Distributed robust adaptive output-feedback synchronization

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Abstract. This paper addresses the output synchronization of heterogeneous uncertain linear systems in the presence of input disturbance. The synchronization problem is solved by introducing the distributed model reference adaptive control. The distinguishing features of the proposed approach are the capability to synchronize the entire network output by using only the control input and the output of neighbouring systems. It also provides synchronization in the presence of bounded control input disturbance. The control input disturbance is handled by extending the proposed method with the sigma-modification. By using the distributed matching condition assumptions, the coupling gains between the agents can be updated adaptively. The distributed robust adaptive controller is first analysed by using the distributed Lyapunov function. Its effectiveness is validated in the simulation of the cart inverted pendulum systems stabilization problem.

Keywords: linear system, synchronization, control input, robust adaptive, Lyapunov system

1. Introduction

In the presence of model dynamics uncertainties, a simple controller cannot stabilize the system. Here, adaptive control can be the solution to control the system with large uncertainties. The stabilization problem arises in many applications such as the inverted pendulum system. The stabilization problem of inverted pendulum systems such as wheeled inverted pendulum [1], double inverted pendulum [2], flying inverted pendulum [3] has been discussed extensively. These stabilization problems represent a potential solution for many applications such as Automated Guided Vehicle (AGV) with heavy payload [4], lower limb exoskeleton for paraplegics [5], stabilization of offshore platform [6], stabilization of unmanned aerial vehicle [7] and a Segway [8].

The systems with adaptive control may lead to an unstable system in the presence of disturbances. A lot of effort has been proposed to handle the disturbance problem, which leads to a research work called robust adaptive control [9]. The stabilization problem becomes more interesting if the systems are distributed [10]. This distributed stabilization problem requires synchronization to achieve its objective. The main objective of the synchronization is to reduce the states’ or the output’s error of the entire agent to zero by the time goes to infinite. The synchronization can be divided into the centralized scheme and distributed scheme. In a centralized scheme, there is a central node that receives all the information of each agent then uses it to control the agents [11–13]. In the distributed scheme, each agent only using the local information, i.e. neighbour information’s control input or states/output [14]. The distributed
synchronization of linear heterogeneous uncertain agents without any sliding mode has been proposed in [15].

In this work, the distributed adaptive reference model control is proposed to converge each uncertain heterogeneous agent’s dynamics to the reference dynamics. This approach only requires the control input and the output of the neighboring agents. Note that the proposed approach does not require any global information and we are assuming a Directed Acyclic Graph (DAG). The bounded input disturbance can be handled by introducing the \( \sigma \)-modification to the proposed control law. The proposed distributed output synchronization with \( \sigma \)-modification shows the synchronization error converges asymptotically to zero. Here, the Lyapunov-based approach is used to analyze the stability of the proposed method.

The simulation of inverted pendulum systems demonstrated the effectiveness of the proposed approach. This manuscript is organized as follows: problem formulation and the system’s dynamics, presents the numerical simulation of the test case: inverted pendulum systems, then concluded and propose directions for further research.

2. Problem formulations

In order to support the proposed method, we propose the communication graph shown in Figure 1 that describes the communication flow in a network. Here, Agent 0, the reference model, does not receive any signal from other agents. Agent 1, the leader, synchronizes its output to the reference model output. Simultaneously, agent 2 and agent 3, synchronizes its output to the predecessor agents, agent 1 and then agent 2.

![Figure 1. DAG of multi-agent system](image)

From the fig. 1, agent 2 and agent 3 synchronize its output to the reference model indirectly by using local information, the output, and the neighboring agent’s input. Note that the communication graph does not contain any cyclic communication, Directed Acyclic Graph (DAG). In cyclic networks, the distributed model reference adaptive control with parameter projection can achieve the synchronization [16]. Then, let us define the agent’s dynamics in the network. The dynamics of the reference model satisfying the following dynamics

\[
y_0 = G_0(s) = k_0 \frac{N_0}{D_0} r
\]

(1)

where \( N_0 \) and \( D_0 \) known monic polynomials, \( y_0 \in \mathbb{R} \) is the output of the reference model, \( r \in \mathbb{R} \) is the reference input, and \( k_0 \) is the high-frequency gain. Then, let us define the dynamics of a leader, and the followers denoted with the subscript 1, 2, and 3 satisfying the following dynamics
where \( u_1, u_2, u_3 \in \mathbb{R} \) are the control inputs and \( y_1, y_2, y_3 \in \mathbb{R} \) are the outputs, \( N_1, N_2, N_3, D_1, D_2, \) and \( D_3 \) are unknown monic polynomials, and \( k_1, k_2, k_3 \) are the high frequency gains. In this work, the \( N_1, N_2, N_3 \) and \( D_1, D_2, D_3 \) may have different dynamics. To achieve of our main objective, we propose the assumptions for the agents and the reference model as follows:

**Reference model Assumptions:**
(i) The Agent 0, the reference model, transfer function, \( G_0 \), is monic Hurwitz polynomials.
(ii) The relative degree of the reference model, \( G_0 \), is the same as the agents’ relative degree, \( G_i \).

**Agents Assumptions:**
(i) The Agent does not have any positive zero.
(ii) The degree of the denominator \( D_i \) is known.
(iii) The sign of the high frequency gains \( k_i, i \in 1,2,3 \) is known.

### 3. Distributed Robust adaptive output-feedback synchronization

#### 3.1. Leader - reference synchronization

This section will discuss the preliminary result of the distributed adaptive controller in line with [17]. It is known that we can synchronize the leader’s output to the reference’s output by using a well-known model reference adaptive control in Chapter 5.3 in [18] which leads to the controller

\[
\dot{u}_1 = f_1' \frac{\beta}{\Lambda(s)} u_1 + g_1' \frac{\beta}{\Lambda(s)} y_1 + h_1 y_1 + l_1 r
\]

where \( f_1', g_1', h_1, \) and \( l_1 \) are the estimates for the ideal gain \( f_1^*, g_1^*, h_1^*, \) and \( l_1^* \), respectively, \( \Lambda(s) \) is a Hurwitz monic polynomial which is defined as

\[
\Lambda(s) = s^{n-1} + \lambda_{n-2}s^{n-2} + \ldots + \lambda_1 s + \lambda_0
\]

and \( \beta \) is defined as

\[
\beta(s) = \begin{bmatrix} s^{n-2} & s^{n-3} & \ldots & s^{n-2} \end{bmatrix} \quad \text{for } n \geq 2
\]

\[
\beta(s) = 0 \quad \text{for } n = 1.
\]

The adaptive law for the leader is defined as follows:

\[
\dot{\theta}_1 = -\Gamma_1 e_{10} \omega_1 sgn\left(\frac{k_0}{k_1}\right)
\]

where

\[
\theta_1' = \begin{bmatrix} f_1' & g_1' & h_1 & l_1 \end{bmatrix}
\]

\[
\omega_1 = \begin{bmatrix} \omega_{u1} & \omega_{y1} & y_1 & r \end{bmatrix}'
\]

\( \Gamma_1 = \Gamma_1' > 0 \) is a positive diagonal matrix, \( e_{10} = y_1 - y_0 \). Then, let us defined ‘\( \omega_{u1}, \omega_{y1}, F, \) and \( d \) as follows

\[
\omega_{u1}' = F\omega_{u1} + Ru_1
\]

\[
F = \begin{bmatrix} -\lambda_{n-2} & \ldots & -\lambda_0 \\ I_{n-2} & 0 \end{bmatrix}
\]

\[
R = \begin{bmatrix} 1 & 0 & \ldots & 0 \end{bmatrix}'
\]

\[
\omega_{y1}' = F\omega_{y1} + Ry_1
\]
By using the control law $u_1$, the convergence of $e_1 \to 0$ for $t \to \infty$ can be achieved. The Lyapunov approach is derived to prove the asymptotic convergence of synchronization error analytically. First, let us rewrite the state–space of the Agent 1 (the leader) in the closed-loop form by adding and subtracting it by the ideal control law:

$$\dot{x}_1 = \tilde{A}_1 \tilde{x}_1 + \tilde{B}_1 u_1 + \tilde{B}_1 (u_1 - \theta'_1 \omega_1) \quad y_1 = \tilde{C}_1 \tilde{x}_1$$

where $\tilde{x}_1 = [x'_1 \quad \omega'_{\phi_1} \quad \omega'_{\psi_1}]^T$. Then $\tilde{A}_1, \tilde{B}_1,$ and $\tilde{C}_1$ are defined as

$$\tilde{A}_1 = \begin{bmatrix} A_1 + B_1 h^*_1 c'_1 & B_1 f^*_1 & B_1 g^*_1 \\ R \tilde{h}^*_1 \tilde{c}'_1 & F + R \tilde{f}^*_1 & R \tilde{g}^*_1 \\ \tilde{R} e'_1 & 0 & F \end{bmatrix}$$

$$\tilde{B}_1 = \begin{bmatrix} B_1 \\ R \\ 0 \end{bmatrix}$$

$$\tilde{C}_1 = \begin{bmatrix} c'_1 & 0 & 0 \end{bmatrix}.$$  \hfill (10)

The transfer function of Agent 1 can be matched to the transfer function of Agent 0 which is defined as follows $\tilde{C}_1(sI - \tilde{A}_1)^{-1} \tilde{B}_1 \tilde{c}_1^* = C_0(sI - A_0)^{-1} B_0$. Thus, the state–space of Agent 1 can be written as follows

$$\dot{x}_{10} = A_0 \tilde{x}_{10} + B_0 r + B_0 \gamma^*_1 (u_1 - \theta'_1 \omega_1) \quad y_{10} = C_0 \omega_{10}$$

where $\gamma^*_1 = \frac{1}{\tau} = \frac{k_0}{k_1}$. Then, we define the state error and the output error between the reference and the leader $x_{10} = \tilde{x}_1 - x_{10}$ and $e_{10} = y_1 - y_{10}$, respectively. The error equation is defined as

$$\dot{\tilde{x}}_{10} = A_0 \tilde{x}_{10} + B_0 \gamma^*_1 \tilde{\theta}'_1 \omega_1 \quad e_{10} = C_0 \omega_{10}$$

where the difference between the estimate and ideal parameters is defined as $\tilde{\theta}'_1 = \theta_1 - \theta'_1$.

The following Lyapunov function is used to proof the asymptotic convergence of the synchronization error between the leader’s (Agent 1) output and the model reference’s (Agent 0) output analytically

$$V_1(\tilde{\theta}_1, \tilde{x}_{10}) = \frac{\tilde{\theta}'_1 \Gamma_1^{-1} \tilde{\theta}_1}{2} |\gamma^*_1| + \frac{\tilde{x}'_{10} P \tilde{x}_{10}}{2}$$

where $P = P^T > 0$ such that

$$PA_0 + A'_0 P = -qq' - wH \quad PB_0 = C_0$$

where $H = H^T > 0$, and $w > 0$. Then, the time derivative of $V_1$ can be defined as follows

$$\dot{V}_1 = -\frac{\tilde{x}'_{10} q \tilde{x}_{10}}{2} - \frac{w}{2} \tilde{x}'_{10} H \tilde{x}_{10} + PB_0 \tilde{x}_{10} \gamma^*_1 \tilde{\theta}'_1 \omega_1$$

$$+ \tilde{\theta}'_1 \Gamma_1^{-1} \tilde{\theta}_1 |\gamma^*_1|.$$  \hfill (15)

Since $PB_0 \omega_{10} = C_0 \omega_{10} = e_{10}$ and $\gamma^*_1 = |\gamma^*_1| \text{sgn}(\gamma^*_1)$, we can define the $\tilde{\theta}'_1$ such that

$$\tilde{\theta}_1 = \dot{\theta}_1 = -\Gamma_1 e_{10} \omega_1 \text{sgn}(\gamma^*_1)$$  \hfill (16)
thus we have
\[
\dot{V}_1 = -\frac{x_{10}'q^T x_{10}}{2} - \frac{w x_{10}' H x_{10}}{2}.
\] (17)

From (17), we obtain that \(V_1\) has a finite limit, so \(\hat{x}_0, \hat{\theta}_1 \in L_e\). We have \(\hat{x}_0 = \hat{x}_1 - x \in \theta_0 \in L_e\), and it is known that \(\hat{x}_0 \in L_e\), therefore we have \(\hat{x}_1 \in L_e\). This implies \(x_1, y_1, \omega_1, \omega_2 \in L_e\). Then, we have \(u_1 \in L_e\). It can be seen that all signals are bounded and leads to \(V_1 \rightarrow 0\) as \(t \rightarrow \infty\). Thus we have \(e_{10} \rightarrow 0\) for \(t \rightarrow \infty\), which concludes the proof.

3.2. Follower-leader robust synchronization

This section’s primary focus is to discover the control law \(u_2\) of the Agent 2 (the follower) that synchronizes its dynamics to the Agent 1 (the leader) under input disturbance. First, let us propose the control law \(u_2\) to match the follower dynamics to the leader dynamics in the absence of input disturbance
\[
u_2 = f_{21}' \frac{\beta}{\Lambda(s)} u_1 + g_{21}' \frac{\beta}{\Lambda(s)} y_1 + h_{21} y_1 + l_{21} u_1
\]
\[
+ f_2' \frac{\beta}{\Lambda(s)} (u_2 - u_1) + g_2' \frac{\beta}{\Lambda(s)} (y_2 - y_1)
\]
\[
+ h_2 (y_2 - y_1)
\] (18)

where the parameter of the control law \(f_{21}', f_2', g_{21}', g_2', h_{21}, h_1,\) and \(l_{21}\) are the estimates for \(f_{21}', f_2', g_{21}', g_2', h_{21}, h_1,\) and \(l_{21}\), respectively. Then let us proposed the following adaptive law
\[
\dot{\theta}_2 = -\Gamma_2 e_{21}^T \omega_2 sgn\left(\frac{k_1}{k_2}\right)
\] (19)

where
\[
\theta_2 = \begin{bmatrix} f_{21}' & g_{21}' & h_{21} & l_{21} \\ f_2' & g_2' & h_2 \end{bmatrix}
\]
\[
\omega_2 = \begin{bmatrix} \omega_{u_1} & \omega_{y_1} & y_1 & u_1 & \omega_{\omega_21} & \omega_{y_21} & y_2 - y_1 \end{bmatrix}^T
\] (20)

\(\Gamma_2 = \Gamma_2 > 0\) is a positive diagonal matrix, and \(e_{21} = y_2 - y_1\). Then, let us defined \(\dot{\omega}_{u_21}\) and \(\dot{\omega}_{y_21}\) as follows
\[
\dot{\omega}_{u_21} = F \omega_{u_21} + R (u_2 - u_1)
\]
\[
\dot{\omega}_{y_21} = F \omega_{y_21} + R (y_2 - y_1)
\] (21)

By using the control law, \(u_2\), we can synchronize the follower’s output to the leader’s output without input disturbance. In the presence of input disturbance, some bounded error will occur in the control law (18). This input disturbance leads to the modification of follower control law \(u_2\) as follows
\[
u_2 = f_{21}' \frac{\beta}{\Lambda(s)} u_1 + g_{21}' \frac{\beta}{\Lambda(s)} y_1 + h_{21} y_1 + l_{21} u_1
\]
\[
+ f_2' \frac{\beta}{\Lambda(s)} (u_2 - u_1) + g_2' \frac{\beta}{\Lambda(s)} (y_2 - y_1)
\]
\[
+ h_2 (y_2 - y_1) + d_{21}
\] (22)

where \(d_{21}\) is the bounded control input error which satisfies \(|d_{21}| < \tilde{d}\) for some unknown \(\tilde{d}\). So that we proposed the adaptive law with leakage or \(\sigma\)-modification as follows
where $\sigma \geq 0$ is a scalar to be designed by the user that allows robustness concerning input disturbance. In order to prove the asymptotic convergence of the synchronization error analytically in the presence of input disturbance, the Lyapunov approach is derived. First, we define the follower, Agent 2, dynamics by adding and subtracting it by the ideal control law

$$\dot{x}_2 = \bar{A}_2 \bar{x}_2 + \bar{B}_2 \bar{u}_2 + \bar{B}_2 (u_2 - \bar{\theta}_2^* \omega_2) \quad y_2 = \bar{C}_2 \bar{x}_2$$

where $\bar{x}_2 = \begin{bmatrix} x'_2 & \omega'_1 & \omega'_y_1 & \omega'_y_2 \end{bmatrix}'$ and $\bar{u}_2 = \begin{bmatrix} u_1 & y_1 \end{bmatrix}'$.

$$\bar{A}_2 = \begin{bmatrix} A_2 + B_2 \bar{h}_2 \bar{c}_2 & B_2 f_1 \bar{c}_2 \bar{c}_2 & B_2 g_1 \bar{c}_2 \bar{c}_2 & B_2 g_2 \bar{c}_2 \bar{c}_2 \\ 0 & F & 0 & 0 \\ Re \bar{c}_2 & 0 & F & 0 \\ Rh \bar{c}_2 & Re \bar{c}_2 & Re \bar{c}_2 & F' \end{bmatrix},$$

$$\bar{B}_2 = \begin{bmatrix} B_2 \\ \frac{B_2}{\bar{c}_2} \\ 0 \\ \bar{R}(1 - \frac{1}{\bar{c}_2}) \frac{B_2}{\bar{c}_2} \end{bmatrix},$$

$$\bar{C}_2 = \begin{bmatrix} c^T_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$ (25)

From Equation (25), the transfer function of Agent 2 can be matched to the transfer function of Agent 1 which is defined as follows $\bar{C}_2 (sI - \bar{A}_2)^{-1} \bar{B}_2 \bar{e}_{21} = \bar{C}_1 (sI - \bar{A}_1)^{-1} \bar{B}_1 \bar{e}_1$. Thus, the state–space of Agent 2 can be written as follows:

$$\dot{x}_2 = A_0 \bar{x}_2 + B_0 \bar{r} + B_0 \gamma^* \bar{e}_{21} (u_2 - \bar{\theta}_2^* \omega_2) \quad y_2 = C_0 \bar{x}_2$$

where $\gamma^* = \frac{1}{\bar{c}_2} = \frac{b_2}{k_2}$.

Then, we define the state error and the output error between the follower and the leader $\tilde{x}_{21} = \bar{x}_2 - x^*_1$, and $\tilde{e}_{21} = y_2 - y_1$, respectively. The error equation is defined as

$$\dot{\tilde{x}}_{21} = A_0 \tilde{x}_{21} + B_0 \gamma^* \bar{e}_{21} (u_2 - \bar{\theta}_2^* \omega_2) \quad \tilde{e}_{21} = C_0 \tilde{x}_{21}$$

where $\gamma^* = \theta_2 - \bar{\theta}_2^*$. The following Lyapunov function is used to prove the asymptotic convergence of the synchronization error between the follower’s (Agent 2) output and the leader’s (Agent 1) output analytically

$$V_2(\tilde{\theta}_2, \tilde{x}_{21}) = \frac{\tilde{x}^T_{21} P \tilde{x}_{21}}{2} + \frac{\tilde{\theta}_2^T \Gamma_2^{-1} \tilde{\theta}_2}{2} |\gamma^*|$$

where $\Gamma_2 > 0$ and $P = P^0 > 0$ such that (14) holds. Then, the time derivative of $V_2$ can be defined as follows

$$\dot{V}_2 = -\frac{\tilde{x}^T_{21} P \tilde{x}_{21}}{2} - \frac{v}{2} \tilde{x}^T_{21} L \tilde{x}_{21} + PB_0 \tilde{x}_{21} \gamma^* \tilde{\theta}_2 \omega_2$$

$$+ \gamma^* \tilde{\theta}_2 \tilde{\theta}_2 \sigma + \tilde{\theta}_2^T \Gamma_2^{-1} \tilde{\theta}_2 |\gamma^*|.$$ (29)

Since $PB_0 \tilde{x}_{21} = C_0 \tilde{x}_{21} = \tilde{e}_{21}$ and $\gamma^* = |\gamma^*| \text{sgn}(\gamma^*)$, we can define $\tilde{\theta}_2$ such that

$$\tilde{\theta}_2 = \tilde{\theta}_2 = -\Gamma_2 \tilde{e}_{21} \omega_2 \text{sgn}(\gamma^*) - \sigma \tilde{\theta}_2 \omega_2$$

thus we have:
From (31), we obtain that $V_2$ has a finite limit, so $\tilde{x}_{21}, \tilde{\theta}_2 \in L_\infty$. We have $\tilde{x}_2 = \tilde{x}_1 - \bar{x}_1 \in L_\infty$ and previously known that $\tilde{x}_1 \in L_\infty$, we have $\tilde{x}_2 \in L_\infty$. This implies $x_2, y_2, \omega_{u1}, \omega_{y1}, \omega_{u21}, \omega_{y21} \in L_\infty$. Then, we have $u_2 \in L_\infty$. It can be seen that all signals are bounded and leads to $V_{21} \to 0$ as $t \to \infty$. Thus, we have $e_{21} \to 0$ for $t \to \infty$ which concludes the proof.

4. Numerical Simulation

This section will consider the synchronization of cart inverted pendulum as a test case for the proposed distributed robust output synchronization control algorithm. Figure 1 shows the communication graph of cart inverted pendulum.

The cart inverted pendulum dynamics are divided into rotational, horizontal, and vertical motions, which are defined as follows:

- Rotational motion of pendulum
  \[ I\ddot{\theta} = f_y l \sin \theta - f_x l \cos \theta \]  
  \[ I = \text{moment of inertia of pendulum}, \quad m = \text{mass of pendulum}, \quad M = \text{mass of cart}, \quad l = \text{length of pendulum}, \quad g = \text{gravitational force}, \quad \theta = \text{pendulum angle}, \quad x = \text{cart position coordinate}, \quad F = \text{external force applied to the cart}, \quad f_x \text{ and } f_y \text{ are the force applied to the pendulum}, \quad b = \text{force caused by friction}. \]

- Horizontal motion of pendulum
  \[ mL^2 \frac{d^2}{dt^2} (x + l \sin \theta) = f_x \]  
  \[ (33) \]

- Vertical motion of pendulum
  \[ mL^2 \frac{d^2}{dt^2} (l \cos \theta) = f_y - mg \]  
  \[ (34) \]

- Horizontal motion of the cart
  \[ M \frac{d^2}{dt^2} = F - f_x - b \dot{x} \]  
  \[ (35) \]

where $I$ is moment of inertia of pendulum, $m$ is the mass of pendulum, $M$ is the mass of cart, $l$ is the length of pendulum, $g$ is gravitational force, $\theta$ is the pendulum angle, $x$ is the cart position coordinate, $F$ is the external force applied to the cart, $f_x$ and $f_y$ are the force applied to the pendulum, and $b$ is force caused by friction.

The cart inverted pendulum is shown in Figure 2. We linearized the cart inverted pendulum dynamics by assuming the angle of the pendulum is slightly change $\sin \theta = \theta$ and $\cos \theta = 1$.

Figure 2. Cart Inverted Pendulum
Therefore, we have the cart inverted pendulum dynamics defined as

\[
\begin{align*}
ML\ddot{\theta} &= (M + m)g\theta + bx' - F \\
Mx' &= F_m - mg\theta - bx.
\end{align*}
\]  
(36)

The representation of cart inverted pendulum dynamics in state-space form can be written as follows

\[
\begin{bmatrix}
\dot{x} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & -\frac{mg}{Ml} & 0 \\
0 & 0 & 0 & 1 \\
0 & \frac{b}{Ml} & \frac{(M+m)g}{Ml} & 0
\end{bmatrix}
\begin{bmatrix}
x \\
\dot{x} \\
\dot{\theta} \\
\dot{\theta}
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
\frac{1}{Ml} \\
0 \\
-\frac{1}{Ml}
\end{bmatrix} F
y = [0 \ 0 \ 1 \ 0].
\]  
(37)

\begin{table}
\centering
\caption{Cart inverted pendulum initial conditions and parameters}
\label{tab:cart_pendulum_initial_conditions}
\begin{tabular}{|c|c|c|c|c|}
\hline
Agent & Initial cond. & mass (m) & length (l) & mass (M) \ Pendulum & Friction \\
(Leader) & x, Pendulum & & & & Cart \\
& & & & & Friction \\
\hline
Agent 1 & [1,0,1,0]' & 0.002 & & 50 & 0.02 & 0.1 \\
(Follower 1) & & & & & & \\
\hline
Agent 2 & [-1,0,-1,0]' & 0.003 & & 75 & 0.0030 & 0.1 \\
(Follower 2) & & & & & & \\
\hline
Agent 3 & [0.5,0,0.5,0]' & 0.003 & & 70 & 0.0030 & 0.1 \\
(Follower 3) & & & & & & \\
\hline
\end{tabular}
\end{table}

Table 1 shows the cart's parameters inverted pendulum, which are used only for the sake of simulations. In practice, the parameters are unknown and heterogeneous among the cart inverted pendulum. The reference model dynamics which satisfied the assumption 1 and the assumption 2 is defined in state-space representation as follows

\[
\begin{bmatrix}
\dot{x} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-1 & -4 & -6 & -4
\end{bmatrix}
\begin{bmatrix}
x \\
\dot{x} \\
\dot{\theta} \\
\dot{\theta}
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
1 \\
0 \\
0
\end{bmatrix} F
y = [0 \ 0 \ 1 \ 0].
\]  
(38)

Figure 3 shows that the synchronization of cart inverted pendulums without input disturbance can be achieved by using control law (18) and the adaptive law (19). Figure 4 shows the synchronization of cart inverted pendulums with input disturbance, \( d = 200 \). It can be seen that the synchronization cannot be achieved if we used the control law (18) and the adaptive law (19).
In the presence of input disturbance, the proposed control law (22) and the adaptive law (23) guarantee the synchronization and the stability of the entire network. Figure 5 shows the synchronization in the presence of input disturbance with $\sigma = 1$. It can be seen that the output synchronization of all agent to the reference model output can be achieved for sinusoidal reference input [19-21].

**Figure 3.** Output Synchronization without input disturbance

![Output Response](image)

**Figure 4.** Output Synchronization with input disturbance, $d\tilde{=} = 200$

![Output Response](image)
Output Response

Figure 5. Output Synchronization with input disturbance, $\bar{d} = 200$ and $\sigma = 1$

5. Conclusion
This work has shown the potential solution of synchronizing the uncertain heterogeneous agents in the presence of input disturbance. The $\sigma$-modification is distributed among the entire network to handle input disturbance. The synchronization is achieved without any global information and applicable only for the DAG network. The Lyapunov-based approach was derived analytically to show that error converges to zero in the presence of input disturbance. Future work will include the distributed $\sigma$-modification for uncertain Euler-Lagrange system that consider the input constraint and the state constraint.

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