Interferometric tunability of the absorption

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We propose an interferometric setup that permits to tune the quantity of radiation absorbed by an object illuminated by a fixed light source. The method can be used to selectively irradiate portions of an object based on their transmissivities or to accurately estimate the transmissivities from rough absorption measurements.

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When an object is illuminated, it will absorb radiation proportionally to its absorption coefficient: Darker portions of the object will absorb more light than more transparent ones. Is there a way around this? In this paper we analyze a setup which uses classical light sources (i.e. coherent beams) and permits to easily tune the quantity of a light absorbed by an object independently on its transparency, by appropriately tuning an interferometer phase. With the same setup, high efficiency measurements of the absorption coefficient can be performed via a feedback mechanism. The only underlying assumption is that the object introduces a negligible dephasing into a probe beam. Since we can employ quasi-monochromatic light, this assumption is met in a variety of systems. Moreover, in the case of objects that have a homogeneous phase image, the dephasing can be easily compensated with the interferometer phase.

Our proposal draws inspiration from the so called “interaction-free-measurement” setups, where a partially transparent object can be discriminated from a totally transparent one with asymptotically negligible radiation absorption.\(^1\)\(^2\)\(^3\)\(^4\)\(^5\)\(^6\)\(^7\) Even though such proposals were originally based on single-photon light pulses, analogous results have been obtained also with classical light.\(^5\)\(^6\)

The layout of the paper follows. We start by describing the proposed interferometric setup. We show how the absorption peak can be tuned and we analyze the irradiation selectivity. We then give the protocol for high efficiency estimation of \(\eta\). We conclude by analyzing inhomogeneous objects, which incorporate different transmissivities. Since the process does not involve any quantum effects (such as entanglement or squeezing) one could also analyze it in terms of a classical theory of radiation, instead of the quantum formalism we use here for rigor.

THE APPARATUS

The proposed apparatus is a modification of the experimental setup of Ref.\(^6\)\(^7\). It is obtained by concatenating a collection of \(N\) Mach-Zehnder (MZ) interferometers and is depicted in Fig. 11. Initially a coherent state \(|\alpha\rangle\) enters through one interferometer port (associated with the annihilation operator \(a_0\)), and no photons enter from the other port (associated with the annihilation operator \(b_0\)). As shown in Fig. 11 after each MZ, one of the two emerging beams (the R beam) is focused on the object. Then the two beams are recombined at the input port of the next MZ. After \(N\) of such steps, the radiation leaves the apparatus at the \(N\)th interferometer outputs \(a_N\) and \(b_N\). As we will show, appropriately tuning the interferometers phase \(\phi\) and the number \(N\) of MZs it is possible to choose the value of the transmissivity \(\eta\) that will absorb the most radiation in the object. The transmissivity \(\eta\) of an object is the probability that a single photon will pass through it or, equivalently, the percentage of the transmitted intensity of an impinging coherent beam. Note that an apparatus employing a single MZ which is crossed \(N\) times by the light can also be employed, where \(N\) can be controlled by appropriately tilting one of the interferometer mirrors or by using an acousto-optics switch.

The input-output relations of the interferometer can be obtained observing that when two coherent states \(|\alpha_n\rangle\) and \(|\beta_n\rangle\) impinge, respectively, into the input ports \(a_n\) and \(b_n\) of the \(n\)-th MZ (see Fig. 2), the corresponding outputs at ports \(a_{n+1}\) and \(b_{n+1}\) are still coherent states of amplitudes \(\alpha_{n+1}\) and \(\beta_{n+1}\), given by

\[
\begin{pmatrix}
\alpha_{n+1} \\
\beta_{n+1}
\end{pmatrix}
= S
\begin{pmatrix}
\alpha_n \\
\beta_n
\end{pmatrix},
\]

with

\[
S = e^{i\phi/2}
\begin{pmatrix}
\cos(\phi/2) & i\sin(\phi/2) \\
i\sin(\phi/2) & \cos(\phi/2)
\end{pmatrix},
\]

where \(\phi\) is the interferometer phase. The output \(a_n\) is directly fed into a port of the MZ number \(n+1\), while the output \(b_n\) first passes through the object and then enters the other port of the same MZ. Since the object absorbs each photon with a probability \(\eta\) without introducing any phase factor, its action on the input coherent state \(|\beta_{n+1}\rangle\)
it exits through the outputs

\[ a \]

\[ \phi \]

the radiation enters from the input

\[ \text{side the MZs and it interacts only with the R beams. Initially} \]

\[ a \]

transforms the annihilation operators

\[ c \]

of the

\[ \text{input annihilation operators} \]

\[ a \]

\[ d \]

FIG. 1. The first of the two 50-50 beam splitters transforms

\[ \text{element in the Mach-Zehnder sequence in the apparatus of} \]

\[ n \]

\[ \text{state} \]

\[ \text{then discards one of the two outputs. As a result the} \]

\[ a \]

\[ \alpha \]

\[ \beta \]

FIG. 2: Mach-Zehnder interferometer constituting the nth element in the Mach-Zehnder sequence in the apparatus of Fig. 1. The first of the two 50-50 beam splitters transforms the input annihilation operators \( a_n, b_n \) into \( c' = (a_n + b_n) / \sqrt{2} \)

\[ \text{and} \]

\[ d = (b_n - a_n) / \sqrt{2} \]

respectively. The second beam splitter transforms the annihilation operators \( c \) and \( d \) into \( a_{n+1} = (c + d) / \sqrt{2} \) and \( b_{n+1} = (d - c) / \sqrt{2} \).

\[ \text{can be modeled as a beam splitter with transmissivity} \]

\[ \eta \]

\[ \text{that couples the input radiation to a vacuum state and} \]

\[ \text{then discards one of the two outputs. As a result the} \]

\[ a \]

\[ a \]

\[ \text{state} \]

\[ \beta \]

\[ \text{transforms into a coherent state of reduced amplitude} \]

\[ \eta \beta_{n+1} \]

\[ a \]

\[ a \]

\[ \text{Thus, in the presence of the absorber, the amplitudes} \]

\[ a \]

\[ \alpha \]

\[ \beta \]

\[ a \]

\[ a \]

\[ \text{ent states at the input of the MZ interferometer number} \]

\[ n+1 \]

\[ \text{is given by} \]

\[ \left( \frac{\alpha_{n+1}}{\beta_{n+1}} \right) = S(\eta) \left( \frac{\alpha_n}{\beta_n} \right), \tag{3} \]

where

\[ S(\eta) = e^{i\phi/2} \left( \begin{array}{cc} \cos(\phi/2) & i \sin(\phi/2) \\ i\sqrt{\eta} \sin(\phi/2) & \sqrt{\eta} \cos(\phi/2) \end{array} \right). \tag{4} \]

Iterating Eq. (3) \( N \) times we can express the amplitude of the coherent states emerging from the whole apparatus as

\[ \left( \frac{\alpha_N}{\beta_N} \right) = S^N(\eta) \left( \frac{\alpha_0}{0} \right). \tag{5} \]

Some examples of such evolution are given in Fig. 3 and an analytic solution can be obtained by diagonalizing \( S(\eta) \). The light absorbed by the object is given by

\[ I_{ab} = |\alpha_0|^2 - (|\alpha_N|^2 + |\beta_N|^2) \equiv r |\alpha_0|^2, \tag{6} \]

i.e. the input intensity \(|\alpha_0|^2\) minus the total output intensity \(|\alpha_N|^2 + |\beta_N|^2\). The quantity \( r \) is a complicated function of \( N, \phi \) and \( \eta \) which can be explicitly computed from Eq. (5). It measures the “effective” absorption constant of the object.

**DISCUSSION**

The possibility of changing the absorption of the illuminated object from its natural value \( 1 - \eta \) to an effective value \( r \approx 0 \) allows one to determine the presence of a completely opaque object (i.e. \( \eta = 0 \)) with only an asymptotically small fraction of the input radiation being absorbed \( \eta = 0 \) (e.g. completely opaque object). In these cases simple analytical solutions can be obtained yielding

\[ \alpha_N = \alpha_0 e^{iN\phi/2} \cos(N\phi/2) \tag{7} \]

\[ \beta_N = i\alpha_0 e^{iN\phi/2} \sin(N\phi/2), \tag{8} \]

for \( \eta = 1 \), and

\[ \alpha_N = \alpha_0 e^{iN\phi/2} \cos^N(\phi/2) \tag{9} \]

\[ \beta_N = 0, \tag{10} \]

for \( \eta = 0 \). By choosing \( \phi = \pi/N \), from Eqs. (8) and (10), it is immediate to see that all radiation exits from the \( b_N \)-port if \( \eta = 1 \) and that most of the radiation (asymptotically all of it for \( N \to \infty \)) exits from the \( a_N \)-port if \( \eta = 0 \). In both cases the light absorption is minimal (i.e. exactly null in the first case and asymptotically null in the second one). Nonetheless they can be discriminated by simply looking from which interferometer ports (e.g. \( a_N \) or \( b_N \)) the light emerges.
The possibility of controlling the effective absorption \( r \) of the object by changing the interferometer parameters is evident from Fig. 4 where we plot \( r \) as a function of the transmissivity \( \eta \) for different values of \( \phi \) (choosing again \( N = \pi/\phi \)). The function \( r \) exhibits a peak for \( \eta = \eta_{\text{max}} \) which increases from \( \eta_{\text{max}} \sim 0 \) to \( \eta_{\text{max}} \sim 1 \) as \( \phi \) decreases. This effect can be explained intuitively as follows. For small values of \( \phi \) (i.e., high values of \( N \)) little radiation is leaked into the R modes at every round trip with the exception of the case when \( \eta \) is high. On the contrary, for small values of \( N \) (i.e., large values of \( \phi \)) a larger amount of radiation is leaked into the R modes at every round trip, so that the absorption peak moves to lower values of \( \eta \). The dependence of the absorption peak maximum as a function of \( \phi \) and \( N \) is depicted in Fig. 4 left. This graph also shows the values of \( \eta_{\text{max}} \) that can be attained in practice: it can be accurately fine-tuned only for \( \eta_{\text{max}} \gtrsim 0.5 \) since only few discrete values of \( \eta_{\text{max}} \) are achievable for low \( N \), whereas high \( \eta_{\text{max}} \sim 1 \) requires large \( N \), which can be difficult to achieve practically. Finally, the selectivity of the absorption, i.e., the width of the effective absorption curve \( r \) as a function of \( \eta \) (see Fig. 4), is not constant when \( \phi \) is varied: The value of the width-at-half-maximum is smaller for absorption curves peaked at \( \eta_{\text{max}} \approx 0.1 \) and larger for \( \eta_{\text{max}} \approx 0.5 \). In the limit \( \phi \to 0 \) the \( r \)-curve becomes a very narrow spike peaked just below \( \eta = 1 \).

![Fig. 3](image3.jpg)

**FIG. 3:** Plot of the (rescaled) output amplitudes of the \( n \)th MZ interferometer \( |\alpha_n/\alpha_0|^2 \) in the \( a \)-modes at L (continuous line) and \( |\beta_n/\alpha_0|^2 \) in the \( b \)-modes at R (dashed line). Initially all the radiation is in mode \( \alpha_0 \), but, as the evolutions progresses, more and more radiation is transferred to the \( b \)-modes, until (for \( n = \pi/\phi \)) the radiation is entirely transferred. Here \( \phi = \pi/10 \) so that the total transfer occurs for \( n = 10 \) (vertical line). Left: The object is completely transparent \( (\eta = 1) \), so that the total energy (dotted line) is constant; Right the object is semi-transparent \( (\eta = 0.9) \), so that the total energy decreases as the evolution progresses.

**HIGH PRECISION \( \eta \)-MEASUREMENTS**

Our scheme can be easily adapted to high precision estimation of the absorption coefficient, starting from low-quality measurements of the absorption \( r \). The main idea is revealed by the lower right graph of Fig. 4. The required iterative procedure is composed by the following steps: i) start by roughly estimating \( \eta \) through an absorption measurement and set the interferometer phase so that the \( r \) curve has a steep slope corresponding to such value of \( \eta \); ii) perform another absorption measurement and estimate a better value of \( \eta \); iii) again tune the interferometer phase, and so on. Since the absorption curve \( r \) for the values of \( \eta \sim 1 \) can be very steep, a very good estimate of these \( \eta \)s can be achieved even when the measurement of \( r \) contains a large error \( \Delta r \) (see Fig. 4). Notice that the high values of \( \eta \sim 1 \) are the
FIG. 5: Left: Maximum \( \eta_{\text{max}} \) (circles) and average value \( \eta_{\text{av}} \) (squares) of the absorption peak \( r \) of Fig. 4 as a function of \( N = \pi/\phi \). (The maximum and the average follow different evolutions because of the asymmetry in the absorption curves). Increasing \( N \) (i.e. decreasing \( \phi \)), the maximum in the absorption peak moves to higher values of \( \eta \). The graph also details which are the actual values of \( \eta \) that can be achieved through the proposed setup as a function of \( N \). The dotted line is the function \((N - 1)/N\) that gives a good interpolation of the peak evolution. Right: Selectivity in the irradiation as a function of the transmissivity peak. The width of the peaks of the dotted line in Fig. 4 is not uniform. Here we plot the Root Mean Square (stars) and the width at half the peaks of the dotted line in Fig. 4 as a function of \( \eta \). Notice that the RMS is almost constant over the whole range.

The absorptivity of the various portions of the sample by conventional imaging techniques, identifying the transmissivity \( \bar{\eta} \) of the region that needs irradiation; ii) tune the phase \( \phi = \pi/N \) so that the absorption is maximized for \( \bar{\eta} \).

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[8] Rigorously speaking, the transformation \( |\beta_{n+1}\rangle \rightarrow |\sqrt{\eta}\beta_{n+1}\rangle \) does not correspond to the linear mapping \( |\beta_{n+1}\rangle \rightarrow |\beta_{n+1}\rangle \) but to its density matrix counterpart: \( |\beta_{n+1}\rangle |\beta_{n+1}\rangle \rightarrow |\sqrt{\eta}\beta_{n+1}\rangle |\sqrt{\eta}\beta_{n+1}\rangle \). The latter accounts for decoherence effect, whereas the former does not. Since in our analysis we are always dealing with factorized states, the two transformations coincide for us.
[9] This effect can be explained intuitively as follows. If the object is transparent, at the first round trip a small amount of radiation leaks into the R modes, at the second round trip a higher amount leaks there and so on constantly increasing through constructive interference until all the radiation moves into such modes after \( N = \pi/\phi \) round trips. If, instead, the object is opaque, the little radiation that has leaked into the R modes at the first round trip is absorbed and does not contribute to the constructive interference that would draw more radiation into these modes at the second round trip. As a result very little radiation transfers and most of it remains in the L modes.