Increasing the effective workspace of the planar 3-RRR mechanism by using kinematic redundancy

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Abstract. A significant issue of parallel mechanisms is the existence of singularities where the end-effector can lose their mobility or cannot be controlled. The closeness to singularities affects the performance of the mechanism, for example, the actuation efforts increase, thus reducing the effective workspace of the mechanism. Kinematic redundancy can be used for avoiding the singularity and increase the effective workspace. The approach is considered on the example of a planar 3-RRR mechanism.

1. Introduction
Parallel mechanisms are widely used in various fields of technology due to their characteristics: increased lift capacity, manipulation speed and precision, less metal consumption [1, 2]. However, their development and universal application are constrained by their significant drawback: the presence of singularities in which the end-effector of the manipulator may lose controllability or mobility [3]. At the same time, negative phenomena arise not only in the zones of singularities, but also at the approach to them, for example, drives speeds or moments of the manipulator increase, and the structural rigidity decreases [4]. Thus, the normal operation of the manipulator is impossible not only in singularities but also in some areas around them. There are many different criteria for closeness to singularities, such as pressure angles, determinant of the Jacobian matrix of mechanism [5]. In this paper, the force criterion presented in [6] will be applied.

The essence of the approach presented in [6] is as follows: at each point of the workspace, a certain force is applied to the end-effector with the worst direction, maximizing the load on the drives. A spur (great) increase in the moments in the drives characterizes the zones of singularities [7]. The effective workspace is defined as the set of points at which the end-effector can be located, according to the design constraints of the real mechanism and the requirements for the necessary orientation of the end-effector, provided that it is possible to maintain at these points certain motion parameters that do not exceed critical, for example, drive moments [8]. Thus, the application of this approach allows us to determine the real size of the effective workspace of the manipulator.

2. Case study
Let’s consider the well-known planar 3-RRR mechanism shown in figure 1(a). This mechanism has three degrees of freedom and can be used as an orientation platform. The following geometric parameters were chosen for the study: $B_C = 0.36$ m, $C_D = 0.3$ m, $A_1A_2 = A_2A_3 = A_3A_1 = 0.6$ m. Stepper motors PL28H28-D5 are used as drives. Two options are considered, first one with the first...
leg’s length $A_iB_i = 0.36$ m and the second one with $A_iB_i = 0.48$ m. The effective workspace of the mechanism is presented in Figure 1 (b,c). Areas near singularities where the drive torque exceeds the maximum permissible torque for the selected motor (0.044 N·m) are not taken into account (white areas). The location of the zones of singularities depends on the geometric parameters of the end-effector configuration and the selected rotation angle $\varphi$, but somehow they cross the workspace and cut large areas. With these parameters, the effective workspace of the mechanism for the first option ($A_iB_i = 0.36$ m) is 0.9339 m$^2$ if a transition through singularities is possible, and 0.5425 m$^2$ if such a transition is not possible. For the second option the effective workspaces are 1.1947 m$^2$ and 0.7841 m$^2$ respectively.

A dynamic method can be used to pass through singularities, in which the end-effector passes through the singularity due to the inertia forces. Drive and kinematic redundancy are also applied. In the first case, an additional chain is introduced into the mechanism (for example, a 3-RRR mechanism becomes a 4-RRR mechanism), which is activated when approaching singularities. This solution complicates the control of the manipulator, limits the workspace and increases the metal content of the structure. Kinematic redundancy occurs when the number of degrees of freedom of the mechanism is
greater than required in the task, thus, when approaching a special position, an excess degree of freedom is used, which changes the configuration of the mechanism, thereby moving the zones of singularities.

Let’s consider a 3-RPRR mechanism with kinematic redundancy in two variants: with an additional drive prismatic pair in one and all kinematic chains (figure 2).

2.1. The partial redundancy
In the first case we consider a 3-RPRR mechanism with one redundant chain in order to verify whether one chain will be enough for a work area free of singularities.

The movement of the prismatic pair is limited by the design features of the mechanism. For this task it is in the range from 0.36 m to 0.48 m.

Next, it is necessary to determine the size of the effective working area of the mechanism with redundancy. For this purpose, an algorithm was developed, which was implemented programmatically using the MATLAB package. The block diagram of the algorithm is presented in Fig. 3. The essence of the algorithm is as follows: at each point of the workspace where the inverse kinematic problem can be solved, a certain force is applied to the end-effector with the worst direction, maximizing the load on the drives, and after that, if the moments in the drives are too big, the additional drive prismatic pair starts to work. If at any position of the prismatic pair the torques are too big, the point excludes from the workspace. The advantage of this algorithm is its convenient scalability and applicability for any type of parallel mechanisms; however, it should be noted that for mechanisms with a number of degrees of freedom more than three and high accuracy, the calculation will be time-consuming.

For a 3-RPRR mechanism with one redundant chain, the effective workspace is 1.1112 m², it is presented in figure 2(a). In this case, there are still zones of singularities in the workspace, but they are no longer closed lines and do not divide the workspace into segments. This case is better than the case without redundancy, but due to the existing gaps in the working area, the application of this mechanism in practice is difficult.

2.2. The full redundancy
Consider the case of full redundancy, which is shown in figure 2 (b). The range of the movement of the prismatic pair is the same as in the first case. The same algorithm was used to determine the effective workspace of this mechanism. For a 3-RPRR mechanism with three redundant chain, the

![Figure 2](attachment:image.png)

**Figure 2.** The effective workspace of 3-RPRR mechanism with one redundant chain (a), with three redundant chains (b)
effective workspace is 2.4685 m² and it’s completely free from singularities, which guarantees the operability of the manipulator.

Figure 3. Algorithm for determining the effective workspace of a mechanism with redundancy

3. Conclusion
The results of mathematical modelling were summarized in Table 1. The main result is an increase by 355% in the size of the effective working area of the mechanism with redundancy compared to the mechanism without redundancy. It can also be concluded that the kinematic redundancy of one chain (the partial redundancy) is enough to break the closed lines of singularities but not enough to make the workspace free from them. Thus, it is shown that kinematic redundancy can be successfully used to increase the effective workspace of a mechanism. The algorithm for determining the effective workspace for mechanisms with kinematic redundancy is presented, which can be used not only for planar mechanisms, but also for any general case.


Table 1. Effective workspace size

| The type of the mechanism                      | Effective workspace size (m²) | Size increase relative to the first option (%) |
|-----------------------------------------------|------------------------------|-----------------------------------------------|
| 3-RRR mechanism with AiBi = 0.36 m           | 0.5425                       | -                                             |
| 3-RRR mechanism with AiBi = 0.48 m           | 0.7841                       | 44.5                                          |
| 3-RPRR mechanism with one redundant chain    | 1.1112                       | 104.8                                         |
| 3-RPRR mechanism with three redundant chain  | 2.4685                       | 355.0                                         |

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