Drag Force in the Vacuum of Confining Gauge Theories

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The complete absence of isolated quarks reaching particle detectors after high energy collisions suggests that some physical mechanism generates resistance to their propagation in the vacuum. In order to reveal such a mechanism, we analyze the fate of an infinitely heavy quark that is initially propagating in the vacuum with inertial motion. The non-perturbative structure of the vacuum is treated here using the gauge/gravity correspondence, the isolated quark on the boundary gauge theory is dual to a trailing string moving in the bulk of a higher dimensional curved space. We find that, for a large class of non-conformal gauge theories with a holographic dual, the geometrical structure of the bulk geometry induces a drag force on the quark that moves in the vacuum. In addition, we show that for these gauge theories there will be the presence of such a drag force due to its vacuum whenever the dual bulk geometry generates a linear potential for a $q\bar{q}$ pair. The relation of the linear $q\bar{q}$ potential with the drag force on the isolated quark is a holographic piece of evidence that both phenomena are different manifestations of the confinement of quarks.

I. INTRODUCTION

In the present paper we discuss the confinement of quarks using holography. We treat the confinement problem from the point of view of a single free quark with infinite mass, considered as a probe particle. The physical picture is set by analyzing the relativistic quark’s motion. This massive probe quark is dual to an open string stretching inside the bulk of the 5-dimensional AdS space, with one of its ends tied to the conformal boundary. The other end is considered moving freely since the scenario is not thermal, thus there is no associated black hole event horizon to tie up this end. This implies that at the initial stage this free end is only affected by the AdS background. This momentum transfer between the background and the string defines the drag experienced by the probe.

The field theory describing strong interactions, the so-called Quantum Chromodynamics (QCD) has two very important features: asymptotic freedom [1] and confinement. The asymptotic freedom was very important for establishing QCD as the formal theory of strong interactions. For the high energy phenomena, this theory is perturbative and leads to a wide range of important results [2, 3]. On the other hand, confinement is harder to deal with its complexity [4, 5], and also because QCD is strongly coupled field theory at the low energy limit, where the perturbative treatment is not possible. This is translated into a non-perturbative vacuum.

There are two definitions of confinement, quark confinement and color confinement. The former establishes that no single quark can be measured as an asymptotic state in any particle detector. Color confinement means that no color charged (or colored) particle state can be measured as an asymptotic state in any particle detector. It happens that color confinement includes quark confinement since quarks are colored particles.

A great difficulty understanding the physics of confinement comes when the strongly coupled interactions at low energy are considered, since confinement is manifest on the observables related with low energy physics, such as the hadronic Regge trajectories and the slope of the $q\bar{q}$ potential at very large separations, implying that theoretical description of confinement is intrinsically non-perturbative.

The AdS/CFT correspondence [6, 7] establishes a connection between the non-perturbative regime of a gauge theory and a gravity theory at weak coupling. The holographic picture of confinement was established by the dependence of the interaction potential of a static quark–anti-quark pair ($q\bar{q}$) with the separation distance between them. The associated gauge theory will be confining if the potential is linear for large separation. In the paper by Kinar et. al. [8] was established a direct mathematical criterion to determine whether the gauge theory is confining or not by finding a sufficient condition for the $q\bar{q}$ potential to be linear at large separations. In this sense, the physical picture of confinement is clear for the static $q\bar{q}$ pair system.

In order to complete the physical picture of confinement we analyse the case of a single quark that is initially moving freely (i.e. not bounded) in the vacuum.
of the gauge theory. To do so, we extend the Gubser’s proposal \cite{9} to the case of zero temperature by providing the dual picture of a free quark on the gauge theory vacuum by an open (trailing) string in the bulk with one endpoint attached on the quark location at the boundary and the other end moves untied. By analyzing such a dual string configuration, we find that for a large class of gauge theories with a holographic dual, there will be a drag force acting on the quark moving in the vacuum if the such theories are confining. Another analogy comes from electricity: conformal vacuum, where the lack of drag force is translated into no confinement, is similar to a \textit{dielectric}, characterized by carriers that cannot move freely. In this simple analogy, confinement and electrical resistant are \textit{emergent properties} arising from the electrical (conformal) properties of the medium. The proposed holographic analysis discussed here provides this nice physical picture for the vacuum of gauge theories experienced by the isolated probe quark. Another possible backgrounds that have been considered in the literature are the finite temperature case \cite{10}, finite chemical potential \cite{11}, and the strong magnetic field \cite{12}.

The presence of a drag force due to the vacuum looks like to be an effect associated with the averaged quantum interactions of the probe quark with the vacuum fluctuations. Effectively the quark motion is damped or equivalently the quark exerts some work on the medium so that its kinetic energy is transferred to the medium. On a phenomenological point of view, when an isolated quark is scattered in the vacuum with sufficiently high energy there will be formed a jet of hadrons centred in the initial direction of the moving quark. This is the jet formation phenomenon \cite{13}. Our hypothesis is that this drag force due to the vacuum is directly related with the jet formation process. In the holographic picture, the long string attached to the quark feels a gravitational inertia (resistance) on the bulk breaking it down into many pieces, each with both ends attached to the boundary in a semi-classical transition, i.e., the free quark interacting with the gauge theory vacuum \textit{hadronizes}. Here we use the results for drag force on the vacuum to estimate the jet energy as a function of the initial quark energy.

The work is organized as follows: in section \textbf{II} we do a summary of the quark-vacuum interaction in the perspective of the fully inclusive quark-jet correlation function and its interpretation in terms of the energy lost by a parton moving in the vacuum. In section \textbf{III} we discuss the drag force in the vacuum of a $\mathcal{N} = 4$ SYM at zero temperature, probing that there is no drag force in such a model. In section \textbf{IV} we discuss the bottom-up of wall model at zero temperature case, demonstrating the existence of drag force attached to the presence of the energy scale that breaks softly the conformal invariance. In section \textbf{V} we discuss the general case given by an AdS-like background. The existence of a drag-force in such backgrounds is conditioned by the existence of a geometric upper bound in the 4-momentum transferred by the string from the boundary to the bulk. In sections \textbf{VI} and \textbf{VII} we focus on the top-down models D3/D7 brane system and the Sakai-Sugimoto configuration. As in the bottom-up case, the drag force is conditioned again by the existence of a confining scales in both models. In section \textbf{VIII} we estimated the fraction of energy loss, finding that the non-conformal systems have associated the so called \textit{dead cone effect}. Finally, in section \textbf{IX} we give some conclusions and final remarks about the phenomenology discussed here.

\section{II. QUARK-VACUUM INTERACTION PHENOMENOLOGY}

One of QCD’s essential properties is confinement, implying that no single-colored parton can exist as a stable free particle. After high energetic collisions, such deep inelastic scattering \cite{14}, free partons travel until they hadronize by mechanisms as the gluon bremsstrahlung \cite{15}. Recall that partons are not measured since they do not reach the detector due to their short mean free path compared to the detector’s distance. In other words, these free partons produce jets of hadrons that will be detected short after. Therefore, talking about hadronization is tightly connected to the partonic propagation mechanism. However, this transition from free partons to bounded hadrons and the jet formation have not been deeply explored. See for example \cite{16}.

One of the forms to address the interaction between free partons and the hadronic jets is the quark-jet correlator, which accounts for the hadron formation in the physical region $x < 1$, where $x$ is the Bjorken variable \cite{17 18}

$$ \Xi_{ij}(k, \hat n) = \frac{1}{N_c} \int d^4 \eta e^{i k_\eta \hat n} Tr_{c}(0 \mid T W_1(\infty, \eta, \hat n) \psi_1(\eta) \times T \bar \psi_2(0) W_2(0, \infty, \hat n) \mid 0) \quad \quad (1) $$

where we have the following definitions: $k$ is the quark four-momentum, $\psi(\eta)$ is the spinor field associated to the quark, $\hat T$ stands for time ordering whereas $T$ is the anti-time ordering and $\mid 0 \rangle$ is the non-perturbative QCD vacuum. Flavor indices are omitted for simplicity. The gauge invariance structure is guaranteed by the Wilson line operators $W(\infty, \hat n, \eta)$, conditioned by choice of the line directions given by the unitary vector $\hat n$. The choice of the Wilsonian path is necessary in order to apply QCD factorization theorems. Finally, the vector $\hat n$ parametrizes the formed jet direction. The trace over color is required since the jets should be colorless when they reach the detectors. This correlator is also called \textit{fully inclusive} since, from the pure diagrammatically point of view, it describes all the hadronic products appearing due to the quark-vacuum interaction without considering jet reconstruction.
This correlator can be parametrized in terms of jet-parton correlation functions using a Lorentz invariant Dirac decomposition, depending on the final hadron twist. For example, in the case of twist-3 order, the jet-quark correlator takes the following spectral form:

$$\Xi(k^2) = \int d\sigma^2 \left[ J_1(\sigma^2) \sigma 1 + J_2(\sigma^2) \gamma_\mu k^\mu \right] \delta(k^2 - \sigma^2)$$

(2)

with $\sigma^2$ defined as the invariant mass of the outgoing jet; the functions $J_i(\sigma^2)$ are the jet mass distributions satisfying

$$J_2(\sigma^2) \geq J_1(\sigma^2) \geq 0 \text{ and } \int d\sigma^2 J_2(\sigma^2) = 1.$$  \hspace{1cm} (3)

imposed by CPT invariance. Notice that $\sigma^2$ can be directly connected, by energy conservation, with the energy lost by a parton moving in the QCD vacuum. The drag force exerted by the vacuum is translated into the quark fragmentation that leads to the production of jets.

Jet formation is the observed phenomenon happening when an isolated quark is scattered away from its bounded partners. This phenomenon is assigned to the interaction of the colored quark with the QCD vacuum. The physical mechanism responsible for transmitting the initial quark in a jet of hadrons has been investigated for a long time \cite{20,22}. The non-perturbative nature of QCD vacuum makes it a challenger and exciting topic which motivated different models such as stochastic instanton configurations \cite{23,24}, chiral disorder \cite{25} and dual superconductors \cite{26,29}. However, a complete description of the non-perturbative QCD vacuum is still lacking.

It is interesting to note that the QCD vacuum also plays a role in a dense medium. The description of the energy loss by gluon radiation of a quark moving in a dense media can be written as an opacity expansion whose zero-order term persists even for zero matter density. It is attributed to the QCD vacuum itself \cite{30,31}. Even that the opacity expansion assumes the presence of a medium, it reveals the vacuum itself acts on the propagating quark damping its motion. It is also remarkable that the vacuum contribution to the energy lost by the heavy quark reproduces the dead cone effect.

In the context of the gauge/ gravity correspondence, the description of quark-vacuum interaction has focused on colorless hadronic systems, especially in the description of mesons. The present paper introduces the analysis of quark-vacuum interaction by extending the Gubser proposal \cite{32} to a vacuum medium. The concept of vacuum in this approach comes from the Minkowski signature of the metric and the absence of a black role in the bulk geometry, which means that the boundary gauge theory is at zero temperature and zero chemical potential. As a result, as we will expose in Section \ref{section}, a non-vanishing drag force is allowed, and consequently, will be there to act on a single quark propagating in the vacuum of the confining gauge theory.

### III. The Vacuum of $\mathcal{N} = 4 \text{ SYM}$

In this section we analyze the vacuum of the conformal dual field theory $\mathcal{N} = 4 \text{ SYM}$, where there is no confinement \cite{32,33}. This conformal theory is constructed as the dual of the geometric background generated by $N_c$ coincident D3-branes, that is $\text{AdS}_5 \times S^5$. See \cite{9}.

Let us focus on the AdS part of such geometry, parametrized by the 5-dimensional Poincaré patch in the following form:

$$dS^2 = \frac{R^2}{z^2} \left[ -dt^2 + d\vec{x}^2 + dz^2 \right], \ z \in (0, \infty).$$  \hspace{1cm} (4)

The isolated heavy quark is dual to an end of an open (trailing) string, attached to the boundary and hanging into the AdS bulk. The other string end just goes to IR sector of AdS$_5$.

The probe quark is assumed to move at constant velocity. Also we adopt the same ansatz used in the finite temperature case for the string profile:

$$X^\mu(t, z) = (t, x(t, z), 0, 0, z), \ x(t, z) = vt + \xi(z),$$

where $v$ is the quark velocity (in the lab frame). For this string configuration the Nambu-Goto action is given by

$$S = \frac{R^2}{2\pi \alpha'} \int dt dz \frac{1}{z^2} \sqrt{1 - v^2 + \xi'^2}.$$  \hspace{1cm} (5)

When we use this ansatz, the Lagrangian density $L = \frac{1}{2\pi \alpha' z^2} \sqrt{1 - v^2 + \xi'^2}$ becomes a function of $z$ and $\xi(z)$ only. Therefore, the equation of motion for the string is expressed as the conservation of $\pi_\xi = \frac{\partial L}{\partial \xi'}$, that is given by

$$\pi_\xi = \frac{R^2}{2\pi \alpha' z^2} \frac{\xi'}{\sqrt{1 - v^2 + \xi'^2}}.$$  \hspace{1cm} (6)

From the expression above, we can solve for $\xi(z)$, obtaining

$$\frac{d\xi}{dz} = \frac{2\pi \alpha' \pi_\xi^2 z^2 \sqrt{1 - v^2}}{\sqrt{R^4 - (2\pi \alpha' \pi_\xi)^2 z^4}}.$$  \hspace{1cm} (7)

The equation above differs from the finite temperature case since there is no blackening factor the numerator inside the square-root is always positive, the isolated quark is a time-like particle, thus we have always $v^2 < 1$. But we still need $\xi(z)$ to be real valued if we want the solution to represent classical string configurations, which requires that the denominator inside the square-root should be always positive: $R^4 - (2\pi \alpha' \pi_\xi)^2 z^4 > 0$ everywhere in the bulk. In this case $z \in (0, \infty)$, the reality condition
for \( \xi' \) requires that \( \pi_\xi \leq \min \frac{R^2}{2\pi\alpha'} \), which constrains the conserved momentum to vanish: \( \pi_\xi = 0 \). Consequently, from eq.\( \text{(7)} \) we find that \( \xi(z) = 0 \) implying that all of the pieces of the string move in parallel with the same velocity.

The drag force on the quark probe is given by the world-sheet momentum \( \Pi^z_x = \frac{\delta S}{\delta \delta z} \). Thus, we find that the drag force vanishes:

\[
F_{\text{Drag}} = \Pi^z_x = -\pi_\xi = 0. \tag{8}
\]

The vanishing of the drag force means that an colored probe quark can move freely in the vacuum of \( \mathcal{N} = 4 \) SYM. This scenario gives us a holographic picture that shows us how the probe quark in this sort of background is not confined. Summarizing, confinement and drag force, on the holographic perspective, are connected, i.e., one is a direct manifestation of the other. This non-confining assertion observed in \( \text{SYM} \). This scenario gives us a holographic picture that requires that \( R^2 e^{2kz^2} - (2\pi\alpha'\pi_\xi)^2 z^4 > 0 \). This condition does not require \( \pi_\xi \) to vanish. It just imposes that \( \pi_\xi \leq \min \left( \frac{R^2 e^{2kz^2}}{2\pi\alpha'} \right) \), implying the only restriction we have is

\[
\pi_\xi \leq \frac{R^2 e k^2}{2\pi\alpha'}. \tag{13}
\]

The conserved momentum is not constrained to vanish. It has a positive upper bound and is allowed to assume any value in the range \( [0, \frac{R^2 e k^2}{2\pi\alpha'}] \). Unfortunately, the condition that \( \xi(z) \) is real everywhere does not determine the conserved momentum as a function of the velocity, as it happens at the finite temperature case. All we have at hand is the range of admissible classical values for the conserved momentum. The existence of this conserved momentum along the holographic coordinate in the bulk is associated with a drag force on the moving probe quark at the boundary. The rate of momentum loss due to the drag force is

\[
\frac{dp}{dt} = -\Pi^z_x = -\frac{\delta S_{sw}}{\delta (\delta z)} = -\pi_\xi, \tag{14}
\]

Consequently, for any non-vanishing momentum \( \pi_\xi \), the quark moving on the vacuum of the boundary theory will be subjected to a drag force. This force acting on the moving probe quark is due to the vacuum itself. This result tells us that, in fact, a confining gauge theory has a mechanism that prevents a single quark moves freely on the vacuum. If the probe quark is thrown into a vacuum, this acts as a viscous medium and exerts a drag force on the quark.

The existence of a drag force at the zero temperature in the context of the soft wall AdS/QCD model is a non-trivial phenomenology associated with the breaking of the conformal symmetry. The dilaton in the soft wall model is responsible for guaranteeing the confinement of
quarks. In the case of a $q\bar{q}$ pair, it makes itself manifest by providing a linear term in the interaction potential \[37\]. But in the present case, it manifests by the presence of a drag force that acts on a quark moving in the vacuum. In this sense, we can expect that in any confining gauge theory an isolated quark moving on the vacuum will be subjected to a drag force, interpreted as the manifestation of confinement. It is important to remark that the quark is assumed to move at a constant velocity and it is infinitely massive. The effects on the energy loss of an isolated quark due to its accelerated motion are discussed in \[37–39\]. The presence of a drag force in the case of constant velocity represents a different mechanism for energy loss of the heavy quark in addition to the radiation emission due to the acceleration.

V. THE GENERAL CASE OF A GAUGE THEORY WITH HOLOGRAPHIC DUAL

The usual approach to discuss confinement is to consider a static quark/anti-quark configuration and to compute the expected value of the Wilson loop operator. The confinement comes from the area law in the Wilson loop operator or equivalently in the linear behavior of the interaction energy as a function of the quark pair separation. In this sense, the seminal work of Sonnenschein \[8\] classify a gauge theory in confining or non-confining according to the geometric structure of its holographic higher dimensional dual description. Here we use an alternative approach to discuss confinement by probing the vacuum of the gauge theory with an isolated quark and looking for a physical mechanism that prevents its free motion.

In a non-confining $\mathcal{N} = 4$ SYM theory, an isolated quark can move on freely on the vacuum while in the confining theory described by the Soft-Wall model an isolated quark moving on the vacuum is subjected to a drag force. These two examples point out a possible relation to the momentum along the holographic direction of the world sheet configuration is as \[18\].

The general structure of the metric components of the bulk geometry is non-vanishing smooth functions, that near the boundary $z \to 0$ satisfy \[18\]. The general structure of the metric in eq.\[18\] represents the zero temperature versions of the bulk metrics that mimic the QCD equation of state at zero chemical potential \[18\]. For these geometries, we find

\[\frac{d\xi}{dz} = \frac{\pi_\xi}{2\alpha'} \sqrt{\frac{G_{00}G_{zz} - G_{xx}G_{zz}v^2}{G_{00}G_{xx}^2 - G_{00}G_{xx}}}. \tag{17}\]

Due to the form of the above equation of motion, we noted that it is not necessary to assume the bulk metric as diagonal. We restrict our discussion to the cases where the bulk geometry admits a Poincaré-like parametrization with Poincaré invariant 4-dimensional slices. This represents most of the AdS/QCD backgrounds at zero temperature. We assume that bulk metric has the following form

\[ds^2 = e^{A(z)}(-dt^2 + dx^2) + e^{B(z)}dz^2. \tag{18}\]

The metric components of the bulk geometry are non-vanishing smooth functions, that near the boundary $z \to 0$ satisfy \[18\]. The general structure of the metric in eq.\[18\] represents the zero temperature versions of the bulk metrics that mimic the QCD equation of state at zero chemical potential \[18\]. For these geometries, we find

\[\frac{d\xi}{ds} = 2\pi\alpha'\pi_\xi e^{\frac{B(z)-A(z)}{2}} \sqrt{1 - \frac{v^2}{e^{2A(z)} - (2\pi\alpha'\pi_\xi)^2}} \tag{19}\]

In order to $\xi(z)$, and consequently $x(t, z)$, be real valued it is necessary and sufficient that $e^{2A(z)} - (2\pi\alpha'\pi_\xi)^2 > 0$. The conserved momentum $\pi_\xi$ is constrained by $\pi_\xi < \frac{1}{2\pi\alpha'} e^{A(z)}$. The constraint on $\pi_\xi$ is satisfied if and only if $0 \leq \pi_\xi \leq \min\left(\frac{A(z)}{2\pi\alpha'}\right)$. Hence, it will be allowed a non-vanishing $\pi_\xi$ whenever the function $f(z) = e^{A(z)}$ have a non-vanishing minimum at some $z_0$, say $f'(z_0) = 0$, $f(z_0) > 0$. In the case where $\pi_\xi \neq 0$, the quark attached at the string endpoint is subjected to drag force $F_{\text{Drag}} = -\pi_\xi$. Thus we find in this form that if $f(z)$ has a non-vanishing minimum then a moving quark in the vacuum of the dual gauge theory will be subjected to a drag force. We also note that, for the classes of bulk geometries we consider, the existence of a non-vanishing minimum for $f(z)$ corresponds precisely to the same requirement that the bulk geometry should satisfy if the dual gauge theory is confining in the infrared, as it was proved in \[8\]. Therefore, we prove that in any confining gauge theory with a holographic dual,
whose bulk geometry has a form eq. (18), there will be a
drag force experienced on any isolated quark that moves
in the vacuum. On the other hand, for a non-confining
gauge theory with a holographic description admitting a
bulk metric with the form of eq. (18), we have (following
[8]) that either \( f(z) \) is not bounded from below or
if it has a minimum at some \( z_0 \) where it vanishes, i.e.,
\( f(z_0) = 0 \). In both cases, the constraint from eq. (19)
requires that the momentum along the string vanishes,
\( \pi_z = 0 \). Consequently, there is no drag force on the mov-
ing quark. This feature is realized in the particular case of
the vacuum of \( \mathcal{N} = 4 \) SYM, discussed in Section , and
is generalized here for the non-confining gauge theories
with bulk geometry in the form of eq. (18).

The physical background of the relation proved here
in a holographic approach is very nice. If the gauge the-
ory is confining then an isolated quark, initially moving
with constant velocity, fells a force due to the vacuum,
and hence the isolated quark cannot exists as a free par-
cle: no forces act in a free particle. However, if there is
no confinement in the gauge theory the isolated moving
quark with constant velocity will not experience absol-
utely any resistance for its motion and it will move as
a free particle. Therefore, it seems that in a confining
gauge theory the concepts of fundamental particle and
free particle do not mix up.

The analysis of the single quark moving across the vac-
uum is not usual in the literature. But some attention
to this issue has been paid in ref. [10] in the context of
the Sakai-Sugimoto model. There the drag force is in-
terpreted as an instability of the system composed of an
isolated quark in the vacuum of such a confining gauge
theory. We also remark that the existence of a drag force
due to the interaction of the quark with the vacuum was
expected in previous holographic analysis to be ruled out,
such as in ref. [41].

In section [10] we have analyzed the soft wall model
[34], that is a bottom-up AdS/QCD model. In the next section,
we will address top-down models, specifically
D3/D7 and Sakai-Sugimoto, that are realizations of the
AdS/CFT correspondence that have been proven to be
confining by the area law of the
AdS/CFT correspondence that have been proven to be

VI. DRAG FORCE IN THE D3/D7 SYSTEM

In the D3/D7 system, we consider the limit \( N_c \gg N_f \),
where a the set of \( N_f \) D7-branes act as probes, implying
that the backreaction of them can be neglected [34]. The
confinement arises from the embedding of the D7 probes
into the D3-background. Quarks in this scheme appear
as the Chan-Paton factors of strings hanging from the
D3 stack to the D7 probe (3-7 string). The conformal
boundary appears in the radial (perpendicular to the D3-
branes) direction of this configuration, thus the confor-
mal boundary will localize at \( r \to \infty \).

Therefore, our trailing heavy quark can be considered
as the D3-brane end of the 3-7 string moving at the con-
formal boundary. Let us describe the geometry for this
configuration.

The starting point is the general Dp-brane metric at
\( T = 0 \), given by the 10-dimensional SUGRA solution

\[
dS^2 = H^{-1/2} \eta_{\mu \nu} \, dx^\mu \, dx^\nu + H^{1/2} \left[ dz^2 + L^2 \Omega_{S^{p-3}}^2 \right],
\]

with

\[
H(r) = \left( \frac{L}{r} \right)^{7-p}.
\]

In the case of \( p = 3 \), \( L \) defines the AdS\(_5\) radius.
The Dq-probe branes are spanned in the
\( \{0, 1, 2, 3, \ldots , p, \ldots , q \} \) directions of the 10-dimensional
flat target space. The set of D3-branes is \emph{embedded}
into the D7-branes, according to the coordinates
\( \nu \in \{0, 1, 2, 3, \ldots , p \} \). The brane intersection defines
a \( \mathcal{N} = 2 \) supersymmetric field theory with fundamental
fermions (quarks) living in \( d + 1 \)-dimensional conformal
boundary. This intersecting geometry is described by a
\emph{wrapped} \( S_n \) sphere inside the \( S_{8-p} \) sphere, where
\( q = d + n + 1 \), with \( d \) the euclidean directions along the
D3-system. In our particular case, considering \( d = 3 \) and
\( p = 4 \) leave us with \( n = 3 \) as the dimension of the \( \text{wrapping sphere and } q = 7 \). This embedding process
defines confinement.

In the case of the D3-brane set embedded into the D7-
brane probe system, we can write the metric as

\[
dS^2 = \left( \frac{L}{z} \right)^2 \eta_{\mu \nu} \, dx^\mu \, dx^\nu + \left( \frac{z}{L} \right)^2 \left[ \frac{L^4}{z^2} \, dz^2 + L^2 \, d\Omega_5^2 \right],
\]

where we have done the transformation \( r = L^2/z \). Now
the boundary lies at \( z \to 0 \). In this chart, the embedding
of the D3 into the D7 can be parametrized by a \emph{azimuthal
function} \( \psi = \psi(z) = \cos \theta \), where \( \theta \) is the azimuthal angle
in the \( S_5 \) sphere. This function controls the perturbations
and will give rise to the meson masses spectrum in the
flavor representation of the D7 [34]. With this choice, we
write the 5-sphere metric as

\[
d\Omega_5^2 = \frac{\psi'^2}{1 - \psi^2} \frac{L^4}{z^4} \, dz^2 + (1 - \psi^2)^2 \, d\Omega_3^2 + \psi^2 \, d\Omega_1^2,
\]

where the prime denotes the \( z \) coordinated derivative, as
we did before. Since we are considering free quarks mov-
ing on the D3-branes, we can keep just with the \( z \)-part
of the $S_5$ sphere. Therefore, the D7 embedded geometry is given by

$$d_{D7}^2 = \left(\frac{L}{z}\right)^2 \eta_{\mu\nu} dx^\mu dx^\nu$$

$$+ \left(\frac{L}{z}\right)^2 \left[1 - \frac{\psi^2 + L^2 \psi'^2}{1 - \psi^2}\right] dz^2. \quad (25)$$

The quark moving on the $3+1$-dimensional boundary can be parametrized by the 3-7 string in the static gauge in the D3-coordinates as $x_1(t, z) = vt + \xi(z)$ with $x_2 = x_3 = 0$, and $\xi(z)$ defines the profile of the 3-7 string. Following the same procedure as we did above with the other geometries, we can construct the worldsheet metric, parametrized as $\sigma_\alpha \in \{t, z\}$, as follows

$$dS_{D7}^2 = dS_{\text{String}}^2 = h_{\alpha\beta} d\sigma^\alpha d\sigma^\beta$$

with

$$dS_{\text{String}}^2 = \left(\frac{L}{z}\right)^2 \left[-(1 - \varphi^2) dt^2 + 2 \varphi \xi' dt dz\right]$$

$$+ \left(\frac{L}{z}\right)^2 \left[1 + \xi'^2 + \frac{L^2 \psi'^2}{1 - \psi^2}\right] dz^2. \quad (26)$$

The Nambu-Goto action in this case is

$$I_{NG} = -\frac{\tau}{2\pi \alpha'} \int_0^{z_{\min}} dz \left(\frac{L}{z}\right)^2 \left[1 - \varphi^2 + \xi'^2ight]$$

$$\quad + \frac{L^2 \psi'^2(1 - \varphi^2)}{1 - \psi^2} \right]^{1/2}, \quad (27)$$

where $z_{\min}$ is defined by the quark mass in the model, when $\psi(z)$ is minimized, i.e., if $\psi'(z_{\min}) = 0$ implies $z_{\min} = \frac{1}{M_g} \frac{44}{43}$. This particular point corresponds with the point where the $S_5$ shrinks. The drag force along the $x_1$-direction is defined as

$$F_{\text{Drag}} = \Pi^x_{x_1}, \quad (28)$$

where $\Pi^x_{x_1}$ is the canonical momentum calculated from the NG action as

$$\Pi^x_\mu = \frac{\delta I_{NG}}{\delta (\partial_\alpha x^\mu)} = -\frac{1}{2\pi \alpha'} \sqrt{-h} \ g_{\mu\nu} \partial_\beta x^\nu \ h^{\alpha\beta}. \quad (29)$$

In our case at hand,

$$2\pi \alpha' \Pi^x_{x_1} = \frac{L^2}{z^2} \sqrt{\frac{\xi'}{\xi'^2 + (1 - \varphi^2) \left(1 - \frac{\psi^2 + L^2 \psi'^2}{1 - \psi^2}\right)}}. \quad (30)$$

From this relation we obtain the expression for $\xi'$ as

$$\xi' = \pm \frac{\sqrt{\left(1 - \varphi^2\right) \left(1 - \psi^2 + L^2 \psi'^2\right)}}{\sqrt{1 - \frac{\varphi^4}{\pi^2} \left(2\pi \alpha' \Pi^x_{x_1}\right)^2}} \frac{z^2}{L^2} (2\pi \alpha' \Pi^x_{x_1}) \quad (31)$$

In Poincaré coordinates it means that the D7 probe brane extends over the range $z \in (0, z_{\min})$. At $z = z_{\min}$ the $S_5$ sphere shrinks and at this point the trails string ends. The $z$ coordinate is bounded from above and it leads to the range of allowed conserved momentum: $0 \leq \Pi^x_{x_1} \leq \frac{M_g^2}{z_{\min}^2}$. In this case where the energy scale $z_{\min}$ is responsible for the confinement at the dual gauge theory in a top-down frame, we also find the presence of a drag force of an isolated quark due to the non-perturbative vacuum of the gauge theory.

VII. DRAG FORCE IN THE SAKAI-SUGIMOTO MODEL

We consider in this section an isolated quark moving on the vacuum of the gauge theory dual to the $D4/D8$ background of the holographic model proposed by Sakai and Sugimoto [46, 47]. As an application of the general result given in section VI the confining bulk geometry will be responsible for the presence of a drag force experienced by the moving quark in the vacuum of the dual gauge theory. The zero temperature geometry of the gravitational background is given by

$$ds^2 = \left(\frac{U}{R}\right)^2 (\eta_{\mu\nu} dx^\mu dx^\nu + f(U) dt^2)$$

$$+ \left(\frac{R}{U}\right)^2 \frac{dU^2}{f(U)} + U^2 d\Omega_4^2, \quad (32)$$

where $x^\mu$ are the coordinates on the 4-dimensional boundary theory. The warp function $f(U)$ is given by

$$f(U) = 1 - \frac{U^3}{U^3_{KK}}. \quad (33)$$

Notice that the radial coordinate $U$ is bounded from below $U \geq U_{KK}$ and the coordinate $\tau$ is periodic with period $\tau_0 = (4\pi/3) \sqrt{R^3/U_{KK}}$.

In this scenario, the dual picture of a free quark moving in the vacuum of the dual gauge theory is a classical string attached to the quark stretching along the radial $U$ direction in the bulk. The Gubser ansatz for the string parametrization gives $x^\mu(t, U) = (t, x(t, U), 0, 0, U)$, where $x(t, U) = vt + \xi(u)$ and the compact directions in the bulk are keep constant. The Nambu-Goto action for this string configuration is given by

$$S_{NG} = \frac{1}{2\pi \alpha'} \int dt dU \sqrt{\frac{1 - \psi'^2}{f(U)} + \frac{U^3}{R^3} \left(\frac{d\xi}{dU}\right)^2}. \quad (34)$$
The conservation of $\pi_\xi = \delta S_{NG}/\delta \xi'(t, U)$ results in the first order differential equation

$$\frac{d\xi}{dU} = 2\pi \alpha' \pi_\xi \left(\frac{R}{U}\right)^{3/2} \sqrt{\frac{1-v^2}{\xi^2 - (2\pi \alpha' \pi_\xi)^2}}. \quad (35)$$

The radial direction is bounded from below as $U \geq U_{KK}$ where the trailing string ends. The above expression will be real valued provided that $\pi_\xi \leq \frac{1}{2\pi \alpha' (\frac{U_{KK}}{R})^{3/2}}$. As it was argued in [48] the limit $U_{KK} \to 0$ corresponds to the supersymmetric case where there is no confinement. On the other hand, we find here that there is a drag force in the vacuum of the dual gauge theory if $U_{KK} > 0$. This means that there will be a drag force at zero temperature even that the gauge theory stays in the confining region. This case represents another example of a top-down model that realizes a direct relation between confinement and drag force at zero temperature.

VIII. ESTIMATING THE FRACTION OF ENERGY LOSS

We have shown that at zero temperature an isolated probe quark falls the effect of a drag force when it is moving in the vacuum of a confining gauge theory. Unfortunately, the usual constraints of classical mechanics do not allow for a complete determination of the magnitude of the drag force as a function of the velocity of the quark. Here we will use some physical arguments to estimate this functional dependence and the fraction of energy transferred to the vacuum by the moving quark.

A naive way to fix $\pi_\xi(v)$ at zero temperature is by considering the finite temperature expression, and then calculating the limit where $T \to 0$. We take, for example, the case of the soft wall model [30], that at finite temperature it is found that

$$\pi_{SW}(v, T) = \frac{\pi^2 T^2 R^2}{2 \alpha'} e^{-\frac{\sqrt{v^2 + \pi^2}}{T}}, \quad (36)$$

The corresponding zero temperature limit is divergent, i.e., $T \to 0 \Rightarrow \pi_{SW} \to \infty$. The expression for $\pi_{SW}(v, T)$ comes from the presence of a black hole in the bulk geometry. But this particular setup is not stable at low temperatures. At some critical temperature $T_c$, the bulk geometry undergoes a Hawking-Page transition from the black hole phase to a thermal AdS phase, where there is no black hole. The critical temperature for the Hawking-Page transition [49] is zero for conformal gauge theories and is non-vanishing for non-conformal gauge theories [30, 51]. In a confining gauge theory, we cannot obtain the zero temperature expression for $\pi_\xi$ by performing the $T \to 0$ limit in the expression obtained from the black hole phase of the bulk geometry.

We assume that if the quark does not move, there is no drag force acting on it, i.e., $\pi_\xi(0) = 0$. The drag force comes from the relative motion between the quark and an external observer that looks at the vacuum state of the gauge theory. For very small velocities, the drag force should be linear in the velocity ($v \ll 1 \Rightarrow \pi_\xi \sim v$).

On the other hand, special relativity imposes that $v < 1$ for a time-like particle, such that the maximum drag will be reached when $v \to 1$ or, equivalently, in the limit $\gamma v \to \infty$, where $\gamma = 1/\sqrt{1-v^2}$ is the Lorentz factor. Thus, we use here an estimation of $\pi_\xi(v)$ given by the following smooth function interpolating both regions, i.e.,

$$\pi_\xi(v) = \pi_{\max}(1-e^{-\gamma v}). \quad (37)$$

In the above expression $\pi_{\max} = \min \left(\frac{A(z)}{\pi z}\right)$, where $A(z)$ is defined in eq. [18]. For each model considered here, we have the following expressions for the transferred momentum

$$\pi_{SW} = \frac{R^2}{2 \pi \alpha'} e^k, \quad (38)$$
$$\pi_{SS} = \frac{1}{2\pi \alpha'} \left(\frac{U_{KK}}{R}\right)^{3/2}, \quad (39)$$
$$\pi_{D3/D7} = \frac{1}{2\pi \alpha'} M_q^2 L^2. \quad (40)$$

The large mass of the heavy quarks allows us to model their classical motion as non-relativistic, since most of its energy will be inertial. Therefore we can take the approximation $\gamma v \ll 1$. In this approximation we have that $e^{-\gamma v} \simeq 1 - \gamma v$ leading to a momentum $\pi_\xi = \pi_{\max} \gamma v$. The holographic dictionary maps this conserved momentum along the holographic coordinate to the momentum loss by the dual heavy quark on the boundary $\frac{dp}{dt} = -\frac{1}{2 \pi \alpha'} \pi_\xi$. Thus, we have that

$$\frac{dp}{dt} = -\frac{\pi_{\max}}{2 \pi \alpha'} \gamma v = -\frac{\pi_{\max}}{2 \pi \alpha' m_q} p(t), \quad (41)$$

where the relativistic momentum is $p = m_q \gamma v$. The above equation is a first-order one and can be directly integrated, obtaining the loss of momentum by the heavy quark moving on the vacuum

$$p(t) = p(0) e^{-\frac{t}{t_0}}, \quad t_0 = \frac{2 \pi \alpha' m_q}{\pi_{\max}}. \quad (42)$$

Notice that in the above equation we have defined the characteristic time $t_0$, which in the finite temperature case, is interpreted as a diffusion time. Nevertheless, in the present case, where the medium of propagation is the vacuum, a different interpretation is required.

Usually, the interpretation is related to the jet formation phenomena as the vacuum analogue of thermal diffusion. We also notice that for an isolated quark $q$ the
time is \( t_0 \sim 1/\pi_{\text{max}} \) with \( \pi_{\text{max}} = f(z_0) \), while for the corresponding \( q \bar{q} \) meson, the string tension (obtained from Wilson Loop calculations) is given by \( \sigma = f(z_0) \). Hence, our estimation of \( \pi_\xi(v) \) reveals that the diffusion time is inversely proportional to the string tension, i.e., \( t_0 \sim \frac{1}{\sigma} \).

The stronger the \( q \bar{q} \) pair is glued together the fast one of them (\( q \) or \( \bar{q} \) will hadronize in the vacuum.

We use the estimation for the time dependence of the quark spatial momentum, eqn. [42], to estimate the fraction of the initial quark energy transferred to the medium (the total energy of the formed jet). The energy of a particle with spatial momentum \( p \) and rest mass \( m \) is given by \( E(p) = \sqrt{m^2 + p^2} \). We suppose that the quark lost energy is until \( t = t_0 \) and let \( p(0) = p_0 \) be the initial (production) quark momentum. Then the fraction of energy lost is estimated

\[
\frac{\delta E}{E} \sim \frac{E(t_0) - E(0)}{E(0)} = -1 + \frac{1}{e} \sqrt{\frac{e^2 + \left(\frac{m}{p_0}\right)^2}{1 + \left(\frac{m}{p_0}\right)^2}}.
\]

(43)

It is remarkable that in the very heavy quark mass limit we have \( \frac{m}{p_0} \rightarrow 0 \), then \( \frac{\delta E}{E} \rightarrow 0 \). While in the limit of very light quarks \( \frac{m}{p_0} \rightarrow \infty \), and hence \( \frac{\delta E}{E} \rightarrow -(e^{-1} - 1) \sim 0.632 \). These simple estimations provide a piece of evidence that heavy quarks will transfer a smaller fraction of its initial energy to the hadronic jet formed in comparison to the energy a light quark transferred to its corresponding hadronic jet.

This phenomenon is known as the dead cone effect [52–58]. This estimation suggests that the dead cone effect is a consequence of the momentum damping and it will be observed given that the final momentum is smaller than the initial momentum. If \( p_f = \frac{p_0}{\Lambda} \), \( \Lambda > 1 \) we get that \( \frac{\delta E}{E} \rightarrow 0 \) in the limit \( \frac{m}{p_0} \rightarrow 0 \) while that in the limit \( \frac{m}{p_0} \rightarrow \infty \) gives \( \frac{\delta E}{E} \rightarrow -\left(\Lambda^{-1} - 1\right) < 0 \). In our case, the existence of a drag force provides the momentum damping of the isolated quark that moves across the vacuum. Consequently, we expect that the dead cone effect will take place as a signature of confining in these holographic dual models.

**IX. CONCLUSIONS AND FINAL REMARKS**

The present results show that the property of confinement in gauge theories can be probed in several forms, making it something multifaceted. The same property that avoids mesons to split into an isolated quark and anti-quark states also prevents them to move freely in the vacuum. However, if any of these two situations (confined or deconfined quarks) are considered to represent the initial conditions, there will be no free quarks moving on the vacuum as the final (asymptotic) state.

On the other hand, it has been shown that for a string representing a meson, with both endpoints attached at the boundary, there is no such resistance [10]. In the bulk, we face that the configuration of many disconnected (infinite) pieces of the string, representing mesons moving with a small fraction of the original momentum, is energetically favored in comparison with an infinitely long string. Consequently, the initial string, attached to the isolated quark, will break into a final state of infinitely many pieces of strings attached to \( q \bar{q} \) pairs. This process gives us a holographic picture of jet formation. The physical picture of jet formation that emerges in this holographic approach matches the early proposal by Field and Feynman [21] and agree qualitatively with the idea that there is a deep connection between quark propagation in vacuum and the hadronization process [59].

An important experimental aspect regarding QGP formation is the back to back jet suppression [60]. This phenomenon works as follows: some high energy virtual photon decays into a \( q \bar{q} \) pair that moves in opposite directions with high momentum. If the pair is created in the QGP fireball neighborhood, one quark will go inward the QGP medium and the other will go outward. The inward quark feels the interactions of the hot medium where it thermalizes and its associated jet is quenched, while the outgoing quark leaves the QGP and penetrates the vacuum with large momentum producing a jet of hadrons. The present results complement the holographic picture of this physical mechanism by discussing the dynamics of the isolated quark that penetrates the vacuum. In the conformal, non-confining, scenario there’s no jet at all since the outgoing quark will move freely at a constant velocity, thus it will not hadronize. Jets in the vacuum will appear at the non-conformal, confining, scenario only.

**Acknowledgments**

We acknowledge the financial support of FONDECYT (Chile) under Grants No. 1180572 (A. V.) and No. 3180592 (M. A. M. C.).

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