Searching for Inflationary B-modes: Can dust emission properties be extrapolated from 350 GHz to 150 GHz?

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ABSTRACT
Recent Planck results have shown that the path to isolating an inflationary B-mode signal in microwave polarization passes through understanding and modeling the interstellar dust polarized emission foreground, even in regions of the sky with the lowest level of dust emission. One of the most commonly used ways to remove the dust foreground is to extrapolate the polarized dust emission signal from frequencies where it dominates (e.g., \(\sim 350\) GHz) to frequencies commonly targeted by cosmic microwave background experiments (e.g., \(\sim 150\) GHz). We show, using a simple 2-cloud model, that if more than one cloud is present along the line-of-sight, with even mildly different temperature and dust column density, but severely misaligned magnetic field, then the 350 GHz polarized sky map is not predictive of that at 150 GHz. This problem is intrinsic to all microwave experiments and is due to information loss due to line-of-sight integration. However, it can be alleviated through interstellar medium tomography: a reconstruction of the dust column and magnetic fields at different distances, which could be achieved through the measurement of dust-absorption-induced polarization properties of starlight from stars at known distances in the optical and infrared.

Key words: cosmology: inflation – polarization – ISM: magnetic fields – ISM: dust – cosmic background radiation – cosmology: observations

1 INTRODUCTION
The recently claimed detection, at high confidence, of B-modes in cosmic microwave background (CMB) polarization that cannot be attributed to lensed E-modes by the BICEP2 experiment (Bicep2 Collaboration 2014) has been greeted with both enthusiasm and caution. If all or part of this B-mode signal is confirmed to be primordial, it would constitute the first detection of a smoking-gun for inflation, a direct probe of yet-unknown physics such as quantum gravity, and thus one of the most important discoveries in astrophysics, cosmology, and high-energy physics in the past several decades. Understanding possible foreground sources that could mimic such a signal has thus become a pressing priority.

Of the possible foreground sources of polarized emission in microwave frequencies, the most likely to cause false positives in the search of inflationary B-modes is emission from interstellar dust (e.g., Planck Collaboration 2014). There are two ways the CMB community is trying to minimize that risk.

The first involves selecting regions of the sky where dust emission, as recorded by past microwave observatories, is at its lowest (typically near the Galactic poles). The BICEP2 experiment focuses on such a “dust-emission hole” in the southern sky. However, the most recent analysis of Planck data has shown that even in such regions, dust emission can be responsible for a B-mode signal comparable to that detected by BICEP2 (Planck Collaboration 2014).

The second way of dust-contamination control involves trying to predict the polarization fraction and polarization direction of the dust emission at frequencies dominated by the CMB and targeted by CMB experiments (60 - 150 GHz), using sky maps at frequencies where dust emission is dominant (e.g., 350 GHz). The idea is that the dust emission spectrum is reasonably well-understood (typically represented by a modified blackbody spectrum); and the polarized emission measured at 350 GHz can be extrapolated to lower frequencies (Planck Collaboration 2014). Additionally, the polarization direction at 350 GHz is dictated by the magnetic field threading the interstellar clouds where the dust resides: if for example all emission originates from a single cloud, then the emission will be partially polarized in a direction perpendicular to the plane-of-the-sky (POS) projection of the magnetic field. The dust emission then at 150 GHz should be polarized in the same direction - since the polarization is generated by that same magnetic field. Cross-correlating...
then the polarization maps at 150 GHz and 350 GHz would reveal whether the 150 GHz signal is entirely due to dust or not.

The validity of such an extrapolation however breaks down whenever more than one “cloud” (more than one dusty region with different temperature and magnetic field) contribute to the integrated signal along the line of sight. The reason is that the total linearly polarized intensity fraction and polarization direction is obtained by algebraic addition of each of the Stokes linear polarization parameters $Q$, $U$, and $I$ along the line of sight. If the relative contribution from two regions with different polarization directions (effectively different magnetic field directions) changes between frequencies, so will the resulting polarization fraction and polarization direction. Indeed, the relative contribution will change between frequencies if the two regions have different temperatures, due to the temperature-dependent black-body part of the dust emission spectrum. Therefore unless all of the line-of-sight signal is due to dust at the same temperature, or due to dust residing in magnetic fields identically oriented on the POS, the resulting polarization direction will change between frequencies.

In this paper we use a simple two-cloud model to demonstrate the potential magnitude of this effect and caution against the extrapolation of polarized dust emission properties from 350 GHz to 150 GHz without proper evaluation of the number of potential contributors along the line of sight.

## 2 TWO-CLOUD MODEL

We consider a simple case where only two “clouds” of temperatures $T_1$ and $T_2$ and dust column densities $\Sigma_1$ and $\Sigma_2$ contribute to the total dust emission along a line of sight (see Fig. 1). The total intensity emitted by each cloud can be generally well described by a modified black-body spectrum \cite{Hildebrand1983},

$$I_\nu = B_\nu(T)\left[1 – \exp(-\kappa\Sigma)\right].$$  \hspace{1cm} (1)

where $B_\nu(T)$ is the blackbody spectral radiance and $\kappa$ is the opacity. For simplicity, we will assume that the opacity as a function of frequency, $\kappa(\nu)$ is the same in the two clouds. Finally, we will assume that the clouds are partially linearly polarized, i.e. that the Stokes parameter $V$ is in both cases equal to zero.

In the case of low optical depth $\kappa\Sigma$ (appropriate for regions of low dust emission, such as the dust-emission “holes” typically targeted by CMB experiments), the ratio $r$ of the total intensities contributed by each cloud can be written as

$$r(\nu) = \frac{I_1}{I_2} = \frac{B_\nu(T_1)\Sigma_1}{B_\nu(T_2)\Sigma_2}. \hspace{1cm} (2)$$

The ratio $r$ will be a function of frequency because of the blackbody part of the emission spectrum.

The polarization state of the dust emission from each cloud can be completely described by the Stokes parameters $Q$, $U$, and $I$ (where $I$ is the total intensity). The polarization degree $p$ and polarization angle $\chi$ of the emission from a single cloud are related to the Stokes parameters through

$$p = \frac{\sqrt{Q^2 + U^2}}{I}, \hspace{0.5cm} \tan 2\chi = \frac{U}{Q}. \hspace{1cm} (3)$$

When both clouds contribute to the emission along a single line of sight, the corresponding Stokes parameters add:

$$Q_{\text{tot}} = Q_1 + Q_2, \hspace{0.5cm} U_{\text{tot}} = U_1 + U_2, \hspace{0.5cm} I_{\text{tot}} = I_1 + I_2 = (1 + r)I_2. \hspace{1cm} (4)$$

The polarization degree $p_{\text{tot}}$ and direction $\chi_{\text{tot}}$ of the total emission then are obtained by supplying $Q_{\text{tot}}$, $U_{\text{tot}}$, and $I_{\text{tot}}$ to Eq. (3).

We will consider two specific cases of relative cloud magnetic field directions to illustrate in an intuitive fashion how the frequency-dependent ratio $r$ enters the expressions of $p_{\text{tot}}$ and $\chi_{\text{tot}}$, and we will then treat the general case.

### 2.1 Magnetic fields parallel to each other

The simplest and least problematic case is the one where the POS magnetic field, and thus the polarization direction, are parallel in the two clouds. Then

$$\frac{U_1}{Q_1} = \frac{U_2}{Q_2} = \tan 2\chi_0 \hspace{1cm} (5)$$

and, assuming $Q_1$ and $Q_2$ are positive,

$$p_1 = \frac{Q_1\sqrt{1 + \tan^2 2\chi_0}}{rI_2}, \hspace{0.5cm} p_2 = \frac{Q_2\sqrt{1 + \tan^2 2\chi_0}}{I_2} \hspace{1cm} (6)$$

The parameters for the total emission then are

$$Q_{\text{tot}} = Q_1 + Q_2, \hspace{0.5cm} U_{\text{tot}} = \tan 2\chi_0(Q_1 + Q_2), \hspace{0.5cm} I_{\text{tot}} = (r + 1)I_2, \hspace{1cm} (7)$$

which yields

$$\tan 2\chi_{\text{tot}} = \tan 2\chi_0, \hspace{0.5cm} p_{\text{tot}} = \frac{r p_1 + p_2}{r + 1} \hspace{1cm} (8)$$

which simplifies to $p_{\text{tot}} = p_1$ if $p_1 = p_2$. If the polarization properties of both clouds are the same, the resulting combined emission will also share these properties.
2.2 Magnetic fields perpendicular to each other

Conversely, the most worrisome case is the one where the POS-projection of the magnetic field in cloud 1 lies in a direction perpendicular to that in cloud 2. The polarization directions will similarly form a 90° angle. Without loss of generality, we can take the magnetic field of cloud 1 to be at an angle $\chi_1 \rightarrow 45^\circ$ from the reference direction, and that of cloud 2 to be at an angle $\chi_2 \rightarrow -45^\circ$. Then $\tan \chi_1 \rightarrow \infty$ and $\tan \chi_2 \rightarrow -\infty$. We can therefore represent this configuration with Stokes parameters

$$U_1 \approx p_1 I_1 = p_1 r I_2, \quad U_2 \approx -p_2 I_2, \quad Q_1, Q_2 \ll U_1, U_2 \quad (9)$$

(where we have taken the Qs to be very small compared to both $U_1$ and $U_2$ but finite to avoid dealing with infinities). For the total signal then we have

$$Q_{\text{tot}} = Q_1 + Q_2, \quad U_{\text{tot}} = I_2 (p_1 r - p_2), \quad I_{\text{tot}} = (1 + r) I_2. \quad (10)$$

If we further sacrifice some more generality for the sake of clarity and consider the degree of polarization of the thermal dust emission, $p$, to be equal in the two clouds and equal to $p_0$, and the Stokes parameters $Q$ in the two clouds to be equal and equal to $Q_0$, we obtain

$$Q_{\text{tot}} = 2Q_0, \quad U_{\text{tot}} = I_2 p_0 (r - 1), \quad I_{\text{tot}} = (1 + r) I_2, \quad (11)$$

and

$$\tan 2\chi_{\text{tot}} = \frac{2p_0 (r - 1)}{2Q_0}, \quad p_{\text{tot}} = p_0 \frac{\sqrt{(r - 1)^2 + 4Q_0^2 / I_2^2 p_0^2}}{r + 1} \quad (12)$$

As long as $r - 1 \gg 2Q_0 / I_2 p_0$, then $p_{\text{tot}} \approx p |r - 1| / (r + 1)$. The polarization angle $\tan 2\chi_{\text{tot}} \rightarrow \infty$ and $\chi \rightarrow 45^\circ$ if $r > 1$ (cloud 1 dominates); conversely, $\tan 2\chi_{\text{tot}} \rightarrow -\infty$ and $\chi \rightarrow -45^\circ$ if $r < 1$ (cloud 2 dominates). If $r - 1$ retains the same sign between two frequencies, the resulting polarization angles will be the same. If $r - 1$ changes sign, the resulting polarization angles will abruptly change from one frequency to the other by 90°.

2.3 Magnetic fields at some angle $\alpha$

Finally, we consider the general case where the POS-projection of the magnetic field in cloud 1 lies in a direction forming an angle $\alpha$ with that of cloud 2. The polarization directions will similarly form an angle $\alpha$. Without loss of generality, we can take the magnetic field of cloud 1 to be at an angle $\chi_1 = \alpha / 2$ from the reference direction, and that of cloud 2 to be at an angle $\chi_2 = -\alpha / 2$. Then $\tan 2\chi_1 = \tan \alpha$, $\tan 2\chi_2 = -\tan \alpha$, and $p_1 = |Q_1| / \sqrt{1 + \tan^2 \alpha / I_1}$ in both cases ($i = 1, 2$). We additionally make the same simplification $p_1 = p_2 = p_0$ as above. We can therefore represent this configuration with Stokes parameters

$$I_1 = r I_2, \quad Q_1 = \frac{p r I_2}{\sqrt{1 + \tan^2 \alpha}}, \quad U_1 = \frac{\tan \alpha p r I_2}{\sqrt{1 + \tan^2 \alpha}} \quad (13)$$

and

$$I_2, \quad Q_2 = \frac{p I_2}{\sqrt{1 + \tan^2 \alpha}}, \quad U_2 = \frac{\tan \alpha p I_2}{\sqrt{1 + \tan^2 \alpha}} \quad (14)$$

where we have taken $Q_1, Q_2$ to be positive and $U_1, U_2$ carry the signs of the problem. The Stokes parameters of the total signal will then be

$$Q_{\text{tot}} = \frac{p_0 I_2 (r + 1)}{\sqrt{1 + \tan^2 \alpha}}, \quad U_{\text{tot}} = \frac{\tan \alpha p_0 I_2 (r - 1)}{\sqrt{1 + \tan^2 \alpha}}, \quad I_{\text{tot}} = (1 + r) I_2. \quad (15)$$

The resulting polarization parameters will thus be

$$\tan 2\chi_{\text{tot}} = \frac{\tan \alpha (r - 1)}{(r + 1)}, \quad p_{\text{tot}} = p_0 \frac{1 + \tan^2 \alpha \left(\frac{1}{r + 1}\right)^2}{1 + \tan^2 \alpha} \quad (16)$$

3 RESULTS

In this section we use our simple model to estimate the potential change, between 150 and 350 GHz, of the observed polarization properties of the combined emission from the two clouds. The polarization fraction $p_{\text{tot}}$ and polarization angle $\chi_{\text{tot}}$ of the combined emission depend on the values of the polarization properties of the individual clouds, $p_1$, and $\alpha$, as well as the fraction between the total intensities from each cloud $r = I_1 / I_2$. The latter in turn depends on the temperature of the two clouds and the ratio of the dust column densities, $\Sigma_1 / \Sigma_2$.

The ratio of the two r-values at the two frequencies, $r_{150GHz/350GHz}$, only depends on the temperatures of the two clouds. Using Eq. (2) we obtain

$$\frac{r_{150GHz}}{r_{350GHz}} = \frac{B_{150GHz}(T_1)}{B_{350GHz}(T_2)} \frac{B_{350GHz}(T_2)}{B_{150GHz}(T_2)} \quad (17)$$

Only the blackbody functions enter in this ratio. For temperatures relevant for interstellar dust, both 150 GHz and 350 GHz are below the frequency where $B_\nu$ peaks, and the ratio $B_{150GHz}(T) / B_{350GHz}(T)$ will be larger for the hotter cloud, so $r_{150GHz} / r_{350GHz}$ will be greater than 1 when cloud 2 is hotter. If both clouds get too hot (and both 150 GHz and 350 GHz are well into the Rayleigh-Jeans regime), then $B_\nu(T_1) / B_\nu(T_2)$ approaches independence of frequency and $r_{150GHz} / r_{350GHz}$ approaches 1. However, this is not typical for temperatures relevant for interstellar dust. These effects can be seen in Fig. 2 (upper panel) where we plot $r_{150GHz} / r_{350GHz}$ as a function of $T_2$ for three values of $T_1$ ($10 \text{ K}, 20 \text{ K}, \text{ and } 50 \text{ K}$).

The quantities of interest however are: the ratio of polarization fractions of the combined emission at the two frequencies, $p_{150GHz} / p_{350GHz}$; and the difference in polarization angles of the combined emission at the two frequencies, $\chi_{150GHz} - \chi_{350GHz}$. These do not have a simple dependence on the ratio of $r$ values. Instead, both $r_{150GHz}$ and $r_{350GHz}$ enter the calculation in a non-trivial fashion that preserves the dependence of the problem on the ratio of column densities, $\Sigma_1 / \Sigma_2$. To illustrate how the values of $r$ at the two frequencies depend both on temperature and the ratio of column densities we plot, in Fig. 2 (lower panel), $r_{150GHz}$ and $r_{350GHz}$ as functions of $T_2$ (for $T_1 = 20 \text{ K}$), for eight different values of $\Sigma_1 / \Sigma_2$ (0.5, 0.7, 1, 1.3, 1.5, 2, 3, 4).

The way the degree of polarization measured for the combined signal changes between 350 GHz and 150 GHz is shown in Fig. 3 where the ratio $p_{150GHz} / p_{350GHz}$ is plotted against the temperature of cloud 2. The temperature of cloud 1 is fixed at the average interstellar dust value of 20
Figure 2. Upper panel: ratio of \( r \) values at 350 GHz and 150 GHz as a function of cloud 2 temperature, for three different cloud 1 temperatures. Lower panel: Ratio \( r = I_1/I_2 \) at 150 GHz (black) and 350 GHz (red), as a function of cloud 2 temperature. Cloud 1 temperature is fixed at 20 K. Line type corresponds to different values of \( \Sigma_1/\Sigma_2 \): dot-dash: 0.5; dash: 0.7; dot: 1; solid: 1.3; thick dot-dash: 1.5; thick dashed: 2; thick dot: 3; thick solid: 4.

Figure 3. Ratio of polarization fractions of combined signal as measured at 350 GHz and 150 GHz as functions of cloud 2 temperature. Color corresponds to angle \( \alpha \) between the POS magnetic fields of the clouds: black: 90°; blue: 85°; red: 80°; green: 75°; magenta: 60°. Line types as in Fig. 2 (lower).

Figure 4. Difference of polarization angles of combined signal as measured at 350 GHz and 150 GHz, as a function of cloud 2 temperature. Line types and colors as in Fig. 3.

[Planck Collaboration 2014b]. Different line types correspond to different values of \( \Sigma_1/\Sigma_2 \) as in Fig. 2 (lower), while different colors correspond to different angles between the POS magnetic fields of the two clouds. We show results for \( \alpha \) of 60, 75, 80, 85 and 89 degrees. Similarly, in Fig. 4 we explore the resulting difference of the polarization angle of the combined signal as measured between 150 GHz and 350 GHz. Here, the difference \( |\chi_{\text{tot},150}\text{GHz} - \chi_{\text{tot},350}\text{GHz}| \) is plotted against \( T_2 \) for the same parameter values as in Fig. 3.

Except when the temperatures of the two clouds are the same, severely misaligned cloud magnetic fields will result in significant differences in polarization degree and polarization angle between 150 GHz and 350 GHz for a range of possible parameters of our model. Caution is then warranted when using 350 GHz data to extrapolate dust properties to 150 GHz, as the 150 GHz polarized emission map might look very different from the 350 GHz map. The polarization degree might differ by large factors (more than a factor of 2 for the most misaligned magnetic fields, more than 20% if the POS magnetic field of the two clouds forms an angle higher than 75°). Similarly, the polarization angle can be up to 90° different if the clouds are perfectly misaligned, and certainly higher than 20 degrees for clouds with severely but not perfectly misaligned POS magnetic fields (~ 85°).

On the other hand, there are large parts of the parameter space where the polarization properties of the combined emission would look very similar at the two frequencies: if the POS magnetic fields of the two clouds form an angle of less than 60°, then the difference in polarization fraction is less than 10% and the difference in polarization angle less than 5°, independently of the other parameters of our model.
Extrapolate Dust Polarization from 350 to 150 GHz?

4 DISCUSSION

We have used a simple 2-cloud model to examine whether extrapolating the polarized dust emission signal from frequencies where it dominates (e.g., 350 GHz) to frequencies commonly targeted by CMB experiments (e.g., 150 GHz) yields robust results if more than one emitting region, with different temperature and differently oriented magnetic field, contributes along a given line of sight. We have shown that the magnetic fields are severely misaligned (angle $\theta \geq 75^\circ$), then the 350 GHz polarized sky map is not predictive of that at 150 GHz. The origin of this effect can be traced to the different fractional contributions between the two clouds at different frequencies, which then affect the polarization properties of the combined signal.

Our goal is to demonstrate this potential systematic uncertainty when using 350 GHz measurements as a proxy for dust emission at 150 GHz. For this reason our model is very basic and can be improved in a variety of ways: e.g., more than two contributing clouds along the line of sight, a physical model of dust properties and emission, and constraining the observed combined emission spectrum along a line of sight to agree within uncertainties with the observed one.

In our model, the degree of polarization and the frequency dependence of the dust opacity are identical in the two clouds. Any difference in these quantities will cause further discrepancies between the maps in the two frequencies. This is easily demonstrated in the case of parallel magnetic fields in the two clouds. Under the assumption $p_1 = p_2 = p_0$, the resulting polarization fraction $p_{\text{rot}}$ is equal to $p_0$ and constant at all frequencies. However, if $p_1 \neq p_2$, $p_{\text{rot}}$ does depend on $\tau$ and, through it, on frequency. If additionally the dust properties and the resulting opacity law differ even slightly, this would translate to a change in $\tau$ between frequencies compounded to that predicted from the black-body law. Thus, the differences we have presented between frequencies in $p_{\text{rot}}$ and $\chi_{\text{tot}}$ are lower limits to the true effect.

There is currently no evidence suggesting that this effect would not be of importance in areas of the sky targeted by CMB polarization experiments which are characterized by low dust emission. Even in these cleanest areas of the sky, dust emission is “patchy” on the plane of the sky (Planck Collaboration 2014), suggesting by copernican arguments that it is also patchy along the line of sight, with several distinct clouds, each with its own, potentially different, magnetic field. The existing (albeit sparse) stellar optical-polarimetry at high Galactic latitudes (Berdyugin et al. 2014), which also traces the magnetic field direction in which absorbing interstellar dust is embedded (Hildebrand 1993), shows strong variations in polarization angle between relatively nearby regions, suggesting that the same would be true along the line of sight.

The problem we have discussed here is intrinsic to all microwave experiments and is due to information loss due to line-of-sight integration. Future experiments with greater sensitivity/angular resolution will not avoid this systematic uncertainty. However, if the magnetic fields of the contributing clouds form an angle smaller than 60$^\circ$, then the resulting change in the polarization properties maps from 350 GHz to 150 GHz is minimal (at least under the assumptions of the simple model considered here). If therefore information existed on the number of the dust-emission contributions along the line of sight and the direction of the POS magnetic field in each contributing “cloud”, the magnitude of this effect could be accurately estimated. As a first step, lines of sight towards multiple contributors with severely misaligned magnetic fields could be masked from CMB polarization analyses. With better models and more careful analysis, the effect could even be corrected for.

Information on the magnetic field direction in individual clouds along the line of sight cannot be obtained through microwave experiments. There is, however, an alternative way to trace dust abundance and magnetic field, that does allow for interstellar medium tomography: the study of the polarization of starlight induced by dichroic dust absorption. In a single cloud, polarization of dust emission and dust-absorption-induced polarization of the light from background stars are complementary effects. Both are due to the alignment of grains with the magnetic field in the cloud, and, as a result, trace the same magnetic field. The additional information that can be provided by optical/infrared polarimetry is that of distance: each star’s polarization is only affected by dust between itself and the observer. With enough stars spread out to a variety of distances, the 3-dimensional structure of the intervening interstellar medium could in principle be reconstructed. In the era of Gaia (e.g., Bailer-Jones et al. 2013) which will provide distance measurements to stars down to 20th magnitude, the bottleneck in such an endeavor will be the number of high-accuracy topopolarimetric measurements that can be performed down to very low polarization fractions characteristic of the regions targeted by CMB experiments.

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REFERENCES

Bailer-Jones et al., 2013, A&A, 559, A74
Berdyugin, A., Piirola, V., & Teerikorpi, P. 2014, A&A, 561, A24
Bicep2 Collaboration. 2014, Physical Review Letters, 112, 241101
Hildebrand, R. H. 1983, QJRAS, 24, 267
Hildebrand, R. H. 1999, in Astrophysics and Space Science Library, Vol. 241, Millimeter-Wave Astronomy: Molecular Chemistry & Physics in Space., ed. W. F. Wall, A. Carramiñana, & L. Carrasco, 115
Planck Collaboration. 2014a, A&A, 566, A55
—. 2014b, ArXiv e-prints 1405.0874
—. 2014c, ArXiv e-prints 1409.5738