Time Asymmetry in Quantum Physics - II.
Experimental Demonstration
Using a Single Ion

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Abstract
Quantum physics involves an ensemble of quantum systems, usually one thinks of a large ensemble of identical quantum systems at one single time. In single ion experiments one has a single quantum system at an ensemble of different times. This provides the means of demonstrating the beginning of time of a semigroup evolution for a decaying state.

1 Introduction – states and observables

In part I of this series of papers we have discussed how time asymmetry arose in quantum mechanics. Here we explain the meaning of the time $t = 0$ of preparation of a quantum state. In the present paper we describe an experiment which verifies our concept. It is based on the idea of quantum jumps for a single ion.

In essentially all discussions of the foundation of quantum theory, one distinguishes between states and observables. States, described by a state
vector $\phi^+$ (or a density operator $W$) fulfill the Schrödinger equation

$$i\hbar \frac{d}{dt}\phi^+(t) = H\phi^+(t). \quad (1.1)$$

The state represents the preparation apparatus of an experiment.

Observables, described by operators $A = A^\dagger$ or projection operators $\Lambda$ ($\Lambda = \Lambda^2$) or, in the simplest case, by one-dimensional projections $\Lambda = \lvert\psi^-\rangle\langle\psi^-\rvert$, or vectors (up to a phase factor) $\psi^-$, fulfill the Heisenberg equation

$$i\hbar \frac{d}{dt}\psi^-(t) = -H\psi^-(t) \quad (1.2a)$$

$$i\hbar \frac{d\Lambda(t)}{dt} = -[H, \Lambda(t)], \quad (1.2b)$$

where (1.2b) for an operator $\Lambda(t)$, is the more familiar form of this dynamical equation. The observable represents the registration apparatus (detector) of an experiment.

The experimental quantities are the probabilities $P_W(\Lambda(t))$ of the observable $\Lambda(t)$ in the state $W$. Theoretically, $P_W(\Lambda)$ are calculated as the Born probabilities

$$P_W(\Lambda(t)) = \text{Tr}(\Lambda(t)W) = \text{Tr}(\Lambda W(t)) \quad (1.3)$$

which for the special case of an observable $\lvert\psi^-\rangle\langle\psi^-\rvert = \Lambda$ in a pure state $\lvert\phi^+\rangle\langle\phi^+\rvert = W$ is given by

$$P_{\phi^+}(\psi^-(t)) = |\langle\psi^-(t)|\phi^+\rangle|^2 = |\langle\psi^-|\phi^+(t)\rangle|^2. \quad (1.4)$$

Experimentally the probabilities $P_W(\Lambda)$ are measured as ratios of large detector counts, e.g.,

$$P_W(\Lambda(t)) = \frac{N(t)}{N}, \quad (1.5)$$

where $N(t)$ is the number of detector count at time $t$, and $N$ is the total number of counts.
The solutions of the dynamical differential equations like (1.1) and (1.2) have to be found under boundary conditions. For standard quantum mechanics, the boundary condition is given by the Hilbert space axiom:

The solutions of (1.1) and of (1.2) are elements of the (norm-complete) Hilbert space $\mathcal{H}$.

\[
\text{set of states } \{\phi^+(t)\} = \mathcal{H} = \text{set of observables } \{\psi^-(t)\}. \quad (1.6)
\]

(This means the energy wavefunctions $\psi^-(E) = \langle -E | \psi^- \rangle$, $\phi^+(E) = \langle +E | \phi^+ \rangle$, must be Lebesgue square integrable.)

In paper I [1], it was argued that under the standard axiom (1.6) one could not have a consistent theory of resonance and decay phenomena: If one wants to include states for which the approximate lifetime-width relation of the Weisskopf-Wigner methods, (2.9) of [1], becomes an exact equality, $\tau = h/\Gamma$, in particular if one wants to have Gamow vectors $\phi^G(t)$ with exponential time evolution, then one cannot sustain the standard axiom (1.6). Therefore, we chose a new boundary condition for the old dynamical equations (1.1), (1.2), the Hardy space axiom:

Set of prepared (in-) states $\{\phi^+\} = \Phi_- \subset \mathcal{H} \subset \Phi^\times_-$

Set of detected (out-) observables $\{\psi^-\} = \Phi_+ \subset \mathcal{H} \subset \Phi^\times_+$ \quad (1.7)

where $\Phi^\times_\pm$ are the two different Hardy spaces of the same Hilbert space $\mathcal{H}$, corresponding to the complex energy semiplanes $\mathbb{C}_\pm$. This means the energy wavefunctions $\psi^-(E) = \langle -E | \psi^- \rangle$ of out-observables must be smooth functions that can be analytically continued into the upper complex energy plane $\mathbb{C}_+$, and the wavefunctions $\phi^+(E) = \langle +E | \phi^+ \rangle$ of the in-state must be smooth functions that can be analytically continued into the lower complex energy plane $\mathbb{C}_-$.

Since the in-states are defined by the preparation apparatus (e.g., accelerator) and the out-observables are defined by the registration apparatus (e.g., detector) of an experiment on micro-systems, there is no need to identify $\{\phi^+(E)\} = \{\psi^-(E)\}$ and use the standard axiom (1.6) which in the mathematical description identifies states and observables with the Hilbert space.

2 Time asymmetry

A consequence of the new Hardy space axiom is the time asymmetry $t \geq t_0 = 0$ of the dynamical evolution given by the semigroup solutions (4.6) and (4.7)
of [1]. In particular the Gamow state $\phi^G$ fulfilling $H^\times \phi^G = (E_R - i\Gamma/2)\phi^G$, evolves in time according to (4.6) of [1] as

$$\phi^G(t) = e^{-iH^\times t}\phi^G = e^{-iE_R t}e^{-i(\Gamma/2)t}\phi^G, \quad t \geq 0. \quad (2.1)$$

and exists only for $t \geq t_0 = 0$.

Since from the Hilbert space solution of the dynamical equations (1.1), (1.2), we are used to time symmetric solutions, $-\infty < t < +\infty$, such a time asymmetry may appear surprising. But, from other areas of physics, we are quite familiar with time-asymmetric solutions of time symmetric dynamical equations due to time-asymmetric boundary conditions, e.g., the retarded and the advanced solutions of electrodynamics. The Maxwell equations, like the equations (1.1) and (1.2), are symmetrical in time. The additional axiom which chooses the retarded potential over the advanced potential is called the “radiation arrow of time”; it is formulated as a boundary condition for the Maxwell equations, which excludes the strictly incoming field in every spacetime region (Sommerfeld radiation condition). All fields possess advanced sources: Radiation must be emitted first by a source before it can be detected by a receiver.

A similar “arrow of time” exist for quantum physics: A state must be prepared first by a preparation apparatus at $t_0 = 0$ before at a later time $t > t_0$ an observable can be measured or registered by a registration apparatus. In order to formulate this quantum mechanical arrow of time we speak of states and observables, and we make a distinction between the states $\{\phi^+\}$ and the observables $\{\psi^-\}$. In quantum mechanics in general one always speaks of states and observables, but then using the standard axiom (1.6) one identifies them in the mathematical description. However, as discussed in [1], if we want to derive for the pole term of the $S$-matrix element $(\psi^-, \phi^+)$ in (3.8) of [1], the relation $\Gamma = \hbar/\tau$ between the Lorentzian width $\Gamma$ and the exponential lifetime $\tau$, then we are forced to distinguish also mathematically between states and observables and use the new axiom (1.7). This in turn leads to the semigroup evolution which introduces a “beginning of time”, the semigroup time $t_0 = 0$.

## 3 Demonstrating the quantal beginning of time

The question in quantum physics then is: What is the meaning of a “beginning of time $t_0$”? How does one observe it and why have we not been more
Figure 1: Simplified energy-level scheme of the In$^+$ ion. [5]

Most experiments in quantum physics deal with large ensembles of quantum systems (elementary particles, or atoms, or ions) which are prepared at a variety of times. That means individual atoms of an ensemble are prepared at many different times and there is no way to distinguish which has been prepared at a time, say $t_0^{(1)}$ and which at a time $t_0^{(2)}$ (by the clocks in the lab). If, however, one could work with single quantum systems and identify the preparation time of a single quantum system, then one could consider the quantum mechanical state, say $\phi^G$, whose ensemble is the collections of the single quantum systems prepared at the “ensemble of times” $t_0^{(1)}$, $t_0^{(2)}$, $t_0^{(3)}$, ...

Such experiments on the excited states of an individual quantum system have been performed in experiments with single laser cooled Ba$^+$ ions [2,3], Hg$^+$ ions [4] and In$^+$ ions [5] using Dehmelt’s idea of shelving the single ion on a metastable level [6]. Experiments of this kind require an atom with two excited states both of which are radiatively coupled to the same ground state but have vastly different transition rates. We will discuss here in detail the experiment in [5] with an In$^+$ ion.

For the In$^+$ these two levels are the $^3P_1$ and $^3P_0$ levels, Fig. [1] [5]. We denote the transitions involved by $A_1$, by $A_2$ and $A_0$. One has the following
resonance scattering and decay processes

\[ \gamma + ^1S_0 \xleftrightarrow{\text{fluorescence transition } A_1} ^3P_1 \xrightarrow{\text{laser driven fast transition } A_1} ^3P_1 \rightarrow \gamma' + ^3P_0 \] (3.1)

We focus our attention on the excited (metastable) level \(^3P_0\) which is the state we describe by the Gamow vector \(\phi^G = |z, J, \cdots\rangle \equiv |''3P''_0\rangle\).

The Gamow vector is an eigenket of the total, interaction incorporating Hamiltonian \(H\) with complex energy eigenvalue \(z_R = E_R - i\Gamma_R / 2\) where \(\Gamma_R = \frac{\hbar}{\tau}\), and \(\tau = \tau(3P_0) \approx 0.14\, s\). We use here the standard spectroscopic notation, e.g., \(^3P_0 = 2s + 1(L)J\), with \(s = 1\) and \(J = 0\) for the excited energy levels, but \(H\) is the exact Hamiltonian which includes spin orbit and hyperfine interaction, and \(\phi^G\) is an eigenstate of \(H\) and of total angular momentum-parity, \(J^\pi\). Due to the hyperfine interaction, the state of the \(^3P_0\) level includes also a superposition with a very small \(J = 1\) contribution which in turn is a superposition of a small amount of triplet and singlet states:

\[ (|''3P''_0\rangle = |3P_0\rangle + \alpha|3P_1\rangle + \beta|1P_1\rangle). \]

Due to this \(J = 1\) component there is a “slow” transition \(A_0\).

The transition \(\gamma + ^1S_0 \rightarrow ^3P_1\) is laser driven and the intensity of the fluorescence transition \(A_1:\ ^3P_1 \rightarrow ^1S_0 + \gamma\) (with a lifetime \(\tau(3P_1) \approx 4 \times 10^{-7}\, s\)) is monitored. Occasionally \(^3P_1\) makes the magnetic dipole transition into \(^3P_0\) with a branching ratio \(10^{-8}\) (there can also be a laser induced transition from \(^1S_0\) to \(^3P_0\)). This is the metastable state \(|''3P''_0\rangle \equiv \phi^G\) in which the ion will be shelved for a long time \((\tau(3P_0) \approx 0.14\, s\).

The experiment is done on a single \(In^+\) and while it is shelved it can not participate in the back and forth transitions

\[ ^1S_0 + \gamma \rightsquigarrow ^3P_1 \] (3.2)

so the shelve time is observed as a dark period of the fluorescence transition \(A_1:\ ^3P_1 \rightarrow ^1S_0 + \gamma\).

The state of \(In^+\) could be either the metastable state \(|''3P''_0\rangle = |\phi^G(t)\rangle\) evolving in time by (2.1), or it could go through many \((10^6)\) back-and-forth transitions (3.2) or it could evolve through superpositions of these three states.

\(^1\)due to the magnetic dipole component of the hyperfine interaction [7]
Figure 2: Two examples of observed quantum jumps resulting in dark periods [5]. The sudden drop in fluorescence defines the beginning of time in \(i\)-th individual quantum state \(\mu^3P_0\). The duration of the dark period \(\Delta t^{(i)}\) is the individual lifetime of this \(\mu^3P_0\). The average lifetime \(\tau\) is the weighted average of these \(\Delta t^{(i)}\).

What one observes (Fig. 2 [5]) is a sudden onset of periods of no fluorescence at a time \(t_0^{(1)}\) and a sudden return of the original fluorescence intensity at a later time \(t^{(1)}\). These onsets of dark periods followed by the return of the fluorescence repeat themselves (about 150 times in the experiment [5]): at \(t_0^{(1)}, t_0^{(2)}, \ldots, t_0^{(150)}\) one sees sudden jumps of the fluorescence intensity to 0, followed by a sudden return of the fluorescence radiation at the later times \(t^{(1)}, t^{(2)}, \ldots, t^{(150)}\).

The only reasonable interpretation of this is that during the dark periods the single \(In^+\) ion is shelved on the level \(\mu^3P_0\). While \(In^+\) is “shelved” on the metastable level \(\mu^3P_0\), it cannot participate in the fluorescence transitions \(3P_1 \leftrightarrow 1S_0 + \gamma\) and one obtains dark periods (“shelf level dwell period”) of various durations

\[
\Delta t^{(i)} = t^{(i)} - t_0^{(i)}, \quad \Delta t^{(i)} > 0, \quad t^{(i)} > t_0^{(i)}.
\]
Figure 3: The fit of the number of dark periods shorter than $t$ to the experimental law (4.7) provides an estimate of the average lifetime of the $^3P_0$ level.

Two of these dark periods for the fluorescence transition are shown in Fig. 2. A dark period means that the system is on the metastable level $^3P_0$, which is described by $\phi^G = |^"3P_0\rangle$; this means that at every of the times $t_0^{(i)}$ an individual $^3P_0$ is prepared, it “lives” for the duration $\Delta t^{(i)}$ and decays at the time $t^{(i)}$.

The state vector $\phi^G(t)$ for the meta-stable $^"3P_0$-state therefore represents an ensemble of individual $^3P_0$-levels of $In^+$; the $i$-th member of this ensemble is created (or prepared) at a definite time $t_0^{(i)}$ which is the time $t = 0$ in the life of this $i$-th member. This is also the time $t = 0$ of the state $\phi^G(t)$.

A quantum mechanical state vector $\phi^+$, or $\phi^G$, or state operator $W$, represents an ensemble (a large number) of individual micro-systems. Usually
one thinks of an ensemble of microsystems as of collections of many particles that are present at one particular point of time (in our lives, or by the laboratory clocks); in this experiment one has one single ion in the $^3P_0$ level at a collection of many preparation times \{t_0^{(i)}\}. The preparation time for this ensemble of single shelved $In^+$ is the ensemble of times \{t_0^{(i)}\}. This ensemble of times \{t_0^{(i)}\} is the preparation time $t_0 = 0$ of the state operator $|\phi^G(t)\rangle\langle\phi^G(t)|$. The state vector $\phi^G(t)$ which represents the ensemble of $In^+$ in the $^3P_0$ level has thus a time evolution $0 \leq t < \infty$, and the semigroup time $t = 0$ is the ensemble of times \{t_0^{(i)}\} where the times $t_0^{(i)}$ are the onset times of dark periods, i.e., the times at which the $^3P_1$ level stops participating in the fluorescence transitions (3.2) and prepares the state $\phi^G = |''^3P''_0\rangle$.

Since the lifetime $^3P_1$ is $10^{-6}$ s, and the lifetime of $''^3P''_0$ is $10^{-1}$ s, the onset time of the dark periods $t_0^{(i)}$ are determined with very high accuracy. The same applies to the time $t^{(i)}$, since one observes a sudden return of fluorescence. This is the time at which the $i$-th $''^3P''_0$-level has decayed into $^1S_0 + \gamma$, and the processes (3.2) can resume. The duration of a dark period $\Delta t^{(i)} = t^{(i)} - t_0^{(i)}$ is thus the individual lifetime of the $i$-th $^3P_0$ level, and it is defined to very high accuracy. Two of these individual lifetimes $\Delta t^{(i)}$ of the $^3P_0$ levels are shown by the duration of the dark periods in Fig. 3 [5].

Summarizing, the interpretation of our observation is the following: Since the single $In^+$ can either make the fluorescence monitored transitions (3.2) or be in the quasistable state $|''^3P''_0\rangle$ the time interval $\Delta t^{(i)} = t^{(i)} - t_0^{(i)}$ is the time which the $i$-th single $In^+$ at the $''^3P''_0$ level of $In^+$ has “lived”, from its preparation at $t_0^{(i)}$ to its decay into $^1S_0 + \gamma$ at $t^{(i)}$.

4 The exponential of the average lifetime

The ensemble of the $''^3P''_0$ levels is described by the state vector $\phi^G(t)$. The preparation time $t_0$ of the state $\phi^G(t)$ is the ensemble of times \{t_0^{(i)}\},

$$t_0 \leftrightarrow \{t_0^{(i)}\}.$$  \hspace{1cm} (4.1)

This time $t_0 > -\infty$ is the time zero in the existence of each individual $''^3P''_0$-$In^+$-ion state. We choose it thus as the time $t = 0$ of the time evolution semigroups

$$U^x_\infty(t) = \{e^{-iH_xt} | 0 \leq t < \infty\}$$  \hspace{1cm} (4.2)
for the Gamow state vector $\phi^G(t)$ of (2.1). And similarly one defines the semigroup for any state $\phi^+ \in \Phi_-$ in (4.6) of [1].

Therewith we have identified the experimental definition of the semigroup time $t_0 = 0$, of the state $\phi^G(t)$: it is the ensemble of onset times of the dark periods $\{t^{(i)}_0\}$ when the $In^+$ is shelved on the $^{3}P_0$ level, and where it remains for an ensemble of individual lifetimes $\Delta t^{(i)}$.

The quantities predicted by quantum theory are the Born probabilities and averages (expectation values) not the properties of individual quantum systems like the $\Delta t^{(i)}$. The Born probability for the Gamow state (2.1) (survival probability) is given by the exponential law

$$P(\ {}^{3}P_0(t)) \equiv P_D(t) = e^{-\Gamma t/\hbar} = e^{-t/\tau}, \quad t > 0. \quad (4.3)$$

These probabilities are measured by the counting ratios:

$$e^{-t/\tau} = P_D(t) \simeq \frac{N_D(t : \Delta t^{(i)} > t)}{N_D} = \text{counting ratio} \quad (4.4)$$

where

$$N_D(t : \Delta t^{(i)} > t) = \text{number of dark periods of duration} \quad \Delta t^{(i)} > t,$$

and $N_D$ is the total number of dark period events that are included. We call

$$N(t : \Delta t^{(i)} < t) = \text{number of dark periods of duration} \quad \Delta t^{(i)} \leq t.$$ 

Intuitively, $N_D(t : \Delta t^{(i)} > t)$ is the number of $^{3}P_0$ levels that live longer than $\Delta t^{(i)}$, and $N(t : \Delta t^{(i)} < t)$ is the number of $^{3}P_0$ levels that have already decayed into $^{1}S_0 + \gamma$.

It is clear that for every time $t \geq 0$

$$N(t : \Delta t^{(i)} \leq t) + N_D(t : \Delta t^{(i)} > t) = N_D \quad (4.5)$$

where $N_D(\approx 150)$ is the total number of dark periods that have been observed. To check the exponential law (4.4) and to obtain an estimate of the average lifetime $\tau = \Gamma/\hbar$, the number of dark periods with duration shorter than $t$, $N(t : \Delta t^{(i)} \leq t) \equiv d(t)$ is considered. Only dark periods of duration

\footnote{The sign $\simeq$ in (4.4) and below indicates that this is an equality between theoretical quantities on the left hand side and experimental quantities on the right hand side (which would become an exact equality in the unrealistic case of continuously infinite events).}
larger than $t_s = 70 \, ms$ could be identified in the experiment [5]. Since from (4.5) with (4.4) follows
\begin{align*}
N_D(t : \Delta t^{(i)} > t) &= N_D e^{-t_s/\tau} e^{-(t-t_s)/\tau} = N_D - N(t : \Delta t^{(i)} < t) \\
&= a' e^{-(t-t_s)/\tau} = a - d(t), \quad (4.6)
\end{align*}
the quantities $a - d(t)$ are plotted versus time in a logarithmic plot, Fig. 3. The straight line confirms the exponential decay law and from the slope of the straight line one obtains the lifetime of the $^3P_0$-level as $\tau(^3P_0) \approx 0.14 \, s$.

Since the counting ratios in (4.4) are according to Fig. 3 in agreement with the exponential law, the average of the lifetime is given by $\tau$:
\begin{equation}
\int_0^\infty dt \mathcal{P}_D(t) = \int_0^\infty dt e^{-t/\tau} = \tau. \quad (4.7)
\end{equation}
Since according to (4.4) $\mathcal{P}_D(t)$ is observed as the counting ratio $N_D(t : \Delta t^{(i)} > 0)/N_D$, the lifetime is measured as the weighted average of the dark periods $\Delta t^{(i)}$:
\begin{equation}
\tau \approx \sum_i \Delta t^{(i)} \frac{N_D(t : \Delta t^{(i)} > t)}{N_D}. \quad (4.8)
\end{equation}
The ensemble of quasistable quantum objects in the state $\phi^G$ is an ensemble of individual $^3P_0$'s each of which is prepared at the collection of times $t_0^{(i)}$ and decays at the collections of times $t^{(i)}$ and lives for a duration of time $\Delta t^{(i)} = t^{(i)} - t_0^{(i)}$. Not the values $\Delta t^{(i)}$ are characteristic quantities of the Gamow state but the weighted average (4.8) which corresponds to the theoretical lifetime $\tau$ of the Gamow state in (2.1) with $\tau = \Gamma_h$.

The lifetime $\tau$ is one of the characteristics of the Gamow state (3.12) of [1]. It is defined by the exponential law (4.3) which follows from (2.1). It is given by the $S$-matrix pole position $z_R = E_R - i\Gamma/2$ which defines a complex energy eigenvector (3.14) of [1] which describes a resonance state of Breit-Wigner width $\Gamma$. And the lifetime of the decaying state is related to the width of the resonance by the exact lifetime-width relation $\tau = \frac{\hbar}{\Gamma}$.

5 Conclusion

The time asymmetry of quantum theory which was shown in [1] to be a byproduct of a unified theory of resonance scattering and decay phenomena,
distinguishes a particular time \( t_0 \), the time \( t = 0 \) of a semigroup time evolution. This is the time at which the state (e.g., decaying state) has been prepared and the registration of the observable (e.g., decay products) can begin.

This semigroup \( t = 0 \) is measured as the collection of times \( \{ t_0^{(i)} \} \) at which the individual \( ^3P_0 \)-levels of a single \( In^+ \) were prepared. These times of preparation of \( ^3P_0 \) levels are documented by the sudden onset times \( t_0^{(i)} \) of the dark periods for the single \( In^+ \) ion. Each dark period also has its individual end \( t^{(i)} \) (documented by the end of the dark period). Thus each dark period has its individual lifetime \( \Delta t^{(i)} = t^{(i)} - t_0^{(i)} \). The quantum mechanical state \( \phi^G \) describes the ensemble of individual microsystems created under the same condition at an ensemble of different times \( \{ t_0^{(i)} \} \). This ensemble of preparation times is the preparation time \( t_0 = 0 \) of the state \( \phi^G \): \( t_0 \leftrightarrow \{ t_0^{(i)} \} \).

In traditional quantum mechanics the state would be asymptotically prepared for \( t \to -\infty \) as a state of the \( (1S_0, \gamma) \)-system and evolve in time through a superpositions of vectors representing \( (1S_0, \gamma), \ (1S_0, \gamma'), \ (1S_0, \gamma'') \), \( \cdots \). Using the Hilbert space axiom \( (1.6) \), the probability for \( ^3P_0 \) would be different from zero – at least infinitesimally – at any time \( t \) between \(-\infty < t < \infty \). It could be significantly different from zero at a particular time \( t_0 \), but there could not be sudden jumps as used for the interpretation of the sudden onset of dark periods at \( t_0^{(i)} \).

On the other hand the existence of these quantum jumps is an experimental confirmation of the semigroup time evolution. The new Hardy space hypothesis \( (1.7) \) which was conjectured in order to obtain a consistent theory that unifies Breit-Wigner resonances and exponentially decaying Gamow states also led to a semigroup evolution \( [1] \) and therewith the semigroup time \( t_0 = 0 \). This time \( t_0 \) represents the ensemble of preparation times \( \{ t_0^{(i)} \} \) for the metastable level \( ^3P_0 \) which is described by the Gamow state vector \( \phi^G = | ^{1''}3P_0'' \rangle \).

Quantum theory describes ensembles of (large) numbers of quantum systems. In the usual experiments this means or at least includes a large number of micro-systems at any given time. In the experiments with single ions, it involves only one single ion state which is prepared at a large number of different times \( t_0^{(i)} \) under identical conditions. This exposes the time of preparations for the state of the single quantum system. What is remarkable about these marvelous experiments \( [3–5] \) is that they can measure quantities that the theory cannot predict, like the individual lifetimes of a single excited
ion level.

Acknowledgment

This work is part of a collaboration sponsored by the US National Science Foundation Award No. OISE-0421936.

References

[1] A. Bohm, H. Kaldass and S. Komy, *Time Asymmetry in Quantum Physics - I. Theoretical Conclusion From Resonance and Decay-Phenomenology*.

[2] W. Nagourney, J. Sandberg, and H. Dehmelt, Phys. Rev. Lett. **56**, 2797 (1986); X. Zhao, N. Yu, H. Dehmelt, and W. Nagourney, Phys. Rev. A **51**, 4483 (1995).

[3] Th. Sauter, W. Neuhauser, R. Blatt, and P. E. Toschek, Phys. Rev. Lett. **57**, 1696 (1986); Berrwist et.al. Phys. Rev. Lett. **57**, 1669 (1986).

[4] J. C. Bergquist, R. G. Hulet, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. **57**, 1699 (1986).

[5] E. Peik, G. Hollemann, and H. Walther, Phys. Rev. A **49**, 402 (1994).

[6] H. Dehmelt, Bull. Am. Phys. Soc. **20**, 60 (1975).

[7] R. H. Garstang, J. Opt. Soc. Am. **52**, 845 (1962).