Newtonian, post-Newtonian and Relativistic Cosmological Perturbation Theory

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Abstract

Newtonian cosmological perturbation equations valid to full nonlinear order are well known in the literature. Assuming the absence of the transverse-tracefree part of the metric, we present the general relativistic counterpart valid to full nonlinear order. The relativistic equations are presented without taking the slicing (temporal gauge) condition. The equations do have the proper Newtonian and first post-Newtonian limits. We also present the relativistic pressure correction terms in the Newtonian hydrodynamic equations.

1. Introduction

Cosmological perturbation theory is an important theoretical tool in interpreting cosmological observations like the two-dimensional temperature and polarization anisotropies of the cosmic microwave background radiation, the three-dimensional distribution and motions of galaxies, distorted images of galaxies due to gravitational lensing, etc. The cosmological perturbation equations are well known in the Newtonian context to fully nonlinear order \cite{1}, whereas the counterparts in Einstein’s gravity are known in linear \cite{2, 3} and low-order perturbation approximation \cite{4}. Here, we present a self-contained summary of the basic equations of recently formulated fully nonlinear and exact cosmological perturbation theory in Einstein’s gravity (Section \ref{section}). Comparisons are made with the Newtonian (Sections \ref{section_2} and \ref{section_4}) and the post-Newtonian equations (Section \ref{section_5}). We also present the Newtonian equations in the presence of relativistic pressure (Section \ref{section_6}).

2. Newtonian cosmological perturbation theory

Newtonian cosmological perturbation equations in the spatially homogeneous and isotropic background world model are \cite{1}

\begin{align}
\ddot{\varrho} + 3 \frac{\dot{a}}{a} \dot{\varrho} &= - \frac{1}{a} \nabla \cdot (\varrho \dot{\mathbf{v}}), \\
\dot{\mathbf{v}} + \frac{\dot{a}}{a} \mathbf{v} + \frac{1}{a} \mathbf{v} \cdot \nabla \mathbf{v} &= \frac{1}{a} \nabla U - \frac{1}{a \varrho} \nabla \bar{p}, \\
\frac{\Delta}{a^2} U &= -4 \pi G (\bar{\varrho} - \varrho). \tag{3}
\end{align}

1
These are the mass conservation, the momentum conservation, and the Poisson’s equations, respectively; \( \tilde{\rho}, \tilde{p}, \mathbf{v}, \) and \( U \) are the mass density, the pressure, the peculiar velocity, and the perturbed gravitational potential, respectively; \( a(t) \) is the cosmic scale factor. We decompose the mass density and pressure to the background and perturbed parts as

\[
\tilde{\rho} = \rho + \delta \rho, \quad \tilde{p} = p + \delta p.
\] (4)

Evolution of the background world model is described by equation (21) properly derived in Einstein’s gravity.

3. General relativistic cosmological perturbation theory

We consider the scalar- and vector-type perturbations in a flat background with the metric convention \[3, 5\]

\[
ds^2 = -(1 + 2\alpha)c^2 dt^2 - 2\chi_i dx^i dt + a^2(1 + 2\varphi)\delta_{ij}dx^i dx^j,
\] (5)

where \( \alpha, \varphi \) and \( \chi_i \) are functions of spacetime with arbitrary amplitudes; index of \( \chi_i \) is raised and lowered by \( \delta_{ij} \) as the metric. We ignored the transverse-tracefree (TT) part of the metric which is interpreted as the gravitational waves to the linear perturbation order. The spatial part of the metric is simple because, in addition to ignoring the TT part, we already have taken the spatial gauge condition without losing any generality to the fully nonlinear order \[3, 5\].

We consider a fluid without anisotropic stress. The energy momentum tensor is given as

\[
\tilde{T}_{ab} = \tilde{\rho}c^2 \tilde{u}_a \tilde{u}_b + \tilde{p}(\tilde{g}_{ab} + \tilde{u}_a \tilde{u}_b),
\] (6)

where tildes indicate the covariant quantities; \( \tilde{u}_a \) is the normalized fluid four-vector; \( \tilde{\rho} \) includes the internal energy; in explicit presence of the internal energy we should replace

\[
\tilde{\rho} \rightarrow \tilde{\rho} \left(1 + \frac{1}{c^2} \tilde{\Pi}\right),
\] (7)

where \( \tilde{\rho} \) in the right-hand-side is the rest-mass density \[6\]. We introduce the following definitions of fluid three-velocities

\[
\tilde{u}_i = \frac{\alpha}{c} \tilde{v}_i, \quad \tilde{v}_i = \frac{1}{\mathcal{N}} \left(1 + 2\varphi\right) \mathcal{N}_i - \frac{c}{\alpha} \chi_i,
\] (8)

where \( \tilde{\gamma} \) is the Lorentz factor

\[
\tilde{\gamma} \equiv \sqrt{1 + \frac{\tilde{v}_i \tilde{v}_k}{c^2(1 + 2\varphi)}} = \frac{1}{\sqrt{1 - \frac{\tilde{v}_i \tilde{v}_k}{c^2(1 + 2\varphi)}}} = \frac{1}{\sqrt{1 - \frac{(1 + 2\varphi)}{\mathcal{N}} \left(1 - \frac{\tilde{v}_i \tilde{v}_k}{\alpha(1 + 2\varphi)}\right) \left(\tilde{v}_i \tilde{v}_k - \chi_i \chi_k\right)}},
\] (9)

and \( \mathcal{N} \) is related to the lapse function in \[19\]. The velocities \( \tilde{v}_i \) and \( \mathcal{N}_i \) are more physically motivated ones \[5\]: \( \tilde{v}_i \) is the fluid three-velocity measured by the Eulerian observer, and \( \mathcal{N}_i \) is the coordinate three-velocity of fluid; the indices of \( \nu_i, \tilde{v}_i \) and \( \mathcal{N}_i \) are raised and lowered by \( \delta_{ij} \).

We can decompose \( \chi_i \) and \( \mathcal{N}_i \) into the scalar- and vector-type perturbations to the nonlinear order as \[5\]

\[
\chi_i = \alpha \chi_i^{(v)} + \chi_i^{(v)}, \quad \mathcal{N}_i = \tilde{\mathcal{N}}_i + \mathcal{N}_i^{(v)},
\] (10)

with \( \chi^{(v)}_{ij} \equiv 0 \equiv \nabla^{(v)}_{ij} \). We assign dimensions as

\[
[a] = [\tilde{\mathcal{G}}_{ab}] = [\tilde{u}_a] = [\tilde{\alpha}] = [\varphi] = [\chi] = [\tilde{\gamma}]/[c] = 1, \quad [\chi^i] = L, \quad [\mathcal{N}] = T, \quad [\tilde{\mathcal{N}}]/[c] = L, \quad [\kappa] = T^{-1}, \quad [\tilde{\mathcal{G}}_{ab}] = [\tilde{\alpha}c^2] = [\tilde{p}], \quad [G\tilde{\mathcal{G}}] = T^{-2},
\] (11)

where \( \kappa \), the perturbed part of the trace of extrinsic curvature, will be introduced below.

Here we present the complete set of fully nonlinear perturbation equations without taking the temporal gauge \[5\].
The definition of \( \kappa \):
\[
\kappa \equiv 3 \frac{\dot{a}}{a} \left( 1 - \frac{1}{N} \right) - \frac{1}{N} \left( 1 + 2\varphi \right) \left[ 3\varphi + \frac{c}{a^2} \left( \chi^k + \frac{\chi^k}{1 + 2\varphi} \right) \right].
\]  
(12)

The ADM energy constraint:
\[
-\frac{3}{2} \left( \frac{\dot{a}^2}{a^2} - \frac{8\pi G}{3} \frac{\dot{a}}{a} - \frac{\Lambda c^2}{3} \right) + \frac{\dot{a}}{a} + \frac{c^2 \Delta \varphi}{a^2 (1 + 2\varphi)} = \frac{1}{6} \kappa^2 - 4\pi G \left( \frac{\overline{\varrho} + \overline{\varphi}}{c^2} \right) (\tilde{\gamma}^2 - 1) + \frac{3}{2} \frac{c^2 \Psi' \varphi_i}{c^2} + \frac{c^2}{4} \tilde{K}_i^j \tilde{K}_j^i.
\]  
(13)

The ADM momentum constraint:
\[
\frac{2}{c^2} \kappa_{ij} + \frac{c}{2a^2 N (1 + 2\varphi)} \left( \Delta \chi_i + \frac{1}{3} \chi^k_{,ik} \right) + 8\pi G \left( \frac{\overline{\varrho} + \overline{\varphi}}{c^2} \right) \dot{a} \tilde{\gamma} \tilde{\gamma} = \frac{c}{2a^2 N (1 + 2\varphi)} \left\{ \left( \frac{N}{N} - \frac{\varphi_i}{1 + 2\varphi} \right) \left( \frac{1}{2} \left( \chi_{,i} - \chi_{,i} \right) \right) - \frac{\varphi_{,i}}{1 + 2\varphi} \right\} + \frac{N}{1 + 2\varphi} \nabla_j \left[ \frac{1}{N} \left( \chi_{,j} + \chi_{,j} - \frac{2}{3} \delta_{,j} \chi_{,j} \right) \right].
\]  
(14)

The trace of ADM propagation:
\[
-\frac{3}{N} \left( \frac{\dot{a}}{a} \right)^2 - \frac{3}{2} \dot{a}^2 - 4\pi G \left( \frac{\overline{\varrho} + \overline{\varphi}}{c} \right) (\tilde{\gamma}^2 - 1) - \frac{c}{a^2 N (1 + 2\varphi)} \left[ \chi_{,j} - \chi_{,j} \right] + \frac{c^2 N_{,j}}{1 + 2\varphi} = \frac{c^2}{4} \tilde{K}_i^j \tilde{K}_j^i.
\]  
(15)

The tracefree ADM propagation:
\[
\left( \frac{1}{N} \frac{\partial}{\partial t} + \frac{3}{a} \frac{\dot{a}}{a} - k + \frac{c \chi^k}{a^2 N (1 + 2\varphi)} \nabla_k \right) \left\{ \frac{c}{a^2 N (1 + 2\varphi)} \left[ \frac{1}{2} \left( \chi_{,j} - \chi_{,j} \right) - \frac{2}{3} \delta_{,j} \chi_{,j} \right] - \frac{\kappa}{1 + 2\varphi} \right\} - \frac{c^2}{a^2 (1 + 2\varphi)} \left( 1 - \frac{\partial}{\partial t} - \frac{c \chi^k}{a^2 N (1 + 2\varphi)} \nabla_k \right) \left\{ \frac{1}{2} \left( \chi_{,j} - \chi_{,j} \right) - \frac{2}{3} \delta_{,j} \chi_{,j} \right\} + \frac{c^2}{a^2 N (1 + 2\varphi)} \left( \chi_{,j} \chi_{,j} - \chi_{,j} \chi_{,j} \right)
\]  
(16)

The covariant energy conservation:
\[
\left[ \frac{\partial}{\partial t} + \frac{1}{a (1 + 2\varphi)} \left( \nabla \chi^k + \frac{c}{a} \chi^k \right) \nabla_k \right] \overline{\varrho} + \overline{\varphi} \left( \frac{\overline{\varrho} + \overline{\varphi}}{c^2} \right) \left\{ \frac{1}{2} \left( \chi_{,j} - \chi_{,j} \right) - \frac{2}{3} \delta_{,j} \chi_{,j} \right\} = 0.
\]  
(17)

The covariant momentum conservation:
\[
\left[ \frac{1}{a^2 N} \frac{\partial}{\partial t} + \frac{1}{a (1 + 2\varphi)} \left( \nabla \chi^k + \frac{c}{a} \chi^k \right) \nabla_k \right] \overline{p}_j + \overline{\varphi} \left( \frac{\overline{\varrho} + \overline{\varphi}}{c^2} \right) \left[ \frac{\partial}{\partial t} + \frac{1}{a (1 + 2\varphi)} \left( \nabla \chi^k + \frac{c}{a} \chi^k \right) \nabla_k \right] \overline{p} = 0.
\]  
(18)
where $\vec{K}$ and $N$ are the tracefree part of the extrinsic curvature and the lapse function, respectively, with

$$
\vec{K}/\vec{K}_i = \frac{1}{a^2 N^2 (1 + 2 \varphi)^2} \left[ \frac{1}{2} \chi (\chi_{ij} + \chi_{ji}) - \frac{1}{3} \chi^i \chi^j - \frac{4}{1 + 2 \varphi} \frac{1}{2} \chi^i \varphi^j (\chi_{ij} + \chi_{ji}) - \frac{1}{3} \chi^i \chi^j \varphi^j \right]
+ \frac{2}{(1 + 2 \varphi)^2} \left( \chi^i \chi^j \varphi^i \varphi^j + \frac{1}{3} \chi^i \chi^j \varphi^i \varphi^j \right), \quad N \equiv \sqrt{1 + 2 \alpha + \frac{\Lambda}{a^2 (1 + 2 \varphi)}}.
\tag{19}
$$

Apparently the basic set of perturbation equations in Einstein’s gravity in (12)-(18) looks quite complicated compared with the Newtonian ones in (1)-(3). In fact equations (12)-(18) are redundant; for example, the Einstein’s equations imply the conservation equations. More importantly, in the above equations, we have not taken the temporal gauge condition yet. Equations (12)-(18) are presented without taking the temporal gauge (hypersurface or slicing) condition. As the temporal gauge condition we can impose any one of the following conditions

- comoving gauge: $\tilde{v} \equiv 0$,
- zero-shear gauge: $\chi \equiv 0$,
- uniform-curvature gauge: $\varphi \equiv 0$,
- uniform-expansion gauge: $\kappa \equiv 0$,
- uniform-density gauge: $\delta \equiv 0$,

or combinations of these to each perturbation order; we may call these the fundamental gauge conditions. With the imposition of any of these temporal gauge conditions the remaining perturbation variables are free from the remnant (spatial and temporal) gauge mode, and have unique gauge-invariant combinations. Thus, we can regard each perturbation variable in those gauges as the gauge-invariant one to nonlinear order [3, 5].

To the background order, (13), (15) and (17), respectively, give

$$
\frac{\ddot{a}}{a^2} = \frac{8 \pi G}{3} \phi + \frac{\Lambda c^2}{3}, \quad \frac{\ddot{a}}{a} = -4 \pi G \left( \frac{\dot{\phi} + \frac{p}{c^2}}{3} + \frac{\Lambda c^2}{3} \right), \quad \dot{\phi} + 3 \frac{\dot{a}}{a} (\dot{\phi} + \frac{p}{c^2}) = 0,
\tag{21}
$$

where $\Lambda$ is the cosmological constant. In the Newtonian limit we ignore $p$ compared with $\dot{\phi} c^2$.

4. Newtonian limit

The infinite speed-of-light limit leads to the Newtonian equations [6]. That is, as the Newtonian limit we consider the weak-gravity, the slow-motion, negligible pressure and internal energy compared with the energy density, and the small-scale (subhorizon) limits

$$
\alpha \ll 1, \quad \varphi \ll 1, \quad \frac{\tilde{v}^2}{c^2} \ll 1, \quad \frac{\tilde{p}}{\sqrt{\phi} c^2} \ll 1, \quad \frac{\ddot{v}}{c^2} \ll 1, \quad \frac{\ddot{a}^2}{a^2 H^2} \gg 1,
\tag{22}
$$

where $k$ the comoving wave-number with $\Lambda = -k^2$; $H \equiv \dot{a}/a$; in the presence of the cosmological constant $\Lambda$, we consider $H^2 \sim 8 \pi G \phi$. We identify

$$
\alpha = \frac{1}{c^2U}, \quad \varphi = \frac{1}{c^2}V, \quad \tilde{v} = v,
\tag{23}
$$

where $U$ and $V$ correspond to the Newtonian and the post-Newtonian perturbed gravitational potentials, respectively [6, 7]; equation (15) gives $\varphi = -\alpha$, thus $V = U$. We can show that equations (1)-(3) follow from equations (17), (18), and (15), respectively, in both the zero-shear gauge ($\chi \equiv 0$) and the uniform-expansion gauge ($\kappa \equiv 0$).
Comparing equation (5) in the perturbation theory with equation (24) in the PN approach, to the 1PN order we have

\[ ds^2 = -\left(1 - \frac{1}{c^2} 2U + \frac{1}{c^4} \left(2U^2 - 4\Phi\right)\right) c^2 dt^2 - \frac{1}{c^2} 2aP \cdot c dt dx^i + \alpha \left(1 + \frac{1}{c^2} 2V\right) \delta_{ij} dx^i dx^j, \tag{24} \]

where the index of \( P \) is raised and lowered by \( \delta_{ij} \). Similarly as in the metric of perturbation theory in equation (5), in the spatial part of the metric we already have taken spatial gauge conditions without losing any generality, and have ignored the TT-type perturbation \([7]\). The dimensions are the following

\[ [U] = [V] = c^2, \quad [P] = c^3, \quad [\Phi] = c^4. \tag{25} \]

Comparing equation (5) in the perturbation theory with equation (24) in the PN approach, to the 1PN order we have

\[ \alpha = -\frac{1}{c^2} \left[U - \frac{1}{c^2} \left(U^2 - 2\Phi\right)\right], \quad \varphi = \frac{1}{c^2} V, \quad \kappa = -\frac{1}{c^2} \left(\frac{\partial}{\partial t} + \frac{1}{a} \nabla \cdot \mathbf{v}\right), \quad \chi_i = \frac{1}{c^2} aP_i. \tag{26} \]

Using the identifications between the two approaches made in equation (26), we can derive the 1PN equations from the perturbation equations in (12)–(18); \( \mathbf{v} \) is used as the fluid three-velocity in \([7]\). Equation (15) to 1PN order gives \( V = U \). From equations (17), (18), (15) and (14), respectively, we can show

\[ \frac{1}{a^2} (a^2 \tilde{\gamma}' + \frac{1}{a} \partial \tilde{\gamma}) = -\frac{1}{c} \left[\tilde{\gamma} \left(\frac{\partial}{\partial t} + \frac{1}{a} \nabla \cdot \mathbf{v}\right) \left(\frac{1}{2} \nabla^2 + 3U + \Pi\right) + \left(\frac{3}{a} + \frac{1}{a^2} \nabla \cdot \mathbf{v}\right) \tilde{p}\right], \tag{27} \]

\[ \frac{1}{a} (a \tilde{\gamma})' + \frac{1}{a} \tilde{\gamma} \mathbf{v}' - \frac{1}{a} \tilde{\gamma} U_j + \frac{1}{a^2} \frac{\partial}{\partial t} \tilde{\gamma} = \frac{1}{c^2} \left[\frac{1}{a^2} \nabla^2 U_j + \frac{2}{a} \left(\Phi - U^2\right)_j + \frac{1}{a^2} (aP_j) - \frac{1}{a^2} (P_{ik} - P_{ki}) \right. \]

\[ \left. + \frac{1}{a} \tilde{\gamma} \mathbf{v}' + 4U + \Pi + \frac{\tilde{p}}{a} \mathbf{v}' - \tilde{v}_i \left(\frac{\partial}{\partial t} + \frac{1}{a} \nabla \cdot \mathbf{v}\right) \left(\frac{1}{2} \nabla^2 + 3U \right) - \frac{\tilde{v}_i}{c} \left(\frac{\partial}{\partial t} + \frac{1}{a} \nabla \cdot \mathbf{v}\right) \tilde{p}\right], \tag{28} \]

\[ \frac{\Delta}{a^2} U + 4\pi G (a\tilde{\gamma} - \tilde{p}) = \frac{1}{c^2} \left\{\frac{1}{a^2} \left[2\Delta \Phi - 2U \Delta U + (aP_j)'\right] + 3U + \frac{\alpha}{a} \mathbf{U} + \frac{6\alpha}{a} U \right. \]

\[ + 8\pi G \left[\tilde{p} \mathbf{v}' + \frac{1}{2} \left(\tilde{\gamma} \Pi - \tilde{\gamma} \Pi\right) + \frac{3}{a} \left(\tilde{\gamma} - \tilde{p}\right)\right]\left(\frac{\partial}{\partial t} + \frac{1}{a} \nabla \cdot \mathbf{v}\right), \tag{29} \]

\[ 0 = \frac{1}{a^2} \left(P_{ik} - \Delta P_i\right) - 16\pi G \tilde{\gamma} \mathbf{v}' + \frac{4}{a} \left(U + \frac{\alpha}{a} \mathbf{U}\right), \tag{30} \]

where the right-hand-sides are 1PN order, and we have recovered \( \Pi \) explicitly; \( \mathbf{v}^2 \equiv \mathbf{v} \cdot \mathbf{v} \). These are the same as cosmological 1PN equations in \([4, 7]\). To 1PN order, from equation (30) we have

\[ \tilde{v}_i = \left(1 - \frac{3}{c^2} U\right) \tilde{v}_i + \frac{1}{c} \frac{\alpha}{a} P_i, \quad \tilde{v}_i = \left(1 - \frac{v^2}{2c^2}\right) v_i, \tag{31} \]

where \( v^2 \equiv \mathbf{v} \cdot \mathbf{v} \). Notice that in order to derive the 1PN equations we have not imposed the temporal gauge condition; in the PN expansion \( \chi_i \) in equation (26) is already higher order. In the above equations we can still impose a gauge condition \([7]\)

\[ \frac{1}{a} P_{ik} + n \mathbf{U} + m \frac{\alpha}{a} U = 0, \tag{32} \]

where \( n \) and \( m \) can be arbitrary real numbers. The uniform-expansion gauge takes \( n = 3 = m \), and the transverse-shear gauge takes \( n = 0 = m \). We note that the comoving gauge, the uniform-curvature gauge and the uniform-density gauge are not available in the PN approximation.
6. Relativistic pressure corrections

We can derive relativistic pressure correction terms in equations (1)-(3) by relaxing the conditions $\tilde{\rho} \ll \bar{\rho} c^2$ and $\Pi/c^2 \ll 1$ in equation (22). We take the zero-shear gauge by setting $\chi \equiv 0$. From equations (17), (18), and (15), respectively, we have

$$\dot{\tilde{\rho}} + 3 \frac{\dot{a}}{a} \left( \frac{\tilde{\rho}}{c^2} + \tilde{p} \right) + \frac{1}{a^2} \nabla \cdot \left( \left( \frac{\tilde{\rho}}{c^2} + \tilde{p} \right) \nabla \tilde{v} \right) = \frac{1}{2} \frac{\tilde{p}}{c^2} \nabla \cdot \tilde{p},$$

(33)

$$\dot{\tilde{v}} + \frac{\dot{a}}{a} \tilde{v} + \frac{1}{a^2} \nabla \tilde{v} = -\frac{1}{\tilde{\rho} + \tilde{p}/c^2} \frac{1}{a^2} \nabla \tilde{p} + \frac{1}{a^2} \tilde{v},$$

(34)

$$\frac{\Delta}{a^2} U = -4\pi G \left( \tilde{\rho} - \rho \right),$$

(35)

where $\tilde{\rho}$ now includes the internal energy. We note that the pressure corrections in the above equations differ from the known ones in the literature [8]. The term in the right-hand-side of equation (33) was not known in the literature. Notice, in particular, the absence of pressure correction term in equation (35). The presence of a pressure correction term, $-12\pi G (\tilde{p} - p)$, in the right-hand-side of equation (35) was often suggested in the literature [9, 8].

7. Discussion

The Newtonian equations in (1)-(3) are derived as the $1/c \to 0$ limit of Einstein’s theory in the zero-shear gauge and the uniform-expansion gauge. The 1PN equations in (27)-(30) are derived as a weak gravity limit keeping $c^2$-order terms compared with the Newtonian limit. Both the Newtonian and 1PN equations are correctly recovered from our fully nonlinear and exact formulation of cosmological perturbation in Einstein’s gravity in equations (12)-(18). In all (Newtonian, 1PN, and Einstein’s) cases the equations are fully nonlinear and exact. In the nonlinear cosmological perturbation equations in the Einstein’s gravity and in the 1PN approximation we can still impose one temporal gauge condition suggested in equations (20) and (32), respectively.

As we consider nonlinear perturbations, ignoring the TT part can be regarded as a serious assumption restricting the range of applications of our nonlinear perturbation formulation in Einstein’s gravity. Except for this shortcoming, as we have not decomposed the background parts, the basic set in equations (12)-(18) can also be regarded as an exact one; by contrast, except for the mass conservation equation, the equations in the Newtonian and 1PN approximation are valid only to the perturbed variables in cosmology; the cosmological background equations are subtracted by using the equation derived in Einstein’s gravity [7]. Meanwhile, by setting the background order quantities as $a \equiv 1$ and $\rho \equiv 0 \equiv p$, equations (1)-3 and equations (27)-(30) are valid for the Newtonian and 1PN hydrodynamic equations, respectively, in the Minkowski background [1, 6].

Derivation of the equations in these proceedings with some applications will be presented in [5, 10].

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References

[1] P.J.E. Peebles, *The large-scale structure of the universe*, (Princeton Univ. Press, Princeton, 1980).
[2] E.M. Lifshitz, J. Phys. (USSR) 10 (1946) 116; J.M. Bardeen, Phys. Rev. D 22 (1980) 1882; H. Kodama, M. Sasaki, 1984, Prog. Theor. Phys. Suppl., 78, 1; V.P. Mukhanov, H.A. Feldman, R.H. Brandenberger, Phys. Rep. 215 (1992) 203.
[3] J.M. Bardeen, *Particle Physics and Cosmology*, edited by L. Fang, A. Zee (Gordon and Breach, London, 1988) 1; J. Hwang, Astrophys. J. 375 (1991) 443.
[4] M. Bruni, S. Matarrese, S. Mollerach, S. Sonego, . Class. Quant. Grav. 12 (1995) 2585; H. Noh, J. Hwang, Phys. Rev. D 69 (2004) 104011; K.A. Malik, D. Wands, Phys. Rep. 475 (2009) 1; K. Nakamura, Advances in Astronomy 2010 (2010) 576273.
[5] J. Hwang, H. Noh, (2012) arXiv:1207.0264, Monthly Not. Roy. Astron. Soc. in press (2013).
[6] S. Chandrasekhar, Astrophys. J. 142 (1965) 1488; S. Chandrasekhar, Y. Notsu, Astrophys. J. 158 (1969) 25; L. Kofman, D. Pogosyan, Astrophys. J. 442 (1995) 30.
[7] J. Hwang, H. Noh, D. Puetzfeld, JCAP 03 (2008) 010; H. Noh, J. Hwang, Astrophys. J. 757 (2012) 145.
[8] E.R. Harrison, Ann. of Phys. 35 (1965) 437; P. Coles, F. Lucchin, Cosmology: The origin and evolution of cosmic structure (Wiley, London, 1995); J.A.S. Lima, V. Zanchin, R. Brandenberger, Monthly Not. Roy. Astron. Soc. 291 (1997) L1; T. Harko, Monthly Not. Roy. Astron. Soc. 423 (2011) 3059.
[9] E.T. Whittaker, Proc. Roy. Soc. A. 149 (1935) 384; W.H. McCrea, Proc. Roy. Soc. A. 206 (1951) 562.
[10] J. Hwang, H. Noh, JCAP 04 (2013) 035; H. Noh, J. Hwang [arXiv:1307.6270] JCAP in press (2013).