Separation Problem in Satellite Gravimetry

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Abstract  The problem of separating gravitation from inertia is discussed in very general sense, and the conclusion is positive: man can separate gravitation from inertia, if various observation techniques are applied for. The accelerometer’s position problem in satellite gravimetry is investigated, and the additional acceleration effect due to the position error of an instrument as well as the difference between the mass center and the gravity center is explored.

Keywords  separation of gravitation and inertia; satellite gravimetry; mass and gravity centers; position error; extra acceleration

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Introduction

The problem of separating gravitation from inertia has attracted not only physicists but also geodesists. Might influenced by Fock[1], Synge first pointed out the essential difference between the gravitational field and the inertial field and further stated that[2], Einstein’s equivalence principle, which played a key role in establishing the general theory of relativity, should be given up. Theoretically, the Riemann tensor is intimately related to the gravitational field and is not influenced by the choice of reference system at all. One can judge on the existence or nonexistence of a gravitational field according to a vanishing or non-vanishing Riemann tensor. From time to time Moritz emphasized that, the separation problem of gravitation and inertia plays an important role in kinematical geodesy (including satellite gravimetry, airborne gravimetry, inertial navigation, etc.)[3].

To determine the earth’s gravity field by satellite gravimetry, we need pure gravitational information (boundary values). However, generally, the measured quantities are composed of gravitation and inertial forces. In the case of the “rest” measurement (such as in the case of setting a gravimeter on the ground), the problem of separation of gravitation and inertia can be solved easily: one needs only to subtract centrifugal forces from the measured quantities. But in the case of the “moving” measurement (such as the satellite gradiometry, marine gravimetry, inertial surveying, as well as airborne gravimetry and gradiometry), the problem is not so easy to solve.

According to Einstein’s general theory of relativity, the gravitational mass is equivalent to the inertial mass (equivalence principle), based on which it is inferred that[4], locally, the gravitational field can be compensated by the inertial field, and the inertial field can be compensated by the gravitational field. That means, locally, we have no way to separate gravitation from inertia because of their local “equivalence”. However, this is true only apparently.
In fact, although, locally, gravitational effect and inertial effect are similar, this does not mean that gravitation can be completely compensated by inertial forces, except at one point. Investigating the higher order differential structures of the gravitational field and the inertial field, one would definitely have the following conclusion: essentially, gravitation is different from inertia, i.e., they cannot be completely equivalent. This conclusion was first pointed out by Synge in 1960\cite{2}, and later a series studies occurred with emphasizing the applications in geodesy\cite{3,5-9}.

1 Separation of gravitation and inertia

In satellite gravimetry, a key problem is how to separate gravitation from inertia. Shen and Moritz once gave a result in the frame of general relativity\cite{6}: if the satellite moves freely, one can use gradiometer to determine the gravity gradient directly, and furthermore the earth’s relativistic gravity field could be determined based on the method of the spherical harmonic expansion\cite{10,11}. However, generally, due to the effects of the non-gravitational forces (such as air damping, light radiation pressure, etc), to deal with the separation problem, the following model is suggested: suppose the position of a satellite is precisely determined (for instance by GPS technique), set previously at the gravity center of the satellite an accelerometer, which senses the acceleration just as that caused by the non-gravitational forces (such as air damping and radiation pressure). With this model it is possible to inversely estimate or predict the non-gravitational force models. Of course it might not be realized because of multi-factors. Nevertheless, no matter how many kinds of external forces exist, the model mentioned above is effective, except that the external forces (excluding gravitation) can penetrate through the shield of the satellite so that they directly act on the accelerometer. In practice, the problem of separating gravitation from inertial is complicated.

1.1 Gravitation

Although Newton found the law of gravitation (any body attracts other bodies) as early as in 1666, and 250 years later Einstein generalized Newton’s gravitational theory, the nature of gravitation is still a great mystery. However one fact is definite: gravitational fields originate from gravitational masses, just as electromagnetic fields originate from (electric) charges. Furthermore, it is found that gravitational fields can be expressed geometrically. Gravitation causes the curvature of the spacetime; and the curvature of the spacetime characterizes some aspects of the gravitational field. Note that we cannot say that the spacetime curvature is the gravitation. Essentially, gravitation is one kind of physical phenomenon arising from the mass sources. To characterize gravitation (or gravitational field), we apply many different mathematical entities such as the metric, the Riemann curvature tensor, the Ricci curvature tensor, the connection coefficients, etc\cite{12}.

1.2 Gravity

Gravity is an additional concept, and it is always connected with a certain celestial body (for instance the earth) for reference. Without referring to some celestial body, gravity is meaningless. Hence, whenever we speak of gravity (or gravity field), we refer to the gravity (field) connected with a celestial body, and generally referring to the gravity field of the earth. In Newtonian theory, gravity is defined as follows (the special definition): referring to a celestial body (especially referring to the earth), at an arbitrary point $P$ (no matter outside or inside the body), gravity is the superposition of the gravitation and the centrifugal force caused by the celestial body, provided that the point $P$ is at the moment rigidly fixed to the body.

We note that, the reference system does neither influence gravitational fields, nor the curvature of the spacetime. Reference system effects can be calculated theoretically, provided that a definite reference system is chosen. Hence, once the earth’s gravitational field is determined, the gravity field is determined. Just because of this reason, generally, we only need to deal with the gravitational field, taking for granted that, once the gravitational field is determined, the gravity field is determined.

1.3 Inertia

When a body has an acceleration with respect to an
inertial reference system, it “feels” a force which is opposite to the direction of the acceleration. Based on the well-known rotation experiment of the water bucket, Newton stated that, inertia arises from the acceleration of the body relative to the absolute space, not arising from the relative acceleration with respect to the surrounding matter[4]. However, Mach stated that inertia does not arise from the acceleration of the body relative to the absolute space but the acceleration relative to the distant stars, i.e., inertia arises from the acceleration relative to the matter of the total universe. Which statement is correct?

Let us first consider the following “Gedanken experiment”: assuming that the universe is completely empty (although such an assumption is not correct), we investigate what will happen. If Mach is right, in this case there will not exist any inertial force at all. However, if Newton is right, inertial forces will exist. In fact, once an acceleration reference system is introduced, an inertial force follows; once a rotating reference system is introduced, the centrifugal force, Coriolis force and Euler force (all of these are the inertial forces) are introduced. It seems that the inertial force is not related to the matter of the total universe.

Let us further investigate Einstein’s free fall elevator. In the free fall elevator, an observer cannot feel gravitation, this is due to the “fact” that gravitation is just balanced by the inertial force (note that only in a uniform gravitational field or in a non-uniform gravitational field but considering one point particle, the above conclusion is correct; hence, if it is necessary, we only consider a uniform gravitational field, or we only consider a point particle instead of the body in a general gravitational field). A body falls freely under the action of the gravitation $F$ generated by the total mass of the universe, and at the same time the body “feels” an opposite inertial force $f$ which has the same magnitude as $F$. Hence, the total force acting on the body is zero. In this case we might state that the inertial force $f$ arises from the body’s acceleration relative to the matter of the total universe, because the gravitation $F$ is caused by the matter of the total universe. However, we make a further reasoning: in addition to $F$, we add an extra force $\delta F$ ($\delta F$ is some kind of force that does not belong to gravitation, e.g., force generated by an electric generator), so that the total active force acting on the body is $F+\delta F$. Of course, in this case the body does not have a free fall motion but a general motion. The body “feels” not only the action of the force $F+\delta F$, but also an inertial force $f'$ which is opposite to $F+\delta F$ in direction and equal to that in magnitude. Now we may ask: have we still a reason to state that the inertial force $f'$ arises from the matter of the total universe? To see this clearly, we assume that the total matter of the universe (except for the body considered) reduces gradually until it vanishes, so that $F$ reduces gradually until it vanishes. Then, the active force acting on the body is only $\delta F$, and the inertial force is $f'=\delta F$. Obviously, we have no reason to state that the inertial force $f'$ arises from the total matter of the universe, but state that it is due to the acceleration of the body relative to the “absolute space”, because in this case there does not exist matter in the universe (note: at present even if we do not know what is space, it is most likely that any meaningful space contains plenty of things, and we could never find a really void space[13]). As long as we accept the inertial law (the first Newtonian law), the above argument has a firm foundation. Unfortunately, since there does not exist the empty universe that we have assumed, we cannot make a final judgement: who is correct, Newton or Mach? We have met a dilemma: both might be right or both might be wrong.

### 1.4 Separation problem

At first, we point out that, if we consider an ideal uniform gravitational field, we have no way to separate gravitation from inertia[5]. However, in reality, there does not exist any uniform gravitational field. Let us for instance consider the gravitational field of the earth. In this case, the gravitational field appears approximately like a spherical field. In such a kind of gravitational field, it will be proved that one can separate gravitation from inertia.

For simplicity and without loss of the generality, we consider the gravitational field caused by a static uniform sphere. Intuitively, when one observes two freely falling stones which have a small horizontal distance $l$ at the beginning and start the free fall at the same time, one will find that these two stones go closer and closer as they fall down: they all fall down toward the center of the sphere. Just because of this
phenomenon, by observing the distance change between these two stones (although it is very difficult to observe this change), one can make a definite judgement whether these two stones are freely falling in a gravitational field or moving uniformly in the absence of any gravitational field, provided that the observer himself as well as the two stones are constrained in a closed freely falling system. In fact, this is just the tidal effect arising from the non-uniform gravitational field.

Mathematically, if we investigate the higher order differential structures of the gravitational field and the inertial field, we will find that these structures are different. Roughly speaking, the structure of the inertial field is smoother than that of the gravitational field (except for the trivial case: uniform gravitational field). As mentioned before, essentially, the gravitational field is different from the inertial field. Gravitation, which arises from the mass sources, is a real existence; and inertia is only a deduced quantity, which arises from the relative motion of the chosen reference system with respect to the inertial reference system. Just because of this difference\(^5\), one will find that it is possible to separate gravitation from inertia.

In the case of free motion (such as in the case of satellite), by investigating the well-known geodesic deviation equation\(^6\), one will find that there exists an entity, the Riemann tensor, which expresses the essential character of gravitation and is not influenced by any inertial effect. Some components of the Riemann tensor with respect to a chosen tetrad can be measured by gradiometers attached to the satellite. These components are enough for us to judge whether there exists a gravitational field or not, corresponding to the measured Riemannian components not being zero, or always being equal to zero\(^5\). Since the Riemannian components contain pure gravitational information, it is possible to find the earth’s gravitational field from the measured components.

Suppose that the earth is a regular body (sphere or ellipsoid), then we can determine the gravitational field by using only one set of the locally measured Riemannian components. Actually, the earth is irregular, and its gravitational field is very complicated. Obviously, it is impossible to determine the gravitational field by using only one set of the measured Riemannian components. We should note that, theoretically, this has nothing to do with the problem of separation of gravitation and inertia. The separation problem has been solved, once the Riemannian components are determined, because these quantities are purely gravitational.

Suppose we can measure the pure gravitation at one point on the ground. If we make an assumption that the earth is a uniform sphere, then we can find the earth’s gravitational field by using only the data measured at the point mentioned above. However, it is impossible to precisely determine the actual field of the earth, because the earth is a very complicated body. To precisely determine the earth’s gravitational field, we need as many sets of observing data (at different points on the earth’s surface or in the Earth’s external space) as possible.

So is the case of the measured Riemannian components. To precisely determine the gravitational field, we need the globally observed data: as many sets of the measured Riemannian components as possible\(^5\), and at the same time we need space technique (such as GPS) to determine the position of the instrument (otherwise we do not know which field point is referred).

In fact, in the case of free fall motion (the ideal satellite which is free of non-gravitational forces), the accelerometers (or gravimeters) are useless. However, by using gradiometers (provided that the gradiometers are attached to the satellite), we can get pure gravitational information\(^6\).

In the case of forced motion (e.g., satellite with low orbit, airplane etc.), the situation is much more complicated. Fortunately, we have found an equation, the worldline deviation equation, which is the generalization of the geodesic deviation equation\(^7\). By applying this equation, one will find that it is possible to locally separate gravitation from inertia. To realize this, there are two ways that can be chosen. One way is that one fixes the gradiometers to the cabinet of a satellite, and the measured quantities will include not only gravitational information, but also inertial effects\(^5\). To eliminate the inertial effects from the ob-
served data (by gradiometers), one has not only to use gyroscopes, but also to measure the relative rotations of the gyroscopes with respect to the airplane\cite{5}. Then, by a complicated calculation, one can eliminate the inertial effects from the observed quantities. Another way is that one fixes the gradiometers to an inertial platform, which is set on the airplane; then, the quantities measured by gradiometers will include only gravitational information\cite{5}. No matter which way one chooses (the second way is easier and recommended), gravitation can be separated from inertia.

2 Position problem

Conventionally, the accelerometer is set at the mass center of the satellite system (for instance by GRACE satellite mission\cite{14}). However, generally, the carrier’s mass center does not coincide with its gravity center, except that it is a special symmetric body, such as a spherical or layered spherical body, whose mass density is uniformly or layered uniformly distributed. If the accelerometer is not located at the gravity center of the carrier, it will sense an additional gravitational acceleration, which is just the difference between the gravitational acceleration of a unit mass point at the accelerometer’s position and that of the carrier’s gravity center. In this case, the values measured by an accelerometer contain gravitational effects. If we do not make corrections or we have no way to make corrections, the above mentioned gravitational effects are in fact unknowably combined with the acceleration caused by the external forces (excluding gravitation), and as a result the satellite gravity data do not exactly express the Earth’s real gravity, or saying that, the satellite data so determined are not pure gravitational.

2.1 Difference between mass and gravity centers

The difference amount of the mass center and gravity center depends on the figure and constitute of the spacecraft (satellite). If the spacecraft is spherically uniform or layered uniform, the mass center coincides with the gravity center. Generally however, the spacecraft is not a sphere, but an irregular figure, such as CHAMP or GRACE satellites\cite{14,15}, referring to what more materials are distributed on the bottom shield and top shield (comparing with the surroundings). To calculate the mass or gravity center we meet a difficulty: we do not know the mass distribution of the considered satellite exactly. Hence, in the sequel, we investigate an ideal model: a closed cabinet is constituted by equal-thick uniform layer, and the bottom and top are equal square-form, saying $a \times a$, with its height $h$. Suppose the satellite moves around the earth in such a way that the bottom or top plane is always perpendicular to the direction from the satellite to the earth’s mass center, otherwise the problem is far more complicated. To simplify the problem, the above model can be approximated by the following model: the bottom and the top of the cabinet are equally circular layer, the other characters are the same as described before. As first-order approximation (in the present case it is accurate enough) we take the earth’s gravitational field as the field generated by a uniform sphere:

$$g^i = \frac{GM}{r^3} x^i, \quad i = 1, 2, 3$$

where $g^i$ is the gravity component; $x^i$ are the coordinates of the field point; $G$ is the gravitational constant; and $M$ is the mass of the earth. The mass center depends only on the mass distribution, not related to the gravity field. The mass center can be expressed as:

$$x_{MC}^i = \frac{1}{M} \int \rho d\tau$$

where under the integral symbol, $x^i$ is the position vector of the integration element $d\tau$; $\tau$ is the region occupied by the mass system.

However, the gravity center depends on the gravity field, varies with gravity. In fact, the gravity center is the position at which the the gravity has the average gravity of the system. To define the gravity center, we first seek for the equivalent acceleration (gravity) of the system, which denotes the average gravity of the system, expressed as:

$$g_{GC}^i = \frac{1}{M} \int 1 \rho g d\tau$$

where $g^i$ is the gravity at the position of integration element $d\tau$. The gravity center can be defined as:

$$r_{GC}^i = \frac{1}{g_{GC}^i} \int x^i g d\tau$$

2.1.1 A system of two point masses

Let us first investigate a system of two unit masses:
two unit masses $A$ and $B$ are rigidly connected. Here we consider two situations: parallel and perpendicular to the direction of the radius.

In the case that the connection line coincides with the direction of the radius, the theoretical accelerations of $A$ and $B$ can be expressed respectively as (suppose $A$ is nearer to the earth’s center than $B$, i.e., $r_a \leq r_b$):

$$a_a = -\frac{GM}{r_a^2}, \quad a_b = -\frac{GM}{r_b^2}$$

(5)

where the minus means that the acceleration direction points to the center of the earth. Suppose the distance between $A$ and $B$ is $2l$. Obviously the mass center of the system is just at the center between $A$ and $B$. At the mass center, a unit mass has an acceleration:

$$a_{MC} = -\frac{GM}{r_{MC}^2} = -\frac{GM}{(r_a + l)^2}$$

(6)

Since the system is connected rigidly, the acceleration of the system is expressed as:

$$a_{GC} = -\frac{1}{2}\left[\frac{GM}{r_a^2} + \frac{GM}{(r_a + 2l)^2}\right]$$

(7)

Combining Eqs. (6) and (7), one will find that there is a difference acceleration $\Delta a = a_{GC} - a_{MC}$:

$$\Delta a = -\frac{1}{2}\left[\frac{GM}{r_a^2} + \frac{GM}{(r_a + 2l)^2}\right] + \frac{GM}{(r_a + l)^2}$$

(8)

Accurate to the order $gl^2 / R^2$, we have:

$$\Delta a = \frac{3GMl^2}{r_a^3 r_b^3}$$

(9)

In the case that the connection line is perpendicular to the direction of the radius, the theoretical accelerations of $A$ and $B$ can be expressed respectively as (in this case $A$ is at the same distance to the earth’s center as $B$ does):

$$a'_a = \frac{GM}{r_a^2} x'_a, \quad a'_b = -\frac{GM}{r_b^2} x'_b$$

(10)

The mass center of the system is just at the center between $A$ and $B$. At the system mass center, a unit mass has an acceleration:

$$a_{MC} = -\frac{GM}{r_{MC}^2}$$

(11)

where $r_{MC}$ is the distance between the system mass center and the earth’s center, expressed as:

$$r_{MC} = \sqrt{r_a^2 - l^2}$$

(12)

We know that the system is connected rigidly, and consequently the acceleration of the system is expressed as:

$$a_{GC} = -\frac{1}{2}\left(\frac{GM}{r_a^2} x'_a + \frac{GM}{r_b^2} x'_b\right)$$

(13)

Since $r_a = r_b + \frac{1}{2}(x'_a + x'_b) = x'_MC$, we have:

$$a'_{GC} = -\frac{GM}{r_a^2} x'_MC$$

(14)

Comparing Eq. (14) with Eq.(11), taking account of Eq. (12), one gets:

$$\Delta a = -\frac{GM}{r_a^2} \sqrt{r_a^2 - l^2} + \frac{GM}{r_b^2} - l^2$$

(15)

From the above equation one has (accurate to the order $gl^2 / R^2$):

$$\Delta a = -\frac{3GMl^2}{r_a^3 r_b^3}$$

(16)

From Eqs. (9) and (16) it is known that, the additional acceleration difference due to the non-coincidence of the mass center and the gravity center is around $\frac{GMl^2}{r_a^3 r_b^3}$, which is so small that it can be neglected in general case.

2.1.2 Cylinder

Suppose the spacecraft has a figure like cylinder, the constitution of it is just as described as in the beginning of Section 2.1. The bottom and the top plane are always perpendicular to the direction of the radius. With a good approximation, it is assumed that the surrounding wall is parallel with the direction of the radius. Based on the investigations of the system of two unit masses, we can conclude that the mass center of the cylinder’s surrounding wall coincides with the gravity center of that. Hence, we need only to consider the different accelerations caused by the bottom and top layers. The mass center is just at the center of the cylinder. With a good approximation (because of the symmetry property), at the mass center, a unit mass has an acceleration:

$$a_{MC} = -\frac{GM}{r_{MC}^2}$$

(17)

where $r_b$ is the distance between the bottom layer’s
center and the earth’s center; \( h \) is the height of the cylinder. However, the system is rigidly fixed, and the acceleration of the system can be expressed as:

\[
a_{gc} = -\frac{1}{2} \left( \frac{GM}{r_p^2} + \frac{GM}{(r_p + h)^2} \right)
\]  

(18)

Comparing the Eq.(17) with Eq. (18), one will find that there is a difference acceleration.

### 2.2 Influence due to position error

The influence due to position error refers to two aspects: ① the instrument is located at the mass center, but the mass center does not coincide with the gravity center; ② the instrument is set at the point that is neither the mass center nor the gravity center. No matter in which case, if the position of the instrument does not coincide with the gravity center, there exists an extra effect in the measured data. To get pure gravitational information, we should take into account such an effect. Based on the studies expressed in Section 2.1, the acceleration difference due to the non-coincidence of the mass center and the gravity center can be completely neglected in practice. That means, in practice, we can take granted that the mass center coincides with the gravity center. In the sequel, let us estimate the acceleration effect caused by the non-coincidence of the instrument’s position and the mass center (gravity center).

To simplify the problem and without loss of generality, suppose the instrument (accelerometer) is set at point \( P \) which is just on the line of the radius direction. Further, we assume that the gravity center is also located on the line of the radius direction. If the accelerometer is located at the gravity center, it will not sense any gravitation; otherwise, it will sense a gravitation. In our present case, the acceleration measured by the instrument can be expressed as:

\[
\Delta a = a_{gc} - a_p
\]  

(19)

where \( a_p = -\frac{GM}{r_p^2} \) is the acceleration of a unit mass at point \( P \); and \( r_p \) is the distance between point \( P \) and the earth’s center. Substituting Eq.(18) into Eq.(19) one gets:

\[
\Delta a = -\frac{1}{2} \left( \frac{GM}{r_p^2} + \frac{GM}{(r_p + h)^2} \right) + \frac{GM}{r_p}
\]  

(20)

If the instrument is set at the mass center, we have Eq.(8), i.e.

\[
\Delta a = \frac{1}{2} \left( \frac{GM}{r_p^2} + \frac{GM}{(r_p + h)^2} \right) + \frac{GM}{r_p}
\]  

(21)

or

\[
\Delta a = \frac{1}{2} \left( \frac{GM}{r_p^2} - \frac{GM}{(r_p + h)^2} \right) + \frac{1}{2} \left( \frac{GM}{(r_p + h)^2} - \frac{GM}{(r_{gc} + b)^2} \right)
\]  

(22)

Generally, let us suppose that the distance between the location of the instrument and the gravity center is \( b \), then, based on Eq.(20) we have (assuming that the instrument is a little farther from the earth’s center):

\[
\Delta a = -\frac{1}{2} \left( \frac{GM}{r_p^2} + \frac{GM}{(r_p + h)^2} \right) + \frac{GM}{r_{gc} + b}
\]  

(23)

From Eq.(18) we know that:

\[
a_{gc} = \frac{1}{2} \left( \frac{GM}{r_p^2} + \frac{GM}{(r_p + h)^2} \right) - \frac{GM}{r_p}
\]  

(24)

Then, from Eqs.(23) and (24) we have (accurate to \( gb/R \)):

\[
\Delta a = -\frac{2GMb}{r_{gc}^3}
\]  

(25)

Or, it can be approximately expressed as:

\[
\Delta a = 10^5 \frac{2b}{R} (\mu\text{Gal})
\]  

(26)

Suppose the position error is 1 cm, then from Eq.(26) one knows that the extra contribution to the measurement due to the position error is about 1.6 \( \mu \text{Gal} \).

### 3 Discussions

In satellite gravimetry, although the mass center does not coincide with the gravity center, the influence caused by the non-coincidence can be neglected. Hence, in practice (before launching a satellite for the purpose of gravimetry, such as CHAMP or GRACE), we can just set the instrument (such as accelerometer) at the mass center of the satellite system. However, the problem is: is it possible to precisely set the instrument at the mass center? Obviously, in general it is impossible. What we can make is to set the instrument in the satellite at the position nearest to the mass center of the system. In another aspect, to adjust the status of a satellite during its flying around the earth, the energy is needed from time to time, i.e., the satellite loses masses from time to time; and this will result in the variation of the mass center from time to time. The variation of the mass center might be ob-
vious. However, generally, it is taken granted that the instrument is a priori set at the mass center of the primary satellite system. If we cannot adjust the instrument’s position according to the variation of the satellite system’s mass center, we must correct the measured data by the accelerometer, because the data contains the extra accelerations caused by the instrument’s position error. For this reason, it is suggested that the energy materials should be a priori equally reserved in both the bottom layer and top layer, and the usage of the energy materials should be at the same time with the same quantities from both (bottom and top) energy containers.

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