PPS: Privacy-Preserving Strategyproof Social-Efficient Spectrum Auction Mechanisms

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Abstract—Many spectrum auction mechanisms have been proposed for spectrum allocation problem, and unfortunately, few of them protect the bid privacy of bidders and achieve good social efficiency. In this paper, we propose PPS, a Privacy Preserving Strategyproof spectrum auction framework. Then, we design two schemes based on PPS separately for 1) the Single-Unit Auction model (SUA), where only single channel to be sold in the spectrum market; and 2) the Multi-Unit Auction model (MUA), where the primary user subleases multi-unit channels to the secondary users and each of the secondary users wants to access multi-unit channels either. Since the social efficiency maximization problem is NP-hard in both auction models, we present allocation mechanisms with approximation factors of $(1 + \epsilon)$ and $32$ separately for SUA and MUA, and further judiciously design strategyproof auction mechanisms with privacy preserving based on them. Our extensive evaluations show that our mechanisms achieve good social efficiency and with low computation and communication overhead.

I. INTRODUCTION

The ever-increasing demand for limited radio spectrum resource poses a great challenge in spectrum allocation and usage [26]. Recent years, auction has been widely regarded as a preeminent way to tackle such a challenge because of its fairness and efficiency [14]. In general, bidders in spectrum auctions are the secondary users, while the auctioneer is a primary user in the single-sided spectrum auctions.

In recent years, many strategyproof auction mechanisms, in which bidding the true valuation is the dominant strategy of bidders, have been proposed for solving spectrum allocation issue. Unfortunately, the auctioneer is not always trustworthy. Once the true valuations of bidders are revealed to a corrupt auctioneer, he may abuse such information to improve his own advantage. Besides, the true valuation may divulge the profit of bidders, which is also a commercial secret for each bidder. Therefore, bid privacy preservation should be considered in spectrum auction design. However, only few studies (e.g., [11], [21]) were proposed to protect the bid privacy of bidders.

Allocating channels to the buyers who value them most will improve the social efficiency. There have been many studies devoted to maximizing the social efficiency while ensuring strategyproofness in spectrum auction mechanism design [17], [29], [10], [26], [33]. Unfortunately, none of these auction mechanisms provides any guarantee on bid privacy preservation.

In this paper, we consider the issue of designing strategyproof spectrum auction mechanism which maximizes the social efficiency while protecting the bid privacy of bidders. We propose a Privacy Preserving Strategyproof spectrum auction framework (PPS). Under PPS, we mainly study two models: 1) the Single-Unit Auction model (SUA) and 2) the Multi-Unit Auction model (MUA). In the SUA model, the auction mechanism design only focuses on single channel trading. Multichannel trading is supported in the case of MUA model. Since the maximization of social efficiency problem in both SUA and MUA are NP-hard, we design allocation mechanisms with approximation factors of $(1 + \epsilon)$ and $32$ separately for the SUA and the MUA. We show that the proposed approximation allocation mechanisms are bid-monotone, and further design strategyproof auction mechanisms based on them, which are denoted as PPS-SUA and PPS-MUA respectively. As the PPS-MUA only ensures the worst case performance, we further propose an improved mechanism, denoted by PPS-EMUA, to improve the social efficiency of PPS-MUA. We also show that PPS-EMUA is strategyproof and privacy-preserving.

It is not a trivial job to protect privacy of the true bid values of bidders in the auction mechanisms as auction relies on these bid values to make decision on allocation and payment computation. Notice that, for maximizing social efficiency and computing payment, we need to compute many various bid sums of conflict-free bidders in our allocation mechanisms. However, it is hard for the auctioneer or the bidders to compute these bid sums with privacy preserving since the auctioneer does not know any bidder’s true bid value. To address these challenges, we will first introduce an agent, which is a semi-trusted third party (such as FCC), different from auctioneer. The agent, together with the auctioneer, will execute the auction in PPS. In our design, bidders apply Paillier’s homomorphic encryption to encrypt the bids so agent can perform computation on the ciphertexts, agent then sends the results by adding random numbers and shuffling bidder IDs to auctioneer for making allocation decision, which provides privacy protection without affecting the correctness of the allocation. We will prove that neither the agent nor the auctioneer can infer any true bid value about the bidders without collusion. To the best of our knowledge, PPS is the first privacy preserving spectrum auction scheme that...
maximizes the social efficiency. Note that we did not focus on
protecting the location privacy of bidders in our mechanisms,
as previous schemes (e.g., \cite{17}) can be integrated into our
mechanisms.

The remainder of paper is organized as follows. In Section
\ref{sec:problem} we formulate the spectrum auction and present the frame-
work of PPS. Section \ref{sec:strategy} proposes a strategyproof spectrum
auction mechanism for solving the single-unit auction model. Section
\ref{sec:extension} further extends the auction model with consider-
ation of multiple trading model. Extensive simulation results are evaluated in Section \ref{sec:evaluation}. Section \ref{sec:related} discusses the
related literatures and section \ref{sec:conclusion} concludes the paper.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. Spectrum Auction Model

We model the procedure of secure spectrum allocation as a
sealed-bid auction, in which there is an auctioneer (a.k.a.
primary user), a set of bidders (a.k.a. secondary users) and an
agent. In each round of the auction, the auctioneer subleases
the access right of \( m \) channels to \( n \) bidders. The bidders
first encrypt their bids by using the encryption key of a
homomorphic encryption scheme (e.g., Paillier’s scheme) for
the auctioneer, and submit the encrypted bids to the agent
(not the auctioneer). Here, \( E(m) \) denotes the homomorphic
encryption of message \( m \). Then, the auctioneer and the agent
allocate the channels to the bidders via communicating with
each other. We assume that the agent is a semi-trusted party,
and will not collude with the auctioneer.

We use \( \mathcal{C} = \{c_1, ..., c_m\} \) to denote the set of channels,
and \( \mathcal{B} = \{1, ..., n\} \) to denote the set of bidders. Each bidder
\( i \in \mathcal{B} \) is described as \( i = \{L_i, N_i, b_i, v_i, p_i\} \), where \( L_i \)
is the geographical location of \( i \), \( N_i \) is the number of channels that
bidder \( i \) wants to buy, \( b_i, v_i \) and \( p_i \) separately denote the bid
value, true valuation and payment of \( i \) for all the channels that
he wants to buy. We assume that the interference radii of all
channels are the same, which are equal to \( \frac{1}{2} \) unit. Then, two
bidders \( i \) and \( j \) conflict with each other if the distance between
\( L_i \) and \( L_j \) is smaller than 1 unit. Bidders can share one channel iff
they are conflict free with each other.

In this paper, we study two spectrum auction models. The first
one is that there is only one channel in the spectrum
market, then \( m = 1 \) and \( N_i = 1 \) for each bidder. We call
this model the Single-Unit Auction model (SUA). The second
one is the Multi-Unit Auction model (MUA) which supports
multiple channels trading in the market. In MUA, each bidder
wants to access \( N_i \geq 1 \) channels rather than part of them.

B. Design Targets

Our work is to design social efficient strategyproof spectrum
auction mechanisms with bid privacy preservation. Firstly, we
will allocate channels to the bidders who value them most to
maximize the social efficiency. However, the optimal channel
allocation problem in SUA and MUA are all NP-hard. Thus,
we will design approximation mechanisms instead. Secondly,
our auction mechanisms should be strategyproof, which means
bidding truthfully is the dominant strategy for any bidders.

To achieve this, it is sufficient to show that our allocation
mechanism is bid-monotone, and always charges each winner
its critical value \( \bar{b} \). We say an allocation mechanism is
bid-monotone if bidder \( i \) wins the auction by bidding \( b_i \), he
will always win by bidding \( b_i' > b_i \). And the critical value of
each bidder \( i \) in a bid-monotone allocation mechanism is the
minimum bid that bidder \( i \) will win in the auction. The third
objective is to protect the privacy of the bid values of bidders.
To achieve privacy protection, we will apply homomorphic
encryption to encrypt the bid values using the public key of
auctioneer, and agent will perform the most of the computation
and send the intermediate results to the auctioneer. We will
show that both the auctioneer and the agent cannot get any
information about the true bid values of bidders as long as
they will not collude with each other.

C. A Spectrum Auction Framework with Privacy Preserving

The process of our spectrum auction mechanisms consists of
three steps: bidding, allocation and payment calculation. To
protect the bid values of bidders, we design a strategyproof
spectrum auction framework with privacy preserving, namely
PPS, which is shown in Algorithm \ref{alg:PPS}.

\begin{algorithm}
\caption{PPS: Privacy Preserving Strategyproof Spectrum
Auction Framework}

1: Each bidder \( i \) submits \( E(b_i), N_i \) and \( L_i \) to the agent,
where \( b_i \) is encrypted by using the encryption key of the
auctioneer;
2: The agent and the auctioneer run a bid-monotone al-
location mechanism while protecting the bid privacy
of bidders.
3: The agent and the auctioneer compute a critical value for
each winner with bid privacy preserving.
\end{algorithm}

III. A SINGLE-UNIT SCHEME

In this section, we will present a strategyproof spectrum
auction mechanism for SUA, denoted by PPS-SUA, which
maximizes the social efficiency and preserves the bid privacy.

A. Initialization and Bidding

Before running the auction, the auctioneer generates an
encryption key \( EK \) and a decryption key \( DK \) of Paillier’s
cryptosystem. Then, he announces \( EK \) as the public key, and
keeps \( DK \) in private. Each bidder \( i \) encrypts his bid \( b_i \) by
using \( EK \), and sends \( (E(b_i), L_i) \) to the agent. In the sending
procedure, each bidder keeps his encrypted bidding price as a
secret to the auctioneer.

B. Allocation Mechanism with Privacy Preserving

After receiving the encrypted bids from bidders, the auction-
earer and the agent allocate channels to bidders via communicat-
ing with each other. The goal of our allocation mechanism
is to maximize the social efficiency, which is equal to finding
a group of conflict-free bidders with highest bid sum, which is
a well-known NP-hard problem. To tackle this NP-hardness,
there is at least one disk each bidder is contained in a grid, which can be done easily. We propose a polynomial time approximation scheme (PTAS) based on shifting strategy. This provides an approximation factor of $(1 + \epsilon)$. For completeness of presentation, we first review this PTAS method.

In the PTAS, we first select a positive integer $k$, then, the plane is subdivided into several grids of size at $k \times k$ by a collection of vertical lines $x = r + ik$ and horizontal lines $y = s + jk$, where $0 \leq r, s \leq k - 1$. We call such a subdivision as $(r, s)$-shifting. Here we assume that the conflict radius of all the bidders, and where $OPT(r, s)$ is the set of all optimal solutions of the subdivided grids, and $w(OPT(r, s))$ is the weight of the optimal solution of $(r, s)$-shifting. It can be proven that there is at least one $(r, s)$-shifting, $0 \leq r, s \leq k - 1$, with

$$w(OPT(r, s)) \geq (1 - \frac{1}{k})^2 w(OPT(B))$$

where $OPT(B)$ is the maximum weighted independent set of all the bidders, and $w(OPT(B))$ is the weight of $OPT(B)$. For any given integer $k \geq 1$, there are $k^2$ kinds of different shiftings in total. We will choose the optimal solution of $(r, s)$-shifting’s that with the highest weight as our final approximation solution. Thus, we have a PTAS for optimal channel allocation problem, i.e. setting $k = \frac{1 + \sqrt{1 + 4 \epsilon}}{2}$.

Based on this PTAS we then present our channel allocation mechanism with privacy preserving. Observe that the bidders submit their bids to agent encrypted using the auctioneer’s public key. Following the PTAS protocol, we need to compute a maximum weighted independent set for each grid in the $(r, s)$-shifting, i.e., compare the weights of all independent sets. Clearly, the auctioneer should not access the encrypted bid of any bidder as he has the decryption key. In our protocol, the agent will compute $E(\sum_{i\in S} b_i)$ for each of the maximal independent set contained in a grid, which can be done easily as $E(b_i)$ is computed from homomorphic encryption. For any given grid $g_{r,s}^j$ of the $(r, s)$-shifting, let $D = \{d_{r,s}^1, \ldots, d_{r,s}^l\}$ be the set of maximal independent sets of bidders in $g_{r,s}^j$. We use $OPT(g_{r,s}^j)$ to denote the optimal solution in the grid $g_{r,s}^j$. Clearly $D$ has cardinality of at most $O(k^2)$ and can be enumerated in time $O(nO(k^2))$. In Algorithm 2 we present our method for finding the $OPT(g_{r,s}^j)$ for each subdivided grid $g_{r,s}^j$ with privacy preserving. To hide the true values of $w(d_{r,s}^i)$ (which may break privacy) from the auctioneer, the agent will mask them by using two random values $\delta_1$ and $\delta_2$ as $\delta_1 + \delta_2 \cdot w(d_{r,s}^i)$. Note that the range $[1, 2^{|\sigma_i}|]$ and $[1, 2^{|\sigma_i|}]$ for $\delta_1$ and $\delta_2$ are chosen based on the consideration of the correctness of modular operations: $\delta_1 + \delta_2 \cdot w(d_{r,s}^i)$ should be smaller than the modulo used in Paillier’s system.

Assume that the number of grids that subdivided by $(r, s)$-shifting is $N_{r,s}$, then the optimal solution of $(r, s)$-shifting is $OPT(r, s) = \bigcup_{j \leq N_{r,s}} d_{r,s}^j$. By sending the intermediate results to the auctioneer, the auctioneer can compare and find which independent set will be chosen for each subgrid. Observe that both the auctioneer and the agent will not know the bid values in the independent set. By using the optimal solution of each grid, the agent can calculate the encrypted value $E(w(OPT(r, s)))$, and allocate channels to bidders without leaking the true bid values of bidders. The allocation will be sent to the auctioneer. The details are described in Algorithm 3.

Algorithm 2 Computing the optimal solution for grid $g_{r,s}^j$

1. The agent randomly picks two integers $\delta_1 \in \mathbb{Z}_{2^{\gamma_1}}$, $\delta_2 \in \mathbb{Z}_{2^{\gamma_2}}$, computes and sends $\{E(\delta_1 + \delta_2 w(d_{r,s}^i))\}_{1 \leq i \leq z}$ to the auctioneer, where $E(\delta_1 + \delta_2 w(d_{r,s}^i)) = E(\delta_1)(\prod_{i \in d_{r,s}^i} E(b_i))^\delta_2$

2. The auctioneer decrypts $\{E(\delta_1 + \delta_2 w(d_{r,s}^i))\}_{0 \leq i \leq z}$, and sorts them in non-increasing order. Assume $w(d_{r,s}^i) \geq w(d_{r,s}^j) \geq \ldots \geq w(d_{r,s}^{k_z})$ where $d_{r,s}^{i}$ is the maximum independent set with rank $i$ in the sorted list.

3. The auctioneer sends $\{\sigma(i)\}_{1 \leq i \leq z}$ to the agent.

4. The agent chooses $d_{r,s}^{\sigma(i)}$ as the optimal solution of grid $g_{r,s}^j$.

Lemma 1: Our allocation mechanism for SUA is bid-monotone.

Proof: Without loss of generality, we assume that the bidder $i$ wins by bidding $b_i$ in grid $g_{r,s}^j$. Then, $\sigma_1(1) = r$, $\sigma_2(1) = s$ and bidder $i$ in $d_{r,s}^i$. It is not hard to get that the bidder $i$ is still in $d_{r,s}^i$, when he increases his bid to $b_i' > b_i$. Furthermore, the increased weight of other shiftings is no more than $(r, s)$-shifting when $i$ increases his bid, which indicates that $\sigma_1(1) = r$ and $\sigma_2(1) = s$ still hold. Thus, we can conclude that $i$ will always win by bidding $b_i' > b_i$.

C. Payment Calculation with Privacy Preserving

We have proved that our allocation mechanism is bid-monotone, which indicates that there exists a critical value
Algorithm 3 PTAS with bid privacy preserving

1: The agent randomly picks two integers $\delta_3 \in \mathbb{Z}_{2^{g_1}}, \delta_4 \in \mathbb{Z}_{2^{g_2}}$, computes and sends $E(\delta_3 + \delta_4 w(OPT(r,s)))$ for any $1 \leq r, s \leq k$ to the auctioneer, where $E(\delta_3 + \delta_4 w(OPT(r,s))) = E(\delta_3) \prod_{j \leq N_r,s} E(w(d_{r,s}(1,j)))^4$

2: The auctioneer decrypts and sorts the weights of the optimal solution of different shiftings in non-increasing order.

3: The agent computes $w(OPT(\sigma_1(1),\sigma_2(1))) \geq \ldots \geq w(OPT(\sigma_1(k^2),\sigma_2(k^2)))$ where $OPT(\sigma_1(i),\sigma_2(i))$ is the optimal solution of $(\sigma_1(i),\sigma_2(i))$-shifting with rank $i$ in the sorted list.

4: The agent chooses $OPT(\sigma_1(1),\sigma_2(1))$ as the final solution, and sends the allocation result to the auctioneer.

for each bidder. The bidder $i$ will win the auction by bidding a price which is higher than its critical value, otherwise, bidder $i$ will lose in the auction. To ensure the strategyproofness of our auction mechanism, we will compute the critical value for each winner as the final payment in the following.

Without loss of generality, we also assume that the bidder $i$ wins by bidding $b_i$ in grid $g_j^{r,s}$. We further assume that $d_{r,s}^{i,j}$ is the maximum independent set with highest weight which does not include bidder $i$, and $OPT(\sigma_1(f(i)),\sigma_2(f(i)))$ is the optimal solution of $(\sigma_1(f(i)),\sigma_2(f(i)))$-shifting which has the highest weight and does not include the bidder $i$. We will calculate the critical value of the winner $i$ based on the following considerations.

- The minimum bid price, denoted as $p_i^1$, ensures bidder $i$ win in grid $g_j^{r,s}$. Then, we can get that $p_i^1 = w(d_{r,s}^{i,j}) - w(d_{r,s}^{i,j}) + b_i$

- The minimum bid of bidder $i$ which makes $OPT(r,s)$ always with the highest weight among all the optimal solutions of shiftings including bidder $i$. We use $p_i^1 (p_i^2$ exists $iff f(i) > 2)$ to denote this minimum bid, and set $p_i^2 = w(OPT(\sigma_1(q),\sigma_2(q))) - w(d_{r,s}^{i,j}) + w(d_{r,s}^{i,j}) - w(OPT(r,s)) + b_i$, then $p_i^2 = \max\{p_i^1,\ldots,p_i^{2,f(i)-2}\}$

- The minimum bid of minimum $i$ that ensures $w(OPT(r,s)) \geq w(OPT(\sigma_1(f(i)),\sigma_2(f(i))))$, which is denoted by $p_i^3$. Then, we can get that $p_i^3 = w(OPT(\sigma_1(f(i)),\sigma_2(f(i)))) - w(OPT(r,s)) + b_i$

In conclusion, the critical value of bidder $i$ is $p_i = \max\{p_i^1,p_i^2,p_i^3,0\}$. Since the agent knows the order of all the maximum independent sets of each grid and the order of all the optimal solution of shiftings, he can compute the encrypted value of $p_i^1, p_i^2$ and $p_i^3$ by homomorphic operations, respectively. Then, our payment calculation mechanism with privacy preserving is depicted as follows:

1) The agent computes $E(p_i^1), E(p_i^{2,1}), \ldots, E(p_i^{2,f(i)-2}), E(p_i^3)$, and sends the results to the auctioneer.

2) The auctioneer decrypts the ciphertexts and sets the payment of winner $i$ as $p_i = \max\{p_i^1,p_i^{2,1},\ldots,p_i^{2,f(i)-2},p_i^3,0\}$

It is easy to prove the following theorems.

Theorem 2: PPS-SUA charges each winner its critical value and is strategyproof.

Theorem 3: The computation and communication cost of PPS-SUA are all $O(n^{k^2+1})$.

D. Privacy analysis of PPS-SUA

Theorem 4: PPS-SUA is bid privacy-preserving.

Proof: To confirm the bid privacy, we consider the view of agent and auctioneer, respectively.

During our auction mechanism for SUA, the agent can obtain nothing but the encrypted bids and the sorting results of the weight of each grid and each shifting. Based on the IND-CPA security of homomorphic cryptosystem, the agent cannot learn more information about the bid of any bidder.

The auctioneer holds the decryption key. Nevertheless, he has no direct access to the encrypted bids. While computing the optimal allocation and critical value of winner $i$, the auctioneer can receive the encrypted weight of maximal independent sets in each grid, weight of the optimal solution of each shifting, and $(p_i^1,p_i^{2,1},\ldots,p_i^{2,f(i)-2},p_i^3)$. From the weight of solutions in the grids or shiftings, the auctioneer cannot infer any bid, since they are encrypted by the agent and the auctioneer has no idea about which bidders are in these solutions, except the winning shifting. Consider $(p_i^1,p_i^{2,1},\ldots,p_i^{2,f(i)-2},p_i^3)$, auctioneer can construct the equation of them. However, the bid value of bidder $i$ can still be well preserved, as auctioneer does not know any value of the variables in these equations.

IV. A Multi-Unit Scheme

In this section, we propose a strategyproof auction mechanism for MUA, namely PPS-MUA, which maximizes the social efficiency and protects the bid privacy of bidders. Then, we design an extended version of PPS-SUA, namely PPS-EMUA, to improve the average performance of PPS-MUA.

A. Initialization and Bidding

The initialization and bidding procedure in MUA is similar as that in SUA, which can be referred in section III-A. At last, each bidder $i$ encrypts his bid $b_i$ by using the encryption key of the auctioneer, and only sends $(E(b_i), N_i, L_i)$ to the agent.

B. Allocation Mechanism with Privacy Preserving

Since SUA is a special case of MUA, the optimal allocation issue in MUA is also NP-hard. Thus, we will introduce a simple allocation mechanism which approximates the social efficiency. We first subdivide the plane into grids at size $2 \times 2$, and use the symbol $g'$ to denote the $i$-th $2 \times 2$ grid. It is obvious that there are four $1 \times 1$ grids in each $g'$. These four $1 \times 1$ grids can be categorized into four types as shown in Fig. 2(a). Let $g_r \in \{g_1, g_2, g_3, g_4\}$ be the $1 \times 1$ grid in $g'$ with type $r$. $g_r$ be the set of $1 \times 1$ grids with type $r$. We also assume that the conflict radius of each bidder
is $\frac{1}{2}$ and regard each bidder as a unit disk. Obviously, each bidder located in $g^l_r$ cannot conflict with the bidders located in $g^l_{r'}$ when $l \neq l'$. Let $OPT(g^l_r)$ be the optimal solution of allocation problem in $g^l_r$, $OPT(g_r)$ be the optimal solution of the allocation problem in $g_r$, then $OPT(g_r) = \bigcup_l OPT(g^l_r)$.

Note that we cannot get the optimal solution in each grid $g^l_r$. To tackle this, we further subdivide each $1 \times 1$ grid $g^l_r$ into four $\frac{1}{2} \times \frac{1}{2}$ sub-grids as shown in Fig. 2(b), which are denoted by $g^l_{r,1}$, $g^l_{r,2}$, $g^l_{r,3}$ and $g^l_{r,4}$, separately. Notice that all the bidders located in the same sub-grid $g^l_{r,s}$ conflict with each other. Thus, one channel can only be sold to one bidder in $g^l_{r,s}$.

The optimal allocation problem in each sub-grid $g^l_{r,s}$ can be reduced to a knapsack problem (KP). Although the KP is an NP-hard problem, there exists a PTAS [13], and a greedy allocation mechanism with approximation factor of 2 (the details can be referred to lines 3-5 in Algorithm 4). It is hard to design a privacy preserving version of the PTAS based on dynamic programming, thus, we design our allocation mechanism for MUA based on the greedy allocation mechanism in each sub-grid $g^l_{r,s}$. Assume that $APP(B)$, $APP(g_r)$, $APP(g^l_r)$ and $APP(g^l_{r,s})$ are the approximation solution of the allocation problem in the whole plane, $g_r$, $g^l_r$ and $g^l_{r,s}$, separately. We choose the $APP(g^l_{r,s})$ with biggest weight as the solution of grid $g^l_r$ and the $APP(g_r)$ with the biggest weight as our final solution $APP(B)$ (the details is depicted in Algorithm 4).

**Theorem 5:** Our auction mechanism for MUA has an approximation factor of 32.

**Proof:** Assume that $OPT(B)$ is the optimal solution of our original allocation problem, and $OPT_r(B) = \{i | i \in OPT(B) \text{ and } i \text{ is allocated in } g_r\}$. Then, we can get that

$$w(OPT(B)) = \sum_{1 \leq r \leq 4} w(OPT_r(B)) \leq \sum_{1 \leq r \leq 4} w(OPT(g_r)) \leq 4 \max_{1 \leq r \leq 4} \{w(OPT(g_r))\}$$

where $w(\cdot)$ is an operation to compute the weight of solutions. For each grid $g^l_r$, we can get that

$$w(OPT(g^l_r)) \leq \sum_{1 \leq s \leq 4} w(OPT(g^l_{r,s})) \leq 4 \max_{1 \leq s \leq 4} \{w(OPT(g^l_{r,s}))\} \leq 4 \max \{w(OPT(g^l_{r,s}))\}$$

Since we sort bidders in non-increasing order according to their per-unit bidding prices, so user $i$ has the $i$-th largest value in $\frac{1}{N_i}$ and $\sum_{i=0}^{k-1} N_i > m$, $\sum_{i=0}^{k-1} b_i > w(OPT(g^l_{r,s}))$. Our approximation allocation mechanism sets $APP(g^l_{r,s}) = \{1, 2, ..., k - 1\}$ if $\sum_{i=0}^{k-1} b_i \geq b_k$; otherwise, we set $APP(g^l_{r,s}) = \{k\}$. Thus, $OPT(g^l_{r,s}) \leq 2APP(g^l_{r,s})$. Because we choose the $APP(g^l_{r,s})$ with biggest weight as $APP(g^l_r)$, we can further get that $OPT(g^l_r) \leq 4 \max_{1 \leq r \leq 4} \{OPT(g^l_r)\} \leq 32APP(B)$.

**Algorithm 4** Channel allocation mechanism for MUA

1. for each sub-grid $g^l_{r,s}$ do
2. if The number of channels that all the bidders located in $g^l_{r,s}$ want to buy is larger than $m$ then
3. Sorting the bidders that located in $g^l_{r,s}$ in non-increasing order according to their per-unit bid values $\frac{1}{w_i}$, where $\sigma(i)$ is the bidder with $i$-th per-unit bid value in the sorted list;
4. Find the critical bidder $\sigma(k)$ in the sorted bidder list, which satisfies:

$$\sum_{i=1}^{k} N_{\sigma(i)} \leq m < \sum_{i=1}^{k+1} N_{\sigma(i)}$$

5. Set $APP(g^l_{r,s}) = \{\sigma(1), \sigma(2), ..., \sigma(k - 1)\}$ if $\sum_{i=1}^{k-1} b_{\sigma(i)} \geq b_{\sigma(k)}$; otherwise, set $APP(g^l_{r,s}) = \{\sigma(k)\}$;
6. else
7. Set $APP(g^l_{r,s})$ is all the bidders that located in $g^l_{r,s}$;
8. for each grid $g^l_r$ do
9. Set $s' = \arg \max \{w(APP(g^l_{r,s}))| 1 \leq s \leq 4\}$, where $w(\cdot)$ is an operation to compute the weight of solutions.
10. Set $APP(g^l_r) = APP(g^l_{r,s})$;
11. for $r = 1$ to 4 do
12. Set $APP(g_r) = \bigcup_l APP(g^l_r)$;
13. Set $r' = \arg \max \{w(APP(g_r))| 1 \leq r \leq 4\}$;
14. Return $APP(B) = APP(g_{r'})$ as the final solution.

In order to protect the true bid value of bidders, the agent confuses the ID of bidders by using a permutation $\pi : \mathbb{Z}^n \rightarrow \mathbb{Z}^n$ after receiving the encrypted bid of bidders. Then, the privacy preserving version of our approximation allocation mechanism is depicted in Algorithm 5.

**Lemma 6:** Our allocation mechanism for MUA is bid-monotone.

**Proof:** Assume bidder $i$ is located in grid $g^l_{r,s}$ and wins the auction by bidding $b_i$, then he must be in the solutions $APP(g^l_{r,s})$, $APP(g^l_r)$ and $APP(B)$ at the same time. Thus, we will check if the bidder $i$ still belongs to these solutions when he bids $b'_i > b_i$ in the following.

First, we consider the solution $APP(g^l_{r,s})$. Obviously, the rank of bidder $i$ will not decrease when bidder $i$ increases his bidding value. Thus, $b'_i$ is always larger than the sum bid of the top $k - 1$ bidders when $i = \sigma(k)$, which means $i$ will remain in $APP(g^l_{r,s})$ in this case. In another case, all the bidders with top $(k - 1)$ per-unit bid remains unchanged when $i$ bids.
b'_i > b_i, and thus their sum bid is still larger than the k-th bid. Thus, i will always win the auction when he increases his bid.

Then, we consider the solutions APP(g'_r,s) and APP(B). When i bids b'_i > b_i, the w(APP(g'_r,s)) will increase, and w(APP(g'_r,s)) will keep unchanged if s' ≠ s. Thus, APP(g'_r,s) still has the highest weight and will be selected as APP(g'_r). Similarly, APP(g_r) will be selected as the final allocation APP(B).

Bidder i will always win by bidding b'_i > b_i if he wins by bidding b_i, i.e., our allocation mechanism is bid-monotone.

Algorithm 5 Channel allocation mechanism for MUA with bid privacy

1: for each sub-grid g'_{r,s} do
2: if The number of channels that all the bidders located in g'_{r,s} want to buy is larger than m then
3: The agent randomly chooses two integers \( \delta_{r,1} \in \mathbb{Z}_{2^r}, \delta_{r,2} \in \mathbb{Z}_{2^r} \), computes and sends \((\pi(i), E(\delta_{r,1} b_i + \delta_{r,2}), N_i)\) to the auctioneer if i is located in g'_{r,s};
4: The auctioneer decrypts and sorts the per-unit bids of bidders in non-increasing order;
5: The auctioneer finds the critical bidder \( \sigma(k) \) in the sorted bidder list, and sends \((\{\sigma(i)\}_{i<k}, \sigma(k))\) to the agent;
6: The agent computes and sends \(E(\delta_{r,1} \sum_{i=1}^{k-1} \sigma(i) + \delta_{r,2}))\) to the auctioneer;
7: The auctioneer sends \(\sigma(i)\) to the agent if \(\sum_{i=1}^{k-1} b_{\sigma(i)} \geq b_{\sigma(k)}\); otherwise, he sends \(\sigma(k)\);
8: The agent sets APP(g'_{r,s}) includes all the bidders that the auctioneer sent to him;
9: else
10: The agent sets APP(g'_{r,s}) as all the bidders located in g'_{r,s};
11: for each grid g'_{r,s} do
12: The agent chooses two integers \( \delta_{r,3} \in \mathbb{Z}_{2^r}, \delta_{r,4} \in \mathbb{Z}_{2^r} \), computes \(\{s, E(\delta_{r,3} w(APP(g'_{r,s})) + \delta_{r,4})\}_{1 \leq s \leq 4}\) and sends them to the auctioneer.
13: The auctioneer decrypts the ciphertexts and finds \(s' = \arg\max\{w(APP(g'_{r,s})) | 1 \leq s \leq 4\}\). Then, he sends \(s'\) to the agent.
14: The agent sets APP(g'_{r,s}) = APP(g'_{r,s})
15: for r = 1 to 4 do
16: The agent sets APP(g_r) = \(\bigcup_i APP(g'_i)\);
17: The agent chooses two integers \( \delta_1 \in \mathbb{Z}_{2^r}, \delta_2 \in \mathbb{Z}_{2^r} \), computes \(\{r, E(\delta_1 w(APP(g_r)) + \delta_2)\}_{1 \leq s \leq 4}\) and sends them to the auctioneer.
18: The auctioneer decrypts the ciphertexts and finds \(r' = \arg\max\{w(APP(g_r)) | 1 \leq r \leq 4\}\). Then, he sends \(r'\) to the agent;
19: The agent sets APP(B) = APP(g_r), and sends APP(B) to the auctioneer as the final solution.

C. Payment Calculation with Privacy Preserving

We now consider the procedure of payment calculation for a winner i which is located in grid g'_{r,s}.

Since the bidder i wins the auction, we can conclude that: 1) \(i \in APP(g'_{r,s})\); 2) \(APP(g'_i) = APP(g'_{r,s})\); and 3) \(APP(B) = APP(g_r)\). We first consider the minimum bid value of bidder i, denoted by \(p_i\), with which the bidder i will be put in APP(g'_{r,s}). In the case that all the bidders located in g'_{r,s} win the auction, we set \(p_i = 0\); otherwise, we assume that \(i = \sigma(j)\) in the sorted bidder list of g'_{r,s} when i bids \(b_i\), then the process of \(p_i\) computation is shown in Algorithm 6.

Under the assumption that APP(g'_{r,s}) keeps unchanged, we suppose \(p_i^2\) is the minimum bid value of bidder i that makes \(APP(g'_i) = APP(g'_{r,s})\), \(p_i^3\) is the minimum bid value of bidder i that makes \(APP(B) = APP(g_r)\). Then, we have \(p_i^1 = \max\{w(APP(g'_{r,s})); s' ≠ s\} - w(APP(g'_{r,s})) + b_i\)
\(p_i^2 = \max\{w(APP(g_r)); r' ≠ r\} - w(APP(g_r)) + b_i\)

The critical value of bidder i is \(p_i = \max(p_i^1, p_i^2, p_i^3)\). Next we will show that we can compute the critical value for each winner without leaking the true bid value of bidders.

Algorithm 6 \(p_i^1\) computation for winner i in MUA

1: Set \(j = j + 1\);
2: Set \(b'_i = \frac{b_{\sigma(i)} N_i}{N_{\sigma(i)}}\);
3: Run lines 3 ~ 5 of Algorithm 4 to check if bidder i will win by bidding \(b'_i\);
4: if i wins by bidding \(b'_i\) then
5: Repeat steps 1 ~ 3 until i lose the auction;
6: if i is the k-th bidder when he bids \(b'_i\) then
7: Set \(p_i^1 = \max\{\sum_{q=1}^{k-1} b_{\sigma(q)}, b'_i\}\), where \(\sigma(q)\) is the bidder with \(q\)-th per-unit bid when i bids \(b'_i\);
8: else
9: Set \(p_i^1 = \max(b_{\sigma(k)} + b_i - \sum_{q=1}^{k-1} b_{\sigma(q)}, b'_i)\);

Since the agent can compute \(E(\delta_{r,1} b_i N_{\sigma(j)} + \delta_{r,2} N_{\sigma(i)})\) which is equal to \(E(\delta_{r,1} b_i N_i + \delta_{r,2} N_{\sigma(i)})\), the auctioneer can decrypt and compute the value of \(\delta_{r,1} b_i + \delta_{r,2}\). Thus, the auctioneer and agent can check if bidder i will win the auction by bidding \(b'_i\) as they did in lines 3 ~ 7 of Algorithm 5.

Further, the agent can get \(\max\{w(APP(g'_{r,s})); s' ≠ s\}\) and \(\max\{w(APP(g_r)); r' ≠ r\}\) via communicating with the auctioneer. Thus, the agent can choose two integers \(\delta_1 \in \mathbb{Z}_{2^r}, \delta_2 \in \mathbb{Z}_{2^r}\) and compute the ciphertexts of \(\delta_1 p_i + \delta_2, \delta_1 p_i^2 + \delta_2, \delta_1 p_i^3 + \delta_2\) through homomorphic operations, and sends them to the auctioneer. Then, the auctioneer decrypts these ciphertexts, sets \(\delta_1 p_i + \delta_2 = \max(\delta_1 p_i^1 + \delta_2, \delta_1 p_i^2 + \delta_2, \delta_1 p_i^3 + \delta_2)\) and sends \(\delta_1 p_i + \delta_2\) to the agent. After computing the payment \(p_i\) of each winner i, the agent sends them to the auctioneer.

From above analysis, we can conclude that:

Theorem 7: We charge each winner its critical value in PPS-MUA. PPS-MUA is strategyproofness.
D. Extended Auction Mechanism for MUA

We have designed a simple allocation mechanism for MUA, which provides an approximation factor of 32. However, PPS-MUA only chooses the solution of a $\frac{1}{2} \times \frac{1}{2}$ sub-grid as the final solution of a $2 \times 2$ grid, while dropping all the other bidders that located in other 15 sub-grids. Although the allocation in this way provides a guarantee for the worst case performance, the average performance may be relatively low. To address this issue, we extend our allocation mechanism by supplementing the solution with other bidders as shown in Algorithm 7.

Algorithm 7 Extended Allocation Mechanism PPS-EMUA

1: Run Algorithm 4 to allocate channels to bidders;
2: Sort all the bidders who lose in Algorithm 4 in non-increasing order according to their bid values.
3: for each loser $i$ in the sorted list do
4:    if we can allocate channels to $i$ without interfering with the existing winners then
5:        Set $i$ wins and allocate channels to him;

Lemma 8: The allocation mechanism PPS-EMUA presented in Algorithm 7 is bid-monotone.

Proof: Since we have proved that if the winner $i$ increases his bid in Algorithm 4 he will always win the auction. Here, we only need to concentrate on the winners that lose in Algorithm 4 but will win in the extended version. Suppose such a winner $i$ increases his bid to $b_i'$ which satisfies $b_i' > b_i$, there are two possible cases: 1) $i$ wins in Algorithm 4 and 2) $i$ remains lose in Algorithm 4. In the case that $i$ loses in Algorithm 4 the final allocation of Algorithm 4 is the same as the allocation when $i$ bids $b_i$. Thus, there is no new bidder whose bidding price is higher than $i$ in the sorted loser list of Algorithm 7 after the bidder $i$ increasing his bid. In addition to $i$ wins by bidding $b_i$, we can conclude that the bidder $i$ will also win the auction when he increases his bid.

As this new allocation mechanism is bid-monotone, there exists a critical value for each winner. We use $p_i'$ here to denote the minimum bid value of bidder $i$ with which $i$ will win in Algorithm 4 and $p_i''$ to denote the minimum bid value of winner $i$ with which $i$ will win in the sorted loser list. According to Algorithm 7, $p_i'$ is the critical value of bidder $i$ in Algorithm 4 and $p_i''$ should be smaller than $p_i'.

For each winner $i$, his critical value can be computed as follows:

- If $i$ wins in line 1 of Algorithm 4 and will lose as long as he bids $b_i' < p_i'$, his critical value is equal to $p_i'$.
- Otherwise, his critical value is equal to $p_i''$. Suppose $f(i)$ is the first bidder in the sorted loser list who loses the auction but will win as long as the bidder $i$'s bidding price is smaller than his, then $p_i'' = b_{f(i)}$ if $f(i)$ exits and $p_i'' = 0$ otherwise.

As the extended allocation mechanism is bid-monotone and we always charge each winner its critical value, we have

Theorem 9: PPS-EMUA is strategyproof and social efficient.

In the following, we will show that PPS-EMUA can be performed with privacy preserving. Due to the page limit, we will only briefly introduce our ideas. Algorithm 8 shows the allocation mechanism of PPS-EMUA with bid privacy.

Algorithm 8 PPS-EMUA: Privacy-Preserving Allocation Mechanism

1: The auctioneer and the agent run Algorithm 8 and protect the true bid value of bidders by using the method we have introduced previously. 2) The auctioneer and agent can check if bidder $i$ will lose as long as his bid is smaller than $p_i'$ by running Algorithm 8 and assuming $i$ loses in line 1 of Algorithm 7. 3) In the case that $i$ may win when he bids smaller than $p_i'$, the auctioneer sets $p_i'' = 0$ if $f(i)$ does not exist, and sets $p_i'' = \delta_1 b_{f(i)} + \delta_2$ if $f(i)$ exists. The auctioneer sends $\delta_1 p_i'' + \delta_2$ in the case that $p_i'$ is the critical value of bidder $i$, and $\delta_1 p_i'' + \delta_2$ in other case. 4) With the encrypted critical value, the agent can compute the payment of winner $i$. After obtaining all the payment of winners, the agent will send them to the auctioneer.

Theorem 10: The computation and communication cost are all $O(n^2)$ for PPS-MUA and PPS-EMUA.

E. Privacy Analysis

Theorem 11: PPS-MUA and PPS-EMUA are privacy-preserving for each bidder.

Proof: Here we only prove it for PPS-EMUA as PPS-MUA is a procedure of PPS-EMUA. We first consider the agent. Except the encrypted bids, the agent can only obtain some orders, such as the bidding price of the bidders in each sub-grid, during our auction mechanism of PPS-EMUA. In the process of payment calculation, the agent can get nothing but the auction outcomes and some new orders. Based on the IND-CPA security of homomorphic cryptosystem, the agent cannot learn more information about the bid of any bidder.

Although the auctioneer holds the decryption key, he has no direct access to the encrypted bids. While computing the allocation in each sub-grid $g_{r,s}^*$, the auctioneer can build $|g_{r,s}^*| + 1$ functions that with $|g_{r,s}^*|$ bids and two random numbers, where $|g_{r,s}^*|$ is the number of bidders that located in $g_{r,s}^*$. Since the number of variables is larger than the number of functions, the auctioneer cannot decrypt any true bid value of bidders. In the other parts of our auction mechanism, the auctioneer only receives the weight of solutions. Since the auctioneer has no idea about which bidders are in these solutions, he can also get nothing from them.
V. PERFORMANCE EVALUATIONS

A. Simulation Setup

In our simulations, the number of bidders varies from 50 to 300, and all the bidders are randomly distributed in a square area. The bidding price of each bidder is uniformly generated in $[0, 100]$. We use a 1024-bit length Paillier homomorphic encryption system in the simulation. Thus, we choose $\gamma_1 = 1007$ and $\gamma_2 = 1022$ to ensure the correctness of modular operations. For Multi-Unit Auction (MUA), we assume the channel demand of each bidder is randomly generated from 1 to 4, and there are 4 or 8 available channels in spectrum market.

We mainly study the social efficiency ratio, computation overhead and the communication overhead in our simulations. We define the social efficiency ratio as the ratio between the social efficiency of our approximation mechanism and the optimal one. Since agent and auctioneer are two central party in this paper, we evaluate the computation overhead of them in our design by recording the required processing time, and evaluate the communication overhead through calculating the size of essential information transferred in the auction. All the simulations are performed over 100 runs and the result is the averaged value.

B. Performance of the PPS

In this section, we mainly focus on the performance of social efficiency ratio, auction computation overhead, and communication overhead under different simulation settings.

We first study the social efficiency ratio of our mechanisms under SUA model and MUA model respectively. From Fig. 3(a) and Fig. 4(a) obviously, the social efficiency ratio decreases when the number of bidders increases. This is because the increasing number of bidders will incur a more fierce degree of competition. Therefore, the social efficiency ratio decreases slightly with the increasing number of bidders in both auction models. Fig. 3(a) also shows that the social efficiency ratio increases when $k$ increases, where $k$ is the size of a subdivided grid. From the theoretical analysis, we can learn that when $k$ increases, less unit-disk defined by bidders’ requests are thrown away by using the shifting method. Thus, the social efficiency ratio increases with the increase of parameter $k$. Of course, the performances of our proposed PPS-SUA is always better than the theoretical bound.

TABLE I: Communication Overhead under SUA model (KB)

| $k$ | Number of bidders |
|-----|-------------------|
| 10  | 50 124 233 333 428 521 611 |
| 20  | 50 231 416 601 799 1026 1273 |
| 30  | 50 327 603 926 1312 1779 2619 |

TABLE II: Communication Overhead under MUA model (KB)

| Channel Number | $k$ | 50 | 100 | 150 | 200 | 250 | 300 |
|----------------|-----|-----|-----|-----|-----|-----|-----|
| 4              | 50  | 33.5 | 61.9 | 87.5 | 110.8 | 132.2 | 153.0 |
| 8              | 50  | 34.2 | 63.7 | 90.7 | 117.2 | 140.6 | 164.1 |
| 12             | 50  | 34.4 | 63.8 | 91.1 | 116.7 | 142.0 | 165.1 |

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| 12             | 50  | 34.4 | 63.8 | 91.1 | 116.7 | 142.0 | 165.1 |
a secure spectrum auction to prevent the frauds of the insincere auctioneer. Unfortunately, none of the existing solutions with privacy preserving provides any performance guarantee, such as maximizing the social efficiency which is often NP-hard. Our mechanisms rely on privacy preserving comparison and polynomial evaluations \cite{13}, which is extensively studied topic in secure multi-party computation \cite{1, 5, 8, 28}.

\section{Conclusion}

In this paper, we focused on designing strategyproof auction mechanisms which maximize the social efficiency without leaking any true bid value of bidders, and proposed a framework of PPS for solving this issue. We designed privacy-preserving strategyproof auction mechanisms with approximation factors of \(1 + \epsilon\) and 32 separately for SUA and MUA. Our evaluation results demonstrated that both PPS-SUA and PPS-EMU A achieve good performance on social efficiency, while inducing only a small amount of computation and communication overhead. A future work is to design robust privacy-preserving strategyproof auction mechanisms without inexplicitly requiring the location of bidders. Another future work is to design privacy-preserving auction mechanisms by removing the dependency of third-party agent.

\section*{Acknowledgement}

The research of authors is partially supported by the National Natural Science Foundation of China (NSFC) under Grant No. 61202028, No. 61170216, No. 61228202, and NSFC CNS-0832120, NSF CNS-1035894, NSF ECCS-1247944. Specialized Research Fund for the Doctoral Program of Higher Education (SRFDP) under Grant No. 20123201120010.

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