Mobile impurity in a Bose-Einstein condensate and the orthogonality catastrophe

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We analyze the properties of an impurity in a dilute Bose-Einstein condensate (BEC). First, the quasiparticle residue of a static impurity in an ideal BEC is shown to vanish with increasing particle number as a stretched exponential, leading to a bosonic orthogonality catastrophe. Then we introduce a variational ansatz, which recovers this exact result and describes the macroscopic dressing of the impurity including its back-action onto the BEC as well as boson-boson repulsion beyond the Bogoliubov approximation. This ansatz predicts that the orthogonality catastrophe also occurs for mobile impurities, whenever the BEC becomes ideal. Finally, we show that our ansatz agrees well with experimental results.

A single distinguishable particle interacting with a quantum bath, often referred to as the polaron problem, is one of the simplest realizations of a non-trivial quantum many-body system. Its fundamental nature attracted considerable interest since the early days of quantum mechanics, beginning with Landau’s seminal paper on electron dressing by phonons [1]. Polaron physics received renewed attention with the advent of ultracold atoms experiments, and it has been extensively studied especially in the case of the bath being a Fermi sea, realizing the Fermi polaron. Owing to a close interplay between theoretical advances [2–5] and state-of-the-art experimental observations [6–11], Fermi polarons are now quite well understood even for strong coupling [12–14].

Recently, the corresponding Bose polaron problem in which impurities are immersed in a dilute BEC has been studied experimentally in three seminal works [15–17]. Since a bosonic bath is much more compressible than its Fermi analogue and undergoes a phase transition at low temperatures, the problem of an impurity in a BEC exhibits richer few- and many-body physics. Theoretical works so far focused on the coupling of the impurity to Bogoliubov excitations of the BEC [18–21], few-body bound states [22–25], Quantum Monte-Carlo (QMC) studies of the ground state [26–28], as well as finite temperature effects [29–32]. So far, there has, however, been little focus on one fundamental question: what is the fate of the Bose polaron when the interactions in the BEC vanish so that the Bose gas becomes infinitely compressible, allowing for a macroscopic dressing of the impurity?

In this letter, we carefully analyze this question. First, we show analytically how, for a static impurity in an ideal BEC, the ground state overlap with the non-interacting state vanishes as a stretched exponential with particle number, leading to a bosonic orthogonality catastrophe (OC). We then develop a variational ansatz, which allows for a macroscopic dressing of the impurity, including the back-action on the BEC as well as boson-boson repulsion beyond the Bogoliubov approximation. The ansatz, which recovers the exact result for a static impurity, predicts that the OC also occurs for an impurity with finite mass when the Bose gas becomes non-interacting. A physical picture emerges where the BEC scatters coherently with the impurity, and a large number of bosons builds a macroscopic but very dilute dressing cloud. Intriguingly, the properties of the polaron are demonstrated analytically to be given by expressions similar to those obtained from perturbation theory, even in the regime governed by the OC at small boson-boson repulsion. Finally, we show how our ansatz recovers experimental observations.

A static impurity in an ideal BEC.— We start by analyzing a static impurity at zero temperature in an ideal gas of $N$ identical bosons of mass $m_B$ within a sphere of radius $R$. For vanishing boson-boson interaction the ground state is simply the product $\prod_{j=1}^{N} \psi(r_j)$ of the lowest energy single-particle wave function. In absence of the impurity, the single particle wave function is $\psi_0(r) = (2\pi R)^{-1/2} \sin(k_0 r)/r$ where $k_0 = \pi/R$ and $r = |r|$. Introducing an impurity in the center that interacts with the bosons via a potential of the form $U(r)$ with short range $r_0 \ll R$, the normalized single-particle wave functions for $r \gg r_0$ become

$$\psi_1(r) = \frac{1}{\sqrt{2\pi R}} \frac{\sin \left( k_1 r + \delta \right)}{r},$$

(1)

where $k_1 = k_0 - \delta/R$, and the phase shift $\delta = - \arctan(ak_1)$ is determined by the boson-impurity scattering length $a$. In the thermodynamic limit where
N, \ R \to \infty \ at \ fixed \ density, \ both \ k_0, k_1 \to 0, \ and \ for \ any \ finite \ \alpha < 0 \ one \ has \ \delta \approx -\alpha k_0 \ll 1. \ The \ difference \ \Delta E \ in \ ground \ state \ energies \ defines \ the \ polaron \ energy \ [33]\n
$$\Delta E_0 = N \frac{\hbar^2 (k_1^2 - k_0^2)}{2m_B} = N \frac{\hbar^2 k_0^2 R}{m_B R} = 2\pi \hbar^2 a/m_B n_0, \quad (2)$$

where $n_0 \equiv N |\psi_0(0)|^2 = \pi N/2R^3$ is the BEC density in the center in absence of the impurity.

The overlap between the ground states with and without the impurity is quantified by the residue $Z_0 = |\langle \Psi_0 | \Psi_1 \rangle|^2 = |\langle \psi_0 | \psi_1 \rangle|^{2N}$. Introducing $k_n = (6\pi^2 n_0)^{1/3}$, we obtain for large system sizes

$$Z_0 = \left[ 1 - \alpha (k_n a)^2 / (2N^{2/3}) \right]^{2N} \approx e^{-\alpha N^{1/3}(k_n a)^2}, \quad (3)$$

where $\alpha \equiv (\pi^2/3 + 1/4)/(3\pi^2)^{2/3}$. Thus, the residue vanishes as a stretched exponential with increasing particle number, giving rise to a bosonic orthogonality catastrophe (bosonic OC). This behavior is even more drastic than the one Anderson predicted for a fermionic bath [34], where the overlap vanishes as a slower power law, due to an infinity of particle-hole excitations in the Fermi sea.

The bosonic OC emerges instead because the ideal BEC is infinitely compressible, so that a macroscopic dressing cloud can gather around the impurity.

A mobile impurity in an interacting BEC.– We now explore how a finite impurity mass and boson-boson repulsion affect the OC. Taking a mobile impurity of mass $m_I$, the Hamiltonian is

$$\hat{H} = \int d^3 r \left[ \frac{\hbar^2 \nabla_r^2}{2m_B} + \frac{T_B}{2} \hat{b}_r^\dagger \hat{b}_r - \mu \right] \hat{b}_r$$

$$+ \hat{a}_s^\dagger \left( \frac{\hbar^2 \nabla_s^2}{2m_I} \right) \hat{a}_s + \int d^3 s \ \hat{b}_r^\dagger \hat{b}_r U(\mathbf{r} - \mathbf{s}) \hat{a}_s^\dagger \hat{a}_s, \quad (4)$$

where $\hat{a}_s^\dagger$ and $\hat{b}_r^\dagger$ create an impurity and a boson, respectively, at position $\mathbf{r}$, and $\mu$ is the chemical potential. The interaction between bosons is given by the regularized potential $T_B = 4\pi\hbar^2 a_B/m_B$ where $a_B$ is the boson-boson scattering length, which is consistent as long as $n_0 a_B^3 \ll 1$ for any local density $n(\mathbf{r})$ in the BEC. As before, the interaction between the bath and the impurity is modeled through a short-ranged potential $U(\mathbf{r}) = U(r)$. We now introduce a variational ansatz for the ground state of Eq. (4), which smoothly connects to the exact result for a static impurity in an ideal BEC. The ansatz includes finite mass effects and accounts for the boson-boson repulsion beyond the Bogoliubov approximation. To this end, consider the state

$$|\Psi_0| = \int \frac{d^3 r}{\sqrt{V}} \hat{a}_s^\dagger \exp \left( \int d^3 s \ \phi(\mathbf{r} - \mathbf{s}) \hat{b}_s^\dagger - c.c. \right) |0\rangle$$

$$= \int \frac{d^3 r}{\sqrt{V}} |\mathbf{r}| |\phi(\mathbf{r})\rangle, \quad (5)$$

where $V$ is the system volume. It describes a BEC given by the coherent state $\hat{b}_s^\dagger |\phi(\mathbf{r})\rangle = |\phi(\mathbf{r} - \mathbf{s})| |\phi(\mathbf{r})\rangle$, which adjusts to the position of the impurity. It follows that $\langle \hat{n}_1(\mathbf{r}) \hat{n}_1(\mathbf{r} + \mathbf{s}) \rangle \sim |\phi(\mathbf{s})|^2$, such that $\phi$ can be regarded as the impurity-bath density-density correlation function. Assuming a spherically symmetric ground state $\phi(\mathbf{r}) = |\phi(\mathbf{r})|$, straightforward algebra yields [35]

$$\langle \hat{H} \rangle = \int d^3 r \phi^* (r) \left[ \frac{\hbar^2 \nabla_r^2}{2m_B} + U(r) + \frac{T_B}{2} |\phi(\mathbf{r})|^2 - \mu \right] \phi(r) \quad (6)$$

with $m_r^{-1} = m_B^{-1} + m_I^{-1}$ the reduced mass. Minimizing Eq. (6) gives

$$\left[ -\frac{\hbar^2 \nabla_r^2}{2m_r} + U(r) + T_B |\phi(\mathbf{r})|^2 \right] \phi(r) = \mu |\phi(\mathbf{r})|^2, \quad (7)$$

Equation (7) is remarkably simple: The mobile impurity is described by a modified Gross-Pitaevskii equation (GPE) where the kinetic term contains the reduced mass, and the bosons scatter collectively on the impurity. Note that the back-action on the BEC due to the dressing cloud around the impurity and boson-boson interactions are naturally included in the GPE, an effect that is not fully accounted for in a Bogoliubov approach to the problem that expands in fluctuations around a homogeneous BEC [21, 31]. For infinite impurity mass we have $m_r = m_B$, and Eq. (7) reduces to the standard GPE for a static potential $U(r)$. If, moreover, $a_B = 0$ it reduces to the one-body Schrödinger equation thereby recovering the exact results for the bosonic OC described above.

To investigate the ground state of the impurity, we solve Eq. (7) subject to the condition $\mu = n_0 \xi$, which ensures that the density far away from the impurity converges to the density $n_0$ of the BEC in absence of the impurity. Introducing the dimensionless quantities $x = r/\xi$, $\phi(x) = \phi(x\xi)/\sqrt{n_0}$ and $U(x) = 2m_r \xi^2 U(x\xi)/\hbar^2$ with $\xi = (8\pi n_0 a_B m_r/m_B)^{-1/2}$, Eq. (7) becomes

$$-\nabla_x^2 + U(x) + |\phi(x)|^2 - 1 \phi(x) = 0, \quad (8)$$

showing that the generalized healing length $\xi$ is the natural length scale of the problem.

The polaron energy is the energy shift $\Delta E$ away from the solution with $\bar{U} = 0$. Using Eq. (8) in (6), one finds

$$\Delta E = -\frac{E_\xi}{2} \int d^3 x \ (|\phi(x)|^4 - 1), \quad (9)$$

where $E_\xi = \hbar^2 n_0 \xi / 2m_r$. The polaron quasiparticle weight $Z$ is the overlap between ground states of the gas with and without impurity. With the coherent ansatz one finds

$$Z = |\langle \Psi_0 | \Psi_1 \rangle|^2 = e^{-N_\xi \int d^3 x \ |\phi(x)| - 1|^2}, \quad (10)$$
where we defined \( N_\xi = n_0 \xi^3 \). This shows that changes in the condensate mode cause an exponential suppression of the overlap. Finally, the number \( \Delta N \) of bosons in the dressing cloud around the impurity is

\[
\Delta N = N_\xi \int d^3x \left( |\tilde{\phi}(x)|^2 - 1 \right).
\]

Condensate wave function.— The condensate wave function \( \phi(x) \) is obtained numerically. In our computation we employ three attractive potentials: a square well \( U_w \Theta(1-r/r_0) \), a Gaussian \( U_g \exp(-r/r_0)^2 \), and an exponential \( U_e \exp(-r/r_0) \), which give rise to effective ranges \( r_e \sim r_0 \), mimicking open-channel dominated resonances [36]. We tune their depth \( U \) independently, so that we can model different scattering lengths \( a \) at fixed effective range \( r_e \). With the wide range of parameters explored, the results given by these three potentials differ by less than the width of the lines in all figures, demonstrating an effective two-parameter universality (given by the scattering length \( a \) and range \( r_e \)) governing this problem. Note that in general one cannot use a zero-range potential for both the boson-boson and the boson-impurity interaction, because in that case Eq. (7) admits only a zero-energy polaron solution.

Numerically, the problem needs careful treatment, because next to the impurity the wave function varies on scales comparable to \( r_e \), while further away it evolves on a scale set by \( \xi \gg r_e \). To achieve sufficient accuracy despite this large separation of scales, we discretize the integral in Eq. (6) on a non-uniform grid featuring an exponentially-growing lattice spacing in the outward radial direction containing several thousand points both inside and outside the potential. For all computational results presented here, we used a grid with maximal radius \( R = 100\xi \), and boundary condition \( \phi(R) = \sqrt{n_0} \).

Results.— In Fig. 1 we plot the residue and energy of the polaron as well as the number of particles in its dressing cloud as a function of the boson-boson scattering length \( a_B \) for various impurity-boson interaction strengths. Fig. 1(a) shows that, in contrast to fermions, the OC persists even for mobile impurities when the BEC becomes ideal. A related finding was discussed in Refs. [21, 22]. In this limit, the residue \( Z \) vanishes and the number of particles \( \Delta N \) in the dressing cloud diverges for \( k_\text{n}a_B \to 0^+ \), see Fig. 1(b). The bosonic OC is cured when the bosons start to repel which leads to a suppression of particles in the dressing cloud.

Equation (8) can be solved analytically for \( |a|^2 \ll \xi^2 r_0 \) [37]. Under this condition, which is fulfilled for any finite \( |a| \) when \( a_B \to 0 \), one obtains for \( x \gtrsim r_0/\xi \) the Yukawa solution

\[
\tilde{\phi}(x) = 1 - (a/\xi)e^{-\sqrt{2}x/\xi}.
\]

Using this in Eqs. (9)-(11) gives

\[
\Delta E = 4\pi E_\xi a/\xi = 2\hbar^2 a_{n0}/m_r, \quad (13)
\]

\[
\log Z = -\sqrt{2}\pi N_\xi a^2/\xi^2 = -\sqrt{2}\pi n_0 \xi a^2, \quad (14)
\]

\[
\Delta N = -4\pi N_\xi a/\xi = -am_B/2a_B m_r. \quad (15)
\]

These expressions, which are recovered by our numerical results when \( k_\text{n}a_B \to 0 \) (so that \( a/\xi \to 0 \)) as shown in Fig. 1, analytically describe how the Bose polaron disappears in the limit of an ideal Bose gas. In particular, the residue vanishes exponentially with \( \log Z \propto 1/\xi \), and the number of particles in the polaron cloud grows as \( \Delta N \propto 1/a_B \) when the BEC loses its stiffness, leading to the build-up of a macroscopic screening cloud around the impurity and causing the bosonic OC. Intriguingly, Eqs. (13) and (14) are the same functional form as those obtained from an expansion in \( k_\text{n}a \) [20, 38], even though they are valid close to the bosonic OC, which must be expected to be well beyond the radius of convergence of perturbation theory.

The insets in Fig. 1 display \( |\phi(r)|^2 \) for \( 1/k_\text{n}a = -1/3 \) at boson-boson scattering lengths \( k_\text{n}a_B = 0.001 \) and 0.1.
They show how the bosons pile up around the impurity in a macroscopic dressing cloud of size $\sim \xi$. Importantly, in this case the local gas parameter $|\phi(0)|^2a_B^2$ remains small everywhere, even close to the impurity. In the most strongly interacting case, the unitary limit $|k_0a| \to \infty$, we found it to vanish as $\propto k_0aB(a_B^2/r_e)^{4/3}$ when $k_0aB \to 0$. This ensures that the assumption of a contact potential for the boson-boson interaction in Eq. (4) is consistent. It also shows that the large number of bosons in the dressing cloud is due to a large radius $\sim \xi$ and not to an exceedingly large density.

**Comparison with experiment.**—We now compare the predictions of our ansatz with experiments. Close to a Feshbach resonance, the effective range $r_e$, defined through the low-energy expansion of the phase shift $k\cot\delta = -1/a + r_e k^2/2 + O(k^4)$, varies slowly around its value right at resonance $[36, 39–41]$

$$r_e = -2R^* + 2\Gamma(1/4)^2 R_{vdW}/(3\pi),$$

(16)

Here, $R^* = h^2/2m a_0^2 \mu \Delta B$, $\Gamma(x)$ is the Gamma function, and $R_{vdW}$ is the van der Waals radius. For open-channel dominated resonances, one has $R_{vdW} \gg R^*$ such that $r_e \sim R_{vdW} \ll \xi$ in typical experiments. For example, the experiments in Aarhus [15], JILA [16], and MIT [17], featured $k_0\xi = 21.7$, 8, 9.6, respectively, giving $r_e/\xi = 0.002, 0.02, 0.01$ (using data from Refs. [40–42]).

Our numerical results for $\Delta E$ are shown in Fig. 2 together with the measurements from the experiments reported in Refs. [15, 16], which had mass ratios $m_1/m_B = 1$ and 40/87, respectively. Corresponding QMC results of Refs. [27, 28] are also displayed. The coherent state ansatz shows good agreement with the experimental data, even for impurities with mass comparable to $m_B$ or lighter. Our ansatz predicts large dressing clouds at unitarity, containing 20–180 bosons for the JILA and Aarhus case, respectively. This strong dressing, however, is accompanied by an extremely small residue $z$ at resonance and therefore a very small spectral weight of the ground state in the experimental radio-frequency spectrum at odds with other theories that recover the experimentally measured spectrum [19, 23].

**Contact.**—We finally examine Tan’s contact parameter, which quantifies the short-range correlations between the impurity and the atoms in the BEC. It can be obtained from Tan’s adiabatic theorem [43–47]

$$C = \frac{8\pi m_e}{\hbar^2} \frac{\partial (\Delta E)}{\partial (1/a)} = -4\pi n_0 \xi^2 \frac{\partial (\Delta E/E_\xi)}{\partial (\xi/a)},$$

(17)

Our ansatz gives to leading order in $a/\xi$

$$C_1 = (4\pi a/\xi)^2 n_0 \xi^2 = 16\pi^2 n_0 a^2,$$

(18)

which agrees with the leading order result of perturbation theory. In the inset of Fig. 2 we show the dependence of the contact as function of $-\xi/a$ for ratios $r_e/\xi$ and mass ratios appropriate for the experiment at MIT [17]. In this experiment the contact, shown as a red square, was obtained from the tail of the radio-frequency response at finite temperatures.

**Discussion and outlook.**—Using an ansatz describing the macroscopic dressing of the impurity and the back-action on the BEC including the boson-boson repulsion beyond the Bogoliubov approximation, we carefully analyzed the fate of the polaron with decreasing boson-boson interaction. We showed that the polaron disappears for $a_B \to 0$ resulting in a bosonic orthogonality catastrophe also when it has a finite mass. Strikingly, our ansatz predicts that the properties of the polaron are accurately described by expressions similar to perturbation theory even in a regime where the polaron picture ceases to be valid, and perturbation theory becomes formally invalid. It would be very interesting to examine this experimentally for instance using a Feshbach resonance to tune $a_B$, and employing Rabi [7] or Ramsey [9, 48, 49] spectroscopy. Also, the predicted large dressing clouds suggest potentially strong induced impurity-impurity interactions, which could affect the spectrum even for small impurity concentrations [50, 51].

**Note added.**—During submission of this manuscript,
we learned of interesting parallel theoretical work focusing on the dynamical properties of an impurity, which derived a similar expression for the ground state energy using the Lee-Low-Pines transformation, and introduced a finite range in the bose-bose interaction [52].

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Supplemental Material:

Mobile impurity in a Bose-Einstein condensate and the orthogonality catastrophe

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DERIVATION OF THE MODIFIED GP EQUATION

We start by presenting a step-by-step derivation of the expectation value of the grand potential,

\[ \hat{\Omega} = \int d^3r \left[ \hat{b}_r^\dagger \left( -\frac{\hbar^2 \nabla^2_s}{2m_B} + \frac{T_B}{2} \hat{b}_r \hat{b}_r - \mu \right) \hat{b}_r + \hat{\tilde{a}}_r \left( -\frac{\hbar^2 \nabla^2_s}{2m_I} \right) \hat{\tilde{a}}_r + \hat{b}_r^\dagger \hat{b}_r U(r-s) \hat{\tilde{a}}_s \hat{\tilde{a}}_s \right], \]  
(S.1)

upon the ansatz state

\[ |\Psi\rangle = \int \frac{d^3r}{\sqrt{V}} \hat{a}_r \exp \left( \int d^3s \phi(r-s) \hat{b}_s^\dagger - c.c. \right) |0\rangle = \int \frac{d^3r}{\sqrt{V}} \hat{a}_r \phi(r). \]  
(S.2)

The result contains three contributions, coming from the bath, the impurity, and their mutual interaction. The contribution from the bath is very simple. Using \( \langle r | r' \rangle = \delta(r-r') \) and \( b_s \phi(r) = \phi(r-s) \phi(r) \), one finds:

\[ \langle \hat{\Omega}_B \rangle = \int \frac{d^3r}{V} \frac{d^3s}{V} \phi^*(r-s) \left[ -\frac{\hbar^2 \nabla^2_s}{2m_B} + \frac{T_B}{2} |\phi(r-s)|^2 - \mu \right] \phi(r-s) = \int \frac{d^3r}{V} \phi^*(r) \left[ -\frac{\hbar^2 \nabla^2_s}{2m_B} + \frac{T_B}{2} |\phi(r)|^2 - \mu \right] \phi(r). \]  
(S.3)

Similarly, the bath-impurity interaction term gives:

\[ \langle \hat{\Omega}_{int} \rangle = \int \frac{d^3r}{V} \frac{d^3s}{V} U(r-s) \langle \phi(r) | \hat{b}_s \hat{\tilde{a}}_s | \phi(r) \rangle = \int \frac{d^3r}{V} \frac{d^3s}{V} U(r-s) |\phi(r-s)|^2 = \int \frac{d^3r}{V} U(r) |\phi(r)|^2. \]  
(S.4)

To study the impurity sector, care must be taken when evaluating the Laplacian. A possible approach is working explicitly with a difference quotient

\[ \nabla^2_r \hat{a}_r = \lim_{d \to 0} \sum_{i=(1,2,3)} \frac{\hat{a}_{r+de_i} + \hat{a}_{r-de_i} - 2\hat{a}_r}{d^2}, \]  
(S.5)

and taking the limit \( d \to 0 \) at the end. Here \( e_i \) is any orthonormal basis set. This gives

\[ \langle \hat{\Omega}_I \rangle = -\frac{\hbar^2}{2m_I} \int d^3r \sum_{i=(1,2,3)} \frac{\hat{a}_{r+de_i} + \hat{a}_{r-de_i} - 2\hat{a}_r}{d^2} \]  

\[ = -\frac{\hbar^2}{2m_I} \lim_{d \to 0} \int \frac{d^3r}{Vd^2} \sum_{i=(1,2,3)} \left( \langle \phi(r) | \phi(r+de_i) \rangle + \langle \phi(r) | \phi(r-de_i) \rangle - 2 \frac{\langle \phi(r) | \phi(r) \rangle}{1} \right). \]  
(S.6)

The overlap between different coherent states is

\[ \langle \phi_1 | \phi_2 \rangle = \exp \left( \int d^3r \left( \frac{\phi_1^*(r) \phi_2(r)}{2} - \frac{|\phi_1(r)|^2}{2} - \frac{|\phi_2(r)|^2}{2} \right) \right). \]  
(S.7)
Applied to the last line in Eq. (S.6), this gives
\[
\langle \phi(r) | \phi(r \pm de_i) \rangle = \exp \left[ \int d^3s \left( \phi^*(r - s) \phi(r \pm de_i - s) - \frac{|\phi(r - s)|^2}{2} - \frac{|\phi(r \pm de_i - s)|^2}{2} \right) \right]
\]
\[
= \exp \left[ \int d^3s \left( \phi^*(s) \phi(s \mp de_i) - |\phi(s)|^2 \right) \right]
\]
\[
= 1 + d \int d^3s \phi^*(s) \partial_i \phi(s) + \frac{d^2}{2} \int d^3s \phi^*(s) \partial_i^2 \phi(s) + \left( \int d^3s \phi^*(s) \partial_i \phi(s) \right)^2 + O(d^3), \tag{S.8}
\]
where the first and last term vanish because \( \phi^*(s) \partial_i \phi(s) \) is antisymmetric in \( s \) due to \( \phi \) being a central function, \( \phi(s) = \phi(s) \). Plugging this expression into (S.6) gives:
\[
\langle \hat{\Omega}_I \rangle = \int d^3r \phi^*(r) \left( -\frac{\hbar^2 \nabla^2}{2m_I} \right) \phi(r). \tag{S.10}
\]
Combining all terms, the expectation value of \( \hat{\Omega} \) over the coherent ansatz \( |\Psi\rangle \) reads:
\[
\langle \hat{\Omega} \rangle = \int d^3r \phi^*(r) \left( -\frac{\hbar^2 \nabla^2}{2m_I} + U(r) + T_B |\phi(r)|^2 - \mu \right) \phi(r), \tag{S.11}
\]
where \( m_{r}^{-1} = m_{B}^{-1} + m_{I}^{-1} \) is the reduced mass for one bath boson and one impurity. By minimizing (S.11) with respect to \( \phi^* \), one recovers the equation introduced in the text,
\[
\left[ -\frac{\hbar^2 \nabla^2}{2m_r} + U(r) + T_B |\phi(r)|^2 \right] \phi(r) = \mu \phi(r), \tag{S.12}
\]
which is a modified GP equation (due to the presence of the reduced mass \( m_r \)) describing the coherent dressing of a mobile impurity when it is immersed in a weakly-interacting BEC.