Structure formation with strings plus inflation: a new paradigm

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Abstract. Recent developments in inflation model building, based on supersymmetry, have produced compelling models in which strings are produced at the end of inflation. In such models the cosmological perturbations are seeded both by the defects and by the quantum fluctuations. We show that such models produce qualitatively new and desirable predictions for CMB anisotropies and the CDM power spectrum. This remark should put an end to the long term animosity between defect and inflationary scenarios of structure formation.

I  INTRODUCTION

Structure formation theories fall broadly within two classes: inflation \cite{1} and defects \cite{2}. In inflationary scenarios the structure of the Universe originates from microphysical quantum fluctuations, which get stretched to cosmological scales by inflationary expansion. In topological defect scenarios as the Universe cools down, high temperature symmetries are spontaneously broken. Remnants of the unbroken phase, called topological defects, may survive the transition, and later seed fluctuations in the CMB and CDM.

A major drawback of inflationary theories is that they are far-removed from particle physics models. Attempts to improve on this state of affairs have been made recently, resorting to supersymmetry \cite{3–9}. In these models one identifies flat directions in the potentials, which are enforced by a (super)symmetry. Such flat directions produce “slow-roll inflation”. In order to stop inflation one must tilt the potential, allowing for the fields to roll down. In so-called D-term supersymmetric inflationary scenarios, inflation stops with a symmetry-breaking phase transition, at which a U(1) symmetry is spontaneously broken, leading to the formation of cosmic strings. This is only the most natural of a whole class of models of so-called hybrid inflation. Hence a network of cosmic strings is formed at the end of inflation.

If one takes the standard cosmology (sCDM), of $\Omega = 1$, $\Omega_{\Lambda} = 0$, $\Omega_{b} = 0.05$ and $H_0 = 50$ km s\(^{-1}\) Mpc\(^{-1}\), one finds that neither strings or inflation fit the COBE normalized large scale structure power spectrum. However, the failings of the inflationary and defect sCDM models are to a certain extent complementary, and an obvious question is whether they can help each other to improve the fit to the data.

Using our recent calculations for local cosmic strings \cite{10} and the by now familiar inflationary calculations \cite{11}, we are able to demonstrate that the answer is yes \cite{12}. Even with Harrison-Zeldovich initial conditions and no inflation produced gravitational waves, the large-angle CMB spectrum is mildly tilted, as preferred by COBE data \cite{13}. The CMB spectrum then rises into a thick Doppler bump, covering the region $\ell = 200 – 600$, modulated by soft secondary undulations. More importantly the standard CDM anti-biasing problem is cured, giving place to a slightly biased scenario of galaxy formation. The cosmic string biasing problem is also cured.

Similar results have been reported by two other groups \cite{14,15}.

II  MODEL BUILDING

The general features of structure formation with strings plus inflation do not depend on the concrete underlying inflationary model. We illustrate these models by considering the $D$-term inflation model in which the
strings plus inflation scenario finds an attractive expression.

To begin with, we define the reduced Planck mass \( M = 1/\sqrt{8\pi G} \). We recall that a supergravity theory is defined by two functions of the chiral superfields \( \Phi_i \): the function \( G(\Phi, \bar{\Phi}) \), which is related to the Kähler potential \( K(\Phi, \bar{\Phi}) \) and the superpotential \( W(\Phi) \) by \( G = K + M^2 \ln |W|^2/M^6 \), and the gauge kinetic function \( f_{AB}(\Phi, \bar{\Phi}) \). The scalar potential \( V \) is composed of two terms, the \( F \)-term

\[
V_F = M^2 e^{G/M^2} (G_i(G^{-1}))^j_\ell G^j - 3M^2)
\]

(1)

and the \( D \)-term

\[
V_D = \frac{1}{2}g^2 \text{Re} f_{AB}^\dagger D^A D^B
\]

(2)

where \( g \) is the U(1) gauge coupling, \( G^i = \partial G/\partial \Phi_i \), and \( G_i = \partial G/\partial \Phi^i \). The function \( D^A \) is given by

\[
D^A = G^i (T^A)_i^j \phi_j + \xi^A,
\]

(3)

where the Fayet-Iliopoulos terms \( \xi^A \), which we take to be positive, can be non-zero only for those \( (T^A)_i^j \) which are U(1) generators. We see that in order to have a positive potential energy density, either the \( F \) term or the \( D \) term must be non-zero. In order to have inflation, there must be a region in field space where the slow-roll conditions \( \epsilon \equiv \frac{1}{2} M^2 |V'|/V^2 \ll 1 \) and \( |\eta| \equiv \min_{\text{eig}} M^2 |V'|/V \ll 1 \) are satisfied, where by the notation in the second condition we mean that the smallest eigenvalue of the matrix is much less than unity. In \( D \)-term inflation, the conditions are satisfied because the fields move along a trajectory for which \( \exp(G/M^2), G^i \) and \( G^i (T^A)_i^j \phi_j \) all vanish, leaving a tree-level potential energy density of \( g^2 \xi^A \xi^A/2 \). Thus the potential is completely flat before radiative corrections are taken into account. At the end of inflation, if the fields are to relax to the supersymmetric minimum with \( D^A + \xi^A = 0 \), the U(1) gauge symmetries are necessarily broken, assuming their corresponding Fayet-Iliopoulos terms are non-zero. Thus strings are inevitable: the only question is how much inflation there is before the fields attain the minimum.

III CALCULATIONS

The spectrum of the perturbations from \( D \)-term inflation is calculable [5], and can be expressed in terms of \( N \), the number of \( e \)-foldings between the horizon exit of cosmological scales today and the end of inflation, which occurs at \( |\eta| = 1 \). One finds

\[
\frac{\ell(\ell + 1)C_\ell^4}{2\pi T_{CMB}^2} \approx \frac{1}{4} |\delta_H(k)|^2 \approx \frac{(2N + 1)}{75} \left( \frac{\xi^2}{M^4} \right),
\]

(4)

where \( T_{CMB} = 2.728K \) is the temperature of the microwave background, and \( \delta_H(k) \) is the matter perturbation amplitude at horizon crossing. The corrections to this formula, which is zeroth order in slow roll parameters, are not more than a few per cent. The inflationary fluctuations in this model are almost scale-invariant (Harrison-Zeldovich) and have a negligible tensor component [4].

The string contribution is uncorrelated with the inflationary one, and is proportional to \( (G\mu)^2 \), where \( \mu \) is the string mass per unit length, given by \( \mu = 2\pi \xi \). We can write it as

\[
\frac{\ell(\ell + 1)C_\ell^8}{2\pi T_{CMB}^2} = \frac{A_8(\ell)}{16} \left( \frac{\xi^2}{M^4} \right),
\]

(5)

where the function \( A_8(\ell) \) gives the amplitude of the fractional temperature fluctuations in units of \( (G\mu)^2 \). Allen et al. [16] report \( A_8(\ell) \approx 60 \) on large angular scales, with little dependence on \( \ell \). Our simulations give \( A_8(\ell) \approx 120 \), with a fairly strong tilt. The source of the difference is not altogether clear: our simulations are based on a flat space code which neglects the energy losses of the strings through Hubble damping. The simulations of Allen et al. do include Hubble damping, which would tend to reduce the string density and hence the normalisation. However, they have a problem of lack of dynamic range, and therefore may be missing some power from strings at early times, and therefore higher \( \ell \).

Jeannerot [5] took the Allen–Shellard normalisation and \( N \approx 60 \), and found that the proportion of strings to inflation is roughly \( 3:1 \). With our normalisation, the approximate ratio is \( 4:1 \). In any case this ratio is
FIGURE 1. The CMB power spectra predicted by cosmic strings, sCDM, and by inflation and strings with $R_{SI} = 0.25, 0.5, 0.75$. The large angle spectrum is always slightly tilted. The Doppler peak becomes a thick Doppler bump at $\ell = 200 - 600$, modulated by mild undulations.

far from a robust prediction in strings plus inflation models, as it depends on the number of $e$-foldings, and the string normalisation, both of which are uncertain. We will therefore leave it as a free parameter. For definiteness we shall parametrize the contribution due to strings and inflation by the strings to inflation ratio $R_{SI}$, defined as the ratio in $C_\ell$ at $\ell = 5$, that is $R_{SI} = C^S_5/C^I_5$.

It is curious to note that the number of $e$-foldings required for solving the flatness problem still leaves room for tuning $R_{SI}$ between nearly 0 and 1.

IV RESULTS

In Figs. 1 and 2 we present power spectra in CMB and CDM produced by a sCDM scenario, by cosmic strings, and by strings plus inflation. We have assumed the traditional choice of parameters, setting the Hubble parameter $H_0 = 50$ km sec$^{-1}$ Mpc$^{-1}$, the baryon fraction to $\Omega_b = 0.05$, and assumed a flat geometry, no cosmological constant, 3 massless neutrinos, standard recombination, and cold dark matter. The inflationary perturbations have a Harrison-Zeldovich or scale invariant spectrum, and the amount of gravitational radiation (tensor modes) produced during inflation is assumed to be negligible.

We now summarise the results.

- The CMB power spectrum shape in these models is highly exotic. The inflationary contribution is close to being Harrison-Zeldovich. Hence it produces a flat small $\ell$ CMB spectrum. The admixture of strings, however, imparts a tilt. Depending on $R_{SI}$ one may tune the CMB plateau tilt between 1 and about 1.4, without invoking primordial tilt and inflation produced gravity waves.

- The proverbial inflationary Doppler peaks are transfigured in these scenarios into a thick Doppler bump, covering the region $\ell = 200 - 600$. The height of the peak is similar for sCDM and strings, with standard cosmological parameters. The Doppler bump is modulated by small undulations, which cannot truly be called secondary peaks. By tuning $R_{SI}$ one may achieve any degree of secondary oscillation softening. This provides a major loophole in the argument linking inflation with secondary oscillations in the CMB power spectrum [17,18]. If these oscillations were not observed, inflation could still survive, in the form of the models discussed in this Letter.

- In these scenarios the LSS of the Universe is almost all produced by inflationary fluctuations. However COBE scale CMB anisotropies are due to both strings and inflation. Therefore COBE normalized CDM fluctuations are reduced by a factor $(1 + R_{SI})$ in strings plus inflation scenarios. This is equivalent to multiplying the sCDM bias by $\sqrt{T + R_{SI}}$ on all scales, except the smallest, where the string contribution
FIGURE 2. The power spectra in CDM fluctuations predicted by cosmic strings, sCDM, and by inflation and strings with $R_{SI} = 0.25, 0.5, 0.75$. We have also superposed the power spectrum as inferred from surveys by Peacock and Dodds [19].

may be non negligible. Given that sCDM scenarios produce too much structure on small scales (too many clusters) this is a desirable feature.

V PRAISE FOR THE MODEL

"Strings plus inflation" are interesting first of all as an inflationary model. Its “flat potential” is not the result of a finely tuned coupling constant, but the result of a symmetry. Hence in some sense these models achieve inflation without fine tuning. The only free parameters are the number of inflationary e-foldings, and the scale of symmetry breaking. These parameters also fix the absolute (and therefore relative) normalizations of string and inflationary fluctuations.

"Strings plus inflation" models are also pervaded by a higher component of particle physics, when compared with other inflationary models.

The structure formation paradigm resulting from this scenario is highly exotic and worth considering just by itself. Regarded in abstract, structure formation may be due to two types of mechanism: active and passive perturbations. Passive fluctuations are due to an apparently acausal imprint in the initial conditions of the standard cosmic ingredients, which are then left to evolve by themselves. Active perturbations are due to an extra cosmic component, which evolves causally (and often non-linearly), and drives perturbations in the standard cosmic ingredients at all times. Inflationary fluctuations are passive. Defects are the quintessential active fluctuation. A scenario combining active and passive perturbation would bypass most of the current wisdom on what to expect in either scenario. It is believed that the presence or absence of secondary Doppler peaks in the CMB power spectrum tests the very fundamental nature of inflation, whatever its guise [17]. In the mixed scenarios we shall consider inflationary scenarios could produce spectra with any degree of secondary oscillation softening.

The combination of these two scenarios smoothes the hard edges of either separate component, leaving a much better fit to LSS and CMB power spectra. We illustrated this point in this review, but left out a couple of issues currently under investigation which we now summarise.

The CDM power spectrum in these scenarios has a break at very small scales, when string produced CDM fluctuations become dominant over inflationary ones. This aspect was particularly emphasized in [15], and there is some observational evidence in favour of such a break. An immediate implication of this result is that it is easier to form structure at high redshifts [20,21]. In [22] it is shown that even with Hot Dark Matter, these scenarios produce enough damped Lyman-α systems, to account for the recent high-redshift observations.

Another issue currently under investigation is the timing of structure formation [20]. Active models drive fluctuations at all times, and therefore produce a time-dependence in $P(k)$ different from passive models. The
effect is subtle, but works so as to slow down structure formation. Hence for the same normalization nowadays
there is more structure at high redshifts in string scenarios.

Overall we end up with a picture in which the CMB is produced by both strings and inflation, the current
large scale structure of the Universe is produced by inflation except on the very small scales, but most of the
structure at high redshift is produced by strings.

In such models there would also be intrinsic non-Gaussianity at the scale of clusters, with interesting con-
nections with the work of [23].

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