Experimental investigations on the cross-correlation function amplitude vector of the dynamic strain under varying environmental temperature for structural damage detection

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Abstract
This article focuses on the experimental investigations on the cross-correlation function amplitude vector of the dynamic strain ($\text{CorV}_S$) under varying environmental temperature for structural damage detection. It is verified that under white noise excitation, $\text{CorV}_S$ is only related to the natural frequencies, mode shapes, and damping ratios of structures. The normalized $\text{CorV}_S$ of the undamaged structure maintains a uniform shape. A laboratory experimental investigation based on an end-fixed steel beam shows that $\text{CorV}_S$ can be used for structural damage detection. However, $\text{CorV}_S$ constructed by the dynamic strain of in-situ test varies with time, and the $\text{CorV}_S$ curves do not have the same shape. When the environmental temperature fluctuates significantly, high correlation exists between the dynamic strain and environmental temperature. By analyzing the power spectral density of the signals measured from active and inactive strain gauges, it is found that the signals induced by temperature stress, which do not reflect the dynamic performance of the bridge, exist in the very low-frequency band. To avoid the interference to $\text{CorV}_S$, the temperature effect component is separated from the dynamic strain by analytical mode decomposition method. Then, each $\text{CorV}_S$ curve maintains a uniform shape. The results demonstrate that it is prone to get a misjudgment for the condition of a structure if temperature effect on $\text{CorV}_S$ is ignored. It is necessary to eliminate the environmental temperature effect on $\text{CorV}_S$ for the damage detection of a structure in service.

Keywords
Damage detection, the varying environmental temperature, dynamic strain, the cross-correlation function amplitude vector, analytical mode decomposition

Introduction
Bridges are subjected to the action of various loads, the impact of natural environment, and the deterioration of structural material during the construction and operation of structures. It is necessary to conduct regular inspection or continuous monitoring of important structures to assess their condition by analyzing the changes in structural parameters and to issue the warning message before the disaster.

The structural modal parameter (natural frequency, mode shape, and damping ratio) is the function of structural physical parameter (mass, stiffness, and damping). When structural damage causes change in the physical
parameter, structural modal parameter changes accordingly. The damage can be identified by using the change of the modal parameter. To seek the reasonable damage feature from structural vibration data, which is closely related to the structural dynamic characteristic and sensitive to the structural damage, is the key issue in the damage detection of a bridge. Structural vibration data are often used to identify the natural frequency, mode shape, modal damping ratio, and the other dynamic parameters of a structure. Many damage identification methods based on the modal parameter have been widely investigated.

Extraction of the time-domain damage feature from the structural vibration data is another way for damage detection. It is not necessary to transform signals into the frequency domain, and the processes of signal transformation and modal identification are omitted. For the long-term structural health monitoring, the construction of the time-domain damage feature, which is sensitive to the structural damage, based on the measured structural dynamic responses, is of great advantage. Ruotolo and Surace made singular value decomposition for the response signal matrix to get the matrix rank, and the damage detection was conducted based on the change of rank. Sohn and Farrar adopted the autoregressive (AR) model coefficient of the response signals as the damage index. Mattson and Pandit proposed the vector AR model to determine the damage location of the structure. Yang et al. detected the structural damage by using cross-correlation function amplitude vector (CorV) of the measured vibration responses when the structure was subjected to the steady random excitation with specific frequency spectrum. The cross-correlation function amplitude vector assurance criterion (CVAC) was defined and used to quantify the variation of CorV.

Li and Law constructed a matrix on the covariance formed by the auto-/cross-correlation function of acceleration responses of a structure under white noise ambient excitation. It was found that the component of the covariance matrix was more sensitive to the local stiffness reduction than the first few modal frequencies and mode shapes obtained from ambient excitation. An et al. presented an efficient damage localization method that computes the curvature directly from acceleration signals without identifying modal shapes of the structure. The method confirmed its effectiveness and robustness to measure the noise through the experimental tests.

The change in the dynamic characteristic parameter has been often used in the research of structural damage mechanism and safety condition evaluation. However, temperature variation may lead to the change in the dynamic characteristic parameter of a structure, and the change degree is even greater than that caused by minute structure damage. Therefore, the influence of environmental factors on the health monitoring and safety assessment of a large-scale structure must be considered. Askegaard and Mossing discovered that the characteristic frequency of a monitored three-span pedestrian bridge changed about 10% quarterly in three years. Alampalli found that freezing of the bearings led to a 40% to 50% modal frequency change, and the frequency change caused by man-made damage was 3% to 8%. Farrar et al. published that the primary frequency of Canyon Alamosa Bridge changed about 5% within 24 h. The research of I-40 Bridge showed that the overall flexural rigidity of the bridge decreased by 21%, but the natural frequency did not change obviously. Zhao and Dewolf found that the variation of modal frequency reached 15.4% through two years’ monitoring of a highway steel girder bridge. Xia et al. analyzed the influence of environmental temperature on the vibrational characteristics of a two span reinforced concrete slab. Two years’ monitoring of data analysis showed that the frequency of the slab decreased by 0.2% with 1°C increase in the temperature. Researchers discovered that among the environmental influence factors such as temperature, humidity, and wind speed, temperature is the most significant factor that affected the dynamic characteristic of a structure.

At present, there is no general method for analyzing the temperature effect on a structure. Sohn et al. used a linear regression model to analyze the relationship between the modal frequency and the temperature of Alamosa Canyon Bridge. Hua et al. used a support vector regression to study the relationship between structural modal parameters and temperature variation. Ni et al. adopted back propagation (BP) neural network technique to study the correlation between structural modal frequency and temperature. Ding et al. studied the temperature effect on the modal frequencies of Run Yang Suspension Bridge by using the improved BP neural network. The aforementioned methods aimed to establish a mathematical model that can reflect the relationship between measured temperature data and structural modal parameters and then eliminate temperature effect by using the established model. If temperature distributions are not very complex, these methods can achieve good results. However, due to the influence of the number and placement of temperature sensors, limited temperature sensors do not completely reflect the temperature distribution information of a structure.

Another way to analyze the temperature effect on a structure is to make structural characteristic parameter contain the influence of environmental factors and then adopt the method of anomaly detection of a structure based on statistics to consider the influence. This method does not need to distinguish and measure the
environmental impacts that are embedded in the structural characteristic parameters as influence variables.\textsuperscript{21,22} Principle component analysis (PCA) is a kind of multivariate statistical method. Yan et al. analyzed the frequencies of the structure under varying environmental temperature by PCA\textsuperscript{23,24} and determined the principal components corresponding to the temperature factor. However, it may lead to misdiagnosis of structural operation state when sample data are not enough to completely cover the change in environmental temperature.

According to the multi-component characteristics of signals, the temperature effect component in signals can be eliminated through the signal separation method. Wu et al. studied the multi-scale features of the dynamic strains from two long-span bridges and extracted the strain caused by the temperature change, the train, and the heavy truck, respectively, through the wavelet transform method.\textsuperscript{25} Li et al. analyzed the intrinsic mode functions (IMFs) energies of the strain signals measured from a suspension bridge in time domain and frequency domain. To extract the temperature effect component of the dynamic strain, the threshold of IMF order was obtained with the empirical mode decomposition method. However, the computation costs are large for the long time monitored data.\textsuperscript{26}

The paper is organized as follows. The following section demonstrates that the CorV\textsubscript{S} of a structure is only related to the natural frequency, mode shape, and damping ratio of a structure under white noise excitation. “Experimental investigations” section describes the laboratory experiment of an end-fixed steel beam and the in-situ experiment of a suspension bridge. The constructed CorV\textsubscript{S} curves based on the raw strain data measured from the beam and the bridge are showed in “Analysis of experimental raw data” section. To illustrate the environmental temperature effect on CorV\textsubscript{S}, “Temperature effect analysis on the dynamic strain measured from the suspension bridge” section analyzes the dynamic strain signals in the time domain and the frequency domain. “Elimination of the effect of varying environmental temperature on CorV\textsubscript{S}” section separates the temperature component from the dynamic strain of a suspension bridge and eliminates the environmental temperature effect on CorV\textsubscript{S}. The final section concludes the work.

Cross-correlation function amplitude vector of the dynamic strain

Under the random excitation, the cross-correlation function of the structural responses \( R_{pl}(\tau) \) measured at the \( p \) and \( l \) measurement points is expressed as

\[
R_{pl}(\tau) = \frac{1}{N} \sum_{i=0}^{N-1} x_p(i)x_l(i+\tau)
\]

where \( N \) is the sample number of structural responses, \( \tau \) is the time delay, and when \( p = l \), \( R_{pl}(\tau) \) is the autocorrelation function.

The dynamical equation for an \( N \)-degree-of-freedoms (\( N \)-DOFs) viscous damped structure is given as

\[
M\ddot{x}_i(t) + C\dot{x}_i(t) + Kx_i(t) = v_f(t)
\]

where \( M \), \( C \), and \( K \) are the \( N \times N \) mass, damping, and stiffness matrices, respectively. \( x_i(t) \), \( \dot{x}_i(t) \), \( \ddot{x}_i(t) \) are the \( N \times 1 \) displacement, velocity, and acceleration vectors, respectively. \( v \) is the \( N \times 1 \) mapping vector, and \( f(t) \) is the random excitation. \( v_f(t) \) means that the excitation is applied to the corresponding DOFs of the structure.

Define \( x_i \) as the displacement vector of a node of the \( i \)th element, then one can obtain

\[
x_i = Pa_i
\]

where \( P \) is the displacement function matrix and \( a_i \) is the coefficient vector of a polynomial function. If the position coordinate of a node is determined, the displacement vector of all nodes of the \( i \)th element \( x_{ei} \) is expressed as

\[
x_{ei} = A_i a_i
\]

\[
a_i = A_i^{-1} x_{ei}
\]
where $A_i$ is the numeric matrix. Substituting equation (5) in equation (3), $x_i$ can be rewritten as

$$x_i = PA_i^{-1}x_{e,i}$$

(6)

The strain vector of a node of $i$th element $e_i$ is expressed as

$$e_i = Dx_i = DPA_i^{-1}x_i = e_{e,i} \quad (i = 0, 1, 2, \ldots, m)$$

(7)

where $m$ is the element number and $D$ is the differential operator.

Furthermore, the strain of an element is obtained as

$$e = Bx$$

(8)

where $x$ is the displacement of a node.

In global coordinate, $x$ can be written as

$$x = \beta x_s$$

(9)

where $\beta$ is the coordinate transformation matrix and $x_s$ is the node displacement vector in global coordinate. Equation (8) can be further expressed as

$$e = B\beta x_s = \tilde{B} x_s$$

(10)

According to Li and Lu,27 substituting equation (10) in equation (2), we have

$$M_e \ddot{e}(t) + C_e \dot{e}(t) + K_e e(t) = \tilde{B}^{-T} L f(t)$$

(11)

where $M_e = \tilde{B}^{-T} M \tilde{B}^{-1}$, $C_e = \tilde{B}^{-T} C \tilde{B}^{-1}$, and $K_e = \tilde{B}^{-T} K \tilde{B}^{-1}$.

Strain responses are obtained by the supposition of strain modes, as shown in equation (12). $\Phi^e$ is the $N \times N$ strain mode shape matrix, and $q(t)$ with the size of $N \times 1$ is the generalized coordinate vector of strain mode.

$$e(t) = \Phi^e q(t)$$

(12)

Substituting equation (12) in equation (11), we have

$$M_r \ddot{q}(t) + C_r \dot{q}(t) + K_r q(t) = \psi f(t)$$

(13)

where $\Phi^T M_r \Phi^e = M_r$, $\Phi^T C_r \Phi^e = C_r$, $\Phi^T K_r \Phi^e = K_r$, and $\psi = \Phi^T \tilde{B}^{-T} L$.

The unit impulse response function of location $l$ can be achieved by

$$h_l = \sum_{i=1}^{N} \frac{\Phi_{l,i}^e}{\omega_{di}} e^{-\xi_i \omega_{di} t} \sin(\omega_{di} t)$$

(14)

where $\omega_i$, $\omega_{di}$, and $\xi_i$ are the $i$th undamped modal frequency, damped modal frequency, and damping ratio, respectively. $\Phi_{l,i}^e$ is the $l$th element of $i$th strain mode shape.

The strain response of location $l$ from the structure under excitation $f(t)$ is written as

$$e_l(t) = \int_{-\infty}^{t} h_l(t - \tau) f(\tau) d\tau$$

(15)
The cross-correlation of the dynamic strains from sensors at the location \( p \) and \( l \) of the structure can be expressed as

\[
R_{pl}(\tau) = E \left\{ \int_{-\infty}^{\infty} h_p(t - \tau) \frac{f(\sigma_1) d\sigma_1}{E(\sigma_1)} \int_{-\infty}^{\infty} h_l(t + \tau) \frac{f(\sigma_2) d\sigma_2}{E(\sigma_2)} \right\}
\]  

(16)

If \( f(t) \) is the white noise excitation, the auto-correlation function of \( f(t) \) is expressed as

\[
E(f(\sigma_1)f(\sigma_2)) = S\delta(\sigma_1 - \sigma_2)
\]  

(17)

where \( S \) is a constant which defines the square value of the amplitude of \( f(t) \) when \( \sigma_1 = \sigma_2 \), and \( \delta(t) \) is the Dirac delta function.

\[
\int_{-\infty}^{+\infty} f(t)\delta(t) dt = f(0)
\]  

(18)

Substituting equation (17) in equation (16), we have

\[
R_{pl}(\tau) = S \int_{-\infty}^{\infty} \int_{-\infty}^{+\infty} h_p(t - \tau) h_l(t + \tau) \delta(\sigma_1 - \sigma_2) d\sigma_1 d\sigma_2 = S \int_{0}^{+\infty} h_p(t) h_l(t + \tau) dt
\]  

(19)

Substituting equation (14) in equation (19), \( R_{pl}(\tau) \) is further expressed as

\[
R_{pl}(\tau) = S \sum_{j=1}^{N} \frac{\Phi_{pl,j}' \psi_{j}}{\omega_{di}} e^{-\xi_{i}\omega_{j}\tau} \{ G_{j}\cos(\omega_{dj}\tau) + H_{j}\sin(\omega_{dj}\tau) \}
\]  

(20)

where \( G_{j} = \sum_{i=1}^{N} \frac{\Phi_{pl,i}' \psi_{i}}{\omega_{di}} A_{ij} \) and \( H_{j} = \sum_{i=1}^{N} \frac{\Phi_{pl,i}' \psi_{i}}{\omega_{di}} B_{ij} \)

\[
A_{ij} = \frac{1}{2} \left[ \frac{s_i + s_j}{(s_i + s_j)^2 + (\omega_{di} - \omega_{dj})^2} - \frac{s_i + s_j}{(s_i + s_j)^2 + (\omega_{di} + \omega_{dj})^2} \right]
\]

\[
B_{ij} = \frac{1}{2} \left[ \frac{\omega_{di} + \omega_{dj}}{(s_i + s_j)^2 + (\omega_{di} + \omega_{dj})^2} + \frac{\omega_{di} - \omega_{dj}}{(s_i + s_j)^2 + (\omega_{di} - \omega_{dj})^2} \right]
\]

\[
s_i = \xi_i \omega_i, \quad s_j = \xi_j \omega_j
\]

The absolute value of the cross-correlation function amplitude of the dynamic strains collected from \( p \) and \( l \) measuring points is defined as \( r_{pl} \)

\[
r_{pl} = |R_{pl}(\tau)|
\]  

(21)

where \( p \) is the reference point.

The cross-correlation function amplitude vector of the dynamic strain \( \text{CorV}_S \) is constructed as

\[
\text{CorV}_S = [r_{p,1}, r_{p,2}, \cdots, r_{p,n}]^T
\]  

(22)

As shown in equation (20), \( \text{CorV}_S \) is only related to the natural frequency, mode shape, and damping ratio of a structure under white noise excitation. Through equation (23), the normalized \( \text{CorV}_S \) of an undamaged
structure will keep a uniform shape. Therefore, \( \text{CorV}.S \) can be treated as a generalized mode shape of the structure under white noise excitation.

\[
\text{CorV}.S(i) = \frac{\text{CorV}.S(i)}{\left( \sum_{i=1}^{n} |\text{CorV}.S(i)|^2 \right)^{1/2}}
\]  

(23)

The structural damage may lead to the changes in structural mode parameters, so the ratio of elements in \( \text{CorV}.S \) will change. Define the \( \text{CorV}.S \) of a healthy structure as the baseline which denotes as \( \text{CorV}.S' \). To detect damage that occurred in the structure, the CVAC, which is defined in equation (24), is adopted to evaluate the difference between \( \text{CorV}.S \) and \( \text{CorV}.S' \). When the CVAC is close to 1, the \( \text{CorV}.S \) of the evaluated structure matches well with the \( \text{CorV}.S' \). It indicates that the operational state of the evaluated structure is normal.

\[
\text{CVAC} = \frac{\left( \sum_{i=1}^{n} (\text{CorV}.S(i) \times \text{CorV}.S'(i)) \right)^2}{\sum_{i=1}^{n} |\text{CorV}.S(i)|^2 \times \sum_{i=1}^{n} |\text{CorV}.S'(i)|^2}
\]  

(24)

**Experimental investigations**

*Laboratory experiment of an end-fixed steel beam model*

To verify that \( \text{CorV}.S \) is effective for structural damage detection, an end-fixed beam model was designed and constructed in a laboratory. The model was made of a square hollow steel tube and bolted on two big steel columns. The elastic modulus of the steel is 206 GPa, and the mass density is 7850 kg/m\(^3\). The sectional width and height of the tube are both 30 mm, and the sectional thickness is 1 mm. The calculated span of the model is 3.12 m. The model was excited by a vibration exciter DHJZQ-50 which connects with a signal generator Agilent 33521A and a signal amplifier DH1301. The beam model and experimental instruments are shown in Figure 1.

Five resistance strain gauges were fixed on the top surface of the model and connected to the signal acquisition instrument HBM-MGC plus AB22A, which provided a continuous measurement of the dynamic strain of the model. Figure 2 shows the location of strain gauges. A band-limited white noise, which drove the vibration

![Experimental apparatus](image)

*Figure 1.* Laboratory experiment: (a) measurement instruments and (b) the end-fixed steel beam model.
exciter, covered 0.001–200 Hz, as shown in Figure 3. The sampling frequency for the signal acquisition instrument was set to 500 Hz. The damage was introduced by fixing a mass on the top of the model between the strain gauges 2 and 3. The weight ratios of the mass to the beam model in percentage are listed in Table 1. There were three cases in the laboratory experiment. Every case consisted of four groups of tests, and any one test ran for 2 min at constant interior temperature. The experimental scenarios are described in Table 1.

**In-situ experiment on the Baling River Suspension Bridge**

The Baling River Suspension Bridge, located in Guizhou province, China, is a highway suspension bridge with a main span of 1088 m (Figure 4). A total of six active strain gauges, denoted as S1–S6, were symmetrically installed on upstream and downstream truss members at mid span of the bridge to measure the dynamic strain responses generated by structural stress. The layout of these strain gauges is shown in Figure 5. Due to the inconvenience of construction, active strain gauges had not been installed on the vertical strut and diagonal strut of the downstream truss. Another two inactive strain gauges, denoted as ST1 and ST2, were arranged near S1 and S5 (Figures 5 and 6). The inactive strain gauge did not sense the structural stress-induced strain and just measured the temperature deformation of the strain gauge itself. Data have been continuously collected by the signal acquisition instrumentation.

![Figure 2. Placement of strain gauges and an added mass on the end-fixed beam model.](image)

![Figure 3. Band-limited white noise excitation with the frequency ranges from 0.001 to 200 Hz.](image)

| Table 1. Damage scenarios for the end-fixed steel beam model. |
|-------------------|-------------------|
| Damage scenarios  | Weight ratio of an added mass to the beam model (%) |
| Case 1: No damage | –                 |
| Case 2: Damage scenarios 1 | 4                 |
| Case 3: Damage scenarios 2 | 8                 |

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Figure 4. The Baling River Suspension Bridge.

Figure 5. Layout of strain gauges at mid span of the bridge.

Figure 6. Strain gauges S1 and ST1 arrangement: (a) strain gauges installation on the upstream truss and (b) S1 and ST1.
instrument HBM-MGC plus AB22A for four days in January and six days in April with the sampling frequency of 50 Hz (Figure 7).

**Analysis of experimental raw data**

**Lab test data**

During the test of the end-fixed steel beam model, the indoor temperature variation was slight and could be ignored. First, the random subspace method is used to identify the modal parameters through strain responses (Figure 8) and then $\text{CorV}_S$ is constructed based on strain data.

The identified strain mode frequencies of the intact and damaged beam are shown in Table 2. In cases 2 and 3, the added masses whose weights were 4% and 8% that of the beam model were successively placed on the model to simulate the damage. The first and the second frequencies of the beam in case 2 are reduced by 2.13% and
1.39%, respectively. With the aggravation of damage, the first and the second frequencies decrease by 3.95% and 2.56%, respectively. As shown in Figure 9, the structural damage only lead to small change in the strain mode shape.

For each test, the cross-correlation functions of the dynamic strain at two measurement points are calculated with $\tau = 4$. The amplitudes of cross-correlation functions are at $\tau = 0$. By using the equations (21) and (22), the $\text{CorV}_S$ of every test is obtained. In case 1, four tests had been performed to demonstrate that the $\text{CorV}_S$ curve of an intact structure has a certain fixed shape. Figure 10(a) shows the normalized $\text{CorV}_S$ curves of case 1. The CVAC values between the tests of case 1 (Table 3) illustrate that the curve forms are all coincident. It is indicated

| Damage scenarios | First frequency | Second frequency |
|------------------|-----------------|-----------------|
| Case 1           | 22.188          | 60.570          |
| Case 2           | 21.716          | 59.728          |
| Case 3           | 21.312          | 59.019          |

Figure 9. Strain mode shapes of the beam under different conditions: (a) the first strain mode shape and (b) the second strain mode shape.

Figure 10. Normalized $\text{CorV}_S$ of laboratory experiment: (a) case 1: no damage and (b) cases 2 and 3: damage scenarios.
that the operational state of the beam model did not change during the tests of case 1, and there was no damage in
the model. For cases 2 and 3, the mean value of normalized CorV_Ss of tests have been calculated and that of case
1 is regarded as reference, as shown in Figure 10(b). Compared to the strain mode shape, the damage leads to
more obvious change in the CorV_S curve. The CVAC value between case 1 and case 2 is 0.9966. With the
increase of damage degree, the value between case 1 and case 3 drops to 0.9898. The results demonstrate that the
damage of a structure can be detected efficiently by means of CorV_S.

In-situ test data

Figure 11 shows the environmental temperature variation during the dynamic strain measurement on the bridge.
Figure 12 displays the measured dynamic strain of S1 in January and April. The 10 days’ data collected on
measurement points are all divided into a few 2-h long data segments. Set S1 as the reference point and the cross-
correlation functions (e.g. $R_{S1-S2}(\tau)$) are calculated based on the data segments of two measurement points. Then,
CorV_S is constructed by the maximum absolute values of cross-correlation functions $r_{pl}(\tau)$, as shown below

$$
\text{CorV}_S^{(i)} = \{r_{S1-S1}, r_{S1-S2}, r_{S1-S3}, r_{S1-S4}, r_{S1-S5}, r_{S1-S6}\}^T, \quad i = 1, 2, \cdots, 120
$$

CorV_S curves based on the 24 h raw strain data are different in shape, as shown in Figure 13. It means the
structure may be in novelty condition or damaged. Obviously, the result does not comply with the real operational
condition of the bridge. In order to realize the change of CorV_S in 10 days, CVAC is evaluated. The CorV_S' is
constructed as the baseline by using the raw strain data in the first 2 h of the first day in January, and then the
CorV_S is obtained by using the raw strain data in the rest of the time periods. Figure 14 shows that CVAC values
change about 60%.

| Test no. | 1–2 | 1–3 | 1–4 | 2–3 | 2–4 | 3–4 |
|----------|-----|-----|-----|-----|-----|-----|
| CVAC     | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |

CVAC: cross-correlation function amplitude vector assurance criterion.

Figure 11. Environmental temperature of the bridge site: (a) January and (b) April.

| Table 3. | Values of CVAC between tests in case 1. |
|----------|---------------------------------------|
| Test no. | 1–2  | 1–3  | 1–4  | 2–3  | 2–4  | 3–4  |
| CVAC     | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
Figure 12. Strain data measured from S1: (a) January and (b) April.

Figure 13. Normalized $\text{CorV}_S$ of the bridge (for raw data).

Figure 14. Variation of CVAC value (for raw data). CVAC: cross-correlation function amplitude vector assurance criterion.
Temperature effect analysis on the dynamic strain measured from the suspension bridge

Time-domain analysis

The data of the inactive strain gauge ST1 present the curve which is consistent with that of the environmental temperature data in form, as shown in Figure 15. The data of the active strain gauge S1 show the trend generated by temperature stress, as shown in Figure 12, and display more significant fluctuation in April in comparison with that in January. It illustrates that the larger change in environmental temperature leads to the greater temperature stress in truss. Figure 16 displays an hour’s data of the active strain gauge fixed on the top chord of truss. Strain time curve includes the spike caused by live load in short time and the trend curve generated by temperature load in a longer time period. The strain induced by live load reflects the dynamic characteristic of the bridge. Some of temperature stress-generated strains are even greater than those induced by live load. The data of active strain gauges can be regarded as the superposition of slow-varying component and fast-varying component.

In order to understand the temperature effect on the dynamic strain collected from active strain gauges, the strain data and the environmental temperature are sequentially divided into 1-h long data segments, and the absolute value of the correlation coefficient between the dynamic strain and environmental temperature is calculated based on hourly averages of strain and temperature data segments. The values in Table 4 indicate that the

Figure 15. Strain data measured from ST1: (a) January and (b) April.

Figure 16. One hour of strain data measured from S1.

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The correlation degree between the dynamic strain and temperature in April is greater than that in January. The bridge is subjected to the action of traffic, wind, temperature load, and so on. High correlation coefficients manifest that the temperature strain is the dominant component in the dynamic strain when the daily environmental temperature of the bridge site varies greatly.

**Frequency-domain analysis**

In order to determine the frequency band in which temperature strains mainly locate, the power spectrum density (PSD) of 24 h signals from the active strain gauges (S1–S4) and the inactive strain gauges (ST1) on the upstream members of steel truss are comparatively analyzed from 0 Hz to 25 Hz. Figure 17 shows that the PSD of the signals from ST1 decrease rapidly around 0 Hz. In the range between 0 Hz and 0.6 Hz, PSD continuously decreases about 87 dB. The PSD curve of signals from active strain gauges drops in the extremely low-frequency range. In this frequency band, PSD curve features of signals of the active and inactive strain gauges are similar. The obvious difference for the curves is that the PSD of signals of active strain gauges increase locally in the decay process, and the local peak exists around 0.01 Hz. The result shows that the power distribution difference exists between the signals of active and inactive strain gauges, and the live load-induced strain contributes to the local increase of the PSD.

The power of frequency bands and the power ratios are listed in Table 5. Here, the power ratio is defined as the ratio of power in frequency bands to the total power. The results show that the temperature strain signals mostly locate below 0.001 Hz. Between 0.001 Hz and 0.1 Hz, the power ratios of signals of the active strain gauges are noticeably greater than those of signals of the inactive gauges. The signals induced by the live load may mainly locate above 0.001 Hz.

**Elimination of the effect of varying environmental temperature on CorV_S**

**Analytical mode decomposition method**

Analytical mode decomposition (AMD) method proposed by Chen and Wang is used for extracting strain components. Feldman presented a theoretical interpretation for the AMD method and considered that it was suitable for signal processing as a kind of low-pass filter and required only Hilbert transform (HT).
According to the analysis of power ratio, it is suitable to set cut-off frequency at 0.001 Hz. Here, 0.001 Hz, 0.002 Hz, 0.005 Hz, and 0.01 Hz are, respectively, adopted to investigate the decomposition efficiency. The signals, as shown in Figure 18, were collected in 24 h from the active strain gauge S1. By using the AMD method, the temperature stress-induced strain (the slow-varying component) and the live load-induced strain (the fast-varying component) are separated from strain data, as shown in Figure 19. Figure 20 displays that the burr in the slow-varying component is becoming obvious when cut-off frequency increases to 0.005 Hz.

**Verification of component decomposition for the dynamic strain**

To inspect that if the selection of cut-off frequency is proper and the vehicle load information of strain data are preserved intact in the fast-varying components, the rain-flow counting method is applied to raw strain data and the fast-varying components separated by different cut-off frequencies.

The stress spectrum of 24 h signals of active gauge S1 is obtained through rain-flow counting method. The stress cycles within 2 MPa are more than 2000 times. The stress cycles in the range of 20 to 22 MPa are only a few times, and the amplitude is about 100 με. It should be produced by the heaviest vehicles in 24 h. The stress cycle in

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**Table 5. Power and power ratio of strain signals in frequency bands (10^-12/Hz).**

| Date            | Strain gauge | 0–0.001 Hz          | 0.001–0.1 Hz         | 0.1–25 Hz         | Total power |
|-----------------|--------------|---------------------|----------------------|------------------|-------------|
| 18 January 2010 | ST1          | 5.003 × 10^6 (99.918%) | 1.820 × 10^5 (0.036%) | 2.313 × 10^5 (0.046%) | 5.007 × 10^6 |
|                 | S1           | 1.237 × 10^7 (69.281%) | 5.383 × 10^6 (30.161%) | 0.996 × 10^6 (0.558%) | 1.785 × 10^7 |
|                 | S2           | 6.768 × 10^7 (66.761%) | 3.287 × 10^6 (32.418%) | 0.833 × 10^6 (0.821%) | 1.014 × 10^7 |
|                 | S3           | 3.798 × 10^7 (70.972%) | 0.824 × 10^6 (15.395%) | 0.730 × 10^6 (13.633%) | 5.351 × 10^5 |
|                 | S4           | 2.426 × 10^6 (72.744%) | 7.453 × 10^5 (22.349%) | 1.636 × 10^5 (4.907%) | 3.335 × 10^6 |
| 27 April 2010   | ST1          | 3.992 × 10^7 (99.959%) | 1.610 × 10^6 (0.040%) | 0.262 × 10^6 (0.001%) | 3.994 × 10^9 |
|                 | S1           | 2.730 × 10^7 (99.722%) | 7.309 × 10^6 (0.267%) | 3.061 × 10^6 (0.011%) | 2.738 × 10^9 |
|                 | S2           | 2.048 × 10^7 (99.489%) | 9.730 × 10^6 (0.473%) | 7.982 × 10^6 (0.038%) | 2.058 × 10^9 |
|                 | S3           | 3.373 × 10^7 (99.950%) | 1.484 × 10^6 (0.044%) | 2.083 × 10^6 (0.006%) | 3.375 × 10^9 |
|                 | S4           | 2.789 × 10^7 (99.892%) | 2.442 × 10^6 (0.087%) | 5.764 × 10^6 (0.021%) | 2.792 × 10^9 |

Note: The data within bracket are power ratios.
Figure 18. 24 h of strain data measured from S1.

Figure 19. Decomposition of the dynamic strain measured from S1 (cut-off frequency $f_c = 0.001$Hz): (a) slow-varying component and (b) fast-varying component.

Figure 20. Slow-varying component of the dynamic strain measured from S1 (cut-off frequency $f_c = 0.005$Hz).
The range of 48 to 50 MPa appears only once. The large stress amplitude may be related to the periodic variation in the environmental temperature.

The stress whose amplitude is less than 22 MPa is regarded as the production of live load. For the raw data of S1 and their fast-varying components, the stress cycle numbers in different frequency bands are listed in Table 6. The results illustrate that the data induced by live load in raw data are completely retained in the fast-varying component. Therefore, adopting 0.001 Hz as the cut-off frequency is appropriate.

**Table 6. Stress cycle number of S1 raw data and the fast-varying component.**

| Stress range | Raw data | 0.001 Hz | 0.002 Hz | 0.005 Hz | 0.01 Hz |
|--------------|----------|----------|----------|----------|---------|
| 0–2 MPa      | 2025     | 2022     | 2030     | 2020     | 1996    |
| 2–4 MPa      | 1046     | 1055     | 1054     | 1053     | 1055    |
| 4–6 MPa      | 521      | 519      | 517      | 509      | 533     |
| 6–8 MPa      | 446      | 445      | 446      | 456      | 439     |
| 8–10 MPa     | 289      | 282      | 277      | 272      | 292     |
| 10–12 MPa    | 190      | 191      | 194      | 200      | 198     |
| 12–14 MPa    | 96       | 100      | 99       | 99       | 97      |
| 14–16 MPa    | 47       | 46       | 48       | 43       | 43      |
| 16–18 MPa    | 18       | 20       | 20       | 22       | 20      |
| 18–20 MPa    | 14       | 14       | 14       | 12       | 6       |
| 20–22 MPa    | 5        | 5        | 5        | 7        | 9       |
| 0–22 MPa     | 4697     | 4699     | 4704     | 4693     | 4688    |

**Figure 21.** Normalized CorV_S of the fast-varying component of the dynamic strain.

**Figure 22.** Variation of CVAC value (temperature strain eliminated). CVAC: cross-correlation function amplitude vector assurance criterion.

the range of 48 to 50 MPa appears only once. The large stress amplitude may be related to the periodic variation in the environmental temperature.
CorV_S without temperature effect

After eliminating the slow-varying component (i.e. temperature stress induced strain) from 24 h raw data, CorV_S curves are identical in shape, as shown in Figure 21. The aforementioned CVAC calculations are repeated for the fast-varying component of the 10 days’ raw data. The results indicate that there is only a slight change in the CVAC values, as shown in Figure 22. The slight differences among CVAC values might be generated by the measurement noise and other environmental effects such as humidity, wind, and so on. Therefore, temperature strain is an important factor in the variation of the CorV_S curve.

Conclusions

Under white noise excitation, CorV_S is the function of the natural frequency, mode shape, and damping ratio of a structure. The experiment in a constant temperature lab proves that normalized CorV_S curves of the nondestructive beam are uniform in the shape. The change of CorV_S which can be evaluated based on CVAC indicates the variation of structural operation condition and damage occurrence. The experimental results show that the introduced damage of the beam leads to more obvious change in the CorV_S curve in comparison with the mode shape. CorV_S is proved to be an effective damage feature. In addition, CorV_S is completely constructed in time domain without complicated computation, and no modal parameter identification is needed.

Under the variant temperature condition, the CorV_S curve constructed by raw strain data of the suspension bridge changes over time. It will make us misjudge the real operation state of the structure. To understand the temperature effect on CorV_S, the different components of raw strain data are analyzed in the time domain and the frequency domain. Temperature effect component of the dynamic strain shows the trend form that experiences in a longer period and exists in the extremely low-frequency range. According to the power distribution of signals of the active and inactive strain gauges, 0.001 Hz is regarded as the boundary frequency for the temperature effect component and the live load component of the dynamic strain data and then adopted as cut-off frequency.

Based on the cut-off frequency, the temperature stress-induced strain is effectively separated from the data of active strain gauges through the AMD method, and the live load-induced strain data are remained completely. Then, CorV_S curves of the bridge maintain a uniform shape. It indicates that the bridge is currently in normal condition. Varying temperature stress of the bridge changes the shape of the CorV_S curve. Therefore, the factor of the misjudgment for the bridge operation state is environmental temperature effect, and the temperature effect component in the dynamic strain should be eliminated when CorV_S is applied to structural damage detection.

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