Modelling dynamic compaction of porous materials with the overstress approach

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Abstract. To model compaction of a porous material we need 1) an equation of state of the porous material in terms of the equation of state of its matrix, and 2) a compaction law. For an equation of state it is common to use Herrmann’s suggestion, as in his $P_\alpha$ model. For a compaction law it is common to use a quasi-static compaction relation obtained from 1) a meso-scale model (as in Carroll and Holt’s spherical shell model), or from 2) quasi-static tests. Here we are interested in dynamic compaction, like in a planar impact test. In dynamic compaction the state may change too fast for the state point to follow the quasi-static compaction curve. We therefore get an overstress situation. The state point moves out of the quasi-static compaction boundary, and only with time collapses back towards it at a certain rate. In this way the dynamic compaction event becomes rate dependent. In the paper we first write down the rate equations for dynamic compaction according to the overstress approach. We then implement these equations in a hydro-code and run some examples. We show how the overstress rate parameter can be calibrated from tests.

1. Introduction
To model compaction of a porous material we need:

- An equation of state (EOS) of the porous material in terms of the EOS of its matrix material.
- A compaction law.

For an EOS we use Herrmann's suggestion as in his $P_\alpha$ model [1]. For a compaction law it is common to use a quasi-static law, which can be obtained from:

- A meso-scale model, like Carroll and Holt's spherical shell collapse model [2].
- A quasi-static test, as in the $P_\alpha$ model [1].

We are interested here in dynamic compaction as in a planar impact test. In dynamic situations changes may be too fast for the state point to be able to follow the quasi-static compaction surface, and it may therefore move beyond that surface (overstress). When it does, it tends to fall back towards the quasi-static surface at a rate that increases with overstress. In this way the process of pore collapse, or pore closure, is made rate dependent. We call this the overstress approach, or the overstress concept. In what follows we:

- Write down the rate equations for dynamic compaction, which we implement in a hydro-code.
- Run an example of planar impact on a porous stainless steel target.
- Evaluate the influence of the rate dependence of the compaction process on stress histories down the target.
2. Herrmann's EOS
Herrmann suggested in his Pα compaction model paper [1] a very simple EOS for porous materials:

\[
E(P, V) = E_m(P_m, V_m); \text{ m for matrix}
\]

\[
P_m = \alpha P; \quad \alpha = \frac{V}{V_m} = \frac{\rho_m}{\rho}
\]

\[
E = E_m\left(\alpha P, \frac{V}{V_m}\right); \quad \varphi = 1 - \frac{1}{\alpha}; \quad \alpha = \frac{1}{1 - \varphi}
\]

where E=specific internal energy, P=pressure, V=specific volume, α=distension ratio, and \(\varphi\)=porosity. Herrmann's EOS is usually used for compaction of porous materials, although we've not seen it verified directly or indirectly. Herrmann's EOS is a special case of:

\[
E = E(P, V, \varphi)
\]

Differentiating we get:

\[
\dot{E} = \frac{\partial E}{\partial P} \dot{P} + \frac{\partial E}{\partial V} \dot{V} + \frac{\partial E}{\partial \varphi} \dot{\varphi}
\]

Using the adiabatic condition:

\[
\dot{E} = -(P + q) \dot{V}
\]

where \(q\) is the artificial viscosity, we get:

\[
\frac{\partial E}{\partial P} \dot{P} = \left( P + q + \frac{\partial E}{\partial V} \right) \dot{V} - \frac{\partial E}{\partial \varphi} \dot{\varphi}
\]

We see that to complete the EOS we need a \(\varphi(P, V)\) or a \(\varphi(P, V)\) relation, which is the compaction law. The first alternative is rate independent (or quasi-static), and the second is a rate dependent compaction law. For Herrmann's EOS the three partial derivatives in equation (3) are given by:

\[
\frac{\partial E}{\partial P} = \frac{1}{1 - \varphi} \frac{\partial E_m}{\partial P_m}
\]

\[
\frac{\partial E}{\partial V} = (1 - \varphi) \frac{\partial E_m}{\partial V_m}
\]

\[
\frac{\partial E}{\partial \varphi} = \frac{P}{(1 - \varphi)^2} \frac{\partial E_m}{\partial V_m} - V \frac{\partial E_m}{\partial V_m}
\]

3. Compaction law
Compaction laws are usually defined in the reduced forms \(\varphi(P)\) or \(\varphi(P)\). The simplest, and quite extensively used, rate independent compaction law is: \(P_{qs}(\varphi) = P_c\). This means that the material is compacted immediately in response to \(P > P_c\). Another rate independent compaction law is derived from Carroll and Holt's quasi-static spherical shell model [2], which is:

\[
P_{qs} = \frac{2}{3} Y_m \varphi \left( \frac{1}{\varphi} \right)
\]

where \(Y_m\) is the flow stress of the matrix material. As this curve goes to infinity for zero porosity, we use a correction that makes it finite at zero porosity:

\[
P_{qs} = \frac{2}{3} Y_m \varphi \left( \frac{1}{\varphi + \delta} \right)
\]

where \(\delta\) is a small number like \(10^{-4}\). A general rate dependent compaction law by the overstress approach can take the form:
where $F$ is an increasing function of its argument. We're using here the simplest form of $F$, with a single material parameter:

\[
\phi = -A_\phi [P - P_{qs}(\phi)]; \quad P > P_{qs}(\phi) \\
\phi = 0; \quad P \leq P_{qs}(\phi)
\]  

(10)

where the coefficient $A_\phi$ can be calibrated from tests or from a model on the meso-scale.

4. Simulations

As mentioned above, we implement our compaction model in a hydro-code. We're using the Lagrange processor of the old commercial code PISCES [3]. Equations (5) and (10) make a system of two ODEs to be integrated simultaneously for each computational cell and time step separately. Entering the subroutine that solves the equation of state together with conservation of energy, $V_i$ and $V_f$ (at the start and end of the time step) are known. The average $\bar{V}$ is then:

\[
\bar{V} = (V_f - V_i)/\Delta t
\]  

(11)

Solving the system of two ODEs from the start of the time step to its end, we obtain $P$ and $\phi$ at the end of the time step. Substituting back into the EOS we also get $E$.

For stainless steel (the matrix) we use the Mie Gruneisen EOS with constant $\rho \Gamma$, referenced to the shock Hugoniot, as given by:

\[
E_m - E_{Hugm} = \frac{V_{0m}}{\Gamma_{0m}} (p_m - P_{Hugm})
\]  

(12)

\[
P_{Hugm} = \rho_{0m} (C_{0m})^2 \frac{\varepsilon}{(1-S_m)^2}; \quad \varepsilon = 1 - \frac{V_m}{V_{0m}}
\]

We ignore the strength of the porous material. The parameters of this EOS (with the usual notation)are:

\[
C_{0m} = 4.57 \text{kny/s}; \quad S_m = 1.49 \\
\Gamma_{0m} = 2.2; \quad Y_m = 0.9 \text{GPa}
\]  

(12)

Next we show results of several planar impact 1D runs, with different values of the coefficient $A_\phi$, the initial porosity $\phi_0$, and the incoming shock pressure $P_{in}$. The target is 100 mm long, and the mesh is 10 cells/mm. The incoming shock is applied by a pressure boundary condition at $x=0$, and the right boundary at $x=100$ mm is free.

In figure 1 we show pressure histories at four Lagrange distances into the target every 20 mm. The incoming shock is 20GPa, the initial porosity is 20%, and $A_\phi=0.005$ (GPa*μs)$^{-1}$. We see from figure 1 that like in viscoplastic response, here too there is a precursor decay phenomenon, and the rate of decay depends on the value of the coefficient $A_\phi$. The last two curves show the effect of the release wave from the free boundary. We see that at $x=80$ mm the pores did not quite close before the release wave arrived.

In figure 2 $P_{in}$ and $\phi_0$ are the same, and we show the influence of the rate coefficient $A_\phi$. We see from figure 2 that as $A_\phi$ increases, precursor decay is faster, and the history curves become steeper.

In figure 3 $P_{in}$ is the same, $A_\phi=0.010$ (GPa*μs)$^{-1}$, and the initial porosity is 10, 20 and 30%. We see from figure 3 that as expected, for higher initial porosity the steady precursor is lower, and the rise time is higher.
In figure 4 we go back to 20% porosity, and the incoming shock is 10, 20 and 30 GPa. We see from figure 4 that the stronger the incoming shock, the steeper are the history curves.

Finally we show in figure 5 the U(u) relation obtained for $\phi_0=20\%$ and $A_\phi=0.010 \text{ (GPa*\mu s)}^{-1}$, where $u$ is the particle velocity at the plateau level, and $U$ is the speed of arrival of the plateau level.

**Figure 1.** Pressure histories every 20 mm into the target. $P_{in}=20\text{ GPa}$, $\phi_0=20\%$, $A_\phi=0.005 \text{ (GPa*\mu s)}^{-1}$.

**Figure 2.** Pressure histories at $x=40\text{ mm}$ into the target. $P_{in}=20\text{ GPa}$, $\phi_0=20\%$, $A_\phi=0.002$, 0.005 and 0.010 (GPa*\mu s)$^{-1}$.

**Figure 3.** Pressure histories $x=40\text{ mm}$ into the target. $P_{in}=20\text{ GPa}$, $\phi_0=10$, 20, and 30\%, $A_\phi=0.010 \text{ (GPa*\mu s)}^{-1}$.

**Figure 4.** Pressure histories at $x=40\text{ mm}$ into the target. $P_{in}=10$, 20, and 30 GPa, $\phi_0=20\%$, $A_\phi=0.010 \text{ (GPa*\mu s)}^{-1}$.
Figure 5. U(u) plot for 20% initial porosity and $A_p=0.010$ (GPa*μs)$^{-1}$. u is the particle velocity at the plateau level. U is the speed of the arrival of the plateau level.

5. Summary

We focus on the dynamic compaction of porous materials. We use Herrmann's suggestion to express the EOS of the porous material in terms of the EOS of the matrix material and the porosity. For a compaction law we use the overstress approach, referenced to the rate independent compaction law derived from Carroll and Holt's quasi-static spherical shell model. This makes our compaction law rate dependent.

We express our compaction model equations by rate equations, and implement them into a hydro code. We run examples of planar impact of a porous stainless steel target.

Using pressure history plots at locations down the target we show the following:

- Pore compaction precursor decay.
- Influence of the rate dependence parameter.
- Influence of the initial porosity.
- Influence of the incoming shock level.
- U(u) plot for the plateau arrival.

Our pore compaction model still needs to be validated and calibrated for different materials by appropriate tests.

References

[1] Herrmann W 1969, Constitutive Equation for the Dynamic Compaction of Ductile Porous Materials, J. Appl. Phys. 40, 2490-2499.
[2] Carroll M M and Holt A C 1972 J. Appl. Phys. 43 1626-1636.
[3] van der Heijden A M A and Groenenboom P H L 1990, A Survey of MSC/PISCES Applications The MacNeal-Schwendler Company B V Gouda The Nedelands