QGP Susceptibilities from PNJL Model

Sanjay K. Ghosh, Tamal K. Mukherjee
Department of Physics, Bose Institute, 93/1, A.P.C Road, Kolkata - 700 009, INDIA

Munshi G. Mustafa and Rajarshi Ray
Theory Division, Saha Institute of Nuclear Physics, 1/AF, Bidhannagar, Kolkata - 700 064, INDIA

An improved version of the PNJL model is used to calculate various thermodynamical quantities, viz., quark number susceptibility, isospin susceptibility, specific heat, speed of sound and conformal measure. Comparison with Lattice data is found to be encouraging.

I. INTRODUCTION

Under the extreme conditions of temperature and/or density a phase transition is expected in the QCD phase diagram. In fact, there are two phase transitions at these extreme conditions, namely the confinement to deconfinement phase transition and the chiral phase transition. These two phase transitions are defined in two extreme quark mass limits. For infinite quark mass limit confinement to deconfinement phase transition is well defined, and Polyakov loop acts as the order parameter of the transition corresponding to the \(Z(3)\) (for three colours) global symmetry breaking. In the limit of zero quark mass we have chiral symmetry restoring transition. This phase transition is characterized by the chiral condensate, which acts as the order parameter. But, in the real world where quark masses have a finite value, the nature of these phase transitions is not known. Whether these two transitions will occur simultaneously or one will precedes the other remain an open question. Order of the phase is another aspect of the phase transition which is also being pursued actively.

Here we are concerned with the thermodynamic aspect of the phase transition in terms of some relevant thermodynamic variables such as various quark number and isospin susceptibilities, specific heat, speed of sound etc., to see how the behaviour of these variables change as we go from one phase to the another. Being in the nonperturbative domain the Lattice QCD and the effective models are the only theoretical framework which can be employed to study the phase transition. The effective model we employ here is known as the PNJL (Polyakov loop + Nambu- Jona-Lasinio) model. Within the framework of PNJL model one can study both the confinement to deconfinement transition and the chiral phase transition together. For details of this model and of its various improvements see Refs. [1, 2, 3, 4, 5, 6, 7, 8, 9].

The plan of the paper is as follows. In the next section we briefly outline the theoretical framework. In section 3, our results are discussed and finally we conclude in section 4.

*Electronic address: sanjay@bosemain.boseinst.ac.in
†Electronic address: tamal@bosemain.boseinst.ac.in
‡Electronic address: munshigolam.mustafa@saha.ac.in
§Electronic address: rajarshi.ray@saha.ac.in
II. THEORETICAL FRAMEWORK

The coupling between the chiral and the deconfinement order parameter within the PNJL model allows us to study the thermodynamics of both the transitions inside a single theoretical framework. The thermodynamics of the gauge sector is controlled by the Polyakov loop whereas the NJL model takes care of the thermodynamics of the quark sector. To obtain various thermodynamic quantities we start with the thermodynamic potential per unit volume computed within the PNJL model [3, 10, 11]:

\[
Ω = \mathcal{U}(\Phi, \bar{\Phi}, T) + 2G_1(\sigma_u^2 + \sigma_d^2) + 4G_2\sigma_u\sigma_d - \sum_{f=u,d} 6 \int \frac{d^3 p}{(2\pi)^3} E_f \theta \left( \Lambda^2 - \vec{p}^2 \right) \\
- \sum_{f=u,d} 2T \int \frac{d^3 p}{(2\pi)^3} \left\{ \ln \left[ 1 + 3 \left( \Phi + \bar{\Phi} e^{-(E_f - \mu_f)/T} \right) e^{-(E_f - \mu_f)/T} + e^{-3(E_f - \mu_f)/T} \right] \\
+ \ln \left[ 1 + 3 \left( \Phi + \bar{\Phi} e^{-(E_f + \mu_f)/T} \right) e^{-(E_f + \mu_f)/T} + e^{-3(E_f + \mu_f)/T} \right] \right\},
\]

where \(\sigma_u\) and \(\sigma_d\) are, respectively, two light flavours condensates and the respective chemical potentials are \(\mu_u\) and \(\mu_d\). Note that \(\mu_0 = (\mu_u + \mu_d)/2\) and \(\mu_f = (\mu_u - \mu_d)/2\). The quasi-particle energies are \(E_{u,d} = \sqrt{\vec{p}^2 + m^2_{u,d}}\), where \(m_{u,d} = m_0 - 4G_1\sigma_{u,d} - 4G_2\sigma_{d,u}\) are the constituent quark masses and \(m_0\) is the current quark mass (we assume flavour degeneracy). \(G_1\) and \(G_2\) are the effective coupling strengths of a local, chiral symmetric four-point interaction. We take \(G_1 = G_2 = G/4\), where \(G\) is the coupling used in Ref. [3]. \(\Lambda\) is the 3-momentum cutoff in the NJL model. The form of the effective gauge potential is in the form given by,

\[
\frac{\mathcal{U}(\Phi, \bar{\Phi}, T)}{T^4} = \frac{\mathcal{U}'(\Phi, \bar{\Phi}, T)}{T^4} - \kappa \ln[J(\Phi, \bar{\Phi})],
\]

where \(\mathcal{U}'\) is taken from [3, 4]. It arises because of the transformation from matrix valued field \(L\) to complex valued field \(\Phi\). The role of this second term (known as Vandermonde determinant (VdM) term) is to regulate the behaviour of \(\Phi\) so that its value remains within the group theoretic limit. This term is dropped while calculating the pressure. For details we refer to [4].

Once the thermodynamic potential \(\Omega\) is fixed, we first obtain the mean fields by minimizing \(\Omega\) w.r.t the field variables \(\Phi, \bar{\Phi}, \sigma_u\) and \(\sigma_d\). We then calculate all the relevant thermodynamic quantities by appropriate differentiation of \(\Omega\). Here we are concerned with the various susceptibilities namely the quark number susceptibility (QNS) and isospin number susceptibility (INS). It is also to be noted that by QGP susceptibility we intend to mean these susceptibilities. Susceptibilities are important as they are related to fluctuations which can be measured experimentally.

We computed all these quantities from the Taylor expansion of the pressure, as it is related to the thermodynamical potential via the relation,

\[
P(T, \mu_0) = -\Omega(T, \mu_0) .
\]

For susceptibilities we expand the scaled pressure \((P/T^4)\) in a Taylor series for the quark number and isospin number chemical potentials, \(\mu_0\) and \(\mu_I\) respectively. The coefficients we are interested in are given by,

\[
c_n(T) = \frac{1}{n!} \left. \frac{\partial^n \left( P(T, \mu_0)/T^4 \right) }{\partial \left( \mu_0 \right)^n} \right|_{\mu_0=0} = c_{n0} ,
\]
\[ c_n(T) = \frac{1}{n!} \frac{\partial^n \left( P(T, \mu_0, \mu_I)/T^4 \right)}{\partial \left( \frac{\mu_0}{T} \right)^{n-2} \partial \left( \frac{\mu_I}{T} \right)^2} \bigg|_{\mu_0=0, \mu_I=0} = c_n^{(n-2)} ; \quad n > 1. \]  

The first equation represents the QNS and its higher order derivatives whereas the second one corresponds to the INS and its higher order derivatives. Specific heat and the speed of sound are calculated from the expansion of the pressure with respect to the temperature. Explicit expressions of these quantities are given by:

\[ C_V = \left. \frac{\partial \epsilon}{\partial T} \right|_V = -T \left. \frac{\partial^2 \Omega}{\partial T^2} \right|_V, \]  

\[ v_s^2 = \left. \frac{\partial P}{\partial \epsilon} \right|_S = \frac{\left. \partial P \right|_V}{\left. \partial T \right|_V} = \frac{\left. \partial \epsilon \right|_V}{\left. \partial T \right|_V} = \frac{T \left. \partial^2 \Omega \right|_V}{\left. \partial T^2 \right|_V}. \]

For details of computing these coefficients please refer to [9, 10].

III. RESULT

![Taylor expansion coefficients of the pressure in quark number and isospin chemical potentials as functions of T/T_c. Symbols are LQCD data [12]. Arrows on the right indicate the corresponding ideal gas values.](image)

We present the QNS, INS and their higher order derivatives with respect to \( \mu_0 \) in Fig. 1. We have plotted the LQCD data from Ref. [12] for quantitative comparison. We find, QNS \( c_2 \) (Fig. 1(a)) matches well with the Lattice data up to \( 1.2T_c \) but beyond which it goes to SB limit whereas the Lattice data saturates at 80% of the ideal gas value. Similar trend is observed for INS \( c_2 \). It remains close to LQCD data up to \( 1.2T_c \) and then it saturates at SB limit. However both QNS \( c_2 \) and INS \( c_2 \) agree well with each other which is also observed in Lattice calculation. It is to be noted here the QNS \( c_2 \) matches well with the LQCD data as shown in [10], if we do not include the VDM term.

Variation of next higher order coefficients \( c_4 \) of both QNS and INS with temperature are shown in Fig. 1(b). We find a good agreement for both the coefficients with the Lattice data for whole range of temperatures. If we do not include the VDM term (see [10]) then the QNS \( c_4 \) matches the
Lattice data only upto $T \sim 1.1T_c$. The effect of the VDM term is to bring the $c_2$ and $c_4$ coefficients to their respective SB limit.

In Fig. 2 we have presented our result for specific heat, speed of sound and conformal measure. Compared to our previous observation in Ref. [10] the specific heat $C_V$ differs only marginally at higher temperature. We find a small peak near the transition temperature which in the case of critical end point (CEP) will diverge. At higher temperature it saturates below the ideal gas limit and approaches the conformal limit of $4\epsilon/T^4$ from above. The effect of the VDM term cancels out for both the speed of sound and the conformal measure as they are the ratios of the energy density and pressure.

IV. SUMMARY

Various thermodynamical variables of importance are calculated using an improved version of the PNJL model. All the variables estimated are found to be in reasonable agreement with the LQCD data, though the QNS $c_2$ and the INS $c_2$ on the Lattice are smaller by about 20%. We found all thermodynamical variables approach to their respective ideal gas limit at higher temperature.

References

[1] P. N. Meisinger and M. C. Ogilvie, Phys. Lett. B 379 163 (1996); Nucl. Phys. B (Proc. Suppl.) 47 519 (1996).
[2] K. Fukushima, Phys. Lett. B 591 277 (2004).
[3] C. Ratti, M.A. Thaler and W. Weise, Phys. Rev. D 73 014019 (2006).
[4] S. Roszner, C. Ratti and W. Weise, Phys. Rev. D 75 034007 (2007).
[5] B-J. Schaefer, J. M. Pawlowski and J. Wambach, Phys. Rev. D 76 074023 (2007).
[6] D. Blaschke, M. Buballa, A. E. Radzhabov and M. K. Volkov, hep-ph/0705.0384.
[7] E. Megias, E. R. Arriola, L. L. Salcedo, Phys. Rev. D 74 065005 (2006); ibid Phys. Rev. D 74 114014 (2006).
[8] S. Roessner, T. Hell, C. Ratti, W. Weise, arXiv:0712.3152 [hep-ph].
[9] S. K. Ghosh, T. K. Mukherjee, M. G. Mustafa and R. Ray, arXiv:0710.2790 [hep-ph], Phys. Rev. D (in press, June, 2008).
[10] S. K. Ghosh, T. K. Mukherjee, M. G. Mustafa and R. Ray, Phys. Rev. D 73 114007 (2006).
[11] S. Mukherjee, M. G. Mustafa and R. Ray, Phys. Rev. D 75 094015 (2007).
[12] C. R. Allton et al., Phys. Rev. D 71 054508 (2005).