Higher-derivative non-Abelian gauge fields via the Faddeev-Jackiw formalism

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Abstract

In this paper we analyze two higher-derivative theories, the generalized electrodynamics and the Alekseev-Arbuzov-Baikov’s effective Lagrangian from the point of view of Faddeev-Jackiw symplectic approach. It is shown that the full set of constraint is obtained directly from the zero-mode eigenvectors, and that they are in accordance with known results from Dirac’s theory, a remnant and recurrent issue in the literature. The method shows to be rather economical in relation to the Dirac’s one, obviating thus unnecessary classification and calculations. Afterwards, to conclude we construct the transition-amplitude of the non-Abelian theory following a constrained BRST-method.

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1 Introduction

A standard classical treatment of constrained theories was given originally by Dirac [1], it essentially analyzes the canonical structure of any theory, and it has been widely used in a great variety of quantum systems. However, it could be realized that Dirac’s methodology is unnecessarily cumbersome and can be streamlined. Within this context, Faddeev and Jackiw [2] suggested a sympletic approach for constrained systems based a first-order Lagrangian. This method has some very interesting properties of obviating the constraint classification, unnecessary calculations and the hypothesis of Dirac’s conjecture as well. The Faddeev-Jackiw (FJ) sympletic formalism has been studied in a systematic way in different scenarios, shedding a new light into the research of constrained dynamics.

The basic geometric structure of the Faddeev-Jackiw theory can be read directly from the elements of the inverse sympletic matrix, and coincides with the correspondent Dirac brackets, providing thus a bridge to the commutators of the quantized theory. On the other hand, the results obtained from the Faddeev-Jackiw approach have been compared with the corresponding results of Dirac method in different situations, for unconstrained and constrained systems, but it is still matter of study.

The method has as the key ingredient that these constraints produce deformations in the two-form sympletic matrix in such way that, when all constraints are considered (by means of a Darboux transformation), the sympletic matrix is non-singular. As a result, it was obtained the Dirac brackets. Nevertheless, it is important to emphasize that sometimes, it happens that the (iteratively deformed) two-form matrix is singular and no new constraint is obtained from the corresponding zero-mode. This is the case when one deals with gauge theories. At this point one should introduce convenient gauge (subsidiary) conditions like a constraint and the two-form matrix becomes, therefore, invertible. This extension was proposed and developed by Barcelos-Neto and Wotzasek [3], and studied in several models [4]. It basically followed the spirit of Dirac’s work, with proposal works by imposing the stability of the constraints under time evolution. So, constraints are not solved but embedded in an extended phase-space. This is a more suitable approach when some relevant symmetries must be preserved.

A subtle issue subsequent to the Faddeev-Jackiw method is its equivalence to the Dirac method. Initially the equivalence was discussed in cases when the systems have not constraint [5]; but, in a constrained system, the situation was not completely clear, and some argumentation was provided earlier [6] about the equivalence between the methods. However, recently it was presented a proof [7] that the usual Faddeev-Jackiw method and Dirac method were not completely equivalent; namely, they showed that some constraints calculated in Dirac formalism do not appear in the calculation in Faddeev-Jackiw formalism. And then these would result in the contradiction between the usual Faddeev- Jackiw quantization and Dirac quantization [8].

Higher-derivative Lagrangian functions [9] are a fairly interesting branch of the ongoing effective

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1 Actually the geometric role played by the constraints is to produce a ’deformation’ in the original, singular, sympletic two-form matrix.
theories, and were initially proposed as an attempt to enhance and render a better ultraviolet behavior of physically relevant models. It is known that higher-derivative theories have, as a field theory, better renormalization properties than the conventional ones. These properties have shown to be quite appealing in the attempts to have a quantized and renormalizable theory of gravity \[10\]. The undesired features of the higher-derivative theory is that they possess a Hamiltonian that is not bounded from below and that the process of adding such terms jeopardizes the unitarity of the theory \[11\]. Besides all these motivations we emphasize that, from a theoretical point of view, higher-derivative theories have many interesting features that justify their study by itself.

As it has been pointed out in several works \[12-14\] along the years, it is long clear that Maxwell’s theory is not the only one to describe the electromagnetic field. One of the most successful generalizations is the generalized electrodynamics \[12\]. Actually, Podolsky’s theory is the only one linear, Lorentz, and $U(1)$ invariant generalization of Maxwell’s theory \[14\]. Another interesting feature inherent to Podolsky’s theory is the existence of a generalized gauge condition also, namely, the generalized Lorenz condition: $\Omega[A] = (1 + M^{-2} \Box) \partial_{\mu} A^{\mu}$; considered an important issue, it is only through the choice of the correct gauge condition that we can completely fix the gauge degrees of freedom of a given gauge theory \[13\]. The relative success of these achievements motivated some authors to propose finite extensions of Quantum Chromodynamics (QCD) \[15\] and also to advocate that higher order terms would be able to explain the quark confinement. Our main goal here would be exactly to study both higher-derivative theories, Podolsky’s electrodynamics and a non-Abelian \[16\] extension of the model, also known as the Alekseev-Arbuzov-Baikov’s effective Lagrangian \[17\] in the framework Faddeev-Jackiw sympletic approach. As far we have no knowledge of application of the Faddeev-Jackiw method to higher-derivative theories. Moreover, it may also shed some new light on the issue of whether the accordance between the Dirac and Faddeev-Jackiw methods holds.

In this paper, we discuss the canonical structure of the Podolsky’s electrodynamics and the $SU(N)$ Alekseev-Arbuzov-Baikov’s effective Lagrangian in the light of the Faddeev-Jackiw approach. In Sect\[2\] we start by making a brief review of both the FJ and (constrained) sympletic formalisms. And, as the generalized electrodynamics of Podolsky has been already subject of analysis from the Dirac’s point of view \[13\], we shall study the theory via the FJ method in order to present an exercise of the methodology and also to check its consistency. Next, in Sect\[3\] we discuss and introduce the Alekseev-Arbuzov-Baikov’s effective Lagrangian by discussing the generalized electrodynamics by making use of enlargement of the gauge group to non-Abelian ones. Having defined the Lagrangian density we proceed in presenting the methodology, and obtaining the full set of constraints of the theory. Although an attempt of a path-integral formulation based on the FJ method has been proposed \[18\], there is no conclusive, neither clear argument to show the consistence of the method. Therefore, by means of complementarity of the previous discussion, we conclude the section by constructing the transition-amplitude for the non-Abelian theory via the Batalin-Fradkin-Vilkovisky (BFV) method \[19\], obtaining an important outcome for subsequent analysis in the quantum level. In Sect\[4\] we
summarize the results, and present our final remarks and prospects.

# 2 Generalized electrodynamics via Faddeev-Jackiw formalism

## 2.1 Faddeev-Jackiw sympletic method

Let us start with a first-order in time derivative Lagrangian, which may arise from a conventional second-order one after introducing auxiliary fields. First, one can construct the sympletic Lagrangian

\[ \mathcal{L} = a_i(\xi) \dot{\xi}^i - \mathcal{V}(\xi), \quad \text{(2.1)} \]

with the arbitrary one-form components \( a_i \), with \( i = 1, \ldots, N \). The first-order system is characterized by a closed two-form. If the two-form is non-degenerated, it defines a sympletic structure on the phase space, described by the coordinates \( \xi_i \). On the other hand, if the two-form is singular, with constant rank, it is called a pre-sympletic two-form. Thus, in terms of components, the (pre)sympletic form is defined by

\[ f_{ij} = \frac{\partial}{\partial \xi^i} a_j(\xi) - \frac{\partial}{\partial \xi^j} a_i(\xi). \quad \text{(2.2)} \]

The Euler-Lagrange equations are given by

\[ f_{ij} \dot{\xi}^j = \frac{\partial}{\partial \xi^i} \mathcal{V}(\xi). \quad \text{(2.3)} \]

Now, when the two-form \( f_{ij} \) is nonsingular, it has an inverse \( f^{ij} \), then

\[ \dot{\xi}^i = f^{ij} \frac{\partial}{\partial \xi^j} \mathcal{V}(\xi), \quad \text{(2.4)} \]

and the basic bracket is defined as \( \{ \xi^i, \xi^j \} = f^{ij} \). However, in the case that the Lagrangian \( \text{(2.1)} \) describes a constrained system, the sympletic matrix is singular, and the constraints hidden in the system need to be determined. Let us suppose that the rank of \( f_{ij} \) is \( 2n \), so there is \( N - 2n = M \) zero-mode vectors \( v^\alpha \), \( \alpha = 1, \ldots, M \). The system is then constrained by \( M \) equation in which no time-derivatives appear. Then there will be constraints that reduce the number of degrees of freedom. Thus, multiplying \( \text{(2.3)} \) by the (left) zero-modes \( v^\alpha \) of \( f_{ij} \) we get the (sympletic) constraints in the form of algebraic relations

\[ \Omega^\alpha \equiv v^\alpha_i \frac{\partial}{\partial \xi^i} \mathcal{V}(\xi) = 0. \quad \text{(2.5)} \]

Then, one can give the first-iterated Lagrangian by introducing corresponding Lagrange multipliers of the obtained constraints

\[ \mathcal{L} = a_i^{(1)}(\xi) \dot{\xi}^i + \Omega^\alpha \lambda_\alpha - \mathcal{V}^{(1)}(\xi). \quad \text{(2.6)} \]

\(^2\)In this section we discuss a system with finite degree of freedom. However, an extension to the infinite degree of freedom case can be attained in a straightforward way.
Hence, one may regard the introduced Lagrange multipliers $\lambda$ as sympletic variables and can extend the sympletic variable set. This procedure reduces the number of $\xi$’s. Then the whole procedure can be repeated again until all constraints are eliminated and we are left with a completely reduced, unconstrained, and canonical system. However, it should be remarked that in the case of gauge theories, the zero-mode does not give any new constraint (it still does not give the full rank matrix), and the sympletic matrix remains singular. Thus, we should consider that it is necessary to introduce gauge condition(s) to obviate the singularity. So the work can be finished in expectation in terms of the original variables, and the basic brackets can be determined.

2.2 Generalized electrodynamics

The purpose of the present study is to examine the FJ methodology applied in the analysis of a higher-derivative theory. It is interesting to study first, as a simpler example, an Abelian theory. Therefore, in order to keep the things in a simple realm, we choose the simplest but rather interesting Abelian electrodynamics of Podolsky, whose Lagrangian density is given by

$$L = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2M^2}\partial_\mu F^{\mu\nu} \partial^\lambda F_{\lambda\nu},$$

(2.7)

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and the spacetime metric elements are $\eta_{\mu\nu} = (1, -1, -1, -1)$. It should be mentioned that we shall follow the Ostrogradski approach [9] to deal with higher-derivative terms. Hence, it should be introduced another set of canonical pair $(\Gamma^\mu \equiv \partial_0 A^\mu, \phi_\nu)$ in order to have a correct expanded phase space to thus proceed with the canonical analysis. With this thought in mind one finds then the following Lagrangian [13]

$$L = \frac{1}{2} \left( \nabla \cdot \vec{\Gamma} - \nabla_0 A_0 \right)^2 + \frac{1}{2} \left( \nabla \times \vec{A} \right)^2$$

$$+ \frac{1}{2M^2} \left[ (\nabla \cdot \vec{\Gamma} - \nabla_0^2 A_0)^2 - (\partial_0 \vec{\Gamma} - \nabla_0 \vec{\Gamma} - \nabla \times (\nabla \times \vec{A}))^2 \right].$$

(2.8)

To transform this Lagrangian from second to first-order, we shall use an auxiliary field, that is choose to be the canonical momentum due to an algebraic simplification that it provides. In that case, we should remind that we have additional set of canonical pairs, in particular here, $(A, \pi)$ and $(\Gamma, \phi)$:

$$\phi^\mu = \frac{\partial L}{\partial (\partial_0 \partial_0 A_\mu)},$$

(2.9)

and

$$\pi^\mu = \frac{\partial L}{\partial (\partial_0 A_\mu)} - 2\partial_k \left( \frac{\partial L}{\partial (\partial_0 \partial_k A_\mu)} \right) - \partial_0 \left( \frac{\partial L}{\partial (\partial_0 \partial_0 A_\mu)} \right),$$

(2.10)

resulting into the following expressions

$$\pi^\mu = F^{\mu0} - M^{-2} \left( \eta^{\mu k} \partial_k \partial_\lambda F^{0\lambda} - \partial_0 \partial_\lambda F^{\mu\lambda} \right),$$

(2.11)
and
\[ M^2 \phi^\mu = \left( \eta^{\mu0} \partial_\lambda F^00 - \partial_\lambda F^{\mu0} \right), \]

where
\[ M^2 \phi = \partial_0 \vec{\Gamma} - \nabla \Gamma^0 - \left( \nabla \times \left( \nabla \times \vec{A} \right) \right). \] (2.12)

In order to obtain the quadratic kinetic terms we may make use of the equation of motion for \( \phi \) (2.12) back into the Lagrangian. Therefore, in this case one can cast the Lagrangian density (2.8) as
\[ \mathcal{L} = -\phi_k \dot{\Gamma}_k + \pi_\mu A^\mu - \mathcal{V}(0), \] (2.13)

where the potential density is
\[ \mathcal{V}(0) = \pi_\mu \Gamma_\mu - \frac{1}{2} \left( \vec{\Gamma} - \nabla A_0 \right)^2 - \frac{1}{2} \left( \nabla \times \vec{A} \right)^2 - \frac{M^2}{2} \phi^2 - \vec{\phi} \cdot \left( \nabla \Gamma_0 + \nabla \times \left( \nabla \times \vec{A} \right) \right). \] (2.14)

The initial set of sympletic variables is seen to be \( \xi^i_\alpha = \{ A_k, \pi_k, A_0, \pi_0, \Gamma_k, \phi_k, \Gamma_0 \} \), this permits us to identify the non-null canonical one-form
\[ \Gamma a_i = -\phi_i, \quad A a_i = -\pi_i, \quad A_0 a = \pi_0. \] (2.15)

These previous results lead to the corresponding two-form matrix
\[ (0) f_{ab}(x,y) = \begin{bmatrix} A_{ij} & 0_{4 \times 3} \\ 0_{3 \times 4} & B_{ij} \end{bmatrix} \delta(x,y), \] (2.16)

with
\[ A_{ij} = \begin{bmatrix} 0 & \delta_{ij} & 0 & 0 \\ -\delta_{ij} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B_{ij} = \begin{bmatrix} 0 & \delta_{ij} & 0 \\ -\delta_{ij} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \] (2.17)

it is not complicated to see that the matrix is singular. Moreover, it is easy to determine that the eigenvector with zero eigenvalue is
\[ \nu_\alpha = \left( 0, 0, 0, 0, 0, \nu^7 \right), \] (2.18)

where \( \nu^7 \) is arbitrary and associated to \( \Gamma_0 \). Therefore, from the eigenvector \( \nu_\alpha \) (2.18) we can evaluate the consistence condition as
\[ \int dxdy \nu^7 \frac{\delta}{\delta \Gamma_0 (x)} \mathcal{V}^{(0)}(y) = \int dx \nu^7 \left( \pi_0 + \nabla \cdot \vec{\phi} \right) = 0, \] (2.19)

since \( \nu^7(x) \) is an arbitrary function, we obtain the constraint
\[ \Omega(x) \equiv \pi_0(x) + \nabla \cdot \vec{\phi}(x) = 0. \] (2.20)
Introducing this constraint back into the Lagrangian by means of a Lagrange multiplier $\lambda$:

$$\mathcal{L} = -\phi_k \dot{\Gamma}_k + \pi_\mu \dot{A}^\mu + \dot{\lambda} \left( \pi_0 + \nabla \cdot \vec{\phi} \right) - \mathcal{V}'^{(1)},$$

(2.21)

where the first-iterated potential density is $\mathcal{V}'^{(1)} = \mathcal{V}'^{(0)} \bigg|_{\Omega=0}$ with

$$\mathcal{V}'^{(1)} = -\pi_k \Gamma_k - \frac{1}{2} \left( \vec{\nabla} - \nabla A_0 \right)^2 - \frac{1}{2} \left( \vec{\nabla} \times \vec{A}_0 \right)^2 - \frac{1}{2M^2} \left( \vec{\nabla} \cdot \vec{\Gamma} - \vec{\nabla}^2 A_0 \right)^2 - \frac{M^2}{2} \phi^2 - \dot{\phi} \cdot \left( \vec{\nabla} \times \left( \vec{\nabla} \times \vec{A}_0 \right) \right).$$

(2.22)

From the above Lagrangian we have the following vectors

$$\Gamma^{(1)}_i = -\phi_i, \quad A^{(1)}_i = -\pi_i, \quad A^0^{(1)} = \pi_0, \quad \lambda^{(1)} = \pi_0 + \nabla \cdot \vec{\phi},$$

(2.23)

these results lead to the corresponding two-form matrix

$$(1) f_{ab}(x,y) = \begin{bmatrix} A_{ij} & D_j \\ -D_i & C_{ij} \end{bmatrix} \delta(x,y),$$

(2.24)

with $C_{ab}(x,y)$ and $D_{ab}(x,y)$ being the Abelian version of the non-Abelian expressions $C_{ab}(x,y)$ and $D_{ab}(x,y)$, Eq.(3.19). We obtain once again a singular matrix. From that, we can determine its eigenvector with zero eigenvalue,

$$\nu_\alpha = (0,0,\nu^3,0,\nu^5,0,\nu^7),$$

(2.25)

Therefore, following the routine, from this eigenvector $\nu_\alpha$ (2.25) we can evaluate the consistence condition

$$\int dx \left( \frac{\delta}{\delta A_0(x)} + \frac{\delta}{\delta \Gamma^i(x)} \right) \int dy \mathcal{V}'(y) = \int dx \nu^3 \left( \vec{\nabla} \cdot \vec{\phi} \right)(x) = 0,$$

(2.26)

where in the last equality we have made use of the relation $\nu^3 - \partial_i \nu^3 = 0$. Once again, as $\nu^3$ is an arbitrary function, we obtain a new constraint relation (Gauss’ law)

$$\vec{\Omega} (x) \equiv \left( \vec{\nabla} \cdot \vec{\phi} \right)(x) = 0.$$

(2.27)

Now, following the methodology, the second-iterated Lagrangian reads

$$\mathcal{L} = -\phi_k \dot{\Gamma}_k + \pi_\mu \dot{A}^\mu + \dot{\lambda} \left( \pi_0 + \nabla \cdot \vec{\phi} \right) + \dot{\eta} \left( \vec{\nabla} \cdot \vec{\phi} \right) - \mathcal{V}'^{(2)},$$

(2.28)

whereas the second-iterated potential density is

$$\mathcal{V}'^{(2)} = \left. \mathcal{V}'^{(1)} \right|_{\Omega=0} \mathcal{V}'^{(1)}.$$

(2.29)

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3It should be noted that when the constraint $\Omega$ is imposed the dependence in $\Gamma_0$ naturally disappears.
From the above Lagrangian one finds the following vectors
\[ \Gamma_{a_i}^{(2)} = -\phi_i, \quad A_{a_i}^{(2)} = -\pi_i, \quad A_0 = \pi_0, \quad \lambda a^{(2)} = \pi_0 + \nabla^2 \phi, \quad \eta a^{(2)} = \nabla^2 \pi, \]
these results lead to the corresponding two-form matrix
\[
(2) f_{ab}(x,y) = \begin{bmatrix} A_{ij} & E_{j,x} \\ -E^T_{i,y} & F_{ij} \end{bmatrix} \delta(x,y),
\]
with
\[
E_{i,x} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\partial_i^x \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad F_{ij} = \begin{bmatrix} 0 & \delta_{ij} & 0 & 0 \\ -\delta_{ij} & 0 & -\partial_i^x & 0 \\ 0 & -\partial_i^x & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.
\]
It follows that the second-iterated matrix \( f_{ab}(x,y) \) is also singular. From that one obtain two zero-mode vectors
\[
\tilde{\nu}_\alpha = \begin{bmatrix} \tilde{\nu}_1^1, 0, 0, 0, 0, 0, 0, \tilde{\nu}^8 \end{bmatrix},
\]
and
\[
\overrightarrow{\nu}_\alpha = \begin{bmatrix} 0, 0, \overrightarrow{\nu}_1^1, 0, \overrightarrow{\nu}_1^2, 0, \overrightarrow{\nu}_1^3 \end{bmatrix},
\]
However, the vector \( \overrightarrow{\nu}_\alpha \) generates the constraint \( \nabla \cdot \overrightarrow{\pi} = 0 \). Therefore, it is only the vector \( \tilde{\nu}_\alpha \) of interest. Subsequently, the consistence condition results into
\[
\int dxdy \left[ \frac{\delta}{\delta A^i(x)} + \tilde{\nu}^8(x) \frac{\delta}{\delta \eta(x)} \right] \nu^{(2)}(y) = 0,
\]
Thus, the zero-mode does not generate any new constraints and, consequently, the sympletic matrix remains singular. Being this a imprint characteristic of gauge theories, therefore, the gauge degrees of freedom has to be fixed. We choose the work here with the generalized Coulomb gauge: \( A_0 = 0 \) and \( (1 + M^{-2} \Box) (\nabla \overrightarrow{\pi}) = 0 \). We then obtain a new Lagrangian density
\[
\mathcal{L} = -\phi_k \Gamma_k + \pi_k \overrightarrow{A}^k + \lambda \left( \pi_0 + \nabla^2 \phi \right) + \eta \left( \nabla^2 \pi \right) + \chi \left( 1 - M^{-2} \Box^2 \right) \nabla \cdot \overrightarrow{A} - \nu^{(3)},
\]
where the third-iterated potential density is \( \nu^{(3)} = \nu^{(2)} \bigg|_{\Omega = 0} \) with
\[
\nu^{(3)} = -\pi_k \Gamma_k - \frac{1}{2} \overrightarrow{A}^2 + \frac{1}{2} \overrightarrow{A} \cdot \left( \nabla^2 \overrightarrow{A} \right) - \frac{1}{2M^2} \left( \nabla \cdot \overrightarrow{A} \right)^2 - \frac{M^2}{2} \phi^2 - \phi \cdot \left( \nabla^2 \overrightarrow{A} \right).
\]
and we have absorbed the \( \nabla \cdot \overrightarrow{A} \) terms into the new constraint. It is worth to emphasize that from the expression for the potential \( \nu^{(3)} \) one may reads which are the dynamical variables; for instance, here, it consists into the canonical set \( \{ A_k, \pi^m \} \) and \( \{ \Gamma_k, \phi^m \} \).

\[ \text{It is worth to mention that the complete generalized Coulomb gauge have in addition the condition: } \Gamma_0 = 0, \text{ but as it has already disappeared in the Lagrangian, it is not necessary to impose it.} \]
Nevertheless, from the above Lagrangian follows the vectors

\[
\Gamma_{a}^{(3)} = -\phi_{i}, \quad A_{a}^{(3)} = -\pi_{i}, \quad \lambda_{a}^{(3)} = \pi_{0} + \nabla \cdot \vec{\phi},
\]

\[
\eta_{a}^{(3)} = \nabla \cdot \vec{\pi}, \quad \chi_{a}^{(3)} = \nabla^{2} \nabla \cdot \vec{A},
\]

where \(\nabla^{2} = (1 - M^{-2} \nabla^{2})\), these lead to the corresponding third-iterated symplectic matrix

\[
(3) f_{ab} (x,y) = \begin{bmatrix}
B_{ij} & G_{j,x} \\
-G_{i,y} & \tilde{F}_{ij,x}
\end{bmatrix} \delta (x,y),
\]

with

\[
G_{i,x} = \begin{bmatrix}
0 & 0 & 0 & 0 & -\nabla^{2} \partial x \\
0 & 0 & 0 & -\partial x & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}.
\]

This \( (3) f_{ab} (x,y) \) is clearly a nonsingular matrix and the corresponding inverse is easily obtained by a simple, but rather lengthy calculation. Moreover, we may relabel \( \lambda = \phi_{0}, \eta = \Gamma_{0}, \) and \( \chi = A_{0}. \) Therefore, the generalized brackets between the dynamical variables, the corresponding Dirac brackets in the generalized radiation gauge, are just the elements of the inverse of such a matrix, and reads

\[
\{ A_{k} (x) , \pi^{m} (y) \}^{\star} = \delta_{k}^{m} \delta (x,y) - \nabla^{2} \partial k \partial^{m} G (x,y),
\]

\[
\{ \Gamma_{k} (x) , \phi^{m} (y) \}^{\star} = \delta_{k}^{m} \delta (x,y).
\]

where we have introduced the Green’s function

\[
\nabla^{2} \nabla^{2} G (x,y) = \delta^{(3)} (x,y).
\]

These results are in accordance to those obtained previously through an analysis à la Dirac in [13]. Though we have obtained the correct brackets to the dynamic variables, we are left with the whole canonical variables (including the kinematical ones) without any trace of which variables are in fact dynamical and that, therefore, should be submitted to the quantization (a natural outcome of the Dirac’s theory). Nevertheless, the analysis of this particular theory showed to us that the outcome of both theories match, although both present pros and cons, especially those involving unnecessary calculation and tedious algebraic work.

### 3 SU (N) higher-derivative Yang-Mills-Utiyama theory

In this section we will go a step further from the previous discussion, and consider an non-Abelian extension of the Podolsky’s theory, also known as the Alekseev-Arbuzov-Baikov’s effective

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\[^{5}\tilde{F}_{ij,x} \text{ is equal to the expression of } F_{ij,x} \text{ Eq.(2.32), but with an additional fifth null line and column.} \]
Lagrangian [17]. This theory was originally proposed to eliminate infrared divergences in $SU(N)$ theories [15]. In order to introduce some concepts, let us consider the $U(1)$ electrodynamics in four dimensions (2.7)

$$L = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2M^2} \partial_\mu F^{\mu \nu} \partial^\lambda F_{\lambda \nu}.$$  

(3.1)

Moreover, to make contact to the non-Abelian theory [16], it is interesting to discuss an additional point. It is not difficult to see that it is still possible to add a second higher-derivative term in the Eq.(3.1), but in order to preserve the original dispersion relation

$$k^2 \left( k^2 - M^2 \right) A_\mu(k) = 0,$$

(3.2)

when the generalized condition $\left( k^2 - M^2 \right) k^\mu A_\mu(k) = 0$ holds. Hence, the Lagrangian should be rewritten as

$$L = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{6M^2} \partial_\mu F^{\mu \nu} \partial^\lambda F_{\lambda \nu} + \frac{1}{6M^2} \partial_\lambda F^{\mu \nu} \partial^\lambda F_{\mu \nu},$$

(3.3)

since $(\partial^\lambda F^{\mu \nu})^2 = 2 \left( \partial^\lambda F^{\mu \nu} \right)^2$. Therefore, the starting point of our analysis would be the Lagrangian density (3.3). Thus, to input an internal symmetry group, the original field must change as $A_\mu \rightarrow A_a^\mu$, where $a = 1, \ldots, (N^2 - 1)$, denotes an index belonging to some internal symmetry group introduced into the original theory, in our case $SU(N)$. Assuming that $X = X^a \tau_a$, where $\tau_a$ are the generators of the corresponding Lie algebra, $[\tau_a, \tau_b] = i f^{abc} \tau_c$, and that it transforms as an adjoint representation of the symmetry group, we rewrite the original Lagrangian density

$$L_{AAB} = -\frac{1}{4} W^a_{\mu \nu} W^{a \mu \nu} + \frac{1}{6M^2} \left( D_\mu W^{\mu \nu} \right)^b \left( D^{\sigma} W^{\sigma \nu} \right)^b$$

$$+ \frac{1}{6M^2} \left( D_\sigma W^{\mu \nu} \right)^a \left( D^{\sigma} W^{\mu \nu} \right)^a - \frac{g}{18M^2} f^{abc} W^a_{\mu \nu} W^{b \nu \lambda} W^c_{\lambda \alpha} \eta^{\alpha \mu},$$

(3.4)

where $W^a_{\mu \nu}$ is an non-Abelian stress-tensor with the following form

$$W^a_{\mu \nu} = F^a_{\mu \nu} + gG^a_{\mu \nu},$$

(3.5)

where $g$ is a group parameter and $G^a_{\mu \nu} = f^{abc} A_b^\mu A_c^\nu$, we also have that the covariant derivative is $D^a_{\mu} \equiv \delta^{ac} \partial_\mu + g f^{abc} A_b^\mu$, with $(\tau_a)^{bc} = -i f^{abc}$.

Now, in order to carry out the second step of the method we should first rewrite the Lagrangian density (3.4) in its first-order form. To accomplish that we may use the canonical momenta due to algebraic simplification in this choice. Therefore, from the definition (2.10) one may evaluate

$$3M^2 \phi^{a \mu} = (D_\sigma W^{\sigma \mu})^a - \eta^{a \mu} \left( D_\sigma W^{\sigma 0} \right)^a + 2 \left( D^0 W^{0 \mu} \right)^a,$$

(3.6)

and, it follows

$$3M^2 \phi^{an} = (D_m W^{mn})^a + 3 \left( D_0 W^{0n} \right)^a.$$  

(3.7)
With the above results in hands we may now rewrite the Lagrangian (3.4) in its first-order form\(^6\) as in terms of the time-derivatives of the field potential, one then obtain the first-order Lagrangian

\[
\mathcal{L} = -\phi_k^a \dot{\Gamma}_k^a + \pi_\mu^a A^{a\mu} - \mathcal{V}^{(0)},
\]

whereas the potential density is

\[
\mathcal{V}^{(0)} = \pi_\mu^a \Gamma^{a\mu} - \frac{M^2}{2} \phi_k^a \phi_k^a + \phi_k^a \left( \frac{1}{3} (D^m W_{mk})^a - \partial_k \Gamma_0^a + g f_{abc} \left( \Gamma_0^b A_k^c + A_0^b \Gamma_k^c \right) + g f_{abc} A_0^b W_{0k} \right)
\]

\[
- \frac{1}{6M^2} \left( D_0 W_{nm} \right)^a \left( D^0 W_{nm} \right)^a + \frac{1}{4} W^a_{km} W^{akm} - \frac{1}{2} \left( \Gamma_k^a - \partial_k A_0^a + g f_{abc} A_0^b A_k^c \right)^2
\]

\[
- \frac{1}{6M^2} \left[ (D_n W_{nm})^a (D^r W_{nm})^a + (D_r W^{0r})^a (D^m W_{m0})^a + \frac{2}{3} (D_m W^{mn})^a (D^r W_{rn})^a + 2 (D_m W_{0r})^a (D^m W_{0r})^a \right] + \frac{g}{18M^2} f_{abc} \left[ 3 W^c_{0m} W^b_{m0} W^c_{0k0} - W^a_{km} W^b_{mj} W^c_{jk} \right].
\]

From the above expression (3.8) for the Lagrangian density one may reads the initial set of sympletic variables

\[
\xi_\alpha^{(0)} = \{ A_k^a, \pi_k^a, A_0^a, \pi_0^a, \Gamma_k^a, \phi_k^a, \Gamma_0^a \},
\]

moreover, from (3.8) we can identify the non-null canonical one-form

\[
\Gamma_k^{(0)} = -\phi_k^a, \quad A_0^{(0)} = -\pi_0^a, \quad A_0 A^{(0)} = \pi_0^a.
\]

From these results, we can compute the elements of the sympletic matrix, leading to the corresponding two-form matrix

\[
\begin{bmatrix}
A_{ij}^{ab} & 0_{4 \times 3} \\
0_{3 \times 4} & B_{ij}^{ab}
\end{bmatrix}
\delta (x,y),
\]

with \(A_{ab} (x,y)\) and \(B_{ab} (x,y)\) having the same expression to the \(A (x,y)\) and \(B (x,y)\), Eq.(2.17), with an additional non-Abelian index \(\delta_{ab}\). The matrix \(\begin{bmatrix} 0 \end{bmatrix} f_{ab} (x,y)\) is obviously singular. The eigenvector with zero eigenvalue is

\[
v_\alpha = (0, 0, 0, 0, 0, 0, v^a_\gamma),
\]

where \(v_\gamma\) is an arbitrary function and associated with \(\Gamma_0^a\). Hence, from the eigenvector (3.13) we can calculate the consistence condition

\[
\int dx v^a_\gamma (x) \left[ \pi_0^a + D_k^b \phi_k^b \right] (x) = 0,
\]

and, since \(v^a_\gamma\) is an arbitrary function, we obtain the constraint

\[
\chi^a (x) \equiv \pi_0^a + D_k^b \phi_k^b = 0.
\]

\(^6\)We follow again the Ostrogradski formalism to deal with the higher-derivative terms.
Following the methodology, we should now introduce this constraint back into the Lagrangian by means of a Lagrange multiplier $\lambda$, one then gets

$$\mathcal{L} = -\phi^a_k \Gamma^a_k + \pi^a_{i<l} \dot{A}^{a\mu} + \lambda^a \left( \pi^a_0 + D^a_{k} \phi^b_k \right) - \mathcal{V}^{(1)},$$

(3.16)

where the first-iterated potential density is $\mathcal{V}^{(1)} = \mathcal{V}^{(0)}\left|_{\chi=0} \right.$ From the above Lagrangian we may notice that the field $\Gamma_0$ naturally disappears when the constraint is taken as a strong relation. Now, in the first-iterate case, the sympletic variables are $\xi^{(1)} = \{ A^a_k, \pi^a_k, A^a_0, \pi^a_0, \Gamma^a_k, \phi^a_k, \lambda^a \}$, and we can read the following one-form

$$\Gamma_{a_i}^{(1)} = -\phi_i^a, \quad \Gamma_{a_i}^{(1)} = -\pi_i^a, \quad \lambda_{a}^{(1)} = \pi_0^a + D^a_{k} \phi^b_k.$$  

(3.17)

By evaluating the corresponding matrix elements, the sympletic two-form matrix reads

$$ \begin{pmatrix} \mathcal{A}^{ab}_{ij} & \mathcal{D}^{ab}_{ij} \\ -\left(\mathcal{D}^T\right)^{ba}_{ji} & \mathcal{C}^{ab}_{ij} \end{pmatrix} \delta_{ij},$$

(3.18)

with

$$\mathcal{C}^{ab}_{ij} = \begin{bmatrix} 0 & \delta^{ab} \delta_{ij} & 0 \\ -\delta^{ab} \delta_{ij} & 0 & (D_y)^{ba}_{i} \\ 0 & (D_x)^{ab}_{i} & 0 \end{bmatrix}, \quad \mathcal{D}^{ab}_{ij} = \begin{bmatrix} 0 & 0 & -g f^{abc} \phi^c_f \\ 0 & 0 & 0 \\ 0 & 0 & \delta^{ab} \end{bmatrix}. \quad (3.19)$$

Again, we see that the first-iterated sympletic is singular. In the next step we should determine its eigenvector with zero eigenvalue. From that it follows

$$\bar{\mathcal{V}}_{\alpha} = (0, (\bar{\mathcal{V}}_2)^a_k, (\bar{\mathcal{V}}_3)^a, 0, (\bar{\mathcal{V}}_5)^a_k, 0, (\bar{\mathcal{V}}_7)^a),$$

(3.20)

Therefore, from the eigenvector $\bar{\mathcal{V}}_{\alpha}$ (3.20) we can evaluate the consistence condition and gets

$$\int d^4x \mathcal{V}^{ab}_{ij}(x) \left( (D_k)^{bc} \left( \pi^{ck} - g f^{cde} \phi^{dk} A^{c}_0 \right) + g f^{bde} \phi^{dk} W^{c}_{0k} \right)(x) = 0,$$

(3.21)

and, since $\bar{\mathcal{V}}^{a}_f$ is an arbitrary function, we obtain the constraint

$$\bar{\mathcal{X}}^{b} = (D_k)^{bc} \left( \pi^{ck} - g f^{cde} \phi^{dk} A^{c}_0 \right) + g f^{bdc} \phi^{dk} W^{c}_{0k} = 0,$$

(3.22)

which is nothing more than the non-Abelian version of the Gauss’s law. Proceeding, we should introduce it back to the Lagrangian as a strong relation, thus the second-iterated Lagrangian reads

$$\mathcal{L} = \hat{\mathcal{L}}^{b} \left( (D_k)^{bc} \left( \pi^{ck} - g f^{cde} \phi^{dk} A^{c}_0 \right) + g f^{bdc} \phi^{dk} W^{c}_{0k} \right) - \phi_k \dot{\Gamma}^a_k + \pi^a \dot{A}^{a\mu} + \lambda^a \left( \pi^a_0 + (D_k \phi^b_k)^a \right) - \mathcal{V}^{(2)},$$

(3.23)
whereas the second-iterated potential density is given by $\gamma^{(2)} = \gamma^{(1)} \big|_{\mathcal{Z}=0}$ with

$$
\gamma^{(2)} = -\pi_k^n \Gamma_k^n - \frac{M^2}{2} \phi_k^a \phi_k^a + \phi_k^d \left( \frac{1}{3} (D^m W_{mk})^a + g f^{abc} A_0^b \Gamma_k^c + g f^{abc} A_0^b W_{0k}^c \right) + \frac{1}{4} W_{km}^a W_{akm} - \frac{1}{2} \left( \Gamma_k^a - \partial_k A_0^a + g f^{abc} a^b A_0^c \left( 2 - \frac{1}{6M^2} (D_0 W_{mn})^a (D_0 W_{nm})^a \right) - \frac{1}{6M^2} \left( (D_r W_{mn})^a (D_r W_{mn})^a + (D_r W_{mn})^a (D_r W_{mn})^a + 2 (D_m W_{0n})^a (D_m W_{0n})^a \right) + \frac{2}{3} (D_m W_{mn})^a (D_m W_{mn})^a \right) + \frac{g}{18M^2} f^{abc} \left[ 3 W_{0m}^a W_{mk}^b W_{k0}^c - W_{km}^a W_{mj}^b W_{jk}^c \right].
$$

(3.24)

From the above second-iterated Lagrangian we read the following vectors

$$
\begin{align*}
\Gamma a_i^{(2)} &= -\phi_i, & A a_i^{(2)} &= -\pi_i, & A_0 a_i^{(2)} &= \pi_0, & \lambda^a (2) &= \pi_0^a + (D_k \phi_k)^a, \\
\eta a_i^{(2)} &= (D_k)^{bc} \left( \delta^{ck} - g f_{ced} \phi^{dk} A_0^e \right) + g f^{bdc} \phi_k^c W_{0k},
\end{align*}
$$

(3.25)

these results lead to the corresponding two-form matrix for $\xi_{\alpha}^{(2)} = \{ A_k^a, \pi_k^a, A_0^a, \pi_0^a, \Gamma_k^a, \phi_k^a, \lambda, \eta \}$

$$
(2) f (x, y) = \begin{bmatrix}
A_{ij}^{ab} & E_{ij}^{ab} x \\
-\left( E^T \right)_{i,y} & F_{ij}^{ab}
\end{bmatrix} \delta (x, y),
$$

(3.26)

with

$$
\begin{align*}
E_{ij,x}^{ab} &= \begin{bmatrix}
0 & 0 & -g f^{abc} \phi_i^c \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & \delta^{ab}
\end{bmatrix}, \\
E_{ij}^{ab} &= \begin{bmatrix}
0 & \delta^{ab} \delta_{ij} & 0 & g f^{abc} \phi_i^d \\
\delta^{ab} \delta_{ij} & 0 & (D_y)^{ba} & (2) (\phi, \eta) f^{ab} \\
0 & - (D_x)^{ab} & 0 & 0 \\
-g f^{abc} \phi_i^d & (2) (\eta, \phi) f^{ab} & 0 & 0
\end{bmatrix}.
\end{align*}
$$

(3.27)

(3.28)

We have again obtained a singular matrix, now the second-iterated one $(2) f (x, y)$. Next, we determine the zero-mode vectors, which now consist in a set of two vectors

$$
\begin{align*}
\vec{v}_\alpha &= \left( \left( \overline{\nu}_1 \right)_k^a, 0, 0, \left( \overline{\nu}_3 \right)_k^a, 0, \left( \overline{\nu}_6 \right)_k^a, 0, \left( \overline{\nu}_8 \right)_k^a \right) \\
\check{v}_\alpha &= \left( 0, \left( \overline{\nu}_2 \right)_k^a, \left( \overline{\nu}_3 \right)_k^a, 0, \left( \overline{\nu}_5 \right)_k^a, 0, \left( \overline{\nu}_7 \right)_k^a \right).
\end{align*}
$$

(3.29)

(3.30)

However, the vector $\check{v}_\alpha$ generates the constraint $\check{\Omega}^b (x) \equiv (D_k)^{bc} (\pi^{ck} - g f_{ced} \phi^{dk} A_0^e) + g f^{bdc} \phi_k^d W_{0k} = 0$. Therefore, it is the vector $\check{v}_\alpha$ only of our interest; but this zero-mode does not generate any new constraints and, consequently, the symplectic matrix remains singular. Therefore, there are gauge degrees
of freedom to be fixed. A suitable choice here is the generalized Coulomb gauge \((1 + M^{-2} □) \partial^k A^a_k = 0, \Gamma_0^a = 0 \) and \(A_0^a = 0\). The same set of constraints obtained here was previously found in [20] through an analysis à la Dirac, where it was also discussed the generator of the gauge symmetry as well. This shows again an accordance between the Dirac and Faddeev-Jackiw methods.

### 3.1 Transition-amplitude via BFV

Instead of evaluating the inverse of the third-iterated two-form matrix, and then determine the generalized brackets between the dynamical fields, and proceed to the quantization by the correspondence principle, we shall rather work in a path-integral framework. Therefore, we shall now compute the transition-amplitude through the BFV formalism [19], because, though there is a proposal relating the path-integral to the FJ-method [18], it is not clear that the method works and it is consistent to a gauge theory. We have that the transition-amplitude in our case is written

\[
Z = \int DA^a_k D\pi^a_k D\Gamma^b_m D\phi^b_m D\lambda^c Db^c D\phi D\phi D\phi D\phi (3.31)
\]

\[
\begin{align*}
\times \exp \left[ i \int d^4x \left\{ \pi^a_k \dot{A}^a_k + \phi^a_k \dot{\Gamma}^a_k + c^a \bar{P}^a + (\partial \bar{c})^d P^d + \lambda^a b^a - \gamma^{(3)} \right\} + i \int d\xi \{ \Psi, Q_{BRST} \} \right]
\end{align*}
\]

where \(\gamma^{(3)}\) is recognized as being the canonical Hamiltonian in the first-order approach, and it is given by the Eq. (3.24)

\[
\gamma^{(3)} = \gamma^{(2)} \bigg|_{\Omega = 0},
\]

also, \(\Omega\) consists in the generalized Coulomb gauge

\[
(1 + M^{-2} □) \partial^k A^a_k = 0, \quad \Gamma_0^a = 0, \quad A_0^a = 0.
\]

Furthermore, \((c^a, \bar{P}^a)\) and \((\bar{c}^a, P^a)\), are the pairs of ghost fields and their respective momenta, while \((\lambda^a, b^a)\) is a Lagrange multiplier and its momentum, all satisfying the following Berezin brackets:

\[
\{ \bar{c}^a(z), P^b(w) \}_B = \delta_{ab} \delta(z, w), \quad \{ \bar{P}^a(z), c^b(w) \}_B = -\delta_{ab} \delta(z, w),
\]

\[
\{ \lambda^a(z), b^b(w) \}_B = \delta_{ab} \delta(z, w).
\]

In the expression (3.31) it remains to define two quantities. The first is the BRST charge, which with the full set of constraints, is written

\[
Q_{BRST} = \int d^3x \left[ \pi^a_k A^a_k + \phi^a_k \Gamma^a_k + c^a \bar{P}^a + (\partial \bar{c})^d P^d + \lambda^a b^a - \gamma^{(3)} \right],
\]

whereas we have that the gauge-fixing function \(\Psi\), in the generalized Coulomb gauge, reads

\[
\Psi = \int d^3z \left[ \frac{i\gamma}{2} b^a \bar{c}^a + i\bar{c}^a (1 + M^{-2} □) \partial^k A^a_k - \lambda^a (1 + M^{-2} □)^{-1} \bar{P}^a \right].
\]
From the above expressions, it is not complicated to evaluate
\[
\{\Psi, Q_{\text{BRST}}\} = \int d^3z \left\{ \frac{\xi}{2} b^a b^a + i \partial^k e^c (1 + M^{-2} \Box ) (D_k c)^e + b^a (1 + M^{-2} \Box ) \partial^k A^a_k \\
+ i (1 + M^{-2} \Box )^{-1} P^a P^a + f^{abc} (1 + M^{-2} \Box )^{-1} P^a \lambda^b c^e \\
+ (1 + M^{-2} \Box )^{-1} \lambda^a \left( D_k \pi^k \right)^b + g f^{bde} \phi^d \Gamma^c_k \right\}.
\] (3.37)

Thus, substituting the result (3.37) into the transition-amplitude expression (3.31), and performing
the variables integration and after some algebraic manipulation, one finds
\[
\mathcal{Z} = \int DA^a_\mu Dc^d Dc^d \\
\times \exp \left[ i \int d^4z \left\{ \mathcal{L}_{\text{AAB}} + i \partial^\mu e^c (1 + M^{-2} \Box ) (D^\mu c)^e - \frac{1}{2\xi} \left[ (1 + M^{-2} \Box ) \partial^\mu A^a_\mu \right]^2 \right\} \right].
\] (3.38)

Hence, from the BFV formalism we have obtained directly the desirable covariant expression for the
transition-amplitude. Furthermore, we see that the ghosts fields are coupled from the gauge fields,
and matter fields may also be included.

4 Concluding Remarks

In this paper we have presented a canonical study of higher-derivative theories, the Podolsky’s
electrodynamics and its non-Abelian extension, the Alekseev-Arbuzov-Baikov’s effective Lagrangian,
in the point of view of the sympletic Faddeev-Jackiw approach. Although the Dirac’s method remains
as the standard method to deal with constrained systems, it has been recognized that some calculation
is unnecessarily cumbersome there, and then it is exactly there where the FJ method shows to be an
economical and rich framework for first-order Lagrangian functions, obviating mainly unnecessary
calculations.

At the beginning we have reviewed briefly the main aspects of the sympletic FJ method. Sub-
sequently, we applied the method on studying the generalized electrodynamics. The full set of the
known constraints \[13\] was obtained, and afterwards it was showed that the third zero-mode vector
does not generate any new constraint, however the sympletic matrix remained singular, an imprint
characteristic of gauge theories. Therefore, the gauge had to be fixed, to attain that we had chosen
to work with the generalized Coulomb gauge. From all that we were able to obtain a nonsingular
sympletic matrix, and by evaluating the inverse of such a matrix, we obtained the generalized brackets
between the dynamical fields, in accordance with the previous results of Dirac’s approach in \[13\].
Moreover, next we introduced the AAB’s effective Lagrangian. The lines in studying this non-Abelian
theory followed those presented to the generalized electrodynamics. Again, the known full set of con-
straints was obtained, in accordance with the result from the Dirac’s approach \[20\]. As it happens
in gauge theories, the third zero-mode vector does not generate any new constraint and the symplectic matrix remained singular. However, instead of using the usual prescription and introduce new constraints, in order to fix the gauge degrees-of-freedom, we had chosen to quantize the theory via path-integral methods. Although it is known a proposal of FJ method in the path-integral framework, its content it is not clear to gauge theories. Therefore, we followed the well-known BFV-method to construct the transition-amplitude.

It was successfully showed here that the sympletic approach of Faddeev-Jackiw works perfectly also to higher-derivative theories, and that all the obtained results were in accordance with previous ones when the Dirac’s methodology was applied. This is somehow in contrast with the assertion [7] that the FJ method produces constraints that do not exist in the Dirac’s theory. Furthermore, this emphasizes that in fact the FJ method poses as a good candidate of framework where deeper analysis may be performed, especially in more intriguing theories, such as General Relativity and renormalizable higher-derivative proposals of a quantum theory of Gravity, in different dimensionality, where the constraint analysis is not always easy to accomplish and clear within the Dirac’s methodology. These issues and others will be further elaborated, investigated and reported elsewhere.

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