Surface roughness prediction in ultrasonic vibration-assisted milling

Yixuan FENG*, Fu-Chuan HSU**, Yu-Ting LU**, Yu-Fu LIN**, Chorng-Tyan LIN**, Chiu-Feng LIN**, Ying-Cheng LU**, Xiaohong LU***, and Steven Y. LIANG*

*Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA, 30332, USA
E-mail: yfeng82@gatech.edu

**Metal Industries Research and Development Centre (MIRDC), Kaohsiung, Taiwan

***Key Laboratory for Precision and Non-traditional Machining Technology of Ministry of Education, Dalian University of Technology, Dalian, Liaoning, People’s Republic of China

Received: 22 December 2019; Revised: 26 April 2020; Accepted: 2 June 2020

Abstract
Better surface finish is believed to be one of the most important benefits of ultrasonic vibration-assisted milling. Many studies have shown that this benefit is most significant during slot-milling of a part when the vibration is applied in feed direction. To explicitly explain this phenomenon, an analytical model is proposed to predict surface roughness based on the trajectory of tool and the response of machined surface. The overall machined surface profile under tool trajectory depends on the tool tip movement, in addition to the tool deformation under cutting force and tool tip geometry. The movement of tool tip is governed by feed, spindle rotation, and ultrasonic vibration. The tool deformation depends on the milling force and stiffness. The geometry of tool tip is characterized by the tip radius and angle. Besides surface profile under tool trajectory, the response of machined surface is considered by assuming pure elastic deformation when the actual instantaneous cutting thickness is smaller than a critical value. In that case, part of the material is recovered so the actual machined surface profile is different from the profile under tool trajectory only. Surface roughness is then calculated based on the actual surface profile. Experiments are conducted on Aluminum alloy in both conventional and ultrasonic vibration-assisted milling under different spindle speed, feed, and vibration amplitude. Through the comparison between the analytical predictions and experimental measurements, the proposed model has high accuracy in all cases with average percentage error of 15%.

Keywords: Surface roughness, Modeling, Slot, Ultrasonic vibration-assisted milling, Minimum cutting thickness

Nomenclature

| Symbol | Description |
|--------|-------------|
| $N_t$  | Number of flutes |
| $f_z$  | Feed per tooth |
| $\phi$ | Angle of rotation |
| $R$    | Radius of tool |
| $A_e$  | Vibration amplitude |
| $\omega$ | Angular vibration frequency |
| $F$   | Milling force |
| $x_e$ | Elastic deformation of milling tool |
| $K$   | Stiffness of tool |
| $r_t$ | Tool tip radius |
| $\alpha_e$ | Tool tip angle |
| $t_{\text{min}}$ | Minimum cutting thickness |
| $t_c$ | Radial cutting thickness |
| $R_a$ | Arithmetic average surface roughness |
| $\sigma_{JC}$ | Reference stress |
Intrinsic length scale

\( \sigma_{\text{fracture}} \) Flow stress

Radius of tool edge

Size of deformation zone

Spindle speed

Maximum height of machined surface profile

1. Introduction

Surface roughness is an important indicator of machined surface quality. With the development of vibration-assisted machining and milling, people have been investigating the effects of vibration on surface roughness for years. Abdur-Rasheed (2011) summarized the measurements on machined surface after vibration-assisted machining for various workpiece and tool combinations. The surface roughness drops for all cases when comparing with conventional orthogonal cutting with a decrease percentage varying from 40 to 70%. Razfar et al. (2011) measured surface roughness of AISI 1020 steel after conventional and ultrasonic vibration-assisted milling when the vibration was applied in feed direction. Again, the surface roughness is improved in all vibration cases by 12.9% in average. In addition, the cutting parameters have similar effects on surface roughness, as increasing feed or spindle speed will result in higher surface roughness after both conventional and ultrasonic vibration-assisted milling. The depth of milling has insignificant effect on surface roughness. Besides cutting parameters, tool geometry including tool radius and tool edge radius has a more dominant effect as confirmed by Xiao et al. (2003) and Hsu et al. (2009). In order to maximize the benefit of ultrasonic vibration-assisted milling in terms of reduced surface roughness, the direction of ultrasonic vibration being applied with respect to the workpiece needs to be correctly chosen as it will also influence the resultant surface roughness. According to the measurements obtained by Halim et al. (2017), the surface roughness was higher in ultrasonic vibration-assisted milling when the vibration was applied in axial direction. In addition, Ko et al. (2010, 2011) and Chen et al. (2018) compared machined surface quality after feed directional and cross-feed directional vibration-assisted milling with conventional milling. A better surface quality was reached in feed directional ultrasonic vibration-assisted milling, while the surface roughness became higher in some cases after cross-feed directional vibration-assisted milling. As shown in Fig.1, with the vibration in the same direction as feed movement, the uncut material area due to gap between tool edges is smaller. Gao and Sun (2015) did analytical predictive modeling of surface roughness for two-dimensional vibration-assisted machining based on kinematics and tool geometry analysis, but the effects of forces and workpiece recovery are ignored.

Analytical predictive modeling of surface roughness has been done on other milling processes. Lu et al. (2018) predicted surface roughness in micro-milling. The tool trajectory is described by a two degree of freedom dynamic model, and the response of workpiece is characterized by the elastic recovery in terms of minimum cutting thickness. Feng et al. (2018a, 2019a) later applied a similar modeling method in laser-assisted milling by considering the effect of laser preheating temperature on elastic recovery. In current study, an analytical model of surface roughness on slot in feed directional ultrasonic vibration-assisted milling is proposed. In addition to tool rotation and feed movement, the proposed model predicts the overall movement of the milling tool under vibration in feed direction. The tool deformation is also factored into account based on milling forces (Feng, et al. 2018b; Pan, et al. 2017a, 2017b). The surface roughness of machined surface is then calculated according to profile under tool trajectory and the elastic recovery. Ultrasonic vibration-assisted milling experiments are conducted on Aluminum alloy 2A12 workpiece (Shen, et al. 2011). Surface roughness of slot bottom surface is measured and compared with conventional milling under different feed and spindle speed. And the effects of cutting and vibration parameters on surface roughness are analyzed under three levels of spindle speed, three levels of feed per tooth, and two levels of vibration amplitude. Up to now, this is the first analytical predictive modeling done on surface roughness considering kinematics, tool geometry, ultrasonic vibration, cutting force, and elastic recovery of workpiece.
2. Analytical modeling of surface roughness with ultrasonic vibration

The surface roughness of machined surface depends on the movement of the tool, interaction between tool and workpiece, recovery of machined surface, random vibration such as chatter or higher mode vibration, and inaccuracy of machining system such as runout of spindle. In current study, kinematic analysis is conducted to describe the tool movement. The tool deflection is considered to address the interaction. The minimum cutting thickness is applied to characterize the recovery of machined surface. Random vibration and runout are unpredictable without extra inputs and varied case by case but can be included if corresponding measurements have been collected through sensors.

2.1 Trajectory of milling tool

The machined surface profile is calculated based on the tool trajectory and the workpiece response in this paper. Kinematic analysis is first conducted to describe the trochoidal path of cutting edge. Tool center is under feed movement and ultrasonic vibration. And additional movement of tool rotation is considered for tool tip movement. If the number of flutes is \( N_t \) and feed per tooth is \( f_z \), the feed movement is \( N_t f_z \) by one round. As shown in Fig.2, the position of \( k^{th} \) cutting edge characterized by \( x_f \), \( y_f \), and \( z_f \) is

\[
\begin{align*}
    x_f &= A_v \sin(\varphi t) + f_z \left( \frac{N_t \varphi}{2\pi} \cdot + k - 1 \right) + R \sin \varphi \\
    y_f &= R \cos \varphi \\
    z_f &= 0
\end{align*}
\]

(1)

where \( \varphi \) is the angle of rotation, \( R \) is the radius of tool, \( A_v \) is the vibration amplitude, and \( \varphi \) is angular vibration frequency. As tool chatter and other vibrations are not included, \( z_f \) is assumed to be zero.
Along the slot centerline, the machined surface is at $\varphi = 90^\circ$ or $\pi/2$. Therefore, the tool position is simplified as

$$
\begin{align*}
  x_f &= A_x \sin(\omega t) + f_x(\frac{N_f}{4} + k - 1) + R \\
  y_f &= 0 \\
  z_f &= 0
\end{align*}
$$

(2)

During ultrasonic vibration-assisted milling, the vibration frequency is typically above 15 kHz, while the spindle speed is below 15,000 rpm or 250 Hz. Since the ultrasonic vibration frequency is much higher than tool rotation frequency, it is assumed that $\varphi$ is a constant or the tool is not rotating at each moment, and the cutting edge is only under vibration. Therefore, the resultant tool trajectory is calculated by shifting the tool tip profile by $A_x$ in both positive and negative feed direction. The detailed calculation of tool tip profile is described in section 2.3.

2.2 Elastic deformation of tool

During ultrasonic vibration-assisted milling, the tool could be deflected under cutting force. In current study, the surface roughness is predicted along the centerline of the groove ($\varphi = 90^\circ$), and the machined surface profile is calculated within the x-z plane, so the force in y direction is not considered. In addition, most of the unpredictable vibrations such as chatter occur in z direction, so only the tool deflection in x direction is calculated. Based on the force prediction model (Feng, et al. 2018b; Pan, et al. 2017a, 2017b), the milling force $F$ in feed direction is calculated, which causes the elastic deformation of milling tool $x_e$

$$
x_e = \frac{F}{K}
$$

(3)

where $K$ is the stiffness of tool. The tool deformation is a dynamic process dependent on vibration mode and instantaneous milling force. However, this process is simplified in current study as the surface roughness along the centerline of the groove is only influenced by the tool deformation at $\varphi = 90^\circ$. The average force at $\varphi = 90^\circ$ is calculated based on the predicted instantaneous force (Feng, et al. 2019b). And the dynamic effects at $\varphi = 90^\circ$ are assumed unchanged and considered through a constant coefficient $K$.

2.3 Geometry of tool tip

Besides the cutting edge trajectory and the effect of cutting force, the detailed machined surface profile is largely dependent on geometry or profile of tool tip. The flat-nose end mill is modeled. Because the surface roughness is on $\mu m$ level, the tool tip cannot be assumed perfectly sharp even for flat-nose end mill. In current study, the tool tip geometry is described following the coordinate system in Fig.3. The shape of the tool tip is described by radius $r_e$ and angle $\alpha_e$, and the origin is on front cutting edge and at same depth as the tool tip center.

Under an angular coordinate $\alpha$ between 180° and 360°, the tool tip profile is

$$
x_p = \begin{cases} 
  \sin \varphi \left( -r + \frac{r}{\cos(\frac{3\pi}{2} - \alpha - \alpha_e)} \right) \cos \alpha, & \pi \leq \alpha \leq \frac{3\pi}{2} - \alpha_e \\
  r_e \sin \varphi (\cos \alpha - 1), & \frac{3\pi}{2} - \alpha_e < \alpha \leq 2\pi 
\end{cases}
$$

(4)
\[
\begin{align*}
y_p &= \begin{cases} 
  \cos \varphi \left(-r_f + \frac{r_i}{\cos \left(\frac{3\pi}{2} - \alpha - \alpha_e\right)}\right) \cos \alpha, & \pi \leq \alpha \leq \frac{3\pi}{2} - \alpha_e \\
r_i \cos \varphi(\cos \alpha - 1), & \frac{3\pi}{2} - \alpha_e < \alpha \leq 2\pi 
\end{cases} \\
z_p &= \begin{cases} 
  \frac{r_i}{\cos \left(\frac{3\pi}{2} - \alpha - \alpha_e\right)} \sin \alpha, & \pi \leq \alpha \leq \frac{3\pi}{2} - \alpha_e \\
r_i \sin \alpha, & \frac{3\pi}{2} - \alpha_e < \alpha \leq 2\pi 
\end{cases}
\end{align*}
\]

At slot centerline, \(
\varphi = 90^\circ, \ y_p
\) is always zero, so the tip profile is within the \(x-z\) plane. As discussed in section 2.1, the tool tip profile is shifted by \(2A_x\) to consider the effect of ultrasonic vibration. Therefore, instead of describing the vibration in tool trajectory, the tool tip profile is assumed to be \(2A_x\) wider, and the tool trajectory is simplified as conventional milling. Before \(z_p\) reaches its minimum \(z_{\min}\) or \(\alpha < 270^\circ\), the profile is shifted by \(A_x\) in opposite feed direction. After \(z_p\) reaches its minimum \(z_{\min}\) or \(\alpha > 270^\circ\), the profile is shifted by \(A_x\) in feed direction. The two portions are connected by a horizontal line with length of \(2A_x\) at \(z_p = z_{\min}\). And \(x_p\) is recalculated as \(x_p'\):

\[
\begin{align*}
x_p' &= \begin{cases} 
  \sin \varphi \left(-r_f + \frac{r_i}{\cos \left(\frac{3\pi}{2} - \alpha - \alpha_e\right)}\right) - A_x, & \pi \leq \alpha \leq \frac{3\pi}{2} - \alpha_e \\
r_i \sin \varphi(\cos \alpha - 1) - A_x, & \frac{3\pi}{2} - \alpha_e < \alpha \leq \frac{3\pi}{2} \\
r_i \sin \varphi(\cos \alpha - 1) + A_x, & \frac{3\pi}{2} < \alpha \leq 2\pi 
\end{cases}
\end{align*}
\]
Therefore, the position of any point on cutting edge is determined by $k$ and $\alpha$ as

$$
\begin{align*}
    x &= x_f - A_x \sin(\omega t) + x_e + x_p, \\
    z &= z_f + z_p
\end{align*}
$$

(8)

### 2.4 Response of machined surface

As illustrated in Fig. 4, tool trajectory described by Eq. (8) is shown in dashed line. Due to the gap $X_i$ between $i^{th}$ and $(i+1)^{th}$ flutes or feed per tooth, the actual cutting thickness varies at different tool tip position. It is believed that when the thickness is small enough, elastic deformation will occur instead of chip. The recovered material after machining is indicated as the dashed area in Fig. 4. Therefore, the dependency of surface roughness on the elastic response of machined surface is presented.

![Fig. 4 Difference between tool trajectory (dashed line) and machined surface profile (solid line)](image)

For the sake of simplicity, it is assumed that a critical value called minimum cutting thickness $t_{\text{min}}$ exists as the boundary between shear-dominant and plowing-dominant milling. The radial cutting thickness $t_c$ is defined as the distance between the point on $(i+1)^{th}$ flute and tool tip center of $i^{th}$ flute exceeding tip radius $r_i$

$$
    t_c = \sqrt{(x_{i+1} - x_o)^2 + (z_{i+1} - z_o)^2} - r_i
$$

(9)

The determination of $t_{\text{min}}$ is presented in section 4. In the case of $t_c \geq t_{\text{min}}$, the ultrasonic vibration-assisted milling is shear-dominant with material removal as chip. And the surface roughness is purely dependent on the tool trajectory. In the case of $t_c < t_{\text{min}}$, the ultrasonic vibration-assisted milling is plowing-dominant with only elastic deformation of workpiece. The surface roughness is then calculated based on tool trajectory and the dashed area in Fig. 4. The new profile is determined by partial boundary of dashed area where $t_c = t_{\text{min}}$. Examples of predicted tool trajectory and machined surface profile in conventional and ultrasonic vibration-assisted milling are shown in Fig. 5 and Fig. 6, respectively. Fig. 5 (a) and Fig. 6 (a) show the tool trajectory in large scale, while Fig. 5 (b) and Fig. 6 (b) show the details of machined surface profile. The colored lines indicate the trajectory of flutes, and the black lines indicate the machined surface profile. The differences between colored and black lines represent the elastic recovery of material. And the wider horizontal portions at the bottom of profile in Fig. 6 (b) are due to the ultrasonic vibration in feed direction. The arithmetic average surface roughness $R_a$ is then calculated as the mean deviation from average height of the black line, between 0 and $Z_{\text{max}}$, as

$$
    R_a = \frac{1}{Z_{\text{max}}} \int_0^{Z_{\text{max}}} |z - z_{\text{avg}}| \, dz
$$

[DOI: 10.1299/jamdsm.2020jamdsm0063] © 2020 The Japan Society of Mechanical Engineers
where $m$ is the number of data points within each portion of profile and $\bar{Z}_i$ is the average height of the assessed profile between $i^{th}$ and $(i+1)^{th}$ flutes.

\[
\bar{Z}_i = \frac{\sum_{j=1}^{m} Z_{i,j}}{X_i}
\]

Fig. 5  Predicted (a) tool trajectory and (b) machined surface profile in conventional milling with each different color representing one tool tip contour
3. Validation by experimental measurement

The experiments are conducted under a five-axis machining center by Shen et al. (2011). An ultrasonic vibrator drives the workpiece to vibrate along the feed direction with a frequency of 19.58 kHz. The vibrator is composed of a transducer and a booster. The whole setup is equipped on a DECKEL MAHO five-axis machining center. The workpiece material is Aluminum alloy 2A12. The size of the workpiece is 20×18×6 mm. The milling tool is made of carbide with two flutes and diameter of 2 mm. Through scanning electron microscope, the tool tip radius and angle are measured as 7 µm and 5°, respectively. The axial depth of milling is 0.5 mm. Each combination of cutting parameters was tested twice. In addition, each surface is measured six times by optical interferometer, and the average value is recorded. The optical interferometer is Wyko NT9300 with resolution of 0.1 nm. For the first part of the experiments, both conventional and ultrasonic vibration-assisted milling with an amplitude of 4 µm are conducted. The comparisons
are made under a feed per tooth of 2, 4, 6, and 8 \( \mu m \), when the spindle speed is 5,000 \( \text{rpm} \). In addition, under a constant feed per tooth of 4 \( \mu m \), comparisons are made under spindle speed of 1,000, 5,000, 9,000, and 13,000 \( \text{rpm} \). For the second part of the experiments, the effects of cutting and vibration parameters on surface roughness are analyzed. A total of 18 measurements are collected under spindle speed of 5,000, 9,000, and 13,000 \( \text{rpm} \), feed per tooth of 4, 6, and 8 \( \mu m \), and vibration amplitude of 4 and 7 \( \mu m \). A monitoring system is used, and no significant parasitic vibration is observed. The system also monitors the real-time vibration amplitude through strain gage, and the desired amplitude is maintained during the experiment.

4. Results and discussions

Fig. 7 shows both experimental measurements and analytical predictions from proposed model when comparing surface roughness after conventional milling to surface roughness after ultrasonic vibration-assisted milling. The error bars indicate good repeatability of the measurement as the maximum deviation is less than 8\% of average. Under the spindle speed of 5,000 \( \text{rpm} \), the surface finish has a huge improvement as the surface roughness drops 70\% in average when the vibration is applied, as shown in Fig.7 (a). When the spindle speed increases to 9,000 and 13,000 \( \text{rpm} \), the benefit of ultrasonic vibration is less significant as the tool rotation frequency is getting closer to vibration frequency, but the decrease of surface roughness is still over 40\%, as shown in Fig.7 (b). The predicted \( R_a \) matches the measurements with an average percentage error of 3.6\% and 13.4\% for conventional and ultrasonic vibration-assisted milling, respectively, under a constant spindle speed. These two errors are 6.4\% and 25.7\% under an increasing spindle speed in Fig.7 (b). The predictive model is less accurate in ultrasonic vibration-assisted milling or under a higher spindle speed as both higher vibration and tool rotation frequencies will bring more error sources such as tool chatter during the experiments, which are not accounted for in predictive model.

![Fig. 7](image-url)
Besides showing the benefits of feed directional ultrasonic vibration-assisted milling, the effects of feed per tooth, spindle speed, and vibration amplitude on surface roughness are reflected through additional experiments as summarized in Figs. 8 and 9. When the feed per tooth increases, the gap between two flutes on machined surface is larger, leading to higher surface roughness. For ultrasonic vibration-assisted milling, when the spindle speed increases, the increase of cutting speed results in larger deformation zone, which is considered as a larger $t_{\text{min}}$ in predictive model, and the surface roughness becomes higher. When the vibration amplitude increases, the benefit of ultrasonic vibration is more significant, and the surface roughness gets lower. The proposed predictive model is able to match the trends in all cases. The average percentage error is 17.3% when the vibration amplitude is 4 $\mu m$ as shown in Fig. 8. The average percentage error is 10.9% when the vibration amplitude is 7 $\mu m$ as shown in Fig. 9.

![Measured surface roughness under vibration amplitude of 4 $\mu m$](image)

![Predicted surface roughness under vibration amplitude of 4 $\mu m$](image)

Fig. 8  Comparison of surface roughness between (a) measurements and (b) predictions under vibration amplitude of 4 $\mu m$. 

[DOI: 10.1299/jamdsm.2020jamdsm0063] © 2020 The Japan Society of Mechanical Engineers
Analysis of variance (ANOVA) is conducted on experimental measurements with results listed in Table 1. Based on the percentage of contribution, spindle speed has the most significant effect on surface roughness, followed by vibration amplitude and feed per tooth.

Table 1 ANOVA analysis of measured surface roughness

| Source               | Degree of freedom | Sum of square | Mean square | Percentage of contribution |
|----------------------|-------------------|---------------|-------------|---------------------------|
| Spindle speed        | 2                 | 0.306         | 0.153       | 53%                       |
| Feed per tooth       | 2                 | 0.073         | 0.037       | 13%                       |
| Vibration amplitude  | 1                 | 0.196         | 0.098       | 34%                       |

The selection of minimum cutting thickness $t_{\text{min}}$ is critical in current study in order to predict the machined surface profile with high accuracy. Several methods have been proposed so far to determine $t_{\text{min}}$ based on tool geometry, material properties, and cutting parameters. According to Lu et al. (2017), $t_{\text{min}}$ is

Fig. 9  Comparison of surface roughness between (a) measurements and (b) predictions under vibration amplitude of 7 $\mu$m
\[ t_{\text{min}} = r_e \left[ 1 - \cos \left( \frac{180 \times l}{2 \pi \times r_e (\sigma_{\text{fracture}} / \sigma_{\text{JC}})^2} \right) \right] \]  \hspace{1cm} (12)

where \( \sigma_{\text{JC}} \) is reference stress and \( l \) is the intrinsic length scale, both of which are dependent on material properties. \( \sigma_{\text{fracture}} \) is the flow stress which also depends on cutting parameters. \( r_e \) is the radius of tool edge. According to Wu et al. (2009), \( t_{\text{min}} \) is

\[ t_{\text{min}} = r_e \left[ 1 - \cos \left( \frac{180 \times L}{\pi \times r_e} \right) \right] \]  \hspace{1cm} (13)

where \( L \) is the size of deformation zone. In current study, as the same combination of tool and workpiece is used throughout all experiments, \( t_{\text{min}} \) is only dependent on cutting parameters in conventional milling as

\[ t_{\text{min}} = 0.525 - 4.5 \times 10^{-2} n + 0.19 f_z + 0.035 f_z^2 \]  \hspace{1cm} (14)

where \( n \) is the spindle speed. For ultrasonic vibration-assisted milling, \( t_{\text{min}} \) is also dependent on vibration amplitude as

\[ t_{\text{min}} = -2.7 \times 10^{-4} n + 0.8 f_z + A_x \]  \hspace{1cm} (15)

\( t_{\text{min}} \) is therefore accurately predicted and used as a boundary of shear-dominant and plowing-dominant processes in ultrasonic vibration-assisted milling, as the deflection of machined surface imparted by the tool is a combination of elastic and plastic deformation. The values of \( t_{\text{min}} \), predicted \( R_a \), and predicted maximum height of machined surface profile (\( R_z \)) in all cases are listed in Tables 2 and 3 for reference.

### Table 2 Minimum cutting thickness and surface roughness in predictions from Fig.7

| \( n \) (rpm) | \( f_z \) (\( \mu \)m) | \( A_x \) (\( \mu \)m) | \( t_{\text{min}} \) \( (10^{-2} \mu \)m) | \( R_a \) (\( \mu \)m) | \( R_z \) (\( \mu \)m) |
|---------------|----------------|----------------|----------------|----------------|----------------|
| 5000          | 2              | \( 0 \)         | 0.8            | 1.29           | 3.86           |
| 5000          | 4              | \( 0 \)         | 1.6            | 1.29           | 3.80           |
| 5000          | 6              | \( 0 \)         | 2.7            | 1.57           | 4.79           |
| 5000          | 8              | \( 0 \)         | 4.1            | 1.95           | 6.11           |
| 5000          | 2              | \( 4 \)         | 3.4            | 0.32           | 1.33           |
| 5000          | 4              | \( 4 \)         | 5.0            | 0.45           | 1.99           |
| 5000          | 6              | \( 4 \)         | 6.6            | 0.54           | 2.03           |
| 5000          | 8              | \( 4 \)         | 8.2            | 0.61           | 2.45           |
| 1000          | 4              | \( 0 \)         | 1.8            | 1.62           | 4.78           |
| 1000          | 4              | \( 0 \)         | 1.6            | 1.29           | 3.80           |
| 1000          | 4              | \( 0 \)         | 1.4            | 0.98           | 2.94           |
| 1000          | 4              | \( 0 \)         | 1.3            | 0.74           | 2.25           |
| 1000          | 4              | \( 0 \)         | 4.6            | 0.36           | 1.61           |
| 1000          | 4              | \( 0 \)         | 5.0            | 0.45           | 1.99           |
| 1000          | 4              | \( 0 \)         | 5.4            | 0.55           | 2.36           |
| 13000         | 4              | \( 0 \)         | 5.8            | 0.67           | 2.76           |
| n (rpm) | f (µm) | A (µm) | t_{min} (10^{1} µm) | R_{a}(µm) | R_{z}(µm) |
|---------|--------|--------|----------------------|-----------|-----------|
| 5000    | 4      |        | 5.0                  | 0.45      | 1.99      |
|         | 6      |        | 6.6                  | 0.54      | 2.03      |
|         | 8      |        | 8.2                  | 0.61      | 2.45      |
| 9000    | 4      | 4      | 5.4                  | 0.55      | 2.36      |
|         | 6      |        | 7.0                  | 0.65      | 2.47      |
|         | 8      |        | 8.6                  | 0.72      | 2.51      |
| 13000   | 4      |        | 5.8                  | 0.67      | 2.76      |
|         | 6      |        | 7.4                  | 0.77      | 2.83      |
|         | 8      |        | 9.0                  | 0.87      | 2.97      |
| 5000    | 4      | 7      | 8.0                  | 0.39      | 1.78      |
|         | 6      |        | 9.6                  | 0.42      | 1.79      |
|         | 8      |        | 11.2                 | 0.43      | 1.63      |
| 9000    | 4      |        | 8.4                  | 0.46      | 2.01      |
|         | 6      |        | 10.0                 | 0.48      | 1.99      |
|         | 8      |        | 11.6                 | 0.50      | 1.85      |
| 13000   | 4      |        | 8.8                  | 0.53      | 2.28      |
|         | 6      |        | 10.4                 | 0.55      | 2.19      |
|         | 8      |        | 12.0                 | 0.58      | 2.11      |

5. Conclusions

In current study, the surface roughness of slot after feed directional ultrasonic vibration-assisted milling is analyzed through analytical predictive model. The effect of ultrasonic vibration on surface profile is considered through shift of the tool trajectory by two vibration amplitude along the feed direction. And the difference between machined surface profile and tool trajectory is characterized by the minimum cutting thickness. Experiments are conducted on Aluminum alloy in both conventional and ultrasonic vibration-assisted milling under different spindle speed, feed, and vibration amplitude. Through the comparison between the analytical predictions and experimental measurements, the following conclusions are summarized:

- Under the spindle speed of 5,000 rpm, the surface finish has a huge improvement as the surface roughness drops 70% in average when the vibration is applied. When the spindle speed increases to 9,000 and 13,000 rpm, the benefit of ultrasonic vibration is less significant, but the decrease of surface roughness is still over 40%.
- For ultrasonic vibration-assisted milling, lower feed per tooth, lower spindle speed, or higher vibration amplitude will decrease the surface roughness.
- The proposed model has high accuracy in all cases. For the first group of experiments, the average percentage error is 13.4% under spindle speed of 5,000 rpm. The average percentage error is 25.7% under feed per tooth of 4 µm. For the second group of experiments, the average percentage error is 17.3% when the vibration amplitude is 4 µm. The average percentage error is 10.9% when the vibration amplitude is 7 µm.

The accuracy of the model can be further improved in the future with additional inputs of parasitic vibration and spindle runout.

References

Abdur-Rasheed, A., A Fundamental Study of Vibration Assisted Machining, Advanced Materials Research. Vol. 264-265 (2011), pp. 1702-1707.
Chen, W., Huo, D., Shi, Y. and Hale, J. M., State-of-the-art review on vibration-assisted milling: principle, system
design, and application, The International Journal of Advanced Manufacturing Technology. Vol. 97 (2018), pp. 2033-2049.

Feng, Y., Hung, T.-P., Lu, Y.-T., Lin, Y.-F., Hsu, F.-C., Lin, C.-F., Lu, Y.-C., Lu, X. and Liang, S. Y., Inverse Analysis of Inconel 718 Laser-Assisted Milling to Achieve Machined Surface Roughness, International Journal of Precision Engineering and Manufacturing. Vol. 19 (2018a), pp. 1611-1618.

Feng, Y., Hung, T.-P., Lu, Y.-T., Lin, Y.-F., Hsu, F.-C., Lin, C.-F., Lu, Y.-C., Lu, X. and Liang, S. Y., Surface roughness modeling in Laser-assisted End Milling of Inconel 718, Machining Science and Technology, (2019a), pp. 1-19.

Feng, Y., Lu, Y.-T., Lin, Y.-F., Hung, T.-P., Hsu, F.-C., Lin, C.-F., Lu, Y.-C. and Liang, S. Y., Inverse analysis of the cutting force in laser-assisted milling on Inconel 718, The International Journal of Advanced Manufacturing Technology. Vol. 96 (2018b), pp. 905-914.

Feng, Y., Hsu, F., Lu, Y., Lin, Y., Lin, C., Lin, C., Lu, Y. and Liang, S.Y. Force Prediction in Ultrasonic Vibration-Assisted Milling, Preprints. (2019b), 2019060190.

Gao, Y. and Sun, R. L., Modelling of Theoretical Surface Roughness for Two-Dimensional Vibration-Assisted Machining, in International Conference on Computer Information Systems and Industrial Applications, ed., Atlantis Press (2015).

Halim, N. F. H. A., Ascroft, H. and Barnes, S., Analysis of Tool Wear, Cutting Force, Surface Roughness and Machining Temperature During Finishing Operation of Ultrasonic Assisted Milling (UAM) of Carbon Fibre Reinforced Plastic (CFRP), Procedia Engineering. Vol. 184 (2017), pp. 185-191.

Hsu, C. Y., Tsao, C. C., Huang, C. H. and Lin, Y. C., A Study on Ultrasonic Vibration Milling of Inconel 718, Key Engineering Materials. Vol. 419-420 (2009), pp. 373-377.

Ko, J. H., Shaw, K. C., Chua, H. K. and Lin, R. M., The Effect of One Directional Ultrasonic Vibration Assistance in High Speed Meso-Scale Milling Process, Key Engineering Materials. Vol. 447-448 (2010), pp. 41-45.

Ko, J. H., Shaw, K. C., Chua, H. K. and Lin, R. M., Cusp error reduction under high speed micro/meso- scale milling with ultrasonic vibration assistance, International Journal of Precision Engineering and Manufacturing. Vol. 12 (2011), pp. 15-20.

Lu, X., Hu, X., Jia, Z., Liu, M., Gao, S., Qu, C. and Liang, S. Y., Model for the prediction of 3D surface topography and surface roughness in micro-milling Inconel 718, The International Journal of Advanced Manufacturing Technology. Vol. 94 (2018), pp. 2043-2056.

Lu, X., Zhang, H., Jia, Z., Feng, Y. and Liang, S. Y., Floor surface roughness model considering tool vibration in the process of micro-milling, The International Journal of Advanced Manufacturing Technology. Vol. 94 (2017), pp. 4415-4425.

Pan, Z., Feng, Y., Ji, X. and Liang, S. Y., Turning Force Prediction of AISI 4130 Considering Dynamic Recrystallization, (2017a), pp. V001T02A040.

Pan, Z., Feng, Y., Lu, Y.-T., Lin, Y.-F., Hung, T.-P., Hsu, F.-C., Lin, C.-F., Lu, Y.-C. and Liang, S. Y., Microstructure-sensitive flow stress modeling for force prediction in laser assisted milling of Inconel 718, Manufacturing Rev. Vol. 4 (2017b), pp. 6.

Razfar, M. R., Sarvi, P. and Zarchi, M. M. A., Experimental investigation of the surface roughness in ultrasonic-assisted milling, Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture. Vol. 225 (2011), pp. 1615-1620.

Shen, X.-H., Zhang, J., Xing, D. X. and Zhao, Y., A study of surface roughness variation in ultrasonic vibration-assisted milling, The International Journal of Advanced Manufacturing Technology. Vol. 58 (2011), pp. 553-561.

Wu, J. H. and Liu, Z. Q., Modeling the Minimum Chip Thickness in Orthogonal Micro-Cutting Based on Plastic Strain Gradient, Advanced Materials Research. Vol. 69-70 (2009), pp. 203-208.

Xiao, M., Sato, K., Karube, S. and Soutome, T., The effect of tool nose radius in ultrasonic vibration cutting of hard metal, International Journal of Machine Tools and Manufacture. Vol. 43 (2003), pp. 1375-1382.