BPS Solutions of Noncommutative Gauge Theories in Four and Eight Dimensions

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Abstract

We study the 1/4 BPS equations in the eight dimensional noncommutative Yang-Mills theory found by Bak, Lee and Park. We explicitly construct some solutions of the 1/4 BPS equations using the noncommutative version of the ADHM-like construction in eight dimensions. From the calculation of topological charges, we show that our solutions can be interpreted as the bound states of the $D0$-$D4$-$D8$ with a $B$-field. We also discuss the structure of the moduli space of the 1/4 BPS solutions and determine the metric of the moduli space of the $U(2)$ one-instanton in four and eight dimensions.
1 Introduction

Noncommutative geometry has played an important part in the study of string/M-theory [1]. In particular, D-branes with a constant NS B-field are of interest in the context of understanding the non-perturbative aspects of string theory. The effective world-volume field theory on D-branes with a B-field turns out to be the noncommutative Yang-Mills theory [2], which has an interesting feature that the singularity of the instanton moduli space is naturally resolved [3].

Four dimensional $U(N)$ $k$-instanton is realized as $k$ D0-branes within $N$ D4-branes in type IIA string theory. When we turn on a self-dual constant B-field which preserves 1/4 of supersymmetries, the instanton moduli space is resolved and the D0-branes cannot escape from the D4-branes. From the viewpoint of the D0-brane theory, the Higgs branch of the moduli space coincides with the moduli space of instantons and the self-dual B-field corresponds to the Fayet-Iliopoulos (FI) parameters. If the FI parameters are non-zero, the D0-D4 system can not enter the Coulomb branch through the small instanton singularity.

It is also of interest to generalize the above system to higher dimensions in the context of both D-brane dynamics and the world-volume theories. The bound states of the D0-D6 and the D0-D8 with a B-field were investigated by several authors [9–19]. These systems are equivalent by T-duality to the rotated branes at angles. It has been shown that the D0-D6 system can form bound states only if the value of the B-field are taken appropriately and that in the D0-D8 system there are three cases preserving respectively 1/16, 1/8 and 3/16 of supersymmetries.

These studies reduce to finding the solutions of the higher dimensional analogue of “self-duality” equations found in [3, 8], which are the first order linear relations amongst components of the field strength. The above three cases of the D0-D8 systems correspond to the subgroup $Spin(7)$, $SU(4)$ and $Sp(2)$ of the eight dimensional rotation group $SO(8)$ respectively. Recently all possible “self-duality” equations in the higher dimensional Yang-Mills theories have been classified in [20] and it is shown that there are many kinds of the BPS equations which preserve 1/16, 2/16, . . . , 6/16 of supersymmetries. These include three new cases besides the above three ones. In this paper, we will focus on the case of the 1/4 supersymmetries, construct some explicit solutions of the corresponding BPS equations and give their D-brane interpretations.

This paper is organized as follows. In section 2, we review the BPS equations in eight dimensions which have been derived in [20]. In section 3, we construct some solutions
of the $1/4$ BPS equations in eight dimensions using the noncommutative version of the ADHM-like construction in eight dimensions. We show that our solutions can be interpreted as the bound states of the $D0$-$D4$-$D8$ with a $B$-field from the calculation of their topological charges. We also discuss the structure of the moduli space of the $1/4$ BPS solutions and determine the metric of the moduli space of the $U(2)$ one-instanton in four and eight dimensions. The final section is devoted to discussions.

## 2 BPS equations in eight dimensions

In this section, we briefly review the results of [20], in which all possible BPS equations in six and eight dimensional Yang-Mills theories have been classified. The authors of [20] have investigated the higher dimensional analogue of “self-duality” equations, which are the linear relations amongst components of the field strength with the constant 4-form tensor $T_{abcd}$,

$$F_{ab} + \frac{1}{2} T_{abcd} F_{ad} = 0.$$  \hfill (2.1)

This is a natural generalization of the four dimensional self-duality equation,

$$F_{ab} + \frac{1}{2} \epsilon_{abcd} F_{ad} = 0.$$  \hfill (2.2)

When this equation holds, the equations of motion $D_a F_{ab} = 0$ are automatically satisfied due to the Jacobi identity. We investigate only the eight dimensional case. The results of [20] are written as follows:

- The $1/16$ BPS equations

  $$F_{12} + F_{34} + F_{56} \pm F_{78} = 0,$$
  $$F_{13} + F_{42} + F_{57} \pm F_{86} = 0,$$
  $$F_{14} + F_{23} + F_{76} \pm F_{85} = 0,$$
  $$F_{15} + F_{62} + F_{73} \pm F_{48} = 0,$$
  $$F_{16} + F_{25} + F_{47} \pm F_{38} = 0,$$
  $$F_{17} + F_{35} + F_{64} \pm F_{82} = 0,$$
  $$\pm F_{18} + F_{27} + F_{63} + F_{54} = 0.$$  \hfill (2.3)

These can be expressed compactly using the structure constants $C_{ijk}$ of the octonion,

$$F_{i8} \pm \frac{1}{2} C_{ijk} F_{jk} = 0.$$  \hfill (2.4)
where \(i, j, k = 1, \ldots, 7\). Some solutions of these equations were constructed in [21, 22].

- The 2/16 BPS equations

\[
\begin{align*}
F_{12} + F_{34} + F_{56} &\pm F_{78} = 0, \\
F_{13} + F_{42} & = 0, \quad F_{57} \pm F_{86} = 0, \quad F_{15} + F_{62} = 0, \\
F_{14} + F_{23} & = 0, \quad F_{76} \pm F_{85} = 0, \quad F_{16} + F_{25} = 0, \\
F_{73} \pm F_{48} & = 0, \quad F_{17} \pm F_{82} = 0, \quad F_{35} + F_{64} = 0, \\
F_{47} \pm F_{38} & = 0, \quad F_{18} \pm F_{27} = 0, \quad F_{63} + F_{54} = 0.
\end{align*}
\]

(2.5)

- The 3/16 BPS equations

\[
\begin{align*}
F_{12} + F_{34} & = 0, \quad F_{13} + F_{42} = 0, \quad F_{14} + F_{23} = 0, \\
F_{56} \pm F_{78} & = 0, \quad F_{75} \pm F_{68} = 0, \quad F_{67} \pm F_{58} = 0, \\
F_{15} = F_{26} = F_{37} & = \pm F_{48}, \\
F_{16} = F_{52} = F_{47} & = \pm F_{83}, \\
F_{17} = F_{53} = F_{64} & = \pm F_{28}, \\
\pm F_{18} = F_{72} = F_{36} & = F_{54}.
\end{align*}
\]

(2.6)

Some solutions of these equations and their ADHM-like constructions are discussed in [6, 7, 15, 16, 18, 19].

- The 4/16 BPS equations

\[
\begin{align*}
F_{12} + F_{34} & = 0, \quad F_{13} + F_{42} = 0, \quad F_{14} + F_{23} = 0, \\
F_{56} \pm F_{78} & = 0, \quad F_{75} \pm F_{68} = 0, \quad F_{67} \pm F_{58} = 0, \\
F_{ab} & = 0 \quad \text{for} \quad a \in \{1, 2, 3, 4\} \quad b \in \{5, 6, 7, 8\}.
\end{align*}
\]

(2.7)

These are the subjects of the next section.

- The 5/16 BPS equations

\[
\begin{align*}
F_{12} = F_{43} = F_{65} & = \pm F_{78}, \\
F_{13} = F_{24} = F_{75} & = \pm F_{86}, \\
F_{14} = F_{32} = F_{76} & = \pm F_{58}, \\
F_{ab} & = 0 \quad \text{for} \quad a \in \{1, 2, 3, 4\} \quad b \in \{5, 6, 7, 8\}.
\end{align*}
\]

(2.8)
3 Solutions of the 1/4 BPS equations and the moduli space

In this section, we construct some solutions of the 1/4 BPS equations (2.7) as the noncommutative instantons on $\mathbb{R}^8$ and interpret them as $D$-brane bound states with a $B$-field. This noncommutativity is induced by a constant NS $B$-field on the $D8$-brane. We also discuss some aspects of the structure of the moduli space of the 1/4 BPS equations in four and eight dimensions, and determine explicitly the metric of the moduli space of the $U(2)$ one-instanton.

The 1/4 BPS equations (2.7) are a copy of the 1/4 BPS equations on the four dimensional hyper-planes spanned by $x^1, \cdots, x^4$ and $x^5, \cdots, x^8$ which we call $\mathbb{R}^4$ and $\tilde{\mathbb{R}}^4$ respectively. Therefore we can easily construct the solutions by placing the four dimensional instanton solutions on each $\mathbb{R}^4$ and $\tilde{\mathbb{R}}^4$. In the following, we consider the solutions of the gauge group $U(N)$ with the instanton number $k$ on the first four dimensional hyper-plane $\mathbb{R}^4$ and the instanton number $k'$ on the second hyper-plane $\tilde{\mathbb{R}}^4$, and interpret them as the $D0$-$D4$-$D8$ bound states with a $B$-field.

3.1 ADHM-like construction for the 1/4 BPS equations in eight dimensions

The ADHM construction is a powerful tool to construct the Yang-Mills instantons in four dimensions [23, 24]. Especially, it is well-known that the instanton moduli space and the ADHM moduli space completely coincide. The ADHM construction can be extended to the 3/16 BPS equations in eight dimensions, which was investigated by several authors [3, 13, 16, 18, 19]. It is also possible to extend the ADHM construction to the 1/4 BPS equations on $\mathbb{R}^8$, which is studied in the following.

In order to treat the eight dimensional space, it is useful to regard the coordinates of $\mathbb{R}^8$ as two quaternionic numbers,

$$x = \sum_{\mu=1}^{8} \hat{\sigma}_\mu x^\mu = \begin{pmatrix} z_2 & z_1 \\ -\bar{z}_1 & \bar{z}_2 \end{pmatrix}, \quad x' = \sum_{\mu=1}^{8} \hat{\sigma}_\mu' x'^\mu = \begin{pmatrix} z_4 & z_3 \\ -\bar{z}_3 & \bar{z}_4 \end{pmatrix},$$

(3.1)
where we defined the eight vector matrices,
\[ \tilde{\sigma}_\mu = \begin{pmatrix} i\tau_1, & i\tau_2, & i\tau_3, & 1_2, & 0, & 0, & 0 \end{pmatrix}, \] (3.2)
\[ \tilde{\sigma}'_\mu = \begin{pmatrix} 0, & 0, & 0, & i\tau_1, & i\tau_2, & i\tau_3, & 1_2 \end{pmatrix}, \] (3.3)

and the four complex coordinates,
\[ z_1 = x^2 + ix^1, \quad z_2 = x^4 + ix^3, \quad z_3 = x^6 + ix^5, \quad z_4 = x^8 + ix^7. \] (3.4)

Here \( \tau_1, \tau_2 \) and \( \tau_3 \) are usual Pauli matrices.

Using the \( 2k \times (N + 2k) \) matrices \( a, b \) and the \( 2k' \times (N + 2k') \) matrices \( a', b' \), we define the Dirac-like operator,
\[ D_z = \begin{pmatrix} a + bx & 0 \\ 0 & a' + b'x' \end{pmatrix} \equiv \begin{pmatrix} D_z & 0 \\ 0 & D'_z \end{pmatrix}, \] (3.5)

where we also defined \( D_z \) and \( D'_z \). We can construct the \( U(N) \) gauge field as
\[ A_\mu = \text{tr} \Psi^\dagger \partial_\mu \Psi = \psi^\dagger \partial_\mu \psi + \psi'^\dagger \partial_\mu \psi', \] (3.6)

where \( \Psi \) is the solution of the following Dirac-like equations,
\[ D_z \Psi = D_z \begin{pmatrix} \psi & 0 \\ 0 & \psi' \end{pmatrix} = 0, \] (3.7)

for the \( (N + 2k) \times N \) matrix \( \psi \) and the \( (N + 2k') \times N \) matrix \( \psi' \), which are normalized as
\[ \psi^\dagger \psi = 1_{N \times N}, \quad \psi'^\dagger \psi' = 1_{N \times N}. \] (3.8)

Then using the completeness equation,
\[ 1_{2N + 2(k + k')} = \Psi \Psi^\dagger + D_z^\dagger \frac{1}{D_z D_z^\dagger} D_z, \] (3.9)

we can obtain the “anti-self-dual” gauge field strength, which satisfies the 1/4 BPS equations (2.7) in eight dimensions,
\[ F_{\mu\nu} = \text{tr} \Psi^\dagger \left( \partial_\mu D_z^\dagger \frac{1}{D_z D_z^\dagger} \partial_\nu D_z \right) \Psi. \] (3.10)

This can be confirmed by writing the field strength more concretely,
\[ F_{\mu\nu} = \begin{cases} \psi^\dagger b^\dagger \tilde{\sigma}_{[\mu} \tilde{\sigma}_{\nu]} \left( D_z D_z^\dagger \right)^{-1} b \psi, & \text{for } \mu, \nu = 1, \ldots, 4, \\ \psi'^\dagger b'^\dagger \tilde{\sigma}'_{[\mu} \tilde{\sigma}'_{\nu]} \left( D'_z D'_z^\dagger \right)^{-1} b' \psi', & \text{for } \mu, \nu = 5, \ldots, 8, \\ 0, & \text{for the other components}. \end{cases} \] (3.11)
Here we must require that \( D_z D_z^† \) and \( D_z' D_z'^† \) commute with \( \tilde{\sigma}_\mu \) and \( \tilde{\sigma}_\mu' \) respectively. This is a necessary condition to obtain the “anti-self-dual” gauge field strength on \( \mathbb{R}^8 \). We call the equations corresponding to this condition the ADHM-like equations in eight dimensions for both the commutative and the noncommutative case.

3.2 Noncommutative version of the ADHM-like construction for the 1/4 BPS equations in eight dimensions

As in the four dimensional case, it is also easy to extend the ADHM-like construction in eight dimensions to noncommutative space because of its algebraic nature. Since we define the instantons as “anti-self-dual” configurations, the “self-dual” \( B \)-field is of interest from the viewpoint of the resolution of the instanton moduli space.

In this case the coordinates of \( \mathbb{R}^8 \) become noncommutative as

\[
[z_1, z_1] = [\bar{z}_2, z_2] = \frac{\zeta}{2}, \quad [\bar{z}_3, z_3] = [\bar{z}_4, z_4] = \frac{\zeta'}{2},
\]  

(3.12)

for positive constant parameters \( \zeta \) and \( \zeta' \). These commutation relations can be represented using the creation and annihilation operators which act on the Fock space of harmonic oscillators,

\[
\sqrt{\frac{2}{\zeta}} z_1 |n_1 : n_2 : n_3 : n_4\rangle = \sqrt{n_1 + 1} |n_1 + 1 : n_2 : n_3 : n_4\rangle,
\]

\[
\sqrt{\frac{2}{\zeta}} \bar{z}_1 |n_1 : n_2 : n_3 : n_4\rangle = \sqrt{n_1} |n_1 - 1 : n_2 : n_3 : n_4\rangle,
\]

\[
\sqrt{\frac{2}{\zeta}} z_2 |n_1 : n_2 : n_3 : n_4\rangle = \sqrt{n_2 + 1} |n_1 : n_2 + 1 : n_3 : n_4\rangle,
\]

\[
\sqrt{\frac{2}{\zeta}} \bar{z}_2 |n_1 : n_2 : n_3 : n_4\rangle = \sqrt{n_2} |n_1 : n_2 - 1 : n_3 : n_4\rangle,
\]

\[
\sqrt{\frac{2}{\zeta}} z_3 |n_1 : n_2 : n_3 : n_4\rangle = \sqrt{n_3 + 1} |n_1 : n_2 : n_3 + 1 : n_4\rangle,
\]

\[
\sqrt{\frac{2}{\zeta'}} z_3 |n_1 : n_2 : n_3 : n_4\rangle = \sqrt{n_3} |n_1 : n_2 : n_3 - 1 : n_4\rangle,
\]

\[
\sqrt{\frac{2}{\zeta}} z_4 |n_1 : n_2 : n_3 : n_4\rangle = \sqrt{n_4 + 1} |n_1 : n_2 : n_3 : n_4 + 1\rangle,
\]

\[
\sqrt{\frac{2}{\zeta'}} z_4 |n_1 : n_2 : n_3 : n_4\rangle = \sqrt{n_4} |n_1 : n_2 : n_3 : n_4 - 1\rangle,
\]

(3.13)
where the number operators are defined by
\[ n_1 = \frac{2}{\zeta} \bar{z}_1, \quad n_2 = \frac{2}{\zeta} \bar{z}_2, \quad n_3 = \frac{2}{\zeta'} \bar{z}_3, \quad n_4 = \frac{2}{\zeta'} \bar{z}_4. \]  
\( (3.14) \)

If we also require that \( D_z D_\dagger_z \) and \( D'_z D'_{\dagger z} \) commutes with \( \tilde{\sigma}_\mu \) and \( \tilde{\sigma}'_\mu \) respectively as in the commutative case, we can obtain the “anti-self-dual” gauge field strength on noncommutative \( R^8 \). As in the four dimensional case, there are equivalence relations between different sets of the matrices \( a, b, a', b' \) as
\[ a \sim M a N, \quad b \sim M b N, \quad a' \sim M' a' N', \quad b' \sim M' b' N', \]  
\( (3.15) \)

where \( M \in \text{GL}(2k, \mathbb{C}) \), \( N \in \text{U}(N + 2k) \), \( M' \in \text{GL}(2k', \mathbb{C}) \) and \( N' \in \text{U}(N + 2k') \). Using these relations, the ADHM-like equations for the 1/4 BPS equations on noncommutative \( R^8 \) can be obtained,
\[ \mu_R = \left[ B_1, B_1^{\dagger} \right] + \left[ B_2, B_2^{\dagger} \right] + II^{\dagger} - J^{\dagger} J = \zeta, \]  
\( (3.16) \)
\[ \mu_C = \left[ B_1, B_2 \right] + I J = 0, \]
\[ \mu'_R = \left[ B'_1, B'_1^{\dagger} \right] + \left[ B'_2, B'_2^{\dagger} \right] + I'I'^{\dagger} - J'^{\dagger} J' = \zeta', \]  
\( (3.17) \)
\[ \mu'_C = \left[ B'_1, B'_2 \right] + I' J' = 0, \]

where \( B_1, B_2 \) are \( k \times k \) matrices, \( I, J^{\dagger} \) are \( k \times N \) matrices, \( B'_1, B'_2 \) are \( k' \times k' \) matrices and \( I', J'^{\dagger} \) are \( k' \times N \) matrices. These are nothing but two sets of the four dimensional ADHM equations on \( R^4 \) and \( \tilde{R}^4 \).

### 3.3 \( U(1) \) instantons

In this subsection, we explicitly construct the noncommutative \( U(1) \) instanton solutions of (2.7) in eight dimensions. This can be easily achieved using the noncommutative \( U(1) \) instanton solutions in four dimensions. Compared with the commutative case, the \( U(1) \) instanton is already non-trivial in the noncommutative case.

#### 3.3.1 Four dimensional case

Before discussing the eight dimensional case, we briefly review the \( U(1) \) one-instanton solution in four dimensions. Some relevant references are \([3, 23]\).

In this case, the ADHM-like equations are the same as (3.13) and can be solved by
\[ B_1 = B_2 = J = 0, \quad I = \sqrt{\zeta}. \]  
\( (3.18) \)
Solving the Dirac-like equations and using (3.11), the anti-self-dual field strength can be obtained,

\[ F = \frac{\zeta}{\delta(\delta + \zeta/2)(\delta + \zeta)} \left\{ f_3(dz_2 \wedge d\bar{z}_2 - dz_1 \wedge d\bar{z}_1) + f_+d\bar{z}_1 \wedge dz_2 + f_-d\bar{z}_2 \wedge dz_1 \right\}, \]  
(3.19)

where we defined

\[ \delta = z_1\bar{z}_1 + z_2\bar{z}_2, \quad f_3 = z_1\bar{z}_1 - z_2\bar{z}_2, \quad f_+ = 2z_1\bar{z}_2, \quad f_- = 2z_2\bar{z}_1. \]  
(3.20)

The topological charge density of (3.19) is given by

\[ q = -\frac{1}{8\pi^2} F \wedge F = -\frac{\zeta^2}{\pi^2} \frac{1}{\delta(\delta + \zeta/2)^2(\delta + \zeta)} p, \]  
(3.21)

where we used

\[ dz^1 \wedge d\bar{z}^1 \wedge dz^2 \wedge d\bar{z}^2 = -4(\text{volume form}), \]  
(3.22)

and defined the projection operator,

\[ p \equiv 1 - |0 : 0\rangle\langle 0 : 0|. \]  
(3.23)

The topological charge of the above configuration can be calculated as

\[ \text{Tr}_H q = \left( \frac{\zeta \pi}{2} \right)^2 \sum_{(n_1, n_2) \neq (0, 0)} q \]  
\[ = -4 \sum_{N=1}^{\infty} \frac{1}{N(N + 1)(N + 2)} = -1, \]  
(3.24)

and coincides with the expected number \(-1\), where we used the integration formula,

\[ \int d^4x \mathcal{O}(x) \rightarrow \text{Tr}_H \mathcal{O}(x) \equiv \left( \frac{\pi \zeta}{2} \right) \sum_{(n_1, n_2)} \langle n_1 : n_2 \rangle \mathcal{O}(x) |n_1 : n_2\rangle, \]  
(3.25)

and the following equations,

\[ \delta |n_1 : n_2\rangle = \frac{\zeta}{2} (n_1 + n_2) |n_1 : n_2\rangle, \]  
(3.26)

\[ \sum_{(n_1, n_2) \neq (0, 0)} \langle N|\mathcal{O}(\delta)|N\rangle = \sum_{N=1}^{\infty} (N + 1) \langle N|\mathcal{O}(\delta)|N\rangle. \]  
(3.27)

This fact enables us to interpret the noncommutative \(U(1)\) one-instanton as the bound state of the \(D0\)-brane and the \(D4\)-brane with a \(B\)-field.
3.3.2 Eight dimensional case

If we place the noncommutative instantons in each four dimensional hyper-plane $\mathbb{R}^4$ and $\tilde{\mathbb{R}}^4$, we can easily construct the eight dimensional solutions.

One-instanton solution

First we consider the simplest case where $N = k = k' = 1$. In this case, the ADHM-like equations (3.16)-(3.17) can be solved by

\begin{align}
B_1 &= B_2 = J = 0, \quad I = \sqrt{\zeta}, \\
B'_1 &= B'_2 = J' = 0, \quad I' = \sqrt{\zeta'}.
\end{align}

(3.28) (3.29)

As in the four dimensional case, the anti-self-dual field strength can be obtained as

\begin{equation}
F = f + \tilde{f},
\end{equation}

(3.30)

where we defined $f$ and $\tilde{f}$ by

\begin{align}
f &= \frac{\zeta}{\delta(\delta + \zeta/2)(\delta + \zeta)} \left\{ f_3(dz_2 \wedge d\bar{z}_2 - dz_1 \wedge d\bar{z}_1) + f_+ d\bar{z}_1 \wedge dz_2 + f_- d\bar{z}_2 \wedge dz_1 \right\}, \\
\tilde{f} &= \frac{\zeta'}{\delta'(\delta' + \zeta'/2)(\delta' + \zeta')} \left\{ f'_3(dz_4 \wedge d\bar{z}_4 - dz_3 \wedge d\bar{z}_3) + f'_+ d\bar{z}_3 \wedge dz_4 + f'_- d\bar{z}_4 \wedge dz_3 \right\}.
\end{align}

(3.31) (3.32)

We also defined

\begin{align}
\delta &= z_1 \bar{z}_1 + z_2 \bar{z}_2, \quad f_3 = z_1 \bar{z}_1 - z_2 \bar{z}_2, \quad f_+ = 2z_1 \bar{z}_2, \quad f_- = 2z_2 \bar{z}_1, \\
\delta' &= z_3 \bar{z}_3 + z_4 \bar{z}_4, \quad f'_3 = z_3 \bar{z}_3 - z_4 \bar{z}_4, \quad f'_+ = 2z_3 \bar{z}_4, \quad f'_- = 2z_4 \bar{z}_3.
\end{align}

(3.33)

This configuration has the four-form charges over the corresponding four dimensional hyper-plane,

\begin{equation}
k = -\frac{1}{2!(2\pi)^2} \int_{\mathbb{R}^4} d^4 x \ f \wedge f = -1, \quad k' = -\frac{1}{2!(2\pi)^2} \int_{\tilde{\mathbb{R}}^4} d^4 x \ \tilde{f} \wedge \tilde{f} = -1.
\end{equation}

(3.34)

These charges can be interpreted as the charge of the $D4$-brane bound to the $D8$-brane. This configuration also has the eight-form charge,

\begin{equation}
Q = \frac{1}{4!(2\pi)^4} \int_{\mathbb{R}^4 \times \tilde{\mathbb{R}}^4} d^8 x \ F \wedge F \wedge F \wedge F,
\end{equation}

(3.35)

which can be calculated explicitly as

\begin{equation}
Q = \frac{6 \cdot 16}{4!(2\pi)^4} \left( \frac{\zeta}{2} \right)^2 \left( \frac{\zeta'}{2} \right)^2 \sum \left\{ \frac{-2\zeta^2}{\delta(\delta + \zeta/2)^2(\delta + \zeta)} \right\} \left\{ \frac{-2\zeta'^2}{\delta'(\delta' + \zeta'/2)^2(\delta' + \zeta')} \right\} = 1.
\end{equation}
where we used
\[ dz^1 \wedge d\bar{z}^1 \wedge dz^2 \wedge d\bar{z}^2 \wedge dz^3 \wedge d\bar{z}^3 \wedge dz^4 \wedge d\bar{z}^4 = 16 \text{(volume form)}. \] (3.36)

This charge can be interpreted as the $D0$-brane charge. As a result, we can regard the above configuration as the bound state of the $D0-D4-D8$ with a $B$-field.

**Multi-instanton solutions**

The above construction can be easily generalized to the multi-instanton solutions. If the solution has the $D4$-brane charge $k$ for the first four dimensional hyper-plane $\mathbb{R}^4$ and $k'$ for the second one $\tilde{\mathbb{R}}^4$, this configuration also has the $D0$-brane charge,
\[ Q = kk'. \] (3.37)

Therefore this configuration can be interpreted as the bound states of the $D0-D4-D8$ with a $B$-field.

### 3.4 Structure of the moduli space of the $U(2)$ one-instanton

Similar constructions are also possible for the case of the noncommutative $U(2)$ instanton. But since the constructions are straightforward, we give the results in the case of the $U(2)$ one-instanton in the appendix.

In this subsection, we consider the structure of the moduli space of the noncommutative $U(2)$ instanton solutions. Though the noncommutative $U(1)$ one-instanton has no degrees of freedom except its position, the $U(2)$ one-instanton is the first example which has the non-trivial structure of the moduli space. In the following, we mainly focus on this case.

**Four dimensional case**

To begin with, we describe the procedure to obtain the moduli space metric from the ADHM data. Noncommutativity parameter $\zeta$ deforms the metric from that in the commutative case. In four dimensions, it is well-known that the moduli space of the noncommutative $U(N)$ $k$-instanton is given by the hyper-Kähler quotient \cite{23, 34, 36},
\[ \mathcal{M}(k, N) = \left\{ \mu_{\mathbb{R}}^{-1}(\zeta) \cap \mu_{\mathbb{C}}^{-1}(0) \right\} / U(k), \]
where the moment maps $\mu_R$ and $\mu_C$ are defined as follows,

$$
\mu_R = [B_1, B_1^\dagger] + [B_2, B_2^\dagger] + II^\dagger - J^\dagger J,
$$
(3.38)

$$
\mu_C = [B_1, B_2] + IJ.
$$

Here, $B_1, B_2$ are $k \times k$ matrices and $I, J^\dagger$ are $k \times N$ matrices. When $\zeta = 0$ i.e. the commutative case, this moduli space of dimension $4Nk$ is singular when the instantons shrink to zero size.

The tangent vectors for the ADHM data are a sum of the differentials with the independent parameters and the infinitesimal gauge transformation,

$$
\delta B_1 = dB_1 - i [\alpha, B_1],
$$

$$
\delta B_2 = dB_2 - i [\alpha, B_2],
$$

$$
\delta I = dI - i\alpha I,
$$

$$
\delta J^\dagger = dJ^\dagger - i\alpha J^\dagger,
$$

where $\alpha$ is a hermitian matrix and the differential $d$ acts on all parameters. These tangent vectors must satisfy the linearized ADHM equations,

$$
[\delta B_1, B_1^\dagger] + [B_1, \delta B_1^\dagger] + [\delta B_2, B_2^\dagger] + [B_2, \delta B_2^\dagger] + \delta II^\dagger + I\delta I^\dagger - \delta J^\dagger J - J^\dagger \delta J = 0,
$$

$$
[\delta B_1, B_2] + [B_1, \delta B_2] + \delta IJ + I\delta J = 0,
$$

(3.40)

and the linearized background gauge fixing condition,

$$
[\delta B_1, B_1^\dagger] - [B_1, \delta B_1^\dagger] + [\delta B_2, B_2^\dagger] - [B_2, \delta B_2^\dagger] + \delta II^\dagger - I\delta I^\dagger + \delta J^\dagger J - J^\dagger \delta J = 0.
$$

(3.41)

From the above equations, we can obtain

$$
[\delta B_1, B_1^\dagger] + [\delta B_2, B_2^\dagger] + \delta II^\dagger - J^\dagger \delta J = 0,
$$

(3.42)

which fixes the matrix $\alpha$. Finally the moduli space metric can be obtained from the formula,

$$
ds^2 = \text{tr} \left( \delta B_1 \delta B_1^\dagger + \delta B_2 \delta B_2^\dagger + \delta I\delta I^\dagger + \delta J^\dagger \delta J \right).
$$

(3.43)

In the following, we explicitly follow the above procedure for the case of the $U(2)$ one-instanton. Now $B_1$ and $B_2$ are numbers, the ADHM equations reduce to

$$
II^\dagger - J^\dagger J = \zeta, \quad IJ = 0.
$$

(3.44)
The general solution to these equations is given by

\[ B_1 = c_1, \quad B_2 = c_2, \quad I = \begin{pmatrix} w_1 & w_2 \end{pmatrix}, \quad J^\dagger = B \begin{pmatrix} -\bar{w}_2 & \bar{w}_1 \end{pmatrix}, \tag{3.45} \]

where \( c_{1,2} \) and \( w_{1,2} \) are regarded as independent variables which parameterize the eight dimensional manifold \( \tilde{M}(1,2) \), and we defined \( B \) by

\[ B \equiv \sqrt{1 - \frac{\zeta}{A}}, \quad \text{where} \quad A \equiv |w_1|^2 + |w_2|^2 \geq \zeta. \tag{3.46} \]

Here we have fixed the global \( U(1) \) symmetry. As we will see, \( c_1 \) and \( c_2 \) can be interpreted as the position of the instanton on \( \mathbb{R}^4 \), and \( w_1 \) and \( w_2 \) are the degrees of freedom of the instanton size and the global \( SU(2) \) transformations as discussed in \cite{36}. Since we can make the following identification,

\[ w_1 \cong -w_1, \quad w_2 \cong -w_2, \tag{3.47} \]

the metric of the relative moduli space which is the metric at the center of mass frame is expected to be the Eguchi-Hanson metric. This is also expected from the fact that the relative moduli space of the noncommutative \( U(1) \) two-instanton is the Eguchi-Hanson space \cite{34} and that the dimensions of \( \tilde{M}(1,2) \) and \( \tilde{M}(2,1) \) are the same \cite{37}.

Now \( \delta B_1 = dB_1 = dc_1 \) and \( \delta B_2 = dB_2 = dc_2 \) are nothing but numbers, we must solve (3.42),

\[ \delta II^\dagger = J^\dagger \delta J. \tag{3.48} \]

Then we can determine \( \alpha \) as

\[ \alpha = -i \frac{\zeta}{2A(2A - \zeta)}(\bar{w}_i dw_i - w_i d\bar{w}_i), \tag{3.49} \]

where we defined the differential of \( A \) as

\[ dA = \partial A + \bar{\partial} A, \]

\[ \partial A = \bar{w}_1 dw_1 + \bar{w}_2 dw_2 = \bar{w}_i dw_i, \quad \bar{\partial} A = w_1 d\bar{w}_1 + w_2 d\bar{w}_2 = w_i d\bar{w}_i. \tag{3.50} \]

Since

\[ \delta B_1 \delta B_1^\dagger + \delta B_2 \delta B_2^\dagger = dc_1 d\bar{c}_1 + dc_2 d\bar{c}_2 \tag{3.51} \]

gives the metric of the coordinates of the center of mass, the metric of the relative moduli space can be obtained substituting (3.49) into \( \alpha \),

\[ \delta I \delta I^\dagger + \delta J^\dagger \delta J = (B^2 + 1)dw_i d\bar{w}_i + \left( \frac{\zeta^2}{4A^3 B^2} + \frac{\zeta}{2A^2} \right)(\bar{w}_i dw_i + w_i d\bar{w}_i)^2 \]

\[ + \frac{\zeta^2}{4A^2(2A - \zeta)}(\bar{w}_i dw_i - w_i d\bar{w}_i)^2. \tag{3.52} \]
We can parameterize \( w_1 \) and \( w_2 \) using the Euler angles as

\[
\begin{align*}
    w_1 &= r \cos \left( \frac{\theta}{2} \right) \exp \left\{ \frac{i}{2} (\psi + \varphi) \right\}, \\
    w_2 &= r \sin \left( \frac{\theta}{2} \right) \exp \left\{ \frac{i}{2} (\psi - \varphi) \right\},
\end{align*}
\]

where the ranges of the angular variables are given by

\[
0 \leq \theta \leq \pi, \quad 0 \leq \varphi \leq 2\pi, \quad 0 \leq \psi \leq 2\pi.
\]

In these variables, the metric of the relative moduli space \( ds^2_{\text{rel}} = \delta I \delta I^\dagger + \delta J \delta J \) becomes

\[
ds^2_{\text{rel}} = \left( 2 - \frac{\zeta}{r^2} \right) \left\{ \frac{dr^2}{1 - \zeta/r^2} + \frac{r^2}{4} (\sigma_1^2 + \sigma_2^2) + \frac{r^4 (r^2 - \zeta)}{(2r^2 - \zeta)^2} \sigma_3^2 \right\},
\]

where we used the \( SU(2)_L \) invariant one-forms,

\[
\begin{align*}
    \sigma_1 &= -\sin \psi d\theta + \cos \psi \sin \theta d\varphi, \\
    \sigma_2 &= \cos \psi d\theta + \sin \psi \sin \theta d\varphi, \\
    \sigma_3 &= d\psi + \cos \theta d\varphi,
\end{align*}
\]

which satisfy the \( SU(2) \) Mauer-Cartan equation,

\[
d\sigma_i = \frac{1}{2} \epsilon_{ijk} \sigma_j \wedge \sigma_k.
\]

Near the origin, the metric of the relative moduli space becomes the one for \( \mathbf{R}^2 \times \mathbf{S}^2 \) which is nonsingular. This gives the explicit realization of the resolution of the instanton moduli space \([3]\). If we further change the variable as

\[
u^2 \equiv 2r^2 - \zeta \geq \zeta,
\]

then we can get the standard Eguchi-Hanson metric, which is hyper-Kähler and has the holonomy \( Sp(1) \),

\[
ds^2_{\text{rel}} = ds^2_{\text{EH}} = \frac{du^2}{1 - \zeta^2/u^4} + \frac{u^2}{4} \left\{ \sigma_1^2 + \sigma_2^2 + \left( 1 - \frac{\zeta^2}{u^4} \right) \sigma_3^2 \right\}.
\]

This agrees with the result in \([37, 39]\).
Eight dimensional case

Since the 1/4 BPS equations (2.7) and the ADHM-like equations for them (3.16) (3.17) in the eight dimensional case are nothing but two sets of the four dimensional ones, we can expect that the relative moduli space in the eight dimensional case becomes the direct product of the relative moduli spaces in the four dimensional case. In particular, the metric of the moduli space of the $U(2)$ one-instanton in the eight dimensional case becomes

$$ds_{\text{rel}}^2 = ds_{\text{EH}}^2 + ds_{\text{\tilde{EH}}}^2,$$

with the holonomy $Sp(1) \times Sp(1)$, where $ds_{\text{EH}}^2$ is the metric of the relative moduli space of the $U(2)$ one-instanton on noncommutative $R^4$ and $ds_{\text{\tilde{EH}}}^2$ is the similar metric on $\tilde{R}^4$.

The moduli space metric of the $U(N)$ one-instanton on noncommutative $R^4$ in its center of mass frame was found to be the $4N - 4$ dimensional hyper-Kähler Calabi metric $ds_{\text{Calabi}}^2$ in [38] with one scale parameter and the principal orbit $SU(N + 1)/U(N - 1)$. The authors of [38] studied the noncommutative caloron solution on $R^3 \times S^1$ and took the large radius limit of $S^1$. Although the relations between the ADHM data and the variables in the metric are not yet made clear, it was pointed out in [39] that the above metric becomes the hyper-Kähler Calabi metric $ds_{\text{Calabi}}^2$ constructed by [40]. The only exception is the $N = 2$ case where we have explicitly written down the relations as (3.45), (3.53) and (3.54). Having this result at hand, it is easy to extend (3.61) to

$$ds_{\text{rel}}^2 = ds_{\text{Calabi}}^2 + ds_{\text{\tilde{Calabi}}}^2,$$

where $ds_{\text{Calabi}}^2$ is the metric of the relative moduli space of the $U(N)$ one-instanton on noncommutative $R^4$ and $ds_{\text{\tilde{Calabi}}}^2$ is the similar metric on noncommutative $\tilde{R}^4$.

4 Discussions and Comments

In this paper, we focused on the 1/4 BPS equations of the noncommutative Yang-Mills theory in eight dimensions, found by [20]. We constructed some explicit solutions by means of the noncommutative version of the ADHM-like construction in eight dimensions, and showed that our solutions can be interpreted as the bound states of the $D0$-$D4$-$D8$ with a $B$-field from the calculation of their topological charges. We also discussed the structure of the moduli space of the 1/4 BPS equations and determined the metric of the moduli space of the $U(2)$ one-instanton in four and eight dimensions.
There is however room for the generalization of our ADHM-like construction. It can only produce the field strength $F_{ab} (a, b = 1, \cdots, 4)$ which depends on the coordinates $x^1, \cdots, x^4$, and $F_{ab} (a, b = 5, \cdots, 8)$ which depends on the coordinates $x^5, \cdots, x^8$. Therefore we should seek for the general ADHM-like construction applicable to more complicated configurations. And it should also be checked whether the instanton number is the expected one.

We point out the relation between our ADHM-like construction for the 1/4 BPS solutions and that for the 3/16 BPS solutions which was used in [8, 15, 18, 19]. It is of interest that when $k = k'$ we can obtain the Dirac-like operator $D_z + D'_z$ for the construction of the 3/16 BPS configurations from our Dirac-like operator (3.3). In this construction, the field strength contains the “self-dual” tensor,

$$\bar{\mathcal{N}}_{\mu\nu} = \frac{1}{2} \left( \Sigma_\mu \Sigma_\nu^\dagger - \Sigma_\nu \Sigma_\mu^\dagger \right),$$

with the definition,

$$\Sigma_\mu \equiv \begin{pmatrix} \bar{\sigma}_\mu \\ \bar{\sigma}'_\mu \end{pmatrix},$$

which satisfies the 3/16 BPS equations (2.6). (Strictly speaking, we have to take different choices of the signatures of $\alpha$s in the notation of [20].) This fact will help extend the noncommutative version of the ADHM-like equations (3.16)-(3.17) for the 1/4 BPS solutions to that for the 3/16 BPS solutions in a systematic way. The tentative ADHM-like equations for the 3/16 BPS solutions in noncommutative $\mathbf{R}^8$ have been proposed in [15]. From these considerations, the holonomy group $Sp(1) \times Sp(1)$ obtained in the last subsection can be regarded as the subgroup of the holonomy group $Sp(2)$ of the 3/16 BPS solutions in eight dimensions.

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**A** \textbf{U}(2) one-instanton solution in eight dimensions

In this appendix, we give the results in the case of the $U(2)$ one-instanton solution in eight dimensions. The ADHM-like equations (3.16)-(3.17) can be solved by

$$B_1 = B_2 = 0, \quad I = \left( \sqrt{\rho^2 + \zeta} \ 0 \right), \quad J^\dagger = \left( 0 \ \rho \right), \quad (A.1)$$
\[ B_1' = B_2' = 0, \quad I' = \left( \sqrt{\rho'^2 + \zeta'}, 0 \right), \quad J'^n = \left( 0, \rho' \right), \quad (A.2) \]

where \( \rho \) and \( \rho' \) parameterize the classical size of the instantons on \( \mathbb{R}^4 \) and \( \tilde{\mathbb{R}}^4 \) respectively.

The strength of the gauge field can be explicitly calculated as
\[
F = f + \tilde{f},
\]
where \( f \) and \( \tilde{f} \) are defined as follows,
\[
f = \left( \frac{1}{2} B_1(z_2 \bar{z}_2 - z_1 \bar{z}_1) B_3(\bar{z}_1 \bar{z}_2) - \frac{1}{2} B_2(z_1 z_2) B_4(\bar{z}_1 \bar{z}_2) \right) (dz_1 \wedge d\bar{z}_1 - d\bar{z}_2 \wedge dz_2)
+ \left( -B_1(z_1 \bar{z}_2) - B_2(z_1 z_2) \right) d\bar{z}_1 \wedge dz_2 + \left( B_1(\bar{z}_1 \bar{z}_2) - B_2(\bar{z}_2 \bar{z}_2) \right) dz_1 \wedge d\bar{z}_2.
\]
\[
\tilde{f} = \left( \frac{1}{2} B_1'(z_1 \bar{z}_3 - z_3 \bar{z}_1) B_3'(\bar{z}_3 \bar{z}_4) - \frac{1}{2} B_2'(z_3 \bar{z}_3) B_4'(\bar{z}_3 \bar{z}_4) \right) (d\bar{z}_3 \wedge d\bar{z}_3 - d\bar{z}_4 \wedge dz_4)
+ \left( -B_1'(z_3 \bar{z}_3) - B_2'(z_3 \bar{z}_3) \right) d\bar{z}_3 \wedge dz_4 + \left( B_1'(\bar{z}_3 \bar{z}_3) - B_2'(\bar{z}_3 \bar{z}_3) \right) dz_3 \wedge d\bar{z}_4.
\]

We also defined the following equations,
\[
B_1 = \frac{-2(\rho^2 + \zeta)}{\delta(\delta + \rho^2 + \zeta/2)(\delta + \rho^2 + \zeta)}, \quad (A.4)
\]
\[
B_2 = \frac{2\rho\sqrt{\rho^2 + \zeta}}{\delta(\delta + \rho^2 + \zeta/2)(\delta + \rho^2 + \zeta) \sqrt{\delta + \rho^2 + \zeta}}, \quad (A.5)
\]
\[
B_3 = \frac{-2\rho^2 \sqrt{\delta + \rho^2 + \zeta}}{(\delta + \zeta)(\delta + \rho^2 + \zeta)(\delta + \rho^2 + 3\zeta/2) \sqrt{\delta + \rho^2 + 2\zeta}}, \quad (A.6)
\]
\[
B_4 = \frac{-2\rho^2}{(\delta + \zeta)(\delta + \rho^2 + \zeta)(\delta + \rho^2 + 3\zeta/2)}, \quad (A.7)
\]

and \( B_1', \ldots, B_4' \) are defined similarly by replacing \( \delta, \zeta \) and \( \rho \) with \( \delta', \zeta' \) and \( \rho' \) respectively. It can be easily checked that the above field strength \( (A.3) \) satisfies the 1/4 BPS equations \( (2.7) \) in eight dimensions.

The calculations of the four-form charge \( (3.33) \) are the same as that of \( (2.9) \), and this shows that our solution \( (A.3) \) carries the D4-brane charges. The eight-form charge \( Q \) \( (3.34) \) of our solution \( (A.3) \) can be calculated only numerically. But since it is believed that the topological charge does not depend on the parameters of size \( \rho \) and \( \rho' \), the eight-form charge can be calculated in the limit of \( \rho, \rho' \to 0 \). Since the part of the \( U(1) \) one-instanton only contributes in this limit, the eight-form charge can be calculated as in the case of the \( U(1) \) one-instanton. The result is \( Q = 1 \). From these facts, we can regard the configuration \( (A.3) \) as the bound states of the \( D0-D4-D8 \) with a \( B \)-field.
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