Crack front shapes and stress intensity factors in plates under a pure bending loading that induces partial closure of the crack faces

Silvia Corbani\textsuperscript{a}, Luiz F. Martha\textsuperscript{a,*}, Jaime T. de Castro\textsuperscript{a}, Bruce Carter\textsuperscript{b}, Anthony Ingraffea\textsuperscript{b}

\textsuperscript{a}Pontifical Catholic University of Rio de Janeiro, Rua Marquês de São Vicente 225, Rio de Janeiro 22453-900, Brazil
\textsuperscript{b}Cornell Fracture Group, Cornell University, 041 Rhodes Hall, Ithaca 14853, U.S.A.

Abstract

This paper investigates, both experimentally and numerically, stress intensity factors and evolving geometry of through edge cracks in plates loaded under pure transverse bending; the loading induces partial closure of the crack faces. After performing experiments on steel specimens, it was found that a kink in the crack front forms as the crack grows, changing the initial straight front into an L-shape. To understand the distribution of stress intensity factors along such crack fronts, the observed crack shapes were reproduced in a three-dimensional fracture code (FRANC3D) coupled with a finite element analysis program (ABAQUS). With this coupled system, linear elastic stress analyses were performed considering the nonlinear effects caused by the crack face contact in the compressed region. The proposed methodology computes stress intensity factors along the crack front based on observed crack front shapes obtained from the experimental tests. A method is proposed to treat numerical noise around the crack front kink and at crack front end. In addition, normalized empirical expressions for stress intensity factors are proposed.

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1. Introduction

The propagation of fatigue cracks is a major problem in the aeronautical, naval, and automotive industry. Due to the complexity of the structures and loading, the complete treatment of the problem usually involves a number of...
simplifications, which might include simplifications to geometry (structural model), material behavior, and/or the adopted loading. In the fatigue design of structures, it is common practice to simplify the geometry and type of loading to match a model that has been studied previously, and therefore, is described in manuals. However, even when adopting such simplifications, some problems still have no known solution and need to be investigated. An example is the propagation of fatigue cracks whose faces suffer partial contact, which can occur in pure bending (no applied tension) of cracked plates subjected to cyclic loading (see Fig. 1).

![Fig. 1. Fatigue crack propagation under pure bending loads that induce partial closure of the crack faces.](image)

This paper studies the fatigue propagation of through cracks in plates subjected to pure bending, considering the physical contact between the crack faces. The bending loads induce both tension and compression regions along the crack front. The portion of the crack faces that are under compression are closed by the loading, independent of any other closure mechanism. For the portion of the crack front that is under tension, the crack grows due to the cyclic bending loading where the stress intensity factor (SIF) is higher than the fatigue threshold; however, the front does not grow where the SIF is under compression. The crack face contact leads to a nonlinear problem behavior, even when considering a simplified linear-elastic material behavior.

The main objective here is to describe the stress intensity factor and geometry changes of a crack front that is subjected to pure bending with contact. In the computer simulations, the yielded regions near the crack front are considered to be small (LEFM), and it is assumed that the crack growth occurs only in mode I.

Surface cracks have been widely studied under tensile nominal stress. Local stress intensity factors for them can be estimated from classical solutions of Newman-Raju (1979, 1981). However, these solutions do not apply to the problem investigated in this paper. This problem can only be analyzed numerically, e.g. using three-dimensional finite element models to calculate the SIF distribution along the front of these cracks.

To know the exact geometry of the crack and the effects on the SIF distribution is not a simple task. Young and Sun (1992) studied the influence of contact on the magnitude of the SIF for a through crack under bending. It was found that the closure at the compressive edge of the through crack tends to reduce the crack opening displacement at the tension side, decreasing the stress intensity factors. However, these results were not verified experimentally. In addition, they do not consider partial closure of crack faces. There are only a few experimental studies of plates under pure bending loading that induce partial closure of the crack faces (Wynn and Smith, 1969; Smith and Smith, 1970; Mullinix and Smith, 1974). These authors investigated local stresses in the neighborhood of a crack front, but crack front geometry marks were not presented.

2. Crack shape and stress intensity factor

2.1. Statement of problem and numerical analysis

The numerical simulations were performed using FRANC3D (Wawrzynek et al., 2009), which is specifically designed to simulate 3D crack propagation, and the finite element solver ABAQUS (Hibbitt et al., 1996). Comparisons are made with fatigue tests conducted on three single-edge-notch bending specimens. The dimensions of the specimens are depicted in Fig. 2: length between anvils = 515 mm, width w = 101.4 mm, thickness t = 8.8 mm, and notch a* = 9.95 mm. Results are presented for three different cyclic loads as shown in Fig. 3. The range of applied load ΔP induces bending; consequently, the applied stress range Δσ varies along the thickness. A linear elastic material behavior is used for the numerical simulations, with properties compatible with ASTM A36 steel. The adopted Young’s modulus E and the Poisson’s ratio values are 211 GPa and 0.3, respectively.
2.2. Crack shape from experiments

The initial through crack shapes for the bending tests were obtained from pure fatigue tensile tests with no crack face contact (fatigue pre-crack tests). The notched specimens were pre-cracked at \( K_{I} = 12 \text{ MPa.m}^{1/2}. \) The lengths \( (a) \) of the notch plus pre-crack extension are: 11.89, 13.63 and 14.77 mm for the specimens, SP01, SP02 and SP03, respectively. The bending tests were carried out with the load history shown in Fig. 3.

The crack front geometries form an L-shape, as shown in Fig. 4 and Fig. 5. Empirical non-concentric elliptical expressions combined with straight lines were created to represent the obtained geometries of the crack fronts. These geometries are input into FRANC3D where appropriate finite element meshes are created for ABAQUS; the ABAQUS results are used by FRANC3D to compute SIFs along the crack fronts.

2.3. The numerical results of stress intensity distribution

Figure 6 shows a typical finite element mesh on the crack plane. This figure depicts the distribution of normal stress component in the direction perpendicular to the crack face. It was observed that the kink point between the straight (initial) and curved part of the crack front introduces a numerical singularity in the SIF distribution. For this reason, the crack front is smoothed at this point using a rounding radius. Fig. 7a shows the influence of the rounding radius on the SIF versus normalized crack front length curves, for mark 03b in specimen SP03. Position 0.0 on the x-axis corresponds to the upper surface of the plate (where stress is compressive), and position 1.0 is on the bottom surface of the plate (where stress is tensile). The kink position is at approximately 0.2 of the normalized crack front. Analyzing these results, the best rounding radius value is 1 mm.

As the numerical simulations produce different crack propagation increments compared to the experiments, a methodology to adapt the numerical prediction of SIF is proposed. This methodology eliminates the numerical singularities, arising from the kink and the end of the crack front, in the distribution of the SIF range \( \Delta K_{J} \). Fig. 7b defines three regions at the crack front: \( CP, PQ, QA \). The \( \Delta K_{J} \) values along region \( PQ \) are not influenced by numerical singularities. This is called the trusted region. The \( \Delta K_{J} \) value at point A is neglected with the assumption that the crack grows parallel with the tensile stress gradient. In regions \( QA \) and \( CP \), the \( \Delta K_{J} \) results are extrapolated.

In summary, the procedure neglects regions with singularities and the following steps are adopted: (i) define the trusted region with normalized \( \Delta K_{J} \); (ii) the normalized \( \Delta K_{J} \) is extrapolated to points A and C using a polynomial curve. If the crack is long, the normalized \( \Delta K_{J} \) is assumed constant until the kink.
Fig. 4. Crack marks: (a) SP01; (b) SP02; (c) SP03.

Fig. 5. Crack front used in numerical simulation (dimensions in millimeter): (a) SP01; (b) SP02; and (c) SP03.

Fig. 6. Mesh and normal stress distribution on crack front of mark 03b in specimen SP03.

Fig. 7. SIF distribution along crack front of mark 03b in specimen SP03.
The several $\Delta K_I$ distributions are normalized with

$$F = \frac{\Delta K_I}{\Delta \sigma \sqrt{\pi a}}$$

(1)

where $\Delta \sigma$ is the maximum remote stress due to bending loads and $a$ is the crack length in the depth direction. $F$ is the geometry factor and $\Delta K_I$ is the SIF range. The distribution of normalized $\Delta K_I$ is shown in Fig. 8, Fig. 9 and Fig. 10 for the three specimens for the trusted and extrapolated regions.
3. Empirical stress intensity equations

The results of normalized $\Delta K_I$ can be fitted by empirical equations for the stress intensity factor at points $C$ and $A$, called $\Delta K_{I,c}$ and $\Delta K_{I,a}$, respectively:

$$\Delta K_{I,c} = \Delta \sigma \sqrt{\pi a} f(d/w)$$

where $f(d/w) = 137.45 \left(\frac{d}{w}\right)^4 - 192.27 \left(\frac{d}{w}\right)^3 + 97.99 \left(\frac{d}{w}\right)^2 - 21.50 \left(\frac{d}{w}\right) + 2.18$. And

$$\Delta K_{I,a} = \Delta \sigma \sqrt{\pi a} \left(-0.100 \left(\frac{a}{t}\right) + 0.878\right),$$

for $a/t > 0.4$.

4. Concluding remarks

It was observed experimentally that a through crack in a plate subjected to pure bending loads (that induce partial closure of the crack faces) never grows in the region under compressive stresses. In addition, the crack propagates on a plane normal to the tensile bending stresses. The stress intensity factors along the crack front have been investigated using 3D finite element models. Two SIF singularity regions on the crack front were identified: a kink point between the initial crack and the propagated crack and at the end point on the bottom surface. A methodology to eliminate these singularities in the stress intensity factor distribution was proposed. Normalized empirical expressions for stress intensity factors were proposed.

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