Optimization of design parameters of a limited angle torque motor using analytical hierarchy process and axiomatic design theory

Mina Roohnavazfar\textsuperscript{a}, Mahmoud Houshmand\textsuperscript{a}, Reza Nasiri Zarandi\textsuperscript{b}\textsuperscript{*} and Mojtaba Mirsalim\textsuperscript{b,c}

\textsuperscript{a}Department of Industrial Engineering, Sharif University of Technology, Azadi Ave., Tehran 11365-11155, Iran; \textsuperscript{b}Department of Electrical Engineering, Amirkabir University of Technology (Tehran Polytechnic), Aboureyhan Building, 424, Hafez Ave., Tehran 13597-45778, Iran; \textsuperscript{c}School of Engineering, St. Mary’s University, San Antonio, TX 78228, USA

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The limited angle torque motor (LATM) has been widely used in systems that require limited motion, ranging from the simple ON–OFF servo valves to the accurate tracking of a reference signal. This paper presents the optimum design procedures of a LATM. According to a request from industry, a two-pole LATM with a toroidal armature winding was designed based on analysis of magnetic equivalent circuit of motor structure, and using selected ferromagnetic material and rare-earth permanent magnets. To select the best design solution, the decision-making tools called analytical hierarchy process (AHP) and axiomatic design (AD) methodologies are employed, respectively. Considering the industrial requirements, AD is proposed to minimise the effect of expert’s judgments and pursue the simple design procedure. Comparison of the AD and AHP methodologies shows that the AD has more confirmation with the industrial design ranges. It requires minimum experts’ interventions and reduces the design computation time.

Keywords: decision-making; design optimization; axiomatic design; analytical hierarchy process; limited angle torque motor

1. Introduction

The limited angle torque motor (LATM) is an electromechanical actuator with a limited rotation of a moving part. It produces torque through a limited rotation angle normally less than \( \pm 180^\circ \). It has a structure similar to a typical direct-current brushless motor, which contains two series-connected armature winding on the stator core and two rare-earth permanent magnets (PM) with high flux densities mounted on the rotor. Compared to conventional motors, this structure has many advantages like higher torque/weight ratio, convenient maintenance due to elimination of mechanical commutation and electronic switching, high reliability, low cost, and accurate positioning capability (Tsai, Lin, Huang, & Cheng, 2009; Zarandi, MeshginKelk, Toorani, & Farahmandzad, 2011). Zarandi et al. (2011) have presented a comprehensive design procedure of an LATM based on magnetic equivalent circuit (MEC) analysis. An optimum design procedure is required to reduce the design iterations and computation time. In addition, the optimum design should consider the industrial requirements. Widdowson, Howe, and Evison

*Corresponding author. Email: Rezanasiri.z@aut.ac.ir

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(1991) proposed a computer-aided optimization of an LATM to maximize torque/current and torque/inertia ratios with experimental validation. A wide variety of design solutions are generated by changing the variable parameters. To choose the best design solution, a direct search within the scanned design-solution space, may be the most reliable method, if an effective decision-making tool is selected. The analytical hierarchy process (AHP) is one of the useful multi-criteria decision-making tools developed by Saaty (1977). This methodology is a powerful and flexible weighted scoring decision-making process to set priorities and make the best decision. Generally, implementing the AHP is based on the expert’s knowledge, objective, and subjective evaluation of the goal, criteria, sub-criteria, and alternatives (Ho, 2008). The AHP has been widely used in various fields such as social, engineering, personal, law, and politics (Vaidya & Kumar, 2006), selecting wastewater treatment method for the colored metal industry (Srdjevic, Samardzic, & Srdjevic, 2012), determining the significance of the interaction between the elements of a transport system having a strong influence on traffic safety (Podvezkoa & Sivilevičius, 2013), using AHP for sorting problems (Ishizakaa, Pearmana, & Nemery, 2012). Furthermore, many studies have been conducted in order to develop the theory of this methodology. For instance, Saaty (1977) proposed the Eigenvector method to extract the priorities of criteria and alternatives. Cogger and Yu (1985) described the least-square method, while Saaty (1990), Crawford and Williams (1985), and Fichtner (1983) proposed the logarithmic least-square method. A review on the AHP literature shows that the prioritization of the wide design-solution space by AHP is time-consuming and completely dependent on the expert’s judgments. It also reveals that in the complex decision-making problems, it is difficult to keep the consistency of experts’ judgments. To solve this problem, axiomatic design (AD) approach is proposed to decrease the number of design solutions and minimize the expert’s intervention in pairwise comparisons. The AD methodology is an efficient scientific design and manufacturing approach (Suh, 1990). Up to now, several research works are reported to develop the AD theory and applications. The AD is applied in product design, system design, manufacturing system design, software design, decision-making, and other issues (Kulak, Cebi, & Kahraman, 2010). In recent years, the AD is widely used to solve the multi-criteria decision-making and product design problems. For instance, manufacturing technology selection (Gonçalves-Coelho & Mourão, 2007), multi-attribute comparison of advanced manufacturing systems (Kulak & Kahraman, 2005a), selection of the best transportation companies (Kulak & Kahraman, 2005b), performance evaluation model for docking facilities in shipbuilding industry (Celik & Kahraman, 2009), logistics tool selection with two-phase fuzzy multi-criteria decision-making (Büyüközkan, Arsenyan, & Ruan, 2012), are examples on decision-making issues. Also, Indicator design for passenger car (Cebi & Kahraman, 2010a), evaluation of emergency core cooling systems (Heo & Lee, 2007), design of the integrity of shaft surfaces for rotating lip seals (Brown, 2011), application of AD principles to control complexity dynamics in a mixed-model assembly system (Matta, 2012), and designing a tunable microscope and single-lens-reflex magnifier, and point-and-shoot camcorders (Lo & Helander, 2007), are examples of AD implementations in product design.

In this paper, the AHP methodology is used to select the best design solution among a wide variety of design solutions of an LATM. In this methodology, the criteria and acceptable design solutions are evaluated based on the expert’s judgments. In order to minimize the effect of expert’s intervention and pursue a simple and short design procedure, the AD methodology is performed considering industrial requirements and the desired design values. Finally the AD and AHP methodologies derive the values of
design parameters (DPs). Comparing the derived parameters, the results show that the AD values are more reliable than AHP values.

2. Problem definition

This work proposes the design procedure of an LATM to meet the industrial application requirements. The configuration of the designed LATM is shown in Figure 1.

To develop a two-pole LATM for an industrial control application that requires a ±30° rotation with a peak torque of 2.5 Nm, a toroidally wound-type solid core stator with a rare-earth pole-tip rotor construction is selected. The two DC stator windings are connected in series for single-phase excitation by a simple DC circuitry. The rotor carries field magnets and the stator supports the armature windings. The interaction of these two magnetic fields produces an electromagnetic torque which causes an attraction or repulsion force between the rotor and the stator. The magnitude and direction of armature current determines the magnitude of the electromagnetic torque and rotor movement direction. All of the desired ranges and values of DPs of this application are shown in the third column of Table 5. These intervals and values were determined by experimental experience of LATM designers (Zarandi et al., 2011). Different kinds of PM and ferromagnetic materials can be used in this actuator’s structure. The theoretical torque vs. shaft-position characteristic of LATMs is shown in Figure 2, where \( \theta \), \( p \), and \( T_p \) are, respectively, the angle of rotation, the number of poles, and the peak torque. Manufacturers generally provide a theoretical torque vs. shaft-position curve for LATMs. This curve is represented by the positive lobe of a cosine function; that is,

\[
T = T_p \cos \left( \theta \cdot \frac{p}{2} \right)
\]

The constant torque region of toroidally wound type LATMs depends on the arc length of each sector of armature winding and the pole arc length as follow:

\[
\theta_0 = \frac{\varphi_s - \varphi_r}{2}
\]
where $\theta_0$, $\varphi_s$, and $\varphi_r$ denote the constant torque range, stator winding arc angle, and rotor pole arc angle, respectively. The output torque of an LATM is directly proportional to the armature current. Hence, the torque-current characteristic is a straight line with a slope known as torque sensitivity constant $k_t$.

A comprehensive design has used the MEC model, which extracts the geometric parameter relations (Zarandi et al., 2011). The maximum torque relation in MEC model at position $\theta_0 = 0$ is obtained as:

$$T_p = \frac{p k k_r s D_i^2 B_0^2 g}{\mu_0} \tag{3}$$

where $p$ is the number of poles, $\mu_0$ is free space permeability, $g$ is the length of air-gap, $D_i$ is the inner diameter of stator core, $B_0$ is the air-gap flux density corresponding to the pick torque state, and $k$ and $k_{rs}$ are geometric constants which represent the ratios of $w/D_i$ and $\varphi_r/\varphi_s$, respectively, and $w$ is the axial length of the motor.

If the maximum torque that is generated in a short time is used instead of a continuous torque in the design procedure, the size, weight, and cost of LATMs can be reduced considerably. The continuous torque, $T_c$, is limited by thermal criteria; whereas the maximum torque, $T_p$, is usually limited by magnetic saturation or supply current/voltage capability (Zarandi et al., 2011). Other actuator’s parameters are calculated in the step-by-step design procedure. In the first step, $\varphi_r$, $\varphi_s$, and $p$ are selected according to the shape of motor, angular range of constant torque, and the required maximum acceleration.

The geometric constant, $k$, is usually considered as $(.8–1.6)/p$ for DC machines (Pyrhönen, Jokinen, & Hrabovcová, 2008). The average value of air-gap flux density $B_0$ in DC machines is between .4 and .8 T (Mittle & Mittal, 1996). Depending on the requirements for the application, we may also have limitations on the axial length of LATMs. According to industrial requirements, the motor axial length $w$ is limited between 20 and 80 mm. Hence, by substituting the given parameters $k$, $B_0$, and $w$ in Equation (3), the air-gap length is calculated. Determining the stator and rotor parameters is an important step in LATM design procedure. The selection of an appropriate ferromagnetic material for stator and rotor cores is very important. The rotor structure is composed of two radially magnetized PM and one cylindrical ferromagnetic magnet holder. Proper materials should have low residual magnetism, low hysteresis loss, and a good saturation value of flux density as well as low cost. The geometric parameters of stator and rotor cores are determined after this step. By considering the continuity of the magnetic flux from PM to air-gap, the ampere’s law, and a best linear curve fitting for B-H characteristics of PM assuming no saturation, the magnetic path length in each PM is calculated. Next, electrical and mechanical time constants of the LATM are calculated.

Figure 2. Torque-rotor position characteristic of an LATM.
based on the dimensions and electrical parameters of the motor. Whereas, the three parameters $k$, $w$, and $B_0$ vary between the given upper and lower limits, for each value of these parameters in their intervals, different LATMs can be designed. The constant parameters of the design should be selected under a better knowledge of actuator’s system and experimentations. By changing $k$, $w$, and $B_0$ a wide variety of the design solutions may be expected. This leads us to a methodology that confines and optimizes the design procedure, and makes more available control on the DPs. So, to select and determine the best design solution, the AHP and AD methodologies will be applied as optimization tools, respectively.

3. AHP overview

The AHP proposed by Saaty in 1977, is an effective approach in dealing with multi-criteria decision-making problems. The five main principles are implemented as:

3.1. Develop a hierarchical framework

The AHP breaks down complex problems into parts and then puts them into a hierarchical framework. At the top level of hierarchy lies decision objective or goal, the lower levels of hierarchy contain criteria and sub-criteria, and the last level contains the decision alternatives.

3.2. Construct pairwise comparison matrices

The AHP uses pairwise comparisons to determine the priority of problem parts. To express one’s opinion on only two elements is more accurate and easier than simultaneously on all the elements. A ratio scale with no units is used by AHP in the comparisons. Saaty described verbal scale of nine levels, which is shown in Table 1. The decision-maker provides a numerical judgment based on this verbal scale (Ishizaka & Labib, 2011).

The pairwise comparison matrix is shown in Equation (4). This matrix has diagonal elements equals 1, and $a_{ij}$ defines the relative importance of the $i$th element to the $j$th element, the $1/a_{ij}$ shows the importance of the $j$th element to the $i$th element.

$$A = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ a_{21} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & 1 \end{bmatrix}$$

Table 1. Saaty’s scale for pairwise comparisons.

| Intensity of importance | Definition                                                      |
|-------------------------|----------------------------------------------------------------|
| 1                       | Equal value                                                   |
| 3                       | Slightly more value                                           |
| 5                       | Essential or strong value                                     |
| 7                       | Very strong value                                             |
| 9                       | Extreme value                                                 |
| 2, 4, 6, 8              | Intermediate values between the two adjacent judgments         |
| Reciprocals             | Reciprocals for inverse comparison                            |
A pairwise comparison matrix is consistent if all comparisons respect the following relation: (Ishizaka, Balkenborg, & Kaplan, 2011)

\[ a_{ij} = a_{ik} \cdot a_{kj} \]  

where \( i, j, \) and \( k \) are any elements of the matrix.

### 3.3. Priorities derivation

The priority vector of each matrix is calculated from the pairwise comparison matrix in each level of the hierarchy framework. The priority vector of the comparison matrix \( A \) is \( W = (w_1, w_2, \ldots, w_n) \), such that \( w_i/w_j \) match the comparisons of \( a_{ij} \) in a consistent matrix, and when slight inconsistencies are introduced, priorities should vary only slightly (Ishizaka & Labib, 2011).

In this step, different methods such as Eigenvector, arithmetic mean, least square, and logarithmic least square have been developed to estimate the priority vector. The Eigenvector and the least square methods are the most cited methods in the literatures. Saaty (2003) showed that the priority vector \( W \) in the pairwise comparison matrix of \( A \), met \( AW = \lambda_{\text{max}} W \). It is obvious that \( W \) is the right Eigenvector and \( \lambda_{\text{max}} \) is the biggest Eigenvalue of matrix \( A \). The logarithmic least square method or the geometric mean (GM) is based on Equation (6). In this method, the differences between \( a_{ij} \) and \( w_i/w_j \) are minimized: (Crawford & Williams, 1985).

\[
\begin{align*}
\text{min} \sum_{i=1}^{n} \sum_{j=1}^{n} (\ln a_{ij} - \ln(w_i) - \ln(w_j))^2 \\
\text{s.t.:} \sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0
\end{align*}
\]  

The optimum solution of (6) is shown as:

\[ w_i = \left( \prod_{j=1}^{n} a_{ij} \right)^{1/n}, \quad i, j = 1, 2, \ldots, n \]  

Crawford and Williams (1985), show that GM is superior to the EM method in several important aspects. They derived GM from statistical considerations and it was shown to be optimal when the judge’s errors are multiplicative with a log-normal distribution.

Furthermore, three fundamental consistency axioms were declared by Barzilai, Cook, and Golany (1987), and it was proved that, the only way which meets in these axioms, is through the geometric mean. In this paper, GM is used to estimate the priority vectors.

### 3.4. Synthesizing the priority vectors

In this step, the priority vectors are synthesized to determine the final priority. Equation (8) has been proposed to prevent the rank reversal phenomenon by Barzilai and Golany (1994).

\[ p_i = \prod_{j} l_{i}^{w_j} \]  

where \( p_i \) is the final priority of the alternative \( i \), \( l_{i} \) is the priority vector of alternative \( i \) with respect to the criterion \( j \), and \( w_j \) is the weight of the criterion \( j \).
3.5. Consistency verification of decision-maker’s judgments

Since the comparisons are carried out through expert’s judgments, some degree of inconsistency may occur. AHP accepts some inconsistencies of the judgments, which happens in practice. Saaty (1990) has proposed the consistency ratio (CR). Based on Saaty’s opinion, if the value of CR is less than .1, the judgments will be acceptable, otherwise the matrix will be inconsistent and the judgments should be reviewed and improved.

In addition to the consistency verification, Aguarón and Moreno-Jiménez (2003) suggest that, in the case of GM to derive the priority vectors, the CR is not appropriate to determine the consistency. They formalized the geometric consistency index (GCI) to measure the consistency of matrices and provided the thresholds associated with it.

\[ GCI = \frac{2 \sum_{i<j} \left( \log a_{ij} - \log \frac{w_i}{w_j} \right)^2}{(n-1)(n-2)} \] (9)

GCI and CR are rational to each other. If the comparison matrices are close to consistency, then the two measures are proportional. The determined GCI is related to the permitted CR values as Table 2 (Aguarón & Moreno-Jiménez, 2003).

4. Implementation of the AHP to select the LATM DPs

In this paper, the selection of the best design solution among wide varieties of acceptable design solutions is done by AHP. In this methodology, some effective criteria are considered and they are evaluated based on the expert’s judgments and acceptable design solutions. The pairwise comparison matrices are constructed based on the expert’s judgments. To extract the priority vector, the GM method has been employed. To construct the hierarchical framework, the main goal, desired criteria, and the acceptable design solutions are put in the first, second, and third layers, respectively. Main goal of the problem is to select the best design solution among all the acceptable design solutions. With respect to the specified industrial requirement and constraints, the weight, volume, time constant, mechanical clearance, and copper losses of the LATM have been considered as criteria. In the LATM design procedure, minimizing the weight, volume, time constant, and copper losses, and maximizing the mechanical clearance to facilitate the manufacturing process are desired. The last layer indicates the acceptable design solutions. The hierarchical framework is shown in Figure 3.

These criteria are compared based on the expert’s judgments in the pairwise manner by the Saaty’s scale. In order to determine the priority of each criterion, the GM method is used. The comparison matrix and the priority vector of the criteria are shown in Table 3. Hence, based on the expert’s judgments, the mechanical clearance and the total weight of the LATM have the maximum and the minimum importance, respectively.

The GCI rate of this matrix is equal to .034. With respect to Table 2, the matrix is considered consistent. In order to calculate the design solutions’ priority vector, all

| CR  | GCI (n = 3) | GCI (n = 4) | GCI (n > 4) |
|-----|------------|------------|------------|
| .05 | .1573      | .1763      | .1850      |
| .10 | .3147      | .3526      | .3700      |
| .15 | .4720      | .5289      | .5550      |
design solutions should be compared in pairwised manner with respect to each criterion. If the number of design solutions is \( n \), there will be five \((n \times n)\) comparison matrices with respect to five criteria. Whereas, if there are a high number of design solutions, it is hard to make the consistent judgments based on the Saaty’s scale. To develop the comparison matrices of design solutions with respect to each criterion, the quantitative values of the same criterion is used. For example, where comparing design solutions \( i \) and \( j \), with respect to a criterion such as the volume, weight, copper losses, and time constant, the corresponding element of \( i \)th row and \( j \)th column is equal to the quantitative value of \( j/i \)th criterion ratio. But, in the case of mechanical clearance criterion, this value may be inverted.

Next, the priority vector of each matrix is calculated with the GM method. The final priority vector of design solutions is calculated by multiplicative aggregation rule as Equation (8). The best design solution is related to the maximum value of the final priority vector. Table 5 shows the derived values of the optimum DPs according to the AHP methodology.

5. AD methodology

AD is a scientific foundation for design and manufacturing, in an effective way. This theory was formed and developed in MIT (Oxford University Press, Suh, 1990). In fact, the goal of this theory is the presentation of a scientific methodology for design procedure, such that each product, system, process, or service can be evaluated based on it. According to AD, every design object can be depicted in four design domains: the customer, the functional, the physical, and the process domains as depicted in Figure 4 (Gonçalves-Coelho & Mourão, 2007). Distinct solutions are created by matching the
desired characteristic in a domain with defined parameters in the neighbor domain. These domains are linked through several mappings as shown in Figure 4. There are three types of mapping: (1) mapping between Customer domain and Functional domain is defined as conceptual design, (2) mapping between Functional domain and Physical domain is defined as product design, and (3) mapping between Physical domain and Process domain is defined as process design (Gonçalves-Coelho & Mourão, 2007).

In customer domain, customer’s needs (CN) are determined. In functional domain, proposed needs in customer's domain, interprets to a set of functional requirements (FRs). In fact, the FRs are the CN in scientific and technical language, which define the problem. In order to meet the FRs, the DPs are defined in the physical domain. The set of FRs and DPs and the mapping between them, produce the product design. In process domain, the process variables (PV) are defined for manufacturing and construction of designed product (Oxford University Press, Suh, 2001).

The design procedure for a product includes the selection of a set of appropriate DPs to meet the FRs. The relation between FRs and DPs is defined by the design matrix in Equation (10).

\[
\{\text{FR}\} = [A]\{\text{DP}\}
\]  

(10)

where \([A]\) is the design matrix; \([A]\) is characterizing the design and shows the relations between FRs and DPs. The elements of \([A]\) consist of ‘X’ and ‘0’ elements such that ‘X’ symbolizes the relation between FR and DP; while ‘0’ symbolizes no relation between FR and DP. There are three types of designs with respect to the number of FRs and DPs. The design is named as coupled if the number of DPs is bigger than the number of FRs, The design is named as redundant if the number of DPs is smaller than the number of FRs and the design can be coupled, decoupled, or uncoupled, if the number of DPs is equal to the number of FRs. If the design matrix is diagonal, the design solution is uncoupled, if the design matrix is triangular, the design solution is decoupled. Otherwise, the design solution is named as coupled and unacceptable (Oxford University Press, Suh, 1990) and (Cebi & Kahraman, 2010b).

AD theory includes the independence axiom and information axiom as follows: (Cebi & Kahraman, 2010b).

Axiom 1: The independence axiom maintains the independence of the FRs.

In other words, this axiom implies that FRs must be satisfied by DPs without affecting any other FRs.

Axiom 2: The information axiom minimizes the information content of the design.

In other words, among all the proposed solutions that satisfy independence axiom, the best design is the design that has the minimum information content. If the

![Figure 4](image-url)  

Figure 4. The design process as a mapping.
probability of success for a given FR is $p$, the information content is calculated by Equation (11).

$$I = \log_2 \frac{1}{p}$$  \hspace{1cm} (11)

If there is more than one FR, the information content is calculated as follows:

$$I_{\text{system}} = \sum_{i=1}^{m} \log_2 \frac{1}{p_i}$$  \hspace{1cm} (12)

6. Implementation of AD to select the LATM DPs

A product is designed to meet an overall set of FRs and constraints. In order to design the LATM, FRs and their corresponding DPs are selected, and then the zigzagging between these two domains is established. The minimum set of the independent requirements that completely characterizes the FRs of the LATM are determined as follow. The values of the FRs are determined based on limitations of industrial application of the LATM. For example, the copper losses are the main source of heat generation in the system. The admissible heat in the system is corresponding to 120 Watt of the copper losses. Furthermore, to track the reference signals, system needs to have an acceptable speed. The time constant of the LATM determines the desired speed. Moreover, the industrial application dictates limitation on actuator installation space.

- FR$_1$ = copper losses (CL) must be 120 (W)
- FR$_2$ = time constant (TC) must be 15 (ms)
- FR$_3$ = volume ($V$) must be 400 (cm$^3$)
- FR$_4$ = peak torque ($T$) must be 2.5 (Nm)

Considering the design relation Equation (13), in order to satisfy copper losses (CL), the terminal resistance ($R$) is considered as first DP.

$$\text{CL} = I^2 \cdot R = \frac{T^2}{K_t^2} \cdot R$$  \hspace{1cm} (13)

where $K_t$ is torque/current sensitivity coefficient and it is equal to 1.11 (Zarandi et al., 2011). In accordance with the design relation Equation (14), the moment of inertia ($J$) is considered as second DP to satisfy the time constant (TC).

$$\text{TC} = \frac{R \cdot J}{k_b \cdot k_t}$$  \hspace{1cm} (14)

where induced voltage coefficient $k_b$ is equal to $k_t$. Considering the design relation Equation (15), in order to satisfy volume ($V$), the total air gap ($g$) is considered as third DP.

$$V = \frac{\pi}{4} (D_o + 2(g - MC))^2 (w + 2(g - MC))$$  \hspace{1cm} (15)

where $D_o$ is the outer diameter, and MC is the mechanical clearance. The design relation in Equation (16) is extracted from Equation (3) with substituting the $k_{rs}$ with the \( \phi_s / \phi_s \).
ratio. In order to satisfy peak torque \(T\), the air-gap flux density \(B_0\) is considered as the fourth DP.

\[
T = \frac{pKD^2B_0^2g\phi_r}{\mu_0\phi_s}
\]  

(16)

In order to select the independent FRs, some requirements should be considered as constraints. Table 4 shows the selected constraints in this work.

The relations between FRs and DPs are shown in the design matrix as Equation (17). This matrix includes two independent down-triangular sub-matrices. According to the AD theory, this matrix is decoupled and does not violate the independence axiom. So, it is considered as a quite reasonable design. In order to design the LATM, the values of the DPs can be determined.

\[
\begin{bmatrix}
CL \\
TC \\
V \\
T
\end{bmatrix} =
\begin{bmatrix}
x_{11} & 0 & 0 & 0 \\
x_{21} & x_{22} & 0 & 0 \\
0 & 0 & x_{33} & 0 \\
0 & 0 & x_{43} & x_{44}
\end{bmatrix}
\begin{bmatrix}
R \\
J \\
g \\
B_0
\end{bmatrix}
\]  

(17)

The design equations of the first sub matrix are shown in Equations (18) and (19). Considering these equations, CL is the first FR that should be set by suitable \(R\) value in Equation (18).

\[
CL = x_{11} \cdot R = \frac{T^2}{K_t^2} \cdot R
\]  

(18)

With respect to the values of CL and \(T\) of the actuator as two FRs, the terminal resistance \(R\) is equal to 29.64 (Ω). By having the value of \(R\), in order to satisfy the time constant (TC) value, it is sufficient to determine the moment of inertia \(J\) in Equation (19).

\[
TC = x_{21} \cdot R + x_{22} \cdot J = \frac{R \cdot J}{k_b \cdot k_t}
\]  

(19)

Considering the \(k_b\) and \(k_t\) coefficients, \(J\) is equal to \(6.23 \times 10^{-4} \text{ (kg m}^2\text{)}\).

In accordance with the first row of the second sub-matrix, and the design relation in Equation (15), the relation of the volume \(V\) and total air gap \(g\) are derived as:

\[
V = x_{33} \cdot g = \frac{\pi}{4} (D_o + 2(g - MC))^2 (w + 2(g - MC))
\]  

(20)

| Table 4. The industrial constraints of the LATM. |
|-----------------------------------------------|
| **Industrial constraints**                     | **Value** |
| Number of poles                                | 2         |
| Outer diameter (mm)                            | 100       |
| Axial length (mm)                              | 40        |
| Mechanical clearance (mm)                      | 1.5       |
| \(k\) (axial length/inner diameter)           | .55       |
| Rotor pole arc angle                           | 120°      |
| Stator winding arc angle                       | 180°      |
In order to select the LATM volume as the third FR, the total air gap \((g)\) should be determined. Considering the outer diameter \((D_o)\) and mechanical clearance (MC), the value of \(g\) is equal to 4.325 (mm).

In addition, the relation of air-gap flux density \((B_0)\) and peak torque \((T)\) is depicted as:

\[
T = x_{43} \cdot g + x_{44} \cdot B_0 = \frac{pKD_i^2B_0^2g\phi_r}{\mu_0\phi_s} \tag{21}
\]

In order to select peak torque, it is required to determine the value of \(B_0\). With respect to the values of \(p, k, D_i,\) and \(k_{rs}\) as constraints, the value of \(B_0\) is equal to .43 Tesla. The values of the optimum DPs based on AD methodology are shown in Table 5.

7. **Comparison of AD and AHP methodologies in the LATM design**

The DPs that are generated from both AD and AHP methodologies are presented in Table 5. The last column of this table shows the desired ranges and values of the DPs. Considering the results, the AD’s values have more conformation with the desired ranges of the DPs rather than the AHP methodology. The comparison of the some important and effective DPs, such as mechanical clearance, terminal resistance, volume, and time constant, by both methodologies are shown in Figure 5. The generated designs where axial length is constant and \(k\) and \(B_0\) are changed in specified intervals are shown in Figure 5(a)–(d). The design solutions of AHP and AD methodologies are emphasized in these figures. Although the mechanical clearance generated by AHP is bigger than

| Parameter                      | AHP | AD | Desired ranges and values of the design parameters |
|--------------------------------|-----|----|---------------------------------------------------|
| Peak torque (Nm)               | 2.5 | 2.5| 2.5                                               |
| Number of poles                | 2   | 2  | 2                                                 |
| \(k_t\) (Nm/A)                 | 1.11| 1.11| 1.11                                              |
| Constant torque range (degree) | ±30°| ±30°| ±30°                                              |
| Toroid dimensions              |     |    |                                                   |
| \(k\) (axial length/inner      | .5  | .55| [.4–.8]                                           |
| diameter)                      |     |    |                                                   |
| Outer diameter (mm)            | 110.02| 100| [90–105]                                         |
| Axial length (mm)              | 40  | 40 | [25–50]                                           |
| Total air gap (mm)             | 3.98| 4.32| Greater than 4                                   |
| Mechanical clearance (mm)      | 1.95| 1.50| Greater than 1                                   |
| Winding specifications         |     |    |                                                   |
| Terminal resistance (Ω)        | 30.36| 29.64| Up to 30                                         |
| Terminal inductance (mH)       | 87.25| 64.70| Up to 75                                         |
| Total volume (cm³)             | 450.34| 400| Up to 420                                        |
| Total weight (kg)              | 2.73| 2.35| Up to 3                                          |
| Time constant (m sec)          | 20.07| 15 | Up to 15                                         |
AD, both the values are in the desired range of industrial requirements (see Figure 5(a)). Figure 5(b)–(d) shows that the AD’s output for the other DPs have more desirable values.

8. Conclusion

In the LATM design procedure, there are many design solutions which are generated by varying some DPs within their specified intervals. In order to select the best design solution, the AHP methodology is used as a multi-criteria decision-making tool. The optimum design solution of this methodology is dependent on selecting the criteria and the expert’s judgments. The impact of experts’ judgment and elimination of the trial and error activities in design procedure is achieved by AD methodology. This methodology facilitates and increases the simplicity of design activities. The AD methodology determines the DPs and constants based on specified FRs and constraints. So it has more conformation with the industrial design range limitations. Also, the AD reduces the design iterations and computation time. For further work it is suggested to apply information axiom of AD theory to determine the best LATM design, according to manufacturing technology alternatives. Also to get precise results, the application of zigzag method on breaking down the FR, DP, and PV in the lower levels is suggested for future study.
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