Definition of Contact Stresses and Gaps in the Hinges of Chain Gears

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Abstract. The article discusses the issues of determining contact stresses in the hinges of chain gears. It is necessary to know these stresses to assess the hinges’ wear resistance, since the wear of hinges (joints) is the most common cause of chain gears’ damage. In this regard, the focus was made on determining the necessary gap between the bushing and the roller of the chain gears. This gap significantly affects the largely determines the distribution pattern and magnitude of the stresses in the hinge, and, consequently, the wear resistance of chain gears. A method for calculating the maximum contact stress in the hinges of chain gears has been developed. It was proved that a decrease of the gap in the hinges influence in the following way: the contact angle between the roller and the bushing increases and the maximum contact stresses become lower. However, for real contact angles, the contact stresses greatly exceed the permissible average specific pressure, causing increased wear on the joints (hinges). One of the possible ways to solve this problem is to use transitional fittings in hinges, in which the gaps and tightness are relatively small. When pressing the rollers into the bushings, microroughnesses are cut from the contact surfaces, and the tightness in the connection practically disappears. Some examples of determining gaps and contact stresses in the hinges of chain gears are given.

1. Introduction
Wear of hinges is the most common type of damage of chains in closed and semi-closed gears of machines, engines and general engineering equipment. Therefore, the main calculation of chain gears is carried out on the condition of wear resistance of the chain joints. For the calculation, it is necessary first of all to know the contact stresses between bushings and rollers of the chain. However, the determination of stresses in the contact zones is a rather complicated task. Therefore, the calculation is carried out according to the conditional pressure in the chain hinges under the assumption of a zero gap between the roller and the bushing and a uniform distribution of pressure in the hinge. Moreover, there is no data on the size of the required gap in the chain hinges in State Standard of Russia (GOST) 13568-97 for driving roller chains and in GOST 588-81 for the traction plate chains, as well as in the literature on this issue. This gap mainly determines the pattern of distribution and meanings of contact stresses in the hinge, and thus wear resistance of chain gears.

2. Calculation of contact stresses
For hinges of chain the bore diameter of the bushing is: \[ d_{\text{bore}} = d + S_h, \]

where \( d \) is a roller diameter, \( S_h \) is a gap between a bushing and a roller in a hinge.

Gaps between the bushing and the roller in the chain hinges are very small. In this regard, the calculation can be reduced to the problem of indenting a circular finger into a body with a circular cavity, which, after application of the load, touches the finger over a relatively large area. The conjugation parameters here can be determined on the basis of the solution of the contact task of the theory of elasticity on the internal compression of two cylindrical bodies whose radii are almost equal. The solution to the task of compressing of cylindrical bodies with similar radii taking into account friction can be approximated by the same task without friction, since the friction forces on the contact surface lead to a redistribution of the contact pressure diagram and its displacement in the direction opposite to the rotation of the gear roller. In this case, the displacement angle \( \rho \) of the center of the contact arc is equal with high accuracy to \( \arctan(f) \), and half of the contact angle \( \phi_0 \) quite does not depend on the friction coefficient \( f \) [1, 2, 3, 4].

To determine the stresses in the gear hinges when \( d_{\text{bush}}/d > 1.5 \), where \( d_{\text{bush}} \) is a bushing’s diameter, we take the following pressure distribution on the contact surface:

\[
p(\varphi) = C \cdot \cos\left(\frac{\pi \varphi}{2 \varphi_0}\right),
\]

where \( \varphi \) is an angular current coordinate. Whereas maximum contact stress at the hinge (when \( \varphi = 0 \)) \( \sigma_{\text{max}} = C \). The value \( C \) can be found from the roller equilibrium conditions:

\[
\int_{-\varphi_0}^{\varphi_0} p(\varphi) \cos \varphi d\varphi = \frac{2N_{\text{sp}}}{d},
\]

where \( N_{\text{sp}} = N/l \) is a specific contact load (\( N \) is the total load on a hinge, \( l \) is bushing’s length). That is

\[
\int_{-\varphi_0}^{\varphi_0} C \cdot \cos\left(\frac{\pi \varphi}{2 \varphi_0}\right) \cos \varphi d\varphi = \frac{2N_{\text{sp}}}{d}.
\]

Whence

\[
C \cdot \frac{\pi \cos \varphi_0}{0.25 \pi^2 - \varphi_0^2} = \frac{2N_{\text{sp}}}{d}.
\]

When \( \varphi_0 = 0.5\pi \), that is when the diameters of the roller and the bore of the bushing are equal, the “uncertainty” of the type 0/0 arises on the left side of expression (4). To disclose the uncertainty, we use the Lopital rule:

\[
\lim_{\varphi_0 \to 0.5\pi} \left( \frac{\pi \cos \varphi_0}{0.25 \pi^2 - \varphi_0^2} \right) = \lim_{\varphi_0 \to 0.5\pi} \left( \frac{\left(\pi \cos \varphi_0\right)'}{\left(0.25 \pi^2 - \varphi_0^2\right)'} \right) = \frac{\pi}{2}.
\]

In this case if \( \varphi_0 = 0.5\pi \):

\[
\sigma_{\text{max}} = C = \frac{4N_{\text{sp}}}{\pi d} = \frac{4}{\pi} p,
\]

where \( p = N_{\text{sp}}/d \) is the average specific contact pressure in the hinge.

Expression (6) contains a multiplier \( (4/\pi) \) that takes into account the sickle-shaped distribution of pressure around the circumference of the pressure, the same as when calculating connections with tightness, which perceive the bending moment. From expression (4) we get:
\[ C = \frac{2N_{sp} \cdot 0.25\pi^2 - \phi_0^2}{\pi \cos \phi_0 \phi_0}. \]  

(7)

Now we express the maximum contact stress by the average specific pressure in the hinge:

\[ \sigma_{\text{max}} = C = kp; \quad k = 2 \frac{0.25\pi^2 - \phi_0^2}{\pi \cos \phi_0 \phi_0}. \]  

(8)

The calculation of the maximum contact stress in hinges of a chain gear which was made using formula (8) agrees quite well with the calculation according to the existing methodology for sliding bearings. For comparison, we calculate the coefficient \( k \) using the formula (8) and compare it with a similar coefficient for sliding bearings given in [5]. The calculation results are shown in Fig. 1.

![Figure 1](image_url)

**Figure 1.** The dependence of coefficient \( k \) on half of the contact angle \( \phi_0 \).

Here, the upper line corresponds to the calculation according to formula (8), and the lower one, according to the data of [5]. From Fig. 1 it can be seen that the value of the coefficient \( k \) determined by formula (8) is slightly higher, especially for small contact half-angles \( \phi_0 \). For large half-angles \( \phi_0 \) the calculations practically coincide. The maximum discrepancy does not exceed 20%. However, the dependences for calculating given in [5] are very complex, and their results are presented in a table form. And the resulting formula is simple and convenient for calculation.

3. Determination of gaps in hinges

For calculating the contact stress it is necessary to know the value of the angle \( \phi_0 \). The angle \( \phi_0 \) is a function of the loading coefficient \( \beta \). In this case (when both the roller and the bushing are made of steel) it can be determined by the following formula:

\[ \beta = \frac{2N_{sp} \cdot 1 - \nu^2}{\pi \varepsilon_{\text{av}} E}, \]  

(9)

where \( \varepsilon_{\text{av}} \) is the average radial gap in the hinge; \( E \) and \( \nu \) are modulus of elasticity and Poisson's ratio of steel, respectively. Then the average relative gap clearance \( \psi_{\text{av}} \) in the hinge is equal:
\[
\psi_{av} = \frac{4p(1-\nu^2)}{\pi\beta E}, \quad \psi_{av} = \frac{S_{av} - \varepsilon_{av}}{d/r},
\]

For maximum load:

\[
\psi_{av} = \frac{4[p](1-\nu^2)}{\pi\beta E},
\]

where \([p]\) is the permissible average specific pressure in the hinge.

The permissible average specific pressure in the hinges of roller chains \([6]\) is:

\[
[p] = \frac{[p_0]}{K_{op}},
\]

where \(K_{op}\) is an operating ratio. Pressure \([p_0]\) depends on chain pitch and the frequency of small sprocket’s rotation. The highest value \([p_0]\) = 35 MPa corresponds to the rotation speed of a small sprocket \(n<50\) min\(^{-1}\). The minimum value for the operating coefficient the lubricated transmission coefficient for lubricated gears \(K_{op}=0, 8\). Then \([p]_{\text{max}}=43, 75\) MPa.

As already noted, the angle \(\phi_0\) is a function of the loading coefficient \(\beta\), which is defined graphically in \([5]\). Using this dependence, the average relative gap \(\psi_{av}\) was calculated for \([p]_{\text{max}}=43, 75\) MPa and for various contact half-angles \(\phi_0\). The calculation results are shown in Fig. 2.

\begin{figure}
\end{figure}

For \(\phi_0>45^\circ\) it is technologically difficult to provide the required gaps in the hinge. If we also take into account that for \(p<[p]_{\text{max}}\) or even for \(p<[p]\), the relative gaps in the hinge decrease. Therefore, we will further assume that \(\phi_0\leq45^\circ\). For this range, we can propose the following dependence connecting the contact half-angle \(\phi_0\) with the average relative gap \(\psi_{av}\):

\[
\psi_{av} \cdot 10^5 = \frac{1.8 \cdot 10^6}{\phi_0^{0.53}} + 51.4.
\]

To determine the contact half-angle itself \(\phi_0\) for \(\phi_0\leq45^\circ\), the following dependence can be proposed:

\[
\phi_0 = 94\beta^{0.53}.
\]
In this case, the average diametrical gap is $S_{av} = \psi_{av} d$. For example, we define the contact half-angle $\varphi_0$ and contact stress for the case when $p = 20$ MPa and $S_{av} = 0.006d$.

In this case:

$$\beta = \frac{4\times 20}{\pi \times 0.006 \times 2.1 \times 10^2} = 0.0184$$

and $\varphi_0 = 94 \times (0.0184)^{0.53} = 11.31^\circ$.

The maximum contact stress in the hinge is:

$$\sigma_{max} = C = kp = 2 \times \frac{0.25 \pi^2 \times (11.31 \pi / 180)^2}{\pi \cos(11.31 \pi / 180)} \times 20 = 160 \text{ MPa}.$$

It greatly exceeds the permissible average specific pressure, causing increased wear of the joints. This tendency remains the same for the entire range of angles $\varphi_0 \leq 45^\circ$.

It is possible to reduce the contact stress in the hinges of the chain gear by increasing the half-angle $\varphi_0$ of the contact of the roller with the bushing, i.e. increasing the loading coefficient $\beta$. This is achieved by reducing the average radial gap in the hinges $e_{av}$. In this case, for the range of angles $\varphi_0 > 45^\circ$, we can propose the following dependence for their determination:

$$\varphi_0 = 63.46 \beta^{0.198}. \quad (15)$$

4. Conclusions

The necessary gaps in the hinges are very small, and it is quite difficult to provide them technologically. One of the possible ways to approach to solve this problem is to use transitional tightness in hinges, for example, of the type H7/m6 or H7/k6. In these cases, the probability of obtaining a gap and a preload is approximately the same. Gaps and preloads are relatively small. When pressing the rollers into the bushings, microroughnesses are cut from the contact surfaces, and the tightness in the connection practically disappears, since it is here approximately equal to the correction for a hug (cut) of microroughnesses during assembly.

5. References

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