Giant Graviton and Quantum Stability in Matrix Model on PP-wave Background

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Abstract

We study classical solutions in Berenstein-Maldacena-Nastase (BMN) matrix model. A supersymmetric (1/2 BPS) fuzzy sphere is one of the classical solutions and corresponds to a giant graviton. We also consider other classical solutions, such as non-supersymmetric fuzzy sphere and harmonic oscillating gravitons. Some properties of oscillating gravitons are discussed. In particular, oscillating gravitons turn into usual supergravitons in the limit $\mu \to 0$. Moreover, we calculate the one-loop effective action around the supersymmetric fuzzy sphere by the use of the background field method and show the quantum stability of the giant graviton. Also, the instability of the non-supersymmetric fuzzy sphere is proven.

Keywords: M-theory, matrix model, supermembrane, pp-wave, giant graviton, fuzzy sphere
1 Introduction

Recently, pp-wave backgrounds are so focused and intensively studied. A maximally supersymmetric pp-wave background is a classical solution of the eleven-dimensional supergravity [1] and it is considered as one of the candidates for supersymmetric backgrounds of M-theory [2]. The metric for this solution found by Kowalski-Glikman [1] (often called KG solution) is described by

\[ ds^2 = -2dx^+dx^- + G_{++}(dx^+)^2 + (dx^\mu)^2, \]
\[ G_{++} \equiv -\left[ \left( \frac{\mu}{3} \right)^2 (x_1^2 + x_2^2 + x_3^2) + \left( \frac{\mu}{6} \right)^2 (x_4^2 + \cdots + x_9^2) \right], \]

and the constant 4-form flux for +, 1, 2 and 3 directions,

\[ F_{+123} = \mu, \quad (\mu \neq 0) \]

is equipped. There are many other pp-wave backgrounds. In particular, after the maximally supersymmetric type IIB pp-wave solution has been found [3], it was shown that the type IIB string theory on the pp-wave is exactly-solvable in the Green-Schwarz (GS) formulation with the light-cone gauge fixing [4–6]. The action of the string on the pp-wave acquires mass terms for bosons and fermions, but it is still free theory and exactly-solvable. It should be noted that pp-wave backgrounds are curved and hence are interesting objects as clues to study string theories on the curved backgrounds, which are difficult subjects. The pp-wave background used in the type IIB analysis can be also obtained from the five-dimensional anti-de Sitter space $AdS_5$ through Penrose limit [7, 8]. From this fact, the $AdS$/CFT correspondence has been investigated [8]. It is possible to study the $AdS$/CFT correspondence at the stringy level without reducing the analysis to the supergravity level in the region where the $AdS$ geometry can be approximated by the pp-wave geometry. It might be possibly expected that some new features of string theories could be understood due to this advantage.

It is well-known that the matrix theory approach to the M-theory seems very successful [10]. Motivated by this, we can also study the Matrix theory [9, 12] or the supermembrane [13, 14] on the pp-wave. The matrix model on the eleven-dimensional maximally supersymmetric pp-wave background has been proposed by D. Berenstein, J. Maldacena and H. Nastase [3], which is often referred as the BMN matrix model. This model has mass terms and Myers terms (bosonic 3-point coupling terms) and contains more interesting physics than the flat case. The pp-wave background is curved and seems more complicated than the flat space but there
are some advantages. For an example, let us consider the supermembrane on the pp-wave. It is a well-known problem that a single supermembrane in the flat space is unstable [11]. However, possibly surprisingly, the quantum supermembrane on the pp-wave might be stable since flat directions of the quartic potential are completely removed due to the presence of mass terms. The continuous spectrum of the supermembrane in the flat space is a troubling feature. However the spectrum of the BMN matrix model might be expected to be discrete and lead to an isolated set of the classical supersymmetric vacua. This is convenient because the structure of the ground states is governed by the semi-classical approximation and it is not needed to solve the full quantum mechanical problem of the ground state wave function as in the flat space, which is too difficult. Thus, it can be expected in the BMN matrix model that some difficulties in the flat case are removed.

The BMN matrix model can be also derived from the supermembrane theory on the pp-wave through the matrix regularization [12]. We have discussed that the supermembrane theory on the maximally supersymmetric pp-wave is closely related to the BMN matrix model in the same way as the flat space in our previous works [13,14]. We have calculated the superalgebra in the supermembrane theory on the pp-wave background by using the standard Poisson-Dirac bracket procedure. The result agrees with the superalgebra in the BMN matrix model and confirms the correspondence of the superalgebra between the supermembrane and the matrix model on the pp-wave. (The correspondence in the flat case has been discussed in [15].) We have also obtained the central charges (brane charges) of the superalgebra, some of which exist only in the pp-wave case. Moreover, we have investigated BPS conditions of the supermembrane on the pp-wave and, for an example, constructed a 1/4 BPS solution whose charge equals to the angular momentum. It implies that this solution should be a rotating BPS configuration. BPS multiplets in the BMN matrix model are also intensively discussed in [16].

In this paper we consider classical solutions in the BMN matrix model. A supersymmetric (1/2 BPS) fuzzy sphere (giant graviton), a non-supersymmetric fuzzy sphere and oscillating gravitons are considered. The supergraviton in the flat space corresponds to a collection of harmonic oscillators on the pp-wave that is non-supersymmetric and considered as a non-BPS object. This result comes from the presence of bosonic mass terms. However, it turns into the supergraviton in the flat limit $\mu \to 0$. Moreover, we calculate the one-loop effective action around the supersymmetric fuzzy sphere by using the background field method and prove the quantum stability of the giant graviton. The one-loop corrections are certainly canceled as
expected from the requirement of the supersymmetry. However, it should be noted that its cancellation is non-trivial since the quantum supersymmetric vacua is non-trivial in the light-cone formulation. It depends on the fact that the supercharges do not commute with the Hamiltonian. In particular, the zero-point energy depends on the definition of the vacua. The one-loop contribution in the BMN matrix model is essentially the zero-point energy contribution induced by the pp-wave background, and hence the cancellation of one-loop contributions would be also non-trivial. Also, we show the instability of the non-supersymmetric fuzzy sphere background in order to compare it with the supersymmetric background.

This paper is organized as follows. Section 2 is devoted to the setup in this paper and we provide a brief review of the BMN matrix model. In section 3 we consider various classical solutions on the pp-wave. Besides a supersymmetric fuzzy sphere, we present a non-supersymmetric fuzzy sphere and harmonic oscillating gravitons corresponding to supergravitons in the flat space. In section 4, we introduce the background field method in the BMN matrix model. The quadratic action is available to calculate the one-loop effective action. In section 5, we calculate the one-loop effective action around the supersymmetric fuzzy sphere and prove the quantum stability of the giant graviton. We also discuss the instability of non-supersymmetric fuzzy sphere background. Section 6 is devoted to conclusions and discussions.

2 Berenstein-Maldacena-Nastase (BMN) Matrix Model

In this section we briefly review the matrix theory on the eleven-dimensional maximally supersymmetric pp-wave background (BMN matrix model). The action of the BMN matrix model $S$ is given by

$$S = S_{\text{flat}} + S_\mu,$$

$$S_{\text{flat}} = \int dt \, \text{Tr} \left[ \frac{1}{2R} D_0 X^r D_0 X^r + \frac{R}{4} ([X^r, X^s])^2 + \Psi^r D_0 \Psi + i R \Psi^r \gamma_5 [\Psi, X^r] \right],$$

$$S_\mu = \int dt \, \text{Tr} \left[ -\frac{1}{2R} \left( \frac{\mu}{3} \right)^2 X^r X^r -\frac{1}{2R} \left( \frac{\mu}{6} \right)^2 X^r X^r - \frac{\mu}{3} i \epsilon_{ijk} X^i X^j X^K - \frac{\mu}{4} \Psi^r \gamma_{123} \Psi \right].$$

where the covariant derivative is defined by

$$D_0 X^r \equiv \partial_0 X^r - i [A, X^r] \equiv \dot{X}^r - i [A, X^r].$$

When we shall rescale the gauge field $A$, parameters $t$ and $\mu$ as

$$t \rightarrow \frac{1}{R} t, \quad A \rightarrow RA, \quad \mu \rightarrow R \mu,$
the actions $S_{\text{flat}}$ and $S_{\mu}$ can be rewritten as

$$S_{\text{flat}} = \int dt \text{Tr} \left[ \frac{1}{2} D_0 X^r D_0 X^r + \frac{1}{4} ([X^r, X^s])^2 + \Psi^T D_0 \Psi + i \Psi^T \gamma_r [\Psi, X^r] \right],$$

(2.2)

$$S_{\mu} = \int dt \text{Tr} \left[ - \frac{1}{2} \left( \frac{\mu}{3} \right)^2 X_I^2 - \frac{1}{2} \left( \frac{\mu}{6} \right)^2 X_I^2 - \frac{\mu}{3} \sum_{i=1}^3 X^i X^i - \frac{\mu}{6} \sum_{i'=-4}^9 X^{i'i'i'} \gamma_{123} \right].$$

(2.3)

This theory has 16 dynamical supersymmetries,

$$\delta_\epsilon X^r = 2 \psi^T \gamma^r \epsilon(t), \quad \delta_\epsilon A = 2 \psi^T \epsilon(t),$$

$$\delta_\epsilon \psi = \left[ D_0 X^r \gamma_r + \frac{i}{2} [X^r, X^s] \gamma_{rs} + \frac{\mu}{3} \sum_{i=1}^3 X'^i \gamma_I \gamma_{123} - \frac{\mu}{6} \sum_{i'=4}^9 X'^i \gamma_{i'i'i'} \gamma_{123} \right] \epsilon(t),$$

(2.4)

$$\epsilon(t) = e^{-\frac{\mu}{4} \gamma_{123} t} \epsilon_0, \quad \epsilon_0: \text{constant spinor},$$

and 16 kinematical supersymmetries,

$$\delta_\eta X^r = \delta_\eta A = 0,$$

$$\delta_\eta \psi = \eta(t),$$

$$\eta(t) = e^{\frac{\mu}{4} \gamma_{123} t} \eta_0, \quad \eta_0: \text{constant spinor}.$$

(2.5)

Note that the bosons and fermions in the action $S$ have different masses. Three of bosons have mass $\mu/3$ and the remains have mass $\mu/6$. On the other hand, all the fermions have mass $\mu/4$. This result depends on the fact that the supercharge do not commute with the Hamiltonian. It might be considered that such a situation comes from the light-cone gauge fixing.

### 3 Classical Solutions of BMN Matrix Model

In this section we shall consider classical solutions by solving classical equations of motion under a certain ansatz. In particular, a fuzzy sphere solution preserving a half of supersymmetries (i.e., 1/2 BPS), a non-supersymmetric fuzzy sphere solution, and oscillating gravitons are constructed here.

Let us set fermionic degrees of freedom to zero since we are restricted ourselves to bosonic backgrounds. Taking the variations of $X$’s in the action $S$, we obtain classical equations of

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*The covariant approach is also discussed in Ref. [17].

†Other several interesting solutions have been reported in [18], though we will not consider them in this paper.
\( \dot{X}' = -[X', X'], \quad \dot{X}'' = -[X'', X'] - (\mu/6)^2 X'', \quad (I = 1, 2, 3), \) \hfill (3.1) \\
\( \dot{X}''' = -[X''', X'] - (\mu/6)^2 X''' , \quad (I' = 4, \ldots, 9). \) \hfill (3.2)

In order to show the energy of classical solutions, we utilize the bosonic potential \( V_B \) in the BMN matrix model defined by

\[
V_B(X) = \text{Tr} \left[ -\frac{1}{4}([X^r, X^s])^2 + \frac{1}{2} \left( \frac{\mu}{3} \right)^2 X^2 + \frac{1}{2} \left( \frac{\mu}{6} \right)^2 X'' + \frac{\mu}{3} i\epsilon_{IJK} X^I X^J X^K \right],
\]

\[
= \frac{1}{2} \text{Tr} \left[ \left( \frac{\mu}{3} X^I + i\epsilon_{IJK} X^J X^K \right)^2 \right. \\
\left. + \frac{1}{2} (i[X', X''])^2 + (i[X', X'])^2 + \left( \frac{\mu}{6} \right)^2 (X''')^2 \right]. \hfill (3.3)
\]

The supersymmetry transformations (2.4) and (2.5) are also available to check whether classical solutions are supersymmetric or not.

### 3.1 Giant Graviton and Fuzzy Sphere Solution

We would like to obtain static fuzzy sphere solutions of equations of motion (3.1) and (3.2). To begin, we shall suppose the following ansatz,

\[ X^I = \alpha J^I, \quad X''' = 0, \hfill (3.4) \]

where \( \alpha \) is an arbitrary constant and \( J^I \)'s are \( SU(2) \) generators and satisfy an \( SU(2) \) Lie algebra,

\[ [J^I, J^J] = i\epsilon_{IJK} J^K. \]

In this case, Eq. (3.2) is satisfied automatically. Inserting the ansatz (3.4) into Eq. (3.1), we obtain an algebraic equation,

\[ \alpha \left( \alpha - \frac{\mu}{3} \right) \left( \alpha - \frac{\mu}{6} \right) = 0. \]

The case \( \alpha = 0 \) denotes a trivial classical supersymmetric vacuum, \( X^r = 0 \). The value \( \alpha = \mu/3 \) corresponds to 1/2 BPS fuzzy sphere solution as pointed out in Ref. \[9\]. This solution can be interpreted as a giant graviton, which is the expanded D0-branes due to the Myers effect in the presence of the constant 4-form flux \[19\]. In general a giant graviton has finite energy.
but the energy of this solution is zero as we can easily see by inserting the solution into
the bosonic potential $V_B$. This solution is labelled by all possible ways of dividing an $N$-
dimensional representation of $SU(2)$ into irreducible representations. This number is equal to
that of partitions of $N$. This is also the number of multiple graviton states with zero energy.
Each irreducible representation corresponds to a single graviton and hence several gravitons
exist in the supersymmetric vacua. Thus the structure of the classical supersymmetric vacua
is non-trivial and interesting [12].

Moreover, another fuzzy sphere solution $\alpha = \mu/6$ is allowed as a classical solution of the
equations of motion. This solution is non-supersymmetric and has finite energy,

$$E = \frac{\mu^4}{2592} \text{Tr} \left[(J')^2\right].$$

The above expression of the energy is described by the quadratic Casimir, and hence it depends
on the representations of the generator $J'$. The physical interpretation is unclear but it might
be possibly interpreted as a kind of the extended objects due to the Myers effect. Finally, we
should comment that this non-supersymmetric fuzzy sphere is unstable at the one-loop level,
as we will see later.

3.2 Oscillating Gravitons on the PP-wave

In the flat space it is understood how various M-theory objects, such as M2-brane (supermem-
brane), M5-brane and supergraviton can be constructed in the context of the matrix theory. In
the pp-wave case several additional terms exist in the action and equations of motion are mod-
ified. Thus, the dynamics is drastically changed even at the classical level. We can guess some
modifications for the dynamics of such M-theory objects [‡]. In this subsection we shall consider
an object in the BMN matrix model corresponding to the supergraviton on the flat background.
In the flat space the discrete light-cone quantized (DLCQ) M-theory should have a point-like
state corresponding to a longitudinal graviton with $p^+ = N/R$ and arbitrary transverse mo-
menta $p^r$’s $\{r = 1, 2, \ldots, 9\}$. This is the supergraviton. In the same way, we can consider an
object on the pp-wave corresponding to the supergraviton and it seems an interesting object
to study. We will construct such solutions explicitly below.

One simple family of solutions for these equations of motion (3.1) and (3.2) is the type
with the diagonal form. In this case the term with commutators and Myers term vanish, and

‡ In fact, we have discussed supermembranes in our previous works [13, 14].
§ In this subsection, we use the action (2.1).
equations of motion are reduced to those of the harmonic oscillator. Then this type of solution is given by

\[
X^I = \begin{pmatrix}
    x_1^I \cos \left( \frac{\mu}{3} t \right) + \frac{2}{\mu} v_1^I \sin \left( \frac{\mu}{3} t \right) \\
    \vdots \\
    x_N^I \cos \left( \frac{\mu}{3} t \right) + \frac{2}{\mu} v_N^I \sin \left( \frac{\mu}{3} t \right)
\end{pmatrix},
\tag{3.5}
\]

for \( I = 1, 2, 3, \) and

\[
X'^I = \begin{pmatrix}
    x_1'^I \cos \left( \frac{\mu}{6} t \right) + \frac{6}{\mu} v_1'^I \sin \left( \frac{\mu}{6} t \right) \\
    \vdots \\
    x_N'^I \cos \left( \frac{\mu}{6} t \right) + \frac{6}{\mu} v_N'^I \sin \left( \frac{\mu}{6} t \right)
\end{pmatrix},
\tag{3.6}
\]

for \( I' = 4, \ldots, 9. \) It can be easily shown that this harmonic oscillating solution turns into an \( N \)-supergraviton solution in the flat limit \( \mu \to 0. \) Thus this type of classical solution corresponds to the supergraviton in the flat space and in this sense we refer to this type of solution as the oscillating \( N \)-graviton. One can also show that it is non-supersymmetric and has finite energy, though a supergraviton in the flat space is a supersymmetric (1/2 BPS) object. It would be possibly guessed that a BPS saturated supergraviton might be lifted up to the non-BPS state due to the effect induced by the pp-wave background, such as bosonic mass terms.

Each graviton has respectively the longitudinal momentum \( p_a^+ \) and transverse one \( p_a^r \) \((a = 1, \ldots, N)\) defined by

\[
p_a^+ = \frac{1}{R}, \quad p_a^r = \frac{v_a}{R}, \quad (r = 1, 2, \ldots, 9).
\tag{3.7}
\]

Its associated energy is expressed as

\[
E_a = \frac{1}{2R} (v_a^r)^2 + \frac{1}{2R} \left( \frac{\mu}{3} \right)^2 \sum_{i=1}^{3} (x_a^i)^2 + \frac{1}{2R} \left( \frac{\mu}{6} \right)^2 \sum_{i'=4}^{9} (x_a^{i'})^2
\]

\[
= \frac{1}{2p_a^+} (p_a^r)^2 + \frac{1}{2p_a^+} \left( \frac{\mu}{3} \right)^2 \sum_{i=1}^{3} (x_a^i)^2 + \frac{1}{2p_a^+} \left( \frac{\mu}{6} \right)^2 \sum_{i'=4}^{9} (x_a^{i'})^2. \tag{3.8}
\]

The “mass” is \( p_a^+ \) and its “spring constant” is proportional to \( \mu^2 \) which arises due to the effect of the pp-wave background. In the \( \mu \to 0 \) limit, we can recover the result in the flat space where the supergraviton solution is a non-relativistic free particle with “mass” \( p_a^+ \).

Also, a single classical graviton with the longitudinal momentum \( p^+ = N/R \) can be described by taking

\[
x_1^r = \cdots = x_N^r, \quad v_1^r = \cdots = v_N^r,
\]

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so that the harmonic oscillations of all matrix components are identical.

Finally, we shall briefly comment on the quantum supergraviton. When the quantum supergraviton is clarified, we can know a number of states in the supergravity multiplet. In the case of the flat space, the supergraviton is certainly a member of the supergravity multiplet but has some difficulties such as the ground states problem. Besides such problems, members of the supergravity multiplet on the pp-wave have different masses in contrast with the flat case. This situation arises due to the fact that supercharges do not commute with the Hamiltonian and rather act as raising or lowering operators. It is the reason why the bosons and fermions in the BMN matrix model can have different masses but the energy shifts of the members in the multiplet are still constrained by the supersymmetry algebra. Thus, it might be not surprising that the supergraviton acquires mass at the quantum level. Of course, the massless quantum supergraviton should be recovered in the limit \( \mu \to 0 \).

4 Background Field Method

In this section we explain the background field method in the BMN matrix model in order to study whether the fuzzy sphere background is stable or not.

To begin, let us decompose the \( X^r \) and \( \Psi \) into the backgrounds \( B^r, F \) and fluctuations \( Y^r, \psi \) as

\[
X^r = B^r + Y^r, \quad \Psi = F + \psi, \quad (4.1)
\]

where we take \( F = 0 \) since the fermionic background is not considered in this paper. Next, we shall take the background field gauge using the gauge-fixing terms and Faddeev-Popov ghost terms as

\[
S_{GF+FP} = -\frac{1}{2} \int dt \text{Tr} \left( (D_{0}^{bg}A)^2 + i\bar{C}\partial_t D_0 C + \bar{C}[B^r, [X^r, C]] \right), \quad (4.2)
\]

where \( D_{0}^{bg}A \) is defined by

\[
D_{0}^{bg}A = \partial_t A + i[B^r, X^r]. \quad (4.3)
\]

The background field gauge condition is \( D_{0}^{bg}A = 0 \). The advantage of this gauge choice is that the second order actions with respect to the fluctuations are simplified.
Hereafter, we shall consider in the Euclidean formulation by taking $t \rightarrow i\tau$ and $A \rightarrow -iA$. When we insert the decompositions of the fields (4.1) into Eq. (2.1), the action $S$ is expanded around the background. The resulting action can be written as

$$S = S_0 + S_2 + S_3 + S_4,$$

where $S_i$ is the order $i$ part with respect to fields representing fluctuations. The symbol “dot” ($\cdot$) is redefined by tau ($\tau$) derivative as $\dot{A} \equiv \partial_\tau A$. The additional terms proportional to $\mu$ cannot appear in the quartic terms. The background field $B^r$ is not specified here yet. The classical solutions discussed in the previous section can be taken as the background $B^r$.

5 Quantum Stability of Giant Graviton

We shall investigate whether the supersymmetric fuzzy sphere (giant graviton) background and fuzzy sphere solution with no supersymmetry are stable or not by calculating the one-loop effective actions around these fuzzy spheres solutions.

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*In the case of the usual type IIB matrix model, the fuzzy sphere background has been discussed in [20].*


\section*{5.1 Stability of Giant Graviton}

In this subsection we consider the quantum stability of the giant graviton that is a supersymmetric (1/2 BPS) fuzzy sphere. In this case the background should be taken as

\[ B^I = \frac{\mu}{3} J^I , \quad B^\sigma = 0 . \quad (5.1) \]

We shall restrict ourselves to the \( N = 2 \) case (i.e., \( 2 \times 2 \) matrices) for simplicity, and expand the fluctuations and gauge fields as

\[
Y^\sigma = \frac{1}{\sqrt{2}} Y^\sigma_0 1_2 + \sqrt{2} Y^\sigma_1 J^1 + \sqrt{2} Y^\sigma_2 J^2 + \sqrt{2} Y^\sigma_3 J^3 ,
\]

\[
\psi = \frac{1}{\sqrt{2}} \psi_0 1_2 + \sqrt{2} \psi_1 J^1 + \sqrt{2} \psi_2 J^2 + \sqrt{2} \psi_3 J^3 ,
\]

\[
A = \frac{1}{\sqrt{2}} A_0 1_2 + \sqrt{2} A_1 J^1 + \sqrt{2} A_2 J^2 + \sqrt{2} A_3 J^3 ,
\]

where \( J^I \equiv \sigma^I / 2 , \ (I = 1, 2, 3) \) and \( \sigma^I \)’s are Pauli matrices. The orthonormal relations and quadratic Casimir of the \( SU(2) \) generators \( J^I \)’s are given by

\[
\text{Tr}[J^I J^J] = \frac{1}{2} \delta^{IJ} , \quad \text{Tr}[(J^I)^2] = \frac{3}{2} .
\]

These are useful to calculate the effective action.

Inserting these expansions into the second order action (4.3), we obtain the resulting action described by

\[
\mathcal{S}_2 = \int d\tau \left[ \frac{1}{2} (\dot{Y}^I_0)^2 + \frac{1}{2} \left( \frac{\mu^2}{9} \right) (Y^I_0)^2 + \frac{1}{2} \left( (\dot{Y}^I_1)^2 + (\dot{Y}^I_2)^2 + (\dot{Y}^I_3)^2 \right) \right.
\]

\[
- \frac{\mu^2}{9} (Y^1_1 Y^2_2 + Y^2_1 Y^3_3 + Y^3_1 Y^1_3 - Y^1_3 Y^2_1 - Y^2_3 Y^1_2 - Y^1_2 Y^3_1)
\]

\[
+ \frac{1}{2} \left( \frac{\mu^2}{3} \right) \left( (\dot{Y}^I_1)^2 + (\dot{Y}^I_2)^2 + (\dot{Y}^I_3)^2 \right) + \frac{1}{2} \left( \frac{\mu^2}{4} \right) \left( (Y^I_0)^2 \right)
\]

\[
+ \frac{1}{2} \left( \dot{A}_0 \right)^2 + \frac{1}{2} \left( \dot{A}_1 \right)^2 + \frac{1}{2} \quad \left( \frac{2}{9} \mu^2 \right) \left( (A_1)^2 + (A_2)^2 + (A_3)^2 \right)
\]

\[
+ \frac{1}{2} \left( \dot{C}_0 \dot{C}_0 + \dot{C}_1 \dot{C}_1 + \dot{C}_2 \dot{C}_2 + \dot{C}_3 \dot{C}_3 \right) + \frac{1}{2} \quad \left( \frac{2}{9} \mu^2 \right) \left( (C_1 C)_1 + (C_2 C)_2 + (C_3 C)_3 \right)
\]

\[
+ i \left( \psi_0 \dot{\psi}_0 + \psi_1 \dot{\psi}_1 + \psi_2 \dot{\psi}_2 + \psi_3 \dot{\psi}_3 \right)
\]

\[
- \frac{\mu}{3} \left( \psi_1^\alpha \gamma_2 \psi_3^\beta - \psi_1^\beta \gamma_3 \psi_2^\alpha + \psi_2^\alpha \gamma_3 \psi_1^\beta - \psi_2^\beta \gamma_1 \psi_3^\alpha + \psi_3^\alpha \gamma_1 \psi_2^\beta - \psi_3^\beta \gamma_2 \psi_1^\alpha \right) \right] . \quad (5.2)
\]
The diagonal parts of the action (5.2) can be integrated out, but we must diagonalize the non-diagonal nine components $Y_i$'s, $(i, I = 1, 2, 3)$ and fermions $\psi_1, \psi_2$ and $\psi_3$ in order to integrate out them. The diagonalization of the $9 \times 9$ matrix can be easily done (see, Appendix A for the detailed calculations). The resulting action for these 9 components are evaluated as

$$S_Z = \int d\tau \left[ \frac{1}{2} \left( \dot{Z}^1 \right)^2 + \frac{1}{2} \left( \frac{\mu^2}{9} \right) (Z^1)^2 
+ \sum_{i=2}^{4} \left[ \frac{1}{2} \left( \dot{Z}^i \right)^2 + \frac{1}{2} \left( \frac{2}{9} \mu^2 \right) (Z^i)^2 \right] + \sum_{i=5}^{9} \left[ \frac{1}{2} \left( \dot{Z}^i \right)^2 + \frac{1}{2} \left( \frac{4}{9} \mu^2 \right) (Z^i)^2 \right] \right]. \tag{5.3}$$

Next, let us diagonalize the fermion parts. This can be also done after simple calculations (see, Appendix B for the technical details), and the result is expressed

$$S_\varphi = \int d\tau \left[ i\varphi_0^T \dot{\varphi}_0 + i\varphi_1^T \dot{\varphi}_1 + i\varphi_2^T \dot{\varphi}_2 + i\varphi_3^T \dot{\varphi}_3 
+ \frac{\mu}{4} \varphi_0^T \gamma_{123} \varphi_0 + \frac{7}{12} \mu \varphi_1^T \gamma_{123} \varphi_1 + \frac{7}{12} \mu \varphi_2^T \gamma_{123} \varphi_2 - \frac{5}{12} \mu \varphi_3^T \gamma_{123} \varphi_3 \right], \tag{5.4}$$

where $\varphi_i$'s are real spinors with 16 components.

Finally, we can obtain the following contents of fields\footnote{We consider the case that $\mu$ is positive since we can impose this condition without the loss of generality.}

**Bosonic Fluctuation $Y^r$**

- 4 massive bosons with mass $\frac{\mu}{3}$: $Y_0^i$ and $Z^1$,
- 3 massive bosons with mass $\frac{\sqrt{2}}{3} \mu$: $Z^i$ $(i = 2, 3, 4)$,
- 5 massive bosons with mass $\frac{2}{3} \mu$: $Z^i$ $(i = 5, \ldots, 9)$,
- 6 massive bosons with mass $\frac{\mu}{6}$: $Y_0^r$,
- 18 massive bosons with mass $\frac{\mu}{2}$: $Y_i^{r'}$ $(i = 1, 2, 3)$,

**Gauge Field $A$**

- 1 massless boson: $A_0$,
- 3 massive bosons with mass $\frac{\sqrt{2}}{3} \mu$: $A_i$ $(i = 1, 2, 3)$,
Ghost $C$

- 1 massless complex ghost: $C_0$,
- 3 massive complex ghosts with mass $\sqrt{2}/3\mu$: $C_i$ ($i = 1, 2, 3$),

Fermion $\varphi$

- 16 massive fermions with mass $\frac{\mu}{4}$: $\varphi_0$,
- 32 massive fermions with mass $\frac{7}{12}\mu$: $\varphi_1$ and $\varphi_2$,
- 16 massive fermions with mass $-\frac{5}{12}\mu$: $\varphi_3$.

The one-loop effective action $W$ is given by the path integral of the second order action $S_2$ with respect to fluctuations around the background as

$$W = -\ln \int [dY][dA][d\bar{C}][dC][d\psi] \exp (-S_2).$$

The action $S_2$ has been already diagonalized and so we can easily integrate out the massive degrees of freedom. The contribution of the fluctuations $Y$’s is given by

$$W_Y = -\ln \left[ \det \left( -\partial^2 + \frac{\mu^2}{9} \right)^{-3/2} \det \left( -\partial^2 + \frac{\mu^2}{9} \right)^{-1/2} \det \left( -\partial^2 + \frac{2}{9}\mu^2 \right)^{-3/2} \right] \times \left[ \det \left( -\partial^2 + \frac{4}{9}\mu^2 \right)^{-5/2} \det \left( -\partial^2 + \frac{\mu^2}{36} \right)^{-6/2} \det \left( -\partial^2 + \frac{\mu^2}{4} \right)^{-18/2} \right]. \quad (5.5)$$

The contributions of the gauge field $A$ and the ghost $C$ ($\bar{C}$) are respectively given by

$$W_A = -\ln \det \left( -\partial^2 + \frac{2}{9}\mu^2 \right)^{3/2}, \quad (5.6)$$

$$W_{gh} = -\ln \det \left( -\partial^2 + \frac{2}{9}\mu^2 \right)^3. \quad (5.7)$$

Finally, we shall consider fermion parts. When we note that the $\varphi_i$’s are real, the contributions of fermions are written by using the Pfaffian as follows:

$$W_F = -\ln \left[ \text{Pf} \left[ i\partial \gamma_{i123} \right] \text{Pf} \left[ i\partial + \frac{7}{12}\mu \gamma_{i123} \right]^2 \text{Pf} \left[ i\partial - \frac{5}{12}\mu \gamma_{i123} \right] \right], \quad (5.8)$$

where $[\text{Pf}(B)]^2 = \det(B)$ for an arbitrary matrix $B$. Here, we use the following identity,

$$\ln \det(\cdots) = \text{Tr} \ln \det(\cdots),$$

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where $\text{Det}$, $\text{Ln}$ and $\text{Tr}$ are the functional determinant, logarithm and trace, respectively. By the use of this identity, we can rewrite the expression $\ln \text{Det}[-\partial^2_t + M^2]$ as

$$\ln \text{Det}[-\partial^2_t + M^2] = L \int \frac{dk}{2\pi} \ln (k^2 + M^2) = L(|M| + E_\infty), \quad (5.9)$$

where $L$ is the length of the temporal direction and $E_\infty$ is a divergent constant defined by

$$E_\infty \equiv \int \frac{dk}{2\pi} \ln k^2.$$

Thus, we can easily obtain the one-loop effective potential $V$ by dividing the one-loop effective action $W$ by the length $L$ as $V = W/L$. The contribution of the fluctuations $Y$'s to the effective potential is calculated as

$$V_Y = \left(\frac{22}{3} + \frac{\sqrt{2}}{2}\right) \mu + 18 E_\infty. \quad (5.10)$$

The contribution of $A$'s is evaluated as

$$V_A = \frac{\sqrt{2}}{2} \mu + 3 E_\infty, \quad (5.11)$$

and that of ghosts is expressed as

$$V_{gh} = -\sqrt{2} \mu - 3 E_\infty. \quad (5.12)$$

In the same way, the fermion part can be rewritten as

$$- \ln \text{Pf} \left[i\partial_t + M \gamma_{123}\right] = -\frac{1}{2} \ln \text{Det} \left[i\partial_t + M \gamma_{123}\right]
= -\frac{1}{2} L \int \frac{dk}{2\pi} \ln \det \left(k + M \gamma_{123}\right)
= -\frac{1}{4} L \int \frac{dk}{2\pi} \ln \det \left[k + M \gamma_{123}\right] + \ln \det \left[k - M \gamma_{123}\right]
= -\frac{1}{4} L \int \frac{dk}{2\pi} \ln \det \left[(k^2 + M^2) \mathbb{1}_{16}\right]
= -4L \int \frac{dk}{2\pi} \ln \left[k^2 + M^2\right] = -4L(|M| + E_\infty).$$

Thus, the one-loop contribution from the fermions is obtained as

$$V_F = -\frac{22}{3} \mu - 16 E_\infty. \quad (5.13)$$

Therefore, the net one-loop contribution is exactly canceled as expected from the supersymmetry when we take the contributions of massless bosons and ghosts into account.
Finally, we should remark that the cancellation of the one-loop contributions is non-trivial. The quantum supersymmetric vacua is non-trivial in the light-cone formulation since the super-charges do not commute with the Hamiltonian. In particular, the zero-point energy depends on the definition of the vacua. The one-loop contribution in the BMN matrix model is essentially the zero-point energy induced by the pp-wave background. From such a reason the cancellation of one-loop contributions would be also non-trivial.

5.2 Instability of Fuzzy Sphere

Here, let us discuss the case of a non-supersymmetric fuzzy sphere with finite energy. In this case the background is described by

\[ B^I = \frac{\mu}{6} I^I, \quad B'^I = 0. \tag{5.14} \]

Then, the second order action \( S_2 \) is written as

\[
S_2 = \int d\tau \left[ \frac{1}{2} (\dot{Y}_0)^2 + \frac{1}{2} \left( \frac{\mu^2}{9} \right) (Y_0')^2 + \frac{1}{2} \left( (\dot{Y}_1')^2 + (\dot{Y}_2')^2 + (\dot{Y}_3')^2 \right) \right.
\]

\[
- \frac{\mu^2}{9} (Y_1^2 Y_2^2 + Y_2^3 Y_3^1 + Y_3^2 Y_1^1 - Y_1^2 Y_2^1 - Y_3^2 Y_2^3 - Y_1^3 Y_3^1) + \frac{1}{2} \left( \frac{\mu^2}{6} \right) ((Y_1')^2 + (Y_2')^2 + (Y_3')^2) + \frac{1}{2} (\dot{Y}_0')^2 + \frac{1}{2} \left( \frac{\mu^2}{36} \right) (Y_0')^2
\]

\[
+ \frac{1}{2} \left( (\dot{Y}_1')^2 + (\dot{Y}_2')^2 + (\dot{Y}_3')^2 \right) + \frac{1}{2} \left( \frac{\mu^2}{12} \right) ((Y_1')^2 + (Y_2')^2 + (Y_3')^2)
\]

\[
+ \frac{1}{2} \left( (\dot{A}_0)^2 + (\dot{A}_1)^2 + (\dot{A}_2)^2 + (\dot{A}_3)^2 \right) + \frac{1}{2} \left( \frac{\mu^2}{18} \right) ((A_1)^2 + (A_2)^2 + (A_3)^2)
\]

\[
+ \frac{1}{2} \left( \dot{C}_0 \dot{C}_0 + \dot{C}_1 \dot{C}_1 + \dot{C}_2 \dot{C}_2 + \dot{C}_3 \dot{C}_3 \right) + \frac{1}{2} \left( \frac{\mu^2}{18} \right) (\dot{C}_1 C_1 + \dot{C}_2 C_2 + \dot{C}_3 C_3)
\]

\[
i \left( \psi_0 \dot{\psi}_0 + \psi_1 \dot{\psi}_1 + \psi_2 \dot{\psi}_2 + \psi_3 \dot{\psi}_3 \right) + \frac{\mu}{4} (\psi_0^T \gamma_{123} \psi_0 + \psi_1^T \gamma_{123} \psi_1 + \psi_2^T \gamma_{123} \psi_2 + \psi_3^T \gamma_{123} \psi_3)
\]

\[
- \frac{\mu}{6} (\dot{\psi}_1 \gamma_3 \psi_3 - \dot{\psi}_1 \gamma_3 \psi_3 + \dot{\psi}_1 \gamma_3 \psi_3 - \dot{\psi}_2 \gamma_1 \psi_3 + \dot{\psi}_3 \gamma_1 \psi_3 - \dot{\psi}_3 \gamma_2 \psi_3) \right]. \tag{5.15}
\]

In the same way as the case of the giant graviton, we must diagonalize non-diagonal nine components \( Y_i'^{I's} \), (i, I = 1, 2, 3) and fermions \( \psi_1, \psi_2 \) and \( \psi_3 \) in order to integrate out them.
After the naive diagonalization of the $9 \times 9$ matrix, the resulting action is obtained as

$$S_Z = \int d\tau \left[ \frac{1}{2} (\dot{Z}^1)^2 + \frac{1}{2} \left( -\frac{\mu^2}{18} \right) (Z^1)^2 \right. + \sum_{i=2}^{4} \left[ \frac{1}{2} (\dot{Z}^i)^2 + \frac{1}{2} \left( \frac{\mu^2}{18} \right) (Z^i)^2 \right] + \sum_{i=5}^{9} \left[ \frac{1}{2} (\dot{Z}^i)^2 + \frac{1}{2} \left( \frac{5\mu^2}{18} \right) (Z^i)^2 \right]. \quad (5.16)$$

Then one tachyonic boson appears. This boson implies an unbounded direction in the quantum bosonic potential and this background is unstable. Also the non-diagonal fermionic part can be rewritten as

$$S_\varphi = \int d\tau \left[ i\varphi_0^r \dot{\varphi}_0 + i\varphi_1^r \dot{\varphi}_1 + i\varphi_2^r \dot{\varphi}_2 + i\varphi_3^r \dot{\varphi}_3 + \frac{\mu}{4} \varphi_0^r \gamma_{123} \varphi_0 + \frac{5}{12} \mu \varphi_1^r \gamma_{123} \varphi_1 + \frac{5}{12} \mu \varphi_2^r \gamma_{123} \varphi_2 - \frac{\mu}{12} \varphi_3^r \gamma_{123} \varphi_3 \right]. \quad (5.17)$$

Finally, we can obtain the following contents of fields:

**Bosonic Fluctuation $Y'$**

- 3 massive bosons with mass $\frac{\mu}{3}$: $Y^{i'}_0$,
- 1 tachyonic bosons with imaginary mass, whose absolute value is $\frac{\sqrt{2}}{6} \mu$: $Z^1$,
- 3 massive bosons with mass $\frac{\sqrt{2}}{6} \mu$: $Z^i$ (i = 2, 3, 4),
- 5 massive bosons with mass $\frac{\sqrt{10}}{6} \mu$: $Z^i$ (i = 5, ..., 9),
- 6 massive bosons with mass $\frac{\mu}{6}$: $Y^{i'}_0$,
- 18 massive bosons with mass $\frac{\sqrt{3}}{6} \mu$: $Y^{i'}_i$ (i = 1, 2, 3),

**Gauge Field $A$**

- 1 massless boson: $A_0$,
- 3 massive bosons with mass $\frac{\sqrt{2}}{6} \mu$: $A_i$ (i = 1, 2, 3),

**Ghost $C$**

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• 1 massless complex ghost: $C_0$,

• 3 massive complex ghosts with mass $\frac{\sqrt{2}}{6} \mu$: $C_i$ ($i = 1, 2, 3$),

**Fermion $\varphi$**

• 16 massive fermions with mass $\frac{\mu}{4}$: $\varphi_0$,

• 32 massive fermions with mass $\frac{5}{12} \mu$: $\varphi_1$ and $\varphi_2$,

• 16 massive fermions with mass $-\frac{\mu}{12}$: $\varphi_3$.

Thus, we can integrate out these fields, and one-loop contributions to the effective potential from $Y$’s, $A$’s, $C$’s and $\varphi$’s are respectively given by

\[
V_Y = \left[1 + \frac{\sqrt{2}}{4} + \frac{3}{2} \sqrt{3} + \frac{5}{12} \sqrt{10} + i \frac{\sqrt{2}}{12}\right] \mu + 18 E_\infty, \tag{5.18}
\]

\[
V_A = \frac{\sqrt{2}}{4} + \frac{3}{2} E_\infty, \tag{5.19}
\]

\[
V_{\text{gh}} = -\frac{\sqrt{2}}{2} - 3 E_\infty, \tag{5.20}
\]

\[
V_F = -\frac{14}{3} - 16 E_\infty, \tag{5.21}
\]

where we have used the formula

\[
\int \frac{dk}{2\pi} \ln(k^2 - M^2) = i |M| + E_\infty.
\]

The divergent constant $E_\infty$ is certainly canceled by taking the massless degrees of freedom into account. Thus, the net contribution is complex and is given by

\[
V_{\text{net}} = \left[ -\frac{11}{3} + \frac{3}{2} \sqrt{3} + \frac{5}{12} \sqrt{10} + i \frac{\sqrt{2}}{12} \right] \mu. \tag{5.22}
\]

The imaginary part in the effective action implies the instability of the non-supersymmetric fuzzy sphere background coming from the fact that the potential of bosons has an unbounded direction. In conclusion a non-supersymmetric fuzzy sphere background is unstable at the one-loop level.
6 Conclusions and Discussions

In this paper we have discussed classical solutions in the BMN matrix model. The classical solutions concretely considered here are a supersymmetric fuzzy sphere (giant graviton), a non-supersymmetric fuzzy sphere and oscillating gravitons. In particular, we have calculated the one-loop effective action around the supersymmetric fuzzy sphere (giant graviton) background and explicitly proven the quantum stability of the giant graviton at the one-loop level. The one-loop contributions are distinctly canceled as expected from the requirement of supersymmetries. However, its cancellation is non-trivial since the quantum supersymmetric vacua is non-trivial in the light-cone formulation. It is based on the fact that the supercharges do not commute with the Hamiltonian. In particular, the zero-point energy depends on the definition of the vacua and the one-loop contribution in the BMN matrix model is essentially the zero-point energy induced by the pp-wave background. Thus the cancellation of one-loop contributions would be also non-trivial. In addition, we have discussed the instability of the non-supersymmetric fuzzy sphere background. In this case the one-loop contribution deserves not to be canceled but also a tachyonic boson appears. This result implies that the quantum bosonic potential has an unbounded direction, and so this non-supersymmetric background is unstable at the one-loop level. In this paper, we have considered the $N = 2$ case only. The extension to an arbitrary $N$ is an interesting future work.

Moreover, we have discussed oscillating graviton solutions. Supergravitons in the flat space corresponds to oscillating modes on the pp-wave due to the presence of mass terms. Certainly, oscillating gravitons turn into the usual supergravitons in the flat limit $\mu \to 0$. In particular, oscillating gravitons are non-supersymmetric and non-BPS states, in contrast with the supergravitons in the flat space. Further considerations about these oscillating gravitons, such as scattering amplitudes are most interesting problems.

Other classical solutions have been already known [18]. In particular, the quantum stability of rotating 1/4 BPS solutions is so interesting but it is difficult to treat such a problem in the background field method because of their time dependences. It is also an interesting open problem.

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Appendix

A Determinant of Bosons

We have to calculate determinants of 9 non-diagonal bosons \(Y_1 Y_1, Y_2 Y_2, Y_3 Y_3, Y_1 Y_2, Y_2 Y_3, Y_3 Y_1, Y_1 Y_3, Y_2 Y_1, Y_3 Y_2\). The expression of associated parts for a supersymmetric fuzzy sphere background is given by

\[
S_2 = \int d\tau \left[ \frac{1}{2}(\dot{Y}_1')^2 + \frac{1}{2}(\dot{Y}_2')^2 + \frac{1}{2}(\dot{Y}_3')^2 + \frac{1}{2} \left( \frac{\mu^2}{3} \right) \left( (Y_1')^2 + (Y_2')^2 + (Y_3')^2 \right) \right.
\]

\[
-2 \cdot \frac{\mu^2}{18} \left( Y_1^2 Y_2^2 + Y_2^2 Y_3^2 + Y_3^2 Y_1^2 - Y_1^2 Y_2^1 - Y_2^2 Y_3^3 - Y_3^2 Y_1^1 \right),
\]

and that for a non-supersymmetric background is expressed as

\[
S_2 = \int d\tau \left[ \frac{1}{2}(\dot{Y}_1')^2 + \frac{1}{2}(\dot{Y}_2')^2 + \frac{1}{2}(\dot{Y}_3')^2 + \frac{1}{2} \left( \frac{\mu^2}{6} \right) \left( (Y_1')^2 + (Y_2')^2 + (Y_3')^2 \right) \right.
\]

\[
-\frac{\mu^2}{9} \left( Y_1^1 Y_2^2 + Y_2^2 Y_3^3 + Y_3^3 Y_1^1 - Y_1^2 Y_2^1 - Y_2^2 Y_3^2 - Y_3^2 Y_1^3 \right)
\]

We shall set \(a = -\frac{1}{2} \partial^2 + \frac{\mu^2}{6}\) (for the supersymmetric fuzzy sphere) or \(a = -\frac{1}{2} \partial^2 + \frac{\mu^2}{12}\) (for the non-supersymmetric fuzzy sphere), and \(b = \frac{\mu^2}{18}\) (for both cases). Then we can easily calculate the associated determinant by the use of the Mathematica, and the result is obtained as

\[
\text{Det} \left[ (a - 2b) \cdot (a - b)^3 \cdot (a + b)^5 \right].
\]

B Determinant of Fermions

We shall express the fermionic parts in terms of a matrix representation as

\[
\begin{pmatrix}
\psi_0^T, \psi_1^T, \psi_2^T, \psi_3^T
\end{pmatrix}
\begin{pmatrix}
a & 0 & 0 & 0 \\
0 & a & b\gamma_3 & -b\gamma_2 \\
0 & -b\gamma_3 & a & b\gamma_1 \\
0 & b\gamma_2 & -b\gamma_1 & a
\end{pmatrix}
\begin{pmatrix}
\psi_0 \\
\psi_1 \\
\psi_2 \\
\psi_3
\end{pmatrix},
\]
where $a = i\partial_r + \frac{\mu}{4}\gamma_{123}$, and $b = \frac{\mu}{3}$ for a supersymmetric fuzzy sphere and $b = \frac{\mu}{6}$ for a non-supersymmetric fuzzy sphere.

Let us decompose the above matrix into block parts

$$M \equiv \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & a & b\gamma_3 & -b\gamma_2 \\ 0 & -b\gamma_3 & a & b\gamma_1 \\ 0 & b\gamma_2 & -b\gamma_1 & a \end{pmatrix} \equiv \begin{pmatrix} A & C \\ D & B \end{pmatrix}.$$ 

By the use of the standard formula

$$\text{Det} M = \text{Det} A \cdot \text{Det} (B - DA^{-1}C),$$

(B.1)

$\text{Det} M$ is rewritten as

$$\text{Det} M = \det \left( \begin{array}{c|c} a^2 + b^2 & b\gamma_1 + b^2\gamma_2\gamma_3 \\ \hline \end{array} \right).$$

By the use of the formula (B.1) once again, we obtain the final result as follows:

$$\text{Det} M = \text{Det} \left[ (a^2 + b^2)^2 + (b\gamma_1 + b^2\gamma_2\gamma_3)^2 \right]$$
$$= \text{Det} \left[ a \cdot (a + b\gamma_{123})^2 \cdot (a - 2b\gamma_{123}) \right].$$
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