Coupling of a free wake vortex ring near-wake model with the Jensen and Larsen far-wake deficit models

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Abstract. This paper presents a simple physical model to improve the currently used far-wake deficit models in the wind industry. The main improvement is deemed on the determination of the wake deficit in the near-wake. A Vortex Ring Model (VRM) is used to calculate the induced velocities in the near-wake, which are then coupled to the Jensen far-wake model and the Larsen far-wake model based on the concept of Eddy Viscosity (EV). The inviscid near-wake VRM is based on the shedding of discrete tip vortex rings released from a uniformly loaded actuator disc. The model is validated against wind tunnel measurements from experiments with a two-bladed turbine and a circular metal mesh with a uniform porosity to represent an actuator disc. The VRM shows a good agreement with the experimental data with respect to the wake deficit evolution. The VRM is coupled with two well-known engineering type far-wake models: the Jensen and Larsen wake deficit models. The results of the coupling of the VRM and the more elaborated Larsen far-wake model are compared against a 3D Large Eddy Simulation (LES) CFD model. This comparison shows the effect of different near-wake models on the development of centreline velocities in the far-wake. The centreline velocity deficit predicted by the VRM-Larsen model more closely matches LES calculations in comparison with the reference Larsen model.

1. Introduction
Most well-known engineering type wind farm wake models include a fairly simple approximation of the near-wake. This is often a combination of an assumed velocity deficit profile (e.g. square hat or Gaussian) and empirical data or a simple streamtube approach based on the conservation of momentum. Disadvantages of using empirical based near-wake models is the uncertainty from measurements that are already introduced at the start of the wake field and the requirement of calibrated coefficients corresponding to the applied case. An analytical, physics based model, independent of empirical constants and rotor blade parameters, could provide a better input to far-wake approximations. An approach by Segalini & Alfredsson [1] using a helical vortex approach is promising, but a simpler engineering type solution was sought.

Discrete vortex rings to describe the near-wake have been used by Baldacchino and van Bussel [2] to investigate the effect of yawed inflow and the existence of ground effects on the near-wake development. In the model, the turbine rotor is replaced by an actuator disc, which sheds discrete vortex rings. The rings are free to move, grow and interact. The resulting set of vortex rings can be used to describe a full 3D velocity field.
In the present paper, the vortex ring model (VRM) is combined with the Jensen [3] and Larsen [4] far-wake models. The Jensen model is selected because it is a widely used engineering model and has a relative simple implementation as compared to other wake models. As a second model, Larsen’s Eddy Viscosity (EV) based far-wake model is selected because it is a more advanced model and offers compatibility with the VRM approach. The coupling could lead to a model with improved accuracy over the complete velocity field compared to the streamtube near-wake model, which is used in the original Larsen far-wake model. This paper discusses a stationary wake, but the VRM supports fluctuations in wind speed and wind direction [5], yawed flow [2] and dynamic loads on the actuator disc [6].

The approach to combine the VRM with the Larsen and Jensen far-wake models is discussed in this paper. The near wake vortex ring model is validated against wind tunnel measurements of (i) a two-bladed rotor model and (ii) a porous disc representing an actuator disc. The results of the VRM-Larsen model are compared to a Large Eddy Simulation (LES) CFD model, which is provided by the Energy Research Centre of the Netherlands (ECN). Lastly, a multiple wake simulation is briefly studied (velocity and power deficits) by applying the VRM-Larsen model to a case from the Horns Rev wind farm.

2. Derived numerical models

2.1. The vortex ring model

In the VRM, the wind turbine rotor is replaced by a uniformly loaded actuator disc, which sheds infinitely thin, discrete vortex rings (see figure 1). The rings are assumed to be inviscid and perfectly circular. The vortex rings are placed instantaneously in the free-stream after constant periods of time $\Delta t$. They have mutual influences on their displacement and expansion/contraction.

![Figure 1. Schematic illustration of an actuator disc releasing a vortex ring](image)

![Figure 2. Coordinate system used on a pair of vortex rings in the free-stream](image)

2.1.1. Derivation of the velocity field from a set of vortex rings

The series of vortex rings can be used to calculate an axi-symmetric 3D velocity field. For an arbitrary point $P(r, \theta, z)$, the induced velocity from a vortex ring can be described by the set of equations displayed in equations 1 and 2, derived by Yoon and Heister [7] using the Biot-Savart law. These set of equations are valid on every point in the field, except on the vortex ring itself (where $R_i = r$ and $z = z_i$), which results in a singularity.

$$u_z = \frac{\Gamma_i}{2\pi \sqrt{((z - z_i)^2 + (r + R_i)^2)}} \left[ K(m) + \frac{R_i^2 - r^2 - (z - z_i)^2}{(z - z_i)^2 + (R_i - r)^2} E(m) \right]$$

(1)
which takes into account the ring’s core size by substituting \[2\], the self-induced velocity is derived by determining the velocity at the centre of the ring other vortex rings. Following the approach by Yu et al. [6] and Baldacchino and van Bussel the undisturbed wind speed, the self-induced velocity and the mutually induced velocity from wake, is determined from the axial velocity of the ring. This velocity is derived by superimposing 2.1.3. Numerical implementation of the free wake VRM The position of a vortex ring in the free experiment discussed in section 3 and \(\Delta t = 0.5s\) is used for the large rotor in the LES case discussed in section 5.

2.1.2. An approach to determine vortex ring strength Van Kuik [8] derived a relation between the pressure jump \(\Delta p\) at the edge of an actuator disc and the shedding of vorticity in the flow. He used the continuity and momentum equations for inviscid and incompressible fluid to derive the following equation:

\[
\frac{\partial \Gamma_{\text{edge}}}{\partial t} = \frac{\Delta p}{\rho}
\]

Following the approach of Yu et al. [6], equation 3 is applied on a series of vortex rings. The pressure jump at the actuator disc is described as the thrust divided by the area. Using the definition of \(C_T\), this can be written as:

\[
\Delta p = \frac{T}{A} = \frac{1}{2} \rho C_T U^2 A = \frac{1}{2} \rho C_T U^2 \infty
\]

Substituting this into equation 3 gives the final expression of the vortex ring strength:

\[
\Gamma_i = \frac{1}{2} U^2_\infty C_T \Delta t
\]

Here, \(\Delta t\) represents the period between the shedding of two vortex rings. Initially, \(\Delta t\) was linked to the revolution of a blade in an approach to simulate a realistic tip vortex pitch, constraining the ring radii by following an empirical wake expansion model as in [2]. The expansion model is not in good agreement with results from wind tunnel experiments, so the expansion model was disregarded and induced vortex ring expansion is used instead. This results in a much smaller value for \(\Delta t\) to guarantee the stability of the simulation when the vortex rings start interacting further down-stream. As a reference, a value of \(\Delta t = 0.01s\) is used for the small rotor in the experiment discussed in section 3 and \(\Delta t = 0.5s\) is used for the large rotor in the LES case discussed in section 5.

2.1.3. Numerical implementation of the free wake VRM The position of a vortex ring in the free wake, is determined from the axial velocity of the ring. This velocity is derived by superimposing the undisturbed wind speed, the self-induced velocity and the mutually induced velocity from other vortex rings. Following the approach by Yu et al. [6] and Baldacchino and van Bussel [2], the self-induced velocity is derived by determining the velocity at the centre of the ring by substituting \(z = z_1\) and \(r = 0\) in equations 1 and 2. This results in the simple equation \(u_s = \Gamma/(2R)\). Baldacchino [9] compared this method with the more well-known Kelvin Theorem which takes into account the ring’s core size \(a\). His research showed that for small core sizes \((a/R < 0.01)\), the differences between the two methods is negligible, thus the more simple approach is used since the rings are assumed infinitely thin. The mutually induced velocity of a vortex ring is determined by the total induced axial velocity on an arbitrary point on the vortex ring. An example of a pair of vortex rings is displayed in figure 2. In this case, the induced velocity on vortex ring 2 by vortex ring 1 is computed with:

\[
u_{m,2} = \frac{\Gamma_1}{2\pi \sqrt{[(z_2 - z_1)^2 + (R_2 + R_1)^2]} \cdot \left[ K(m) + \frac{R^2_1 - R^2_2 - (z_2 - z_1)^2}{(z_2 - z_1)^2 + (R_1 - R_2)^2} E(m) \right]}
\]
This procedure is repeated for all \( n \) vortex rings, which concludes the derivation of the translational velocity of a single ring with:

\[
u_i = U_\infty + \nu_s + \nu_m = U_\infty + \frac{\Gamma_i}{2R_i} + \sum_{j=1}^{n} \nu_{m,j} \tag{7}\]

The position of the vortex rings is then updated with a second order update scheme. The wake expansion is modelled by allowing the rings to expand and contract freely. The expansion of a ring is determined with the same method as the mutually induced axial velocity, except the radial velocity is now determined on the ring. The numerical timestep of the simulation is determined as 0.5\( \Delta t \).

The induced velocities result in vortex ring interaction. Figure 3 and 4 illustrate the distribution of vortex rings in the wake for two typical cases. The graphs display the effect of the thrust on vortex ring interaction. As expected, a higher thrust coefficient results in stronger vortex rings, which interact at an earlier stage compared to an actuator with a lower thrust. A converged solution is obtained by allowing the rings to travel far down-stream (> 20\( D \)) whereas only the velocities in the near-wake are analysed, far from the starting vortices.

![Figure 3. Simulated vortex ring distribution with \( C_T = 5/9 \)](image1)

![Figure 4. Simulated vortex ring distribution with \( C_T = 8/9 \)](image2)

2.2. The Large Eddy Simulation model

The coupled VRM and far-wake models are compared with a LES based Energy-Conserving Navier-Stokes (ECNS) solver, which is a CFD code developed by ECN. It discretises the incompressible Navier-Stokes equations on a uniform Cartesian grid and solves them using a Finite-Volume approach. Central to the code is the use of energy-conserving schemes that guarantee the absence of numerical dissipation, not only through spatial-discretisation but also through temporal-integration [10]. With wind turbine or farm simulations, the presence of numerical dissipation is known to spuriously speed-up wake recovery, leading to the under prediction of wake deficit, which is averted in the ECNS code. To model the large, energy-containing scales of the flow, the ECNS uses the Smagorinsky model with the Smagorinsky constant \( C_S \), set to 0.15 [11]. For simulating a single turbine, the inflow is assigned a simple Dirichlet condition, whereas the rest of the boundaries are outflow boundaries that prohibit back-flow. The turbines are modelled as uniformly loaded actuator disks that are introduced in the computational domain as body forces. Furthermore, the time-step is set to 0.01s to ensure that the solution is independent of the time-step. Further details regarding the LES code can be found in Mehta et al. [12].
3. Validation of the VRM with wind tunnel experiments

Experiments with a two-bladed rotor and an actuator disc represented by a porous mesh were carried out by Lignarolo et al. [13]. They provided the velocity field of the wake of the experiment for a comparison with the VRM. The experiments were conducted in the Open Jet Facility (OJF) at the Delft University of Technology. The experiments were conducted with a high thrust coefficient of $C_T = 0.93$, which is above normal operational conditions. This relatively high load stresses the model and approaches the limit. Higher values for $C_T$ will result in instabilities with heavy ring interaction and rings travelling upstream. An overview of remaining parameters of the experiment is presented in table 1. Figure 5 shows velocity profiles at various distances down-wake from the rotor disk. Each graph shows the actuator and rotor measurements from the wind tunnel and the simulation results, in which a distinction is made between time-averaged (TA) and instantaneous (I) results. The time-averaged results show velocities averaged over two time periods $2\Delta t$, with a sample rate of 500 samples per period. This is done to verify if the instantaneous velocity field is representative for a time independent period. A condition is set to disregard a ring when it is within $0.02D$ of the axial position where induced velocities are determined to avoid the ring singularities. The instantaneous results were taken at the end of the simulation, where the vortex rings have reached a minimum distance of $20D$ and the exact axial position was taken between the closest two vortex rings. Wake deficits and expansion rates are in reasonable agreement between the VRM and the experimental results. The results of the instantaneous velocity field is in accordance with the time-averaged values. The instantaneous velocity field simulation has a much shorter computational period which is a significant advantage over the time-averaged method. Vortex ring expansion and interaction can be seen in figures 6 and 7, where the positions of the vortex rings at a fixed time instant are overlaid over the average velocity field of the experimental turbine and porous disc respectively. The display of strong vortex ring interaction seems to correspond with the destabilisation and growth of the wake shear layer, where the near-wake transitions to the turbulent far-wake. Further investigation would however be required to understand the relation between the mutual instabilities predicted by the dynamic (inviscid) vortex model and wake breakdown observations. Theoretically, the VRM should show a closer relation to the actuator disc when compared to the rotor, since the vortex rings are shed from an actuator disc and wake rotation is not modelled in the VRM. Figure 5 shows that the velocities from the porous disc experiment are closer to the VRM as compared to the 2-bladed turbine, but the deficit is still a bit underestimated.

Table 1. Parameters of the wind tunnel experiment from Lignarolo et al. [13]

| Parameter                      | Wind Turbine | Actuator |
|-------------------------------|--------------|----------|
| Diameter                      | $D$          | 0.6 m    | 0.6 m    |
| Free-stream velocity          | $U_\infty$   | 4.7 m/s  | 4.7 m/s  |
| Rotational frequency          | $\omega$     | 109.3 rad/s | -        |
| Reynolds number (root)        | $Re_{cr}$    | 32,000   | -        |
| Reynolds number (tip)         | $Re_{ct}$    | 96,000   | -        |
| Reynolds number (diameter)    | $Re_D$       | 188,000  | 188,000  |
| Thrust coefficient            | $C_T$        | 0.93     | 0.93     |
| Tip speed ratio               | $\lambda$    | 6.97     | -        |
| Turbulence Intensity          | $TI$         | 0.5%     | 0.5%     |
4. Coupling the near and far-wake models
The VRM is coupled with the Jensen and Larsen wake deficit models and discussed in the following two sections. Results of the coupled simulation are discussed in section 5.

4.1. Coupling of the VRM with the Jensen model
The Jensen or PARK model is a engineering type far-wake model based on a linear expanding wake and the conservation of momentum [3], although Bastankhah and Porté-Agel have shown that it is actually only based on mass continuity [14]. The wake expansion is prescribed by the wake decay coefficient (WDC) $k$, which depends on the free-stream atmospheric turbulence and terrain roughness. The Jensen model does not distinguish the near- and far-wake. Although limited in physical representation, the model is popular due to its simplicity and low computational cost. For these reasons, the VRM is coupled to the Jensen model as a preliminary
investigation. The VRM is used to determine the axial velocities at the end of the near-wake. It is found that at a distance of 2D, the initial velocity deficit is converged when using $C_T = 8/9$, but this distance is of course sensitive to the operating $C_T$. This velocity deficit is then used to determine the disc averaged axial induction input to the Jensen model, resulting in an equal velocity at 2D. The Jensen model is used to calculate the far-wake from 2D onwards, with the solved value for $\alpha$ and the given value for $k$. The initial expansion of the wake is fitted to match the expansion from the VRM. In summary, the VRM is used to determine the initial induction velocities and expansion for the Jensen model.

4.2. Coupling of the VRM with the Larsen EV model

The VRM was coupled to the wake deficit model of the Dynamic Wake Meandering (DWM) model introduced by Larsen et al [4]. The structure of the deficit model introduced in the original DWM, consists of three parts: a uniform inflow or start deficit from the upstream turbine, an initial expansion and wake deficit, determined by a near-wake model and finally, a far-wake model based on the EV turbulent mixing concept by Ainslie [15]. In the original model, near-wake velocity deficits and edge expansion is determined with a streamtube momentum model or a blade element momentum (BEM) model, if blade characteristics are known. This results in an initial velocity deficit at a position where the pressure has fully recovered, often taken to be 2D downstream. The disadvantage of the streamtube model is that the deficit is only determined at the end of the near-wake at an assumed position. Simultaneously, a hat profile is taken as an initial far-wake profile, whereas experimental results clearly show substantial thickening of the shear layer towards the end of the near-wake. The BEM model results in a more realistic wake profile but requires blade parameters and pitch angles of the modelled wind turbine.

The Larsen model is well suited for coupling to the VRM, since the model explicitly distinguishes the near- and far-wake. In the present approach, the streamtube model is replaced by the VRM, from which the axial and radial velocities were determined at a position of 2D. This velocity field is then extracted as an initial condition for the EV far-wake. In a multiple wake simulation, an area weighted average of the velocity on the down-wake rotor is determined from the results of the far-wake simulation. This weighted average is subsequently used to determine the strength of the vortex rings released from the second turbine. The induced velocities from the series of vortex rings are then added to the already calculated velocity field from the previous turbine in a region from $-2D$ towards $+2D$ with respect to the down-wake turbine. This results in a two-way interaction of the VRM and the Larsen EV model.

In Larsen’s far-wake model, the turbulent mixing process is described with rotationally symmetric Navier-Stokes equations, where the pressure is assumed to be constant. The thin shear layer approximation is applied to the far-wake flow, i.e. radial flow derivatives are considered to be much larger than axial derivatives. This results in the momentum and continuity equations displayed in equation 8 and 9 in cylindrical coordinates.

$$\frac{U}{dz} + V \frac{dU}{dr} = -\frac{1}{r} \frac{d}{dr}(r \overline{uv})$$

$$\frac{V}{r} + \frac{dV}{dr} + \frac{dU}{dz} = 0$$

In these equations, $U$ represents the velocity in the axial direction ($z$) and $V$ the velocity in the radial direction ($r$). $\overline{uv}$ denote the temporal averaged fluctuating velocity components. Using the concept of EV, these Reynolds stresses are expressed as:

$$-\overline{uv} = \nu_t \frac{dU}{dr}$$
The EV term \( \nu_t \) consists of a laminar part, driven by the velocity difference across the shear layer and a turbulent part, based on the turbulence intensity, TI. It further includes filter functions for the build-up of turbulence and empirical constants. The latest derivation by Madsen et al. [16] and Keck et al. [17] was used in the present implementation of the Larsen model. The equations are discretised on a staggered uniform grid with a finite difference scheme, using an upwind scheme for the axial derivatives and a central scheme for the radial derivatives. This results in two non-homogeneous linear systems of equations for a single radial position.

5. Results of a VRM-Jensen and VRM-Larsen comparison with LES

The results for a single turbine case in laminar flow from the VRM-Larsen model are compared with the results from the LES model. For a single turbine, the domain is \( 12D, 8D, 8D \) in size and uses 120, 80, 80 grid points with the actuator disk located at \( 4D \), to ensure a stable inflow. The simulation was run with a laminar inflow, i.e. turbulence in the domain was solely due to the actuator disks, in accordance with the tests carried out with the VRM-Larsen model. The LES simulations were carried out on the Delft University of Technology Beowulf cluster.

![Figure 8. Normalised axial velocities along the centreline of the wake from the Jensen, Larsen, and LES simulations](image)

![Figure 9. Wake profiles at various positions in the wake from the Larsen, VRM-Larsen and LES simulations](image)

To approach the laminar inflow condition best in the Jensen model, a value for \( k \) of 0.03 was used, which is lower than the recommended low turbulence offshore value of 0.04. A value of 0 would simply result no wake recovery. The results can be seen in figure 8. The VRM-Jensen model generally results in a more gradual wake recovery as compared with the reference Jensen model. A lower value for \( k \) would perhaps result in closer fit with the LES, to which the model could be tuned to make it more suitable for laminar inflow. The coupling with the Jensen model was considered as a concept study and thus no further investigations were performed with it. A follow-up coupling with a more advanced model based on Jensen (e.g. the model derived by Bastankhah and Porté-Agel [14]) might lead to more favourable results.

The VRM-Larsen model shows a reasonable resemblance with the LES simulation in the near- and far-wake, as can be seen in figures 8 and 9. The streamtube near-wake model which was used in the Larsen model seems to overpredict the wake losses in the near-wake, which results in a lower centreline velocity in the far-wake.

The LES results show some unnatural behaviour along the edge of the rotor, where the velocity in the near-wake experiences a sudden drop which is visible in the first three wake
Figure 10. Velocities along the centreline for row E at Horns Rev in a full parallel wind direction

This is the result of the implementation of a circular actuator disc on a Cartesian grid of limited resolution. This could be circumvented by using a Gaussian distribution to apply the force more gradually on finite volumes, but this is not yet implemented in the ECNS code. A simulation with an increase in resolution on a smaller domain was carried out and showed a reduction of this effect. An increase of resolution on the same domain was not possible with the available computational power. The LES results also show a contraction of the wake at $8D$, which was not observed by the other models. This could be a side-effect of using laminar inflow or the used Smagorinsky model, which is dissipative in nature and possibly resulting in a quick dissipation of the turbulence created by the actuator disc.

The results from the LES calculations show that wake recovery due to turbine-generated turbulence in the far-wake starts later than the previously assumed distance of $2D$. Ideally, the VRM near-wake should be extended to a position between $2D - 4D$, however strong vortex ring interaction after $2D$ prohibited a converging velocity field in this region.

6. Discussion and recommendations from the LES results

The VRM-Larsen model has shown a reasonable accordance to a single wake LES simulation in laminar flow, with less than 1/100th of the required computational time. This encouraged further testing of the model and a multiple wake simulation was performed and compared with power deficit data from Horns Rev. A benchmarking case from Gaumond et al. [18] is used as a reference. The case involves a wind direction parallel to a wind farm row ($270^\circ$ West), with an uncertainty in the 10 minute average wind direction measurements of $\pm 2.5^\circ$. The array consists of 10 Vestas V80 wind turbines, spaced at $7D$. The thrust coefficient of the wind turbines is taken as $C_T = 0.805$ in accordance with the wind turbine parameters from Hansen et al. [19] and is assumed constant between 5 and 9 m/s. The undisturbed wind speed was measured as $8m/s \pm 0.5m/s$.

The velocities along the centreline through the array of wind turbines is displayed in figure 10 for the Larsen and VRM-Larsen models. The figure shows influence of the VRM on the wake of the upwind turbine. The model captures the deceleration of the flow in front of the rotor, whereas the Larsen model only carries information down-stream.

Figure 11 shows the normalised power output of the turbines for the specified case. Included
are the field measurements, results from the Jensen and the Larsen models as implemented by Gaumond and the VRM-Larsen and streamtube-Larsen models as implemented by the author. All models underpredict the power output of the wind turbines. Gaumond attributes this to uncertainty in the wind direction of the data, which is highly likely since it is 10 minute averaged and the western met mast is located relatively far from the wind farm, compared to the eastern met mast. The difference between Larsen by Gaumond and Streamtube-Larsen is initially surprising, since both models are based on the same EV model. However, the difference can be attributed to a number of uncertainties which are not addressed in detail in the paper by Gaumond. It is likely that the BEM near-wake model is used in Gaumond’s Larsen simulation since the turbine parameters were known. In this near-wake model, the hub is disregarded and the airflow is allowed to pass without interruption, which results in higher velocities in the centre of the wake. The exact empirical constants used in Gaumond’s EV-term are also unclear. These constants determine the relative influence from ambient turbulence in wake recovery and the results were found to be rather sensitive to variations in these empirical constants. The Larsen model by Gaumond might have been supplemented by correction models for turbine added turbulence or wake meandering from the DWM, which would have an effect on the power prediction. Lastly, the power prediction is perhaps averaged over different wind directions (±2.5°), which is not implemented in the streamtube-Larsen and VRM-Larsen models.

Due to these uncertainties, the comparison was found to be inconclusive and further analyses of the model is required. Ongoing work will investigate a two-turbine case and compare the VRM-Larsen model with the LES calculations, in order to better understand discrepancies in the models and observations. This will also make for a fairer comparison, since all the parameters used in the LES are known. Such an approach also presents the opportunity to implement yawed inflow cases, ground effects and wake meandering.

7. Conclusion
The near-wake model based on ideal inviscid vortex rings has been validated against wind tunnel measurements. The model shows a reasonable to good agreement with the actuator disc represented by a porous mesh in terms of development of wake deficit and expansion.

The model is successfully coupled to the Jensen and Larsen far-wake models. The VRM-Jensen model shows a more gradual wake recovery and was developed as a proof of concept.

The VRM provides a good alternative for the streamtube momentum model that is used in the Larsen DWM model. The VRM provides the ability to describe an axi-symmetric near-wake velocity field. The resulting thicker shear layer and different velocities at the end of the near-wake influence the far-wake in a positive way with respect to LES results in a laminar flow, single turbine case. The VRM-Larsen model approaches the LES, whilst maintaining an engineering-type computational cost.

In future work, a two-turbine case will be designed for a LES comparison. This provides the opportunity to implement other wake effects in the near-wake, e.g. yawed inflow and ground effect or transfer functions from the LES or DWM, e.g. meandering in the far-wake and rotor added turbulence.

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