Can R-parity violation explain the LSND data as well?

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Abstract

The recent Super-Kamiokande data now admit only one type of mass hierarchy in a framework with three active and one sterile neutrinos. We show that neutrino masses and mixings generated by R-parity-violating couplings, with values within their experimental upper limits, are capable of reproducing this hierarchy, explaining all neutrino data particularly after including the LSND results.

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The vacuum oscillation interpretation of solar neutrino data requires $\Delta m_{\text{solar}}^2 \sim 10^{-10}$ eV$^2$, while the matter enhanced MSW solution prefers the range $\Delta m_{\text{solar}}^2 \sim (10^{-5} - 10^{-4})$ eV$^2$ [1]. The atmospheric neutrino anomaly can be explained by $\Delta m_{\text{atm}}^2 \sim (2 - 5) \times 10^{-3}$ eV$^2$ (with $\sin^2 2\theta_{\text{atm}} > 0.88$) [2, 3]. In the standard three neutrino framework, which offers two independent mass differences, it is possible to conceive of mass hierarchies which can explain both solar and atmospheric results. If in addition the LSND data, carrying a positive indication of $\nu_\mu \leftrightarrow \nu_e$ oscillation with $\Delta m_{\text{LSND}}^2 \sim (0.3 - 1)$ eV$^2$ [4], are also sought to be simultaneously explained, then one has to expand the standard three neutrino scenario as it falls short of one independent mass difference. In fact, within this three neutrino framework one can fit any two of the three $\Delta m^2$ mentioned above. Under the circumstances, one often leaves the LSND data out of consideration pending further confirmation from the MiniBooNE [5] at FNAL or MINOS long baseline [6] experiments, since KARMEN [7, 8] does neither confirm nor exclude the LSND results. On the other hand, if one includes the LSND results to be explained together with the solar and atmospheric data, the minimal extension of the standard scenario that needs to be done is to add one sterile neutrino ($\nu_s$) [9, 10] to the list of three active states. The resulting four neutrino picture can explain all the data [11].

What are the possible choices of mass hierarchies among these four neutrinos? The data suggest that the choices are very limited. First, we divide them into two types, I and II. In type I, there are three almost degenerate states explaining the solar and atmospheric results, with a fourth state separated by a large gap. This hypothesis cannot explain the LSND results, since the LSND data require that the separated fourth state will have to be either $\nu_e$ or $\nu_\mu$, but each of them will have to be closely spaced with a third state to explain the solar and atmospheric results. If one disregards the LSND results till further confirmation, then of course type I scenario is allowed with the isolated state either $\nu_s$ or $\nu_\tau$. In type II, there are two pairs of approximately degenerate states separated by a large gap. The mixing between the two pairs is very small. This scenario can explain all the data, or in other words, the large gap could be the LSND gap. Two cases may arise in this framework. In type IIa scenario, $\langle \nu_\tau - \nu_e \rangle$ form one pair which explains the solar neutrino data, while $\langle \nu_\mu - \nu_s \rangle$ form the other pair maximally mixed to explain the
atmospheric anomaly. In type IIb, \((\nu_e-\nu_\mu)\) form the pair that explains the atmospheric anomaly, while \((\nu_s-\nu_\tau)\) pair explains the solar data\(^1\). The most recent Super-Kamiokande (SK) data rule out the \(\nu_\mu \leftrightarrow \nu_\tau\) interpretation of atmospheric neutrino anomaly at 99\% CL \(^2\), thus strongly disfavouring the type IIa scenario. We are then left with type IIb as the only surviving option, shown in Fig. 1\(^3\). It should be noted that the oscillation data cannot discriminate between cases that occur by interchanging either the members within a given pair or the relative spacing of the two pairs. These cases are not separately shown.

Under the circumstances, it is a timely exercise to identify those scenarios which reproduce the type IIb spectrum. Do the R-parity-violating \((R)\) supersymmetric models \(^4\) fall in that category? In this note we seek to find an answer to this question. Defined in terms of lepton number \((L)\), baryon number \((B)\) and spin \((S)\) of the particle as \(R = (-1)^{3B+L+2S}\), this discrete parity is +1 for all SM particles and -1 for their superpartners. Since neither \(L\)- nor \(B\)-conservation is ensured by gauge invariance or any such fundamental principles, R-parity is an \textit{ad hoc} symmetry put in by hand. Although there is no experimental confirmation yet in favour of non-vanishing \(R\) interactions, the neutrino oscillation data are somehow suggestive that it would be premature to abandon those couplings, as the origin of neutrino masses and mixings could be traced to some of these non-vanishing \(L\)-violating couplings. In order not to allow rapid proton decay we do not switch on \(L\) and \(B\) violations simultaneously. The following \(L\)-violating terms in the superpotential are then allowed: (a) \(\lambda_{ijk} L_i L_j E_k^c\), (b) \(\lambda'_{ijk} L_i Q_j D_k^c\), and (c) \(\mu_{ij} L_i H_u\). In these expressions, \(L_i\) and \(Q_i\) are SU(2)-doublet lepton and quark superfields, \(E_k^c\) and \(D_k^c\) are SU(2)-singlet charged lepton and down quark superfields, and \(H_u\) is the Higgs superfield that gives masses to up-type quarks. Stringent experimental constraints on these \textit{a priori} independent parameters exist in the literature \(^5\).

Attempts to fit the observed neutrino data by masses and mixing angles generated by \(R\) couplings have been done in the past, both in the context of bilinear \(^6\) as well as trilinear \(^7\) \(L\)-violating parameters. Most of the analyses have been carried out in the three-neutrino picture. A few remarks on those analyses are in order. Most of them discarded the LSND data as something not so reliable, and confined the discussions to the possibility of fitting only solar and atmospheric data. Adhikari and Omanovic \(^8\), on the other hand, tried to fit solar, atmospheric \((\text{preliminary SK})\) and LSND all together in a three-neutrino picture and that too considering only trilinear \(R\) couplings. But they did not consider the SK zenith angle dependence and also assumed an energy independent solar neutrino solution by ignoring the Chlorine data of the Homestake mine experiment. In fact, if one takes all \textit{present} data into consideration, it is not enough just to add bilinear \(R\) terms to their analysis – the data compel one to go for a four-neutrino picture. Ref. \(^9\) deals with the simultaneous presence of bilinear and trilinear parameters. Although the emphasis in ref. \(^10\) is mostly on finding an explanation for the solar and atmospheric neutrino data in a three-neutrino framework, a qualitative discussion on how to simultaneously explain the LSND data by admitting a fourth sterile state has also been presented.

In the present work, we perform a numerical study to examine whether R-parity violation, with bilinear and trilinear parameters together, can reproduce the type IIb spectrum. Even though the data allow an interchange of the location of the two pairs, we work in a situation where \((\nu_e-\nu_\tau)\) form the heavier pair\(^11\). We assume that the masses of all active neutrinos are generated by R-parity violation. Since the mixing between the two pairs is small, we can, for all practical purposes, focus on the heavier pair and work in the \((\nu_\mu-\nu_\tau)\) subspace, always assuming

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\(^1\)\(\nu_e \leftrightarrow \nu_s\) solar neutrino oscillation will be tested by the SNO experiment soon \(^12\).

\(^2\)This framework is consistent with the Big-Bang Nucleosynthesis constraints on sterile - active mixings \(^13\).

\(^3\)This hierarchy is stable under radiative corrections \(^14\).
that the parameters $\lambda'_{ijk}$ and $\mu_1$, responsible for the $\nu_e$ mass generation, are much smaller. We also assume that the sterile state receives mass of the order of the $\nu_e$ mass from some other source. More specifically, we concentrate on those parameters which generate the masses and mixing angles in the above $(2 \times 2)$ subsector, keeping in mind that each of the two absolute masses should be of the order of the LSND gap. In other words, we parametrize the $\nu_\mu$-$\nu_\tau$ mass matrix in terms of the $R$ couplings, vary them within physical ranges, and then observe whether there exist solutions that simultaneously satisfy the following experimental constraints:

$$
\begin{align*}
\Delta m_{\text{L}}^2 &\sim m_4^2 \sim m_3^2 \sim (0.3 - 1) \text{ eV}^2, \\
\Delta m_{\text{atm}}^2 &= m_4^2 - m_3^2 \sim (2 - 5) \times 10^{-3} \text{ eV}^2, \\
\sin^2 2\theta_{\text{atm}} &\equiv \sin^2 2\theta_{\text{}4} > 0.88.
\end{align*}
$$

(1)

Now we turn our attention to the analytic expressions of the neutrino masses induced by bilinear and trilinear couplings. We write the mass matrix as $M_{\nu} = M_{\nu}^\text{tree} + M_{\nu}^\text{loop}$. The bilinear couplings contribute to the tree mass \[16\]. In a basis where there are no sneutrino vacuum expectation values (VEVs) \[20\], this can be expressed as\[4\]

$$
M_{\nu_{\mu'}}^\text{tree} = \frac{g_2}{4 \det M} \frac{(M_1 + \tan^2 \theta_W M_2)}{\mu_4} v_d^2 \equiv \alpha_{\mu_{i'i'}} (v_d = (H_d^0)),
$$

where $M_{1,2}$ are the gaugino masses, and $\det M$ is the determinant of the $(4 \times 4)$ neutralino mass matrix in the R-parity-conserving case. Considering only the $\lambda'$ couplings, the one-loop squark-mediated contribution to the neutrino mass, in the same basis as before, can be written as \[17\]

$$
M_{\nu_{\mu'}}^\text{loop} \simeq \frac{N_e}{16 \pi^2} \lambda'_{ijk} \lambda'_{jkl} v_d \left[ \frac{f(m_{d_i}^2/m_{d_k}^2)}{m_{d_k}} + \frac{f(m_{d_k}^2/m_{d_i}^2)}{m_{d_i}} \right],
$$

(3)

where $f(x) = (x \ln x - x + 1)/(x - 1)^2$. Here, $m_{d_i}$ is the down quark mass of the $i$th generation, $m_{d_k}$ is an average of $d_{Li}$ and $d_{Ri}$ squark masses, and $N_e = 3$ is the colour factor. While writing Eq. (3), we assumed that the left-right squark mixing terms are family-diagonal and are proportional to the corresponding quark masses, i.e., $m_{d_i}^2(i) = m_{d_i} m_{d_i}$. The expression of the $\lambda'$-induced slepton-mediated contributions to the neutrino mass is similar to Eq. (3), and we do not display it here.\[3\]

In the basis ($\nu_\mu$, $\nu_\tau$), the mass matrix can be parametrized as

$$
M_{\nu} = \begin{pmatrix}
K \lambda_2^2 + \alpha \mu_3^2 & K \lambda_2 \lambda_3 + \alpha \mu_2 \mu_3 \\
K \lambda_2 \lambda_3 + \alpha \mu_2 \mu_3 & K \lambda_3^2 + \alpha \mu_3^2
\end{pmatrix},
$$

(4)

where $\alpha$ is given by Eq. (2), $\lambda_{2,3}$ are two generic trilinear couplings, and $K$ captures the loop factors that enter into Eq. (3).

We observe that the determinant of the mass matrix with only bilinear or with only trilinear couplings is identically zero\[3\]. But this yields a big hierarchy between $m_3$ and $m_4$, contrary to

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4In principle, one can rotate away the bilinear $\mu_i$ terms from the superpotential, but still having the $\Delta L = 2$ effects via the presence of sneutrino VEVs after the minimization of the scalar potential. One can as well work in a framework where there are no sneutrino VEVs but the $\mu_i$ parameters are present in the theory \[20\]. We work in the latter basis. We emphasize that going from one basis to another, the parametrization would change, but the general conclusion we draw at the end remains unaffected. For a discussion on the basis-independent parametrizations of R-parity violation, see refs. \[21\].

5We neglect the one-loop diagrams induced by the product of bilinear and trilinear couplings \[22\].

6If we have more than one combination of trilinear couplings, then although the determinant of the mass matrix with trilinear couplings alone will not be zero, still much more fine-tuning may be necessary in order to arrive at any solution with only trilinear couplings.
the requirement that these two states should be approximately degenerate. Only after taking the bilinear and trilinear parameters together we obtain solutions that pass the test in Eq. (1). Actually we need at least four independent input parameters in the mass matrix in order to fit the data. In this case, they are the two trilinear couplings ($\lambda_2$, $\lambda_3$) and the two bilinear mass parameters ($\mu_2$, $\mu_3$). Notice that these four parameters as well as $\alpha$ can take either sign, while $K$ is always positive. We point out that the tree and loop contributions may have the same or opposite signs, and this relative sign plays a crucial role in deciding which combinations of parameters are allowed by the data. After diagonalizing the mass matrix in Eq. (4), we demand that the eigenvalues $m_{3,4}$ and the mixing angle $\theta_{34} = \theta_{\text{atm}}$ satisfy Eq. (1). In Figs. 2a and 2b we have displayed only a part of the solutions by plotting the acceptable mass spectrum as a function of some allowed input parameters, just to demonstrate that the mechanism of R-parity violation works as a viable explanation\(^7\). The above parametrization is rather general as one can apply it for any $\lambda'$ (or $\lambda$, for that matter) couplings irrespective of the second and third generation indices. Also at this stage one need not specify the squark (or slepton) and gaugino masses as they are absorbed in $K$ and $\alpha$ respectively. We observe that the 'filter' of Eq. (1) prefers a negative $\alpha$, which implies that one or more gaugino mass parameters could be negative (see Eq. (2)). We stress though that we do get some solutions with positive values of $\alpha$ as well.

Now assuming, as an illustrative example, that $\lambda'_{233}$ and $\lambda'_{333}$ are the only dominant trilinear couplings, the factor $K$ turns out to be $K \sim N_c m_b^2/8\pi^2 m_{\tilde{b}}^2$. Taking the squarks and gauginos to be approximately at the 300 GeV scale, one obtains $\alpha \sim 2 \times 10^{-4}$ GeV$^{-1}$, and $K \sim 1 \times 10^{-3}$ GeV. The minimum and maximum values of the input parameters that pass this test turn out to be

$$
\lambda'_{233} \sim \lambda'_{333} : [-1.3 \times 10^{-3}, 1.3 \times 10^{-3}]; \\
\mu_{2,3} \text{ (GeV)} : [-5.0 \times 10^{-3}, 1.0 \times 10^{-3}].
$$

These values are consistent with the existing constraints on the above parameters \(^{15, 23}\).

To conclude: If R-parity violation has to explain all neutrino data, it is essential to have both trilinear and bilinear $R$ terms in addition to having a sterile neutrino in the model. Then it is possible to generate maximally mixed $\nu_\mu$ and $\nu_\tau$ with their absolute masses in the eV range and mass-squared difference in the milli-eV$^2$ range, as required by the data. The quoted ranges of the $R$ parameters within which we obtain solutions are based on certain simple-minded but plausible approximations made for the ease of presentation. The bottom line of our analysis is that we provide an affirmative answer to the question we have asked in the title.

\textit{Note added:} While we were finishing this note, we became aware of a preliminary SK solar neutrino analysis update \(^{24}\) disfavouring a pure $\nu_e \leftrightarrow \nu_s$ solar neutrino oscillation at 95% CL. First, this is only a 2-$\sigma$ result which is not enough to exclude a model. Second, it has been claimed that a rate + spectrum combined analysis in a full four-flavour scenario exhibits an allowed zone \(^{25}\). So the fate of the sterile state may not be that dwindling, and we must wait till the SNO experiment tests this option.

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\(^7\)In principle, the allowed spectrum should have been displayed as a five-dimensional plot where all the four input parameters are varying. Mostly for the purpose of a simplified presentation, merely to point out that there indeed exist solutions satisfying Eq. (1), we plotted the spectrum in two dimensions in Figs. 2a and 2b. In each plot, the 'other three' parameters are also varying.
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Figure 1: The type IIb four-neutrino mass pattern. Interchange of the members in a given pair or the relative location of the two pairs may be allowed.

Figure 2: Mass spectrum as a function of the (a) trilinear and (b) bilinear R-parity-violating parameters.