Theoretical Rates for Direct Detection of SUSY Dark Matter

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Exotic dark matter together with the vacuum energy (associated with the cosmological constant) seem to dominate in the Universe. Thus its direct detection is central to particle physics and cosmology. Supersymmetry provides a natural dark matter candidate, the lightest supersymmetric particle (LSP). Furthermore from the knowledge of the density and velocity distribution of the LSP, the quark substructure of the nucleon and the nuclear structure (form factor and/or spin response function), one is able to evaluate the event rate for LSP-nucleus elastic scattering. The thus obtained event rates are, however, very low. So it is imperative to exploit the modulation effect, i.e. the dependence of the event rate on the Earth’s motion and the directional signature, i.e. the dependence of the rate on the direction of the recoiling nucleus. In this paper we study such experimental signatures employing a supersymmetric model with universal boundary conditions at large \( \tan \beta \).

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I. Introduction

In recent years the consideration of exotic dark matter has become necessary in order to close the Universe. The COBE data suggest that CDM (Cold Dark Matter) is at least 60%. On the other hand evidence from two different teams, the High-z Supernova Search Team and the Supernova Cosmology Project suggests that the Universe may be dominated by the cosmological constant \( \Lambda \). Thus the situation can be adequately described by a baryonic component \( \Omega_B = 0.1 \) along with the exotic components \( \Omega_{CDM} = 0.3 \) and \( \Omega_\Lambda = 0.6 \) (see next section for the definitions). In another analysis Turner gives \( \Omega_m = \Omega_{CDM} + \Omega_\Lambda = 0.4 \). Since the non exotic component cannot exceed 40% of the CDM there is room for the exotic WIMP’s (Weakly Interacting Massive Particles). In fact the DAMA experiment has claimed the observation of one signal in direct detection of a WIMP, which with better statistics has subsequently been interpreted as a modulation signal.

In the most favored scenario of supersymmetry the LSP can be simply described as a Majorana fermion, a combination of the neutral components of the gauginos and Higgsinos.
II. An Overview of Direct Detection - The Allowed SUSY Parameter Space.

Since this particle is expected to be very massive, \( m_\chi \geq 30\text{GeV} \), and extremely non relativistic with average kinetic energy \( T \leq 100\text{keV} \), it can be directly detected mainly via the recoiling of a nucleus \((A,Z)\) in elastic scattering. In order to compute the event rate one needs the following ingredients:

1) An effective Lagrangian at the elementary particle (quark) level obtained in the framework of supersymmetry as described, e.g., in Refs.\(^1\)\(^{14}\).

2) A procedure in going from the quark to the nucleon level, i.e. a quark model for the nucleon. The results depend crucially on the content of the nucleon in quarks other than \( u \) and \( d \). This is particularly true for the scalar couplings as well as the isoscalar axial coupling \(^{18}\)\(^{20}\).

3) Compute the relevant nuclear matrix elements\(^{22}\)\(^{23}\) using as reliable as possible many body nuclear wave functions. The situation is a bit simpler in the case of the scalar coupling, in which case one only needs the nuclear form factor.

Since the obtained rates are very low, one would like to be able to exploit the modulation of the event rates due to the earth's revolution around the sun\(^{24}\)\(^{25}\)\(^{27}\). To this end one adopts a folding procedure assuming some distribution\(^{1}\)\(^{25}\)\(^{27}\) of velocities for the LSP. One also would like to know the directional rates, by observing the nucleus in a certain direction, which correlate with the motion of the sun around the center of the galaxy and the motion of the Earth\(^{11}\)\(^{28}\).

The calculation of this cross section has become pretty standard. One starts with representative input in the restricted SUSY parameter space as described in the literature\(^{12}\)\(^{14}\). We will adopt a phenomenological procedure taking universal soft SUSY breaking terms at \( M_{\text{GUT}} \), i.e., a common mass for all scalar fields \( m_0 \), a common gaugino mass \( M_{1/2} \) and a common trilinear scalar coupling \( A_0 \), which we put equal to zero (we will discuss later the influence of non-zero \( A_0 \)'s). Our effective theory below \( M_{\text{GUT}} \) then depends on the parameters:

\[
\begin{align*}
    m_0, & \ M_{1/2}, \ \mu_0, \ \alpha_G, \ M_{\text{GUT}}, \ h_t, \ , \ h_b, \ , \ h_\tau, \ \tan \beta,
\end{align*}
\]

where \( \alpha_G = g^2_G / 4\pi \) (\( g_G \) being the GUT gauge coupling constant) and \( h_t, h_b, h_\tau \) are respectively the top, bottom and tau Yukawa coupling constants at \( M_{\text{GUT}} \). The values of \( \alpha_G \) and \( M_{\text{GUT}} \) are obtained as described in Ref\(^{1}\). For a specified value of \( \tan \beta \) at \( M_S \), we determine \( h_t \) at \( M_{\text{GUT}} \) by fixing the top quark mass at the center of its experimental range, \( m_t(m_t) = 166\text{GeV} \). The value of \( h_\tau \) at \( M_{\text{GUT}} \) is fixed by using the running tau lepton mass at \( m_Z \), \( m_\tau(m_Z) = 1.746\text{GeV} \). The value of \( h_b \) at \( M_{\text{GUT}} \) used is such that:
\[ m_b(m_Z)_{\text{SM}} = 2.90 \pm 0.14 \text{ GeV}. \]

after including the SUSY threshold correction. The SUSY parameter space is subject to the following constraints:

1.) The LSP relic abundance will satisfy the cosmological constrain:

\[ 0.09 \leq \Omega_{\text{LSP}} h^2 \leq 0.22 \]  

(1)

2.) The Higgs bound obtained from recent CDF and LEP2, i.e. \( m_h > 113 \text{ GeV} \).

3.) We will limit ourselves to LSP-nucleon cross sections for the scalar coupling, which gives detectable rates

\[ 4 \times 10^{-7} \text{ pb} \leq \sigma_{\text{nucleon}}^{\text{scalar}} \leq 2 \times 10^{-5} \text{ pb} \]  

(2)

We should remember that the event rate does not depend only on the nucleon cross section, but on other parameters also, mainly on the LSP mass and the nucleus used in target. The condition on the nucleon cross section imposes severe constraints on the acceptable parameter space. In particular in our model it restricts \( \tan \beta \) to values \( \tan \beta \simeq 50 \). We will not elaborate further on this point, since it has already appeared [31].

### III. Expressions for the Differential Cross Section

The effective Lagrangian describing the LSP-nucleus cross section can be cast in the form [3]

\[ L_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \{(\bar{\chi}_1 \gamma^\lambda \gamma_5 \chi_1) J_\lambda + (\bar{\chi}_1 \chi_1) J\} \]  

(3)

where

\[ J_\lambda = \bar{N} \gamma_\lambda (f_0^V + f_1^V \tau_3 + f_0^A \gamma_5 + f_1^A \gamma_5 \tau_3) \]  

\[ J = \bar{N} (f_0^V + f_1^V \tau_3) \]  

(4)

We have neglected the uninteresting pseudoscalar and tensor currents.

With the above ingredients the differential cross section can be cast in the form [3,23,24]

\[ d\sigma(u, v) = \frac{du}{2(\mu_r \mu_b)^2} \left[ (\Sigma_S + \Sigma_V \frac{v^2}{c^2}) F^2(u) + \Sigma_{\text{spin}} F_{11}(u) \right] \]  

(5)

\[ \Sigma_S = \sigma_0 \left( \frac{\mu_r(A)}{\mu_r(N)} \right)^2 \left( A^2 \left[ (f^S_0 - f^S_1 A - 2Z)^2 \right] \right) \approx \sigma_{p, \chi^0} A^2 \left( \frac{\mu_r(A)}{\mu_r(N)} \right)^2 \]  

(6)
\[ \Sigma_{\text{spin}} = \sigma_{p,\chi}^{\text{spin}} \zeta_{\text{spin}} \], \quad \zeta_{\text{spin}} = \frac{(\mu_r(A)/\mu_r(N))^2}{3(1 + f_A^2)} S(u) \quad (7) \]

\[ S(u) = \left[ \left( \frac{f_0}{f_A} \Omega_0(0) \right)^2 \frac{F_{00}(u)}{F_{11}(u)} + 2 \left( \frac{f_0}{f_A} \Omega_0(0) \Omega_1(0) \frac{F_{01}(u)}{F_{11}(u)} + \Omega_1(0) \right)^2 \right] \quad (8) \]

\[ \bar{\Sigma}_V = \sigma_{p,\chi}^V \zeta_V \quad (9) \]

\[ \zeta_V = \frac{(\mu_r(A)/\mu_r(N))^2}{(1 + f_V^2)^2} A^2 \left( 1 - \frac{f_V^2}{f_A^2} \frac{A - 2Z}{A} \right)^2 \left( \frac{\nu_0}{c} \right)^2 \left[ 1 - \frac{1}{(2\mu_r b)^2} (1 + \eta)^2 \left( \frac{2u}{c} \right)^2 \right] \quad (10) \]

\( \sigma_{p,\chi}^i = \text{proton cross-section, } i = S, \text{spin, } V \) given by:

\( \sigma_{p,\chi}^S = \sigma_0 (f_S^0)^2 \left( \frac{\mu_r(N)}{m_N} \right)^2 \) (scalar), \( \text{(the isovector scalar is negligible, i.e.} \sigma_{p,\chi}^S = \sigma_{p,\chi}^S \text{)} \)

\( \sigma_{p,\chi}^{\text{spin}} = \sigma_0 \left( f_A^0 + f_A^1 \right)^2 \left( \frac{\mu_r(N)}{m_N} \right)^2 \) (spin), \( \sigma_{p,\chi}^V = \sigma_0 \left( f_V^0 + f_V^1 \right)^2 \left( \frac{\mu_r(N)}{m_N} \right)^2 \) (vector)

where \( m_N \) is the nucleon mass, \( \eta = m_e/m_N A \), and \( \mu_r(A) \) is the LSP-nucleus reduced mass, \( \mu_r(N) \) the LSP-nucleon reduced mass and

\[ \sigma_0 = \frac{1}{2\pi} (G_F m_N)^2 \simeq 0.77 \times 10^{-38} \text{cm}^2 \quad (11) \]

\[ Q = Q_0 u \quad Q_0 = \frac{1}{Am_N b^2} = 4.1 \times 10^3 A^{-4/3} \text{ KeV} \quad (12) \]

where \( Q \) is the energy transfer to the nucleus and \( F(u) \) is the nuclear form factor.

In the present paper we will concentrate on the coherent mode. For a discussion of the spin contribution, expected to be important in the case of the light nuclei, has been reviewed elsewhere.[31]

**IV. Expressions for the Rates.**

The non-directional event rate is given by:
\[ R = R_{\text{non-dir}} = \frac{dN}{dt} = \frac{\rho(0)}{m_N} \frac{m}{A m_N} \sigma(u, v)|v| \]  
(13)

Where \( \rho(0) = 0.3\text{GeV/cm}^3 \) is the LSP density in our vicinity and \( m \) is the detector mass. The differential non-directional rate can be written as

\[ dR = dR_{\text{non-dir}} = \frac{\rho(0)}{m_N} m \sigma(u, v)|v| \]  
(14)

where \( d\sigma(u, v) \) was given above.

The directional differential rate in the direction \( \hat{e} \) is given by:

\[ dR_{\text{dir}} = \frac{\rho(0)}{m_N} m \sigma(u, v)|v| \frac{1}{2\pi} d\sigma(u, v) \]  
(15)

where \( \hat{e} \) is the direction to the Earth, \( H(\hat{e}) \) is the Heaviside step function. The factor of \( 1/2\pi \) is introduced, since the differential cross section of the last equation is the same with that entering the non-directional rate, i.e. after an integration over the azimuthal angle around the nuclear momentum has been performed. In other words, crudely speaking, \( 1/(2\pi) \) is the suppression factor we expect in the directional rate compared to the usual one. The precise suppression factor depends, of course, on the direction of observation. The mean value of the non-directional event rate of Eq. (14), is obtained by convoluting the above expressions with the LSP velocity distribution \( f(v, v_E) \) with respect to the Earth, i.e. is given by:

\[ \langle \frac{dR}{du} \rangle = \frac{\rho(0)}{m_N} m \sigma(u, v) \int f(v, v_E)|v| \frac{d\sigma(u, v)}{du} d^3v \]  
(16)

The above expression can be more conveniently written as

\[ \langle \frac{dR}{du} \rangle \approx \frac{\rho(0)}{m_N} m \sigma(u, v) \frac{d\Sigma}{du} \]  
(17)

where \( \alpha \) is the phase of the Earth (\( \alpha = 0 \) around June 2nd) and \( Q_{\text{min}} \) is the energy transfer cutoff imposed by the detector. In the above expressions \( \bar{R} t \) is the rate obtained in the conventional approach by neglecting the folding with
the LSP velocity and the momentum transfer dependence of the differential cross section, i.e. by

$$R = \frac{\rho(0)}{m_\chi} \frac{m}{Am} \sqrt{\langle v^2 \rangle} \left[ \Sigma_S + \Sigma_{\text{spin}} + \frac{\langle v^2 \rangle}{c^2} \Sigma_V \right]$$  \hspace{1cm} (19)$$

where $\Sigma_i$, $i = S, V, \text{spin}$ contain all the parameters of the SUSY models. The modulation is described by the parameter $h$.

The total directional event rates can be obtained in a similar fashion by by integrating Eq. (15) with respect to the velocity as well as the energy transfer $u$. We find

$$R_{\text{dir}} = \bar{R} \left[ \frac{t_{\text{dir}}}{2\pi} \left[ 1 + (h_1 - h_2) \cos \alpha \right] + h_3 \sin \alpha \right]$$ \hspace{1cm} (20)$$

where the quantity $t_{\text{dir}}$ provides the unmodulated amplitude, while $h_1, h_2$ and $h_3$ describe the modulation. They are functions of the angles $\Theta$ and $\Phi$, which specify the direction of observation $\hat{e}$, as well as the parameters $a$ and $Q_{\text{min}}$. The effect of folding with LSP velocity on the total rate is taken into account via the quantity $t_{\text{dir}}$, which depends on the LSP mass. All other SUSY parameters have been absorbed in $\bar{R}$. In the special case previously studied, i.e. along the coordinate axes, we find that: a) in the direction of the sun’s motion $h_2 = h_3 = 0$, b) along the radial direction (y axes) $h_3 = 0$ and c) in the vertical to the galaxy $h_2 = 0$. Instead of $t_{\text{dir}}$ itself it is more convenient to present the reduction factor of the unmodulated directional rate compared to the usual non-directional one, i.e.

$$f_{\text{red}} = \frac{R_{\text{dir}}}{R} = \frac{t_{\text{dir}}}{(2\pi t)} = \frac{\kappa}{(2\pi)}$$ \hspace{1cm} (21)$$

It turns out that the parameter $\kappa$, being the ratio of two rates, is less dependent on these parameters. Given the functions $h_l(a, Q_{\text{min}})$, $l = 1, 2, 3$, one can plot the the expression in Eqs (18) and 20 as a function of the phase of the earth $\alpha$.

V. The Scalar Contribution- The Role of the Heavy Quarks

The coherent scattering can be mediated via the the neutral intermediate Higgs particles (h and H), which survive as physical particles. It can also be mediated via s-quarks, via the mixing of the isodoublet and isosinglet s-quarks of the same charge. In our model we find that the Higgs contribution becomes dominant and, as a matter of fact the heavy Higgs H is more important (the Higgs particle $A$ couples in a pseudoscalar way, which does not lead to coherence).
is well known that all quark flavors contribute, since the relevant couplings are proportional to the quark masses. One encounters in the nucleon not only the usual sea quarks ($\bar{u}u, \bar{d}d$ and $ss$) but the heavier quarks $c, b, t$ which couple to the nucleon via two gluon exchange, see e.g. Drees et al. and references therein.

As a result one obtains an effective scalar Higgs-nucleon coupling by using effective quark masses as follows

$$m_u \rightarrow f_u m_N, \quad m_d \rightarrow f_d m_N, \quad m_s \rightarrow f_s m_N$$

$$m_Q \rightarrow f_Q m_N, \quad (\text{heavy quarks } c, b, t)$$

where $m_N$ is the nucleon mass. The isovector contribution is now negligible.

The parameters $f_q$, $q = u, d, s$ can be obtained by chiral symmetry breaking terms in relation to phase shift and dispersion analysis. Following Cheng and Cheng we obtain:

$$f_u = 0.021, \quad f_d = 0.037, \quad f_s = 0.140 \quad \text{(model B)}$$

$$f_u = 0.023, \quad f_d = 0.034, \quad f_s = 0.400 \quad \text{(model C)}$$

We see that in both models the s-quark is dominant. Then to leading order via quark loops and gluon exchange with the nucleon one finds:

$$f_Q = \frac{2}{27}(1 - \sum f_q), \quad \text{i.e.} \quad f_Q = 0.060 \quad \text{(model B)}, \quad f_Q = 0.040 \quad \text{(model C)}$$

There is a correction to the above parameters coming from loops involving s-quarks and due to QCD effects. Thus for large $\tan\beta$ we find

$$f_c = 0.060 \times 1.068 = 0.064, \quad f_t = 0.060 \times 2.048 = 0.123, \quad f_b = 0.060 \times 1.174 = 0.070 \quad \text{(model B)}$$

$$f_c = 0.040 \times 1.068 = 0.043, \quad f_t = 0.040 \times 2.048 = 0.082, \quad f_b = 0.040 \times 1.174 = 0.047 \quad \text{(model B)}$$

For a more detailed discussion we refer the reader to Refs.\cite{18,19,20}.

### VI. Results and Discussion

The three basic ingredients of our calculation were the input SUSY parameters (see sect. 1), a quark model for the nucleon (see sect. 3) and the velocity distribution combined with the structure of the nuclei involved (see sect. 2). We will focus our attention on the coherent scattering and present results for the popular target $^{127}$I. We have utilized two nucleon models indicated by B and C which take into account the presence of heavy quarks in the nucleon. We also considered energy cut offs imposed by the detector, by considering two typical cases $Q_{\text{min}} = 0, 10 \text{ KeV}$. The thus obtained results for the un modulated non directional event rates $\bar{R}$ in the case of the symmetric isothermal model for a typical SUSY parameter choice are shown in Fig. 7.
FIG. 1.: The Total detection rate per \((kg - target)yr\) vs the LSP mass in GeV for a typical solution in our parameter space in the case of \(^{127}I\) corresponding to model B (thick line) and Model C (fine line). For the definitions see text.

The two relative parameters, i.e. the quantities \(t\) and \(h\), are shown in Fig. 2 and Figs 3,4 respectively in the case of isothermal models.

FIG. 2.: The dependence of the quantity \(t\) on the LSP mass for the symmetric case (\(\lambda = 0\)) on the left as well as for the maximum axial asymmetry (\(\lambda = 1\)) on the right in the case of the target \(^{127}I\). For orientation purposes two detection cutoff energies are exhibited, \(Q_{min} = 0\) (thick solid line) and \(Q_{min} = 10\) keV (thin solid line). As expected \(t\) decreases as the cutoff energy and/or the LSP mass increase. We see that the asymmetry parameter \(\lambda\) has little effect on the un modulated rate.
FIG. 3.: The same as in Fig. 2 for the modulation with $\lambda = 0$. We see that the modulation is small and decreases with the LSP mass. It even changes sign for large LSP mass. The introduction of a cutoff $Q_{\text{min}}$ increases the modulation (at the expense of the total number of counts).

FIG. 4.: The same as in Fig. 3 for $\lambda = 1$. We see that the modulation increases with the asymmetry parameter $\lambda$.

The case of non isothermal models, e.g. caustic rings, is more complicated and it will not be further discussed here.

It is instructive to examine the reduction factors along the three axes, i.e. along $+z, -z, +y, -y, +x$ and $-x$. Since $f_{\text{red}}$ is the ratio of two parameters, its dependence on $Q_{\text{min}}$ and the LSP mass is mild. So we present results for $Q_{\text{min}} = 0$ and give an average as a function of the LSP mass (see Table
As expected the maximum rate is along the sun’s direction of motion, i.e. opposite to its velocity (−⟨z⟩) in the Gaussian distribution and +⟨z⟩ in the case of caustic rings. In fact we find that κ(⟨z⟩) is around 0.5 (no asymmetry) and around 0.6 (maximum asymmetry, λ = 1.0). It is not very different from the naively expected \( f_{\text{red}} = 1/(2\pi) = \kappa = 1 \). The asymmetry \(|R_{\text{dir}}(−) - R_{\text{dir}}(+)\|/(R_{\text{dir}}(−)+R_{\text{dir}}(+))\) is quite large in the isothermal model and smaller in caustic rings. The rate in the other directions is quite a bit smaller (see Table 4).

As we have seen the modulation can be described in terms of the parameters \( h_i, \ i = 1, 2, 3 \) (see Eq. (24)). If the observation is done in the direction opposite to the sun’s direction of motion the modulation amplitude \( h_1 \) behaves in the same way as the non directional one, namely \( h \). It is instructive to consider directions of observation in the plane perpendicular to the sun’s direction of motion (\( \Theta = \pi/2 \)) even though the un modulated rate is reduced in this direction. Along the −⟨y⟩ direction (\( \Phi = (3/2)\pi \)) the modulation amplitude \( h_1 - h_2 \) is constant, −0.20 and −0.30 for \( \lambda = 0, 1 \) respectively. In other words it large and leads to a maximum in December. Along the +⟨y⟩ direction the modulation is exhibited in Figs 5 and 6.

FIG. 5.: The quantity \( h_1 - h_2 \) in the direction +⟨y⟩ for \( \lambda = 0 \) (thick line) and \( \lambda = 1 \) thin line. In the −⟨y⟩ direction this quantity is constant and negative, −0.20 and −0.30 for \( \lambda = 0 \) and 1 respectively. As a result the modulation effect is opposite (minimum in June the 3rd).
FIG. 6.: The same as in Fig. 5 for the modulation amplitude in the direction $+x$, which is essentially $h_3$, since $|h_1| << |h_3|$. Thus in this case the maximum occurs around September the 3rd and the minimum 6 months later. The opposite is true in the $-x$ direction.

TABLE I.: The ratio $\kappa$ of the un modulated directional rate along the three directions to the non-directional one: $z$ is in the direction of the sun’s motion, $x$ is in the radial direction and $x$ is perpendicular to the axis of the galaxy. The asymmetry is also given. $Q_{\text{min}} = 0$ was assumed.

| $\lambda$ | dir. | +    | -    | asym | +    | -    | asym |
|-----------|------|------|------|------|------|------|------|
| 0         | z    | 0.02 | 0.50 | 0.92 | 0.75 | 0.25 | 0.50 |
| 0         | y    | 0.16 | 0.16 | 0.22 | 0    | 1.00 |
| 0         | x    | 0.16 | 0.16 | 0    | 0.37 | 0.24 | 0.21 |
| 1         | z    | 0.04 | 0.58 | 0.90 | -    | -    | -    |
| 1         | y    | 0.12 | 0.12 | 0    | -    | -    | -    |
| 1         | x    | 0.17 | 0.17 | 0    | -    | -    | -    |
VII. Conclusions

In the present paper we have discussed the parameters, which describe the event rates for direct detection of SUSY dark matter. Only in a small segment of the allowed parameter space the rates are above the present experimental goals. We thus looked for characteristic experimental signatures for background reduction, i.e. a) Correlation of the event rates with the motion of the Earth (modulation effect) and b) the directional rates (their correlation both with the velocity of the sun and that of the Earth.)

A typical graph for the total unmodulated rate is shown Fig. 1. The relative parameters in the case of non directional experiments are exhibited in Fig. 2 and Figs 3 and 4. We must emphasize that these two graphs do not contain the entire dependence on the LSP mass. This is due to the fact that there is the extra factor \( \mu^2 r \) in Eq. (2.10) but mainly due to the fact that the nucleon cross section depends on the LSP mass. Fig 2 and Figs 3 and 4 were obtained for the scalar interaction, but do not expect a very different behavior in the case of the spin contribution. The overall spin contribution, however, is going to be very different, but such considerations are beyond the goals of the present paper. We should also mention that in the non directional experiments the modulation \( 2h_1 \) is small, i.e. for \( \lambda = 0 \) less than 4% for \( Q_{min} = 0 \) and 12% for \( Q_{min} = 10 \text{ KeV} \) (at the expense of the total number of counts). For \( \lambda = 1 \) there in no change for \( Q_{min} = 0 \), but it can go as high as 24% for \( Q_{min} = 10 \text{ KeV} \).

For the directional rates It is instructive to examine the reduction factors along the three axes, i.e along \( +z, -z, +y, -y, +x \) and \( -x \). These depend on the nuclear parameters, the reduced mass, the energy cutoff \( Q_{min} \) and \( \lambda \). Since \( f_{red} \) is the ratio of two parameters, its dependence on \( Q_{min} \) and the LSP mass is mild. So we present results for \( Q_{min} = 0 \) and give their average as a function of the LSP mass (see Table I). As expected the maximum rate is along the sun’s direction of motion, i.e opposite to its velocity (\( -z \)) in the Gaussian distribution and \( +z \) in the case of caustic rings. In fact we find that \( \kappa (z) \) is around 0.5 (no asymmetry) and around 0.6 (maximum asymmetry, \( \lambda = 1.0 \)). It is not very different from the naively expected \( \kappa = 1/(2\pi) = \kappa = 1 \). The asymmetry along the sun’s direction of motion, \( \text{asym} = |R_{dir}(-) - R_{dir}(+)/|/(R_{dir}(-) + R_{dir}(+)) \) is quite characteristic, i.e. quite large in the isothermal models and smaller in caustic rings. The rate in the other directions is quite a bit smaller (see Table I). But, as we have seen, in the plane perpendicular in the sun’s velocity we have very large modulation, which may overcompensate for the large reduction factor. It is interesting to note that the dependence on the phase of the Earth \( \alpha \) changes substantially with the direction of observation.

In conclusion in the case of directional un modulated rates we expect...
characteristic and pretty much unambiguous correlation with the motion on the sun, which can be explored by the experimentalists. The reduction factor in the direction of the motion of the sun is approximately only $1/(4\pi)$ relative to the non directional experiments. In the plane perpendicular to the motion of the sun we expect interesting modulation signals, but the reduction factor becomes worse. A more complete discussion will be given elsewhere.

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