Symmetry Energy and Structure of Exotic Nuclei

M K Gaidarov1, P Sarriguren2, A N Antonov1, E Moya de Guerra3
1Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia
1784, Bulgaria
2Instituto de Estructura de la Materia, IEM-CSIC, Serrano 123, E-28006 Madrid, Spain
3Departamento de Física Atomica, Molecular y Nuclear, Facultad de Ciencias Físicas,
Universidad Complutense de Madrid, E-28040 Madrid, Spain
E-mail: gaidarov@inrne.bas.bg

Abstract. The symmetry energy, the neutron pressure and the asymmetric compressibility of
spherical Ni, Sn, Pb and deformed Kr, Sm neutron-rich even-even nuclei are calculated within
the coherent density fluctuation model using the symmetry energy as a function of density within
the Brueckner energy-density functional. The correlation between the thickness of the neutron
skin and the characteristics related with the density dependence of the nuclear symmetry energy
is investigated for isotopic chains of these nuclei in the framework of the deformed self-consistent
mean-field Skyrme HF+BCS method. The mass dependence of the nuclear symmetry energy
and the neutron skin thickness are also studied together with the role of the neutron-proton
asymmetry. The studied correlations reveal a smoother behavior in the case of spherical nuclei
than for deformed ones. We also notice that the neutron skin thickness obtained for 208Pb with
SLy4 force is found to be in a good agreement with the recent data. In addition to the interest
that this study may have by itself, we give some numerical arguments in proof of the existence
of peculiarities of the studied quantities in Ni and Sn isotopic chains that are not present in the
Pb chain.

1. Introduction
Great attention has been paid to the nuclear equation of state of isospin asymmetric nuclear
matter (ANM), in particular the nuclear matter symmetry energy. The density-dependent
symmetry energy governs numerous isospin-dependent properties of nuclei such as the binding
energy, the location of the drip lines, the density distributions, as well as the reactions: giant
resonances, heavy ion collisions, isospin diffusion, and multifragmentation. At the same time, the
nuclear symmetry energy is crucial in the astrophysical calculations of neutron stars, supernova
explosions and stellar nucleosynthesis. The neutron skin thickness is one of the observables
where symmetry energy shows up in the ground state of nuclei [1]. Neutron skin defined through
the rms radii of protons and neutrons depends on the properties of the nuclear surface. The
relative differences of the neutron and the proton distributions in this region are sensitive to the
symmetry energy at the subsaturation densities.

The main aim of this work is to investigate the relation between the neutron skin thickness and
some nuclear matter properties in finite nuclei, such as the symmetry energy at the saturation
point \( s \), symmetry pressure \( p_0 \) (proportional to the slope of the bulk symmetry energy), and
asymmetric compressibility \( \Delta K \), considering nuclei in given isotopic chains and within a certain
theoretical approach. For this purpose, the Brueckner energy density functional for infinite
nuclear matter [2, 3] was applied to calculate \( s \), \( p_0 \), and \( \Delta K \) of medium-heavy and heavy spherical Ni \((A = 74 - 84)\), Sn \((A = 124 - 152)\), Pb \((A = 202 - 214)\) nuclei [4] and deformed Kr \((A = 82 - 120)\), Sm \((A = 140 - 156)\) isotopes [5] that include surface effects. A theoretical approach that combines the deformed HF+BCS method with Skyrme-type density-dependent effective interactions (e.g., [6]) and the coherent density fluctuation model (CDFM) [7] was used. We would like to note the capability of the CDFM to be applied as an alternative way to make a transition from the properties of nuclear matter to the properties of finite nuclei. Finally, we have analyzed in details the existence of kinks in Ni and Sn isotopic chains and the lack of such a kink for the Pb chain. The kinks are produced because of the sensitivity of the symmetry energy and neutron pressure to the shell structure (see, for instance, the discussion in Refs. [4, 8]).

2. Theoretical framework

The quantity \( s^{ANM}(\rho) \), which refers to the infinite system and therefore neglects surface effects, is related to the second derivative of the energy per particle \( E(\rho, \delta) \) using its Taylor series expansion in terms of the isospin asymmetry \( \delta = (\rho_n - \rho_p)/\rho \) \((\rho, \rho_n\) and \( \rho_p \) being the baryon, neutron and proton densities, respectively) (see, e.g., [4]):

\[
s^{ANM}(\rho) = \frac{1}{2} \left. \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2} \right|_{\delta=0} = a_4 + \frac{p_0^{ANM}}{\rho_0^2} (\rho - \rho_0) + \frac{\Delta K^{ANM}}{18\rho_0^2} (\rho - \rho_0)^2 + \cdots . \tag{1}
\]

In Eq. (1) the parameter \( a_4 \) is the symmetry energy at equilibrium \((\rho = \rho_0)\). In ANM the pressure \( p_0^{ANM} \), the ”slope” parameter \( L^{ANM} \) and the curvature \( \Delta K^{ANM} \) are:

\[
p_0^{ANM} = \rho_0 \left. \frac{\partial s^{ANM}(\rho)}{\partial \rho} \right|_{\rho=\rho_0}, \quad L^{ANM} = \frac{3p_0^{ANM}}{\rho_0}, \quad \Delta K^{ANM} = 9\rho_0^2 \left. \frac{\partial^2 s^{ANM}(\rho)}{\partial \rho^2} \right|_{\rho=\rho_0}. \tag{2}
\]

For instance, using the Brueckner theory, the symmetry energy \( s^{ANM}(x) \) has the form:

\[
s^{ANM}(x) = 41.7\rho_0^{2/3}(x) + b_4\rho_0(x) + b_5\rho_0^{4/3}(x) + b_6\rho_0^{5/3}(x). \tag{3}
\]

Under some approximation the symmetry energy for finite nuclei and the related quantities can be written within the CDFM as infinite superpositions of the corresponding ANM quantities weighted by \(|f(x)|^2\):

\[
s = \int_0^{\infty} dx |f(x)|^2 s^{ANM}(x), \tag{4}
\]

\[
p_0 = \int_0^{\infty} dx |f(x)|^2 p_0^{ANM}(x), \quad \Delta K = \int_0^{\infty} dx |f(x)|^2 \Delta K^{ANM}(x). \tag{5}
\]

The neutron skin thickness is usually estimated as the difference of the rms radii of neutrons and protons:

\[
\Delta R = < r_n^2 >^{1/2} - < r_p^2 >^{1/2}. \tag{6}
\]

In our calculations the following Skyrme force parametrizations are used: SLy4, Sk3, SGII, and LNS (see, e.g., [4]). Also, we use the proton and neutron densities obtained from self-consistent deformed Hartree-Fock calculations with density-dependent Skyrme interactions [6] and pairing correlations.
3. Results and discussion

The correlation of the neutron skin thickness $\Delta R$ in the Ni isotopic chain with the $s$, $p_0$ and $\Delta K$ parameters is shown in Fig. 1. It is seen from Fig. 1(a) that there exists an approximate linear dependence between $\Delta R$ and $s$ for the even-even Ni isotopes with $A = 74 - 84$. We observe a smooth growth of the symmetry energy till the double-magic nucleus $^{78}\text{Ni}$ ($N = 50$) and then a linear decrease of $s$ while the neutron skin thickness of the isotopes increases. This behavior is valid for all Skyrme parametrizations used in the calculations, in particular, the average slope of $\Delta R$ for various forces is almost the same. We also find a similar approximate linear correlation for Ni isotopes between $\Delta R$ and $p_0$ [Fig. 1(b)] and less strong correlation between $\Delta R$ and $\Delta K$ [Fig. 1(c)]. As in the symmetry energy case, the behavior of the curves drawn in these plots shows the same tendency, namely the inflexion point transition at the double-magic $^{78}\text{Ni}$ nucleus. We would like to note that the predictions for the difference $\Delta R$ between the rms radii of neutrons and protons with Skyrme forces obtained in Ref. [9] exhibited a steep change at the same place in which the number of neutrons starts to increase in the chain of nickel isotopes.

In our work we consider a chain of deformed neutron-rich even-even Kr isotopes (including, as well, the case of some extreme neutron-rich nuclei up to $^{120}\text{Kr}$). The results for the symmetry energy $s$ as a function of the mass number $A$ for the whole Kr isotopic chain are presented in Fig. 2. We observe peaks of the symmetry energy at specific Kr isotopes, namely at semi-magic $^{86}\text{Kr}$ ($N = 50$) and $^{118}\text{Kr}$ ($N = 82$) nuclei. In addition, a flat area is found surrounded by transitional regions $A = 88 - 96$ and $A = 110 - 116$. The results shown in Fig. 2 are closely related to the evolution of the quadrupole parameter $\beta$ as a function of the mass number $A$ that is presented in Fig. 3. First, one can see from Fig. 3 that the semi-magic $A = 86$ and $A = 118$ Kr isotopes are spherical, while the open-shell Kr isotopes within this chain possess two equilibrium shapes, oblate and prolate. In the case of open-shell isotopes, the oblate and prolate minima are very close in energy and the energy difference is always less than 1 MeV. Nevertheless, we specify in Fig. 3 which shape corresponds to the ground state of each isotope by encircling them. Thus, the trend that the evolution of the symmetry energy shown in Fig. 2 follows can be clearly understood. The peaks of the symmetry energy correspond to the closed-shell nuclei that are spherical. Mid-shell nuclei ($A = 96 - 110$) are well deformed and exhibit a stabilized behavior with small values of $s$. The transitional regions from spherical to well deformed shapes correspond to transitions from the peaks to the valley in the symmetry energy.
4. Conclusions
The results of the present work can be summarized as follows:

i) There exists an approximate linear correlation between $\Delta R$ and $s$, as well as $p_0$, while the relation between $\Delta R$ and $\Delta K$ is less pronounced. A behavior containing an inflexion point transition at specific shell closure is observed for these correlations ($^{78}$Ni, $^{132}$Sn, $^{86}$Kr, and $^{144}$Sm).

ii) Our HF+BCS calculations lead to $s$ in the range of 27–30 MeV, which is in agreement with the empirical value of 30±4 MeV. The calculated values of $p_0=1.36–1.68$ MeV/fm$^3$ lead to values of the slope parameter $L=26–32$ MeV, in agreement with other theoretical predictions.

iii) The kinks displayed by the Ni and Sn can be understood as consequences of particular differences in the structure of these nuclei and the resulting densities and weight functions. It is shown that for the Pb isotopes the different signs of the relative deviations corresponding to the range of integration on $x$ that contains the peak of $|f(x)|^2$ (being sensitive to the different density profiles of the considered isotopes around double-magic ones) is in favor of the absence of kink in the Pb chain.

Concluding, we would like to note that the capability of the present method can be further demonstrated by taking into consideration Skyrme-type and relativistic nuclear energy density functionals.

References
[1] Brown B A 2000 Phys. Rev. Lett. 85 5296
[2] Brueckner K A, Buchler J R, Jorna S and Lombard R J 1968 Phys. Rev. 171 1188
[3] Brueckner K A, Buchler J R, Clark R C and Lombard R J 1969 Phys. Rev. 181 1543
[4] Gaidarov M K, Antonov A N, Sarriguren P and Moya de Guerra E 2011 Phys. Rev. C 84 034316
[5] Gaidarov M K, Antonov A N, Sarriguren P and Moya de Guerra E 2012 Phys. Rev. C 85 064319
[6] Moya de Guerra E, Sarriguren P, Caballero J A, Casas M and Sprung D W L 1991 Nucl. Phys. A 529 68
[7] Antonov A N, Hodgson P E and Petkov I Zh 1988 Nucleon Momentum and Density Distributions in Nuclei (Clarendon Press, Oxford); 1993 Nucleon Correlations in Nuclei (Springer-Verlag, Berlin-Heidelberg-New York)
[8] Warda M, Viñas X, Roca-Maza X and Centelles M 2010 Phys. Rev. C 81 054309
[9] Sarriguren P, Gaidarov M K, Moya de Guerra E and Antonov A N 2007 Phys. Rev. C 76 044322