Hysteretic ac losses in a superconductor strip between flat magnetic shields

Y A Genenko and H Rauh

Institut für Materialwissenschaft, Technische Universität Darmstadt, 64287 Darmstadt, Germany

E-mail: hera@tgm.tu-darmstadt.de

Received 8 March 2010
Published 7 June 2010
Online at stacks.iop.org/SUST/23/075007

Abstract

Hysteretic ac losses in a thin, current-carrying superconductor strip located between two flat magnetic shields of infinite permeability are calculated using Bean’s model of the critical state. Exact numerical calculations and approximate analytical forms delineating the penetration of magnetic flux and the energy dissipated during a cycle of the ac transport current, per unit length of the strip, are derived. For the shields oriented parallel to the plane of the strip, the penetration of the self-induced magnetic field is found to be enhanced, with the analytical current dependence of the ac loss conditionally resembling that of an isolated superconductor slab; for the shields oriented perpendicular to the plane of the strip, the penetration of the self-induced magnetic field is found to be impaired, with the analytical current dependence of the ac loss conditionally duplicating that of a regular set of curved superconducting tapes extending longitudinally around a surface of cylindrical shape. Thus, hysteretic ac losses can strongly augment or, respectively, wane when the shields approach the strip.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The problem of reducing hysteretic ac losses has lately become a major issue for large-scale superconductor applications [1–3]. In planar superconductors such as thin superconductor strips, a promising way to curtail the dissipation of electromagnetic energy exploits the shielding effect of magnetically susceptible environments, thereby controlling the distributions of the transport current and the magnetic field [4–8]. The quest to minimize hysteretic ac losses in superconductor/magnet composites, on the other hand, arises naturally because of the wide practical use of soft-magnetic substrates for the fabrication of superconductor-coated tapes. Despite great effort directed at numerical simulations of composites of the above sort [9–12], a theoretical analysis which would allow simple estimates of the penetration of magnetic flux and the consequential hysteretic ac losses in thin superconductor strips for basic configurations like, e.g., bilayer superconductor/magnet heterostructures [13], has not come forth yet. Here, therefore, we present such calculations in the case of a magnetically shielded superconductor strip for two fundamental shielding geometries, namely bulk flat magnets oriented parallel or, respectively, perpendicular to the plane of the strip.

2. Theoretical model

We consider an infinitely extended type-II superconductor strip of width $2w$ and thickness $d \ll w$ limited by the range $-w \leq x \leq w$ and located between two infinitely extended, homogeneous soft magnets, the direction of translational invariance of this heterostructure being parallel to the $z$-axis of a Cartesian coordinate system $x$, $y$, $z$, with vertical or, respectively, horizontal distance $a$ between the surfaces of the magnets and the centre of the strip, as depicted in figure 1. The magnets are understood to reveal permeability $\mu \rightarrow \infty$; an idealization which has proven representative for real magnets with relative permeability exceeding about two hundred [14]. The strip is supposed to carry a longitudinal transport current that changes periodically with time, at fixed amplitude $I$, in the absence of an externally applied magnetic field. By virtue of the restriction concerning the dimensions of the strip, spatial variations of the current and the self-induced magnetic field on
current, changes sign as the current alternates between a length scale less than \(d\) may be ignored and, for mathematical convenience, the strip regarded as infinitesimally thin, enabling the physical state of the strip to be characterized by the sheet current \(J\) as a function of \(x\) alone.

Implementing Bean’s model of the critical state duly adapted to the geometry of the strip [15, 16], we assume that the dynamics of the magnetic flux is controlled by the field-independent critical sheet current \(J_c = dJ_c\) with the critical current density \(j_c\). As long as the amplitude of the ac transport current \(I\) stays below the maximum loss-free current \(I_c = 2wJ_c\), a flux-free region of half-width \(b < w\) prevails in the central part of the strip, \(-b \leq x \leq b\), where the normal component of the magnetic field \(H_n\) disappears, while in the marginal, flux-penetrated parts of the strip, \(-w \leq x \leq -b\) and \(b \leq x \leq w\), the sheet current \(J\) equals \(J_c\). Like the tangential component of the magnetic field, the sheet current is continuous over the width of the strip [14].

The scenario of entry and exit of magnetic flux in the presence of magnetic shields is essentially the same as for an unshielded strip [15, 16]: in particular, the distribution of the magnetic field along the strip, emerging during the gradual increase of the transport current from the virgin state of the strip up to the state associated with the maximum value of the current, changes sign as the current alternates between \(I\) and \(-I\). Accordingly, by resorting to a quasistatic approach, the energy dissipated during a cycle of the ac transport current, per unit length of the strip, amounts to

\[
U_{ac} = 8\mu_0 J_c \int_b^w dx \int_b^x d'x' H_n(x')
\]

with the vacuum permeability \(\mu_0\).

2.1. Longitudinal shielding geometry

Following previous analysis, for the bulk flat magnets oriented parallel to the plane of the strip as shown in figure 1(a), the normal component of the magnetic field in the flux-filled margins of the strip, \(-w \leq x \leq -b\) and \(b \leq x \leq w\), reads [14]

\[
H_n(x) = H_c \text{sgn}(x) \frac{\sqrt{u^2(x) - p^2}}{q^2 - p^2},
\]

where \(H_c = J_c/\pi\), apart from \(u(x) = \tanh(\pi x/2a)\) and \(p = \tanh(\pi b/2a)\) as well as \(q = \tanh(\pi w/2a)\). Herein, since the half-width of the flux-free zone \(b\) depends on the transport current \(I\) itself,

\[
p = \sqrt{1 - \cosh^2 \left( \frac{\pi w}{2a} \frac{I}{I_c} \right) \sech^2 \left( \frac{\pi w}{2a} \right)}.\]

The variation of the normalized component of the magnetic field \(H_n/H_c\) over the width of the strip, calculated from equation (2) for a set of values of the normalized vertical distance between the surfaces of the magnets and the centre of the strip \(a/w\), adopting the normalized current \(I/I_c = 0.8\), is displayed in figure 2. Evidently, while at large \(a/w\), the field profile virtually reproduces that of an isolated strip [15], it straightens and augments in strength while showing a reduced flux-free zone, as \(a/w\) abates, reminiscent of the field distribution in an isolated superconductor slab [17], for which the strip together with its magnetic mirror images effectively assembles into a regular stack of flat superconducting films [18, 19].

Substituting the normal component of the magnetic field, equation (2), into (1) and changing the variables \(x\) and \(x'\) to \(u = \tanh(\pi x/2a)\) and \(u' = \tanh(\pi x'/2a)\), respectively, yields the energy dissipated during a cycle of the ac transport current, per unit length of the strip, for the longitudinal shielding geometry,

\[
U_{ac} = U_0 \left( \frac{2\mu_0}{\pi w} \right)^2 \int_p^q \frac{du}{1-u^2} \int_p^u \frac{du'}{1-u'^2} \arctan \sqrt{\frac{u'^2-p^2}{q^2-p^2}}.
\]

where \(U_0 = 2\mu_0 I_c^2/\pi\). In the limit of the magnets situated close to the strip, \(d \ll a \ll w\), equation (4) may be approximated with high accuracy by the expression

\[
U_{ac} \cong U_0 \left[ \frac{\pi w}{12a} \left( \frac{l}{w} \right)^3 + \frac{\ln 2}{2} \left( \frac{l}{w} \right)^2 \right],
\]

Figure 1. Cross-sectional view of a superconductor strip of width \(2w\) (dark shading) located between two bulk flat magnets (light shading) extending infinitely in the \(z\)-direction of a Cartesian coordinate system \(x, y, z\), adapted to the strip for (a) the longitudinal shielding geometry and (b) the transverse shielding geometry. The definition of the distance \(a\) between the surfaces of the magnets and the centre of the strip is indicated for either magnet configuration.
introducing the flux penetration depth $l = w - b$, where $b$ is related to $p$ from equation (3) as given above. A basically equivalent representation uncovering the dependence on the current $I$, albeit confined to the range $d/w \leq a/w \leq I/I_c \leq 1 - a/w$, obtains from equation (5), since under these circumstances the approximations

$$p \cong 1 - \frac{1}{2} \exp\left(-\frac{\pi w}{a}\left(1 - \frac{I}{I_c}\right)\right),$$

$$q \cong 1 - 2 \exp\left(-\frac{\pi w}{a}\right), \quad b \cong a\left(\frac{2 \ln 2}{\pi}\right) + \left(1 - \frac{I}{I_c}\right)$$

hold, so that the simple universal form

$$U_{ac} \cong U_0\left(\frac{2a}{\pi w}\right)^2 f\left(\frac{\pi w}{2a}, \frac{1}{I_c}\right), \quad f(i) = \frac{1}{6}i^3 - \frac{(\ln 2)^2}{2}i$$

ensues. The cubic term herein, which dominates in the high-current regime, describes the hysteretic ac loss as for an isolated superconductor slab [17].

The dependence of the normalized hysteretic ac loss $U_{ac}/U_0$ on the normalized transport current $I/I_c$, calculated numerically from equation (4) for the above set of values of the normalized distance $a/w$, is portrayed in figure 3 (solid lines) and contrasted with dependencies due to equation (5) (dashed line) and, respectively, the cubic approximation of equation (7) (dotted line). This reveals that, while at large $a/w$, the ac loss is practically like for an unshielded strip [15], it increases substantially, as $a/w$ abates, displaying a current variation like for a regular stack of flat superconducting films [19], the predictions of equation (5) differing indiscernibly from the exact results within its limitation.

2.2. Transverse shielding geometry

Following previous analysis, for the bulk flat magnets oriented perpendicular to the plane of the strip as shown in figure 1(b), the normal component of the magnetic field in the flux-filled margins of the strip, $-w < x < -b$ and $b < x < w$, reads [14]

$$H_a(x) = H_c \text{sgn}(x) \text{arctanh}\left(\sqrt{\frac{\nu^2(x) - r^2}{s^2 - r^2}}\right),$$

where $H_c = J_c/\pi$, apart from $\nu(x) = \tan(\pi x/2a)$ and $r = \tan(\pi b/2a)$ as well as $s = \tan(\pi w/2a)$. Herein, since the half-width of the flux-free zone $b$ depends on the transport current $I$, respectively.

$$r = \sqrt{\cos^2\left(\frac{\pi w}{2a}\frac{1}{I_c}\right) \sec^2\left(\frac{\pi w}{2a}\right) - 1},$$

The variation of the normalized component of the magnetic field $H_a/H_c$ over the width of the strip, calculated from equation (8) for a set of values of the horizontal distance between the surfaces of the magnets and the edges of the strip, $c = a - w$, normalized by the half-width of the strip $w$, adopting the normalized current $I/I_c = 0.8$, is displayed in figure 4. Evidently, while at large $c/w$, the field profile virtually reproduces that of an isolated strip [15], it steepens and weakens in strength while showing an enlarged flux-free zone, as $c/w$ abates, reminiscent of the field distribution in a regular array of superconductor strips [19].

Substituting the normal component of the magnetic field, equation (8), into (1) and changing the variables $x$ and $x'$ to $v = \tan(\pi x/2a)$ and $v' = \tan(\pi x'/2a)$, respectively, yields the energy dissipated during a cycle of the ac transport current, per unit length of the strip, for the transverse shielding.
The dependence of the normalized hysteretic ac loss $U_{ac}/U_0$ on the normalized transport current $I/I_c$ is portrayed in figure 5 (solid lines) and contrasted with the dependence due to equation (13) (dashed line). This reveals that, while at large $c/w$, the ac loss is practically like for an unshielded strip [15], it decreases substantially, as $c/w$ abates, displaying a current variation like for a single gap in a flat superconducting sheet [21] or for a regular set of curved superconducting tapes extending longitudinally around a surface of cylindrical shape [22], the predictions of equation (13) differing indiscernibly from the exact results within its limitations.

### 3. Summary and conclusion

By resorting to a quasistatic approach, we have presented exact numerical calculations and approximate analytical forms delineating the penetration of magnetic flux and hysteretic ac losses in a thin, current-carrying type-II superconductor strip located between two flat magnetic shields. For the shields oriented parallel or, respectively, perpendicular to the plane of the strip, our results predict a strong increase or, respectively, decrease of the hysteretic loss, when the shields approach the strip. In the limit of the magnets situated close to the strip, the current dependence of the hysteretic loss follows that known for such diverse objects as a regular stack of flat superconducting films or a regular set of curved superconducting tapes extending longitudinally around a surface of cylindrical shape; a fact which demonstrates how basic superconductor/magnet heterostructures like the ones studied here may be used for complicated superconductor arrangements instead. The simple analytical forms derived on the assumption of infinite permeability, furthermore, can serve as guides to estimating hysteretic ac losses in practically relevant configurations involving magnetic shields of finite permeability too. This requires bearing in mind that the idealizations underlying the shielding geometries at hand imply overestimates of the redistribution of the magnetic flux due to the magnetic shielding effect, resulting in overestimates of the hysteretic loss as compared to realistic bilayer geometries and in underestimates of the hysteretic loss as compared to realistic transverse geometries.
References

[1] Gómory F, Šouc J, Vojenčiak M, Seiler E, Klinčok B, Ceballos J M, Pardo E, Sanchez A, Navau C, Farinon S and Fabbricatore P 2006 Supercond. Sci. Technol. 19 S60
[2] Malozemoff A P, Snitchler G and Mawatari Y 2009 IEEE Trans. Appl. Supercond. 19 3115
[3] Clem J R 2009 Phys. Rev. B 80 184517
[4] Majoros M, Glowacki B A and Campbell A M 2000 Physica C 334 129
[5] Genenko Y A 2002 Phys. Stat. Sol. (a) 189 469
[6] Genenko Y A and Rauh H 2007 Physica C 460 1264
[7] Aladyshkin A Y, Silhanek A V, Gillijns W and Moshchalkov V V 2009 Supercond. Sci. Technol. 22 053001
[8] Genenko Y A, Rauh H, Krüger P and Narayanan N 2009 Supercond. Sci. Technol. 22 055001
[9] Alamgir A K M, Gu Z and Han Z 2005 Physica C 432 153
[10] Gómory F 2006 Appl. Phys. Lett. 89 072506
[11] Gu Z, Alamgir A K M, Qu T and Han Z 2007 Supercond. Sci. Technol. 20 133
[12] Gómory F, Vojenčiak M, Pardo E and Šouc J 2009 Supercond. Sci. Technol. 22 034017
[13] Mawatari Y 2008 Phys. Rev. B 77 104505
[14] Genenko Y A, Snezhko A and Freyhardt H C 2000 Phys. Rev. B 62 3453
[15] Norris W T 1970 J. Phys. D: Appl. Phys. 3 489
[16] Brandt E H and Indenbom M 1993 Phys. Rev. B 48 12893
[17] Campbell A M and Evetts J E 1972 Critical Currents in Superconductors (London: Taylor and Francis)
[18] Mawatari Y 1996 Phys. Rev. B 54 13215
[19] Müller K H 1997 Physica C 289 123
[20] Genenko Y A 2004 Physica C 401 210
[21] Majoros M 1996 Physica C 272 62
[22] Mawatari Y 2009 Phys. Rev. B 80 184508