Flavor-Changing Neutral Currents Induced by
the Democratic Seesaw Mass Matrix

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Abstract

Flavor-changing neutral currents (FCNC) are studied on the basis of a “democratic seesaw” mass matrix model, which yields a singular enhancement of the top-quark mass $m_t$ and can give reasonable quark masses and CKM matrix elements. The most exciting aspect of the model is that the structure of the $6 \times 6$ right-handed fermion mixing matrix in the up-quark sector, $U_R^u$, shows an abnormal structure in contrast to that of $U_L^u$. This causes characteristic effects on the right-handed FCNC concerned with top quark. A single top-quark production at future $e^+ e^-$ colliders, $e^+ + e^- \rightarrow Z_R \rightarrow t + \bar{c}$ ($\bar{t} + c$), is discussed.

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1. Introduction

Recently, in order to understand why the observed top-quark mass \( m_t \) is so enhanced in contrast to the other quark masses, i.e., \( m_t \gg m_b \), while \( m_u \sim m_d \), Fusaoka and the author [1] have proposed a “democratic seesaw” mass matrix model for quarks and leptons \( f_i \) (\( f = u, d, \nu, e; \ i = 1, 2, 3 \)). The 6 \times 6 mass matrix \( M \) for the fermions \( (f,F) \) (\( F_i \) are hypothetical heavy fermions corresponding to the conventional quarks and leptons \( f_i \)) has the form [2]

\[
M = \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} = m_0 \begin{pmatrix} 0 & Z \\ \kappa Z & \lambda O_f \end{pmatrix}, \tag{1.1}
\]

where the structure of the heavy fermion mass matrix \( M_F \) has a form [3] \([(\text{unit matrix}) + (\text{democratic matrix})]\) and is controlled by a \( f \)-dependent (complex) parameter \( b_f e^{i\beta_f} \) as

\[
O_f = 1 + 3b_f e^{i\beta_f} X , \tag{1.2}
\]

\[
1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{1.3}
\]

while the matrix \( m_0 Z = m_L = m_R / \kappa \) is universal for all fermion sectors \( (f,F) \), i.e.,

\[
m_0 Z = m_L = \frac{1}{\kappa} m_R = m_0 \begin{pmatrix} z_1 & 0 & 0 \\ 0 & z_2 & 0 \\ 0 & 0 & z_3 \end{pmatrix}, \tag{1.4}
\]

with \( z_1^2 + z_2^2 + z_3^2 = 1 \). For \( \lambda^2 \gg \kappa^2 \gg 1 \), the mass matrix (1.1) leads to the well-known “seesaw” form [4] of the 3 \times 3 light-fermion mass matrix:

\[
M_f \simeq m_L M_F^{-1} m_R = \frac{\kappa}{\lambda} m_0 Z O_f^{-1} Z. \tag{1.5}
\]

Note that the inverse matrix of \( O_f \) defined in (1.2), \( O_f^{-1} \), is given by

\[
O_f^{-1} = 1 + 3a_f e^{i\alpha_f} X , \tag{1.6}
\]

with

\[
a_f e^{i\alpha_f} = -\frac{b_f e^{i\beta_f}}{1 + 3b_f e^{i\beta_f}} , \tag{1.7}
\]
so that the limit of \( b_f e^{i\beta_f} \rightarrow -1/3 \) leads to \( a_f e^{i\alpha_f} \rightarrow \infty \). On the other hand, a democratic mass matrix [5] makes only one family heavy. Therefore, a choice \( b_u = -1/3 \) and \( b_d \neq b_u \) (but \( b_d \sim b_u \)) can provide that \( m_t \gg m_b \) with keeping \( m_u \sim m_d \). By assuming that the parameter \( b_f \) takes the value \( b_e = 0 \) in the charged lepton sector, they have fixed the parameters \( z_i \) as
\[
\frac{z_1}{\sqrt{m_e}} = \frac{z_2}{\sqrt{m_\mu}} = \frac{z_3}{\sqrt{m_\tau}} = \frac{1}{\sqrt{m_e + m_\mu + m_\tau}}
\]
(1.8)
from \( M_e = m_0(\kappa/\lambda)Z \cdot 1 \cdot Z \). Then, by choosing
\[
b_u e^{i\beta_u} = -1/3 \quad \text{and} \quad b_d e^{i\beta_d} = -e^{-i18^\circ}
\]
for the quark sectors together with \( \kappa/\lambda = 0.02 \), they have obtained reasonable quark mass ratios and Cabibbo-Kobayashi-Maskawa (CKM) [6] matrix parameters. Furthermore, recently, the author [7] has pointed out that the choice \( b_f \simeq -1/2 \) is favorable to understanding a large neutrino mixing which has been suggested from the atmospheric neutrino data [8].

Thus, the democratic seesaw mass matrix model can give favorable results in the phenomenology of the quark- and lepton-flavor physics, although the theoretical background of the model is still unclear: why the heavy fermion mass matrices \( M_F \) take the form \([\text{unit matrix})+(\text{democratic matrix})]\); what is the origin which yields \( m_R = \kappa m_L \); why the nature chooses \( b_u = -1/3, b_d \simeq -1, b_\nu \simeq -1/2 \) and \( b_e = 0 \); and so on.

The purpose of the present paper is not to answer these theoretical questions. This democratic seesaw mass matrix model brings many interesting new aspects in the quark and lepton phenomenology, e.g., the predictions [1] of the considerably light mass of the fourth up-quark \( t' (\equiv u_4) \) compared with the other heavy fermions, and so on. In order for the present model to be taken seriously, we need more studies on the phenomenological characteristics of the model in contrast to the conventional mass matrix models.

As one of such characteristic features of the model, in the present paper, we will point out that the structure of the \( 6 \times 6 \) right-handed fermion mixing matrix in the up-quark sector, \( U^R_u \), takes a peculiar structure in contrast to that of \( U^L_u \): For convenience, we denote the \( 6 \times 6 \) mixing matrix \( U \) in terms of \( 3 \times 3 \) matrices \( U_{ab} \) \((a, b = f, F)\)
\[
U = \begin{pmatrix}
U_{ff} & U_{fF} \\
U_{Ff} & U_{FF}
\end{pmatrix}
\]
(1.9)
In a sector which satisfies the seesaw approximation (1.5), for example, for the down-quark sector, the mixing matrices \( U_{fF}^L \) and \( U_{Ff}^L \) \((U_{fF}^R \text{ and } U_{Ff}^R)\) are suppressed
by a factor $1/\lambda (\kappa/\lambda)$ compared with $U_{ff}^L = U_{ff}^{R*}$ and $U_{FF}^L = U_{FF}^{R*}$. However, in the up-quark sector, in which the seesaw approximation (1.5) is not valid any more, the mixing matrix elements $(U_{UU}^R)_{3i}$ and $(U_{UU}^R)_{1i}$ ($i = 1, 2, 3$) do not suffer such suppression, and, instead, $(U_{UU}^R)_{3i}$ and $(U_{UU}^R)_{1i}$ are suppressed by a factor $\kappa/\lambda$ (see (3.8) later). This abnormal structure is due to the enhancement of the top-quark mass $m_t \equiv m_3^u$ (and the suppression of the fourth up-quark mass $m_4^u$) as stated in Sec.3. This will cause characteristic effects on the right-handed flavor-changing neutral currents (FCNC) concerned with top quark.

In Sec.2, we give an outline of the model. In Sec.3, we give the $6 \times 6$ mixing matrices $U_f^L$ and $U_f^R$, and in Sec.4, we give the induced right-handed FCNC structure. In Sec.5, as an example of the observable effects of the FCNC, a single top-quark production in future $e^+e^-$ colliders is discussed.

2. Outline of the model

In the present model, quarks and leptons $f_i$ belong to $f_L = (2,1)$ and $f_R = (1,2)$ of $\text{SU}(2)_L \times \text{SU}(2)_R$ and heavy fermions $F_i$ are vector-like, i.e., $F_L = (1,1)$ and $F_R = (1,1)$. The vector-like fermions $F_i$ acquire masses $M_F$ at an energy scale of the order $\lambda m_0$. Note that in our model, there is no Higgs boson with $(2,2)$ of $\text{SU}(2)_L \times \text{SU}(2)_R$ differently from the standard $\text{SU}(2)_L \times \text{SU}(2)_R$ model [9]. The $\text{SU}(2)_L$ and $\text{SU}(2)_R$ symmetries are broken by Higgs bosons $\phi_L = (\phi_L^+, \phi_L^0)$ and $\phi_R = (\phi_R^+, \phi_R^0)$, which belong to $(2,1)$ and $(1,2)$ of $\text{SU}(2)_L \times \text{SU}(2)_R$, respectively. We assume that these Higgs bosons couple to the fermions universally:

$$H_{\text{Yukawa}} = \sum_{i=1}^{3} y_{Li} \left[ (\bar{u} \ d)_{Li} \begin{pmatrix} \phi_L^+ \\ \phi_L^0 \end{pmatrix} D_{Ri} + (\bar{u} \ d)_{Li} \begin{pmatrix} \phi_R^- \\ \phi_R^0 \end{pmatrix} U_{Ri} \right] + \text{h.c.} + (L \leftrightarrow R) + [(u, d, U, D) \to (\nu, e, N, E)] ,$$

(2.1)

where $y_{Li}$ and $y_{Ri}$ are real parameters, and they are universal for all the fermion sectors. Therefore, the mass matrix which is sandwiched by $(f_L, \overline{F}_L)$ and $(f_R, \overline{F}_R)^T$ is given by the $6 \times 6$ matrix (1.1), where $m_L = y_{Li}(\phi_L^0)$.

As seen in Ref.[1], phenomenologically, up- and down-quark masses are well described by choosing $b_u = -1/3$ and $b_d \simeq -1$:

$$m_u \simeq \frac{3m_e \kappa}{4m_\tau \lambda} m_0 , \quad m_c \simeq \frac{2m_\mu \kappa}{m_\tau \lambda} m_0 , \quad m_t \simeq \frac{1}{\sqrt{3}} m_0 ,$$

$$m_4^u \simeq \frac{1}{\sqrt{3}} \kappa m_0 , \quad m_5^u \simeq m_6^u \simeq \lambda m_0 ,$$

(2.2)
\[
m_d \simeq \frac{1}{2|\sin(\beta_d/2)|} \frac{m_c \kappa}{m_t \lambda} m_0 \ , \quad m_s \simeq 2 \left| \sin \frac{\beta_d}{2} \right| \frac{m_u \kappa}{m_c \lambda} m_0 \ , \quad m_b \simeq \frac{1}{2} \kappa m_0 \ ,
\]
\[
m_d^4 \simeq m_b^d \simeq \lambda m_0 \ , \quad m_6^d \simeq 2\sqrt{1 + 3 \sin^2(\beta_d/2)} \lambda m_0 \ .
\]

(2.3)

The observed quark mass ratios \(m_c/m_t\) and \(m_d/m_s\) require \(\kappa/\lambda \simeq 0.02\) and \(|\beta_d| \simeq 18^\circ\), respectively. These input values can give reasonable values of the CKM mixings \(|V_{ij}|\).

Note that only the fourth up-quark mass \(m_4^u\) is remarkably light compared other heavy fermions. The enhancement of the top-quark mass \(m_t\) \((\equiv m_3^u)\) is caused at the cost of the lightening of \(u_4\) \((\equiv U_1)\). We speculate \(m_4^u/m_3^u \simeq \kappa \sim m_{W_R}/m_{W_L}\), i.e., \(m_4^u \sim 10^3\) GeV. We can expect the observation of the fourth up-quark \(u_4\) at an energy scale at which we can observe the right-handed weak bosons \(W_R\).

3. Peculiar structure of the mixing matrix \(U_u^R\)

In the Ref.[1], of the 6\times6 mixing matrices, only the left-handed light-fermion-mixing part \(U_{ff}^L\) have been studied. In the present paper, we investigate the 6\times6 mixing matrix (1.9) and will find that right-handed mixing matrix \(U_u^R\) for the up-quark sector \((u, U)\) has a peculiar structure in contrast to \(U_u^L\), although the mixing matrix \(U_d^R\) for the down-quark sector \((d, D)\) has a similar structure to \(U_u^L\).

For the case where the seesaw expression (1.1) is a good approximation, the 6\times6 mixing matrices \(U^L\) and \(U^R\) for the fermions \((f_L, F_L)\) and \((f_R, F_R)\) are given by

\[
U^L \simeq \begin{pmatrix} U_f^L & 0 \\ 0 & U_F^L \end{pmatrix} \begin{pmatrix} 1 & -m_{LM_F^{-1}} \\ M_{F}^{-1}m_L & 1 \end{pmatrix} = \begin{pmatrix} U_f^L & -U_f^L m_{LM_F^{-1}} \\ U_F^L M_{F}^{-1} m_L & U_F^L \end{pmatrix},
\]

(3.1)

\[
U^R \simeq \begin{pmatrix} U_f^R & 0 \\ 0 & U_F^R \end{pmatrix} \begin{pmatrix} 1 & -m_{RM_F^{-1}} \\ M_{F}^{-1}m_R & 1 \end{pmatrix} = \begin{pmatrix} U_f^R & -U_f^R m_{RM_F^{-1}} \\ U_F^R M_{F}^{-1} m_R & U_F^R \end{pmatrix},
\]

(3.2)

where \(U_f^L, U_f^R, U_F^L\) and \(U_F^R\) are defined by

\[
-U_f^L m_{LM_F^{-1}} m_R U_f^R = D_f ,
\]

(3.3)

\[
U_F^L M_F U_F^R = D_F ,
\]

(3.4)
$D_f = \text{diag}(m^f_1, m^f_2, m^f_3)$ and $D_F = \text{diag}(m^F_1, m^F_2, m^F_3) \equiv \text{diag}(m^F_2, m^F_2, m^F_3)$. The mixing matrix $U^R$ is related to $U^L$ as follows:

$$
(U^R_{ff})_{ij} = (U^L_{ff})_{ij}^*, \quad (U^R_{FF})_{ij} = (U^L_{FF})_{ij}^*, \quad (U^R_{fF})_{ij} = \kappa(U^L_{fF})_{ij}^*, \quad (U^R_{Ff})_{ij} = \kappa(U^L_{Ff})_{ij}^*,
$$

(3.5)

where $i = 1, 2, 3$. For example, the explicit numerical result of $U^L_d$ without the seesaw approximation is given by

$$
|U^L_d| = \begin{pmatrix}
0.9772 & 0.2061 & 0.0506 & 0.0490 + 0.0007 i & 4 \times 10^{-5} + 0.0007 i \\
0.2117 & 0.9540 & 0.2124 & 0.2063 + 0.0646 i & 0.0035 + 0.0035 i \\
0.0137 & 0.2179 & 0.9759 & 0.4335 + 0.4809 i & 0.5251 + 0.5251 i \\
0.0118 + 0.1649 i & 0.0209 + 0.0209 i & 0.7176 + 0.6961 i & 0.0215 \\
0.0064 + 0.1011 i & 0.7927 + 0.7927 i & 0.3895 + 0.4268 i & 0.8162 \\
0.0046 + 0.0660 i & 0.2706 + 0.2706 i & 0.5773 + 0.5773 i & 0.5774 \\
\end{pmatrix}, \quad (3.6)
$$

where we have used the input values $b_d = -1$, $\beta_d = -18^\circ$, $\kappa/\lambda = 0.02$ and $\kappa = 10$ according to Ref. [1].

However, for the up-quark sector with $b_f e^{i\beta_f} = -1/3$, the relations (3.5) are not valid any longer. For up-quark sector, the $6 \times 6$ left-handed mixing matrix $U^L_u$ is given by

$$
U^L_u = \begin{pmatrix}
+0.9994 & -0.0349 & -0.0084 & -0.0247 + 6 \times 10^{-5} i + 4 \times 10^{-6} i \\
+0.0319 & +0.9709 & -0.2373 & -0.2051 + 0.4346 i + 0.0259 i \\
+0.0165 & +0.2369 & +0.9714 & +0.8990 + 0.8431 i - 0.0444 i \\
+0.0934 + 0.1114 i & +0.1114 + 0.1114 i & -1.0365 + 0.5774 i + 0.5774 + 0.5774 \\
-0.0118 + 0.1649 i & +0.0209 + 0.0209 i & -0.7176 + 0.6961 i + 0.0215 \\
-0.0064 + 0.1011 i & +0.7927 + 0.7927 i & -0.3894 + 0.4267 + 0.8163 \\
\end{pmatrix},
$$

(3.7)
while the right-handed mixing matrix $U^R_u$ is given by

$$
U^R_u = \begin{pmatrix}
+0.9994 & -0.0349 & -0.0084 & -0.0247 \frac{\kappa}{\lambda} + 6 \times 10^{-5} \frac{\kappa}{\lambda} + 4 \times 10^{-6} \frac{\kappa}{\lambda} \\
+0.0319 & +0.9709 & -0.2373 & -0.2051 \frac{\kappa}{\lambda} - 0.4346 \frac{\kappa}{\lambda} + 0.0259 \frac{\kappa}{\lambda} \\
+0.0256 \frac{\kappa}{\lambda} & +0.3459 \frac{\kappa}{\lambda} & -0.0747 \frac{\kappa}{\lambda} & +0.5773 + 0.5773 + 0.5774 \\
+0.0165 & +0.2369 & +0.9713 & +0.3274 \frac{\kappa}{\lambda} + 0.2716 \frac{\kappa}{\lambda} - 0.6160 \frac{\kappa}{\lambda} \\
-0.0118 \frac{\kappa}{\lambda} & +0.1649 \frac{\kappa}{\lambda} & +0.0209 \frac{\kappa}{\lambda} & -0.7176 + 0.6961 + 0.0215 \\
-0.0064 \frac{\kappa}{\lambda} & -0.1011 \frac{\kappa}{\lambda} & +0.7929 \frac{\kappa}{\lambda} & -0.3894 - 0.4267 + 0.8161
\end{pmatrix}
$$

Note that $U^R_u$, (3.8), shows a peculiar structure as if the third and fourth rows of the matrix $U^R_u$ are exchanged each other in contrast to the mixing matrices (3.6) and (3.7). As seen in Sec. 4, for the FCNC phenomenology, the structure of $U^R_uU_u$ plays an essential role. The $3 \times 3$ matrix $U^R_u$ is analytically given by

$$
U^R_{uU} \approx \begin{pmatrix}
\frac{3}{2} z_1 \frac{\kappa}{\lambda} & \frac{3}{2} z_2^{3} \frac{\kappa}{\lambda} & \frac{3}{2} z_3^{3} \frac{\kappa}{\lambda} \\
-\frac{z_2}{z_1} \frac{\kappa}{\lambda} & -2 z_2 \frac{\kappa}{\lambda} & 2 z_2^{3} \frac{\kappa}{\lambda} \\
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}
\end{pmatrix} .
$$

This peculiar structure can be understood from the following situation. For convenience, we transform the heavy fermion basis in which $M_F = m_0 \lambda O_f$ is given by (1.2) into a basis in which the heavy fermion mass matrix $M_F$ is diagonalized as

$$
AM_F A^{-1} = \tilde{M}_F = m_0 \lambda \text{diag}(1 + 3 b f e^{i \beta_f}, 1, 1) .
$$

Then, the $6 \times 6$ mass matrix for up-quark sector is transformed as

$$
\tilde{M} = \begin{pmatrix}
1 & 0 \\
0 & A
\end{pmatrix} M \begin{pmatrix}
1 & 0 \\
0 & A^T
\end{pmatrix} = \begin{pmatrix}
0 & \tilde{m}_L \\
\tilde{m}_R & \tilde{M}_U
\end{pmatrix} = m_0 \begin{pmatrix}
0 & \tilde{Z}^T \\
\kappa \tilde{Z} & \lambda \tilde{O}_u
\end{pmatrix} ,
$$

(3.10)
where

\[
\tilde{Z} = AZ = \begin{pmatrix}
\frac{1}{\sqrt{3}} z_1 & \frac{1}{\sqrt{3}} z_2 & \frac{1}{\sqrt{3}} z_3 \\
-\frac{1}{\sqrt{2}} z_1 & \frac{1}{\sqrt{2}} z_2 & 0 \\
-\frac{1}{\sqrt{6}} z_1 & -\frac{1}{\sqrt{6}} z_2 & 2 z_3
\end{pmatrix}, \quad (3.12)
\]

\[
\tilde{O}_u = AO_uA^T = \text{diag}(0, 1, 1) . \quad (3.13)
\]

The mixing matrices \( U^L \) and \( U^R \) are given as matrices which diagonalize Hermitian matrices \( H_L \equiv \tilde{M}\tilde{M}^\dagger \) and \( H_R \equiv \tilde{M}^\dagger\tilde{M} \), respectively:

\[
H_L = \tilde{M}\tilde{M}^\dagger = m_0^2 \begin{pmatrix}
\tilde{Z}^T\tilde{Z} & \lambda\tilde{Z}^T\tilde{O}_u \\
\lambda\tilde{O}_u\tilde{Z} & \lambda^2\tilde{O}_u^2 + \kappa^2\tilde{Z}\tilde{Z}^T
\end{pmatrix}, \quad (3.14)
\]

\[
H_R = \tilde{M}^\dagger\tilde{M} = m_0^2 \begin{pmatrix}
\kappa^2\tilde{Z}^T\tilde{Z} & \kappa\lambda\tilde{Z}^T\tilde{O}_u \\
\kappa\lambda\tilde{O}_u\tilde{Z} & \lambda^2\tilde{O}_u^2 + \tilde{Z}\tilde{Z}^T
\end{pmatrix}. \quad (3.15)
\]

In our scheme, the numbering of the fermions \( f_i \) \((i = 1, 2, \ldots, 6)\) is defined as \( m_1^f < m_2^f < m_3^f < m_4^f < m_5^f < m_6^f \). Since \((\tilde{O}_u^2)_{11} = 0\), we see that \((H_L)_{33} = m_0^2(\tilde{Z}^T\tilde{Z})_{33}\) and \((H_L)_{44} = m_0^2\kappa^2(\tilde{Z}\tilde{Z}^T)_{11}\), i.e., \((H_L)_{33} \ll (H_L)_{44}\), while \((H_R)_{33} = m_0^2\kappa^2(\tilde{Z}^T\tilde{Z})_{33}\) and \((H_R)_{44} = m_0^2(\tilde{Z}\tilde{Z}^T)_{11}\), i.e., \((H_R)_{33} \gg (H_R)_{44}\). This causes the exchange \( U_{3i} \leftrightarrow U_{4i} \). On the other hand, for the ordinary case \( b_f e^{i\beta_f} \neq -1/3\), both \((H_L)_{44}\) and \((H_R)_{44}\) are of the order of \(m_0^2\lambda^2\), i.e., \((H_L)_{33} \ll (H_L)_{44}\) and \((H_R)_{33} \ll (H_R)_{44}\), so that such an exchange \( U_{3i} \leftrightarrow U_{4i} \) is not caused.

As a result, the \(6 \times 6\) mixing matrix \( V^R\) for the right-handed charged currents shows an abnormal structure in contrast to the CKM mixing matrix \( V^L\): the magnitudes of \( V^R_{tq} \) \((q = d, s, b)\) are suppressed by the order of \(\kappa/\lambda\), while the magnitudes of \(V^R_{t'q} \) \((t' \equiv u_4)\) are given by

\[
|V^R_{t'q}| \simeq |V^L_{tq}|, \quad (q = d, s, b), \quad (3.16)
\]

If we suppose \(\kappa \sim 10\), the mass of the fourth up-quark \(t'\) is of the order of \(10^3\) GeV, so that we can expect observation of a single \(t'\)-production via \(W_R\)-exchange,
$u + d \rightarrow d + t'$, at LHC, because of $|V_{ud}^R| \simeq 1$ and $|V_{td}^R| \sim 10^{-2}$.

4. Structure of FCNC

When the mass matrix $M$ given in (1.1) is transformed as

$$\bar{\psi}_L M \psi_R + h.c. = \bar{\psi}'_L D \psi'_R + h.c., \quad (4.1)$$

where $\psi = (f, F)^T$, and $\psi' = U \psi$ is the mass-eigenstates, the vertex $\bar{\psi}_A \Gamma^{AB} \psi_B$ ($A, B = L, R$) is also transformed into $\bar{\psi}'_A \Gamma'^{AB} \psi'_B$, where

$$\Gamma'^{AB} = U_A \Gamma^{AB} U_B^\dagger. \quad (4.2)$$

For simplicity, hereafter, we drop the indices $A, B$. Correspondingly to (1.9), we denote the $6 \times 6$ matrix $\Gamma$ in terms of $3 \times 3$ matrices $\Gamma_{ab} (a, b = f, F)$ as

$$\Gamma = \begin{pmatrix} \Gamma_{ff} & \Gamma_{fF} \\ \Gamma_{Ff} & \Gamma_{FF} \end{pmatrix}. \quad (4.3)$$

Our interest is in the physical vertex $\Gamma'^{ff}$ which is given by

$$\Gamma'^{ff} = \sum_a \sum_b U_{fa} \Gamma_{ab} U_{fb}^\dagger, \quad (4.4)$$

where $U_{ab}^\dagger \equiv (U_{ab})^\dagger = (U^\dagger)_{ba}$, because $(\Gamma'^{ff})_{ij}$ with $i \neq j$ mean transitions between $f_i$ and $f_j$, i.e., appearance of the FCNC.

In our SU(2)$_L \times$SU(2)$_R \times$U(1)$_Y$ gauge model, the neutral currents $J^\mu_L = g_L^2 \bar{\psi}_L \Gamma^\mu L \psi$, which couple with the left-handed weak boson $Z^\mu_L$, are given by

$$\Gamma^\mu_L = \begin{pmatrix} c^f_L 1 & 0 \\ 0 & c^F_L 1 \end{pmatrix} \cdot \frac{1}{2} \gamma^\mu (1 - \gamma_5) + \begin{pmatrix} d^f_L 1 & 0 \\ 0 & d^F_L 1 \end{pmatrix} \cdot \frac{1}{2} \gamma^\mu (1 + \gamma_5), \quad (4.5)$$

where

$$c^f_L = \pm \frac{1}{2} - \sin^2 \theta_L Q_f, \quad c^F_L = - \sin^2 \theta_L Q_F, \quad (4.6)$$

$$d^f_L = \pm \frac{1}{2} h_L - \sin^2 \theta_L Q_f, \quad d^F_L = - \sin^2 \theta_L Q_F, \quad (4.7)$$

$$\sin^2 \theta_L = 1 - m^2_{W_L}/m^2_{Z_L}. \quad (4.8)$$
\[
\begin{align*}
\epsilon &= m_{W_L}^2/m_{W_R}^2, \\
\epsilon &= \frac{\sin^2 \theta_L}{1 - \epsilon/\cos^2 \theta_L \cos^2 \theta_L}, \quad (4.9)
\end{align*}
\]

the factor \(\pm \frac{1}{2}\) takes \(+\frac{1}{2}\) and \(-\frac{1}{2}\) for up- and down-fermions, respectively, and \(Q_f\) (\(Q_F\)) is charge of the fermion \(f\) (\(F\)). Using the unitary condition for \(U_{ab}\), \(U_{ff}U_{ff}^\dagger + U_{fF}U_{fF}^\dagger = 1\), we can express the physical vertex \(\Gamma_{Lff}'\) as

\[
\begin{align*}
\Gamma_{Lff}' &= \left( c_L^f U_{ff}^L U_{ff}^L + c_L^F U_{fF}^L U_{fF}^L \right) \cdot \frac{1}{2} \gamma^\mu (1 - \gamma_5) \\
&+ \left( d_L^f U_{ff}^R U_{ff}^R + d_L^F U_{fF}^R U_{fF}^R \right) \cdot \frac{1}{2} \gamma^\mu (1 + \gamma_5) \\
&= \left( c_L^f 1 - (c_L^f - c_L^F) U_{fF}^L U_{fF}^L \right) \cdot \frac{1}{2} \gamma^\mu (1 - \gamma_5) \\
&+ \left( d_L^f 1 - (d_L^f - d_L^F) U_{fF}^R U_{fF}^R \right) \cdot \frac{1}{2} \gamma^\mu (1 + \gamma_5).
\end{align*}
\]

Similarly, for the neutral current \(J_R^\mu = g_R^Z \overline{\psi} \Gamma_R^\mu \psi\), which couples with the right-handed weak boson \(Z_L\), we obtain

\[
\begin{align*}
\Gamma_{Rff}' &= \left( c_R^f 1 - (c_R^f - c_R^F) U_{fF}^R U_{fF}^R \right) \cdot \frac{1}{2} \gamma^\mu (1 + \gamma_5) \\
&+ \left( d_R^f 1 - (d_R^f - d_R^F) U_{fF}^L U_{fF}^L \right) \cdot \frac{1}{2} \gamma^\mu (1 - \gamma_5),
\end{align*}
\]

where

\[
\begin{align*}
c_R^f &= \pm \frac{1}{2} - \sin^2 \theta_R Q_f, \\
c_R^F &= -\sin^2 \theta_R Q_F, \quad (4.13) \\
d_R^f &= \pm \frac{1}{2} h_R - \sin^2 \theta_R Q_f, \\
d_R^F &= -\sin^2 \theta_R Q_F, \quad (4.14) \\
\sin^2 \theta_R &= 1 - m_{W_R}^2/m_{Z_R}^2, \\
h_R &= -\frac{\sin^2 \theta_R}{1 - \epsilon \cos^2 \theta_R}, \quad (4.15) \\
g_R^Z &= -g_Z^L \frac{\sin \theta_L}{\sin \theta_R \cos \theta_R} \sqrt{\frac{1 - \epsilon \cos^2 \theta_R}{1 - \epsilon/\cos^2 \theta_L}} \\
&= \frac{e}{\cos \theta_L \sin \theta_R \cos \theta_R} \sqrt{\frac{1 - \epsilon \cos^2 \theta_R}{1 - \epsilon \cos^2 \theta_R/\cos^2 \theta_L}}. \quad (4.17)
\end{align*}
\]
Note that the FCNC are induced by the second terms $U_{fF}U_{fF}^{\dagger}$ with magnitude $(c^f - c^F) [(d^f - d^F)]$. The numerical results of $\xi^L \equiv U_{fF}U_{fF}^{\dagger}$ and $\xi^R \equiv U_{fF}U_{fF}^{\dagger}$ are as follows:

\[
\xi_u^L = \begin{pmatrix} 2.43 \times 10^{-9} & 2.01 \times 10^{-8} & -8.85 \times 10^{-8} \\ 2.01 \times 10^{-8} & 9.26 \times 10^{-7} & -2.21 \times 10^{-6} \\ -8.85 \times 10^{-8} & -2.21 \times 10^{-6} & 6.08 \times 10^{-6} \end{pmatrix}, \quad (4.18)
\]

\[
\xi_u^R = \begin{pmatrix} 2.43 \times 10^{-7} & 2.01 \times 10^{-6} & -2.84 \times 10^{-4} \\ 2.01 \times 10^{-6} & 9.26 \times 10^{-5} & -7.09 \times 10^{-3} \\ -2.84 \times 10^{-4} & -7.09 \times 10^{-3} & 1.000 \end{pmatrix}, \quad (4.19)
\]

\[
|\xi_d^L| = |\xi_d^R| = \begin{pmatrix} 9.61 \times 10^{-9} & 4.03 \times 10^{-8} & 8.52 \times 10^{-8} \\ 4.03 \times 10^{-8} & 1.87 \times 10^{-7} & 3.51 \times 10^{-7} \\ 8.52 \times 10^{-8} & 3.51 \times 10^{-7} & 2.78 \times 10^{-6} \end{pmatrix}, \quad (4.20)
\]

where, for simplicity, for $\xi_d$, we have denoted only the magnitudes.

As seen from (4.18)–(4.20), the magnitudes of $(\xi_u^L)_{12}$ and $(\xi_u^L)_{13}$ are sufficiently small, so that the contributions to $D^0\bar{D}^0$ and $K^0\bar{K}^0$ mixings and the rare decays of $D^0$ and $K^0$ are safely negligible. Although the value $(\xi_u^L)_{33} \approx 1$ is noticeable, it is hard to observe the effects.

We give attention to the magnitudes $\xi_{ut}^R \equiv |(\xi_u^R)_{13}| = 2.8 \times 10^{-4}$ and $\xi_{ct}^R \equiv |(\xi_u^R)_{23}| = 0.0071$, which are considerably large compared with the other off-diagonal elements. The mixing $u \leftrightarrow t$ can contribute to a single top-quark production, $e^- + u \to e^- + t$, at HERA. However, because of the smallness of the value $\xi_{ut}^R$, the cross section $\sigma(e^- + p \to e^- + t + X)$ is of the order of $10^{-8}$ pb for $m_{Z_R} \simeq 0.9$ TeV, so that it is hard to observe the single top-quark production at HERA. On the other hand, the mixing $c \leftrightarrow t$ can contribute to a single top-quark production, $e^+ + e^- \to Z_R \to t + \tau (\bar{t} + c)$, at super $e^+e^-$ colliders. In the next section, we will discuss a possibility of the observation of $e^+ + e^- \to t + \tau (\bar{t} + c)$.

### 5. Single top-quark production at future $e^+e^-$ colliders

The matrix element of the reaction $e^+ + e^- \to t + \bar{\tau}$ is given by

\[
\mathcal{M} = G_L(\bar{u}_e\gamma_\mu(v_L - a_L\gamma_5)v_e) \frac{g^{\mu\nu} - q^{\mu}q^{\nu}/m_{Z_L}^2}{q^2 - m_{Z_L}^2 + im_{Z_L}\Gamma_{Z_L}}(\bar{u}_t\gamma_\nu(1 + \gamma_5)v_t)
\]

\[
+(L \to R), \quad (5.1)
\]
where we have neglected the $\xi_{ct}^L$-term and

$$G_L = -\frac{1}{2} \varepsilon \tan^2 \theta_L (g_Z^L)^2 \xi_{ct}^R, \quad G_R = \frac{1}{2} (g_Z^R)^2 \xi_{ct}^R,$$

(5.2)

$$v_L = \frac{1}{4} (1 - 4 \sin^2 \theta_L) , \quad a_L = \frac{1}{4}$$

$$v_R = \frac{1}{4} (1 - 5 \sin^2 \theta_R) , \quad a_R = \frac{1}{4} (1 + \sin^2 \theta_R) ,$$

(5.3)

for $|\varepsilon| \ll 1$. For $\sin^2 \theta_R \sim \sin^2 \theta_L$, the second term in (5.1), the $Z_R$-exchange term, is dominated. When we neglect the contribution of the $Z_L$-exchange term, we obtain

$$\sigma(e^+e^- \to t\bar{c}) \simeq G_R^2 \frac{v_R^2 + a_R^2}{2\pi} \frac{(s - m_t^2)^2 (2s + m_t^2)}{s^2 [(s - m_{Z_R}^2)^2 + (m_{Z_R} \Gamma_{Z_R})^2]} .$$

(5.4)

Here, the decay width $\Gamma_{Z_R}$ is given by

$$\Gamma_{Z_R} = \frac{(g_{Z_R}^R)^2}{12\pi} \sum_f (v_R^2 + a_R^2) m_{Z_R} ,$$

(5.5)

where the sum is taken over all quarks and leptons, so that $\sum(v_R^2 + a_R^2) = 3 - 3\sin^2 \theta_R + 2 \sin^4 \theta_R$. In order to give rough estimates, we take the inputs $\sin^2 \theta_R = \sin^2 \theta_L = 0.23$, $\alpha^{-1} = 120$, $m_{Z_R} = 10 m_{Z_L} = 0.9$ TeV, and $m_t = 0.18$ TeV, and we obtain $\Gamma_{Z_R} = 0.049 m_{Z_R}$ and

$$\sigma = 6.0 \times 10^{-7} \text{ pb} \quad \text{at} \ \sqrt{s} = 0.2 \text{ TeV} ,$$

$$\sigma = 3.1 \times 10^{-5} \text{ pb} \quad \text{at} \ \sqrt{s} = 2m_t = 0.36 \text{ TeV} ,$$

$$\sigma = 1.1 \times 10^{-4} \text{ pb} \quad \text{at} \ \sqrt{s} = 0.5 \text{ TeV} ,$$

$$\sigma = 7.5 \times 10^{-4} \text{ pb} \quad \text{at} \ \sqrt{s} = 0.7 \text{ TeV} ,$$

(5.6)

where $\sigma = \sigma(t\bar{c}) + \sigma(c\bar{t})$. The value of $\sigma$ is highly dependent on the choice of the value $m_{Z_R}$ (the value of $\sigma$ is roughly proportional to $m_{Z_R}^4$). For example, if we take $m_{Z_R} = 0.5$ TeV, the values of $\sigma$ given in (5.6) become large by a factor 11 times. Therefore, the values given in (5.6) should not be taken rigidly. However, even taking such ambiguity into consideration, the value of $\sigma$ at $\sqrt{s} = 0.2$ TeV is too small to observe the single top-quark production at LEP.

In order to contrast the $t + \bar{c}$ production to the ordinary $t + \bar{t}$ production, it is favorable that the observation of $t + \bar{c}$ is carry out at an $e^+e^-$ energy $\sqrt{s}$ which
is slightly smaller than \( \sqrt{s} = 2m_t \). However, even if we have an \( e^+e^- \) collider with \( L = 10^{34} \text{ cm}^{-2}\text{s}^{-1} \), the value of \( \sigma L \) is, at most, \( \sigma L = 0.026 \text{ day}^{-1} \) at \( \sqrt{s} = 2m_t \), so that it is not so easy to detect the single top-quark production. For example, a future \( e^+e^- \) collider JLC is designed as \( L = 8 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1} \) at \( \sqrt{s} = 0.5 \text{ TeV} \) in JLC [10]. This collider parameter gives \( \sigma L = 0.078 \text{ day}^{-1} \) (i.e., one event/two weeks). Therefore, the single top-quark production will be barely detectable at such a future collider.

If we can observe the direct production of \( Z_R \) at future \( e^+e^- \) colliders, then we will reach the observation of the single top quark production at \( \sqrt{s} = m_{Z_R} \): for example, for \( m_{Z_R} = 0.9 \text{ TeV} \), we obtain \( \sigma = 0.085 \text{ pb} \), which gives \( \sigma L = 3.9 \text{ hour}^{-1} \) for \( L = 1.26 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1} \) at JLC [10].

6. Summary

In conclusion, we have pointed out that the right-handed flavor-mixing matrix in the democratic seesaw mass matrix model takes a peculiar structure for the up-quark sector. The \( 6 \times 6 \) mixing matrix \( U^R_u \) takes an abnormal structure as if the third and fourth rows are exchanged. This is due to the top-quark-mass enhancement and the fourth up-quark-mass suppression in the model.

We have found that the fourth up-quark \( t' \) is considerably light compared with the other heavy fermions (\( m_{t'} \sim \) a few TeV in contrast to the other \( m_F \sim \) a few hundred TeV) and it can couple to the right-handed weak boson \( W_R \) with a sizable magnitude \( |V^R_{t'd}| \), i.e., \( |V^R_{t'd}| \approx |V^L_{t'd}| \). Therefore, we can expect the observation of the single \( t' \)-production, \( u + d \rightarrow d + t' \), at LHC.

We have investigated possible FCNC effects within the framework of \( \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_Y \) gauge model. We have estimated the cross section of the single top-quark production \( e^+e^- \rightarrow \tau \tau \) through FCNC and obtained \( \sigma(e^+e^- \rightarrow \tau \tau + c\bar{c}) \approx 1.1 \times 10^{-4} \text{ pb} \) at \( \sqrt{s} = 0.5 \text{ TeV} \), so that the single top-quark production can be barely detected at future \( e^+e^- \) colliders with high luminosity such as JLC.

Thus, the exciting aspect of the present model is that the right-handed fermion mixing matrix \( U^R_u \) in the up-quark sector has a peculiar structure. We hope that non-standard effects from such an abnormal structure of \( U^R_u \) will be observed at future colliders such as JLC.

Acknowledgments

A series of works based on the democratic seesaw mass matrix model was first started in collaboration with H. Fusaoka. The author would like to thank H. Fusaoka for his enjoyable collaboration. The author also thank R. Hamatsu.
and A. Miyamoto for information on the collider parameters of HERA and JLC, respectively. This work was supported by the Grant-in-Aid for Scientific Research, the Ministry of Education, Science and Culture, Japan (No.08640386).

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