Crystalline Anisotropic Topological Superconductivity in Planar Josephson Junctions

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We theoretically investigate the crystalline anisotropy of topological phase transitions in phase-controlled planar Josephson junctions (JJs) subject to spin-orbit coupling and in-plane magnetic fields. It is shown how topological superconductivity (TS) is affected by the interplay between the magnetic field and the orientation of the junction with respect to its crystallographic axes. This interplay can be used to electrically tune between BDI and D symmetry classes in a controlled fashion and thereby optimize the stability and localization of Majorana bound states in planar Josephson junctions. Our findings can be used as a guide for achieving the most favorable conditions when engineering TS in planar JJs and can be particularly relevant for setups containing non-collinear junctions which have been proposed for performing braiding operations on multiple Majorana pairs.

Introduction.—Majorana bound states (MBS) are localized zero-energy quasiparticle excitations at the boundaries of topological superconductors [11-14]. These states are not only of tremendous interest for fundamental research, but also because their non-Abelian statistics makes them ideal building blocks for fault tolerant quantum computation [5, 7]. Realizations [8-12] of topological superconductors hosting MBS are usually sought in materials with proximity-induced s-wave pairing and a nontrivial spin structure, typically provided by spin-orbit coupling (SOC) and/or magnetic textures [13, 22]. In the pursuit of topological superconductivity (TS), early experimental efforts have focused mostly on one-dimensional (1D) systems such as hybrid structures of superconductors and semiconductor nanowires [8, 11] or atomic chains [12]. Although there is mounting evidence pointing to the appearance of MBS in such 1D systems, a major challenge in the field is to find flexible alternative platforms that do not require fine-tuning of parameters, can be easily scaled to large numbers of states, and enable the implementation of braiding protocols.

A promising route to address these issues is to go to two-dimensional (2D) geometries, especially in light of the remarkable experimental progress in proximity-inducing superconductivity in 2D systems and surface states [23-30]. Among the various proposals for 2D setups hosting MBS [13, 31-37], those based on phase-controlled planar Josephson junctions (JJs) [Fig. 1(a)] appear particularly auspicious [13, 35, 36]. Planar JJs formed in 2D electron gases (2DEGs) [35, 36] further benefit from long established techniques of controlling semiconductor quantum wells. In fact, there is already tentative evidence for a topological phase transition and TS in such semiconductor-based JJs [35, 36]. There has, however, been no conclusive experimental evidence of MBS in planar JJs yet. Hence, finding conditions under which well-localized MBS form in planar JJs is a topic that is vigorously studied [41-47].

Most theoretical works [35, 36, 42-46, 48, 49] on planar JJs have considered the effects of Rashba SOC resulting from structure inversion asymmetry [50, 51] but have ignored Dresselhaus SOC intrinsically present in non-centrosymmetric semiconductors due to the lack of bulk inversion symmetry [50, 51]. Without Dresselhaus SOC, the Rashba SOC field exhibits a $C_\infty$ (or $C_4$ if contributions cubic in momentum are considered) symmetry. However, the presence of both Dresselhaus and Rashba SOCs lowers the symmetry to $C_{2v}$, resulting in various magnetoanisotropic phenomena in both the normal [52-54] and superconducting [55-58] states. Magnetoanisotropic effects due to the co-existence of Rashba and Dresselhaus SOC in planar JJs and their relevance for the realization of TS have recently been theoretically investigated [47].

In addition to magnetoanisotropy, crystalline anisotropic effects have also been observed in systems with coexisting Rashba and Dresselhaus SOCs in the normal state [59]. Here we theoretically investigate crystalline anisotropic TS (CATS) in a planar JJ, that is, how TS is affected by the orientation of the junction with respect to a fixed crystallographic axis. The realization of TS strongly depends on both the crystallographic orientation of the junction and the
direction of the applied magnetic field. Therefore, understanding the properties of CATS is crucial for the optimal experimental design of planar JJs. Furthermore, in dependence of the crystallographic orientation, a top gate tuning the Rashba SOC strength can be used for controlling TS. In particular, we propose that CATS provides a natural tool to optimize the stability and localization of MBS in planar JJs by manipulating the symmetry class of the system.

Model.—We consider a planar JJ composed of a 2DEG formed in a non-centrosymmetric semiconductor and subject to an in-plane magnetic field $B$ [Fig. 1]. Superconducting regions (S) are induced in the 2DEG by proximity to a superconducting cover, such as Al or Nb, while the uncovered region remains in the normal (N) state. The system is described by the Bogoliubov-de Gennes (BdG) Hamiltonian

$$H = H_0 \tau_z - \frac{g^* \mu_B}{2} B \cdot \Sigma + \Delta(x) \tau_x + \Delta^*(x) \tau_-, \quad (1)$$

where

$$H_0 = \frac{p^2}{2m^*} + V(x) - (\mu_S - \varepsilon) + \frac{\alpha}{\hbar} (p_x \sigma_x - p_z \sigma_y) + \frac{\beta}{\hbar} [(p_x \sigma_x - p_y \sigma_y) \cos 2\theta_c - (p_y \sigma_y + p_y \sigma_y) \sin 2\theta_c],$$

and $\sigma_{x,y,z}$ and $\tau_{x,y,z}$ represent Pauli matrices in spin and Nambu space respectively with $\tau_{\pm} = (\tau_x \pm \tau_y)/\sqrt{2}$. Here $p$ is the momentum, $m^*$ the electron effective mass, $\alpha$ and $\beta$ are, respectively, the Rashba and Dresselhaus SOC strengths, $\theta_c$ characterizes the junction orientation with respect to the [100] crystallographic direction of the semiconductor [Fig. 1(b)], and $V(x) = (\mu_S - \mu_N)\Theta(W/2 - |x|)$ describes the difference between the chemical potentials in the N ($\mu_N$) and S ($\mu_S$) regions. The chemical potentials are measured with respect to the minimum of the single-particle energies, $\varepsilon = -m^* \alpha^2 (1 + |\sin 2\theta_c|)/2h^2$, where we have used the SOC parametrization,

$$\alpha = \lambda \cos \theta_{so}, \quad \beta = \lambda \sin \theta_{so}, \quad \lambda = \sqrt{\alpha^2 + \beta^2}. \quad (3)$$

The second contribution to the BdG Hamiltonian in Eq. (1) with the vector of spin Dirac matrices $\Sigma = \sigma \tau_0$ represents the Zeeman splitting due to an external magnetic field, in which the JJ reference frame is determined by $B = |B| (\cos(\theta_B - \theta_c), \sin(\theta_B - \theta_c), 0)^T$, with $\theta_B$ denoting the angle of the magnetic field with respect to the [100] crystallographic direction [Fig. 1(b)]. The spatial dependence of the superconducting gap amplitude and the corresponding phase difference $\phi$ is described by $\Delta(x) = \Delta_0 \text{sgn}(x) \varepsilon / 2 \Theta(|x| - W/2)$, where $W = 2W_S + W_N$ is the total width of the JJ.

Symmetries.—The BdG Hamiltonian (1) anticommutes with the charge conjugation operator $C = \sigma_y \tau_y K$ ($C^2 = 1$), as a manifestation of the particle-hole symmetry. The presence of $B$ and/or $\phi$ breaks the conventional time-reversal symmetry and $[H, T] \neq 0$, where $T = -i\sigma_y K$ and $K$ indicates complex conjugation. Therefore, Eq. (1) belongs, generically, to the symmetry class D. However, under some conditions a transition to the higher BDI symmetry class can occur when a new effective time-reversal symmetry emerges in the system. The effective time-reversal operator can be constructed as

$$T = i (n \cdot \Sigma) R_x T = i (\cos \varphi \Sigma_x + \sin \varphi \Sigma_y) R_x T, \quad (4)$$

where $R_x = (x \to -x)$ is the reflection operator with respect to the $yz$ plane and $(n \cdot \Sigma)$ is the spin reflection with respect to the plane normal to $n = (\cos \varphi, \sin \varphi, 0)^T$ (parametrized by the angle $\varphi$). Since $T^2 = 1$, by requiring $[H, T] = 0$ one can determine the regions of the $(\alpha, \beta, \theta_c, \theta_B)$ parameter space for which Eq. (1) belongs to the BDI symmetry class, independently of $\varphi$. When the Hamiltonian belongs to the BDI symmetry class, it also possesses chiral symmetry, $\{S, H\} = 0$, characterized by the chiral operator $S = CT$ ($S^2 = 1$). The results of the symmetry analysis are shown in Table I.

The presence of SOC leads to the magnetoanisotropy of the topological state and the BDI class emerges only for specific directions of $B$ with respect to the junction orientation. Furthermore, when only one type of SOC is present, the BDI symmetry class can always be achieved (as long as the magnetic field is properly oriented) independently of the junction orientation. In particular, when $\alpha \neq 0$, $\beta = 0$, and $\theta_c = 0$ we recover the results reported in Ref. [35]. However, the coexistence of both Rashba and Dresselhaus SOC results in crystalline anisotropy and reduces the parameter space of the BDI class, which in such circumstances can only occur when $\theta_c$ equals an odd multiple of $\pi/4$, that is, when the junction orientation is aligned with one of the symmetry axes of the total SOC field pointing along the [110] and [101] crystallographic directions of the proximitized semiconductor. This is a distinctive property of CATS which, as explained below, can be used for removing inconvenient BDI subclasses from the class D phase.

Topological gap and topological charge.—To better understand the magneto-crystalline anisotropy of the TS phase and the symmetry classes, we calculate the topological gap,

$$\Delta_{top} = \min_{k_y} |E(k_y)|, \quad (5)$$

for a system with translational invariance along the junction direction (that is, the $y$ direction). In such a system

| $\alpha$  | $\beta$  | $\theta_c$  | $\theta_B$  | $\varphi$  |
|----------|----------|-------------|-------------|------------|
| $\neq 0$ | $0$      | any         | $\theta_c + \frac{(2n+1)\pi}{4}$ | $n\pi$ |
| $0$      | $\neq 0$ | any         | $\theta_c - \frac{(2n+1)\pi}{4}$ | $2\theta_c - 2n\pi$ |
| $\neq 0$ | $0$      | $\frac{(2n+1)\pi}{4}$ | $\theta_c + \frac{(2n+1)\pi}{4}$ | $n\pi$ |
the momentum component $p_y$ can be substituted by $h k_y$ in the BdG Hamiltonian (1) and we compute its Andreev spectrum $E(k_y)$ numerically for all $k_y$. Then $\Delta_{\text{top}}$ is obtained as the eigenenergy closest to zero, as indicated by Eq. (5). The size of $\Delta_{\text{top}}$ determines the degree of topological protection of the TS state and can be related to the localization of the MBS that would emerge if the system was also confined to finite length in the junction direction.

Complementary to $\Delta_{\text{top}}$, we also calculate the topological charge $Q$ (that is, the $Z_2$ topological index associated to the D symmetry class),

$$Q = \text{sgn} \left[ \frac{\text{Pf}\{H(k_y = \pi)\sigma_y \tau_y\}}{\text{Pf}\{H(k_y = 0)\sigma_y \tau_y\}} \right],$$

(6)

where $\text{Pf}\{\ldots\}$ denotes the Pfaffian [60]. $Q$ determines whether the system is in a trivial ($Q = 1$) or topological ($Q = -1$) phase.

**Numerical simulations**—For illustration, we performed numerical simulations using the Kwant package [61] and a discretized version of Eq. (1) with lattice constant $a = 20$ nm. We chose system parameters similar to those found in Al/InAs$_1$–Sb$_2$ JJs [62], namely: $m^* = 0.013 m_0$ (with $m_0$ the bare electron mass), $\Delta = 0.21$ meV, $g^2 = -20$, and $\lambda = 15$ meV nm. Moreover, $B = 0.6$ T, $\mu_S = \mu_N = 2$ meV, $W_S = 350$ nm, and $W_N = 100$ nm.

The dependence of $\Delta_{\text{top}}$ on $\theta_c$ and $\phi$ is shown in Fig. 2.

**FIG. 2.** Topological gap $\Delta_{\text{top}}$ as a function of $\theta_c$ and $\phi$ for (a) $\theta_{so} = 0$ (only Rashba SOC) and $\theta_B = 0$, (b) $\theta_{so} = \pi/4$ and $\theta_B = 0$, (c) $\theta_{so} = \pi/2$ (only Dresselhaus SOC) and $\theta_B = 0$, (d) $\theta_{so} = \theta_B = \pi/8$, (e) $\theta_{so} = \pi/8$ and $\theta_B = \pi/4$, and (f) $\theta_{so} = \theta_B = \pi/4$. Gray-shaded and non-shaded areas represent trivial ($Q = 1$) and class D TS ($Q = -1$) respectively, except along the thin vertical traces appearing in (a), (c), (e), and (f), which correspond to the BDI topological phase. The BDI class emerges when the conditions in Table I are fulfilled.

for different $\theta_B$ and different ratios of Rashba vs Dresselhaus, parametrized by the angle $\theta_{so}$ (cot $\theta_{so} = \alpha/\beta$).

**FIG. 3.** Topological gap $\Delta_{\text{top}}$ as a function of $\theta_B$ and $\theta_{so}$ for $\theta_c = \pi/8$ and (a) $\phi = 0$ and (b) $\phi = \pi$. Gray-shaded areas correspond to trivial regions ($Q = 1$). The symbols indicate points at which the conditions for the BDI symmetry class (see Table I) are fulfilled and correspond to cases in which only Rashba (circles) or only Dresselhaus (diamonds) are present.
cal phase transitions between class D and class BDI TS can be achieved with CATS by electrically tuning the strength of \( \alpha \) (and thus \( \theta_{so} \)). This can be inferred from Fig. 3 where \( \Delta_{top} \) is plotted as a function of \( \theta_B \) and \( \theta_{so} \) for fixed \( \theta_c \) and different \( \phi \). At \( \phi = 0 \), topological phase transitions between BDI and gapped D classes can be realized by just tuning \( \theta_{so} \) [Fig. 3(a)]. However, at \( \phi = \pi \) it is more convenient to tune both \( \theta_{so} \) and \( \theta_B \), as shown in Fig. 3(b).

In the absence of crystalline anisotropy a small deviation of \( \theta_B \) from the BDI-class conditions in Table I leads to a quick decrease of \( \Delta_{top} \) [Figs 2(a,c,e,f)]. However, in the CATS phase, the BDI class is limited to single points in the phase diagram and a gapped D class can be achieved within a finite range of magnetic field orientations. Thus, using \( \theta_{so} \) and \( \theta_B \) as control knobs for the CATS phase can enhance the tunability of topological phase transitions between trivial, class BDI and gapped class D phases in planar JJs.

**Optimal magnetic field orientation**—Achieving a finite \( \Delta_{top} \) that is as large as possible is crucial for finding well-localized MBS. Since \( \Delta_{top} \) strongly depends on the junction crystallographic orientation, the SOC relative strength, and the magnetic field direction, it is important to find the relation between \( \theta_B \), \( \theta_c \), and \( \theta_{so} \) that leads to the most favorable experimental design for realizing a gapped TS phase. This condition can be found by looking at the SOC field. For the system considered here, the SOC Hamiltonian with translational invariance along the \( y \) direction can be written as \( W(p_x, k_y) \cdot \sigma \), where the SOC field \( W(p_x, k_y) \) is split into separate \( p_x \) and \( k_y \)-dependent parts. Requiring the \( k_y \)-dependent part of the SOC field, \( W(k_y) = k_y(\alpha - \beta \sin 2\theta_c, -\beta \cos 2\theta_c, 0)^T \), to be perpendicular to \( B \) [47] then yields the optimal orientation of \( B \),

\[
\theta_B = \theta_c + \arctan(\cot \theta_{so} \sec 2\theta_c - \tan 2\theta_c) . \tag{7}
\]

This relation can be used to estimate the angle \( \theta_B \) that could lead to the best topological protection in dependence of \( \theta_{so} \) and \( \theta_c \).

For illustration Fig. 4 shows \( \theta_B \), computed by Eq. (7), as a function of \( \theta_c \), for different values of \( \theta_{so} \). Without crystalline anisotropy (\( \theta_{so} = n\pi/2 \)) with an integer \( n \), that is, if only Rashba or only Dresselhaus SOC is present, the optimal orientation of \( B \) coincides with the BDI condition [Fig. 4(a)]. This in turn implies that one cannot easily get free of the BDI class without quickly reducing \( \Delta_{top} \). In the presence of crystalline anisotropy, however, the optimal \( \theta_B \) differs from the BDI condition [dashed lines in Fig. 4(b)], enabling a pure class D phase with finite \( \Delta_{top} \). In the special cases \( \theta_{so} = (2n + 1)\pi/4 \) (i.e., \( \alpha = \pm \beta \)) the optimal \( \theta_B \) is independent of \( \theta_c \), therefore when the magnetic field is detuned from the optimal direction the spectrum becomes gapless, independently of the junction crystallographic orientation, which can be seen in Fig. 2(b).

This is corroborated by Fig. 5 which shows numerical calculations of \( \Delta_{top} \) at fixed \( \phi \) that are in excellent agreement with the analytical predictions [7] shown in Fig. 4. Figure 5 further shows that for a fixed orientation of \( B \) TS can be still controlled in junctions with different crystallographic orientations by electrically tuning Rashba SOC (and thereby \( \theta_{so} \)). This can be particularly relevant in more complex geometries like zigzag-junctions [43], as well as in tree-junctions [48, 63, 64], and X-junctions [49], which have been proposed for performing fusion and braiding operations on multiple Majorana pairs.

**Conclusions**—Crystalline anisotropic topological superconductivity thus presents a promising path for manipulating Majorana bound states in phase-controlled
planar Josephson junctions. The interplay between the magnetic field, spin-orbit coupling and the orientation of the junction with respect to its crystallographic axes allows for tuning between BDI and D symmetry classes in a controlled fashion. Our analytical formula for the optimal magnetic field orientation, confirmed by numerical simulations, can prove particularly interesting to future experiments seeking stable and well-localized Majorana bound states.

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