SUPER-RADIANCE:
FROM NUCLEAR PHYSICS TO PENTAQUARKS *

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The phenomenon of super-radiance in quantum optics predicted by Dicke 50 years ago and observed experimentally has its counterparts in many-body systems on the borderline between discrete spectrum and continuum. The interaction of overlapping resonances through the continuum leads to the redistribution of widths and creation of broad super-radiant states and long-lived compound states. We explain the physics of super-radiance and discuss applications to weakly bound nuclei, giant resonances and widths of exotic baryons.

1. Introduction

Traditionally nuclear theory is divided into nuclear structure and nuclear reactions. Being naturally related by physics of nuclei as the subject of research, these fields are still significantly different in their methods, quality and justification of approximations and level of understanding. Moving away from the line of nuclear stability, we have to overcome this barrier. Only the consistent consideration of microscopic structure together with the response of the system to external fields, including various reaction amplitudes, is capable of developing the general picture of loosely bound nuclei. Such systems which become open even under weak excitations have their specifics in extreme sensitivity of internal properties to the proximity of continuum. Similar problems emerge in atomic and molecular physics as well as in mesoscopic condensed matter physics and quantum optics.

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In a weakly bound quantum system, couplings of intrinsic dynamics with reaction channels can crucially determine many important physical properties. Above threshold, intrinsic energy levels become resonances embedded in the continuum with the lifetime typically shortening as excitation energy increases. The relevant physical parameter here is $\kappa = \gamma / D$, the ratio of the characteristic partial width (for a given channel) of a resonance to the spacing between the resonances. The presence of overlapping resonances, $\kappa > 1$, is usually associated with a very complicated and unpredictable pattern of Ericson fluctuations that can only be considered in statistical terms of random amplitudes. It turns out however that different, less known, dynamics can take place in the region of a relatively small number of open channels. New collective phenomena are possible leading to the redistribution of the widths (and of corresponding time scales) and creation of short-lived (super-radiant) as well as long-lived (trapped) structures. Being a consequence of the unitarity of the dynamics in the continuum, this restructuring appears in any consistent theory that considers the bound states and continuum on equal footing. In particular, it follows from classical Fano theory widely used in quantum optics.

The term super-radiance (SR) refers to the discovery by Dicke who 50 years ago has shown that, among $2^N$ states of a system of $N$ two-level atoms confined to the volume of size smaller than the radiation wavelength between the two levels, one state exists that radiates very fast and coherently so that its width is close to $\Gamma = N \gamma$, where $\gamma$ is the width of an individual isolated atom, and the intensity is proportional to $N^2$. The remaining states live for a long time having very small widths. The SR is observed in the laser pulse transmission through a resonant medium. It is important that the coherent coupling of the atoms is reached due to their interaction through the common radiation field, independently of the direct atom-atom interaction.

The analog of the SR emerges in many-body quantum systems, Fig. 1, where $N$ intrinsic states of the same symmetry are coupled to common decay channels as was seen in numerical simulations for the nuclear continuum shell model. At $\kappa \sim 1$, the system undergoes a phase transition from the separated narrow resonances to the width accumulation by the SR states; their number correlates with that of the open channels. The theory of the phenomenon was given in, where the analogy to the Dicke SR state was pointed out; for various aspects of theory see.

Due to the very general character of the SR, it was observed in many
different situations in atomic, molecular, condensed matter and nuclear physics. We present a brief review of the theory and discuss selected applications to nuclear physics of low and intermediate energies, including the suggestions for the SR as a reason for the narrow width of exotic baryons.

2. Ingredients of the theory

1. The convenient approach to the unified theory of intrinsic states coupled to the continuum is provided by the Feshbach projection techniques\textsuperscript{12}. The full system is described by the Hermitian Hamiltonian $H$. The Hilbert space is decomposed into two classes, internal, $Q \equiv \{|1\rangle\}$, and external, $P = \{|c; E\rangle\}$, where $c$ marks the continuum channels. The total eigenfunction at energy $E$ contains the contributions of both classes; they are coupled by the matrix elements $\langle c; E | H | 1 \rangle$. Eliminating the external states by the projection at given energy, one comes to the effective eigenvalue problem in the intrinsic space, where the effective Hamiltonian is given by

$$ H_{QQ}(E) = H_{QQ} + H_{QP} \frac{1}{E - H_{PP} + i0} H_{QP}. \quad (1) $$

2. The internal energy-independent Hamiltonian $H_{QQ}$ determines the eigenvalues $\epsilon_n$ of the states which would be bound without the continuum coupling. This coupling converts at least some of them into resonances with complex energies $E_j = E_j - (i/2)\Gamma_j$.

3. The effective Hamiltonian (1) depends on running energy $E$. The continua $c$ are started at threshold energies $E_c$. At $E > E_c$, the denominator of the propagator in eq. (1) contains singular terms corresponding to the real (on-shell) decay into the channel $c$ with energy conservation. The real part $\Delta$ of the propagator corresponds to the principal value of the singular terms and describes the virtual (off-shell) processes of coupling through all (closed and open) channels, while only open channels contribute to the imaginary part $(-i/2)W$ that makes the effective Hamiltonian non-Hermitian.
4. The anti-Hermitian part of the effective Hamiltonian is factorized into a number of terms equal to the number \( k \) of channels open at energy \( E \). Thus, the general form of \( \mathcal{H} \) is

\[
\mathcal{H}_{12} = H_{12} + \Delta_{12}(E) - \frac{i}{2} W_{12}(E), \quad W_{12} = \sum_{c:\text{open}} A_{1}^{c} A_{2}^{c*},
\]

(2)

where the amplitudes \( A_{1}^{c} \) for the coupling of the intrinsic state \( |1\rangle \) to the channel \( c \) are proportional to the original matrix elements \( \langle 1|H|c\rangle \).

5. The principal part \( \Delta \) renormalizes the intrinsic Hamiltonian \( H_{QQ} \). In many situations it is of minor significance being small and weakly dependent on energy. The anti-Hermitian part is the driving force for new physics.

The amplitudes \( A_{1}^{c} \) should vanish at threshold energy \( E_{c} \), and this energy dependence is crucial for weakly bound systems.

6. The diagonalization of \( \mathcal{H} \), eq. (2), determines the complex eigenvalues \( \mathcal{E}_{j} = \hat{E}_{j} - (i/2)\Gamma_{j} \) and the (biorthogonal) set of the eigenfunctions \( |j\rangle \) of quasistationary states depending on running energy \( E \). The resonance centroid \( E_{j} \) on the real axis is self-consistently determined by \( \hat{E}_{j}(E = E_{j}) = E_{j} \).

7. The same formalism determines the reaction cross sections at a given energy with the aid of the scattering matrix

\[
S_{ab}(E) = s_{a}^{1/2} \left\{ \delta_{ab} - \sum_{12} A_{1}^{a*} \left( \frac{1}{E - \mathcal{H}_{12}} \right) A_{2}^{b} \right\} s_{b}^{1/2}.
\]

(3)

Here the “potential” phases \( s_{a} = \exp(2i\delta_{a}(E)) \) describe the contributions of remote resonances which usually are not taken into account explicitly.

The full effective Hamiltonian \( \mathcal{H} \), eq. (2) in the denominator in eq. (3) includes the same amplitudes \( A_{1}^{c} \) as in the numerator, and this makes the \( S \)-matrix explicitly unitary.

3. SR phase transition

At energy below the lowest threshold, we have the Hermitian Hamiltonian \( H + \Delta \). This determines the discrete spectrum of bound states \( \epsilon_{n} \) and the stationary wave functions \( |\psi_{n}\rangle \). The unitary transformation to this basis of the internal representation keeps the anti-Hermitian part factorized so that at energy in the continuum

\[
\mathcal{H}_{nn'} = \epsilon_{n} \delta_{nn'} - \frac{i}{2} W_{nn'}, \quad W_{nn'} = \sum_{c:\text{open}} A_{n}^{c} A_{n'}^{c*}.
\]

(4)
The strength of the continuum coupling in channel $c$ is measured by the ratio $\kappa_c = \gamma_c/D$ of the typical partial width $\gamma_c = |A|^2$ to the mean level spacing $D$ (we consider the class of intrinsic states with the same exact quantum numbers). At small $\kappa_c$, we have in this channel isolated narrow resonances with the widths $\sim \gamma_c$.

As $\kappa_c$ increases, the role of the off-diagonal damping increases. When $\kappa_c$ is getting close to 1, the percolation happens: decay of a level $n$ with return to the overlapping level $n'$ becomes likely, and all levels are coherently coupled through the continuum, similarly to the Dicke coherent state. Now we are close to another limit where the dynamics are defined by $W_{nn'}$. At strong coupling one can start with the doorway representation of the eigenstates of $W$. Due to the factorization of $W$, the rank of this matrix is equal to the number $k$ of open channels, and there are $k$ doorway states with non-zero widths. The remaining $N - k$ states are weakly coupled to the continuum only through the doorways. The eigenstates of $H$ are divided into two groups: the SR states sharing almost all summed width of the $N$ states, and the trapped states which are very long-lived.

In the simplest limit of a single open channel and degenerate intrinsic spectrum, $\epsilon_n \equiv \epsilon$, the whole width $\Gamma = \text{Tr} W$ is concentrated in one SR state. If the intrinsic levels $\epsilon_n$ are not degenerate but their spread $\delta \epsilon \sim ND \ll \Gamma$, the sum $\tilde{\Gamma}$ of all small $N - 1$ widths is

$$\tilde{\Gamma} \approx 4 \frac{(\Delta \epsilon)^2}{\Gamma}.$$  \hspace{1cm} (5)

The continuum coupling in the limit of $\kappa \sim (\Gamma/ND) \gg 1$ implements the segregation of distinct time scales in the reaction process:

- the shortest time $\tau_d \sim \hbar/\Gamma$ corresponds to the fast direct reaction through the SR intermediate state;
- fragmentation time $\tau_f \sim \hbar/\Delta \epsilon \sim \kappa \tau_d$ is what is necessary for the internal damping of the original excitation of one of the intrinsic states;
- Weisskopf time $\tau_W \sim \hbar/D \sim N \kappa \tau_d$ is the recurrence time of the wave packet (intrinsic equilibration);
- compound lifetime of trapped states $\tau_c \sim \hbar/(\tilde{\Gamma}/N) \sim \kappa \tau_W$ that allows for the full exploration of intrinsic space.

4. Some applications

The universality of the SR mechanism guarantees its appearance in any situation with a not very large number of open channels (otherwise the off-diagonal damping is quenched by the Ericson fluctuations of amplitudes.
corresponding to different channels) and the necessary ingredients present. There are many examples of the manifestation of the SR in molecular, atomic and condensed matter physics, see for instance 14,15,16. Below we shortly discuss some examples from nuclear physics.

### 4.1. Two collectivities

Giant resonances (GR) in the response function of the nucleus to the multipole excitation appear as a result of the coherent coupling of simple particle-hole excitations. By our terminology, they are generated by the intrinsic interaction that accumulates the multipole strength at some energy shifted from the interval $\Delta \epsilon$ of the unperturbed intrinsic states. The strength collectivization, however, is not what is seen in the reaction. The observed cross sections are determined by the partial widths of unstable intrinsic states with respect to a given channel.

The simple model 17,18 with the effective Hamiltonian

$$\mathcal{H}_{nn'} = \epsilon_n \delta_{nn'} + \lambda d_n d_{n'} - \frac{i}{2} A_n A_{n'},$$

contains unperturbed energies, real multipole interaction and interaction through the continuum. The dynamics here are determined by two multidimensional vectors $\mathbf{d} = \{d_n\}$ and $\mathbf{A} = \{A_n\}$. The multipole interaction creates the GR shifted by $\Omega = \lambda \mathbf{d}^2$ along the real energy axis from the unperturbed centroid $\bar{\epsilon}$ and accumulating the multipole strength. The continuum interaction creates the SR state shifted by $\Gamma = \mathbf{A}^2$ along the imaginary energy axis to the lower part of the complex plane and accumulating almost all available decay width.

The interplay of the collective effects depends on the “angle” $\phi$ between the vectors $\mathbf{d}$ and $\mathbf{A}$. In the degenerate case, $\epsilon_n = \epsilon$, the reaction amplitude looks like

$$T(E) = \frac{(E - \epsilon - \Omega)\Gamma + \lambda (\mathbf{A} \cdot \mathbf{d})^2}{(E - \epsilon - \Omega)[E - \epsilon + (i/2)\Gamma] + (i/2)\lambda (\mathbf{A} \cdot \mathbf{d})^2}.$$  

In the limits of parallel internal and external couplings, $\phi = 0^\circ$, we have

$$T(E) = \frac{\Gamma}{E - \epsilon - \Omega + (i/2)\Gamma}.$$  

Here the SR and GR collectivization are combined, and the experiment will reveal the shifted “Giant Dicke resonance” with full strength and full width. This is what we expect to see in gamma scattering, where both intrinsic excitation and decay have the same multipole nature. In the opposite case of
orthogonal couplings, $\phi = 90^\circ$, the result is $T(E) = \Gamma / |E - \epsilon - (i/2)\Gamma|$. Now the experiment would show only the unshifted SR resonance with vanishing multipole strength but broad width (the decay channel has another nature, for example results from evaporation). The GR state is dark with collective strength but no access to the continuum. Fig. 2 shows a more realistic case of non-degenerate intrinsic states when both the strength and the width are shared between the displaced GR and the SR in the region of unperturbed excitation energies. As the coupling with continuum increases, the low-energy branch is expected to develop into what is called pigmy giant resonance. We hope to present the study of the pigmy resonance elsewhere but the mechanism shown here is quite universal. Because of complicated interference between the two types of collectivity, the observed picture should differ in different channels.

Figure 2. The scattering amplitude in the model of eq. (6) with 20 states with spacing between $\epsilon_n$ set as a unit of energy; the parameters are $\Omega = \Gamma = 80$. The panels show the evolution of scattering as a function of increasing angle between the multidimensional vectors $d$ and $A$.

4.2. Loosely bound nuclei

These applications stimulated the renewed interest to the SR physics. As we have discussed at the previous Seminar, in the proximity of thresholds the coupling of intrinsic states with and through the continuum dominates the dynamics. The shell model machinery can be generalized to include the effective non-Hermitian energy-dependent Hamiltonian. In particular, one can consistently consider systems like $^{11}$Li where the single-particle
states are unbound but additional correlations bring paired states back from the continuum. In all such cases the correct energy dependence of the amplitudes $A_i$ near threshold is crucial. However, this dependence is sensitive to the absolute position of thresholds. To solve the problem in a nucleus $A$ with one- and two-body decay channels, we need to know the spectrum of the $(A-1)$ and $(A-2)$ nuclei, and the whole chain of daughter nuclei. The plausibility of such a program was demonstrated for the chain of oxygen isotopes $^{16}$O. Fig. 3 shows full shell model calculation for the few oxygen nuclei above the $^{16}$O core and comparison with data; this will be further discussed in a forthcoming publication.

Figure 3. Continuum shell model calculation for the oxygen isotopes with $A = 16$ to $19$. One and two-neutron emission processes are considered. On a single energy scale stable nuclear states are shown with solid lines and neutron-unstable states by dashed lines. The experimentally observed states of $^{19}$O are shown to the right on the same energy scale. The scarce experimental data for energies and neutron decay widths are listed in the table along with theory predictions. Correspondingly, in the level scheme the arrows indicate the decay transitions of these states.

5. **Widths of exotic baryons**

The SR mechanism works in hadron physics. A baryon resonance $R$ created, for example, in a photonic experiment mixes with the $RN^{-1}$ states of $R$-(nucleon hole) nature. The baryon resonances have excitation energies starting from few hundred MeV and typical vacuum widths exceeding 100 MeV. The continuum mixing with the $RN^{-1}$ states of the same symme-
try produces an SR state and few trapped states which are observed as narrow resonances on the broad SR background. This pattern was seen in the $^{12}\text{C}(e,e'p\pi^-)^{11}\text{C}$ Mainz experiment \cite{21} at the $\Delta$-isobar region and attributed \cite{13} to the SR mechanism. In terms of two collectivities, the SR state here is a good candidate for the parallel case. As shown by old RPA calculations \cite{22}, the pionic coherent state (similar to the nuclear GR) accumulates the pion decay width as well.

Figure 4. The two-state model for the pentaquark in comparison with the results of the CLAS experiment $\gamma d \rightarrow pK^-(K^+n)$ \cite{24}. The $p$-wave ($K^+n$) continuum channel is considered with kinematical dependence $A(E) \sim E^{3/4}$ near threshold, solid curve. For two other curves, the amplitude $A(E)$ at high energy is damped as $A(E) \sim [E^3/(1 + E/\Lambda)^3]^{1/4}$ and the cutoff parameters $\Lambda$ are 300 MeV, dashed curve, and 500 MeV, dotted curve.

We have applied \cite{23} the same idea to recent observations by different groups of the $\Theta^+$ resonance with strangeness $+1$. The resonance coined as pentaquark has a very narrow width smaller than few MeV. In the experiments on nuclei the situation should be quite similar to that in the $\Delta$-case. In \cite{23} the arguments are given that in the deuteron and proton experiments the same mechanism can still be at work because the intrinsic QCD dynamics create few states of different nature but the same symmetry which are coupled through the continuum. It is sufficient to have just two such states, for example (but not necessarily) a standard quark bag and a large size quasimolecular state. Fig. 4 shows the result for a simple two-state model with naturally chosen parameter values where the narrow resonance on the broad background emerges due to the SR mechanism.

6. Conclusion

The phenomenon of SR is very general; it has to appear, provided all necessary ingredients are in place, in any theory that describes physics on the borderline of discrete and continuum spectrum and agrees with the unitarity requirements. The segregation of physical time scales is accompanied by the appearance of long-lived states on a broad background. One can expect the most spectacular manifestations in many-body systems (au-
toionizing atomic states, chemical reactions, loosely bound nuclei, collective dynamics in the continuum and analogous physics on a quark level). There are important ramifications for quantum chaos and its experimental tests, mesoscopic condensed matter physics, studies of entanglement, decoherence and quantum information.

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