Semileptonic $B$ to Scalar meson Decays in the Standard Model with Fourth Generation

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Abstract

We study the effects of the fourth generation of quarks on the total branching ratio and the lepton polarizations in $\bar{B}_0 \rightarrow K^*_0(1430)\ell^+\ell^-$ ($\ell = \mu, \tau$) decay. Taking fourth generation quark mass $m_{t'}$ of about 400 to 600 GeV with the mixing angle $|V_{t'b'}V_{t's'}|$ in the range $(0.05 - 1.4) \times 10^{-2}$ and using the phase to be $80^\circ$, it is found that the branching ratio and lepton polarizations are quite sensitive to these fourth generation parameters. In future the experimental study of this decay will give us an opportunity to study new physics effects, precisely, to search for the fourth generation of quarks ($t', b'$) in an indirect way.

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I. INTRODUCTION

The CP violation through the CKM paradigm \[1,2\] in the SM has been extremely successful in explaining most of the experimental data. However in the past few year a lot more data were accumulated from the two B-factories and also the improvement in the accuracy of some of the theoretical calculations led us to understand that several of the experimental results are difficult to explain with the SM with three generations (SM3) \[3–5\]. This leads us the think about some beyond the SM3 scenarios and among them the simplest one is the Standard Model with fourth generation (SM4). In this model the SM is enlarged by a complete sequential 4th family of quarks and leptons: a new \((t', b')\) and \((\nu', \ell')\) which are the heavy chiral doublets. Review and the summary statements of SM4 can be found in \[6\].

During the last years, a number of analysis were published with the goal of investigating the impact of the existence of a fourth generation on Higgs physics \[7–9\], electroweak precision tests \[7, 9–13\], renormalisation group effects \[14, 15\] and flavor physics \[16–31\]. In addition to this the detailed analysis of supersymmetry in the presence of a fourth generation have recently been performed in \[32, 33\].

In flavor Physics the importance of SM4 is in the Flavor Changing Neutral Currents (FCNC) which lies in the fact that on one hand it contains much fewer parameters than other New Physics (NP) scenarios like the Littlest Higgs model with T-parity (LHT), Randall-Sundrum (RS) models or the general Minimal Supersymmetric Standard Model (MSSM) and on the other hand there is the possibility of having simultaneously sizable NP physics effects in the \(K\) and \(B\) systems compared to above mentioned NP models. Moreover, having the same operator structure as of the SM3, it implies that the non-perturbative uncertainties in the SM4 are at the same level as in the SM3. Recently, Buras et al. \[29\] have performed a detailed analysis of non-Minimal Flavor Violating (MFV) effects in the \(K\), \(B_d\) and \(B_s\) system in the SM4 where they paid particular attention to the correlation between flavor observables and addressed with in this framework a number of anomalies present in the experimental data. In addition to this they have also studied the \(D^0 - \bar{D}^0\) mixing in the SM4 where they calculated the size of allowed CP violation which is found at the observable level well beyond anything possible with CKM dynamics \[34\].

In this work we investigate the possibility of searching for NP in the \(\bar{B}_0 \rightarrow K^*_0(1430) l^+ l^- (l = \mu, \tau)\), where \(K^*_0(1430)\) is a scalar meson, using the fourth generation of quarks \((t', b')\). At quark level this decay is governed by \(b \rightarrow s\) transitions which are in forefront of indirect investigation of fourth generation. In these FCNC transitions the fourth generation quark \((t')\), like \(u, c, t\) quarks, contributes at loop level. Therefore, it modifies the corresponding Wilson coefficients which may have effects on branching ratio and lepton polarization asymmetries of \(\bar{B}_0 \rightarrow K^*_0(1430) l^+ l^-\) decay. Now the main job of investigating the semi-leptonic \(B\) meson decay is to properly evaluate the hadronic matrix elements for \(B \rightarrow K^*_0(1430)\), namely the transition form factors, which are governed by the non-perturbative QCD dynamics. Several methods exist in the literature to deal with this problem, such as simple quark model, light-front approach, QCD Sum Rules (QCDSR), light-cone QCD sum rules (LCSR) and perturbative QCD factorization approach (PQCD).

In our numerical analysis for \(\bar{B}_0 \rightarrow K^*_0(1430)\) decays, we shall use the results of the form factors calculated by LCSR approach in Ref. \[35\], and explore the effects of fourth generation parameters \((m_{t'}, V_{t'b} V_{t's})\) on branching ratios and lepton polarization asymmetries. By incorporating the recent constraints \(m_{t'} = 400 – 600\) GeV and \(V_{t'b} V_{t's} = (0.05 – 1.4) \times 10^{-2}\) \[28\] our results show that the decay rates are quite sensitive to these parameters. Now the forward-backward asymmetry is zero in the SM3 for these decays because of the absence of the scalar type coupling, and it remains zero in SM4 as there is no new operator in addition to the SM3 operators. The hadronic uncertainties associated with the form factors and other input parameters have negligible effects on the lepton polarization asymmetries and this makes them the efficient way in establishing the NP. Here, we have also studied these asymmetries in the SM4 found that the effects of fourth generation parameters are quite significant in some regions of parameter space of SM4.

The paper is organized as follows. In Sec. II, we present the effective Hamiltonian for the semileptonic decay \(\bar{B} \rightarrow K^*_0 l^+ l^-\) Section III contains the parameterizations and numbers of the form factors for the said decay using the LCSR approach. In Sec. IV we present the basic formulas of physical observables like decay rates and polarization...
asymmetries of final state lepton for the said decay. Section V is devoted to the numerical analysis where we study the sensitivity of these physical observables on fourth generation parameter \((m_{t'}, V_{tb}^* V_{ts})\). The main results are summarized in Sec. VI.

II. EFFECTIVE HAMILTONIAN

At quark level the decay \(B \rightarrow K_0^*(1430)l^+l^-\) is governed by the transition \(b \rightarrow sl^+l^-\) for which the effective Hamiltonian can be written as

\[
H_{\text{eff}} = -\frac{4GF}{\sqrt{2}} V_{tb}^* V_{ts} \sum_{i=1}^{10} C_i(\mu) O_i(\mu),
\]

where \(O_i(\mu) (i = 1, \ldots, 10)\) are the four-quark operators and \(C_i(\mu)\) are the corresponding Wilson coefficients at the energy scale \(\mu\) and the explicit expressions of these in the SM3 at NLO and NNLL are given in [36–46]. The operators responsible for \(B \rightarrow K_0^*(1430)l^+l^-\) are \(O_7, O_9\) and \(O_{10}\) and their form is given by

\[
\begin{align*}
O_7 &= \frac{e^2}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu}P_R b) F^{\mu\nu}, \\
O_9 &= \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu l), \\
O_{10} &= \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu\gamma_5 l),
\end{align*}
\]

with \(P_{L,R} = (1 \pm \gamma_5)/2\). Now, the fourth generation comes into the play just in the same way as the three generation SM, i.e. the full set of operators remains the same as in the SM3. Therefore, the effect of fourth generation displays itself by changing the values of Wilson coefficients \(C_7(\mu), C_9(\mu)\) and \(C_{10}\) via the virtual exchange of fourth generation up-type quark \(t'\) which then takes the form;

\[
\lambda_tC_i \rightarrow \lambda_tC_i^{SM} + \lambda_{t'}C_i^{new},
\]

where \(\lambda_f = V_{tb}^* V_{ts}\) and the explicit forms of the \(C_i\)'s can be obtained from the corresponding expressions of the Wilson coefficients in SM3 by substituting \(m_t \rightarrow m_{t'}\). By adding an extra family of quarks, the CKM matrix of SM3 is extended by another row and column which now becomes \(4 \times 4\). The unitarity of which leads to

\[
\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0.
\]

Since \(\lambda_u = V_{ub}^* V_{us}\) has a very small value compared to the others, therefore, we will ignore it. Then \(\lambda_t \approx -\lambda_c - \lambda_{t'}\) and from Eq. (3) we have

\[
\lambda_tC_i^{SM} + \lambda_{t'}C_i^{new} = -\lambda_c C_i^{SM} + \lambda_{t'} \left(C_i^{new} - C_i^{SM}\right).
\]

One can clearly see that under \(\lambda_{t'} \rightarrow 0\) or \(m_{t'} \rightarrow m_t\) the term \(\lambda_{t'} \left(C_i^{new} - C_i^{SM}\right)\) vanishes which is the requirement of GIM mechanism. Taking the contribution of the \(t'\) quark in the loop the Wilson coefficients \(C_i\)'s can be written in the following form

\[
\begin{align*}
C_7^{\text{tot}}(\mu) &= C_7^{SM}(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_7^{new}(\mu), \\
C_9^{\text{tot}}(\mu) &= C_9^{SM}(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_9^{new}(\mu), \\
C_{10}^{\text{tot}} &= C_10^{SM} + \frac{\lambda_{t'}}{\lambda_t} C_{10}^{new},
\end{align*}
\]

where we factored out \(\lambda_t = V_{tb}^* V_{ts}\) term in the effective Hamiltonian given in Eq. (1) and the last term in these expressions corresponds to the contribution of the \(t'\) quark to the Wilson Coefficients. \(\lambda_{t'}\) can be parameterized as:

\[
\lambda_{t'} = |V_{t'b}V_{ts}| e^{i\phi_{ts}}.
\]
In terms of the above Hamiltonian, the free quark decay amplitude for $b \rightarrow s l^+ l^-$ in SM4 can be derived as:

$$
\mathcal{M}(b \rightarrow s l^+ l^-) = -\frac{G_F a_{ql}}{\sqrt{2} \pi} V_{tb} V_{ts}^* \left\{ C_9^{\text{tot}}(s \gamma_\mu P_L b)(\bar{l} \gamma_\mu l) + C_1^{\text{tot}}(s \gamma_\mu P_L b)(\bar{l} \gamma_\mu \gamma_5 l) - 2m_b C_7^{\text{tot}}(\bar{s} \sigma_{\mu \nu} q^\nu P_R b)(\bar{l} \gamma_\mu l) \right\},
$$

(7)

where $q^2$ is the square of momentum transfer. The operator $O_{10}$ cannot be induced by the insertion of four-quark operators because of the absence of the $Z$-boson in the effective theory. Therefore, the Wilson coefficient $C_{10}$ does not renormalize under QCD corrections and hence it is independent on the energy scale. In addition to this, the above quark level decay amplitude can receive contributions from the matrix element of four-quark operators, $\sum_{i=1}^{6} (l^+ l^- s) O_i(b)$, which are usually absorbed into the effective Wilson coefficient $C_9^{\text{SM}}(\mu)$ and can usually be called $C_9^{\text{eff}}$, that one can decompose into the following three parts

$$
C_9^{\text{SM}} = C_9^{\text{eff}}(\mu) = C_9(\mu) + Y_{SD}(z, s') + Y_{LD}(z, s'),
$$

where the parameters $z$ and $s'$ are defined as $z = m_c/m_b$, $s' = q^2/m_b^2$. $Y_{SD}(z, s')$ describes the short-distance contributions from four-quark operators far away from the $c\bar{c}$ resonance regions, which can be calculated reliably in the perturbative theory. The long-distance contributions $Y_{LD}(z, s')$ from four-quark operators near the $c\bar{c}$ resonance cannot be calculated from first principles of QCD and are usually parameterized in the form of a phenomenological Breit-Wigner formula making use of the vacuum saturation approximation and quark-hadron duality. We will neglect the long-distance contributions in this work because of the absence of experimental data on $B \rightarrow J/\psi K_0^*(1430)$. The manifest expressions for $Y_{SD}(z, s')$ can be written as

$$
Y_{SD}(z, s') = h(z, s')(3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + C_5(\mu) + C_6(\mu))
- \frac{1}{2} h(1, s')(4C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu))
- \frac{1}{2} h(0, s')(C_5(\mu) + 3C_4(\mu)) + \frac{2}{9} (3C_3(\mu) + C_4(\mu) + C_5(\mu) + C_6(\mu)),
$$

(8)

with

$$
h(z, s') = \frac{8}{9} \ln z + \frac{8}{27} + \frac{4}{9} x - \frac{2}{9} (2 + x) |1 - x|^{1/2} \left\{ \ln \frac{\sqrt{1 - x} + 1}{\sqrt{1 - x} - 1} - i \pi \right\} \quad \text{for } x \equiv 4z^2/s' < 1,
- \frac{2}{9} \arctan \frac{1}{\sqrt{1 - x}} \quad \text{for } x \equiv 4z^2/s' > 1,
$$

$$
h(0, s') = \frac{8}{27} - \frac{8}{9} \ln \frac{m_b}{\mu} - \frac{4}{9} \ln s' + \frac{4}{9} i \pi.
$$

(9)

Apart from this, the non-factorizable effects from the charm loop can bring about further corrections to the radiative $b \rightarrow s\gamma$ transition, which can be absorbed into the effective Wilson coefficient $C_7^{\text{eff}}$. Specifically, the Wilson coefficient $C_7^{\text{eff}}$ is given by

$$
C_7^{\text{SM}}(\mu) = C_7^{\text{eff}}(\mu) = C_7(\mu) + C_{b \rightarrow s\gamma}(\mu),
$$

with

$$
C_{b \rightarrow s\gamma}(\mu) = i \alpha_s \left[ \frac{2}{9}\eta^{14/3} (G_1(x_t) - 0.1687) - 0.03 C_2(\mu) \right],
$$

(10)

$$
G_1(x_t) = \frac{x_t(x_t^2 - 5x_t - 2)}{8(x_t - 1)^3} + \frac{3\eta^2 \ln^2 x_t}{4(x_t - 1)^4},
$$

(11)

where $\eta = \alpha_s(m_W)/\alpha_s(\mu)$, $x_t = m_t^2/m_W^2$, $C_{b \rightarrow s\gamma}$ is the absorptive part for the $b \rightarrow s\bar{c} \rightarrow s\gamma$ rescattering and we have dropped out the tiny contributions proportional to CKM sector $V_{ub} V_{us}^*$. In addition, $C_7^{\text{new}}(\mu)$ can be obtained by replacing $m_t$ with $m_{\mu}$ in the above expression. Similar replacement ($m_t \rightarrow m_{\mu}$) has to be done for the other Wilson Coefficients $C_9^{\text{eff}}$ and $C_{10}$ which have too lengthy expressions to give here and their explicit expressions are given in refs. [36, 40].
III. PARAMETERIZATIONS OF MATRIX ELEMENTS AND FORM FACTORS IN LCSR

With the free quark decay amplitude available, we can proceed to calculate the decay amplitudes for semi-leptonic decays of $\bar{B}_0 \to K_0^* (1430) l^+ l^-$ at hadronic level, which can be obtained by sandwiching the free quark amplitudes between the initial and final meson states. Consequently, the following two hadronic matrix elements need to be computed as can be observed from Eq. (11). Generally, the above two matrix elements can be parameterized in terms of a series of form factors as

$$\langle K_0^*(p) | \bar{s} \gamma_\mu \gamma_5 b | B_{q'}(p + q) \rangle, \quad \langle K_0^*(p) | \bar{s} \sigma_{\mu\nu} \gamma_5 q^\nu b | B_{q'}(p + q) \rangle$$

need to be computed as can be observed from Eq. (1). Generally, the above two matrix elements can be parameterized in terms of a series of form factors as

$$\langle K_0^*(p) | \bar{s} \gamma_\mu \gamma_5 b | B_{q'}(p + q) \rangle = -i[f_+(q^2)p_\mu + f_-(q^2)q_\mu], \quad \langle K_0^*(p) | \bar{s} \sigma_{\mu\nu} \gamma_5 q^\nu b | B_{q'}(p + q) \rangle = -\frac{1}{m_B + m_{K_0^*}} \left[ (2p + q)_\mu q^2 - \left(m_B^2 - m_{K_0^*}^2\right) q_\mu \right] f_T(q^2). \quad \text{(12)}$$

The form factors are the non-perturbative quantities and to calculate them one has to rely on some non-perturbative approaches. Considering the distribution amplitudes up to twist-3, the form factors at small $q^2$ for $\bar{B}_0 \to K_0^* l^+ l^-$ have been calculated in [35] using the LCSR. The dependence of form factors $f_i(q^2)(i = +, -, T)$ on momentum transfer $s$ are parameterized in either the single pole form

$$f_i(q^2) = \frac{f_i(0)}{1 - a_i q^2/m_{B_0}^2},$$

or the double-pole form

$$f_i(q^2) = \frac{f_i(0)}{1 - a_i q^2/m_{B_0}^2 + b_i q^4/m_{B_0}^4},$$

in the whole kinematical region $0 < q^2 < (m_{B_0} - m_{K_0^*})^2$ while non-perturbative parameters $a_i$ and $b_i$ can be fixed by the magnitudes of form factors corresponding to the small momentum transfer calculated in the LCSR approach. The results for the parameters $a_i$, $b_i$ accounting for the $q^2$ dependence of form factors $f_+$, $f_-$ and $f_T$ are grouped in Table 1.

| | $f_i(0)$ | $a_i$ | $b_i$ |
|---|---|---|---|
| $f_+$ | $0.97^{+0.20}_{-0.22}$ | $0.86^{+0.19}_{-0.18}$ | |
| $f_-$ | $0.073^{+0.02}_{-0.02}$ | $2.50^{+0.44}_{-0.47}$ | $1.82^{+0.69}_{-0.76}$ |
| $f_T$ | $0.60^{+0.04}_{-0.13}$ | $0.69^{+0.26}_{-0.27}$ | |

IV. FORMULA FOR PHYSICAL OBSERVABLES

In this section, we are going to perform the calculations of some interesting observables in phenomenology like the decay rates, forward-backward asymmetry as well as the polarization asymmetries of final state lepton. From Eq. (7), it is straightforward to obtain the decay amplitude for $\bar{B}_0 \to K_0^* l^+ l^-$ as

$$\mathcal{M}_{\bar{B}_0 \to K_0^* l^+ l^-} = \frac{G_F}{\sqrt{2}} \alpha V_{tb} V_{ts}^* \left[ T_{1\mu}^1 (\bar{l} \gamma_\mu l) + T_{1\mu}^2 (\bar{l} \gamma_\mu \gamma_5 l) \right], \quad \text{(16)}$$

where the functions $T_{1\mu}^1$ and $T_{1\mu}^2$ are given by

$$T_{1\mu}^1 = iC_9^{\text{tot}} f_+(q^2) p_\mu + \frac{4 i m_b}{m_B + m_{K_0^*}} C_7^{\text{tot}} f_T(q^2) p_\mu, \quad \text{(17)}$$
where $K$.

The auxiliary functions are defined as $u = iC_{10} \ell^2 (f_+ (q^2) p_\mu + f_- (q^2) q_\mu)$.

Due to the equation of motion for lepton fields, the terms proportional to $q_\mu$ in $T^1$, namely $f_- (q^2)$ do not contribute to the decay amplitude.

A. The differential decay rates and forward-backward asymmetry of $\bar{B}_0 \to K^*_0 (1430) l^+ l^-$

The semi-leptonic decay $\bar{B}_0 \to K^*_0 (1430) l^+ l^-$ is induced by FCNCs. The differential decay width of $\bar{B}_0 \to K^*_0 (1430) l^+ l^-$ in the rest frame of $\bar{B}_0$ meson can be written as:

$$\frac{d\Gamma (\bar{B}_0 \to K^*_0 (1430) l^+ l^-)}{dq^2} = \frac{1}{(2\pi)^3 32m_{\bar{B}_0}} \int_{u_{\text{min}}}^{u_{\max}} |\bar{M}_{\bar{B}_0 \to K^*_0 (1430) l^+ l^-}|^2 du,$$  \hspace{1cm} (18)

where $u = (p_{K^*_0 (1430)} + p_{l^-})^2$ and $q^2 = (p_{l^+} + p_{l^-})^2; p_{K^*_0 (1430)}, p_{l^+}$ and $p_{l^-}$ are the four-momenta vectors of $K^*_0 (1430), l^+$ and $l^-$ respectively; $|\bar{M}_{\bar{B}_0 \to K^*_0 (1430) l^+ l^-}|^2$ is the squared decay amplitude after integrating over the angle between the lepton $l^-$ and $K^*_0 (1430)$ meson. The upper and lower limits of $u$ are given by

$$u_{\max} = (E^*_{K^*_0 (1430)} + E^*_{l^+})^2 - (\sqrt{E^*_{K^*_0 (1430)}^2 - m_{K^*_0 (1430)}^2} - \sqrt{E_{l^+}^2 - m_{l^-}^2})^2,$$

$$u_{\min} = (E^*_{K^*_0 (1430)} + E^*_{l^+})^2 - (\sqrt{E^*_{K^*_0 (1430)}^2 - m_{K^*_0 (1430)}^2} + \sqrt{E_{l^+}^2 - m_{l^-}^2})^2;$$  \hspace{1cm} (19)

where the energies of $K^*_0 (1430)$ and $l^-$ in the rest frame of lepton pair $E^*_{K^*_0 (1430)}$ and $E^*_{l^+}$ are determined as:

$$E^*_{K^*_0 (1430)} = \frac{m^2_{\bar{B}_0} - m^2_{K^*_0 (1430)} - q^2}{2\sqrt{q^2}}, \hspace{1cm} E^*_{l^+} = \frac{q^2}{2\sqrt{q^2}}.$$  \hspace{1cm} (20)

Collecting everything together, one can write the general expression of the differential decay rate for $\bar{B}_0 \to K^*_0 (1430) l^+ l^-$ as:

$$\frac{d\Gamma}{dq^2} = G_F^2 \alpha^2 |V_{tb} V_{tb}^*|^2 \sqrt{1 - \frac{4m^2_{l^+}}{q^2}} \sqrt{\lambda (m^2_B, m^2_{K^*_0}, q^2) \times \left\{ |A|^2 \left( 2m^2_{l^+} + q^2 \right) \lambda + 12q^2 m^2_{l^+} \left( m^2_B - m^2_{K^*_0} - q^2 \right) (C^* + C B) + 12m^2 q^4 |C|^2 \right.}

+ \left. |B|^2 \left( 2m^2_{l^+} + q^2 \right) \left( m^4_B - 2m^2_B m^2_{K^*_0} - 2q^2 m^2_{K^*_0} \right) + \left( m^2_{K^*_0} - q^2 \right)^2 + 2m^2 \left( m^4_{K^*_0} + 10m^2_{K^*_0} q^2 \right) \right\}},$$  \hspace{1cm} (21)

where

$$\lambda = \lambda (m^2_B, m^2_{K^*_0}, q^2) = m^4_B + m^4_{K^*_0} + q^4 - 2m^2_B m^2_{K^*_0} - 2m^2_{K^*_0} q^2 - 2q^2 m^2_B.$$  \hspace{1cm} (22)

The auxiliary functions are defined as:

$$A = iC_9 \ell^2 f_+ (q^2) + \frac{4im_b}{m_B + m_{K^*_0}} C_7 \ell^2 f_T (q^2)$$

$$B = iC_{10} \ell^2 f_+ (q^2)$$

$$C = iC_{10} \ell^2 f_- (q^2)$$  \hspace{1cm} (23)

Just to make a comment, the form factor $f_- (q^2)$ is an order of magnitude smaller than the form factors $f_+ (q^2)$ and $f_T (q^2)$, therefore, the value of auxiliary function $C$ is suppressed by the same magnitude compared to $A$ and $B$. 

6
B. Lepton Polarization asymmetries of $\bar{B}_0 \to K^*_0(1430)l^+l^-$

In the rest frame of the lepton $l^-$, the unit vectors along longitudinal, normal and transversal component of the $l^-$ can be defined as [53]:

$$s_{L}^{\mu} = (0, \vec{e}_{L}) = \left(0, \frac{\vec{p}_-}{|\vec{p}|-1}\right),$$

$$s_{N}^{\mu} = (0, \vec{e}_{N}) = \left(0, \frac{\vec{p}_{K_0} \times \vec{p}_-}{|\vec{p}_{K_0}| \times |\vec{p}_-|}\right),$$

$$s_{T}^{\mu} = (0, \vec{e}_{T}) = \left(0, \frac{\vec{e}_N \times \vec{e}_L}{|\vec{e}_N \times \vec{e}_L|}\right),$$

where $\vec{p}_-$ and $\vec{p}_{K_0}$ are the three-momenta of the lepton $l^-$ and $K^*_0(1430)$ meson respectively in the center mass (CM) frame of $l^+l^-$ system. Lorentz transformation is used to boost the longitudinal component of the lepton polarization to the CM frame of the lepton pair as

$$\left(s_{L}^{\mu}\right)_{CM} = \left(\frac{|\vec{p}_-|}{m_l}, \frac{E_l |\vec{p}_-|}{m_l |\vec{p}_-|}\right)$$

(25)

where $E_l$ and $m_l$ are the energy and mass of the lepton. The normal and transverse components remain unchanged under the Lorentz boost.

The longitudinal ($P_L$), normal ($P_N$) and transverse ($P_T$) polarizations of lepton can be defined as:

$$P_i^{(\mp)}(q^2) = \frac{d\Gamma}{dq^2}(\xi^{\mp} = \epsilon^{\mp}) - \frac{d\Gamma}{dq^2}(\xi^{\mp} = -\epsilon^{\mp})$$

$$+ \frac{d\Gamma}{dq^2}(\xi^{\mp} = \epsilon^{\mp}) + \frac{d\Gamma}{dq^2}(\xi^{\mp} = -\epsilon^{\mp})$$

(26)

where $i = L, N, T$ and $\xi^{\mp}$ is the spin direction along the leptons $l^\mp$. The differential decay rate for polarized lepton $l^\mp$ in $\bar{B}_0 \to K^*_0(1430)l^+l^-$ decay along any spin direction $\xi^{\mp}$ is related to the unpolarized decay rate [15] with the following relation

$$\frac{d\Gamma}{dq^2} = \frac{1}{2} \left(\frac{d\Gamma}{dq^2}\right) [1 + (P_L^\mp \epsilon^\mp + P_N^\mp \epsilon^\mp + P_T^\mp \epsilon^\mp) \cdot \xi^\mp].$$

(27)

We can achieve the expressions of longitudinal, normal and transverse polarizations for $\bar{B}_0 \to K^*_0(1430)l^+l^-$ decays as collected below. The longitudinal lepton polarization can be written as [53]

$$P_L(q^2) = (1/\frac{d\Gamma}{dq^2}) \frac{\alpha^2 G_F^2 |V_{tb} V_{ts}|^2 \lambda^{3/2} (m_B^2, m_{K_0}^2, q^2)}{3072m_B^3\pi^5} (1 - \frac{4m_l^2}{q^2})(A^*B + A^*B).$$

(28)

Similarly, the normal lepton polarization is

$$P_N(q^2) = (1/\frac{d\Gamma}{dq^2}) \frac{\alpha^2 G_F^2 |V_{tb} V_{ts}|^2 m_l}{4096m_B^3\pi^4 \sqrt{q^2}} \left[1 - \frac{4m_l^2}{s} \left(m_B^2 - m_{K_0}^2 + q^2)(AB^* + AB^*) - 2q^2(A^*C + AC^*)\right]\right].$$

(29)

and the transverse one is given by

$$P_T(q^2) = (1/\frac{d\Gamma}{dq^2}) \frac{-i\alpha^2 G_F^2 |V_{tb} V_{ts}|^2 \lambda^{1/2}(m_B^2, m_{K_0}^2, q^2)}{2048m_B^3\pi^4} m_l (1 - \frac{4m_l^2}{q^2})(m_B^2 - m_{K_0}^2 + q^2)(B^*C - BC^*).$$

(30)

The $\frac{d\Gamma}{dq^2}$ appearing in the above equation is the one given in Eq. (21) and $\lambda(m_B^2, m_{K_0}^2, q^2)$ is the same as that defined in Eq. (22).
V. NUMERICAL ANALYSIS

In this section we will analyze the dependency of the total branching ratios and different lepton polarizations on the fourth generation SM parameters i.e. fourth generation quark mass \( m_{t'} \) and to the product of quark mixing matrix \( V^\ast_{t'b} V_{t's} = |V^*_{t'b} V_{t's}| e^{i\phi_{bs}} \). One of the main input parameters is the form factors which are the non-perturbative quantities and one needs some model to calculate them. Here, we will use the form factors that were calculated using the LCSR \[35\] and their dependence on \( q^2 \) is given in Section II and the corresponding values of different parameters is listed in Table I. Also we use the next-to-leading order approximation for the Wilson coefficients \( C^{SM}_i \) and \( C^{new}_i \) at the renormalization point \( \mu = m_b \). It has already been mentioned that besides the short distance contributions in the \( C^{eff}_i \) there are the long distance contributions resulting from the \( c\bar{c} \) resonances like \( J/\Psi \) and its excited states. In the present study we do not take these long distance effects into account and also we use the central value of the form factors and the other input parameters given in Table I.

\[
\begin{align*}
\frac{d}{dq^2} BR_{B_0 \to K^0(1430)\mu^+\mu^-} (GeV^2) & \\
0 & \quad 10^5 \\
0 & \quad 10^3 \\
0 & \quad 10^5 \\
0 & \quad 10^3 \\
\frac{d}{dq^2} BR_{B_0 \to K^0(1430)\mu^+\mu^-} (GeV^2) & \\
0 & \quad 10^5 \\
0 & \quad 10^3 \\
0 & \quad 10^5 \\
0 & \quad 10^3 \\
\end{align*}
\]

FIG. 1: The dependence of branching ratio of \( B_0 \to K^0(1430)\mu^+\mu^- \) on \( q^2 \) for different values of \( m_{t'} \) and \( |V^*_{t'b} V_{t's}| \). \( |V^*_{t'b} V_{t's}| = 0.002, 0.006, 0.009 \) and \( 0.014 \) in (a), (b), (c) and (d) respectively. In all the graphs, the solid line corresponds to the SM, dashed line, dashed-dotted and long dashed lines are for \( m_{t'} = 200 \) GeV, 400 GeV and 600 GeV, respectively.


In order to perform quantitative analysis of physical observables, it is necessary to have the numerical values of the new parameters \( m_{\ell'} \), \( |V_{\ell'b}V_{\ell's}|, \phi_{sb} \). In the forthcoming analysis we use the constraints of Ref. 28 on the fourth generation parameters, where it is found that \( m_{\ell'} \) varies from 400 – 600 GeV with the mixing angle \( |V_{\ell'b}V_{\ell's}| \) in the range of about \( (0.05 \text{ to } 1.4) \times 10^{-2} \) and the value of CP-odd phase is from 0° to 80°. Keeping the value of the phase \( \phi_{sb} = 80^\circ \) and for different values of \( m_{\ell'} \) and \( |V_{\ell'b}V_{\ell's}| \) we will plot the physical observables with square of the momentum transfer \( q^2 \) to see their effects at small and large value of \( q^2 \).

The numerical results for the decay rates and polarization asymmetries of the lepton are presented in Figs. 1-7. Figs. 1 and 2 describes the differential decay rate of \( B \to K_0^*(1430)\ell^+\ell^- \), from which one can see that the fourth generation effects are quite distinctive from that of the SM3 both in the small and large momentum transfer region. At small value of \( s \) the dominant contribution comes from \( C_7^{tot} \) where as at the large value of \( q^2 \) the major contribution is from the \( Z \) exchange i.e \( C_{10}^{tot} \) which is sensitive to the mass of the fourth generation quark \( m_{\ell'} \). Furthermore, for both the channels, the branching ratios are enhanced sizably in terms of \( m_{\ell'} \) and \( |V_{\ell'b}V_{\ell's}| \) and for \( m_{\ell'} = 600 \) and \( |V_{\ell'b}V_{\ell's}| = 1.4 \times 10^{-2} \) the branching ratios are increased by an order of magnitude.
As an exclusive decay, there are different source of uncertainties involved in the calculation of the above said decay. The major uncertainties in the numerical analysis of $B^0 \to K_0^*(1430)l^+l^-$ decay originated from the $B^0 \to K_s^*(1430)$ transition form factors calculated in the LCSR approach as shown in Table I, which can bring about almost 40% errors to the differential decay rate of above mentioned decay, which showed that it is not a very suitable tool to look for the new physics. The large uncertainties involved in the form factors are mainly from the variations of the decay constant of $K_0^*(1430)$ meson and the Gegenbauer moments in its distribution amplitudes. There are also some uncertainties from the strange quark mass $m_s$, which are expected to be very tiny on account of the negligible role of $m_s$ suppressed by the much larger energy scale of $m_b$. Moreover, the uncertainties of the charm quark and bottom quark mass are at the 1% level, which will not play significant role in the numerical analysis and can be dropped out safely. It also needs to be stressed that these hadronic uncertainties almost have no influence on the various asymmetries including the lepton polarization asymmetry on account of the serious cancelation among different polarization states and this make them the best tool to look for physics beyond the SM. This has already been described in ref.[35] and was shown in Fig. 3(a,b) for the longitudinal and normal lepton polarization asymmetries.

Fig. 4(a,b,c,d) shows the dependence of longitudinal lepton polarization asymmetry for the $B \to K_0^* l^+l^-$ decay on the square of momentum transfer for different values of $m_t$ and $|V_{tb}^* V_{ts}|$. The value of longitudinal lepton polarization for muon is around $-1$ in the SM3 and we have significant deviation in this value in SM4. Just in the case of $m_t = 600$ and $|V_{tb}^* V_{ts}| = 1.4 \times 10^{-2}$ the value of the longitudinal lepton polarization becomes $-0.6$ which will help us to see experimentally the SM4 effects in these flavor decays. In large $q^2$ region, the longitudinal lepton polarization approaches to zero both in the SM3 and SM4 which is due to the factor $\lambda(m_B^2, m_{K_s^*}^2, m_{l^+}^2)$ that approaches to zero at large value of $q^2$. Similar effects can be seen for the final state tauon (c.f. Fig. 5) but the value for this case is too small to measure experimentally.

The dependence of normal lepton polarization asymmetries for $B \to K_0^* l^+l^-$ on the momentum transfer square are presented in Figs. 6 and 7. In terms of Eq. (24), one can see that it is proportional to the mass of the final state lepton and for $\mu$ its values is expected to be small and Fig. 6(a,b,c,d) displays it in the SM3 as well as SM4 for the different value of fourth generation parameters. In SM4, one can see a slight shift from the SM3 value which, however, is too small to measure experimentally. Now, for the $\tau^+\tau^-$ channel, Eq. (24) we will have a large value of normal lepton polarization compared to the $\mu^+\mu^-$ case in the SM3. Fig. 7 shows that there is a significant decrease in the value of $P_N$ in SM4 compared to SM3 and its experimental measurement will give us some clue about the fourth generation of quarks.

FIG. 3: The dependence of Longitudinal (Fig. 3a) and Normal lepton polarization (Fig. 3b) of $B_0 \to K_0^*(1430)\mu^+\mu^-$ on $q^2$ for different values of the input parameters. Solid value corresponds to the central value, dotted line is for maximum value and long dashed line is for minimum value of input parameters.
![Graphs showing the dependence of Longitudinal lepton polarization on $q^2$ for different values of $m_{t'}$ and $|V^*_{tb}V_{t's}|$. The values of fourth generation parameters and the legends are same as in Fig.1.]

**FIG. 4:** The dependence of Longitudinal lepton polarization of $\bar{B}_0 \to K_0^*(1430)\mu^+\mu^-$ on $q^2$ for different values of $m_{t'}$ and $|V^*_{tb}V_{t's}|$. The values of fourth generation parameters and the legends are same as in Fig.1.

Now, from Eq. (30) we can see that it is proportional to the lepton mass as well as to the form factor $f_-(q^2)$ which is an order of magnitude smaller than the $f_+(q^2)$ and $f_T(q^2)$. This makes the transverse lepton polarization asymmetry almost to be zero in the SM3 as well as in the SM4 and it is non-zero only in the models where we have new operators, e.g. scalar type operators in the MSSM.

**VI. CONCLUSIONS**

We have carried out the study of invariant mass spectrum and polarization asymmetries of semileptonic decays $\bar{B}_0 \to K_0^*(1430)l^+l^-$ ($l = \mu, \tau$) decays in SM4. Particularly, we analyzed the sensitivity of these physical observables on the fourth generation quark mass $m_{t'}$ as well as the the mixing angle $|V^*_{tb}V_{t's}|$ and the main outcomes of this study can be summarized as follows:
The dependence of Longitudinal lepton polarization of \( \bar{B}_0 \to K^*_0(1430)\tau^+\tau^- \) on \( q^2 \) for different values of \( m_{t'} \) and \( |V^*_{tb}V_{ts}| \). The values of fourth generation parameters and the legends are same as in Fig.1

- The differential decay rates deviate sizably from that of the SM especially both in the small and large momentum transfer region. These effects are significant and the branching ration increases by an order of magnitude for \( m_{t'} = 600 \) GeV and \( |V^*_{tb}V_{ts}| = 1.4 \times 10^{-2} \).

- It has been shown in the literature \[35\] that the value of the forward-backward asymmetry for \( \bar{B}_0 \to K^*_0(1430)\ell^+\ell^- \) is non zero only in the models where we have the scalar type operators (like SUSY models). Now, due to the absence of scalar type operators in the SM3 as well as in SM4 the forward-backward asymmetry for the decay \( \bar{B}_0 \to K^*_0(1430)\ell^+\ell^- \) is zero in both these models.

- The longitudinal, normal and transverse polarizations of leptons are calculated in the SM4. It is found that the SM effects are very promising which could be measured at future experiments and shed light on the new physics beyond the SM. It is hoped that this can be measurable at future experiments like LHC and BTeV machines where a large number of \( b\bar{b} \) pairs are expected to be produced.
FIG. 6: The dependence of Normal lepton polarization of $\bar{B}_0 \rightarrow K^*_0(1430)\mu^+\mu^-$ on $q^2$ for different values of $m_{t'}$ and $|V_{t'b}V_{t's}|$. The values of fourth generation parameters and the legends are same as in Fig.1.

In short, the experimental investigation of observables, like decay rates, forward-backward asymmetry, lepton polarization asymmetries and the polarization asymmetries of $\bar{B}_0 \rightarrow K^*_0(1430)l^+l^-$ ($l = \mu, \tau$) decay will be used to search for the SM4 effects and will help us to put constraints on fourth generation parameters in an indirect way.

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FIG. 7: The dependence of Normal lepton polarization of $B_0 \rightarrow K_\tau^0 (1430) \tau^+ \tau^-$ on $q^2$ for different values of $m_t'$ and $|V^*_t V_{ts}|$.

The values of fourth generation parameters and the legends are same as in Fig.1.

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