The theoretical basis of the method of optimizing parameters of the propulsion multipurpose wheeled vehicles

Valery Guskov1,*, Tatiana Mikulic2, Victoria Pavlova3, and Alexey Sochnev1

1Belarusian National Technical University, Department "Tractors", 220013 Minsk, Nezavisimosti, 65, Belarus
2Belarusian National Technical University, Department "Details of machines", 220013 Minsk, Nezavisimosti, 65, Belarus
3Belarusian National Technical University, Department "Transport systems and technologies", 220013 Minsk, Nezavisimosti, 65, Belarus

Abstract. In the design of the wheel machine there are problems of selecting the parameters of its engine at a given load on the wheel or determining the optimal normal load at a given wheel parameters. The method of optimization of parameters of a wheel and its loading proceeding from an integral criterion of efficiency is offered. As a criterion, such an indicator as the efficiency of the driving wheel is taken. The article offers mathematical dependences for determining the efficiency, which take into account the wheel slipping, the resistance force due to the soil crumpling and the formation of a track, as well as the tangential force of traction realized by the wheel.

1 Introduction

Questions related to the study of traction qualities of wheeled vehicles are considered in many works [1-3]. It should be noted that for the machine as a whole, traction and dynamic qualities are usually evaluated by the traction efficiency and dynamic factor [3-4].

However, each of the wheels operates under certain conditions in terms of vertical load, torque input, driving conditions. In addition, for example, for tractors, the wheels of the front and rear axles have different sizes. In this regard, of particular interest is the assessment of the contribution of each individual wheel to the overall traction dynamics of a multi-axle wheeled vehicle.

Traction properties of the wheel depend on a large number of parameters. These include design parameters, geometric dimensions, and operating conditions [1-2].

When assessing the efficiency of the wheel in the traction mode during mathematical modeling, as well as in experimental studies, it is advisable to use some integral index, taking into account the variety of factors acting on the wheel. This will allow the design stage and research to choose the optimal mode of operation of the wheel or to choose the appropriate tire for certain operating conditions of the wheel machine. The paper proposes to use as such an indicator the coefficient of performance, which takes into account the wheel slipping, the force of resistance due to the crushing of the soil by the driver and the formation of a track, as well as the tangential force of traction realized by the wheel.

2 Main part

As mentioned above, the design of the wheel machine raises questions about the choice of parameters of its engine at a given load on the wheel or determining the optimal normal load at a given wheel parameters.

* Corresponding author: POVAREKHO@bntu.by

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The variety of structural and layout solutions chassis of wheeled vehicles (Figures 1, 2) complicates the selection of tires and load modes of the wheels of individual bridges.

Fig. 1. Some layouts wheeled chassis.

Fig. 2. The circuit chassis of wheeled tractors and tractor units.

Layout solutions may differ in the number of wheel axle, the number of driving and driven, controlled and not controlled bridges, the nature of their location along the base of the machine. Double and built trolleys can be used. All this leads to the fact that the load on the wheels and the conditions of their movement are different. Therefore, the choice of the wheel scheme of the chassis, the nature of the load being transported, the load distribution along the cargo platform must be made from the condition of ensuring the greatest efficiency of the wheels of the drive axles in terms of the best implementation of traction and dynamic qualities. All this will improve the traction quality of the machine, fuel efficiency and performance in General.

Depending on the type of torque supply to the drive wheels (differential or locked), driving conditions (straight or cornering), the design features of the transmission wheel can operate in traction or brake modes. In this paper, the wheel movement in the traction mode is considered.

In General, the wheel slip characterizes the speed loss due to the displacement of the instantaneous center of speeds relative to the contact spot and is calculated by the expression:
The variety of structural and layout solutions of wheeled vehicle chassis (Figures 1, 2) complicates the selection of tires and load modes of the wheels of individual bridges. Figure 1 shows some layouts of wheeled chassis. Figure 2 illustrates the circuit of a chassis of wheeled tractors and tractor units. Layout solutions may differ in the number of wheel axle, the number of driving and driven, controlled and not controlled bridges, the nature of their location along the base of the machine. Double- and built trolleys can be used. All this leads to the fact that the load on the wheels and the conditions of their movement are different. Therefore, the choice of the wheel scheme of the chassis, the nature of the load being transported, and the load distribution along the cargo platform must be made from the condition of ensuring the greatest efficiency of the wheels of the drive axles in terms of the best implementation of traction and dynamic qualities. All this will improve the traction quality of the machine, fuel efficiency and performance in general.

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\[
\delta = \frac{\omega\cdot r_k - v}{\omega\cdot r_k} = 1 - \frac{v}{\omega\cdot r_k},
\]

where \(\omega\) – the angular velocity of the wheel; \(r_k\) – the kinematic radius of the wheel; \(v\) – the linear velocity of the wheel center.

The settlement scheme of movement of the drive wheel taking into account influencing forces is shown in Figure 3.

![Diagram of forces and moments applied to the drive wheel moving along a horizontal ground surface in a steady state.](image)

In this figure, the following designations are accepted: \(r_o\), \(r_s\), \(r_d\) – nominal, static and dynamic wheel radiiuses; \(c\) – coordinate of the point \(O''\) of application of the reaction of a soil surface; \(M_k\) – torque applied to the center \(O\) of the wheel; \(h\) and \(h_{sh}\) – rut depth and deformation of the tire under normal load \(G\).

The analysis of the driving wheel working processes during its movement allowed us to make an assumption that the coefficient of efficiency of the driving wheel can be used as a complex indicator of the efficiency of the traction properties. The efficiency of the drive wheel \(\eta_k\) is determined according to the expression:

\[
\eta_k = (1 - \delta) \cdot \left(1 - \frac{F_f}{F_k}\right),
\]

where \(F_f\) - the strength of the resistance due to the soil crumpling and the formation of a track; \(F_k\) - tangential thrust force (Figure 3).

The authors propose a method for optimizing the parameters of the wheel and its load, when as a criterion of efficiency (objective function) takes the coefficient of efficiency of the driving wheel \(\eta_k\).

To determine the parameters included in the formula for determining the efficiency of the wheel, a calculation scheme was used (Figure 3). On the basis of the D’Alembert principle, the equations of equilibrium of forces and moments applied to the driving wheel moving along the horizontal ground surface in an unsteady mode were compiled:

\[
\begin{align*}
\Sigma X &= x - F_j - F_w = 0 \\
\Sigma Y &= y - G = 0 \\
\Sigma M_0 &= y \cdot c + x \cdot r_d + M_j - M_k = 0
\end{align*}
\]
where $M_k$ – torque applied to drive wheel; $G$ – normal load acting on the wheel; $R$ – the reaction of soil; $x$ and $y$ – the horizontal and vertical components of reaction $R$; $F_f$ and $M_t$ – force and moment of inertia; $F_w$ – the force of air resistance.

Taking into account the designations given in Figure 3, the force of resistance to movement due to the crumpling of the ground by the wheel and the formation of a track is equal to:

$$F_f = y \cdot \frac{M_k}{r_d} = G \cdot \frac{c}{r_d} = f \cdot G,$$

(4)

where $f$ – coefficient of resistance to movement.

The reaction $x$ represents the driving force, which is denoted as $F_k$.

To determine the values included in the system of equations (1), Professor V.V. Guskov [6] developed the theoretical basis of the interaction of the driving wheel with the ground surface. They are based on the modern provisions of the theory of soil mechanics in the application of dynamic loads and the dependence of the resistance of the soil compression and shear, proposed by Professor V.V. Katsygin [7].

In particular, the compressive stress of the soil is determined by the formula:

$$\sigma = \sigma_0 \cdot \frac{k}{\sigma_0} \cdot h,$$

(5)

where $\sigma_0$ – soil bearing capacity; $k$ – the coefficient of volumetric collapse of the soil; $h$ – the depth of immersion of the stamp.

Shear stress arising from the deformation of the soil:

$$\tau = f_{sk} \cdot q_x \cdot \left(1 + \frac{f_n}{c h k_t} \right) \cdot \frac{\Delta}{k_t},$$

(6)

where $q_x$ – wheel pressure on the ground; $f_{sk}$ – the coefficient of sliding friction; $f_n$ – the coefficient of static friction; $k_t$ – the coefficient of deformation of the soil; $\Delta$ – shear deformation.

The graphic image of the dependence of the ground compression stresses is shown in Figure 4.

Fig. 4. Compression stress-strain dependence ($k = t g \alpha$).

Figure 4 shows that there are three areas of this dependence: the first section reflects the elastic deformation; the second – plastic; the third – the flow of the soil.
The graphic image of the dependence of shear stresses arising from the deformation of the soil is shown in Figure 5.

**Fig. 5.** The dependence of shear stress against deformation.

Figure 5 shows that shear stresses reach a maximum at some strain $\Delta_0$ and then decrease. This phenomenon is explained by the fact that on the site I the soil is compacted (static friction), and on the site II – is shifted (sliding friction).

On the basis of the provisions of soil mechanics obtained mathematical expressions for determining the power parameters acting on the wheel. These indicators are proposed to determine according to the developed algorithm of the interaction of the drive wheel with the ground surface, proposed by Professor V.V. Guskov [6].

Resistance to movement:

$$ F_r = \int_{0}^{h_0} b \cdot \sigma_0 \cdot t h \left[ \frac{k}{4b \cdot \sigma_0} \cdot D_{pr} \cdot \ln \left( \frac{D_{pr} - h}{D_{pr} - h_0} \right) \right] dh, $$

where $b$ – wheel width; $D_{pr}$ – led wheel diameter; $h_0$ – the soil deformation for the corresponding vertical loads.

Vertical load, leading to soil deformation on the value of $h_0$:

$$ G = \int_{0}^{h_0} b \cdot \sigma_0 \cdot \frac{(D_{pr} - 2 \cdot h)}{2 \cdot \sqrt{D_{pr} \cdot h - h^2}} \cdot t h \left[ \frac{k}{4b \cdot \sigma_0} \cdot D_{pr} \cdot \ln \left( \frac{D_{pr} - h}{D_{pr} - h_0} \right) \right] dh. $$

**Fig. 5.** The dependence of shear stress against deformation.

Tangential traction force developed by the wheel:

$$ F_k = \int_{0}^{L_{pr}} 2 \cdot b \cdot f_{sk} \cdot q_x \cdot L_{pr} \cdot \delta_x \left( 1 + \frac{f_n}{ch \frac{\delta_x L_{pr}}{k_t}} \right) \cdot t h \frac{\delta_x L_{pr}}{k_t} dL, $$

where $L_{pr}$ – the length of the bearing surface of the wheel.

The following assumptions were made in obtaining these equations.

1. The wheel moves under the action of the applied torque $M_k$ in a steady state on a horizontal surface.
2. When considering the process of interaction of the driving wheel with the ground surface, mechanical characteristics of soils are used to determine which mathematical dependences proposed by Professor V. V. Katsygin are used [7].
3. The nominal diameter $D_0$ of the real wheel is replaced by the reduced diameter $D_{pr}$ of the hard wheel according to the dependence [5]:

$$ D_{pr} = D_0 + \frac{h_{sh}}{h} (D_0 - 2 \cdot h - h_{sh}). $$

4. The actual length of the reference surface is calculated by the formula:
\[ L_{pr} = \frac{D_0}{2} \cdot \arctg \left( \frac{D_{pr} \cdot h^2}{2 \cdot D_{pr} \cdot h - h} \right) + \sqrt{D_{pr} \cdot h}. \] (11)

Thus, on the basis of the proposed mathematical models, the assessment of the optimal conditions for the operation of the wheel machine engine can be solved in two directions.

1. The optimal vertical load on the wheel is determined for its specified design parameters, i.e. the equation of the form is solved:
   \[ \frac{d\eta_k}{dG} \rightarrow 0. \] (12)

2. The design parameters of the wheel at a given load are determined:
   \[ \frac{\partial \eta_k}{\partial(D_0, b_0, p_w, \ldots)} \rightarrow 0, \] (13)

where \( p_w \) – the air pressure in the tire.

To verify the proposed method, an example of solving the problem of optimizing the normal \( G \) load on the drive wheel (bus 580/70R42) with dimensions is given: \( D_0 = 1.9 \) m; \( b_0 = 0.714 \) m a pressure of \( p_w = 0.2 \) MPa, moving along the stubble of a loam of normal humidity (\( w = 16\% \)). The soil has the following physical and mechanical properties: \( f_s = 0.76; \ f_n = 0.79; \ k = 0.04 \) m; \( \sigma_0 = 1.58 \times 10^6 \) N/m².

Figure 6 shows the results of the calculation of the traction-chain properties of the specified tire depending on the normal load.

![Fig. 6. Traction properties of the tires 580/70R42 depending on the vertical load.](image)

In this figure \( F_{kr} \) - hook effort. The hook force is the useful force realized by the wheel and is determined by:
\[ F_{kr} = F_k - F_f \] (14)

As can be seen from the presented graphic dependences, with the growth of the vertical load on the wheel, there is an increase in the tangential thrust force \( F_k \), the force of resistance to the movement of the wheel \( F_f \) and, accordingly, the realized hook load \( F_{kr} \). The nature of these parameters changes is close to linear.

The dependence of the efficiency of the wheel \( \eta_k \) has a pronounced maximum corresponding to the vertical load \( G_{opt} = 32.5 \) kN.

3 Conclusion

As a result of the research the following results were obtained.

1. The proposed method of optimization when used as a criterion for the efficiency of the traction coefficient of efficiency of the wheel allows to determine the rational parameters of the designed wheel drive with high efficiency and efficiency.
2. This method was introduced into the practice of designing the perspective wheeled tractors of the family "Belarus" Minsk tractor plant.

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