Coulomb interaction in a non-uniform dielectric medium \cite{1,2}, is one of the central points in investigations of large classes of nanoscale materials, such as, for example, graphene\cite{3,4} and other atomically thin crystals including their heterostructures\cite{5}, as well as colloidal nanoplatelets\cite{6}, and two-dimensional perovskites\cite{7,8}. This problem has been, in recent years, particularly largely discussed in reference to a vast amount of investigations of excitons in monolayers (MLs) of semiconducting transition metal dichalcogenides (S-TMDs)\cite{9-13}. Surprisingly, at first sight, the Rydberg series of $s$-type excitonic states in these archetypes of two-dimensional (2D) semiconductors, does not follow the model system of a 2D hydrogen atom\cite{14-16}, with its characteristic energy sequence, $\sim 1/(n-1/2)^2$, of states with a principal quantum number $n$. The main reason for that is a dielectric inhomogeneity of the 2D S-TMD structures, i.e., MLs surrounded by (deposited/encapsulated on/in) alien dielectrics. At large electron-hole ($e$-$h$) distances, the Coulomb interaction scales with the dielectric response of the surrounding medium whereas it appears to be significantly weakened at short $e$-$h$ distances by the usually stronger dielectric screening in the 2D plane. A common approach to account for the excitonic spectra of S-TMD MLs refers to the numerical solutions of the Schrödinger equation, in which the $e$-$h$ attraction is approximated by the Rytova-Keldysh (RK) potential\cite{11,12}. The RK approach has been used to explain a number of excitonic features in S-TMD MLs\cite{17}. However, it is only solvable numerically. A more phenomenological and intuitive approach, presented below, might be an optional solution to this problem.

In this Letter, we demonstrate that the energy spectrum, $\epsilon_n$ ($n=1, 2, \ldots$), of Rydberg series of $s$-type excitonic states in S-TMD MLs may follow a simple energy ladder: $\epsilon_n = -Ry^*/(n+\delta)^2$. From magneto-optical investigations of a WSe$_2$ ML encapsulated in hexagonal boron nitride (hBN), we accurately establish that $Ry^*$=140.5 meV and $\delta$=-0.083 in this particular S-TMD system. The $\epsilon_n$ spectrum, with $\delta\sim 0$, turns out to closely reflect the characteristic spectrum of a three-dimensional (3D) hydrogen atom. The $\epsilon_n$ spectrum, close to zero for WSe$_2$ as well as for MoS$_2$ monolayers, what implies that the energy ladder of excitonic states in these two-dimensional structures resembles that of Rydberg states of a three-dimensional hydrogen atom.

To accurately determine the characteristic ladder of $s$-type excitonic resonances in the experiment, we profited of a particularly suitable for this purpose method of magneto-optical spectroscopy\cite{19,20}. The active part of the structure used for these experiments was a WSe$_2$ ML embedded in between hBN layers. More details on samples’ preparation and on the experimental techniques can be found in the Supplemental Materials (SM). We measured the (circular) polarization resolved magneto-photoluminescence (PL) at low temperatures (4.2 K) and in magnetic fields up to 14 T, applied in the direction perpendicular to the monolayer plane. Here we focus on magneto-PL spectra of our WSe$_2$ ML, observed in the spectral range from $\sim$1.7 to $\sim$1.9 eV. As shown in Fig. 1(a) and (b), these spectra are composed of up to five PL peaks, which are clearly resolved in the range
of high magnetic fields. Following a number of previous investigations\textsuperscript{17,21-24} on similar structures, the observed PL peaks are identified with a series of excitonic resonances forming the 1\textit{s}, 2\textit{s}, \ldots, 5\textit{s} Rydberg series of the so-called A exciton\textsuperscript{11,12}. Each \textit{ns} PL peak, \textit{n}=1, 2, \ldots, 5, demonstrates the valley Zeeman effect. This is illustrated in Fig. 1(c) in which the energies of the \(\sigma^+\) and \(\sigma^-\) polarized PL peaks are plotted as a function of the magnetic field. In accordance with previous reports we extract \(g=−4.1\) for valley \(g\)-factor of the 1\textit{s} resonance, but read a significantly stronger valley Zeeman effect for all excited states \(g\sim−4.8\). The later observation is intriguing and should be investigated in more details, which is, however, beyond the scope of the present paper.

The magnetic field evolution of the mean energies of \(\sigma^+\) and \(\sigma^-\) PL peaks is illustrated in Figs 1(d) and (e). These energies, \(E_{\text{ns}}\), are plotted as a function of the magnetic field \(B\) in Fig. 1(d), and as a function of \(B^2\) in Fig. 1(e), which illustrates the characteristic but distinct behavior of \(\text{ns}\) states in the so-called low- and high-field regime\textsuperscript{14,20}. The high field limit, for a given \(\text{ns}\) resonance, appears when \(l_B\ll r_{\text{ns}}\), or conversely when binding energy of the \(\text{ns}\) state \(E^{\sigma\text{s}}_b\ll\hbar\omega^*_c/2\). Here \(r_{\text{ns}}\) and \(E^{\sigma\text{s}}_b\) denote, correspondingly, the mean lateral extension and the binding energy \(E^{\sigma\text{s}}_b=\hbar\omega^*_c\delta\) of a given \textit{ns} state at \(B=0\), \(\hbar\omega^*_c=\hbar eB/\mu\), and other symbols have their conventional meaning. In the high-field limit, the energies of \(E_{\text{ns}}\) resonances approach a linear dependence upon \(B\), with a slope given by \((n-1/2)\hbar\omega^*_c\). In the low field limit \((l_B\gg r_{\text{ns}}, E^{\sigma\text{s}}_b>>\hbar\omega^*_c/2)\), the \(\text{ns}\) resonances display the diamagnetic shifts: \(E_{\text{ns}}(B)=E_{\text{ns}}(B=0)+\sigma B^2\), where \(\sigma = [(e\rho_{\text{ns}})^2/8\mu]\) is the diamagnetic coefficient. The 1\textit{s} and 2\textit{s} resonances follow the low-field regime in the entire range of the magnetic fields investigated, see Fig. 1(e). The high field regime is approached for the 5\textit{s} resonances with an approximate linear dependence of \(E_{5\text{s}}\) with \(B\), in the range above \(\sim8\) T. This linear dependence, marked with a solid line in Fig. 1(d), displays a slope of 2.1 meV/T, which if compared to \(9/2\hbar\omega^*_c\) dependence, provides an estimate of 0.25 \(m_0\) for the reduced mass in the WSe\textsubscript{2} ML. However, one may also argue that working with magnetic fields up to 14 T only, the high field limit is still barely developed even for the 5\textit{s} state. In this context, our estimation of the reduced effective mass should be seen as its upper bound and, in the following we assume \(\mu=0.2\) \(m_0\) for the WSe\textsubscript{2} ML, following the results of experiments performed in fields up to 60 T\textsuperscript{17}.

In the following we focus on the energy sequence \(E_{\text{ns}}\) of 1\textit{s}, 2\textit{s}, \ldots, 5\textit{s} excitonic resonances as they appear in the absence of magnetic field. As shown in Fig. 1(e), the apparent \(E_{\text{ns}}\) values are accurately determined with linear extrapolations of \(E_{\text{ns}}\) versus \(B^2\) dependences to \(B=0\). Next, we put forward a hypothesis that the energy sequence \(E_{\text{ns}}\) obeys the following rule:

\[
E_{\text{ns}} = E_g - \frac{Ry^*}{(n+\delta)^2},
\]

where, at this point, \(E_g\), \(Ry^*\), and \(\delta\) should be regarded as unknown adjustable parameters. To test the above formula against experimental data, we note that Eq. 1 implies that (for example) the ratio \((E_{3\text{s}}-E_{1\text{s}})/(E_{2\text{s}}-E_{1\text{s}})\) only depends on \(\delta\), and, reading this ratio from the experiment, we extract \(\delta=−0.083\). With this value we find (see Fig. 2) that our experimental \(E_{\text{ns}}\) series perfectly matches Eq. 1, and, at the same time, we determine two other parameters, \(E_g=1.873\) eV and \(Ry^*=-140.5\) meV.
modified Kratzer potential\textsuperscript{18}

\[
U_{\text{ext}}(r) = -\frac{e^2}{r_0} \left[ \frac{r_0^*}{r} - \frac{g^2 r_0^*}{r^2} \right],
\]

where \( r_0^* = r_0/\varepsilon \) is the reduced screening length and \( g \) is a tunable parameter. For the case of \( g^2 = 0.21 \), \( U_{\text{ext}}(r) \) fits \( U_{\text{RK}}(r) \) in the region \( r > r_{\text{min}} = 0.46 r_0^* \) with the relative deviation less than 5%. For the WSe\textsubscript{2} ML encapsulated in hBN, the minimal distance \( r_{\text{min}} = 4.6 \text{ Å} \) is comparable with the lattice constant \( a = 3.28 \text{ Å} \) of WSe\textsubscript{2} (see Fig 3 for comparison).

The Schrödinger equation with the Kratzer potential (2) is exactly solvable providing the excitonic spectrum of the \( s \)-type states (see SM for details):

\[
\epsilon_n = -\frac{Ry^*}{(n + g\kappa - 1/2)^2},
\]

in which \( \kappa^2 = 2r_0^*/a_B^* \) and \( a_B^* = \hbar^2/\varepsilon e^2 \) is the effective Bohr radius. The effective Rydberg constant \( Ry^* = e^2/2\varepsilon a_B^* \) sets the energy scale in the system, while \( \delta = g\kappa - 1/2 \) defines the relative positions of the energy levels in the spectrum. Since \( g\kappa \propto \sqrt{\mu r_0}/\varepsilon \), the parameter \( \delta \) is system dependent and its value can be tuned, in particular, by modifying the dielectric constant \( \varepsilon \). We note that for a given material, \( Ry^* \propto 1/\varepsilon^2 \) and \( \delta + 1/2 \propto 1/\varepsilon \). Such scaling laws as well as the energy sequence (3) can be derived from numerical simulations with \( U_{\text{RK}}(r) \) potential at relatively large \( \varepsilon \) (see SM for details). Note that Eq. 3 is an analogous of our experimentally found relation given by Eq. 1.

In the following, we introduce \( U_{\text{cor}}(r) \) which replaces the Kratzer potential at small distances \( r \), comparable with the lattice constant of WSe\textsubscript{2} in our particular case. We choose the constant attractive potential \( U_{\text{cor}}(r) = V_0 \). Below we demonstrate, that \( U_{\text{cor}}(r) \) does not change \( \propto (n + \delta)^{-2} \) behaviour of the spectrum and modifies only \( \delta \) parameter.

![FIG. 2. Experimentally obtained transition energies for the exciton states as a function \( 1/(n + \delta)^2 \) for \( \delta = -0.083 \). The black line shows a fit of the data to the model described by Eq. 1. The grey lines denote the band-gap (\( E_g \)) and excitonic binding (\( E_b \)) energies.](image)

![FIG. 3. Rytova-Keldysh (purple curve), Coulomb (blue curve) and Kratzer potential with \( g^2 = 0.21 \) (red curve), as a function of dimensionless parameter \( r/r_0^* \). The energy scale is measured in units of \( U_0 = e^2/r_0 \). The grey rectangular depicts the region of distances smaller than the lattice constant \( a = 3.28 \text{ Å} \) of WSe\textsubscript{2} ML encapsulated in hBN (\( r_0^* = 10 \text{ Å} \)).](image)
The energy to mimic the spectrum of Coulomb bound excitonic states was derived using the aid of Eq. 1, leading to

\[
\epsilon_n = -134 \text{ meV}/(n - 0.099)^2. \tag{5}
\]

Both found values of \(Rg^* = 134 \text{ meV}\) and \(\delta = -0.099\) match their experimentally obtained counterparts (with the aid of Eq. 1) 140.5 meV and -0.083, respectively.

The model proposed above accounts well for the experimental results obtained for the WSe\(_2\) monolayer and it is obviously interesting to test this model for other S-TMD materials, as well. Unfortunately, the observation of the rich Rydberg spectrum of excitonic states in S-TMD MLs seems to be, so far, uniquely reserved for WSe\(_2\) MLs. Nevertheless, for all other S-TMD MLs studied, i.e., MoS\(_2\), WS\(_2\), and MoSe\(_2\) MLs encapsulated in hBN, we do experimentally observe the 2s in addition to the 1s excitonic resonance (PL and reflectance contrast spectra), see Fig. 4 and Fig. S5 in SM. The energy positions, \(E_{1s}\) and \(E_{2s}\), of, correspondingly, the 1s and 2s resonances (of an exciton) are directly read from the data shown in Fig. 4. Of interest is the energy difference \((E_{2s} - E_{1s}) = \Delta E_{2s-1s}\) listed in Table I, for all four MLs investigated. As shown in Fig. 4, the PL peaks associated with the excited excitonic states are followed by noticeable PL tails developed at higher energies. We believe that these PL tails penetrate above the band-gap energies which are, however, not spectacularly marked in the spectra. We note, however, that in the case of our exemplary WSe\(_2\) ML, the PL intensity at the band-gap energy (accurately estimated from magneto-PL data and marked with a pink arrow in Fig. 4) consists of 5% of the intensity of the 2s exciton PL peak. Applying the same convention to all spectra presented in Fig. 4, we estimate the band gap in the three other MLs, as illustrated with pink arrows in this figure. Most critical is the estimation of the band gap in MoSe\(_2\) ML, which requires a convolution of the PL spectra due to an additional signal associated with the B-exciton resonance (see SM for details).

With estimation of the band gap and reading the energies of 1s excitonic resonances directly from the spectra (see Fig. 4), we extract exciton binding energies \(E_{1s}^{exp} = (E_{1s} - E_{1s}^\text{exp})\) and show their values in Table I. Having estimated \(\Delta E_{2s-1s}^{exp}\) and \(E_{2s}^{exp}\) parameters, and following our predictions that \(E_{n s} = E_g - Rg^*/(n + \delta)^2\), where \(Rg^* = Rg \times \mu/((\varepsilon^2_{hBN} m_0)\), we derive the \(\delta^{exp}\) and \(\mu^{exp}\) parameters for all MLs studied, see Table I. We found very good agreement between our estimations and results of DFT calculations\(^{29}\) for the reduced masses in WS\(_2\) and MoS\(_2\) MLs, while we note an apparent discrepancy for WSe\(_2\) and MoSe\(_2\) MLs.

Concluding, the presented experimental and theoretical study let us proposed that the ns Rydberg series of excitonic states in S-TMD monolayers encapsulated in hBN follows a simple energy ladder: \(\epsilon_n = -Rg^*/(n + \delta)^2\). \(\epsilon_n^* = Rg^*\), and \(\delta = 0.21\), where \(\epsilon_n^*\) is the Rydberg energy, \(\mu\) denotes the reduced e-\(h\) mass, and \(\varepsilon\) is the dielectric constant of the surrounding material. The dielectric polarizability \(\chi_{2D}\) of a monolayer is only encoded in \(\delta\) which, in the first approximation, is given by \(\delta = 0.21\), where \(\kappa^2 = 2\mu_\infty \varepsilon^2/\hbar^2\) and \(\tau_0 = 2\pi \chi_{2D}\) is a characteristic 2D screening length. Strikingly, \(\delta\) is found to be close to zero for WSe\(_2\) and MoS\(_2\) ML whose \(\epsilon_n^*\) spectrum resembles that of a 3D hydrogen atom. The proposed model may be applicable to other Coulomb bound states (e.g., donor and/or acceptor states), also to other systems, such as colloidal platelets\(^6\) or 2D perovskites\(^7\). Finally, we note that our \(\epsilon_n = -Rg^*/(n + \delta)^2\) solution coincides with that expected for a hypothetical hydrogen atom in fractional dimension \(N\), \((N = 2\delta + 3)\), which was indeed speculated\(^{16}\) to mimic the spectrum of Coulomb bound...
TABLE I. Series of parameters ($E_{b}^\text{exp}$, $\Delta E_{2s+1\ell}$, $\delta^\text{exp}$, $\mu^\text{exp}$) obtained from the analysis of PL spectra shown in Fig.4, compared with results of DFT calculations ($\mu^\text{DFT}$ ($m_0$))$^{26}$.

| Monolayer | $E_{b}^\text{exp}$ (meV) | $\Delta E_{2s+1\ell}$ (meV) | $\delta^\text{exp}$ | $\mu^\text{exp}$ ($m_0$) | $\mu^\text{DFT}$ ($m_0$) |
|-----------|---------------------|---------------------|-----------------|-----------------|---------------------|
| WSe$_2$   | 167                 | 130                 | -0.083          | 0.21            | 0.16                |
| MoSe$_2$  | 216                 | 153                 | 0.174           | 0.44            | 0.27                |
| WS$_2$    | 174                 | 141                 | -0.229          | 0.15            | 0.15                |
| MoS$_2$   | 217                 | 168                 | -0.095          | 0.26            | 0.24                |

states in low-dimensional semiconductor structures.

The work has been supported by the EU Graphene Flagship project (no. 785219), the ATOMOPTO project (TEAM programme of the Foundation for Polish Science, co-financed by the EU within the ERDF-Fund), the National Science Centre, Poland (grants no. DEC-2013/10/M/ST3/00791, UMO-2017/24/C/ST3/00119), and the Nanofab facility of the Institut Néel, CNRS UGA, and the LNCMI-CNRS, a member of the European Magnetic Field Laboratory (EMFL). K.W. and T.T. acknowledge support from the Elemental Strategy Initiative conducted by the MEXT, Japan, and the CREST (JPMJCR15F3), JST.

1. N. S. Rytova, Proc. MSU, Phys. Astron. 3, 308 (1967).
2. L. V. Keldysh, JETP Lett. 29, 658 (1979).
3. V. N. Kotov, B. Uchoa, V. M. Pereira, F. Guinea, and A. H. Castro Neto, Rev. Mod. Phys. 84, 1067 (2012).
4. C. Faugeras, S. Berciaud, P. Leszczynski, Y. Henri, K. Nogajewski, M. Orlita, T. Taniguchi, K. Watanabe, C. Fosythe, P. Kim, R. Jalil, A. K. Geim, D. M. Basko, and M. Potemski, Phys. Rev. Lett. 114, 126804 (2015).
5. A. K. Geim and I. Grigorieva, Nature 499, 419 (2013).
6. S. Ithurria, M. D. Tessier, B. Mahler, R. P. S. M. Lobo, and A. L. Dubertret, B. and Efros, Nature Materials 10, 936 (2011).
7. C. C. Stoumpos, D. H. Cao, D. J. Clark, J. Young, J. M. Rondinelli, J. I. Jang, J. T. Hupp, and M. G. Kanatzidis, Chemistry of Materials 28, 2852 (2016).
8. J.-C. Blancan, A. V. Stier, W. Tsai, I. Land Nie, C. C. Stoumpos, B. Trazoré, L. Pedesseau, M. Kepenekian, F. Katsutomi, G. T. Nee, J. Kono, S. Tretiak, S. A. Crooker, C. Katán, M. G. Kanatzidis, J. J. Crochet, J. Even, and A. D. Mohite, Nature Communications 9, 2254 (2018).
9. A. Chernikov, T. C. Berkelbach, H. M. Hill, A. Rigosi, Y. Li, O. B. Aslan, D. R. Reichman, M. S. Hybertsen, and T. F. Heinz, Phys. Rev. Lett. 113, 076802 (2014).
10. A. Raja, A. Chaves, J. Yu, G. Arefe, H. M. Hill, A. F. Rigosi, T. C. Berkelbach, P. Nagler, C. Schüller, T. Korn, C. Nuckolls, J. Hone, L. E. Brus, T. F. Heinz, D. R. Reichman, and A. Chernikov, Nature Communications 8, 15251 (2017).
11. M. Koperski, M. R. Molas, A. Arora, K. Nogajewski, A. Slobodenjuk, C. Faugeras, and M. Potemski, Nanophotonics 6, 1289 (2017).
12. G. Wang, A. Chernikov, M. M. Glazov, T. F. Heinz, X. Marie, T. Amand, and B. Urbanzek, Rev. Mod. Phys. 90, 021001 (2018).
13. M. Trushin, M. O. Goerbig, and W. Belzig, Phys. Rev. Lett. 120, 187401 (2018).
14. A. H. MacDonald and D. S. Ritchie, Phys. Rev. B 33, 8336 (1986).
15. E. S. Koteles and J. Y. Chi, Phys. Rev. B 37, 6332 (1988).
16. P. Christol, P. Lefebvre, and H. Mathieu, Journal of Applied Physics 74, 5626 (1993).
17. A. V. Stier, N. P. Wilson, K. A. Velizhanin, J. Kono, X. Xu, and S. A. Crooker, Phys. Rev. Lett. 120, 057405 (2018).
18. A. Kratzer, Zeitschrift für Physik 3, 289 (1920).
19. O. Akimoto and H. Hasegawa, Journal of the Physical Society of Japan 22, 181 (1967).
20. M. Potemski, L. Viña, G. E. W. Bauer, J. C. Maan, K. Ploog, and G. Weimann, Phys. Rev. B 43, 14707 (1991).
21. M. Manca, M. M. Glazov, C. Robert, F. Cadiz, T. Taniguchi, K. Watanabe, E. Courtade, T. Amand, P. Renucci, X. Marie, G. Wang, and B. Urbanzek, Nature Communications 8, 14927 (2017).
22. C. M. Chow, H. Yu, A. M. Jones, J. Yan, D. G. Mandrus, T. Taniguchi, K. Watanabe, Y.-W. Yao, and X. Xu, Nano Letters 17, 1194 (2017).
23. S.-Y. Chen, T. Goldstein, J. Tong, T. Taniguchi, K. Watanabe, and J. Yan, Phys. Rev. Lett. 120, 046402 (2018).
24. E. Liu, J. van Baren, T. Taniguchi, K. Watanabe, Y.-C. Chang, and C. H. Lui, arXiv e-prints (2019), arXiv:1901.11044 [cond-mat.mtrl-sci].
25. R. Geick, C. H. Perry, and G. Rupprecht, Phys. Rev. 146, 543 (1966).
26. A. Kormányos, G. Burkard, M. Gmitra, J. Fabian, V. Zólyomi, N. D. Drummond, and V. Fal’ko, 2D Materials 2, 022001 (2015).
27. F. Cadiz, E. Courtade, C. Robert, G. Wang, Y. Shen, H. Cai, T. Taniguchi, K. Watanabe, H. Carrere, D. Lagarde, M. Manca, T. Amand, P. Renucci, S. Tongay, X. Marie, and B. Urbanzek, Phys. Rev. X 7, 021026 (2017).
28. J. Wierzbowski, J. Klein, F. Sigger, C. Straubinger, M. Kremser, T. Taniguchi, K. Watanabe, U. Wurstbauer, A. W. Holleitner, M. Kaniber, K. Müller, and J. J. Finley, Scientific Reports 7, 12383 (2017).
29. E. Courtade, M. Semina, M. Manca, M. M. Glazov, C. Robert, F. Cadiz, G. Wang, T. Taniguchi.
Supplemental Material:

Energy spectrum of two-dimensional excitons in a non-uniform dielectric medium

M. R. Molas, A. O. Slobodeniuk, K. Nogajewski, M. Bartos, L. Bala, A. Babinski, K. Watanabe, T. Taniguchi, C. Faugeras, and M. Potemski

1 Laboratoire National des Champs Magnétiques Intenses, CNRS-UGA-UPS-INSA-EMFL, 25, avenue des Martyrs, 38042 Grenoble, France
2 Faculty of Physics, University of Warsaw, ul. Pasteura 5, 02-093 Warszawa, Poland
3 National Institute for Materials Science, 1-1 Namiki, Tsukuba 305-0044, Japan

This supplemental material provides: S1 description of preparation of the studied samples and used experimental setups, S2 excitonic spectrum and eigenfunctions in the Kratzer potential, S3 numerical analysis of the excitonic spectrum in the Rytova-Keldysh potential, S4 derivation of the excitonic spectrum in WSe2 monolayer encapsulated in hBN, S5 dependence of excitonic diamagnetic coefficients in WSe2 monolayer, S6 low temperature reflectance contrast spectra of the investigated monolayers, S7 estimation of the band-gap energy in MoSe2 monolayer.

S1. Samples and experimental setups

The active parts of our samples consist of a monolayer (ML) of semiconducting transition metal dichalcogenides (S-TMD), i.e. WS2, MoS2, WS2, and MoSe2, which has been encapsulated in hexagonal boron nitride (hBN) and deposited on a bare Si substrate. They were fabricated by two-stage polydimethylsiloxane (PDMS)-based exfoliation of S-TMD and hBN bulk crystals.

The µ-photoluminescence (µ-PL) and µ-reflectance contrast (µ-RC) experiments were performed using a λ=515 nm CW laser diode and a 100 W tungsten halogen lamp, respectively.

Micro-magneto-PL measurements were performed in the Faraday configuration using an optical-fiber-based insert placed in a superconducting magnetic coil producing magnetic fields up to 14 T. The sample was mounted on top of an x − y − z piezo-stage kept in gaseous helium at T = 4.2 K. The excitation light was coupled to an optical fiber with a core of 5 µm diameter and focused on the sample by an aspheric lens (spot diameter around 1 µm). The signal was collected by the same lens, injected into a second optical fiber of 50 µm diameter, and analyzed by a 0.5 m long monochromator equipped with a CCD camera. A combination of a quarter wave plate and a polarizer are used to analyze the circular polarization of signals. The measurements were performed with a fixed circular polarization, whereas reversing the direction of magnetic field yields the information corresponding to the other polarization component due to time-reversal symmetry.

Investigations at zero magnetic field were carried out with the aid of a continuous flow cryostat mounted on x − y motorized positioners. The sample was placed on a cold finger of the cryostat. The excitation light was focused by means of a 50x long-working distance objective with a 0.5 numerical aperture producing a spot of about 1 µm. The signal was collected via the same microscope objective, sent through a 0.5 m monochromator, and then detected by a CCD camera.

S2. Excitonic spectrum and eigenfunctions in the Kratzer potential

We solve two-dimensional (2D) Schrödinger equation with the Kratzer potential \( U_{\text{Kr}} = -\frac{\epsilon^2}{r_0^2} + \frac{g^2 \xi^2}{r^2} \) for wave-function \( \psi(r) = \psi(r, \phi) \)

\[
\left\{ -\frac{\hbar^2}{2\mu} \left[ \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right] + U_{\text{ext}}(r) - \epsilon \right\} \psi(r, \phi) = 0, \tag{6}
\]

in which \( U_{\text{ext}}(r) = -\frac{\epsilon^2}{r_0^2} - \frac{g^2 \xi^2}{r^2} \) is the modified Kratzer potential. \( r \) is in-plane electron-hole distance, \( \mu \) denotes the reduced electron-hole mass, \( \varepsilon \) represents the dielectric constant of the material surrounding the monolayer, \( r_0^2 = r_0^2 / \varepsilon \) is the reduced screening length, and \( g \) is a tunable parameter. Taking \( \psi_m(r, \phi) = e^{im\phi} \phi_m(r) / \sqrt{2\pi} \) and introducing the new variable \( \xi = r / r_0^* \) (\( r_0^* = r_0^2 / \varepsilon \) is the reduced screening length), we obtain the equation

\[
\left\{ \frac{d^2}{d\xi^2} + \frac{1}{\xi} \frac{d}{d\xi} - \frac{-k^2 \xi^2 + \kappa^2 \xi - g^2 \kappa^2 - m^2}{\xi^2} \right\} \phi_m(\xi) = 0 \tag{7}
\]

with \( k^2 = -2\mu r_0^2 / \hbar^2 > 0 \) and \( \kappa^2 = 2\mu r_0^2 / \hbar^2 > 0 \). The solution to this eigenvalue problem is

\[
\phi_{n, m}(r) = \frac{\beta_{n, m}}{\sqrt{2n + 2\delta_m - 1}} \sqrt{(n + |m| - 1)!} \Gamma(n + |m| + 2\delta_m) \times (\beta_{n, m} r)^M e^{-\beta_{n, m} r^2 / 2L_{2n-|m|-1}^M} \left( \beta_{n, m} r \right)^M L_{2n-|m|-1}^M \tag{8}
\]
with \( M = \sqrt{m^2 + g^2 \kappa^2} \), \( \delta_m = M - |m| \) and \( \beta_{n,m} = 2 \mu e^2 / h^2 \varepsilon (n + \delta_m - 1/2) \), respectively. \( n=1, 2\ldots \) is a principal quantum number, \( m=0, \pm 1, \pm 2\ldots \) is an angular momentum quantum number, and \( L^m_n(x) \) is the modified Laguerre polynomial. The energy spectrum for such system is described with:

\[
\epsilon_{n,m} = \frac{-\mu e^4}{2\hbar^2 \varepsilon^2} \frac{1}{(n + \delta_m - \frac{1}{2})^2}.
\]

For \( \rho=0 \), our result coincides with 2D Hydrogen model\(^{40} \). In the case of \( s \)-type states \((n=1, 2\ldots \) and \( m=0)\), the excitonic spectrum simplifies to

\[
\epsilon_n = \frac{-\mu e^4}{2\hbar^2 \varepsilon^2} \frac{1}{(n + g \kappa - \frac{1}{2})^2}.
\]

We mention the following consequences of this model: (i) the energy scale (prefactor in \( \epsilon_n \)) does not depend on the screening length \( r_0 \). It coincides with the Rydberg constant \( Ry^* \) for an exciton with reduced mass \( \mu \) in an environment with dielectric constant \( \varepsilon \), \textit{i.e.} \( Ry^* = \mu e^4 / 2\hbar^2 \varepsilon^2 \); (ii) the information about relative positions of the energy levels of the system is encoded in the denominators in Eq. 9 and 10; (iii) the Kratzer potential lifts the Coulomb degeneracy of the \( s \) \((m=0)\) and \( p \)-type \((m=\pm 1)\) states, as one can be noticed from Eq. (9); (iv) since \( Ry^* \propto 1/\varepsilon^2 \) and \( \delta + 1/2 \propto 1/\varepsilon \), the energy ladder of the excitons can be progressively tuned by changing the dielectric constant \( \varepsilon \) of the surrounding medium. Surprisingly, the results of numerical simulations performed in the Rytova-Keldysh potential\(^{1,2} \), discussed in the next section, demonstrate the similar behavior. This fact can be interpreted as the indirect confirmation, that the Kratzer potential is a good approximation for the considered model.

Using the wave-functions obtained in Eq. (8), we calculate the mean value of \( r^2 \), which can be useful for analysis of diamagnetic shift of excitons, which reads

\[
\langle r^2 \rangle_{n,m} = \frac{2}{(\beta_{n,m})^2} [3 - 3m^2 + 5n(n - 1) - 5\delta_m - 6|m|\delta_m + 2\delta_m(5n + \delta_m)].
\]

For \( s \)-type states characterized by \( m=0 \), it takes the form

\[
\langle r^2 \rangle_{n,0} = \frac{2}{(\beta_{n,0})^2} [2g^2 \kappa^2 + 10g\kappa n - 5g\kappa + 5n^2 - 5n + 3].
\]

Moreover, for the special case \( g\kappa = 1/2 \), while Eq. 10 resembles the three-dimensional (3D) hydrogen model, the mean value of the \( \tilde{r}^2 \) parameter is given by

\[
\langle r^2 \rangle_{n,0}|_{g\kappa = 1/2} = (a_0^*)^2 n^2 (5n^2 + 1)/2 \approx 5(a_0^*)^2 n^4/2,
\]

where \( a_0^* = \hbar^2 \varepsilon / \mu e^2 \) is the effective Bohr radius. It is interesting to note that the latter formula coincides with the mean value of \( r^2 \) for 3D hydrogen atom\(^{41} \).

The eigenfunctions of the Schrödinger Hamiltonian with the Kratzer potential (8) tend to zero at \( r \to 0 \). This is the consequence of the repulsive part of the potential at short distances. Therefore, such solutions can not be good approximation for exciton wave-functions at small distances, since the Rytova-Keldysh potential is attractive. In order to improve the current result, one needs to modify the Kratzer potential at small distances.

**S3. Numerical analysis of the excitonic spectrum in the Rytova-Keldysh potential**

We solve numerically the eigenvalue problem for 2D Schrödinger Hamiltonian with the Rytova-Keldysh potential\(^{1,2}\). We analyse the scale laws for the spectrum both as a function of principal number \( n \) and dielectric constant of surrounding medium \( \varepsilon \). We start from the radial equation on wave function \( \phi(r) \) for the \( s \)-type states characterized by zero angular momentum:

\[
\left\{- \frac{\hbar^2}{2\mu} \frac{1}{r} \frac{d}{dr} \left[ r \frac{d}{dr} \right] - \frac{\pi e^2}{2r_0} \left[ H_0 \left( \frac{r\varepsilon}{r_0} \right) - Y_0 \left( \frac{r\varepsilon}{r_0} \right) \right] - \epsilon \right\} \phi(r) = 0,
\]

where \( H_0(x) \) and \( Y_0(x) \) are the zeroth order Struve and Neumann functions. Introducing new variables \( \xi = r\varepsilon / r_0 = r/r_0^* \) and \( \epsilon = (\mu e^4 / 2\hbar^2 \varepsilon^2)W = W Ry^* \), we rewrite the equation

\[
\left\{- b^2 \frac{1}{\xi} \frac{d}{d\xi} \left[ \xi \frac{d}{d\xi} \right] - \pi b \left[ H_0(\xi) - Y_0(\xi) \right] - W \right\} \phi(\xi) = 0
\]

\[ (15) \]
FIG. S1. Interpolation lines for \((-W_n)^{-1/2}\) for \(n=1, 2, \ldots, 5\) and different values of the dielectric constant from \(\varepsilon_{\text{min}} = 1\) (top blue line) until \(\varepsilon_{\text{max}} = 5\) (bottom purple line) with step \(\Delta\varepsilon = 0.5\). Blue, yellow, and green circles represent the values of function \((-W_n)^{-1/2}\) for different \(n\), when \(\varepsilon=1, 1.5, 2\), respectively.

with \(b = \hbar^2\varepsilon^2/(\mu\varepsilon^2 r_0) = a_B^*/r_0^*\) - the ratio of the natural scales in the system. We derive the spectrum of this differential equation as a function of \(\varepsilon\) and \(n\).

According to our hypothesis described in the main text, the excitonic spectrum should be described with:

\[
\varepsilon_n = R_Y^* W_n = -\frac{R_Y^*}{(\alpha n + \beta)^2} = -\frac{R_Y^* \alpha^{-2}}{(n + \delta)^2},
\]

(16)

where \(\alpha \simeq 1\), while \(\beta\) (and hence \(\delta = \beta/\alpha\)) strongly depends on \(\varepsilon\). Therefore, we estimate the linear behaviour of \((-W_n)^{-1/2}\) with \(n\).

The results of numerical simulations (performed in "Mathematica") for the case of WSe_2 (\(\mu=0.2 m_0\)\(^{17}\), \(r_0 = 45 \text{ Å}\)\(^{42}\)) are presented in Figs S1 and S2. Indeed, the linear growth of \((-W_n)^{-1/2}\) as a function of \(n\) for different values of \(\varepsilon\) can be appreciated in Fig. S1. The Fig. S2 qualitatively confirms that \(\alpha \simeq 1\). The precision of this result (relative deviation of the curve \((-W_n)^{-1/2} - n\) from its average value) becomes higher for larger values of \(\varepsilon\).

FIG. S2. Interpolation lines for \((-W_n)^{-1/2} - n\) for \(n=1, 2, \ldots, 5\) and different values of the dielectric constant from \(\varepsilon_{\text{min}} = 1\) (top blue line) until \(\varepsilon_{\text{max}} = 5\) (bottom purple line) with step \(\Delta\varepsilon = 0.5\). Blue, yellow, and green circles represent the values of function \((-W_n)^{-1/2} - n\) for different \(n\), when \(\varepsilon=1, 1.5, 2\), respectively.
We consider the solution of 2D Schrödinger equation in the potential, defined in the main text, which reads

\[ U_{\text{app}}(\xi) = \begin{cases} -U_0 \left( \frac{1}{\xi} - \frac{0.22}{\xi^2} \right), & \text{for } \xi > \xi_0; \\ -1.71134 U_0, & \text{for } \xi < \xi_0. \end{cases} \]  

We restrict our consideration only to the \( s \)-type states characterized by zero angular momentum. The regular radial solution of the Schrödinger equation in the region \( \xi < \xi_0 \) is

\[ \phi_1(\xi) \sim J_0(\kappa \sqrt{v_0 - |E|} \xi), \]  

where \( J_0(x) \) is the zero-order Bessel function of the first kind, \( E = \epsilon / U_0 \) and \( v_0 = 1.71134 \). The solution for the region \( \xi > \xi_0 \) has the form

\[ \phi_2(\xi) \sim e^{-k\xi} \Psi \left( -\frac{\kappa^2}{2k} + g\kappa + \frac{1}{2}, 1 + 2g\kappa; 2k\xi \right) \]  

Introducing the normalized logarithmic derivatives for both solutions \( f_n(\mathcal{E}) = \kappa^{-1}[d \ln \phi_n(\xi)/d\xi]_{\xi=\xi_0}, n = 1, 2 \)

\[ f_1(\mathcal{E}) = -\sqrt{v_0 - |\mathcal{E}|} \frac{J_1(2g^2\kappa \sqrt{v_0 - |\mathcal{E}|})}{J_0(2g^2\kappa \sqrt{v_0 - |\mathcal{E}|})}, \]  

\[ f_2(\mathcal{E}) = -\sqrt{|\mathcal{E}|} + \frac{1}{2g} + \left[ \kappa - (2g\kappa + 1) \sqrt{|\mathcal{E}|} \right] \times \frac{\Psi \left( -\frac{\kappa}{2\sqrt{|\mathcal{E}|}} + g\kappa + \frac{3}{4}, 2 + 2g\kappa; 4g^2\kappa \sqrt{|\mathcal{E}|} \right)}{\Psi \left( -\frac{\kappa}{2\sqrt{|\mathcal{E}|}} + g\kappa + \frac{3}{4}, 1 + 2g\kappa; 4g^2\kappa \sqrt{|\mathcal{E}|} \right)}, \]  

one derives the energy spectrum from continuity equation \( f_1(\mathcal{E}) = f_2(\mathcal{E}) \). We solve the latter equation for WSe\(_2\) ML encapsulated in hBN with a set of parameters: \( \mu = 0.2 \, m_0 \), \( \varepsilon = 4.5 \), and \( r_0 = 45 \, \text{Å} \). The derivatives as a function of \( |\mathcal{E}| \) are presented in Fig. S3 and the excitonic spectrum is defined by the intersection points of \( f_1(\mathcal{E}) \) and \( f_2(\mathcal{E}) \) curves.

FIG. S3. The normalized logarithmic derivatives for core solution \( f_1(\mathcal{E}) \) (red curve), external solution \( f_2(\mathcal{E}) \) (purple curve) and for regular solution \( f_3(\mathcal{E}) \) for the Kratzer potential with \( g^2 = 0.21 \) (blue curve) as a function of dimensionless parameter \( |\mathcal{E}| = |\epsilon| / U_0 \). The dashed purple lines represent the asymptotes of \( f_2(\mathcal{E}) \).
The excitonic spectrum, obtained within the described above method, can be presented in the same form as for the Kratzer potential (compare with Eq. 5 in the main text) and is given by

$$\epsilon_n = -\frac{134 \text{ meV}}{(n-0.099)^2}. \quad (23)$$

This result demonstrates the good coincidence with the excitonic spectrum reported in Ref. 17, with relative errors 8% for \(n = 1\), 3.5% for \(n = 2\) and less than 2% for higher excited states. In order to check the precision of the graphical method, we applied it for the case, when the core potential is described by the Kratzer one with the same parameter \(g^2 = 0.21\). For the Kratzer potential, graphical solution provides the excitonic spectrum, which follows Eq. 10. In this case, the logarithmic derivative is

$$f_3(\mathcal{E}) = -\sqrt{|\mathcal{E}|} + \frac{1}{2g} - \frac{\kappa - (2g\kappa + 1)\sqrt{|\mathcal{E}|}}{1 + 2g\kappa} \times \frac{1}{1} F_1 \left( \frac{-n}{2\sqrt{|\mathcal{E}|}} + g\kappa + \frac{3}{2}, 2 + 2g\kappa; 4g^2\kappa\sqrt{|\mathcal{E}|} \right), \quad (24)$$

where \(1F_1(a, c; z)\) is the confluent hypergeometric function of the first kind, and corresponds to the blue curve in Fig. S3.

One can mention, that the calculation with the potential \(U_{\text{app}}(\xi)\) does not approximate perfectly the 1s-exciton state. From the technical point of view such discrepancy can be the consequence of the modification of the Rytova-Keldysh potential at small distances. In order to check this hypothesis, we estimate the ground state energy of excitons using the Ritz variational procedure. We take the variational wave-function in the form \(\psi_0(r) = \beta \exp(-\beta r/2)/\sqrt{2\pi}\) and evaluate the average of the Hamiltonian

$$H = -\frac{\hbar^2}{2\mu} \Delta_{2D} - \frac{\pi e^2}{2r_0} \left[ H_0 \left( \frac{r\xi}{r_0} \right) - Y_0 \left( \frac{r\xi}{r_0} \right) \right]. \quad (25)$$

The kinetic energy can be calculated directly with

$$T_0(\beta) = -\frac{\hbar^2}{2\mu} \int_0^\infty dr \psi_0(r) \frac{d}{dr} \left[ \frac{d\psi_0(r)}{dr} \right] = \frac{\hbar^2 \beta^2}{8\mu}. \quad (26)$$

To determine the potential energy, a few steps need to be performed. First, we present the Keldysh potential in the integral form \(^4\)

$$U(r) = -\frac{e^2}{2\pi \varepsilon} \int_0^{2\pi} d\theta \int_0^\infty dk \frac{e^{ikr \cos \theta}}{1 + kr_0^*}, \quad (27)$$

Then, we substitute it in the expression for the average potential energy

$$U_0(\beta) = -\frac{e^2 \beta^2}{2\pi \varepsilon} \int_0^\infty dk \int_0^{2\pi} d\theta \int_0^\infty dr e^{ikr \cos \theta - \beta r}. \quad (28)$$

Consequently, after evaluation of integrals and adding the kinetic energy part, we determine formula

$$\epsilon(a) = \frac{e^2}{r_0^*} \left[ \frac{\hbar^2 \varepsilon^2}{8\mu_0 \varepsilon^2} a^2 + af(a) \right], \quad (29)$$

where \(a = \beta r_0^*\) is the dimensionless parameter and

$$f(a) = \frac{(a - 1)\sqrt{1 + a^2} - 2a^2 \text{Arctanh} \left( \frac{1 + a}{\sqrt{1 + a^2}} \right)}{(1 + a^2)^{3/2}}. \quad (30)$$

The minimum of \(\epsilon(a)\) can be found straightforwardly (with the help of ”Mathematica”, for example). For the case of \(\mu = 0.2 m_0\) \(^7\), we got the ground state energy of exciton, \(\epsilon_0 = 157\) meV, which is in good agreement with the results of numerical simulation reported in Ref. 17, obtained for the same values of parameters. Moreover, the relative deviation of the variational exciton ground-state energy, calculated for the different values of \(\varepsilon\), deviates from the numerical results discussed in Ref. 17 less than 2%.

Surprisingly, the numerical simulations for WSe\(_2\) monolayer encapsulated in hBN with an effective mass \(\mu = 0.21 m_0\) are in better agreement with the experimentally obtained excitonic spectrum, \(\epsilon_n = -140.5\) meV/(\(n - 0.083\))^2 (see the main text), than for \(\mu = 0.20 m_0\) and leads to the energy ladder of excitons given by

$$\epsilon_n = -141\text{ meV}/(n - 0.087)^2. \quad (31)$$

The excitonic binding energy, \(E_b = 162\) meV, calculated with the aid of the Ritz variational method also nicely matches to the experimental one, \(E_b = 167\) meV, obtained in the main text.
S5. Dependence of excitonic diamagnetic coefficients in WSe$_2$ monolayer

To test our assumption that the excitonic spectrum in the WSe$_2$ ML encapsulated in hBN resembles a 3D hydrogen atom ($\sim -1/n^2$), we investigate dependence of the obtained diamagnetic coefficients $\sigma$ of excitonic states, $n_s$, in this system. We found theoretically (see Eq. 13) that the mean value of $r^2$ calculated with the aid of our Kratzer potential approach approximate the one for 3D hydrogen atom, in which $r^2$ parameters of excitonic states scales with $n^4$. With the aid of Eq. 13 and $\sigma = (er)^2/8\mu$, we calculate theoretical $\sigma$ values of excitons for WSe$_2$ ML encapsulated in hBN with parameters: $\mu=0.2\,m_0^{17}$ and $\varepsilon = 4.5^{25}$. The theoretical values are compared with the experimental diamagnetic coefficients in Fig. S4. The theoretical dependence fits very well the experimental data up to the 4s state. This additionally confirms that the excitonic spectrum of the WSe$_2$ monolayer encapsulated in hBN corresponds to that of a 3D hydrogen atom. The apparent discrepancy between the theory and the experiment for the 5s state results in our opinion from the small range of the low-field limit, which affects the determined $\sigma$ value for this state.

![Graph showing the dependence of excitonic diamagnetic coefficients](image)

FIG. S4. Diamagnetic coefficients $\sigma$ for the excitonic states as a function $n^4$ obtained (crosses) experimentally due to the analysis performed in the main text and (circles) theoretically using Eq. 13. The black line connects theoretical points as a guide to the eye.

S6. Low temperature reflectance contrast spectra of S-TMD monolayers

The low temperature RC spectra of WSe$_2$, MoS$_2$, WS$_2$, and MoSe$_2$ encapsulated in hBN are presented in Fig. S5. We define the RC spectrum as $RC(E) = \frac{R(E) - R_0(E)}{R(E) + R_0(E)} \times 100\%$, with $R(E)$ and $R_0(E)$, respectively, the reflectance of the dielectric stack composed of a monolayer encapsulated in hBN supported by a bare Si substrate and of the two alone layers of hBN on top of Si substrate. Note that the presented spectra correspond to the PL ones shown in Fig. 5 in the main text. First, the spectra display two pronounced resonances labelled $1s_{A,B}$ which arise from the ground state of the so-called A and B excitons$^{45-48}$. In addition to them, less pronounced features, labelled $2s_{A,B}$ and $3s_A$, appear at about 200 meV higher in energy as compared to the $1s_{A,B}$ ones. The assignment of the $2s_A$ and $3s_A$ features to the first and the second excited state of the A exciton is straightforward and corresponds to many other investigations on S-TMD MLs encapsulated in hBN$^{17,33,49-51}$. The origin of the $2s_B$ is less clear, as it has not been reported so far. Due to the similar energy separation between $1s_B$ and $2s_B$ as compared with the $1s_A$ and $2s_A$, we ascribed tentatively the $2s_B$ features to the first excited states of the B exciton, which, however, requires further investigations.
FIG. S5. Low temperature RC spectra of S-TMD monolayers measured at $T=5$ K. The spectral regions around the $1s_A$ resonance are scaled and shifted for clarity.

S7. Estimation of the band-gap energy in MoSe$_2$ monolayer

FIG. S6. Low temperature photoluminescence spectrum of MoSe$_2$ monolayer at $T=5$ K limited to the high energy PL signal composed of the $2s_A$ line of A exciton. The blue, green, and red curves display fits of Gaussian profiles to the corresponding $1s_A$, $n s_A$, and $1s_B$ lines. The pink vertical arrow denotes the estimated band-gap energy $E_g$.

As it has been discussed in the main text, the estimation of band-gap energy is essential for our analysis of excitonic ladder in S-TMD MLs. As it is discussed in the main text, the estimation of band-gap energy can be easily carried out for WSe$_2$, MoS$_2$, and WS$_2$ MLs, however, this issue is more complex for the MoSe$_2$ one. This is, because PL
related to the ground state of B exciton, $1s_B$, appears in the spectral range of the emissions related to the $2s$ and higher $ns$ states of the A exciton, labelled as $1s_A$ and $ns_A$ in Fig. S6. To determine the band-gap energy of MoSe$_2$ ML, we use the procedure described in the main text. The PL intensity at the band-gap energy of WSe$_2$ ML equals 5% of the intensity of the $2s$ exciton PL peak. In order to apply the same approach for MoSe$_2$ ML, we deconvolute the spectrum shown in Fig. S6 with three Gaussian profiles (the PL due to the $ns_A$ and $1s_B$ are resolved spectrally).

We set the linewidth of the $1s_B$ emission equals to 28 meV, as obtained from the upconversion PL spectrum of MoSe$_2$ ML reported in Ref. 49. The result of the procedure is presented in Fig. S6. The band-gap energy in MoSe$_2$ ML determined using the procedure equals 1.861 eV and is marked in Fig. S6 with pink arrows.

---

1. A. Castellanos-Gomez, M. Busecema, R. Molenaar, V. Singh, L. Janssen, H. S. J. van der Zant, and G. A. Steele, 2D Materials 1, 011002 (2014).
2. A. Kratzer, Zeitschrift für Physik 3, 289 (1920).
3. X. L. Yang, S. H. Guo, F. T. Chan, K. W. Wong, and W. Y. Ching, Phys. Rev. A 43, 1186 (1991).
4. N. S. Rytova, Proc. MSU, Phys. Astron. 3, 308 (1967).
5. L. V. Keldysh, JETP Lett. 29, 658 (1979).
6. H. A. Bethe and E. M. Salpeter, Quantum mechanics of one and two-electron atoms (Springer-Verlag Berlin, 1957).
7. A. V. Stier, N. P. Wilson, K. A. Velizhanin, J. Kono, X. Xu, and S. A. Crooker, Phys. Rev. Lett. 120, 057405 (2018).
8. T. C. Berkelbach, M. S. Hybertsen, and D. R. Reichman, Phys. Rev. B 88, 045318 (2013).
9. H. Bateman and A. Erdelyi, Higher Transcendental Functions, Volume 1 (Mc Graw-Hill book Co. New York, 1953).
10. R. Geick, C. H. Perry, and G. Rupprecht, Phys. Rev. 146, 543 (1966).
11. P. Cudrazo, I. V. Tokatly, and A. Rubio, Phys. Rev. B 84, 085406 (2011).
12. Y. Li, A. Chernikov, X. Zhang, A. Rigosi, H. M. Hill, A. M. van der Zande, D. A. Chenet, E.-M. Shih, J. Hone, and T. F. Heinz, Phys. Rev. B 90, 205422 (2014).
13. A. Arora, M. Koperski, K. Nogajewski, J. Marcus, C. Faugeras, and M. Potemski, Nanoscale 7, 10421 (2015).
14. A. Arora, K. Nogajewski, M. Molas, M. Koperski, and M. Potemski, Nanoscale 7, 20769 (2015).
15. M. R. Molas, K. Nogajewski, A. O. Slobodeniuk, J. Binder, M. Bartos, and M. Potemski, Nanoscale 9, 13128 (2017).
16. C. Robert, M. A. Semina, F. Cadiz, M. Manca, E. Courtade, T. Taniguchi, K. Watanabe, H. Cai, S. Tongay, B. Lassagne, P. Renucci, T. Amand, X. Marie, M. M. Glazov, and B. Urbaszek, Phys. Rev. Materials 2, 011001 (2018).
17. B. Han, C. Robert, E. Courtade, M. Manca, S. Shree, T. Amand, P. Renucci, T. Taniguchi, K. Watanabe, X. Marie, L. E. Golub, M. M. Glazov, and B. Urbaszek, Phys. Rev. X 8, 031073 (2018).
18. A. O. Slobodeniuk, L. Bala, M. Koperski, M. R. Molas, P. Kossacki, K. Nogajewski, M. Bartos, K. Watanabe, T. Taniguchi, C. Faugeras, and M. Potemski, arXiv e-prints (2018), arXiv:1810.00623 [cond-mat.mes-hall].
19. I. C. Gerber, E. Courtade, S. Shree, C. Robert, T. Taniguchi, K. Watanabe, A. Balocchi, P. Renucci, D. Lagarde, X. Marie, and B. Urbaszek, Phys. Rev. B 99, 035443 (2019).