First results from the asymmetric $\mathcal{O}(a)$ improved Fermilab action

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We present first results from calculations using an $\mathcal{O}(a)$ improved (FNAL) space-time asymmetric fermion action on a $12^3 \times 24$ quenched lattice at $\beta = 5.7$ and with $c_{SW} = 1.57$. The mass dependent asymmetry parameter $\zeta$ is determined non-perturbatively from the energy-momentum dispersion relation. Calculations have been made in the charm and bottom quark mass sectors in order to test the $\zeta$ dependence of the spectrum, since it is at these heavier masses that the asymmetry is expected to be most relevant.

1. The Fermilab improved action

For full details of this fermionic improvement scheme the reader is referred to [1]; here we merely note that it results in an action with mass dependent coefficients which is asymmetric in space and time.

The lattice dispersion relation may be written in the form

$$E^2(p^2) = M_1^2 + \frac{M_1}{M_2} p^2 + O(p^4),$$

defining the static mass $M_1 = E(0)$ and the kinetic mass $M_2 = \left. \left(\frac{\partial^2 E}{\partial p^2}\right) \right|_{p=0}$.

Lattice discretisation effects mean than in general $M_1 \neq M_2$ at $\mathcal{O}(a_{\text{quark}})$, and restoring the relativistic dispersion relation to this order is the first stage in the improvement program. This is achieved by introducing an asymmetry in the temporal and spatial quark propagation and adjusting it until $M_1 = M_2$, which then constitutes the first improvement condition of this scheme.

To this end we define the action

$$S_0 = \sum_x \left\{ \bar{\psi}_x \psi_x - \kappa_t [ \bar{\psi}_x (1-\gamma_0) U_{0x} \psi_{x+0} 
+ \bar{\psi}_x (1+\gamma_0) U_{0x-0} \psi_{x-0} ] 
- \kappa_s \sum_i [ \bar{\psi}_x (1-\gamma_i) U_{i,x} \psi_{x+i} 
+ \bar{\psi}_x (1+\gamma_i) U_{i,x-i} \psi_{x-i} ] \right\},$$

(2)

It is helpful to parameterise this asymmetry by defining $\zeta = \kappa_s / \kappa_t$, in terms of which the quark mass is

$$M_0 = \frac{1}{2\kappa_t} - 3\zeta - 1 - \left( \frac{1}{2\kappa_{\text{crit}}} - 4 \right).$$

(3)

At some value $\zeta = \zeta_{NP}$, which we attempt here to find at various quark masses, $M_1 = M_2$.

In order to remove $\mathcal{O}(a)$ artifacts from the action the terms

$$S_E = i\kappa_x c_E \sum_{x,i} \bar{\psi}_x \sigma_{0i} F_{0i}(x) \psi_x$$

(4)

and

$$S_B = i\kappa_x c_B \sum_{x,i<j} \bar{\psi}_x \sigma_{ij} F_{ij}(x) \psi_x$$

(5)
2. Non-perturbative tuning of $\zeta$

The strategy employed here was to tune $\zeta$ by requiring that $M_1 = M_2$ for the spin-averaged $1S$ quarkonium state. The results obtained from this non-perturbative tuning can be compared where appropriate with tree-level perturbation theory and with results obtained using the symmetric ($\zeta = 1$) action.

These calculations were performed on 100 quenched $12^3 \times 24$ configurations at $\beta = 5.7$ with $c_E = c_B = 1.57$, the tree-level tadpole improved perturbative value on this lattice.

To find $M_1$ and $M_2$, the ground state energy $E(p)$ was computed at five momenta using a two-state fit to a matrix of smeared correlators (as described in [4]). $M_1$ is simply $E(0)$ and $M_2$ was extracted from the coefficient $a_1$ obtained by fitting the dispersion relation to the function

$$E(p^2) = a_0 + a_1 p^2 + a_2 (p^2)^2 + a_3 \sum p_i^4.$$  \hfill (6)

We obtain a graph (figure 1) of the non-perturbatively tuned $\zeta_{NP}$ as a function of $M_0$ which we compare with tree level perturbation theory. The extent to which $\zeta_{NP}$ satisfies the improvement condition can be judged from figure 2, where for comparison the corresponding points previously obtained using the symmetric action on the same lattice are also plotted.

While figure 2 presents the dependence of $\zeta_{NP}$ upon $M_0$, it is $M_2$ that emerges as the physically significant mass parameter in the heavy quark expansion. Therefore it is useful to know how $M_2$ depends on $M_0$ once $\zeta$ has been tuned, and this is shown in figure 3.

3. Quarkonium spectrum

The values of $aM_2$ that correspond on this lattice to the $c\bar{c}$ and $b\bar{b}$ mesons are known from the previous calculations with the symmetric action, and parameters of the asymmetric action yielding values of $aM_2$ comparable to these were found (see figures 3 and 4). To verify that the asymmetric action reproduces the same physics a spectral calculation was performed at these parameters on 300 configurations. $2S$ states were obtained using a three-state fit to the full correlator matrix. The scale was set from the spin-averaged $1P-1S$ splitting.

Figure 2. $M_2$ and $M_1$ at $\zeta_{NP}$ compared with the $M_1 = M_2$ line and the corresponding results from the symmetric action.

Figure 3. The variation of $M_2$ with $M_0$ for the tuned action, compared with the symmetric action and tree-level perturbation theory.
mass. The hyperfine splitting is shown in figure 5 as a function of $M^2$ for all the data sets examined (i.e. with untuned asymmetric actions as well as at $\zeta_{NP}$ and $\zeta = 1$).

4. Conclusions

We have demonstrated the feasibility of a non-perturbative tuning of the first parameter of the Fermilab improved action, and find this value to be rather higher than the tree level prediction, although the mass dependence is already qualitatively predicted at tree level. The bare quark mass required to reach a particular physical régime is found to be much greater than for the symmetric action (again this behaviour is qualitatively predicted at tree level) which can have algorithmic implications in the quark propagator computation.

The quarkonium spectra (and a similar analysis of the hyperfine, fine structure and the $2S-1S$ splittings) show that the tuned action produces the expected physics, and supports the use of $M_2$ as the physically relevant mass scale in computations where $\zeta$ is not tuned. Further confirmation comes from figure 5 where it can be seen that the hyperfine splittings lie on the same curve regardless of whether the action is tuned or not, indicating that it is dependent on $M_2$ only.

Acknowledgements

This work was carried out in collaboration with Aida El-Khadra, Jim Simone, Andreas Kronfeld and Paul Mackenzie.

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