Magnetic draping of merging cores and radio bubbles in clusters of galaxies

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ABSTRACT

Sharp fronts observed by Chandra satellite between dense cool cluster cores moving with near-sonic velocity through the hotter intergalactic gas, require strong suppression of thermal conductivity across the boundary. This may be due to magnetic fields tangential to the contact surface separating the two plasma components. We point out that a super-Alfvenic motion of a plasma cloud (a core of a merging galaxy) through a weakly magnetized intercluster medium leads to "magnetic draping", formation of a thin, strongly magnetized boundary layer with a tangential magnetic field. For supersonic cloud motion, $M_s \geq 1$, magnetic field inside the layer reaches near-equipartition values with thermal pressure. Typical thickness of the layer is $\sim L/M_A^2 \ll L$, where $L$ is the size of the obstacle (plasma cloud) moving with Alfvén Mach number $M_A \gg 1$. To a various degree, magnetic draping occurs both for sub- and supersonic flows, random and ordered magnetic fields and it does not require plasma compressibility. The strongly magnetized layer will thermally isolate the two media and may contribute to the Kelvin-Helmholtz stability of the interface. Similar effects occur for radio bubbles, quasi-spherical expanding cavities blown up by AGN jets; in this case the thickness of the external magnetized layer is smaller, $\sim L/M_A^3 \ll L$.

1. Introduction

Chandra observations of intergalactic medium (IGM) in clusters of galaxies often show sharp discontinuities in gas density, that separate dense cool gas moving with near-sonic velocity through the hotter gas (Markevitch et al. 2000; Vikhlinin et al. 2001). These fronts result from cluster mergers, when a cold subcluster core moves through a hot IGM at transonic velocities. The front is very sharp. For example, in the case of Abell A3667 the thickness of the front may be as thin as $\sim 5$ kpc, (Vikhlinin et al. 2001).

This sharpness is surprising since heat conduction together with development of the Kelvin-Helmholtz instability from tangential motion of gas are expected to result in much broader transitions. Subtle geometrical and kinetic effects may stabilize the contact (Churazov & Inogamov 2004) but this does not alleviate the problem of heat conduction. For typical cluster parameters,
the Spitzer mean free path in unmagnetized plasma is $\sim 30$ kpc, much larger than the thickness of the transition layer. Thermal conductivity in unmagnetized gas should lead to cloud evaporation on time scale of $\sim 10^7$ years (Ettori & Fabian 2000). Suppression of conductivity due to saturation of heat flux, when electron mean free path is comparable to the scale length of the temperature gradient (Cowie & McKee 1977), is not sufficient. This prompted Ettori & Fabian (2000) to argue that classical conductivity has to be reduced by a factor of between 250 and 2500.

Magnetic field is the prime suspect in reducing the conductivity across the front, since Larmor radius, typically in thousands of kilometers, is many orders of magnitude smaller than both electron mean free path and the scale of the temperature gradient. It was suggested that fields turbulent on small scales may do the job, reducing Spitzer conductivity by a large factor (Chandran & Cowley 1998). On the other hand, Narayan & Medvedev (2001) argued that if the scale of magnetic turbulence is smaller than the mean free path, and if the fluctuation spectrum extends over several decades in wave-vector, thermal conductivity is strongly enhanced almost up to the Spitzer value. A possible caveat in this argument in application to cold fronts is that it assumes that magnetic fields in the two media do intertwine. Vikhlinin et al. (2001) suggested that cross-boundary drift, heat conduction and KH instabilities are all suppressed by a large, $\geq 10 \mu G$, magnetic field along the contact surface. Such large, near-equipartition (with thermal pressure) magnetic fields must be local to the front in view of the estimates of sub-microgauss fields by Faraday rotation measurements in radio sources seen through the IGM (e.g. Kim et al. 1991), (see reviews by Carilli & Taylor 2002; Enßlin et al. 2005; Govoni 2006). Vikhlinin et al. (2001) suggested that a magnetic field parallel to the contact surface arises due to shearing of an initially turbulent magnetic field.

In this letter we discuss a straightforward dynamical effect that leads to formation of a narrow layer of tangential, near-equipartition magnetic field at the contact discontinuity. Such region forms regardless of however small the magnetic field is in the bulk of a cluster. This effect is well-known in space physics, and sometimes is called magnetic draping or magnetic barrier. It is mostly pronounced for the interaction of the Solar wind with Venus and Mars, planets that do not have their own magnetic field. It is also related to the so-called plasma depletion layer, where the plasma density is depressed with respect to values in the rest of the magnetosheath (Zwan & Wolf 1976; Paschmann et al. 1978).

2. Properties of the transition layer

2.1. Formation: divergence of a magnetic field on the contact in kinetic approximation

To explain the effect of magnetic draping in its simplest form, consider an interaction of a cloud moving through a weakly magnetized medium parametrized by the ratio of total plasma to magnetic pressure $\beta = 8\pi p / B^2 \gg 1$. Since the magnetic field is weak, one may be prompted to neglect its dynamical effects on the flow completely. It turns out, that such approximation is not
self-consistent and should break down close to the contact discontinuity separating two fluids.

As a first step, which will be shown to be faulty, let us consider magnetic field in kinetic approximation, when dynamical effects of the magnetic field on the flow are neglected and the field lines are just advected with the flow satisfying the froze-in condition. For simplicity, let us first assume that magnetic field in bulk is large scale and is directed along \( y \) axis, orthogonally to the direction of cloud motion in \( z \) direction (see Fig. 1). In addition, we assume that the cloud is axially symmetric around the stagnation line. The motion of magnetic field generates an inductive electric field, which under ideal approximation is \( \mathbf{E} = -\mathbf{v} \times \mathbf{B} \). Combining induction equation \( \partial_t \mathbf{B} = -\text{curl} \mathbf{E} \) with continuity equation \( \partial_t \rho + \text{div} \rho \mathbf{v} = 0 \) gives (Alfvén 1942)

\[
(\partial_t + (\mathbf{v} \cdot \nabla)) \frac{\mathbf{B}}{\rho} = \frac{d}{dt} \frac{\mathbf{B}}{\rho} = \left( \frac{\mathbf{B}}{\rho} \cdot \nabla \right) \mathbf{v} \tag{1}
\]

On the other hand if \( \delta \mathbf{l} \) is an infinitesimal vector connecting two fluid elements along the flow line, its evolution with time is described by the same equation \( \frac{d}{dt} \delta \mathbf{l} = (\delta \mathbf{l} \cdot \nabla) \mathbf{v} \). This shows that quantity \( \frac{\mathbf{B}}{\rho} \) evolves according to the length along flow line. Now, it is easy to see that as field line is wrapped around the contact, its length increases in proportion to the radius of the magnetosphere \( \varpi \) at a given location, \( B/\rho \sim (B/\rho)_0(\varpi/\varpi_0) \) where \( \varpi_0 \) is the initial "impact parameter" of a given field line. On the contact \( \varpi_0 \to 0 \) and as a consequence the ratio \( (B/\rho) \to \infty \) (Pudovkin & Semenov 1985).

To understand this in a different way, note that Eq. (1) can be solved using Cauchy’s integral (e.g. Stern 1966):

\[
\frac{\mathbf{B}}{\rho} = \frac{\mathbf{B}_0 \cdot \nabla^{(0)} \mathbf{r}}{\rho_0} \tag{2}
\]

where \( \nabla^{(0)} \mathbf{r} \) is a derivative of the coordinate of a flow element \( \mathbf{r}(\mathbf{r}_0, t) \) with respect to initial coordinate \( \mathbf{r}_0 \), \( \mathbf{B}_0 \) is magnetic field at infinity. In curvilinear coordinates with scale factors \( h_i \) Eq. (2) becomes

\[
\frac{B_i}{\rho h_i} = \frac{B_{0,j}}{\rho_0 h_{0,j}^{(0)}} \frac{\partial x_i}{\partial x_j^{(0)}} \tag{3}
\]

(summation on \( j \) only). For an axially symmetric flow the scale factor \( h_\phi^{(0)} \) corresponding to the cyclic variable \( \phi \) is proportional to \( \varpi_0 \), initial radial cylindrical coordinate, which is equal to the distance from the symmetry axis. For toroidal component of magnetic field, Eq. (3) gives

\[
\frac{B_\phi}{\rho} = \left( \frac{B_\phi}{\rho} \right)_0 \left( \frac{\varpi}{\varpi_0} \right) \tag{4}
\]

where we used the fact that the azimuthal angle remains constant, \( \phi_0 = \phi \), so that \( \frac{\partial \phi}{\partial \varpi_0} = 1 \). Eq. (4) shows that \( B_\phi \) tends to infinity on the contact surface corresponding to \( \varpi_0 \to 0 \).

Divergence of the toroidal component of magnetic field can also be shown explicitly by rewriting \( \phi \) component of the Eq. (1) (in spherical or cylindrical coordinate system) as

\[
(\partial_t + (\mathbf{v} \cdot \nabla)) \frac{B_\phi}{\varpi \rho} = 0 \tag{5}
\]
implying that $B_\phi/\varpi\rho$ remains constant along the flow lines and is equal to $B_{0,\phi}/\varpi_0\rho_0$ (e.g., for a constant magnetic field at infinity along $y$ direction $B_{0,\phi} = B_0 \sin \phi$).

To illustrate the divergence of the other component of the magnetic field parallel to the contact (besides $B_\phi$) we will assume a particular form of the contact and a type of the fluid motion. As an example, consider a spherical body of radius $R_0$ moving with velocity $v_0$ subsonically through an incompressible fluid. The velocity flow in the frame associated with the body at a point defined by unit vector $\mathbf{n}$ is given by $\mathbf{v} = R_0^3(3\mathbf{n}(\mathbf{nv}_0 - \mathbf{v}_0))/(2r^3) - v_0$ where $r$ is a distance to the center of the body (e.g. Landau & Lifshits 1975). We can find an equation for flow lines $\varpi/\varpi_0 = 1/\sqrt{1 - (R_0/r)^3}$, which gives $\partial \theta / \partial \theta_0 = (\varpi/\varpi_0)(z_0/z) \sim \theta/\theta_0$ (last relation assumes small $\theta$ and $\theta_0$). Here $\theta$ is a polar angle of a spherical system of coordinates aligned with the direction of the motion, Fig. 1. Thus, $\partial \theta / \partial \theta_0$ diverges on the contact $\varpi_0 \to 0$ (or $\theta_0 \to 0$) implying that the component $B_\theta$ of the magnetic field diverges on the contact. Explicit calculations of the magnetic field structure for this problem confirm this conclusion (Bernikov & Semenov 1980). Incompressible motion past an axially symmetric body of arbitrary shape can be obtained from the solution for a spherical body through conformal mapping of the boundaries.

It is easy to see that the divergence of $\partial \theta / \partial \theta_0$ at $\theta_0 \to 0$ occurs more generally than just in the case of an incompressible flow considered above. It follows from the fact that near stagnation line radial (in cylindrical coordinates) velocity increases linearly with distance from the line, $v_\varpi \propto \varpi$, which implies that $\partial \theta / \partial \theta_0 \sim \theta/\theta_0$.

The above derivations simplify along the stagnation line $\varpi = 0$. Then the induction equation gives $B_\varpi v_z = \text{const} = B_0 v_0$. Since at the stagnation point $v_z = 0$, it follows that $B_\varpi \to \infty$.

So far we have assumed that there are no discontinuities (shocks) in the flow. In case of a supersonic motion the toroidal component of the magnetic field will experience a jump at the shock and, in addition, the flow lines will generally experience a bend. This will introduce correction factors to the above relations but will not remove the divergence of a magnetic field on the contact. The subsonic incompressible flow considered above to prove the divergence of the tangential to the contact component of a magnetic field should be a reasonable approximation near the contact surface far downstream of a possible forward shock.

The derivations above show that magnetic field is dynamically amplified on the contact. This is generally independent of whether the motion is supersonic or subsonic and is independent of plasma compressibility, though details of magnetic field amplification will be somewhat different in these cases. In addition, since governing equations can be written in terms of total derivatives along flow lines and thus are independent on the global structure of a magnetic field, tangled fields are also subject to the same amplification of the component parallel to the contact. The amplification occurs due to longitudinal stretching of magnetic field lines. Thus, a magnetized boundary layer is created in which magnetic field may reach near-equipartition values (for supersonic bulk motion) regardless of its value in the bulk (it should be non-zero, though). This effect is called magnetic draping or magnetic barrier.
Note also that the 2-D case is very different from 3-D case. For example, in 3-D, the induction equation for stationary flow gives

\[(\mathbf{v} \cdot \nabla)(r \sin \theta E_\phi) = 0\] (6)

so that the quantity \(r E_\phi\) is constant along the flow line, \(\varpi E_\phi = \varpi_0 E_{0,\phi}\). This implies that toroidal electric field vanishes on the contact, \(\varpi_0 = 0\). Contrary to this, in 2-D the electric field is constant everywhere, including on the contact, \(\mathbf{E} = (B_l v_n - B_n v_l) \mathbf{e}_x\), where coordinate \(l\) is along the generators of the contact surface from the axis of the symmetry and \(n\) is orthogonal to it. Divergence of magnetic field on the contact then follows from the requirement that normal components of both the velocity and magnetic field should vanish, \(v_n = B_n = 0\), implying \(B_l \to \infty\) (cf. Parker 1973, model for planar flows).

For super-Alfvenic motion, \(M_A \geq 1\), the only case when a magnetized layer does not form near the contact is when a cloud moves along the external magnetic field. In this case both \(B_\phi = 0\) and \(E_\phi = 0\) everywhere. In an axially symmetric, incompressible, stationary flow, velocity and magnetic field may be expressed in terms of flux functions \(\Psi\) and \(P\), \(\mathbf{v} = \nabla \Psi / \varpi\), \(\mathbf{B} = \nabla P / \varpi\) where \(P = A_\phi \varpi\) and \(A_\phi\) is a toroidal component of vector potential. Condition \(E_\phi = 0\) then gives (e.g. in spherical coordinates)

\[\partial_\theta P \partial_r \Psi - \partial_\theta \Psi \partial_r P = 0\] (7)

implying that \(P = f(\Psi)\) and \(B_r = v_r f',\ B_\theta = v_\theta f'\). In particular, if magnetic field at infinity is constant in space then \(f' = \text{constant}\), so that two flux functions are linearly related, \(P \propto \Psi\). Since for a spherical obstacle \(v_r = 0\) and \(v_\theta\) is finite on the contact, magnetic field remains finite everywhere.

The divergence of magnetic field on the contact is, of course, a consequence of the kinetic approximation. In full MHD case magnetic field is expected to be amplified to the point when its dynamical influence cannot be neglected anymore. For sonic and supersonic motion this corresponds to near-equipartition (with total pressure, not just those of relativistic particles) fields. The layer itself must be modeled using MHD, not fluid equations. Inside the layer non-isotropic magnetic pressure will break the axial symmetry of the flow even for an axially symmetric obstacle. Stagnation point flow changes into stagnation line flow, where stream lines branch off not only at the stagnation point, but also at all points along the symmetry line (\(x = 0\) plane in Fig. 1). In case of a flow of plasma cloud, since position of the contact is determined by the pressure balance between two media, this will lead to deformation of the contact surface (by a fraction \(\sim 1/M_A^2\)).

A magnetized layer should also be associated with depletion of plasma. To see the reason for this, note that the total pressure, which is a sum of magnetic and gas pressures, should remain approximately constant well behind a possible forward shock (neglecting possible temperature anisotropy, see below). Increasing magnetic pressure requires a decrease in gas pressures, which in a

\(\text{Tangential component of electric field is continuous across tangential discontinuities, like shocks.}\)
polytropic fluid with adiabatic index $\gamma$ (e.g. for isentropic ideal gas) is accompanied by a decrease in density (approximately as $\rho \propto (\beta/(1 + \beta))^{1/\gamma}$, decreasing with decreasing $\beta$). Similarly, the plasma temperature should decrease as well, $T \propto (\beta/(1 + \beta))^{(\gamma-1)/\gamma}$. Thus, for a supersonic motion, the plasma density first increases at the forward shock and then decreases inside the magnetic barrier close to the contact (Wu 1992).

Reality is a bit more complicated than this simple estimate since several competing plasma physics effects come into play. First, compression of plasma in a magnetic field creates anisotropy of plasma pressure due to conservation of adiabatic invariants (double adiabatic model of Chew et al. 1956). When compressed normally to magnetic field lines, conservations of the adiabatic invariant $p_\perp/(B\rho) = \text{const}$ and effective polytropic index $\gamma_\perp = 2$ would lead to density and transverse temperature $T_\perp$ increasing with a magnetic field. But strong plasma anisotropy leads to expulsion of plasma from high magnetic field regions due to mirror forces (magnetic bottling, reflection of particles from regions of high magnetic field, is the best known example of mirror forces). Second, onset of plasma instabilities may limit the pressure anisotropy (e.g. to $T_\perp \sim T_\parallel$). In case of planets interacting with the Solar wind, typically isotropic MHD model give a reasonable fit to magnetic field, pressure and temperature profiles (Denton & Lyon 1996; Pudovkin et al. 1999; Song & Russel 2002). Behavior of the magnetized boundary layer in some aspects is opposite to the purely fluid case. For example, the former leads to density minimum on the contact while the later predicts density maximum (Lees 1964), (see Song & Russel 2002, for a recent review).

Historically, magnetic draping effect was somewhat a surprise in modeling of Solar wind interaction with planets. It was expected that for small magnetization the flow may be computed from purely hydrodynamical equations, and a magnetic field may be added later using frozen-in condition (Spreiter et al. 1966). Using this prescription Alksne (1967) found that the magnetic field goes to infinity at the contact, especially strongly at plane containing the symmetry axis and magnetic field ($y = 0$ plane in our notations). Zwan & Wolf (1976) (see also Southwood & Kivelson 1995) calculated in details the properties of the magnetic barrier and predicted that it should be associated with depletion of plasma density, as magnetic field lines are stretched along the contact surface and plasma is allowed to flow along stretched magnetic field lines and, in addition, developing of temperature anisotropy leads to magnetic mirror forces pushing plasma away from regions of high magnetic field.

These theoretical ideas have been generally confirmed by direct satellite observations of the Terrestrial (e.g. Paschmann et al. 1978; Crooker 1979; Kallio et al. 1994; Wang et al. 2003), Cytherean (Biernat et al. 1999) and Martian (Öieroset et al. 2004) depletion layers. Overall, observations seem to be consistent with modern full MHD models (e.g. Kallio et al. 1998; Erkaev et al. 2000, and preceding references). Magnetic draping also occurs at the outer heliosphere and may be related to low frequency, $\sim 3$ kHz, radiation observed by the Voyager spacecraft (Cairns 2004). In astrophysical setting these ideas have been touched upon theoretically by Kulcsrud et al. (1965); Rosenau & Frankenthal (1976) for the case of supernova expansion and by Lyutikov (2002) in the case of relativistic GRB outflows. A number of numerical experiments also saw formation
of the magnetic barrier (Mac Low et al. 1994; Jones et al. 1996; Gregori et al. 2000; Asai et al. 2004, 2005), producing results in agreement with the above theoretical estimates. In particular, in case of merging cluster cores, the work of Asai et al. (2005) clearly shows formation of magnetic barrier and its larger thickness in 2-D, as expected (see section 2.2).

2.2. Thickness of the magnetic barrier

For low magnetization, $\beta \gg 1$, a flow may be separated in two regions: a bulk, where motion is nearly hydrodynamic, and a boundary layer, where effects of a magnetic field are important. Flows in the two regions must be matched at their boundaries. Thus, magnetic draping almost does not affect, for example, the location of the forward shock in front of the obstacle.

Next we estimate a thickness of the transition layer. Both for subsonic and supersonic motion of the cloud through IGM, the motion near the critical point is strongly subsonic and can be considered incompressible. In this case the velocity field is $v_\infty = -(3V_0/2)(\varpi/L)$, $v_z = 3V_0(z/L)$ (Landau & Lifshits 1975). Then, along the stagnation line $\varpi = 0$ magnetic field evolves according to

$$B_\varpi + 2z\partial_z B_\varpi = 0$$

(8)

giving $B_\varpi \propto 1/\sqrt{z}$. To estimate the thickness of the magnetized layer, note that inside the layer magnetic pressure becomes of the order of ram pressure, $B^2 \sim 8\pi \rho v_0^2$ (at this point magnetic forces will strongly affect the plasma flow). If the typical size of the plasma cloud is $L$ and $M_A = v_\infty/v_{A,\text{inf}}$ is Alfvén Mach number defined in terms of Alfvén velocity $v_{A,\text{inf}}$ at infinity, then, using Eq (8) we find

$$\frac{\Delta r}{L} \sim \frac{1}{M_A^2} \ll 1$$

(9)

The value of plasma $\beta$ inside the magnetized sublayer is $\beta_{in} \sim (1 + M_s^2)/M_s^2$, where $M_s = v_\infty/c_s$ is sonic Mach number at infinity. Thus, for supersonic motion, $M_s \geq 1$, a near-equipartition layer, $\beta \sim 1$, forms (see also Zwan & Wolf 1976). In case of subsonic motion plasma $\beta$ inside the sublayer is much smaller than in the bulk, $\beta_{in}/\beta \sim 1/M_A^2 \ll 1$ (so that plasma is more strongly magnetized). Both for subsonic and supersonic motion, the thickness of the magnetized boundary layer is much smaller that the size of the cloud (for $M_A \geq 1$). Note, the dimensionality of the problem is an important issue. Repeating the above estimates for a 2-D flow, the thickness of the magnetized layer is $\Delta r/L \sim 1/M_A$ (Erkaev et al. 1995), much larger than in the 3-D case.

The time it takes to form the layer may be estimated from a condition that a swept-up magnetic flux is of the order of the magnetic flux through the layer. A layer forms rather quickly, after the cloud traversed a length $l \sim L/M_A \leq L$. An exception is when the cloud moves very slowly, sub-Alfvenically $M_A \leq 1$, in which case no magnetized layer forms anyway.
3. Application to cluster cold fronts

A well-studied case is Abell 3667, which we use below for numerical estimates (Vikhlinin & Markevitch 2002). In this case the density of the hot component is \( n_h \sim 8 \times 10^{-4} \text{ cm}^{-3} \), its temperature \( T_h \sim 8 \text{ keV} \), front velocity \( \sim 1500 \text{ km s}^{-1} \), corresponding to sonic Mach number \( M_s \sim 1 \). The Spitzer Coulomb free path is \( \lambda_e \sim 30 \text{ kpc} \) (Ettori & Fabian 2000). If an average magnetic field in the cluster is \( \sim 1 \mu\text{G} \), the electron Larmor radius is only \( \sim 4 \times 10^8 \text{ cm} \). Thus, the Larmor radius is many orders of magnitude smaller than the collision length, so that even in case of highest possible cross-field diffusion (Bohm diffusion), cross-field conductivity is virtually zero.

Assuming \( B = 1 \mu\text{G} \), the plasma \( \beta \) parameter in the bulk is then \( \beta \sim 400 \), Alfvén Mach number is \( M_A \sim \sqrt{\gamma \beta / 2 M_s} \sim 20 \), so that for the core of \( L \sim 500\text{kpc} \) the thickness of the strongly magnetized boundary layer is \( \sim L / M_A^2 \sim 1.25\text{kpc} \). This is somewhat smaller than Chandra resolution for a typical cluster, but since the thickness depends strongly on the assumed IGM magnetic field, \( \propto B^2 \), it is possible that in some cases the layer may be resolved.

If the IGM is permeated by large scale magnetic field, the magnetized layer provides a contribution to rotation measure of the order of

\[
RM \sim 15 \text{rad/m}^2 \left( \frac{M_s}{1 \mu\text{G}} \right)^{-1} \left( \frac{B_\infty}{1 \mu\text{G}} \right)^2 \left( \frac{n}{8 \times 10^{-4}} \right)^{1/2} \left( \frac{T}{8\text{keV}} \right)^{-1/2} \left( \frac{L}{500\text{kpc}} \right)
\]

This is a relatively small value. In addition, since only the component of magnetic field along the line of sight contributes to the rotation measure, this estimate is subject to strong geometrical variations over the contact.

4. Radio bubbles

Magnetic draping should also be important for stabilization of rising radio bubbles, X-ray emission voids of up to 30 kpc in size, against Rayleigh-Taylor and Kelvin-Helmholtz instabilities and, similar to merging cores, suppression of thermal conductivity across the front. For example, (Robinson et al. 2004) (see also (Churazov et al. 2001; Brüggen & Kaiser 2002; Jones & De Young 2005)) showed that in the absence of magnetic field, bubbles are disrupted by hydrodynamic instabilities and effects of thermal conduction. In order to stabilize the contact, locally magnetic field should be of the order of equipartition field, while the field in bulk is much smaller. The effect of magnetic draping provides exactly what is needed for stability and suppression of conductivity: near-equipartition magnetic field tangential to the contact (in case of nearly sonic or supersonic expansion). Hydrodynamically, the problem of expanding cavity blown by an AGN jet is similar to the one considered by Kulsrud et al. (1965) of a supernova remnant expanding into ISM (see also De Young 2003).

Consider an impermeable sphere expanding radially with radius \( R(t) \) into incompressible medium permeated by constant magnetic field \( B_0 \). From the incompressibility condition we find
\( v_r = R'(t)(R(t)/r)^2 \). Inductive electric field is in \( \phi \) direction and the induction equation gives

\[
\begin{align*}
\partial_t B_\theta &= \frac{\partial_r (B_\theta v_r)}{r} \\
\partial_t B_r &= -\frac{\partial_\theta (\sin \theta B_\theta v_r)}{r \sin \theta}
\end{align*}
\]  

(11)

Assuming that \( B_\theta \) depends on self-similar variable \( \xi = r/R(t) > 1 \), equation for \( B_\theta \) becomes

\[
B_\theta' (\xi^3 - 1) + \frac{B_\theta}{\xi} = 0
\]

(12)

which gives

\[
\begin{align*}
B_\theta &= \frac{\sin \theta}{(1 - \xi^{-3})^{1/3}} B_0 \\
B_r &= -\cos \theta (1 - \xi^{-3})^{1/3} B_0
\end{align*}
\]  

(13)

This implies that tangential component of magnetic field diverges on the contact \( \xi = 1 \) as \( \propto (3(\xi - 1))^{-1/3} \). Using Eq. (13) we can find equation for field lines, \( (\xi^3 - 1) \sin \theta = \varpi_0/R(t) \), see Fig. 2. Similarly to the case of translational motion, amplification of magnetic field on the contact will lead to formation of a magnetized boundary layer which thickness now is smaller by a factor \( M_A: \Delta r/R \sim M_A^{-3} \) (cf. Kulsrud et al. 1965). MHD simulations generally confirm this picture: Robinson et al. (2004) and Jones & De Young (2005) find that modest IGM magnetic fields can suppress thermal conductivity and stabilize the rising bubbles against disruption by fluid instabilities.

Presence of a large scale magnetic field inside a bubble, a leftover from AGN pumping, may ease constraints on stability. An internal magnetic field may also be needed to compensate for high external pressure since bubbles appear to be in pressure equilibrium, but the absence of X-ray emission argues for a lower temperature in comparison with external IGM. An additional stabilizing effect may come from a pile-up of magnetic field inside the contact discontinuity. Typically, the entropy of gas in the cores is lower than at the outskirts, resulting in motion of gas between the core and the contact. This will similarly create a magnetized layer on the inside of the contact.

5. Conclusion

We point out that presence of a dynamically unimportant magnetic field in the bulk of the IGM leads to formation of a strongly magnetized boundary layer which undoubtedly affects the mechanical and thermodynamical coupling between plasmas of the cold merging cores or rising AGN blown bubbles on the one side and hot IGM plasma on the other side. To a different degree this occurs both around sub-sonically and super-sonically moving flows, for random and large-scale magnetic fields and does not require plasma compressibility. Primarily, magnetic draping leads to strong suppression of thermal conductivity across the contact and may explain observed narrowness
of the transition layers. A boundary layer of near-equipartition magnetic field (for supersonic motion $M_s \gg 1$) is also expected to stabilize the KH instability of the contact, especially close to the critical point of the flow (in case of merging cores).

Thus, even weak bulk magnetization strongly affects the interaction of a flow with an obstacle. This runs contrary to expectations that a weak magnetic field in the bulk should not affect much overall dynamics of a merging cluster. For example, Heinz et al. (2003) (see also Nagai & Kravtsov 2003; Bialek et al. 2002) carried out hydrodynamical simulations of the interaction of cold subcluster plasma and hot ambient matter neglecting heat conduction and magnetic fields. When Asai et al. (2004) repeated simulations of Heinz et al. (2003), they found that when the Spitzer conductivity is adopted, the subcluster evaporates rapidly and the cold front is not formed.

The main prediction of the model is that magnetic field may reach near-equipartition values and be directed along the contact separating two fluids. This is best tested with high resolution polarization radio observation. If high-energy non-thermal particles are accelerated locally and produce synchrotron emission, the direction of the magnetic field may be then inferred from linear polarization. For example, radio observations of NGC 4522, a prominent galaxy in the Virgo cluster, indeed show magnetic fields along the front (Vollmer et al. 2004). Parallel magnetic fields are also observed in the case of peripheral features seen in Abell 2256 (Rottgering et al. 1994; Clarke 2001). In addition, a magnetized layer may contribute to the rotation measure of background radio sources. Naturally, in both cases we can estimate only average values of magnetic field along the line of sight and it’s not easy to single out contributions from a narrow, strongly magnetized layer.

A number of observations may already be interpreted in the framework of the model: enhanced RMs are often seen at the edges of structural features in clusters. E.g., Carilli et al. (1988) see enhanced RMs at the edges of hot spots in Cygnus A (Carilli et al. 1988), Taylor et al. (1992) find an enhanced RM at the edge of one of the hot spots in 3C 194. Though our estimates in application to Abell 3667 show that variations of RM across the front are typically small (and even smaller for expanding bubble due to smaller thickness in that case, Section 4), total RM is a strong function of assumed magnetic field in the bulk, $\sim B^2$. The Expanded Very Large Array (EVLA) will be most instrumental in mapping magnetic fields in clusters of galaxies.

Another prediction of the model is that due to the depletion of plasma from the magnetized sheath we may observe a narrow layer $\sim 1\text{kpc}$ thick of suppressed X-ray emission on the outside edge of cold fronts. Naturally, observations needed to test this are very challenging.

Dynamic amplification of an external magnetic field on both sides of the contact creates conditions favorable for reconnection between external and internal magnetic fields (e.g. Pudovkin et al. 2002). As a result of reconnection, normal components of velocity at the contact surface may be non-zero, determined either by stripping or by physics of a reconnection layer. Reconnection is known to be an efficient site of particle acceleration which may produce observable radio and non-thermal X-ray signals (cf. a model of non-thermal emission in young supernova remnants of Lyutikov & Poln 2004). Wrapping of magnetic field lines may create conditions favorable for
reconnection in the wake of the core, similar to reconnection in the Earth magneto-tail (Galeev 1979).

Numerical simulations required to correctly describe the structure of the magnetic barrier are bound to be complicated. First, they must be done in full 3-D using MHD approximation: fluid models lead to qualitatively different results, while 2-D MHD simulations grossly overestimate the thickness of the magnetic barrier (by a factor $M_A \gg 1$ if compared with the 3-D case). In addition, since it is expected that plasma inside the magnetic barrier becomes anisotropic, simulations should account for possible temperature anisotropy, e.g. within the framework of Chew-Goldberger-Low (Chew et al. 1956) theory. Finally, kinetic effects, like self-limiting temperature anisotropy, may be important as well.

Finally we note that the effect of magnetic draping may be important in other astrophysical applications, in particular in case of suppressed conductivity between dense cold HI ISM clouds and a lower density warmer medium (e.g. review by Bregman 2004). Supernova shocks and outflows from OB associations induce a large scale motion in the warm medium which will lead to magnetic blanketing of cold clouds. This effect should be especially important for High-Velocity Clouds.

I would like to thank Andrei Kravtsov, Jean Eilek, Maxim Markevitch, Christoph Pfrommer, Dmitry Uzdensky and Alexey Vikhlinin for comments and discussions.

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Fig. 1.— Draping of magnetic field lines around an axially symmetric obstacle. $B_0$ is magnetic field at infinity along $y$ direction, a cloud is moving along $z$ direction, dashed line is a flow line of a fluid element, $\varpi_0$ is the initial distance from the symmetry axis, $\theta$ is a polar angle between a unit radius-vector from the origin $\mathbf{n}$ to the current position of a fluid element and $z$ axis.
Fig. 2.— Magnetic field lines in the $z - y$ plane for subsonic incompressible expansion of a sphere into a medium with a constant magnetic field along $z$ axis at infinity, kinematic approximation. Note the strong compression of field lines on the contact $r = R(t)$. 