Energy Distribution of $\phi$ in Pure Penguin Induced $B$ Decays

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Abstract

We study the energy distribution of $\phi$ in pure penguin induced $B \to X_s\phi$ taking into account the fermi motion of $b$ inside $B$ meson for $b \to s\phi$ and also modification due to gluon bremsstrahlung process $b \to s\phi g$. We find that the contribution to $B \to X_s\phi$ from $b \to s\phi g$ is less than 3%. This study provides a criterion for including most of the $\phi$'s produced in a penguin process.

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Rare $B$ decays, particularly pure penguin decays, have been a subject of considerable theoretical and experimental interest recently [1]. The photonic penguin processes have been observed by the CLEO collaboration [2] in both the exclusive mode $B \rightarrow K^{*}\gamma$ and in the inclusive mode $B \rightarrow X_{s}\gamma$. The Standard Model (SM) is consistent with experimental data [3]. A signature of pure penguin hadronic processes are the exclusive modes $B \rightarrow K\phi$, $K^{*}\phi$ etc. or the inclusive mode $B \rightarrow X_{s}\phi$ [4,5,6] and other modes resulting from processes like $b \rightarrow \bar{s}ss$. The search for exclusive processes has not yet led to a definite observation. The inclusive mode with a larger branching ratio would be a complementary way of searching for penguin processes. At the quark level $B \rightarrow X_{s}\phi$ results from $b \rightarrow s\phi$, just as $B \rightarrow X_{s}\gamma$ results from $b \rightarrow s\gamma$. In both cases the energy spectrum of $\phi$ or $\gamma$ are not monenergetic as a result of two effects. First, because $b$ quark is in the $B$ meson, its fermi momentum smears the energy spectrum of $\phi$ or $\gamma$. Therefore the distribution of energy depends to some extent on the choice of the wave function. Second effect arises from the process $b \rightarrow sg\gamma$ in the photonic case, and $b \rightarrow sg\phi$ in the hadronic case. The photonic case has been discussed in a series of papers by Ali and Greub [7]. They find that the dominant contribution to the $\gamma$ spectrum comes from the wave function effect. We shall perform a similar calculation for the hadronic case. The wave function effect is treated with a Monte-Carlo simulation of decays. The gluonic correction is carried out in a simple effective Hamiltonian approximation. We find that the second effect is negligible in our case.

The QCD corrected $H_{\Delta B=1}$ relevant to us can be written as follows [8]:

$$H_{\Delta B=1} = \frac{G_F}{\sqrt{2}} [V_{ub}V_{us}^{\ast} (c_1 O_1^u + c_2 O_2^u) + V_{cb}V_{cs}^{\ast} (c_1 O_1^c + c_2 O_2^c) - V_{tb}V_{ts}^{\ast} \sum c_i O_i] + H.C. , \quad (1)$$

where the Wilson coefficients (WCs) $c_i$ are defined at the scale of $\mu \approx m_b$; and $O_i$ are defined as

$$O_1^q = \bar{s}_\alpha \gamma_{\mu}(1 - \gamma_5)q_\beta \bar{q}_\beta \gamma^\mu(1 - \gamma_5)b_\alpha , \quad O_2^q = \bar{s}_\alpha \gamma_{\mu}(1 - \gamma_5)q_\beta \bar{q}_\beta \gamma^\mu(1 - \gamma_5)b_\alpha ,$$
$$O_{3,5} = \bar{s}_\alpha \gamma_{\mu}(1 - \gamma_5)b \sum_{q'} q'_\gamma_{\mu}(1 \mp \gamma_5)q' , \quad Q_{4,6} = \bar{s}_\alpha \gamma_{\mu}(1 - \gamma_5)b_\beta \sum_{q'} q'_\beta \gamma_{\mu}(1 \mp \gamma_5)q'_\alpha ,$$
$$O_{7,9} = \frac{3}{2} \bar{s}_\alpha \gamma_{\mu}(1 - \gamma_5)b \sum_{q'} e_{q'} q'_\gamma_{\mu}(1 \pm \gamma_5)q' , \quad Q_{8,10} = \frac{3}{2} \bar{s}_\alpha \gamma_{\mu}(1 - \gamma_5)b_\beta \sum_{q'} e_{q'} q'_\beta \gamma_{\mu}(1 \pm \gamma_5)q'_\alpha . \quad (2)$$
The Wilson coefficients at $\mu = m_b$ at the next-to-leading order have been evaluated in Refs. [6,8,9]. For $m_t = 176$ GeV and $\alpha_s(m_Z) = 0.117$, we find

$$
c_1 = -0.307, c_2 = 1.147, c_3 = 0.017, c_4 = -0.037, c_5 = 0.010
$$
$$
c_6 = -0.045, c_7 = 1.2 \times 10^{-5}, c_8 = 3.8 \times 10^{-4}, c_9 = -0.010, c_{10} = 2.1 \times 10^{-3}.
$$

We shall consider the effect due to fermi momentum in $b \to s\phi$ process, which we assume has the same $\phi$ energy distribution as $B \to X_s\phi$, in section (a) below. In section (b) we shall consider the effect due to $b \to s\phi g$ process.

**a. $b \to s\phi$**

Using $H_{B=1}$ in Eq.(1), we obtain the decay amplitude for $B \to X_s\phi$

$$
A(b \to s\phi) = -a\bar{s}\gamma_{\mu}(1 - \gamma_5)b\phi^\mu,
$$

where $\epsilon^\mu$ is the polarization of the $\phi$ particle; $a = (g_{\phi}G_FV_{tb}V_{ts}^*/\sqrt{2})[c_3 + c_4 + c_5 + \xi(c_3 + c_4 + c_6) - (c_7 + c_8 + c_9 + \xi(c_8 + c_9 + c_{10}))/2]$ with $\xi = 1/N_c$, and $N_c$ is the number of colors. The coupling constant $g_{\phi}$ is defined by $<\phi|\bar{s}\gamma_{\mu}s|0> = ig_{\phi}\epsilon^\mu$. From the experimental value for $Br(\phi \to e^+e^-)$ [10], we obtain $g_{\phi}^2 = 0.0586$ GeV$^4$. The branching ratio for $b \to s\phi$ is predicted to be $1.7 \times 10^{-4}$ [6] for $\alpha_s(m_Z) = 0.117$.

The decay rate is given by

$$
\Gamma(b \to s\phi) = \frac{|a|^2m_\phi^3}{8\pi m_\phi^2}\lambda_{s\phi}^{3/2}[1 + \frac{3}{\lambda_{s\phi}}m_\phi^2(1 - \frac{m_\phi^2}{m_b^2} + \frac{m_\phi^2}{m_b^2})],
$$

where $\lambda_{ij} = (1 - m_j^2/m_b^2 - m_\phi^2/m_b^2)^2 - 4m_\phi^2m_j^2/m_b^4$.

To study the energy distribution of $\phi$, we adopt the model in Ref. [11] in which the $b$ quark is not at rest inside $B$ but with a fermi momentum $p_b$ according to a Gaussian distribution,

$$
\Phi(\vec{p}_b) = \frac{4}{\sqrt{\pi p_f^3}}e^{-\vec{p}_b^2/p_f^2},
$$

3
where the parameter $p_f$ is determined from experimental data to be between 0.21 to 0.39 GeV \cite{12}. The $b$ quark mass expressed in terms of the $B$ meson mass $m_B$ and the spectator quark mass $m_q$ in the rest frame of $B$, is given by

$$m_b^2 = m_B^2 + m_q^2 - 2m_B\sqrt{\vec{p}_b^2 + m_q^2}. \quad (7)$$

In the rest frame of the $B$, the $b \rightarrow s\phi$ decay width $\Gamma(m_b)$ is given by $(m_b/E_b)\Gamma(b \rightarrow s\phi)$. Its contribution to the decay width $\Gamma_s\phi$ for $B \rightarrow X_s\phi$ is averaged over all allowed momenta $\vec{p}_b$. We have

$$\Gamma_{s\phi}(B \rightarrow X_s\phi) = \frac{\int_{P_{max}}^{P_{max}} \Phi(\vec{p}_b)\Gamma(m_b)d|\vec{p}_b|}{P_{max}} = \sqrt{\frac{(m_B^2 + m_q^2 - (m_\phi + m_s)^2)^2}{4m_B^2} - m_q^2}. \quad (8)$$

Due to the finite momentum distribution, the energy of $\phi$ from $b$ quark decay is no longer monoenergetic, instead there will be a distribution. The $\phi$ energy spectrum generated from a Monte-Carlo simulation of the decay is shown in Figure 1. In the figures we have used the constituent mass of 0.3 GeV and 0.5 GeV for the spectator quark and the $s$-quark, respectively. From Fig.1, we see indeed that there is a spread in the $\phi$ energy with the maximum located at about 2.55 GeV.

b. $b \rightarrow s\phi g$

The quark level effective Hamiltonian responsible for $b \rightarrow s\phi g$ is complicated. We use the simplified effective Hamiltonian in Eq.(4) to obtain the $\phi$ energy distribution for the process $b \rightarrow s\phi g$ by attaching a gluon on either the initial $b$ or the final $s$ quarks. This is expected to be a good approximation because the dominant effect comes from the bremsstrahlung of gluon emission from the external light quark. We obtain

$$A(b \rightarrow s\phi g) = ag_s\left(\frac{\bar{s}\gamma_\mu(2p_\nu - \not\!p_g\gamma_\nu)T^a(1 - \gamma_5)b}{(p_b - p_g)^2 - m_b^2} + \frac{\bar{s}(2p_\nu + \gamma_\nu \not\!p_g)\gamma_\mu T^a(1 - \gamma_5)b}{(p_s + p_g)^2 - m_s^2}\right)G^{a\mu}_\nu\phi^\mu, \quad (9)$$
where $p_b, p_s,$ and $p_g$ are the b-quark, s-quark and gluon momenta, respectively. From above we have the following $\phi$ energy spectrum

$$\frac{d\Gamma(b \to s\phi g)}{dE_\phi} = \frac{|a|^2\alpha_s}{32\pi^2m_b^2N_c^2N_c^2} \int_{t_{\text{min}}}^{t_{\text{max}}} dt \left[ \frac{1}{1+Y}(4+(1+Y)^2+(1-Y)^2\frac{1+\mu_s^2}{2\mu_\phi^2}) \right.$$  

$$+ \frac{2}{X^2(1+Y)}(1-2\mu_\phi^2-X-Y+XY-2\mu_\phi^2\frac{1-Y}{1+Y})(\frac{1-\mu_s^2}{\mu_\phi^2}-2\mu_\phi^2+1+\mu_s^2) \right], \quad (10)$$

where

$$X = \frac{s+t-m_s^2-m_\phi^2}{m_b^2}, \quad Y = \frac{s+t-m_s^2-m_\phi^2}{s-t-m_s^2+m_\phi^2},$$

$$s = m_b^2 + m_\phi^2 - 2m_b E_\phi, \quad \mu_{s,\phi} = \frac{m_{s,\phi}}{m_b},$$

$$t_{\text{max, min}} = \frac{(s-m_s^2)}{2s}((m_b^2-s-m_\phi^2) \pm \sqrt{(m_b^2-s-m_\phi^2)^2 - 4m_b^2s}) + m_\phi^2. \quad (11)$$

The energy distribution in Eq.(10) has the well-known infrared divergence due to the zero mass of the gluon. To regulate the infrared divergence, we assign an effective gluon mass of about $2m_\pi$ which represents the lowest invariant mass of the gluon. The $\phi$ energy distribution for $b \to s\phi g$ is shown in Figure 2. Here we have neglected the effect due to non-zero $\vec{p}_b$ discussed in the previous section which is small and approximated the $b \to s\phi g$ contribution to $B \to X_s\phi$ by Eq.(10). We find that the effect of $b \to s\phi g$ on $B \to X_s\phi$ is small because $BR(b \to s\phi g)/BR(b \to s\phi)$ is only about 3%. The total energy distribution is shown in Figure 3.

The spectrum of $\phi$ should approximate the spectrum that arises from the decays $B \to K\pi\phi, K\pi\pi\phi,$ etc. Of course the monoenergetic $\phi$’s that arise from two body modes like $B \to K\phi$ or $K^*\phi$ are included in an average sense. The specific two body modes are not expected to be more than 10% of the inclusive $X_s\phi$ production \[6\]. In this paper we have not discussed the $\phi$ energy spectrum from the decay of the dominant non-penguin processes which are expected to have a much softer spectrum since they always arise from decay of charmed states. We assume that this experimentally well known contribution has been substracted in the region of interest. If in addition to the selection criterion on $\phi$ discussed in this letter, $X_s$ will be experimentally shown to include an odd number of kaons, then the penguin process will be even more enhanced \[4\].
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FIG. 1. $E_\phi$ distribution for $b \to s\phi$ for $p_f = 0.3$ GeV.

FIG. 2. $E_\phi$ distribution for $b \to s\phi g$. 
FIG. 3. $E_\phi$ distribution for $b \rightarrow s\phi + s\phi g$. The solid, dotted and dashed lines are for $p_f = 0.39$ GeV, 0.30 GeV and 0.21 GeV, respectively.