New supersymmetric generalization of the Liouville equation

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Abstract

We present new $n = (1, 1)$ and $n = (1, 0)$ supersymmetric generalization of the Liouville equation, which originate from a geometrical approach to describing the classical dynamics of Green–Schwarz superstrings in $N = 2$, $D = 3$ and $N = 1$, $D = 3$ target superspace. Considered are a zero curvature representation and Bäcklund transformations associated with the supersymmetric non–linear equations.

PACS: 11.15-q, 11.17+y

Keywords: Superstrings, non–linear equations, worldsheet supersymmetry, Bäcklund transformations.

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†Work supported in part by the International Science Foundation under the grant N RY 9200, by the State Committee for Science and Technology of Ukraine under the Grant N 2/100 and by the INTAS grants 93–127, 93–493, 93–633, 94-2317

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1 Introduction

The dynamics of some physical systems is described by non–linear equations, which are exactly solvable. Solitons, monopoles, instantons and relativistic strings are examples of such systems.

For instance, the classical dynamics of a free relativistic string in $D=3$ space–time is governed by the Liouville equation [1, 2, 3] and it is described by the complex Liouville equation in $D=4$ space–time [4, 5]. The Lagrangian, from which the Liouville equation can be obtained arises also when one calculates the conformal anomaly in the quantum theory of non–critical ($D \neq 26$) bosonic strings [5]. The calculation of the superconformal anomaly of a relativistic spinning string in $D \neq 10$ results in an effective Lagrangian, which produces an exactly solvable $n = (1, 1)$ supersymmetric generalization of the Liouville equation [6] considered earlier in [7]. There has also been an activity in looking for and studying other completely–integrable supersymmetric systems (see [8]–[13] and refs. therein). As further development of this research it seems of interest to find and analyse supersymmetric non–linear equations that describe Green–Schwarz superstrings.

The purpose of this letter is to present an $n = (1, 0)$ and $n = (1, 1)$ worldsheet supersymmetric generalization of the Liouville equation which describe the classical dynamics of Green–Schwarz superstrings in $N = 1, D = 3$ and $N = 2, D = 3$ target superspace, respectively. The worldsheet supersymmetry is a counterpart of the fermionic $\kappa$–symmetry of the Green–Schwarz superstrings in a doubly–supersymmetric approach (see [14] and refs. therein).

The $n = (1, 1)$ super–Liouville equation we obtained turns out to be different from and cannot be transformed into the conventional one [7] by any local change of variables. The superfield form of this new $n = (1, 1)$ superfield generalization of the Liouville equation admits Bäcklund transformations of variables, and is exactly solvable in a “trivial” way, since at the component level the bosonic and fermionic part of the $n = (1, 1)$ superfield equation are reduced, respectively, to the bosonic Liouville equation and the free chiral fermion equations, which are completely disconnected from each other. We would like to thank F. Toppan for pointing us this reduction.

As to the $n = (1, 0)$ case, it can be obtained as a reduction of both, the standard and the new $n = (1, 1)$ supersymmetric Liouville system.

To begin with, in Section 2 we shall briefly discuss how the Liouville equation is extracted from the equations of motion of a bosonic string in a form of the zero curvature representation, which allows one to naturally get Bäcklund transformations for the Liouville variable. In Section 3 we shall apply the same technique for analysing the new $n = (1, 1)$ and $n = (1, 0)$ supersymmetric generalization of the Liouville equation.

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1 We denote the number of left and right supersymmetries on the worldsheet by small $n$, and by capital $N$ the number of supersymmetries in target space.
2 The Liouville equation and the $D = 3$ bosonic string

The reduction of the string equations of motion to the Liouville equation is achieved by changing string variables and solving for the Virasoro constraints in such a way that only the variables which correspond to the independent physical degrees of freedom of the string remain and the latter satisfy the Liouville equation [1, 2, 3, 15]:

$$\partial_{++}\partial_{--}w = \exp\{2w\},$$

where $\partial_{\pm\pm} \equiv \partial/\partial\xi^{\pm\pm}$, $\xi^{\pm\pm} = 2^{-1/2}(x^0 \pm x^1)$ are light–cone coordinates on the worldsheet, and the worldsheet function $w(\xi)$ is real for the $D = 3$ bosonic string and complex for the $D = 4$ bosonic string. (Each plus (minus) denotes the left (right) Majorana–Weyl spinor representation of $SO(1, 1)$, and the couples of pluses and minuses denote the $SO(1, 1)$ vector representation).

The equations of motion of a bosonic string in space–time of dimension greater than four were transformed into a system of non–linear equations in [15, 3].

A natural way of getting the nonlinear equations from the string equations is to use a geometrical approach ([1, 2, 3, 15] and references therein) which is based on the theory of surfaces embedded into a target space. From the geometrical point of view the functions which enter the non–linear equations determine the geometrical objects on the worldsheet, such as a metric and connection forms, induced by the embedding. For instance, the $SO(1, 1)$ spin connection has the form

$$\Omega^{(0)} = (d\xi^{++}\partial_{++} - d\xi^{--}\partial_{--})w - dl,$$

where $l(\xi)$ corresponds to a local $SO(1, 1)$ transformation in the space tangent to the worldsheet [1].

In the geometrical approach the form (2) together with another two differential one–forms

$$\Omega^{\pm\pm} = -2d\xi^{\mp\mp}\exp\{w \mp l\},$$

is part of an $SL(2, \mathbb{R})$ connection

$$\Omega^{(0)} = \frac{1}{2} \begin{pmatrix} \Omega^{(0)} & \Omega^{(-)} \\ \Omega^{(+)} & -\Omega^{(0)} \end{pmatrix}, \quad (\alpha, \beta = 1, 2),$$

which satisfies the $SL(2, \mathbb{R})$ Maurer–Cartan equation

$$d\Omega^{(0)} - \Omega^{(0)} \Omega^{(0)} - \Omega^{(0)}\Omega^{(0)} = 0.$$

Note that $SL(2, \mathbb{R})$ is isomorphic to the Lorentz group in $D = 3$ target space.

Eq. (6) contains the Liouville equation (1) as the differential form equation

$$d\Omega^{(0)} - 1/2 \Omega^{--}\Omega^{++} = 0,$$

Remind that the bosonic string has one independent transversal degree of freedom in $D = 3$ and two in $D = 4$. In (2) and below the the external differential $d$ and external product are used in such a way that for a $p$–form $\omega_p$ and a $q$–form $\omega_q$ we have $\omega_p\omega_q = (-1)^{pq}\omega_q\omega_p$, $d(\omega_p\omega_q) = \omega_p d\omega_q + (-1)^q d\omega_p\omega_q$. 

3 In (2) and below the the external differential $d$ and external product are used in such a way that for a $p$–form $\omega_p$ and a $q$–form $\omega_q$ we have $\omega_p\omega_q = (-1)^{pq}\omega_q\omega_p$, $d(\omega_p\omega_q) = \omega_p d\omega_q + (-1)^q d\omega_p\omega_q$. 


called the Gauss equation in surface theory (see \([1, 2, 3]\)).

Another two equations, which enter \((5)\), are so called Peterson–Codazzi equations

\[
d\Omega^{\pm\pm} \mp \Omega^{(0)} \Omega^{\pm\pm} = 0,
\]

which are identically satisfied when \(\Omega^{(0)}, \Omega^{\pm\pm}\), have the form \([2], [3]\).

The system of equations \((5)\) (or \((3), (4)\)) describes the classical motion of the \(D = 3\) bosonic string in the geometrical approach \([1, 2, 3]\). It has the form of the zero curvature condition for the connection \((4)\) and can be considered as the integrability condition for an associated system of linear equations

\[
d\Psi_{\alpha} - \Omega_{\alpha}^{\beta} \Psi_{\beta} = 0.
\]

The invariance of the theory under the local \(SO(1, 1)\) transformations, which is reflected in the presence in eqs. \([2], [3]\) of an arbitrary field \(l(\xi)\), allows one to reproduce a Bäcklund transformation \([16, 7, 10]\) which relates a solution of the Liouville equation \((1)\) with a solution of the free field equation

\[
\partial_{++} \partial_{--} l = 0.
\]

This is achieved by further specifying the \(\Omega^{(0)}\) form in \((2)\) by requiring its \(d\xi^{++}, d\xi^{--}\) coefficients to be, respectively,

\[
\partial_{++} w - \partial_{++} l = \frac{1}{b} \exp\{w + l\},
\partial_{--} w + \partial_{--} l = b \exp\{w - l\},
\]

where \(b\) is an arbitrary constant.

Eqs. \((10)\) are the Bäcklund transformations being a useful tool in studying exactly solvable systems \([7, 16\) and references therein]. The self–consistency conditions for \((10)\) are the Liouville equation \((1)\) for \(w(\xi)\) and eq. \((9)\) for \(l(\xi)\).

Let us turn now to the consideration of an \(n = (1, 0)\) and \(n = (1, 1)\) supersymmetric generalization of the Liouville equation.

### 3 Worldsheet supersymmetric generalization of the Liouville equation and \(D = 3\) Green–Schwarz superstrings

Because of the Virasoro conditions and fermionic constraints related to the local \(\kappa–\)symmetry of the Green–Schwarz strings \([18]\) the classical dynamics of superstrings in \(D = 3\) superspace with \(N=1,2\) Grassmann spinor coordinates is described by one independent bosonic variable and one (or two) independent fermionic variables, respectively.

A doubly supersymmetric generalization \([13]\) of the geometrical approach \([1, 2, 3]\) to the consideration of superstrings and supermembranes as supersurfaces embedded into a target superspace allows one to reduce \(N = 1, 2, D = 3\) Green–Schwarz superstring equations.
of motion to a system of equations for the independent superstring variables \cite{4}. In the case of the $N = 1, D = 3$ Green–Schwarz superstring we get the Liouville equation for the bosonic string variable $w(\xi)$ and a chirality condition for the fermionic string variable $\psi_L(\xi)$:

\begin{equation}
\partial_+ \partial_- w = \exp\{2w\},
\end{equation}

\begin{equation}
\partial_- \psi_L = 0.
\end{equation}

In the case of the $N = 2, D = 3$ Green–Schwarz superstring the system of equations for the bosonic variable $w(\xi)$ and fermionic variables $\psi_R(\xi), \psi_L(\xi)$ of opposite worldsheet spinor chirality has the form

\begin{equation}
\partial_+ \partial_- w = \exp\{2w\},
\end{equation}

\begin{equation}
\partial_+ \psi_R = 0, \quad \partial_- \psi_L = 0.
\end{equation}

Eqs. (12) reduce to (11) when $\psi_R \equiv 0$. The integrability of eqs. (11), (12) is obvious.

Eqs. (11) and (12) possess, respectively, $n = (1,0)$ and $n = (1,1)$ supersymmetry (which replace $\kappa$–symmetry) on the worldsheet, and, in fact, were derived from a superfield system of equations which we shall write down below.

But firstly, let us present the conventional $n = (1,1)$ supersymmetric Liouville equation \cite{5} which arises in the quantum theory of non–critical fermionic strings \cite{5}:

\begin{equation}
D_- D_+ \Phi = 2ie^\Phi,
\end{equation}

where $\Phi = \phi + i\eta^+ \psi_+ + i\eta^- \psi_- + i\eta^- F$ is a superfield in $n = (1,1)$ worldsheet superspace parametrized by bosonic coordinates $\xi^{\pm\pm}$ and fermionic coordinates $(\eta^+, \eta^-)$, and $D_\pm = \frac{\partial}{\partial \eta^\pm} + 2i\eta^\pm \partial_{\pm\pm}$ are supercovariant derivatives which form the flat $n = (1,1)$ superalgebra

\begin{equation}
D_+ D_+ = 2i\partial_{++}, \quad \{D_+, D_-\} = 0, \quad D_- D_- = 2i\partial_{--}.
\end{equation}

From eq. (13) one gets the following system of equations for the components of $\Phi(\xi, \eta)$

\begin{equation}
\partial_+ \partial_- \phi = -e^\phi(e^K + \frac{1}{2}i\psi_- \psi_+), \quad F = 2e^K,
\end{equation}

\begin{equation}
\partial_+ \psi_- = -e^\phi \psi_+, \quad \partial_- \psi_+ = e^\phi \psi_-.
\end{equation}

It is obvious that eqs. (12) and (15) cannot be transformed into each other by any local change of variables. However, eqs. (11) can be obtained from (13) by a truncation of the latter to an $n = (1,0)$ supersymmetric system. This truncation is performed by imposing an additional condition on $e^{-\phi} \psi_- \equiv \psi_L$ to be a chiral field $\partial_- \psi_L = 0$. Then from (13) it follows that $\psi_+$ is not an independent field anymore since its equation of motion becomes a consequence of the two other. identically satisfied. Thus, we get (11) upon redefining $\phi \rightarrow \phi' = \phi - \frac{i}{4}\psi_L \partial_+ \psi_L$ and replacing $\partial_- \rightarrow -\partial_-$. 

\footnote{the details of this reformulation will be published elsewhere}

\footnote{$n = (2,2)$ and $n = (4,4)$ supersymmetric generalization of the Liouville equation were constructed in \cite{10}}
In contrast to (13) the bosonic and fermionic variables of (11) and (12) do not form a single superfield and transform non-linearly under supersymmetry. Each of them (upon some field redefinition) is the leading component of a corresponding bosonic $W(\xi, \eta)$ or fermionic $\Psi_L(\xi, \eta), \Psi_R(\xi, \eta)$ superfields.

$\Psi_L(\xi, \eta)$ and $\Psi_R(\xi, \eta)$ are chiral superfields:

$$D^-\Psi_L = 0 \quad \Rightarrow \quad \partial_-\Psi_L = 0$$ (16)

$$D^+\Psi_R = 0 \quad \Rightarrow \quad \partial_+\Psi_R = 0,$$ (17)

and are connected with $W(\xi, \eta)$ through the following relations:

$$D_+(e^{3W}\Psi_L) = e^{3W} \quad \Rightarrow \quad D_+\Psi_L + 3D_+W\Psi_L = 1$$ (18)

$$D_-(e^{3W}\Psi_R) = e^{3W} \quad \Rightarrow \quad D_-\Psi_R + 3D_-W\Psi_R = 1.$$ (19)

The superfield form of eq. (12) in $n = (1, 1)$ worldsheet superspace $(\xi^{\pm\pm}, \eta^+, \eta^-)$ is

$$D_-D_+W = 4e^{2W}\Psi_L\Psi_R;$$ (20)

and the superfield form of (11) in $n = (1, 0)$ worldsheet superspace $(\xi^{\pm\pm}, \eta^+)$ is

$$\partial_-D_+W = 2ie^{2W}\Psi_L$$ (21)

where $W$ and $\Psi_L$ are taken to be independent of $\eta_-$ and $\Psi_R$ is put equal to zero.

The conditions (16)–(19) can be explicitly solved with $W$, $\Psi_L$ and $\Psi_R$ having the following component form

$$W = w + \frac{2i}{3}\eta^+e^{-3w}\partial_+(e^{3w}\psi_L) + \frac{2i}{3}\eta^-e^{-3w}\partial_-(e^{3w}\psi_R) + 4\eta^+\eta^-\psi_L\psi_R\partial_+\partial_-, \quad (22)$$

$$\Psi_L = \psi_L(\xi^{++}) + \eta^+(1 - 2i\partial_+\psi_L\psi_L), \quad (23)$$

$$\Psi_R = \psi_R(\xi^{--}) + \eta^-(1 - 2i\partial_-\psi_R\psi_R). \quad (24)$$

In the doubly–supersymmetric geometrical approach the system of equations (16)–(20) arises as the zero curvature condition (5) for an $SL(2, \mathbb{R})$ connection composed out of differential one–superforms

$$\Omega^{--} = \exp(W + L)(-2e^{++}(1 - D_+W\Psi_L) - 4ie^+\Psi_L), \quad (25)$$

$$\Omega^{++} = \exp(W - L)(-2e^{--}(1 - D_-W\Psi_R) - 4ie^-\Psi_R), \quad (26)$$

$$\Omega^{(0)} = (e^+D_+ + e^{++}\partial_+ + e^-D_- - e^{--}\partial_-)W - dL, \quad (27)$$

and the system of equations (16), (18), (21) is part of the zero curvature condition (5) for $SL(2, \mathbb{R})$ one–superforms

$$\Omega^{--} = \exp(W + L)(-2e^{++}(1 - D_+W\Psi_L) - 4ie^+\Psi_L), \quad (28)$$

$$\Omega^{++} = -2e^{--}\exp(W - L) \quad (29)$$
\[ \Omega^{(0)} = (e^+ D_+ + e^{++} \partial_{++} - e^{-} \partial_{--}) W - dL. \]  

(30)

In \((25)\)–\((30)\)

\[ e^\pm \equiv d\eta^\pm, \quad e^{++} \equiv d\xi^{++} - 2id\eta^+ \eta^+, \quad e^{-} \equiv d\xi^{-} - 2id\eta^- \eta^- \quad \text{(for } n = (1, 1)\) 

\[ e^{-} \equiv d\xi^{-} \quad \text{(for } n = (1, 0)\) 

are the basic supercovariant one–forms in \(d = 2\) superspace.

One can check that the system of \(n = (1, 1)\) superfield equations \((16)\)–\((20)\) is part of the Maurer–Cartan equations \((5)\) for the forms \((25)\)–\((27)\), and the system of \(n = (1, 0)\) superfield equations \((16)\), \((18)\), \((21)\) is part of the Maurer–Cartan equations \((5)\) for the forms \((28)\)–\((30)\).

In the geometrical approach to Green–Schwarz superstrings \([14]\) the Maurer–Cartan equations for the forms \((25)\)–\((27)\) and \((28)\)–\((30)\) describe, respectively, the classical motion of the \(N = 2\) and \(N = 1, D = 3\) superstring, the \(\kappa\)–symmetry being replaced by \(n = (1, 1)\) and \(n = (1, 0)\) worldsheet supersymmetry, respectively.

\(n = (1, 1)\) Bäcklund transformations

\[ D_+ W - D_+ L = \frac{2i}{b} \exp\{W + L\} \Psi_L, \]
\[ D_- W + D_- L = 2ib \exp\{W - L\} \Psi_R, \]  

(31)

and \(n = (1, 0)\) Bäcklund transformations

\[ D_+ W - D_+ L = \frac{2i}{b} \exp\{W + L\} \Psi_L, \]
\[ \partial_- W + \partial_- L = b \exp\{W - L\}, \]  

(32)

which relate the superfields \(W\) and \(L\) are obtained as additional conditions imposed on the components of \(\Omega^{(0)}\) forms \((27)\), \((30)\).

The integrability conditions for \((31)\) and \((32)\) are, respectively, the non–linear equations \((24)\) and \((27)\) for \(W\), and the linear superfield equations for \(L\):

\[ D_+ D_- L = 0 \quad \text{(for } n = (1, 1)) \]
\[ D_+ \partial_- L = 0 \quad \text{(for } n = (1, 0)). \]  

(33)

For checking this one has to take into account eqs. \((16)\)–\((19)\).

4 Conclusion and discussion

We have presented the new exactly solvable \(n = (1, 1)\) and \(n = (1, 0)\) supersymmetric generalization of the Liouville equation, which originate from the geometrical approach to describing \(N = 2, 1, \) superstrings in \(D = 3\) \([14]\). At the component level they split into the purely bosonic Liouville equation and the free fermion equations.

The \(n = (1, 0)\) supersymmetric system of eqs. \((11)\) (or \((16), (18), (21)\)) can be obtained (by a truncation) of both, the standard \(n = (1, 1)\) supersymmetric generalization \((15)\) (or
and the new $n = (1, 1)$ supersymmetric generalization (12) (or (16)–(20)) of the Liouville equation.

However, the two $n = (1, 1)$ supersymmetric versions of the Liouville equation are not connected with each other by any local transformation of variables. To see this one can check, that (without reducing the model to $n=(1,0)$ case) it is not possible, by use of local operations, to construct chiral fermions from the components (or the superfield $W$) of the standard supersymmetric Liouville equation (15), (13), while the chiral fermions are part of the new $n = (1, 1)$ supersymmetric system of eqs. (12), (16)–(20). The indirect way to connect the two equations is to make the Bäcklund transformation (31) from a solution of the new super–Liouville equation to a solution of the free differential equation (33) and then to make the Bäcklund transformation from the solution of (33) to a solution of the standard super–Liouville equation (13) [4].

The problem of the relationship between the two $n = (1, 1)$ supersymmetric non–linear systems seems to be also connected with the problem of classical (and quantum) equivalence of (non–critical) Green–Schwarz and relativistic spinning strings [18, 19, 20].

To trace this relationship one should compare the effective Liouville Lagrangian originated from the quantum theory of spinning strings [6] with an effective Lagrangian which should arise as a result of the computation of anomalies in non–critical Green–Schwarz superstrings, or compare the $n = (1, 1)$ supersymmetric generalization of the Liouville equation describing the classical $N = 2$, $D = 3$ Green–Schwarz superstring with an $n = (1, 1)$ supersymmetric system of equations to which one should reduce the equations of motions of an $n = (1, 1)$, $D = 3$ spinning string. As far as we know, neither the effective Lagrangian for the non–critical Green–Schwarz superstrings, nor the $n = (1, 1)$ supersymmetric non–linear equations describing the classical dynamics of the $n = (1, 1)$, $D = 3$ spinning string have been obtained yet.

Another difference between the standard and the new $n = (1, 1)$ supersymmetric Liouville system is that the zero curvature representation for the former is associated with an $OSp(1|2)$ connection [7, 11, 12], while in the case considered above it is associated with a connection of $SL(2, \mathbb{R})$ which is the structure group of the flat $D = 3$ target superspace. This simpler group structure resulted in a “trivial” component form (12) of our super–Liouville system in comparison with the standard one (13).

Thus, the two ways of supersymmetrizing the Liouville equation are essentially different at this point and remind the situation with the non–linear Schrödinger equation for which supersymmetric versions based on $OSp(1|2)$ [12] and $SL(2, \mathbb{R})$ [13] are known to be different.

To conclude, we would like to note that the geometrical approach which has been used above for obtaining non–linear equations describing strings is applicable to studying p–branes ($p > 1$) as well [3]. The equations of motion of p–branes can also be rewritten as a zero curvature representation treated as a self–consistency condition for an associated system of linear equations.

In a recent paper [21] non–linear equations of motion of a bosonic $(D – 2)$–brane

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\[6\text{we thank E. Ivanov for pointing our attention to this relationship}\]
moving in D–dimensional space–time were reduced to a form of the zero curvature representation by fixing a gauge with respect to the worldsurface diffeomorphisms and the Lorentz transformations. In [21] the zero curvature representation and an associated system of linear equations obtained this way were used, in particular, for finding explicit solutions and constructing non–local conservation charges describing a classical motion of \((D – 2)\)–branes.

The geometrical approach [1, 2, 3, 14, 14], upon which the present letter is heavily based, produces a Lorentz–covariant counterpart of the construction of ref. [21] for any bosonic p–branes.

Acknowledgements The authors would like to thank P. Howe, E. Ivanov, S. Krivonos, M. Mukhtarov, P. Pasti, A. Sorin, M. Tonin and F. Toppan for interest to this work and fruitful discussion. Special thanks are to F. Toppan who helped us to analyse the component structure of the non–linear equations. I.B. and D.V. are grateful to Paolo Pasti and Mario Tonin for hospitality at the Padova Section of the INFN and Physics Department of Padova University, where part of this work was carried out.

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