An analysis of the distribution of background star polarization in dark clouds

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ABSTRACT
The polarization observed for stars background to dark clouds (Bok Globules) is often used as diagnostic to study the ongoing star formation processes in these clouds. Such polarization maps in the optical have been reported for eight nearby clouds CB3, CB25, CB39, CB52, CB54, CB58, CB62 and CB246 in one of our previous work (Sen et al 2000). With a view to understand the origin of this polarization, in the present work attempts are made to look for any possible relation between this observed polarization and other physical parameters in the cloud (like temperature, turbulence etc.). The observed polarization does not seem to be clearly related to the dust and gas temperatures ($T_d$ and $T_g$) in the cloud as expected from Davis-Greenstein grain alignment mechanism (Davis & Greenstein 1952). However, the average observed polarization ($p_{av}$) appears to be related to the turbulence $\Delta V$ (measured by $^{12}$CO line width) by the mathematical relation $p_{av} = 2.95 \exp(-0.24\Delta V)$. The possible relation between the direction of polarization vector and other physical parameters are also discussed. For this analysis in addition to the data on above eight dark clouds, the data on CB4 (Kane et al. 1995) are also included for comparison.

In order to study the spatial distribution of the degree of polarization and position angles across the different parts of the cloud a simple model is proposed, where the cloud has been assumed to be a simple dichroic polarizing sphere and the light from the background star first passes through the IS medium and then through the cloud, before reaching the observer. One finds this simple model can explain to a reasonable extent the observed spatial (radial) dependence of the value of $p$ for two of the clouds (CB25, CB39), but for rest of the clouds the model fails. However, through this model one can explain why the polarization ($p$) need not always increase with total extinction $A_v$ as one moves in the deeper interior part of the cloud.

Key words: stars: formation – ISM : dust, extinction, clouds, globules – polarization

1 INTRODUCTION
The small compact dark clouds or 'Bok Globules' as they are also known as, are believed to be the ideal sites for star formation (Bok & Reily 1947). Such clouds have been catalogued by Bernard (1927), Lynds (1962) and more recently by Clemens & Barvains (1988).

These clouds are undergoing gravitational collapse and eventually may form stars. The ambient magnetic field plays a key role in the collapse dynamics by directing the outflows, impeding the plasma movement across magnetic field and in many other ways. Owing to this, there have been several attempts in past to measure strength and geometry of the magnetic field

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within the cloud. Astronomers have been using background star polarimetry as a tool to understand the ambient magnetic field and study the star formation dynamics in the cloud (Vrba et al. 1981; Joshi et al. 1985; Goodman et al. 1989; Myers & Goodman 1991; Kane et al. 1995; Sen et al. 2000, to mention a few). This technique has an underlying assumption that, the light from the background stars are scattered in the forward direction by the magnetically aligned dichroic dust grains in the cloud. Davis & Greenstein (1952) first worked out a procedure showing how grain alignments are possible by magnetic field. Several modifications of this mechanism and various other alignment mechanisms are presently discussed in the literature (for a detail review on this please see Lazarian et al. (1997)).

It is normally expected that, grains which cause polarization, should also be responsible for the extinction observed for the background stars. However, Goodman et al. (1995) observed a lack of dependence of polarization with extinction and this has questioned the validity of polarization as a tracer of magnetic field in these clouds. More recently Sen et al. (2000) have mapped eight star forming clouds CB3, CB25, CB39, CB 52, CB54, CB58, CB62 and CB246 in white light polarization and commented on the possible star formation dynamics there.

With the above background, in this paper a detail analysis of the polarization images of the above eight clouds is carried out. Attempts were made to understand whether the ambient physical parameters like temperature and turbulence have any role on the observed polarization value. Further the projected angular distance (henceforth 'radial distance') of the background stars from the cloud center were estimated, for the eight clouds as observed by Sen et al. (2000). Hence the data was analysed, to find whether the polarization values observed for these stars are anyway related with these distances?

## 2 THE STATISTICAL DISTRIBUTION OF THE DEGREE OF POLARIZATION AND POSITION ANGLE IN A GIVEN CLOUD

It is well known that the observed polarization is a positive definite quantity and instead of Gaussian distribution it follows Ricean distribution given by (Simon & Stewart 1984)

\[
F(p, p_0) = \frac{p}{\sigma_p} I_0\left(\frac{pp_0}{\sigma_p^2}\right) \exp\left(\frac{p^2 + p_0^2}{2\sigma_p^2}\right) \exp\left(-\frac{p^2}{2\sigma_p^2}\right)
\]

(1)

where \(p_0\) is the true value of fractional polarization being estimated by \(p\) and \(I_0\) is the modified Bessel function of order zero. There are several schemes available for de-biasing these data, however none of these schemes are fully satisfactory. There is a Rice factor \(1 - (\sigma_p^2/p^2)^{1/2}\) which is often used to de-bias such data. By multiplying each observed polarization value by the Rice factor, the polarization values are corrected for their non-Gaussian nature. In Fig 1 histogram plots showing number of stars within a given range of Rice corrected polarization values for each cloud are made. The position angle or direction vector of observed polarization (\(\theta\)) values of all stars can also be considered for a similar analysis. It is normally assumed that the polarization is caused due to ambient magnetic field in the cloud with the direction of polarization lying along the direction of the magnetic field in the cloud. The direction of ambient magnetic field in a given cloud can be assumed as the direction of the projection of galactic plane in that part of the cloud (denoted by \(\theta_C\)). Now in order to study the distribution of observed \(\theta\) values in different clouds, in Fig. 2 similar histogram plots are made showing number of stars observed in a given range of \(\theta\) values.

As can be seen from Fig. 1, the clouds CB3, CB52, CB58 and CB246 show a tendency for bimodal polarization (Rice corrected) distributions. For other clouds only one peak in the number distribution is observed. The bimodal distribution can be explained in a number of ways. As discussed in detail by Vrba et al. (1988) and also commented by Myers & Goodman (1995) (for a similar study on CB4), the low polarization component may be arising out of the foreground stars and the high polarization component may be due to the background stars. However, in a dark cloud with a given sample of stars, some stars are neither foreground nor background to the cloud, but lying a bit outside the periphery of the cloud. This happens because shape of the cloud is mostly irregular and does not evenly cover the area of the rectangular detector. So these stars also contribute to the polarization data, and represent simple interstellar polarization. Such stars probably contribute largely to the second Gaussian component, as in the present case all the clouds are quite nearby and one should not have many stars foreground to the clouds. It is also likely that (i) the polarization produced within the cloud has direction different from that produced in the interstellar medium. In the IS medium one should have polarization mostly aligned along the direction of galactic magnetic field (coinciding with the direction of galactic plane) or (ii) within the same cloud itself there may be no-uniform magnetic fields. Such features can be studied from the histogram plot of \(\theta\). Myers & Goodman (1995) made a very detailed analysis on the dispersion in the direction of polarization for 15 dark clouds, five clusters and six complexes. It was shown that the bimodal distributions can be explained, through a model, where there exist two components in magnetic field one uniform and another non-uniform. The non-uniform part has an isotropic probability distribution of direction, a Gaussian distribution of amplitudes and N correlation lengths along the line of sight. This model was applied to the cloud L1755 by Goodman et al. (1995) to explain the distribution of direction of polarization vectors.
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Figure 1. Histogram showing the number ($N_{\text{stars}}$) distribution of stars having Rice corrected polarization ($p_{\text{rice}}$) values in different ranges for various clouds.

Figure 2. Histogram showing the number ($N_{\text{stars}}$) distribution of stars having position angle ($\theta$) values in different ranges for various clouds.

As can be seen from Fig 2, almost all the clouds show a single peak in the distribution of $\theta$ values, which is somewhat very close to the direction of galactic magnetic field ($\theta_G$). In clouds CB52 and CB58 there may be small exception showing two peaks in $\theta$ distribution, but it is not very significant. A closer look at the histogram depicts that the peak in $\theta$ values for all the clouds lies within $1\sigma_\theta$ (as listed in Table 1) from the direction of galactic plane $\theta_G$ (representing magnetic field). A similar conclusion can also be arrived at by looking at Table 1, where one finds the $\sigma_\theta$ and $|\theta_G - \theta|$ values are very close to each other. This observation suggests that the direction of IS magnetic field and that of the magnetic field within the cloud (responsible for grain alignment) may be the same or these two differ only within $1\sigma$. The field responsible for the alignment of grains, therefore, can be assumed to be related to the galactic magnetic field.
3 OBSERVED POLARIZATION AND AMBIENT PHYSICAL CONDITIONS IN THE CLOUD

3.1 The dependence of observed polarization on dust and gas temperature

The light from stars background to the cloud is generally found to be polarized. This happens due to the scattering of the light from the background stars by the aligned dichroic grains present in the cloud. It is believed that the alignment is resulted from an interaction between the rotational dynamics of the grains and the ambient magnetic field. This mechanism called paramagnetic relaxation was originally suggested by Davis & Greenstein (1951). It can be shown that the percentage of polarization ($p\%$) as expected by this mechanism can be expressed as (Vrba et al. 1981):

$$p(\%) = 67F A_v$$  \hspace{1cm} (2)

where $A_v$ is total visual extinction and the expression for $F$ can be found from Jones & Spitzer (1967):

$$F = \frac{\chi'' B^2}{75\alpha \omega n} \left[ \frac{2\pi}{mkT_d} \right]^{3/2} (\gamma - 1)(1 - T_d/T_g)$$  \hspace{1cm} (3)

where $\chi''$ is the imaginary part of the complex susceptibility of the grains, $\omega$ is the angular velocity of rotation, $T_d$ and $T_g$ are the dust and gas (kinetic) temperatures, $B$ is magnetic field, $n$ is gas density in the vicinity of grain (generally taken as Hydrogen gas density), $m$ is gas molecular mass, $k$ is Boltzman constant, $\gamma = (1/2)[(b/a)^2 + 1]$, $b$ and $a$ are short and long axes of the grains. Further it is known that (Davis & Greenstein 1951; Purcell 1979):

$$\frac{\chi''}{\omega} = 2.6 \times 10^{-12} T_d^{-1}$$

Therefore one can write a simplified expression for $p(\%)$ as:

$$p(\%) \sim \frac{B^2}{n} \frac{1}{\sqrt{T_g}} \left[ \frac{1}{T_d} - \frac{1}{T_g} \right] A_v$$  \hspace{1cm} (4)

The total extinction $A_v$ in a cloud can be related to the gas (hydrogen) density in the cloud. The relation $A_v \sim \sigma_H$, with $\sigma_H$ as gas column density, seems to be true for all parts of the cloud and this relation has been experimentally verified except at very high opacities (Jenkins & Savage 1974). Subsequently many authors (Dickman 1978; Gerakins et al. 1995) used such a relation to study various physical parameters of clouds. Thus following Vrba et al. (1981) one may write

$$n(\text{atoms} \ cm^{-3}) = 532 \times A_v/l (\text{pc})$$

where $l$ is the path length from the background star to the observer. Therefore assuming classical Davis & Greenstein mechanism one may obtain from the Eqn. (4):

$$p(\%) \sim \frac{B^2}{\sqrt{T_g}} \left[ \frac{1}{T_d} - \frac{1}{T_g} \right] l$$  \hspace{1cm} (5)

The background stars have been randomly selected in the present sample. Therefore, one may assume the path lengths from the various background stars to the clouds to be of same order. Further these clouds are very near, so one may also assume that the path lengths (between the cloud and the observer) will be very small and will vary little from cloud to cloud. Under these circumstances one may write:

$$p(\%) \sim \text{constant} + \frac{B^2}{\sqrt{T_g}} \left[ \frac{1}{T_d} - \frac{1}{T_g} \right] b$$

where the 'constant' term above is equivalent to the contribution in polarization of the path length in the region outside the cloud. This contribution has been assumed to be a constant, as these path lengths are of same order and the variations in temperatures of the interstellar medium outside the various clouds will be very small. The other quantity $b$ is the path length through the cloud and assuming the cloud to be spherical in shape, it can be expressed by the relation $b = 2\sqrt{R_0^2 - r^2}$, where $R_0$ is the projected angular radius of the cloud and $r$ is the projected angular distance of the background star from the cloud centre (details of this geometry are also discussed in section 4.3). To find the average of observed polarization in a cloud one has to consider the average value of $b$ in a cloud, and it can be easily shown than $b_{av} \sim R_0$. Hence one will write

$$p_{av}(\%) \sim \text{constant} + \frac{B^2}{\sqrt{T_g}} \left[ \frac{1}{T_d} - \frac{1}{T_g} \right] R_0$$

The above equation has been obtained with several assumptions and may represent a highly simplistic and ideal situation. Also the above equation can not be simplified any further. However, in order to explore a possible dependence of observed polarization on temperature, as a very crude approximation one may further assume that, the clouds are of same size and the strengths of magnetic field within different clouds are of same order. As a result one may write:
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\[ p_{av}(\%) \sim \text{constant} + \frac{1}{\sqrt{T_d}} \left( \frac{1}{T_d} - \frac{1}{T_g} \right) \tag{6} \]

However, the classical Davis & Greenstein Mechanism has undergone many modifications and various other grain aligning mechanisms are now being used to explain the background star polarization (Cugnon 1985; Lazarian 1997; Lazarian et al. 1997). In the present work restricting oneself to the simplest classical model of Davis & Greenstein, one should get the polarization observed in a cloud to be related to the dust and gas temperature \((T_d \text{ and } T_g)\) by the Eqn.(6).

In the present analysis average polarization values for nine (=8+1) clouds are available. One can study the dependence of these polarization values on the dust and gas temperatures \((T_d \text{ and } T_g)\). The average polarization values as observed for different clouds \(p_{av}\) are listed in Table 1. The values of \(T_d, T_g\), as obtained from Clemens et al. (1991) are also listed in Table 1. These authors used deep IRAS image analysis and \(^{12}\)CO spectroscopy to calculate dust and gas temperatures. They calculated fluxes at 12, 25, 60 and 100 \(\mu\)m bands and the spectrum was not found to fit a single black body. This resulted different temperatures for different band pairs, which was explained as the IR emissions coming from many different dust populations each at somewhat different temperatures. This according to the authors may be expected, as the shape of the interstellar extinction curve justifies a range of dust grains sizes as shown by Mathis et al. (1977). One can see from Eqn (6) that \(p_{av}\) has a linear dependence on \(1/(T_d)\). Thus if different dust populations give rise to different values of polarization, one may consider an effective value of polarization, calculated based on the harmonic mean of three dust temperatures. The values of harmonic mean of the three dust temperatures \((T_{12}/25), T_{(25/60)}, T_{(60/100)}\) as listed by Clemens et al. (1991) are calculated and these mean values are actually listed as \(T_d\) in Table 1.

The same authors from their CO spectroscopy have also determined the radiation temperature \((T_R)\) for all the CB clouds, which have been converted into gas kinetic temperature in some cases following the procedure as laid out by Dickman (1978). By following the same procedure the gas kinetic temperatures \((T_g\) as in Eqn. (6)) for the present sample of clouds were calculated and the values are listed in Table 1. From the values of \(T_d\) and \(T_g\) the value of the expression \(\frac{1}{\sqrt{T_g}} \left( \frac{1}{T_d} - \frac{1}{T_g} \right)\) is also evaluated and denoted by \(T1\).

In order to calculate the average of polarization and position angle values, one can estimate the weighted mean, where the weights are inverse of the square of errors \(e_p\) and \(e_\theta\) respectively. However one may note that, stars which are background to the cloud are fainter (due to extinction) and thereby will have higher values of \(e_p\). On the other hand the foreground stars will have lower values of \(e_p\). Therefore if one weights the data with inverse of \(e_p\) or \(e_\theta\), then attempt to model fit the cloud polarization will give more emphasis on the foreground stars, rather than the background ones. These will clearly defeat our purpose of analysing the polarizing properties of the cloud. With this justification only simple un-weighted averages of \(p\) and \(\theta\) values are considered for this analysis.

In Fig. 3, the average polarization \((p_{av})\) values are plotted against the \(T1\) values. For CB62 and CB246, \(T1\) values were not calculated as data was not available from Clemens et al. (1991). As can be clearly seen from Fig. 3, the plot does not suggest any relation between \(p_{av}\) and \(T1\) as is expected from Eqn. (6). However, with the present condition that \(T_g < T_d\), one expects the long axis of grains to be aligned parallel to the magnetic field. The negative polarization values as obtained through the present analysis (for the given set of gas and dust temperature values) indicate such a geometry of alignment.

Lazarian et al. (1997) in their work on the dark cloud L1755 tried to explain the polarimetric data in terms of grain alignment mechanisms other than Davis & Greenstein. Considering the grains to be of super-paramagnetic material (with an justification from Goodman & Whittet 1995), the authors suggested that the degree of Davis & Greenstein alignment should depend on \((T_d/T_m)\) where \(T_m = (T_d + T_g)/2\) or the average of of dust and gas temperatures. Thus one should have

\[ p(\%) \sim \left( \frac{T_d}{T_d + T_g} \right) \]

Based on above a plot of \(p_{av}\) versus \(\frac{T_d}{(T_g + T_d)}\) ( denoted by \(T2\)) is made as in Fig. 4. This plot also does not show any systematic dependence of polarization on temperature (in terms of the parameter \(T2\)). However, if one excludes the data corresponding to CB4, it appears that a straight line \((p \sim T2)\) may be fitted. At least compared to Fig. 3, the present plot in Fig. 4 (excluding CB4) shows some indications for a dependence of \(p\) on \(T2\), as expected. Also there can be reasons for the exclusion of data corresponding to CB4, where polarization values were obtained in V filter rather than in white light as in all other cases. The polarization in white light is always lower than what is observed through band pass filters (as polarization at different wavelengths combine vectorially to give lower net average polarization). At this stage it may be also noted that, there are mechanisms other than Davis- Greenstein one, which are now being used by several authors to explain polarization caused by aligned grains. These include Purcell alignment, alignment by radiation torque, mechanical alignment of superthermally rotating grains (for a detailed review please see Lazarian et al 1997).

We may further note that, about half of the cloud sample here have active ongoing star formation present within these clouds. Embedded stars warm their local dust and thereby dominate the luminosity budget of their clouds. Thus the IRAS fluxes measured in such clouds reflect the dust heated by the embedded stars and not the quiescent cloud material. Also one
may note, star formation affects the gas in a cloud in many important ways. It can similarly heat the gas traced by CO and it will effect the line width.

Thus there may be several reasons, due to which the above observed average polarization $p_{av}$ fails to show any systematic dependence on dust and grain temperatures.

### 3.2 The dependence of polarization on the turbulence in the cloud

In the present analysis it is observed that the average polarization $p_{av}$ varies substantially from cloud to cloud which have different physical conditions as listed by Clemens et al. (1991). These authors also listed $^{12}$CO line width (in terms of $\Delta V$ km sec$^{-1}$) values, which are assumed to be good indicators of turbulence within the cloud (listed in Table 1). Based on this the authors have also classified the clouds into three groups: A ($T < 8.5^\circ K$ and $\Delta V < 2.5$ km sec$^{-1}$), B ($T > 8.5^\circ K$) and C ($T < 8.5^\circ K$ and $\Delta V > 2.5$ km sec$^{-1}$).

One may expect the turbulence to have its influence on the grain alignment. However, as pointed out by the anonymous referee of this paper, turbulence does not disturb the grain alignment on the scale one is dealing with here. As a result of fast Larmor precession the grain follows the local direction of magnetic field if the time scale for change of magnetic field direction is longer than the time of Larmor precession. Turbulence gives rise to variations in the local magnetic field direction along the line of sight and the alignment takes place in respect to the local magnetic field. In the line of sight there may be several independent directions of alignment causing a net depolarization and resulting a low value of observed polarization.

In this scenario an empirical relation of the type $p = a \, \exp(-\Delta V b)$ may be used to analyse the present situation. Here as the turbulence becomes too high, one should get a zero value for polarization, even if other parameters (contained in $a$) are favourable to produce high polarization. On the other hand if no turbulence is present, one can not get 100% polarization as the other parameters will decide the minimum observable polarization (decided by the value of $a$). The above equation $p = a \, \exp(-\Delta V b)$ can be linearised and by the method of least-square one may fit the following curve to data

$$ln(p) = 1.0831 - 0.2424 \, \Delta V$$

or

$$p = 2.95\exp(-0.24 \, \Delta V)$$

The above relation suggests a maximum value 2.95% for background star polarization. Fig 5. shows a plot of $ln(p)$ versus $\Delta V$ along with the above line of best fit (Eqn. (7)) for all the clouds except CB62 for which data is not available. The data on CB4 is also included from Kane et al. 1995 in this plot. In Fig 5. a clear trend is observed where the average polarization decreases with increase in turbulence $\Delta V$. This can be clearly explained, as one knows the turbulence present in the cloud can be held responsible for lowering of polarization values. However, in Fig. 3 and 4 where $p_{av}$ has been plotted across some meaningful function of $T$, no such clear relation exists. But one may note that, if $p_{av}$ has a stronger dependence on $\Delta V$ as compared to $T$, then one can not explore the relation between $p_{av}$ and functions of $T$ from Fig. 3 and 4. To analyse this situation little further, one may study the relation by plotting data points $ln(p_{av}/T_x)$ against $\Delta V$, which will remove the effect of temperature from the observed polarization. A new straight line $ln(p_{av}/T_x) = 1.1484 - 0.2457 \times \Delta V$ may be fitted on this new set of data with no further reduction in fitting error. Also one may note in the present case, corresponding to CB246 there will be no data point. Therefore, this situation is not considered any further.

The dispersion ($\sigma_\theta$) in the direction of polarization vector should be indicative of the non-uniformity in the direction of the magnetic field over different parts of the cloud. Now since this non-uniformity is observed in a plane perpendicular to the line of sight, one can also expect non-uniformity of same type to be present along the line of sight. However, as pointed out by the anonymous referee, one notes that the non-uniformity in a plane perpendicular to the line of sight is expected to be smaller than the 3-D non-uniformity along the line of sight. The projection diminishes the plane-of-sky non-uniformity. A random variation in the direction of magnetic field should reduce the value of observed polarization, averaged over the line of sight. The dust particles in a cloud can in general be expected to be aligned by uniform interstellar magnetic field. In that case one should have lower value of variance in the direction of polarization. Also the average value of polarization in the cloud should be close to the interstellar polarization value in that part of the sky. But in situations where there exist additional aligning mechanisms (operating within the cloud like molecular outflow, etc.) one should get a higher dispersion (variance) in the direction of polarization. This will happen because the interstellar magnetic field will act in vectorial combination with those additional aligning forces in the cloud. A higher value of dispersion in $\theta$ is also possible if there are irregularities (complexities) in the magnetic field structure, caused due to mechanisms intrinsic to the cloud.

Thus a high value of $\sigma_\theta$ in some clouds, should indicate that there are irregularities in the structure of aligning forces. Also under such cases in these clouds, one should get lower values of average polarization. In Fig 8. $p_{av}$ has been plotted against $\sigma_\theta$, where one can see tendencies for a decrease in $p_{av}$ with the increase in $\sigma_\theta$. In fact the data seem to be distributed into two clusters, where two different straight lines may be fitted. The data points corresponding to the clouds CB4, CB39, CB52 and CB58 may be fitted into a separate straight line as compared to other clouds. These two groups of data points
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Figure 3. The average of observed polarization ($p_{av}$) versus $T_1(=\frac{1}{\sqrt{T_g^{(1)}}(T_d^{(1)}-\frac{T_g^{(1)}}{T_g^{(1)}}))}$ for various clouds.

Figure 4. The average of observed polarization ($p_{av}$) versus $T_2(=\frac{T_d}{(T_g+T_d)})$ for various clouds.

However, do not systematically fall in any of the groups A, B, C as discussed earlier (Cf. Table 1). One may try to explore the physical properties of the two sets of clouds which are responsible for this observed behaviour. This is planned for future work.

If grains are aligned by galactic magnetic field, one should find average $p_{av}$ in a cloud to be higher where the difference between the galactic plane direction $\theta_G$ and $\theta_{av}$ i.e. $|\theta_G - \theta_{av}|$ is lower. To establish this idea in Fig. (9), $p_{av}$ has been plotted across $|\theta_G - \theta_{av}|$ for different clouds. Primarily it appears to be a scatter plot, but one can find tendencies for the increase in $p_{av}$ with the decrease in $|\theta_G - \theta_{av}|$ as expected. Here also one can notice that, the data points corresponding to the clouds CB4, CB52 and CB58 can be fitted into a separate straight as compared to the other clouds which fit in a second straight line. One may note that CB52 and CB58 are two such clouds which showed bimodal distributions for $\theta$ (Cf. Fig 2. and Section 2), as compared to others.

From the above discussions and discussions in Sect 3.1 and 3.2, it appears that the presence of turbulence lowers the polarization observed in a cloud and thus can be accepted as one of the factors responsible for the non-uniformity in magnetic
The log of average of observed polarization $ln(p_{av})$ are plotted against the turbulence $\Delta V$ for various clouds. The line of best fit $ln(p) = 1.0831 - 0.2424\Delta V$ is shown along with.

The dispersion in the direction of polarization vectors ($\sigma_\theta$) are plotted against gas temperatures ($T_g$) for different clouds. As a result it can be concluded that polarization observed for stars background to a given cloud, is not independent of the physical properties of that cloud. The work by Goodman et al. (1995) expressed concern that, it seems the polarization observed for stars background to a cloud is independent of the cloud and not produced within the cloud.

Table 1. For various CB clouds, the number of stars, average polarization, average position angle, dispersion, dust and gas
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Figure 7. The dispersion in the direction of polarization vectors \((\sigma_\theta)\) are plotted against amount of turbulence \((\Delta V)\) for different clouds.

Figure 8. The average of observed polarization \((p_{av})\) are plotted against variance \((\sigma_\theta)\) in the direction of polarization vector.

| Name of the cloud | No. of stars | \(p_{av}\) | \(\theta_{av}\) | \(\sigma_\theta\) | \(T_d(0^0K)\) | \(T_g(0^0K)\) | \(\Delta V(kms^{-1})\) | \(|\theta_G - \theta_{av}|\) | Cloud Group |
|------------------|--------------|---------------|---------------|----------------|----------------|----------------|----------------|----------------|-------------|
| CB3              | 31           | 1.41          | 65.43         | 5.46           | 56             | 11.27          | 3.08            | 23.57          | C           |
| CB25             | 21           | 2.35          | 150.89        | 9.90           | 45             | 8.90           | 0.70            | 5.89           | A           |
| CB39             | 21           | 1.95          | 77.81         | 43.35          | 66             | 9.45           | 2.05            | 0.73           | A           |
| CB52             | 16           | 1.27          | 101.06        | 51.95          | 58             | 11.08          | 4.51            | 36.04          | C           |
| CB54             | 48           | 0.86          | 115.96        | 37.85          | 58             | 11.22          | 1.67            | 49.94          | C           |
| CB58             | 29           | 1.81          | 151.06        | 43.01          | 58             | 11.22          | 1.67            | 49.94          | C           |
| CB62             | 13           | 0.70          | 67.64         | 45.80          | 51             | –              | –               | 22.36          | ?           |
| CB246            | 14           | 1.92          | 67.43         | 19.48          | –              | 9.50           | 1.62            | 14.57          | A           |
| CB4              | 80           | 2.84          | 70.55         | 25.71          | 55             | 11.77          | 0.57            | 19.45          | A           |
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4 THE SPATIAL DISTRIBUTION OF THE POLARIZATION AND POSITION ANGLE VALUES

4.1 A simple model for the polarization introduced by the cloud

The clouds which have been observed by Sen et al (2000) are nearby with distances less than 600 pc (Clemens & Barvainis (1988)). As a result the polarization observed for these background stars can be assumed to consist of only two components:

1) polarization introduced by the interstellar (IS) medium background to the cloud. For the foreground stars as the cloud is nearby, one can neglect the polarization caused due to the IS medium between the cloud and the observer.

2) polarization introduced by the cloud itself, which is believed to be optically thicker.

It is assumed that the polarization properties of the IS medium can be approximated by a simple dichroic sheet polarizer (aligned in an arbitrary direction by angle $\phi$ with respect to some reference direction) can be mathematically represented by the following Mueller Matrix (Kliger et al 1990; Shurcliff 1962)

$$A(\phi) = \frac{1}{2} \begin{pmatrix}
(k_1 + k_2) & (k_1 - k_2)c_2 & (k_1 - k_2)s_2 & 0 \\
(k_1 - k_2)c_2 & (k_1 + k_2)c_2^2 + 2s_2^2k & (\sqrt{k_1 - k_2})^2c_2s_2 & 0 \\
(k_1 - k_2)s_2 & (\sqrt{k_1 - k_2})^2c_2s_2 & (k_1 + k_2)s_2^2 + 2c_2^2k & 0 \\
0 & 0 & 0 & 2k
\end{pmatrix}$$

where $c_2 = \cos(2\phi)$; $s_2 = \sin(2\phi)$; $k = \sqrt{k_1k_2}$ and $k_1$ and $k_2$ are the transmission coefficients for light when the electric vector is parallel and perpendicular to the optic axis of the polarizer.

It is assumed that the light from the background star is initially unpolarized and so it can be represented by the Stokes column matrix (Shurcliff 1962; Stoke 1852) $S = \{I \ 0 \ 0 \ 0\}$. If one assumes the IS medium to be a dichroic polariser with optic axis making an angle 0 (zero) with the reference direction then one can represent its polarizing properties by the Mueller matrix $A(0)$. Similarly the cloud can be represented by the matrix $A'(\phi)$ with transmission coefficients $k'_1$ and $k'_2$. Now let the light reaching the observer after it comes out of the cloud, be represented by the Stokes column matrix $S' = \{I' \ Q' \ U' \ V'\}$. Therefore one may write

$$[S'] = [A'(\phi)][A(0)][S]$$

After the appropriate matrix multiplication one gets

$$\begin{pmatrix}
I' \\
Q' \\
U' \\
V'
\end{pmatrix} = \frac{1}{4} \begin{pmatrix}
I(k_1 + k_2)(k'_1 + k'_2) + I(k_1 - k_2)(k'_1 - k'_2)c_2 & I(k_1 - k_2)(k'_1 - k'_2)c_2 + 2s_2^2k' \\
I(k_1 + k_2)(k'_1 - k'_2)c_2 + I(k_1 - k_2)(k'_1 + k'_2)c_2^2 + 2s_2^2k' & (\sqrt{k_1 - k_2})^2c_2s_2 \\
I(k_1 + k_2)(k'_1 - k'_2)s_2 + I(k_1 - k_2)(\sqrt{k_1 - k_2})^2c_2s_2 & 0
\end{pmatrix}$$

where $k' = \sqrt{k'_1k'_2}$. After simplification the above Eqn.(10) reduces to

Figure 9. The average of observed polarization ($p_{av}$) are plotted against $|\theta_{av} - \theta|$.
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\[
\begin{pmatrix}
I' \\
Q' \\
U' \\
V'
\end{pmatrix} = \frac{I_{k_1}k_2'(1 + f)}{4} \begin{pmatrix}
(1 + f') + p(1 - f')c_2 \\
(1 - f')c_2 + p((1 + f')c_2^2 + 2s_2^2\sqrt{f'}) \\
(1 - f')s_2 + p(1 - \sqrt{f'})c_2s_2 \\
0
\end{pmatrix}
\]

(11)

where \(f = k_2/k_1\) and \(f' = k_2'/k_1\).

Since the interstellar polarization \(p = (k_1 - k_2)/(k_1 + k_2)\), one may also write

\[f = (1 - p)/(1 + p)\]

(12)

4.2 A model for the transmission coefficients of the cloud:

In general one may assume the cloud is spherical in shape, with radius \(R_0\). Now as shown in Fig 10, the background star is seen through the cloud, at a radial distance \(r\) from the center of the cloud. Therefore, the starlight passes a distance \(2h\) through the cloud, where \(h \sim \sqrt{R_0^2 - r^2}\). One may assume that the starlight passes through \(2h\) number of layers through the cloud and the polarization effect of each layer is equivalent to \(c\) (some arbitrary constant) times that of the IS medium, where the later has transmission coefficients \(k_1\) and \(k_2\). This also amounts to the assumption that the composition (characterised by \(k_1\) and \(k_2\)) of the dusts in the cloud and the IS medium are the same. However, within the cloud the grains may have higher number density or may be better aligned.

The estimated (or expected) value of polarization \(p_e\) present in the light coming out of the cloud can be expressed as

\[p_e = \sqrt{Q'^2 + U'^2}.\]

At this stage one can consider two special cases when \(\phi = 0\) & 90 degrees and these cases are represented by the following two equations:

\[p_e(\phi = 0) = \frac{(1 - f') + p(1 + f')}{(1 + f') + p(1 - f')},\]

\[p_e(\phi = 90) = \frac{- (1 - f') + p(1 + f')}{(1 + f') - p(1 - f')}\]

However in general when \(p \ll 1\), one can use the approximation \(f' = f^{2h} = ((1 - p)/(1 + p))^{2h} \approx (1 - 2hp)/(1 + 2hp)\) and from Eqn. (11) one can write

\[p_e \approx \frac{\sqrt{(2hpc_2 + p(c_2^2 + s_2^2(1 - hp)(1 + 2hp)/(1 + hp))c_2^2 + (2hps_2 + p(1 - (1 - hp)(1 + 2hp)/(1 + hp))c_2s_2)^2}}}{1 + 2hp^2c_2^2}\]

4.3 Fitting the observed polarization for radial distance from cloud centre

For all the eight clouds observed, the radial distances \(r\) (in arc sec) have been estimated from the co-ordinates (RA and DEC) of each star available in Sen et al. (2000). The polarizations observed for such field stars in white light are plotted against \(r\), in Figs. (11) and (12). In some cases there is a trend (example CB25, CB39), where as one moves away from the cloud center the polarization decreases and then attains a minimum value somewhere between 150-250 arc sec. After that as one moves toward the periphery of the cloud, the polarization value increases and reaches the IS polarization value as one finally moves out of the cloud.

In order to find some estimate for the interstellar polarization for the nearby region of the cloud, one can take a vectorial average of all the polarization values (taking into account the associated position angles in the measurements) that have been observed for the outer most part of the cloud and assume that value to be representative of the IS polarization value (denoted by \(p\), cf. Table 2). The same Table also lists the distances corresponding to the outer most star, which is also assumed to be roughly the dimension \(R_0\) of the cloud.

The values of \(r\) and \(h\) assumed by us, in principle should be proportional to the actual distances within the cloud. The quantity \(c\) in the expression \(h = c\sqrt{R_0^2 - r^2}\) (as was introduced earlier in Section 4.2) can act as a proportionality constant to normalise such distances. The value of \(c\) will be optimised later during model fitting the data.

After finding out \(r\) (and hence \(h\)) for each field star, one estimates the polarization \(p_e\), using relation (13) and minimize
the value of quantity $\chi^2 = \sum (pe - po)^2$ with $c$ and $\phi$ as fitting parameters (where $po$ is the observed polarization). While minimising the value of $\chi^2$, the data was not weighted with $1/(ep)^2$, the justifications of which have been already discussed in Section 3.1. In Table 2. one finds the optimized values of $c$ and $\phi$, along with the corresponding minimised value of $\chi^2$. In this connection it may be mentioned that, while minimising the values of $\chi^2$, it was noticed that only at a unique set of values for $(c$ and $\phi)$, the minimum value for $\chi^2$ can be obtained.

Fig. 11 shows a plot of the polarization values ($po$) observed for all the field stars in the clouds CB3, CB25, CB39 and CB52, along with a curve representing the polarization ($pe$) values as estimated using Eqn. (13) and after best values of $c$, $\phi$ have been obtained by $\chi^2$ minimisation technique. Fig 12 represents a similar plot for CB54, CB58, CB62 and CB246.

From these two set of figures one finds that for some clouds like CB3, CB25 and CB39 the polarization value is relatively high at near the center region and then reaches a minimum at a distance varying between 150-250 arc sec. After this it increases again as one moves out of the cloud and reaches the region of IS medium. At least for the clouds CB25 and CB39 this feature is more clearly seen. This happens when the relative angle between the magnetic field in cloud (related to the direction of optic axis) and that of IS medium, $\phi$ is close to 90°.

For the other clouds CB52, CB54, CB58, CB62 and CB246 one can see this ideal model does not fit to the data. Thus once can rule out the possibility that these clouds can be represented by a simple sphere containing an uniformly directed magnetic field, responsible for the alignment of grains. However, even an ideal cloud of above type may not fit to the observed data due to any (or all) of the following reasons:

1) There may be always some stars, which are foreground to the cloud. They will however, exhibit very little polarization, unless they are intrinsically polarized.

2) Some of the background stars, even may show high difference in polarization from the model curve, if the stars themselves are intrinsically polarized.

3) All the stars, background to the cloud are not placed at same distances behind the cloud. As a result they are passing through different distances through the IS medium and will have different values of IS polarization in them.

4) If the shape of the cloud is different from an ideal sphere.

As at present it is not possible to distinguish the background stars from the foreground ones, the 'goodness of fit' can not be improved any further.

The goodness of our fit could have been also improved, if one could have measured polarization values sharply at a particular wavelength (say through narrow band filters), rather than white light. This is because the polarization produced by passage through dichroic polarizer, as discussed above, has strong wavelength dependence and stars are of various spectral types.

However, based on the present analysis one can not claim that a uniformly directed magnetic field (for that matter any aligning force) exists throughout the entire cloud which is assumed to be spherical, with exceptions like in CB3, CB25 and CB39. In clouds CB25 and CB39 (and to some extent for CB3) the magnetic field appears to be quite uniform.

Further based on the present analysis one can also show that, the curve relating $pe$ with $r$ can assume different shapes, according to different values of $\phi$. So it is not always necessary that, as one moves towards the centre of the cloud, the polarization should also increase. Goodman et al. (1995), had questioned the validity of background star polarimetry as a tool to study the cloud properties. The main concern expressed by the authors was that as one moves towards the interior (center) of the cloud the total extinction ($A_v$) increases, but the polarization does not increase as expected. However, with the help of present analysis one can show that, the observed polarization depends largely on the geometry of the magnetic field (as aligning force) within the cloud and as a result it does not always increase with $A_v$.

| Cloud  | $R_0$ (arc sec) | $p$ (in %) | $\phi$ (in degrees) | $c$ | $\chi^2$ |
|--------|----------------|------------|---------------------|-----|----------|
| CB3    | 190            | 1.92       | 69                  | 0.003| 11       |
| CB25   | 208            | 3.32       | 70                  | 0.003| 8        |
| CB39   | 228            | 2.56       | 75                  | 0.004| 4        |
| CB52   | 247            | 0.65       | 10                  | 0.003| 12       |
| CB54   | 379            | 0.40       | 10                  | 0.002| 23       |
| CB58   | 279            | 1.90       | 60                  | 0.002| 62       |
| CB62   | 207            | 0.40       | 0                   | 0.002| 8        |
| CB246  | 191            | 1.84       | 60                  | 0.004| 13       |
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Figure 10. A model for cloud with the light from background star passing through it

Figure 11. Observed Polarization versus radial distance plot for the clouds CB3, CB25, CB39 and CB52. The curves joining the $\triangle$, represent our proposed model.

5 CONCLUSIONS

The polarization observed for stars background to eight clouds (from Sen et al. 2000) and one from Kane et al. (1995) have been analysed and some of the major conclusions are summarised below:

1. A histogram plot showing Rice corrected polarization values against number of stars, shows bimodal distribution with two peaks in polarization values, for some of the clouds. A possible interpretation in terms of a mixture of polarization due to IS medium and that due to cloud are discussed.

2. A similar histogram plot with position angle ($\theta$) values, also shows some indications for bimodal distribution, which can be explained in terms of the inhomogenities in magnetic field geometry. However, the average direction of polarization vector and that of the interstellar magnetic field seem to be the same.

3. The observed average polarization in a cloud does not appear to be related to the dust and gas temperatures as expected from Davis & Greenstein (1951) mechanism.
4. The observed average polarization \( p \) and turbulence \( \Delta V \) present in the cloud, can be related by a line of best fit

\[
\ln(p) = 1.083 - 0.2424\Delta V.
\]

This finding bears importance as one can show that physical conditions within the cloud can influence the polarization which one observes for stars background to the cloud.

5. By assuming a given cloud to be a simple dichroic sphere, one can calculate the expected polarization values for stars at different projected distances from the cloud center. This model can explain to a reasonable extent the spatial distribution of observed polarization in CB25 and CB39 (and to some extent CB3). But for other clouds the model fails.

However, based on this model one can explain why polarization always does not increase with total extinction \( A_v \) as one moves towards the center of the cloud.

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