Intermediate psuedoscalar resonance contributions to $B \to X_s \gamma \gamma$

Mohammad R. Ahmady$^a$ *, Emi Kou$^b$ † and Akio Sugamoto$^b$ ‡

$^a$LINAC Laboratory, The Institute of Physical and Chemical Research (RIKEN)
2-1 Hirosawa, Wako, Saitama 351-01, Japan

$^b$ Department of Physics, Ochanomizu University
1-1 Otsuka 2, Bunkyo-ku, Tokyo 112, Japan

(August 1997)

Abstract

We calculate the decay rate $\Gamma(B \to X_s \gamma \gamma)$ via intermediate psuedoscalar charmonium $\eta_c$. This process is thought to be the main long-distance contribution which dominates the corresponding inclusive rare B decay. We point out that once the momentum dependence of $\eta_c \to \gamma \gamma$ conversion strength, due to off-shellness of the intermediate $\eta_c$, is taken into account, the rate of this decay mode is reduced. The change in differential decay rate in region of spectrum immediately below $m_{\eta_c}$ is more significant. On the other hand, we point out that the unexpectedly large branching ratio for $B \to X_s \eta'$ which is recently observed by CLEO could indicate a potentially larger long-distance contribution via $\eta'$ resonance.

*Email: ahmady@riken.go.jp
†Email: kou@fs.cc.ocha.ac.jp
‡Email: sugamoto@phys.ocha.ac.jp
The rare decays of B meson are quite interesting since, among other reasons, their measurement could point to some clues on physics beyond Standard Model. For this purpose, however, it is essential to isolate the short-distance (SD) contributions from the long-distance (LD) background as only the former is sensitive to virtual exotic particles from "new physics". For example, the rare dileptonic B decay \( B \to X_s \ell^+\ell^- (\ell = \mu, e) \) in which the total decay rate is dominated by the LD resonance contribution from intermediate \( \psi \) and \( \psi' \) vector mesons, much theoretical attentions have been focused on searching for observables and regions of various decay distributions where one can probe SD physics without significant LD interference [1,2]. Therefore, for the purpose of extracting reliable conclusions on parameters of Standard Model and beyond from rare B decays, careful consideration of LD effects is of utmost importance.

In this paper, we examine the pure LD contribution to \( B \to X_s \gamma\gamma \) via intermediate pseudoscalar \( \eta_c \). This is expected to be the main process dominating the corresponding flavor changing neutral current (FCNC) decay channel [3]. However, we show that a careful consideration of the off-shellness of the intermediate \( \eta_c \) could reduce the contribution of this LD mode. In fact, a model calculation of \( \eta_c \)-photon-photon coupling reveals a drastic suppression of this vertex form factor \( f(q^2) \) when \( q^2 \), the invariant mass of the two photons, is small as compared to when \( q^2 \approx m_{\eta_c}^2 \). Indeed, similar suppression of \( \psi \)-\( \gamma \) conversion strength on photon mass shell as compared to when \( \psi \) is on its mass-shell was shown to be responsible for significant reduction of resonance to non-resonance interference in dileptonic rare decays \( B \to X_s \ell^+\ell^- \) [2]. Inclusion of this form factor for \( \eta_c \to \gamma\gamma \) in the evaluation of \( B \to X_s \gamma\gamma \) via \( \eta_c \) is our main point in this paper. We also comment on the possibility that the recent CLEO measurement of a large \( B \to X_s \eta' \) decay rate could indicate a potentially larger LD contribution to \( B \to X_s \gamma\gamma \) via \( \eta' \).

We start by writing the effective Hamiltonian for LD \( B \to X_s \gamma\gamma \) via \( \eta_c \) which is approximated by quark level decay \( b \to s\eta_c \to s\gamma\gamma \). The \( \eta_c \to \gamma\gamma \) transition is modeled by a triangle quark loop (Fig. 1) where the coupling of the pseudoscalar meson \( \eta_c \) to charm quarks is taken to be a constant:
where $p_1$ and $p_2$ are the four-momenta of the photons and $q = p_1 + p_2$. The form factor $f(q^2)$ is obtained from the quark loop calculation:

$$f(q^2) = \int_0^1 dx \int_0^{1-x} dy \frac{1}{m_c^2 - q^2 x y}$$

$$= \begin{cases}
-\frac{2}{q^2} \arcsin^2 \sqrt{\frac{q^2}{4m_c^2}} & 0 \leq q^2 \leq 4m_c^2 \\
\frac{2}{q^2} \left[ \ln \left( \sqrt{\frac{q^2}{4m_c^2}} + \sqrt{\frac{q^2}{4m_c^2} - 1} \right) - \frac{I\pi}{2} \right]^2 & 4m_c^2 \leq q^2
\end{cases}$$

where $m_c$ is the charm quark mass. The constants are all swept into the factor $N$ which can be obtained using the requirement that for $q^2 = m_{\eta_c}^2$ eqn. (1) should yield the experimentally measured decay rate $\Gamma(\eta_c \to \gamma\gamma)$. Consequently, we obtain the following form for the $\eta_c \to \gamma\gamma$ transition amplitude:

$$A(\eta_c \to \gamma\gamma) = \frac{16i}{\pi^{3/2}} \frac{\sqrt{m_{\eta_c}} \Gamma(\eta_c \to \gamma\gamma)}{f(q^2)\epsilon_{\mu}(p_1)\epsilon_{\nu}(p_2)p_1\alpha p_2\beta}.$$  (3)

$\epsilon(p_i)$ is the polarization of the photon with momentum $p_i$. Throughout this paper, we assume weak binding for charmonium and therefore $m_{\eta_c} \approx 2m_c$ is used in our calculations.

Neglecting penguin operators, the relevant effective Hamiltonian for $B \to X_s \eta_c$ can be written as:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} \left[ \left( C_2(\mu) + \frac{1}{3} C_1(\mu) \right) \bar{s} \gamma^\mu (1 - \gamma_5) b^\dagger \bar{c} \gamma_\mu (1 - \gamma_5) c^j \right.$$  (4)

$$+ C_1(\mu) \bar{s} \gamma^\mu (1 - \gamma_5) T_{a}^{i} b^a \bar{c} \gamma_\mu (1 - \gamma_5) T^{a}_{jm} c^m \right] + H.C.,$$

where $i$ and $j$ are color indices, $T^a$ ($a = 1..8$) are generators of $SU(3)_{\text{color}}$, and $C_1(\mu)$ and $C_2(\mu)$ are QCD improved Wilson coefficients. Consequently, assuming factorization, the matrix element for the underlying quark level decay can be simplified as follows:

$$< \eta_c | H_{\text{eff}} | b > = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} \left( C_2(\mu) + \frac{1}{3} C_1(\mu) \right) f_{\eta_c} \bar{s} \gamma^\mu (1 - \gamma_5) b q_\mu$$

$$= \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} \left( C_2(\mu) + \frac{1}{3} C_1(\mu) \right) f_{\eta_c} \left[ -m_s \bar{s}(1 - \gamma_5) b + m_b \bar{s}(1 + \gamma_5) b \right].$$  (5)

In writing (5), the following definition for pseudoscalar decay constant has been utilized:
Using (3) and (5), the amplitude for $B \to X_s \gamma \gamma$ via $\eta_c$ can be expressed as:

$$A_{LD}(\eta_c)(B \to X_s \gamma \gamma) = C f(q^2) \bar{s}(\eta_c) (1 - \gamma_5) \Gamma_{\eta_c}(\eta_c \to 2) V_{cb}^* V_{cs} \left( C_2(\mu) + \frac{1}{3} C_1(\mu) \right) f_{\eta_c},$$

where

$$C = \frac{16iG_F}{\sqrt{2} \pi^{3/2}} \sqrt{m_{\eta_c}} \sqrt{\Gamma_{\eta_c}(\eta_c \to 2)} V_{cb}^* V_{cs} \left( C_2(\mu) + \frac{1}{3} C_1(\mu) \right) f_{\eta_c},$$

and $\Gamma_{\eta_c}$ is the total decay width of $\eta_c$. It is then straightforward to calculate the differential decay rate from (7):

$$d\Gamma^{LD(\eta_c)}(B \to X_s \gamma \gamma) = \frac{m_b}{512 \pi^3} |C|^2 |\bar{f}(s)|^2 \frac{s^2}{(s - y)^2 + y^2 \frac{f_{\eta_c}(y)}{m_{\eta_c}^2}} \left[ (1 - x)^2 - s(1 + x) \right] (1 - x),$$

where

$$|\bar{f}(s)|^2 = \frac{4}{s^2} \begin{cases} \arcsin^4 \frac{s}{y} & 0 \leq s \leq y \\ \ln^2 \left( \sqrt{\frac{s}{y} + \sqrt{\frac{s}{y} - 1} + \frac{s^2}{4}} \right) & y \leq s \end{cases}. \tag{10}$$

The dimensionless quantities $s, x$ and $y$ are $q^2/m_b^2, m_s^2/m_b^2$ and $m_{\eta_c}^2/m_b^2$, respectively. In Fig. 2, the form factor $|\bar{f}(s)|^2$ normalized to its value on $\eta_c$ mass-shell $|\bar{f}(y)|^2$ is depicted. We notice that for values of $q^2$ immediately below $m_{\eta_c}^2$, $|f(q^2)|^2$ decreases steeply from $|f(q^2 = m_{\eta_c}^2)|^2$. In fact, $|f^2(0)|^2/|f^2(q^2 = m_{\eta_c}^2)|^2 \approx 0.16$ which indicates that the momentum dependence of this form factor is quite significant. As we mentioned before, a similar mechanism for intermediate vector mesons $\psi$ and $\psi'$ is believed to suppress the LD contributions to rare decay $B \to X_s \gamma \ell^+ \ell^-$ and significantly reduces the resonance to nonresonance interference in dileptonic rare B decays $B \to X_s \ell^+ \ell^-$. In our numerical calculations $m_b, m_s$ and $f_{\eta_c}$ are taken as 4.8, 0.5 and 0.48 GeV, respectively, and $C_1(\mu) + 1/3 C_2(\mu) = 0.155$ for $\mu \approx m_b$ is adopted from next-to-leading order calculation. Using
from semileptonic $B$ decay $B \rightarrow X_c \ell \bar{\nu}_\ell$, we obtain

$$BR^{LD(\eta_c)}(B \rightarrow X_s \gamma \gamma) = 9.1 \times 10^{-7},$$

which is of the same order of magnitude as the estimated $(2 - 8) \times 10^{-7}$ SD contributions [3]. At this point, we would like to remark that the total LD branching ratio (via $\eta_c$) is dominated by the peak at $s = m_{\eta_c}^2/m_b^2$. However, had we inserted the constant form factor $f(q^2 = m_{\eta_c}^2)$ in (9) rather than $f(q^2)$, a larger branching ratio $BR^{LD(\eta_c)}(B \rightarrow X_s \gamma \gamma) = 10.1 \times 10^{-7}$ would have been resulted. In fig. 3, the invariant mass distribution of the decay rate has been shown (eqn. (9)) and for comparison, the case where $f(q^2 = m_{\eta_c}^2)$ replaces $f(q^2)$ in (9) is depicted as well. We observe that due to the significant decrease in $\eta_c$-photon-photon coupling strength for $q^2$ values immediately below $m_{\eta_c}^2$, the LD differential decay rate in this region is further reduced. For example, even at $s = 0.2$ which is not too close to the resonance, the reduction factor is around $1/3$. This means that a wider range of invariant mass spectrum below $\eta_c$ resonance $0 \leq s \leq 0.39$ is available for probing SD physics.

Now, we turn to another potentially large source of LD contribution to $B \rightarrow X_s \gamma \gamma$. Recently, CLEO discovered an unexpectedly large branching ratio for $B \rightarrow X_s \eta'$ [6]

$$BR(B \rightarrow X_s \eta') \quad 2.2 \leq E_{\eta'} \leq 2.7 \text{GeV} = (7.5 \pm 1.5 \pm 1.1) \times 10^{-4}.$$  

This decay mode can contribute to LD $B \rightarrow X_s \gamma \gamma$ by subsequent $\eta'$ decay to two photons. To make a rough estimate of this contribution, we compare the relevant branching ratios for $\eta'$ and $\eta_c$ cases:

$$\frac{BR(B \rightarrow X_s \eta')BR(\eta' \rightarrow \gamma \gamma)}{BR(B \rightarrow X_s \eta_c)BR(\eta_c \rightarrow \gamma \gamma)} \approx 6,$$

where $BR(B \rightarrow X_s \eta_c) \approx 8.7 \times 10^{-3}$ has been used [4]. This could be an indication that the LD $B \rightarrow X_s \gamma \gamma$ via $\eta'$ may surpass that of $\eta_c$. However, to calculate this contribution more accurately, we need to know the mechanism of such a large $\eta'$ production in nonleptonic $B$
decay. Unfortunately, so far a clear process for $B \to X_s \eta'$ has not been established yet and there are various reservations with regard to suggested mechanisms \cite{8}. In any case, a large peak at low $s = m_{\eta'}/m_b^2 \approx 0.04$ is expected.

In conclusion, we calculated the LD decay process $B \to X_s \gamma\gamma$ via $\eta_c$ taking into account the momentum dependence of $\eta_c$-photon-photon vertex for off mass-shell $\eta_c$. We indicated that a wider range of the invariant mass spectrum below $\eta_c$ resonance could be available for probing SD physics. However, the decay mode via $\eta'$ could be a larger source of LD background to $B \to X_s \gamma\gamma$ rare decay which ought to be investigated once the mechanism for $B \to X_s \eta'$ is established.

Acknowledgement

The authors would like to thank T. Morozumi for useful discussions. M. A. acknowledges support from the Science and Technology Agency of Japan under an STA fellowship.
REFERENCES

[1] See for example, A. Ali, T. Mannel and T. Morozumi, Phys. Lett. B237 (1991) 505;
P. J. O’Donnell, M. Sutherland and H. K. K. Tung, Phys. Rev. D46 (1992) 4091.

[2] M. R. Ahmady, Phys. Rev. D53 (1996) 2843.

[3] L. Reina, G. Ricciardi and A. Soni, hep-ph/9706253.

[4] N. G. Deshpande, Xiao-Gang He and J. Trampetic, Oregon University Report No. OITS-564 (unpublished);
   G. Eilam, A. Ioannissian, R. R. Mendel and P. Singer, Phys. Rev. D53 (1996) 3629.

[5] A. J. Buras, Nucl. Phys. B434 (1995) 606.

[6] P. Kim (CLEO), talk at FCNC 97, Santa Monica CA (Feb, 1997).

[7] M. R. Ahmady and E. Kou, hep-ph/9701224, to appear in Z. Phys. C.

[8] D. Atwood and A. Soni, Phys. Lett. B405 (1997) 150;
   I. Halperin and A. Zhitnitsky, hep-ph/9705251;
   W.-S. Hou and B. Tseng, hep-ph/9705304;
   A. L. Kagan and A. A. Petrov, hep-ph/9707354.
Figure Captions

**Figure 1**: The triangle quark loop diagram for $\eta_c$-photon-photon coupling.

**Figure 2**: The variation of the form factor $|\tilde{f}(s = q^2/m_b^2)|^2$ normalized to $|\tilde{f}(y = m^2_{\eta_c}/m_b^2)|^2$ as a function of $s$.

**Figure 3**: The invariant mass spectrum of the two photons in $B \to X_s \gamma \gamma$ decay for momentum dependent (solid line) and constant (dashed line) $\eta_c$-photon-photon coupling. The differential branching ratio is in units of $10^{-7}$.
Figure 1

\[\eta_c \quad q = p_1 + p_2\]
Figure 3

\[ \frac{dBR}{ds} \times 10^7 \]