Implications of the $R_K$ and $R_{K^*}$ anomalies

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We discuss the implications of the recently reported $R_K$ and $R_{K^*}$ anomalies, the lepton flavor non-universality in the $B \rightarrow K\ell^+\ell^-$ and $B \rightarrow K^{*}\ell^+\ell^-$. Using two sets of hadronic inputs of form factors, we perform a fit of the new physics to the $R_K$ and $R_{K^*}$ data, and significant new physics contributions are found. We suggest to study the lepton flavor universality in a number of related rare $B$, $B_s$, $B_c$ and $\Lambda_b$ decay channels, and in particular we give the predictions for the $\mu$-to-$e$ ratios of decay widths with different polarizations of the final state particles, and of the $b \rightarrow d\ell^+\ell^-$ processes which are presumably more sensitive to the structure of the underlying new physics. With the new physics contributions embedded in Wilson coefficients, we present theoretical predictions for lepton flavor non-universality in these processes.

I. INTRODUCTION

The standard model (SM) of particle physics is now completed by the discovery of Higgs boson. Thus the focus in particle physics has been gradually switched to the search for new physics (NP) beyond the SM. This can proceed in two distinct ways. One is the direct search at the high energy frontier, in which new particles beyond the SM are produced and detected directly. The other is called indirect search, which is at the high intensity frontier. The new particles will presumably manifest themselves as intermediate loop effects, and might be detectable by low-energy experiments with high precision.

In flavor physics, the $b \rightarrow s\ell^+\ell^-$ process is a flavor changing neutral current (FCNC) transition. This process is of special interest since it is induced by loop effects in the SM, which leads to tiny branching fractions. Many extensions of the SM can generate sizable effects that can be experimentally validated. In particular, the $B \rightarrow K^*(\rightarrow K\pi)\mu^+\mu^-$ decay offers a large number of observables to test the SM, ranging from the differential decay widths, polarizations, to a full analysis of angular distributions of the final state particles, for an incomplete list one can refer to Refs. [1–21] and many references therein.

In the past few years, quite a few observables in the channels mediated by $b \rightarrow s\ell^+\ell^-$ transition have exhibited deviations from the SM expectations. The LHCb experiment has first observed the so-called $P_5'$ anomaly, a sizeable discrepancy at $3.7 \sigma$ between the measurement and the SM prediction in one bin for the angular observable $P_5'$ [22]. This discrepancy was reproduced in a later LHCb analysis for the two adjacent bins at large $K^*$ recoil [23]. To accommodate this discrepancy,
considerable attentions have been paid to explore new physics contributions (see Refs. [24–31] and references therein), while at the same time, this has also triggered the thoughts to revisit the hadronic uncertainties [32, 33].

More strikingly, the LHCb measurement of the ratio [34]:

$$R_K[q^2_{\text{min}}, q^2_{\text{max}}] = \frac{\int_{q^2_{\text{min}}}^{q^2_{\text{max}}} dq^2 d\Gamma(B^+ \rightarrow K^+ \mu^+ \mu^-)/dq^2}{\int_{q^2_{\text{min}}}^{q^2_{\text{max}}} dq^2 d\Gamma(B^+ \rightarrow K^+ e^+ e^-)/dq^2},$$  \hspace{1cm} (1)

gives a hint for the lepton flavour universality violation (LFUV). A plausible speculation is that deviations from the SM are present in $b \rightarrow s \mu^+ \mu^-$ transitions instead in $b \rightarrow s e^+ e^-$ ones. Very recently the LHCb collaboration has found sizable differences between $B \rightarrow K^* e^+ e^-$ and $B \rightarrow K^* \mu^+ \mu^-$ at both low $q^2$ region and central $q^2$ region [35]. Results for ratios

$$R_{K^*}[q^2_{\text{min}}, q^2_{\text{max}}] = \frac{\int_{q^2_{\text{min}}}^{q^2_{\text{max}}} dq^2 d\Gamma(B \rightarrow K^{*} \mu^{+} \mu^{-})/dq^2}{\int_{q^2_{\text{min}}}^{q^2_{\text{max}}} dq^2 d\Gamma(B \rightarrow K^{*} e^{+} e^{-})/dq^2},$$  \hspace{1cm} (2)

are given in Tab. I, from which we can see the data showed significant deviations from unity. These interesting results have subsequently attracted many theoretical attentions [36–59].

| Observable          | SM results     | Experimental data         |
|---------------------|----------------|---------------------------|
| $R_K : q^2 = [1,6]$ GeV$^2$ | $1.00 \pm 0.01$ [60] | $0.745^{+0.090}_{-0.074} \pm 0.036$ [34] |
| $R_{K_{\text{low}}} : q^2 = [0.045, 1.1]$ GeV$^2$ | $0.920^{+0.007}_{-0.006}$ [39] | $0.66^{+0.11}_{-0.07} \pm 0.03$ [35] |
| $R_{K^*_{\text{central}}} : q^2 = [1.1, 6]$ GeV$^2$ | $0.996 \pm 0.002$ [39] | $0.69^{+0.11}_{-0.07} \pm 0.05$ [35] |

The statistics significance in the data is low at this stage, about $3\sigma$ level. In order to obtain more conclusive results, one should measure the muon-versus-electron ratios in the $B \rightarrow K\ell^+\ell^-$ and $B \rightarrow K^*\ell^+\ell^-$ more precisely, meanwhile one should also investigate more channels with better sensitivities to the structures of new physics contributions. In this paper, we will focus on the latter. To do so, we will first discuss the implications of the $R_K$ and $R_{K^*}$ anomalies in a model-independent way, where the new particle contributions are parameterized in terms of effective operators. Since there is lack of enough data, we analyze their impact on the Wilson coefficients of SM operators $O_{9,10}$. We then propose to study the lepton flavor universality in a number of rare $B, B_s, B_c$ and $\Lambda_b$ decay channels. Incorporating the new physics contributions, we will present the predictions for the muon-versus-electron ratios in these channels, making use of various updates of form factors [61–66]. We will demonstrate that the measurements of lepton flavor non-universality with different polarizations of the final state hadron, and in the $b \rightarrow d\ell^+\ell^-$ processes are of great value to decode the structure of the underlying new physics.

The rest of this paper is organized as follows. In the next section, we will use a model-independent approach and quantify the new physics effects in terms of the short-distance Wilson...
coefficients. In Section III, we will study the LFUV in various FCNC channels. Our conclusion is given in the last section.

II. IMPLICATIONS FROM THE $R_K$ AND $R_{K^*}$

In this section, we will first study the impact of the $R_K$ and $R_{K^*}$ data. In the SM, the effective Hamiltonian for the transition $b \to s \ell^+\ell^-$

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu)$$

involves the four-quark and the magnetic penguin operators $O_i$. Here $C_i(\mu)$ are the Wilson coefficients for these local operators $O_i$. $G_F$ is the Fermi constant, $V_{tb}$ and $V_{ts}$ are CKM matrix elements. The dominant contributions to $b \to s \ell^+\ell^-$ come from the following operators:

$$O_7 = \frac{em_b}{8\pi^2} \bar{s} \sigma_{\mu\nu}(1+\gamma_5)b F_{\mu\nu} + \frac{em_s}{8\pi^2} \bar{s} \sigma_{\mu\nu}(1-\gamma_5)b F_{\mu\nu},$$

$$O_9 = \frac{\alpha_{em}}{2\pi}(\bar{l} \gamma_{\mu} l) s \gamma^\mu(1-\gamma_5)b, \quad O_{10} = \frac{\alpha_{em}}{2\pi}(\bar{l} \gamma_{\mu} \gamma_5 l) s \gamma^\mu(1-\gamma_5)b. \quad (3)$$

The above effective Hamiltonian gives the $B \to K\ell^+\ell^-$ decay width as:

$$\frac{d\Gamma(B \to K\ell^+\ell^-)}{dq^2} = \frac{G_F^2 \sqrt{\lambda} \alpha_{em}}{1536m_B^2\pi^5} |V_{tb} V_{ts}^*|^2 \times \left[ \lambda(1 + 2\hat{m}_t^2) \left| C_9 f_+(q^2) + C_7 \frac{2m_{b}f_T(q^2)}{m_B + m_K} \right|^2 \\
+ \lambda \beta_t^2 |C_{10}|^2 f_+^2(q^2) + 6\hat{m}_t^2 |C_{10}|^2 (m_B^2 - m_K^2)^2 f_0^2(q^2) \right], \quad (4)$$

where $\hat{m}_t = m_t/\sqrt{q^2}$, $\beta_t = \sqrt{1 - \hat{m}_t^2}$, $\lambda = (m_B^2 - m_K^2 - q^2)^2 - 4m_K^2q^2$, and $f_+, f_0$ and $f_T$ are the $B \to K$ form factors. In the above expression, we have neglected the non-factorizable contributions which are expected to be negligible for $R_K$.

The decay width for $B \to K^{*\ell^+\ell^-}$ can be derived in terms of the helicity amplitude $^{67}[71]$. The differential decay width is given as

$$\frac{d\Gamma(B \to K^{*\ell^+\ell^-})}{dq^2} = \frac{3}{4} (I_1^* + 2I_1^2) - \frac{1}{4} (I_2^* + 2I_2^2), \quad (5)$$

with

$$I_1^* = (|A_{L0}|^2 + |A_{R0}|^2) + 8\hat{m}_t^2 \text{Re}[A_{L0}^* A_{R0}] + 4\hat{m}_t^2 |A_t|^2,$$

$$I_2^* = (3/4 - \hat{m}_t^2) (|A_{L\perp}|^2 + |A_{L\parallel}|^2 + |A_{R\perp}|^2 + |A_{R\parallel}|^2) + 4\hat{m}_t^2 \text{Re}[A_{L\perp}^* A_{R\perp} + A_{L\parallel}^* A_{R\parallel}],$$

$$I_2^* = -\beta_t^2 (|A_{L0}|^2 + |A_{R0}|^2),$$

$$I_2^* = \frac{1}{4} \beta_t^2 (|A_{L\perp}|^2 + |A_{L\parallel}|^2 + |A_{R\perp}|^2 + |A_{R\parallel}|^2). \quad (6)$$

The handedness label $L$ or $R$ corresponds to the chirality of the di-lepton system. Functions $A_{L/Ri}$
can be expressed in terms of $B \to K^*$ form factors

\[
A_i^1 = 2\sqrt{N_{K_j}N_1} C_{10} \frac{\sqrt{\lambda}}{\sqrt{q^2}} A_0(q^2),
\]

\[
A_{L0}^1 = \frac{N_1 \sqrt{N_{K_j}^2}}{2m_{K_j} \sqrt{q^2}} \left[ (C_9 - C_{10})(m_B^2 - m_{K^*}^2 - q^2)(m_B + m_{K^*}) A_1 - \frac{\lambda}{m_B + m_{K^*}} A_2 \right]
+ 2m_\theta C_7[(m_B^2 + 3m_{K^*}^2 - q^2)T_2 - \frac{\lambda}{m_B^2 - m_{K^*}^2}T_3],
\]

\[
A_{L_\perp}^1 = -\sqrt{2N_{K_j}} N_1 \left[ (C_9 - C_{10})C_{10} \frac{\sqrt{\lambda V}}{m_B + m_{K^*}} + \frac{2m_\theta C_7}{q^2} \sqrt{\lambda T_1} \right],
\]

\[
A_{L||}^1 = \sqrt{2N_{K_j}} N_1 \left[ (C_9 - C_{10})(m_B + m_{K^*}) A_1 + \frac{2m_\theta C_7}{q^2} (m_B^2 - m_{K^*}^2) T_2 \right],
\]

with $N_1 = \frac{iG_F\alpha_{em}}{4\sqrt{2}} V_{tb} V^*_{ts}$, $N_{K_j} = 8/3\sqrt{\lambda q^2} \beta_1/(256\pi^3 m_B^2)$ and $\lambda \equiv (m_B^2 - m_{K^*}^2 - q^2)^2 - 4m_{K^*}^2 q^2$.

The right-handed decay amplitudes are obtained by reversing the sign of $C_{10}$:

\[
A_{Ri} = A_{Li}|_{C_{10} \to -C_{10}}.
\]

Within the SM, one can easily find that results for $R_K$ and $R_{K^*}$ are extremely close to 1 and thus deviate from the experimental data. If new physics is indeed present, it can be in $b \to s\mu^+\mu^-$ and/or $b \to s\ell^+\ell^-$ transitions. In order to explain the $R_K$ and $R_{K^*}$ data, one can enhance the partial width for the electronic mode or reduce the one for the muonic mode. It seems that the SM result for the $B \to K e^+e^-$ is consistent with the data, and thus here we will adopt the strategy that the muonic decay width is reduced by new physics.

After integrating out the high scale intermediate states the new physics contributions can be incorporated into the effective operators. As there is lack of enough data that shows significant deviations with SM, we will assume that NP contributions can be incorporated into Wilson coefficients $C_9$ and $C_{10}$. For this purpose, we define

\[
\delta C^\mu_9 = C^\mu_9 - C^SM_9, \quad \delta C^\mu_{10} = C^\mu_{10} - C^{SM}_{10}.
\]

The $O_7$ contribution to $b \to s\ell^+\ell^-$ arises from the coupling of a photon with the lepton pair. On one hand, this coupling is highly constrained from the $b \to s\gamma$ data. On the other hand, this coefficient is flavor blinded and thus even if NP affect $C_7$, the $\mu$-to-$e$ will not be affected.

For the analysis, we adopt three scenarios,

1. Only $C_9$ is affected with $\delta C_9^\mu \neq 0$.
2. Only $C_{10}$ is affected with $\delta C_{10}^\mu \neq 0$.
3. Both $C_9$ and $C_{10}$ are affected in the form: $\delta C_9^\mu = -\delta C_{10}^\mu \neq 0$.

Using the $R_K$ and $R_{K^*}$ data, we show our results in FIG. I. The left panel corresponds to scenario 1, and the middle panel corresponds to the constraint on $\delta C_{10}^\mu$, the last one corresponds to the
FIG. 1: Impact of $R_K$ and $R_{K^*}$ data on the $\delta C^\mu_9$ (left panel), $\delta C^\mu_{10}$ (central panel) or $\delta C^\mu_9 - \delta C^\mu_{10}$ (right panel). The dependence of the total $\chi^2$ for all data in Tab. I on Wilson coefficients is shown as the solid (red) and dashed (blue) curves, which correspond to the form factors from LQCD $^{65, 75}$ and LCSR $^{72, 73}$, respectively. Removing the low-$q^2$ data for $B \to K^*\ell^+\ell^-$, the results are shown as dotted (black) and and dot-dashed (green) curves.

scenario 3 with a nonzero $\delta C^\mu_9 - \delta C^\mu_{10}$. In this analysis, we have used two sets of $B \to K$ and $B \to K^*$ form factors. One is from the light-cone sum rules (LCSR) $^{72, 74}$, corresponding to the dashed curves. The other is from Lattice QCD (LQCD) $^{65, 75}$, which gives the solid curves. As one can see clearly from the figure, the results are not sensitive to the form factors, and this also partly validate the neglect of other hadronic uncertainties like non-factorizable contributions.

Using the LQCD set of form factors $^{65, 75}$ and the data in Tab. I, we found the best-fitted central value and the 1σ range for $\delta C^\mu_9$ in scenario 1 as

$$\delta C^\mu_9 = -1.83, \quad -2.63 < \delta C^\mu_9 < -1.25.$$  

(13)

For scenario 2, we have

$$\delta C^\mu_{10} = 1.43, \quad 1.04 < \delta C^\mu_{10} < 1.89,$$  

(14)

while for the $\delta C^\mu_9 = -\delta C^\mu_{10}$, we obtain

$$\delta C^\mu_9 - \delta C^\mu_{10} = -1.47, \quad -1.89 < \delta C^\mu_9 - \delta C^\mu_{10} < -1.08.$$  

(15)

A few remarks are given in order.

- Since the Wilson coefficient in the electron channel is unchanged, the $\delta C^\mu_9$ and $\delta C^\mu_{10}$ could be viewed as the difference between the Wilson coefficients for the lepton and muon case.

- We have found the largest deviation between the fitted results and the data comes from the low-$q^2$ region. Removing this data, we show the $\chi^2$ in FIG. 1 as dotted and dot-dashed...
curves, where the $\chi^2$ has been greatly reduced. The reason is that in low-$q^2$ region, the dominant contribution to $R_{K^*}$ arises from the transverse polarization of $K^*$. From Eq. (9) and (10), one can see this contribution is dominated by $O_7$ and less sensitive to $O_{9,10}$. A light mediator that only couples to the $\mu^+\mu^-$ is explored for instance in Refs. [47, 52, 54].

- For the $R_{K}$ and $R_{K^*}$ predictions in Refs. [39, 60], theoretical errors are typically less than one percent, while Ref. [76] gives the prediction with even smaller uncertainty $R_K = 1.0003 \pm 0.0001$. However it is necessary to stress that these results did not consider the electromagnetic corrections properly. We give the Feynman diagrams in Fig. 2. Fig. 2(a) is the typical Sudakov form factor, which usually introduces a double logarithm in terms of $\alpha/\pi \ln(q^2/m^2_{\ell})$. The difference between the double logarithms for the electron and muon mode is about 3%. A complete analysis requests the detailed calculation of all diagrams in Fig. 2 and analyses can be found in Ref. [77]. The nonfactorizable corrections to the amplitude can be found in Ref. [78].

- It is necessary to point out that there are a number of observables in $B \to K \mu^+\mu^-$ and $B \to K^* \mu^+\mu^-$ that have been experimentally measured. These observables are of great values to provide very stringent constraints on the Wilson coefficients in the factorization approach. On the other hand, most of these observables in $B \to K \mu^+\mu^-$ and $B \to K^* \mu^+\mu^-$ are not sensitive to the flavor non-universality coupling since only the mu lepton is involved. The exploration of the $\mu$-to-$e$ ratios will be able to detect the difference in the new physics
couplings to fermions. It is always meaningful to conduct a comprehensive global analysis and incorporate as many observables as possible. At this stage, the study of flavor non-universality in flavor physics is at the beginning, and we believe measuring more $\mu$ to $e$ ratios (for instance the ones in Table II shown in the following section) will be helpful.

- For a more comprehensive analysis, one may combine various experimental data on the flavor changing neutral current processes for instance in Refs. \[36–40\]. We quote the results in scenario I in Ref. \[36\],

$$ \delta C_{\mu}^9 = -1.58 \pm 0.28, \quad \delta C_{\mu}^e = -0.10 \pm 0.45, $$

from which we can see that the results are close to our scenario 1. This implies that for the determination of flavor dependent Wilson coefficient, the $R_K$ and $R_K^*$ are dominant. From a practical viewpoint, since the main purpose of this paper is to explore the implications of the large lepton flavor non-universality, we will use our fitted results to predict the lepton flavor non-universality for a number of other channels.

Explicit models which can realize these scenarios include the flavor non-universal $Z'$ model, leptoquark model and vector-like models, see, e.g., Refs. \[79–108\] and many references therein. Their generic contributions are shown in FIG. \[3\]. Taking the $Z'$ model as an example, the SM can be extended by including an additional $U(1)'$ symmetry, which can leads to the Lagrangian of $Z'b\bar{s}$ couplings

$$ \mathcal{L}_{\text{FCNC}}^{Z'} = -g'(B^L_{sb}\bar{s}L\gamma_\mu b_L + B^R_{sb}\bar{s}R\gamma_\mu b_R)Z'^\mu + \text{h.c.}. $$

It contributes to the $b \to s\ell^+\ell^-$ decay at tree level

$$ \mathcal{H}_{\text{eff}}^{Z'} = \frac{8G_F}{\sqrt{2}}(\rho_{sb}^L\bar{s}L\gamma_\mu b_L + \rho_{sb}^R\bar{s}R\gamma_\mu b_R)(\rho_{ll}^L\bar{\ell}L\gamma_\mu \ell_L + \rho_{ll}^R\bar{\ell}R\gamma_\mu \ell_R), $$

where the coupling is

$$ \rho_{ff'}^{L,R} = g'\frac{M_{Z'}}{gM_{Z'}}B_{ff'}^{L,R}, $$

where the $g$ standard model $SU(2)_L$ coupling. For simplicity, one can assume that the FCNC couplings of the $Z'$ and quarks only occur in the left-handed sector: $\rho_{sb}^R = 0$. Thus in this case the effects of the $Z'$ will modify the Wilson coefficients $C_9$ and $C_{10}$:

$$ C_{9}' = C_9 - \frac{4\pi}{\alpha_{em}}\frac{\rho_{sb}^L(\rho_{ll}^L + \rho_{ll}^R)}{V_{tb}V_{ts}^*}, \quad C_{10}' = C_{10} + \frac{4\pi}{\alpha_{em}}\frac{\rho_{sb}^L(\rho_{ll}^L - \rho_{ll}^R)}{V_{tb}V_{ts}^*}. $$

From this expression, we can see that the $\delta C_{9}^\mu$ and $\delta C_{10}^\mu$ are not entirely correlated. This corresponds to the scenario 1 and 2 in our previous analysis.

The impact in a leptoquark model has been discussed for instance in Ref. \[43\], where the NP contribution satisfies

$$ \delta C_{9}^{LQ,\mu} = -\delta C_{10}^{LQ,\mu}. $$

This corresponds to the scenario 3.
FIG. 3: New physics scenarios that can contribute to $b \to s\mu^+\mu^-$. The panel (a) shows a $Z'$, and in the other four diagrams $\Delta$ denotes a leptoquark with different spins and charges.

III. LEPTON FLAVOR UNIVERSALITY IN FCNC CHANNELS

In this section, we will study the $\mu$-to-$e$ ratios of decay widths in various FCNC channels. Since the three scenarios considered in the last section describe the data equally well, we will choose the first one for illustration in the following. We follow a similar definition

$$R_{B,M}[q_{\text{min}}^2, q_{\text{max}}^2] \equiv \frac{\int_{q_{\text{min}}^2}^{q_{\text{max}}^2} dq^2 d\Gamma(B \to M\mu^+\mu^-)/dq^2}{\int_{q_{\text{min}}^2}^{q_{\text{max}}^2} dq^2 d\Gamma(B \to Me^+e^-)/dq^2},$$

(22)

where $B$ denotes a heavy particle and $M$ denotes a final state. The channels to be studied include $B \to K_0^{*}(1430)\ell^+\ell^-$, $B_s \to f_0(980)\ell^+\ell^-$, $B \to K_1(1270)\ell^+\ell^-$, $B_s \to f_2(1525)\ell^+\ell^-$, $B \to \phi\ell^+\ell^-$, $B_c \to D_s\ell^+\ell^-$, $B_c \to D_s^{*}\ell^+\ell^-$. The expressions for their decay widths have been given in the last section. In addition, we will also analyze on the $R$ ratio for the baryonic decay $\Lambda_b \to \Lambda\ell^+\ell^-$. The differential decay width for $\Lambda_b \to \Lambda\ell^+\ell^-$ is given as

$$\frac{d\Gamma}{dq^2}[\Lambda_b \to \Lambda\ell^+\ell^-] = 2K_{1ss} + K_{1cc},$$

(23)

where

$$K_{1ss}(q^2) = \frac{1}{4} \left[ |A_{\perp 1}|^2 + |A_{\parallel 1}|^2 + 2 |A_{\perp 0}|^2 + 2 |A_{\parallel 0}|^2 + (R \leftrightarrow L) \right],$$

$$K_{1cc}(q^2) = \frac{1}{2} \left[ |A_{\perp 1}|^2 + |A_{\parallel 1}|^2 + (R \leftrightarrow L) \right].$$

(24)
The functions $A$ are defined as

$$A_{\perp_1}^{L(R)} = \sqrt{2}N \left[ (C_9 + C_{10}) H_\perp^V - \frac{2m_b C_7}{q^2} H_\perp^T \right], \quad A_{\parallel_1}^{L(R)} = -\sqrt{2}N \left[ (C_9 + C_{10}) H_\parallel^A + \frac{2m_b C_7}{q^2} H_\parallel^{T5} \right],$$

$$A_{\perp_0}^{L(R)} = \sqrt{2}N \left[ (C_9 + C_{10}) H_\perp^0 - \frac{2m_b C_7}{q^2} H_\perp^0 \right], \quad A_{\parallel_0}^{L(R)} = -\sqrt{2}N \left[ (C_9 + C_{10}) H_\parallel^A + \frac{2m_b C_7}{q^2} H_\parallel^{T5} \right],$$

where the normalization factor $N$ is

$$N = G_F V_{tb} V_{ts}^\ast \alpha_{\text{em}} \sqrt{\frac{q^2}{3} \cdot \frac{\lambda(m_{\Lambda_b}^2, m_{\Lambda}^2, q^2)}{2^{11} m_{\Lambda_b}^3 \pi^5}}.$$  \hspace{1cm} (25)

The helicity amplitudes are given by

$$H_0^V = f_0^V(q^2) \frac{m_{\Lambda_b} + m_{\Lambda}}{\sqrt{q^2}} \sqrt{s_-}, \quad H_0^A = -f_0^A(q^2) \sqrt{2s_-},$$

$$H_1^V = -f_1^V(q^2) \sqrt{q^2} \sqrt{s_-}, \quad H_1^A = -f_1^A(q^2) \sqrt{2s_-},$$

$$H_5^T = f_5^T(q^2) \sqrt{q^2} \sqrt{s_+}, \quad H_5^{T5} = -f_5^{T5}(q^2) (m_{\Lambda_b} - m_{\Lambda}) \sqrt{2s_+},$$

where $s_\pm = (m_{\Lambda_b} \pm m_{\Lambda})^2 - q^2$. The $f_i^{0/1}$ with $i = V, A, T, T5$ are the $\Lambda_b \to \Lambda$ form factors.

The $B_s \to \phi \ell^+\ell^-$ and $\Lambda_b \to \Lambda$ form factors are used from LQCD calculation in Refs. [63, 110], respectively. The $B \to K_0^*(1430)$ and $B_s \to f_0(980)$ form factors are taken from Ref. [111]. The $B \to K_1(1270)$ form factors are calculated in the perturbative QCD approach [63], and the mixing angle between $K_1(1^{++})$ and $K_1(1^{+-})$ is set to be approximately 45°. In this case the $B \to K_1(1400)\ell^+\ell^-$ is greatly suppressed [112]. The $B \to K_2$ and $B_s \to f_2(1525)$ form factors are taken from Ref. [64]. The $B_c \to D_s/D_{s}^\ast$ form factors are provided in light-front quark model [62], and in this work we calculated the previously-missing tensor form factors. Using the Wilson coefficient $\delta C_9^b$ in Eq. (13), we present our numerical results for $R_{B,M}$ in TABLE III.

Three kinematics regions are chosen in the analysis: low $q^2$ with [0.045, 1] GeV$^2$, central $q^2$ with [1, 6] GeV$^2$ and high $q^2$ region with [14 GeV$^2$, $q_{\text{max}}^2 = (m_B - m_M)^2$]. For a vector final state, the longitudinal and transverse polarizations are separated and labeled as $L$ and $T$, respectively. For $\Lambda_b \to \Lambda\ell^+\ell^-$, a similar decomposition is used, in which the superscript 0 means the $\Lambda_b$ and $\Lambda$ have the same polarization while 1 corresponds to different polarizations. The SM predictions for these ratios are listed in Tab. III.

A few remarks are given in order.

- From the decay widths for $B \to K^*\ell^+\ell^-$, we can see that in the transverse polarization, the contribution from $O_7$ is enhanced at low $q^2$, and thus the $R_{B,M}^T$ is less sensitive to the NP in $O_{9,10}$. Measurements of the $\mu$-to-$e$ ratio in the transverse polarization of $B \to V\ell^+\ell^-$ at low $q^2$ can tell whether the NP is from the $q^2$ independent contribution in $C_{9,10}$ or the $q^2$ dependent contribution in $C_7$. 

TABLE II: Theoretical results for the $\mu$-to-$\epsilon$ ratio $R_{B,M}$ of decay widths as defined in Eq. 13 in various $b \to s\ell^+\ell^-$ channels. Three kinematics regions are chosen: low, central and high $q^2$ regions. Wilson coefficient $C_9$ is used as in Eq. 13 based on the analysis of $R_K$ and $R_{K^*}$. For a vector final state, the longitudinal and transverse polarizations are separated and labeled as $L$ and $T$, respectively. For $\Lambda_b \to \Lambda^+\ell^-$, a similar decomposition is used: the superscript 0 means that the $\Lambda_b$ and $\Lambda$ have the same polarization, while 1 corresponds to different polarizations.

| Observable       | Low $q^2$ : $[0.045,1]$GeV$^2$ | Central $q^2$ : $[1,6]$GeV$^2$ | High $q^2$ : $[14]$GeV$^2$, $q_{\text{max}}^2$ |
|------------------|---------------------------------|---------------------------------|-----------------------------------------------|
| $R_{B,K_s^0(1430)}$ | $0.688^{+0.075}_{-0.073}$      | $0.702^{+0.076}_{-0.075}$      | $0.721^{+0.074}_{-0.074}$                     |
| $R_{B_s,1f(980)}$  | $0.687^{+0.074}_{-0.074}$      | $0.700^{+0.076}_{-0.076}$      | $0.707^{+0.075}_{-0.074}$                     |
| $R_{B_s,D_s}$      | $0.686^{+0.075}_{-0.075}$      | $0.699^{+0.077}_{-0.077}$      | $0.706^{+0.076}_{-0.076}$                     |
| $R_{B_s,\phi}$     | $0.863^{+0.016}_{-0.010}$      | $0.772^{+0.051}_{-0.040}$      | $0.710^{+0.071}_{-0.067}$                     |
| $R_{B_s,\phi}^L$   | $0.697^{+0.074}_{-0.074}$      | $0.701^{+0.076}_{-0.076}$      | $0.706^{+0.073}_{-0.073}$                     |
| $R_{B_s,\phi}^T$   | $0.975^{+0.024}_{-0.034}$      | $1.059^{+0.049}_{-0.108}$      | $0.712^{+0.070}_{-0.065}$                     |
| $R_{B_s,\phi}^{T,0}$ | $0.926^{+0.006}_{-0.012}$     | $0.940^{+0.003}_{-0.012}$      | $0.749^{+0.056}_{-0.041}$                     |
| $R_{B_s,D_s}^L$     | $0.704^{+0.066}_{-0.059}$      | $0.719^{+0.067}_{-0.060}$      | $0.736^{+0.060}_{-0.049}$                     |
| $R_{B_s,D_s}^T$     | $0.956^{+0.015}_{-0.035}$      | $1.289^{+0.113}_{-0.182}$      | $0.756^{+0.037}_{-0.037}$                     |
| $R_{B_s,K_s^0}$     | $0.881^{+0.017}_{-0.011}$      | $0.759^{+0.055}_{-0.044}$      | $0.718^{+0.068}_{-0.062}$                     |
| $R_{B_s,K_s^0}^L$   | $0.675^{+0.075}_{-0.076}$      | $0.696^{+0.077}_{-0.077}$      | $0.713^{+0.065}_{-0.065}$                     |
| $R_{B_s,K_s^0}^T$   | $0.983^{+0.026}_{-0.038}$      | $1.051^{+0.049}_{-0.109}$      | $0.721^{+0.066}_{-0.059}$                     |
| $R_{B_s,f_2}$       | $0.858^{+0.014}_{-0.008}$      | $0.767^{+0.052}_{-0.040}$      | $0.720^{+0.067}_{-0.060}$                     |
| $R_{B_s,f_2}^L$     | $0.675^{+0.075}_{-0.075}$      | $0.697^{+0.076}_{-0.076}$      | $0.716^{+0.069}_{-0.063}$                     |
| $R_{B_s,f_2}^T$     | $0.982^{+0.026}_{-0.037}$      | $1.063^{+0.052}_{-0.114}$      | $0.723^{+0.058}_{-0.058}$                     |
| $R_{B_s,K_s^0(1270)}$ | $0.909^{+0.008}_{-0.004}$    | $0.880^{+0.002}_{-0.000}$      | $0.714^{+0.069}_{-0.065}$                     |
| $R_{B_s,K_s^0(1270)}^L$ | $0.751^{+0.085}_{-0.094}$  | $0.717^{+0.088}_{-0.100}$      | $0.712^{+0.071}_{-0.067}$                     |
| $R_{B_s,K_s^0(1270)}^T$ | $0.978^{+0.025}_{-0.036}$  | $1.078^{+0.056}_{-0.118}$      | $0.714^{+0.069}_{-0.064}$                     |
| $R_{\Lambda_s,\Lambda}$ | $0.931^{+0.014}_{-0.007}$   | $0.773^{+0.051}_{-0.039}$      | $0.712^{+0.071}_{-0.068}$                     |
| $R_{\Lambda_s,\Lambda}^L$ | $0.708^{+0.073}_{-0.070}$  | $0.705^{+0.074}_{-0.072}$      | $0.707^{+0.073}_{-0.072}$                     |
| $R_{\Lambda_s,\Lambda}^T$ | $1.071^{+0.023}_{-0.032}$  | $1.104^{+0.060}_{-0.124}$      | $0.715^{+0.070}_{-0.065}$                     |

- In the central $q^2$ region, the operators $O_7$ and $O_{9,10}$ will contribute destructively to the transverse polarization of $B \to V\ell^+\ell^-$. Reducing $C_9$ with $\delta C_9^\mu < 0$ will affect the cancellation, and as a result the decay width for the muonic decay mode will be enhanced. Thus instead of having a ratio smaller than 1, one will obtain a surplus for this ratio.

- Results for $\Lambda_b \to \Lambda$ with different polarizations are similar, but it should be pointed out that differential decay widths in Eq. 23 have neglected the kinematic lepton mass corrections. Thus the results in the low $q^2$ region are not accurate.

- For the $B \to K_{0,2}(1430)\ell^+\ell^-$ and $B_c \to D_s^*$, the high $q^2$ region has a limited kinematics, and thus the results are difficult to be measured.
TABLE III: Theoretical results for the \( \mu \)-to-\( e \) ratio \( R_{B,M} \) of decay widths as defined in Eq. (22) in various \( b \rightarrow s\ell^+\ell^- \) channels in the SM. Three kinematics regions are chosen: low, central and high \( q^2 \) regions. For a vector final state, the longitudinal and transverse polarizations are separated and labeled as \( L \) and \( T \), respectively. We do not present the results \( \Lambda_b \rightarrow \Lambda\ell^+\ell^- \) since the lepton mass effects are not included in Eq. (25).

| Observable | Low \( q^2 : [0.045, 1]\text{GeV}^2 \) | Central \( q^2 : [1, 6]\text{GeV}^2 \) | High \( q^2 : [14\text{GeV}, q_{\text{max}}^2] \) |
|------------|-------------------------------------|-------------------------------------|-------------------------------------|
| \( R_{B,K_1(1430)} \) | 0.980 | 1.001 | 1.029 |
| \( R_{B_s,f_0(980)} \) | 0.980 | 1.000 | 1.004 |
| \( R_{B_s,D^*} \) | 0.981 | 1.001 | 1.006 |
| \( R_{B_s,\phi} \) | 0.937 | 0.998 | 0.993 |
| \( R_{B_s,\phi}^L \) | 0.991 | 1.001 | 0.999 |
| \( R_{B_s,\phi}^T \) | 0.902 | 0.985 | 0.997 |
| \( R_{B_s,D} \) | 0.917 | 0.995 | 0.997 |
| \( R_{B_s,D}^L \) | 0.978 | 0.997 | 0.997 |
| \( R_{B_s,D}^T \) | 0.908 | 0.990 | 0.997 |
| \( R_{B_s,K_0} \) | 0.932 | 0.996 | 0.997 |
| \( R_{B_s,K_0}^L \) | 0.971 | 0.998 | 0.998 |
| \( R_{B_s,K_0}^T \) | 0.902 | 0.985 | 0.997 |
| \( R_{B_s,f_2} \) | 0.930 | 0.995 | 0.998 |
| \( R_{B_s,f_2}^L \) | 0.971 | 0.998 | 0.998 |
| \( R_{B_s,f_2}^T \) | 0.902 | 0.985 | 0.997 |
| \( R_{B_s,\Lambda_{(1270)}} \) | 0.950 | 1.015 | 0.998 |
| \( R_{B_s,\Lambda_{(1270)}}^L \) | 1.064 | 1.039 | 0.999 |
| \( R_{B_s,\Lambda_{(1270)}}^T \) | 0.901 | 0.985 | 0.997 |

- Among the decay processes involved in Table II, a few of them have been experimentally investigated: the branching fractions of \( B_s \rightarrow \phi\ell^+\ell^- \) [113, 114], \( \Lambda_b \rightarrow \Lambda\ell^+\ell^- \) [115] and \( B_s \rightarrow f_0(980)\ell^+\ell^- \) [116] have been measured. So for these channels, the measurement of the \( \mu \)-to-\( e \) ratio will be straightforward when enough statistical luminosity is accumulated.

For the other channels, we believe most of them except the \( B_c \) decay might also be experimentally measurable, especially at the Belle-II with the designed 50\( ab^{-1} \) data and the high luminosity upgrade of LHC.

- In FIG. 3 a new particle like \( Z' \) or leptoquark can contribute to the \( R_K \) and \( R_{K^*} \). The coupling strength is unknown, and in principle it could be different from the CKM pattern. In the SM, the \( B \rightarrow \pi\ell^+\ell^- \) and \( B_s \rightarrow K\ell^+\ell^- \) have smaller CKM matrix elements. Thus if the NP contributions had the same magnitude as in \( b \rightarrow s\ell^+\ell^- \), their impact in \( B \rightarrow \pi\ell^+\ell^- \) and \( B_s \rightarrow K\ell^+\ell^- \) would be much larger. But in many frameworks, the new physics in \( b \rightarrow d\ell^+\ell^- \)
is suppressed compared to those in $b \to s\ell^-\ell^-$, for recent discussions see Ref. [117]. This can be resolved by experiments in the future.

- The weak phases from $Z'$ and leptoquark can be different from that in $b \to s\mu^+\mu^-$ or $b \to d\mu^+\mu^-$, which may induce direct CP violations. In the $b \to d\mu^+\mu^-$ process, the current data on $B \to \pi\mu^+\mu^-$ contains a large uncertainty [118]

$$A_{CP}(B^+ \to \pi^+\mu^+\mu^-) = (-0.12 \pm 0.12 \pm 0.01).$$

(28)

This can be certainly refined in the future. It should be noticed that the SM contribution may also contain CP violation source [119, 120] since the up-type quark loop contributions are sizable.

IV. CONCLUSIONS

Due to the small branching fractions in the SM, rare decays of heavy mesons can provide a rich laboratory to search for effects of physics beyond the SM. Up to date, quite a few quantities in $B$ decays have exhibited moderate deviations from the SM. This happens in both tree operator and penguin operator induced processes. The so-called $R_{D(D^*)}$ anomaly gives a hint that the tau lepton might have a different interaction with the light leptons. The $V_{ub}$ and $V_{cb}$ puzzles refer to the difference for the CKM matrix elements extracted from the exclusive and inclusive decay modes. In the $b \to s\ell^+\ell^-$ mode, the $P_5^\prime$ in $B \to K^{*}\ell^+\ell^-$ has received considerable attentions on both the reliable estimates of hadronic uncertainties and new physics effects. In addition, LHCb also observed a systematic deficit with respect to SM predictions for the branching ratios of several decay modes, such as $B_s \to \phi\mu^+\mu^-$ [113, 114]. Though the statistical significance is low, all these anomalies indicate that the NP particles could be detected in flavor physics.

In this work, we have presented an analysis of the recently observed $R_K$ and $R_{K^*}$ anomalies. In terms of the effective operators, we have performed a model-independent fit to the $R_K$ and $R_{K^*}$ data. In the analysis, we have used two sets of form factors and found the results are rather stable against these hadronic inputs. Since the statistical significance in $R_K$ and $R_{K^*}$ is rather low, we proposed to study a number of related rare $B, B_s, B_c$ and $\Lambda_b$ decay channels, and in particular we have pointed out that the $\mu$-to-$e$ ratios of decay widths with different polarizations of the final state particles, and in the $b \to d\ell^+\ell^-$ processes are likely more sensitive to the structure of the underlying new physics. After taking into account the new physics contributions, we made theoretical predictions on lepton flavor non-universality in these processes which can stringently examined by experiments in future.
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Appendix A: Definitions of $R_{L,T}$ and $R_{0,1}$

For $B$ decays to vector final state, we define the longitudinal and transverse ratios $R_{L}$ and $R_{T}$ as

$$R_{V}^{L,T}[q_{\text{min}}^{2}, q_{\text{max}}^{2}] = \frac{\int_{q_{\text{min}}^{2}}^{q_{\text{max}}^{2}} dq^{2} d\Gamma^{L,T}(B \rightarrow V \mu^{+} \mu^{-})/dq^{2}}{\int_{q_{\text{min}}^{2}}^{q_{\text{max}}^{2}} dq^{2} d\Gamma^{L,T}(B \rightarrow Ve^{+}e^{-})/dq^{2}},$$

(A1)

where the longitudinal and transverse differential widths are defined by

$$d\Gamma^{L}(B \rightarrow V \mu^{+} \mu^{-})/dq^{2} = \frac{3}{4}I_{1}^{ss} - \frac{1}{4}I_{2}^{ss},$$

(A2)

$$d\Gamma^{T}(B \rightarrow V \mu^{+} \mu^{-})/dq^{2} = \frac{3}{2}I_{1}^{ss} - \frac{1}{2}I_{2}^{ss},$$

(A3)

$V$ denotes a vector final state. The expressions for $I_{1}^{c,s}$ and $I_{2}^{c,s}$ are given by Eq. (6).

For $\Lambda_{b} \rightarrow \Lambda \ell^{+} \ell^{-}$ decays, we define ratios with equal or different polarization as

$$R_{0,1}^{\ell^{+}\ell^{-}}[q_{\text{min}}^{2}, q_{\text{max}}^{2}] = \frac{\int_{q_{\text{min}}^{2}}^{q_{\text{max}}^{2}} dq^{2} d\Gamma^{0,1}(\Lambda_{b} \rightarrow \Lambda \mu^{+} \mu^{-})/dq^{2}}{\int_{q_{\text{min}}^{2}}^{q_{\text{max}}^{2}} dq^{2} d\Gamma^{0,1}(\Lambda_{b} \rightarrow \Lambda e^{+}e^{-})/dq^{2}},$$

(A4)

the superscript 0 means that the $\Lambda_{b}$ and $\Lambda$ have the same polarization, while 1 corresponds to different polarizations. The expressions for $d\Gamma^{0,1}/dq^{2}$ are

$$d\Gamma^{0}(\Lambda_{b} \rightarrow \Lambda \mu^{+} \mu^{-})/dq^{2} = 2K_{1ss}^{0},$$

(A5)

$$d\Gamma^{1}(\Lambda_{b} \rightarrow \Lambda \mu^{+} \mu^{-})/dq^{2} = 2K_{1ss}^{1} + K_{1cc}^{1},$$

(A6)

$$K_{1ss}^{0,1}$$ and $K_{1cc}^{1}$ are defined by

$$K_{1ss}^{0} = \frac{1}{2}(|A_{R}^{L}|^{2} + |A_{0}^{L}|^{2} + |A_{\perp 0}^{L}|^{2} + |A_{\parallel 0}^{L}|^{2}),$$

(A8)

$$K_{1ss}^{1} = \frac{1}{4}(|A_{R}^{L}|^{2} + |A_{0}^{L}|^{2} + |A_{\perp 1}^{L}|^{2} + |A_{\parallel 1}^{L}|^{2}),$$

(A9)

$$K_{1cc}^{1} = \frac{1}{2}(|A_{R}^{R}|^{2} + |A_{0}^{R}|^{2} + |A_{\perp 1}^{R}|^{2} + |A_{\parallel 1}^{R}|^{2}).$$

(A10)
The $A$ functions have already been defined in Eq. (25).

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