Improvement of Phase Noise Performance in Tracking Array of UAV Signal Based on Mixed Phased/Retrodirective Array

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Abstract—An improved mixed phased/retrodirective array is presented. The phase conjugation technique will be achieved in base band instead of intermediate frequency (IF) band. Canceling the need to the intermediate frequency stage in the receiver will reduce the complexity and cost of the system. The ability to entire processing of the tracking array function to be applied using software defined radio (SDR) system is added. The effect of the phase errors at each channel is compensated, and the noise performance of the tracking array is improved. Also an expanded analytical study of the noise performance of the array to include the impact of the phase errors on the array performance is presented. The proposed equivalent one-channel model of the N-channel array model provides a clear and efficient way to characterize the noise performance of array receiver systems with any amplitude tapering and also considering the phase errors. The improvement provided by the mixed phased/retrodirective array compared to the traditional phased array is evaluated. The effect of array size on the tracking array performance in the presence of phase error is discussed. A monopulse tracking array is taken as an example.

1. INTRODUCTION

Phased array is recently used in the tracking system of an unmanned aerial vehicle (UAV) signal to obtain a high gain datalink between the UAV and ground station [1], where phased array has the ability to achieve high speed tracking and avoid the problems of mechanical movement of the antenna in traditional systems [2, 3]. However, phased array is sensitive to phase errors caused by increased sources of noise in the phased array [4–6]. These phase errors will affect the coherency required to achieve the array factor. On the other hand, achieving tracking array based on a phased array requires high calibration of the array, where the algorithms used in the tracking array (direction finding and tracking algorithms) are sensitive to the receiver noise and phase errors among array elements [7, 8], and these algorithms also increase the computational cost of the system [7, 9]. Phase error’s effect on the pattern can include loss in gain, increased sidelobe levels, and increased beam pointing errors (BPE) [10]. Many of these phase error sources are random and cannot be compensated for using pre-calibration or adaptive signal processing techniques [10, 11]. A lot of studies have been done on phase errors to characterize the performance of an array due to these errors and to aid array designers in setting acceptable tolerance limits for these types of errors [6, 10, 11]. Receiver noise is another concern that affects array performance [12, 13]. Ref. [12] presented an analytical study for the improvement of noise performance in phased array receivers compared to that of each individual array channel, where the multi-channel system of the array was converted to its equivalent one-channel system and then used the defined effective gain, noise, and signal to noise ratio (SNR) to evaluate the noise performance of the array, but that study did not consider the phase errors caused by different array elements.

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We presented, in a previous work [14], a structure of a tracking array that was a mixed phased/retrodirective array. This array had the ability to auto-track the UAV signal without the need of the complex processing algorithms used in a smart antenna, where the phase conjugation technique and complex vector multiplication are used together to generate the geometric phase of each channel. Due to using the phase conjugation technique in this tracking array, the effect of phase errors is reduced to the case of a single antenna system. The phase conjugation technique was achieved using the digital heterodyne mixing technique in IF band. That approach has disadvantage of the need to use a digital local oscillator of double IF frequency [15–17], so a high speed digital signal processor is needed, and on the other hand, relatively high speed analog to digital converters are needed, which increase the cost of the system.

In this paper, we present an improved mixed phased/retrodirective array, where the need for intermediate frequency stage is eliminated by achieving the phase conjugation technique in software, and thus the entire processing of the tracking array function can be applied using software defined radio (SDR) system. This improvement makes this tracking array applicable to any smart antenna system based on SDR without any modification in its basic function [18, 19]. Then to evaluate the noise performance of this proposed tracking array, we will expand the analytical study presented in [12] to include the impact of the phase errors on the array performance, where we will redefine the equivalent model parameters to be used in comparing the noise performance of the proposed array, with the noise performance of the traditional phased array, after taking the phase errors resulting from different sources of noise into consideration.

To expand the analytical study on the noise performance of the phased array, phase errors will be considered. At first we will define the model of the received signal on each array channel with phase error, then we will redefine parameters of the equivalent one-channel system based on this signal model.

We will evaluate the noise performance of the traditional phased array and discuss the limitations of using it in building the tracking system based on this expanded analytical study, where a monopulse tracking array based on a traditional phased array is taken as an example. Then we will evaluate the noise performance of the proposed array, based on this expanded analytical study and compare it with the noise performance of the traditional phased array, to show the improvement provided by the proposed array.

This paper is organized as follows. In Section 2, we recalculate effective parameters of the equivalent model after considering phase errors, and then analytical study to the noise performance of the traditional phased array is done. Section 3 presents the improved tracking array. In Section 4, we make analytical study on the noise performance of the proposed tracking array. In Section 5, the simulation and discussion are presented. Section 6 presents the conclusion.

2. EFFECTIVE ONE-CHANNEL MODEL WITH PHASE ERROR

We will expand the analytical study on the noise performance of the phased array done in [12] to include the impact of the phase errors on the array performance. At first we will write the model of the signal at the output of each array channel with phase errors, then we will recalculate the effective parameters of the equivalent model according to this signal model.

2.1. Signal Model with Phase Error

Almost all array elements (local oscillators, quadrature modulators/demodulators) and element dislocation contribute, directly or indirectly, to increasing the phase errors [11]. For the purpose of analysis, we will express the total phase errors as one random variable that will appear in the signal as random phase shift. So the phase of the received signal at the output of each channel in the array will be shifted by the geometric phase of this channel and the phase error, where the geometric phase of each channel is related to the geometry of array elements and the direction of arrival of the received signal [3–5]. Figure 1 shows a single channel in a phased array receiver.

Equation (1) shows the model of the received narrow band signal at the output of the antenna (input of the low noise amplifier (LNA)).

\[ S_{lk} = S_e^{j\phi_{lk}} \] (1)
where \( k \) is the channel number \( k = 1, 2, \ldots, N \) with \( N \) being the size of the array, \( \phi_{gk} \) the geometric phase of the \( k \)th channel, and \( S \) the original signal.

Then the signal at the output of the \( k \)th channel in Figure 1 (before multiplying by the complex weight) will have additional random phase shift due to the phase errors caused by the channel components, and it can be expressed as:

\[
S_{chk} = Se^{j(\phi_{gk} + \varepsilon_k)}
\]

(2)

where \( \varepsilon_k \) is the total phase error caused by the channel components in the block “\( Ch_k \)” in Figure 1.

Then the output signal after being multiplied by the complex weight (the input of the array combiner shown in Figure 2) will be:

\[
S_{ok} = w_k^* S_{chk}
\]

(3)

where \( w_k^* = e^{-j\phi_k} \) is the complex weight of the corresponding channel.

**Figure 1.** Single channel in a phased array receiver.

**Figure 2.** (a) \( N \)-channel phased array. (b) One-channel equivalent system [12].
Using Eq. (2) in Eq. (3), the signal at the output of the \( k \)th channel will be:

\[
S_{ok} = S e^{j(\phi_gk + \varepsilon_k) - \phi_k}
\]  

(4)

In order to direct the array response towards the source of the signal, the phase of the complex weight of each channel must be adjusted to be equal to the geometric phase of this channel \( w_k^* = e^{-j\phi_gk} \). Then, the output signal will be:

\[
S_{ok} = S e^{j\varepsilon_k}
\]  

(5)

2.2. Equivalent Model

In this section, we will convert the \( N \)-channel array system to its equivalent one-channel system. Figure 2(a) shows the general configuration of a phased-array receiver with size \( N \).

We will calculate parameters of the equivalent model (Figure 2(b)) according to the signal model with phase error presented in Section 2.1, but in this section we will represent symbols \( s_{1k} \) and \( s_I \) as the power of the signal for simplicity. The received signal power at the output of the antenna of each channel in Figure 2(a) is

\[
s_{1k} = \left| \sqrt{W_I A e^{j\phi_gk}} \right|^2 = W_I A e = s_I
\]  

(6)

where \( A_e \) is the effective area of each antenna, \( W_I \) the incident power density, and \( s_I \) the power of the signal at the output of the antenna of each channel considering that all channels are identical.

The signal power at the output of the array (the output of the combiner) will be

\[
s_o = \left| \sum_{k=1}^{N} \left( \sqrt{W_I A e G_k e^{j(\phi_gk + \varepsilon_k) - \phi_k}} a_k \right) \right|^2
\]  

(7)

where \( G_k \) is the gain of the \( "k" \)th channel, \( \phi_k \) the phase of the complex weight, and \( a_k \) the combining coefficient which in general reflects the weighted amplitude tapering in the beamforming network [12]. In this equation, we take the general case before adjusting the array to be directed toward the received signal.

The thermal noise power at the antenna terminal [12, 13] is given by:

\[
n_I = k_B T_a B
\]  

(8)

where \( T_a \) is the antenna noise temperature, \( B \) the operating bandwidth, and \( k_B \) the Boltzmann constant.

Then the noise power at the output of each channel will be:

\[
n_{Rk} = n_1 G_k F_k
\]

\[
= k_B T_a B G_k F_k
\]  

(9)

where \( F_k \) is the noise figure of the \( "k" \)th channel.

Assuming that the noise components from different array channels are mutually uncorrelated to each other, we will use the calculated noise power at the array output in [12] as:

\[
n_o = \sum_{k=1}^{N} \left( k_B T_a B G_k F_k |a_k|^2 \right)
\]  

(10)

Now we will calculate effective parameters of the equivalent model of Figure 2(b), where the effective single antenna of the equivalent one-channel system is supposed to represent the entire antenna array in the original \( N \)-channel array system, so we can consider that \( A_{e, eff} = N A_e \) [12]. Therefore, the effective received signal power and noise power at the antenna terminal are given by:

\[
s_{1, eff} = W_1 A_{e, eff} = NW_1 A_e
\]

\[
= NS_I
\]  

(11)

\[
n_{I, eff} = k_B T_a B
\]  

(12)
The one-channel equivalent model should give the same output of the signal and noise power as the original N-channel array system, under the same input conditions. So considering Eqs. (7) and (11) we can find the effective gain as:

\[
G_{\text{eff}} = \frac{s_{\text{O}}}{s_{\text{I,eff}}} = \frac{1}{N} \left| \sum_{k=1}^{N} \left( \sqrt{G_k e^{j(\phi_{gk} + \varepsilon_k - \phi_k)}} a_k \right) \right|^2
\]

Then the effective noise figure is found from Eqs. (10), (12), and (13) to be:

\[
F_{\text{eff}} = \frac{n_{\text{O}}}{G_{\text{eff}} n_{\text{I,eff}}} = \frac{N \sum_{k=1}^{N} \left( G_k F_k |a_k|^2 \right)}{\left| \sum_{k=1}^{N} \left( \sqrt{G_k e^{j(\phi_{gk} + \varepsilon_k - \phi_k)}} a_k \right) \right|^2}
\]

We can see that the recalculated effective gain and effective noise figure in Eqs. (13) and (14) provide a clear and efficient way to characterize the noise performance of array receiver systems with any amplitude tapering and also in the presence of the phase errors of the array elements.

To take a special case for this study, we will assume that channels of the array are identical, and the amplitude \(a_k = 1\). So when adjusting the array to be directed toward the received signal, the effective gain and noise figure will be as follows:

\[
G_{\text{eff}} = \frac{G}{N} \left| \sum_{k=1}^{N} e^{j\varepsilon_k} \right|^2
\]

\[
F_{\text{eff}} = F \cdot N^2 \frac{1}{\left| \sum_{k=1}^{N} e^{j\varepsilon_k} \right|^2}
\]

Then the effective SNR can be defined as:

\[
\text{SNR}_{\text{O,eff}} = \frac{s_{\text{I,eff}}}{n_{\text{I,eff}}} F_{\text{eff}}
\]

Using Eqs. (11), (12), and (16) in Eq. (17) the effective SNR will be:

\[
\text{SNR}_{0,\text{eff}} = \text{SNR}_O \frac{1}{N} \left| \sum_{k=1}^{N} e^{j\varepsilon_k} \right|^2
\]

where \(\text{SNR}_O\) is the SNR at the output of each individual channel.

It is clear from Eqs. (15), (16), and (18) that the phase error will affect the effective parameters in the equivalent model of the traditional phased array system, where it will decrease the effective gain and increase effective noise figure, and as a result, the signal to noise ratio will be decreased, so the sensitivity of the receiver will be affected. In the next section, we will characterize the loss in the signal to noise ratio due to the phase error in the traditional phased array.

### 2.3. SNR Loss

Equation (18) defines the SNR at the output of the phased array receiver as a function of the phase error which is a random variable. So it is better to make a statistical study to evaluate the effect of the phase error on the SNR. We can define the SNR loss due to the phase error compared to the ideal case when there is no phase error as:

\[
L_{\text{SNR}} = \frac{\text{SNR}_{0,\text{eff}}}{\text{SNR}_{\text{id}}} = \frac{\text{SNR}_O \frac{1}{N} \left| \sum_{k=1}^{N} e^{j\varepsilon_k} \right|^2}{\text{SNR}_O \frac{1}{N} \left| \sum_{k=1}^{N} e^{j\varepsilon_k} \right|^2} = \frac{N}{N^2}
\]
where SNR_{id} is the ideal signal to noise ratio and given by Eq. (18) when there is no phase error.

From Eq. (19), we can see that the SNR loss is a random variable, so we will find its expectation. To simplify the analysis, we take the assumption that the phase error is a random variable with uniform distribution of the form:

$$\varepsilon_k \sim U[-\delta_{\text{max}}, \delta_{\text{max}}]$$  \hspace{1cm} (20)

where $0^\circ \leq \delta_{\text{max}} \leq 180^\circ$ is the upper bound on the amplitude of phase deviation.

Then the expectation of the SNR loss is expressed as:

$$E[L_{\text{SNR}}] = \left| \mathbb{E} \left[ \sum_{k=1}^{N} e^{j\varepsilon_k} \right]^2 \right| / N^2$$  \hspace{1cm} (21)

By calculating the term $E[\sum_{k=1}^{N} e^{j\varepsilon_k}]^2$ for the uniform distribution of the random phase error variable and using it in Eq. (21), the SNR loss will be expressed as follows:

$$E[L_{\text{SNR}}] = \frac{\sin^2(\delta_{\text{max}})}{\delta_{\text{max}}^2} + \frac{1}{N} \left( 1 - \frac{\sin^2(\delta_{\text{max}})}{\delta_{\text{max}}^2} \right)$$  \hspace{1cm} (22)

### 3. IMPROVED MIXED PHASED/RETRODIRECTIVE TRACKING ARRAY

This tracking array is based on the proposed tracking array presented in [14] which is based on the mixing between the phased array and retrodirective array to find the geometric phase of each array channel automatically, where the phase conjugation technique is achieved in IF band.

In this improved tracking array, the phase conjugation technique will be achieved in base band instead of IF band, so canceling the need for the intermediate frequency stage in the receiver, which means that we do not need to use a double IF frequency digital local oscillator anymore, thus reducing the complexity and cost of the system. Figure 3 shows the block diagram of the improved tracking array.

From the block diagram, we can see that the process of generating the phase conjugated version of the received signal on the array channels is simplified to a big degree by achieving the phase conjugation technique in baseband instead of IF band, so the function of the tracking array can be achieved in baseband after generating the IQ signals.

Canceling the need to use the digital heterodyne mixing technique in IF band to achieve the phase conjugation technique gives the ability to use either a simple FPGA module, where there is no need to high speed processor, or applying the entire tracking function on an SDR system. We will discuss these two choices separately.

First choice: Due to canceling the processing in the IF band, there is no need to use a high speed processor, where the process of generating the phase conjugated version of the received signal is simply achieved by multiplying the $Q$ component of each signal by factor $(-1)$. We can represent this process mathematically by writing the received signals in a complex form.

The complex form of the received signal on the reference channel is:

$$IQ_{ref} = I_{ref} + jQ_{ref}$$  \hspace{1cm} (23)

where $I_{ref}$ and $Q_{ref}$ are the quadrature and in phase components at the output of the quadrature demodulator of the reference channel.

The complex form of the received signal on the $k$th channel is:

$$IQ_k = I_k + jQ_k$$  \hspace{1cm} (24)

where $I_k$ and $Q_k$ are the quadrature and in phase components at the output of the quadrature demodulator of the $k$th channel.

Then phase conjugated version of the received signal on each channel is obtained from Eq. (24) as:

$$IQ_k^* = I_k + j(-1)Q_k = I_k - jQ_k$$  \hspace{1cm} (25)
The goal of generating the phase conjugated version of the received signal on each channel is to find the geometric phase of this channel so finding the required complex weight to direct the array response towards the signal source based on the complex vectors multiplication. Figure 4 shows the representation of these complex vectors.

We can see that phase $\phi_k$ of the complex vector of the received signal on each channel is shifted from phase $\phi_{ref}$ of the received signal on the reference channel by geometric phase $\phi_{gk}$ of this channel. Then by multiplying the complex vector of the phase conjugated version of the received signal on each channel with the complex vector of the received signal on the reference channel, we will get a complex vector with phase equal to the conjugation of the geometric phase of the corresponding channel which is the required complex weight of this channel.

Then the complex vector of the required complex weight of each channel will be given by multiplying Eq. (23) with Eq. (25) as:

$$w_k^* = IQ_{ref} \times IQ_k^* = (I_{ref}I_k + Q_{ref}Q_k) + j(I_kQ_{ref} - I_{ref}Q_k)$$

So we can see that the process of generating the complex weight of each channel is simplified to the product and sum operations on the “$I$” and “$Q$” components of received signals.

Second choice: If the output of the IQ generator in Figure 3 is sent to a computer device, then the whole tracking process can be achieved using software. A software radio environment like GNU Radio software, which is used in building the SDR systems [20], can be used to achieve the whole process of tracking. Using IQ components of the received signals, we can represent the received signal on each
channel in its complex form and then collect them in one matrix as:

\[ V = \begin{bmatrix} 
e^{j\phi_{\text{ref}}} 
e^{j(\phi_{\text{ref}}+\phi_{g_1})} \cdots 
e^{j(\phi_{\text{ref}}+\phi_{g_{N-1}})} \end{bmatrix} \]  

(27)

Then the phase conjugated version of the received signal at each channel can be simply generated by conjugating the matrix in Eq. (27) using GNU radio functions:

\[ V^\ast = \begin{bmatrix} 
e^{-j\phi_{\text{ref}}} 
e^{-j(\phi_{\text{ref}}+\phi_{g_1})} \cdots 
e^{-j(\phi_{\text{ref}}+\phi_{g_{N-1}})} \end{bmatrix} \]  

(28)

Then the required complex weights to direct the array response towards the received signal will be:

\[ C = e^{j\phi_{\text{ref}}} \times V^\ast \\
= e^{j\phi_{\text{ref}}} \begin{bmatrix} 
e^{-j\phi_{\text{ref}}} 
e^{-j(\phi_{\text{ref}}+\phi_{g_1})} \cdots 
e^{-j(\phi_{\text{ref}}+\phi_{g_{N-1}})} \end{bmatrix} \\
= \begin{bmatrix} 
e^{j0} 
e^{-j\phi_{g_1}} \cdots 
e^{-j\phi_{g_{N-1}}} \end{bmatrix} \\
= W^H \]  

(29)

where \( W^H \) is the Hermitian of the required complex weight matrix, which is used to direct the array response towards the received signal.

Using Eqs. (27) and (29), we can get the array response:

\[ B = W^H V \\
= Ne^{j\phi_{\text{ref}}} \]  

(30)

This is equal to the peak of the array factor, so having a permanent high gain reception beam.

### 3.1. Reduce the Effect of Phase Error

To find the effect of the phase error on the proposed tracking array performance, we will find the array response based on the signal model with phase error presented in Section 2.1.

By considering the signal model with phase error in Eq. (2), the matrix of the received signals in Eq. (27) will be modified to be:

\[ V = \begin{bmatrix} 
e^{j(\phi_{\text{ref}}+\varepsilon_0)} 
e^{j(\phi_{\text{ref}}+\phi_{g_1}+\varepsilon_1)} \cdots 
e^{j(\phi_{\text{ref}}+\phi_{g_{N-1}}+\varepsilon_{N-1})} \end{bmatrix} \]  

(31)

Then the phase conjugated version of the received signal at each channel will become:

\[ V^\ast = \begin{bmatrix} 
e^{-j(\phi_{\text{ref}}+\varepsilon_0)} 
e^{-j(\phi_{\text{ref}}+\phi_{g_1}+\varepsilon_1)} \cdots 
e^{-j(\phi_{\text{ref}}+\phi_{g_{N-1}}+\varepsilon_{N-1})} \end{bmatrix} \]  

(32)
So complex weights of the array channels will become:

\[
C = e^{j(\phi_{ref}+\varepsilon_0)} * V^* \\
= e^{-j(\phi_{ref}+\varepsilon_0)} \left[ e^{-j(\phi_{ref}+\varepsilon_1)} e^{-j(\phi_{ref}+\phi_2+\varepsilon_2)} \ldots e^{-j(\phi_{ref}+\phi_{N-1}+\varepsilon_{N-1})} \right] \\
= \left[ e^{j0} e^{j(\phi_{ref}-\varepsilon_1+\varepsilon_0)} e^{j(\phi_{ref}-\varepsilon_2+\varepsilon_0)} \ldots e^{j(\phi_{ref}-\varepsilon_{N-1}+\varepsilon_{N-1})} \right] \\
= W^H 
\]

We can see that the phase error of each channel is included in its corresponding complex weight, so the phase error of each channel will be compensated in the array response. Using Eqs. (31) and (33), we can get the array response as:

\[
B = W^H V \\
= Ne^{j\phi_{ref}} e^{j\varepsilon_0} 
\]

So the effect of the phase errors is reduced to the case of a single antenna system, where the phase error simply rotates the phase of the received signal while the signal amplitude is not affected.

### 3.2. Noise Performance of the Improved Tracking Array

To analyze the noise performance of the mixed phased/retrodirective tracking array, we will calculate effective parameters of the equivalent model of Figure 2(b) based on the resulting complex weight in Eq. (33).

Using Eqs. (2) and (33) in Eq. (3), the model of the signal at the output of each channel in Figure 2(a) will be expressed as follows:

\[
S_{ok} = w_k^* S_k = Se^{j\varepsilon_0} 
\]

Taking the assumption that channels of the array are identical, the signal power at the output of the array (the output of the combiner) will be

\[
s_o = \left| \sum_{k=1}^{N} \left( \sqrt{W_I A_e Ge^{j\varepsilon_0}} \right) \right|^2 
\]

Using Eqs. (11) and (36), we will get the effective gain:

\[
G_{eff} = \frac{s_o}{s_{1.eff}} = \frac{1}{N} \left[ \sum_{k=1}^{N} (\sqrt{Ge^{j\varepsilon_0}}) \right]^2 
\]

\[
= NG 
\]

Then the effective noise figure can be found from Eqs. (10), (12), and (37) to be represented as:

\[
F_{eff} = \frac{n_o}{G_{eff} s_{1.eff}} = \frac{N \sum_{k=1}^{N} (GF) \left( \sum_{k=1}^{N} (\sqrt{Ge^{j\varepsilon_0}}) \right)^2}{F} 
\]

Using Eqs. (11), (12), and (38) in Eq. (17), the effective SNR will be:

\[
SNR_{o.eff} = N * SNRO 
\]

Using Eq. (39) in Eq. (19), we can find its SNR loss as:

\[
L_{SNR} = \frac{SNR_{o.eff}}{SNR} = 1 
\]

From Eqs. (37), (38), and (39), we can see that the mixed phased/retrodirective tracking array cancels the effect of the phase noise on its performance, compared with Eqs. (15), (16), and (18) where the performance of the traditional phased array is a function of the phase error.

Table 1 shows the comparison of the equivalent parameters of the traditional phased array and the mixed phased/retrodirective array.
Table 1. Comparison of the equivalent parameters.

| Used Array    | $F_{\text{eff}}$ | $G_{\text{eff}}$ | $L_{\text{SNR}}$ |
|---------------|------------------|------------------|------------------|
| Mixed array   | $F$              | $N \cdot G$     | 1                |
| Phased array  | $F \cdot N^2 \frac{1}{\sum_{k=1}^{N} e^{j\varepsilon_k}}$ | $G \frac{\left(\sum_{k=1}^{N} e^{j\varepsilon_k}\right)^2}{\sum_{k=1}^{N} e^{j2\varepsilon_k}}$ | $\frac{\left(\sum_{k=1}^{N} e^{j\varepsilon_k}\right)^2}{N^2}$ |

4. SIMULATION AND RESULTS

In this section, the noise performance improvement of the proposed mixed array and its ability to eliminate the impact of the phase error is evaluated compared with the conventional phased array, where the effect of phase error on the effective parameters of the equivalent one-channel model due to increasing the number of array elements is tested.

Using the traditional phased array in achieving tracking array is then discussed, and the monopulse tracking array is taken as an example.

Using Eqs. (15) and (37) and supposing that the array response is directed towards a received signal at the angular position 45°, the effect of the array size on the effective gain in the presence of the phase error is tested, where the phase error is supposed to have a uniform distribution. Figure 5 shows the effect of phase error on the gain increments as a function of the array size, where the gain increment is defined as

$$G_{\text{inc}} = 10 \log \left( \frac{G_{\text{eff}}}{G} \right)$$

(41)

Figure 5. The effect of phase error on the gain increments.

Because the mixed array eliminates the effect of the phase error, its gain increment has one curve in Figure 5 for all values of the phase error deviation, where its gain increases proportionately to the number of elements, while for the traditional phased array, the phase error reduces the gain, for the same number of elements. On the other hand, phase error will limit the effect of increasing the array size.
Antenna arrays are usually used in reception to enhance the signal from the desired direction against the noise and thus improve the signal to noise ratio at the output of the array receiver. However, this performance will be affected due to phase error. Figure 6 shows the loss of signal-to-noise ratio on the output of the array due to phase error for different values of array size.

Note that there is no loss in the SNR on the output of the mixed array, and therefore this SNR will be proportional to the SNR on the output of the single channel in the array by the number of elements of the array, while for the traditional phased array, the phase error will result in a decrease in the signal-to-noise ratio for the same number of elements. On the other hand, the increase in the number of elements will increase the impact of the phase error on the loss of signal-to-noise ratio. Hence, the use of large arrays requires high calibration of the array elements to reduce the phase error.

The decrease in the signal-to-noise ratio in conventional phased array adds additional concern when it is used in the construction of a tracking array, since the maximum range of tracking is related to the signal to noise ratio by the following [21].

\[
R_{\text{max}}^2 = \frac{P_T G_T G_R \lambda^2}{(4\pi)^2 (\text{SNR}) L_s k_B T_s B} \frac{1}{T_s B} \tag{42}
\]

where \(G_T\) is the transmitter gain, \(G_R\) the receiver gain, and \(\lambda\) the wavelength of the received signal.

Thus, increasing the maximum range due to the decrease in signal-to-noise ratio may lead to loss of the datalink between the UAV and the ground station and thus failure in tracking.

A tracking array using the monopulse algorithm for tracking based on the traditional phased array [2] is taken as an example. Figure 7 shows the comparison of the response of the mixed tracking array with the tracking array based on the monopulse algorithm for different values of the phase error deviation and for a different values of array size, when tracking the moving UAV during its movement within the field of view \([-45^\circ + 45^\circ]\).

We can see that the performance of the mixed array is stable against the phase error, and therefore, increasing the number of elements will improve the signal to noise ratio and thus improve the sensitivity of the receiver. For the monopulse tracking array based on the traditional phased array, the phase error will lead to a deviation of the array performance, and on the other hand, the increase in the number of elements in the presence of the phase error will lead to the deviation of the array performance due to the loss in the signal-to-noise ratio. From Eq. (16), it is clear that the effective noise figure will be increased due to the phase error as well as the array size, so a high noise level will be at the output, which will affect the tracking accuracy of the monopulse algorithm [2].
Figure 7. Array response of mixed tracking array and monopulse array.

5. CONCLUSION

Canceling the need for the intermediate frequency stage in the receiver will reduce the complexity and cost of the system, and also give the ability to the entire processing of the tracking array function to be applied using software defined radio (SDR) system. Thus this improvement makes the tracking array applicable to any smart antenna system based on SDR without any modification in its basic function.

Due to using the phase conjugation technique, the effect of phase errors at each channel will be compensated, so improving noise performance of the array.

The recalculated effective parameters of the equivalent model provide a clear and efficient way to characterize the noise performance of array receiver systems with any amplitude tapering and also considering the phase errors of the array elements, so it can be used to compare the performance of the array with that of an individual array channel and to compare the performance of different array systems.

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