Chiral aspects of hadron structure

A.W. Thomas$^1$ and G. Krein$^2$

$^1$ Department of Physics and Mathematical Physics and Special Research Center for the Subatomic Structure of Matter, University of Adelaide, SA 5005, Australia

$^2$ Instituto de Física Teórica, Universidade Estadual Paulista
Rua Pamplona, 145 - 01405-900 São Paulo, SP, Brazil

Abstract

Chiral loop corrections for hadronic properties are considered in a constituent quark model. It is emphasized that the correct implementation of such corrections requires a sum over intermediate hadronic states. The leading non-analytic corrections are very important for baryon magnetic moments and explain the failure of the sum rule $(\mu_{\Sigma^+} + 2\mu_{\Sigma^-})/\mu_A = -1$ predicted by the constituent quark model.

PACS NUMBERS: 11.30.Rd, 12.39.Jh, 12.39.Fe, 12.40.Yx, 13.40.E

KEYWORDS: Chiral symmetry, quark model, potential models, hadron spectrum, magnetic moments
The role of chiral symmetry in hadron structure, spectroscopy and hadron-hadron interactions is a recurrent theme in modern strong interaction physics. Even lattice QCD cannot avoid the issue. Current lattice simulations for light current quark masses are computationally intensive and present computer limitations mean that lattice simulations are restricted to relatively large quark masses. In order to make contact with the physical world extrapolation schemes must be devised. Guidance from quark models which are able to interpolate between the correct chiral and heavy quark limits of QCD is of importance for such extrapolations.

Extrapolations of lattice data which respect both the chiral behavior of QCD and the heavy quark limit have recently been developed for baryon masses [1] and magnetic moments [2]. The cloudy bag model [3] (CBM) proved extremely useful as a framework for exploring such problems. As the CBM is based upon the MIT bag as a model for the underlying quark structure, it would clearly be desirable to carry out similar studies with alternative models. However, the essential feature of the CBM, which must be retained in any treatment of hadron structure or hadronic interactions in order to be consistent with the chiral structure of QCD, is that in calculating chiral loops one must project onto intermediate hadronic states [4]. One must not calculate chiral loops at the quark level, independent of the hadronic environment in which the quark is found.

This particular point is of importance for the problem recently raised by Lipkin [5] concerning the sum rule for the ratio of the $\Sigma^+$ and $\Lambda$ hyperons

$$R_{\Sigma/\Lambda} \equiv \frac{\mu_{\Sigma^+} + 2\mu_{\Sigma^-}}{\mu_\Lambda}. \quad (1)$$

The quark model prediction is $R_{\Sigma/\Lambda}^{QM} = -1$. Using the experimental values from hyperon magnetic moments, one obtains $R_{\Sigma/\Lambda}^{Exp} = -0.23$. In the following we show that the leading non-analytic (LNA) chiral corrections are large and explain why the sum rule fails. Moreover, we explain why the correct model independent LNA behavior is not obtained from chiral loops on single quarks.

We start by reviewing the basic equations and concentrate on pionic corrections; the extension to include kaon corrections is straightforward. The Hamiltonian of a chiral quark model can quite generally be written as

$$H = H_0 + H_\pi + W \quad (2)$$

where $H_0$ describes the bare quark states $|B^{(0)}_\alpha\rangle$ of the system.
\[ H_0|B^{(0)}_\alpha\rangle = E^{(0)}_\alpha |B^{(0)}_\alpha\rangle. \] (3)

In this, \( \alpha \) represents the set of spatial, spin and isospin quantum numbers of the baryons. \( H_0 \) is the Hamiltonian for non-interacting pions and \( W \) is the pion-quark interaction vertex. Note that in principle \( H_0 \) might contain not only the confinement interaction, but also hyperfine interactions, such as one-gluon exchange.

Pionic corrections are calculated by projecting the Hamiltonian onto the single-baryon states \( |B^{(0)}\rangle \equiv B^\dagger|0\rangle \), where \( B^{(0)} \) is the creation operator of the bare baryon and \( |0\rangle \) is the vacuum state. The resulting effective baryon-pion Hamiltonian can be written schematically as

\[
H = \sum_\alpha E^{(0)}_\alpha B^\dagger_\alpha B_\alpha + \sum_j \omega_j a^\dagger_j a_j + \sum_{j\alpha\alpha'} W^{ij}_{\alpha\alpha'} B^\dagger_{\alpha'} B_\alpha a_j + \text{h.c.} \] (4)

where \( a_j^\dagger \) and \( a_j \) are pion creation and annihilation operators and \( j \) indicates isospin and spatial variables. (Note that we have taken bare baryon states with different \( \alpha \) indices to be orthogonal.)

The physical baryon mass \( M_B \) (where \( B \) stands for e.g. \( N, \Delta, \Sigma, \Lambda \) etc.) can be computed by dressing the bare (quark model) baryon with its meson cloud in a straightforward way:

\[
M_B = M_B^{(0)} + \Sigma(M_B) \] (5)

where \( M_B^{(0)} \) is the bare baryon mass (i.e. without pionic corrections) and the self-energy, \( \Sigma(E) \), is given as

\[
\Sigma(E) = \langle B_0|W^\dagger \frac{1}{E - H_0} W|B_0\rangle. \] (6)

Here \( W \) is the effective pion-nucleon vertex and \( H_0 \) is the single-particle Hamiltonian, evaluated with the physical masses of the baryons and the pion \[3\]. Insertion of a sum over intermediate baryon-pion states in Eq. (6) leads to

\[
\Sigma(E) = \sum_n \langle B_0|W^\dagger |n\rangle \frac{1}{E - E_n} \langle n|W|B_0\rangle. \] (7)

\[1\] One can also systematically choose the level of sophistication of the underlying quark model by subdividing the space of bare hadron states into a \( P \)-space which is dealt with explicitly and a \( Q \)-space whose effects are parametrized \[3\].
The structure vertex-propagator-vertex $W^{\dagger}(E - \tilde{H}_0)^{-1}W$ in Eq. (3) is an effective quark-quark interaction. For a quark-quark interaction mediated by pion exchange, it is crucial that it includes processes where the pion is emitted and re-absorbed by the same quark, as well as those where it is exchanged between a pair of quarks. As explained in Ref. [4], any treatment that misses diagrams where the pion is emitted and absorbed by the same quark line in the hadron leads to incorrect conclusions concerning hadron properties. Such diagrams are essential for the correct spin-isospin dependence of the corrections and, in particular, to yield the correct leading non-analytic contributions (LNA). On the other hand, in order to obtain the correct LNA corrections to hadron masses it is not sufficient to consider just loop diagrams on a quark line, independent of its environment.

Let us be more specific. Consider an interaction of the form (we restrict ourselves to the SU(2) case, but the argument is general):

$$
\sum_{i<j} \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j V_{ij},
$$

(8)

where $V_{ij}$ is the radial part, not restricted to be a contact interaction. Such an interaction is the basis of the calculations in several quark models, in particular in the model of Glozman and Riska [5]. The overall strength (in hadron $|H\rangle$) from the interaction of Eq. (8) is given by the spin-isospin ($SI$) matrix element

$$
\langle SI \rangle_H = \langle H| \sum_{i<j} \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j |H\rangle,
$$

(9)

which yields, for the $N$ and the $\Delta$, $\langle SI \rangle_N = 30$ and $\langle SI \rangle_\Delta = 6$. These lead to the relations

$$
M_N = M_0 - 15P_{\pi 00}^\pi
$$

(10)

$$
M_\Delta = M_0 - 3P_{\pi 00}^\pi,
$$

(11)

where $M_0$ is the corresponding unperturbed energy and $P_{\pi 00}^\pi \simeq 30$ MeV is the fitting parameter corresponding to the radial matrix element of Eq. (8), in the lowest-energy unperturbed shell of the 3-quark system.

On the other hand, the correct field theoretic self-energy calculation leads to

$$
M_N = M_0 - \frac{25}{2} P_{\pi 00}^\pi - 16P_{\pi N\Delta}^\pi
$$

(12)

$$
M_\Delta = M_0 - 4P_{\pi \Delta N}^\pi - \frac{25}{2} P_{\pi 00}^\pi,
$$

(13)
where \( P_{N\Delta}^\pi \) and \( P_{\Delta N}^\pi \) differ from \( P_{00}^\pi \) by a factor \( \Delta M = M_\Delta - M_N \) in the energy denominator (see Eq. (7) and Fig. 1). Note that the form of these equations is general, in the sense that the chiral limit does not alter this structure. Also, it should be noted that because of the dependence on \( \Delta M = M_\Delta - M_N \) of the self-energies \( P_{N\Delta}^\pi \) and \( P_{\Delta N}^\pi \), these equations go beyond simple perturbation theory (recall the difference between Rayleigh-Schrödinger and Wigner-Brillouin methods).

\[
\Delta M = M_\Delta - M_N
\]

FIGURE 1. One-loop pion self-energy corrections to the nucleon (\( N \)) and delta (\( \Delta \)).

Suppose for the moment that \( P_{N\Delta}^\pi = P_{\Delta N}^\pi \). Then we find

\[
M_\Delta - M_N = 12P_{N\Delta}^\pi.
\]

Of course, \( P_{N\Delta}^\pi \) can now be fitted to the experimental value of \( M_\Delta - M_N \), and nothing apparently changes with respect to the result of Eqs. (10) and (11). But this is not the entire story, since this then implies a huge nucleon self energy. To estimate this, suppose \( P_{N\Delta}^\pi = P_{00}^\pi \), which is equivalent to setting the \( \Delta-N \) mass difference to zero in the radial integrals. Then, the nucleon self-energy would be given by \(-57/2P_{00}^\pi\), instead of \(-15P_{00}^\pi\). This would imply a total nucleon self-energy of -855 MeV; a pretty big self-energy indeed! The situation becomes even worse in practice, because the \( \Delta-N \) mass difference is quite large and therefore \( P_{00}^\pi > P_{N\Delta}^\pi \). Of course, it is very hard to justify nucleon self-energies of the order of the nucleon rest mass in a non-relativistic framework. In addition, in the process of calculating self-energy corrections for the low-lying and excited states, the incorrect treatment of intermediate hadronic states will lead to fatally incorrect systematics.
Let us now consider the problem raised by Lipkin [5]. The magnetic moment of a baryon $B$ is defined by $\mu_B = G_M^B(0)$, where $G_M^B(q^2)$ is the magnetic form factor. In Fig. 1 we show the different contributions to $G_M^B(q^2)$. The calculation of $G_M^B(q^2)$ proceeds on the same lines as for the cloudy bag model [3]; the explicit expressions (including both octet and decuplet baryons in the intermediate states) were derived in Ref. [7]. The leading non-analytic contributions to $\mu_B$ come from intermediate states with the same quantum numbers as the external ones, i.e. from the terms $C = B$ in Fig. 1(c). The explicit form of this contribution is

$$G_M^{1(c)}(0) = 2m_N \frac{1}{16\pi^2 f_\pi^2} \beta_{BB\pi} \int dk \frac{k^4 u^2(k)}{w^4(k)}$$

(15)

where the $\beta_{BB\pi}$ are given by SU(3). For the case of interest here they are given by

$$\beta_{NN\pi} = (F + D)^2 \langle N|\tau_3|N\rangle$$

$$\beta_{\Sigma^-\Sigma^-\pi} = -\beta_{\Sigma^+\Sigma^+\pi} = 2 \left( \frac{1}{3}D^2 + F^2 \right)$$

$$\beta_{\Lambda\Lambda\pi} = 0$$

(16)

where $F$ and $D$ are usual SU(3) axial couplings.

![Diagram](image)

**FIGURE 1.** The various contributions to the magnetic moment of a baryon.

The LNA contribution for $G_M(0)$ is easily obtained from Eq. (15) and is given by

$$\mu_B^{LNA} = \frac{m_N m_\pi}{8\pi f_\pi^2} \beta_{BB\pi} \equiv \alpha_B m_\pi.$$

(17)

Note that this is precisely what is obtained in Refs. [8] [9] with chiral perturbation theory – as it must, because both calculations are based upon chiral symmetry. The LNA behavior does not necessarily dominate the physics of hadronic properties in general. However, in the
case of the magnetic moments the fact that the LNA term is also the first term in a power series for $\mu_B$ as a function of $m_\pi$ and that the numerical value of the LNA coefficient is large means that it is phenomenologically important there. Over the range $m_\pi \in (0, 2m_{\pi}^{\text{phys}})$ it is quite a good approximation to write the baryon magnetic moments as:

$$\mu_B = \mu_B^0 + \alpha_B m_\pi + O(m_\pi^2).$$

Applying this to the $\Sigma^\pm$ and $\Lambda$ magnetic moments, with the one loop chiral coefficients of Ref. [8] one finds:

$$\mu_{\Sigma^+} \simeq 2.80 - 2.46 m_\pi$$
$$\mu_{\Sigma^-} \simeq -1.50 + 2.46 m_\pi$$

with $m_\pi$ in GeV. The $\Lambda$ magnetic moment is just equal to the experimentally measured moment up to order $m_\pi^2$. We stress that the coefficient of the $m_\pi$ term is model independent and that the non-analytic terms are large – for example, roughly $1/3$ of the measured $\Sigma^-$ magnetic moment at the physical pion mass.

Using these expressions we can now evaluate the ratio of magnetic moments considered by Lipkin, as a function of the pion mass:

$$\frac{\mu_{\Sigma^+} + 2\mu_{\Sigma^-}}{\mu_\Lambda} \simeq 0.33 - 0.56 \frac{m_\pi}{m_{\pi}^{\text{phys}}} + O(m_\pi^2).$$

This expression makes it exceptionally clear why the sum-rule fails. It is extremely sensitive to the value of the pion mass because it involves a LNA piece of the $\Sigma$ magnetic moment that has not been arranged to cancel in some way. While the ratio is only -0.23 at the physical pion mass, way below the naive expectation of -1, and in the chiral limit it even has the opposite sign (+0.33), at just above twice the physical pion mass it would take the expected value. We note that such behavior could never be reproduced within a constituent quark model, where the constituent mass would vary by a mere 10-20 MeV as $m_\pi$ varies over $(0, 2m_{\pi}^{\text{phys}})$.

Next we examine whether the LNA contributions can be equivalently included in the constituent quark mass [10]. The essence of chiral perturbation theory, as a phenomenological implementation of the chiral symmetry of QCD, is that there are certain non-analytic terms in the exact expression for any hadronic property, as a function of the current quark masses, which are model independent. These terms are determined by just a few gross hadron properties, including their axial charges. For the nucleon mass the leading non-analytic (LNA) term is:
\[ \delta M_N = -\frac{3}{32\pi f_{\pi}^2} g_A^2 m_N^3. \]  

This comes from the loop diagram in Fig. 1(a), with the pion-nucleon coupling given by PCAC as \( \frac{g_A}{f_{\pi}} \). In terms of quarks this diagram necessarily involves pion exchange between \( i = j \) and \( i \neq j \), the essential point being that the 3-quark intermediate state is a nucleon and hence degenerate with the initial baryon state.

In Ref. [10] it is argued that the LNA contribution comes from quark self-energy loops. It is emphasized that the non-relativistic quark model connection between \( g_A \) (nucleon understood) and \( g_A^q \) (the quark axial charge), namely \( g_A^q = \frac{3}{5} g_A \), satisfies:

\[ 3 [g_A^q]^2 = \frac{27}{25} g_A^2 \approx g_A^2. \]  

(23)

This argument, which attributes the discrepancy to the use of exact SU(6) is misleading. We note that Eq. (22) is model independent – it cannot depend on the model used to describe nucleon structure. As an extreme example, we note that if the relativistic quark model were used to relate \( g_A^q \) and \( g_A \) we would find:

\[ 3 [g_A^q]^2 = \frac{27}{25(0.65)} g_A^2 \approx 2.6 g_A^2, \]  

(24)

where the factor of 0.65 is the well known relativistic correction for massless quarks given by (for example) the MIT bag model [11]. The discrepancy in this case is now 260%.

To summarize, the suggestion that one can incorporate the chiral loop on the individual quark lines into the definition of the constituent quark mass is inappropriate for many reasons. Firstly, if one were to use such a mass to define a Dirac magnetic moment for the quark one would build in an incorrect LNA contribution. Secondly, for an unstable particle like the \( \Delta \) the mass has an imaginary piece arising from the decay to \( N\pi \). In order to obtain the correct imaginary piece (i.e., the correct width of the \( \Delta \)) one must include the imaginary part associated with the pion being emitted and absorbed by the same quark – an imaginary part that is omitted in a naive calculation which ignores the environment in which the constituent quark is sitting. Finally, if one were to compute chiral corrections to the magnetic moments of the quarks themselves they would be model dependent, for the reasons explained around Eqs. (23) and (24). This is in contradiction with QCD, within which the LNA correction should be model independent.

In conclusion we have shown that the LNA corrections to hadron properties have important practical consequences. In particular, they explain the reason for the failure of the
sum-rule for hyperon magnetic moments noted by Lipkin. We have also shown that the correct LNA contributions to hadronic properties such as masses and magnetic moments \textit{cannot} be obtained by calculating loops at the quark level, independent of the hadronic environment.

This work was supported in part by the Australian Research Council and CNPq (Brazil).
REFERENCES

[1] D. B. Leinweber, A. W. Thomas, K. Tsushima, S. V. Wright, hep-lat/9906027, to appear in Physical Review D.

[2] Derek B. Leinweber, Ding H. Lu, Anthony W. Thomas, Phys. Rev. D 60 (1999) 034014.

[3] S. Théberge, A.W. Thomas and G.A. Miller, Phys. Rev. D 22 (1980) 2838; Erratum-ibid. Phys. Rev. D 23 (1981) 2106; A.W. Thomas, Adv. Nucl. Phys. 13 (1984) 1; G.A. Miller, Int. Rev. Nucl. Phys. 2 (1984) 190.

[4] A.W. Thomas and G. Krein, Phys. Lett. B 456 (1999) 5.

[5] H. J. Lipkin, hep-ph/9911261.

[6] L. Ya. Glozman and D. O. Riska, Phys. Rep. 268 (1996) 263.

[7] S. Thèberge, G.A. Miller and A.W. Thomas, Can. J. Phys. 60 (1982) 59.

[8] E. Jenkins, M. Luke, A. V. Manohar, and M. J. Savage, Phys. Lett. B 302 (1993) 482.

[9] L. Durand and P. Ha, Phys. Rev. D 58 (1998) 013010.

[10] L. Ya. Glozman, Phys. Lett. B 459 (1999) 589.

[11] T. de Grand, R.L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D 12 (1975) 2060.