A repeat-until-success quantum computing scheme

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Abstract. Recently we proposed a hybrid architecture for quantum computing based on stationary and flying qubits: the repeat-until-success (RUS) quantum computing scheme. The scheme is largely implementation independent. Despite the incompleteness theorem for optical Bell-state measurements in any linear optics set-up, it allows for the implementation of a deterministic entangling gate between distant qubits. Here we review this distributed quantum computation scheme, which is ideally suited for integrated quantum computation and communication purposes.

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1. Introduction

One of the fascinating aspects of quantum computers is the potential ability to process efficiently certain computational tasks that are deemed intractable using classical computer technology [1]. Existing experiments in quantum information theory have focused on the implementation of sequential interaction of particles (quantum bits or qubits) based on a model of a quantum computer as a network of quantum logic gates [2, 3]. Although the basic features of a quantum computer have been demonstrated to some extent, certain currently insurmountable challenges exist: scalability and decoherence, for instance.

In a network of logic gates, a quantum computer proceeds reversibly. A different model of a scalable quantum computer has recently been proposed [4] in which the entire resource for the quantum computation rests initially in the form of a specific entangled state (a so-called cluster state) of a large number of qubits. Information is then written onto the cluster, processed, and read out from the cluster by single-qubit measurements only. Thus, unlike the usual paradigm of assembling logic devices for computation, this scheme is essentially irreversible. The entangled state of the cluster serves as a universal substrate for any quantum computation. One way to create cluster states is to employ systems with a quantum Ising-like interaction.

Photons are often regarded as a favourite qubit due to their long decoherence times and speed as well as their ability to distribute themselves in an optical network. However, photons cannot interact directly with each other. Without non-linear effects [5, 6], photons can only become entangled via post-selected measurements and local operations [7]–[10]. Moreover, linear optics alone does not permit complete Bell measurements [11]. Thus, such entangling operations between photons are necessarily probabilistic. Obtaining success probabilities close to unity requires therefore the presence of highly entangled ancilla states and quantum teleportation [7] as a universal quantum primitive [12]. First experiments demonstrating the feasibility of proposed schemes have already been performed [13]–[15].

Photons can be transmitted easily from point to point. Hence they are often regarded as ‘flying’ qubits. However, there is a trade-off to this advantage of easy distribution in quantum computing: it is generally difficult to store them and to use them as quantum memory. ‘Stationary’ qubits on the other hand, i.e. qubits realized through atoms or ions, provide good quantum memory due to the relatively long decoherence times of their internal ground states. For stationary qubits, it is relatively easy to implement single qubit rotations and to read out information with a very high precision. Experiments done in Innsbruck and Boulder have already demonstrated the feasibility of two-qubit gates for ion trap quantum computing [16, 17]. However, ion trap quantum computing with more than 10 qubits remains challenging due to the relatively high vulnerability of two-qubit gate operations to decoherence.

It is therefore natural to consider a hybrid platform based on both flying and stationary qubits. Numerous such schemes have already been explored [18]–[28], exploiting the benefits of stationary atomic and flying photonic qubits. Here, we discuss as an example of such a robust and scalable hybrid quantum computing scheme, namely our recently proposed repeat-until-success (RUS) quantum computing scheme [27, 28]. Under real conditions, when photon loss is a possibility the scheme can be used for the efficient build up of cluster states for one-way quantum computing, as we describe in section 3. Finally, we summarise our results in section 4.
2. Repeat-until-success scheme

The repeat-until-success (RUS) quantum computing scheme is based essentially on an atom–photon interaction. There are inherently three possible schemes for entangling the single atoms in distant cavities using a generic atom–photon interaction (see figure 1). The first scheme involves sending a photon or photons from the first atom to the second one through an optical fibre or free space, thereby giving rise to an effective interaction between the atoms [22, 24] with or without the need of measurement. The second scheme [19, 23], [26]–[28] relies on photon emissions from each atom, whereby the state of each newly generated photon depends on the state of its respective source. In this case, an entangling measurement is performed on the photons, which subsequently creates entanglement between the atoms. Our RUS scheme falls into this class. In a third scheme, entangled photons from the common source are sent to each atom simultaneously [21, 25]. If the photons are entangled, then this entanglement can be transferred onto the atoms via the cavity modes.

For the RUS quantum computing scheme, we need to focus a bit more on the second scheme. The main idea behind the RUS scheme is shown in figure 2. Consider two isolated single atom–cavity systems A and B interacting with two photons P and Q. System A gets entangled to system P (or B to Q) through an effective Hamiltonian so that

\[
(\alpha|0\rangle + \beta|1\rangle)|\xi\rangle \rightarrow \alpha|0\rangle|h\rangle + \beta|1\rangle|v\rangle,
\]

(1)

where |0\rangle and |1\rangle are the states of the atom systems, |h\rangle and |v\rangle describe a horizontally and a vertically polarised photon, and |\xi\rangle is the vacuum state of the photon systems. At this stage the systems A–P and B–Q remain separable. But if an appropriate measurement is made on the two photons P and Q, the two atoms A and B can become entangled. Indeed, the state of the whole

\[Figure 1.\] Three different schemes to entangle atoms in different cavities.
system (A–P and B–Q) at this stage is

\[
|\psi_{\text{enc}}\rangle = (\alpha|0\rangle_A|h\rangle_P + \beta|1\rangle_A|v\rangle_P)(\alpha'|0\rangle_B|h\rangle_Q + \beta'|1\rangle_B|v\rangle_Q)
= \alpha\alpha'|00\rangle|hh\rangle + \alpha\beta'|01\rangle|hv\rangle + \beta\alpha'|10\rangle|vh\rangle + \beta\beta'|11\rangle|vv\rangle, \tag{2}
\]

if the initial state of the two atomic qubits equalled

\[
|\psi_{\text{in}}\rangle = (\alpha|0\rangle + \beta|1\rangle)(\alpha'|0\rangle + \beta'|1\rangle). \tag{3}
\]

The subscript refers to the atom systems A and B, and the photon systems P and Q. If we now project the states of the photon on one of the four Bell states, namely \(|hh\rangle \pm |vv\rangle)/\sqrt{2}\) and \(|hv\rangle \pm |vh\rangle)/\sqrt{2}\), we easily see that the atom states (A and B) become entangled. If the system P and Q are atom systems, then a complete ‘Bell states’ measurement is possible [29]. However, if P and Q are photons, a complete Bell state measurement is not possible, using using only passive linear optics elements [11]. Fortunately, we can still perform a partial Bell state measurement. The key point for the RUS computing concept is that partial Bell state measurements can be found such that the intended qubit gate operation is accomplished, if a Bell state is measured. However, if a product state is measured, the original qubit state is nevertheless unmodified, up to known local unitary operations. This is the concept of ‘failure with insurance’.

To see how a partial Bell state measurement is done, we consider again two stationary qubits, like the atom-cavity systems shown in figure 1, which are initially prepared in the product state (3). Then two photons are created simultaneously, as described by the operation (1), which entangles the state of each atom with the state of a newly generated photon. For simplicity, we now write the state of the combined atom–cavity–photon systems in equation (2) as

\[
|\psi_{\text{enc}}\rangle = \alpha\alpha'|00\rangle(|hh\rangle + |hv\rangle + |vh\rangle + |vv\rangle) + \frac{1}{2}\alpha\beta'|01\rangle(|hh\rangle - |hv\rangle + |vh\rangle - |vv\rangle) + \frac{1}{2}\beta\alpha'|10\rangle
\times (|hh\rangle + |hv\rangle - |vh\rangle - |vv\rangle) + \frac{1}{2}\beta\beta'|11\rangle(|hh\rangle - |hv\rangle - |vh\rangle + |vv\rangle). \tag{4}
\]

We now allow each photon to go through a Hadamard beam splitter, which transforms \(|h\rangle\) into \((|h\rangle + |v\rangle)/\sqrt{2}\) and \(|v\rangle\) into \((|h\rangle - |v\rangle)/\sqrt{2}\). This results in the transformation of the state (4) into

\[
|\psi'_{\text{enc}}\rangle = \frac{1}{2}\alpha\alpha'|00\rangle(|hh\rangle + |hv\rangle + |vh\rangle + |vv\rangle) + \frac{1}{2}\alpha\beta'|01\rangle(|hh\rangle - |hv\rangle + |vh\rangle - |vv\rangle) + \frac{1}{2}\beta\alpha'|10\rangle
\times (|hh\rangle + |hv\rangle - |vh\rangle - |vv\rangle) + \frac{1}{2}\beta\beta'|11\rangle(|hh\rangle - |hv\rangle - |vh\rangle + |vv\rangle). \tag{5}
\]
Figure 3. Realisation of the partial Bell measurement for RUS quantum computing.

The photons are then directed into a polarizing beam splitter (PBS) which transmits photons with state \( |h \rangle \) but reflects photons with state \( |v \rangle \). Thus photons with the state \( |hh \rangle \) and \( |vv \rangle \) maintain separate paths after the PBS (anti-bunching) while photons with state \( |hv \rangle \) and \( |vh \rangle \) leave the setup together (bunching). The schematic setup for the process and the following detection of the photons is shown in figure 3.

It is instructive to see that the state of the atom–cavity systems after the passing of the photons through the PBS can be written as

\[
|\psi''\rangle_{\text{enc}} = \frac{1}{2} (\alpha \alpha'|00\rangle + \beta \beta'|11\rangle)(|hh\rangle + |vv\rangle) + \frac{1}{2} (\alpha' \beta'|01\rangle + \beta \alpha'|10\rangle)(|hh\rangle - |vv\rangle)
+ \frac{1}{2} (\alpha|0\rangle + \beta|1\rangle)(\alpha'|0\rangle - \beta'|1\rangle)|hv\rangle + \frac{1}{2} (\alpha|0\rangle - \beta|1\rangle)(\alpha'|0\rangle + \beta'|1\rangle)|vh\rangle.
\]  

(6)

For simplicity we have neglected the indices, which indicate the respective output ports of the photons. As mentioned above, if both photons have the same polarisation, they leave the setup through different output ports. If both photons are of different polarisation, they leave the setup through the same output port.

Detecting the photons in a rotated basis, which does not distinguish the polarisations \( |h \rangle \) and \( |v \rangle \) it is thus possible to perform a measurement which distinguishes between two Bell and two product states. For example, measuring the states \( (|h\rangle \pm |v\rangle)/\sqrt{2} \) in each output port of the PBS, corresponds to a measurement of the two-photon states

\[
|\Phi_1\rangle \equiv \frac{1}{\sqrt{2}}(|hh\rangle + |vv\rangle), \quad |\Phi_2\rangle \equiv \frac{1}{\sqrt{2}}(|hh\rangle - |vv\rangle), \quad |\Phi_3\rangle \equiv |hv\rangle, \quad |\Phi_4\rangle \equiv |vh\rangle.
\]  

(7)
Figure 4. A unitary operation on a single qubit. The states at the various nodes in the quantum network shown are state 1 = $X^\beta HZ_\eta X^\alpha HZ |\psi\rangle$, state 2 = $X^\gamma HZ_\theta$ state 1 and state 3 = $X^\delta HZ_\phi$ state 2 = $(-1)^{x+y+z} X^\beta+\delta Z^{x+y} X_\phi Z_\eta X_\gamma Z |\psi\rangle$. Since the results of the measurements are known, by Euler decomposition, one can perform any arbitrary unitary operation on an unknown state $|\psi\rangle$.

on the atom–photon state in equation (5). Such a measurement is called an incomplete Bell state measurement. Thus, if the outcome is $|\Phi_1\rangle$ or $|\Phi_2\rangle$, the stationary qubits become maximally entangled. If the outcome of the measurement is $|\Phi_3\rangle$ or $|\Phi_4\rangle$, the atoms can be reset to their original state by appropriate known local operations. The latter property is crucial for the realisation of repeat-until-success quantum gate operations as arbitrary measurements may not necessarily recover the original state of the atoms. Since each possible measurement outcome occurs with probability $1/4$ and the probability for a successful Bell state detection is $1/2$, we need only to perform an average of two measurements to complete a gate. The above measurement basis is chosen as an example to illustrate the principles of the scheme and is not restrictive. See [28] for general guidelines for the measurement basis selection and the realisation of maximally-entangling deterministic two-qubit gate operations for arbitrary initial states.

3. One-way quantum computer

In classical computation, it is possible to assemble a network of logic gates to perform various tasks. Examples of logic gates are the AND, OR, NOT, NAND and NOR gates. It has been shown that all of the above logic gates can be implemented by various configurations of one or more NAND (or NOR) gates. The negation is essential as two NOR gates can give an OR (positive variant) gate but not two OR gates in concatenation. Thus, we say that a NAND gate is a universal gate.

Just as a NAND gate constitutes a universal gate for classical computation, the set of all one-qubit gates and any two-qubit entangling gate forms the set of universal gates for quantum computation. However, different from classical computations, these gates do not have to be performed successively. Instead of performing entangling quantum gate operations, a quantum computer can be prepared in a highly entangled state, a so-called cluster state. Once a cluster state has been generated, single-qubit rotations and single-qubit measurements are sufficient to simulate the performance of universal gate operations and to realise any possible quantum algorithm [4]. The simplest version is the linear chain for an arbitrary unitary operation on a qubit shown in figure 4.
One of the main applications of the RUS scheme is to create a cluster state. In our scheme, cluster states can be generated by attempting to bond qubits using the above described entangling operation. Under realistic conditions, where photon loss is a possibility, this operation has three possible outcomes: ‘success’, ‘repeatable’ or ‘insurance’, and ‘failure’. As we have seen earlier, observation of photon antibunching (or bunching) corresponds to a ‘success’ (or ‘insurance’). Observing fewer than two photons denotes a failure. In this case, the static qubits are left in an unknown state. However, this damage can be repaired as follows. Firstly, each of the two qubits involved in the failed gate can be measured in the computational basis to determine the nature of the error. If either qubit was already part of a cluster state, the bonds to its neighbours within the cluster are also destroyed. However, the remainder of the cluster state can be recovered by applying appropriate single-qubit operations to these neighbouring qubits, conditional on the outcome of the measurement on the qubit involved in the failed controlled-phase (CZ) gate. Therefore, the cluster state can grow, shrink, or remain the same size, depending on whether the CZ operation was successful, failed, or failed with insurance. The possibility of the ‘failed with insurance’ scenario offered by the RUS scheme helps boost the growth rate of the cluster state [28]. The key to scalably generating cluster states is to attempt CZ operations between qubits such that the cluster state grows on average. Indeed, this can be accomplished efficiently with an appropriate cluster state growing strategy [26, 28], [30]–[32].

4. Concluding remarks

In this review article, we have described a proposed hybrid architecture for quantum computing using stationary and flying qubits. The scheme is largely implementation independent: it could be a single atom–cavity system or a system based on dipole induced transparency. Moreover, quantum electrodynamic (QED) cavity implementations do not need to operate in the strong coupling regime. We showed that, despite the incompleteness theorem for optical Bell-state measurements, it is in principle possible to implement a deterministic gate between distant qubits [27, 28].

For non-negligible noise, the gate becomes necessarily probabilistic. In order to achieve robustness against general decoherence, we construct cluster or graph-states using the two-qubit RUS quantum gate. Even in the presence of photon loss, distributed quantum computation can still be performed with high fidelity. Our entangling operation, which produces the bonds in the graph states, is not limited to physically adjacent matter qubits. As a consequence, no extensive swapping operations need to be taken into account in the production of nontrivial graph states. This architecture for quantum computation is inherently distributed, and hence it is ideally suited for integrated quantum computation and communication purposes.

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