Research Article

Numerical Simulation of Corroded Reinforced Concrete Beam Strengthened by a Steel Plate with Different Strengthening Schemes

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This paper proposes 3D nonlinear finite element (FE) models to predict the response of corroded reinforced concrete (RC) beam strengthened using a steel plate. Five FE models are developed based on the tests carried out by the authors in a previous investigation, in which three models are used to simulate the corroded RC beams with different schemes. The FE models use the coupled damaged-plasticity constitutive law for concrete in tension and compression and consider the bond-slip between the corroded tensile steel bar and concrete. The cohesive element is also used to model the cohesive bond between the steel plate and concrete. The FE results of load-deflection and the crack distribution are compared with the test data. The FE results are consistent with the test results. The influence of the thickness of the steel plate, the thickness, and location of the U-shaped steel strip on the bearing capacity of the strengthened corroded beam is analyzed through FE models. The results show that the thickness of the steel plate on the bottom surface should not exceed 4 mm for the flexure-strengthened and combined strengthened beams with a 10% corrosion rate. It is most reasonable to improve the bearing capacity using the 3 mm and 2 mm of thick U-shaped steel strips for the shear-strengthened and combined strengthened beams, respectively. The most reasonable location of the U-shaped steel plate is at the end of the steel plate for beams with a 10% corrosion rate.

1. Introduction

The total number of highway bridges has already reached up to 0.80 million at the end of 2018 in China [1], in which the number of reinforced concrete (RC) bridges accounts for 90%. However, more than 15% of these RC bridges need to be maintained and rehabilitated due to the corrosion of the steel bar. The corrosion of the steel bar has already become the main reason for the performance deterioration of RC bridges, and many scholars have already studied the mechanical property of corroded structure through theoretical and experimental research [2–7].

There are many strengthening technologies to repair the degraded RC structures, in which the strengthening technologies of bonding fibre-reinforced polymers (FRPs) and external steel plates are broadly applied. The FRP is the brittle material, while the low-carbon (mild) steel used in the steel plate contributes to the overall ductility of an externally plated structure. Moreover, the cost of strengthening project using FRP is still large in China. However, the external plating is a relatively convenient method to strengthen RC beams compared with other strengthening methods because it is inexpensive and convenient in construction and has little effect on the overall dimensions of structure [8]. Therefore, the external steel plate system is still one of the main strengthening schemes for corroded RC bridges in China.

There was already much experimental and theoretical research focusing on the structural performance of externally plated beam [9–18]; at the same time, the finite element
(FE) modeling was also used. Ziraba and Baluch [19] presented a nonlinear finite element model for the flexure-shear response of RC beam strengthened by epoxy-bonded steel plates. The model includes a special interface element to simulate the thin epoxy adhesive layer which allows for the metamorphosis of failure mode from plate yielding to separation as the plate thickness is increased. Taljsten [20] proposed a numerical finite element model to verify the derivation of shear and peeling stresses in the adhesive layer of a beam with a strengthening plate bonded to its soffit and loaded with an arbitrary point load. Adhikary and Mutsuyoshi [21] investigated the effectiveness of web-bonded continuous steel plates for shear-strengthening of RC beams having internal stirrups through the two-dimensional nonlinear finite element model. Arslan et al. [22] built a three-dimensional nonlinear finite element model to support the experiments for the flexural strengthening of RC beams using epoxy-bonded continuous horizontal steel plates. Abu-Obeidah et al. [23] presented the development of a 3D nonlinear FE model to capture and predict the response of shear deficient simply supported RC beams strengthened externally with aluminum alloy plates. Though the strengthening material was aluminum plate, the method of developing model was similar to the previous scholars’ modeling.

The above studies mainly focused on the numerical analysis of noncorroded RC structures strengthened by steel plate. So far, though there are already many studies focusing on the structural properties of corroded RC elements externally bonded FRP laminates through the experiment and FE analysis [24–31], little experimental, theoretical, and numerical research focuses on the structural performance of corroded beam strengthened by steel plate. Huang et al. and Zhang et al. proposed the calculation model of corroded beam strengthened by steel plate and analyzed the different mechanical performance of corroded beams strengthened by steel plate with different strengthening schemes [32, 33]. The numerical simulation method of bonding behavior between the concrete and corroded steel bar is different from that in the noncorroded beam because of steel bar corrosion, which leads to a different structural performance of corroded RC beam. In addition, in Chinese code for design of strengthening concrete structure [34], the clauses and calculation method of strengthening design mainly direct at the noncorroded beams. There are no specific design clauses for the corroded beams. The strengthening scheme for non-corroded beams may be not suitable for corroded beams. Therefore, it is necessary to continually research the structural behavior of corroded beam strengthened by steel plate.

Compared with FE analysis, the experimental testing could not provide the full field of results due to the limited amount of measurement equipment, for example, strain gauges and LVDTS. At the same time, experimental programs are expensive, time-consuming, and limited by the specific conditions, while the FE model can overcome the above disadvantages. The aim of this study is to develop FE models to investigate the performance of corroded RC beams strengthened by steel plate with different strengthening schemes (flexure-strengthening scheme, shear-strengthening scheme, and combined-strengthening scheme). Based on the previous test work conducted by the authors [32], the FE models are built using the finite element software ABAQUS version 6.14 [35]. The FE models use the coupled damaged-plasticity constitutive law for concrete in tension and compression and consider the bond-slip between the corroded tensile steel bar and concrete. The models are verified by comparing the predicted load-deflection response results with test results at each level of loading. The failure modes of beams in FE models are compared with test results. The FE models are also used to investigate the influence thickness of steel plate, the location, and thickness of the U-shaped steel strip on the load-deflection response through sensitivity analysis, and several design advices for strengthening corroded beam are presented.

2. Summary of the Test Program

2.1. Dimensions, Strengthening, and Materials. In this study, a total of five rectangular RC beams are tested. The length, width, and height of beams are 1800 mm, 150 mm, and 300 mm (see Figure 1). All of the steel bars are hot rolled ribbed steel bars. The beams are all reinforced in flexure with two 22 mm steel bars located at a depth of 30 mm from the bottom surface of the beam. The compressive steel bars in which the diameter is 14 mm are 30 mm from the top face of the beam. The diameter of the stirrup is 8 mm. The spaces between the stirrups are 100 mm at the middle and 70 mm at the ends along beam length. The steel bar details of the beam are shown in Figure 1.

The designed cube compressive strength of concrete is 30 MPa. The concrete of all beams was from the same batch, the practical cube compressive strength of concrete was 30.2 MPa through material strength standard test based on the standard for test methods of concrete physical and mechanical properties [36]. The standard test was using a concrete cube compressive strength tester to measure the concrete strength. The dimension of cube concrete was 150 mm × 150 mm × 150 mm, and the concrete was curing in humid air with 20 °C and relative humidity of 95%. The steel bar is the HRB335 grade. The steel plates are bonded to the tensile face of beam specimens using the structural adhesive and chemical bolt. The compressive strength of the adhesive is 89.5 MPa, and the tensile bond strength is 21.0 MPa, the type of steel plate is Q235 grade, and the yield strength is 235 MPa; the above two materials’ mechanical properties are obtained from the supplier.

2.2. Steel Bar Corrosion and Strengthening Schemes. The beam descriptions are summarized in Table 1. The beams C0C30 and C10C30 represent control beams that were not strengthened with steel plate, but beam C10C30 was subjected to 10% mass loss corrosion in the tensile steel bar through the electrochemical corrosion in the laboratory (see Figure 2(a)). In order to make sure the corrosion rate was the same and corrosion was uniform, the NaCl solution
concentration (3%) and electricity in corrosion progress were kept the same for all beams. The corrosion rate was measured by comparing the quality of corroded steel bar with noncorroded steel bar. Figure 2(b) shows the corrosion statue of steel bar; it can be found that the corrosion of steel bar has already reached the generalized corrosion stage and the corrosion of most parts of the steel bar is uniform.

Beams S10C30, U10C30, and SU10C30 were all strengthened with Scheme 1, Scheme 2, and Scheme 3, respectively, and the tensile steel bars in these three beams were all subjected to 10% mass loss corrosion using the same method as beam C10C30. Scheme 1 was flexure-strengthening in which the beam was strengthened using a steel plate on the bottom surface. Scheme 2 was shear-strengthening in which the beam was strengthened by U-shaped steel strips at the end of the beam. Scheme 3 was the combined-strengthening scheme which combined Scheme 1 and Scheme 2.

All strengthening schemes are designed according to the code for the design of strengthening concrete structures, the thickness, and location of steel plate; grinding number and location are in accordance with this code’s stipulations. The strengthening process included surface preparation by grinding the bottom and side faces of the beam using a sander to ensure adequate bond with the surfaces of steel plate or U-shaped steel strip. The steel plate and U-shaped steel strip were also ground to create a rough surface. Once the rough surfaces were prepared, the structural adhesives were applied to bond the steel plate or U-shaped steel strip to the concrete surface. At last, the chemical bolts were used to fasten the steel plate. The steel sheet was used to fasten the U-shape steel strip. The details of the steel plate and U-shaped steel strips are present in Figure 3. Figures 3(a) and 3(b) present the details of steel plates used to strengthen beams S10C30 and SU10C30, respectively. Figure 3(c) presents the details of the U-shaped steel strip and steel...
sheet. Figure 4 depicts the schematic of the strengthened beam.

All beams were tested under a three-point bending system, and the ultimate load, load-deflection, and crack distribution were measured in the test.

3. Finite Element (FE) Model

3.1. Model Description. All of the beams in the test are simulated by the FE modeling using finite element software ABAQUS. In order to save the computational time and volume of results files, except for beam C0C30, only one quarter of the beam was modeled for other beams due to the symmetry in geometry and loading conditions. Figure 5 shows the developed FE models of all beams in the test. It can be observed from Figure 5(b) that the FE model has two symmetry planes which are perpendicular to the \( x \)-axis and \( z \)-axis, respectively.

3.2. Element Types. Several elements types are used to simulate the FE models. The 3-dimension 8-node linear brick, reduced integration element (C3D8R) [35], is used to simulate the concrete, steel plate, support, and bearing plate. The 2-node linear 3-dimension (T3D2) truss element is adopted for all the steel bars. Just only the stress along the length direction can be computed for this element, and every node in this element only has the translational degrees of freedom. The adhesive is simulated using the 8-node three-dimensional cohesive element (COH3D8).

3.3. Material Constitutive

3.3.1. Steel Bar. The mass loss of the corroded steel bar is simulated through decreasing the sectional area in the FE model. The tensile capacity curve of the corroded steel bar was obtained from the material test after the test finished. The density of the steel bar is \( 7.8 \times 10^{-3} \text{kg/m}^3 \) and Poisson’s ratio is 0.3. It is needed to change the nominal stress (strain) values in the material test to the real stress (strain) values. At the same time, the yield strains in ABAQUS are obtained by all the plastic stains subtracting the peak elastic strain value. Figure 6 presents the stress-strain relationship of all steel bars in ABAQUS.

3.3.2. Concrete. The equivalent uniaxial stress-strain relationship of concrete is needed in the FE model. The concrete stress-strain model in Chinese code for design of concrete...
structures \[37\] is adopted as the concrete uniaxial stress-strain relationship, as described as follows:

Uniaxial compression:

\[
\sigma_c = (1 - d_c)E_c \varepsilon_c,
\]

\[
d_c = \begin{cases} 
1 - \frac{\rho_c n}{n - 1 + x}, & x \leq 1, \\
1 - \frac{\rho_c}{\alpha_c (x - 1)^2 + x}, & x > 1,
\end{cases}
\]

Uniaxial tension:

\[
\sigma_t = (1 - d_t)E_t \varepsilon_t,
\]

\[
d_t = \begin{cases} 
1 - \rho_t \left[1.2 - 0.2x^5\right], & x \leq 1, \\
1 - \frac{\rho_t}{\alpha_t (x - 1)^{1/5} + x}, & x > 1,
\end{cases}
\]

where \(\sigma_c\) and \(\sigma_t\) are compressive and tensile stress of concrete, respectively. \(\varepsilon_c\) and \(\varepsilon_t\) are the compressive and tensile strains of concrete, respectively. \(E_c\) is the initial (undamaged) modulus of the material. \(d_c\) and \(d_t\) are the compressive and tensile damage evolution parameters, respectively. \(f_{cm}\) and \(f_{tm}\) are the mean values of the compressive and tensile
strength of concrete. $\varepsilon_{c0}$ and $\varepsilon_{t0}$ are peak strains corresponding to the mean values of the compressive and tensile strength of concrete, respectively. $\alpha_c$ and $\alpha_t$ are the parameter values of decrease parts of compressive and tensile stress-strain curves, respectively.

In this study, a brief presentation of the damaged-plasticity model from ABAQUS is presented. The damaged-plasticity model of concrete assumes the two kinds of failure mode: concrete cracking and compressive crushing. The evaluation of the failure surface of the concrete is controlled by two hardening variables, $\varepsilon_{pl}^c$ and $\varepsilon_{pl}^t$, which present the compressive and tensile equivalent plastic strain of concrete, respectively.

The definitions of cracking strain and compressive inelastic strain of concrete are shown in Figures 7 and 8, respectively. The degradation of elastic stiffness is characterized by two damage variables, $d_t'$ and $d_c'$, which are assumed to be the functions of plastic strains, temperature, and field variables.

In Figure 7, the curve includes two parts: the elastic part and the softening branch. The uniaxial tensile stress-strain relationship can be presented as follows:

$$\sigma_t = (1 - d_t')E_c\left(\varepsilon_t - \varepsilon_{pl}^t\right).$$

(3)

In reinforced concrete, the specification of postfailure behavior generally means giving the postfailure stress based on the tensile cracking strain $\varepsilon_{cr}^k$. The tensile cracking strain is defined as follows:

$$\varepsilon_{cr}^k = \varepsilon_t - \varepsilon_{el0}^t, \quad \varepsilon_{el0}^t = \frac{\sigma_t}{E_c},$$

(4)

where $\varepsilon_{el0}^t$ is the tensile elastic strain.

ABAQUS automatically converts the tensile cracking strain values to equivalent tensile plastic strain values using the relationship

\[\text{Figure 5: Developed FE models: (a) beam C0C30, (b) beam C10C30, (c) beam S10C30, (d) beam U10C30, and (e) beam SU10C30.}\]
Similarly, in Figure 8, the uniaxial compressive stress-strain relationship can be presented as follows:

\[
\sigma_c = (1 - d')E_c (\varepsilon_c - \varepsilon_{el}^0),
\]

where \( \varepsilon_{in}^c = \varepsilon_c - \varepsilon_{el}^c \), \( \varepsilon_{el}^c = \frac{\sigma_c}{E_c} \)

\[
\varepsilon_{pl}^c = \varepsilon_{in}^c - \frac{\sigma_c}{E_c} \frac{d'}{1 - d'}.
\]

(5)

According to experimental results from existing research [38], the chloride ion erosion has little influence on the concrete compressive strength when the concrete is in 3% NaCl solution and the soak time is not beyond 90 days. The corrosion time is not beyond 90 days for all beams, so the concrete compressive strength in FE model is the same as the value from the material test.

3.3.3. Adhesive. The traction-separation description is used to define the constitutive response of the cohesive element in ABAQUS. The typical traction-separation response model is illustrated in Figure 9: \( t_{in}, t_{in}^0, t_{in}^1 \) and \( t_{in}^1 \) are peak values of nominal stress when the deformation is either purely normal to interface or purely in the first or second shear direction, respectively. The corresponding separations are denoted by \( \delta_{in}, \delta_{in}^0, \) and \( \delta_{in}^1 \). \( K_b \) is the initial slope (i.e., the stiffness) of the model. \( G_b \) is the cohesive fracture energy. In Figure 8, the
analytical expression of the bilinear traction-separation model in the three directions is given as follows:

\[
\text{traction} = \begin{cases} 
K_b \cdot \delta_n(\delta_s, \delta_t), \\
t_n(t_s, t_t) \cdot \left( \delta_n^0(\delta_s^0, \delta_t^0) - \delta_n(\delta_s, \delta_t) \right) / \delta_n'(\delta_s', \delta_t') - \delta_n'(\delta_s^0, \delta_t^0), \\
0, 
\end{cases}
\]

\[
G_b = 0.5 t_n(t_s, t_t) \cdot \delta_n'(\delta_s', \delta_t').
\]

3.3.4. Steel Plate, U-Shaped Steel Strip, Support, and Bearing Plate. The steel plate, U-shaped steel strip, support, and bearing plate are all grade Q235 steel. The constitutive models of steel plate, U-shaped steel strip, support, and bearing plate are presented in Figure 10. \(\sigma_y\) and \(\varepsilon_y\) are the yield strength and yield strain, respectively. The values of \(\sigma_y\) and \(\varepsilon_y\) are 235 MPa and 0.001175, respectively.

3.4. Modeling of Bolt. In order to simplify the calculation and increase the possibility of convergence, there is no real part of the chemical bolt in the FE model. A multipoint constraint (MPC) is used to model the bolt. The MPC provides a rigid link between two nodes to constrain the displacement and rotation [35]. The two nodes are the reference points (RP) at the center of the drill holes of concrete and the steel plate, respectively. The constraints between the two reference points and holes are the coupling constraint. The beam type multipoint constraint is illustrated in Figure 11.

3.5. Modeling of Bond Slip Caused by Corrosion of the Steel Bar. According to the experiment results of relevant research [39, 40], the corrosion-induced bond failure between steel bar and concrete is the main reason causing performance deterioration of corroded beam, so the bond-slip behavior between corroded steel bar and concrete is considered to simulate the influence of steel bar corrosion on the bearing capacity in FE model. The bond failure is mainly caused by the splitting mechanism due to the fact that corrosion decreases the confinement between the steel bar and concrete. Therefore, the splitting failure mode is the main mode of corrosion-induced bond failure. The FIB 2010 [41] developed a nonlinear model to determine the bond-slip between noncorroded steel bar and concrete, as shown in Figure 12. For the pull-out failure, the curve includes three stages. The first stage has a nonlinear increase of bond stress until the maximum bond stress \(\tau_{max}\). In the second stage, the bond stress then keeps constant for the certain range from slip \(s_1\) to \(s_2\) under confined conditions. If there is a lack of confinement, this horizontal line will become inclined, indicating a splitting failure rather than a pull-out failure. After that, the
bond stress decreases until a lower constant level \( \tau_{bf} \) in the third stage. For the splitting failure, there is no constant stage for bond stress, and the bond strength is much lower than the pull-out failure. Figure 12 illustrates the bond-slip curves of splitting failure under the condition of unconfined and stirrup confining.

In Figure 12, the nonlinear increase for bond-slip curve in the first stage can be expressed as follows:

\[
\tau = \tau_{\text{max}} \left( \frac{s}{s_1} \right)^a, \quad 0 \leq s \leq s_1,
\]

(8)

where all of the parameters are defined in Table 2. In this study, the tensile steel bar is confined by the stirrups, so the parameters defining under the condition of unconfined are not showed in Table 2, where \( \tau_{bu,\text{split}} \) is the peak value of bond strength in a splitting failure, \( c \) is the clear distance between ribs of steel bar, and \( f_{cm} \) is the mean cylinder concrete compressive strength.

The maximum bond stress between steel bar and concrete will change due to the corrosion of the steel bar according to the test results of relevant research. Based on the experimental results of deterioration of maximum bond strength [42], Li et al. [42] proposed the calculation mode of corrosion-affected bond strength \( \tau_{c,\text{max}} \):

\[
\tau_c = \tau_{c,\text{max}} \left( \frac{s}{s_1} \right)^a, \quad 0 \leq \rho \leq 4 \text{ (percentage)},
\]

or

\[
\tau_c = \tau_{c,\text{max}} \left( 0.1887e^{-0.0069\rho} + 9.662e^{-0.5552\rho} \right) \tau_{\text{max}}, \quad 4 \text{ (percentage)} \leq \rho \leq 80 \text{ (percentage)}.
\]

(9)

By substituting equation (7) into equation (8), \( \tau_c \) can be rewritten as follows:

\[
\tau_c = \tau_{c,\text{max}} \left( \frac{s}{s_1} \right)^a.
\]

(10)

The bond-slip model proposed in equation (9) and the FE model also suppose the corrosion of the steel bar is uniform. Based on equation (8) for the noncorroded steel bar, the corrosion-induced bond stress \( \tau_c \) can be proposed as follows:
The above bond-slip model is a one-dimensional response; the bond stress is parallel to the steel bar. The simplest and most effective approach to define the interface is to use zero thickness spring-like elements that characterize a one-dimensional stress-strain response [43]. This means that the bond-slip response can be considered only in the direction parallel to the steel bar. The Spring 2 element in ABAQUS can model this bond-slip between the concrete and corroded steel bar as shown in Figure 5. The stress-slip results can be calculated using equation (9), which is imported into the ABAQUS. The relative displacement $\Delta u$ (slip) across a Spring 2 element is the difference between the $i$th component of displacement of the spring’s first node and the $j$th component of displacement of the spring’s second node:

$$\Delta u = u_i^1 - u_j^2,$$

where $i$ and $j$ are connected to concrete and steel bar, respectively. The configuration of Spring 2 element is set such that only the displacement in the x-axis is enabled, while the stiffness of spring in other directions ($y$-axis and $z$-axis) is set very large to restrain the displacement. In this study, in order to set bond-slip values in ABAQUS conveniently, the x-axis direction in the local coordinate system is the same as the x-axis direction in the global coordinate system.

4. Results and Discussion

4.1. Model Validation. In this study, the FE models are validated through comparing the load-deflection curves of FE models with those of the obtained test data. The advantages of displacement control versus load control are to overcome both the convergence difficulties and the rigid body modes when two bodies are disconnected in contact pairs and follow/obtain the descending branch of stress-strain curve/load-deflection curve [44]. Therefore, the load-deflection curves of FE model are obtained using the
displacement control method which is done by applying the coupling constraint between a reference point and loading plate, and then the displacement is applied to the reference point. The load is obtained by calculating the related force of the reference point.

Figure 13 depicts the load-deflection curves comparison between test data and FE models. Table 3 shows the validation results between the test and FE models. When the load significantly decreases, the failure of FE model is defined. It can be observed from Figure 13 and Table 3 that there is a good correlation between the FE and test results at all phases of loading still failure. Table 3, $\mu$ is the modulus of toughness (MOT), which is the total area compared with load-deflection curve and coordinate axis. This value can contain energy-absorption of the material in elastic and plastic range; it is a more appropriate and comprehensive measure for assessing the energy-absorption capacities and ductilities of beams compared with curvature [41]. In order to calculation MOT, the decline curve of load-deflection obtained from test and FE models is shown in Figure 13. Table 3 shows that the difference percent of the ultimate loads of FE models to that of the obtained from test data ranges from $-7.1\%$ to $4.2\%$, the Mean Absolute Percent Error (MAPE) of ultimate load for all FE models is 3.3\%. Moreover, the difference percent of ultimate deflections and MOT of all FE models to those obtained from test data ranges from $-2.3\%$ to $4.5\%$ and $4.0\%$ to $19.1\%$, respectively; the MAPE of ultimate deflection and MOT value are $2.2\%$ and $10.1\%$, respectively. Thus, the FE models are able to precisely predict the bearing capacity of tested beams.

The concrete damaged-plasticity model assumes that the crack appears when the maximum principal plastic strain is larger than zero. The orientation of cracks is considered to be perpendicular to the maximum principal plastic strains. Therefore, the maximum principal plastic strain nephogram can be used to predict the crack distribution. Figure 14 depicts the maximum principal plastic strain nephograms and crack distribution of the
Figure 13: Comparison of the load-deflection curve between the test and FE model: (a) beam C0\textunderscore C30, (b) beam C10\textunderscore C30, (c) beam S10\textunderscore C30, (d) beam U10\textunderscore C30, and (e) beam SU10\textunderscore C30.
strengthened beams at the ultimate load. Based on the test results in [32], the single shear-strengthening scheme induced flexural failure. The flexure-strengthening scheme resulted in diagonal tensile failure. The combined-strengthening schemes ended up with support crush failure. Compared with crack distribution of beam S10C30, the maximum principal plastic strains nephogram also creates the diagonal tensile cracking which extends from the end of steel plate to the near area of loading point; this leads to the diagonal tensile failure mode of beam S10C30. The failure mode of beam U10C30 is a flexural failure caused by the flexural cracks on the bottom face of beam. The maximum principal plastic strains nephogram of beam U10C30 also shows that the maximum principal plastic strains on the bottom face of the beam exceed the tensile strength of concrete, and then the concrete cracks. The failure mode of beam SU10C30 is the support crushing. The maximum principal plastic strain nephogram presents that the area in which the maximum principal stress exceeds the concrete strength is concentrated in the near region of support. It can be concluded that the FE models are accurately capable of predicting the failure mode and the crack distribution of the tested beam.

5. Analysis of the Strengthening Design Method for the Corroded Beam

The Chinese Code for the design of strengthening concrete structure stipulates that the thickness of the steel plate on the bottom surface of the beam should not exceed 5 mm for handwork. The FE model and test results show that the failure modes of beams S10C30 and SU10C30 are diagonal tension failure and support crushing, respectively [8]. The load-deflection curves of beams S10C30 and SU10C30 in Figure 13 indicate that these failure modes are brittle and lead to the ductility decrease. The reasons causing these failure modes may be that the thickness of the steel plate on the bottom surface is over thick for beams S10C30 and SU10C30, and the location and thickness of the U-shaped steel strip are not suitable for beam SU10C30. In this paper, the influence of the thickness of the bottom steel plate, the location, and the thickness of the U-shaped steel strip on the bearing capacity of the tested beams is studied through the FE models.

### 5.1. Influence of Thickness of the Steel Plate on the Bottom Surface of the Beam

Figure 15 presents the load-deflection curves of beams S10C30 and SU10C30 strengthened by different thicknesses of steel plate on the bottom surface. Table 4 shows the ultimate load, ultimate deflection, and MOT of FE models. The thickness of the U-shaped steel strip of beam SU10C30 is kept in 3 mm.

For beam S10C30, the ultimate load of beam increases from 165.5 kN to 188.9 kN as the thickness of the steel plate increases, while the MOT value decreases from 1944.2 joules to 1308.0 joules as shown in Table 5. At the same time, the ultimate loads of beam strengthened by 4 mm and 5 mm thick steel plates are close; however, the MOT value of beam strengthened by 4 mm thick steel plate is 32% larger than beam strengthened by 5 mm thick steel plate. This means that the ductility of the beam strengthened by 5 mm thick steel plate significantly decreases. It can be concluded that the thickness of the steel plate for a corroded beam with 10% corrosion rate strengthened by a flexure-strengthening scheme should not exceed 4 mm.

For beam SU10C30, it can be observed from Table 5 that the ultimate deflection and MOT value decrease from 1256.7 mm to 1129.3 mm at the same time, the ultimate loads of beam strengthened by 4 mm and 5 mm thick steel plates are close; however, the MOT value of beam strengthened by 4 mm thick steel plate is 32% larger than beam strengthened by 5 mm thick steel plate. This means that the ductility of the beam strengthened by 5 mm thick steel plate significantly decreases. It can be concluded that the thickness of the steel plate for a corroded beam with 10% corrosion rate strengthened by a flexure-strengthening scheme should not exceed 4 mm.

### Table 3: Validation results between the test and FE model.

| Beam      | Results | \( P_u \) (mm) | \( \Delta_u \) (mm) | \( \mu \) (kN-mm) | Failure mode |
|-----------|---------|----------------|-------------------|------------------|-------------|
| C0C30     | FE      | 139.3          | 8.3               | 1342.4           | CC          |
|           | Test    | 150.0          | 8.4               | 1126.3           | CC          |
|           | Difference (%) | –7.1        | –2.3              | 19.1             | –           |
| C10C30    | FE      | 148.0          | 12.9              | 1456.4           | CC          |
|           | Test    | 142.0          | 12.8              | 1351.7           | CC          |
|           | Difference (%) | 4.2          | 0.7               | 7.7              | –           |
| S10C30    | FE      | 188.9          | 10.0              | 1308.0           | DT          |
|           | Test    | 185.0          | 9.7               | 1155.7           | DT          |
|           | Difference (%) | 2.1          | 0.3               | 3.2              | –           |
| U10C30    | FE      | 169.7          | 12.4              | 1745.5           | FFT         |
|           | Test    | 173.0          | 12.4              | 1818.7           | FFT         |
|           | Difference (%) | –1.9        | 0                 | 4.0              | –           |
| SU10C30   | FE      | 188.7          | 5.7               | 1256.7           | SC          |
|           | Test    | 190            | 5.4               | 1129.3           | SC          |
|           | Difference (%) | –1.0        | 4.5               | 6.7              | –           |

\( P_u \) = the ultimate load at the ultimate statue, \( \Delta_u \) = the ultimate deflection value before the load significantly decrease, \( \mu \) = modulus of toughness (MOT), CC = concrete crushing at compressive zone, DT = diagonal tension failure, FFT = flexural failure at tensile zone, and SF = support crushing.
Figure 14: Continued.
the largest ultimate bearing capacity, its ductility decreases by 15% compared with the beam strengthened by 3 mm thick steel plate. Therefore, it can be concluded that 3 mm thick steel plate on the bottom surface can most effectively increase the bearing capacity of combined strengthened beam SU10C30.

5.2. Influence of Thickness of U-Shaped Steel Strip. The Chinese code for the design of strengthening concrete structure also stipulates that the thickness of the U-shaped steel strip should not be less than 4 mm for the combined strengthened beam. In this paper, the influence of the thickness of the U-shaped steel strip on the bearing capacity
of beams U10C30 and SU10C30 is studied through the FE models. The thickness of the bottom steel plate may be too thick, which is one reason leading to the failure mode of tested beam SU10C30 being support failure [32], at the same time, the conclusion has been shown in Section 5.1 that the 3 mm thick steel plate on the bottom surface can most effectively increase the bearing capacity of combined strengthened beam, so the thickness of steel plate on the bottom surface is 3 mm for beam SU10C30 in FE models.

**Table 4: Ultimate load, ultimate deflection, and MOT value of beams U10C30 and SU10C30 with different DUMBs.**

| Beam     | DUMB (mm) | $P_u$ (mm) | $\Delta_u$ (mm) | $\mu$ (kN-mm) |
|----------|-----------|------------|-----------------|----------------|
| U10C30   | 450       | 169.7      | 12.4            | 1745.5         |
|          | 400       | 167.7      | 11.8            | 1656.1         |
|          | 350       | 143.5      | 12.4            | 1435.6         |
|          | 300       | 148.6      | 6.8             | 753.4          |
| SU10C30  | 450       | 188.7      | 5.7             | 1256.7         |
|          | 400       | 166.6      | 6.4             | 1082.4         |
|          | 350       | 180.9      | 5.3             | 621.9          |
|          | 300       | 169.7      | 4.5             | 700.1          |

**Table 5: Ultimate load, ultimate deflection, and MOT value of beams S10C30 and SU10C30 strengthened by the different thicknesses of the steel plate.**

| Beam     | Thickness of steel plate (mm) | $P_u$ (mm) | $\Delta_u$ (mm) | $\mu$ (joule) |
|----------|-------------------------------|------------|-----------------|---------------|
| S10C30   | 2                             | 165.5      | 14.0            | 1944.2        |
|          | 3                             | 178.6      | 13.0            | 1918.4        |
|          | 4                             | 185.6      | 11.9            | 1728.6        |
|          | 5                             | 188.9      | 10.0            | 1308.0        |
| SU10C30  | 2                             | 178.8      | 9.9             | 1568.2        |
|          | 3                             | 185.5      | 8.7             | 1446.1        |
|          | 4                             | 182.8      | 7.6             | 1289.8        |
|          | 5                             | 188.7      | 5.7             | 1256.7        |

**Figure 16:** Load-deflection curves of the beams U10C30 and SU10C30 strengthened by the different thicknesses of U-shaped steel strip: (a) U10C30 and (b) SU10C30.
1 mm to 5 mm. Figure 16 shows the load-deflection curves of beams U10C30 and SU10C30 strengthened by the different thicknesses of the U-shaped steel strip. The ultimate load, ultimate deflection, and MOT of beams U10C30 and SU10C30 strengthened by the different thicknesses of the U-shaped steel strip are given in Table 6.

For beam U10C30, it can be obtained from Figure 16(a) and Table 6 that the ultimate load of the beam slightly decreases when the thickness of the U-shaped steel strip exceeds 3 mm, and a similar situation also appears for the MOT value. Therefore, it can be concluded that it is most reasonable to improve the bearing capacity of the shear-strengthened beam with 10% corrosion rate using the 3 mm thick U-shaped steel strip. The ultimate load and the ductility of the shear-strengthened beam will not be significantly improved when the thickness of the U-shaped steel strip exceeds 3 mm.

For beam SU10C30, it can be observed from Figure 16(b) and Table 6 that the difference value of ultimate load between beams strengthened by 3 mm, 4 mm, and 5 mm thick U-shaped strip is less than 6 kN. Moreover, the MOT value of those beams has decreased by 22.6%, 16.4%, and 19.5%, respectively, compared with the beam strengthened by 2 mm thick U-shaped steel strip. Therefore, it can be concluded that the advice in Chinese code that the thickness of the U-shaped steel strip at the end of the beam should exceed 4 mm is not suitable for the combined strengthened beam with 10% corrosion rate. The ductility of the combined strengthened beam will decrease significantly when the thickness of the U-shaped steel strip exceeds 2 mm.

5.3. Influence of Location of U-Shaped Steel Strip. The influence of the location of the U-shaped steel strip on the bearing capacity of the tested beam is also analyzed in this paper. Figure 17 presents the load-deflection of beams U10C30 and SU10C30 with the different distances of the inner side of the U-shaped steel strip to the middle of the beam (DUMB). Table 4 shows the ultimate load, ultimate

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**Table 6: Ultimate load, ultimate deflection, and MOT value of beams U10C30 and SU10C30 strengthened by the different thicknesses of the U-shaped steel strip.**

| Beam   | Thickness of U-shaped steel strip (mm) | \( P_u \) (mm) | \( \Delta_u \) (mm) | \( \mu \) (kN-mm) |
|--------|----------------------------------------|---------------|-----------------|-----------------|
| U10C30 | 1                                      | 159.6         | 11.9            | 1414.7          |
|        | 2                                      | 166.7         | 12.8            | 1719.0          |
|        | 3                                      | 169.7         | 12.4            | 1745.5          |
|        | 4                                      | 168.9         | 13.1            | 1734.3          |
|        | 5                                      | 167.6         | 13.2            | 1691.2          |
| SU10C30| 1                                      | 173.3         | 10.8            | 1501.2          |
|        | 2                                      | 179.7         | 11.0            | 1571.9          |
|        | 3                                      | 185.5         | 8.7             | 1217.3          |
|        | 4                                      | 189.0         | 9.2             | 1314.8          |
|        | 5                                      | 191.3         | 8.9             | 1265.6          |

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**Figure 17:** Load-deflection curves of the beam SU10C30 strengthened by the different DUMBs. (a) Beam U10C30 and (b) Beam SU10C30.
deflection, and MOT value of beams U10C30 and SU10C30 with the different DUMBs. The DUMB of the tested beam is 450 mm.

For the beam U10C30, the ductility of the beam in which the DUMB is 300 mm is only 43.1% of the tested beam, and the MOT value decreases as the location of the U-shaped steel strip is closer to the middle of the beam, which indicates that the ductility of the beam decreases as DUMB decreases. At the same time, the ultimate load of the beam decreases significantly when the DUMB is less than 400 mm. It can be concluded that using a U-shaped steel strip at the end of the corroded beam can most effectively improve the bearing capacity of the corroded beam strengthened by the shear-strengthening scheme.

For the beam SU10C30, though the MOT value of the beam in which DUMB is 400 mm is close to the tested beam, the ultimate load of the beam in which DUMB is 400 mm has decreased by 22.1 kN. At the same time, the MOT values of beams in which the DUMBs are 350 mm and 300 mm, respectively, have decreased by 50.5% and 44.3% compared to the FE models of the tested beam, respectively. It can be also concluded that it is most reasonable to improve the bearing capacity of the combined strengthened beam through using the U-shaped steel strip to anchor the end of the steel plate at the bottom surface.

6. Conclusions and Comments

This paper has presented five 3D nonlinear finite element (FE) models for predicting the bearing capacity of two nonstrengthened and three corroded strengthened beams designed by the authors in the previous test. The accuracy of five FE models is validated by comparing the predicted load-deflection curves and the crack distribution of FE models with the measured test data. Based on the results of FE models, the influence of the thickness of the steel plate, the location, and thickness of the U-shaped steel strip on the bearing capacity of the strengthened beam is investigated, and several design advices for strengthened corroded beams are proposed. The following can be concluded from this study:

(i) The load-deflection curves of the developed FE models and the cracking distribution are in good agreement with the measured test data.
(ii) For 1.8 m long flexure-strengthened corroded beam with 10% corrosion rate, the thickness of the steel plate should not exceed 4 mm.
(iii) For the 1.8 m long shear-strengthened corroded beam with 10% corrosion rate, it is most reasonable to improve bearing capacity using the 3 mm thick U-shaped steel strip. The ultimate load and ductility of the shear-strengthened beam will not be significantly improved when the thickness of the U-shaped steel plate exceeds 3 mm. The most reasonable location of the U-shaped steel strip is at the end of the beam.
(iv) For 1.8 m long combined-strengthened corroded beam with 10% corrosion rate, the 3 mm thick steel plate on the bottom surface can most effectively increase the bearing capacity. The ductility of the combined strengthened beam will decrease significantly when the thickness of the U-shaped steel strip exceeds 2 mm. The most reasonable location of the U-shaped steel plate is also at the end of the steel plate.

This paper mainly focuses on the influence of different strength schemes on the mechanical behavior of the corroded beam based on small size beam, and corresponded conclusions can provide some useful information for the strengthening of real bridge structures. However, the real bridges are large size structures and carry live loads, and their structural behaviors are more complex compared with the tested beams. Thus, associated further research is needed to consider these effects.

The corrosion rate of all tests is 10%, larger corrosion rate will lead to bond deterioration between steel bar and concrete, the mechanical performance of the strengthened beam will be different, associated further research has been carried out, and the results will be written in the further papers.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

Data curation was performed by Jianxin Peng and Xinhua Liu; formal analysis was carried out by Jianxin Peng; funding acquisition was carried out by Jianren Zhang; software was provided by Huang Tang and Linfa Xiao; writing of the original draft was performed by Huang Tang; and reviewing and editing were done by Huang Tang and Jianxin Peng.

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