Stability of magnetic vortex in soft magnetic nano-sized circular cylinder

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(Dated: July 13, 2021)

The small magnetic nano-scale cylinders made of soft magnetic materials recently gained attention due to the progress in fabrication and observation techniques and also because of their possible applications in magnetic random access memory (MRAM) devices. Within such cylinders of circular shape in a certain range of sizes the magnetic vortices are frequently observed (see e.g. [1, 2, 3, 4]). For applications, which usually try to avoid vortex formation (such as MRAM cells, [5]), it is important to know the sizes of the cylinder where the vortices do not form.

In thin ferromagnetic cylinders with the thickness \( L \) of the order of a few \( L_E \) (where \( L_E = \sqrt{C/M_S} \) is the exchange length, \( C \) is the exchange constant of the material, \( M_S \) is the saturation magnetization) distribution of the magnetization vector can be assumed uniform along the cylinder axis. Then, there are two characteristic sizes of the cylinder important for the presence of the vortex state versus the uniformly magnetized one in zero applied magnetic field. The first is the single-domain radius \( R_{EQ} \), which is defined as a radius of the cylinder (at a given thickness) in which the energies of the uniformly magnetized state and the state with the vortex are the same \( \tilde{E} \). In cylinders with radii \( R < R_{EQ} \) the uniformly magnetized state has a lower energy than the vortex state.

However, the metastable vortices still may be present in cylinders with radii below \( R_{EQ} \). There is another characteristic radius, the absolute single domain radius \( R_S \) of the cylinder, which is obtained from the requirement that the vortex is unstable (therefore is absolutely prohibited) in cylinders with \( R < R_S \).

The rigorous calculation of \( R_S \) requires evaluating the second variation of the energy functional including the long-range dipolar interactions, which is currently beyond possibilities of analytical methods. The other way to estimate the stability radius is to assume the precise way (mode) the vortex loses its stability and then to calculate the stability radius with respect to that process. There is an infinite set of possible candidate modes. Provided calculation of the energies is rigorous, the result for a particular candidate is a lower bound for the stability radius. The “true” \( R_S \) is the highest of all the lower bounds for all possible modes. This is also easy to understand from a thought experiment of having a cylinder with vortex and slowly shrinking its radius, as soon as the largest absolute single domain radius is reached the vortex loses stability according to that particular mode.

The first estimate of the absolute single-domain radius \( R_S \) was obtained for the circular cylinder by Usov and coworkers [6] for the mode corresponding to the uniform translation of the vortex. Such a translation is accompanied by formation of magnetic charges on the cylinder sides.

In small and flat ferromagnetic cylinders of isotropic (ideally soft) material the following hierarchy of energies is present [7] by decreasing their importance for minimization: exchange energy, energy of magnetic charges of the cylinder faces (because the cylinder is flat), energy of magnetic charges on cylinder sides, energy of volume magnetic charges. The energy of volume charges is the least important one, because of a general amplification of the effects associated with surface compared to the ones associated with volume, as the size of the particle decreases. Thus, it may be expected (and further calculations in this work support this conjecture) that a mode with non-uniform vortex deformation having no side magnetic charges (but having volume ones) may be more favorable for the vortex stability loss.

To find such a mode is not an easy task because once the vortex is deformed the problem loses the cylindrical symmetry and becomes two-dimensional (leading to a system of partial differential integral equations) instead of one-dimensional (described by a single ordinary differential equation) if the axial symmetry is present. It is possible to write the solution for the displaced magnetic vortex with no side magnetic charges and very small face...
charges at the vortex center exactly minimizing the exchange energy functional in the following form:

\[ f(z) = \frac{1}{c} \left( iz + \frac{1}{2} (a - \bar{a} z^2) \right), \tag{1} \]

where \( z = X + iY, \ t = \sqrt{-1}, \ X \) and \( Y \) are Cartesian coordinates in the cylinder plane, so that the axis \( Z \) is parallel to the cylinder axis, the complex parameter \( a \) describes the vortex displacement (for \( a = 0 \) the vortex is centered), the real parameter \( c \) is related to the vortex radius, and over a variable denotes the complex conjugation, the components of the magnetization vector \( \vec{M}(X,Y) \) are given by \( M_X + i M_Y = M_S 2 w/(1 + w f) \) and \( M_Z = M_S (1 - w f)/(1 + w f) \). A model of configuration \( \vec{M}(X,Y) \) is shown in Fig. 1. In the case of no applied field (the case when the field is non-zero was considered elsewhere) the phase of the complex number \( a \) is not important, this parameter will be considered real in the rest of this work. The expression (1) is not arbitrary, it follows from the analysis performed in [8], that it is the only way to displace the vortex center so that its structure keeps minimizing the exchange energy functional exactly and has no surface magnetic charges on the cylinder side. The magnetization distribution (1) has surface magnetic charges on the faces of the cylinder localized near the vortex center, which is a topological singularity. Unlike side charges, the charges on the faces can not be avoided if the cylinder is in a state of non-uniform magnetization (topologically charged state), such as vortex.

The values of the parameters \( a \) and \( c \) need to be found from minimization of the magnetostatic energy for the configuration (1). It is clear that in absence of magnetic field, the equilibrium \( a = 0 \) due to symmetry, in this case (1) reduces to the structure of an axially symmetric centered vortex exactly coinciding with the one studied by Usov [8]. The parameter \( c = R v / R \), where \( R v \) is the equilibrium vortex radius calculated in [8].

The stability radius can be found from the condition

\[ \frac{\partial^2}{\partial a^2} (E_{EX}(a) + E_{MS}(a)) \bigg|_{a=0} = 0, \tag{3} \]

which means that the energy minimum on the variable \( a \) turns to maximum. In this expression \( E_{EX} \) and \( E_{MS} \) are the total exchange and magnetostatic energy of the cylinder respectively.

The normalized exchange energy of the particle with magnetic configuration (1) is

\[ E_{EX} = \frac{E_{EX}}{4 \pi M_S^2 V} = \frac{L}{4 \pi R^2} (2 - \log \frac{2 c}{1 + \sqrt{1 - a^2}}), \tag{4} \]

where \( V = L \pi R^2 \) is the particle volume.

Due to the lack of space the change (with \( a \)) of the vortex core magnetostatic energy (face charges) will be neglected in this paper. This approximation is equivalent to neglecting terms of the order \( c^3 \ll 1 \). The density of volume magnetic charges (also neglecting the fact that inside the vortex core \( |f(z)| < 1 \) they are reduced, equivalent to throwing out terms of the order \( c^3 \ll 1 \)) is

\[ \nabla \cdot \vec{M}(r) = -\frac{a M_S}{R} \cos \phi + O(a^2), \tag{5} \]

in the polar coordinate system \( r, \phi \) centered at the particle axis. Their normalized energy is

\[ e_{vol}^{MS} = \frac{E_{MS}^{vol}}{4 \pi M_S^2 V} = \frac{R a^2}{2 L} F_1^V \left( \frac{L}{R} \right) + O(a^3) \tag{6} \]

where straightforwardly

\[ F_1^V(x) = \frac{1}{(2 \pi)^2} \int_0^x \int_0^x \int_0^1 \int_0^1 \int_0^{2 \pi} \int_0^{2 \pi} \frac{r_1 r_2 \cos \phi_1 \cos \phi_2 d z_1 d z_2 d r_1 d r_2 d \phi_1 d \phi_2}{\sqrt{(z_1 - z_2)^2 + r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\phi_1 - \phi_2)}}, \tag{7} \]
FIG. 2: The phase diagram of nano-scale cylinders. The roman numbers mark the regions separated by the solid lines corresponding to different ground states of the particle [6,7]: I – vortex state, II – magnetized uniformly in-plane, III – magnetized uniformly parallel to the cylinder axis. The shaded region marks the range of cylinder’s geometrical parameters where the vortices are metastable, below that region no complete vortices can be present. The previous estimation of $R_S$ corresponds to the uniform vortex displacement [7].

which can be simplified into

$$F^V_1(x) = \int_0^\infty \frac{dk}{k} \frac{x}{k} \left(1 - \frac{1 - e^{-kx}}{kx}\right) \left[\int_0^1 \rho J_1(k\rho) d\rho\right]^2.$$  \hspace{1cm} (8)

Equation (3) was then solved numerically for $R_S/L_E$ at different $L/L_E$, the result is shown in Fig. 2. It is possible to obtain the asymptotic of this dependence at $L \ll L_E$ analytically by expanding $F^V_1(x<<1) = kx^2 + O(x^3)$, where $k = 0.0882686 \ldots$ we get

$$\frac{R_S}{L_E} = \frac{1}{8\pi k(L/L_E)}, \quad L \ll L_E. \hspace{1cm} (9)$$

This asymptotic was used to verify the results of the numerical computation and is also plotted in Fig. 2. Neglecting the vortex core is not such a bad approximation as it seems, in fact, it only breaks when $c = R_V/R$ is close to one, which happens near the intersection of the $R_S$ and $R_V$ lines in Fig. 2. Including the core into consideration is straightforward but involves more complicated formulas. Such more detailed calculation will be published in a forthcoming paper.

This work was supported in part by the Grant Agency of the Czech Republic under projects 202/99/P052, 101/99/1662, the INTAS Grant 31-311 and the Korea Institute for Advanced Study. The authors would like to thank Ivan Tomáš for reading the manuscript and many valuable discussions.

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