Scaling Behavior and Variable Hopping Conductivity in the Quantum Hall Plateau Transition

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We have measured the temperature dependence of the longitudinal resistivity $\rho_{xx}$ of a two-dimensional electron system in the regime of the quantum Hall plateau transition. We extracted the quantitative form of scaling function for $\rho_{xx}$ and compared it with the results of ordinary scaling theory and variable range hopping based theory. We find that the two alternative theoretically proposed scaling functions are valid in different regions.

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The study of the transition regions separating adjacent quantum Hall (QH) states is an active topic of research in the field of two dimensional electron systems\cite{1, 2}. Several experimental groups have studied the temperature dependence of the half width for the longitudinal resistance $\rho_{xx}$, and of the maximum slope in the Hall resistance $\rho_{xy}$ between neighboring Hall plateaus\cite{3, 4}. A remarkable quantum phase transition has been demonstrated by showing a scaling behavior in temperature: $\frac{d\rho_{xx}}{dT} \propto T^{-\kappa}$ and $\Delta B \propto T^\kappa$ with critical exponent $\kappa$. However, in sharp contrast to short range alloy potential scattering in InGaAs/InP samples, AlGaAs-GaAs heterostructures has long range Coulomb scattering which results in nonuniversality of the temperature exponent $\kappa$\cite{5, 6}. Nevertheless, recently Li et al. realized several Al$_x$Ga$_{1-x}$As-GaAs samples where dominant contribution of the disorder is from short range random alloy potential fluctuation by varying Al concentration $x$ and observed a perfect scaling with $\kappa = 0.42$\cite{7}.

There are extensive theoretical works to understand this quantum critical phenomena and the actual value of $\kappa$. The easiest way to see this is to consider a experiment where the magnetic field $B$ is being varied to make the chemical potential move through $E_c$ to cause the delocalization-localization transitions. In analogy with other localization transitions, one expects a power-law divergence of localization length $\xi \propto |B - B_c|^{-v}$, where $v$ is the localization critical exponent. Since the experimental data were measured at finite temperature, the quantum phase coherence length sets an effective finite system size $L$. Usually $L$ scales with temperature as $L \propto T^{-\beta} \propto T^{-\frac{p}{2}}$, where $p$ or $z$ is called the dynamical exponent. In the critical region, resistance tensor scales as $\rho_{xy} = \rho_c f(B/L)$, where $f$ is a scaling function which can derived from the microscopic calculation. Then one obtains $\rho_{xx} = \rho_c f(|B - B_c| T^{-\kappa})$, the scaling function of both the longitudinal resistance $\rho_{xx}$ and the Hall resistance $\rho_{xy}$, where the exponent is expressed as $\kappa = \frac{p}{2v}$.

Most of the experimental work on the plateau transition in QH effect can be quantitatively understood on the basis of the scaling law defined above, which has sometimes been referred to as the one-parameter scaling theory\cite{8}.

Along this way, a great deal of theoretical work has been carried out to study the quantity $v$\cite{9} and $p$\cite{10, 11, 12, 13, 14}, using different models and different calculation techniques. Several years ago, Sheng and Weng\cite{15} consider the Anderson Hamiltonian subjected to a strong magnetic field, use Kubo formula, and find that the longitudinal resistivity in the critical region well follows a simple exponential form:

$$\rho_{xx} = \rho_c \exp(-s), s = \left(\frac{c_0 |i - i_c|^v}{T^\frac{p}{2}}\right),$$

(1)

where $\rho_c$ is the resistivity of critical point, $s$ is a scaling variable, $z$ is the dynamical exponent, $v$ is the localization exponent, $i = ln/eB$ is the filling factor, is the critical point and $c_0$ is a constant.

On the other hand, an alternative way to obtain this scaling phenomena is to argue that the transport in the
plateau transition is dominated by variable range hopping (VRH) in the presence of Coulomb interaction. By assuming VRH transport responsible for broadening of the \( \sigma_{xx} \) peaks, Polyakov and Shklovskii also arrives at an explicit expression of scaling function

\[
\sigma_{xx} = \sigma_0 \exp(-\sqrt{\frac{T_0}{T}})
\]

with a temperature dependent prefactor \( \sigma_0 \sim 1/T \). Using an appropriate definition of scaling variable, we obtain

\[
\sigma_{xx} = \sigma^* x \exp(-\sqrt{T^* x}), x = \frac{|i - i_c|^v}{T^*}
\]

where \( x \) is a scaling variable, \( \sigma^* \) and \( T^* \) is a constant. Why such hopping-based strong localization theory should have validity in the scaling regime is not clear. However the above two approaches looks very different, we are not aware of a direct measurement to verify them in the same sample.

In a former paper, we have reported the plateau-to-plateau transitions and corresponding exponents \( \kappa \). In the present paper, we focus on the quantitative form of the scaling function \( f \) in the transition regime and compare it with the above two theoretical results. The sample we measured was grown by molecular-beam epitaxy and consists of a 25 nm wide GaAs well bounded on each side by undoped and Si \( \delta \)-doped layers of Al\(_{0.35}\)Ga\(_{0.65}\)As. It has a low-temperature mobility \( \mu = 2.1 \times 10^5 \text{ cm}^2/\text{V s} \) and the electron density \( N_e \) is fixed at \( 2.8 \times 10^{11} \text{ cm}^{-2} \). The sample is of high quality as it shows very strong inte-

FIG. 1: The plateau to plateau transition in longitudinal resistivity and Hall resistivity vs magnetic field.

For simplicity, we focus on the transition from \( i = 3 \) to \( i = 4 \) integer QH states. In Fig.(1) we plot \( \rho_{xy} \) and \( \rho_{xx} \) vs \( B \) at different temperature from 1.79K to 2.69K. A \( \rho_{xx} \) peak that widens with \( T \) and accompanying step in \( \rho_{xy} \) clearly show the transition between the neighboring plateaus around filling factor \( i = 3, 4 \). Next, we convert the \( \rho \)'s to conductivity \( \sigma \)'s using the standard matrix conversion,

\[
\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}, \sigma_{xy} = \frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}
\]

and plot the \( \sigma \) traces in Fig.(2).

One way of extracting quantitative informations from the transition is by conducting a scaling analysis of the data. In Fig.(3), \( \rho_{xx} \) is plotted as a function of scaling variable \( s \) defined in Eq.(1). While the critical point \( i_c = h n/eB_c \approx 3.4 \) can be directly obtained form the data, we vary the value of \( z \) until we obtain the optimal collapse of the \( \rho_{xx} \) traces obtained at different \( T \)'s. We find that the longitudinal resistance exhibits an exponential dependence such as Eq.(1) in the critical region covering a range for \( i_c < i < 3.6 \). The corresponding scaling functions start to deviate from the exponential form beyond the critical region. The resulting values of the critical exponent, \( \kappa = 1/zv = 0.69 \), are indeed close to the values obtained from ordinary approaches using the relations \( \frac{d\rho_{xx}}{dB} \propto B_\nu \propto T^{-\kappa} \) and \( \Delta B \propto T^\nu \).
FIG. 2: Longitudinal conductivity and Hall conductivity vs magnetic field.

FIG. 3: $\rho_{xx}$ as a function of the scaling variable $s$. We use the data of 3-4 transition. Dashed line is the function of Eq.(1) of variable $s$.

In Fig.(4), $\sigma_{xx}$ is plotted as a function of scaling variable $x$ defined in Eq.(2). We vary the value of $v$ until all experiment points of $\sigma_{xx}$ fall onto a straight line as represented by the scaling function in Eq.(3). We find that the longitudinal conductivity $\sigma_{xx}$ exhibits a scaling function dependence such as Eq.(3) in the transition region covering a wide range from $3.5 < i < 3.8$. The corresponding scaling functions start to deviate from the form of Eq.(3) beyond the above region. The resulting values of the critical exponent, $v = 2.35$, are indeed close to the values obtained from Koch. et al.\cite{5, 19}.

In our experiment, we obtain $\rho_{xx}$ or $\sigma_{xx}$ data to study the plateau transition in the region $3 \leq i \leq 4$. From the above analysis, we find that both two scaling form are valid for different regions of plateau transition (one is near the critical point $i_c \approx 3.4$, $3.4 < i < 3.6$, the other is slightly far from the critical point $3.5 < i < 3.8$). Here we proposed an possible physical explanation to support this difference. In integer QH effect, since the Fermi energy is in a region of localized states when the Hall resistivity is quantized and the longitudinal resistivity vanishes, it can be concluded from the wide plateau that most of the electron states are localized at low temperatures. Actually Paalanen et al. estimated that, in an AlGaAs-GaAs heterostructures at 50mK, 95% of the states in each Landau level are localized\cite{20}. The mechanism for the conductivity is variable range hopping (VRH) in the presence of Coulomb interaction. Thus we may expect that VRH can dominates the conductivity in the transition regime, when the localization length $\xi$ becomes much smaller than the effective temperature length $L$. This means Eq.(3) is valid in the region which is slightly far from the critical point $i_c$ (eg. $3.5 < i < 3.8$ in our sample). On the other hand, metal-insulator transition dominates the conductivity in the transition regime, when the
localization length $\xi$ becomes much larger than the effective temperature length $L$. Then Eq.(1) is valid in the region which is near the critical point $i_c$ (e.g. $i_c < i < 3.6$ in our sample). Thus the two different scaling function are valid in different regions in the plateau transition.

In conclusion, we have measured the temperature dependence of the longitudinal resistivity $\rho_{xx}$ of a two-dimensional electron system in the regime of the quantum Hall plateau transition. We extracted the quantitative form of scaling function for $\rho_{xx}$ and compared it with two alternative theoretically proposed scaling functions. Further we determine different regions where the two alternative theoretically proposed scaling functions are valid in the same sample. However we focus on the behavior at high temperatures, similar experiment in lower temperature (several mK-1K region) would be discussed in the near future.²¹

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