On the kinematic limit of the charm production in fixed-target experiments with the intrinsic charm from the target

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Abstract

In this note we provide a detailed derivation of the kinematic limit of the charm production in fixed-target experiments with the intrinsic charm coming from the target. In addition, we discuss the first measurement of charm quark production in the fixed-target configuration at the LHC.

1 Introduction

The production of charmed hadrons via the intrinsic charm mechanism from the target was proposed in Ref. [1] and Ref. [2]. Utilizing this idea, in Refs. [3, 4] production of double charmed baryons and the J/ψ meson from the target was investigated at the STAR and LHCb/SMOG fixed-target programs [5, 6]. By a straightforward calculation it was shown that in case of the production of intrinsic charm from the target, charm hadrons have a maximum momentum in the laboratory frame of only a few GeV/c and a rapidity \( y < 2 \). This result is in perfect agreement with predictions from the intrinsic charm model about the \( \Delta y \) region for the charm state, \( \Delta y = y - y_{\text{tar}} (y_{\text{tar}} = 0) < 2 \).

\(^1\text{see chapter 6.1}\)
Recently, the LHCb collaboration presented the first measurement of charm production in the fixed-target configuration at the LHC \cite{7}. The production of $J/\psi$ and $D^0$ mesons is studied with beams of protons of different energies colliding with gaseous targets of helium and argon with nucleon–nucleon centre-of-mass energies of $\sqrt{s} = 86.6$ and 110.4 GeV, respectively. The $J/\psi$ and $D^0$ production cross-sections in $p\text{He}$ collisions in the rapidity range $[2, 4.6]$ are found to be $\sigma(J/\psi) = 652\pm33\pm42\text{ nb per nucleon}$ and $\sigma(D^0) = 80.8\pm2.4\pm6.3\mu\text{b per nucleon}$, where the first uncertainty is statistical and the second is systematic.

Using an approximation for the fraction $x$ of the nucleon momentum carried by the target parton \footnote{We provide the equation in the form it was used in the LHCb publication \cite{7}. However, it is easy to see that this equation is not fully correct. Using the standard Feynman-$x$ notation in the center-of-mass system $x \approx \frac{2m_c}{\sqrt{s_{NN}}} \sinh(y^*)$ (cf. Eq. (47.43) in Ref. \cite{8}) and taking into account that $\sinh(y^*) \approx -\exp(-y^*)/2$ for large negative values of $y^*$, one finally obtains $x \approx -\frac{m_c}{\sqrt{s_{NN}}} \exp(-y^*)$.}

\begin{equation}
  x \approx \frac{2m_c}{\sqrt{s_{NN}}} \exp(-y^*),
\end{equation}

where $m_c = 1.28\text{ GeV}/c^2$ is the mass of the $c$ quark and $y^*$ is centre-of-mass rapidity. Providing an analysis of the production of $D^0$ mesons with the intrinsic charm from the target, LHCb reported no evidence for a substantial intrinsic charm content of the nucleon \cite{7}.

In this short note we provide a detailed derivation of the kinematic limit of the charm production in the fixed-target experiments with the intrinsic charm coming from the target. In addition, we show that based on a misunderstanding of the kinematics of the intrinsic charm from the target in the laboratory frame the LHCb/SMOG result is in contradiction to the properties of the intrinsic charm model.
2 Derivation of the kinematic limit

2.1 Calculation of the Center of Mass System energy

Given an invariant energy $\sqrt{s}$, one can easily calculate the energies in the laboratory frame, the center-of-mass frame and of course in a lot of different other frames. Starting from

$$s = (p_P + p_T)^2 = p_P^2 + 2p_P p_T + p_T^2 = m_P^2 + 2p_P p_T + m_T^2$$

we first can calculate in the laboratory system where $p_T = (m_T; 0, 0, 0)$. One obtains

$$s = m_P^2 + m_T^2 + 2E_{\text{lab}} p_T$$

For the center-of-mass system, however, we have $\vec{p}_P + \vec{p}_T = \vec{0}$ ($\Rightarrow \vec{p}_P = -\vec{p}_T =: \vec{p}$) and, therefore, $\sqrt{s} = E_P + E_T$ where $E_P = (m_P^2 + \vec{p}^2)^{1/2}$, $E_T = (m_T^2 + \vec{p}^2)^{1/2}$. Inserting these energies we obtain

$$\sqrt{m_P^2 + \vec{p}^2} + \sqrt{m_T^2 + \vec{p}^2} = \sqrt{s},$$

and this equation can be solved by two-fold squaring,

$$2\sqrt{m_P^2 + \vec{p}^2}\sqrt{m_T^2 + \vec{p}^2} = s - m_P^2 - m_T^2 - 2\vec{p}^2$$

$$4(m_P^2 + \vec{p}^2)(m_T^2 + \vec{p}^2) = (s - m_P^2 - m_T^2 - 2\vec{p}^2)^2$$

$$4\vec{p}^4 + 4\vec{p}^2(m_P^2 + m_T^2) + 4m_P^2 m_T^2 = (s - m_P^2 - m_T^2)^2 - 4\vec{p}^2(s - m_P^2 - m_T^2) + 4\vec{p}^4$$

$$4s\vec{p}^2 = (s - m_P^2 - m_T^2)^2 - 4m_P^2 m_T^2 = \lambda(s,m_P, m_T),$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is Källén’s function. Given $s$ by Eq. (3), one can proceed to calculate the square of the three-momentum to be

$$\bar{p}^2 = \lambda(m_P^2 + m_T^2 + 2E_{\text{lab}} m_T, m_P^2, m_T^2) = \frac{4((E_{\text{lab}})^2 - m_P^2)m_T^2}{4(m_P^2 + m_T^2 + 2E_{\text{lab}} m_T)} = \frac{(E_{\text{lab}})^2 m_T^2}{m_P^2 + m_T^2 + 2E_{\text{lab}} m_T} \approx \frac{(E_{\text{lab}})^2 m_T^2}{2E_{\text{lab}} m_T} = \frac{1}{2} E_{\text{lab}} m_T.$$
2.2 Lorentz transformations

In order to get from one system to another, four-vectors have to be multiplied by Lorentz matrices. The Lorentz matrix which puts the general target four-vector $p_T = (E_T; -\vec{p})$ in the CMS to rest is given by

$$(\Lambda_{\nu}^{\mu}(E_T; -\vec{p})) = \Lambda(p_T) = \begin{pmatrix} E_T/m_T & \vec{p}^T/m_T \\ \vec{p}/m_T & 1 + \vec{p}\vec{p}^T/m_T/(E_T + m_T) \end{pmatrix}$$  \hspace{1cm} (7)$$

where $m^2_T = E^2_T - \vec{p}^2$. The upper index $T$ stands for transposition of the (column) three-vector $\vec{p}$. It can easily be seen that $\Lambda(p_T)p_T = (m; 0)^T$. Assuming that $J/\psi$ takes over a ratio $x \in [0, 1]$ of the three-momentum $\vec{p}$ (for $x > 0$ from the projectile, for $x < 0$ from the target) and applying the Lorentz transformation to the corresponding four-vector $p_\psi = (E_\psi; x\vec{p})$ with $E^2_\psi = m^2_\psi + x^2\vec{p}^2$, one obtains a value for the four-vector in the laboratory frame, i.e. the frame where $p_T$ is at rest. The result reads

$$E^\text{lab}_\psi = \frac{1}{m_T} \left( \sqrt{\vec{p}^2_T + m^2_T} \sqrt{x^2\vec{p}^2_T + m^2_\psi} - x\vec{p}_T^2 \right),$$  

$$\vec{p}^\text{lab}_\psi = \frac{\vec{p}_T}{m_T} \left( \frac{x^2\vec{p}^2_T + m^2_\psi - x\vec{p}_T^2}{m_T(m_T + \sqrt{\vec{p}^2_T + m^2_T})} \right).$$  \hspace{1cm} (8)$$

Taking into account that the three-momenta are usually much larger than the masses, we can expand into $m^2_T/\vec{p}^2$ and $m^2_\psi/\vec{p}^2$ to $x < 0$ (target) and obtain the maximum values of energy and momentum of the charm in the laboratory frame

$$E^\text{lab}_\psi = \frac{1}{2m_T} \left( m^2_\psi + m^2_T \right), \quad \vec{p}^\text{lab}_\psi = \frac{\vec{p}}{2m_T\vec{p}} \left( m^2_\psi - m^2_T \right)$$  \hspace{1cm} (9)$$

This last expression depend solely on the two masses $m_\psi$ and $m_T$ and no longer on the beam energy.

3 A comment on the LHCb/SMOG result

Let us remind the reader that in the fixed-target configuration, the LHCb acceptance gives access to the large Bjorken-$x$ region of the nucleon target (up to $x \sim 0.37$ for $D^0$ mesons)
and to rapidity region $2 < y < 4.6$ in the laboratory frame. Using Eq. (9) we can obtain the kinematic limit (i.e. $x_F = 1$) of the charm produced by the intrinsic charm mechanism from the target in the laboratory frame, $y < 0.6$ and $p_L \sim 1.2 - 1.3$ GeV/$c$. In other words, the LHCb collaboration provided an analysis outside the signal region.

## 4 Conclusion

In this note we provide a detailed derivation of the kinematic limits of the charm production in fixed-target experiments with the intrinsic charm coming from the target. We have also shown that the LHCb/SMOG result is based on a misunderstanding of the kinematics of the intrinsic charm. Moreover, the investigation of the production of charmed hadrons with intrinsic charm from the target at the LHCb/SMOG fixed-target program is fundamentally unfeasible.

### Acknowledgements

We would like to thank Stanley J. Brodsky for pointing out the LHCb/SMOG result to us and stimulating discussions. We would also like to thank Ramona Vogt, Paul Hoyer and Jean-Philippe Lansberg for useful comments. This research was supported by the Estonian Research Council under Grants No. TK133 and PRG356.

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