Abstract

The main argument against the reality of the electromagnetic quantum vacuum fluctuations is that they do not activate photon detectors. In order to met this objection I propose a model of photocounting which, in the simple case of a light signal with constant intensity, predicts a counting rate proportional to the intensity, in agreement with the standard quantum result.

PACS numbers: 42.50.Dv, 42.50.Lc.
I. Introduction

The existence of vacuum fluctuations is a straightforward consequence of field quantization. In addition, quantum vacuum fluctuations have consequences which have been tested empirically. For instance, the vacuum fluctuations of the electromagnetic field (or zeropoint field, ZPF) give rise to the main part of the Lamb shift and to the Casimir effect. The ZPF was proposed in 1912 by Planck when he wrote the radiation law in the form

\[ \rho(\omega, T) = \frac{\omega^2}{\pi^2 c^3} \left[ \frac{\hbar \omega}{\exp(\hbar \omega/k_B T) - 1} + \frac{1}{2} \hbar \omega \right], \quad (1) \]

where the second term represents the ZPF. That the thermal spectrum contains an \( \omega^3 \) term has been proved by experiments measuring current fluctuations in circuits with inductance at low temperature. Of course, the ZPF term is ultraviolet divergent so that some cutoff should be assumed, likely at about the Compton wavelength, where the fluctuations of the Dirac electron-positron sea become important.

It is believed that the ZPF cannot be interpreted as a real random electromagnetic field because it does not activate photodetectors in the absence of signals. (There is also a gravitational problem because, if the quantum vacuum fluctuations are at the origin of the cosmological constant as is usually assumed, the constant should be many orders of magnitude larger than the observed value. But we shall not be concerned with gravitational effects in this paper.) A common explanation of the fact that the ZPF does not activate photodetectors is to say that the ZPF is not real, but virtual. However replacing a word, real, by another one, virtual, with a less clear meaning is not a good solution. In the present article I shall show that the behaviour of photodetectors can be explained without renouncing to the reality of the ZPF. The proof goes via constructing an explicit model of detector producing a counting rate proportional to the intensity of the signal, that is able to subtract efficiently the ZPF.

II. Stochastic properties of the zeropoint field

The vacuum field (i.e. the last term of eq. (1)) contains an average energy \( \frac{1}{2} \hbar \omega \) per normal mode of the electromagnetic radiation. According to quantum mechanics it is impossible by any (controllable) means to reduce
that energy. Indeed, any reduction would lead to a violation of the Heisenberg (uncertainty) relations. Therefore we should assume that vacuum field energy cannot be changed by the presence of optical devices like mirrors, lenses, beam-splitters, etc. In fact, they cannot reduce the vacuum energy, as said above, but they cannot increase it either by conservation of energy. What is possible is to change the structure of the normal modes and this is what happens in the Lamb shift (where normal modes are modified by the presence of an atom) or the Casimir effect (normal modes are changed by macroscopic objects).

That the vacuum energy cannot be changed by any controllable method also follows from thermodynamic considerations. In fact the vacuum field exists even at zero Kelvin, as shown by eq.(1) and, by the second law, no useful energy could be extracted from it. We might imagine a Maxwell demon able to change the energy content of some modes of the vacuum field. But that change would necessarily be uncontrollable. In summary, we must assume that no action may change the stochastic properties of the vacuum field (that is, the joint probability distribution of the energy in the normal modes).

Now we shall derive some relevant properties of the ZPF in free space, that is far from any material body. The ZPF is characterized by the electric field, \( \mathbf{E}(\mathbf{r}, t) \), and the magnetic field, \( \mathbf{B}(\mathbf{r}, t) \). For our purposes the most relevant quantity is the radiation intensity represented by the Poynting vector,

\[
\mathbf{S}(\mathbf{r}, t) = \frac{1}{\mu_0} \mathbf{E}(\mathbf{r}, t) \times \mathbf{B}(\mathbf{r}, t).
\]

At every point in space the components of the Poynting vector of the ZPF may be considered three independent stochastic processes. Every process should be stationary with zero mean by symmetry considerations. The autocorrelation might be derived from the spectrum of the process, which could be easily related to the last term of (1).

Thus the most relevant properties of the vector stochastic process \( \mathbf{S}(\mathbf{r}, t) \), at any point in space \( \mathbf{r} \), may be written

\[
\langle \mathbf{S}(\mathbf{r}, t) \rangle = 0, \quad \langle S_j(\mathbf{r}, t)S_k(\mathbf{r}, t') \rangle = \delta_{jk}F(t' - t),
\]

where \( \delta_{jk} \) is the Kronecker delta and \( F(t' - t) \) the autocorrelation function.

Now we shall study the situation where we have a light signal superimposed to the ZPF. The signal energy is concentrated within a narrow region in momentum space. However the ZPF would contain an average energy \( \frac{1}{2}\hbar \omega \)
in every mode because that energy cannot be reduced as explained above. The important result is that the signal frequency bandwidth is substantially more narrow than the ZPF bandwidth, the latter covering the whole spectrum until a cut-off. This leads us to approximate the spectrum of the ZPF by a white noise, which gives an autocorrelation function in the form of a Dirac’s delta. Thus the properties of the Poynting vector of the light beam will be (compare with (2))

\[ \langle S_j(r,t) \rangle = \delta_{j3} I_s, \langle S_j(r,t)S_k(r,t') \rangle = \delta_{j3}\delta_{k3} I_s^2 + \sigma^2 \delta_{jk} \delta(t' - t), \]  

(3)

where \( S_3 \) is the component of the Poynting vector in the direction of the beam, \( I_s \) the signal intensity and \( \sigma \) is a constant. This equation will be the basis of our subsequent study but I point out that the approximation (3) may be too crude for some applications. Possible improvements will be considered elsewhere.

III. Detection model

Several models of photodetection have been proposed recently resting upon the idea that there exists a ”detection time”, \( T \), independent of the light intensity and such that the probability of a count depends on the radiation (including the ZPF) which enters the detector during the time \( T \). I have shown elsewhere \( ^6 \) that those models are not compatible with empirical evidence.

Instead of fixing the detection time, \( T \), I shall assume that a count is produced when the radiation energy accumulated in the detector surpasses some threshold. This means that once the photocounter is ready to detect (which will happen some ”dead time” after a count is produced, but we will neglect the dead time here), the detector begins to accumulate the radiation energy entering in it. If \( I(t) \) is the total intensity (we are using the word intensity for ”component of the Pointing vector in the direction of the beam”) entering the detector at time \( t \), the accumulated energy at time \( T \) will be

\[ E(T) = A \int_0^T I(t) dt, \]

(4)

where \( A \) is the entrance area of the detector (in the following we shall put \( A = 1 \) for the sake of simplicity).
The essential assumption of our model is that a detection event is produced at a time \( T \), after the previous count, when \( T \) is such that

\[
E(T) \equiv \int_0^T I(t) \, dt = E_m, \tag{5}
\]

where \( I(t) \) is the radiation intensity entering the detector and \( E_m \) is a parameter characteristic of the detector.

The use of eq. (5) may be cumbersome due to the fluctuations of the ZPF and the signal. Indeed constructing a detailed detection model on the basis of that equation would require using the theory of "first passage time" for the stochastic process \( I(t) \), which has a finite, nonzero, correlation time. However the problem is dramatically simplified if we assume that \( I(t) \) is a white noise (having a null correlation time) superimposed to a deterministic signal with constant intensity \( I_s \), as in eq. (3), so that the stochastic process \( E(T) \) (see (5)) is a Wiener (Brownian motion) process.

The calculation of the first-passage time is now easy. We shall begin solving the diffusion equation

\[
\frac{\partial \rho}{\partial t} = \frac{1}{2} \sigma^2 \frac{\partial^2 \rho}{\partial E^2} - I_s \frac{\partial \rho}{\partial E} \tag{6}
\]

with an absorbing barrier at \( E = E_m \). The result is

\[
\rho(E,t) = \frac{1}{\sigma \sqrt{2\pi t}} \left\{ \exp \left[ -\frac{(E - I_s t)^2}{2\sigma^2 t} \right] - \exp \left[ \frac{2E_m I_s}{\sigma^2} - \frac{(E - I_s t - 2E_m)^2}{2\sigma^2 t} \right] \right\}. \tag{7}
\]

Hence, if we have a detection event at time \( t = 0 \), the probability that the next detection event takes place before time \( t \) is

\[
P(t) = 1 - \int_{-\infty}^{E_m} \rho(E,t) \, dE. \tag{8}
\]

The integration is straightforward and we get the following distribution of first-passage times

\[
P(t) = \frac{1}{2} \text{erfc} \left( \frac{E_m - I_s t}{\sigma \sqrt{2t}} \right) + \frac{1}{2} \exp \left( \frac{2E_m I_s}{\sigma^2} \right) \text{erfc} \left( \frac{E_m + I_s t}{\sigma \sqrt{2t}} \right),
\]

the corresponding density being

\[
\frac{dP(t)}{dt} = \frac{E_m}{\sigma \sqrt{2\pi t^3}} \exp \left[ -\frac{(E_m - I_s t)^2}{2\sigma^2 t} \right].
\]
Our aim is calculating the detection rate, which is the inverse of the mean first passage time, that is

\[ < t > = \int_0^\infty t \frac{dP(t)}{dt} dt. \]  

(9)

The proof that this average gives the inverse of the detection rate is as follows. We consider that the detector is active during a very large time interval. Within it we will have a large number of detection events. Let us assume, for the sake of clarity, that the time intervals between two detection events form a discrete sequence \( t_1, t_2, \ldots, t_j, \ldots \). If we have \( N_j \) time intervals of duration \( t_j \), then the detection rate will be

\[ R = \frac{\sum N_j}{\sum N_j t_j} = \frac{1}{\sum P_j t_j} = \frac{1}{< t >}, \]  

(10)

where \( P_j \) is the probability that a time interval between two detection events has duration \( t_j \). If we pass to the continuous, we shall replace the summation by an integral, giving a rate \( R \) equal to the inverse of \( < t > \), which completes the proof.

The integral in (9) may be reduced to standard form with the change of variables \( x = \frac{E_m}{\sigma \sqrt{2}}, \ y = \frac{I_s}{\sigma \sqrt{2}}, \ u = \sqrt{t}, \) and we obtain

\[ < t > = \frac{2x}{\sqrt{\pi}} \int_0^\infty du \exp \left[ -\left( \frac{x}{u} - yu \right)^2 \right] = \frac{x}{y}. \]

Hence eq.(10) gives

\[ R = \frac{I_s}{E_m}. \]  

(11)

It is remarkable that we obtain a perfect subtraction of the ZPF, a result in agreement with the quantum mechanical prediction. The result may be generalized to the case where the signal intensity is not a constant, but a known function of time. It would be enough to substitute \( \int_0^t I_s(t')dt' \) for \( I_s t \) in the above equations, although the integrals would be more involved. More difficult would be to treat the common case where the signal itself fluctuates (with a correlation time of the order of the inverse of the frequency bandwidth). We shall study that problem elsewhere.

We may now analyze coincidence counts in two detectors when the incoming beams, with intensities \( I_1(t) \) and \( I_2(t) \) above the ZPF, are correlated.
The calculation is not difficult if the correlation time of the signal is of the order of the typical time interval between detection events, or larger. In these conditions we may assume that eq. (10) is still valid for each detector and the coincidence rate, with a time delay $\tau$, will be

$$R_{12} \propto \langle I_1(t)I_2(t+\tau) \rangle,$$

again in agreement with the quantum prediction. However the current situation may not be that (see next section.) In practice the crosscorrelation time of the signals is much shorter than the inverse of the detection rate. Again the calculation in these conditions will be rather involved and shall not be considered here.

IV. Discussion

Our analysis shows that quantum vacuum fluctuations of the electromagnetic field (or ZPF) may be efficiently subtracted by a model which assumes that the radiation is a classical (Maxwell) field including a fluctuating ZPF, provided that the fluctuations of the signal have a large enough correlation time in comparison with the correlation time of the ZPF. This is usually the case in astronomical observations. In contrast, in standard quantum optical experiments the fluctuations of the signal may have a rather short correlation time. If the correlation time of the signal does not fulfil the assumptions of the previous section, the presence of the ZPF will probably give rise to departures from the standard quantum predictions eqs. (10) and (??), that is they will produce some nonidealities in the behaviour of optical photon counters. This is specially important when it is necessary to measure coincidence counting rates with short time windows, as is frequent in quantum optical experiments (e.g. optical tests of Bell’s inequality). If this is the case, our approach may provide an explanation for the difficulties of performing loophole-free tests of Bell’s inequality using optical photons. As is well known all performed experiments suffer from the ”detection loophole” and I conjecture that the cause might be the existence of fundamental nonidealities in the behaviour of photon counters.

I emphasize that, although our model is semiclassical, probably the main properties of the model would be reproduced by a more rigorous quantum treatment. Furthermore the difficulties for reaching an intuitive picture of how detectors subtract the ZPF probably do not derive from quantum theory.
itself, but from the use of approximations like first-order perturbation theory or taking the limit of time $t \to \infty$ in calculating the probability of photon absorption per unit time. Indeed I have conjectured elsewhere that excessive idealizations might be at the origin of the difficulties for understanding intuitively the paradoxical aspects of quantum physics.\footnote{8} Although simplifications are extremely useful for calculations, they tend to obscure the physics.

Acknowledgement. I acknowledge financial support from DGICYT, Project No. PB-98-0191 (Spain). I thank Trevor W. Marshall for pointing out that the solution of eq.(6) was wrong in a previous version of the article.
References

1P. W. Milonni, *The quantum vacuum* (Academic Press, San Diego, 1994)

2Prof. Lamb has expressed the opinion that his experiment provided an empirical proof that the vacuum is not empty. This is related to his statement that the photon is the quantum of the electromagnetic field, but not a particle, see W. E. Lamb Jr., *Appl. Phys. B* 60, 77 (1995). The point is important for the present paper because it stresses the difference between photon counting and detection of true particles, like electrons or atoms. Indeed it is not difficult to detect atoms with practically 100% efficiency and no noise, but this is not the case with photons.

3See, e.g., G. Bressi, G. Carugno, R. Onofrio and G. Rouso, *Phys. Rev. Lett.* 88, 041804-1 (2002) and references therein.

4R. H. Koch, D. J. van Harlingen and J. Clarke, *Phys. Rev. Lett.* 45, 2132 (1980); 47, 1216 (1981); *Phys. Rev. B* 26, 74 (1982).

5K. Dechoum, L. de la Peña and E. Santos, *Found. Phys. Lett.* 113, 253 (2000); A. Casado, R. Ramon-Risco and E. Santos, *Z. Naturforsch.* 56a, 178 (2001); A. Casado, T. W. Marshall, R. Risco-Delgado and E. Santos, arXiv.org/quant-ph/0202097.

6E. Santos, arXiv.org/quant-ph/0206161.

7I. S. Gradshteyn and I. M. Ryzhik (A. Jeffrey, Editor), *Tables of Integrals, Series and Products*, Academic Press, Boston, 1994. Integral nº 3.324.

8See, e.g., F. Laloe, *Am. J. Phys.* 69, 655 (2001).

9E. Santos, arXiv.org/quant-ph/0103062.