Maximum mass of compact stars from gravitational wave events with finite-temperature equations of state

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We conjecture and verify a set of relations between global parameters of hot and fast-rotating compact stars which do not depend on the equation of state (EoS), including a relation connecting the masses of the mass-shedding (Kepler) and static configurations. We apply these relations to the GW170817 event by adopting the scenario in which a hypermassive compact star remnant formed in a merger evolves into a supramassive compact star that collapses into a black hole once the stability line for such stars is crossed. We deduce an upper limit on the maximum mass of static, cold neutron stars 2.15_{-0.18}^{+0.16} \leq M_{\text{TOV}} \leq 2.24_{-0.44}^{+0.45} for the typical range of entropy per baryon 2 \leq S/A \leq 3 and electron fraction Y_e = 0.1 characterizing the hot hypermassive star. Our result implies that accounting for the finite temperature of the merger remnant relaxes previously derived constraints on the value of the maximum mass of a cold, static compact star.

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I. INTRODUCTION

Neutron (or compact) stars, containing matter at densities exceeding that at the centers of atomic nuclei, represent unique laboratories to probe the matter under extreme conditions. Considerable effort is underway to pin down the dense matter equation of state (EoS) as present in neutron stars, which is pressed ahead by many recent observations and the prospects opened by the dawn of multi-messenger astrophysics. Among these are the precise pulsar mass determinations from the pulsar timing analysis [1,2], measurements of compact star masses and radii through the x-ray observations of their surface emission [3,4] in particular, the results of the NICER experiment [5,6], and the gravitational wave detection of binary neutron star (BNS) mergers by the LIGO-Virgo collaboration [7,8]. Among the events in the last category, the GW170817 event is currently outstanding, since it has been possible to measure not only the neutron star tidal deformability during inspiral, but also electromagnetic counterparts [9,10]. As a result, the GW170817 event has triggered a large number of works which are aimed at constraining neutron star properties and the EoS, either from the analysis of the tidal deformability alone (see for example [11,12]), or from a combination of tidal deformability and the electromagnetic signal [13–18]. Including the information from the electromagnetic signal requires as an input numerical modeling of the merger process, which introduces additional uncertainties but, at the same time, broadens the experimental base of the analysis.

Another interesting event is GW190814, where the mass of the lighter object has been determined (at 90% credible level) to be 2.50–2.67 M_⊙ [19]. In the standard interpretation [20,24] this is either the most massive neutron star observed to date or is a black hole that is located in the so-called mass-gap. Other, more exotic models include for example a strange star [25,26] or a compact star in an alternative theory of gravity [27]. The neutron star interpretation of the light companion in the GW190814 challenges our current understanding of the EoS, even if one assumes that this star is rotating very rapidly [20–24].

An important aspect of the merger process is that before the merger the two stars are well described by a one-parameter EoS of cold matter in weak (β-)equilibrium, which typically relates pressure to (energy) density. This means that the measured tidal deformabilities and masses of the two merging stars essentially concern this cold EoS of dense matter in β-equilibrium. In contrast, after the merger the evolution of the post-merger remnant (if there is no prompt black hole formation) requires as an input an EoS at non-zero temperature and out of (weak) β-equilibrium, i.e., the pressure becomes a function of three thermodynamic parameters [32–34]. Most commonly, these are chosen to be baryon number density, n_B, temperature T and charge fraction Y_Q = n_Q/n_B, where n_Q is defined as the total hadronic charge density [35]. The electron fraction Y_e = Y_Q due to electrical charge neutrality. In the following, when referring to cold compact stars, we will assume that they are in β-equilibrium. Small deviations from β-equilibrium, which can lead to some kinematical effect (bulk viscosity, etc) will be neglected.
Alongside full-fledged hydrodynamics simulations of the post-merger phase, different studies focused on stationary solutions for compact star configurations, which give, among other things, hints on the magnitude of the maximum mass supported by a post-merger object and thus the conditions for the formation of a black hole. As evidenced by numerical simulations, post-merger objects are rapidly rotating and support significant internal flows. Therefore, to assess the stability of the post-merger object rapidly and differentially rotating configurations of compact stars should be studied.

Universal relations, i.e., relations between different global quantities of the star found empirically to be independent of the EoS, have attracted much attention in this context. Such relations have been established for both uniformly \[48, 51\] and differentially rotating stars \[52, 54\] in the case of cold stars, described by zero-temperature EoS with the matter under $\beta$-equilibrium. However, for the merger remnant the thermal effects cannot be ignored and can influence, among other observables, the maximum mass of a static or rapidly rotating star \[52, 56\] as well as the applicability of universal relations. In Refs. \[55, 57, 58\], it has been shown that thermal effects induce deviations from the universal relations obtained for $\beta$-equilibrated matter at zero temperature. Subsequently, Ref. \[59\] demonstrated that universality is restored if finite-temperature configurations with the same entropy per baryon and electron fraction are considered. Here we will extend the study of Ref. \[59\] which has focused on non-rotating or slowly rotating stars to rapid rotation.

As a consequence of our findings on universality for hot stars, we revisit the inference of the maximum mass of a compact star from the analysis of the GW170817 event. This problem has been addressed by several authors, see Refs. \[24, 28–30\] using the scenario of the formation of a hypermassive compact star in the merger event and its delayed collapse to a black hole close to the neutral stability line for supramassive compact stars. Some of these authors employed the universality of the linear relation between the maximum gravitational mass for uniformly rotating stars at the Kepler limit, $M_K^\star$, and the same quantity for a non-rotating star $M_{\text{TOV}}^\star = \max (M_{\text{TOV}}) \[48, 51, 60\]$

$$M_K^\star = C_M^\star M_{\text{TOV}}^\star \, . \tag{1}$$

Here and below the superscript $\star$ refers to quantities characterizing the maximum mass objects. The employed value for $C_M^\star \approx 1.2 \[48, 51, 60\]$, relating $M_K^\star$ and $M_{\text{TOV}}^\star$ has, however, been determined assuming that the star rotating at Kepler frequency is cold and in $\beta$-equilibrium, which is not necessarily the case for the merger remnant. Therefore we will revisit this question and will determine the impact of nonzero temperature and matter out of $\beta$-equilibrium on the value of $C_M^\star$.

This paper is organized as follows. In Sec. III we investigate different universal relations for fast-rotating stars. Section IV is devoted to the discussion of the maximum mass of fast-rotating compact stars. We derive a new upper limit on $M_{\text{TOV}}^\star$ using the universal relations in Sec. V. Our conclusions are collected in Sec. VI. Throughout this paper we use natural units with $c = \hbar = k_B = G = 1$.

II. SETUP

This section is devoted to a description of our strategy to solve for the structure of a hot rapidly and rigidly rotating relativistic star. More details on the formalism can be found in \[55, 61, 62\]. Combined Einstein and equilibrium equations are solved, assuming stationarity and axisymmetry. Besides, we assume the absence of meridional currents such that the energy-momentum tensor fulfills the circularity condition, i.e. there is no convection. An EoS is needed to close the system of equations. In neutron stars older than several minutes matter is cold, neutrino-transparent, and in (approximate) $\beta$-equilibrium. Its EoS is barotropic, i.e. it depends only on one variable, which commonly is chosen as baryon number density $n_B$. In contrast, the merger-remnant matter is hot and not necessarily in $\beta$-equilibrium, such that the EoS depends in addition to $n_B$ on temperature $T$ and electron fraction $Y_e = n_e/n_B$ or thermodynamically equivalent variables. Under the above-mentioned assumptions, in particular stationarity, the most general solution for the star’s structure becomes again barotropic, i.e., the electron fraction and the temperature need to be related to $n_B$ \[61, 63\]. To fulfill this requirement, we consider below stars characterized by constant entropy per baryon $S/A$ and some fixed value of the electron fraction or constant electron lepton fraction $Y_L = (n_e + n_\nu) / n_B = n_L/n_B$ ($n_e$ and $n_L$ being the neutrino and electron lepton number densities, respectively). It should be stressed that this simplified setup does not reflect realistic conditions in the merger remnant. A variation of the values of $S/A$ and $Y_e$ or $Y_L$ should nevertheless allow us to cover the relevant conditions and thus to estimate the sensitivity of the universal relations and those observables needed to place limits on $M_{\text{TOV}}^\star$ on the thermal and out of $\beta$-equilibrium effects and to give an uncertainty range.

A. Numerical models of rapidly rotating hot stars

For computing numerical models of hot rapidly rotating stars, we have used the LORENE library \[64\].

1 If the assumption of rigid rotation is relaxed, then stationary solutions can be constructed with non-barotropic equations of state, see for example, Refs. \[64, 65\].

2 \url{https://lorene.obspm.fr}
LORENE is a set of C++ classes developed for solving problems in numerical relativity. It contains tools for computing equilibrium configurations of relativistic rotating bodies \cite{67} for which combined Einstein and equilibrium equations are solved assuming stationarity, axial symmetry, asymptotic flatness, and circularity.

Using a quasi-isotropic gauge, the line element expressed in spherical-like coordinates reads \cite{67}
\[
ds^2 = -N^2 dt^2 + A^2 (dr^2 + r^2 d\theta^2)
+ B^2 r^2 \sin^2 \theta (d\varphi^2 + N^2 dt)^2,
\]
with $N, N^r, A,$ and $B$ being functions of coordinates $(r, \theta)$. Under the present symmetry assumptions, Einstein equations for the four metric potentials reduce to a set of four elliptic (Poisson-like) partial differential equations, in which source terms contain both contributions from the energy-momentum tensor (matter) and nonlinear terms with non-compact support, involving the gravitational field itself. More details and explicit expressions can be found in \cite{67}.

The matter is assumed to behave as a perfect fluid such that the energy-momentum tensor can be written as
\[
T^\alpha\beta = \left(\varepsilon + p\right) u^\alpha u^\beta + p g^\alpha\beta,
\]
where $\varepsilon$ is the total energy density (including rest mass), $p$ the pressure, and $u^\alpha$ the fluid four-velocity. The angular velocity of the fluid then becomes $\Omega := \dot{u}_\varphi / u^t$. Equilibrium equations are derived from energy and momentum conservation, $\nabla \cdot T^{\alpha\beta} = 0$, and become within the present setup\cite{55, 61, 68}
\[
\partial_t \left( H + \ln N - \ln \Gamma \right) =
\frac{e^{-H}}{m_B} \left[ T \partial_t (S/A) + \mu_L \partial_t Y_L \right] - u_\varphi u^t \partial_t \Omega,
\]
where $\Gamma = N u^t$ is the Lorentz factor of the fluid with respect to the Eulerian observer and $S/A$ the entropy per baryon ($k_B = 1$),
\[
H = \ln \left( \frac{\varepsilon + p}{m_B n_B} \right),
\]
is the pseudo-log enthalpy with $m_B$ being a constant of the dimension of a mass\cite{3}. Since in this work we consider only uniform rotations with $\Omega = \text{const}$, constant $S/A$, and constant $Y_e$ with $\mu_L = 0$ or constant $Y_L$, the right hand side of Eq. (4) vanishes and the equilibrium equation takes the same form as in the zero temperature and $\beta$-equilibrium case.

Upon computing models of rotating stars, at finite temperature, an additional difficulty arises from the fact that the surface of the star is no longer well defined since an extended dilute atmosphere can form, see for instance

\[\text{the discussion in } 59, 68. \text{ For simplicity, we assume that the surface corresponds to the density } n_B = 10^{-6} \text{ fm}^{-3} \text{ for all EoS models and any considered combinations of } S/A \text{ and } Y_e/Y_L. \text{ We have checked that our conclusions do not depend on the choice of the definition of the surface, see Appendix A.}

Employing LORENE, we find global stellar parameters such as gravitational, $M_G$, and baryon, $M_B$, mass, and equatorial circumferential radius $R$. We additionally compute the angular momentum, the moment of inertia, and the quadrupole moment. The corresponding expressions for the quadrupole moment can be found in \cite{12} and \cite{73}. For our setup with constant $S/A$, the star's total entropy is simply given by $S/A M_B$.

\[\text{B. Equations of state} \]

The system of equations for solving for the star's structure discussed in the preceding section is closed by an EoS. To ensure that our results are not an artifact of a particular choice of EoS model, we have performed the same calculations for a set of different EoS models. There exists a large number of EoS models obtained for cold matter in compact stars. The number of EoS covering the regimes of finite temperature and varying electron fraction is however small. These are mostly based on density functional theory. Here we choose a set of EoS models that are based either on relativistic density functional theory with various parameterizations or a non-relativistic model based on Skyrme functional and an empirical extension of a variational microscopic model. These models are reasonably compatible with existing constraints from nuclear experiments, theory, and astrophysics, in particular, they predict maximum masses above $2 M_\odot$ \cite{1–3, 7–4} or at least marginally consistent with this value.
TABLE I. Global parameters of cold neutron stars (first four columns) for EoS considered in this work. These columns list (from left to right) the EoS model acronym, maximum gravitational and baryonic masses, radius of a 1.4\(M_\odot\) star and the tidal deformability \(\Lambda\) range for the GW170817 event. The latter quantity is calculated assuming for the merger the masses \(m_1 \in (1.36,1.60)M_\odot\) and \(m_2 \in (1.16,1.36)M_\odot\), which corresponds to the mass ratio range \(0.73 \leq q = m_2/m_1 \leq 1\). The remaining columns list properties of symmetric nuclear matter at saturation density according to the employed EoS model:

| Model       | \(M_{\text{TOV}}^\star\) (\(M_\odot\)) | \(M_B^\star\) (\(M_\odot\)) | \(R_{1.4}\) (km) | \(\Lambda\) (MeV) | \(n_s\) (fm\(^{-3}\)) | \(K\) (MeV) | \(E_s\) (MeV) | \(L\) (MeV) |
|-------------|---------------------------------------|-----------------------------|------------------|------------------|----------------|-------------|-------------|-------------|
| FSU2H NY    | 2.35                                 | 2.77                        | 13.82            | 499-530          | -16.40         | 0.155       | 231.3       | 47.2        |
| SFHo        | 1.99                                 | 2.34                        | 11.9             | 366-401          | -16.19         | 0.158       | 245.4       | 47.1        |
| NL3-\(\omega\rho\) | 2.75                               | 3.39                        | 13.82            | 1042-1051        | -16.24         | 0.148       | 271.6       | 55.5        |
| SFU2H       | 2.39                                 | 2.86                        | 13.28            | 635-655          | -16.28         | 0.150       | 238.0       | 44.5        |
| SRO(APR)    | 2.17                                 | 2.66                        | 11.33            | 271-295          | -16.00         | 0.160       | 231.3       | 47.2        |
| BHBA\(\sigma\) | 2.10                               | 2.45                        | 13.22            | 754-789          | -16.00         | 0.149       | 242.6       | 55.0        |

To be specific, we consider one non-relativistic density-functional (DFT) model, RG(SLY4); five variants of relativistic DFT, one with density-dependent couplings, HS(DD2); five with non-linear couplings, HS(IUF); SFHo, NL3-\(\omega\rho\); and FSU2H; as well as the SRO(APR) model.

The latter is based on the APR EoS, which itself is partly adjusted to the variational calculation of \(\Lambda\). If available, we compare the above purely nucleonic EoS models with the corresponding EoS allowing for the presence of hyperons. These are BHBA\(\sigma\) the extension of HS(DD2); SFHoY, an extension of SFHo, \(\Lambda\)-hyperons; and FSU2HY, an extension of FSU2H. For NL3-\(\omega\rho\); and FSU2HY we adopt the parameterizations in [92], but disregard the \(\sigma^*\)-meson field. Except for FSU2HY, \(\Lambda\)-hyperons and NL3-\(\omega\rho\), EoS data are publicly available on the COMPARE data base [93]. Key properties of our collection of the EoS are summarized in Table I together with present constraints. The value for the tidal deformability of NL3-\(\omega\rho\) lies above the 90% confidence interval given by the GW170817 event [68], but in view of the large uncertainty we feel it premature to exclude a certain EoS model and keep the NL3-\(\omega\rho\) model as representative of a large deformability in our EoS sample. In Fig. 1 we show the pressure as a function of energy density for cold, \(\beta\)-equilibrated matter.

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\(^4\) The EoS model BHBA\(\sigma\) contains only \(\Lambda\)-hyperons and not the full baryon octet. There exists a version, DD2Y [55], based on the same nucleonic HS(DD2) EoS, which contains the full baryon octet. For the present purpose, both give very similar results.

\(^5\) https://compose.obspm.fr

### III. UNIVERSAL RELATIONS FOR FAST ROTATING STARS AT FINITE TEMPERATURE

Although the properties of static and rotating stars depend strongly on the EoS, a series of “universal” relations have been found between global parameters of static stars which are almost EoS independent (see for a review [94]). These were later extended to slowly and maximally fast-rotating stars [95, 101]. The practical importance of such relations resides in their potential to provide constraints on quantities that are difficult to access experimentally.

It was previously shown that most of the universal relations for slowly rotating stars remain valid at finite temperature if the same thermodynamic conditions are maintained (for example by fixing \(S/A\) and \(Y_e\) [59]). Here we extend this investigation to rapidly rotating hot stars. In Sec. III A we first address the universal relations between global properties of non-rotating and Keplerian configurations for stars with constant \(S/A\) and \(Y_e\). In the subsequent Sec. III B we address the universal relations among the normalized moment of inertia, quadrupole moment, and the compactness for the maximum mass configuration of a compact star at the Kepler limit.

#### A. Relations between global properties of non-rotating and Keplerian configurations

In this subsection, we are interested in a particular class of universal relations, among the parameters of non-rotating and maximally rotating (at the mass-
TABLE II. Fitting parameters entering Eqs. (8), (9), (10) and their standard errors (in parenthesis), under different thermodynamic conditions specified in the first column.

| Thermo. cond. | $C_0^M$ | $C_1^M$ | $C_0^R$ | $C_1^R$ | $C_0^f$ | $C_1^f$ |
|--------------|----------|----------|----------|----------|----------|----------|
| $T = 0$, $\beta$-eq. | 1.2187 (0.0064) | 1.3587 (0.0104) | 1259.63 (9.72) | 1795.30 (4.35) | 1.2187 (0.0064) | 1.3587 (0.0104) |
| $S/A = 2$, $Y_e = 0.1$ | 1.1617 (0.0032) | 1.3459 (0.0051) | 1237.68 (5.47) | 1798.92 (2.17) | 1.1617 (0.0032) | 1.3459 (0.0051) |
| $S/A = 2$, $Y_e = 0.4$ | 1.1894 (0.0029) | 1.3282 (0.0038) | 1201.11 (5.60) | 1795.30 (4.35) | 1.1894 (0.0029) | 1.3282 (0.0038) |
| $S/A = 3$, $Y_e = 0.1$ | 1.1181 (0.0026) | 1.3593 (0.0051) | 1210.11 (5.60) | 1798.92 (2.17) | 1.1181 (0.0026) | 1.3593 (0.0051) |
| $S/A = 3$, $Y_e = 0.4$ | 1.0877 (0.0023) | 1.3506 (0.0063) | 1199.01 (7.10) | 1798.92 (2.17) | 1.0877 (0.0023) | 1.3506 (0.0063) |

shifting limit) stars. The original motivation for studying these relations was to constrain on the stellar radii using the measurements of masses and frequencies of sub-millisecond pulsars \cite{32, 37}. The non-observation of a rapidly rotating pulsar in the remnant of SN1987A led to a declining interest in these relations, although searches of sub-millisecond pulsars continued \cite{101}. The fastest rotating pulsar observed to date \cite{102} rotates at 716 Hz, which is still far from Kepler frequencies predicted by the various EoS of dense matter. The gravitational wave event GW170817 and the attempt to deduce a maximum mass constraint for a non-rotating cold neutron star stimulated several recent studies of rigidly \cite{101} and differentially rotating stars \cite{52, 53}. Furthermore, the GW190814 event rekindled the interest in the subject within the scenario in which the light component of this merger event is a maximally rotating compact star \cite{32–38, 41}.

Equation \( (1) \) which expresses the maximum gravitational mass of the Keplerian configuration as a function of the maximum mass of a non-rotating star is an example of such relations. It was initially proposed in \cite{48, 60} and later on confirmed by extensive computations in \cite{51}. Other examples are a relation between the circumferential equatorial radius of the maximum mass configuration at the Kepler limit and the circumferential radius of the maximum mass static configuration \cite{48, 60}.

\[ R_K^* = C_R^* R_{\text{TOV}}^*, \quad (6) \]

and the dependence of the rotation frequency of this maximum mass configuration at the Kepler limit on mass and radius of the non-rotating maximum mass configuration \cite{95, 98}.

\[ f_K^* = C_f^* x_{\text{TOV}}^*, \quad (7) \]

where \( x_{\text{TOV}}^* = (M_{\text{TOV}}^*/M_\odot)^{1/2}, \) \((10 \text{ km}/R_{\text{TOV}}^*)^{3/2}\). This functional form is actually identical to the Newtonian expression for the mass shedding frequency of a rotating sphere, see also the discussion in \cite{97} about its justification in the relativistic case.

Motivated by the findings of Ref. \cite{59} we reinterpret Eqs. (1), (2) and (3) as relations between properties of maximum mass Keplerian and static configurations with identical thermodynamic conditions

\[ M_K^*(S/A, Y_e) = C_M^*(S/A, Y_e) M_{\text{TOV}}^*(S/A, Y_e), \quad (8) \]

\[ R_K^*(S/A, Y_e) = C_R^*(S/A, Y_e) R_{\text{TOV}}^*(S/A, Y_e), \quad (9) \]

\[ f_K^*(S/A, Y_e) = C_f^*(S/A, Y_e) x_{\text{TOV}}^*(S/A, Y_e), \quad (10) \]

which implies that the coefficients \( C_i^*, \ i \in M, R, f \) depend on two additional thermodynamic parameters, which are chosen here to be \( S/A \) and \( Y_e \). The subscript \( S \) refers to static, hot configurations and the subscript “TOV” refers to cold static stars in \( \beta \)-equilibrium.

The relations \( (8), (9), (10) \) are shown in Fig. 2 for various combinations of \( S/A = 2, 3 \) and \( Y_e = 0.1, 0.4 \) and eleven different EoS models. Nature does of course not supply us with hot stars under these idealized conditions with constant \( S/A \) and \( Y_e \). For the sake of the argument, we have chosen these values from the typical range of values we encounter in the central part of hot stars, i.e. proto-neutron stars or the binary merger remnants. For completeness, we show also the results corresponding to cold stars. The values of \( C_i^* \) obtained by a fit to these results are provided in Table II for each considered thermodynamic condition. In the bottom panel of Fig. 2 the dependence of \( f_K^* \) on \( x_{\text{TOV}}^* \) is considered, too. As a trivial consequence of the linear dependencies in Eqs. \( (8), (9), (10) \) one finds again a linear relation \( f_K^* = C_f^* x_{\text{TOV}}^* \).

Our results show that universality holds reasonably well for hot rapidly rotating stars as well if the same constant \( S/A \)- and \( Y_e \)-values are considered. Similar results were obtained and discussed for non-rotating in Ref. \cite{59}. Moreover, since our set of EoS models contains purely nucleonic models as well as models with hyperons, we conclude that these relations are insensitive to the baryonic composition of matter, be it purely nucleonic or with an admixture of hyperons. As mentioned above, the proportionality coefficients depend, however, on the thermodynamic conditions. A small residual dependence of \( C_R^* \) on the EoS remains. It arises, as previously discussed for cold stars \cite{61}, from a weak dependence of the maximum mass static configuration on the compactness \( \Xi_S = M_S^*/R_S^* \). The \( \Xi_S \)-dependence of \( C_M^* \) and \( C_R^* \) is depicted in Fig. 3 for the same EoS models and thermodynamic conditions as in Fig. 2.

Refs. \cite{97, 99, 103} suggested that relations analogous to Eqs. \( (6) \) and \( (7) \) hold for stars with the same gravitational mass (and not only at the maximum of a sequence). These can again be generalized to configurations with the fixed \( S/A \) and \( Y_e \) to find

\[ R_K(M) = C_R R_S(M), \quad (11) \]

\[ f_K(M) = C_f x_S(M), \quad (12) \]
FIG. 2. Top panel: maximum gravitational mass at the Kepler limit \(M_K\) vs. maximum gravitational mass of a static star \(M_\odot\), Eq. 8; middle panel: equatorial circumferential radius of the maximum mass Keplerian configuration \(R_K\) vs. circumferential radius of the maximum mass static configuration \(R_S\), Eq. 9; bottom panel: rotation frequency of the maximum mass configuration at the Kepler limit \(f_K\) as function of \(x_S\) or \(x_K\), i.e., for the maximum mass static \((S)\) and Keplerian \((K)\) configurations, see Eq. (10). The results correspond to eleven EoS models and different thermodynamic conditions expressed in terms of \(S/A\) and \(Y_e\). Results for cold stars are shown for comparison.

FIG. 3. The dependence of \(C_M^*\) (top) and \(C_R^*\) (middle) on the compactness of the maximum mass static configuration. The same thermodynamic conditions and EoS models as in Fig. 2 are considered.

| Thermo. cond.          | \(a_1\)     | \(a_2\)     |
|------------------------|-------------|-------------|
| \(T = 0, \beta\)-eq.  | 0.9398 (0.1093) | 0.1246 (0.0277) |
| \(S/A = 2, Y_e = 0.1\) | 1.1632 (0.0788) | 0.0379 (0.0180) |
| \(S/A = 2, Y_e = 0.4\) | 1.2474 (0.0867) | 0.0130 (0.0178) |
| \(S/A = 3, Y_e = 0.1\) | 1.1458 (0.0976) | 0.0159 (0.0190) |
| \(S/A = 3, Y_e = 0.4\) | 1.1777 (0.0844) | 0.0109 (0.0153) |

TABLE III. Fitting parameters entering Eq. (13), and their standard errors (in parenthesis), under different thermodynamic conditions specified in the first column.

where \(x_S = \left[(M/M_\odot) \cdot (10 \text{ km}/R_S(M))\right]^{1/2}\).

Eqs. (11) and (12) are plotted in Figs. A and B for our collection of eleven EoS. The same for a cold star as well as for stars with \((S/A = 2, Y_e = 0.1)\) are also plotted. It can be seen that the relation (11) holds, but the proportionality constant \(C_R^*\) slightly depends on the EoS for finite \(S/A\). The relation (12) is confirmed too. The observed deviations occur only for \(M_S \gtrsim 0.7-0.8M_\odot\), in agreement with previous findings [99]. We thus find again that the different thermodynamic conditions lead to different values for the proportionality coefficients in Eqs. (11) and (12), but the linear relationships remain well fulfilled.

B. Relations between global parameters of the maximum mass configuration at the Kepler limit

For cold compact stars in \(\beta\)-equilibrium numerous other universal relations between global properties have been found, notably the so-called “I-Love-Q” relations [104, 105] between the moment of inertia \((I)\), the tidal deformability \((\Lambda)\), and the quadrupole moment \((Q)\). In this context, different relations expressing global properties in terms of the star’s compactness \(\Xi\) have received much attention, too [68, 51, 106, 107].

Here, we will consider as an example two such relations and investigate whether they hold for rapidly rotating hot
compact stars. These are \( \bar{I} = I/M^3 \) and \( \bar{Q} = QM/J^2 \), with \( J \) standing for the angular momentum, expressed as polynomials of \( \Xi^{-1} \):

\[
\bar{I} = a_1 \Xi^{-1} + a_2 \Xi^{-2},
\]

\[
\bar{Q} = b_1 \Xi^{-1} + b_2 \Xi^{-2}.
\]

Slightly different polynomial expressions of \( \bar{I} \) and \( \bar{Q} \) in terms of \( \Xi^{-1} \) have been previously proposed in \[51\], who have also shown that they are universal for rigidly and slowly rotating cold, \( \beta \)-equilibrated stars. In Ref. \[59\] these relations were shown to be universal also for hot stars, as long as the same pair of constant \( S/A \) and \( Y_e/Y_L \) is considered.

More specifically, we will investigate the behavior of the different quantities taken for the maximum mass Keplerian configuration, \( i.e. \), we study \( \bar{I}_K \) and \( \bar{Q}_K \) as a function of \( \Xi^*_K = M_K/R_K^* \). Note that because of rotational stretching of the star, the equatorial and polar radii are different; we recall that \( R_K^* \) refers to the equatorial circumferential one. Fig. 5 depicts these relationships. Each symbol indicates a particular EoS model and the different colors differentiate different thermodynamic conditions among \( S/A = 2, 3 \) and \( Y_e = 0.1, 0.4 \). Results for cold stars in \( \beta \)-equilibrium are shown by black symbols for comparison. Results of fits using Eqs. (13) and (14) are illustrated with lines in Fig. 5; values of the fitting parameters entering eqs. (13) and (14) are provided in Tables III and IV. These fits reproduce the exact results with good accuracy; the reduced \( \chi^2 \)-values are of the order of \( 10^{-3}(10^{-2}) \) for \( \bar{Q}^* \) vs. \( \Xi^* \) (\( \bar{I} \) vs. \( \Xi^* \)) are and slightly increasing with \( S/A \). Although some scattering is seen in Fig. 5, a functional form similar to the one obtained for slowly rotating stars applies reasonably well to the maximum mass configuration at the Kepler limit, too, and universality is again reasonably well fulfilled. However, the relative displacement of points corresponding to a given combination of entropy and electron fraction indicates that the values of the parameters \( a_i, b_i \) entering Eqs. (13), and (14) depend on thermodynamic conditions, as expected.
As well-known, for cold compact stars the value of $M_{K}^{\star}$, is 20% larger than $M_{\text{TOV}}^{\star}$, independent of the EoS [48, 51, 60]. As seen in the previous section, the value of $C_{M}^{\star}$ ≈ 1.2 is, however, only valid if both, $M_{\text{TOV}}^{\star}$ and $M_{K}^{\star}$, are computed for cold, $\beta$-equilibrated, stars. The assumption of a cold star fails for the merger remnant, as the EoS obtains significant thermal corrections [59, 109], too. The purpose of this section is to investigate the relation between $M_{K}^{\star}$ for various thermodynamic conditions and the cold $M_{\text{TOV}}^{\star}$ to verify to which extent thermal and out of equilibrium effects can change the estimated value of $M_{\text{TOV}}^{\star}$.

What are the effects of finite-temperature EoS on the maximum masses of a static and a rapidly rotating star, respectively? First, compact stars expand due to thermal effects (e.g. [59, 108]), therefore a same-mass hot star will have a larger radius than its cold counterpart. Consequently, the larger centrifugal force acting on particles on the stellar surface will be larger and, therefore, the Keplerian limit will be achieved for smaller frequencies, which will result in smaller masses at the Kepler limit. Second, the thermal pressure adds to the degeneracy pressure which means that a larger mass can be supported against the gravitational pull. Thus we see that there is an interplay between two competing effects. Fig. 7 shows the variation of $C_{M}^{\star}$ with $S/A$ for different purely nucleonic EoS and a constant electron fraction of $Y_{e} = 0.1$. The value of the Keplerian maximum mass $M_{K}^{\star}$ is normalized to that of the TOV maximum mass $M_{\text{TOV}}^{\star}$ computed for a cold star. An inspection of Fig. 7 shows that one EoS model (RG(SLy4)) manifests a monotonic increase of $C_{M}^{\star}$ over the considered $S/A$ range while the remaining six models show a non-monotonic behavior; the position of the minimum value of $C_{M}^{\star}$ for the latter category of models is situated in the domain $1 \leq S/A \leq 3.5$. This variety of behaviors is associated with the interplay between the effects of the increase of the pressure due to the thermal contribution and expansion of the star with temperature and the associated reduction of the Keplerian frequency.

To disentangle the different effects discussed above, Fig. 8 shows $M_{K}^{\star}$ this time normalized to the maximum mass of a non-rotating configuration with the same val-
are observed to almost linearly decrease with $R^{0.25}$ as an example, the results in the maximum masses at Kepler frequency nor-

respectively; the values of the fitting parameters are provided in the legend. The lines correspond to Eqs. (13) and (14), models and various thermodynamic conditions as indicated in Table III and IV.

As discussed above, the value of the electron fraction influences maximum masses and radii and thus our results. Also, at the center of the merger remnant, neutrinos are trapped at least during early post-merger such that also, at the center of the merger remnant, neutrinos are trapped at least during early post-merger such that

To examine the dependence on $Y_e$ and $Y_L$, we show in Fig. 11 the maximum masses at Kepler frequency nor-

| $S/A$ | $M_k$ | $M_B^*$ | $R_K^*$ | $\Xi$ | $n_B^{(0)}$ |
|-------|-------|---------|--------|------|----------|
| 1     | 2.92  | 3.44    | 16.0   | 0.27 | 0.72     |
| 2     | 2.84  | 3.27    | 17.1   | 0.25 | 0.72     |
| 3     | 2.79  | 3.09    | 19.7   | 0.21 | 0.65     |
| 4     | 2.84  | 3.01    | 26.3   | 0.16 | 0.46     |

FIG. 6. Relations between global properties of maximum mass configurations at the Kepler limit: normalized moment of inertia $I$ as function of the star’s compactness (bottom) and normalized quadrupole moment $Q$ as function of compactness (top). Compactness is here defined with the equatorial radius. The results correspond to eleven different EoS models and various thermodynamic conditions as indicated in the legend. The lines correspond to Eqs. (13) and (14), respectively; the values of the fitting parameters are provided in Tables III and IV.

FIG. 7. Dependence of $C_M^*$ [see Eq. (1)] on entropy per baryon $S/A$ for fixed $Y_e = 0.1$ and for various nucleonic EoS as labeled.

FIG. 8. Same as in Fig. 7 except that the normalization is done by the maximum gravitational mass of the non-rotating star with the same $S/A$ and $Y_e$ values.

| $S/A$ | $M_k$ | $M_B^*$ | $R_K^*$ | $\Xi$ | $n_B^{(0)}$ |
|-------|-------|---------|--------|------|----------|
| 1     | 2.92  | 3.44    | 16.0   | 0.27 | 0.72     |
| 2     | 2.84  | 3.27    | 17.1   | 0.25 | 0.72     |
| 3     | 2.79  | 3.09    | 19.7   | 0.21 | 0.65     |
| 4     | 2.84  | 3.01    | 26.3   | 0.16 | 0.46     |

TABLE V. Dependence on $S/A$ of some global properties of the maximum mass configuration of stars at Kepler limit for the HS(DD2) EoS [77, 78] and for fixed $Y_e = 0.1$. Listed are gravitational and baryonic masses, equatorial circumferential radius, compactness of the non-rotating configuration and central baryonic number density.
nalized to the non-rotating maximum mass as function of $S/A$ for different values of constant $Y_e$ and $Y_L$. The SFHoY EoS model $[91]$, has been chosen for that purpose, we have checked that other EoS models behave qualitatively similarly.

First, since neutrinos themselves only contribute weakly to the EoS at high density and therefore only have a very small impact on maximum masses, we observe that the main difference between choosing $Y_e$ or $Y_L$ arises from the fact that the electron fraction is equal to the hadronic charge fraction $Y_Q$, whereas due to the presence of neutrinos $Y_L \neq Y_Q$. This shift in $Y_Q$ induces a different behavior of the hadronic part of the EoS which is well visible in the maximum masses. This implies, too, that for our study it is sufficient to vary either $Y_e$ or $Y_L$ if the range is chosen large enough. Second, since a higher electron/lepton fraction increases the star’s radius, the Kepler frequency is lower and the supported mass, too. Thus the ratio of the Kepler maximum mass $M_K^*$ and the static one $M_S^*$ decreases with increasing $Y_e/Y_L$ with the most pronounced reduction observed at low entropies per baryon, where thermal effects are small. A related question is whether the presence of muons would change our results. It is obvious that in equilibrium, for the thermodynamic conditions considered here, charged muons will be abundant. In contrast to core-collapse supernovae, where there are no muons in the progenitor star and complete equilibrium has to be reached by dynamical reactions (see e.g. $[111]$), the two neutron stars before merger contain already muons, such that the merger remnant should indeed contain a non-negligible fraction of muons. The EoS itself is, however, still dominated by the hadronic part, such that again the influence of muons on our results would manifest itself only by a potential shift in the hadronic charge fraction since in the presence of charged muons we have $Y_Q = Y_e + Y_\mu$. In the following discussion we choose $Y_e = 0.1$, which should be close to the conditions in the central part of the merger remnant, see e.g. Ref. $[112]$, keeping in mind that, if the electron fraction in the merger remnant is higher, then $C_M^*$ is reduced.

### A. Comparison between nucleonic and hyperonic equations of state

So far, when selecting the EoS of dense matter, we assumed that neutron star matter contains nucleons and leptons. At densities exceeding several times the nuclear saturation density, non-nucleonic degrees of freedom, such as hyperons, meson-condensates, and even quark matter may appear $[113]$. Below we explore the effect of different compositions on the observables discussed by comparing the results for purely nucleonic EoS with those obtained in the models allowing for the presence of hyperons. In the present context, the focus will be on the changes in the composition of matter at finite temperature favoring the onset of hyperons $[114, 115]$, which is expected to change the value of $C_M^*$ at high entropies.

Fig. $[10]$ depicts this comparison in detail. The bottom panels display $C_M^*$ vs $S/A$ for four different EoS models and their hyperonic counterparts. Although qualitatively the behavior for all the EoS models is the same, a quantitative difference exists between the purely nucleonic models and those with an admixture of hyperons. More precisely, for low $S/A$-values the hyperonic models start with higher values of the ratio $C_K^*/C_{TOV}^*$ and manifest a much stronger decrease of $C_M^*$ with $S/A$ than the nucleonic models. To understand this, different effects have to be considered. First, $M_{TOV}^*$ for hyperonic models is much smaller than for purely nucleonic models, since the presence of hyperons softens the EoS. Second, this softening reduces the radius, thus leading to a comparatively higher rotation frequency and supported mass at Kepler limit, see the upper panels in Fig. $[10]$. The increasing abundance of hyperons with increasing $S/A$ leads to a less pronounced increase in the supported mass due to thermal effects, which explains the more pronounced decrease in $M_K^*/M_{TOV}^*$ as $S/A$.

### V. Maximum TOV Mass from GW170817

The event GW170817 and its electromagnetic counterpart have been used by several authors to place an upper limit on the value of the maximum mass of static cold compact star configurations, $M_{TOV}^* \leq 2.17M_\odot$ was inferred from conservative estimates of energy deposited into the short-gamma-ray burst and kilonova ejecta. Ref. $[28]$ used the universal relation between the mass of Keplerian configurations and static ones, derived for cold compact stars, see Eq. $[11]$, to place a limit $M_{TOV}^* \leq 2.16^{+0.17}_{-0.15}M_\odot$ consistent with the one derived in Ref. $[24]$. A weaker constraint $M_{TOV}^* \leq 2.3M_\odot$ was found in Ref. $[24]$, who used EoS
based on ad-hoc piecewise polytropic parameterization in combination with the angular momentum conservation and numerical simulation to show that the merger remnant at the onset of collapse to a black hole needs not to rotate rapidly. Note that Ref. [116] derived a lower (instead of an upper) limit on $M^*_{\text{TOV}}$ from straightforward numerical simulations, i.e. without use of universal relations, on the basis of the fact that no prompt BH collapse has been observed.

The physical picture of the GW170817 event that underlies the argumentation for placing the upper limit on $M^*_{\text{TOV}}$ is as follows [28–30]. Initially, the merger leaves behind a hypermassive neutron star (HMNS) which is differentially rotating. The HMNS star spins-down by losses to gravitational and neutrino radiation, as well as mass ejection, whereas the internal dissipation leads to vanishing internal shears and eventually to uniform rotation. (The magneto-dipole radiation due to the star’s B-field can be neglected over the time-scales of 10 ms.) At this stage, the star is in the region of stability of supramassive neutron stars, which support themselves against gravitational collapse due to uniform rotation. Subsequently, the star crosses the stability line beyond which it is unstable to collapse. While in principle the star may cross this line (which connects $M^*_{\text{TOV}}$ and $M^*_K$) at any point, it has been argued that the dynamics of the merger suggest that this crossing occurs in the vicinity of $M^*_K$ (see, however, Ref. [29], where this assumption has been questioned and the resulting corrections to the limits have been explored. Since the slower rotation implies a larger maximum mass limit, one should keep in mind that our estimate below may be relaxed somewhat.)

The extraction of the upper limit circumvents the full dynamical study and uses the baryon mass conservation between the instances of creation of HMNS in the merger (hereafter $t = 0$) and the moment of collapse to a black-hole ($t = t_c$), which reads

$$M_B(t_c, S/A, Y_e) = M_B(0) - M_{\text{out}} - M_{\text{ej}},$$

where $M_{\text{out}}$ refers to the baryon mass of the torus formed around the black-hole, after the merger and $M_{\text{ej}}$ refers to the baryon mass of the ejecta. The left-hand-side of (15) refers here to a hot supramassive compact star at the instance of collapse, $M_B(0)$ is the baryonic mass of the HMNS formed in the merger at the initial time $t = 0$.

As already mentioned in the introduction, the previous estimates of the $M^*_{\text{TOV}}$ were based on EoS of cold baryonic matter, i.e. they do not account for the thermal pressure in the BNS merger remnant and consider in particular the cold mass on the left-hand side of Eq. (15). Numerical simulations, however, show evidence that the BNS merger remnant is heated up to temperatures of the order of tens of MeV. Thus, it is necessary to carry out the analysis of the post-merger remnant taking into account the finite-temperature EoS of baryonic matter.
In the left-hand side of Eq. (18) we now substitute
\[ M_B(t_\odot, S/A, Y_\odot) = \eta(S/A, Y_\odot)M(t_\odot, S/A, Y_\odot) = \eta(S/A, Y_\odot)M^*_K(S/A, Y_\odot), \]  
where the second equality assumes that at the instance of collapse the star is rotating at the maximum of its rotational speed, consistent with Ref. [25], but see also Ref. [29]. The coefficient \( \eta(S/A, Y_\odot) \) relates the baryonic and gravitational masses of the hot compact star at the instance of collapse and is an EoS-dependent quantity. On the right-hand side of Eq. (15) we introduce the same quantity for the newly formed object via \( M_B(0) = \eta(0)M(0) \), where \( M(0) = 2.73M_\odot \) [14] is the gravitational mass of the merger as measured during inspiral for the GW170817 event, i.e. for cold stars. Thus, the mass conservation equation (15) can be rewritten as
\[ M^*_K(S/A, Y_\odot) = \frac{1}{\eta(S/A, Y_\odot)}[\eta(0)M(0) - M_{\text{out}} - M_\odot]. \]  
(17)

It has been estimated from the analysis of GW170817 that \( M_\odot \approx 0.03 - 0.05 M_\odot \) and \( 0.02 \leq M_{\text{out}} \leq 0.1 M_\odot \) [29]. Taking \( M_{\text{out}} = 0.06 \pm 0.04 M_\odot \) and \( M_\odot = 0.04 \pm 0.01 M_\odot \) we have \( M_{\text{out}} + M_\odot = 0.1 \pm 0.041 \). Thus, the masses on the right-hand side are fixed within the given limits and the knowledge of the two \( \eta \)-coefficients allows one to estimate the Keplerian maximum mass of a hot supramassive compact star on the left-hand side of Eq. (17).

As illustrated in Fig. 11 for cold compact stars based on our collection of EoS we have \( \eta(0) \approx 1.120 \pm 0.002 \) for \( M = 1.6 M_\odot \) and \( \eta(0) \approx 1.085 \pm 0.001 \) for \( M = 1.2 M_\odot \). The chosen values of gravitational masses bracket the range \( 1.2 \leq M_\odot \leq 1.6 \) from which the masses of two stars are drawn to add up to the gravitational mass \( 2.73_{-0.04}^{+0.04} M_\odot \) of the merger remnant at \( t = 0 \) [14]. For our estimates we adopt the value \( \eta(0) \approx 1.1004_{-0.0003}^{+0.0014} \) leading to \( M_B(0) = 3.00_{-0.05}^{+0.05} M_\odot \). We extract values of \( \eta(S/A, Y_\odot) \) for two values of entropy as given in Fig. 11 assuming that the star is rotating at the Keplerian frequency. We then find that \( \eta(2, 0.1) \approx 1.193 \pm 0.004 \) and \( \eta(3, 0.1) \approx 1.099 \pm 0.003 \). For the quantity \( (M_{\text{out}} + M_\odot)/\eta(S/A, Y_\odot) \) we obtain \( 0.087 \pm 0.030 \) and \( 0.091 \pm 0.037 \) for \( S/A = 2 \) and \( 3 \) and \( Y_\odot = 0.1 \), respectively. Substituting the numerical values we find
\[ M^*_K(2, 0.1) = 2.55_{-0.04}^{+0.06}; \quad M^*_K(3, 0.1) = 2.64_{-0.04}^{+0.06}. \]  
(18)

It was shown recently that several universal relations hold for hot, isentropic stars out of \( \beta \)-equilibrium [58], if thermodynamic conditions in terms of entropy per baryon and electron/lepton fraction are fixed. In Section III we have extended these findings to relations between stars rotating at Kepler frequency and non-rotating ones. The above limits can thus be used to set a limit on the maximum mass of non-rotating hot compact stars, using Eq. (8) and fitting parameters in Table II We find
\[ M^*_S(2, 0.1) = 2.19_{-0.03}^{+0.05}; \quad M^*_S(3, 0.1) = 2.36_{-0.04}^{+0.05}. \]  
(19)

We can also use the limits (18) in combination with the results shown in Fig. 7 to deduce an upper limit on the maximum mass of cold compact stars. Let us stress that in this case universality is lost, and \( C^*_M \) assumes values in a range \( 1.15 < C^*_M < 1.23 \) (\( S/A = 2 \)) and \( 1.10 < C^*_M < 1.29 \) (\( S/A = 3 \)) for the eleven EoS models considered here. The average values \( C^*_M = 1.19 \pm 0.04 \) for \( S = 2 \) and \( C^*_M = 1.18 \pm 0.11 \) for \( S = 3 \) can now be used to obtain, respectively,
\[ M^*_\text{TOV} = 2.15_{-0.07-0.16}^{+0.09+0.16}; \quad M^*_\text{TOV} = 2.24_{-0.07-0.44}^{+0.10+0.14}. \]  
(20)
deviation. Here and in the formulas for the masses above the error propagation for the upper and lower limits was computed independently. The first uncertainty thereby stems from the propagation of errors from $M_K^*$, whereas the second part indicates the EoS dependence in $C_M^*$.

When comparing the limits [20] with those of previous works [28, 29], one should keep in mind that we used a (recent) value for the mass $M(0)$, which is slightly lower than the value of 2.74$M_\odot$ [118] used in these studies. Our limits on $M_{\text{TOV}}^*$ would have been higher had we adopted the larger value of $M(0)$. It is seen that, if just before collapse the supermassive neutron star has average entropy per baryon $S/A = 3$, then the estimate of the TOV mass is significantly relaxed compared to the bound placed in Refs. [21, 28, 30]. According to the discussion in Sec. IV, a higher electron fraction in the merger remnant would further relax the bound on the TOV mass. Please note that we have considered stars at constant entropy per baryon and constant electron fraction, whereas a realistic merger remnant shows in particular strong entropy gradients [63, 112]. The values of $S/A = 2$ and 3 can be roughly taken as typical average values for the inner part of the merger remnant, thus most relevant for the mass. As is obvious from the difference in the results for $S/A = 2$ and $S/A = 3$, the detailed entropy profile influences the final limit for $M_{\text{TOV}}^*$. These profiles cannot be measured and the exact entropy distribution in the remnant depends on many parameters, among others the EoS. Including the uncertainty on the exact entropy profiles would considerably increase the global uncertainty and further relax the limits.

The limit we found is similar to the one in Ref. [29] but for a physically different reason. The last fact indicates that lifting the assumption that the star rotates at the Keplerian frequency would further loosen the bound on the TOV mass. Let us, however, stress the fact that universality is lost when extracting the cold TOV mass limits [20] from the information on the hot merger remnant, independent of the assumption about rotation at collapse, i.e., these final limits become EoS dependent.

VI. SUMMARY AND CONCLUSIONS

In this work, we have addressed two interrelated topics that rely on the knowledge of finite temperature EoS of dense matter. First, we have extended the universal relations, previously found for hot slowly rotating compact stars, to rapidly rotating stars. In particular we considered in detail the mass-shedding (Keplerian) limit. Secondly, we discussed an improvement of the previous mass limits for non-rotating compact stars obtained from the GW170817 event in the scenario where the merger remnant is a hypermassive compact star that collapses to a black hole upon crossing the neutral stability line as a supramassive (uniformly rotating) compact star.

Our analysis was carried out using a variety of finite-temperature EoS. The collection used includes relativistic density functional theory based EoS with nucleonic degrees of freedom as well as EoS models allowing for the presence of hyperons. These EoS satisfy the astrophysical constraints on neutron stars and nuclear data (nuclear binding energies, rms radii, etc). As an alternative to the covariant description, we used a non-relativistic model based on a Skyrme-type functional and a parametrization of a microscopic model. In this way, we were able to bracket the range of possible predictions for the observables stemming from various EoS with different underlying methods of modeling.

When considering universal relations, we followed the strategy of Ref. [59] to search universality under the same thermodynamical conditions, meaning that we compare observables of the same star or various rotating and non-rotating configurations at the same fixed entropy per baryon $S/A$ and electron fraction $Y_e$. Specifically, we considered a class of relations which connect the Keplerian configurations with their non-rotating counterparts given by Eqs. [8, 10] generalizing the earlier zero-temperature studies to the finite-temperature case. We find that these relations are universal (in the sense of independence on the EoS) to good accuracy. Similarly, finite-temperature universality propagates beyond zero-temperature results for the relations connecting radii and frequencies of the same mass Keplerian and non-rotating stars, see Eqs. [11 and 12]. Finally, we have verified (partially) the validity of the $I$-Love-$Q$ relations by computing the first and the last quantity of the triple, specifically, $I = EM^3$ and $Q = EM^2$ for maximum-mass Keplerian configurations. We find that the functional dependence of these quantities for the maximum mass configurations at the Kepler limit on the compactness of the star is similar to the one obtained for slowly rotating stars.

The relation between the maximum masses of non-rotating and Keplerian sequences is an important link needed for placing limits on the maximum mass of a cold, non-rotating star from studies of the millisecond pulsars or gravitational wave analysis of binary neutron star mergers. We have explored this relation for finite-temperature stars finding that there are two competing effects: one is the thermal expansion of the star, which reduces the Kepler frequency and, implicitly, the star’s mass at this limit and the additional thermal pressure which makes a star of a given mass more stable against collapse. If the static and maximally rotating configurations are taken at the same values of $S/A$ and $Y_e$, then we find universality of the coefficient relating their masses, see Fig. 8.

The second important application of our analysis concerns the upper limit on the maximum mass of a non-rotating cold compact star. Several works, using various methods and scenarios, claimed that this maximum mass can be tightly constrained using the GW170817 event [24, 28, 30] to the range $M_{\text{TOV}}^* \leq 2.17 - 2.3M_\odot$, where the upper range in this limit arises when consider-
ing below-Keplerian rotations, instead of Keplerian ones. We have improved on the previous analysis by extracting the ratio of the baryonic to gravitational masses for hot compact stars of given $S/A$ and $Y_e$ and applying this to the same scenario. Our central finding is that the upper limit on the maximum mass of static, cold neutron stars is
\[ 2.15^{+0.09+0.16}_{-0.07-0.16} \leq M_{\text{TOV}}^\star \leq 2.24^{+0.10+0.44}_{-0.07-0.44} \]
for a typical parameter range $2 \leq S/A \leq 3$ and $Y_e = 0.1$ of the hot merger remnant. Note that the large error in the case of $S = 3$ is dominated by the non-universal behavior $C_M^*$ as displayed in Fig. 7. We thus conclude that accounting for the finite temperature of the merger remnant relaxes the derived constraints on the maximum mass of the cold, static compact star, obtained in Refs. 24, 28–30. In particular, universality is lost and the final number becomes EoS dependent due to the EoS dependence of $C_M^*$. In case the collapse to a black hole does not occur at the maximum possible mass of supramassive compact stars 29, as we assumed here, the upper limit will become less stringent.

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Appendix A: Influence of the surface definition on results

In this appendix, we discuss the sensitivity of our results on the density at which the surface is located. This is essential for establishing the validity of our results and conclusions. The available data for most finite-temperature EoS models are limited to temperatures above $T = 0.1$ MeV, such that for a range of entropy per baryon, no solution for the EoS at very low densities can be found. In practice, for the values of $S/A$ considered, many of the EoS models used did not have solutions for densities below roughly $(10^{-8}-10^{-7}$ fm$^{-3}$). This calls for an extrapolation of the required thermodynamic quantities from the densities where solutions were available to lower densities. Extrapolation of thermodynamic quantities introduces an error in the EoS. To avoid the above-stated extrapolation we define the surface of the star at $n_B = 10^{-5}$ fm$^{-3}$ uniformly in our modelling. This surface definition allows us to use the data provided for every EoS model in the parameter range used in our calculations.

To gauge the amount by which the value of the maximum mass changes with a variation of the location of the surface, we refer to the results for $M_K^*$ in Section 15. We verified that changing the surface density from $n_B = 10^{-7}$ fm$^{-3}$ to $10^{-5}$ fm$^{-3}$ resulted in a change of the value of $M_K^*$ only in the third decimal. The extrapolation has thereby been performed assuming linear dependencies of $\log \varepsilon$ and $\log p$ on $\log n_B$ with parameters calculated over the densities covering the lowest available data, $10^{-5} \leq n_B \leq 10^{-4}$ fm$^{-3}$. The small change in $M_K^*$ can be understood from the fact that the maximum mass is sensitive only to the high-density physics.

To quantify the uncertainties on the results in Section 13, we consider again two different values of the density at which we define the surface of the star. This time, in addition to the value of $n_B = 10^{-5}$ fm$^{-3}$ for the surface density, we take a surface at $n_B = 10^{-8}$ fm$^{-3}$, implying again an extrapolation of EoS data over the domain for which data are not available. We find that the extension of the surface by locating it at a lower density diminishes the maximum rotation frequency and that the higher the entropy per baryon the larger the induced differences in all studied quantities. However, neither the Kepler frequency, nor the quadrupole moment, the moment of inertia or the values of the gravitational mass in the ranges discussed in Section 13 are modified by more than a few per mille upon varying the location of the surface. We, therefore, conclude that we can safely define the surface at $n_B = 10^{-5}$ fm$^{-3}$.

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