CP-violating asymmetry in chargino decay into neutralino and W boson

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Abstract

In the MSSM with complex parameters, loop corrections to $\tilde{\chi}_i^\pm \rightarrow \tilde{\chi}_j^0 W^\pm$ lead to a CP violating asymmetry $A_{\text{CP}} = (\Gamma(\tilde{\chi}_i^+ \rightarrow \tilde{\chi}_j^0 W^+) - \Gamma(\tilde{\chi}_i^- \rightarrow \tilde{\chi}_j^0 W^-))/\Gamma(\tilde{\chi}_i^+ \rightarrow \tilde{\chi}_j^0 W^+) + \Gamma(\tilde{\chi}_i^- \rightarrow \tilde{\chi}_j^0 W^-))$. We calculate this asymmetry at full one-loop level. We perform a detailed numerical analysis for $\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 W^\pm$ and $\tilde{\chi}_2^\pm \rightarrow \tilde{\chi}_1^0 W^\pm$ analyzing the dependence on the parameters and phases involved. Asymmetries of several percent are obtained. We also discuss the feasability of measuring these asymmetries at LHC.
It is well known that supersymmetric models contain new sources of CP violation if the parameters are complex. In the Minimal Supersymmetric Standard Model (MSSM), the $U(1)$ and $SU(2)$ gaugino mass parameters $M_1$ and $M_2$, respectively, the higgsino mass parameter $\mu$, as well as the trilinear couplings $A_f$ (corresponding to a fermion $f$) may be complex. Usually, $M_2$ is made real by redefining the fields. Non-vanishing phases of $M_1$ and $\mu$ cause CP-violating effects already at tree-level in the chargino and neutralino production and decay [1, 2, 3]. In case the trilinear couplings of the third generation ($A_t, A_b, A_\tau$) are complex not only the stop, sbottom, and stau sectors [4] are strongly affected but also the Higgs sector [5, 6]. The three neutral Higgs bosons are no more CP eigenstates.

Although new phases in addition to the CKM in the Standard Model (SM) are desirable to explain baryogenesis, there are severe constraints on the phase of $\mu$ from the experimental limits on the electric dipole moments (EDMs) of the electron, neutron and Hg. For example, in the constraint MSSM $|\phi_\mu|$ has to be small [7, 8] for a SUSY particle spectrum of the order of a few TeV.

In this note, we study CP violation in the decays $\tilde{\chi}_i^+(k_2) \rightarrow \tilde{\chi}_j^0(k_1) + W^+(p)$ and $\tilde{\chi}_i^-(k_2) \rightarrow \tilde{\chi}_j^0(-k_1) + W^-(p)$ in the MSSM with complex parameters by calculating the CP-violating asymmetry

$$A_{CP} = \frac{\Gamma_+(\tilde{\chi}_i^+ \rightarrow \tilde{\chi}_j^0 W^+) - \Gamma_-(\tilde{\chi}_i^- \rightarrow \tilde{\chi}_j^0 W^-)}{\Gamma_+(\tilde{\chi}_i^+ \rightarrow \tilde{\chi}_j^0 W^+) + \Gamma_-(\tilde{\chi}_i^- \rightarrow \tilde{\chi}_j^0 W^-)}$$

at full one-loop order. The asymmetry is zero if CP is conserved and also vanishes at tree-level in case of CP violation. In Fig. 1 we show the graphs which contribute to this asymmetry at one-loop level. Of course, they give a contribution to $A_{CP}$ only if they have an absorptive part, i.e. some decay channels of $\tilde{\chi}_i^\pm$ must be open in addition to that into $\tilde{\chi}_j^0 W^\pm$.

This asymmetry was already calculated in [9] considering only the third generation quarks and squarks in the vertex graphs. We have improved this calculation in several points. First, we performed a full one-loop calculation. In particular, we also calculated the contributions from self-energies of the charginos. It turns out, that these are important. (The self-energies of the neutralinos do not contribute due to their Majorana nature and the $W^\pm - H^\pm$ transition vanishes for on-shell $W$-bosons.) In addition, we take the Yukawa couplings running, which also gives a sizeable effect. Moreover, we take into account that the neutral Higgs bosons ($h^0, H^0, A^0$) mix if the SUSY parameters mentioned are complex. In our case, this influence is, however, very small. As a loop-level quantity the asymmetry $A_{CP}$ depends on the phases of all complex parameters involved. One, however, expects that the dependence on the phases of $M_1$ and $A_{t,b}$ is strongest (taking $\mu$ real). There is even a strong correlation between them. Therefore, a measurement of this asymmetry represents not only a test of CP violation in chargino decay, but can also be used for the determination of the phases of $M_1$ and $A_{t,b}$.
The widths $\Gamma_{(\pm)}$ can be written as $\Gamma_{(\pm)} \propto |\mathcal{M}_{\text{tree}}^{(\pm)}|^2 + 2 \text{Re}[\mathcal{M}_{\text{tree}}^{(\pm)} \mathcal{M}_{\text{loop}}^{(\pm)}]$.

The Feynman amplitudes for the tree- and one-loop level, $\mathcal{M}_{\text{tree}}^{(\pm)}$ and $\mathcal{M}_{\text{loop}}^{(\pm)}$, are given by

\[
\mathcal{M}_{\text{tree}}^{(\pm)} = i \bar{u}_{\tilde{\chi}_i^0}(k_1)\gamma^\mu (O^R P_R + O^L P_L) u_{\tilde{\chi}_i^{+}}(k_2)\epsilon_\mu^*(-p),
\]
\[
\mathcal{M}_{\text{tree}}^{(-)} = i \bar{v}_{\tilde{\chi}_i^0}(-k_2)\gamma^\mu (O^R v_R + O^L v_L) v_{\chi_j^0}(k_1)\epsilon_\mu^*(p),
\]
\[
\mathcal{M}_{\text{loop}}^{(\pm)} = i \bar{u}_{\tilde{\chi}_i^0}(k_1)[(\gamma^\mu \Lambda_{(+)}^R + k_2 R_{(+)}^L) P_R + (\gamma^\mu \Lambda_{(+)L} + k_2 R_{(+)}^R) P_L] u_{\tilde{\chi}_i^+}(k_2)\epsilon_\mu^*(-p),
\]
\[
\mathcal{M}_{\text{loop}}^{(-)} = i \bar{v}_{\tilde{\chi}_i^0}(k_2)[(\gamma^\mu \Lambda_{(-)}^R + k_2 R_{(-)}^L) P_R + (\gamma^\mu \Lambda_{(-)L} + k_2 R_{(-)}^R) P_L] v_{\chi_j^0}(k_1)\epsilon_\mu^*(p). \tag{2}
\]

Since $|\mathcal{M}_{\text{tree}}^{(\pm)}|^2 = |\mathcal{M}_{\text{tree}}^{(-)}|^2$, and assuming, that the one-loop contribution is small compared...
to the tree-level one, the CP-violating asymmetry $A_{\text{CP}}$ takes the form

$$A_{\text{CP}} = \frac{\text{Re}[M_{\text{tree}}^{(+)} M_{\text{loop}}^{(+)}] - \text{Re}[M_{\text{tree}}^{(-)} M_{\text{loop}}^{(-)}]}{|M_{\text{tree}}|^2},$$

(3)

with the squared tree-level amplitude

$$|M_{\text{tree}}|^2 = \rho(|O_R|^2 + |O_L|^2) - 12m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0} \text{Re}[O_R^* O_L],$$

(4)

and the one-loop contributions

$$\text{Re}[M_{\text{tree}}^{(+)} M_{\text{loop}}^{(+)}] = \rho \text{Re}[(\lambda_{(+)}^R O^{R*} + \lambda_{(+)}^L O^{L*}) - 6m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0} \text{Re}[(\lambda_{(+)}^R O^{L*} + \lambda_{(+)}^L O^{R*})]
\frac{\lambda}{2m_W^2}(m_{\tilde{\chi}_j^0} \text{Re}[\Pi_{(+)}^R O^{R*} + \Pi_{(+)}^L O^{L*}] + m_{\tilde{\chi}_i^0} \text{Re}[\Pi_{(+)}^L O^{L*} + \Pi_{(+)}^R O^{R*}]),$$

(5)

$$\text{Re}[M_{\text{tree}}^{(-)} M_{\text{loop}}^{(-)}] = \rho \text{Re}[(\lambda_{(-)}^R O^{R*} + \lambda_{(-)}^L O^{L*}) - 6m_{\tilde{\chi}_i^0}m_{\tilde{\chi}_j^0} \text{Re}[(\lambda_{(-)}^R O^{L*} + \lambda_{(-)}^L O^{R*})]
\frac{\lambda}{2m_W^2}(m_{\tilde{\chi}_j^0} \text{Re}[\Pi_{(-)}^R O^{R*} + \Pi_{(-)}^L O^{L*}] + m_{\tilde{\chi}_i^0} \text{Re}[\Pi_{(-)}^L O^{L*} + \Pi_{(-)}^R O^{R*}]),$$

(6)

with the kinematic factor $\lambda = \lambda(m_{\tilde{\chi}_i^0}^2, m_{\tilde{\chi}_j^0}^2, m_W^2)$, $\lambda(x, y, z) = (x - y - z)^2 - 4yz$, and $\rho = \frac{\lambda}{m_W^2} + 3(m_{\tilde{\chi}_i^0}^2 + m_{\tilde{\chi}_j^0}^2 - m_W^2)$.

The chargino-neutralino-W coupling parameters $O^{L,R}$, defined by the Lagrangian

$$\mathcal{L}_{W\tilde{\chi}_i^0\tilde{\chi}_j^{\pm}} = W^{-\tilde{\chi}_j^{\mp}}_\mu \gamma^\mu (O^{R}_{ji} P_R + O^{L}_{ji} P_L) \tilde{\chi}_i^+ + W^{\tilde{\chi}_j^{+}}_\mu \gamma^\mu (O^{R*}_{ji} P_R + O^{L*}_{ji} P_L) \tilde{\chi}_j^0,$$

(7)

are

$$O^{R}_{ji} = gZ_{ji}^* U_{i1} + \frac{g}{\sqrt{2}} Z_{j3}^* U_{i2} \quad \text{and} \quad O^{L}_{ji} = gZ_{ji} V_{i1}^* - \frac{g}{\sqrt{2}} Z_{j4} V_{i2}^*,$$

(8)

where $U$, $V$, and $Z$ are the matrices diagonalizing the chargino and neutralino system (see eqs. (A.16) and (A.17)). $\Lambda$ and $\Pi$ are form factors which are given in the Appendix A. We only give the form factors for $\tilde{\chi}^+$ and not for $\tilde{\chi}^-$, so that $\Lambda, \Pi$ always stands for $\Lambda_{(+)}, \Pi_{(+)}$. The form factors $\Lambda_{(-)}$ and $\Pi_{(-)}$, belonging to the $\tilde{\chi}^-$ decay, can be easily obtained by conjugating all couplings.

In Appendix A we present all formulas for the vertex contributions with the $\tilde{t}\tilde{b}$ and $b\tilde{b}$ loops and the chargino self-energy contribution with the $\tilde{b}$ loop, see graphs $SF_1 F_2$, $FS_1 S_2$, and $SF$ of Fig. 1. The complete analytical formulas will be given in [10]. All individual one-loop graphs were numerically checked using the packages FeynArts, FormCalc, and LoopTools [11], and FF [12]. We included the CP-violating mixing of the neutral Higgs bosons by writing our own FeynArts model file. For the numerical program we used FeynHiggs [13].
1 Numerical results

We present numerical results for the decay rate asymmetries $A_{\text{CP}}$ according to eq. (1) $	ilde{\chi}_i^\pm \to \tilde{\chi}_j^0 W^\pm$, for $i = 1, 2$ and $j = 1$. A discussion of the other channels will be given in [10]. For the SM input parameters we take $m_Z = 91.1875$ GeV, $m_W = 80.45$ GeV, $\cos \theta_W = m_W/m_Z$, $\alpha(m_Z) = 1/127.9$, the on-shell parameters $m_t = 178$ GeV, and $m_\tau = 1.777$ GeV. For the bottom mass, our input is the $\overline{\text{MS}}$ value $m_b(m_b) = 4.2$ GeV. For the values of the Yukawa couplings of the third generation quarks ($h_t$, $h_b$), we take the running ones at the scale of the decaying particle mass. In principle, the parameters $A_f$, the U(1) gaugino mass parameter $M_t$ of the neutralino sector, and $\mu$ can be complex. We assume that $|M_t| = M_2/2$. In general, there are 15 independent sfermion-breaking mass parameters. We take $M_{\tilde{Q}}$ as input and assume the MSUGRA inspired ratios $m_{\tilde{q}} : M_{\tilde{Q}} : m_{\tilde{t}} = 3 : 2 : 1$ with $m_{\tilde{q}} = M_{\tilde{Q}_{1,2}} = M_{\tilde{U}_{1,2}} = M_{\tilde{D}_{1,2}}$, $M_{\tilde{Q}_{3}} = M_{\tilde{U}_{3}} = M_{\tilde{D}_{3}}$, and $m_{\tilde{t}} = M_{\tilde{L}_{1,2,3}} = M_{\tilde{E}_{1,2,3}}$. In order to reduce the number of input parameters further, we use $A_t = A_h = A_\tau = : A$. In all figures we take $M_{A_0} = 300$ GeV, $\tan \beta = 10$, and $\phi_\mu = \pi/10$.

For the decay $\tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 W^\pm$, the total one-loop asymmetry $A_{\text{CP}}$ is shown in Fig. 2a and the tree-level branching ratio (BR) in Fig. 2b, for $M_2 = 500$ GeV, $|A| = 400$ GeV, $\phi_A = -\pi/4$, $\phi_{M_1} = 3\pi/4$, and three values of $M_{\tilde{Q}}$ as a function of $|\mu|$. $|A_{\text{CP}}|$ increases for increasing values of $|\mu|$ because the tree-level decay width of $\tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 W^\pm$ goes to zero, as $\tilde{\chi}_1^0$ becomes almost a pure bino which does not couple to $W^\pm$. Therefore, for $|\mu| \gtrsim 550$ GeV the branching ratio drops below 1%. The higher the value of $M_{\tilde{Q}}$ the heavier becomes the stop mass. Hence $A_{\text{CP}}$ goes down but the branching ratio in (b) increases.

![Figure 2](image)

**Figure 2:** For $M_2 = 500$ GeV, $|A| = 400$ GeV, $\phi_A = -\pi/4$, $\phi_{M_1} = 3\pi/4$, and three values of $M_{\tilde{Q}}$, (a) the asymmetry $A_{\text{CP}}$ and (b) the tree-level branching ratio BR are given as functions of $|\mu|$.

Fig. 3 shows the dependence of $A_{\text{CP}}$ on $\phi_A$ for $M_2 = 500$ GeV, $|\mu| = 600$ GeV, $|A| = 400$ GeV, $M_{\tilde{Q}} = 400$ GeV, and various $\phi_{M_1}$. $A_{\text{CP}}$ has its maximum at $|\phi_A| \sim \pi/2$ and is
larger at large negative values of the phase $\phi_{M_1}$.

![Diagram](image.png)

Figure 3: The dependence of $A_{CP}$ on $\phi_A$, and various values of $\phi_{M_1}$, with $M_2 = 500$ GeV, $|\mu| = 600$ GeV, $|A| = 400$ GeV, and $M_{\tilde{Q}} = 400$ GeV.

Now we discuss the asymmetry $A_{CP}$ for $\tilde{\chi}_2^\pm \rightarrow \tilde{\chi}_1^0 W^\pm$. Fig. 4a shows the dependence of the asymmetry $A_{CP}$ on the gaugino mass parameter $M_2$ for various values of $|A|$, $\phi_{M_1} = \pi$, $M_{\tilde{Q}} = 300$ GeV, $\phi_A = -\pi/4$, and $|\mu| = 200$ GeV. For $M_2 > 200$ GeV, the lighter chargino and the two lighter neutralinos have dominating higgsino components and the heavier chargino is mostly gaugino-like ($>90\%$). The bigger $|A|$, the bigger is the mixing in the squark sector and hence $A_{CP}$. Around $M_2 \sim 450$ GeV the $\tilde{\chi}_2^+$ becomes massive enough so that the channels into $b\tilde{t}_2$ and $\tilde{t}_1 \tilde{b}_2$ open. For $M_2 \gtrsim 250$ GeV, the third generation (s)quark contributions clearly dominate the asymmetry, the self-energy contribution being bigger than the vertex contribution. For $M_2 < 680$ GeV, the vertex and the self-energy contributions for the third generation (s)quarks have opposite signs and cancel each other to a high degree. Nevertheless, they remain the dominant contributions in a large part of Fig. 4b.

Various pseudothresholds are visible in Fig. 5a, where the squark mass parameter $M_{\tilde{Q}}$ is varied. The parameter set $M_2 = 450$ GeV, $\phi_{M_1} = \pi$, and $|\mu| = 200$ GeV gives the masses $m_{\tilde{\chi}_2^+} = 468.55$ GeV and $m_{\tilde{\chi}_1^0} = 185.66$ GeV. The strong dependence of $A_{CP}$ on the phase is clearly visible. Fig. 5b illustrates the dependence on the phase $\phi_A$ for $M_{\tilde{Q}} = \{230, 300, 400\}$ GeV. That $A_{CP}$ does not factorize into a $\phi_A$ dependent and a $\phi_A$ independent part can be seen from the fact that the three curves do not meet in a single point. The other phases $\phi_\mu$ and $\phi_{M_1}$ distort the factorization.

The $|\mu|$ dependence of the decay $\tilde{\chi}_2^\pm \rightarrow \tilde{\chi}_1^0 W^\pm$ is shown in Fig. 6 for the same parameter set as used in Fig. 2. The total one-loop asymmetry $A_{CP}$ is shown in Fig. 6a and the tree-level branching ratio (BR) in Fig. 6b. In the region $|\mu| \sim 400$ GeV to 600 GeV the character of the $\tilde{\chi}_2^+$ and $\tilde{\chi}_1^0$ changes, for $\tilde{\chi}_2^+$ from gaugino to higgsino and for $\tilde{\chi}_1^0$ from higgsino to mainly bino. Therefore, one has a strong dependence in $A_{CP}$ and BR there. The dependence on $M_{\tilde{Q}}$ is analogous to that in Fig. 2. For $|\mu| \gtrsim 600$ GeV, the mass of $\tilde{\chi}_2^+ \sim |\mu|$ and $\tilde{\chi}_1^0 \sim M_2/2 = 250$ GeV. Therefore, the decay width of $\tilde{\chi}_2^\pm \rightarrow \tilde{\chi}_1^0 W^\pm$
Figure 4: The dependence of $A_{CP}$ on $M_2$ for $\phi_{M_1} = \pi$, $M_Q = 300$ GeV, $\phi_A = -\pi/4$, $|\mu| = 200$ GeV,
(a) the total asymmetry $A_{CP}$ for various values of $|A|$. 
(b) the different contributions to the asymmetry for $|A| = 400$ GeV: the vertex contribution with the third generation (s)quarks in the loop in black (1); the chargino self-energy contribution with the third generation (s)quarks in the loop in red (2); vertex and self-energy corrections with all other (s)fermions in the loop in blue (3); all remaining corrections in green (4).

Figure 5: For $M_2 = 450$ GeV, $\phi_{M_1} = \pi$, $|A| = 400$ GeV, $|\mu| = 200$ GeV,
(a) the total asymmetry depending on $M_Q$ for various values of $\phi_A$.
(b) the total asymmetry depending on $\phi_A$ for various values of $M_Q$.

increases with $|\mu|$ and $A_{CP}$ goes to zero. The hump in Fig. 6b at $|\mu| \sim 600$ GeV for $M_Q = 500$ GeV is due to the opening of the $\tilde{t}_2 b$ channel.

It is known that the electric dipole moments (EDM) of the electron, the neutron and mercury strongly depend on the phase of $\mu$ for a light SUSY spectrum [14]. The experimental constraints for the EDMs of the electron [15], the neutron [16], and mercury [17] can
be fulfilled by heavy sfermions of the first generations [18] or if cancellations of different contributions occur [8]. We checked for all plots all three EDMs and found always (small) values of $\phi_\mu$ that fulfill all EDM constraints.

Finally, we want to comment on the measurability of this asymmetry. At LHC charginos are mainly produced in the cascade decays of gluinos and squarks so that the production rate strongly depends on their masses. If the gluino and squark masses are about the same, the gluino production cross section is far the dominant one. With $m_\tilde{g} \sim m_\tilde{q} = 750$ GeV, we expect roughly $2.4 \times 10^5$ events containing $\tilde{\chi}_1^\pm$ (one has the same amount of $\tilde{\chi}_1^+$ and $\tilde{\chi}_1^-$ in the case where they originate from gluinos or from a gluon-gluon process), assuming a luminosity of $10^5$ pb$^{-1}$ and a branching ratio of a gluino decaying into a $\tilde{\chi}_1^\pm$ of 40%.

Taking into account the branching ratio for $\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 W^\pm$, one can measure $A_{CP}$ for this decay with a statistical significance of $\sim 2$ (confidence level of 95%). For measuring $A_{CP}$ for $\tilde{\chi}_2^\pm \rightarrow \tilde{\chi}_1^0 W^\pm$, assuming $5 \times 10^4$ events containing a $\tilde{\chi}_2^+$ or $\tilde{\chi}_2^-$, one gets a similar statistical significance.

2 Conclusions

We have calculated the CP-violating asymmetry between the partial decay rates $\Gamma(\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^0 W^+)$ and $\Gamma(\tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 W^-)$ due to phases in the MSSM. It is a pure loop effect. We have calculated this asymmetry at full one-loop order. We have given numerical results for $\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 W^\pm$ and $\tilde{\chi}_2^\pm \rightarrow \tilde{\chi}_1^0 W^\pm$. The respective asymmetries are of the order of several percent, depending on the values of parameters and phases involved. In order to have reasonable branching ratios for the decays the $\tilde{\chi}_1^0$ must not be very bino like. We also discussed the feasibility of measuring such an asymmetry at LHC. It might be possible to measured it with a confidence level of 95%. 

Figure 6: For $M_2 = 500$ GeV, $|A| = 400$ GeV, $\phi_A = -\pi/4$, $\phi_{M_1} = 3\pi/4$, and three values of $M_3$, (a) the asymmetry $A_{CP}$ and (b) the tree-level branching ratio BR are given as functions of $|\mu|$. 

\begin{align*}
A_{CP} &\quad [\%] \\
\begin{array}{c}
|\mu| [\text{GeV}] \\
\end{array} \\
\begin{array}{c}
200 & 300 & 400 & 500 & 600 & 700 & 800 \\
\end{array}
\end{align*}

\begin{align*}
\text{BR} &\quad [\%] \\
\begin{array}{c}
|\mu| [\text{GeV}] \\
\end{array} \\
\begin{array}{c}
200 & 300 & 400 & 500 & 600 & 700 & 800 \\
\end{array}
\end{align*}
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The relevant parts of the Lagrangian are

\[
\mathcal{L} = -\frac{g}{\sqrt{2}} W^-_\mu \bar{b} \gamma^\mu P_L t - \frac{g}{\sqrt{2}} W^+_\mu \bar{t} \gamma^\mu P_L b \\
+ ic_{\mu n} W^-_\mu \bar{b} \gamma^\mu \frac{\partial}{\partial t_n} + ic_{\mu n} W^+_\mu \bar{t} \gamma^\mu \frac{\partial}{\partial t_n}
\]

\[
+ \bar{t} \left( \lambda^b_{i m} P_R + k^b_{i m} P_L \right) \tilde{b}_m + \bar{t} \left( \lambda^b_{i m} P_L + k^b_{i m} P_R \right) \tilde{t}_m
\]

\[
+ \bar{b} \left( \lambda^b_{i m} P_R + k^b_{i m} P_L \right) \tilde{b}_m + \bar{b} \left( \lambda^b_{i m} P_L + k^b_{i m} P_R \right) \tilde{t}_m
\]

\[
+ \bar{t} \left( a^{i j}_{m n} P_R + b^{i j}_{m n} P_L \right) \tilde{b}_n + \bar{b} \left( a^{i j}_{m n} P_L + b^{i j}_{m n} P_R \right) \tilde{t}_n
\]

\[
+ \bar{b} \left( a^{b}_{m j} P_R + b^{b}_{m j} P_L \right) \tilde{b}_n + \bar{b} \left( a^{b}_{m j} P_L + b^{b}_{m j} P_R \right) \tilde{t}_n,
\]

with the coupling parameters to the \( W \) boson,

\[
c^+_m = -\frac{g}{\sqrt{2}} R^b_{m L} R^{i*}_{n L},
\]

the chargino,

\[
\bar{t}^{i}_{m i} = -gV_{i l} R^{i*}_{l L} + gh_{t} V_{i 2} R^{i*}_{n R}, \quad \lambda^{b}_{i m} = gh_{b} U^{i}_{2 2} R^{i*}_{n L},
\]

and the neutralino,

\[
a^{i}_{n j} = g R^{i}_{n L} f^{i}_{L j} - gh_{t} R^{i*}_{n R} Z_{j 4}, \quad b^{i}_{n j} = -gh_{t} R^{i*}_{n L} Z_{j 4} + g R^{i*}_{n R} f^{i}_{L j},
\]

\[
a^{b}_{m j} = g R^{b*}_{m L} f^{b}_{L j} - gh_{b} R^{b*}_{m R} Z_{j 3}, \quad b^{b}_{m j} = -gh_{b} R^{b*}_{m L} Z_{j 3} + g R^{b*}_{m R} f^{b}_{L j},
\]

with the gaugino components of the neutralino

\[
f^{b}_{R j} = \sqrt{2} \epsilon_{q} \tan \theta_{W} Z_{j 1}, \quad f^{i}_{L j} = -\sqrt{2} \left( (\epsilon_{q} - I_{3L}^{q}) \tan \theta_{W} Z_{j 1} + I_{3L}^{q} Z_{j 2} \right),
\]

and the Yukawa couplings

\[
h_{t} = \frac{m}{\sqrt{2} m_{W} \sin \beta}, \quad \text{and} \quad h_{b} = \frac{m}{\sqrt{2} m_{W} \cos \beta}.
\]

The charge and the isospin of the quark \( q \) are given by \( \epsilon_{q} \) and \( I_{3L}^{q} \), \( g \) is the \( SU(2) \) coupling parameter.

The mixing matrices are defined as

\[
U^{0}_{j \alpha} \mathcal{M}^{q}_{\alpha \beta} V^{*}_{k \beta} = \delta_{j k} m^{q}_{\alpha \beta},
\]

\[
Z^{0}_{j \alpha} \mathcal{M}^{q}_{\alpha \beta} Z^{*}_{k \beta} = \delta_{j k} m^{0}_{\alpha \beta} \quad \text{for the basis } \{ \tilde{b}, \tilde{\omega}^{3}, \tilde{b}^{0}, \tilde{b}^{0}_{2} \},
\]

\[
R^{q}_{j \alpha} (\mathcal{M}^{2})^{q}_{\alpha \beta} R^{*}_{k \beta} = \delta_{j k} m^{2}_{\alpha \beta}.
\]
with the mass matrices

$$\mathcal{M}^{\tilde{\chi}^+} = \begin{pmatrix} M_2 & \sqrt{2} m_W \sin \beta \\ \sqrt{2} m_W \cos \beta & \mu \end{pmatrix},$$  \hspace{1cm} (19)

$$\mathcal{M}^{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0 \end{pmatrix},$$  \hspace{1cm} (20)

$$\left(\mathcal{M}^2\right)^{\tilde{q}} = \begin{pmatrix} m^2_{\tilde{q}L} & a_q^* m_q \\ a_q m_q & m^2_{\tilde{q}R} \end{pmatrix},$$  \hspace{1cm} (21)

where the abbreviations $s_W = \sin \theta_W$, $c_W = \cos \theta_W$, $s_\beta = \sin \beta$, $c_\beta = \cos \beta$, and

$$m^2_{\tilde{q}L} = M^2_Q + m^2_Z \cos 2\beta (\tilde{I}^q_{3L} - e_q \sin^2 \theta_W) + m^2_q,$$  \hspace{1cm} (22)

$$m^2_{\tilde{q}R} = M^2_{\tilde{q}R} + e_q m^2_Z \cos 2\beta \sin^2 \theta_W + m^2_q,$$  \hspace{1cm} (23)

$$a_q = A_\eta - \mu^* (\tan \beta)^{-2} q_R = |a_q| e^{i\phi_q},$$  \hspace{1cm} (24)

are introduced for a more convenient notation. $M^2_{\tilde{q}R} = M^2_D (M^2_D)$ is the soft breaking mass parameter for the right stops (sbottoms).

The form factors giving the major contribution for most of the parameter regions studied, can be split into the contributions $(\tilde{t}\tilde{b})$, $(\tilde{b}\tilde{t})$, and $(\tilde{b}\tilde{b})$:

$$\Lambda^{L,R}_{(\tilde{t}\tilde{b})} = \Lambda^{L,R}_{(\tilde{b}\tilde{t})} + \Lambda^{L,R}_{(\tilde{b}\tilde{b})},$$  \hspace{1cm} (25)

$$\Pi^{L,R}_{(\tilde{t}\tilde{b})} = \Pi^{L,R}_{(\tilde{b}\tilde{t})} + \Pi^{L,R}_{(\tilde{b}\tilde{b})},$$  \hspace{1cm} (26)

with

$$\Lambda^R_{(\tilde{t}\tilde{b})} = -\frac{3}{16\pi^2} \sum_{n=1}^2 \sum_{m=1}^{2} a^n_{ij} m_{\tilde{\chi}^0} (C_0 + C_1) s_{ij} + b^n_{ij} C_1 m_{\tilde{\chi}^+} + b^n_{ij} C_2 m_{\tilde{\chi}^+},$$  \hspace{1cm} (27)

$$\Lambda^L_{(\tilde{t}\tilde{b})} = \frac{3}{8\pi^2} \sum_{n=1}^{2} a^n_{ij} (B_0 - 2C_{00} + C_{01} m^2_{\tilde{t}j} + C_{11} m^2_{\tilde{t}j} + C_{12} m^2_{\tilde{t}j}) + b^n_{ij} C_1 m_{\tilde{\chi}^0} + a^n_{ij} C_2 m_{\tilde{\chi}^+},$$  \hspace{1cm} (28)

$$\Pi^R_{(\tilde{t}\tilde{b})} = -\frac{3}{8\pi^2} \sum_{n=1}^{2} \sum_{m=1}^{2} a^n_{ij} (k^n m_{\tilde{t}j} + l^n m_{\tilde{t}j} (C_2 + C_{12} + C_{22}) m_{\tilde{\chi}^+}) + b^n_{ij} C_1 m_{\tilde{\chi}^0} + a^n_{ij} C_2 m_{\tilde{\chi}^+},$$  \hspace{1cm} (29)

$$\Pi^L_{(\tilde{t}\tilde{b})} = -\frac{3}{8\pi^2} \sum_{n=1}^{2} \sum_{m=1}^{2} a^n_{ij} (C_1 + C_{11} + C_{12}) m_{\tilde{\chi}^0} + b^n_{ij} C_1 m \tilde{t} j,$$  \hspace{1cm} (30)

$$\Lambda^R_{(\tilde{b}\tilde{b})} = \frac{3}{8\pi^2} \sum_{m,n=1}^{2} b^m_{ij} m_{\tilde{\chi}^+} C_{00},$$  \hspace{1cm} (31)
\[ \Lambda_{(bb)}^L = \frac{3}{8\pi^2} \sum_{m,n=1}^2 \bar{b}_{m}^{\hat{b}} (\hat{b}_{m}^{\hat{b}}) c_{mn}^i C_{00}, \tag{32} \]

\[ \Pi_{(bb)}^R = \frac{3}{8\pi^2} \sum_{m,n=1}^2 \bar{c}_{mn}^i (\hat{a}_{mn}^i) (C_2 + C_{12} + C_{22}) m_{\tilde{\chi}_i^+} + \bar{b}_{m}^{\hat{b}} (\hat{b}_{n}^b) (C_1 + C_{11} + C_{12}) m_{\tilde{\chi}_j^0}, \tag{33} \]

\[ \Pi_{(bb)}^L = \frac{3}{8\pi^2} \sum_{m,n=1}^2 \bar{c}_{mn}^i (\hat{a}_{mn}^i) (C_1 + C_{11} + C_{12}) m_{\tilde{\chi}_j^0} + \bar{b}_{m}^{\hat{b}} (\hat{b}_{n}^b) (C_2 + C_{12} + C_{22}) m_{\tilde{\chi}_i^+}, \tag{34} \]

\[ \Lambda_{(ib)}^R = -\frac{3}{16\pi^2} \frac{O_{R(3-i)}^{(3-i)} \bar{m}_{\tilde{\chi}_i^+} - m_{\tilde{\chi}_j^0}^2}{m_{\tilde{\chi}_j^0}^2} \sum_{n=1}^2 B_0 m_b (k_{n(3-i)}^i \tilde{m}_{n_{\tilde{\chi}_i^+}} + \tilde{m}_{n_{\tilde{\chi}_j^0}}^i m_{\tilde{\chi}_j^0}^+ + l_{n(3-i)}^i \tilde{m}_{n_{\tilde{\chi}_j^0}}^i m_{\tilde{\chi}_j^0}^+), \tag{35} \]

\[ \Lambda_{(ib)}^L = -\frac{3}{16\pi^2} \frac{O_{L(3-i)}^{(3-i)} \bar{m}_{\tilde{\chi}_i^+} - m_{\tilde{\chi}_j^0}^2}{m_{\tilde{\chi}_j^0}^2} \sum_{n=1}^2 B_0 m_b (l_{n(3-i)}^i \tilde{m}_{n_{\tilde{\chi}_i^+}}^i m_{\tilde{\chi}_i^+} + k_{n(3-i)}^i \tilde{m}_{n_{\tilde{\chi}_j^0}} m_{\tilde{\chi}_j^0}^+ + l_{n(3-i)}^i \tilde{m}_{n_{\tilde{\chi}_j^0}}^i m_{\tilde{\chi}_j^0}^+), \tag{36} \]

where

\[ C_{X(\hat{i}b)} = C_X(m_{\tilde{\chi}_{i,j}^0}^2, m_{\tilde{\chi}_{i,j}^0}^2, m_{\tilde{\chi}_{i}^+}^2, m_{\tilde{\chi}_{i}^+}^2, m_{\tilde{\chi}_{j}^0}^2, m_{\tilde{\chi}_{j}^0}^2) \tag{37} \]

\[ B_{0(\hat{i}b)} = B_0(m_{\tilde{\chi}_{i}^+}^2, m_{\tilde{\chi}_{j}^0}^2, m_{\tilde{\chi}_{j}^0}^2) \tag{38} \]

\[ C_{(\hat{i}b)} = C_X(m_{\tilde{\chi}_{i,j}^0}^2, m_{\tilde{\chi}_{i,j}^0}^2, m_{\tilde{\chi}_{i}^+}^2, m_{\tilde{\chi}_{i}^+}^2, m_{\tilde{\chi}_{j}^0}^2, m_{\tilde{\chi}_{j}^0}^2) \tag{39} \]

\[ B_{X(\hat{i}b)} = B_X(m_{\tilde{\chi}_{i}^+}^2, m_{\tilde{\chi}_{j}^0}^2, m_{\tilde{\chi}_{j}^0}^2) \tag{40} \]

The $B$- and $C$-functions are given in the notation of [19].

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