Are there the VP couplings in the $\psi(3770)$ non-charmed decays hidden behind the current measurements?

D. Zhang, G. Rong, J.C. Chen

Institute of High Energy Physics, Beijing 100049, China

Abstract

A global analysis of the full amplitudes for $e^+e^- \rightarrow$ VP (Vector and Pseudoscalar) channels at $\sqrt{s}$ =3.773 GeV and 3.670 GeV, which were measured by the CLEO-c Collaboration, shows that those measurements are essentially nontrivial for searching for the $\psi(3770)$ non-$D\bar{D}$ decays. Unlike the nearly negative verdict on the $\psi(3770)$ strong decays to the VP channels in the original analysis of the CLEO-c data, there exist some unusual solutions that predict the remarkable strength of $SU(3)$ symmetry VP decay of $\psi(3770)$ resonance, which give some clue to understand the mechanism of $\psi(3770)$ non-$D\bar{D}$ decays and to reexplain the well-known $\rho - \pi$ puzzle in the $J/\psi$ and $\psi(3686)$ decays.

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*Electronic address: zhangdh@mail.ihep.ac.cn
I. MOTIVATION

There is a long-standing puzzle in understanding the existence measurements for \(\psi(3770)\) and \(D \bar{D}\) production cross sections at the peak of \(\psi(3770)\) production in \(e^+e^-\) annihilation \[1, 2\]. Potential Models predict that \(\psi(3770)\) decays into \(D \bar{D}\) with branching fraction of \(\sim 100\%\). Recently careful investigation shows that the branching fraction of \(\psi(3770)\) non-\(D \bar{D}\) decay would be up to more than \(10\%\) \[3, 4\]. It is very interesting to know what are the exclusive non-\(D \bar{D}\) final states of \(\psi(3770)\) decays. Except the electromagnetic transitions and hadronic transitions of \(\psi(3770)\) to lower charmonium states, are there indeed other significant exclusive non-charmed decay modes from \(\psi(3770)\) decays?

In the charmonium decays, there is another long-standing puzzle in understanding the \(\rho \pi\) decays of \(J/\psi\) and \(\psi(3686)\). The partial widths of \(\rho \pi\) channel and other VP channels in the \(\psi(3686)\) decays are unexpectedly lower than those in \(J/\psi\) decays. This is so called "\(\rho \pi\)" puzzle. Are the \(J/\psi\) decay rates enhanced by some unknown mechanism or the \(\psi(3686)\) decay rates are suppressed abnormally? To investigate the possible source of this puzzle, it is also important to measure the \(\psi(3770)\) VP decay amplitude.

Recently, the BES Collaboration \[3\] observed a large production cross section for \(e^+e^- \rightarrow K^*(892)^0\bar{K}^0 +\text{c.c.}\)

\[
\sigma(e^+e^- \rightarrow K^*(892)^0\bar{K}^0 +\text{c.c.}) = (15.0 \pm 4.6 \pm 3.3) \text{ pb},
\]

at center-of-mass energy of \(\sqrt{s}=3.773\) GeV and found that the \(K^*(892)K^\mp\) production is suppressed. Taking into account the possible interference between the strong decay amplitude and the continuum production amplitude at \(\sqrt{s}=3.773\) GeV, the BES Collaboration set an upper limit on the strong decay partial width for \(\psi(3770) \rightarrow K^*(892)\bar{K} +\text{c.c.}\) to be

\[
\Gamma(\psi(3770) \rightarrow K^*(892)\bar{K} +\text{c.c.}) < 29.0 \text{ keV}
\]

at 90% confidence level.

The CLEO-c Collaboration made more careful studies of twelve exclusive VP decay channels for \(\psi(3770) \rightarrow \rho \pi, K^*(892)\bar{K} +\text{c.c.}, \omega \pi^0, \rho \eta, \rho \eta', \omega \eta, \omega \eta', \phi \eta, \phi \eta'\) and \(\phi \pi^0\) reported in Ref. \[6\]. The CLEO-c Collaboration measured the cross sections for all of the channels at the energies \(\sqrt{s}=3.773\) GeV and \(\sqrt{s}=3.670\) GeV. The CLEO-c results show that the measured cross sections at \(\sqrt{s}=3.773\) GeV are almost equal to or even less than the ones
measured at \( \sqrt{s}=3.670 \) GeV, which mean that the net cross sections for the \( \psi(3770) \) decays are consistent with zero except only for the channel \( \psi(3770) \rightarrow \phi\eta \). The negative results about the \( \psi(3770) \) strong VP decays led people ignoring the important strong decay component existing in \( \psi(3770) \) and only focusing their attention on the form factors of those channels as well as the isospin violation in electromagnetic interaction [6].

In this paper, we develop a model to account for both the amplitudes of electromagnetic (E-M) production and \( \psi(3770) \) strong decay in the process of \( e^+e^- \rightarrow \text{VP} \). By analyzing the cross sections for the exclusive VP channels, which were measured by the CLEO-c Collaboration, we extract out the branching fractions for \( \psi(3770) \) decay to these VP final states.

II. THE MODEL AND THE FORMULAE

In the \( \psi(3686) \) decay sector, because of the smallness of the strong VP decay coupling, the E-M decay component as well as the continuum (E-M) component of the VP channel would be no longer the small amounts comparing with those of strong decay. People have to deal with the two components properly [7]. At the resonance peak, the production amplitude consists of two parts, one is the decay amplitude of charmonium resonance and the other is continuum E-M production amplitude. In the resonance decay part, there are two components as well. They are the E-M decay amplitudes and the strong decay amplitude. Totally, there are three components involved in the \( e^+e^- \) annihilation process at the resonance peak, which are the strong decay component, the E-M decay component and the continuum production component.

Unlike the VP decays of \( J/\psi \) and \( \psi(3686) \), the E-M decay amplitudes of \( \psi(3770) \) can be neglected due to the little tiny dileptonic decay branching fraction. There are only the strong decay amplitude and the continuum production amplitude in the \( e^+e^- \) collision production at \( \sqrt{s}=3.773 \) GeV. Typically, according to the conventional point of view, the partial widths of the \( \psi(3770) \) VP decay channels could be up to keV order of magnitude, like their cousins in \( J/\psi \) decays. However, due to the large width of \( \psi(3770) \) the decay amplitudes of those channels can not get large amplification as the ones at the narrow resonance states. Associated with the measurements of the form factors of channel \( \omega\pi^0 \) at the energies of \( \psi(3686) \) and \( \psi(3770) \) resonance vicinities [6, 8, 9] the decay process with
only a few keV partial width of the rather wide resonance is really hard to be measured if one does not consider the interference between the amplitudes of the strong decay and the continuum production. As the measurements by CLEO-c [6], both the evident yield excess of channel $\phi\eta$ and the rather large yield deficit of the $\rho\pi$ channel at the resonance peak hint that there must be rather complex interference between the two kinds of amplitudes acting globally on the VP channels. Some destructive interference just shows up at $\rho\pi$ channel in the “deficit” way. And more complex interferences cause the $\phi\eta$ yields enhanced at resonance peak. In fact, in such complicate interference case the decay contributions may easily be covered up by the continuum contribution and the interference contributions. If one completely neglects the buried decay contribution, the single E-M amplitude assumption would not describe the measured cross sections well. In practice, it is dangerous to measure the branching fractions for the $\psi(3770)$ non-$D\overline{D}$ decays by simply considering the net yields for the channels observed at the peak of $\psi(3770)$ over that at the nearby off resonance region. In this analysis we introduce the strong decay amplitudes in the analysis formalism to see how the strong decay affects the VP production at the $\psi(3770)$ resonance peak.

We describe the global decay of $\psi(3770)$ and the continuum production process still based on the flavor $SU(3)$ invariant model, which was developed thirty years ago [10]. In this model the strange quark mass correction in both of the strong coupling and the E-M coupling, the wave function nonet symmetry breaking and the double Okubo-Zweig-Iizuka (DOZI) suppression effects are all taken into account. As for the continuum production at the two energy points $\sqrt{s}=3.773$ GeV and 3.670 GeV, except the coherent strong decay amplitudes from the $J/\psi$ and $\psi(3686)$ tails, which can safely be neglected from the calculations, there is only the continuum E-M amplitudes itself. In addition, we guess that the incoherent component contributions which are mainly from the initial state radiative (ISR) return to $J/\psi$ and $\psi(3686)$ resonances have efficiently been rejected in the work reported in Ref. [6] and can be neglected in our analysis too.

Following the convention given in Ref. [11], we define that $g$ represents the VP strong decay amplitude in the flavor $SU(3)$ symmetry limit; $g_s$ represents the strong decay amplitude from $s$ quark,

$$s_g = 1 - (g_s/|g|)/2,$$

characterizes the $SU(3)$ mass violation, which is as the same as the parameter “$s$” given in the Tab. VIII of Ref. [11]; $\theta_P$ represents the $\eta-\eta'$ mixing angle; the product $r \cdot g$ represents
the amplitude correction of the SU(3) nonet symmetry violation with the factor \((1 - s_p)\) for a strange pseudoscalar production and with the factor \((1 - s_V)\) for a strange vector production. If \(s_V = s_p = 0\), (exactly \(s_V + s_p = 0\)), \(r \cdot g\) measures the pure DOZI amplitude correction. Unlike the case in Ref. \[11\], because the E-M amplitude is no longer small comparing with the strong amplitude, we have to consider both the isoscalar and isovector components of the E-M amplitude. We define the E-M amplitude in the form of SU(3) octet matrix representation as

\[
E = e_1 \cdot I_3 + e_0 \cdot Y
\]

in which \(e_0\) and \(e_1\) are the isoscalar and isovector components, respectively, and \(I_3\) and \(Y\) are, respectively, the isospin third component and the hypercharge matrices in flavor SU(3) octet space. We define \(\theta_0\) as the phase of \(e_0\) relative to \(g\), \(\delta_1\) as the phase shift difference of \(e_1\) to \(e_0\) and a factor \((1/2 - s_e)\) as the correction for strange quark coupling to E-M isoscalar part \(e_0\). We assume that the couplings \(e\)'s and their phases do not change in the all VP channels, and their moduli at the two different energy points \(\sqrt{s}=3.773\) GeV and \(\sqrt{s}=3.671\) GeV only change with a \(1/s^3\) dependence. If \(e_1 = e_0\), we return to common definition as Refs. \[10, 11\] did.

For the channels \("ch"\), \((ch=\rho^0\pi^0, K^{*0}\overline{K}^0 + c.c., etc)\) at energy \(\sqrt{s}\), \(M_{res,\sqrt{s}}^{ch}\) denotes the resonance decay amplitude and \(M_{ctm,\sqrt{s}}^{ch}\) denotes the continuum production amplitude. The total production amplitudes are then written as

\[
M_{\sqrt{s}}^{ch} = M_{res,\sqrt{s}}^{ch} + M_{ctm,\sqrt{s}}^{ch},
\]

in which

\[
M_{res,3670}^{ch} = 0,
\]

and

\[
M_{ctm,3670}^{ch} = M_{ctm,3770}^{ch} \cdot f_d
\]
at the energy of \(\sqrt{s}=3.670\) GeV, where \(f_d = 3.773^3/3.670^3\) is the scaling factor for the \(1/s^3\) energy dependence of the cross section. The amplitudes \(M_{ctm,3770}^{ch}\) and \(M_{res,3770}^{ch}\) defined at \(\sqrt{s}=3.773\) GeV for all of the channels are listed in Tab. \[\]

In the table, \(X_\eta = \cos(54.736^o + \theta_p)\), \(Y_\eta = \sin(54.736^o + \theta_p)\), \(X_\eta' = -\sin(54.736^o + \theta_p)\) and \(Y_\eta' = \cos(54.736^o + \theta_p)\), which are the same as those given in Ref. \[11\]. Those amplitudes completely control the correlations among the VP channel productions. If any significant decay amplitude \(M_{res,3773}^{ch}\) given in
TABLE I: The Amplitudes for VP production in $e^+e^-$ annihilation at $\sqrt{s} = 3.773$ GeV. The coupling $g_K$ defined in channels $K^*\bar{K} + \text{c.c.}$ can be considered as a free parameter if one of $g$ and $g_s$ is fixed.

| Channel(ch)         | $M_{res,3770}^ch$ | $M_{ctm,3770}^ch$ |
|---------------------|-------------------|-------------------|
| $\rho^0\pi^0, \rho^\pm\pi^\mp$ | $g$              | $e_0$             |
| $\omega\eta$       | $gX_\eta + \sqrt{2}rg[\sqrt{2}X_\eta + (1 - s_\rho)Y_\eta]$ | $e_0X_\eta$ |
| $\phi\eta$         | $g_sY_\eta + r(g_1 - s_\rho)[\sqrt{2}X_\eta + (1 - s_\rho)Y_\eta]$ | $-2e_0(1 - s_\rho)Y_\eta$ |
| $\omega\eta'$      | $gX_{\eta'} + \sqrt{2}rg[\sqrt{2}X_{\eta'} + (1 - s_\rho)Y_{\eta'}]$ | $e_0X_{\eta'}$ |
| $\phi\eta'$        | $g_sY_{\eta'} + r(g_1 - s_\rho)[\sqrt{2}X_{\eta'} + (1 - s_\rho)Y_{\eta'}]$ | $-2e_0(1 - s_\rho)Y_{\eta'}$ |
| $\omega\pi^0$      | $0$               | $3e_1$            |
| $\phi\pi^0$        | $0$               | $0$               |
| $\rho^0\eta$       | $0$               | $3e_1X_\eta$      |
| $\rho^0\eta'$      | $0$               | $3e_1Y_{\eta'}$   |
| $K^{*0}\bar{K}^0 + \text{c.c.}$ | $g_K = (g + g_s)/2$ | $-e_0(1/2 - s_\rho) - 3/2e_1$ |
| $K^{*\pm}\bar{K}^\mp$ | $g_K = (g + g_s)/2$ | $-e_0(1/2 - s_\rho) + 3/2e_1$ |

Tab. I has been measured to be non-zero, which means that the measured couplings $g$, $g_s$ etc. are non-zero, this indicates that $\psi(3770)$ has a significant branching fraction for decay to the non-charmed channel “ch”.

We start the analysis from the observed numbers of the events for the VP channels, their errors and their corresponding detection efficiencies at the energies of $\sqrt{s} = 3.773$ GeV and 3.670 GeV, which were published by CLEO-c Collaboration [6]. The numbers and errors of the events are obtained in both the signal windows and the side bands. Tab. II shows those numbers and detection efficiencies which are given in Ref. [6]. Taking the numbers of the events from Tab. II we obtain the numbers,

$$N_{\sqrt{s},\text{ch}}^{\text{obs}} = (N_{sw}^{\sqrt{s}} - N_{sb}^{\sqrt{s}})_{\text{ch}}, \quad (3)$$

of the observed events at $\sqrt{s}$ for the channel “ch”. Usually the determinations of the detection efficiencies and the ISR corrections are all energy dependent and relate to the production line shapes for those channels. We guess that the determinations of the detection efficiencies
TABLE II: The numbers of the events observed by CLEO-c, the subscripts $sw$ and $sb$ indicate the signal window and the side band window, respectively; the upper script 3.67 and 3.77 indicate the c.m. energies.

| Channel $(ch)$ | $N^{3.67}_{sw}$ | $N^{3.67}_{sb}$ | $N^{3.77}_{sw}$ | $N^{3.77}_{sb}$ | $\epsilon(\%)$ |
|---------------|----------------|----------------|----------------|----------------|--------------|
| $\rho\pi$     | 43             | 5.4            | 314            | 44.8           | 26.3         |
| $\rho^0\pi^0$ | 21             | 3.4            | 130            | 33.0           | 32.5         |
| $\rho^+\pi^+$ | 22             | 2.0            | 184            | 11.8           | 23.1         |
| $\omega\pi^0$ | 54             | 6.2            | 696            | 39.2           | 19.0         |
| $\phi\pi^0$   | 1              | 1.6            | 2              | 40             | 16.5         |
| $\rho^0\eta$  | 36             | 3.1            | 508            | 31.0           | 19.6         |
| $\omega\eta$  | 4              | 0.0            | 15             | 6.0            | 9.9          |
| $\phi\eta$    | 5              | 1.0            | 132            | 15.9           | 11.0         |
| $\rho^0\eta'$ | 1              | 0.0            | 27             | 0.9            | 2.9          |
| $\omega\eta'$ | 0              | 0.0            | 2              | 0.0            | 1.5          |
| $\phi\eta'$   | 0              | 0.0            | 9              | 2.0            | 1.2          |
| $K^{*0}\bar{K}^0$ + c.c. | 38 | 0.4           | 501            | 18.1           | 8.8          |
| $K^{*\pm}\bar{K}^\mp$ | 4 | 1.0 | 36 | 32.4 | 16.0 |

given in Ref. [6] were done under the assumption of that the continuum cross section line shape is in the $1/s^3$ energy dependence and the energy cut is at $\sqrt{s} \geq J/\psi$ mass, where $\sqrt{s}$ is the center of mass energy of the ISR return system. It should be stressed that, for the ISR and FSR (Final State Radiative) corrections in the continuum processes, our calculation gives $\eta_{ctm} = 1.19$ at $\sqrt{s} = 3773$ GeV and $\eta_{ctm} = 1.11$ at $\sqrt{s} = 3670$ GeV, while Ref. [6] gives $\eta_{ctm} = 1/1.20 = 0.833$ at both of the two energy points. As for the resonance decay, the ISR correction is quite different from the one for the continuum process. In our calculation, the ISR correction factor for the resonance is $\eta_{res} = 0.824$ including the FSR correction at $\sqrt{s} = 3.773$ GeV for all channels. In our ISR correction calculations, the $v^3$ phase space dependences have been taken into account, where $v = \sqrt{[1 - (m_V + m_P)^2/s][1 - (m_V - m_P)^2/s]}$ is the velocity of the vector daughter in the CM decay system. However, in the calculation of $\eta_{ctm}$’s, a mean production threshold of the channels has been set to serve as the
common threshold for all of those channels. For this reason, the channel dependences of the corrections $\eta_{ctm}$’s are ignored in this analysis. For the calculation of the contribution of the interference terms among the amplitudes describing different processes, we have to know their own detection efficiencies and ISR corrections. In the analysis we simply take the geometric average of the related coefficients as the effective ones.

Using the amplitudes of VP channels in Eq.(2) and Tab. I, the efficiencies for those channels, the ISR and FSR correction as given above, the $v^3$ phase space dependences and the luminosities $L_{\sqrt{s}}$ accumulated at the two collision energy points of $\sqrt{s} = 3.773$ GeV and 3.670 GeV, we can calculate the expected numbers $N_{3670}^{\text{exp,}ch}$ and $N_{3773}^{\text{exp,}ch}$ of the event yields for those channels under the simplification assumption concerning the interference terms mentioned above. For channel “$ch$”

$$N_{3670}^{\text{exp,}ch} = L_{3670} \cdot v^3_{3670} \cdot \epsilon_{ctm}^{ch} \cdot \eta_{ctm,3670} \cdot |M_{ctm,3670}^{ch}|^2$$

and

$$N_{3773}^{\text{exp,}ch} = L_{3773} \cdot v^3_{3773} \times \left| \sqrt{\epsilon_{res}^{ch} \cdot \eta_{res}^{ch} \cdot M_{res,3773}^{ch}} \right|^2 \times \left| \sqrt{\epsilon_{ctm}^{ch} \cdot \eta_{ctm}^{ch} \cdot M_{ctm,3773}^{ch}} \right|^2.$$

Comparing the numbers $N_{\sqrt{s}}^{\text{obs,}ch}$ defined in Eq.(3) with the expected one $N_{\sqrt{s}}^{\text{exp,}ch}$, we get the equation set

$$N_{\sqrt{s}}^{\text{exp,}ch} = N_{\sqrt{s}}^{\text{obs,}ch}$$

in which $ch = \rho \pi$, $K^*(892) K^0 c.c.$, $\omega \pi^0$, $\rho \eta$, $\rho \eta'$, $\omega \eta$, $\omega \eta'$, $\phi \eta$, $\phi \eta'$ and $\phi \pi^0$, $\sqrt{s} = 3.670$ and 3.773 GeV. Because of the zero observation and zero expectation for the $\phi \pi^0$ channel, listed in Tab. II and Tab. I we can get rid of this channel in our analysis. So we only focus our attention on the rest eleven channels.

In the maximum likelihood fit, leaving the parameters $g$, $g_s$, $e_0$, $e_1$, $r$, $s_e$, $s_N$, $s_P$, $\theta_P$, $\delta_1$ and $\cos \theta_0$ free, we can solve the Eq.(4) by maximizing the probability function

$$Prob = \prod_{ch,\sqrt{s}} P_{ch}(N_{\sqrt{s}}^{\text{obs,}ch}, N_{\sqrt{s}}^{\text{exp,}ch})$$

where $P_{ch}(N_{\sqrt{s}}^{\text{obs,}ch}, N_{\sqrt{s}}^{\text{exp,}ch})$ is the probability of finding $N_{\sqrt{s}}^{\text{obs,}ch}$ events with the assumed mean number $N_{\sqrt{s}}^{\text{exp,}ch}$ at the energy $\sqrt{s}$, and get the solution of Eq.(4) with most probable values of the parameters which control the VP channel production at $\sqrt{s} = 3.773$ and 3.670 GeV.
From the solutions of $g$, $r$, $e_0$ and/or $e_1$ etc, one can get the Born cross sections for the VP channel production at $\sqrt{s}=3.773$ or 3.670 GeV. For example, the Born cross section for decay channel $\psi(3770) \to \text{"ch"}$ at $\sqrt{s}=3.773$ GeV can be written as

$$\sigma_{\text{res}}^{\text{ch}} = v_{3773}^3 \cdot |M_{\text{res,3773}}^{\text{ch}}|^2. \quad (6)$$

The total Born cross section for $e^+e^-$ annihilation to the channel $\text{"ch"}$ at $\sqrt{s}=3.773$ GeV is given by

$$\sigma_T^{\text{ch}} = v_{3773}^3 \cdot |M_{\text{res,3773}}^{\text{ch}} + M_{\text{ctm,3773}}^{\text{ch}}|^2. \quad (7)$$

While the Born cross section for the channel $\text{"ch"}$ in continuum production at $\sqrt{s}$ is then given by

$$\sigma_{\text{ctm,}\sqrt{s}}^{\text{ch}} = v_{\sqrt{s}}^3 \cdot |M_{\text{ctm,}\sqrt{s}}^{\text{ch}}|^2. \quad (8)$$

III. THE RESULTS

According to the different coupling configurations of the amplitudes for the VP channels, shown in Eq.(2) and Tab. I, the twelve VP channels can be divided into three sub-sets. The first one consists of the channels without E-M isovector components, such as $\rho\pi^0$, $\omega\eta$, $\phi\eta$, $\omega\eta'$ and $\phi\eta'$. The second one consists of the E-M isovector component only, which is the pure E-M channels $\omega\pi^0$, $\rho\eta$, and $\rho\eta'$. The third subset includes only the channels $K^+\overline{K} + c.c.$, for which the amplitudes involve all of the two E-M coupling parts and the strong coupling component as well as their interferences in the VP production. Since there is no common coupling parameter in the first two sub-sets despite of the pseudoscalar mixing angle $\theta_P$, one can simply try to solve Eq.(4) separately in the two sub-sets at first.

A. The channels without isovector E-M amplitude

We start the analysis from the first set in which the channels are without the isovector E-M amplitude contribution. From the numbers of events, $N_{\text{obs,}\sqrt{s}}^{\text{ch}}$, observed at the two energy points of $\sqrt{s}=3.671$ GeV and 3.773 GeV for the six channels $\rho^0\pi^0$, $\rho^{\pm}\pi^\mp$, $\omega\eta$, $\phi\eta$, $\omega\eta'$ and $\phi\eta'$, we solve the Eq.(4). Leaving all of the related parameters, such as $|g|$, $g_s$, $|e_0|$, $\cos\theta_0$, $s_e$, $\theta_P$, $r$, $s_V$ and $s_P$ free, the fitting yields $|g| = 2.752 \pm 0.291$. 


\(g_s = 1.279 \pm 0.706,\)
\(\cos \theta_0 = -0.935 \pm 0.141,\)
\(|e_0| = 1.495 \pm 0.126,\)
\(s_e = 0.264 \pm 0.247,\)
\(r = -0.236 \pm 0.070,\)
\(\theta_P = (-17.83 \pm 12.12)^\circ,\)
\(s_V = 0.092 \pm 0.941\)
and
\(s_P = -0.091 \pm 0.587.\)

Indeed, as hinted by the measurements from the CLEO-c \([6]\) and guessed above, the solution of \(\cos \theta_0 = -0.935 \pm 0.141\) shows that \(g\) and \(e_0\) are almost opposite in the VP production at \(\psi(3770)\) resonance peak. Associated with earlier measurements in the \(J/\psi\) production and decays, in which the E-M decay amplitudes and the strong decay amplitude are more likely with the phase difference of \(\phi_{e,\text{res}} - \phi_g \sim +90^\circ\) \([12, 13]\), and the phase difference between the continuum amplitude and the one of resonance E-M decay at the resonance peak is \(\phi_{e,\text{ctm}} - \phi_{e,\text{res}} = +90^\circ\) too \([14, 15]\), the measurement of the phase difference \(\theta_0 = \phi_{e,\text{ctm}} - \phi_g = (160^{+20}_{-17})^\circ\) between \(e_0\) and \(g\) here is reasonable. The 90\(^\circ\) phase difference between the E-M decay amplitudes and the strong decay amplitude shown in \(J/\psi\) decays \([12, 13]\) is mainly due to the exist of an original 90\(^\circ\) phase difference from the short distance force range in the on-shell three gluon annihilation of \(1^-\) quarkonium states. This argument should be kept in the \(\psi(3770)\) decays. Out of the hard quark-gluon interaction level the strong phase shift of long distance final state interaction should essentially be small \([16]\).

The resolutions of \(s_V = 0.092 \pm 0.941\) and \(s_P = -0.091 \pm 0.587\), which are consistent with zero, seem to mean the small nonet symmetry breaking between the singlet and octet wave functions. So we can assume that the nonet symmetry maintains for the wave functions, which means that the nonet symmetry violation is only in the DOZI coupling. If we fix \(s_V = s_P = 0\) in the fit, we get the parameters \(|g|, g_s, |e_0|, \cos \theta_0, s_e, \theta_P\) and \(r\), which are listed in the second column ("Solution 1") of Tab. \(\text{TIII}\), where \(\theta_P = (-18.73^{+5.8}_{-4.7})^\circ\) indicates

\[X_\eta = Y_{\eta'} = 0.809^{+0.045}_{-0.064} \text{ and } Y_\eta = -X_{\eta'} = -0.588^{+0.068}_{-0.079}.\]
TABLE III: The fitted parameters.

|                      | Solution 1 | Solution 2 | Solution 3 | Solution 4 |
|----------------------|------------|------------|------------|------------|
|                      | the channels | the channels | all channels | all channels |
|                      | without isovector | with isovector | with | without |
|                      | E-M component | E-M component | counter terms | counter terms |
| | | | | |
| $|g|$ | $2.64^{+0.41}_{-0.54}$ | $2.64$(fixed) | $2.67^{+0.38}_{-0.32}$ | $2.43^{+0.31}_{-0.29}$ |
| $g_s$ | $1.23^{+0.07}_{-0.74}$ | $-1.79^{+0.42}_{-0.36}$ | $1.18^{+0.71}_{-0.78}$ | $-1.23^{+0.52}_{-0.49}$ |
| $s_g$ | $0.27^{+0.14}_{-0.13}$ | $0.84^{+0.17}_{-0.20}$ | $0.28 \pm 0.16$ | $0.75 \pm 0.42$ |
| $g_s^{add}$ | | $-1.51^{+0.43}_{-0.40}$ | | $0.0$(fixed) |
| $\cos\theta_0$ | $-0.91^{+0.12}_{-0.13}$ | $-0.91$(fixed) | $-0.91 \pm 0.07$ | $-0.86^{+0.06}_{-0.04}$ |
| $|e_0|$ | $1.49^{+0.12}_{-0.13}$ | $1.49$(fixed) | $1.49^{+0.12}_{-0.13}$ | $1.44^{+0.13}_{-0.14}$ |
| $s_e$ | $0.25^{+0.19}_{-0.22}$ | $-0.35 \pm 0.11$ | $0.20^{+0.20}_{-0.23}$ | $-0.29^{+0.13}_{-0.14}$ |
| $a^\mu$ | | $-0.55^{+2.38}_{-0.22}$ | | $0.0$(fixed) |
| $|e_1|$ | $1.223^{+0.016}_{-0.018}$ | $1.226^{+0.017}_{-0.019}$ | $1.22 \pm 0.02$ | |
| $\delta_1$ | | $(7.39^{+5.90}_{-7.69})^o$ | $(6.70^{+6.59}_{-8.19})^o$ | $(7.62^{+8.88}_{-6.02})^o$ |
| $\theta_p$ | $(-18.7^{+5.8}_{-4.7})^o$ | $(-23.2 \pm 2.2)^o$ | $(-22.5^{+1.9}_{-2.0})^o$ | $(-24.1^{+2.1}_{-2.3})^o$ |
| $r$ | $-0.24 \pm 0.06$ | $-0.26 \pm 0.06$ | $-0.32^{+0.09}_{-0.08}$ | |
| $s_V$ | $0.0$(fixed) | $0.0$(fixed) | $-0.42^{+0.36}_{-0.59}$ | |
| $s_P$ | $0.0$(fixed) | $0.0$(fixed) | $0.45 \pm 0.70$ | |
| $\chi^2/n_{dof}$ | $4.63/5=0.93$ | $3.98/5=0.80$ | $9.26/11=0.84$ | $13.2/11=1.20$ |

The fit gives $\chi^2/n_{dof} = 4.63/5 = 0.93$, which is also listed in Tab. III With the $g$ and $g_s$, we obtain the $SU(3)$ mass correction

$$s_g = 0.268^{+0.140}_{-0.127}.$$  

Except for the three coupling strengths, $|e_0|$, $|g|$ and $|g_s|$, which have their own dynamics in the higher energy position, the other relative correction parameters, $s_g$, $s_e$, $r$ and the mixing angle $\theta_p$ obtained from the fit are all reasonable comparing with those obtained from $J/\psi$ decays measured by Mark-III and DM2 [17]. However, from Eqs.(2) and (6), we find that the strong decay coupling $|g|$ gives quite large cross section, branching fraction and the
partial width for \( \psi(3770) \rightarrow \rho\pi \) decay, which are

\[
\begin{align*}
\sigma_{\text{res}}^{\rho\pi} &= (18.2_{-6.70}^{+6.11}) \text{ pb} \\
B^{\rho\pi} &= (1.83_{-0.67}^{+0.62}) \times 10^{-3} \\
\Gamma^{\rho\pi} &= 49.7_{-18.3}^{+16.9} \text{ keV}.
\end{align*}
\]

The partial width is almost in two order of magnitude higher than that of the conventional typical partial width of the \( J/\psi \) VP decays. The latter one is at the order of 1 keV. If we assume that the fraction of the width of \( \psi(3770) \rightarrow \rho\pi \) to the width of \( \psi(3770) \rightarrow \text{light hadrons} \) is roughly as the same as the one in the \( J/\psi \) decays, the huge partial width of \( \psi(3770) \rightarrow \rho\pi \) would predict that about 10\% of \( \psi(3770) \) decays to non-\( D\bar{D} \) final states. This predicted branching fraction for \( \psi(3770) \rightarrow \text{non-} D\bar{D} \) is almost as the same as the one measured by BES-II Collaboration \[3, 4\].

The large cross section \( \sigma_{\text{res}}^{\rho\pi} \) as shown in Eq. (9) is a factor of more than 3 of the total \( \rho\pi \) production cross section,

\[
\sigma_T^{\rho\pi} = 5.35_{-1.58}^{+1.45} \text{ pb},
\]

obtained from Eq. (7). This cross section is consistent with the measurement reported in Ref. \[6\], (see Tab. IV). In the Tab. IV we list all of the production cross sections and the branching fractions for \( \psi(3770) \rightarrow \text{VP} \) predicted in this work, and also listed the production cross sections of the VP channels given in Ref. \[6\] as the comparison.

As for the \( \rho\pi \) production cross section at \( \sqrt{s} = 3.670 \text{ GeV} \) in this measurement, we get

\[
\sigma_{T,3670}^{\rho\pi} = 6.34_{-1.03}^{+1.05} \text{ pb},
\]

which is indeed higher than the one given in Eq. (10) at resonance peak. Because of the different determinations of the ISR corrections in Ref. \[6\] and in our work, the decrease of the Born cross section of \( \rho\pi \) channel at \( \psi(3770) \) resonance peak is not so large as that obtained by CLEO-c Collaboration \[6\] (see also Tab. IV). Owing to the large cancellation between the two amplitudes \( g \) and \( e_0 \), the large cross section \( \sigma_{\text{res}}^{\rho\pi} \) for \( \psi(3770) \rightarrow \rho\pi \) disappeared without the global amplitude analysis.

**B. The pure E-M channels \( \omega\pi^0, \rho^0\eta \) and \( \rho^0\eta' \) plus the channel \( K^+\bar{K}+\text{c.c} \)**

We consider together the last two sub-sets of the channels in which the amplitudes of E-M production contain isovector component. The isoscalar E-M component and the strict
decay coupling only serve in the channel $K^*\overline{K}+c.c.$, and the strong coupling here is with a combined form $g_K = (g + g_s)/2$. Inserting the ten numbers of the events observed at the two energy points of 3.670 GeV and 3.773 GeV for the rest five channels into Eq. (4), leaving $e_1, \delta_1, s_e, \theta_P$ and $g_K$ free (or instead of $g_K$, leaving $g_s$ free but fixing $g$ at some reasonable value such as $|g|=2.635$ obtained from last solution independently), and fixing $|e_0|=1.494$ and $\cos\theta_0=-0.907$ obtained also from last solution in assumption of that there is no more correction added to the couplings $g$ and $|e_0|$ measured in last solution in the channel $K^*\overline{K}$, we fit the numbers of events observed in the five channels and obtain the solution of the free parameters. The results are listed in the third column (“Solution 2”) of Tab. III. The fit gives $\chi^2/n_{dof} = 3.98/5 = 0.80$.

From above two solutions, we see that the measured pseudoscalar mixing angles $\theta_P$ in the two independent measurements are consistent with each other. The isovector E-M component, $e_1$, is really split from the isoscalar one, $e_0$, with almost 2$\sigma$ deviation in magnitude and with a non-zero phase shift difference. However, we note that the $s$ quark strong decay coupling $g_s$ and its E-M coupling correction $s_e$ in the solution 2 are both with the negative values, while they are supposed to be positive in the conventional $SU(3)$ invariant model with simple static mass corrections. The minus $s_e$ is formally very likely to be some anomalous “magnetic moments” term added to the $s$ quark E-M coupling. As for the minus $g_s$, which is obviously irrelevant to the three pure E-M channels, it seems that the $s$ quark strong coupling undergo almost 180$^o$ phase shift from the conventional $SU(3)$ strong interaction wave function. The odd behavior of the minus $g_s$ as well as the E-M “anomalous magnetic moments” term for the $s$ quark indicate that there might be some other dynamic sources or more complicated interaction correction contributing to the production of $K^*\overline{K}+c.c.$ in $e^+e^-$ annihilation.

Owing to the cancellation of $g$ and the opposite $g_s$ (small $g_K$), the decay cross section, branching fraction and partial width

$$\begin{align*}
\sigma_{res}^{K^*\overline{K}} &= (0.558^{+0.702}_{-0.379}) \text{ pb} \\
B^{K^*\overline{K}} &= (0.56^{+0.70}_{-0.38}) \times 10^{-4} \\
\Gamma^{K^*\overline{K}} &= (1.52^{+1.92}_{-1.03}) \text{ keV,}
\end{align*}$$

for $\psi(3770) \to K^*\overline{K}+c.c$ in this solution are quite small comparing with that of $\psi(3770) \to \rho\pi$ measured in last solution given in Eq.(9). These results are also listed in Tab. IV.
The remarkable increase of the E-M coupling $e_0$ by negative correction $s_e$ and the large interferences between the two E-M amplitudes and the strong amplitudes result a serious asymmetry between the total production cross sections of channels $K^{*0}\overline{K}^0$ +c.c. and $K^{*\pm}K^{\mp}$. For example, at $\sqrt{s} = 3.773$ GeV the cross sections

$$\sigma_K^{K^{*0}\overline{K}^0 +\text{c.c.}} = 19.30^{+3.55}_{-1.85} \text{ pb}$$

$$\sigma_K^{K^{*\pm}K^{\mp}} < 0.50 \text{ (pb, at 90% confidence level)},$$

are consistent with the observed values reported in Refs. [6] and [9]. Using the parameters $|e_1|$ and $\theta_P$, and from Eq.(8) we get the production cross sections of the three pure E-M channels,

$$\sigma_{\omega\pi^0}^{\text{ctm}} = (11.74^{+0.31}_{-0.34}) \text{ pb}$$

$$\sigma_{\rho\eta}^{\text{ctm}} = (8.00 \pm 0.22) \text{ pb}$$

$$\sigma_{\rho\eta'}^{\text{ctm}} = (2.57 \pm 0.28) \text{ pb}$$

These are also listed in Tab. [IV]. The E-M production cross sections and the total production cross sections obtained in this subsection are all systematically lower than those measured by CLEO-c Collaboration [6] by about 30%, (see Tab. [IV]). Those differences are also due to the different determinations of ISR corrections in the two works as mentioned above.

C. The global fit including all of the measured VP channels

We can introduce two additional effective counter terms, $g^{\text{add}}_s$ and $a^\mu$, to compensate the odd behavior appeared in the channel $K^*(892)\overline{K}^0$ +c.c. for both the strong coupling and E-M coupling of $s$ quark. We can simply assume that $g^{\text{add}}_s$ and $g$, $a^\mu$ and $e_0$ are collinear, respectively. In this case, we define the amplitudes of channels $K^*(892)\overline{K}^0$ +c.c. as

$$M_{\text{res,3770}}^{K^{*+}K^{*-}K^+} = (g + g_s)/2 + g^{\text{add}}_s,$$

$$M_{\text{ctm,3770}}^{K^{*+}K^{*-}K^+} = -e_0(1/2 - s_e) + 3/2e_1 + a^\mu,$$

$$M_{\text{res,3770}}^{K^{*0}\overline{K}^0 +\text{c.c.}} = (g + g_s)/2 + g^{\text{add}}_s,$$

$$M_{\text{ctm,3770}}^{K^{*0}\overline{K}^0 +\text{c.c.}} = -e_0(1/2 - s_e) - 3/2e_1 + a^\mu,$$

as given in Tab. [I]. With those amplitudes we globally solve the Eq.(11) with all of the observed channels. Fixing $s_V = s_P = 0$ and leaving all of the other parameters including $g^{\text{add}}_s$ and $a^\mu$ free, the fit gives the most probable values of parameters $|g|$, $g_s$, $g^{\text{add}}_s$, $\cos\theta_0$, $|e_0|$, etc.
\(s_e, a^u |e_1|, \delta_1, r, \theta_p\). Those results are listed in the fourth column ("Solution 3") of Tab. III with \(\chi^2/n_{\text{dof}}=9.26/11=0.84\). From the parameters of solution 3 and Eqs.\((2),(6),(7),(8)\) and \((15)\), we can get the production cross sections of the twelve VP channels, including the zero measurement of channel \(\phi \pi^0\). For example, for the channels \(\rho \pi\) and \(K^*(892)\bar{K}+c.c.\), the decay cross sections from \(\psi(3770)\) resonance, and their decay branching fractions and partial widths can be calculated as

\[
\begin{align*}
\sigma_{\text{res}}^{\rho\pi} &= (18.68^{+4.12}_{-4.19}) \text{ pb} \\
B^{\rho\pi} &= (1.87^{+0.41}_{-0.42}) \times 10^{-3} \\
\Gamma^{\rho\pi} &= 51.1^{+11.2}_{-11.5} \text{ keV} \\
\sigma_{\text{res}}^{K^*\bar{K}} &= (0.56^{+1.28}_{-0.22}) \text{ pb} \\
B^{K^*\bar{K}} &= (0.56^{+1.28}_{-0.22}) \times 10^{-4} \\
\Gamma^{K^*\bar{K}} &= (1.52^{+3.50}_{-0.60}) \text{ keV}.
\end{align*}
\]

And the production cross sections of the three pure E-M channels can be calculated as

\[
\begin{align*}
\sigma_{\text{ctm}}^{\omega\pi^0} &= (11.79^{+0.34}_{-0.36}) \text{ pb} \\
\sigma_{\text{ctm}}^{\rho\eta} &= (7.92 \pm 0.41) \text{ pb} \\
\sigma_{\text{ctm}}^{\rho\eta'} &= (2.69 \pm 0.29) \text{ pb}.
\end{align*}
\]

The measured values in Eqs.\((16),(17)\) are all consistent with those measured in the last two subsections. As for the measurements of other channels, we have

\[
\begin{align*}
B^{\omega\eta} &= (2.12^{+0.75}_{-0.48}) \times 10^{-4} \\
B^{\omega\eta'} &< 0.25 \times 10^{-4}, \ (\text{ at } 90\% \ \text{c.l.}) \\
B^{\phi\eta} &= (0.87^{+0.57}_{-0.58}) \times 10^{-4} \\
B^{\phi\eta'} &< 0.60 \times 10^{-4}, \ (\text{ at } 90\% \ \text{c.l.}),
\end{align*}
\]

obtained in the assumption of that there exist the counter terms \(g_{s}^{\text{add}}\) and \(a^u\) and \(s_{V}=s_{P}=0\).

If we fix the strong couplings \(g\) and \(g_s\) to be the values which force the partial widths of \(\rho\pi\) and \(K^*\bar{K}\) etc. to be at the order of 1 keV which is at the same order of the \(J/\psi\) VP decay coupling dynamics, the fitted \(\chi^2\) is 47.27 for 15 degree of freedom which corresponds to 5.3 standard deviation worse than that of solution 3.

If there essentially were no the two counter terms specially for channel \(K^*\bar{K}+c.c.\) \((g_{s}^{\text{add}} = a^u = 0)\), i.e. there existed the negative coupling \(g_s\) and negative correction \(s_e\) universally
allowed on all other relative channels with the value $s$ quark, the fit gives a somehow poor solution with large $\chi^2=19.2$ for 13 degree of freedom, which gives 2.7 standard deviation away from the counter term assumption (solution 3). However, in this case, the effects of the opposite $g_s$ and negative $s_e$ in the related channels of $\phi\eta$ and $\phi\eta'$ might be compensated by somehow larger nonet symmetry breaking with non-zero $s_V$ and $s_P$. Ignoring the terms $g_{s_{\text{add}}}$ and $a^\mu$ used in the last solution and leaving the nonet symmetry breaking parameters $s_V$ and $s_P$ free, we solve the Eq. (11) again to obtain a new solution. The results are listed in the fifth column ("Solution 4") of Tab. III. The fit gives $\chi^2/n_{\text{d.o.f.}}=13.2/11=1.2$, which is a little bit higher than those in other solutions. However, the solution still has the statistical significance more than 5.6 standard deviation to the one obtained in the assumption that only the E-M components (i.e. the $e_0$, $e_1$ and $\delta_1$, and the $\eta - \eta'$ mixing angle) act on the VP channel production. As for the solution 4, we obtain the unusual solution again, which is like solution 2 with the negative $g_s$ and negative E-M correction $s_e$ for the $s$ quark couplings. The parameters of $r$, $s_V$ and $s_P$ which associate with the nonet symmetry breaking measurements are now with unexpected larger values. Nevertheless, as guessed above, those unusual numbers may suggest more complicate dynamics for the $s$ quark production either in the channel $K^*\overline{K}$ or in all of the relative channels, involving value $s$ quark production. This solution predicts the decay cross sections, branching fractions and decay width of channels $\rho\pi$ and $K^*(892)\overline{K}$ and the production cross sections of three pure E-M production channels $\omega\pi^0$, $\rho\eta$ and $\rho\eta'$,}

\[
\begin{align*}
\sigma_{\rho\pi}^{\text{res}} &= (15.50^{+4.21}_{-3.52}) \text{ pb} \\
B_{\rho\pi} &= (1.55^{+0.42}_{-0.35}) \times 10^{-3} \\
\Gamma_{\rho\pi} &= 42.4^{+11.3}_{-9.6} \text{ keV} \\
\sigma_{K^*\overline{K}}^{\text{res}} &= (1.14^{+1.12}_{-0.57}) \text{ pb} \\
B_{K^*\overline{K}} &= (1.14^{+1.12}_{-0.57}) \times 10^{-4} \\
\Gamma_{K^*\overline{K}} &= (3.11^{+3.06}_{-1.56}) \text{ keV} \\
\sigma_{\omega\pi^0}^{\text{ctm}} &= (11.67^{+0.35}_{-0.38}) \text{ pb} \\
\sigma_{\rho\eta}^{\text{ctm}} &= (8.10^{+0.60}_{-0.61}) \text{ pb} \\
\sigma_{\rho\eta'}^{\text{ctm}} &= (2.43^{0.39}_{-0.37}) \text{ pb},
\end{align*}
\] (19)

which are consistent with those obtained in the last solution with the counter terms. How-
ever, for the channels $\omega\eta$, $\omega\eta'$, $\phi\eta$ and $\phi\eta'$, we obtain the branching fractions of

\[
\begin{align*}
B^{\omega\eta} &= (0.92^{+0.75}_{-0.92}) \times 10^{-4} \\
B^{\omega\eta'} &< 1.66 \times 10^{-4}, \text{ (at 90\% c.l.)} \\
B^{\phi\eta} &< 1.57 \times 10^{-4}, \text{ (at 90\% c.l.)} \\
B^{\phi\eta'} &= 3.46^{+8.93}_{-1.75} \times 10^{-4},
\end{align*}
\]

which are quite different comparing with those given in Eq. (18). As for the branching fractions for $\psi(3770) \rightarrow \phi\eta$, the measured values obtained by this analysis are also somehow inconsistent with that measured by CLEO-c Collaboration [6], (see Tab. IV). However both the results show the large $\psi(3770)$ VP decay couplings. The summed total cross sections over all VP channels of $\sigma_T^{\text{ch}}$ at both energy points of $\sqrt{s}=3.670$ GeV and 3.773 GeV are listed in Tab. IV. The summed values of solution 3 and solution 4 as well as the results of CLEO-c measurement show in almost equals at the two energy points. The real resonance decays have been “hidden”. This is mainly owing to the destructive nature of interference between the couplings $g$ and $e_0$.

IV. DISCUSSION AND SUMMARY

From above global analyses of the production cross sections for twelve channels of $e^+e^- \rightarrow \text{VP}$ measured by CLEO-c Collaboration at the $\psi(3770)$ resonance peak of $\sqrt{s} = 3.773$ GeV and at the energy of $\sqrt{s}=3.670$ GeV in the continuum region, we obtained four different solutions of the parameters to control the VP channel production. From Tab. IV we see that the measured branching fractions or the production cross sections except for the channels involving the nonet symmetry breaking are all consistent with each other in the four solutions.

It is remarkable that the large $SU(3)$ symmetry strong decay strength $|g|$ leads to the huge branching fraction of a level of $10^{-3}$ for the typical channel $\rho\pi$, which corresponds to the decay width of two order of magnitude higher than that in $J/\psi$ decays. The large strong decay coupling hidden behind the VP channel production in the $e^+e^-$ annihilation at the $\psi(3770)$ resonance peak might help people to understand the sources of the $\psi(3770)$ non-$D\bar{D}$ decays and give people some useful information to reexplain the long-standing $\rho\pi$ puzzle in the $1^{--}$ charmonium state VP decays. Furthermore, the large strong decay coupling with
TABLE IV: Summary of the results. The errors for our measurements are statistical only. The branching fraction of channel $\phi\eta$ measured by CLEO-c Collaboration is directly from work $[6]$ and the upper limits of other channels measured by CLEO-c Collaboration are from the “Method II” of paper $[6]$. 

| & Solution 1 & Solution 2 & Solution 3 & Solution 4 & CLEO-c |
|---|---|---|---|---|---|
| $\rho\pi$ | $\sigma_{3670}$ (pb) | $6.86_{-1.41}^{+2.15}$ | $6.89_{-1.60}^{+1.58}$ | $6.43_{-1.35}^{+3.91}$ | $8.0_{-0.9}^{+0.5}$ |
| & $\sigma_{T}$ (pb) | $5.35_{-1.74}^{+1.45}$ | $5.42_{-2.60}^{+3.38}$ | $5.08_{-1.28}^{+3.57}$ | $4.4_{-0.5}^{+0.3}$ |
| & $\sigma_{R}$ (pb) | $18.24_{-6.70}^{+6.11}$ | $18.68_{-3.96}^{+5.46}$ | $15.50_{-2.96}^{+5.04}$ | $< 0.04$ |
| & Br $10^{-3}$ | $1.83_{-0.67}^{+0.61}$ | $1.87_{-0.34}^{+0.05}$ | $1.55_{-0.30}^{+0.05}$ | $< 0.004$ |
| $K^*\eta K^0 + e.c.$ | $\sigma_{T}$ (pb) | $19.30_{-1.45}^{+1.66}$ | $19.34_{-1.66}^{+1.11}$ | $19.44_{-1.66}^{+3.61}$ | $23.3_{-1.1}^{+1.3}$ |
| & $\sigma_{R}$ (pb) | $0.28_{-0.11}^{+0.06}$ | $0.28_{-0.11}^{+0.06}$ | $0.57_{-0.29}^{+0.05}$ | $< 20.8$ |
| & Br $10^{-4}$ | $0.28_{-0.11}^{+0.06}$ | $0.28_{-0.11}^{+0.06}$ | $0.57_{-0.29}^{+0.05}$ | $< 20.8$ |
| $K^+ K^0\pi$ | $\sigma_{T}$ (pb) | $< 0.502$ | $< 2.59$ | $< 0.77$ | $< 0.6$ |
| & $\sigma_{R}$ (pb) | $0.28_{-0.11}^{+0.06}$ | $0.28_{-0.11}^{+0.06}$ | $0.57_{-0.29}^{+0.05}$ | $< 0.1$ |
| & Br $10^{-4}$ | $0.28_{-0.11}^{+0.06}$ | $0.28_{-0.11}^{+0.06}$ | $0.57_{-0.29}^{+0.05}$ | $< 0.1$ |
| $\omega\eta$ | $\sigma_{T}$ (pb) | $11.74_{-0.34}^{+0.33}$ | $11.79_{-0.36}^{+0.33}$ | $11.67_{-0.38}^{+0.36}$ | $14.6_{-0.6}^{+0.5}$ |
| & $\sigma_{R}$ (pb) | $0.0$ | $0.0$ | $0.0$ | $< 0.06$ |
| & Br $10^{-4}$ | $0.0$ | $0.0$ | $0.0$ | $< 0.06$ |
| $\rho^0\pi$ | $\sigma_{T}$ (pb) | $8.00_{-0.22}^{+0.22}$ | $7.92_{-0.41}^{+0.41}$ | $8.10_{-0.44}^{+0.44}$ | $10.3_{-0.5}^{+0.1}$ |
| & $\sigma_{R}$ (pb) | $0.0$ | $0.0$ | $0.0$ | $< 1.3$ |
| & Br $10^{-4}$ | $0.0$ | $0.0$ | $0.0$ | $< 1.3$ |
| $\rho^0\eta$ | $\sigma_{T}$ (pb) | $2.57_{-0.78}^{+0.87}$ | $2.69_{-0.29}^{+0.29}$ | $2.43_{-0.32}^{+0.32}$ | $3.8_{-0.8}^{+0.9}$ |
| & $\sigma_{R}$ (pb) | $0.0$ | $0.0$ | $0.0$ | $< 0.4$ |
| & Br $10^{-4}$ | $0.0$ | $0.0$ | $0.0$ | $< 0.4$ |
| $\omega\eta$ | $\sigma_{T}$ (pb) | $2.19_{-0.77}^{+1.33}$ | $2.12_{-0.21}^{+0.21}$ | $0.92_{-0.92}^{+0.92}$ | $< 0.1$ |
| & $\sigma_{R}$ (pb) | $0.45_{-0.10}^{+0.67}$ | $0.38_{-0.20}^{+0.62}$ | $0.32_{-0.42}^{+0.42}$ | $0.4_{-0.2}^{+0.2}$ |
| & Br $10^{-4}$ | $2.19$ | $2.12_{-0.21}^{+0.21}$ | $0.92_{-0.92}^{+0.92}$ | $< 0.1$ |
| $\omega'\eta$ | $\sigma_{T}$ (pb) | $0.44_{-0.30}^{+0.46}$ | $0.59_{-0.39}^{+0.50}$ | $0.44_{-0.41}^{+1.84}$ | $0.6_{-0.3}^{+0.6}$ |
| & $\sigma_{R}$ (pb) | $< 0.33$ | $< 0.27$ | $< 0.47$ | $< 1.9$ |
| & Br $10^{-4}$ | $< 0.33$ | $< 0.27$ | $< 0.47$ | $< 1.9$ |
| $\phi\pi$ | $\sigma_{T}$ (pb) | $3.91_{-1.27}^{+2.75}$ | $3.70_{-1.02}^{+2.75}$ | $4.5_{-0.5}^{+1.0}$ | $4.5_{-0.5}^{+1.0}$ |
| & $\sigma_{R}$ (pb) | $0.84_{-0.58}^{+0.79}$ | $0.87_{-0.52}^{+0.85}$ | $1.45$ | $2.4_{-0.7}^{+0.6}$ |
| & Br $10^{-4}$ | $0.84_{-0.58}^{+0.79}$ | $0.87_{-0.52}^{+0.85}$ | $1.45$ | $3.1_{-0.7}^{+0.6}$ |
| $\phi'\eta$ | $\sigma_{T}$ (pb) | $2.83_{-1.68}^{+4.95}$ | $2.29_{-1.76}^{+4.26}$ | $1.69_{-1.35}^{+2.35}$ | $2.5_{-0.4}^{+1.1}$ |
| & $\sigma_{R}$ (pb) | $< 0.52$ | $< 0.66$ | $3.46_{-1.76}^{+4.90}$ | $< 3.8$ |
| & Br $10^{-4}$ | $< 0.52$ | $< 0.66$ | $3.46_{-1.76}^{+4.90}$ | $< 3.8$ |

All “ch” summed | $\sigma_{3670}^{\text{eff}}$ (pb) | $58.3_{-6.2}^{+6.2}$ | $62.3_{-6.9}^{+5.0}$ | $64.2_{-6.8}^{+15.8}$ | $< 3$ |
| $\sigma_{3770}^{\text{eff}}$ (pb) | $54.4_{-4.1}^{+13.0}$ | $52.9_{-3.6}^{+6.2}$ | $64.6_{-4.3}^{+14.5}$ | $< 4.3$ |

the opposite s-quark strong coupling and the minus E-M correction $s_e$ or equivalently, the s-quark “anomalous magnetic moments” $a^\mu$, required by the $K^*\bar{K}$ production as presented in those solutions with different treatments and different assumptions in this work are all unusual comparing with the conventional hard gluon annihilation picture plus single $\psi(3770)$ resonance assumption. If those measurements and analyses are all correct, one has to re-understand the strong interaction dynamics which leads to the large Okubo-Zweig-Iizuka rule violation in the vector meson $\psi(3770)$ decays and the strange behavior of the s quark couplings to the light hadron production in the energy region around $\psi(3770)$ resonance. It
seems that people has to seriously consider the role of the long distance strong interaction corrections including the $D(D_s)$ meson exchange scheme to describe those anomalous phenomena and the large Okubo-Zweig-Iizuka breaking in this energy region, like many authors did [18, 19].

The destructive interferences mechanism might manifest the possibility of the significant existences of buried $\psi(3770)$ non-$D\overline{D}$ decays. However, how do people understand the large net inclusive non-$D\overline{D}$ hadron branching fraction of $\psi(3770)$ decays measured by BES Collaboration [3, 4] recently. Phenomenologically, for example, if the behavior of the reversed $s$ quark strong coupling appeared in VP channel is still maintained in those cases, it would lead to constructive interference with the parallel continuum E-M amplitudes and cause abundant strangeness meson production at the $\psi(3770)$ peak, resulting in the large net cross section excess. The one of the exceptions is the channel $K^+\overline{K}+c.c.$, in which the E-M production is due to the magnetic moments coupling. The minus E-M coupling correction for $s$ quark or the counter term $a^\mu$ as a special “anomalous magnetic moments” enhances the E-M production at the continuum region and leads to the observation of the equal cross sections of channel $K^+\overline{K}+c.c.$ at the two energy points $\sqrt{s}=3.670$ and 3.773 GeV in work [6]. This argument can be cleared up by coming more precisely experimental measurements. Of course, another probable outlet would be that there are more complicated structures or contents in the $\psi(3770)$ resonance scope which are evident in a measurement of cross sections for $e^+e^- \rightarrow$ hadrons by the BES Collaboration [20, 21]. This measurement of the cross section indicates that there are somehow complicate “di-resonance” structure instead of the conventional single $\psi(3770)$ resonance assumption [20, 22]. The exist of the extra substances might respond to the unusual behavior of $\psi(3770)$ VP decays and the measured large non-$D\overline{D}$ branching fraction of $\psi(3770)$ decays [3, 4].
V. ACKNOWLEDGMENTS

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