Mie Coefficients in Double-layered Sphere Irradiated by a Planar Wave

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Both the Mie coefficients and the electromagnetic field distributions in the two-layered sphere containing left-handed media irradiated by a planar electromagnetic wave are presented.
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I. INTRODUCTION

More recently, a kind of artificial composite media (the so-called left-handed media) having a frequency band where the effective permittivity ($\varepsilon$) and the effective permeability ($\mu$) are simultaneously negative attracts attention of many researchers in various fields such as materials science, condensed matter physics, optics and electromagnetism [1–5]. Veselago first considered this peculiar medium and showed that it possesses a negative index of refraction [6]. It follows from the Maxwell’s equations that in this medium the Poynting vector and wave vector of electromagnetic wave would be antiparallel, i.e., the vector $k$, the electric field $E$ and the magnetic field $H$ form a left-handed system; thus Veselago referred to such materials as “left-handed”, and correspondingly, the ordinary medium in which $k$, $E$ and $H$ form a right-handed system may be termed the “right-handed” medium. Other authors call this class of materials “negative-index media (NIM)”, “double negative media (DNM)” [5] and Veselago’s media. It is readily verified that in such media having both $\varepsilon$ and $\mu$ negative, there exist a number of peculiar electromagnetic properties, for instance, many dramatically different propagation characteristics stem from the sign change of the group velocity, including reversals of both the Doppler shift and the Cherenkov radiation, anomalous refraction, modified spontaneous emission rates and even reversals of radiation pressure to radiation tension [2]. In experiments, this artificial negative electric permittivity media may be obtained by using the array of long metallic wires (ALMWs), which simulates the plasma behavior at microwave frequencies, and the artificial negative magnetic permeability media may be built up by using small resonant metallic particles, e.g., the split ring resonators (SRRs), with very high magnetic polarizability [3,7–9].

The extinction properties of a sphere (single-layered) with negative permittivity and permeability is investigated by Ruppin. Since recently Wang and Asher developed a novel method to fabricate nanocomposite SiO$_2$ spheres ($\sim 100$ nm) containing homogeneously dispersed Ag quantum dots ($2 \sim 5$ nm) [10], which may has potential applications to design and fabrication of photonic crystals, it is believed that the absorption and transmittance of double-layered sphere deserves consideration. In this paper, we present Mie coefficients of double-layered sphere and consider the scattering problem, including the topics on field distribution, electromagnetic cross section, extinction spectra as well as some potential peculiar properties arising from the presence of left-handed media. The formulation presented here can be easily generalized to cases of multiple-layered spheres.

II. METHODS

It is well known that in the absence of electromagnetic sources, the characteristic vectors such as $E$, $B$, $D$, $H$ and Hertz vector in the isotropic homogeneous media agree with the same differential equation

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1Note that, in the literature, many authors mention the year when Veselago suggested the left-handed media by mistake. They claim that Veselago proposed the concept of left-handed media in 1968. On the contrary, the true fact is as follows: Veselago’s excellent paper was first published in July, 1964 [Usp. Fiz. Nauk 92, 517-526 (1964)]. In 1968, this original paper was translated into English by W. H. Furry and published again in the journal of Sov. Phys. Usp. [6].
\[ \nabla \nabla \cdot \mathbf{C} - \nabla \times \nabla \times \mathbf{C} + k^2 \mathbf{C} = 0. \]  \hspace{1cm} (2.1)

The three independent vector solutions to the above equation is [11]

\[ \mathbf{L} = \nabla \psi, \quad \mathbf{M} = \nabla \times \mathbf{a} \psi, \quad \mathbf{N} = \frac{1}{k} \nabla \times \mathbf{M} \]  \hspace{1cm} (2.2)

with \( \mathbf{a} \) being a constant vector, where the scalar function \( \psi \) satisfies \( \nabla^2 \psi + k^2 \psi = 0 \). It is verified that the vector solutions \( \mathbf{M} \), \( \mathbf{N} \) and \( \mathbf{L} \) possess the following mathematical properties

\[ \mathbf{M} = \mathbf{L} \times \mathbf{a} = \frac{1}{k} \nabla \times \mathbf{N}, \quad \mathbf{L} \cdot \mathbf{M} = 0, \quad \nabla \cdot \mathbf{L} = \nabla^2 \psi = -k^2 \psi, \quad \nabla \cdot \mathbf{M} = 0, \quad \nabla \cdot \mathbf{N} = 0. \]  \hspace{1cm} (2.3)

Set \( \mathbf{M} = \mathbf{m} \exp(-i\omega t) \) and \( \mathbf{N} = \mathbf{n} \exp(-i\omega t) \), the vector wave functions \( \mathbf{M} \) and \( \mathbf{N} \) can be expressed in terms of the following spherical vector wave functions [11]

\[ m_{e}^{m_{m}} = \mp \frac{m}{\sin \theta} z_n(kr) \frac{P_n^m(\cos \theta)}{\sin \theta} \sin m \phi i_2 - z_n(kr) \frac{\partial P_n^m(\cos \theta)}{\partial \theta} \sin \theta m \phi i_3, \]
\[ n_{e}^{m_{m}} = \frac{n(n+1)}{kr} z_n(kr) P_n^m(\cos \theta) \sin \theta m \phi i_1 + \frac{1}{kr} \frac{\partial}{\partial r}[z_n(kr)] \frac{\partial P_n^m(\cos \theta) \sin \theta m \phi i_2}{\partial \theta} \]
\[ \mp \frac{m}{kr \sin \theta} \frac{\partial}{\partial r}[z_n(kr)] P_n^m(\cos \theta) \sin \theta m \phi i_3. \]  \hspace{1cm} (2.4)

In what follows we treat the Mie coefficients of double-layered sphere irradiated by a plane wave.

A. Definitions

We consider a double-layered sphere with interior radius \( a_1 \) and external radius \( a_2 \) having relative permittivity (permeability) \( \epsilon_1 (\mu_1) \) and \( \epsilon_2 (\mu_2) \), respectively, placed in a medium having the relative permittivity \( \epsilon_0 \) and permeability \( \mu_0 \). Suppose that the double-layered sphere irradiated by the following plane wave with the electric amplitude \( E_0 \) along the \( \hat{z} \)-direction of Cartesian coordinate system [11]

\[ \mathbf{E}_i = a_x E_0 \exp(ik_0 z - i\omega t) = E_0 \exp(-i\omega t) \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left( m_{o1n}^{(1)} - i n_{e1n}^{(1)} \right), \]
\[ \mathbf{H}_i = a_y \frac{k_0}{\mu_0 \omega} E_0 \exp(ik_0 z - i\omega t) = -i \frac{k_0}{\mu_0 \omega} E_0 \exp(-i\omega t) \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left( m_{e1n}^{(1)} + i n_{o1n}^{(1)} \right), \]  \hspace{1cm} (2.5)

where

\[ m_{o1n}^{(1)} = \pm \frac{1}{\sin \theta} j_n(k_0r) P_n^1(\cos \theta) \sin \theta \phi i_2 - j_n(k_0r) \frac{\partial P_n^1(\cos \theta)}{\partial \phi} \sin \theta \phi i_3, \]
\[ n_{e1n}^{(1)} = \frac{n(n+1)}{k_0r} j_n(k_0r) P_n^1(\cos \theta) \sin \theta \phi i_1 + \frac{1}{k_0r} \frac{\partial}{\partial r}[j_n(k_0r)] \frac{\partial P_n^1(\cos \theta) \sin \theta \phi i_2}{\partial \phi} \pm \frac{1}{k_0r \sin \theta} \frac{\partial}{\partial r}[j_n(k_0r)] \frac{P_n^1(\cos \theta) \sin \theta \phi i_3}{\partial \phi} \]  \hspace{1cm} (2.6)

with the primes denoting differentiation with respect to their arguments and \( k_0 = \sqrt{\epsilon_0 \mu_0 \omega} \). In the region \( r < a_1 \), the wave function is expanded as the following series

\[ \mathbf{E}_i = E_0 \exp(-i\omega t) \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left( a_n^{(1)} m_{o1n}^{(1)} - ib_n^{(1)} n_{e1n}^{(1)} \right), \]
\[ \mathbf{H}_i = -i \frac{k_1}{\mu_1 \omega} E_0 \exp(-i\omega t) \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left( b_n^{(1)} m_{e1n}^{(1)} + ia_n^{(1)} n_{o1n}^{(1)} \right). \]  \hspace{1cm} (2.7)

Note that here the propagation constant in \( m_{o1n}^{(1)} \) and \( n_{e1n}^{(1)} \) which have been defined in (2.6) is replaced with \( k_1 \), i.e., \( \sqrt{\epsilon_1 \mu_1 \omega} \). In the region \( a_1 < r < a_2 \), one may expand the electromagnetic wave amplitude as
where the unit vectors of Cartesian coordinate system agree with

\[ m_{i}^{(1)} \]

In the similar manner, it follows from (2.6) with the Hankel functions \( h_{n}^{(1)} \).

Note that here in order to obtain \( m_{i}^{(3)} \) and \( n_{i}^{(3)} \), the spherical Bessel functions \( j_{n} \) in the spherical vector function \( m_{i}^{(1)} \) and \( n_{i}^{(1)} \) is replaced by the Hankel functions \( h_{n}^{(1)} \), namely, the explicit expressions for \( m_{i}^{(3)} \) and \( n_{i}^{(3)} \) can also be obtained from (2.6), so long as we replace the spherical Bessel functions \( j_{n} \) in (2.6) with the Hankel functions \( h_{n}^{(1)} \).

Apparently, here the propagation constant in \( m_{i}^{(1)} \) and \( n_{i}^{(1)} \) should be replaced with \( k_{2} \), i.e., \( \sqrt{\varepsilon_{2} / \mu_{2} c} \).

In the region \( r > a_{2} \) the reflected wave is written [11]

\[ E_{r} = E_{0} \exp(-i\omega t) \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left( a_{n}^{m} m_{i}^{(3)}(n_{i}^{(3)}) - ib_{n}^{m} n_{i}^{(3)}(n_{i}^{(3)}) \right) \],

\[ H_{r} = -\frac{k_{0}}{\mu_{0} \omega} E_{0} \exp(-i\omega t) \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left( b_{n}^{m} m_{i}^{(3)}(n_{i}^{(3)}) + ia_{n}^{m} n_{i}^{(3)}(n_{i}^{(3)}) \right) \].

where the propagation vector in \( m_{i}^{(3)} \) and \( n_{i}^{(3)} \) is replaced by \( k_{0} \), i.e., \( \sqrt{\varepsilon_{0} / \mu_{0} c} \).

B. Boundary conditions

At the boundary \( r = a_{1} \), the boundary condition is given as follows

\[ i_{1} \times E_{m} = i_{1} \times E_{e}, \quad i_{1} \times H_{m} = i_{1} \times H_{e}, \]

(2.10)

where the unit vectors of Cartesian coordinate system agree with \( i_{1} \times i_{2} = i_{3}, i_{2} \times i_{3} = i_{1}, i_{3} \times i_{1} = i_{2} \).

It follows from \( i_{1} \times E_{m} = i_{1} \times E_{e} \) that

\[ a_{n}^{m} j_{n}(N_{2} \rho_{1}) + a_{n}^{m} h_{n}^{(1)}(N_{2} \rho_{1}) = a_{n}^{m} j_{n}(N_{1} \rho_{1}), \]

\[ N_{1} b_{n}^{m} [N_{2} b_{n} j_{n}(N_{2} \rho_{1})]^{'} + N_{1} b_{n}^{m} [N_{2} b_{n} h_{n}^{(1)}(N_{2} \rho_{1})]^{'} = N_{2} b_{n}^{m} [N_{1} b_{n} j_{n}(N_{1} \rho_{1})]^{'} \]

(2.11)

where \( \rho_{1} = k_{0} a_{1}, N_{1} = \frac{k_{0}}{\mu_{1}} \).

In the similar manner, it follows from \( i_{1} \times H_{m} = i_{1} \times H_{e} \) that

\[ b_{n}^{m} j_{n}(N_{2} \rho_{1}) + b_{n}^{m} h_{n}^{(1)}(N_{2} \rho_{1}) = b_{n}^{m} j_{n}(N_{1} \rho_{1}), \]

\[ N_{2} b_{n}^{m} [N_{2} b_{n} j_{n}(N_{2} \rho_{2})]^{'} + N_{2} b_{n}^{m} [N_{2} b_{n} h_{n}^{(1)}(N_{2} \rho_{2})]^{'} = N_{2} b_{n}^{m} [N_{1} b_{n} j_{n}(N_{1} \rho_{2})]^{'} \]

(2.12)

where \( \rho_{2} = k_{0} a_{2}, N_{2} = \frac{k_{0}}{\mu_{2}} \).

In the meanwhile, it follows from \( i_{1} \times (H_{e} + H_{s}) = i_{1} \times H_{m} \) that

\[ j_{n}(\rho_{2}) + a_{n}^{m} h_{n}^{(1)}(\rho_{2}) = a_{n}^{m} j_{n}(N_{2} \rho_{2}) + a_{n}^{m} h_{n}^{(1)}(N_{2} \rho_{2}), \]

\[ N_{2} [N_{2} j_{n} j_{n}(N_{2} \rho_{2})]^{'} + N_{2} b_{n}^{m} [N_{2} b_{n} h_{n}^{(1)}(N_{2} \rho_{2})]^{'} = b_{n}^{m} [N_{2} j_{n} h_{n}^{(1)}(N_{2} \rho_{2})]^{'} + b_{n}^{m} [N_{2} h_{n}^{(1)}(N_{2} \rho_{2})]^{'} \]

(2.13)

where \( \rho_{2} = k_{0} a_{2}, N_{2} = \frac{k_{0}}{\mu_{2}} \).

In the meanwhile, it follows from \( i_{1} \times (H_{e} + H_{s}) = i_{1} \times H_{m} \) that

\[ b_{n}^{m} [N_{2} j_{n} h_{n}^{(1)}(N_{2} \rho_{2})]^{'} + b_{n}^{m} [N_{2} h_{n}^{(1)}(N_{2} \rho_{2})]^{'} = \mu_{0} a_{n}^{m} [N_{2} j_{n} j_{n}(N_{2} \rho_{2})]^{'} + a_{n}^{m} [N_{2} h_{n}^{(1)}(N_{2} \rho_{2})]^{'} \]

\[ \mu_{2} b_{n}^{m} [N_{2} j_{n} h_{n}^{(1)}(N_{2} \rho_{2})]^{'} + b_{n}^{m} [N_{2} h_{n}^{(1)}(N_{2} \rho_{2})]^{'} = N_{2} \mu_{0} b_{n}^{m} [N_{2} j_{n} j_{n}(N_{2} \rho_{2})]^{'} + N_{2} \mu_{0} b_{n}^{m} [N_{2} h_{n}^{(1)}(N_{2} \rho_{2})]^{'} \]

(2.14)
C. Calculation of Mie coefficients

1. Mie coefficient $a_n^r$

According to the above boundary conditions, one can arrive at the following matrix equation

$$\begin{pmatrix}
    -j_n(N_1\rho_1) & j_n(N_2\rho_1) & h_n^{(1)}(N_2\rho_1) & 0 \\
    -\mu_2[N_1\rho_1 j_n(N_1\rho_1)]' & \mu_1[N_2\rho_1 j_n(N_2\rho_1)]' & \mu_1 h_n^{(1)}(N_2\rho_2) & 0 \\
    0 & j_n(N_2\rho_2) & h_n^{(1)}(N_2\rho_2) & -h_n^{(1)}(\rho_2) \\
    0 & \mu_0[N_2\rho_2 j_n(N_2\rho_2)]' & [N_2\rho_2 h_n^{(1)}(N_2\rho_2)]' & -\mu_2[\rho_2 h_n^{(1)}(\rho_2)]'
\end{pmatrix}
\begin{pmatrix}
    a_n^r \\
    a_n^m \\
    a_n^m \\
    a_n^m
\end{pmatrix}
= 
\begin{pmatrix}
    0 \\
    0 \\
    j_n(\rho_2) \\
    \mu_2[\rho_2 j_n(\rho_2)]'
\end{pmatrix}.
$$

(2.15)

For convenience, the $4 \times 4$ matrix on the left hand side of (2.15) is denoted by

$$A = 
\begin{pmatrix}
    A_{11} & A_{12} & A_{13} & 0 \\
    A_{21} & A_{22} & A_{23} & 0 \\
    0 & A_{32} & A_{33} & A_{34} \\
    0 & A_{42} & A_{43} & A_{44}
\end{pmatrix}
$$

(2.16)

whose determinant is

$$\det A = A_{11}A_{22}A_{33}A_{44} - A_{11}A_{22}A_{34}A_{43} - A_{11}A_{32}A_{23}A_{44} + A_{11}A_{42}A_{23}A_{44}
- A_{21}A_{12}A_{33}A_{44} + A_{21}A_{12}A_{34}A_{43} + A_{21}A_{32}A_{13}A_{44} - A_{21}A_{42}A_{13}A_{44}.
$$

(2.17)

So, the Mie coefficient $a_n^r$ is of the form

$$a_n^r = \frac{\det A_r}{\det A}
$$

(2.18)

with

$$A_r = 
\begin{pmatrix}
    A_{11} & A_{12} & A_{13} & 0 \\
    A_{21} & A_{22} & A_{23} & 0 \\
    0 & A_{32} & A_{33} & j_n(\rho_2) \\
    0 & A_{42} & A_{43} & \mu_2[\rho_2 j_n(\rho_2)]'
\end{pmatrix}
$$

(2.19)

whose determinant is

$$\det A_r = A_{11}A_{22}A_{33}\mu_2[\rho_2 j_n(\rho_2)]' - A_{11}A_{22}j_n(\rho_2)A_{43} - A_{11}A_{32}A_{23}\mu_2[\rho_2 j_n(\rho_2)]' + A_{11}A_{42}A_{23}j_n(\rho_2)
- A_{21}A_{12}A_{33}\mu_2[\rho_2 j_n(\rho_2)]' + A_{21}A_{12}j_n(\rho_2)A_{43} + A_{21}A_{32}A_{13}\mu_2[\rho_2 j_n(\rho_2)]' - A_{21}A_{42}A_{13}j_n(\rho_2).
$$

(2.20)

2. Mie coefficient $b_n^r$

According to the above boundary conditions, one can arrive at the following matrix equation

$$\begin{pmatrix}
    -N_2[N_1\rho_1 j_n(N_1\rho_1)]' & N_1[N_2\rho_1 j_n(N_2\rho_1)]' & N_1[2\rho_1 h_n^{(1)}(N_2\rho_1)]' & 0 \\
    -N_1\mu_2 j_n(N_1\rho_1) & N_2\mu_1 j_n(N_2\rho_1) & N_2\mu_1 h_n^{(1)}(N_2\rho_1) & 0 \\
    0 & [N_2\rho_2 j_n(N_2\rho_2)]' & [N_2\rho_2 h_n^{(1)}(N_2\rho_2)]' & -N_2[\rho_2 h_n^{(1)}(\rho_2)]'
\end{pmatrix}
\begin{pmatrix}
    b_n^r \\
    b_n^m \\
    b_n^m \\
    b_n^m
\end{pmatrix}
= 
\begin{pmatrix}
    0 \\
    0 \\
    N_2[\rho_2 j_n(\rho_2)]' \\
    \mu_2 j_n(\rho_2)
\end{pmatrix}.
$$

(2.21)

For convenience, the $4 \times 4$ matrix on the left hand side of (2.21) is denoted by
Thus be discussed in detail, which are now under consideration and will be submitted elsewhere for the publication.

by a plane wave. The scattering and absorption properties of double-layer sphere containing left-handed media can

\[ B = \begin{pmatrix} B_{11} & B_{12} & B_{13} & 0 \\ B_{21} & B_{22} & B_{23} & 0 \\ 0 & B_{32} & B_{33} & B_{34} \\ 0 & B_{42} & B_{43} & B_{44} \end{pmatrix} \] (2.22)

whose determinant is

\[ \det B = B_{11}B_{22}B_{33}B_{44} - B_{11}B_{22}B_{34}B_{43} - B_{11}B_{23}B_{32}B_{44} + B_{11}B_{24}B_{32}B_{43} 
- B_{21}B_{12}B_{33}B_{44} + B_{21}B_{12}B_{34}B_{43} + B_{21}B_{13}B_{32}B_{44} - B_{21}B_{14}B_{32}B_{43}. \] (2.23)

So, the Mie coefficient \( b_n^r \) is of the form

\[ b_n^r = \frac{\det B_r}{\det B} \] (2.24)

with

\[ B_r = \begin{pmatrix} B_{11} & B_{12} & B_{13} & 0 \\ B_{21} & B_{22} & B_{23} & 0 \\ 0 & B_{32} & B_{33} & N_2 \rho_2 j_n(\rho_2) \\ 0 & B_{42} & B_{43} & \mu_2 j_n(\rho_2) \end{pmatrix} \] (2.25)

whose determinant is

\[ \det B_r = B_{11}B_{22}B_{33}\mu_2 j_n(\rho_2) - B_{11}B_{22}N_2[\rho_2 j_n(\rho_2)]^r B_{43} - B_{11}B_{23}B_{32}\mu_2 j_n(\rho_2) + B_{11}B_{24}B_{32}N_2[\rho_2 j_n(\rho_2)]^r 
- B_{21}B_{12}B_{33}\mu_2 j_n(\rho_2) + B_{21}B_{12}N_2[\rho_2 j_n(\rho_2)]^r B_{43} + B_{21}B_{13}B_{32}\mu_2 j_n(\rho_2) - B_{21}B_{14}B_{32}N_2[\rho_2 j_n(\rho_2)]^r. \] (2.26)

III. DISCUSSION OF SOME TYPICAL CASES WITH LEFT-HANDED MEDIA INVOLVED

In this section we briefly discuss several cases with left-handed media involved.

(i) If the following conditions \( h_n^{(1)}(N_2\rho_1) = 0 \), \( h_n^{(1)}(N_2\rho_2)^\prime = 0 \), \( h_n^{(1)}(N_2\rho_2) = -h_n^{(1)}(\rho_2) \), \( h_n^{(1)}(N_2\rho_2) = -[h_n^{(1)}(\rho_2)]^\prime \), \( N_2 = \mu_2 = \epsilon_2 = -1 \) and \( \mu_0 = +1 \) are satisfied, then it is easily verified that

\[ \det A = 0, \quad \det B = 0 \quad \text{and} \quad a_n^r = \infty, \quad b_n^r = \infty. \] (3.1)

Thus in this case the extinction cross section of the two-layered sphere containing left-handed media is rather large.

(ii) If the following conditions \( h_n^{(1)}(N_2\rho_1) = 0 \), \( [h_n^{(1)}(N_2\rho_1)]^\prime = 0 \), \( h_n^{(1)}(N_2\rho_2) = j_n(\rho_2) \), \( [h_n^{(1)}(N_2\rho_2)]^\prime = [j_n(\rho_2)]^\prime \), \( N_2 = \mu_2 = \epsilon_2 = -1 \) and \( \mu_0 = +1 \) are satisfied, then it is easily verified that

\[ \det A_r = 0, \quad \det B_r = 0 \quad \text{and} \quad a_n^r = 0, \quad b_n^r = 0. \] (3.2)

Thus in this case the extinction cross section of the two-layered sphere containing left-handed media is negligibly small.

(iii) The case with \( N_1 = \mu_1 = \epsilon_1 = -1, \ N_2 = \mu_2 = \epsilon_2 = 0, \ N_0 = \mu_0 = \epsilon_0 = +1 \) is of physical interest, which deserves consideration by using the Mie coefficients presented above.

Based on the calculation of Mie coefficients, one can treat the scattering problem of double-layered sphere irradiated by a plane wave. The scattering and absorption properties of double-layered sphere containing left-handed media can thus be discussed in detail, which are now under consideration and will be submitted elsewhere for the publication.

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**APPENDICES**

5
**Appendix A: Reduced to the case of single-layered sphere**

In order to see whether the above Mie coefficients in the case of double-layered sphere is correct or not, we consider the reduction problem of double-layered case to the single-layered one when the following reduction conditions are satisfied: \( A_{11} = -A_{12}, A_{21} = -A_{22} \) \( N_1 = N_2, \rho_1 = \rho_2, \mu_1 = \mu_2 \).

By lengthy calculation, we obtain

\[
\text{det} A = (A_{11}A_{23} - A_{21}A_{13})(A_{42}A_{34} - A_{32}A_{44})
= (A_{11}A_{23} - A_{21}A_{13}) \left\{ j_n(N_2\rho_2)\mu_2 \left[ \rho_2 h_n^{(1)}(\rho_2) \right]' - \mu_0 [N_2\rho_2 j_n(N_2\rho_2)]' h_n^{(1)}(\rho_2) \right\}
\] (A1)

and

\[
\text{det} A_r = (A_{11}A_{23} - A_{21}A_{13}) \left\{ \mu_0 [N_2\rho_2 j_n(N_2\rho_2)]' j_n(\rho_2) - j_n(N_2\rho_2)\mu_2 [\rho_2 j_n(\rho_2)]' \right\}.
\] (A2)

Thus

\[
a_n^{\prime} = \frac{\mu_0 [N_2\rho_2 j_n(N_2\rho_2)]' j_n(\rho_2) - j_n(N_2\rho_2)\mu_2 [\rho_2 j_n(\rho_2)]'}{j_n(N_2\rho_2)\mu_2 \left[ \rho_2 h_n^{(1)}(\rho_2) \right]' - \mu_0 [N_2\rho_2 j_n(N_2\rho_2)]' h_n^{(1)}(\rho_2)},
\] (A3)

which is just the Mie coefficient \( a_n^{\prime} \) of single-layered sphere [11].

In the same fashion, when the reduction conditions \( B_{11} = -B_{12}, B_{21} = -B_{22} \) \( N_1 = N_2, \rho_1 = \rho_2, \mu_1 = \mu_2 \) are satisfied, one can arrive at

\[
\text{det} B = (B_{11}B_{23} - B_{21}B_{13})(B_{42}B_{34} - B_{32}B_{44})
= (B_{11}B_{23} - B_{21}B_{13}) \left\{ [N_2\rho_2 j_n(N_2\rho_2)]' \mu_2 h_n^{(1)}(\rho_2) - N_2^2 \mu_0 j_n(N_2\rho_2) \left[ \rho_2 h_n^{(1)}(\rho_2) \right]' \right\}
\] (A4)

and

\[
\text{det} B_r = (B_{11}B_{23} - B_{21}B_{13}) \left\{ N_2^2 \mu_0 j_n(N_2\rho_2) [\rho_2 j_n(\rho_2)]' - [N_2\rho_2 j_n(N_2\rho_2)]' \mu_2 j_n(\rho_2) \right\}.
\] (A5)

So,

\[
b_n^{\prime} = \frac{N_2^2 \mu_0 j_n(N_2\rho_2) [\rho_2 j_n(\rho_2)]' - [N_2\rho_2 j_n(N_2\rho_2)]' \mu_2 j_n(\rho_2)}{[N_2\rho_2 j_n(N_2\rho_2)]' \mu_2 h_n^{(1)}(\rho_2) - N_2^2 \mu_0 j_n(N_2\rho_2) \left[ \rho_2 h_n^{(1)}(\rho_2) \right]'},
\] (A6)

which is just the Mie coefficient \( b_n^{\prime} \) of single-layered sphere [11].

This, therefore, means that the Mie coefficients of double-layered sphere presented here is right.

**Appendix B: All the Mie coefficients**

In this appendix, all the Mie coefficients in the double-layered sphere are given. The results are as follows:

**A. Mie coefficients \( a_n^{\prime}, a_n^{\prime\prime} \) and \( a_n^{\prime\prime\prime} \)**

The 4 \times 4 matrix \( A_4 \) is

\[
A_4 = \begin{pmatrix}
0 & A_{12} & A_{13} & 0 \\
0 & A_{22} & A_{23} & 0 \\
j_n(\rho_2) & A_{32} & A_{33} & A_{34} \\
\mu_2 [\rho_2 j_n(\rho_2)]' & A_{42} & A_{43} & A_{44}
\end{pmatrix}
\] (B1)
whose determinant is
\[
\det A = j_n(\rho) A_{12} A_{23} A_{44} - j_n(\rho) A_{22} A_{13} A_{44} - \mu_2 [\rho_j j_n(\rho)]' A_{12} A_{23} A_{34} + \mu_2 [\rho_j j_n(\rho)]' A_{22} A_{13} A_{44}. \tag{B2}
\]
So,
\[
a_n^t = \frac{\det A_t}{\det A}. \tag{B3}
\]

The 4 × 4 matrix \(A_m\) is
\[
A_m = \begin{pmatrix}
A_{11} & 0 & A_{13} & 0 \\
A_{21} & 0 & A_{23} & 0 \\
0 & j_n(\rho) & A_{33} & A_{34} \\
0 & \mu_2 [\rho_j j_n(\rho)]' & A_{43} & A_{44}
\end{pmatrix} \tag{B4}
\]
whose determinant is
\[
\det A_m = -A_{11} j_n(\rho) A_{23} A_{44} + A_{11} \mu_2 [\rho_j j_n(\rho)]' A_{23} A_{34} + A_{21} j_n(\rho) A_{13} A_{44} - A_{21} \mu_2 [\rho_j j_n(\rho)]' A_{13} A_{44}. \tag{B5}
\]
So,
\[
a_m = \frac{\det A_m}{\det A}. \tag{B6}
\]

The 4 × 4 matrix \(A_{\bar{m}}\) is
\[
A_{\bar{m}} = \begin{pmatrix}
A_{11} & A_{12} & 0 & 0 \\
A_{21} & A_{22} & 0 & 0 \\
0 & j_n(\rho) & A_{32} & A_{34} \\
0 & \mu_2 [\rho_j j_n(\rho)]' & A_{42} & A_{44}
\end{pmatrix} \tag{B7}
\]
whose determinant is
\[
\det A_{\bar{m}} = A_{11} A_{22} j_n(\rho) A_{44} - A_{11} A_{22} A_{44} \mu_2 [\rho_j j_n(\rho)]' - A_{21} A_{12} j_n(\rho) A_{44} + A_{21} A_{12} A_{44} \mu_2 [\rho_j j_n(\rho)]'. \tag{B8}
\]
So,
\[
a_{\bar{m}} = \frac{\det A_{\bar{m}}}{\det A}. \tag{B9}
\]

It is readily verified that under the reduction conditions \(A_{11} = -A_{12}, A_{21} = -A_{22}, N_1 = N_2, \rho_1 = \rho_2, \mu_1 = \mu_2\), one can arrive at
\[
\det A_t = \det A_m, \quad a_n^t = a_m^t, \quad \det A_{\bar{m}} = 0, \quad a_{\bar{m}} = 0, \tag{B10}
\]
which means that the above Mie coefficient can be reduced to the those of single-layered sphere.

### B. Mie coefficients \(b_n^t, b_m^t\) and \(b_{\bar{m}}^t\)

The 4 × 4 matrix \(B_t\) is
\[
B_t = \begin{pmatrix}
0 & B_{12} & B_{13} & 0 \\
0 & B_{22} & B_{23} & 0 \\
N_2 [\rho_j j_n(\rho)]' & B_{32} & B_{33} & B_{34} \\
\mu_2 j_n(\rho) & B_{42} & B_{43} & B_{44}
\end{pmatrix} \tag{B11}
\]
whose determinant is
\[
\det B_t = N_2 [\rho_2 j_n(\rho_2)]' B_{12} B_{23} B_{44} - N_2 [\rho_2 j_n(\rho_2)]' B_{23} B_{13} B_{44} - \mu_2 j_n(\rho_2) B_{12} B_{23} B_{34} + \mu_2 j_n(\rho_2) B_{22} B_{13} B_{34}. \quad (B12)
\]

So,
\[
b_t^n = \frac{\det B_t}{\det B}. \quad (B13)
\]

The 4 × 4 matrix \(B_m\) is
\[
B_m = \begin{pmatrix}
B_{11} & B_{12} & 0 & 0 \\
B_{21} & B_{22} & 0 & 0 \\
0 & N_2 [\rho_2 j_n(\rho_2)]' & B_{33} & B_{34} \\
0 & \mu_2 j_n(\rho_2) & B_{43} & B_{44}
\end{pmatrix}
\]
whose determinant is
\[
\det B_m = -B_{11} N_2 [\rho_2 j_n(\rho_2)]' B_{23} B_{44} + B_{11} \mu_2 j_n(\rho_2) B_{23} B_{34} + B_{21} N_2 [\rho_2 j_n(\rho_2)]' B_{13} B_{44} - B_{21} \mu_2 j_n(\rho_2) B_{13} B_{34}. \quad (B15)
\]

So,
\[
b_m^n = \frac{\det B_m}{\det B}. \quad (B16)
\]

The 4 × 4 matrix \(B_{\bar{m}}\) is
\[
B_{\bar{m}} = \begin{pmatrix}
B_{11} & B_{12} & 0 & 0 \\
B_{21} & B_{22} & 0 & 0 \\
0 & B_{32} & N_2 [\rho_2 j_n(\rho_2)]' & B_{34} \\
0 & B_{42} & \mu_2 j_n(\rho_2) & B_{44}
\end{pmatrix}
\]
whose determinant is
\[
\det B_{\bar{m}} = B_{11} B_{22} N_2 [\rho_2 j_n(\rho_2)]' B_{44} - B_{11} B_{22} B_{34} \mu_2 j_n(\rho_2) - B_{21} B_{12} N_2 [\rho_2 j_n(\rho_2)]' B_{44} + B_{21} B_{12} B_{34} \mu_2 j_n(\rho_2). \quad (B18)
\]

So,
\[
b_{\bar{m}}^n = \frac{\det B_{\bar{m}}}{\det B}. \quad (B19)
\]

It is readily verified that under the reduction conditions \(B_{11} = -B_{12}, \ B_{21} = -B_{22}, \ N_1 = N_2, \ \rho_1 = \rho_2, \ \mu_1 = \mu_2\), one can arrive at
\[
\det B_t = \det B_m, \quad b_t^n = b_m^n, \quad \det B_{\bar{m}} = 0, \quad \bar{b}_{\bar{m}}^n = 0, \quad (B20)
\]
which means that the above Mie coefficient can be reduced to the those of single-layered sphere.

[1] Smith, D. R., Padilla, W. J., Vier, D. C. et al., Phys. Rev. Lett. 84, 4184 (2000).
[2] Klimov, V. V., Opt. Comm. 211, 183 (2002).
[3] Pendry, J. B., Holden, A. J., Robbins, D. J. and Stewart, W. J., IEEE Trans. Microwave Theory Tech. 47, 2075 (1999).
[4] Shelby, R. A., Smith, D. R. and Schultz, S., Science 292, 77 (2001).
[5] Ziolkowski, R. W., Phys. Rev. E 64, 056625 (2001).
[6] Veselago, V. G., Sov. Phys. Usp. 10, 509 (1968).
[7] Pendry, J. B., Holden, A. J., Stewart, W. J. and Youngs, I., Phys. Rev. Lett. 76, 4773 (1996).
[8] Pendry, J. B., Holden, Robbins, D. J. and Stewart, W. J., J. Phys. Condens. Matter 10, 4785 (1998).
[9] Maslovski, S. I., Tretyakov, S. A. and Belov, P. A., Inc. Microwave Opt. Tech. Lett. 35, 47 (2002).
[10] Wang, W. and Asher, S. A., J. Am. Chem. Soc. 123, 12528 (2001).
[11] Stratton, J. A., Electromagnetic theory (New York: McGraw-Hill) (1941).