Study of Rupture Mechanism in Concrete Girder Strengthened by External Fiber Reinforced Polymer Using Crack Analysis

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Abstract. In the concrete beam strengthened by using fiber-reinforced polymer (FRP) sheet, the failure of the beam in the design standard is considered as the delamination of the FRP sheet at the end of the concrete beam or break of the FRP sheet when its maximum strain reaches. However, as cracks always happen in bending concrete beam, the failure is predicted to occur between two adjacent cracks. This failure also occurs in concrete near the adhesive zone instead of delamination of the FRP sheet. By analyzing the crack formation of a concrete beam under flexure, the average strain of the FRP sheet at failure is assumed as the ratio of crack width and distance between two adjacent cracks. This assumption is consistent with the comparison of experimental data from the literature. The proposed maximum strain value from assumption has a lower value than the strain of FRP sheet specified in design specification ACI 440-2R which is determined from pulling tests. Tests of FRP strengthened concrete beam under flexure rupture were conducted in the laboratory. The failure morphologies confirm that cracks of concrete beam affect significantly rupture type.

1. Introduction
In recent times, with the advent of aged concrete structure, strengthening technics have received an increasing amount of attention. The technics of using fiber-reinforced polymer (FRP) composite in sheet-shaped has become renowned for its ease of implementation. Some recent studies reveal that when the concrete beam under flexure reaches its ultimate strength, the morphologies of rupture are diverse. The rupture occasionally occurs as concrete crush, de-bonding or break of FRP sheet. Furthermore, FRP strengthening makes concrete structure brittle, thereby may not meet the ductility requirements. This ductile issue mainly relates to the cohesion quality between the FRP sheet and the concrete beam. However, detail requirements for this matter remain unclear in design specification and thus attract great attention recently.

To determine the cohesion properties between FRP and concrete, pulling experiments of an FRP sheet adhered to the concrete beam in the pure shear state (Figure 1.a) are common. In the test, the stresses of FRP were measured to investigate the rupture mechanism. Results show that, along with the FRP sheet, the stresses are not linear but are local concentration. Besides, the stress concentrates on the part near the end of the FRP plate. The mobilized adhesion length is only effective in a short-range near the end of the concrete block and the shear stress of concrete at rupture is about C=5MPa [2]. This rupture occurs in the FRP sheet ($\varepsilon_{cr}$) with a length of less than 125mm [1][2][3][4]. The destruction of the reinforced structure begins at the highest stress concentration location where the concrete reaches the tensile strain threshold $\varepsilon = 0.0035$. After that, it continues to mobilize the tensile
strength of concrete at the adjacent zone within the length of $\omega_0$ (Figure 1.b). In case mechanical anchors devices are used, the pulling force can increase the strength by 11% compared to conventional adhesion solutions [4]. Otherwise, the force in the FRP sheet depends only on the tensile strength of the concrete within a certain length range (strength development length).

Chen et al. [5] conducted experimental studies on 50 laboratory samples in combination with the theory of fracture mechanics applied to the adhesion strength. Results reveal that the strength of concrete is the decisive factor affects the development length of an FRP sheet. The results of Chen et al is being used in ACI 440.2R standard [7]. The important result is the strength development length of the FRP sheet as in Eq.1.

$$L_0 = \frac{E_f t_f}{\sqrt{f'_c}}$$ (mm)  \hspace{1cm} (1)

Where: $E_f$ (MPa), $t_f$ (mm), $f'_c$ (MPa) are FRP elastic modulus, FRP thickness and the compressive strength of concrete respectively.

The width of the FRP sheet is a key factor in the cohesion of the FRP sheet with the concrete beam. Some authors used similar pulling experiments to find the influence of FRP sheet width on the cohesion strength [8]. The principal finding is large width better reduces the cohesion of FRP sheet width with concrete beam than narrower ones do. Furthermore, the destructive form is mainly the separation of FRP sheet at the mid-span position [9] where the strain of concrete is largest. The separation then spreads to adjacent areas [10]. Neto et al. [1] conducted a 3-D mathematical model to

**Figure 1.** Pulling experiment for stress distribution on the FRP sheet [1]
analyze the cohesion between the FRP sheet and concrete. The effective width of the destructive surface is in Figure 1.c. The value of this width is shown in Eq.2.

\[ b_{ef} = \left( 2 - \frac{b_f}{b_s} \right) b_f \text{ (mm)} \] (2)

As the reviews above, cohesion and rupture length are mainly based on the experiments of pulling the FRP sheet directly out of the concrete surface. Working conditions of FRP sheet is different to pulling test. Generally, the strengthened beams are under flexural and shear force. As concrete beam always has cracks under that bending condition, these cracks may affect the cohesion of FRP sheet with the concrete beam. Therefore, the above research models may not be similar to the actual working conditions of the strengthened beam. These effects of crack formation have not been addressed. This paper aims to explore effects of the crack to cohesion of FRP sheet with beam and to propose a new approach to predict the rupture of strengthened concrete beam. The work also assesses the effect of crack width on strain of FRP sheet at beam failure as well as the delamination of reinforced FRP sheet from concrete. Flexure tests are conducted until failure to compare the rupture types as model’s validation. The results of this study support the view that cracks in concrete beam are not negligible.

2. Theoretical study

2.1. Review for determining crack distance and crack width

Calculation of crack distance and crack width in a concrete beam under flexure can follow various guidelines [7][11]. This subsection aims to review the guidelines as the material for further analyses.

Crack width is determined as per FIB model code [11]:

\[ w_d = 2l_{ls,max} \left( \varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs} \right) \text{ (mm)} \] (3)

Where: \( w_d \) (mm), \( l_{ls,max} \) (mm), \( \varepsilon_{sm} \), \( \varepsilon_{cm} \), \( \varepsilon_{cs} \) are crack width, distance between cracks, average deformation of steel within \( l_{ls,max} \), average deformation of concrete within \( l_{ls,max} \), average axial deformation of concrete due to shrinkage respectively.

The distance between cracks can be determined as per Eq.4.

\[ l_{ls,max} = k_c + \frac{1}{4} \frac{f_{ctm}}{\tau_{lima}} \frac{\varphi_f}{\rho_f e_f} \text{ (mm)} \] (4)

Where: \( k \) is an experimental coefficient, taken as \( k=1 \); \( c \) is the concrete cover thickness; \( \tau_{lima} \) is the average adhesive stress between concrete and steel. This value is calculated as \( \tau_{lima} = 1.8 f_{ctm} (t) \) per sect. 7.6.2 of FIB model code [11]; \( f_{ctm} \) is the tensile strength at time \( t \) of concrete.

The value of average deformation in Equation (3) as per Eq.5 can be calculated as below:

\[ \left( \varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs} \right) = \frac{\sigma_s - \beta \sigma_{sr} - \eta_t e_{sh}}{E_s} \] (5)

Where: \( \sigma_s \) is the stress in reinforcements at crack location. The basic steps in guidelines [7][11] include:
- Check if the cross-section cracks or not.
- Determine the geometric characteristics of the cracked section.
- Calculate the stress in reinforcement on the cracked section.

In Eq.5, \( \sigma_{sr} \) is the maximal stress in reinforcement at the formation of crack in pure tension.
\[
\sigma_{\text{ut}} = \frac{f_{cu}}{\rho'_{s,\text{ef}}} \left(1 + \alpha_e \rho_{s,\text{ef}} \right) \quad \text{(N/mm}^2) \tag{6}
\]

Where: \( \rho_{s,\text{ef}} = A_s / A_{c,\text{ef}} \) is the ratio of area of reinforcements in tension (\( A_s \)) and area of concrete in tensile zone (\( A_{c,\text{ef}} \)) having the same center of gravity as the reinforcement. The ratio (\( \alpha_e \)) between the elastic modules of steel and concrete:

\[ \alpha_e = E_s / E_c \; ; \]

Coefficient \( \beta \) in Eq.5 is an experimental value and taken as 0.6 according to [11]. \( n_r \) is the experimental coefficient which accounts for the effect of shrinkage. \( \varepsilon_{\text{sh}} \) is the shrinkage deformation of concrete.

2.2. Proposed peeling force of FRP sheet at failure

In the RC beam structure under flexure, from experimental morphologies of girder rupture, the first cracks appear in areas with large bending moments (mid-span). As the load increases, crack width and crack depth increase accordingly. When the mid-span reaches a yielded moment, the cracks will spread to both ends of the beam. The expansion of the crack affects the deformation of the FRP sheet. The larger the crack, the greater the deformation of the sheet. The smaller the crack gap, the shorter the adhesion length of the FRP sheet to the concrete. The behaviors in the FRP sheet will not simply be proportioned to the behavior of the concrete under the assumption of a flat section. There are many values for the tensile strength limit of concrete depending on the working status and adhesion. Supposedly, the concrete of the beam will crack when it reaches the tensile limit \( f_n = 0.25 \sqrt{f_c} \) [12]. According to the statistics from the experiments in [1,2,4,5] failure shall occur in the concrete area near the FRP sheet. When the cracks appear, the length of failure between FRP and concrete rupture will occur on the length equal to the distance between two cracks \( (l_{y,\text{max}}) \). According to the reciprocal stress rule, the shear stress is equal to the tensile stress of concrete in the direction of removing concrete from the reinforcement within two consecutive cracks. This assumption is presented in Figure 2.
The ultimate tensile force corresponding to the destruction of concrete is predicted to be the tensile strength of concrete \(f_n\) on the area limited by two cracks \(L_{s,max}\) and the effective width of concrete \(b_{ef}\). This proposed model is expressed in Eq.7.

\[
N_u = 0.25 \sqrt{f_n b_{ef} L_{s,max}} \quad \text{(N)} \tag{7}
\]

2.3. Predict the strength development length of FRP sheet

From the analysis of the cohesion failure of the FRP sheet and concrete surface, the range of failure is the gap between two cracks on a flexural beam structure \(l_{ls,max}\) as in Eq.8.

\[
L_E = L_{ls,max} \quad \text{(mm)} \tag{8}
\]

2.4. Strain of FRP sheet at rupture

The limit of strain in FRP sheet is specified in ACI 440.2R [7] and is expressed in Eq.9.

\[
\varepsilon_{f\text{ld}} = 0.41 \sqrt{\frac{f_c}{nE_f t_f}} \leq 0.9 \varepsilon_{f\text{u}} \tag{9}
\]

However, calculated data as well as experimental ones from the literature [1][2][3] showed that FRP strain was about 1% when flexure rupture occurred whereas the ultimate strain of FRP sheet was higher (around 3%). Based on the test beams outlined in experimental data of Chen et al, 2001 [5], it can be assumed that the thickness of the concrete cover is not too large. The value of concrete cover varies at about 30mm. Therefore, the crack width at failure is normally proportional to the stress in the tensile reinforcement. The average value of crack width is 0.354mm at flexure rupture. The average strain of the FRP sheet at rupture is determined by the ratio of crack width and distance between two cracks. As a result, this strain value is an average strain and can be calculated as Eq.10.

\[
\varepsilon_{f\text{ld}} = \frac{W_d}{l_{ls,max}} \tag{10}
\]

3. Comparison of proposed theory to experiments

To validate the proposed theory, the difference between experimental data from literature and the theory is investigated. The values for comparison are the tensile force of the pulling test shown in Figure 1. The proposed development lengths are compared to data results of the work of Chen et al. [5] as well as ACI 440.2R [7]. Tests are also conducted to show the rupture morphology of concrete beam strengthened by using various types of FRP.
3.1. Comparison to experimental data from literature

The ultimate tensile force in the FRP sheet is calculated according to Eq. 7 then compared with 32 experimental results of [5]. The results are shown in Figure 3.

Figure 3. Comparing of cohesion force between proposed theory and experiments

A Comparison in Figure 3 shows that the prediction of failure force is close to the published experimental data. The peeling force of the FRP sheet from concrete in the failure state accords to the assumption of tensile rupture of concrete within the length between two cracks.

3.2. Comparison to other results about development length

A development length of the FRP sheet is calculated by using Eq. 8 and then compared with the research results of Chen et al. in Table 1 of reference [5] as well as results from Eq.1. The results, in Figure 4, indicate that the development length, proposed in Eq.8, is shorter than the theoretical basis of [5] as well as ACI 440.2R.

Figure 4. Comparing the anchor length of FRP sheet
3.3. Strain of FRP sheet at rupture

A strain of the FRP sheet at rupture is calculated following the proposed model at Eq.10. Comparison between calculated strain and experimental data of Reed et al. [6] (ACI 440.2R standard in Eq. 9 [7]) and Reed et al, 2016 [6]. The results are shown in Figure 5.

![Figure 5. Strain of FRP sheet at beam failure](image)

Figure 5 shows that the proposed model in Eq.10 is approximate to the experimental data of [6]. Beam fails almost immediately when the strain of the FRP sheet is still low. According to the ACI 440.2R standard [7], the allowable strain in the FRP sheet is higher while the proposed model is closer to experimental data of Reed et al [6]. As experiments of Reed et al [6] were conducted in a flexure beam, the result of Reed et al reflects closer working condition than those from pulling test of Chen et al 2001 [5] and thus better describe the actual mechanism of the concrete beam at rupture. Consequently, the strain at failure of the FRP sheet proposed in Eq.10 reflects closer results to [6]. Furthermore, strain in FRP sheets is still low compared to the results of [5][7]. This can be explained that due to the early destruction of concrete, beam damage appears earlier than expected and the strain of FRP sheet is lower than in requirement of ACI440.02R.

3.4. Experiments on the actual failure of beams

Tests were conducted to verify the accordance of the rupture morphology to the assumption. The tested beams were reinforced concrete beams which dimensions were \(L*h*b = 2500*250*100\) (mm). These beams were strengthened by using one to two layers of carbon fiber composite sheet or one to five layers of glass fiber reinforced composite. Four-point bending test was conducted until failure at Structural Engineering Laboratory at the University of Transport and Communications. The rupture morphologies are shown in Figure 6.
The failures of the beam in Figure 6 show that the distance between the cracks is similar to the theoretical calculation of the crack length. I.e. within a range of approximately 120 mm. Concrete popped out between the reinforcement layer and the FRP sheet in the middle span of the beam. This morphology shows that the possibility of rupture of concrete beam strengthened with FRP happens in mid-span of beam rather than delamination of FRP ends at anchorage zone.

3.5. Discussion

Following the results of theoretical studies and experimental data, the location of rupture appears at the concrete zone between the FRP sheet and the concrete beam. The rupture happens in the cracked zone rather than dragging at the ends of the FRP sheet. The strain of the FRP sheet at rupture remains low while the surrounding concrete fails already. The minimum tensile strength of adhesive such as epoxy resin is normally more than 15MPa, which is much greater than the tensile strength of concrete (about 3.5 MPa). Therefore, the failure of concrete at the contact zone hardly happens. When concrete beams are subjected to bending and cracks occur, the gap between the cracks is smaller than the strength development length and thus the beam rupture comes earlier than the prediction of current design code [7]. Consequently, the average strain of the FRP sheet at rupture reduces and is hard to reach the

Figure 6. Failure of concrete strengthened FRP sheet in concrete near adhesive zone between two cracks
maximum strain limit in [7]. The maximum tensile stress focuses on the mid-span area of the strengthened beam under flexure. Assuming that the shear failure of the beam does not occur, the first failure point shall be in the middle of the span where the bending moment is at the highest value. To ensure the strengthening requirements, the development length at the ends of the FRP sheet may not be enough. The requirement of ACI 440.2R of six inches is appropriate compared to this study. However, in the case of shear force is insignificant, the failure occurs in the middle span of the beam where the bending moment is maximal.

Notwithstanding that cracks affect the rupture mechanism of the flexure beam, the rupture types of strengthened concrete beam remain unsolved. Further research should be undertaken to investigate the effects of anchoring solutions of FRP sheet, beam length, beam height, and loading condition to the strengthened beam. This study addresses the roles of cracks, which happen in bending concrete beam. As a result, strengthening the cohesion of the FRP sheet to concrete in the flexure zone (mid-span) is sometimes more important. Some necessary solutions may include U-wrapping around the beam at the high bending moment area. This solution ensures the working conditions of the beam following the rupture model of ACI 440.2R design standard. This study helps to improve the brittle destructive requirements of beams reinforced with FRP sheet and thus meets the ductility requirement.

4. Conclusions
Based on the analysis of the crack mechanism of reinforced concrete beams strengthened by the FRP sheet, the failure of concrete near the FRP sheet at the zone of high moment value may occur first. This rupture appears at the concrete zone near the adhesive zone of FRP and between two adjacent cracks. In this case, the maximum strain of the FRP sheet at rupture does not reach the limit strain specified in the design specification of the ACI committee. Development length resulted from pulling test in 2001, should be updated following the same bending condition.

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