Small Cell Association with Networked Flying Platforms: Novel Algorithms and Performance Bounds

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Abstract

Fifth generation (5G) and beyond-5G (B5G) systems expect coverage and capacity enhancements along with the consideration of limited power, cost and spectrum. Densification of small cells (SCs) is a promising approach to cater these demands of 5G and B5G systems. However, such an ultra dense network of SCs requires provision of smart backhaul and fronthaul networks. In this paper, we employ a scalable idea of using networked flying platforms (NFPs) as aerial hubs to provide fronthaul connectivity to the SCs. We consider the association problem of SCs and NFPs in a SC network and study the effect of practical constraints related to the system and NFPs. Mainly, we show that the association problem is related to the generalized assignment problem (GAP). Using this relation with the GAP, we show the NP-hard complexity of the association problem and further derive an upper bound for the maximum achievable sum data rate. Linear Programming relaxation of the problem is also studied to compare the results with the derived bounds. Finally, two efficient (less complex) greedy solutions of the association problem are presented, where one of them is a distributed solution and the other one is its centralized version. Numerical results show a favorable performance of the presented algorithms with respect to the exhaustive search and derived bounds. The computational complexity comparison of the algorithms with the exhaustive search is also presented to show that the presented algorithms can be practically implemented.

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Index Terms

5G, backhaul/fronthaul network, binary integer linear program, drones, networked flying platforms (NFPs), small-cell networks, Unmanned aerial vehicles (UAVs)

I. INTRODUCTION

The advancement of technology (such as video services) and a rapid growth in the number of cellular users (such as mobile devices and tablets etc.) have been pushing the limits of wireless communication systems. Next generation systems expect coverage and capacity enhancements along with the consideration of limited power, cost and spectrum. To cater for these demands, a ten-fold increase in the radio spectrum is required [1]. Therefore, researchers both in academia and industry are looking towards latest wireless technologies such as millimeter-Wave (mmWave) and free space optics (FSO), as they can provide hundreds of megahertz of bandwidth for wireless transmission. However, these wireless technologies under the usual power constraints have a limited range as the signal degrades due to environmental effects. This transmitter-receiver distance reduction and the growing cellular user crowds lead to the idea of small cell (SC) densification. This densification of SCs (e.g., pico and femto cells) is being considered as a corner stone of fifth generation (5G) and beyond-5G (B5G) cellular networks.

China Mobile in 2011 [2] proposed a cloud radio access network (C-RAN) architecture, which is considered as a promising paradigm for 5G and B5G cellular systems, as it can resolve the backhaul traffic limitations by providing a fronthaul link. Due to the dense deployment of SCs, fronthaul links demand a high capacity of more than 2.5 Gbps with a low latency of around $100\ \mu s$ or less [3]. In terms of wired technology, these demands can be fulfilled only by fiber optical links as they offer an abundant bandwidth with low latency data transfer. However, such fiber links deployment results in high capital expenditure (CAPEX) as compared to wireless fronthaul links [4]. Wireless fronthaul links can be realized using microwave bands for non-line-of-sight (NLoS) case or mmWave/FSO for line-of-sight (LoS) case. Microwave links can cover a wide area but suffer from low data rates as currently available commercial products provides a maximum of 2 Gbps throughput [4]. FSO and mmWave based fronthaul links have attracted an eye of various researchers as they meet the capacity requirements of 5G and B5G systems and they are light-weight and easy to install. However, mmWave/FSO suffer from susceptibility to weather conditions [5] and require a LoS connection, which is a main hurdle in urban regions due to few available ground locations. Recently, a scalable idea was presented in [5] that utilizes
networked flying platforms (NFPs) as a wireless fronthaul hub point between SCs and core network. These NFP-hubs provide a possibility of wireless LoS fronthaul link to utilize radio frequency (RF), mmWave and FSO technologies, and thus, overcomes the limitations of few available wireless NLoS ground fronthaul links.

Recently, both academia and industry started taking interest in utilizing NFPs such as unmanned aerial vehicles (UAVs), drones and unmanned balloons for wireless communications. These NFPs can be manually controlled but mainly designed for autonomous pre-determined flights. Latest NFPs are capable of carrying RF/mmWave/FSO payloads along with an extended battery life [6]. On the basis of their flying altitude range, NFPs are categorized into low-altitude platform (LAP) (less than 5km), medium-altitude platform (MAP) (between 5km to 10km) and high-altitude platform (HAP) (greater than 10km).

In this work, we employ NFPs as aerial hubs to provide fronthaul connectivity to a network of SCs. We define the association problem of SCs and NFPs, present its performance bounds, then propose novel efficient (less computationally complex) centralized and distributed greedy algorithms for its solution.

A. Related Work

With the popularity of NFPs, a widely used air-to-ground (ATG) propagation model was presented in [7]. This model considers the aerial communication between NFPs and terrestrial nodes. Later on, a closed form expression of the path loss and the effect of change of altitude of the NFP over the coverage area was presented in [8]. For the case of two NFPs, the coverage area was analyzed by varying the distance between the NFPs and their altitudes in [9].

In the literature, 3D placement of NFPs and a related research problem of the association of NFPs and users were studied by a few researchers [10]–[18]. In all of those works, the NFPs were used as flying base stations (BSs) to provide wireless connectivity to the ground users. In [10], authors have designed the 3D placement problem of a single NFP BS considering only the signal-to-noise ratio (SNR) as a quality-of-service (QoS) parameter and studied the coverage region of the NFP BSs for different urban environments. A number of constraints including backhaul data rate, maximum bandwidth of a single NFP and path loss were taken into consideration for joint 3D placement and association problem of a single NFP BS in [11]. However, a computationally-expensive and not practically-implementable exhaustive search method was used in both [10] and [11] to solve the designed problems. In [12], the 3D placement problem was decoupled into
first finding the optimal altitude and then using circle placement problem to optimize the 2D placement of a single NFP BS in order to maximize the number of users in a covered region.

For the case of multiple NFPs, association of NFP BSs and users on the basis of SINR parameter was presented in [13], then the 3D placement problem is solved using particle swarm optimization (PSO) algorithm. The work in [14] and [15] dealt with the 3D placement of the NFPs considering only the SINR constraint, where the former used circle packing theory to enhance the coverage performance with minimum power, while the later used entropy and network bargaining approaches to enhance the capacity and coverage area. In [16] and [17], a delay-sensitive cell association problem was designed for multiple NFP BSs that co-exist with terrestrial BSs and optimal packing theory was used to solve the designed problem. A Linear Programming (LP) relaxation along with rounding was used in [18] to solve the association problem and then the PSO algorithm was utilized to solve the 3D placement problem. However, due to LP relaxation and then rounding, a number of constraints of the association problem may not be satisfied exactly.

Since NFPs gain popularity in communication systems, they have been studied as either repeaters or BSs to enhance the network coverage and signal strength mainly in hard to reach areas. To the best of our knowledge, there is only one work, [5], in the literature apart from our recently published conference papers, [19] and [20], that uses NFPs as hub points to provide fronthaul connectivity. The work in [5] was limited to the feasibility study of using NFPs as fronthaul hubs, design of backhaul framework, investigation about the effect of weather conditions on the system and evaluation of the implementation cost of the proposed system as compared to other wired/wireless fronthaul/backhaul links. In [19] and [20], we formulated and analyzed the association problem of SCs and NFPs. Further, instead of using exhaustive search methods, we presented efficient greedy algorithms.

**B. Contributions**

This work is an extension of our work in [19] and [20]. Here, we reconsider the mathematical problem formulation for the association of SCs and NFPs. We also study in detail the effect of a number of practical constraints on the association problem, where the constraints are related to 5G and B5G systems and NFPs, such as backhaul data rate, NFP’s bandwidth and number of links limitations. We further modify our previously proposed greedy algorithms to achieve enhanced problem solutions.
In this work, we propose an analytical framework for the analysis of the association problem of SCs and NFPs. On top of the proposed framework, the main contributions of the paper are summarized as follows:

i) We show the relevance of the association problem of SCs and NFPs with the generalized assignment problem (GAP). Using this relevance, for the first time in the literature to the best of the authors’ knowledge, we show that the association problem is at least NP-hard.

ii) Again, capitalizing on the relation with the GAP problem, we present an analytical derivation of the upper bound for the association problem with some relaxations. This follows the same framework used for a well known branch and bound (B&B) method for GAP [21].

iii) We present efficient (less complex) greedy solutions for the association problem as opposed to the exhaustive search presented in literature for related problems.\(^1\)

C. Paper Organization and Notations

The rest of the paper is organized as follows. In section II, a system model and the association problem are presented. Section III includes the relation of association problem with the GAP, its approximation and the upper bounds using the same relevance. Section IV presents two efficient greedy solutions proposed for the association problem. Numerical results and related discussions

\(^1\)The algorithms presented here are further modified and enhanced versions of our algorithms presented in [19] and [20].
are presented in section \[ \text{V} \] Computational complexity of the algorithms is discussed in section \[ \text{VI} \] and finally section \[ \text{VII} \] concludes the paper.

Following notations are used in the paper. A constant number is denoted as either \( x \) or \( X \). The matrix and its \((i,j)\)-th entry are denoted by \( X \) and \( x_{ij} \) or \( X_{ij} \), respectively. A set or a list is represented as \( \mathcal{X} \). Notation \( 0_{A \times B} \) represents an \( A \times B \) matrix of all 0 entries.

II. SYSTEM MODEL AND PROBLEM FORMULATION

This section first presents the NFP connected SCs system model under consideration. An ATG channel model is provided that highlights mainly the path loss parameter for the communication between NFPs and SCs. Finally, the association problem of SCs and NFPs is emphasized and mathematically formulated for the user-centric case, where the objective is to maximize the overall sum data rate.

A. System Model

Consider a heterogeneous network (HetNet) (e.g., a 5G or B5G system) as shown in Fig. [1] that consists of three classes of wireless nodes: i) ground SCs, ii) NFP-hubs, and iii) ground core network gateway. SCs accumulate and route the downlink/uplink traffic between cellular users and core network using fronthaul links. NFPs act as hub points to provide fronthaul connectivity between SCs and core network. For brevity, NFP-hubs will be referred to as NFPs. NFPs are distributed in a two-level hierarchy, where a number of NFPs spread over a region up to an altitude of 5km, i.e., LAP, and NFPs are connected to a mother NFP placed at an altitude of higher than 5km, i.e., either MAP or HAP [6].

NFPs are connected to each other and mother-NFP through FSO links, where we neglect the FSO link losses in this work. NFPs can share the control information such as bandwidth, data rate and other requirements with each other as well as mother-NFP, however, all the data information can only be shared with mother-NFP. We assume that the distribution of SCs and NFPs does not change for time duration \( T \), and thus, we study their association considering the active SCs and NFPs during the time interval \( [0 \ T] \).

B. Air-to-Ground Path Loss Model

For the communication between NFPs and SCs, we have adopted a widely used ATG path loss model presented in [7] and [8]. The model is based on the proposition supported by the statistical
derivation in [7] that the ATG communications may belong to one of the two propagation groups: i) LoS receivers, and ii) NLoS receivers. The first propagation group includes the receivers placed in LoS or near-LoS conditions, however the NLoS receivers rely on the coverage via reflections and diffractions only. The radio signals first propagate through the free space incurring free-space path loss (FSPL) and then reaches the receivers either directly (i.e., LoS receivers) or incur scattering and shadowing because of man-made structures (i.e., NLoS receivers). The two propagation groups result in a path loss (referred as excessive path loss that is additional to FSPL) following a Gaussian distribution [8]. The considered model in [8] deals with its mean value instead of its random behavior.

An important factor of the mean value of the excessive path loss is the probability of LoS ($P(\text{LoS})$), that depends on the considered environment (such as rural, urban, or others) and the orientation of NFPs and ground SCs and it was formulated in [7] and [8] as

$$P(\text{LoS}) = \frac{1}{1 + \alpha \exp \left\{ -\beta \left( \frac{180}{\pi} \theta - \alpha \right) \right\}}$$

(1)

where $\alpha$ and $\beta$ are parameters with constant values that depend on the specific environment. The elevation angle from the ground SC to the NFP is represented by $\theta = \arctan \left( \frac{h_D}{s} \right)$, where $s = \sqrt{(x-x_D)^2 + (y-y_D)^2}$ denotes the horizontal distance between SC and NFP. The positions of SCs and NFPs in a cartesian coordinate system with respect to the origin are denoted by $(x, y)$ and $(x_D, y_D, h_D)$, respectively. The mean path loss is presented as

$$PL(dB) = 10 \log \left( \frac{4\pi f_c d}{c} \right)^\gamma + P(\text{LoS})\eta_{\text{LoS}} + P(\text{NLoS})\eta_{\text{NLoS}}$$

(2)

where the first term represents the FSPL that depends on carrier frequency $f_c$, speed of light $c$, path loss exponent $\gamma$ and the distance $d = \sqrt{h_D^2 + s^2}$ between NFP and SC. Variables $\eta_{\text{LoS}}$ and $\eta_{\text{NLoS}}$ represent additional losses for LoS and NLoS links, respectively and $P(\text{NLoS}) = 1 - P(\text{LoS})$. All parameters in (2) depend on the environment. It can be noticed from (2) that for a known distribution of SCs and NFPs, if we fix the PL then we can estimate the geographical area covered by the NFP [8].

C. Problem Formulation

The communication of the user data between SCs and the core network depends on the fronthaul link of the NFPs. An intelligent association and placement of the NFPs can provide efficient throughput, widespread connectivity and result in a better QoS. In this work, we fix
the height of the NFPs and consider a random 2D placement of NFPs and SCs. Thus, our main focus is the association aspect of the problem.

Consider the system shown in Fig. 1 where $N_{SC}$ SCs are distributed randomly over a square region of area $A_S$. Over the same region, $N_D$ NFPs are distributed randomly in a horizontal plane at an altitude of $h_D$ above the ground level. Mother-NFP is placed at a height greater than $h_D$, so it can have a direct LoS connection with the $N_D$ NFPs. Using a stochastic-geometry approach, the random distribution of both SCs and NFPs follows a Matérn type-I hard-core process \[22\] with the same average density of $\lambda$ per m$^2$ having a minimum separation of $s_{SC}^{\min}$ and $s_D^{\min}$ with their neighbors, respectively. Note that, the average number of SCs and NFPs in a given area is equal to their average density multiplied by the size of the area such that

$$N_{\text{avg}}^k = \lambda \exp(-\lambda A_{k_{\text{sep}}}) A_S,$$

where the subscript $k \in \{SC, D\}$ refers to SCs and NFPs, respectively. $A_{k_{\text{sep}}}$ is the area of the separation, i.e., $A_{k_{\text{sep}}} = s_{k_{\text{min}}}^2$ for a square region. Let us denote the random distribution points of both SCs and NFPs as $(x_i, y_i)$ and $(x_{Dj}, y_{Dj}, h_{Dj})$, respectively, where $i \in \mathcal{N} = \{1, \ldots, N_{SC}\}$ and $j \in \mathcal{M} = \{1, \ldots, N_D\}$.

First of all, before studying the objective of the association problem, the limiting factors (related to the available resources) that affect the communication between SCs and NFPs are discussed below. Three major limiting factors have a direct effect on the association of SCs and NFPs, namely, backhaul data rate, bandwidth, and the number of links of the NFPs. Thus, the association between SCs and NFPs varies with changes in the limiting factors.

The backhaul link between the core network and the mother-NFP, i.e., hop A, limits the maximum allowable data rate of the network, that is referred here as backhaul data rate $R$. This means that the sum of the data rate for all the NFP and SC pairs cannot exceed the backhaul data rate $R$. Let us denote the requested data rate of $i$-th SC associated with $j$-th NFP by $r_{ij}$, then this constraint can be written as \[5b\], where $A_{ij}$ is an entry of an $N_{SC} \times N_D$ association matrix $A$ that shows the association of SCs and NFPs as

$$A_{ij} = \begin{cases} 
1, & \text{if } i\text{-th SC is connected with } j\text{-th NFP}, \\
0, & \text{otherwise}. 
\end{cases} \quad (3)$$

The next limitation is posed by the fronthaul FSO link in the hop B, i.e., from mother-NFP to each NFP. Depending upon the quality of the FSO link, the $j$-th NFP is allocated a maximum bandwidth, $B_j$, that can be distributed among associated SCs. This limits the sum of requested bandwidth of all SCs associated with $j$-th NFP and it can be mathematically represented as \[5c\].
The allocated bandwidth $b_{ij} = \frac{r_{ij}}{\eta_{ij}}$ of the $i$-th SC and the $j$-th NFP pair depends on $r_{ij}$ and the spectral efficiency $\eta_{ij} = \log_2 (1 + \text{SINR}_{ij})$, where the SINR can be expressed as

$$\text{SINR}_{ik} = \frac{P_{rik}}{\sum_{j=1, j\neq k}^{N_D} P_{r_{ij}} + \sigma}$$

(4)

Here, $P_{r_{ij}}$ represents the received power from the $j$-th NFP to the $i$-th SC and $\sigma$ represents the noise floor of each receiver.

In the next hop, i.e., hop C between the $j$-th NFP and the $i$-th SC, the RF fronthaul link should satisfy a QoS requirement. Every NFP can serve SCs placed inside a specific area computed using (2) for fixed positions of NFPs, SCs and a maximum path loss [8] and [9]. This maximum path loss is dictated by the minimum required SINR to serve a SC via RF link. Thus, each NFP-SC pair link should satisfy a minimum SINR QoS requirement that can be written as (5d).

Considering all the above mentioned constraints, for fixed positions of NFPs and SCs, Our objective is to find the best possible association of the SCs with the NFPs such that the sum data rate of the overall system is maximized. Such a problem can be formulated as

$$\max_{\{A_{ij}\}} \sum_{i=1}^{N_{SC}} \sum_{j=1}^{N_{D}} r_{ij} \cdot A_{ij}$$

subject to

$$\sum_{i=1}^{N_{SC}} \sum_{j=1}^{N_{D}} r_{ij} \cdot A_{ij} \leq R,$$  

(5b)

$$\sum_{i=1}^{N_{SC}} b_{ij} \cdot A_{ij} \leq B_j, \quad \text{for } j \in M,$$  

(5c)

$$\frac{1}{\text{SINR}_{ij}} \cdot A_{ij} \leq \frac{1}{\text{SINR}_{\text{min}}}, \quad \text{for } i \in N \& j \in M,$$  

(5d)

$$\sum_{i=1}^{N_{SC}} A_{ij} \leq N_{l_j}, \quad \text{for } j \in M,$$  

(5e)

$$\sum_{j=1}^{N_{D}} A_{ij} \leq 1, \quad \text{for } i \in N,$$  

(5f)

$$A_{ij} \in \{0, 1\}, \quad \text{for } i \in N \& j \in M.$$  

(5g)

Constraint (5e) shows that the $j$-th NFP can establish a maximum of $N_{l_j}$ links with the SCs as per the number of transceivers. Further, each SC can be associated to a maximum of one NFP that is included in constraint (5f).
III. PROBLEM APPROXIMATION AND UPPER BOUNDS

This section presents the analysis of the association problem (5). First of all, we show that if some of the constraints of the problem in (5) are relaxed, then it exactly maps to the GAP. Then, using the relation with GAP, it is shown that problem (5) is at least NP-hard. Further, an analysis for the upper bound of the association problem (5) without constraints (5b) and (5e) is presented. Furthermore, we study the LP relaxation of the association problem (5) to obtain another upper bound. In addition, to get a tighter upper bound, a B&B method is used for the association problem (5) without neglecting any constraints. Finally, the bounds and relaxed solutions are numerically compared to one another, as well as to those proposed in the next section.

A. Relation to the GAP

Here, first of all, we define the GAP and then show its relevance with the association problem (5). Consider $n$ tasks to be assigned to $m$ agents, where $j$-th agent can complete $i$-th task as per its own capability. This means that the $j$-th agent can complete the $i$-th task with a cost/weight $w_{ij}$ that then returns a utility/profit $p_{ij}$. Thus, the weights and profits are dependent on the $j$-th agent for the $i$-th task. The objective is to maximize the overall profit by assigning each task to exactly one agent without exceeding the capacity of the $j$-th agent, $c_j$. Such a problem is known as GAP [23] and can be written as

$$\max \sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij} \cdot x_{ij}$$

subject to

$$\sum_{i=1}^{n} w_{ij} \cdot x_{ij} \leq c_j, \quad \text{for } j \in \mathcal{M},$$

$$\sum_{j=1}^{m} x_{ij} = 1, \quad \text{for } i \in \mathcal{N},$$

$$x_{ij} \in \{0, 1\}, \quad \text{for } i \in \mathcal{N} \& j \in \mathcal{M},$$

where

$$x_{ij} = \begin{cases} 
1, & \text{if } i\text{-th task is assigned to the } j\text{-th agent}, \\
0, & \text{otherwise.}
\end{cases}$$
and the following are the usual restrictions on the GAP variables

\[ p_{ij}, \ w_{ij} \ \& \ c_j \text{ are positive integers,} \quad \text{(8)} \]
\[ |\{j : w_{ij} \leq c_j\}| \geq 1 \quad \text{for } i \in \mathcal{N}, \quad \text{(9)} \]
\[ c_j \geq \min_{i \in \mathcal{N}} \{w_{ij}\} \quad \text{for } j \in \mathcal{M}. \quad \text{(10)} \]

If the weights are fractional, thus condition (8) is violated, then it can be handled by multiplication of weights with a proper factor. If \( i \)-th task does not satisfy condition (9), then that task cannot be assigned to any agent and GAP instance is infeasible. However, another variant of GAP, known as LEGAP always admits a feasible solution as in its definition the equality in constraint (6c) is replaced with an inequality, such that \( \sum_{j=1}^{m} x_{ij} \leq 1, \forall i \in \mathcal{N} \), that allows \( i \)-th agent to be un-associated under certain conditions [23]. The agents that violates condition (10) can be removed from the problem.

Comparison of GAP in (6) with the association problem (5) shows that if the constraints (5b), (5d) and (5e) are neglected, then the resulting relaxed association problem is equivalent to GAP. Note that, the association problem (5) without constraints (5b) and (5e) will be referred to as the relaxed association problem. Below, we show the relevance of the relaxed association problem with GAP and then discuss the effect of neglecting constraints (5b) and (5e). Later on, we also incorporate the constraint (5d) in the derivation of the upper bound of the association problem (5).

The objective variables \( x_{ij} \) in (6a) and \( A_{ij} \) in (5a) exactly match each other, where the subscripts \( i \) denoting tasks in (6a) is equivalent to SCs in (5a) and \( j \) denoting agents in (6a) is equivalent to NFPs in (5a). This means that assigning \( i \)-th task to \( j \)-th agent is the same as associating the \( i \)-th SC to the \( j \)-th NFP. The variables \( p_{ij}, w_{ij} \) and \( c_j \) in (6) depend on both tasks and agents, which is the same as of \( r_{ij}, b_{ij} \) and \( B_j \) in association problem (5) that depend on both SCs and NFPs; these variables are equivalent, respectively. The constraint (5c) that keeps track of the bandwidth limit of the \( j \)-th NFP is the same as of constraint (6b) that tracks the maximum capacity of \( j \)-th agent, thus they are equivalent as well. The variables in association problem (5) satisfy the restrictions of the GAP variables given in (8) to (10). For some cases, if the variables in association problem (5) are in fractions then they can be converted to integers with a multiplication of an appropriate factor.

The above discussion shows that the relaxed association problem (5) without constraints (5b) and (5e) is equivalent to GAP.
B. Complexity of the Association Problem

The relevance of association problem (5) with the GAP can be exploited to study its complexity.

**Proposition 1.** Association problem (5) is at least NP-hard.

*Proof.* As association problem (5) is a subset of the relaxed association problem, thus to show that the association problem (5) is at least NP-Hard, it would be enough to show that the relaxed association problem is NP-hard. It is shown in section III-A that the relaxed association problem is equivalent to GAP, where GAP is an NP-hard problem [24]. Thus, as GAP is NP-hard, so the equivalent relaxed association problem is also NP-hard. This shows that the association problem (5), which is a subset of the relaxed association problem, must be at least NP-hard. \qed

C. Upper Bound of the Optimization Problem

**Remark 1.** Neglecting constraints (5b) and (5e) of the association problem (5) results in a relaxed upper bound that is not lower than the original optimal objective function in (5).

*Proof.* Knowing that problem (5) is a maximization problem, enlarging the feasible set by removing constraints can only increase the objective. \qed

As per Remark 1, the relaxed association problem provides a higher sum data rate than the association problem (5). Thus, the upper bound of the relaxed association problem can be regarded as the upper bound for the association problem (5). Considering the relevance of our relaxed association problem with the GAP as shown in section III-A and using the capacity relaxation procedure adopted in [21], in the following, we derive an upper bound for the relaxed association problem.

In the relaxed association problem, the bandwidth constraint (5c) that is equivalent to the capacity constraint (6b) of GAP is relaxed such that

\[ b_{ij} \cdot A_{ij} \leq B_j, \quad \text{for } i \in \mathcal{N} \& j \in \mathcal{M} \]  

Now, we are relaxing (5c), unlike what is stated in Remark 1. After the above relaxation, the resulting problem has an optimal solution \( \hat{A} \) that is obtained by determining \( j \)-th NFP for \( i \)-th SC such that

\[ j(i) = \arg \max \{ r_{ij} : j \in \mathcal{M}, b_{ij} \leq B_j, \ \text{SINR}_{ij} \geq \text{SINR}_{\text{min}} \} \quad \text{for } i \in \mathcal{N} \]  

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and setting $\hat{A}_{i,j(i)} = 1$ and $\hat{A}_{ij} = 0$ for all $j \in \mathcal{M}\{j(i)\}$. This results in an upper bound

$$U_0 = \sum_{i=1}^{N_{SC}} r_{i,j(i)}$$ (13)

which is then improved as follows. Consider $\mathcal{L}_j$ to be the list for the $j$-th NFP that consists of SCs associated with it and $O_j$ to be the overload indicator for the $j$-th NFP such that $O_j > 0$ shows that the $j$-th NFP has exceeded the bandwidth limit $B_j$ and the list and indicator are defined as

$$\mathcal{L}_j = \left\{ i : \hat{A}_{ij} = 1 \right\}, \text{ for } j \in \mathcal{M}$$ (14)

$$O_j = \sum_{i \in \mathcal{L}_j} b_{ij} - B_j, \text{ for } j \in \mathcal{M}$$ (15)

Let us define a set $\hat{\mathcal{M}}$ consisting of those NFPs for which the relaxed constraint (5c) is violated and consider $\hat{\mathcal{L}}$ to be the list of SCs associated with those NFPs that violates constraint (5c). As per the definitions, these sets can be written as

$$\hat{\mathcal{M}} = \{ j : O_j > 0 \}$$ (16)

$$\hat{\mathcal{L}} = \bigcup_{j \in \hat{\mathcal{M}}} \mathcal{L}_j$$ (17)

If the $i$-th SC that is currently associated with the $j$-th NFP is reassigned to the other NFP such that it results in second maximum data rate, then the resulting minimum penalty is given as

$$q_i = r_{i,j(i)} - \max \{ r_{ij} : j \in \mathcal{M} \{ j(i) \}, b_{ij} \leq B_j, \text{ SINR}_{ij} \geq \text{ SINR}_{\min} \}, \text{ for } i \in \hat{\mathcal{L}}$$ (18)

This results in a lower bound on the maximum achievable sum data rate in order to satisfy constraint (5c), due to penalty $q_i$. Now, for each NFP $j \in \hat{\mathcal{M}}$, the objective is to minimize the reassignment penalty by solving the 0-1 single knapsack problem that can be written as

$$\min v_j = \sum_{i \in \mathcal{L}_j} q_i y_{ij}$$ (19a)

subject to

$$\sum_{i \in \mathcal{L}_j} b_{ij} y_{ij} \geq O_j,$$ (19b)

$$y_{ij} \in \{0,1\}, \text{ for } i \in \mathcal{L}_j,$$ (19c)
where \( y_{ij} = 1 \) if and only if the \( i \)-th SC is dissociated from the \( j \)-th NFP. The resulting bound is thus

\[
U_1 = U_0 - \sum_{j \in \mathcal{M}} v_j.
\] (20)

Using the bound \( U_1 \), we follow a B&B method presented in [21], where each branch is bounded by \( U_1 \). This will provide a solution of the relaxed association problem that can be used as an upper bound of the association problem (5).

D. LP relaxation and bound

In this section, we will use the LP relaxation on the association problem (5). This means that, we will relax the binary constraint on the association matrix \( A \) as defined in (5g). So, now the entries of the association matrix can vary between 0 and 1 such that \( 0 \leq A_{ij} \leq 1 \). Note that, this will each SC to be associated with multiple NFPs, that will result in relaxation of the constraint (5f). Note that, constraint (5f) along with the binary constraint (5g) previously restricted each SC to be associated to a maximum of one NFP only. However, such a relaxation allows us to solve the association problem using convex programming tools. Such a solution can be regarded as a bound of the optimization problem (5), which considers all the constraints except the constraint (5g), i.e., binary condition over the association matrix.

IV. PROPOSED SOLUTION

It is shown in section III-B that the association problem (5) is at least NP-hard. It is well known that there exists no standard method to solve such an NP-hard problem [25] and [26]. Therefore, we have presented two simple bounds of the problem to show the closeness of our proposed greedy solutions with the bounds. Further, we use B&B method to get the exact solution of the association problem (5), where B&B being an exhaustive search is considered here as a benchmark solution of the association problem (5). We call it an exact solution as it does not involve any relaxation of the constraints as compared to the solution bounds presented in sections III-C and III-D.

To get an efficient and less computationally-complex solution of the NP-hard association problem (5), we present here two greedy solutions that are designed to solve (5) without relaxing any constraints. One of them is designed for the case where the processing power of SCs and NFPs is utilized and is named as Modified Distributed Maximal Demand Minimum Servers...
(M(DM)^2S) algorithm. The other greedy solution is designed for the case of C-RAN architecture, where SCs and NFPs lack the processing power, and thus, the algorithm runs at the mother-NFP or baseband unit (BBU) pool. Therefore, it is named as Centralized Maximal Demand Minimum Servers (CMDMS) algorithm.

Since our scope of work does not focus on finding an optimal 3D placement of NFPs, the random positions of SCs and NFPs are given as a realization of the Matérn type-I process as indicated earlier. First of all, we present a system initialization algorithm that requires some of the known system parameters as an input and provide a random distribution of SCs and NFPs in a specified rectangular region of area $A_S$. Using this we can generate a number of random scenarios with varying positions of SCs and NFPs to evaluate the association algorithms under different scenarios.

A. System Initialization

The algorithm Initialization mainly consists of two steps. One of which deals with the the SCs, while the other accounts for the distribution of the NFPs. For the distribution of the SCs, as a system parameter, the algorithm needs to know about the rectangular area, $A_S$, average density of SCs, $\lambda$ per m$^2$, the minimum separation between them, $s_{\text{SC}}^{\text{min}}$ in meters, and the number of SCs, $N_{\text{SC}}$. Using these parameters, SCs are distributed randomly using Matérn type-I hard-core process. This provides $N_{\text{SC}}^{\text{avg}} = \lambda \exp(-\lambda s_{\text{SC}}^{\text{min}}^2) A_S$ average number of random distribution points in the rectangular area, $A_S$. Finally, we pick $N_{\text{SC}}$ points out of the generated points that provides the 2D locations of $N_{\text{SC}}$ SCs as $(x_i, y_i)$, where $i \in \mathcal{N}$.

For the NFPs, we assume symmetry in the case of number of links and bandwidth such that each NFP can support a fixed number of links $N_l$ and fixed maximum bandwidth $B$, i.e., $N_{lj} = N_l$ and $B_j = B \forall j \in \mathcal{M}$. The algorithm Initialization either has the information of the number of NFPs, $N_D$ as an input parameter or it can be computed as follows. To compute the minimum number of required NFPs, the algorithm uses the input information $B, b_i$ and $N_l$ and computes the maximum number of SCs that can be associated with a single NFP, $N_{\text{SC}}^D$ depending on the bandwidth information such that

$$N_{\text{SC}}^D = \left\lfloor \frac{B}{b_{\text{avg}}} \right\rfloor,$$  \hfill (21)
Algorithm Initialization

System Initialization

**Input:** \( \lambda, A_S, s_{\text{min}}^{\text{BS}}, h_{\text{max}}, \text{PL}_{\text{max}}, \alpha, \beta, \eta_{\text{LoS}}, \eta_{\text{NLoS}}, N_{\text{SC}}, N_D \)

**Output:** \((x_i, y_i), (x_{Dj}, y_{Dj}, h_D)\)

1: **Distribution of SCs:**
2: \((x_i, y_i) \leftarrow \text{Matérn Process}(A_S, \lambda, s_{\text{min}}^{\text{SC}})\)
3: \((x_i, y_i) \leftarrow \text{Randomly select } N_{\text{SC}} \text{ number of points out of } (x_i, y_i)\)
4: **Distribution of NFPs:**
5: Compute \(s_{\text{min}}^{\text{D}}\) using (1), (2), \(\text{PL}_{\text{max}}, \alpha, \beta, \eta_{\text{LoS}}, \eta_{\text{NLoS}}\)
6: if \(N_D = 0\) then
7: Compute \(N_D\) using (22)
8: end if
9: \((x_{Dj}, y_{Dj}) \leftarrow \text{Matérn Process}(A_S, \lambda, s_{\text{min}}^{\text{D}})\)
10: \((x_{Dj}, y_{Dj}) \leftarrow N_D \text{ points out of } (x_{Dj}, y_{Dj}) \text{ and } h_{Dj} = h_{\text{max}}\)

where \(b_{\text{avg}} = \frac{1}{N_{\text{SC}}} \sum_{i \in \mathcal{N}} b_i\) is the average bandwidth required by a SC. Now, the minimum number of required NFPs is computed as

\[
N_D = \left\lceil \frac{N_{\text{SC}}}{\text{min}\{N_l, N_{\text{D}}^{\text{SC}}\}} \right\rceil. \tag{22}
\]

Now, again we use another symmetry for the NFPs with the assumption that the height of every NFP is fixed to a maximum defined height \(h_{\text{max}}\) such that \(h_{Dj} = h_{\text{max}}\), where \(h_{\text{max}}\) is obtained by the initialization algorithm as an input parameter. Next, using \(h_{\text{max}}\) and considering a fixed maximum path loss \(\text{PL}_{\text{max}}\) in (2), we can compute \(s\) that corresponds to the maximum distance covered by \(j\)-th NFP. This maximum distance is equivalent to the required minimum separation between NFPs \(s_{\text{D}}^{\text{min}}\). Finally, we distribute the NFPs using Matérn type-I hard-core process with all the parameters same as of the distribution of SCs except the separation distance being equal to \(s_{\text{D}}^{\text{min}}\). This provides 3D locations of the NFPs as \((x_{Dj}, y_{Dj}, h_D)\). The above mentioned procedure is summarized in Algorithm Initialization.

Next, the SINR parameter (4) for each pair of SC and NFP is computed using a snapshot of the above distribution of SCs and NFPs combined with the bandwidth and data rate requirements of SCs, i.e., \(b_{ij}\) and \(r_{ij}\), respectively. All this information is then passed to the below presented algorithms to find the association of SCs and NFPs by solving the association problem (5).

**B. Modified Distributed Maximal Demand Minimum Servers Algorithm**

This algorithm is designed to use the processing power of three network nodes including SCs, NFPs and mother-NFP. Therefore, it is divided into three steps that are distributed among those three nodes, i.e., first step at SCs, second step at NFPs and third step at mother-NFP. Mainly,
the second step that is divided among NFPs speeds up the optimization process. Each step takes care of one or more constraints of the association problem (5).

1) **Step 1**: This step is performed at the $i$-th SC individually. The $i$-th SC uses SINR parameter from (4) and compares it with the minimum SINR requirement $\text{SINR}_{\text{min}}$ satisfying constraint (5d). This provides a list of possible pairs with NFPs for the $i$-th SC, $i \in \mathcal{N}$. Out of its list, the $i$-th SC picks the $j$-th NFP, $j \in \mathcal{M}$ that results in maximum value of the decision ratio $\frac{r_{ij}}{b_{ij}}$ and sends the association request to only the selected NFP. As each SC selects only one NFP, this procedure takes care of the constraint (5f). Note that the decision ratio is designed keeping in view the objective function (5a) and constraint (5c), where we want to maximize the data rate $r_{ij}$ and minimize the bandwidth $b_{ij}$.

2) **Step 2**: At this step, the $j$-th NFP uses information about its maximum number of supported links and bandwidth, and initializes counters $C_{N_j}^j = 0$ and $C_{b}^j = 0$. The $j$-th NFP, $j \in \mathcal{M}$ receives a number of association requests from a group of SCs. The $j$-th NFP goes through its own list of association requests and selects, one by one in an ascending order, the SCs resulting in the maximum decision ratio $\frac{r_{ij}}{b_{ij}}$ till the end of the list, or till it reaches its bandwidth/links capacity, whichever occurs earlier. Before associating the selected request of the $i$-th SC. The $j$-th NFP verifies the constraints for the maximum number of supported links $N_l$ and maximum bandwidth $B$ as follows. NFP verifies if it has remaining resources to serve the selected request of SC i.e., $C_{N_j}^j < N_l$ and $C_{b}^j + b_{ij} \leq B$. If those two conditions are satisfied then it associates the $i$-th SC and updates its respective association entry $A_{ij}$ and related counters as $C_{N_j} = C_{N_j} + 1$ and $C_{b}^j = C_{b}^j + b_{ij}$. If any of the two limits including the number of links and the bandwidth for the $j$-th NFP is reached, i.e., $C_{N_j}^j \not\leq N_l$ and $C_{b}^j \not\leq B$, then the association process for this NFP ends. Thus, at this step, the $j$-th NFP takes care of the two constraints including the maximum number of links, i.e., (5e) and the maximum bandwidth, i.e., (5c). Furthermore, in case if all the requests of SCs are entertained already and no further request is remaining for the $j$-th NFP, then the process at this step completes for the $j$-th NFP.

This step is designed to use the processing power of NFPs and further it is distributed among them in such a way so it can be performed in parallel. This distribution and parallel processing speeds up the overall association process. Also, note that until this step, we have satisfied constraints (5c) to (5g) only. We have also used the information of constraint (5b) but has

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2This decision ratio is inspired by the optimal solution for the Knapsack problem, which has similarities to our problem.
not verified it yet, as all the information is distributed between NFPs and SCs, so there is no way to collectively keep track of the combined data rate information.

3) **Step 3:** All of the information at step 2 is shared with the mother-NFP. Mother-NFP generates the association matrix $A$, where $j$-th column has $N_l$ number of ones at maximum. It initializes the data rate counter $C_r$ with the currently assigned total data rate of the associated SCs and keep track of the sum data rate as per the association matrix. Thus, at this step, mother-NFP verifies the constraint (5b) as follows.

If the backhaul data rate limit is not reached yet, i.e., $C_r < R$, then mother-NFP goes through the association matrix to look for the SCs not associated with any NFP. For those remaining SCs, mother-NFP creates a list of possible NFP-SC pairs. Out of the list, the NFP-SC pairs not satisfying the SINR constraint (5d) are discarded out of the list. Then, mother-NFP selects the NFP-SC pair with maximum decision ratio $r_{ij}/b_{ij}$. For the selected $j$-th NFP, mother-NFP first verifies the number of links and bandwidth resources, i.e., $C_{N_l}^j < N_l$ and $C_b^j < B$. In case either of the number of links or bandwidth limits have been reached, all the respective NFP-SC pairs of the $j$-th NFP are discarded from the list. Otherwise, the backhaul data rate and bandwidth constraints, i.e., (5b) and (5c), are verified for the selected NFP-SC pair such that $C_r + r_{ij} \leq R$ and $C_b^j + b_{ij} \leq B$. If the constraints are verified then mother-NFP associates the NFP-SC pair and updates the association matrix $A$, and the related counters such that $C_{N_l}^j = C_{N_l}^j + 1$, $C_b^j = C_b^j + b_{ij}$ and $C_r = C_r + r_{ij}$. Also, for the selected SC associated with the $k$-th NFP, other possible links are neglected, i.e., links with the $j$-th NFPs, $j \in \mathcal{M}$, where $j \neq k$ are discarded from the list to satisfy constraint (5f). Then, mother-NFP selects the next NFP-SC pair resulting in next maximum decision ratio and keeps associating the remaining SCs until the resources are fully utilized or all of the SCs gets associated.

The other case that needs to be checked is when the backhaul data rate limit has been exceeded, i.e., $C_r > R$. This may happen due to the distributed nature of the algorithm, as until step 2 there is no centralized tracking of the sum data rate for all of the NFP-SC associations. For this case, mother-NFP goes through the association matrix $A$ to find out the associated NFP-SC pairs. Out of those pairs, it selects the one that results in minimum value of data rate $r_{ij}$. Then, it disassociates the selected pair and updates the association matrix index $A_{ij}$ and the counters as $C_{N_l}^j = C_{N_l}^j - 1$, $C_b^j = C_b^j - b_{ij}$ and $C_r = C_r - r_{ij}$. Same procedure is followed until the backhaul data rate limit is satisfied.

Throughout this algorithm, priority is given to the NFP-SC pairs resulting in maximum decision
ratio which means that the algorithm is designed to increase the data rate under the bandwidth limit mainly. This is in accordance with the objective function (5a) and thus this algorithm focuses on user-centric case where SCs with users who demand high data rate are given priority. This algorithm provides an efficient solution in three simple steps with less computational complexity as compared to B&B method and is summarized in Algorithm $\text{M(DM)}^2S$. Note that, we had presented a similar algorithm named Distributed Maximal Demand Minimum Servers ((DM)$^2$S) in [19], where a different decision ratio was used and the case of $C_r < R$ at step 3 was not considered.

C. Centralized Maximal Demand Minimum Servers Algorithm

This algorithm is designed for the C-RAN architecture where the processing takes place mainly at the baseband unit (BBU) pool. Thus, here we consider that the SCs and NFPs only carry the control information and all the data processing takes place at either the mother-NFP and the BBU pool. Both the mother-NFP or he BBU pool receive all the necessary information from the SCs and NFPs. Similar to the distributed algorithm, this one is designed also for the user-centric case where priority is given to the SCs demanding a high data rate and keeping in view the bandwidth constraint. Thus, we use the same decision ratio in this algorithm as used in Algorithm $\text{M(DM)}^2S$.

Mother-NFP receives the necessary information about the SCs and NFPs such as SINR of the NFP-SC links $\text{SINR}_{ij}$, minimum SINR requirement of the system $\text{SINR}_{\text{min}}$, demanded bandwidth $b_{ij}$ and data rate $r_{ij}$ of SCs, number of links $N_{lj}$ and bandwidth $B$ limits of the NFPs and backhaul data rate limit $R$. Using all of the above control information, mother-NFP creates a list of NFP-SC pairs that satisfy the SINR constraint (5d). It also initializes the counters form zero for the number of links $C^j_{N_l}$ assigned to the $j$-th NFP, assigned bandwidth $C^j_{b}$ to the $j$-th NFP and assigned sum data rate $C_r$ of all the NFPs. The backhaul data rate limit $R$, i.e., constraint (5b) is verified by mother-NFP such as $C_r < R$. If the verification fails, the algorithm terminates. Otherwise, the association proceeds as follows. Mother-NFP goes through the list of NFP-SC pairs and selects the pair that provides maximum decision ratio $r_{ij}/b_{ij}$. For the selected $j$-th NFP, it verifies the number of links $N_l$ and bandwidth $B$ limits such as $C^j_{N_l} < N_l$ and $C^j_{b} < B$. If any one of the two limits is exceeded, it means the selected NFP cannot provide further resources for the association of SCs. Thus, the selected and remaining pairs related to the selected $j$-th NFP are discarded from the list. If the two constraints are satisfied, then mother-NFP verifies the
Algorithm M(DM)^2S Modified Distributed Maximal Demand Minimum Servers Algorithm

Input: \( N_{SC}, N_D, \text{SINR}_{\text{max}}, N_l, B, R, \text{SINR}_{ij}, r_{ij}, b_{ij} \)  
Output: \( A \)

1: Initialize: \( A = 0_{N_{SC} \times N_D} \)
2: **Step 1:** at each SC
3: for \( i = 1 \) to \( N_{SC} \) do
4: Create a list of NFPs satisfying \( \text{SINR}_{ij} \geq \text{SINR}_{\text{min}} \)
5: Out of the list, select and request \( j \)-th NFP with max. \( r_{ij}/b_{ij} \)
6: end for
7: **Step 2:** at each NFP
8: Initialize counters: \( C_j^N_l = 0, C_j^b = 0 \) for \( j \in \mathcal{M} \)
9: for \( j = 1 \) to \( N_D \) do
10: while \( C_j^N_l < N_l \) AND \( C_j^b < B \) AND List for the \( j \)-th NFP not empty do
11: Select \( i \)-th SC with max. \( r_{ij}/b_{ij} \)
12: if \( C_j^b + b_{ij} \leq B \) then
13: Update \( A_{ij} = 1, C_{N_l}^j = C_{N_l}^j + 1 \) and \( C_j^b = C_j^b + b_{ij} \)
14: end if
15: end while
16: end for
17: **Step 3:** at mother-NFP
18: Initialize: \( C_r \) as total data rate of associated SCs
19: Create a list of un-associated SCs by scanning \( A \)
20: Initialize counters \( C_j^N_l \) and \( C_j^b \) by scanning \( A \)
21: while \( C_r < R \) AND List of un-associated SCs not empty do
22: Out of the list, discard NFP-SC pairs with \( \text{SINR}_{ij} < \text{SINR}_{\text{min}} \)
23: Select NFP-SC pair with max. \( r_{ij}/b_{ij} \)
24: if \( C_j^N_l < N_l \) AND \( C_j^b < B \) then
25: if \( C_r + r_{ij} \leq R \) AND \( C_j^b + b_{ij} \leq B \) then
26: Update \( A_{ij} = 1, C_{N_l}^j = C_{N_l}^j + 1, C_r = C_r + r_{ij} \) and \( C_j^b = C_j^b + b_{ij} \)
27: Discard other pairs of the \( i \)-th SC from the list
28: end if
29: else
30: Discard all pairs of the \( j \)-th NFP from the list
31: end if
32: end while
33: while \( C_r > R \) do
34: Create a list of associated NFP-SC pairs
35: Select the pair with min. \( r_{ij} \)
36: De-associate the selected pair and update \( A_{ij} = 0, C_{N_l}^j = C_{N_l}^j - 1, C_r = C_r - r_{ij} \) and \( C_j^b = C_j^b - b_{ij} \)
37: end while

sum data rate and bandwidth constraints (5b) and (5c) respectively, using demanded data rate and bandwidth information of the selected NFP-SC pair such as \( C_r + r_{ij} \leq R \) and \( C_j^b + b_{ij} \leq B \). If the constraints are satisfied the mother-NFP associates the selected pair by modifying the association matrix entry as \( A_{ij} = 1 \) and updates the respective counters accordingly. Further,
Algorithm **CMDMSA** Centralized Maximal Demand Minimum Servers Algorithm

**Input:** \(N_{SC}, N_{D}, N_{l}, \text{SINR}_{\text{min}}, B, R, \text{SINR}_{ij}, r_{ij}, b_{ij}\)

**Output:** \(A\)

1: Create a list of NFP-SC pairs satisfying \(\text{SINR}_{ij} \geq \text{SINR}_{\text{min}}\)
2: Initialize counters: \(C_{N_{l}}^{j} = 0, C_{b}^{j} = 0\) for \(j \in \mathcal{M}\) and \(C_{r} = 0\)
3: while List of NFP-SC pairs is not empty AND \(C_{r} < R\) do
4: Select NFP-SC pair with max. \(r_{ij}/b_{ij}\)
5: if \(C_{N_{l}}^{j} < N_{l}\) AND \(C_{b}^{j} < B\) then
6: if \(C_{r} + r_{ij} \leq R\) AND \(C_{b}^{j} + b_{ij} \leq B\) then
7: Update \(A_{ij} = 1, C_{N_{l}}^{j} = C_{N_{l}}^{j} + 1, C_{r} = C_{r} + r_{ij}, \) and \(C_{b}^{j} = C_{b}^{j} + b_{ij}\)
8: Discard other pairs of SC from the list
9: else
10: Discard selected NFP-SC pair from the list
11: end if
12: else
13: Discard all pairs of the \(j\)-th NFP from the list
14: end if
15: end while

**TABLE I: Simulation Parameters**

| Parameter | Value   | Parameter | Value   |
|-----------|---------|-----------|---------|
| \(\alpha\) | 9.61    | \(\beta\) | 0.16    |
| \(\eta_{\text{LoS}}\) | 1 dB | \(\eta_{\text{NLoS}}\) | 20 dB |
| \(f_{c}\) | 2 GHz  | \(P_{t}\) | 5 Watts |
| \(\text{SINR}_{\text{min}}\) | -5 dB | \(\text{PL}_{\text{max}}\) | 115 dB |
| \(\lambda\) | \(5 \times 10^{-6} \text{ m}^{-2}\) | \(h_{D_{\text{max}}}\) | 300 meters |
| \(N_{SC}\) | 30      | \(N_{D}\) | 3       |
| \(r_{SC}\) | \{30, 60, 90, 120, 150\} Mbps |

After the association, the remaining links of the \(i\)-th SC with other NFPs except the selected NFP are discarded from the list. In case the constraints are not satisfied, the selected pair is discarded from the list. The algorithm terminates if either the sum data rate limit is reached or the list of NFP-SC pairs to be associated ends. The whole procedure is summarized in Algorithm **CMDMSA**.

**V. Numerical Results**

Consider a system as shown in Fig. [1] where we analyze the association problem of SCs and NFPs distributed over a square region of area \(A_{S} = 16 \text{ km}^{2}\). For the distribution of SCs and NFPs, we use Algorithm **Initialization**. The same average density \(\lambda\) is used in *Matèrn* type-I
hard-core process for the distribution of both SCs and NFPs. Neighbouring SCs are separated by maintaining a minimum separation of \( s_{\text{SC}}^{\text{min}} = 300 \) meters. For the distribution of NFPs, we compute the minimum separation \( s_{D}^{\text{min}} \) between neighbouring NFPs using maximum path loss \( \text{PL}_{\text{max}} \) as shown in Table I. Next, we assign the data rate to the SCs randomly from a pre-defined vector \( r_{\text{SC}} \), where it is assumed that the \( i \)-th SC will demand same data rate from any of the \( j \)-th NFP, \( j \in \mathcal{M} \) such that \( r_{ij} = r_i, \forall j \in \mathcal{M} \). Then, using the parameters defined in Table I the demanded bandwidth \( b_{ij} \) of the SCs and SINR\(_{ij} \) of the NFP-SC pairs are computed. A snapshot is taken of the current scenario and all the relevant information is passed to the algorithms to find the best possible association of SCs and NFPs. We consider a number of scenarios with varying distributions of NFPs and SCs to analyze the performance of association algorithms over different scenarios.

Fig. 2 depicts one of the considered scenarios for the distribution of SCs and NFPs. Here, only 2D view is shown as the \( N_D \) NFPs are placed at the same height. Fig. 2a and 2b show the association computed using GAP bound and LP relaxation, respectively. Further, the association computed using B&B exhaustive search that considers all the constraints without any relaxation is shown in Fig. 2c. As the association of our greedy algorithms is same for the considered case, so the association is shown jointly in 2d. It can be noticed that all the algorithms and even as per the GAP bound, we are unable to associate \( N_{SC} \) SCs. This is due to the constraints (5b), (5c) and (5e). Thus, the applied backhaul data rate \( R = 2.3 \) Gbps, bandwidth \( B = 0.4 \) GHz and number of links \( N_l = 10 \) limits are less than the system requirements. In Fig. 2a, although the GAP bound does not consider the data rate and number of links constraints, 3 SCs remain un-associated due to the bandwidth limit only. In the LP-relaxed solution, if the association matrix entry is greater than zero then the related NFP-SC pair is shown as associated. However, as LP-relaxed solution considers all constraints, 3 SCs remains un-associated due to limited resources. The B&B method takes care of all the constraints without any relaxation and thus, 5 SCs are not associated to satisfy the constraints. Our greedy algorithms also take care of all the constraints and has same number of unassociated SCs as B&B but works in a different fashion, thus, result in a different association.

To get more insights into the presented algorithms and bounds, we run a few experiments in the following to study the effect of various limitations due to constraints (5b), (5c) and (5e) on the objective function (5a), i.e., sum data rate of the overall system. Moreover, we discuss the effect of the constraints on the number of associated SCs.
A. Experiment 1: Effect of backhaul data rate

Fig. 3 plots the sum data rate of the associated SCs versus $R_r$, which is a ratio of the backhaul data rate limit $R$ to the sum of the demanded data rate by the $N_{SC}$ SCs. For a single value of $R_r$, we have generated 100 different scenarios and then averaged the associated data rate of SCs. For various scenarios, the ratio $R_r$ is kept the same by changing the backhaul data rate limit according to the demanded sum data rate of the SCs. Further, the other limits such as those of the bandwidth and the number of links are relaxed by providing more resources than required. This is done so that the effect of the backhaul data rate can be observed easily. We can notice that for every algorithm except the GAP bound, the sum data rate increases with the increase in
ratio $R_r$ until the ratio reaches one i.e., $R_r = 1$. Then, the sum data rate remains the same even with the increase in $R_r$ because beyond $R_r = 1$ algorithm have already associated all the SCs and thus providing extra resources is unnecessary. Further, we can observe that the performance of our greedy algorithms is very close to the exhaustive B&B method and LP relaxation. The GAP bound remains the same as the maximum achievable data rate, as the GAP bound does not take care of the constraint (5b).

Fig. 4 depicts the effect of the data rate constraint on the number of associated SCs. It can be noticed that the number of associated SCs by our greedy algorithms is nearly same as that of LP-relaxed solution. The performance of B&B method is close to the LP-relaxed solution, however, it deteriorates when the rate ratio ranges between 0.4 to 0.6. In all conditions the worst performance is obtained by (DM)$^2$S algorithm.

**B. Experiment 2: Effect of bandwidth limit**

Fig. 5 shows the sum data rate of SCs versus the bandwidth limit applied to NFPs. We can notice that the sum data rate increases with the increase in bandwidth limit, which is intuitive as NFPs will be able to associate more SCs due to more bandwidth. Here, we have supplied more than required backhaul data rate limit $R$ and number of links limit $N_l$, just to observe the effect of bandwidth limit alone. In terms of bounds, we can notice that our derived GAP bound

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3 It must be noted that B&B is still an upper-bound for our algorithms if for maximizing the sum data rate objective, while in this figure the number of associated SCs is considered.
Fig. 4: Number of associated SCs vs. $R_r$ for the association algorithms with restrictions $B = 5$ Gbps, $N_l = 30$ averaged over 100 different scenarios.

provides a more tight bound as compared to LP-relaxed bound. In this case, the GAP bound and the B&B method provide the same result due to the fact that both take care of the bandwidth limit and the remaining constraints are already relaxed. This shows that the GAP bound whose run-time speed is better than exhaustive B&B method can be used to study the comparison of the proposed algorithms if we neglect the backhaul data rate and number of links limits. Otherwise, if we include the other two constraints then the GAP bound provides an upper limit on the sum data rate of the SCs. Moreover, we can see the performance of our proposed greedy algorithms $\text{M(DM)}^2\text{S}$ and CMDMSA is very close to the exhaustive B&B method and better than $(\text{DM})^2\text{S}$ algorithm.

Fig. 6 depicts the effect of the bandwidth constraint on the number of associated SCs. It can be noticed that the maximum number of SCs associated by LP-relaxed solution matches that achieved by our designed greedy algorithms. In case of less available bandwidth, the GAP bound and the B&B method associated fewer SCs than the LP solution, however the gap in the performance decreases with the increase in bandwidth. The $(\text{DM})^2\text{S}$ algorithm results in the worst performance.

C. Experiment 3: Effect of number of links limit

In Fig. 7, we plot the sum data rate versus the number of links limit $N_l$, where $N_l$ increases from 2 to 30 links. The backhaul data rate $R$ and bandwidth $B$ limits are provided such that they do not effect the association of SCs, so the effect of the number of links $N_l$ can be observed
Fig. 5: Sum data rate of SCs vs. bandwidth limit $B$ for the association algorithms with restrictions $R = 5$ Gbps, $N_l = 30$ averaged over 100 different scenarios.

Fig. 6: Number of associated SCs vs. bandwidth limit $B$ for the association algorithms with restrictions $R = 5$ Gbps, $N_l = 30$ averaged over 100 different scenarios.

exclusively. Again, in this case, the GAP bound remains unaffected as it does not consider the number of links constraint (5e). There is a performance gap between the exhaustive B&B method and our presented algorithms in this case. This is due to the fact that our greedy algorithms M(DM)$^2$S and CMDMSA do not take an intelligent decision on the basis of the number of links. For example, there can be a situation, where due to less number of available links, we should give priority to some SCs to be associated to a certain NFP, however our algorithms lack this decision power. Note that, such a situation occurs in a case when very few links $N_l$ are available, as it can be seen from Fig. 7 that the performance gap between the B&B method and our proposed algorithms decreases with the increase in $N_l$. For the case of few available links
Fig. 7: Sum data rate of SCs vs. number of links limit $N_l$ for the association algorithms with restrictions $R = 5$ Gbps, $B = 5$ GHz averaged over 100 different scenarios.

$N_l$, one can use the $(DM)^2S$ algorithm, whose performance is close to the B&B method.

In Fig. 8, we study the effect of the number of links constraint (5c) on the number of associated SCs. It can be observed that the performance of all the considered algorithms is the same in case the links $N_l$ are either scarce or abundant. The performance of the algorithms varies in a region where the number of links ranges between 5 to 15. LP-relaxed solution again leads the algorithms followed by a very close performance between B&B and our presented greedy algorithms. Meanwhile, the $(DM)^2S$ algorithm again results in the minimum number of associated SCs. The GAP bound is not affected by the change in the number of links and provides the upper limit due to the relaxed bandwidth limit.

VI. COMPUTATIONAL COMPLEXITY

In this section, we provide the worst case complexity and average run time comparison of the algorithms. The B&B method has the same complexity as that of Brute-force in the worst case [27] and [28]. Table II compares, in terms of the number of flops, the worst case complexity of the presented algorithms with the Brute-force and $(DM)^2S$ algorithm. It can be noticed that the presented algorithms are computationally less expensive than the B&B algorithm in the worst case and slightly more expensive than the $(DM)^2S$ algorithm, however, provides the same performance as can be seen from the numerical results.

Table III compares the run time speed of the proposed algorithms $(DM)^2S$ and CMDMSA with the GAP bound and the exhaustive B&B method. By observing the Figures 3, 5 and 7 and
Fig. 8: Number of associated SCs vs. number of links limit $N_l$ for the association algorithms with restrictions $R = 5$ Gbps, $B = 5$ GHz averaged over 100 different scenarios.

TABLE II: Computational complexity of the algorithms.

| Algorithm   | Complexity Order |
|-------------|------------------|
| Brute-force | $O\left(N_l^3 N_{SC}^{N_l+2}\right)$ |
| M(DM)$^2$S | $O\left(N_l^2 N_{SC}^2\right)$ |
| CMDMSA     | $O\left(N_l^2 N_{SC}^2\right)$ |
| (DM)$^2$S  | $O\left(N_l N_{SC}\right)$ |

TABLE III: Run Time comparison of the algorithms for the system with constraints $R = 2.3$ Gbps, $B = 0.6$ GHz and $N_l = 8$ averaged over 100 different scenarios.

| Method    | Elapsed Time (seconds) |
|-----------|------------------------|
| GAP bound | 0.036                  |
| B&B       | 45.145                 |
| M(DM)$^2$S| 0.000445               |
| CMDMSA    | 0.000523               |

as per the constraints (5b), (5c) and (5e), all the limits are applied simultaneously. The results are averaged over 100 different scenarios. It can be noticed from Table III that the proposed algorithms are faster than the GAP bound and B&B method. Therefore, the proposed algorithms are computationally less expensive and are practically applicable.
VII. Conclusions

This work considered the use of NFPs as fronthaul hubs to provide backhaul connectivity to an ultra dense network of SCs. An association problem of SCs and NFPs is formulated in order to maximize the sum data rate of the overall SC network along with the consideration of backhaul and NFP-related limitations such as backhaul data rate, bandwidth and number of links limits of NFP. In the literature, the association problem of SCs and NFPs was claimed to be NP-hard, however, in this work, we have shown it to be NP-hard by relating it with the GAP. Then, using this relevance, a performance bound is derived and verified by a numerical comparison with LP relaxed solution as well as B&B exhaustive search. Further, two efficient (less complex) greedy algorithms are designed to solve the problem. The performance of the presented algorithms is same as of exhaustive B&B search as well as the presented bounds, both in terms of sum data rate and number of associated SCs. However, the presented algorithms has a lower complexity than all the other methods, thus they can be practically implemented. In future work, we will consider another aspect of the problem where the objective is to serve maximum number of users, i.e., network centric approach. Similar techniques can be used to derive related bounds and also similar greedy algorithms can be presented for such a case.

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