Arrangement Method of Vibration Dampers Considering Frequency Response Physical Characteristics and Stiffness of Conductor

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Abstract. In order to study the calculation method of installation distance of vibration dampers with higher precision, so as to maximize its anti-vibration performance on the conductor. The arrangement method of dampers based on the frequency response characteristics of conductors is proposed, the dynamic model of conductor considering stiffness is established. The model is solved by finite difference method, the natural frequency and vibration mode of conductor in case of Aeolian vibration are obtained, and the wavelength of conductor vibration is obtained according to its mode, so as to calculate the installation distance of vibration dampers. The calculation results of the new method are compared with those of the traditional method, which proves the advantage of the new method in calculation accuracy. The new method has a good effect on improving the calculation accuracy of the installation distance of the vibration dampers on the conductors, which is greatly influenced by its stiffness. At the same time, the frequency response of the conductor should be considered as much as possible in the calculation of the installation distance of the damper.

Keywords: Cconductor, vibration damper, frequency response characteritics, stiffness, arrangement method.

1. Introduction

In the uniform laminar wind field, the Aeolian vibration of conductors is occurred due to the periodic shedding of Karman vortex. Long term Aeolian vibration is very easy to cause fatigue damage to the conductors. Vibration dampers is widely used in overhead transmission lines because of its good anti-vibration performance. Many scholars have studied the system of the conductor with vibration dampers. Andre Leblond [1] deals with a method for evaluating the flow of vibration power in single overhead conductors along a full-scale test line. The method leads to the determination of vibration power flow as well as the spectral content of these waves. Whenever a damping device is attached to the conductor, an average traveling wave reflection coefficient can be calculated, yielding the damping device efficiency. Li Li [2] used cable element to simulate transmission line based on energy equivalence principle. The magnetic damping of transmission line and damper is regarded as viscous damping, and the wind input power is equivalent to the Aeolian excitation force acting on the element node. According
to the relationship between amplitude and curvature, the distribution law of dynamic bending strain amplitude along the transmission line is deduced. Ye Zhixiong and Fu Lei [3, 4] established the mechanical model of the interaction between overhead transmission line and vibration damper. The calculation equation of energy dissipation rate of vibration damper which can consider the installation position of vibration damper, transmission line tension and bending stiffness of transmission line is derived, based on the propagation principle of vibration wave and the mechanical impedance of vibration damper. Wang Feng [5] designed and manufactured the Aeolian vibration test model of long-span transmission line with vibration dampers, studied the vibration response of damper transmission line system under different tension, and analyzed the influence of the weights orientation of FR-4 damper and the mixed installation of FR-4 and FDN dampers on the vibration of long-span conductor.

The quality, quantity and installation position of vibration dampers play an important role in the prevention and control of Aeolian vibration of overhead lines. The traditional arrangement method of vibration dampers does not consider the conductor stiffness, and the wind speed involved in the calculation of strand Aeolian vibration prevention frequencies are empirical values. In this paper, a new method of damper arrangement is proposed, and the mathematical model of overhead line considering stiffness is established. Based on the frequency response characteristics and the results of modal analysis of a type of ground wire, the installation position of its vibration dampers is calculated.

2. Traditional method and its limitation

When the uniform laminar wind blows through the conductor, as shown in Figure 1, alternating falling vortices are generated on the leeward side of it, generating periodic excitation in the up and down directions, causing the conductor Aeolian vibration. When the vibration enters a stable state, the vortex shedding frequency $f_w$ is equal to the conductor vibration frequency $f_c$, and the expressions of $f_w$ and $f_c$ are shown in equations (1) and (2) [6].

\[ f_w = S_t \frac{v}{D} \]  

\[ f_c = \frac{1}{\lambda} \sqrt{\frac{T}{m}} \]  

In equation (1), $S_t$ is the Strouhal constant, generally $0.185 \sim 0.2$; $v$ is the wind speed, in m/s; $D$ is the conductor diameter, in m.
In equation (2), \( T \) is the conductor tension, in N; \( m \) is the mass per unit length, in kg/m; \( \lambda \) is the length of the vibration wave, m.

The traditional layout methods of vibration damper in the world include equidistant and unequal arrangement. The vibration dampers are installed in the maximum and minimum wavelength range to achieve the anti-vibration effect. The lower limit of wind speed used for wavelength calculation is 0.5 m/s and the upper limit is 5~6 m/s. Traditional methods have the following limitations:

1. For the ground wire with small diameter, the wind speed value method has the problem of inaccurate value, as described in part 3;
2. The equation used in the calculation of strand wavelength ignores the influence of the stiffness on its vibration wavelength [7].

The above limitations will lead to inaccurate calculation of the vibration wavelength, which will weaken the damping effect of dampers arrangement scheme or poor the anti-vibration performance of high-frequency vibration.

3. Theory of the new method
The method includes measuring the frequency response characteristics, then establishing a model considering the conductor stiffness, calculating the vibration wavelength range combined with the measurement results of the frequency response of the conductor, and reasonably arranging the installation position of the dampers, so as to consume the vibration energy of the wire to the greatest extent.

3.1. Theory of frequency response characteristics measurement of the conductor
The frequency response characteristic of the conductor is the ability of the it to resist Aeolian vibration without additional anti-vibration devices. Because the overhead transmission line conductor is twisted by multiple single wires, its frequency response has strong nonlinearity. The energy balance method is often used to measure the frequency response of conductor in the world. The common test device for measuring the frequency response characteristics of the conductor is shown in Figure 2.

![Figure 2. Schematic diagram of conductor frequency response measuring device.](image)

The experiment was conducted according to reference [8], the exciting force \( F \), exciting speed \( V \), the phase angle \( \phi \) between exciting force and speed, the amplitude \( A \) of the first free half wave of the conductor at the excitation end and the dynamic bending strain \( \varepsilon \) of the conductor at the outlet of the clamp on the left side in Figure 2 are measured. In the test, there is no other damping device on the conductor, so all the energy input to it is absorbed, that is,

\[ P_w = P_s \] (3)
In equation (3),

\[
P_w = F \cdot v \cdot \cos \phi \\
P_s = \phi(f, A, D) = 10^\alpha \left(\frac{A}{D}\right)^\beta
\]

In equations (4), \( f \) is the vibration frequency of the conductor, in Hz; \( \alpha \) and \( \beta \) are the so-called frequency response coefficient \[9\]. The amplitude \( A_b \) of the conductor under the condition \( P_w = P_s \) is calculated by equations (3) and (4). The strain value \( \varepsilon_b \) at the outlet of the clamp when the amplitude is \( A_b \) at each frequency \( f \) can be obtained from the corresponding relationship \( \varepsilon - A \), which is measured in the test, by interpolation. Thus, the corresponding relationship between \( \varepsilon_b \) and \( f \) is established, and that is the frequency response characteristic of the conductor.

3.2. Dynamic model considering conductor stiffness and its solution

Assuming that the conductor is homogeneous, the damping is ignored. When Aeolian vibration occurs by excitation of wind, the micro-element mechanical analysis is shown in Figure 3.

The following equation can be obtained from the force and moment balance conditions in the \( y \) direction in Figure 3,

\[
\begin{align*}
mdx \frac{\partial^2 y}{\partial t^2} + M + \frac{\partial M}{\partial x} dx &= T_{\beta} \\
q(x, t) + \frac{\partial Q}{\partial x} dx &= 0
\end{align*}
\]

Figure 3. Micro-element stress analysis of the conductor.

In equations (5), \( Q \) is the shear force on the micro-element of the conductor, in N; \( M \) is the bending moment of micro-element, in N•m; \( q(x,t) \) is the excitation force, in N/m. The differential equation of conductor free vibration shown in equation (6) can be obtained by equation (5),

\[
EI \frac{\partial^4 y}{\partial x^4} - T \frac{\partial^3 y}{\partial x^3} + m \frac{\partial^2 y}{\partial t^2} = 0
\]
In equation (6), $EI$ is the dynamic bending stiffness of the conductor, in N•m². In this study, half of the maximum dynamic bending stiffness is taken [10], and the calculation method of the maximum dynamic bending stiffness is referred to in reference [11].

The conductor is erected between two towers, and its mechanical model is shown in Figure 4. It is assumed that the conductor is a beam hinged at both ends. If the conductor is divided into $n$ elements, a total of $n-1$ element nodes are generated, and the finite difference method [12] is used to solve equation (6).

![Figure 4. Mechanical model of overhead line.](image)

The displacement function of stable Aeolian vibration of conductor is shown as follows,

$$y = y_0e^{i\omega t}$$  \hspace{0.2cm} \text{(7)}

The dimensionless parameters $Y = y_0/D$ and $X = x/L$ are introduced. $L$ is the span length, in m. Equation (6) can be optimized into dimensionless form,

$$\frac{EI}{mL^2} \frac{d^4 Y}{dx^4} - \frac{T}{mL^2} \frac{d^2 Y}{dx^2} - \omega^2 Y = 0$$  \hspace{0.2cm} \text{(8)}

The fourth-order central difference method is used to obtain the relationship as follows,

$$\left( \frac{d^4 Y}{dx^4} \right)_i \approx \frac{1}{6\Delta x^4} \left( -Y_{i+3} + 12Y_{i+2} - 39Y_{i+1} + 56Y_i - 39Y_{i-1} + 12Y_{i-2} - Y_{i-3} \right)$$  \hspace{0.2cm} \text{(9)}

$$\frac{d^2 Y}{dx^2} \approx \frac{1}{12\Delta x^2} \left( Y_{i+2} + 16Y_{i+1} - 30Y_i + 16Y_{i-1} - Y_{i-2} \right)$$  \hspace{0.2cm} \text{(10)}

In equations (9) and (10), $i \in [0, n]$, $\Delta x$ is the unit length, in m.

Substituting equations (9) and (10) into (8):

$$-aY_{i+3} + (12a + b)Y_{i+2} - (39a + 16b)Y_{i+1} + (56a + 30b - \omega^2)Y_i$$  
$$- (39a + 16b)Y_{i-1} + (12a + b)Y_{i-2} - aY_{i-3} = 0$$  \hspace{0.2cm} \text{(11)}

In equation (11),

$$a = \frac{EI}{6mL^2\Delta x^4}$$  \hspace{0.2cm} \text{(12)}

$$b = \frac{T}{12mL^2\Delta x^2}$$  \hspace{0.2cm} \text{(13)}

The two ends of the conductor are hinged, so the boundary conditions are:
\[ \begin{align*}
Y_0 &= 0 \\
Y_n &= 0
\end{align*} \tag{14} \]

\(Y_2, Y_3, Y_{n+1}, Y_{n+2}\) are used in equation (11), and the following assumptions are made,

\[ Y_2 = Y_2, \ Y_3 = Y_3, \ Y_{n+1} = Y_{n+1}, \ Y_{n+2} = Y_{n+2} \tag{15} \]

The characteristic equation of the system can be obtained from equations (11), (14) and (15), shown as follows,

\[ [K - M \omega^2]Y = 0 \tag{16} \]

In equation (16),

\[ Y = [Y_1 \ Y_2 \ \cdots \ Y_{n+1}]^T, \quad M = E_{n+1} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1 \end{bmatrix} \tag{17} \]

\[
K_{(n-1)\times(n-1)} = \begin{bmatrix}
k_1 + k_3 & k_2 + k_4 & k_3 & k_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
k_2 + k_4 & k_1 & k_2 & k_3 & k_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
k_3 & k_2 & k_1 & k_2 & k_3 & k_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
k_4 & k_3 & k_2 & k_1 & k_2 & k_3 & k_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & k_4 & k_3 & k_2 & k_1 & k_2 & k_3 & k_4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k_4 & k_3 & k_2 & k_1 & k_2 & k_3 & k_4 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & k_4 & k_3 & k_2 & k_1 & k_2 & k_3 & k_4 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & k_4 & k_3 & k_2 & k_1 & k_2 & k_3 & k_4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & k_4 & k_3 & k_2 & k_1 & k_2 & k_3 & k_4 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & k_4 & k_3 & k_2 & k_1 & k_2 & k_3 & k_4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & k_4 & k_3 & k_2 & k_1 & k_2 & k_3 + k_4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_4 & k_3 & k_2 & k_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_4 & k_3 & k_2 + k_4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_4 & k_3 + k_4 \\
\end{bmatrix} \tag{18} \]

In this matrix,

\[
\begin{align*}
k_1 &= 56a + 30b \\
k_2 &= -39a - 16b \\
k_3 &= 12a + b \\
k_4 &= -a
\end{align*} \tag{19} \]

The eigenvalues and eigenvectors of coefficient matrix (16) are the natural frequencies of the system and its corresponding main vibration modes.
The main vibration mode of each order is the relative displacement of each element node at different resonance frequencies. The vibration wavelength can be obtained by using the vibration mode of the conductor. When the conductor resonates, the wavelength is the length of a complete sine wave in the direction of the conductor axis.

4. Application of the new method

The new method is used to arrange the vibration dampers of ground wire in a project, and the characteristics of this method are discussed. The project uses concentric-lay-stranded aluminum-clad steel conductor JLB20A-150 as the ground wire, and its technical parameters are shown in Table 1.

Table 1. Technical parameters of ground wire JLB20A-150.

| Diameter of conductor (mm) | 15.75 |
|---------------------------|-------|
| Mass per unit length (kg/m) | 0.9894 |
| Tension (kN) | 33.93 |
| Diameter of steel (mm) | 3.15 |
| Number of steel | 19 |
| Dynamic bending stiffness (N·m²) | 414.14 |

The frequency response characteristics of the ground wires produced by two manufacturers are tested, and the results are shown in Table 2.

Table 2. Test results of frequency response characteristics of two ground wires.

| Frequency (Hz) | Dynamic bending strain (με) | Frequency (Hz) | Dynamic bending strain (με) |
|---------------|----------------------------|---------------|----------------------------|
| 15.57         | 245.234                    | 18.73         | 496                        |
| 20.75         | 336.933                    | 23.99         | 582                        |
| 26.04         | 434.616                    | 29.33         | 439                        |
| 31.53         | 377.165                    | 36.16         | 417                        |
| 36.82         | 347.254                    | 41.22         | 367                        |
| 42.13         | 347.59                     | 46.42         | 344                        |
| 47.39         | 299.987                    | 51.57         | 333                        |
| 52.74         | 276.451                    | 56.67         | 310                        |
| 58.08         | 262.538                    | 61.87         | 279                        |
| 63.51         | 230.507                    | 65.32         | 271                        |
| 68.99         | 212.638                    | 70.92         | 267                        |
| 74.13         | 203.421                    | 75.29         | 249                        |
| 79.63         | 178.401                    | 80.66         | 220                        |
| 85.21         | 169.193                    | 84.21         | 205                        |
| 90.8          | 145.509                    | 91            | 169                        |
| 96.54         | 148.188                    | -             | -                          |
| 102.26        | 136.778                    | -             | -                          |

It can be seen that the frequency response characteristics of the same type of ground wires may be different due to different manufacturers or production time. As shown in Figure 5, the upper limit frequencies of ground wire JLB20A-150 produced by the two manufacturers are 84.21Hz and 74.13Hz respectively. The wind speed calculated by equation (1) is 7.16m/s and 6.31m/s. Obviously, the traditional method is not accurate enough to take the upper limit wind speed of 5~ 6m/s for this kind of strand.
Taking conductor 1# as an example, assuming that the span length is 100m, the conductor is equally divided into 500 units, and the natural frequency and vibration mode of the conductor are obtained by using the method described in subsection 2.2. The lower limit of the anti-vibration frequency of the ground wire is 5.56Hz and the upper limit is 84.42Hz. The wavelength is obtained by using the vibration mode of the conductor, where the minimum ($\lambda_m$) is 3.38m and maximum ($\lambda_M$) is 33.44m. The equidistant installation distance $b=1.11m$, unequal installation distance $b_1=1.31m$ and $b_2=0.89m$ are calculated according to the calculation method of installation distance of vibration damper in preference [6]. The traditional method does not consider the frequency response characteristics and stiffness of ground wire, and the empirical value is used for wind speed. The lower limit is 0.5m/s and the upper limit is 6m/s. Therefore, the maximum wavelength $\lambda_M$ is 31.54m, corresponding to the lower limit of ground wire vibration frequency (5.87Hz), and the the minimum wavelength $\lambda_m$ is 2.62m, corresponding to upper limit (70.48Hz). Using the same method above, the equidistant installation distance $b=1.21m$, and unequal installation distance $b_1=1.44m$, $b_2=0.98m$. Thus, the maximum error that using the traditional method to calculate the installation position of the vibration dampers of JLB20A-150 ground wire can reach 10%.

5. Conclusions

(1) For the ground wire, the empirical wind speed value is used when calculating the vibration wavelength. The upper limit frequency of the conductor that needs anti-vibration may be lower than the actual value. Therefore, the frequency response characteristics of the conductor shall be fully considered when arranging the vibration damper;

(2) There is a certain error between the vibration wavelength calculated by the traditional method and the dynamic model considering stiffness, and the error will further increase when the span decreases or the strand section increases.

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