$K^0$ form factor and charge radius in a covariant Salpeter model

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Abstract

The electromagnetic form factor for the $K^0$ is calculated in a covariant formulation of the Salpeter equation for $q\bar{q}$-bound states, which has been presented recently for the mass spectrum, decay properties and form factors of the light pseudoscalar and vector mesons. The $K^0$ charge radius dependence on the difference between strange and down constituent quark mass is discussed.

I. INTRODUCTION

It has been proposed by Magahiz et al. [1] to observe the reaction $p(e, e'K^0)\Sigma^+$ with the CLAS Large Acceptance Spectrometer at CEBAF to gain insight into strangeness electro-production of nuclei. If the longitudinal and transverse part of the differential cross section could be separated, the t-channel reaction would allow the measurement of the $K^0$ electromagnetic form factor which due to the mass difference between the strange and down quarks does not vanish. In view of this proposed experiment predictions for the $K^0$ form factor have been recently published and it commonly turned out that due to the accessible values of momentum transfer up to few (GeV)$^2$ a covariant description of the underlying dynamics is mandatory. Cardarelli et al. investigated a relativistic constituent quark model based on the light front formalism [2]. Therein they made use of an interaction kernel motivated by an effective $q\bar{q}$-Hamiltonian which has been developed by Godfrey and Isgur [3]. In another paper Buck, Williams and Ito [4] calculated the $\pi$ and $K$ form factors by employing a model described in [5], with a separable ansatz including symmetry breaking effects.

In two previous papers [6,7] we presented a covariant quark model based on the Salpeter equation and used it to compute (transition) form factors between the light pseudoscalar and vector mesons [8]. This brief report shall serve as an extension of the latter to the neutral strange meson.

II. THE MODEL

Starting from the Bethe-Salpeter equation, we use a $q\bar{q}$-interaction assumed instantaneous in the rest frame of the bound state and free effective quark propagators to arrive...
at the Salpeter equation, which is expressed as an eigenvalue problem for the bound state mass and solved numerically [3]. In addition to the calculation of mass spectra we have presented there a method to reconstruct the four-dimensional Bethe-Salpeter amplitude from the equal-time Salpeter amplitudes.

In our model the interaction consists of a confining potential which is linearly rising in coordinate space, and an instanton-induced interaction derived by ’t Hooft (see [7] and references therein) as a possible solution of the $U_A(1)$-problem. We would like to emphasize that the potential and mass parameters used in our model have been fixed in [7] to obtain a reasonable agreement with the experimental mass spectrum of the low lying pseudoscalar and vector mesons as well as the leptonic $\pi$ and $\rho$ decay widths. The electromagnetic form factors as well as the other decay widths (e.g. $M \rightarrow M' \gamma$) are consistently obtained by employing the Mandelstam formalism to the formerly calculated Bethe-Salpeter amplitudes. No additional parameter is used to calculate the current matrix elements. In lowest order we find for the electromagnetic current coupling to the quark:

$$\langle P' \mid j_\mu^{(1)}(x) \mid P \rangle = -e_1 \int \frac{d^4p}{(2\pi)^4} \text{tr} \left\{ [(i\partial^\mu - m_2)\bar{\chi}_P(x,y)] \gamma_\mu \chi_P(x,y) \right\}$$

which is formally analogous to the results obtained by Buck et al. [4] except for the inner structure of the amputated Bethe-Salpeter amplitude, which in our model contains in general eight amplitudes for fixed spin and parity [8] and thus reflects the full Dirac structure of the $q\bar{q}$-system.

### III. RESULTS: THE $K^0$ ELECTROMAGNETIC FORM FACTOR AND CHARGE RADIUS

Our results for the $K^0$ form factor $f(Q^2)$ are shown in Fig. 1 and 2 for small and large momentum transfer (in Fig. 4 we plotted $Q^2 \cdot f(Q^2)$). Our calculation agrees remarkably well with the prediction of Buck et al. [4], where the parameter have been fixed to reproduce the $\pi^+$ and $K^+$ charge radii and decay constants. However, our maximum of $Q^2 \cdot f(Q^2)$ appears at a smaller momentum transfer of approximately 2 GeV$^2$.

As our calculation has been performed in the framework of a covariant quark model which includes confinement, and therefore is able to describe not only the masses and decay properties of the pseudoscalars but also of the vector mesons, the $K^0$ form factor calculation is put on a more general basis than in the work of Buck et al. [4], although our results do not differ significantly.

The $K^0$ charge radius, as has been discussed e.g. in [4], is most sensitive to the mass difference between the strange and down quark mass. We have estimated the charge radius by a least-square fit of a quadratic function to our form factor below 0.1 GeV$^2$ and studied its dependence on the differences of the constituent quark masses $m_s - m_d$ keeping the sum of them fixed to our original value $m_d + m_s = 170\text{MeV} + 390\text{MeV} = 560\text{MeV}$ [7]. The results plotted in Fig. 3 indeed shows a strong dependence on the quark mass difference, as long as it is smaller than 250 MeV.

An experimental measurement of the $K^0$ charge radius therefore would be an interesting opportunity to determine the difference between strange and nonstrange constituent quark mass, alternatively to estimates from purely spectroscopic quark model calculations.
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FIGURES

FIG. 1. The $K^0$ form factor at small momentum transfer

FIG. 2. The $K^0$ form factor times $Q^2$ at large momentum transfer
FIG. 3. The mean squared charge radius as a function of the difference between strange and down constituent quark mass. The dot indicates the prediction of our original model from \[7\].