DOUBLE BETA DECAY IN DEFORMED NUCLEI

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Abstract

The pseudo SU(3) approach has been used to describe many low-lying rotational bands, as well as BE(2) and B(M1) intensities in rare earth and actinide nuclei, both with even-even and odd-mass numbers \cite{1,2,3,4,5,6}. The $\beta\beta$ half lives of some of these parent nuclei to the ground and excited states of the daughter ones were evaluated for the two and zero neutrino emitting modes \cite{7,8,9,10,11} using the pseudo SU(3) scheme. The predictions were in good agreement with the available experimental data for $^{150}$Nd and $^{238}$U.

The double electron capture half-lives of $^{156}$Dy, $^{162}$Er and $^{168}$Yb were also studied using the same formalism \cite{12}. The first nuclei was found to be the best candidate for experimental detection, with a half-life of $\approx 10^{24}$ yr to the first excited $0^+$ state in $^{156}$Gd.

There are strong selection rules which restrict the two neutrino mode of the $\beta\beta$ decay in some other nuclei, including $^{160}$Gd. Experimental limits for the $\beta\beta$ decay of $^{160}$Gd have been reported \cite{13,14}. Recently it was argued that the strong cancellation of the $2\nu$ mode in the $\beta\beta$ decay of $^{160}$Gd would suppress the background for the detection of the $0\nu$ mode \cite{15}.

In the present contribution we extend the previous research \cite{7,8,9,10,11} evaluating the $\beta\beta$ half lives of $^{160}$Gd using the pseudo SU(3) model. While the $2\nu$ mode is forbidden when the most probable occupations are considered, states with different occupation numbers can be mixed through the pairing interaction. The amount of this mixing is evaluated using perturbation theory. The possibility of observing the $\beta\beta$ decay in $^{160}$Gd is discussed for both the $2\nu$ and $0\nu$ modes.

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1 Introduction

The calculation of two neutrino double beta decay matrix elements has proven to be extremely sensitive to the details of the wave functions of the initial and final nuclei \cite{16}. While QRPA calculations are easy to perform, the uncertainties in the residual particle-particle proton-neutron interaction strongly limit their predictive power \cite{16}. Shell model studies in the full $fp$ shell provide reliable matrix elements for $^{48}$Ca \cite{17} and other light nuclei \cite{18}. Matrix elements
for $^{76}$Ge, $^{82}$Se and $^{136}$Xe were obtained using very large shell model spaces, which are however strongly truncated [19]. For heavier nuclei standard shell model calculations are impracticable.

The pseudo SU(3) shell model [20, 21] is a microscopic model which allows the description of heavy deformed nuclei in the laboratory frame through the use of a fermionic many-particle basis with good angular momentum. The microscopic Hamiltonian employed includes single-particle energies as well as pairing and quadrupole-quadrupole interactions. Most of the Hamiltonian parameters are fixed by known systematics. Three “rotor terms” allow a fine tuning of the energy spectra and are fitted for each nuclei. The pseudo SU(3) model has been used to describe many low-lying rotational bands, as well as B(E2) and B(M1) intensities in rare earth and actinide nuclei, both with even-even and odd-mass numbers. It was exhibited as a powerful and predictive tool in the description of heavy deformed nuclei.

The $\beta\beta$ half lives of some of these parent nuclei to the ground and excited states of the daughter ones were evaluated for the two and zero neutrino emitting modes using the pseudo SU(3) scheme. The predictions were in good agreement with the available experimental data for $^{150}$Nd and $^{238}$U. Predictions for double electron capture were also performed. These results are briefly reviewed in the present contribution.

Extending the previous research the $\beta\beta$ half lives of $^{160}$Gd are evaluated using the pseudo SU(3) model. While the $2\nu$ mode is forbidden when the most probable occupations are considered, states with different occupation numbers can be mixed through the pairing interaction. The amount of this mixing is evaluated using perturbation theory. The possibility of observing the $\beta\beta$ decay in $^{160}$Gd is discussed for both the $2\nu$ and $0\nu$ modes.

2 The pseudo SU(3) formalism

In the pseudo SU(3) shell-model coupling scheme [20], normal parity orbitals $(\eta, l, j)$ are identified with orbitals of a harmonic oscillator of one quanta less $\tilde{\eta} = \eta - 1$. The set of orbitals with $j = j = \tilde{l} + \tilde{s}$, pseudo spin $\tilde{s} = 1/2$, and pseudo orbital angular momentum $\tilde{l}$, define the so-called pseudo space. The orbitals with $j = \tilde{l} \pm 1/2$ are nearly degenerate. For configurations of identical particles occupying a single $j$ orbital of abnormal parity, a convenient characterization of states is made by means of the seniority coupling scheme.

The many-particle states of $n_\alpha$ nucleons in a given shell $\eta_\alpha$, $\alpha = \nu$ or $\pi$, can be defined by the totally anti-symmetric irreducible representations $\{1^{\nu_\alpha}\}$ and $\{1^{\pi_\alpha}\}$ of unitary groups. The dimensions of the normal (N) parity space is $\Omega^N_\alpha = (\tilde{\eta}_\alpha + 1)(\tilde{\eta}_\alpha + 2)$ and that of the unique (A) space is $\Omega^A_\alpha = 2(\eta_\alpha + 2)$ with the constraint $n_\alpha = n^N_\alpha + n^A_\alpha$. Proton and neutron states are coupled to angular momentum $J^N$ and $J^A$ in both the normal and unique parity sectors, respectively. The wave function of the many-particle
state with angular momentum $J$ and projection $M$ is expressed as a direct product of the normal and unique parity ones, as:

$$|JM> = \sum_{J^N J^A} [|J^N> \otimes |J^A>]_M.$$  \hspace{1cm} (1)

Since we are interested in describing low-lying energy states, only pseudo spin zero configurations are taken into account in the normal parity space and only seniority zero configurations in the abnormal parity space. This simplification implies that $J^A = J^N = 0$. This is a strong assumption, but one that is physically motivated and very useful for simplifying the calculations.

Double beta decay, when described in the pseudo SU(3) scheme, is strongly dependent on the occupation numbers for protons and neutrons in the normal and abnormal parity states: $n^N, n^\nu_N, n^A, n^\nu_A$. These numbers are determined by filling the Nilsson levels from below, as discussed in [7].

In the first series of papers [7, 8, 9, 10, 11] we evaluated the $\beta\beta$ matrix elements by taking into account only the leading SU(3) coupled proton-neutron irrep, which in recent calculation was shown to represent around 60% of the wave function in even-even Dy and Er isotopes [6].

### 3 Allowed two neutrino $\beta^-\beta^-$ decays

In all of the calculations only one active shell was allowed for protons, and likewise, only one for neutrons. This is a very strong truncation. For the $\beta\beta_{2\nu}$ decay this implies that only one uncorrelated Gamow-Teller transition is allowed: that which removes a neutron from a normal parity state with maximum angular momentum and creates a proton in the intruder shell ($h_{9/2}^n \rightarrow h_{11/2}^p$ in rare earth nuclei, $i_{11/2}^n \rightarrow i_{13/2}^p$ in actinides). This unique Gamow-Teller transition controls the $\beta\beta_{2\nu}$ decay. Under these assumptions, if the occupation of the Nilsson levels is such that the number of protons in the abnormal states does not change for the initial and final state configurations, the decay is forbidden.

The published results for the six allowed two neutrino $\beta^-\beta^-$ emitters are given in Table 1.

| Transition | $\tau^{1/2}_{2\nu}[yr]$ | $\tau^{1/2}_{0\nu}[yr]$ |
|------------|------------------------|------------------------|
| $^{146}$Nd $\rightarrow ^{146}$Sm | $2.1 \times 10^{31}$ | $1.18 \times 10^{28}$ |
| $^{148}$Nd $\rightarrow ^{148}$Sm | $6.0 \times 10^{20}$ | $6.75 \times 10^{24}$ |
| $^{150}$Nd $\rightarrow ^{150}$Sm | $6.0 \times 10^{18}$ | $1.05 \times 10^{24}$ |
| $^{186}$W $\rightarrow ^{186}$Os | $6.1 \times 10^{24}$ | $5.13 \times 10^{25}$ |
| $^{192}$Os $\rightarrow ^{192}$Pt | $9.0 \times 10^{25}$ | $3.28 \times 10^{26}$ |
| $^{238}$U $\rightarrow ^{238}$Pt | $1.4 \times 10^{21}$ | $1.03 \times 10^{24}$ |

Table 1: The calculated double beta half-lives for the two-neutrino and the zero-neutrino

The agreement between the theoretical two neutrino half lives with the available data for $^{150}$Nd ($\tau^{1/2}_{2\nu} = 9(17) \times 10^{18}$ yr) and $^{238}$U ($\tau^{1/2}_{2\nu} = 2 \times 10^{21}$ yr)
yr) is good. For the theoretical $\beta\beta_0\nu$ half lives we assumed $< m_\nu > = 1$eV.

4 Allowed double electron capture with two neutrino emission

The $2\nu$ double electron capture (ECEC)

$$2e^- + (A, Z + 2) \rightarrow (A, Z) + 2\nu$$ (2)

has larger Q-values than the concurrent $\beta^+\beta^+$ and $\beta^+\text{EC}$ processes, and has no Coulomb suppression but is very difficult to detect, because only two X rays are emitted together with the neutrinos.

The double electron capture decay to excited states in the final nuclei

$$(A, Z + 2) + 2e^- \rightarrow (A, Z)^* + 2\nu \rightarrow (A, Z) + 2\gamma$$ (3)

has been proposed as a good candidate to be measured \cite{22, 23}. The two gammas are far easier to detect than the X rays. A sensitivity close to $\sim 10^{22}$yr has been estimated for this type of experiments \cite{24}.

| Transition                  | $\tau^{1/2}$ (yr)    |
|-----------------------------|----------------------|
| $^{156}\text{Dy} \rightarrow ^{156}\text{Gd}$ | $0^+ \rightarrow 0^+ (\text{g.s.})$ | $2.74 \times 10^{22}$ |
|                            | $0^+ \rightarrow 0^+ (1)$ | $8.31 \times 10^{24}$ |
|                            | $0^+ \rightarrow 0^+ (2)$ | $1.08 \times 10^{25}$ |
| $^{162}\text{Er} \rightarrow ^{162}\text{Dy}$ | $0^+ \rightarrow 0^+ (\text{g.s.})$ | $2.85 \times 10^{22}$ |
|                            | $0^+ \rightarrow 0^+ (1)$ | $3.70 \times 10^{27}$ |
| $^{168}\text{Yb} \rightarrow ^{168}\text{Er}$ | $0^+ \rightarrow 0^+ (\text{g.s.})$ | $2.00 \times 10^{23}$ |
|                            | $0^+ \rightarrow 0^+ (1)$ | $5.36 \times 10^{33}$ |

Table 2: Half-lives for the ECEC$_{2\nu}$ decay to the ground and excited states of the final nuclei.

In Table 2 we present the ECEC$_{2\nu}$ decay of $^{156}\text{Dy}$, $^{162}\text{Er}$ and $^{168}\text{Yb}$ to the ground and excited states of $^{156}\text{Gd}$, $^{162}\text{Dy}$ and $^{168}\text{Er}$ respectively \cite{24}. The kinematical factors $G_{2\nu}(\sigma)$ were evaluated following the prescriptions given in \cite{24}. When the energy released in the decay to an excited stated ($2E_\nu$) is small the available phase space $G_{2\nu}$ is strongly reduced, and the half life could be very large. It is the case in the double electron capture decay to the first excited $0^+$ state in $^{168}\text{Er}$. The nuclear matrix elements associated with the decay to the ground state of the final nuclei have values close to 0.05 - 0.06, a factor of 5 smaller than the assumption of Barabash \cite{24}, and are similar for the three nuclei studied. The nuclear matrix elements to excited states show a wide spread, being close to those of the ground state.
for $^{156}$Dy, suppressed by a factor 5 for $^{162}$Er and by a factor 80 for $^{168}$Yb. While in general it is confirmed that deformed nuclei have smaller nuclear matrix elements than spherical $^{156}$Dy appears to be the best candidate of this group for experimental detection, with a half-life around $10^{24}$ years for the double electron capture to the first excited $0^+$ state.

5 Forbidden two neutrino $\beta^-\beta^-$ decays

As mentioned above, the number of nucleons in normal and unique parity orbitals is determined by the filling of the deformed Nilsson orbitals. In this way the theory predicts the complete suppression of the $\beta\beta_{2\nu}$ decay for the following five nuclei: $^{154}$Sm, $^{160}$Gd, $^{176}$Yb, $^{232}$Th and $^{244}$Pu. It was expected that these forbidden decays would have, in the best case, matrix elements that would be no greater than 20% of the allowed ones, resulting in at least one order of magnitude reduction in the predicted half-life.

Experimental limits for the $\beta\beta_{2\nu}$ decay of $^{160}$Gd have been reported. Recently it was argued that the strong cancellation of the $2\nu$ mode in the $\beta\beta$ decay of $^{160}$Gd would suppress the background for the detection of the $0\nu$ mode.

Assuming a slightly larger deformation for $^{160}$Dy, the most probable occupations for 16 valence protons are 10 in normal and 6 in unique parity orbitals, and for the 12 valence neutrons are 6 in normal and 6 in unique parity orbitals. The dominant component of the wave function is

$$|^{160}\text{Gd}, 0^+\rangle = |(h_{11/2})^6_\pi, J^A_\pi = 0; (i_{13/2})^6_\nu, J^A_\nu = 0 > A \rangle$$

$$\{2^4\}^*(10, 4)_\pi; \{2^4\}_\nu(18, 4)_\nu; 1(28, 4)K = 1, J = 0 > N \rangle .$$

The two neutrino double beta decay is allowed to these excited state.

The two neutrino double beta operator annihilates two neutrinos and creates two protons with the same spatial quantum numbers. It cannot connect the states $|^{160}\text{Gd}, 0^+\rangle$ and $|^{160}\text{Dy}, 0^+(a)\rangle$ and the transition is forbidden.

However, the pairing interaction allows mixing between different occupations. In the deformed single particle Nilsson scheme it takes about $\Delta E = 1.50$ MeV to promote a pair of protons from the last occupied normal parity orbital to the next intruder orbital. These excited state has 8 protons in normal and 8 in unique parity orbitals. Its wave function has the form

$$|^{160}\text{Dy}, 0^+\rangle = |(h_{11/2})^8_\pi, J^A_\pi = 0; (i_{13/2})^6_\nu, J^A_\nu = 0 > A \rangle$$

$$\{2^4\}^*(10, 4)_\pi; \{2^4\}_\nu(18, 0)_\nu; 1(28, 4)K = 1, J = 0 > N \rangle .$$

The two neutrino double beta decay is allowed to these excited state.
Using perturbation theory the $^{160}$Dy wave function is

$$|^{160}\text{Dy}, 0^+\rangle = a |^{160}\text{Dy}, 0^+(a)\rangle + b |^{160}\text{Dy}, 0^+(b)\rangle,$$

with $a^2 + b^2 = 1$ and

$$b = \langle |^{160}\text{Dy}, 0^+(b)\rangle | H_{\text{pairing}} |^{160}\text{Dy}, 0^+(a)\rangle / \Delta E.$$

Using standard SU(3) techniques we obtain $a = 0.972$, $b = 0.233$. For the two neutrino mode the half-life is $\tau_{2\nu} = 1.65 \times 10^{22}$ yr. It is delayed by one order of magnitude (compared with other nuclei with similar $Q_{\beta\beta}$ values) due to the presence of $b^2$ in the nuclear transition matrix element. The zero neutrino double beta transition is allowed to both states in $^{160}$Dy due to the presence of the neutrino potential in the transition operator. Assuming again $< m_\nu > = 1$eV the half-life is $\tau_{0\nu} = 3.96 \times 10^{25}$ yr.

The suppression of the $2\nu$ mode would facilitate the search for the zero neutrino mode in the double beta decay of $^{156}$Dy, as it was mentioned in [15]. However, the value of the zero neutrino half-life is larger than the one expected in this reference.

6 Conclusions

The pseudo SU(3) shell model allows for a fully microscopic description of heavy deformed nuclei in the laboratory frame, in a basis having good angular momentum and particle number. It has proven to be a powerful and predictive technique to describe the low energy spectra and electromagnetic transitions in rare earths and actinide isotopes. It has also been applied to the description of the double beta decay in these nuclei.

The model Hamiltonian includes realistic single particle energies, quadrupole-quadrupole and pairing interactions, whose strengths are fixed using known systematics. It also has three rotor terms which allow a fine tuning of the energy spectra. The wave functions mix different SU(3) irreps, while in even-even nuclei there is always one leading irrep which contributes with about 60% of the total wave function.

Using only these leading irreps the double beta transition matrix elements were evaluated. Both the two neutrino and zero neutrino mode were studied, involving decays to the ground state and to excited states in the daughter nuclei. It was found that, at first order, the two neutrino double beta decay is forbidden for many candidates. The double electron capture was also studied for three nuclei, concluding that the ECEC of $^{156}$Dy could be detected in future experiments.

Including different occupation numbers in normal and unique parity orbitals for protons and neutrons through pairing mixing it was possible to evaluate the forbidden half life of $^{160}$Gd, which is suppressed by a factor 10 (compared with other nuclei with similar $Q_{\beta\beta}$ values). It could offer a window to detect the zero neutrino mode which, having more decay channels, is less suppressed by deformation.
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