The one loop renormalization of the effective Higgs sector and its implications

Qi-Shu YAN∗ and Dong-Sheng Du†
Theory division, Institute of high energy physics, Chinese academy of sciences, Beijing 100039, Peoples’ Republic of China

We study the one-loop renormalization the standard model with anomalous Higgs couplings (O(p^2)) by using the background field method, and provide the whole divergence structure at one loop level. The one-loop divergence structure indicates that, under the quantum corrections, only after taking into account the mass terms of Z bosons (O(p^2)) and the whole bosonic sector of the electroweak chiral Lagrangian (O(p^4)), can the effective Lagrangian be complete up to O(p^4).

I. INTRODUCTION

In the last paper [1], we have considered the one-loop renormalization of the electroweak chiral Lagrangian (EWCL) without Higgs boson up to O(p^4) and have derived its renormalization group equations. The real world might exist a light Higgs boson, as preferred by the supersymmetric models and indicated at the LEP. So to take into account a light Higgs boson in the EWCL O(p^4) is necessary in phenomenologies.

In this paper, we study the one loop divergence structures of the effective Higgs sector up to O(p^2) [2], so as to ascertain the set of complete operators up to O(p^4). We also intend to provide a reference to the renormalization of the extended EWCL with effective Higgs sector.

The parameterization of the effective Higgs sector has been conducted in the work of R. S. Chivukula and V. Koulovassilopoulos [2]. The phenomenologies of those anomalous couplings of Higgs sector has been considered in [3]. Below just for the sake of convenience, we formulate the effective Higgs sector as

∗E-mail Address: yanqs@mail.ihep.ac.cn
†E-mail Address: duds@mail.ihep.ac.cn
\[ L = -H_1 - H_2 - k'(v_1 + h)^2 \frac{1}{4} \text{tr}[DU^\dagger \cdot DU] - \frac{v_2^2}{4} \text{tr}[DU^\dagger \cdot DU] \]
\[ -\frac{1}{2} \partial h \cdot \partial h - \frac{m_H^2}{2} h^2 - \frac{\lambda_3}{3!} h^3 - \frac{\lambda_4}{4!} h^4. \] (1)

\[ H_1 = \frac{1}{4} W_{\mu\nu} W^{\mu\nu}, \] (2)

\[ H_2 = \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \] (3)

This way of parameterization is equivalent to that one given in \cite{2}, and the relations between our parameterization and the one in \cite{2} is

\[ v_1 = V_0 \frac{k}{k'}, \quad v_2 = V_0 \sqrt{1 - \frac{k^2}{k'^2}}, \quad V_0^2 = k' v_1^2 + v_2^2. \] (4)

Where \( v_1 \) just indicates that part of the vacuum expectation value yields by the Higgs potential of \( h \), while the \( v_2 \) parameterize the extra origin of vacuum expectation value different from the contribution of \( \langle h \rangle \).

The convention of this paper is as the same as given in \cite{1}, and we will conduct all the calculations in the Euclidean space. The related definitions omitted here can be found in \cite{1}.

The structure of this paper is organized as following. In the section II, we will introduce the background field gauge. In the section III, we provide the quadratic terms relevant to the master formula of the one loop counter term. In the section IV, we provide the necessary steps as how to extract the relevant divergences. In the section V, we use the Schwinger proper time \cite{6} and short distance expansion \cite{7} to extract the desired divergences. We end this paper with some discussions and conclusions.

**II. THE BACKGROUND FIELD METHOD AND THE GAUGE FIXING TERMS**

In the spirit of the background field gauge quantization \cite{4}, we can decompose the Goldstone field into the classic part \( \overline{U} \) and quantum part \( \xi \) as

\[ U \rightarrow \overline{U} \hat{U}, \quad \hat{U} = \exp\left\{ \frac{i2\xi}{\sqrt{k'V}} \right\}, \] (5)

where \( V = v_1 + \overline{h} \). To parameterize the quantum Goldstone field in the above form is to simplify the presentation of the standard form of quadratic terms. The vector fields in the mass eigenstates are split as
$V_\mu \to \nabla_\mu + \tilde{V}_\mu,$  

(6)

where $\nabla_\mu$ represents the classic background vector fields and $\tilde{V}_\mu$ represents the quantum vector fields.

By using the Stueckelberg transformation [5] for the background vector fields, we have

$$\mathcal{W}^a \to U^\dagger \mathcal{W} U + iU^\dagger \partial_\mu U, \quad \mathcal{B} \to \mathcal{B}, \quad \mathcal{\hat{W}}^a \to U^\dagger \mathcal{\hat{W}} U, \quad \hat{B} \to \hat{B},$$  

(7)

so the background Goldstone fields can completely be absorbed by redefining the background vector fields, and will not appear in the one-loop effective Lagrangian.

The Stueckelberg fields is invariant under the gauge transformation of the background gauge fields, such a property guarantees that the following computation is gauge invariant with respect to the background gauge transformation from the beginning if we can express all effective vertices into the Stueckelberg fields. After the loop calculation, by using the inverse Stueckelberg transformation, the Lagrangian can be restored to the form represented by its low energy degrees of freedom.

Similarly, the Higgs scalar is split as

$$h = \overline{h} + \hat{h},$$  

(8)

where the $\overline{h}$ and $\hat{h}$ represent the background and quantum Higgs, respectively.

The equation of motion of the background vector fields is given as

$$D_\mu \widehat{W}^{\mu \nu} = -\sigma_{0,V} V^\nu,$$  

(9)

$$\sigma_{0,V} = k R_2 \frac{1}{4} \text{dia}\{0, G^2, g^2, g^2\} V^2.$$  

(10)

with $\mathcal{\hat{W}}^{\mu \nu,T} = \{ A^{\mu \nu} + ieF_\mathcal{Z}^{\mu \nu}, Z^{\mu \nu} - i\frac{g}{2} F_\mathcal{Z}^{\mu \nu}, \mathcal{\hat{W}}^{+, \mu \nu}, \mathcal{\hat{W}}^{-, \mu \nu}\}$. The $\sigma_{0,V}$ is the mass matrix of the vector boson. The EOM of vector bosons derives the following relations

$$\partial \ln V \cdot Z = -\frac{R_2}{2} \partial \cdot Z,$$  

(11)

$$\partial \ln V \cdot W^+ = -\frac{R_2}{2} \left[ d \cdot W^+ + i\frac{1}{2} \frac{g^2}{G} Z \cdot W^+ \right],$$  

(12)

$$\partial \ln V \cdot W^- = -\frac{R_2}{2} \left[ d \cdot W^+ - i\frac{1}{2} \frac{g^2}{G} Z \cdot W^+ \right],$$  

(13)

$$R_2 = \frac{k' \overline{\mathcal{h}}^2 + 2V_0^2 k' \mathcal{h} + V_0^2}{k' V^2} = 1 + \frac{v_0^2}{k' V^2}.$$  

(14)

These relations are helpful to eliminate terms, like $\partial \overline{h} \cdot Z$, in the loop calculations.

The equation of motion of the background Higgs field is determined as
\[
\partial^2 \mathcal{H} = V[k^1/4 \mathcal{O}_{WZ} + \mathcal{E}_1 + \mathcal{E}_3 V^2] + \mathcal{E}_2 V^2 + \mathcal{E}_0,
\]
(15)

\[
\mathcal{E}_0 = -m_H^2 v_1 + \frac{\lambda_3 v_1^2}{2} - \frac{\lambda_4}{6} v_1^3
\]

\[
\mathcal{E}_1 = \frac{\lambda_4}{2} v_1^2 - \lambda_3 v_1 + m_H^2
\]

\[
\mathcal{E}_2 = \frac{\lambda_3 - \lambda_4 v_1}{2}
\]

\[
\mathcal{E}_3 = \frac{\lambda_4}{6}
\]

\[
\mathcal{O}_{WZ} = G^2 Z \cdot Z + 2g^2 W^+ \cdot W^-.
\]
(16)

When \( \mathcal{E}_0 \) and \( \mathcal{E}_2 \) vanish and \( k' = k = 1 \), the equation of motion of the background Higgs field restores to that of the SM.

To guarantee the quadratic terms to have the standard form given in Eq. (26), the gauge fixing terms of the quantum fields are chosen as

\[
\mathcal{L}_{GF,A} = -\frac{1}{2} (\partial \cdot \hat{A} - ie(\hat{W}^- \cdot \hat{W}^+ - \hat{W}^+ \cdot \hat{W}^-))^2,
\]
(17)

\[
\mathcal{L}_{GF,Z} = -\frac{1}{2} (\partial \cdot \hat{Z} - \frac{1}{2} \sqrt{k'GV R_2} \xi_Z + i \frac{\xi}{3} (\hat{W}^- \cdot \hat{W}^+ - \hat{W}^+ \cdot \hat{W}^-))^2,
\]
(18)

\[
\mathcal{L}_{GF,W} = -(d \cdot \hat{W}^+ + \frac{1}{2} g \sqrt{k'GV R_2} \xi_W + i \frac{\xi}{3} (\hat{W}^+ - \hat{W}^- + i e \hat{W}^+ \cdot \hat{A}))
\]

\[
(d \cdot \hat{W}^- + \frac{1}{2} g \sqrt{k'GV R_2} \xi_W - i \frac{\xi}{3} (\hat{W}^- + i g^2 G \hat{W}^+ \cdot \hat{Z} + ie \hat{W}^+ \cdot \hat{A})).
\]
(19)

These gauge fixing terms are the 't Hooft and Feynman gauge in the background gauge method, and will make the massive vector bosons, the corresponding Goldstones and ghost, to have the same masses.

### III. THE QUADRATIC FORMS OF THE ONE-LOOP LAGRANGIAN

Up to one loop level, only the quadratic terms of the quantum fields are relevant, and they can be cast into the following standard form

\[
\mathcal{L}_{quad} = \frac{1}{2} \nabla^\mu a \nabla^\nu b \hat{V}^a \nabla^\nu a + \frac{1}{2} \xi^i \hat{\xi}^j + \bar{c} a \nabla^\mu a \xi^j + \nabla^\mu h \nabla^\mu \hat{h}
\]

\[
+ \frac{1}{2} \hat{V}^a \nabla^\mu a \xi^j + \frac{1}{2} \xi^i \hat{X}^a \nabla^\mu \hat{V}^a
\]

\[
+ \frac{1}{2} \hat{V}^a \nabla^\mu a \hat{h} + \frac{1}{2} \hat{h} \nabla^\mu a \hat{V}^a + \frac{1}{2} \xi^i \hat{X}^a \hat{V}^a + \frac{1}{2} \hat{h} \hat{X}^i \hat{\xi}^i,
\]
(20)

\[
\nabla^\mu \nabla^\nu = D^2 ab g^{\mu \nu} + \sigma^{ab}_{0,VV} g^{\mu \nu} + \sigma^{ab}_{2,VV},
\]
(21)
\[ \square^{ij}_{\xi\xi} = R_2 \left[ d^{2,ij} + \sigma^{ij}_{0,\xi\xi} + \sigma^{ij}_{2,\xi\xi} + \sigma^{ij}_{2,\xi\xi} \right], \tag{22} \]

\[ \square_{hh} = \partial^2 + \sigma_{hh}, \tag{23} \]

\[ \square^{ab}_{\xi\xi} = D^{2,ab} + \sigma^{ab}_{0,V^V}; \tag{24} \]

\[ X_h^{\alpha} = X_h^{\alpha,i} d_{\alpha} + X_h^{\alpha,2}; \tag{25} \]

\[ X_{ih} = X_{ih}^{\alpha,i} \partial_{\alpha} + X_{ih}^{\alpha,2}; \tag{26} \]

where \( V^V = (A, Z, W^{-}, W^{+}) \) and \( \xi^V = (\xi_Z, \xi^-, \xi^+) \).

And the covariant differential operators \( D = \partial + \Gamma_V \) and \( d = \partial + \Gamma_{\xi} \). The gauge connection of vector bosons \( \Gamma_V \) is defined as

\[
\Gamma^{ab}_{V,\mu} = \begin{pmatrix}
0 & 0 & i e W^\mu_+ & -i e W^\mu_-
0 & -i \frac{g}{2} W^{\mu}_- & i \frac{g}{2} W^{\mu}_+
& & & \\
i e W^{\mu}_- & -i \frac{g}{2} W^{\mu}_+ & -i e A_{\mu} + i \frac{g}{2} Z_{\mu} & 0
& & & \\
-i e W^{\mu}_+ & i \frac{g}{2} W^{\mu}_- & 0 & i e A_{\mu} - i \frac{g}{2} Z_{\mu}
\end{pmatrix}.
\]

The gauge connection of Goldstone bosons \( \Gamma_{\xi} \) is defined as

\[
\Gamma^{\mu}_{\xi,\mu} = \begin{pmatrix}
0 & i \frac{g}{2} W^{\mu}_- & -i \frac{g}{2} W^{\mu}_+
& & & \\
i \frac{g}{2} W^{\mu}_+ & -i e A_{\mu} & 0
& & & \\
-i \frac{g}{2} W^{\mu}_+ & 0 & i e A_{\mu}
\end{pmatrix}.
\]

The mass matrix of the Goldstone bosons have the following form

\[ \sigma_{0,\xi\xi}^{ij} = k' R_2 \frac{1}{4} \text{dia}\{G^2, g^2, g^2\} V^2. \tag{27} \]

Due to the gauge fixing terms we have chosen, the massive vector bosons have the same mass with their corresponding Goldstone bosons.

The matrix \( \sigma_{2,V^V}^{\mu\nu,ab} \) reflects that the vector bosons are the spin 1 particles, and is given below as

\[
\sigma^{\mu\nu,ab}_{2,V^V} = \begin{pmatrix}
\sigma^{\mu\nu}_{2,AA} & \sigma^{\mu\nu}_{2,AZ} & \sigma^{\mu\nu}_{2,AW^+} & \sigma^{\mu\nu}_{2,AW^-}
\sigma^{\mu\nu}_{2,ZA} & \sigma^{\mu\nu}_{2,ZZ} & \sigma^{\mu\nu}_{2,ZW^+} & \sigma^{\mu\nu}_{2,ZW^-}
\sigma^{\mu\nu}_{2,W^-A} & \sigma^{\mu\nu}_{2,W^-Z} & \sigma^{\mu\nu}_{2,W^-W^+} & \sigma^{\mu\nu}_{2,W^-W^-}
\sigma^{\mu\nu}_{2,W^+A} & \sigma^{\mu\nu}_{2,W^+Z} & \sigma^{\mu\nu}_{2,W^+W^+} & \sigma^{\mu\nu}_{2,W^+W^-}
\end{pmatrix},
\]

and the components read

\[
\sigma^{\mu\nu}_{2,AA} = \sigma^{\mu\nu}_{2,AZ} = \sigma^{\mu\nu}_{2,ZA} = \sigma^{\mu\nu}_{2,ZZ} = 0,
\sigma^{\mu\nu}_{2,AW^+} = -\sigma^{\mu\nu}_{2,W^+A} = 2 i e \tilde{W}^{-\mu\nu},
\]

\[
\sigma^{\mu\nu}_{2,AW^-} = \sigma^{\mu\nu}_{2,W^-A} = 0,
\sigma^{\mu\nu}_{2,ZW^+} = \sigma^{\mu\nu}_{2,W^-Z} = 0,
\sigma^{\mu\nu}_{2,ZW^-} = \sigma^{\mu\nu}_{2,W^-W^+} = 0,
\]

\[
\sigma^{\mu\nu}_{2,W^-W^-} = \sigma^{\mu\nu}_{2,W^+W^+} = 0.
\]
\[ \sigma_{2,AW} = -\sigma_{2,W-A} = -2ie\bar{W}^{+}, \mu \nu, \]
\[ \sigma_{2,ZW} = -\sigma_{2,W+Z} = -2g^2 G \bar{W}^{-}, \mu \nu, \]
\[ \sigma_{2,W} = 2i \bar{W}^{+}, \mu \nu, \]
\[ \bar{W}^{-} = 0, \quad (28) \]

There is no deviation from the gauge theory without spontaneous symmetry breaking if the anomalous operators \( O(p^4) \) have not been taken into account.

The matrix \( \sigma_{2,\xi} \) indicates that Goldstone bosons are spin 0 particles, and is given as

\[
\sigma_{2,\xi}^{ij} = \begin{pmatrix}
\sigma_{2,\xi+\xi+} & \sigma_{2,\xi+\xi-} & \sigma_{2,\xi-\xi-} \\
\sigma_{2,\xi-\xi+} & \sigma_{2,\xi-\xi+} & \sigma_{2,\xi+\xi+} \\
\sigma_{2,\xi+\xi-} & \sigma_{2,\xi+\xi+} & \sigma_{2,\xi+\xi-}
\end{pmatrix},
\]

and its components read

\[
\sigma_{2,\xi+\xi+} = \frac{g^2}{2} W^{+} \cdot W^{-}, \\
\sigma_{2,\xi+\xi-} = -\frac{g^2}{4} W^{-} \cdot W^{-}, \\
\sigma_{2,\xi-\xi-} = -\frac{g^2}{4} W^{+} \cdot W^{+}, \\
\sigma_{2,\xi+\xi+} = \sigma_{2,\xi+\xi+} = \frac{gG}{4} W^{-} \cdot Z, \\
\sigma_{2,\xi+\xi-} = \sigma_{2,\xi+\xi-} = \frac{gG}{4} W^{+} \cdot Z, \\
\sigma_{2,\xi+\xi+} = \sigma_{2,\xi+\xi+} = \frac{g^2}{4} W^{+} \cdot W^{-} + \frac{G^2}{4} Z \cdot Z .
\]

(29)

The matrix \( \sigma_{2,\mathcal{H}_K,\xi} \) includes the terms related with the equation of motion of the background Higgs field, which has the form \(-\sigma_{2,\mathcal{H}_K} \mathbf{1}_{3 \times 3} \), while \( \sigma_{2,\mathcal{H}_K} \) is given as

\[
\sigma_{2,\mathcal{H}_K} = \sigma_{2,\mathcal{H}_K,0} - \frac{k'}{4} \mathcal{O}_{WZ} - \frac{R_2 - 1}{R_2} \partial^2 \ln V, \\
\sigma_{2,\mathcal{H}_K,0} = -\frac{1}{4} \mathcal{O}_{WZ} - \mathcal{E}_1 - \mathcal{E}_3 V^2 - \mathcal{E}_2 V - \frac{\mathcal{E}_0}{V},
\]

(30) (31)

where \( \sigma_{2,\mathcal{H}_K,0} \) represent the part of the SM, while the rest of terms indicate the deviation due to the anomalous couplings in the Higgs sector if the \( \mathcal{E}_0 \) and \( \mathcal{E}_1 \) vanish.

The \( \sigma_{hh} \) is given as
\[ \sigma_{hh} = -\frac{k'}{4}O_{WZ} - 2\mathcal{E}_2V - 3\mathcal{E}_3V^2 - \mathcal{E}_1. \]  

(32)

Due to the fact that the Higgs scalar is a singlet of the gauge symmetry, there is no vector fields in the \( \sigma_{hh} \).

The mixing terms between the vector and Goldstone bosons are determined as

\[
\vec{X}^\mu,,a_h = \sqrt{k'} R_2 \left( \begin{array}{ccc} 0 & -i\frac{g'^2}{2}V W^- & i\frac{g'^2}{2}V W^+ \\ G\partial\mu \bar{T} & \frac{g}{2G}(g^2 - g'^2) V W^- & -i\frac{g}{2G}(g^2 - g'^2)V W^+ \\ -i\frac{g^2}{2}V W^+ & -g\partial\mu \bar{T} - \frac{1}{2}igGV Z \mu & 0 \\ i\frac{g^2}{2}V W^- & 0 & -g\partial\mu \bar{T} + \frac{1}{2}igGV Z \mu \end{array} \right),
\]

While the matrix \( \vec{X}^\mu,,a_h \) is just the rearrangement of the \( \vec{X}^\mu,,a \), and here we do not rewrite it. The mixing terms between vector and Higgs bosons are determined as

\[
\vec{X}_h^\mu,a = \frac{k'}{2} \left\{ 0, -G^2 V Z \mu, -g^2 V W^+, -g^2 V W^- \right\},
\]

(33)

The mixing terms \( \vec{X}_h^\mu,a \) is the rearrangement of the \( \vec{X}_h^\mu,a \). The mixing terms between Higgs and Goldstone bosons are determined as

\[
X_{h\xi}^{\alpha,i} = \sqrt{k'} \left\{ -GZ^\alpha, gW^-,-a, gW^+,-a \right\},
\]

(34)

\[
X_{h\xi,2}^i = \sqrt{k'} R_2 \left\{ -\frac{G}{2} \partial \cdot Z, \left[ \frac{g}{2}d \cdot W^- + i\frac{1}{2}g'^2 W^- \cdot Z \right] \right\},
\]

\[
\left[ \frac{g}{2}d \cdot W^+ - i\frac{1}{2}g'^2 W^+ \cdot Z \right] \]

(35)

The terms \( X_{\xi h}^{\alpha,i} \) and \( X_{\xi h,0}^i \) are omitted here.

**IV. EVALUATING THE TRACES AND LOGARITHMS**

By diagonalizing the quantum fields, we can integrate the quadratic terms of the Lagrangian by using the Gaussian integral. And the \( \mathcal{L}_{1\text{-loop}} \) can be expressed as the traces and logarithms

\[
S_{1\text{-loop}} = Tr \ln \bar{c}c - \frac{1}{2} \left[ Tr \ln \bar{V} + Tr \ln \bar{\xi} + Tr \ln \bar{h}h \right],
\]

where
\[ \Box'_{\xi \xi} = \Box_{\xi \xi} - \vec{X}_\xi \Box^{-1} \vec{X}_\xi, \]  
\[ \Box'_{hh} = \Box_{hh} - \vec{X}_h \Box^{-1} \vec{X}_h, \]  
\[ \Box''_{hh} = \Box'_{hh} - X_{h\xi} \Box^{-1} X_{\xi h}, \]  
\[ X'_{h\xi} = X_{h\xi} - \vec{X}_h \Box^{-1} \vec{X}_\xi, \]  
\[ X'_{\xi h} = X_{\xi h} - \vec{X}_\xi \Box^{-1} \vec{X}_h. \]  

Expanding the \( \text{Tr} \ln \Box'_{\xi \xi} \) and \( \text{Tr} \ln \Box''_{hh} \) with the following relations

\[ \text{Tr} \ln \Box'_{\xi \xi} = \text{Tr} \ln \Box_{\xi \xi} + \text{Tr} \ln \left( 1 - \vec{X}_\xi \Box^{-1} \vec{X}_\xi \right), \]  
\[ \text{Tr} \ln \Box''_{hh} = \text{Tr} \ln \Box'_{hh} + \text{Tr} \ln \left( 1 - X_{h\xi} \Box^{-1} X_{\xi h} \Box^{-1} \right). \]

Since we consider the renormalization of the theory, so we are only interested in those divergent terms, which can be expressed as

\[ \int L_{1\text{-loop}} = \text{Tr} \Box_{\xi \xi} - \frac{1}{2} \left[ \text{Tr} \ln \Box_{V} + \text{Tr} \ln \Box_{\xi \xi} + \text{Tr} \ln \Box_{hh} ight. \\
- \left. \text{Tr} (\vec{X}_\xi \Box^{-1} \vec{X}_\xi \Box^{-1}) - \text{Tr} (\vec{X}_h \Box^{-1} \vec{X}_h \Box^{-1}) \\
- \frac{1}{2} \text{Tr} (X_{h\xi} \Box^{-1} X_{\xi h} \Box^{-1}) + \cdots \right]. \]

Due to the property of the \( \text{Tr} \), we know that the above equation is independent of the sequence of integrating-out quantum fields. The omitted terms are finite and will not contribute to the one-loop divergence structures. Below we will use the Schwinger proper time and short distance expansion method to extract the related terms from the compact form of one loop effective Lagrangian given in Eq. (44).

To extract the counter terms, the following two counting rules are necessary: one is the auxiliary counting rule, which reads

\[ [\bar{W}_{\mu}]_a = [\partial_\mu]_a = [D_\mu]_a = 1, \quad [v_1]_a = 0. \]

From this rule, we know

\[ [\sigma_{2,VV}]_a = [\sigma_{2,\xi\xi}]_a = [\sigma_{2,h\xi}]_a = [\sigma_{2,hh}]_a = [X'_{h\xi}]_a = [X'_{\xi h}]_a = 2, \]

\[ [\vec{X}_{h}]_a = [X_{h}]_a = [\vec{X}_{\xi}]_a = [X_{\xi}]_a = [X_{h\xi}]_a = [X_{\xi h}]_a = 1. \]

This rule is intended to count the dimensions of operators with background fields and their differentials.
The other one is the divergence counting rule, which reads

\[ [z^\mu]_d = 1, \quad [\tau]_d = -2. \quad (48) \]

By using the Schwinger proper time htkl and short distance expansion method \[\square\], after integrating over mediate coordinate spaces and proper times, we can exact all the related divergence structures yielded by the radiative correction. In the Schwinger proper time method, the standard propagators can be expressed as

\[
\langle x|\Box^{-1,ab}_{\nu V;x\mu}|y\rangle = \int_0^\infty \frac{d\tau}{(4\pi\tau)^{\frac{d}{2}}} \exp\left\{ -\epsilon_F \tau \right\} \exp\left( -\frac{z^2}{4\tau} \right) H^{\mu\nu,ab}_{V V}(x, y; \tau),
\]

\[
\langle x|\Box^{-1,ab}_{\xi\xi;x\mu}|y\rangle = \int_0^\infty \frac{d\tau}{(4\pi\tau)^{\frac{d}{2}}} \exp\left\{ -\epsilon_F \tau \right\} \exp\left( -\frac{z^2}{4\tau} \right) H^{\xi\xi,ab}_{\xi\xi}(x, y; \tau),
\]

where the \(\epsilon_F\) is the Feynman prescription which will be taken to be zero in the final step, and the \(z\) is the distance of two event points \(y\) and \(x\) which satisfies \(z = y - x\). The integral over the proper time \(\tau\) and the factor \(1/(4\pi\tau)^{\frac{d}{2}} \exp\left(-z^2/(4\tau)\right)\) conspire to separate the divergent part of the propagator. And the \(H(x, y; \tau)\) is analytic with reference to the arguments \(z\) and \(\tau\), which means that \(H(x, y; \tau)\) can be analytically expanded with reference to both \(z\) and \(\tau\). Then we have

\[
H(x, y; \tau) = H_0(x, y) + H_1(x, y)\tau + H_2(x, y)\tau^2 + \cdots, \quad (51)
\]

\[
H_i(x, y) = H_i(x, y)|_{x=y} + z^\alpha \partial_\alpha H_i(x, y)|_{x=y} + \frac{1}{2} z^\alpha z^\beta \partial_\alpha \partial_\beta H_i(x, y)|_{x=y} + \cdots \quad (52)
\]

where \(H_0(x, y)\), \(H_1(x, y)\), and \(H_2(x, y)\) are the Silly-De Witt coefficients. The coefficient \(H_0(x, y)\) is the Wilson phase factor, which indicates the phase change of a quantum state from the point \(x\) to the point \(y\) and reads

\[
H_0(x, y) = \exp \int_x^y \Gamma(z) \cdot dz,
\]

where \(\Gamma(z)\) is the affine connection defined on the coordinate point \(z\). Higher order coefficients are determined by the lower ones by the following recurrence relation

\[
(1 + n + z^\mu D_{\mu,x}) H_{n+1}(x, y) + (D^2 + \sigma) H_n(x, y) = 0. \quad (54)
\]

All these Silly-De Witt coefficients are gauge covariant with respect to the gauge transformation.

For the case in the following section, here we consider the case \(Tr \ln(1 - \tilde{X} \Box^{-1}_{vec} \tilde{X} \Box^{-1}_{scal})\) with \(\tilde{X} = \tilde{X}_B D_\alpha + \tilde{X}_C\) and \(\tilde{X} = \tilde{X}_B D_\alpha + \tilde{X}_C\). The \(\tilde{X}_B\), \(\tilde{X}_B\), \(\tilde{X}_C\), and \(\tilde{X}_C\) have the following dimensions in the auxiliary counting rule,
\[ [\hat{X}_B^\alpha] = [\hat{X}_B^\alpha] = 1, \quad [\hat{X}_C] = [\hat{X}_C] = 2. \] (55)

By using the dimensional regularization and the modified minimal subtraction scheme, we can extract the divergences of the general term \( Tr \ln (1 - \hat{X} \hat{\Box}^{-1} \hat{X} \hat{\Box}^{-1}) \) which have the following forms

\[
Tr \ln (1 - \hat{X} \hat{\Box}^{-1} \hat{X} \hat{\Box}^{-1}) = -\frac{1}{\epsilon} [t_{BB} + t_{BC} + t_{CC} + t_{BBBB}],
\]

\[
t_{BB} = -\frac{g_{\alpha\beta} \alpha' \beta'}{12} X_B D^\alpha D^\beta' \hat{\Box}^{-\alpha'} \hat{\Box}^{-\beta'} - \frac{1}{4} \hat{X}_B \Gamma_{\alpha'} \hat{X}_B - \frac{1}{4} \hat{X}_B X_B \Gamma_{\alpha' \alpha'},
\]

\[
t_{BC} = \frac{1}{2} [\hat{X}_B D_\alpha \hat{X}_C - \hat{X}_C D_\alpha \hat{X}_B],
\]

\[
t_{CC} = \hat{X}_C \hat{X}_C,
\]

\[
t_{BBBB} = \frac{g_{\alpha\alpha'} \beta \beta'}{24} X_B X_B X_B X_B .
\] (56)

where \( H_{vec,a} \) and \( H_{scal,1} \) are the second Silly-De Witt coefficients, and they might have their Lorentz indices, so the Lorentz indices should be understood as being contracted (by multiplying metric tensors). The covariant differential operators are defined as

\[
\hat{X}_D \hat{X} = \hat{X}_D \hat{X} + \hat{X}_\Gamma_w \hat{X} - \hat{X}_\Gamma_\xi \hat{X},
\]

\[
\hat{X}_D D \hat{X} = \hat{X}_D \hat{X} + \hat{X}_\Gamma_w \hat{X} + \hat{X}_\Gamma_\xi \hat{X} - 2 \hat{X}_\Gamma_w \hat{X} \hat{X} + 2 \hat{X}_\Gamma_w \hat{X} \hat{X} \hat{X}_\Gamma_\xi + \hat{X}_\Gamma_\xi \hat{X}_\Gamma_\xi \hat{X} - \hat{X}_\Gamma_\xi \hat{X} \hat{X} \hat{X}_\Gamma_\xi .
\] (57)

And the high rank tensor \( g_{\alpha\alpha' \beta \beta'} \) is defined as

\[
g_{\alpha\beta \gamma \delta} = g_{\alpha\beta} g_{\gamma \delta} + g_{\alpha \gamma} g_{\beta \delta} + g_{\alpha \delta} g_{\beta \gamma} .
\] (58)

It is symmetric under the change of its Lorentz indices.

**V. COUNTER TERMS**

By using the heat kernel method, the variant of the Schwinger proper time method, the determinant of the D’Alambert operators can be easily evaluated and the related divergences can be quickly extracted out. The divergences of the \( Tr \ln \hat{\Box} \) in the Eq. (44) are given as

\[
\frac{1}{2} Tr \ln \hat{\Box} = -\frac{20}{3} H_1 - \frac{(g^2 + G^2)}{16} V^4 k^2 R^2 ,
\] (59)
\[ \frac{1}{2} \varepsilon Tr \ln \Box_{\xi \xi} = -\frac{g^2}{12} H_1 + \frac{g^2}{12} H_2 + \frac{1}{12} \mathcal{L}_1 - \frac{1}{24} \mathcal{L}_2 - \frac{1}{24} \mathcal{L}_3 \]
\[ -\frac{1}{12} \mathcal{L}_4 + \frac{1}{48} \mathcal{L}_5 - k'R_2 \frac{G^2 g^2}{32} V^2 Z \cdot Z \]
\[ + k'R_2 \frac{g^2 + G^2}{32} V^2 \mathcal{O}_{WZ} - k'R_2 \frac{2g^2 + G^2}{64} V^4 \]
\[ - \frac{3}{4} \sigma_2, \mathcal{H} + \sigma_2, \mathcal{H} \left[ k'R_2 \frac{2g^2 + G^2}{8} V^2 - \mathcal{O}_{WZ} \right], \quad (60) \]
\[ \frac{1}{2} \varepsilon Tr \ln \Box_{hh} = -\frac{k'^2}{16} L_5 - \frac{k'}{16} \left[ \lambda_4 V^2 + 4\mathcal{E}_2 V + 2\mathcal{E}_1 \mathcal{O}_{WZ} \right] \]
\[ - \frac{\lambda^2}{16} V^4 - \frac{\lambda_4}{2} \mathcal{E}_2 V^3 - \frac{4\mathcal{E}_2^3 + \mathcal{E}_0 \lambda_4}{4} V^2 - \mathcal{E}_0 \mathcal{E}_1 V - \frac{\mathcal{E}_2^2}{4}, \quad (61) \]
\[ -\varepsilon Tr \ln \Box_{cc} = -\frac{2}{3} H_1 + \frac{g^2 + 2g^2}{32} k'^2 R_2^2 V^4. \quad (62) \]

The divergence terms from the mixing terms need some labor. But to conduct the calculation in the coordinate space can simplify the calculation to a considerable degree. With the formula given in the last section, contributions of the mixing terms with two propagators can be easily extracted as

\[ -\frac{\varepsilon}{2} Tr (\hat{X}_\xi \Box^{-1}_{V^V} \hat{X}_\xi \Box^{-1}_{\xi^1}) = \frac{G^2 g^2}{8} (v^2 + k' V^2) Z \cdot Z V^2 - \frac{g^2 + G^2}{8} (v^2 + k' V^2) \mathcal{O}_{WZ} \]
\[ - k'R_2 \frac{1}{2} (2g^2 + G^2) \partial \mathcal{H} \cdot \partial \mathcal{H}, \quad (63) \]
\[ -\frac{\varepsilon}{2} Tr (\hat{X}_h \Box^{-1}_{V^V} \hat{X}_h \Box^{-1}_{hh}) = - k'^2 \frac{g^2 + G^2}{8} Z \cdot Z V^2 - k'^2 \frac{g^2}{8} V^4 \mathcal{O}_{WZ}, \quad (64) \]
\[ -\frac{\varepsilon}{2} Tr (X_{h \xi} \Box^{-1}_{\xi^1} X_{h \xi} \Box^{-1}_{\xi^1}) = -\frac{1}{2} (t_{BB1} + t_{BB2} + t_{BC1} + t_{BC2} + t_{CC}), \quad (65) \]
\[ t_{BB1} = \frac{k'}{R_2} \left\{ \frac{g^2}{6} H_1 - \frac{g^2}{6} H_2 + \frac{1}{6} \mathcal{L}_1 + \frac{1}{6} \mathcal{L}_2 + \frac{1}{6} \mathcal{L}_3 + \frac{1}{6} \mathcal{L}_4 \right\} \]
\[ - \frac{1}{6} \mathcal{L}_5 - \frac{1}{2} \frac{g'^4}{G^4} \left[ \mathcal{L}_6 - \mathcal{L}_a \right] \]
\[ - k'R_2 \frac{G^2}{4} (\partial \cdot Z)^2 - \frac{g^2}{2R_2} (d \cdot W^+) (d \cdot W^-) \]
\[ - i k \frac{g^2 + G^2}{G R_2} [(d \cdot W^+) (W^- \cdot Z) - (d \cdot W^-) (W^+ \cdot Z)], \quad (66) \]
\[ t_{BB2} = \frac{k'}{R_2} \left\{ \frac{1}{4} \mathcal{L}_2 + \frac{1}{4} \mathcal{L}_3 + \frac{1}{2} \mathcal{L}_4 \right\} \]
\[ - \frac{(1 - \frac{k'}{4}) \mathcal{L}_5 - \frac{k'^2 G^2 g'^2}{16} V^2 Z \cdot Z. \]
The divergences of the four propagators term is given as

\[\begin{align*}
&\frac{k'}{16}(k'R_2g^2 + 2\lambda_4)V^2 - \frac{k'}{4R_2}(2\varepsilon_2V + \varepsilon_1)]\mathcal{O}_{WZ} \\
&+ \frac{k'}{4R_2}\sigma_2,\mathcal{H}_{\mathcal{K}}\mathcal{O}_{WZ} ,
\end{align*}\]

(67)

\[t_{BC1} = \frac{k'}{R_2}g'^4\mathcal{L}_a - \frac{k'}{R_2}G^2(\partial \cdot Z)^2 + g^2(d \cdot W^+(d \cdot W^-)
\]

\[\begin{align*}
&-i \frac{k'}{R_2}g^2g' \quad [(d \cdot W^+(W^- \cdot Z) - (d \cdot W^-)(W^+ \cdot Z)].
\end{align*}\]

(68)

\[t_{BC2} = \frac{g'^4}{G^4}k'R_2(R_2 - 1)^2(\mathcal{L}_a - \mathcal{L}_a)
\]

\[\begin{align*}
&+ \frac{1}{2}k'R_2(R_2 - 1)^2\left[G^2(\partial \cdot Z)^2 + 2g^2(d \cdot W^+(d \cdot W^-)
\right]
\]

\[\begin{align*}
&+i \frac{g^2g'^2}{G}k'R_2(R_2 - 1)^2\left[(d \cdot W^+(W^- \cdot Z) - (d \cdot W^-)(W^+ \cdot Z)
\right]
\end{align*}\]

(69)

The sum over all contributions yields the following total divergence structures as

\[\begin{align*}
-\frac{\bar{\varepsilon}}{4}Tr(Xh_{\xi\xi} \square_{\xi\xi} Xh_{\xi\xi} \square_{\xi\xi} Xh_{\xi\xi} \square_{\xi\xi} Xh_{\xi\xi}) = -\frac{k'^2}{24R_2^2}(2\mathcal{L}_a + \mathcal{L}_a) .
\end{align*}\]

(70)

The sum over all contributions yields the following total divergence structures as

\[\begin{align*}
\bar{\varepsilon} &\quad D_{tot} = \bar{\varepsilon}(D_{tot}^{KR} + D_{tot}^{AN})
\end{align*}\]

(71)

\[D_{tot}^{KR} = D_{\sigma}^{KR} + D_{\bar{\sigma}}^{KR}
\]

\[D_{\sigma}^{KR} = -\frac{43}{6}g^2H_1 + \frac{1}{6}g'^2H_2
\]

\[\begin{align*}
&- \frac{2g^2 + G^2}{32}V^2\mathcal{O}_{WZ} - \frac{2g^2 + G^2}{2}\partial \bar{\mathcal{H}} \cdot \partial \mathcal{H} - \frac{\varepsilon_1^2}{4} \\
&- \frac{1}{2}(2\varepsilon_2 + 3\varepsilon_3)V^2 - \frac{3}{64}\left[(2g^2 + G^2) + 48\varepsilon_3^2\right]V^4 \\
&- \frac{3}{16}\mathcal{L}_5 - \varepsilon_2\varepsilon_1V - \frac{1}{2}\varepsilon_2\varepsilon_3V^3
\end{align*}\]

(72)

\[\begin{align*}
D_{\bar{\sigma}}^{KR} &\quad = -\frac{3}{4}\sigma_{2,\mathcal{H}_0} + \varepsilon_2(2g^2 + G^2)V^2 - 3\mathcal{O}_{WZ} ,
\end{align*}\]

(73)

\[D_{tot}^{AN} = D_{tot}^{AN} + D_{\sigma}^{AN} + D_{\bar{\sigma}}^{AN} + D_{total}^{AN} + D_{\sigma}^{AN} + D_{V}^{AN}
\]

\[D_{L}^{AN} = \left(\frac{k'}{R_2} - 1\right)\left\{\frac{g^2}{12}H_1 + \frac{g'^2}{12}H_2 - \frac{1}{12}\mathcal{L}_1 + \frac{1}{24}\mathcal{L}_2 + \frac{1}{24}\mathcal{L}_3 - \left(\frac{k'}{R_2} - 1\right)\frac{1}{12}\mathcal{L}_4\right\}
\]

12
each terms of the limit, some terms dependent on the anomalous couplings of the Higgs potential. In the following potential vanish, the anomalous couplings of the Higgs potential. When the anomalous couplings of the Higgs potential vanish, the term includes all terms independent of \( \sigma_2, \delta \). The above equations are exact, and have not expanded around the total vacuum expectation value, \( V_0 \). The \( D_{\text{tot}}^{KR} \) contains all terms independent of \( k' \) and \( R_2 \), but dependent on the anomalous couplings of the Higgs potential. When the anomalous couplings of the Higgs potential vanish, the \( D_{\text{tot}}^{KR} \) just reduce the one loop divergence structure of the SM. The \( D_{\text{tot}}^{AN} \) includes all terms with \( k' \) and \( R_2 \) expand around units of the SM, and it also contains some terms dependent on the anomalous couplings of the Higgs potential. In the following limit,

\[
R_2 \to 1, \ k' \to 1.
\]

(75)

each terms of the \( D_{\text{tot}}^{AN} \) vanishes.

By using the equation of motion of the background Higgs field, we get

\[
\begin{align*}
D_{\sigma}^{KR} &= \frac{3}{16} \mathcal{L}_5 - \frac{2g^2 + G^2}{32} V^2 \mathcal{O}_{WZ} - \frac{3 \mathcal{E}_0^2}{4V^2} - \frac{3}{2} \mathcal{E}_0 \mathcal{E}_1 \mathcal{E}_2 \left[ \mathcal{E}_2^2 + 2\mathcal{E}_0 \mathcal{E}_2 \right] \\
&- \frac{1}{8} \left[ \mathcal{E}_0 (2g^2 + G^2) + 12 \mathcal{E}_1 \mathcal{E}_2 + 12 \mathcal{E}_0 \mathcal{E}_3 \right] V \\
&- \frac{1}{8} \left[ 2\mathcal{E}_1 (2g^2 + G^2) + 6 \mathcal{E}_2^2 + 12 \mathcal{E}_1 \mathcal{E}_3 \right] V^2 \\
&- \frac{1}{8} \mathcal{E}_2 (2g^2 + G^2 + 12 \mathcal{E}_3) V^3 - \frac{1}{8} \mathcal{E}_3 (2g^2 + G^2 + 6 \mathcal{E}_3) V^4.
\end{align*}
\]

(76)
The $\mathcal{L}_5$ in $D^K_{\sigma R}$ just cancel that in the Eq. (72). When the anomalous couplings in the Higgs potential vanish, the $D^K_{\sigma R}$ just the master equation of the SM, and no extra divergence will appear.

The $D^A_L$, $D^A_N$, $D^A_{SF}$, and $D^A_{GF}$ contains the anomalous operators of the vector boson sector up to $O(p^4)$. The $D^A_{SF}$ and $D^A_{GF}$ indicate that the anomalous operators $L_{11}$, $L_{12}$, and $L_{13}$, should be included when there are anomalous couplings in Higgs sector, and the equation of motion of vector bosons can not eliminate out these terms. As pointed out in the literature [8], these three anomalous operators are just related with the equation of motion of vector bosons can not eliminate out these terms. As pointed out in the literature [8], these three anomalous operators are just related with the $R_b$, $B^0 - \bar{B}^0$ mixing, and $b \to s$ transitions. The $D^A_{Mass}$ will contribute the parameter $\rho$, and such a fact indicates to formulate a set of complete operators, the mass term of $Z$ bosons with the following form

$$\frac{3}{4} m_Z^2 \left( h \cdot h + v_1 \right)^2$$

must be added to the effective Lagrangian at the $O(p^2)$ order.

The $D^A_\sigma$ is very complicated, so here we expand it and keep only the terms up to $O(p^4)$, which read

$$D^A_\sigma = D^A_L + D^A_W + D^A_{HK} + D^A_{HP}$$
$$D^A_L = -\frac{1}{16} \left[ k^4 - 4(2k' + 1)k^2 + 12k'^2 + 8k' - 9 \right] \mathcal{L}_5$$
$$D^A_W = [D^A_{WZ,0} + D^A_{WZ,1} + D^A_{WZ,2}]\mathcal{O}_{WZ}$$
$$D^A_{WZ,0} = V_0^2 \frac{2g^2 + G^2}{32k^2} \left( k^2 (k^2 + 1) - 2k'^2 \right)$$
$$D^A_{WZ,1} = 2V_0 \overline{\mathcal{H}} \left[ \frac{(2g^2 + G^2)(1 - k^2)}{32k'} + \frac{m_H^2}{8k^2V_0^2} \left( (k' - 1)k^4 + (5k' + 1)k^2 \right) \right]$$
$$D^A_{WZ,2} = k' \overline{\mathcal{H}} \left[ - \frac{(2g^2 + G^2)(k'^2 - 1)}{32k'} + \frac{\lambda_3}{8kk'V_0} \left( (k' - 1)k^4 + (5k' + 1)k^2 \right) \right]$$
$$D^A_{HK} = \frac{(2g^2 + G^2)(k^2 - k')(2k^2 - k')}{4k^2} \partial \overline{\mathcal{H}} \cdot \partial \overline{\mathcal{H}}$$

$^1$here the mass eigenstates should be understood as the combination of the Stueckelberg fields
\[
D_{\text{HP}}^{AN} = D_{\text{HP,1}}^{AN} + D_{\text{HP,2}}^{AN} + D_{\text{HP,3}}^{AN} + D_{\text{HP,4}}^{AN}
\]
\[
D_{\text{HP,1}}^{AN} = \frac{(2g^2 + G^2)(k^2(k' + 1) - 2k'^2)m_H^2}{8kk'}\lambda_3^2
\]
\[
D_{\text{HP,2}}^{AN} = \left\{ \left[ k^4(4k' - 3) - 4k'k^2(2k'^2 - 3k' - 2) + k'^2(-12k'^2 - 16k' + 15) \right] \frac{m_H^4}{4k^2V_0^2} \right. \\
+ \frac{2g^2 + G^2}{8k^2} \left[ (2k'^3 + k^2(-4k'^2 + k' + 1))m_H^2 \right] \\
+ \frac{2g^2 + G^2}{16kk'} \left[ (k^2(k' + 1) - 2k'^3)V_0\lambda_3 \right] \right\}
\]
\[
D_{\text{HP,3}}^{AN} = \left\{ \frac{2g^2 + G^2}{48kk'} \left[ k^2(k' + 1) - 2k'^3 \right] \lambda_4 V_0 + \left[ \frac{m_H^4}{2k^3V_0^3} \left( k^6(6 - 8k') + k'k^4(8k'^2 - 8k' - 11) + k'^3(12k'^2 + 16k' - 15) \right) \right] \\
+ \frac{(2g^2 + G^2)m_H^2}{4k^2V_0^3} \left[ (k^2 - k')k' \right] \\
+ \frac{(2g^2 + G^2)\lambda_3}{16k^2} \left[ 2k'^3 + k^2(-4k'^2 + k' + 1) \right] \\
+ \frac{\lambda_3 m_H^2}{4k^2V_0^2} \left[ k^4(4k' - 3) - 4k'k^2(2k'^2 - 3k' - 2) + k'^2(-12k'^2 - 16k' + 15) \right] \right\}
\]
\[
D_{\text{HP,4}}^{AN} = \left\{ \frac{2g^2 + G^2}{48k^2} \left[ 2k'^3 + k^2(-4k'^2 + k' + 1) \right] \lambda_4 \\
+ \left[ \frac{m_H^4}{4k^4V_0^4} \left( 12k^8(4k' - 3) + 2k'k_6(-16k'^2 + 4k' + 31) + k'^2k^4(8k'^2 - 8k' - 11) - 3k'^3(12k'^2 + 16k' - 15) \right) \right] \\
+ \frac{m_H^2}{12k^3V_0^3} \left[ k^4(2g^2 + G^2)(k' - k^2) + k^2\lambda_4[k^4(4k' - 3) - 4k'k^2(2k'^2 - 3k' - 2) + k'^2(-12k'^2 - 16k' + 15))] \right] \\
+ \left[ \frac{\lambda_3^2}{16k^2V_0^2} \left( k^4(4k' - 3) - 4k'k^2(2k'^2 - 3k' - 2) + k'^2(-12k'^2 - 16k' + 15) \right) \right] \\
+ \frac{(2g^2 + G^2)\lambda_3}{8k^3V_0^3} \left( k'^3(k^2 - k') \right) \right\} \\
+ \frac{\lambda_3 m_H^2}{2k^3V_0^3} \left[ k^6(6 - 8k') + k^4k'(8k'^2 - 8k' - 11) \right]
\]
\[ +k'^2(12k^2 + 16k' - 15) \] \( \bar{h}^4 + \cdots \). (78)

In this step, we have expanded \( 1/V \) and \( R_2 \) and one of the important expansions is provided below

\[
\frac{k'}{R_2} - 1 = (k^2 - 1) - 2k(k^2 - k')\frac{\bar{h}}{V_0} + (k^2 - k')(4k^2 - k')\frac{\bar{h}^2}{2V_0^2} + \cdots. \tag{79}
\]

The \( D_{WZ,0}^{AN} \), \( D_{WZ,1}^{AN} \) and \( D_{WZ,2}^{AN} \) determine the renormalization constants of the vacuum expectation value \( V_0 \), the anomalous couplings \( k \) and \( k' \), respectively. One feature is remarkable, that the mass of the Higgs scalar contributes to the renormalization constants of these two anomalous couplings.

The \( D_{HP,i}^{AN} \) are related with the renormalization of the Higgs potential. Again, the mass of the Higgs scalar contributes to both the triple and quartic couplings of Higgs sector.

As careful readers have noted, we have not specify the dimension of the background Higgs field in our auxiliary counting rule. Due to the fact that the vacuum and the background Higgs field are originated from the same source, it is difficult to determine a consistent Higgs field’s dimension, the traditional momentum counting rule is also invalid in this case. Here after expanding the \( R_2 \), we omit those operators proportional to \( \bar{h}/V_0 \) and only keep those operators with mass dimension 4—i.e. operators up to the marginal operators in the Wilsonian renormalization framework \cite{10}.

Expanding \( 1/V \) around \( R_2 \) with respect to \( \bar{h}/V_0 \), some terms in \( D_{KR}^{AN} \) and all terms of \( D_{\sigma}^{AN} \) will induce infinite divergence tower with infinite \( \bar{h} \). Like those in the bosonic sector \( O(p^4) \) \cite{1}, the anomalous couplings of the Higgs sector up to \( O(p^2) \) also induce infinite divergence structures, even though at the one loop level. Such a fact is not a surprise, as these anomalous couplings violate the requirement of the renormalizability of the theory. However, as the basic idea of effective theory prescribed \cite{11}, these operators still respect the symmetry of the dynamic system. If we include all possible operators permitted by the specified symmetries (Lorentz symmetry and gauge symmetries), these infinite operators still might form a set of complete operators, and can be regarded as renormalizable, according to \cite{12}, though infinite renormalization conditions are needed in order to determine the infinite couplings of the theory, though the prediction power of the theory is as good as the ordinary renormalizable with finite number of couplings.
VI. DISCUSSIONS AND CONCLUSIONS

We have studied the one loop divergence structures of the effective Higgs sector up to $O(p^2)$. Some features of the divergences of this unrenormalizable theory are revealed. We have found that, up to $O(p^4)$, the mass term of $Z$ boson and the whole EWCL of bosonic sector are necessary to define a set of complete operators at the $O(p^4)$ order, even those terms which might be eliminated by the equation of motion of vector bosons in the EWCL $O(p^4)$ should be included.

We will study the renormalization and provide the complete one loop renormalization group equations of the extended EWCL with effective Higgs sector up to $O(p^4)$ [9].

ACKNOWLEDGMENTS

The work of Q. S. Yan is supported by the Chinese Postdoctoral Science Foundation and the CAS K. C. Wong Postdoctoral Research Award Foundation. The work of D. S. Du is supported by the National Natural Science Foundation of China.

[1] Q. S. Yan and D. S. Du, hep-ph/0212367.
[2] R. S. Chivukula and V. Koulovassilopoulos, Phys. Lett. B 309 (1993) 371.
[3] H. J. He, et. al. hep-ph/0211229.
[4] B.S. DeWitt, Phys. Rev. 162(1967)1195, 1239.
[5] E. C. G. Stueckelberg., Helv. Phys. Acta 11 (1938) 299; 30 (1956) 209; T. Kunimasa and T. Goto, Prog. Theor. Phys. 37 (1967) 524.
[6] Schwinger, Phys. Rev. 82 (1951) 664; I. G. Avramidi, Lecture Notes in Physics: N.s. M. Monograph; 64 Heat Kernel and Quantum Gravity, Springer, 2000; R. D. Ball, Phys. Rep. 182 (1989) 1.
[7] B. S. Dewitt, Phys. Rept. 19 C (1975) 295; L.S. Brown, Phys. Rev. D 15 (1977) 1469; C. Lee, Nucl. Phys. B 207 (1982) 157; A. Nyffeler, and A. Schenk, hep-ph/9409043.
[8] J. Berabéu, D. Comelli, and A. Pich and A. Santamaria, Phys. Rev. Lett. 78 (1997) 2002.
[9] Q. S. Yan and D. S. Du, in preparation.
[10] K. G. Wilson and J. Kogut, Phys. Repts. 12 C, (1974) 75; J. Polchinski, Nucl. Phys. B 231 (1984) 269.

[11] S. Weinberg, Physica 96 A (1979) 327; H. Leutwyler, Ann. Phys. NY 235 (1994) 165.

[12] S. Weinberg, The Quantum Theory of Fields, Vol. I, Published by Cambridge University Press, Chapter 12.