Analysis of the strong vertices of $\Sigma_cN^D^*$ and $\Sigma_bN^B^*$ in QCD sum rules

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(Dated: March 1, 2022)

The strong coupling constant is an important parameter which can help us to understand the strong decay behaviors of baryons. In our previous work, we have analyzed strong vertices $\Sigma_cN^D$, $\Sigma_cN^B$, $\Sigma_cN^D^*$, $\Sigma_cN^B^*$ in QCD sum rules. Following these work, we further analyze the strong vertices $\Sigma_cN^D^*$ and $\Sigma_bN^B^*$ using the three-point QCD sum rules under Dirac structures $\hat{\gamma}_\alpha \hat{p}$ and $\hat{\psi} \hat{p}_\alpha$. In this work, we first calculate strong form factors considering contributions of the perturbative part and the condensate terms $\langle \bar{q}q \rangle$, $\langle \bar{\psi}_s \pi GG \rangle$, and $\langle \bar{q}g_s \sigma Gq \rangle$. Then, these form factors are used to fit into analytical functions. According to these functions, we finally determine the values of the strong coupling constants for these two vertices $\Sigma_cN^D^*$ and $\Sigma_bN^B^*$.

PACS numbers: 13.25.Ft; 14.40.Lb

1 Introduction

In the past 20 years, we have witnessed the baryon spectrum been established step by step with the cooperative efforts from both experimentalists and theorists. Up to now, about 20 charmed baryon candidates have been discovered by different experimental collaborations. Besides, many bottom baryons, e.g. $\Lambda_b$, $\Xi_b$, $\Sigma_b$, $\Sigma_b^*$, and $\Omega_b$, have also been announced by CDF and LHCb collaborations. In 2017, LHCb Collaboration reported the observation of the doubly charmed baryon $\Xi^{++}$ in the $\Lambda^+_c K^- \pi^+ \pi^+$ mass spectrum, which has became a new motivation for researchers to devote themselves to studying the properties of these heavy baryons.

These charmed and bottom baryons, which contain at least a heavy quark, provide a unique system for testing models of quantum chromodynamics (QCD), the theory that describes the strong interaction. In other words, these special baryons can be looked as a particular laboratory for studying dynamics between light quarks and heavy ones, and also as an excellent ground for testing validity of the quark model and heavy quark symmetry. The properties of these baryons such as the mass spectrum, the magnetic moments, the strong, electromagnetic and weak decay behaviors have been studied with a variety of theoretical models. As an important parameter, the strong coupling constant can not only help us to know about the strong decay behaviors of baryons but also play
an essential role for understanding its inner structure. Thus, people calculated some of the strong coupling constants $g_{\Sigma_{c}^{*}}^{\phi}$, $g_{\Xi_{c}^{*}}^{\Xi_{c}^{*}}$, $g_{\Xi_{b}^{*}}^{\Xi_{b}^{*}}$, $g_{\Sigma_{c}^{*} \Xi_{c}^{*}}$, $g_{\Xi_{b}^{*} \Xi_{b}^{*}}$, $g_{\Lambda_{b}^{*} B^{*}}$, $g_{\Lambda_{c}^{*} B^{*}}$, $g_{\Lambda_{c}^{*} D^{*}}$, $g_{\Lambda_{c}^{*} D^{*}}$, $g_{\Xi_{b}^{*} N^{*}}$, and $g_{\Sigma_{c}^{*} N^{*}}$, etc.\[32–48\].

To calculate the strong coupling constant, we can adopt several theoretical models including perturbative and non-perturbative methods. The QCD sum rules, proposed by Shifman, Vainshtein, and Zakharov\[30\], connects hadron properties and QCD parameters\[31\]. It has been widely used to study the properties of the hadrons\[32–48\]. In our previous work, we have analyzed the strong vertices $\Sigma_{c}^{*} N D$, $\Sigma_{b}^{*} N B$, $\Sigma_{c} N D$ and $\Sigma_{b} N B$ in QCD sum rule framework\[27, 28\]. As a continuation of this work, we analyze the strong vertices $\Sigma_{c} N D^{*}$ and $\Sigma_{b} N B^{*}$ using the three-point QCD sum rules under the Dirac structures $\bar{q}p \gamma_{\alpha}$ and $\bar{q}g_{\alpha}$. This paper is organized as follows. After the Introduction, we present details of the analysis of vertices $\Sigma_{c} N D^{*}$ and $\Sigma_{b} N B^{*}$. In Sec.3, we present the numerical results and discussions. Finally, the paper ends with the Conclusion.

2 QCD sum rules for $\Sigma_{c} N D^{*}$ and $\Sigma_{b} N B^{*}$

In order to obtain the strong coupling constants of vertices $\Sigma_{c} N D^{*}$ and $\Sigma_{b} N B^{*}$, we write out the following three-point correlation function,

$$\Pi_{\alpha}(p, p', q) = i^{2} \int d^{4}x d^{4}y e^{-ip.x} e^{ip'.y} \langle 0 | T \left( J_{N}(y) J_{D^{*}[B^{*}]}^{\alpha}(0) \mathcal{T}_{\Sigma_{c}[\Sigma_{b}]}(x) \right) | 0 \rangle,$$

where $J_{N}$, $J_{D^{*}[B^{*}]}$ and $J_{\Sigma_{c}[\Sigma_{b}]}$ denote interpolating currents of $N$, $D^{*}[B^{*}]$ and $\Sigma_{c}[\Sigma_{b}]$, and $\mathcal{T}$ is the time ordered product. Currents are composite operators made of quark and gluon fields that can create the studied hadrons from vacuum. It has the same quantum numbers with these hadrons\[32, 49\]. In this paper, the interpolating currents are written as,

$$J_{\Sigma_{c}[\Sigma_{b}]}(x) = \epsilon_{ijk} \left( u^{T}(x) C \gamma_{\mu} d^{j}(x) \right) \gamma_{5} \gamma^{\mu} c[b]^{k}(x)$$

$$J_{N}(y) = \epsilon_{ijk} \left( u^{T}(y) C \gamma_{\mu} w^{j}(y) \right) \gamma_{5} \gamma^{\mu} d^{k}(y)$$

$$J_{D^{*}[B^{*}]}^{\alpha}(0) = \pi(0) \gamma_{\alpha} c[b](0) \quad (2)$$

In QCD sum rule framework, there is a region of $p$ where correlation function can be equivalently described at both hadron and quark sector. The former is called the phenomenological side and the latter is called QCD or operator product expansion (OPE) side. Matching these two sides of the sum rule, we can obtain information about hadron properties.

2.1 The phenomenological side

On the phenomenological side, we insert a complete set of intermediate hadronic states into the correlation $\Pi_{\alpha}(p, p', q)$. These intermediate states have the same quantum numbers as the current operators $J_{N}$, $J_{D^{*}[B^{*}]}$ and $J_{\Sigma_{c}[\Sigma_{b}]}$. Isolation of ground-state contributions results in the following
expression,

\[ \Pi_{\alpha}^{HAD}(p, p', q) = \frac{\langle 0| J_{N}^{\alpha}| p' \rangle \langle 0| J_{D^{*} \cdot [B^{*}]}^{\alpha}| D^{*} [B^{*}] (q) \rangle \langle \Sigma_{c}^{\alpha} | \Sigma_{b} \rangle (p), \mathcal{F}_{\Sigma_{c}^{\alpha}} | 0 \rangle}{(p^{2} - m_{\Sigma_{c}^{\alpha}}^{2}) (q^{2} - m_{N}^{2}) (\Sigma_{c}^{\alpha} - \Sigma_{b}^{\alpha}) (q^{2} - m_{D^{*} \cdot [B^{*}]}^{2})} \]

\[ \langle N(p') D^{*} [B^{*}] (q) | \Sigma_{c}^{\alpha} | \Sigma_{b} \rangle (p) \rangle + \cdots \tag{3} \]

Here, ellipsis denotes the contributions from higher resonances and continuum states. We substitute the matrix elements appearing in Eq. (3) with the following parameterized equations,

\[ \langle 0| J_{N}^{\alpha}| p', s' \rangle = \lambda_{N} u_{N}(p', s'), \]

\[ \langle 0| J_{D^{*} \cdot [B^{*}]}^{\alpha}| D^{*} [B^{*}] (q) \rangle = m_{D^{*} \cdot [B^{*}]} f_{D^{*} [B^{*}]} \varepsilon^{*}, \]

\[ \langle \Sigma_{c}^{\alpha} | \Sigma_{b} \rangle (p), \mathcal{F}_{\Sigma_{c}^{\alpha}} | 0 \rangle = \lambda_{\Sigma_{c}^{\alpha}} \Sigma_{b} \langle \mu_{\Sigma_{c}^{\alpha}} | (p, s) \rangle, \]

\[ \langle N(p') D^{*} [B^{*}] (q) | \Sigma_{c}^{\alpha} | \Sigma_{b} \rangle (p) \rangle = \varepsilon_{\beta} u_{N}(p', s') [g_{1} \gamma_{\beta} - g_{2} \frac{i \sigma_{\beta \nu}}{m_{\Sigma_{c}^{\alpha}} + m_{N}} q_{\nu}] u_{\Sigma_{c}^{\alpha}} | (p, s) \rangle. \tag{4} \]

In the hadron degrees of freedom, the correlation function \( \Pi_{\alpha}(p, p', q) \) is finally decomposed into the following different dirac structures,

\[ \Pi_{\alpha}^{HAD}(p, p', q) = \frac{m_{D^{*} \cdot [B^{*}]} f_{D^{*} [B^{*}]} \lambda_{N} \lambda_{\Sigma_{c}^{\alpha}}}{(p^{2} - m_{\Sigma_{c}^{\alpha}}^{2}) (q^{2} - m_{N}^{2}) (\Sigma_{c}^{\alpha} - \Sigma_{b}^{\alpha}) (q^{2} - m_{D^{*} \cdot [B^{*}]}^{2})} \]

\[ \times \left\{ (g_{1} + g_{2}) (m_{N} - m_{\Sigma_{c}^{\alpha}}) \hat{\gamma}_{\alpha} + \left( g_{1} + g_{2} \right) m_{\Sigma_{c}^{\alpha}} (m_{N} + m_{\Sigma_{c}^{\alpha}}) \hat{\gamma}_{\alpha} \right\} \]

\[- \left( g_{1} + g_{2} \right) \hat{\gamma}_{\alpha} \hat{p}_{\alpha} + \left( g_{1} + g_{2} \right) m_{\Sigma_{c}^{\alpha}} (m_{N} + m_{\Sigma_{c}^{\alpha}}) \hat{\gamma}_{\alpha} \hat{p}_{\alpha} \]

\[- 2 g_{1} \hat{p}_{\alpha} + 2 \left( g_{1} + g_{2} \right) m_{N} + m_{\Sigma_{c}^{\alpha}} \hat{\gamma}_{\alpha} \hat{p}_{\alpha} \]

\[- 2 \left( g_{1} m_{N} + g_{2} (m_{N} - m_{\Sigma_{c}^{\alpha}}) \right) \hat{p}_{\alpha} \]

\[ + \left[ g_{1} m_{\Sigma_{c}^{\alpha}} - m_{N}^{2} \right] \hat{q}_{\alpha} + \left[ g_{1} m_{\Sigma_{c}^{\alpha}} (m_{N} - m_{\Sigma_{c}^{\alpha}}) / q^{2} + g_{2} m_{N} + m_{\Sigma_{c}^{\alpha}} / q^{2} \right] \hat{q}_{\alpha} \]

\[ + \left[ (2 g_{1} + g_{2}) m_{\Sigma_{c}^{\alpha}} / q^{2} + g_{1} m_{\Sigma_{c}^{\alpha}} (m_{N} - m_{\Sigma_{c}^{\alpha}}) / q^{2} \right] q_{\alpha} \]

\[ + \cdots \tag{5} \]

If all criteria of the QCD sum rules are satisfied, each dirac structure in Eq.(5) can be used to carry out the calculation. It is true that people indeed had different choice about this problem in the similar work[19, 50, 51]. And these researches indicated that different Dirac structures can really lead to compatible results. For simplicity, we choose \( \hat{\gamma}_{\alpha} \hat{p}_{\alpha} \) and \( \hat{\gamma}_{\alpha} \hat{p}_{\alpha} \) Dirac structures to perform our analysis.

2.2 The OPE side

Considering all possible contractions of the quark fields with Wick’s theorem, we write the correla-
tation function as follows,

\[
\Pi_{\alpha}^{OPE}(p, p', q) = i^2 \int d^4x \int d^4y e^{-ip.x} e^{ip'.y} \epsilon_{abc} \epsilon_{ijk} \\
\times \left\{ \gamma_5 \gamma_\mu S_{d}^{ij}(y - x) \gamma_\nu C S_{u}^{jT}(y - x) C \gamma_\mu S_{u}^{h}(y) \gamma_\alpha S_{c}^{hT}(x - y) \gamma_\beta S_{u}^{h}(x) \right\} (x - y) \gamma_\gamma \gamma_5 \\
- \gamma_5 \gamma_\mu S_{d}^{ij}(y - x) \gamma_\nu C S_{u}^{T}(y - x) C \gamma_\mu S_{u}^{h}(y) \gamma_\alpha S_{c}^{hT}(x - y) \gamma_\beta S_{u}^{h}(x) \right\} \right\} (6)
\]

Here, \( S_{q[i]} \) stands for up- and down-quark, or charm- and bottom-quark propagators which will be replaced by the following propagators \[17\] [48].

\[
S_{u[d]}^{mn}(x) = \frac{i}{2\pi^2 x^4} \delta_{mn} - \frac{m_{u[d]}(y)}{4\pi^2 x^2} \delta_{mn} - \frac{g_{A} G_{A}^{\alpha}}{32\pi^2 x^2} \left\{ f_{\alpha} \right\} + \cdots ,
\]

\[
S_{c[b]}^{mn}(x) = \frac{i}{2(2\pi)^4} \int d^4 k e^{-ik.x} \left\{ \delta_{mn} - \frac{g_{A} G_{A}^{\alpha}}{4} \right\} \left\{ \frac{m_{c[b]}(y)}{k - m_{c[b]}(y)} + (k + m_{c[b]}(y)) \sigma_{\alpha}\beta \right\} \\
\times \left\{ \frac{k^2 + m_{c[b]}(y)}{(k^2 - m_{c[b]}(y))^2} \right\} + \cdots \right\} (7)
\]

After a lengthy derivation, which need us to carry out the process of Fourier transformation, Feynman parametrization etc, we can obtain the same Dirac structures as the phenomenological side in Eq.(5). One can consult reference 27 for technical details of these processes. For each Dirac structure, the correlation function can be decomposed into two parts, perturbative term and non-perturbative term,

\[
\Pi_{\alpha}^{OPE} = \Pi_{\alpha}^{pert} + \Pi_{\alpha}^{non-pert} \]

where the latter is composed of condensate terms, and \( i \) stands for different Dirac structure. Using dispersion relation, the correlation function for a special Dirac structure can be written as,

\[
\Pi_{\alpha}^{OPE}(q^2) = \int_{s_1}^{s_0} ds \int_{u_1}^{u_0} du \rho_{\alpha}^{pert}(s, u, q^2) + \rho_{\alpha}^{non-pert}(s, u, q^2) \\
\times \left\{ \frac{m_u(m_b x - m_u y) - m_d(m_b x + m_u(2x + y - 2))}{x + y - 1} \right\} dy, \]

(10)

Here, \( \rho_{\alpha}^{pert/\rho_{\alpha}^{non-pert}} \) is spectral density which is obtained from the imaginary part of correlation function. During these derivations, we set \( s = p^2, u = p'^2 \) and \( q = p - p' \) in the spectral densities. For dirac structures \( \gamma \gamma \gamma \alpha \) and \( \gamma \gamma \gamma \alpha \), its perturbative term are written as,

\[
\rho_{\gamma \gamma \gamma \alpha}^{pert}(s, u, q^2) = \frac{3}{32\pi^4} \int_0^1 dx \int_0^{1-x} \left\{ \frac{m_u(m_b x - m_u y) - m_d(m_b x + m_u(2x + y - 2))}{x + y - 1} \right\} \theta[H(s, u, q^2)] dy,
\]

\[
\rho_{\gamma \gamma \gamma \alpha}^{pert}(s, u, q^2) = \frac{3}{32\pi^4} \int_0^1 dx \int_0^{1-x} \left\{ \frac{2[m_b x + y(m_u(2x + y - 3) - 3m_d(x + y - 1))]}{x + y - 1} \right\} \theta[H(s, u, q^2)] dy,
\]

I

(11)

(12)
where $H[s, u, q^2] = x(m^2_{c[b]} - q^2 y) + sx(x + y - 1) + uy(x + y - 1)$ and $\Theta$ stands for a unit-step function. Considering these limits, the integral limit for parameter $q$ where $\delta = \frac{(s+u-\Delta^2)}{2u}$ and $\Delta = [(s + u - q^2)^2 - 4su]x^2 - 2u[(s + u - q^2) + 2m^2_{c[b]}]x + 4su + u^2$

For non-perturbative terms, its spectral densities are written as,
\[
\rho_{\text{non-pert}}^{\text{non-pert}}(s, u, q^2)
\]
\[
= \frac{\langle \gamma q \rangle}{64\pi^2} \int_0^1 \left\{ -16\left[ m_d + m_u \right] (x + y - 1) \right\} dx
- \frac{\langle \gamma q \rangle}{16\pi} \left\{ \frac{[m_d - m_u] [m_b (m_u - 2m_b) + 2q^2]}{(m^2_b - q^2)^2} \Theta[u] - \frac{2m_am_u^2}{m^2_b - q^2} \delta[u] \right\}
\]
\[
+ \frac{\langle \gamma q \rangle}{16 \times 64\pi^2} \int_0^1 \left\{ 4\frac{u_m (m_u - m_d) (3x^3 - 2x^2)}{\sqrt{x} (u + 2y - 1) + (s - q^2)^2} \right\} dx
- \frac{m^2_0 \langle \gamma q \rangle}{64 \times 4\pi^2} \left\{ \frac{8 \left[ m_d [m_b (6m_d - 3m_u) + 2m_u (m_u - m_d)] + 4m^2_b q^2 [m_u - m_d] + q^4 (m_d - m_u) \right]}{(m^2_b - q^2)^4} \Theta[u] \right\}
\]
\[
- \frac{m^2_0 \langle \gamma q \rangle}{64 \times 4\pi^2} \left\{ \frac{8 \left[ (m^2_b - q^2) (6m_d - 3m_u) + 2m_am_u + m_b m_u (m_d - m_u) \right]}{3(m^2_b - q^2)^3} \Theta[u] \right\}
\]
\[
- \frac{m^2_0 \langle \gamma q \rangle}{64 \times 4\pi^2} \left\{ \frac{8 \left[ 9m^2_d [2m_d - m_u] + 3m^2_b m_u [m_d - m_u] + m_b m_u [m_d - m_u] [2s + q^2] \right]}{3(m^2_b - q^2)^3} \right\} \delta[u]
\]

where $\delta$ stands for Delta function.

### 3 The results and discussions

To calculate strong form factor, we match Eq.(10) with the hadronic representation Eq.(5), invoking the quark-hadron duality. After that, we make the change of variables $p^2 \rightarrow -P^2$, $p'^2 \rightarrow -P'^2$,
$q^2 \to -Q^2$ and perform a double Borel transformation in $P^2$, $P'^2$, introducing two Borel parameters $M_1$ and $M_2$. After these preformation, the strong form factors can be written as,

$$g_{2\Sigma_c N D^* [\Sigma_b N B^*]}(Q^2) = \frac{(m_N + m_{\Sigma_c} [\Sigma_b]) (Q^2 + m_{D^* [B^*]}^2)}{2m_{D^* [B^*]} f_{D^* [B^*]} \lambda_N \lambda_{\Sigma_c} [\Sigma_b]} \ e^{-\frac{m_N^2}{M_1^2}} e^{-\frac{m_{D^* [B^*]}^2}{M_2^2}}$$

$$\times \int_{(m_c [u] + m_u + m_d)^2}^{u_0} ds \int_{(2m_u + m_d)^2}^{u_0} du \left[ \rho_{\#_{\Sigma_c} \alpha}^{pert} (s, u, Q^2) + \rho_{\#_{\Sigma_c} \alpha}^{non-pert} (s, u, Q^2) \right] e^{-\frac{m_N^2}{M_1^2}} e^{-\frac{m_{D^* [B^*]}^2}{M_2^2}}$$

(15)

$$g_{1\Sigma_c N D^* [\Sigma_b N B^*]}(Q^2) + g_{2\Sigma_c N D^* [\Sigma_b N B^*]}(Q^2) = \frac{Q^2 + m_{D^* [B^*]}^2}{m_{D^* [B^*]} f_{D^* [B^*]} \lambda_N \lambda_{\Sigma_c} [\Sigma_b]} \ e^{-\frac{m_N^2}{M_1^2}} e^{-\frac{m_{D^* [B^*]}^2}{M_2^2}}$$

$$\times \int_{(m_c [u] + m_u + m_d)^2}^{u_0} ds \int_{(2m_u + m_d)^2}^{u_0} du \left[ \rho_{\#_{\Sigma_c} \alpha}^{pert} (s, u, Q^2) + \rho_{\#_{\Sigma_c} \alpha}^{non-pert} (s, u, Q^2) \right] e^{-\frac{m_N^2}{M_1^2}} e^{-\frac{m_{D^* [B^*]}^2}{M_2^2}}$$

(16)

In order to eliminate the contributions from excited and continuum states at OPE side, two continuum threshold parameters, $s_0$ and $u_0$, are adopted as the upper limits of integrals in Eqs.(15) and (16). Commonly, the values of these parameters are employed as $s_0 = (m_i + \Delta_i)^2$ and $u_0 = (m_o + \Delta_o)^2$, where $m_i$ and $m_o$ are ground state masses of the in-coming and out-coming baryons. In addition, the values of $s_0$ and $u_0$ in general are expected to be close to the mass squared of the first excited state of these in-coming and out-coming baryons, which will lead $\Delta_i$ and $\Delta_o$ to be about about 0.3GeV$^2 \sim 0.5$GeV$^2$. As for the other parameters in Eqs.(15) and (16), e.g. the masses of the hadrons and the quarks, the decay constants, the vacuum condensates, their values are all listed in Table 1.

Physical properties extracted from sum rules must be independent of Borel parameters $M_1^2$ and $M_2^2$. The assumption is that there exist a region for these parameters, called Borel window in which two sides have a overlap and information on the lowest state can be extracted. Minimum and maximum values for the Borel window can be determined according to two criterion of QCD sum rules, pole dominance and OPE convergence. That is to say, pole contribution should be as large as possible.
comparing with contributions of higher and continuum states. Meanwhile, we should also ensure OPE convergence and the stability of our results. After a comprehensive consideration, the continuum threshold parameters are chosen to be $u_0 = 2.07 GeV^2$ and $s_0 = 8.72 GeV^2 [39.90 GeV^2]$ for vertex $\Sigma c N D^* \Sigma_b NB^*$. Besides, the Borel windows that we choose are listed in Figs.1-8. From these figures we can see the weak dependence of the results on Borel parameter.

These results are obtained in deep space-like region $q^2 \to -\infty$, where the intermediate mesons $D^*$ and $B^*$ are off-shell. In order to obtain strong coupling constants, we must extrapolate these results into deep time-like region. This extrapolation to deep time-like region is mode-dependent, thus there are no specific expressions for the dependence of the strong form factors on $Q^2$. Our analysis indicates that this dependence can be appropriately fitted into the following exponential function,

$$g_{\Sigma c N D^* \Sigma_b NB^*}(Q^2) = A exp[BQ^2]$$

(17)

The fitted results for parameters $A$ and $B$ in this equation are listed in Table II. In Figs.9-12, we also
The strong form factors $g_{1\Sigma_c,NB^*}$ and $g_{2\Sigma_c,NB^*}$ on Borel parameter $M^2_1$.

show the dependence of the strong form factors on $Q^2$ for the QCD sum rules and the corresponding fitting results, in which it is marked as Central value and Fitted curve of Central value. The values of the strong coupling constants can be obtained from the fitting function at $Q^2 = -m^2_{D^*[B^-]}$, which are

\begin{align*}
g_{1\Sigma_c,ND^*}(Q^2 = -m^2_{D^*}) & = 13.69 \pm 2.92 \\
g_{2\Sigma_c,ND^*}(Q^2 = -m^2_{D^*}) & = 15.34 \pm 3.19 \\
g_{1\Sigma_c, NB^*}(Q^2 = -m^2_{B^*}) & = 2.93 \pm 0.75 \\
g_{2\Sigma_c, NB^*}(Q^2 = -m^2_{B^*}) & = 3.61 \pm 0.82
\end{align*}

The errors appearing in these above results come from the uncertainties of the fitting parameters $\delta A$ and $\delta B$ which are also listed in Table II. Besides, uncertainties of results coming from input parameters can theoretically be estimated with uncertainty transfer formula $\delta = \sqrt{\sum_i (\frac{\partial f}{\partial x_i})^2 (x_i - \bar{x}_i)^2}$, where $f$ denotes the strong form factors in Eqs.(15) and (16), and $x_i$ denotes input parameters $m_{\Sigma^*}, m_{\Sigma_c}, m_b, m_c, \lambda_{\Sigma^*}, \lambda_{\Sigma_c}, \langle \bar{q} q \rangle, \ldots$. For simplicity, the values of the upper and lower limits of the
strong form factors are approximated by taking \( f_{\text{upper(lower)}} = f(\pi_i \pm \Delta x_i) \), which are marked as Upper bound and Lower bound in Figs.9-12. After these approximations, these results are also fitted into the same kind of analytical function with Eq.(17) and are also extrapolated into the physical regions in order to get the uncertainties of the strong coupling constants. Finally, we obtain the strong coupling constants,

\[
\begin{align*}
g_1 \Sigma_{c ND}^*(Q^2 = -m_{D^*}^2) &= 13.69^{+62.92}_{-6.10} \pm 2.92 \\
g_2 \Sigma_{c ND}^*(Q^2 = -m_{D^*}^2) &= 15.34^{+7.17}_{-6.77} \pm 3.19 \\
g_1 \Sigma_{b NB}^*(Q^2 = -m_{B^*}^2) &= 2.93^{+10.08}_{-1.77} \pm 0.75 \\
g_2 \Sigma_{b NB}^*(Q^2 = -m_{B^*}^2) &= 3.61^{+12.85}_{-2.68} \pm 0.82
\end{align*}
\]

where the first part of the uncertainties in the results comes from the input parameters, \( m_{\Sigma^*_c}, m_{\Sigma^*_b}, m_{\pi}, m_{\Sigma^*_c}, \lambda_{\Sigma^*_c}, \lambda_{\Sigma^*_b}(\bar{q}q), \cdots \) and the second part originates from the fitting parameters.

4 Conclusion
TABLE II: Input parameters used in this analysis.

|                | A               | B               |
|----------------|-----------------|-----------------|
| $g_{\Sigma_c N D^*}(Q^2)$ | 18.95 ± 3.82    | 0.08069 ± 0.01737 |
| $g_{\Sigma_b N B^*}(Q^2)$ | 21.18 ± 4.16    | 0.08014 ± 0.01695 |
| $g_{\Sigma_b N B^*}(Q^2)$ | 13.52 ± 1.2     | 0.05393 ± 0.00843 |
| $g_{\Sigma_c N D^*}(Q^2)$ | 15.45 ± 1.23    | 0.05125 ± 0.00749 |

In this paper, we perform a systematic analysis on strong vertices $\Sigma_c N D^*$ and $\Sigma_b N B^*$ with QCD sum rules. We firstly calculate strong form factors in space-like regions ($q^2 < 0$). Then, the form factors are fitted into analytical functions which are used to extrapolate into time-like regions ($q^2 > 0$) to obtain strong coupling constants. These results will be valuable for studying the strong decay behavior of the charmed and bottom baryons in the future.

Acknowledgment

This work has been supported by the Fundamental Research Funds for the Central Universities, Grant Number 2016MS133, Natural Science Foundation of HeBei Province, Grant Number A2018502124.

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