Is the Preferred Basis selected by the environment?

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We show that in a quantum measurement, the preferred basis is determined by the interaction between the apparatus and the quantum system, instead of by the environment. This interaction entangles three degrees of freedom, one system degree of freedom we are interested in and preserved by the interaction, one system degree of freedom that carries the change due to the interaction, and the apparatus degree of freedom which is always ignored. Considering all three degrees of freedom the composite state only has one decomposition, and this guarantees that the apparatus would end up in the expected preferred basis of our daily experiences. We also point out some problems with the environment-induced super-selection (Einselection) solution to the preferred basis problem, and clarifies a common misunderstanding of environmental decoherence and the preferred basis problem.

I. INTRODUCTION

The preferred basis problem (PBP) is one of the most fundamental problems in the foundations of quantum theory\cite{1,2,4}. It has been one of the key problems associated with the interpretation of quantum mechanics, in particular the many world interpretation\cite{1,2}. The suggestion that the preferred basis problem is solved by the “environment induced superselection (Einselection)”\cite{4}, was developed by Zurek \cite{6,7}. This theory maintains that the preferred basis is selected by the environment. However, the introduction of the environment is not really necessary to solve the PBP. In addition it can lead to several issues. In this paper we suggest a different solution to the PBP, that the preferred basis is determined by the configuration of the measurement apparatus, and the interaction between the apparatus and the environment only determines whether the correlation between the system and apparatus is preserved.

As we know from Newton’s third law of motion, when one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction to that of the first body. (This gives us momentum conservation, which can equivalently be formulated in Lagrangian or Hamiltonian approach.) However, when talking about the quantum measurement process, beginning with von Neumann’s contributions, it is generally supposed that the interaction between the quantum system and classical apparatus only changes the state of the apparatus, while leaving the state of the system untouched\cite{1,6}.

\begin{equation}
|s_n\rangle|a_0\rangle \xrightarrow{H_{za}} |s_n\rangle|a_n\rangle
\end{equation}

Here $|s_n\rangle$ and $|a_n\rangle$ represent the quantum system and the classical apparatus. This comes from the third postulate of quantum mechanics, that repetitive measurements yield the same results\cite{5}. However, this directly conflicts with the Newton’s third law, and the fundamental momentum conservation law that can be derived from that. To solve such a contradiction between classical and quantum mechanics there are two possible solutions. The first is to regard the effect on the quantum system is being too small and thus the state of the system can be treated as unchanged. However, this approximation is problematic. During the interaction between two objects, the change of the motion of the object with the smaller mass will be larger than the the more massive one, as result of the conservation of momentum. In the quantum measurement scenario, when an interaction is strong enough to alter the classical apparatus, which is assumed to be far larger than the quantum system, so as to present a different reading, it is difficult to believe that the state of the quantum system will be unchanged.

The second solution, as will be discussed in detail in the next section, is that the quantum system is described by two uncoupled degrees of freedom. The interaction only changes one degree of freedom, while the other degree of freedom—the one we want to measure, is preserved. Actually it turns out that the ordinary examples of quantum measurement such as Stern-Gerlach (S-G) experiment fall into this category. However, the degree of freedom describing the macroscopic apparatus is usually ignored. In our approach this hidden degree of freedom solves the PBP.

This paper is organized in the following method: Section II introduces the hidden degree of freedom. Section III briefly introduces the PBP, the Einselection solution, and our solution that the preferred basis is determined by the configuration of the measurement apparatus. Section IV compares the two solutions, pointing out the advantages of our solution, and clarifies a common misunderstanding of decoherence and the role of environment. Finally Section V analyzes the role of the environment in PBP and the relationship with decoherence.

II. THE HIDDEN DEGREE OF FREEDOM

We begin with the archetype of quantum measurement—the S-G experiment. A spin one-half
particle interacts with a macroscopic set of magnets. The particle is described by two uncoupled degrees of freedom, the spin degree of freedom \( |s_n\rangle \), which is preserved by the interaction with the magnet, and the momentum (spatial) dependent degree of freedom \( |p_n\rangle \), which changes in accordance with \( |s_n\rangle \), in that when the particles pass through the magnet, spin up particles will be deflected upwards, and spin down particles deflected downwards, both by specific amounts. (It should be noted that \( |p_n\rangle \) are not the exact momentum eigenstates.) However, according to Newton’s third law the interaction between the particle and magnet should not only change the state of the particle, but also the state of the magnet, which is always ignored in the description of this experiment.

Usually, the position of the particle is considered as the apparatus. However, according to the measurement scheme, the apparatus should be some macroscopic object, the change of which can be read by us easily, namely, we do not need to perform an extra quantum measurement to determine the state of the apparatus. From the point of view of the foundations of quantum mechanics, the apparatus is usually illustrated as a panel with a pointer, which can be read by a human observer. In the S-G experiment, it is the macroscopic magnet that should be regarded as the apparatus. The interaction between the particle and the magnet changes the direction of the velocity of the particle according to which spin it carries. The same interaction will also have an effect on the magnet: the magnet is subject to a torque whose direction and magnitude depend on the particle’s spin as it interacts with the magnet. If the system and apparatus are well separated from the environment, and the magnet is delicate enough, the magnet will rotate in different directions with different spin inputs. The spatial degree of freedom of the particle is actually the one that carries the change due to the interaction, while the spin state is the one we are interested in and is preserved in the measurement. The composite system of the particle and the magnet before the measurement will be

\[
\frac{1}{\sqrt{2}}(|s_+\rangle + |s_-\rangle)|p_0\rangle|a_0\rangle
\]

Here \( s \) describes different spin, \( p \) and \( a \) describes the spatial state of the particle and the state of the magnet respectively. It will evolve according to

\[
\frac{1}{\sqrt{2}}(|s_+\rangle + |s_-\rangle)|p_0\rangle|a_0\rangle \xrightarrow{H_{pa}} \frac{1}{\sqrt{2}}(|s_+\rangle|p_+\rangle|a_+\rangle + |s_-\rangle|p_-\rangle|a_-\rangle)
\]

This is similar to the case in quantum optics, when the Polarizing Beam Splitter (PBS) is used to entangle the polarization and the trajectory of a photon. As we know, a Wollaston prism type PBS will deflect the horizontally polarized photon toward the left and vertically polarized photon toward the right, as shown in Fig 1. From momentum conservation we can see that the PBS will undergo a momentum transfer depending on the different polarization of the photons deflected. In this case, the polarization and position state are the states of the two degrees of freedom of the system, which are preserved and changed respectively. Thus for an apparatus consisting of a PBS, its momentum which will change differently according to the polarization of the photon interacting with it. (It should be noted that a successful PBS that serves the goals for quantum optics should be large enough so that \( a_H|a_V\rangle \approx 1 \).

One may argue that the perturbation of the apparatus (whether a magnet or a PBS) is so small that it is less than their self uncertainty, and the states are basically the same as before the measurement. This involves a deep issue of how to interpret the quantum state and uncertainty of the apparatus, a macroscopic object, which yet to be clarified (the standard Copenhagen interpretation simply states that classical objects are not described by QT). In principle, \( a_H \) and \( a_V \) should not be exactly the same before and after the interaction, although the overlap is nearly, but not exactly, 1. This tiny difference has profound implications and provides us the solution of the preferred basis problem.

III. PREFERRED BASIS PROBLEM

The ideal measurement scheme was first introduced by von Neumann in his masterpiece Mathematische Grundlehren der Quantenmechanik. A typical microscopic
system $S$, described by basis vectors $\{s_n\}$ in a Hilbert space $\mathcal{H}_S$, interacts with a measuring apparatus $A$, represented by basis vectors $\{a_n\}$ spanning a Hilbert space $\mathcal{H}_A$, where the $a_n$ are assumed to correspond to macroscopically distinguishable “pointer” positions that in turn correspond to the outcome of a measurement if $S$ is in the state $|s_n\rangle$. Now consider the micro-system in an arbitrary state $\{s_n\}$ interacting with the apparatus in the initial state $|a_0\rangle$. After the measurement interaction, the final state of the apparatus will become $|a_n\rangle$, while leaving the state of the system unchanged.

$$|s_n\rangle|a_0\rangle \xrightarrow{H_{SA}} |s_n\rangle|a_n\rangle$$  
(4)

Here the $|a_n\rangle$ are assumed to correspond to macroscopically distinguishable “pointer” positions, $\langle a_m|a_n\rangle = \delta_{mn}$. Since $\{s_n\}$ spans a complete Hilbert space, any arbitrary state $|\psi\rangle$ can be expressed as $\sum_n c_n |s_n\rangle$ where $c_n = \langle s_n|\psi\rangle$. This can be understood as the system’s state being in a superposition of different $|s_n\rangle$, which is possible for microscopic system. If such a system interacts with the measuring apparatus, according to the linearity of the Schrödinger equation the total system $SA$ will evolve as

$$\left( \sum_n c_n |s_n\rangle \right)|a_0\rangle \xrightarrow{H_{SA}} \sum_n c_n |s_n\rangle|a_n\rangle.$$  
(5)

The final state of the total system suggests that the system-apparatus is now in a superposition state. However, our experience tells us that this macroscopic superposition has never been observed. Either there should be some physical process that destroys the superposition or a proper interpretation concerning this superposition is required. This is the essence of the quantum measurement problem, which should be divided into two subproblems, namely the definite outcome problem and the preferred basis problem.

The definite outcome problem mainly deals with why we get one definite single outcome after a measurement instead of observing the total system in superposition state. Associated with this problem are the questions: why is there a non-unitary measurement process? how does one interpret this indeterministic process? how does one understand the probability associated with the outcomes? Even after some 90 years of debate, there is still no consensus on the answers to these questions. It should be noted that the definite outcome problem has not been solved by the decoherence program, which mainly deals with how the system couples to the environment and how the environment states evolve into a complete set of orthogonal states. It explains how the states of the system lose coherence but not how we get one definite state out of all possible states.

While the definite outcome problem is usually regarded as the measurement problem that the majority of the interpretations of quantum mechanics is aimed at explaining, the preferred basis problem is one that receives relatively less attention.

The preferred basis problem demonstrates that after the system and apparatus interact they become entangled

$$|\Psi\rangle = \sum_n c_n |s_n\rangle|a_n\rangle,$$  
(6)

When $c_n$ are not distinct, following from the biorthogonal decomposition theorem we can in general rewrite the state in terms of different state vectors,

$$|\Psi\rangle = \sum_n c_n' |s_n'\rangle|a_n'\rangle,$$  
(7)

such that the same post measurement state seems to correspond to two different measurements, that is, of the observables $A = \sum_n \lambda_n |s_n\rangle\langle s_n|$ and $B = \sum_n \lambda_n' |s_n'\rangle\langle s_n'|$ of the system, respectively, although in general $A$ and $B$ do not commute.

It thus seems that from quantum mechanics we cannot tell which observable(s) of the system is (are) being recorded, via the formation of quantum correlations, by the apparatus. This can be stated in a general theorem \cite{PSB}: When quantum mechanics is applied to an isolated composite object consisting of a system $S$ and an apparatus $A$, it cannot determine which observable of the system has been measured—in obvious contrast to our experience of the workings of measuring devices that seem to be “designed” to measure certain quantities.

The PBP has been a profound challenge to the interpretation of quantum mechanics. For example, in von Neumann’s “collapse interpretation”, the question that remains is why should the composite system collapse into $|s_n\rangle|a_n\rangle$ and not $|s_n'\rangle|a_n'\rangle$? Alternatively, in the “many world interpretation”, one must ask why we observe a universe containing the composite system in $|s_n\rangle|a_n\rangle$ instead of a universe containing $|s_n'\rangle|a_n'\rangle$?

It is often believed that the preferred basis problem can be solved by the Environment induced superselection theory (Eisenselection) by Zurek \cite{Zurek, Zurek2, Zurek3, Zurek4}. Zurek introduces the state of the environment $E$ which interacts with the composite system $SA$. The system-apparatus-environment state then evolves in the following manner:

$$|\Phi\rangle = \left( \sum_n c_n |s_n\rangle|a_n\rangle \right)|e_0\rangle \xrightarrow{H_{SE}} \sum_n c_n |s_n\rangle|a_n\rangle|e_n\rangle$$  
(8)

where $\langle a_m|a_n\rangle = \delta_{mn}$. According to the tridecompositional uniqueness theorem \cite{Zurek}, even if the $c_n$ are not distinct, the decomposition of the total system $|\Phi\rangle$ is unique, as long as $\{|s_n\rangle\}$, $\{|a_n\rangle\}$ are linearly-independent bases and $\{|e_n\rangle\}$ is noncolinear. In other words, $|\Phi\rangle$ cannot be decomposed into another basis

$$|\Phi\rangle = \sum_n c_n |s_n\rangle|a_n\rangle|e_n\rangle \neq \sum_n c_n' |s_n\rangle|a_n\rangle|e_n'\rangle$$  
(9)

The preferred basis of the apparatus is chosen by the interaction with the environment, in that the projection $P_n = |a_n\rangle\langle a_n|$ should commute with the interaction.
Hamiltonian \( [H_{ce}, \hat{P}_z] = 0 \), in order that any correlation of the measured system with the eigenstates of a preferred apparatus observable \( \hat{O} = \sum_n \lambda_n |a_n\rangle\langle a_n| \) is preserved.

The key point in solving the PBP is the use of the tridecompositional uniqueness theorem. However, this does not necessarily require introducing the environment. From the previous discussion we see that to conserve total momentum, as long as a measurement type interaction happens, there should always be another degree of freedom of the system \( |p\rangle \) that carries the change due to the interaction,

\[
|\Phi\rangle = \sum c_n |s_n\rangle|p_0\rangle|a_0\rangle \xrightarrow{H_{ce}} \sum c_n |s_n\rangle|p_n\rangle|a_n\rangle \quad (10)
\]

where \( |s_n\rangle, |p_n\rangle \) are the states corresponding to the two degrees of the system, and \( |a_n\rangle \) is the state of the macroscopic system. In realistic cases \( |p\rangle \) is usually used as the the apparatus, since the difference between \( |a_0\rangle \) and \( |a_n\rangle \) is negligible. The point here is that, as long as the overlap is not exactly one, namely, \( \{ |a_n\rangle \} \) are not collinear, the tridecompositional uniqueness theorem can be used. This means that before the interaction between the apparatus and the environment, the preferred basis has already been determined by the interaction between the system and apparatus. The preferred basis is selected by the intrinsic mechanism of the measurement interaction, or in other words, the internal configuration of the measurement apparatus, instead of the environment. We name this solution as “interaction induced superselection”, (Inselection).

IV. INSELECTION VS EINSELECTION

The advantage of Inselection over Einselection is significant. First of all, Inselection is simpler, and more natural. When designing a quantum measurement experiment, what we actually do is to find a mechanism such that, if the input quantum state is \( |s_1\rangle \), the output state of the classical apparatus is \( |a_1\rangle \), if input \( |s_2\rangle \), the output is \( |a_2\rangle \), etc, and all these \( |a_n\rangle \) can be differentiated without an additional quantum measurement. The preferred bases in the PBP are just the states corresponding to the different final configuration of the apparatus after coupling to different states of the quantum system, as we expected. In order to abide by the third postulate of QT, we choose this mechanism in which there will be a third degree of freedom that carries the change due to the interaction between the apparatus and the system. The three degrees of freedom become entangled during this interaction so that the global state has only one decomposition, and that guarantees we will end up with the preferred basis of the apparatus. The preferred basis is already selected by this measuring interaction, without the need to introduce the environment. The role of the environment is to break the quantum coherence of the different states of the apparatus, when they are coupled to the environment, rather than to select the preferred basis. There is only one interaction in Inselection after which the preferred basis is readily determined, while in Einselection, which ignores the third degree of freedom, the preferred basis is not determined until the second interaction with the environment.

Here it will be beneficial to clarify a common misunderstanding of decoherence and Einselection for the PBP, which are closely related. Decoherence is a highly successful theory, which explains how a quantum system loses its quantum coherence due to its interaction with the environment. Decoherence deals with the system-environment interaction. The global system includes only the system and the environment and there is no pre-defined apparatus. There is also a pointer basis in decoherence, but that is the basis of the environmental states entangled with the system, which quickly become orthogonal and remains stable under the interaction with other parts of the environment. On the contrary, in the PBP Einselection serves to determine the preferred basis of the apparatus, by introducing a third party, the environment, so that the tridecompositional uniqueness theorem can be used to rule out other possible decompositions. In that case the preferred basis is chosen by the interaction between the apparatus and the environment.

The introduction of the environment to select the preferred basis is, not only redundant, but problematic. We should note that the PBP only arises when the expansion coefficients \( c_n \) are the same, otherwise the decomposition is unique. This means, according to Einselection, the preferred basis is determined by the interaction between the system and environment only for the special cases when \( c_n \) are the same, while for all other cases when the \( c_n \) are distinct, the preferred basis is determined by another interaction, the interaction between the apparatus and system. Thus the physical mechanism determining the preferred basis is different only because the superposition coefficients of the system are different. However, in real world, there is hardly any difference between \( c_n \) and \( c_n + \delta \), when \( \delta \) is very small. In Inselection, on the other hand, the preferred basis is determined by the interaction between the system and apparatus for all cases, which is more consistent.

Moreover, Einselection implies that if we could control the interaction between the apparatus and the environment, we can actually select a different preferred basis than what naturally arises from the mechanism of the measurement. This leads to serious problems. Let us consider a common example in literature introducing the PBP\[4\] (this example actually has a problem, see discussion later). Suppose we have a spin in the state \( |z-\rangle_1 = \frac{1}{\sqrt{2}}(|x+\rangle_1 - |x-\rangle_1) \). Now this becomes entangled with another spin state, using the second spin as the measurement apparatus. The composite system is
We have shown that in a quantum measurement, the interaction between the apparatus and the quantum system will entangle three degrees of freedom, one system degree of freedom that we are interested in and it is preserved by the interaction, one system degree of freedom that carries the change due to the interaction, and the apparatus degree of freedom which is always ignored. Considering all three degrees of freedom the composite state only has one decomposition, and this guarantees the apparatus will end up in the preferred basis which is what we expect in daily experience. We also point out some problems with the previous Einselection solution to the PBP, and this clarifies a common misunderstanding of decoherence and the preferred basis problem.

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