Temporal imaging for ultra-narrowband few-photon states of light: supplementary material

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Light-atoms interaction

To describe interaction between light and atomic coherence we use three level atom model with adiabatic elimination. The most comfortable coordinate system runs in time with beam $t \rightarrow t + z/c$. We explicitly make the control Rabi frequency $\Omega(t)$ time-dependent, as this is directly controlled in the experiment. Notably, $\Omega(t)$ represents the slowly-varying amplitude of this control field. Furthermore, we write the equations in terms of demodulated zero-spatial-frequency coherence $\rho_{0g}(z,t) = \rho_{0g}(z,t) e^{i \omega_0 z - i \Delta \omega_0 t}$, where $\rho_{0g}(z,t)$ is the actual ground-state coherence, $\Delta \omega_0 \approx 2 \pi \times 6.8$ GHz is the hyperfine splitting between levels $|g\rangle$ and $|h\rangle$ and $K_{gh} = \sqrt{\omega_0^2/c^2 - k_y^2 - k_z^2 - \omega_C/c}$ ($\omega_C$ - coupling field frequency, $k_x$, $k_y$, $k_z$ - transverse spatial components of the signal beam with respect to the coupling beam; for our case $k_y = 0$ and $ck_z/\omega_0 \approx 8$ mrad). Then, the light-coherence evolution is given by following coupled equations written in the frame of reference co-moving with the optical pulse:

$$\frac{\partial \rho_{0g}(z,t)}{\partial t} = i \frac{\Omega(t)}{\hbar} dA(z,t) - \frac{1}{2\tau} \rho_{0g}(z,t) + i \delta_{\text{tot}}(z,t) \rho_{0g}(z,t),$$

(S1)

$$\frac{\partial A(z,t)}{\partial z} = -i \frac{\hbar \Omega(t) \rho_{0g}(z,t) / d + A(z,t) / 2 \Delta + \Gamma}{2 \Delta + \Gamma} \delta n(z),$$

(S2)

$$\frac{\partial \rho_{0g}(z,t)}{\partial z} = -i \frac{\hbar \Omega(t) \rho_{0g}(z,t) / d + A(z,t) / 2 \Delta + \Gamma}{2 \Delta + \Gamma} \delta n(z),$$

where $1/(2\tau) = |\Omega(t)|^2 \Gamma / (8\Delta^2 + 2\Gamma^2)$ is decoherence caused by radiative broadening, $\delta_{\text{tot}} = \delta_0 + \delta_{acS} + \delta_{SSM} + \delta_Z$ is total two-photon detuning including ac-Stark shift caused by control beam $\delta_{acS} = |\Omega(t)|^2 \Delta / (4\Delta^2 + \Gamma^2)$, SSM and spatially varying Zeeman shift $\delta_Z = \mu_0 g_F m_B \beta_z$, with $F = 2, m = 2$ and $g_{F=2} = 1/2$, caused by linearly varying external magnetic field $B = B_0 + \frac{\beta_z}{\mu_0} z$, where $\mu_0$ is the Bohr magneton. The atomic concentration is denoted by $n(z)$ and we define the ensemble optical depth as $OD = \int gn(z)dz$. In practice the value of $\delta_0$ is chosen to cancel out the light shift caused by the control field: $\delta_0 = -\delta_{acS}$.

Spectral resolution

The spectral resolution of the device is limited by finite duration $T$ of the measurement window combined with the exponential decay of the atomic coherence caused by the control field. One could consider that upper limit for $T$ is given by combination of the bandwidth $\mathcal{B}$ and the control field chirp $\alpha$ by $T_{\text{max}} = \mathcal{B}/\alpha$ as for $\alpha T > \mathcal{B}$ a monochromatic input field $A(\omega) = \delta(\omega)$ lies outside the inhomogeneously broadened absorption spectrum. However, in the usual operation regime we set $\alpha \ll \mathcal{B}^2$ and to maintain high initial efficiency we always have $\tau < T_{\text{max}}$. In this regime the finite atomic coherence lifetime $\tau$ limits the available measurement time $T$ which we set to be $T = \tau$ to maintain high overall efficiency $\tilde{\eta}$. To estimate the resolution accounting for both $\tau$ and $T$ we calculate the power spectrum of a monochromatic input pulse with exponentially decaying amplitude $A(t) = (\Theta(t) - \Theta(t - \tau)) \exp(-t/\tau)$:

$$|\tilde{A}(\omega)|^2 \approx 1 + 2e^{\pi \omega^2 / \tau^2}.\cos(\tau \omega / 2\pi)^2,$$

and define the spectral resolution $\delta\omega$ as FWHM of the power spectrum $|\tilde{A}(\omega)|^2$. We numerically find $\delta\omega/2\pi \approx 0.78/\tau$.

Group-delay dispersion estimate

By imposing the parabolic phase shift onto the atomic ensemble, we imitate temporal imaging setups that use group-delay dispersion in chirped fiber Bragg gratings (CFBG), or just fibers, to achieve large group delays. The temporal propagation length we achieve in our setup amounts to $f_1 = 9600$ s which corresponds to a GDD of $25 \mu s^2$ over our 1 MHz bandwidth. To achieve such GDD, one would need $10^{12}$ km of typical telecom fiber (GDD...
when the control field is not applied the value of the magnetic field gradient depends only on the speed of shifting the atomic cloud. Generated by a pair of rounded square-shaped coils in external constant $\sim 1$ G magnetic field along the cloud, generated by external Helmholtz coils. The gradient of magnetic field for GEM is generated by a pair of rounded square-shaped coils in opposing configuration connected in series. The coils are made of 9 turns of copper wire wound over a base with a side length \( d = 10 \) cm. The separation between the coils is \( L = 17 \) cm. This configuration provides almost linear magnetic field gradient of value 0.08 G/cm over the 10 mm long cloud. The coils current and thus the gradient can be quickly switched to the opposite value using an electronic switch capable of connecting either constant current source (typ. 15A) in either direction or opposing configuration connected in series. The coils are made of 9 turns of copper wire wound over a base with a side length \( d = 10 \) cm. The separation between the coils is \( L = 17 \) cm. This configuration provides almost linear magnetic field gradient of value 0.08 G/cm over the 10 mm long cloud. The coils current and thus the gradient can be quickly switched to the opposite value using an electronic switch capable of connecting either constant current source (typ. 15A) in either direction or providing 160V for fast current reversal. This is accomplished using MOSFETs in H-bridge topology. The high switching speed (4.3 A/\( \mu s \) corresponding to 0.35 G/cm/\( \mu s \)) equals 160 V divided by inductance of the coils.

The instantaneous current in the coils is measured using LEM-LA 100-P current transducer with sub \( \mu s \) response time. The apparent overshoot after the current reversal is in fact a start of a combination of oscillation of LC circuit formed by the coils inductance and parasitic capacitance of the switch and an exponential decay of the current to a new steady state value. When the control field is not applied the value of the magnetic field gradient \( \beta \) controls only the speed of shifting the atomic coherence in the \( K_z \) space. Therefore, by applying the control after a short delay from the end of the switching state (when the overshoot is mostly present) we avoid the unwanted effect of readout efficiency change due to different instantaneous bandwidth \( B \). The residual variation of \( \beta \) during the readout process slightly affects the temporal profile of readout speed, which manifests as a chirp of the expected intensity oscillations of the output signal, as in Fig. 3(d) of the Article.

The magnetic gradient overshoot is in reality much smaller than estimated from the instantaneous current. We attribute this to eddy currents created in the metal parts of the magneto-optical trap apparatus occurring during switching of the coils which virtually compensate the amount of gradient change due to the overshoot.

**Supplementary Material**

**Experimental setup details**

**Magnetic field**

To determine the quantization axis the atomic cloud is kept in external constant $\sim 1$ G magnetic field along the cloud, generated by external Helmholtz coils. The gradient of magnetic field for GEM is generated by a pair of rounded square-shaped coils in opposing configuration connected in series. The coils are made of 9 turns of copper wire wound over a base with a side length \( d = 10 \) cm. The separation between the coils is \( L = 17 \) cm. This configuration provides almost linear magnetic field gradient of value 0.08 G/cm over the 10 mm long cloud. The coils current and thus the gradient can be quickly switched to the opposite value using an electronic switch capable of connecting either constant current source (typ. 15A) in either direction or providing 160V for fast current reversal. This is accomplished using MOSFETs in H-bridge topology. The high switching speed (4.3 A/\( \mu s \) corresponding to 0.35 G/cm/\( \mu s \)) equals 160 V divided by inductance of the coils.

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**Filtering system and noise characterization**

To minimize the noise we need to efficiently filter signal photons from the control beam and other noise. For this purpose, we have built a multi-stage filtering system (Fig. S2). Firstly, we filter most of control beam photons using far field aperture. Next, we use the fact that they have orthogonal polarization to the signal photons and we filter them using quarter-wave plate and Wollaston polarizer. After that we use near field aperture to remove photons scattered in other parts of MOT. Later, the glass cell containing Rubidium-87 pumped to 5S1/2, \( F = 1 \) state with 780 nm laser and buffer gas (nitrogen) is used to filter out stray control beam light while preserving the multimode nature of our device as compared to the cavity based filtering. Finally we use a 795 nm interference filter to remove other frequency photons.

**Fig. S1.** Simulated comparison of the output signal, as in Fig. 3(c) of the main Article with and without the chirp of the readout field. We observe no more than 1% difference, varying that omitting the final chirp for the purpose of observing proper intensity profile at the output is well justified.

25 ps\(^2\)/km or billions of commercially available CFBGs (GDD $\sim 10^4$ ps\(^2\)).

**Simplified protocol**

In the main Article we mentioned that in practice the temporal far-field imaging sequence we do not implement the chirp at readout, as it would not change the final intensity. While this is true in the ideal case, here we also want to argue that in particular, then only reason the difference may arise is a relative change in the single-photon detuning $\Delta$. Here we simulate the protocol with and without the chirp and in Fig. S1 we observe that the difference is minimal, staying always below 1%.

**Fig. S2.** Filtering setup. The signal separated from the coupling laser light using a sequence of far field apertures, Wollaston polarizer, near field aperture, optically pumped atomic filter and interference filter. Transmission of the signal photons through this system amounts about 60%.

**Fig. S3.** SPCM noise photons count rate versus decay rate of atomic coherence. The slope of the fitted line amounts to 0.023 and can be interpreted as average number of noise photons registered during readout process.
coming mainly from the filter pump.

To estimate the amount of noise present in our experiment we use the same experimental sequence as in Fig. 3 but without any optical field at the input. We perform the experiment for different values of the control laser power $P$ and calculate the noise count rate for each point. In Figure S3 we present measured noise count rate as a function of atomic coherence decay rate $1/\tau$, which is proportional to the power of the control beam $P$ (see Fig. 4(d) in the Article). The slope of the fitted line can be interpreted as average number of noise photons registered during the typical $\tau$-long readout process. Note that as we increase the coupling laser intensity, we register more noise photons yet during a shorter window. This gives us a constant mean photon number per readout. Thanks to our filtering system we achieved the value of $\bar{n}_{\text{noise}} = 0.023$ which means, that we register approximately 1 noise photon per 40 single experiments. Simultaneously, the transmission of the signal photons amounts to about 60%, while the detection efficiency is ~65%. With a typical memory process efficiency of 25%, we obtain noise per single photon sent to the device $\mu_1 = 0.23$, which corresponds to $\mu_1 = 0.016$ per single mode (i.e. in a single temporal mode storage experiment). The main limitation is still filtering of coupling light, as witnessed by removing the atomic ensemble and still observing the same noise level. That could be improved further by coupling the signal to a single-mode fibre or using more efficient filtering.

**Optical depth**

![Fluorescence image of atomic cloud from the side.](image)

**Fig. S4.** Fluorescence image of atomic cloud from the side. Center blue lines corresponds to its integrals along $y$ and $z$ axis. Red area, to the right marks the part of atomic cloud illuminated with signal laser. The orange line above presents concentration distribution along the $z$ axis corresponding to the spectral profile in presence of magnetic field gradient.

Figure S4 shows the image of atomic cloud from the side. Atoms are formed into pencil shape area with diameter of about 0.5 mm. The signal laser diameter amounts to about 0.1 mm and illuminates the middle of the atomic cloud, where the optical depth is the highest. Figure S5 presents single photon absorption profile of the signal. Fitting the saturated Lorentz profile, we estimated that optical depth amounts to about 76.

![Single photon absorption profile of the signal laser.](image)

**Fig. S5.** Single photon absorption profile of the signal laser. Fitted red line corresponds to OD=76.