Application of hyper-chaotic Lorenz system for data transmission

A V Kondrashov, M S Grebnev, A B Ustinov and V V Perepelovskii
Department of physical electronics and technology, St. Petersburg Electrotechnical University, St. Petersburg, Russia

e-mail: avkondrashov@etu.ru

Abstract. A novel method of digital data transmission is developed and experimentally investigated. The main idea of the method is to convert digital data into a chaotic spectrum, mix it with a background signal and transmit to receive using TCP/IP protocol. The hyper-chaotic Lorenz system of differential equations is used as a source for carriers of the chaotic spectrum. Chaotic synchronization is not used in the method, that allows to improve stability of data transmission. Human voice is used as the background signal. Therefore, the method allows one to use single communication channel for transmission of different signals without losses. Numerical investigations approved that the chaotic signal transformation does not change chaotic nature of the Lorenz system. Thus, developed method demonstrates high privacy level.

1. Introduction
Dynamic chaos attracts special interest of scientists and engineers because it could be used in development of the novel communication systems [1], radar systems [2-5], random number generators [6,7] and in radio-frequency illumination sources [8]. The main advantages of chaos-based telecommunication systems are high level of privacy and information capacity. However, such systems have various disadvantages.

Chaotic synchronization is the main effect which is used for development of telecommunication systems based on the dynamic chaos. One of the first chaos based telecommunication methods was the chaotic cloaking [1], which allows to transmit analogue signals. Digital signals could be transmitted by the chaos shift keying method or by modulation of chaos generator parameters [9-11]. But these methods are highly sensitive to communication circuit noises. Another disadvantage is the necessity to construct two identical generators for the receiver and transmitter. Solving of this problem is usually quite complicate. In addition, there are numerous methods allowing to reconstruct parameters of the transmitter chaos generator using a signal in a communication circuit, and decode information data. Mentioned problems could be solved by using general synchronization for development of the telecommunication system [12,13]. But in this case the process of data decoding in the receiver becomes more complicated. Search of the new methods which allows to solve these problems is of a great interest.

In this paper we demonstrate a new method of digital data transmission based on the dynamic chaos. In contrast to the existing methods, dynamic chaos is used neither as a carrier nor as a cloaking signal.

2. Operating principals
Experimental prototypes of the transmitter and receiver are made in LabView Software. Any type of a computer network based on the TCP/IP protocol could be used as a communications circuit. Using of
the TCP/IP protocol allows to exclude an influence of noise in communication circuit. Both the transmitter and receiver are based on the two hyper-chaotic systems. The modified Lorenz chaotic system (1) is used as the hyper-chaotic system [15]:

\[
\begin{align*}
\dot{x}_1 &= \sigma(x_2 - x_1), \\
\dot{x}_2 &= r x_1 + x_2 - x_1 x_3 - x_4, \\
\dot{x}_3 &= x_1 x_2 - d x_3, \\
\dot{x}_4 &= k x_2 x_3,
\end{align*}
\]

Values of \(\sigma, r, d,\) and \(k\) in the model are 10, 28, 2.67, and 0.1, correspondingly. The Euler method is used for solving the chaotic system during a transmission process.

The transmission process consists of two parts. The first part is an onset of synchronization of the chaotic systems of the transmitter and receiver. In this work, synchronization achieves through the onset of the same initial conditions for both couples of the Lorenz systems. The synchronization is achieved by solving of the transmitter and receiver chaotic systems in the opposite directions of time. In the beginning of the synchronization process initial conditions are different in the transmitter and receiver. Eight values of variables \(x_n\) are determined on each iteration of the solving process. The results obtained in the transmitter are sent to the receiver. The results obtained in the receiver are sent to the transmitter. Both the receiver and transmitter simultaneously compare the obtained and received results. If they are the same, then these results become the new initial conditions for the second part of the data transmission process.

A general idea of the second part of the transmission process is to mix an information data signal and an analogue signal in the transmitter and decompose them after receiving. In other words, a transmitter circuit should work with two signal sources: a continuous analogue signal source and digital information data source. A human voice is used as the analogue signal in the experimental investigation. Binary ASCII-characters are used as information data. Information data is transmitted step-by-step. Figure 1 shows a block-diagram of a single step.

![Figure 1. Transmitter algorithm block-diagram.](image)

In the beginning of the transmission process, the two signals come to a transmitter input. One is information data consisting of eight bits \(b_n\), where \(n\) is an index of a bit in the transmitted character. Another signal is a short record of the human voice, which could be described as a function \(A(t)\). A transmitter model makes the Fourier transformation of the analogue signal obtaining its frequency spectrum \(A(f)\). Next, transmitter model makes several iterations of solving of the hyper-chaotic systems to determine eight unique values of the variables \(x_n\). Then model scales the obtained values of \(x_n\) in the way that the scaled values \(f_n\) get into the analogue signal frequency range. After scaling, amplitudes of Fourier-components \(A(f_n)\) change their values according to bit values. In doing so, 1-bit Fourier amplitude is found as an average value of the nearest frequency amplitudes. An amplitude corresponding
to 0-bit is found by multiplication of 1-bit amplitude by some coefficient. Therefore, one obtains a significantly transformed analogue signal spectrum which is transmitted to the receiver. All following characters of information data are transmitted in the same way but the values of $f_n$ are changed chaotically during the transmission.

Figure 2 represents the examples of human voice spectra before the information data mixing (see figure 2(a)) and after it (see figure 2(b)). The values of $f_n$ in this example are 104 Hz, 150 Hz, 210 Hz, 275 Hz, 300 Hz, 365 Hz, 410 Hz, 450 Hz. One can see that the spectra are almost the same. It is worth mentioning that a repetition of some word or sentence by the same person changes the voice spectrum more significant. Therefore, the addition of informational data has no noticeable effect on human voice.

![Figure 2](image_url)

**Figure 2.** Fourier spectra of human voice spectra before information data mixing (a) and after it (b).

Decomposition of information data in the receiver goes in a similar way. The receiver determines the same values of $x_n$ and $f_n$ as the transmitter. After receiving of the incoming signal, it measures the amplitudes of $A(f_n)$, compares them with an average value, and determines bits’ value. Thus, the described method allows one to transmit several signals in the same transmission channel without interfering between them.

3. **Analysis of chaotic dynamics stability**

Above described algorithm of signals mixing and decomposing assumes that the transformations of $x_n$ do not change their chaotic nature. Therefore, it is necessary to obtain long realizations of the hyper-chaotic Lorenz system, then to transform them according to above described algorithm, and to analyze the obtained solutions. Note that a broadband spectrum is a special property of the chaotic signal. Figure 3 shows the Fourier-spectra of $x_i(t)$ component before transformation, after exclusion of duplicated values of $x_i$, and after scaling of $x_i$. One can see that the form of the spectra changes. An influence of exclusion is similar to frequency filtration. Due to this fact, the lower frequencies are suppressed (see figure 3(b)). The scaling procedure is equivalent to modulation of some carrier by chaotic signal; therefore, the spectrum is shifted to a higher frequency range, which is closer to a human voice spectrum (see figure 3(c)). However, all three spectra are broadband and correspond to the chaotic signals.

![Figure 3](image_url)

**Figure 3.** Fourier spectra of $x_i$ variable before transformation (a), after exclusion of duplicated values (b), and after scaling (c).
Analysis of the hyper-chaotic attractor of the Lorenz system should be done for a more accurate investigation. The four differential equations allow to construct the phase portrait of the system in the four-dimension phase space $x_1, x_2, x_3, x_4$. Figure 4 demonstrates three 3D projections of the hyper chaotic phase portraits before transformation, after exclusion, and after scaling. One can see that the topologies of all phase portraits are similar. The unmodified phase portrait consists of two branches connected to each other. An exclusion of the duplicates leads only to reduction of density of phase portrait points, but keeps the form of the phase portrait. Such a transformation does not lead to noticeable changes of dynamics and stays chaotic. After scaling of the variables values, one of the phase portrait branches folded with the other. The values of variables also shifted into the frequency range of the human voice. This linear transformation does not affect noticeable on the dynamic regime.

As is known, the values of the fractal dimension and Lyapunov exponent numerically characterize dynamics and allow one to determine whether it is chaotic or periodic one.

![Figure 4. 3D projections of hyper chaotic Lorenz system before transformation (a) after exclusion of duplicated values (b) and after scaling (c), after exclusion and scaling (d).](image)

An estimation of the fractal dimension was done using the standard algorithm of Grassberger - Procaccia [16]. The method developed in paper [17] was used for calculation of the Lyapunov exponents. It turned out that a fractal dimension of the hyper-chaotic Lorenz system is 3.1. an exclusion of the duplicates does not affect this value. The scaling process reduces the fractal dimension down to 1.7. This reduction is caused by the folding of two branches. The values of the Lyapunov exponents after all transformations are positive. The results of calculations approve that the dynamics of the hyper-chaotic Lorenz system remain chaotic. Therefore, the values of $f_n$ change aperiodically during transmission and never repeat during one transmission cycle, but remain deterministic. Onset of the same initial conditions allows keeping synchronization of the chaotic systems without coupling.

4. Conclusion
In conclusion, the novel algorithm of the data transmission using digital networks was obtained. The developed methods are based on the hyper-chaotic Lorenz system of differential equation, but do not use the chaotic synchronization, that solves the problems described in the introduction. The numerical
investigation demonstrates that all transformations of the chaotic signal do not change its deterministic nature. Therefore, the transmitter and receiver work synchronously.

Acknowledgments
The work on the development of the experimental prototypes of transmitter and receiver was supported by Ministry of Education and Science of Russian Federation (Project "Goszadanie"). The work on numerical analysis of chaotic dynamics stability was supported by a grant of the President of the Russian Federation for young scientists and PhDs MK-2531.2019.8.

References
[1] Eisencraft M, Attux R and Suyama R 2017 Chaotic Signals in Digital Communications (Boca Raton: CRC Press)
[2] Flores B C, Solis E A and Thomas G 2002 International Society for Optics and Photonics 4727 100-11
[3] Ashtari A, Thomas G, Garces H and Flores B C 2007 International Waveform Diversity and Design Conf. (Pisa) vol 1 pp 353-57
[4] Liu Z, Zhu X, Hu W and Jiang F 2007 International Journal of Bifurcation and Chaos 17 1735-39
[5] Lin F Y and Liu J M 2004 IEEE J. Quantum Electron 40 815-20
[6] Argyris A, Deligiannidis S, Pikasis E, Bogris A and Syvridis D 2010 Optics express 18 18763-68
[7] Akgul A and Li C, Pehlivan I 2017 J. Circuit syst. comp. 26 1750190
[8] Dmitriev A S and Efremova E V 2017 Tech. Phys. Lett. 43 42-45
[9] Dedieu H, Kennedy M P and Hasler M 1993 IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing 40 634-42
[10] Yang T and Chua L O 1996 IEEE Trans. Circuits Syst.: I Regular Papers 43 817–19
[11] Cuomo K and Oppenheim A US Patent No. 5291555 date of patent 1.03.1994
[12] Terry J R and VanWiggeren G D 2001 Chaos, Solitons and Fractals 12 145-52
[13] Koronovskii A A, Moskalenko O I and Hramov A E 2009 Advances in Physical Sciences 52 1213–39
[14] Cuomo K M, Oppenheim A V and Strogatz S H 1993 IEEE Trans. Circuits Syst. II 40 626–33
[15] Gao M, Chen G, Chen Z and Cang S 2007 Physics Letters A 361 78-86
[16] Grassberger P and Procaccia I 1983 Phys. Rev. Lett. 50 346-48
[17] Wolf A, Swift Q, Swinney H L and Vastano J A 1985 Physica D: Nonlinear Phenomena 16 285-17