Multi peripheral Model for the Nonforward $M^2$ Absorptive Part in the Double-Regge Region

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The forward absorptive part in $M^2$ of $a+b +c$ elastic scattering has been related by Mueller to the inclusive single-particle spectrum for $a+b+c$. The nonforward $a+b+c$ 3-3 $M^2$ absorptive part in the double-Regge region also has several physical applications. The $M^2$ absorptive part of this amplitude is calculated in the region of double-Regge exchange using a multiperipheral model with exponential damping in momentum transfer and Regge behavior in inclusive subenergies. The absorptive part is then extended to an analytic function in $M^2$.

I. INTRODUCTION

Recent investigations of inclusive spectra have given importance to the three-particle to three-particle (3-3) amplitude. This is due to the optical theorem of Mueller which relates the $M^2 = (p_a - p_b + p_c)^2$ absorptive part of the forward 3-3 amplitude for $a+b+c$ to the single-particle inclusive cross section for $a+b+c$. The nonforward 3-3 amplitude also plays an important role in the analysis of inclusive reactions. In order to fully study the analytic properties of the forward 3-3 absorptive part it is necessary to examine analytic continuations in two-particle subenergies and in the trajectories by using the nonforward 3-3 amplitude. This was first done by Halliday and Parry in $\varphi^3$ theory to demonstrate the factorization of the $M^2$ discontinuity.

The nonforward multiperipheral amplitude that we calculate here is used to study the relation between Pomeronchukon coupling strengths in inclusive and exclusive experiments. The six-point amplitude with double Pomeronchukon exchange was fitted to the pionization spectra in the 3-3 region to determine the double Pomeronchukon coupling strength. The nonforward six-point amplitude was then analytically continued to the forward 2-4 double Pomeronchukon exchange production region and compared with experimental data to demonstrate the need for Pomeronchukon decoupling.

Another application of the nonforward 3-3 absorptive part is to inclusive experiments on nuclei as studied by Bander using the eikonal approach. In the usual eikonal computation of a total cross section on a nucleus from the optical theorem, one uses the amplitude $S(k)$ for nucleon-nucleon elastic scattering at an impact parameter $b$, which requires a full knowledge of its nonforward transform $S(q^2)$. In the eikonal treatment of the inclusive 3-3 amplitude it is similarly necessary to know it at nonforward momentum transfers and energies in order to compute the scatterings with the nucleons located at various impact parameters and parallel displacements.

The 3-3 absorptive part calculated in this paper is also evaluated at the forward point to give the pionization spectrum. The method given here is a simpler and more direct way to get this pionization spectrum for the exponentially damped multiperipheral models. This result has recently been used in generalizing to arbitrary forms of damping in momentum transfer, thereby allowing fits to be obtained for pionization spectra over all $q_i$.5

In this paper we compute the $M^2$ absorptive part of the nonforward 3-3 amplitude in the double-Regge region for a simple multiperipheral model with Regge behavior and exponential damping in momentum transfer. After computing the $M^2$ absorptive part we then extend it to an analytic function.

The model for the absorptive part in the inclusive central plateau region consists of the production of the observed particle $c$ with a summation over numbers of particles emitted faster or slower than $c$, Fig. 1. The summation over both of these sets of particles is similar to the summation in a total cross section and is assumed to produce a Regge behavior, $s_1, s_2$ as in the multiperipheral model. In order to get the observed exponential falloff in $(p_i)^2$ in the central region, we include factors for exponential damping in the internal momentum transfers $e^{\alpha_i t_1}, e^{\alpha_i t_2}$. This model has been formulated by Caneschi and Pignotti and analytically computed by Silverman and Tan and others.

By squaring this amplitude and integrating over the inclusive phase space, we get a form for the
single-particle spectrum or equivalently the $M^2$ absorptive part of the forward 3-3 amplitude in the central inclusive region. In this paper we use this model to compute the nonforward 3-3 $M^2$ absorptive part in the double-Regge region, Fig. 2. By the double-Regge region we mean that subenergies $s_1$, $s_1'$, $s_2'$, and $s_2$ are large enough to be in the Regge region, Fig. 3. The calculations are carried out in the nonforward region adjacent to the forward central plateau region so that $p_1^2 - p_c$ has negative energy $p_1^2 < -M^2$. The absorptive part is then used to construct the analytic nonforward 3-3 amplitude in the double-Regge region.

As a special point we will present the forward 3-3 $M^2$ absorptive part for double Pomeranchukon exchange $\alpha_1 = 1$, $\alpha_2 = 1$, which agrees with the previously calculated single-particle spectrum in this model. The method employed in this paper gives a simpler and more direct calculation, however, as well as extending the calculation to trajectories of arbitrary intercept.

In Sec. II we formulate the model and calculate part of the intermediate-state integration as the quasi-two-body phase space for producing quasi-particles of intermediate masses squared $s_1$ and $s_2$, Fig. 2. The integration over the quasiparticle masses squared $s_1$ and $s_2$ is carried out in Sec. III in the double-Regge region. The results are presented in terms of the external invariants in Sec. IV. In Sec. V, we extend the absorptive part to an analytic function for the nonforward 3-3 amplitude in the double-Regge region.

II. CALCULATION OF $M^2$ ABSORPTIVE PART

The $M^2$ absorptive part for Fig. 2 can be computed by first performing the phase-space integrations over all of the particles in the sets labeled by momenta $p_1$ and $p_2$. We assume this is done (Ref. 8, Appendix) and produces Regge behavior in the momenta squared $s_1 = p_1^2$, $s_1' = p_1'^2$. The remaining integrations over $p_1$ and $p_2$ can be performed by considering these sets as intermediate particles of masses squared $s_1$, $s_2$, and then doing the simple two-body phase-space integral to get a function $A'$. Finally, the intermediate particle masses squared $s_1$, $s_2$, are integrated over their allowed values to get the absorptive part $A$.

The two-body phase-space integral over the exponentially damped momentum transfers for intermediate-particle-masses squared $s_1$, $s_2$, is (Fig. 2)

\[
A' = \int d^4p_1 \delta^+(p_1^2 - s_1) \int d^4p_2 \delta^+(p_2^2 - s_2) \delta^4(p_1 + p_2 - p_a - p_b - p_c) \\
\times \exp\left[\Omega_1\left[(p_1^2 - p_a^2) + (p_2^2 - p_b^2)\right] + \Omega_2\left[(p_1^2 - p_c^2) + (p_2^2 - p_d^2)\right]\right].
\]

(2.1)

To compute the two-body phase space we work in the c.m. frame of $p_1$ and $p_2$ where the kinematics are:

\[
\begin{align*}
\bar{p}_1 + \bar{p}_2 &= 0 = \bar{p}_a + \bar{p}_b + \bar{p}_c = \bar{p}_1 + \bar{p}_2', \\
M &= (p_1 + p_2)^2 = (p_1 + p_2)^2, \\
p_a^2 + p_b^2 &= M, \quad M = \sqrt{M^2}, \\
P &= \left|\bar{p}_1\right| = \left|\bar{p}_2\right| = \frac{\Delta^{1/2}(M^2, s_1, s_2)}{2M}, \\
\Delta(x, y, z) &= x^2 + y^2 + z^2 - 2xy - 2yz - 2zx, \\
p_1^2 &= \frac{M^2 + s_1 - s_2}{2M}, \quad p_2^2 = \frac{M^2 + s_2 - s_1}{2M}.
\end{align*}
\]

(2.2)

The integrals give the usual two-body phase-space result of an integral over the angles of $\bar{p}_1$

\[
A' = \frac{P}{4M} \int d\Omega_1 \exp\left[\Omega_1\left[2m_a^2 + 2s_1 - 2(p_a^2 + p_b^2)p_1^2 + 2(p_1^2 + p_2^2)p_1 \cdot p_1' + \Omega_2\left[2m_2^2 + 2s_2 - 2(p_c^2 + p_d^2)p_2^2 - 2(p_1' + p_2')p_1 \cdot p_1'\right]\right]\right].
\]

(2.3)

We define a vector which is fixed by external momenta

\[
Q = \Omega_1(p_2' + p_1') - \Omega_2(p_2 + p_1), \quad Q = |Q|,
\]

(2.4)
and choose the $z$ axis along this vector. The angular integrals are then easily done. The result for the two-body phase space is

$$A' = \frac{\pi}{2MQ} \sinh(2QP) \exp\{Q[2m_q^2 + 2s_1 - 2(p_0^2 + p_0^2)] + Q[2m_s^2 + 2s_2 - 2(p_0^2 + p_0^2)]\}.$$  \hfill (2.5)

III. INTEGRATION OVER SUBENERGIES

We now integrate over the subenergies $s_1, s_2$ to get the nonforward $M^2$ absorptive part $A$. We assume Regge behavior for the absorptive parts of $p_0 + Q_1 - p_0' + Q_1'$ and $p_s + Q_2 - p_s' + Q_2'$ and include dependence on the external momentum transfers $t_1 = (p_s' - p_s)^2$, $t_2 = (p_0' - p_0)^2$.

$$A = c\frac{\beta_0(t_1)\beta_0(t_2)}{s_1 s_2} \int ds_1 ds_2 (s_1)^{\alpha'\beta_1} (s_2)^{\alpha'\beta_2} A'.$$  \hfill (3.1)

Since the over-all factors $c, \beta_0, \beta_0$ do not enter into the integrals we neglect them for now.

The dependence on $s_1$ and $s_2$ enters explicitly and through $P, p_0', p_0''$ in Eq. (2.5). Defining

$$Q^0 = \Omega_0(p_0^2 + p_0^2) - \Omega_0(p_s^2 + p_s^2),$$

we have explicitly

$$A = \frac{\pi}{2MQ} \exp\{Q[2m_q^2 - M(p_s^2 + p_s^2)] + Q[2m_s^2 - M(p_0^2 + p_0^2)]\}$$

$$\times \int ds_1 ds_2 \sinh\left(\frac{Q}{M} \Delta^{1/2}(M^2, s_1, s_2)\right)(s_1)^{\alpha'\beta_1}(s_2)^{\alpha'\beta_2} \exp[2\Omega_0 s_1 + 2\Omega_0 s_2 - Q^0/(s_1 - s_2)].$$  \hfill (3.3)

The boundary is given by the condition that the c.m. momentum be positive, and that the subenergies be above threshold

$$\Delta(M^2, s_1, s_2) > 0, \quad s_1 > s_0^2, \quad s_2 > s_0^2.$$  \hfill (3.4)

To do the integration we substitute

$$x = \frac{1}{M}(p_0^2 - P), \quad y = \frac{1}{M}(p_s^2 - P)$$

which gives

$$\frac{s_1}{M^2} = x(1 - y), \quad \frac{s_2}{M^2} = y(1 - x), \quad ds_1 ds_2 = (M^2)^2(1 - x - y) dx dy$$

and simplifies

$$\Delta^{1/2}(M^2, s_1, s_2) + M^2(1 - x - y).$$

We then split up $\sinh[QM(1 - x - y)]$ into its two exponentials. In the second exponential we substitute

$$x = 1 - y', \quad y = 1 - x', \quad 1 - x - y' = -(1 - x' - y').$$

Then both exponentials are identical and can be combined to give

FIG. 2. Multiperipheral diagram for the $M^2$ absorptive part of nonforward $abc$ scattering.

FIG. 3. External invariants for the nonforward $a\bar{b}c$ scattering.
\[
A = (M^2)^{2\alpha_1} \rho_0 \frac{2}{4\sqrt{M^2 - 2M}\pi^{1/2}} \exp\left[\frac{2m_s^2 - M(p_s + p_0^2)}{\Omega_1} + \frac{2m_b^2 - M(p_b^2 + p_0^2)}{\Omega_2} + M]^Q\right] \\
\times \int_0^1 dx \int_0^1 dy (1 - x - y) [x(1 - x)]^\alpha [y(1 - y)]^\beta \\
\times \exp\left[-x(M(q + Q') - 2\Omega_1 M^2) - y(M(q - Q') - 2\Omega_2 M^2) - 2xy(\Omega_1 + \Omega_2 M^2)\right].
\]

(3.8)

We have taken the limit \( M^2 \to \infty \) here and \( s_1, s_r, s'_1, s'_r \to \infty \) in order to make the thresholds reduce to the limits 0 to 1.

In Sec. IV, Eq. (4.9), we show that the coefficients of \( x \) and \( y \) in the exponential are proportional to

\[ s_1 + s'_1 \text{ and } -s_r + s'_r, \]

respectively. We evaluate the integral for very large magnitudes of these external subenergies, that is, the double-Regge region. Then \( x \) and \( y \) terms may be dropped with respect to 1 in the factors, and the limits of \( x \) and \( y \) extended to infinity. Defining

\[ x' = x[M(q + Q') - 2\Omega_1 M^2], \quad y' = y[M(q - Q') - 2\Omega_2 M^2], \]

(3.9)

the integration becomes

\[ \int_0^1 dx' \int_0^1 dy'(x')^\alpha (y')^\beta e^{-xy' / \kappa} = \Gamma(\alpha_1 + 1) \Gamma(\alpha_2 + 1) \kappa^{\alpha_1 + 1} U(\alpha_2 + 1, -\alpha_1 + \alpha_2 + 1, \kappa), \]

(3.10)

where \( U \) is the confluent hypergeometric function\(^{10}\) (also called \( \Psi \))\(^{11}\) and

\[ \kappa = \frac{[M(q + Q') - 2\Omega_1 M^2][M(q - Q') - 2\Omega_2 M^2]}{2M^2(\Omega_1 + \Omega_2)}. \]

(3.11)

The result of the phase-space integrations in the double-Regge region is

\[
A = (M^2)^{2\alpha_1} \rho_0 \frac{2}{4\sqrt{M^2 - 2M}\pi^{1/2}} \exp\left[\frac{2m_s^2 - M(p_s + p_0^2)}{\Omega_1} + \frac{2m_b^2 - M(p_b^2 + p_0^2)}{\Omega_2} + M]^Q\right] \\
\times \exp\left[\frac{2m_s^2 - M(p_s + p_0^2)}{\Omega_1} + \frac{2m_b^2 - M(p_b^2 + p_0^2)}{\Omega_2} + M]^Q\right] \Gamma(\alpha_1 + 1) \Gamma(\alpha_2 + 1) \kappa^{\alpha_1 + 1} U(\alpha_2 + 1, -\alpha_1 + \alpha_2 + 1, \kappa).
\]

(3.12)

IV. KINEMATICS AND THE DOUBLE-REGGE RESULT

It remains to express the c.m. momentum components in terms of the invariants, Fig. 7,

\[
s_1 = (p_s + p_b)^2, \quad s'_1 = (p_s + p_b')^2,
\]

\[
s_r = (p_s + p_b)^2, \quad s'_r = (p_s + p_b')^2,
\]

\[
s = (p_s + p_b)^2, \quad s' = (p_s + p_b')^2.
\]

(4.1)

In the nonforward \( ab \) region, \( p_e = -p_0 \), and \( p_b' \) \(< -m_c \), so that the subenergies \( s_1, s'_1, s_r, s'_r \) are negative. The time components in the c.m. system are found by scalar products with the vector \( p_s + p_b + p_e = (M, 0) \). This gives

\[
2M^2 = s + s_1 - m_s^2 - m_b^2 - m_e^2,
\]

\[
M^2 = s' + s'_1 - m_s^2 - m_b^2 - m_e^2.
\]

(4.3)

To calculate \( Q = |Q| \), we first calculate \( Q_s Q_r \), and then

\[ Q^2 = (Q_s)^2 - Q_r Q_t. \]

(4.4)

Using (3.2) and (2.4) along with

\[ p_s^2 - p_s + p_b' = p_b - p_b', \]

\[ \tau = (p_e^2 - p_e'^2), \]

(4.5)

we find

\[ -Q_s Q_t^2 = 2\Omega_1 \Omega_2 (2M^2 - s - s'_1 - s_r - s'_r) + (\Omega_1 \Omega_2 + \Omega_2 \Omega_2 - \Omega_1 \Omega_1) \tau \]

\[ -4\Omega_1^2 m_s^2 - 4\Omega_2 m_b^2 + 4\Omega_1 \Omega_2 m_e^2. \]

(4.6)

Now using (4.4), (4.2), and (4.6) we compute \( Q \) in the double-Regge limit

\[
M^2 \to \infty, \quad |s_1|, |s'_1| \to \infty, \quad |s_r|, |s'_r| \to \infty,
\]

\[
t_1, t_2, \tau, \frac{(s_1 + s'_1)(s_r + s'_r)}{M^2} \text{ all fixed.}
\]

(4.7)

The result including terms of \( O(m^2) \) is
\[ 2MQ = 2M^2(\Omega_1 + \Omega_r) - \Omega_1(s_1 + s'_1 - t_1 + 2m^2) \]
\[ - \Omega_1(s_1 + s'_1 - t_4 + 2m^2) \]
\[ - \frac{\Omega_1}{\Omega_1 + \Omega_r} \left[ \frac{(s_1 + s'_1)(s_1 + s'_1)}{M^2} + \tau - 4m^2 \right]. \]

(4.8)

For the coefficients of \( x \) and \( y \) in the exponent of (3.8) and in the powers of (3.12) we have
\[
A = c_0(t_1)B_0(t_2) = \frac{\pi \Gamma(\alpha_0 + 1) \Gamma(\alpha_0 + 1) \Gamma(\alpha_0 + 1)}{8(\Omega_1 + \Omega_r)^{\alpha_0 + \alpha_0 + 2}} \left[ - \frac{1}{2} \Omega_1(s_1 + s') \right]^{\alpha_0} \left[ - \frac{1}{2} \Omega_1(s_1 + s') \right]^{\alpha_0} 
\times \exp \left[ - \frac{2\Omega_1\Omega_r}{\Omega_1 + \Omega_r} \left( \frac{1}{2} - m^2 \right) + \frac{1}{2}(\Omega_1(t_1 + t_2)) \right] e^{-\kappa \pi} \kappa^{-\alpha_0} U(\alpha_2 + 1, - \alpha_2 + \alpha_2 + 1; \kappa), \]

(4.11)

where \( \alpha_0 = \alpha_0(t_1), \alpha_0 = \alpha_0(t_2) \).
The result (4.11) also holds in the forward direction, where
\[
p_{\text{s}} = p_{\text{s}}, \quad p_{\text{t}} = p_{\text{t}}, \quad p_{\text{t}} = p_{\text{t}} = p_{\text{t}}, \quad s_{\text{t}} = s_{\text{l}}, \quad s_{\text{t}} = s_{\text{l}}, \quad s = s, \quad t_1 = 0, \quad t_4 = 0, \quad \tau = 0, \]
(4.12)

and
\[
\kappa = \frac{2\Omega_1\Omega_r}{\Omega_1 + \Omega_r} \left[ \left( p_{\text{t}}^2 \right)^2 + m^2 \right]. \]

(4.13)

For the case of double Pomeron exchange \( \alpha_0(0) = 1, \alpha_0(0) = 1 \), we obtain for the single-particle spectrum or \( M^2 \) absorptive part in the central plateau region
\[
A_{pp} = c_0(0)B_0(0) \frac{\pi M^2}{16(\Omega_1 + \Omega_r)^2} \times \exp \left[ \frac{2\Omega_1\Omega_r}{\Omega_1 + \Omega_r} m^2 \right] e^{-\kappa \pi} U(2, 1, \kappa). \]

(4.14)

This is the same as the earlier result\(^3\) if we note that
\[
e^{-\kappa \pi} U(2, 1, \kappa) = (1 + \kappa)E_1(\kappa) - e^{-\kappa}, \]
(4.15)

where \( E_1(\kappa) \) is the exponential integral function.

V. EXTENSION TO ANALYTIC FUNCTION IN \( M^2 \)

The absorptive part in \( M^2 \) calculated in (4.11) can be extended to an analytic function by a procedure equivalent to using a Cauchy integral or dispersion relation in \( M^2 \). Of course this does not give the entire 3-3 amplitude but only that part arising from the \( M^2 \) cut, or equivalently, from diagrams like Fig. 2.

We note that the only occurrence of \( M^2 \) in the result (4.11) is in the variable \( \kappa(\pi) \) (4.10). Keeping \( (s_1 + s'_1) \) and \( (s_1 + s'_1) \) fixed and negative, but very
\[
[M(\pi + \pi') - 2M_r M^2] = -\Omega_1(s_1 + s'1) + O(m^2), \]
\[
[M(\pi - \pi') - 2M_r M^2] = -\Omega_1(s_1 + s'1) + O(m^2). \]

(4.9)

For \( \kappa \) of Eq. (3.11) we then have
\[
\kappa = \frac{2\Omega_1\Omega_r}{\Omega_1 + \Omega_r} \left[ (s_1 + s'1)(s_1 + s'1) \right] \frac{4M^2}{\Omega_1 + \Omega_r}. \]

(4.10)

Our final result from (3.12) for the 3-3 nonforward absorptive part in the double-Regge region is
\[
\text{large in magnitude, the } M^2 \text{ cut from threshold to infinity is mapped into a cut in } \kappa \text{ for } 0 < \kappa < \infty. \]

Therefore we make the analytic continuation in the variable \( \kappa \). The function of \( \kappa \) in (4.11) can be written in terms of entire functions\(^3\) \( M \) (also called \( \Phi \)).\(^11\)
\[
\text{Imf}(\kappa) = e^{-\frac{\kappa}{\pi}} \kappa^{-\alpha_2} \frac{M(\alpha_2 + 1, - \alpha_2 + \alpha_2 + 1, \kappa)}{\sin \pi \alpha_2} 
\times \left[ \frac{\Gamma(\alpha_2 + 1) \Gamma(\alpha_2 + 1)}{\Gamma(\alpha_2 + 1) \Gamma(\alpha_2 + 1)} \right] 
- \kappa^{-\alpha_2} M(\alpha_2 + 1, - \alpha_2 + \alpha_2 + 1, \kappa), \]

(5.1)

Since the \( M \) functions are entire, we can extend \( \text{Imf}(\kappa) \) to an analytic function with a \( 0 < \kappa < \infty \) cut by replacing \( \kappa^{-\alpha_2} \) by \( (-\kappa)^{-\alpha_2} / \sin \pi \alpha_2 \):
\[
\text{Imf} \left[ \frac{-\kappa - \pi}{\sin \pi \alpha_2} \right] = e^{-\frac{\kappa}{\pi}}, \quad \kappa^{-\alpha_2} \]

(5.2)

and similarly for \( \kappa^{-\alpha_2} \).

Using
\[
\frac{\pi}{\sin \pi \alpha_2} = - \Gamma(1 + \alpha_2) \Gamma(- \alpha_2) \]

(5.3)

and the Kummer transformation\(^10\)
\[
e^{-\frac{\kappa}{\pi}} \text{M}(\alpha, \beta, \kappa) = M(\beta - \alpha, \alpha, - \kappa), \]

(5.4)

we have
\[
f(\kappa) = \frac{-1}{\pi} \Gamma(\beta - \alpha, - \alpha_2) \Gamma(- \alpha_2) \left[ \frac{\pi}{\sin \pi \alpha_2} \right] 
\times \left[ \frac{(-\kappa)^{-\alpha_2}}{\Gamma(\alpha_2 + 1)} \Gamma(- \alpha_2) \Gamma(- \alpha_2 + 1) \right] 
- \kappa^{-\alpha_2} M(\alpha_2 + 1, - \alpha_2 + \alpha_2 + 1, - \kappa), \]

(5.5)
This may be recomposed to give the analytic function

\[ f(\kappa) = -\frac{1}{\pi} \Gamma(-\alpha_1) \Gamma(-\alpha_2) (-\kappa)^{-\alpha_1} \times U(-\alpha_1, -\alpha_1 + \alpha_3 + 1, -\kappa). \]  

(5.6)

The same result is obtained by performing the Cauchy integral, and is unique up to an arbitrary polynomial in \( \kappa \).

Including (5.6) with the remainder of (4.11) gives the analytic extension in \( M^2 \) or \( \kappa \). The signature structure of the Regge exchanges \( \alpha_1 \) and \( \alpha_2 \) can be found by adding diagrams like Fig. 2 but with exchanged \( (a \to \bar{a}', b \to \bar{b}') \) and \( (b \to \bar{b}', a \to \bar{a}') \). Care must be taken to establish that the diagram of Fig. 2 and the exchanged diagrams from \( (a \to \bar{a}') \) and \( (b \to \bar{b}') \) are evaluated above the \( \kappa \) cut, but the diagram from the exchange \( (a \to \bar{a}', b \to \bar{b}') \) is evaluated below the \( \kappa \) cut. This has been carried out in Ref. 3.

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A Statistical Theory of Particle Production*

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A statistical theory of particle production is formulated in analogy with the generalized Ginzburg-Landau theory of phase transitions in superconductors and intensity fluctuations in lasers.

I. INTRODUCTION

At present accelerator energies the number of secondaries produced in hadron-hadron collisions is large enough so that one can consider statistical theories of particle production. In this paper we shall present such a theory.¹

Our basic approach is as follows: We do not attempt to treat in detail the fundamental dynamics underlying particle production. Instead, we imagine integrating out the microscopic degrees of freedom and representing them in terms of a small number of phenomenological parameters. We are then left with a statistical theory of the macroscopic observables.

A well-known example of this approach is the Ginzburg-Landau theory of superconductivity.² In its generalized form² it provides a statistical theory of the superconducting order parameter which depends upon a small set of experimentally determined parameters. Once a microscopic theory of superconductivity was formulated by BCS,³ it was possible to derive the Ginzburg-Landau theory from it and to obtain expressions for the phe-