UNITARIZING NON-MINIMAL INFLATION VIA
A LINEAR CONTRIBUTION TO THE FRAME FUNCTION

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ABSTRACT: We show that non-minimal inflation, based on the \( \phi^4 \) potential, may be rendered unitarity conserving and compatible with the Planck results for \( 4.6 \cdot 10^{-3} \leq r_{21} = c_{2R}/c_{1R} \leq 1 \), if we introduce a linear contribution \( (c_{1R} \phi) \) to the frame function which takes the form \( f_R = 1 + c_{1R} \phi + c_{2R} \phi^2 \). Supersymmetrization of this model can be achieved by considering two gauge singlet superfields and combining a linear-quadratic superpotential term with a class of logarithmic or semi-logarithmic Kähler potentials with prefactor for the logarithms including the inflaton field \( -2(n+1) \) where \( -0.01 \leq n \lesssim 0.013 \).

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I. INTRODUCTION

Although excellently compatible with data [1–3], (quartic) non-minimal inflation (nMI) based on the potential

\[ V_{\text{CI}}(\phi) = \lambda^2 \phi^4 / 4 \]  

and realized [4, 5] thanks to the presence of a non-minimal coupling function

\[ f_R(\phi) = 1 + c_R \phi^2 \]  

between the inflaton \( \phi \) and the Ricci scalar \( \mathcal{R} \), suffers from an inconsistency [6, 7] with the validity of the effective theory. Indeed, the establishment of the inflationary stage with \( \phi \leq 1 \) – in the reduced Planck units with \( m_p = 1 \) – requires large \( c_R \) values which drive the Einstein frame (EF) inflationary scale

\[ \hat{V}_{\text{CI}}^{1/4} = V_{\text{CI}}^{1/4} / f_R^{1/2} \]  

to values well above the Ultraviolet (UV) cut-off scale

\[ \Lambda_{\text{UV}} = m_p / c_R \]

of the effective theory which, thereby, breaks down above it – a criticism of these results may be found in Ref. [8]. Several ways have been proposed to surpass this inconsistency. E.g., incorporating new degrees of freedom at \( \Lambda_{\text{UV}} \) [9], or assuming additional interactions [10], or introducing a sizable kinetic mixing in the inflaton sector which dominates over \( f_R \) [11-14] or even adding an \( \mathcal{R}^2 \) term [15]. In the case of high-scale nMI this problem can be also eluded invoking a large inflaton vacuum expectation value (v.e.v) \( \langle \phi \rangle \) as in Refs. [16–19]. Unitarity-conserving nMI can be also achieved by considering a scalar field, which exhibits a linear contribution to its non-minimal coupling to gravity, in addition to the Standard Model (SM) Higgs field which is coupled quadratically to gravity [20, 21].

In a recent paper [22] we propose a novel solution to the aforementioned problem which works only in the context of Supergravity (SUGRA) and is relied on the adoption of an exclusively linear non-minimal coupling to gravity in conjunction with an appropriate selection of the prefactor of the logarithm of the Kähler potential. We here suggest an alternative solution, operative also in non-SUSY settings. It is tied to the presence of a subdominant linear term into \( f_R \), which thereby takes the form

\[ f_R(\phi) = 1 + c_{1R} \phi + c_{2R} \phi^2. \]  

In a such case, the canonically normalized inflaton \( \hat{\phi} \) is related to the initial field \( \phi \) as \( \hat{\phi} \sim c_{1R} \phi \) at the vacuum of the theory, in sharp contrast to what we obtain for \( f_R \) in Eq. (2) where \( \hat{\phi} \approx \phi \). Indeed, \( \hat{\phi} \) is given in terms of \( \phi \) using the formula [5]

\[ \frac{d\hat{\phi}}{d\phi} = \sqrt{\frac{1}{f_R} + \frac{3}{2} \left( \frac{f_{R,\phi}}{f_R} \right)^2} \]

where the symbol \( \phi \) as subscript denotes derivation with respect to \( (w.r.t) \) the field \( \phi \). From Eq. (6) we can easily infer that if \( f_R \) is linear [7, 23] or if it includes a linear contribution \( f_{R,\phi} \neq 0 \) and so \( \hat{\phi} \neq \phi \) at the vacuum of the theory which typically is given by the condition \( \langle \phi \rangle = 0 \). As a consequence, the small-field series of the various terms of the action, expressed in terms of \( \hat{\phi} \), contain powers of the ratio \( r_{21} = c_{2R}/c_{1R}^2 \) and not of the parameters \( c_{1R} \) or \( c_{2R} \) appearing in the right-hand side (r.h.s) of Eq. (5) – preventing, thereby, the reduction of \( \Lambda_{\text{UV}} \) below \( m_p \) for \( r_{21} \leq 1 \), despite the fact that \( c_{1R} \) and \( c_{2R} \) may be large.

Although the present proposal is "tailor-made" for the non-SUSY regime of the quartic nMI, we prefer to investigate the relevant setting in the context of SUGRA in order to enrich the parameter space of the model and highlight its differences with the proposal of Ref. [22]. Indeed, the emergent picture here is radically different from that found in Ref. [22]. Namely, the inflationary potential is of Starobinsky type and the observables crucially depend on the ratio \( r_{21} \) which is an extra parameter w.r.t those employed in Ref. [22]. On the other hand, we do not consider any mixing of the inflaton with other fields as in Ref. [21] and so our setting is considerably simplified.

Below, in Sec. II, we describe how we can formulate this kind of unitarity-safe nMI both within a SUSY and non-SUSY framework. The dynamics of the resulting inflationary models is studied in Sec. III and these are tested against observations in Sec. IV. Finally, we analyze the UV behavior of the models in Secs. V and summarize our conclusions in Sec. VI.
II. SUSY VERSUS non-SUSY FRAMEWORK

Here we shortly remind the establishment of nMI within a non-SUSY framework – in Sec. II A – and then in the context of SUGRA – see Sec. II B.

A. Non-SUSY Setting

Non-Minimal Inflation (i.e., nMI) is formulated in the Jordan frame (JF) where the action of the inflaton \( \phi \) is given by

\[
S = \int d^4x \sqrt{-g} \left( -\frac{f_R}{2} \mathcal{R} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_{\text{Cl}}(\phi) \right).
\]

(7a)

Here \( g \) is the determinant of the background Friedmann-Robertson-Walker metric, \( g^{\mu\nu} \) with signature \(+,+,+,−\) whereas \( V \) is given by Eq. (1). By performing a conformal transformation [5] according to which we define the EF metric \( \tilde{g}_{\mu\nu} = f_R g_{\mu\nu} \) with determinant \( \tilde{g} \), we can write \( S \) as follows

\[
S = \int d^4x \sqrt{-\tilde{g}} \left( -\frac{1}{2} \tilde{\mathcal{R}} + \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} - \tilde{V}_{\text{Cl}}(\tilde{\phi}) \right),
\]

(7b)

where \( \tilde{\mathcal{R}} \) is the EF Ricci scalar curvature, \( \tilde{V}_{\text{Cl}} \) is given as a function of \( \phi \) by the virtue of Eq. (3) with \( f_R \) defined by Eq. (5). If we wish to couple this model to the SM, we have to assume that the SM fields are minimally coupled to gravity and the potential mixing of \( \phi \) to the SM Higgs field is very weak – cf. Ref. [21].

B. Supergravity Embeddings

A convenient implementation of nMI in SUGRA is achieved by employing two singlet superfields \( z^\alpha = \Phi, S \), with \( \Phi (\alpha = 1) \) and \( S (\alpha = 2) \) being the inflaton and a “stabilizer” field respectively. We below describe the salient feature of our SUGRA setting in Sec. II B 1 and outline the derivation of the inflationary potential in Sec. II B 2.

1. Set-up

The EF action for \( z^\alpha \)'s can be written as [24]

\[
S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} \mathcal{R} + K_{\alpha\beta} \tilde{g}^{\mu\nu} \partial_\mu z^\alpha \partial_\nu z^\beta - \tilde{V} \right),
\]

(8)

where the summation is taken over the scalar fields \( z^\alpha \). \( K_{\alpha\beta} = \tilde{K}_{\alpha\beta} \) with \( \tilde{K}_{\alpha\beta} = \tilde{\delta}_{\alpha\beta} \) and \( \tilde{V} \) is the EF F-term SUGRA scalar potential which can be extracted once the superpotential \( W \) and the Kähler potential \( K \) have been selected, via the formula

\[
\tilde{V} = e^K \left( K_{\alpha\beta} D_\alpha W D_\beta W^* - 3|W|^2 \right).
\]

(9)

where \( D_\alpha W = W, z^\alpha + \tilde{K}_{\alpha z^\beta} W \) is the Kähler covariant derivative.

The presence of the stabilizer field \( S \) facilitates the reproduction of Eq. (3) from Eq. (9) by placing \( S \) at the origin. Then, the only surviving term in Eq. (9) is

\[
\tilde{V}_{\text{Cl}} = e^K \left( S^2 |W|^2 \right),
\]

(10)

and the numerator in Eq. (3), originating from Eq. (1), can be derived if we adopt the following superpotential

\[
W = \lambda S \Phi^2.
\]

(11)

\( W \) can be uniquely determined if we impose two symmetries: (i) an \( R \) symmetry under which \( S \) and \( \Phi \) have charges \( 1 \) and \( 0 \); (ii) a global \( U(1) \) symmetry with assigned charges \( -1 \) and \( 1 \) for \( S \) and \( \Phi \). The derivation of the denominator in Eq. (3), with \( f_R \) defined in Eq. (5), can be obtained, though, by violating the latter symmetry as regards \( \Phi \). Indeed, we employ one of the Kähler potentials below

\[
K_1 = -N \ln (1 + F_R + F_R^2 - F_-/N + F_{1S}), \quad (12a)
\]

\[
K_2 = -N \ln (1 + F_R + F_R^2 - F_-/N + F_{2S}), \quad (12b)
\]

where \( N > 0 \) and the functions \( F_R \) and \( F_- \) are defined as

\[
F_R = c_1 R \Phi/\sqrt{2} + c_2 R \Phi^2 \quad \text{and} \quad F_- = -\frac{1}{2} (\Phi - \Phi^*)^2.
\]

(13)

From these, the first one allows for the introduction of the polyvonic non-minimal coupling \( f_R \) in Eq. (5), whereas the second one assures canonical normalisation of \( \Phi \) without any contribution to the non-minimal coupling along the inflationary path – cf. Ref. [13]. On the other hand, the functions \( F_{IS} \) with \( l = 1, 2 \) are defined as

\[
F_{1S} = -\ln (1 + |S|^2/N), \quad (14a)
\]

\[
F_{2S} = N \ln \left( 1 + |S|^2/N_S \right), \quad (14b)
\]

and offer canonical normalisation and safe stabilization of \( S \) during and after nMI [25]. We avoid here to study \( K \)'s obtained by placing \( F_- \) outside the argument of \( \ln \) since the resulting models exhibit a more complicated inflationary dynamics which leads to observables drastically deviating from those in non-SUSY case, as shown in Ref. [22].

The construction of Eq. (7a) can be obtained within SUGRA if we perform the inverse of the conformal transformation described above Eq. (7b) with

\[
f_R = -\Omega/N,
\]

(15)

and specify the following relation between \( K \) and \( \Omega \),

\[
-\Omega/N = e^{-K/N} \Rightarrow K = -N \ln (-\Omega/N).
\]

(16)

Working along the lines of Refs. [23, 24] we arrive at the JF action

\[
S = \int d^4x \sqrt{-g} \left( \frac{\Omega}{2N} \mathcal{R} + \omega_{\alpha\beta} \partial_\mu z^\alpha \partial_\nu z^\beta - V \right.
\]

\[
- \frac{27}{N^3} \Omega A_\mu A^\mu \right) \quad \text{with} \quad \omega_{\alpha\beta} = \Omega_{\alpha\beta} + \frac{3 - N \Omega_{\alpha \Omega_{\beta}}}{N}.
\]

(17)
Here $V = \Omega^2 \hat{V} / N^2$ is the JF potential and $A_\mu$ is [24] the purely bosonic part of the on-shell value of the auxiliary field found to be

$$A_\mu = -iN \left( \Omega_\alpha \partial_\mu z^\alpha - \Omega_\alpha \partial_\mu z^\alpha \right) / 6\Omega. \quad (18)$$

We note that, contrary to the non-SUSY case – see Eq. (7a) –, we obtain a kinetic mixing in Eq. (17) which, along the inflationary trough $\text{Im}\Phi = S = 0$, can be cast in the form

$$\omega_{\alpha\beta} = \begin{pmatrix} 1 + (N - 3) & 0 \\ 0 & f_R K_{SS} \end{pmatrix}, \quad (19a)$$

where

$$K_{SS} = \begin{cases} 1/f_R & \text{for } K = K_1, \\ 1 & \text{for } K = K_2. \end{cases} \quad (19b)$$

From Eq. (19a) we see that canonical kinetic terms arise for $N = 3$ and $K = K_1$. On the contrary, for $N \neq 3$ there is some kinetic mixing which, however, does not disturb essentially the realization of nMI and allows us to obtain adjustable inflationary observables – see Sec. III.

2. Inflationary Potential

We here verify that the proposed $W$ and $K$‘s in Eqs. (11) and (12a) or (12b) result to an inflationary model approaching $\hat{V}_{CI}$ in Eq. (3) with $f_R$ defined in Eq. (5) and $\phi$ found by Eq. (6). Indeed, computing $\hat{V}_{CI}$ in Eq. (10), along the direction

$$\Phi = \Phi^* \text{ and } S = 0, \text{ or } s = s = \theta = 0, \quad (20)$$

if we express $\Phi$ and $S$ according to the parametrization

$$\Phi = \phi e^{i\theta} / \sqrt{2} \text{ and } S = (s + is) / \sqrt{2}, \quad (21)$$

we can extract the final form of $\hat{V}_{CI}$ in Eq. (10)

$$\hat{V}_{CI} = \frac{\lambda^2 \phi^4}{4 f_R^4} = \frac{\lambda^2 \phi^4}{4 f_R^4}, \quad \begin{cases} f_R & \text{for } K = K_1, \\ 1 & \text{for } K = K_2. \end{cases} \quad (22)$$

Here we take into account Eq. (19b) and introduce $n$ through the relation

$$N = \begin{cases} 2n + 3 & \text{for } K = K_1, \\ 2(n + 1) & \text{for } K = K_2. \end{cases} \quad (23)$$

For $n = 0$, $\hat{V}_{CI}$ reduces to the one obtained in Eq. (3) with $f_R$ shown in Eq. (5). This choice is special since it yields integer $N$‘s which are more friendly to string theory. However, non-integer $N$‘s are also acceptable [12, 14, 18, 26] and assist us to cover the whole allowed domain of the observables. More specifically, for $n < 0$, $V_{CI}$ remains an increasing function of $\phi$, whereas for $n > 0$, it develops a local maximum $\hat{V}_{CI}(\phi_{\text{max}})$ where

$$\phi_{\text{max}} = \frac{4}{c_1 R} \left( n - 1 + \sqrt{(n - 1)^2 + 16n c_1} \right)^{-1}. \quad (24)$$

In a such case we are forced to assume that hilltop [27] nMI occurs with $\phi$ rolling from the region of the maximum down to smaller values.

To specify the EF canonically normalized inflaton, we note that, for both $K$‘s in Eqs. (12a) and (12b), $K_{\alpha\beta}$ along the configuration in Eq. (20) takes the form

$$(K_{\alpha\beta}) = \text{diag} \left( K_{\Phi^*}, K_{SS} \right), \quad (25)$$

where $K_{SS}$ is given by Eq. (19b) and

$$K_{\Phi^*} = (2f_R + N c_1^2 R + 4N c_2^2 R \phi^2 + 4N c_1 c_2 R \phi) / 2f_R^2. \quad (26)$$

Therefore, the EF canonically normalized fields, denoted by hat, are defined via the relations

$$\frac{d\hat{\phi}}{d\phi} = \sqrt{K_{\phi^*}} = J, \quad \hat{\phi} = J\phi \text{ and } (\hat{s}, \hat{\bar{s}}) = \sqrt{K_{SS}} (s, \bar{s}) \quad (27)$$

and the spinors $\psi_S$ and $\psi_{\phi^*}$ associated with the superfields $S$ and $\Phi$ are normalized similarly, i.e., $\psi_S = \sqrt{K_{SS}^{-1}} \psi_S$ and $\psi_{\phi^*} = J \psi_{\phi^*}$. For $N = 3$ the leftmost equality in Eq. (27) reduces to Eq. (6) which is valid in the non-SUSY regime.

Taking the limit $c_1 R \ll c_2 R$ we can verify that the configuration in Eq. (20) is stable w.r.t the excitations of the non-inflaton fields, finding the expressions of the masses squared $\hat{m}_\phi^2$ (with $\alpha^2 = \theta$ and $s$) arranged in Table I, which approach rather well the quite lengthy, exact expressions taken into account in our numerical computation. These expressions assist us to verify that the positivity of $\hat{m}_\phi^2$ requires $1 \leq N \leq 6$ for $K = K_1$ and $N_S \leq 6$ for $K = K_2$. Moreover, for both masses squared we obtain $\hat{m}_\phi^2 \gg \hat{H}_{CI}^2 / \hat{V}_{CI}$ for $\phi < \phi_{\text{max}}$ (where $\phi$ and $\phi_{\text{max}}$ are the values of $\phi$ when $c_4 = 0.05/\text{Mpc}$ crosses the horizon of nMI and at its end correspondingly. In Table I we display the masses of the corresponding fermions too. The derived mass spectrum can be employed in order to find the one-loop radiative corrections, $\Delta \hat{V}_{CI}$, to $\hat{V}_{CI}$. The resulting $\Delta \hat{V}_{CI}$ lets intact our inflationary outputs, provided that the renormalization-group mass scale $\Lambda$, is determined by requiring $\Delta \hat{V}_{CI}(\phi_{\text{crit}}) = 0 \text{ or } \Delta \hat{V}_{CI}(\phi_{\text{end}}) = 0$. The possible dependence of our findings on the choice of $\Lambda$ can be totally avoided if we confine ourselves to $0 < N_S < 6$ resulting to $\Lambda \simeq (1.1 - 2.5) \cdot 10^{-3}$ for $K = K_1$ or $\Lambda \simeq (1.72 - 2.9) \cdot 10^{-5}$ for $K = K_2$. Under these circumstances, our inflationary predictions can be exclusively reproduced by using $\hat{V}_{CI}$ in Eq. (22) – cf. Ref. [12].

**Table I:** Mass squared spectrum along the path in Eq. (20).

| FIELDS | EIGEN-STATES | $K = K_1$ | $K = K_2$ |
|--------|--------------|------------|------------|
| 1 real scalar | $\theta$ | $6(1 - 1/N)H_{C1}^2$ | $6H_{C1}^2$ |
| 2 real scalars | $\tilde{\alpha}, \tilde{\bar{\alpha}}$ | $6c_2^2 R \phi^2 / N$ | $6c_2^2 R / N_S$ |
| 2 Weyl spinors | $\hat{s}_n \phi^2 / Nc_1$ | $(4 - c_1 R(N - 4) - 2N c_2 R \phi^2) / 2f_R^2$ | $3H_{C1}^2 / Nc_2^2 R \phi^4$ |
| | $N = N - 3$ | $N = N - 2$ | |
III. INFLATION ANALYSIS

From the setting of our models we can easily deduce that their free parameters, for fixed $n$, are $r_{21} = c_{2R}/c_{1R}^2$ and $\lambda/c_{1R}^2$ and not $c_{1R}$, $c_{2R}$ and $\lambda$ as naively expected. In fact, if we perform a rescaling $\phi = \hat{\phi}/c_{1R}$, Eq. (7a) preserves its form replacing $\phi$ with $\hat{\phi}$ where $f_R$ and $V_{Cl}$, respectively, read

$$f_R = 1 + \hat{\phi} + r_{21} \hat{\phi}^2$$

and $V_{Cl} = \lambda^2 \hat{\phi}^4/4c_{1R}^4$, (28)

which, indeed, depend only on $r_{21}$ and $\lambda/c_{1R}^2$. Note that here we have the same number of parameters with those employed in the models of Refs. [12–14] and one parameter more than those in Ref. [22].

These parameters may be constrained by applying the inflationary criteria. In particular, the period of slow-roll nMI is determined in the EF by the condition [28]

$$\max[\hat{c}^2(\phi) , |\hat{\eta}(\phi)|] \leq 1,$$  (29a)

where the slow-roll parameters $\hat{c}$ and $\hat{\eta}$ read

$$\hat{c} = \left( \hat{V}_{Cl,\hat{\phi}}/\sqrt{2V_{Cl}} \right)^2 \text{ and } \hat{\eta} = \hat{V}_{Cl,\hat{\phi}\hat{\phi}}/\hat{V}_{Cl},$$  (29b)

and can be derived by employing $\hat{V}_{Cl}$ in Eq. (22) and $J$ in Eq. (27), without express explicitly $\hat{V}_{Cl}$ in terms of $\phi$. In the limit $c_{1R} \ll c_{2R}$, the numerator in Eq. (26) is dominated by $4Nc_{2R}^2 \phi^2$ whereas the denominator by $2c_{2R}^2 \phi^4$ and so we can achieve the approximate formula

$$J \approx 2\sqrt{2N}/\phi,$$  (30)

which turns out to be rather accurate. Inserting this into Eq. (29b) we arrive at the following results

$$\sqrt{\hat{c}} = \frac{1}{\sqrt{Nf_R}} \left( 2nc_{2R}\phi^2 - (n - 1)c_{1R}\phi - 2 \right)$$  (31a)

and

$$\hat{\eta} = \frac{1}{Nf_R} \left( 8 + c_{1R}\phi \left( 7 - 9n + (n(8n - 9) - 1)c_{2R}\phi^2 \right) \right.$$

$$+ \phi^2 (2c_{1R}^2 (1 - n)^2 + 4c_{2R} (n(2c_{2R}n\phi^2 - 5) - 1)) \right).$$  (31b)

We can numerically verify that Eq. (29a) is saturated for $\phi = \phi_\epsilon \ll 1$, which is found from the condition

$$\hat{\eta}(\phi_\epsilon) \approx 1 \Rightarrow \phi_\epsilon \approx \frac{1 + 9n}{Nc_{1R}r_{21}},$$  (32)

where we keep from the expression in Eq. (31b) the most significant terms for $\phi_\epsilon \ll 1$.

The number of e-foldings $\hat{N}_*$ that the scale $k_\epsilon = 0.05/$Mpc experiences during this nMI and the amplitude $A_\phi$ of the power spectrum of the curvature perturbations generated by $\phi$ can be computed using the standard formulae

$$\hat{N}_* = \int_{\phi_\epsilon}^{\phi_*} d\phi_{\hat{\phi}} \hat{\phi}\hat{V}_{Cl,\hat{\phi}}/\hat{V}_{Cl,\hat{\phi}} \text{ and } A_\phi = \frac{1}{12\pi^2} \left[ \frac{\hat{V}_{Cl}^2}{\hat{V}_{Cl,\hat{\phi}}^2} \right]_{\hat{\phi}=0},$$  (33)

where $\phi_\epsilon [\phi_*]$ is the value of $\phi [\hat{\phi}]$ when $k_\epsilon$ crosses the inflationary horizon. Taking into account Eq. (30) and $\phi_* \gg \phi_\epsilon$, from Eq. (33a) we find

$$\hat{N}_* \approx - \frac{N(1 + n)}{2n(1 - n)} \ln \left( 1 - \frac{2nc_{1R}r_{21}\phi_*}{1 - n} \right).$$  (34a)

Solving the equation above w.r.t $\phi_*$ we find

$$\phi_* \approx \frac{(1 - e_n)(1 - n)}{2nr_{21}c_{1R}} \text{ where } e_n = e^{-2(1-n)n\hat{N}_*(1+n)}.$$  (34b)

Taking the limit $n \to 0$ of the results above, we obtain

$$\hat{N}_* \approx Nc_{1R}r_{21}\phi_* \Rightarrow \phi_* \approx \hat{N}_*/Nc_{1R}r_{21}.$$  (34c)

From the last expressions above we easily infer that there is a lower bound on $c_{1R}$, e.g. for $n = 0$ we find $c_{1R} \gtrsim \hat{N}_*/nr_{21}$ - above which $\phi_* \leq 1$ and so, our proposal can be stabilized against corrections from higher order terms.

Inserting Eq. (34b) into Eq. (33b) we can derive a constraint on $\lambda/c_{1R}^2$ for chosen $r_{21}$, i.e.,

$$\frac{\lambda}{c_{1R}^2} \approx 2^{z - 2\pi n_e} \left( \frac{3A_\phi}{N} \right)^{n_e} \frac{1 + e_n + 1 - e_n}{n} \right)^n \frac{(1 - n)^n}{1 - e_n}.$$  (35a)

Substituting $e_n$ from its definition in Eq. (34b) and computing the limit $n \to 0$, the above expression may be simplified as

$$\frac{\lambda}{c_{1R}^2} \approx 4\sqrt{6NA_\phi}c_{1R}c_{2R}r_{21}/\hat{N}_*.$$  (35b)

Taking into account the definition of $r_{21}$, we infer that $\lambda$ is proportional to $c_{2R}$ for $n = 0$ similarly to the original nMI [4, 5], where $\lambda$ is proportional to $c_{1R}$.

The remaining inflationary observables are found from the relations

$$n_s = 1 - 6\epsilon_* + 2\eta_* \text{, } r = 16\epsilon_*,$$  (36a)

$$a_s = 2 \left( 4\eta_*^2 - (n - 1)^2 \right)/3 - 2\epsilon_*,$$  (36b)

where the variables with subscript $*$ are evaluated at $\phi = \phi_* \text{ and } \epsilon_* = \hat{V}_{Cl,\hat{\phi}}/\hat{V}_{Cl}$. Inserting $\phi_*$ from Eq. (34b) into Eqs. (31a) and (31b) and then into equations above we can obtain some analytical estimates. These become more meaningful, expanding successively the results for low $r_{21}, n$ and $1/\hat{N}_*$. Our final, quite accurate expressions are

$$n_s \approx 1 - \frac{2}{N_s - N_*} - \frac{2}{N_*^2 - N_*} - 16nr_{21}/\hat{N}_*,$$  (37a)

$$r \approx - \frac{32n}{N_s} + \frac{16N_s}{N_s^2} - \frac{32nN_s}{N_s^2}(1 + 2r_{21})$$

$$+ \frac{64N_s^2}{N_s^3}(1 + 8n)nr_{21},$$  (37b)

$$a_s \approx - \frac{2}{N_s} - \frac{4n}{N_s^2} - \frac{12N}{N_s^3}.$$  (37c)
Due to the approximations made, the results for \( n = 0 \) are not obtained by taking the relevant limit of the expressions above. Repeating the procedure, i.e., plugging Eq. (34e) into Eqs. (31a) and (31b) and expanding successively the results for low \( r_{21} \) and \( 1/\tilde{N}_e \) we find

\[
\begin{align*}
n_s &\approx 1 - \frac{2}{N_e^2} + \frac{2N}{N_e^2} - \frac{8N^2}{N_e^2} r_{21}, \\
r &\approx \frac{16N^2 - 32N^2}{N_e^2} + \frac{64N^2}{N_e^2} r_{21}, \\
a_s &\approx -\frac{2}{N_e^2} + \frac{6N}{N_e^2} - \frac{20N}{N_e^2} r_{21}.
\end{align*}
\]  

(38a), (38b), and (38c)

We remark a weak dependence of the results on \( n \) and \( r_{21} \) which may deviate from the ones obtained in the contemporary nMI [4, 5].

**IV. NUMERICAL RESULTS**

Our analytic findings above can be verified numerically and employed in order to delineate the available parameter space of the models. In particular, we confront the quantities in Eq. (33) with the observational requirements [22]

\[
\begin{align*}
\tilde{N}_e &\approx 61.3 + \frac{1}{2} \ln \left( \frac{\tilde{G}_{\text{CI}}(\phi_e)}{\tilde{G}_{\text{FK}}(\phi_e)} \right)^{1/2} \lesssim 58 - 60 \quad (39a)
\end{align*}
\]

and \( A_s \approx 2.105 \times 10^{-9} \),

(39b)

where we assume that nMI is followed in turn by an oscillatory phase, with mean equation-of-state parameter \( w_{\text{rh}} \approx 1/3 \), radiation and matter domination. Also \( g_{\text{bh}} = 228.75 \) or 106.75 is the energy-density effective number of degrees of freedom which corresponds to the Minimal SUSY SM or SM spectrum respectively.

Enforcing Eqs. (39a) and (39b) we can restrict \( \lambda/\hat{c}_2^2 \) \( \tilde{N}_e \), and \( \phi_e \) and compute the models’ predictions via Eqs. (36a) and (36b), for any selected \( r_{21} \) and \( n \). The outputs, encoded as lines in the \( n_s - r_{0.002} \) plane, are compared against the observational data [1, 3] in Fig. 1 for \( K = K_1 \) – here \( r_{0.002} = 16\tilde{G}(\phi_h, 0.002) \) where \( \phi_h, 0.002 \) is the value of \( \phi \) when the scale \( k = 0.002/\text{Mpc} \), which undergoes \( \tilde{N}_{0.002} = (\tilde{N}_e + 3.22) \) e-foldings during nMI, crosses the horizon of nMI. We draw dot-dashed, double dot-dashed, solid, dotted and dashed lines for \( n = 0 \), \pm 0.005, 0, \pm 0.005 and \pm 0.01 respectively and show the variation of \( r_{21} \) along each line. We take into account the data from Planck and Baryon Acoustic Oscillations (BAO) and the BK14 data taken by the Bicep2/Keck Array CMB polarization experiments up to and including the 2014 observing season. Fitting the data above [2, 3] with \( \Lambda \)CDM + r we obtain the marginalized joint 68% [95%] regions depicted by the dark [light] shaded contours in Fig. 1. Approximately we may write

\[
\begin{align*}
n_s &\approx 0.967 \pm 0.0074 \quad \text{and} \quad r \lesssim 0.07,
\end{align*}
\]  

(40)

at 95% confidence level (c.l.) with \( |a_s| \ll 0.01 \). The constraint on \( |a_s| \) is readily satisfied within the whole parameter space of our models.

**Fig. 1:** Allowed curves in the \( n_s - r_{0.002} \) plane for \( K = K_1 \), \( n = 0 \), \pm 0.005, \pm 0.01 (using the line types shown in the plot legend) and various \( r_{21} \)'s indicated on the lines. The marginalized joint 68% [95%] regions from Planck, BK14 and BAO data [2] are depicted by the dark [light] shaded contours. The minimal and maximum \( r \)'s (corresponding to the minimal \( r_{21} \)'s) for the \( n \)’s shown in the plot are shown in the table for \( K = K_1 \) or \( K_2 \).

From Fig. 1 we observe that the allowed \( n_s \) and \( r \) values increase as \( n \) decreases. More interestingly, for any selected \( n \) there is a lower \( (r_{21})_{\text{min}} \) and an upper \( (r_{21})_{\text{max}} \) bound on \( r_{21} \), which is translated correspondingly to an upper \( (r_{21})_{\text{max}} \) and a lower \( (r_{21})_{\text{min}} \) bound on \( r \). Namely, the origin of \( (r_{21})_{\text{max}} \) comes from the requirement that \( \Lambda \) has to remain within the domain of the validity of the perturbation theory and so it has to be lower than about \( \sqrt{\frac{4\pi}{3}} \approx 3.5 \). On the other limit, the various lines terminate for \( r_{21} = 1 \), beyond which the effective theory ceases to be well defined – see Sec. V. The bounds provided by these constraints together with the \( r_{21} \) values are listed in the Table of Fig. 1. These values are found not only for \( K = K_1 \) but also for \( K = K_2 \) for the sake of comparison. Indeed, had we employed \( K = K_2 \) the various lines in Fig. 1 would have been remained almost intact with the same \( r_{21} \) and the \( (r_{21})_{\text{min}} \) acquiring the values arranged in the Table.

From our findings we see that, for \( K = K_1 \), a little larger \( r \)'s are achieved, in accordance with our analytic expressions in Eqs. (37b) and (38b). Since \( r \geq 0.0032 \), our models are expected to be testable by the forthcoming experiments [29], which are expected to measure \( r \) with an accuracy of \( 10^{-3} \).

For \( n = 0 \) and \( N = 3 \) – recall Eq. (26) – we obtain the results for the non-SUSY regime. Moreover, this \( n \) value results to integer \( N \)'s, – via Eq. (23) – in the SUSY regime, which may be regarded as the theoretically most favored. In
FIG. 2: Allowed (shaded) region in the $n - r_{21}$ plane for $K = K_1$. The constraint fulfilled along each line is also shown on it.

In particular, for $K = K_1$ and $N = 3$ we get

$$9.67 \lesssim \frac{n_s}{0.1} \lesssim 9.69$$

whereas for $K = K_2$ and $N = 2$ we get the same $n_s$ interval with $r$ ranging between the two values indicated in the Table of Fig. 1. In both cases we have $|n_s| \simeq 0.00049 - 0.00054$. Therefore, the compatibility of these outputs with the observational values in Eq. (40) is certainly impressive.

Varying continuously $n$ we can identify the allowed region in the $n - r_{21}$ plane – as in Fig. 2. The allowed (shaded) region is bounded by the solid black line, which corresponds to $r_{21} \simeq 1$, the dashed black line which originates from the bound $\lambda \lesssim 3.5$ and the dot-dashed and dashed gray lines along which the lower and upper bounds on $n_s$ in Eq. (40) are saturated respectively. We remark that increasing $n_s$ with fixed $r_{21}$, $n_s$ decreases, in accordance with our findings in Fig. 1.

Fixing $n_s$ to its central value in Eq. (40), we obtain the gray solid line along which we get clear predictions for $n$ and $r$. Namely,

$$2 \lesssim \frac{n}{0.001} \lesssim 5, \quad 0.0046 \lesssim r_{21} \lesssim 1 \quad \text{and} \quad 9.8 \gtrsim \frac{r}{0.001} \gtrsim 6.4,$$

with $n_s/10^{-4} \simeq (4.5 - 4.9)$. Had we employed $K = K_2$, the allowed region in Fig. 2 would have been remained very similar whereas Eq. (42a) would have been modified as follows

$$0.3 \lesssim \frac{n}{0.001} \lesssim 2, \quad 0.0085 \lesssim r_{21} \lesssim 1 \quad \text{and} \quad 7.8 \gtrsim \frac{r}{0.001} \gtrsim 5.6.$$

We complete our numerical analysis by studying the structure of $\hat{V}_{CI}$. We fix $K = K_1$, $\phi_s = 1$ and $r_{21} = 0.05$ and draw $\hat{V}_{CI}$ in Eq. (22) (gray, black and light gray lines) as a function of $\phi$ for $n = -0.01, 0$ and $0.01$ respectively. The corresponding values of $\lambda$, $c_{1R}$, $n_s$ and $r$ are listed in the second, third, fourth and fifth leftmost columns of the Table below the graph, not only for $K = K_1$ but also for $K = K_2$ for comparison purposes. In all cases $a_s \simeq -5 \cdot 10^{-4}$. These results are obtained by our numerical code taking into account exact expressions for $\hat{V}_{CI}$, $J$ and the other observables – i.e., Eqs. (22), (27), (33), (36a) and (36b). These values are also consistent with those obtained by employing the formulas of Sec. III – i.e., Eqs. (34a), (35a) and (37a) – (38c) – and displayed in the four rightmost columns of the Table in Fig. 3. Note that in the case of analytic expressions we prefer to compare $\phi_s$ derived by Eq. (34b) with $\phi_s = 1$, used in all cases numerically, and let $c_{1R}$ as input parameter. Moreover, we observe that $\hat{V}_{CI}$ is a monotonically increasing function of $\phi$ for $n \leq 0$ whereas it develops a maximum at $\phi_{\text{max}} = 2.96$, for $n = 0.01$, which leads to a mild tuning of the initial conditions of nMI since $\phi_s \ll \phi_{\text{max}}$. It is also remarkable that $r$ increases with the inflationary scale, $\lambda^{-1/4}$, which in all cases is roughly of the order of 0.01$m_p$. Since $\hat{V}_{CI}^{1/4} \ll m_p$ and $m_p$ is the UV cutoff scale of the theory – as we show in Sec. V –, the classical approximation, used in our analysis is perfectly valid. Finally, it is worth emphasize that $\hat{V}_{CI}$ is of Starobinsky-type although $\langle \phi \rangle = 0$ in sharp contrast to the models of induced-gravity inflation [16, 18, 19] where $\langle \phi \rangle \gg 0$.  

| $n_s$ | $c_{1R}/10^3$ | $n_s$ | $r/0.01$ | $n_s$ | $r/0.01$ |
|-------|----------------|-------|-----------|-------|-----------|
| $K = K_1$ | $-1$ | 0.625 | 5.3 | 0.973 | 1.7 | 0.67 | 0.97 | 0.975 | 1.5 |
| $K = K_1$ | 0 | 0.355 | 4.2 | 0.967 | 1.2 | 0.397 | 0.95 | 0.968 | 1.1 |
| $K = K_1$ | 1 | 0.204 | 3.35 | 0.96 | 0.78 | 0.23 | 0.95 | 0.963 | 0.64 |
| $K = K_1$ | | | | | | | |
| $K = K_2$ | $-1$ | 1.56 | 8.9 | 0.975 | 1.4 | 1.67 | 0.97 | 0.975 | 1.4 |
| $K = K_2$ | 0 | 0.64 | 6.3 | 0.967 | 0.78 | 0.73 | 0.95 | 0.968 | 0.79 |
| $K = K_2$ | 1 | 0.275 | 4.55 | 0.956 | 0.43 | 0.32 | 0.96 | 0.955 | 0.34 |
V. EFFECTIVE CUT-OFF SCALE

The main motivation of the nMI proposed in this work is that it is unitarity-safe, despite the fact that its implementation with subplanckian $\phi$ values requires relatively large $c_{1R}$ and $c_{2R}$ values – see, e.g., the Table of Fig. 1. To show that this fact is valid we extract below the UV cut-off scale, $\Lambda_{UV}$, expanding the action in the JF – see Sec. V.A – and in the EF – see Sec. V.B.

A. Jordan Frame Computation

Thanks to the special dependence of $f_R$ on $\phi$ in Eq. (5), the interaction between the excitation of $\phi$ about $\langle \phi \rangle = 0$, $\delta \phi$, and the graviton, $h_{\mu\nu}$ preserves the perturbative unitarity for $r_{21} \leq 1$. Indeed, expanding $g_{\mu\nu}$ about the flat spacetime metric $\eta_{\mu\nu}$ and the inflaton $\phi$ about its v.e.v.,

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu} \quad \text{and} \quad \phi = 0 + \delta \phi,$$  

and retaining only the terms with two derivatives of the excitations, the part of the lagrangian corresponding to the two first terms in the r.h.s of Eq. (7a) takes the form [8, 18]

$$\delta \mathcal{L} = \left( \frac{f_R}{\Lambda_{UV}} \right)^2 \frac{1}{2} G_{\mu\nu} h_{\mu\nu} + \frac{1}{4} \left( f_R \phi \right)^2 \delta \phi^2$$

$$= \left( \frac{f_R}{\Lambda_{UV}} \right)^2 \left( \eta_{\mu\nu} + h_{\mu\nu} \right)$$

$$= \frac{1}{8} \left( \frac{f_R}{\Lambda_{UV}} \right)^2 \left( \eta_{\mu\nu} + h_{\mu\nu} \right) + \frac{1}{8} \left( \frac{f_R}{\Lambda_{UV}} \right)^2 \delta \phi^2$$

$$= \frac{1}{8} \left( \frac{f_R}{\Lambda_{UV}} \right)^2 \left( \eta_{\mu\nu} + h_{\mu\nu} \right) + \frac{1}{8} \left( \frac{f_R}{\Lambda_{UV}} \right)^2 \delta \phi^2 + \frac{1}{4} \left( \frac{f_R}{\Lambda_{UV}} \right)^2 \left( \eta_{\mu\nu} + h_{\mu\nu} \right) \delta \phi + \cdots$$

(44)

where $\langle f_R \rangle = 1$ for the non-SUSY case and $\langle f_R \rangle = 1 + (N - 3)c_{1R}^2 / 2$ – see Eq. (19a) – for our SUGRA scenario. Therefore, for $N = 3$ the results below reduce to that obtained in the non-SUSY regime. The functions $G_{\mu\nu}$ and $G_{\phi}$ are identical to the functions $F_{\mu\nu}$ and $F_{\phi}$ defined in Ref. [18]; $h_{\mu\nu}$ and $\delta \phi$ are the JF canonically normalized fields defined by the relations

$$\delta \phi = \sqrt{\frac{f_R}{\langle f_R \rangle}} \delta \phi \quad \text{with} \quad f_R = f_K f_R + \frac{3}{2} \frac{f_R}{f_R}. $$

and

$$h_{\mu\nu} = \sqrt{\frac{f_R}{\langle f_R \rangle}} h_{\mu\nu} + \frac{f_R}{f_R} \eta_{\mu\nu} \delta \phi.$$  

(45)

Taking into account that $\langle \phi \rangle = 0$ we find $\langle f_R \rangle = 1$ and $\langle f_R \rangle = 1 + N c_{1R}^2 / 2 \simeq N c_{1R}^2 / 2$.

The possible problematic process, which causes [6] concerns about the unitarity-violation, is the $\delta \phi - \delta \phi$ scattering process via s-channel graviton, $\tilde{h}$, exchange originating from the last term in the r.h.s of Eq. (44) with $\tilde{h} = \tilde{h}_{\mu\nu}$. The UV cut-off scale $\Lambda_{UV}$ is identified as follows

$$\Lambda_{UV}^{-1} \simeq c_{2R} \sqrt{\frac{f_R}{\langle f_R \rangle}} \frac{2 r_{21}}{N} \Rightarrow \Lambda_{UV} \sim r_{21}^2.$$  

(46)

Therefore, the theory retains the perturbative unitarity up-to $m_P$, for $r_{21} \leq 1$.

B. Einstein Frame Computation

Alternatively, $\Lambda_{UV}$ can be determined in EF, following the systematic approach of Ref. [7]. We concentrate here on the SUGRA versions of our model. The transition to the non-SUSY case can be easily achieved setting $n = 0$ or $N = 3$. The EF (canonically normalized) inflaton is

$$\tilde{\delta} \phi = \langle J \rangle \delta \phi \quad \text{with} \quad \langle J \rangle = \sqrt{1 + c_{1R}^2} \frac{N}{2} \simeq c_{1R} \sqrt{\frac{N}{2}}.$$  

(47)

From the last expression, we can clearly appreciate the importance of the linear term in $f_R$, Eq. (5), to distinguish $\tilde{\delta} \phi$ from $\delta \phi$ – recall that in the standard non-minimal Higgs inflation [6, 8] $\tilde{\delta} \phi = \delta \phi$. As anticipated in Sec. I, this fact implies that our models are valid up to $m_P$. To prove it, we focus on the second term in the r.h.s of Eq. (8) for $\mu = \nu = 0$ and we expand it about $\langle \phi \rangle = 0$ in terms of $\delta \phi$. Our result is written as

$$J^2 \delta \phi = \left( 1 - 2(1 - 2r_{21}) \right) \frac{2}{N} \tilde{\delta} \phi + \left( 3 - 10r_{21} \right) \frac{2}{N} \tilde{\delta} \phi$$

$$- (2 - 9r_{21}) \frac{4}{N} \sqrt{\frac{2}{N}} \tilde{\delta} \phi^3 + \cdots$$

(48a)

where we neglect terms suppressed by powers of $r_{21}$ and inverse powers of $c_{1R}$ – since $r_{21} \leq 1$ and $c_{1R} \gg 1$. Expanding similarly $\tilde{V}_{C1}$, see Eq. (22), in terms of $\tilde{\delta} \phi$ we have

$$\tilde{V}_{C1} \approx \frac{\lambda^2 \tilde{\delta} \phi}{N^2 c_{1R}^2} \left( 1 - 2(1 + n) \sqrt{\frac{2}{N}} \tilde{\delta} \phi + \left( 3 + 5n + 2(1 + n)r_{21} \right) \frac{2}{N} \tilde{\delta} \phi^3 \right.$$

$$- \left( 2 + 13 \frac{4}{3} - n - (3 + 5n)r_{21} \right) \frac{4}{N} \sqrt{\frac{2}{N}} \tilde{\delta} \phi^3 + \cdots \right).$$  

(48b)

Consequently, we verify again that our models preserve the perturbative unitarity up to $m_P$ for $r_{21} \leq 1$.

VI. CONCLUSIONS AND PERSPECTIVES

We presented a unitarized version of non-minimal inflation (i.e. nMI) which fits the Planck data very well. The main novelty of our proposal is the consideration of a linear term into the frame function, Eq. (5), – involving the parameter $c_{1R}$ – apart from the usual quadratic term proportional to $c_{2R}$ and the quartic potential in Eq. (1). This setting can be elegantly implemented not only in non-SUSY regime but also within SUGRA, employing the super- and Kähler potentials given in Eqs. (11) and (12a) or (12b) and extending the parameter space of the model by one parameter $n$ defined in Eq. (23). Our investigation reveals that $n$ has to be tuned into the interval $(-0.01, 0.013)$. Confining ourselves to the most natural value $n = 0$, we achieved observational predictions
which may be tested in the near future and converge towards the “sweet” spot of the present data for $r_{21} = c_{2R}/\ell_{
abla}^2$ into the range $(4.6 \times 10^{-4} − 1)$ – see Fig. 1. Thanks to the presence of the non-vanishing $c_{1R}$, no problem with the perturbative unitarity arises for $r_{21} \leq 1$, although the attainment of $\text{nMI}$ with subplanckian values requires relatively large $c_{1R}$’s (and $c_{2R}$’s). It is gratifying, finally, that the allowed parameter space of our models can be studied analytically and rather accurately. As a last remark, we would like to point out that, although we have restricted our discussion to a gauge singlet inflaton, the applicability of our proposal can be easily extended to gauge non-singlet fields. Indeed, the unitarization of non-minimal Higgs inflation based on the potential $V_{\text{H}} = \lambda^2 (\Phi^\dagger \Phi)^2/4$ – where $\Phi$ is now a Higgs field in the fundamental representation of an $SU(N)$ gauge group –, according to our suggestion here, requires the consideration of the frame function $f_R = 1 + c_{1R} \sqrt{\Phi^\dagger \Phi} + c_{2R} \Phi^\dagger \Phi - \cdots$ cf. Ref. [5, 21]. The second non-analytic term in the r.h.s of the expression above, although unusual, is perfectly acceptable. In this case, the inflationary predictions are expected to be quite similar to the ones obtained here, although the parameter space may be further restricted from the data on the Higgs mass. Indeed, we should take into account the renormalization-group running of the various parameters from the inflationary up to the electroweak scale in order to connect convincingly the high- with the low-energy phenomenology – cf. Ref. [30]. Since our main aim here is the demonstration of the modification on the observables of nMI due to the introduction of the linear term in $f_R$, we opted to utilize just a gauge-singlet inflaton.

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