Universality classes of quantum chaotic dissipative systems

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Abstract – We study the ensemble of complex symmetric matrices. The ensemble is useful in the study of the effect of dissipation on systems with time-reversal invariance. We consider the nearest-neighbor spacing distribution and spacing ratio to investigate the fluctuation statistics and show that these statistics are similar to that of dissipative chaotic systems with time-reversal invariance. We show that, unlike cubic repulsion in eigenvalues of Ginibre matrices, this ensemble exhibits a weaker repulsion. The nearest-neighbor spacing distribution exhibits $P(s) \propto -s^3 \log s$ for small spacings. We verify our results for quantum kicked rotor with time-reversal invariance. We show that the rotor exhibits similar spacing distribution in dissipative regime. We also discuss a random matrix model for transition from the time-reversal invariant to the broken case.

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Introduction. – The quantum-mechanical behavior of dissipative quantum systems is of great interest [1–4]. For quantum chaotic systems, ensembles of asymmetric complex matrices (the Ginibre matrices) are helpful to study the effect of dissipation on the statistical properties. We will consider the random symmetric complex matrices and their application in the study of the effect of dissipation in quantum chaotic systems with time-reversal invariance (TRI).

There has been a lot of work on Hermitian and unitary random matrices [5–11]. In recent times also, these matrices have had important applications in the study of Majorana fermions and superconductivity [12], open quantum systems [13,14], information scrambling [15,16], out-of-time-order correlators [17–20], etc. The various ensembles of Hermitian matrices, viz., Gaussian Orthogonal Ensembles (GOE), Gaussian Unitary Ensemble (GUE), and Gaussian Symplectic Ensemble (GSE), give real eigenvalues and are applicable in the study of the Hamiltonians of conservative quantum chaotic systems. GUE is applicable when TRI is broken. GOE is applicable when TRI and rotational symmetry are both preserved. When TRI is preserved but rotational symmetry is broken, GOE and GSE are applicable for systems with integral and half-integral spins, respectively. A similar classification applies to the ensemble of unitary matrices, viz., Circular Orthogonal Ensembles (COE), Circular Unitary Ensemble (CUE), and Circular Symplectic Ensemble (CSE). They are used in the study of the evolution operators for quantum chaotic maps which arise from time-periodic Hamiltonians. The system of Quantum Kicked Rotor (QKR) provides a nice demonstration of COE and CUE [21]. The above three classes of ensembles of both types are invariant under orthogonal, unitary, and symplectic transformations, respectively. Moreover, in both types, the matrices are symmetric, asymmetric, and quaternion self-dual, respectively. These are characterized by the Dyson parameter $\beta$ with values 1, 2, and 4, respectively. These ensembles provide universal eigenvalues fluctuation statistics. For example, the nearest-neighbor spacing distribution (nnsd), viz., the distribution of consecutive eigenvalues and eigenangles for both types of ensembles, shows Wigner distribution with linear, quadratic, and quartic level repulsions for the three $\beta$ classes, respectively. In contrast, quantum integrable systems show Poisson statistics where level clustering is observed [10,11,22,23]. The Poisson distribution may be interpreted as the $\beta = 0$ case.

Quantum Chaotic Dissipative Systems (QCDs) are studied in the framework of Ginibre Ensembles (GinE) [11,24–26]. These ensembles do not follow any hermiticity or unitarity, but consist of matrices with general complex
elements. Such matrices are useful in the study of several phenomena, e.g., dissipative quantum maps [27], disordered systems [28,29], quantum chromodynamics at finite chemical potential [30,31], fractional quantum Hall effect [32], biological and neural networks [33–35], S-matrix poles for chaotic systems [36], etc. Eigenvalues for such ensembles lie in the complex plane [10,24,37]. The imaginary part of the eigenvalues and the eigenvectors are considered as a manifestation of dissipation in the system. The spacing distribution for the Ginibre ensemble shows cubic repulsion in the eigenvalues [38] and is verified in Dissipative Quantum Kicked Rotor (DQKR) without TRI [27,38–41].

In this paper, we consider the fluctuation statistics of DQKR when TRI is preserved. The Quantum Kicked Rotor (QKR) with TRI preserved and TRI broken is modeled by COE and CUE, respectively [21,42,43]. In a similar way, we introduce the ensemble of symmetric-Ginibre matrices (symm-GinE) as a random matrix model to study the TRI case of DQKR. We will show that the fluctuation statistics obtained in DQKR is different from the TRI breaking case.

Four classes of complex random matrices. — Analogously to the above four cases of the circular and Hermitian random matrix ensembles, we have four classes for dissipative systems. Analogous to the Poisson statistics is the distribution of uncorrelated complex numbers. The corresponding nnsd exhibits the Wigner distribution with linear repulsion [38]. The dissipative systems with no time-reversal invariance are represented by complex asymmetric matrices (the Ginibre ensemble) and the corresponding nnsd exhibits universal cubic repulsion. We will show that the effect of dissipation on time-reversal invariant systems can be studied with the ensemble of complex symmetric matrices. We also believe that the ensemble of complex quaternion self-dual matrices will be applicable in the study of dissipation in time-reversal invariant systems. The difference between these two is decided by rotational symmetries in the above Gaussian and circular ensembles. We again represent the four classes by the parameter $\beta$. The parameter has the value $\beta = 0$ for complex diagonal matrices, $\beta = 1$ for symmetric-Ginibre matrices, $\beta = 2$ for general complex matrices (the Ginibre matrices) and $\beta = 4$ for the self-dual complex quaternion matrices.

We consider ensembles of $N$-dimensional matrices $M$ with elements distributed as complex Gaussian variables of zero mean and variances $\nu^2$ for both real and imaginary parts. The joint probability distribution (jpd) of these matrices can be written as

$$P(M) \propto \exp\left[\frac{1}{2\nu^2} (\text{Tr} \, M^\dagger M)\right]$$

$$= \exp\left[\frac{1}{2\nu^2} \sum_{j,k} |M_{j,k}|^2\right],$$

where the $M_{jk}$ (j, k = 1, ..., N) are complex numbers for $\beta = 0, 1, 2$ and complex quaternion numbers for $\beta = 4$. For $\beta = 0$, we have $M_{ij} = M_{ji} = 0$, for $\beta = 1$, we have $M_{ij} = M_{ji}$, and for $\beta = 2$, $M_{ij}$ and $M_{ji}$ are independent. For $\beta = 4$, $M_{jk}$ are the quaternions with the property $M_{jk}^\dagger = M_{kj}$, where $D$ represents the dual of the quaternion. A quaternion number $q$ is written as $q = q_0 e_0 + q_1 e_1 + q_2 e_2 + q_3 e_3$, where $e_0, e_1, e_2, e_3$ are the quaternion units. The dual of $q$ is given by $q^* = q_0^* e_0 + q_1^* e_1 + q_2^* e_2 + q_3^* e_3$. In the matrix representation, the quaternions are replaced by their 2-dimensional matrices [10].

Two-dimensional random matrices. — We first consider the spacing distribution for various universality classes in two-dimensional complex random matrices ($N = 2$). The spacing $s = |z_1 - z_2|$ of the eigenvalues $z_1$ and $z_2$ can be written in terms of the matrix elements as

$$s_M = \sqrt{[(M_{11} - M_{22})^2 - 4M_{12}M_{21}]}.$$  \hspace{1cm} (2)

The spacing distribution $p(s, \beta)$ for $\beta$ is given by

$$p(s, \beta) \propto \int \ldots \int \delta(s - s_M) P(M) dM,$$

with $\nu^2$ chosen such that the average spacing is unity. $s_M$ can be written in terms of the matrix elements as

$$s_M = \sqrt{[(M_{11} - M_{22})^2 - 4M_{12}M_{21}]}.$$  \hspace{1cm} (3)

The compact expression for the spacing distribution can be derived for $\beta = 0, 1, 2$ from (4). For the $\beta = 0$ case, we get the Wigner distribution

$$p(s, 0) = \frac{\pi}{2} s \exp(-\pi/4s^2).$$

For the $\beta = 1$ case, we obtain

$$P(s, 1) = c_1 s^4 K_0(c_2 s^2),$$

with

$$c_1 = \frac{1}{213} \left[ \Gamma\left(\frac{1}{4}\right)\right]^8, \quad c_2 = 2 \left[ \Gamma\left(\frac{5}{4}\right)\right]^4.$$  \hspace{1cm} (5)

Here $K_0(s)$ is the zeroth-order modified Bessel function of the second kind [44],

$$K_0(s) = \int_s^\infty \frac{1}{\sqrt{x^2 - s^2}} e^{-x} dx.$$  \hspace{1cm} (6)
Fig. 1: Nearest-neighbor spacing distribution for two-dimensional matrices. Theory is from (5), (6), (9) and numerical results are from simulation of two-dimensional matrices.

For the $\beta = 2$ case we have [38]

$$p(s, 2) = 2 \left( \frac{9\pi}{16} \right)^2 s^3 \exp \left( -\frac{9\pi}{16} s^2 \right). \quad (9)$$

We have scaled the variance in all four cases so as to get the normalized spacing distribution with the mean spacing one. For small spacing we see from (6), (9) that the Ginibre ensemble exhibits cubic repulsion, $P(s) \propto s^3$, whereas the ensemble of symmetric-Ginibre matrices follows $P(s) \propto (-s^3 \log(s))$ [44]. The nearest-neighbor spacing distribution for all four classes is shown in fig. 1.

Unlike the Gaussian ensemble for conservative systems, the spacing distribution for large-dimension matrices is not similar to that of the two-dimensional case except for small spacings, but exhibits universality in their respective classes. The $\beta = 0$ case, however, remains in the same large-dimensional matrices.

**Ginibre ensemble for large dimensions — brief review.** The Ginibre ensemble consists of asymmetric matrices with complex entries. The matrix elements follow the Gaussian distribution. The jpdf for the Ginibre matrices is given by (1) with $v^2 = 1/2$:

$$P(M) \propto \exp(-\text{Tr} M^\dagger M). \quad (10)$$

The eigenvalues of such matrices lie in the complex plane. In order to obtain eigenvalue jpdf, the matrix is transformed to an eigenvalue-eigenvector space followed by the integration over eigenvector variables. The eigenvalue jpdf for the Ginibre ensemble is given by [10,24,38],

$$P(z_1, z_2, \ldots, z_N) = C \prod_{1 \leq j < k \leq N} |z_j - z_k|^2 e^{-\sum_{j=1}^N |z_j|^2}. \quad (11)$$

For the Ginibre ensemble, the spectral density is constant for large values of $N$ and is given by

$$R_\beta(z) = \begin{cases} 1/\pi, & |z| \leq \sqrt{N}, \\ 0, & |z| > \sqrt{N}. \end{cases} \quad (12)$$

The spacing distribution can also be evaluated from jpdf of eigenvalues. One defines $p(s, \beta)$ to represent the spacing distribution of the nearest-neighbor distance for each particle in the complex plane, i.e., for each eigenvalue $z_0$ one can find an eigenvalue $z_1$ for which $s = |z_0 - z_1|$ is minimum. For large $N$ the nsd for the Ginibre ensemble is given by [38]

$$p_N(s, 2) = -\frac{d}{ds} \int_0^\infty e^{-x^2} x^2 \frac{e^{-3x}}{2} \prod_{n=1}^\infty \right( e^{-6x^2} \right), \quad (13)$$

where

$$e_n(x) = \sum_{k=0}^\infty \frac{x^k}{k!}. \quad (14)$$

For small spacings, $p_N(s, 2)$ can be written as

$$p_N(s, 2) = 2s^3 - s^3 + \frac{1}{3} s^7 - \frac{11}{12} s^9 + \ldots. \quad (15)$$

Thus, the nearest-neighbor spacing distribution exhibits cubic repulsion for small spacings.

**Numerical results for large $N$.** For numerical results we use (10) for all three $\beta$ and consider ensembles of 10000 matrices with $N = 500$. For $\beta = 1$, we construct symm-GinE by considering the ensemble of complex symmetric matrices with real and imaginary entries of the off-diagonal matrix elements which are independently distributed as Gaussian variables with mean 0 and variance 1/2. In this case the diagonal matrix elements have variance twice that of the off-diagonal elements. For $\beta = 2$, every element is a complex Gaussian variable with mean and variance 1/2. For $\beta = 4$, we need one symmetric and three anti-symmetric complex matrices with the same mean and variance as above.

We diagonalize the matrices using standard LAPACK routines [45]. The eigenvalues are uniformly distributed in a circle of radius $\sqrt{N}$ for both symmetric-Ginibre and Ginibre matrices. For $\beta = 4$, there are $N$-distinct eigenvalues, each doubly degenerate, and are distributed uniformly in a circle of radius $2\sqrt{N}$. In fig. 2, we show the eigenvalues scatter plot for $\beta = 1, 2$. The eigenvalue distribution is isotropic. We also plot the radial density $R_\beta(|z|)$, normalized to $N$ (i.e., $\int_0^\infty 2\pi r R_\beta(r) dr = N$), in the same figure. The density for $\beta = 2$ can be evaluated from the eigenvalue jpdf [10,24]. For $\beta = 1$, the density can be evaluated through the supersymmetry approach [36] or through the electrostatic analogue [46].

We evaluate the nearest-neighbor spacing distribution for both systems. The solid lines in fig. 4 show the nearest-neighbor and next-nearest-neighbor spacing distributions for $\beta = 1, 2$ and 4. It may be noted that this letter is concerned with $\beta = 1, 2$ and we show the spacing distribution for $\beta = 4$ for completeness. There exist systems where spectral density may not be uniform [39] and one requires the unfolding method to remove the global variations. In order to unfold the spectra of such cases, we scale each spacing, $s$, by $s/R_\beta$ to get the spectral density similar to that of the Ginibre ensemble, where $R_\beta$ is the average spectral density around the eigenvalue pair. We will deal with non-uniform density in the dissipative quantum kicked rotor discussed ahead.
Quantum kicked rotor. – The Hamiltonian for the kicked rotor is defined as

\[ H = \frac{(p + \gamma)^2}{2} + \kappa \cos(\theta + \theta_0) \sum_{n=-\infty}^{\infty} \delta(t - n), \quad (16) \]

where \( \gamma \) and \( \theta_0 \) are time-reversal and parity breaking parameters. For sufficiently large values of the kicking parameter, \( \kappa \geq 10 \), the classical kicked rotor exhibits chaotic motion. The quantum-mechanical analogue of classical chaotic motion can be studied using the time evolution operator given by \( U = BG \), where \( B = \exp[-i(\kappa \cos(\theta + \theta_0))/\hbar] \) and \( G = \exp[-i(p + \gamma)^2/(2\hbar)] \) with \( \theta, p \) the position and momentum operators, respectively. For values of \( \kappa^2/N \) of the order \( O(10) \), the classical system becomes chaotic but the quantum system shows Poisson statistics because of the localization effect [42,43,47,48]. For sufficiently large values of \( \kappa^2/N \), i.e., \( O(1000) \), the corresponding quantum system displays chaos and follows the circular ensemble models [21,42,43].

We apply torus boundary conditions so that both \( \theta \) and \( p \) are discrete. We set \( \hbar = 1 \) and consider the \( N \)-dimensional model. In the position representation, the \( B \) operator is given by

\[ B_{mn} = \exp \left[ -i\kappa \cos \left( \frac{2\pi m}{N} + \theta_0 \right) \right] \delta_{mn}, \quad (17) \]

and the \( G \) operator is given by

\[ G_{mn} = \frac{1}{N} \sum_{l=-N'}^{N'} \exp \left[ -i \left( \frac{l^2}{2} - \gamma l + 2\pi l \left( \frac{m-n}{N} \right) \right) \right]. \quad (18) \]

Here \( N' = (N - 1)/2 \) and \( m, n = -N', -N' + 1, \ldots, N' \). Thus, the evolution operator can be written in the position basis as [21]

\[
U_{mn} = \frac{1}{N} \exp \left[ -i\kappa \cos \left( \frac{2\pi m}{N} + \theta_0 \right) \right] \times \sum_{l=-N'}^{N'} \exp \left[ -i \left( \frac{l^2}{2} - \gamma l + 2\pi l \left( \frac{m-n}{N} \right) \right) \right]. \quad (19)
\]

The above evolution operator is unitary. In the chaotic regime, the nnsd for this operator is similar to that of COE (CUE) for \( \gamma = 0 \) (0 < \( \gamma < 1 \).

Dissipative quantum kicked rotor. – We introduced a dissipation term in the quantum kicked rotor. The dissipation operator, \( D \), is given by \( D(\alpha) = e^{-\alpha p^2} \), where \( \alpha \) is a control parameter for dissipation. The evolution operator for the dissipative kicked rotor can be written as \( U(\alpha) = BGD \) and the corresponding matrix elements for the Floquet operator in the position basis can be written as

\[
F_{mn}(\alpha) = \frac{1}{N} \exp \left[ -i\kappa \cos \left( \frac{2\pi m}{N} + \theta_0 \right) \right] \times \sum_{l=-N'}^{N'} \exp \left[ -i \left( \frac{l^2}{2} - \gamma l + 2\pi l \left( \frac{m-n}{N} \right) \right) \right]. \quad (20)
\]

It is worthwhile to mention that \( U(\alpha) \) is not unitary for non-zero values of \( \alpha \). Any other ordering of operators \( B, G \) and \( D \), e.g., \( U(\alpha) = DBG \), in general, gives different results from \( U(\alpha) = BGD \) but produces similar statistics for spacing distributions. Since, the Floquet operator \( F \) is no longer unitary, the eigenvalues start falling towards the center and constitute a ring-like structure. We have studied the time-reversal broken \( (\gamma \neq 0) \) case for this system in [39].

The time-reversal preserved case corresponds to \( \gamma = 0 \). We construct the spectra using (20) with \( \gamma = 0 \) and \( N = 501 \). The spectral density is not uniform in this case as shown in fig. 3. To avoid the errors in unfolding due to non-uniform density, we consider the nearly uniform part of the spectra, viz., the spectra in a ring of inner and outer radius 0.255 and 0.520, respectively, i.e., considering approximately 50% eigenvalues of spectra. We thus calculate the nearest-neighbor spacing distribution. The nnsd and next nnsd are in excellent agreement with the spacing distribution for operator \( U(\alpha) = DGB \) in the chaotic regime, the nnsd for this operator is similar to that of COE (CUE) for \( \gamma = 0 \) (0 < \( \gamma < 1 \).
and $\gamma$ of symmetric-Ginibre matrices (the best-fit curve for the $\beta$ operators spacing distribution for two types of the time evolution operators). Fig. 4: (a) The nearest-neighbor and (b) next-nearest-neighbor spacings for (a) the time-reversal invariant ($\gamma = 0$) case and (b) the time-reversal non-invariant ($\gamma = 0.7$) case. The density profiles for both the cases are shown in (c) and (d), respectively.

![Fig. 3: Scatter plot for the eigenvalues of the Floquet operator for (a) the time-reversal invariant ($\gamma = 0$) case and (b) the time-reversal non-invariant ($\gamma = 0.7$) case. The density profiles for both the cases are shown in (c) and (d), respectively.](image)

Table 1: Comparison of mean and variance of the ratio of spacings for DQKR with that of the corresponding random matrix ensembles. The DQKR with TRI (without TRI) is in good agreement with symmetric-Ginibre (Ginibre) random matrices.

| Type-1          | Type-2          |
|-----------------|-----------------|
| $m$             | $\sigma \times 10^2$ | $m$             | $\sigma \times 10^2$ |
|-----------------|-----------------|-----------------|-----------------|
| DQKR $\gamma = 0.0$ | 0.7232          | 3.8789          | 0.8995          | 3.1567          |
| symm-Ginibre    | 0.7213          | 3.9046          | 0.8990          | 3.1701          |
| DQKR $\gamma = 0.7$ | 0.7397          | 3.5108          | 0.9084          | 2.6903          |
| Ginibre         | 0.7371          | 3.5415          | 0.9086          | 2.6857          |

Fig. 4: (a) The nearest-neighbor and (b) next-nearest-neighbor spacing distribution for two types of the time evolution operators $U(\alpha) = BGD$ and $U(\alpha) = DBG$ of the kicked rotor. We consider both the cases, viz., with $\gamma = 0$ (TRI preserved) and $\gamma = 0.7$ (TRI broken) and show their agreement with that of symmetric-Ginibre matrices ($\beta = 1$) and Ginibre matrices ($\beta = 2$), respectively. The dashed lines in the inset of (a) show the best-fit curve for the $\beta = 1$ and 2 cases.

ensemble of symmetric-Ginibre matrices ($\beta = 1$) exhibits $P(s) \propto -s^3 \log(s)$ [38,46] and DQKR with TRI broken as well as the corresponding ensemble of Ginibre matrices ($\beta = 2$) exhibits $P(s) \propto s^3$ [38].

**Ratio test.** When eigenvalues lie on the real line or circle, the spacing distribution of the ensembles can be computed relatively easily. This is due to the unfolding procedure which works quite well in one-dimensional spectra. In the case of the Ginibre ensemble and symmetric-Ginibre ensemble the spectra we obtained are two-dimensional. Due to the limitations of the unfolding procedure we are constrained to use a short range of spectra of the ensemble. In order to use a large range of spectra to study the distribution we take the ratios of the spacings, and evaluate the spacing ratio in two ways. In the first case we take the ratio of nearest and next-nearest spacing of the spectra and call it type-I ratio. In the second case we take the ratio of the spacing of the nearest neighbor of a spectra and the spacing of the nearest neighbor from the said nearest neighbor and call it type-II ratio. In both cases we consider the spectra in a ring of inner and outer radius 0.203 and 0.601, respectively, i.e., we consider about 87% eigenvalues of spectra. The average ($m$) and variance ($\sigma$) of the ratio we obtained are shown in table 1. We again see excellent agreement of the spacing ratio for the quantum kicked rotor with that of random matrix ensemble for both TRI preserved (corresponding to $\beta = 1$) and TRI broken (corresponding to $\beta = 2$) cases.

**Intermediate ensembles and their relation with the dissipative quantum kicked rotor.** The intermediate cases of the kicked rotor with time-reversal invariance weakly broken can be modeled with the linear combination of symmetric and anti-symmetric matrices which act as a crossover between the symmetric-Ginibre ensemble and the Ginibre ensemble. The intermediate matrices $M$ can be defined as

$$M(\eta) = \frac{1}{\sqrt{1 + \eta^2}}S + \frac{\eta}{\sqrt{1 + \eta^2}}A,$$

where $S$ and $A$ are symmetric-Ginibre and complex anti-symmetric matrices, respectively. For $\eta = 0$, we get symmetric-Ginibre matrices and for $\eta = 1$ we get Ginibre matrices. Note that the variance of the distribution for the elements of matrix $M$ is independent of $\eta$. We show in fig. 5 the spacing distribution for DQKR with various values of the TRI breaking parameter $\gamma$ and also find the corresponding best suitable value for the crossover parameter $\alpha$. For a quantitative analysis we show here the mean
and variance of different plots in table 2. Here $m$ and $\sigma$ represent the mean and variance of the spacing distribution. The subscripts 0, 1 represent the nsd and next nsd, respectively.

**Conclusion.** – We have studied the ensembles of complex symmetric matrices. We have shown that the spacing distribution for symm-Ginibre ensembles is different from that of Ginibre ensembles. The Ginibre ensemble is applicable to explore the universal aspects of chaotic dissipative systems without time-reversal invariance. We have shown that symm-Ginibre ensembles are useful to study dissipative systems with time-reversal invariance. In this connection, we have considered a system of the quantum kicked rotor which is a prototype model of chaos and works excellently with the theoretical predictions. We have verified the nearest-neighbor spacing distribution for the quantum kicked rotor with time-reversal invariance and have shown its agreement with that of the symm-Ginibre ensemble. We have also introduced the complex matrices that are useful in the study of the dissipative systems with TRI weakly broken.

A natural extension of this work is to explore the effect of dissipation on systems with time-reversal invariance but without rotational symmetry and we believe them to belong to the $\beta = 4$ class. The suitable models to explore the universal features under such symmetry are quantum kicked tops [38,49] and symplectic kicked rotor, obtained after coupling spin-(1/2) degree of freedom with the standard kicked rotor [50,51].

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