Uemura plot as a certificate of two-dimensional character of superconducting transition for quasi-two-dimensional HTS

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Abstract

For quasi-two-dimensional HTS in superconducting state the dimensional crossover, $3D \rightarrow 2D$ is studied. With using general properties of superconducting state for 2D systems the universal temperature dependence of the relation of the penetration lengths of magnetic field along axis $\hat{c}$, $\lambda^2(0)/\lambda^2(T/T_c)$, is found out. This dependence evidences about two-dimensional character of superconducting transition for quasi-two-dimensional HTS and leads to the Uemura plot for quasi-two-dimensional HTS.

1. Introduction. Quasi-two-dimensional (quasi 2D) or highly anisotropic underdoped HTS are exhibiting such out of the ordinary properties as "semiconducting-like" c-axis resistivity, $\rho_c(T)$, near $T_c$, big interval of two-dimensional superconducting fluctuations, $\Delta^N_{2D} \sim T_c$, and pseudo-gap states at $T \lesssim T^*$, where $T^*$ is the temperature of charge ordering. These properties lead to the discussion of the association of superconducting transition with Berezinskii-Kosterlitz-Thouless (BKT) transition in CuO$_2$ planes at $T_{BKT} < T_c$, where $T_{c0}$ is the temperature of two-dimensional superconducting transition in the mean-field theory (see, for example, review [1] and the references there). Despite the observations of two-dimensional character of superconducting fluctuations and the evidences for a BKT transition have been reported in most of quasi 2D HTS [2,3], up to now the question whether a BKT transition is observable in bulk cuprates with taken into account interlayer coupling in these materials [4,5] is under the discussion.
Interlayer coupling can lead to the dimensional crossovers both as at \( T > T_c \) (2D \( \rightarrow \) 3D), so in superconducting state near \( T_c \) (3D \( \rightarrow \) 2D) [4-6]. Theoretical model of such superconducting transition is well known [7-9]: at enough small probability \( t_c \) of charge tunnelling between \( CuO_2 \) planes transition has two-dimensional character with finite region \( \Delta_{3D} \) of three-dimensional superconducting fluctuations. Anisotropy of exchange interactions in \( CuO_2 \) planes and along axis \( \hat{c} \) in spin-fluctuational pairing model assumes big values of \( \Delta_{2D}^N \) and \( T_{c0} \) and the distinction of temperatures \( T_c << T_{c0} \). For quasi 2D HTS in ref. [10,11] it was shown that at \( T > T_c \) two-dimensional character of superconducting fluctuations leads to the temperature dependence of the probability of charge tunnelling \( t_c(T) \), and to the exasperation of semiconducting-like of \( \rho_c(T) \) near \( T_c \), so that 2D \( \rightarrow \) 3D crossover occurs at \( T_c^0 > T_c > T_{BKT} \) before BKT transition. This point out to the two-dimensional character of superconducting transition, which develops under Kats scenario [7] at enough small value of probability \( t_c(T_c^0) \):

\[
T_c^0/\varepsilon_F < t_c(T_c^0),
\]

where \( \varepsilon_F \) is Fermi energy, values \( T_c^0 \) and the temperature of three-dimensional transition in the mean field theory are the same order values. The interval \( \Delta_{3D}^N \) of three-dimensional fluctuations in normal state can be found out from the measurements of resistivity \( \rho_c(T) \)

\[
\Delta_{3D}^N \simeq T_c^0 - T_c << \Delta_{2D}^N
\]

It is known that at \( T < T_c \) back crossover 3D \( \rightarrow \) 2D occurs at \( T_{cr} \), which value depends on correlation length \( \xi_c(T) \) along axis \( \hat{c} \) [6, 12]. This paper is devoted to the studying of the dimensionality of the superconducting state of quasi 2D HTS and to the determination of the values \( T_{cr} \). With using general properties of superconducting state for 2D systems the universal temperature dependence of the relation of the penetration lengths of magnetic field along axis \( \hat{c} \), \( \lambda^2(0)/\lambda^2(T/T_c) \), is found out.

2. **Universal dependence of** \( T_c(\lambda^{-2}(0)) \). The penetration lengths of magnetic field along axis \( \hat{c} \), \( \lambda(T) \) determines by London formula \( \lambda(T) \simeq n_{s,3}^{-1/2} \), where \( n_{s,3} \) is the three-dimensional superfluid density. The density \( n_{s,3} \) evidently interconnects with two-dimensional superfluid density \( n_{s,3} = n_s(T)\nu/l \), where \( \nu \) is number of layers, \( l \) is the lattice constant. V.Pokrovskii shown that for quasi 2D HTS the penetration length \( \lambda(T) \) and two-dimensional superfluid density \( n_s(T) \) is connected by the expression

\[
\lambda^2(0)/\lambda^2(T) = n_{s,3}(T)/n_{s,3}(0) = n_s(T)/n_s(0),
\]

Here it will shown that for quasi 2D HTS (3) leads to universal dependence of \( T_c(\lambda^{-2}(0)) \),
which was called "Uemura plot" and was discovered at the measurements of muon relaxation rate [14].

For plane system (3) can be written as ratio \( \rho_s(T/T_c) = n_s(T/T_c)/n_s(0) \), where \( \rho_s(T/T_c) \) is dimensionless hardness and satisfy to universal dependence [15]

\[
\rho_s(T/T_c) = \exp\left(-\frac{T e^{-1}}{T_c \rho_s(T/T_c)}\right)
\]  

(4)

The decision of (4) was received in ref.[15]: \( \rho_s(0) = 1 \), and at \( T = T_c \)

\[
\rho_s(T/T_c)|_{T=T_c} = e^{-1}
\]  

(5)

Expressions (3)-(4) results to universal character of temperature dependence of ratio

\[
\lambda^2(0)/\lambda^2(T/T_c) = \rho_s(T/T_c)/\rho_s(0) = \exp\left(-\frac{T e^{-1}\lambda^2(T/T_c)}{T_c \lambda^2(0)}\right),
\]  

(6)

and simple relation between values \( \lambda^2(T) \) and \( n_s(T) \) at \( T = T_c \) and at \( T = 0 \):

\[
\lambda^2(0)/\lambda^2(T_c) = n_s(T)/n_s(0) = e^{-1}
\]  

(7)

Using this relation and Kosterlitz-Thouless-Nelson formula [16]

\[
k_B T_c = \frac{\hbar^2}{32 \pi m} n_s(T_c),
\]  

(8)

we can receive the universal relation between the density \( n_s(0) \) at \( T = 0 \) and transition temperature \( T_c \):

\[
T_c = \frac{k^2 e^{-1}}{32 k_B \pi m} n_s(0),
\]  

(9)

where \( k_B \) is Boltzman constant. For two-dimension superconductor the role of effective penetration length acts magnetic screening length

\[
L_s(T) = \frac{mc^2}{2 \pi n_s(T) e^2},
\]  

(10)

which is connected with the bulk London penetration length

\[
L_s = 2d^{-1} \lambda^2,
\]  

(11)

where \( d \) - thickness of \( CuO_2 \) plane.

It is seen from (7-11) that the transition temperature is proportional to \( \lambda^{-2}(0) \)

\[
T_c = k \lambda^{-2}(0),
\]  

(12)

where

\[
k = \frac{\epsilon^2 \hbar^2 e^{-1} d}{64 k_B \pi^2 e^2},
\]  

(13)
depends only from universal constants and from thickness of CuO$_2$ plane. The temperature of dimensional crossover, $T_{cr} < T_c$, depends on correlation length $\xi_c(T)$, and can be determined at the measurement of penetration length as the boundary of two-dimensional region, where at $T > T_{cr}$ the measurement values of $\lambda^2(0)/\lambda^2(T/T_c)$ are diverging from universal relation (6). Knowing $T_{cr}$ let us to determine the interval $\Delta_{3D}^S \approx (T_c - T_{cr})$ of three-dimensional fluctuations at $T < T_c$. Full interval is equal

$$\Delta_{3D} = \Delta_{3D}^N + \Delta_{3D}^S = T^0_c - T_{cr}$$

(14)

does not depend on the exactness of the measurement of $T_c$, and determines as the difference between the temperatures of two dimensional crossovers: in normal state, $T^0_c$, and in superconducting state, $T_{cr}$. So, for $La_{1.85}Sr_{0.15}CuO_4$ the measurements of resistivity $\rho_c(T)$ [17] and of the penetration length [18] let us to determine the temperature $T^0_c \sim 41.5K$, and $T_{cr} \sim 26K$. This lead to the value $\Delta_{3D} \sim 15.5K$. We see that the region $\Delta_{3D}$ of three-dimensional superconducting fluctuations is finite in the concrete, that evidences about the two-dimensional character of superconducting transition.

Thus, here it is shown that expression (12), which was found out at the measurements of nuon relaxation rate [14] for quasi 2D HTS, really is universal and is the sequence of general consistent pattern (4), (8) of superconducting state for two-dimensional systems and must to fulfil for HTS with two-dimensional superconducting state at $T < T_{cr}$. This means that for quasi 2D HTS superconducting transition has two-dimensional character and develops on Kats scenario [7], and at $T < T_{cr}$ the temperature dependence of $\lambda^2(0)/\lambda^2(T/T_c)$ also must to be universal (6).

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