Quasiparticles in high temperature superconductors: consistency of angle resolved photoemission and optical conductivity

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The consistency of angle-resolved photoemission and optical conductivity experiments on high temperature superconductors is examined. In the limit (apparently consistent with angle-resolved photoemission data) of an electron self energy with a weak momentum dependence and a strong frequency dependence formulae are derived which directly related quantities measured in the two experiments. Application of the formulae to optimally and overdoped Bi₂Cu₂Sr₂Cu₂O₈₊δ shows that the total self energy inferred from photoemission measurements cannot be interpreted as a transport scattering rate (in agreement with work of Varma and Abrahams), but that the inelastic part may be so interpreted, if Landau parameter effects are non-negligible.

I. INTRODUCTION

The fundamental concept underlying the modern understanding of the physics of metals is the quasiparticle concept, the idea that the crucial low energy eigenstates of interacting electron systems are sufficiently similar to conventional electrons that they may be used in standard ways to calculate transport and other quantities. The utility of the quasiparticle concept in the case of the high temperature copper oxide superconductors remains the subject of controversy. Some authors argue that an intrinsically non-fermi-liquid picture involving unconventional excitations or a nontrivial critical point is needed. Others assert that a more or less conventional picture of electrons scattered by some (perhaps somewhat unconventional) scattering mechanism suffices. Intermediate views exist also.

Recent improvements in angle-resolved photoemission experiments (for reviews see) have provided new insight into this issue. Detailed measurements of the momentum, temperature and energy dependence of the electron spectral function have demonstrated the existence, near the fermi surface of optimally doped materials, of reasonably well defined peaks. It seems natural to interpret the peak position in terms of a quasiparticle scattering rate. The question of the relation between the scattering rate and dispersion deduced from angle resolved photoemission experiments and those deduced from the frequency and temperature dependence of the electrical conductivity immediately arises. A similar question concerns the relation between the optical and photoemission data and the predictions of specific models, for example various versions of the ‘spin fermion’ model of carriers interacting with spins or the ‘marginal fermi liquid’ model of Varma and co-workers. In the existing literature, it is generally assumed that knowledge of the (low frequency) self energy measured e.g. via photoemission or calculated via a model determines the frequency dependent conductivity.

In this paper we examine the issue in more detail. We show how to compare, with minimal assumptions, the photoemission and optical data. We show that the scattering rates and mass enhancements deduced from photoemission do not by themselves describe the high-Tc optical data in optimally doped cuprates: first, some portions of the self energy inferred from data must be discarded (as noted earlier by Varma and Abrahams) and even after this is done an extra modification (in fermi liquid language a Landau parameter) must be introduced. Implications of these results for our physical understanding of the cuprates and for attempts at modeling cuprate properties are outlined.

The rest of this paper is organized as follows. Section II summarizes the formalism needed to discuss the photoemission data, section III presents the theory for the conductivity and section IV relates the theory to available data. Section V is a conclusion.

II. PHOTOEMISSION: THEORY

The propagation of an electron in a solid is described by the electron Green function

\[ G(r, t) = \langle T_\psi(r, t)\psi^+(0, 0) \rangle \]  

where \( \psi \) is the electron annihilation operator. Photoemission measurements allow the determination of the imaginary part of the Fourier transform of \( G \), i.e. the spectral function

\[ A(p, \omega) = ImG(p, \omega) \]  

These measurements are typically interpreted in terms of differences between the actual \( G \) and that corresponding to a reference system in which the electron propagates without scattering according to some reference dispersion \( \varepsilon_p \). One defines the self energy \( \Sigma \) via

\[ \Sigma(p, \omega) = \omega - \varepsilon_p - G^{-1}(p, \omega) \]
The self energy has both real ($\Sigma'$) and imaginary ($\Sigma''$) parts. The real part depends on the choice of reference energy $\varepsilon_p$, and the condition $\varepsilon_p + \Sigma(p,\omega = 0) = 0$ defines the fermi surface. A common choice for reference dispersion is the dispersion $\varepsilon_{p,band}$ predicted by the local density approximation to the Kohn-Sham band theory equations; when band theory results are needed we use the tight binding parametrization of the LDA band structure determined by Andersen et al. and presented in more detail in the Appendix.

The high $T_c$ superconductors have a fundamentally two dimensional dispersion and a topologically simple fermi surface, so it is convenient to parametrize momentum by the reference energy $\varepsilon_p$ and an angular coordinate $\theta$ describing position on a surface of constant reference energy. The observed photoemission spectra involve mainly energies $\omega \lesssim 0.2eV$ in which range the calculated band dispersion is (especially in high symmetry directions) linear in momentum (for the direction perpendicular to the fermi surface). For these energies the observed spectral functions display a reasonably well defined, reasonably symmetrical peak of approximately Lorentzian form, if $A$ is measured as a function of $p$ at constant $\omega, \theta$, (‘MDC’) but a rather broad, asymmetric structure if $A$ is measured as a function of $\omega$ at constant $\theta, \varepsilon_p$ (EDC). The sharpness of the observed MDC curves implies that in the $\omega < 0.2eV$ region where angle resolved photoemission data are available, the imaginary part of the self energy is small compared to the range over which $\varepsilon_p$ varies. This will be important in the theory of the optical conductivity to be discussed below. Further, the data suggest that in the range $\omega < \omega_c \lesssim 0.2eV$, $\Sigma''$ depends reasonably strongly on $\omega$ and $\theta$ and reasonably weakly on $\varepsilon_p$. However, as we shall see it is likely that at higher energies, beyond the measurement range, $\Sigma$ depends more strongly on $\varepsilon_p$.

It is convenient to introduce a frequency $\omega_c$ separating high and low energy scales and to write

$$\Sigma(p,\omega) = \Sigma_{low}(\omega,\theta) + \Sigma_{high}(\omega,p)$$

with $Im\Sigma_{low}(\omega) = Im\Sigma(\omega)$ for $|\omega| < \omega_c$, $Im\Sigma_{low} = Im\Sigma(\omega_j)$ for $|\omega| > \omega_c$ and $Re\Sigma_{low}$ the Kramers-Kronig transform of $Im\Sigma_{low}$. For energies well below $\omega_c$ and momenta not too far from $p_F$ we linearize $\Sigma_{high}(\omega,p)$ in $\omega$ and $p - p_F$. (This procedure is unfortunately clumsy–it leads at intermediate stages in calculations to non-analytic behavior at $\omega = \omega_c$, which of course cancels from physical quantities but we shall not need to consider $\omega = \omega_c$ in this paper).

In the low energy region one may therefore write

$$G(p,\omega) = \frac{1}{(Z(\theta) - v_0(p - p_F) - \Sigma_{low}(\omega,\theta))}$$

with

$$Z(\theta) = \left(1 - \frac{\partial \Sigma_{high}(\omega,\theta)}{\partial \omega}\right)^{-1} \bigg|_{\omega = 0}$$

the quasiparticle weight coming from contributions to $\Sigma$ at $\omega > \omega_c$, and

$$v_0 = \frac{\partial (\varepsilon_p + \Sigma_{high}(\omega << \omega_c, p))}{\partial p}|_{p_F}$$

a (possibly angle-dependent) velocity which may differ from the band velocity if $\Sigma_{high}$ has significant momentum dependence.

Eq. implies that if measured as a function of $p$, $A$ has a peak centered at a momentum $p_\omega$ set by

$$\varepsilon_{p,\omega} + \Sigma_{high}(\omega << \omega_c, p_\omega) = \frac{\omega - \Sigma'_{low}}{Z(\theta)}$$

If the peak is not too far from the fermi level then use of the low energy approximations shows that the peak position $\delta p_\omega$ is given by

$$\delta p_\omega = \frac{\omega - \Sigma'(\theta,\omega)}{v_0 Z(\theta)}$$

. If $\Sigma''$ is not too large then one may linearize $\varepsilon_p$ near $p_\omega$, and if $\omega$ is not too large then $p$ is close to $p_F$ so that we may linearize everything about the fermi surface, obtaining $(\varepsilon_p + \Sigma_{high}(\omega << \omega_c, p)) = \frac{\omega - \Sigma'_{low}}{Z(\theta)} + v_0(p - p_\omega)$, so that (in agreement with data) $A$ measured as a function of momentum (MDC) has an approximately Lorentzian peak of half width

$$W(\omega) = \Sigma'_{low}(\theta,\omega)$$

Similarly, the slope $v_\omega^* = \partial \omega / \partial p_\omega$ of the dispersion curve $\omega = \varepsilon_{p,\omega}$ at $p = p_\omega$ is given by

$$v_\omega^* = \frac{v_0}{(1 - \partial \Sigma'_{low}/\partial \omega)}$$

The quantities $W$ and $v_\omega^*$ are directly measurable, and are independent of the choice of reference dispersion and of the behavior of $\Sigma_{high}$. In the analysis of optical conductivity presented in the next section we shall require their $\omega \to 0$ limit.

III. OPTICAL CONDUCTIVITY: THEORY

The conductivity is given by

$$\sigma_{xx}(i\Omega_n, T) = \frac{\chi_{ij}^{GI}(i\Omega_n, T)}{i\Omega_n}$$

where $\chi_{ij}^{GI}$ is the gauge invariant current-current correlation function, which may be expressed in the usual way in terms of electron Green functions and a vertex operator, $T$. The photoemission data discussed above indicate that at least in the $\omega < 0.2eV$ energy range, the self energy has negligible dependence on the magnitude of the momentum and is small compared to the range over
which $\varepsilon_p$ varies. In this case, the usual arguments of fermi liquid theory may be applied and in particular by integrating over the magnitude of the energy first one finds ($\omega_{n+} = \omega_n + \Omega_n$)

$$\chi^{Gf}_{ij}(i\Omega_n, T) = \pi T \sum_{\omega_n} \int \frac{pF(\theta)d\theta}{(2\pi)^2} N_0 v_x(\theta) \frac{(sgn(\omega_n) - sgn(\omega_{n+})) T^{Gf}_x(\theta, \omega_n)}{i\Omega_n - \Sigma(\theta, i\omega_n) + \Sigma(\theta, \omega_n)}$$

(3)

Here we note that the momentum-independence of the self-energy (in the energy range of interest) implies that the 'bare' current operator is the 'bare' velocity $v_0$ defined above in Eq. (bare' in quotes because $v_0$ does depend on $\Sigma_{high}$ and the choice of reference dispersion). $T$ is the vertex operator, defined by

$$T^\Omega_x(\theta, \omega) = v_x(\theta) + T \sum_{\omega'} \int \frac{d\theta'}{2\pi} T^\Omega(\theta, \theta'; \omega, \omega')(14a)$$

$$\frac{(sgn(\omega') - sgn(\omega')) T^\Omega(\theta', \omega)}{i\Omega_n - \Sigma(\theta, i\omega_n) + \Sigma(\theta, \omega')}$$

with $I$ the generalization to nonzero frequencies of the usual Landau interaction function (into which we have absorbed the factors of $v$ and $p_F$).

The vertex function has two purposes: it converts the 'single-particle' scattering rate and mass enhancements described by $\Sigma$ to a transport rate and mass enhancement described by a new function $\Sigma_{tr}$ by suppressing the contribution from 'forward scattering' and it expresses the 'backflow' arising because in an interacting system motion of one electron affects the motion of others, so the current is not given accurately by the single particle velocity. Note that if $\partial\Sigma/\partial p = 0$ at all frequencies and momenta then the current operator is given by the derivative of the reference dispersion $\varepsilon_p$ and the backflow part, $\Lambda$, of the vertex correction vanishes identically. However, a $\Lambda$ which is nonnegligible in the frequency range of interest may arise from a $\partial\Sigma/\partial p$ which is non-negligible only at frequencies beyond the frequency range of interest. For an explicit example see Ref. 15.

It is perhaps instructive to restate this conclusion in the language of the quantum Boltzmann equation. There are two nontrivial terms in this equation: one gives the interaction induced 'feedback' of the excitation of one particle-hole pair on the behavior of others (i.e. accounts for backflow); the other is the collision term representing scattering of a quasiparticle from one state to another. The collision term involves a probability $W(p, p')$ for scattering an electron from state $p$ to state $p'$ and the resulting conductivity depends on the structure of $W(p, p')$: for example, if the scattering is mostly forward ($W(p, p')$ appreciable only for $p$ near $p'$) then the scattering will have little effect on the conductivity. The self energy is proportional to $\int (dp') W(p, p')\zeta(p')$ with $\zeta$ a factor relating to the probability that state $p'$ is available as a final state, that any other excitation needed in the scattering process can be created, etc. Measurement of the self energy by itself thus does not contain enough information to reconstruct $W(p, p')$; and in diagrammatic language this information is contained in the vertex function.

Eq. (14) cannot be analysed without further assumptions. We shall assume that the self energy has two contributions: one coming from low energies and essentially observable by present-day angle-resolved photoemission experiments (this is the 'quasiparticle part' of the electron Green function) and one coming from high energies, not directly observable in present-day angle-resolved photoemission experiments but contributing indirectly to low energy physics via the Landau parameter and the velocity renormalization. We shall further assume, following Ref. 15, that the low energy contribution to $\Sigma$ consists of two parts: an inelastic part with a negligible momentum dependence but a significant frequency dependence and one arising from a quasistatic scattering highly peaked in the forward direction. Thus

$$\Sigma(\theta, \omega) = isgn(\omega)\Gamma_{forward}(\theta) + \Sigma_{inel}(\omega) + \Sigma_{high}(p, \omega)$$

(15)

We shall now write an expression for the low frequency conductivity which separates the effects of the 'quasiparticle' and high energy contributions. Eq. (15) implies that the Landau interaction function consists of two parts: one from the forward scattering contribution, of the form $2\pi I_{forward}(\theta; \Omega)\phi(\theta - \theta')$ (independent of $\omega, \omega'$ because the scattering is taken to be quasistatic, and with peakedness in the forward direction specified by $\phi$) and one, coming from high energies, which is independent of $\Omega, \omega, \omega'$ in the frequency range of interest. We define

$$B(\theta, i\Omega) = T \sum_{\omega} \frac{i\pi (sgn(\omega) - sgn(\omega + i\Omega))}{i\Omega_n - \Sigma(\theta, i\omega + i\Omega) + \Sigma(\theta, \omega)}$$

(16)

and $B', T'$ by

$$B'^{-1} = B^{-1} - I_{forward}$$

(17)

$$T' = (1 - I_{forward})B$$

(18)

The vertex $T'$ satisfies the simple integral equation

$$T'_x(\theta, \Omega) = v_x(\theta) + \int \frac{d\theta'}{2\pi} I_{high}(\theta, \theta') B'(\theta', \Omega) T'_x(\theta', \Omega)$$

(19)

while the conductivity per CuO$_2$ plane becomes

$$\sigma(i\Omega_n, T) = \frac{1}{i\Omega} \int \frac{pF(\theta)d\theta}{2\pi v_F(\theta)} v_x(\theta) B'(\theta, \Omega) T'_x(\theta, \Omega)$$

(20)

We think of the function $B'$ as the function $B$ with the forward scattering contributions removed.

We now consider the low frequency expansion of $\chi_{jj}$.

We expect

$$B'(\theta, \Omega) = \frac{i\Omega}{\Gamma(\theta, T)} + \frac{\Omega^2\Lambda(T)}{\Gamma(\theta, T)^2}$$

(21)
For example, if the self energy is momentum-independent then (here $\Sigma_{\pm} = \Sigma(\theta, \varepsilon \pm \Omega/2)$

$$B_{m-i}^{*}(\theta, \Omega) = \int \frac{d\varepsilon}{\pi} \frac{f(\varepsilon_{-}) - f(\varepsilon_{+})}{\Omega - (\Sigma'_{+} - \Sigma'_{-}) - i(\Sigma''_{+} + \Sigma''_{-})}$$

so that

$$\Gamma_{m-i}^{-1} = \int \frac{d\varepsilon - \partial f(\varepsilon)/\partial \varepsilon}{2\Sigma''(\varepsilon)}$$

$$\Lambda_{m-i} = \Gamma_{m-i}^{2}$$

Eq. 24 implies

$$T'(\theta, \Omega) = v_{x}(\theta) + i\Omega \int \frac{p_{F}(\theta')d\theta'}{2\pi} \frac{I_{\text{high}}(\theta, \theta')v_{x}(\theta')}{\Gamma_{tr}(\theta')}$$

Combining Eqs 21 and 25 yields a low frequency expansion for the conductivity of the form

$$\sigma(\Omega) = \sigma_{qp}(\Omega) + \sigma_{LP}(\Omega)$$

with

$$\sigma_{qp}(\Omega) = \frac{2e^{2}}{h} \int \frac{p_{F}(\theta)d\theta}{(2\pi)^{2}v_{F}(\theta)} \left[ \frac{v_{x}^{2}(\theta)}{\Lambda(\theta)} + \frac{i\Omega\Lambda(T)v_{x}^{2}(\theta)}{\Gamma(\theta, T)^{2}} \right]$$

$$\sigma_{LP}(\Omega) = \frac{2e^{2}}{h} \int \frac{p_{F}(\theta)d\theta}{(2\pi)^{2}v_{F}(\theta)} v_{x}(\theta)$$

$$\int \frac{p_{F}(\theta')d\theta'}{(2\pi)^{2}} \frac{I_{\text{high}}(\theta, \theta')v_{x}(\theta')}{\Gamma(\theta')}$$

where $\sigma_{qp}$ is the contribution obtained from the quasiparticle scattering and dispersion and $\sigma_{LP}$ arises from the Landau or backflow renormalization. Observe that the Landau renormalization affects the first frequency correction to the conductivity, but not the dc value.

It is very convenient to write this expression in terms of an inverse ‘transport’ mean free path $W$ and a transport velocity defined analogously to Eq. 14

$$W_{tr}(\theta) = \Gamma(\theta)/v_{F}(\theta)$$

$$v_{x}^{*}(\theta) = \frac{v_{x}(\theta)}{\Lambda(\theta)}$$

Then one has

$$\sigma_{qp}(\Omega, T) = \frac{2e^{2}}{h} \int \frac{p_{F}(\theta)d\theta}{(2\pi)^{2}} \left( \frac{v_{x}(\theta)}{v(\theta)} \right)^{2}$$

$$\left( \frac{1}{W_{tr}(\theta)} + \frac{i\Omega}{v^{*}(\theta)W_{tr}(\theta)^{2}} \right)$$

$$\sigma_{LP} = i\Omega \int \frac{p_{F}(\theta)d\theta}{(2\pi)^{2}} \frac{p_{F}(\theta')}d\theta' \left( \frac{p_{F}(\theta')}{(2\pi)^{2}} \right)^{2}$$

$$\frac{v_{x}(\theta)I_{\text{high}}(\theta, \theta')v_{x}(\theta')}{W_{tr}(\theta)W_{tr}(\theta')}$$

In particular, the dc limit of the conductivity is

$$\sigma_{dc}(T) = \frac{2e^{2}}{h} \int \frac{p_{F}(\theta)d\theta}{(2\pi)^{2}} \left( \frac{v_{x}(\theta)}{v(\theta)} \right)^{2} \frac{1}{W_{tr}(\theta)}$$

and is given entirely in terms of fermi surface geometry and the transport mean free path.

In the absence of Landau renormalization, the imaginary part $\sigma'' \rightarrow \sigma''_{qp}$ given by

$$\lim_{\Omega \rightarrow 0} \sigma''_{qp}(\Omega) = \frac{2e^{2}}{h} \int \frac{p_{F}(\theta)d\theta}{(2\pi)^{2}} \left( \frac{v_{x}(\theta)}{v(\theta)} \right)^{2} \frac{i\Omega}{v^{*}(\theta)W_{tr}(\theta)^{2}}$$

In experimental analyses of optical conductivity it is conventional (see, e.g. [12]) to define an optical mass and scattering rate via

$$\Gamma_{opt}(\Omega) = KRe\sigma(\Omega)^{-1}$$

$$m^{*}/m_{opt} = -\frac{KIm\sigma(\Omega)^{-1}}{\Omega}$$

where $K$ is a constant related to the optical spectral weight in the frequency range of interest. The values of $\Gamma$ and $m^{*}/m_{opt}$ depend on the value used for $K$, leading to ambiguity in the values of $\Gamma_{opt}$ and $m^{*}/m_{opt}$ similar to the ambiguity in the single-particle self energy arising from uncertainty as to the correct choice of reference velocity. One quantity which is independent of the choice of $K$ is the ratio

$$\lim_{\Omega \rightarrow 0} \Gamma_{opt}^{*} = \frac{i\OmegaRe\sigma}{Im\sigma}$$

The formulae given above involve quantities defined in the $\Omega \rightarrow 0$ limit. One may consider extending the analysis to higher frequencies, but our lack of knowledge of the vertex function renders such an analysis problematic.

Eqs 21-24 are our principal results. They show that measurement of the transport mean free path and quasiparticle velocity predict the dc conductivity and its first frequency derivative only if Landau parameter effects are negligible. Thus if measurements of $W$ and $v^{*}$ are available, comparison of Eq 21 to data will show whether Landau parameter effects are important.

IV. OPTICAL AND PHOTOEMISSION DATA IN HIGH T$_{c}$ SUPERCONDUCTORS: ANALYSIS AND CONSISTENCY WITH QUASIPARTICLE PICTURE

In this section we use the formulae derived above to investigate the relation between the photoemission and optical spectra in optimally and over-doped Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$, and in particular to determine whether the scattering rates inferred from photoemission experiments may be interpreted as transport rates, and to estimate the value of any Landau renormalization.
We have selected this material because extensive photoemission and optical data are available. In other high-\textit{T}_c materials insufficient information exists to perform the analysis at present.

Experiments show that in optimally doped BSCCO the fermi surface is to reasonable approximation a circle of radius \( p_F = 0.71\AA^{-1} \) centered at the \((\pi, \pi)\) point. The quasiparticle velocity \( v^* = 1.8eV\cdot\AA\) with negligible variation around the fermi surface (in the normal state), and the 'MDC full width' \( 2W(\theta, T, \varepsilon) \) is reasonably well represented by\cite{Varma87}

\[
2W(\theta, T, \varepsilon) = \Gamma_0 \max(\varepsilon, \pi T) + \Gamma_1 (1 + \cos(4\theta)) + \Gamma_2 \quad (38)
\]

with \( \Gamma_0 = 8 \times 10^{-5} [\text{Å}^{-1}K^{-1}] \), \( \Gamma_1 = 0.05\text{Å}^{-1} \) \( \Gamma_2 = 0.01\text{Å}^{-1} \) and \( \theta = 0 \) at the antinodal point\((0, \pi)\). Comprehensive data from other groups are not available as of this writing but we note that the zone-diagonal \( \omega = 0 \) MDC widths reported in Ref\cite{Fischer89} are very close to the zone diagonal numbers obtained from the formula above.

Let us first make the assumption that vertex corrections are negligible. Then from Eqs\cite{Varma87} \( \Gamma_0 \) and \( \Gamma_1 \) we obtain \( c \) is the mean interplane distance

\[
\sigma_{dc} = \frac{e^2 p_F}{\hbar c 2\pi} \int d\varepsilon \frac{\partial f}{\partial \varepsilon} I_1(\varepsilon, T) \quad (39)
\]

\[
\lim_{\Omega \to 0} \frac{\sigma_{dc}^\omega(\Omega)}{\Omega} = \frac{e^2 p_F}{\hbar c 2\pi v^*} \int d\varepsilon \frac{\partial f}{\partial \varepsilon} I_2(\varepsilon, T) \quad (40)
\]

The two integrals are

\[
I_1 = \int \frac{d\theta}{2\pi} \frac{1}{2W(\theta, T, \varepsilon)} = \frac{1}{\sqrt{(\Gamma_0 \max(\varepsilon, \pi T) + \Gamma_2)^2 + 2(\Gamma_0 \max(\varepsilon, \pi T) + \Gamma_2) \Gamma_1}} \Gamma_1 \quad (41)
\]

\[
I_2 = \int \frac{d\theta}{2\pi} \frac{1}{2W(\theta, T, \varepsilon)^2} = \left(\frac{\Gamma_0 \max(\varepsilon, \pi T) + \Gamma_2}{\Gamma_0 \max(\varepsilon, \pi T) + \Gamma_2 + 2\Gamma_1}\right)^{3/2} \left(\frac{\Gamma_0 \max(\varepsilon, \pi T) + \Gamma_2}{\Gamma_0 \max(\varepsilon, \pi T) + \Gamma_2 + 2\Gamma_1}\right)^{3/2} \quad (42)
\]

We now proceed further with the analysis and consider the leading correction to the imaginary part of the conductivity. These data are conveniently presented in terms of the ‘optical scattering rate \( \Gamma^*_{opt} \). Numerical evaluation of Eq\cite{Varma87} assuming a velocity which is temperature and frequency independent lead to the results shown in Table II. Photoemission information on the temperature dependence of the velocity is lacking, but available data\cite{Fischer89} do not suggest a strong frequency dependence in the low frequency regimes relevant to this calculation We note that the results are somewhat sensitive to the precise assumptions made. To demonstrate this point we consider two sub cases of ‘Case B: above–where we retain only the inelastic portion of the scattering rate and (Case B’”) where we retain also the \( \Gamma_2 = 0.01A^{-1} \) angle-independent offset. Table II summarizes our calculated results.

| \( \rho [\mu\Omega \text{ cm}] \) calculated for optimally doped Bi\(_2\)Sr\(_2\)Ca\(_2\)Cu\(_{2}\)O\(_{8+\delta}\) at temperatures indicated, using photoemission data\cite{Fischer89} assuming (A) directly measured MDC width (B) Only \( T \) and \( \omega \)-linear parts, and compared to data\cite{Gao91}. |
|-----------------|--------|--------|--------|
|                 | 100K   | 200K   | 300K   |
| \( \rho : \text{Case A} \) | 162    | 232    | 300    |
| \( \rho : \text{Case B} \)   | 60     | 120    | 180    |
| \( \rho : \text{Data} \)     | 75     | 130    | 240    |

| \( \Gamma^*_{opt} \) [meV] calculated for optimally doped Bi\(_2\)Sr\(_2\)Ca\(_2\)Cu\(_{2}\)O\(_{8+\delta}\) from photoemission data\cite{Fischer89} assuming (A) the entire photoemission scattering rate contributes to the conductivity (B) only the inelastic part contributes and (B’) that both the inelastic and the offset at the zone diagonal \( \Gamma_2 \) parts contribute, and compared to data\cite{Gao91}. |
|-----------------|--------|--------|--------|
| \( \Gamma^*_{opt} : \text{Case A} \) | 100    | 160    | 210    |
| \( \Gamma^*_{opt} : \text{Case B} \) | 46     | 92     | 140    |
| \( \Gamma^*_{opt} : \text{Case B’} \) | 64     | 110    | 155    |
| \( \Gamma^*_{opt} : \text{Data} \)   | 20     | -      | 80     |

Let us suppose first (‘Case A’) that the entire observed photoemission linewidth may be interpreted as a transport scattering rate. Then from Eq\cite{Varma87} we obtain the resistivities listed in the second row of Table I. These are plainly higher than the measured resistivities, listed in the third row of Table I. As an alternative assumption we may follow Varma and Abrahams\cite{Varma87} and argue that only the inelastic (\( \Gamma_0 \)) should be interpreted as a contribution to the transport rate. In this case (‘Case B’) we obtain the numbers shown in the second row of Table I, which as previously noted by Varma and Abrahams\cite{Varma87} are in reasonably good agreement with data\cite{Gao91}.

The dc resistivities in ‘Case A’ are much too large to be relevant to optimally doped BSCCO. We conclude that at least the strongly angle-dependent contribution to \( W \) cannot correspond to a transport rate. This conclusion was previously stated by Varma and Abrahams\cite{Varma87}, who attributed the \( \Gamma_3 \) term to impurities situated far from the \( CuO_2 \) planes. The ubiquity of the large MDC widths in this region of the fermi surface suggests to us that this explanation is untenable; however the conclusion that the width does not correspond to a transport rate seems inescapable. Interestingly, the dc conductivities in Case B seem to agree reasonably well with the observed resistivity, suggesting that the ‘inelastic’ part of the angle resolved photoemission MDC width does correspond to a transport rate. However, using these parameters in the quasiparticle formulæ strongly overestimates the renormalized optical scattering rate, especially at lower \( T \), suggesting that there is a significant Landau renormalization of the
TABLE III: ρ[μΩ cm] calculated for overdoped Bi$_2$Ca$_2$SrCu$_2$O$_{8+δ}$ at temperatures indicated, using directly measured MDC width and compared to resistivity data from the same paper.

|          | 70K | 120K | 160K |
|----------|-----|------|------|
| ρ - A    | 43  | 85  | 121  |
| ρ - data | 58  | 70  | 88   |

TABLE IV: Effective scattering rate Γ* [meV] calculated for overdoped Bi$_2$Sr$_2$CaCu$_2$O$_{8+δ}$ from measured MDC width and compared to data.

|          | 70K | 120K | 160K |
|----------|-----|------|------|
| Γ* - A   | 36  | 72   | 102  |
| Γ* - data| 19  | 28   | 35   |

conductivity.

For other doping levels the comparison is more difficult to undertake at this stage, because the photoemission data are less extensive. A recent paper presents evidence that in an overdoped sample of Bi$_2$Sr$_2$CaCu$_2$O$_{8+δ}$, W(θ, ω = 0) is only weakly dependent on angle, but varies more rapidly than linearly with temperature, being about W = 0.02A$^{-1}$ at 70K rising to 0.04A$^{-1}$ at 120K and 0.057A$^{-1}$ at 160K. The fermi surface radius (measured from the (π, π) point) is slightly larger (0.78A$^{-1}$ vs 0.72A$^{-1}$ in the optimally doped material studied in the previous work). The frequency dependence of W is still found to be linear (at least at very low T). Converting from the units of $[\text{meV}]$ calculated for overdoped Bi$_2$Sr$_2$CaCu$_2$O$_{8+δ}$ to the conventions of this paper yields, in convenient units

$$W_{OD}(\omega, T \to 0) \left[A^{-1}\right] = 1.2 \times 10^{-5} \omega [\text{K}]$$ (43)

The crossover between ω-dominated and T-dominated regimes is not discussed, however as can be seen from Eq. (5) for a frequency corresponding to 600K the frequency dependent contribution is only $\Delta W_{OD} = 7.2 \times 10^{-3}A^{-1}$ negligible compared to the dc scattering rate. Therefore we may to reasonable accuracy simply neglect the ω-dependence of the scattering rate, and use a Drude model. Our results are given in Tables III and IV.

Here, as noted by the authors of Ref. 19, the photoemission and resistivity data appear to have an inconsistent temperature dependence. Also, the optical scattering rate is again underpredicted, suggesting the importance of a Landau parameter.

For underdoped materials sufficient photoemission data does not yet exist to make the comparison feasible. Determining the behavior of the Landau parameter with doping would be very important.

V. CONCLUSION

We have presented a precise and reasonably model-independent method for comparing the photoemission and optical scattering rates and mass enhancements, and have applied the method to optimally doped and overdoped Bi$_2$Ca$_2$SrCu$_2$O$_{8+δ}$. Our method provides relations between directly measured quantities and therefore provides an unambiguous test of whether the ‘MDC width’ measured in angular resolved photoemission experiments corresponds to a transport mean free path. In agreement with previous authors, we find that it does not. The discrepancy is particularly severe in the case of optimally doped Bi$_2$Ca$_2$SrCu$_2$O$_{8+δ}$ where use of the full MDC width grossly overpredicts the resistivity. We conclude, in agreement with previous authors, that the broadening of the photomission spectra in the vicinity of the (0, π) point of the fermi surface should not be regarded as a contribution to the transport part of the self energy. The authors of Ref. 15 argued that the large broadening in this part of the zone arises from elastic scattering by out of plane impurities. In our view the ubiquity of the zone-corner broadening in cuprate materials argues instead in favor of an intrinsic, probably many-body origin to the phenomenon; understanding why it does not remain a very challenging theoretical problem. However, assuming that this broadening enters transport in the usual way is inconsistent with data. In our view the apparent irrelevance of the large zone-corner self-energy to the low frequency transport casts doubt on the attempts to describe transport and optical propeties with a ‘spin-fermion’ model, because in these models it is precisely the zone-corner scattering rate which is taken to be crucial for the conductivity. However, it is possible to find parameter regimes in spin-fermion models for which the scattering is not so strongly angle-dependent and reasonable (modulo Landau-parameter effects) fits to the conductivity may be achieved.

Our main new finding is that even if one is selective in the part of the photoemission data one interprets as giving rise to a transport rate (for example by selecting only the ‘inelastic’ part, agreement between calculation and experiment cannot be obtained unless a ‘Landau parameter’ (corresponding to an interaction-induced vertex correction to the conductivity) is introduced. The importance of this vertex correction casts doubt on most of the existing calculations of the frequency and temperature dependent conductivity, which neglect vertex corrections. This conclusion may be stated in a different way. If (as, for example, was very elegantly done in Ref. 15) a model self energy is constructed which reproduces (without vertex corrections) the conductivity spectrum, this self energy will necessarily fail to fit the photoemission spectrum. Determining the doping dependence of the vertex correction factor is an important topic for future research. We suspect that this must be large, because the low frequency optical spectral weight (which is closely related to the Γ* discussed above, dis-
plays a strong doping dependence whereas the observed low energy photoemission velocity does not. We also note that information on the temperature and frequency dependence of the photoemission-determined quasiparticle velocity would considerably help in making this comparison precise.

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Appendix: Band Theory

The natural choice for the reference dispersion is that given by band theory. There is general agreement that the dispersion in a single zone-diagonal velocity is reasonably robust, varying between about 3.8 – 4.1 eV – A depending on calculation and precise doping. We adopt here \( t = 0.38eV \), \( t’ = 0.32t \) and \( t'' = 0.5 \) implying a zone-diagonal velocity \( \partial\varepsilon_{LDA,p}/\partial p = 3.9 – 4.1 \) eV – A with the variation arising mainly from nonlinearities in the dispersion. The fermi line parameter \( p_{FS} \approx 0.7A^{-1} \). It is also sometime convenient to define the kinetic energy \( K \) via

\[
K = 2 \int \frac{dp}{(2\pi)^2} v^2_{p,z} \delta(\varepsilon_p - \mu) = v_{p_{FS}}/2\pi \quad (45)
\]

The best choice of the parameters is a subtle issue

\[
\varepsilon_{LDA}(p) = -2t(\cos(p_xa) + \cos(p_ya)) + 4t’ \cos(p_xa) \cos(p_ya) - 2t''(2\cos(2p_xa) + \cos(2p_ya))
\]

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