Spinning particle approach to higher spin field theory

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Abstract. We shortly review on the connection between higher-spin gauge field theories and supersymmetric spinning particle models. In such approach the higher spin equations of motion are linked to the first-class constraint algebra associated with the quantization of particle models. Here we consider a class of spinning particle models characterized by local \( O(N) \)-extended supersymmetry since these models are known to provide an alternative approach to the geometric formulation of higher spin field theory. We describe the canonical quantization of the models in curved target space and discuss the obstructions that appear in presence of an arbitrarily curved background. We then point out the special role that conformally flat spaces appear to have in such models and present a derivation of the higher-spin curvatures for maximally symmetric spaces.

1. Introduction
Particle models –through the worldline formalism– provide an efficient alternative method, to conventional second-quantized field theories, for the computation of quantum field theory Feynman diagrams and effective actions. Moreover some spinning particle models can also be applied to the first quantization of higher spin fields. In particular, from the canonical quantization of \( O(N) \)-extended locally super-symmetric particle models, the first-class constraints yield the equations of motions for free spin-\( \frac{N}{2} \) fields in terms of the corresponding linearized curvatures, that in this approach are the wave functions of the particle models. In other words the quantization of \( O(N) \)-extended susy particle models is related to the so-called ”geometric formulation” of higher spin field theory. Such formulation was proposed in [1, 2] (using the the linearized higher spin geometry developed in [3, 4]) in order to relax algebraic constraints present in the conventional formulation given in terms of higher spin potentials, and investigate alternatives to introduce interactions in the higher spin fields, a problem that still constitutes an open field of research [5, 6].

Toward an alternative approach to the geometric formulation we studied the quantization of \( O(N) \) spinning particle models [7, 8, 9, 10]

\[
S = \int dt \left[ p_\mu \dot{\psi}^\mu + \frac{1}{2} \psi_i \dot{\psi}_i^\mu - e \left( \frac{1}{2} p_\mu p^\mu \right) - i \chi_i \left( p_\mu \psi_i^\mu \right) - i a_{ij} \left( \psi_i^\mu \psi_j^\mu \right) \right], \quad i = 1, \ldots, N \tag{1}
\]
both using canonical quantization [11, 12] and path integrals [13]. Above \((x^\mu, p_\mu)\) are bosonic canonical coordinates, \(\psi^i_{\mu}\) their fermionic partners and \((e, \chi_i, a_{ij})\) are the gauge fields of the local worldline-Poincaré symmetry, local extended supersymmetry and local \(SO(N)\) gauge symmetry respectively. The canonical quantization of the flat target space models described by (1) and their curved deformations is analyzed below. In particular for conformally flat spaces it is shown that the constraint algebra closes, although being non linear and for a further restricted class of backgrounds, characterized by maximal symmetry, the quadratically deformed constraint algebra is solved and linearized higher-spin curvatures are found, for an arbitrary \(N\) and an arbitrary (even) space-time dimension \(D = 2d\); for \(D = 2d + 1\) and \(N > 2\) these models are quantum mechanically trivial due to a global anomaly [8, 9]. The path integral quantization is very interesting in that the worldline on the circle of the above action yields the effective action of quantum higher spin fields coupled to external backgrounds. In [13] we computed the worldline path integral on the circle with flat background and we found that it correctly reproduces the number of degrees of freedom of the propagating higher spin field, that is the information included in the flat space one-loop effective action. Extensions to curved space backgrounds are under study and will be published shortly.

In the present report we restrict our study to \(O(N)\) spinning particle models. However it is worth mentioning other particle models relevant to higher spin field physics, such as twistor-like superparticles [14, 15], \(U(N)\) spinning particle [16, 17], and \(Sp(2r)\) models [18].

### 2. Symmetries of the models

The above flat space action (1) is invariant under the local symmetry generated by the first class constraints \((H, Q_i, J_{ij})\) associated to the aforementioned gauge fields. Transformations of the canonical variables can be obtained with the help of elementary Poisson brackets \(\{x^\mu, p_\nu\}_PB = \delta^\mu_\nu\) and \(\{\psi^i_{\mu}, \psi^j_{\nu}\}_PB = -i\delta^{ij}\eta_{\mu\nu}\), whereas transformations of the gauge fields can be obtained using the first class algebra

\[
\begin{align*}
\{Q_i, Q_j\}_PB &= -i2\delta_{ij}H, \\
\{J_{ij}, Q_k\}_PB &= \delta_{jk}Q_i - \delta_{ik}Q_j, \\
\{J_{ij}, J_{kl}\}_PB &= \delta_{jk}J_{i\ell} - \delta_{ik}J_{j\ell} + \delta_{j\ell}J_{i\ell} - \delta_{i\ell}J_{j\ell}
\end{align*}
\]  

(2)

and imposing the invariance of the action under the local transformation generated by \(G = \xi H + i\epsilon_i Q_i + \frac{1}{2}\alpha_{ij} J_{ij}\). One thus gets

\[
\begin{align*}
\delta x^\mu &= \{x^\mu, G\}_PB = \xi p^\mu + i\epsilon_i \psi^i_{\mu}, \\
\delta p_\mu &= \{p_\mu, G\}_PB = 0, \\
\delta \psi^i_{\mu} &= \{\psi^i_{\mu}, G\}_PB = -\epsilon_j p^\mu + \alpha_{ij} \psi^j_{\mu}, \\
\delta \epsilon_i &= \hat{\xi} + 2i\chi_i \epsilon_i, \\
\delta \chi_i &= \hat{\epsilon}_i - a_{ij} \epsilon_j + \alpha_{ij} \chi_j, \\
\delta a_{ij} &= \hat{\alpha}_{ij} + \alpha_{im} a_{mj} + \alpha_{jm} a_{im}
\end{align*}
\]  

(3)

The rigid symmetries involve transformations under the target space Poincaré group that ensures the relativistic invariance of the model. In addition the model is conformally invariant. To prove conformal invariance we can first obtain the background symmetries of the model where background fields, such as the metric, transform as well. In order to do so we rewrite the flat space action (1) using arbitrary coordinates

\[
S = \int dt \left[ p_\mu \dot{x}^\mu + \frac{i}{2} \psi_{\mu} \dot{\psi}^\mu - e \left( \frac{1}{2} \pi_{\mu} \pi^\mu \right) - i \chi_i \left( \psi^a_{\mu} \psi_a^\mu \pi^\mu \right) - \frac{i}{2} a_{ij} \left( \psi^a_{\mu} \psi^a_{\nu} \pi_\mu \pi_\nu \right) - \frac{i}{2} \right]
\]  

(4)

where tangent space indices are used for the fermions and a vielbein is accordingly introduced, \(\psi^a_{\mu} = e^a_\mu(x) \psi^a_{\mu}\), and for later convenience the hamiltonian has been renamed \(H_0\). A spin
connection $\omega_{\mu ab}$ can be uniquely constructed and in turn the covariant momenta read

$$\pi_\mu = p_\mu - \frac{i}{2} \omega_{\mu ab} \psi^a_i \psi^b_i$$

that satisfy the bracket relations

$$\{\pi_\mu, \pi_\nu\}_P = \frac{i}{2} R_{\mu \nu ab} \psi^a_i \psi^b_i.$$

It is thus now easy to check that action (4) is invariant under (i) target space diffeomorphisms, (ii) local Lorentz transformations and (iii) Weyl rescalings. As a consequence of the latter background symmetries there exist conformal Killing vectors that leave the metric invariant. Hence the model (4) is invariant under rigid symmetries belonging to the conformal group $SO(D, 2)$ [19, 20]. Moreover background Weyl symmetry guarantees that the spinning particle is consistent in any conformally flat space.

2.1. First class algebra on conformally flat spaces

It is instructive to compute the constraint algebra on an arbitrarily curved background. With the only help of the elementary Poisson brackets discussed above, for the nonvanishing brackets one obtains (the $SO(N)$ subalgebra generated by $J_{ij}$ holds unchanged)

$$\{Q_i, Q_j\}_P = -2i \delta_{ij} H_0 - \frac{i R}{(D - 1)(D - 2)} J_{ik} J_{jk} - \frac{R_{ab}}{D - 2} \left( \psi^a_i \psi^b_j + (i \leftrightarrow j) \right)$$

$$\{Q_i, H_0\}_P = -\frac{i}{2} \pi^a R_{abcd} \psi^b_i \psi^c_i \psi^d$$

that generically fails to be first class. A possible option, as suggested by the results obtained in the previous section, is to restrict attention to conformally flat spaces. In such a case, the Weyl tensor identically vanishes and the Riemann tensor can be written as combination of Ricci and scalar tensor, so that the above algebra reduces to

$$\{Q_i, Q_j\}_P = -2i \delta_{ij} H_0 - \frac{i R}{(D - 1)(D - 2)} J_{ik} J_{jk} - \frac{R_{ab}}{D - 2} \left( \psi^a_i \psi^b_j + (i \leftrightarrow j) \right)$$

$$\{Q_i, H_0\}_P = 0$$

which becomes first class, though with structure functions rather than structure constants.

The above algebra further simplifies if we consider maximally symmetric spaces -that are a subclass of conformally flat spaces- for which the Riemann tensor is uniquely fixed in terms of the curvature scalar $R_{abcd} = b(\eta_{ac} \eta_{bd} - \eta_{ad} \eta_{bc})$ with $b \equiv \frac{R}{\eta_{ij}(D - 1)}$. Hence (8) reduces to

$$\{Q_i, Q_j\}_P = -2i \delta_{ij} H + ib \left( J_{ik} J_{jk} - \frac{1}{2} \delta_{ij} J_{kl} J_{kl} \right), \quad \{Q_i, H\}_P = 0$$

where we have introduced the improved hamiltonian

$$H = H_0 + \Delta H = \frac{1}{2} \pi^a \pi_a - \frac{b}{4} J_{kl} J_{kl}.$$
3. Canonical quantization

In order to canonically quantize the models we need to promote (bosonic and fermionic) phase space variables to operators: the fundamental (anti) commutations relations read

$$[x^\mu, p_\nu] = i \delta^\mu_\nu, \quad \{\psi^a_i, \psi^b_j\} = \eta^{ab} \delta_{ij}$$

so that the fermionic variables give rise to a (multi)-Clifford algebra and can be represented in terms of (tensor products of) gamma matrices and in turn the wave function can be written as a multispinor $R_{\alpha_1 \ldots \alpha_N}$. We also need to define constraints with suitable ordering, so that the algebra remains first class at the quantum level. The ordering is again immediate for the $\mathfrak{so}(N)$ subalgebra, $J_{ij} = \frac{i}{2}[\psi^a_i, \psi^b_j]$. The ordering in the susy operators is uniquely fixed by replacing the covariant momentum with covariant derivatives acting on multispinorial wave functions, $Q_i = \psi^a_i e^\mu_a \left( p_\mu - \frac{i}{2} \omega_{\alpha \beta} \psi^b_k \psi^b_k \right)$. The non vanishing part of the algebra thus reads

$$[J_{ij}, J_{kl}] = SO(N) \text{ algebra}$$

$$\{Q_i, Q_j\} = 2 \delta_{ij} H - \frac{b}{2} \left( J_{ik} J_{jk} + J_{jk} J_{ik} - \delta_{ij} J_{kl} J_{kl} \right)$$

$$[J_{ij}, Q_k] = i \delta_{jk} Q_i - i \delta_{ik} Q_j$$

where $H = \frac{1}{2}(\pi^a a_a - i\omega_{ab} a^b) - \frac{b}{7} J_{kl} J_{kl} - b A(D)$, with $A(D) = (2 - N) \frac{D}{\pi} - \frac{D^2}{2}$. These constraints are to be imposed à la Dirac-Gupta-Bleuler onto the wave function. Hence, in the above Clifford basis, the susy constraints are nothing but $N$ Dirac equations and along with the $SO(N)$ constraints give rise (in flat 4d space) to the so-called Bargmann-Wigner equations for the spin-$\frac{N}{2}$ field. Below we make use of a different basis for the Grassmann variables in which the higher spin geometry emerges in a more direct way.

3.1. Integer higher-spin fields in (A)dS space

We consider the case when the number of susy charges is even $N = 2S, \ S \in \mathbb{N}$. In this case we can complexify the fermions

$$\psi^a_i = \frac{1}{\sqrt{2}}(\psi^a_i + i\psi_{i+S}), \quad \psi^{aI} = \frac{1}{\sqrt{2}}(\psi^a_i - i\psi_{i+S}), \quad i, I = 1, \ldots, S$$

so that

$$\{\psi^a_i, \psi^{bJ}\} = \eta^{ab} \delta^J_I$$

and we can represent them using a coherent state basis $\langle \psi \rangle$ where $\psi^a_i$ act multiplicatively whereas $\psi^{aI} = \frac{\partial}{\partial \psi_{aI}}$ act as left derivatives. Therefore the generic wave function will be a combination of antisymmetric (multi) forms

$$\langle \psi | \otimes \langle x | R = \sum_{A_i=0}^{D} R_{\mu_1 \ldots \mu_{A_1} \ldots \mu_{A_S}}(x) \psi^{\mu_{A_1}}_1 \cdots \psi^{\mu_{A_1}}_1 \cdots \psi^{\nu_{A}}_S \cdots \psi^{\nu_{A}}_S.$$  

All other operators involving fermions get complexified accordingly, $Q_i \rightarrow (Q_I, Q^I), \ J_{ij} \rightarrow (J^I, K_{IJ}, K^{IJ})$, with

$$Q_I = \psi^a_I e^\mu_a \pi_\mu, \quad Q^I = \psi^{aI}_I e^\mu_a \pi_\mu$$

$$K_{IJ} = \psi_I \cdot \psi_J \quad K^{IJ} = \psi^I \cdot \psi^J, \quad J^I = \psi_I \cdot \psi^J - d \delta^I_J$$

(18)  

(19)
and we can select as set of independent constraints \((J^I_L, Q_I, K^{IJ})\). In the above complex basis once this set of constraints is satisfied the remaining constraints are satisfied automatically thanks to the first class algebra; here we will not report the whole algebra written in the complex basis and address the interested reader to the original papers where it is explicitly worked out [11, 12].

The \(J^I_L\) constraints are irreducibility constraints in that they select out of the above sum a tensor that transforms irreducibly under \(GL(D)\). Such a tensor is described by a Young tableau with \(d\) rows (note that when \(I = J\) the operator selects tensors that have \(d\) indices in the \(I\) group) and \(S\) columns

\[
R \sim d \left\{ \begin{array}{c} \hline \hline \end{array} \right\} S \quad (20)
\]

that is the linearized higher spin curvature; note that for \(S = 2\) and \(D = 4\) \((d = 2)\) it has the symmetry of the Riemann tensor. For generic \(S\) and \(D\) it is a higher spin curvature.

The \(Q_I\) constraints are integrability constraints: their solution leads to write the curvature in terms of a potential (and the integrability conditions convert into differential Bianchi identities)

\[
Q_I |R\rangle = 0 \quad \rightarrow \quad |R\rangle = q|\phi\rangle \quad (21)
\]

where \(q\) is an \(S\)-th order differential operator constructed out of \(Q_I\’s\) and \(K^{IJ}\’s\)

\[
q = \sum_{n=0}^{[S/2]} (-b)^n r_n(S) q_n(S), \quad q_n(S) = \frac{1}{S!} \epsilon^{I_1...I_S} K_{I_1I_2}...K_{I_{2n-1}I_{2n}} Q_{I_{2n+1}}...Q_{I_S} \quad (22)
\]

and the numerical coefficients \(r_n(S)\) are uniquely fixed [11] by imposing conditions (21) and read

\[
r_n(S) = \frac{1}{2n} \sum_{k=1}^{n} r_{n-k}(S) a_{2k}(S - 2(n - k) + 1), \quad r_0(S) \equiv 1 \quad (23)
\]

where \(a_{2k}(S) = f_k P(S, 2k)\), with \(P(S, m) = S(S-1)...(S-m)\) being the Pochhammer function and \(f_k\) the coefficients generated by the Taylor expansion of the tangent function \(\tan(z) = \sum_k f_k z^{2k+1}\). Above the relevant part of the algebra is

\[
\{Q_I, Q_J\} = b(K_{IJ}J^L_J + K_{JL}J^L_I) \quad (24)
\]

and the commutators among \(J^I_L\) and other operators, that can be summarized as \([J^I_L, U^{JK}_K] = \delta^K_I U^{LJ}_K - \delta^K_L U^{IJ}_K\). Note that in flat space \(Q_I\’s\) anticommute among themselves and the above operator simply reduces to \(q_0 = Q_1...Q_S\), that correctly reproduces the flat space curvature whereas in (A)dS, again due to (24), corrections appear as powers of the curvature \(b\). The potential is also an irreducible representation of \(GL(D)\), that satisfies \(J^I_L|\phi\rangle = -\delta^I_J|\phi\rangle\) and is described by

\[
\phi \sim d - 1 \left\{ \begin{array}{c} \hline \hline \end{array} \right\} S \quad (25)
\]

so that for \(D = 4\) it describes a rank-\(S\) totally symmetric tensor, the (metric-like) spin-\(S\) irreducible representation of the Poincaré group. For \(D > 4\) there is no unique massless
irreducible representation of the Poincaré group: all mixed symmetry tensors represented by Young tableaux with $S$ columns are spin-$S$ irreducible representations. In the present formalism we obtain the curvature and (below) the equation of motion for the mixed-symmetry tensor described by (25).

Finally the trace constraints $K^{IJ}$ lead to the equation of motion for the higher spin potential

$$0 = K^{IJ}|R⟩ = K^{IJ}q|φ⟩ = q^{IJ}G|φ⟩. \quad (26)$$

Here the operator $G$ is a second order differential operator named Fronsdal-Labastida [21, 22, 23] operator

$$G = -2H_0 + Q_IQ^I + \frac{1}{2}Q_IQ_JK^{IJ} - bK_{IJ}K^{IJ} + bα_S(D) \quad (27)$$

where $α_S(D) = S^2 + A_S(D)$, whereas $q^{IJ}$ is an $(S - 2)$-th order operator. The higher derivative equation of motion (26) can be cast into a second-order equation of motion by introducing a compensator field as nontrivial kernel of operator $q^{IJ}$

$$G|φ⟩ = |C⟩, \quad q^{IJ}|C⟩ = 0. \quad (28)$$

Both the higher derivative equation (26) and the compensated equation (28) are gauge invariant under the transformation $δ|φ⟩ = Q_KV^K|ξ⟩$ with unconstrained gauge parameter $|ξ⟩$; in (28) the gauge transformation of the compensator is tuned to be equal to the (generically nonvanishing) transformation of the l.h.s. and schematically reads $Gδ|φ⟩ = δ|C⟩ \propto K^{IJ}|ξ⟩$. It is thus possible to perform a partial gauge fixing in order to gauge away the compensator and obtain the Fronsdal-Labastida equation of motion

$$G|φ⟩ = 0 \quad (29)$$

that is gauge invariant provided the parameter is (for $S > 2$) traceless, $K^{IJ}|ξ⟩ = 0$, and the potential is (for $S > 3$) double-traceless, $K^{IJ}K^{MN}|φ⟩ = 0$.

3.2. Half-integer higher-spin fields in (A)dS space

The results of the previous section can be extended to the case of half-integer spin corresponding to an odd number of supersymmetry charges, $N = 2r + 1, r ∈ N$ and $S = r + 1/2$. Now we consider the complexification of the even part of the fermionic fields and represent the Grassmann variables labelled with $N$ as gamma matrices

$$ψ^a_I = \frac{1}{\sqrt{2}}(ψ^a_i + iψ_{i+r}), \quad ψ^a_I = \frac{1}{\sqrt{2}}(ψ^a_i - iψ_{i+r}), \quad i, I = 1, \ldots, r$$

$$ψ^a_N = \frac{1}{\sqrt{2}}γ^a$$

so that now the wave function will be a spinor tensor

$$⟨ψ| ⊗ ⟨χ_α| ⊗ ⟨x|R⟩ = \sum_{A_i=0}^D R_{μ_1⋯μ_A_1⋯μ_1⋯μ_A_r,α}(x) ψ^{μ_1}_1 ⋯ ψ^{μ_A_1}_1 ⋯ ψ^{r}_r ⋯ ψ^{μ_A_r}_r. \quad (31)$$

where $|χ_α⟩$ is a generic state in the fermionic space $γ^a|χ_α⟩ = γ^a_a_α|χ_α⟩$. In turn the first class constraints split as $Q_i → (Q_I, Q^I, Π)$, $J_{ij} → (J_{IJ}, K_{IJ}, K^{IJ}, L_I, L^I)$, where

$$Π = γ^a e^a_α π_μ, \quad L^I = γ^aψ^a_I, \quad L_I = γ^aψ_a$I$. \quad (32)
Here the first term is the Dirac operator and the second is the "gamma trace" operator. The first class algebra allows us to choose \((Q_I, J^I, L^I)\) as an independent set of constraints, since \([L^I, L^J] = -2K^{IJ}\). Again the \(J^I\) constraints are irreducibility constraints picking a single (spinor)-tensor out of the above sum, the one whose tensorial part is described by (20). The integrability constraints now satisfies a slightly more complicated algebra, namely

\[
\{Q_I, Q_J\} = b(K_{II}J^I J^\perp + K_{JI}J^I L^J) + \frac{b}{4}(L_1L_J + L_J L_I). \tag{33}
\]

However it is possible to rotate the irreducibility operators as

\[
Q^\pm_I = Q_I \pm \frac{\sqrt{b}}{2} L_I \tag{34}
\]

and select \((Q^\pm_I, J^I, L^I)\) as independent set of constraints. These operators satisfy the relations

\[
Q^\pm_I Q^\pm_J + Q^\pm_I Q^{\mp_j} = b(K_{II}J^I J^\perp + K_{JI}J^I L^J) = Q^\pm_I Q^\pm_J + Q^\pm_I Q^{\mp_j} \tag{35}
\]

that is the same as (24). Hence, the integrability conditions in terms of \(Q^\pm_I\)

\[
Q^\pm_I |R\rangle = 0 \rightarrow |R\rangle = q|\phi\rangle \tag{36}
\]

are solved by

\[
q = \sum_{n=0}^{[r/2]} (-b)^n r_n(r)q_n(r) \tag{37}
\]

\[
q_n(r) = \frac{1}{r!} e^{I_1 \cdots I_r} \times \left\{ \begin{array}{ll} K_{I_1I_2} \cdots K_{I_{2n-1}I_{2n}} Q^{(+)I_{2n+1}} Q^{(-)I_{2n+2}} \cdots Q^{(-)I_r}, & r = 2p \\ K_{I_1I_2} \cdots K_{I_{2n-1}I_{2n}} Q^{(+)I_{2n+1}} Q^{(-)I_{2n+2}} \cdots Q^{(+)I_r}, & r = 2p + 1 \end{array} \right. \tag{38}
\]

where \(r_n(r)\) are the same numerical coefficients of the previous section.

Finally the gamma trace constraints lead to the equation of motion for the higher spin potential

\[
L^I |R\rangle = L^I q|\phi\rangle = q^I \mathcal{F}_r |\phi\rangle = 0 \tag{39}
\]

where

\[
\mathcal{F}_r = (-)^{r-1}\left(\mathbb{1} + Q_K L^K\right) + \frac{\sqrt{b}}{2} L^K L_K \tag{40}
\]

is the so-called Fang-Fronsdal operator [24, 25]. Similarly to the integer case, the higher-derivative equation (38) can be cast into a linear differential equation by introducing a compensator field that belongs to the kernel of operator \(q^I\); namely

\[
\mathcal{F}_r |\phi\rangle = |C\rangle \tag{41}
\]

and again both the compensated linear equations and (38) are gauge-invariant, with unconstrained parameter, under the following transformation

\[
\delta|\phi\rangle = \left\{ \begin{array}{ll} Q^{(+)I_1} V^I_1 |\xi\rangle, & r = 2p \\ Q^{(-)I_1} V^I_1 |\xi\rangle, & r = 2p + 1 \end{array} \right. \tag{42}
\]

upon which \(\mathcal{F}_r |\phi\rangle = \delta(C) \propto L^I |\xi\rangle\). Hence, by gauging away the compensator one gets the Fang-Fronsdal equation (generalization of Dirac’s and Rarita-Schwinger’s) \(\mathcal{F}_r |\phi\rangle = 0\) that is gauge-invariant (for \(S > 3/2\)) if \(|\xi\rangle\) is gamma-traceless and (for \(S > 5/2\)) \(|\phi\rangle\) is triple gamma-traceless.
4. Conclusions
We described an alternative approach to the study of higher spin fields, based on locally supersymmetric $O(N)$-extended spinning particle models. In particular, we showed that the first class constraints lead to the geometric formulation of higher spin fields and we computed linearized curvatures, in $D$-dimensional maximally symmetric spaces and discussed gauge invariance and linearized free equations of motion associated to the vanishing of the (gamma) trace of the curvature. The models discussed above describe a set of mixed-symmetry higher spin potentials corresponding to Young tableaux with $D-1$ rows and $S$ columns, thus reducing to totally symmetric tensors in $D = 4$. We also showed that the constraint algebra remains first class for the whole class of conformally flat spaces.

References
[1] Francia D and Sagnotti A 2002 Phys. Lett. B543 303–310 (Preprint hep-th/0207002)
[2] Francia D and Sagnotti A 2003 Class. Quant. Grav. 20 S473–S486 (Preprint hep-th/0212185)
[3] Weinberg S 1965 Phys. Rev. 138 B988–B1002
[4] de Wit B and Freedman D Z 1980 Phys. Rev. D21 358
[5] Vasiliev M A 2004 Fortsch. Phys. 52 702–717 (Preprint hep-th/0401177)
[6] Sorokin D 2005 AIP Conf. Proc. 767 172–202 (Preprint hep-th/0405069)
[7] Gershun V D and Tkach V I 1979 JETP Lett. 29 288–291
[8] Howe P S, Penati S, Pernici M and Townsend P K 1988 Phys. Lett. B215 555
[9] Howe P S, Penati S, Pernici M and Townsend P K 1989 Class. Quant. Grav. 6 1125
[10] Marnelius R 2009 (Preprint 0906.2084)
[11] Bastianelli F, Corradini O and Latini E 2008 JHEP 11 054– (Preprint 0810.0188)
[12] Corradini O 2010 JHEP 09 113 (Preprint 1006.4452)
[13] Bastianelli F, Corradini O and Latini E 2007 JHEP 02 072– (Preprint hep-th/0701055)
[14] Bandos I A, Lukierski J and Sorokin D P 2000 Phys. Rev. D61 045002 (Preprint hep-th/9904109)
[15] Bandos I, Bekaert X, de Azcarraga J A, Sorokin D and Tsulaia M 2005 JHEP 05 031 (Preprint hep-th/0501113)
[16] Bastianelli F and Bonezzi R 2009 JHEP 03 063 (Preprint 0901.2311)
[17] Bastianelli F and Bonezzi R 2010 JHEP 05 020 (Preprint 1003.1046)
[18] Bastianelli F, Corradini O and Waldron A 2009 JHEP 05 017– (Preprint 0902.0530)
[19] Marnelius R 1979 Phys. Rev. D20 2091
[20] Siegel W 1988 Int. J. Mod. Phys. A3 2713–2718
[21] Fronsdal C 1979 Phys. Rev. D20 848–856
[22] Fronsdal C 1978 Phys. Rev. D18 3624
[23] Labastida J M F 1989 Nucl. Phys. B322 185
[24] Fang J and Fronsdal C 1978 Phys. Rev. D18 3630
[25] Fang J and Fronsdal C 1980 Phys. Rev. D22 1361

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