Current–Voltage Characteristics of Two–Dimensional Vortex Glass Models

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Abstract

We have performed Monte Carlo simulations to determine current–voltage characteristics of two different vortex glass models in two dimensions. The results confirm the conclusions of earlier studies that there is a transition at \( T = 0 \). In addition we find that, as \( T \to 0 \), the linear resistance vanishes exponentially, and the current scale, \( J_{nl} \), where non-linearities appear in the \( I-V \) characteristics varies roughly as \( T^3 \), quite different from the predictions of conventional flux creep theory, \( J_{nl} \sim T \). The results for the two models agree quite well with each other, and also agree fairly well with recent experiments on very thin films of YBCO.

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I. INTRODUCTION

Fluctuation effects are much larger in high $T_c$ superconductors than in conventional materials [1], so there has been considerable effort, recently, to go beyond the traditional mean field approximation in describing the superconducting phase transition. The effects of fluctuations, and also of disorder, are particularly strong in a magnetic field. In fact much of the $H - T$ phase diagram of high $T_c$ materials is occupied by a “vortex liquid” regime in which the resistance has dropped because superconducting short range order has formed, so flux lines exist locally, but the resistance is not yet zero because the flux lines move under the action of a Lorentz force due to the current, and hence give rise to a voltage [2]. An important question is whether, at lower $T$, the flux lines can be collectively pinned by defects sufficiently strongly that they have no linear response to the Lorentz force, which implies a vanishing linear resistance. In the presence of disorder, there is no long range order in the arrangement of vortices [3]. Nonetheless, it is argued [4] that a transition to a state with vanishing linear resistance can occur. Such a state is called the vortex glass.

While there is theoretical [5,6] and experimental [7] evidence for a finite vortex glass transition temperature, $T_c$, in bulk superconductors, several simulations [8,6] have clearly shown that $T_c = 0$ in two-dimensional systems, and rigorous analytic arguments [9] have established that there is no vortex glass order at finite $T$ in 2-d.

Recent work [10] has shown that inclusion of gauge field fluctuations (ie, screening) changes the universality class, but one still has $T_c = 0$ in two dimensions. Even though $T_c = 0$, the correlation length diverges as $T \to 0$, which leads to observable consequences at finite temperatures, as we shall discuss in detail here. In contrast, a different simulation of the non-linear IV characteristic of the d=2 gauge glass model found evidence for a finite $T_c$ [11]. However behavior consistent with $T_c = 0$ has recently been observed [12] in experiments on very thin, 16Å, films of YBCO.

Although the experiments [12] on the 2-d samples are in quite good agreement with theory, they measure different quantities from what has been calculated so far, and a scaling
hypothesis is needed to make a comparison. The experiments determined current–voltage (I–V) characteristics while the simulations investigated the size–dependence of the rigidity of the system with respect to a twist in the phase of the condensate. It therefore seems worthwhile to make a direct comparison by calculating I–V characteristics from the simulations on 2-d systems. As an additional benefit, we study two models which are somewhat different microscopically: the gauge glass, defined in Eq. (I) below, which has effectively random fluxes penetrating the sheet, and a more realistic model, defined in Eq. (II) below, which has a net uniform field penetrating the sheet and a random pinning potential for the vortices. We find that universal properties are the same for these two models. Since the gauge glass has been extensively used for numerical studies of the vortex glass transition it is reassuring that it gives the same results as a model with a net field, at least in two–dimensions.

II. THE MODELS

The first model that we study is the gauge glass [5,6,8], whose Hamiltonian is

$$\mathcal{H}_{gg} = -\sum_{\langle i,j \rangle} \cos(\phi_i - \phi_j - A_{ij}) .$$

(1)

The phase of the condensate, $\phi_i$, is defined on each site, $i$, of a square lattice, with $N = L^2$ sites. The sum is over all nearest neighbor pairs on the lattice. The effects of the external magnetic field and disorder are represented by the quenched vector potentials, $A_{ij}$, which are taken to be independent random variables with a uniform distribution between 0 and $2\pi$.

To compute the I–V characteristics we need to incorporate dynamics into the model. The standard way of doing this is to view the model as a set of coupled Josephson junctions [13,14]. Josephson’s and Kirchoff’s equations for the current are then

$$I_{ij} = \frac{V_{ij}}{R_0} + I_c \sin(\phi_i - \phi_j - A_{ij}) + \eta_{ij}(t)$$

(2)

$$V_{ij} = \frac{\hbar}{2e} \frac{d}{dt}(\phi_i - \phi_j)$$

(3)
\[ \sum_j I_{ij} = I_{i;\text{ext}}, \quad (4) \]

where \( i \) and \( j \) are nearest neighbor pairs. Eq. (3) expresses the sum of the current from site \( i \) to neighboring site \( j \) as the sum of a resistive current given by \( V_{ij}/R_0 \), a Josephson current, and a Langevin current noise source \( \eta_{ij}(t) \). \( I_c \) is the maximum Josephson current of the nearest neighbor pair, and \( R_0 \) is the shunt resistance of the pair. The thermal noise has a Gaussian distribution with the following properties:

\[ \langle \eta_{ij}(t) \rangle = 0, \quad (5) \]

\[ \langle \eta_{ij}(t) \eta_{kl}(t') \rangle = \frac{2k_B T}{R_0} \delta_{ij,kl} \delta(t - t'), \quad (6) \]

which ensures that the system comes to thermal equilibrium at temperature \( T \). Eq. (3) is the Josephson relation connecting the voltage \( V_{ij} \) with the time derivative of the phase difference, \( \phi_i - \phi_j \). Eq. (4) is Kirchoff’s law expressing current conservation at site \( i \). \( I_{i;\text{ext}} \) is the external current at site \( i \). This is zero except for sites on the top row where an external current \( J \equiv I/L \), is fed in, and sites on the bottom row where the same current is extracted. The total current through the sample is then \( I \). The average voltage across the system, \( V \), is given by

\[ V = \frac{1}{L^2} \frac{\hbar}{2e} \sum_{i \in \text{bottom}} \sum_{j \in \text{top}} \frac{d}{dt} \langle \phi_i - \phi_j \rangle \quad (7) \]

where the brackets, \( \langle \ldots \rangle \), denote a time average. The average electric field is then obviously given by \( E = V/L \). For this model we work in units where \( \hbar/(2e) = R_0 = I_c = 1 \).

Throughout the paper we also set Boltzmann’s constant to be unity. The equations of motion are integrated using a first order approximation with a time step of \( \delta \tau = 0.05 \tau \), where \( \tau = \hbar/(2eR_0I_c) \) is the basic unit of time (which is set equal to unity). The scale of \( \tau \) is the typical time for a neighboring pair of sites to accumulate a relative phase of order unity.

We have also studied an equivalent form for the gauge glass written in terms of vortices. To obtain this we replace the cosine by the periodic Gaussian (Villain) function [16], and perform standard manipulations [16][17], obtaining
\[
\mathcal{H}_\text{gg}^V = -\frac{1}{2} \sum_{i,j}(n_i - b_i)G(i - j)(n_j - b_j),
\]
where the \(\{n_i\}\) run over all integer values, subject to the “charge neutrality” constraint \(\sum_i n_i = 0\), and are interpreted as the vortex “charges”, and \(G(i - j)\) is the vortex interaction,
\[
G(i - j) = \left(\frac{2\pi}{L}\right)^2 \sum_{k \neq 0} \frac{1 - \exp[i \mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)]}{4 - 2 \cos k_x - 2 \cos k_y}.
\]
At large distance, \(G(i - j) \rightarrow 2\pi \log |\mathbf{r}_i - \mathbf{r}_j|\). The vortices sit on the sites of the dual lattice, which lie in the centers of the squares of the original lattice. The magnetic fluxes \(b_i\) are the lattice curl of the vector potential and are given by \((1/2\pi)\) times the directed sum of the quenched vector potentials on the links of the original lattice which surround the site \(i\) of the dual lattice.

We study the I–V characteristics of this vortex model by using Monte Carlo dynamics. That is, we equate Monte Carlo ‘time’ and real time, an approximation which is expected to be good in the limit of overdamped dynamics and which has proven reasonable in other simulations. Choosing a nearest neighbor pair, \(i, j\) at random, we try to increase \(n_i\) by 1 and decrease \(n_j\) by 1, thus transferring a unit vortex from \(j\) to \(i\). If the change in energy is \(\Delta E\), the move is accepted with the probability \(1/(1 + \exp(\beta \Delta E))\) appropriate to the “heatbath” algorithm. An applied current density \(J\) gives a Lorentz force of \(Jh/2e\) on a unit vortex. This can be incorporated into the Monte Carlo moves by adding to \(\Delta E\) an amount \(Jh/(2e)\) if the vortex moves in the opposite direction to the Lorentz force, subtracting this amount if it moves in the same direction, and making no change in \(\Delta E\) if it moves in a perpendicular direction. Biasing the moves in this way takes the system out of equilibrium and causes a net flux of vortices in a direction perpendicular to the current. This then generates a voltage \(V\), where
\[
V = \frac{h}{2e} \langle I^V(t) \rangle,
\]
with
\[
I^V(t) = \frac{1}{L\Delta t} \sum_i \Delta Q^V_i(t)
\]
the vortex current. Here \( t \) denotes a Monte Carlo “time” (incremented by \( \Delta t \) after each attempted move), and \( \Delta Q_{V}^{V}(t) = 1 \) if a vortex at site \( i \) moves one lattice spacing in the direction of the Lorentz force at time \( t \), \( \Delta Q_{V}^{V}(t) = -1 \) if the vortex moves in the direction opposite to the Lorentz force, and \( \Delta Q_{V}^{V}(t) = 0 \) otherwise. We set \( \Delta t = 1/4N \) so that an attempt is made to move each vortex once in each direction, on average, per unit time. We shall use units where \( h/2e = 1 \) when dealing with vortex models.

The linear resistance can also be obtained from the Kubo formula for fluctuations in the voltage in the absence of any net current. This is exact for discrete time Monte Carlo dynamics provided the sum over time is made symmetrical about \( t = 0 \) \[19\] i.e.

\[
R_{\text{lin}} = \frac{1}{2T} \sum_{t=-\infty}^{\infty} \Delta t \left\langle V(t)V(0) \right\rangle, \tag{12}
\]

which, in our units, can be expressed in terms of the vortex current as

\[
R_{\text{lin}} = \frac{1}{2T} \sum_{t=-\infty}^{\infty} \Delta t \left\langle I^{V}(t)I^{V}(0) \right\rangle. \tag{13}
\]

Using Monte Carlo dynamics should be a good approximation near a critical point, where the vortex motion is slow and overdamped. However, because of discretization of time, and the fact that the fastest that a vortex can move is one lattice spacing per time step, it is not very satisfactory for large currents or high temperatures, where the vortex motion is ballistic. For example, at high temperatures with no bias current, a vortex moves in the \( \pm \) direction with probability, \( 1/4 \), so, from Eq. (13), the resistance is given by \( R_{\text{lin}} = 1/(2T) \) and tends to zero, which is unphysical.

The gauge glass represents a system with random fluxes penetrating the film but with zero net field. It is a convenient model to study but it should be verified that it is in the same universality class as experimental systems which have a net uniform field penetrating the film and a random pinning potential for the vortices. We have therefore also studied a random pinning model with the following Hamiltonian,

\[
\mathcal{H}_{rp} = -\frac{1}{2} \sum_{i,j} n_{i}G(i-j)n_{j} - \sum_{i} v(i)n(i)^{2}, \tag{14}
\]
where the \( n(i) \) are restricted to the values \( 0, \pm 1 \), \( G(i - j) \) is given by Eq. (9), and \( v(i) \) is a random pinning potential, uniformly distributed in the interval \( -\Delta < v(i) < \Delta \). We set \( \Delta = \pi \) and fix the net filling, \( f (\equiv (1/N) \sum n_i ) \), which effectively determines the magnetic field, to be 1/4. We obtain I–V characteristics from Monte Carlo dynamics. At each Monte Carlo time we try to insert a +1, −1 pair at a randomly chosen pair of sites \( i \) and \( j \). The analysis is then precisely the same as described above for the vortex representation of the gauge glass.

III. SCALING THEORY

To analyze the results it is necessary to understand how the I–V characteristics vary in the vicinity of a second order phase transition. A detailed scaling theory has been developed \[1\] and we now summarize the results of this for the case of a zero temperature transition, where the correlation length diverges as

\[ \xi \sim T^{-\nu} \]  

and the relaxation time, \( \tau \), also diverges. Normally one defines a dynamic exponent, \( z \), by \( \tau \sim \xi^z \), but since the transition is at \( T = 0 \), the relaxation has an activated form and diverges exponentially as \( T \to 0 \). Formally this corresponds to \( z = \infty \).

The vector potential, \( \mathbf{A} \), enters the Hamiltonian in Eq.(1) in the dimensionless form \( A_{ij} \sim \int_j^i \mathbf{A}(r) \cdot dr \). Therefore \( \mathbf{A} \) scales as \( 1/\xi \). The electric field is given by \( \mathbf{E} = -\partial_t \mathbf{A} \) and so scales as \( 1/(\xi \tau) \). Now \( JE \) is the energy dissipated per unit volume per unit time. The natural unit of energy is \( k_B T \) so \( \mathbf{E} \cdot \mathbf{J} \) scales like \( k_B T/(\xi^d \tau) \) and hence \( J \) scales like \( k_B T/\xi^{d-1} \). It is important to keep the factors of \( T \) because \( T_c = 0 \). Combining these results we obtain (for \( k_B = 1 \))

\[ T^2 E \cdot \frac{\tau}{\xi^{d-2}} = g \left( \frac{J \xi^{d-1}}{T} \right) , \]  

where \( g \) is a scaling function. In \( d = 2 \) with \( T_c = 0 \) this becomes
\[ T^* \frac{E}{J} = g \left( \frac{J}{T^{1+\nu}} \right) . \]  

From Eq. (17) one sees that the characteristic current scale, \( J_{nl} \), at which non-linear behavior sets in, varies with \( T \) as

\[ J_{nl} \sim T^{1+\nu} . \]  

(18)

Now the linear resistance is just

\[ R_{lin} = \lim_{J \to 0} \frac{E}{J} , \]  

(19)

and \( g(0) \) must be a constant, which we take to be unity, so we can write

\[ \frac{E}{JR_{lin}} = g \left( \frac{J}{T^{\nu+1}} \right) , \]  

(20)

and

\[ TR_{lin} = \frac{1}{\tau} = A \exp \left( -\Delta E(T)/T \right) , \]  

(21)

where \( \Delta E(T) \) is the typical barrier that a vortex has to cross to move a distance \( \xi \). One conventionally defines a barrier exponent \( \psi \) by \( \Delta E \sim \xi^\psi \sim T^{-\psi \nu} \) in terms of which

\[ TR_{lin} \sim \exp \left( -C/T^{1+\psi \nu} \right) . \]  

(22)

It has been suggested [1] that \( \psi \) may be zero in \( d = 2 \), leading to barriers which are either finite as \( T \to 0 \) or diverge logarithmically. In the latter case the linear resistance would vary as \( \exp \left( -C |\log(T)|^{\mu}/T \right) \), where \( \mu \) is another exponent.

In Eq. (21), the linear resistance is seen to vanish exponentially fast as \( T \to 0 \). In these circumstances it is generally more difficult to estimate the form of possible power law prefactors than the form of the leading exponential dependence. We have incorporated a factor of \( T \) on the LHS of Eq. (21) because it emerged naturally from the scaling ansatz. This factor of \( T \) also looks reasonable when compared with the Kubo formula, Eq. (13), since it is \( TR_{lin} \) which is given by the voltage fluctuations. However, it is possible that additional factors of \( T \) could be present in the scaling region.
In a finite system, the I–V characteristics will also depend on the size of the system when the bulk correlation length, $\xi$, becomes comparable with $L$. According to finite size scaling [20] only the ratio $L/\xi$ is important and so, from Eq. (15), one can generalize Eq. (20) to

$$\frac{E}{JR_{\text{lin}}} = \tilde{g} \left( \frac{J}{T^{1+\nu}}, \frac{L^{1/\nu} T}{\nu} \right).$$  \hspace{1cm} (23)

The non-linear behavior in the finite-size regime, described by Eq. (23), is rather complicated because it involves a function of two variables. We have therefore studied non-linear behavior either in the range where finite size corrections are negligible or, having already determined $\nu$, choose $L$ and $T$ such that the second argument, $L^{1/\nu} T$, is constant.

**IV. RESULTS**

In Fig. 1 we show some of the experimental data of Dekker et al. [12] on 16 Å films of YBCO. For small current densities, the data is flat, indicating Ohm’s law, with a linear resistivity which decreases rapidly with decreasing temperature. As $J$ is increased the data starts to curve upwards, indicating non-linear response. The current scale, $J_{\text{nl}}$, at which non-linear behavior sets in, is also seen to decrease with decreasing temperature. Analyzing all their data, Dekker et al. find the linear resistivity varies as

$$R_{\text{lin}} \sim \exp(-C/T^p)$$  \hspace{1cm} (24)

with $p \simeq 0.6$. This variation, which is less rapid than an Arrhenius form, $p = 1$, is difficult to understand from the classical models which we study here. It may therefore indicate that quantum tunneling of vortices is important at the lowest temperatures. On the other hand, $J_{\text{nl}}$ is found to vary with $T$ like $T^3$, which, from Eq. (18) implies $\nu \simeq 2$, in agreement with the earlier simulations [8,6] which studied a classical model.

We next discuss our numerical results for the I-V characteristics of the gauge glass in the vortex representation. Data for $T$ times the linear resistance against $1/T$ is shown in
Fig. 2 on a log-linear plot. A simple Arrhenius form, i.e. a temperature independent barrier height, $\Delta E$, would correspond to a straight line. In fact for the largest size, the data is very close to a straight line indicating that the barrier exponent, $\psi$, in Eq. (22) is zero or close to it. These results are consistent the suggestion [1] of a logarithmically increasing barrier, but this weak dependence will be difficult to see on data which are over a modest range of temperature in finite size systems.

Data for the non-linear response at finite $J$ is shown in Fig. 3. The data for the smallest current density, $J$, was actually obtained for $J = 0$ from the Kubo formula, Eq. (13). As in the experimental results in Fig. 1, one sees Ohm’s law at small $J$ (where each dataset is horizontal) with a linear resistance which decreases rapidly with temperature, as shown in more detail in Fig. 2, but deviations from Ohms law occur at a scale, $J_{nl}$, where the data start to curve upwards. One sees that $J_{nl}$ decreases with decreasing temperature, as expected. From Fig. 2, finite size effects appear quite small for $L = 16$ at $T = 0.5$ and 0.8. Assuming $\nu \approx 2$, the data for $L = 32$ in Fig. 2 at $T = 0.35$ should also not be significantly affected by finite size effects. We have therefore analyzed the data in Fig. 3 according to the expected result for bulk behavior, Eq. (20). The scaling plot is shown in Fig. 4. The data scales reasonably well with $\nu = 1.8$ which is in quite good agreement with other estimates [8,6]. The values of $R_{lin}$ in this plot were obtained from the Kubo formula.

Next we discuss the data obtained for the phase representation of the gauge glass. Results for the linear resistance are shown in Fig. 5. The data is consistent with an Arrhenius form for the largest sizes, as was found for the vortex representation (Fig. 4). Results for non-linear current voltage characteristics for the gauge glass model in the phase representation are shown in Fig. 6. Unlike the vortex representation, which has discrete time and so a maximum vortex velocity, the dynamics of the phase representation uses continuous time [21] and so can sensibly be applied for large values of $J$ (and also high-$T$). In Fig. 6 one clearly sees a “flux-flow” regime for large $J$, where the Lorentz force is sufficient to overcome pinning, and the only hindrance to vortex motion comes from friction. This leads to a resistance which is roughly independent of $J$ and also only rather weakly dependent
on temperature. For small $J$, however, the vortices are pinned by defects and move only by activation over barriers. The linear resistance in this ‘flux-creep’ region, is therefore much smaller than that observed for larger $J$ and is also strongly temperature dependent.

We next describe our results for the random pinning potential model in Eq. (14). The linear resistance is shown in Fig. 7. As for the gauge glass, the data for the larger sizes seem to be tending towards an Arrhenius behavior.

The non-linear behavior of the random pinning potential model is shown in Fig. 8. As for the gauge glass results in Fig. 6 one sees a “flux-flow” regime for large $J$, and a “flux-creep” region at small $J$ where the linear resistance is small and strongly temperature dependent. Deviations from the Ohm’s law behavior in the flux creep regime occur at a current scale, $J_{nl}$, which decreases as $T$ decreases. For sufficiently large $J$, $E$ will saturate, because there is a maximum vortex velocity when each vortex hops every step, and so the ratio $E/J$ will decrease. This (unphysical) behavior, which is just visible in the figure at the largest values of $J$, is caused by discretization of time in the Monte Carlo simulations. This discretization is, however, not expected to affect universal critical properties near the $T = 0$ critical point.

A scaling plot of the non-linear I-V characteristics of the random pinning potential model is shown in Fig. 9. Since there are finite-size effects within the range of accessible sizes, we assume that $\nu \simeq 2$, and choose sizes and temperatures such that $LT^2$ is constant. In this way, the second argument in the scaling function in Eq. (23) is constant and $E/(JR_{lin})$ should only be a function of $J/T^3$. The data is seen to scale very well over a wide range. These results provide strong evidence that the gauge glass and random pinning potential models are in the same universality class with a correlation length exponent at the $T = 0$ transition of $\nu \simeq 2$.

Finally, in Fig. 10 we compare the non-linear current-voltage characteristics from the simulations on the random pinning potential and gauge glass models with the experimental results shown in Fig. 1. The same value of $\nu = 2$ was used for all the data, and, for each system, a temperature scale, $T_0$, was adjusted to get the best scaling. The two sets of simulation data agree quite well with each other. While the gauge glass data were obtained
in the region where $\xi \ll L$, as deduced from the results for $R_{lin}$, the random pinning potential model data was in the finite size region and so the data were taken at fixed $LT^2$ (or equivalently at fixed, but not very small, $\xi/L$). The good agreement between the two sets of data indicates that finite size corrections are not very important for the I-V characteristics. There is, however, a discrepancy between the numerical results and experiment, for which we do not have an explanation.

V. CONCLUSIONS

We have studied the I-V characteristics of two models for vortex glass behavior in two dimensions. For the first model, the gauge glass, our results confirm earlier studies [6,8] which found a zero temperature transition with a correlation length exponent $\nu \simeq 2$. This behavior has also been seen experimentally [12] on very thin films of YBCO, though the detailed form of the I-V scaling function is somewhat different between theory and experiment. We also find that the linear resistance varies with an Arrhenius form as $T \to 0$, indicating that the barrier exponent, $\psi$, is either zero or close to zero. Experimentally the resistance appears to vanish at low $T$ less rapidly than an Arrhenius form, which may indicate that quantum fluctuations of the vortices play a role in the YBCO films. The other model which we study, the random pinning potential model, is rather more realistic in that it describes a system with a uniform applied magnetic field perpendicular to the film, as opposed to the gauge glass which has random fluxes. Nonetheless, the random pinning potential model is found to be in the same universality class as the gauge glass, since they both have a zero temperature transition with $\nu \simeq 2$ and an Arrhenius behavior for the linear resistance. Hence these two models are equivalent in $d = 2$. It can also be shown that in $6 - \epsilon$ dimensions, the gauge glass transition is in the same universality class as the vortex glass transition in a disordered model with a net magnetic field. It is therefore quite plausible that the two models have the same critical behavior in three dimensions, though we are not aware of a direct demonstration of this.
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[21] Of course our numerical implementation of the continuous time dynamics uses a discrete time step, but this time step is much smaller than the characteristic microscopic time, $\tau$, which corresponds to the time of one sweep in the Monte Carlo simulation.
FIGURES

FIG. 1. A plot of some of the experimental data, at a field of 0.5 T, on very thin, 16 Å, films of YBCO from Ref. [12]. The numbers denote the temperatures, in Kelvin, at which the different data sets were taken.

FIG. 2. A plot of $T$ times the linear resistance, on a logarithmic scale, against $1/T$ for different sizes for the gauge glass in the vortex representation. The data for the largest size, $L = 32$, is well approximated by a straight line, indicating a temperature independent barrier height, i.e. Arrhenius behavior.

FIG. 3. A log-log plot of the non-linear current voltage characteristics of the gauge glass model in the vortex representation. The data shown for the smallest current density, $J$, is actually for $J = 0$ and was obtained from the Kubo formula, Eq. (13). One sees Ohm’s law at small $J$ (where each dataset is horizontal) with a linear resistance which decreases rapidly with temperature, as shown in more detail in Fig. 2. Deviations from Ohm’s law, where the data start to curve, occur at a current scale, $J_{nl}$, which decreases with decreasing temperature.

FIG. 4. A scaling plot of the data in Fig. 3, assuming that finite-size corrections are small and hence that the scaling form expected for bulk behavior, Eq. (20), is appropriate. The value $\nu = 1.8$ obtained from this fit, is in reasonable agreement with other estimates.

FIG. 5. A plot of the linear resistance, on a logarithmic scale, against $1/T$ for different sizes for the gauge glass in the phase representation. The data for the largest size, $L = 16$, is fairly close to a straight line, indicating a temperature independent barrier height, i.e. Arrhenius behavior.

This is similar to what was observed in the vortex representation, Fig. 2.
FIG. 6. Results for non-linear current voltage characteristics for the gauge glass model in the phase representation. One sees a “flux-flow” regime for large $J$, where the resistance is largely independent of $J$ and $T$, and a “flux-creep” region at small $J$ where the linear resistance is small and strongly temperature dependent. Deviations from the Ohm’s law behavior seen at small $J$ occur at a current scale, $J_{nl}$, which decreases as $T$ decreases.

FIG. 7. A plot of the linear resistance, on a logarithmic scale, against $1/T$ for different sizes for the random pinning potential model in Eq. (14). The data for the largest sizes seems to be tending to a straight line, indicating a temperature independent barrier height, i.e. Arrhenius behavior. This is similar to what was observed for the gauge glass, see Figs. 2 and 5.

FIG. 8. Results for non-linear current voltage characteristics for the random pinning potential model in Eq. (14). For each size, $L$, the temperature has been chosen so that $LT^2 = 2$, in order to keep the second argument of the finite-size scaling function in Eq. (23) constant. The net filling, $f \equiv (1/N) \sum_i n_i$, is equal to 1/4. As for the gauge glass results presented in Fig. 6 one sees a “flux-flow” regime for large $J$, and a “flux-creep” region at small $J$ where the linear resistance is small and strongly temperature dependent. Deviations from the Ohm’s law behavior seen in the flux creep regime occur at a current scale, $J_{nl}$, which decreases as $T$ decreases.

FIG. 9. A scaling plot of the non-linear current-voltage characteristics of the random pinning potential model. Since there are finite-size effects in this data, we assume that $\nu \simeq 2$, and choose sizes and temperatures such that $LT^2$ is constant. In this way, the second argument in the scaling function in Eq. (23) is constant and $E/(JR_{lin})$ should only be a function of $J/T^3$. The data is seen to scale very well to this expected form over a wide range.
FIG. 10. A scaling plot combining the non-linear current-voltage characteristics of both the experimental data in Fig. 1 (lines) and simulations (points). The data points with $LT^2 = 2$ were obtained for the random pinning potential model and the other two sets of data points, at $T = 0.35$ and 0.50, were for obtained for the gauge glass model in the vortex representation. The value $\nu = 2$ was used for all the data. The temperature scale was set by $T_0 = 1$ (random pinning potential model), $T_0 = 1.15$ (gauge glass), and $T_0 = 12K$ (experiment). The two sets of data from the simulations agree quite well with each other, but the experimental results lie lower than theory at large values of $J/T^{1+\nu}$. 