Static quark correlators and quarkonium properties at non-zero temperature

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Abstract. We discuss different static quark correlators, including Wilson loops in 2+1 flavor QCD at non-zero temperature and their relation to in-medium quarkonium properties. We present lattice results on static correlation functions obtained with highly improved staggered fermion action and their implications for potential models.

1. Introduction
At sufficiently high temperatures strongly interacting matter undergoes a transition to a new state, often called quark-gluon plasma that is characterized by chiral symmetry restoration and color screening (see e.g. Ref. [1] for a current review). Experimentally such state of matter can be studied in relativistic heavy collisions. There was considerable interest in the properties and the fate of heavy quarkonium states at finite temperature since the famous conjecture by Matsui and Satz [2]. It has been argued that color screening in medium will lead to quarkonium dissociation above deconfinement, which in turn can signal quark-gluon plasma formation in heavy ion collisions. The basic assumption behind the conjecture by Matsui and Satz was the fact that medium effects can be understood in terms of a temperature-dependent heavy-quark potential. Color screening makes the potential exponentially suppressed at distances larger than the Debye radius and therefore it cannot bind the heavy quark and anti-quark once the temperature is sufficiently high. Based on this idea potential models at finite temperature with different temperature dependent potentials have been used over the last two decades to study quarkonium properties at finite temperature (see e.g. Refs. [3, 4] for recent reviews). It is not clear if and to what extent medium effects on quarkonium binding can be encoded in a temperature dependent potential. Effective field theory approach, namely the so-called thermal pNRQCD, can provide an answer to this question [5]. The notion of the potential can be defined using EFT approach both at zero and non-zero temperature. Thermal pNRQCD that will be discussed in the next section is based on the weak-coupling techniques. To understand the non-perturbative aspects of color screening as well as to test the reliability of the weak-coupling approach lattice calculations of the correlation functions of static quarks are needed. The correlation functions of static quarks that propagate around the periodic time direction $\tau = 1/T$ are related to the free energy of a static quark anti-quark pair. We will see that pNRQCD is a useful tool in understanding the temperature dependence of the static correlators. We also consider Wilson loops evaluated at time extent $\tau < 1/T$. They are naturally related to the static energy at non-zero temperature.
In principle, it is possible to study the problem of quarkonium dissolution without any use of potential models. In-medium properties of different quarkonium states and/or their dissolution are encoded in spectral functions. Spectral functions are related to Euclidean meson correlation functions which can be calculated on the lattice. Reconstruction of the spectral functions from the lattice meson correlators turns out to be very difficult, and the corresponding results remain inconclusive. We will discuss the calculation of the spectral functions using potential models in the light of lattice calculations of Wilson loops.

2. pNRQCD at finite temperature
There are different scales in the heavy quark bound state problem related to the heavy quark mass $m$, the inverse size $\sim mv \sim 1/r$ and the binding energy $mv^2 \sim \alpha_s/r$. Here $v$ is the typical heavy quark velocity in the bound state and is considered to be a small parameter. Therefore it is possible to derive a sequence of effective field theories using this separation of scales (see Refs. [6, 7] for recent reviews). Integrating out modes at the highest energy scale $\sim m$ leads to an effective field theory called non-relativistic QCD or NRQCD, where the pair creation of heavy quarks is suppressed by powers of the inverse mass and the heavy quarks are described by non-relativistic Pauli spinors [8]. At the next step, when the large scale related to the inverse size is integrated out, the potential NRQCD or pNRQCD appears. In this effective theory the dynamical fields include the singlet $S(r, R)$ and octet $O(r, R)$ fields corresponding to the heavy quark anti-quark pair in singlet and octet states respectively, as well as light quarks and gluon fields at the lowest scale $\sim mv^2$. The Lagrangian of this effective field theory has the form

$$\mathcal{L} = -\frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu} + \sum_i \bar{q}_i i D q_i + \int d^3 r \left[ S^\dagger \left[i \partial_0 + \frac{\nabla^2}{m} - V_s(r) \right] S + O^\dagger \left[i \partial_0 + \frac{\nabla^2}{m} - V_o(r) \right] O + V_A \left\{ O^\dagger \vec{r} \cdot g\vec{E} S + S^\dagger \vec{r} \cdot g\vec{E} O \right\} + \frac{V_B}{2} \left\{ O^\dagger \vec{r} \cdot g\vec{E} O + O \vec{r} \cdot g\vec{E} \right\} + \ldots \right] \quad (1)$$

Here the dots correspond to terms which are higher order in the multipole expansion [6]. The relative distance $r$ between the heavy quark and anti-quark plays a role of a label, the light quark and gluon fields depend only on the center-of-mass coordinate $R$. The singlet $V_s(r)$ and octet $V_o(r)$ heavy quark potentials appear as matching coefficients in the Lagrangian of the effective field theory and therefore can be rigorously defined in QCD at any order of the perturbative expansion. At leading order

$$V_s(r) = -\frac{4}{3} \frac{\alpha_s}{r}, \quad V_o(r) = \frac{1}{6} \frac{\alpha_s}{r} \quad (2)$$

and $V_A = V_B = 1$. One can generalize this approach to finite temperature. However, the presence of additional scales makes the analysis more complicated [5]. The effective Lagrangian will have the same form as above, but the matching coefficients may be temperature-dependent. In the weak coupling regime there are three different thermal scales: $T$, $gT$ and $g^2 T$. The calculations of the matching coefficients depend on the relation of these thermal scales to the heavy quark bound-state scales [5]. To simplify the analysis the static approximation has been used, in which case the scale $mv$ is replaced by the inverse distance $1/r$ between the static quark and anti-quark. The binding energy in the static limit becomes $V_o - V_s \approx N\alpha_s/(2r)$. When the binding energy is larger than the temperature the derivation of pNRQCD proceeds in the same way as at zero temperature and there is no medium modifications of the heavy quark potential [5]. But bound state properties will be affected by the medium through interactions with ultra-soft gluons, in particular, the binding energy will be reduced and a finite thermal width will
appear due to medium induced singlet-octet transitions arising from the dipole interactions in the pNRQCD Lagrangian [5] (c.f. Eq. (1)). When the binding energy is smaller than one of the thermal scales the singlet and octet potential will be temperature-dependent and will acquire an imaginary part [5]. The imaginary part of the potential arises because of the singlet-octet transitions induced by the dipole vertex as well as due to the Landau damping in the plasma, i.e. scattering of the gluons with space-like momentum off the thermal excitations in the plasma. In general, the thermal corrections to the potential go like \((rT)^n\) and \((m_D r)^n\) [5], where \(m_D\) denotes the Debye mass. Only for distances \(r > 1/m_D\) there is an exponential screening. In this region the singlet potential has a simple form

\[
V_s(r) = -\frac{4}{3} \frac{\alpha_s}{r} e^{-m_D r} + i \frac{4}{3} \alpha_s T \frac{2}{r m_D} \int_0^\infty dx \frac{\sin(m_D x)}{(x^2 + 1)^2} - \frac{4}{3} \alpha_s (m_D + iT),
\]

The real part of the singlet potential coincides with the leading order result of the so-called singlet free energy [9]. The imaginary part of the singlet potential in this limit has been first calculated in [10]. For small distances the imaginary part vanishes, while at large distances it is twice the damping rate of a heavy quark [11]. This fact was first noted in Ref. [12] for thermal QED.

The effective field theory at finite temperature has been derived in the weak-coupling regime assuming the separation of different thermal scales as well as \(\Lambda_{QCD}\). In practice, the separation of these scales is not evident and one needs lattice techniques to test the approach. Lattice QCD is formulated in Euclidean time. Therefore the next section will be dedicated to the study of static quarks at finite temperature in Euclidean time formalism.

3. Static meson correlators in Euclidean time formalism

Consider static (infinitely heavy) quarks. The position of heavy quark anti-quark pair is fixed in space and propagation happens only along the time direction. With respect to the color a static quark anti-quark (\(QQ\)) pair can be in a singlet or in an octet state. Therefore we can define the following \(QQ\) (meson) operators

\[
J(\vec{x}, \vec{y}; \tau) = \bar{\psi}(\vec{x}, \tau) U(\vec{x}, \vec{y}; \tau) \psi(\vec{y}, \tau),
\]

\[
J^a(\vec{x}, \vec{y}; \tau) = \bar{\psi}(\vec{x}, \tau) U(\vec{x}, \vec{0}; \tau) T^a U(\vec{0}, \vec{y}; \tau) \psi(\vec{y}, \tau),
\]

for singlet and octet state respectively. Here \(U(\vec{x}, \vec{y}; \tau)\) are the spatial gauge transporters connecting \(\vec{x}\) and \(\vec{y}\), \(\vec{x}_0\) is the coordinate of the center of mass of the meson and \(T^a\) are the \(SU(3)\) group generators. We consider the correlation function of singlet and octet meson operators at maximal Euclidean time \(\tau = 1/T\):

\[
G_1(r, T, \tau = 1/T) = \langle J(x, y, \tau = 1/T) J^\dagger(x, y; 0) \rangle,
\]

\[
G_8(r, T, \tau = 1/T) = \frac{1}{8} \langle J^a(x, y, \tau = 1/T) J^{a\dagger}(x, y; 0) \rangle.
\]

Integrating out the static quark fields \(\psi\) and replacing the quark propagators by temporal Wilson lines \(L(\vec{x}) = \prod_{n=0}^{N_x-1} U_0(\vec{x}, \tau)\) with \(U_0(\vec{x}, t)\) being the temporal links, we get the following expression for the above correlators:

\[
G_1(r, T) = \frac{1}{3} \langle \text{Tr} \left[ L^\dagger(\vec{x}) U(\vec{x}, \vec{y}; 0) L(\vec{y}) U^\dagger(\vec{x}, \vec{y}, 1/T) \right] \rangle,
\]

\[
G_8(r, T) = \frac{1}{8} \langle \text{Tr} L^\dagger(x) \text{Tr} L(y) \rangle - \frac{1}{24} \langle \text{Tr} \left[ L^\dagger(x) U(x, y; 0) L(y) U^\dagger(x, y, 1/T) \right] \rangle,
\]

\[
r = |\vec{x} - \vec{y}|.
\]
Figure 1. The singlet free energy (left) and the screening function (right) as function of the distance \( r \) at different temperatures calculated with the HISQ action.

The correlators depend on the choice of the spatial transporters \( U(\vec{x}, \vec{y}; \tau) \). Typically, a straight line connecting points \( \vec{x} \) and \( \vec{y} \) is used as a path in the gauge transporters, i.e. one deals with time-like rectangular cyclic Wilson loops. In the special gauge, where \( U(\vec{x}, \vec{y}; \tau) = 1 \) the above correlators give the standard definition of the singlet and octet free energies of a static \( Q\bar{Q} \) pair

\[
\exp(-F_1(r, T)/T) = \frac{1}{3} \langle \text{Tr}[L^\dagger(x)L(y)] \rangle,
\]

\[
\exp(-F_8(r, T)/T) = \frac{1}{8} \langle \text{Tr}[L^\dagger(x)L(y)] \rangle - \frac{1}{24} \langle \text{Tr}[L^\dagger(x)L(y)] \rangle.
\]

One can also fix the Coulomb gauge and define the interpolating meson operators without the spatial transporters, and use the above expression to define the singlet and octet correlators.

The correlator \( G(r, T) = \frac{9}{8} \langle \text{Tr}[L^\dagger(x)L(y)] \rangle \) gives the free energy \( F(r, T) = -T \ln G(r, T) \) of static quark anti-quark pair separated by distance \( r \) \[13\]. It can be expressed in terms of energy levels \( E_n(r) \) of static quark anti-quark pair at \( T = 0 \) \[14\]

\[
G(r, T) = \sum_{n=1}^{\infty} e^{-E_n(r)/T}.
\]

It is tempting to rewrite Eq. (9) or Eq. (11) as

\[
\exp(-F(r, T)/T) = \frac{1}{9} \exp(-F_1(r, T)/T) + \frac{8}{9} \exp(-F_8(r, T)/T)
\]

and interpret this expression as the decomposition of the free energy of static \( Q\bar{Q} \) pair into singlet and octet contributions \[13, 15, 16, 17\]. This decomposition is intuitively very appealing and should be valid in perturbation theory. However, it is problematic as \( G_1(r, T) \) is path- or gauge-dependent. The problem is also evident if one writes the spectral decomposition of \( G_1(r, T) \) \[14\]:

\[
G_1(r, T) = \sum_{n=1}^{\infty} c_n(r) e^{-E_n(r)/T}.
\]

The coefficients \( c_n(r) \) are different from unity and are path- or gauge-dependent. The EFT approach can help to resolve this puzzle. One can use pNRQCD also in Euclidean time formulation \[18\] and study the Polyakov loop correlator in this framework. The Polyakov loop correlator can be written in terms of correlation function of singlet and octet fields \[18\]

\[
G(r, T) = Z_s(r) \langle S(r, \tau = 1/T) S^\dagger(r, 0) \rangle + Z_o(r) \langle O^a(r, \tau = 1/T) O^{a\dagger}(r, 0) \rangle.
\]
Figure 2. The free energy of static $Q\bar{Q}$ pair (left) and the difference $F(r, T) - F_\infty(T)$ (right) calculated with HISQ action as function of the distance $r$ at different temperatures. In the right panel the filled symbols correspond to the lattice data, while the open symbols correspond to the values reconstructed from the singlet free energy. The legend in the left panel is the same as in Fig. 1 (left).

For $rT \ll 1$ one can use the zero temperature version of pNRQCD where the singlet and octet potentials are known up to 2-loop order. One can then show that $Z_s = Z_o = 1/9$ and thus in this limit the conjectured decomposition of the Polyakov loop correlator in terms of singlet and octet contribution is justified [18]

$$G(r, T) = \frac{1}{9} \exp(-V_s(r)/T) + \frac{8}{9} \exp(-V_o(r)/T).$$  (16)

The singlet and octet contributions are gauge-independent in this framework. When the binding energy $E_{\text{bin}} \sim \alpha_s/r$ is the largest scale in the problem the free energy of $Q\bar{Q}$ pair is dominated by the singlet contribution and is equal to the zero temperature potential [18] up to the term $T \ln 9$ coming from the normalization constant. When the temperature is much larger than the binding energy, i.e. $\alpha_s/(rT) \ll 1$ the exponentials in Eq. (16) can be expanded and we get

$$F(r, T) = \frac{\alpha_s^2}{r^2 T}.$$  (17)

We see that despite no $T$-dependence of the potential in this limit, the free energy is strongly temperature dependent and it is very different from the potential. The complete next-to-leading order result can be found in Ref. [18].

Similarly, for the singlet correlator one can write

$$G_1(r, T) = \tilde{Z}_s(r) \langle S(r, \tau = 1/T) S^\dagger(r, 0) \rangle$$  (18)

At leading order $\tilde{Z}_s(r) = 1$ and $F_1(r, T) \approx V_s(r)$. At very high temperatures for $r \sim 1/m_D$ with $m_D = gT \sqrt{3/2}$ being the leading order Debye mass, the singlet and octet correlators can be calculated in the hard thermal loop (HTL) approximation [9]

$$F_1(r, T) = -\frac{4}{3} \frac{\alpha_s}{r} \exp(-m_D r) - \frac{4\alpha_s m_D}{3}, \quad F_8(r, T) = \frac{1}{6} \frac{\alpha_s}{r} \exp(-m_D r) - \frac{4\alpha_s m_D}{3}.$$  (19)

The singlet and octet free energies are gauge-independent at this order. At large distances the singlet and octet free energies approach a constant value $-\frac{4\alpha_s m_D}{3}$. This constant corresponds
to the leading order result for the free energy of two isolated static quarks $F_{\infty}$, which has been also calculated to next-to-leading order [19, 18]. The next-to-leading corrections are small and do not change the qualitative behavior of $F_{\infty}(T)$ which decreases with increasing temperatures. At leading order we have $(F_1(r,T) - F_{\infty}(T))/(F_8(r,T) - F_{\infty}(T)) = -8$.

The free energy of static $Q\bar{Q}$ pair was calculated at leading order long time ago [13, 15, 16]

$$F(r,T) = -\frac{1}{9} \frac{\alpha_s^2}{r^2} \exp(-2m_D r) - \frac{4\alpha_s m_D}{3}.$$  \hspace{1cm} (20)

The above expression can also be obtained by inserting Eqs. (19) into Eq. (13) and expanding the exponentials to order $\alpha_s^2$ thus confirming the validity of the decomposition and the partial cancellation of the singlet and octet contributions at leading order. The free energy was calculated at next-to-leading order for $r \approx 1/m_D$ [16] but the decomposition into singlet and octet contributions was not shown. Because of the partial cancellation of the singlet and octet contributions for $r \sim 1/m_D$ we expect that $F(r,T) - F_{\infty}(T) \ll F_1(r,T) - F_{\infty}(T)$ at sufficiently high temperatures. In summary, the partial cancellation of the singlet and octet contribution to the free energy happens both at short and long distances and leads to its strong temperature dependence.

4. Lattice results on the free energy and the singlet free energy

We calculated Polyakov loop correlators as well as singlet correlators on the lattice in 2+1 flavor QCD using Highly Improved Staggered Quark (HISQ) action [20] on $24^3 \times 6$ and $16^3 \times 4$ lattices. The strange quark mass $m_s$ was fixed to its physical value, while for the light quark masses we used $m_l = m_s/20$ that corresponds to the pion mass of about 160 MeV. The detailed choice of the lattice parameters is discussed in Ref. [20]. To calculate the singlet free energy we used the Coulomb gauge. The free energy and the singlet free energy have an additive divergent part that has to be removed by adding a normalization constant determined from the zero temperature potential. We used the normalization constants from Ref. [20]. The numerical results for the singlet free energy are shown in Fig. 1. At short distances the singlet free energy agrees with the zero temperature potential calculated in Ref. [20], while at large distances it approaches a constant value $F_{\infty}(T)$ equal to the excess free energy of two isolated static quarks. As the temperature increases the deviation from the zero-temperature potential shows up at shorter and shorter distances as the consequence of color screening. To explore the screening behavior in Fig. 1 we also show the combination $S(r,T) = r \cdot (F(r,T) - F_{\infty}(T))$ which we call the screening function. The screening function should decay exponentially. We indeed observe the exponential decay of this quantity at distances larger than $1/T$. Thus at high temperatures the behavior of the singlet free energy expected from the weak-coupling calculations seems to be confirmed by lattice QCD, at least qualitatively. Let us also mention that at high temperatures the behavior of the singlet free energy is similar to that observed in pure gauge theory [21, 22].

In Fig. 2 we show our results for the free energy of static $Q\bar{Q}$ pair as function of the distance at different temperatures. At short distances and low temperatures the free energy is expected to be dominated by the singlet contribution and we expect it to be equal to the zero temperature potential up to the term $T \ln 9$ coming from the normalization, see the discussion in the previous section. Therefore in the figure the numerical results have been shifted by $-T \ln 9$. Indeed, for the smallest temperature and the shortest distances $F(r,T) - T \ln 9$ is equal to the zero temperature potential shown as the dashed black line. At higher temperature $F(r,T)$ is very different from the zero-temperature potential. At large distance the free energy approaches a constant value $F_{\infty}(T)$ that decreases with increasing temperatures as expected (see discussions above). The temperature dependence of $F(r,T)$ is much larger than that of the singlet free energy. This is presumably due to the partial cancellation of the singlet and octet contribution discussed above. To verify this assertion we calculated $F(r,T) - F_{\infty}(T)$ using the numerical data for
Figure 3. The effective potential $V_{eff}(r, \tau)$ as function of $r T$ calculated for $48^3 \times 16$ lattice and $\beta = 7.5$ (left), $48^3 \times 12$ lattice and $\beta = 7.28$ (middle), and $48^3 \times 12$ lattice and $\beta = 7.5$ (right). The left, middle and right panels correspond to temperatures of 225 MeV, 249 MeV and 300 MeV respectively.

Figure 4. The effective potential $V_{eff}(r, \tau)$ as function of $\tau T$ calculated on $24^3 \times 6$ lattices for three different temperatures. The upper panels correspond to $r T = 1/2$ while the lower panels correspond to $r T = 1$. Filled and open diamonds correspond to the Coulomb gauge results at finite and zero temperature respectively.

$F_1(r, T) - F_{\infty}(T)$ and the leading order relation $(F_1(r, T) - F_{\infty}(T))/(F_8(r, T) - F_{\infty}(T)) = -8$. The corresponding results are shown in the right panel of Fig. 2. As one can see from the figure the numerical data for $F(r, T)$ are in reasonable agreement with the ones reconstructed from this procedure. The reconstruction works better with increasing temperature. Thus the expected cancellation of the singlet and octet contributions to the free energy of static $Q\bar{Q}$ pair seems to be confirmed by lattice calculations.

5. Wilson loops at non-zero temperature

In the previous section we considered correlation function of static $Q\bar{Q}$ pair evaluated at Euclidean time $t = 1/T$. These correlators are related to the free energy of static $Q\bar{Q}$ pair.
One can consider Wilson loops for Euclidean times $t < 1/T$ which have no obvious relation to the free energy of a static $Qar{Q}$ pair. Wilson loops at non-zero temperature have been first studied in Refs. [23, 24] in connection with heavy quark potential at non-zero temperature and a spectral decomposition has been conjectured for the Wilson loops

$$W(r, \tau) = \int_0^\infty \sigma(\omega, r, T)e^{-\omega \tau}. \quad (21)$$

At zero temperature the spectral function is proportional to sum of delta functions $\sigma(r, \omega) = \sum_n c_n \delta(E_n(r) - \omega)$ and thus the spectral decomposition is just the generalization of Eq. (14). At high temperatures the spectral function will be proportional to sum of smeared delta functions and the position and the width of the lowest peak are related to real and imaginary part of the potential, respectively [23, 24]. Motivated by this we calculated Wilson loops on finite temperature lattices in 2+1 flavor QCD using the HISQ action with physical strange quark mass and light quark masses $m_l = m_s/20$. We performed calculations using $48^3 \times 16$ and $48^3 \times 12$ at $\beta = 7.5$ as well as $48^3 \times 12$ lattices at $\beta = 7.28$ ($\beta = 10/g^2$). These correspond to temperatures 225 MeV, 300 MeV and 249 MeV respectively. In addition we performed calculations using $24^3 \times 6$ lattices for the lattice parameters discussed in the previous section. One of the problems in extracting physical information from the Wilson loops on the lattice is large noise associated with them. To reduce the noise smeared gauge fields are used in the spatial gauge transporters $U(\vec{x}, \vec{y}; \tau)$ that enter the Wilson loops. Alternatively one can fix the Coulomb gauge and omit the spatial gauge connections, i.e calculate correlation function of Wilson lines of extent $t < 1/T$. This method was used by the MILC collaboration [25] as well as by the HotQCD collaboration [20] to calculate the static potential at zero temperature. We used both approaches. If the Wilson loop is dominated by the ground state for some value of $\tau$ we may try to extract the static energies at non-zero temperature from single exponential fits or from the ratio of the Wilson loops at two neighboring time-slices separated by single lattice spacing $a$

$$aV_{\text{eff}}(r, \tau) = \ln W(r, \tau/a)/W(r, \tau/a + 1). \quad (22)$$

At zero temperature for sufficiently large $\tau$ the effective potential $V_{\text{eff}}(r, \tau)$ should reach a plateau. For non-zero temperature the situation is more complicated due to the backward propagating contribution. Lattice calculations of the Wilson loops at non-zero temperature in SU(3) gauge theory show exponential decay in $\tau$ but at distance around $\tau T = 1$ the Wilson loops increase again [23, 24]. Similar behavior was observed in 2+1 flavor QCD [26]. While no temporal boundary conditions are imposed on static quarks the gluon fields are periodic in time and this may give rise to a contribution that propagates backward in time. Such backward propagating contribution was also observed in the study of bottomonium spectral function in NRQCD at non-zero temperature [27]. In Fig. 3 we show the effective potential calculated on $N_\tau = 12$ and 16 lattices as function of the distance $r$ for different $\tau$. For $N_\tau = 16$ lattice that corresponds to the temperature of 225 MeV plateau seems to be reached for $\tau T \leq 1/2$. For these values of $\tau$ the backward propagating contribution is expected to be small. However, the statistical errors are very large for $rT > 1$. For $N_\tau = 12$ the effective potential does not reach a plateau for $rT \leq 1/2$. We do not consider larger values of $\tau$ because of the backward propagation contribution. We attempted to extract the static energy by removing the backward propagating contribution and fitting the remainder by single exponential. The results are shown in Fig. 3 as open symbols and agree quite well with $V_{\text{eff}}(r, \tau = 5/(12T))$. Thus it is reasonable to assume that the static energy is well approximated by $V_{\text{eff}}(r, \tau = 5/(12T))$ at these temperatures. For $rT > 1$ the static energy is larger than the free energy. These findings are in agreement with earlier findings based on $24^3 \times 6$ lattices [26].

The above analysis as well as the analysis performed in Ref. [26] is based on using the Coulomb gauge. It is important to check how the results depend on the choice of the static
meson operator. In addition it is interesting to study the onset of medium effects as function of $\tau$. We calculated rectangular Wilson loops using smeared gauge fields in the spatial gauge transporters. To reduce the noise we used several iterations of APE [28] and HYP [29] smearings. Namely, we used 5, 10 and 20 steps of APE smearing and 1, 2 and 5 steps of HYP smearing. The numerical results are shown in Fig. 4. As expected the HYP smearing is more efficient than APE smearing but for the coarse lattices used in our study the difference is not that large. One needs 5 steps of HYP smearing or 10 steps of APE smearing to get results comparable to the Coulomb gauge results. Fig. 4 shows that except for the lowest temperature and distances $rT \leq 1/2$ the static potential is affected by the medium. While $V_{\text{eff}}$ seems to reach a plateau at zero temperature no plateau is observed at finite temperature. Overall, the behavior of the Wilson loops with smeared spatial links is similar to the Coulomb gauge result if sufficient number of smearing steps is used.

6. Quarkonium spectral functions

Heavy meson correlation functions in Euclidean time $G(\tau,T)$ are related to the meson spectral functions $\sigma(\omega,T)$

$$G(\tau,T) = \int_0^\infty d\omega \sigma(\omega,T) \frac{\cosh(\omega(\tau - 1/(2T)))}{\sinh(\omega/(2T))}. \quad (23)$$

Attempts to reconstruct quarkonium spectral function from lattice QCD using the above equation and Maximum Entropy Method have been presented in Ref. [30, 31, 32]. In these studies it was concluded that charmonium ground state may survive in the deconfined medium up to the temperature 1.6 times the transition temperature or maybe even higher contrary to the expectations based on color screening. However, the reconstruction of meson spectral functions from Euclidean correlator is very difficult [33], and the spectral functions are also strongly modified by cutoff effects [34]. The suggested survival of quarkonium states in the deconfined medium is closely related to the weak temperature dependence of the Euclidean time quarkonium correlators [35]. Using potential models, quarkonium spectral functions have been calculated and it was shown that Euclidean correlation functions do not show significant temperature dependence even if bound states are dissolved due to the limited Euclidean time extent [36]. This seems to be confirmed by the study of spatial charmonium correlators which indicate the dissolution of the ground state for $T > 300$ MeV [37] as well as by the study of P-wave bottomonium correlators using lattice NRQCD [27], where larger values of the Euclidean time can be used.

Potential models can be related to pNRQCD. As the temperature increases the binding energy becomes smaller and eventually will be the smallest scale in the problem: $E_{\text{bin}} \ll \Lambda_{\text{QCD}} \ll T, m_D, 1/r$, and all the other scales in the problem can be integrated out [38]. In this case the potential will be equal to the static energy. The real part of the static energy can be estimated using lattice QCD. In Ref. [38] a phenomenological form based on lattice QCD calculations of the singlet free energy was used for the real part of the potential while for the imaginary part of the potential the hard thermal loop result [10] was used. The spectral functions calculated in this approach show that most quarkonia states melt at temperatures $T > 250$ MeV, while ground state bottomonium melts at temperature $T > 450$ MeV [38]. The estimates of the static energy obtained from Wilson loops and discussed in the previous section turn out to be very close to the phenomenological potential used in Ref. [38]. Therefore the above estimates of the maximal temperatures that permit the existence of quarkonium states still hold.

7. Conclusion

Color electric screening in high-temperature QCD can be studied using correlation functions of static mesons that go around the periodic Euclidean time direction. These are related to the free
energy of static quark anti-quark pair. Determination of quarkonium spectral functions from the meson correlation functions calculated on the lattice is very difficult. The study of Wilson loops at non-zero temperature offers the possibility to extract the potential that can be used in potential model calculations to extract the quarkonium spectral functions.

Acknowledgements

This work was supported by U.S. Department of Energy under Contract No. DE-AC02-98CH10886. Computations have been performed on BlueGene/L computers of the New York Center for Computational Sciences (NYCCS) at Brookhaven National Laboratory and on clusters of the USQCD collaboration in JLab and FNAL.

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