Neural forecasting: Introduction and literature overview

Konstantinos Benidis, Syama Sundar Rangapuram, Valentin Flunkert, Bernie Wang, Danielle Maddix, Caner Turkmen, Jan Gasthaus, Michael Bohlke-Schneider, David Salinas, Lorenzo Stella, Laurent Callot, Tim Januschowski

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Abstract

Neural network based forecasting methods have become ubiquitous in large-scale industrial forecasting applications over the last years. As the prevalence of neural network based solutions among the best entries in the recent M4 competition shows, the recent popularity of neural forecasting methods is not limited to industry and has also reached academia. This article aims at providing an introduction and an overview of some of the advances that have permitted the resurgence of neural networks in machine learning. Building on these foundations, the article then gives an overview of the recent literature on neural networks for forecasting and applications.

1 Introduction

Over 20 years ago, Zhang et al. [203] reviewed the state-of-the-art in forecasting with artificial neural networks in this very journal. Since this publication, forecasting with neural networks (NNs) has indeed become the state-of-the-art for many practical tasks.

*Email addresses: {kbenidis, rangapur, flunkert, yuyawang, dmmaddix, atturkm, gasthaus, bohlikem, dsalina, stellalo, lcallot, tjnsch}@amazon.com
This is particularly true for the class of operational forecasting problems, a class of problems in which forecasts are used as inputs to automated decision making systems at scales and frequencies that do not permit systematic manual auditing [92]. Examples include forecasting the demand for items sold by large online retailers [192, 162, 142, 11], traffic [117, 133, 126] or energy [52, 173, 124]. Industrial applications have been front-runners for the use of NNs in forecasting, with academia being more reserved at first [136]. However, the undeniable success of NNs [172] in competitions such as the M4 [137] have led to a shift in thinking in the academic forecasting community.

Neural networks have a long history, starting in 1957 with Rosenblatt [156] and in 1964 with [90] for forecasting specifically. They have long entered the canon of standard techniques for forecasting and as such are a natural part of good forecasting textbooks (e.g., [92]). Until recently, however, results obtained with NNs for forecasting were mostly mixed as reflected in the widely cited review by Zhang et al. [203].

The purpose of this paper article is to explain the reasons behind the recent success of NNs and why we believe that this success will be sustained. We proceed in four steps. In section 3 we review popular NN architectures and discuss recent advances that transformed NNs to a state-of-the-art solution for a wide range of applications. We then provide an overview of the literature on neural forecasting models (Section 4) with a particular focus on work following the deep learning revolution started in 2012 [111]. Third, we provide an overview of the range of applications of neural forecasting methods (Section 5), and finally we speculate on potentially fruitful areas for future research (Section 6). We start with a brief overview on the history of NNs (Section 2).

2 A brief history of neural networks

The origins of NNs trace back in the 1950’s where Rosenblatt [156] conceived the idea of the perceptron, a simple mathematical model of how the neurons in our brains operate. In the decades that followed, various extensions of the

1Kaggle competitions such as [https://www.kaggle.com/c/web-traffic-time-series-forecasting](https://www.kaggle.com/c/web-traffic-time-series-forecasting) foreshadowed the results of the M4 competition.

2NNs were invented much before other well-known ML techniques such as Support Vector Machines [23] or Random Forests [81].
perceptron and several training rules were explored \cite{195}. Since Rosenblatt’s seminal work up to the present day interest in NNs has oscillated heavily, with upsurges in attention from the scientific community attributable to major breakthroughs.

For example, Rumelhart et al. \cite{158, 159} popularized the training of multilayer perceptrons (MLPs) using back-propagation to tackle complex learning problems. Significant advances were made in the years following these publications, among which: the proof that MLPs are universal function approximators \cite{88}, the first widely commercial use of NNs in handwritten zip code recognition \cite{120}, the use of convolutional NNs (CNNs) \cite{119} achieving state-of-the-art results, and Long Short Term Memory (LSTM) \cite{83} cells that address the issue of recurrent NNs’ (RNNs) training, just to name a few.

Despite these advances, NNs remained hard to train and difficult to work with. Methods such as Support Vector Machines (SVMs) \cite{23} and Random Forests (RFs) \cite{81} were developed in the 1990s. These methods proved to be highly effective (Lecun et al. \cite{121} found that SVMs were as good as the best designed NNs available at the time) and were supported by attractive mathematical theory. This shifted the interest of researchers away from NNs.

The breakthrough that marked the dawn of the deep learning era came in 2006 when Hinton et al. \cite{79} showed that it was possible to train NNs with a large number of layers (deep) if the weights are initialized appropriately rather than randomly. The key insight was to start by pre-training each layer in an unsupervised way and then proceed with standard supervised training, using the weights of the pre-trained layers as initial values. Glorot and Bengio \cite{65} proposed an efficient weight initialization scheme, commonly known as Xavier initialization, that can be used directly without unsupervised pre-training and has become the standard in deep learning. Glorot and Bengio \cite{65} also showed that the choice of the non-linear activation function in a NN has a big impact on training and performance. This spurred new research dedicated to finding suitable activation functions and resulting in the well known rectified linear unit (ReLU) \cite{98, 143, 66}.

Deep learning emerged as the result of the combination of the key insights on weight initialization and activation functions discussed above as well as effective optimization algorithms such as \cite{103} with well-known concepts such as back-propagation, MLPs, CNNs and RNNs. These breakthroughs, along with the explosion of available data and computational power in the last decade, led to remarkable results in multiple areas \cite{47, 111, 50, 186, 172} in which deep learning models are now unrivalled.
In parallel to these scientific advances, elegant general-purpose open-source frameworks [1, 149, 33] and application-specific toolkits [78, 48, 13, 4] are available to researchers and practitioners alike. These frameworks have shortened the cycle between scientific breakthroughs and widespread application of new discoveries while they simplified the use of deep learning models. As a result, the literature on deep learning is exploding (e.g., more than one thousand articles at NeurIPS 2018 and 2019).

The explosion of deep learning has also had an impact on time series forecasting [116]. New models and architectures specifically designed for forecasting tasks are being proposed, taking advantage of deep learning to supercharge classical forecasting models or developing entirely novel approaches. These new models deliver significant performance improvement compared to the restrictive single layer MLP architecture that was dominant (mainly due to practical restrictions) in previous decades [203] and are becoming the state-of-the-art in time series forecasting applications [137]. One aim of this article is to review and popularize the recent developments in forecasting driven by machine learning for the readers of the IJF and the wider forecasting community.

3 An overview of modern neural networks

In this section we review the main neural network structures that are the building blocks of all contemporary architectures. We then present current training techniques and highlight the advances that made the training of large NNs possible. Finally, we discuss existing software frameworks and application-domain specific toolkits for creating and training NNs.

While NN architectures for modern applications have increased in complexity, they continue to be composed of combinations of basic structures such as MLPs, RNNs and CNNs that are well-known and have been explored for decades (a significant exception is the use of the ReLU). The term modern NNs refers to techniques such as principled weight initialization, careful activation function selection, and learning rate optimization that have allowed for the exploration and efficient training of increasingly complex architectures. These new architectures have encountered great success in complicated modeling tasks which resulted in the popularity that NNs now enjoy.
3.1 Architectures

Neural networks are compositions of differentiable black-box functions formed from simple building blocks in order to learn an approximation of some unknown function from data. The architectures of neural networks and their building blocks are loosely inspired from biological neural networks. A neural network is a directed acyclic graph consisting of interconnected nodes. The connections, or edges, between the nodes contain weights (also called parameters) that are learned from the data. Here we briefly review the most widely used neural network architectures.

3.1.1 Multilayer perceptrons (MLP)

Multilayer perceptrons (MLPs) or, alternatively and equivalently feed-forward neural networks, is the most basic form of artificial NNs. The name feed-forward comes from the fact that the inputs are fed forward through the network to outputs in one direction.

The simplest instantiation of an MLP is a model that consists of a single layer. A layer is a set of nodes which apply an affine transformation followed by a nonlinear activation (see Section 3.2.1) on the common among all nodes– set of inputs they receive. However, the weights of each node are different which allows them to represent possibly a different function. The affine transformation of each node is learnable, in the sense that the weights of the transformation are learned during training. For more clarity, in Figure 1 we illustrate the simple structure of a single node. In the case of the single-layer perceptron the only layer is called the output layer and each of its nodes...
is fully connected to all input features. Note that a fully connected layer is traditionally called *dense* layer and the input features are denoted as the *input* layer (although not a layer in the strict sense). Figure 2a illustrates the structure of a single layer perceptron with 1 output node.

A nonlinear activation function at the output nodes is typically used to constrain the output to an appropriate target domain, while the number of nodes in the output layer depends on the particular learning task. For example, in the case of regression and binary classification, the output layer consists of a single node that outputs a real value and a probability, respectively. A logistic regression model can be seen as a special case of a single layer perceptron with one output node that applies the sigmoid function (see Section 3.2.1) as a nonlinear activation to produce probabilistic outputs. Learning the parameters of a Gaussian distribution requires two outputs: one for the mean defined on the real line, and one for the variance using an activation function constraining the parameter to be strictly positive. For more general learning problems, like multi-class classification and language translation, the number of output nodes can be much larger.

In the more general case of multi-layer perceptrons, the layers are stacked on top of each other in order to learn more complex nonlinear representations of the data. The intermediate layers (between the input and output layers)
are called *hidden* layers. The nodes in each layer of the network are fully connected to all the nodes in the previous layer. The output of the last hidden layer can be seen as some nonlinear feature representation obtained from the inputs of the network. The output layer, which is essentially a single layer perceptron, then learns a mapping from these nonlinear features to the actual target. Learning with MLPs, and more generally with NNs, can be thought of as the process of learning a nonlinear feature map of the inputs as well as learning the relationship between this feature map and the actual target. Figure 2b illustrates the structure of an MLP with two hidden layers.

One of the main limitations of MLPs is that they do not exploit the structure often present in the data in applications such as computer vision, natural language processing and time series forecasting. Moreover, the number of inputs and outputs is fixed making them inapplicable to problems with varying input and output sizes as in time series forecasting. In the following sections, we discuss more complex architectures that overcome these limitations, for which MLPs are often used as the basic building blocks.

### 3.1.2 Convolutional neural networks (CNN)

Convolutional neural networks \[\text{[118]}\] (CNN or ConvNet, for short) are a special class of NNs that are designed for applications where inputs have a known ordinal structure such as images and time series \[\text{[70]}\]. CNNs are *locally* connected NNs that use *convolutional* layers to exploits the structure present in the input data.

A convolutional layer applies a convolution function to smaller neighborhoods of the input data. Convolution here refers to the process of computing moving weighted sum by sliding the so-called *filter* or *kernel* over different parts of the input data. The size of the neighborhood as well as how the filter is slid across the input are part of the hyper-parameters of the model. A nonlinear activation, typically ReLU (see Section 3.2.1), is then applied to the output of the convolution operation after adding a bias term.

The filter contains weights to be learned. The aim is that these weights are adjusted in such a way that the filter extracts relevant features from the raw input data. For instance in the case of image processing applications, the filter could learn to detect edges in the given images.

Since the precise location of the feature being extracted in the input data is not relevant, the same filter is used to convolve different parts of the input to extract a single feature. This sharing of weights in the filter drastically
reduces the number of free parameters compared to the dense layers of MLPs and results in better generalization performance, especially in the case of image data where the number of inputs is very large. Moreover, since the given input data might have various useful features relevant for the task at hand, more than one filter is typically learned in a convolutional layer.

In addition to convolutional layers, CNNs also use a pooling layer to reduce the size of the feature representation as well as to make the features extracted from the convolutional layer more robust. For example, a commonly used max-pooling layer, which is applied to the output of convolutional layer, extracts the maximum value of the features in a given neighborhood. Similarly to the convolution operation, the pooling operation is applied to smaller neighborhoods by sliding the corresponding filter over the input. A pooling layer, however, does not have any learnable weights and hence both the convolution and the pooling layer are counted as one layer in CNNs.

One can further extend this notion of feature extraction by stacking several convolutional layers on top of each other which corresponds to combining low level features extracted in previous layers in order to derive higher order features. Such hierarchical extraction of features have made CNNs hugely successful in many image processing and computer vision applications [111].

3.1.3 Recurrent neural networks (RNN) and variants

Recurrent neural networks (RNNs) are neural networks specifically designed to handle sequential data that arise in applications such as time series, natural language processing and speech recognition.

Classical MLPs can be adapted to address the sequential nature of the data by treating time as an explicit part of the input. However, such an approach has inherent difficulties, namely the inability to process sequences of varying length and to detect time invariant patterns in the data [57]. A more direct approach is to use recurrent connections that connect the neural networks hidden units back to themselves with a time delay [99, 100].

Since hidden units learn some kind of feature representations of the raw input, feeding hidden units back to themselves in each time step can be interpreted as providing the network with a dynamic memory. A simple RNN was proposed by [57] based on this idea where, at each time step $t$, the network receives external input for time $t$ and the output of the hidden units from the previous time step $t - 1$. One crucial detail here is that the same network is used for all time steps; i.e., the weights of the network are shared.
across time steps. Figure 3 illustrates the general structure of an RNN.

This weight-sharing idea is similar to that of CNNs where the same filter is used across different parts of the input. This allows the RNNs to handle sequences of varying length during training and, more importantly, generalize to sequence lengths not seen during training.

Although RNNs have been widely used in practice, it turns out that training them is quite difficult given that they are typically applied to very long sequences of data. A common issue while training very deep neural networks by gradient-based methods using back-propagation (see Section 3.2) is that of vanishing or exploding gradients [147] which renders learning impossible.

Long short-term memory networks (LSTM) were proposed [83] to address this problem. Instead of using a simple network at each time step, LSTMs use a more complicated architecture composed of a cell and gates that control the flow of input to the cell as well as decide on what information should be kept inside the cell and what should be propagated to the next time step. The cell has a memory state which is propagated across time steps along with the output of the LSTM unit, which is itself a function of the cell state. Unlike the output of the LSTM unit, the cell state undergoes minimal changes across time steps thus the derivative with respect to the cell state does not decay or grow exponentially [14]. Consequently, there is at least one path where the gradient does not vanish or explode making LSTMs suitable for processing long sequences. LSTMs have seen huge success in a wide range of applications such as forecasting, machine translation, speech processing, and handwriting recognition. There are several variants of LSTMs that are widely used in practice. Gated recurrent units (GRU) [36] are a simplification of LSTMs that do not use a separate memory cell and consequently they are computationally more efficient while still being comparable to LSTMs [40].

3.2 Training a neural network: old and new

Stochastic gradient descent is the current default method to train a neural network, using back-propagation to compute the gradients.

Back-propagation is effectively a recursive application of the chain rule along the network’s computational graph in order to compute the gradients with respect to the parameters. Using a gradient-based optimization algorithm, the network’s weights are updated to decrease the loss until the network converges to a state that cannot improve further.
Figure 3: Structure of an RNN. At each time step $t$ the network receives an external input $x_t$ and the output of the hidden units from the previous time step $h_{t-1}$. The internal state of the network is updated to $h_t$ that is going to play the role of the previous state in the next time step $t + 1$. Finally, the network outputs $y_t$ which is a function of $x_t$ and $h_t$.

Back-propagation was not always the standard way to train NNs; many other training rules have been proposed in the early days [195]. Back-propagation was derived by multiple researchers in the early 60’s, implemented to run on computers in 1970 [128], and proposed as a training method for NNs a few years later [193]. However, it suffered from a lack of academic interest and it was not until 1986 that this approach was popularized by Rumelhart et al. [158, 159] and became the standard approach for training NNs.

Despite the efficiency of back-propagation, several issues kept NNs from achieving their full potential. In particular, the issue of vanishing or exploding gradients [15, 82] that appears after chaining multiple gradients in large architectures, made NNs hard to train and slow to converge. These issues were compounded by the lack of computational power available at the time. In the following decades, several techniques were proposed that alleviated or eliminated these issues.

In this section we discuss the recent advances that had a significant impact in the performance of neural networks. Table 1 gives an overview of these changes.

### 3.2.1 Activation functions

The choice of the activation function in a NN has a huge impact on the way the network is trained [65]. Some of the most common activation function
|                      | Old                          | Current                                      |
|----------------------|------------------------------|----------------------------------------------|
| **Structure**        | MLP, RNN and CNN with limited (1-2) layers | Many more layers, attention architecture, dilated convolution |
| **Activations**      | tanh, sigmoid                | ReLU and variants                            |
| **Initialization/normalization** | Small random numbers        | Xavier, batch normalization                   |
| **Gradient update rules** | Stochastic gradient descent  | ADAM, gradient clipping                       |
| **Regularization**   | $\ell_2$, $\ell_1$, elastic net | dropout                                      |
| **Computation power** | CPU                          | Multi-core CPUs, two orders of magnitude faster GPUs |
| **Data**             | Restricted to small datasets | Large datasets with millions of examples     |

Table 1: Summary of the evolution of NNs.

Historically, the most common nonlinear activation function used in NNs was the sigmoid function, which squashed numbers in the $[0, 1]$ range. Although a popular choice for many decades, the sigmoid function suffers from three basic problems: First, it exacerbates the vanishing gradient problem since nodes that are saturated (i.e., they have values far from zero) have nearly zero gradient. Second, the sigmoid outputs are not zero-centered which has an impact in the convergence of the training. Finally, the exponential function is more expensive to compute compared to other alternatives. A strictly better choice than the sigmoid function is the tanh function since it suffers from the same problems but it addresses the issue of zero-centered outputs.
The breakthrough in activation functions came with the usage of the rectified linear unit (ReLU) \cite{98, 143, 66}. ReLU does not saturate in the positive region, which limits the vanishing gradient problem, and it is much more computationally efficient since it is just a thresholding operation. In practice, it allows for faster convergence than the sigmoid or tanh functions \cite{111}. Although the output of the ReLU is not zero-centered and it still suffers from the vanishing gradient problem in the negative region, its use greatly improved the training of neural networks.

Several variations of the ReLU were proposed to address the remaining issues. Two of the most important variations are the Leaky ReLU$^3$ and the exponential linear unit (ELU) \cite{42}. The Leaky ReLU addresses the vanishing gradient issue in the negative region by introducing a slope, resulting in better convergence behaviour than the plain ReLU \cite{198}. The ELU tries to address the vanishing gradient issue and improve on the zero mean output by introducing an exponential decay in the negative region. However, this improvement comes with the cost of computing the exponential function. Nowadays, ReLU and its variants are the default choices as NN activation functions.

\footnote{$^3$First proposed and used in the Kaggle NDSB competition.}
3.2.2 Weight initialization and normalization

One important reason hindering the efficient training of complex neural network architectures in the early days was the naive way in which the weights (parameters) of the networks were initialized.

Initialization was usually done with random small numbers, which contributed to the vanishing gradient problem making it impossible to train a network with many layers. This issue was addressed by Glorot and Bengio [65] with a weight normalization rule dependent on the size of the inputs. This initialization rule yields a constant distribution (standard normal) on the output values of each layer. It is now known as Xavier initialization.

The attractive properties of the Xavier rule were derived for linear activations. The rule worked in practice for tanh activation functions but not for the newly popular ReLU. An important improvement to the Xavier rule that accounted for the lost variance of the weights caused by the thresholding applied by the ReLU was proposed in [75].

In addition to these principled initialization rules, [94] introduced batch normalization. The aim of batch normalization is similar to that of the Xavier rule: keep a standard normal distribution of the output values of each layer. With this method, an additional batch normalization layer is inserted between the existing layers of the network to normalize the intermediate values. It is well documented that batch normalization improves the stability and training speed of neural networks, though the theoretical reasons behind this remain under discussion.

3.2.3 Gradient update rules

Neural networks are usually trained using stochastic gradient descent, a method by which the gradient is computed for a small, randomly selected subset of the data (mini-batch) before updating the weights of the model. Using this method, neural network training was notoriously slow to converge.

Some enhancements to simple stochastic gradient descent using momentum [150] or Nesterov’s accelerated momentum [144] have been used for decades, but their impact on convergence speed was marginal. During the 2010s, a lot of research was dedicated to finding better gradient update rules to increase the convergence rate of NNs. This yielded notable improvements such as Adagrad [56], RMSProp [181], and Adam [103]. The latter has arguably become the default optimizer. A complete overview of the most
important update rules can be found in [137].

In complement to these improved optimization algorithms and motivated by the vanishing and exploding gradient issue, Pascanu et al. [138] proposed a gradient clipping method to prevent exploding gradients and a gradient regularizer to prevent vanishing gradients. These methods have made the training of complex neural architectures considerably faster and more stable.

3.2.4 Regularization

Regularization techniques have been widely used in the machine learning and statistics literature. In regression or classification models a common approach is to include a regularization term in the objective that restricts the parameter space and improves the generalization capabilities of the model by avoiding overfitting. Regularization functions such as the $\ell_2$-norm [84] (“ridge”), the $\ell_1$-norm [180] (“lasso”), and the elastic net [206], which is a linear combination of the $\ell_1$ and $\ell_2$-norms, have been popular in classical time-series and forecasting [28, 105, 26, 25]. $\ell$-norm regularization has been widely used in neural networks for the same purpose as in regression models.

A regularization method that is specifically tailored to NNs, dropout, was proposed in [80] and [176]. The idea of dropout is very simple: in order to prevent overfitting, some nodes are randomly masked out of the network at each pass during training. Perhaps the most intuitive way to understand why dropout helps is to see it as a very efficient means of implementing the powerful approach of ensemble learning. Training many NNs to create an ensemble is possible but is computationally expensive. Dropout achieves essentially the same objective and significantly improves generalization performance without increasing computational costs.

3.2.5 Computation power and data corpora

The increase in the processing power in modern computers has undoubtedly played a key role in the recent success of neural networks.

Rough comparisons between the most powerful computer processors available at the birth of neural networks in the late 50’s with those available today reveals a twelve orders of magnitude increase in processing power. The use of multi-core central processing units (CPUs) and especially the use of graphics processing units (GPUs) for parallel computation has significantly decreased the time required to train large-scale models [153]. This opened the way for
the exploration of ever more complex architectures that were impractical to train due to time constraints.

Simultaneously to the increase in computation power came an increase in the amount of data available to train models. This led to the creation of large datasets with millions of examples publicly available to researchers [49]. The public availability of these massive datasets allowed research to take advantage of the methodological advances discussed above to train models with increasingly complex architectures and better generalization properties.

3.3 Software frameworks

The availability of high-quality, open-source software frameworks has been a key contributor to the spread of deep learning. Some of the most popular frameworks are TensorFlow [1], PyTorch [149] and MXNet [33]. These frameworks have converged on similar concepts and allow engineers and scientists to share ideas in code easily and to effectively build on top of each others contributions.

Key ingredients common to these frameworks include abstractions to express the building blocks of neural networks such as convolutions and LSTM cells, common optimization algorithms and initialization schemes, and regularization or dropout.

Perhaps most importantly, these framework provide auto-differentation along the computation graph. This automates the computation of higher-order gradients, thereby lifting much of the implementation burden from researchers. Computational efficiency is largely inherited by using vectorized calculation which can take full advantage of GPU processing.

These frameworks mainly focus on general-purpose components and, while it is possible to use them for forecasting or other specialized tasks, considerable time investment is needed to do so. This has led to the development of specialized software packages on top of these frameworks, such as Sockeye for Machine Translation [78] or the GluonNLP and GluonCV packages for natural language processing and computer vision respectively [73, 76, 204].

Forecasting has trailed this trend, but recently a number of specialized packages have been released. Tensorflow contains components to build Bayesian structural time series models [318] PyTorch offers sample implementations [https://www.tensorflow.org/probability/api_docs/python/tfp/sts]
tations such as Deep4Cast\footnote{https://github.com/MSRDL/Deep4Cast} but, to the best of our knowledge at the time of writing, no specialized or integrated packages are available. Arguably the most comprehensive solution is GluonTS \cite{GluonTS} which contains abstractions for building (probabilistic) time series models as well as tooling for tasks such as backtesting and evaluation. An overview of available forecasting tools, including but not restricted to neural networks can be found in Januschowski et al. \cite{Januschowski2017}, Kolassa and Hyndman \cite{Kolassa2019}.

Most of the above software is targeted towards the Python programming language which has been the language of choice in deep learning. Other NN implementations exist, for example the nfor package in the R programming language\footnote{https://kourentzes.com/forecasting/2017/02/10/forecasting-time-series-with-neural-networks-in-r/} but these lag state-of-the-art in deep learning. Commercial software is also available for neural forecasting models such as DeepAR in SageMaker \cite{AWSDeepAR}, Amazon Forecast\footnote{https://aws.amazon.com/forecast/} or software by BlueYonder\footnote{http://www-ekp.physik.uni-karlsruhe.de/~feindt/ISC2013.pdf}.

## 4 Neural forecasting models

In this section we provide a literature overview of the neural network-based forecasting models. First, we introduce some basic notation: scalars are denoted by lower-case letters and vectors by lower-case bold letters, i.e., $x$ and $\mathbf{x}$, respectively. We denote by $\mathcal{Z} = \{z_{i,1:T_i}\}_{i=1}^N$ a set of $N$ univariate time series, where $z_{i,1:T_i} = (z_{i,1}, \ldots, z_{i,T_i})$ and $z_{i,t} \in \mathbb{R}$ is the value of the $i$-th time series at time $t$. The set of time series is associated with a set of time-varying covariate vectors denoted by $\mathcal{X} = \{x_{i,1:T_i}\}_{i=1}^N$, with $x_{i,t} \in \mathbb{R}^D$. The set of parameters of a NN is denoted by $\Phi$ and the set of output parameters of a neural network as $\Theta$. The probability distribution of a random variable $Z$ is denoted as $p(z)$, its conditional distribution as $p(z|x)$ and a distribution parametrized by a set of parameters $\Theta$ as $p(z; \Theta)$, while $\hat{z}_t$ denotes an estimate of $z_t$. Finally, $\tau$ is the forecast horizon of a model.

Since the onset of the deep learning revolution, a large number of neural forecasting models have been proposed using a wide range of paradigms, architectures, or objective functions. For the purpose of this overview, we
categorize these models in two simple model classes defined based on the nature of the forecasts and the prediction horizon.

**Point vs probabilistic forecasts**  Point forecast models produce point estimates of the future values of the time series given the past values and relevant covariates, i.e., \( \hat{z}_{i,T_i+\tau} = f(z_{i,1:T_i}, x_{i,1:T_i+\tau}; \Phi) \), where \( f(\cdot) \) denotes a model with some parametric structure with unknown parameters \( \Phi \), and \( \tau \) defines the prediction horizon. The value predicted is most commonly the expected value of the time series at some future point, but it can also be a median or some other quantile.

Probabilistic forecast models predict the conditional distribution of the future values of the time series given relevant covariates (generative models) and/or past values of the time series (discriminative models) \[145\]. Discriminative models are more flexible than generative models as they make fewer structural assumptions, making them applicable to a broader class of application domains. Further distinctions within the subspace of discriminative models are discussed in \[58\].

| Forecast Type | Model |
|---------------|-------|
| Point         | \( \hat{z}_{i,T_i+1:T_i+\tau} = f(z_{i,1:T_i}, x_{i,1:T_i+1}; \Phi) \) |
| Probabilistic | \( p(z_{i,T_i+1:T_i+\tau} | z_{i,1:T_i}, x_{i,1:T_i+1}; \Phi) = f(z_{i,1:T_i}, x_{i,1:T_i+1}; \Phi) \) |

Table 2: Classes of neural forecasting models based on the forecast type.

**One-step vs multi-step forecasts**  One-step forecasting models predict only the next time step of the time series, i.e., \( \tau = 1 \). In models with recurrent structures, predictions for longer horizons can be generated using the *teacher forcing* method, i.e., recursively supply the observed values as inputs during training and feed the network with its own one-step ahead estimates when generating out of sample predictions to achieve multi-step forecasts \[71\]. These models do not predict directly a multi-step horizon, and each prediction of the future horizon depends on the predicted values of the previous time steps.

Sequence-to-sequence (seq2seq) models directly generate multi-step predictions \[37\] \[177\] \[139\]. Given an input sequence they directly generate predictions for a complete sequence of future values for a range of horizons.
Most studies of neural time series forecasting models in the 80’s and 90’s were based on point-forecast MLP networks [194, 16, 31, 122, 32, 35, 178], although there were some limited attempts to use RNN architectures [43, 112]. Most of these works used networks with a single hidden layer and were generally limited to one-step forecasts [203, Table 1]. Moreover, the activation functions of these networks were restricted to sigmoids and hyperbolic tangents (tanh).

The architectures of these early neural forecasting models were constrained by the limitations of neural networks available at the time. The advances of the last two decades discussed in the previous section have relaxed many of these constraints, leading to the development of models with richer and more creative architectures combining classical and new ideas. In the following we propose a unified overview of these modern neural forecasting models. We begin by discussing individual models grouped in 5 families. We then summarize the comparison in Table 3.

4.1 One-step prediction models

Recurrent models reduce the task of predicting a sequence of observations to a sequence of one-step-ahead predictions. During training, forecasts for the whole sequences are generated and the error is aggregated over the sequence for the model update. When generating out-of-sample forecasts, the model is rolled forward recursively by feeding in the model’s own predictions for the previous steps as past values.

In 2017 Salinas et al. [162] proposed DeepAR, an RNN based forecasting model using LSTM or GRU cells. At each time step, DeepAR takes as input the previous time points and covariates, and estimates the distribution of the value of the next time point. This is done via the estimation of the parameters of a pre-selected parametric distribution such as the negative binomial. Training and prediction follow the general approach for autoregressive models. Mukherjee et al. [142] used this model structure with a mixture of Gaussian’s as the distribution and domain specific feature processing blocks.

While DeepAR only learns a univariate distribution, Toubeau et al. [182] and Salinas et al. [160] combined RNN based-models with copulas to model multivariate probability distributions. The model in [182] uses a nonparametric copula to capture the multivariate dependence structure. In contrast, the work in [160] uses a Gaussian copula process approach. The authors use a low-rank covariance matrix approximation to scale to thousands of
dimensions. Additionally, the model implements a non-parametric transformation of the marginals to deal with varying scales in the dimensions and non-Gaussian data.

Another line of work has focussed on how the probabilistic output is modeled. The DeepAR model [162] assumes a parametric form for the output distribution \( p(z_{i,t} | x_{i,t}, \Theta_t) \). Typical choices include the Gaussian, Student-\( t \), or negative-binomial distributions. More flexible families, such as mixtures of Gaussians, have also been proposed [142, 20]. A popular alternative to this approach is based on quantile regression [107, 106]. In these approaches, the model predicts one or more quantiles of the output distribution directly.

The model proposed in [199] extends the quantile autoregression framework [108, 61] with a flexible, neural-network-based function estimator [30].

The spline quantile function RNN model (SQF-RNN) proposed in [62] uses the same basic RNN architecture as DeepAR, but uses a spline-based parametrization of the quantile function of the output distribution instead of a parametric form, leading to improved empirical performance and greater robustness. The model is trained by optimizing the CRPS [68] directly, instead of the more common maximum likelihood procedure.

LSTNet [114] is a model using a combination of CNN and RNN. Targeting at multivariate time series, LSTNet uses a convolution network (without pooling) to extract short-term temporal patterns as well as correlations among variables. The output of the convolution network is fed into a recurrent layer and a temporal attention layer which, combined with the autoregressive component, generates the final forecast.

In [205], the authors use an LSTM with Monte Carlo dropout as both the encoder and decoder. However, unlike other models that directly use RNN to generate forecasts, the learned embedding at the end of the decoding step is fed into a MLP prediction network and combined with other external features to generate the forecast. Along a similar line, [117] employs LSTM as a feature extractor (LSTM autoencoder), and uses the extracted features, combined with external inputs to generate the forecasts with another LSTM.

Qiu et al. [151] proposed an ensemble of deep belief networks for time series forecasting. The outputs of all the networks is concatenated and fed into a support vector regression model (SVR) that gives the final prediction. The neural networks and the SVR are not trained jointly though.

Hsu [89] proposed an augmented LSTM model which combines autoencoders with LSTM cells. The input observations are first encoded to latent variables, which is equivalent to feature extraction, and are fed into the
LSTM cells. The decoder is an MLP which maps the LSTM output into the predicted values.

Building upon the success of CNNs in other application domains, Borovykh et al. [21] proposed an adjustment to the well-known WaveNet architecture [185] (see also Section 4.5) that makes it applicable to conditional time series forecasting tasks. They tested their model on various datasets with mixed results, concluding that it can serve as a strong baseline and that various improvements could be made. In a similar vein, inspired from the Transformer architecture [186] (see also Section 4.5), Song et al. [175] proposed an adjustment that makes the architecture applicable to time series. Their method is applied to both regression and classification tasks.

In a purely local setting, Zhang [202] proposed a hybrid model of a NN with ARIMA to capture the nonlinear and linear patterns of the time series, respectively. ARIMA is initially used to model the linear component of the data and subsequently the NN is used to model the residuals of the ARIMA to learn the nonlinear patterns for each time series. The final model is the sum of the nonlinear and linear components. Khashei and Bijari [101] proposed a different kind of hybrid model, extending the work of Zhang [202]. In their approach, a classifier based on a binary NN is applied to the residuals of an ARIMA model, classifying them as either “negative trend”, “no trend”, or “positive trend”. These pointwise trend predictions are then used to construct a trend correction that is additively combined with the ARIMA model. In experiments on three time series the authors demonstrate that their hybrid model outperforms both pure ARIMA and NN models, as well as the hybrid approach of Zhang [202].

In contrast to pure deep learning methods to time series forecasting, Ranganapuram et al. [154] propose to combine classical state space models (SSM) with deep learning. The main goal here is to bridge the gap between SSMs that provide a principled framework for incorporating structural assumptions but fail to learn patterns across a collection of time series, and deep learning based methods that are capable of extracting higher order features but results in models that are hard to interpret. Their method parametrizes a linear SSM using an RNN. The parameters of the RNN are learned jointly from a dataset of raw time series and associated covariates. Instead of learning the state space model parameters $\Theta_{1:1:T}$ for each time series individually, the model learns a globally shared mapping from the covariates associated with each target time series to the parameters of linear state space model.
This mapping

\[ \Theta_{i,t} = \Psi(x_{i,1:t}; \Phi), \quad i = 1, \ldots, N, \quad t = 1, \ldots, T_i + \tau \]

is implemented by an RNN with weights \( \Phi \) which are shared across different time series as well as different time steps. Note that \( \Psi \) depends on the entire covariate time series up to time \( t \) as well as the set of shared parameters \( \Phi \). Since each individual time series \( i \) is modeled using an SSM with parameters \( \Theta_i \), assumptions such as temporal smoothness in the forecasts are easily enforced.

The shared model parameters \( \Phi \) are learned by maximizing the likelihood given the observations \( Z = \{z_{i,1:T_i}\}_{i=1}^N \). The likelihood terms for each time-series reduce to the standard likelihood computation under the linear-Gaussian state space model, which can be carried out efficiently via Kalman filtering [12]. Once the parameters \( \Phi \) are learned, it is straightforward to obtain the forecast distribution via the SSM parameters \( \Theta_{i,T_i+1:T_i+\tau} \). One major limitation of their method is that the data is assumed to follow a Gaussian distribution. It is not clear how to extend their method to arbitrary distributions.

### 4.2 Sequence-to-sequence models

Sequence-to-sequence (seq2seq) models directly map input sequences to output sequences that may have different length. This style of model was originally introduced in machine translation where the original and translated sequences usually have different length [177, 37]. Additionally, seq2seq models typically require the complete encoding of the input sequence before the output sequence is generated. The general architecture of a seq2seq model follows an encoder-decoder pattern: the encoder maps the input sequence to a latent state vector and the decoder generates the output sequence from this state vector. Encoders and decoders can, in principle, be any NN architecture.

In the context of forecasting, this architecture maps the past target values and covariates to the future target values. The main advantage of seq2seq models over autoregressive models is that the decoder architecture can be chosen to output all future target values at once. This removes the need of autoregressive models to unroll over the forecast horizon which can lead to error accumulation because early forecast errors propagate through the
forecast horizon. Thus, the decoder of seq2seq forecasting models is typically an MLP while other architectures are also used for the encoder [192] [146].

Wen et al. [192] proposed a seq2seq architecture for probabilistic forecasting. Their model uses an RNN or dilated causal convolution as the encoder and an MLP as the decoder (MQ-RNN / MQ-CNN). The MLP is constructed to output a set of quantile levels for the entire forecast horizon. The advantage of this architecture is that the simultaneous output of all quantile levels for the entire forecast horizon is insensitive to error accumulation. However, the model needs to be retrained if other quantile levels than the trained one are required by the user. The authors show that dilated convolutions can also be used as the encoder [192] [185]. Wen and Torkkola [190] extended the MQ-CNN model with a generative quantile copula. This model learns the conditional quantile function that maps the quantile index, which is a uniform random variable conditioned on the covariates, to the target. During training, the model draws the quantile index from a uniform distribution. This turns MQ-RNN into a generative, marginal quantile model. The authors combine this approach with a Gaussian copula to draw correlated marginal quantile index random values. The authors show that the Gaussian copula component improves the forecast at the distribution tails.

Chen et al. [34] proposed DeepTCN, another seq2seq model where the encoder is the dilated causal convolution with residual blocks, and the decoder is simply a MLP with residual connections. Structure-wise, DeepTCN is almost the same as the basic structure of MQ-CNN [191], i.e., without the local MLP component that aims to model spikes and events.

N-BEATS [146] uses a deep, residual stack of MLP layers to do point forecasts. The basic building block in this architecture is a forked MLP stack that takes the block input and feeds the intermediate representation into separate MLPs to learn the parameters of the context (the authors call it backcast) and forecast time series models. The authors propose a deep, residual architecture that removes the part of the context signal it can explain well before passing to the next block and adds up the forecasts. The learned time series model can have free parameters or be constrained to follow a particular, functional form. The authors show that constraining the model to trend and seasonality functional forms does not have a big impact on the error and generates models whose stacks are interpretable, because the trend and seasonality components of the model can be separated and analyzed.

In Lv et al. [133], the authors propose a stacked autoencoder (SAE) architecture to learn features from spatio-temporal traffic flow data. On top
of the autoencoder, a logistic regression layer is used to output predictions of the traffic flow at all locations in a future time window. The resulting architecture is trained layer-wise in a greedy manner on publicly available data from the California department of transportation. The experimental results show that the method significantly improves over other shallow architectures, suggesting that the SAE is capable of extracting latent features regarding the spatio-temporal correlations of the data.

In the same context of spatio-temporal forecasting and under the seq2seq framework, Li et al. [125] proposed the Diffusion Convolutional Recurrent NN (DCRNN). Diffusion convolution is employed to capture the dependencies on the spatial domain, while an RNN is utilized to model the temporal dependencies. Finally, Asadi and Regan [5] proposed a framework where the time series are decomposed in an initial preprocessing step to separately feed short-term, long-term, and spatial patterns into different components of a neural network. Neighbouring time series are clustered based on their similarity of the residuals as there can be meaningful short-term patterns for spatial time series. Then, in a CNN based architecture, each kernel of a multi-kernel convolution layer is applied to a cluster of time series to extract short-term features in neighbouring areas. The output of the convolution layer is concatenated by trends and is followed by a convolution-LSTM layer to capture long-term patterns in larger regional areas.

Bandara et al. [9] addressed the problem of predicting a set of disparate time series, which may not be well captured by a single global model. For this reason, the authors propose to cluster the time series according to a vector of features extracted using the technique from [93] and the Snob clustering algorithm [188]. Only then, an LSTM RNN is trained per-cluster, after having decomposed the series into trend, seasonality and residual components. The RNN is followed by an affine neural layer to project the cell outputs to the dimension of the intended forecast horizon. This approach is applied to publicly available datasets from time series competitions, and appears to consistently improve against learning a single global model. In subsequent work Bandara et al. [10] continued to mix heuristics, in this instance seasonality decomposition techniques, known from classical forecasting methods with standard NN techniques. Their aim is to improve on scenarios with multiple seasonalities such as inter and intra daily seasonalities. The findings are that for panels of somewhat unrelated time series, such decomposition techniques help global models whereas for panels of related or homogeneous time series this may be harmful. The authors do not attempt to integrate these steps
into the NN architecture itself, which would allow for end-to-end learning.

Finally, Cinar et al. [41] proposed a content attention mechanism that seats on top of any seq2seq RNN. The idea is to select a combination of the hidden states from the history and combine them using a pseudo-period vector of weights to the predicted output step.

### 4.3 Local models and global models

With *local models*, the free parameters of the model are learned individually for each series in a collection of time series. Classical local time series models such as state space models (SSMs), ARIMA, and exponential smoothing (ETS) [91] excel at modeling the complex dynamics of individual time series given a sufficiently long history. Other local models include Gaussian SSMs, which are computationally efficient, e.g., via a Kalman filter, and Gaussian Processes (GPs) [155, 165, 64, 24]. Both methods provide uncertainty estimates, which are critical for optimal downstream decision making. Since these methods are local, they learn one model per time series and cannot effectively extract information across multiple time series. These methods are unable to address cold-start problems where there is a need to generate predictions for a time series with little or no observed history.

With *global models*, also known as panel models in econometrics and statistics, the free parameters of the model are learned jointly on every series in a collection of time series. Deep learning models have proven particularly well suited at taking advantage of large amounts of data to learn parameters of a single global model over an entire collection of time series [162, 62, 154, 192, 117]. These methods are able to extract patterns from collections of irregular time series even when these patterns would not be distinguishable using a single series.

Local-global models have been proposed to combine the advantages of both local and global models into a single model. Examples include mixed effect models [45], which consist of two kinds of effects: fixed (global) effects that describe the whole population, and random (local) effects that capture the idiosyncratic of individuals or subgroups. A similar mixed approach is used in Hierarchical Bayesian [63] methods, which combine global and local models to jointly model a population of related statistical problems. In [3, 131], other combined local and global models are detailed.

A recent local-global family of models, Deep Factors [189] provide an alternative way to combine the expressive power of NNs with the data ef-
iciency and uncertainty estimation abilities of classical probabilistic local models. Each time series, or its latent function for non-Gaussian data, is represented as the weighted sum of a global time series and a local model. The global part is given by a linear combination of a set of deep dynamic factors, where the loading is temporally determined by attentions. The local model is stochastic. Typical choices include white noise processes, linear dynamical systems, GPs [135] or RNNs. The stochastic local component allows for the uncertainty to propagate forward in time, while the global NN model is capable of extracting complex nonlinear patterns across multiple time series. The local-global structure extracts complex nonlinear patterns globally while capturing individual random effects for each time series locally.

The Deep Global Local Forecaster (DeepGLO) [167] is a method that thinks globally and acts locally to forecast collections of up to millions of time series. It is a hybrid model that uses a global matrix factorization model [201] regularized by a temporal deep leveled network and a local temporal deep level network to capture patterns specific to each time series. Each time series is represented by a linear combination of \( k \) basis time-series, where \( k \ll N \) and \( N \) is the total number of time series. The global and local models are combined through data-driven attention for each time series. A type of temporal convolution (leveled network) is used, that can be trained across a large amount time series with different scales without the need for normalization or rescaling.

Combining forecasts from different models can often increase forecast accuracy [13]. The so called “hybrid” models [202, 101, 102] go one step further and directly integrate aspects of different model classes into a single model. One such hybrid model within the local-global model class is the ES-RNN model proposed by Smyl [172] that has recently attracted attention by winning the M4 competition [137] by a large margin on both evaluation settings. In the ES-RNN model, locally estimated level and trend components are multiplicatively combined with an RNN model. In particular, the \( h \)-step ahead prediction \( y_{t+1:t+h} = l_t \cdot s_{t+1:t+h} \cdot \exp(\text{RNN}(x_t)) \) consists of a level \( l_t \) and a seasonal component \( s_t \) obtained through local exponential smoothing, and the output of a global RNN model \( \text{RNN}(x_t) \) trained on features \( x_t \) extracted from de-seasonalized and normalized time series \( x_t = \log(y_{t-K:t}/(s_{t-K:t}l_t)) \) cut in a window of length \( K+1 \). The RNN models are composed of dilated LSTM layers with additional residual connections. The M4-winning entry used slightly different architectures for the different type of time series in the competition.
4.4 NNs for intermittent demand

Neural forecasting models had a major impact on operational forecasting problems where intermittent time series occur regularly [22]. Research on NNs for intermittent time series forecasting has been an active area. Salinas et al. [162] propose a standard RNN architecture with a negative binomial likelihood to handle intermittent demand similar to [174] in classical methods. To the best of our knowledge, other likelihoods that have been proposed for intermittent time series in classical models, e.g., by [166], have not yet been carried over to NNs.

The seminal paper on intermittent demand forecasting is [46]. Croston separates the data in a sequence of observed non-zero demands and a sequence on time intervals between positive demand observations, and run exponential smoothing separately on both series instead of smoothing the original sparse time series. A comparison of NNs to classical models for intermittent demand first appeared in Gutierrez et al. [74], where the authors compare the performance of a shallow and narrow MLP with Croston’s method. They find NNs to outperform classical methods by a significant margin.

Kourentzes [110] proposes two MLP architectures for intermittent demand, taking demand sizes and intervals as inputs. As in Gutierrez et al. [74], the networks are shallow and narrow by modern standards, with only a single hidden layer and 3 hidden units. The difference between the two architectures is in the output. In one case interval times and non-zero occurrences are output separately, while in the other a ratio of the two is computed. The approach proposed by Kourentzes [110] outperforms other approaches primarily with respect to inventory metrics, but not forecasting accuracy metrics, challenging previous results in [74]. It is unclear whether the models are used as global or local models. However, given the concern around overfitting and regularization, we assume that these models were primarily used as local models in the experiments.

Both approaches of [74, 110] only offer point forecasts. This shortcoming is addressed by the approach that appeared recently in [183], where the authors propose renewal processes as natural models for intermittent demand forecasting. Specifically, they use RNNs to modulate both discrete-time and continuous-time renewal processes, using the simple analogy that RNNs can replace exponential smoothing in [46].

Finally, a recent trend in sequence modeling employs NNs in modeling discrete event sequences observed in continuous time [55, 140, 196, 168, 184].
Notably, Xiao et al. [197] use two RNNs to parameterize a probabilistic “point process” model. These networks consume data from an asynchronous event sequences and uniformly sampled time series observations respectively. Their model can be used in forecasting tasks where time series data can be enriched with discrete (i.e., continuous-time) event observations.

4.5 New architectures for forecasting

Sequence modeling is the central task not only in forecasting but also in many other areas of machine learning such as natural language processing, speech and audio synthesis, and speech recognition. Deep learning has proven to be very successful in these disciplines and has largely replaced traditional models in many applications.

Until recently, most deep learning models for sequences were based on RNNs. While RNNs have been extremely successful they often come with practical challenges: they can be hard to train and may require extensive tuning of the training parameters to achieve good results; they struggle with learning long range dependencies which is a big problem for e.g., speech synthesis; and their sequential nature means they cannot exploit highly parallel computer architectures such as GPUs very well making them relatively long to train.

Recently, new architectures have become popular that avoid some of these restrictions. Van Den Oord et al. [185] introduced the WaveNet architecture for speech synthesis, which uses the so-called dilated causal convolutions to learn the long range dependencies important for audio signals. Since this architecture is based on convolutions, training is very efficient on GPUs – prediction is still sequential and further changes are necessary for fast inference.

New model architectures based on the so-called attention mechanism have been gaining importance especially in seq2seq learning [38, 39, 186, 123]. Typically, models with encoder-decoder architectures are used for this problem setting, where the encoder (an RNN), learns a representation of the input sequence while the decoder (another RNN) is trained to predict the target sequence one step at a time using the representation learned by the encoder. Given that these models use a fixed length representation on the encoder side (usually the output of the last time step of the encoder), often inferior performance has been noted for varying the length and for very long sequences [6]. The authors of [6, 7] suggest learning more than one representations on the
encoder side (e.g., one for each time step of the input sequence) and using a weighted sum of these encoder representations in the decoder. These weights (normalized so that they can be treated as probabilities) are jointly learned along with the encoder and decoder. This is essentially training the decoder to learn which parts of the input sequence it should pay attention in order to produce a correct prediction at the current time step. This attention mechanism has been shown to be quite useful in speech recognition and machine translations tasks [38, 39].

Inspired by the success of attention models, the authors of [186] develop the so-called Transformer model which gets rid of RNNs altogether and uses attention alone, in combination with feed-forward NNs to achieve state-of-the-art results. The main idea is to extend the encoder-decoder attention mechanism of [6] to intra- or self-attention within the encoder and decoder to learn where to focus in order to get good feature representations. That is, the encoder in the transformer model finds suitable positions (or context) in the whole input sequence to compute feature embeddings for each element in the sequence, thus making it undirectional compared to RNNs. Only feed-forward NNs are used for learning these representations and hence all the related computations can be done in parallel for all time steps, making the overall model training orders of magnitude faster than the existing models. To account for the sequential nature of the input, a separate positional encoding for the time index is learned using sinusoidal functions.

Similarly, the decoder learns self-attention (by using elements only up to the current time point to avoid data leak) as well as encoder-decoder attention similar to [6]. For efficient implementation, the attention function is described as mapping a query and a set of key-value pairs to an output, where the query, keys, values, and output are all vectors. During prediction, the model generates a key for each historical point in the sequence and a query for the point that will be predicted. The dot product between query and key represents the probability of attending to that point, i.e., using it for prediction. The Transformer model has become quite popular and has been successfully used in learning language representations that achieved state-of-the-art results on eleven natural language processing tasks [50].

These new architectures have recently been applied to time series forecasting. Adaptations of WaveNet for forecasting are available [19, 4]. Li et al. [123] introduce two modifications to the transformer architecture to improve its performance for forecasting. First, they include causal convolutions in the attention to make the key and query context dependent, which
makes the model more sensitive to local contexts. Second, they introduce a sparse attention, meaning the model cannot attend to all points in the history, but only to selected points. Through exponentially increasing distances between these points, the memory complexity can be reduced from quadratic to $O(L(\log L)^2)$, where $L$ is the sequence length, which is important for long sequences that occur frequently in forecasting.

4.6 Summary

The previous sections introduced a large number of neural forecasting models, which we summarize in Table 3. The list below provide keys to reading the table.

- **Forecast** distinguishes between probabilistic (Prob.) and point forecasts.

- **Horizon** indicates whether the model predicts a single step (noted 1) in which case multi-step ahead forecasts are obtained recursively, or whether it directly predicts a whole sequence ($\geq 1$).

- **Loss** and **Metrics** specifies the loss used for training and metrics used for evaluation. Here, we only provide an explanation of the acronyms and not the definition of each metric which can be easily found in the corresponding papers: negative log-likelihood (NLL), quantile loss (QL), continuous ranked probability score (CRPS), (normalized) (root) mean squared error (NRMSE, RMSE, MSE), root relative squared error (RRSE), relative geometric RMSE (RGRMSE), weighted absolute percentage error (WAPE), normalized deviation (ND), mean absolute deviation (MAD), mean absolute error (MAE), mean relative error (MRE), (weighted) mean absolute percentage error (wMAPE, MAPE), mean absolute scaled error (MASE), overall weighted average (OWA), mean scaled interval score (MSIS), Kullback-Leibler divergence (KL), Value-at-Risk (VaR), expected shortfall (ES), empirical correlation coefficient (CORR), area under the receiver operating characteristic (AUROC), percentage best (PB).

Table 3 illustrates the wealth of neural forecasting methods now available. Some of these methods hybridize or build upon some of the most successful classical methods to enable their use at scales or on use cases hitherto
unachievable. Others exploring entirely new directions by leveraging architectures developed in other fields of machine learning. The variety of use cases to which these papers have been successfully applied demonstrates the benefit that neural forecasting methods bring to the forecasting community and why they should now constitute an integral part of the toolset of any researcher in the field.
| Study                  | Structure      | Forecast  | Horizon | Loss | Metrics                  | Data Types                 | Comments                                                                 |
|-----------------------|----------------|-----------|---------|------|--------------------------|----------------------------|--------------------------------------------------------------------------|
| DeepAR [162]          | RNN            | Prob.     | 1       | NLL  | Coverage, QL, ND, NRMSE  | demand, traffic, electricity | Learns parametric distributions                                          |
| Toubeau et al. [182]  | RNN/CNN        | Prob.     | 1       | NLL/QL | RMSE, price              | electricity                |                                                                          |
| Salinas et al. [160]  | RNN            | Prob      | 1       | NLL  | QL, MSE                  | electricity, traffic, exchange rate, solar, taxi, wiki                | Learns multivariate model via low-rank Gaussian copula processes           |
| ARMDN [142]           | RNN            | Prob.     | 1       | NLL  | wMAPE                    | demand                     | Like [162], but using mixture of Gaussian’s and domain specific feature processing |
| QARNN [199]           | MLP            | Prob.     | 1       | QL   | VaR, ES                  | finance                    |                                                                          |
| SQF-RNN [62]          | RNN            | Prob.     | 1       | CRPS | QL, MSIS, NRMSE, OWA    | demand, traffic, count data, electricity, exchange rate, finance, M4 | Models non-parametric distributions with splines                          |
| LSTNet [114]          | CNN + RNN + MLP| Point     | 1       | $\ell_1$ | RRSE, CORR               | traffic, solar, electricity, exchange rate | Extacts short and long temporal patterns with a CNN and RNN, respectively |
| Zhu and Laptev [205]  | RNN + MLP      | Prob.     | 1       | -    | sMAPE, calibration       | daily trips                | Fits an encoder (RNN) that constructs an embedding state, which is fed to a prediction network (MLP) |
| Method                        | Type                  | Objective | Loss Function | Regularizer | Error Metrics | Forecasting Targets |
|-------------------------------|-----------------------|-----------|---------------|-------------|---------------|---------------------|
| Laptev et al. [117]           | RNN                   | Prob. 1   | MSE           | sMAPE       | traffic, M3   |                     |
| MLP + SVR [151]              | Point 1               | 1         | $\ell_2$ for MLP, SVR objective | RMSE, MAPE | energy, housing |                      |
| A-LSTM [89]                  | RNN + MLP             | Point 1   | $\ell_2$, $\ell_2$ regularizer | RMSE       | electricity consumption | Combination of LSTM with autoencoders |
| CNN [21]                     | Point 1               | 1         | $\ell_1$, $\ell_2$ regularizer | RMSE, MASE, HITS | index forecasting, exchange rate | WaveNet [185] based model adjusted for time series forecasting |
| SAnD [175]                   | MLP + Attention       | Point 1   | $\ell_2$, cross-entropy, multi-label classification loss | AUROC, MASE, MSE | clinical | Transformer [123] based model adjusted for time series forecasting |
| MLP [202]                    | Point 1               | 1         | MSE           | MSE, MAD    | sunspot, lynx, exchange rate | Hybrid local model that uses ARIMA to capture the linear component and a NN for the nonlinear residuals |
| MLP [101]                    | Point 1               | 1         | MSE           | MAE, MSE    | sunspot, lynx, exchange rate | Hybrid local model that uses ARIMA and a NN for trend correction |
| Deep State Space [154]       | RNN + State Space     | Prob $\geq$ 1 | NLL           | P50, P90 quantile loss | traffic, electricity, tourism, M4 | RNN parameterized a state space model |
| Method                 | Model Components | Objective Function | Evaluation Measures | Additional Info |
|-----------------------|------------------|--------------------|---------------------|-----------------|
| MQ-RNN/CNN + RNN/CNN + MLP | Prob $\geq 1$ QL, QL, calibration, sharpness | QL, QL, inverse reconstruction loss, NLL | demand for demand Learns pre-specified grid of quantiles |
| CNN + MLP | Prob $\geq 1$ QL | QL, quantile crossing, QL over sum of future intervals | demand | Combines model in MQ-RNN/CNN + RNN/CNN + MLP with Gaussian copula |
| DeepTCN | CNN + MLP | Prob $\geq 1$ QL | QL | retail demand | Learns pre-specified grid of quantiles |
| N-BEATS | MLP | Point $\geq 1$ sMAPE, MASE, MAPE | sMAPE, MASE, OWA | M4 | Deep, residual MLP that learns interpretable trend and seasonality function |
| Lv et al. | Stacked autoencoder | Point $\geq 1$ MSE, KL sparsity constraint | MAE, MRE, RMSE | traffic |
| DCRNN | RNN | Point $\geq 1$ NLL | MAE, MAPE, RMSE | traffic |
| CNN + RNN | Point $\geq 1$ $\ell_2$ | MAE, RMSE | traffic | Decomposition-based model for spatio-temporal forecasting |
| RNN + Classical Decomposition | Point $\geq 1$ | sMAPE | CIF2016, NN5 | Clusters time series based on set of features and train one model per cluster |
| Method | Type | Loss Function | Metrics | Notes |
|--------|------|---------------|---------|-------|
| LSTM-MSNet | Point | $\ell_1$ | sMAPE, MASE | Decomposition based model with multiple seasonal patterns |
| RNN + Classical Decomposition | Point | $\ell_1, \ell_2$ regularizer | MSE, sMAPE | energy, max temperature, CPU usage, air quality |
| RNN + Attention | Prob | $\ell_1$ | QL, MAPE | electricity, traffic, uber |
| Deep Factors | CNN | $\ell_2$ | WAPE, MAPE, sMAPE | electricity, traffic, wiki |
| DeepGLO | CNN | $\ell_2$ | WAPE, MAPE, sMAPE | Global matrix factorization regularized by a deep leveled network |
| ES-RNN | CNN | $\ell_2$ | QL | MASE, sMAPE, MSIS |
| ES-RNN | CNN | $\ell_2$ | QL | MASE, sMAPE, MSIS |
| Attentional Twin RNN | Prob | $\ell_2$ | MAE | point process data |
| Attentional Twin RNN | Point | $\ell_2$ | MAE | intermittent demand |
| Attentional Twin RNN | Point | $\ell_2$ | MAE | intermittent demand |
| Attentional Twin RNN | Point | $\ell_2$ | MAE | intermittent demand |

**Notes:**
- **Deep Factors:** Global RNN and a local GP
- **DeepGLO:** Global matrix factorization regularized by a deep leveled network
- **ES-RNN:** Locally estimated seasonality and trend and global RNN.
| Model               | Type  | Prob | Loss | Quantile | NLL  | Grids/Usage   |
|---------------------|-------|------|------|----------|------|---------------|
| Deep Renewal Process | RNN   | Prob | ≥ 1  | P50, P90 | NLL  | intermittent demand |
|                     |       |      |      |          |      | RNN-based intermittent demand model inspired by point processes |
| WaveNet             | CNN   | Prob | ≥ 1  | mean opinion score | NLL  | traffic, electricity, M4 |
|                     |       |      |      |          |      | Diluted causal convolutions |
| Transformer         | MLP   | Point | 1    | QL       | NLL  | electricity, traffic, wind, M4, solar |
|                     |       |      |      |          |      | Transformer with causal convolutions and sparse attention. |

Table 3: Summary of modern neural forecasting models.
5 Applications

Application domains of neural forecasting span most of the traditional areas of application of general forecasting methods and open up new ones. A notable difference between classical and neural methods is the amount of data needed to train a model. This makes neural networks best suited when large corpora of time series are available and less useful in situations where data is limited, such as macro-economic time series forecasting\[44\]. In what follows, we discuss selected applications for neural forecasting. This list is subjective and we do not claim comprehensiveness.\[10\]

5.1 Retail demand forecasting

Forecasting the demand of multiple products offered by retailer for the purpose of inventory management is one area where neural forecasting methods have delivered substantial improvements over classical ones. This problem has a long history in forecasting due to its enormous impact in supply chain optimization, see for instance \[46\] for early work on intermittent demand forecasting. In such a problem, the demand for a large number of products is observed over periods of several years, yielding vast amounts of data \[162, 191, 11, 142\].

An advantage of using NNs in this context is the relative ease with which metadata associated to each product can be included in the models as part of the input vector, for instance by providing the category of products as in \[162\]. Another important advantage is that such methods can often be used to predict items for which little to no history is available as shown in Figure 5, which depicts predictions of \[162\] on products with little history. In such cases, neural network models can predict seasonality for new items since these global models learn patterns jointly over the whole dataset and can extrapolate these learned regularities to new series.

\[10\] Notable omissions include weather forecasting (e.g., \[67, 72\]) and anomaly detection \[2\].

\[11\] Not many datasets are available for public research. One exception is \url{https://research.chicagobooth.edu/kilts/marketing-databases/dominicks}. The upcoming M5 competition has the potential to change this.
5.2 Energy

The energy domain offers a number of important forecasting applications such as supply and demand, peak load, and price forecasting for electricity as a whole or by source, for crude oil, or for other energy sources. The GEFCom competitions \cite{85,86,87} have put these into the spotlight and neural methods have been applied to these problems. Dimoulkas et al. \cite{52} use an ensemble of feed-forward NNs (with five hidden layers) and an unorthodox approach to obtain probabilistic predictions. They rely on temperature as a feature and choose different temperature scenarios to obtain different (point) forecasts. Smyl and Hua \cite{173} use a global (deep) feed-forward NN with bypass layers to circumvent the vanishing gradient problem. However, due to the instability of the approach, Smyl and Hua \cite{173} decided to remove the NN from their ensemble forecast model.

Outside of the GEFCom competitions, Saxena et al. \cite{163} proposed an ensemble method for predicting peak electricity demand consisting of a shallow MLP and classical time series models showing favorable results (which is expected as little data for a single time series is available). Marcjasz et al. \cite{138} use a nonlinear autoregressive model (NARX), an alternative to LSTMs.

For crude oil price forecasting, \cite{124} use a CNN in a model pipeline to predict crude oil prices. The CNN is used to extract features from textual data from a news outlet, and not directly on time series information. Electricity load forecasting benefit from general methodological advances and energy consumption \cite{51} is often used for benchmarking \cite{162,189,154}. It points to
the possibility of including the hierarchical aspects of series as group features \cite{162} or of modeling more complex multivariate distributions \cite{182,161}.

5.3 Further applications

Traffic forecasting \cite{54,187} is a popular application domain for NNs \cite{117,126,133}. Forecasting can be used in multiple ways, for instance to determine pricing policies or to assist decision making \cite{187}. Neural forecasting methods can be interesting in this context as they permit the combination of spatial and temporal information when available \cite{126} as one can typically combine convolution in the spatial domain to RNN modeling in the time dimensions. Other approaches \cite{133} use stacked auto-encoders to represent spatio-temporal traffic flow features. As for energy, many general purpose neural forecasting models are benchmarked on traffic datasets \cite{51,179}.

Finance is typically more challenging than other application domains as the signal is removed by each participant in order to make profit: this is in contrast to demand forecasting where seasonality still exists even as the forecasting methods continue to improve. In addition, estimating the covariance matrix of a large number of series is crucial in this domain, a task which has proven hard for classical methods \cite{29,27}. Neural forecasting methods are used to handle complex signals that challenge the assumptions of some standard models such as linearity or Gaussian noise. Finance In this context, NN models have been applied to several areas, for instance to model long-tail distributions \cite{200} or volatility \cite{132,8} with RNNs. For example, \cite{8} propose a single-hidden layer NN for forecasting a volatility index.

In complex computer systems such as databases or cloud services, NNs are commonly used to forecast the state of the system and support automated decisions. Products such as AWS Auto Scaling\footnote{https://aws.amazon.com/autoscaling/} offers predictive scaling of compute resources. Overviews are available \cite{152,130,141,104,115,134}. A popular dataset for this class of tasks is the wikipedia website hits \cite{62}.

5.4 Remarks

Table \ref{tab:literature_overview} summarizes our literature overview on NNs. For many of the applications listed in the table, NNs are competitive, but determining whether they represent the state-of-the-art requires (much) further work.
The most important point about applications is perhaps the amount of data required to estimate the parameters of most neural networks. How much data does one need for a given application? Several important points should be discussed on this question. First, the amount of data is often misunderstood as the number of series but in reality the amount of data typically relates to the number of observations. For instance, one may have only one time series but many thousands of observations such as the case of a real-time sensor time series where measurements happens every second for a year allowing to fit a complex NN [2]. Second, it is probably better to see the amount of data in terms of information quantity. For instance, in finance the amount of information of many millions of hourly transactions is limited given the very low signal-to-noise ratio. In contrast to a retailer whose products follow clear seasonality and patterns, making it easier to apply NN methods.

To summarize, the amount of data is best understood as the number of observations but some domains may contain much less information, challenging the use of NNs compared to more robust linear models. On a practical point of view, NNs have been reported to outperform demand forecasting baselines starting from 50000 observations in [162] and from a few hundred observations in load-forecasting [154, 189]. Understanding better these limitation, both theoretically and empirically, is an area of current research and is not yet well understood. See [113] for some current theoretical work on sample complexity of local-global approaches for instance.

6 A look into the future

Having presented an overview of the current state of NNs for forecasting, in this section we distill some open questions and promising research directions. First, we presents questions and themes that are directly related to NNs in forecasting, then more general questions that go beyond NNs but that the success of NNs emphasises and finally, applications.

6.1 Research directions specific to NNs

The current research in NNs for forecasting can be roughly classified in (i) hybridizing existing time series techniques with NNs, or, (ii) bringing innovations from other, related areas or general purpose technique to forecast-
We expect both (i) and (ii) to continue to bear much fruit in the short to mid-term. For (i), NNs help to alleviate the often cumbersome work of feature engineering and can be useful tools to relax assumptions such as linearity or Gaussianity. One illustration of this line of work is the combination of state space models with NNs, which has seen continued attention \cite{154,172,62}. Other probabilistic models, such as renewal or point processes are only at the beginning of being integrated with NNs for forecasting \cite{183}.

For (ii), sequence models from areas such as Natural Language Processing and Machine Translation have natural extensions to forecasting and these will continue to be explored. Attention-based models have been considered in forecasting (e.g., \cite{50,123}), but given the overwhelming success of BERT \cite{50} in language-related areas, we expect this to be only the beginning. Other topics such as transfer learning or continual learning are seeing much attention in the larger ML community. They have natural applications in industrial forecasting systems and are currently under-explored. If the past is an indication for the future, then non-trivial extensions of existing techniques are needed to apply them to forecasting.

General challenges for NNs, such as data effectiveness, are important in forecasting and likely need a special treatment (see \cite{60} for an approach in time series classification with transfer learning). Other topics of general ML interest such as interpretability, explainability and causality (e.g., \cite{17,129,164}) are challenging in general for NNs. They have particular practical importance in the forecasting setting since the typical interaction with a forecasting system is in a business-to-business scenario and not in an end-consumer-facing scenario. In the scenarios important for forecasting, business analysts scrutinize forecasts carefully and they typically ask for interpretability and explainability right after an accurate forecasting method.

Beyond these organic developments, it is our hope that original methods such as new NN architectures will be pioneered in the time series prediction sector \cite{146} is a good example) and that those will then feed back into the general NN literature to help solve problems in other disciplines. Given the richness and challenging nature of industrial applications in forecasting and the inherent need for probabilistic forecasts, the time is right for truly new models to appear in the forecasting domain. Applications that could inspire such developments include spatio-temporal and hierarchical forecasting problems, where hierarchies can be temporal, spatial, or defined by other metadata. These applications are challenging enough to require and inspire
truly original methodologies that could find their way back into the NN mainstream.

Finally, gaining an empirical and systematic understanding of how NN architectures deliver increased accuracy as a function of properties of the data and of the forecasting task at hand would open the way for a better use of neural forecasting methods. Hewamalage et al. [77] provide first steps in this directions, focused primarily on RNNs.

6.2 General research directions emphasized by NNs

NNs have been successful in Forecasting when deployed as global models. However, many other methods can be used as global models, such as random forests. Traditional forecasting models, in contrast, are mostly used as local models. One clear research directions is to gain a better understanding of this contrast between global and local models. How much of the success of NNs for forecasting can be attributed to NNs being used as global models? Also, studying the interplay between global and local models is an important area. The goal should be to get the best of both local and global models. Wang et al. [189], Sen et al. [167] have started exploring this question, but this is an area which needs more attention.

NNs have contributed to spectacular success in reinforcement learning (e.g., [170] [171]). With reinforcement learning, there is an ML paradigm available which allows the consolidation of two-step approaches used in many applications for which forecasting is deployed. Often, forecasting merely serves as input to downstream decision problems (often mixed-integer non-linear stochastic optimization problems), for example to address problems such as restocking decisions. The forecast is therefore only indirectly interesting and the primary object of interest are the decisions. Since forecasting accuracy is only a proxy for overall improvements, we are bound to make suboptimal decisions. Reinforcement learning provides a framework to solve the decision problem directly [96]. It will be interesting to see whether reinforcement based approaches can improve decision making – and how good forecasting models could help improve reinforcement approaches.

6.3 Application areas

We believe that many improvements to forecasting methods, in particular NNs for forecasting, will come as a consequence of increasing the number
and complexity of forecasting applications. Many potential applications of forecasting methods are underexplored. To pick areas that are close to the authors’ interests, in database management, cloud computing, and system operations a host of applications would greatly benefit from the use of principled forecasting methods. Examples include predictive auto-scaling to manage computing resources needed for a system to run nominally, anomaly detection for cloud services (forecasting methods have been explored in this area but we believe we have only scratched the surface of what is possible) or access pattern forecasting in database. While the literature is heavy on problem descriptions, it is light on forecasting methodologies [141, 104, 115, 134, 169]. Commonly ad-hoc (and non-probabilistic) forecasting methods and evaluation metrics are used. Forecasting can also be used to help solve core ML tasks such as hyper-parameter optimization (e.g., [53]) and we expect more applications to open up in this area.

7 Conclusions

This article has attempted to provide an overview of the use of neural networks for forecasting. We began by providing a panorama over some of the core concepts in the modern literature on neural network chosen by their degree of relevance for forecasting. We then reviewed recent advances in neural forecasting models and presented a broad range of application areas in forecasting where these methods have proven their effectiveness.

Neural forecasting methods excel at addressing forecasting problems with many related time series and at extracting weak signals and complex patterns from large amounts of data. The availability of efficient programming frameworks help to alleviate many of the pain points that practitioners experience with other methods such as manual feature engineering or the need to derive gradients.

Neural networks are not a silver bullet. For many important classes of forecasting problems such as long-range macro-economomic forecasts or other problems requiring external domain knowledge not learnable from the data, neural forecasting methods are not the most appropriate choice and will likely never be. Still, it is our firm belief that neural networks belong to the toolbox of every forecaster, in industry and academia.
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