Effect of wall-slip on natural convection in a square cavity

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Abstract. Numerical simulations are performed to investigate the effect of wall-slip on natural convection inside a square cavity for Rayleigh number (Ra) = $10^5$. The bottom wall is heated and top wall is maintained at a cold temperature keeping the other two vertical walls adiabatic. The analysis is conducted by varying the Slip factor (SF) from 0 (no-slip) to 1 (full-slip) in steps of 0.2. For each value of SF, the flow patterns are visualised using streamlines and heat transfer behaviour is examined using isotherms. Results are quantified using local Nusselt number ($Nu_l$) and average Nusselt number ($Nu_{avg}$) along the hot wall. It is observed that for SF=0.8 and SF=1 configurations, the natural convection patterns and heat transfer characteristics undergo notable changes.

1. Introduction
Natural convection in a square cavity is a highly researched topic because of the simple geometry and wide range of applications. Numerous studies have been conducted to understand the importance of convective heat transfer in the fields of atmospheric sciences to real world engineering problems like design of nuclear reactors and cooling of electronic devices.

One of the pioneer researchers, Vahl Davis[1] numerically investigated buoyancy-driven flow in a square cavity with differentially heated vertical walls. Wan et al.[2] employed a high-accuracy discrete singular convolution (DSC) method for simulating natural convection in a square cavity in the range $10^3 \leq Ra \leq 10^8$. These investigations are considered to be benchmarks for natural convection in a square enclosure in the laminar regime. Buoyancy driven laminar and turbulent heat transfer in a square cavity with differentially and partially heated walls was studied by Markatos and Pericleous[3], the study being important in understanding the thermal and dynamic effects of natural convection in many practical problems. Valencia and Frederick[4] performed investigations on natural convection in a square enclosure with half heated and half insulated vertical walls and observed that at higher Ra, flow features near to the unheated vertical walls exhibited significant changes owing to the absence of buoyancy forces.

Laje and Bejan[5] investigated the phenomenon of natural convection in a square cavity for a range of Prandtl number (0.01 $\leq$ Pr $\leq$ 10) and Rayleigh number ($10^2 \leq Ra \leq 10^{11}$). In this study, attention was given to find out the highest Ra where inertial fluctuations were possible in the steady laminar regime. Hasnaoui et al.[6] discussed buoyancy driven flow in an inclined cavity bounded by a porous layer. Another study by Dalal and Das[7] focused on the behaviour of flow in the laminar regime with sinusoidally heated bottom wall and other walls kept at constant cold temperature. Basak et al.[8] studied laminar natural convection flow with uniform and non-uniform heating of the bottom wall. Numerical investigation of steady natural convection in a square cavity was conducted by Sathiyamoorthy et al.[9] with insulated top wall and sinusoidally heated bottom wall and linearly heated vertical walls. They
compared the results by varying the Prandtl number (Pr) and aspect ratio (AR) and found out that multiple secondary circulations were formed at lower Pr values whereas Pr ≥ 0.71 witnessed only a pair of secondary circulations. Kane et al.[10] studied the behaviour of boundary layer in natural convection in a square cavity. The square enclosure they considered had the central portion of the vertical walls kept at different temperatures. The remaining parts of the vertical walls and the top and bottom walls were kept insulated and the effect of Rayleigh number was studied.

Various other studies (Devaraj et al.[11], Zhang et al.[12], Cibik and Kaya[13]) have been conducted on natural convection in a square enclosure applying different boundary conditions. However, attention is not given to understand the influence of more complex boundary conditions such as slip boundary conditions, hence it is essential to carry out such a study. The application of this problem can be extended to industries that work with hydrophobic surfaces and modern-day engineering applications like MEMS devices. This paper discusses the effect of slip boundary condition on natural convection in a square cavity at Rayleigh number \(10^5\) by varying the slip boundary condition from no-slip (SF=0) to full-slip (SF=1).

The structure of the presentation of our observations in the rest of the paper is as follows. Solution method in Section 2 describes the solver details, computational domain and the boundary conditions. The results are explained in detail in Section 3 which contains grid sensitivity studies followed by code validation. Flow and thermal patterns are visualised using streamlines and isotherms and heat transfer characteristics are quantified using local Nusselt number \(Nu_l\) and average Nusselt number \(Nu_{avg}\) for different configurations of slip conditions. Conclusions in Section 4 summarises the overall numerical findings.

2. Solution method
2.1. Numerical methodology
Numerical simulations are carried out using buoyantBoussinesqPimpleFoam, the generic solver available in the open source computational fluid dynamics toolbox OpenFOAM. A second order central differencing Gauss linear scheme is used for spatial discretization of convective and diffusive terms. The pressure-velocity coupling at each iteration is handled by PIMPLE algorithm. With the Boussinesq approximation, fluid properties are assumed to be constant except the fluid density which is obtained by \(1.0-\beta(T-T_{ref})\) where, \(\beta\) is the coefficient of thermal expansion, \(T\) is the temperature and \(T_{ref}\) being...
the reference temperature.

The system is governed by the PDEs corresponding to mass, momentum and energy conservation, given as follows:

Continuity:
\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \]  \hspace{1cm} (1)

X-momentum:
\[ U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \]  \hspace{1cm} (2)

Y-momentum:
\[ U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + RaPr \Theta \]  \hspace{1cm} (3)

Energy:
\[ U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} = \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} \]  \hspace{1cm} (4)

The equations are non-dimensionalised as below:

\[ X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{uL}{\alpha}, \quad V = \frac{vL}{\alpha}, \quad \Theta = \frac{T - T_c}{T_h - T_c}, \quad P = \frac{pL^2}{\rho \alpha^2} \]  \hspace{1cm} (5)

Here, \( X \) and \( Y \) represent non-dimensional horizontal and vertical coordinates. Similarly \( U \) and \( V \) are horizontal and vertical non-dimensional velocities. \( \Theta \) is the non-dimensional temperature, \( T_c \) is the cold wall temperature and \( T_h \) is the temperature of the hot wall. \( \alpha \) represents the thermal diffusivity, \( L \) denotes the length of the cavity and \( P \) is used to represent the non-dimensional pressure.

Rayleigh number based on the enclosure width is defined as \( Ra = \frac{g\beta L^3(T_h - T_c)}{\nu \alpha} \) and Prandtl number as \( Pr = \frac{\nu}{\alpha} \) where, \( \nu \) is the kinematic viscosity of the fluid, \( g \) is the acceleration due to gravity acting downwards in the negative \( y \) direction.

The rate of heat transfer is computed at each wall and is expressed in terms of local Nusselt number (\( Nu_l \)) and surface-averaged Nusselt number (\( Nu_{avg} \)).

\[ Nu_l = \left. \frac{\partial \Theta}{\partial n} \right|_{\text{wall}}, \quad Nu_{avg} = \frac{1}{W} \int_0^W Nu_l \, dS \]  \hspace{1cm} (6)

where \( n \) is the unit normal to the wall, \( dS \) is the small elemental area and \( W \) is the surface area of the wall.

2.2. Computational domain and boundary conditions

Figure 1(a) illustrates the schematic of the computational domain which consists of a square enclosure with width \( L \). The top and bottom walls are differentially heated and maintained at cold temperature (\( \Theta = 0 \)) and hot temperature (\( \Theta = 1 \)) respectively whereas the vertical walls are adiabatic.

Slip conditions on the walls are varied using the \textit{partialSlip} boundary condition (BC) available in OpenFOAM. \textit{partialSlip} represents a mixed boundary condition which is an implementation of the Navier slip law as expressed below. For a 2-D flow, the first order approximation for the slip-velocity is given by

\[ U + \lambda \frac{dU}{dY} = 0 \]  \hspace{1cm} (7)

here \( \lambda \) is the slip length as defined in Legendre et al.[14]
Let $U_0$ be the tangential velocity on the wall face boundary and $U_1$ the cell center velocity at the boundary cell. Equation (7) can be re-written in terms of $U_0$, $U_1$ and $D$ as follows

$$U_0 + \lambda \frac{(U_0 - u_1)}{D/2} = 0$$

(8)

where $D$ is the size of the adjacent cell. The above equation after re-arranging becomes

$$U_0 = \frac{2\lambda D}{D + 2\lambda} U_1$$

(9)

Here, we introduce a new term *Slip factor* ($SF$) defined as

$$SF = \frac{2\lambda D}{D + 2\lambda}$$

(10)

Hence equation (9) becomes

$$U_0 = (SF) U_1$$

(11)

For $\lambda \to 0$, we have $SF = 0$, which indicates no-slip condition.

For $\lambda \to \infty$, we have $SF = 1$, which indicates full-slip condition.

![Figure 1: Schematic of (a) computational domain (b) grid structure used for the present study.](image)

Conditions imposed at the walls other than the slip boundary conditions are as follows:

- **Left wall**: Neumann boundary condition, $\frac{\partial \Theta}{\partial Y} = 0$
- **Top wall**: Dirichlet boundary condition, $\Theta = 0$
- **Right wall**: Neumann boundary condition, $\frac{\partial \Theta}{\partial Y} = 0$
- **Bottom wall**: Dirichlet boundary condition, $\Theta = 1$
3. Results and discussion

3.1. Grid sensitivity

Grid sensitivity studies are performed to optimise the grid size for accurate results and to minimise the computational time. The analysis is conducted for grid sizes of $21 \times 21$, $41 \times 41$, $61 \times 61$, $81 \times 81$ and $101 \times 101$, for Rayleigh number $10^5$ with no-slip boundary condition. Table 1 given below summarises the grid independence results for the average Nusselt number ($N_{u_{avg}}$) at the hot wall. The percentage deviation (shown in brackets) reduces with grid size up to $81 \times 81$, and further increase in grid size to $101 \times 101$ is accompanied by an increase in percentage difference. It can be attributed to the increase in numerical errors (truncation error, round-off error and iterative convergence error) with increase in grid size. Hence, a grid size of $81 \times 81$ is employed throughout our computations.

| Grid size     | $N_{u_{avg}}$   |
|---------------|-----------------|
| $21 \times 21$| 4.820 (-)       |
| $41 \times 41$| 4.607 (4.419%)  |
| $61 \times 61$| 4.565 (0.912%)  |
| $81 \times 81$| 4.559 (0.131%)  |
| $101 \times 101$| 4.534 (0.548%) |

3.2. Validation

The computational code used for the present study is validated using the benchmark solutions available in the existing literature.

Figures 2(a) and (b) illustrate the comparison of velocity distribution along the horizontal and vertical axes passing through the centre of the domain.

**Figure 2:** Comparison of (a) horizontal velocity at $X=L/2$ (b) vertical velocity at $Y=L/2$. 
Table 2: Validation study: Comparison of $N_{u_{avg}}$ values with the existing literature; The percentage deviation (absolute value) of calculated values with the literature is indicated in brackets.

| Ra  | Present Study | Wan et al.[2] | Vahl Davis[1] | Tang & Tsang[15] | le Quere & Roquefort[16] |
|-----|---------------|---------------|---------------|----------------|-------------------------|
| $10^3$ | 1.109 (0.716%) | 1.117 (0.716%) | 1.12 (0.982%) | 1.118 (0.805%) | 1.117 (0.716%) |
| $10^4$ | 2.246 (0.355%) | 2.254 (0.355%) | 2.243 (0.134%) | 2.25 (0.178%) | 2.238 (0.357%) |
| $10^5$ | 4.559 (0.848%) | 4.598 (0.848%) | 4.519 (0.885%) | 4.52 (0.863%) | 4.519 (0.885%) |

Table 2 compares the results obtained for average Nusselt number ($N_{u_{avg}}$) on the hot wall for the present study with the data in previous literature and the maximum deviation is found to be less than 1%. The above observations conclude that the present results are in good agreement with the existing data.

3.3. Streamlines

Figure 3 illustrates the streamlines for $Ra = 10^5$ for varying slip conditions. At no-slip condition (SF = 0), a unicellular primary vortex rotating clockwise is seen to occupy majority of the domain. It is also observed that in addition to the primary vortex, two tiny secondary vortices inhabit the top-left and bottom-right corners of the domain. These secondary vortices are seen to be circulating in a direction opposite to the primary vortex. As the SF value is increased, the recirculating secondary vortices are seen to gradually reduce in size giving way to the dominance of the primary vortex and this continues upto SF=0.6.

![Streamlines for various slip conditions](image-url)
At SF=0.8, the primary vortex suddenly reverses the direction of circulation and begins to rotate in the counter-clockwise direction. At the same time, the tiny secondary vortices change its position as well as direction and appear in the top-right and bottom-left corners. The size of the secondary vortices is also seen to decrease compared to the previous slip configurations. The influence of abrupt change in the flow fields bring about significant changes in the temperature fields which will be discussed in the next section.

At full-slip (SF=1), the primary vortex splits into two counter rotating vortices that is seen to occupy the entire domain and the secondary vortices cease to exist. The streamline pattern at SF=1 is similar to the pattern reported for a square enclosure by Basak et al.\[17\] with cold vertical walls, hot bottom wall and insulated top wall however, the case used no-slip boundary condition on the walls. The existence of similar flow fields for entirely different boundary conditions requires further exploration.

### 3.4. Isotherms

Existing literature report strong convection currents at Ra=10^5 and discrete heating of top and bottom walls causes strong density variations in the domain. The strong convection currents influence the separation of boundary layer on the bottom wall and subsequent development of plumes. These hot plumes ascend and impinge on the cold wall and get distorted. Visual inspection reveals that for no-slip condition (SF=0), the plumes get initialised near to the edges of the bottom wall and ascend through the vertical adiabatic wall. Impingement of these hot plumes on the cold top wall ejects the cold sub-layers

![Isotherms](image.png)

**Figure 4:** Isotherms for various slip configurations plotted at Ra =10^5.
which gives rise to a cold plume which later moves down towards the hot bottom wall. The combined influence of viscous forces and strong convection currents promote continuous advancement of the hot plume through the left adiabatic wall which pushes down the cold plume through the right adiabatic wall. The same phenomenon is seen to occur with increase in SF upto 0.6 (Figures 4(a)-(d)). At SF=0.8, similar to the flow reversal observed while visualising streamlines, the thermal fields too undergo an abrupt transition. As evidenced in Figure 4(e), the advancement of hot plume through the right adiabatic wall distorts the cold plume and paves way for its descend through the left wall. For the full-slip (SF=1) configuration, the hot plumes symmetrically advance along the vertical walls thus giving uniformity to the thermal fields.

3.5. Variation in Nusselt number

Nusselt number is an important non-dimensional parameter that describes the prominent mode of heat transfer; conductive or convective.

Figure 5(a) elucidates the variation in local Nusselt number ($N_u$) along the hot bottom wall. Moving from A to B, for SF=0, the variation in $N_u$ is found to be increasing upto X=0.8 after which it decreases towards the end. This is the general trend for slip configurations upto 0.6. As evidenced in the isotherms (Figures 4(a)-(d)), the boundary layer becomes thinner from left to right and thereafter it thickens slightly towards the end. The maximum value of local Nusselt number is registered where the boundary layer thickness is minimum. The trend reverses when SF=0.8 as the boundary layer is stuffed closer to each other near to the left corner, A. The boundary layer thickness increases while moving towards B which is accompanied by a decrease in heat transfer rate as evidenced in the $N_u$ curve. This variation can be related to the reversal in flow and thermal fields as illustrated in Figures 3(f) and 4(f). At full-slip condition, the $N_u$ curve becomes symmetric to the vertical axis passing through the centre of the domain which is due to the uniformity in flow and thermal fields on either side of the vertical axis. Also, the peak value of is observed at mid-plane. Maximum value for local Nusselt number ($N_u$) is observed at full-slip boundary condition (SF=1) and as SF increases, the maximum value of $N_u$ also increases. It is observed that the average Nusselt number ($N_{uavg}$) along the hot wall of the enclosure increases with increase in slip. This may be a result of the reducing influence of viscous forces thereby creating strong

![Figure 5: Nusselt number variation : (a) $N_u$ variation along the hot wall and (b) $N_{uavg}$ variation with slip factor.](image)
convection currents in the domain. The variation in $\overline{Nu_{avg}}$ is gradual and increasing when the slip factor is increased to 0.6. Between SF = 0.6 and 0.8, there is appreciable rise in average Nusselt number and further increase in slip (SF = 0.8 to SF = 1.0) registers a steep increase in the $\overline{Nu_{avg}}$ as the viscous effects become negligible. Between SF=0 (no-slip) and SF=1 (full-slip) there is an enhancement of nearly 117% in average Nusselt number.

4. Conclusions
Numerical investigations are conducted to analyse the effect of varying slip boundary condition on natural convection in a square cavity. It is observed that changing the slip factor (SF) from 0 to 0.6 brings marginal variation in flow and thermal fields as well as the heat transfer characteristics. For SF=0.8 and full-slip (SF=1) condition, significant changes are observed in the convection patterns. The other inferences are summarised as follows.

- For the slip configurations ranging from SF=0 (no-slip) to SF=0.6, visual inspection reveals a clockwise rotating primary vortex along with two secondary vortices at the top-left and bottom-right corners. For SF=0.8, this primary vortex reverses its direction and the secondary vortices shift to the top-right and bottom-left corners.
- At full-slip condition (SF=1), the primary vortex breaks into two counter rotating vortices and the secondary vortices disappear.
- Between SF=0 and SF=0.6, the combined influence of convection currents and viscous forces promote the advancement of hot plume along the left adiabatic wall. At SF=0.8, the thermal fields undergo a sudden transition causing the hot plume to ascend through the right adiabatic wall.
- For full-slip (SF=1) configuration, the hot plumes symmetrically advance along the vertical walls thus giving uniformity to the thermal fields.
- Between SF=0 and SF=0.8, the variation in local Nusselt number is asymmetric over the hot wall and the maximum value of $\overline{Nu_{l}}$ increases with increase in slip factor. Symmetric variation is observed only at SF=1 (full-slip) for which the peak value is the maximum among all the slip configurations considered.
- The results obtained for average Nusselt number ($\overline{Nu_{avg}}$) for varying slip conditions show a marginal increase between SF=0 and SF=0.6, becomes appreciable towards SF=0.8 followed by a steep rise for the full-slip configuration.

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