SOME NEW HERMITE–HADAMARD TYPE INEQUALITIES VIA CAPUTO $k$–FRACTIONAL DERIVATIVES CONCERNING $(n + 1)$–DIFFERENTIABLE GENERALIZED RELATIVE SEMI–$(r; m, h_1, h_2)$–PREINVEX MAPPINGS

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Abstract. In this article, we first presented a new identity concerning $(n + 1)$–differentiable mappings defined on $m$–invex set via Caputo $k$–fractional derivatives. By using the notion of generalized relative semi–$(r; m, h_1, h_2)$–preinvexity and the obtained identity as an auxiliary result, some new estimates with respect to Hermite–Hadamard type inequalities via Caputo $k$–fractional derivatives are established. It is pointed out that some new special cases can be deduced from main results of the article.

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