Determining of the characteristics of nonlinear resistive-capacitive and resistive-inductive elements

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Abstract. We present a method for finding the dependency of resistance and inductance on the current for the series connection of resistive and inductive nonlinear elements and finding the dependency of resistance and capacitance on the applied voltage for the parallel connection of resistive and capacitive nonlinear elements at a constant frequency. The initial data are current and voltage oscillograms measured for different moments of time. Examples of determination of the specified dependencies are given, estimates of the absolute and relative error of the approach are given.

1. Introduction

Building a mathematical model of active-reactive nonlinear elements from experimental data is a complex and unsolved problem of theoretical electrical engineering. General approaches to its solution based, for example, on Volterra–Picard series or polynomials [1–4], are very complex and work effectively for sufficiently smooth nonlinearities. At the same time, modelling of processes in circuits with unloaded transformers and chokes with saturation requires calculation of losses. Such calculations require that the active (generally nonlinear) component be extracted. A similar situation arises in the modelling of processes in ferroelectrics used in the production of modern ferroelectric capacitors. Here the greatest interest is the dependency of the capacitance on the voltage [5–9]. In both applications, the measurement of the current nonlinearity in the operating mode is also of interest.

The proposed approach is focused on the determination of the current-voltage characteristics (IVC) of a nonlinear device when it operates in a steady mode. In this case, the measurement of voltage and current (for example, using a current transformer) can be done during operation.

2. The method of determining the IVC

Consider a nonlinear resistive-inductive element. Its mathematical model is a series connection of nonlinear \( R=R(i) \) and \( L=L(i) \). The problem is to find the dependencies \( R=R(i) \) and \( L=L(i) \) during the experiment.

Let us connect the test element to an AC voltage source \( u(t) = U_m \sin \omega t \) and obtain the current and voltage oscillograms on the period \( T = \frac{2\pi}{\omega} = 0.02 \) s. Using a piecewise polynomial approximation, we obtain analytical dependences \( u(t) \) and \( i(t) \).

The mathematical model of the device discussed has the form:
\[ u(t) = R(i) i(t) + L(i) \frac{di(t)}{dt} \]  \hspace{1cm} (1)

Suppose that it is possible to divide the period \( T \) into two segments as follows: on one of them the function \( i=i(t) \) increases monotonously, and on the other—decreases monotonously. This will allow us to go from dependencies \( i=i(t) \) and \( u=u(t) \) to \( t_1=t_1(i) \), \( t_2=t_2(i) \), and, accordingly, \( u_1=u_1(i) \), \( u_2=u_2(i) \). Thus, for each value \( i \), in the interval \([ -I_m, I_m] \), there is a pair of voltage values \( u_1 \) and \( u_2 \) and a pair of time values (for this period) \( t_1 \) and \( t_2 \). Then from (1) we get:

\[
\begin{align*}
  u_1(i) &= R(i) + \frac{L(i)}{dt_1(i)/di}, \\
  u_2(i) &= R(i) + \frac{L(i)}{dt_2(i)/di},
\end{align*}
\]

therefore

\[
\begin{align*}
  L(i) &= \frac{u_1(i) - u_2(i)}{dt_1(i)/di - dt_2(i)/di}, \\
  R(i) &= \frac{1}{i} [u_1(i) - L(i) di/dt_1(i)].
\end{align*}
\]  \hspace{1cm} (2)

Since \( u(t) = U_m \sin \omega t \), it is possible avoid piecewise polynomial approximation. Instead, \( u_1(i) \) and \( u_2(i) \) can be substituted in (2) as \( u(t_1) \) and \( u(t_2) \). Equations (2) give us a method to calculate the required dependencies. The accuracy of the definitions of \( R(i) \) and \( L(i) \) will be discussed below.

Let us consider the parallel connection of nonlinear \( R(u) \) and \( C(u) \). The problem is to find these dependencies. Connect the elements to an AC voltage source and obtain the oscillograms of the input current and voltage. The mathematical model of the device has the form:

\[ i(t) = \frac{u(t)}{R(u)} + C(u) \frac{du(t)}{dt} \]  \hspace{1cm} (3)

Divide period \( T \) into two segments. On one of them the function \( u(t) \) is increasing monotonously and the other—decreasing monotonously. Thus, each value \( u \) on the interval \([ -U_m, U_m] \) will correspond to a pair of values \( t_1(u), t_2(u) \), and, respectively, \( i_1(u) \) and \( i_2(u) \). Then one can rewrite (4) as a system:

\[
\begin{align*}
  i_1(u) &= \frac{u}{R(u)} + \frac{C(u)}{dt_1(u)/du}, \\
  i_2(u) &= \frac{u}{R(u)} + \frac{C(u)}{dt_2(u)/du},
\end{align*}
\]

therefore

\[
\begin{align*}
  C(u) &= \frac{i_1(u) - i_2(u)}{dt_1(u)/du - dt_2(u)/du}, \\
  R(u) &= \frac{u}{i_1(u) - C(u) du/dt_1(u)}. \\
\end{align*}
\]  \hspace{1cm} (4)

Equation (3) gives us the method of calculation of the dependencies \( R(u) \) and \( C(u) \).

3. The results of the study of the accuracy of the method and their discussion
Consider a nonlinear RL circuit in which \( L(i) = 0.2/(1+i^2) \), \( R(i) = 1+100 \cdot i^2 \). Let us connect the circuit to a sinusoidal voltage source \( u(t) = 100\sqrt{2} \sin 2\pi 50t \) (V) and obtain the current and voltage oscillograms. Let us keep 100, 150 and 200 points on the same period. We obtain functions \( i(t) \) and \( u(t) \) by interpolating the experimental points with segments of cubic polynomials. After that, we perform transformations (2-3) and obtain dependencies \( L(i) \) and \( R(i) \). The found dependency \( L(i) \) is shown in Figure 1, and the dependency \( R(i) \)—in Figure 2. The figures under the letters (a), (b) and (c) correspond to 100, 150 and 200 experimental points. The dashed line on the chart shows the original function, the solid line—the result of the calculation.
As we can see, when keeping 200 experimental points, the initial and calculated characteristics practically coincide. When keeping 100 points there is a discrepancy in $L(i)$ at the ends of the interval. When keeping 150 points, $R(i)$ diverges in the vicinity of zero. This shows the sensitivity of this method to the amount of input data.

The relative error for the case of 200 experimental points is shown in Figure 3 (a) and (b) for $L(i)$ and $R(i)$, respectively.

$$\delta L(i) = \frac{L(i) - 0.2/(1 + i^2)}{\int_{-1.11}^{1.11} 0.2/(1 + i^2) di} \times 100\%,$$

$$\delta R(i) = \frac{R(i) - 1 - 100\cdot i^2}{\int_{-1.11}^{1.11} 1 + 100\cdot i^2 di} \times 100\%.$$
As we can see, at most of the interval the error of $L(i)$ does not exceed 0.08%, but increases at the ends of the interval. The maximum error in $R(i)$ is approximately 0.17%.

Consider the parallel connection of nonlinear resistive and capacitive elements. Let it be $C(u) = 16 \cdot 10^{-6} / (1 + u^210^{-4})$, $R(u) = 50 + 5 \cdot 10^{-3} u^2$.

Let us connect these elements to a similar voltage source and obtain the oscillograms of the input current and voltage. After that let us keep 200 points for the period and make transformations (2-3). The results of the definitions of $C(u)$ and $R(u)$ are shown in Figure 4, (a) and (b) respectively. The dashed line on the chart shows the original function, the solid line—the result of the calculation.

![Figure 3](image_url)  
**Figure 3.** The relative error of $L(i)$ and $R(i)$.

![Figure 4](image_url)  
**Figure 4.** The dependencies $C(u)$ and $R(u)$. 
As we can see in Figure 4, (b), the function $R(u)$ diverges in the vicinity of zero. This problem can be solved by replacing the obtained function in a given area with a smooth polynomial.

The relative error of $C(u)$ and $R(u)$ is represented by the functions:

$$
\delta C(u) = \frac{C(u) - 16 \cdot 10^{-6} / (1 + u^2 10^{-4})}{16 \cdot 10^{-6} / (1 + u^2 10^{-4})} \cdot 100\%,
\delta R(u) = \frac{R(i) - 50 - 5 \cdot 10^{-3} \cdot i^2}{50 + 5 \cdot 10^{-3} \cdot i^2} \cdot 100\%.
$$

The relative error for the case of 200 experimental points is shown in figure 5, (a) and (b) for $C(u)$ and $R(u)$ respectively.

![Figure 5](image)

Figure 5. The relative error of $C(u)$ and $R(u)$.

As we can see in figure 5, the relative error of the function $C(u)$ for most of the interval does not exceed 0.000012% but increases sharply at the end of the interval. The relative error of $R(u)$ function determination does not exceed 0.001% at most of the interval but increases sharply at the end of the interval and in the vicinity of zero.

4. Conclusions

The proposed method allows us to find the characteristics of the parameters of the equivalent circuit of a nonlinear element for 1-2 periods of power frequency. This makes it possible to dampen the interfering effect of changes in the electromagnetic environment and changes in the temperature of the element during the experiment. The use of modern digital measurement technology allows us to achieve high accuracy in the application of this approach. The approach is also easy to implement.

References

[1] Nitsch J, Korovkin N and Solovyeva E 2004 Examination of the demodulation effect of two-tone disturbances on nonlinear elements Advances in Radio Science 2 51–6
[2] Nitsch J B, Solovyeva E B, Korovkin N V and Scheibe H J 2008 Compensation of low-frequency disturbances by means of linearization of the electronic systems characteristic IEEE Transactions on Electromagnetic Compatibility 50(4) 887–94
[3] Bondarenko A V, Hayakawa M, Korovkin N V and Selina E E 2005 A general modeling method of synthesis of complex technical and biological systems IEEJ Transactions on Fundamentals and Materials 125(7) 577–82
[4] Korovkin N V and Solovyeva E B 2005 Synthesis of nonlinear compensators on basis of operational equations for electronic devices protected from low-frequency noises *IEEE 6th International Symposium on Electromagnetic Compatibility and Electromagnetic Ecology Proceedings* 2005 1513088 153–6

[5] Emel’yanov O A and Plotnikov A P 2017 Determining the dependence of the capacitance of ferro-ceramic capacitors on voltage by the pulse discharge method *Measurement Techniques* 60(9) 922–7

[6] Pan M-J and Randall Clive A 2010 A brief introduction to ceramic capacitors *IEEE Elect. Insul. Mag.* 26(3) 44–50

[7] Adalev A S, Korovkin N V and Hayakawa M 2006 Identification of electric circuits described by ill-conditioned mathematical models *IEEE Transactions on Circuits and Systems I: Regular Papers* 53(1) 78–91

[8] Adalev A S, Hayakawa M and Korovkin N V 2005 Identification of electric circuits: Problems and methods of solution accuracy enhancement *Proceedings - IEEE International Symposium on Circuits and Systems* 1464754 980–3

[9] Korovkin N V and Pankin A M 2017 Diagnostics indicators in monitoring the technical state of direct charge sensors in nuclear reactors *Atomic Energy* 122(4) 284–9