Fractality feature in oil price fluctuations

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The scaling properties of oil price fluctuations are described as a non-stationary stochastic process realized by a time series of finite length. An original model is used to extract the scaling exponent of the fluctuation functions within a non-stationary process formulation. It is shown that, when returns are measured over intervals less than 10 days, the Probability Density Functions (PDFs) exhibit self-similarity and monoscaling, in contrast to the multifractal behavior of the PDFs at macro-scales (typically larger than one month). We find that the time evolution of the distributions are well fitted by a Lévy distribution law at micro-scales. The relevance of a Lévy distribution is made plausible by a simple model of nonlinear transfer.

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I. INTRODUCTION

Many natural or man-made phenomena, such as turbulence flows, fluctuations in finance (stock market), seismic recording, internet traffic, climate change, etc., are characterized by randomness or stochasticity [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. Analysis of non-stationary stochastic processes referring to quantities which fluctuate widely and are uncertain has been a problem of fundamental interest for a long time. Over the past two decades, oil price has increased very sharply, rising from $25 per barrel in January 1986 to a peak of close to $122 per barrel in the last week of July 2008. The effects of oil price fluctuations on the world economy are undeniable and particularly evident from the international reports. Oil price data as a time series exhibit complex patterns and seemingly appear to be a chaotic system. Indeed, the behavior of oil price fluctuations can be efficiently modelled by the Ising model which was proposed for stock-price fluctuations [19] and by the cascade model developed based on fractal concepts which was used for hydrodynamics and magnetohydrodynamic turbulence [20, 21, 22]. Here, we employ the latter technique to characterize the statistical properties of the oil-price time series, which sensitively distinguish between self-similarity and multi-fractality in a time series. The model is based on two-point increments of oil price, yielding a comprehensive and scale-dependent characterization of the statistical properties of the system via an associated Probability Density Function (PDF). It is necessary to stress that the data series is represented by a finite number of records which do not constitute a stationary process. The effect of non-stationarity on the detrended fluctuation analysis has been investigated in Ref. [23]. To eliminate the effect of sinusoidal trend, we apply the Fourier Detrended Fluctuation Analysis (FDFA). After the elimination of the trend we use the Multifractal Detrended Fluctuation Analysis (MF- DFA) to analysis the data set. The MF- DFA methods are the modified version of detrended fluctuation analysis (DFA) to detect multifractal properties of time series. The detrended fluctuation analysis (DFA) method introduced by Peng et al. [24] has became a widely-used technique for the determination of (multi)fractal scaling properties and the detection of long-range correlations in noisy, non-stationary time series [23, 24]. It has successfully been applied to diverse fields such as DNA sequences [25, 26], heart rate dynamics [10, 27], neuron spiking [28], human gait [29], long-time weather records [30, 31], cloud structure [32, 33], geology [34], ethnology [35], economical time series [36], solid state physics [37], sunspot time series [38], and cosmic microwave background radiation [11, 12].

We propose a method for generating a stationary process analysis out of a non-stationary process. The fact is that stationary stochastic systems often show scaling in a statistical sense, coincident with non-Gaussian leptokurtic (heavy-tailed) statistics. Importantly, identification of the associated scaling exponents implies the ability to interpret and estimate the behavior of the fluctuations as well as the detection of long-range correlations. A self-similar Brownian walk with Gaussian PDFs, which has scaling exponent 1/2, is a good example of the process where shows uncorrelation on all temporal scales. We try to determine the scaling properties of the PDFs that are leptokurtic at micro-scales. The scaling exponents can be determined through the scaling behavior of the moments, usually characterized by computing structure functions. It is said that the fluctuations are self-similar.
(monofractal) if scaling exponents of the moments exhibit a linear power-law dependence. In contrast a nonlinear dependence infers to multifractal scaling, which is caused by intermittent small-scale structures of oil price fluctuations. A similar feature has been found in physical systems for example, in velocity and magnetic fields of the solar wind \cite{37,38,39} as well as in magnetohydrodynamic turbulence studied via direct numerical simulations \cite{22}. Finally, distributed price changes are characterized by a stable Lévy distribution which will be discussed in Section V. Finally, in Section VI we will summarize all results discussed throughout this paper.

The paper is structured as follows. In Section II we describe our data set. The MF-DFA method is briefly presented in Section III and shown that the scaling exponent determined via the MF-DFA method are identical to those obtained by the standard multifractal formalism based on PDF analysis. In Section IV we employ a recently developed technique \cite{22,37,38} that sensitively distinguishes between self-similarity and multifractality in times series. By analyzing the temporal evolution of price dynamics, we demonstrate the strongly the non-Gaussian behavior of the returns of oil price and scale-dependent behavior of the PDFs. Also we explain the Hurst exponents analysis. The micro-scale PDFs resemble leptokurtic Lévy distribution which will be discussed in Section V. Finally, in Section VI we will summarize all results discussed throughout this paper.

II. THE DATA

Over a twelve-year period, on average, the price of oil has increased from $25 per barrel in January 1986 to a peak of close to $122 per barrel in the last week of July 2008. Oil price as recorded in international markets \cite{40} offer us an almost unique possibility to gain information on the stochastic dynamics state in a very large scale range, say 1 day up to 200 days. Our database consists of about 5695 daily price values which seem to provide a set of data points which will be sufficient to obtain the scaling properties the system. Fig. 1 presents the daily fluctuations in oil price $p(t)$ in the period 1986-2008. It is evident from the figure that the fluctuations do not constitute a stationary process, for instance one can show that the variance of the signal in some window does not remain stable upon increasing the window size. Let us introduce the increments (or returns) $\delta p(t,\tau)$ defined by, $\delta p(t,\tau) = p(t+\tau)-p(t)$. The resulting series for $\delta p(\tau)$ is shown in the inset graph of Fig. 1. It is straightforward to show, by measuring the average and the variance of $\delta p(t,\tau)$ in a moving window, that $\delta p(t,\tau)$ is stationary. Upon initiating the analysis of the distribution of oil price returns, the mean, standard deviation, skewness, and kurtosis of the return series are calculated (see Table I). It is easy to show that the skewness of a Gaussian function is zero so that the negative value of skewness, $\lambda = -0.606749$, is a hallmark of departure of the PDFs from the Gaussian distribution (such as a leptokurtic distribution), which confirms the existence of intermittency in the fluctuations. On the other hand the large value of kurtosis, $\kappa = 9.55225$, with respect to Gaussian kurtosis ($\kappa = 3$), show that the tails of the return distribution are fatter than the Gaussian ones.

III. THE MF-DFA ANALYSIS

The MF-DFA method is a modified version of detrended fluctuation analysis to detect multifractal properties of a time series. Omitting unnecessary details, a brief summary of the method for calculating MF-DFA based on fractal concepts can be formulated in five steps. We take the price series $p_k$ with the size of $N$ and follow the steps as follow:

- **step 1:** Determine the ”profile”

$$Y(i) = \sum_{k=1}^{i} [p_k - \langle p \rangle], \quad i = 1, \ldots, N$$

where $\langle p \rangle$ is the mean of the series. Subtraction of the mean $\langle p \rangle$ is not compulsory, since it would be eliminated by the detrending later in the third step.

- **step 2:** Divide the profile $Y(i)$ into $N_s \equiv int(N/s)$ nonoverlapping segments of equal lengths $s$. Since the length $N$ of the series is often not a multiple of the considered time scale $s$, a short part at the end of the profile may remain. In order not to disregard this part of the series, the same procedure is repeated starting from the opposite end.

- **step 3:** Calculate the local trend for each of the $2N_s$ segments by a least-square fit of the series. Then deter-
The Hurst exponent is called the scaling exponent or correlation function, so, 0 \leq \alpha < \infty. As a result, the fluctuation function \( \mathcal{F}_q(s) \) shows a scaling behavior, \( \alpha(2) \), which is identical to the well-known Hurst exponent \( H \). The Hurst exponent is called the scaling exponent or correlation exponent, and represents the correlation properties of the signal. If \( H = 0.5 \), there is no correlation and the signal is uncorrelated, if \( H > 0.5 \), there is positive correlation in the signal. We obtain the following estimate for the Hurst exponent, \( H = 0.52 \pm 0.02 \) as we can see in Fig. 2. Since \( H > 0.5 \) it is concluded that oil price returns show persistence, i.e., a certain correlation among consecutive increments. For \( s \sim 150 \) the empirical data deviate from the initial scaling behavior, as we can see in Fig. 2. This indicates that oil price tends to lose its memory after a period of the order of 200 days or less.

## IV. STATISTICAL SELF-SIMILARITY

A set of time series \( \delta p(t, \tau) \) is obtained for each value of nonoverlapping time lag \( \tau \). The return of the stochastic variable \( \delta p(t, \tau) \) is said to be self-similar with parameter \( \alpha \) (\( \alpha \geq 1 \)), if for any \( \lambda \)

\[
\delta p(\tau) \sim \lambda^\alpha \delta p(\lambda \tau). \tag{5}
\]

The relation (5) is interpreted as an equality in law (EL), that is the two sides of the equation have the same statistical properties. For the associated cumulative probability distribution, it follows that

\[
\varphi(\delta p(\tau) \leq \rho) = \varphi(\lambda^\alpha \delta p(\lambda \tau) \leq \rho), \tag{6}
\]

for any real \( \rho \). This implies for the probability density \( P \)

\[
P[\delta p(\tau)] = \lambda^{-\alpha} P_s[\lambda^{-\alpha} \delta p_s], \tag{7}
\]

introducing the master PDF \( P_s \) with \( \delta p_s = \delta p(\lambda \tau) \). According to Eq. (7), there is a family of PDFs that can be collapsed to a single curve \( P_s \), if \( \alpha \) is independent of \( \tau \). This is known as monoscaling in contrast to multifractal scaling observed, e.g., for oil price returns at macro-scales.

To characterize quantitatively the observed stochastic process, we measure the PDF \( P(\delta p) \) of the price fluctuations for \( \tau \) ranging from 1 to 200 days. The number of data in each set decreases from the maximum value of 5695 (\( \tau = 1 \)) to the minimum value of 5495 (\( \tau = 200 \)). In Fig. 3, we show the four selected PDFs (normalized with the variance \( < \delta p(\tau)^2 >^{1/2} \)) for \( \tau = 1, \tau = 20, \tau = 60, \) and \( \tau = 200 \) from the top (right side) to the bottom (right side) respectively. The distributions lose their leptokurtic shape, as \( \tau \) increases. Due to the lack in correlation among distant fluctuations, the associated distributions become approximately Gaussian at macro-scale. The scaling behavior of the distribution at coarser time scales has two different regimes. At micro-scales (typically shorter than 10 days), correlations between successive price changes are dominant. This may be due to several reasons, such as oil pipe line damage, weather changes, or local variations in the internal (US) oil availability. Interestingly, the PDFs at the micro-scales have the same leptokurtic shape, exhibit monoscaling, and do not change fundamentally, and resemble closely Lévy distributions, see Fig. 4. On the other hand, at macro-scale (typically larger than one month) permanent crisis
The analysis which follows is also valid for the moments; however, structure functions are typically calculated for a data series. The arguments do not apply to structure functions. The generalized structure function of order \( n \) are simply defined as

\[
\langle |\delta p|^n \rangle = \int_{-\infty}^{\infty} |\delta p|^n P(\delta p, \tau) d(\delta p). \tag{8}
\]

The analysis which follows is also valid for the moments; however, structure functions are typically calculated for a data series. The arguments do not apply to structure functions of odd order, which not only may have negative coefficients, but could in fact even change the sign of the scaling range. The proof will, however, remain valid for odd orders if the structure functions are defined with the absolute value of the returns. Using the relation (7) we obtain

\[
\langle |\delta p|^n \rangle = \lambda^{\zeta_n} S^n_s(\delta p_s; \pm \infty), \quad \tag{9}
\]

where the linear function \( \zeta_n = \alpha n \) refer to the statistical self-similarity, monoscaling case. On the contrary, in some cases, one may observe multifractality scaling, in the sense that a nonlinear dependence is observed on \( n \) where \( \zeta_n = n\alpha(n) \) a convex function of \( n \) and \( \zeta_{n+1} > \zeta_n \forall n \). This deviation from strict self-similarity over all time scale \( \tau \), also termed multifractal scaling, is caused by the intermittent micro-scale structure of turbulence.

To test if the above-mentioned interesting observations in oil price are a phenomenon related to inherent properties of stochastic processes, structure functions \( S^n(\tau) \) for different \( \tau \), given by Eq. (9), are computed. A difficulty that can arise in the experimental determination of the \( \zeta_n \) is that for a finite length times series, the integral Eq. (8) is not sampled over the range \((-\infty; +\infty)\), rather the outlying measured values of \( y \) determine the limit, \([-y; +y]\). Fig. 6 shows some selected \( S^n(\tau) \), according to Eq. (8). The slope of the curve gives scaling exponent \( \zeta_n \) which can be obtained by fitting a straight line to a log-log plot in the interval \( l \in [1, 100] \). Because of increasing of the statistical errors at the higher orders, such a fitting becomes rather arbitrary. In Fig. 7 we report the scaling exponents extracted from the structure functions. The behavior of \( \zeta_n \) against \( n \) shows that scaling exponents have nonlinear behavior at all scales, say, they are different from the usual linear law.

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FIG. 3: The PDFs of the normalized oil price fluctuations for four different scales, (a): \( \tau = 1 \), (b): \( \tau = 20 \), (c): \( \tau = 60 \), and (d): \( \tau = 200 \). For comparison, the Gaussian distribution with the same variance is depicted(solid line).

FIG. 4: The PDFs of the oil price fluctuations on micro-scales. We can see that the shape of the PDFs do not change fundamentally as a sequent of the monofractality.

FIG. 5: The PDFs of the oil price fluctuations on macro scales. The PDFs become Gaussian as the time scales is increased. This is coincide with multifractality feature.
FIG. 6: The structure functions are depicted, as computed from Eq. (8). In order to obtain the scaling exponents, we take the logarithmic slope of linear least-square fits (solid line).

FIG. 7: The scaling exponents \( \zeta_n \) of the structure functions are depicted versus the corresponding order \( n \). The nonlinear behavior is obvious in the figure.

To apply the rescaling procedure given by Eq. (7) the exponent \( \alpha \) is extracted from the underlying data by an independent technique. The standard deviation which is defined by the root of the second-order structure function, \( \sigma(\tau) = [S^2(\tau)]^{1/2} \) and has the minimum of statistical error, exhibits power-law behavior with respect to the increment distance, \( \sigma(\tau) \sim \tau^\alpha \) as depicted in Fig. (8).

A linear least-square fit is carried out to obtain \( \alpha \). The characteristic exponent deduced in this way is \( \alpha = 0.52 \pm 0.03 \) which is in good agreement with the value of \( H \) obtained in Section (III). Fig. (9) and Fig. (10) show the rescaled sets of PDFs on the micro- and macro-scales respectively. Evidently the PDFs at micro-scales are self-similar and collapse with weak scattering on the master PDF, \( P_x \), when using the characteristic exponents given above. The corresponding increment distances \( \tau \) are all shorter than 10 days. We may model this PDFs by a Lévy distribution, which thus turns out to be a successful fit to the distribution of oil price fluctuations. On the other hand, at macro-scale the PDFs do not show a self-similar behavior and rather constitute a multiscascade process. This occurs at scales larger than the one month. Because of the resulting multifractal scaling of the PDFs it is evident that they can not collapse onto a single curve, see Fig. (10).

V. LÉVY DISTRIBUTION MODEL

Lévy-stable laws are a rich class of probability distributions that comprise fat tails and have many intriguing mathematical properties. They have been proposed as models for many types of physical and economic systems. There are several reasons for using Lévy-stable laws to describe complex systems. First of all, in some cases there are solid theoretical reasons for expecting a non-Gaussian Lévy stable model, which can be a good fitting to experimental and numerical results. The second reason is the Generalized Central Limit Theorem which states that the only possible non-trivial limit of normalized sums of independent identically distributed terms is Lévy-stable. (Recall that the classical Central Limit Theorem states that the limit of normalized sums of independent identically distributed terms with finite variance is Gaussian.) The third argument for modeling with Lévy-stable distributions is empirical; many large data sets exhibit fat tails (or heavy tails); for a review see [42, 13]. Such data sets were described by a Casting model based on the lognormal ansatz (in terms of the variance of the Gaussian distribution). To confirm that model it is convenient to summarize the basic
and the symmetric Lévy distribution becomes

\[
L_\mu(\delta x_{\Delta s}) \equiv \frac{1}{\pi} \int_0^\infty \exp(-\gamma \Delta s q^\mu) \cos(q\delta x_{\Delta s}) dq, \tag{12}
\]

where the increment is \( \delta x = x_s - x_{s-\Delta s} \); here, \( 0 < \mu < 2 \), and \( \gamma > 0 \) is a scale factor. The maximum event probability leads to

\[
P(0) = L_\mu(0) = \frac{\Gamma(1/\mu)}{\pi \mu (\gamma \Delta s)^{1/\mu}}. \tag{13}
\]

The exponent \( \mu \) of the best fits is constant at the microscales range and amounts approximately to \( \mu \sim 1.92 \) which is \( \mu = 1/\alpha \). Natural phenomena also investigated where similar findings have been reported include, for example, financial systems e.g. the Tehran price stock market, where \( \mu \sim 1.36 \) \cite{4}, and also physical systems such as the solar wind, where \( \mu \sim 3.3 \) \cite{44}. From Fig. 9 we conclude that a central section of Lévy distributions describe very well the dynamics of the PDFs of oil price fluctuations at micro-scales. One can see that the rescaled PDFs are definitely non-Gaussian.

VI. SUMMARY

In this paper, we have presented a statistical analysis of oil price in the United States for the period of 1986 to 2008. We have applied a generic MF-DFA method to extract scaling exponent of the fluctuation functions, in particular, relying on the second order \( F_2(s) \) which was used in the rescaling procedure. However, the simple scaling properties that we have found via an analysis of the PDFs, allow us to detect mono(multi) fractality feature over all time scales. The presence of intermittency in oil price fluctuations is manifested by the leptokurtic nature of the PDFs which show increased probability of large fluctuations compared to that of the Gaussian distribution. Fluctuations on the macro temporal scales, \( \tau > 10 \) day, converge toward a Gaussian distribution and are an uncorrelated signal. The reason may be due to unstable conditions in the Middle East or OPEC’s decisions oil production. The critical macro scale which was obtained is different for some financial and physical systems; see for example in Refs. \cite{4, 10}. We have also obtained a good collapse onto a single curve for \( \tau < 10 \), according to the rescaling procedure \cite{7}. The proximity of the PDFs to Lévy distributions is made plausible by a simple model mimicking nonlinear spectral transfer.
