Attractive and repulsive dipolar interaction in bilayers of indirect excitons

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We explore attractive dipolar interaction in indirect excitons (IXs). For one layer of IXs in a single pair of coupled quantum wells (CQW), the out-of-plane IX electric dipoles lead to repulsive dipolar interaction between IXs. The attractive dipolar interaction between IXs is realized in a 2-CQW heterostructure with two IX layers in two separated CQW pairs. We found both in experimental measurements and theoretical simulations that increasing density of IXs in one layer causes a monotonic energy reduction for IXs in the other layer. We also found an in-plane shift of a cloud of IXs in one layer towards a cloud of IXs in the other layer. This behaviour is qualitatively consistent with attractive dipolar interaction. The measured IX energy reduction and IX cloud shift are higher than the values given by the correlated liquid theory.

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A spatially indirect excitons (IX), also known as an interlayer exciton, is a bound pair of an electron and a hole confined in separated layers. Due to the electron-hole separation, IXs have built-in electric dipole moment \( ed \), where \( d \) is the distance between the electron and hole layers and \( e \) electron charge. Furthermore, due to the electron-hole separation, IXs have long lifetimes within which they can cool below the temperature of quantum degeneracy [1]. These properties make IXs a platform for exploring quantum gases with dipolar interaction.

IXs can be realized in a pair of quantum wells separated by a narrow tunneling barrier. For one layer of IXs in a single pair of coupled quantum wells (CQW), the out-of-plane IX electric dipoles lead to repulsive dipolar interaction between side-to-side IX dipoles (Fig. 1). This configuration is extensively studied both theoretically and experimentally. The phenomena originating from the repulsive dipolar interaction in a single IX layer include the enhancement of IX energy with density that has been known since early studies of IXs [2–6], screening of in-plane disorder potential by repulsively interacting IXs [7–10] that leads to IX delocalization and long-range IX transport [7–18], strong correlations [10, 19–26], and predicted crystal phases [27–32].

Quantum gases with dipolar interaction are also explored with cold atoms. In these systems, dipolar interactions lead to droplet structures with spatial ordering and coherence [36–40], few-body complexes [41, 42], and pair superfluid and crystal phases in bilayers of dipoles [43–46].

The specific property of dipolar interaction is its anisotropy. For instance, for two parallel dipoles tilted at angle \( \theta \) relative to the line connecting them, the interaction at \( r \gg d_1, d_2 \) is given by \( \nu(r) \sim e^2 d_1 d_2 (1 - 3 \cos^2 \theta)/er^3 \), where \( e \) is the dielectric constant of the material and \( p_{1,2} = ed_{1,2} \) the dipole moments [47]. For the out-of-plane IX dipoles in a single IX layer this expression reduces to \( \nu(r) \sim e^2 d^2/er^3 \) describing the repulsive dipolar interaction between IXs.

The other specific property of dipolar interaction for the IXs is the induced orientation of IX dipoles. The heterostructure design and/or applied voltage, which produces the electric field in the heterostructure, determine the quantum well layers where electrons and holes are confined: Exchanging the quantum wells by the electron and the hole, i.e. flipping the IX dipole, is energetically unfavorable. Furthermore, tilting the IX dipole relative to the \( z \) direction causes an in-plane separation of the electron and the hole in the IX and, as a result, reduces...
the IX binding energy. This induces the orientation of IX dipoles in the direction normal to the QW plane.

The induced orientation of the IX dipoles and the repulsive dipolar interaction for a single IX layer make challenging exploring the attractive dipolar interaction in IX systems. The studies of IX dipoles have been concentrated on the case of repulsively interacting IXs [1–31]. However, the angle-dependent dipolar IX interaction and, in particular, dipolar attraction gives an access to new phenomena in quantum dipolar gases. For instance, the dipolar attraction leads to the phenomena in cold atoms outlined above [36–46]. IX attraction can be realized by extending IX heterostructures beyond a single CQW design and studies of attractively interacting IX dipoles were recently started in two stacked CQW pairs [48–50].

In this work, we explore the attractive dipolar interaction between IXs in a 2-CQW heterostructure with two IX layers in two separated CQW pairs (Fig. 1). The intra-CQW interaction between IX side-to-side dipoles is repulsive similar to single CQW heterostructures [1–31]. The inter-CQW interaction between IX head-to-tail dipoles is attractive. It changes to repulsive with increasing in-plane separation between the IXs and, in turn, $\theta$ following the anisotropy of dipolar interaction outlined above. Both our experimental measurements and theoretical simulations show (i) a monotonic energy reduction for IXs in one layer with increasing density of IXs in the other layer and (ii) an in-plane shift of a cloud of IXs in one layer towards a cloud of IXs in the other layer. This behavior is qualitatively consistent with attractive dipolar interaction, however, the measured IX energy reduction and IX cloud shift are higher than the values given by the correlated liquid theory.

Experiment. The studied 2-CQW heterostructure (Fig. 1a) is grown by molecular beam epitaxy. Indirect excitons IX$_2$ form in CQW$_2$, indirect excitons IX$_1$ in CQW$_1$. CQW$_2$ consist of two 15 nm GaAs QWs separated by 4 nm Al$_{0.33}$Ga$_{0.67}$As barrier, CQW$_1$ consists of two 12 nm GaAs QWs separated by 4 nm Al$_{0.33}$Ga$_{0.67}$As barrier. CQW$_2$ and CQW$_1$ are separated by 12 nm Al$_{0.33}$Ga$_{0.67}$As barrier, narrow enough to allow substantial inter-layer interaction between IX$_2$ and IX$_1$, yet wide enough to suppress tunneling of electrons and holes between CQW$_2$ and CQW$_1$. $n^+$-GaAs layer with $n_{6m} \sim 10^{18}$ cm$^{-3}$ serves as a bottom electrode. The CQW pair is positioned 100 nm above the $n^+$-GaAs layer within undoped 1 $\mu$m thick Al$_{0.33}$Ga$_{0.67}$As layer. The two CQW pairs are positioned closer to the homogeneous bottom electrode to suppress the fringing in-plane electric field in excitonic devices [51]. The top semitransparent electrode is fabricated by applying 2 nm Ti and 7 nm Pt on 7.5 nm GaAs cap layer. Applied gate voltage $V_g = -2$ V creates electric field in the z-direction.

The IX$_2$ energy is lower than the IX$_1$ energy. This energy difference gives an opportunity to selectively gen-

![Graphs and diagrams showing energy levels and intensity changes for IXs in different conditions.](image-url)
erate IX$_2$ by optical excitation. Excitons are generated by semiconductor lasers L$_2$ and L$_1$ at the energies 1.532 and 1.541 eV resonant to spatially direct excitons (DXs) in CQW$_2$ and CQW$_1$, respectively. The resonant to DX excitation increases the light absorption and, in turn, IX density for a given laser power [52]. L$_2$ generates IX$_2$. L$_1$ generates IX$_1$ and also roughly 2 times smaller concentration of IX$_2$ due to a weaker nonresonant absorption of L$_1$ light in CQW$_2$. L$_2$ and L$_1$ excitations are focused to ~5 µm hwhm spots, which are separated by 50 µm. This configuration allows exploring the effects of IX interactions on the IX cloud position. IX photoluminescence (PL) is measured in a 20 ns time window starting 20 ns after the end of the L$_1$ and/or L$_2$ excitation pulses. This allows for studying of only long-lived IXs after DXs recombined. Both IX$_2$ and IX$_1$ have long lifetime in the range of hundreds of ns (~800 ns for IX$_2$ and ~260 ns for IX$_1$) allowing them to travel over long distances reaching hundreds of microns.

Time-resolved imaging experiments are performed with a pulse laser pulse duration 2000 ns, period 4000 ns, and edge sharpness ~2 ns. The rectangular-shaped pulses are realized by a pulse generator driving the semiconductor lasers. The pulse duration and period are optimized to allow the IX PL image to approach equilibrium during the laser excitation and decay between laser pulses. The PL images are captured using a PicoStar HR TauTec time-gated intensifier. The PL passes through a spectrometer with a resolution of 0.18 meV before entering the intensifier coupled to a liquid-nitrogen-cooled CCD. The measurements are performed at $T_{\text{bath}} = 1.7$ K.

To analyze the attractive inter-layer IX interaction in the IX bilayer we measure how the selective generation of IX$_2$ affects the energies and cloud position of IX$_1$. Figure 1b presents the differential x-energy image obtained by subtracting the x-energy images created by only L$_1$ on [Fig. S5c in supporting information (SI)] and by only L$_2$ on (Fig. S5b) from the x-energy image created by both lasers on simultaneously (Fig. S5a). The differential x-energy image shows an increase of IX$_2$ energy, a decrease of IX$_1$ energy, and a spatial shift of the IX$_1$ cloud towards the IX$_2$ cloud. These phenomena are detailed below.

First, we consider the IX energy variations. Figures 2a–c show that the increase of L$_2$ power ($P_{L2}$) and, in turn, IX$_2$ density ($n_2$) leads to a monotonic decrease of IX$_1$ energy. An energy decrease corresponds to attractive IX$_1$–IX$_2$ interaction. In comparison, when only IX$_2$ are present in the system (L$_1$ is off), the increase of $P_{L2}$ and, in turn, $n_2$ leads to a monotonic increase of IX$_2$ energy (Fig. 2d–f). An energy increase corresponds to repulsive IX$_2$–IX$_2$ interaction, which has been extensively studied in single layers of IXs [1–31].

Figures 2c,f also show that the increase of $P_{L1}$ leads to a monotonic increase of both IX$_1$ and IX$_2$ energies. L$_1$ generates both IX$_1$ and IX$_2$, therefore, the effect of increasing $P_{L1}$ on IX$_1$ (or IX$_2$) energy is a combined effect of attractive IX$_1$–IX$_2$ and repulsive IX$_1$–IX$_1$ (or IX$_2$–IX$_2$) interactions. The monotonic increase of both IX$_1$ and IX$_2$ energies with $P_{L1}$ indicates that the repulsive interaction is stronger. This is consistent with the relative strength of the attractive (Fig. 2a) and repulsive (Fig. 2d) interaction in the experiments with increasing $P_{L2}$ which increase only IX$_2$ density.

In the mean-field approximation the repulsive interaction between IXs in a single layer increases the IX energy by $\Delta E = 4\pi e^2 dn/\varepsilon$. This equation known as the “plate capacitor” formula provides a qualitative explanation for the observed monotonic increase of $\Delta E$ with the exciton density $n$ [2]. However, the capacitor formula can significantly overestimate $\Delta E(n)$ due to the IX correlations [10, 19–26]. To compare the attractive and repulsive dipolar interactions, avoiding the complexity of the relation between $\Delta E$ and $n$, Fig. 2g presents the change in IX$_1$ energy $\Delta E_1$ vs the change of IX$_2$ energy $\Delta E_2$ for the data in Fig. 2c,f. The energy shifts $\Delta E$ are measured relative to the IX energies at the lowest $n$. Figure 2g shows that for all studied $P_{L1}$, the increase of $P_{L2}$ and, in turn, $n_2$ leads to $\Delta E_2$ larger in absolute value than $\Delta E_1$, indicating that the repulsive IX$_2$–IX$_2$ interaction is stronger than the attractive IX$_2$–IX$_1$ interaction.

We also consider the spatial shift of the IX$_1$ cloud toward the IX$_2$ cloud [53]. Figure 3 shows that the IX$_1$ cloud attracts to the IX$_2$ cloud. With increasing $P_{L2}$, the spatial shift is nonmonotonic. This behaviour is observed for different $P_{L1}$. A larger spatial shift, reaching ~10 µm, is observed at low $P_{L1}$ (Fig. 3b).

**Theory.** We analyze the dipolar interaction in IX bilayers theoretically and compare the experimental data with theoretical simulations. The numerical simulation of such two-species many-body system is done through the Hyper-netted Chain (HNC) formalism [54]. The intra- and inter-layer interaction is modelled by assum-
FIG. 4: Decrease and increase of IX energy due to attractive and repulsive dipolar IX interaction: Theory. (a) The decrease of IX energy with increasing IX density. \( n_1 = 1.5 \times 10^{10} \text{ cm}^{-2} \). An energy decrease corresponds to attractive IX1 – IX2 interaction. (b) IX1 – IX2 density correlation function. \( n_1 = n_2 = 10^{10} \text{ cm}^{-2} \). (c) IX1 energy as a function of both IX1 and IX2 density. (d) The increase of IX2 energy with increasing IX1 density. \( n_1 = 0 \). An energy increase corresponds to repulsive IX2 – IX2 interaction. (e) IX2 – IX1 (red solid line) and IX1 – IX1 (black dashed line) density correlation functions. \( n_1 = n_2 = 10^{10} \text{ cm}^{-2} \). (f) IX2 energy as a function of both IX1 and IX2 density. (g) The change in IX1 energy vs the change of IX2 energy. Each line corresponds to increasing IX2 density. For the blue, orange, green, red, and purple lines, \( n_1 = 0.5, 1, 1.5, 2, \) and \( 2.5 \times 10^{10} \text{ cm}^{-2} \), respectively.

While both the experimental measurements and theoretical simulation show (i) a monotonic IX energy reduction and increase of IX energy due to attractive and repulsive dipolar IX interaction: Theory. The density correlation functions for the cases of attractive IX1 – IX2 and repulsive IX1 – IX1 and IX2 – IX2 interactions are presented in Fig. 4b and 4e, respectively. For the attractively interacting IXs (Fig. 4b), the correlation function enhancement above 1, the mean-field value, increases the interaction energy compared with the vanishing interlayer interaction in the mean-field approximation. On the contrary, the repulsively interacting IXs avoid each other (Fig. 4e) that lowers the intralayer IX interaction energy compared with the uncorrelated state assumed in the mean-field approximation.

As for the experimental data, we compare the attractive and repulsive dipolar interactions in a graph showing \( \Delta E_1 \) vs \( \Delta E_2 \). Figure 4g shows that for all studied \( n_1 \), the increase of IX2 leads to calculated \( \Delta E_2 \) larger in absolute value than \( \Delta E_1 \), in qualitative agreement with the experimental data in Fig. 2g.

We also simulated the spatial shifts of the IX1 cloud toward the IX2 cloud. The simulations of the IX spatial profiles are based on the IX generation, diffusion, and recombination and are outlined in SI. Figure 5 shows that the IX1 cloud attracts to the IX2 cloud in the simulations, in qualitative agreement with the attraction observed in the experiment (Fig. 3). In comparison, both our experimental measurements (Fig. S3) and theoretical simulations (Fig. S4) show that two clouds of repulsively interacting IX2 repel each other, see SI.

While both the experimental measurements and theoretical simulation show (i) a monotonic IX1 energy reduc-
Fig. 5: Attraction of the IX$_1$ cloud to the IX$_2$ cloud: Theory. (a) The IX$_1$ cloud profiles simulated for different $P_{L1}$ and, in turn, IX$_2$ densities. $P_{L1} = 250 \, \mu W$. The profiles of L$_1$ and L$_2$ laser excitation spots are shown by black and purple lines, respectively. Dashed line indicates the center of L$_1$ excitation spot. (b) The center of mass position of the IX$_1$ cloud as a function of $P_{L2}$ for different $P_{L1}$.

The interaction- and/or disorder-induced IX mass enhancement may be one possible reason for this discrepancy. The magnitude of IX$_1$ energy reduction, $\Delta E_1$, scales with the strength of interlayer IX$_1$ – IX$_2$ dipolar attraction (Fig. S1). For the case of a single IX$_1$ – IX$_2$ pair, $\Delta E_1$ can be estimated as the binding energy of IX$_1$ – IX$_2$ biexciton state, $E_b$. For a bare IX mass, $m_{IX} \approx 0.2 m_0$ [56], $E_b \approx 0.3$ meV (Fig. S2). Higher $E_b$ can be achieved for higher IX masses, and, e.g. for the reduced IX mass enhanced to 2$m_0$, $E_b$ reaches $\sim 1.2$ meV (Fig. S2), making the IX$_1$ – IX$_2$ interaction scale comparable to the experiment (Fig. 2). A mass enhancement can be caused by interaction, however, only a relatively weak interaction-induced mass enhancement, up to $\sim 25\%$, was observed in electron-hole systems in single QWs [57]. The studies of effects of interaction and/or disorder on the IX mass can be a subject for future work.

In summary, we presented experimental and theoretical studies of attractive dipolar interaction in IX bilayers. We found that increasing density of IXs in one layer causes a monotonic energy reduction for IXs in the other layer. We also found an in-plane shift of a cloud of IXs in one layer towards a cloud of IXs in the other layer. This behaviour is qualitatively consistent with attractive dipolar interaction. The measured IX energy reduction and IX cloud shift are higher than the values given by the correlated liquid theory.

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Supporting information: Attractive and repulsive dipole-dipole interaction between indirect excitons

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EXCITON INTERACTION

Interaction potentials

We modeled IXs as composite bosonic particles with a rigid internal charge distribution. The interactions of such particles can be specified in terms of potentials $u_{ij}(r)$, $1 \leq i, j \leq 2$, which are functions of pairwise in-plane distances $r$ of the excitons. Here and below we use subscripts 1 and 2 to label the CQWs (12 nm- and 15 nm-wide, respectively). To compute these interaction potentials, we assumed that the charge distributions of all the electrons and holes are spherically symmetric Gaussians. The radius $a$ of the Gaussians is our adjustable parameter that accounts for the width of the quantum wells and the internal motion of particles about the center of mass of each exciton. We computed these interaction potentials by taking the convolutions of the Coulomb kernel $e^2/\sqrt{r^2 + z^2}$ with the charge densities of the interacting particle pairs. The result for the inter-CQW potential $u_{12}(r) = u_{21}(r)$ is

$$u_{12}(r) = \sum_{\sigma=\pm} \sum_{\tau=\pm} V(\sqrt{r^2 + z_{\sigma\tau}^2}), \quad V(r) = \frac{e^2}{\epsilon r} \text{erf}\left(\frac{r}{2a}\right), \quad z_{\sigma\tau} = \tau \frac{d_1 - \sigma d_2}{2} + D,$$

where erf($x$) is the error function and $D$ is the z-axis distance between the CQW centers. The intra-CQW potentials $u_{kk}(r)$, $k = 1$ or 2, are given by the same equation with $D = 0$ and the electron-hole separations $d_1, d_2$ replaced by $d_k$. The plots of these potentials for parameter values representative of our experimental device are shown in Fig. S1. At $r \ll a$ all of them approach constant finite values and at large $r$, these potentials behave as $1/r^3$. Potentials $u_{11}(r)$ and $u_{22}(r)$ are strictly repulsive. Potential $u_{12}(r)$ is attractive in the range of distances $r$ selected for the plot. At larger $r$, it eventually becomes repulsive but it is already very small as such $r$.

FIG. S1: Model interaction potentials: $u_{22}$ (top) and $u_{11}$ (middle) are the intra-CQW potentials for IX$_2$ and IX$_1$, respectively; $u_{12}$ (bottom curve) is the inter-CQW interaction potential. Parameters: $d_1 = 20$, $d_2 = 25$, $D = 43$, $a = 5$ (all in nm); $\epsilon = 13$. 

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The potentials $u_{ij}(r)$ serve as inputs to our computer program that computes many-body properties within the zero-temperature hypernetted chain (HNC) formalism. Another input parameter is the effective mass $m_{IX}$ of the excitons, which we took to be 0.2 of the free electron mass $m_0$. Our implementation of the HNC method is based on Ref. [1]. The output of these calculations include the pair correlation functions $G_{ij}(r)$, the energy density $\varepsilon = \varepsilon(n_1, n_2)$, and the chemical potentials

$$\mu_j = \frac{\partial \varepsilon}{\partial n_j}$$

of the excitons as functions of their number densities $n_1$ and $n_2$ in the CQWs. If the shake-up effects, i.e., many-body relaxation processes following the exciton recombination, can be neglected, then the exciton emission energies (or exciton “single-particle energies”) $E_j$ should coincide with their chemical potentials:

$$E_j \approx \mu_j.$$ 

Based on this assumption, we have constructed the plot of $\Delta E_1$ vs. $\Delta E_2$ shown in Fig. 4g of the main text. Representative intra-CQW and inter-CQW pair-correlation functions are plotted in Fig. 4b,e. At short distances, these functions show a deep “correlation hole” for excitons of the same CQW and a strong correlation peak for excitons of different CQWs.

**Biexciton binding energy**

From previous theoretical work on double-layer bosonic systems with repulsive intra-layer and attractive inter-layer dipolar interactions [2–5], we expect that the exciton system at low enough equal densities $n_1 = n_2$ should be made of bound pairs, the inter-CQW biexcitons. If $n_1 \neq n_2$, then biexcitons and unpaired excitons may co-exist. A rough estimate of the required density is given by the Mott criterion stating that the biexcitons should appear when the dimensionless parameter $\min(n_1, n_2) b^2$ is less than some critical number, which is usually numerically small, perhaps, 0.02. Here $b$ is the spatial size of the biexciton. Within our rigid-body approximation, the biexciton bound state can be easily found numerically. In the relative coordinates, this problem reduces to solving a Schrödinger equation for a particle of reduced mass $m_{IX}/2$ subject to the confining potential $u_{12}(r)$. For the same parameters as in Fig. S1, we obtained the binding energy to be $E_b = 0.286$ meV. From Fig. S2 we deduce the spatial size of the biexciton to be $b \sim 30$ nm, so that the Mott critical density for biexcitons is of the order of $10^{10}$ cm$^{-2}$, not too far from the exciton densities realized in our experiment.
The following argument suggests that $E_b$ is in fact the maximum possible shift of the single-particle energies due to the inter-CQW attraction. Indeed, in the limit of high densities, where average intra-CQW exciton separation is smaller than $D_j$, correlations are negligible. At intermediate densities, where HNC should be accurate, the dependence of say $E_1 = E_1(n_1, n_2)$ on $n_2$ with $n_1$ held fixed is either monotonic or flat within computational accuracy, see Fig. 4c,f. Therefore, the asymptotic limit $n_1 = n_2 \to 0$, where all excitons are paired and

$$E_1 = E_2 = -E_b$$

should correspond to the largest possible attraction effect. Note that the HNC method reproduces this asymptotic limit only approximately. The tendency toward pairing is manifested in the aforementioned peak in the pair-correlation function $G_{12}(r)$ at $r = 0$. The shape of this peak computed by the HNC resembles the probability distribution of the biexciton, cf. Figs. 4b and S2. The integrated weight $N = n_2 \int G_{12}(r)d^2r$ of the peak (where the integration extends up to $r \sim b$) is the average number of excitons of CQW$_1$ attracted to an exciton in CQW$_2$. When the biexcitons form, $N$ should approach unity. Yet within our HNC calculations $N$ keeps increasing as $n_1 = n_2$ decreases. This suggests that the standard HNC method is inadequate in the low-density regime where we should instead use Eq. (4).

We found both in experimental measurements (Fig. 2a,c) and theoretical simulations (Fig. 4a,c) that increasing density of IXs in one layer causes a monotonic energy reduction for IXs in the other layer. These results differ with the results of Refs. [6, 7] where a nonmonotonic dependence on the density was reported. The nonmonotonic dependence on the density was attributed to many-body polaron effects in Refs. [6, 7]. Our simulations show no indication for the non-monotonic dependence on the density.

The experiment still poses a challenge for the theory because the shift of $E_1$ has been observed to routinely exceed the computed $E_b = 0.286$ meV, see Fig. 2a. To identify a possible reason for the discrepancy, we examined this important parameter more critically. First, we tested the validity of the rigid-body approximation. We used a previously developed computational tool [8] to accurately solve for the exciton and biexciton binding energies as two-body and four-body problems, respectively. For the parameters of Fig. S1 we obtained $E_b = 0.33$ meV. Hence, the rigid-body approximation is not the major source of the discrepancy. Next, we noticed that $E_b$ is greatly reduced compared to the depth $\approx 2$ meV of the potential well $n_1(r)$. This reduction is due to the zero-point motion. An illustration of how this quantum motion may affect the binding energy, we recalculated $E_b$ and the wavefunctions of biexcitons for reduced masses enhanced two- and twenty-fold. In the latter case, the binding energy rises to 1.16 meV, see Fig. S2, which is close to the experimentally measured shifts of $E_1$ we attributed to the inter-CQW attraction. It is hard to expect that the exciton mass is indeed enhanced by such an enormous factor due to the interaction alone. (As a point of reference, only a relatively weak interaction-induced mass enhancement, up to 25%, was observed in electron-hole systems in single QWs [9].) However, the suppression of the zero-point motion of an exciton pair may in principle be facilitated by disorder in the system that traps the excitons close together in deep potential wells.

**DYNAMICS OF EXCITON DENSITY DISTRIBUTION**

In this section we summarize the set of equations we used to model the macroscopic dynamics of excitons. To simplify the modeling, we assumed that the exciton densities $n_k$ and currents $j_k$ were functions of a single spatial coordinate $x$. These quantities obey the continuity equation

$$\partial_t n_k(x, t) = \partial_x j_k + g_k(x, t) - n_k/\tau_k,$$  \hspace{1cm} (5)

where $\tau_k$ is the lifetime of the excitons in $k$th CQW, which is known from the experiment, and $g_k$ is the generation rate proportional to the local laser power. To represent the exciton currents, we used the drift-diffusion approximation,

$$j_k(x, t) = -D_k \partial_x n_k(x, t) - B_k n_k(x, t) \partial_x \mu_k,$$  \hspace{1cm} (6)

where $D_k$ and $B_k = D_k/\tau$ are the diffusion coefficient and the drift mobility, respectively. Finally, to simplify the treatment of interaction effects, we linearized the density dependence of the exciton chemical potentials, such that

$$\mu_1 = \gamma_{11} n_1 + \gamma_{12} n_2, \hspace{1cm} \mu_2 = \gamma_{22} n_2 + \gamma_{12} n_1,$$  \hspace{1cm} (7)

where $\gamma_{ij}$ are interacting constants. Based on the simulations presented in Figs. 4a and 4d of the main text, we set the constants to be $\gamma_{11} = 9.3, \gamma_{22} = 11$, and $\gamma_{12} = -0.2$, all in units of $10^{-11}$ meV cm$^{-2}$. [Note that the intra-CQW coupling constants are 1/3 of the “plate capacitor” values, i.e., $\gamma_{kk} = (1/3) \times (4\pi e^2 d_k/e)$]. We developed a computer program that solves these coupled equations on a discrete grid of $x$ as a function of the time variable $t$, starting from
As outlined in the main text, the simulations show that the IX₁ cloud attracts to the IX₂ cloud (Fig. 5), in qualitative agreement with experimental observations. The IX₂ cloud is shown in blue, while the IX₁ cloud is shown in orange. Dashed lines indicate the centers of the excitation spots. The width of the IX₂ cloud obtained as the sum of simulated IX₂ cloud profiles is wider than the sum of simulated IX₁ cloud profiles, indicating a repulsion between the IX₂ clouds.

Initial conditions $n₁ = n₂ ≡ 0$. To get a relation between the laser powers and the generation rates, we fitted the shifts $E_k = μ_k$ of the exciton emission energies measured as functions of the laser power to the results of these simulations. We estimated the diffusion coefficients $D_k$ by fitting the calculated width of the IX₁ and IX₂ exciton clouds to the measured widths of these clouds generated selectively by L₁ or L₂.

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FIG. S5: **Position-energy images of IX luminescence.** (a) Both L₂ and L₁ lasers are on. (b) Only L₂ is on. (c) Only L₁ is on. The arrows indicate the excitation spot positions of L₂ and L₁ lasers resonant to direct excitons in 15 nm CQW and 12 nm CQW, respectively. L₂ generates IX₂. L₁ generates IX₁ and also a smaller concentration of IX₂. The laser powers $P_{L1} = 10 \mu W$, $P_{L2} = 250 \mu W$. Gate voltage $V_g = -2.0 V$, temperature $T = 1.7 K$.

agreement with the attraction observed in the experiment (Fig. 3). In comparison, both our experimental measurements (Fig. S3) and theoretical simulations (Fig. S4) show that two clouds of repulsively interacting IX₂ repel each other.

**POSITION-ENERGY LUMINESCENCE IMAGES**

The differential $x$-energy image (Fig. 1b) is obtained by subtracting the $x$-energy images created by only L₁ on (Fig. S5c) and by only L₂ on (Fig. S5b) from the $x$-energy image created by both lasers on simultaneously (Fig. S5a).

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