Brane Solutions with/without Rotation in PP-wave Spacetime

Rashmi R. Nayak\textsuperscript{a}\textsuperscript{*}, Kamal L. Panigrahi\textsuperscript{b}\textsuperscript{†}, and Sanjay Siwach\textsuperscript{c}\textsuperscript{‡}

\textsuperscript{a}Institute of Physics, Bhubaneswar 751 005, INDIA

\textsuperscript{b}Dipartimento di Fisica, Universita’ di Roma “Tor Vergata”
INFN, Sezione di Roma “Tor Vergata”, Via della Ricerca Scientifica 1
00133 Roma, ITALY

\textsuperscript{c}Institute of Mathematical Sciences, CIT Campus, Taramani,
Chennai, 600 113, INDIA

Abstract

We present two classes of brane solutions in pp-wave spacetime. The first class of branes with a rotation parameter are constructed in an exact string background with NS-NS and R-R flux. The spacetime supersymmetry is analyzed by solving the standard Killing spinor equations and is shown to preserve the same amount of supersymmetry as the case without the rotation. This class of branes do not admit regular horizon. The second class of brane solutions are constructed by applying a null Melvin twist to the brane solutions of flat spacetime supergravity. These solutions admit regular horizon. We also comment on some thermodynamic properties of this class of solutions.

\textsuperscript{*}e-mail: rashmi@iopb.res.in
\textsuperscript{†}e-mail: Kamal.Panigrahi@roma2.infn.it, INFN fellow
\textsuperscript{‡}e-mail: sanjay@imsc.res.in
1 Introduction

Study of string theory in plane wave (or pp-wave) background has drawn lots of attention in the last couple of years, in search of establishing AdS/CFT like dualities. These backgrounds can be seen as a small deformation of ten dimensional Minkowski spacetime [1]. Plane wave (or pp-wave) spacetime qualifies, the most, for analyzing certain issues of quantum gravity and give a consistent background for studying string theory in light-cone gauge. These backgrounds are obtained by applying Penrose-Guven limit on $\text{AdS}_p \times S^q$ type of geometries and also from the near horizon geometries of various supergravity solutions in diverse dimensions. Of particular interest, is the maximally supersymmetric pp-wave background which is obtained from the near horizon geometry of coincident D3-branes in ten-dimensional spacetime in the Penrose limit. String theory in this background is exactly solvable in light-cone gauge and is shown to be dual to $\mathcal{N} = 4$ super Yang-Mills theory in large R charge sector [2]. The PP-wave/CFT dualities have been analyzed (See [3] for the updated references on this subject), and some speculations have been made regarding the ‘holography’ in plane wave backgrounds. Whereas the above issues are slightly more clear in backgrounds with NS-NS 3-form flux [4–9] (e.g. the Nappi-Witten backgrounds and $\text{AdS}_3 \times S^3$ spacetime), they are not very profound in the case of maximally supersymmetric plane wave background with R-R flux.

Plane wave spacetime in the presence of non-constant flux is also an interesting background to study string theory, as it provides examples of integrable models on the worldsheet [10, 11]. These backgrounds can also be interpreted as the deformation of the flat spacetime and are supported by null matter fields. The corresponding worldsheet theory is described by the nonlinear sigma model [10–13]. and represent the nontrivial examples of interacting theories in light-cone gauge. The pp-wave backgrounds with non-constant 3-form NS-NS ($H_3$) and R-R ($F_3$) flux do not admit, in contrast to their 5-form R-R flux ($F_5$) counterpart, the linearly realized ‘supernumerary’ killing spinors. Moreover these theories are closely related to the closed strings in a constant magnetic field, in the presence of antisymmetric tensor fields and a non-trivial metric [14], which also provides an example of $\alpha'$ exact string theory background. These backgrounds are known to be homogeneous plane wave backgrounds and string theory in these type of spacetime has been analyzed in great detail [15,16].

In recent years, several important aspects of string dualities have been revealed and D-branes have played important role in these developments. While D-branes can be treated as the black branes in supergravity theories, the effective field theory on the brane are of super Yang Mills type theories. So the study of D-branes in various non trivial and non generic backgrounds with/without flux give ideas about the structures of string theory as well as the related gauge theories. D-branes in various curved back-
ground have generated renewed interest in the context of plane wave background for various reasons. First, these nonperturbative objects are easily tractable in pp-wave background and second, the supergravity solutions can be constructed with not much efforts as compared to its AdS counterpart. Various D-brane supergravity solutions in maximally and less than maximally supersymmetric plane wave backgrounds have been analyzed in the past couple of years [17–28]. In this context various attempts have also been made in finding out black hole/brane solutions with regular horizon in plane wave space time. The analysis of [25,29] shows strong evidence in favour of the non-existence of horizon in the spacetime with covariantly constant and null Killing vectors, proposed in [30]. Moreover, the brane solutions seem to be singular. In [31], however, a consistent method for obtaining a black string solution with regular horizon has been discussed, which relies on a solution generating technique [32] known as null Melvin twist (NMT). This particular mechanism, transforms a flat spacetime to a plane wave spacetime. So the natural guess would be to start with a black brane solution in flat spacetime and apply NMT to obtain a solution in plane wave spacetime which preserves horizon. As proved in [31], this particular transformation indeed preserve the horizon and that the area of the horizon remains the same even after the NMT.

Motivated by the recent interest in finding out exact string backgrounds and the classical solutions of branes and their bound states, in various nontrivial backgrounds with/without flux, in this paper, we present some Dp and Dp-Dp’ branes in homogeneous pp-wave background with non-constant flux and with a rotation parameter. This class of solutions are seen not to admit a regular horizon. We also examine another class of D-branes which are obtained by applying a solution generating technique known as null Melvin twist. These class of solutions do keep the asymptotic of the spacetime as that of the plane waves but don’t give rise to the null matter content of the theory. The rest of the paper, is organized as follows. In section-2, after a small digression for the discussion of the homogeneous plane wave space time, we present the classical solutions of some Dp as well as Dp-Dp’ branes in this background with an explicit inclusion of the rotation parameter. We keep the fluxes completely general and show that they solve supergravity field equations. In section-3, we analyze the supersymmetry of the background and the branes in this background by solving the Killing spinor equations. The solutions of the Killing spinor equations are shown to constrain the structure of the 3-form fluxes. We also make some remarks on the properties of horizon of these brane solutions. In section-4, we present classical solutions of black branes in asymptotically plane wave spacetime by applying the null Melvin twist on the non-extremal brane solutions of flat spacetime supergravity. We also compute the horizon area and temperature of these black branes. In section-5, we conclude with some remarks and discussions.
2 Branes with rotation in pp-wave background

2.1 The Background

As a warm up exercise, below, we recapitulate few basic facts about the homogeneous plane wave background discussed in [16], which will be helpful in fixing the notations etc. The most general null Brinkmann metric in d-dimensions with flat transverse space is given by:

\[ ds^2 = 2dudv + \mathcal{H}(u, x)du^2 + 2A_i(u, x)dx^i du + dx^i dx_i \]  

(2.1)

Exact string backgrounds, with this metric, can be constructed by switching on the appropriate field strengths and the dilaton:

\[ B_{iu}^{NS} = B_i(u, x), \quad \phi = \phi(u). \]  

(2.2)

For the above ansatz, the one-loop conformal invariance, or in other words, the type II supergravity field equations give the following conditions:

\[ -\frac{1}{2} \Box \mathcal{H} + \partial_u \partial^i A_i + \frac{1}{4} F_{ij} F^{ij} - \frac{1}{4} H_{ij} H^{ij} + 2\partial_u^2 \phi = 0, \]

\[ \partial^i F_{ij} = 0, \quad \partial^i H_{ij} = 0, \]  

(2.3)

where \( F_{ij} = \partial_i A_j - \partial_j A_i \) and \( H_{ij} = \partial_i B_j - \partial_j B_i \). In principle, the general solutions to these equations do not define an exact background. Some of the special cases where it does, has been discussed in [16].

In the present paper, we shall be interested in the supergravity background with the metric, NS-NS 2-form \( (\mathcal{B}) \) and R-R two form \( (\mathcal{B}') \) (and a constant dilaton):

\[ ds^2 = 2dudv + \mathcal{H}(x_i)du^2 + 2JA_i(x_i)dx^i du + \sum_{i=1}^4 (dx^i)^2 + \sum_{a=5}^8 (dx^a)^2, \]

\[ \mathcal{B} = b_i(x_i)du \wedge dx^i, \quad H_3 = H_{ij}(x_i) du \wedge dx^i \wedge dx^j, \]

\[ \mathcal{B}' = J A_i(x_i) du \wedge dx^i, \quad F_3 = J F_{ij}(x_i) du \wedge dx^i \wedge dx^j, \]  

(2.4)

where \( H_3 \) and \( F_3 \) are the field strengths associated with the \( \mathcal{B} \) and \( \mathcal{B}' \) respectively: \( F_{ij} = \partial_i A_j - \partial_j A_i \) and \( H_{ij} = \partial_i B_j - \partial_j B_i \). \( J \) is an arbitrary constant parameter. Note that we are considering the case in which the above metric functions are independent of the light-cone time \( u \). We keep the most general form of the NS-NS and R-R field strengths, but the restrictions on them would be imposed by the requirement of supersymmetry, as we will see in the subsequent analysis. Few remarks regarding the
structure of $A(x^i)$ are in order now. If we restrict: $J A_i dx^i du = J \epsilon_{ij} x^i dx^j du$, where $\epsilon_{12} = \epsilon_{34} = 1$, then this can be interpreted as the rotation in the $x^i$ space of solutions, parametrized by $J$. That would further restrict the background fields turned on, which in turn play an important role in the analysis of Killing spinor equations. We will come back to this issue later on.

To be a consistent solution of supergravity, the above ansatz should be supplemented by the constraints:

$$\Box^{(i)} H(x_i) = -(\partial_i b_j)^2, \quad \partial^k F_{k \ell} = 0, \quad \partial^k H_{k \ell} = 0. \quad (2.5)$$

We have checked that the solution (2.4) supplemented by the conditions (2.5) satisfy all the type IIB field equations and the Bianchi identities. We shall be interested in this background for the subsequent analysis of the present paper. As can be seen from the metric (2.4) that switching off the gauge field ($A$) we get back to the pp-wave metric of [11] with non-constant NS flux.

### 2.2 Supergravity solutions

In this section, we present classical solutions of branes in the above background supported by NS-NS ($H_3$) and R-R ($F_3$) flux in the transverse direction of the branes. We start by writing down the supergravity solution of $N$ D-strings lying on top of each other in this background. The metric, the dilaton, and the field strengths of such a configuration is given by:

$$ds^2 = f_1^{-1/2} \left( 2 du dv + H(x_i) du^2 + 2 J A_i(x_i) du dx^i \right) + f_1^{1/2} \left( \sum_{i=1}^{4} (dx^i)^2 \right) + \sum_{a=5}^{8} (dx^a)^2, \quad H_3 = H_{ij}(x_i) \ du \wedge dx^i \wedge dx^j, \quad e^{2\phi} = f_1,$$

$$B' = (f_1^{-1} - 1) \ du \wedge dv + \frac{J}{f_1} A_i(x_i) \ du \wedge dx^i. \quad (2.6)$$

where $f_1 = 1 + \frac{Q_1}{r^6}$ is the harmonic function in the transverse space of the D-string and $B'$ is the Ramond-Ramond potential. We have checked that the above solution solves type IIB field equations provided the constraints (2.5) are also imposed. Similarly, a

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1Similar analysis have also been performed in the context of closed strings in the presence of magnetic field, and proved to be conformally invariant background [14]
D5-brane solution is given by:

\[
\begin{align*}
 ds^2 &= f_5^{-\frac{1}{2}} \left( 2dudv + \mathcal{H}(x_i)du^2 + 2JA_i(x_i)dudx^i + \sum_{a=5}^{8} (dx^a)^2 \right) \\
 &\quad + f_5^{-\frac{1}{2}} \sum_{i=1}^{4} (dx^i)^2, \\
 H_3 &= H_{ij}(x_i) \, du \wedge dx^i \wedge dx^j, \quad e^{2\phi} = f_5^{-1}, \\
 F_3 &= JF_{ij}(x_i) \, du \wedge dx^i \wedge dx^j, \quad F_{ijk} = \epsilon_{ijk} \partial_i f_5,
\end{align*}
\]

where \( f_5 = 1 + \frac{Q_5}{r^2} \) is the harmonic function in the transverse 4-space. \( F_3 \) and \( F_{ijk} \) are the Ramond-Ramond field strengths. We have once again checked that the above ansatz solves type IIB field equations provided the constraints (2.5) are also imposed. The above solutions reduce to those presented in [24] for \( J = 0 \). Other Dp-brane (for \( p \geq 2 \)) solutions can also be constructed first by smearing \( a = x^5, \ldots x^8 \) directions and then by applying T-dualities along those.

Now we present the classical solution of D1-D5 system, as an example of Dp-D(p+4) brane bound state in the background (2.4). The metric, dilaton, NS-NS and R-R fields of such a configurations are given by:

\[
\begin{align*}
 ds^2 &= (f_1 f_5)^{-\frac{1}{2}} \left( 2dudv + \mathcal{H}(x_i)du^2 + 2JA_i(x_i)dudx^i \right) + \left( \frac{f_1}{f_5} \right)^{\frac{1}{2}} \sum_{a=5}^{8} (dx^a)^2 \\
 &\quad + \left( \frac{f_1}{f_5} \right)^{\frac{1}{2}} \sum_{i=1}^{4} (dx^i)^2, \\
 H_3 &= H_{ij}(x_i) \, du \wedge dx^i \wedge dx^j, \quad e^{2\phi} = f_5^{-1}, \\
 B' &= (f_1^{-1} - 1) \, du \wedge dv + \frac{J}{f_1} A_i(x_i) \, du \wedge dx^i, \\
 F_{ijk} &= \epsilon_{ijk} \partial_i f_5,
\end{align*}
\]

where \( f_{1,5} = 1 + \frac{Q_{1,5}}{r^2} \) are the harmonic functions of D1 and D5 brane in the common transverse 4-space. We have once again checked that the above ansatz solves type IIB field equations provided the identities (2.5) are also imposed. This solution reduces to the D1-D5 brane solution of [24] for \( J = 0 \). We would like to point out that a similar solution has already been presented in [26]. However the choice of the background flux turned on to compensate the effect of \( \mathcal{H} \) is different here. The corresponding differential equations for \( \mathcal{H} \) is given by (2.5) and is independent of the parameter \( J \) for all the solutions. The explicit solution can be read off from the reference [25] for the branes presented here.
3 Supersymmetry

The supersymmetry variation of dilatino and gravitini fields of type IIB supergravity in ten dimensions, in string frame, is given by [33, 34]:

\[
\delta \lambda_\pm = \frac{1}{2} (\Gamma^\mu \partial_\mu \phi \mp \frac{1}{12} \Gamma^{\mu \nu \rho} H_{\mu \nu \rho}) \epsilon_\pm + \frac{1}{2} e^{\phi} (\pm \Gamma^M F_M^{(1)} + \frac{1}{12} \Gamma^{\mu \nu \rho} F^{(3)}_{\mu \nu \rho}) \epsilon_\mp, \quad (3.1)
\]

\[
\delta \Psi_\mu^\pm = \left[ \partial_\mu + \frac{1}{4} (w_{\mu \hat{a} \hat{b}} \mp \frac{1}{2} H_{\mu \hat{a} \hat{b}}) \Gamma^{\hat{a} \hat{b}} \right] \epsilon_\pm
+ \frac{1}{8} e^\phi \left[ \mp \Gamma^\lambda F_\lambda^{(1)} - \frac{1}{3!} \Gamma^{\lambda \nu \rho} F^{(3)}_{\lambda \nu \rho} + \frac{1}{2.5!} \Gamma^{\lambda \nu \rho \alpha \beta} F^{(5)}_{\lambda \nu \rho \alpha \beta} \right] \Gamma_\mu \epsilon_\mp, \quad (3.2)
\]

where we have used (\mu, \nu, \rho) to describe the ten dimensional space-time indices, and hats represent the corresponding tangent space indices.

3.1 Background supersymmetry

Before analyzing the supersymmetry of the rotating Dp and Dp-Dp’ brane solutions in homogeneous plane wave background, let’s first discuss the supersymmetry of the background itself. The dilatino (3.1) and gravitino (3.2) variations impose nontrivial conditions on the spinor \( \epsilon_\pm \). First the dilatino variation gives:

\[
\mp \Gamma^{\hat{a} \hat{b}} H_{\hat{a} \hat{b}} \epsilon_\pm + J \Gamma^{\hat{a} \hat{b}} \mathcal{F}_{\hat{a} \hat{b}} \epsilon_\mp = 0. \quad (3.3)
\]

Similarly, from the gravitini variations, we get the following conditions on the spinors to have nontrivial solutions:

\[
\delta \psi_\pm^u \equiv \left( \partial_u + \frac{J}{8} \mathcal{F}_{\hat{a} \hat{b}} \Gamma^{\hat{a} \hat{b}} \mp \frac{1}{4} H_{\hat{a} \hat{b}} \Gamma^{\hat{a} \hat{b}} \right) \epsilon_\pm - \frac{J}{8} \mathcal{F}_{\hat{a} \hat{b}} \Gamma^{\hat{a} \hat{b}} \Gamma^\theta \epsilon_\mp = 0,
\]

\[
\delta \psi_\pm^v \equiv \partial_v \epsilon_\pm = 0, \quad \delta \psi_\pm^a \equiv \partial_a \epsilon_\pm = 0, \quad \delta \psi_\pm^i \equiv \partial_i \epsilon_\pm = 0. \quad (3.4)
\]

In writing down the above supersymmetry variations we have made use of the standard supersymmetry condition\(^2\)

\[
\Gamma^\hat{u} \epsilon_\pm = 0. \quad (3.5)
\]

After imposing this condition the dilatino variation, above, is satisfied. One notices that for the variations of the remaining terms in the gravitino variation \( \delta \psi_\pm^u \), for a constant spinor \( \epsilon_0 \), we need to restrict the structure of the background flux \( \mathcal{F}_{\hat{a} \hat{b}} \) and

\(^2\)this condition does not depend on the details of the pp-wave background that we are considering.
also \( H_{ij} \). One such possibility has been discussed in [24]. For the case: \( F_{u12}^{(3)} = F_{u34}^{(3)} \) and \( H_{u12}^{(3)} = H_{u34}^{(3)} \), with all other components of \( F_{ij} \) and \( H_{ij} \) set to zero, we have to impose the condition:

\[
(1 - \Gamma^{1234}) \epsilon_\pm = 0. \tag{3.6}
\]

So the amount of supersymmetry preserved, after imposing the above two conditions, (3.5) and (3.6) is 1/4 of the original one. This fact has also been shown in [35]. We would like to point out that the addition of the rotation \( J \), does not destroy more supersymmetry compared to the case without \( J \). So the natural guess would be that the fate of the remaining supersymmetry in the presence of D-branes will be the same as the case without \( J \), that has been explained in [24, 27]. We will examine this fact by giving examples of D-branes that we have considered in the previous section.

### 3.2 D-brane supersymmetry

In this section, we analyze the supersymmetry conditions for the D-string (2.6) and the D1-D5 brane bound state (2.8) solutions presented in the previous section. First, the dilatino variation equation for the D-string solution presented in (2.6) gives:

\[
\left( \Gamma^\dot{\alpha} \epsilon_\pm - \Gamma^{\dot{\alpha} \dot{\alpha}} \epsilon_\pm \right) \partial_{\delta} f_1 \mp \frac{f^2_1}{4} \Gamma^{\dot{\alpha} \dot{\alpha}} H_{\dot{\alpha} \dot{\alpha}} \epsilon_\pm + \frac{J}{4} f^2_1 \Gamma^{\dot{\alpha} \dot{\alpha}} F_{\dot{\alpha} \dot{\alpha}} \epsilon_\pm = 0, \quad \alpha = 1, \ldots, 8. \tag{3.7}
\]

On the other hand, the gravitini variations gives the following conditions on the spinors to have nontrivial solutions:

\[
\begin{align*}
\delta \psi^\pm_u &\equiv \left( \partial_u + \frac{J}{4} f_1 \Gamma^{\dot{\alpha} \dot{\alpha}} F_{\dot{\alpha} \dot{\alpha}} \right) \epsilon_\pm = 0, \\
\delta \psi^\pm_v &\equiv \partial_v \epsilon_\pm = 0, \\
\delta \psi^\pm_i &\equiv \left( \partial_i + \frac{1}{8} \frac{\partial_i f_1}{f_1} \right) \epsilon_\pm = 0, \\
\delta \psi^\pm_a &\equiv \left( \partial_a + \frac{1}{8} \frac{\partial_a f_1}{f_1} \right) \epsilon_\pm = 0. \tag{3.8}
\end{align*}
\]

In writing down the above gravitini variations, we have made use of the D-string supersymmetry condition:

\[
\epsilon_\pm - \Gamma^{\dot{\alpha} \dot{\alpha}} \epsilon_\pm = 0, \tag{3.9}
\]

in addition to the necessary condition (3.5). By imposing (3.5), the dilatino variation is satisfied. The gravitini variations \( \delta \psi^\pm_v, \delta \psi^\pm_i \) and \( \delta \psi^\pm_a \), solve for the spinor:
\[ \epsilon_{\pm} = \exp \left(-\frac{1}{8} \ln f_1 \right) \epsilon_{\pm}^0, \] with \( \epsilon_{\pm}^0 \) being a spinor which can depend on the coordinate \( u \), leaving the following equations to have a nontrivial solution:

\[
\left( \frac{J}{4} f_1^{-\frac{1}{2}} F_{ij} \Gamma^{ij} + \frac{1}{4} H_{ij} \Gamma^{ij} \right) \epsilon_{\pm}^0 - \frac{J}{8} f_1^{\frac{1}{2}} F_{ij} \Gamma^{\dot{a}\dot{b}} \Gamma^{\dot{a}} \epsilon_{\mp}^0 = 0, \tag{3.10}
\]

and

\[
\partial_u \epsilon_{\pm}^0 = 0. \tag{3.11}
\]

One can see that the existence of the solution to the above equations can be obtained by restricting the form of the functions \( F_{ij} \) and \( H_{ij} \). By making the choice: \( F_{u12} = F_{u34} \) and \( H_{u12} = H_{u34} \), we get an additional condition on the spinors \( \epsilon_{\pm}^0 \):

\[
(1 - \Gamma^{12\bar{3}4}) \epsilon_{\pm}^0 = 0. \tag{3.12}
\]

Coming back to the counting of the surviving supersymmetry (which are of ‘standard type’ only), it is easy to see that the D-string solution (2.6), after imposing the conditions (3.9), (3.12) along with (3.5), preserves 1/8 of the original supersymmetry. Therefore, even with the addition of the rotation \( J \), the D-string supersymmetry remains the same as in the non-rotating case, discussed in [24]. One can also show in a similar way that the D5-brane solution presented in (2.7) preserves 1/8 of the supersymmetries.

Now let’s analyze the supersymmetry of the D1-D5 brane bound state solution presented in (2.8). First the dilatino variation gives:

\[
\delta \lambda_{\pm} \equiv \left( \Gamma^i \epsilon_{\pm} + \Gamma^{\dot{a}\dot{b}} \epsilon_{\mp} \right) \frac{f_{1,i}}{f_1} - \left( \Gamma^i \epsilon_{\pm} - \frac{1}{3!} \epsilon_{\mp}^{ij} \Gamma^{jkl} \epsilon_{\mp} \right) \frac{f_{5,ij}}{f_5} + (f_{1} f_{5}) \frac{i}{4} \Gamma^{\dot{a}\dot{b}} H_{ij} \epsilon_{\pm} \]

\[
+ J \left( \frac{f_{5}^{\frac{1}{2}}}{f_5} \right) \Gamma^{\dot{a}\dot{b}} F_{ij} \epsilon_{\mp} = 0. \tag{3.13}
\]

The gravitino variations are very similar to those presented for the D-string case. Therefore we skip the detailed expressions for those. After imposing respectively the D-string and the D5-brane supersymmetry conditions:

\[
\Gamma^i \epsilon_{\pm} - \Gamma^{\dot{a}\dot{b}} \epsilon_{\mp} = 0, \tag{3.14}
\]

and

\[
\Gamma^i \epsilon_{\pm} - \frac{1}{3!} \epsilon_{\mp}^{ij} \Gamma^{jkl} \epsilon_{\mp} = 0, \tag{3.15}
\]

\(^3\)since \( \epsilon_{\pm}^0 \) is a function of \( u \) only and both \( \mathcal{F}_{ij} \) and \( H_{ij} \) are functions of \( x^i \) only.
along with the standard supersymmetry condition (3.5), the dilatino variation is fully satisfied. The gravitini variation equations, however, would require, an additional condition:

\[(1 - \Gamma^{1234})\epsilon^0_{\pm} = 0,\]  

for the existence of a constant spinor \((\epsilon^0_{\pm})\) solution similar to that presented for the D-string case. Let’s now count the amount of supersymmetry preserved after imposing all these conditions. First of all, \(\Gamma^a\epsilon_{\pm} = 0\) breaks half of the supersymmetries. The fate of the remaining supersymmetries can be found out by examining the conditions (3.14)-(3.16). It is easy to see however that they indeed are only two independent conditions on the spinor \(\epsilon\). So the D1-D5 solution presented in (2.8) preserves 1/8 of the supersymmetries.

Few remarks are in order now. As can be seen from the supersymmetry analysis of the D-branes in the pp-wave background, there is always a ‘decoupling’ between the standard D-brane supersymmetry conditions and the supersymmetry condition imposed by the pp-wave (which in turn comes from the light-cone gauge fixing). We would like to note that the branes that we are considering here are longitudinal branes [19] (all the light-cone directions fall into the worldvolume directions of the branes and other pp-wave directions are transverse to the brane).

### 3.3 Horizon/No horizon

The plane wave spacetime with covariantly constant and null Killing isometries, as such do not admit horizon [30]. Relaxing the covariant constancy condition raised some hope that there might exist horizon in spacetime admitting null killing isometry only (e.g. p-brane solutions in pp-wave spacetime) [29]. However, the analysis of [25, 29] shows strong evidence in favour of the non-existence of regular horizon for p-branes in pp-wave spacetime.

To examine the issue that the solutions presented in previous section admit regular horizon or not, one would like to see how the curvature tensors behave in the near horizon limit. Without addition of the rotation term, it has been noticed earlier that the potential divergent quantities are the components of the Riemann tensor [25,36]. An invariant measure of the divergence are the Riemann tensor components as measured in an orthonormal frame. A natural choice for it is the parallel transported frame as emphasized in [36]. One can show that in the parallel transported frame some of the Riemann tensors diverge in the near horizon geometry, thereby showing the appearance of pp-curvature singularities [25,36]. Hence their doesn’t exist the regular event horizon. Generalization to the non-extremal solutions along the lines
of [25,29] doesn’t improve the situation. It is not difficult to see that the addition of rotation term doesn’t change the situation in both extremal and non-extremal cases.

However it has been argued that the pp-singularities close-off of spacetime near the horizon [36], thereby acting as the boundary of the spacetime and if one accepts this interpretation, the issue that the horizon is a singular surface becomes less important and the solutions can be thought of well behaved.

4 PP-wave branes from flat spacetime brane solutions

We start by writing down the most general non-extremal p-branes in ten dimensions. The metric, dilaton and the field strengths of such non-extremal p-branes in ten dimensional spacetime is given by:

\[
\begin{align*}
    ds^2 &= H_p^{\frac{1}{p}} \left( - f(r) dt^2 + \sum_{\alpha=1}^{p} (dy^\alpha)^2 \right) + H_p^{\frac{1}{p}} (f^{-1}(r) dr^2 + r^2 d\Omega_{d+1}^2), \\
    e^\phi &= H_p^{\frac{3}{4-p}}, \quad E^{(p+1)} = \tilde{Q} f(r) dt \wedge dy^1 \wedge dy^2 \wedge ... \wedge dy^p, \\
    H_p &= 1 + \frac{Q_p}{r^d}, \quad f(r) = 1 - \frac{\mu}{r^d}. 
\end{align*}
\]

(4.1)

Where \( H_p \) is the harmonic function for the \( p \)-branes which satisfies the Green function equation in the transverse \( d+2 = 9-p \) space and \( f(r) \) is the nonextremal parameter.

Now we apply the solution generating technique, NMT as described in [31] on (4.1) to generate black \( p \)-branes in the plane wave spacetime. The first step involves a boost along \( y_1 \) (which is one of the isometry directions along the brane). The resulting metric and field strength of the boosted \( p \)-brane becomes:

\[
\begin{align*}
    ds^2 &= H_p^{\frac{1}{p}} \left( - \hat{K}^{-1}(r) f(r) dt^2 + \hat{K}(r)[d\tilde{y}_1 + A(r)d\tilde{y}]^2 + \sum_{2}^{p} (dy^\alpha)^2 \right) \\
    &+ H_p^{\frac{1}{p}} (f^{-1}(r) dr^2 + r^2 d\Omega_{d+1}^2), \\
    e^\phi &= H_p^{\frac{3}{4-p}}, \quad E^{(p+1)} = \tilde{Q} f(r) \gamma d\tilde{t} \wedge d\tilde{y}^1 \wedge dy^2 \wedge ... \wedge dy^p, \\
    dt &= \cosh \gamma d\tilde{t} - \sinh \gamma d\tilde{y}^1, \quad dy^1 = - \sinh \gamma d\tilde{t} + \cosh \gamma d\tilde{y}^1, 
\end{align*}
\]

(4.2)

where:
The next and the vital step of the NMT is to apply a twist \( \sigma \rightarrow \sigma + 2\alpha d\tilde{y}_1 \), where the one form \( \sigma \) is defined such that \( d\Omega^2_{d+1} = \frac{1}{4}\sigma^2 + d\Sigma^2_d \). The spacetime, after this twist becomes:

\[
\begin{align*}
\hat{K}(r) &= 1 + \frac{Q}{r^d}, \quad A(r) = \frac{Q}{r^d} \hat{K}^{-1}, \quad \hat{Q} = \mu \sinh^2 \gamma, \quad Q = \mu \sinh \gamma \cosh \gamma. \quad (4.3)
\end{align*}
\]

The next step involves a T-duality along \( \tilde{y}^1 \). The resulting solution is a boosted \((p - 1)\)-brane in ten dimensional spacetime with the following form of the metric, dilaton and the other fields:

\[
\begin{align*}
ds^2 &= H_{p-1}^{-\frac{1}{2}} \left( - \hat{K}^{-1}(r) f(r) dt^2 + \sum_{\alpha=2}^p (dy^\alpha)^2 \right) + H_{p-1}^\frac{1}{2} \left( \hat{K}^{-1}(r) (dy^1)^2 \right) \\
&\quad + f^{-1}(r) dr^2 + r^2 d\Omega^2_{d+1}, \\
e^\phi &= \frac{H_{p-1}^{\frac{3-(p-1)}{2}}}{\sqrt{\hat{K}(r)}}, \quad B = A(r) \ dt \wedge d\tilde{y}_1, \\
E^{(p)} &= \hat{Q} \frac{f}{H_{p-1}} \ dt \wedge dy^2 \wedge \ldots \wedge dy^{(p-1)}, \quad (4.4)
\end{align*}
\]

Next one T-dualises back along \( \tilde{y}^1 \) to get back a \( Dp \)-brane solution and apply the inverse boost along \( \tilde{y}_1 \). The purpose of inverse boost is to cancel the boost charge as in step one. One gets the following configuration of metric and other fields:

\[
\begin{align*}
ds^2 &= H_p^{-\frac{1}{2}} \left[ - \left( \hat{K}^{-1}(r) f(r) \cosh^2 \gamma + \frac{(A(r) \cosh \gamma + \sinh \gamma)^2}{(\hat{K}^{-1}(r) + r^2 \alpha^2)} \right) dt^2 \\
&\quad + 2 \left( - \hat{K}^{-1}(r) f(r) \sinh \gamma \cosh \gamma + \frac{1}{K^{-1}(r) + r^2 \alpha^2} (A^2(r) \sinh \gamma \cosh \gamma \\
&\quad + A(r) (\cosh^2 \gamma + \sinh^2 \gamma) + \sinh \gamma \cosh \gamma) \right) dt dy_1
\end{align*}
\]
\[ + \left( -\frac{1}{\tilde{K}(r)} f(r) \sinh^2 \gamma + \frac{(A \sinh \gamma b + \cosh \gamma)^2}{(\hat{K}^{-1}(r) + r^2 \alpha^2)} \right) dy_1^2 + \sum_{a=2}^p dy_a^2 \]

\[ + H_p \int fdr^2 + \frac{1}{4} \frac{r^2 \tilde{K}^{-1}(r)}{(\hat{K}^{-1}(r) + r^2 \alpha^2)} \sigma^2 + r^2 d\Sigma_2^2, \]

\[ B = \frac{1}{2} \frac{r^2 \alpha}{K^{-1}(r) + r^2 \alpha^2} \left[ (\sinh \gamma + A(r) \cosh \gamma) dt \right. \]

\[ \left. + (\cosh \gamma + A(r) \sinh \gamma) dy_1 \right] \wedge d\sigma \]

\[ e^\phi = \frac{H_p^{(3-p)}}{\sqrt{\tilde{K}(r) (\hat{K}^{-1}(r) + r^2 \alpha^2)}}, \quad E^{(p+1)} = \tilde{Q} \frac{f}{H_p} dt \wedge dy_1 \ldots \wedge dy_p, \quad (4.6) \]

The final step is to take the limit \( \alpha \to 0 \) and \( \gamma \to \infty \) while keeping \( \beta = \frac{1}{2} \alpha e^\gamma \) fixed. The net effect of this step is to make the twist null hence the name null twist.

\[ ds^2 = H_p^{-\frac{1}{2}} \left[ -f(r) \left( 1 + \frac{\beta^2 r^2}{k(r)} \right) dt^2 - \frac{2 \beta^2 r^2 f(r)}{k(r)} dt dy_1^1 + \left( 1 - \frac{\beta^2 r^2}{k(r)} \right) (dy_1^1)^2 \right. \]

\[ \left. + \sum_{a=2}^p dy_a^2 \right] + H_p^{\frac{1}{2}} \left[ fdr^2 - \frac{1}{4} \frac{\beta^2 r^4 (1 - f(r))}{k(r)} \sigma^2 + r^2 d\Omega_{d+1}^2 \right], \]

\[ e^\phi = \frac{H_p^{(3-p)}}{\sqrt{k(r)}}, \quad B = -\frac{1}{2} \frac{r^2 \beta}{k(r)} \left( f(r) dt + dy_1^1 \right) \wedge \sigma, \]

\[ E^{(p+1)} = \tilde{Q} \frac{f}{H_p} dt \wedge dy_1 \ldots \wedge dy_p, \quad k(r) = 1 + \frac{\beta^2 \mu}{r^{d-2}}. \quad (4.7) \]

The above solution reduces to the black p-brane in flat spacetime (4.1) in \( \beta \to 0 \) limit and goes to the plane wave metric in ten-dimensions asymptotically. For \( \beta = 0 \), it is well known that the above metric admits a regular event horizon at

\[ r_+ = \mu^{1/\tilde{d}}. \]

It was noticed by the authors of [31] in the context of black string solution in asymptotically plane wave spacetime that the solution obtained by NMT also admits regular horizon at \( r_+ \). It also remains true for the black brane solutions constructed above. Moreover the area of event horizon remains invariant under NMT i.e. it is independent of \( \beta \). The area of event horizon (as measured in Einstein frame) is given by:

\[ A = \sqrt{(k(r) - \beta^2 r^2) H} r^{d+1} \Omega_{d+1}. \]
\[ H(r_+) \mu \frac{d+1}{d}. \] (4.8)

One quarter of the area of event horizon measured in Plank units furnish the statistical entropy of the black branes according to the laws of black hole thermodynamics. Defining \( 1 - \mu r^{-d} = \rho^2 \), and Euclidean time \( i\tau = t \), the metric can be put in the following form:

\[
d s^2 = \frac{4r_+^2}{d^2} H^{d/8} (d \rho^2 + \frac{d^2}{4r_+^2} H^{-1} \rho^2 d\tau^2 + ...). \] (4.9)

The temperature of the black brane solutions is given by the inverse periodicity of the Euclidean time:

\[
T = \frac{\tilde{d}}{4\pi r_+} H^{-\frac{1}{2}}(r_+), \] (4.10)

which is also independent of \( \beta \).

5 Discussion

In this paper we have presented two classes of brane solutions. The first one represent a class of rotating Dp-branes in homogeneous plane wave spacetime with both NS-NS and R-R flux. We discussed the supersymmetry of these branes and their bound states by solving the type-II Killing spinor equations explicitly. We briefly reviewed the possibility of having a regular horizon for this class of branes. The worldsheet analysis the background and the rotating branes presented in section-2 is rather straightforward. Following [11, 14], one can write down the bosonic sigma model action in the presence of the non-constant R-R and NS-NS flux. By using the D-brane boundary conditions it is easy to write down the mode expansions and thereby the classical Hamiltonian of the system. So we skip the details here. Regarding the event horizon, it has been shown in [26], in the context of Godel background, that one indeed find out a regular horizon by applying suitable T-duality/dimensional reduction as that removes the pp-singularities. One could possibly try to find out the Godel type solutions from the branes solutions presented here and analyze the properties of horizon, the holographic screens and the closed timelike curves. The gauge theory duals of these branes could also be found out by identifying the supergravity modes with the corresponding operators in the boundary theory. In the line of the supergravity theories, an interesting exercise would be to find out the most general structure of
the metric and the other field strengths and try to solve the field equations. The later would impose certain constraints of the structure of the flux compatible with the supersymmetry of the background.

The second class of branes were constructed from the nonextremal branes in the flat spacetime by applying a NMT along the translational isometry of the brane solutions. This class of branes admit regular horizon and the corresponding thermodynamical quantities like the entropy and temperature are computed. They are found to be independent of the parameter $\beta$ that defines the plane wave spacetime. In summary, the generic solution obtained by null Melvin twist inherit their thermodynamic properties from the parent solution in flat spacetime. It would be interesting to apply the procedure to construct general rotating and charged black brane solutions using this procedure.

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