Abstract—Federated learning (FL) has attracted much attention as a privacy-preserving distributed machine learning framework, where many clients collaboratively train a machine learning model by exchanging model updates with a parameter server instead of sharing their raw data. Nevertheless, FL training suffers from slow convergence and unstable performance due to stragglers caused by the heterogeneous computational resources of clients and fluctuating communication rates. This paper proposes a coded FL framework to mitigate the straggler issue, namely stochastic coded federated learning (SCFL). In this framework, each client generates a privacy-preserving coded dataset by adding additive noise to the random linear combination of its local data. The server collects the coded datasets from all the clients to construct a composite dataset, which helps to compensate for the straggling effect. In the training process, the server as well as clients perform mini-batch stochastic gradient descent (SGD), and the server adds a make-up term in model aggregation to obtain unbiased gradient estimates. We characterize the privacy guarantee by the mutual information differential privacy (MI-DP) and analyze the convergence performance in federated learning. Besides, we demonstrate a privacy-performance tradeoff of the proposed SCFL method by analyzing the influence of the privacy constraint on the convergence rate. Finally, numerical experiments corroborate our analysis and show the benefits of SCFL in achieving fast convergence while preserving data privacy.

Index Terms—Federated learning (FL), coded computing, stochastic gradient descent (SGD), mutual information differential privacy (MI-DP).

I. INTRODUCTION

The recent development of deep learning (DL) has led to main breakthroughs in various domains, including healthcare [1], autonomous vehicles [2], and the Internet of Things (IoT) [3]. These applications in turn lead to an unprecedented volume of data generated at the wireless network edge by massive end devices. To utilize these data for DL model training, the traditional approach is to directly upload them to a cloud server. However, such a centralized approach may raise severe privacy concerns, as the local data collected by devices usually contain private and sensitive information [4].

To resolve this issue, federated learning (FL) [5] was proposed by Google to collaboratively learn a global model without sharing local data. A canonical FL system consists of a centralized parameter server and a large number of clients (e.g., IoT and mobile devices). In the training process, the clients perform training locally on their private data and upload the model updates to the server. After receiving the model updates from the clients, the server aggregates them into a new global model by weighted averaging. One of the main challenges in FL is the straggler effect, where a small number of significantly slower devices or unreliable wireless links may drastically prolong the training time. Since the client data in FL are non-independent and identically distributed (non-IID) [6], simply ignoring the stragglers may degrade the training efficiency and model performance.

Motivated by the coded computing techniques [7]–[11], coded federated learning (CFL) [12]–[14] has been recently proposed to mitigate the stragglers in federated linear regression by constructing the coded datasets. In CFL [12], [13], each client generates a coded dataset by applying random linear projection on their weighted local data, which is uploaded to a centralized server before training starts. During training, the server uses these coded datasets to compute the coded gradients in order to compensate for the missing gradients from the straggling clients. However, as the CFL-FB [12] framework performs gradient descent in a full-batch (FB) manner, the training process is computationally expensive. A variant of CFL-FB, namely CodedFedL, was investigated in [13], which adopts the mini-batch stochastic gradient descent (SGD) [15] algorithm to improve the training efficiency. Although the convergence of CodedFedL was analyzed in [13], it relies on simplified assumptions by neglecting the variance from mini-batch sampling. Moreover, the interplay between privacy leakage in coded data sharing and convergence performance of CFL is not well understood. Most recently, a differentially private coded federated learning (DP-CFL) scheme that adds Gaussian noise to the coded data was proposed in [14] for a better privacy guarantee. Nevertheless, DP-CFL restricts the gradient computation on the server once the coded datasets are collected, which fails to exploit the local computational resources at the clients for fast distributed training.

In this paper, we propose a stochastic coded federated learning (SCFL) framework for efficient federated linear regression. Specifically, each client generates a privacy-preserving coded dataset by adding Gaussian noise to the
random linear combination of its local data. The server collects the coded datasets from all the clients to construct a composite dataset, which helps to compensate for the straggling effect. In the training process, the server as well as clients compute the stochastic gradients on a batch of samples, and the server adds a make-up term in model aggregation to obtain unbiased gradient estimates. We characterize the privacy guarantee in coded data sharing using the mutual information differential privacy (MI-DP) method \cite{Liu2018} and analyze the convergence performance of SCFL. Besides, we theoretically demonstrate the privacy-performance tradeoff of the proposed SCFL framework by analyzing the influence of the privacy constraint on the convergence rate. Numerical experiments demonstrate such a tradeoff and show the benefits of SCFL in achieving fast convergence while preserving data privacy.

II. SYSTEM MODEL

A. Federated Learning for Linear Regression

We consider an FL system with a centralized server and \( n \) clients. They collaborate to train a model \( \mathbf{W} \in \mathbb{R}^{d \times o} \) where \( d \) and \( o \) are respectively the input and output dimensions. We focus on the linear regression problem over the training dataset \( (\mathbf{X}, \mathbf{Y}) \), where \( \mathbf{X} \in \mathbb{R}^{m \times d} \) concatenates the \( d \)-dimensional features of \( m \) data samples, and \( \mathbf{Y} \in \mathbb{R}^{m \times o} \) represents the corresponding labels. We formulate the following empirical risk minimization problem to optimize \( \mathbf{W} \):

\[
\min_{\mathbf{W}} J(\mathbf{W}) = \frac{1}{2} \| \mathbf{XW} - \mathbf{Y} \|^2_F.
\]  

Since the data samples are stored locally, each client \( i \) can only access \( l_i \) local data samples, denoted as \( \mathbf{X}^{(i)} = \mathbf{Z}^{(i)} \mathbf{X}, \mathbf{Y}^{(i)} = \mathbf{Z}^{(i)} \mathbf{Y} \), where the matrix \( \mathbf{Z}^{(i)} \in \mathbb{R}^{l_i \times m} \) represents the data availability on client \( i \) and \( \mathbf{Z}^{(i)}[i,0] = l_i \). Specifically, \( z_{j,j'}^{(i)} = 1 \) if the \( j \)-th data sample in the dataset corresponds to the \( j' \)-th data sample at client \( i \); otherwise \( z_{j,j'}^{(i)} = 0 \). We assume that the local datasets are disjoint across clients. Accordingly, we have \( \mathbf{X} = [\mathbf{X}^{(1)T}, \mathbf{X}^{(2)T}, \ldots, \mathbf{X}^{(n)T}]^T, \mathbf{Y} = [\mathbf{Y}^{(1)T}, \mathbf{Y}^{(2)T}, \ldots, \mathbf{Y}^{(n)T}]^T \), and \( m = \sum_{i=1}^n l_i \).

In canonical FL systems, the centralized server updates the global model based on the uploaded gradients from the clients. However, the training efficiency is critically affected by the delays in computing and communicating the gradients. Particularly, the server may need to wait for a few straggling clients in each training epoch. To characterize this phenomenon, we describe the computation and communication models in the next subsection.

B. Computation and Communication Models

Following previous works on CFL \cite{Agarwal2017, Bottou2018}, we consider clients with different processing rates and link constraints. In each epoch, the time needed for client \( i \) to complete local training and gradient exchange with the server is expressed as \( t_i(b_i) = t_{ci}(b_i) + t_{ui} + t_d \). Here, \( t_{ci}(b_i) = \frac{b_i}{\text{MACR}_i} \) denotes the time for computing the gradient on a batch of \( b_i \) data samples, where MACR\(_i\) denotes the Multiply-Accumulate (MAC) rate, and \( N_{\text{MAC}} \) is the number of MAC operations required for processing one data sample. We assume the server follows the same computation model as the clients but with a higher MAC rate. Besides, \( t_{ui} \) denotes the gradient uploading time for client \( i \), and \( t_d \) is the model downloading time. As the uplink transmission exhibits strong stochastic fluctuations in the link quality, we model the uplink data transmission of the \( i \)-th client by a data rate \( r_{\text{uplink}}^{(i)} \) and a link erasure probability \( p_i \) \cite{Ho2000}, i.e., the expected number of transmissions attempt \( N_i \) required before successful communication from the \( i \)-th client to the server follows a geometric distribution. In addition, we assume the server broadcasts the model to all the clients reliably with rate \( r_{\text{downlink}} \), attributed to more capable downlink radio resources. Therefore, the arrival probability that the model update of client \( i \) is received by the server within time \( T \) is given by \( p_i(T, b_i) = \text{Prob}\{t_i(b_i) \leq T\} \).

III. STOCHASTIC CODED FEDERATED LEARNING

In this section, we introduce the proposed Stochastic Coded Federated Learning (SCFL) framework.

A. Coded Data Preparation

Client \( i \) generates the coded data locally using the random projection matrix (denoted as \( \mathbf{G}_i \in \mathbb{R}^{c \times l_i} \)) and the additive Gaussian noise (denoted as \( \sigma \mathbf{N}_i \in \mathbb{R}^{c \times d} \)), where \( c \) is the amount of generated coded data, and \( \sigma \geq 0 \) controls the noise level. In particular, each entry of \( \mathbf{G}_i \) and \( \mathbf{N}_i \) is independently sampled from the standard normal distribution \( \mathcal{N}(0,1) \). The coded dataset is computed according to \( \tilde{\mathbf{X}}^{(i)} = \mathbf{G}_i \mathbf{X}^{(i)} + \sigma \mathbf{N}_i \) and \( \tilde{\mathbf{Y}}^{(i)} = \mathbf{G}_i \mathbf{Y}^{(i)} \). As a result, the coded dataset on the server can be expressed as:

\[
\tilde{\mathbf{X}} = \mathbf{G} \mathbf{X} + \sigma \mathbf{N}, \quad \tilde{\mathbf{Y}} = \mathbf{G} \mathbf{Y},
\]

where \( \mathbf{X} \triangleq \sum_{i=1}^n \mathbf{X}^{(i)}, \mathbf{G} \triangleq [\mathbf{G}_1, \mathbf{G}_2, \ldots, \mathbf{G}_n], \mathbf{N} \triangleq \sum_{i=1}^n \mathbf{N}_i \), and \( \tilde{\mathbf{Y}} \triangleq \sum_{i=1}^n \tilde{\mathbf{Y}}^{(i)} \). Note that the server only has access to the coded dataset \((\tilde{\mathbf{X}}, \tilde{\mathbf{Y}})\) without knowing matrices \( \mathbf{G}, \mathbf{N}, \mathbf{X} \) and \( \mathbf{Y} \). It is noteworthy that the above operations only occur once before training starts and thus incur negligible communication overhead.

B. Stochastic Gradient Computation

In the \( r \)-th training epoch of SCFL, to compensate the potential straggling clients, the server samples a random batch of the coded data with size \( b_i \) and computes the gradient \( g^{(r)}_i(\mathbf{W}^{(r)}) \) on \( \tilde{\mathbf{X}}^{(r)} \) and \( \tilde{\mathbf{Y}}^{(r)} \) as \( S^{(r)}_i \mathbf{X} + \mathbf{Y}^{(r)} \), where \( S^{(r)}_i \triangleq \text{diag}(s^{(r)}_{i,1}, s^{(r)}_{i,2}, \ldots, s^{(r)}_{i,c}) \in \mathbb{R}^{c \times c} \) is a diagonal matrix denoting the result of uniform sampling without replacement, and each diagonal entry \( s^{(r)}_{j,j} \) follows a Bernoulli distribution, i.e., \( s^{(r)}_{j,j} \sim \text{Bernoulli}(\frac{1}{c}) \). Specifically, \( s^{(r)}_{j,j} = 1 \) if the \( j \)-th coded data sample is selected in the \( r \)-th epoch; otherwise \( s^{(r)}_{j,j} = 0 \). Similarly, client \( i \) samples a batch of its local data with size \( b_i \) to compute the gradient \( g_i^{(r)}(\mathbf{W}^{(r)}) \). Denote the sampling matrix as \( S^{(r)}_j \triangleq \text{diag}(s^{(r)}_{j,1}, s^{(r)}_{j,2}, \ldots, s^{(r)}_{j,c}) \in \mathbb{R}^{l_i \times c} \), where each diagonal element \( s^{(r)}_{j,j} \) follows a Bernoulli distribution, i.e., \( s^{(r)}_{j,j} \sim \text{Bernoulli}(\frac{1}{c}) \). The mini-batch data
sampled in this epoch is expressed as $\hat{X}^{(i,r)} = S^{(i,r)}X^{(i)}$ and $\hat{Y}^{(i,r)} = S^{(i,r)}Y^{(i)}$. After stochastic gradient computation, we have $g_r(W^{(r)}) = \frac{1}{r} \nabla X^{(r)}T(\hat{X}^{(i,r)}W^{(r)} - \hat{Y}^{(i,r)})$ and $g_0(W^{(r)}) = \frac{1}{b_i} \nabla (\hat{X}^{(i,r)}W^{(r)} - \hat{Y}^{(i,r)})$. In every epoch, the server waits for a duration of time $T$, and aggregates the received stochastic gradients as follows:

$$g(W^{(r)}) = \frac{1}{2} \left[ \sum_{i=1}^{n} \hat{g}_i(W^{(r)}) + g_r(W^{(r)}) + g_0(W^{(r)}) \right] \tag{3}$$

where $\hat{g}_i(W^{(r)}) \triangleq \frac{2}{\mu_i r} \nabla g_r(W^{(r)}) \mathbb{1}\{t_i(b_i) \leq T\}$ denotes the arrived gradient from client $i$, $\mathbb{1}\{t_i(b_i) \leq T\}$ represents the arrival status, and $g_0(W^{(r)}) \triangleq -n\sigma^2 W^{(r)}$. We assign a higher weight $\frac{1}{\mu_i r}$ to the clients with lower arrival probabilities, such that $\mathbb{E}[\hat{g}_i(W^{(r)})] = g_i(W^{(r)})$. Besides, as adding Gaussian noise leads to a bias in the gradient estimation, the make-up term $g_0(W^{(r)})$ erases the bias incurred by data coding. Thus, such an aggregate scheme guarantees that the aggregated gradient $g(W^{(r)})$ is an unbiased gradient of the empirical risk function in (1), i.e., $\nabla f(W^{(r)}) \triangleq X^T(WX - Y)$. In the training process, the server updates the global model according to $W^{(r+1)} = W^{(r)} - \eta_r g(W^{(r)})$ where $\eta_r$ denotes the learning rate in the $r$-th epoch. After that, the server transmits the model to the clients and maintains a local copy. We adopt the average of these models over the entire training process as the learned model, i.e., $\frac{1}{T} \sum_{r=1}^{T} W^{(r)}$, where $R$ is the number of training epochs.

IV. THEORETICAL ANALYSIS

A. Convergence Analysis

We first present the following assumptions [13], [18] to facilitate the convergence analysis:

**Assumption 1.** The maximum absolute value of entries in $X$ is upper bounded by 1.

**Assumption 2.** There exist constants $\{\alpha_i\}$’s, $\{\zeta_i\}$’s, $\{\kappa_i\}$’s, and $\phi$ such that $\alpha_i^2 \leq \|X^{(i)}\|_F^2 \leq \zeta_i^2$, $\|X^{(i)}W^{(r)} - Y^{(i)}\|_F \leq \kappa_i^2$, and $\|W^{(r)}\|_F^2 \leq \phi^2$.

We are now to elaborate that the aggregated stochastic gradient in (3) is an unbiased estimate (i.e., Lemma 2) of the gradient $\nabla f(W^{(r)})$ with the bounded variance (i.e., Lemma 3). We first derive some important properties of the predefined matrices in the following lemma.

**Lemma 1.** Matrices $G$, $N$, $S^{(r)}$, and $\{S^{(i,r)}\}$’s have the following properties:

- $\mathbb{E}[\|\frac{1}{2}G^T G\|_F^2] = I_m$ and $\mathbb{E}[\|\frac{1}{2}G^T G - I_m\|_F^2] = \frac{m \cdot m^2}{c}$.
- $\mathbb{E}[\|\frac{1}{2}NTN\|_F^2] = \|N\|_F \|N\|_F - n\|N\|_F^2 = \frac{(d + d^2 \ln n)}{c}$.
- $\mathbb{E}[\|S^{(r)}T S^{(r)}\|_F] = I_c$ and $\mathbb{E}[\|\frac{1}{2}S^{(r)}T S^{(r)} - I_c\|_F^2] = \frac{c(c-b_c)}{b_c}$.
- $\mathbb{E}[\|\frac{1}{b_i}S^{(i,r)}T S^{(i,r)}\|_F^2] = I_c$ and $\mathbb{E}[\|\frac{1}{b_i}S^{(i,r)}T S^{(i,r)} - I_c\|_F^2] = \frac{c(c-b_c)}{\sum_{i}(l_i(b_i))}$.

**Proof.** According to the definition of $G$, $\frac{1}{2}G^T G$ follows the Wishart distribution, i.e., $\frac{1}{2}G^T G \sim W(c, I_m)$. Besides, since matrix $S^{(r)}$ is symmetric and diagonal, each entry of $\frac{1}{2}S^{(r)}T S^{(r)}$ has a unit mean. The proofs for $N$ and $\{S^{(i,r)}\}$ can be obtained similarly, which are omitted for brevity.

**Lemma 2.** The aggregated gradient in (3) is an unbiased estimate of the global gradient, i.e., $\mathbb{E}[g(W^{(r)})] = \nabla f(W^{(r)})$.

**Proof.** The result directly follows the properties derived in Lemma 1.

**Lemma 3.** The variance of stochastic gradients is bounded as follows:

$$\mathbb{E}
\left[
\left\|g(W^{(r)}) - \nabla f(W^{(r)})\right\|_F^2
\right] \leq \rho,$$

where $\rho \triangleq \frac{1}{c} \alpha_i^2 \kappa_i^2 + \frac{1}{c} (m + m^2) \zeta_i^2 + \frac{d}{c} \phi^2 + \frac{d m a^2}{c} (\zeta_i^2 + \kappa_i^2) + \frac{1}{c} \sum_{i=1}^{n} \frac{1}{\mu_i r} \sum_{b_i} \kappa_i^2 \zeta_i^2 + \frac{1}{c} \sum_{i=1}^{n} \alpha_i^2 \zeta_i^2 + \kappa_i^2 \zeta_i^2 + \frac{c}{c} \zeta_i^2$,

**Proof.** Please refer to the Appendix.

With Lemmas 2 and 3, we establish the convergence of SCFL in the following theorem.

**Theorem 1.** Define the optimality gap after a duration of time $T_{ot}$ as $G(T_{ot}) \triangleq \mathbb{E}[f\left(\frac{1}{T} \sum_{r=1}^{T} W^{(r)}\right)] - \min_W f(W)$, where $R = \left\lfloor \frac{T_{ot}}{T} \right\rfloor$. With Assumption 2, if the learning rate is chosen as $\eta_r = \frac{1}{c + \frac{1}{T}}$ and $\gamma = \sqrt{\frac{4 \rho^2 \phi}{R}}$, we have:

$$G(T_{ot}) \leq \sqrt{\frac{4 \rho^2 \phi}{R}} + 2 \phi^2 \gamma.$$
Theorem 2. With Assumption\cite{13} the privacy budget of client $i$ is given as follows:
\[
\epsilon_i = \frac{1}{2} \log_2 \left( 1 + \frac{c}{h^2(X^{(i)}) + \sigma^2} \right),
\]
where $h(X^{(i)}) \doteq \min_{k_2} \sum_{k_1=1}^{t_i} [X^{(i)}]_{k_1,k_2}^2 - \max_{k_3 \in I_i} [X^{(i)}]_{k_3,k_2}^2$ with $X^{(i)}_{j,k}$ denoting the $(j,k)$-th entry of matrix $X^{(i)}$. In particular, we select $\epsilon \doteq \max_i \epsilon_i$ as the privacy budget for coded data sharing.

Proof. The proof is similar to that of Theorem 2 in \cite{18}. \qed

Remark 2. The privacy budget in CodedFedL \cite{13} can be viewed as a special case of Theorem 2 with $\sigma = 0$. By adding Gaussian noise to the coded data, the proposed SCFL provides better privacy protection than CodedFedL.

Remark 3. (Privacy-performance tradeoff) According to Theorems\cite{7} and\cite{2} there is a tradeoff between privacy protection and convergence performance. Particularly, increasing the coded data size $c$ or the additive noise level $\sigma$ leads to a smaller optimality gap, but it results in more severe privacy leakage.

V. NUMERICAL EXPERIMENTS

In this section, we evaluate the performance of the proposed SCFL framework on two image classification tasks.

A. Experimental Setup

1) Wireless Edge Environment: We consider a wireless network with a server and $n = 20$ edge devices, using the delay model described in Section II-B to compute the overall training time. The downlink data rate of each client is set to $r_{\text{downlink}} = 1$ Mbps, and the uplink data rate of device $i$ is $r_{\text{uplink},i} = \mu_{\text{uplink},i} \times 1$ Mbps, where $\mu_{\text{uplink},i}$ is sampled from a uniform distribution $U(0.3, 1)$. The transmission failure probability $p_i$ is assumed to be 0.1. Besides, we randomly generate 20 MAC rates, i.e., $\text{MACR}_i = \mu_{\text{comp},i} \times 1,536$ KMAC per second, where $\mu_{\text{comp},i}$ is sampled from a uniform distribution $U(0.1, 1)$. The computation rate of the server is set as 15, 360 KMAC per second \cite{12}.

2) Baselines: We compare SCFL with the following baseline FL methods:

- **FL-PMA ($\psi$):** In the partial model aggregation (PMA) strategy\cite{6}, clients compute the stochastic gradient over the local mini-batches, and the server aggregates the gradients received from the first-arrived $(1 - \psi)n$ clients. In particular, setting $\psi = 0$ corresponds to FedAvg\cite{5} that aggregates all the gradients in each training round.

- **CFL-FB\cite{12} and CodedFedL\cite{13}:** Before training starts, the clients generate coded datasets based on the weighted local datasets and share them with the server. In each training epoch, the server and clients compute the gradients based on their data samples. After waiting for a duration of time, the server aggregates the received gradients to update the global model. Particularly, CodedFedL computes the stochastic gradients on a batch of samples, while the CFL-FB method trains model in a full-batch manner.

- **DP-CFL\cite{14}:** Each client first generates a coded dataset by perturbed random linear combinations of its data for uploading to the server. The server performs gradient descent based on the coded datasets with no further communication with the clients.

3) Dataset: We consider two benchmarking datasets, i.e., the MNIST\cite{20} and CIFAR-10\cite{21} datasets, to conduct the experiments. To simulate the non-IID data distribution, we adopt the skewed label partition\cite{22} to shuffle the MNIST and CIFAR-10 datasets. Specifically, we sort a dataset by the labels, divide it into 20 shards with identical sizes, and assign one shard to each client. Following\cite{13}, we leverage the random Fourier feature mapping (RFFM)\cite{23} to transform the MNIST classification task into a linear regression problem. Each transformed vector has a size of 2,000. Besides, each image in CIFAR-10 is represented by a 4096-dimensional feature vector extracted by a pretrained VGG model\cite{24}. We perform mini-batch SGD to achieve efficient federated learning, where each local dataset is partitioned into 30 subsets on MNIST and 20 subsets on CIFAR-10. The CodedFedL method obtains the optimal client batch sizes and coded datasets by solving the optimization problem of (23) in\cite{13}. For fair comparisons, we implement the same batch sizes for the SCFL, CodedFedL, DP-CFL, and FL-PMA methods.

B. Convergence Rate

In this subsection, we compare the convergence rates of different methods by setting the additive noise level to zero. The results in Fig. 1 show that the CFL-FB method exhibits high training latency, which demonstrates the effectiveness of mini-batch sampling in speeding up convergence. We also see that the conventional FL scheme (i.e., FL-PMA ($\psi = 0$)) converges slower than the SGD-based CFL methods (i.e., SCFL, CodedFedL, and DP-CFL) due to the straggling effect. Although FL-PMA can improve the convergence speed by dropping more stragglers (i.e., increasing the value of $\psi$), a larger dropout rate $\psi$ leads to more severe performance degradation especially in the non-IID scenario. Besides, the DP-CFL method prolongs the training process compared

![Fig. 1. Convergence time of different FL methods on (a) the MNIST dataset and (b) the CIFAR-10 dataset](image-url)
with SCFL. This is because DP-CFL restricts the gradient computation on the server without utilizing the clients’ computational resources. Moreover, the CodedFedL method has a comparable convergence rate to our method, but it does not provide an effective mechanism to adjust the privacy budget in coded data sharing. We investigate the privacy-performance tradeoff in the next subsection.

C. Privacy-Performance Tradeoff

We compare the learned model performance of SCFL, CodedFedL, and DP-CFL subject to different privacy budgets $c$, where $c$ is adjusted by varying the additive noise level. As observed in Fig. 2, reducing the privacy budget (i.e., the privacy constraint becoming more restrictive) degrades the model performance, which is consistent with the analysis in Remark 3. Besides, SCFL achieves a better privacy-performance tradeoff than other baseline CFL methods. As the privacy budget reduces, larger additive noise leads to biased gradient estimates in the SCFL and CodedFedL methods. In comparison, our gradient aggregation scheme in (3) adds a make-up term to mitigate the variance in model updating.

VI. CONCLUSIONS

In this paper, we proposed a novel algorithm to alleviate the straggler issue in federated learning, namely stochastic coded federated learning (SCFL). SCFL enjoys high training efficiency without impairing model accuracy by adopting a mini-batch SGD algorithm. We provided both the convergence and privacy analysis for SCFL, which showed a tradeoff between model performance and privacy. Simulations verified this tradeoff and demonstrated that SCFL achieves fast convergence while preserving privacy. For future works, it is worth investigating how to extend SCFL to other learning tasks.

APPENDIX

Define the full-batch gradient on the coded dataset $(\tilde{X}, \tilde{Y})$ as $\nabla f_s(W(r)) = \frac{1}{c} \tilde{X}^T(XW(r) - \tilde{Y})$. Given the independent error sources (i.e., mini-batch sampling in (3) and data coding in (3)), we decompose the variance caused by the server side as a summation of the following inequalities:

$$
E[\|g_b(W(r)) - \nabla f_s(W(r))\|_F^2] \leq \left(\frac{c}{b_s}\right)^2 E\left[\frac{c}{b_s}S_s(r) - I\right] \|XW(r) - \tilde{Y}\|_F^2 & \leq \left(\frac{c}{b_s}\right)^2 \|XW(r) - \tilde{Y}\|_F^2 \leq \left(\frac{c}{b_s}\right)^2 \zeta_c \kappa^2, \tag{8}
\right.

$$

and

$$
E[\|\nabla f_s(W(r)) + g_b(W(r)) - \nabla f(W(r))\|_F^2] \leq 4 \|X_T\|_F^2 E\left[\left(\frac{1}{c}G^T G - I\right)^2\right] \|XW(r) - Y\|_F^2 + 4\sigma^4 E\left[\left(\sum_{i=1}^n N_i \sum_{i=1}^n N_i - nI\right)^2\right] \|W(r)\|_F^2 + 4\sigma^2 E\left[\|G^T N\|_F^2\right] \|XW(r) - Y\|_F^2 \leq \frac{4c^2}{(m + m^2)\zeta_c \kappa^2 + \frac{4c^2}{(d + d^2)n\sigma^4 \phi^2 + \frac{4d^2m^2n^2\sigma^4}{c^2} (\zeta^2 \phi^2 + \kappa)}, \tag{9}
\right.

$$

where (a) and (c) follow the inequality $\|ABx\|_F^2 \leq \|A\|_F^2 \|B\|_F^2 \|x\|_2^2$ for any compatible matrices $A, B$ and vector $x$. (b) and (d) hold due to Lemma 1 and the fact that the expected values of $\|N^T G\|_F^2$ and $\|G^T N\|_F^2$ equal $dnm$. Defining the full-batch gradient on client $i$ as $\nabla f_i(W(r)) = X^{(r)^T}(X^{(r)}W(r) - Y^{(r)})$, we characterize the client-side variance as follows:

$$
E\left[\left\|\sum_{i=1}^n g_i(W(r)) \frac{1}{p_i(T, b_i)} \{T_i(b_i) \leq t \} - \nabla f_i(W(r))\right\|_F^2\right] \leq 2E\left[\left\|\sum_{i=1}^n \frac{1}{p_i(T, b_i)} \{T_i(b_i) \leq t \} - g_i(W(r))\right\|_F^2\right] + 2E\left[\left\|\sum_{i=1}^n g_i(W(r)) - \nabla f(W(r))\right\|_F^2\right] \leq 2E\left[\left\|\sum_{i=1}^n \frac{1}{p_i(T, b_i)} \{T_i(b_i) \leq t \} - g_i(W(r))\right\|_F^2\right] + 2E\left[\left\|X^{(r)^T}\left(\sum_{i=1}^n \frac{l_i}{b_i} S^{(i,r)} - I\right)(X^{(r)}W(r) - Y^{(r)})\right\|_F^2\right] \leq 2 \sum_{i=1}^n \frac{1}{p_i(T, b_i)} \zeta^2 c_i^2 \kappa_i^2 + 2 \sum_{i=1}^n \frac{l_i(1 - p_i(T, b_i))}{b_i} \zeta^2 c_i^2 \kappa_i^2, \tag{10}
\right.

$$

where (e) follows the Jensen’s inequality, (f) holds since $\sum_{i=1}^n Z^{(r)^T} Z^{(r)} = I_m$ and $\sum_{i=1}^n \nabla f_i(W(r)) = \nabla f(W(r))$. The first term in (g) follows $E[\|T_i(b_i) \leq t \} - p_i(T, b_i)| = p_i(T, b_i)(1 - p_i(T, b_i))$. Besides, the proof for the second term in (g) is similar to (8) and thus omitted. Then by summing up (8)-(10) we conclude the proof.
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