A Bias Correction Method in Meta-analysis of Randomized Clinical Trials with no Adjustments for Zero-inflated Outcomes: Supplementary Materials

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Appendix A. Analogue between the ZIBC method and EM algorithm

We can obtain Equation (5) following the idea of the EM algorithm. Denote $\log L_{CV}(\beta) \triangleq \sum_{i=1}^{n} \{-\exp(x_i^t \beta) + y_i x_i^t \beta\}$ as the log-likelihood function of $\beta$ under conventional model minus a constant, the analogous expectation and maximization steps are shown as follows.

- (Analogous) expectation-step:

  Taking the expectation of $\log L_{CV}(\beta)$, we have

  $$
  \mathbb{E}[\log L_{CV}(\beta)] = \sum_{i=1}^{n} \{-\exp(x_i^t \beta) + \mathbb{E}[y_i] \cdot x_i^t \beta\}
  = \sum_{i=1}^{n} \{-\exp(x_i^t \beta) + (1-\pi_i) \exp(x_i^t \beta_0) \cdot x_i^t \beta\}
  \approx \sum_{i=1}^{n} \{-\exp(x_i^t \beta) + (1-\bar{\pi}) \exp(\bar{x}_i^t \beta_0) \cdot x_i^t \beta\}
  \triangleq Q(\beta).
  $$

- (Analogous) maximization step:

  To maximize $Q(\beta)$, we derive its first derivative with respect to $\beta$, which is given by

  $$
  \frac{\partial Q(\beta)}{\partial \beta} = \sum_{i=1}^{n} \{-\exp(x_i^t \beta) + (1-\pi) \exp(\bar{x}_i^t \beta_0)\} x_i
  = \left[(1-\bar{\pi}) \exp(\bar{x}_i^t \{\beta_0 - \beta\}) - 1\right] \sum_{i=1}^{n} \exp(x_i^t \beta) x_i.
  $$

  Note that $\beta^*$ is the solution of $S_{CV}(\beta) = 0$. By plugging $\beta^*$ in $\frac{\partial Q(\beta)}{\partial \beta}$, we obtain Equation (5).
Appendix B. Proof of Lemma 1

Proof. Consider three “average” subjects in the control group, intervention group, and the overall sample (denoted as \{average, C\}, \{average, T\}, and \{average\}), with \(x_{\text{average, }C,p-2} = \bar{x}_{C,p-2}\), \(x_{\text{average, }T,p-2} = \bar{x}_{T,p-2}\), and \(x_{\text{average, }p-2} = \bar{x}_{p-2}\), respectively. Without loss of generality, assuming that observed covariates excluding the intervention assignment are grand mean centered before data analysis, we have \(x_{\text{average, }p-2} = \bar{x}_{p-2} = 0\). Since \(\bar{x}_{C,p-2} = \bar{x}_{T,p-2}\), we also have \(x_{\text{average, }C,p-2} = x_{\text{average, }T,p-2} = x_{\text{average, }p-2} = 0\). Therefore, we have

\[
\log(\mu_{\text{average}, C}) = \beta_{0,C}^0 \\
\log(\mu_{\text{average}, T}) = \beta_{0,T}^0 + \beta_{1,T}^0 \\
\log(\mu_{\text{average}}) = \beta_{0}^0 + \beta_{1}^0 \{A_{\text{average}} = T\}
\]

under the true method. If an average subject in the overall sample belongs to the control group, then

\[
\log(\mu_{\text{average}}) = \log(\mu_{\text{average}, C}) \\
\Rightarrow \beta_{0,C}^0 = \beta_{0}^0 \\
\Rightarrow \hat{\beta}_{0,C,MLE} \approx \hat{\beta}_{0,MLE}.
\]

Similarly, if an average subject belongs to the intervention group, then

\[
\log(\mu_{\text{average}}) = \log(\mu_{\text{average}, T}) \\
\Rightarrow \beta_{0,T}^0 + \beta_{1,T}^0 = \beta_{0}^0 + \beta_{1}^0 \\
\Rightarrow (\overline{\beta_0 + \beta_1})_{T,MLE} \approx \hat{\beta}_{0,MLE} + \hat{\beta}_{1,MLE}.
\]

Under similar arguments, for the conventional method, we have

\[
\hat{\beta}_{0,C, CV} \approx \hat{\beta}_{0, CV}.
\]
and

\[(\hat{\beta}_0 + \hat{\beta}_1)_{T, CV} \approx \hat{\beta}_{0, CV} + \hat{\beta}_{1, CV}. \]  \hspace{1cm} (A.4)

Plug Equations (A.1) and (A.3) into Equation (8), and plug Equations (A.2) and (A.4) into Equation (9), we have

\[
\hat{\beta}_{0, MLE} \approx \hat{\beta}_{0, CV} - \log(1 - \bar{\pi}_C)
\]

\[
\hat{\beta}_{0, MLE} + \hat{\beta}_{1, MLE} \approx \hat{\beta}_{0, CV} + \hat{\beta}_{1, CV} - \log(1 - \bar{\pi}_T),
\]

which directly gives Equation (10).
Figure S1: Coverage rates and MSE values of the true (blue dashed line), ZIBC (red dotted line) and conventional (black solid line) methods from 1000 replications (K=16)
Figure S2: Coverage rates and MSE values of the true (blue dashed line), ZIBC (red dotted line) and conventional (black solid line) methods from 1000 replications (K=20)