A new learning trajectory on the pyramid volume for secondary school

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Abstract. This research aims to design Learning Trajectory (LT) for pyramid volume learning. This LT was designed based on curriculum review on the volume of pyramid in secondary schools and study of learning obstacles based on didactic, ontogenic, and epistemological aspects identified through diagnostic tests, interviews with mathematics teachers, and analysis of lesson plan used by the teacher in learning then analyzed using Didactical Design Research. This study involved 27 8th grade students and one mathematics teacher. The results showed that the concept of Pythagoras and volume of cube with learning media support played an important role in the learning trajectory of the pyramid volume.

1. Introduction
The part of mathematics that is often present in everyday life is geometry [1]. Geometry material in secondary school mathematics includes lines, angles, plane, congruence, space constructions, and Pythagoras [2]. One of the disadvantages of students in the field of geometry is shown by the inability of students to recognize wake from flat side space [3]. The inability of parts in geometry can have an impact on the inability of other parts in geometry because there are many discussion points in interconnected geometry [4]. This inability is also caused by students having difficulty concentrating due to a lack of understanding of basic geometry [5].

The obstacles encountered by students related to didactic factors are referred to as learning obstacles [6]. In daily practice, every child naturally experiences situations of learning obstacles [6 - 8]. Regarding constraints in geometry, didactic problems can also occur when students do not do geometric work exactly as expected by the teacher [9]. Some obstacles in the concept of the volume of the pyramid were raised by the researchers. Generally states that the difficulty found by students is not understanding the purpose of the problem because there are other concepts such as Pythagoras, not knowing the nature and elements of the pyramid, as well as miscalculations [2, 10, 11] and learning methods used by the teacher [12, 13]. This didactic obstacle often arises when the learning path created by the teacher is not following the learning trajectory that students should go through.

Learning trajectory is a sequence in the delivery of material following students' thinking abilities [14, 15]. The learning trajectory set forth as a learning plan as a prediction of students' thinking paths is referred to as a hypothetical learning trajectory (HLT) [16, 17]. In the process of learning activities, teachers are required to have appropriate actions to anticipate any mental activity that arises from students, but still pay attention to the objectives of learning itself so that HLT can be interpreted as a prediction of learning activities, such as what students think and how student understanding develops [16]. HLT consists of three components: learning objectives that determine the direction, goals of learning activities and hypothetical-predictive learning processes of how students think and understand.
will develop in the context of learning activities [16 - 18]. This learning trajectory is then much forgotten in teaching materials or textbooks used by the teacher as a guide to carry out learning in class.  

We know that each student has their characteristics and must be studied first by the teacher. The learning flow that is implemented is not enough based only on the standardized syntax. The lesson plan must be following the needs of students, students' thinking character, and the nature of the concept of the material itself. Before applying a learning method, it is necessary to first examine the students' abilities and teaching materials that are following the conditions and needs of students [19]. Thus the teacher can create the right didactic situation for students [10, 18]. Besides, a teacher needs to have the ability to create didactic relationships between students and teaching materials to create an ideal learning trajectory for students [18].  

Several studies related to the volume of the pyramid were conducted through PMRI or RME learning [10, 20], through cooperative learning [21], and the use of three-dimensional media in learning [22]. The research is generally successful and can improve student achievement in pyramid volume material. The learning steps carried out in class are done based on the learning steps that have been determined based on the theory. Likewise the trajectory of learning the volume of a pyramid that has been arranged in the curriculum [23] which is then poured in textbooks and used by teachers [24]. What can be seen from the learning in the textbook? Most learning trajectories in textbooks do not begin with an analysis of student needs first [13]. While learning must be in accordance with the analysis of student needs, how the level of student thinking and what concepts have been understood by students related to the concepts that are learned [6, 25]. In this case, there is no test to find out the obstacles faced by students regarding the concepts that underlie the volume of pyramid and the student's initial knowledge. In addition, the existing learning flow does not facilitate diverse student learning styles. This is what researchers see as a shortcoming of the learning path that has existed so far.

Learning obstacle and learning trajectory make a teacher need to master teaching material, and have other knowledge related to students and be able to create a didactical design that can encourage the learning process optimally [18, 26]. Based on this, we conducted a series of tests to identify learning obstacles that occurred related to the volume of the pyramid on 27 students of class VIII. The average student can understand the purpose of the problem and can identify the problem correctly but makes mistakes when determining the correct concept to answer the problem. The concept of graduation that has been obtained in elementary schools has become a particular obstacle for students even up to college [27]. Pythagoras is also a problem in the material volume of the pyramid, as students only know the formula but do not know its application in squares and triangles [28, 29]. Based on this Pythagoras concept becomes an important concern in the preparation of HLT pyramid volume. Based on our identification, learning obstacles in the material volume of the pyramid occur in 1) the concept of Pythagoras; 2) the concept of area and volume units; 3) the properties of the pyramid, and 4) the concept of rank. The use of media is also a concern for us in this HLT. Other researchers use prisms to construct the concept of pyramid volume [12], whereas we use the cube as a medium to construct the concept of pyramid volume.

This article focuses on how learning trajectory learning volume of the pyramid is based on the stages of learning in didactic situation theory. The purpose of this study is to create or design a Learning Trajectory in learning the pyramid volume based on learning obstacles and student characteristics.

2. Method
This research is part of the Didactical Design Research in the teaching experiment and retrospective analysis stages. Data obtained from the observations, interviews, documentation, final test, and learning video recordings. This test is based on learning obstacles that have been identified with the aim of whether learning obstacles can be minimized. The main instrument in this study was HLT which was tested on a group of eighth-grade students. Participants in this study were 25 eighth-grade students in a secondary school in Palembang, Indonesia. HLT is based on curriculum review and study of learning obstacles in the pyramid volume. HLT is then described as a didactical design that refers to the stages of learning in the Theory of Didactic Situations, namely: 1) action, 2) formulation, and 3) validation [6, 30] (Table 1).
Table 1. Hypothetical learning trajectory of pyramid volume.

| Learning Objectives | Student Activities | Mathematical Hypothesis |
|---------------------|-------------------|-------------------------|
| Prerequisite concept| 1. Recalling the concept of rank | Students understand the concept of prerequisite material |
|                     | 2. Recalling the concept of pythagoras | |
|                     | 3. Recalling the concept of area and volume units | |
|                     | 4. Recalling the concept of the pyramid | |
| Determine the pyramid volume formula | 1. Identify the volume of the cube | Students know that in one cube there are six layers |
| Calculate pyramid using the pyramid volume formula | 2. Looking for cube and pyramid relationships | |
| Prerequisite concept| 5. Recalling the concept of rank | Students understand the concept of prerequisite material |
|                     | 6. Recalling the concept of pythagoras | |
|                     | 7. Recalling the concept of area and volume units | |
|                     | 8. Recalling the concept of the pyramid | |

Table 1 is an HLT that is compiled based on the study of students’ learning obstacles to the volume of pyramid and is implemented in the classroom as a teaching experiment. The process in this study is a metaphorical process, a process to show the interaction between student teachers and material and their relationships with each other [18]. HLT serves as a guide for educators (teachers) and researchers in teaching activities, interviews, and observations. The trial results were examined based on the didactical situation theory as the basic framework for the preparation of this learning design.

3. Result and Discussion

3.1. Teaching Experiment

In the trials of the HLT, one of the researchers acts as a model teacher and the mathematics teacher observes the learning activities and then provides suggestions and conducts discussions with the researchers after the end of the lesson.

The first stage is to recall the concept of rank with the activities of students who are given is giving questions about the appointment using student activity sheets and supported by instructional media following the concept of rank to answer questions. by giving examples of rank numbers, such as 5, 7 and so on, students are expected to be able to recall the concept of rank. In the initial stage, most students are still confused in determining the basic numbers but already understand where the numbers are, but still confused when asked about numbers. then most students can make examples of numbers of numbers even though some children still cannot make examples of numbers of numbers. then there is the problem of completing a set of numbers partially able to solve as again students are still confused in completing it.

The next step is to remember the concept of Pythagoras. The student activity is the teacher giving questions related to the Pythagorean concept and supported by learning media in the form of a right triangle (Figure 1).
Figure 1 is a media in the form of a right triangle to remember the concept of Pythagoras. In this situation, most students already understand the location of right angles and the right symbols, and also students already know where the slashes are on the learning media provided. When asked what formula is suitable for finding the hypotenuse only one student answers. This is due to students not understanding the concept of Pythagoras because so far the concept of Pythagoras is only known through formulas. As didactic anticipation of the students' responses, the teacher reminded them about the concept of Pythagoras through the media of images and gave various problems related to right triangles.

The third stage is to remember the concept of area and volume units with student activities that are given giving questions about the unit area and volume using student activity sheets and students directly mention examples of unit and volume area, and it is hoped students can remember the concept of area and volume units. Most students have been able to distinguish units of area and volume and mention them but when answering questions sometimes students are still wrong in writing the unit and not even writing the units. At this stage, students begin to be enthusiastic in answering each question given.

Next, the fourth step is to recall the concept of a pyramid with student activity which is given giving questions about pyramid using student activity sheets and is supported by learning media with various forms of the pyramid from a triangular pyramid, rectangular pyramid, and pentagon pyramid (Figure 2). Thus students are expected to remember the concept of the pyramid.

Figure 2 is a learning media in the form of the pyramid that is used by students to identify the types of the pyramid. When asked the question from which can determine the types of the pyramid, only one student who answered correctly is the base of the pyramid. most students already know the position from the top, height and peak point on the pyramid, but almost all students do not know where the diagonal plane and the side plane of the pyramid are. To respond the obstacles such as this, the teacher asks students to identify what the differences are for each pyramid that is displayed, model the diagonal in the classroom as an example of building space, and what about the diagonal in the pyramid.

The fifth stage has a learning goal to find the formula for the volume of the pyramid with the activities given giving questions about the relationship of the cube with the pyramid so that the students find the formula for the volume of the pyramid from the volume formula of the cube and supported by cube-
shaped learning media consisting of six rectangular pyramid. It is expected that students can find the formula for the volume of the pyramid independently with the help of questions and learning media that have been prepared. Six pyramids that are half the height of the side of the cube are arranged so that they form a whole cube (Figure 3).

Figure 3. Find the volume of pyramid formula.

Figure 3 is a learning media in the form of six pyramids that can form a cube. When in class, only one group out of five groups know the cube volume formula, and the other groups just give up to find the pyramid volume formula, while students give up so the teacher gives the motivation to write the answers they know. Then the teacher and students discuss to get the formula for the volume of the pyramid. However, only one group out of five groups knew the formula for pyramid volume, and the other group gave up just like that to find the formula for pyramid volume. The teacher then provides activities and provides opportunities for students to determine the volume of the pyramid through its way. Next, through teaching aids, teachers and students discuss to get the formula for the volume of the pyramid. Through this activity, students are expected to be able to formulate that:

\[ 6 \times \text{pyramid volume} = \text{Cube volume} \]
\[ \text{Pyramid volume} = \frac{1}{6} \times \text{Cube volume} = \frac{1}{6} \times s \times s \times s \]

Because the height of the pyramid \( t \) is \( \frac{1}{2} \) the height of the cube \( s \), and then \( s = 2t \). So that
\[ \text{Pyramid volume} = \frac{1}{6} \times s \times s \times 2t = \frac{1}{3} \times s^2 \times t \]

The sixth or final stage has the goal of students being able to calculate the volume of the pyramid with the activities given giving questions about the volume of the pyramid using the activity sheet and it is hoped that students are able to calculate the volume of the pyramid using the formula for the volume of the pyramid they have found. At this stage students can use the formula that has been found. There are students’ answers in different ways but they are correct in answering questions. Students look very enthusiastic in answering questions and discussing the answers presented in front of the blackboard.

Based on the results of the final test, seen several changes that occur in students' understanding of the volume of the pyramid. This can be seen from 79.2% of students can write the formula for the volume of the pyramid, as well as calculate the volume of the pyramid correctly and write down the volume unit correctly. This result is better than before, where 89% of students know the pyramid volume formula but cannot use it because they do not understand the Pythagorean concept. Meanwhile, there are more than 70% of students who are wrong in the volume formula and determine the volume of the pyramid because they do not know the nature and characteristics of the pyramid. In the final test it was found that 83.3% were able to understand the concept of the pyramid, were able to complete the rank, and students were able to solve with the Pythagorean formula and correct in area units.

3.2. Retrospective Analysis

Based on the identification of learning obstacles, the lack of student knowledge about the concept of rank brings its impact when determining the volume of the pyramid. Therefore learning begins by recalling the concept of rank. This stage is an adaptation stage, where students begin to be conditioned to be ready to enter and develop certain concepts [31]. This stage bridges students' initial knowledge with didactic learning stages. This activity continues on the delivery of the Pythagorean concept and
unit of area and volume. This concept has been studied before at the elementary school level, but it leaves epistemological obstacles when dealing directly with problems related to right triangles. This adaptation process makes students ready to receive and build new knowledge [31, 32] through the acculturation process. This method is in line with the Theory Didactical Situation construction which consists of two types of processes, namely: adaptation through the understanding of a didactic and milieu situations, while acculturation through understanding didactical situations and didactical contracts [32].

The a-didactic situation starts when it enters the core material, the volume of the pyramid. This concept was built using pyramid-shaped media. The use of suitable media also plays a role in improving students' abilities [33]. This stage, the students are given six pyramids that can be arranged into a cube and students are asked to find the volume formula cube in their way. The teacher provides the media as a means for students to learn independently and there is no direct interaction between the teacher and students is one of the stages of scaffolding [34]. This is what is called the action stage. At this stage, students are not directly confronted with the construction of abstract concept algebra but are involved in a situation that can create an impetus for students to construct their knowledge. Learning trajectory based on existing textbooks uses the properties of algebraic operations in each step of its construction. Without being connected with concrete objects students are required to use algebraic properties in an abstract [13]. This is what is then anticipated by researchers by providing appropriate activities.

An individual develops from using natural thinking in using his logical thinking, which is related to mathematical reasoning, accompanied by the process of construction, rejection, and use of a method [6]. Various responses emerge as part of the activity of thinking. It aims to achieve new knowledge that comes from the ability to adapt to new situations. New stimuli and reactions to these conditions are evidence that the learning process has occurred [8]. The results of the analysis of the learning trajectory in existing textbooks obtained the ability to think needed in the construction of the volume of the pyramid in the book turned out to be higher than the ability possessed by students [13]. This shows that the volume of pyramid construction activities is not following the natural development of students. In other words, the pyramid volume construction process presented is not following the student learning trajectory. This was then revised by the researcher by presenting a learning trajectory that could construct students to develop their knowledge through prior knowledge, namely by constructing the volume of the pyramid through the concept of cube volume.

The situation of formulation and validation of the concept occur when students test the truth of the formulas they have obtained. The teacher gives a problem that can prove that the volume of pyramid is 1/6 of the volume of the cube and students with their creations try various ways to prove this. The existence of didactic intervention is a means of communicating explicitly for students and also as a means of learning to build logical thinking based on evidence [6]. Such a process can lead to validation actions, i.e. the teacher starts with what is known and ends with mathematical knowledge through the construction process [30].

The last stage is the giving of questions relating to the volume of the pyramid and related to daily life. The process of this situation gives the value of the truth of science learned in class, usually related to concepts, symbols and knowledge that may be used at different times and for purposes [26]. The situation of validation is an important stage because in this stage students apply the concepts that they get to solve a problem and is a stage that students can apply these concepts in a context. Learning trajectories that pay attention to this are often ignored. This can be seen from the questions given by most teaching materials and learning methods only emphasize the use of formulas [12, 24]. Learning trajectory developed previously directly on the concept of the pyramid, such as the elements of the pyramid, building space, pyramid volume construction, and various types of pyramid volumes [13]. Meanwhile, learning trajectory, which was designed by researchers, is based on a comprehensive identification of learning obstacles that starts from the concept of volume units, Pythagorean theorems, space constructions, and pyramid volumes. However, learning activities that occur are usually still dominated by teachers and the use of innovative learning media is still limited but, geometry problems require certain visualizations to solve problems and emphasize the optimal use of innovative learning media [27]. In the learning process that a teacher must be able to encourage students to be able to construct their knowledge through a didactic situation that suits the learning needs of students [25].
The final test has shown many changes that are better than the initial conditions of students that b students can understand the concept of the pyramid, know the elements of the pyramid, and can determine the volume of the pyramid appropriately. Even though there was a calculation error, students understood what strategy was used to solve the problem.

4. Conclusion
Learning Trajectory based on learning obstacle on the pyramid volume material has learning objectives: 1) remembering the concept of rank; 2) remember the concept of Pythagoras, 3) remembering the concept of area and volume units; 4) the nature and characteristics of the pyramid; 5) find the pyramid volume formula; 6) use the formula to determine the volume of the pyramid.

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