Simplified selection of optimal shell in shape of translational surface

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Abstract. The translational shells are arised by moving generatrix line along directrix line. Four shells of similar overall size and rising height are modelled in the present job, each generatrix coincide with directrix, the symmetric square plane is considered. The surface types are: surface of translation of circle along circle, catenary along catenary, sinus along sinus and ellipse along ellipse. The behaviour under equally distributed load like self-weight of 4 types of shells is considered, equally distributed load case while supporting on four corner points is considered. The solution is obtained by means of finite element analysis. The ratio of bending and membrane stresses is of interest as the criterion of optimal operation under load. The equalent von Mises stresses for abstract material with characteristics of reinforced concrete are obtained to reveal the most dangerous zones and evaluate the overall stress distribution. The study is of interest in view of perspectives to introduction into practise of structure elements of such shape.

Keywords: translational surface, thin elastic shell, stress-strain state, finite element method.

1. Introduction
The shells in shape of transition surfaces worth consideration in aspect of their introduction in modern architecture because of their low weight and volume combined with aesthetical properties [1-2].

The optimal choice of variable geometrical parameters and similar or close overall dimensions and rising heights is of interest for mechanical engineers since the times of the first shell researchers. The large majority of such investigations take into consideration domes and vaults of traditional, well-known shapes, for example, domes on the base of different rotation surfaces [3-9]. The shells of less widespread non-traditional shapes of shells in form of revolution surfaces are concerned in papers [10-12]. Some criteria of optimality for revolution surface shells are developed and applied, they are specified in [11]. Quite less attention of researchers was focused on translation surface shells.

Four shells in shape of translational surfaces on square plane are modeled in the present job: surface of translation of circle along circle, catenary along catenary, and ellipse along ellipse, sinusoid along sinusoid. The structure elements can be created by spraying mortar on the steel or fiber reinforcement grid of the specified shape. All the shapes proposed can be fabricated from bars, bar mats or grids by flexure.

The subject of this job is comparison and choosing the optimal shape of the roof of the square plane among translational surfaces. The optimality criterion is minimum stress and strain. The model of 6x6 m square plane roof of reinforced concrete was investigated, with four point hinge supports in corners.

The peculiarities of four type of shells stress-strain state can be evaluated and clearly shown on such an example. The most reasonable variant of roof shape can be recommended for introduction into practice, the probable difficulties and ways to meet them while design and construction are to be discussed.
2. Methods

2.1. Geometrical modeling

The models for comparison were created using analytic expressions for the corresponding surfaces. The equations of the translational surfaces are of common knowledge. They are provided in [13].

2.1.1. Surface of translation of catenary curve along catenary curve.

The surface equation:

\[
  z = -a \cdot \cosh((x - b) / a) + a \cdot \cosh(b / a)) - d \cdot \cosh((y - c) / d) + d \cdot \cosh(c / d),
\]

(1)

where \( e = b = 3 \text{ m} \) is half of the span along x and y coordinate lines respectively, \( a = d = 4.657746 \text{ m} \) are parameters relative to rising height. The rising height is calculated within \( 10^6 \text{ m} \), because its derivation is not explicit.

2.1.2. Surface of translation of circle along circle (toroid)

The surface equation:

\[
  z = \left[ r_1^2 - \left( x + \frac{a}{2} \right)^2 \right]^{0.5} + \left[ r_2^2 - \left( y - \frac{b}{2} \right)^2 \right]^{0.5} - r_1^2 - r_2^2 = 6\text{ m} \quad \text{is span along the coordinate lines x, y respectively,}
\]

\( r_1 = r_2 = 5\text{ m} \) are radii of directrix and generatrix circles respectively.

\[
  a = b = 6\text{ m}
\]

(2)

2.1.3. Surface of translation of ellipse along ellipse.

The surface equation:

\[
  z = f_1 - \frac{b}{a} \cdot \left( a^2 - \left( x - \frac{c}{2} \right)^2 \right)^{1/2} + f_2 - m + \frac{m}{n} \cdot \left( n^2 - \left( y - \frac{d}{2} \right)^2 \right)^{1/2},
\]

(3)

where \( a = n = 3\text{ m}, b = m = 1\text{ m}, c = d = 6\text{ m}, f_1 = f_2 = 3\text{ m} \).

Where \( f_1 \) is rising height of the ellipses in planes \( y = 0 \) and \( y = d \); \( f_2 \) is rising height of the ellipses in planes \( x = 0 \) and \( x = c \). \( c, d \) are dimensions in plane; \( a, b \) are halves of axis of ellipses, got by section the surface by plane \( y = \text{const} \); \( m, n \) are halves of axis of ellipses, got by section the surface by plane \( x = \text{const} \).
2.1.4. Surface of translation of sinusoid curve along sinusoid.
The surface equation:

\[ z = c \cdot \sin \frac{\pi x}{a} + d \cdot \sin \frac{\pi y}{b}, \]  

Where \(a=b=6\) m is overall dimensions of flat contour in plan, lengths of one half-wave of the sinusoid along \(x\) and \(y\) direction, respectively, \(c=d=1\) m are amplitudes of the sinusoids. Geometrical parameters of the model in the present job: \(f=2\) m, \(a=6\) m, \(b=6\) m.

2.2. Numerical study
Four finite element models were investigated, each structure element has dimensions in plane 6×6 m, rising height of 2 m, thickness of 0.08 m, material characteristics \(E=325\) MPa, \(\nu=0.17\). The four corners points of the shell have hinged supports. The equally distributed vertical load like self-weight of 10 KN/m² is applied on upper surface. Static structural analysis was conducted by FEM software Ansys APDL. The shells were meshed by quadrilateral elements shell181 [14]. Element side size is 0.25 m, the model has 27×27 nodes and 676 elements, that is enough for the purpose of qualitative evaluation of stress-strain state. For the similar purpose the approximate location of problem zones with high stresses was obtained by calculating the equivalent stresses in abstract material without exact reinforcement scheme.

3. Results
As a result of analysis of the models created, the stress-strain state parameters for each shell structure were obtained. The isofields of stresses, forces and moments, displacements and also illustrative visual graphs in middle section were got by finite element analysis in Ansys APDL. The summary table allows to evaluate the differences between each structure’s behavior under load. The figures and summary table are represented below.

3.1. Shell in shape of surface of translation of catenary curve along catenary curve:
3.2. Shell in shape of surface of translation of circle curve along circle
Figure 6 a) $N_{11}$, N/m.

Figure 6 b) $N_{22}$, N/m.

Figure 6 c) $Q_{23}$, N/m.

Figure 6 d) $M_{11}$, N·m/m.

Figure 6 e) $M_{22}$, N·m/m.

Figure 6 f) $u_x$ displacement, m.

Figure 6 g) von Mises stress, N/m².
3.3. Shell in shape of surface of translation of ellipse along ellipse

Figure 7 a) $N_{11}$, N/m.

Figure 7 b) $N_{22}$, N/m.

Figure 7 c) $N_{12}$, N/m.

Figure 7 d) $M_{11}$, N·m/m.

Figure 7 e) $M_{22}$, N·m/m.

Figure 7 f) $Q_{23}$, N/m.

Figure 7 g) $u_z$ displacement, m.

Figure 7 h) von Mises stress, N/m².
3.4. Shell in shape of surface of translation of sinusoid along sinusoid

Figure 8a) $N_{11}$, N/m.

Figure 8b) $N_{22}$, N/m.

Figure 8c) $N_{12}$, N/m.

Figure 8d) $M_{11}$, N·m /m.

Figure 8e) $M_{22}$, N·m /m.

Figure 8f) $Q_{23}$, N·m.

Figure 8g) $u_z$ displacement, m.

Figure 8h) von Mises stress N/m$^2$. 
Table 1 Maximum parameters of stress-strain state.

| Generatrix curve type | N_{11}, N/m | N_{22}, N/m | N_{12}, N/m | Q_{23}, N/m | M_{11}, N·m/m | M_{22}, N·m/m | u_z, m |
|-----------------------|--------------|--------------|--------------|-------------|----------------|----------------|--------|
| catenary              | 2401/2501    | -1841/1841   | -170/170     | -160/160    | 38/-38        | 1.3×10^4       |        |
| circle                | 4598/-4170   | -1969/-192   | -224/-192    | 54/-207     | 1.5×10^4       |               |        |
| ellipse               | 3523/-1158   | 312/-198     | 456/207      | 532/207     | 3.54×10^4     |               |        |
| sinusoid              | -3077/-1460  | 2/-6.23      | 5.692/2.74   | 2/4.94      | 4.9×10^5      |               |        |

4. Discussion

The results of this investigation have shown that the most advantageous behavior (for reinforced concrete) while the considered scheme of supports and loading is demonstrated by the shell which shape is based on sinusoid curve, the worst one is that of the shell of shape based on the circle (it has maximum stresses). The behavior of translational surface shells while supporting on 4 corner points does not strictly correspond to the behavior of archs of the similar curves, but some analogy can be made. So it can be supposed that tensile part of common two-pinched archs in shape of circle or parabola corresponds to the lower part of the translational shells of positive curvature, unlike the sinusoid curve, which curvature and convexity is changing so that the stresses keep compressive. Translational shells of circle, catenary and ellipse base need supporting on fixed diaphragm plates like cylindrical ones. Without diaphragms they have remarkable zones of tension. It is clear that the corners of all such shells need extra reinforcement and thickening and modeling of such a corner by one point is a simplifying assumption, relevant only for preliminary theoretical qualitative analysis.

Thus, the use of sinusoidal translational shells once again confirms its feasibility. It is also interesting to compare the results obtained in this paper with the results of the study of dome shells of revolution [11], for which the catenoid and parabola were the most rational forms for generatrix curve. In the scope of this short article, only one calculation scheme is considered, and it is quite abstract. It is intended to continue this research and study the behavior of shells for other static schemes with other boundary conditions, as well as try to design real structural elements.

5. Conclusions

The special case of shells in form of translational surfaces on square plane under equally distributed load with hinged supports in four corner points is considered. This job is closely connected to paper [15], where load case with fixed supports along all sides is considered. The particularities of shells behavior are revealed, the facilities for practical application are estimated. While this load case the shell in shape of ‘sinusoid along sinusoid’ surface turned out to have almost only compression stresses, that is an advantage for reinforced concrete structures. It also has small values of bending moments and minimum deflection. The shells in shape of ‘catenary along catenary’, ‘circle along circle’ show the resembling behavior in the sense that they both have zones of tension and compression, moreover, the axial forces and bending moments of ‘catenary along catenary’ are less than those of ‘circle along circle’ shell. The shell in shape of ‘ellipse along ellipse’ surface has high values of bending moments and, in addition, remarkable shear forces. It means that such shell is needs reinforcement most of all.

The prospect of this investigation is to estimate not only the static structural, but also buckling and dynamic analysis of the shells of non-traditional shapes with different load cases.
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