Ab Initio Study of Phase Stability in Doped TiO₂

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Abstract:

Ab-initio density functional theory (DFT) calculations of the relative stability of anatase and rutile polymorphs of TiO₂ were carried using all-electron atomic orbitals methods with local density approximation (LDA). The rutile phase exhibited a moderate margin of stability of ~ 3 meV relative to the anatase phase in pristine material. From computational analysis of the formation energies of Si, Al, Fe and F dopants of various charge states across different Fermi level energies in anatase and in rutile, it was found that the cationic dopants are most stable in Ti substitutional lattice positions while formation energy is minimised for F doping in interstitial positions. All dopants were found to considerably stabilise anatase relative to the rutile phase, suggesting the anatase to rutile phase transformation is inhibited in such systems with the dopants ranked F>Si>Fe>Al in order of anatase stabilisation strength. Al and Fe dopants were found to act as shallow acceptors with charge compensation achieved through the formation of mobile carriers rather than the formation of anion vacancies.

Keywords: Solar Energy, Titanium Dioxide, Photocatalysis, Density Functional Theory
1. Introduction

Over the last 4 decades, titanium dioxide has been attracting significant research interest in the field of renewable energy. Owing to the particular energetic levels of its conduction and valence bands, titanium dioxide is able to function as a semiconductor photocatalyst and facilitate reactions involved in applications including hydrogen production through water splitting (solar hydrogen) [1-4], sunlight-driven water purification [5-8], self cleaning coatings [9-12] and self sterilizing coatings [13-15]. These applications are of great interest in facilitating environmental remediation and in enabling the conversion of solar energy to a storable medium. Of further interest in the field of renewable energy generation, TiO$_2$ is also of great promise in photovoltaic applications and is widely used for the fabrication of photoanodes in Dye Sensitized Solar Cells (DSSCs) [16-19].

Titanium dioxide, as with other semiconductor photocatalysts, facilitates reactions through the photo-generation of electron-hole pairs (excitons) by irradiation exceeding the material’s band gap. These electron-hole pairs tend to undergo rapid recombination, a phenomenon which can be observed through photoluminescence, [20, 21] or alternatively, can react with surface adsorbed species in order to directly or indirectly facilitate desired reactions [22-24].

Titanium dioxide is commonly used in its two main phases, the equilibrium rutile phase and the metastable anatase phase, which transforms to rutile through thermal treatment. Although rutile exhibits a narrower band-gap than anatase, ~3.0eV compared with ~3.2eV, [25-27] anatase is generally considered to exhibit superior photocatalytic performance, owing to higher levels of surface area and thus higher activity [28].

Owing to a more flexible assembly of 4-edge-sharing TiO$_6$ octahedra, anatase, rather than rutile, is frequently the first crystalline phase formed in many synthesis routes. [29, 30] Mixed phase compositions of anatase and rutile are widely accepted to exhibit improved exciton separation, through a trapping of conduction band electrons in the rutile phase and valence band holes in the anatase phase, and consequently superior photocatalytic activity as well as superior performance in photovoltaic applications [31-36]. A third metastable phase, brookite, has also been reported to exhibit photocatalytic activity [37, 38], and indeed mixtures of anatase rutile and brookite have been reported to exhibit good levels of activity[38, 39]. However, this phase is of lesser interest in energy applications owing to the complexity in its synthesis.

The application of titanium dioxide in photocatalysis has been drawing increasing scientific interest over recent years and many studies have focused on enhancing the performance of TiO$_2$ photocatalysts. In essence, the enhancement of the activity of TiO$_2$ photocatalysts is attempted through the enhancement of exciton generation, and/or the reduction of exciton recombination.
The addition of anionic and cationic dopants is a common method of controlling the properties of titanium dioxide and enhancing the material’s performance as a photocatalyst. Dopant elements can be added to TiO$_2$ to achieve various outcomes:

- Formation of new valence states in TiO$_2$ [40-43]
- Creation of charge carrier trapping sites [44-47]
- Band gap reduction [48-50]
- Control of phase transformation behaviour [51-54]
- Surface area enhancement [53, 55]

In particular, dopants, or unintentional impurities, have a pronounced effect on the anatase to rutile phase transformation. Certain dopants are reported to promote the anatase to rutile phase transformation, allowing this reconstructive process to occur at lower temperatures and take place more rapidly. Conversely, dopants are often reported to inhibit the anatase to rutile phase transformation, imparting greater stability to the anatase phase. The effects of dopants on the anatase to rutile phase transformation have been reviewed comprehensively elsewhere. [29] Briefly, the promotion of the anatase to rutile phase transformation is generally reported to occur through an easing of the atomic rearrangement involved in the transformation, often as the result of an increase in the density of anion vacancies. This may occur when cationic dopants of low valence substitute for Ti in the anatase lattice [29, 56, 57]. Conversely, the inhibition of the anatase to rutile phase transformation is accepted to occur through a restriction of the atomic rearrangement involved in the phase transformation. This may occur as the result of the substitution of Ti$^{4+}$ with cations of equal or higher valence, or through the presence of dopant elements located in interstices or at grain boundaries. Although the effects of dopants on the anatase to rutile phase transformation can ostensibly be predicted from consideration of ionic radii and valence, some controversy exists as numerous dopants are reported both as inhibitors and as promoters of the phase transformation and the mechanisms through which doping with such elements takes place and through which the phase transformation kinetics are altered remain unclear. The ambiguity regarding the effects of certain dopants on the anatase to rutile phase transformation may stem from the sensitivity of the material to experimental conditions.

In the present work we aim to shed light on the location of dopant elements by examining the role of sole-dopants from an energetics perspective through computer simulation of the formation energies of dopants in substitutional and interstitial positions. The effects of dopant elements on the stability of the anatase phase is predicted by the analysis of total free energy of doped TiO$_2$ in anatase and rutile phases. An understanding of the fundamental effects of dopants on phase stability and transformations in TiO$_2$ is of great importance for the use of this material in energy applications.
2. Methodology

First-principles density functional theory (DFT) enables the calculation and prediction of material properties directly from quantum mechanical considerations, without the aid of phenomenological parameters. DFT methods evaluate the electronic structure by constructing a potential acting on a system’s electrons. The DFT potential is a sum of external potentials \( V_{\text{ext}} \) which is exclusively determined by the geometry and the chemical composition of the system, and an effective potential \( V_{\text{eff}} \) that represents the inter-electronic interactions. As a result, a DFT problem for a system with \( n \) electrons is a set of \( n \) one-electron Schrödinger-like equations which are known as Kohn-Sham equations [58]

\[
H\Psi_n = \left(-\frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}} + V_{\text{eff}}\right)\Psi_n = \varepsilon_n \Psi_n
\]  

(1)

Here, \( \Psi_n \) are the \( n \) one-electron wavefunctions, \( V_{\text{ext}} \) is the external potential of the nuclei, \( V_{\text{eff}} \) the effective potential, \(-\frac{\hbar^2}{2m} \nabla^2\) is the kinetic energy operator and \( \varepsilon_n \) is the DFT eigenvalue. \( V_{\text{eff}} \) contains terms that govern the electronic exchange and correlation \( (V_{\text{xc}}) \) which are usually approximated by various formalisms, which have been reviewed elsewhere [59]. Since the exchange energy is better known and characterized for uniform electrons, from a computational point of view, it is practical to approximate the exchange-correlation energy terms of any system by the one of the uniform electronic gas. This procedure is called Local Density Approximation (LDA) and offers a very popular and cost-effective method for conducting DFT calculations [60].

2.1. Computational Settings

\textit{Ab initio} calculations were performed with the use of DMol\(^3\), a density functional theory based software package developed by Accelrys [61, 62]. Here a spin unrestricted method was utilized where spin-up and spin-down electrons were considered independently to allow accurate calculations of spin related phenomena. This is important for transition elements where the d orbitals are partially filled and spin distribution over the orbitals is asymmetric. The \textit{Double-numeric plus polarization} (DNP) basis set was used to construct the molecular orbitals. The chosen basis set gives an accurate description of bonding and is generally more reliable than DMOL\(^3\)’s other available sets such as the minimal basis set which are often are inadequate for quantitative analyses.

Local density approximation \((LDA)\) based on Perdew-Wang formalism was applied to approximate the exchange-correlation energy [63]. Here \( V_{\text{xc}} \) was calculated from the DFT total energy \( (E_{\text{LDA}}) \) according to the standard LDA procedure shown in following equation [64]:

\[
V_{\text{xc}}^{\text{LDA}}(r) = \frac{\delta E_{\text{LDA}}}{\delta \rho(r)}
\]

(2)
where the exchange-correlation potential \((V_{xc})\) was directly related to the electronic density \(\rho(r)\) at point \(r\). Real-space global cutoff radii were set at 0.52 nm for all elements and thus beyond that distance the wave functions of all elements were assumed to be zero. This allows the mathematical convergence of the calculated wave-functions without significantly compromising the accuracy of the calculation. The value of 0.52 nm for real-space global cut-off radii has been demonstrated to be adequate for metal oxide DFT calculations [61].

Since periodic boundary conditions were imposed during the calculation, Bloch’s theorem [65] was applicable to the calculations and provided a great reduction in the required computational resources. Since electronic states are only allowed at a set of \(k\)-points, determined by the periodic boundary conditions, the infinite number of electrons in the periodic solid are accounted for by an infinite number of \(k\)-points. Bloch’s theorem changes the problem of calculating an infinite number of electronic wavefunctions to one of calculating a finite number of wavefunctions at an infinite number of \(k\)-points confined to the first Brillouin zone. Since the electronic wavefunctions at \(k\)-points that are very close together vary very smoothly, DFT expressions that contain a sum over \(k\)-points, such as eigenvalues, can be efficiently evaluated using a numerical scheme that performs summation over a small number of special points in the Brillouin zone. In the present work, Brillouin zone sampling was carried out by choosing a \(k\)-point set generated by the Monkhorst-Park scheme with grid spacing of \(~0.2\) nm\(^1\) between \(k\)-points for all studied configurations and the total energy \(E_{total}\) in the system was determined by integrating the wave function over the \(k\)-space [66]. For verification purposes, convergence testing was performed, first by increasing \(k\)-point density and then by increasing the real space global cut-off radii; it was found that the total energy differs less by \(10^{-5}\) eV/atom. Thus the results are considered well converged.

For geometry optimization, both DFT total energy and its first derivatives, with respect to ionic coordinates, are minimized by the displacement of ionic positions. The derivatives of the total energy with respect to the ionic positions, shown in Eq. 3, are termed Hellman Feynman forces, and in equilibrium should equal zero [67].

\[
\vec{F}_i = -\frac{\partial E_{total}}{\partial \vec{r}_i} \tag{3}
\]

Minimizing \(E_{total}\) and \(\vec{F}\) with respect to the ionic coordinates \((\vec{r}_i)\) is a commonplace optimization problem, and in DMOL\(^3\) a standard Broyden–Fletcher–Goldfarb–Shanno method, also known as the BFGS algorithm, [68] is utilized iteratively for this purpose. DFT methods based on this algorithm have been widely employed in similar studies and are described in greater detail elsewhere [69, 70].

The convergence thresholds for energy, Cartesian components of internal forces acting on the ions, and displacement values were set to be \(10^{-5}\) eV/atom, 0.5 eV/nm, and \(10^{-6}\) nm, respectively. To avoid artificial hydrostatic pressure during the simulation, supercell lattice constants were fixed to the theoretical values and only internal coordinates were allowed to relax to a minimum energy configuration.
For doped systems, the formation energy ($E^f$) of each dopant in both substitutional and interstitial sites is calculated as function of Fermi level energy ($E_{\text{Fermi}}$). Then the formation enthalpies ($\Delta^F$) of the impurity doped, n-type rutile and anatase TiO$_2$ are compared to probe the effect of the dopants in anatase phase stabilisation.

Calculations were performed for pristine TiO$_2$, and TiO$_2$ doped with Si, Al, Fe, and F. This dopant range was selected as it represent dopants which are commonly found (although often unappreciated) as impurities in TiO$_2$.

- Si and Al are common cationic impurities that may derive from the glass and single crystal substrates commonly used in thin film fabrication
- Fe is a common cationic impurity, frequently used as a dopant in many materials and the interpretation of its effects on the anatase to rutile phase transformation is controversial.
- F$^-$ derives from fluorine doped tin oxide conducting substrates and it is a strongly electronegative anionic impurity/dopant.

3. Results

3.1. Pristine TiO$_2$

The optimized crystal structures, including lattice parameters, and the formation enthalpy ($\Delta^F$) of pristine rutile and anatase TiO$_2$ were determined by allowing all lattice constants and internal coordinates to relax as presented in Table 1. $\Delta^F$ was calculated according to the following formula [71]:

$$\Delta^F (\text{TiO}_2) = E^{\text{total}} (\text{TiO}_2) - E^{\text{total}} [\text{Ti(metal)}] - E^{\text{total}} [\text{O}_2(g)]$$

(1)

where $E^{\text{total}}$ is the density functional theory (DFT) total energy of TiO$_2$, metallic Ti and O$_2$. The calculated lattice parameters for a-TiO$_2$ are $a = 0.375$ nm, $c = 0.961$ nm and $u = 0.206$ (u is the fractional $z$ component of the oxygen position in TiO$_2$). For r-TiO$_2$, the calculated lattice parameters were found to be $a = 0.457$ nm and $c = 0.294$ nm. All of these parameters are in good agreement with prior observations differing from the experimental values by less than 1% [72]. The difference in lattice parameters is consistent with the well-known bond softening effect of LDA functions [59]. The $\Delta^F$ of a-TiO$_2$ and r-TiO$_2$ were calculated to be -9.860 eV/f.u. and -9.863 eV/f.u. respectively which are in reasonable agreement with previous DFT calculations based on plane-wave pseudopotential methods.[73] In this work, $\Delta E$ is defined as $\Delta^F (\text{r-TiO}_2) - \Delta^F (\text{a-TiO}_2)$ and is an indicator of the phase stability in TiO$_2$. Positive $\Delta E$ implies that a-TiO$_2$ is more energetically favourable over r-TiO$_2$. The presented DFT calculations indicate that, at standard conditions of temperature and pressure, in pristine TiO$_2$, r-TiO$_2$ is slightly more stable than a-TiO$_2$, as $\Delta E$ equals only to -3 meV. Consequently, such a small stability margin leaves the possibility of a-TiO$_2$ stabilization, depending on
conditions of crystal growth and the presence of impurities. It should be noted that since the r-TiO$_2$ is more stable than a-TiO$_2$, the phase transformation from a-TiO$_2$ to r-TiO$_2$ is irreversible, as is widely reported in the literature [74, 75].

Table 1. Lattice parameters and formation enthalpy of anatase and rutile. Calculated values and experimental values reported in the literature [72]. (f.u.=formula unit)

| Property | Anatase TiO$_2$ | Rutile TiO$_2$ |
|----------|----------------|---------------|
|          | Calculated     | Experimental  | Calculated | Experimental |
| a (nm)   | 0.375          | 0.378         | 0.457      | 0.45936      |
| c (nm)   | 0.961          | 0.952         | 0.294      | 0.296        |
| u        | 0.206          | 0.208         | -          | -            |
| $\Delta F$ (eV/f.u.) | -9.860 | - | -9.863 | - |
| $\Delta F$ (KJ/mole) | -951.345 | - | -951.634 | - |

3.2. Doped TiO$_2$

To explore the strategies to stabilize a-TiO$_2$ over r-TiO$_2$ by doping, the formation energy of various charge states of the Si, Al, Fe and F dopants in both a-TiO$_2$ and r-TiO$_2$ systems were calculated. Both substitutional and interstitial sites of both TiO$_2$ polymorphs were considered for impurity doping. Figure 1(a) and (b) schematically represent the lattice sites of the substitutional and interstitial dopants in a-TiO$_2$ and r-TiO$_2$, respectively. The formation energy of doping dopant was calculated according to the following formula: [76]

$$E_f = E^{\text{total}}(\text{TiO}_2:M) - E^{\text{total}}(\text{TiO}_2) - \mu_N + \mu_M + qE_{\text{Fermi}}$$

Here, $E^{\text{total}}(\text{TiO}_2:M)$ is the DFT total energy of the doped system, $E^{\text{total}}(\text{TiO}_2)$ is the total energy of the undoped system, $\mu_N$ is the chemical potential of the substituted element, if any, $\mu_M$ is the chemical potential of the impurity and $q$ is the total charge dopant is has a positive (negative) values for donors (acceptors). $E_{\text{Fermi}}$ is the Fermi level energy with respect to the valence band maximum (VBM). There are number of factors that can affect the position of $E_{\text{Fermi}}$ in an oxide, including (a) carrier concentration, temperature, presence of co-dopants and unintentional charge compensators. However, TiO$_2$ is known to exhibits persistent n-type conductivity due to its metal excess nature which implies that is located near the conduction band maximum (CBM) edge. The chemical potentials of metallic elements,
following common practice, [73, 77] were calculated from the DFT total energy of the corresponding metal oxide. For gaseous elements (O and F) the chemical potential was set to equal the molecule’s total energy per element, thus reflecting the availability of single oxygen atoms (or fluorine) for the formation of the oxide [76].

Using the obtained formation energies ($E^f$), one can calculate the transition level, \( \varepsilon(q/q') \), of each dopant which is defined as the Fermi level \( (E_{Fermi}) \) position where the charge states of a dopant \( q \) and \( q' \) have equal formation energy, and is calculated for a dopant as follows:

\[
\varepsilon(q/q') = \frac{E^f(TiO_2:M,q) - E^f(TiO_2:M,q')}{q - q'}
\]

The experimental significance of the \( \varepsilon(q/q') \) value is that for \( E_{Fermi} \) positions below \( \varepsilon(q/q') \) charge state \( q \) is stable, while for \( E_{Fermi} \) positions above \( \varepsilon(q/q') \) charge state \( q' \) is relatively stable. In other words, \( \varepsilon(q/q') \) can determine the electrical activity of a dopant. If a dopant’s \( \varepsilon(q/q') \) is positioned within few meV of the band edges (valence-band maximum (VBM) for an acceptor, conduction-band minimum (CBM) for a donor), such a dopant is likely to be thermally ionized at room temperature and the dopant is called a shallow donor/acceptor. If \( \varepsilon(q/q') \) is located far from the band edges, it is unlikely to be ionized at room temperature, and thus it constitutes a deep level dopant.

In calculations for all doped systems, a supercell of \( 3a \times 2a \times 2c \) relative to the primitive cell, containing 72 ions, was considered for both TiO\(_2\) polymorphs. By introducing one dopant atom per supercell, one obtains a cation doping concentration of \( \sim 4.17 \text{ at}\% \) for dopants substituting a Ti ion and an anion doping concentration of \( \sim 2.08 \text{ at}\% \) for elements substituting an O ion. To examine the suitability of this supercell for accurate \( E^f \) calculations, test calculations were performed on the r-TiO\(_2\):Si\(_{Ti}\) (rutile doped substitutionally with Si) system as a selected test system. The \( E^f \) values of neutral Si\(_{Ti}\) in as \( 3a \times 2a \times 2c \) and \( 3a \times 3a \times 2c \) and \( 3a \times 3a \times 3c \) were calculated to be 0.369 eV, 0.371 eV and 0.372 eV respectively, indicating only a very minor change in formation energy in larger supercells. Thus it can be concluded that a \( 3a \times 2a \times 2c \) supercell size is adequate for the purpose of calculating \( E^f \).

Calculated dopant formation energies in anatase and rutile and \( \Delta E \) values are shown in Table 2.
Figure 1. Schematic representation of interstitial and substitutional dopant positions in (a) anatase and (b) rutile TiO$_2$ with arrows showing cation-dopant to oxygen distances.

**TiO$_2$:Si**

Si has two common oxidation states, i.e. Si$^{2+}$ and Si$^{4+}$. When a Si$^{2+}$ substitutes a Ti$^{4+}$ (Si$_{Ti}^{2+}$) in TiO$_2$’s host lattice, it leaves two holes in the valance band, while a Si$_{Ti}^{4+}$ is isovalent to Ti$^{4+}$ and has no electrical activity. For interstitial Si (Si$_{Int}$), Si$_{Int}^{0}$ is a neutral dopant, while Si$_{Int}^{2+}$ and Si$_{Int}^{4+}$ are double and quadruple donors, respectively. The $E'$ of Si in a-TiO$_2$ and r-TiO$_2$ are presented in Figure 2(a) and (b) respectively. For a-TiO$_2$, Si$_{Ti}$ is more stable than Si$_{Int}$ for the entire range of the band gap with $\varepsilon(4+/2+) = 1.439$ eV, indicating that Si$_{Ti}$ is a deep double acceptor. Here, when the Fermi level is located at the VBM ($E_{Fermi} = VBM$), that is p-type TiO$_2$, the $E'$ of both Si$_{Ti}$ and Si$_{Int}$ is $\sim 0.400$ eV. However as Fermi level moves to the CBM as the case of n-type TiO$_2$, Si$_{Int}^{2+}$ become more stable than Si$_{Ti}^{2+}$ by a large margin of 11.623 eV. In r-TiO$_2$, when the Fermi level is positioned near CBM, Si$_{Int}^{4+}$ is more stable than Si$_{Ti}^{4+}$ by 2.935 eV. However at $E_{Fermi} = 0.826$ eV , the $E'$ of Si$_{Ti}^{4+}$ and Si$_{Int}^{4+}$ become equal. When $E_{Fermi}$ moves towards the CBM (as in n-type r-TiO$_2$) the Si$_{Int}^{2+}$ form becomes more stable than Si$_{Ti}^{0}$ by 8.742 eV, indicating that Si$_{Ti}$ is more stable than Si$_{Int}$ for n-type r-TiO$_2$. The transition level $\varepsilon(4+/2+)$ for Si$_{Ti}$ is 1.821 eV. For n-type TiO$_2$, $\Delta E$ ($[\Delta' (r - TiO_2 : Si) - \Delta' (a - TiO_2 : Si)]/f.u.$) is 32.43 meV which indicates the Si dopant acts as an inhibitor to anatase to rutile phase transformation.

**TiO$_2$:Al**

Al usually occurs in 3+ charge state. In TiO$_2$ host lattice environment substitutional Al$^{3+}$ (Al$_{Ti}^{3+}$) acts as single acceptor while interstitial Al$^{3+}$ (Al$_{Int}^{3+}$) acts as triple donor. The $E'$ of
$Al^{3+}_{\text{Ti}}$ in a-TiO$_2$ is presented in Figure 2(c). It is evident that $Al^{3+}_{\text{Ti}}$ is more stable than $Al^{4+}_{\text{Int}}$ for the entire range of the band gap in anatase TiO$_2$. Additionally $Al^{3+}_{\text{Ti}}$ is always more stable than $Al^{4+}_{\text{Ti}}$ which is neutral with respect to the host crystal environment, indicating that Al is a shallow acceptor in a-TiO$_2$. For r-TiO$_2$, as in Figure 2(d), when the Fermi level is located at CBM, $E'_{\text{F}}$ of $Al^{4+}_{\text{Ti}}$ is $-1.593$ eV, lower than the one of $Al^{3+}_{\text{Ti}}$ which is $0.519$ eV. As the Fermi level moves to VBM, $Al^{3+}_{\text{Ti}}$ becomes more stable than $Al^{0}_{\text{Int}}$. When the Fermi Level equals to VBM $Al^{3+}_{\text{Ti}}$ is more stable than $Al^{0}_{\text{Int}}$ by $7.258$ eV. The $\varepsilon(4+/3+)$ and $\varepsilon(3+/0)$ for $Al^{3+}_{\text{Ti}}$ and $Al^{0}_{\text{Int}}$ is $0.519$ eV and $2.192$ eV respectively, indicating that the first is a deep donor while the latter is deep triple acceptor in r-TiO$_2$’s host lattice.

As $Al^{3+}_{\text{Ti}}$ is substantially more stable in n-type TiO$_2$ for both of its polymorphs the $A'$ per formula of a-TiO$_2$:Al$_{\text{Ti}}$ and r-TiO$_2$:Al$_{\text{Ti}}$ for $E_{\text{Fermi}} = \text{VBM}$ were compared. It was found that normalized $A'(\text{a-TiO}_2:\text{Al})$/f.u. was lower than $A'(\text{r-TiO}_2:\text{Al})$/f.u. by $17.68$ meV. The results indicate that Al dopants act as weak inhibitors of the anatase to rutile phase transformation.

**TiO$_2$:Fe**

Fe usually occurs in 2+ and 3+ oxidation states. $Fe^{2+}_{\text{Ti}}$ and $Fe^{3+}_{\text{Ti}}$ act as double and single acceptors in TiO$_2$ while $Fe^{2+}_{\text{Int}}$ and $Fe^{3+}_{\text{Int}}$ act as double and triple donors in TiO$_2$. As shown in Figure 2(e), in a-TiO$_2$, $Fe^{2+}_{\text{Ti}}$ is more stable than $Fe^{3+}_{\text{Int}}$ for entire range of $E_{\text{Fermi}}$ over the band gap. When the Fermi level is located at VBM, the $E'_{\text{F}}$ of $Fe^{4+}_{\text{Ti}}$ is $0.020$ eV and is slightly lower than the one of $Fe^{3+}_{\text{Int}}$ with $E'_{\text{F}}$ of $0.021$ eV. The stability of $Fe^{4+}_{\text{Ti}}$ at $E_{\text{Fermi}} = \text{VBM}$ indicates that $Fe^{2+}_{\text{Ti}}$ is not electrically active in p-type TiO$_2$. The first transition level $\varepsilon(4+/3+)$ for $Fe^{3+}_{\text{Ti}}$ occurs when $E_{\text{Fermi}} = 0.880$ eV while the second transition level $\varepsilon(3+/2+)$ occurs when $E_{\text{Fermi}} = 1.315$ eV. Both of these levels are located in the middle of the band gap and indicate that $Fe^{3+}_{\text{Ti}}$ is a deep acceptor in a-TiO$_2$. In the case of n-type a-TiO$_2$ $Fe^{2+}_{\text{Ti}}$ with $E'_{\text{F}} = -4.225$ eV is more stable than $Fe^{0}_{\text{Int}}$ by $8.669$ eV implying that $Fe^{2+}_{\text{Int}}$ is unlikely to exist in considerable concentrations. In r-TiO$_2$, according to Figure 2(f), When the $E_{\text{Fermi}}$ equals to VBM, $Fe^{3+}_{\text{Int}}$ with $E'_{\text{F}} = -2.381$ eV is more stable than $Fe^{4+}_{\text{Ti}}$ of which $E'_{\text{F}} = -0.372$ eV. However after $E_{\text{Fermi}} = -0.669$ eV the $E'_{\text{F}}$ of $Fe^{3+}_{\text{Ti}}$ becomes more stable than the one of $Fe^{4+}_{\text{Ti}}$. When $E_{\text{Fermi}}$ is located at CBM as the case of n-type TiO$_2$, the $Fe^{2+}_{\text{Ti}}$ with $E'_{\text{F}} = -3.821$ eV is more stable than the $Fe^{0}_{\text{Int}}$ by $7.539$ eV. As a result, one can say that generally $Fe^{3+}_{\text{Ti}}$ is more stable than $Fe^{3+}_{\text{Int}}$ in r-TiO$_2$ for most of the allowable range of $E_{\text{Fermi}}$. Similar to the case of a-TiO$_2$, $Fe^{3+}_{\text{Ti}}$ possesses two transition levels in band gap of r-TiO$_2$: $\varepsilon(4+/3+) = 1.270$ eV and $\varepsilon(3+/2+) = 1.682$ eV. These transition levels are slightly closer to the CBM when compared to the ones of a-TiO$_2$, indicating that $Fe^{3+}_{\text{Ti}}$ constitutes a shallower acceptor in r-TiO$_2$. 

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However these transition levels are still considered so deep in the band gap that \( Fe_{ni}^{2+} \) is not expected to be thermally ionized in r-TiO\(_2\).

Since \( Fe_{ni}^{2+} \) is the most stable in both a- and r-TiO\(_2\) when \( E_{Fermi} = CBM \) as in the case of \( n \)-type TiO\(_2\), the total energy per unit formula of these two systems were compared together. It was found that \( \Delta E'(a\text{-TiO}_2;Fe)/f.u. \) was lower than \( \Delta E'(r\text{-TiO}_2;Fe)/f.u. \) by 18.47 meV, indicating Fe dopants act as phase transformation inhibitors in TiO\(_2\).

\textbf{TiO}_2:F

F occurs in 1- oxidation state. When F’ substitutes an O\(^{2-}\), it forms a single donor. While \( F_{int}^- \) constitutes a single acceptor when located in TiO\(_2\) host lattice. The \( E' \) of both \( F_O^- \) and \( F_{int}^- \) in a-TiO\(_2\) is demonstrated in Figure 2(g). When the Fermi level is located at VBM, \( F_O^- \) with \( E' = -1.980 \text{ eV} \) is more stable than \( F_{int}^- \) that has \( E' = -0.269 \text{ eV} \). For \( E_{Fermi} > 0.856 \text{ eV} \), \( F_{int}^- \) becomes more stable than \( F_O^- \). When \( E_{Fermi} \) is located at CBM, \( E' \) of \( F_{int}^- \) and \( F_O^{2-} \) equals to \(-3.469 \text{ eV} \) and \(-0.629 \text{ eV} \) respectively, indicating the stabilization of \( F_{int}^- \) over \( F_O^- \)

\textit{by} -2.840 \text{ eV in n-type a-TiO}_2. The behaviour of F in r-TiO\(_2\) is demonstrated in Figure 2(h). For \( E_{Fermi} = VBM \), \( F_O^- \) with \( E' = -2.716 \text{ eV} \) is substantially more stable than \( F_{int}^0 \) with \( E' = 1.158 \text{ eV} \). \( F_O^- \) remains the most stable form of F in r-TiO\(_2\)’s host lattice for most of the band gap region when its transition level \( \epsilon(1/-2-) \) occurs at \( E_{Fermi} = 1.825 \text{ eV} \) where \( F_O^{2-} \) becomes more stable. 2- oxidation state for \( F_{O}^{2-} \) implies that the extra electron from neighbouring Ti ions remains localized on an F site and does not participate in conduction mechanism. For \( E_{Fermi} > 2.621 \text{ eV} \), the ionized interstitial F (\( F_{int}^- \)) becomes the most stable form of F in r-TiO\(_2\) with its \( E' = -1.470 \text{ eV} \) for \( E_{Fermi} = CBM \).

Since \( F_{int}^- \) is the most stable in both a- and r-TiO\(_2\) when \( E_{Fermi} = CBM \) as in the case of \( n \)-type TiO\(_2\), the total energy per unit formula of these two systems were compared together. It was found that \( \Delta E'(a\text{-TiO}_2;F)/f.u. \) was lower than \( \Delta E'(r\text{-TiO}_2;F)/f.u. \) by 84.78 meV, indicating F dopants act as inhibitors in TiO\(_2\).

\textbf{Table 2. Stable dopant configurations and formation energies in anatase and rutile TiO}_2 \textbf{and total free energy differentials}

| System  | Most stable configuration in a-TiO\(_2\) | Most stable configuration in r-TiO\(_2\) | \( \Delta E \) (meV) |
|---------|------------------------------------------|------------------------------------------|---------------------|
| TiO\(_2\):Si | \( Si_{ni}^{2+} \) | \( Si_{ni}^{2+} \) | 32.43 |
| \( n \)-type | \( -3.126 \) | \( -2.386 \) |          |
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|         |      |      |      |      |      |
|---------|------|------|------|------|------|
| TiO₂:Al | Al⁺³ | -2.658 | Al⁺³ | -2.272 | 17.68 |
| TiO₂:Fe | Fe⁺² | -4.225 | Fe⁺² | -3.820 | 18.47 |
| TiO₂:F  | F⁻⁻  | -3.468 | F⁻⁻  | -1.470 | 84.78 |
Figure 2. Formation energies \( E_f \) of interstitial (red) and substitutional (blue) dopants in anatase (a, c, e, g) and rutile (b, d, f, h) titanium dioxide.
4. Discussion

4.1. Pristine TiO₂

Previous studies have examined the phase stability of pristine anatase and rutile from experimental and first principles approaches [78]. In the present work, comparable results were obtained for the lattice parameters for these two important phases of TiO₂. However, thermodynamic data at room temperature differs from what has been reported to date.

A comparison of the calculated formation enthalpies of rutile and anatase yields a $\Delta E$ value of just 0.289 KJ mol$^{-1}$ for the theoretical transformation of pristine anatase to rutile at room temperature, although, as the result of an energetic barrier, this transformation does not take place at room temperature. This value suggests a narrow room temperature margin of stability and is lower than reported values to date, which have reported values between 1.7 and 12 KJ mol$^{-1}$ [74, 78-80]. The discrepancies between calculated margins of stability, in this paper and in previous reports, is not unusual and is likely the result of approximated local density functional used [78].

4.2. Doped TiO₂

The current work presents a novel Ab-initio study of dopant formation enthalpies in interstitial and substitutional positions and a total free-energy analysis of doped a-TiO₂ and r-TiO₂. From the calculations shown here it is apparent that doping with Si, Al, Fe and F should bring about a stabilization of the anatase phase.

As shown in Figure 2, from consideration of formation energies, substitutional Si$^{2+}$ doping is energetically favoured in both a-TiO₂ and r-TiO₂. This is in contrast with some reports which suggest Si doping inhibits the phase transformation by occupying interstices in TiO₂,[81, 82] and other reports which have suggested silicon doping occurs by substitutional Si$^{4+}$ accompanied by a lattice parameter contraction. [83] In general, the calculated inhibition of the anatase to rutile phase transformation by Si dopant, shown in Table 2, is in good agreement with reported experimental findings in the literature [62, 83-86].

While doped anatase has, in the past, been assumed to be kinetically stabilized, calculated results from the present work show that doped anatase phase TiO₂ is thermodynamically the most stable phase, at standard conditions of temperature and pressure, exhibiting lower $\Delta F$.

As with Si doping, the calculated stabilization of the anatase phase by Al-doping is in agreement with previous experimental findings. The results presented here show that substitutional Al$^{3+}$ doping, which is energetically favoured over interstitial Al$^{3+}$, doping does not bring about an increase in anion vacancies rather charge compensation for Al$^{3+}$ substituting for Ti$^{4+}$ occurs through the presence of mobile charge carriers or other unintentional shallow donors. This contrasts with the previous assumptions that Al doping stabilized anatase by occupying interstices and grain boundaries, and that substitution of Ti$^{4+}$ by cations of lower valence would necessarily increase oxygen vacancy levels and enhance...
the phase transformation. [29, 57] This finding may help explain the observation of anatase stabilization by low valence dopants [87].

Fe doping of TiO$_2$ has been reported to bring about mixed effects on the anatase to rutile phase transformation with some reports suggesting an inhibiting effect [88] and others suggesting a promotion of the transformation [9, 56, 89]. Calculations from an energetic perspective suggest Fe is present in TiO$_2$ as substitutional Fe$^{2+}$ in Ti lattice positions and brings about a stabilization of the anatase phase. It should be noted that the computational approach considers the effect of single impurity dopant while in experimental situation inhibiting Fe ions may compete against other promoting dopants which might be the cause of observed controversy. [90-93]

The only anion dopant for which calculations were carried out in this study, was fluorine. The stabilization of anatase was strongest in this case with the order of stabilization being F>Si>Fe>Al as indicated by a comparison of $\Delta F$ (a-TiO$_2$:F) with $\Delta F$ (r-TiO$_2$:F) which gave $\Delta E$ values of 84.78, 32.43, 18.46 and 17.68 meV respectively. The inhibition of rutile formation by F doping is supported by previously reported experimental results [94, 95].

While from energetic considerations, the studied systems are most stable with substitutional doping in the anatase phase, restricted elemental diffusion, morphological aspects and reactions of dopant compound are likely to bring about significant deviation from calculated behaviour.

### 5. Conclusions

Based on DFT calculations, all investigated cationic dopants are more stable in Ti substitutional lattice sites and are predicted to stabilise the anatase phase relative to rutile under standard conditions of pressure and temperature when present as sole-dopants.

Anionic doping with Fluorine exhibits lower formation energy in interstitial lattice sites and is predicted to bring about a significant inhibition of the anatase to rutile transformations.

The order in which anatase is stabilised was F>Si>Fe>Al in order of stabilisation strength.

Substitutional doping with cations of valence lower than 4+ does necessarily not bring about an increase in anion vacancies, rather charge compensation is predicted to occur through the formation of free holes, deep or shallow.

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