Large-$N$ analysis of $(2 + 1)$-dimensional Thirring model

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Abstract

We analyze $(2 + 1)$-dimensional vector-vector type four-Fermi interaction (Thirring) model in the framework of the $1/N$ expansion. By solving the Dyson-Schwinger equation in the large-$N$ limit, we show that in the two-component formalism the fermions acquire parity-violating mass dynamically in the range of the dimensionless coupling $\alpha$, $0 \leq \alpha \leq \alpha_c \equiv \frac{1}{16}\exp(-\frac{N\pi^2}{16})$. The symmetry breaking pattern is, however, in a way to conserve the overall parity of the theory such that the Chern-Simons term is not induced at any orders in $1/N$. $\alpha_c$ turns out to be a non-perturbative UV-fixed point in $1/N$. The $\beta$ function is calculated to be $\beta(\alpha) = -2(\alpha - \alpha_c)$ near the fixed point, and the UV-fixed point and the $\beta$ function are shown exact in the $1/N$ expansion.

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Recently, there has been a resurgence of interest in four-Fermi interaction partly due to the extraordinary heaviness of the top-quark, compared to other quarks and leptons \[1\]. One of the key ideas in this approach is that the 4-Fermi interaction, introduced in the standard electro-weak theory as a low energy effective interaction, becomes a relevant operator as the ultraviolet cutoff, $\Lambda$, goes to $\infty$, due to a strong interaction among fermions. When the 4-Fermi coupling is larger than a critical value, the 4-Fermi interaction induces the condensation of the top-quark, as shown in the original Nambu-Jona-Lasinio model \[2\]. Thus the top-quark gets a large mass, and the electro-weak symmetry breaks dynamically. As described below, similar dynamical behavior occurs in the $(2 + 1)D$ Thirring model.

The $(2 + 1)$-dimensional Thirring model is given in the Euclidean version by

$$L = i\overline{\psi}_i \partial_i \psi_i + \frac{g}{2N} \left( \overline{\psi}_i \gamma_\mu \psi_i \right) \left( \overline{\psi}_j \gamma^\mu \psi_j \right),$$

where $\psi_i$ are two-component spinors and $i, j$ are summed over from 1 to $N$. The $\gamma$ matrices are defined as

$$\gamma_3 = \sigma_3, \quad \gamma_1 = \sigma^1, \quad \gamma_2 = \sigma^2,$$

where $\sigma$’s are the Pauli matrices. Since the four-Fermi coupling $g$ has a mass-inverse dimension, the model is not renormalizable in ordinary (weak) coupling expansion. But, it has been shown renormalizable for $(2 + 1)$-dimensions in the large flavor ($N$) limit \[3\]. It is therefore sensible to analyze the $3D$ Thirring model in the large-$N$ expansion.

There are at least two ways of viewing the $3D$ Thirring model in treating the dimensional coupling constant $g$. One is taking $g$ as a genuine dimensional parameter that sets the natural scale of the theory; for example, the dynamically generated fermions, if any, will be proportional to this scale; $m_{\text{dyn}} \sim 1/g$. The another one is to take the dimensional parameter, $1/g$, as the UV cutoff of the theory; $g \equiv \frac{1}{\alpha N}$, where
Λ is the UV cutoff and α is a dimensionless coupling. Therefore, in this case, the only dimensional parameter in the model is the ultraviolet cutoff. In the continuum limit, the four-Fermi operator (together with the ultraviolet cutoff), \[ \frac{1}{\Lambda} \left( \bar{\psi} \gamma_{\mu} \psi \right)^2, \]
becomes a relevant operator in the large-\(N\) approximation. In this approach, if dynamical mass is generated, it will be independent of the ultraviolet cutoff, \(\Lambda\); it will be the one introduced in trade of the dimensionless parameter, \(\alpha\), by the so-called dimensional transmutation, which happens in any renormalizable theories.

The first viewpoint is taken by several authors. For instance, it has been shown in [4] that the 3D Thirring model is UV-finite at all orders of \(1/N\) since the scale \(1/g\) is negligible in the deep UV region. And also Gomes et. al. [5] found in this viewpoint that the model behaves similarly to QED\(_{2+1}\) [6], which also has a dimensional parameter, \(e\), the electric charge; in both models, the fermion mass is generated when \(1/N > 1/N_c\). But, as we shall see later, it is in the second viewpoint that 3D Thirring model is similar to 3D Gross-Neveu model [7]. Namely, 3D Thirring model has a two-phase structure, parity-broken and parity-unbroken, and the fermion acquires dynamical mass for strong coupling, \(g > g_c\) (or \(0 \leq \alpha \leq \alpha_c\)). Though the model is still UV-finite perturbatively in \(1/N\) expansion, there exists non-perturbative (in \(1/N\)) renormalization for \(\alpha\). The coupling is running, \(\beta(\alpha) = -2(\alpha - \alpha_c)\), for the same reason as in the Gross-Neuve model. The UV-fixed point \(\alpha_c\) is found to be \(\frac{1}{16} \exp(-\frac{N\pi^2}{16})\) in the \(1/N\) expansion. The UV-fixed point and the \(\beta\) function does not change at all, even if one includes the higher order corrections, due to the Ward-Takahashi identity and the UV structure of the theory. This is in contrast with the result in [4] presenting vanishing \(\beta\) function. Therefore, we see that two viewpoints are in many ways different from each other.

Now we start with the effective theory with a UV cutoff. Introducing an auxiliary
field $A_\mu$ to facilitate the $1/N$ expansion, we can rewrite Eq. (1) as

$$L = i\overline{\psi}_i \partial_\mu \psi_i - \frac{1}{\sqrt{N}} A_\mu \left( \overline{\psi}_i \gamma_\mu \psi_i \right) + \frac{1}{2} \alpha \Lambda A_\mu^2,$$

(3)

where $\alpha \Lambda = \frac{1}{g}$. As was mentioned in [5], the theory is consistent for positive $\alpha$. As we shall see later, for negative $\alpha$, the theory is unstable showing tachyons in the four-point fermion Green’s function. Eq. (3) is not gauge invariant under the usual gauge transformation on $\psi$ and $A_\mu$. However, as was claimed in ref. [5], Eq. (3) with a gauge fixing term has a restricted gauge symmetry. In this paper we choose to work in the Landau gauge. The Thirring model with $N$ two-component complex spinors has $U(N)$ global symmetry and parity also. Under $U(N)$,

$$\psi_i \mapsto \psi'_i = g_i^j \psi_j,$$

(4)

and under parity, $P : x = (x, y, t) \mapsto x' = (-x, y, t)$, the fermion fields transform as

$$\psi(x) \mapsto \psi'(x') = e^{i\delta} \sigma^1 \psi(x)$$

(5)

One can see that the fermion mass term is parity-odd. When the number of fermion flavors is even, the model has another obvious discrete $Z_2$ symmetry, which interchanges half of the fermions with the other half: $Z_2$ mixes the fermion fields as, for $i = 1, \cdots, \frac{N}{2},$

$$\psi_i(x) \mapsto \psi_{\frac{N}{2}+i}(x)$$

$$\psi_{\frac{N}{2}+i} \mapsto \psi_i(x)$$

(6)

We define a new parity $P_4$ which combines the parity for the two-component spinor with $Z_2$, $P_4 \equiv P Z_2$ [8]. As described below, in the $(2 + 1)D$ Thirring model, it is $P$ (not $P_4$) that is spontaneously broken. The fermion mass is dynamically generated in such a way $P_4$ is conserved. When $P_4$ is not broken, the Chern-Simons term is not induced.
Now we will examine the pattern of the spontaneous breaking of parity. An order
parameter for the spontaneous breaking of parity is the vacuum condensate of the
fermion bilinear, \( \langle \bar{\psi} \psi(x) \rangle \), which will be determined once one finds the (asymptotic)
behavior of the fermion propagator.\[\text{(3)}\]

In the \( 1/N \) expansion one has the following Dyson-Schwinger gap equation,
\[
- (Z(p) - 1) \not{p} + \Sigma(p) = \frac{1}{N} \int_{-\Lambda}^{\Lambda} \frac{d^3k}{(2\pi)^3} D_{\mu\nu}(p-k) \gamma_\nu \frac{Z(k) k^2 - \Sigma(k)}{Z^2(k)k^2 + \Sigma^2(k)} \Gamma_\mu(k, p-k; p),
\]
where \( D_{\mu\nu} \) is the photon propagator, \( \Sigma \) is the fermion self energy, \( Z \) is the fermion
wave function renormalization constant, and \( \Gamma_\mu \) is the vertex function. In the Landau
gauge, the photon propagator is given by
\[
D_{\mu\nu} = \frac{1}{p^2} \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \Pi_1(p^2) + \epsilon_{\mu\nu\rho} \frac{p_\rho}{|p|^2} \Pi_2(p^2),
\]

where \( \Pi_1 \) and \( \Pi_2 \) are given by
\[
\Pi_1(p^2) = \frac{\alpha \Lambda + \Pi_e}{(\alpha \Lambda + \Pi_e)^2/p^2 + \Pi_o^2},
\]
\[
\Pi_2(p^2) = \frac{\Pi_o}{(\alpha \Lambda + \Pi_e)^2/p^2 + \Pi_o^2},
\]
The resummation technique of the \( 1/N \) expansion results in the nontrivial photon
propagator as given above. \( \Pi_e \) [\( \Pi_o \)] in Eq. (9) [\( \Pi_{10} \)] is the even (odd) part of the
vacuum polarization which will be determined once we solve the above coupled
Dyson-Schwinger equations, Eq.’s (7) and (8).

Since \( Z(p) = 1 + O(1/N) \) and \( \Gamma_\mu = \gamma_\mu + O(1/N) \), we may take, at the leading
order in \( 1/N \), \( Z(p) = 1 \) and \( \Gamma_\mu = \gamma_\mu \) consistently in Eq.(7). Then, taking trace over
the gamma matrix, we get
\[
\Sigma(p) = \frac{1}{N} \int_{-\Lambda}^{\Lambda} \frac{d^3k}{(2\pi)^3} \frac{2\Pi_1(p-k) \cdot k \Sigma(k)}{(p-k)^2 + \Sigma^2(k)} + \frac{1}{N} \int_{-\Lambda}^{\Lambda} \frac{d^3k}{(2\pi)^3} \frac{(p-k) \cdot k \Pi_2(p-k)}{|p-k|^3 k^2 + \Sigma^2(k)}
\]
\( \text{(11)} \)
The magnitude of dynamically generated mass must be small, compared to the
cutoff, $\Lambda$, of the theory, in the $1/N$ approximation [6]. We may therefore assume
$\Sigma(p) \ll p \ll \Lambda$. The vacuum polarization tensor takes then a simple form;

$$\Pi_e(p) = \frac{|p|}{16},$$

$$\Pi_o(p) = \frac{1}{N} \sum_{i=1}^{N} M_i \frac{1}{4 |p|},$$

where $M_i \simeq \Sigma_i(0)$, the mass of the $i$-th fermion, In general, it is hard to find $M_i$ by
solving the gap equations directly. But, following the same argument of Coleman
and Witten [10], one can easily show that the magnitude of $M_i$ is independent of
$i$ in the large $N$ limit [11]. Therefore, it is reasonable to assume that $M_i = M$ for
$i = 1, ..., N - L$ and $M_i = -M$ for $i = N - L + 1, ..., N$, as is done in [6]. For
momenta $p$ such that $M \ll p \ll \Lambda$,

$$\frac{\Pi_1(p)}{p^2} = \frac{1}{\alpha \Lambda + \frac{|p|}{16}},$$

$$\frac{\Pi_2(p)}{p^2} = \frac{M \theta}{4 |p|} \frac{1}{\alpha \Lambda + \frac{|p|}{16}},$$

where $\theta = 1 - \frac{2L}{N}$, a parameter characterizing the parity($P_4$) violation of the theory.
Now we will show that $\theta = 0$ admits a consistent solution of the gap equation. Taking
the fermion self energy at zero momentum from Eq. (7) and letting $M_i \simeq \Sigma_i(0)$, we
find that

$$M_i = \frac{2}{N} \int_{\Lambda}^{\Lambda} \frac{d^3k}{(2\pi)^3} \frac{M_i}{k^2 + M^2} \frac{1}{\alpha \Lambda + \frac{|k|}{16}} - \frac{1}{N} \int_{\Lambda}^{\Lambda} \frac{d^3k}{(2\pi)^3} \frac{k^2}{k^2 + M^2} \frac{M \theta}{4 |k|} \frac{1}{\alpha \Lambda + \frac{|k|}{16}}.$$
mass. This phase factor increases the ground state energy, and so the model with the lowest ground state energy should be the one in which the overall phase is minimized. Since $\text{Det}(\mathcal{J} + M + i\mathcal{A})$ is the complex conjugate of $\text{Det}(\mathcal{J} - M + i\mathcal{A})$, the overall phase vanishes provided the number of positive mass fermions is equal to the number of negative ones, which is possible only when $N$ is even.

Actually, one can show further that $\theta = 0$ at all orders of $1/N$ using non-renormalization theorem of Coleman and Hill [13]. The requirement of this non-renormalization theorem is the gauge invariance and analyticity of the one-loop $n$-photon functions, which is fulfilled in the $\alpha \leq \alpha_c$ in this model. In this range of the coupling $\alpha$, the Chern-Simons term is not subject to the radiative corrections beyond one loop, thus the one loop result is exact. One speculates, however, finite radiative correction to the Chern-Simons term in the other phase due to the lack of analyticity of the fermionic loops in the infrared region [14].

Then above Vafa and Witten’s argument ensures the cancellation of the Chern-Simons term generated from each fermion and this explains the absence of the corrections at all.

Following the Cornwall, Jackiw, and Tomboulis formalism [15], we calculate the effective potential of the operator expectation value $\langle \bar{\psi}\psi(x) \rangle$. At the extrema it is found to be

$$V = \frac{N}{2\pi^2} \int^\Lambda dp p^2 \left[ \frac{\Sigma^2}{p^2 + \Sigma^2} - \ln \left( 1 + \frac{\Sigma^2}{p^2} \right) \right].$$

(17)

This is the same expression as in QED$_{2+1}$ in the $1/N$ expansion [3]. It can be easily seen from Eq. (17) that any nontrivial solution has a lower energy than the perturbative vacuum solution, $\Sigma(p) = 0$. Therefore, once such a parity-breaking solution is found, it is always energetically favored to the symmetric one. Our solution to the gap equation has thus lower vacuum energy than the trivial solution.
Rewriting the Eq. (16) when $\theta = 0$,

$$M = \frac{2}{N} \int^{\Lambda} \frac{d^3k}{(2\pi)^3} \frac{M}{k^2 + M^2} \frac{1}{\alpha \Lambda + \frac{|k|}{16}}. \quad (18)$$

The above equation indicates that there is a two-phase structure. When $\alpha > \alpha_c$, the parity symmetry is manifest, where the critical value $\alpha_c$ is defined by the following equation

$$1 = \frac{2}{N} \int^{\Lambda} \frac{d^3k}{(2\pi)^3} \frac{1}{k^2} \frac{1}{\alpha_c \Lambda + \frac{|k|}{16}}. \quad (19)$$

If $\alpha \leq \alpha_c$, non-trivial parity-violating fermion mass is generated in a way to preserve the total parity symmetry of the theory.

From Eq. (19), one obtains

$$\alpha_c = \frac{1}{16} \exp\left(-\frac{1}{16} N \pi^2\right). \quad (20)$$

From the above equation, one sees the non-perturbative nature in the phase transition point. The $1/N$ factor in the gap equation (18) has been traded to the non-analytic factor $N$ in $\alpha_c$. The similar phenomena can be seen in the exponential hierarchy between the dynamically generated fermion mass and the cutoff in QED$_{2+1}$ [4]. The parity violating region, $0 \leq \alpha \leq \alpha_c$ is very small, so the theory has the parity symmetry for almost all the positive region of $\alpha$.

The cutoff dependence of the bare coupling is determined by the requirement that the gap equation, Eq. (18), be independent of the UV cutoff, $\Lambda$, as taken to $\infty$. In the vicinity of $\alpha_c$, we get the $\beta$ function for $\alpha$ given by

$$\beta(\alpha) \equiv \Lambda \frac{\partial \alpha}{\partial \Lambda} = -2(\alpha - \alpha_c). \quad (21)$$

$\alpha$ increases (decreases) to $\alpha_c$ when $\alpha < \alpha_c$ ($\alpha > \alpha_c$). $\alpha_c$ is the UV-fixed point of both phases of the Thirring model. The above equation shows that $\alpha_c$ is a UV-fixed point which is again non-perturbative in $1/N$ (it has nonanalytic dependence on
Therefore, it is not feasible to study the theory in the vicinity of $\alpha_c$ with the perturbative $1/N$ expansion. To observe the effects of the running of the coupling $\alpha$ as in Eq. (21), one need to go beyond the perturbative expansion; one need to solve, for example, the Dyson-Schwinger equation, just as is done in this paper.

As we shall see later, the theory is UV-finite at any finite orders of $1/N$. Within the $1/N$ expansion the UV behavior of the theory is more improved than the one in the Gross-Neveu type model. In the latter, the resummation technique of the $1/N$ expansion results in the UV dimension of the auxiliary field $\sigma = \bar{\psi}\psi$ being 1 (2 classically). This is a key point of the renormalizability of the theory. In the Thirring model the mass of the photon propagator in Eq.(8) should not be neglected at high momenta. It renders all the integration UV-finite [16].

This can be checked by a direct calculation. For instance, the fermion wave function renormalization $Z$ at $O(1/N)$ is given by Eq.(7). It is not difficult to see that the integration is finite. This (perturbative) UV finiteness of the theory is consistent with the non-perturbative nature of the UV-fixed point $\alpha_c$.

As we mentioned in the introductory part of this paper, the mass $M$ is that it is not a value which can be determined as in QED$_{2+1}$ [6] but a parameter as in Gross-Neveu type model [3]. $M$ is a physical quantity, in fact it is the pole mass of the fermions, and therefore it should be independent of $\Lambda$. As in the case of Gross-Neveu model, one may interpret Eq. (18) as fine-tuning the coupling $g$ in order to have $M \ll \Lambda$. In other words, for $M \ll \Lambda$, the coupling is tuned to be very close, or equal, to the critical value Eq. (20).

The four-point fermionic Green’s function, in leading order, is given by the photon propagator in Eq. (8). We see that there are no tachyons for $g > 0$, so the theory is consistent in that region. For $g < 0$, the Green’s function shows the existence of tachyons.
Finally, we show that the features of the 3D Thirring model we have found are exact in the $1/N$ expansion even if one includes the higher order corrections. To go beyond the leading order, one should solve the Dyson-Schwinger equation keeping the higher order corrections in the propagators and the vertex function. But, by following general arguments, the Dyson-Schwinger equation takes a simpler form. Firstly, since $\theta = 0$ in all orders in $1/N$ as we showed earlier, we can set $\Pi_\nu(p) = 0$ in the photon propagator. Secondly, beyond the leading order, the magnitude of $M_i$ is same for all flavors, $|M_i| = M$, which can be seen easily by an argument similar to that of Coleman and Witten \cite{coleman}. Therefore, the gap equation Eq.(18) becomes

$$M = \frac{2}{N} \int \frac{d^3k}{(2\pi)^3} \frac{M}{Z(k)k^2 + M^2 \alpha \Lambda + \Pi_e(k)} \Gamma(k,0;k),$$

(22)

where $\Gamma(k,0;k) = \frac{1}{4} g^{\mu\nu} \text{Tr} \gamma_\mu \Gamma_\nu(k,0;k)$. Keeping terms up to $O(1/N)$, we find by explicit calculations that

$$Z(k) = 1 - \frac{16}{3N\pi^2} \ln \left( \frac{16\alpha + 1}{16\alpha} \right) + O \left( \frac{1}{\Lambda} \right),$$

(23)

$$\Gamma_\mu(k,0;k) = \gamma_\mu \left( 1 - \frac{16}{3N\pi^2} \ln \left( \frac{16\alpha + 1}{16\alpha} \right) \right) + O \left( \frac{1}{\Lambda} \right).$$

(24)

Similarly, the even part of the vacuum polarization is, up to the terms in $1/N$,

$$\Pi_e(k) = \frac{|k|}{16} + \text{const.} \times \frac{1}{N} \cdot \frac{k^2}{\alpha \Lambda} \ln \frac{\Lambda}{M} + O \left( \frac{1}{\Lambda} \right).$$

(25)

where $\text{const.}$ is a pure number. The Feynman diagrams relevant to the corrections are shown in Figure 1.

Above results, Eq.’s (23)-(25), show that the next-to-leading corrections are either finite or suppressed by $1/\Lambda$; the $1/N$ corrections are UV-finite. By dimensional counting, one can easily show that the 3D Thirring model is in fact UV-finite in all orders in the $1/N$ expansion \cite{17}.

This is the same result as in \cite{4}, but the reason is quite different. In our case, because of $\alpha \Lambda$ in the photon propagator, the loop integrations are UV-finite.
The dangerous terms in deriving the UV-fixed point $\alpha_c$ will be the terms which are not suppressed as $\Lambda \to \infty$. But, such terms do not occur in any orders in $1/N$ because of the UV finiteness of the theory. Since the higher order corrections to the vacuum polarization are suppressed by $1/\Lambda$, the equation defining $\alpha_c$ becomes now

$$1 = \frac{2}{N} \int_0^\Lambda \frac{d^3 k}{(2\pi)^3} \frac{1}{(1 - F(\alpha_c)) k^2} \frac{1}{\alpha_c \Lambda + \frac{|k|}{16}} \cdot (1 - F(\alpha_c)) + O\left(\frac{1}{\Lambda}\right)$$

where $F(\alpha_c) = \frac{16}{3N\pi^2} \ln \left(\frac{16\alpha_c + 1}{16\alpha_c}\right) + O(1/N^2)$.

We see that in the above equation the contributions from the $1/N$ corrections in Eq.’s (23) and (24) cancel each other exactly. This is due to the Ward-Takahashi identity of the gauge-invariance in the 3D Thirring model. The equation defining $\alpha_c$ is therefore same for all orders in $1/N$, and the UV-fixed point and the $\beta$ function we found earlier are in fact exact in the $1/N$ expansion.

In summary, we have analyzed the (2+1)-dimensional Thirring model in the $1/N$ expansion. It has been shown that the fermions acquire parity-violating mass in the way to conserve the total parity symmetry of the theory. The Chern-Simons term is not generated at any finite orders in $1/N$. We have also shown that the theory has a two-phase structure and a UV-fixed point as in the Gross-Neveu model, albeit at non-perturbative order in $1/N$. The UV-fixed point, $\alpha_c = \frac{1}{16} \exp\left(-\frac{N\pi^2}{16}\right)$, and the $\beta$ function, $\beta(\alpha) = -2(\alpha - \alpha_c)$, which we have found by solving the Dyson-Schwinger, are shown exact in the $1/N$ expansion.

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[16] In [5], the same theory has been analyzed keeping the photon mass as a parameter, which leads to the similar behavior of $\Sigma(p)$ to the one in QED.

[17] Suppose the photon mass in the propagator is negligible. Then, the most divergent terms are logarithmic, $\sim \ln \Lambda$, since $3D$ QED is renormalizable in the $1/N$ expansion. But, because the photon mass term is proportional to $\Lambda$, any internal photon line decreases the UV divergence by dimension one, and thus the logarithmic divergence is absent in all orders in $1/N$; $\int_{\Lambda}^{\Lambda} \frac{dk}{\Lambda+k} = \ln 2$, while $\int_{\Lambda}^{\Lambda} \frac{dk}{k} = \ln \Lambda$. 
Figure Caption

**Figure 1:** The Feynman diagrams relevant to the $1/N$ corrections (wavy line - photon, solid line - fermion).