Permeability Estimation of Rock Reservoir Based on PCA and Elman Neural Networks

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Abstract: An intelligent method which based on fuzzy neural networks with PCA algorithm, is proposed to estimate the permeability of rock reservoir. First, the dimensionality reduction process is utilized for these parameters by principal component analysis method. Further, the mapping relationship between rock slice characteristic parameters and permeability had been found through fuzzy neural networks. The estimation validity and reliability for this method were tested with practical data from Yan’an region in Ordos Basin. The result showed that the average relative errors of permeability estimation for this method is 6.25%, and this method had the better convergence speed and more accuracy than other. Therefore, by using the cheap rock slice related information, the permeability of rock reservoir can be estimated efficiently and accurately, and it is of high reliability, practicability and application prospect.

1. Introduction

Reservoir permeability includes porosity in rock parameters, and there are mainly two kinds of determination methods: direct determination method and indirect determination method in laboratory. Indirect measurement method is affected by collection error and other factors, the error is greater than[1]. In recent years, many scholars have studied the permeability measurement of rock reservoirs, in particular, the use of more advanced artificial neural networks and other methods. Fuzzy control and artificial neural network belong to the artificial intelligence technology, each of them has the advantage that the other party does not have, and has complementary nature. The specialty of fuzzy control is the ability of logical reasoning, which can simulate the ability of human judgment and decision. Artificial neural networks, have the ability of nonlinear mapping, learning ability, parallel processing and fault tolerance. Therefore, fuzzy neural network (FNN) combined with fuzzy theory and artificial neural network can improve the defect of neural network and broaden the range and ability of neural network to process information[2]. So far, this paper presents an intelligent calculation of permeability of rock reservoir based on PCA and fuzzy neural network. Through the collected rock thin section identification report[3] of Yanchang Formation reservoir in Yanan area, Ordos basin, Feature screening was performed. As a result of the thin sections of rock, the characteristic parameters of each sample is as many as dozens, principal component analysis (PCA) dimension reduction method is introduced to reduce the feature parameters of rock slice effectively, then, the fuzzy neural network is used to calculate the permeability of rock reservoir intelligently. Finally, the results are compared with those obtained by the direct determination method in laboratory.[4, 5].
2. Fuzzy Neural Network

Fuzzy neural network is based on the advantages of fuzzy theory and neural network, and its theoretical basis is the learning mechanism of fuzzy mathematics and neural network. Fuzzy mathematics is used to describe, study and deal with things with fuzzy characteristics of mathematics, the basic concepts are membership degree and fuzzy membership function. Among them, membership degree refers to the degree of membership of element u belonging to fuzzy subset f, it is represented by a \( \mu_f(u) \). It is a number between \([0,1]\). The \( \mu_f(u) \) is closer to 0, the smaller the extent that u belongs to the fuzzy subset f; the closer to 1, the greater the degree of u belonging to f. Fuzzy membership function is used to quantitatively calculate the membership function of the elements, generally including trigonometric functions, trapezoidal functions and normal functions, such as \( ^{[2]} \).

2.1 T-S fuzzy model

T-S fuzzy system is a kind of fuzzy system with strong adaptive ability. The model can not only update automatically, but also modify the membership function of fuzzy subset constantly. The T-S fuzzy system is defined by the following “if-then” rule form. In the case of rule Ri, the fuzzy inference is as follows:

\[
R_i: \text{if } x_1 \text{ is } A_{1i}, x_2 \text{ is } A_{2i}, \ldots, x_k \text{ is } A_{ki}, \text{ then } y_i = p_{0i}^j + p_{1i}^j x_1 + \ldots + p_{ki}^j x_k
\]

Among them, \( A_j \) is fuzzy set of fuzzy system; \( P_I \) is fuzzy system parameter; \( Y_i \) is output according to fuzzy rule. The input part (i.e., the if part) is fuzzy, and the output part (i.e., the then part) is determined. The fuzzy inference represents the output as the linear combination of the input. For input vectors \( x = [x_1, x_2, \ldots, x_k] \), Firstly, the membership degree of each input vector \( X_i \) is calculated according to fuzzy rules:

\[
\mu_{X_i} = \exp(-(x_i - c_j^i)^2 / b_j^i)
\]  \hspace{1cm} (1)

Type medium, \( j = 1, 2, \ldots, k \), \( i = 1, 2, \ldots, n \), \( c_j^i \) and \( b_j^i \) respectively for the center and width of the membership function; \( K \) as input parameters for the \( N \) number; fuzzy subset number. The membership degree of fuzzy calculation, using fuzzy operator for Liancheng operator:

\[
\omega^j = u_{A_{j_1}}(x_1) \ast u_{A_{j_2}}(x_2) \ast \ldots \ast u_{A_{j_k}}(x_k)
\]  \hspace{1cm} (2)

Among which, \( i = 1, 2, \ldots, n \). The output value of the fuzzy model is calculated according to the fuzzy calculation results:

\[
y_i = \sum_{j=1}^{n} \omega^j (p_{0i}^j + p_{1i}^j x_1 + \ldots + p_{ki}^j x_k) / \sum_{j=1}^{n} \omega^j
\]  \hspace{1cm} (3)

2.2 T-S Fuzzy Neural Network

![Figure 1](image-url)

Figure 1 Topological Structure of T-S Fuzzy Neural Network

T-S fuzzy neural network is divided into four layers: input layer, fuzzification layer, fuzzy rule calculation layer and output layer, as shown in Figure 2. The input layer is connected to the input vector \( x \), and the number of nodes is the same as the dimension of the input vector. The fuzzy layer adopts the membership function formula (1) to fuzzification the input value, and the fuzzy membership value is obtained. Fuzzy rules with fuzzy multiplication formula calculation layer (2)
are obtained. The output layer Omega by formula (3) to calculate the output value of the fuzzy neural network.

The learning algorithm of FNN is as follows:

1. Error calculation

\[ e = \frac{1}{2} (y_d - y_c)^2 \]  

In the formula, \( y_d \) is the expected output of the network; \( y_c \) is the actual output of the network; \( e \) is the error between the expected output and the actual output.

2. Coefficient correction

\[ p'_j(k) = p'_j(k-1) - \alpha \frac{\partial e}{\partial p'_j} \]  

\[ \frac{\partial e}{\partial p'_j} = (y_d - y_c)w_j / \sum_{i=1}^{m} w_j \cdot x_j \]

In the formula, \( p'_j \) is the neural network coefficient; the alpha is the network learning rate; the \( x_j \) is the network input parameter; the \( w_j \) is the continuous product of the membership degree of the input parameter.

3. Parameter modification

\[ c'_j(k) = c'_j(k-1) - \beta \frac{\partial e}{\partial c'_j} \]  

\[ b'_j(k) = b'_j(k-1) - \beta \frac{\partial e}{\partial b'_j} \]

In the formula, \( c'_j \) and \( b'_j \) are the center and width of the membership function respectively.

3. Experimental Results

The experiment first uses the PCA to reduce the dimension of the characteristic parameters, and then uses the fuzzy neural network to train and calculate the data after dimensionality reduction. Finally, the experimental results are analyzed and compared.

3.1 Data Reduction

The experiment uses 41 reservoir rock samples of Yanchang group from the Yanan area were tested, as mentioned above, each sample data were reported in 13 slice identification parameters and permeability, the 13 character parameters of the fuzzy neural network is trained, for calculating the corresponding permeability.

In order to prevent a single variable from influencing other variables and to obtain a comparable range of values, linear transformation of the 13 characteristic parameters is carried out, and the range of variables is [0.01, 0.99]. Then the 41 groups of rock sample data by principal component analysis, princomp function application in MATLAB PCA processing, after calculation, the cumulative contribution of sample characteristic data of the first 6 principal components was over 90%, therefore, the two dimensions will feature parameters from 13 dimensions to 6 dimensions for subsequent conversion, fuzzy neural network network training and testing.

3.2 Fuzzy Neural Network Trainings

The structure of PCA-FNN is 5-20-3-1, that is, there are 20 membership functions, and the fuzzy membership function center C and width b are obtained randomly. The 41 sets of rock data samples were randomly divided into two parts: training sets and test sets, in which the training sets were 35 groups, and the test sets were the remaining 6 groups. Experiments based on the fuzzy neural network toolbox in MATLAB[4], neural network has been stabilized after 130 trainings(Figure 2), the training
network is used for test set for testing.

![Figure 2](image1)

![Figure 3](image2)

3.3 Interpretation of Result

The trained fuzzy neural network is used to predict the 6 sets of data in the test set. In the calculation results of permeability, the average absolute error is 0.0355mD, the relative error is 2.32%, the average relative error is 6.25%, and the error of the 6 groups of test results is less than 10% (Table 1). Combined with the above, the intelligent calculation of permeability of rock reservoir by using PCA and fuzzy neural network has good effect.

| Number | Measured permeability/mD | Computing result/mD | Absolute error/mD | Relative error/% |
|--------|--------------------------|---------------------|-------------------|-----------------|
| 1      | 0.7211                   | 0.6640              | 0.0571            | 7.92            |
| 2      | 0.9976                   | 0.9427              | 0.0549            | 5.50            |
| 3      | 0.155                    | 0.1514              | 0.0036            | 2.32            |
| 4      | 0.15                     | 0.1566              | 0.0066            | 4.42            |
| 5      | 0.4452                   | 0.4062              | 0.0390            | 8.77            |
| 6      | 0.6014                   | 0.5499              | 0.0515            | 8.56            |

Contrast curve is calculated based on the PCA fuzzy neural network and the permeability of the result and the real data (Figure 3), the network is consistent with the measured data for calculation of reservoir permeability of rock, reflect the fuzzy neural network learning ability and the ability of calculation indicate that the application of the method of intelligent rock reservoir permeability is feasible. Study on calculation. However, there is still some error between the results and the real data, which may be due to the insufficient number of samples.

3.4 Method Contrast

In order to test the application effect of PCA and fuzzy neural network, the sample data set without dimensionality reduction is selected to calculate the permeability of rock reservoir. The test results of the above method are compared with the experimental results of PCA and fuzzy neural network (Table 2). The training set of the above method is the same as that of PCA and fuzzy neural network.

| Comparison | PCA dimension reduction | Dimensionality reduction |
|------------|-------------------------|-------------------------|
| Input vector number | 5          | 13                      |
| Hidden node number   | 20         | 52                      |
| Convergence period   | 130        | 445                     |
| Relative error of permeability | 6.25%     | 32.14%                 |

According to table 2, the PCA method can effectively reduce the number of input vectors and the number of hidden nodes, simplify the structure of fuzzy neural network, and improve the convergence speed of network training. In comparison of relative error of permeability, PCA and fuzzy neural network method could greatly improve the calculation accuracy compared with other methods. It shows that PCA and fuzzy neural network are feasible for the intelligent calculation of permeability of
rock reservoir. At the same time, PCA and fuzzy neural network still have some shortcomings, for example, there are some errors in the calculation results.

4. Conclusion
In this paper, an intelligent calculation method based on PCA and fuzzy neural network is proposed, which is applied to the calculation of permeability of rock reservoir. Test slice identification and physical properties of reservoir rock sample data analysis method to prolong Ordos basin of Yanan area, the average relative error of the calculated permeability is 6.25%, the test results show that the PCA and the fuzzy neural network for intelligent reservoir permeability of rock can be calculated is feasible. Moreover, the method is superior to other methods in the convergence speed and accuracy of network training. Therefore, based on PCA and fuzzy neural network, the permeability of rock reservoir can be calculated quickly and accurately, which can help reservoir analysts to determine the reservoir parameters. Future work is to collect a large number of rock samples thin section identification and physical analysis data for subsequent testing, in order to improve the calculation accuracy of the method.

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