Forward Displacement Analysis of a Non-plane Two Coupled Degree Nine-link Barranov Truss Based on the Hyper-chaotic Least Square Method

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Abstract The hyper-chaotic least square method for finding all of the real solutions of nonlinear equations was proposed and the following displacement analysis on the 33rd non-plane 2-coupled-degrees nine-link Barranov truss was completed. Four constrained equations were established by a vector method with complex numbers according to four loops of the mechanism, and four supplement equations were also established by increasing four variables and the relation of the sine and cosine functions. The established eight equations are those of the forward displacement analysis of the mechanism. In combining the least square method with hyper-chaotic sequences, a hyper-chaotic least square method based on utilizing a hyper-chaotic discrete system to obtain and locate initial points so as to find all the real solutions of the nonlinear questions was proposed. A numerical example was given. A comparison was also done with another means of finding a solution method. The results show that all of real solutions were quickly obtained, and it proves the correctness and validity of the proposed method.

Keywords Nine-link Barranov truss; forward displacement; hyper-chaotic sequences; least square method

1. Introduction

In the kinematic analysis and the innovative design of the plane mechanism, the planar basic kinematic chain is analyzed as an independent structure unit; in particular, the displacement analysis forms the most basic work. Yang [1] pointed out that the basic kinematic chain is a Barranov truss by definition. According to the numbers of a basic loop and topology, there are 33 species of basic kinematic chains which are from one to four loops, where these is a 9-link Barranov truss from the 6th to the 33rd species. The coupling coefficients of the Barranov truss
are, from the 6th to 29th species, 1, while the others are 2. It is difficult to research the displacement analysis of a 9-link Barranov truss; the more difficult work is involved in the displacement analysis of the 9-link Barranov truss, for which the coupling coefficient is 2. As a solution to this problem, the displacement analysis of the mechanism is usually summarized as the problem of solving simultaneous nonlinear equations. Broadly, there are two groups of algorithms: the numerical method and analytical method. The numerical method usually takes the homotopy continuation algorithm for obtaining the most solutions or all solutions, but the construction of the initial equations is very difficult, and the efficiency is low [1]. Hang [2] calculated the assembly configuration numbers of all 33 species of basic kinematic chains, whereas the results of the 25th and 31st types are imprecise. Typical analytical methods mainly include the Wu algorithm, the resultant elimination method, and the Groebner algorithm. The analytical method can obtain all of the solutions; however, the middle expansion items are too large, the computational time is long, and the multidimensional problems cannot be solved adequately. With the resultant elimination method it is easy to produce the extraneous roots, so a mass of techniques and experience are used for solving this problem, and the result should be verified by the numerical method [3-8].

The researched 9-link Barranov truss is the 33rd non-planar basic kinematic chain in [1], which has a symmetrical structure, and the variables of position-closed equations are 3 or 4, so the elimination is very difficult. If the Sylvester algorithm is directly applied in the elimination, the displacement analysis is not finished due to the variables of every equation being 2. If we adopt the Wu algorithm or the Groebner algorithm separately, the speed and memory of the computer is not sufficient. Wang Pin [9] solved this problem by using the Dixon method, and analyzed the reason of producing an extraneous root. Unfortunately, the computational process is very complicated. In conclusion, the problem of how to quickly acquire all of the real solutions is important in this field, and it is also one of the most basic problems of another 9-link Barranov truss.

Chaos marks one of the most important achievements of the 20th Century. The solution to the engineering problem by using chaos is a challenge for the modern theory of mechanisms. Luo Yonxin [10] proposed a method of solving 6-SPS within the range of real numbers. In this method, the concentrated points of Julia using the Newton Method appear in the neighbourhood unions of the Jacobian gram determinant (the value is 0) of the set of equations to be solved, but it is not proven and the process of solving the multivariable Jacobian gram determinant is very complicated. The chaotic sequence is a new method for obtaining all of the solutions for the real numbers of the mechanism by using the Newton iterative starting points of chaotic and hyper-chaotic systems [11-13]. When the Newton and the quasi-Newton method are not convergent, the mathematical programming method [14] is adopted. However, the computational efficiency of solving the 33rd Barranov truss using the hyper-chaotic system mathematical programming method is low. The Newton method makes higher demands for the initial value, while the least square method can expand the range of choosing the initial value. Furthermore, for a given initial value, the Newton method is not convergent; while the least square method may be convergent. Combining the Newton downhill method with hyper-chaotic sequences, a hyper-chaotic Newton-downhill method based on utilizing a hyper-chaotic discrete system to obtain and locate initial points in order to find all of the real solutions of the nonlinear questions was proposed [15]. In this paper, the hyper-chaotic sequence and the least square method is combined. The initial value is gained by adopting the hyper-chaotic Hénon map, which is applied in the least square method for solving the 33rd Barranov truss. The calculation example shows that the proposed method is correct and effective.

2. The Hénon hyper-chaotic system

The Lyapunov exponent is one of the most effective methods which describes the chaotic character of nonlinear systems. The numbers of the Lyapunov exponent are equal to the dimensions n of a system state space, and if there is a Lyapunov exponent larger than zero, the system is chaotic; if there are two Lyapunov exponents larger than zero, the system is hyper-chaotic. The more Lyapunov exponents there are, the more unstable the system will be [16]. Generally, the state variable number is more (e.g. a discrete high-dimensional system whose n is larger than 2), the probability of unstable state is higher.

There is a generalized Hénon map in [17]:

\[
\begin{align*}
    x_{i,k+1} &= a - x_{i-1,k}^2 - b x_{n,k} \\
    x_{i,k+1} &= x_{i-1,k}
\end{align*}
\]

(1)

Where \(i=2,3,...,n\), \(m\) is the dimension of system; \(k\) is the discrete time; and \(a\) and \(b\) are the adjustable parameters. When \(i=2\), the above map is a Hénon map. When \(a=1.76\) and \(b=0.1\), calculating the Lyapunov exponents of the systems whose dimensions form 2 to 10, we find the relationship between the numbers \(n\) of the Lyapunov exponents and the dimensions \(m\) of system is as follows: \(n_i = n + 1\), i.e. when the dimension of system is larger than 2, the system is hyper-chaotic [17]. When \(n > 10\), the simulation is conducted in [14], and the result is also \(n_i = n + 1\).
As illustrated in Fig. 1, when $n_i = 13$, twelve positive Lyapunov exponents are derived from the simulation experiment.

![Lyapunov exponent of Henon maps with n=13](image)

**Figure 1.** Lyapunov exponent of Henon maps with n=13

3. The least square method of nonlinear equations

There is a nonlinear equation:

$$f(x) = [f_1(x), ..., f_m(x)] = 0$$  \(\text{(2)}\)

Its solution is $x = [x_1, x_2, ..., x_n]^\top$, i.e. $J_i = \frac{\partial f(x_i)}{\partial x_i}$. The iteration method of the least square method is described as follows:

1. Choosing primarily $x_{0i}$;
2. Executing the iteration based on the formula (3):
   $$x_{i+1} = x_i - (J_i^T J_i)^{-1} J_i^T f(x_i)$$  \(\text{(3)}\)

Where, $f(x_i)$ is the value of $f(x)$ at the point $x_i$; $J_i$ is the Jacobi matrix of $f(x)$ at the point $x_i$. Generally, $J_i^T J_i$ is the symmetric positive semi-definite matrix and its inverse matrix $(J_i^T J_i)^{-1}$ is also existent, but the value of the determinant $\det(J_i^T J_i)$ is very small, the pathological phenomenon is serious.

In order to solve the above problem, many scholars proposed some improved algorithms; the most famous one is the damping least square method (L-M algorithm). Based on this algorithm, Zhan Chongxi presented a new method which is more effective than the L-M algorithm, and the convergent rate is faster. Adopting this method, the basic idea is as follows:

Let $A_i = J_i^T J_i$, the $A_i$ can be divided into $A_i = L_i D_i L_i^T$, where $L_i$ is the lower triangular matrix; $D_i$ is the diagonal matrix, so the formula (3) can be rewritten into $L_i D_i L_i^T (x_{i+1} - x_i) = -J_i^T f(x_i)$; then, the damping is placed on $D_i$, the formula (4) is derived as follows:

$$L_i (D_i + \mu I) L_i^T (x_{i+1} - x_i) = -J_i^T f(x_i)$$  \(\text{(4)}\)

Where, $I$ is an $n$ order identity matrix; $\mu_i > 0$ is the damping factor, whose selection algorithm and convergence iteration are described in [18].

4. The hyper-chaotic least square method of nonlinear equations

The procedure for solving the nonlinear equations set based on the hyper-chaotic least square method is as follows:

1. Constructing the chaotic set $x_i(i+1, j)$ ($i = 1, 2, ..., n$, $n + 1$ is the number of variables of a hyper-chaotic system, $n$ is the number of positive Lyapunov exponents, $j = 1, 2, ..., N$, $N$ is the size of the hyper-chaotic set) according to the formula (1);
2. Setting $x_i(0, j)$ as the initial value of the least square method, then, the formula (3) is executed three times in gaining all of the real solutions of the formula (2).

5. Mathematical model

Fig. 2 is the configuration of a 9-link Barranov truss. The length of every link is $l_1, l_2, ..., l_9$ respectively, the angle of any two links is $\alpha$, $\beta$, $\gamma$, and the positions of the fixed hinge are $B$, $C$, and $F$. Due to the topological structure of the Barranov truss, the links $KG$ and $IJ$ are intersectional, and the special non-planer 9-link Barranov truss has three types (the 28th, 29th and 33rd). Now, we need to calculate the angles $\theta$, $\theta_2$, $\theta_3$, $\theta_4$ and the positions of the other hinges.

![Structure of a nine-link Barranov truss](image)

**Figure 2.** Structure of a nine-link Barranov truss
Four vector equations are obtained according to the vector relationship of the figure:

\[
\begin{align*}
JH &= JF + FC + CD + DH \\
GK &= GC + CF + FL + LK \\
EI &= ED + DC + CF + FL + LI \\
BA &= BC + CD + DE + EA
\end{align*}
\] (5)

The above four equations are rewritten in the complex exponential form:

\[
\begin{align*}
JH &= -l_4e^{i\theta} + FC + l_1e^{i\alpha_1} + l_1e^{i\theta_1} + l_2e^{i\alpha_2} \\
GK &= -l_4e^{i\alpha_1} - FC + l_1e^{i\theta_1} + l_2e^{i\theta_2} + l_3e^{i\alpha_3} \\
EI &= -l_4e^{i\theta_2} - l_2e^{i\theta_1} - FC + l_1e^{i\theta_3} + l_3e^{i\theta_4} \\
BA &= BC + l_1e^{i\theta_3} + l_2e^{i\theta_2} + l_2e^{i\theta_1} + l_3e^{i\alpha_4}
\end{align*}
\] (6)

The polynomial expression (7) which has four unknown variables \( \theta_1, \theta_2, \theta_3, \theta_4 \) is obtained by making \( JH = l_4, GK = l_1, EI = l_3, BA = l_5 \) substituted into the formula (6).

\[
\begin{align*}
f_1(\theta_1, \theta_2, \theta_3, \theta_4) &= 0 \\
f_2(\theta_2, \theta_3, \theta_1, \theta_4) &= 0 \\
f_3(\theta_3, \theta_1, \theta_2, \theta_4) &= 0 \\
f_4(\theta_4, \theta_1, \theta_2, \theta_3) &= 0
\end{align*}
\] (7)

Let \( x_i = \sin \theta_i, x_i = \cos \theta_i \). Then the polynomial expression (7) is converted into the formula (8):

\[
\begin{align*}
f_1(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) &= 0 \\
f_2(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) &= 0 \\
f_3(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) &= 0 \\
f_4(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) &= 0
\end{align*}
\] (8)

According to the mathematical property \( \sin^2 \theta + \cos^2 \theta = 1 \), there are four equations as follows:

\[
x_{2i-1}^2 + x_{2i}^2 = 1 (i = 1, 2, 3, 4)
\] (9)

The formulas (8) and (9) constitute an equation set which has the solution \( x = [x_1, x_2, \cdots, x_8]^T \), then when calculating the \( \theta_1, \theta_2, \theta_3, \theta_4 \) the positions of every link are derived from the vector relationship.

Putting the following half-angle formulas

\[
y_i = \tan \frac{\theta_i}{2}, \sin \theta_i = \frac{2y_i}{1+y_i^2}, \cos \theta_i = \frac{1-y_i^2}{1+y_i^2} (i = 1, 2, 3, 4)
\]

into the formula (7), the formula (10) is gained.

\[
\begin{align*}
F_1(y_1, y_2, y_3, y_4) &= 0 \\
F_2(y_1, y_2, y_3, y_4) &= 0 \\
F_3(y_1, y_2, y_3, y_4) &= 0 \\
F_4(y_1, y_2, y_3, y_4) &= 0
\end{align*}
\] (9)

If \( y_1, y_2, y_3, y_4 \) is obtained, \( \theta_1, \theta_2, \theta_3, \theta_4 \) can be gained, and then the positions of every link are derived from the vector relationship.

6. Calculation experiment

As illustrated in Fig. 2, let the positions of the fixed hinge be \( B(7, -211/3), C(24, -32), F = (5, -61) \) the length of the links be \( l_1 = 40, l_4 = 40, l_3 = 15, l_5 = 26 \), \( l_{10} = 15, l_{11} = 2\sqrt{135}, l_{12} = 20, l_{13} = \sqrt{661}, l_{14} = 5\sqrt{6610}/3 \), \( l_{15} = \sqrt{6610}/3, l_{16} = 7\sqrt{65}, l_{17} = 25, l_{18} = 25 \), and let the angles \( \alpha_1, \alpha_2, \alpha_3, \alpha_4 \) be 0.5325, 0.2838, 0.9273, 0.9273 and 0.6435 respectively. The starting points of hyper-chaotic set are produced by random numbers, the positive Lyapunov exponent hyper-chaotic sequence with \( n = 8 \) is derived from the hyper-chaotic variable with \( n_t = 9 \), which are gained by the generalized Hénon hyper-chaos of formula (1). Where the length is \( N = 100 \), if the length of a hyper-chaotic sequence \( N \) is too large, the computational time is long; conversely, if the length of a hyper-chaotic sequence \( N \) is too small, not all of the real solutions will be obtained. Therefore, for this kind of problem, a small value is used in trial, and then a larger value is used. Generally, the value is equal to \( (8-12) \times \) the numbers of variables. As can be seen in Table 1, the solution of eqs.(7) and (8) \( x = [x_1, x_2, \cdots, x_8]^T \) is obtained by the iteration with a time of 51s, and then the solution is converted into \( \theta_1, \theta_2, \theta_3, \theta_4 \) in Table 2, and the fourteen configurations of 9-link Barranov truss is similar to Fig.2, with different \( \theta_1, \theta_2, \theta_3, \theta_4 \). We find that the result is the same as with [7] and [15]. If the hyper-chaotic Newton method is utilized in [11], and the hyper-chaotic sequence is derived from the generalized Hénon hyper-chaos of formula (1), only 13 solutions are gained and with a longer executing time of 6950s due to the partial iteration divergence or matrix singularity. In addition, the numbers for the solution and the executing time are variable every time it turns on. If the length of the hyper-chaotic sequence is increased, all of the solutions may be gained, but the computational time is much longer. While the mathematical programming method in [14] is used, 14
solutions can be derived from the fmincon function with an executing time 8451s; if all of the solutions are gained, a longer time is required. If the formula (10) is used, not all of the solutions can be derived from any method, because, *inter alia*, of the matrix singularity and the iteration divergence, and the executing time is longer than other methods, with the acquired solutions containing extraneous roots. As such, in order to avoid the formula singularity, the equations set should not be established by the universal substitution of the half angle formula when solving this mechanism.

| NO | $x_1$  | $x_2$  | $x_3$  | $x_4$  |
|----|--------|--------|--------|--------|
| 1  | 0.4536 | -0.8912| -0.7684| 0.6399 |
| 2  | 0.8086 | 0.5884 | 0.8708 | -0.4917|
| 3  | 0.7574 | -0.6530| 0.7788 | -0.6273|
| 4  | -0.0199| -0.9998| 0.9189 | -0.3944|
| 5  | -0.0574| 0.9983 | -0.6707| -0.7417|
| 6  | 0.4708 | -0.8822| 0.7819 | -0.6234|
| 7  | -0.1489| 0.9889 | 0.3483 | -0.9374|
| 8  | -0.8421| 0.5393 | -0.8772| -0.4800|
| 9  | 0.4028 | 0.9153 | 0.9956 | 0.0932 |
| 10 | 0.8946 | 0.4469 | -0.6758| 0.7370 |
| 11 | 0.9343 | 0.3566 | 0.9824 | 0.1865 |
| 12 | 0.0301 | 0.9995 | 0.3159 | 0.9488 |
| 13 | 0.3080 | -0.9514| 0.8451 | 0.5346 |
| 14 | -0.6697| 0.7426 | 0.9975 | -0.0701|

| NO | $x_5$  | $x_6$  | $x_7$  | $x_8$  |
|----|--------|--------|--------|--------|
| 1  | 0.2407 | 0.9706 | -0.8992| 0.4376 |
| 2  | 0.1457 | 0.9893 | 0.9365 | 0.3507 |
| 3  | -0.7219| 0.6920 | -0.2374| 0.9714 |
| 4  | -0.4047| -0.9145| -0.8357| -0.5493|
| 5  | 0.8594 | -0.5113| 0.9739 | 0.2269 |
| 6  | 0.9983 | 0.0576 | 0.6880 | -0.7257|
| 7  | 0.9993 | -0.0384| 0.9749 | 0.2228 |
| 8  | -0.7861| -0.6180| -0.8269| -0.5623|
| 9  | 0.0590 | 0.9983 | 0.9565 | 0.2918 |
| 10 | 0.3933 | -0.9194| 0.8166 | -0.5772|
| 11 | 0.9343 | 0.3566 | 0.8933 | -0.4494|
| 12 | -0.9874| -0.1581| -0.2726| 0.9621 |
| 13 | 0.2475 | 0.9689 | 0.5130 | -0.8584|
| 14 | -0.8808| -0.5977| 0.7255 | -0.6882|

Table 1. The computing results of the variables

### Table 2. The computing results of angle variables

| NO | $\theta_1$  | $\theta_2$  | $\theta_3$  | $\theta_4$  |
|----|-------------|-------------|-------------|-------------|
| 1  | 156.3458    | -73.7573    | 98.3196     | 159.4094    |
| 2  | 147.3418    | -49.2588    | 12.8164     | 121.9834    |
| 3  | 138.2602    | -31.0945    | -20.5792    | 75.1222     |
| 4  | 126.8699    | -143.1301   | 126.8699    | 90.0015     |
| 5  | 107.9217    | -151.5386   | 71.8525     | 96.4338     |
| 6  | 107.2288    | -92.7704    | 150.8555    | 45.2570     |
| 7  | -13.5675    | -146.5848   | 66.8780     | -33.5839    |
| 8  | -25.2822    | -148.0106   | 33.0251     | -49.9194    |
| 9  | -28.7645    | 115.3412    | 1.5132      | 14.9492     |
| 10 | -44.0398    | 116.9741    | 71.2226     | -11.2244    |
| 11 | -116.2190   | 45.9586     | 91.5228     | -14.7186    |
| 12 | -148.5141   | -22.8839    | 43.1758     | -42.2220    |
| 13 | -149.8547   | 108.8990    | 69.2612     | 101.7434    |
| 14 | -160.6921   | -32.2923    | -43.7392    | -13.6843    |

7. Conclusions

The four constraint equations of the 9-link Barranov truss of the 33rd non-planer dual coupling are established by the combination of the vector method and the complex method, with four additional equations gained by increasing four variables based on the trigonometric function, and then the equations set of the mechanism which has 8 variables is obtained. The hyper-chaotic sequence which is produced by the hyper-chaotic system is regarded as the initial value of the least square method, a method of solving all of the real solutions of the nonlinear equations set which is called the least square method based on the hyper-chaotic sequence. The correct position of the 33rd 9-link Barranov truss is calculated, and the procedure is given. The method researched overcomes the defect of non-convergence and low computational efficiency using the Newton iteration method, the quasi-Newton method, and the hyper-chaotic mathematical programming method.

The example calculation shows that the proposed method is correct and effective, and the efficiency of solving real solutions is high and the result is the same as the similar method. The new idea presented can solve the position of the Barranov truss, the Assur group and a partial parallel mechanism. The position analysis of the Barranov truss affords a good foundation for Kinematics and dynamics.

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