A Simplified James Webb Space Telescope Effective Radius Deep Field Simulation Using a Geometric-Focused Ensemble Approach

Matthew Sailer

1 Department of Space Studies, American Military University, Charles Town, West Virginia, United States

E-mail: matthew.sailer@mycampus.apus.edu; mwsailer12@gmail.com

Received xxxxxxx
Accepted for publication xxxxxxx
Published xxxxxxx

Abstract

The James Webb Space Telescope (JWST) is expected to observe objects as far as \( z = 15 \) with a sensitivity 100-times greater than the Hubble Space Telescope (HST). Recent studies predict the characteristics of JWST’s deep field image using a deterministic approach based on recent observational measurements with corresponding ranges of uncertainty. This study presents a simplified geometric-focused deep field simulation of galaxy effective radius using an ensemble approach to demonstrate the high variability in results due to the uncertainty ranges of observational measurements. This study conducted two ensemble simulations: a parameter sensitivity ensemble, where each initial condition was perturbed individually, and a 1000-member full ensemble where all initial conditions were perturbed simultaneously. A mean galaxy coverage percentage was calculated for each ensemble as an output variable to objectively compare model runs. This study found the uncertainty in the estimated number of unseen galaxies in the HUDF provides the largest variability of results. The 1000-member ensemble resulted in a standard deviation of \( \pm 30.30\% \). The Apparent Galaxy Wall (AGW) effect is introduced and defined as \( \geq 50\% \) area of a deep field image occupied by galaxies. A one-way one-sample t-test was conducted on the 1000-member ensemble, and this study concluded the JWST is likely to observe the AGW effect with an estimated galaxy coverage percentage of \( 55.12 \pm 30.30\% \). A discussion is included on the potential impacts of the AGW effect being observed and its potential to form a pseudo-cosmological horizon that may inhibit the effectiveness of future observatories.

Keywords: JWST, Effective Radius, Simulation, Galaxy, Ensemble, Redshift

1. Introduction

After years of delays, the James Webb Space Telescope (JWST) is expected to launch before the end of 2021, and countless scientists, researchers, and space enthusiasts eagerly look forward to the release of its first images (Fisher 2021). The four primary research areas of JWST are defined as (i) The End of the Dark Ages: The First Light and Reionization, (ii) The Assembly of Galaxies, (iii) The Birth of Stars and Protoplanetary Systems, and (iv) The Planetary Systems and the Origins of Life (Gardner et al. 2006). The JWST has been described as the successor to the Hubble Space Telescope (HST) and designed to observe in longer wavelengths to probe deeper into the observable universe (Gardner et al. 2006; NASA 2020). The primary research areas and status as the successor to HST indicate the JWST is likely to produce some of the furthest-reaching deep field images ever captured with distances expected to reach redshifts as large as \( z = 15 \) with the NIRCam instrument (Gardner et al. 2006).

One of the JWST mission objectives is to study the Hubble Ultra Deep Field (HUDF) and The Great Observatories Origins Deep Survey (GOODS) region...
in greater depth with its highly sensitive infrared imager Mid-InfraRed Instrument (MIRI) and Near-Infrared Camera (NIRCam) respectively (Villard 2017).

Given the NIRCam specifications, a numerical simulation can shed light on the expected JWST deep field images before release. This study develops a novel simulation of the JWST deep field images using a heavy emphasis on general characteristics of the telescope and recent observational measurements of the universe. The python code written for this study can be downloaded from the Astrophysical Source Code Library (Sailer 2021).

1.1 Necessity for Study

Although several studies predict the characteristics of the JWST deep field images from accepted methods, it is impossible to determine a method’s accuracy until the images are released. Despite the completed studies and simulations, alternate and novel methods of simulation provide added benefit to existing simulations and contribute to a broader range of potential outcomes. This study aims to simulate the JWST deep field images by focusing on the typical galaxy’s apparent two-dimensional geometry through high-z interpolation of recent observation-based studies and using an ensemble method through the perturbation of the initial conditions. This study’s approach offers another factor with which to consider the future of high-z focused observatories through the prediction of galaxy number density saturation and the Apparent Galaxy Wall (AGW) effect.

1.2 The Apparent Galaxy Wall (AGW) Effect

One of the unique predictions resulting from this study is an apparent wall of galaxies resulting from a large galaxy number density and increased apparent angular diameters of high-z galaxies due to the expansion of the universe as predicted by general relativity. Under the right conditions, from the perspective of an observatory, a given field of view may become saturated with spatially magnified high-z galaxies resulting in a visually obstructing barrier acting as a pseudo-cosmological horizon. This effect is distinct from the confusion limit as the confusion limit occurs when the Point Spread Functions (PSF) in an image under go significant overlapping (Vaisanen et al 2001). The AGW effect occurs when the resolved images of galaxies overlap due to increased apparent angular sizes due to the universe’s expansion. In the same way, this effect is distinct from the Extragalactic Background Light (EBL) as the AGW effect is defined by the effective radii of resolved galaxies (Cooray 2016). As observatories increase in sensitivity and probe deeper into the early universe where galaxy number density is greater, it is increasingly likely that galaxy number density saturation may be observed. The AGW effect is defined in this study as occurring when ≥ 50% of a given area is visually obstructed by galaxies due to galaxy number density saturation. This study predicts a high probability of the JWST observing the AGW effect in its deep field images.

1.3 Scope of Research

This research is not a full simulation of the JWST and only focuses on a simplistic study of the increase of detected galaxy effective radii due to the universe’s expansion in its deep field image. Because of this, the JWST’s NIRCam PSF is not considered. This simplification is further supported since high-z galaxy effective radii produced in this simulation are not yet points, rather they are resolved shapes under the constraints of the JWST’s spatial resolution. An added major simplification assumes the sizes of galaxies at each redshift are uniform and each galaxy is disk-like. This study acknowledges that in nature this will not be the case as luminosity and size depends on the type of galaxy simulated. This distinction between galaxies is neglected in this study due to the current lack of high-z observations in the high redshifts expected to be reached by the JWST. This study assumes for simplicity that the effective radius of the average galaxy follows an effective radius-redshift relationship presented by Allen et al. (2017). Additionally, since this study focuses primarily on effective radius, the Sérsic profile is intentionally neglected in the simulations.

This study acknowledges that the results will vary based on the filters used by the JWST NIRCam. Because this study is focused on the AGW Effect as a theoretical concept, there is less of a focus on the specific filters, but a high focus on the general specifications of the JWST’s capabilities outlined in Gardner et al. (2016). This simulation will assume the JWST is capable of reaching an AB magnitude of 31 through 100-200 hours of exposure per filter to resolve objects z ≥ 15. Finally, this simulation assumes the variability of galaxy characteristics is small at z ~ 15 which may not be the case in nature. This assumption is made from the general principle that as the age of the universe approaches 0, the universe approaches a
generally isotropic state. The same is assumed for galaxy variability. Very few observations of galaxies at these extreme redshifts exist, so the accuracy of this assumption cannot be known outside of theoretical work.

Using these simplifications, an ensemble approach is taken to simulate the JWST deep field images. Because of this study’s simplifications, an original code was written using the angular-diameter-redshift relation and general geometric trends created from recent high-z galaxy measurements. The simplifications allowed for an efficient simulation that can be extensively perturbed to study the effects of the uncertainty in the initial conditions. Using this method, the consequences of the variability of recent measurements can be fully appreciated through an ensemble simulation. As discussed above, the initial results of this simulation may not be visually accurate to the actual JWST deep field images, but they serve as an estimate of the average galaxy’s two-dimensional geometries through random distributions and orientations.

2. Background

2.1 Previous Deep Field Images

The HUDF image is arguably the most famous deep field image in astronomy with an estimated 10,000 galaxies in one image and often serves as a comparison for deep field images and early universe analysis (Beckwith et al. 2006; Inami et al. 2017; NASA 2018). Other notable deep field images include the Extreme Deep Field, the Spitzer Cosmic Assembly Near-Infrared Deep Extragalactic Survey (SCANDELS), and the GOODS Re-ionization Era wide-Area Treasury from Spitzer (GREAT) survey to name a few (Illingworth et al. 2013; Ashby et al. 2015; De Barros et al. 2019). Because of the substantial analysis performed on the HUDF, it was chosen as the calibration source in this study. The JWST is expected to see further than ever before and generally outperform previous deep field images due to its large primary mirror, advanced detectors, near-IR wavelengths detection, and high planned exposure time (Gardner et al. 2006; NASA 2020).

2.2 Previous Simulations

There have been several simulations of the upcoming JWST deep field images generated in recent years that do not necessarily depict the AGW effect occurring (ESA 2002; Gardner et al. 2006; ESO 2011; Kauffmann et al. 2020). Additionally, some catalogs have been created and surveys performed that predict the characteristics of distant galaxies expected to be observed by the JWST or relating effective radius to redshift. These catalogs and surveys are based on collected data from the JAdes extraGalactic Ultradeep Artificial Realizations (JAGUAR) catalog, the FourStar Galaxy Evolution Survey (ZFOURGE), and the Multi-Unit Spectroscopic Explorer survey (MUSE) to name a few (Spitler 2012; Herenz et al. 2017; Williams et al. 2018). No research has predicted galaxy number density saturation from the results of these surveys and catalogs at the time of writing. While these catalogs are built from recent observations and offer valuable insight, this study opted to build a simulation using recent measurements of the evolution of effective radius alone while calculating the number density using a unique approach.

Apart from the JWST predictions based on catalogs and surveys, several visual simulations are presented on several organizational websites including the European Space Agency and the European Southern Observatory (ESA 2002; ESO 2011). While mock catalogs can provide simulated characteristics of galaxies in the early universe, it is the visual simulations of these mock catalogs that provide a clear representation of what may be seen in the upcoming JWST deep field images. These visual simulations appear to be based on some of these surveys, but little supporting documentation or publications on how these simulations were calculated were found online apart from the ESA’s (2002) being simulated from Andrew Benson’s theoretical model of the universe and post-processed by the Space Telescope Science Institute to add effects from the instruments.

Additionally, a commonly used programme called SkyMaker has been used as an accepted simulation programme for deep field images (Bertin 2008). The resulting simulations assume a universe of flat geometry with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$. Previous simulations which use SkyMaker have not demonstrated the AGW effect.

While there are many simulations of the early universe, not all of them are focused on the upcoming JWST deep field images. Only simulations that focus on the JWST will be discussed in this project, and other simulations which are not directly aimed at predicting the JWST like the EAGLE Project and the
The most recent simulation with substantial documentation was published by Kauffmann et al. (2020) building off the mock catalog and visual simulation by Williams et al. (2018). Kauffmann et al. (2020) produced a deep field simulation using the JAGUAR catalog and assumed \( \Omega_m = 0.3, \Omega_{\Lambda} = 0.7, H_0 = 70 \frac{\text{km}}{\text{s-Mpc}} \), a resolution of 0.031′/pixel, a ~29 AB mag (5σ), and an area of 4.5 arcmin².

### 2.3 The Limitations of Current Simulations

While Kauffmann et al.’s (2020) simulation excels in many areas and produced a great visual simulation, this study does not use Kauffmann et al.’s (2020) approach for the following reasons. First, Kauffmann et al. (2020) use the JAGUAR catalog for mock sources which is created from recent observations. The results of this catalog imply that the JWST is only likely to see tens of galaxies at \( z > 10 \) (Williams et al. 2017; Kauffmann et al. 2020). In both publications, a similar estimate is made which appears to contradict expected numbers when considering the HUDF. The current record for the highest redshift measured in the HUDF region is \( z \sim 11 \) (Oesch et al. 2016; Jiang et al. 2020; Schauer, Drory & Bromm 2020). When considering the HUDF MUSE survey successfully measured the redshifts of ~15% of the galaxies in the HUDF, and considering closer galaxies are generally easier to obtain a redshift value than galaxies at high-redshifts due to a generally greater photon flux, it can be reasonably concluded that a significant number of galaxies imaged in the HUDF have redshifts of \( z > 10 \) (Inami et al. 2017). Similarly, when considering the JWST is expected to see objects 100 times fainter than the HST, and that early galaxies are theorized to contain hot and bright massive stars, it can be subjectively concluded that tens of galaxies with redshifts of \( z > 10 \) is a substantial underestimate (Gardner et al. 2006; Larson & Bromm 2009; Benson 2010; Hashimoto et al. 2018; Inami et al. 2017). Further evidence for this can be found when considering the deep field image captured by the Spitzer Space Telescope depicting hints of the AGW effect with the appearance of a high galaxy number of density at large redshifts (NASA 2019). This appearance, however, is due to the confusion limit where the PSF of galaxies up to \( z \sim 3 - 3.5 \) is overlapping due to the wavelengths detected by Spitzer in the 3-10 μm range (Vaisanen et al 2001).

Second, several articles theorize that there may exist 2 - 10 times as many galaxies as seen in the HUDF’s field of view that were not detected due to the limitations of the HST (Hille 2016; Conselice et al. 2016; Lauer et al. 2021). Because of the difficulty of modeling an uncertain galaxy number density, galaxy catalogs alone may not offer the best foundation on which to build an accurate simulation of the early universe.

Third, the functions used to interpolate backward in time heavily rely on the observations of previous observatories which Williams et al. (2018) admit are limited by spatial coverage, sensitivity, and atmospheric effects of the HST, SST, and ground-based observatories, respectively. Relying on observations alone may not yield the most accurate galaxy number density estimates for JWST simulations. Using a general fundamental approach using Comoving Volume may offer a more accurate representation of galaxy numbers.

Finally, the major constants that describe the universe are assumed as constant values for these simulations with no perturbations to explore the effects of changes to these constants. Since many of the universe’s constants contain uncertainty ranges, the value of the initial conditions will affect the result of the simulation. This study perturbs the initial conditions to generate the full range of possible results to demonstrate the importance of an ensemble approach.

Because of these considerations, this study opted to construct a novel simulation, in python, assuming the general specifications of the JWST as outlined by Gardner et al. (2006) and focuses on the expected two-dimensional geometries of the imaged high-z galaxies. The two-dimensional geometries are calculated using generalizations concluded from recent studies combined with fundamental equations, like comoving volume, that describe the evolving universe.

### 3. Method

An original code was written in python to produce a geometric-focused JWST deep field simulation that can be run with an ensemble method. A randomly generated deep field image is produced from calculations based on general equations derived from recent observations. The resulting simulation
represents galaxies as randomly oriented ellipses with both random angular orientation and random edge-on
to top-down orientation. In addition to a deep field
image, the code produces a calculation of the percentage of background space obstructed by
foreground galaxies and a few other supporting figures
that will be discussed in later sections. The percentage
of background space obstructed by foreground
galaxies by area will be referred to as “galaxy
coverage percentage” throughout the rest of this paper.
The code’s effectiveness was tested by inputting initial
conditions of the HST and successfully producing a
similar-looking image to the HUDF to demonstrate the
effectiveness of the methods used in this study. There
are eight perturbed initial conditions with 100
perturbations values. Each value was run 10 times with
random ellipse orientations to produce 1000 unique
results for a 1000-member ensemble. The galaxy
coverage percentages associated with each unique result were averaged and compared to the AGW
definition of $\geq 50\%$ using a one-way one-sample t-test.
For simplicity, this study ignored the effects of
gravitational lensing events. By ignoring these effects,
this model underestimates the galaxy coverage
percentage calculations since gravitational lensing
would further obscure the image background due to
the distorted spacetime.

3.1 Estimates and Initial Conditions

Several general estimates were made to create a
simple, yet effective, simulation. The generally
accepted $\Lambda$CDM model was assumed for this
simulation since it holds up against many observational tests including CMB, supernovae,
galaxy clustering, and weak lensing observations
(Conley et al. 2011; Zehavi et al. 2011; Suzuki et al.
2012; Bennett et al. 2013; Heymans et al. 2013; Planck
Collaboration et al. 2016; Hildebrandt et al. 2017;
Wang et al. 2018). Even though the $\Lambda$CDM model was
assumed, this study did not assume specific values for
constants, but rather, the constants describing the
$\Lambda$CDM model were perturbed through their ranges of
uncertainty from recent measurements.

3.1.1 JWST Characteristics

The JWST’s NIRCam characteristics can be
found in its user documentation and summary
publications. The following specifications of the
JWST were incorporated into the calculations
involved in this simulation. NIRCam was designed to
have a sensitivity reaching an AB magnitude of 31
from 100 - 200 hours of exposure per filter resulting in
the ability to detect sources 100 times fainter than the
HST and reaching a redshift of $z > 15$ (Gardner et al.
2006). Additionally, STScI (2016) and Villard (2017)
provides an overview of the JWST NIRCam
instrument from the James Webb Space Telescope
User Documentation and specifies it will have a spatial
resolution as low as 0.031 arcsec per pixel for the
NIRCam. This high spatial resolution and sensitivity
are in part due to a significantly larger segmented
primary mirror allowing for 6.25 times more light-
collecting area with highly sensitive sensors (NASA
2020). While the JWST is equipped to capture images
at many exposures and distances, this simulation will
focus only on JWST deep field images from NIRCam
and assume an observing time of 100-200 hours.

3.1.2 General cosmological considerations

The early universe is difficult to thoroughly
study through observations alone due to our
observational limitations (Larson & Bromm 2009).
Because of this, this study made the following
generalizations given the respective supporting
publications. (i) All simulated galaxies are assumed to
be disk-like in shape. This generalization is made for
simplicity and supported by Benson’s (2010) study
which argues the first galaxies were likely disk-shaped
due to the conservation of angular momentum with
elliptical galaxies forming only after a merger event.
Krogager et al.’s (2013) quasar study appears to
support this idea as a galaxy at $z = 2.35$ appears to be
edge-on in orientation. (ii) Galaxy formation is
assumed to have occurred between $20 < z < 50$ as
described by Benson (2010). This code assumes no
galaxy formation occurred outside of this redshift
range when estimating galaxy number densities. (iii)
The first stars are estimated to have a large mass and
high luminosity (Gardner et al. 2006) resulting in the
first galaxies also having a high luminosity (Benson
2010). Evidence for this can be found in several
studies like the Butcher-Oemler effect (Butcher &
Oemler 1984), the brighter-than expected high-z
galaxies captured by SST (De Barros et al. 2019), and
higher than expected galaxy maturity found in the
ALPINE-ALMA $\text{[CII]}$ survey (Le Fèvre et al. 2020).
This study makes the general assumption that most
average-sized galaxies between $0 < z < 15$ will be
detected to some degree in the JWST’s deep field
images due to the high sensitivity of the JWST
and expected characteristics of early galaxies.
3.1.3 Previous Measurements of $H_0$, $\Omega_m$, $\Omega_\Lambda$, $\Omega_k$, and the Dark Energy Equation of State

Numerous studies have calculated estimates for $H_0$, $\Omega_m$, $\Omega_\Lambda$, $\Omega_k$, and the Dark Energy Equation of State. This study gathered many recent measurements of these constants to determine a range of possible values using their uncertainties.

The Hubble Constant, $H_0$, has been measured to be: $70.4^{+1.4}_{-1.4}$ km/s/Mpc, $73 \pm 2$ (random) ± 4 (systematic) km/s/Mpc, $68.5 \pm 3.5$ km/s/Mpc, $68.3^{+2.7}_{-2.6}$ km/s/Mpc in a spatially flat $\Lambda$CDM model and $68.4^{+3.3}_{-3.3}$ km/s/Mpc in a non-spatially flat $\Lambda$CDM model, $73.52 \pm 1.62$ km/s/Mpc, $72.56 \pm 1.5$ km/s/Mpc, $69.8 \pm 0.8$ (±1.1% stat) ±1.7 (±2.4% sys) km/s/Mpc, $68.20 \pm 0.81$ km/s/Mpc, and $67.4 \pm 0.5$ km/s/Mpc (Benson 2010; Freedman & Madore 2010; Verde, Protopapas & Jimenez 2014; Chen, Kumar & Ratra 2017; Riess et al. 2018; Capozziello, Ruchika & Sen 2019; Freedman et al. 2019; Jee et al. 2019; Alam et al. 2020; Planck Collaboration et al. 2020). In recent years, estimates for $H_0$ appear to diverge depending on the method used to estimate it. These 2 values are $\sim 67$–73 km/s/Mpc (Castelvecchi 2020). The mass density of the universe, $\Omega_m$, has been measured to be: $0.2726 \pm 0.0014, 0.295 \pm 0.034$ (stat+sys), 0.32 ± 0.05, and 0.315 ± 0.007 (Benson 2010; Betoule et al. 2014; Verde et al. 2014; Planck Collaboration et al. 2020). The dark energy density of the universe, $\Omega_\Lambda$, has been measured to be: $0.728^{+0.015}_{-0.016}$ and 0.6847 ± 0.0073 (Benson 2010; Planck Collaboration et al. 2020). The total energy density of the universe, $\Omega_k$, has been measured to be: $-0.02 \pm 0.24, -0.0023 \pm 0.0022$, and $0.001 \pm 0.002$ (Wei & Wu 2017; Alam et al. 2020; Planck Collaboration et al. 2020). The dark energy equation of state, $w$, has been measured to be: $w = -1, -1.027 \pm 0.055, -1, -0.912 \pm 0.081$, and $-1.03 \pm 0.03$ (Huterer & Cooray 2005; Betoule et al. 2014; Moews et al. 2019; Alam et al. 2020; Planck Collaboration et al. 2020). These measurements were split into their values and uncertainty ranges and presented in Table 1. This extensive review was necessary to determine the perturbation range used in the simulation. Each parameter in Table 1 was averaged together to obtain a mean estimate which served as a baseline value for the simulation. Only $H_0$ estimates based on high-z observations are included in the Maximum and Minimum row since focusing on measurements within a small distance from the Milky Way can yield inaccurate results (Alam et al., 2020). The maximum and minimum values of the ranges of uncertainties were identified and used as the perturbation ranges in the ensemble runs.

**Proper Length Estimates**

The edge of a galaxy is difficult to define when measuring its length and width due to star density generally decreasing as radius increases. The brightness of a disk-like galaxy near its edge can be defined using a Sérsic profile with an index of 4 as a solution to De Vaucouleurs’ Law and can be seen in Equation 1 where $I(r)$ represents the intensity at radius $r$, $r_e$ is the effective radius, and $I_0$ is the brightness at $r_e$ (de Vaucouleurs 1948; Mazure & Capelato 2002).

$$I(r) = I_0 e^{-7.666 \left( \left( \frac{r}{r_e} \right)^{1/4} - 1 \right)}$$ (1)

This study estimates the proper length of a galaxy by assuming the effective radius can be considered a measurement for proper length. Although several recent studies were conducted that estimate the average effective radii of high-z galaxies, there is a wide range of results. Ono et al. (2013) estimate the effective radii of galaxies between $7 < z < 12$ are 0.3–0.4 kpc and evolve proportionally to $(1+z)^{1.28 \pm 0.13}$ from observations of the HUDF. Allen et al. (2017) estimate an effective radius evolution seen in Equation 2 from stellar-mass log ($M_*/M_\odot$) > 10 from redshifts of 1 – 7.2 using samples from star-forming galaxies in the FourStar Galaxy Evolution Survey (ZFOURGE).

$$r_e = 7.07(1+z)^{-0.89 \pm 0.07} \text{ kpcs}$$ (2)

Williams et al. (2018) offer a proper length estimate for high-z galaxies; however, the estimate is split into several different types of galaxies. For simplicity, this study assumes Inami et al.’s general estimate $1 < z < 7.2$. For galaxies with redshifts of $z < 1$, the effective radius is calculated using the effective radius found when $z = 1$ assuming more consistent effective radii at low-redshifts.

### 3.1.4 Galaxy Merger Rate

Because this study uses the comoving volume to estimate the galaxy number density, the galaxy merger rate must be included. The galaxy merger rate is difficult to determine since the large-scale motion of
Table 1. A consolidation of numerous measured values for $H_0$, $\Omega_m$, $\Omega_\Lambda$, $\Omega_k$, and $w$ over the past 2 decades. For simplicity, this table presents the highest of the two uncertainties when publications present an asymmetric uncertainty range and presents the value associated with a spatially flat universe if multiple values are presented. Only $H_0$ estimates based on high-z observations are included in the Maximum and Minimum row since focusing on measurements within a small distance from the Milky Way can yield inaccurate results (Alam et al., 2020).

| Author                  | $H_0$ | $H_0 \pm$ | $\Omega_m$ | $\Omega_m \pm$ | $\Omega_\Lambda$ | $\Omega_\Lambda \pm$ | $\Omega_k$ | $\Omega_k \pm$ | $w$   | $w \pm$ |
|-------------------------|-------|-----------|------------|-----------------|-------------------|---------------------|-----------|----------------|-------|---------|
| Huterer & Cooray (2005) | -     | -         | -          | -               | -                 | -                   | -1        | -              | -     | -       |
| Benson (2010)           | 70.4  | $\pm1.4$  | 0.273      | $\pm0.014$     | 0.728             | $\pm0.016$         | -         | -              | -     | -       |
| Freedman & Madore (2010)| 73    | $\pm2$    | -          | -               | -                 | -                   | -         | -              | -     | -       |
| Betoule et al. (2014)   | -     | -         | 0.295      | $\pm0.034$     | -                 | -                   | -         | $-1.027$       | $\pm0.055$ |         |
| Verde, et al. (2014)    | 68.5  | $\pm3.5$  | 0.32       | $\pm0.05$      | -                 | -                   | -         | -              | -     | -       |
| Chen et al. (2017)      | 68.3  | $\pm2.7$  | -          | -               | -                 | -                   | -         | -              | -     | -       |
| Wei & Wu (2017)         | -     | -         | -          | -               | -                 | $-0.02$            | $\pm0.24$ | -              | -     | -       |
| Riess et al. (2018)     | 73.5  | $\pm1.62$ | -          | -               | -                 | -                   | -         | -              | -     | -       |
| Capozziello et al. (2019)| 72.6 | $\pm1.5$  | -          | -               | -                 | -                   | -         | -              | -     | -       |
| Freedman et al. (2019)  | 69.8  | $\pm0.8$  | -          | -               | -                 | -                   | -         | -              | -     | -       |
| Jee et al. (2019)       | 82    | $\pm8.4$  | -          | -               | -                 | -                   | -         | -              | -     | -       |
| Moews et al. (2019)     | -     | -         | -          | -               | -                 | -                   | -         | -1             | -     | -       |
| Alam et al. (2020)      | 68.2  | $\pm0.81$ | -          | -               | -                 | $-0.0023$           | $\pm0.002$ | $-0.912$       | $\pm0.081$ |         |
| Planck Collaboration et al. (2020) | 67.4 | $\pm0.5$ | 0.315     | $\pm0.007$ | 0.685           | $\pm0.0073$     | 0.001        | $\pm0.002$ | $-1.03$ | $\pm0.003$ |
| Maximum                 | 73.5  | 1.62      | 0.32       | 0.05            | 0.728             | 0.016               | 0.001       | 0.002         | $-0.912$ | 0.081 |
| Minimum                 | 67.4  | -0.5      | 0.273      | -0.014          | 0.684             | -0.0073            | -0.02       | -0.024        | -1.03 | -0.003 |
| Mean                    | 71.4  | -         | 0.301      | -               | 0.706             | -                   | $-0.0071$   | -              | $-0.994$ | -           |
the universe is minuscule compared to the average human lifetime and limitations of current observatories, but estimates can still be made from observations (Conselice et al. 2016). The universe has evolved into a state of higher structure since the Big Bang due to the gravitational attraction of galaxies into clusters and superclusters, and many of the galaxies today, including the Milky Way, are believed to have undergone several merger events in the past (Lotz et al. 2011). This adds another level of complexity in determining the galaxy merger rate (Postman 2001). Conselice et al. (2016) estimate the total number of galaxies in the universe declines with time proportionally to \( \sim \frac{4}{t} \) where \( t \) is the age of the universe in Gyrs from observations out to \( z = 8 \). For simplicity, this study will assume this estimate for all values of \( z \).

### 3.1.5 HUDF Unseen Galaxies

A deep field image cannot capture all existing galaxies due to the variation of galaxy sizes and luminosities. Recent studies estimate there exists between 2 - 10 times as many galaxies as seen in the HUDF with redshifts \( 0 < z < 8 \) which are unable to be detected by the HST (Hille 2016; Conselice et al. 2016; Lauer et al. 2021). The most recent estimate is from the New Horizons’ Cosmic Optical Background (COB) that estimates only two times as many galaxies exist (Lauer et al. 2021). This study will assume the JWST is likely to see many of these unseen galaxies due to its ability to see objects 100 times fainter than the HST (Gardner et al. 2006), the unexpectedly high luminosity of galaxies in the early universe (Butcher & Oemler 1984; De Barros et al. 2019), and the higher than expected galaxy maturity in the early universe (Le Fèvre et al. 2020).

### 3.1.6 Anisotropy of Dark Energy

Recent studies have suggested the nature of dark energy may be anisotropic. (Kolatt & Lahav 2001; Schwarz & Weinhorst 2007; Kalus et al. 2013; Campanelli et al. 2011; Mariano & Perivolaropoulos 2012; Cai et al. 2013; Cooke & Lynden-Bell 2010; Antoniou & Perivolaropoulos 2010; Heneka et al. 2014; Zhao et al. 2013; Yang et al. 2013; Javanmardi et al. 2015; Lin et al. 2015; Chen & Chen 2019; Colin et al. 2019; Migkas et al. 2020; Porter & Watzke 2020). The anisotropy of dark energy was considered in this study with the ratio of anisotropy being perturbed as an initial condition in the ensemble simulation. This study assumed the anisotropy of dark energy is consistent in nature by generalizing the rate of expansion as proportional to a direction-dependent Hubble constant for simplicity.

### 3.2 General Approach

This study’s code perturbed the following eight initial conditions: (i) the Hubble constant \( (H_0) \), (ii) the mass density \( (\Omega_m) \), (iii) the dark energy density \( (\Omega_X) \), (iv) the Dark energy equation of state parameter \( (w) \), (v) the number of unseen galaxies by the HST \( (N_{\text{Unseen}}) \), (vi) the increase in the percentage of each galaxy the JWST will be able to see from an increase in sensitivity compared to HST \( (R_{\text{Increase}}) \), (vii) the ratio of the Hubble constant in the \( y \)-axis to the Hubble constant in the \( x \)-axis due to the anisotropy of dark energy \( (DE_{\text{Ratio}}) \), and (viii) the maximum redshift expected to be measured by the JWST \( (Z_{\text{MAX}}) \). A parameter-sensitivity ensemble was run by choosing a baseline set of initial conditions and perturbing each initial condition individually through its corresponding observation-based uncertainty range before calculating the corresponding change to the galaxy coverage percentage. The standard deviation of each initial condition’s perturbation result was used as a measure of its sensitivity with the most sensitive parameters having the highest standard deviation. A full ensemble was run by perturbing all initial conditions simultaneously through their observation-based uncertainty ranges and calculating the corresponding galaxy coverage percentages. A one-way one-sample t-test was used to analyse the resulting 1000 galaxy coverage percentages compared with the AGW effect’s definition of \( \geq 50\% \) coverage.

This study’s simulation approach aimed to take advantage of the powerful ensemble approach to computation. Since computing has been described as the third pillar of science, the method of computation was a vital consideration for this study’s effectiveness, so an ensemble approach was chosen (Skuse 2019; Winsberg 2019). The ensemble approach tends to be underused in science but is extensively used in numerical weather prediction with effective results (Cahir n.d.; NOAA n.d.; JMA n.d.; Smagorinsky 1983; Gneiting & Raftery 2005; Environment Canada 2013; Andersson 2014; Slater, Villarini & Bradley 2016; Kikuchi et al. 2017). Additionally, the ensemble approach has been effectively used in multiple studies including water resource studies, space weather, and gene expression to name a few demonstrating its effectiveness as a computational method (Knipp 2016;...
3.2.1 Parameter-Sensitivity Ensemble

A preliminary ensemble simulation was run to test the sensitivity of each initial condition. A baseline set of initial conditions was defined as a control. The baseline was chosen to be $H_0 = 71.4$, $\Omega_m = 0.301$, $\Omega_\Lambda = 0.706$, $w = -0.994$, $N_{\text{Unseen}} = 6$, $R_{\text{Increase}} = 130\%$, DE_Ratio = 0.925, ZMAX = 16. Each initial condition was run through a range of 10 values with each value run 10 times while the rest of the initial conditions were held constant. The resulting galaxy coverage percentages were averaged together for each initial condition parameter and the standard deviation was calculated. The parameters were ranked from most sensitive to least sensitive based on their standard deviations.

3.2.2 Ensemble Simulation

A full ensemble simulation was run perturbing all initial conditions simultaneously to calculate the range of possible resulting galaxy coverage percentages. Since the initial condition ranges are defined by the uncertainties of recent measurements, this ensemble represents the full range of possible JWST deep field outcomes. The resulting galaxy coverage percentages were saved in a list, and the mean and standard deviation of this list was calculated.

3.2.3 T-test Analysis

A one-way one-sample t-test was used to compare the resulting mean and standard deviation from the ensemble simulation with the AGW definition of ≥ 50% galaxy coverage percentage. The statistical significance was calculated for this prediction. A one-way one-sample t-test was used due to its simplicity, robustness, and ease of calculation (Jackson 2016; UCLA 2016; Flom 2018; UCLA 2019). For this test, with a mean galaxy coverage percentage of < 50%, it could be confidently concluded that the AGW effect is not likely to be observed by the JWST. Additionally, if the final galaxy coverage percentage mean is ≥ 50%, then the one-tailed one-sample t-test can be conducted to test for 95% confidence. For this t-test, the alternative hypothesis ($H_a$) was defined as: The JWST will observe a galaxy coverage of ∼ 50%, and the null hypothesis ($H_0$) was defined as: The JWST will observe a galaxy coverage of < 50%.

3.3 Calculations

The following section discusses the calculations made by this study’s code to create the JWST Deep Field simulation. The following modules were necessary for this code to operate: numpy, quad from scipy.integrate, math, matplotlib, Ellipse from matplotlib.patches, and cv2.

3.3.1 Control Panel

The control panel defines the values of the initial conditions to include image resolution, image dimensions, the speed of light, the maximum redshift detected in the HUDF, all perturbed initial conditions used for the ensemble simulation, and an arbitrary length and redshift used to test and plot the angular-diameter-redshift relation.

3.3.2 Preliminary Calculations

The following preliminary calculations are made before any simulation is attempted. First $\Omega_k$ is calculated according to Equation 3 (Weinberg 1972; Weedman 1988; Sahni & Starobinsky 2000; Carroll & Ostlie 2018; Peebles 2020). Second, $m$ is calculated according to the dark energy equation of state in Equation 4 as discussed by Moews et al. (2019) where $w$ is the dark energy equation of state parameter. In this study, $w$ is assumed to be constant. Third, the universe’s age ($t$) is calculated according to Equations 5 and 6 (Hogg 2000; Sahni & Starobinsky 2000; Croton 2013; Chen et al. 2017; Balakrishna Subramani et al. 2019).

$$\Omega_k = \Omega_A + \Omega_m$$

$$m = 3(1+w)$$

$$E(z) = \sqrt{(1-\Omega_k)(1+z)^2 + \Omega_m(1+z)^3 + \Omega_\Lambda(1+z)^{3m}}$$

$$t = \frac{1}{H_0} \int_0^\infty \frac{dz}{(1+z)^3E(z^2)}$$

Fourth, the galaxy number density is estimated using comoving volume (Hogg, 2000) and Conseilce et al.'s (2016) merger rate estimate. A list of redshifts is calculated in intervals of 0.5 Gylrs out to the redshift defined in the ZMAX initial condition. A galaxy number density is calculated for each redshift using the following method.

The integral seen in Equation 7 is constructed to calculate the number of galaxies between 2 redshifts.
where “Galaxy Number” is the number of galaxies in an arbitrary field of view between redshifts of \( z_1 \) and \( z_2 \). \( q \) is an integration constant, \( (1+z') \) is the galaxy merger rate estimated by Conselice et al. (2016), and \( V_c(z') \) is the comoving volume as described by Hogg (2000) and defined in Equations 8-11.

\[
\text{Galaxy Number} = \int_{z_1}^{z_2} q(1+z') V_c(z') \, dz'
\] (7)

\[
V_c = \begin{cases} 
\frac{4\pi}{\Omega_c} D_H^3 & \text{for } \Omega_k > 1 \\
\frac{4\pi}{\Omega_c} D_M^3 & \text{for } \Omega_k = 1 \\
\frac{4\pi}{\Omega_c} D_C^3 & \text{for } \Omega_k < 1
\end{cases}
\] (8)

\[
D_M = \begin{cases} 
D_H \frac{1}{\sqrt{\Omega_k}} \sinh \left( \sqrt{\Omega_k} \frac{D_M}{D_H} \right) & \text{for } \Omega_k > 1 \\
D_C \frac{1}{\sqrt{\Omega_k}} \sin \left( \sqrt{\Omega_k} \frac{D_M}{D_C} \right) & \text{for } \Omega_k = 1 \\
D_H \frac{1}{\sqrt{\Omega_k}} \sin \left( \sqrt{\Omega_k} \frac{D_M}{D_H} \right) & \text{for } \Omega_k < 1
\end{cases}
\] (9)

\[
D_C = D_H \int_{0}^{z_2} \frac{dz''}{E(z'')}
\] (10)

\[
D_H = \frac{c}{H_0}
\] (11)

By setting “Galaxy Number” = 10,000 according to the HUDF estimate (Villard 2017; NASA 2018), \( z_1 = 0 \), and \( z_2 = 11 \) according to the maximum redshift calculated in the HUDF (Oesch et al. 2016; Jiang et al. 2020; Schauer et al. 2020), a value of \( q \) is calculated. Equation 7 is then used to calculate a galaxy number density between each previously chosen redshift. These galaxy number densities are saved in a separate list and associated with the corresponding \( z_2 \) used in the calculation thus producing a slight underestimate for galaxy number densities. Each galaxy number density is then divided by the total area of the HUDF to calculate the density per arcmin\(^2\). If the resulting galaxy number densities are less than 1 galaxy per arcmin\(^2\), then the value is assigned 1. This estimation is made for the following reasons. (i) In future calculations, if a galaxy number density is less than 1 galaxy per arcmin\(^2\), the simulation tends to generate no galaxies due to whole number rounding. (ii) Due to the inherent structure of the universe, there is a higher-than-average local density of galaxies. This approach inherently assumes a uniform galaxy distribution that is not observed in nature. By rounding the low-redshift galaxy number densities up to 1-galaxy per arcmin\(^2\), a higher local galaxy density is simulated with little effect to the results since few low-redshift galaxies appear in deep field images compared to high-z galaxies.

This approach assumes the volume of each 0.5 Gyr bin is equivalent to the same arcmin\(^2\) field of view. This assumption is made because of the small angular dimensions in a deep field image resulting in similar volumes, however, this does produce another underestimate at high-redshifts and overestimate at the lowest redshift. When testing this simulation with the HUDF parameters, it is evident this assumption does not make a significant difference to the result and produces an accurate image.

Fifth, after a galaxy number density is calculated for each redshift, a proper length is calculated in a separate list for each redshift according to Allen et al.’s (2017) effective radius estimate presented in Equation 2. These effective radii are doubled to represent an effective diameter used for the semi-major axis length of the ellipses generated in each bin. Allen et al.’s (2017) equation was found using observations of galaxies between redshifts of \( 1 < z < 7.2 \) with a mass of \( \log \left( \frac{M_\ast}{M_\odot} \right) > 10 \). For \( z \leq 1 \), a value of \( z = 1 \) is assumed since galaxy growth has been observed to be higher at high-redshifts, and the full galaxy shape appears to be visually resolved at closer distances in deep field images (Illyngworth et al. 2013; Ashby et al. 2015; NASA 2018; De Barros et al. 2019; Allam et al. 2020). This assumption was found to not significantly affect the result and produces an accurate image when using HUDF parameters. Additionally, for \( z \leq 1 \), the corresponding galaxy number densities are multiplied by \( N_{\text{Unseen}} \) to account for the total number of galaxies in existence since most low-redshift galaxies are likely detected by the HST. For \( z > 1 \), if the HST parameters are used in the initial conditions, no further galaxy number densities are adjusted. If the JWST parameters are used in the initial conditions, the rest of the galaxy number densities are multiplied by \( N_{\text{Unseen}} \) due to the significantly increased sensitivity of the JWST’s NIRCam. Admittedly, this is a general approximation that assumes most high-redshift galaxies are similar in characteristics due to the young age of the universe at the time of photon departure and detectable by the JWST, but because of the small amount of available high-redshift observations, it is difficult to know with certainty how many galaxies will be detected by the JWST.

Sixth, for all proper length estimates associated with redshifts of \( z > 1 \), the \( R_{\text{Increase}} \) percentage is multiplied to determine a new proper length since a larger portion of each high-redshift galaxy is likely to be seen due to the increased sensitivity of the JWST.
compared to HST. Finally, a value of k is chosen based on the value of \( \Omega_k \) where k = 0 if \( \Omega_k = 1 \), k = 1 if \( \Omega_k > 1 \), and k = -1 if \( \Omega_k < 1 \). The value of k is used in a future calculation.

### 3.3.3 First and Second Figure Generation

This simulation first creates an image plotting angular diameter vs redshift for an object with a proper length equal to the test length out to the test redshift specified in the control panel. This image is used to test and display the angular-diameter-redshift relation used in this simulation according to Equation 12 where \( \theta \) is the angular diameter, \( D \) is the proper length, \( c \) is the speed of light, and \( S \) is defined in Equation 13 (Weinberg, 1972; Weedman, 1988; Sahni & Starobinsky, 2000; Balakrishna Subramani et al., 2019; Peebles, 2020).

\[
\theta = \begin{cases} 
\frac{DH_0(1+z)}{cS} & \text{for } k = 1 \\
\frac{DH_0(1+z)\sqrt[4]{[D_A + D_m - 1]}}{\sinh(\sqrt{D_A + D_m - 1})} & \text{for } k = 0 \\
\frac{DH_0(1+z)\sqrt[4]{[D_A + D_m - 1]}}{\sinh(\sqrt{D_A + D_m - 1})} & \text{for } k = -1 
\end{cases} 
\]  

(12)

\[
S = \int_0^z \sqrt{\frac{dz}{\sqrt{1 + (1 - \Omega_m - \Omega_A)(1+z)^2 + \Omega_A(1+z)^3}}} 
\]

(13)

A second image is generated in the same manner using the basic trigonometry in Equation 14 instead of Equations 12 and 13. This image represents the expected angular diameters if the universe was not expanding and is used to compare the effects of expansion on the apparent angular diameter of high-redshift objects.

\[
\theta = \arctan\left( \frac{\text{Proper Distance}}{\text{Proper Length}} \right) 
\]

(14)

Figs. 1 and 2 present examples of the first and second images generated by this simulation respectively. This comparison is useful to visually see the difference between the expected angular size of a high-z galaxy and its apparent angular size based on the selected mass and energy densities. At high redshifts, galaxies appear to be several times larger than expected.

### 3.3.4 Third Figure: The Deep Field Simulation

The third figure displays a visual representation of the deep field simulation. The code runs through each galaxy number density bin from the highest redshift bin to the lowest redshift bin and generates ellipses on a two-dimensional plane for each bin. Since the galaxy number density estimates describe the number of galaxies per arcmin$^2$, the number of galaxies is multiplied by the total area in arcmin$^2$ according to the specifications in the control panel. For each bin, a matrix of equally spaced locations is generated according to the control panel’s deep field image dimensions and corresponding new galaxy number density. An ellipse with a random angular orientation is generated at each location with a semi-major axis defined by the angular-diameter-redshift
relation calculated with the bin’s corresponding proper length. The semi-minor axis is calculated by multiplying the semi-major axis by a random number between 0 and 1 to simulate a random orientation from edge-on to top-down. Each ellipse’s angular orientation, semi-major axis, and semi-minor axis are adjusted according to the effects of dark energy anisotropy that will be discussed in a later section. The ellipses’ locations are randomly chosen within a square with a side length equal to the distance between adjacent locations. This allows random ellipse placement on any location in the image. An ellipse color is chosen on a scale from blue to red proportional to its distance where the furthest galaxies are fully red ellipses, and the closest galaxies are fully blue ellipses. This helps visualize the three-dimensional nature of a deep field image. After a plane of ellipses is generated for each bin, the planes are added together to form one deep field image and displayed as the third figure. An example of the third generated figure can be seen in Fig. 3.

![Deep Field Simulation](image)

**Figure 3.** An example of the third image generated by this simulation. This image displays a visual depiction of the JWST deep field simulation with randomly generated positions and orientations of galaxies represented by ellipses. The ellipse colors are chosen to change from blue to red based on redshift with the closest galaxies being blue. This image assumed 82,807 galaxies are observable by the JWST with a furthest measurable redshift of $z = 15.3636$.

### 3.3.5 Fourth and Fifth Figures: Reference Figures

The fourth and fifth figures are generated to produce an image with a 100% galaxy coverage percentage and a 0% galaxy coverage percentage respectively. These figures do not represent a deep field simulation of any kind and are only used to calculate the galaxy number density of the third figure. The number of white pixels in each reference figure is calculated using the cv2 module. The number of pixels in the empty image is subtracted from the full image to determine the total number of pixels in the image’s field of view. The deep field image’s number of white pixels is calculated in the same manner and subtracted from the empty deep field image to determine how many pixels are not covered by ellipses. When this value is divided by the total number of pixels in the field of view and multiplied by 100%, a galaxy coverage percentage is found between 0% and 100%. This value is displayed in python’s console after the code completes execution. Since these figures simply produce solid red and white squares respectively, examples are not shown in this paper.

### 3.3.6 Sixth Figure: Anisotropy of Dark Energy

As discussed above, the anisotropy of dark energy is defined in the initial conditions and perturbed in the ensemble. This simulation assumes the anisotropy of dark energy can be generalized as a direction-dependent Hubble constant representing a universe expanding at a different rate along one axis than the others. Previous studies have typically investigated the anisotropy of dark energy using large samples of redshifts to measure the expansion rate in the radial axis. This study considered the effects of dark energy anisotropy on the angular-diameter-redshift relation where dark energy anisotropy would distort the apparent shapes of high-redshift galaxies.

This distortion occurs when the apparent angular diameter of an object is calculated using Equations 12 and 13 with a different $H_0$ in the x-axis than in the y-axis. Fig. 4 presents the expected change to a high-$z$ galaxy’s appearance where the gray shaded ellipse represents the appearance in a non-expanding universe, the blue ellipse represents the apparent size of the same galaxy caused by isotropic universe expansion, and the red ellipse represents an anisotropic universe expansion where $H_0$ in the y-axis is less than $H_0$ in the x-axis.

The change in a high-redshift galaxy’s expected shape takes the form of not just a smaller image but a decreased angle of orientation in the direction of the axis of larger $H_0$. While this effect would be indiscernible in the image of a single galaxy, with a large enough sample of galaxies, an angle bias may appear towards the axis of larger $H_0$.

Once an ellipse is randomly generated in the deep field image, its semi-major axis, semi-minor axis, and angle of orientation are adjusted for the dark energy anisotropy ratio defined in the initial conditions using trigonometry. These adjustments can be seen in Equations 15-17 where $a_{old}$ is the apparent length of the semi-major axis in arcmin of a high-$z$ galaxy in a...
non-expanding universe, $b_{\text{old}}$ is the length of the semi-minor axis in arcmin in a non-expanding universe, $H_{0,y}$ is the Hubble constant in the y-axis, $H_{0,x}$ is the Hubble constant in the x-axis, $a$ is the apparent length of the semi-major axis in arcmin in an expanding universe, $\theta_{\text{new}}$ is the galaxy’s new angle with respect to the x-axis in a universe with anisotropic expansion, and $\theta$ is the galaxy’s angle with respect to the x-axis in a universe with isotropic expansion.

$$\theta_{\text{new}} = \arctan \left( \frac{2a_{\text{old}} \sin(\theta) \cdot \frac{H_{0,y}}{H_{0,x}}}{2a \cos(\theta)} \right)$$  \hspace{1cm} (15)$$

$$2a = \frac{H_{0,y} \cos(\theta)}{H_{0,x} \cos(\theta_{\text{new}})}$$  \hspace{1cm} (16)$$

$$2b = \frac{H_{0,y} \sin(\theta_{\text{new}})}{H_{0,x} \sin(\theta)}$$  \hspace{1cm} (17)$$

Equations 15-17 are valid for all $\theta$’s between 0 and 90 except for $\theta = 90$ and $\theta = 0$ where the limit must be taken to yield the correct result. This study applies Equations 15-17 to each simulated galaxy and creates a new ellipse based on its redshift and orientation. These new ellipses are used for the final plot. Equations 15 is adjusted when $\theta > 90$ to ensure $\theta_{\text{new}}$ is always closer to the x-axis than $\theta$. This adjustment can be seen in Equations 18-20.

if $\theta \leq 180$, then $\theta = 180 - \theta$ and $\theta_{\text{new}} = 180 - \theta_{\text{new}}$  \hspace{1cm} (18)$$

if $\theta \leq 270$, then $\theta = \theta - 180$ and $\theta_{\text{new}} = 180 + \theta_{\text{new}}$  \hspace{1cm} (19)$$

if $\theta \leq 360$, then $\theta = 360 - \theta$ and $\theta_{\text{new}} = 360 - \theta_{\text{new}}$  \hspace{1cm} (20)$$

Every $\theta_{\text{new}}$ is saved in a list used in the final figure. Each $\theta_{\text{new}}$ in this list is divided into two new lists depending on $\theta$ being a positive slope or negative slope. All three lists are individually averaged into the average positively oriented ellipses, negatively oriented ellipses, and total ellipses. The average positively and negatively oriented angles are plotted as vectors with a length of 2 arbitrary units in the sixth figure. The average of all angles is displayed in the command line. Isotropic universe expansion yields an average angle of 45 degrees for positively oriented ellipses and 135 degrees for negatively oriented ellipses. An anisotropic universe expansion yields an average angle of < 45 degrees for positively oriented ellipses and > 135 degrees for negatively oriented ellipses. An example of the sixth figure produced by this simulation is shown in Fig. 5. Simulations resulting from Equations 15-17 with varying values of $H_{0,x}$, $H_{0,y}$, and $\theta$ with $\frac{b_{\text{old}}}{a_{\text{old}}} = 0.5$ and $b_{\text{old}} = 0.5$ arcmin are shown in Fig. 6 with the same visual convention used in Fig. 4.

![Figure 4](image-url)  

**Figure 4.** A high-z galaxy with a semi-major axis of 0.5 arcmin is represented by 3 ellipses. The gray shaded ellipse represents the galaxy’s appearance in a non-expanding universe, the blue ellipse represents the increase in apparent angular size due to the Angular-Diameter-Redshift relation from isotropic expansion of dark energy, and the red ellipse represents the distortion from anisotropic universe expansion. In this image, the expansion of the universe, and the value of $H_0$, is greater in the x-axis than in the y-axis where $H_{0,x}$ and $H_{0,y}$ represents the expansion rates of the universe in the x-axis and y-axis respectively, and where $H_{0,y} = 0.25H_{0,x}$.

![Figure 5](image-url)  

**Figure 5.** An example of the sixth figure produced by this simulation. This figure was produced with DE_Ratio = 0.5. The red arrow represents the average angle of orientation of all positively oriented ellipses and the blue arrow represents the angle of orientation of all negatively oriented galaxies. The red and blue arrows are <45 degrees and >135 degrees due to the anisotropy of dark energy defined in this instance.
Figure 6. Simulations resulting from Equations 15-17 with varying values of $H_{0,x}$, $H_{0,y}$, and $\theta$ with $\frac{b_{old}}{b_{uld}} = 0.5$ and $b_{old} = 0.5$ arcmin. Rows A, B, C, D, and E represent a simulated high-z galaxy at various angular orientations and DE Ratios. In these figures, the gray ellipse represents the galaxy’s angular size and shape if it was seen at the same distance in a non-expanding universe. The blue dotted ellipse represents the galaxy’s apparent angular size due to isotropic expansion of the universe according to the angular-diameter-redshift relation described by Equations 12 and 13. The red dotted ellipse represents the galaxy’s new apparent angular size, shape, and angular orientation if the universe was undergoing an overall spatially anisotropic expansion where the expansion rate differs between the x-axis and y-axis. In this figure, the ellipses are at an angle $\theta$ between the semi-major axis and the x-axis. Row A represents a high-z galaxy where $H_{0,y} = H_{0,x}$ and $\theta = 90, 67.5, 45, 22.5, and 0$ from left to right respectively. Row B represents a high-z galaxy where $H_{0,y} = 0.75H_{0,x}$ and $\theta = 90, 67.5, 45, 22.5, and 0$ from left to right respectively. Row C represents a high-z galaxy where $H_{0,y} = 0.5H_{0,x}$ and $\theta = 90, 67.5, 45, 22.5, and 0$ from left to right respectively. Row D represents a high-z galaxy where $H_{0,y} = 0.25H_{0,x}$ and $\theta = 90, 67.5, 45, 22.5, and 0$ from left to right respectively. Row E represents a high-z galaxy where $H_{0,y} = 0$ and $\theta = 90, 67.5, 45, 22.5, and 0$ from left to right respectively.
4. Results

4.1 Code Verification with a HUDF Simulation

This study used a novel approach to simulate deep field images instead of traditional interpolation from observations. While observations were still the basis for this study’s simulation, the interpolation to high-redshifts was different from previous studies. Because of this, a test image was created to simulate the HUDF and test the validity of the simulation approach used in this study. The following initial conditions were chosen to produce a simulation of the HUDF image: $H_0 = 71.4$, $\Omega_m = 0.301$, $\Omega_L = 0.706$, $w = -0.994$, N_Unseen = 6, R_Increase = 100%, DE_Ratio = 1, and ZMAX = 11. Additionally, to match the HUDF, the image’s dimensions were defined to produce an image size of $3.1 \times 3.1$ arcmin$^2$.

For this HUDF image, the N_Unseen parameter was only applied to redshift planes with $z < 1$ since the unseen galaxies in the HUDF should remain unseen at high-redshifts. A redshift value of 1 was chosen from a subjective estimate of the distance where some galaxies start to become unseen in the HUDF image from Inami et al.’s (2017) galaxy number count data. This is admittedly a rough estimate, however, an estimate had to be made as determining the number of unseen galaxies is an extremely difficult task as can be seen by the large range of estimates (Hille 2016; Conselice et al. 2016; Lauer et al. 2021).

The simulation was run with these parameters and compared to the HUDF. A negative of the HUDF was generated using the open-source programme imgonline.com (Imgonline n.d.). This programme generated a negative of the HUDF using all color channels resulting in an image with a white background like the simulation’s output. The results of the simulation and the HUDF negative are shown side-by-side in Fig. 7.

For this HUDF image, the N_Unseen parameter was only applied to redshift planes with $z < 1$ since the unseen galaxies in the HUDF should remain unseen at high-redshifts. A redshift value of 1 was chosen from a subjective estimate of the distance where some galaxies start to become unseen in the HUDF image from Inami et al.’s (2017) galaxy number count data. This is admittedly a rough estimate, however, an estimate had to be made as determining the number of unseen galaxies is an extremely difficult task as can be seen by the large range of estimates (Hille 2016; Conselice et al. 2016; Lauer et al. 2021).

Several notable differences can be seen between these two images. First, the ellipses in the simulation are uniformly shaded with a distinct edge while galaxies tend to decrease in brightness with an increase in radius. This causes some simulated ellipses to appear slightly larger than the galaxies in the HUDF. Second, a slight excess of foreground galaxies can be seen in the simulation when compared to the HUDF. This over-estimate of foreground galaxies is due to the larger value of N_Unseen. A value of 6 was used since 6 is the halfway point of the range of unseen galaxy measurements (Hille 2016; Conselice et al. 2016; Lauer et al. 2021). N_Unseen can be adjusted to fine-tune the number of foreground galaxies. Third, the spatial distributions of galaxies differ due to the random distribution of ellipses used in the code.

Subjectively, the HUDF simulation appears to match well with the HUDF image. Similar numbers of galaxies were generated in each image with similar apparent angular sizes, both images appear to have similarly random spatial and angular distributions, and both images appear to have similarly random edge-on to top-down views. From a successful HUDF replication, the methods used in this study appear to produce valid results.

4.2 Parameter Sensitivity Ensemble

A 100-member parameter sensitivity ensemble was run for each parameter as described in section 3.2.1 to determine the most sensitive parameter to changes across its range of uncertainty. Each
parameter was perturbed individually across the range of recent observation-based estimates while holding the rest of the initial conditions constant with their respective baseline values. The galaxy coverage percentages were averaged for each parameter and the corresponding standard deviation was found. The parameters were ordered by standard deviation from largest to smallest and can be seen in Table 2.

Table 2. The model results are presented showing all 8 perturbation ensembles used to determine the relative sensitivity of the initial conditions. The parameters are listed from highest to lowest standard deviation. The “N_Unseen” parameter was found to be most sensitive to small changes due to a large standard deviation while the dark energy equation of state parameter, w, was found to be the least sensitive. All parameters were found to have a positive correlation to the resulting changes in the resulting value of galaxy coverage percentage.

| Parameter       | Standard Deviation | Correlation |
|-----------------|--------------------|-------------|
| “N Unseen”      | 16.9%              | Positive    |
| “R_Increase”    | 9.76%              | Positive    |
| \(\Omega_m\)   | 4.31%              | Positive    |
| \(H_0\)         | 4.22%              | Positive    |
| “ZMAX”          | 3.88%              | Positive    |
| \(\Omega_\Lambda\) | 1.72%            | Positive    |
| “AnisotropyRatio” | 1.26%            | Positive    |
| w               | 0.34%              | Positive    |

The results of the parameter sensitivity ensemble should not be interpreted as determining the most sensitive parameters to change in general, but rather, the most sensitive to change from the uncertainty range of their measurements. In other words, a 5% change in the dark energy equation of state parameter (w) may be a larger change than a 5% change in the N_Unseen parameter but will not be reflected in this parameter sensitivity ensemble since the value of w has been measured to within a much smaller range of values than the number of unseen galaxies.

The results of the parameter sensitivity ensemble show that the parameter most sensitive to changes within its measured value is the N_Unseen parameter, the number of unseen galaxies in the HUDF, with a standard deviation of ± 16.9056% galaxy coverage percentage. The dark energy equation of state parameter is the least sensitive with a standard deviation of just ± 0.3420% galaxy coverage percentage. It is worthy of note that the maximum distance reached by JWST has a smaller standard deviation than H0 and \(\Omega_m\) indicating these values have a surprisingly large impact on the results of a simulation.

4.3 Ensemble Simulation

The 1000-member ensemble was run as described in section 3.2 to determine the expected galaxy coverage percentage in the JWST deep field images. The dimensions of the ensemble simulation were selected to produce an image of the same dimensions and pixel size as specified in the JWST documentation, however, both NIRCam images were combined into one rectangular image (STScI, 2016). The initial condition perturbation ranges are shown in Table 3.

Table 3. The perturbation range used for each initial condition in this study’s ensemble. For each parameter’s perturbation range, a list was generated of 100 equally spaced values spanning the range. Using each set of list members, the ensemble was run 10 times to account for the randomness of ellipse placement and orientation inherent in the simulation. This resulted in a 1000-member ensemble.

| Parameter       | Minimum | Maximum |
|-----------------|---------|---------|
| “N Unseen”      | 2.00    | 10.0    |
| “R_Increase”    | 100     | 160     |
| \(\Omega_m\)   | 0.2586  | 0.370   |
| \(H_0\)         | 66.9    | 75.12   |
| “ZMAX”          | 15.0    | 17.0    |
| \(\Omega_\Lambda\) | 0.677   | 0.744   |
| “AnisotropyRatio” | 0.850   | 1.00    |
| w               | -1.033  | -0.831  |

A list of 100 values evenly spaced between the range for each initial condition was generated. Each set of initial condition values was run 10 times to produce a 1000-member ensemble. Fig. 8 presents a sample of the results from 10 ensemble members sorted from low to high initial condition values. In Fig. 8, it is evident that a wide variety of results is obtained from the initial condition perturbations. The AGW effect is quickly evident in the last 5 samples with more than half of the white background being covered by galaxies. The galaxy number densities in Fig. 8 range from 54,389 to 323,126. The resulting mean galaxy coverage percentage was 55.12 ± 30.30%.
A Sample of 10 Ensemble Members

Figure 8. The results from 10 ensemble members. The wide variety of results from the initial condition perturbations can be seen with this ensemble member sample. The ensemble members range from 54,389 to 323,126 simulated ellipses in a deep field image. The high galaxy number densities seen in some of these figures depict the AGW effect. Sections A-J depict the 5th, 15th, 25th, 35th, 45th, 55th, 65th, 75th, and 95th set of initial conditions respectively. Sections A-J depict ten ensemble members from the 1000-member ensemble and show how the result changes from the initial condition perturbations.

4.4 T-Test Results

The results from the Ensemble Simulation were analysed using a one-way one-sample t-test as discussed in section 3.2.3. A t-score of 5.344 was obtained corresponding to a significance level of $5.646 \times 10^{-8}$. This significance level exceeds the 5-σ result as is commonly used in physics (Chandler, 2012). While a 5-σ result is often used as the threshold for which a discovery is made, this study did not take any measurements, and so this high significance level should not be viewed as a discovery.
4.5 Monochrome Simulation

One final simulation was made as a visual prediction of the JWST's deep field image. The simulation was run again with the following initial conditions: $H_0 = 71.4$, $\Omega_m = 0.301$, $\Omega_\Lambda = 0.706$, $w = -0.994$, $N_{\text{Unseen}} = 2$, $R_{\text{Increase}} = 130\%$, $DE_{\text{Ratio}} = 1$, and $Z_{\text{MAX}} = 15$. The image dimensions were chosen to match the NIRCam image dimensions including the 44-arcsec gap between the two NIRCam modules (STScI 2016). For this image, the Tolman Surface Brightness was used to adjust the brightness of each ellipse according to Equation 2.

$$\alpha = \begin{cases} \left(\frac{1+\frac{z}{2.64}}{1+z}\right)^3, & z \geq 5.264 \\ 1, & z < 5.264 \end{cases}$$

An exponent of 3 is estimated as observations have found the Tolman Surface Brightness to be between approximately 2.6 and 3.4 depending on the frequency band (Lubin & Sandage 2001). Ellipses at distances of $z < 5.264$ have an alpha value equal to 1 with more distant ellipse alpha values decreasing to 0 proportional to the Tolman Surface Brightness. A value of 5.264 was calculated using Equation 2 where $z = 0.35$ is subjectively estimated as the distance where most galaxies in the HUDF are no longer saturated. A factor of 100 is used to calculate a new saturation redshift to estimate the JWST’s 100 times sensitivity. This estimate is meant to be rough due to the difficulty inherent in this estimate and may not fully represent what will be observed in nature.

$$100 \times (1 + 0.35)^3 = (1 + z)^3$$

The monochrome simulation can be seen in Fig. 9.

5. Discussion

The 1000-member ensemble simulation resulted in an estimated 55.12% galaxy coverage percentage. This percentage was tested for statistical significance above the AGW effect definition of $\geq 50\%$ using a one-way one-sample t-test and found an extremely high statistical significance. While the statistical significance is high, the standard deviation of $\pm 30.30\%$ for the ensemble demonstrates the large variability of results due to the uncertainties of the initial conditions.

This is demonstrated further in the parameter sensitivity ensemble where several of the individual parameters have a significant range of estimated values. The majority of previous JWST simulations assumed specific values for $H_0$, $\Omega_m$, and $\Omega_\Lambda$. The parameter sensitivity ensemble demonstrated the variability in results from the uncertainty in these measurements alone. Notably, $H_0$ and $\Omega_m$ alone each resulted in a standard deviation of $> 4\%$. This study concludes the assumed universe model has a significant effect on the results of a simulation, and previous simulations may not adequately represent the full range of possible results.

Despite the high statistical significance, this prediction should be understood as a significant likelihood that the JWST will observe the AGW effect due to the large uncertainty range of the initial conditions. This study ultimately concludes the JWST is likely to observe the AGW effect with greater than 50% of the background being obstructed by galaxies.

The Extremely Large Telescope (ELT) is currently under construction at the time of this research and maybe the first telescope to have trouble observing objects beyond the AGW if the AGW does occur in nature (ESO 2014).

Should the AGW effect be observed, a pseudo-cosmological horizon would be found. An obstruction caused by foreground galaxies may limit the potential of future observatories aiming to reach further distances due to the obstruction. Additionally, this effect may have the potential to adversely affect the funding and scientific goals of larger and more sensitive observatories.

Future work could include accounting for the distortion effects from gravitational lensing when two galaxies at different distances line up. Additionally, the distance to the AGW in terms of redshift could be found through optimization techniques where a 50%
galaxy coverage percentage is achieved. Additionally, once the JWST deep field images are released, a comparison can be made to determine the accuracy of all previous simulations. Finally, the approach used in this study to account for the effects from dark energy anisotropy may be useful in detecting anisotropy in the normal plane to the line of sight in future deep field images and may be worthy of further investigation in a separate study.

6. Conclusion

Throughout this study, it is evident that simulations of deep field images are difficult to confidently predict due to the number of assumptions required and wide ranges in the uncertainty of many constants governing the evolution of the universe. There have been several previous simulations of the JWST’s deep field image that used assumptions of the universe’s geometry, the galaxy number densities, and the evolutions of high-z galaxies to make their predictions. While these assumptions are built off recent observations, the corresponding wide ranges of uncertainty can lead to a wide range of initial conditions resulting in substantially different results as this study’s parameter sensitivity ensemble demonstrated. This study conducted a novel geometric-focused deep field simulation of the expected JWST future deep field images using an ensemble approach. The AGW effect was introduced and defined as $\geq 50\%$ of a deep field image occupied by galaxies with larger than expected apparent angular sizes due to the universe’s expansion. This study ran a 1000-member ensemble by perturbing the initial conditions through ranges of uncertainty obtained from recent estimates. A galaxy coverage percentage was calculated for each ensemble member, averaged together, and analysed using a one-way one-sample t-test to determine whether the JWST is likely to observe the AGW effect. This study concluded the JWST is likely to observe the AGW effect with a galaxy coverage percentage of 55.12 $\pm$ 30.30% and found the most sensitive parameter to changes within its range of uncertainty was the estimated number of unseen galaxies in the HUDF. This study discussed the potential impacts of the AGW effect and its potential to form a pseudo-cosmological horizon that may inhibit the effectiveness of future observatories.

Acknowledgments

I thank my thesis advisor Dr. Dmitry Bizyaev and colleague Dr. Heide Doss for their indispensable support, encouragement, and expertise in this study. Additionally, I thank the American Military University for allowing me to conduct this study for partial fulfillment of the requirements for the degree of Master of Science in Space Studies. Finally, I thank friends and family for their continued support throughout this endeavor.

Data Availability

The software written for this study is free and accessible through the Astrophysical Source Code Library (Sailer 2021). Please report any bugs to the lead author using the contact information listed on the first page of this article.

References

Alam, S. et al., 2021, Phys. Rev. D, 103, 8
Andersson, E., 2014, ECMWF, available at https://www.ecmwf.int/en/forecasts/documentation-and-support/medium-range-forecasts
Antoniou, L. & Perivolaropoulos, L., 2010, J. Cosmol. Astropart. Phys., 2010, 012
Ashby, M. L. N. et al., 2015, Astrophys. J., Suppl. Ser., 218, 33
Balakrishna Subramani, V., Kroupa, P., Shenavari, H., & Muralidhara, V., 2019, MNRAS, 488, 3876–3883
Beckwith, S. V. W. et al., 2006, AJ, 132, 1729–1755
Benson, A. J., 2010, Phys. Rep., 495, 33–86
Bertin, E., 2008, Mem. S.A.It, 75, 422
Betoule, M. et al., 2014, A&A, 568, A22
Butcher, H., & Oemler, A., Jr., 1984, ApJ, 285, 426
Cahir, J., n.d., Encyclopedia Britannica, available at https://www.britannica.com/science/weather-forecasting/Numerical-weather-prediction-NWP-models
Cai, R.-G., Ma, Y.-Z., Tang, B., & Tuo, Z.-L., 2013, Phys. Rev. D, 87, 123522
Campanelli, L., Cea, P., Fogli, G. L., & Marrone, A., 2011, Phys. Rev. D, 83, 103503
Canada, E., 2013, Weather.gc.ca. available at https://weather.gc.ca/en/ensemble/index_e.html
Capozziello, S., Ruchita, & Sen, A. A., 2019, MNRAS, 484, 4484–4494
Carroll, B. W., & Ostlie, D. A., 2018, An introduction to modern astrophysics (2nd ed.), Cambridge University Press
Castelvachchi, D., 2020, Nature, 583, 500–501
Chandler, D., 2012, MIT News | Massachusetts Institute of Technology, available at https://news.mit.edu/2012/explained-sigma-
02098–text/nbr/520most%20cases%20%20five
Chen, C.-T., & Chen, P., 2019, Eur. Phys. J. C., 79, 649
Chen, Y., Kumar, S., & Ratra, B., 2017, ApJ, 835, 86
Colin, J., Mohayee, R., Rameez, M., & Sarkar, S., 2019, A&A, 631, L13
Conselice, C. J., Wilkinson, A., Duncan, K., & Mortlock, A., 2016, ApJ, 830, 83
Cooke, R., & Lynden-Bell, D., 2010, MNRAS, 401, 1409–1414
Cooray, A., 2016, R. Soc. Open Sci., 3, p.150555
Crain, R. A. et al., 2015, MNRAS, 450, 1937–1961
Croton, D. J., 2013, Publ. Astron. Soc. Aust., 30, e052
De Barros, S., Oesch, P. A., Labbé, I., Stefanon, M., González, V., Smit, R., Bouwens, R. J., & Illingworth, G. D., 2019, MNRAS, 489, 2355–2366
de Vaucouleurs, G., 1948, Annales d’Astrophysique, vol. 11, p. 247
ESA, 2002, available at https://esahubble.org/images/oph0220a/
ESO, 2011, available at https://esahubble.org/images/jwst_simulation_halosize/
ESO, 2014, available at https://www.eso.org/sci/facilities/eelt/science/
Fisher, A., 2021, NASA available at https://www.nasa.gov/press-release/goddard/2021/nasa-statement-on-james-webb-space-telescope-launch-readiness
Flom, P., 2018, Scienсing, available at https://scienсing.com/advantages-using-independent-group-тетш-8647277.html
Freedman, W. L., & Madore, B. F., 2010, Annu. Rev. Astron. Astrophys., 48, 673–710
Freedman, W. L. et al., 2019, ApJ, 882, 34
Gardner, J. P. et al., 2006, Space Sci. Rev., 123, 485–606
Gettings, T., 2005, J. Sci., 310, 248–249
Hencea, C., Marra, V., & Amendola, L., 2014, MNRAS, 439, 1855–1864
19
