The Sequential Algorithm for Combined State of Charge and State of Health Estimation of Lithium Ion Battery based on Active Current Injection

Ziyou Song\textsuperscript{a}, Xiaogang Wu\textsuperscript{b}, Xuefeng Li\textsuperscript{b}, Jun Hou\textsuperscript{c*}, Heath Hofmann\textsuperscript{c}, and Jing Sun\textsuperscript{a}

\textsuperscript{a} Department of Naval Architecture and Marine Engineering, University of Michigan, Ann Arbor, MI 48109, USA

\textsuperscript{b} College of Electrical and Electronics Engineering, Harbin University of Science and Technology, Harbin 150080, China

\textsuperscript{c} Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI 48109, USA

Abstract—When State of Charge (SoC), State of Health (SoH), and parameters of the Lithium-ion battery are estimated simultaneously, the estimation accuracy is hard to be ensured due to uncertainties in the estimation process. To improve the estimation performance a sequential algorithm, which uses the frequency-scale separation and estimates the parameters/states sequentially by injecting currents with different frequencies, is proposed in this paper. Specifically, by incorporating a high-pass filter, the ohmic resistance and the RC pair can be independently characterized by injecting high-frequency and medium-frequency currents, respectively. Using the above estimated parameters, the battery capacity and SoC can then be estimated concurrently. Experimental results show that the estimation accuracy of the proposed sequential algorithm is much better than the concurrent algorithm where all parameters/states are estimated simultaneously, and the computational cost can also be reduced. Finally, the experiments are conducted under different temperatures to verify the effectiveness of the proposed algorithm for various battery capacities.

Keywords—Lithium ion battery; SoC/SoH estimation; Sequential algorithm; Active current injection

1. Introduction

Accurate estimation of critical states, such as the state of charge (SoC) and state of health (SoH), play an important role in ensuring safe, reliable, and efficient operation of lithium-ion batteries [1].
Estimation of SoC and SoH is generally intertwined with estimation of battery parameters, which significantly vary with battery aging and changes in operating conditions [2] and are difficult to be sufficiently calibrated offline [3]. However, when all states/parameters are estimated simultaneously, substantial uncertainties are introduced in the estimation process and the inaccurate estimate of any parameter or state will dramatically impair the overall estimation performance. It has been proven that the error variance of multi-parameter estimation, where all states and parameters are estimated simultaneously, is no less than that of single-parameter estimation where only one parameter or state is estimated and all other parameters are well known [4]. Moreover, multi-parameter estimation imposes a critical constraint on the battery current profile, as a persistently exciting (PE) input condition is required to guarantee convergence of the estimated parameters/states [5]. Generally speaking, to achieve PE the excitation should contain one frequency component for every two parameters to be estimated [6]. Therefore, it is worthwhile to investigate new algorithms which can separate the estimation of battery states from its parameters, and therefore improve the estimation performance.

Model-based algorithms have been widely used for battery state estimation [7]. Commonly used models include the open circuit voltage (OCV) model [8], equivalent circuit model (ECM) [9], neural network model [10], and electrochemical model [11]. A previous study has shown that the first-order ECM is an acceptable choice for lithium-ion batteries due to its adequate fidelity and low computational cost [12]. The first-order ECM adopted in this paper is shown in Fig. 1, where an ohmic resistor $R_s$, an RC pair ($R_t/C_t$), and a DC voltage source $v_{oc}$ are connected in series [13]. In practical applications, these parameters along with the SoC and SoH should be estimated online [2].
Fig. 1. The first-order equivalent circuit model for battery

For SoC estimation, the standard approach is coulomb counting [15], but this open-loop method is sensitive to an inaccurate initial SoC and measurement noise [16]. The most widely used algorithms for battery SoC estimation include, but are not limited to, the extended Kalman filter (EKF) [17], unscented Kalman filter [18], \( H_\infty \) observer [19], sliding mode observer [20], and fuzzy logic based method [21]. SoH is generally defined as the ratio of the remaining capacity to the original capacity [22]. The SoH estimation problem is essentially a parameter (i.e., capacity) estimation problem; applicable algorithms include Kalman filters [23], least-squares-based methods [24], moving-horizon observers [25], and Lyapunov-based methods [26]. In addition, combined SoC/SoH estimation has also been investigated. For example, the standard dual-extended Kalman filter (DEKF) has been developed to identify SoC and SoH concurrently [27].

In addition to the estimation algorithm, the input-output data used in the estimation process, i.e., the battery current and voltage, also dramatically affects the estimation accuracy [28]. However, none of the aforementioned studies considers the optimization of the battery current waveform to enhance signal richness and hence parameter identifiability. An over-actuated system, such as a battery/supercapacitor hybrid energy storage system, offers an additional degree of freedom which provides the opportunity to inject sufficiently rich input signals while achieving output regulation.
objectives simultaneously [29]. To improve estimation performance when all parameters/states need to be estimated a sequential algorithm, which uses frequency-scale separation and estimates the parameters/states sequentially through active current injection, is proposed in this paper.

The proposed sequential algorithm consists of three main steps: To exploit frequency-scale separation of different dynamics associated with different parameters and states the first-order ECM, which is governed by the initial SoC, the SoC variation, the ohmic resistance $R_o$, and the RC pair, is studied in simulation. It can be found that the initial SoC dynamics can be removed from the battery voltage dynamics with a high-pass filter. In addition, simulation results show that for the 18650 Lithium ion battery studied in this work, the SoC variations can be neglected as long as the battery current frequency is not extremely low (i.e., below 0.001 Hz), and the ECM dynamic behavior is dominated by the resistance $R_s$ when the battery current frequency is high (i.e., above 0.1 Hz). Consequently, in Step #1, the proposed sequential algorithm estimates the ohmic resistance $R_s$ independently by injecting a high-frequency current and incorporating a high-pass filter, since the RC pair can be regarded as a short-circuit under these conditions. In Step #2, based on the estimated $R_s$, the RC pair can be characterized (i.e., the diffusion resistance $R_t$ and the time constant $\tau$) by injecting a medium-frequency current. Finally, in Step #3, the battery capacity and SoC can be estimated concurrently based on the above estimated parameters. The EKF is adopted in Steps #1 and #2 to estimate battery parameters, and a DEKF is adopted in Step #3 to estimate the battery SoC and SoH. The experimental results verify the effectiveness of the proposed sequential algorithm, which significantly increases the estimation accuracy when compared to the case where all parameters/states are estimated simultaneously.

The rest of the paper is organized as follows. In Section 2, the battery model is described and
the dynamics of the first-order ECM are analyzed. In Section 3, the sequential algorithm is proposed. In Section 4, experimental results are provided for validation. Conclusions are given in Section 5.

2. System description

2.1 The first-order equivalent circuit model

Defining the battery terminal voltage as $v_b$ and the battery current as $i_b$ (positive for discharging and negative for charging), as shown in Fig. 1, the ECM dynamics can be presented as follows according to Kirchhoff’s laws:

$$\begin{align*}
\dot{v}_c &= -\frac{1}{C_r R_C} v_c + \frac{1}{C_c} i_b, \\
v_b &= v_{OC} - R_i i_b - v_c
\end{align*}$$

(1)

where $v_c$ is the voltage over the RC pair and $v_{OC}$ is the open circuit voltage (OCV). The OCV-SoC relation can be expressed as [30]

$$v_{OC}(z) = K_0 - \frac{K_1}{z} - K_2 z + K_3 \ln(z) + K_4 \ln(1 - z),$$

(2)

where $K_{0-4}$ are the model parameters and $z$ is the normalized SoC, and the SoC dynamic is given by [31]

$$z = z_0 - \int_{t_0}^{t} \eta \frac{i_b(t)}{Q_b} dt,$$

(3)

where $z_0$ is the initial SoC, $\eta$ is the charging/discharging efficiency, $t_0$ is the start time, and $Q_b$ is the battery capacity. It has been shown that the OCV-SoC relationship doesn’t significantly change with battery degradation level and operating condition [31], [32], which means that the OCV can be accurately estimated by Eq. (2) as long as the SoC is known. The slope of the OCV-SoC curve is constant for most battery chemistries within the normal operating range. For example, the OCV slope is nearly constant in the range of 10%-90% SoC, e.g., 0.65 V/100% for LiNiMnCo batteries and 0.17V/100% for LiFePO$_4$ battery [9], [32]. As a result, Eq. (2) can be linearized to simplify the
analysis.

$$v_{oc}(t) = a \left( z_0 - \int_b^t \frac{\eta_i}{Q_b} \, dv \right) + b,$$  \hspace{1cm} (4)

where \(a\) and \(b\) are the coefficients of the linearized OCV-SoC function. Note that the ECM parameters, including \(R_s\), \(R_t\), and \(\tau\), are significantly influenced by the battery degradation and the operating condition, and are therefore difficult to calibrate for all practical scenarios [23]. Especially, the influence of degradation on the battery characteristics is almost impossible to be entirely investigated offline [33]. As a result, battery parameters should be estimated online along with battery SoC and SoH. Since battery parameters vary most slowly than the battery SoC [23], two assumptions are made in the estimation process:

1) The initial value of \(v_C\) is assumed to be 0, and

2) The parameters of \(R_s\), \(R_t\), and \(\tau\) are assumed to be constant.

Under these assumptions, based on Eqs. (1) and (4), the transfer function from \(i_b\) to \(v_b\) can be derived using the Laplace transform:

$$v_b(s) = \frac{ae_{\theta}}{s} + \frac{b}{s} - \frac{a}{s} \frac{\eta}{Q_b} i_b(s) - R_i i_b(s) - \frac{R_s}{1 + \tau s} i_b(s),$$  \hspace{1cm} (5)

where \(s\) is the complex Laplace variable. There are therefore three parameters (i.e., \(R_s\), \(R_t\), and \(C_i\)) and two states (i.e., \(z_0\) and \(Q_b\)) in Eq. (5) that should be estimated.

We point out that, when compared to the first-order ECM adopted in this paper, higher-order models are more accurate to represent the battery dynamics. However, the parameter estimation of higher-order models has much higher computational cost and a more critical requirement on the battery current waveform. For example, when the second-order ECM is adopted, there are 7 parameters to be estimated, consequently four frequency components are required to persistently excite the battery [6] which is difficult to realize in practical applications. The choice of the model
is the result of a trade-off between accuracy and simplicity, so the first-order ECM is used in this paper.

2.2 The analysis of the first-order ECM dynamics

As shown in Eq. (5), the battery terminal voltage dynamics includes four parts. The first part \( \left( \frac{az_0 + b}{s} \right) \) is constant and related to the initial SoC. The second \( -\frac{a}{s} \eta i_a(s) \) is related to the SoC variation and is significantly influenced by the battery capacity. The third \( -R_i i_b(s) \) is related to the ohmic resistance, and the fourth \( -\frac{R}{1 + \tau s} i_b(s) \) is related to the RC pair. Since the first part is constant, it can be removed by a high-pass filter. A first-order high-pass filter is applied to Eq. (5), which yields:

\[
V_{fb}(s) = \frac{(az_0 + b)T_c}{1 + T_c s} \frac{a}{s} \eta i_a(s) - R_i i_a(s) - \frac{R}{1 + \tau s} i_a(s),
\]

where

\[
\begin{align*}
V_{fb}(s) &= \frac{T_c s}{1 + T_c s} v_b(s) \\
I_{fb}(s) &= \frac{T_c s}{1 + T_c s} i_b(s)
\end{align*}
\]

\( T_c \) is the time coefficient of the high-pass filter, and \( v_{fb} \) and \( i_{fb} \) are the filtered terminal voltage and current, respectively. The dynamics of the filtered system can be presented in the time domain using the inverse Laplace transform \( \mathcal{L}^{-1} \). The effects of the initial SoC are given as

\[
\mathcal{L}^{-1}\left\{ \frac{(az_0 + b)T_c}{1 + T_c s} \right\} = (az_0 + b) e^{-\frac{T_c}{T_c}}.
\]

which will decay exponentially in time due to the high-pass filter, at the rate defined by \( T_c \).

To evaluate the effects of the high-pass filter on separating the battery dynamics, we consider the Samsung 18650 Lithium ion batteries. The parameters of an 18650 Lithium ion battery are specified in Table 1. The coefficients \( K_{0-4} \) of the OCV-SoC for the adopted cell are 2.6031, 0.0674,
-1.527, 0.6265, and -0.0297, respectively. The initial SoC dynamics vanish more quickly as $T_c$ decreases (i.e., the cut-off frequency increases). The initial SoC dynamics can be removed from the battery terminal voltage after ~6 minutes when $T_c$ is 80s. Therefore the initial SoC can be neglected in the filtered system and it will not influence the estimation of the other parameters/states.

**Table 1**

Specifications for the Samsung 18650 battery cell

| Parameter                           | Value  |
|-------------------------------------|--------|
| Nominal Voltage (V)                 | 3.63   |
| Cell Capacity (Ah)                  | 2.47   |
| Cell Weight (g)                     | 45     |
| Current limitations (A)             | -2.6-5.2 |
| Ohmic Resistance $R_s$ (mΩ)         | ~100   |
| Diffusion Resistance $R_t$ (mΩ)     | ~30    |
| Time Constant $\tau$ (s)            | ~15    |
| Discharge/Charge Efficiency $\eta$ (%) | 98     |
| OCV-SoC slope $a$ (mV/100%)         | ~8.845 |
| Standard Deviation of Voltage Measurement Noise $\sigma_V$ (mV) | 20     |
| Battery Current Amplitude $M$ (A)   | 1      |

As shown in Eq. (6), except for the initial SoC response, the other responses are significantly influenced by the filtered current $i_{bf}$. To quantify the influence of the current’s frequency on the voltage dynamics a sinusoidal current, which is frequently used for parameter estimation, is considered in this paper:

$$
\begin{align*}
  i_b(t) &= M \cos(\omega t) \\
  i_b(s) &= \frac{Ms}{s^2 + \omega^2} 
\end{align*}
$$

(8)

where $M, \omega$ are the current magnitude and frequency, respectively. Therefore, the asymptotically
converged filtered current is given as

\[
\begin{align*}
    i_{\text{ref}}(t) &\rightarrow -\frac{MT_c \omega^2 \cos(\omega t) - MT_c \omega \sin(\omega t)}{1+T_c^2 \omega^2} \\
    i_{\text{ref}}(s) &\rightarrow \frac{Ms T_s}{s^2 + \omega^2 + 1 + T_c s}
\end{align*}
\]

(9)

Similarly, the SoC variation dynamics, ohmic resistance \( R_s \) dynamics, and RC pair dynamics asymptotically converge to

\[
\begin{align*}
    L^{-1}\left\{ -\frac{a}{s} \frac{\eta}{Q_b} i_{\text{ref}}(s) \right\} &\rightarrow -\frac{\eta T_c^2 \omega \sin(\omega t) - \eta T_c \cos(\omega t)}{Q_b \left(1+T_c^2 \omega^2\right)} \\
    L^{-1}\left\{ -R_i_{\text{ref}}(s) \right\} &\rightarrow -\frac{M R T_c^2 \omega^2 \cos(\omega t) - M R T_c \omega \sin(\omega t)}{1+T_c^2 \omega^2} \\
    L^{-1}\left\{ -\frac{1}{1+\tau s} i_{\text{ref}}(s) \right\} &\rightarrow -\frac{M R T_c \omega \left[T_c \omega \cos(\omega t) - \sin(\omega t) + \omega \tau \cos(\omega t) + T_c^2 \omega^2 \tau \sin(\omega t)\right]}{(1+T_c^2 \omega^2)(1+\tau^2 \omega^2)}
\end{align*}
\]

(10)

To clearly show the influence of current frequency, the battery voltage responses when the current frequency is 0.4Hz, 0.004Hz, and 0.0004Hz are shown in Fig. 2, where \( T_c \) is fixed at 80s.

When the current frequency is 0.4Hz, as shown in Fig. 2 (a), the battery terminal voltage is governed by the ohmic resistance component, and the SoC variation component as well as the RC pair component can be neglected. The RC pair can be regarded as a short circuit when the current frequency is sufficiently high. In addition, the SoC variation component is small because high-frequency current does not induce a significant change in the battery SoC, as shown in Eq. (10).

This means that, based on the filtered signals, \( R_s \) can be estimated independently by injecting high-frequency current regardless of the other parameters. As shown in Fig. 2 (b), the RC pair component is comparable with the ohmic resistance component when the current frequency decreases to 0.004Hz, while the SoC variation dynamics can still be neglected. Assuming that the parameters don’t change quickly [23], the estimated \( R_s \) can be used to estimate \( R_t \) and \( \tau \) when the current frequency is around 0.004Hz (i.e., medium-frequency current). As the current frequency further
decreases to 0.0004Hz, as shown in Fig. 2 (c), none of three components can be neglected, which means that the SoC variation component should be considered when the current frequency is very low. This analysis forms the laws of the sequential estimation algorithm presented in the next section.
11

(c) \( \omega \) is 0.0004Hz

Fig. 2. The battery voltage component due to various current frequencies

3. The sequential algorithm for combined SoC and SoH estimation

To reduce the uncertainty in the estimation process and therefore increase the estimation accuracy, a sequential algorithm is proposed in this paper, as shown in Fig. 3. A high-pass filter is used to block the constant initial SoC component, and a high-frequency current is injected to estimate the ohmic resistance \( R_s \). Using the estimated \( R_s \), a medium-frequency current is injected to estimate the diffusion resistance \( R_t \) and the time constant \( \tau \). After obtaining the estimated \( R_s, R_t, \) and \( \tau \), the SoC and SoH can be estimated finally.
The sequential algorithm, which estimates the battery parameters and states separately, can reduce the computational cost when compared to the case when all states and parameters are estimated simultaneously [34]. More importantly, the sequential algorithm can improve the estimation accuracy since it exploits the frequency spectrum separation and eliminates the interactions of parameter and state estimation [28]. Three steps are involved in the sequential estimation, and the associated algorithms are presented herein.

3.1 A review of EKF and DEKF

The EKF is used in Steps #1 and #2 of the sequential algorithm to estimate the battery parameters. The EKF determines the optimal feedback gain to suppress both process noise and measurement noise [35]. The general discrete time state-space equation of the process for which EKF is applied can be expressed as

$$
\begin{align*}
\theta_{k+1} &= \theta_k + r_k \\
X_{k+1} &= H(X_k, \theta_k, u_k) + w_k, \\
Y_{k+1} &= G(X_k, \theta_k, u_k) + v_k
\end{align*}
$$

(11)

where $k$ is the time index, $X_k$ is the state vector, $\theta_k$ is the parameter vector, $u_k$ is the input vector, $Y_k$ is the output vector, $r_k$ is the process noise for parameters, $w_k$ is process noise for states, and $v_k$ is the measurement noise. In Steps #1 and #2, only the battery parameters are estimated, and the battery states are not involved. The calculation process of the EKF is summarized in Table 2.

Table 2

| Initialization: |
|-----------------|
| $\hat{\theta}_0 = E[\theta_0]$ |
| $\Sigma_{\theta_0} = E[(\theta_0 - \hat{\theta}_0)(\theta_0 - \hat{\theta}_0)^T]$ |

(12)
where $\sum_{\theta_i}$ is the covariance matrix of parameter estimation error.

Parameter prediction:

$$\begin{align*}
\hat{\theta}_k &= \hat{\theta}_{k-1} \\
\Sigma_{\theta_k} &= \Sigma_{\theta_{k-1}} + \Sigma_{n_{k-1}},
\end{align*}$$  \hspace{1cm} (13)

where $\Sigma_{n_{k-1}}$ is the covariance matrix of process noise.

Parameter update:

$$\begin{align*}
K_k^\theta &= \Sigma_{\theta_{k-1}} \left( C_{k-1}^\theta \right)^T \left[ \left( C_{k-1}^\theta \right) \Sigma_{\theta_{k-1}} \left( C_{k-1}^\theta \right)^T + \Sigma_{n_{k-1}} \right] \\
\hat{\theta}_k &= \hat{\theta}_{k-1} + K_k^\theta \left[ Y_k - G \left( X_{k-1}, \hat{\theta}_{k-1}, u_k \right) \right], \\
\Sigma_{\theta_k} &= \left( I - K_k^\theta C_{k-1}^\theta \right) \Sigma_{\theta_{k-1}}
\end{align*}$$

where $C_{k-1}^\theta = \frac{\partial G(X_{k-1}, \theta, u_k)}{\partial \theta}|_{\theta = \hat{\theta}_{k-1}}$.

In Step #3, the combined SoC and SoH estimation is conducted based on the original system. We point out that the high-order OCV-SoC relationship (see Eq. (2)) is used to estimate SoC and SoH, and the linear relation (Eq. (4)) is just used to simplify the analysis rather than the algorithm design process. The remaining battery capacity, which is also one of the battery parameters, is estimated to show the SoH. As a result, both the parameter (i.e., battery capacity) and the state (i.e., SoC) need to be estimated in Step #3. The DEKF method is a commonly used technique to simultaneously estimate states and parameters [36]. In the DEKF, two EKFs are run concurrently and, at every time step when the observation (i.e., output) is available, the state EKF estimates the states using the latest model parameter estimates from the parameter EKF. Meanwhile, the parameter EKF estimates the model parameters using the latest state estimates from the state EKF [37]. Based on Eq. (11), the detailed calculation process of the DEKF is specified in Table 3.

**Table 3**

The DEKF calculation process

**Initialization:**
\[
\begin{align*}
\dot{\theta}_0 &= E[\theta_0] \\
\Sigma_{\theta_0} &= E[(\theta_0 - \hat{\theta}_0)(\theta_0 - \hat{\theta}_0)^T] \\
\dot{X}_0 &= E[X_0] \\
\Sigma_{X_0} &= E[(X_0 - \hat{X}_0)(X_0 - \hat{X}_0)^T]
\end{align*}
\]  

(15)

where \( \Sigma_{X_0} \) is the covariance matrix of state estimation error.

Parameter prediction:

\[
\begin{align*}
\dot{\theta}_k &= \hat{\theta}_{k-1} \\
\Sigma_{\theta_k} &= \Sigma_{\theta_{k-1}} + \Sigma_{u_k}
\end{align*}
\]  

(16)

State prediction:

\[
\begin{align*}
\dot{X}_k &= H(\hat{X}_{k-1}, \hat{\theta}_k, u_k) \\
\Sigma_{X_k} &= A_k \Sigma_{X_{k-1}} A_k^T + \Sigma_{u_k}
\end{align*}
\]  

(17)

where \( A_k = \left. \frac{\partial H(X, \hat{\theta}_k, u_k)}{\partial X} \right|_{x-X_k} \).

State update:

\[
\begin{align*}
K^X_k &= \Sigma_{X_k} \left( C^X_k \right)^T \left( C^X_k \Sigma_{X_k} \left( C^X_k \right)^T + \Sigma_{u_k} \right) \\
\dot{X}_k &= \hat{X}_k + K^X_k \left( Y_k - G(\hat{X}_k, \hat{\theta}_k, u_k) \right) \\
\Sigma_{X_k} &= (I - K^X_k C^X_k) \Sigma_{X_k}
\end{align*}
\]  

(18)

where \( C^X_k = \left. \frac{\partial G(X, \hat{\theta}_k, u_k)}{\partial X} \right|_{x-X_k} \).

Parameter update:

\[
\begin{align*}
K^\theta_k &= \Sigma_{\theta_k} \left( C^\theta_k \right)^T \left( C^\theta_k \Sigma_{\theta_k} \left( C^\theta_k \right)^T + \Sigma_{\theta_k} \right) \\
\dot{\theta}_k &= \hat{\theta}_k + K^\theta_k \left( Y_k - G(\hat{X}_k, \hat{\theta}_k, u_k) \right) \\
\Sigma_{\theta_k} &= (I - K^\theta_k C^\theta_k) \Sigma_{\theta_k}
\end{align*}
\]  

(19)

where \( C^\theta_k = \left. \frac{dG(\hat{X}_k, \theta, u_k)}{d\theta_i} \right|_{\theta-\hat{\theta}_k} \).

3.2 The sequential algorithm

Incorporating the algorithms described in section 3.1, the sequential parameter and SoC/SoH estimation approach is formulated as follows:

**Step #1:**

The first step of the sequential algorithm is estimating the ohmic resistance by using a high-
pass filter and injecting high-frequency current. Based on Eq. (6), the battery terminal voltage can be simplified as

\[ V_{st}(s) = -R_{st}(s) . \quad (20) \]

Therefore, the discrete time state-space equation (11) for estimating the ohmic resistance \( R_s \) using the EKF can be given as

\[
\begin{align*}
R_s(k) &= R_s(k-1) + r_k \\
V_{st}(k) &= -R_{st}(k) + v_k + R_s(k-1)
\end{align*}
\quad (21)
\]

**Step #2:**

When the medium-frequency current is injected, the battery terminal voltage is governed by ohmic resistance dynamics and RC pair dynamics, so Eq. (6) can be simplified as

\[ V_{st}(s) = -R_{st}(s) - \frac{R_s}{1 + \tau_s} i_{st}(s) . \quad (22) \]

The estimated ohmic resistance obtained in Step #1 is used in the Step #2, and the bilinear transformation is used to discretize Eq. (22). Consequently, the discrete time state-space equation (11) for estimating the \( R_t \) and \( \tau \) is given as

\[
\begin{align*}
\theta_x(k) &= \theta_x(k-1) + r_k \\
V_{st}(k) &= -\hat{R}_t i_{st}(k) - R_t i_z(k) + v_k + \theta_x(k)
\end{align*}
\quad (23)
\]

where

\[
\begin{align*}
\theta_x(k) &= \begin{bmatrix} R_t(k) \\ \tau(k) \end{bmatrix} \\
i_z(k) &= \frac{T_s}{T_s + 2\tau} \left[ \hat{i}_{st}(k) + i_{st}(k-1) \right] - \frac{T_s - 2\tau}{T_s + 2\tau} i_z(k-1)
\end{align*}
\]

and \( T_s \) is the sampling period (i.e., 1s).

**Step #3:**

Given that \( R_o, R_t, \) and \( \tau \) are estimated in Steps #1 and #2, the SoC and SoH can be simultaneously estimated in Step #3. Based on Eqs. (1)-(3), the discrete time state-space equation.
for estimating the SoC and SoH estimation is shown as

\[
Q_n(k) = Q_n(k-1) + r_n
\]

\[
X_i(k) = \begin{bmatrix} e^{-\frac{\tau_i}{T_i}} & 0 \\ 0 & 1 \end{bmatrix} X_i(k-1) + \begin{bmatrix} \hat{R} \left( 1 - e^{-\frac{\tau_i}{T_i}} \right) \\ -\frac{\eta T_i}{Q_i} \end{bmatrix} i_i(k), \quad (24)
\]

\[
V_n(k) = \text{OCV}(z(k)) - v_C(k) - \hat{R}(k) i_i(k)
\]

where

\[
X_i(k) = \begin{bmatrix} v_C(k) \\ z(k) \end{bmatrix}^T.
\]

As shown in Eq. (24), the voltage of the RC pair \(v_C\) is also estimated for better performance [2], and the DEKF presented in Section 3.1 is adopted. Since SoC and SoH are estimated together, the inaccurate estimation of either SoC or SoH will influence the other. However, the estimation accuracy can still be significantly improved for the proposed algorithm, since less uncertainties are involved in this case when compared to the case where all battery parameters/states are estimated simultaneously, which will be verified by the experimental results later.

For different battery cells, which are manufactured by different companies or have different chemistries, the battery parameters may vary significantly. As a result, the frequency of the injected current and the cut-off frequency of the high-pass filter should be adapted to different battery cells. At any rate, the proposed sequential algorithm is a general method for battery parameter/state estimation.

4. Experimental results

Experimental testing is focused on an 18650 Lithium ion battery, whose specifications are listed in Table 1. In the experiments, error caused by ECM inaccuracy is introduced. An Arbin battery test system (BT2000) was used in the following experiments [38]. The initial SoC is 80% for all three experiments, and they are illustrated as follows.
**Experiment #1:** Validation of the sequential algorithm under 20°C.

In Experiment #1, the initial guesses of the estimated parameters are $\begin{bmatrix} \hat{R}_s(0) & \hat{R}_t(0) & \hat{\tau}(0) & \hat{\nu}_c(0) \end{bmatrix} = [0.02, 0.03, 15, 2]$, and the initial values of estimated SoC and $\nu_c$ are 50% and 0V, respectively.

- As shown in Fig. 4(a), in Step #1, the 0.5Hz sinusoidal current with amplitude of 0.5C is injected and the first-order butterworth high-pass filter has 3dB bandwidth of 0.05Hz. The estimation result of $R_s$ is shown in Fig. 4(b), which indicates that the estimated value can accurately track the real value, which is obtained from the HPPC test.

- In Step #2, the 0.02Hz sinusoidal current with amplitude of 0.5C is added on a base current (0.004Hz sinusoidal current with amplitude of 0.5C) and the first-order butterworth high-pass filter has 3dB bandwidth of 0.002Hz. The estimated $R_s$ is used in Step #2, and the estimated parameters can track the real values for $R_t$ and $\tau$, but there is a slight static error in the $R_t$ estimation result, as shown in Fig. 4(c). Therefore, the first-order ECM has limitations representing the studied 18650 Lithium ion battery, since the parameters of the RC pair are difficult to estimate. Since the sampling frequency in Step #1 is high, we need to manually control the Arbin test system and switch to Step #2 in the experiment. As a result, there is a transient period from 200s to 287s, which is shown in Fig. 4(a). The estimation process of Step #2 is from 288s to 1187s, which includes a hold-up time of 400s to avoid initial SoC dynamics. Step #3 starts from 1200s.

- The estimated $R_s$, $R_t$, and $\tau$ are used in Step #3 to estimate SoC and SoH simultaneously. A scaled New European Driving Cycle (NEDC) is used in Step #3 to represent a practical profile for batteries. As shown in Fig. 4(d), the SoC estimation performance is satisfactory and the estimated error is below 1% under a significant initial condition error (30%).
- As shown in Fig. 4(e), the estimated battery capacity converges to the real value obtained from the static capacity test after 1800s, and there is no significant error after convergence.

- The estimated voltage shown in Fig. 4(f) tracks the actual terminal value well. As a result, the effectiveness of the proposed sequential algorithm is verified experimentally.
**Experiment #2**: Validation of the sequential algorithm under 40°C.

In Experiment #2, the temperature is increased to 40°C, and the battery parameters slightly change when compared to the those under 20°C according to the battery static capacity test [39]. Especially, the battery capacity increases from 2.47Ah to 2.62Ah. The initial guesses of the estimated parameters and states are similar to the ones in Experiment #1. The same current profile, as shown in Fig. 4(a), as well as the same high-pass filters are used.

- As shown in Fig. 5(a) and (b), the parameter estimation performance is satisfactory and similar to the results of Experiment #1.
- The SoC estimation error is less than 1% when the initial guess error is 30%, as shown in Fig. 5(c).
- For the battery capacity (i.e., SoH), the convergence time is about 600s, and no significant static error exists after convergence, as shown in Fig. 5(d).

The proposed algorithm not only decreases the computational cost when compared to the case where all parameter/states are estimated simultaneously, but also improves the estimation...
performance. This conclusion can be theoretically proven using Cramer-Rao (CR) bound analysis, which quantifies the fundamental relationship between estimation accuracy and measurement data [40]. It has been shown that the estimation error of multi-parameter estimation, where all states and parameters are estimated simultaneously, is no less than that of single-parameter estimation where only one parameter or state is estimated, given that either all other parameters are well known or their dynamics can be removed (like the proposed sequential algorithm performs) [4]. Due to the space limitations, this analysis is not extended further. Experiment #3 is conducted to verify that the proposed sequential algorithm.

![Graphs showing estimation results](image)

(a) $R_s$ estimation result (Step #1)  
(b) $R_t$ and $\tau$ estimation result (Step #2)  
(c) SoC estimation result (Step #3)  
(d) $Q_b$ estimation result (Step #3)
Experiment #3: Performance of concurrent parameter/state estimation under 20°C.

The multi-scale EKF has been validated to be effective to estimate all parameters/states simultaneously [2], [36]. The detailed design process for a multi-scale EKF, which adapts the slowly time-varying parameters on the macro time-scale and estimates the fast time-varying states on the micro time-scale, is provided in Ref. [36].

- Considering the influence of battery current on the estimation performance, the optimal current profile, including three sine waves with frequencies of 0.01Hz, 0.05Hz, and 0.1Hz, is used in Experiment #3, as shown in Fig. 6(a). It has been verified that the optimal current profile achieves much better estimation performance than the scaled NEDC does [41].

- The parameter estimation results are shown in Fig. 6(b). It can be seen that the estimation performance of $R_s$ and $Q_b$ (i.e., SoH) are satisfactory.

- The estimated $R_t$ and $\tau$ cannot track the actual values even though the optimal current profile, which can theoretically achieve the best estimation performance, is used.

- As shown in Fig. 6(c), the estimated SoC needs a longer time to converge to the real value.
as compared to the sequential algorithm. Moreover, the static error of the estimated SoC is around 2%.

- As the estimation processes of different states/parameters will influence each other and bring more uncertainty, the estimation error of the battery terminal voltage is enlarged, as shown in Figs. 4(f), 5(e), and 6(d).

![Images of graphs and plots](image1.png)

Fig. 6. Experimental results of estimating all parameters/states simultaneously

Based on the above experimental results, it is shown that the sequential algorithm, which separates the estimation process, can achieve a better estimation performance when compared to the case where all parameters/states are estimated simultaneously.

We would like to point out that the implications of injecting a current signal for active
parameter estimation could be complicated for general application (e.g., electric vehicles). However, an over-actuated system like the battery/supercapacitor (SC) hybrid energy storage system [42] or hybrid electric vehicle [43] provides an opportunity to achieve sufficiently rich input signals and output regulation objectives simultaneously [44]. Therefore, the proposed sequential algorithm can be directly used, and the potential negative influence of injecting the current on the system performance (i.e., system efficiency and power supply quality) can be minimized given the over-actuated nature. Specifically, for any power demand $P_d$, we have $P_d = P_{s1} + P_{s2}$, where $P_{s1}$ and $P_{s2}$ are the power from source 1 (i.e., battery) and source 2 (i.e., SC). The required current for battery parameters/states estimation can be injected directly, while the supercapacitor can be used to deliver the power demand and eliminate the negative consequences. In addition, when the battery is used as the sole energy source, the proposed algorithm also can be used if the battery charging current can be changed and therefore the required excitation can be added. The influence of temperature is investigated in the experiment, and a remark can be given to address the influence of battery degradation.

**Remark.** As a battery ages, its parameters will change, and the change will be reflected in the estimation results. One of the main goals for online parameter estimation is to detect the aging for condition monitoring. It has been proven that the proposed sequential algorithm can accurately estimate the battery parameters when they change via temperature and SoC. Therefore, the proposed algorithm can effectively detect the battery degradation in practical applications.

5. **Conclusion**

When battery states/parameters are estimated simultaneously, substantial uncertainties are introduced in the estimation process, and inaccurate parameters will therefore impair the state
estimation performance. To improve the estimation performance, the sequential algorithm, which uses frequency-scale separation and estimates the parameters/states sequentially by injecting the current with different frequencies, is proposed in this paper. Specifically, by using a high-pass filter, the ohmic resistance $R_s$ can be estimated independently via injecting a high-frequency current. Then, using the estimated $R_s$, the RC pair can be estimated by injecting a medium-frequency current. Finally, based on the above estimated parameters, the battery SoC and SoH can be estimated simultaneously. Experimental results show that the estimation accuracy of the proposed sequential algorithm is satisfactory and better than the case where all parameters/states are estimated simultaneously, although the optimal current profile is used. The proposed algorithm can be implemented online when the battery is used in over-actuated systems, which offer an additional degree of freedom to inject optimal current signals.

**Acknowledgement**

This work is supported by the National Natural Science Foundation of China (Grant No. 51877057).

**References**

[1] Zhang, J., & Lee, J. (2011). A review on prognostics and health monitoring of Li-ion battery. *Journal of Power Sources, 196*(15), 6007-6014.

[2] Xiong, R., Sun, F., Chen, Z., & He, H. (2014). A data-driven multi-scale extended Kalman filtering based parameter and state estimation approach of lithium-ion olymer battery in electric vehicles. *Applied Energy, 113*, 463-476.

[3] Hu, X., Sun, F., & Zou, Y. (2010). Estimation of state of charge of a lithium-ion battery pack for electric vehicles using an adaptive Luenberger observer. *Energies, 3*(9), 1586-1603.
[4] Lin, X., & Stefanopoulou, A. G. (2015). Analytic bound on accuracy of battery state and parameter estimation. *Journal of The Electrochemical Society, 162*(9), A1879-A1891.

[5] Gurjar, M., & Jalili, N. (2007). Toward ultrasmall mass detection using adaptive self-sensing piezoelectrically driven microcantilevers. *IEEE/ASME Transactions on mechatronics, 12*(6), 680-688.

[6] Ioannou, P. A., & Sun, J. (1996). *Robust adaptive control* (Vol. 1). Upper Saddle River, NJ: PTR Prentice-Hall.

[7] Han, X., Ouyang, M., Lu, L., Li, J., Zheng, Y., & Li, Z. (2014). A comparative study of commercial lithium ion battery cycle life in electrical vehicle: Aging mechanism identification. *Journal of Power Sources, 251*, 38-54.

[8] Çadırcı, Y., & Özkazanç, Y. (2004). Microcontroller-based on-line state-of-charge estimator for sealed lead–acid batteries. *Journal of Power Sources, 129*(2), 330-342.

[9] Lin, X., Perez, H. E., Mohan, S., Siegel, J. B., Stefanopoulou, A. G., Ding, Y., & Castanier, M. P. (2014). A lumped-parameter electro-thermal model for cylindrical batteries. *Journal of Power Sources, 257*, 1-11.

[10] Charkhgard, M., & Farrokhi, M. (2010). State-of-charge estimation for lithium-ion batteries using neural networks and EKF. *IEEE transactions on industrial electronics, 57*(12), 4178-4187.

[11] Gu, W. B., & Wang, C. Y. (2000). Thermal-electrochemical modeling of battery systems. *Journal of The Electrochemical Society, 147*(8), 2910-2922.

[12] Hu, X., Li, S., & Peng, H. (2012). A comparative study of equivalent circuit models for Li-ion batteries. *Journal of Power Sources, 198*, 359-367.

[13] Zheng, Y., Ouyang, M., Lu, L., Li, J., Han, X., Xu, L., ... & Freyermuth, V. (2013). Cell state-of-charge inconsistency estimation for LiFePO4 battery pack in hybrid electric vehicles using mean-difference model. *Applied energy, 111*, 571-580.

[14] Zou, Y., Hu, X., Ma, H., & Li, S. E. (2015). Combined state of charge and state of health estimation over lithium-ion battery cell cycle lifespan for electric vehicles. *Journal of Power Sources, 273*, 793-803.
[15] Ng, K. S., Moo, C. S., Chen, Y. P., & Hsieh, Y. C. (2009). Enhanced coulomb counting method for estimating state-of-charge and state-of-health of lithium-ion batteries. *Applied energy, 86*(9), 1506-1511.

[16] Zheng, Y., Ouyang, M., Lu, L., Li, J., Zhang, Z., & Li, X. (2015). Study on the correlation between state of charge and coulombic efficiency for commercial lithium ion batteries. *Journal of Power Sources, 289*, 81-90.

[17] Plett, G. L. (2004). Extended Kalman filtering for battery management systems of LiPB-based HEV battery packs: Part 3. State and parameter estimation. *Journal of Power sources, 134*(2), 277-292.

[18] Sun, F., Hu, X., Zou, Y., & Li, S. (2011). Adaptive unscented Kalman filtering for state of charge estimation of a lithium-ion battery for electric vehicles. *Energy, 36*(5), 3531-3540.

[19] Zhang, F., Liu, G., Fang, L., & Wang, H. (2012). Estimation of Battery State of Charge With H∞ Observer: Applied to a Robot for Inspecting Power Transmission Lines. *IEEE Transactions on Industrial Electronics, 59*(2), 1086-1095.

[20] Kim, I. S. (2006). The novel state of charge estimation method for lithium battery using sliding mode observer. *Journal of Power Sources, 163*(1), 584-590.

[21] Zenati, A., Desprez, P., & Razik, H. (2010, November). Estimation of the SOC and the SOH of Li-ion Batteries, by combining Impedance Measurements with the Fuzzy Logic Inference. In *IECON 2010-36th Annual Conference on IEEE Industrial Electronics Society* (pp. 1773-1778). IEEE.

[22] Rong, P., & Pedram, M. (2006). An analytical model for predicting the remaining battery capacity of lithium-ion batteries. *IEEE Transactions on Very Large Scale Integration (VLSI) Systems, 14*(5), 441-451.

[23] He, H., Xiong, R., Zhang, X., Sun, F., & Fan, J. (2011). State-of-charge estimation of the lithium-ion battery using an adaptive extended Kalman filter based on an improved Thevenin model. *IEEE Transactions on Vehicular Technology, 60*(4), 1461-1469.

[24] Rahimi-Eichi, H., Baronti, F., & Chow, M. Y. (2014). Online adaptive parameter identification and state-of-
charge coestimation for lithium-polymer battery cells. *IEEE Transactions on Industrial Electronics*, 61(4), 2053-2061.

[25] Suthar, B., Ramadesigan, V., Northrop, P. W., Gopaluni, B., Santhanagopalan, S., Braatz, R. D., & Subramanian, V. R. (2013, June). Optimal control and state estimation of lithium-ion batteries using reformulated models. In *American Control Conference (ACC), 2013* (pp. 5350-5355). IEEE.

[26] Dey, S., Ayalew, B., & Pisu, P. (2015). Nonlinear robust observers for state-of-charge estimation of lithium-ion cells based on a reduced electrochemical model. *IEEE Transactions on Control Systems Technology*, 23(5), 1935-1942.

[27] Wan, E. A., & Nelson, A. T. (2001). Dual extended Kalman filter methods. *Kalman filtering and neural networks*, 123-173.

[28] Lin, X. (2017). Analytic Analysis of the Data-Dependent Estimation Accuracy of Battery Equivalent Circuit Dynamics. *IEEE Control Systems Letters*, 1(2), 304-309.

[29] Song, Z., Hou, J., Xu, S., Ouyang, M., & Li, J. (2017). The influence of driving cycle characteristics on the integrated optimization of hybrid energy storage system for electric city buses. *Energy*, 135, 91-100.

[30] Weng, C., Cui, Y., Sun, J., & Peng, H. (2013). On-board state of health monitoring of lithium-ion batteries using incremental capacity analysis with support vector regression. *Journal of Power Sources*, 235, 36-44.

[31] Wei, J., Dong, G., & Chen, Z. (2018). Remaining Useful Life Prediction and State of Health Diagnosis for Lithium-Ion Batteries Using Particle Filter and Support Vector Regression. *IEEE Transactions on Industrial Electronics*, 65(7), 5634-5643.

[32] Samad, N. A., Siegel, J. B., & Stefanopoulou, A. G. (2014, October). Parameterization and validation of a distributed coupled electro-thermal model for prismatic cells. In *ASME 2014 Dynamic Systems and Control Conference* (pp. V002T23A006-V002T23A006). American Society of Mechanical Engineers.
[33] Hu, Z., Xu, L., Huang, Y., Li, J., Ouyang, M., Du, X., & Jiang, H. (2018). Comprehensive analysis of galvanostatic charge method for fuel cell degradation diagnosis. *Applied Energy, 212*, 1321-1332.

[34] Hannan, M. A., Lipu, M. S. H., Hussain, A., & Mohamed, A. (2017). A review of lithium-ion battery state of charge estimation and management system in electric vehicle applications: Challenges and recommendations. *Renewable and Sustainable Energy Reviews, 78*, 834-854.

[35] Lee, J. H., & Ricker, N. L. (1994). Extended Kalman filter based nonlinear model predictive control. *Industrial & Engineering Chemistry Research, 33*(6), 1530-1541.

[36] Hu, C., Youn, B. D., & Chung, J. (2012). A multiscale framework with extended Kalman filter for lithium-ion battery SOC and capacity estimation. *Applied Energy, 92*, 694-704.

[37] Haykin, S. S. (Ed.). (2001). *Kalman filtering and neural networks* (pp. 221-269). New York: Wiley.

[38] "Bt2000 battery testing system," Arbin Inc., Tech. Rep., 2009

[39] Song, Z., Hofmann, H., Li, J., Hou, J., Zhang, X., & Ouyang, M. (2015). The optimization of a hybrid energy storage system at subzero temperatures: Energy management strategy design and battery heating requirement analysis. *Applied energy, 159*, 576-588.

[40] Rothenberger, M. J., Docimo, D. J., Ghanaatpishe, M., & Fathy, H. K. (2015). Genetic optimization and experimental validation of a test cycle that maximizes parameter identifiability for a Li-ion equivalent-circuit battery model. *Journal of Energy Storage, 4*, 156-166.

[41] Song, Z., Wu, X., Li, X., Hofmann, H., Sun, J., & Hou, J. (2018). Current Profile Optimization for Combined State of Charge and State of Health Estimation of Lithium Ion Battery based on Cramer-Rao Bound Analysis. *IEEE Transactions on Power Electronics*.

[42] Song, Z., Li, J., Hou, J., Hofmann, H., Ouyang, M., & Du, J. (2018). The Battery-Supercapacitor Hybrid Energy Storage System in Electric Vehicle Applications: A Case Study. *Energy, 154*, 433-441.
[43] Du, J., Chen, J., Song, Z., Gao, M., & Ouyang, M. (2017). Design method of a power management strategy for variable battery capacities range-extended electric vehicles to improve energy efficiency and cost-effectiveness. *Energy, 121*, 32-42.

[44] Reed, D. M., Sun, J., & Hofmann, H. F. (2017). Simultaneous identification and adaptive torque control of permanent magnet synchronous machines. *IEEE Transactions on Control Systems Technology*, 25(4), 1372-1383.