with this process. Modeling adiabatic spectral changes will require to follow the evolution of the spectral CR distribution in space and time in a galaxy simulation, which we postpone to future work (Girichidis et al. 2020). Furthermore, we have to specify the characteristic timescales of losses due to escape. The residence time of CRs is determined by diffusion and advection, i.e.,

$$\tau_{\text{esc}}^{-1} = \tau_{\text{diff}}^{-1} + \tau_{\text{adv}}^{-1}. \quad (6)$$

We estimate the diffusion timescale by

$$\tau_{\text{diff}} = \frac{L_{\text{CR}}^2}{D(E)} \propto E^{-0.5}, \quad (7)$$

with the energy-dependent diffusion coefficient $D(E) = D_0 (E/E_0)^{0.5}$ as inferred from observed beryllium isotope ratios (Evoli et al. 2020a), where $D_0 = 10^{28}$ cm$^2$ s$^{-1}$, $E_0 = 3$ GeV, and we adopt the diffusion length in each cell, $L_{\text{CR}} = \varepsilon_{\text{CR}} |\nabla \varepsilon_{\text{CR}}|$. The characteristic timescale of advection is calculated as

$$\tau_{\text{adv}} = \frac{L_{\text{CR}}}{v_z}. \quad (8)$$

Assuming that in our cell-based steady-state approximation the azimuthal fluxes in and out of the cell compensate each other, we only take the velocity $v_z$ of the cell in $z$-direction perpendicular to the disc into account for the estimation of the advection timescale. To justify this assumption, we show in Fig. 6 a map of the azimuthal velocity field $v_\phi$ in the disc that only varies smoothly on large scales. This is quantified by calculating the corresponding gradient map, $|\nabla v_\phi| = |v_\phi - (v_\phi)_\text{neighbours}|$, which we normalise by $|v_\phi|$ and $|v_z|$, respectively (two right-hand panels in Fig. 6). Here, $(v_\phi)_\text{neighbours}$ denotes the mean azimuthal velocity of the neighbouring cells within twice the radius of the cell. This histogram shows that the velocity of a computational Voronoi cell does not differ significantly from the mean azimuthal velocity value of the adjacent cells. On average, the deviation is of order $\sim 3 \times 10^{-3}$ times smaller than the azimuthal velocity of the cell itself. Similarly, $|\nabla v_\phi|$ typically amounts to 10 per cent of $|v_z|$, and the azimuthal velocity difference is below the absolute value of the vertical velocity in most cells ($|\nabla v_\phi| < |v_z|$) except for a small subset of cells at spiral density waves. Hence, this statistically justifies our assumption underlying Eq. (8).

Note that radial CR transport via advection and anisotropic diffusion is also strongly suppressed because of the largely toroidal magnetic field configuration in the disc (Pakmor & Springel 2013; Pakmor et al. 2016c) and because circular rotation dominates the kinetic energy density (Pfrommer et al. 2021). Any residual CR fluxes not explicitly modeled in our steady-state approach need to be simulated by evolving the CR electron and proton spectra in our MHD simulations (Winner et al. 2019, 2020; Girichidis et al. 2020).

Electrons. High-energy CR electrons lose mainly their energy via radiation processes. CR electrons with Lorentz factor $\gamma_e$ and normalised velocity $\beta_e = v_e/c$ suffer synchrotron and IC losses at rates given by (see e.g. Blumenthal & Gould 1970)

$$b_{\text{syn}} = \frac{4}{3} \sigma_T c \beta_e^2 \gamma_e^2 \epsilon_B, \quad (9)$$

and

$$b_{\text{IC}} = \frac{4}{3} \sigma_T c \beta_e^2 \gamma_e^2 \epsilon_{\text{ph}}, \quad (10)$$

where $\epsilon_B = B^2/(8\pi)$ is the magnetic energy density, $B$ is the root-mean square magnetic field, and we assume the Thomson-limit for IC scattering, which holds if $\gamma_e \nu < m_e c^2$, where $\nu$ is the frequency of the incoming photon. The photon energy density $\epsilon_{\text{ph}}$ includes photons from the CMB as well as stellar radiation. To account for the latter, we assume that the UV light emitted by young stellar populations is re-emitted in the FIR so that we are able to infer the FIR luminosity of each cell from its current SFR, where we adopt the relation obtained by Kennicutt (1998):

$$\frac{\text{SFR}}{M_\odot \text{yr}^{-1}} = \epsilon \times 4.5 \times 10^{-44} \frac{L_{\text{FIR}}}{\text{erg s}^{-1}} = \epsilon \times 1.7 \times 10^{-10} \frac{L_{\text{FIR}}}{L_\odot}. \quad (11)$$

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The parameter $\epsilon = 0.79$ follows from assuming a Chabrier (2003) initial mass function (Crain et al. 2010), yielding

$$\frac{L_{\text{FIR}}}{L_\odot} = 7.4 \times 10^9 \frac{\text{SFR}}{M_\odot \text{yr}^{-1}}.$$  \hspace{1cm} (12)

We assume a Planck distribution corresponding to the FIR regime, i.e. wavelengths ranging from 8 – 1000 $\mu$m with a typical warm dust temperature of $\sim$ 20 K (Calzetti et al. 2000). The evaluation of the energy loss rate in each cell is then performed by summing over the flux arriving from all other cells $i$ with SFR $> 0$ at a distance $R_i$, i.e.

$$\epsilon_i = \sum_j \frac{L_{\text{FIR}}}{4\pi R_j^2 c}$$  \hspace{1cm} (13)

and use $R_i = [3V_i/(4\pi)]^{1/3}$ as the distance if the considered cell is actively star forming, where $V_i$ denotes the cell’s volume. Because the computational cost of this sum would otherwise be proportional to the square of the cell number, we accelerate its computation with a tree code. We use $\epsilon_{i*}$ as the incident radiation field in Eq. (10), together with the CMB, i.e., $\epsilon_{\text{ph}} = \epsilon_{i*} + \epsilon_{\text{CMB}}$. For these assumptions, the effect of the Klein-Nishina suppression of the IC emission is expected to be negligible because it is only relevant if the energy of the incoming photon becomes comparable to the electron rest mass. In the rest frame of the electron this amounts to $\gamma_{e\text{ph}} \approx mc^2$. For IR photons ($h\nu \approx 10^{-2}$ eV), the Klein-Nishina suppression would thus only become relevant above $\gamma_{e} \approx 5 \times 10^5$ (or $E_{e} \approx 25$ TeV). Because this is larger than the cut-off in the primary electron spectrum, Klein-Nishina effects can only become relevant for secondary electrons, which can also be produced at higher energies. Additionally, including UV radiation for the incoming photon field, with typical temperatures around $10^4$ K (or $h\nu \approx 2.4$ eV) would become relevant for electrons with $\gamma_{e} \approx 2 \times 10^5$ (or $E_{e} \approx 100$ GeV) and could change the detailed shape of the electron spectrum, as recently pointed out by Evoli et al. (2020b).

Third, for losses due to bremsstrahlung emission, we assume a fully ionized medium (see Eq. A4). Besides the energy loss processes that lead to the emission of photons, Coulomb interactions with the ambient medium have to be taken into account (see Eq. A6). They typically affect the low-energy part of the electron spectrum.

The discussed energy loss processes for CR electrons occur on characteristic timescales

$$\tau_{\text{loss}} = \frac{E}{b_1(E)},$$  \hspace{1cm} (14)

where $b_1$ denotes the various CR electron cooling rates. They allow us to determine the importance of each energy loss process for a given energy. In Fig. 7 we analyse our fiducial galaxy with a halo mass of $10^{12} M_\odot$ at $t = 1.1$ Gyr (i.e., identical to the simulations shown in Fig. 1) and show maps of the ratio of different cooling timescales at an energy of $E = 10$ GeV, averaged over a thin slice around the mid-plane of the disc with a thickness of 500 pc. The total electron cooling timescale $\tau_{e}$, which includes all electron cooling processes as discussed above, is shortest in the central kpc in the disc and increases outwards up to cooling times $\tau_e \sim 100$ Myr. Figure 7 shows that synchrotron losses of electrons only dominate in the very central regions, where the magnetic field is strongest. Otherwise, losses due to IC scattering occur on the shortest timescale in the disc and a few kpc above it, before escape processes take over and dominate over radiative losses.

In the case of CR protons, the shortest timescale within the disc is the hadronic timescale because of its dependence on gas density. It is acting on typical timescales of a few tens to a few hundred Myrs. In regions of lower gas density, in the vicinity of SNRs and in outflows, escape processes are faster than hadronic interactions. Comparing the two escape losses considered here, we find that losses due to advection are predominantly occurring within the outflow, where $v_z$ is large and therefore $\tau_{\text{adv}} < \tau_{\text{diff}}$, while diffusion dominates elsewhere. Due to their energy dependence, we expect diffusion losses to gain importance at higher proton energies.

### 3.1.2 Normalization of the CR spectra and the determination of $R_{\text{ep}}^{\text{ini}}$

After solving the spectral transport equation for CR protons and electrons (Eq. 3) in each Voronoi cell of our MHD simulation, we need to re-normalise the resulting steady-state spectra. The steady-state CR proton spectrum can be re-normalised to reproduce the simulated CR energy density in each cell. However, to obtain the correct normalization of the primary electron spectrum, we need a relation between the source functions of primary electrons and
Figure 7. We show maps of characteristic timescales and their ratios at an energy of 10 GeV for our fiducial galaxy at a simulation time of 1.1 Gyr. The upper panels show the total electron cooling timescale \( \tau_e \) (left), that includes all cooling processes. The middle and right panels show the IC-to-synchrotron cooling time and escape-to-IC cooling time. The lower panels show maps of the total cooling timescale of protons \( \tau_p \) (left), the time scale ratios of escape-to-hadronic cooling (middle) and of diffusion-to-advection losses (right). All ratios are averaged over slices with a thickness of 500pc.
protons, which we can apply in each cell. To this end, we normalise the simulated electron spectrum of a Milky Way-like galaxy, averaged around the solar galactocentric radius to the observed electron-to-proton ratio at a kinetic energy of 10 GeV, which is given by $K_{ep}^{obs} = 10^{-2}$ (Cummings et al. 2016). We use this information to infer the corresponding injection spectrum of primary electrons, $Q_e^{prim}$, and the injection ratio $K_{ep}^{inj}$.

In this way, we obtain a ratio of injected electrons to protons in each cell. By construction, the mean of this distribution averaged around the Solar circle reproduces the observed value of the electron-to-proton ratio after taking into account all cooling processes. Naturally, for a specific value of $K_{ep}^{obs}$ we will thus get a distribution of injection ratios, $K_{ep}^{inj}$. The individual steps of this procedure are explained in detail in Appendix A2.

Assuming that this injection ratio is universal, we can then apply it to the remaining part of the galaxy and to other simulated galaxies with different masses. Note that conceptually, in this framework, injection relates to effective injection $q(E)$ into a computational Voronoi cell and should not be confused with instantaneous CR injection at an individual SNR. Therefore, we do not aim to reproduce observed ratios at SNRs with our resulting value for $K_{ep}^{inj}$.

In order to quantify this dispersion of $K_{ep}^{inj}$ in our simulated galaxy, we use Eq. (A16), where we only consider CR spectra in a galactocentric ring at the solar radius and re-normalise the proton injection function $q_p$ to the CR energy density in each cell (see Eq. A8) and the primary electron injection function $q_e^{prim}$ to the observed value $K_{ep}^{obs}$ (see Eq. A14). Figure 8 shows a histogram of $K_{ep}^{inj}$, around the solar radius with $5\text{kpc} < r < 11\text{kpc}$ and $h < 1\text{kpc}$, for two snapshots at 4 and 5 Gyr that exhibit global SFRs of $2.4\text{M}_\odot\text{yr}^{-1}$ and $1.7\text{M}_\odot\text{yr}^{-1}$, respectively. Hence, their SFRs are comparable to the observationally inferred galactic value of $1.9\text{M}_\odot\text{yr}^{-1}$ (Chomiuk & Povich 2011). Despite the rather short electron cooling timescales, we obtain a narrow distribution in $K_{ep}^{inj}$ with mean $<K_{ep}^{inj}> = 0.02$ in both snapshots and apply this value to all evolutionary states of our simulated galaxies. This enables us to obtain the normalisation of the primary electron spectrum in other galaxies or outside the solar circle.

### 3.1.3 Applicability of the steady-state assumption

In order to scrutinise our steady-state assumption and identify its caveats, we compare the change of total CR energy density in each simulation cell over a global timestep (of 0.76 Myrs) and infer a corresponding timescale $\tau_{CR}$, i.e., the characteristic timescale of the change in total energy density of CRs. The purpose of $\tau_{CR}$ is to provide an instantaneous time-scale (consistent with the numerical discretisation of the time integration used in the simulation) on which we report the relative change in CR proton energy density. Changes in this quantity on longer time intervals would probe CR evolution and would not represent instantaneous changes. On smaller time intervals, the hydrodynamic quantities would not represent a consistent state for Voronoi cells on the largest timesteps by construction. All cooling processes in the diffusion-loss equation should be of the same order or faster than that timescale, such that a steady state can be maintained, i.e., $\tau_{all} < \tau_{CR}$. Here, $\tau_{all}$ is the combined rate of all relevant cooling processes at a given energy and the diffusion loss rate, i.e., $\tau_{all}^{-1} = \tau_{cool}^{-1} + \tau_{diff}^{-1}$. As demonstrated in Fig. 7, the advection time-scale is larger than the diffusion time-scale with the exception of CR-driven galactic outflows. Thus, this justifies our neglect of considering the advection process in $\tau_{all}$.

In Fig. 9 we show the ratio of these timescales at 10 GeV. The upper panel shows maps centered on the mid-plane of the disc, in which we average over slices with a thickness of 500 pc. We obtain $\tau_{all} < \tau_{CR}$, the condition for a steady-state, predominantly inside the disc. This is owing to the short hadronic timescales in regions of high gas density in combination with large diffusive losses in the central region. Still, there remain some areas, where the steady-state assumption breaks down. This occurs either in regions of low gas density, where hadronic losses are weak, and/or in the vicinity of SN explosions, where CRs are freshly injected, leading to a sudden change in the CR energy density and disturbing the steady-state configuration.

Nevertheless, the cells contributing predominantly to non-thermal emission processes respect the steady-state assumptions. This can be deduced from the lower panels of Fig. 9, where we show normalised histograms of the ratios $\tau_{CR}/\tau_{all}$ of all cells, weighted with the CR energy density (left-hand panel), the synchrotron emission (middle panel) and the hadronic $\gamma$-ray emission resulting from neutral pion decay (right-hand panel), see Paper II and Paper III for the description of the emission processes. Clearly, weighting the timescale ratios by the non-thermal emission of each cell, either synchrotron or hadronic gamma-ray emission, leads to a shift in the ratios $\tau_{CR}/\tau_{all}$ towards higher values, indicating that the steady-state assumption is justified in cells that dominate the non-thermal emission. Yet, there is a non-negligible fraction of cells that do not obey the steady-state criterion and demand a more sophisticated treatment of the time evolution of CR spectra in three-dimensional MHD simulations, which we will examine in future work using new algorithms to follow CR electron and proton spectra (Winner et al. 2019; Girichidis et al. 2020; Ogrodnik et al. 2020).
3.2 Secondary electrons and positrons

In addition to primary CR electrons that are accelerated at sources such as SNRs, pulsar-wind nebulae, or gamma-ray binaries, CR electrons can also be produced in inelastic collisions of CR protons with protons and other nuclei in the ambient ISM. Such hadronic reactions produce charged pions that decay into secondary electrons (and neutrinos) and the neutral pions into \( \gamma \)-rays.

For the calculation of the production spectrum of secondary electrons and positrons, parametrizations of the cross sections of pion production are required. In the following, we adopt the model by Yang et al. (2018) for the low energy range \( T_p < 10 \text{ GeV} \) (see equations B1, B2 and B5), the description by Kelner et al. (2006) for \( T_p > 100 \text{ GeV} \) (see Eq. B6) and a cubic spline interpolation in between. We describe the calculation of the production spectrum of secondary particles in Appendix B1 in more detail. Furthermore, we provide our own parametrization of the total cross section of charged pion production at low proton energies in Appendix B2. We compare our approach to an analytical approximation in Appendix B3, that will be useful in the following Section.

3.3 Ratio of primary to secondary electrons

To complement the numerical analysis of our work, we derive here an analytical approximation for the ratio of secondary electrons and...
positrons to primary electrons which helps to understand the physics underlying our simulation results. For simplicity, we assume that the injection spectral indices of CR electrons and protons are identical. Our analytical insight can be used to determine the relevance of each population to CR observables and quantify each contribution to the non-thermal emission, and eventually compare it to simulations. The steady-state spectrum after taking into account all cooling losses resulting from a source function $q$ is approximately given by

$$f_{e/p} = q_{e/p} \tau_{e/p},$$  \hfill (15)

where $\tau_{e/p}^{-1} = \tau_{\text{esc}}^{-1} + \tau_{\text{loss},e/p}^{-1}$ and this equation is valid for protons, primary and secondary electrons in this analytical approximation. Furthermore, the spectrum of secondary particles (before undergoing cooling processes) is connected to the source function of secondary particles via

$$f_{e^\ast}^{\sec,\text{ancooled}} = q_{e^\ast}^{\sec} \tau_{\pi},$$  \hfill (16)

where $\tau_{\pi}$ is the characteristic timescale of pion production or hadronic interactions of CRs with the ISM (Eq. B18). Therefore, the steady-state spectrum of secondary electrons/positrons is given by

$$f_{e^\ast}^{\sec} = q_{e^\ast}^{\sec} \tau_{e} = \frac{f_{e^\ast}^{\sec,\text{ancooled}}}{\tau_{\pi}} \tau_{e}.$$  \hfill (17)

Consequently, the ratio of primary to secondary electrons can be expressed as

$$\frac{f_{e}^{\sec,\text{ancooled}}}{f_{e}} = \frac{f_{e}^{\sec}}{f_{e}^{\sec,\text{ancooled}}} = \frac{f_{e}^{\sec}}{f_{e}^{\sec,\text{ancooled}}} \frac{\tau_{\pi}}{\tau_{e}}.$$  \hfill (18)

where $f_{e}^{\sec} = f_{e}^{\sec} + f_{e}^{\sec}$ is the total steady-state distribution of secondary electrons and positrons. Adopting the analytical approximation for $f_{e}^{\sec}/f_{p}$ (Eq. B19) at a fixed physical momentum $P_{0}$, we get:

$$\frac{f_{e}^{\sec}}{f_{p}} = \frac{f_{e}^{\sec}}{f_{p}} \frac{1}{P_{0}} \frac{1}{(m_{e}c)^{3}} \frac{1}{16 \alpha_{p} m_{e}} \approx \frac{128}{3} \frac{1}{16 \alpha_{p} m_{e}}.$$  \hfill (19)

Combining Eqs. (15), (18), and (19) and evaluating the spectra at 10 GeV, such that the primary electron and proton source functions are linked by $K_{\text{ep}}^{\inj}$ (see A15), yields

$$f_{e}^{\sec} = f_{e}^{\sec} = K_{\text{ep}}^{\inj} \frac{3}{128} \frac{1}{16 \alpha_{p} m_{e}} \tau_{\pi} \tau_{p}$$  \hfill (20)

$$= K_{\text{ep}}^{\inj} \frac{3}{128} \frac{1}{16 \alpha_{p} m_{e}} \left( 1 + \frac{\tau_{\pi}}{\tau_{\text{esc}}} \right)$$  \hfill (21)

$$\approx 0.48 \left( 1 + \frac{\tau_{\pi}}{\tau_{\text{esc}}} \right)$$  \hfill (22)

and $K_{\text{ep}}^{\inj} \approx 0.02$. Here, we assume that losses due to Coulomb interactions are negligible in comparison to hadronic losses, which is reasonable at and above the considered energies.

This implies in the fully calorimetric limit, where hadronic losses dominate over escape losses, i.e., $\tau_{\pi} \ll \tau_{\text{esc}}$ (which is the case in the dense ISM, see Fig. 7), that the ratio of primary to secondary electrons depends only on $K_{\text{ep}}^{\inj}$ and the spectral index of the cooled proton spectrum $\alpha_{p}$. In particular, we find that in this limit, secondary electrons dominate above the primary electron population. On the other hand, primary electrons dominate over secondary electrons as soon as escape losses are comparable to or larger than pionic losses, i.e., $\tau_{\pi} \gtrsim \tau_{\text{esc}}$. In particular, as soon as (energy dependent) diffusive losses are important and steepen the proton spectra, the resulting secondary electron spectrum will be steeper than the primary one and hence, the ratio of secondaries to primaries will decrease at higher energies.

4 RESULTS

4.1 CR spectra and maps

In order to obtain a representative spectrum of a certain region of our simulated galaxies, we average over our cell-based steady-state spectra. In Fig. 10 we show the CR proton spectra of our fiducial galaxy, as well as primary and secondary electrons averaged over galacto-centric rings with different radii as indicated by the colors. The height over which the spectra are averaged is the scale height of the gas density, which in this case is $h_{D} = 0.72$ kpc.

We assume an injection spectral index of $\alpha_{\text{inj}} = 2.2$ for protons and electrons. Energy-dependent diffusion dominates CR transport at high energies. With our assumed diffusion coefficient $D \propto E^{0.5}$, the proton spectra soften to an asymptotic spectral index of $\alpha_{p} = 2.7$ (shown with a grey-dashed line in the left-hand panel of Fig. 10, where the spectral change at 1 GeV is due to the relativistic dispersion relation). While this asymptotic spectrum is realised at small and large galactic radii, it is not achieved at intermediate galactocentric radii. The dominant loss process of protons at low energies is Coulomb cooling that causes the spectra to turn down in comparison to a pure momentum power-law spectrum shown with a grey-dashed line in Fig. 10. This effect of Coulomb cooling below $\sim$ 1 GeV becomes more pronounced towards the denser central regions as evidenced by the stronger spectral cutoff.

While Coulomb interactions similarly affect the spectrum of primary electrons at energies $\leq$ 1 GeV, diffusion plays a subdominant role in the central galactic regions at high energies where radiative losses due to synchrotron and IC interactions dominate. This causes the injection spectral index to steepen by one, to asymptotically arrive at the steady-state electron spectrum

$$f_{e}^{\sec} (E_{e}) \propto \frac{1}{b_{\text{IC}} + b_{\text{syn}}} \int_{0}^{E_{e}} dE' q_{e}^{\sec} (E') dE' \propto E_{e}^{-(\alpha_{\text{inj}} + 1)}.$$  \hfill (23)

However, in contrast to CR protons, which only show mild spectral index variations with radius at energies larger than $\sim$ 10 GeV (see left-hand panel of Fig. 10), the primary electrons undergo a change from the radiative loss-dominated regime in the centre to a more diffusion dominated regime in the outskirts of the galaxy which implies a hardening of the spectra by 0.5, yielding a spectral index of primary electrons $\alpha_{\text{prim},e} = 2.7$ (see middle panel of Fig. 10).

By contrast, $\alpha_{\text{sec},e}$, the spectral index of secondary electrons, is $\gtrsim \alpha_{\text{prim},e}$ because they originate from the steady-state CR proton population, which exhibits a steepened spectral index due to diffusive losses. After suffering radiative losses the high-energy spectral index of a steady-state secondary electron population asymptotically approaches $\alpha_{\text{sec},e} = 3.7$, independent of galactocentric radius (right-hand panel of Fig. 10).

We also show maps of the CR spectra at 10 GeV in the upper panels of Fig. 11, averaged over thin slices of 300 pc. Similarly, the lower panels in Fig. 11 show averages over 300 pc of the spectral index at 10 GeV of the different CR populations. We only consider cells that sum up to 99 per cent of the total energy density of the simulations in order to speed up the calculations.\footnote{This includes all cells within the disc that are relevant for our results.} Whereas the primary CR electrons reside in the same spatial regions as the CR protons, the
secondary CR electron population is more concentrated towards the
disc, where the gas density is large enough to yield a sufficiently
large hadronic production rate. As expected, these regions coincide
with those of short hadronic timescales shown in Fig. 7.

The spectral index analysis is performed at 10 GeV mainly for
observational reasons: (i) hadronic interactions of 10 GeV protons
produce ∼ 1 GeV photons resulting from the decay of neutral pions
that are well observed by Fermi and (ii) because this is the typical
electron momentum that contributes to the synchrotron radiation that
is observed at 1.4 GHz, assuming typical magnetic field strengths
of ∼ 1 µG. Figure 11 shows little variation in the proton spectral
index which would translate into little variation of the gamma ray
spectral index provided the pion decay is the dominant gamma-
ray channel. We also observe a similar contribution of primary
and secondary electrons in the galactic mid-plane implying an
insignificant contribution of secondaries at higher energies because
of their steeper spectra.

Moreover, secondary electrons show a more compact spatial
distribution surrounding the mid-plane. Considering the more
extended magnetic field distribution, we expect the primary
synchrotron emission to match the secondary emission in the
mid plane and to dominate over the secondary emission at
higher frequencies and larger galactic heights. Similar arguments
hold for the IC emission. For an incident radiation field that
peaks at FIR frequency corresponding to a Planckian photon
spectrum characterised by temperature 20 K, electrons with \( p_e \sim
10 \text{ GeV}/(m_e c^2) \) are able to Compton up-scatter these photons to
\( \sim 10 \text{ MeV} \), where we expect a similar contribution of secondaries in
the mid-plane, but a subdominant contribution elsewhere because the
radiation field is usually also more extended than the gas distribution.
The spectral index of different CR species at 10 GeV (lower panels in
Fig. 11) clearly show that the advection timescale dominates in the
outflow regions (see also Fig. 7) so that the spectral index of protons
and primary electrons reflects the index of the injected spectrum,
\( \alpha_{\text{inj}} = 2.2 \).

These considerations are summarised in Fig. 12 that demonstrates
that the ratio of primary to secondary electrons is \( \gtrsim 1 \) so that
primaries are dominant for most parts of the galaxy. Only within the
disc where the gas density is high, secondary electrons are roughly
three times more abundant than primary electrons at 10 GeV. This
is in accordance with Eq. (21): using \( \tau_{\text{ad}}^{[0]} = 0.02, \alpha_p = 2.2 \)
and assuming that \( \tau_{\pi} \ll \tau_{\text{esc}} (\tau_{\pi} \approx \tau_{\text{esc}}) \), which is required in order
to produce secondaries efficiently, we obtain for the ratio in the
analytical approximation \( f_{\text{sec}}^{[0]} / f_{\text{sec}}^{[0]} \approx 0.2 \) (0.4). A steeper spectral
index of the cooled proton spectrum \( \alpha_p > 2.2 \) would decrease the
production rate of secondaries, which can be inferred both from
our expression in Eq. (21) and from the fact that a steeper spectral
index signals a dominant role of energy dependent diffusion losses
in comparison to hadronic losses, making secondary production less
efficient. Hence, at higher energies, the importance of primary versus
secondary electrons decreases further (see middle and right-hand
panel of Fig. 12).

4.2 Comparison to observations
4.2.1 Proton and electron spectra
Since crossing the heliopause in August 2012, Voyager 1 has been
observing CRs in the local interstellar medium at energies below
\( \sim 1 \text{ GeV} \), which are not subject to solar modulation effects (Cummins
et al. 2016). Complementary, AMS-02 observed CR spectra at larger
energies (Aguilar et al. 2014a; Aguilar et al. 2015). We see that CR
proton spectra are affected by solar modulation at energies \( \lesssim 10 \text{ GeV} \)
(Potgieter 2013).

In Fig. 13 we show both sets of observational data together with
our spectra of CR protons and electrons of a Milky Way-mass galaxy
\( M_{200} = 10^{12} M_\odot \) at 5 Gyr, averaged over a ring at \( r = 8 \pm 1 \text{kpc} \)
muonic the conditions of the galactocentric orbit of the Sun, and
different heights above and below the midplane, i.e. \( \pm h \), as indicated
in the plot. We normalise our spectra to the observations at 10 GeV,
which are unaffected by solar modulation (for details, see Table 2).
This accounts for differences to the recent star formation history
of our simulations and the Milky Way, which determines the CR
injection rate and the fact that the Milky Way is under-luminous in
gamma rays in comparison to the far infrared-gamma-ray relation
by a factor 2.9 (using the relation of Ajello et al. 2020, but see also
Ackermann et al. 2012b; Rojas-Bravo & Araya 2016; Pfommer et al.
2017b). The decreasing gas density with height over the disc causes
Figure 11. We show slices of the spectral CR density at 10 GeV (top six panels for protons, primary and secondary electrons from left to right) and spectral indices (lower six panels) of CR protons ($\alpha_p$), primary and secondary electrons ($\alpha_{\text{prim}, e}$ and $\alpha_{\text{sec}, e}$), each averaged over thin (300 pc) slices for our fiducial galaxy (see Table 1).
The deviation of the ratio $A_{e}/A_{p}$ from unity is below 1 per cent for the mean electron densities of interest here ($n_e = 0.02 - 0.2 \text{ cm}^{-3}$).
Figure 13. Spectra of CR protons and electrons at $t = 5$ Gyr, averaged over rings at $r = 8 \pm 1$ kpc with different heights. For comparison, we show data of Voyager 1 (Cummings et al. 2016) and AMS-02 for electrons (Aguilar et al. 2014a) and protons (Aguilar et al. 2015). The AMS-02 data affected by solar modulation, i.e. below 10 GeV, is shown with grey symbols. The simulated CR spectra are normalised to the observations at 10 GeV to account for deviations of our simulated from the observed SFR (see Table 2 for details).

Table 2. Summary of the normalization factors used for the CR proton and electron spectra in Fig. 13, respectively, averaged over different heights $h$, in order to match the data at 10 GeV.

| $h$ [kpc] | norm. factor protons | norm. factor electrons |
|----------|----------------------|------------------------|
| 0.5      | 0.18                 | 0.20                   |
| 1.0      | 0.21                 | 0.26                   |
| 2.0      | 0.25                 | 0.37                   |
| 3.5      | 0.31                 | 0.50                   |

4.2.2 Positron ratio

Several experiments reported the positron ratio that decreases with energy until $\sim 8$ GeV at which point it starts to rise again, including TS93 (Golden et al. 1996), Wizard/CAPRICE (Boezio et al. 2001), HEAT (Beatty et al. 2004), AMS-01 (Aguilar et al. 2007), PAMELA (Adriani et al. 2009), Fermi (Ackermann et al. 2012a), AMS-02 (Aguilar et al. 2013; Aguilar et al. 2014b). If positrons solely arise from hadronic interactions of CR protons with the ISM, the positron fraction decreases with energy because of the steeper spectrum of secondaries in comparison to primaries. Thus, the observed rise of the positron fraction has been either attributed to annihilating/decaying dark matter particles (e.g. Yin et al. 2009; Cholis & Hooper 2013; Feng et al. 2020), or local astrophysical sources such as pulsars or SNe (e.g. Serpico 2012; Di Mauro et al. 2017; Hooper et al. 2009; Mertsch et al. 2020). We obtain in Fig. 14 the expected behaviour of a decreasing positron fraction with energy, coinciding with observations up to $\sim 8$ GeV. The overall normalization though depends on the height over which we average the CR electron spectra, as depicted in the legend. This is due to the fact that the decreasing gas density with height leads to a less efficient production of secondaries and therefore to a decrease in the fraction of positrons in comparison to all leptons, the latter being dominated by primary electrons.

Note that we do not simultaneously reproduce the observations of CR spectra and the positron fraction within one model, when averaged over the same height. This is consistent with the fact that we somewhat overproduce the proton spectrum at energies $\sim 100$ GeV in comparison to the observations (see Fig. 13), which produces secondary electrons at around 6 GeV. In consequence, our positron fraction tends to be higher by a factor $\approx 1.9$ in comparison to the observed values.

Another uncertainty in the calculation of the positron fraction is our assumption about the nuclear enhancement factor, that accounts for heavier nuclei in the composition of CRs and the ISM (see Appendix B1), for which we adopt the wounded nucleon model. However, Kafexhiu (2016) recently analysed the impact of different effects such as sub-threshold pion production on the cross section.
of secondary particle production. In particular at low energies, the wounded nucleon model seems to break down. For example, in collisions of protons with carbon nuclei, electrons and positrons are shown to be created in equal amount, in contrast to pp-collisions, where more positrons than electrons are produced at low energies. Yang et al. (2018) apply those considerations to solar abundances and quantify the effect on the gamma-ray spectrum. Still, the exact effect on the electron and positron spectra for solar abundances has (to our knowledge) not yet been analysed.

5 SUMMARY AND CONCLUSIONS

For the first time, we calculate steady state CR spectra in three-dimensional MHD simulations that self-consistently include CRs. We model their spectra with a cell-based steady-state approximation, including hadronic and Coulomb losses for CR protons and radiative losses due to synchrotron, bremsstrahlung and IC losses for CR electrons. In addition, we estimate losses due to advection and diffusion, while assuming an energy-dependent diffusion coefficient. Furthermore, we carefully calculate the production of secondary electrons and positrons by combining existing models from the literature (Yang et al. 2018; Kelnner et al. 2006) at intermediate energies and provide our own parametrisation of the total cross sections of negatively and positively charged pion production, respectively (Eqs. B9 and B10).

Intriguingly, while our modelling of CR sources and transport in a self-consistently evolving galaxy does not involve fine-tuning of parameters, the emerging steady-state spectra convincingly reproduce observational CR features measured locally in the Milky Way. These include on the one hand measured CR proton and electron spectra in the ISM by AMS-02 (Aguilar et al. 2014a; Aguilar et al. 2015) and Voyager 1 (Cummings et al. 2016), for which we reproduce the inversion of proton and electron spectra so that the latter dominates the total particle spectrum at low energies. We attribute this to steady-state Coulomb cooling and analytically verify this behavior. On the other hand, our steady-state spectra also match the observed shape of the positron fraction up to energies of ~8 GeV, before the excess of positrons (attributed to additional sources such as pulsars) comes into effect. Solely the total normalization of the CR spectra and positron fraction is not reproduced. However, this is not too surprising, considering the fact that our simulations are not designed to fully reproduce a realistic Milky Way analogue with a simultaneous match of current SFR, halo mass, as well as the potentially complex star-formation history. This could be addressed in future work by applying our model to cosmological simulations, where galaxies can be found that resemble the Milky Way in more detail. Summarising, the match of simulations and observational data indicates that our modelling apparently does not miss critical physics ingredients and is able to provide physics insights into observational findings that are very complementary to state-of-the-art approaches of CR transport analyses.

Our model enables us to obtain spatial and spectral information of CRs in simulated galaxies with different galaxy sizes and injection efficiencies of CRs. We use this approach as a starting point to analyze the non-thermal emission processes arising from CRs in star-forming galaxies in two accompanying publications. Paper II focuses on gamma-ray emission processes, examines the influence of CR transport models on the total gamma-ray luminosity and emission spectra, whereas Paper III is aimed towards understanding the radio emission of star-forming galaxies. In particular, our analytical modelling of the ratio of secondary-to-primary electrons enables us to constrain their relative contribution to the non-thermal emission processes.

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DATA AVAILABILITY

The simulations and data analysis scripts underlying this article will be shared on reasonable request to the corresponding author. The AREPO code is publicly available.

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Figure 14. We show the positron fraction $f_e / (f_e + f^-)$, averaged over different heights, at $t = 5$ Gyr (as in Fig. 13). The data points indicate observations performed by AMS-02 (Accardo et al. 2014). The rise of the the positron fraction above ~8 GeV is due to additional positron sources that are not modelled here (see Section 4.2.2).
APPENDIX A: LOSS RATES AND NORMALIZATION OF CR SPECTRA

A1 Loss rates of CR protons and electrons

Protons. CR protons with an energy above the threshold energy of pion production $E > E_{\pi^0} = 1.22 \text{ GeV}$ lose energy at a loss rate given by

$$b_\pi = K_\pi T_p \sigma_{pp} n_N.$$  (A1)

where the inelasticity of the pp-interaction is $K_\pi = 1/2$ (Mannheim & Schlickeiser 1994). $T_p$ is the particle kinetic energy, $\sigma_{pp}$ is the cross section of proton-proton collisions (given by Eq. B14) and $n_N = n_H + 4n_{He} = ((X_H + 1 - X_H)/2)p/m_p = r/p$ is the number density of target nucleons, where $X_H = 0.76$ denotes the hydrogen fraction and $p$ is the gas density.

Additionally, CR protons lose energy through Coulomb interactions (Gould 1972a), given by

$$b_{\text{Coul,p}} = \frac{3\sigma_{Tne}n_em_e^3}{2\beta} \left[ \ln \left( \frac{2ym_e^2\beta^2}{\hbar\omega_p} \right) - \frac{\beta^2}{2} \right]$$  \hspace{1cm} (A2)

$$\equiv \frac{3\sigma_{T}\eta n_em_e^3}{2\beta} A_p,$$  \hspace{1cm} (A3)

where $n_e = n_H + 2n_{He} = ((X_H + (1 - X_H)/2)p/m_p = 0.88p/m_p$ is the electron number density, $\hbar$ is the reduced Planck constant, $e$ the elementary charge, $c$ the speed of light and the plasma frequency is defined as $\omega_p = \sqrt{4\pi n_e^2 e/m_e}$. Furthermore, $\sigma_T = 8\sigma_{}\beta^2/3$ is the Thomson cross-section, where $\sigma_T = e^2/(m_e c^2)$ denotes the classical electron radius, and $A_p$ defines the Coulomb logarithm and the velocity correction term in the bracket. The Lorentz factor $\gamma$ and the normalised velocities $\beta = v/c$ without subscripts refer to protons (electrons are denoted with a subscript $e$).

Electrons. Besides losses due to IC and synchrotron emission, CR electrons lose energy due to bremsstrahlung emission. Following Blumenthal & Gould (1970), this yields in the case of highly relativistic electrons the expression

$$b_{\text{brem,s}} = 4\alpha r^2_G n_p \beta e \gamma_e \left[ \ln(2\gamma_e) - \frac{1}{3} \right] m_e c^2.$$  \hspace{1cm} (A4)

where we assume a fully ionized medium with $n_p = 0.88p/m_p$ and $\alpha$ the fine structure constant. Additionally, we take Coulomb losses of CR electrons into account. The expression for the energy loss rate has been derived by Gould (1972b) and reads

$$b_{\text{Coul,e}} = \frac{3\sigma_T n_e m_e c^3}{2\beta e} \left[ \ln \left( \frac{m_e c^2 \beta e \sqrt{\gamma_e - 1}}{\hbar \omega_p} \right) - \ln(2) \left( \frac{\beta_e^2}{2} + \frac{1}{\gamma_e} + \frac{1}{2} + \left( \gamma_e - 1 - \frac{4}{4\gamma_e} \right)^2 \right) \right]$$  \hspace{1cm} (A5)

$$\equiv \frac{3\sigma_T n_e m_e c^3}{2\beta e} A_e,$$  \hspace{1cm} (A6)

where $A_e$ defines the Coulomb logarithm and various correction terms in the bracket.

A2 Normalization of CR spectra

In this section, we explain the detailed procedure of obtaining the normalization of the primary electron and proton spectra, that allow us to infer an injected ratio of electrons to protons $K_{ep}^{\text{inj}}$.

A priori, we do not know the normalization of the injection spectra in each cell so that we first assume $C_{i,\gamma} = 1$ in Eq. (4), for protons and electrons. We calculate the steady-state spectrum $f_{\gamma,0}$ that results from a given injection spectrum and all energy loss processes, and then re-normalise the source function and spectral density via the CR.
energy density $\varepsilon_{\text{CR}}$ in every cell:

$$f_p(p) = f_{P,0}(p) \frac{\varepsilon_{\text{CR}}}{\varepsilon_{\text{CR,0}}}$$  \hspace{1cm} (A7)

and

$$q_p(p) = q_{P,0}(p) \frac{\varepsilon_{\text{CR}}}{\varepsilon_{\text{CR,0}}}.$$  \hspace{1cm} (A8)

where

$$\varepsilon_{\text{CR,0}} = \int T_p(p) f_{P,0}(p) \, dp_p$$  \hspace{1cm} (A9)

and $T_p(p) = \left( \frac{3}{2} + 1 - \frac{1}{m_p^2} \right) m_p e^2$. Similarly, for the primary electrons we calculate a steady-state spectrum $q_{e,0}^{\text{prim}}(p_e)$ from the injection spectrum $q_{e,0}^{\text{inj}}(p_e)$, and re-normalise it as

$$f_e^{\text{prim}}(p_e) \, dp_e = A_{\text{norm}} f_{e,0}^{\text{prim}}(p_e) \, dp_e.$$  \hspace{1cm} (A10)

To identify $A_{\text{norm}}$, we relate the steady state spectra of all electrons (primary plus secondary) to protons via the observed ratio of electrons to protons $K_{\text{prim}}^{\text{obs}}$ at a kinetic energy of 10 GeV, or equivalently at the corresponding normalised momenta, $p_{1,10\text{GeV}}$, i.e.

$$f_e^{\text{prim+sec}}(p_{e,10\text{GeV}}) \, dp_e = K_{\text{prim}}^{\text{obs}} f_p^{\text{obs}}(p_{p,10\text{GeV}}) \, dp_p.$$  \hspace{1cm} (A11)

Because we have already normalised the proton spectrum $N_p$, we are able to obtain the primary electron spectrum via

$$f_e^{\text{prim}}(p_e) = A_{\text{norm}} f_{e,0}^{\text{prim}}(p_e) = \frac{K_{\text{prim}}^{\text{obs}} f_p^{\text{obs}}(p_{p,10\text{GeV}}) m_e}{m_p - f_{e,0}^{\text{sec}}(p_{e,10\text{GeV}})},$$  \hspace{1cm} (A12)

where we account for the normalised momenta $dp_p = dP_m e/m_p$ and use the fact that the normalised and un-normalised primary electron spectra are self-similar. Equivalently, we determine the normalization of the electron injection spectrum $q_{e,0}^{\text{prim}}(p_e)$, that is linearly related to $f_e^{\text{prim}}(p_e)$:

$$q_e^{\text{prim}}(p_e) = A_{\text{norm}} q_{e,0}^{\text{prim}}(p_e).$$  \hspace{1cm} (A13)

In order to infer the ratio of injected electrons to protons $K_{\text{ep}}^{\text{inj}}$, we compare the electron and proton injection spectrum at the same (physical) momentum $P_0$:

$$q_e^{\text{prim}}[P_0/(m_e c)] \, dp_e = K_{\text{ep}}^{\text{inj}} q_p[P_0/(m_p c)] \, dp_p.$$  \hspace{1cm} (A14)

which yields after inserting our definition of the source functions the following expression:

$$K_{\text{ep}}^{\text{inj}} = \frac{q_e^{\text{prim}}[P_0/(m_e c)] m_e}{q_p[P_0/(m_p c)] m_p} = C_e \left( \frac{m_p}{m_e} \right)^{1-\alpha_p}.$$  \hspace{1cm} (A15)

In this way, we obtain in each cell a ratio of injected electrons to protons that eventually reproduces the observed value after taking into account all cooling processes. If the electrons and protons cool on the same timescales (at the considered energy of 10 GeV), we obtain $K_{\text{ep}}^{\text{inj}} \approx K_{\text{ep}}^{\text{obs}}$. On the other hand, $K_{\text{ep}}^{\text{inj}} < K_{\text{ep}}^{\text{obs}}$ (or $K_{\text{ep}}^{\text{inj}} > K_{\text{ep}}^{\text{obs}}$) implies that the timescales of the hadronic cooling processes are smaller (larger) than the leptonic ones.

In the literature, the normalization is often defined differently, in terms of injected energy into CR protons and electrons, i.e., $\varepsilon_{\text{CR}}^{\text{inj}}$:

$$\zeta_{\text{prim}}^{\text{inj}} \int_0^\infty T_p(p) q_p(p) \, dp_p = \int_0^\infty T_e q_e^{\text{prim}}(p_e) \, dp_e.$$  \hspace{1cm} (A16)

Assuming the same injected spectral index of electrons and protons, $2 < \alpha_{\text{inj}} < 3$, and a lower momentum cutoff that is much smaller than $m_p c (m_e c)$ for protons (electrons), it can be shown that this is related to $K_{\text{ep}}^{\text{inj}}$ in the following way:

$$K_{\text{ep}}^{\text{inj}} = \zeta_{\text{prim}}^{\text{inj}} \left( \frac{m_p}{m_e} \right)^{2-\alpha_p}. $$  \hspace{1cm} (A17)

This implies that our approach, where we find $K_{\text{ep}}^{\text{inj}} \approx 0.02$, this corresponds to a primary electron-to-proton energy fraction of $\zeta_{\text{prim}}^{\text{inj}} \approx 9\%$ (assuming $\alpha_p = 2.2$), which is consistent with the parameters used in other models, such as the one-zone steady-state models by Lacki et al. (2010).

APPENDIX B: ELECTRON SOURCE FUNCTION AND PARAMETRIZATION OF THE PION CROSS SECTION

Here, we describe numerical algorithms for computing the electron source function and charged pion cross section in Appendix B1, before we detail our parametrisation of the total cross section of pion production, $\sigma_\pi$, in Appendix B2. Finally, we derive an analytical approximation for the secondary electron source function in Appendix B3.

B1 Production of Secondary Electrons and Positrons

The minimum total proton energy required to produce a pion is $E_p^{\text{min}} = 1.22 \text{ GeV}$. The production spectrum, i.e., the number of produced secondary particles per energy, time and volume, or source function $q_s$, of a secondary particle species $s = \gamma, e^-, e^+$ for a given CR proton distribution is given by

$$q_s(E_s) = c n_H \int_0^{E_s^{\text{min}}} \frac{dE_p}{E_p} \frac{d\sigma_s(E_s, E_p)}{dE_s}.$$  \hspace{1cm} (B1)

The differential cross section of the secondary particle species $s$ can be calculated by means of the differential cross section for the production of a pion with energy $E_\pi$ from the collision of a proton with energy $E_p$, i.e. $d\sigma_s(E_p, E_\pi)/dE_\pi$. Then, we can solve the integral

$$\frac{d\sigma_s(E_s, E_p)}{dE_s} = \int_{E_s^{\text{min}}}^{E_s^{\text{max}}} \frac{dE_\pi}{E_\pi} \frac{d\sigma_s(E_p, E_\pi)}{dE_\pi} f_{s,\pi}(E_s, E_\pi),$$  \hspace{1cm} (B2)

where $f_{s,\pi}(E_s, E_\pi)$ is the normalised probability distribution for the production of a secondary particle $s$ from a single pion energy $E_\pi$. Here, we are interested in electrons and positrons ($s = e^-, e^+$), and thus only in charged pions, while we will consider the production of neutral pions and gamma rays in Paper II. For the normalised electron/positron energy distribution $f_{s,\pi}(E_s, E_\pi)$, we use the expressions derived by (Dermer 1986b), assuming a mono-energetic, unpolarized, isotropic distribution of pions, to eventually calculate $d\sigma_s(E_s, E_p)/dE_s$ from Eq. (B2). In the literature, there are different parametrisations for the corresponding terms of Eq. (B1).
They relate to the definition of the pion source function in the following way

\[ q_\pi(E_\pi) = c n_H \int_{E_{\text{min}}}^{E_{\text{max}}} \, dE_p f_p(E_p) \frac{d\sigma_\pi(E_p, E_\pi)}{dE_\pi}, \tag{B3} \]

so that the source function of a secondary particle species is

\[ q_s = \int_{E_{\text{min}}}^{E_{\text{max}}} \, dE_\pi q_\pi(E_\pi) f_\pi(E_s, E_\pi). \tag{B4} \]

The differential cross section of charged pion production or the electron source function can be obtained from simulations of pp-interactions, e.g. Pythia (Sjöstrand et al. 2006, 2008), SIBYLL (Fletcher et al. 1994), QGSJET (Kalmykov & Ostapchenko 1993; Kalmykov et al. 1997; Ostapchenko 2006) and Geant4 (Agostinelli et al. 2003; Allison et al. 2006). At low proton energies near the kinematic threshold \((T_p < 10 \text{ GeV})\), we adapt the approach given by Yang et al. (2018). They utilised the hadronic interaction model of the Geant4 Toolkit (Agostinelli et al. 2003; Allison et al. 2006) to provide a parametrization for the normalised pion energy distribution \(f(x, T_p)\), that reads

\[ \frac{d\sigma_{\pi}}{dx} = \sigma_{\pi} \times f(x, T_p). \tag{B5} \]

Yang et al. (2018) provide analytical formulae for \(f(x, T_p)\), with \(x = T_p/T_p^{\text{max}}\). The parametrization of the total cross section of pion production \(\sigma_{\pi}\) is described in Section B2, where we provide our own fit to the data, shown by the solid lines in Fig. B1 and compare it to models in the literature.

In the high-energy range of protons with \(T_p > 100 \text{ GeV}\), we use an analytical parametrization provided by Kelner et al. (2006), where they give production rate of secondary electrons from the SIBYLL code (Fletcher et al. 1994). It is given in terms of a distribution \(F_{e^\pm}(x, E_p)\), such that \(F_{e^\pm}(x, E_p) \, dx\) describes the number of produced electrons and positrons per collision in the interval \((x, x + dx)\) with \(x = E_{e^\pm}/E_p\). Assuming that the production of secondary positrons is equal to the production of secondary electrons in this energy regime,

\[ \sigma_{\pi} = c n_H \int_{E_{\text{min}}}^{E_{\text{max}}} \, dE_p f_p(E_p) \frac{d\sigma_{\pi}(E_p)}{dE_\pi} \cdot \frac{dE_p}{E_p}. \tag{B6} \]

Here, \(\sigma_{\pi}^{\text{inel}}(E_p) f_p(E_p) F_{e^\pm}(E_{e^\pm}, E_p) \, \frac{dE_p}{E_p}\) is the corresponding production rate by \(\sigma_{\pi}^{\text{inel}}(E_p) f_p(E_p) F_{e^\pm}(E_{e^\pm}, E_p) \, \frac{dE_p}{E_p}\). (B6)

Figure B2 shows the parametrizations of the differential cross sections for different electron energies. We use a cubic spline to interpolate between the Yang et al. (2018) model at low proton energies \((T_p < 10 \text{ GeV})\) and the Kelner et al. (2006) model at high proton energies \((T_p > 100 \text{ GeV})\).

So far, those parametrizations only consider pp-interactions. In addition, interactions of heavier CR nuclei with the ISM have to be taken into account. Following the wounded nucleon model (Biallas et al. 1976), one can rewrite the pion production so that it can be considered to be produced in pp-collision that are enhanced by a multiplicative factor (the ‘nuclear enhancement factor’). Based on parametrizations by Lebedev et al. (1963) and Orth & Buffington (1976), Dermer (1986a) finds a value of 1.39, that increases to 1.45 if heavier nuclei than helium are included, whereas Stephens & Badhwar (1981) estimate it to 1.6 ± 0.1. Hence, we adopt the geometric mean of 1.5 and note on the relevance for studying those effects of heavier nuclei on the resulting spectrum of secondary electrons and positrons in more detail in the future (using e.g., the parametrizations of sub-threshold pion production given by Kafexhiu et al. 2014).

### B2 Parametrizations for \(\sigma_{\pi}\)

For the total cross section of pion production, \(\sigma_{\pi}\), Yang et al. (2018) use their own fit to the experimental data below 2 GeV and a different prescription given in Golokhvastov (2001) at higher proton energies (see red dashed lines in Fig. B1). For \(T_p > 2 \text{ GeV}\), they express the cross section as

\[ \sigma_{\pi} = \sigma_{\pi}^{\text{inel}} \langle n_{\pi} \rangle. \tag{B7} \]
Here, the pion average yield is parametrized as \( \langle n_\pi \rangle = 0.78(w - 2)^{3/4}w^{-1/4} - 1/2 + \varepsilon \) where \( w = \sqrt{(3/n_p)c^2} \), \( s \) denotes the square of the total energy in the CMS, i.e., \( \sqrt{s} = [2m_p c^2(E_p + m_p c^2)]^{1/2} \), and \( \varepsilon = 0 \) for \( \pi^- \), \( 1/3 \) for \( \pi^0 \) and \( 2/3 \) for \( \pi^+ \). This fit is based on experimental data by Golokhvastov (2001). The resulting curves of the total cross sections are shown in Fig. B1, where the red dashed lines show the parametrizations used in Yang et al. (2018) and the points are the experimental data that they refer to. In the case of negative pions, one can clearly see the discontinuity at \( T_p = 2 \) GeV, where the description changes from a fit to the data the pion average yield by Golokhvastov (2001) and the total inelastic cross section from Kafeixiu et al. (2014):

\[
\sigma_{pp}^{inel}(T_p) = 30.7 - 0.96 \log \left( \frac{T_p}{T_p^h} \right) + 0.18 \log^2 \left( \frac{T_p}{T_p^h} \right) \times \left[ 1 - \frac{T_p}{T_p^h} \right]^{1.913} \text{mbarn.} \quad (B8)
\]

The black dashed line shows the approach by Dermer (1986b), that only starts at \( T_p = 0.95 \) GeV for \( \sigma_{pp}^\pi \) and an extrapolation to lower energies would lead to an underestimate of the cross section in comparison to the experimental data. Because Yang et al. (2018) do not provide an expression for their fit to the data points below 2 GeV, we determine our own fit to the data points up to \( T_p \leq 1.1 \) GeV and use the parametrization by Dermer (1986b) for higher proton energies, which leads to the orange solid line in Fig. B1. For the cross section of positively charged pion production, we fitted the curve from Fig. 4 in Yang et al. (2018), that is a sum of all channels leading to the production of positively charged pions. In this case, it connects relatively smoothly to the description of the cross section for \( T_p > 2 \) GeV from Eq. (B7). We fit the cross sections for charged pions as follows.

The total cross section for negatively charged pions is fit by \( \sigma_{pp}^\pi = \text{mbarn} = \exp (f_1(x)) \) with

\[
f_1(x) = a_1 \ln(x/c_1) + b_1 \ln^2(x/c_1) \quad \text{(B9)}
\]

where \( x = T_p/\text{GeV}, \) for \( T_p < 1.1 \) GeV, which corresponds to a momentum of \( P_p = 1.8 \) GeV/c. The parameters \( a_1, b_1, \) and \( c_1 \) are shown in Table B1. Therefore, we can use the parametrization by Dermer (1986b) for higher momenta, which is valid for values above \( P_p > 1.65 \) GeV/c but deviates from the data below \( T_p < 1.1 \) GeV, see the black dashed line for \( \sigma_{pp}^\pi \) in Fig. B1.

Furthermore, we provide our parametrizations for \( \sigma_{pp}^\pi = \text{mbarn} = f_2(x) \) by

\[
f_2(x) = [a_2 - b_2 \ln(x/c_2) + d_2 \ln^2(x/c_2)] + [c_2 - f_2 \ln(x/c_2) + d_2 \ln^2(x/c_2)] \quad \text{(B10)}
\]

where \( x = T_p/\text{GeV}, \) valid for \( T_p < 1.95 \) GeV. For higher energies, we apply the parametrization by Yang et al. (2018).

### B3 Analytical approximation for the source function of secondary electrons

Following Pfrommer & Enlin (2004), here we derive an analytical approximation for the source function of secondary electrons, which provides physical insight into our numerical approach of the hadronic reaction. Adapting a delta approximation for the production of pions, the differential cross section of pion production reads

\[
\frac{d\sigma(\pi^+ E_p)}{dE_{\pi^+}} = \frac{d\sigma(\pi^- E_p)}{dE_{\pi^-}} \delta(\pi^+ - \langle \pi^+ \rangle) \theta(E_p - E_{th}). \quad (B11)
\]

#### Table B1. Shown are the fit parameters for equations (B9) and (B10).

| \( a_1 \) | \( b_1 \) | \( c_1 \) |
|-----|-----|-----|
| 5.4868 | -10.4440 | 1.3219 |

| \( a_2 \) | \( b_2 \) | \( c_2 \) |
|-----|-----|-----|
| -1.5997 \times 10^{-1} | -1.3570 \times 10^{-1} | 2.7219 \times 10^{-1} |

| \( d_2 \) | \( e_2 \) | \( f_2 \) |
|-----|-----|-----|
| -2.9436 \times 10^{-2} | -5.5311 \times 10^{-2} | -5.3490 \times 10^{-2} |

Assuming isospin symmetry, i.e., that the multiplicity of neutral pions is half that of charged pions, \( \langle n_\pi \rangle = (\langle n_{\pi^+} + \langle n_{\pi^-} \rangle)/2 \), this yields the pion source function

\[
q_{\pi^+}(E_{\pi}) + q_{\pi^-}(E_{\pi}) = 2q_{\pi}(E_{\pi}) \quad \text{(B12)}
\]

from a proton energy distribution \( f_p(E_p) \). At high energies, one can furthermore assume a constant pion multiplicity \( \xi = 2 \), following the model by Pfrommer & Enlin (2004), as well as a mean pion energy \( \langle E_{\pi} \rangle(E_p) \approx K_pP_p/\xi \approx T_p/(2E_p) \), where the inelasticity \( K_p \) was assumed to be 1/2 (Mannheim & Schlickeiser 1994). In the high-energy limit, the proton power-law distribution in momentum is also a power-law distribution in energy since \( \gamma_p = E_p/(m_p c^2) = \sqrt{1 + p_p^2/m_p^2} \), with \( p_p \gg 1 \) and furthermore, \( T_p/(m_p c^2) = \gamma_p - 1 \approx \gamma_p \). If the energy distribution is given by a power-law with spectral index \( \alpha_p \) and normalization factor \( \tilde{C}_p \), we obtain the expression

\[
q_{\pi^+}(E_{\pi}) + q_{\pi^-}(E_{\pi}) = \frac{4}{3} \xi^2 - a_p \frac{C_p}{m_p c^2} n_{\pi}(E_{\pi}) \left( \frac{2E_{\pi}}{m_p c^2} \right)^{-\alpha_p} \quad \text{(B13)}
\]

In this approximation, the effective inelastic cross section \( \sigma_{pp}^\pi \) was modeled by Pfrommer & Enlin (2004), which also accounts for kaon decay modes. It reads

\[
\sigma_{pp}^\pi(E_p) = 32 \times [0.96 + \exp(4.4 - 2.4\alpha_p)] \text{mbarn.} \quad \text{(B14)}
\]

Transforming the distribution of pions into a distribution of electrons/positrons, i.e., \( q_{\pi^+} dE_{\pi^+} = q_{e^+} dE_{e^+} \), and estimating the mean energy of the produced electrons or positrons from the decay channel \( \pi^\pm \rightarrow e^\pm + 3\nu \) to be \( \langle E_{e^\pm} \rangle = \langle E_{\pi^\pm} \rangle / 4 \) (Mannheim & Schlickeiser 1994) yields

\[
q_{e^+} \text{sec}(E_{e^+}) = q_{\pi^+} \{ E_{\pi^+}(E_{e^+}) \} \frac{dE_{\pi^+}}{dE_{e^+}} = 4q_{\pi^+}(4E_{e^+}). \quad \text{(B15)}
\]

Combining this with Eq. (B13) gives

\[
q_{e^+} \text{sec}(E_e) = q_{e^+} \text{sec}(E_e) + q_{e^+} \text{sec}(E_e) = \frac{64}{3} n_{\pi} \frac{C_p}{m_p c^2} \sigma_{pp}^\pi(E_p) \left( \frac{16E_e}{m_p c^2} \right)^{-\alpha_p}. \quad \text{(B16)}
\]

Note that \( \tilde{C}_p \) denotes the normalization of the CR distribution function in units of cm\(^{-3}\) sec\(^{-1}\) while \( C_p \) is the normalization of the CR source function in units of cm\(^{-3}\) sec\(^{-1}\).
Figure B.3. Ratio of secondary electrons and positrons to protons in our approach (solid lines) compared to the analytical model (dashed lines, Pfrommer & Enßlin 2004), for different values of the spectral index of the proton spectrum $\alpha_p$.

The resulting secondary electron/positron energy distribution can be inferred from the fact that the source function is a production rate that is acting on a characteristic timescale of pp-interactions $\tau_\pi$:

$$f_{e^\pm,\text{uncooled}}^{\text{sec}}(E_{e^\pm}) = q_{e^\pm}(E_{e^\pm})\tau_\pi,$$  \hspace{1cm} (B17)

$$\tau_\pi = \frac{1}{c n_H K_p \sigma_{pp}},$$ \hspace{1cm} (B18)

provided there are no other cooling processes. Figure B3 shows the ratio of the resulting spectrum of secondary electrons plus positrons to the underlying proton spectrum as a function of energy, $E$, i.e. in the analytical model

$$\frac{f_{e^\pm,\text{uncooled}}^{\text{sec}}(E)}{f_p(E)} = \frac{2 f_{e^\pm,\text{uncooled}}^{\text{sec}}(E)}{f_p(E)} = \frac{128}{3} 16^{-\alpha_p}.$$ \hspace{1cm} (B19)