Electromagnetic form factors and polarizations of non-Dirac particles with rest spin 1/2

L.M. Slad

Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Moscow 119991, Russian Federation

Abstract

We consider one aspect of the theoretical foundations of polarization experiments on elastic scattering of electrons on protons yielding form factor ratios incompatible with those that are extracted from nonpolarization experiments. We analyze the consequences of abandoning the assumption that the nucleon is a Dirac particle. We show that the process of elastic scattering of electrons on nucleons is described by the same formulas, irrespective of the proper Lorentz group representation associated with the nucleon as a particle with the rest spin 1/2.

1. Introduction

A number of amazing experimental results on scattering of electrons on protons have recently been obtained [1]–[7]. Their totality gives evidence of an incompatibility between the values of the same quantity, the ratio of the electric form factor $G_E$ to the magnetic form factor $G_M$ of the proton, obtained using two different methods.

One of the methods is based on extracting the form factor ratio from the Rosenbluth formula [8], which describes the scattering of nonpolarized electrons on nonpolarized protons in the laboratory reference frame,

$$
\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E'^2}{4E^3 \sin^4(\theta/2)} \left[ \frac{G_E^2 + \tau G_M^2}{1 + \tau} + \frac{2\tau G_M^2 \tan^2 \frac{\theta}{2}}{2} \right],
$$

where $E$ and $E'$ are the respective energies of the electron in the initial and final states, $\theta$ is the electron scattering angle, $Q^2$ is the squared transferred momentum, $\tau = Q^2 / 4M^2$, and $M$ is the proton mass. Recent high-precision experiments [6], [7] in the domain $Q^2 \leq 5.5$ (GeV/c)$^2$ yield the $R = \mu_p G_E / G_M$ value (where $\mu_p$ is the magnetic moment of the proton) close to unity. This conclusion agrees with the outcome of Arrington’s global analysis [9], to which he subjected the results of numerous previous experimental works.

The second method for obtaining the ratio of the electromagnetic form factors of the proton is based on polarization measurements. Specifically, if longitudinally polarized electrons are scattered on nonpolarized protons at rest, then the polarization of recoil protons has a transverse (in the plane of all the particle momenta) component $P_x$ and the longitudinal component $P_z$, and the relation [10]

$$
\frac{G_E}{G_M} = -\frac{P_x}{P_z} \cdot \frac{E + E'}{2M} \tan \frac{\theta}{2}
$$

holds, where $P_x$ and $P_z$ pertain to the rest frame of the final proton. Using this formula, the authors of [1]–[5] concluded that there is an almost linear decrease in $R$ as $Q^2$ increases, from a value close to unity for small $Q^2$ to a value approximately equal to 0.3 at $Q^2 = 5.6$ (GeV/c)$^2$.

This discrepancy in the values of $R$ has caused a number of new publications [11]–[15] analyzing the contribution of the two-photon exchange between electrons and protons to the
corrections to formulas (1) and (2). We note that radiation corrections to Rosenbluth formula (1), with the two-photon exchange included, have been the subject of multiple investigations since times of old. In due time, the line was drawn here in [16], which is still used in processing experimental data on the electron scattering on nucleons (e.g., by Arrington [9]). In [11]–[15], the dominant opinion is that if the structure of virtual hadron states in the processes involving a two-photon exchange is taken into account, then this reduces the discrepancy in the values of \( R \) obtained from polarization and nonpolarization measurements several times but does not eliminate it.

The question of the source of the discrepancies related to \( R \) seems not to have yet obtained a clear and unambiguous theoretical resolution within the study of only radiation corrections. We believe it is also necessary to analyze all aspects of the theoretical models and assumptions used in the course of obtaining the final result in the works on scattering of electrons on protons. So, conclusions regarding the polarization of recoil protons are usually obtained based, first, on theoretically modeling the azimuthal asymmetry [17] occurring because of the spinorbit coupling in secondary scattering of protons on a carbon target and, second, on the phenomenological Bargmann–Michel–Telegdi formula [18] describing the spin rotation of a relativistic particle in a homogeneous magnetic field. Finally, both formulas (1) and (2) were originally obtained under the assumption that the proton is a Dirac particle. This work is devoted to a thorough analysis of a number of consequences of dropping this assumption. We establish that formulas (1) and (2) are universal, i.e., they are applicable to both a Dirac and a non-Dirac nucleon; we thus reduce the number of theoretical aspects of polarization experiments on the electron scattering on protons, which, in our opinion, would be worth comprehensively reconsidering.

2. Electromagnetic current and wave vectors for a non-Dirac particle with rest spin 1/2

Assigning a nucleon the Dirac representation of the proper Lorentz group \( L^1_+ \) entirely agreed with the early views of the inner structure of the nucleon based on the meson model, where the physical nucleon consists of a bare (Dirac) nucleon and a cloud of pseudoscalar pions. This assignment implies the most general form of the conserved nucleon electromagnetic current in the momentum space [19], [20]

\[
\begin{align*}
  j^\mu(p,p_0) &= ie\bar{u}(p) \left[ \gamma^\mu F_1(Q^2) + \frac{i\kappa}{2M}\sigma^{\mu\nu}q_\nu F_2(Q^2) \right] u(p_0),
\end{align*}
\]

which is a polar four-vector of the orthochronous Lorentz group \( L^1 \). Here, \( \bar{u} \) and \( u \) are Dirac spinors for the nucleon, \( \kappa \) is the anomalous magnetic moment of the nucleon expressed in nuclear magnetons, \( q = p - p_0 \), and \( Q^2 = -q^2 \). The Dirac form factor \( F_1 \) and the Pauli form factor \( F_2 \) in (3) are related to the Sachs form factors \( G_E \) and \( G_M \) in (1) and (2) as [20]

\[
G_E = F_1 - \kappa \tau F_2, \quad G_M = F_1 + \kappa F_2.
\]

The quark model of hadrons and then of quantum chromodynamics altered our perception of the essence of the inner structure of the nucleon. It is now assumed that the nucleon consists of three valence quarks and a sea of quark antiquark pairs and gluons. Hence, its vector-valued wave function of parton coordinates transforms under some representation of the proper Lorentz group, a representation belonging to the direct product of a countable set of the Dirac representations assigned to the quarks and a countable set of the four-vector representations assigned to the gluons. This product decomposes into a direct sum of all finite-dimensional irreducible representations of the proper Lorentz group with half-integer spins. This conclusion concerns all nucleon states, both the ground state and the resonance states. Regarding the
nucleon as the ground state, we must take into account that the nucleon has a certain spin in the rest frame. Therefore, each irreducible \( L^\uparrow \) representation involved in describing the nucleon must contain its rest spin 1/2. Therefore, in our opinion, the modern parton model of hadrons and group theory arguments give evidence for assigning the nucleon the infinite-dimensional representation\(^1\)

\[
S^{3/2} = \bigoplus_{n=0}^{+\infty} \left( \left( -\frac{1}{2}, \frac{1}{2} + n \right) \oplus \left( \frac{1}{2}, \frac{1}{2} + n \right) \right).
\]

We already dealt with this representation in [23]–[26] in studying the theory of ISFIR-class fields, i.e., the fields transforming under the \( L^\uparrow \) representations decomposable into an infinite direct sum of finite-dimensional irreducible representations. A physical implication established in this theory, the characteristics of its mass spectra [26], agrees well with the experimental picture.

We nevertheless note that all the arguments and derivations here except formula (65) have the same force for representation (5), for any finite direct sum of finite-dimensional irreducible \( L^\uparrow \) representations containing spin 1/2, for any finite or infinite direct sum of infinite-dimensional irreducible representations of the form

\[
S^{k_1,K} = \bigoplus_{n \in K} \left( \left( -\frac{1}{2}, k_1 + n \right) \oplus \left( \frac{1}{2}, k_1 + n \right) \right),
\]

where \( K \) is a subset of the integers and \( k_1 \) is a real number such that \( |k_1| \in (0,1/2) \cup (1/2,1) \), and for the infinite-dimensional representations \( S^{1/2} = (-1/2,1/2) \oplus (1/2,1/2) \) and \( S^{\text{unit}} = (-1/2,i\rho) \oplus (1/2,i\rho) \), where \( \rho \) is a positive number. If \( k_1 \) were complex in formula (6), \( \text{Re} k_1 \neq 0 \) and \( \text{Im} k_1 \neq 0 \), then a nonvanishing relativistic-invariant bilinear form and a free-field Lagrangian would not exist. And if we set \( k_1 = 0 \) in formula (6), then the corresponding free-field theory would not be CPT invariant [27], [28]. We recall that a theory of fields transforming as a finite direct sum of infinite-dimensional irreducible representations of the proper Lorentz group has several properties, listed in the introduction to [24], that make it unacceptable for particle physics.

If the nucleon field \( \psi \) is assigned some non-Dirac representation of the \( L^\uparrow \) group, then there are no reasons to restrict the Lagrangian to only the minimal coupling to the electromagnetic field \( A_\mu \) and the lowest nonminimal coupling involving the gauge-invariant antisymmetric tensor \( F^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \), as is necessarily the case with a Dirac particle. Such a field \( \psi \) in the general case must be assigned the electromagnetic current with a countable set of terms of the form

\[
J^\mu(p,p_0) = i e \left( \psi(p), \left[ \Gamma^\mu K_0(Q^2) + \Gamma^{\mu\nu}q_\nu K_1(Q^2) + \Gamma^{\mu\nu_1...\nu_n}q_{\nu_1}q_{\nu_2}K_2(Q^2) + \ldots \right. \right. \left. \left. + \Gamma^{\mu\nu_1...\nu_n}q_{\nu_1}q_{\nu_2} \ldots q_{\nu_n}K_{n+1}(Q^2) + \ldots \right] \psi(p_0) \right),
\]

where \( \Gamma^{\mu\nu} \) and \( \Gamma^{\mu\nu_1...\nu_n} \), \( n = 1, 2, \ldots \), are matrix tensor operators of the Lorentz group and are antisymmetric in \( \mu \) and \( \nu \). This antisymmetry ensures that the current \( J^\mu \) is conserved and the corresponding Lagrangian is gauge invariant. The needed information about the structure of any four-vector operator \( \Gamma^\mu \) and the details concerning the relativistic-invariant bilinear form of the \( (\psi_1, \psi_2) \) type, which we consider nondegenerate, can be found in [21], [22]. The description

\(^1\)For irreducible \( L^\uparrow \) representations \( \tau \), we use the notation of Gelfand and Yaglom [21], [22]: \( \tau = (\ell_0, \ell_1) \), where \( 2\ell_0 \) is an integer and \( \ell_1 \) is an arbitrary complex number. The canonical basis of the representation space is related to the \( SO(3) \) subgroup and is denoted by \( \xi_{(\ell_0,\ell_1)m} \), where \( \ell \) is the spin and \( m \) is its projection on the third axis with \( m = -\ell, -\ell + 1, \ldots, \ell \) and \( \ell = |\ell_0|, |\ell_0| + 1, \ldots \). In the general case, the last sequence is infinite. The representation \( (\ell_0, \ell_1) \) is finite-dimensional, and the above sequence of spins terminates if \( 2|\ell_0| + 2|\ell_1| \) are integers of the same parity and \( |\ell_1| > |\ell_0| \). The pairs \( (\ell_0, \ell_1) \) and \( (-\ell_0, -\ell_1) \) describe the same representation. The Dirac representation is \((-1/2,3/2) \oplus (1/2,3/2)\).
of the general structure of the matrix operators $\Gamma^{\mu\nu}$ and, even more so, of the operators $\Gamma^{\mu\nu_1...\nu_n}$ in (7) is still not available in the literature; nor is it needed for our purposes here. In what follows, we need the properties of tensors and tensor operators of the Lorentz group with respect to the spatial reflection. Unless stated otherwise, we assume that they are everywhere the same as for the appropriate tensor product of polar four-vectors and polar vector operators of the orthochronous Lorentz group $L^\uparrow$.

Of all the properties of the infinite sum of operators in current (7) involved in describing a nonminimal electromagnetic coupling, we select only two: the antisymmetry of the resulting operator and its dependence on the components of the transferred momentum four-vector. We express this using the notation

$$\Lambda^{\mu\nu}(q) \equiv \Gamma^{\mu\nu} K_1(Q^2) + \Gamma^{\mu\nu_1} q_\nu K_2(Q^2) + \ldots + \Gamma^{\mu\nu_1...\nu_n} q_\nu \ldots q_\nu K_{n+1}(Q^2) + \ldots$$

In what follows, in describing the elastic scattering of electrons on nucleons, we use the laboratory frame with the recoil nucleon momentum directed along the third coordinate axis (the $Z$ axis) and the initial electron momentum lying in the $XZ$ plane. Hence,

$$p_0 = \{M, 0, 0, 0\}, \quad p = \{E_N, 0, 0, p_N\}, \quad q = p - p_0 = \{q^0, 0, 0, q^3\}.$$ (9)

In calculating the characteristics of the elastic scattering of an electron on a non-Dirac particle with the rest spin $1/2$, we need commutation relations of the operators $\Lambda^{\mu\nu}(q)$ with the infinitesimal operator $I^{12}$ of the rotation group $^2$. In the chosen reference frame, they are given by

$$[I^{12}, \Lambda^{\mu\nu}(q)] = -g^{1\mu} \Lambda^{2\nu}(q) + g^{1\nu} \Lambda^{2\mu}(q) + g^{2\mu} \Lambda^{1\nu}(q) - g^{2\nu} \Lambda^{1\mu}(q),$$ (10)

where $g^{00} = -g^{11} = -g^{22} = -g^{33} = 1$, $g^{\rho\sigma} = 0$, and $\rho \neq \sigma$.

In fact, declaring that the operator $\Lambda^{\mu\nu}(q)$ is an antisymmetric tensor operator means that the quantity $(\psi_1, \Lambda^{\mu\nu}(q)\psi_2)$ transforms under the group $L^\uparrow$ as an antisymmetric tensor and that the contraction $(\psi_1, \Lambda^{\mu\nu}(q)\eta_{\mu\nu}\psi_2)$, where $\eta_{\mu\nu}$ is any antisymmetric second-rank tensor, is a relativistic-invariant nondegenerate bilinear form. If we assume that the field vectors $\psi_1$ and $\psi_2$ belong to the representation space $S(g)$ of the orthochronous Lorentz group, then we have

$$[S(g)]^{-1} \Lambda^{\mu\nu}(q') \eta_{\mu\nu} S(g) = = \Lambda^{\mu\nu}(q) \eta_{\mu\nu}$$ (11)

and

$$\chi'_{\mu} = [l(g)]_{\mu}^{\rho} \chi_{\rho},$$

$$\eta'_{\mu\nu} = [T(g)]_{\mu\nu}^{\rho\sigma} \eta_{\rho\sigma} = \frac{1}{2} \{[l(g)]_{\mu}^{\rho} [l(g)]_{\nu}^{\sigma} - [l(g)]_{\mu}^{\sigma} [l(g)]_{\nu}^{\rho}\} \eta_{\rho\sigma},$$ (12)

where $\chi$ is an arbitrary polar four-vector. We consider an infinitesimal rotation $g_1$ about the third coordinate axis. Then $q' = q$, and also

$$\chi' = \{\chi_0, \chi_1 - \epsilon_1 \chi_2, \chi_2 + \epsilon_1 \chi_1, \chi_3\}, \quad S(g_1) = 1 + \epsilon_1 I^{12}.$$ (13)

Using (13), (12), and (11), we obtain commutation relations (10). Under other infinitesimal rotations or proper Lorentz transformations, the four-vector $q$ changes, and formula (11) cannot lead to commutation relations of the operators $\Lambda^{\mu\nu}(q)$ with any of the other ($\{\rho, \sigma\} \neq \{1, 2\}$) infinitesimal operators $I^{\rho\sigma}$ of the $L^\uparrow$ group.

$^2$The information extracted from [21], [22] about Lorentz group representations and relativistic-invariant equations was briefly outlined in [24], but the action of the infinitesimal operators of the rotation group on the basis vectors $\xi_{\tau l m}$ was not written there, because they were not needed in that paper (in contrast to the present paper). We introduce the notation $H^3 = iI^{12}$, $H^- = iI^{23} + I^{31}$, and $H^+ = iI^{23} - I^{31}$. Then $H^3 \xi_{\tau l m} = m \xi_{\tau l m}$, $H^- \xi_{\tau l m} = \sqrt{(l-m+1)(l+m)} \xi_{\tau l m-1}$, and $H^+ \xi_{\tau l m} = \sqrt{(l-m)(l+m+1)} \xi_{\tau l m+1}$.
We assume that the wave function of a non-Dirac nucleon $\psi(p)$, as well as of a Dirac nucleon, transforming under one of the above $L^\dagger_\pm$-representations $S_0$, satisfies some definite relativistic-invariant equation

$$(\Gamma^\mu p_\mu - R)\psi(p) = 0,$$  \hspace{1cm} (14)

where $R$ is a scalar matrix operator.

Because the matrix of the $\Gamma^0$ and $R$ operators are diagonal with respect to the spin $l$ and to its projection $m$ on the third axis and because their elements are independent of $m$ [21], [22], it follows that each particle described by some solution $\psi(p)$ of (14) in the particle rest frame can be assigned a definite spin and $m$-independent wave function components. In particular, for a spin-$1/2$ particle, two of its independent states in the rest frame can be assigned the vectors

$$\psi_m(p_0) = \sum_{(l_0,l_1) \in S_0} u_{(l_0,l_1)\frac{1}{2}m}(0)\xi_{(l_0,l_1)\frac{1}{2}m},$$  \hspace{1cm} (15)

where $m = -1/2, 1/2$ and

$$u_{(l_0,l_1)\frac{1}{2}m}(0) = u_{(l_0,l_1)\frac{1}{2}m}(0).$$  \hspace{1cm} (16)

Because (14) is invariant under spatial reflections, each particle state vector in the particle rest frame has a certain $P$-parity, namely,

$$P\psi_m(p_0) = r\psi_m(p_0), \quad u_{(-\frac{1}{2},l_1)\frac{1}{2}m}(0) = ru_{(\frac{1}{2},l_1)\frac{1}{2}m}(0),$$  \hspace{1cm} (17)

where $r = +1$ or $r = -1$.

At the transition from the particle rest frame to the chosen laboratory frame corresponding to the element $g_0$ of the proper Lorentz group, the transformations in the space of wave vectors $\psi$ are given by the operator $S(g_0) = \exp(\alpha I^{03})$, where $\tanh\alpha = v = p_N/E_N$. Matrix elements of this operator are diagonal with respect to the spin projection on the third axis:

$$\exp(\alpha I^{03})\xi_{(l_0,l_1)lm} = \sum_{\nu} A^{(l_0,l_1)}_{\nu m,lm}(\alpha)\xi_{(l_0,l_1)\nu m}.$$  \hspace{1cm} (18)

Acting with this operator on both sides of (15), we obtain the vectors

$$\psi_m(p) = S(g_0)\psi_m(p_0) = \sum_{(l_0,l_1) \in S_0} \sum_{l} u_{(l_0,l_1)lm}(\alpha)\xi_{(l_0,l_1)lm},$$  \hspace{1cm} (19)

where

$$u_{(l_0,l_1)lm}(\alpha) = A^{(l_0,l_1)}_{lm,\frac{1}{2}m}(\alpha)u_{(l_0,l_1)\frac{1}{2}m}(0).$$  \hspace{1cm} (20)

In what follows, we use the general relations

$$A^{(l_0,l_1)}_{l-m,\frac{1}{2}-m}(\alpha) = A^{(-l_0,l_1)}_{lm,\frac{1}{2}m}(\alpha) = \left[A^{(l_0,l_1)}_{lm,\frac{1}{2}m}(-\alpha)\right]^*,$$  \hspace{1cm} (21)

between matrix elements of finite transformations of the $L^\dagger_\pm$ group. They easily follow from the result of acting by the infinitesimal operator $I^{03}$ on canonical basis vectors (see formulas (1.7) and (1.9))\textsuperscript{3}.

3. Elastic scattering cross section of a nonpolarized electron on a nonpolarized non-Dirac particle with the rest spin 1/2

\textsuperscript{3}Here and hereafter, a reference of the form (I.N) is to formula (N) in [24].

5
If the electromagnetic current of a nucleon is given by formulas (7) and (8), then the elastic scattering of an electron on this nucleon with fixed values of its spin projection on the third axis in the initial and final states corresponds to the matrix element

\[
\mathcal{M}_{m_2m_1} = \frac{e^2}{q^2} \bar{u}_{e'}(k')\gamma_\mu u_e(k) \cdot \left( \psi_{m_2}(p), [\Gamma^\mu K_0(Q^2) + \Lambda^{\mu\nu}(q)q_\nu] \psi_{m_1}(p_0) \right),
\]

(22)

where \(k\) and \(k'\) are the respective four-momenta of the incident and scattered electrons described by the Dirac spinors \(u_e(k)\) and \(\bar{u}_{e'}(k')\).

In calculating the squared modulus of the matrix element \(\overline{M}^2\) summed over the polarizations of final-state particles and averaged over polarizations of the initial-state particles, we can use well-known formulas for the electron, which is a Dirac particle. Specifically [29],

\[
\frac{1}{2} \sum_{\text{polar. } e,e'} [\bar{u}_{e'}(k')\gamma_\mu u_e(k)][\bar{u}_{e'}(k')\gamma_\mu u_e(k)]^* = l_{\mu\nu} = 4k_\mu k_\nu + g_{\mu\nu} + 2(k_\mu q_\nu + k_\nu q_\mu).
\]

(23)

This equality allows eliminating the wave vectors of electrons, their Dirac spinors, from our calculations. But in the general case, in finding the quantity \(\overline{M}^2\), we must constantly deal with wave vectors of non-Dirac nucleons. To represent this quantity most simply, similar in structure to the form of the Dirac nucleon, we must carefully establish a number of general relations for elements of its electromagnetic current. We take up this problem now.

First, we use the relation reflecting the relativistic invariance of the bilinear form,

\[
(I^{\mu\nu}\psi_1, \psi_2) = -(\psi_1, I^{\mu\nu}\psi_2),
\]

(24)

and its corollary [21], [22]

\[
(\xi_{\tau'\tau'} \psi_{m'\tau'}, \xi_{\tau m}) = \delta_{\tau'\tau'} \delta_{\tau m'} a_{\tau'\tau}(l),
\]

(25)

where \(\tau^* = (l_0, -l_1) \sim (-l_0, l_1)\) if \(\tau = (l_0, l_1)\), and also relations (I.14), (I.17), (14), (15), and (19). We first establish the equality

\[
(\Gamma^\mu \psi_1, \psi_2) = (\psi_1, \Gamma^\mu \psi_2),
\]

(26)

for all values of the index \(\mu\), and then

\[
(\psi_{m_2}(p), \Gamma^0 \psi_{m_1}(p_0)) = \delta_{m_2m_1} \frac{1}{M} (\psi_{m_1}(p), R\psi_{m_1}(p_0)),
\]

(27)

\[
(\psi_{m_2}(p), \Gamma^3 \psi_{m_1}(p_0)) = \delta_{m_2m_1} \frac{q^0}{q^0 M} (\psi_{m_1}(p), R\psi_{m_1}(p_0)).
\]

Second, we note that while the action of the operator \(\Gamma^0\) on a vector of the canonical basis does not change the quantum number \(m\) of the vector, the infinitesimal operators \(I^{10}\) and \(I^{20}\) (see relations (I.7), (I.9), and (I.10)) and hence the operators \(\Gamma^1\) and \(\Gamma^2\) found in accordance with (I.17) change this quantum number by \(\pm 1\). Also, based on formula (10) and the explicit form of the operator \(I^{12}\) in the canonical basis, it is easy to establish that the two components \(\Lambda^{12}(q)\) and \(\Lambda^{30}(q)\) of the antisymmetric tensor \(\Lambda^{\mu\nu}(q)\) do not change the number \(m\) in the vectors \(\xi_{\tau m}\), and the other components change it by \(\pm 1\). All this, taken together with (15) and (19), where \(m = \pm 1/2\), and with (25), yields

\[
(\psi_{m_2}(p), U_1 \psi_{m_1}(p_0)) = \delta_{m_2m_1} (\psi_{m_1}(p), U_1 \psi_{m_1}(p_0)),
\]

\[
(\psi_{m_2}(p), U_2 \psi_{m_1}(p_0)) = \delta_{m_2-m_1} (\psi_{-m_1}(p), U_2 \psi_{m_1}(p_0)),
\]

(28)
if \( U_1 \in \{ \Lambda^{12}(q), \Lambda^{30}(q) \} \) and \( U_2 \in \{ \Gamma^1, \Gamma^2, \Lambda^{23}(q), \Lambda^{31}(q), \Lambda^{10}(q), \Lambda^{20}(q) \} \).

Third, we verify the intuitive expectation that two quantities \((\psi_{m_2}(p), W\psi_{m_1}(p_0))\) and \((\psi_{-m_2}(p), W\psi_{-m_1}(p_0))\), where \( W \) is some scalar or four-vector or tensor operator of the Lorentz group, have the same value, at least up to a phase factor. We immediately note that in a sufficiently general case involving elements of current (7), (8), the required proof is nontrivial. It is based on the fact that the parameters \( \varepsilon_{\mu\nu} \) of proper transformations of the orthochronous Lorentz group constitute an antisymmetric tensor of this group. Let the field \( \psi \) be a vector in the space \( L \) of an \( L^\dagger \)-representation \( S(g) \). The vector \( \psi' = \exp(\varepsilon_{\mu\nu} I^{\mu\nu}/2) \psi \) belonging to the space \( L \) can be subjected to the transformation from the group \( L^\dagger \) generated by \( g \) in two equivalent ways, represented by the left- and right-hand sides of the equality

\[
S(g)\psi' = \exp\{[T(g)]_{\mu\nu} \varepsilon_{\rho\sigma} I^{\mu\nu}/2\} S(g)\psi. \tag{29}
\]

Hence,

\[
S(g)\exp(\varepsilon_{\mu\nu} I^{\mu\nu}/2) = \exp\{[T(g)]_{\mu\nu} \varepsilon_{\rho\sigma} I^{\mu\nu}/2\} S(g). \tag{30}
\]

Let the element \( g_2 \) of the orthochronous Lorentz group correspond to rotation through the angle \( \pi \) about the first axis (the \( X \) axis) followed by a spatial reflection. Under this transformation, which leaves the polar four-vector \( q \) unchanged, we have

\[
(\{\eta_{12}, \eta_{23}, \eta_{31}, \eta_{01}, \eta_{02}, \eta_{03}\} = \{-\eta_{12}, -\eta_{23}, -\eta_{31}, -\eta_{01}, \eta_{02}, \eta_{03}\}, \quad S(g_2) = P \exp(\pi I^{23}). \tag{31}
\]

In view of the relativistic invariance of the bilinear form, relations (11), (30), (31), and (15), the explicit form of the operator \( I^{23} \) in the canonical basis, and relations (16) and (17), we obtain

\[
(\psi_{m_2}(p), \Lambda^{\rho\sigma}(q)\eta_{\rho\sigma}\psi_{m_1}(p_0)) =
\]

\[
= (S(g_2) \exp(\alpha I^{03})\psi_{m_2}(p_0), S(g_2)\Lambda^{\rho\sigma}(q)\eta_{\rho\sigma}\psi_{m_1}(p_0)) = \]

\[
= (\exp(\alpha I^{03})S(g_2)\sum_{\tau} u_{\tau + \frac{1}{2}m_2}(0)\xi_{\tau \frac{1}{2}m_2}, \Lambda^{\rho\sigma}(q)\eta_{\rho\sigma}S(g_2)\sum_{\tau} u_{\tau + \frac{1}{2}m_1}(0)\xi_{\tau \frac{1}{2}m_1}) = \]

\[
= (\exp(\alpha I^{03})P\sum_{\tau} u_{\tau + \frac{1}{2}m_2}(0)(-i)\xi_{\tau \frac{1}{2} - m_2}, \Lambda^{\rho\sigma}(q)\eta_{\rho\sigma}P\sum_{\tau} u_{\tau + \frac{1}{2}m_1}(0)(-i)\xi_{\tau \frac{1}{2} - m_1}) = \tag{32}
\]

\[
= (\exp(\alpha I^{03})P\sum_{\tau} u_{\tau \frac{1}{2} - m_2}(0)\xi_{\tau \frac{1}{2} - m_2}, \Lambda^{\rho\sigma}(q)\eta_{\rho\sigma}P\sum_{\tau} u_{\tau \frac{1}{2} - m_1}(0)\xi_{\tau \frac{1}{2} - m_1}) = \]

\[
= (\psi_{-m_2}(p), \Lambda^{\rho\sigma}(q)\eta_{\rho\sigma}\psi_{-m_1}(p_0)).
\]

Hence, the relation

\[
(\psi_{m_2}(p), \Lambda^{\mu\nu}(q)\psi_{m_1}(p_0)) = \kappa_1(\psi_{-m_2}(p), \Lambda^{\mu\nu}(q)\psi_{-m_1}(p_0)) \tag{33}
\]

holds, where \( \kappa_1 = 1 \) if neither of the two indices \( \mu \) and \( \nu \) is equal to unity and \( \kappa_1 = -1 \) otherwise. Similarly, we find that

\[
(\psi_{m_2}(p), \Gamma^\mu\psi_{m_1}(p_0)) = \kappa_0(\psi_{-m_2}(p), \Gamma^\mu\psi_{-m_1}(p_0)), \tag{34}
\]

where \( \kappa_0 = 1 \) if \( \mu = 0, 2 \) or \( 3 \) and \( \kappa_0 = -1 \) if \( \mu = 1 \).

\(^4\)Relation (30) is quite possibly new in group theory. A similar relation can also be given for any Lie group \( G \), taking into account that its parameters are components of vectors in the adjoint representation space of \( G \).
Fourth, using relations (28), (1.15), (10), (24), (15), and (19) and the explicit form of the infinitesimal operator $I^{12}$, we obtain

\[
\begin{align*}
(\psi_{-m_1}(p), \Gamma^2\psi_{m_1}(p_0)) &= 2i m_1 (\psi_{-m_1}(p), \Gamma^1\psi_{m_1}(p_0)), \\
(\psi_{-m_1}(p), \Lambda^{2\sigma}\psi_{m_1}(p_0)) &= 2i m_1 (\psi_{-m_1}(p), \Lambda^{1\sigma}\psi_{m_1}(p_0)),
\end{align*}
\]

where $\sigma = 0$ or $\sigma = 3$.

Only now, based on relations (22), (23), (27), (28), and (33)–(35), do we find the simplest expression for the squared modulus of the matrix element $M_2$ describing the process of elastic scattering of nonpolarized electrons on nonpolarized nucleons. Next, using the standard formula (see, e.g., [30]) expressing the differential cross section of the process in terms of $M_2$, we obtain a distribution over the solid angle of scattered electrons. We verify that this distribution is described by exactly Rosenbluth formula (1). The role of the electromagnetic form factors $G_E$ and $G_M$ is then played by the quantities

\[
G_E = \frac{C}{\sqrt{\tau + 1}} (\psi_{+1/2}(p), [K_0(Q^2)R - Mq^3\Lambda^{03}(q)]\psi_{+1/2}(p_0)),
\]

\[
G_M = MC \sqrt{\tau} (\psi_{+1/2}(p), [K_0(Q^2)\Gamma^1 + q^0\Lambda^{10}(q) - q^3\Lambda^{13}(q)]\psi_{-1/2}(p_0)),
\]

where

\[
C = (\psi_{+1/2}(p_0), R\psi_{+1/2}(p_0))^{-1}.
\]

4. Polarization of recoil nucleons in elastic scattering of polarized electrons on nonpolarized non-Dirac nucleons

The calculations taking the polarization of an electron with the mass $m_e$ into account are based on the relation [29]

\[
u_e(k)\bar{u}_e(k) = \frac{1}{2}(k + m_e)(1 - \gamma^5\hat{a}),
\]

where the components of the axial four-vector $a$ are

\[
a^0 = \frac{k\xi}{m_e}, \quad \mathbf{a} = \xi + \frac{(k\xi)\mathbf{k}}{m_e(m_e + E)}
\]

with $\xi$ being the unit vector of the electron polarization in the rest frame of the electron. We recall that the polarization of a spinor particle in some state is twice the average value of spin in this state.

Summing over the polarizations of the scattered relativistic electron, using formulas (39) and (40), and neglecting terms of the order of $m_e/E$, one obtains [10]

\[
\sum_{\text{polar. } e'} [\bar{u}_{e'}(k')\gamma_\mu u_e(k)] [\bar{u}_{e'}(k')\gamma_\nu u_e(k)]^* = l_{\mu\nu} + 2i\xi_\parallel \varepsilon_{\mu\nu\rho\sigma}q^0k^\sigma,
\]

where $l_{\mu\nu}$ is given by relation (23) and $\xi_\parallel$ denotes the longitudinal polarization of the initial-state electron, $-1 \leq \xi_\parallel \leq +1$. We note that the transverse polarization of the initial electron, being nonzero as long as $|\xi_\parallel| \neq 1$, makes a negligibly small contribution $O(m_e/E)$ to the right-hand side of (41).

The analogue of (39) and (40) is an essential element in the standard procedure for calculating the polarization of Dirac particles produced in some process [29]. It was used in [10] for
recoil nucleons in the scattering of polarized electrons on a nonpolarized target. Considering non-Dirac particles, we lose this procedure together with formulas (39) and (40).

Our method for finding the polarization of recoil nucleons is based on the explicit form of their state vectors $\Psi_{m_1}^{\lambda'}$, each of which is a superposition of two state vectors (19) with definite spin projections on the third axis, and the coefficients in this superposition are given by the appropriate matrix elements of the electron scattering on nucleons, namely,

$$\Psi_{m_1}^{\lambda'} = N_{m_1}^{\lambda'} \left[ M_{\lambda' m_1}^{\lambda'} \psi_{-\frac{1}{2}} (p) + M_{\lambda' m_1}^{\lambda'} \psi_{\frac{1}{2}} (p) \right],$$

(42)

where

$$N_{m_1}^{\lambda'} = (|M_{\lambda' m_1}^{\lambda'}|^2 + |M_{\lambda' m_1}^{\lambda'}|^2)^{-1/2}$$

(43)

and $\lambda'$ is the polarization (helicity) of the scattered electron (it is not written explicitly in formula (22)).

The average value $b$ of a quantity characterizing recoil nucleons and described by an operator $B$ is found by the formula

$$b = D_0^{-1} \sum_{\lambda', m_1} (\Psi_{m_1}^{\lambda'}, B \Psi_{m_1}^{\lambda'}) w_{m_1}^{\lambda'},$$

(44)

where

$$D_0 = (\psi_m (p_0), \psi_m (p_0))$$

(45)

and $w_{m_1}^{\lambda'}$ is the probability that the created nucleon and the electron have one of the two possible values of the spin projection $m_1$ on the third axis and one of the two possible values of polarization (helicity) $\lambda'$. Obviously,

$$w_{m_1}^{\lambda'} = N^2 (N_{m_1}^{\lambda'})^{-2},$$

(46)

where

$$N = (\sum_{\lambda', m_1, m_2} |M_{m_1 m_2}^{\lambda'}|^2)^{-1/2}.$$  

(47)

In the relativistic quantum theory, the spin operator is identified with the antisymmetric tensor operator $iI^{\mu\nu}$, whose components are generators of the proper Lorentz group. We are primarily interested in the average values of purely spatial components of spin, constituting a vector $s$. It is essential that in the rest frame of a Dirac or non-Dirac spinor particle, for any state with a definite $P$-parity, the average values of the other components of the spin tensor are equal to zero:

$$(\psi (p_0), I^{0i} \psi (p_0)) = 0,$$

(48)

where

$$\psi (p_0) = c_{-1/2} \psi_{-1/2} (p_0) + c_{1/2} \psi_{1/2} (p_0),$$

(49)

$i = 1, 2, 3$, and $c_m$ are arbitrary superposition coefficients. Indeed, using relations (49) and (17), the invariance of the bilinear form under spatial relations, and relation (11) with the replacement of $\Lambda^\mu
\nu (q)$ with $I^{\mu
\nu}$ and $S(g)$ with $P^{-1}$, we obtain

$$(\psi (p_0), [I^{0i} \eta_{0i} + I^{jk} \eta_{jk}] \psi (p_0)) = (P \psi (p_0), [I^{0i} \eta_{0i} + I^{jk} \eta_{jk}] P \psi (p_0)) =$$

$$(\psi (p_0), P^{-1} [I^{0i} \eta_{0i} + I^{jk} \eta_{jk}] P \psi (p_0)) =$$

$$(\psi (p_0), [I^{0i} \eta_{0i} + I^{jk} \eta_{jk}] \psi (p_0)).$$

(50)

Interestingly, Frenkel, who suggested a relativistic description of the classical spin as an antisymmetric tensor prior to the appearance of the Dirac equation, formulated the rule that
in the rest frame of a particle, only purely spatial components of its spin tensor can be nonzero
[31] (a review of Frenkel’s works on the classical theory of spin can be found in [32]).

Using relation (48) and the known formulas for a Lorentz transformation of the components
of an antisymmetric tensor in passing from one inertial reference frame to another, we obtain

\[ s_x = \cosh \alpha \cdot s_{0x}, \quad s_y = \cosh \alpha \cdot s_{0y}, \quad s_z = s_{0z}, \tag{51} \]

where the vector \( s_0 \) refers to the rest frame of the final nucleon and the vector \( s \) refers to our
chosen laboratory frame.

Consecutively replacing the operator \( B \) in formula (44) with the operators \( iI_{23}, iI_{31}, \) and
\( iI_{12} \), recalling their action on the canonical basis vectors, and taking (42), (45), (46), (19)–(21),
and (17) into account, we obtain

\[
\begin{align*}
    s_x &= N^2 \sum_{\lambda',m_1} \Re \left[ (M^\lambda_{1/2,m_1}^*) M^\lambda_{-1/2,m_1} \right] D(\alpha), \\
    s_y &= N^2 \sum_{\lambda',m_1} \Im \left[ (M^\lambda_{1/2,m_1}^*) M^\lambda_{-1/2,m_1} \right] D(\alpha), \\
    s_z &= \frac{N^2}{2} \sum_{\lambda',m_1} \left[ |M^\lambda_{1/2,m_1}|^2 - |M^\lambda_{-1/2,m_1}|^2 \right],
\end{align*}
\]

where

\[
D(\alpha) = D_0^{-1} \sum_{\tau',\tau} \sum_{l} \left( u_{\tau'l}^2(\alpha) \xi_{\tau'l} \right) (l + 1/2) u_{\tau'-l}^2(\alpha) \xi_{\tau-l}. \tag{53}\]

Calculations based on (52), (22), (41), (23), (27), (28), and (33)–(37) give the results

\[
\begin{align*}
    s_x &= 8N^2 \xi_{\parallel} \left( \frac{e^2}{q^2} \right)^2 \frac{\tau}{C^2 \sqrt{1 + \tau}} \cot \frac{\theta}{2} G_E G_M D(\alpha), \\
    s_y &= 0, \\
    s_z &= -4N^2 \xi_{\parallel} \left( \frac{e^2}{q^2} \right)^2 \left( \frac{E + E'}{MC^2} \right) \tau \sqrt{1 + \tau} G_M^2.
\end{align*}
\]

Using formulas (51) and expressing the polarization vector in terms of the rest spin of a fermion
(\( P = 2s_0 \)), we hence obtain

\[
\frac{G_E}{G_M} = \frac{P_x}{P_z} \cdot \frac{E + E'}{2M} \cdot \frac{\cosh \alpha}{D(\alpha) \tan \frac{\theta}{2}}. \tag{55}\]

We now prove that the equality

\[
D(\alpha) = \cosh \alpha \tag{56}\]

holds for any of the \( L_{+}^\uparrow \)-representations \( S_0 \) that we consider.

We introduce a four-vector operator \( L^\mu \) such that it has a single coupling to each irreducible
representation \((\pm 1/2, l_1)\) belonging to \( S_0 \), namely, to the representation \((\mp 1/2, l_1)\). We set all
the arbitrary constants assigned to the operator \( L^0 \) in the general case equal to unity. Then
(see formulas (I.20) and (I.21))

\[
L_0^\xi(\pm \frac{1}{2}, l_1)lm = (l + 1/2) \xi_{(\mp \frac{1}{2}, l_1)lm}. \tag{57}\]
We also introduce a polar four-vector $\zeta$ such that only its time component is nonzero in the chosen laboratory frame: $\zeta_0 = 1$, $\zeta_i = 0$, $i = 1, 2, 3$. In the rest frame of the final-state nucleon, passing to which corresponds to the element $g_0^{-1}$ of the $L^\dagger$ group, we have

$$\zeta' = \{\zeta'_0, \zeta'_1, \zeta'_2, \zeta'_3\} = \{\cosh \alpha, 0, 0, \sinh \alpha\}. \quad (58)$$

Turning to relations (53), (57), (25), (16), (20), and (21) with the relativistic invariance of the bilinear form and relations (I.13) and (58) taken into account, we obtain the chain of equalities

$$D(\alpha) = D_0^{-1} \sum_{l_0, l_1} \sum_{l_0 = -1/2}^{1/2} \sum_{l, t, t'} \left( u_{(0, 0', 0')}^{(l_0, l_t', l_t)} \xi_{(l_0, l_t')}^{(l_0, l_t')} r_{(0, 1)}^{(l_0, l_t', l_t)} \frac{1}{2} \right) =$$

$$= D_0^{-1} r \sum_{l_0, l_1} \sum_{l_0 = -1/2}^{1/2} \sum_{l, t, t'} \left( u_{(0, 0', 0')}^{(l_0, l_t', l_t)} \xi_{(l_0, l_t')}^{(l_0, l_t')} r_{(0, 1)}^{(l_0, l_t', l_t)} \frac{1}{2} \right) =$$

$$= D_0^{-1} r \sum_{l_0, l_1} \sum_{l_0 = -1/2}^{1/2} \sum_{l, t, t'} \left( u_{(0, 0', 0')}^{(l_0, l_t', l_t)} \xi_{(l_0, l_t')}^{(l_0, l_t')} r_{(0, 1)}^{(l_0, l_t', l_t)} \frac{1}{2} \right) =$$

$$= D_0^{-1} r (\psi_m(p), \mu \zeta_\mu \psi_m(p)) =$$

$$= D_0^{-1} r (\psi_m(p_0), S(g_0^{-1}) \mu \zeta_\mu S^{-1}(g_0^{-1}) \psi_m(p_0)) =$$

$$= D_0^{-1} r (\psi_m(p_0), \mu \zeta_\mu \psi_m(p_0)) =$$

$$= D_0^{-1} r (\psi_m(p_0), [\cosh \alpha \cdot L^0 + \sinh \alpha \cdot L^3] \psi_m(p_0)).$$

In addition, the next-to-last relation in (59) and formula (17), taken together with the invariance of the bilinear form under spatial reflections, relation (I.13) with $S(g) = P^{-1}$, and Eq. (58), imply that

$$D(\alpha) = D_0^{-1} r (P \psi_m(p_0), \mu \zeta_\mu P \psi_m(p_0)) =$$

$$= D_0^{-1} r (\psi_m(p_0), P^{-1} \mu \zeta_\mu P \psi_m(p_0)) =$$

$$= D_0^{-1} r (\psi_m(p_0), [L^0 \zeta'_0 - L^3 \zeta'_3] \psi_m(p_0)) =$$

$$= D_0^{-1} r (\psi_m(p_0), [\cosh \alpha \cdot L^0 - \sinh \alpha \cdot L^3] \psi_m(p_0)).$$

Comparing the final results in chains (59) and (60), we have

$$\psi_m(p_0), L^3 \psi_m(p_0) = 0. \quad (61)$$

Therefore, recalling (45), we obtain

$$D(\alpha) = \cosh \alpha \cdot D_0^{-1} r (\psi_m(p_0), L^0 \psi_m(p_0)) =$$

$$= \cosh \alpha \cdot D_0^{-1} \sum_{l_0, l_1} \sum_{l_0 = -1/2}^{1/2} \sum_{l} \left( u_{(0, 0', 0')}^{(l_0, l_t', l_t)} (0) \xi_{(l_0, l_t')}^{(l_0, l_t')} (0) r_{(0, 1)}^{(l_0, l_t', l_t)} \frac{1}{2} \right) =$$

$$= \cosh \alpha \cdot D_0^{-1} \sum_{l_0, l_1} \sum_{l_0 = -1/2}^{1/2} \sum_{l} \left( u_{(0, 0', 0')}^{(l_0, l_t', l_t)} (0) \xi_{(l_0, l_t')}^{(l_0, l_t')} (0) r_{(0, 1)}^{(l_0, l_t', l_t)} \frac{1}{2} \right) =$$

$$= \cosh \alpha \cdot D_0^{-1} (\psi_m(p_0), \psi_m(p_0)) = \cosh \alpha,$$

which was to be proved.
Therefore, the same formula (2) (obtained previously for Dirac nucleons) holds regardless of which non-Dirac representation $S_0$ describes the nucleon.

We consider the particular case of equality (56) where the representation $S_0$ is the direct sum $\tau^* \oplus \tau = (-1/2, 1/2+n) \oplus (1/2, 1/2+n)$ of two irreducible finite-dimensional representations of the proper Lorentz group, where $n$ is any positive integer. We can set [24]

$$a_{\tau \tau}(l) = (-1)^{l-1/2}.$$  \hspace{1cm} (63)

In accordance with (16) and (17), we fix the values of the components of the state vectors $\psi_m(p_0)$ as

$$u_{(\frac{1}{2}, \frac{1}{2}+n)}(0) = u_{(\frac{1}{2}, \frac{1}{2}+n)}\frac{1}{4}(0) = ru_{(-\frac{1}{2}, \frac{1}{2}+n)}\frac{1}{4}(0) = ru_{(-\frac{1}{2}, \frac{1}{2}+n)}\frac{1}{2}(0) = 1/\sqrt{2}. \hspace{1cm} (64)$$

Then $D_0 = r$, and relation (56) with (53), (45), (25), (63), (20), and (21) taken into account becomes

$$n^{-1/2} \sum_{l=1/2}^{n-1/2} (-1)^{l-1/2}(l + 1/2) \left[ |A_{(-\frac{1}{2}, \frac{1}{2}+n)}(\alpha)|^2 + |A_{\frac{1}{2}, \frac{1}{2}+n}(-\alpha)|^2 \right] = 2 \cosh \alpha. \hspace{1cm} (65)$$

We first discovered and verified this identity with $n = 1, 2, \ldots, 5$ for finite transformations of the proper Lorentz group directly, knowing the explicit form of the appropriate matrix elements.

5. Concluding remarks

The obtained results concerning the polarizations of a non-Dirac particle with the rest spin 1/2 are rather counterintuitive. Our original expectations were based on the fact that the state vector of such a particle, referred to the $L_{\perp}^+$-representation $S^{3/2}$ given by Eq. (5), for example, should have all half-integer spin values in the laboratory frame. It would seem that the average value of the spin projection onto any direction noncoincident with the particle momentum direction, also being the same in both the rest frame and the laboratory frame, should depend on both the particle velocity and the specific distribution of the components of its state vector (15) over the irreducible representations involved in $S^{3/2}$. These a priori statements are nullified by the remarkable relation (56), whose existence could not have been foreseen without a detailed analysis.

With this paper, we have fully clarified one aspect of the theoretical foundations of experiments on elastic scattering of electrons on protons by establishing that the freedom inherent in assigning one proper Lorentz group representation or another to the nucleon has no relation whatsoever to the contradiction encountered in the results of polarization and nonpolarization experiments, the results gleaned using the respective formulas (2) and (1).

Elsewhere, we may perhaps sketch a number of problems relating to another aspect of the theoretical foundations of polarization experiments [1]–[5]: the validity of the Bargmann–Michel–Telegdi equation for the spin rotation of a relativistic nucleon in its motion in a homogeneous magnetic field. The magnetic field between two targets is a necessary ingredient for measuring the longitudinal component of the recoil proton polarization because the secondary proton scattering itself allows extracting only the transverse polarization components. But because of the spin rotation, the longitudinal polarization component is manifested as a certain change in the transverse component $P_x$ (in the plane of the momenta of the recoil proton and electrons) and as the appearance of the $P_y$ component (in the direction perpendicular to the above plane), which would have been zero after the scattering of an electron (see the second equality in (54)).

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