Chiral nucleon dynamics

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ABSTRACT

These lectures give an introduction to baryon chiral perturbation theory. I show in detail how to construct the chiral effective pion–nucleon Lagrangian in the one loop approximation. Particular emphasis is put on the physics related to electromagnetic probes, as manifest in pion photo- and electroproduction, Compton scattering off nucleons and the electroweak form factors of the nucleon. Other topics discussed in some detail are the meaning of low–energy theorems, pion–nucleon scattering, the reaction $\pi N \rightarrow \pi \pi N$ and isospin violation in the pion–nucleon system. The chiral predictions are confronted with the existing data. Some remaining problems and new developments are outlined.

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1 Introduction

Over the last few years, a tremendous amount of very precise data probing the structure of the nucleon at low energies has become available. With the advent of CEBAF at Jefferson Lab, this data base can increase significantly if the low energy region is given sufficient attention. These data encode information about the nucleon in the non-perturbative regime, i.e. at typical momentum scales were straightforward perturbation theory in
the running strong coupling constant $\alpha_s(Q^2)$ is no longer applicable. At present, there exist essentially two approaches to unravel the physics behind the wealth of empirical information. On one hand, one uses models which stress certain aspects of the strong interactions (but tend to neglect or forget about others). The other possibility is to make use of the symmetries of the Standard Model (SM) and formulate an effective field theory (EFT) to systematically explore the strictures of these symmetries. In the case of QCD and in the sector of the three light quarks u, d and s we know that there exist an approximate chiral symmetry which is spontaneously broken with the appearance of eight Goldstone bosons, the pions, kaons and the eta. These pseudoscalar mesons are the lightest strongly interacting particles and their small but non-vanishing masses can be traced back to the fact that the current masses $m_u$, $m_d$ and $m_s$ are small compared to any typical hadronic scale, like e.g. the proton mass. In the meson sector, the EFT is called chiral perturbation theory and is well developed and applied successfully to many reactions (for reviews, see e.g. (3) (4) (5)). Of course, there is also the lattice formulation of QCD which over the years has shown great progress but is not yet in the status to discuss in detail what will be mostly the topic here, namely the baryon structure as accurately probed with real or virtual photons. Clearly, the baryons (and in particular the nucleons I will mostly focus on) are not related directly to the spontaneous chiral symmetry breakdown. However, their interactions with pions and among themselves are strongly constrained by chiral symmetry. This is, of course, known since the sixties (see e.g. the lectures by Coleman [6] and references therein). However, to go beyond the current algebra or tree level calculations, one needs a systematic power counting scheme as it was first worked out for the meson sector by Weinberg [7]. As shown by Gasser et al. [8], the straightforward generalization to the baryon sector leads to problems related to the non-vanishing mass of the baryons in the chiral limit, i.e. one has an extra large mass scale in the problem. Stated differently, baryon four-momenta are never small compared to the chiral symmetry breaking scale $\Lambda, m_B/\Lambda \sim 1$. This can be overcome by a clever choice of velocity-dependent fields which allows to transform the baryon mass term in a string of $1/m_B$ suppressed interaction vertices. Then, a consistent power counting scheme emerges where the expansion in small momenta and quark masses can be mapped one-to-one on a (Goldstone boson) loop expansion. A nice introduction to some of the topics, in particular of the formal aspects, has been given by Ecker [10]. His lectures should be consulted for some details not covered here.

These lectures will be organized as follows. After a short review of the construction of the EFT in the presence of matter fields, I will give a sample calculation (of the pion cloud contribution to the nucleon mass). I elaborate in some detail on the structure of the next-to-leading order terms of dimension two and three in the effective Lagrangian. In particular, I show how certain operators with fixed coefficients arise in the heavy fermion formalism. This is followed by a study of the numerical values of the low-energy constants appearing at second order and their phenomenological interpretation in terms of resonance saturation. Consequently, I construct the complete dimension three effective Lagrangian. Then, I will discuss the status of the low-energy theorems in pion photo- and electroproduction and Compton scattering. In the following sections, chiral predictions are confronted with the data. Some new developments and open problems are touched upon in section 12.
2 Chiral perturbation theory with baryons

Chiral perturbation theory (CHPT) is the EFT of the SM at low energies in the hadronic sector. Since as an EFT it contains all terms allowed by the symmetries of the underlying theory [7], it should be viewed as a direct consequence of the SM itself. The two main assumptions underlying CHPT are that

(i) the masses of the light quarks u, d (and possibly s) can be treated as perturbations (i.e., they are small compared to a typical hadronic scale of 1 GeV) and that

(ii) in the limit of zero quark masses, the chiral symmetry is spontaneously broken to its vectorial subgroup. The resulting Goldstone bosons are the pseudoscalar mesons (pions, kaons and eta).

CHPT is a systematic low–energy expansion around the chiral limit [7] [1] [2] [11]. It is a well–defined quantum field theory although it has to be renormalized order by order. Beyond leading order, one has to include loop diagrams to restore unitarity perturbatively. Furthermore, Green functions calculated in CHPT at a given order contain certain parameters that are not constrained by the symmetries, the so–called low–energy constants (LECs). At each order in the chiral expansion, those LECs have to be determined from phenomenology (or can be estimated with some model dependent assumptions). For a review of the wide field of applications of CHPT, see, e.g., Ref. [3].

In the baryon sector, a complication arises from the fact that the baryon mass $m_B$ does not vanish in the chiral limit [8]. Stated differently, only baryon three–momenta can be small compared to the hadronic scale. To see this in more detail, let us consider the two–flavor case with $m_u = m_d = \hat{m}$ and collect the proton and the neutron in a bi-spinor

$$\Psi = \begin{pmatrix} p \\ n \end{pmatrix},$$

which transforms non–linearly under chiral transformations,

$$\Psi \rightarrow \Psi' = K(L, R, U(x)) \Psi.$$

Here, $K(L, R, U(x))$ is a non–linear function of the meson fields collected in $U(x)$ and of $L, R \in SU(2)$ [12] [13]. It is defined via

$$u' = LuK\dagger = KuR\dagger,$$ with $U = u^2$, $u'^2 = U'^2$.

The unimodular unitary matrix $U$ ($U^\dagger U = UU^\dagger = 1$, det$(U) = 1$) transforms linearly under chiral transformations,

$$U \rightarrow U' = LUR\dagger.$$

It is most convenient to parametrize $U$ as follows

$$U = (\sigma + i\vec{\tau} \cdot \vec{\pi})/F, \quad \sigma^2 + \vec{\pi}^2 = F^2,$$
with $F$ the pion decay constant in the chiral limit, $F_\pi = F [1 + \mathcal{O}(\hat{m})] = 92.4$ MeV. It is now straightforward to write down chiral covariant derivatives and construct the lowest order effective Lagrangian

$$L_{\text{eff}} = L^{(1)}_{\pi N} + L^{(2)}_{\pi\pi}$$

$$L^{(1)}_{\pi N} = \bar{\Psi} \left( i \gamma_\mu D^\mu - \hat{m} + \frac{1}{2} \hat{g}_A \gamma_5 u_\mu \right) \Psi$$

$$L^{(2)}_{\pi\pi} = \frac{F^2}{4} \langle \nabla^\mu U \nabla^\mu U^\dagger \rangle + \frac{F^2 M^2}{4} \langle U + U^\dagger \rangle$$

with $u_\mu = i u_\dagger \nabla_\mu U u_\dagger$ and $\langle \ldots \rangle$ denotes the trace in flavor space. Here, the superscript ‘⊙’ denotes quantities in the chiral limit, i.e. $Q = \hat{Q} [1 + \mathcal{O}(\hat{m})]$ (with the exception of $M$ which is the leading term in the quark mass expansion of the pion mass and $F$), $g_A$ is the axial–vector coupling constant measured in neutron $\beta$–decay, $g_A = 1.26$ and $m$ denotes the nucleon mass. For the three flavor case, one has of course two axial couplings. The chiral dimension (power) of the respective terms is denoted by the superscripts ‘⊙’ (i=1,2). The pertinent covariant derivatives are

$$\nabla_\mu U = \partial_\mu U - ie A_\mu [Q, U]$$

$$D_\mu \Psi = \partial_\mu \Psi + \frac{1}{2} \{ u_\dagger (\partial_\mu - ie A_\mu Q) u + u (\partial_\mu - ie A_\mu Q) u_\dagger \} \Psi = \partial_\mu \Psi + \Gamma_\mu \Psi$$

with $Q = \text{diag}(1,0)$ the (nucleon) charge matrix and I only consider external vector fields, i.e. the photon $A_\mu$. $D_\mu$ transforms homogeneously under chiral transformation, $D'_\mu = KD_\mu K^\dagger$, and $\Gamma_\mu$ is the connection [12]. To show the strength of the effective Lagrangian approach, let me quickly derive the so–called Goldberger–Treiman relation (GTR) [15]. For that, I set $A_\mu = 0$ and expand the pion–nucleon Lagrangian to order $\pi$,

$$L^{(1)}_{\pi N} = \bar{\Psi} \left( i \gamma_\mu \partial^\mu - \hat{m} \right) \Psi - \frac{\hat{g}_A}{F} \bar{\Psi} \gamma_5 \gamma_5 \frac{\vec{r}}{2} \partial_\mu \vec{r} + \ldots$$

from which we read off the $NN\pi$ vertex in momentum space

$$V_{NN\pi} = \frac{\hat{g}_A}{2F} \gamma_\mu q^\mu \gamma_5 \vec{r}$$

where the momentum $q_\mu$ is out–going. The transition amplitude for single pion emission off a nucleon takes the form

$$T_{NN\pi} = -i \bar{u}(p') V_{NN\pi} u(p) = i \frac{\hat{g}_A}{F} \hat{m} \bar{u}(p') \gamma_5 u(p) \tau^i$$

where I have used the Dirac equation $\bar{u}(p') \gamma_\mu q^\mu \gamma_5 u(p) = -2 \hat{m} \bar{u}(p') \gamma_5 u(p)$ with $q = p' - p$. Comparing eq. (2.13) with the canonical form of the transition amplitude

$$T_{NN\pi} = i \hat{g}_{\pi N} \bar{u}(p') \gamma_5 u(p)$$

(2.14)
one arrives directly at the GTR,
\[ g_{\pi N} = \frac{g_A \hat{m}}{F}, \]
which is fulfilled within a few \% in nature. This relation is particularly intriguing because it links the strong pion–nucleon coupling constant to some weak interaction quantities like \( g_A \) and \( F_\pi \) as a consequence of the chiral symmetry. I will discuss this relation and the deviation thereof in more detail in section 11.6. Finally, if one wants to discuss processes with two (or more) nucleons in the initial and final state, one has to add a string of terms of the type
\[ \mathcal{L}_{\bar{\Psi} \Psi \bar{\Psi} \Psi} + \mathcal{L}_{\bar{\Psi} \Psi \bar{\Psi} \Psi \bar{\Psi} \Psi} + \ldots \]
which are also subject to a chiral expansion and contain LECs which can only be determined in few or many nucleon (baryon) processes.

Clearly, the appearance of the mass scale \( \hat{m} \) in eq.(2.7) causes trouble. To be precise, if one calculates the self–energy of the nucleon mass to one loop, one encounters terms of dimension zero, i.e. in dimensional regularization one finds a term of the type \[ \mathcal{L}_{\pi N}^{(0)} = c_0 \bar{\Psi} \Psi, \quad c_0 \sim \left( \frac{\hat{m}}{F} \right)^2 \frac{1}{d-4} + \ldots, \]
where the ellipsis stands for terms which are finite as \( d \to 4 \). Such terms clearly make it difficult to organize the chiral expansion in a straightforward and simple manner. They can only be avoided if the additional mass scale \( \hat{m} \sim 1 \text{ GeV} \) can be eliminated from the lowest order effective Lagrangian. (Notice here the difference to the pion case - there the mass vanishes as the quark masses are sent to zero.) To do that, consider the mass of the nucleon large compared to the typical external momenta transferred by pions or photons and write the nucleon four–momentum as
\[ p_\mu = m v_\mu + \ell_\mu, \quad p^2 = m^2, \quad v \cdot \ell \ll m. \]
Notice that to this order we do not have to differentiate between \( m \) and \( \hat{m} \) and \( v_\mu \) is the nucleon four–velocity (in the rest–frame, we have \( v_\mu = (1, \vec{0}) \)). In that case, we can decompose the wavefunction \( \Psi \) into velocity eigenstates \[ \Psi(x) = \exp[-i \hat{\bar{m}} \cdot v \cdot x] [H(x) + h(x)] \]
with
\[ \not{v} H = H, \not{v} h = -h, \]
or in terms of velocity projection operators
\[ P_v^+ H = H, \quad P_v^- h = h, \quad P_v^\pm = \frac{1}{2} (1 \pm \not{v}), \quad P_v^+ + P_v^- = 1. \]
One now eliminates the 'small' component \( h(x) \) either by using the equations of motion or path–integral methods. The Dirac equation for the velocity–dependent baryon field
\[ H = H_v \] (I will always suppress the label 'v') takes the form \( iv \cdot \partial H_v = 0 \) to lowest order in \( 1/m \). This allows for a consistent chiral counting as described below and the effective pion–nucleon Lagrangian takes the form:

\[
\mathcal{L}^{(1)}_{\pi N} = \bar{H} \left( iv \cdot D + \partial \right) S \cdot u H + \mathcal{O} \left( \frac{1}{m} \right),
\]

(2.22)

with \( S_\mu \) the covariant spin–operator

\[
S_\mu = \frac{i}{2} \gamma_5 \sigma_{\mu\nu} v^\nu, \quad S \cdot v = 0, \quad \{ S_\mu, S_\nu \} = \frac{i}{2} (v_\mu v_\nu - g_\mu\nu), \quad \{ S_\mu, S_\nu \} = \frac{i}{2} \epsilon_{\mu\nu\gamma\delta} v^\gamma S^\delta,
\]

(2.23)

in the convention \( \epsilon^{0123} = -1 \). There is one subtlety to be discussed here. In the calculation of loop graphs, divergences appear and one needs to regularize and renormalize these. That is done most easily in dimensional regularization since it naturally preserves the underlying symmetries. However, the totally antisymmetric Levi-Civita tensor is ill-defined in \( d \neq 4 \) space–time dimensions. One therefore has to be careful with the spin algebra. In essence, one has to give a prescription to uniquely fix the finite pieces. The mostly used convention to do this is to only use the anticommutator to simplify products of spin matrices and only take into account that the commutator is antisymmetric under interchange of the indices. Furthermore, \( S^2 \) can be uniquely extended to \( d \) dimensions via \( S^2 = (1 - d)/4 \). With that in mind, two important observations can be made. Eq. (2.22) does not contain the nucleon mass term any more and also, all Dirac matrices can be expressed as combinations of \( v_\mu \) and \( S_\mu \) \( \square \),

\[
\bar{H} \gamma_\mu H = v_\mu \bar{H} H, \quad \bar{H} \gamma_5 H = 0, \quad \bar{H} \gamma_\mu \gamma_5 H = 2 H S_\mu H,
\]

(2.24)

\[ \bar{H} \sigma_{\mu\nu} H = 2 \epsilon_{\mu\nu\gamma\delta} \bar{H} S^\delta H, \quad \bar{H} \gamma_5 \sigma_{\mu\nu} H = 2i \bar{H} (v_\mu S_\nu - v_\nu S_\mu) H, \]

to leading order in \( 1/m \). More precisely, this means e.g. \( \bar{H} \gamma_5 H = \mathcal{O}(1/m) \). We read off the nucleon propagator,

\[
S_N(\omega) = \frac{i}{\omega + i\eta}, \quad \omega = v \cdot \ell, \quad \eta > 0,
\]

(2.25)

and the Feynman insertion for the emission a pion with momentum \( \ell \) from a nucleon is

\[
\frac{g_A}{F_\pi} \tau^a S \cdot \ell.
\]

(2.26)

It is also instructive to consider the transition from the relativistic fermion propagator to its counterpart in the heavy fermion limit. Starting with \( S_N(p) = i/(p - m) \), one can project out the light field component,

\[
S_N = \begin{array}{c}
P_v^+ \frac{i}{p^2 - m^2} \quad P_v^+ = i \frac{p \cdot v + m}{p^2 - m^2} \quad \frac{2m + v \cdot \ell}{2mv \cdot \ell + \ell^2} \quad \frac{P_v^+}{P_v^+} \\
\equiv \frac{i}{v \cdot \ell + \ell^2/2m - (v \cdot \ell)^2/2m + \mathcal{O}(1/m^2)} \quad \frac{P_v^+}{v \cdot \ell} \sim \mathcal{O}(1/m) \end{array}
\]

(2.27)

which in fact shows that one can include the kinetic energy corrections to be discussed in more detail below already in the propagator. Notice that from now on I will not always distinguish between the observables and their chiral limit values (although that distinction should be kept in mind). Before proceeding with some actual calculations in heavy baryon CHPT (HBCHPT), let me outline the chiral power counting which is used to organize the various terms in the energy expansion.

6
3 Chiral power counting

To calculate any process to a given order, it is useful to have a compact expression for the chiral power counting \[7\] \[10\]. First, I will restrict myself to purely mesonic or single–baryon processes. Since these arguments are general, I will consider the three flavor case. Any amplitude for a given physical process has a certain chiral dimension \( D \) which keeps track of the powers of external momenta and meson masses (collectively labelled \( d \)). The building blocks to calculate this chiral dimension from a general Feynman diagram in the CHPT loop expansion are (i) \( I_M \) Goldstone boson (meson) propagators \( \sim 1/(\ell^2 - M^2) \) (with \( M = M_{\pi,K,N} \) the meson mass) of dimension \( D = -2 \), (ii) \( I_B \) baryon propagators \( \sim 1/v \cdot \ell \) (in HBCHPT) with \( D = -1 \), (iii) \( N_d^M \) mesonic vertices with \( d = 2, 4, 6, \ldots \) and (iv) \( N_d^{MB} \) meson–baryon vertices with \( d = 1, 2, 3, \ldots \). Putting these together, the chiral dimension \( D \) of a given amplitude reads

\[
D = 4L - 2I_M - I_B + \sum_d d(N_d^M + N_d^{MB})
\]  

(3.1)

with \( L \) the number of loops. For connected diagrams, one can use the general topological relation

\[
L = I_M + I_B - \sum_d (N_d^M + N_d^{MB}) + 1
\]  

(3.2)

to eliminate \( I_M \):

\[
D = 2L + 2 + I_B + \sum_d (d - 2)N_d^M + \sum_d (d - 2)N_d^{MB}.
\]  

(3.3)

Lorentz invariance and chiral symmetry demand \( d \geq 2 \) for mesonic interactions and thus the term \( \sum_d (d - 2)N_d^M \) is non–negative. Therefore, in the absence of baryon fields, Eq. (3.3) simplifies to

\[
D = 2L + 2 + \sum_d (d - 2)N_d^M \geq 2L + 2.
\]  

(3.4)

To lowest order \( p^2 \), one has to deal with tree diagrams \( (L = 0) \) only. Loops are suppressed by powers of \( p^{2L} \).

Another case of interest for us has a single baryon line running through the diagram (i.e., there is exactly one baryon in the in– and one baryon in the out–state). In this case, the identity

\[
\sum_d N_d^{MB} = I_B + 1
\]  

(3.5)

holds leading to

\[
D = 2L + 1 + \sum_d (d - 2)N_d^M + \sum_d (d - 1)N_d^{MB} \geq 2L + 1.
\]  

(3.6)

Therefore, tree diagrams start to contribute at order \( p \) and one–loop graphs at order \( p^3 \). Obviously, the relations involving baryons are only valid in HBCHPT.
Let me now consider diagrams with $N_\gamma$ external photons. Since gauge fields like the electromagnetic field appear in covariant derivatives, their chiral dimension is obviously $D = 1$. One therefore writes the chiral dimension of a general amplitude with $N_\gamma$ photons as

$$D = D_L + N_\gamma \, ,$$

where $D_L$ is the degree of homogeneity of the (Feynman) amplitude $A$ as a function of external momenta ($p$) and meson masses ($M$) in the following sense (see also [16]):

$$A(p, M; C_i^r(\lambda), \lambda/M) = M^{D_L} A(p/M, 1; C_i^r(\lambda), \lambda/M) \, ,$$

where $\lambda$ is an arbitrary renormalization scale and $C_i^r(\lambda)$ denote renormalized LECs. From now on, I will suppress the explicit dependence on the renormalization scale and on the LECs. Since the total amplitude is independent of the arbitrary scale $\lambda$, one may in particular choose $\lambda = M$. Note that $A(p, M)$ has also a certain physical dimension (which is of course independent of the number of loops and is therefore in general different from $D_L$). The correct physical dimension is ensured by appropriate factors of $F_\pi$ and $m$ in the denominators.

Finally, consider a process with $E_n$ ($E_n = 4, 6, \ldots$) external baryons (nucleons). The corresponding chiral dimension $D_n$ follows to be [17]

$$D_n = 2(L - C) + 4 - \frac{1}{2} E_n + \sum_i V_i \Delta_i \, , \quad \Delta_i = d_i + \frac{1}{2} n_i - 2 \, ,$$

where $C$ is the number of connected pieces and one has $V_i$ vertices of type $i$ with $d_i$ derivatives and $n_i$ baryon fields (these include the mesonic and meson-baryon vertices discussed before). Chiral symmetry demands $\Delta_i \geq 0$. As before, loop diagrams are suppressed by $p^{2L}$. Notice, however, that this chiral counting only applies to the irreducible diagrams and not to the full S–matrix since reducible diagrams can lead to IR pinch singularities and need therefore a special treatment (for details, see refs. [17] [18]).

I will now briefly discuss the general structure of the effective Lagrangian based on these power counting rules, restricting myself again to the two–flavor case and processes with one nucleon in the asymptotic in– and out–states. While the lowest order Lagrangian eq.(2.22) has $D = 1$, one can construct a string of local operators with $D = 2, 3, 4, \ldots$. One–loop diagrams start at order $p^3$ if one only uses insertions from $\mathcal{L}^{(1)}_{\pi N}$. Two–loop graphs are suppressed by two more powers of $p$ so that within the one–loop approximation one should consider tree diagrams from

$$\mathcal{L}_{\pi N} = \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N} \, ,$$

and loop diagrams with insertions from $\mathcal{L}^{(1,2)}_{\pi N}$. It is important to stress that not all of the terms in $\mathcal{L}^{(2,3,4)}_{\pi N}$ contain LECs, but some of the coefficients are indeed fixed for kinematical or similar reasons. I will discuss the general structure of $\mathcal{L}^{(2)}_{\pi N}$ in section 5. It should also be stressed that although there are many terms in $\mathcal{L}^{(2,3,4)}_{\pi N}$, for a given process most of them do not contribute. As an example let me quote the order $q^4$ calculation of the nucleons’ electromagnetic polarizabilities [19] which involves altogether four LECs from $\mathcal{L}^{(2)}_{\pi N}$ and four from $\mathcal{L}^{(4)}_{\pi N}$, a number which can certainly be controlled. To all of this, one has of course to add the purely mesonic Lagrangian $\mathcal{L}^{(2)}_{\pi \pi} + \mathcal{L}^{(4)}_{\pi \pi}$ [1] [2].
4 A simple calculation

In this section, I will present a typical calculation, namely the nucleon mass shift from the pion loop (in the one–loop approximation). The full result can e.g. be found in refs.\[20\] [14]. A similar sample calculation has been given in the lectures by Jenkins and Manohar\[20\], but I will use another method which is easier to generalize to processes with external photons.

Consider the Feynman diagram where the nucleon emits a pion of momentum $\ell$ and absorbs the same pion (which is incoming with momentum $-\ell$). Using eqs.(2.25,2.26) and the relativistic propagator for the pion, the mass shift $\delta m$ is given by

$$\delta m = i \frac{3g_A^2}{F_F^2} \int \frac{d^d\ell}{(2\pi)^d} \frac{i}{\ell^2 - M_\pi^2 + i\eta} \frac{i}{v \cdot \ell + i\eta} S \cdot (-\ell) S \cdot \ell, \quad (4.1)$$

making use of $\tau^a \tau^a = 3$. From the anti–commutation relation of two spin matrices, eq.(2.23), and by completing the square we have

$$S \mu S \nu \ell^\mu \ell^\nu = \frac{1}{4} \left( v \cdot \ell + M_\pi^2 - \ell^2 - M_\pi^2 \right), \quad (4.2)$$

so that

$$\delta m = i \frac{3g_A^2}{4F_F^2} J(0) M_\pi^2 \quad (4.3)$$

To calculate this integral, we make use of dimensional regularization. The first term in the square brackets vanishes in dimensional regularization (see e.g. ref.[21]) and the second one is odd under $\ell \rightarrow -\ell$, i.e. it also vanishes. So we are left with

$$\delta m = \frac{3g_A^2}{4F_F^2} J(0) M_\pi^2 \quad (4.4)$$

$$J(0) = \frac{1}{i} \int \frac{d^d\ell}{(2\pi)^d} \frac{1}{(M_\pi^2 - \ell^2 - i\eta)(v \cdot \ell - i\eta)} \quad (4.5)$$

The remaining task is to evaluate $J(0)$. For that, we use the identity

$$\frac{1}{AB} = \int_0^\infty dy \frac{2}{[A + 2yB]^2} \quad (4.6)$$

define $\ell' = \ell - yv$, complete the square and use $v^2 = 1$,

$$J(0) = \frac{1}{i} \int_0^\infty dy \int \frac{d^d\ell'}{(2\pi)^d} \frac{1}{\left[M_\pi^2 + y^2 + \ell'^2 - i\eta\right]^2} \quad (4.7)$$

$$= \frac{2}{(2\pi)^d} \int_0^\infty dy \int_0^\infty d\ell' \frac{(\ell')^{d-1}}{\left[M_\pi^2 + y^2 + \ell'^2\right]^2} \frac{2\pi^{d/2}}{\Gamma(d/2)} \quad (4.8)$$

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where we have performed a Wick rotation, $\ell_0 \to i\ell_0$ and dropped the $i\eta$. The last factor in eq.(4.8) is the surface of the sphere in $d$ dimensions. Introducing polar coordinates, $y = r \cos \phi$ and $\ell' = r \sin \phi$ and noting that the Jacobian of this transformation is $r$, we have

$$ J(0) = \frac{4(4\pi)^{-d/2}}{\Gamma(d/2)} \int_0^\infty dr \frac{r^d}{(r^2 + M_\pi^2)^{d/2}} \int_0^{\pi/2} d\phi \,(\sin \phi)^{d-1}. \tag{4.9} $$

We then perform the further substitution $r = M_\pi \tan \Phi$ in the $r$–integral. Then, both integrals appearing in eq.(4.9) can be expressed in terms of products of $\Gamma$ functions with the result

$$ J(0) = M_\pi^{d-3} (4\pi)^{-d/2} \Gamma \left( \frac{1}{2} \right) \Gamma \left( \frac{3 - d}{2} \right) = -\frac{M_\pi}{8\pi}, \tag{4.10} $$

where in the last step I have set $d = 4$. This leads us to

$$ \delta m = -\frac{3g_\Lambda^2 M_\pi^3}{32\pi F_\pi^2}. \tag{4.11} $$

The pion loop leads to a self–energy $\Sigma$ which shifts the pole of the nucleon propagator by $\delta m$, i.e.

$$ m = \bar{m} + \Sigma(0) = \bar{m} + \delta m. \tag{4.12} $$

There are a few important remarks concerning eq.(4.11). First, the pion loop contribution is non–analytic in the quark masses since

$$ M_\pi^2 = 2 \bar{m} B \left[ 1 + \mathcal{O}(m_{\text{quark}}) \right]. \tag{4.13} $$

The constant $B$ is related to the scalar quark condensate and is assumed to be non–vanishing in the chiral limit (supported by lattice data), $B = |\langle 0 | \bar{q} q | 0 \rangle| / F^2$ and large, $B \sim 1.5 \text{ GeV}$. (For a different scenario, see e.g. refs.[23]). Second, the pion cloud contribution is attractive, i.e. it lowers the nucleon mass, and third, it vanishes in the chiral limit, i.e it has the expected chiral dimension of three (since it is a one–loop graph with insertions from the lowest order effective Lagrangian). More detailed studies of the baryon masses and $\sigma$–terms can be found in refs.[24][20][25][26][27][28]. Here, I only wish to state that at present it is not known whether the scalar three–flavour sector (i.e. the baryon masses and the $\sigma$–terms) can be described consistently within CHPT. The first complete calculation to order $p^4$ has been presented in [24], but the appearing LECs had to be modeled using resonance saturation including Goldstone boson loops. I will later return to the questions surrounding the three flavor sector and only remark here that much more work needs to be done to achieve a consistent picture in the presence of kaon loops, the main problem being that the kaon loop corrections are rather large (since $(M_K/M_\pi)^3 \sim 46$) and the corresponding LECs have therefore large coefficients to compensate for the meson cloud contribution. It might, however, well be that the corrections beyond leading order are much smaller. This can only be decided upon a series of complete calculations to order $p^4$. 

10
5 The structure of $\mathcal{L}^{(2)}_{\pi N}$

In this section, I will first write down the order $p^2$ effective Lagrangian, $\mathcal{L}^{(2)}_{\pi N}$, and then discuss some of its peculiarities. Allowing for the moment for $m_u \neq m_d$, its most general form is (I only consider external scalar and vector fields, the generalization to pseudoscalar and axial–vector ones is straightforward):

$$\mathcal{L}^{(2)}_{\pi N} = \mathcal{H} \left\{ \frac{1}{2m} (v \cdot D)^2 - \frac{1}{2m} D^2 - i \frac{g_A}{2m} \{ S \cdot D, v \cdot u \} + c_1 \langle \chi \rangle + \left( \frac{c_2 - \frac{g_A^2}{8m}}{2} \right) (v \cdot u)^2 \right\}$$

$$+ c_3 u \cdot u + \left( c_4 + \frac{1}{4m} \right) [S^\mu, S^\nu] u_\mu u_\nu + c_5 \tilde{\chi} = \chi - \frac{1}{2} \langle \chi \rangle, \quad f_{\mu\nu} = u^T F_{\mu\nu} u + u F_{\mu\nu} u^T. \quad (5.1)$$

Here, $\chi = 2BM$ ($\mathcal{M}$ is the quark mass matrix) and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ the canonical photon field strength tensor. The term proportional to $c_5$ vanishes in the isospin limit $m_u = m_d$. One could further reduce the number of terms by using the nucleon’s equation of motion (eom), $v \cdot D H = 0$ \cite{gap}, but that is not necessary and sometimes complicates matters. In what follows, I will always include these eom terms in the effective Lagrangian. One observes that some of the terms in eq.(5.1) have no LECs but rather fixed coefficients. The origin of this is clear, these terms stem from the expansion of the relativistic form is (I only consider external scalar and vector fields, the generalization to pseudoscalar and axial–vector ones is straightforward):

$$\mathcal{H} = \mathcal{H}(\pi N). \quad (5.2)$$

$$\text{with} \quad \chi = \chi^0 \pm u \chi^\dagger u, \quad \tilde{\chi} = \chi - \frac{1}{2} \langle \chi \rangle, \quad f_{\mu\nu} = u^T F_{\mu\nu} u + u F_{\mu\nu} u^T. \quad (5.3)$$

$$\text{to construct} \quad \mathcal{L}^{(2)}_{\pi N} = \frac{1}{2m} \mathcal{H} (i \not\partial + \frac{g_A}{2} \not\gamma_5 \frac{1 - \gamma^\dagger \gamma}{2} (i \not\partial + \frac{g_A}{2} \not\gamma_5 \gamma) \mathcal{H}. \quad (5.4)$$

Altogether, we have four different products of terms in eq.(5.4). Let me consider the one proportional to $\not\partial \not\partial$, the other contributions can be calculated in a similar fashion:

$$\frac{i^2}{2m} \mathcal{H} \not\partial \frac{1 - \gamma^\dagger \gamma}{2} \not\partial \mathcal{H} = - \frac{1}{2} \frac{1}{m} D^\mu D^\nu \mathcal{H} \left[ \gamma_\mu \frac{1 - \gamma^\dagger \gamma}{2} \gamma_\nu \right] \mathcal{H}. \quad (5.5)$$

Straightforward application of the $\gamma$–matrix algebra allows us to write the term in the square brackets on the r.h.s. as

$$\mathcal{H}[\ldots] \mathcal{H} = \mathcal{H}[g_{\mu\nu} - i \sigma_{\mu\nu} - \gamma_\mu v_\nu] \mathcal{H}, \quad (5.6)$$

where I have used that $P^+ \mathcal{H} = \mathcal{H}$. Collecting pieces, we have

$$\frac{i^2}{2m} \mathcal{H} \left\{ g_{\mu\nu} D^\mu D^\nu - 2i \epsilon_{\mu\nu\alpha\beta} D^\mu D^\nu v^\alpha S^\beta - v_\mu D^\mu v_\nu D^\nu \right\} \mathcal{H}. \quad (5.7)$$
This simplifies further since $\epsilon_{\mu\nu\alpha\beta}$ and $D^\mu D^\nu$ are anti– and symmetric under $\mu \leftrightarrow \nu$, respectively,

$$\frac{1}{2} \bar{\Psi} \left\{ -\frac{g^2}{m} \{ -D^2 + (v \cdot D)^2 + i\epsilon_{\mu\nu\alpha\beta} [D^\mu, D^\nu] v^\alpha S^\beta \} \right\} \Psi .$$

(5.8)

Finally, the commutator of two chiral covariant derivatives is related to the chiral connection $\Gamma_\mu$ via

$$[D_\mu, D_\nu] = \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu + [\Gamma_\mu, \Gamma_\nu] .$$

(5.9)

We can work this out for the explicit form of the connection given in eq. (2.10) and find

$$[D_\mu, D_\nu] = -i \frac{f^{+}_{\mu\nu}}{2} + \frac{1}{4} [u_\mu, u_\nu] .$$

(5.10)

Putting everything together, we find that the first three terms in eq.(5.1) plus the piece proportional to $g_A^2 / (8 \bar{m})$ in the fifth and the piece proportional to $1/(4 \bar{m})$ in the seventh term are generated by expanding the operator $\bar{\Psi} \Phi$. The first two contributions are corrections to the kinetic energy and contain (besides others) a two–photon–nucleon seagull which leads to the correct LET (low–energy theorem) for low–energy Compton scattering (see section 9). Similarly, the third term has no free coefficient since it gives the leading term in the quark mass expansion of the electric dipole amplitude $E_{0+}$ in neutral pion photoproduction off protons, $E_{0+} \sim M_\pi / m$. These terms have no direct relativistic counter parts but are simply due to the expansion of the relativistic pion–nucleon effective Lagrangian. Such constraints were e.g. not accounted for in eq.(17) of ref.[9]. The finite coefficients $c_{1,2,3,4}$ can be determined from the pion–nucleon $\sigma$–term, $S$– and $P$–wave $\pi N$ scattering lengths and subthreshold parameters [14] [12] [31] [32]. The last two terms in eq.(5.1) are easy to pin down, they are related to the isoscalar and isovector anomalous magnetic moment of the nucleon (in the chiral limit) [14]

$$c_6 = \kappa_V \quad c_7 = \frac{1}{2} (\kappa_S - \kappa_V) .$$

(5.11)

Furthermore, the LEC $c_5$ can be determined from the strong contribution to the neutron–proton mass difference. The resulting values are summarized in table 1 (for more details, please consult ref.[32]) together with the results based on resonance saturation as explained in the next section.

The important lesson to be learned from this discussion is that in HBCHPT we find in $\mathcal{L}_{\pi N}^{(2,3,\ldots)}$ terms which have no LECs but rather coefficients which are fixed. This is an artefact of the dual expansion in small momenta $p$ versus the chiral symmetry breaking scale and versus inverse powers of the nucleon mass, i.e.

$$\frac{p}{\Lambda_\chi} , \quad \frac{p}{m} .$$

(5.12)

In practice, since $m \sim \Lambda_\chi$, one has essentially one expansion parameter besides the one due to the effect of the finite quark masses. Since loops only appear at $D = 3$, all the LECs in $\mathcal{L}_{\pi N}^{(2)}$ are finite (as already should have become clear from the previous discussion). It appears that the numerical values of the dimension two LECs can be understood from resonance exchange as will be shown next.
Table 1: Values of the LECs $c_i$ in GeV$^{-1}$ and the dimensionless couplings $c'_i$ for $i = 1, \ldots, 5$. The LECs $c_6, 7$ are dimensionless. Also given are the central values (cv) and the ranges for the $c_i$ from resonance exchange as detailed in section 5. The * denotes an input quantity.

| $i$ | $c_i$      | $c'_i = 2m c_i$ | $c_i^{\text{Res}}$ | cv  | $c_i^{\text{Res}}$ ranges |
|-----|------------|-----------------|---------------------|-----|--------------------------|
| 1   | $-0.93 \pm 0.10$ | $-1.74 \pm 0.19$ | $-0.9^*$            |      |                          |
| 2   | $3.34 \pm 0.20$  | $6.27 \pm 0.38$  | $3.9$               | $2 \ldots 4$ |                          |
| 3   | $-5.29 \pm 0.25$ | $-9.92 \pm 0.47$ | $-5.3$              | $-4.5 \ldots -5.3$ |                          |
| 4   | $3.63 \pm 0.10$  | $6.81 \pm 0.19$  | $3.7$               | $3.1 \ldots 3.7$ |                          |
| 5   | $-0.09 \pm 0.01$ | $-0.17 \pm 0.02$ | \(\text{--}\)       | \(\text{--}\)       |                          |
| 6   | $5.83$        | \(\text{--}\)    | $6.1$               | \(\text{--}\)       |                          |
| 7   | $-2.98$       | \(\text{--}\)    | $-3.0$              | \(\text{--}\)       |                          |

6 Phenomenological interpretation of the LECs

In this section, we will be concerned with the phenomenological interpretation of the values for the LECs $c_i$. For that, guided by experience from the meson sector [33], we use resonance exchange. Let me briefly elaborate on the Goldstone boson sector. In the meson sector at next–to-leading order, the effective three flavor Lagrangian $L^{(4)}$ contains ten LECs, called $L_i$. These have been determined from data in Ref.[2]. The actual values of the $L_i$ can be understood in terms of resonance exchange [33]. For that, one constructs the most general effective Lagrangian containing besides the Goldstone bosons also resonance degrees of freedom. Integrating out these heavy degrees of freedom from the EFT, one finds that the renormalized $L_i^\prime(\mu = M_\rho)$ are practically saturated by resonance exchange ($S, P, V, A$). In some few cases, tensor mesons can play a role [34]. This is sometimes called *chiral duality* because part of the excitation spectrum of QCD reveals itself in the values of the LECs. Furthermore, whenever vector and axial resonances can contribute, the $L_i^\prime(M_\rho)$ are completely dominated by $V$ and $A$ exchange, called *chiral VMD* [35]. As an example, consider the finite (and thus scale–independent) LEC $L_9$. Its empirical value is $L_9 = (7.1 \pm 0.3) \cdot 10^{-3}$. The well–known $\rho$–meson (VMD) exchange model for the pion form factor (neglecting the width),

$$F_\pi^V(q^2) = \frac{M_\rho^2}{M_\rho^2 - q^2} = 1 + \frac{q^2}{M_\rho^2} + \ldots$$

leads to $L_9 = F_\pi^2/(2M_\rho^2) = 7.2 \cdot 10^{-3}$, by comparing with the small momentum expansion of the pion form factor, $F_\pi^V(q^2) = 1 + \langle r^2 \rangle_{\pi} q^2/6 + \mathcal{O}(q^4)$. The resonance exchange result is in good agreement with the empirical value. Even in the symmetry breaking sector related to the quark mass, where only scalar and (non-Goldstone) pseudoscalar mesons can contribute, resonance exchange helps to understand why SU(3) breaking is generally
of $O(25\%)$, except for the Goldstone boson masses. Consider now an effective Lagrangian with resonances chirally coupled to the nucleons and pions. One can generate local pion–nucleon operators of higher dimension with given LECs by letting the resonance masses become very large with fixed ratios of coupling constants to masses, symbolically

$$\tilde{\mathcal{L}}_{\text{eff}}[U, M, N, N^*] \rightarrow \mathcal{L}_{\text{eff}}[U, N] , \quad (6.2)$$

That procedure amounts to decoupling the resonance degrees of freedom from the effective field theory. However, the traces of these frozen particles are encoded in the numerical values of certain LECs. In the case at hand, we can have baryonic ($N^*$) and mesonic ($M$) excitations,

$$c_i = \sum_{N^*=\Delta,R,...} c_i^{N^*} + \sum_{M=S,V,...} c_i^M , \quad (6.3)$$

where $R$ denotes the Roper $N^*(1440)$ resonance. Consider first scalar ($S$) meson exchange. The SU(2)$_{\pi\pi}$ interaction can be written as

$$\mathcal{L}_{\pi S} = S [\bar{c}_m \text{Tr}(\chi +) + \bar{c}_d \text{Tr}(u_\mu u^\mu)] . \quad (6.4)$$

From that, one easily calculates the $s$–channel scalar meson contribution to the invariant amplitude $A(s,t,u)$ for elastic $\pi\pi$ scattering,

$$A^S(s,t,u) = \frac{4}{F_\pi^4(M^2_S - s)} [2\bar{c}_m M^2_\pi + \bar{c}_d(s - 2M^2_\pi)]^2 + \frac{16\bar{c}_m M^2_\pi}{3F_\pi^4M^2_S} [\bar{c}_m M^2_\pi + \bar{c}_d(3s - 4M^2_\pi)] . \quad (6.5)$$

Comparing with the SU(3) amplitude calculated in [36], we are able to relate the $\bar{c}_{m,d}$ to the $c_{m,d}$ of [33] (setting $M_S = M_{S_8} = M_S$ and using the large–$N_c$ relations $\bar{c}_{m,d} = c_{m,d}/\sqrt{3}$ to express the singlet couplings in terms of the octet ones), $\bar{c}_{m,d} = c_{m,d}/\sqrt{2}$, with $|c_m| = 42$ MeV and $|c_d| = 32$ MeV [33]. Assuming now that $c_1$ is entirely due to scalar exchange, we get

$$c_1^S = -\frac{g_S \bar{c}_m}{M^2_S} . \quad (6.6)$$

Here, $g_S$ is the coupling constant of the scalar–isoscalar meson to the nucleons, $\mathcal{L}_{SN} = -g_S NN S$. What this scalar–isoscalar meson is essentially doing is to mock up the strong pionic correlations coupled to nucleons. Such a phenomenon is also observed in the meson sector. The one loop description of the scalar pion form factor fails beyond energies of 400 MeV, well below the typical scale of chiral symmetry breaking, $\Lambda_\chi \simeq 1$ GeV. Higher loop effects are needed to bring the chiral expansion in agreement with the data [37]. Effectively, one can simulate these higher loop effects by introducing a scalar meson with a mass of about 600 MeV. This is exactly the line of reasoning underlying the arguments used here (for a pedagogical discussion on this topic, see [38]). It does, however, not mean that the range of applicability of the effective field theory is bounded by this mass in general. In certain channels with strong pionic correlations one simply has to work harder than in the channels where the pions interact weakly (as demonstrated in great
detail in [37]) and go beyond the one loop approximation which works well in most cases. For \( c_1 \) to be completely saturated by scalar exchange, \( c_1 \equiv c_1^S \), we need
\[
\frac{M_S}{\sqrt{g_S}} = 180 \text{ MeV} \quad .
\] (6.7)

Here we made the assumption that such a scalar has the same couplings to pseudoscalars as the real \( a_0(980) \) resonance. It is interesting to note that the effective \( \sigma \)-meson in the Bonn one–boson–exchange potential [31] with \( M_S = 550 \text{ MeV} \) and \( g_S^2/(4\pi) = 7.1 \) has \( M_S/\sqrt{g_S} = 179 \text{ MeV} \). This number is in stunning agreement with the the value demanded from scalar meson saturation of the LEC \( c_1 \). With that, the scalar meson contribution to \( c_3 \) is fixed including the sign, since \( c_m c_d > 0 \) (see ref.[33]),
\[
c_3^S = -2 \frac{g_s \bar{c}_d}{M_S^2} = 2 \frac{c_d}{c_m} c_1 = -1.40 \text{ GeV}^{-1} \quad .
\] (6.8)

The isovector \( \rho \) meson only contributes to \( c_4 \). Taking a universal \( \rho \)-hadron coupling and using the KSFR relation, we find
\[
c_4^\rho = \frac{\kappa_\rho}{4m} = 1.63 \text{ GeV}^{-1} \quad ,
\] (6.9)

using \( \kappa_\rho = 6.1 \pm 0.4 \) from the analysis of the nucleon electromagnetic form factors, the process \( \bar{N}N \to \pi\pi \) [40] [41] and the phenomenological one–boson–exchange potential for the NN interaction. I now turn to the baryon excitations. Here, the dominant one is the \( \Delta(1232) \). Using the isobar model and the SU(4) coupling constant relation (the dependence on the off–shell parameter \( Z \) has already been discussed in [5]), the \( \Delta \) contribution to the various LECs is readily evaluated,
\[
c_2^\Delta = -c_3^\Delta = 2c_4^\Delta = \frac{g_\Delta^2 (m_\Delta - m)}{2[(m_\Delta - m)^2 - M_\pi^2]} = 3.83 \text{ GeV}^{-1} \quad .
\] (6.10)

These numbers are taken as the central values of the \( \Delta \) contribution in what follows. Unfortunately, there is some sizeable uncertainty related to these. Dropping e.g. the factor \( M_\pi^2 \) in the denominator of eq.(6.10), the numerical value decreases to 2.97 GeV\(^{-1}\). Furthermore, making use of the Rarita–Schwinger formalism and varying the parameter \( Z \), one can get sizeable changes in the \( \Delta \) contributions ( e.g. \( c_2^\Delta = 1.89, c_3^\Delta = -3.03, c_4^\Delta = 1.42 \) in GeV\(^{-1}\) for \( Z = -0.3 \)). From this, we deduce the following ranges: \( c_2^\Delta = 1.9 \ldots 3.8, c_3^\Delta = -3.8 \ldots -3.0, c_4^\Delta = 1.4 \ldots 2.0 \) (in GeV\(^{-1}\)). The Roper \( N^*(1440) \) resonance contributes only marginally, see ref.[32] Putting pieces together, we have for \( c_2, c_3 \) and \( c_4 \) from resonance exchange (remember that \( c_1 \) was assumed to be saturated by scalar exchange)
\[
\begin{align*}
    c_2^{\text{Res}} &= c_2^\Delta + c_2^R = 3.83 + 0.05 = 3.88 \
    c_3^{\text{Res}} &= c_3^\Delta + c_3^S + c_3^R = -3.83 - 1.40 - 0.06 = -5.29 \
    c_4^{\text{Res}} &= c_4^\Delta + c_4^\rho + c_4^R = 1.92 + 1.63 + 0.12 = 3.67
\end{align*}
\] (6.11)
with all numbers given in units of GeV\(^{-1}\). Comparison with the empirical values listed in table 1 shows that these LECs can be understood from resonance saturation, assuming only that \(c_1\) is entirely given by scalar meson exchange. As argued before, the scalar meson parameters needed for that are in good agreement with the ones derived from fitting NN scattering data and deuteron properties within the framework of a one–boson–exchange model. We stress again that this \(\sigma\)–meson is an effective degree of freedom which parametrizes the strong \(\pi\pi\) correlations (coupled to nucleons) in the scalar–isoscalar channel. It should not be considered a novel degree of freedom which limits the applicability of the effective field theory to a lower energy scale. As pointed out before, there is some sizeable uncertainty related to the \(\Delta\) contribution as indicated by the ranges for the \(c_{\text{Res}}^i\) in table 1. It is, however, gratifying to observe that the empirical values are covered by the band based on the resonance exchange model. The LECs \(\kappa_\sigma = -0.12\) and \(\kappa_v = 5.83\) can be estimated from neutral vector meson exchange using Eq.(5). For the values from \([40]\), \(\kappa_\omega = -0.16 \pm 0.01\) and \(\kappa_\rho = 6.1 \pm 0.4\), we see that the isoscalar and isovector anomalous magnetic moments in the chiral limit can be well understood from \(\omega\) and \(\rho^0\) meson exchange. It is amusing that the isovector pion cloud of the nucleon calculated to one loop allows to explain the observed difference between \(\kappa_\rho\) and \(\kappa_v\). In strict vector meson dominance these would be equal. It is well known \([42]\) that the low energy part of the nucleon isovector spectral functions can not be understood in terms of the \(\rho\)–resonance alone (see also section 10).

7 Construction and structure of \(\mathcal{L}_{\pi N}^{(3)}\)

At the next order, \(p^3\), divergences appear. Ecker \([13]\) has first calculated the full determinant using heat–kernel methods and given the divergent terms,

\[
\mathcal{L}_{\pi N}^{(3)} = \frac{1}{(4\pi)^2} \sum_{i=1}^{22} b_i H(x) O_i(x) H(x) \tag{7.1}
\]

with

\[
b_i = b_i'(\lambda) + \Gamma_i L(\lambda) , \quad L = \mu^{d-4} \left\{ \frac{1}{16\pi^2} \left( \ln(4\pi) + \Gamma'(1) + 1 \right) \right\} . \tag{7.2}
\]

The \(O_i\) are monomials in the fields and have dimension three. Their explicit forms together with the values of the \(\Gamma_i\) can be found in ref.\([13]\). Only a few of the finite \(b_i\) have either been determined from phenomenology or estimated from resonance exchange \([23]\) \([14]\) \([31]\) \([59]\). Furthermore, Ecker and Mojžiš \([30]\) have constructed all finite terms and used field redefinitions to find a minimal set of independent operators at dimension three. I will now outline how one arrives at this list and, in addition, show how one arrives at the \(1/m\) and \(1/m^2\) corrections to the various counterterms not listed in \([30]\). Such corrections can of course be absorbed in the numerical values of the corresponding LECs, but for the reasons discussed before, I prefer to keep them explicitly. The construction of \(\mathcal{L}_{\pi N}^{(3)}\) is done in five steps, which are: (1) enumeration of the building blocks for relativistic spin–1/2 fields chirally coupled to pions and external sources, (2) saturation of the free Lorentz indices by elements of the underlying Clifford algebra and other operators of chiral dimension zero (and one), (3) construction of the overcomplete relativistic Lagrangian, (4) reduction of terms by use of various relations and (5) performing the non–relativistic limit and working out the \(1/m\) corrections by use of the path integral \([14]\). I will now outline these steps, for more details I refer to the thesis of Steininger \([44]\).
Step 1: We start with the fully relativistic theory, i.e. any covariant derivative $D_\mu$ counts as order $p^0$. Formally, one could thus construct terms of the type $\bar{\Psi} \ldots D^{38} \Psi$. To avoid this, consider the covariant derivative only as a building block when it acts on operators sandwiched between the spinors (the only exception to this rule is the lowest order Lagrangian). The pertinent terms involving $D_\mu$ acting on the nucleon fields, which are all of chiral dimension zero (or one), are given together with the elements of the Clifford algebra in step 2 since they are used to saturate the free Lorentz indices of the building blocks. In this way, one avoids from the beginning all terms with an arbitrary large number of $D_\mu$'s, which are formally allowed. With that in mind, the possible building blocks at orders $p$, $p^2$ and $p^3$ are (also given are the respective charge conjugation, $C$, and parity, $P$, assignments):

| Operator | $C$ | $P$ | Operator | $C$ | $P$ |
|----------|-----|-----|----------|-----|-----|
| $u_\mu$  | +   | −   | $[D_\mu, <\chi_+>]$ | +   | +   |
| $\mathcal{O}(p^2)$ | $<\hat{\chi}^- u_\mu>$ | + | + |
| $\hat{\chi}^+$ | + | + | $<\chi^- u_\mu>$ | + | + |
| $<\chi_+>$ | + | + | $[\chi^-, u_\mu]$ | − | + |
| $\hat{\chi}^-$ | + | − | $[D_\mu, \hat{\chi}^-]$ | + | − |
| $<\chi_->$ | + | − | $[D_\mu, <\chi_->]$ | + | − |
| $<u_\mu u_\nu>$ | + | + | $<u_\mu u_\nu u_\lambda>$ | + | − |
| $[u_\mu, u_\nu]$ | − | + | $<u_\mu [u_\nu, u_\lambda]>|$ | − | − |
| $[D_\mu, u_\nu]$ | + | − | $< [D_\mu, u_\nu] u_\lambda>$ | + | + |
| $\hat{F}^{+}_{\mu\nu}$ | − | + | $[[D_\mu, u_\nu], u_\lambda]$ | − | + |
| $<F^{+}_{\mu\nu}>$ | − | + | $[D_\mu, [D_\nu, u_\lambda]]$ | + | − |
| $\mathcal{O}(p^3)$ | $<\hat{\mathcal{F}}^{+}_{\mu\nu} u_\lambda>$ | − | − |
| $<\hat{\chi}^+ u_\mu>$ | + | − | $<F^{+}_{\mu\nu}> u_\lambda$ | − | − |
| $<\chi_+> u_\mu$ | + | − | $[\hat{F}^{+}_{\mu\nu}, u_\lambda]$ | + | + |
| $[\chi_+, u_\mu]$ | − | − | $[D_\lambda, \hat{F}^{+}_{\mu\nu}]$ | − | + |
| $[D_\mu, \hat{\chi}^+]$ | + | + | $[D_\lambda, <F^{+}_{\mu\nu}>]$ | − | + |

Here, the definition $\hat{A} = A - \langle A \rangle / 2$ is used. Of course, hermiticity has to be assured by appropriate factors of $i$ and combinations of terms. Note that the terms of orders $p$ and $p^2$ are also needed since they can come in with appropriate powers of $1/m$.

Step 2: Now we have to construct the elements of the Clifford algebra and all other operators of dimension zero/one to contract the Lorentz indices of the building blocks. First,
I give all operators constructed from $\gamma$ matrices, the metric tensor and the totally antisymmetric tensor in $d = 4$ according to the number of free indices, called $N_I$:

\begin{align}
N_0 & : 1, \gamma_5 ; \\
N_1 & : \gamma_\mu, \gamma_\mu \gamma_5 ; \\
N_2 & : g_{\mu\nu}, g_{\mu\nu} \gamma_5, \sigma_{\mu\nu}, \sigma_{\mu\nu} \gamma_5 ; \\
N_3 & : g_{\mu\nu}\gamma_\lambda, g_{\mu\nu} \gamma_\lambda \gamma_5, \epsilon_{\mu\nu\lambda\gamma}\gamma_5, \epsilon_{\mu\nu\lambda\gamma}\gamma_5 \gamma_5 .
\end{align}  \tag{7.4}

Similarly, the terms involving the covariant derivative read

\begin{align}
N_1 & : D_\mu ; \\
N_2 & : \{D_\mu, D_\nu\} ; \\
N_3 & : D_\mu D_\nu D_\lambda + \text{permutations} .
\end{align}  \tag{7.5}

It is important to note that $\gamma_5$ and $g_{\mu\nu} \gamma_5$ have chiral dimension one because they only connect the large to the small components and thus appear first at order $1/m$ (as discussed before). Terms with more $D_\mu$’s can always be reduced to the terms listed in eqs.(7.4,7.5) or only contribute to higher orders by use of the baryon eom,

$$\not{D} \Psi = \left(m + \frac{g_A}{2} \gamma_5 \not{u}\right) \Psi + \mathcal{O}(p^2) .$$ \tag{7.6}

Let me give one example. Be $O$ some operator of dimension three constructed from the above building blocks and properly contracted to be a scalar with $PC = ++$ contributing as $\bar{\Psi} O \Psi$ to the effective Lagrangian. So what happens to a term like $\bar{\Psi} O D^2 \Psi + h.c.$ which is also of dimension three and commensurate with all symmetries? It contributes only to higher orders as can be seen from the following chain of manipulations:

\begin{align}
\bar{\Psi} O D^2 \Psi &= \bar{\Psi} O g_{\mu\nu} D^\mu D^\nu \Psi \\
&= \bar{\Psi} O \not{D} \not{D} \Psi - i \bar{\Psi} O \sigma^{\mu\nu} [D_\mu, D_\nu] \Psi \\
&= \bar{\Psi} O \not{D} \not{D} \Psi - i \frac{3}{2} \bar{\Psi} O \sigma^{\mu\nu} [D_\mu, D_\nu] \Psi .
\end{align} \tag{7.7}

The first term on the r.h.s. of eq.(7.7) amounts to a repeated use of the nucleon eom, i.e. to three terms with coefficients $m^2$, $m g_A$ and $g_A^2$, in order. While the first of these can simply be absorbed in the LEC accompanying the original term $\bar{\Psi} O \Psi$, the other two start to contribute at order four and five, respectively. Similarly, the second term on the r.h.s. of eq.(7.7) leads to a dimension five operator by use of eq.(5.10).

Step 3: It is now straightforward to combine the building blocks with the operators enumerated in step 2 to construct the effective Lagrangian. One just has to multiply the various operators so that the result has $J^{PC} = 0^{++}$. This gives

$$L_{\pi N}^{(3)} = \sum_{i=1}^{51} \bar{\Psi} O_i^{(3)} \Psi \tag{7.8}$$

where the $O_i^{(3)}$ are monomials in the fields of chiral dimension three. The full list of terms is given in [44] and I have made explicit only three terms for the later discussion. At this stage, the Lagrangian is over–complete. All terms are allowed by the symmetries, but they are not all independent. While one could work with such an over–complete set of terms, it is more economical to reduce them to the minimal number of independent ones.
Step 4: Let us now show how one can reduce the number of terms in $\mathcal{L}^{(3)}_{\pi N}$. To be specific, consider the third term on the r.h.s. of eq. (7.8). Performing partial integrations, we can reshuffle the covariant derivative to the extreme left and and right,

$$\bar{\Psi} \gamma_5 \gamma_\mu [D_\mu, \chi_-] \Psi = -\bar{\Psi} \hat{D}_\mu \gamma_5 \gamma_\mu \chi_- \Psi - \bar{\Psi} \gamma_5 \gamma_\mu \chi_- D_\mu \Psi.$$  \hspace{1cm} (7.9)

Using now the eom, eq. (7.6), and the one for $\bar{\Psi}$, this can be cast into the form

$$\bar{\Psi} \gamma_5 \gamma_\mu [D_\mu, \chi_-] \Psi = -2m \bar{\Psi} \gamma_5 \chi_- \Psi - g_A \bar{\Psi} [\gamma_5, \chi_-] \Psi + \mathcal{O}(p^4)$$  \hspace{1cm} (7.10)

and these are terms which already exist at dimension three, i.e. the term under consideration can be absorbed in the structure of these two terms. By similar methods, one can reduce the 51 terms down to 24 independent ones.

Step 5: As a final step, we now use the rules derived before, see eq. (2.24), to go to the non–relativistic limit. For example, the first two terms given in eq. (7.8) take the

$$H S^\nu \langle \chi^+ u_\mu \rangle H + H S \cdot u \langle u^2 \rangle H.$$  \hspace{1cm} (7.11)

In a similar fashion, all other terms can be reduced and the $1/m$ corrections can be worked out along the lines spelled out in appendix $A$ of ref. [14]. Let me discuss as an example the part of the third order action which has the form [14]

$$S'_{\pi N} = -\frac{1}{(2m)^2} \int d^4 x \bar{H} (\gamma_0 B^{(1)} \gamma_0)(iv \cdot D + g_A S \cdot u) B^{(1)} H,$$  \hspace{1cm} (7.12)

which translates into the following piece of the third order effective Lagrangian,

$$\mathcal{L}^{(3)'}_{\pi N} = -\frac{1}{4m^2} \bar{H} \left[ (iv \cdot D)^3 + D_\mu v \cdot DD^\mu - 2[S_\mu, S_\nu] D^\mu v \cdot DD^\nu \right] H.$$  \hspace{1cm} (7.13)

One has now to express the various products of derivatives, $\gamma$ matrices and so on in terms of non–relativistic four–vectors like e.g. for the product of the first terms in the three square brackets,

$$\bar{H} \not{D} \not{v} D \not{D} H = \bar{H} ((v \cdot D)^3 + D_\mu v \cdot DD^\mu - 2[S_\mu, S_\nu] D^\mu v \cdot DD^\nu) H.$$  \hspace{1cm} (7.14)

so that

$$\mathcal{L}^{(3)'}_{\pi N} = -\frac{i}{4m^2} \bar{H} \left[ (v \cdot D)^3 - D_\mu v \cdot DD^\mu + 2[S_\mu, S_\nu] D^\mu v \cdot DD^\nu + \ldots \right] H.$$  \hspace{1cm} (7.15)

With 8 other relations of this type one can fill in the ellipsis. Similarly, the other pieces contributing at this order can be worked out. Putting pieces togethers, the complete third order Lagrangian takes the form

$$\mathcal{L}^{(3)}_{\pi N} = \mathcal{L}^{(3), \text{fixed}}_{\pi N} + \sum_{i=1}^{24} b_i \bar{H} \hat{O}_i H,$$  \hspace{1cm} (7.16)

where the first term in Eq. (7.16) subsumes all terms with fixed coefficients of lower chiral dimension, i.e. the ones of the types $g_A/m^2$, $1/m^2$ and $c_i/m$, respectively. The $b_i$ are the third order LECs. The complete Lagrangian in this basis can be found in [14] and is already given in a different basis in [30].

For $\mathcal{L}^{(4)}_{\pi N}$, no systematic study enumerating all terms exists so far but some dimension four operators have been used and their corresponding LECs fixed $[19]$ $[38]$. Furthermore, the renormalization of the generating functional to order $p^4$ has been worked out by Müller [15] and will be available soon [33].

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8 Isospin symmetry and virtual photons

Up to now, I have mostly treated pure QCD in the isospin limit. We know, however, that there are essentially two sources of isospin symmetry violation (ISV). First, the quark mass difference \( m_d - m_u \) leads to strong ISV. Second, switching on the electromagnetic (em) interaction, charged particles are surrounded by a photon cloud making them heavier than their neutral partners. An extreme case is the pion, where the strong ISV is suppressed (see below) and the photon cloud is almost entirely responsible for the charged to neutral mass difference. Matters are different for the nucleon. Here, pure em would suggest the proton to be heavier than the neutron by 0.8 MeV - at variance with the data. However, the quark mass (strong) contribution can be estimated to be \((m_n - m_p)^{\text{str}} = 2.1\) MeV. Combining these two numbers, one arrives at the empirical value of \(m_n - m_p = 1.3\) MeV.

Consider first the strong interactions. The symmetry breaking part of the QCD Hamiltonian, i.e. the quark mass term, can be decomposed into an isoscalar and an isovector term

\[
H_{\text{QCD}}^{sb} = m_u \bar{u}u + m_d \bar{d}d = \frac{1}{2}(m_u + m_d)(\bar{u}u + \bar{d}d) + \frac{1}{2}(m_u - m_d)(\bar{u}u - \bar{d}d) .
\]

The quark mass ratios can easily be deduced from the ratios of the (unmixed) Goldstone bosons, in particular, \(m_d/m_u = 1.8 \pm 0.2\) \[17\]. Therefore, \((m_d - m_u)/(m_d + m_u) \simeq 0.3\) and one could expect large isospin violating effects. However, the light quark masses are only about 5...10 MeV (at a renormalization scale of 1 GeV) and the relevant scale to compare to is of the order of the proton mass. This effectively suppresses the effect of the sizeable light quark mass difference in most cases, as will be discussed below. We notice that in the corresponding meson EFT, Eq.(2.8), the isoscalar term appears at leading order while the isovector one is suppressed. This is essentially the reason for the tiny quark mass contribution to the pion mass splitting. On the other hand, in the pion-nucleon system, no symmetry breaking appears at lowest order but at next-to-leading order, the isoscalar and the isovector terms contribute. These are exactly the terms \(\sim c_1\) and \(\sim c_5\) in Eq.(5.1).

In his seminal paper in 1977, Weinberg pointed out that reactions involving nucleons and neutral pions might lead to gross violations of isospin symmetry \[18\] since the leading terms from the dimension one Lagrangian are suppressed. In particular, he argued that the mass difference of the up and down quarks can produce a 30% effect in the difference of the \(\pi^0p\) and \(\pi^0n\) S-wave scattering lengths while these would be equal in case of isospin conservation. This was later reformulated in more modern terminology \[19\]. To arrive at the abovementioned result, Weinberg considered Born terms and the dimension two symmetry breakers. However, as shown in ref. \[31\], at this order there are other isospin-conserving terms which make a precise prediction for the individual \(\pi^0p\) or \(\pi^0n\) scattering length very difficult. Furthermore, there is no way of directly measuring these processes. On the other hand, there exists a huge body of data for elastic charged pion–nucleon scattering \((\pi^\pm p \rightarrow \pi^\pm p)\) and charge exchange reactions \((\pi^- p \rightarrow \pi^0 n)\). In the framework of some models it has been claimed that the presently available pion–nucleon data basis exhibits isospin violation of the order of a few percent \[50\] \[51\]. What is, however, uncertain is to what extent the methods used to separate the electromagnetic from the strong ISV effects match. To really pin down isospin breaking due to the light quark mass difference,
one needs a machinery that allows to simultaneously treat the electromagnetic and the strong contributions. Here, CHPT comes into the game since one can extend the effective Lagrangian to include virtual photons as I will show now.

Consider now the photons as dynamical degrees of freedom. To do this in a systematic fashion, one has to extend the power counting. A very natural way to do this is to assign to the electric charge a chiral dimension one, based on the observation that

$$\frac{e^2}{4\pi} \sim \frac{M_\pi^2}{(4\pi F_\pi)^2} \sim \frac{1}{100} \quad .$$

This is also a matter of consistency. While the neutral pion mass is unaffected by the virtual photons, the charged pions acquire a mass shift of order $e^2$ from the photon cloud. If one would assign the electric charge a chiral dimension zero, then the power counting would be messed up. This is discussed in more detail in Gasser’s lectures [52]. The extension of the meson Lagrangian is standard, I only give the result here and refer to refs. [33] [53] [54] for all the details. Also, I consider only the two flavor case,

$$\mathcal{L}^{(2)}_{\pi\pi} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{2} (\partial_\mu A_\mu)^2 + \frac{F_\pi^2}{4} (\nabla_\mu U \nabla^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U) + C \langle QUQU^\dagger \rangle \quad ,$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ the photon field strength tensor and $\lambda$ the gauge–fixing parameter (from here on, I use the Lorentz gauge $\lambda = 1$). Also,

$$\nabla_\mu U = \partial_\mu U - i (v_\mu + a_\mu + QA_\mu) U + i U (v_\mu - a_\mu + QA_\mu) \quad ,$$

is the generalized pion covariant derivative containing the external vector ($v_\mu$) and axial–vector ($a_\mu$) sources. It is important to stress that in refs. [33] [53] [54], $Q$ denotes the quark charge matrix. To make use of the nucleon charge matrix commonly used in the pion–nucleon EFT, one can perform a transformation of the type $Q \rightarrow Q + \alpha e 1$, with $\alpha$ a real parameter. One observes that $d \langle QUQU^\dagger \rangle / d\alpha \sim e^2 1$, i.e. to this order the difference between the two charge matrices can completely be absorbed in an unobservable constant term. To use the higher order terms constructed in [55], one would have to rewrite them in terms of the nucleon charge matrix. In what follows, I will not need these terms. It is advantageous to work in the $\sigma$–model gauge for the pions, Eq.(2.5). In that case, the last term in $\mathcal{L}^{(2)}_{\pi\pi}$ leads only to a term quadratic in pion fields. Consequently, the LEC $C$ can be calculated from the neutral to charged pion mass difference since this term leads to $(\delta M_\pi^2)_{\text{em}} = 2e^2 C / F_\pi^2$. This gives $C = 5.9 \cdot 10^{-5}$ GeV$^4$. Extending this unique lowest order term to SU(3), one can easily derive Dashen’s theorem. To introduce virtual photons in the effective pion–nucleon field theory, consider the nucleon charge matrix $Q = e \text{diag}(1,0)$. Note that different to what was done before, the explicit factor of $e$ is subsumed in $Q$ so as to organize the power counting along the lines discussed before. For the construction of chiral invariant operators, let me introduce the matrices

$$Q_\pm = \frac{1}{2} (u Q u^\dagger \pm u^\dagger Q u) \quad , \quad \hat{Q}_\pm = Q_\pm - \frac{1}{2} \langle Q_\pm \rangle \quad .$$

(8.5)
By construction, the $\hat{Q}_\pm$ are traceless. Under chiral SU(2)$_L \times$SU(2)$_R$ symmetry, the $Q_\pm$ transform as

$$Q_\pm \rightarrow K Q_\pm K^\dagger.$$  \hspace{1cm} (8.6)

Furthermore, under parity ($P$) and charge conjugation ($C$) transformations, one finds

$$P Q_\pm P^{-1} = \pm Q_\pm , \quad C Q_\pm C^{-1} = \pm Q_\pm^T ,$$  \hspace{1cm} (8.7)

where $Q^T$ is the transposed of the matrix $Q$. For physical processes, only quadratic combinations of the charge matrix $Q$ (or, equivalently, of the matrices $Q_\pm$) can appear. The following relations are of use,

$$\langle Q_- \rangle = \langle Q_- Q_+ \rangle = 0 , \quad \langle [iD_\mu, Q_\pm] \rangle = 0 ,$$  \hspace{1cm} (8.8)

together with the SU(2) matrix relation $\{A, B\} = \langle AB \rangle + A \langle B \rangle + B \langle A \rangle - \langle A \rangle \langle B \rangle$. It is now straightforward to implement the (virtual) photons given in terms of the gauge field $A_\mu$ in the effective pion–nucleon Lagrangian. Starting from the relativistic theory and decomposing the spinor fields into light (denoted $N$) and heavy components (velocity eigenstates), one can proceed as discussed before. In particular, to lowest order (chiral dimension one)

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N} \left( i v \cdot \tilde{D} + g_A S \cdot \tilde{u} \right) N ,$$  \hspace{1cm} (8.9)

with

$$\tilde{D}_\mu = D_\mu - i Q_+ A_\mu , \quad \tilde{u}_\mu = u_\mu - 2 Q_- A_\mu .$$  \hspace{1cm} (8.10)

At next order (chiral dimension two), we have

$$\mathcal{L}_{\pi N, em}^{(2)} = \bar{N} F_\pi^2 \left\{ f_1 \langle Q_+^2 - Q_-^2 \rangle + f_2 \hat{Q}_+ \langle Q_+ \rangle + f_3 \langle Q_+^2 + Q_-^2 \rangle + f_4 \langle Q_+ \rangle^2 \right\} N .$$  \hspace{1cm} (8.11)

For the electromagnetic terms, we have written down the minimal number allowed by all symmetries. Note that the last two terms in $\mathcal{L}_{\pi N, em}^{(2)}$ are proportional to $e^2 \bar{N} N$. This means that they only contribute to the electromagnetic nucleon mass in the chiral limit and are thus not directly observable. However, this implies that in the chiral two–flavor limit ($m_u = m_d = 0$, $m_s$ fixed), the proton is heavier than the neutron since it acquires an electromagnetic mass shift. Only in pure QCD ($e^2 = 0$), this chiral limit mass is the same for the two particles. Since at present only calculations to order $q^3$ have been performed, one can always use the physical nucleon mass, denoted $m$, and do not need to bother about its precise value in the chiral limit. The numerical values of the electromagnetic LECs $f_i$ and $\hat{f}_i$ will be discussed below. The normalization factor of $F_\pi^2$ in the electromagnetic pion–nucleon Lagrangian is introduced so that the $f_i$ have the same dimension as the strong LECs $c_i$.

To go beyond tree level, we have to construct the terms of order $p^3$. From the building blocks at our disposal, we can construct 17 independent terms which are compatible with all symmetries, following ref. [56]. The number of possible terms is limited due to the fact that at least two charge matrices must appear. In particular, charge conjugation allows to sort out terms which would otherwise be allowed. In table 2, I write down a
Table 2: Construction of the dimension three operators including virtual photons. Each of these operators represents a class of operators with the same behavior under parity and charge conjugation.

A typical set of operators, starting from the relativistic theory and then transforming into the heavy baryon formalism. The Dirac matrix $\Gamma_\mu$ has to be chosen such that a) the indices get contracted and b) the corresponding operator has $PC = ++$. In some case, denoted by 'no', it is not possible to achieve this with either $\gamma_\mu$, $\gamma_5$ or $\gamma_\mu\gamma_5$ (these are the only Dirac matrices which can be used to construct a dimension three term fulfilling the aforementioned requirements). Some of these terms are accompanied by finite LECs whereas the others absorb the divergences appearing at one loop with LECs that are only finite after renormalization. The renormalization procedure to render these finite is spelled out in detail in ref. [57]. The electromagnetic part of the dimension three Lagrangian takes the form (after combining all finite terms with the ones obtained after renormalization)

$$\mathcal{L}^{(3)}_{\pi N, \text{em}} = \sum_{i=1}^{17} g_i \bar{N} \mathcal{O}_i N,$$

(8.12)

with the $\mathcal{O}_i$ monomials in the fields of dimension three. The low–energy constants $g_i$ absorb the divergences in the standard manner,

$$g_i = \kappa_i L + g_i^r(\mu),$$

(8.13)

with $\mu$ the scale of dimensional regularization. The explicit expressions for the operators $\mathcal{O}_i$ and their $\beta$–functions $\kappa_i$ are collected in table 3. The $g_i^r(\mu)$ are the renormalized, finite and scale–dependent low–energy constants. These can be fixed by data or have to be estimated with the help of some model. They obey the standard renormalization group equation,

$$g_i^r(\mu_1) = g_i^r(\mu_2) + \kappa_i \log \frac{\mu_2}{\mu_1},$$

(8.14)
Consider now the neutron-proton mass difference as an instructive example. It is given by a strong insertion $\sim c_5$ and an electromagnetic insertion $\sim f_2$,

$$m_n - m_p = (m_n - m_p)_{\text{str}} + (m_n - m_p)_{\text{em}} = 4 c_5 B (m_u - m_d) + 2 e^2 F_\pi^2 f_2 + O(p^4). \quad (8.15)$$

Note that one-loop corrections only start at order $p^4$. This can be traced back to the fact that the photonic self-energy diagram of the proton (on-shell) at order $p^3$ vanishes since it is proportional to $\int d^4k [k^2 v \cdot k]^{-1}$. Such an integral vanishes in dimensional regularization. At chiral dimension three, the electromagnetic LEC $f_2$ can therefore be fixed from the electromagnetic proton mass shift, $(m_n - m_p)_{\text{em}} = -(0.7 \pm 0.3)\text{MeV}$, i.e.
\[ f_2 = -(0.45 \pm 0.19) \text{GeV}^{-1}. \] The strong contribution has been used in [32] to fix the LEC \( c_5 = -0.09 \pm 0.01 \text{GeV}^{-1}. \) Note that it is known that one–loop graphs with an insertion \( \sim m_d - m_u \) on the internal nucleon line, which in our counting appear at fourth order, can contribute sizeably to the strong neutron-proton mass difference [58]. This underlines the need for a complete \( \mathcal{O}(p^4) \) calculation.

9 The meaning of low–energy theorems (LETs)

In this section, I will briefly discuss the meaning of the so–called low–energy theorems. More details are given in ref.[60]. Let us first consider a well–known example of a LET involving the electromagnetic current. Consider the scattering of very soft photons on the proton, i.e., the Compton scattering process \( \gamma(k_1) + p(p_1) \rightarrow \gamma(k_2) + p(p_2) \) and denote by \( \varepsilon(\varepsilon') \) the polarization vector of the incoming (outgoing) photon. The transition matrix element \( T \) (normalized to \( d\sigma/d\Omega = |T|^2 \)) can be expanded in a Taylor series in the small parameter \( \delta = |\vec{k}_1|/m \). In the forward direction and in a gauge where the polarization vectors have only space components, \( T \) takes the form

\[
T = c_0 \varepsilon' \cdot \bar{\varepsilon} + i c_1 \delta \bar{\sigma} \cdot (\varepsilon' \times \bar{\varepsilon}) + \mathcal{O}(\delta^2). \tag{9.1}
\]

The parameter \( \delta \) can be made arbitrarily small in the laboratory so that the first two terms in the Taylor expansion (9.1) dominate. To be precise, the first one proportional to \( c_0 \) gives the low–energy limit for the spin–averaged Compton amplitude, while the second \( \sim c_1 \) is of pure spin–flip type and can directly be detected in polarized photon proton scattering (to my knowledge, such a test has not yet been performed). The pertinent \( \text{LETs} \) fix the values of \( c_0 \) and \( c_1 \) in terms of measurable quantities [61],

\[
c_0 = -\frac{Z^2 e^2}{4\pi m}, \quad c_1 = -\frac{Z^2 e^2 \kappa_p^2}{8\pi m} \tag{9.2}
\]

with \( Z = 1 \) the charge of the proton. To arrive at Eq. (9.2), one only makes use of gauge invariance and the fact that the \( T \)–matrix can be written in terms of a time–ordered product of two conserved vector currents sandwiched between proton states. The derivation proceeds by showing that for small enough photon energies the matrix element is determined by the electromagnetic form factor of the proton at \( q^2 = 0 \) [61].

Similar methods can be applied to other than the electromagnetic currents. In strong interaction physics, a special role is played by the axial–vector currents. The associated symmetries are spontaneously broken giving rise to the Goldstone matrix elements

\[
\langle 0 | A_{\mu}^a(0) | \pi^b(p) \rangle = i\delta^{ab} F_\pi p_\mu \tag{9.3}
\]

where \( a, b \) are isospin indices. In the chiral limit, the massless pions play a similar role as the photon and many \( \text{LETs} \) have been derived for “soft pions”. In light of the previous discussion on Compton scattering, the most obvious one is Weinberg’s prediction for elastic \( \pi p \) scattering [32]. We only need the following translations :

\[
\langle p | T \sum_{\mu} J_{\mu}^{em}(x) J_{\mu}^{em}(0) | p \rangle \rightarrow \langle p | T A_{\mu}^{+}(x) A_{\mu}^{-}(0) | p \rangle , \tag{9.4}
\]

25
\[ \partial \mu J^{\text{em}}_{\mu} = 0 \rightarrow \partial \mu A^\pi_\mu = 0 . \] (9.5)

In contrast to photons, pions are not massless in the real world. It is therefore interesting to find out how the LETs for soft pions are modified in the presence of non–zero pion masses (due to non–vanishing quark masses). In the old days of current algebra, a lot of emphasis was put on the PCAC (Partial Conservation of the Axial–Vector Current) relation, consistent with the Goldstone matrix element (9.3),

\[ \partial \mu A^a_\mu = M^2_{\pi} F_\pi \pi^a . \] (9.6)

Although the precise meaning of (9.6) has long been understood \[6\], it does not offer a systematic method to calculate higher orders in the momentum and mass expansion of LETs. The derivation of non–leading terms in the days of current algebra and PCAC was more an art than a science, often involving dangerous procedures like off–shell extrapolations of amplitudes. In the modern language, i.e. the EFT of the Standard Model, these higher order corrections can be calculated unambiguously and one correspondingly defines a low–energy theorem via:

\[ \mathbf{L(OW)} \mathbf{E(NERGY)} \mathbf{T(HEOREM)} \mathbf{OF} \mathbf{O(}\bar{p}^n\mathbf{)} \]

\[ \equiv \text{GENERAL PREDICTION OF CHPT TO } O(p''^n) . \] (9.7)

By general prediction I mean a strict consequence of the SM depending on some LECs like \( F_\pi, m, g_A, \kappa_\rho, \ldots \), but without any model assumption for these parameters. This definition contains a precise prescription how to obtain higher–order corrections to leading–order LETs.

The soft–photon theorems, e.g., for Compton scattering \[61\], involve the limit of small photon momenta, with all other momenta remaining fixed. Therefore, they hold to all orders in the non–photonic momenta and masses. In the low–energy expansion of CHPT, on the other hand, the ratios of all small momenta and pseudoscalar meson masses are held fixed. Of course, the soft–photon theorems are also valid in CHPT as in any gauge invariant quantum field theory. To understand this difference of low–energy limits, I will now rederive and extend the LET for spin–averaged nucleon Compton scattering in the framework of HBCHPT \[14\]. Consider the spin–averaged Compton amplitude in forward direction (in the Coulomb gauge \( \varepsilon \cdot v = 0 \))

\[ e^2 \varepsilon^\mu \varepsilon'^\nu \frac{1}{4} \text{Tr} \left[ (1 + \gamma_\lambda v^\lambda) T_{\mu\nu}(v, k) \right] = e^2 \left[ \varepsilon^2 U(\omega) + (\varepsilon \cdot k)^2 V(\omega) \right] \] (9.8)

with \( \omega = v \cdot k \) (\( k \) is the photon momentum) and

\[ T_{\mu\nu}(v, k) = \int d^4k \ e^{ikx} \left( N(v) | j^{\text{em}}_{\mu}(x) j^{\text{em}}_\nu(0) | N(v) \right) . \] (9.9)

All dynamical information is contained in the functions \( U(\omega) \) and \( V(\omega) \). We only consider \( U(\omega) \) here and refer to Ref. \[14\] for the calculation of both \( U(\omega) \) and \( V(\omega) \). In the Thomson limit, only \( U(0) \) contributes to the amplitude. In the forward direction, the only quantities with non–zero chiral dimension are \( \omega \) and \( M_\pi \). In order to make this
dependence explicit, we write \( U(\omega, M_\pi) \) instead of \( U(\omega) \). With \( N_\gamma = 2 \) external photons, the degree of homogeneity \( D_L \) for a given CHPT contribution to \( U(\omega, M_\pi) \) follows from Eq. (3.6):

\[
D_L = 2L - 1 + \sum_d (d - 2) N_d^M + \sum_d (d - 1) N_d^{MB} \geq -1 .
\]  

(9.10)

Therefore, the chiral expansion of \( U(\omega, M_\pi) \) takes the following general form:

\[
U(\omega, M_\pi) = \sum_{D_L \geq -1} \omega^{D_L} f_{D_L}(\omega/M_\pi) .
\]  

(9.11)

The following arguments illuminate the difference and the interplay between the soft–photon limit and the low–energy expansion of CHPT. Let us consider first the leading terms in the chiral expansion (9.11):

\[
U(\omega, M_\pi) = \frac{1}{\omega} f_{-1}(\omega/M_\pi) + f_0(\omega/M_\pi) + \mathcal{O}(p) .
\]  

(9.12)

Eq. (9.10) tells us that only tree diagrams can contribute to the first two terms. However, the relevant tree diagrams do not contain pion lines. Consequently, the functions \( f_{-1}, f_0 \) cannot depend on \( M_\pi \) and are therefore constants. Since the soft–photon theorem \([61]\) requires \( U(0, M_\pi) \) to be finite, \( f_{-1} \) must actually vanish and the chiral expansion of \( U(\omega, M_\pi) \) can be written as

\[
U(\omega, M_\pi) = f_0 + \sum_{D_L \geq 1} \omega^{D_L} f_{D_L}(\omega/M_\pi) .
\]  

(9.13)

But the soft–photon theorem yields additional information: since the Compton amplitude is independent of \( M_\pi \) in the Thomson limit and since there is no term linear in \( \omega \) in the spin–averaged amplitude, we find

\[
\lim_{\omega \to 0} \omega^{n-1} f_n(\omega/M_\pi) = 0 \quad (n \geq 1)
\]  

(9.14)

implying in particular that the constant \( f_0 \) describes the Thomson limit:

\[
U(0, M_\pi) = f_0 .
\]  

(9.15)

Let me now verify these results by explicit calculation. In the Coulomb gauge, there is no direct photon–nucleon coupling from the lowest–order effective Lagrangian \( \mathcal{L}_{\pi N}^{(1)} \) since it is proportional to \( \varepsilon \cdot v \). Consequently, the corresponding Born diagrams vanish so that indeed \( f_{-1} = 0 \). On the other hand, I had argued in section 5 that the heavy mass expansion of the relativistic \( \pi N \) Dirac Lagrangian leads to a Feynman insertion of the form (from the first two terms in eq.(5.1)):

\[
\frac{ie^2}{m} \frac{1}{2} (1 + \tau_3) \left[ \varepsilon^2 - (\varepsilon \cdot v)^2 \right] = \frac{ie^2 Z^2}{m} \varepsilon^2
\]  

(9.16)

producing the desired result \( f_0 = Z^2/m \), the Thomson limit.
At the next order in the chiral expansion, \( \mathcal{O}(p^3) \) \((D_L = 1)\), the function \( f_1(\omega/M_\pi) \) is given by the finite sum of 9 one–loop diagrams \[63\] \[14\]. According to Eq. (9.14), \( f_1 \) vanishes for \( \omega \to 0 \). The term linear in \( \omega/M_\pi \) yields the leading contribution to the sum of the electric and magnetic polarizabilities of the nucleon, defined by the second–order Taylor coefficient in the expansion of \( U(\omega, M_\pi) \) in \( \omega \):

\[
f_1(\omega/M_\pi) = -\frac{11 g_A^2 \omega}{192\pi F_\pi^2 M_\pi} + \mathcal{O}(\omega^2) .
\]

(9.17)

The \( 1/M_\pi \) behaviour should not come as a surprise – in the chiral limit the pion cloud becomes long–ranged (instead of being Yukawa–suppressed) so that the polarizabilities explode. This behaviour is specific to the leading contribution of \( \mathcal{O}(p^3) \). In fact, from the general form (9.13) one immediately derives that the contribution of \( \mathcal{O}(p^n) \) \((D_L = n-2)\) to the polarizabilities is of the form \( c_n M_\pi^{n-4} \) \((n \geq 3)\), where \( c_n \) is a constant that may be zero. One can perform a similar analysis for the amplitude \( V(\omega) \) and for the spin–flip amplitude. We do not discuss these amplitudes here but refer the reader to Ref. \[14\] for details.

Next, let us consider the processes \( \gamma N \to \pi^0 N \) \((N = p, n)\) at threshold, i.e., for vanishing three–momentum of the pion in the nucleon rest frame. At threshold, only the electric dipole amplitude \( E_{0+} \) survives and the only quantity with non–zero chiral dimension is \( M_\pi \). In the usual conventions, \( E_{0+} \) has physical dimension \( -1 \) and it can therefore be written as

\[
E_{0+} = \frac{e g_A}{F} A \left( \frac{M_\pi^2}{m}, \frac{M_\pi}{F} \right) .
\]

(9.18)

The dimensionless amplitude \( A \) will be expressed as a power series in \( M_\pi \). The various parts are characterized by the degree of homogeneity \((in M_\pi) \) \( D_L \) according to the chiral expansion. Since \( N_\gamma = 1 \) in the present case, we obtain from Eq. (3.6)

\[
D_L = D - 1 = 2L + \sum_d (d-2)N_d^M + \sum_d (d-1)N_d^{MB} .
\]

(9.19)

For the LET of \( \mathcal{O}(p^3) \) in question, only lowest–order mesonic vertices \((d = 2)\) will appear. Therefore, in this case the general formula for \( D_L \) takes the simpler form

\[
D_L = 2L + \sum_d (d-1)N_d^{MB} .
\]

(9.20)

I will not discuss the chiral expansion of \( E_{0+} \) step by step, referring to the literature \[64\] \[65\] \[14\] for the actual calculation and for more details to the Comment \[60\]. Up–to–and–including order \( \mu^2 \), one has to consider contributions with \( D_L = 0, 1 \) and 2. In fact, for neutral pion photoproduction, there is no term with \( D_L = 0 \) since the time–honored Kroll–Ruderman contact term \[66\] where both the pion and the photon emanate from the same vertex, only exists for charged pions. For \( D_L = 2 \) there is a one–loop contribution \((L = 1)\) with leading–order vertices only \((N_d^{MB} = 0 \) \((d > 1)\)). It is considerably easier to work out the relevant diagrams in HBCHPT \[14\] than in the original derivation \[64\] \[65\]. In fact, at threshold only the so–called triangle diagram (and its crossed partner) survive out of some 60 diagrams. The main reason for the enormous simplification in HBCHPT
is that one can choose a gauge without a direct $\gamma NN$ coupling of lowest order and that there is no direct coupling of the produced $\pi^0$ to the nucleon at threshold. Notice that the loop contributions are finite and they are identical for proton and neutron. They were omitted in the original version of the LET and in many later derivations. The full LETs of $O(p^3)$ are given by

$$E_{0+}(\pi^0 p) = -\frac{eg_A}{8\pi F_\pi} \left[ \frac{M_\pi}{m} - \frac{M_\pi^2}{2m^2} (3 + \kappa_p) - \frac{M_\pi^2}{16F_\pi^2} + O(M_\pi^3) \right]$$  \hspace{1cm} (9.21)

$$E_{0+}(\pi^0 n) = -\frac{eg_A}{8\pi F_\pi} \left[ \frac{M_\pi^2}{2m^2} \kappa_n - \frac{M_\pi^2}{16F_\pi^2} + O(M_\pi^3) \right]$$  \hspace{1cm} (9.22)

We note that the electric dipole amplitude for neutral pion production vanishes in the chiral limit. By now, even the terms of order $M_\pi^3$ have been worked out, see ref.[59]. The derivation of LETs sketched above is based on a well–defined quantum field theory where each step can be checked explicitly. Nevertheless, the corrected LETs have been questioned by several authors. A detailed discussions of the various assumptions like e.g. analyticity in the pion mass, which do not hold in QCD, can be found in ref.[60]. Even better, by now the data support the CHPT predictions, see section 11.1.

The last example I want to treat is the case of two–pion photoproduction. At threshold, the two–pion photoproduction current matrix element can be decomposed into amplitudes as follows (to first order in $e$ and in the gauge $\epsilon_0 = 0$):

$$T \cdot \epsilon = \chi^i_f \left\{ i\not\sigma \cdot (\hat{\epsilon} \times \hat{k}) [M_1 \delta^{ab} + M_2 \delta^{ab} \tau^3 + M_3 (\delta^{a3} \tau^b + \delta^{b3} \tau^a)] \right\} \chi_i ,$$  \hspace{1cm} (9.23)

with $\chi_{i,f}$ two–component Pauli and isospinors. Here, I am only interested in the first non–vanishing contributions to $M_{2,3}$, given by some tree diagrams. So we have $N_\gamma = 1$, $L = 0$ and only lowest order mesonic vertices ($d = 2$), i.e.

$$D_L = \sum_d (d - 1) N_d^{MB} .$$  \hspace{1cm} (9.24)

Tree diagrams with lowest order vertices from $\mathcal{L}_{\pi N}^{(1)}$ all vanish due to the threshold selection rules $\epsilon \cdot v = \epsilon \cdot q_i = 0$, $S \cdot q_i = 0$ and $v \cdot (q_1 - q_2) = 0$. The first non–vanishing contribution comes from tree diagrams with insertions from $\mathcal{L}_{\pi N}^{(2)}$, in particular the expansion of $f_{\mu\nu}^+$ from eq.(7.1) leads to a $\gamma\pi\pi NN$ vertex proportional to $\bar{k}_V$ so that

$$M_2 = -2M_3 = \frac{\epsilon}{4} \left( 2 \hat{g}_A^2 - 1 - \bar{k}_V \right) + O(p) = \frac{\epsilon}{4} \left( 2g_A^2 - 1 - \bar{k}_V \right) + O(p) .$$  \hspace{1cm} (9.25)

The full one–loop results can be found in ref.[69]. It is amusing to note that this particular vertex has been overlooked in other derivations of these LETs simply because not the most general effective $\pi N$ Lagrangian at order $p^3$ was used.
10 Spectral functions of electroweak form factors

As a first application, I will now take a closer look at the spectral functions of the nucleon's electroweak form factors at low $t$ since CHPT can be applied to extract some useful information. The structure of the nucleon as probed with virtual photons is parametrized in terms of four form factors (here, $N$ is the fully relativistic spin–1/2 field),

$$< N(p') | J_\mu | N(p) > = e \bar{u}(p') \left\{ \gamma_\mu F_1^N(t) + \frac{i\sigma_{\mu\nu}k^\nu}{2m} F_2^N(t) \right\} u(p), \quad N = p, n, \quad (10.1)$$

with $t = k_\mu k^\mu = (p' - p)^2$ the invariant momentum transfer squared and $J_\mu$ the em current related to the photon field. In electron scattering, $t < 0$ and it is thus convenient to define the positive quantity $Q^2 = -t > 0$. $F_1$ and $F_2$ are called the Pauli and the Dirac form factor (ff), respectively, with the normalizations $F_p^1(0) = 1$, $F_n^1(0) = 0$, $F_p^2(0) = \kappa_p$, and $F_n^2(0) = \kappa_n$. Also used are the electric and magnetic Sachs ff's,

$$G_E = F_1 \tau F_2, \quad G_M = F_1 + F_2, \quad \tau = Q^2/4m^2. \quad (10.2)$$

In the Breit–frame, $G_E$ and $G_M$ are nothing but the Fourier–transforms of the charge and the magnetization distribution, respectively. There exists already a large body of data for the proton and also for the neutron. In the latter case, one has to perform some model–dependent extractions to go from the deuteron or $^3$He to the neutron. More accurate data are soon coming (ELSA, MAMI, TJNAF, . . .). There are also data in the time–like region from the reactions $e^+e^- \rightarrow p\bar{p}, n\bar{n}$ and from annihilation $p\bar{p} \rightarrow e^+e^-$, for $t \geq 4m^2$. It is thus mandatory to have a method which allows to analyse all these data in a mostly model–independent fashion. That’s were dispersion theory comes into play. Although not proven strictly (but shown to hold in all orders in perturbation theory), one writes down an unsubtracted dispersion relation for $F(t)$ (which is a generic symbol for any one of the four ff’s),

$$F(t) = \frac{1}{\pi} \int_{t_0}^\infty dt' \frac{\text{Im} F(t)}{t' - t}, \quad (10.3)$$

with $t_0$ the two (three) pion threshold for the isovector (isoscalar) ff's. $\text{Im} F(t)$ is called the spectral function. It is advantageous to work in the isospin basis, $F_i^{s,v} = (F_i^p \pm F_i^n)/2$, since the photon has an isoscalar ($I = s$) and an isovector ($I = v$) component. These spectral functions are the natural meeting ground for theory and experiment, like e.g. the partial wave amplitudes in $\pi N$ scattering. In general, the spectral functions can be thought of as a superposition of vector meson poles and some continua, related to n-particle thresholds, like e.g. $2\pi$, $3\pi$, $KK$, $NN$ and so on. For example, in the Vector Meson Dominance (VMD) picture one simply retains a set of poles. It is important to realize that there are some powerful constraints which the spectral functions have to obey. Consider the spectral functions just above threshold. Here, unitarity plays a central role. As pointed out by Frazer and Fulco [72] long time ago, extended unitarity leads to a drastic enhancement of the isovector spectral functions on the left wing of the $\rho$ resonance. This is due to a logarithmic singularity on the second Riemann sheet at the anomalous threshold

$$t_c = 4M_\pi^2 - \frac{M_\pi^4}{m_\pi^2} = 3.98M_\pi^2, \quad (10.4)$$

with $t_c$ the critical momentum transfer squared and $m_\pi$ the pion mass.
very close to the normal threshold $t_0 = 4M_\pi^2$. Leaving out this contribution from the two–pion cut leads to a gross underestimation of the isovector charge and magnetic radii. This very fundamental constraint is very often overlooked. In the framework of chiral perturbation theory, this enhancement is also present at the one–loop level as first shown in ref.\[8\]. Let me briefly explain how one can find $t_c$ by using the so–called Landau equations (for an introduction, see e.g. the textbook by Itzykson and Zuber \[73\]). For the pertinent one loop graph shown in fig. 1, we have to analyze the imaginary part. For that, one sets the particles in the loop on their mass shell, $q_1^2 = q_2^2 = M_\pi^2$, $q_3^2 = m^2$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{triangle_diagram.png}
\caption{The so-called triangle diagram leading to the anomalous threshold in the isovector spectral functions. Solid, dashed and wiggly lines denote nucleons, pions and photons, in order.}
\end{figure}

Furthermore, the loop momenta must be linearly dependent, $\alpha_1q_1 + \alpha_2q_2 + \alpha_3q_3 = 0$ which translates into the condition that the determinant formed from the scalar products must vanish, $\det(q_i \cdot q_j) = 0$ $(i, j = 1, 2, 3)$, i.e.

$$\frac{m^2}{4} t \left(4M_\pi^2 - \frac{M_\pi^4}{m^2} - t\right) = 0 ,$$

leading to the anomalous threshold. The subdeterminant in the space of the meson momenta $(i, j = 1, 2)$ is given by $t(4M_\pi^2 - t)/4$ and thus leads to the normal threshold. Furthermore, the imaginary part of the triangle diagram shown in fig. 1, denoted $\mathcal{G}$, is readily worked out,

$$\operatorname{Im} \mathcal{G} = \frac{1}{8\pi \sqrt{t(4m^2 - t)}} \arctan \frac{\sqrt{(t - 4M_\pi^2)(4m^2 - t)}}{t - 2M_\pi^2} .$$

At $t = t_c$, the argument of the arctan is $i$ and since $2\arctan(i x) = i \ln[(1 + x)/(1 - x)]$, we recover the announced logarithmic singularity. Recently, the question whether a similar phenomenon appears in the isoscalar spectral function has been answered \[74\]. For that, one has to consider two–loop graphs as shown in fig.2. Although the analysis of Landau equations reveals a branch point on the second Riemann sheet,

$$\sqrt{t_c} = M_\pi \left(\sqrt{4 - M_\pi^2/m^2} + \sqrt{1 - M_\pi^2/m^2}\right) \to t_c = 8.9 M_\pi^2 ,$$

31
close to the threshold $t_0 = 9M_\pi^2$, the three-body phase factors suppress its influence in the physical region. Consequently, the spectral functions rise smoothly up to the $\omega$ pole and the common practise of simply retaining vector meson poles at low $t$ in the isoscalar channel is justified. Similarly, the three-pion contribution to the nucleon isovector axial form factor is numerically very small and can safely be neglected (as compared e.g. to the correlated $\pi\rho$-exchange).

![Figure 2](image.png)

**Figure 2:** Isoscalar spectral functions weighted with $1/t^2$ for the electric (lower) and magnetic (upper line) Sachs ff (right panel). In the left panel the underlying two–loop graphs are shown (solid, dashed, wiggly lines: Nucleons, pions and photons).

## 11 Confronting the data

As already stated in the introduction, most of the precision data on the nucleon at low energies come from processes involving real or virtual photons such as pion photo– and electroproduction as well as Compton scattering. This is mostly due to the advent of the CW accelerators such as MAMI at Mainz and improved detector technology. It is worth to stress that there are on–going activities in these fields also at LEGS (Brookhaven), SAL (Saskatoon), BATES (MIT), ELSA (Bonn) and other places. Clearly, CEBAF at Jefferson Lab will significantly contribute accurate data to a large variety of processes. Other precise data come from atomic energy shifts at PSI (Villigen) and there is also a tremendous amount of threshold data for $\pi N \rightarrow \pi\pi N$ from such places like TRIUMF (Vancouver). Just from the beginning I would like to stress that one should not see these different experiments and data in isolation but that they are rather intimately connected. For example, the imaginary parts of the various multipoles in pion photo– and electroproduction are proportional to the respective pion–nucleon scattering phase shifts via the Fermi–Watson final state theorem. Furthermore, various contact terms show up in different reactions. For example, the LECs $c_{1,2,3}$ which can be determined in $\pi N \rightarrow \pi N$ [31] show up in the order $p^4$ calculation of the nucleons’ electromagnetic polarizabilities [19] from non–vanishing insertions of $\mathcal{L}_{\pi N}^{(2)}$. This should always be kept in mind. For a nice and instructive flow–chart giving the links between low–energy pion,
experiments I refer the reader to Fig.7.33–1 in the monograph of de Benedetti [75]. A remark on the pion–nucleon coupling constant is in order. In most of the calculations shown, the Karlsruhe–Helsinki value of $g_{\pi N} = 13.4$ was used. There seem, however, to be indications from various sources that the value is smaller, $g_{\pi N} = 13.05$. This can be accounted for by simply plugging in this value into the formulae given in the respective papers. I will come back to this point when discussing the so–called Goldberger–Treiman discrepancy. For the illustrative purpose of these lectures, however, the results based on the larger value of $g_{\pi N}$ are perfectly suitable.

11.1 Charged and neutral pion photoproduction

Charged pion photoproduction at threshold is well described in terms of the Kroll–Ruderman contact term, which is non–vanishing in the chiral limit. All chiral corrections including the third order in the pion mass have been calculated [76]. The chiral series is quickly converging and the theoretical error on the CHPT predictions is rather small, see table 4. The LECs have been determined from resonance exchange. Notice that these uncertainties do not account for the variations in pion–nucleon coupling constant. The available threshold data are quite old, with the exception of the recent TRIUMF experiment on the inverse reaction $\pi^- p \rightarrow \gamma n$ and the SAL measurement for $\gamma p \rightarrow \pi^+ n$. While the overall agreement is quite good for the $\pi^+ n$ channel, in the $\pi^- p$ channel the CHPT prediction is on the large side of the data. Clearly, we need more precise data to draw a final conclusion. It is, however, remarkable to have predictions with an error of only 2%. The threshold production of neutral pions is much more subtle since the corresponding electric dipole amplitudes vanish in the chiral limit. Space does not allow to tell the tale of the experimental and theoretical developments concerning the electric dipole amplitude for neutral pion production off protons, for details see [82]. Even so the convergence for this particular observable is slow, a CHPT calculation to order $p^4$ does allow to understand the energy dependence of $E_{0+}$ in the threshold region once three LECs are fitted to the total and differential cross section data [83]. The threshold value agrees with the data, see table 5. More interesting is the case of the neutron. Here, CHPT predicts a sizeably larger $E_{0+}$ than for the proton (in magnitude). The CHPT predictions for $E_{0+}(\pi^0 p, n)$ in the threshold region clearly exhibit the unitary cusp due to the opening of the secondary threshold, $\gamma p \rightarrow \pi^+ n \rightarrow \pi^0 p$ and $\gamma n \rightarrow \pi^- p \rightarrow \pi^0 n$, respectively. Note, however, that while $E_{0+}(\pi^0 p)$ is almost vanishing after the secondary threshold, the neutron electric dipole amplitude is sizeable ($-0.4$ compared to $2.8$ in units of $10^{-3}/M_{\pi^+}$).

| $E_{0+}^{th}(\pi^+ n)$ | CHPT [79] | Order | Experiment |
|------------------------|-----------|-------|------------|
| 28.2 ± 0.6             | $p^4$     | 27.9 ± 0.5, 28.8 ± 0.7, 27.6 ± 0.6 [79] |
| $E_{0+}^{th}(\pi^- p)$ | $p^4$     | −31.4 ± 1.3, −32.2 ± 1.2 [80], −31.5 ± 0.8 [81] |

Table 4: Predictions and data for the charged pion electric dipole amplitudes in $10^{-3}/M_{\pi^+}$. 

The question arises how to measure the neutron amplitude? The natural neutron target is the deuteron. The corresponding electric dipole $E_d$ amplitude has been calculated to order $p^4$ in ref. [84]. It was shown that the next-to-leading order three-body corrections and the possible four-fermion contact terms do not induce any new unknown LEC and one therefore can calculate $E_d$ in parameter-free manner. Furthermore, the leading order three-body terms are dominant, but one finds a good convergence for these corrections and also a sizeable sensitivity to the elementary neutron amplitude. Note also that neutral pion production off deuterium has recently been measured at SAL. Still, there is more interesting physics in these channels. Quite in contrast to what was believed for a long time, there exist a set of LETs for the slopes of the $P$-waves $P_{1,2} = 3E_{1+} \pm M_{1+} \mp M_{1-}$ at threshold, e.g.

$$\frac{1}{|q|} P_{1,\text{thr}}^{\pi_0 p} = \frac{eg_{\pi N}}{8\pi m^2} \left( 1 + \kappa_p + \mu \left[ -1 - \frac{\kappa_p}{2} + \frac{g_{\pi N}^2}{48\pi} (10 - 3\pi) \right] + O(\mu^2) \right).$$

(11.1)

Numerically, this translates into

$$\frac{1}{|q|} P_{1,\text{thr}}^{\pi_0 p} = 0.512 (1 - 0.062) \text{ GeV}^{-2} = 0.480 \text{ GeV}^{-2},$$

(11.2)

which is given in table 5 in units which are more used in the literature. The agreement with the data is stunning. A theoretical uncertainty on this order $p^3$ calculation can only be given when the next order has been calculated. That calculation is underway. Soon, there will also be an experimental value for $P_2$ at threshold once the MAMI data on $\gamma p \to \pi^0 p$ have been analyzed.

| Order | Experiment | Units |
|-------|------------|-------|
| $E_{0+}^{\text{thr}}(\pi^0 p)$ | $-1.16 [84]$ | $p^4$ |
| $P_{1}^{\text{thr}}(\pi_0 p)$ | $10.3 [78]$ | $p^3$ |
| $E_{0+}^{\text{thr}}(\pi^0 d)$ | $-1.8 \pm 0.2 [84]$ | $p^4$ |

Table 5: Predictions and data for neutral pion S- and P-wave multipoles.

### 11.2 Pion electroproduction

Producing the pion with virtual photons offers further insight since one can extract the longitudinal S-wave multipole $L_{0+}$ and also novel P-wave multipoles. Data have been taken at NIKHEF [89] [90] and MAMI [91] for photon virtuality of $k^2 = -0.1 \text{ GeV}^2$. In fact, it has been argued previously that such photon four-momenta are already too large for CHPT tests since the loop corrections are large [92]. However, these calculations were performed in relativistic baryon CHPT and thus it was necessary to redo them in the heavy fermion formalism. This was done in [93]. The abovementioned data for
differential cross sections were used to determine the three novel S–wave LECs. I should mention that one of the operators used is of dimension five, i.e. one order higher than the calculation was done. This can not be circumvented since it was shown that the two S–waves are overconstrained by a LET valid up to order $p^4$. The resulting S–wave cross section $a_0 = |E_{0+}|^2 + \varepsilon_L |L_{0+}|^2$ shown in fig. 3 is in fair agreement with the data. Note also that it is dominated completely by the $L_{0+}$ multipole (upper dot-dashed line) since $E_{0+}$ passes through zero at $k^2 \approx -0.04 \text{ GeV}^2$. However, in agreement with the older (and less precise) calculations, the one loop corrections are large so one should compare at lower photon virtualities. In ref. [93], many predictions for $k^2 \approx -0.05 \text{ GeV}^2$ are given. At MAMI, data have been taken in this range of $k^2$ and we are looking forward to their analysis, in particular it will be interesting to nail down the zero–crossing of the electric dipole amplitude and to test the novel P–wave LETs [94].

Figure 3: The S–wave cross section $a_0$ compared to the data for various photon polarizations. The upper (lower) dash–dotted line gives the contribution of the longitudinal (electric) dipole amplitude to $a_0$ for $\varepsilon = 0.79$ (solid line).

11.3 Compton scattering

As discussed before, the low–energy Compton scattering amplitude can be decomposed in a spin–independent and a spin–dependent part. In the former case, the nucleon structure is encoded in the so–called electromagnetic polarizabilities. These have been measured over the years at Illinois, Mainz, Moscow, Oak Ridge and Saskatoon. Calculations have been performed to fourth order with some LECs determined from data and others from resonance saturation. A summary is given in table 6. While in case of the proton a consistent picture is emerging [19], the only published empirical values deduced from scattering slow neutrons on heavy atoms have recently been put into question [100]. This experimental problem remains to be sorted out. The most promising result is the almost complete cancellation of a negative non–analytic pion loop contribution with the large positive contribution of tree level $\Delta$–exchange in case of the protons magnetic polarizability [19]. This is the first time that a consistent picture of the para– and diamagnetic
contributions to $\beta_p$ has been found and it underlies the importance of chiral, i.e. pion loop, physics in understanding the nucleon structure as revealed in Compton scattering. Furthermore, there exist chiral predictions for the spin-dependent polarizabilities. As of today, no direct measurements exist but some indirect information based on multipole analysis points towards the important role of the $\Delta(1232)$ as an active dof to understand these quantities. I refer to the Mainz chiral dynamics proceedings [95] for a more detailed discussion.

| Prediction | Order | Ref. | Data | Ref. | Units |
|------------|-------|------|------|------|-------|
| $\bar{\alpha}_p$ | $p^4$ | [19] | 10.4 $\pm$ 0.6 | [96] [97] [98] | $10^{-4}$ fm$^3$ |
| $\bar{\beta}_p$ | $p^4$ | [19] | 3.8 $\mp$ 0.6 | [96] [97] [98] | $10^{-4}$ fm$^3$ |
| $\bar{\alpha}_n$ | $p^4$ | [19] | 12.3 $\pm$ 1.3 | [99] | $10^{-4}$ fm$^3$ |
| $\bar{\beta}_n$ | $p^4$ | [19] | 3.5 $\mp$ 1.3 | [99] | $10^{-4}$ fm$^3$ |

Table 6: Chiral predictions for the electric ($\bar{\alpha}$) and magnetic ($\bar{\beta}$) polarizabilities of the proton and the neutron. Note that a recent reanalysis of the Oak Ridge experiment does not lead to the same values as the published ones[100].

### 11.4 Pion–nucleon scattering

There is, of course, ample of data on elastic pion–nucleon scattering in the threshold region. The one-loop contribution to the $\pi N$-scattering amplitude to order $p^3$ in HBCHPT has first been worked out by Mojžíš [101]. Here, I follow ref. [32] in which certain aspects of pion–nucleon scattering have also been addressed. In the center-of-mass frame the $\pi N$-scattering amplitude $\pi^a(q) + N(p) \rightarrow \pi^b(q') + N(p')$ takes the following form:

$$T^{ba}_{\pi N} = \delta^{ba} \left[ g^+((\omega, t) + i\vec{\sigma} \cdot (\vec{q}' \times \vec{q}) h^+((\omega, t)) + i\epsilon^{bac} \left[ g^-((\omega, t) + i\vec{\sigma} \cdot (\vec{q}' \times \vec{q})) h^-((\omega, t)) \right] \right]$$  \hspace{0.5cm} (11.3)

with $\omega = v \cdot q = v \cdot q'$ the pion cms energy and $t = (q - q')^2$ the invariant momentum transfer squared. $g^+(\omega, t)$ refers to the isoscalar/isovector non-spin-flip amplitude and $h^+(\omega, t)$ to the isoscalar/isovector spin-flip amplitude. After renormalization of the pion decay constant $F_\pi$ and the pion-nucleon coupling constant $g_{\pi N}$, one can give the one-loop contributions to the cms amplitudes $g^\pm((\omega, t)$ and $h^\pm((\omega, t)$ at order $p^3$ in closed form, see ref. [32]. In table 7, I show the predictions for the remaining S, P, D and F-wave threshold parameters which were not used in the fit to determine the LECs. In some cases, contributions from the dimension three Lagrangian appear. The corresponding LECs have been estimated using resonance exchange. In particular, the 10% difference in the P–wave scattering volumina $P_1^-$ and $P_2^-$ is a clear indication of chiral loops, because nucleon and $\Delta$ Born terms give the same contribution to these two observables. Note also that the eight D– and F–wave threshold parameters to this order are free of contributions from dimension three and thus uniquely predicted. The overall agreement of the predictions...
Table 7: Threshold parameters predicted by CHPT. The order of the prediction is also given together with the “experimental values”. Only $a^\pm$ can be measured from pionic atoms. The other values come from the Karlsruhe–Helsinki dispersive analysis.

with the existing experimental values is rather satisfactory. Still, a complete calculation to next order is called for. In ref. [104], it was shown that pion scattering off deuterium can give some bounds on the isoscalar $\pi N$ scattering length and therefore a particular combination of the LECs $c_1, c_2$ and $c_3$. The resulting value is consistent with the one found before. The formalism spelled out in section 8 has been applied in ref. [57] to calculate the isospin violating corrections to neutral pion scattering off protons and neutrons to $O(p^3)$. It substantiates Weinberg’s claim made in 1977 [48] that there are sizeable effects of isospin violation, stemming in part from the light quark mass difference and also from electromagnetism.

11.5 The reaction $\pi N \to \pi\pi N$

Single pion production off nucleons has been at the center of numerous experimental and theoretical investigations since many years. One of the original motivations of these works was the observation that the elusive pion–pion threshold S–wave interaction could be deduced from the pion–pole graph contribution. A whole series of precision experiments at PSI, TRIUMF and CERN has been performed over the last decade and there is still on–going activity. On the theoretical side, chiral perturbation theory has been used to consider these processes. Beringer considered the reaction $\pi N \to \pi\pi N$ to lowest order in

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Obs.} & \text{CHPT} & \text{Order} & \text{Ref.} & \text{Exp. value} & \text{Ref.} & \text{Units} \\
\hline
a^- & 9.2 \pm 0.4 & p^4 & [102] & 8.4 \ldots 10.4 & [103] & 10^{-2} M_{\pi}^{-1} \\
b^- & 2.01 & p^3 & [12] & 1.32 \pm 0.62 & [42] & 10^{-2} M_{\pi}^{-1} \\
P_1^- & -2.44 \pm 0.13 & p^3 & [12] & -2.52 \pm 0.03 & [42] & M_{\pi}^{-3} \\
P_2^+ & -2.70 \pm 0.12 & p^3 & [12] & -2.74 \pm 0.03 & [42] & M_{\pi}^{-3} \\
a_2^{++} & -1.83 & p^3 & [12] & -1.8 \pm 0.3 & [42] & 10^{-3} M_{\pi}^{-5} \\
a_2^{+-} & 2.38 & p^3 & [12] & 2.20 \pm 0.33 & [42] & 10^{-3} M_{\pi}^{-5} \\
a_2^{-+} & 3.21 & p^3 & [12] & 3.20 \pm 0.13 & [42] & 10^{-3} M_{\pi}^{-5} \\
a_2^-- & -0.21 & p^3 & [12] & 0.10 \pm 0.15 & [42] & 10^{-3} M_{\pi}^{-5} \\
a_4^{++} & 0.29 & p^3 & [12] & 0.43 & [42] & 10^{-3} M_{\pi}^{-7} \\
a_4^{+-} & 0.06 & p^3 & [12] & 0.15 \pm 0.12 & [42] & 10^{-3} M_{\pi}^{-7} \\
a_4^{-+} & -0.20 & p^3 & [12] & -0.25 \pm 0.02 & [42] & 10^{-3} M_{\pi}^{-7} \\
a_4^-- & 0.06 & p^3 & [12] & 0.10 \pm 0.02 & [42] & 10^{-3} M_{\pi}^{-7} \\
\hline
\end{array}
\]
chiral perturbation theory \[105\]. Low–energy theorems for the threshold amplitudes \(D_1\) and \(D_2\) were derived in \[106\],

\[
D_1 = \frac{g_A}{8F^2_\pi} \left(1 + \frac{7M_\pi}{2m}\right) + \mathcal{O}(M^2_\pi) = 2.4 \text{ fm}^3,
\]

\[
D_2 = -\frac{g_A}{8F^2_\pi} \left(3 + \frac{17M_\pi}{2m}\right) + \mathcal{O}(M^2_\pi) = -6.8 \text{ fm}^3.
\]

These are free of unknown parameters and not sensitive to the \(\pi\pi\)–interaction beyond tree level. A direct comparison with the threshold data for the channel \(\pi^+p \rightarrow \pi^+\pi^+n\), which is only sensitive to \(D_1\), leads to a very satisfactory description whereas in case of the process \(\pi^-p \rightarrow \pi^0\pi^0n\), which is only sensitive to \(D_2\), sizeable deviations are found for the total cross sections near threshold. These were originally attributed to the strong pionic final–state interactions in the \(I_{\pi\pi} = 0\) channel. However, this conjecture turned out to be incorrect when a complete higher order calculation of the threshold amplitudes \(D_{1,2}\) was performed \[107\]. In that paper, the relation between the threshold amplitudes \(D_1\) and \(D_2\) for the reaction \(\pi N \rightarrow \pi \pi N\) and the \(\pi \pi\) S–wave scattering lengths \(a^0_0\) and \(a^2_0\) in the framework HBCHPT to second order in the pion mass was worked out (for details, I refer to that paper). Notice that the pion loop and pionic counterterm corrections only start contributing to the \(\pi\pi N\) threshold amplitudes at second order. One of these counterterms, proportional to the low–energy constant \(\ell_3\), eventually allows to measure the scalar quark condensate, i.e. the strength of the spontaneous chiral symmetry breaking in QCD. However, at that order, the largest contributions to \(D_{1,2}\) stem indeed from insertions of the dimension two chiral pion–nucleon Lagrangian, which is characterized by the LECs constants called \(c_i\). In particular this is the case for the amplitude \(D_2\). To be specific, consider the threshold amplitudes \(D_{1,2}\) calculated from the relativistic Born graphs (with lowest order vertices) and the relativistic \(c_i\)–terms expanded to second order in the pion mass. This gives

\[
D^\text{Born}_1 + D^c_i = (2.33 + 0.24 \pm 0.10) \text{ fm}^3 = (2.57 \pm 0.10) \text{ fm}^3, 
\]

\[
D^\text{Born}_2 + D^c_i = (-6.61 - 2.85 \pm 0.06) \text{ fm}^3 = (-9.46 \pm 0.06) \text{ fm}^3,
\]

which are within 14% and 5% off the empirical values,

\[
D^\text{exp}_1 = (2.26 \pm 0.10) \text{ fm}^3, \quad D^\text{exp}_2 = (-9.05 \pm 0.36) \text{ fm}^3,
\]

respectively. It appears therefore natural to extend the same calculation above threshold and to compare to the large body of data for the various reaction channels that exist \[108\]. It was already shown by Beringer \[105\] that taking simply the relativistic Born terms does indeed not suffice to describe the total cross section data for incoming pion energies up to 400 MeV in most channels. Such a failure can be expected from the threshold expansion of \(D_2\), where the Born terms only amount to 73% of the empirical value. We therefore expect that the inclusion of the dimension two operators, which clearly improves the prediction for \(D_2\) at threshold, will lead to a better description of the above threshold data. In particular, it will tell to which extent loop effects are necessary (and thus testing the sensitivity to the pion–pion interaction beyond tree level) and to which extent one
has to incorporate explicit resonance degrees of freedom like the Roper and the Δ–isobar as well as heavier mesons (σ, ρ, ω) as dynamical degrees of freedom (as it is done in many models, see e.g. [109] [110]). Since the LECs \( c_i \) have previously been determined, all our results to this order are based on a truly parameter–free calculation. One finds that (a) for pion energies up to \( T_\pi = 250 \text{ MeV} \), in all but one case the inclusion of the contribution \( \sim c_i \) clearly improves the description of the total cross sections (solid versus dashed line), most notably in the threshold region for \( \pi^- p \rightarrow \pi^0 \pi^0 n \). Up to \( T_\pi = 400 \text{ MeV} \), the trend of the data can be described although some discrepancies particularly towards the higher energies persist, and (b) double differential cross sections for \( \pi^- p \rightarrow \pi^+ \pi^- n \) at incident pion energies below \( T_\pi = 250 \text{ MeV} \) are well described. In fig. 4, the total cross section for \( \pi^- p \rightarrow \pi^+ \pi^- n \) is shown in comparison to the data [111]. In particular, the novel TRIUMF threshold data [112] between threshold and \( T_\pi \) show an excellent agreement with the previously published prediction.

![Figure 4: Total cross section for \( \pi^- p \rightarrow \pi^+ \pi^- n \). Left panel: The threshold region. The new TRIUMF data are depicted by the diamonds and have significantly smaller error bars than the older data. Right panel: Comparison to the data up to pion kinetic energy of 400 MeV.](image)

### 11.6 The Goldberger–Treiman discrepancy revisited

I briefly want to return to the Goldberger–Treiman relation, Eq.(2.15). In nature, it is not exactly fulfilled and one considers the so–called Goldberger–Treiman discrepancy (GTD),

\[
\Delta_{\pi N} \equiv 1 - \frac{m g_A}{F_\pi g_{\pi N}} .
\]  

(11.9)

In fact, at this point the precise values of the axial–vector coupling, the pion decay and, in particular, the pion–nucleon coupling constant come into play. To third order, the GTD is given in terms of the LEC \( b_{23} \) (see [3] for a precise definition),

\[
\Delta_{\pi N} = -\frac{M_\pi^2}{8\pi^2 F_\pi^2} b_{23} .
\]  

(11.10)
Furthermore, if one describes the whole GTD by a form factor effect and chooses one particular form for the pion interpolating field, one can translate the value of $b_{23}$ into the cut–off of a monopole pion–nucleon form factor via (for details, see [3])

$$\Lambda = \frac{4\pi F_\pi}{\sqrt{-2b_{23}}}.$$  \hspace{1cm} (11.11)

In table 8, I have summarized the status for various input values. The first two values for $g_A$ refer to the PDG 1996 value [113] and the next two to the most recent Grenoble measurement [114]. I use throughout the PDG value for $F_\pi$ and give results for the two values of $g_{\pi N}$ discussed in the introduction of this section. It should be noted that for the smaller value of $g_{\pi N}$, which seems to be most widely accepted, the GTD is less than 2% and the resulting form factor is no longer as soft as claimed previously.

| $g_A$ | $F_\pi$ [MeV] | $g_{\pi N}$ | $\Delta_{\pi N}$ [%] | $b_{23}$ | $\Lambda$ [MeV] |
|-------|---------------|-------------|----------------------|---------|-----------------|
| 1.260 | 92.42         | 13.40       | 4.5                  | -1.57   | 656             |
| 1.260 | 92.42         | 13.05       | 2.0                  | -0.68   | 994             |
| 1.266 | 92.42         | 13.40       | 4.1                  | -1.41   | 691             |
| 1.266 | 92.42         | 13.05       | 1.5                  | -0.52   | 1135            |

Table 8: The Goldberger–Treiman discrepancy $\Delta_{\pi N}$ for various input parameters. Also given are the LEC $b_{23}$ and the pion–nucleon cut–off as explained in the text.

12 Problems, open questions and omissions

Here, I will list a few topics which deserve further study. This list should neither be considered complete nor does the ordering imply any priority. I also mention a few interesting and important developments which were not discussed in any detail.

(i) More precise data to which HBCHPT can be applied are needed. This would then allow for a systematic study of the LECs appearing in $L_{\pi N}^{(3,4)}$ and to judge the validity (quality) of the resonance saturation principle which is often used to get a handle on the LECs. This calls for a joint effort of the experimenters and the theoreticians. Experimental activities are under way for double neutral pion production off nucleons, ordinary and radiative muon capture on protons, charged and neutral pion electroproduction off nucleons as well as kaon photoproduction off protons and deuterium.
In the three flavor sector, we do not yet have a consistent picture. The problems here are related to the facts that a) the expansion parameter $M_K/(4\pi F_K) = 0.4$ is not that small and b) there are in some channels even subthreshold resonances which make a direct application of CHPT problematic. Only a few complete (and in some cases accurate) calculations exist, i.e. we can not yet draw decisive conclusions. A status report is given in [115]. There exist many data and more are coming, as an example let me just mention the accurate threshold kaon photoproduction ones from ELSA or the proposals to measure the hyperon polarizabilities at Fermilab and CERN and, of course, the kaon production data off nucleons and deuterium from Jefferson Lab. More theoretical effort is needed to clarify the situation.

Jenkins and Manohar first advocated to supplement the EFT of the ground state baryons and Goldstone bosons by the spin-3/2 decuplet [116] [20]. This approach has been taken up in quite a few papers thereafter. If one thinks of extending calculations like $\pi N \rightarrow \pi N$ through the $\Delta$ region, this is certainly unavoidable. Recently, Hemmert et al.[117] have developed a framework to consistently treat the residual octet–decuplet mass difference in a systematic fashion extending the path integral formalism of [14]. This so–called "small scale" expansion is not exactly a low–energy expansion since in the chiral limit, the $N\Delta$ splitting remains finite. What is missing is a set of complete calculations for observables from which one could assess the accuracy of the approach. In particular, it remains to be seen that in cases where the conventional picture fails, the small scale expansion indeed works better. Work along these lines is underway for Compton scattering [118], pion photoproduction in the $\Delta$ region and other processes.

The extension of the CHPT approach to systems of two or more nucleons has only begun. Despite some theoretical problems (the power counting only applies to the subset of irreducible diagrams), it seems to shed some light on the phenomenology of nuclear forces like e.g. the smallness of three–, four–, . . . body forces and the masking of isospin violation in these systems [17] [18] [119] [120] [121] [122]. However, it is mandatory to perform these calculations with all the input which is available from the single baryon sector. This has not yet been done. Most promising seems to be a mixed approach, in which one uses CHPT to calculate the kernel of the process under consideration and sews that together with precise wave functions from boson–exchange models [123]. The most precise calculation so far concerns neutral pion production off deuterium discussed before. A review is given in [124].

Throughout these lectures, I have been rather casual in discussing the precise relation between S–matrix elements calculated in the heavy fermion approach and the relativistic version. This has been worked out in detail in ref. [125]. As an example, consider the tree graphs with fixed LECs contributing to the threshold parameters in pion–nucleon scattering. In ref. [32], these were calculated by working out the relativistic Born graphs and then expanding the resulting expressions in powers of $M_\pi/m$. On the other hand, the same calculation in ref. [101] was performed entirely in the heavy baryon framework with no recourse to the relativistic theory. Of course, these approaches have to be completely equivalent. This can be seen if one studies carefully the wave function renormalization in HBCHPT, which shows some intriguing features. For details, I refer to [125]. For purely calculational purposes, I recommend the procedure used e.g. in [39] or [32].
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