Photon-propagation model with random background field:
Length scales and Cherenkov limits

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Abstract

We present improved experimental bounds on typical length scales of a photon-propagation
model with a frozen (time-independent) random background field, which could result from anom-
alous effects of a static, multiply connected spacetime foam.

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I. INTRODUCTION

In a previous article [1], we have proposed a simple photon-propagation model to describe the potential effects of a static spacetime foam composed of identical, randomly-distributed defects (e.g., microscopic wormholes) embedded in Minkowski spacetime. For this particular model, a modified photon dispersion law was derived in the long-wavelength limit,

\[
\omega^2 \sim (1 - A^2 \gamma_1) c^2 k^2 - A^2 \gamma_1^2 \frac{e^2}{l^2} k^4,
\]

where \( k \equiv |\vec{k}| \) is the photon wave number and \( \omega \) the frequency, \( A \) the amplitude of the frozen (time-independent) random background field \( g_1(\vec{x}) \), \( l_\gamma \) a characteristic length scale of \( g_1(\vec{x}) \), \( \gamma_1 \) a nonnegative dimensionless coefficient, and \( c \) a fundamental constant tracing back to the Minkowski line element (see also below). An upper bound \( l_\gamma < 1.6 \times 10^{-22} \text{ cm} \), for \( A = \alpha \approx 1/137 \), was then obtained from observations of a particular TeV flare in an active galactic nucleus.

In this Brief Report, we use recent results on ultra-high-energy cosmic rays [2, 3] to improve our previous bound on \( l_\gamma \). In addition, we give a careful discussion of the possible relation between the photonic length scale \( l_\gamma \) and the characteristic length scales of the microscopic spacetime structure.

In the following, we will use standard natural units with \( \hbar = c = 1 \), except when stated otherwise. For the physical situation discussed in the next section, the operational definition of the velocity \( c \) is the maximum attainable velocity of the proton. (Further discussions on Lorentz noninvariance can be found in, e.g., Refs. [2, 3, 4, 5] and references therein.)

II. PHOTON PROPAGATION

Assuming a modified photon dispersion law with a negative dimensionful coefficient \( K_{1\text{neg}} \),

\[
E_\gamma \sim k + K_{1\text{neg}} k^3,
\]

and an unchanged (ultrarelativistic) proton dispersion law,

\[
E_p \sim k,
\]

the Cherenkov-like proton process \( p \rightarrow p + \gamma \) becomes kinematically allowed [2]. From observations of ultra-high-energy cosmic rays, Gagnon and Moore [3] obtain the following bound:

\[
0 \leq -K_{1\text{neg}} < (4 \times 10^{22} \text{ GeV})^{-2}.
\]
There is also a bound on the difference between the maximum attainable velocities of particles with spin 1 and spin \(1/2\). For a modified photon dispersion law

\[ E_\gamma = c (1 + \epsilon) k \]  

(5)

and an unmodified fermion dispersion law (3), the authors of Ref. [3] obtain the bound

\[ |\epsilon| < 1.6 \times 10^{-23} . \]  

(6)

We now turn to a simple photon-propagation model [1] with a fixed random background field \(g_1(x)\) and an action given by

\[ S_{\text{photon}} = - \frac{1}{4} \int_{\mathbb{R}^4} \text{d}^4 x \left( F_{\mu\nu}(x) F_{\kappa\lambda}(x) \eta^{\mu\nu} \eta^{\kappa\lambda} + g_1(x) F_{\kappa\lambda}(x) \tilde{F}^{\kappa\lambda}(x) \right) , \]  

(7)

in terms of the standard Maxwell field strength tensor \(F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu\), the dual tensor \(\tilde{F}^{\kappa\lambda} \equiv \frac{1}{2} \epsilon^{\kappa\lambda\mu\nu} F_{\mu\nu}\), and the inverse Minkowski metric \(\eta^{\mu\nu}\). The random background field \(g_1(x)\) is assumed to be time-independent,

\[ g_1(x^0, x^1, x^2, x^3) = g_1(x^1, x^2, x^3) \equiv g_1(\vec{x}) , \]  

(8)

and to fluctuate around a value zero with amplitude \(A\); see Sec. IV of Ref. [1] for further properties. The random background field \(g_1(\vec{x})\) in Eq. (7) can be seen to act as a variable coupling constant, with spacetime taken to be perfectly smooth (manifold \(M = \mathbb{R}^4\) and metric \(g_{\mu\nu}(x) = \eta_{\mu\nu}\)).

For the photon-propagation model (7), we have calculated in Sec. V of Ref. [1] the dispersion law (1), with \(l_\gamma\) and \(\gamma_1\) determined in terms of the autocorrelation function of \(g_1(\vec{x})\),

\[ l_\gamma = l_\gamma [g_1] , \quad \gamma_1 = \gamma_1 [g_1] . \]  

(9)

The dispersion law (1) gives then the following photon energy:

\[ E_\gamma \sim k \left( 1 - \frac{1}{2} A^2 \gamma_1 - \frac{1}{2} A^2 l_\gamma^2 k^2 \right) , \]  

(10)

for parametrically small amplitude \(A\) or for \(\gamma_1\) and \(l_\gamma^2 k^2\) much less than unity.

From the experimental bounds (4), (6) and the relation (10), we obtain

\[ l_\gamma < (2 \times 10^{20} \text{GeV})^{-1} \left( \alpha A^{-1} \right) \approx (1.0 \times 10^{-34} \text{cm}) \left( 1/137 A^{-1} \right) , \]  

(11a)

\[ \gamma_1 < (6 \times 10^{-19}) (\alpha/A)^2 , \]  

(11b)

where \(\alpha \approx 1/137\) is the fine-structure constant (the possible relation \(A \sim \alpha\) will be discussed in the next section). Note that the bound (11a) is 12 orders of magnitude better than the one given in Sec. VI of Ref. [1], where \(l_\gamma\) was called \(l_{\text{foam}}\). This bound on \(l_\gamma\) for \(A \sim \alpha\) is, in fact, of the order of the Planck length,

\[ l_{\text{Planck}} \equiv \sqrt{\frac{G \hbar}{c^3}} \approx 1.6 \times 10^{-33} \text{ cm} , \]  

(12)

which may determine the fine-scale structure of spacetime itself [6].
III. SPACETIME STRUCTURE

In order to connect the photon parameters $\gamma_1 \left[ g_1 \right]$ and $l_\gamma \left[ g_1 \right]$ derived from the effective action (7) to the microscopic structure of spacetime, we introduce the following definitions:

$$l_\gamma \equiv l_{\text{wormhole}} \left( \frac{l_{\text{wormhole}}}{l_{\text{separation}}} \right)^{3/2}, \quad (13a)$$

$$\gamma_1 \equiv \left( \frac{l_{\text{wormhole}}}{l_{\text{separation}}} \right)^3. \quad (13b)$$

These definitions are motivated by a very simple spacetime model [1] consisting of static, randomly-distributed wormholes [6] embedded in Minkowski spacetime. This toy model has, by definition, a preferred frame of reference. The length $l_{\text{wormhole}}$ would then correspond to an appropriate characteristic dimension of an individual wormhole (e.g., the average width of the two mouths or the long distance between the centers of the mouths, where both lengths are measured in the Minkowski part of spacetime and the short distance through the wormhole throat is assumed to be zero). The length $l_{\text{separation}}$ would correspond to the average separation between different wormholes (the wormhole density is $n_{\text{wormholes}} = l_{\text{separation}}^{-3}$).

The anomaly calculation reported in the Appendix of Ref. [1], specialized to the case $l_h = \delta$ and with notations ($l_{\text{foam}}$, $d$, $a$) for ($l_\gamma$, $l_{\text{wormhole}}$, $l_{\text{separation}}$) here, gave $A = \alpha$ in the effective action (7) and extra factors 0.18 and 0.15 on the right-hand sides of Eqs. (13a) and (13b), respectively. This calculation was, however, based on several simplifying assumptions and is, therefore, not absolutely rigorous. The two most important results would be that there are no extremely small or large factors on the right-hand sides of Eqs. (13ab) and that the effective amplitude $A$ is of order $\alpha$. The physical interpretation of the quantities $l_{\text{wormhole}}$ and $l_{\text{separation}}$, defined mathematically by Eqs. (13ab), would be that they emerge directly from the underlying spacetime structure. Indeed, a successful calculation would relate the “randomness” of the couplings $g_1(\vec{x})$ in the effective action (7) to the (as of yet, unknown) microscopic structure of spacetime. (An entirely different origin for the variable couplings $g_1(x)$ of a $F_{\kappa\lambda} \tilde{F}^{\kappa\lambda}$ term in the effective action is, of course, not excluded; see, e.g., Ref. [5] and references therein.)

A concrete example of this particular spacetime model with permanent wormholes would then have

$$l_{\text{wormhole}} \approx 10 \times l_{\text{Planck}}, \quad l_{\text{separation}} \gtrsim 10^8 \times l_{\text{Planck}}, \quad (14)$$

in order to be consistent with the bounds (11a) and (11b) for $A = \alpha$. More generally, Fig. 1 shows which combinations of values of $l_{\text{wormhole}}$ and $l_{\text{separation}}$ are allowed or excluded, assuming $A = \alpha$. For $l_{\text{wormhole}} \lesssim 1.3 \times 10^{-25} \text{ cm}$ (or $l_{\text{separation}} \lesssim 1.5 \times 10^{-19} \text{ cm}$) the bound (11b) is seen to be the stronger one and for the other case the bound (11a). Without further input, we cannot say anything about $l_{\text{wormhole}}$ and $l_{\text{separation}}$ individually.
FIG. 1: Excluded region [shaded region above the solid curves] for photon-propagation length scales \( l_{\text{wormhole}} \) from cosmic-ray bounds \( l_{\text{separation}} \) for \( A = \alpha \). These length scales can perhaps be interpreted as corresponding to a static spacetime model which consists of identical, randomly-distributed wormholes with a length \( l_{\text{wormhole}} \) for the characteristic dimension of an individual wormhole and \( l_{\text{separation}} \) for the average separation between the different wormholes (see text).

IV. DISCUSSION

Using experimental bounds \( \text{[3]} \) on possible Lorentz-violating modifications of the photon dispersion law from the absence of Cherenkov-like processes for high-energy cosmic rays, we have obtained bounds on the length scales of a photon-propagation model \( \text{[7]} \) with time-independent random background field, which could result from a static, multiply connected spacetime foam \( \text{[1]} \). Even though the effective length scale \( l_\gamma \) which enters the photon dispersion law is constrained to be below the Planck length \( l_{\text{Planck}} \) for \( A = \alpha \), these bounds do not rigorously exclude a foamlike structure of spacetime with length scales \( l_{\text{wormhole}} \) and \( l_{\text{separation}} \) at or even above the Planck length (see Fig. 1).

On the other hand, it would perhaps not be unreasonable to expect \( \text{[6]} \) some remnant “quantum-gravity” effect with both length scales \( l_{\text{wormhole}} \) and \( l_{\text{separation}} \) of the order of the Planck length \( \text{[12]} \), even for a time-independent model with corresponding preferred frame of reference. But the static wormhole gas with \( l_{\text{wormhole}} \sim l_{\text{separation}} \sim l_{\text{Planck}} \approx 10^{-33} \text{ cm} \) and \( A \sim \alpha \) is ruled out by the bounds \( \text{[11]ab} \) in terms of \( \text{[13]ab} \); cf. Fig. 1. \( \text{[The crucial assumption here is that the (static) spacetime foam gives rise to an effective theory \( \text{[7]} \) with } g_1(\vec{x}) \text{ amplitude } A \text{ of order } \alpha. \text{ If, for some reason, } A \text{ would be very much smaller than } \alpha, \text{ the bounds \( \text{[11]ab} \) become essentially inoperative. As mentioned in the previous section, the preliminary calculations of Ref. \( \text{[1]} \) do suggest } A \sim \alpha, \text{ but this remains to be confirmed.} \)

The tentative conclusion is, therefore, that a preferred-frame graininess of space with a single length scale \( l_{\text{Planck}} \) may be hard to reconcile with the current experimental bounds from
cosmic-ray physics. Without fine-tuning, such a graininess of space can also be expected to show up in “low-energy” physics (i.e., $\sqrt{s} \ll E_{\text{Planck}} \equiv \sqrt{\hbar c^5/G} \approx 1.2 \times 10^{19}$ GeV) with powers of the coupling constants as the only suppression factor, an example being the linear term of Eq. (10) with $A^2 \gamma_1 \sim \alpha^2$. One possible solution would have gravity as an emergent phenomenon and the Lorentz-violation scale moved to trans-Planckian energies [8]. But, this is only one out of many suggestions and the puzzle of the apparent smoothness of space remains unsolved.

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