Efficiency limit factor analysis for the Francis-99 hydraulic turbine

Y Zeng¹, L X Zhang², J P Guo³, Y K Guo⁴, Q L Pan⁵, J Qian⁶

¹. Faculty of Metallurgical and Energy Engineering, Kunming University of Science and Technology, Kunming 650093, China. Email: zengyun001@163.com
². Department of Engineering Mechanics, Kunming University of Science and Technology, Kunming 650500, China. Email: zlxzcc@126.com
³. Yunnan Institute of Water Resources and Hydropower Engineering Investigation, Kunming 650053, Yunnan, China. Email: okgjp@126.com
⁴. School of Engineering, University of Bradford, BD7 1DP, UK. Email: y.guo16@bradford.ac.uk
⁵. Sinohydro Bureau 14 Co. LTD, Kunming 650500, China. Email: 13888359265@163.com
⁶. College of Electric Power Engineering, Kunming University of Science and Technology, Kunming 650500, China. Email: qj0117@163.com

Abstract: The energy loss in hydraulic turbine is the most direct factor that affects the efficiency of the hydraulic turbine. Based on the analysis theory of inner energy loss of hydraulic turbine, combining the measurement data of the Francis-99, this paper calculates characteristic parameters of inner energy loss of the hydraulic turbine, and establishes the calculation model of the hydraulic turbine power. Taken the start-up test conditions given by Francis-99 as case, characteristics of the inner energy of the hydraulic turbine in transient and transformation law are researched. Further, analyzing mechanical friction in hydraulic turbine, we think that main ingredients of mechanical friction loss is the rotation friction loss between rotating runner and water body, and defined as the inner mechanical friction loss. The calculation method of the inner mechanical friction loss is given roughly. Our purpose is that explore and research the method and way increasing transformation efficiency of water flow by means of analysis energy losses in hydraulic turbine.

Keywords: inner energy loss; power model; Francis-99; mechanical friction; efficiency

List of symbols

\[ c_1, c_2 \quad \text{undetermined coefficient} \]
\[ Q_z \quad \text{flow discharge at the peak efficiency point in m}^3\text{s}^{-1} \]
\[ C_f \quad \text{viscosity resistance coefficient} \]
\[ R_e \quad \text{Reynolds number} \]
1. Introductions

It is an effective path that research energy loss characteristics of hydraulic turbine, then take steps to decrease loss and improve transfer efficiency of water energy. Loss characteristics of hydraulic turbine is one of research hotspot. Usually, the inner energy loss of hydraulic turbine is divided into the volume loss, the hydraulic loss and the mechanical friction loss. The physical conception volume loss is the most clear. However, effect of the volume loss on efficiency of hydraulic turbine is usually ignored due to its value too small. The mechanical friction loss includes two parts, one is the external mechanical friction loss produced by friction resistance between bearing and shafting of hydraulic turbine generating unit, another is the internal mechanical friction loss generated by rotating resistance, which is the resistance between water body and rotating runner of turbine in [1,2]. Although calculation external mechanical friction is difficult, it could be obtained by test and measurement, such as the external mechanical friction torque is provided in Francis-99 in [3]. However, the friction resistance between rotating runner and water body is called as the internal mechanical friction loss, its action mechanism is different from the disk loss of centrifugal pump, and its calculation and test method is not reported in the literature. Taken the hydraulic loss as core, calculation and test research in [4,5] are the most active research field. Especially, with the development of computational fluid dynamics (CFD), all aspects of hydraulic turbine characteristics, such as numerical simulation of the hydraulic turbine passageway in
[6,7,8], optimization design in [9,10,11], performance prediction in [12,13,14], and character change of runner blade in [15,16,17], have obtained many achievement. On account of the limitation of computing resource and inner flow regime complexity of turbine runner, the CFD using to research transient process of hydraulic turbine still has some limitations in [18].

A calculation method of inner energy loss of hydraulic turbine is proposed in Ref.[19], which can be extracted loss characteristics of hydraulic turbine from test data, and it has been obtained new extensive application in [20]. Based on this method, using data provided by the Francis-99 test platform, the energy loss characteristics of hydraulic turbine are investigated. Meanwhile, calculation method of the internal mechanical friction loss is proposed, although it is an attempt and exploration.

2. Power calculation model of hydraulic turbine

2.1 Basic model

Refer to the turbine model with the inner energy loss characteristics in [19], the inner energy of the hydro turbine composition can be written as following:

\[ P_i = \gamma QH_t - \Delta P_h - \Delta P_v - \Delta P_m - \Delta P_i \]  

(1)

Where \( \gamma QH_t \) is the water flow power of hydraulic turbine, \( \Delta P_v = \gamma QH_t k_v \) is the volume loss power, the \( \Delta P_h = k_h Q^3 \) is turbine loss power in the flow channel.

At the optimum operation of the hydro turbine, there is no water flow impacting on the inlet of the turbine runner while water outflow moves along the normal direction at the outlet of the turbine runner. Under non-optimum operation, there exists the impact loss at both the inlet and outlet of the turbine runner. From the efficiency curve of the hydro turbine, the impact loss is almost the same for the same deviation of the flow rate away from the peak efficiency point. Therefore, it is possible to combine the impact loss at the inlet and the outlet, the effect of the draft tube and other losses except clear losses in Eq.(1) into the impact loss power \( \Delta P_i \). As such, \( \Delta P_i \) is a function of flow and can be expressed as \( \Delta P_i = F[(Q - Q_z)] \).

The power expression of hydraulic turbine can be written as follow:

\[ P_t = \gamma H_t Q(1 - k_v) - k_v Q^3 - \Delta P_m - F[(Q - Q_z)] \]  

(2)

**Remark 1**: The head of hydraulic turbine \( H_t \) in Eq.(2) can be calculated by using the form given by the IEEE Working Group in [22], that is \( H_t = H_0(1 - f_p q^2 - \Delta h_q) \), \( f_p q^2 \) is the friction loss head in pipeline in relative, in steady state, \( \Delta h_q = 0 \). Thus the Eq.(2) is suitable for both the large and small fluctuations under arbitrary head.

2.2 Computation of characteristic parameters

Using the data under steady state operation to calculate the characteristic parameters. Let \( h_q = 0 \); the coefficient of volume loss \( k_v \) can be selected according to the seal characteristics of the turbine. Usually, \( k_v \) is taken as 0.0025~0.005.

(1) Determination of parameter \( k_h \)

Based on the definition of the impact loss power in Remark 1, assume that the impact loss be zero at the peak efficiency point for the same head and the impact loss be the same for the same flow deviation away from the peak efficiency point. Selecting the efficiency curve at some water head and taking a point at each side of the maximum efficiency point, the flow discharge satisfies the following.

\[
\begin{align*}
Q_i &= Q_z + \Delta Q \\
Q_2 &= Q_z - \Delta Q
\end{align*}
\]  

(3)

The impact loss at these two points is equal to each other. From Eq. (2), one can obtain:
\[ k_h = \frac{\gamma H_t (Q_1 - Q_2)(1 - k_1) - P_{t1} + P_{t2}}{Q_1^3 - Q_2^3} \]  \tag{4}

where \( P_{t1} = 9.81Q_1\eta_1 \), \( P_{t2} = 9.81Q_2\eta_2 \), \( \eta_1 \) and \( \eta_2 \) is the efficiency corresponding to flow \( Q_1 \) and \( Q_2 \), respectively.

(2) Mechanical friction loss
Assume the impact loss \( \Delta P = 0 \) at the maximum efficiency point, the loss power due to the mechanical friction at the maximum efficiency point can be calculated as:
\[ \Delta P_m = \gamma H_\eta (1 - k_1) - k_2Q_1^3 - P_{t0} \]  \tag{5}

(3) Impact loss
The impact loss power can be written as:
\[ \Delta P = \gamma Q H_\eta - \Delta P_m - \Delta P_h - \Delta P_f - P_t \]  \tag{6}

Given flow discharge \( Q \), the impact loss power can be calculated by combining Eq.(6) with interpolation calculation of P-Q curve. Taking \((Q - Q_0)\) as variable, the impact loss power is further fitted to impact loss function \( F(Q - Q_0) \).

3. Francis-99 example
Whole data of four operation points provided by the Francis-99 test case are list as table 1.

| Parameter                  | PL  | BEP | HL   | Min load |
|----------------------------|-----|-----|------|----------|
| Guide vane angle \( \alpha \) (°) | 6.72| 9.84| 12.43| 0.80     |
| Net head \( H_t \) (m)       | 11.87| 11.94| 11.88| 12.14    |
| Discharge \( Q \) (m\(^3\)s\(^{-1}\)) | 0.140| 0.200| 0.242| 0.022    |
| Torque to generator \( T_t \) (Nm) | 426.39| 616.13| 740.54| 11.16    |
| Friction torque \( T_f \) (Nm) | 4.40| 4.52| 3.85| 4.66     |
| Runner rotation speed \( n \) (rpm) | 332.84| 332.59| 332.59| 332.80   |
| Hydraulic efficiency \( \eta \) (%) | 90.13| 92.39| 91.71| 20.94    |

The \( g \) is the gravity, \( g = 9.882(\text{m} \cdot \text{s}^{-2}) \), \( \gamma = 999.8(\text{kg} \cdot \text{m}^{-3}) \) is the water density, \( \gamma = 9.88(\text{kN} \cdot \text{m}^{-3}) \). The PL is the operation point of part load, the BEP is the operation point of the peak efficiency point, and the HL is the operation point of high load.

Using data of four operation points, power model and power loss of hydraulic turbine will be calculated in next sections.

**Remark 2.** The classical calculation equation of hydraulic turbine is \( P_t = \gamma Q H_\eta (\text{kW}) \), \( \gamma = 9.81(\text{kN} \cdot \text{m}^{-3}) \). In the Francis-99 test case, the \( \gamma \) is 9.88(\text{kN} \cdot \text{m}^{-3})\. If power of hydraulic turbine is calculated by the equation \( P_t = 9.88Q H_\eta (\text{kW}) \), while test power is calculated by the equation \( P_t = n \pi T_t / 30000 \) (kW), maximum power error at the peak efficiency point is 1.58%. Analysis revealed, the maximum power error at the peak efficiency point is 0.86%, where it uses the \( \gamma = 9.81(\text{kN} \cdot \text{m}^{-3}) \). Therefore, we use \( \gamma = 9.81(\text{kN} \cdot \text{m}^{-3}) \) to calculate in follow-up sections.

Hydraulic turbine power changing with flow discharge is shown as Figure 1.
In figure 1, the sign * denotes test data, while the solid line is fitting curve. Efficiency curve can be obtained by test data at PL, BEP, HL and Min load. In order to decrease error, one uses test data at PL, BEP and HL to obtain the fitting curve of efficiency top parts. Using the top curve calculation of interpolation to obtain new data, and combining with data at Min load to compose new data group, and then the efficiency curve can be obtained by fitting curve. The efficiency curve is shown as figure 2.

In figure 2, the sign * denotes test data, while the solid line is fitting curve. 

**Remark 3:** The purpose of forming efficiency curve is using the classical equation $P_t = \gamma QH\eta$ to calculate two side power at the peak efficiency point, and then the coefficient $k_h$ can be calculated by the Eq.(4). However, there are only test data of four operation point in the Francis-99 test case. Obviously, using test data of four point to fit the efficiency curve has certain subjectivity, and maybe exist large error. For this reason, the efficiency curve is not applied to use in later calculation in this paper. Form the Eq.(4), the calculation of coefficient $k_h$ only requires to two side power at the peak efficiency point. From figure 1, the power of hydraulic turbine changing with the flow discharge is near to straight line. The turbine power expression can be obtained by combining polynomial of degree 1 and degree 2. So, the power fitting curve as figure 1 is used to calculate power of hydraulic turbine near to the peak efficiency point, while calculates the coefficient $k_h$.

4 **Francis-99 model of power calculation**
In order to analyze relative change of parameters, according to data given by Ref.[5] determine parameters basic value system as, $H_b=12.00$ m, $Q_b=0.20$ m$^3$s$^{-1}$, $P_{t0}=22.00$ kW, $\omega_b=333$ rpm. In fact, purpose of selection basic value system is that reflect relative change of calculation parameters, and change is more perceptual intuition.

The volume loss coefficient of hydro turbine $k_v$ is near selected as zero, that is $k_v=0$.

In order to improve the computational accuracy, ten unit flow increments are selected as $\Delta Q=\pm 0.001, 0.002, 0.003, 0.005, 0.01$ m$^3$s$^{-1}$. The corresponding coefficient $k_h$ is calculated using the ten increments. The mean value is $k_h=60.4641$.

Assumption the impact loss power at the peak efficiency point is zero, the mechanical friction loss power can be obtained from Eq.(5), $\Delta P_m=1.4823$ kW, or $6.74\% P_{t0}$.

According to the Q-P curve shown as figure 1, and given flow discharge $Q$, the impact loss power can be extracted by Eq.(6). The impact loss power changing with flow discharge is shown as figure 3.

\[
F(Q-Q_{BEF}) = -60.4641(Q-Q_{BEF})^3 + 5.6980(Q-Q_{BEF})^2 + 0.0017(Q-Q_{BEF})
\]  

Figure 3 shows that impact loss power is direct relation to degree of deviation peak efficiency point. Especially, the proportion of impact loss in low load regions is large.

Thus, the power expression of hydraulic turbine Eq.(2) can be rewritten as following.

\[
P_t = 9.81H_fQ - 60.4641Q^3 - 1.4823 - F(Q-Q_{BEF})
\]

5. Energy loss characteristics of hydraulic turbine

Taken data the Min Load to PL as case to calculation, its sample rate is 5 kS/s, but its data fluctuation is large. To this reason, after analyzing various digital filtering method, this paper uses the polynomial fitting of degree 30 to smoothness sample data. The head of hydraulic turbine change in transient is shown as figure 4(a), and the flow discharge in transient is shown as figure 4(b).
Figure 4  Sampling data pretreatment

In figure 4, solid line is smoothing data, which shows that using this kind of higher degree polynomial fitting method to smooth sample data is feasible.

Using test data of head and flow discharge in transient, the power of hydraulic turbine can be calculated by Eq.(8), it is shown as Figure 5.
Figure 5 Turbine power changes in transient
Taken the $P_{t0}=22$ kW as basic value, the energy loss change in startup process is shown as Figure 6.

![Figure 6. Characteristics of power loss](image)

Remark 4. In other calculation example, the minimum value of total loss power is near the peak efficiency point. In Francis-99 case, the flow discharge in the peak efficiency point is 0.2 m$^3$/s, where the minimum value of total loss is not near it. The reason may include two parts, (1) amplitude of head variation in test is small, thus its impact loss is low; (2) The hydraulic loss defined in model (1) is the hydraulic loss of runner passageway loss. The runner diameter of Francis-99 is 349 mm, its size is small. With flow discharge increase, the hydraulic loss in runner regional sharply increase, its maximum is about 2.06%$P_{t0}$, means the hydraulic loss is leaning large.

Remark 5. The loss energy of runner passageway increase with flow discharge, and it relate with the three power of flow discharge. Therefore, by means of optimization design of runner passageway and elaborate processing to decrease loss is an effective measure, especially in high load zone.

6 Mechanical friction loss analysis

6.1 Loss composition

The mechanical friction loss power defined in Eq.(1) is direct related to rotational speed of hydraulic turbine generating units. Under units connecting to power network, fluctuation amplitude of rotated speed of units is very small. As a result, the mechanical friction loss power in transient is approximately invariant, it is about 6.74%$P_{t0}$. It accounts for a large proportion, and remains invariant within the scope of the whole power change.

There are three kinds of mechanical friction in runner zone of hydraulic turbine.

(1) The friction torque in different operation point in table 1 are 4.40, 4.52, 3.85 and 4.66 Nm respectively. Transforming them into power and using relative value form, they are 0.70, 0.72, 0.61, 0.74 %$P_{t0}$ respectively.

Obviously, friction torque are obtain by measuring thrust bearing and guide bearing in Francis-99 test, which is the friction torque between rotation system of units and support system of bearing, that is external mechanical friction torque.

(2) One of link generating power loss is the sealing ring of runner. Its calculation power loss is complex, thus it is not calculated in this paper. Total loss in sealing ring is about 0.5%～0.8%$P_{t0}$ refer to Ref.[21], so it is approximately selected as 0.8% $P_{t0}$ in our calculation.

(3) Internal mechanical friction loss. The internal mechanical friction loss is that the friction between rotating runner and water body. In this case, total mechanical friction loss power is 6.74%$P_{t0}$.
so the internal mechanical friction loss power can be indirectly obtained, that is 5.24%$P_{t0}$ = 6.74%$P_{t0}$ - 0.70%$P_{t0}$ - 0.8%$P_{t0}$.

From view of loss numerical value, the internal mechanical friction loss is main ingredients of total loss, it need to be analyzed in the furture.

6.2 Internal mechanical friction loss

The wheel as a whole, the rotation friction surface of wheel rotating part and water body include four parts, inlet edge of blade, outlet edge of blade, runner cone and the parts region of bottom ring, as Figure 7.

Figure 7. Schematic diagram of rotation friction surface

Due to edge of rotation surface is irregular shape, rotation friction force should be calculated by selecting infinitesimal element and integrating method. Instant motion of infinitesimal element is similar to motion form of fluid flowing through the stationary flat. Therefore, refer to Prandtl-Schlichting equation, the friction resistance coefficient of rotation motion can be approximately expressed as following.

\[
C_f = \frac{c_1}{(\lg Re)^2} \tag{9}
\]

\[
R_f = \frac{v^2 \omega}{\mu} \tag{10}
\]

where $r$ is the distance from infinitesimal element to axes.

\[
R_f = \frac{1}{2} C_f v^2 S \tag{11}
\]

where $v$ is the speed, the dynamic viscosity in water temperature 20°C is $\mu = 1.005 \times 10^{-3}$ N·s·m$^{-2}$.

**Remark 6:** The Eq.(9) is obtained by refer to the Prandtl-Schlichting equation, where coefficient $c_1$ and $c_2$ is undetermined coefficient. Currently, there are not systematic test analysis and reference for the rotation resistance of hydraulic turbine. According to the empirical formula in planar turbulent boundary layer, let $c_1 = 0.455$ and $c_2 = 2.58$. We think that rotation motion of turbine runner is similar to motion of rotor disc, so calculation in this paper uses $c_1 = 0.455$ and $c_2 = 2.58$.

Friction resistance of any infinitesimal element in rotation edge can be expressed as following.

\[
dR_f = \frac{1}{2} C_f v^2 dS = \frac{1}{2} C_f v^2 r^2 dS \tag{12}
\]
And friction torque is

\[ M_f = \int \int \int r \, dr \, d\theta \, dz = \frac{1}{2} \int \int C_f \gamma \omega^2 r^3 \, dS \]  

(13)

In inlet water edge, \( r = R_1 \), the wet area of infinitesimal element is \( dS = R_1 \, d\theta \, dh_1 \). \( \theta_1 \) is the angle of infinitesimal element motion, \( h_1 \) is the height of infinitesimal element. Assume that the Reynolds number in inlet water edge is constant, the resistance torque in inlet water edge of runner can be obtained by integral operation.

\[ M_{f1} = \int \int r \, dr \, d\theta = \frac{1}{2} C_f \gamma \omega^2 \int \int d\theta \, dh_1 = C_f \pi \gamma \omega^2 H_1 R_1^4 \]  

(14)

Lower edge in outlet water of the bottom ring and runner cone can be approximately account as rotation cylinder. Their friction torque can be calculated using the Eq.(14).

At radius \( r \) of outlet water edge, the wet area of infinitesimal element is \( dS = r \, d\theta \, dh_4 \). Shape of outlet water edge is considered as straight line approximately. Its rotation friction torque is.

\[ M_{f4} = \int \int r \, dr \, d\theta = \frac{1}{2} C_f \gamma \omega^2 \int \int r^4 \, d\theta \, dh_4 = \frac{1}{5} \pi C_f \gamma \omega^2 \frac{h_4}{R_{r2} - R_{r1}} (R_{r2}^3 - R_{r1}^3) \]  

(15)

Total rotation friction torque can be written as following.

\[ M_f = M_{f1} + M_{f2} + M_{f3} + M_{f4} \]

\[ = C_f \pi \gamma \omega^2 \left[ H_1 R_1^4 + H_2 R_2^4 + H_3 R_3^4 + \frac{1}{5} \frac{h_4}{R_{r2} - R_{r1}} (R_{r2}^3 - R_{r1}^3) \right] \]  

(16)

**Remark 7.** The Eq.(16) shows that internal mechanical friction loss of the Francis turbine is closely related to geometric structure of runner and blade. This feature provides important basis for decreasing internal mechanical friction loss and increase efficiency of hydraulic turbine.

Some of structure sizes can be obtained from runner draw of Francis-99, \( H_1 = 0.057m \), \( R_1 = 0.315m \), \( H_2 = 0.020m \), \( R_2 = 0.175m \), \( H_3 = 0.070m \), \( R_3 = 0.050m \), \( H_4 = 0.095m \), \( R_4 = 0.175m \), \( H_4 = 0.165m \), \( R_4 = 0.065m \). Ignoring influence of splitter blades, the equivalent radius of outlet water edge of runner is \( (R_{r2}^2 - R_{r1}^2)^{0.5} = 0.1625m \).

Rotation resistance torque of each parts can be calculated by the Eq.(14) and Eq.(15), and are listed as table 2.

| Table 2. Rotation resistance torque of each parts |
|-----------------------------------------------|
| Rotary surface | 1 | 2 | 3 | 4 |
| Radius (m) | 0.3150 | 0.1750 | 0.0500 | 0.1625 |
| Reynolds number \( \times 10^6 \) | 3.4422 | 1.0624 | 0.0867 | 0.9161 |
| \( C_f \times 10^{-3} \) | 3.5839 | 4.4206 | 7.3895 | 4.5448 |
| \( M_f \) (Nm) | 7.6822 | 0.3167 | 0.0123 | 0.7458 |
| Ratio of resistance power (%Pt0) | 1.2177 | 0.0502 | 0.0020 | 0.1182 |

**Remark 8.** Total resistance power by summing losses of four parts is 1.3881%\( P_{t0} \). It is less than the 5.24%\( P_{t0} \) obtained in section 5.2. However, we find that \( 4 \times 1.3881\% P_{t0} = 5.5524\% P_{t0} \), it is close to the 5.24%\( P_{t0} \) obtained in section 5.2. Why is this so? Although we don’t still give reasonable excuse, but we think that it is related to the calculation method of the coefficient \( C_f \). For example, the inlet water edge is accounted as a whole surface of rotation, and it’s the coefficient \( C_f \) is calculated by means of the pannel viscosity resistance method. So calculation of the coefficient \( C_f \) should be improved in the future.
Table 2 reveals composition ratio of the internal mechanical friction loss power in the Francis-99, which provides more intuitional calculation basis and reference for optimization design based on geometric structure of turbine runner.

7. Conclusions

By subdividing internal energy loss of hydraulic turbine and taking Francis-99 as case, detailed calculation and extraction method of energy loss of hydraulic turbine are given. Using algebra model form of hydraulic turbine power research changes of energy loss characteristic in transient, that provides a theoretical basis for analysis affect factors and limitation conditions of hydraulic turbine efficiency.

According to analysis examples of Francis-99, two points conclusions can be given as following.

(1) Using proposed method decomposing internal energy loss of hydraulic turbine and investigating its changes in transient process, can more clear reveal ratio of losses and their transformation law, which provides more direct reference for optimization design of hydraulic turbine.

(2) Rotation friction loss of hydraulic turbine has larger ratio in energy loss, and relates to geometric structure closely, that is worth attention. Derivation of rotation friction loss given by this paper is just a attempt, in which certain parameters selection and approximate treatment not yet sufficient theoretical foundation, they need further research.

Acknowledgments

The research reported here is financially supported by the National Natural Science Foundation of China under Grant No. 51579124, 51469011,51279071. The constructive and insightful comments and suggestions made by two anonymous Reviewers have significantly improved the quality of the final manuscript.

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