Can supersymmetric loops correct the fermion mass relations in SU(5)?

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We investigate three different possibilities for improving the fermion mass relations that arise in grand unified theories (GUTs). Each scenario relies on supersymmetric loop effects alone, without modifying the naive Yukawa unification. First, we consider $A$-terms that follow the usual proportionality condition. In this case SUSY effects can improve the mass relations, but not completely. Interestingly, imposing Yukawa coupling unification for two families greatly constrains the range of parameters in the MSSM. Secondly, we employ a new ansatz for the tri-linear $A$-terms that satisfies all experimental and vacuum stability bounds, and can successfully modify the mass relations. Finally, we investigate the use of general (non-proportional) $A$-terms, with large off-diagonal entries. In this case flavor changing neutral current (FCNC) data present an important constraint. We do not pretend to present a complete, motivated theory of fermion masses. Rather this paper can be viewed as an existence proof, serving to show that Yukawa coupling unification can occur even within the framework of minimal GUTs.

I. INTRODUCTION

Unified theories that incorporate supersymmetry have met with great success [1]. Included in the attractive features of such theories are: the possibility of ameliorating the hierarchy problem, a natural radiative mechanism for electroweak symmetry breaking, and a candidate for dark matter. One way to solve this problem is to invoke higher-dimensional operators [4], without modifying the naive Yukawa unification. As a result, we can have a framework that essentially postpones the problem to energies beyond $M_{GUT}$. Another approach involves the use of non-minimal representations for the Higgs sector. For instance, in the context of a SU(5) GUT, by enlarging the Higgs sector with a $45$ representation, Georgi and Jarlskog [5] obtained the relations $m_d = 3m_e$ and $m_s = m_{\tau}/3$, which correctly reproduces the observed fermion masses when evolved down to low energies.

In this paper we present a simpler approach which has been overlooked thus far. Instead of adding extra matter or higher-dimensional operators to the theory, we work within the MSSM framework. We use a radiative mechanism to correct the light generation mass relations, i.e. $m_d = m_e$, $m_s = m_{\mu}$, in a manner that respects the successful relation $m_\tau = m_{\mu}$. It is an exciting possibility that one could accommodate the unification of the Yukawa couplings within the simplest of SUSY theories, the MSSM.

The correction of fermion masses by SUSY loops has been known for a while [6]. These corrections have been applied in an attempt to generate fully the quark and lepton masses [7,8]. The idea here is to use these radiative effects in a somewhat more modest way, to fix up the GUT relations, a task which we find to be non-trivial. We will investigate this idea within three contexts. First, we investigate the case of proportional $A$-terms. Then, we examine two cases with non-minimal $A$-terms. In our second approach, we use a particular ansatz that achieves unification for all three families. In the final approach, we recognize that $A$-terms that are slightly more generic than the proportional case can also help remedy the mass relations. That is to say, if one allows off-diagonal $A$-terms, one can remedy the Yukawa relations. It appears that such a scenario is still consistent with flavor-changing neutral current (FCNC) bounds.

II. STANDARD A-TERM CORRECTIONS

We are interested in evaluating the corrections to fermion masses coming from SUSY loops. First, we consider the standard case where we have $A$-terms
that are proportional to the Yukawa matrices. The dominant corrections are the flavor conserving (FC) gluino and Higgsino-mediated loops. We include only these corrections in our analysis. Since these corrections depend on only a few MSSM parameters it is feasible to perform a detailed numerical analysis. Because these corrections also modify the CKM mixing matrix, we prefer to follow a procedure consistent with all low-energy data. Our chosen method is essentially a bottom-up approach, and consists of these steps:

- Working in a basis where lepton and u-type quark mass matrices are diagonal, we obtain the non-diagonal d-quark mass matrices \( m_d \), at the \( M_Z \)-scale. If we assume that the mass matrix is symmetric, we can write:

\[
m_d = V_{CKM} \cdot m^{\text{diag}}_d \cdot V_{CKM}^T,
\]

where \( V_{CKM} \) denotes the quark mixing matrix. The lepton mass matrix, \( m_l \), is diagonal. We can extract the quark mass matrix at \( M_Z \) utilizing the quark masses in Table I, by taking into account the QCD renormalization effects.

- We then modify the mass matrix of the down-type quarks by subtracting the contributions of the SUSY loops. The SUSY corrections to the lepton mass matrix are negligible. The flavor conserving (FC) corrections to the down quarks includes the gluino and Higgsino loops, namely \( (\delta m_d)^{\text{FC}} = (\delta m_d)^g + (\delta m_d)^{\tilde{H}} \).

- The Yukawa couplings are simply related to the mass matrix through the vacuum expectation value of the down-type Higgs. Namely,

\[
Y^d = \frac{m_d}{v \sin \beta},
\]

where \( v = 174 \text{ GeV} \). We convert the Yukawa couplings from the \( \overline{MS} \) scheme to the \( \overline{DR} \) scheme, using the dictionary of Martin and Vaughn. This is necessary to be compatible with the renormalization group equations used in the next step.

- Once we have the Yukawa couplings of both quarks and leptons at \( M_Z \) in hand, we evolve the Yukawa matrices up to the GUT scale \( (M_{GUT}) \), using the renormalization group equations of the MSSM. This yields the Yukawa matrices at the GUT scale \( Y^d(M_{GUT}) \) and \( Y^l(M_{GUT}) \).

- The next step is to diagonalize the Yukawa matrix, \( Y^d(M_{GUT}) \), and compare its eigenvalues with the lepton Yukawa couplings at the GUT scale, \( Y^l(M_{GUT}) \), to see whether unification has occurred. To quantify this unification we define

\[
\epsilon_i = |(Y_{ii}^d - Y_{ii}^l)/Y_{ii}^l|.
\]

- We repeat the above steps, scanning the parameter space of the MSSM, to search for those regions where \( \epsilon_i \) is close to zero.

Now that we have outlined the steps in the analysis, we note the explicit corrections to the Yukawa couplings. The gluino loop, \( (\delta m_d)^g \), contributes to all the diagonal elements \( m_{dd,ss,bb} \). We work in the limit when \( \mu \tan \beta \) is much larger than the contribution from the tri-linear scalar coupling. In this limit, the gluino loop contribution may be expressed as:

\[
(\delta m_d)^g = \frac{2\alpha_s \mu M_X}{3\pi} I(x) \tan \beta m_d,
\]

The Higgsino-mediated loop \( (\delta m_d)^{\tilde{H}} \), is only relevant for the third family due to the suppression from small Yukawa couplings. Its contribution is given by:

\[
(\delta m_d)^{\tilde{H}} = \frac{Y_{ii} \mu A_t}{4\pi M_X} I(x) \tan \beta \delta_{i3} \delta_{j3}.
\]

As usual, \( M_X \) denotes the gluino mass, \( \mu \) is the parameter for the Higgs term in the superpotential, and \( A_t \) denotes the tri-linear scalar term for top. \( M_X \) denotes the largest mass appearing in the loop, and \( I(x) \) denotes the loop integral, given by

\[
I(x) = \frac{-1 + x - x \log(x)}{(1 - x)}.
\]

where \( x = m_\nu^2/m_q^2 \) for the gluino loop, and \( x = \mu^2/m_\nu^2 \) for the higgsino loop.

The key point in this analysis is to recognize that the higgsino and gluino pieces come with an opposite sign. As a result these two contributions may nearly cancel each other for the third generation, thereby preserving the \( m_b = m_t \) relation to some extent. However, since the higgsino piece only contributes to the third generation, the gluino piece remains uncanceled for the other two families and can be utilized to fix up a mass relation of

\[
\begin{array}{|c|c|}
\hline
\text{Input} & \text{Value (\( \overline{MS} \) scheme)} \\
\hline
\alpha_s(M_Z) & 0.119 \\
\sin \theta_W & 0.23124 \\
\alpha_s(M_Z) & 127.943 \\
m_t(m_t) & 165 \text{ GeV} \\
m_b(m_b) & 4.25 \text{ GeV} \\
m_c(m_c) & 1.15 \text{ GeV} \\
m_s (2 \text{ GeV}) & 115 \text{ MeV} \\
m_d (2 \text{ GeV}) & 6 \text{ MeV} \\
m_u (2 \text{ GeV}) & 3 \text{ MeV} \\
m_s (m_s) & 1.777 \text{ GeV} \\
m_b (m_b) & 105 \text{ MeV} \\
m_c (m_c) & 0.511 \text{ MeV} \\
\hline
\end{array}
\]
course, it can only be used for one of the two families. It will exacerbate the problem for the remaining family.

The results for the parameter \( \epsilon_i \) for the 3 generations are shown in FIG. 1-3. These graphs correspond to \( m_\beta = m_\tilde{q} = 500 \text{ GeV} \), and \( \mu = -600 \text{ GeV} \). Notice that there is a limited region in the \( (A_t, \tan \beta) \) plane where unification (say \( |\epsilon| < .2\% \)) occurs for both the the first and third generation. In fact, if we assume that this approach is part of the solution to remedying the mass relations, we can view the results of this analysis as making predictions for \( \tan \beta \) and \( A_t \). We simply mention here that the other sign of \( \mu \) allows one to achieve unification for the second and third generation. Though this appears to be more difficult, and requires large values of \( \tan \beta \) where the perturbativity of the b-quark Yukawa coupling is an issue.

One could view this scenario with some wariness, as it remedies the mass relation for one generation at the expense of exacerbating it for the other generation. Nevertheless, this sort of scenario allows one to get the correct fermion spectrum by, for instance, introducing just one higher-dimensional operator involving the remaining family. In some sense, we have achieved “flavor from no flavor”.

**III. A NON-MINIMAL ANSATZ FOR THE \( A \)-TERMS.**

Since our initial efforts with proportional \( A \)-terms did not succeed in achieving complete unification, we are forced to resort to a more general form for the \( A \)-terms, thereby opening the door for a larger number of free-parameters. This makes it more difficult to perform a detailed numerical analysis. Instead, we shall be content to present an “existence proof” of the viability of these mechanisms. In the following, we shall employ a top-down approach, by evolving the Yukawa couplings from the GUT-scale to the weak-scale, and then including the threshold corrections.

We shall begin by considering a particular ansatz for the tri-linear terms, aimed at correcting the Yukawa...
matrices for the light generations. In this section, we say nothing about $b$-$\tau$ unification, since that relation is already close, and presumably can be fixed up by methods similar to those used in the previous section. So, let us start by writing the down-type fermion mass matrix of first and second generations as:

$$M^0 = \begin{pmatrix} 0 & \lambda m \\ \lambda m & m \end{pmatrix}$$

(6)

Thus, at GUT scale one has $m^0_{\mu} = m_{\tau}^0 = m$, and $\lambda^2 = m^0_{\mu}/m^0_{\tau} = m^0_{d}/m^0_{u}$. The superscript 0 will indicate fermion masses evaluated at the GUT scale. Now, we shall include the correction coming from the gaugino-fermion loops, using the an ansatz for the $A$-terms that results in the following contribution to the squark mass matrices:

$$\delta m^2_{LR} \supset \begin{pmatrix} 0 & 0 \\ 0 & X \end{pmatrix} v \cos \beta + \begin{pmatrix} 0 & \lambda \\ \lambda & 1 \end{pmatrix} AY_{s}v \cos \beta$$

(7)

Here $Y_{s}$ is the Yukawa coupling for the strange quark. We have separated out the first piece to emphasize the largeness of the tri-linear term, $X$, that is necessary as we will see below. The grand unification of the term $X$ to be common to both charged leptons and down-type quarks. We note that in the mass-eigenstate basis for the fermions, the first matrix on the right-hand side becomes off-diagonal, which can lead to FCNC problems. We will address this issue shortly. Despite our concerns about FCNCs, and the fact that we do not yet have a microscopic theory to motivate our ansatz, we will boldly forge ahead. We now show that this form for the $A$-terms proves to be phenomenologically very interesting in repairing the mass relations.

The dominant correction to the quark masses arises from the gluino-mediated loop. This is larger than the bino-loop contribution due to the difference in the gauge couplings. Therefore, for questions of unification, we can again neglect the corrections to lepton masses. Moreover, because of the form chosen for the tri-linear coupling terms, Eqn. (6), the gluino-loop contributes mostly to the $(2,2)$ entry of the quark mass matrix. For simplicity, we neglect all corrections but this one. After this correction, the lepton mass matrix approximately retains the form of Eqn. (6), while the quark mass matrix becomes:

$$M_{d} = \begin{pmatrix} 0 & \lambda m \\ \lambda m & m + \delta m_{s} \end{pmatrix}$$

(8)

The basic idea will be to utilize the gluino loop correction, $\delta m_{s}$, to cancel a substantial portion of the muon mass, $m$, derived from the Yukawa coupling.

To determine the precise amount of cancellation necessary, one must look at the experimental data at low energy. In particular, given the mass matrix of the form in Eqn. (6), one generates a Cabibbo angle, $\lambda_{c}$ given by the relationship:

$$\lambda_{c} = \frac{m_{\mu}}{m_{\tau}}$$

We note that the masses in this equation are the physical masses, which yields $\lambda_{c} \approx 0.22$. This value is consistent with other determinations of $\lambda_{c}$, a well-known fact. Now, utilizing the diagonalized form of Eqn. (6) for the corrected mass matrix, we can write:

$$\lambda_{c} \approx \frac{\lambda m_{s}}{(m_{s}^2 + \delta m_{s})/(1 + \lambda_{c}^2)}$$

(10)

We also note that the ratio $\lambda^2 = m^0_{\mu}/m^0_{\tau}$ is independent of scale (to one loop), so we can fix the value of $\lambda$ at the weak-scale using the physical lepton masses, namely $\lambda \sim 0.07$. This gives us the relation:

$$1 + \frac{\delta m_{s}}{m_{s}^2} \approx \frac{1}{3}$$

(11)

Also, we note that, after RGE running, we have the approximate relation, $m_{\tilde{g}}^2 \approx 2.1 m_{\mu}$.

Finally, we wish to determine the constraint on the SUSY parameters. To do this, we need to relate the SUSY parameters to $\delta m_{s}$. The correction due to the gluino loop can be written as [13]:

$$\delta m_{s} = \frac{2\alpha_{s} m_{LR}^{2} M_{X}}{3\pi M_{X}^{2}} f(x)$$

(12)

where $m_{LR}^{2}$ is given in Eqn. (6). We assume that $m_{LR}^{2}$ is completely dominated by the piece proportional to $X$. Here, $M_{X}$ denotes the largest mass appearing in the loop, and $f(x)$ denotes the loop integral, given in Eqn. (6), which becomes close to 1 in magnitude at small $x$. Indeed, we will be considering small $x$, as we will see $m_{\tilde{g}}^2 \approx m_{\tilde{g}}^2 \gg M_{g}^2$, is necessitated by the FCNC data coupled with concerns about naturalness. In the following, we denote the mass of all heavy scalar particles by $m_{\tilde{g}}$.

Therefore, combining Eqs. (6), (11), and (12), we find that the supersymmetric parameters must satisfy the following constraint if we are remedy the mass relations:

$$\frac{M_{g} X \cos \beta}{m_{\tilde{g}}^2} \approx 0.035$$

(13)

As we intimated earlier, it is a non-trivial task to reconcile this requirement with FCNC data. In particular, a strong constraint comes from the $\mu \rightarrow e\gamma$ limit [13]. We find that once a gluino mass is chosen, Eq. (13), along with the limit from $\mu \rightarrow e\gamma$, sets the mass scale for the sleptons. For the gluino, we choose a mass of 600 GeV, and utilize the relation $M_{g} \approx 7 M_{1}$ that arises from the unification of gaugino masses. A rotation of the matrix of Eq. (6) to the diagonal fermion mass basis generates a $\delta_{12}$ in the lepton sector given by
\[
(\delta_{12})_{LR} \sim \frac{X \cos \beta}{m^2} \frac{\lambda v}{M_\tilde{g}} = \left( \frac{M_\tilde{g} X \cos \beta}{m^2} \right) \frac{\lambda v}{M_\tilde{g}} = 7.1 \times 10^{-4}.
\]

Here, we have inserted the value for \( \lambda \) and used Eqn. \([13]\). Using this value for \((\delta_{12})_{LR}\) and the constraint on the \( \mu \to e\gamma \) branching ratio, one can find \( \tilde{m} \gtrsim 4.4 \text{ TeV} \).

It then remains to check whether a scenario with 4.4 TeV sleptons and squarks satisfies the remaining FCNC data. Note that the expression for \((\delta_{12})_{LR}\) is identical to Eq. \((14)\), with the replacement \( \lambda \to \lambda_c \). Using the analysis of \([13]\), which includes NLO QCD effects, we have checked that such a scenario satisfies the bounds from \( \Delta M_K \) by about a factor of 5. However, constraints from \( \epsilon \) \([13]\), and \( \epsilon'/\epsilon \) \([14]\) \([13]\), are not satisfied unless the phase of \((\delta_{12})_{LR}\) is less than about 0.2. We have also checked, adapting the analysis of \([13]\), that this scenario is safe from measurements of the anomalous magnetic moment of the muon \([20]\).

Incidentally, considering the large value of the tri-linear coupling required in this scenario, one might be concerned about charge and color breaking minima of the scalar potential. For the values of \( M_3 \) and \( \tilde{m} \) described above, with \( \tan \beta \sim 2 \), we have \( X \approx 2500 \). Although it is possible that such a scenario might not have the correct electroweak symmetry breaking (EWSB) vacuum as an absolute minimum, \( X \approx \tilde{m} \) does allow the EWSB vacuum satisfying the less stringent criteria of meta-stability \([21]\). That is to say, we succeed in producing a EWSB vacuum possessing a lifetime greater than the age of the universe.

Finally, we wish to comment on proton decay in this scenario. One might expect the proton decay rate to be significantly affected on two counts. First of all, the first and second generation scalar masses are pushed to several TeV. However, increasing the masses for the first and second generation superpartners is not enough to decrease the proton decay rate predicted by SU(5) GUTs, which admittedly have difficulty accommodating the experimental bounds.

Finally, it should be noted that the unification of fermion masses with this \textit{ansatz} requires both scalars of a few TeV, while gauginos may remain relatively light. Many possibilities exist for such scenarios \([24]\). However, none of these scenarios generates a large enough tri-linear coupling, \( X \). Until a micro-physical motivation can be found for this case, the above discussion must be simply regarded as an existence proof.

**IV. CORRECTIONS FROM FLAVOR-CHANGING GLUINO INTERACTIONS**

Finally, let us consider a scenario in which we allow effects from flavor violating (FV) gluino-quark-squark interactions to enter into our analysis. Such contributions are known to induce potentially dangerous FCNC effects. Therefore, we must be careful to satisfy such bounds.

In this case, the mass-insertion method proves somewhat complicated, as several insertions are necessary. Therefore, we find it convenient to write the vertex \( \tilde{g} - \tilde{d}_L - d_L - \tilde{d}_R - d_R \), in terms of \( 3 \times 3 \) mixing matrices, \( W_{L,R} = V^d_{L,R} V^{d_1}_{L,R} \), where \( V^d_{L,R} \) denote the rotation matrices required to diagonalize the fermion (sfermion LL,RR) mass matrices. A discussion of the bounds on the \( W_{L,R} \) matrices is presented in \([10]\).

In general, the corrections to fermion masses will include corrections from both gluino and Higgsino mediated loops. In the large \( \tan \beta \) limit, it is reasonable to include the Higgsino loop only for the heavier fermions (third generation), whereas the gluino flavor conserving (FC) loop contributes to all diagonal elements of the fermion mass matrices. The size of the gluino loop is proportional to the fermion mass itself. Thus, we can write the correction to quark masses as: \( \delta m_q = (\delta m_d)_{FC} + (\delta m_d)^{FV} \). For the quarks, \( (\delta m_d)^{FV} \) is given in the previous section in Eqn. \((12)\), the FV gluino contribution is given by \([10]\),

\[
(\delta m_d)^{FV} = 0.7 m_b \frac{\mu}{M_\tilde{g}} \frac{\tan \beta}{60} \frac{\chi^{1/2} \tilde{H}}{0.5} T_{3i}^d
\]

where \( T_{3i}^d = W^d_{L,R} W_{R,3i}^{d_1} \). \( \tilde{H} \) denotes a combination of loop integrals, analogous to Eqn. \((3)\), and in general \( \tilde{H} \approx 0.5 \). Note that this equation is proportional to \( m_b \), which will allow these corrections to reach substantial values, a prerequisite for correcting the GUT relations.

In order to obtain the required values for the fermion masses, it seems convenient to fix the values of the FC and FV contributions, in such a way that their contribution to \( m_b \) cancels. Again, we neglect the contribution to the leptons. In this case, the ratios \( m_{d_i}/m_{l_i} \) can be written as:

\[
\frac{m_{d_i}}{m_{l_i}} = \left( 1 + \frac{\delta m_{d_i}}{m_{d_i}} \right) \frac{m_{d_i}^0}{m_{l_i}^0}.
\]
Here, the 0 superscript again denotes evaluation at the GUT-scale. In the above expression, $\delta m_{d, s}/m_{d, s}$ includes contributions from both FC and FV pieces. In order to generate the correct mass ratios, $\delta m_{d, s}/m_{d, s}$ must in turn satisfy:

$$
\left( \frac{\delta m_{d}}{m_{d}} \right)^{FC} + \left( \frac{\delta m_{d}}{m_{b}} \right)^{FV} \frac{m_{b}}{m_{d}} = 2 \quad (17)
$$

$$
\left( \frac{\delta m_{s}}{m_{s}} \right)^{FC} + \left( \frac{\delta m_{s}}{m_{b}} \right)^{FV} \frac{m_{b}}{m_{s}} = -\frac{2}{3} \quad (18)
$$

After substituting the expressions for the factors resulting from Eqn. (8), one finds that the correct mass ratios require: $T_{31}^{d} T_{32}^{d} \approx 10^{-5}$ (for $\tan \beta = 45$). This solution is in agreement with current bounds on FCNC phenomenology, which imply: $T_{31}^{d} T_{32}^{d} < 2 \times 10^{-5}$ \[10\]. Thus, the quark-lepton mass relations for all three families can be remedied, using a large effect from both FC and FV corrections.

There could be some concern about whether the Yukawa coupling of the b quark would remain perturbative in such a large $\tan \beta$ scenario, or whether this solution could survive after more stringent bounds on FCNC are applied. In the face of such objections, it should be noted that including the FV corrections to the lepton masses (which will be proportional to the tau mass and could conceivably be non-negligible), or by scanning the relevant parameters, one should be able to retain this (or a similar) solution.

V. CONCLUSIONS

The main result of this paper is that the “wrong fermion mass relations” of the minimal SUSY GUTs can be greatly ameliorated by employing radiative corrections due to the superpartners of the SM particles.

Employing the conventional $A$-terms, we can obtain correct unification relations for the Yukawa couplings for two families, depending on the sign of the $\mu$ parameter. We call this scenario “flavor from no flavor.” This scenario effectively limits the parameter space of the MSSM where two families correctly unify. However, to unify the remaining family a higher dimensional operator (or a more complicated higgs sector) is required.

We have also presented a non-minimal ansatz for the $A$-terms, that fixes the relations $m_{d} = m_{e}$, $m_{s} = m_{\mu}$, while retaining the successful relation $m_{b} = m_{\tau}$. This ansatz is barely consistent with current FCNC bounds.

As a final approach to modify the mass relations, we considered Flavor violating (FV) gluino-quark-squark interactions. The values of the $A$-matrix required barely satisfy all phenomenological and stability bounds of the MSSM.

Although this paper does not present a theory of fermion masses, it does presents three cases that can be considered as “counter-examples” to the usual claim that the SU(5) GUT model is excluded because of the wrong fermion mass relations. We stress our result: fermion mass relations can work for all three families even in the minimal SU(5) GUT.

VI. ACKNOWLEDGMENTS

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