Study of high-strength steel fiber concrete strength characteristics under elevated temperatures using mathematical modelling methods

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Abstract. This article presents the study of the various factors influence on the high-strength steel fiber concrete strength characteristics using a mathematical model that was obtained on the basis of experimental data. At the same time, the steel-fiber concrete reinforcement, the scale factor, the temperature and the duration of heating are related to the factors under study. A fundamental approach to building a model is to use exponential functions. This function allows to move from the product of the studied factors to their sum, which greatly simplifies the further use of the dependencies obtained in engineering practice. Separately, it should be noted that one of the studied factors (the duration of exposure to temperatures) is set by the result of its impact on the final response function. That is, taking into account the destructive processes in the structure of concrete with short-term heating and structural – with long-term. In addition, the optimization of the proposed mathematical model is performed, as a result of which it is established that the maximum values of the strength of the steel fiber concrete is achieved at the values of the percentage of reinforcement of HSFRC.

1. Introduction

High rates of residential construction and industrial buildings with complex architectural forms, the construction of special structures of large-span bridges, skyscrapers, offshore oil platforms, tanks for liquids and gases storage, nuclear power plants protective shells, etc., require the development of new effective concretes. Dispersion-reinforced high-strength concrete is one of such materials. Fiber reinforcement is an effective means of increasing concrete strength and deformability under compression and tension, crack resistance and reinforced concrete structures rigidity. This is especially important for heavily loaded structures of high-rise buildings and structures, as well as for structures exposed to variable temperature and humidity effects. However, the usage of high-strength steel fiber concrete (HSFRC) for structures subjected to thermal effects, is constrained by insufficient knowledge of the elevated temperatures influence and duration of their action on strength characteristics HSFRC.
Thus, studies of the physics and mechanical properties of steel fiber concrete and methods for calculating and designing structures without considering the effects of elevated temperatures are presented in the works of S Caprielov [1], A P Krichevsky [2], L G Kurbatov [3, 4], I A Lobanov [5], V I Korsun [6, 7], V I Morozov [8], Yu V Pukharenko [9], F P Rabinovich [10], A E Sargsyan [11], K V Talantova [12], A Kelly [13]. On the other hand, sufficiently complete experimental data on the effect of high temperatures (till +200°C) on the main characteristics of the strength and deformation properties, but not on the HSFRC, and on reinforced concrete structures under uneven heating conditions, were obtained in the works of V I Veretennikov [14], M A Ivanov [15], V I Korsun [16, 17], K D Nekrasov [18], M Collegadi [19], C Galle [20], V M Malhotr [21], A P Krichevsky [22], A F Milovanov [23], N So Tupov [24], S L Fomin [25].

Based on this, the study of high-strength steel fiber concrete strength characteristics at elevated temperatures can be considered an urgent scientific task of sectoral importance. However, such studies require special equipment, are long in time, and their conduct is not rational without the use of mathematical and computer modeling tools.

2. Results of experimental research

Experimental studies of the physics and mechanical properties of HSFRC were performed on samples that were made of concrete modified with the help of an organic and mineral modifier (OMM) with the addition of steel fiber. Composition of high-strength concrete: Portland cement M500 – 545 kg/m³; quartz sand (MK = 1.9) – 660 kg/m³; crushed granite fraction 5...20 mm – 870 kg/m³; organic and mineral modifier (OMM) – 190 kg; water – 153 l/m³.

Fiber – made of steel with curved ends. Production: Khartsyzsk branch of PJSC «Production Association «STALKANAT-SILUR». It has the following characteristics: length \( l = 60.0 \pm 6.0 \) mm, diameter \( d = 0.75 \pm 0.07 \) mm; the length and height of the bent-end, respectively, \( l_1 = 5.0 \pm 1.0 \) mm, \( h_1 = 2.9 \pm 0.5 \) mm; temporary resistance gap – 1160-1290 MPa.

3 series of samples with fiber weight content per 1 m of concrete (in % of volume) were tested, respectively: series I – 0 kg (0%); series II – 50 kg/m (0.6%); series III – 200 kg/m³ (2.5%).

The research program included three groups of experiments:

- research of HSFRC temperature and shrinkage deformations. Study of their strength and deformation properties characteristics under conditions of axial compression and tension in the temperature range from +20°C to +200°C;
- investigation of shrinkage deformations dependence, strength and deformation properties characteristics of modified concrete on the size (scale) of prototypes;
- investigation of indirect mesh and dispersed reinforcement effect on the modified concrete prism samples strength and deformation under axial compression.

As a result, the following experimental data were obtained (table. 1).

3. A mathematical model of the strength characteristics ITSFM at compression in conditions of heating up to +200°C

For the analytical description of this dependence, we use a polynomial of the 3rd degree, which is able to provide the inflection point. In General, its equation can be represented as:

\[
y = ax^3 + bx^2 + cx + d,
\]

where \( a, b, c, d \) – are the coefficients of the equation determined on the basis of the available experimental data..

Similarly, we determine the dependence on the percentage of reinforcement and the scale factor, respectively, with the help of parabolic and linear dependences. It should be noted that the percentage of reinforcement is an order of magnitude less than other factors affecting the physical and mechanical properties of steel fiber concrete. When erected in the 3rd stage, this difference increases many times. To avoid this effect and to improve the accuracy of the simulation at a lower rounding of the values,
we use for modeling relative values reduced to natural values by the first values of the corresponding series.

### Table 1. The original experimental data.

| Relative characteristics of steel fiber concrete | Reinforcement, $\mu$, % | Scale factor, $M$ | Temperature, °C | Strength, $R_{stb}$ | $R_b$ | Elastic modulus, $E_{stb}$ | $E_b$ | Limiting the compressibility, $\varepsilon_{stb}$ | $\varepsilon_b$ | Coefficient of transverse deformation, $\eta_{stb}$ | $\eta_b$ |
|-------------------------------------------------|------------------------|------------------|-----------------|----------------------|------|-----------------------------|------|-----------------------------|------|-----------------------------|------|
|                                                 |                        |                  | 20              | 1,00 / 0,95          | 1,00 / 1,05 | 0,9 / 0,95                  | 1,06 / 1,34 | 1,00 / 1,21                    | 1,17 / 1,38 | 1,00 / 0,79                   | 0,76 / 0,99 |
| Strength, $R_{stb}$                             |                        |                  | 90              | 1,11 / 0,95          | 1,00 / 1,09 | 1,00 / 1,05                 | 1,14 / 1,31 | 1,06 / 1,25                    | 1,15 / 1,32 | 0,9 / 0,75                    | 0,7 / 0,99 |
|                                                 |                        |                  | 150             | 1,19 / 0,95          | 1,00 / 1,12 | 1,00 / 1,05                 | 1,06 / 1,34 | 1,06 / 1,25                    | 1,16 / 1,42 | 0,9 / 0,75                    | 0,7 / 0,99 |
|                                                 |                        |                  | 200             | 1,21 / 0,95          | 1,00 / 1,12 | 1,00 / 1,05                 | 1,06 / 1,34 | 1,06 / 1,25                    | 1,16 / 1,42 | 0,9 / 0,75                    | 0,7 / 0,99 |

By analogy with the research [26,27], we represent the mathematical model in the form of coefficients product affecting the desired strength characteristic:

$$
\frac{R_{stb}}{R_b} = \gamma_i \gamma_{stb} \gamma_{\mu},
$$

where $\gamma_i$ – factors that may affect the desired structural behavior.

To reduce the product of coefficients to the sum of polynomials, we define the coefficients $\gamma_i$ using an exponential function with polynomials of varying degrees:
\[ \gamma_i = e^{a_i \left( \frac{t}{20} \right)^3 + b_i \left( \frac{t}{20} \right)^2 + c_i \frac{t}{20} + d_i}; \]
\[ \gamma_\mu = e^{a_\mu \mu^2 + b_\mu \mu + c_\mu}; \]
\[ \gamma_M = e^{a_M \frac{M}{30} + b_M}; \]

(2)

where \( \gamma_i \) – coefficient corresponding to the temperature change of concrete;
\( \gamma_\mu \) – coefficient depending on the degree of reinforcement of concrete;
\( \gamma_M \) – scale factor.

Considering that all coefficients (2) have the same base, when substituting in (1) we get:
\[ \frac{R_{shb}}{R_b} = e^{a_i \left( \frac{t}{20} \right)^3 + b_i \left( \frac{t}{20} \right)^2 + c_i \frac{t}{20} + d_i + a_\mu \mu^2 + b_\mu \mu + a_M \frac{M}{30} + d_M}. \]

(3)

It should be noted that when adding polynomials, we get the free term \( d = d_i + c_\mu + b_M \).

Considering that all three coefficients are equal, it is possible in the future to divide the obtained value of the free term into three equal parts for each of the coefficients.

\[ d_i = c_\mu = b_M = \frac{d}{3}. \]

Let's take a logarithm both sides of equation (3):
\[ a_i \left( \frac{t}{20} \right)^3 + b_i \left( \frac{t}{20} \right)^2 + c_i \frac{t}{20} + d_i + a_\mu \mu^2 + b_\mu \mu + a_M \frac{M}{30} = \ln \frac{R_{shb}}{R_b}. \]

(4)

Thus, we obtain an equation with 7 unknown coefficients. Next, we use the least squares method to determine the values of polynomial coefficients at which the sum of the regression residuals squares takes the minimum value.

As a result, a mathematical model with a coefficient of determination was obtained for short-term heating. \( R^2 = 0.973 \):
\[ \left( \frac{R_{shb}}{R_b} \right)_{ST} = e^{-0.003 \left( \frac{t}{20} \right)^3 + 0.05 \left( \frac{t}{20} \right)^2 - 0.245 \frac{t}{20} - 0.044 \mu^2 + 0.185 \mu - 0.129 \frac{M}{30} + 0.342}. \]

(5)

Accordingly, for a long-term heating, a similar equation is obtained with the coefficient of determination \( R^2 = 0.892 \):
\[ \left( \frac{R_{shb}}{R_b} \right)_{LT} = e^{-0.002 \left( \frac{t}{20} \right)^3 + 0.02 \left( \frac{t}{20} \right)^2 - 0.127 \frac{t}{20} + 0.112 \mu^2 + 0.392 \mu - 0.07 \frac{M}{30} + 0.125}. \]

(6)

In accordance with the research of Professor V Korsun. [17], expressions of temperature influence functions and heating duration on the strength concrete characteristics under conditions of exposure to elevated temperatures reflect mainly destructive processes in the structure of concrete during short-term heating and mainly structural processes during long-term heating:
\[ \frac{R_{shb}}{R_b} = 1 - \left( \frac{R_{shb}}{R_b} \right)_{ST} + \left( \frac{R_{shb}}{R_b} \right)_{LT}. \]

(7)
Considering the duration of heating, we obtain a general mathematical model of the steel-fiber-concrete strength:

\[ \frac{R_{sh}}{R_b} = 1 - e^{-0.003\left(\frac{t}{20}\right)^2 - 0.05\left(\frac{t}{20}\right)^2 - 0.245\mu^2 + 0.185\mu^2 - 0.129M + 0.342} + e^{-0.002\left(\frac{t}{20}\right)^2 + 0.02\left(\frac{t}{20}\right)^2 - 0.127\mu^2 + 0.392\mu^2 - 0.071M + 0.525}. \]

It should be noted that the increase in the polynomial coefficients number does not affect the accuracy of the simulation results.

4. Research of strength characteristics of steel fiber concrete on the basis of the mathematical model

To analyze the resulting model, it is convenient to use its graphical display. However, the model of dependence of the strength factor \( \frac{R_{sh}}{R_b} \) from 4 factors is a complex geometric object belonging to a multidimensional space, which is difficult to visualize on a plane. In addition, one of the factors - the duration of the impact of temperatures is implicitly stated not by specific values, but by the result of its impact on the final response function. Proceeding from this, we will present graphical visualization of simulation results in the form of two off-click surfaces for the values of the scale factor \( M = 30 \) and \( M = 16 \).

We use the «solution Search» add-in of MS Excel office Suite to optimize simulation results. As a result, the maximum value of steel fiber concrete strength is equal \( \frac{R_{sh}}{R_b} = 1.345 \) achieved when \( t = 200°C \), \( M = 30 \) и \( \mu = 1,613 \). It should be noted that the increase in the strength factor is observed with increasing temperature \( \frac{R_{sh}}{R_b} \), but the optimal percentage of HSFRC reinforcement in terms of strength at different temperatures is the interval \( \mu = 1,5 - 1,7% \).

![Graphical visualization of the steel fiber concrete strength modelling results.](image-url)

Figure 1. Graphical visualization of the steel fiber concrete strength modelling results.

Similarly, we study the influence of factors on other physics and mechanical characteristics of steel fiber concrete. At the same time, the mathematical model itself will remain unchanged; only the coefficients of the polynomial dependencies of equation (3), which are obtained on the basis of experimental data, will change.
Let's perform a graphical visualization of the obtained models.

\[
\text{Elastic modulus } \frac{E_{eb}}{E_b}
\]

\[
\text{Ultimate compressibility } \frac{\varepsilon_{eb}}{\varepsilon_b}
\]

\[
\text{Lateral strain factor } \frac{\eta_{eb}}{\eta_b}
\]
Figure 2. Graphic visualization of the results of modelling the physics and mechanical properties of steel fiber concrete.

5. Conclusion
In this paper, we proposed an approach to the creation of the HSFRC physics and mechanical characteristics mathematical model under prolonged temperature exposure. In this case, three factors (the percentage of reinforcement, scale factor and temperature) are included in the final model in an explicit form. The fourth factor (the duration of exposure to temperature) is implicitly specified not by specific values, but by the result of its impact on the final response function (considering destructive processes in the structure of concrete during short-term heating and constructive - with long-term). On the basis of the obtained mathematical model, the interval of optimal HSFRC percent reinforcement values were selected. It allows to reach the maximum steel fiber concrete strength values. In addition, the resulting mathematical model can be applied to the calculation of any material characteristics, which will require only the recalculation of polynomial coefficients obtained on the basis of experimental data.

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