Is there a radio excess from the decoupling of pre-recombination bremsstrahlung?

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Recently it has been suggested that thermal bremsstrahlung emission, when it decouples prior to recombination, creates an excess over the Planck cosmic microwave background spectrum at sub-GHz frequencies. Remarkable by itself, this would also explain a long-standing unexplained deficit in the predictions of the extragalactic radio background. In this brief note we reiterate that no such non-thermal component can arise by itself when matter and radiation remain kinetically coupled.

I. INTRODUCTION

Can a substantial spectral distortion in the cosmic microwave background (CMB) radiation develop from the decoupling of a thermal process that helped maintain the blackbody form in the first place? It was recently suggested that this may indeed happen by the free-free emission process, Bremsstrahlung, in the pre-recombination early Universe at redshifts of a few thousand$^1$. An excess of photons for comoving frequencies of 10 MHz to 1 GHz in the Rayleigh-Jeans tail of the CMB is found, surpassing the Planckian occupation number. This would not only amount to a remarkable addition to our understanding of CMB formation, but may also explain a to-date unexplained tension in the extragalactic radio background. When expectations based on faint source counts are confronted with observations, a deficit in the predictions is found$^2,3$. For example, at a frequency of $\nu = 200$ MHz the differential Planckian photon number density is 0.01 cm$^{-3}$ GHz$^{-1}$. Filling the gap and explaining the radio data requires an excess $\Delta n/n_{\text{Planck}}|_{200\ \text{MHz}} = 100\%$, where it is surmised that the “non-Planckian” build-up of photons from of the free-free process happens from $z = 2150$ until recombination. Finally, an extra abundance of photons in the purported frequency range would also amplify$^4,5$ the predictions for the much sought-after cosmological 21 cm signal$^6$, especially given the recently claimed observations$^7$.

Evidently, the claim of a guaranteed non-thermal $O(1)$ distortion of the CMB warrants scrutiny. It is the purpose of this short note to recall standard arguments from kinetic theory and to clarify that no such build-up from thermal initial conditions is possible.

II. BOOTSTRAP SPECTRAL DISTORTION?

We now follow through a chain of standard arguments that shows that a large spectral distortion cannot be seeded by thermal pre-recombination bremsstrahlung alone. For this purpose we ignore any effects that may originate from the mismatch of photon and electron temperatures. Compton scattering ensures their equivalence, $T_e = T$, to high precision until and during recombination.

Denote the isotropic and homogeneous energy-differential number density of photons by $dn/d\omega$. The associated occupation number is denoted by $f(\omega)$ with $dn/d\omega = (\omega/\pi)^2 f(\omega)$. If a change in $n$ is due to the emission and absorption of single quanta in process $A$, we may write

$$\frac{\partial}{\partial t} \frac{dn}{d\omega} \bigg|_A = \frac{d\Gamma}{d\omega dV} (1 + f) - \frac{dn}{d\omega} \Gamma_{\text{abs}}. \quad (1)$$

Here, $d\Gamma/d\omega dV$ is the spontaneous emission rate per volume and energy, corrected for stimulated emission by the factor $(1 + f)$. The second term on the right hand side accounts for absorption with total rate $\Gamma_{\text{abs}}(\omega)$. In thermal equilibrium, Eq. (1) must vanish identically by the principle of detailed balancing. Hence, plugging in the equilibrium distribution $f_T = [\exp(\omega/T) - 1]^{-1}$ yields the relation between forward and inverse process,

$$\Gamma_{\text{abs}} = \frac{\pi^2}{\omega^2} \frac{\omega}{T} \frac{d\Gamma}{d\omega dV}. \quad (2)$$

In an expanding FRW Universe, the Boltzmann equation for the evolution of the photon occupation number $f(\omega)$ under the influence of $A$ is given by

$$\frac{\partial f}{\partial t} - H \frac{\omega}{T} \frac{\partial f}{\partial \omega} = \frac{\partial f}{\partial \omega} \bigg|_A. \quad (3)$$

Introducing the dimensionless and comoving variable $x = \omega/T$ scales out the expansion term on the left-hand-side when we are to consider the evolution of $f(x)$. A photon number changing process such as double Compton scattering or bremsstrahlung, together with Compton scattering, will bring the photon spectrum to a blackbody form when the rates are faster than the Hubble rate; for an explicit demonstration, see Figs. 1 and 2 of$^8$.

We are interested in a deviation from the Planck spectrum that may develop as the Universe cools down. To this end, we write

$$f(x) = f_T(x) + \Delta f(x) \quad (4)$$

where $\Delta f > 0$ would mean an excess of photons compared to a blackbody. By construction, terms with $f_T$
drop out of (3), and one is left with an equation that describes the departure from the blackbody spectrum,

$$\frac{\partial \Delta f}{\partial t} = \frac{\pi^2}{x^2 T^3} \frac{d\Gamma}{dx dV} \Delta f (1 - e^\gamma).$$  \hspace{1cm} (5)$$

We may now focus on the pre-recombination era at redshift $z \lesssim 10^4$ where it is claimed that a large deviation from bremsstrahlung decoupling may be imprinted onto the CMB. This epoch is commonly referred to y-distortion era where Compton and double Compton scattering have become inefficient in changing photon number and momenta, respectively \textcircled{3}. The remaining channel for photon emission is then non-relativistic Bremssstrahlung. Here, the dominant contribution is dipole emission in the collision of electrons with protons, $e^p \rightarrow e^p \gamma$ with an $O(1)$ correction from helium \textcircled{10–12}; the quadrupole process from electron scattering, $ee \rightarrow ee \gamma$, is suppressed \textcircled{13, 14}. The emission rate in a Maxwellian plasma where matter is in equilibrium with radiation with common temperature $T$ is given by

$$\frac{d\Gamma}{dx dV} = \frac{16}{3} \sqrt{\frac{2 \pi}{3}} \frac{\alpha^3 n_e n_p}{m_e^3 T^{3/2}} \langle g_{\text{ff}} \rangle.$$  \hspace{1cm} (6)$$

Here, $\alpha$ is the fine-structure constant, $n_e$ is the electron mass, $n_e \approx n_p$ are the (approximate) electron and proton number densities and $\langle g_{\text{ff}} \rangle$ is the thermally averaged Gaunt factor; see \textcircled{15} for an expression for $g_{\text{ff}}$ and definition of $\langle g_{\text{ff}} \rangle$ that covers all non-relativistic kinematic regimes. We note that the emission process is not in the Born regime and results exact to all orders in the Coulomb interaction of the colliding particle pair must be used \textcircled{16}; for quadrupole emission cf. \textcircled{13, 14}. To see this, one evaluates the Sommerfeld parameter for a typical relative initial velocity $\eta = Z^2 \alpha / v \approx Z^2 \alpha \sqrt{m_e / T} = 6 \times 10^4$ to 10(40) for redshifts $z = 3000$ to 1000 for protons (fully ionized helium).

The solution of (3) is readily obtained and given by

$$\Delta f(x, z = 0) = \Delta f(x, z_{\text{high}}) e^{-\tau_{\text{ff}}(z_{\text{high}})}$$  \hspace{1cm} (7)$$

where $z_{\text{high}}$ is some pre-recombination redshift and $\tau_{\text{ff}}$ is the optical depth due to bremsstrahlung,

$$\tau_{\text{ff}}(z) = \int_0^z \frac{1}{(1 + z) H} \frac{\pi^2}{x^2 T^3} \frac{d\Gamma}{dx dV} (1 - e^\gamma)$$  \hspace{1cm} (8)$$

which is a strictly positive quantity. For example, a contribution at $x = 200$ MHz (1 GHz) requires emission with $x = 0.02$ ($x = 0.1$). Indeed, the associated optical depth is small $\tau_{\text{ff}}(z_{=2000}) = 0.003 \times 10^{-5}$ and the Universe is already transparent for frequencies $\nu \gtrsim 10$ MHz.

However, as is evident from (4), a thermal plasma in equilibrium with matter cannot, by itself, develop a deviation from the Planck spectrum. Free-free emission in a medium where photons and electrons share the same temperature will only reduce any preexisting spectral distortion. Indeed, only if some non-thermal deviation in form of $\Delta f(x, z_{\text{high}}) \neq 0$ at initial redshift $z_{\text{high}}$ was already present—not produced by thermal bremsstrahlung—may such distortion survive until today. Prerequisite for it is that $\tau_{\text{ff}} \ll 1$.

The statements made here are not new, but appear in various and explicit forms in a broad and long history of studies on thermalization and spectral distortions of the CMB, see, e.g., \textcircled{8, 9, 17, 18} and references therein. The spectral distortions that are predicted from the standard recombination cosmic history, e.g., induced by the late-time mismatch of electron and photon temperature, are, compared to the claim made in \textcircled{9}, minute.

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\textbf{References}

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\footnote{The thermally averaged Gaunt factor $\langle g_{\text{ff}} \rangle(x)$ is often written in the form $e^{-\eta} g_{\text{ff}}(x)$ where the exponential is then part of the thermally averaged Kramers emissivity.}
