The inhomogeneous quark condensate in compressed skyrmion matter

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The inhomogeneous quark condensate, responsible for the dynamical chiral symmetry breaking in the cold nuclear matter, is studied by putting skyrmions onto the face-centered cubic crystal and treating the skyrmion matter as a nuclear matter. By varying the crystal size, we explore the effect of density on the local structure of the quark-antiquark condensate. By endowing the light vector mesons ρ and ω with hidden local symmetry and incorporating a scalar meson as a dilaton of spontaneously broken scale symmetry, we uncover the intricate interplay of heavy mesons in the local structure of quark condensate in dense baryonic matter described in terms of skyrmion crystal. It is found that the inhomogeneous quark density persists to as high a density as ~ 4 times nuclear matter density. The difference between the result from the present approach and that from the chiral density wave ansatz is also discussed.

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While the observation of the Higgs boson is considered to account for the masses of “elementary constituents” of visible matter, i.e., quarks and leptons, the bulk of the mass of the proton, the constituents of which are three light quarks, i.e., two up quarks and one down quark, remains more or less unexplained. The quark masses account for only ~ 1% of the proton mass. This is in marked contrast to the next scale hadron, nucleus. The mass of a nucleus of mass number A is given almost entirely, say, ~ 98%, by the sum of the masses of A nucleons. It is generally accepted, although not proven rigorously, that the nucleon mass arises from the non-perturbative dynamics of strong interactions encoded in QCD. What is involved is quark confinement and chiral symmetry spontaneously broken by the vacuum. It is not established but generally believed that the two are related. It is easier to address the latter in effective theories, so we will focus on it in what follows.

In QCD, the order parameter of the chiral symmetry breaking is the pion decay constant$f_π$ and, in terms of the fundamental QCD quantities, the quark condensates, typically the two-quark condensate$⟨\bar{q}q⟩$. The two-quark condensate in cold/hot dense matter has been studied for a long time. In Refs. [1, 2] it was found that in the neutron matter at the neutron star density, both the$σ$(the isoscalar component of the chiral four-vector) and pion condensates exist and because of the chiral invariance of the system, these two condensates are linked to each other through a chiral rotation. Later, the approach put forward in Ref. [1] was investigated in more detail and it was found that both condensates,$σ$ and pion, could depend on the position, that is, the condensates are inhomogeneous [3]. In a phenomenological model with nucleons as explicit degrees of freedom which are put on a crystal lattice, Pandharipande and Smith found that in solid neutron matter, given an enhanced attractive tensor force, which is stronger than that given by standard one-pion exchange, the inhomogeneous pion condensate could take place [4]. Since then, the inhomogeneity of these condensates, now termed as chiral density wave (CDW), has been extensively studied in various different effective theory approaches modelling low-energy QCD, see e.g., Refs. [5, 6] and references therein.

In this article, continuing our effort to understand hadronic matter at high density, we approach this problem by putting skyrmions, representing nucleons, on a face-centered cubic (FCC) crystal lattice. This has the advantage of enabling one to describe, on the same footing, both the basic structure of the nucleon and the properties of nuclear matter, thereby probing the modification of the vacuum in which the nucleons are propagating. By squeezing the crystal lattice, density effect is simulated and the properties of the condensates are modified. The model we will employ is the “generalized” skyrmion model that incorporates, in addition to the pion field carrying topology, the vector mesons$ρ$ and $ω$ introduced as hidden local symmetry fields and the dilaton$χ$ associated with the broken scale symmetry of QCD. We shall call this “dHLS skyrmion” to be distinguished from the Skyrme model with pion field only. It turns out that each of these fields plays an essential role in the structure of the nucleon as well as dense matter. In this article, we shall study the matter density effect and hadron resonance effect on the quark-antiquark condensates at low and high density of nuclear matter. The details of what we obtained before are involved and given elsewhere [7, 8] to which we refer the readers. We summarize the key points that are relevant to what we need for the discus-
1. There is a topology change from skyrmions to half-skyrmions at a density $n_{1/2}$ with or without non-pion fields. It might involve no symmetry change of QCD and hence no order parameter like what happens in condensed matter physics \[9\]. The location of $n_{1/2}$ depends, however, on the presence of the non-pion fields. Phenomenologically, it lies at $n_{1/2} \sim 2n_0$ where $n_0$ is the normal nuclear matter density.

2. The quark-antiquark condensate $\langle \bar{q}q \rangle \neq 0$ for $n < n_{1/2}$ as is appropriate for Nambu-Goldstone phase. At $n_{1/2}$ the two-quark condensate on average in a unit cell vanishes, so $\langle \bar{q}q \rangle = 0$ for $n \geq n_{1/2}$ (here and in what follows $\langle \rangle$ stands for the space average in a unit crystal cell). However the pion decay constant does not vanish, with massive hadrons excited, until a higher density $n_c > n_{1/2}$ with $n_c$ being the density at which the chiral symmetry is restored. What happens is that the two-quark condensate is not locally zero but vanishes on average in the unit cell. Thus chiral symmetry is not actually restored. This means that the two-quark condensate is not an order parameter for chiral symmetry in this model. There must be different order parameters for chiral symmetry, perhaps of multi-quark nature, with the Gell-Mann-Oakes-Renner relation still holding in the phase with possible multi-quark condensates as order parameter. The locally non-vanishing two-quark condensate immediately suggests the existence of an inhomogeneity in the half-skyrmion phase.

3. The nucleon mass can be characterized by two components as $m_N = m_0 + M(\langle \bar{q}q \rangle)$ where $M(\langle \bar{q}q \rangle) \to 0$ as $\langle \bar{q}q \rangle \to 0$ and $m_0 \neq 0$. Here $m_0$ resembles the chiral invariant mass term in the parity-doubled nucleon model \[6\]. Furthermore as will be mentioned later, parity doubling does take place in hadron spectrum in cold/hot dense medium. However in the dHLS model, the $m_0$ term which is related to the chiral doubling structure of nucleons does not figure explicitly in the Lagrangian, so it most likely reflects an emerging symmetry induced by the medium.

4. There is an intricate interplay between the isovector vector meson $\rho$, the isoscalar vector meson $\omega$ and the scalar (dilaton) meson in dense baryonic matter. Suppose one generalizes the model used here by putting the infinite tower of isovector vector mesons $\rho, \rho'$ etc and $\sigma_1, \sigma'_1$ etc. in the skyrmion model while dropping all isoscalar vector mesons and scalar mesons. Then one can arrive in flat space at many-nucleon systems – and infinite matter – that are described by BPS skyrmions with vanishing binding energies \[10\]. This is close to what is observed in experiments for binding energies of medium and heavy nuclei. What one notes is that each member of the tower of the isovector mesons tends to move the system closer to a matter of BPS skyrmions. In the model used in this article, however, this tendency with the $\rho$ is spoiled by the isoscalar vector meson $\omega$ \[10\].

In an effective model of QCD with linear chiral symmetry, the chiral four-vectors in the meson fields $M$ are decomposed as

$$M(x) = \sigma(x) + i\tau_a \pi_a(x), \quad \text{with} \quad a = 1, 2, 3,$$

with $\tau^a$ as Pauli matrices. In the Nambu-Goldstone phase the chiral four-vectors are constrained by $\langle \sigma^2 + \pi^2 \rangle_{\text{QCD}} = f_\pi^2$ (here and in what follows $\langle \rangle_{\text{QCD}}$ stands for the medium-free QCD vacuum) in the QCD vacuum. Moreover, due to the parity invariance of the QCD vacuum, one normally has $\langle \sigma \rangle_{\text{QCD}} = f_\pi \propto \langle \bar{q}q \rangle_{\text{QCD}}$ and $\langle \pi_a \rangle_{\text{QCD}} \propto \langle \bar{q}i\gamma_5 q \rangle_{\text{QCD}} = 0$. However, in medium, due to the interaction between the mesons and baryonic matter, both $\langle \sigma \rangle$ and $\langle \pi_a \rangle$ could be nonzero and position dependent \[1, 6\].

In the nonlinear realization of chiral symmetry, the two-quark condensate is normalized as $\langle U \rangle_{\text{QCD}} = \langle M \rangle_{\text{QCD}}/f_\pi = 1$ in the QCD vacuum with $U = \exp(i\pi/2\pi)$. However, in the nuclear matter medium, there can be deviation due to the interaction between the pion and the matter medium and the value of the deviation from 1 accounts for the deviation of the two-quark condensate from its vacuum value.

To explore the density dependence of the quark-antiquark condensate in a theory with the nonlinear realization of chiral symmetry, it is convenient to decompose the field $U(x)$ as in \[10\],

$$U(x) = \phi_0(x) + i\tau_a \phi_a(x), \quad \text{with} \quad a = 1, 2, 3.$$ 

In the ground state of the system, $\phi_0(x)$ accounts for the isoscalar quark condensate while $\phi_a(x)$ accounts for the isovector quark condensate which, in the QCD vacuum, have values $\phi_0(x) = 1$ and $\phi_a(x) = 0$ but in the nuclear matter medium, might have $\phi_0(x) \neq 1$ and $\phi_a(x) \neq 0$ due to the matter effect. And, because of the cluster structure of the nuclear matter, the quark condensates are expected to be functions of space coordinates $x$, i.e. inhomogeneous.

In the present work, to simulate the nuclear matter environment, among all the approaches to the nuclear matter, we adopt the skyrmion matter approach and regard

\[ \text{[1]} \]

\[ \text{[6]} \]

\[ \text{[10]} \]
the skyrmion matter as nuclear matter \[14, 15\]. Since in each crystal cell, the baryon number is fixed, the density effect is simulated by changing the crystal size. Here, we will adopt the face-centered cubic (FCC) crystal \[12, 17\] for which the nuclear matter density \( n \) and crystal size \( L \) is related through relation \( n = 4/(2L)^3 \). As stated at the beginning, in the skyrmion crystal approach, one can treat both the nuclear matter and medium modified hadron properties in a unified way \[18\].

To simulate nuclear matter from FCC crystal, it is convenient to define the unnormalized quantities \( \bar{\phi}_0, \bar{\phi}_1, \bar{\phi}_2, \bar{\phi}_3 \) for which the nuclear matter density \( n \) and crystal size \( L \) is related through relation \( n = 4/(2L)^3 \). As stated at the beginning, in the skyrmion crystal approach, one can treat both the nuclear matter and medium modified hadron properties in a unified way \[18\].

In the crystal lattice, the unnormalized fields have the Fourier series expansions as

\[
\begin{align*}
\bar{\phi}_0(x, y, z) &= \sum_{a,b,c} \beta_{abc} \cos(a \pi x/L) \cos(b \pi y/L) \cos(c \pi z/L), \\
\bar{\phi}_1(x, y, z) &= \sum_{h,k,l} \alpha_{hkl} \sin(h \pi x/L) \cos(k \pi y/L) \cos(l \pi z/L), \\
\bar{\phi}_2(x, y, z) &= \sum_{h,k,l} \alpha_{hkl} \cos(l \pi x/L) \sin(h \pi y/L) \cos(k \pi z/L), \\
\bar{\phi}_3(x, y, z) &= \sum_{h,k,l} \alpha_{hkl} \cos(k \pi x/L) \cos(l \pi y/L) \sin(h \pi z/L).
\end{align*}
\]

By varying the Fourier coefficients \( \alpha \) and \( \beta \), one obtains the minimal energy, interpreted as the ground state of the nuclear matter, for a specified crystal size \( L \). The inhomogeneous quark condensates with respect to their values in vacuum can then be calculated by substituting the values of \( \alpha \) and \( \beta \) so determined into Eq. \( 3 \). In addition, by varying the crystal size \( L \), the density effect on the quark condensate can be accessed. Note that due to the FCC structure and the arrangement of the nearest neighbor skyrmions to yield the strongest attractive interaction, the modes appearing in the above equation are not independent \[18\].

To explore the \( \pi, \rho \) and \( \omega \) meson effect on the inhomogeneous quark condensate, we use the skyrmion model gotten from the hidden local symmetry approach given up to the next-to-leading order in chiral expansion including the homogeneous Wess-Zumino term. The degrees of freedom involved are then the pion, the lowest-lying \( \rho \) and \( \omega \) as employed in \[7, 20\] to explore the structure of single baryon as well as baryonic matter. Explicitly, we will consider the models HLS(\( \pi \), HLS(\( \pi, \rho \)) and HLS(\( \pi, \rho, \omega \)) which are explicitly defined in Refs. \[7, 20\]. In addition to the hidden local symmetry degrees of freedom, to explore the light scalar meson (i.e., dilaton) effect on nuclear matter, we apply the model dHLS-II(\( \pi, \rho, \omega \)) defined in Ref. \[8\] in which the chiral restoration can be realized at some high density \( n_c \). For simplicity, we will not present the explicit forms of the model but refer to Refs. \[7, 8, 20\] for details.

From Eq. \( 3 \) we see that, in the skyrmion crystal approach, all the quark condensates are functions of the three-dimension coordinates. In Fig. \( 1 \) we illustrate the crystal size dependence of the quark-antiquark condensates \( \bar{\phi}_0(x, z) \propto \langle \bar{q}q(x, z) \rangle^* \) and \( \bar{\phi}_1(x, z) \propto \langle \bar{q} \tau^a \gamma_5 q(x, z) \rangle^* \) at \( z = 0 \) plane using the model HLS(\( \pi, \rho, \omega \)) in which all the \( \pi, \rho \) and \( \omega \) meson contributions are included and a self-consistent power counting mechanism exists. These plots tell us that in both the skyrmion and half-skyrmion phase, the chiral symmetry is locally broken since the quark condensates could be locally nonzero.

Let us first look at the first row of Fig. \( 1 \) from Fig. \( 1 \( a \) ) one sees that the large area is occupied by white color so that at this density the space average of \( \bar{\phi}_0 \) is approximately its value in the QCD vacuum, i.e., \( \sim 1 \) which agrees with the result of Ref. \[2\]. By comparing Fig. \( 1 \( a \) ) and Fig. \( 1 \( b \) ), we see that a larger space is occupied by negative values of \( \bar{\phi}_0 \), indicating that the space average of \( \bar{\phi}_0 \) becomes smaller as density increases. The comparison of Fig. \( 1 \( b \) ) – which is in skyrmion phase – and Fig. \( 1 \( c \) ) – which is in half-skyrmion phase – indicates that the changeover from the skyrmion phase to the half-skyrmion phase could be smooth. Finally, the close resemblance of Figs. \( 1 \( c \) ) and \( (d) \) reflects the fact that even in the half-skyrmion phase the chiral symmetry is still locally broken, though the condensate is zero averaged in the unit cell and the order parameter for chiral symmetry breaking in medium \( f_\pi^* \) is nearly constant independent of density.

Next we consider the second row of Fig. \( 1 \) which describes the density dependence of the inhomogeneous parity-odd isospin nonsinglet quark condensate \( \langle \bar{q} \tau^a \gamma_5 q \rangle^* \). We first find that this inhomogeneous condensate is an odd function of the coordinate \( x \) because in the skyrmion crystal approach the flipping of the coordinate \( x \) is accompanied by the flipping of the corresponding component of \( \bar{\phi}_i \) and, because of this property, we conclude the space average of the quantity \( \bar{\phi}_i \) is zero. The figures in this row also shows the density dependence of the component \( \bar{\phi}_1 \) in the skyrmion phase and its nearly density independence in the half-skyrmion phase.

We plot in Fig. \( 2 \) the effect of the \( \omega \) meson and the scalar (dilaton) meson on the quark condensates in the \( x-y \)-plane. In this plot, we take the typical crystal size \( L = 1.5 \text{ fm} \) at which the system is in the skyrmion phase for all four models. Compared to the model with pion only, HLS(\( \pi \)), we find that the plot from HLS(\( \pi, \rho \)) is occupied by less blue (soliton matter) space which indicates that the quark condensates appear in a smaller area due to the attractive force from the \( \rho \) meson. Similarly, the subtle difference between Figs. \( 2 \( c \) ) and \( (d) \) tells us that the inclusion of the dilaton field associated with the dynamical breaking of the scale symmetry slightly shrinks the area of the inhomogeneous quark condensates due to attractive force from dilaton. However, the dramatic difference between Figs. \( 2 \( b \) ) and \( (c) \) results from HLS(\( \pi, \rho \)) and HLS(\( \pi, \rho, \omega \)) shows the crucial effect on the quark condensates from the \( \omega \) meson. The repulsive force com-
ing from the $\omega$ exchange causes the inhomogeneous quark condensates to expand. This may explain the deviation from the BPS structure caused by the presence of the $\omega$ meson noted in the item $[\text{III}]$.

As mentioned, in our model, the chiral condensate is locally broken in both the skyrmion phase and half-skyrmion phase although in the latter phase it is restored globally. There is a basic difference between this skyrmion prediction and the CDW phase discussed in Ref. $[\text{II}]$. First of all, in $[\text{II}]$, the quark-antiquark condensate accounted for the isoscalar tetraquark state or a pion-pion resonance, whereas our dilaton is $f_0(500)$ which can be associated with a pseudo-Golstone boson in the vicinity of a QCD IR fixed point $[\text{II}1]$. In our scheme, there is no low-lying two-quark configuration fluctuation at low density corresponding to the fourth component of the chiral four vector in the linear realization of chiral symmetry.

It is however interesting to make a comparison between the two models. To do so, we take the Dautry-Nyman ansatz $[\text{II}]$ as done in $[\text{II}]$

$$\phi_0(x) = \cos(2fx); \quad \phi_3 = \sin(2fx).$$

In this form, the parameter $f$ is an indicator for the inhomogeneous quark condensates. It was found in $[\text{II}]$ that, at low chemical potential, $f = 0$, i.e., there are no inhomogeneous quark condensates, but at high chemical potential, $f$ becomes a nearly non-zero constant which indicates the onset of the inhomogeneous quark condensates. In terms of the chiral field $U$, the indicator $f$ in $[\text{II}]$ is simply

$$f = \frac{i}{4} \text{Tr} \left( \tau_3 U_0^\dagger \partial_x U_0 \right).$$

However in the skyrmion crystal approach, the situation becomes somewhat complicated because of both the three-space dependence and the three-isospin dependence locked into hedgehog. Here the relevant quantity is

$$f_{ij}(x, y, z) = \frac{i}{2} \text{Tr} \left( \tau_i U_0^\dagger \partial_j U_0 \right) = \frac{1}{2} \left[ \phi_0 \partial_j \phi_i - \phi_i \partial_j \phi_0 + \epsilon_{iab} \phi_a \partial_j \phi_b \right],$$

which is a three-by-three matrix in the spin-isospin space and each matrix element depends on the coordinates $x, y,$ and $z$. An explicit numerical simulation shows that in the FCC crystal, the quantities $f_{ij}(x, y, z)$ vanishes in neither skyrmion phase nor half-skyrmion phase so that the inhomogeneous quark condensate exists locally in both the skyrmion phase and half-skyrmion phase. The inhomogeneity with $f \neq 0$ in the skyrmion phase could be due to the fact that at low density the classical skyrmion configuration in crystalline form is not in Fermi liquid state. The homogeneity may be recovered by collective quantization to restore spin and isospin. The half-skyrmion phase at $n > n_{1/2}$, however, is likely non-Fermi liquid in a crystalline form.

In the CDW calculation studied in Ref. $[\text{II}]$, the indicator of the homogeneous quark condensate is the quantity $\phi$ defined there related to the pion decay constant via $f_\pi = \phi/Z$ with $Z$ being wave function renormalization constant of the pseudoscalar mesons arising from the removing of the $\pi$-a$_1$ mixing terms. It was found that in the low chemical potential region, the homogeneous quark condensate $\phi$ decreases slightly with the increase of chemical potential while in the high chemical potential region it stays as a constant with about 25% of the vacuum. However, in the FCC skyrmion crystal approach, in all the models considered in the present work, the homogeneous quark condensate accounted for

\[\text{FIG. 1. (Color online) The density effect on } \sigma(x, y, 0) \text{ (1st row) and } \pi_1(x, y, 0) \text{ (2nd row). The half-skyrmion phase appears at } L = 1.45 \text{ fm.}\]
FIG. 2. (Color online) The $\rho$, $\omega$ and $\chi$ meson effects on $\sigma(x, y, 0)$ (1st row) and $\pi_1(x, y, 0)$ (2nd row) in the skyrmion phase at $L = 1.5$ fm.

by the space averaged $\sigma$ decreases with increasing density in the skyrmion phase and vanishes in the half-skyrmion phase $[7, 8]$. Although the inhomogeneity in the quark-antiquark condensate persists in both the skyrmion phase and the half-skyrmion phase with no obvious phase transitions, there is a drastic change in nuclear physics across the density $n_{1/2}$ $[22]$. As mentioned, there seems to emerge the parity-doubling in hadronic spectrum suggested in the parity-doubling effective Lagrangian model $[23]$ and lattice studies $[21]$. Indeed, it was found very recently in Ref. $[25]$ that when the heavy-light mesons with chiral partner structure coupled to the HLS($\pi$) up to one-derivative terms are put on the crystal, their masses, split in the skyrmion matter, become degenerate in the half-skyrmion phase. It would be interesting to study similar phenomena in the light meson sector.

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