OPTIMAL CONTROL STRATEGIES FOR AN ONLINE GAME ADDICTION MODEL WITH LOW AND HIGH RISK EXPOSURE

YOUMING GUO AND TINGTING LI*

School of Science, Guilin University of Technology
Guilin, Guangxi 541004, China

(Communicated by Hao Wang)

ABSTRACT. In this paper, we establish a new online game addiction model with low and high risk exposure. With the help of the next generation matrix, the basic reproduction number $R_0$ is obtained. By constructing a suitable Lyapunov function, the equilibria of the model are Globally Asymptotically Stable. We use the optimal control theory to study the optimal solution problem with three kinds of control measures (isolation, education and treatment) and get the expression of optimal control. In the simulation, we first verify the Globally Asymptotical Stability of Disease-Free Equilibrium and Endemic Equilibrium, and obtain that the different trajectories with different initial values converges to the equilibria. Then the simulations of nine control strategies are obtained by forward-backward sweep method, and they are compared with the situation of without control respectively. The results show that we should implement the three kinds of control measures according to the optimal control strategy at the same time, which can effectively reduce the situation of game addiction.

1. Introduction. On May 25, 2019, the World Health Organization held the 72nd World Health Assembly, at which the 11th revision of the international classification of diseases was adopted[37]. For the first time, the addictive behavior of video games called “game barrier” was listed as disease. The design of online games basically has the following characteristics: rich imagination, fair competition, rich game rewards for inviting new players, etc. Compared with some boring daily life, it is full of thrill and excitement. For some people with weak self-control, it is very easy to indulge and spread the game. Therefore, the online game has a certain degree of infectivity in the population, especially in young people[30].

Online games are called electronic heroin by many people[21]. Because online game is a double-edged sword, it is easy to indulge in it when people use it for leisure and entertainment. Addicting to online games will bring a lot of negative effects to individual psychology, physiology, family and society. All over the world, there are different degrees of online game addiction. According to the 43rd statistics of China Internet Network Information Center (CNNIC) in 2019[31], there are 829

2020 Mathematics Subject Classification. Primary: 34D23; Secondary: 49J15.
Key words and phrases. Game addiction model, globally asymptotically stable, Lyapunov function, forward-backward sweep method, optimal control.

The second author is supported by the Basic Competence Promotion Project for Young and Middle-aged Teachers in Guangxi, China (2019KY0269).
* Corresponding author: Tingting Li.
million internet users in mainland China, and about 580 million users often play online games.

Many scholars refer to the research methods of infectious diseases to study infectious problems in many other fields, such as: information diffusion, drinking, financial crimes, game addiction, etc. Zhang et al.\cite{39} reviewed the new development of network information dissemination, evaluated the existing models and algorithms in the field of biological mathematics, and discussed the future development direction. Bonyah et al.\cite{4} modelled the effects of heavy alcohol consumption on the transmission dynamics of gonorrhea, the authors considered the impact of alcoholism with time dependent control. Akanni et al.\cite{2} analysed the dynamics and the optimal control measures of the financial crime model. Viriyapong and Sookpiam\cite{34} studied the stability and qualitative behavior of an online game addiction model in Thailand, the results of numerical simulation show that both the education campaign and the family understanding are important factors in reducing $R_0$. Wang et al.\cite{36} researched the dynamic properties of an online game model with age structure. Unfortunately, these references do not take into account that game addicts and people who are game professionals are different. The latter also have a great passion for the game, and spend a long time in the game world every day. But these people usually have professional operation team, scientific guidance and training, and will not bring some potential risks to the society. Through the study of game addiction, Li and Guo\cite{19} put forward an online game addiction model with a warehouse of professional game people for the first time.

In recent years, optimal control theory has been widely used in biological mathematics to find the optimal control results, which provided suggestions for public health departments to control the epidemic situation\cite{40, 16, 38, 33, 1, 35, 12, 23, 11, 9, 22}. For example, Zhou et al.\cite{40} presented the optimal media reporting intensity of an emerging infectious disease. The authors extended existing models through considering a new media function, which was a function related to the number of infected people and the strength of the media. Khan et al.\cite{16} showed a dynamical model of asymptomatic carrier Zika virus model with optimal control strategies. Yıldız and Karaoğlu\cite{38} studied the optimal strategies for an extended tuberculosis model by incorporating exogenous reinfection. Ullah et al.\cite{33} explored the mathematical formulation of hepatitis B virus with optimal control analysis. Agusto and Khan\cite{1} proposed a mathematical model to analyse the transmission dynamics of Dengue fever in Pakistan. The imperfect vaccine was incorporated into the model and the best control strategy to curtail the disease was obtained by applying optimal control theory.

Based on such a fact, it takes a period of time for a susceptible person to change into a addict. Therefore, in this paper we consider the exposed population as a separate warehouse. According to relevant research findings, people between the ages of 12 and 28 are more likely to be addicted to online games than other age groups, and men are more likely to be addicted to games than women. So it is reasonable for us to divide the exposed population into low-risk exposed $E_1$ and high-risk exposed $E_2$, where $E_2$ represents the male aged 12-28 in the exposed population.

Inspired by the above literature, we set up a new extended online game addiction model with exposed warehouses, which are divided into low-risk exposed and high-risk exposed warehouse. We first make a qualitative analysis of its stability, and then make a quantitative numerical simulation.
The organizational structure of this paper is as follows: an extended online game addiction model and some of its basic properties are shown in Section 2. The basic reproduction number and the existence of equilibria are obtained in Section 3. The stability analysis of equilibria are presented in Section 4. The optimal control problem is analyzed in Section 5. The results of the numerical simulation with detailed discussion are shown in Section 6. Finally, the paper ends with a summary conclusion in Section 7.

2. The model formulation.

2.1. System description. On April 15, 2007, the General Administration of Press and Publication of China issued the Real Name Certification Scheme for Online Game Anti-addiction System. In this mechanism, it is clearly defined that it is unhealthy to play games more than 5 hours a day. To some degree, it has a negative impact on people’s daily life.

In 2003, the China National Sports Bureau approved E-sports as a national sport, and the national team of E-sports was established in 2013[28]. From then on, an important group with game as its profession were born. Although they play games more than five hours a day, they have scientific planning, training and guidance, which will not cause obstacles to their daily life. So they are different from addicted people. We summarize this kind of people who take game as their profession into a warehouse, which is called professional (P). So we take people who are not game professionals and play more than five hours a day as infected (I). After contacting with the infected person, the susceptible person becomes the exposed person. There are two different types of people in the exposed group, low-risk ($E_1$) and high-risk ($E_2$). Their infection rates are different.

The total population denoted by $N(t)$ is partitioned into six compartments, namely, the susceptible (S), low-risk exposed ($E_1$), high-risk exposed ($E_2$), infected (I), professional (P) and quitting (Q). Thus, the total population is given by:

$$N(t) = S(t) + E_1(t) + E_2(t) + I(t) + P(t) + Q(t).$$

The population flow among those compartments is shown in Figure 1.

The transfer diagram leads to the following system of ordinary differential equations:

$$\begin{align*}
S'(t) &= \mu N - \mu S - \frac{\beta SI}{N} \\
E_1'(t) &= \theta \frac{\beta SI}{N} - (v_1 + v_2 + \mu)E_1 \\
E_2'(t) &= (1 - \theta) \frac{\beta SI}{N} - (w_1 + w_2 + \mu)E_2 \\
I'(t) &= v_1 E_1 + w_1 E_2 - (k_1 + k_2 + \mu)I \\
P'(t) &= v_2 E_1 + w_2 E_2 + k_2 I - (\delta + \mu)P \\
Q'(t) &= k_1 I + \delta P - \mu Q
\end{align*}$$

In system (2), $\mu$ is the natural birth rate and death rate; $\theta$ denotes proportion of individuals who become low-risk exposed $E_1$; $\beta$ is the basic contact transmission rate; $\delta$ denotes the quitting rate of $P$; $v_1$ represents the proportion of $E_1$ who become infected $I$; $v_2$ represents the proportion of $E_2$ who become infected $P$; $w_1$ represents the proportion of $E_2$ who become infected $I$; $w_2$ represents the proportion of $E_2$ who become infected $P$; $k_1$ represents the proportion of $I$ who become quitting $Q$; $k_2$ represents the proportion of $I$ who turns out to be a professional player $P$. 
2.2. Positivity and boundedness of solutions. Since the number of people in each warehouse is non-negative for all $t > 0$, we should first prove the positive solution of the system (2). System (2) can be rewritten in the form of the following matrix

$$X' = G(X)$$

where $X = (S, E_1, E_2, I, P, Q)^T \in \mathbb{R}^6$ and $G(X)$ is given by

$$G(X) = \begin{pmatrix}
G_1(X) \\
G_2(X) \\
G_3(X) \\
G_4(X) \\
G_5(X) \\
G_6(X)
\end{pmatrix} = \begin{pmatrix}
\frac{\mu N - \mu S - \frac{\beta S I}{N}}{1 - \theta} - \left( v_1 + w_1 + \mu \right)E_1 \\
\frac{\theta \frac{\beta S I}{N}}{N} - \left( w_1 + w_2 + \mu \right)E_2 \\
v_1 E_1 + w_1 E_2 - (k_1 + k_2 + \mu)I \\
v_2 E_1 + w_2 E_2 + k_2 I - (\delta + \mu)P \\
k_1 I + \delta P - \mu Q
\end{pmatrix}. \quad (4)$$

From the result on page 4 of [5], we have,

$$G_i(X)|_{X_i(t) = 0, X_i \in C_+} \geq 0, \quad i = 1, 2, 3, 4, 5, 6. \quad (5)$$

Since every component of $G$ is greater than or equal to 0 and the system (2) has a non-negative initial value, all the solutions of the model are remaining in a positive region.
Because of \( \Sigma_{i=1}^{6} G_i(x) = 0 \), \( N(t) \) is a constant denoted by \( N \). We set
\[
\Omega = \{(S, E_1, E_2, I, P, Q) \in \mathbb{R}^6_+ \mid S + E_1 + E_2 + I + P + Q = N\}.
\] (6)
It is a positive invariant set of system (2). The dissipative and the global attractor are still in \( \Omega \).

3. The basic reproduction number and existence of Endemic Equilibrium.

3.1. The basic reproduction number. The basic reproduction number \( R_0 \) represents ‘the average number of new infections directly caused by an infected case during his entire infectious period, in a wholly susceptible population’. It is a key concept in epidemiology, and is inarguably ‘one of the foremost and most valuable ideas that mathematical thinking has brought to epidemic theory’[13].

Obviously, system (2) has the following solution, which is recorded as Disease-Free Equilibrium
\[
D_0 = (N, 0, 0, 0, 0, 0).
\] (7)
Then, we use the classical method of next generation matrix to solve the basic regeneration number \( R_0 \). (For details, please refer to page 33 of reference [7]). Letting \( x = (E_1, E_2, I, P, Q, S)^T \), then system(2) can be written as
\[
\frac{dx}{dt} = \mathcal{F}(x) - \mathcal{V}(x).
\] (8)
where
\[
\mathcal{F}(x) = \begin{pmatrix}
\frac{\beta SI}{N} \\
(1 - \theta) \frac{\beta SI}{N} \\
0 \\
0 \\
0 \\
0
\end{pmatrix}, \quad \mathcal{V}(x) = \begin{pmatrix}
(v_1 + v_2 + \mu)E_1 \\
(w_1 + w_2 + \mu)E_2 \\
-v_1E_1 - w_1E_2 + (k_1 + k_2 + \mu)I \\
-v_2E_1 - w_2E_2 - k_2I + (\delta + \mu)P \\
-k_1I - \delta P + \mu Q \\
\mu(S - N) + \frac{\beta SI}{N}
\end{pmatrix}.
\]
The Jacobian matrices of \( \mathcal{F}(x) \) and \( \mathcal{V}(x) \) at the Disease-Free Equilibrium \( D_0 \) are
\[
D \mathcal{F}(D_0) = \begin{pmatrix} F_{3 \times 3} & 0 \\ 0 & 0 \end{pmatrix}, \quad D \mathcal{V}(D_0) = \begin{pmatrix} V_{3 \times 3} & 0 \\ J_1 & J_2 \end{pmatrix}.
\]
where
\[
F_{3 \times 3} = \begin{pmatrix} 0 & 0 & \theta \beta \\ 0 & 0 & (1 - \theta) \beta \\ 0 & 0 & 0 \end{pmatrix},
\]
\[
V_{3 \times 3} = \begin{pmatrix} v_1 + v_2 + \mu & 0 & 0 \\ 0 & w_1 + w_2 + \mu & 0 \\ -v_1 & -w_1 & k_1 + k_2 + \mu \end{pmatrix},
\]
\[
J_1 = \begin{pmatrix} -v_2 & -w_2 & -k_2 \\ 0 & 0 & -k_1 \\ 0 & 0 & \beta \end{pmatrix}, \quad J_2 = \begin{pmatrix} \delta + \mu & 0 & 0 \\ -\delta & \mu & 0 \\ 0 & 0 & \mu \end{pmatrix}.
\]
Following Driessche et al.[7], the basic reproduction number, denoted by $R_0$, is given by

$$R_0 = \rho(FV^{-1}) = \frac{\beta[(1 - \theta)w_1(\mu + v_1 + v_2) + \theta v_1(\mu + w_1 + w_2)]}{(k_1 + k_2 + \mu)(v_1 + v_2 + \mu)(w_1 + w_2 + \mu)},$$

where $\rho(A)$ denotes the spectral radius of a matrix $A$.

3.2. Existence of Endemic Equilibrium. The Endemic Equilibrium $D^* = (S^*, E_1^*, E_2^*, I^*, P^*, Q^*)$ of system (2) is determined by equations:

\begin{align*}
\mu N - \mu S - \beta \frac{SI}{N} &= 0 \\
\theta \beta \frac{SI}{N} &= (v_1 + v_2 + \mu)E_1 \\
(1 - \theta)\beta \frac{SI}{N} &= (w_1 + w_2 + \mu)E_2 \\
v_1E_1 + w_1E_2 &= (k_1 + k_2 + \mu)I \\
v_2E_1 + w_2E_2 + k_2I &= (\delta + \mu)P \\
k_1I + \delta P &= \mu Q
\end{align*}

By solving the above equations, we get the unique expression of $D^*$. In combination with the formula (7) obtained above, we obtain the following theorem.

**Theorem 3.1.** In the system (2), there is always a Disease-Free Equilibrium $D_0 = (N, 0, 0, 0, 0, 0, 0)$. When $R_0 > 1$, the system has a unique Endemic Equilibrium $D^* = (S^*, E_1^*, E_2^*, I^*, P^*, Q^*)$, where

\begin{align*}
S^* &= \frac{N}{R_0} \\
E_1^* &= \frac{\theta \mu N(R_0 - 1)}{R_0(v_1 + v_2 + \mu)} \\
E_2^* &= \frac{(1 - \theta)\mu N(R_0 - 1)}{R_0(w_1 + w_2 + \mu)} \\
I^* &= \frac{N \mu (R_0 - 1)}{\beta} \\
P^* &= \frac{1}{\delta + \mu} \left[ \frac{\theta \mu N(R_0 - 1)}{R_0(v_1 + v_2 + \mu)} + \frac{(1 - \theta)\mu N(R_0 - 1)}{R_0(w_1 + w_2 + \mu)} \right] + k_2 \frac{N \mu (R_0 - 1)}{\beta} \\
Q^* &= k_1 \frac{N (R_0 - 1)}{\beta} + \frac{\delta}{\delta + \mu} \left[ \frac{\theta N(R_0 - 1)}{R_0(v_1 + v_2 + \mu)} + \frac{(1 - \theta)N(R_0 - 1)}{R_0(w_1 + w_2 + \mu)} \right] + k_2 \frac{N (R_0 - 1)}{\beta}
\end{align*}

4. Stability analysis of equilibria. We denote a vector $x = (E_1, E_2, I, P, Q, S)^T$ and

$$f(x) = \begin{pmatrix}
\theta \frac{SI}{N} - (v_1 + v_2 + \mu)E_1 \\
(1 - \theta) \frac{SI}{N} - (w_1 + w_2 + \mu)E_2 \\
v_1E_1 + w_1E_2 - (k_1 + k_2 + \mu)I \\
v_2E_1 + w_2E_2 + k_2I - (\delta + \mu)P \\
k_1I + \delta P - \mu Q \\
\mu N - \mu S - \frac{\beta SI}{N}
\end{pmatrix}.$$
So the Jacobian matrix of $f(x)$ about vector $x$ is as the following:

$$J = \frac{\partial f(x)}{\partial x} = \begin{pmatrix} -a_1 & 0 & \theta \beta & 0 & 0 & 0 \\ 0 & -a_2 & (1-\theta)\beta & 0 & 0 & 0 \\ v_1 & w_1 & -a_3 & 0 & 0 & 0 \\ v_2 & w_2 & k_2 & -(\delta + \mu) & 0 & 0 \\ 0 & 0 & k_1 & \delta & -\mu & 0 \\ 0 & 0 & -\frac{\beta S}{N} & 0 & 0 & -\frac{\beta I}{N} - \mu \end{pmatrix},$$

where $a_1 = v_1 + v_2 + \mu, a_2 = w_1 + w_2 + \mu, a_3 = k_1 + k_2 + \mu$.

**Theorem 4.1.** For the system (2), the Disease-Free Equilibrium $D_0$ is Locally Asymptotically Stable (LAS) if $R_0 < 1$.

**Proof.** Since

$$J(D_0) = \begin{pmatrix} -a_1 & 0 & \theta \beta & 0 & 0 & 0 \\ 0 & -a_2 & (1-\theta)\beta & 0 & 0 & 0 \\ v_1 & w_1 & -a_3 & 0 & 0 & 0 \\ v_2 & w_2 & k_2 & -(\delta + \mu) & 0 & 0 \\ 0 & 0 & k_1 & \delta & -\mu & 0 \\ 0 & 0 & -\frac{\beta S}{N} & 0 & 0 & -\frac{\beta I}{N} - \mu \end{pmatrix}.$$

It is easily known that the eigenvalues of $J_4$ are $\lambda_1 = -(\delta + \mu), \lambda_2 = \lambda_3 = -\mu$.

The characteristic equation of characteristic matrix of $M$ is

$$\lambda^3 + (a_1 + a_2 + a_3)\lambda^2 + [a_2a_3 + a_1a_2 + a_1a_3 - w_1(1-\theta)\beta - v_1\theta\beta]\lambda$$

$$+ [a_1a_2a_3 - a_1w_1(1-\theta)\beta - a_2v_1\theta\beta] = 0.$$

Due to $0 < a_i < 1, (i = 1, 2, 3)$ and $0 < R_0 < 1$,

$$R_0 = \frac{\beta[(1-\theta)w_1(\mu + v_1 + v_2) + \theta v_1(\mu + w_1 + w_2)]}{(k_1 + k_2 + \mu)(v_1 + v_2 + \mu)(w_1 + w_2 + \mu)}$$

$$= \frac{\beta[(1-\theta)w_1a_1 + \theta v_1a_2]}{a_1a_2a_3}$$

$$= \frac{\beta(1-\theta)w_1}{a_2a_3} + \frac{\beta v_1}{a_1a_3}.$$

so we have

$$a_2a_3 + a_1a_2 + a_1a_3 - w_1(1-\theta)\beta - v_1\theta\beta > 0, \quad a_1a_2a_3 - a_1w_1(1-\theta)\beta - a_2v_1\theta\beta > 0.$$

So the real part of all eigenvalues of $M$ are negative, the DFE $D_0$ is LAS. The proof is completed.

**Theorem 4.2.** For the system (2), the Disease-Free Equilibrium $D_0$ is Globally Asymptotically Stable (GAS) if $R_0 < 1.$
Proof. Consider the subsystem of (2) as follows:

\[ E_1'(t) = \frac{\theta \beta SI}{N} - (v_1 + v_2 + \mu)E_1 \]
\[ E_2'(t) = (1 - \theta)\frac{\beta SI}{N} - (w_1 + w_2 + \mu)E_2 \]
\[ I'(t) = v_1E_1 + w_1E_2 - (k_1 + k_2 + \mu)I \]

For \( S \leq N \),
\[
\begin{pmatrix}
E_1' \\
E_2' \\
I'
\end{pmatrix} \leq \begin{pmatrix}
\theta \beta I - (v_1 + v_2 + \mu)E_1 \\
(1 - \theta)\beta I - (w_1 + w_2 + \mu)E_2 \\
v_1E_1 + w_1E_2 - (k_1 + k_2 + \mu)I
\end{pmatrix}
\]
\[ = M \begin{pmatrix}
E_1 \\
E_2 \\
I
\end{pmatrix} \]

Since the real parts of all eigenvalues of matrix \( M \) are less than 0, the system (2) is stable when \( R_0 < 1 \). So \( (E_1, E_2, I) \rightarrow (0,0,0) \) as \( t \rightarrow \infty \). With the help of comparison theorem, we can get that \( (S, E_1, E_2, I, P, Q) \rightarrow (N,0,0,0,0,0) \) as \( t \rightarrow \infty \). So when \( R_0 \) is less than 1, \( D_0 \) is Globally Asymptotically Stable (GAS).

**Theorem 4.3.** For the system (2), the Endemic Equilibrium \( D^* = (S^*, E_1^*, E_2^*, I^*, P^*, Q^*) \) is Globally Asymptotically Stable if \( R_0 > 1 \).

Proof. Because \( N = S + E_1 + E_2 + I + P + Q \) is a constant, we introduce the fractions of them:

\[ s = \frac{S}{N}, \quad e_1 = \frac{E_1}{N}, \quad e_2 = \frac{E_2}{N}, \quad i = \frac{I}{N}, \quad p = \frac{P}{N}, \quad q = \frac{Q}{N}. \]

with \( s + e_1 + e_2 + i + p + q = 1 \). We obtain the following relations from system (2) at \( D^* \) as

\[
\begin{align*}
\mu &= \beta s^*i^* + \mu s^* \\
\theta \beta s^*i^* &= (v_1 + v_2 + \mu)e_1^* \\
(1 - \theta)\beta s^*i^* &= (w_1 + w_2 + \mu)e_2^* \\
v_1e_1^* + w_1e_2^* &= (k_1 + k_2 + \mu)i^* \\
v_2e_1^* + w_2e_2^* + k_2^* &= (\delta + \mu)p^* \\
k_1i^* + \delta p^* &= \mu q^*
\end{align*}
\]

We introduce the Lyapunov function \( V \) as follows:

\[ V = x_1[s - s^* - s^*ln(\frac{s}{s^*})] + x_2[e_1 - e_1^* - e_1^*ln(\frac{e_1^*}{e_1})] + x_3[e_2 - e_2^* - e_2^*ln(\frac{e_2^*}{e_2})] \\
+ x_4[i - i^* - i^*ln(\frac{i}{i^*})] + x_5[p - p^* - p^*ln(\frac{p^*}{p})] + x_6[q - q^* - q^*ln(\frac{q^*}{q})], \]

where \( x_i \) is the undetermined coefficients. Applying the system (16),

\[ V' = x_1[0 - \frac{s^*}{s}s - s^* + \frac{e_1^*}{e_1}e'] + x_2[0 - \frac{e_2^*}{e_2}e'] + x_3[0 - \frac{i^*}{i}i'] + x_4[0 - \frac{p^*}{p}p'] + x_6[0 - \frac{q^*}{q}q'] \\
= x_1[s^*i^* + \mu s^* - \beta s i - \mu s - \frac{s^*}{s}\beta s^*i^* + \frac{s^*}{s}\mu s^* + \beta s^*i + \mu s^*] \\
+ x_2[\theta \beta s i - (v_1 + v_2 + \mu)e_1 - \frac{e_1^*}{e_1}\theta \beta s i + (v_1 + v_2 + \mu)e_1^*] \]
AN ADDICTION MODEL WITH EXPOSURE 9

\[+x_3[(1-\theta)\beta s i - (w_1 + w_2 + \mu)e_2 - \frac{c^2}{e_2}(1-\theta)\beta s i + e_2^*(w_1 + w_2 + \mu)]\]
\[+x_4[v_1e_1 + w_1e_2 - (k_1 + k_2 + \mu)i - \frac{i}{i}v_1e_1 - \frac{i}{i}w_1e_2 + i(k_1 + k_2 + \mu)]\]
\[+x_5[v_2e_1 + w_2e_2 + k_2i - (\delta + \mu)p - \frac{p}{p}v_2e_1 - \frac{p}{p}w_2e_2 - \frac{p}{p}k_2i + p^*(\delta + \mu)]\]
\[+x_6[k_1i + \delta p - \mu q - \frac{q^*}{q}k_1i - \frac{q}{q}i\delta p + p^*\mu]\]

\[= x_1\mu s^*(2 - \frac{s}{s^*} - \frac{s^*}{s}) + si[-x_1\beta + x_2\theta + x_3(1-\theta)\beta] + i[x_1\beta s^* \]
\[-x_4(k_1 + k_2 + \mu) + x_5k_2 + x_6k_1 + e_1[-x_2(v_1 + v_2 + \mu) + 4x_1v_1 + x_5v_2\]
\[+e_2[-x_3(v_1 + w_2 + \mu) + 4x_1w_1 + x_5w_2] + p[-x_5(\delta + \mu) + x_6\delta] + q(-x_6\mu)\]
\[+[x_1\beta s^*i^* + x_2(v_1 + v_2 + \mu)e_1 + x_3e_2^*(w_1 + w_2 + \mu)\]
\[+x_4i^*(k_1 + k_2 + \mu) + x_5p^*(\delta + \mu) + x_6^*\mu]\]
\[=[x_1\beta s^*i^* = x_6\delta = 0] - x_6\mu = 0\]

In order to eliminate these uncertain items \( si, i, e_1, e_2, p, q \), we let:

\[-x_1\beta + x_2\theta + x_3(1-\theta)\beta = 0\]
\[x_1\beta s^* - x_4(k_1 + k_2 + \mu) + x_5k_2 + x_6k_1 = 0\]
\[-x_2(v_1 + v_2 + \mu) + 4x_1v_1 + x_5v_2 = 0\]
\[-x_3(v_1 + w_2 + \mu) + 4x_1w_1 + x_5w_2 = 0\]
\[-x_5(\delta + \mu) + x_6\delta = 0\]

So we have:

\[x_1 = 1\]
\[x_2 = \frac{v_1\beta s^*}{(v_1 + v_2 + \mu)(k_1 + k_2 + \mu)}\]
\[x_3 = \frac{w_1\beta s^*}{(w_1 + w_2 + \mu)(k_1 + k_2 + \mu)}\]
\[x_4 = \frac{\beta s^*}{k_1 + k_2 + \mu}\]
\[x_5 = 0\]
\[x_6 = 0\]

Thus

\[V' = \mu s^*(2 - \frac{s}{s^*} - \frac{s^*}{s}) + V'_1 - V'_2\]

where

\[V'_1 = \beta s^*i^* + x_2(v_1 + v_2 + \mu)e_1^* + x_3e_2^*(w_1 + w_2 + \mu) + x_4i^*(k_1 + k_2 + \mu)\]
\[= \beta s^*i^* + x_2\theta \beta s^*i^* + x_3(1-\theta)\beta s^*i^* + \beta s^*i^*\]
\[= 3\beta s^*i^*\]
By the arithmetic-geometric mean inequality,
\[ V'_{21} \geq 3 \sqrt[3]{x_2 \theta \frac{s^*}{s} \beta s^* i^* x_2 \theta \frac{e_1}{e_1} \beta s^* i^* e_1 x_2 (v_1 + v_2 + \mu)} = 3 x_2 \theta \beta s^* i^* \]
\[ V'_{22} \geq 3 \sqrt[3]{x_3 (1 - \theta) \frac{s^*}{s} \beta s^* i^* x_3 (1 - \theta) \frac{e_2}{e_2} \beta s^* i^* e_2 x_3 (w_1 + w_2 + \mu)} = 3 x_3 (1 - \theta) \beta s^* i^* \]
Therefore \( V'_2 \geq 3 \beta s^* i^* \), and we can get the conclusion that \( V'(t) \leq 0 \). Through the LaSalle Invariance Principle, the Endemic Equilibrium \( D^* \) is Globally Asymptotically Stable.

5. Optimal control. In this section, we expand system (2) by adding some conventional controls. We consider four control variables: \( u_1 \) for the isolation ratio of \( I \); \( (u_2, u_3) \) for the education ratio of \( E_1, E_2 \); \( u_4 \) for the treatment ratio of \( I \). The goal of control is not only to minimize the number of addicts, but also to minimize the cost of control. In order to find the optimal control strategy, we study the optimal control effect via the following objective function
\[ J = \int_0^{t_f} [A_1 E_1 + A_2 E_2 + A_3 I + C_1 \frac{u_1^2}{2} + C_2 \frac{u_2^2}{2} + C_3 \frac{u_3^2}{2} + C_4 \frac{u_4^2}{2}] dt \] (17)
where \( A_1, A_2, A_3 \) are the weight coefficients relate to the exposed and infected people. The constants \( C_1, C_2, C_3, C_4 \) are the weight coefficients of the control variables \( u_1, u_2, u_3 \) and \( u_4 \).

Thus, the state system becomes
\[
\begin{align*}
S'(t) & = \mu N - \mu S - (1 - u_1) \frac{\beta SI}{N} \\
E_1'(t) & = (1 - u_1) \theta \frac{\beta SI}{N} - [(1 - u_2) v_1 + v_2 + \mu] E_1 \\
E_2'(t) & = (1 - u_1)(1 - \theta) \frac{\beta SI}{N} - [(1 - u_3) w_1 + w_2 + \mu] E_2 \\
I'(t) & = (1 - u_2) v_1 E_1 + (1 - u_3) w_1 E_2 - (k_1 + k_2 + \mu + u_4) I \\
P'(t) & = v_2 E_1 + w_2 E_2 + k_3 I - (\delta + \mu) P \\
Q'(t) & = (k_1 + u_4) I + \delta P - \mu Q
\end{align*}
\] (18)

Then we look for an optimal control such that
\[ J(u_1^*, u_2^*, u_3^*, u_4^*) = \min J(u_1, u_2, u_3, u_4), \quad u_1, u_2, u_3, u_4 \in U \] (19)
Here, \( u_i(t) \in [0, 1] \), for all \( t \in [0, t_f] \), \( i = 1, 2, 3, 4 \). The control set is defined as
\[ U = \{(u_1, u_2, u_3, u_4) | u_i(t) \text{ is Lebesgue measurable on } [0, 1], \ i = 1, 2, 3, 4 \}. \] (20)

According to the Pontryagin’s maximum principle, we set the Hamiltonian function as follows
\[
H = A_1 E_1 + A_2 E_2 + A_3 I + \frac{1}{2} (C_1 u_1^2 + C_2 u_2^2 + C_3 u_3^2 + C_4 u_4^2) \\
+ \lambda_1 [\mu N - \mu S - (1 - u_1) \frac{\beta SI}{N}] \\
+ \lambda_2 [(1 - u_1) \theta \frac{\beta SI}{N} - [(1 - u_2) v_1 + v_2 + \mu] E_1]
\]
where \( \lambda_i \) (\( i = 1, 2, 3, 4, 5, 6 \)) are the adjoint variables.

**Theorem 5.1.** Given optimal control pairs \((u_1^*, u_2^*, u_3^*, u_4^*)\) and solutions \(S(t), E_1(t), E_2(t), I(t), P(t), Q(t)\) of the state system (18), there exist adjoint variables \(\lambda_i (i = 1, 2, 3, 4, 5, 6)\), satisfying the following adjoint system

\[
\begin{align*}
\lambda'_1 &= \lambda_1 \mu + (1 - u_1) \frac{\beta SI}{N} - [(1 - u_2)w_1 + w_2 + \mu]E_2 \\
+ \lambda_3((1 - u_2)v_1 + (1 - u_3)w_1 - (k_1 + k_2 + \mu + u_4)I) \\
+ \lambda_5[v_2E_1 + w_2E_2 + k_2I - (\delta + \mu)P] \\
+ \lambda_6[(k_1 + u_4)I + \delta P - \mu Q]
\end{align*}
\]

\(\lambda_i\) is the adjoint variable corresponding to the optimal control function \(u_i\). The terminal condition of adjoint equations is given by

\[
\lambda_i(t_f) = 0, \quad i = 1, 2, 3, 4, 5, 6.
\]

and the optimal control functions are given by

\[
\begin{align*}
u_1^* &= \max\{0, \min\{1, \frac{\beta SI}{\lambda_1 + \lambda_2 \theta - \lambda_3 (1 - \theta)}\}\} \\
u_2^* &= \max\{0, \min\{1, \frac{\beta SI}{\lambda_1 + \lambda_2 \theta - \lambda_3 (1 - \theta)}\}\} \\
u_3^* &= \max\{0, \min\{1, \frac{\beta SI}{\lambda_1 + \lambda_2 \theta - \lambda_3 (1 - \theta)}\}\} \\
u_4^* &= \max\{0, \min\{1, \frac{\beta SI}{\lambda_1 + \lambda_2 \theta - \lambda_3 (1 - \theta)}\}\}
\end{align*}
\]

**Proof.** According to the Pontryagin’s Maximum Principle, first we differentiate the Hamiltonian operator \(H\). The adjoint system can be written as

\[
\begin{align*}
\lambda'_1(t) &= -\frac{\partial H}{\partial S}(t) \\
\lambda'_2(t) &= -\frac{\partial H}{\partial E_1}(t) \\
\lambda'_3(t) &= -\frac{\partial H}{\partial E_2}(t) \\
\lambda'_4(t) &= -\frac{\partial H}{\partial I}(t) \\
\lambda'_5(t) &= -\frac{\partial H}{\partial P}(t) \\
\lambda'_6(t) &= -\frac{\partial H}{\partial Q}(t)
\end{align*}
\]

and \((u_1^*, u_2^*, u_3^*, u_4^*)\) satisfy the condition

\[
\frac{\partial H}{\partial u_i} = 0
\]

where \(i = 1, 2, 3, 4\). By solving the above equations, the proof is completed.
6. Numerical simulation.

6.1. The simulation of state system without control. In order to compare with the optimal system, we first simulate the situation without human control, so we set \( u_i = 0, i = 1, 2, 3, 4 \). The other parameters of this section are shown in Table 1.[34, 19]

| Parameters | Descriptions                                      | Values          |
|------------|---------------------------------------------------|-----------------|
| \( \mu \)  | Natural supplementary and death rate               | 0.05 per week   |
| \( \theta \) | Proportion of individuals who became low risk exposed | 0.4 per week    |
| \( \beta \) | Contact transmission rate                         | 0.1~0.8 per week|
| \( v_1 \)  | Proportion of \( E_1 \) who become infected       | 0.2 per week    |
| \( v_2 \)  | Proportion of \( E_1 \) who become professional   | 0.2 per week    |
| \( w_1 \)  | Proportion of \( E_2 \) who become infected       | 0.3 per week    |
| \( w_2 \)  | Proportion of \( E_1 \) who become professional   | 0.1 per week    |
| \( k_1 \)  | Proportion of \( I \) who become quitting        | 0.05 per week   |
| \( k_2 \)  | Proportion of \( I \) who become professional     | 0.1 per week    |
| \( \delta \) | Proportion of \( P \) who become quitting        | 0.5 per week    |
| \( u_1 \)  | The decreased proportion by isolation             | Variable        |
| \( u_2 \)  | The decreased proportion in \( E_1 \) by prevention| Variable        |
| \( u_3 \)  | The decreased proportion in \( E_2 \) by prevention| Variable        |
| \( u_4 \)  | The decreased proportion in \( I \) by treatment   | Variable        |

The research object of this paper is the long-term residents in mainland China. According to some relevant survey data[31], the initial population of each warehouse is as follows: \( S(0) = 580, E_1(0) = 38, E_2(0) = 70, I(0) = 50, P(0) = 20, Q(0) = 71 \) (units in million).

First, we select \( \beta = 0.2 \), and we solve the state system under this initial value and obtain the curve of population change in each warehouse, as shown in Figure 2.

In Figure 2, we can see that the change of population in each warehouse will eventually tend to a stable state, which is consistent with the content of Theorem 2 in the theoretical proof part when \( R_0 \) is less than 1. In this case, game addiction will eventually disappear and will not break out. Further, we increased the contact infection rate \( \beta \) to 0.8. We solve the state system under this initial value and obtain the curve of population change in each warehouse similarly, as shown in Figure 3. At this point, the value of \( R_0 \) is 2.3111 and game addiction will continue. The simulation results are shown in Figure 3, which also confirm the correctness of Theorem 4.

Next, in order to explore the influence of initial infectious number on the system, we choose the following initial values: \( x_{01} = (580, 38, 70, 50, 20, 71), x_{02} = (580, 38, 70, 55, 18, 68), x_{03} = (580, 38, 70, 60, 16, 65), x_{04} = (580, 38, 70, 65, 14, 62), x_{05} = (580, 38, 70, 45, 22, 74), x_{06} = (580, 38, 70, 40, 24, 77) \). In Figure 4, if we take different initial values, the number of infected people will all converge to 0, and the addicted people will disappear when \( R_0 = 0.5778 \) and \( \beta = 0.2 \). In Figure 5, we can see that when we take different initial values, the number of infected people will all converge to 67.916 million, and the addicted people will continue when \( R_0 = 2.3111 \) and \( \beta = 0.8 \).

From Figure 5, we can see that in the absence of human intervention, the situation of game addiction is still relatively serious. About 67.916 million people are addicted.
6.2. The simulation of optimal control. In this section, we use the forward-backward sweep method to solve the optimal system. In recent years, this method has been widely used in the solution of optimal system. First, we choose the appropriate initial value within the allowable range, and use the forward fourth-order Runge-Kutta method to solve the state system. Then, the results are substituted into the adjoint system, and the adjoint variables are solved by the backward fourth-order Runge-Kutta method. The new state variables and adjoint state variables are selected as the new initial values, and the first two steps are repeated until the two adjacent state variables are close enough. Please refer to references [9, 22] for more details.

Considering that in real world the specific implementation process of the control strategy has a wide range and involves many people, the control effect is very difficult to achieve the ideal effect, so it is more reasonable to set the upper limit of the control variable to 0.8. The weight factors we select $C_1 = 10000$, $C_2 = 1000$, $C_3 = 1000$, $C_4 = 10000$, $A_1 = 250$, $A_2 = 250$ and $A_3 = 10$. The values of the remaining parameters are shown in Table 1.

In reality, the main control measures are: isolation, education and treatment. We do not consider a single control strategy here, because the effect of multiple
control measures combination will be better in real world. In order to study the effectiveness of different control strategies, we consider the following combination strategies:

Scenario 1: Coupled control strategies
Strategy A: Isolation + treatment \((u_1(t), u_4(t))\).
Strategy B: Isolation + education \((u_1(t), u_2(t), u_3(t))\).
Strategy C: Education + treatment \((u_2(t), u_3(t), u_4(t))\).

Scenario 2: Threefold control strategies
Strategy D: Isolation + education + treatment \((u_1(t), u_2(t), u_3(t), u_4(t))\).
Strategy E: Isolation + education for \(E_1\) + treatment \((u_1(t), u_2(t), u_4(t))\).
Strategy F: Isolation + education for \(E_2\) + treatment \((u_1(t), u_3(t), u_4(t))\).

Scenario 3: Constant control strategies
Strategy G: High intensity \((u_1(t) = u_2(t) = u_3(t) = u_4(t) = 0.8)\).
Strategy H: Low intensity \((u_1(t) = u_2(t) = u_3(t) = u_4(t) = 0.2)\).
Strategy I: Middle intensity \((u_1(t) = u_2(t) = u_3(t) = u_4(t) = 0.5)\).

Strategy A: Isolation \((u_1)\) and treatment \((u_4)\)
In strategy A, we consider the isolation and treatment measures implemented together, so the control variable \(u_1 \neq 0\), \(u_2 = u_3 = 0\) and \(u_4 \neq 0\). The comparison between the result of strategy A and the one without control is shown in Figure 6. In
Figure 4. Dynamical behavior of infected when $R_0 = 0.5778$ and $\beta = 0.2$.

Figure 5. Dynamical behavior of infected when $R_0 = 2.3111$ and $\beta = 0.8$. 
Figure 6. Graphical results for strategy A.

Figure 6 (a), the number of susceptible population under strategy A is significantly higher than that of population without control. In Figure 6 (b) - (f), the population of each warehouse under strategy A is smaller than that without control. In figure 6 (g), the isolation measure is set to a maximum of 0.8 from the beginning and kept for 4 weeks, then gradually reduced to 0. In Figure 6 (j), the intensity of the treatment is set to 0.65 on the first week, then gradually reduced to 0. Figure 6 shows that through the implementation of control strategy A, the number of addicts is indeed reduced, which is what we hope.
Strategy B: Isolation ($u_1$) and education ($u_2, u_3$)

In strategy B, we consider the isolation and education measures implemented together, so the control variable $u_1 \neq 0, u_2 \neq 0, u_3 \neq 0$ and $u_4 = 0$. The comparison between the result of strategy B and that without control is shown in Figure 7. We can see from Figure 7 (a)-(f) that, it can reduce the number of people addicted to the game, and its control effect is similar to figure 6. Therefore, in order to further distinguish the effect of each strategy, we will compare each control strategy in the

Figure 7. Graphical results for strategy B.
end. It is worth noting that the decrease rate of the number of exposed and infected compartments are slower than that in Figure 6, which is not a satisfactory result.

From Figure 7 (g)-(i) we know that if there is no treatment, isolation and education measures will continue to maintain a very high intensity. It can be seen that the treatment measures are very important to control the game addiction.

**Strategy C: education** \((u_2, u_3)\) **and treatment** \((u_4)\)

In strategy C, we consider the education and treatment measures implemented together, so the control variable \(u_1 = 0\), \(u_2 \neq 0\), \(u_3 \neq 0\) and \(u_4 \neq 0\). The comparison between the result of strategy C and that without control is shown in Figure 8. As it can be seen from Figure 8, the number of addicts has also decreased a lot, and the control effect is also very good. The values of control variables are shown in Figure 8 (g)-(j).

From the Figure 8, we can get the conclusion that education and treatment play a very important role in controlling game addiction, which also inspires researchers to study more efficient education and treatment programs.

**Strategy D: Isolation** \((u_1)\), **education** \((u_2, u_3)\) **and treatment** \((u_4)\)

In strategy D, we consider all measures implemented together. The comparison between the result of strategy D and that without control is shown in Figure 9. From the Figure 9 (a)-(f), it can be seen that the number of exposed and addicted people decrease a lot. The good results are similar to those in Figure 6. From these pictures, the number of exposed and infected people in strategy A and strategy D seems to be less than that under strategies B and C. We will further determine this from accurate data.

**Strategy E: Isolation** \((u_1)\), **education for** \(E_1\) \((u_2)\) **and treatment** \((u_4)\)

In strategy E, we consider the isolation, education for \(E_1\) and treatment measures implemented together. So the control variable \(u_1 \neq 0\), \(u_2 \neq 0\), \(u_3 = 0\) and \(u_4 \neq 0\). The comparison between the result of strategy E and that without control is shown in Figure 10. From the Figure 10 (a)-(f), it can be seen that the number of exposed and addicted people decrease a lot. Compared with strategy A, education measure \(u_2\) for \(E_1\) is added in strategy E. But the results of strategy E and strategy A are very similar, and the difference between them may not be easy to compare graphically.

**Strategy F: Isolation** \((u_1)\), **education for** \(E_2\) \((u_3)\) **and treatment** \((u_4)\)

In strategy F, we consider the isolation, education for \(E_2\) and treatment measures implemented together. So the control variable \(u_1 \neq 0\), \(u_2 = 0\), \(u_3 \neq 0\) and \(u_4 \neq 0\). The comparison between the result of strategy F and that without control is shown in Figure 11. From the Figure 11 (a)-(f), we can see the trend of population change in each warehouse, which is also generally in line with our expectations.

**Strategy G: High intensity control** \((u_1(t) = u_2(t) = u_3(t) = u_4(t) = 0.8)\).

In order to reduce the number of addicts as quickly as possible, we will consider the maximum intensity control strategy. So the control variable \(u_1(t) = u_2(t) = u_3(t) = u_4(t) = 0.8\). The comparison between the result of strategy F and that without control is shown in Figure 12. As can be seen from Figure 12 (d), there is a very sharp decrease in the number of addicts, which is caused by the continuous maximum intensity of control. Although this is a very good result, when the number of addicts is effectively controlled, there is no need to maintain the highest intensity of control, which will inevitably waste a lot of human and financial resources. Therefore, we need to further evaluate the strategy from the objective function \(J\).

**Strategy H: Low intensity control** \((u_1(t) = u_2(t) = u_3(t) = u_4(t) = 0.2)\).
In strategy H, we consider that the intensity of all control variables should be kept at a low level to observe the population change trend of each warehouse, so the control variable \( u_1(t) = u_2(t) = u_3(t) = u_4(t) = 0.2 \). The comparison between the result of strategy H and that without control is shown in Figure 13. From the Figure 13 (b)-(d), we can also see that the number of exposed and addicted people decrease a lot. Compared with other control strategies, the decrease rate of exposed and addicted warehouse is very slow, which is caused by the lack of power due to
continuous low intensity control. This shows that although the low intensity control may waste less human and financial resources than the high intensity control, the control effect is also not ideal.

**Strategy I:** Middle intensity control \((u_1(t) = u_2(t) = u_3(t) = u_4(t) = 0.5)\).

In view of the above two control strategies, we continue to consider the middle intensity control strategy. In strategy I, the control variable \(u_1(t) = u_2(t) = u_3(t) = u_4(t) = 0.5\). The comparison between the result of strategy I and that without control is shown in Figure 14. Compared with the case of without control, the
number of exposed and addicted under this strategy also decreased rapidly. From the graphic observation, we also get an ideal result.

Our goal is not only to minimize the number of exposed and infected people, but also to minimize the cost of control. In order to further distinguish the strategies, we get the total number of infections under each strategy, the number of people who avoid infection due to the implementation of the control strategy, and the objective function $J$. They are shown in Table 2 respectively.

**Figure 10.** Graphical results for strategy E.
From the data of strategy A, B and C, we can see that in strategy B, which lacks treatment, the number of addicts is much higher than that of other strategies. This shows that if only isolation and education are used to control addiction, the effect is very unsatisfactory, which also highlights the importance of treatment measures.

From the data in Table 2, we can obtain that strategy D is the optimal control strategy no matter which term it is from. If we follow the three control measures of strategy D, the number of addicts can be greatly reduced. From the data of strategy
E and strategy F, we can see that the implementation of educational measures on high risk exposure is more efficient than that on low risk groups. At the same time, we can also get that high risk exposure will have more influence than low risk exposure.

From the data of constant control strategy G, H and I, we know that although high intensity control strategy G can relatively reduce the number of infected people, it will produce very large cost consumption. Low intensity control strategy H will
not only increase the number of infected people, but also produce relatively large cost consumption. Comparing the middle intensity control strategy I with the strategy D, E and F, we find that its control effect is not very good. It can be seen that the strength of control does not need to be kept at the highest intensity, which can save more human and financial resources.

In summary, we propose to use three control strategies as shown in Figure 9 (g). It will effectively alleviate the current serious situation of game addiction.
Figure 14. Graphical results for strategy I.

7. Conclusion. The increasingly serious situation of online game addiction has brought many negative effects to individuals and society. Some countries have begun to take some measures to prevent and control it. In this paper, an ordinary differential equations system was established to simulate and analyze the situation of game addiction. The main findings were as follows:

(i) A model with low-risk exposed and high risk exposed individuals was established, and the expression of the basic reproduction number was solved by the
method of next generation matrix. The disease-free equilibrium and endemic equilibrium of the system were solved, and the global stability was proved by constructing appropriate Lyapunov function.

(ii) In this paper, we considered the three most commonly used control measures (isolation, education and treatment) in real life. The optimal control system (composed of state system and adjoint system) was established, and the expression of the optimal solution was obtained.

(iii) In the simulation, we chose a suitable initial value, got the global stability of DFE and EE, and proved the correctness of theorems 4.1 and 4.2. At the same time, we also chose different initial values and got the same conclusion.

(iv) The control problem of game addiction was also constructed and solved, and different combination strategies of control measures were shown. The optimal control strategy was obtained by comparing different combination strategies. The results showed that every control measure is very important when the addictions broke out. The best control results could be achieved when isolation measure were taken together with education and treatment.

Through the study of these findings, if we control the game addiction according to the optimal control solution, the serious situation of game addiction will be greatly reduced. At the same time, these results can also be used as suggestions for the public health management departments.

As Professor Kuang[17] said: “Time delay is almost everywhere in the objective world”. There are some differences between game addiction and infectious diseases. In this model, it takes a long time to change from an exposed player to an addicted player, so there is a time delay in this process. In the next step, we will consider introducing time delay into the model, then modeling and analyzing. By comparing their results, then we discuss the importance of time delay.

Acknowledgments. The authors are grateful to the editor and the reviewers for their valuable comments and suggestions. This work was supported by Basic Competence Promotion Project for Young and Middle-aged Teachers in Guangxi, China (2019KY0269).

Disclosure statement. There are no conflicts of interest by the authors.
REFERENCES

[1] F. B. Agusto and M. A. Khan, Optimal control strategies for dengue transmission in pakistan, *Math. Biosci.*, **305** (2018), 102–121.

[2] J. O. Akanni, F. O. Akinpelu, S. Olaniyi, A. T. Oladipo and A. W. Ogunsola, Modelling financial crime population dynamics: Optimal control and cost-effectiveness analysis, *Int. J. Dyn. Control*, **8** (2020), 531–544.

[3] A. Barrea and M. E. Hernández, Optimal control of a delayed breast cancer stem cells nonlinear model, *Optimal Control Appl. Methods*, **37** (2016), 248–258.

[4] E. Bonyah, M. A. Khan, K. O. Okosun and J. F. Gómez-Aguilar, Modelling the effects of heavy alcohol consumption on the transmission dynamics of gonorrhea with optimal control, *Math. Biosci.*, **309** (2019), 1–11.

[5] D. K. Das, S. Khajanchi and T. K. Kar, The impact of the media awareness and optimal strategy on the prevalence of tuberculosis, *Appl. Math. Comput.*, **366** (2020), 124732, 23 pp.

[6] C. Ding, Y. Sun and Y. Zhu, A schistosomiasis compartment model with incubation and its optimal control, *Math. Methods Appl. Sci.*, **40** (2017), 5079–5094.

[7] P. van den Driessche and J. Watmough, Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission, *Math. Biosci.*, **180** (2002), 29–48.

[8] G. Fan, H. R. Thieme and H. Zhu, Delay differential systems for tick population dynamics, *J. Math. Biol.*, **71** (2015), 1017–1048.

[9] W. H. Fleming and R. W. Rishel, *Deterministic and Stochastic Optimal Control*, Springer-Verlag, New York, 1975.

[10] D. Gao and N. Huang, Optimal control analysis of a tuberculosis model, *Appl. Math. Model.*, **58** (2018), 47–64.

[11] Y. Guo and T. Li, Optimal control and stability analysis of an online game addiction model with two stages, *Math. Method App. Sci.*, **43** (2020), 4391–4408.

[12] K. Hattaf, Optimal control of a delayed HIV infection model with immune response using an efficient numerical method, *ISRN Biomathematics*, (2012), Article ID215124.

[13] J. M. Heffernan, R. J. Smith and L. M. Wahl, Perspectives on the basic reproductive ratio, *J. R. Soc. Interface*, **2** (2005), 281–293.

[14] H.-F. Huo, F.-F. Cui and H. Xiang, Dynamics of an SAITS alcoholism model on unweighted and weighted networks, *Physica A*, **496** (2018), 249–262.

[15] H.-F. Huo and X.-M. Zhang, Complex dynamics in an alcoholism model with the impact of Twitter, *Math. Biosci.*, **281** (2016), 24–35.

[16] M. A. Khan, S. W. Shah, S. Ullah and J. F. Gómez-Aguilar, A dynamical model of asymptomatic carrier zika virus with optimal control strategies, *Nonlinear Anal. Real World Appl.*, **50** (2019), 144–170.

[17] Y. Kuang, *Delay Differential Equations with Application in Population Dynamics*, Academic Press, Inc., Boston, MA, 1993.

[18] V. Lakshmikantham, S. Leela and A. A. Martynyuk, *Stability Analysis of Nonlinear Systems*, Marcel Dekker, Inc., New York, 1989.

[19] T. Li and Y. Guo, Stability and optimal control in a mathematical model of online game addiction, *Filomat*, **33** (2019), 5691–5711.

[20] Z. Lin and H. Zhu, Spatial spreading model and dynamics of West Nile virus in birds and mosquitoes with free boundary, *J. Math. Biol.*, **75** (2017), 1381–1409.

[21] Z. Lu, From E-Heroin to E-sports: The development of competitive gaming in China, *The International Journal of the History of Sport*, **33** (2017), 2186–2206.

[22] D. L. Lukes, *Differential Equations: Classical to Controlled*, Mathematics in Science and Engineering, Academia Press, New York, 1982.

[23] M. McAsey, L. Mou and W. Han, Convergence of the forward-backward sweep method in optimal control, *Comput. Optim. Appl.*, **53** (2012), 207–226.

[24] K. O. Okosun, M. A. Khan, E. Bonyah and O. O. Okosun, Cholera-schistosomiasis coinfection dynamics, *Optim. Contr. Appl. Met.*, **40** (2019), 703–727.

[25] K. A. Pawelek, A. Oeldorf-Hirsch and L. Rong, Modeling the impact of Twitter on influenza epidemics, *Math. Biosci. Eng.*, **11** (2014), 1337–1356.

[26] M. Sana, R. Saleem, A. Manaf and M. Habib, Varying forward backward sweep method using Runge-Kutta, Euler and Trapezoidal scheme as applied to optimal control problems, *Sci.Int.(Labore)*, **27** (2015), 839–843.
[27] O. Sharomi and A. B. Gumel, Curtailing smoking dynamics: A mathematical modeling approach, *Appl. Math. Comput.*, **195** (2008), 475–499.

[28] Statistical Classification of Sports Industry, 2019. Available from: [http://www.stats.gov.cn/tjgz/tzgb/201904/t20190409_1658556.html](http://www.stats.gov.cn/tjgz/tzgb/201904/t20190409_1658556.html).

[29] X. Sun, H. Nishiura and Y. Xiao, Modeling methods for estimating HIV incidence: A mathematical review, *Theor. Biol. Med. Model.*, **17** (2020), 1–14.

[30] C. S. Tang, Y. W. Koh and Y. Q. Gan, Addiction to internet use, online gaming, and online social networking among young adults in China, Singapore, and the United States, *Asia Pac. J. Public. He.*, **29** (2017), 673–682.

[31] The 43rd Statistical Report on Internet Development in China, 2019. Available from: [http://www.cac.gov.cn](http://www.cac.gov.cn).

[32] X. Tian, R. Xu and J. Lin, Mathematical analysis of a cholera infection model with vaccination strategy, *Appl. Math. Comput.*, **361** (2019), 517–535.

[33] S. Ullah, M. A. Khan and J. F. Gómez-Aguilar, Mathematical formulation of hepatitis B virus with optimal control analysis, *Optim. Contr. Appl. Met.*, **40** (2019), 529–544.

[34] R. Viriyapong and M. Sookpiam, Education campaign and family understanding affect stability and qualitative behavior of an online game addiction model for children and youth in Thailand, *Math. Method App. Sci.*, **42** (2019), 6906–6916.

[35] X. Wang, M. Shen, Y. Xiao and L. Rong, Optimal control and cost-effectiveness analysis of a Zika virus infection model with comprehensive interventions, *Appl. Math. Comput.*, **359** (2019), 165–185.

[36] X. Wang, Y. Shi, D. Wang and C. Xu, Dynamic Analysis on a Kind of Mathematical Model Incorporating Online Game Addiction Model and Age-Structure, *Journal of Beijing University of Civil Engineering and Architecture*, **2** (2017), 54–58.

[37] World Health Statistics 2019, 2019. Available from: [https://www.who.int/data/gho/publications/world-health-statistics](https://www.who.int/data/gho/publications/world-health-statistics).

[38] T. A. Yıldız and E. Karaoğlu, Optimal control strategies for tuberculosis dynamics with exogenous reinfections in case of treatment at home and treatment in hospital, *Nonlinear Dynam.*, **97** (2019), 2643–2659.

[39] Z.-K. Zhang, C. Liu, X.-X. Zhan, X. Lu, C.-X. Zhang and Y.-C. Zhang, Dynamics of information diffusion and its applications on complex networks, *Phys. Rep.*, **651** (2016), 1–34.

[40] W. Zhou, Y. Xiao and J. M. Heffernan, Optimal media reporting intensity on mitigating spread of an emerging infectious disease, *Plos. One.*, **3** (2019), E0213898.

Received May 2020; revised August 2020.

E-mail address: kyoisgood@163.com
E-mail address: wsltt0621@163.com