CHARMED QUARK AND $J/\Psi$ PHOTOPRODUCTION IN THE SEMIHARD APPROACH OF QCD AT HERA ENERGIES

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**ABSTRACT**

We compare our theoretical results for $c\bar{c}$—quark and $J/\Psi$—photoproduction cross sections obtained in the semihard approach of QCD with H1 and ZEUS experimental data.

1. Introduction

The observation of heavy quark and quarkonium photoproduction offers a unique opportunity to probe the gluon distribution in proton at small $x$ by measurements of the total cross sections of these processes or of their differential distributions.

It is known that at HERA energies and beyond heavy quark and quarkonium photoproduction processes are of the so-called semihard type. In such processes by definition a hard scattering scale $Q$ (or heavy quark mass $M$) is large as compared to the QCD parameter, but $Q$ is much smaller than the total center of mass energies: $\Lambda_{QCD} \ll Q \ll \sqrt{s}$. The last condition implies that the processes occur in small $x$ region: $x \sim M^2/s \ll 1$. In such a case the perturbative QCD expansion has large coefficients $O(\ln^n(s/M^2)) \sim O(\ln^n(1/x))$ besides the usual renormalization group ones which are $O(\ln^n(Q^2/\Lambda^2))$. It means that in the small $x$ region ($x \sim M^2/s \ll 1$) we should first of all sum up all the terms of the type $(\alpha_s \ln s/M^2)^n$ and also the terms $(\alpha_s \ln Q^2/Q_0^2 \ln s/M^2)^n$ (moreover $\alpha_s \ln Q^2/Q_0^2$ in DIS). The second problem that appears at small $x$ is that of the violation of unitarity due to the increase of the gluon density at $x \to 0$. The latter one is reduced to taking into account the absorption (screening) corrections which should stop the growth of the cross section at $x \to 0$ (in accordance with the unitarity condition). Both these problems were resolved in the so-called semihard approach (SHA) by L. Gribov, E. Levin, M. Ryskin. Therefore we used this SHA for the calculations of the cross sections of $J/\Psi$— and $c\bar{c}$—photoproduction. In present paper the result of these calculations will be compared with new H1 and ZEUS experimental data at HERA energies.
2. Heavy quark photoproduction cross section in the semihard approach

In the SHA heavy quark photoproduction is determined by the contribution of two photon-gluon fusion diagrams with "unusual" properties of the gluons in proton. These gluons are the off mass shell ones, their virtuality \( q^2 = -q_T^2 \), and they possess the property of the alignment of their polarization vector along their transverse momentum such as \( \epsilon_\mu = q_{T\mu}/|q_T| \). Their distributions in \( x \) and \( q_T \) in proton is given by the unintegrated gluon structure function \( \phi(x, q_T^2) \), which is connected with the usual gluon distribution function \( G(x, Q^2) \) by the following relation

\[
\int_0^{Q^2} \phi(x, q_T^2) dq_T^2 = xG(x, Q^2). \tag{1}
\]

The exact expression for the function \( \phi(x, q^2) \) can be obtained as a solution of the evolution equation, which, contrary to the parton model case, is nonlinear due to interactions between the gluons in small \( x \) region.

In our calculations we used the following phenomenological parametrization \( \phi \):

\[
\phi(x, q_T^2) = C \frac{0.05}{x + 0.05} (1 - x)^3 f(x, q_T^2), \tag{2}
\]

where

\[
\begin{align*}
     f(x, q_T^2) & = 1, \quad q_T^2 \leq q_0^2(x) \\
     f(x, q_T^2) & = \left(\frac{q_0^2(x)}{q_T^2}\right)^2, \quad q_T^2 > q_0^2(x),
\end{align*}
\]

and \( q_0^2(x) = Q_0^2 + \Lambda^2 \exp(3.56 \sqrt{\ln(x_0/x)}) \), \( Q_0^2 = 2GeV^2 \), \( \Lambda = 56 \text{ MeV} \), \( x_0 = 1/3 \). The parameter \( q_0^2(x) \) can be treated as a new infrared-cutoff, which plays the role of a typical transverse momentum of partons in the parton cascade of the proton in semihard processes. The behaviour of \( q_0(x) \) was discussed in \( \phi \). It increases with \( \ln(1/x) \) and at \( x = 0.01 - 0.001 \) the values of \( q_0(x) \) are about \( 2 - 4GeV \).

For relatively small virtuality \( Q^2 \leq q_0^2(x) \) the gluon function behaves as \( xG(x, Q^2) \sim CQ^2 \). So we have the saturation of the gluon density at these values \( Q^2 \).

The normalization factor \( C \approx 0.97 \text{ mb} \) in (2) was obtained in \( \phi \) where \( b\bar{b} \)-pair production at Tevatron energy was described.

Thus, in the case of real transverse polarized photon the heavy quark photoproduction cross section at \( x \ll 1 \) can be expressed as

\[
\frac{d\sigma}{d^2p_{1T}}(\gamma p \to Q\bar{Q}X) = \int dy_1^* d^2q_T \frac{\phi(x, q_T^2)}{\pi} \frac{|M|^2}{16\pi^2 (sx)^2 \alpha}, \tag{4}
\]

where \( p_T, y_1^* \) are transverse momentum and rapidity (in the center of mass frame of colliding particles) of heavy quark and \( \alpha = 1 - \alpha_1 \) with \( \alpha_1 = M \exp(y_1^*)/\sqrt{s} \).

The matrix element \( M \) for a subprocess \( \gamma g^* \to c\bar{c} \) depends on the virtuality of the gluon and differs from the one of the usual parton model. For the square of this matrix element we used the following form:

\[
|M|^2 = 16\pi^2 e_Q^2 \alpha_s \alpha_{em}(xs)^2 \left[ \frac{\alpha_1^2 + \alpha^2}{(t-M^2)(u-M^2)} + \frac{2M^2}{q_T^2} \left( \frac{\alpha_1}{u-M^2} - \frac{\alpha}{t-M^2} \right)^2 \right], \tag{5}
\]
where \( \hat{s}, \hat{t}, \hat{u} \) are usual Mandelstam variables of partonic subprocess \( \gamma g^* \to c\bar{c} \).

### 3. Quarkonium photoproduction cross section in SHA of QCD and in colour singlet model

We also used similar formulas for heavy quarkonium photoproduction. In the case of real transverse polarized photon, the heavy quarkonium photoproduction cross section at \( x \ll 1 \) in SHA is expressed in the form (we consider here \( J/\Psi \) - photoproduction):

\[
\sigma(\gamma p \to J/\Psi X) = \int \frac{dz}{z(1-z)} \int dp_T^2 \int d\phi \frac{\varphi(x, q^2_T)}{2\pi} \frac{1}{16\pi(s + q^2_T)^2} \sum |\vec{M}(\gamma g^* \to J/\Psi g)|^2, \quad (6)
\]

where (in the lab. frame) \( z = E/E_\gamma, \ s = 2m_p E_\gamma \) and \( p = (E, \vec{p}_T, p_z) \) is 4–momentum of the quarkonium, \( \phi \) is the angle between initial gluon and quarkonium transverse momenta \( \vec{q}_T \) and \( \vec{p}_T \). In (6) \( \sum \) indicates an average over two photon polarizations and colours of initial gluon as well as a sum over polarizations of final particles.

If we take the limit of zero \( q^2_T \), and if we average over the transverse directions of \( \vec{q}_T \), we obtain the formula of standard parton model (SPM):

\[
\sigma(\gamma p \to J/\Psi X) = \int \frac{dz}{z(1-z)} \int dp_T^2 \frac{xG(x, Q^2)}{16\pi(x^2)} \sum |M_{part}(\gamma g \to J/\Psi g)|^2, \quad (7)
\]

where \( \sum \) now indicates an average over colours and transverse polarizations of real initial gluon and photon as well as a sum over polarizations of final particles. We averaged over the transverse directions of \( \vec{q}_T \) using expression:

\[
\int d\vec{q}_T^2 \int d\phi \frac{\varphi(x, \vec{q}_T)}{\vec{q}_T^2} \sum |\vec{M}|^2 = xG(x, Q^2) \sum |M_{part}|^2, \quad (8)
\]

where

\[
\int_0^{2\pi} \frac{d\phi q_T^\mu q_T^\nu}{\vec{q}_T^2} = \frac{1}{2} g_{\mu\nu}. \quad (9)
\]

Within the framework of perturbative QCD the photoproduction of \( J/\Psi \) particles is described by subprocess \( \gamma g \to J/\Psi g \). In this approach, so called ”colour singlet model” (CSM), the quarkonium is represented by a define wave function so that the final \( c\bar{c} \) system are colour singlet, \( J^p = 1^- \) state of specified mass. At not very large \( p_T \) the contribution of these subprocesses are significantly greater than others, such as: \( \gamma g \to b\bar{b} \) with \( b \to J/\Psi X \). That is why we took into account only this.

The matrix element \( \vec{M} \) of the process \( \gamma g^* \to J/\Psi g \) was calculated using the sum of six diagrams according to the CSM. We makes summation over spins and colours of final gluon, \( J/\Psi \) and photon as in SPM. In the case of initial off shell gluon with transverse momentum \( \vec{q}_T \) we takes polarization tensor in the form:

\[
d_{\mu\nu}(q) = \varepsilon_\mu(q)\varepsilon_\nu(q) = \frac{q_T^\mu q_T^\nu}{\vec{q}_T^2}. \quad (10)
\]
The calculation of $\sum |\vec{M}|^2$ was made analytically by "REDUCE" system and result can be expressed in the following form:

$$\sum |\vec{M}|^2 = \frac{Bx^2}{q_T^2} \sum_{i=1}^{6} F_i(z, q_T^2, \hat{t}, \hat{u}),$$

(11)

where

$$B = \frac{32\pi^2}{3\alpha_s(Q^2)\alpha_s(q^2)\Gamma_{ee}M},$$

(12)

$Q^2 = M^2 + p_T^2$, $M$ is the mass of quarkonium, $\Gamma_{ee}$ is the leptonic quarkonium width, $\hat{t}$ and $\hat{u}$ are ordinary Mandelstam variables, $\hat{t} = M^2 - (M^2 + p_T^2)/z$, $\hat{u} = M^2 - q_T^2 - xzs + 2p_Tq_T\cos\phi$ and $x = (M^2 - \hat{u} - \hat{t})/s$. The explicit form of functions $F_i$ are given in \cite{9}.

We would like to note the limits of applicability of CSM: $z \leq 0.8$ and $p_T^2 \geq 0.1M^2$ \cite{8}, where $z = E_{J/\Psi}/E_{\gamma}$ (in lab. frame) and $p_T$ is the $J/\Psi-$ transverse momentum. These limits correspond to the region of H1 experimental data \cite{4}.

4. Results

The results of our calculations for cross sections of $\gamma p \rightarrow c\bar{c}X$ and $\gamma p \rightarrow J/\Psi X$ processes \cite{1, 2} are compared (in Figs. 1, 2) with H1 and ZEUS experimental data \cite{3, 4, 5}. (The curves in Fig.1 correspond to different values of $c-$quark mass: solid - $m_c = 1.3$ GeV, dashed - 1.4 GeV and dash-dotted - 1.5 GeV.)

In Fig. 3 the $z$-distribution of $J/\Psi-$mesons \cite{5} is shown (solid curve) in comparison with H1 experimental data \cite{4} and colour octet model (COM) result (dash curve) \cite{11}.
5. Conclusions

We see that our theoretical curves obtained in \ref{1}, \ref{2} describe very well the new H1 and ZEUS data, and the strong increase of z-distribution obtained in COM is not confirmed by the experimental data \ref{4} and our CSM results \ref{1}.

Thus it means that the results obtained in the semihard approach of QCD can be used for extraction of the effective gluon distribution in proton from the H1 and ZEUS experimental data for heavy quark and quarkonium photoproduction at HERA energies.

6. References

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