Lorentz-invariant and Lorentz-non-invariant aspects of a scalar tachyon field Lagrangian and the scalar tachyon Feynman propagator

V.F. Perepelitsa
ITEP, Moscow

Abstract
As it is well known, a consistent theory of faster-than-light particles (tachyons) can be built replacing the standard Lorentz-invariant approach to the quantum field theory of tachyons by the Lorentz-covariant one, invoking a concept of the preferred reference frame. This is a mandatory condition imposed by the requirement of the causality conservation. In this note some features of a Lorentz-violating (but Lorentz-covariant) Lagrangian of a scalar tachyon field are considered. It is shown that the equation of motion and the Feynman propagator resulting from it are Lorentz-invariant, while the Lorentz symmetry of the suggested tachyon field model can be defined as spontaneously broken.
1 Introduction

The generally accepted opinion existing inside the physics community is that faster-than-light signals and particles (tachyons, \[1, 2\]) would lead inevitably to causality violations. Another serious problem related to the tachyons is the presumable instability of the tachyon vacuum. Sometimes also the violation of the unitarity by tachyons interacting with ordinary particles is declared \[4, 5\].

Meanwhile it is known since the 1970’s that tachyons do not violate causality if one postulates the existence of a preferred reference frame in which the propagation of free tachyons is ordered by retarded causality (see e.g. \[6, 7\]).

With similar ideas it has been shown recently \[3\] that the tachyon hypothesis, if one treats tachyons properly, does not lead to the appearance of causal paradoxes, while the causality principle has to be redefined as a requirement of the absence of causal loops, i.e. the impossibility of a transfer of information to the past of an observer. The resolution of causal paradoxes comes, indeed, from a trivial observation that fast tachyons can probe cosmological distances. This leads to a conclusion that the completely correct description of tachyon behaviour can be made within general relativity only, while the Lorentz symmetry has to be considered as an approximate one when applied to tachyons. This makes the use of tachyons for the construction of the causal loops impossible (the reasons for this are formulated in the next paragraph). It has been shown in \[3\] that the correct approach to the tachyon theory can be achieved only within the postulate of a tight association of the tachyon preferred reference frame with the comoving frame of relativistic cosmology (see the definition of the latter, for example, in \[8\]). This is the absolute rest frame \[9\] in which our universe is embedded. In particular, the distribution of matter in the universe is isotropic in this frame only, the same is true for the relic black body radiation. The causality protection formula, valid in all inertial frames, was formulated in \[3\] as follows:

\[ Pu \geq 0, \]

(1.1)

where \( P \) is a 4-momentum of particles transferring a signal and \( u \) is a 4-velocity of the preferred reference frame with respect to (any particular) inertial observer. It is a boundary condition which should be imposed on solutions of any tachyon equation of motion.

Next step, the introduction of the concept of the preferred reference frame into the Minkowski space, can be considered as an action approximating that space to the real world space-time. It does not destroy the mathematical perfectionness of the Lorentz group since the derivation of the Lorentz transformations is based on the requirement of the invariance of the interval between world points when passing from one inertial frame to another, and the presence or the absence of the preferred reference frame among the frames under consideration does not affect the derivation to any extent. In view of this one can retain the Lorentz group (as well as the Poincaré group) when treating tachyons. Together with this the introduction of the preferred reference frame into the tachyon theory removes the problem of the instability of the tachyon vacuum \[3\]. The space of the preferred frame

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1Often causality violation is related intrinsically to the mere possibility of existence of tachyons by pointing out that the positive time interval between events connected by a space-like world line can be converted to a negative time interval by a suitable Lorentz transformation. Such a change of the event time order can indeed take place in the case of tachyon signals, but it does not mean yet the causality violation. The causality violation appears only when a causal loop can be constructed, i.e. a sending by an observer a signal to its own past, see \[3\].
turns out to be spanned by the continuous background of free, zero-energy on-mass-shell tachyons propagating isotropically, i.e. the eigenvalues of the tachyon Hamiltonian are restricted from below, in this frame, by zero value. This excludes the possibility of the construction of the causal loops using tachyons since for such a construction negative energy tachyons (propagating backward in time) are necessary. In the frames moving with respect to the preferred one negative energy tachyons can appear due to Lorentz boosts, but causal loops cannot, since the presence or the absence of the causal loops is an invariant property of the relativistic theory describing tachyons. For example, the energy boundaries of the tachyon vacuum, which has to be defined in tachyon quantum field theories by the tachyon vacuum gauge

\[ Pu = 0, \]  

(1.2)

in the frames moving with respect to the preferred one are given by expressions

\[ E_0^+ = \frac{\mu |u|}{\sqrt{1 - u^2}}, \]  

(1.3)

for the direction coinciding with the preferred frame velocity \( u \) and by

\[ E_0^- = -\frac{\mu |u|}{\sqrt{1 - u^2}}. \]  

(1.4)

for the opposite direction, as Fig. 1 illustrates.

Simultaneously it turns out that in any reaction in which tachyons participate asymptotic “in” and “out” tachyonic Fock spaces are unitarily equivalent, which solves the unitarity problem.

As “toy” models the Lorentz-covariant quantum field models of scalar tachyons\(^\text{2}\) were considered in ref. [3]. They are based on Lorentz-covariant scalar tachyon Lagrangians with spontaneously broken Lorentz symmetry, so the Lorentz invariance violation appears to be restricted to the tachyon sector only, affecting the asymptotic tachyon states and leaving the sector of ordinary particles within the Standard Model untouched, at least up to possible small radiative corrections. For example, the Hermitian tachyon field operator with the causal \( \Theta \)-function accounting for the boundary condition (1.1), \( \Theta(\mu u) \), reads as follows:

\[ \Phi(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^4k \left[ a(k) \exp(-ikx) + a^+(k) \exp(ikx) \right] \delta(k^2 + \mu^2) \Theta(\mu u), \]  

(1.5)

where \( k \) is a tachyon four-momentum, \( a(k), a^+(k) \) are annihilation and creation operators with bosonic commutation rules, annihilating or creating tachyonic states with 4-momentum \( k \), and \( \mu \) is a tachyon mass parameter. As can be seen, the expression (1.5)

\(^\text{2}\)It was argued in [3] that a realistic model of a tachyon theory should be built upon the infinite-dimensional unitary irreducible representations of the Poincaré group (so called “infinite spin” tachyons). Within the conjecture that elementary particles are realisations of the unitary irreducible representations of the Poincaré group the only alternative to the infinite spin tachyon models is a scalar tachyon model. However this model cannot represent tachyons at a fundamental level since it possesses several diseases; in particular, such a model would lead to the instability of photons via their decay to tachyon-antitachyon pairs [3]. Note that decays of photons to the infinite spin tachyons are forbidden by the angular momentum conservation combined with kinematic restrictions imposed on this process.
is explicitly Lorentz-covariant. This covariance includes the invariant meaning of the creation and annihilation operators defined in the preferred frame; thus, for example, an annihilation operator $a(k)$ remains an annihilation operator $a(k')$ in the boosted frame, even if the zero component of $k'$ may become negative. This is because the one-sheeted tachyon mass-shell hyperboloid is divided by the covariant boundary $\Theta(ku)$ into two parts separated in an invariant way. This results, in particular, in a possibility of the standard operator definition of the invariant vacuum state $|0\rangle$ via the annihilation operators $a(k)$, $a(k)|0\rangle = 0$ for all $k$ such that $|k| > \mu$, because the vacuum state energy and, as a consequence, the field Hamiltonian turn out to be bounded from below in any reference frame, see [3] and formulae (2.5), (2.6) below.

![Diagram](image_url)

Fig. 1. The tachyon vacuum energy levels as seen $a)$ from the preferred reference frame $A$ and $b)$ from a frame moving with respect to the preferred one with 3-velocity $u$. The direction of the preferred frame motion as seen from the (moving) frame $B$ is indicated by an arrow in the top part of $b)$. $E_0^+$ and $E_0^-$ mark the “forward” and the “backward” tachyon vacuum energy levels in the moving frame given by (1.3) and (1.4). The vertical axes on both figures are for tachyon energies, with the hatched regions to be excluded domains for asymptotic tachyon states.
In paper [10] a formal way of introducing the causal $\Theta$-function into the tachyon field operator (1.5) has been suggested. In this note we continue the consideration of Lorentz-invariant and Lorentz-non-invariant properties of the tachyon field models mentioned above, including an important element of the models such as the Lorentz-invariance of the Feynman propagator of the tachyon scalar fields.

The overall approach to tachyon field models emerging from this consideration possesses the following attractive features:

1. Its main concept is based on the experimentally proved phenomenon [9]: the existence of a preferred reference frame which is the comoving frame of the relativistic cosmology [8].

2. The introduction of this concept into the tachyon theory can be done in a covariant way which leads to the result that the Lorentz symmetry of the tachyon theory appears to be spontaneously broken.

3. The spontaneous breaking of the Lorentz symmetry allows one to avoid introducing “by hand” Lorentz-violating terms into Lagrangians as it is suggested by authors of the Standard Model Extension (SME) [11], which would lead to the Lorentz-non-invariant tachyon propagators making tachyons subject to strict restrictions imposed on the Lorentz violation by a variety of experiments (see [12, 13]).

4. Faster-than-light velocities appear to be allowed in a wide range of $c < v < \infty$ as distinct to the standard models of the Lorentz invariance violation (i.e. SME) in which only tiny positive deviations from the velocity of light are permitted for particles having energies below the Plank mass scale [11, 14, 15].

5. So formulated the tachyon hypothesis results in the possibility of the existence of a new world of elementary particles residing beyond the light barrier as distinct to the SME in which the Lorentz violating particles are assumed to be (some of the) already known particles, e.g. neutrinos, electrons, muons, pions, etc.

6. The velocity of light remains an invariant quantity to be a barrier between the tachyon world of particles and that of subluminal particles which can be considered as a justification of a fundamental character of this velocity.

This note is organized as follows. In Section 2 we suggest a Lorentz-non-invariant, but Lorentz-covariant modification of the scalar tachyon Lagrangian which leads to a tachyon Hamiltonian possessing Lorentz-non-invariant boundaries of the tachyon vacuum. The modified Lagrangian leaves however the tachyon equation of motion unchanged which leads to the Lorentz invariance of the tachyon Feynman propagator considered in Section 3. Representations of this propagator in the configuration space are given in Section 4. Section 5 contains the note conclusion. The main text of the note is supplemented with an Appendix in which some Lorentz-invariant and Lorentz-non-invariant two-point functions of scalar tachyon fields are presented.

In formulae used in this note the velocity of light $c$ and the Planck constant $\hbar$ are taken to be equal to 1.
2 Proposed Lorentz-non-invariance of the tachyon Lagrangian and some remarks on it

We start with a Lorentz-invariant Lagrangian of a free scalar tachyon field

\[ L = \frac{1}{2} \int d^3x \left[ \dot{\Phi}^2(x) - \left( \nabla \Phi(x) \right)^2 + \mu^2 \Phi^2(x) \right] \] (2.1)

Let us consider a possible modification of the Lagrangian (2.1) by adding to it a Lorentz-non-invariant, but Lorentz-covariant term proportional to the 4-velocity \( u \) of the preferred reference frame:

\[ L = \frac{1}{2} \int d^3x \left[ \dot{\Phi}^2(x) - \left( \nabla \Phi(x) \right)^2 + \mu^2 \Phi^2(x) + \lambda u^\mu \partial_\mu \Phi(x) \right]. \] (2.2)

where \( \lambda \) has the dimensionality of the mass squared. For the suggested tachyon field model viability it is important that the additional term does not change the equation of motion:

\[ \left( \frac{\partial^2}{\partial t^2} - \partial_i \partial^i - \mu^2 \right) \Phi(x) = 0, \quad i = 1, 2, 3. \] (2.3)

Choosing \( \lambda = \mu^2 \) one gets the corresponding Hamiltonian

\[ H = \int d^3x \left[ \dot{\Phi}^2(x) + \left( \nabla \Phi(x) \right)^2 - \mu^2 \Phi^2(x) + \frac{\mu^2 u \sqrt{1 - u^2}}{\sqrt{1 - u^2}} \nabla \Phi(x) \right]. \] (2.4)

Thus, the additional term in the integrand of (2.4) shifts the tachyon vacuum energy boundaries depending on the direction of the 3-velocity of the preferred reference frame \( u \) with respect to the tachyon source (illustrated by formulae (1.3), (1.4) and by Fig. 1), which was just the aim of the introduction of this term.

After second quantization procedure the Hamiltonian reads (see [3]):

\[ H = \int \left| \mathbf{k} \right| > \mu, \omega > | \mathbf{k} \rangle \langle \mathbf{k} | \left( 2\pi \right)^3 \omega / \sqrt{1 - \mathbf{u}^2} a_\mathbf{k}^+ a_\mathbf{k}. \] (2.5)

Thus the Hamiltonian is bounded from below and is Hermitian. In the preferred reference frame

\[ H = \int \left| \mathbf{k} \right| > \mu, \omega > 0 \left( 2\pi \right)^3 \omega a_\mathbf{k}^+ a_\mathbf{k} \] (2.6)

having non-negative eigenvalues.

To conclude this section we formulate its result: the Lagrangian (2.2) differs from the Lagrangian (2.1) by a Lorentz-non-invariant term presented in the former; and since this additional term, written down as \( \lambda \partial_\mu F^\mu(x) \), where \( F^\mu(x) \equiv u^\mu \Phi(x) \), is proportional to the total divergence of the 4-vector \( F^\mu(x) \), the two Lagrangians, with and without the additional term, are physically equivalent since the term with \( \partial_\mu F^\mu(x) \) does not contribute to physical quantities, excepting those related to the tachyon vacuum.

Furthermore, as mentioned above, the additional term does not change the tachyon equation of motion (2.3). Therefore within our approach the Lorentz invariance can be defined as spontaneously broken and its violation appears to be restricted to the asymptotic-tachyon-states sector only, even in the case of presumed tachyon interactions with ordinary particles, as noted in the following section.
3 Lorentz invariance of tachyon Feynman propagator

Considering a tachyon propagator in momentum space as an inverse of a Fourier transform of the wave equation (2.3), we can write down, for example, the Feynman propagator as

$$\tilde{D}_F(k) = \frac{i}{k^2 + \mu^2 + i\epsilon}$$  \hspace{1cm} (3.1)

to be used in Feynman diagrams describing tachyon interactions, of course, only within our toy model of scalar tachyons. In the configuration space

$$D_F(x - y) = \int_{|k| \geq \mu} \frac{d^4k}{(2\pi)^4} \frac{i \exp[-i k (x - y)]}{k^2 + \mu^2 + i\epsilon}. $$  \hspace{1cm} (3.2)

One can see that the tachyon Feynman propagator defined by formula (3.2) is explicitly Lorentz-invariant since all ingredients in this formula, including integration limits, are Lorentz-invariant (in particular, if the integration limit $|k| \geq \mu$ holds in some inertial frame it holds in any such frame). Let us note that, as a consequence, the same invariance holds also for virtual tachyons and tachyon loops appearing in Feynman diagrams of reactions containing only ordinary particles in the initial and final states. Representations of the tachyon Feynman propagator (3.2) in configuration space are given in the next section.

It is not a problem to obtain (3.2) as a time-ordered product of the tachyon field operators (1.5), taken at points $x$ and $y$ and averaged over the tachyonic vacuum, i.e. as an amplitude of the tachyon transition from $x$ to $y$ or vice versa:

$$D_F(x - y) \equiv \langle 0 | T\Phi(x)\Phi(y) | 0 \rangle = \begin{cases} D(x - y) & \text{if } x^0 > y^0 \\ D(y - x) & \text{if } x^0 < y^0 \end{cases},$$  \hspace{1cm} (3.3)

where $D(x - y)$ is a correlation function of the tachyon field operators $\Phi(x), \Phi(y)$:

$$D(x - y) \equiv \langle 0 | \Phi(x)\Phi(y) | 0 \rangle = \int_{|k| \geq \mu} \frac{d^3k}{(2\pi)^3} \frac{\exp[-i k (x - y)]}{2\omega}$$  \hspace{1cm} (3.4)

(for the proof see Appendix).

According to the Lehman-Symanzik-Zimmerman reduction formula, in the computation of the $S$-matrix elements using the Feynman diagrams the Feynman propagators should be attached to the internal lines of the diagrams only. Therefore the Feynman rules for the construction of Feynman diagrams which include tachyons appear to remain standard, with a minor exception: to each external tachyon leg (asymptotic tachyon state) the causal $\Theta$-function $\Theta(ku)$ should be attached.

We observe thus that the operation of the time ordering of field operators in (3.3) realises, in general, the same function as causal $\Theta$-terms in the field operators (1.5). Therefore we can interpret heuristically the obtained results as follows.

The standard $i\epsilon$ prescription, aimed at the definition of the integration contour on the complex plane of $k^0$, allows virtual tachyons (as well as virtual ordinary particles) to avoid causal restrictions, which are otherwise (i.e. in the case of real faster-than-light particles) imposed by the causal $\Theta$ function on the propagation of free tachyons. In other words, the $i\epsilon$ prescription, ensuring the time ordering of the tachyon field operators which contain
the $\Theta$ functions apparently breaking the Lorentz invariance, makes the virtual tachyons insensitive to the existence of the preferred reference frame (which indeed is intuitively obvious), and this results in the Lorentz invariance of the tachyon Feynman propagator, similarly to the Feynman propagators of ordinary particles.

The Lorentz invariance of the tachyon Feynman propagator, obtained in our model with the spontaneously broken Lorentz invariance, means, together with the assumed Lorentz invariance of the Feynman propagators of all other particles, that the speed of light remains a unique, universal velocity constant which limits particle velocities on both sides of the light barrier, bounding the maximum attainable velocities of ordinary particles and restricting the tachyon velocities from below. In particular, an explicit breaking of the Lorentz symmetry by adding to the Lagrangian the Lorentz-violating terms which affect the particle propagators, suggested by the SME [11] (see also [14, 15]), which lead to individual maximum attainable velocity for each fundamental field, differing from the velocity of light, is not relevant to our approach. For the same reason the strong restrictions on multiple Lorentz-violating coefficients compiled in the “Data Tables for Lorentz and CPT violation” [13] are not applicable to our considerations.

4 The Feynman propagator for scalar tachyons in the configuration space

Let us obtain $D_F(x - y)$ explicitly:

\[
D_F(x - y) = \int_{|k| \geq \mu} \frac{d^4k}{(2\pi)^4} \frac{i \exp [-ik(x - y)]}{k^2 - \mu^2 + i\epsilon}
= \int_{|k| \geq \mu} \frac{d^3k}{(2\pi)^4} \frac{i \exp [-ik_0\Delta t + i k(x - y)]}{k^2 - \mu^2 + i\epsilon}
= \int_{|k| \geq \mu} \frac{d^3k}{(2\pi)^3} \exp [-i\omega|\Delta t| + i k(x - y)],
\]

where $\Delta t = x^0 - y^0$, and we have used the integral representation

\[
\exp \left( -i\omega|\Delta t| \right) = \frac{i\omega}{\pi} \int_{-\infty}^{+\infty} dk_0 \exp \left( -ik_0\Delta t \right) \frac{1}{k_0^2 - \omega^2 + i\epsilon}, \quad \epsilon \to 0^+.
\]

Integration of (4.1) over the angles of $k$ gives

\[
D_F(x - y) = \frac{1}{4\pi^2} \int_{\mu}^{\infty} dk \frac{k^2}{\omega} \sin(\|k\|\| x - y \|) \cdot \exp \left( -i\omega|\Delta t| \right),
\]

(4.3)

With the definition of the interval

\[
s \equiv \Delta t^2 - (x - y)^2 = \Delta t^2 - r^2,
\]

(4.4)

where $r \equiv \|x - y\|$, we can investigate the behaviour of the $D_F$ outside and inside the light cone. For spacelike intervals, $s < 0$, we can put $\Delta t = 0$ to obtain

\[
D_F(r) = \frac{1}{4\pi^2 r} \int_{\mu}^{\infty} \frac{|k|dk}{\sqrt{k^2 - \mu^2}} \sin |k|r = -\frac{\mu}{8\pi r} \, Y_1(\mu r),
\]

(4.5)
where $Y_1$ is the Bessel function of the second kind; it represents an outgoing wave for large $r$. To compare: for an ordinary scalar particle with the mass $m$ the corresponding Feynman propagator

$$D_F^{\text{ord}}(r) = \frac{m}{4\pi^2 r} K_1(mr),$$

(4.6)

where $K_1$ is the modified Bessel function of the second kind \cite{16}; it dumps exponentially for large $r$, the characteristic damping length being the particle Compton length $\lambda = 1/m$.

For timelike intervals, $s > 0$, we can put in (4.3) $r = 0$:

$$D_F(|\Delta t|) = \frac{1}{4\pi^2} \int_\mu^\infty \frac{k^2 dk}{\sqrt{k^2 - \mu^2}} \exp\left(-i\sqrt{k^2 - \mu^2} |\Delta t|\right) = \frac{\mu}{4\pi^2 |\Delta t|} K_1(\mu|\Delta t|),$$

(4.7)

i.e. it dumps exponentially for large $|\Delta t|$, with the characteristic damping time being the tachyon Compton length $\lambda = 1/\mu$. For an ordinary scalar particle

$$D_F^{\text{ord}}(|\Delta t|) = \frac{im}{8\pi |\Delta t|} H_1^{(1)}(m|\Delta t|),$$

(4.8)

where $H_1^{(1)}$ is the Hankel function of the first kind which represents an outgoing wave for large $|\Delta t|$ \cite{16}.

5 Conclusion

A modification of a scalar tachyon field Lagrangian by adding to it a Lorentz-non-invariant, but Lorentz-covariant term is suggested, aimed at the conservation of causality, with the Lorentz invariance of the tachyon Feynman propagator being conserved. It is shown that the standard $i\epsilon$ prescription in the Feynman propagator realises the same function over virtual particles (having $k^2 \geq 0$ as well as $k^2 < 0$) as the causal $\Theta$ function in the quantum field operators of free tachyons (asymptotic tachyon states). The result is a possibility to have a tachyon model free of causal paradoxes, tachyon vacuum instability, and Lorentz-violating radiative corrections coming from virtual tachyons.

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Appendix. Lorentz-invariant and Lorentz-non-invariant two-point functions of scalar tachyon fields

The correlation function $D(x - y)$ can be represented as

$$D(x - y) \equiv \langle 0|\Phi(x)\Phi(y)|0 \rangle = \left( \int \frac{d^4k}{(2\pi)^3} \exp[-ik(x - y)] \delta(k^2 + \mu^2) \Theta(ku) \right)_{\text{Evac}}$$

$$= \left( \int_{|k|\geq\mu,\omega\geq ku} \frac{d^3k}{(2\pi)^3} \frac{\exp[-i\omega\Delta t + ik(x - y)]}{2\omega} \right)_{\text{Evac}} (A.1)$$

where $\Delta t = x^0 - y^0$ and the angular brackets $\langle \rangle_{Evac}$, surrounding the integrals in (A.1), denote the averaging over the tachyon vacuum energy boundaries. Such an averaging is a necessary action since the tachyon vacuum energy boundaries are not, in general (in the frames moving with respect to the preferred one), rotationally invariant.

Generally speaking, the calculation of tachyon vacuum expectation values of any combination of tachyon operators requires such an averaging as distinct to the calculation of analogous vacuum expectation values in the case of ordinary particles, when such calculations result in Lorentz-invariant c-number functions due to Lorentz-invariance of the ordinary particle vacuum. In our case (with tachyons) the expression inside the vacuum brackets $\langle \rangle_{Evac}$ in (A.1) is a Lorentz-non-invariant c-number function due to the Lorentz-non-invariant (though Lorentz-covariant) integration limits, $\omega \geq ku$ in (A.1), coming from the causal $\Theta$-function (these limits are illustrated, in particular, by formulae (1.3), (1.4) and by Fig. 1). Thus, an observer in a frame moving with respect to the preferred one can detect that the energy boundaries of the tachyon vacuum are different in the forward and backward hemispheres of his motion\(^3\), the situation which has to be taken into account when calculating any tachyon-involved vacuum expectation value.

Fortunately, the averaging of the expression inside the vacuum brackets in (A.1) over the tachyon vacuum energy boundaries contracts the above Lorentz-non-invariance since these boundaries are governed by formula (1.2), the same one which imposes those integration limits. This occurs owing to the fact that the boundaries are symmetric with respect to the zero energy level in the preferred reference frame, depending on the direction of the observer motion. The statement about Lorentz-invariance of the correlation function $D(x - y)$ can be proved as follows.

First of all, we note that the averaging over the tachyonic vacuum energy boundaries can be done for each individual direction of $k$, i.e. for fixed values of the angles $\theta$ and $\phi$, the polar and azimuthal angles with respect to the directions of $k$ and $x - y$. Changing the first integration over $d^4k$ in (A.1) from $d\omega$ to $d|k|$ we can rewrite it as

$$D(x - y) = \int \left[ \frac{d\cos \theta d\phi}{2(2\pi)^3} \right. \times \left. \int_{|\omega|\geq\sqrt{\omega^2 + \mu^2}|u|\cos \psi} d\omega \sqrt{\omega^2 + \mu^2} \exp(-i\omega\Delta t + i\sqrt{\omega^2 + \mu^2}|x - y|\cos \theta) \right]_{\text{Evac}} (A.2)$$

Here $\psi$ is the angle between the directions of $k$ and $u$. Obviously, the expression enclosed by the vacuum brackets in (A.2) can be written as a sum

$$\frac{1}{2} \left( I_{FW}(x - y, u) + I_{BW}(x - y, u) \right), \quad (A.3)$$

\(^3\)An attempt to detect these boundaries was undertaken in [7].
where
\[ I(x - y, u) = \int_{\omega_{c}}^{\omega_{f}} d\omega \sqrt{\omega^2 + \mu^2} \exp(-i\omega\Delta t + i\sqrt{\omega^2 + \mu^2}|x - y| \cos \theta), \]

and the subscripts FW and BW define the corresponding integrals in the forward and backward hemispheres of the vector \( u \), where \( \cos \psi > 0 \) and \( \cos \psi < 0 \), respectively.

Each of the hemisphere integrals \( I(x - y, u) \) can be written down as a sum of two terms, one of which does not depend on the vector \( u \), and another which does. So,
\[ I_{FW}(x - y, u) = \int_{0}^{\infty} d\omega \sqrt{\omega^2 + \mu^2} \exp(-i\omega\Delta t + i\sqrt{\omega^2 + \mu^2}|x - y| \cos \theta) \]
- \( \int_{E_0^+ (|u|, \psi)} d\omega \sqrt{\omega^2 + \mu^2} \exp(-i\omega\Delta t + i\sqrt{\omega^2 + \mu^2}|x - y| \cos \theta), \]

where
\[ E_0^+ (|u|, \psi) = \frac{\mu |u| \cos \psi}{\sqrt{1 - u^2 \cos^2 \psi}}, \quad \cos \psi > 0. \]

Analogously,
\[ I_{BW}(x - y, u) = \int_{0}^{\infty} d\omega \sqrt{\omega^2 + \mu^2} \exp(-i\omega\Delta t + i\sqrt{\omega^2 + \mu^2}|x - y| \cos \theta) \]
+ \( \int_{E_0^- (|u|, \psi)} d\omega \sqrt{\omega^2 + \mu^2} \exp(-i\omega\Delta t + i\sqrt{\omega^2 + \mu^2}|x - y| \cos \theta), \]

where
\[ E_0^- (|u|, \psi) = \frac{\mu |u| \cos \psi}{\sqrt{1 - u^2 \cos^2 \psi}}, \quad \cos \psi < 0. \]

As follows from (A.6), (A.8), \( |E_0^- (|u|, \psi)| = E_0^+ (|u|, \psi) \). Therefore
\[ \int_{E_0^+ (|u|, \psi)} d\omega \sqrt{\omega^2 + \mu^2} \exp(-i\omega\Delta t + i\sqrt{\omega^2 + \mu^2}|x - y| \cos \theta) = \int_{E_0^- (|u|, \psi)} d\omega \sqrt{\omega^2 + \mu^2} \exp(-i\omega\Delta t + i\sqrt{\omega^2 + \mu^2}|x - y| \cos \theta). \]

As a result
\[ \frac{1}{2} \left( I_{FW}(x - y, u) + I_{BW}(x - y, u) \right) = \int_{0}^{\infty} d\omega \sqrt{\omega^2 + \mu^2} \exp(-i\omega\Delta t + i\sqrt{\omega^2 + \mu^2}|x - y| \cos \theta). \]

Collecting all the ingredients of (A.2) and reverting back from the integration over \( d\cos \theta d\Phi \omega d\omega \) to the integration over \( d^3 k \) we obtain finally
\[ D(x - y) = \int_{|k| \geq \mu} \frac{d^3 k}{(2\pi)^3} \frac{\exp(-i\omega\Delta t + i\mu(k \cdot (x - y))}{2\omega}, \]

\(^4\)Probably, it is worth noting that the exponential terms in (A.9) below are identical even though the integration domains over the variable \( \omega \) in them are quite different: this is due to the fact that exponential indices, containing the \( \omega \), are indeed the scalar products of the 4-vectors \( k \) and \( x - y \), i.e. they are Lorentz scalars by definition.
which does not depend on $u$, i.e. it is manifestly Lorentz-invariant, as has been stated above.

This function can be used to construct the Feynman propagator

$$D_F(x - y) = \begin{cases} D(x - y) & \text{if } x^0 > y^0 \\ D(y - x) & \text{if } x^0 < y^0 \end{cases} = \int_{|k| > \mu} \frac{d^3k}{(2\pi)^3} \frac{\exp[-i\omega|\Delta t| + ik(x - y)]}{2\omega}$$  \hspace{0.5cm} (A.12)

in which we have changed the integration variable from $k$ to $-k$ when replacing $\Delta t$ by $|\Delta t|$ in the case of $x^0 < y^0$. The resulting expression in (A.12) coincides with the last line of (4.1) which proves (3.3).

The commutator of scalar tachyon fields reads

$$\Delta(x - y) \equiv [\Phi(x), \Phi(y)] = \int \frac{d^3k}{(2\pi)^3} \left\{ \exp[-ik(x - y)] - \exp[(ik(x - y))] \right\} \delta(k^2 + \mu^2) \Theta(ku)$$  \hspace{0.5cm} (A.13)

which is not automatically zero at $(x - y)^2 < 0$ as distinct to the field commutators of ordinary particles. Let us consider it in the preferred reference frame:

$$[\Phi(x), \Phi(y)] = \frac{1}{(2\pi)^3} \int_{|k| > \mu, \omega > 0} \frac{d^3k}{2\omega} \left\{ \exp[-i\omega\Delta t + ik(x - y)] - \exp[i\omega\Delta t - ik(x - y)] \right\}$$  \hspace{0.5cm} (A.14)

If $\Delta t \neq 0$ the commutator does not vanish (excepting the case of $\omega = 0$ corresponding to the exchange of vacuum tachyons). However, the commutator $\Delta(x - y)$ vanishes at $\Delta t = 0$ in the preferred reference frame, which is obvious from (A.14); the same is true for the correspondingly Lorentz-shifted $x - y$ in boosted frames (due to the covariance of the expression (A.13)).

Though being Lorentz-covariant, the commutator (A.13) is not Lorentz-invariant since it depends on the value of $|u|$ and on the angle between the directions of $u$ and $x - y$ in the frames moving with respect to the preferred one. In other words, though all terms in the integrand of (A.13), except $\Theta(ku)$, are Lorentz invariant, the integration limits in this expression, imposed by the causal $\Theta$-function, are rotationally invariant in the preferred reference frame only, as can be seen from (A.14). Just this general rotational non-invariance of the integration limits results in the Lorentz-non-invariance of the commutator (A.13). This means that the amplitude of propagation of a tachyon from $x$ to $y$ is not equal to the amplitude of propagation of the same tachyon from $y$ to $x$, the latter being the complex conjugate of the former.

However, being averaged over the tachyonic vacuum, the commutator

$$\langle 0 | [\Phi(x), \Phi(y)] | 0 \rangle$$  \hspace{0.5cm} (A.15)

becomes Lorentz-invariant since the rotational non-invariance mentioned in the previous paragraph is cancelled by the averaging. Simultaneously, this leads to the vanishing of the commutator (A.15) at spacelike separations of the $x$ and $y$ positions:

$$\langle 0 | \Delta(x - y) | 0 \rangle \equiv \langle 0 | [\Phi(x), \Phi(y)] | 0 \rangle = D(x - y) - D(y - x) = 0 \quad ((x - y)^2 < 0)$$  \hspace{0.5cm} (A.16)

(A.16) can be considered as a limiting case of a causal loop construction with the use of tachyons. Its vanishing means a principal impossibility of such loops expressed in terms of quantum field theory, i.e. (A.16) is the micro-causality condition of a tachyon theory.

\footnote{In the case of a complex tachyon field the propagation of the tachyon from $y$ to $x$ would be replaced by the propagation of an antitachyon in that direction.}
References

[1] Bilaniuk O. M. P., Deshpande V. K., Sudarshan E. C. G., “Meta”-relativity, Amer. J. Phys. 30, 718-723 (1962).

[2] Feinberg G., Possibility of faster-than-light particles, Phys.Rev. 159, 1089-1105 (1967).

[3] Perepelitsa V. F., Looking for a theory of faster-than-light particles, arXiv:gen-ph/14073245 (2014).

[4] Boulware D. G., Unitarity and interacting tachyons, Phys. Rev. D 1, 2426-2427 (1970).

[5] Jacobson T., Tsamis N. C., Woodard R. P., Tachyons and perturbative unitarity, Phys. Rev. D 38, 1823-1834 (1988).

[6] Sigal R., Shamaly A., Tachyon behavior in general relativity, Phys. Rev. D 10, 2358-2361 (1974).

[7] Perepelitsa V. P., Tachyon Michelson experiment, Phys. Lett. B 67, 471-473 (1977).

[8] Landau L. D., Lifshitz E. M., Theoretical Physics, vol.2, Classical Theory of Fields, Sect. 112, Addison-Wesley, New York, 1985.

[9] Patrignani C. et al. (Particle Data Group), Chin. Phys. C 40, Sect. 27.3.2 (2016).

[10] Perepelitsa V. F., How to implant a causal $\Theta$-function into a tachyon field operator, arXiv:gen-ph/1512.07921 (2015).

[11] Colladay D., Kostelecký V. A., Lorentz-violating extension of the standard model, Phys. Rev. D 58, 116002 (1998), and references therein.

[12] Mattingly D., Modern tests of Lorentz invariance, Living Rev. Rel., 5, 5-84; arXiv:gr-qc/0502097 (2005).

[13] Kostelecký V. A., Russel N., Data tables for Lorentz and CPT violation, Rev. Mod. Phys., 83, 11-31 (2011) (arXiv:0801.0287v9).

[14] Coleman S., Glashow S. L. Cosmic ray and neutrino tests of special relativity, Phys. Lett. B 405, 249-252 (1997).

[15] Coleman S., Glashow S. L. High-energy tests of Lorentz invariance, Phys. Rev. D 59, 116008-1–116008-14 (1999).

[16] Huang K., Quantum Field Theory: from Operators to Path Integrals, Sect. 2.9, John Wiley and Sons, New York, Chichester, Weinheim, Brisbane, Singapore, Toronto, 1998.