Hierarchical axioms for quantum mechanics

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The origin of nonclassicality in quantum mechanics (QM) has been investigated recently by a number of authors with a view to identifying axioms that would single out quantum mechanics as a special theory within a broader framework such as convex operational theories. In these studies, the axioms tend to be logically independent in the sense that no specific ordering of the axioms is implied. Here, we identify a hierarchy of five nonclassical features that separate QM from a classical theory: (Q1) Incompatibility and indeterminism; (Q2) Contextuality; (Q3) Entanglement; (Q4) Nonlocality and (Q5) Indistinguishability of identical particles. Such a hierarchy isn’t obvious when viewed from within the quantum mechanical framework, but, from the perspective of generalized probability theories (GPTs), the later axioms can be regarded as further structure introduced on top of earlier axioms. Relevant toy GPTs are introduced at each layer when useful to illustrate the action of the nonclassical features associated with the particular layer.

I. INTRODUCTION

What exactly makes quantum mechanics (QM) nonclassical? This question has been answered in different ways in quantum optics, in quantum information and the foundations of QM [1]. For example, in quantum optics, a state is considered to be nonclassical if its Glauber-Sudarshan P function [2] cannot be described as classical probability distribution function [2], i.e., it takes negative values. Other quasi-probability distributions that are used similarly to characterize nonclassical properties of light include the Wigner distribution W(x, p) and the Husimi Q distribution [3]. In quantum information theory, we associate nonclassicality with bi-partite or multi-partite quantum correlations that correspond to nonlocality [5], entanglement [6, 7], the weaker condition of non-vanishing discord [8] and Einstein-Podolsky-Rosen steering.

Quantum correlations are nonclassical when nonlocal, in that they can violate Bell-type inequalities [9, 10] that classical correlations cannot. However, the converse is not true as there exists nonclassical states which are local. In what follows, this point will be illustrated though a proposed hierarchical structure of the axioms of QM. Correlation inequalities for temporal situation can be proposed based on the assumption of realism and non-invasiveness [11].

The assumptions behind the derivation of a Bell-type inequality are localism and realism. A classical theory is necessarily local and realist. Consequently, a violation of Bell’s inequality essentially implies non-classicality. Likewise, as a classical theory is necessarily non-contextual and realist, a violation of a contextuality inequality would also imply non-classicality, but as before the converse is not true.

In the context of multi-partite systems, the relation between local properties and its non-local nature has been extensively studied by various authors [12–19]. In the inverse direction, bounds on nonlocality in nonsignalling theories have been derived by assumptions about monopartite system properties like uncertainty [20] and complementarity [21]. In Ref. [10], it’s shown that any theory which cannot be ascribed a simplex state space structure, with pure states being one-shot distinguishable, has a no-cloning theorem. Monopartite nonclassical systems has been considered in the ontological framework by Spekkens in Ref. [22, 23]. An argument for the classicality for discrete physical theories satisfying an information theoretic axiom is presented in Ref. [24].

In this work, we identify five basic elements that separate QM from classical physics: (Q1) Incompatibility and indeterminism; (Q2) Contextuality; (Q3) Quantum entanglement; (Q4) Nonlocality; (Q5) Indistinguishability of identical particle. A toy theory is associated with each axioms, and we list the relevant information theoretical tasks that can be achievable in those theories.

In the foundations of quantum mechanics, one often studies nonclassical features in the framework of generalized probability theories (GPTs) [12–27], with the aim to identify the minimal set of axioms to guarantee the nonclassical properties in QM. In this approach, QM, classical theory and a set of other nonclassical probabilistic theories can be considered as special cases in a framework of GPTs. To be precise, by QM we mean the operational formulation of QM in terms of measurements, probabilities and correlations as would be observed in a laboratory experiment, and states are considered to be the lists of probabilities of outcomes, with state spaces being the convex set of such operational states. This formulation avoids terminology such as Hilbert spaces or phase that can’t be directly observed. Note that, mathematically, quantum mechanics can be viewed as a non-classical probability calculus based on a non-classical propositional logic, such that quantum states are measures on a suitably defined non-Boolean...
(non-distributive), orthocomplemented lattice.

One usually associates nonclassicality of QM with features like fundamental indeterminacy \cite{28}, Heisenberg uncertainty, monogamy of nonlocal correlations \cite{29}, privacy of nonlocal correlations \cite{30}, and the impossibility of perfect cloning \cite{31,32}. However, to the best of our knowledge, till now no hierarchical understanding of quantum nonclassicality is known, such that certain features are understood to be built on top of the others. This is of course because within QM, the features are mathematically interdependent, with no obvious ordering discernible. Here, we address this issue, by presenting a hierarchy of quantum features, that are inspired by GPT considerations.

There are certain details to which we will return elsewhere: underlying the hierarchy is the assumption of linearity, which is not specific to quantum mechanics, and exists already in classical mechanics; the hierarchical transition from (Q2) to (Q3) assumed a tensor product structure between the state space, which itself is not a nonclassical feature; (Q3) to (Q4) must include classical notions of entanglement and quantum steering. Let us elucidate on the axioms (Q2)- (Q5), and the toy theories valid in each layer. Quantum information processing tasks that can be performed in a layer is also discussed. Finally, the paper is concluded in Section VII.

II. INCOMPATIBILITY

A fundamental feature of quantum mechanics is the incompatibility of two observables, which, for projective measurements coincides with non-commutativity. In the context of GPTs, the idea of incompatibility is formalized by the smallest value of mixing or “unsharpness” for which two measurements will be jointly measurable, in the sense that they can be obtained as marginals of a master observable \cite{33}. The cryptographic power of incompatibility, without any other nonclassical feature being assumed, can be illustrated via the BB84-like key distribution protocol, which we call “local key distribution” (LKD).

LKD works as follows. Alice and Bob each have two copies of the key to a strongbox. Alice opens the box and leaves a random bit \( \kappa \in \{0, 1\} \) for Bob to read, and then locks the box. Bob comes along later, open the box with his key, and receives his message.

In the real world, a classical implementation of LKD is not unconditionally secure, since classical laws do not preclude that an eavesdropper can break open the box, read the secret bit \( \kappa \) and then rebuild the box, without Alice and Bob knowing about it.

Consider the two-input-two-output operational theory \( \mathcal{T} \), with two dichotomic measurements \( X \) and \( Z \). The pure states of the theory \( \mathcal{T} \), which forms the state space \( \Sigma \), are:

\[
\begin{align*}
\psi_X^+ &= (1, 0 | \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \\
\psi_X^- &= (0, 1 | \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \\
\psi_Z^+ &= (\frac{1}{2}, \frac{1}{2} | 1, 0) \\
\psi_Z^- &= (\frac{1}{2}, \frac{1}{2} | 0, 1).
\end{align*}
\]

With the nonsimpliciality condition,

\[
\frac{1}{2}(\psi_X^+ + \psi_X^-) = \frac{1}{2}(\psi_Z^+ + \psi_Z^-) = \left(\frac{1}{2}, \frac{1}{2} | \frac{1}{2}, \frac{1}{2}\right).
\]

\( \mathcal{T} \) is nonclassical.

Now consider the following protocol implemented in the nonclassical but noncontextual theory given by \( \mathcal{T} \). It is the LKD version of the Bennett-Brassard 1984 (BB84) quantum key distribution (QKD) protocol \cite{34}, and we may refer to it as BB84 LKD \cite{35}.

1. Alice randomly prepares \( n \) particles in one of the four states \( |\psi_X^+\rangle, |\psi_X^-\rangle \) given by \( \mathcal{T} \) by measuring \( X \) or \( Z \) on each particle. She transmits them to Bob.

2. Bob measures the particles randomly in the basis \( X \) or \( Z \). He notes the outcomes \( \tau \).

3. On the key string so extracted, Alice and Bob publicly discuss to retain only those outcomes where their bases agree; this forms their raw key string;

4. They agree on certain coordinates and announce the outcomes on those coordinates. If too many of them are mismatched, they deem the protocol round secure and abort the round.

5. Else Alice and Bob proceed to classically extract a secure, smaller key from the remaining bits.

Security of the above protocol originates from the fact that although Eve can deterministically extract the encoded bit by measurement if she measures in the right basis, she will produce measurement disturbance if she gets the basis wrong, which can be detected in Step (3).

Suppose Eve implements such an interpret-resend attack, by measuring \( X \) or \( Z \) on \( m \) gbits from a total of \( n \) particles transmitted. She will be able to extract \( I(A:E) = \frac{m}{2} \) bits of information on average. Let \( f = \frac{m}{n} \).

On average, Alice and Bob will check the basis not used by Eve half the times, and half of these times, they would obtain the answer not encoded by Alice. On the remaining fraction, their measured outcome will be consistent
with Alice’s encoded value. Thus, the error observed by Alice and Bob is \( e = \frac{1}{4} \), so that on average Bob receives

\[ I(A : B) = \left(1 - h\left(\frac{1}{4}\right)\right) \text{ bits of information per transmitted gbit.} \]

The protocol can be shown to be secure if \( I(A : B) \geq I(A : E) \) \cite{36}, which in this case becomes

\[ \left(1 - h\left(\frac{f}{4}\right)\right) \geq \frac{f}{2} \] \hspace{1cm} (3)

or \( f \approx 0.68 \), so that the tolerable error rate \( e_{\text{max}} = 0.48/4 = 17\% \). The probability that Eve is not detected on an attacked particle is \( \frac{4}{5} \). Therefore the probability that she escapes detection on all the \( m \) bits she attacks is \( (3/4)^m \), which falls exponentially with security parameter \( m \). This exponential drop characterizes unconditional security.

However, the security described above is not device independent (cf. \cite{37}, and references therein). Alice and Bob implicitly assume in the BB84 LKD protocol that the preparation and measurement devices are trustworthy. Now suppose that the device has been manufactured by Eve such that each actual particle is replaced by a pre-existing bit is presented, and similarly for if she measures \( Z \). If (subsequently) Bob measures the same observable, he would obtain the same bit as obtained by Alice, else an uncorrelated bit. This reproduces the BB84 statistics. It is entirely insecure once Eve learns about their respective bases during their public discussion. This is just the higher-dimension attack \cite{38} adapted from BB84 to the present protocol. Thus, BB84 LKD is not secure in the device independent scenario.

### III. CONTEXTUALITY

Now we consider a nonclassical theory with nontrivial congruence structure, characterized by five observables \( V, W, X, Y, Z \) and the cyclic \( R_2 \)-chain: \( R_2(V, W), R_2(W, X), R_2(X, Y), R_2(Y, Z), R_2(Z, V) \), and every other pair being incongruent. By the assumption of tomographic separability, an arbitrary state is assumed to be completely specified by the five fiducial probabilities \( P_V, P_W, P_X, P_Y \) and \( P_Z \), which in turn determine \( P_{VW}, P_{VX}, P_{VY}, P_{VZ} \) and \( P_{WX} \), assumed to be consistent with contextual no-signaling. We shall refer to this theory (fragment) as \( T_{\text{KCBS}} \), in view of the work where the contextuality of such correlations was studied \cite{39}.

Consider a state \( \rho \) in this contextual theory, where all the above pairs produce perfectly random but anticorrelated outcomes, i.e., 01 or 10. We can check directly that there is no way to assign values 0 and 1 to these five observables (indeed, any odd number of observables) in such a way as to satisfy this requirement, because there would be a clash of values on at least one observable. That is, if \( A \) has value \( a \), then \( B \) has \( \overline{a} \), \( C \) has \( a \), \( D \) has \( \overline{a} \), \( \overline{a} \) so that \( E \) has \( a \) requiring \( A \) to have \( \overline{a} \), contrary to assumption. Thus, \( \rho \) does not correspond to a state that has a JD over the five variables, where they take definite values. This is witnessed by the violation of the KCBS inequality

\[ \langle VW \rangle + \langle WX \rangle + \langle XY \rangle + \langle YZ \rangle + \langle ZA \rangle \geq -3, \] \hspace{1cm} (4)

where \( V, W, X, Y, Z = \pm 1 \) \cite{39}.

Now consider the following protocol implemented in \( T_{\text{KCBS}}: \) KCBS LKD.

1. Alice prepares \( n \) particles in state \( \rho \), which she leaves at a pre-agreed location, after measuring each using one of the five observables \( V, W, X, Y, Z \);

2. Later Bob arrives at the location and measures the particles randomly in any one of these five bases.

3. Alice and Bob announce their measurement bases and throw away the (approximately 40\%) data corresponding to instances where their bases aren’t either identical or juxtaposed;

4. They publicly agree on certain coordinates of particles, and disclose their measurement outcomes for these. If their selected bases are identical (resp., juxtaposed), they verify that the outcomes are random and identical (resp., anti-correlated). If too many of them fail this criterion, then they abort the protocol.

5. Else Alice and Bob proceed to classically distil a secure, smaller key from the remaining bits.

In this case note that no pre-existing record can reproduce the KCBS perfect correlations. Thus, any passive cheat device of the type mentioned above will fail to pass the KCBS test, giving rise to a kind of device independence. Note that an active cheat device, meaning one that allows memory to be carried forward in time (which is a kind of local signaling), can defeat the protocol. For example, the device produces an arbitrary output when Alice measures. Her basis and outcome information are retained in the system’s memory, so that if Bob measures in the basis as she did, then the device produces the identical (resp., anticorrelated) outcome if his basis matches (resp., is juxtaposed to) hers, and a random outcome otherwise. Note that the memory corresponds to a kind of signal formally, but which is not prohibited by special relativity, since the correlation is local. We shall refer to this scenario where Eve that is restricted from memory attacks in a local protocol, as bounded device independence.

Thus, contextuality can provide security to LKD in the restricted device-independent scenario where memorylessness (with regard to Alice’s input) is assumed, but a nonclassical theory without contextuality lacks security even in this memoryless device-independent scenario. Note that in the case where device independence is based on nonlocality, the memory of Alice’s actions (and
IV. ENTANGLEMENT

Quantum entanglement, an important element which deviates our classical world view, acts as a resource for many information theoretic as well as many computational advantages, providing a dominant non-classical feature \( [40] \). Even though, as few results claim, the existence of entanglement in classical optics, it lack any information theoretic advantages. An important task which reveals entanglement and also of necessary is teleportation. It is been proved that entanglement is sufficient for teleportation in any GPTs. The important relation revealed by Spekkens toy model \( [23] \) as well as the toy model proposed by Hardy shows the difference between entanglement and non-locality \( [11] \). They show that both entanglement and teleportation is possible in local theory, thus seperating nonlocality from entanglement.

V. NONLOCALITY

In a scheme for QKD, suppose at the end of the quantum part, Alice’s and Bob’s joint probability \( P(ab|xy) \) is described by:

\[
P(ab|xy) = \sum_\mu p(\mu), P(a|x, \mu)P(b|y, \mu), \tag{5}
\]

with \( p(\mu) \) being a probability distribution over parameter \( \mu \). Since this is potentially preparation information with Eve, the condition of security in the device-independent sense is that \( P(ab|xy) \) shouldn’t have the above form, i.e., should be a nonlocal correlation \( [10] \). Thus, including the axiom of nonlocal correlations being allowed in the theory enables DI security in the usual paradigm of cryptography.

Indeed, in any non-signaling theory, nonlocality can be the basis for distilling shared secret randomness \( [12] \).

VI. INDISTINGUISHABILITY OF IDENTICAL PARTICLES

Our final axiom is indistinguishability, relatively less studied aspect of quantum nonclassicality both in quantum information processing and the GPT framework. A task that separates quantum mechanics from classical mechanics, that wouldn’t be possible even with our above axioms (i.e., quantum mechanics based purely on distinguishable particles), is boson sampling, a task that becomes easy when quantum indistinguishability is included.

Boson sampling, introduced in Ref. \( [43] \) and experimentally realized in Refs. \( [44, 45] \) and references therein, is the task of exactly or approximately sampling from the probability distribution of identical bosons scattered by a linear interferometer. It’s widely believed that this task is intractable in the classical world (i.e, intractable for classical computers), but can be solved efficiently in the quantum world. In other words, inclusion of \( (Q6) \) in a nonclassical theory provides us the ability of solving boson sampling problem. The appearance of the permanent in the outcome statistics of single-photon measurements makes the task computationally hard, whereas the corresponding linear optics uses only polynomial resources to implement it practically. Here, we note that the permanent of a square matrix in linear algebra is a determinant-like function of the matrix, and a special case of immanant, a more general matrix function.

The reason this is interesting from a computational perspective is that the probability distribution that boson sampling device is required to sample from, which as noted above is connected to the permanent of a complex matrix. Computing the permanent, as well as approximating it within multiplicative error, are known in the general case to be in the \#P-hard computational complexity class. Therefore, just as quantum nonlocality suggests a “behind-the-scenes” super-classical communication, so does bosonic sampling suggest a “behind-the-scenes” super-classical computing, that in some ways is more spectacular than the quantum speedup witnessed in Shor’s prime factorization algorithm.

VII. CONCLUSION

We have proposed a hierarchy of axioms for quantum mechanics, meant to bring out the increasing structure in the theory as a departure from classical mechanics, rather than to derive quantum mechanics per se. These axioms, inspired by considerations from GPTs or convex operational theories, are in their proper order given by: \( (Q1) \) Incompatibility (or complementarity) and indeterminism; \( (Q2) \) Contextuality; \( (Q3) \) Entanglement; \( (Q4) \) Nonlocality and \( (Q5) \) Indistinguishability of identical particles. The axioms are illustrated either through a quantum information processing task that wouldn’t be possible without it, or a task from a GPT. We hope that this work should light on the question of what makes quantum mechanics special to be singled out by Nature.

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