Viscous corrections to anisotropic flow and transverse momentum spectra from transport theory

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Viscous hydrodynamics is commonly used to model the evolution of the matter created in an ultra-relativistic heavy-ion collision. It provides a good description of transverse momentum spectra and anisotropic flow. These observables, however, cannot be consistently derived using viscous hydrodynamics alone, because they depend on the microscopic interactions at freeze-out. We derive the ideal hydrodynamic limit and the first-order viscous correction to anisotropic flow (v2, v3 and v4) and momentum spectrum using a transport calculation. The linear response coefficient to the initial anisotropy, \( v_n(p_T)/\varepsilon_n \), depends little on \( n \) in the ideal hydrodynamic limit. The viscous correction to the spectrum depends not only on the differential cross section, but also on the initial momentum distribution. This dependence is not captured by standard second-order viscous hydrodynamics. The viscous correction to anisotropic flow increases with \( p_T \), but this increase is slower than usually assumed in viscous hydrodynamic calculations. In particular, it is too slow to explain the observed maximum of \( v_n \) at \( p_T \sim 3 \text{ GeV}/c \).

I. INTRODUCTION

Relativistic viscous hydrodynamics is the state of the art for describing the evolution of the strongly-coupled system formed in an ultrarelativistic heavy-ion collision at RHIC or LHC. It has long been realized that ideal hydrodynamics naturally explains the large magnitude of elliptic flow. However, the system formed in such a collision is so small that deviations from local thermal equilibrium are sizable, resulting in the inclusion of viscosity in hydrodynamic calculations. Viscosity typically reduces the magnitude of elliptic flow by 20%. Viscous effects on higher harmonics of anisotropic flow, such as triangular flow, are even larger.

Even though there is a consensus that viscosity matters, the calculation of viscous corrections to observables is not yet under control. The reason is that viscosity affects not only the space-time history of the fluid, but also the momentum distribution of particles at “freeze-out”, which has an off-equilibrium part proportional to viscosity. Viscous hydrodynamics itself does not fully specify this off-equilibrium part. The only requirement is that the system of particles should generate the same energy-momentum tensor as the fluid just before freeze out. This requirement, however, does not constrain the dependence of the relative deviation to equilibrium on the momentum \( p \) in the rest frame of the fluid, which is essentially a free function \( \chi(p) \). This function is not universal, and involves the differential cross sections between constituents. It is typically put by hand in hydrodynamic calculations.

The common lore is that effects of viscosity are more important for particles with larger transverse momenta. This is due to the fact that most hydrodynamic calculations use the “quadratic” ansatz \( \chi(p) \propto p^2 \). While this choice generally results in a improved description of experimental data, it is not supported by any theoretical argument. We evaluate viscous corrections to observables (specifically, transverse momentum spectra and anisotropic flow) by solving numerically a relativistic Boltzmann equation. We simulate relativistic particles undergoing elastic collisions with a total cross section \( \sigma_{\text{tot}} \). In the limit \( \sigma_{\text{tot}} \to +\infty \), a generic observable \( f(\sigma) \) can be expanded in powers of \( 1/\sigma_{\text{tot}} \):

\[
f(\sigma_{\text{tot}}) \approx f^{(0)} + \frac{1}{\sigma_{\text{tot}}} \delta f + O\left(\frac{1}{\sigma_{\text{tot}}^2}\right).
\]

The leading term \( f^{(0)} \) is the limit of infinite cross section, which corresponds to ideal hydrodynamics in the limit of a vanishing freeze-out temperature. The next-to-leading term \( \delta f \) is a viscous correction: since the shear viscosity \( \eta \) scales like \( 1/\sigma_{\text{tot}} \), this correction is proportional to \( \eta \).

We evaluate \( f^{(0)} \) and \( \delta f \) by solving numerically the relativistic Boltzmann equation for several large values of the cross section \( \sigma_{\text{tot}} \). Our primary goal is to illustrate by an explicit calculation how the viscous correction to anisotropic flow depends on transverse momentum \( p_T \), and to what extent this dependence is sensitive to the structure of the differential cross section. We do not mean here to carry out a full realistic simulation of a heavy-ion collision. In particular, for sake of simplicity, our transport calculation uses massless particles which supply the possibility of having only shear viscosity with no bulk viscosity. The resulting equation of state is harder than that of QCD near the deconfinement crossover. This results in larger shear and harder \( p_T \) spectra.

We also study the dependence of observables on initial conditions. In second-order viscous hydrodynamics,
II. INITIAL CONDITIONS AND EVOLUTION

Initial conditions follow Bjorken’s boost-invariant prescription [25], but with a finite extent in space-time rapidity \(-2.5 < \eta < 2.5\). The evolution is started at time \(\tau_0 = 0.6\, \text{fm}/c\) [26] after the collision. The initial conditions of the Boltzmann equation are specified by the one-body density \(f(x, p)\) in coordinate \((x)\) and momentum \((p)\) space at time \(\tau_0\).

The initial density profile in transverse coordinate space \((x, y)\) is taken from an optical Glauber [27] calculation for a central Au-Au collision at \(\sqrt{s} = 200\, \text{GeV}\), corresponding to the top RHIC energy. This initial density is azimuthally symmetric, so that anisotropic flow vanishes by construction. We introduce anisotropy artificially by deforming the initial distribution, thus mimicking an initial state fluctuation [11].

In hydrodynamics, one typically deforms the initial energy density profile [11, 28]. In a transport calculation, where the initial conditions are specified by the initial positions of particles, it is simpler to just shift these positions by a small amount. Introducing the complex notation \(z = x + iy\), in order to generate flow in harmonic \(n\), we shift \(z\) according to

\[
z \rightarrow z + \delta z \equiv z - \alpha z^n - 1,
\]

where \(\bar{z} \equiv x - iy\), and \(\alpha\) is a real positive quantity chosen in such a way that the correction is small. This transformation is invariant under the change \(z \rightarrow e^{2\pi n/z}, \) i.e., it has \(2\pi/n\) symmetry. Therefore, to leading order in \(\alpha\), the only nonvanishing anisotropic flow coefficient is \(v_n\) [11].

The initial eccentricity in harmonic \(n\) is defined for \(n \geq 2\) by [23, 30]

\[
\varepsilon_n \equiv -\frac{\sum_{j}(z_j + \delta z_j)^n}{\sum_{j}|z_j + \delta z_j|^n},
\]

where the sum runs over all particles with initial position \(z_j\). Inserting Eq. (2) into Eq. (3), and using the fact that the distribution of \(z_j\) is azimuthally symmetric, one obtains, to leading order in \(\alpha\) and for a large number of particles

\[
\varepsilon_n \simeq -\frac{\sum_j n z_j^n - 1 \delta z_j}{\sum_j |z_j|^n} = n\alpha \frac{\sum_j |z_j|^{2(n-1)}}{\sum_j |z_j|^n} \\
\simeq n\alpha \frac{\left\langle z^{2(n-1)} \right\rangle}{\left\langle z^n \right\rangle},
\]

where, in the right-hand side, angular brackets denote an average taken with the optical Glauber profile, and \(r = |z| = \sqrt{x^2 + y^2}\). We carry out simulations for \(n = 2, 3, 4\). We fix \(\alpha\) in such a way that \(\varepsilon_n = 0.2\). We have checked that this value is sufficiently small that the response is linear [11]. In hydrodynamics with fluctuating initial conditions, \(v_2\) and \(v_3\) are determined to a good approximation by linear response to the eccentricity in the corresponding harmonic [31, 32]. On the other hand, \(v_4\) is the superposition of a linear term [33], and a nonlinear term induced by \(v_2\) [34]. The present study only addresses the linear term.

In momentum space, we consider two different types of initial conditions. The first case is a thermal Boltzmann distribution

\[
dN/d^3p \propto e^{-p/T}.
\]

In addition, one requires that the temperature \(T\) and the local density \(n\) be such that \(n/T^3\) is a constant throughout the transverse plane, as in a thermal gas of massless particles with zero chemical potential. Thus this initial condition is that of a hydrodynamic calculation with zero chemical potential [23]. The initial temperature at the center of the fireball is \(T_0 = 340\, \text{MeV}\). The second case is a constant distribution:

\[
dN/d^3p \propto \theta(p_0 - p).
\]

The maximum momentum \(p_0\) and the proportionality constant are chosen such that the particle density \(n\) and the energy density \(\varepsilon\) are the same as with the previous initial condition. Similar initial conditions have been previously used in transport calculations in order to mimic the effect of saturation in high-energy QCD [35, 36]. Both types of initial conditions have exactly the same energy-momentum tensor.

The evolution of the system is determined by the relativistic classical Boltzmann equation. We use a relativistic transport code developed to study heavy-ion collisions at RHIC and LHC energies [19, 37, 41], which uses the test-particle method. The collision integral is solved by using Monte Carlo methods based on the stochastic interpretation of transition amplitudes [13, 37, 12]. The total cross section is fixed throughout the evolution. The differential cross section is

\[
\frac{d\sigma}{dt} \propto \frac{1}{(t - m_D^2)^2}.
\]
where \( t \) is the usual Mandelstam variable. This differential cross section is typically used in parton cascade approaches \([37, 38, 42–44]\) and by symmetry the \( u \)-channel is included. The limit \( m_D \to \infty \) corresponds to an isotropic cross section. The opposite limit, where \( m_D \) is smaller than the typical particle energy, corresponds to a forward-peaked cross section.

In a transport approach, one can follow the evolution of the system until the last collision, but this is numerically expensive. Instead, we choose to follow the system until a fixed final time, and check stability of our results with respect to this final time. The calculations presented in this paper are carried out with a final time \( t_f = 12 \text{ fm}/c \), but we have checked that the momentum spectra are unchanged if we extend the final time to \( t_f = 12 \text{ fm}/c \).

Throughout this paper, we carry out four sets of calculations: Three sets with the thermal initial distribution \([6]\) and the values \( m_D = 0.3 \text{ GeV} \), \( m_D = 0.7 \text{ GeV} \) and an isotropic cross section, and a fourth set with the constant initial distribution \([6]\) and an isotropic cross section. Thus we study how results depend on the initial distribution and on the differential cross section.

For each set of parameters, we perform different calculations for the following set of total cross sections: \( \sigma_{\text{tot}} = 20, 25, 30 \) and \( 35 \text{ mb} \). Results for anisotropic flow \( v_n \) \([1]\) are shown in Fig. 1. The dependence of these observables on \( 1/\sigma_{\text{tot}} \) is essentially linear, corresponding to the regime where viscous hydrodynamics applies. In order to improve the accuracy, we fit these results with a polynomial of order 2 in \( 1/\sigma_{\text{tot}} \) and extract the ideal hydrodynamic limit and the first viscous correction using Eq. (13): specifically, the ideal hydrodynamic limit \( f^{(0)} \) is the extrapolation to \( 1/\sigma_{\text{tot}} \to 0 \) and the viscous correction \( \delta f \) is the slope at the origin.

### III. IDEAL HYDRODYNAMICS

We first study the ideal hydrodynamic limit, defined by the extrapolation \( \sigma_{\text{tot}} \to +\infty \) in the Boltzmann equation. Ideal hydrodynamics corresponds to thermal local equilibrium \([23]\). One expects thermalization to wash out details of initial conditions, so that observables should not depend on the initial momentum spectrum provided the energy density is fixed. Similarly, the momentum distribution in thermal equilibrium is universal, therefore one expects observables to be independent of the differential cross section in this limit.

Figure 2 displays the ideal hydrodynamic limit for the \( p_T \) spectrum in the rapidity interval \( |y| < 0.5 \). The spectra have been divided by \( dN/dy \) so that they are normalized to unity.

![FIG. 1. Left panel: \( v_n(p_T) \) as a function of \( 1/\sigma_{\text{tot}} \) (thermal initial distribution, isotropic scattering cross section) in the rapidity interval \( |y| < 0.5 \). The \( y \)-intercept is the ideal hydrodynamic limit \( v_n^{(0)} \), while the slope corresponds to the viscous correction \( \delta v_n \), as defined by Eq. (13).](image1)

![FIG. 2. Ideal hydrodynamic limit for the \( p_T \) spectrum in the rapidity interval \( |y| < 0.5 \). The spectra have been divided by \( dN/dy \) so that they are normalized to unity.](image2)
It would be interesting to reproduce these results using ideal hydrodynamics with the same ideal gas equation of state (constant sound velocity $c_s = 1/\sqrt{3}$) and a small freeze-out temperature.

**IV. VISCOS CORRECTION**

We now present results for the first-order viscous correction to observables, corresponding to the term $\delta f$ in Eq. (1). We scale the viscous correction $\delta f$ by the ideal hydrodynamic limit $f^{(0)}$. Eq. (1) shows that $\delta f/f^{(0)}$ has the dimension of the cross section $\sigma_{\text{tot}}$. The relative viscous correction is $\delta f/f^{(0)}$ divided by the total cross section $\sigma_{\text{tot}}$. As a rule of thumb, viscous hydrodynamic applications if $\delta f/f^{(0)}$ is significantly smaller than $\sigma_{\text{tot}}$ in absolute value.

Figure 4 displays $\delta f/f^{(0)}$ for the transverse momentum spectrum. Viscous effects result in a particle excess at large $p_T$, corresponding to an increase of the average $p_T$, that is, a higher temperature. The reason is that viscosity decreases the longitudinal pressure, thereby reducing longitudinal cooling.

Unlike the ideal hydrodynamic limit, the first-order viscous correction depends on the differential cross section. It is larger for smaller values of $m_D$. This is due to the fact that the scattering is forward peaked and less efficient in thermalizing the system. For an isotropic cross section, the relative viscous correction is almost linear in $p_T$: $\delta f/f^{(0)} \propto p_T^{0.98}$. This result is consistent with the results obtained in [48] in the Chapman-Enskog approximation.

Surprisingly, the viscous correction also depends on the initial momentum distribution. Two initial conditions with exactly the same energy-momentum tensor $T^{\mu\nu}$ lead to different first-order viscous corrections to observables, at variance with usual viscous hydrodynamics [12]. Figure 4 shows that for a constant initial momentum distribution, Eq. (1), $\delta f$ is smaller at high $p_T$ than with a thermal initial distribution. This depletion at high $p_T$ can be understood as a memory of the initial conditions, where all particles initially have momenta below a threshold $p_0$.

Finally, we study the viscous correction to anisotropic flow $v_n(p_T)$. Figure 4 displays $-\delta v_n/v_n^{(0)}$ as a function of transverse momentum $p_T$. Viscous effects decrease anisotropic flow [12], therefore the correction is shown with a minus sign. The viscous correction to $v_n$ increases as a function of harmonic order $n$, as already observed in viscous hydrodynamic calculations [11]. At large transverse momentum, it scales approximately like the order $n$ [10], $-\delta v_n(p_T)/v_n^{(0)} \propto n$. This dependence is weaker than the $n^2$ dependence reported in previous studies [33, 50, 52].

As expected, the viscous correction depends on the differential cross section. As for the spectrum, it is larger with a forward-peaked cross section ($m_D = 0.3$ GeV) than with an isotropic cross section. Results with the thermal distribution [9] and with the constant distribution [9] (not shown) are consistent within error bars.
Boltzmann equation and studying the limit of large scattering and anisotropic momentum (the first-order viscous correction for the transverse momentum) currently used in most section, but it is still much slower than linear. Thus increase with \( n = 2, 3 \) and 4 respectively.

The ideal hydrodynamic limit is found to be independent of microscopic details, as expected from the universality of thermodynamic behavior. The linear response coefficients \( v_n(p_T)/\varepsilon_n \) depends little on harmonic \( n \) in the ideal hydrodynamic limit.

The first order viscous corrections to observables, on the other hand, are not universal. As expected \([10]\), they depend on the differential cross section. For all the differential cross sections investigated in this paper, we find that the relative viscous correction to anisotropic flow, \( v_n \), does not increase significantly with \( p_T \) at large \( p_T \). Our results suggest that first-order viscous corrections do not explain the decrease of \( v_n \) at high \( p_T \), at variance with common lore \([2]\), and that a different mechanism, such as jet quenching \([33]\), is needed at high \( p_T \).

The viscous correction to \( v_n \) increases linearly with \( n \). The stronger \( n^2 \) dependence typically found in hydrodynamics \([33]\) leads to negative values of \( v_4 \) and \( v_6 \) at large \( p_T \) \([14]\), even for small viscosities. A weaker dependence on harmonic \( n \) is therefore likely to improve agreement with experimental data.

Finally, our results clearly show that usual, second-order relativistic hydrodynamics \([2, 12]\) is unable to capture the first-order viscous correction to observables. Two initial conditions with exactly the same energy-momentum tensor, which would therefore yield the exact same hydrodynamical flow, are found to yield different momentum spectra at the end of the evolution. Specifically, the first-order viscous correction is found to retain the memory of the initial condition. It has already been pointed out that the relaxation-time approximation \([54]\), on which usual hydrodynamic equations are based, is not justified. Within a strong coupling calculation, a proper treatment of the underlying microscopic degrees of freedom leads to an evolution equation which is second-order rather than first-order \([53]\), so that the solution is not solely determined by the initial value \( T^\mu_\nu \). Our calculation provides an explicit illustration of this point within a weak-coupling calculation.

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