An experimental study of strings, woodwinds (organ pipe, flute, clarinet, saxophone and recorder), and the voice was undertaken to illustrate the basic principles of sound production in music instruments. The setup used is simple and consists of common laboratory equipment. Although the canonical examples (standing wave on a string, in an open and closed pipe) are easily reproduced, they fail to explain the majority of the measurements. The reasons for these deviations are outlined and discussed.

INTRODUCTION

Being a clarinet enthusiast, I wanted to share the richness of music (and the physicist’s way of approaching it) to engineering students within the frame of their general physics classes. The basics are reported in nearly every physics textbook and many details are provided in more specialized contributions [1–5]. I could however not find a comparison of the waveforms and spectra of different instruments in any of these sources. I therefore measured and compared this data by myself.

The whole orchestra being too ambitious for a start, I restricted myself to two instrument families, namely strings and woodwinds (and the former are given a rather perfunctory treatment) and included the voice. The waveform and spectra presented in the first part illustrate the behavior of standing waves on strings and in closed and open pipes (variations 1–2), as well as highlight the basic differences between the timbre of the instruments (variations 3–5). In the second part, we will note that although the instruments are easy to identify, variations between models or performers remain hard to assess (variation 6). Furthermore, the simple models fail to explain the characteristics of some instruments (variations 6–8).

THEME

Physicists have been studying the spectrum of musical instruments at least since the 1940s. The first available methods were based on heterodyne analysis [7] or on sonographs [8]. The development of the Fast Fourier Transform (FFT) algorithm coupled with the apparition of fast and relatively cheap microprocessors has greatly facilitated the task of the musically inclined physicists. Quite sophisticated analysers have been realized [9] but setups based on commercial instruments work just as well for basic analysis [10].

The waveforms have been acquired with the setup presented in Figure 1. It consists of a condenser microphone connected directly to a 125 MHz LeCroy9400 digital oscilloscope. The spectrum was calculated by the data acquisition program LabView by FFT (this task can be directly performed on most modern oscilloscopes). On most figures, the time axis of the waveforms has been scaled and aligned for easier comparison. The spectra are given over 10 or 20 harmonics with ticks corresponding to multiples of the fundamental frequency.

FIG. 1. The experimental setup.

VARIATION 1: VIBRATING STRINGS

A string instrument in its simplest form is composed of a stretched string over a resonance body that transmits the vibration of the string to the air. Despite the simplicity of the vibrating system string instruments show a phenomenal diversity of timbre [12]. This arises from the variety of excitation as the string can be plucked (guitar, harp, harpsichord), bowed (violin, viola...), or struck (piano). The resonance body plays also a great role as is attested by the timbre difference between a guitar and a banjo.

When a continuous transverse wave is generated on a stretched string of length \( l \), a standing wave forms following the superposition of the waves after reflection at the stops. Simple considerations show that the only allowed modes of vibration correspond to wavelengths of \( \lambda = 2l/n \) where \( n \geq 1 \) [4,5], which forms a harmonic series [3] (inset of Figure 2). The vibration of a string will be a superposition of the different modes with varying...
amplitudes determined by the mode of excitation, time since the excitation etc.

One of the most simple string instrument, the string sonometer, is formed of a simple stretched string over a box-shaped resonance body. Figure 2 shows the sound produced by a plucked and bowed sonometer tuned to A2 and demonstrates the richness of the sound produced by a vibrating string as many intense upper harmonics are detected. During bowing for example, the 5th, 16th and 23rd harmonics are stronger than the fundamental.

The point and manner of excitation along the string (what physicists call the initial conditions) influences decisively the timbre: the spectra displayed in Figure 2 differ markedly, especially in the higher modes. By plucking a string, we impose an initial condition such that the shape of the string is triangular with zero and maximal displacement at the stops and at the position of the finger, respectively. The relation between position and intensity of the excited harmonics can be easily predicted (but is not easy to reproduce experimentally). This relation is not as simple for the bowed string, since the excitation velocities (what physicists call the initial conditions) influences the timbre: the spectra displayed in Figure 2 differ markedly, especially in the higher modes.

Finally, we always consider that the properties of the vibrating string are ideal. A real string has however some stiffness, which causes an increase of the frequency of the higher modes with respect to an ideal string. This can be detected on Figure 2, especially for the plucked string (presumably because of the larger displacement). As a consequence, the harmonics of a string are systematically sharp, be it for plucked, bowed or struck strings such as in a piano.

FIG. 2. Waveforms and corresponding spectra of a sonometer (A2, 110 Hz) plucked and bowed at 1/10 of the string length, with in inset the first three vibration modes. The thick line above the waveforms indicates the period of an oscillation.

VARIATION 2: VIBRATING AIR COLUMNS

The principle of wind instruments is a bit more complicated than that of strings. The vibrating medium is the air inside a pipe that acts as a resonator where a standing wave can form. The ends of the pipe determine the boundary conditions. At a closed end, the amplitude of vibration is zero and the pressure variation is maximal (the displacement of the air and the resulting variation of pressure are in anti-phase). Conversely, the pressure will remain constant at the end of an open pipe and the standing wave shows a pressure node and a displacement antinode. This is schematically shown in the insets of Figure 3.

As a consequence, a complete series of harmonics can form in a pipe of length l open at both ends with wavelengths equal to 2·l/n with n ≥ 1. If the pipe is closed at one end, the wavelength of the fundamental corresponds to four times the length of the pipe and only the odd harmonics of wavelengths equal to 4·l/(2n + 1) with n ≥ 0 are allowed.

The vibration of the air can be excited by different means. The most simple one is to produce an edge tone by steering an airjet over an edge [like the top of a bottle or the edge of the organ flue pipe shown in Figure 3(a)]. The edge forms an obstacle for the jet and generates periodic vortices at the mouth of the instrument. The vortices produce in turn periodic displacements of the air molecules. When the edge forms the upper portion of a pipe, the edge tone is locked by resonance to the modes of the pipe. The pitch can then only be changed by increasing the frequency of the vortices (i.e., by blowing faster) to lock the edge tone in a higher mode (which is exactly how flautists change the register and reach higher octaves with their instruments). Such an edge excitation acts like an open end, as the vortices induce air displacement but no pressure variations.

The other means of excitation in wind instruments involve a mechanical vibration: that of the performer’s lips for brass instruments or of a reed for woodwinds. Similarly to the edge tones, the vibration of the lips or of the reed is locked to the resonances of the pipe. The simple reed of the clarinet [see Figure 3(b)] and of the saxophone, and the double reed of the oboe and bassoon, acts actually as a pressure regulator by admitting periodically air packets into the pipe. A reed is therefore equivalent to a closed end as it produces a pressure antinode.

As the frequency of the vortices increases, the excited harmonics evolve constantly. The odd harmonics increase in intensity and the even harmonics decrease. The same thing happens when the length of the pipe is varied. For brass instruments, the reed is usually fastened to the instrument by a means of an edge tone acts like a closed end.

FIG. 3. Excitation systems for woodwinds: (a) the edge of a flue organ pipe, and (b) the reed and mouthpiece of a clarinet.

FIG. 4. Waveform and corresponding spectra of a closed and open organ flue pipe (B♭3, 235 Hz and B♭4, 470 Hz, respectively). The timescale of the upper waveform has been divided by two with respect to the lower waveform. The insets show the first three vibration modes for the variation of the pressure.

We can verify the above principles with a square wooden flue organ pipe of 0.35 m length. The excitation system is reproduced in Figure 3(a) and acts as an open end, and the other end can be either closed or open. As shown on Figure 4, the fundamental of the closed pipe is found at 235 Hz, which corresponds well to a wavelength of \( \lambda = v/f = 4 \cdot 0.35 = 1.4 \) m with \( v = 330 \) m/s. The waveform is nearly triangular, and the even harmonics are far weaker than the odd. The same pipe with its end open sounds one full octave higher (the wavelength of the fundamental is shorter by a factor of 2) and displays a complete series of harmonics.
VARIATION 3: TUTTI

FIG. 5. Waveform of a violin, recorder, flute, clarinet, saxophone and of the author singing the French vowel “aa” (as in sat) (A₄, 440 Hz for the former and A₃, 220 Hz for the latter three instruments). The timescale of the upper waveforms has been divided by two with respect to the lower waveforms.

FIG. 6. Spectra corresponding to the waveforms of Figure 5.

We are now ready to study the behaviour of most strings and woodwinds. Figures 5 and 6 show the waveforms and spectra of six different instruments. Table I also summarizes the characteristics of the woodwinds studied here. A quick glance shows numerous disparities between the instruments, and we will try now to understand these timbre variations and their origin in more detail.

TABLE I. Characteristics of the woodwinds studied in this work.

| instrument   | bore          | excitation |
|--------------|---------------|------------|
| flute        | cylindrical   | edge       |
| clarinet     | cylindrical   | single reed|
| saxophone    | conical       | single reed|
| recorder     | cylindrical   | edge       |

VARIATION 4: THE VIOLIN

The violin produces a very rich sound with at least 20 strong harmonics and complex waveforms, as was the case for the string sonometer in Figure 2. The strongest mode is not the fundamental, but the 7th harmonic in the case of Figure 3.

VARIATION 5: THE FLUTE

As can be seen on Figure 3, woodwinds show simple waveforms and spectra when compared to string instruments. The flute (flauto traverso) is a textbook example of a wind instrument with open ends as the pipe is (nearly) cylindric over the whole length. The most salient feature of the flute is the limited number of harmonics (~7) with an intensity that decreases monotonously [18,19]. The timbre is also very similar for the first two registers (not shown here).

VARIATION 6 (MENUETTO): THE CLARINET

We have seen that a pipe closed at one end shows only odd harmonics: the clarinet, with its simple reed and (nearly) cylindric bore, should be a prototype of such a pipe.

At first sight, this is indeed the case. In the low register (Figure 5 for an A₃ [21]), the odd harmonics are clearly the strongest modes. The even harmonics, although present, are strongly attenuated (at least up to the 6th harmonic). There are other marked differences with the flute. First, the sound is far richer in higher harmonics. Second, the waveform varies considerably with the pitch as displayed on Figure 7. The contents of higher harmonics strongly decreases from 20 for the A₃, to 9 and 5 for the A₄ and A₅. The contribution of the even harmonics becomes also increasingly important. The third mode remains more intense than the second for the A₄, but this is not the case anymore for A₅.

FIG. 7. Waveform and corresponding spectra of a clarinet (A₃, A₄, A₅ at 220, 440 and 880 Hz, respectively). The timescale of the second and third waveform have been divided by two and four with respect to the lower waveform.

The clarinet shows thus a fascinating behavior: it responds like a pipe closed at one end in the lower register but gives a sound with strong even harmonics in the higher registers. The timbre varies therefore as the pitch is increased, with a very distinctive sound for each register. This is due to several facts. First, the bore of the clarinet is not perfectly cylindric but has tapered and slightly conical sections [2]. Second, the flared bell, the constricting mouthpiece [Figure 3(b)] and the toneholes
(even if they are closed) perturb significantly the standing waves. Finally, for wavelengths comparable to the diameter of the toneholes, the sound wave is no longer reflected at the open tonehole but continues to propagate down the pipe. This corresponds a frequency of ∼1500 Hz in typical clarinets [22], and the sound will show increasing amounts of even harmonics with increasing pitch, as found on Figure 7.

Figure 6 leaves out one important feature. The clarinet does not change from the first to the second register by an octave (i.e., by doubling the frequency), but by a duodecime (tripling the frequency). This feature is due to the excitation system alone (the reed acts as a closed end), as can be easily demonstrated by replacing the mouthpiece of a flute (or of a recorder) with a clarinet mouthpiece mounted on a section of pipe such that the overall length of the instrument remains identical. The instrument sounds a full octave lower and changes registers in duodecimes, not in octaves. The reverse effect can be demonstrated by mounting a flute mouthpiece on a clarinet.

**Trio I: timbre quality**

It appears from Figure 3 that it is quite easy to recognize an instrument family by its waveform or spectra. It would be tantalizing if one could also recognize one clarinet from another, for example to choose and buy a good instrument.

Figure 8 shows the spectra of my three clarinets playing the same written note [23], with the same mouthpiece, reed, embouchure and loudness. The upper curve correspond to my first instrument, a cheap wooden B♭ student model. The two lower curves were obtained with professional grade B♭ and A clarinets. I can identify each instrument by playing a few notes from the produced sound and from muscular sensations in the embouchure and respiratory apparatus.

At first glance, the spectra of the three clarinets are readily comparable. Closer inspection shows that the spectra begin to differ from the 10th harmonic on! There are actually far less variations in relative intensities between the two B♭ instruments than between the two pro clarinets. The pro B♭ seems to be slightly richer in harmonics than the student model. The A has no strong harmonics beyond the 11th. This leads to the conclusion that the B♭ and A clarinets are (slightly) different instruments (many clarinetists will agree with that point). The measured differences between two B♭ clarinets remain however quite subtle despite the huge and easily audible difference in timbre.

**Trio II: tone quality**

Is it possible to tell apart a good from a bad tone? This question is of utmost importance for every musician to obtain the desired tone quality. Figure 6 shows two clarinet tones obtained on the same instrument, with the same mouthpiece and reed. The first is a good tone: one could describe it as fullbodied, agreeable to the ear. The second is a beginner’s tone: emitted with a closed throat and weak. The difference is instantly audible but difficult to quantify from Figure 6. The variations appear again in the higher harmonics: the bad tone show no harmonics beyond the 12th, which is at least five modes less than the good tone.

**VARIATION 7: THE SAXOPHONE**

Can one predict the spectra of the saxophone, a single reed instrument with a truncated conical bore, by extrapolation from the previous observations? The sax is a wind instrument, which would imply a limited number of harmonics, and a spectra mainly composed of odd harmonics because of the reed. A short glance at Figures 6 and 7 shows that both predictions are wrong. The even harmonics are as strong as the odd [24]. The sound remains very rich in harmonics even in the higher registers, far more than for the clarinet, and the timbre changes only slightly between the first and the second register. The saxophone does not behave at all like a clarinet!

The main reason is the form of the bore: in a cone, the standing waves are not plane but spherical [23–25]. This has profound implications for the standing wave pattern [24]. In short, the intensity of a wave travelling down or up the pipe is in first approximation constant along the pipe, which implies that the amplitude scales with
the inverse of the distance to the cone apex. The waves interfer to form a spherical standing wave with pressure nodes separated by the usual half-wavelength spacing, but with an amplitude that varies as the inverse of the distance to the cone apex. This is true for a closed as well as an open end \[24\]. A conical bore shows therefore a complete harmonic series, be it excited with a reed, the lips or an edge \[22\]. It would seem also that the conical pipe of the saxophone favors the higher harmonics as compared to the cylindric bore of the clarinet.

**VARIATION 8: THE RECORDER**

The predictions for the saxophone were wrong, so let’s try again with another instrument – the recorder for example. That should be easy: the bore is nearly cylindric, it is excited by an edge and should therefore be similar to the open organ flue pipe. I expected a limited number of harmonics and a full harmonic series. Figure 5 shows that I was wrong again and this puzzled me greatly. The alto recorder indeed has a limited number of harmonics, and a similar timbre in the two registers. But it shows the spectrum of a closed pipe – the even harmonics are more suppressed than for the clarinet – and despite that it changes registers in octaves!

What is the explanation for the odd behaviour of the recorder? The player generates an airjet by blowing into a rectangular windcanal, which is then cut by the edge (see Figure 3). It appears from calculations that the position of the edge relative to the airjet influences critically the intensity of the different harmonics \[26\]. When the edge cuts the side of the jet, the full harmonic series is observed. The even harmonics are however completely absent when the edge is positioned in the center of the jet, as is the case for most modern recorders (among those the one I used). This of course does not affect the modes of resonance of the instrument: the second harmonic can be excited easily by increasing the speed of the airjet, which raises the pitch by an octave. It follows also that I have been very lucky with the open organ flue pipe which follows the expected behaviour shown on Figure 1! thanks to a favorable position of the edge with respect to the airjet \[24\].

**VARIATION 9: A CAPELLA**

We perform frequently with a peculiar and versatile musical instrument, namely our voice. Few instruments have such varied expressive possibilities and ability to change the timbre and loudness. From the point of view of musical acoustics, the voice is a combination of a string and a wind instrument. A pipe, the vocal tract, is excited by the vibration of the vocal cords that generate a complete harmonic series as is usual for vibrating strings (see Figure 10). The timbre is however determined by the shape of the vocal tract that acts as a resonator. Depending on the position of the tongue and on the mouth opening, the position and width of the formants of the vocal tract (the broad resonances, indicated in Figure 10) can be varied and some harmonics produced by the vocal cords are favored with respect to others \[22\]. Note that vocal tract and vocal cords are independent of each other, which implies that the timbre of the voice will change with the pitch for a given tract shape as the harmonics are shifted towards higher frequencies while the position of the formants remains constant.

The effect of the vocal tract shape is displayed in Figure 10 for three vowels sung at the same pitch (the formants are also indicated). The tongue is placed closed to the palate to produce the “ii”: it is nearly sinusoidal with weak upper harmonics. The first formant peaks around 200 Hz and decreases rapidly. The second and third formant around 2000 and 3000 Hz are however easily visible. The “ou” results from a single formant with a maximum around 300 Hz and a slowly decreasing tail: the waveform is more complex and richer in higher harmonics, giving a flute-like sound. The “aa” is obtained with an open tract and is far more complex. The most intense harmonic is the third because of the relatively high position (~800 Hz) and large width of the first formant. The second and the third formant are as intense as the first and give a significant amount of higher harmonics to the sound.

**FIG. 10.** Waveform and corresponding spectra of the author singing an A₃ (220 Hz) on three different vowels: the french “ii” (as in this), “ou” (as in shoe) and “aa” (as in sat). The formants are indicated for each spectrum by a dotted line in linear scale.

**FINALE**

We have seen that the physicist’s approach to musical instruments opens fascinating and complex possibilities. The classical examples (closed and open pipe, for example) are easy to reproduce, but one steps quickly into territory uncharted by the classical physics textbook, which makes the exploration all the more exciting. It remains also that instruments are easy to identify by their timbre, but that it is quite difficult to tell two different models from one another and to classify the quality of the produced sound. It may be even more difficult (not to say impossible) to examine the quality of an interpretation and to understand why well-played music touches us so deeply.

Musical acoustics is a beautiful subject to teach at every level. Music appeals to everybody and a lot of students play or have played at some stage an instrument: this makes often for lively demonstrations in front of the
class. It involves both wave mechanics and fluid mechanics in quite complex ways, and a simple experimental setup can offer direct and compelling insights in the physics of sound production. I hope that this excursion in the basic physics of musical instruments will motivate some of the readers to include the subject in their curriculum and that it may provide helpful material for those who already do.

ACKNOWLEDGMENTS

I thank heartily the different people that either lent me their instrument or that took some time to come and play in the lab: Ariane Michellod (flute), Séverine Michellod (recorders), Stephan Fedrigo (violin – handcrafted by the performer!) and Lukas Bürgi (saxophone). I am also greatly indebted to Paul Braissant, Bernard Egger and Yvette Fazan, who maintain and expand an impressive collection of physics demonstration experiments at EPFL, and who are never put off by the sometimes strange requests of physics teachers.

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