BARROW’S INEQUALITY AND SIGNED ANGLE BISECTORS

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Abstract. In this paper we give one extension of Barrow’s type inequality in the plane of the triangle $\triangle ABC$ introduce signed angle bisectors.

1. Introduction

Let triangle $\triangle ABC$ be given in Euclidean plane. Denote by $R_A, R_B$ and $R_C$ the distances from the arbitrary point $M$ in the plane of $\triangle ABC$ to the vertices $A$, $B$ and $C$ respectively, and denote by $\ell_a = |MA|$, $\ell_b = |MB|$ and $\ell_c = |MC|$ the length of angle bisectors of $\angle BMC$, $\angle CMA$ and $\angle AMB$ from the point $M$ respectively (Fig. 1).

Barrow’s inequality [2]:

$$R_A + R_B + R_C \geq 2 (\ell_a + \ell_b + \ell_c)$$

(1)

is true when $M$ is arbitrary point in the interior of triangle $\triangle ABC$. The equality holds iff triangle $ABC$ is equilateral and point $M$ is its circumcenter. In this paper we consider a Barrow’s type inequality when $M$ is arbitrary point in the plane of the triangle $\triangle ABC$ introduce signed angle bisectors. Let us notice that inequalities with angle bisectors recently are considered in papers [1], [6], [7], [15].

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{Barrow.pdf}
\caption{Barrow’s inequality (point $M$ into $\triangle ABC$)}
\end{figure}

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Inequality of Erdös-Mordell [4]:

\[ R_A + R_B + R_C \geq 2(r_a + r_b + r_c) \]  

(2)

is a consequence of inequality of Barrow, where \( r_a, r_b \) and \( r_c \) are distances of interior point \( M \) of triangle to the sides \( BC, CA \) and \( AB \) respectively.

Let us notice that topic of the Erdös-Mordell inequality is current, as it has been shown in recent papers. V. Pambuccian proved that, in the plane of absolute geometry, the Erdös-Mordell inequality is an equivalent to non-positive curvature [12]. In the paper [11] is given an extension of the Erdös-Mordell inequality on the interior of the Erdös-Mordell curve. In relation to the Erdös-Mordell inequality N. Der gia des in the paper [3] proved one extension of the Erdös-Mordell type inequality

\[ R_A + R_B + R_C \geq \left( \frac{c}{b} + \frac{b}{c} \right) r'_a + \left( \frac{a}{c} + \frac{c}{a} \right) r'_b + \left( \frac{b}{a} + \frac{a}{b} \right) r'_c \]  

(3)

where \( r'_a, r'_b \) and \( r'_c \) are signed distances of arbitrary point \( M \) in the plane triangle to the sides \( BC, CA \) and \( AB \) respectively.

2. The Main Results

Proof of Barrow’s inequality in the paper of Z. Lu [10] is based on the next theorem.

**Statement 1.** Let \( p, q, r \geq 0 \) and \( \alpha + \beta + \gamma = \pi \). Then we have the inequality

\[ p + q + r \geq 2\sqrt{qr} \cos \alpha + 2\sqrt{pr} \cos \beta + 2\sqrt{pq} \cos \gamma. \]  

(4)

Peculiarity of Barrow’s and Lu’s proofs are, that is, primarily algebraic. In Lu’s proof, Barrow’s inequality follows from positivity of quadratic function \( f(x) = x^2 - 2(\sqrt{r} \cos \beta + \sqrt{q} \cos \gamma) x + q + r - 2\sqrt{qr} \cos \alpha \) in the point \( x = \sqrt{p} \) with an appropriate geometric interpretation for \( p, q, r \) and \( \alpha, \beta, \gamma \) (for details see [10]).

In this paper we also give one algebraic proof with geometric interpretation for points outside of the triangle \( \triangle ABC \). The following theorems are true.

**Statement 2.** Let \( p, q, r \geq 0 \) and \( \alpha = \beta + \gamma \). Then we have the inequality

\[ p + q + r \geq -2\sqrt{qr} \cos \alpha + 2\sqrt{pr} \cos \beta + 2\sqrt{pq} \cos \gamma. \]  

(5)

**Proof.** Let us consider the quadratic function

\[ g(x) = x^2 - 2(\sqrt{r} \cos \beta + \sqrt{q} \cos \gamma) x + q + r + 2\sqrt{qr} \cos \alpha. \]  

(6)

Then a quarter of the discriminant is

\[ \frac{1}{4} \delta = (\sqrt{r} \cos \beta + \sqrt{q} \cos \gamma)^2 - (q + r + 2\sqrt{qr} \cos \alpha). \]  

(7)
Based on $\alpha = \beta + \gamma$ we have $\cos \alpha = \cos (\beta + \gamma) = \cos \beta \cos \gamma - \sin \beta \sin \gamma$ and hence
\[
\frac{1}{4} \delta = r \cos^2 \beta + q \cos^2 \gamma + 2\sqrt{rq} \cos \beta \cos \gamma - q - r - 2\sqrt{rq} \cos \alpha
\]
\[
= r \cos^2 \beta + q \cos^2 \gamma + 2\sqrt{rq} \cos \beta \cos \gamma - q - r - 2\sqrt{rq} \cos (\beta + \gamma)
\]
\[
= -r \sin^2 \beta - q \sin^2 \gamma + 2\sqrt{rq} \cos \beta \cos \gamma - 2\sqrt{rq} \cos \beta \cos \gamma + 2\sqrt{rq} \sin \beta \sin \gamma.
\]
Using previous identity we obtained
\[
\delta = -4 \left( \sqrt{r} \sin \beta - \sqrt{q} \sin \gamma \right)^2 < 0,
\]
hence $g(x) \geq 0$. Finally, letting $x = \sqrt{\rho}$ we obtained (5). □

**Remark 1.** Let us emphasize that for term $A = p + q + r + 2\sqrt{qr} \cos \alpha - 2\sqrt{pq} \cos \beta - 2\sqrt{pq} \cos \gamma$, when $\gamma = \alpha - \beta$, follows inequality
\[
A = \left( \sqrt{r} - \sqrt{p} \cos \beta + \sqrt{q} \cos \alpha \right)^2 + \left( \sqrt{p} \sin \beta - \sqrt{q} \sin \alpha \right)^2 \geq 0,
\]
alogously using the LAGRANGE's complete square identity from [8], [9]. Therefore we have second proof of inequality (5).

**Statement 3.** Let $p, q, r \geq 0$ and $\alpha = \beta + \gamma$. Then we have the inequality
\[
p + q + r \geq 2\sqrt{qr} \cos \alpha - 2\sqrt{pq} \cos \beta - 2\sqrt{pq} \cos \gamma.
\] (8)

**Proof.** Let us consider the term $A = p + q + r + 2\sqrt{qr} \cos \alpha + 2\sqrt{pq} \cos \beta + 2\sqrt{pq} \cos \gamma$, for $\gamma = \alpha - \beta$. Notice that for the term $A$, by the LAGRANGE's complete square identity, the following two representations are true.

1° If $\frac{\pi}{2} \leq \alpha < \pi$, then $\cos \alpha \leq 0$, and therefore
\[
A = \left( \sqrt{r} + \sqrt{p} \cos \beta + \sqrt{q} \cos \alpha \right)^2 + \left( \sqrt{p} \sin \beta + \sqrt{q} \sin \alpha \right)^2 - 4\sqrt{qr} \cos \alpha \geq 0. \tag{9}
\]

2° If $0 < \alpha < \frac{\pi}{2}$, then $\cos \alpha > 0$. From $\alpha = \beta + \gamma$ follows $\cos \beta > 0$, and therefore
\[
A = \left( \sqrt{r} - \sqrt{p} \cos \beta - \sqrt{q} \cos \alpha \right)^2 + \left( \sqrt{p} \sin \beta + \sqrt{q} \sin \alpha \right)^2 + 4\sqrt{pq} \cos \beta \geq 0. \tag{10}
\]

Let us introduce the division of the plane of triangle $\triangle ABC$ to following areas $\lambda_0 = (+, +, +)$, $\lambda_1 = (-, +, +)$, $\lambda_2 = (+, -, +)$, $\lambda_3 = (+, +, -)$, $\lambda_4 = (+, -, -)$, $\lambda_5 = (-, +, -)$, $\lambda_6 = (-, -, +)$, (Fig. 2), via signs of homogenous barycentric coordinates of a point as given in the paper [14] (see also the Section 7.2 in [5]). Then $\lambda_0$ is the interior area of the triangle $\triangle ABC$. Let us notice that $(\lambda_0 \cup \lambda_1) \cup (BC)$ is the interior area of the angle $\angle A$, and $\lambda_4$ is the interior area of the opposite angle. Analogously $(\lambda_0 \cup \lambda_2) \cup (AC)$ is the interior area of the angle $\angle B$, $\lambda_5$ is the interior area of the opposite angle and $(\lambda_0 \cup \lambda_3) \cup (AB)$ is the interior area of the angle $\angle C$, $\lambda_6$ is the interior area of the opposite angle.
The following auxiliary statement is true.

**Lemma 0.** Let $B$ and $C$ be fixed points in the plane and let $M$ be arbitrary point in the plane. For $\ell$ length of angle bisector of $\angle BMC$ from point $M$ following formulas are true:

$$\ell = \frac{2R_BR_C}{R_B + R_C} \cos \frac{\alpha_M}{2} = \frac{\sqrt{R_BR_C}}{R_B + R_C} \sqrt{(R_B + R_C)^2 - |BC|^2},$$  \hspace{2cm} (11)

where $R_B = |MB|$, $R_C = |MC|$ and $\alpha_M = \angle BMC$. Especially, for $\varphi$ line throughout points $B$ and $C$ is true:

$$\ell = \begin{cases} 0 & : M \in [BC], \\ \frac{2R_BR_C}{R_B + R_C} & : M \in \varphi \setminus [BC]. \end{cases}$$  \hspace{2cm} (12)

In further considerations let $p = R_A$, $q = R_B$, $r = R_C$. Then, Z. Lu, in the paper [10], proved the following Barrow’s type inequality.

**Theorem 0.** [10] In the area $\lambda_0$ the following inequality is true:

$$R_A + R_B + R_C \geq \left( \frac{\sqrt{R_C}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_C}} \right) \ell_a + \left( \frac{\sqrt{R_C}}{\sqrt{R_A}} + \frac{\sqrt{R_A}}{\sqrt{R_C}} \right) \ell_b + \left( \frac{\sqrt{R_A}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_A}} \right) \ell_c.$$  \hspace{2cm} (13)

**Remark 2.** Barrow’s inequality is a consequence of the previous inequality.

From previous Lemma follows next auxiliary statement.

**Lemma 1.** (i) If $M = A$, i.e. $R_A = 0$ then:

$$R_B + R_C \geq \left( \frac{\sqrt{R_C}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_C}} \right) \ell_a.$$  \hspace{2cm} (14)

(ii) If $M = B$, i.e. $R_B = 0$ then:

$$R_A + R_C \geq \left( \frac{\sqrt{R_C}}{\sqrt{R_A}} + \frac{\sqrt{R_A}}{\sqrt{R_C}} \right) \ell_b.$$  \hspace{2cm} (15)

(iii) If $M = C$, i.e. $R_C = 0$ then:

$$R_A + R_B \geq \left( \frac{\sqrt{R_B}}{\sqrt{R_A}} + \frac{\sqrt{R_A}}{\sqrt{R_B}} \right) \ell_c.$$  \hspace{2cm} (16)
Denote with \( \text{cl} \) closure of a plane set. The following theorem is true.

**Theorem 1.** In the area \( \text{cl} (\lambda_1) \setminus \{B, C\} \) the following inequality is true:

\[
R_A + R_B + R_C \geq \left( \frac{\sqrt{R_C}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_C}} \right) (\ell_a) + \left( \frac{\sqrt{R_C}}{\sqrt{R_A}} + \frac{\sqrt{R_A}}{\sqrt{R_C}} \right) \ell_b + \left( \frac{\sqrt{R_A}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_A}} \right) \ell_c. \tag{17}
\]

**Proof.** Let \( M \in \text{cl} (\lambda_1) \setminus \{B, C\} \), then \( \alpha_M = \beta_M + \gamma_M \) i.e. \( \frac{\alpha_M}{2} = \frac{\beta_M}{2} + \frac{\gamma_M}{2} \) (Fig. 3).

![Figure 3: Extension of the Barrow’s inequality](image)

Based on the Statement 2 the following inequality holds

\[
R_A + R_B + R_C \geq -2 \sqrt{R_B R_C} \cos \frac{\alpha_M}{2} + 2 \sqrt{R_A R_C} \cos \frac{\beta_M}{2} + 2 \sqrt{R_A R_B} \cos \frac{\gamma_M}{2} \tag{18}
\]

and based on the Lemma from previous inequality we obtained

\[
R_A + R_B + R_C \geq - \frac{R_B + R_C}{\sqrt{R_B R_C}} \ell_a + \frac{R_A + R_C}{\sqrt{R_A R_C}} \ell_b + \frac{R_A + R_B}{\sqrt{R_A R_B}} \ell_c \\
= \left( \frac{\sqrt{R_C}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_C}} \right) (-\ell_a) + \left( \frac{\sqrt{R_C}}{\sqrt{R_A}} + \frac{\sqrt{R_A}}{\sqrt{R_C}} \right) \ell_b + \left( \frac{\sqrt{R_A}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_A}} \right) \ell_c. \tag{19}
\]

Next two theorems are direct consequence of the Statement 2 by following cyclic replacements \( \alpha_M \mapsto \beta_M, \beta_M \mapsto \gamma_M, \gamma_M \mapsto \alpha_M \) and \( R_A \mapsto R_B, R_B \mapsto R_C, R_C \mapsto R_A \) respectively.

**Theorem 2.** In the area \( \text{cl} (\lambda_2) \setminus \{A, C\} \) the following inequality is true:

\[
R_A + R_B + R_C \geq \left( \frac{\sqrt{R_C}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_C}} \right) \ell_a + \left( \frac{\sqrt{R_C}}{\sqrt{R_A}} + \frac{\sqrt{R_A}}{\sqrt{R_C}} \right) (\ell_b) + \left( \frac{\sqrt{R_A}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_A}} \right) \ell_c. \tag{20}
\]
THEOREM 3. In the area $\text{cl} \left( \lambda_3 \right) \setminus \{A, B\}$ the following inequality is true:

$$R_A + R_B + R_C \geq \left( \frac{\sqrt{R_C}}{\sqrt{R_A}} + \frac{\sqrt{R_B}}{\sqrt{R_A}} \right) \ell_a + \left( \frac{\sqrt{R_C}}{\sqrt{R_B}} + \frac{\sqrt{R_A}}{\sqrt{R_B}} \right) \ell_b + \left( \frac{\sqrt{R_A}}{\sqrt{R_B}} + \frac{\sqrt{R_{10}}}{{\sqrt{R_A}}} \right) (-\ell_c). \quad (21)$$

The following theorem is true.

THEOREM 4. In the area $\lambda_4$ the following inequality is true:

$$R_A + R_B + R_C \geq \left( \frac{\sqrt{R_C}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_C}} \right) \ell_a + \left( \frac{\sqrt{R_C}}{\sqrt{R_A}} + \frac{R_A}{\sqrt{R_C}} \right) (-\ell_b)_c + \left( \frac{\sqrt{R_A}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_A}} \right) (-\ell_c). \quad (22)$$

Proof. Let $M \in \lambda_4$, then $\alpha_M = \beta_M + \gamma_M$ i.e. $\frac{\alpha_M}{2} = \frac{\beta_M}{2} + \frac{\gamma_M}{2}$. Based on the Statement[3] the following inequality is true

$$R_A + R_B + R_C \geq 2 \sqrt{R_BR_C} \cos \frac{\alpha_M}{2} - 2 \sqrt{R_AR_C} \cos \frac{\beta_M}{2} - 2 \sqrt{R_AR_B} \cos \frac{\gamma_M}{2}. \quad (23)$$

Substitutions

$$\ell_a = |MA'| = 2 \frac{R_BR_C}{R_B + R_C} \cos \frac{\alpha_M}{2}, \quad (24)$$
$$\ell_b = |MB'| = 2 \frac{R_AR_C}{R_A + R_C} \cos \frac{\beta_M}{2}, \quad (25)$$
$$\ell_c = |MC'| = 2 \frac{R_AR_B}{R_A + R_B} \cos \frac{\gamma_M}{2} \quad (26)$$

in (23) give

$$R_A + R_B + R_C \geq \frac{R_B + R_C}{\sqrt{R_B R_C}} \ell_a - \frac{R_A + R_C}{\sqrt{R_A R_C}} \ell_b - \frac{R_A + R_B}{\sqrt{R_A R_B}} \ell_c$$

$$= \left( \frac{\sqrt{R_C}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_C}} \right) \ell_a + \left( \frac{\sqrt{R_C}}{\sqrt{R_A}} + \frac{\sqrt{R_A}}{\sqrt{R_C}} \right) (-\ell_b) + \left( \frac{\sqrt{R_A}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_A}} \right) (-\ell_c). \quad □ \quad (27)$$

Next two theorems are direct consequence of the Statement[3] by following cyclic replacements $\alpha_M \mapsto \beta_M$, $\beta_M \mapsto \gamma_M$, $\gamma_M \mapsto \alpha_M$ and $R_A \mapsto R_B$, $R_B \mapsto R_C$, $R_C \mapsto R_A$ respectively.

THEOREM 5. In the area $\lambda_5$ the following inequality is true:

$$R_A + R_B + R_C \geq \left( \frac{\sqrt{R_C}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_C}} \right) (-\ell_a) + \left( \frac{\sqrt{R_C}}{\sqrt{R_A}} + \frac{\sqrt{R_A}}{\sqrt{R_C}} \right) \ell_b + \left( \frac{\sqrt{R_A}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_A}} \right) (-\ell_c). \quad (28)$$

THEOREM 6. In the area $\lambda_6$ the following inequality is true:

$$R_A + R_B + R_C \geq \left( \frac{\sqrt{R_C}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_C}} \right) (-\ell_a) + \left( \frac{\sqrt{R_C}}{\sqrt{R_A}} + \frac{\sqrt{R_A}}{\sqrt{R_C}} \right) (-\ell_b) + \left( \frac{\sqrt{R_A}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_A}} \right) \ell_c. \quad (29)$$
Now, we give definition of the signed angle bisector for the point \( M \) in the plane of the triangle \( \triangle ABC \). Let be \( A \) fixed vertex and let \( p \) be line through vertices \( B \) and \( C \). Denote \( d = |MA| \) distance of the point \( M \) to the line \( p \) and let \( \ell = |MA'| \) be length of the bisector of the angle \( \angle BMC \). If \( d' \) be signed distance of the point \( M \) to the line \( p \) related to the vertex \( A \) [13] (p. 308.), then \( d' = +d \) if \( M \) and \( A \) with same side of line \( p \), otherwise \( d' = -d \). Let us define signed angle bisector \( \ell' \) analogously \( \ell' = +\ell \) if \( M \) and \( A \) with same side of line \( p \), otherwise \( \ell' = -\ell \) (Fig. 4). In the case \( M \in p \) then \( d' = 0 \) and then \( \ell' \) given by formula (12).

\[
\begin{align*}
    d_a' &= +d_a \\
    \ell_a' &= +\ell_a
\end{align*}
\]

\[
\begin{align*}
    d_b' &= -d_a \\
    \ell_b' &= -\ell_a
\end{align*}
\]

\[
\begin{align*}
    d_c' &= +d_a \\
    \ell_c' &= +\ell_a
\end{align*}
\]

Figure 4: Signed distances and signed angle bisectors

Let us denote \( \mu_1 = \text{cl}(\lambda_1) \setminus \{B,C\} \), \( \mu_2 = \text{cl}(\lambda_2) \setminus \{A,C\} \), \( \mu_3 = \text{cl}(\lambda_3) \setminus \{A,B\} \), \( \mu_4 = \lambda_4 \), \( \mu_5 = \lambda_5 \) and \( \mu_6 = \lambda_6 \). Then \( \bigcup_{i=1}^{6} \mu_i \cup \{A,B,C\} \) is a complete division of the plane of the triangle \( \triangle ABC \). Finally, analogously to DERGIADIES extension of the Erdős-Mordell inequality [3], from previous theorems, an extension of BARROW’s type inequality (13) is obtained by the following theorem.

**Statement 4.** For the point \( M \in \bigcup_{i=1}^{6} \mu_i \) the following inequality is true:

\[
R_A + R_B + R_C \geq \sum_{i=1}^{6} \left( \frac{\sqrt{R_C}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_C}} \right) \ell'_a + \left( \frac{\sqrt{R_C}}{\sqrt{R_A}} + \frac{\sqrt{R_A}}{\sqrt{R_C}} \right) \ell'_b + \left( \frac{\sqrt{R_A}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_A}} \right) \ell'_c ; \quad (30)
\]

otherwise for points \( M = A, M = B, M = C \) following inequalities [14], [15], [16] are true respectively.

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