A Study of Dirac Fermionic Dark Matters

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Abstract

We study pure weak eigenstate Dirac fermionic dark matters (DM). We consider WIMP with renormalizable interaction. According to results of direct searches and the nature of DM (electrical neutral and being a pure weak eigenstate), the quantum number of DM is determined to be $I_3 = Y = 0$. There are only two possible cases: either DM has non-vanishing weak isospin ($I \neq 0$) or it is an isosinglet ($I = 0$). In the first case, the Sommerfeld enhancement is sizable for large $I$, producing large $\chi^0\chi^0 \rightarrow VV$ rates. In particular, we obtain large $\chi\bar{\chi} \rightarrow W^+W^-$ cross section, which is comparable to the latest bounds from indirect searches and $m_\chi$ is constrained to be larger than few hundred GeV to few TeV. It is possible to give correct relic density with $m_\chi$ higher than these lower bounds. In the second case, to couple DM to standard model (SM) particles, a SM-singlet vector mediator $X$ is required from renormalizability and the SM gauge quantum numbers. To satisfy the latest bounds of direct searches and to reproduce the DM relic density at the same time, resonant enhancement in DM annihilation diagram is needed. Thus, the masses of DM and the mediator are related.
I. INTRODUCTION

It is known that the discrepancy in speed of galaxy in our universe between observation and prediction from Newtonian gravitation theory indicate that there must be something “dark” there. These so called dark matter (DM), according to the observation of Wilkinson Microwave Anisotropy Probe (WMAP) and Planck, supply about 23% of composition to our universe [1, 2]. Dark matter cannot be observed from measuring their luminosity. Then, would it be possible that they are something like black hole, neutral star, brown dwarf, etc., which can only emit little or even no electro-magnetic radiation. Big-Band nucleosynthesis (BBN) provides powerful constraints on this account. From predictions of the abundances of the light elements, D, 3He, 4He, etc., one can evaluate the value of relic blackbody photon density as \( \eta \equiv n_b/n_\gamma \approx (5.1 - 6.5) \times 10^{-10} \) [3]. The measurements can be converted to the baryonic fraction of critical density, \( \Omega_b = \rho_b/\rho_{\text{crit}} \approx (0.019 - 0.04)h^{-2} \), where \( h = 0.72 \pm 0.08 \) is the present Hubble parameter. The resulting baryonic fraction \( \Omega_b \) is much smaller than the latest result on cold DM faction, \( \Omega_{DM}h^2 = 0.1187 \pm 0.0017 \) [2]. It tells us that, in the standard model (SM) of particle physics, there is no candidate for DM. Therefore, one has to extend the SM to account for the DM.

To construct a DM model, there are some basic requirements on DM. DM must be stable, charge neutral and have non-negligible mass. “Stable” means that it should live long enough that we can still observe their relic. “Neutral” is to avoid DM to shine and “non-negligible mass” means that the DM can gather gravitationally on small scales and so seed galaxy formation. There are many DM candidates such as weakly interactive massive particles (WIMP), axions, Kaluza-Klein mode in extra dimensions, etc. For a recent review of dark matter, see [4].

In this study, we will only consider the WIMP scenario. DM only interact through the gravity and weakly interacting force with interaction cross-sections basically not higher than the weak scale. We investigate a renormalizable DM model by introducing a pure weak eigenstate Dirac fermion as a DM candidate. We do not consider scalar or Majorana DM, which have been discussed intensively in the literature (see, for example, [5–11]). There are some Dirac fermionic DM models being considered in past years [12–17]. For instance, fermionic DM contributing to indirect precesses [12, 13], fermionic DMs with a charged scalar particle as a mediator to couple to SM particles through renormalizable terms are discussed in [14], while some use vector bosons, such as \( Z' \), to mediate interactions with SM particles [15, 17].

The lay out of this work is as following. In the next section, we introduce a weak eigenstate Dirac fermionic DM model with renormalizable interaction. We try to develop the model logically with a bottom-up approach. We then constrain the model using relic density, direct and indirect detection experiments. Numerical results are presented in Sec. III, which follows by discussion and conclusion in Sec. IV. Some formulas are collected in the Appendix.
II. FRAMEWORK

In WIMP scenario one can write down a simple DM model by adding on SM a single Dirac fermionic multiplet $\chi$ with the Lagrangian such as:  

$$L = L_{SM} + \bar{\chi}(i\gamma_\mu D^\mu - m_\chi)\chi,$$

where the covariant derivative $D_\mu$ contains the known electroweak gauge couplings to the vector bosons of the SM such that

$$D_\mu = \partial_\mu + i\frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) + i\frac{1}{\sqrt{g^2 + g'^2}}Z_\mu(g^2 T^3 - g'^2 Y) + i\frac{gg'}{\sqrt{g^2 + g'^2}}A_\mu Q,$$

Here, in the second line we have used the condition “electric charge neutrality”, $Q = T^3 + Y = 0$, and the definition of weak mixing angle, $\cos \theta_W = g/\sqrt{g^2 + g'^2}$. Note that in this work we only consider renomalizable interactions. Therefore, the DM cannot couple to Higgs. Furthermore, we may assign some $Z_2$ symmetry to maintain the stability of the DM.

In the Lagrangian, the $Z$ boson interaction term will produce a tree-level spin independent elastic cross sections with a nucleus $N$:  

$$\sigma^{SI}_{A}(\chi N \rightarrow \chi N) = \frac{L_N^2}{4\pi} \left( \frac{g}{\cos \theta_W M_Z} \right)^4 I_3^2 \left[ -\frac{1}{4} (A - Z) + \frac{1}{4} - \sin \theta_W^2 Z \right]^2,$$

where $Z$ and $A$ are the number of protons and of nucleons in the target nucleus, $I_3$ is the weak isospin quantum number and $\mu_N$ is the reduce mass of DM and nucleus. The above formula gives a normalized cross section (see Appendix A)

$$\sigma_N^Z \simeq c I_3^2 \times 10^{-40} \text{cm}^2,$$

for $m_\chi$ ranges from few GeV to few TeV. Therefore, the magnitude of the cross section exceeds most of the experimental upper bounds which obtained from direct detection searches for $m_\chi \gtrsim 10$ GeV [20].

The situation forces us to consider two cases of heavy DM with different quantum numbers: (i) $I \neq 0, I_3 = Y = 0$, and (ii) $I = Y = 0$.

Before we proceed to these two cases, it will be useful to recall some basics formulas. To obtain the thermal relic density for DM, we must solve the Boltzmann equation, which control the evolution of the DM abundance,

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_{\text{ann}}v \rangle [n_\chi n_{\bar{\chi}} - n_{\chi}^{\text{eq}}n_{\bar{\chi}}^{\text{eq}}],$$

where $H \equiv \dot{a}/a = \sqrt{4\pi^2 g_*(T)T^4/(45M_{\text{Pl}}^2)}$ is the Hubble parameter, $M_{\text{Pl}}$ is the Planck mass, $g_*$ is the total effective numbers of relativistic degrees of freedom [22, 23] and $n_\chi(n_{\bar{\chi}})$ is the number density of DM (anti-DM).

1 The mass of the fermionic DM should be larger than GeV, which is known as the Lee-Weinberg limit [19].

2 In fact, the case of DM with non-vanishing $T_3$ is still allowable for light WIMP candidates by only consider the constraint from direct detection searches. There are also some efforts are devoted to searching for DM with mass of order $\lesssim 10$ GeV [21]. But, here we do not consider light DM case.
FIG. 1: The Feynman diagrams of DM annihilation for $W^+W^-$ channel.

Following the standard procedure [22] to solve Eq. (5) approximately, we obtain the relations:

$$\Omega_{\text{DM}} h^2 \approx 1.04 \times 10^9 \frac{\text{GeV}^{-1}}{M_{\text{Pl}} \sqrt{g_* (T_f) J(x_f)}}$$

(6)

$$x_f \approx \ln \left[ \frac{2 \times 0.038 m_x M_{\text{Pl}} \langle \sigma_{\text{ann}} v \rangle}{\sqrt{g_* (T_f) x_f^{1/2}} } \right]$$

(7)

where we have

$$J (x_f) \equiv \int_{x_f}^{\infty} \frac{\langle \sigma_{\text{ann}} v \rangle}{x^2} dx$$

(8)

with $x_f$ defined as $m_x/T_f$ and $T_f$ being the freeze-out temperature, and the thermal averaged annihilation cross section $\langle \sigma_{\text{ann}} v \rangle$ with $v$ the “relative velocity” is defined as

$$\langle \sigma_{\text{ann}} v \rangle \equiv \frac{3 \sqrt{6}}{\sqrt{\pi} v_0^3} \int_0^{\infty} dv v^2 (\sigma_{\text{ann}} v) e^{-3v^2/2v_0^2}$$

$$= \frac{x_f^{3/2}}{2 \sqrt{\pi}} \int_0^{\infty} dv v^2 (\sigma_{\text{ann}} v) e^{-vx^2/4},$$

(9)

where we define $v_0 \equiv (v^2)^{1/2}$ and $v_0 = \sqrt{6/x_f}$ has been used in the last expression. It is straightforward to obtain

$$J (x_f) = \int_{x_f}^{\infty} \frac{\langle \sigma_{\text{ann}} v \rangle}{x^2} dx = \int_0^{\infty} dv (\sigma v) v \left[ 1 - \text{erf} \left( v \sqrt{x_f}/2 \right) \right].$$

(10)

We can now turn to the formalisms for the two above mentioned cases.

I. $I \neq 0, I_3 = Y = 0$ case

In this case, the DM possesses non-vanishing weak isospin $I$ but with zero hypercharge. The constraint condition, $I_3 = 0$, indeed avoids the troublesome $Z$ diagram. However, the contribution from the $W$ boson interaction needs to be investigated as well. Note that this case was also studied in [18, 24]. For completeness, we shall include them in this analysis. In fact, this work differs from the previous studies in several aspects. We focus on the Dirac Fermionic DM case. We are interested in finding the direct consequences of Eq. (1) instead of completing the model by adding other ingredients. Therefore, all isospin assignments are kept. As we will discuss later, the
Sommerfeld effects applicable to any isospin assignment will also be given. Furthermore, we are in a position that new data, such as galactic annihilation rate [25], is available and can be compared to.

The DM pair can annihilate into a $W$ boson pair (see Fig. 1) and then can contribute to the relic density of DM and indirect processes from milky way satellites. The $\chi^0\chi^0 \rightarrow W^+W^-$ annihilation cross section contributed from Fig. 1 for case I is calculated to be

$$\sigma_{\text{ann}}v = \left[I(I+1)\right]^2 \frac{g^4}{32\pi s^{3/2}} \frac{(2m_W^2 - s)}{(s - 2m_W^2)} \left\{ \frac{(m_W^2)}{m_W^2 (s - 4m_W^2) + m_W^4} \right\} \left\{ \frac{(4m_W^2 (s - 2m_W^2) - 8m_W^4 + 4m_W^4 + s^2)}{\sqrt{(s - 4m_W^2) (s - 4m_W^2)}} \right\} \log \left[ \frac{\sqrt{s} - 4m_W^2}{\sqrt{s - 4m_W^2} (s - 4m_W^2) + 2m_W^2 - s} \right] \}.$$  

(11)

After substituting $s = 4m_W^2 + m_X^2 v^2$ into the above equation and expanding around $v^2$, one obtains:

$$\langle \sigma_{\text{ann}}v \rangle = \langle a^{+-} + b^{-+} v^2 + O(v^4) \rangle,$$

(12)

where we have

$$a^{+-} = \left[I(I+1)\right]^2 \frac{g^4 (m_X^2 - m_W^2)^{3/2}}{8\pi m_X (2m_X^2 - m_W^2)^2},$$

$$b^{-+} = \left[I(I+1)\right]^2 \frac{g^4 (m_X^2 - m_W^2)^{1/2}}{192\pi m_X (2m_X^2 - m_W^2)^4} \left( 24m_X^6 + 28m_X^4 m_W^2 - 36m_X^2 m_W^4 + 17m_W^6 \right).$$

(13)

with $g = e/\sin \theta_W$. We find that neglecting $v^4$ and higher order terms is a good approximation. In fact, substituting $v^2 = \langle v^2 \rangle$ into $\sigma v$ almost gives identical results to the above approximated results. For thermal relic abundance, we have $\langle v^2 \rangle = 6x_f^{-1}$ from Eq. (9), and, consequently,

$$\langle \sigma_{\text{ann}}v \rangle \simeq a^{+-} + 6 \frac{b^{-+}}{x_f}, \quad J(x_f) \simeq \frac{a^{+-} + 3b^{-+}/x_f}{x_f}.$$ 

(14)

Note that for $\langle \sigma_{\text{ann}}v \rangle$ of indirect processes from milky way satellites, we have the thermal average quantity $\langle v^2_{1,2} \rangle = v_0^2/2$, where $v_0$ is chosen to be the canonical value $270\sqrt{2}$ km/s [26].

It is known that we need to take into account Sommerfeld enhancement effect, when the velocity is very small [27]. In the elastic scattering case, the cross-section receives Sommerfeld enhancement as

$$\sigma v = \langle \sigma v \rangle_0 S,$$

(15)

where $\langle \sigma v \rangle_0$ corresponds to the perturbative result and $S$ is the Sommerfeld enhancement factor. Equivalently, the amplitude receives a $S^{1/2}$ factor. For a force carrier with mass $m_\phi$ and couplings $\alpha$, the Sommerfeld factor is given by [29]

$$S(\alpha) = \frac{\pi}{e^\nu} \frac{\sinh \left( \frac{2\pi e^\nu}{\pi^2 e^\phi/6} \right)}{\cosh \left( \frac{2\pi e^\nu}{\pi^2 e^\phi/6} \right) - \cos \left( 2\pi \frac{1}{\pi^2 e^\phi/6} - \frac{e^{\nu/2}}{(\pi^2 e^\phi/6)^{3/2}} \right)},$$

(16)

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3 Some authors also called this as Sakharov effect [28].
with
\[ \epsilon_v \equiv \frac{v}{\alpha}, \quad \epsilon_\phi \equiv \frac{m_\phi}{\alpha m_\chi}, \]

(17)

Note that we have \( S > 1 \) for \( \alpha > 0 \) and vise versa.

The Sommerfeld enhancement in the present case is rather involved, since the \( \chi^0 \chi^0 \) state can rescatter into other states, such as \( \chi^+ \chi^- \) and so on, through \( t \)-channel diagrams by exchanging \( W \) and \( Z \) with the rescattered state annihilated into \( W^+ W^- \). To simplify the calculation we follow \cite{24,30} to consider the SU(2) symmetric limit. For a generic isospin \( I \), scatterings \( \chi^i \chi^j \to \chi^0 \chi^0 \) (with \( i, j = -I, -I + 1, \ldots, I - 1, I \) produce a potential \( V_{ij} = -|V_W| \sum_{c=1,2,3} T^c_{ij} T^c_{ji} \) with \( |V_W| = \alpha_We^{-m_W xe r/r} \). \[ \]

To proceed we use a procedure that is similar to those used in the study of final state interaction \cite{31}. We note that the potential can be diagonalized into several irreducible representations: \[ \]

\[ V_{ij} = \sum_{L=1}^{2I} (U^T)_{iL} \{ -[I(I+1) - L(L+1)/2]|V_W| \} U_{Lj}, \]

(18)

with
\[ U_{Lj} = (-1)^j \langle Ij I(-j)|L0 \rangle, \]

(19)

where \( \langle Ij I(-j)|L0 \rangle \) is the Clebsch-Gordan coefficient (in the \( \langle j_1 j_2 m_2 |JM \rangle \) notation). The irreducible parts of \( V \) do not mixed in further rescattering as it is easy to see that \( (V^n)_{ij} = \sum_L U^T_{iL} \{ -[I(I+1) - L(L+1)/2]|V_W| \} U_{Lj} \). The Sommerfeld enhancement factor of the irreducible parts can be obtained as the elastic case and, consequently, we have
\[ S_{ij} = \sum_{L=1}^{2I} U^T_{iL} S([I(I+1) - L(L+1)/2] \alpha W) U_{Lj}, \]

(20)

where \( S(\alpha) \) is given by Eq. \( \cite{16} \) but with \( m_\phi = m_{W,Z} \). The \( \chi^0 \chi^0 \to W^+ W^- \) amplitude with Sommerfeld enhancement, \( A_S \), is now given by
\[ A_S(\chi^0 \chi^0 \to W^+ W^-) = \sum_i A(\chi^i \chi^i \to W^+ W^-) S^{1/2}_{i0}, \]

(21)

where \( i \) is summed over all \( \chi^i \chi^i \) states. Therefore, the Sommerfeld enhanced \( s \)-wave part of \( \sigma v \) is given by
\[ a^{++}_S = \sum_{i,j} S^{1/2}_{bi} a_{ij}^{++} S^{1/2}_{j0}, \]

(22)

where \( i \) and \( j \) are summed over \( \chi^i \chi^i \) and \( \chi^j \chi^j \) states, respectively, and \( a_{ij}^{++} \) corresponds to the contribution from the \( A^*(\chi^i \chi^i \to W^+ W^-) A(\chi^j \chi^j \to W^+ W^-) \) part.

Note that we differ from \cite{24,30} as we do not consider \( \chi^0 \) to be identical to \( \chi^{-i} \). Therefore we do not have the factor of \( \sqrt{2} \) on the \( \chi^0 \chi^0 \) state (for \( i \) or \( j = 0 \) from the identical particle effect and we have a \( V \) matrix with larger dimension.

The expression is obtained with the help of \( \sum_c T^c_{ij} T^c_{ji} = - \sum_c T^c_{ij} T^c_{-i-j} (-)^{i-j} \) and the standard method of addition of angular momentum.
It is straightforward to obtain
\[ a_{ij}^{\pm} = \frac{g^4(m_\chi^2 - m_W^2)^{3/2}}{16\pi m_\chi(4m_\chi^2 - m_W^2)(2m_\chi^2 - m_W^2)} \{ 2[I(I + 1) - i^2][I(I + 1) - j^2](4m_\chi^2 - m_W^2)^2 + ij(4m_\chi^2 + 20m_\chi^2 m_W^2 + 3m_W^4) \} \] (23)
and, consequently,
\[ a_{-}^{\pm} = a^{\pm} - \frac{1}{9} \left[ 2S^{1/2}(I(I + 1)\alpha_W) + S^{1/2}([-3 + I(I + 1)]\alpha_W) \right]^2. \] (24)
Note that a similar expression holds for the Sommerfeld enhanced b term \((b_S)\). Finally, we obtain
\[ \langle \sigma^+ v \rangle = \left\langle (\sigma^+ v)_{ij}^{\pm} \right\rangle = \frac{1}{9} \left[ 2S^{1/2}(I(I + 1)\alpha_W) + S^{1/2}([-3 + I(I + 1)]\alpha_W) \right]^2 \]. (25)
In the \( S \to 1 \) limit \( \langle \sigma^+ v \rangle \) reduces to the one given in Eq. (12). Furthermore, if the eigenvalues of \( V_{ij} \) were degenerate, we return to the elastic result as in Eq. (15).

Note that through rescattering we can also have \( \chi^0_\chi^0 \to Z^0 Z^0, Z^0 \gamma, \gamma \gamma \) annihilations, with
\[ A_S(\chi^0_\chi^0 \to Z^0 Z^0) = \sum_i A(\chi^i_\chi^i \to Z^0 Z^0)S_{i0}^{1/2} \]
\[ A_S(\chi^0_\chi^0 \to Z^0 \gamma) = \sum_i A(\chi^i_\chi^i \to Z^0 \gamma)S_{i0}^{1/2}, \]
\[ A_S(\chi^0_\chi^0 \to \gamma \gamma) = \sum_i A(\chi^i_\chi^i \to \gamma \gamma)S_{i0}^{1/2}, \] (26)
and, consequently,
\[ a_{ij}^{00,0\gamma,\gamma} = \sum_{i,j} S_{i0}^{1/2} a_{ij}^{00,0\gamma,\gamma} S_{j0}^{1/2}, \] (27)
with
\[ a_{ij}^{00} = \frac{g^4 \cos^4 \theta_W (m_\chi^2 - m_Z^2)^{3/2} i^2 j^2}{4\pi m_\chi(2m_\chi^2 - m_Z^2)^2}, \quad a_{ij}^{0\gamma} = \frac{e^2 g^2 \cos^2 \theta_W (4m_\chi^2 - m_Z^2) i^2 j^2}{32\pi m_\chi^4}, \quad a_{ij}^{0\gamma} = \frac{e^4 i^2 j^2}{16\pi m_\chi^2}, \] (28)
and similar expressions for \( b \) terms. There processes also contribute to the relic density and are the inevitable consequences and signatures of inelastic Sommerfeld effects.

We obtain the annihilation cross sections for \( \chi^0_\chi^0 \to Z^0 Z^0, Z^0 \gamma, \gamma \gamma \) as
\[ \langle \sigma^\alpha v \rangle = \left\langle (a^\alpha + b^\alpha v^2) \frac{1}{9} \left[ S^{1/2}(I(I + 1)\alpha_W) - S^{1/2}([-3 + I(I + 1)]\alpha_W) \right] \right\rangle, \] (29)
with \( \alpha = 00, 0\gamma, \gamma \gamma \) and
\[ (a^{00}, b^{00}) = 2(a^{++}, b^{++})|_{g \to g \cos \theta_W, m_W \to m_Z}, \]
\[ (a^{0\gamma}, b^{0\gamma}) = 2(a^{++}, b^{++})|_{g \to e, m_W \to 0}, \]
\[ a^{0\gamma} = [I(I + 1)]^2 \frac{e^2 g^2 \cos^2 \theta_W (4m_\chi^2 - m_Z^2)}{32\pi m_\chi^4}, \]
\[ b^{0\gamma} = [I(I + 1)]^2 \frac{e^2 g^2 \cos^2 \theta \{ 12m_\chi^4 + 13m_\chi^2 m_Z^2 - m_Z^4 \}}{96\pi m_\chi^4 (4m_\chi^2 - m_Z^2)}. \] (30)
It is clear that these \( \langle \sigma^\alpha v \rangle \)s go to zero in the \( S \to 1 \) or in the degenerate limit. Note that we do not include loop contribution in these modes, since in most cases the contributions form inelastic rescattering parts are larger than the perturbative ones. We are ready to perform numerical study, where results will be given in the next section.
II. I = Y = 0 case

In this case, the DM candidate is a pure weak isospin singlet Dirac fermion. The case that DM is a scalar has been discussed by others [5, 7]. To reproduce the observed relic density, we need to couple $\chi$ to SM fermions $f$. We consider renormalizable interaction only. Therefore, an additional particle $X$ is necessary to mediate the $\chi \bar{\chi} \to f \bar{f}$ annihilation process. Since the DM is a weak isospin singlet, the mediator can only be a singlet and the $f \bar{f}$ bilinear term that couple to $X$ should be a singlet as well. It is easy to see that the $f \bar{f}$ bilinear term can only take the forms of $\bar{f}_L \gamma_\mu f_L$ and $\bar{f}_R \gamma_\mu f_R$, and hence the mediator particle $X$ should be a vector boson, if only renormalizable interaction is allowed.  

The Lagrangian involving $\chi$, $f$ and $X$ is given by:

$$\mathcal{L} = \mathcal{L}_{SM} + \bar{\chi} \left(i\partial^\mu - m_\chi\right)\chi + \sum_f \left(\bar{f}_L i\gamma^\mu f_L + \bar{f}_R i\gamma^\mu f_R - \lambda_f \bar{f}_L H f_R - \lambda_{\bar{f}} \bar{f}_R H^\dagger f_L\right)$$

$$- \frac{1}{4} \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu} + \frac{1}{2} M_X^2 X^\mu X^\mu, \quad (31)$$

with

$$D_\mu^{(L,R)} f = \partial_\mu + ig_f^{(L,R)} X^\mu f. \quad (32)$$

and $\mathcal{F}_{\mu\nu} = \partial_{\mu} X_{\nu} - \partial_{\nu} X_{\mu}$ and the SM fermions $f$s pick up masses from Higgs mechanism (using the Higgs doublet $H$) as usual. Here $g_\chi$, $g_f^{(L,R)}$ are corresponding coupling constants and $f^{(L,R)}$ is left (right) fermion. For simplicity, we only consider a vector-type interaction, $g_f^V = g_f^R$. The interaction term of the Lagrangian can be recast as:

$$\mathcal{L}_{int} = -g_\chi \bar{\chi} \gamma^\mu \chi X^\mu - \sum_f g_f^V \bar{f} \gamma^\mu f X^\mu \quad (33)$$

with $g_f^V = \frac{1}{2}(g_f^L + g_f^R)$. In order to determine the relic density of DM particles, we need to calculate the cross section of DM annihilation to fermion pairs. The result is given by

$$\sigma_{\text{ann}} = \frac{M_X}{\sqrt{s}} \times \frac{g_\chi^2}{(s - M_X^2)^2 + M_X^2 \Gamma_{\text{tot}}^2} \sum_f \frac{\Gamma(\tilde{X} \to \bar{f}f)}{\sqrt{s - 4m_\chi^2}} \left(s + 2m_\chi^2\right), \quad (34)$$

where $s = 2m_\chi^2(1 + 1/\sqrt{1 - v^2})$ is the square of the center-of-mass energy; $\Gamma(\tilde{X} \to \bar{f}f)$ is the decay width of “virtual” $X$ with mass $M_{\tilde{X}} = \sqrt{s}$ and $N_f$ is the number of color of the $f$-fermion.

We have to calculate $\langle \sigma_{\text{ann}} v \rangle$ numerically, since the standard method (Taylor expand) gives extremely poor results near the pole, even producing negative cross section [33]. We can determine the validity parameter space of $g_\chi$ and $g_f^V$ using the constraint from thermal relic abundance and direct detection for any given values of $m_\chi$, $M_X$. Note that is this case, the Sommerfeld

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6 We are different from [32] in this respect, where they have scalar mediator.

7 It is noted that the original integrated upper limit is infinity. Here we modified it to 1, because the relative velocity $v$ cannot be larger than light speed.
FIG. 2: (a) Predicted relic density fractions for $I = 1, 2, 3$ compared to the data, $\Omega_m h^2 = 0.1187 \pm 0.0017$ [2]. The solid (dashed) lines are with (without) the Sommerfeld factor. (b) to (d): The galactic DM annihilation cross sections for $W^+W^-, Z^0Z^0, Z^0\gamma, \gamma\gamma$ channels for different $I = 1, 2, 3$ cases. The solid (dashed) lines are the results with (without) the Sommerfeld factor. The $W^+W^-$ data is from[25] with both ends extrapolated.

enhancement in irrelevant. As we shall see, we need to make use of the resonant effect to give viable results on relic density without violating the direct search data. In that region ($m_\chi \sim m_X/2$), the Sommerfeld factor $S$ as given by Eq. (16) is very close to unity.

III. RESULTS AND DISCUSSIONS

I. $I \neq 0, I_3 = Y = 0$ case

We first give the results of case I. In Fig. 2(a) we show our results on relic abundance for $I = 1, 2, 3$ and compare to the experimental result $\Omega_m h^2 = 0.1187 \pm 0.0017$ [2]. We take $x_f \simeq 24$ to simplify the calculations. Solid (dashed) lines are results with (without) the Sommerfeld factor. We see that the observed relic density can be reproduced in all three cases with TeV DM masses.


TABLE I: $m_\chi$ lower limits ($m_\chi^{\text{LL}}$) obtained from Fermi-Lat constraints on $\chi\bar{\chi} \to W^+W^-$ rates, $m_\chi$ required to give correct thermal relic and the Galactic $\langle \sigma v \rangle$ at the corresponding dark matter masses are shown. Dark matter masses are shown in TeV, while $\langle \sigma v \rangle$ in cm$^3$/s. Values in parenthesis are obtained without using the Sommerfeld enhancement factors.

| Isospin $m_\chi^{\text{LL}}$ (Indirect) | $m_\chi$ (Relic) | $\langle \sigma v \rangle(W^+W^-)$ | $\langle \sigma v \rangle(Z^0Z^0)$ | $\langle \sigma v \rangle(Z^0\gamma)$ | $\langle \sigma v \rangle(\gamma\gamma)$ |
|---------------------------------|-----------------|-------------------------------|-------------------|-------------------|-------------------|
| $I = 1$ | 0.23(0.22) | 2.42 ± 0.02(1.98 ± 0.01) | 8.1 × 10$^{-25}$ | 2.2 × 10$^{-25}$ | 1.2 × 10$^{-25}$ | 1.8 × 10$^{-26}$ |
| $I = 2$ | 1.54(0.45) | 11.06 ± 0.07(6.13 ± 0.04) | 6.6 × 10$^{-24}$ | 4.5 × 10$^{-25}$ | 2.6 × 10$^{-25}$ | 3.7 × 10$^{-26}$ |
| $I = 3$ | 5.52(0.69) | 30.92 ± 0.22(12.26 ± 0.09) | 2.2 × 10$^{-24}$ | 1.1 × 10$^{-27}$ | 6.2 × 10$^{-28}$ | 8.8 × 10$^{-29}$ |

*Inferred by comparing to the extrapolated Femi-LAT data.

(see also the third column of Table I). Without the Sommerfeld factor, the masses scale as $I(I + 1)$. The Sommerfeld enhancement become more prominent in the large $I$ case, and, consequently, the mass grows faster than the simple scaling. From the figure one may easily infer that the DM masses to give correct DM relic density are larger than 50 TeV for $I > 4$ and, hence, for practical purpose we shall restrict $I$ up to 3.

In Fig. 2(b) to (d) we show the results of galactic $\langle \sigma v \rangle$ on WIMP annihilation for $\chi^0\bar{\chi}^0 \to W^+W^-, Z^0Z^0, Z^0\gamma, \gamma\gamma$ channels for WIMP candidates with different isospin ($I = 1, 2, 3$) and compare them to the milky way satellites data on the $W^+W^-$ rate $^{25}$. We see that, when the Sommerfeld factor are removed, the $W^+W^-$ data constraints the DM masses to be heavier than few hundred GeV. However, except for $I = 1$, all DM with sub-TeV mass are excluded when the Sommerfeld enhancements are included (see also the second column of Table I). The signatures of the enhancement are sizable $Z^0Z^0, Z^0\gamma, \gamma\gamma$ rates. It will be interesting to search for these processes.

In Table I results on $m_\chi$ lower limits ($m_\chi^{\text{LL}}$) obtained from Fermi-Lat constraints on $\chi\bar{\chi} \to W^+W^-$ rates, $m_\chi$ required to give correct thermal relic and the Galactic $\langle \sigma v \rangle$ at the corresponding dark matter masses are collected. Note that these $\langle \sigma v \rangle$ are different from and, in fact, much than their counter part in the $x_f = 24$ period as the Sommerfeld factors are more effective here. Note that our results on $I = 2$ are similar to those in $^{18}$ $^{24}$.

In this case we do not consider direct search as there is no data on the interesting mass regions to give the correct relic density in present and near future experiments.

II. $I = 0, I_3 = Y = 0$ case

We now turn to case II. We shall discuss the valid parameter space first. To simplify the numerical analysis, instead of solving Eq. (7) directly, we set the parameter value $x_f = 24$, which is checked to be a good approximation, and assume the coupling constant $g_f^Y$ to be proportional to $g_\chi$ with $n \equiv g_f^Y / g_\chi$. The proportionality of couplings may come from some underlying gauge symmetries, which we will not go further into.

For given values of mediator mass $M_X$ and coupling ratio $n$, we can solve $g_\chi$ numerically by substituting Eq. (34) into Eq. (6) and (8). The results are shown in Fig. 3. In Fig. 3(a)-(c), we show the allowable range for the parameter $g_\chi$ as a function of the DM mass with $\Omega_{\text{abm}}h^2 =
FIG. 3: (a)-(c) The $g_\chi$ function for $M_X = 1000, 800$ and $600$ GeV separately. In each figure, from top to bottom, we adopt $g_Y^V = 0.1g_\chi$, $g_\chi$ and $2g_\chi$ to show the allowed range with $\Omega_{\text{DM}}h^2 = 0.1187 \pm 0.0017$. The shadow region, from top to bottom, also corresponding to $g_\chi$ constrained by Xenon100 experiment [34] for different $n$ . (d) Combining (a)-(c) results using the coupling constant $G_\chi$. The black (short dashed) line is corresponding to contact interaction. The blue (solid) and green (dot-dashed) lines are for containing BW resonance effect. The light gray shadow region was obtained by extrapolating the Xenon100’s result.

$0.1187 \pm 0.0017$ for different mediator mass, namely $M_X = 1000, 800$ and $600$ GeV. In each figure, the curve from top to bottom, we adopt $n = 0.1, 1, 2$. The shaded region are the allowed region of $g_\chi$ (with increasing $n$ from top to bottom), which constrained by the Xenon100 results [34] of the spin-independent DM-nucleon elastic scattering process. We note that the $g_\chi$ curve was bent down to Xenon100 allowable region around resonance point. It is this Breit-Wigner (BW) resonance effect that make the model to survive from the Xenon100 experimental bound.

To further explore the physical meaning, we define a new coupling constant $G_\chi$ such as

$$G_\chi \equiv \frac{g_\chi g_Y^V}{M_X^2}.$$  

In Fig. 3(d), we combing results in Fig. 3(a)-(d) using $G_\chi$. We also plot the $G_\chi$ of the contact interaction.
FIG. 4: (a) The magnitude of elastic scattering cross section of our model for $M_X = 600, 800, 1000$ GeV with $n = 1$ and direct detection results which obtained from different experimental group [20]. We see that the BW effect indeed affect the behavior of the curve in the resonant region. The result of contact interaction also be showed. It reveals that the contact interaction model can only survive at large DM mass region ($m_\chi \gtrsim 3$ TeV).(b) The DM annihilation cross section of indirect search for the $\mu^+\mu^-$ channel. The blue(dot-dashed), orange(dashed) and purple(solid) curve are corresponding to $g_V^f = 0.1g_\chi$, $g_\chi$ and $2g_\chi$ separately with $M_X = 1000$ GeV and are compared to the black solid line corresponding to the FermiLAT result [25].

For example, for $m_X = 1000$ GeV with $n = 2, 1$ and $0.1$, dark matters having $m_\chi = 500^{+19.78}_{-56.12}$, $500^{+19.78}_{-66.12}$ and $500^{+19.78}_{-81.79}$ GeV, respectively, can evade the direct search bound. The corresponding typically elastic cross section $\sigma_{ZN}$ for DM and nuclei is normalized to DM-
FIG. 5: The band corresponds to the expected parameter space to saturate $\Delta a_\mu$. The shaded region in the lower right corner is the allowed region from $eq$ contact interaction.

proton elastic cross section $\sigma_p$ such that (see Appendix [A])

$$\sigma_N^Z = \sigma_p = \frac{9\mu_p^2}{\pi} G_\chi^2$$

with reduced mass $\mu_p = m_\chi m_p / (m_\chi + m_p)$. The result is shown in Fig. 4(a). In the figure we demonstrate the elastic scattering cross section curves of our model for $M_X = 600, 800, 1000$ GeV with $n = 1$ and the contact interaction model. As mentioned with the resonance effect, the model can survive from the direct search bound.

In addition to the direct search result, we also calculate the DM annihilation cross section of indirect search for the $\mu^+\mu^-$ channel. The result is showed in Fig. 4(b). The blue(dot-dashed), orange(dashed) and purple(solid) curves are corresponding to $M_X = 1000$ GeV with $g^V_f = 0.1g_\chi, g_\chi$ and $2g_\chi$, respectively.

IV. DISCUSSION AND CONCLUSIONS

The muon $g - 2$ puzzle could be a hint for some unknown contributions from physics beyond the SM. It will be interesting to explore the connection with the DM sector. In Fig. 5 we show the expected parameter space to saturate $\Delta a_\mu$ [3]. Since the expected parameter space is excluded by $(g^V_f / M_X)^2 = \eta^0_{\chi^-} \lesssim 5.01 \times 10^{-8}$ [35], we conclude that our model is not sufficient to explain the deviation.

In conclusion, we study pure weak eigenstate Dirac fermionic dark matters. We consider WIMP with renormalizable interaction. According to results of direct searches and the nature of DM, the quantum number of DM is determined to be $I_3 = Y = 0$. There are only two possible cases: either DM has non-vanishing weak isospin ($I \neq 0$) or it is an isosinglet ($I = 0$). In the first case, we find that the Sommerfeld enhancement is sizable for large $I$ DM, producing large $\chi^0 \bar{\chi}^0 \rightarrow W^+ W^-$, $Z^0 Z^0$, $Z^0 \gamma$, $\gamma \gamma$ rates. In particular, we obtain large $\chi \bar{\chi} \rightarrow W^+ W^-$ cross section, which is comparable to the latest bounds from indirect searches and $m_\chi$ is constrained to be larger than few hundred GeV to few TeV. It is possible to give correct relic density with $m_\chi$ higher than these lower bounds. In the second case, to couple DM to standard model particles, a SM-singlet
vector mediator $X$ is required from renormalizability and SM gauge quantum numbers. To satisfy
the latest bounds of direct searches and to reproduce the DM relic density at the same time,
resonant enhancement in DM annihilation diagram is needed. Thus, the masses of DM and the
mediator are related. Our model is not sufficient to explain the $\Delta a_\mu$ deviation.

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Appendix A: DM-nucleon elastic cross section

In this appendix, to avoid confusion, we will give some definitions to different DM scattering
cross section. The first is DM-nucleus zero momentum transfer spin-independent(SI) cross section,
\[
\sigma_{\text{SI}}^A = \frac{m_\chi^2 m_A^2}{\pi (m_\chi + m_A)^2} \times \frac{g^2_\chi}{M_X^4} [f_p Z + f_n (A-Z)]^2
\]
where $m_A$ is the mass of target nucleus with $Z$ protons and $A-Z$ neutrons and $f_{p,n}$ are the
couplings to protons and neutrons with $f_{p,n} = 2g_{u,d}^V + g_{d,u}^V = 3g_f^V$ in this model. By above equation,
for $Z = A = 1$, we then have DM-proton cross section
\[
\sigma_p = 9 \mu_p^2 (g_\chi g_f^V)^2 / (\pi M_X^4) = 9 \mu_p^2 G_\chi^2 / \pi.
\]
The third is total cross section $\sigma_t = \sum_i \eta_i \sigma_{A_i}$. Here the summation is over isotopes $A_i$ with
fractional number abundance $\eta_i$ since it is usually to include the possibility of multiple isotopes for
each detector in laboratory. To conciliate results from different detectors, one usually normalize
the total cross section to one nucleon cross section $\sigma_N^Z$ such as
\[
\sigma_N^Z = \sigma_t / N = \frac{\sigma_p}{N \mu_p^2} \sum_i \eta_i (\mu_{A_i}/\mu_p)^2 \left[ Z + (A_i - Z) f_n / f_p \right]^2.
\]
with $N$ is normalization constant. For $f_p/f_n = 1$ (isospin symmetry), it is easy to obtain $\sigma_N^Z = \sigma_p \sum_i \eta_i (\mu_{A_i}/\mu_p)^2 / N$. Because for one isotope dominated detector, say, with proton as target,
the normalized cross section should be equal to DM-proton cross section. We then have $N = \sum_i \eta_i (\mu_{A_i}/\mu_p)^2$ and hence $\sigma_N^Z = \sigma_p$ for different detector.

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