\(\chi_c\) production in modified NRQCD

Sudhansu S. Biswal\(^1\), Sushree S. Mishra\(^1\)\(^\dagger\) and K. Sridhar\(^2\)\(^\ddagger\)

1. Department of Physics, Ravenshaw University, Cuttack, 753003, India.
2. School of Arts and Sciences, Azim Premji University, Sarjapura, Bangalore, 562125, India.

Abstract

In a previous paper, we had modified Non-Relativistic QCD as it applies to quarkonium production by taking into account the effect of perturbative soft-gluon emission from the colour-octet quarkonium states. We tested the model by fitting the unknown non-perturbative parameter in the model from Tevatron data and using that to make parameter-free predictions for \(J/\psi\) and \(\psi'\) production at the LHC. In this paper, we study \(\chi_c\) production: we fit as before the unknown matrix-element using data from Tevatron. We, then, extend the results of the previous paper for \(J/\psi\) production by calculating the effect of \(\chi_c\) feed-down to the \(J/\psi\) cross-section, which, by comparing with CMS results at \(\sqrt{s} = 13\) TeV, we demonstrate to be small. We have also computed \(\chi_1^c\) and \(\chi_2^c\) at \(\sqrt{s} = 7\) TeV and find excellent agreement with data from the ATLAS experiment.

The effective theory for studying heavy quarkonium physics, Non-Relativistic Quantum Chromodynamics (NRQCD) \[1\], found much success in explaining the systematics of charmonium production at the Fermilab Tevatron \[2\] in contrast to the then existing model of quarkonium production – the colour-singlet model \[3\]. But while NRQCD predicted \[4, 5\] a fully transversely polarised \(J/\psi\) at large \(p_T\) the Tevatron experiments found no evidence for this \[6\]. These and other problems like the \(\eta_c\) cross-section measured in the LHCb experiment \[7\] suggest that NRQCD may require some modification to address quarkonium production fruitfully.

One approach exploits the fact that the colour-singlet model predicts zero polarisation and attempts to increase the colour-singlet contribution by invoking higher-order effects in the singlet channel \[8, 9\]. \[\dagger\] For this approach to work one has to show why colour-octet operators are small in the regions of phase space under consideration.

\(^{\dagger}\)E-mail: sudhansu.biswal@gmail.com
\(^{\ddagger}\)Email: sushreesimran.mishra97@gmail.com
\(^{\ast}\)E-mail: sridhar.k@apu.edu.in
\(^{1}\)For reviews of the status of these calculations and their experimental consequences, see Refs. \[10, 11\].
In NRQCD, colour-singlet or colour-octet $c\bar{c}$ states are produced at the perturbative level and these states then transform into the physical charmonium state by means of a non-perturbative transition. The separation of the short-distance perturbative process from the non-perturbative part is given by the factorization theorems proved within NRQCD. The dynamics of NRQCD also helps us keep track of the quantum numbers of the colour-octet (or singlet) $c\bar{c}$ state that transforms into the physical charmonium state.

The cross section for production of a quarkonium state $H$ is given as

$$\sigma(H) = \sum_{n=\{\alpha,S,L,J\}} \frac{F_n}{M_Q d_n} \langle O_n^{(2S+1)LJ} \rangle,$$  \hspace{1cm} (1)

where $F_n$'s are the short-distance coefficients and $O_n$ are operators of naive dimension $d_n$, describing the long-distance effects. These non-perturbative matrix elements are guaranteed to be energy-independent due to the NRQCD factorization formula, so that they may be extracted at a given energy and used to predict quarkonium cross-sections at other energies. Other than prediction of the energy-dependence of the $J/\psi$ $p_T$ distribution, several independent tests of the effective theory have been proposed [12].

In a recently proposed modification of NRQCD [13], we had suggested that the colour-octet $c\bar{c}$ state can radiate several soft perturbative gluons – each emission taking away little energy but carrying away units of angular momentum. In the multiple emissions that the colour-octet state can make before it makes the final NRQCD transition to a quarkonium state, the angular momentum and spin assignments of the $c\bar{c}$ state changes constantly.

For $J/\psi$, for example, the NRQCD cross-section formula which was given as follows when written down explicitly in terms of the octet and singlet states

$$\sigma_{J/\psi} = \hat{F}_{3S_1^{[1]}} \times \langle \mathcal{O}(3S_1)^{[1]} \rangle + \hat{F}_{3S_1^{[8]}} \times \langle \mathcal{O}(3S_1)^{[8]} \rangle + \hat{F}_{1S_0^{[8]}} \times \langle \mathcal{O}(1S_0)^{[8]} \rangle + \frac{1}{M^2} \left[ \hat{F}_{3P_J^{[8]}} \times \langle \mathcal{O}(3P_J)^{[8]} \rangle \right],$$  \hspace{1cm} (2)

gets modified to the following in the modified NRQCD with perturbative soft gluon emission:

$$\sigma_{J/\psi} = \left[ \hat{F}_{3S_1^{[1]}} \times \langle \mathcal{O}(3S_1)^{[1]} \rangle \right] + \left[ \hat{F}_{3S_1^{[8]}} + \hat{F}_{1P_J^{[8]}} + \hat{F}_{1S_0^{[8]}} + \langle \hat{F}_{3P_J^{[8]}} \rangle \right] \times \left( \frac{\langle \mathcal{O}(3S_1)^{[1]} \rangle}{8} \right) + \left[ \hat{F}_{3S_1^{[8]}} + \hat{F}_{1P_J^{[8]}} + \hat{F}_{1S_0^{[8]}} + \langle \hat{F}_{3P_J^{[8]}} \rangle \right] \times \langle \mathcal{O} \rangle,$$  \hspace{1cm} (3)
where
\[
\langle O \rangle = \times \left[ \langle O (3S_1^{[8]}) \rangle + \langle O (1S_0^{[8]}) \rangle + \frac{\langle O (3P_j^{[8]}) \rangle}{M^2} \right].
\] (4)

In contrast to the usual case, where we needed to fix three non-perturbative parameters to get the \( J/\psi \) cross-section, in our case it is the sum of these parameters: so we have a single parameter to fit.

In Ref. [13], \( J/\psi \) and \( \psi' \) production in modified NRQCD was already studied. For both these charmonium states, the non-perturbative parameter (the single one that we needed to fit) was fitted from the old Tevatron data [2] and we used the fitted parameter to make predictions for \( J/\psi \) and \( \psi' \) production at the LHC and our predictions were in excellent agreement with the data. For \( J/\psi \) production at the LHC, the CMS experiment [14] does not distinguish between direct \( J/\psi \) and those coming from \( \chi_c \) states. We had in that case only made a rough estimate of the magnitude of the \( \chi_c \) contribution and were convinced that it was small. So the theoretical results we compared with CMS data in Ref. [13] were the direct production ones.

To do the full \( J/\psi \) production at LHC we need to also include the result of the feed-down from the \( \chi_c \) states and we undertake this task in this paper. To do this we need to extract the non-perturbative parameter for \( \chi_c \) production from a fit to Tevatron data and then use that to compute the \( \chi_c \) production at LHC and its contribution to the \( J/\psi \) cross-section. The ATLAS experiment has also measured and presented results on \( \chi_c^1 \) and \( \chi_c^2 \) production [15] at 7 TeV energy. We are in a position to also compare our results with these data.

For \( \chi_c \) production the cross-section expression, similar to Eq. [3] is given by:

\[
\sigma_{\chi_c} = \left[ \hat{F}_{3P_{j}^{[1]}} \times \langle O^x (3P^{[1]}) \rangle \right]
+ \left[ \hat{F}_{3S_{1}^{[8]}} + \hat{F}_{1P_{j}^{[8]}} + \hat{F}_{1S_{0}^{[8]}} + \hat{F}_{3P_{j}^{[8]}} \right] \times \left( \langle O^x (3P_{j}^{[1]}) \rangle / 8 \right)
+ \left[ \hat{F}_{3S_{1}^{[8]}} + \hat{F}_{1P_{j}^{[8]}} + \hat{F}_{1S_{0}^{[8]}} + \hat{F}_{3P_{j}^{[8]}} \right] \times \langle O^x \rangle,
\] (5)

where
\[
\langle O^x \rangle = \times \left[ M^2 \langle O (3S_1^{[8]}) \rangle + M^2 \langle O (1S_0^{[8]}) \rangle + \langle O (3P_j^{[8]}) \rangle \right].
\] (6)

The cross-section kinematics are familiar:
\[
\frac{d\sigma}{dp_T} (p\bar{p} \to c\bar{c} \, [2S+1L_j^{[1,8]}] \, X) = \sum \int dy \int dx_1 \, x_1 \, G_{a/p}(x_1) \, x_2 \, G_{b/p}(x_2) \, \frac{4p_T}{2x_1 - x_T} \, e^y \, \frac{d\tilde{\sigma}}{d\ell} (ab \to c\bar{c} \, [2S+1L_j^{[1,8]}] \, d),
\] (7)
where the summation is over the partons \((a, b)\), \(G_{a/p}\), \(G_{b/p}\) are the distributions of partons \(a\) and \(b\) in the protons and \(x_1, x_2\) are the respective momentum they carry. In the above formula, 
\[
x_T = \sqrt{x^2_T + 4\tau} \equiv 2M_T/\sqrt{s} \quad \text{with} \quad x_T = 2p_T/\sqrt{s} \quad \text{and} \quad \tau = M^2/s.
\]
\(\sqrt{s}\) is the center-of-mass energy, \(M\) is the mass of the resonance and \(y\) is the rapidity at which the resonance is produced. The matrix elements for the subprocesses can be found in Refs. [16, 17, 18].

We use, as we did with \(J/\psi\) and \(\psi'\) in Ref. [13], the Tevatron data to determine the non-perturbative parameter for \(\chi_c\) production. However, the CDF experiment at the Tevatron does not give us the individual cross-sections for the three states \(\chi_c^{0, 1, 2}\) but only the sum of all three resonances all decaying into the \(J/\psi\). We assume that the non-perturbative parameters for the three states are equal and with this assumption we need to extract only a single parameter from the \(\chi_c\) \(p_T\) distribution that the CDF experiment has made available [2]. The fit to the CDF \(\chi_c\) distribution is shown in Fig. 1.

![Graph](image)

**Figure 1**: Predicted differential cross section fitted to the data on \(\chi_c\) production from the CDF experiment at Tevatron at \(\sqrt{s} = 1.8\) TeV.

With the non-perturbative parameter so obtained from the CDF experiment, we are now in a position to calculate the \(p_T\) distributions for each of the \(\chi_c\) states at LHC energies and, after folding in the branching ratios for these states to decay into a \(J/\psi\), we are able to calculate the inclusive \(J/\psi\) \(p_T\) distribution with the \(\chi_c\) contribution included and compare with the 13 TeV data from the CMS experiment (see Fig. 2). The experiment has made measurements in five different rapidity intervals and we have
also carried out our computation for all five intervals. In Fig. 2, where we have shown the comparison of our results with those of the experiment, we have the theoretical results for inclusive as well as direct $J/\psi$ production. True to the estimates we had made earlier, the contribution to the $J/\psi$ cross-section from $\chi_c$ feed-down is very small.

Figure 2: Predicted differential distributions for full $J/\psi$ production at the LHC running at 13 TeV compared with data from the CMS experiment.

The ATLAS experiment at the LHC has data on $\chi_1^c$ and $\chi_2^c$ production at at a $\sqrt{s}$ of 7 TeV. A comparison of our theoretical predictions with these states provides the most direct check on our theoretical model. In Fig. 3, we show the comparison of

Figure 3: Predicted differential distributions for $\chi_1^c$ and $\chi_2^c$ production at the LHC running at 7 TeV compared with data from the ATLAS experiment.
our predictions with the ATLAS data for both $\chi^1_c$ and $\chi^2_c$ and a very good agreement between our model predictions and the data is seen to result.

To conclude, following up on the success of the modified NRQCD that we proposed in Ref. [13] in predicting the data on $J/\psi$ and $\psi'$ at the LHC, we calculated the cross-section for $\chi_c$ production in this paper. Using the Tevatron data to fit the single non-perturbative parameter that we need, we then used it to compute inclusive $J/\psi$ production at $\sqrt{s}=13$ TeV and compared it with the data from the CMS experiment. Our results show that the contribution to the $J/\psi$ cross-section from $\chi_c$ feed-down is not significant. We have also computed $\chi^1_c$ and $\chi^2_c$ at $\sqrt{s} = 7$ TeV and a comparison with the data from the ATLAS experiment are seen to be very good.

Acknowledgments

We would like to thank Vaia Papadimitriou for comments and discussions.

References

[1] G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D 51, 1125 (1995) [Erratum-ibid. D 55, 5853 (1997)] [arXiv:hep-ph/9407339].

[2] F. Abe et al. [CDF], Phys. Rev. Lett. 79, 572 (1997); F. Abe et al. [CDF], Phys. Rev. Lett. 79, 578 (1997).

[3] R. Baier and R. Ruckl, Z. Phys. C 19, 251 (1983).

[4] P. L. Cho and M. B. Wise, Phys. Lett. B 346, 129 (1995) [arXiv:hep-ph/9411303].

[5] M. Beneke and M. Kramer, Phys. Rev. D 55, 5269 (1997) [arXiv:hep-ph/9611218].

[6] A. A. Affolder et al. [CDF Collaboration], Phys. Rev. Lett. 85, 2886 (2000) [arXiv:hep-ex/0004027]; A. Abulencia et al. [CDF Collaboration], Phys. Rev. Lett. 99, 132001 (2007) [arXiv:0704.0638 [hep-ex]].

[7] R. Aaij et al. [LHCb], Eur. Phys. J. C 75, no.7, 311 (2015) [arXiv:1409.3612 [hep-ex]]; R. Aaij et al. [LHCb], Eur. Phys. J. C 80, no.3, 191 (2020) [arXiv:1911.03326 [hep-ex]].

[8] B. Gong, X. Q. Li and J. X. Wang, Phys. Lett. B 673, 197 (2009) [arXiv:0805.4751 [hep-ph]].
[9] P. Artoisenet, J. Campbell, J. P. Lansberg, F. Maltoni and F. Tramontano, Phys. Rev. Lett. **101**, 152001 (2008) [arXiv:0806.3282 [hep-ph]].

[10] J. P. Lansberg et al., AIP Conf. Proc. **1038**, 15 (2008) [arXiv:0807.3666 [hep-ph]].

[11] J. P. Lansberg, Eur. Phys. J. C **61**, 693 (2009) [arXiv:0811.4005 [hep-ph]].

[12] M. Cacciari and M. Kramer, Phys. Rev. Lett. **76**, 4128 (1996) [arXiv:hep-ph/9601276]; J. Amundson, S. Fleming and I. Maksymyk, Phys. Rev. D **56**, 5844 (1997) [arXiv:hep-ph/9601298]; S. Gupta and K. Sridhar, Phys. Rev. D **54**, 5545 (1996) [arXiv:hep-ph/9601349]; Phys. Rev. D **55**, 2650 (1997) [arXiv:hep-ph/9608433]; M. Beneke and I. Z. Rothstein, Phys. Rev. D **54**, 2005 (1996) [Erratum-ibid. D **54**, 7082 (1996) J [arXiv:hep-ph/9603400]; W. K. Tang and M. Vanttinen, Phys. Rev. D **54**, 4349 (1996) [arXiv:hep-ph/9603266]; E. Braaten and Y. Q. Chen, Phys. Rev. Lett. **76**, 730 (1996) [arXiv:hep-ph/9508373]; K. M. Cheung, W. Y. Keung and T. C. Yuan, Phys. Rev. Lett. **76**, 877 (1996) [arXiv:hep-ph/9509308]; P. L. Cho, Phys. Lett. B **368**, 171 (1996) [arXiv:hep-ph/9509355]; K. M. Cheung, W. Y. Keung and T. C. Yuan, Phys. Rev. D **54**, 929 (1996) [arXiv:hep-ph/9602423]; P. Ko, J. Lee and H. S. Song, Phys. Rev. D **53**, 1409 (1996) [arXiv:hep-ph/9510202]; G. T. Bodwin, E. Braaten, T. C. Yuan and G. P. Lepage, Phys. Rev. D **46**, 3703 (1992) [arXiv:hep-ph/9208254]; K. Sridhar, A. D. Martin and W. J. Stirling, Phys. Lett. B **438**, 211 (1998) [arXiv:hep-ph/9806253]; K. Sridhar, Phys. Rev. Lett. **77**, 4880 (1996) [arXiv:hep-ph/9609285]; K. Sridhar, Phys. Lett. B **674**, 36 (2009) [arXiv:0812.0474 [hep-ph]]; E. Braaten, B. A. Kniehl and J. Lee, Phys. Rev. D **62**, 094005 (2000) [arXiv:hep-ph/9911436]; S. S. Biswal and K. Sridhar, J. Phys. G **39**, 015008 (2012) [arXiv:1007.5163 [hep-ph]].

[13] S. S. Biswal, S. S. Mishra and K. Sridhar, Phys. Lett. B **832**, 137221 (2022) [arXiv:2201.09393 [hep-ph]].

[14] A. M. Sirunyan et al. [CMS], Phys. Lett. B **780**, 251 (2018) [arXiv:1710.11002 [hep-ex]].

[15] G. Aad et al. [ATLAS], JHEP **07**, 154 (2014) [arXiv:1404.7035 [hep-ex]].

[16] P. L. Cho and A. K. Leibovich, Phys. Rev. D **53**, 6203 (1996) [arXiv:hep-ph/9511315].

[17] R. Gastmans, W. Troost and T. T. Wu, Nucl. Phys. B **291**, 731 (1987).

[18] P. Mathews, P. Poulose and K. Sridhar, Phys. Lett. B **438**, 336 (1998) [arXiv:hep-ph/9803424].