Test of $CPT$ Symmetry in $CP$-violating $B$ Decays

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Abstract

Considering that the existing experimental limit for $CPT$ violation is still poor, we explore various possible ways to test $CPT$ symmetry in $CP$-violating $B$ decays at $e^+e^-$ $B$ factories. We find that it is difficult to distinguish between the effect of direct $CP$ violation and that of $CPT$ violation in the time-integrated measurements of neutral $B$ decays to $CP$ eigenstates such as $\psi K_S$ and $\pi^+\pi^-$. Instead, a cleaner signal of small $CPT$ violation may appear in the time-integrated $CP$ asymmetries for a few non-$CP$-eigenstate channels, e.g., $B^0_\text{d}/\bar{B}^0_\text{d} \to D^{\pm}\pi^\mp$ and $D^{(*)0}_\text{d} K_S$. The time-dependent measurements of $CP$ asymmetries are available, in principle, to limit the size of $CPT$-violating effects in neutral $B$-meson decays.

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Tests of the fundamental symmetries and conservation laws have been an important topic in particle physics. Recently the necessity of testing $CPT$ symmetry has been emphasized by several authors [1-3]. On the experimental side, the existing evidence for $CPT$ invariance is rather poor. The limit for the strength of $CPT$-violating interaction in the $K^0 - \bar{K}^0$ system is about 10% of that of $CP$-violating interaction [2-4]. This means that $CPT$ symmetry is tested only at the 10% level. On the theoretical side, the universality of $CPT$ theorem is questionable since it is proved in local renormalizable field theory with a heavy use of the properties of asymptotic states [5]. This proof may not be applicable for QCD, where quarks and gluons are confined other than in a set of asymptotic states [3].

The simplest test of $CPT$ invariance is to examine the equality of the masses and lifetimes of a particle and its antiparticle. Beyond the $K$-meson system, Kobayashi and Sanda are the first to suggest some ways to check $CPT$ symmetry in $B$-meson decays at the future $B$ factories [3]. With the assumption that the amplitudes for semileptonic decays satisfy the $\Delta Q = \Delta B$ rule and $CPT$ invariance, they relaxed $CPT$ symmetry for the mass matrix of the $B_d$ mesons. They found that both $B_d^0 - \bar{B}_d^0$ mixing and $CP$ asymmetries in neutral $B$ decays will get modified if $CPT$ violation is present. A key point of their work is that $CP$ asymmetries in nonleptonic $B$ decays such as $B_d^0/\bar{B}_d^0 \to \psi K_S$ may be more sensitive to the presence of small $CPT$-violating effects than the dileptonic decay rates of $B_d^0\bar{B}_d^0$ pairs.

In this work, we shall make an instructive analysis of the effects of $CPT$ violation on $CP$-violating asymmetries in neutral $B$-meson decays. Both time-dependent and time-integrated $CP$ asymmetries are calculated to meet various possible measurements at $e^+e^-$ $B$ factories. We suggest several ways for distinguishing $CPT$ violation from direct $CP$ violation in $B$ decay amplitudes and indirect $CP$ violation via interference between decay and mixing. We show that it is difficult to extract the $CPT$-violating information from the time-integrated measurements of neutral $B$ decays to $CP$ eigenstates such as $B_d^0/\bar{B}_d^0 \to \psi K_S$ and $\pi^+\pi^-$. Instead, cleaner signals of small $CPT$ violation may appear in the time-integrated $CP$ asymmetries of some non-$CP$-eigenstate channels, e.g., $B_d^0/B_d^0 \to D^{\pm}\pi^{\mp}$ and $(\bar{D}^{(*)})^0 K_S$. Measurements of the time development of $B_d^0$ vs $\bar{B}_d^0$ decays are available, in principle, to limit the size of $CPT$-violating effects on $CP$ asymmetries.

We begin with the mass eigenstates of the two $B_d$ mesons [6]:

$$|B_1> = \frac{1}{\sqrt{|p_1|^2 + |q_1|^2}} \left( p_1|B_d^0> + q_1|\bar{B}_d^0> \right),$$

$$|B_2> = \frac{1}{\sqrt{|p_2|^2 + |q_2|^2}} \left( p_2|B_d^0> - q_2|\bar{B}_d^0> \right),$$

(1)
where \( p_{1,2} \) and \( q_{1,2} \) are parameters of the \( B_d^0 \bar{B}_d^0 \) mass matrix elements. For convenience, the ratios of \( q_{1,2} \) to \( p_{1,2} \) can be further written as

\[
\frac{q_1}{p_1} = e^{i\phi} \tan \frac{\theta}{2}, \quad \frac{q_2}{p_2} = e^{i\phi} \cot \frac{\theta}{2},
\]

where \( \theta \) and \( \phi \) are generally complex [6]. CPT symmetry requires \( q_1/p_1 = q_2/p_2 = e^{i\phi} \) or \( S \equiv \cot \theta = 0 \), and CP invariance requires that both \( S = 0 \) and \( \phi = 0 \) hold. Furthermore, the proper-time evolution of an initially \( (t = 0) \) pure \( B_d^0 \) or \( \bar{B}_d^0 \) is given by

\[
|B_d^0(t)\rangle = e^{-(i m + \frac{\Delta m}{2}) t} \left[ g_+(t)|B_d^0\rangle + \bar{g}_+(t)|\bar{B}_d^0\rangle \right], \quad |\bar{B}_d^0(t)\rangle = e^{-(i m + \frac{\Delta m}{2}) t} \left[ \bar{g}_-(t)|B_d^0\rangle + g_-(t)|\bar{B}_d^0\rangle \right],
\]

where

\[
g_\pm(t) = \cos^2 \frac{\theta}{2} e^{\pm (i \Delta m - \frac{i \Delta \Gamma}{2}) t} + \sin^2 \frac{\theta}{2} e^{\mp (i \Delta m - \frac{i \Delta \Gamma}{2}) t},
\]

\[
\bar{g}_\pm(t) = \sin \frac{\theta}{2} \cos \frac{\theta}{2} \left[ e^{(i \Delta m - \frac{i \Delta \Gamma}{2}) t} - e^{-(i \Delta m - \frac{i \Delta \Gamma}{2}) t} \right] e^{\pm i \phi}.
\]

In Eqs. (3) and (4) we have defined \( m \equiv (m_1 + m_2)/2 \), \( \Gamma \equiv (\Gamma_1 + \Gamma_2)/2 \), \( \Delta m \equiv m_2 - m_1 \), and \( \Delta \Gamma \equiv \Gamma_1 - \Gamma_2 \), where \( m_{1,2} \) and \( \Gamma_{1,2} \) are the mass and width of \( B_{1,2} \).

Concentrating only on checking CPT symmetry in the mass matrix of the \( B_d \) mesons, here we assume that semileptonic \( B \) decays satisfy the \( \Delta Q = \Delta B \) rule and CPT invariance. We also neglect CPT violation in the transition amplitudes for nonleptonic \( B \) decays. Relaxing these limitations can certainly be done, but the results will become too complicated. To obtain simple and instructive results, we further assume that

\[
\frac{\Delta \Gamma}{\Gamma} = 0, \quad \text{Im} \phi = 0, \quad \text{Im} \theta = 0,
\]

and \( S = \cot \theta \) is small and real. As examined in Ref. [3], large \( \text{Im} \phi \) and \( |S| \) should be measured in the dileptonic decay ratios of \( B_d^0 \bar{B}_d^0 \) pairs at the \( \Upsilon(4S) \). On the other hand, a model-independent analysis shows \( \Delta \Gamma/\Gamma \leq 10^{-2} \), which has little effect on the time-integrated CP asymmetries in \( B_d \) decays [7]. In this work, we proceed with the above approximations to calculate the decay probabilities of \( B_d^0 \) and \( \bar{B}_d^0 \) mesons and explore the CPT-violating effects on CP asymmetries instructively. The more precise results will be presented elsewhere.

The unique experimental advantages of studying \( b \)-quark physics at the \( \Upsilon(4S) \) resonance is well known [8]. Both symmetric and asymmetric \( B \) factories will be built in the near future based on such threshold \( e^+e^- \) collisions [9]. At the \( \Upsilon(4S) \) resonance, the \( B \)'s are produced in
a two-body ($B^+_d B^-_u$ and $B^0_d \bar{B}^0_d$) state with definite charge parity. The two neutral $B$ mesons mix coherently until one of them decays. Thus one can use the semileptonic decays of one meson to tag the flavor of the other meson decaying into a flavor-nonspecific hadronic final state. The time-dependent wave function for a $B^0_d \bar{B}^0_d$ pair produced at the $\Upsilon(4S)$ can be written as

$$
\frac{1}{\sqrt{2}} \left[ |B^0_d(k, t) > \otimes |\bar{B}^0_d(-k, t) > + C|B^0_d(-k, t) > \otimes |\bar{B}^0_d(k, t) > \right],
$$

where $k$ is the three-momentum vector of the $B_d$ mesons, and $C = \pm$ is the charge parity of the $B^0_d \bar{B}^0_d$ pair. Supposing one $B_d$ meson decaying into a semileptonic state $|l^+ X^+ >$ at (proper) time $t_1$ and the other into a nonleptonic state $|f >$ at time $t_2$, the time-dependent probabilities for such joint decays are given by

$$
\text{Prob}(l^+ X^-, t_1; f, t_2)_C \propto |A_t|^2 |A_f|^2 e^{-\Gamma(t_1 + t_2)} |g_+(t_1)[\bar{g}_-(t_2)] + \zeta_f g_-(t_2))] + C\bar{g}_-(t_1)[g_+(t_2) + \zeta_f \bar{g}_+(t_2)]|^2,
$$

$$
\text{Prob}(l^- X^+, t_1; f, t_2)_C \propto |A_t|^2 |A_f|^2 e^{-\Gamma(t_1 + t_2)} |\bar{g}_+(t_1)[g_-(t_2)] + \zeta_f g_-(t_2)] + Cg_-(t_1)[\bar{g}_+(t_2) + \zeta_f \bar{g}_+(t_2)]|^2,
$$

where

$$
A_f \equiv < f |H|B^0_d >, \quad \bar{A}_f \equiv < f |H|\bar{B}^0_d >, \quad \zeta_f \equiv \frac{\bar{A}_f}{A_f},
$$

and $A_t \equiv < l^+ X^- |H|B^0_d >^{CPT} = < l^- X^+ |H|\bar{B}^0_d >$. By applying Eq. (5), Eq. (7) is further simplified as

$$
\text{Prob}(l^\pm X^\mp, t_1; f, t_2)_C \propto |A_t|^2 |A_f|^2 e^{-\Gamma(t_1 + t_2)} \left\{ (1 + |\xi_f|^2) \mp (1 - |\xi_f|^2) \cos[\Delta m(t_2 + C t_1)] \right\}
$$

$$
\pm 2\text{Im} \xi_f \sin[\Delta m(t_2 + C t_1)]
$$

$$
\pm 2\text{Re} \xi_f \left[ C - (1 + C) \cos(\Delta m t) + \cos[\Delta m(t_2 + C t_1)] \right],
$$

where $\xi_f \equiv e^{i\phi} \zeta_f$, $|\xi_f| = |\zeta_f|$, and those $O(S^2)$ terms have been neglected. Clearly a $CPT$-violating term proportional to $S$ appears.

Within limits of our present detector technology, we have to consider the feasibility for an $e^+e^-$ collider to measure the time development of the decay probabilities and $CP$ asymmetries. For a symmetric collider running at the $\Upsilon(4S)$ resonance, the mean decay length of $B$’s is insufficient for the measurement of $(t_2 - t_1)$ [8]. On the other hand, the quantity $(t_2 + t_1)$ cannot be measured in a symmetric or asymmetric storage ring operating at the $\Upsilon(4S)$, unless the bunch lengths are much shorter than the decay lengths [8,9]. Therefore,
only the time-integrated measurements are available at a symmetric $B$ factory. Integrating

\[ \text{Prob}(l^\pm X^\mp, t_1; f, t_2)_C \] over $t_1$ and $t_2$, we obtain

\[ \text{Prob}(l^\pm X^\mp, f)_- \propto |A_l|^2 |A_f|^2 \left[ \frac{1 + |\xi_f|^2}{2} \mp \frac{1 - |\xi_f|^2}{2} \mp \frac{x_d^2 S \Re \xi_f}{1 + x_d^2} \right] \]  

(10)

and

\[ \text{Prob}(l^\pm X^\mp, f)_+ \propto |A_l|^2 |A_f|^2 \left[ \frac{1 + |\xi_f|^2}{2} \mp \frac{1 - |\xi_f|^2}{2} \mp \frac{2x_d \Im \xi_f}{(1 + x_d^2)^2} S \Re \xi_f \pm \frac{x_d^2 (1 - x_d^2)}{(1 + x_d^2)^2} S \Re \xi_f \right] . \]  

(11)

For an asymmetric collider running in this energy region, one might want to integrate Eq. (9) only over $(t_2 + t_1)$ in order to measure the development of the decay probabilities with

\[ \Delta t \equiv (t_2 - t_1) \] [8]. In this case, we obtain

\[ \text{Prob}(l^\pm X^\mp, f; \Delta t)_- \propto |A_l|^2 |A_f|^2 e^{-i|\Delta t|} \left\{ \frac{1 + |\xi_f|^2}{2} \mp \frac{1 - |\xi_f|^2}{2} \cos(\Delta m\Delta t) \right. \]

\[ + \left. \pm \Im \xi_f \sin(\Delta m\Delta t) \mp S \Re \xi_f [1 - \cos(\Delta m\Delta t)] \right\} \]

(12)

and

\[ \text{Prob}(l^\pm X^\mp, f; \Delta t)_+ \propto |A_l|^2 |A_f|^2 e^{-i|\Delta t|} \left\{ \frac{1 + |\xi_f|^2}{2} \mp \frac{1 - |\xi_f|^2}{2} \cos(\Delta m\Delta t + \phi_{xd}) \right. \]

\[ + \left. \pm \frac{1}{\sqrt{1 + x_d^2}} \Im \xi_f \sin(\Delta m\Delta t + \phi_{xd}) \right\} \]

\[ \mp S \Re \xi_f \left[ \frac{4 - x_d^2}{4 + x_d^2} - \frac{1}{\sqrt{1 + x_d^2}} \cos(\Delta m\Delta t + \phi_{xd}) \right] \],

(13)

where $\phi_{xd} \equiv \arctan x_d$.

For the case that one neutral $B$ meson decays into $|l^\mp X^\pm >$ at time $t_1$ and the other decays into $|f >$ (the $CP$-conjugate state of $| f >$) at time $t_2$, the corresponding decay probabilities

\[ \text{Prob}(l^\mp X^\pm, t_1; \bar{f}, t_2)_C \] can be obtained from Eq. (9) by the replacements $A_f \rightarrow \bar{A}_f$, $\zeta_f \rightarrow \bar{\zeta}_f$, and $\xi_f \rightarrow \bar{\xi}_f$, where

\[ \bar{A}_f \equiv < \bar{f}|H|B^0_d > , \quad A_f \equiv < \bar{f}|H|B^0_d > , \quad \bar{\zeta}_f \equiv \frac{A_f}{\bar{A}_f} , \quad \bar{\xi}_f \equiv e^{-i\phi} \bar{\xi}_f . \]  

(14)

In a similar manner, one can obtain $\text{Prob}(l^\mp X^\pm, \bar{f})_\pm$ and $\text{Prob}(l^\mp X^\pm, \bar{f}; \Delta t)_\pm$ straightforwardly from Eqs. (10-13).

The difference between the decay probabilities associated with $B^0_d \rightarrow f$ and $\bar{B}^0_d \rightarrow \bar{f}$ is a basic signal for $CP$ violation. Corresponding to the possible measurements for joint $B^0_d \bar{B}^0_d$
decays at symmetric (S) and asymmetric (A) $e^+e^-$ $B$ factories, we define the $CP$-violating asymmetries as

$$A^S_C = \frac{\text{Prob}(l^-X^+, f)_C - \text{Prob}(l^+X^-, \bar{f})_C}{\text{Prob}(l^-X^+, f)_C + \text{Prob}(l^+X^-, \bar{f})_C}$$

and

$$A^A_C(\Delta t) = \frac{\text{Prob}(l^-X^+, f; \Delta t)_C - \text{Prob}(l^+X^-, \bar{f}; \Delta t)_C}{\text{Prob}(l^-X^+, f; \Delta t)_C + \text{Prob}(l^+X^-, \bar{f}; \Delta t)_C}.$$  \hspace{1cm} (15)

In the following, we calculate the asymmetries for two categories of neutral $B$ decays and discuss the small $CPT$-violating effects on them.

We first consider the $B^0_d$ and $\bar{B}^0_d$ decays to $CP$ eigenstates (i.e., $|\bar{f}| = \pm |f|$) such as $\psi K_S, \pi^+\pi^-$, and $\pi^0 K_S$. With the phase convention $CP|B^0_d > = |\bar{B}^0_d >$ and the approximations in Eq. (5), we have $A_f = \pm A_f, \bar{A}_f = \pm \bar{A}_f, \zeta_f = 1/|\zeta_f|$, and $\xi_f = 1/|\xi_f|$. For convenience, we define three characteristic quantities:

$$U = \frac{1 - |\zeta_f|^2}{1 + |\zeta_f|^2}, \quad V = \frac{-2\text{Im}\xi_f}{1 + |\xi_f|^2}, \quad W = \frac{2\text{Re}\xi_f}{1 + |\xi_f|^2}. \hspace{1cm} (16)$$

Nonvanishing $U$ and $V$ imply the $CP$ violation in the decay amplitude and the one from interference between decay and mixing [10], respectively. $W$ (proportional to $S$) is a measure of $CPT$ violation in the mass matrix of the neutral $B$ mesons.

For symmetric and asymmetric $e^+e^-$ collisions at the $\Upsilon(4S)$, the corresponding $CP$ asymmetries in $(B^0_d\bar{B}^0_d)_C \rightarrow (l^\pm X^\mp)f$ are given by

$$A^S_- = \frac{\frac{1}{1+x_d^2}U + \frac{x_d^2}{1+x_d^2}W}{}, \quad A^S_+ = \frac{1-x_d^2}{1+x_d^2})U + \frac{2x_d}{1+x_d^2})V + \frac{x_d^2(1-x_d^2)}{(1+x_d^2)^2}W; \hspace{1cm} (17)$$

and

$$A^A(\Delta t) = U \cos(\Delta m \Delta t) + V \sin(\Delta m \Delta t) + W[1 - \cos(\Delta m \Delta t)],$$

$$A^A_+(\Delta t) = \frac{1}{\sqrt{1+x_d^2}}[U \cos(\Delta m \Delta t + \phi_{x_d}) + V \sin(\Delta m \Delta t + \phi_{x_d})]$$

$$+ W \left[\frac{4-x_d^2}{4+x_d^2} + \frac{1}{\sqrt{1+x_d^2}} \cos(\Delta m \Delta t + \phi_{x_d}) \right]. \hspace{1cm} (18)$$

From the above equations we observe that all the $CP$ asymmetries get modified if $CPT$ violation is present. Two remarks are in order.

(1) $A^S_-$ is neither a pure measure of direct $CP$ violation in the decay amplitude (i.e., $|\xi_f| \neq 1$) nor that of small $CPT$ violation in the mass matrix, but a combination of both of
them. If $S$ happens to take the special value
\[
S = \frac{1}{2x_d^2} \left| \frac{\zeta_f}{\Re \xi_f} \right|^2 - 1,
\]
the effects of $CP$ and $CPT$ violation will completely cancel out in $A_S^S$. In $A_S^S$, the $CP$ asymmetry from the interference between decay and mixing is commonly dominant. A combination of measurements of $A_S^S$ and $A_S^C$ can in principle determine $V$ or $\Im \xi_f$ unambiguously, as the relation
\[
V = \frac{(1 + x_d^2)^2}{2x_d} \left[ A_S^S - \frac{1 - x_d^2}{1 + x_d^2} A_S^C \right]
\]
holds.

(2) $CPT$-violating effects can be probed by measuring the time development of the $CP$ asymmetries at the $\Upsilon(4S)$. With the help of
\[
A^A_\pm(\Delta t) = \frac{1}{\sqrt{1 + x_d^2}} A^A_\pm \left( \Delta t + \frac{\phi_{x_d}}{\Delta m} \right) + \left( \frac{4 - x_d^2}{4 + x_d^2} - \frac{1}{\sqrt{1 + x_d^2}} \right) W,
\]
the magnitude of $W$ or $S$ may be definitely limited by the data on $A^A_\pm(\Delta t)$ and $A^A_\pm(\Delta t)$. In addition, nonvanishing signals of direct $CP$ violation and $CPT$ violation can appear on some special points of $A^A_\pm(\Delta t)$. For example,
\[
U = A^A_- \left( \frac{2n\pi}{\Delta m} \right)
\]
and
\[
2W = A^A_+ \left( \frac{2n\pi}{\Delta m} \right) + A^A_- \left( \frac{(2n + 1)\pi}{\Delta m} \right),
\]
where $n = 0, \pm 1, \pm 2,$ and so on.

It is worthwhile at this point to emphasize that measuring the time-integrated asymmetry $A_S^S$ for $B^0_d$ vs $\bar{B}^0_d \to \psi K_S$ might not be very good to probe $CPT$ violation in the $B_d$ mass matrix. As discussed in Ref. [11], the so-called hairpin channel may contribute to these two decay modes. Thus a small deviation of $|\xi_{\psi K_S}|$ from unity is possible. With the help of two-loop effective weak Hamiltonians and factorization approximations, the ratio of hairpin to tree-level amplitudes is estimated to be as large as 8% [11]. Therefore, direct $CP$ violation should be taken into account when we study the fine effect of $CPT$ violation in $B^0_d/\bar{B}^0_d \to \psi K_S$ and other neutral $B$ decays.

Now we consider the case that $B^0_d$ and $\bar{B}^0_d$ decay to a common non-$CP$ eigenstate (i.e., $|f \neq f \rangle$ but their amplitudes $A_f$ ($A_f$) and $\bar{A}_f$ ($\bar{A}_f$) contain only a single weak phase. Most of such decays occur through the quark transitions $b \to u\bar{c}$ ($q\bar{q}$) and $c\bar{u}$ ($q\bar{q}$).
(with \( q = d, s \)), and the typical examples are \( B^0_d / \bar{B}^0_d \to D^\pm \pi^\mp \) and \((\bar{D}^{(*)0})_S\). In this case, no measurable direct \(CP\) violation arises in the decay amplitudes since \(|\tilde{A}_f| = |A_f|, |\bar{A}_f| = |A_f|, |\tilde{\xi}_f| = |\xi_f|, \) and \(|\xi_f| = |\xi_f|\) [10,12]. For convenience, we define

\[
\tilde{U} = \frac{1 - |\xi_f|^2}{1 + |\xi_f|^2}, \quad \tilde{V}_\pm = \frac{-\text{Im}(\xi_f \pm \bar{\xi}_f)}{1 + |\xi_f|^2}, \quad \bar{W}_\pm = \frac{\text{SRe}(\xi_f \pm \bar{\xi}_f)}{1 + |\xi_f|^2}.
\]

(25)

Note that here a nonzero \(\tilde{U}\) does not mean \(CP\) violation in the decay amplitude. \(\tilde{V}_-\) and \(\bar{W}_-\), which imply indirect \(CP\) violation and \(CPT\) violation, will contribute to the \(CP\) asymmetries in this kind of decay modes.

For symmetric and asymmetric \(e^+e^-\) collisions at the \(\Upsilon(4S)\) resonance, the corresponding \(CP\) asymmetries in the decay modes in question are given as

\[
A^S_- = \frac{x_d^2 \bar{W}_-}{1 + x_d^2 + \tilde{U} + x_d^2 \bar{W}_+},
\]

\[
A^S_+ = \frac{2x_d \tilde{V}_- + x_d^2 (1 - x_d^2) \bar{W}_-}{(1 + x_d^2)^2 + (1 - x_d^2) \tilde{U} + 2x_d \tilde{V}_+ + x_d^2 (1 - x_d^2) \bar{W}_+};
\]

and

\[
A^A_-(\Delta t) = \frac{\tilde{V}_- \sin(\Delta m \Delta t) + \bar{W}_- [1 - \cos(\Delta m \Delta t)]}{1 + \tilde{F}(\Delta m \Delta t)},
\]

\[
A^A_+(\Delta t) = \frac{\tilde{V}_- \sin(\Delta m \Delta t + \phi_{x_d}) + \bar{W}_- \left[ \frac{4 - x_d^2}{4 + x_d^2} \sqrt{1 + x_d^2 - \cos(\Delta m \Delta t + \phi_{x_d})} \right]}{\sqrt{1 + x_d^2 + \tilde{F}(\Delta m \Delta t + \phi_{x_d}) + \bar{W}_+ \left[ \frac{4 - x_d^2}{4 + x_d^2} \sqrt{1 + x_d^2 - 1} \right]}},
\]

(26)

(27)

where \(\tilde{F}\) is a function defined as

\[
\tilde{F}(y) \equiv \tilde{U} \cos y + \tilde{V}_+ \sin y + \bar{W}_+(1 - \cos y).
\]

(28)

It should be noted that, in contrast with the asymmetry \(A^S_-\) in \(B^0_d / \bar{B}^0_d\) decays into \(CP\) eigenstates (see Eq. (18)), here \(A^S_-\) is a pure measure of \(CPT\) violation only if \(\text{Re}\bar{\xi}_f \neq \text{Re}\xi_f\). There is a handful of neutral B decays, e.g., \(B^0_d \to \bar{D}^{0(*)}_S, D^{(*)\pm} \pi^\mp\), and \(D^{(*)0} \pi^0\), which satisfy the conditions \(|\tilde{\xi}_f| = |\xi_f|\) and \(\text{Re}\xi_f \neq \text{Re}\bar{\xi}_f\). Taking \(B^0_d / \bar{B}^0_d \to D^+ \pi^-\) for example, an isospin analysis shows that

\[
\xi_{D^+ \pi^-} = \frac{V_{cb} V^*_{ub}}{V_{cd} V^*_{ub}} \left[ \frac{\tilde{a}_{3/2} e^{i\delta_{3/2}} + \sqrt{2} \tilde{a}_{1/2} e^{i\delta_{1/2}}}{\tilde{a}_{3/2} e^{i\delta_{3/2}} - \sqrt{2} \tilde{a}_{1/2} e^{i\delta_{1/2}}} \right],
\]

\[
\bar{\xi}_{D^- \pi^+} = \frac{V_{ud} V^*_{cb}}{V_{ub} V^*_{cd}} \left[ \frac{\tilde{a}_{3/2} e^{i\delta_{3/2}} + \sqrt{2} \tilde{a}_{1/2} e^{i\delta_{1/2}}}{\tilde{a}_{3/2} e^{i\delta_{3/2}} - \sqrt{2} \tilde{a}_{1/2} e^{i\delta_{1/2}}} \right],
\]

(29)

where \(V_{ij}\) (\(i = u, c, t; j = d, s, b\)) are the Cabibbo-Kobayashi-Maskawa matrix elements; \(\tilde{a}_{3/2}\) and \(\tilde{a}_{1/2}\) are the isospin amplitudes for a \(B^0_d / \bar{B}^0_d\) decaying into \(D^+ \pi^-\) (\(D^- \pi^+\))
and its CPT-conjugate process [12], and $\delta_{3/2}$ and $\delta_{1/2}$ are the corresponding strong phases. Obviously $|\bar{\xi}_{D-\pi^+}| = |\xi_{D+\pi^-}|$ holds, but $\bar{\xi}_{D-\pi^+} \neq \xi_{D+\pi^-}$ because of $\delta_{3/2} \neq \delta_{1/2}$. As a result, measurements of $A^S$ in such decay modes may serve as a good test of CPT symmetry in the $B$-meson decays.

From Eq. (26) we see that $A^S_\pm$ is also modified in the presence of nonvanishing $\tilde{W}_-$ or $S$. However, it is difficult to extract any information on CPT violation from them. In principle, measurements of the time-dependent asymmetries $A^A_\pm(\Delta t)$ are possible to probe $S$ with less ambiguity. For example, nonvanishing CPT violation can be extracted from

$$A^A_\pm \left(\frac{(2n+1)\pi}{\Delta m}\right) = \frac{2\tilde{W}_-}{1 - \Upsilon + 2\tilde{W}_+},$$

where $n = 0, \pm 1, \pm 2,$ and so on.

In order to probe the sources of CP and CPT violation in neutral $B$-meson decays, we have explored various possible measurements at $e^+e^-$ $B$ factories. It is shown that CPT-violating effects can in principle be distinguished from direct and indirect CP-violating effects by measuring the time development of CP asymmetries $A^A_\pm(\Delta t)$. A clean signal of CPT violation may appear in the time-integrated asymmetries $A^S_\pm$ for some non-CP-eigenstate decay modes, e.g., $B^0_d/\bar{B}^0_d \to D^{\pm}\pi^\mp$ and $\bar{D}^{(*)0}\psi K_S$. In contrast, measuring $A^S_\pm$ in the CP-eigenstate decays such as $B^0_d/\bar{B}^0_d \to \psi K_S$ and $\pi^+\pi^-$ cannot provide a good limit on CPT violation, since direct CP violation may compete with or dominate over CPT violation in them.

In keeping with the current interest in tests of discrete symmetries and conservation laws in the $K$-meson system [13], the parallel approaches are worth pursuing, especially on investigating CP violation and checking CPT invariance, for weak decays of $B$ mesons. With the development in building high-luminosity $B$ factories [8,9], it is possible to observe CP (or T) violation in neutral $B$ decays in the near future. The fine effects of CPT violation, if they are present, may be probed in the second-round experiments at a $B$ factory.

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