The GPD $\bar{H}$ and spin correlations in wide-angle Compton scattering

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Abstract

Wide-angle Compton scattering (WACS) is discussed within the handbag approach in which the amplitudes are given by products of hard subprocess amplitudes and form factors, specific to Compton scattering, which represent $1/x$-moments of generalized parton distributions (GPDs). The quality of our present knowledge of these form factors and of the underlying GPDs is examined. As will be discussed in some detail the form factor $R_A$ and the underlying GPD $\bar{H}$ are poorly known. It is argued that future data on the spin correlations $A_{LL}$ and/or $K_{LL}$ will allow for an extraction of $R_A$ which can be used to constrain the large $-t$ behavior of $\bar{H}$.

1 Introduction

Hard exclusive processes have extensively be investigated, experimentally as well as theoretically, over the last twenty years. It is fair to say that some understanding of these processes have been achieved so far. Thus, it is clear now that the handbag graph shown in Fig. 1, controls the two complementary processes - deeply virtual (DVCS) [1, 2, 3] and wide-angle Compton scattering [4, 5] even for kinematics accessible at the Jefferson Lab. DVCS is characterized by small momentum transfer from the initial to the final

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state proton and a large photon virtuality. The amplitudes in this case are
given by convolutions of hard subprocess amplitudes and soft proton matrix
elements parametrized as GPDs. As derived in [4, 5], for large Mandelstam
variables, $s, -t, -u$, the WACS amplitudes are given by products of hard
subprocess amplitudes and form factors, specific to WACS, which represent
$1/x$-moments of GPDs. The handbag approach can be generalized to deeply
virtual meson electroproduction (DVMP) and to wide-angle photoproduction
of mesons. It turned out that in both cases the numerical estimations
of cross sections fail by order of magnitude in comparison with experiment
at least for Jefferson Lab kinematics [6, 7, 8]. For pion electroproduction
an explanation of this failure has been found: lacking contributions from
transversity GPDs, formally of twist-3 nature, which are strongly enhanced
by the chiral condensate [9, 10, 11]. Inclusion of these contributions leads
to reasonable agreement with experiment [12, 13]. Whether this mechanism
also explains the underestimate of the pion-photoproduction cross section is
not yet clear [14].

With regard to planned experiments at Jefferson Lab it seems timely to
have a fresh look at WACS within the handbag approach. As compared to
the situation around 2000 there is new aspect: we have now a fair knowledge
of the GPDs $H$ and $E$ at large $-t$, underlying the Compton form factors $R_V$
and $R_A$, respectively, from an analysis of the nucleon form factors [15]. On
the other hand, our present knowledge of the GPD $\tilde{H}$ at large $-t$, related
to the form factor $R_A$, is still poor due to the very limited experimental in-
formation available on the isovector axial form factor of the nucleon in that
kinematical range. It is important to realize that for known Compton form
factors, evaluated for instance from given GPDs, the WACS cross section as
well as spin-dependent observables can be computed free of any adjustable
parameter. However, because of the poor knowledge of $\tilde{H}$, the present nu-
merical computations of the form factor $R_A$ suffer from large uncertainties
and, therefore, predictions for WACS observables which are sensitive to $R_A$,
too. With regard to the interest in the GPD $\tilde{H}$ at large $-t$ for studying
the impact parameter distribution of quarks with definite helicity it will be
proposed in this article to turn the strategy around and to extract the form
factor $R_A$ from spin correlations like $A_{LL}$ or $K_{LL}$. Accurate data on such
spin correlations at sufficiently large $-t$ and $-u$ will provide a set of values
on $R_A(t)$ which subsequently can be used as a constraint on $\tilde{H}$ in addition
to the data on the axial form factor. Also for this form factor more and
better data are to be expected from the FNAL MINERVA experiment in the
Figure 1: The handbag contribution to WACS. The particle momenta and helicities are quoted. The horizontal lines represent any number of spectator partons.

near future. Constraining \( \tilde{H} \) by data on \( F_A \) and \( R_A \) will also allow for a more accurate flavor separation as was possible up to now. It even might be possible to say something about \( \tilde{H} \) for sea quarks at large \(-t\).

The plan of the paper is the following: In Sect. 2 the handbag approach to WACS will be recapitulated and in Sect. 3 the properties of the zero-skewness GPDs at large \(-t\) will be discussed. Sect. 4 is devoted to a discussion of spin correlations with regard to their sensitivity to \( R_A \). A summary is given in Sect. 5. In the appendices different conventions for the spin observables are presented and compared to each other.

2 The structure of the handbag mechanism

For the description of the handbag contribution to WACS we follow Ref. [5]. It is assumed that the Mandelstam variables \( s, -t \) and \( -u \) are much larger than \( \Lambda^2 \) where \( \Lambda \) is a typical hadronic scale of order 1 GeV. It is of advantage to work in a symmetric frame where the momenta of the initial \((p)\) and final \((p')\) state protons are parametrized as

\[
p'(\gamma) = \left[ p^+, \frac{m^2 - \frac{\Delta^2}{4}}{2p^+}, \frac{1}{2} \Delta_\perp \right]
\]

in light-cone coordinates; \( m \) denotes the proton mass. The photon momenta \((q\) and \(q')\) are defined analogously. In this frame the plus and minus light-cone components of the momentum transfer, \( \Delta = p' - p \), are zero implying
\[ t = -\Delta^2 \] as well as a vanishing skewness parameter \( \xi = (p - p')^+/(p + p')^+ \). The handbag contribution, shown in Fig. 1, is then defined through the assumption that the soft hadronic wave functions occurring in the Fock decomposition of the proton state are dominated by parton virtualities in the range \( |k_{i}^2| \lesssim \Lambda^2 \) and by intrinsic transverse momenta, \( k_{i}^\perp \), that satisfy \( k_{i,\perp}^2/x_i \lesssim \Lambda^2 \) where \( x_i = k_i^+/p^+ \) is the usual light-cone momentum fraction.  

In frames with non-zero skewness there are additional contributions. On these presuppositions one can show that the photon-parton scattering is hard and the momenta of the active partons, \( k_j, k'_j \), are approximately on-shell, collinear with their parent hadrons and with momentum fractions, \( x_j, x'_j \), close to 1. A consequence of these properties is that the Mandelstam variables in the photon-parton subprocess, \( \hat{s}, \hat{t}, \hat{u} \), and in the overall photon-proton reaction, \( s, t, u \), are approximately equal up to corrections of order \( \Lambda^2/t \). The propagator poles of the handbag graphs at \( \hat{s} = 0 \) (in the graph shown in Fig. 1) and \( \hat{u} = 0 \) (for the graph with the two photons crossed) are thus avoided. The physical picture is that of a hard photon-parton scattering and soft emission and re-absorption of partons by the proton.

The (light-cone) helicity amplitudes for WACS in the symmetric frame are then given by [5, 16]

\[
\mathcal{M}_{\mu' + \mu} (s, t) = 2\pi \alpha_{em} [\mathcal{H}_{\mu' + \mu} (\hat{s}, \hat{t}) (R_V(t) + R_A(t)) \\
+ \mathcal{H}_{\mu' - \mu} (\hat{s}, \hat{t}) (R_V(t) - R_A(t))],
\]

\[
\mathcal{M}_{\mu' - \mu} (s, t) = \pi \alpha_{em} \frac{\sqrt{-t}}{m} [\mathcal{H}_{\mu' + \mu} (\hat{s}, \hat{t}) + \mathcal{H}_{\mu' - \mu} (\hat{s}, \hat{t})] R_T(t). \tag{2}
\]

The amplitudes are subject to uncontrolled corrections of order \( \Lambda^2/t \). Explicit helicities are labeled only by their signs. In the Compton amplitudes, \( \mathcal{M} \), the explicit helicities refer to those of the protons, in the subprocess amplitudes, \( \mathcal{H} \), to those of the active partons.

The soft proton matrix elements \( R_i \ (i = V, A, T) \), appearing in (2), represent new types of proton form factors specific to Compton scattering. These Compton form factors are defined as \( 1/x \)-moments of zero-skewness GPDs.

\[ ^{2} \text{A restriction of transverse momenta by } k_{i,\perp}^2 \lesssim \Lambda^2 \text{ instead fails to ensure small parton virtualities in the proton. At least one parton virtuality would be of order } \Lambda \sqrt{-t} \text{ and not } \Lambda^2. \]
for \( -t \gg \Lambda^2 \). For active quarks of flavor \( a \) (\( u, d, \ldots \)) they read

\[
R^a_V(t) = \int_{-1}^{1} \frac{dx}{x} H^a(x, t), \\
R^a_A(t) = \int_{-1}^{1} \frac{dx}{x} \text{sign}(x) \tilde{H}^a(x, t), \\
R^a_T(t) = \int_{-1}^{1} \frac{dx}{x} E^a(x, t).
\]  

(3)

The full form factors in (2) are given by the sum

\[
R_i(t) = \sum_a e_a^2 R^a_i(t),
\]

(4)

e\_a being the charge of the quark \( a \) in units of the positron charge. In principle there is a fourth form factor, related to the GPD \( \tilde{E} \), which however decouples in the symmetric frame. The flavor form factors (3) also appear in wide-angle photoproduction of mesons [8].

The hard scattering amplitudes \( H_{\mu'\lambda',\mu\lambda} \) are to be calculated perturbatively. To leading order (LO), obtained from the graph shown in Fig. 1 and the one with the two photons crossed, they read

\[
H_{++;++}^{LO} = 2\sqrt{\frac{s}{-\hat{u}}}, \quad H_{++;++}^{LO} = 2\sqrt{\frac{-\hat{u}}{s}} , \quad H_{--;++}^{LO} = 0.
\]  

(5)

Since the quarks are taken as massless there is no quark helicity flip to any order of \( \alpha_s \). In [16] the next-to-leading order (NLO) corrections have also been calculated. They provide phases and logarithmic corrections to the LO amplitudes and generate a non-zero photon helicity-flip amplitude

\[
H_{--;++}^{NLO} = -\frac{\alpha_s}{2\pi} C_F \left( \sqrt{\frac{s}{-\hat{u}}} + \sqrt{\frac{-\hat{u}}{s}} \right).
\]  

(6)

The matching of the subprocess and the full Mandelstam variables is simple if the mass of the proton can be neglected. In this case

\[
\text{scenario 1 : } \quad \hat{s} = s , \quad \hat{t} = t , \quad \hat{u} = u.
\]  

(7)

\(^3\)Since we only consider zero-skewness GPDs their skewness argument is dropped unless stated otherwise.

\(^4\)For a discussion of the scale-dependence of the Compton form factors see [17].

\(^5\)Since in WACS \(-\hat{t}\) and \(-\hat{u}\) are of order \( \hat{s} \) there are no large logarithms in the NLO amplitudes.
In order to estimate the influence of the proton mass two more scenarios have been introduced in [18]:

\text{scenario 2} : \hat{s} = s - m^2, \quad \hat{t} = t, \quad \hat{u} = u - m^2, \\
\text{scenario 3} : \hat{s} = s - m^2, \quad \hat{t} = -\frac{\hat{s}}{2}(1 - \cos \theta_{cm}), \quad \hat{u} = -\hat{s} - \hat{t}. \quad (8)

The center-of-mass-system (c.m.s.) scattering angle is denoted by $\theta_{cm}$. In the latter two scenarios one has $\hat{s} + \hat{t} + \hat{u} = 0$ in contrast to scenario 1. As advocated for in [18] WACS observables are calculated in the three scenarios and the differences are considered as uncertainties of the predictions.

To NLO there are also contributions from the gluonic subprocess $\gamma g \rightarrow \gamma g$. They are in general small and we only take into account the most important one arising from the gluonic GPD $H^g$:

$$2\pi\alpha_{em} \left[ \mathcal{H}^g_{\mu',\mu} + \mathcal{H}^g_{\mu',-\mu} \right] R^g_V(t) \quad (9)$$

which is to be added to the proton helicity non-flip amplitude in (2). The explicit helicity labels refer to the gluon helicity now. The form factor $R^g_V$ is given by

$$R^g_V(t) = \sum_a e_a^2 \int_0^1 \frac{dx}{x^2} H^g(x, t). \quad (10)$$

The additional factor $1/x$ as compared to the form factors (3) is conventional, it appears as a consequence of the definition of the gluon GPD whose forward limit is

$$xg(x) = H^g(x, 0). \quad (11)$$

We refrain from quoting the NLO subprocess amplitudes here; they can be found in [16].

The last issue to be discussed in this section is the choice of the helicity basis. The derivation of the amplitudes (2) naturally requires the use of light-cone helicities. However, for comparison with experiment the use of the ordinary helicities is more convenient. Diehl [19] has given the transformation from on basis to the other. The standard helicity amplitudes, $\Phi_{\mu';\nu',\mu\nu}$, (the notation of the helicity labels are kept) are related to the light-cone amplitudes (2) by

$$\Phi_{\mu';\nu',\mu\nu} = \mathcal{M}_{\mu';\nu',\mu\nu} + \frac{\eta}{2} \left[ (-1)^{1/2-\nu'} \mathcal{M}_{\mu'-\nu',\mu\nu} + \mathcal{M}_{\mu';\nu',\mu-\nu'} \right] + \mathcal{O}(\Lambda^2/t) \quad (12)$$
where
\begin{equation}
\eta = \frac{2m}{\sqrt{s}} \frac{\sqrt{-t}}{\sqrt{s + \sqrt{-u}}}.
\end{equation}

In principle there are 16 amplitudes for Compton scattering. However parity and time-reversal invariance lead to relations among them \(^6\)
\begin{equation}
\Phi_{-\mu'-\nu',-\mu-\nu} = \Phi_{\mu\nu,\mu'\nu'} = (-1)^{\mu-\nu-\mu'+\nu'} \Phi_{\mu'\nu',\mu\nu}.
\end{equation}

With the help of (14) one sees that there are only 6 independent amplitudes for which we choose \(^6\)
\begin{align}
\Phi_1 &= \Phi_{++;++}, \quad \Phi_3 = \Phi_{-+;++}, \quad \Phi_5 = \Phi_{-+,+-}, \\
\Phi_2 &= \Phi_{-+;++}, \quad \Phi_4 = \Phi_{+-;++}, \quad \Phi_6 = \Phi_{+++-}.
\end{align}

Inspection of (2) and (12) reveals that
\begin{equation}
\Phi_2 = -\Phi_6 + \mathcal{O}(\Lambda^2/t) .
\end{equation}

This relation is a robust property of the handbag mechanism which is difficult to change. The photon helicity-flip amplitudes \(\Phi_i, \ i = 2, 3, 6\) are related to \(\mathcal{H}_{NLO}^{++}\) and, hence, they are of order \(\alpha_s\) (see (6)).

3 The GPDs at large \(-t\)

The sum rules for the Dirac \((i = 1)\) and Pauli \((i = 2)\) form factors of the nucleon read
\begin{equation}
F_i^{p(n)}(t) = e_u F_i^{u(d)}(t) + e_d F_i^{d(n)}(t)
\end{equation}
with the flavor form factors defined by
\begin{equation}
F_i^{a}(t) = \int_0^1 dx K_{iv}^a(x, t)
\end{equation}
where \(K_{iv}^a\) denotes the relevant proton GPD, either \(H_v^a\) for the Dirac form factor or \(E_v^a\) for the Pauli one. A valence-quark GPD is defined by the combination
\begin{equation}
K_{iv}^a(x, t) = K_{iv}^a(x, t) + K_{iv}^a(-x, t).
\end{equation}

\(^6\)Analogous relations for the other set of amplitudes, \(\mathcal{M}\) and \(\mathcal{H}\)
In [15] the GPDs $H$ and $E$ for valence quarks have been extracted from the data on the magnetic form factor of the proton and the neutron and from the ratios of electric and magnetic form factors exploiting the sum rules (17) with the help of a parametrization of the zero-skewness GPDs:

$$K^a_{ia}(x,t) = k^a_i(x) \exp[tf^a_i(x)].$$

(20)

In [15, 17] it is advocated for the following parametrization of the profile function

$$f^a_i(x) = (\alpha^a_i \ln (1/x) + B^a_i)(1-x)^3 + A^a_i x(1-x)^2$$

(21)

which differs from the Regge-like parametrization

$$f ^{a}_{iR}(x) = \alpha^a_i \ln (1/x) + B^a_i$$

(22)

frequently used in DVCS and DVMP. The above parametrization refers to a definite factorization scale $\mu_F$ for which we take 2 GeV throughout this work. The forward limit of the GPD $H^a$ is given by the flavor-$a$ parton densities, $q^a(x)$, for which the ABM11 densities [21], evaluated at the scale $\mu_F$, are used in [15]. The forward limit of $E^a$ which is not accessible in deep-inelastic scattering and is, therefore, to be determined in the form factor analysis, too. It is parametrized like the parton densities

$$e^a(x) = N_a x^{\alpha^a}(1-x)^{\beta^a}(1+\gamma^a\sqrt{x}).$$

(23)

The normalization, $N_a$, is obtained from the contribution of quarks of flavor $a$ to the anomalous magnetic moment of the nucleon ($\kappa_u = 1.67, \kappa_d = -2.03$)

$$\kappa_a = \int_0^1 dx E^a(x,0).$$

(24)

The parameters of the profile functions as well as the additional ones for $e^a(x)$ are fitted to the nucleon form factor data in [15] and from the resulting GPDs the Compton form factors $R_V$ and $R_T$ are subsequently evaluated. These form factors will be used in the following without exception.

The sum rule for the isovector axial form factor reads

$$F_A(t) = \int_0^1 dx [\tilde{H}^u_i(x,t) - \tilde{H}^d_i(x,t)] + 2 \int_0^1 dx [\tilde{H}^u(-x,t) - \tilde{H}^d(-x,t)].$$

(25)
In contrast to the electromagnetic form factors the sea quark contributions do not drop in this case. The GPD $\tilde{H}$ for valence quark is parametrized in the same fashion as $H$ with the unpolarized parton densities replaced by the polarized ones
\[
\tilde{H}_v^a(x, t) = \Delta q^a(x) \exp \left[ t \tilde{f}^a(x) \right].
\] (26)
The profile function is parametrized as in (21). In [15] a fit of the parameters for $\tilde{H}$ has not been attempted since the data basis for the axial form factor is meager. The only set of data that covers a fairly large range of $-t (\leq 3 \text{ GeV}^2)$ is that one measured by Kitagaki et al [22]. These data are presented in a form of a dipole fit with a mass parameter $M_A = (1.05^{+0.12}_{-0.16})$ GeV and are shown in Fig. 2. More accurate data on $F_A$ are to be expected from the FNAL MINERVA experiment. Therefore, with regard to the current data situation, it has been assumed in [15] that $\tilde{f}^a$ is the same as the profile function, $f_1^a$, for $H_v^a$. The sea quark contribution has been neglected. It is very small at large $-t$ as estimated in [15] (see also the discussion below). The polarized parton densities of [23] were taken for the forward limit at the scale 2 $\text{GeV}$. The results on $F_A$ and $R_A$ are displayed in Figs. 2 and 3, respectively. The axial form factor, although in agreement with experiment, lies at the lower edge of the data. The parameters of the profile function $\tilde{f}^a$ used in [15] are quoted in Tab. 1.

Let us consider the Fourier transformation of the zero-skewness valence quark GPDs to the impact parameter plane
\[
k_i^a(x, b^2) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-ib\cdot\Delta_\perp} K_i^a(x, t = -\Delta_\perp^2).
\] (27)
Explicitly the parametrization (20) leads to
\[
k_i^a(x, b^2) = \frac{1}{4\pi} \frac{k_i^a(x)}{f_1^a(x)} \exp \left[ -\frac{b^2}{4f_1^a(x)} \right].
\] (28)
The sum and difference of the unpolarized and the longitudinally polarized impact parameter distributions
\[
q_i^a(x, b^2) = \Delta q_i^a(x, b^2)
\] (29)
possess a density interpretation [24] which implies the bound [17]
\[
\tilde{f}^a(x) \leq f^a(x)
\] (30)
in the region where antiquarks can be neglected. Taking \( \tilde{f}^a(x) < f^a(x) \) instead of \( \tilde{f}^a(x) = f^a(x) \) increases the flavor form factor
\[
\tilde{F}^a(t) = \int_0^1 dx \tilde{H}^a_v(x, t),
\]
for large \(-t\). As we will discuss in Sect. 4 the data on the helicity correlation \( K_{LL} \) [25, 26], although measured at values of \(-t\) or \(-u\) being not sufficiently large for an application of the handbag approach, have rather large values. This may be taken as a hint at larger values of \( R_A \) than quoted in [15], see Fig. 3.

In order to understand the reason for choosing the complicated profile function (21) let us discuss the general behavior of the GPDs (20). They behave Regge-like for \( x \to 0 \), i.e. for \( \tilde{H} \)
\[
\tilde{H}^a \sim x^{-\tilde{\alpha}_a - t\tilde{\alpha}'_a}
\]
where the power \( \tilde{\alpha}_a \), the intercept of a Regge-like trajectory, is hidden in the polarized parton densities. For \( u \)-valence quarks its value is about 0.1 as obtained from a fit to the DSSV PDFs [23] in the range \( 10^{-3} < x < 10^{-2} \).

\[^7\]An analogous discussion can be established for the other GPDs.
\( \Delta q^d(x) \) behaves similar but with very large uncertainties. I.e. at small \(-t\), the GPDs are singular for \( x \to 0 \) as the parton densities. With increasing \(-t\) the singularity becomes milder and turns into a zero for

\[ -t > -t_0 = \tilde{\alpha}_a/\tilde{\alpha}'_a. \]  

(33)

With \( \tilde{\alpha}'_a \simeq 0.9 \text{ GeV}^{-2} \), \( t_0 \) is about \(-0.1 \text{ GeV}^2\). While the flavor form factors (31) exist for all \( t \), the \( 1/x \)-moments are singular for \( -t < -t_0 \). This is unproblematic since the Compton form factors are only defined for \( -t \gg \Lambda^2 \).

In order to achieve larger values for \( R_A \) at intermediate \(-t\) it is plausible to use smaller values of \( \tilde{\alpha}'_a \) and, hence, a larger value of \(-t_0\) closer to the \( t \)-range of applicability of the handbag approach. For this purpose we consider two examples: For the case \#1 we take \( \simeq \tilde{\alpha}'_a/2 \), characteristic of a Regge cut, and for case \#2 \( \tilde{\alpha}'_u = 0.144 \) but leaving \( \tilde{\alpha}'_d \) unchanged, see Tab. 1. In both cases the other parameters in the profile function are fitted to the \( F_A \) data (for case \#1 common factors for the \( u \) and \( d \) quark parameters \( A \) and \( B \) are used). The parameters of \( f^a \) are compiled in Tab. 1. In all cases the bound (30) is respected for all \( x \).

|         | \( \tilde{f}^u \) |          | \( \tilde{f}^d \)          |          |
|---------|------------------|----------|--------------------------|----------|
|         | \( \tilde{\alpha}'_u \) | \( \tilde{B}_u \) | \( \tilde{A}_u \) | \( \tilde{\alpha}'_d \) | \( \tilde{B}_d \) | \( \tilde{A}_d \) |
| [15]    | 0.961            | 0.545    | 1.264                    | 0.861    | 0.333 | 4.198 |
| \#1     | 0.432            | 0.654    | 1.239                    | 0.387    | 0.400 | 4.284 |
| \#2     | 0.144            | 1.100    | 1.150                    | 0.861    | 0.333 | 4.198 |

Table 1: Parameters of the profile function \( \tilde{f} \) for \( u \) and \( d \) valence quarks. All quantities are given in units of \( \text{GeV}^{-2} \).

The results on \( F_A \) for the cases \#1 and \#2 are also shown in Fig. 2. Both the examples are well in agreement with the data but substantially larger for \( \sqrt{-t} \) in the range \( 1 - 2 \text{ GeV} \) than the form factor proposed in [15] from the assumption \( \tilde{f}^a = f^a \) for \( \sqrt{-t} \simeq 2 \text{ GeV} \). At large \( \sqrt{-t} \) all three cases are very close to each other. A similar behavior is seen in Fig. 3 for the form factor \( R_A \). The GPD \( \tilde{H}^a \) for the three cases are displayed in Fig. 4. The GPD exhibits a pronounced maximum (minimum) at a large value of \( x \), its position
Figure 4: Left: The zero-skewness GPD \( \tilde{H} \) versus \( x \) at the scale 2 GeV. The solid (dashed, dotted) line represents the result obtained from [15] (#1, #2).

Figure 5: Right: The ratio of \( q_u^v(x, b) \) for the Regge-like profile function (22) and for (21) at \( x = 0.05(0.2, 0.6) \) solid (dashed, dotted) line. The scale is 2 GeV.

moves towards larger values of \( x \) for increasing \( -t \) and becomes narrower. For the three cases \( \tilde{H} \) looks very similar, there are only small differences in the position and the height of the maximum respective minimum. The above considerations tell us that there is a strong \( x - t \) correlation in the GPDs (20).

Due to the \( x - t \) correlation the flavor form factors calculated from the GPDs (20), (26) are under control of large \( x \) at large \( -t \). The polarized and unpolarized parton densities behave as \( (1 - x)^\beta \) for \( x \rightarrow 1 \) and an analogous behavior holds for \( E \), see (23). For valence quarks the powers \( \beta \) are obtained from fits to the large \( x \) behavior \( (0.65 < x < 0.85) \) of the ABM [21] and DSSV [23] PDFs. The corresponding powers of \( E \) are determined in the nucleon form factor analysis performed in [15]. The various powers of the
valence-quark GPDs are compiled in Tab. 2. With the help of the saddle point method one can show [15] that the flavor form factors behave as 

\[ F_i(t) \sim 1/(-t)^{(1+\beta)/2} \]  

(34)

at sufficiently large \(-t\) (the powers \((1 + \beta/2)\) are also quoted in Tab. 2). One also sees that the saddle point lies in the so-called soft region

\[ 1 - x \sim \Lambda/\sqrt{-t} \]  

(35)

where, again at sufficiently large \(-t\), the active parton carries most of the proton’s momentum while all spectators are soft. This is the region of the Feynman mechanism, discussed already by Drell and Yan [28].

The GPD parametrizations (20) and (26) may analogously be extended to sea quarks and gluons. For the corresponding flavor form factors, i.e. their lowest moments, the power behavior (34) holds too. The powers \(\beta\) of the PDFs for sea quarks and gluons are \(\gtrsim 5\). This is in agreement with perturbative QCD considerations in the limit \(x \to 1\) [29]. Similar powers are expected for \(E\) for gluons and sea quarks. The corresponding flavor form factors are therefore strongly suppressed, the valence-quark form factors dominate at large \(-t\). Since for \(x \to 1\) the \(1/x\)-factor in the Compton form factors can be neglected, these form factors are also dominated by the valence quark contributions and in particular by the \(u\)-valence quark one. This has the following consequences which hold at sufficiently large \(-t\):

\[ R_V \simeq e_u^2 F_1^u, \quad R_A \simeq e_u^2 \tilde{F}_1^u, \quad R_T/R_V \simeq 1/|t|^{0.6}, \]  

(36)

and for the ratio of the form factors for Compton scattering off neutrons and off protons one has

\[ R_n^i/R_p^i \simeq e_d^2/e_u^2. \]  

(37)

Another issue is the role of the third term in (21) which in fact is responsible for the large \(-t\) behavior of the GPDs. This term cannot be fixed from DVCS and DVMP data since they are typically measured at rather small values of \(-t\) (note that factorization of these processes requires \(-t \ll Q^2\)). Therefore, the Regge-like profile function (22) is frequently used in the analysis of these processes. As discussed in [17] the Regge-like profile function

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\(^8\)A power law behavior of form factors and other exclusive observables have also been obtained from soft physics, namely from overlaps of light-cone wave functions, in [27].
although it is a reasonable approximation at low $x$ (low $-t$), is unphysical at large $x$ (large $-t$). The transverse distance between the active parton and the cluster of spectators, say for the GPD $H$, is given by

$$d_a(x) = \frac{\sqrt{\langle b^2 \rangle_a}}{1-x} = 2 \frac{\sqrt{f^a_1(x)}}{1-x}$$

(38)

for the parametrization (20). In the limit $x \to 1$, the profile function (21) leads to a finite result for the distance $d_a(x)$ ($\sim 2\sqrt{A^a}$) while for (22) the distance $d_a$ becomes singular ($\sim 2\sqrt{B^a/(1-x)}$). This consideration makes it clear that the Regge-like profile function does not allow for an investigation of the localization of the partons in the impact parameter plane. This is also obvious from Fig. 5 where the ratio of $q^u_v(x, b^2)$ evaluated from the profile functions (22) and (21) is displayed for several values of $x$. While there is not much difference between the two distributions, $q^u_R(x, b^2)$ (evaluated from (22)) and $q^u_{DK}(x, b^2)$ (evaluated from (21)) at small $x$, the Regge-like profile function leads to a much wider distribution at large $x$ than the profile function (21).

Finally, we want to discuss the impact-parameter distribution of valence quarks with definite helicities, defined by

$$q^a_{v\pm}(x, b^2) = \frac{1}{2}[q^a_v(x, b^2) \pm \Delta q^a_v(x, b^2)].$$

(39)

Here, $q_{v\pm}(x, b^2)$ is the distribution of quarks with helicity parallel (+) or anti-parallel (−) to the proton's helicity. These distributions are axially symmetrical around the direction of the proton momentum. In Figs. 6 and 7, we show these distributions versus $b$ at $x = 0.05$ and 0.6, respectively. By far the most important distribution is the $u$-quark one with parallel helicity. At large $x$ in particular the other three distributions can be neglected. At low $x$ the second largest distribution is the $d$-quark one with anti-parallel helicity.

### Table 2: The powers $\beta$ and $(1 + \beta)/2$ for the valence quark GPDs.

|       | $H^u_v$ | $H^d_v$ | $E^u_v$ | $E^d_v$ | $\bar{H}^u_v$ | $\bar{H}^d_v$ |
|-------|---------|---------|---------|---------|---------------|---------------|
| $\beta$ | 3.50    | 5.00    | 4.65    | 5.25    | 3.43          | 4.22          |
| $(1 + \beta)/2$ | 2.25    | 3.00    | 2.83    | 3.12    | 2.22          | 2.61          |
Figure 6: Left: The impact parameter distributions for valence quarks with definite helicities at $x = 0.05$ (in fm$^{-2}$). $\Delta q(x, b^2)$ is evaluated from the profile function for example #1. The PDFs are taken at the scale 2 GeV.

Figure 7: Right: as Fig. 6 but for $x = 0.6$.

The dominance of the $u$-quark distribution with parallel helicity at large $x$ is expected from perturbative QCD [29]. On the other hand, the behavior of the $d$-quark distribution does not match the perturbative QCD predictions at the current experimentally accessible range of $x$ - the helicity-parallel distribution does not dominate since $\Delta q^d$ is negative. One also observes from Figs. 6 and 7 the typical behavior of the impact-parameter distribution: a very broad distribution at low $x$ which becomes narrower with increasing $x$, i.e. for $x \to 1$ the active parton is close to the proton’s center of momentum [24].
4 Spin correlations

All numerical results for WACS observables are evaluated to order $\alpha_s$ and terms of order $\eta^2$ are neglected throughout. The Compton form factors $R_V$ and $R_T$ are taken from [15]; for the third form factor, $R_A$, the three examples are considered which are shown in Fig. 3 and for which the different profile functions for $\bar{H}$ are quoted in Tab. 1. The gluonic form factor $R^g_V$ is also taken into account in addition to the valence quark Compton form factors. Numerical values for this form factor are taken from [8] where this form factor is modeled as a light-cone wave function overlap.

The unpolarized differential cross section

$$\frac{d\sigma}{dt} = \frac{1}{32\pi(s-m^2)^2} \left[ |\Phi_1|^2 + |\Phi_2|^2 + 2|\Phi_3|^2 + 2|\Phi_4|^2 + |\Phi_5|^2 + |\Phi_6|^2 \right]$$  \hspace{1cm} (40)

evaluated for the two examples of $R_A$, #1 and #2, does not differ much from the result given in [15] since the $R_A$ contribution is suppressed by $t^2/(s-u)^2$ in comparison to the $R_V$ one. We therefore refrain from showing results on the cross section but refer to [15] and concentrate ourselves in this article on spin effects. \(^9\)

For later use we also quote the LO handbag result for the cross section:

$$\frac{d\sigma}{dt} = \frac{\pi\alpha^2_{em}}{(s-m^2)^2} \left( \frac{s-u}{-us} \right)^2 (1 + \kappa^2_T) R^2_V(t) \left[ 1 + \frac{t^2}{(s-u)^2} \frac{R^2_A(t)}{R^2_V(t)} \frac{1}{1 + \kappa^2_T} \right]$$  \hspace{1cm} (41)

where

$$\kappa_T(t) = \frac{\sqrt{-t}}{2m} \frac{R_T(t)}{R_V(t)}. \hspace{1cm} (42)$$

This quantity acquires values of between about 0.3 and 0.6 and depends on $t$ only mildly at large $-t$, see Tab. 2.

The first observables we are going to discuss is the helicity ($L$-type, see App. A) correlation between the initial state photon and proton defined in terms of cross sections $d\sigma(\mu\nu)$ by

$$A_{LL} = \frac{d\sigma(++) - d\sigma(+-)}{d\sigma(++) + d\sigma(+-)},$$  \hspace{1cm} (43)

\(^9\)Tables with numerical results for the observables can be obtained from the author on request.
leading to

\[
A_{LL} \frac{d\sigma}{dt} = \frac{1}{32\pi(s - m^2)^2} \left[ |\Phi_1|^2 + |\Phi_2|^2 - |\Phi_5|^2 - |\Phi_6|^2 \right]
\]  

(44)

in terms of the helicity amplitudes (15). The analogous correlation between the helicities of the incoming photon and the outgoing proton reads

\[
K_{LL} = \frac{d\sigma(++) - d\sigma(+-)}{d\sigma(++) + d\sigma(+-)}
\]

(45)

in terms of the cross sections \(d\sigma(\mu\nu')\). Expressed through the c.m.s. helicity amplitudes it reads

\[
K_{LL} \frac{d\sigma}{dt} = \frac{1}{32\pi(s - m^2)^2} \left[ |\Phi_1|^2 - |\Phi_2|^2 - |\Phi_5|^2 + |\Phi_6|^2 \right].
\]

(46)

Since \(\Phi_2 = -\Phi_6\), see (16), one arrives at

\[
A_{LL} = K_{LL}.
\]

(47)

As mentioned in Sect. 2 this is a robust prediction of the handbag approach. However, using massive point-like quarks, \(A_{LL}\) and \(K_{LL}\) differ from each other in the backward hemisphere where \(A_{LL}\) becomes smaller than \(K_{LL}\) [18, 30]. Handbag results on \(A_{LL} = K_{LL}\) are shown in Fig. 8 in a kinematical region where \(-t\) and \(-u\) are at least larger than about 2.5 GeV². The bands in the plot represent the results evaluated from example #1 and are displayed for several values of \(s\). The widths of the bands indicate possible kinematical corrections due to the mass of the proton [18]. For \(s\) below about 10 GeV² the uncertainty due to the proton mass corrections are rather large but become tolerable above 10 GeV². Results on \(A_{LL} = K_{LL}\) evaluated from example #2 and from \(R_A\) as quoted in [15] are also shown at \(s = 7.8\) and 15 GeV². At fixed \(t\) a strong energy dependence is to be noticed. The available data on \(K_{LL}\) are also displayed in Fig. 8. They are measured at c.m.s. scattering angles of \(\theta_{cm} = 70^\circ\) [26] and 120° [25] at \(s = 7.8\) and 6.9 GeV², respectively. Both the data points are not compatible the prerequisite for the application of the handbag approach, namely \(s, -t, -u \gg \Lambda^2\). For the data point at 70° \(t\) is \(-2.1\) GeV² and at 120° \(u\) is only \(-1.04\) GeV².

\[10\] It can be shown that in this kinematical range the present cross section data [31] are compatible with factorization of the RCS amplitudes in subprocess amplitudes and form factor which only depend on \(t\).
As one sees from the plot the helicity correlation parameter is very sensitive to the actual value of $R_A$. It seems possible, as our analysis reveals, to achieve values for of $K_{LL}$ as large as the data [26, 25] indicate. The sensitivity of $A_{LL}$ and $K_{LL}$ on the form factor $R_A$, or, strictly speaking, on the ratio $R_A/R_V$ is obvious from a comparison with the LO result:

$$K_{LL} = 2 \frac{-t}{s-u} \frac{R_A}{R_V} \frac{1 + \eta \kappa_T}{1 + \kappa_T^2} \left[ 1 + \frac{t^2}{(s-u)^2} \frac{R_A^2}{R_V^2} \frac{1}{1 + \kappa_T^2} \right]^{-1}. \quad (48)$$

In a somewhat rough approximation the helicity correlation is given by the
Klein-Nishina helicity correlation for massless quarks

\[ A_{LL}^{KN} = K_{LL}^{KN} = \frac{s^2 - u^2}{s^2 + u^2} \]  

(49)
diluted by the ratio of axial-vector over vector form factor, \( R_A / R_V \). Hence, accurate data on \( A_{LL} \) and/or \( K_{LL} \) would allow for a determination of that ratio and, subsequently, for an extraction of \( R_A \) for a given vector form factor.

Somewhat similar is the correlation between the helicity of the incoming proton and the sideway polarization (\( S \)-type, see App. A) of the incoming proton defined by the following ratio of the cross sections \( d\sigma(\mu \rightarrow) \)

\[ A_{LS} = \frac{d\sigma(\rightarrow) - d\sigma(\rightarrow)}{d\sigma(\rightarrow) + d\sigma(\rightarrow)} \]  

(50)
which can be expressed as

\[ A_{LS} \frac{d\sigma}{dt} = \frac{1}{16\pi(s - m^2)} \text{Re} \left[ (\Phi_1^* - \Phi_5^*)\Phi_4 - (\Phi_2^* + \Phi_6^*)\Phi_3 \right]. \]  

(51)
For the correlation between the helicity of the incoming photon and the sideway polarization of the outgoing proton we analogously find

\[ K_{LS} \frac{d\sigma}{dt} = \frac{1}{16\pi(s - m^2)} \text{Re} \left[ (\Phi_1 - \Phi_5)\Phi_4^* + (\Phi_2 + \Phi_6)\Phi_3^* \right]. \]  

(52)
With \( \Phi_2 = -\Phi_6 \) we have

\[ A_{LS} = -K_{LS} \]  

(53)
in the handbag approach.  \(^{11}\) The LO result for \( K_{LS} \) reads:

\[ K_{LS} = 2\frac{-t}{s - u} \frac{R_A \kappa_T - \eta}{R_V 1 + \kappa_T^2} \left[ 1 + \frac{t^2}{(s - u)^2} \frac{R_A^2 1}{R_V^2 1 + \kappa_T^2} \right]^{-1}. \]  

(54)
Thus, as \( K_{LL} \), it is roughly given by the Klein-Nishina helicity correlation (49) diluted by the ratio of \( R_A \) and \( R_V \) but additionally multiplied by the factor \( \kappa_T - \eta \). This factor, for which \( \kappa_T \) arises from the proton helicity flip amplitude (2) and \( \eta \) from the change of the helicity basis (13), makes

\(^{11}\) As explained in App. B a different convention is chosen in \([32]\): A lab frame is used and the \( L \) and \( S \) spin directions for the incoming proton are opposite to the ones described in App. A.
$K_{LS}$ very small for $s$ of about 8 GeV$^2$. For larger $s$ $K_{LS}$ becomes large in particular at large $-t$ as can be seen from Fig. 9. In general the factor $\kappa_T - \eta$ makes $K_{LS}$ less suitable for an extraction of $R_A$ than $K_{LL}$ or $A_{LL}$.

The predictions for $A_{LS} = -K_{LS}$ evaluated from the three examples quoted in Tab. 2 are shown in Fig. 9 and compared to the data [26]. The data point published in [25] is not shown in the plot. Its value is

$$K_{LS} = 0.114 \pm 0.078 \pm 0.04 \quad s = 6.9 \text{ GeV}^2 \quad t = -4.0 \text{ GeV}^2 \quad (55)$$

and is a bit more than 1$\sigma$ away from the prediction.

Many more spin correlation observables can be defined. Most of them are difficult to measure. Several of them are zero due to parity invariance, e.g. $A_{LN}, A_{\parallel N}, A_{\perp N}$ (and the analogous $K_{ij}$ observables). Others are of order $\alpha_s$.

An example of this is the correlation between a linearly polarized incoming photon, perpendicular to the scattering plane, and an $N$-type polarization of the incoming proton

$$A_{\perp N} = \frac{d\sigma(\perp \uparrow) - d\sigma(\perp \downarrow)}{d\sigma(\uparrow \uparrow) + d\sigma(\downarrow \downarrow)} \quad (56)$$

which in terms of helicity amplitude reads

$$A_{\perp N} \frac{d\sigma}{dt} = \frac{1}{16\pi(s - m^2)^2} \text{Im} \left[ (\Phi_1^* + \Phi_3^*) (\Phi_2 + \Phi_4) + (\Phi_3^* + \Phi_5^*) (\Phi_4 - \Phi_6) \right]. \quad (57)$$

From Eqs. (2) and (12) follows

$$A_{\perp N} \frac{d\sigma}{dt} \sim 2\kappa_T R^g V \frac{s - u}{-u s} \text{Im} (\mathcal{H}_{++}^g + \mathcal{H}_{+-}^g). \quad (58)$$

The NLO subprocess amplitudes, derived in [16], provide

$$\text{Im} \left[ \mathcal{H}_{++}^g + \mathcal{H}_{+-}^g \right] = -\alpha_s C_F \sqrt{\frac{\hat{s}}{-\hat{u}}} \left[ -\frac{2\hat{t} - \hat{u}}{2\hat{s}} + (1 + \frac{\hat{t}^2}{-\hat{u}\hat{s}}) \ln \frac{-\hat{t}}{\hat{s}} \right]. \quad (59)$$

A non-zero result on $A_{\perp N}$ requires proton helicity flip which is provided by the handbag approach through $R_T$, and phase differences which are obtained from the NLO corrections. Exactly the same result is found for the transverse target polarization, $P$, [16]. Predictions for $A_{\perp N}$ are of the order of 10% with rather large uncertainties because of badly known gluon form factor $R^g_V$. 

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An example of an observables for which only corrections of order $\alpha_s^2$ lead to a non-trivial result, is the helicity transfer from the incoming to the outgoing photon \[ D_{LL} = 1 + \mathcal{O}(\alpha_s^2). \] (60)

A deviation from 1 requires photon helicity flip which is of order $\alpha_s$ in the handbag approach, see (6). The deviation from 1 due to the NLO photon helicity flip are tiny, of the order of 1%.

One may also consider spin correlations between the final state photon and proton ($C_{ij}$) or between the final state photon and the initial state proton. These observables are similar to the $A_{ij}$ and $K_{ij}$ ones. For instance,

\[ C_{LL} = A_{LL}, \quad C_{LS} = A_{LS}. \] (61)

The last observable we want to comment on is the incoming photon asymmetry, $\Sigma$, defined as

\[ \Sigma = \frac{d\sigma(\bot) - d\sigma(\|)}{d\sigma(\bot) + d\sigma(\|)}, \] (62)

which at least at low energies and/or small $-t$ has been measured or in photoproduction of pions, e.g. [33]. This observable can be expressed by

\[ \Sigma \frac{d\sigma}{dt} = \frac{1}{16\pi(s - m^2)^2} \text{Re} \left[ (\Phi_1 + \Phi_3)\Phi_3^* + (\Phi_2 - \Phi_6)\Phi_4^* \right]. \] (63)
It is of order $\alpha_s$ and can be expressed by

$$\Sigma = -\frac{\alpha_s}{\pi}C_F \left[ 1 + \frac{i^2}{(s-u)^2} \frac{R_A^2}{R_V} \frac{1}{1 + \kappa_T^2} \right]^{-1} \simeq -\frac{\alpha_s}{\pi}C_F. \quad (64)$$

Predictions for $\Sigma$ are depicted in Fig. 10 at $s = 15$ GeV$^2$. This asymmetry is only mildly dependent on the Compton form factors.

Before closing this section a remark is in order concerning other models for WACS. Above we have already mentioned the handbag model proposed by Miller [30]. In this model massive, point-like quarks are used and the form factors are evaluated from wave-function overlaps. Due to the quark masses $A_{LL} \neq K_{LL}$ and $D_{LL}$ deviates from unity in the backward hemisphere. In the model invented by the authors of [34] the leading contribution is the same as our LO result with, however, $R_V = R_A$ and $R_T = 0$. The form factor $R_V$ is not related to GPDs but determined from a fit to the differential cross section. Corrections to the leading contribution are calculated from the soft collinear effective theory. This model leads to $A_{LL} \simeq K_{LL}$ with values similar to our results. Dagaonkar [35] proposes an endpoint model for WACS which bears similarity to the approach discussed in [5]. In [35] spin effects are not discussed.

5 Summary

In the handbag approach the amplitudes for WACS are composed of products of subprocess amplitudes for Compton scattering off massless quarks and form factors that represent $1/x$-moments of GPDs. The relevant GPDs, $H$ and $E$, for the form factors $R_V$ and $R_T$, are reasonably well known for valence quarks from an analysis of the nucleon form factors [15]. In consequence of the scarce experimental information available for the isovector axial form factor of the nucleon the GPD $\tilde{H}$ and, hence, the form factor $R_A$, is poorly known. Therefore, predictions on the spin correlations which are sensitive to $R_A$, on which the interest is focused in this work, suffer from large uncertainties, in particular for $s$ smaller than about 10 GeV$^2$ where also proton mass corrections of kinematical and dynamical origin are rather large.

Therefore, one may turn around the strategy and extract $R_A$ from WACS data at sufficiently large Mandelstam variables, as for instance from the spin.
correlations $A_{LL}$ or $K_{LL}$, and use the results as an additional constraint in the analysis of $\tilde{H}$ with the help of the sum rule for the axial form factor. This constraint will also improve the flavor separation of $\tilde{H}$. With only data on $F_A$ on disposal the flavor separation requires assumptions. Measurements of spin correlations in photoproduction of pions may provide further constraints on $\tilde{H}$. Results similar to (48), (54) hold for photoproduction [8, 14] with, of course, flavor compositions of the form factors that differ from (4). Such an analysis will likely improve our knowledge of $\tilde{H}$ for valence quarks at large $-t$ substantially and allow for a reliable investigation of the impact-parameter distribution of valence quarks with definite helicity.

Acknowledgements: The author would like to thank Bogdan Wojtsekhowski and Dustin Keller for discussions on the spin correlation observables.

A The c.m.s. convention

Here, in this appendix we define various polarization states of the involved particles in the c.m. system. We define a unit vector perpendicular to the scattering plane by

$$N = \frac{p \times p'}{|p \times p'|} = \frac{q \times q'}{|q \times q'|}$$

(65)

where the momenta denote the 3-momenta of the particles involved. As longitudinal, $L$, and sideway, $S$, spin directions we define

$$L^{(i)} = \frac{p^{(i)}}{|p^{(i)}|}, \quad S^{(i)} = \frac{N \times L^{(i)}}{|N \times L^{(i)}|}$$

(66)

for the incoming and outgoing protons. The vectors $N$, $L^{(i)}$ and $S^{(i)}$ form a right-handed system. The $L$ and $S$ directions for the photons are defined analogously.

We use the convention advocated for by Bourrely, Leader and Soffer [36] and define the rotation of a vector through an azimuthal angle $\varphi$ and a polar angle $\vartheta$ by the matrix $R(\varphi, \vartheta, 0)$ with the Pauli matrices, $\sigma_i$ for the protons and the spin-1 matrices for photons and momenta. The different polarization states of the proton - $L$, $S$ and $N$ - are defined as spin eigenstates of $A \cdot \sigma$ where $A$ is one of the unit vectors (65) and (66). For the $L$-type polarizations the eigenstates are just the usual helicity states whereas for the
$S$-type polarization with the eigenvalue 1/2 of the operator $S^\mu \cdot \sigma/2$ is

$$| \rightarrow \rangle = \frac{1}{\sqrt{2}} [ | + \rangle - | - \rangle ] .$$  \hspace{1cm} (67)$$

In terms of helicity amplitudes an amplitude for sideway polarization of the initial state proton reads

$$\Phi_{\mu'\nu',\mu \rightarrow} = \frac{1}{\sqrt{2}} ( \Phi_{\mu'\nu',\mu+} - \Phi_{\mu'\nu',\mu-} ) .$$  \hspace{1cm} (68)$$

For the $N$-type polarization with positive and negative eigenvalue of $N \cdot \sigma/2$ one finds

$$\Phi_{\mu'\nu',\mu \uparrow\downarrow} = \frac{1}{\sqrt{2}} ( \Phi_{\mu'\nu',\mu+} + i \Phi_{\mu'\nu',\mu-} ) .$$  \hspace{1cm} (69)$$

Analogously relations hold for the final state proton.

For the photons the $N$ and $S$-type polarization correspond to linear photon polarizations. They are usually denoted by $\perp$ and $\parallel$, respectively. For the initial state photon the amplitudes for linear photon polarization read

$$\Phi_{\mu'\nu',\perp\nu} = \frac{i}{\sqrt{2}} ( \Phi_{\mu'\nu',\nu+} + \Phi_{\mu'\nu',\nu-} ) ,$$

$$\Phi_{\mu'\nu',\parallel\nu} = -\frac{1}{\sqrt{2}} ( \Phi_{\mu'\nu',\nu+} - \Phi_{\mu'\nu',\nu-} ) .$$  \hspace{1cm} (70)$$

Again, analogous relations hold for the final state photon.

B The lab system convention

In [32] the lab system is considered for the definition of spin directions. For both the photons as well as for the final state proton spin directions are defined which, after boosting from the lab system to the c.m. system, fall together with our conventions, see (65), (66). However, for the initial state proton, being at rest in the Lab system, the same spin directions as for the initial state photon are chosen in [32]. After a boost to the c.m. system one sees that this choice implies differences as compared to [16]:

$$L^{BGLMN} = -L^{HKM} , \quad S^{BGLMN} = -S^{HKM} , \quad N^{BGLMN} = N^{HKM} .$$  \hspace{1cm} (71)$$

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Suppose the scattering plane is the $e_1 - e_3$ plane and $N = e_2$. Then the corresponding spin states for the $L$ and $S$ directions are the eigenstates of $\sigma_3/2$ and $\sigma_1/2$ respectively instead of $-\sigma_3/2$ and $-\sigma_1/2$ as is the case for the conventions discussed in App. A. In terms of proton helicity the spin state with the eigenvalue 1/2 of the operator $\sigma_3/2$ corresponds to negative helicity. For the sideway polarization with eigenvalue 1/2 of the operator $\sigma_1/2$ is

$$| \to \rangle^{BGLMN} = \frac{1}{\sqrt{2}} [ |+\rangle + |-\rangle ]. \quad (72)$$

For a helicity amplitude this implies

$$\Phi_{\nu'\mu'\mu \to}^{BGLMN} = \frac{1}{\sqrt{2}} (\Phi_{\nu'\mu'\mu +} + \Phi_{\nu'\mu'\mu -}) \quad (73)$$

instead of (68). Hence,

$$A_{LL}^{BGLMN} = -A_{LL}^{HKM}, \quad A_{LS}^{BGLMN} = -A_{LS}^{HKM} \quad (74)$$

but

$$K_{LL}^{BGLMN} = K_{LL}^{HKM}, \quad K_{LS}^{BGLMN} = K_{LS}^{HKM} \quad (75)$$

References

[1] A. V. Radyushkin, Phys. Rev. D 56, 5524 (1997).
[2] X. D. Ji and J. Osborne, Phys. Rev. D 58, 094018 (1998).
[3] J. C. Collins and A. Freund, Phys. Rev. D 59, 074009 (1999).
[4] A. V. Radyushkin, Phys. Rev. D 58, 114008 (1998).
[5] M. Diehl, T. Feldmann, R. Jakob and P. Kroll, Eur. Phys. J. C 8, 409 (1999).
[6] S. V. Goloskokov and P. Kroll, Eur. Phys. J. C 50, 829 (2007).
[7] K. Kumericki, T. Lautenschlager, D. Mueller, K. Passek-Kumericki, A. Schaefer and M. Meskauskas, arXiv:1105.0899 [hep-ph].
[8] H. W. Huang and P. Kroll, Eur. Phys. J. C 17, 423 (2000).
[9] S. V. Goloskokov and P. Kroll, Eur. Phys. J. C 65, 137 (2010).
[10] S. V. Goloskokov and P. Kroll, Eur. Phys. J. A 47, 112 (2011).
[11] G. R. Goldstein, J. O. Gonzalez Hernandez and S. Liuti, Phys. Rev. D 91, no. 11, 114013 (2015).
[12] I. Bedlinskiy et al. [CLAS Collaboration], Phys. Rev. C 90, no. 2, 025205 (2014) Addendum: [Phys. Rev. C 90, no. 3, 039901 (2014)].
[13] M. Defurne et al. [Jefferson Lab Hall A Collaboration], Phys. Rev. Lett. 117, no. 26, 262001 (2016).
[14] H. W. Huang, R. Jakob, P. Kroll and K. Passek-Kumericki, Eur. Phys. J. C 33, 91 (2004).
[15] M. Diehl and P. Kroll, Eur. Phys. J. C 73, no. 4, 2397 (2013).
[16] H. W. Huang, P. Kroll and T. Morii, Eur. Phys. J. C 23, 301 (2002) Erratum: [Eur. Phys. J. C 31, 279 (2003)].
[17] M. Diehl, T. Feldmann, R. Jakob and P. Kroll, Eur. Phys. J. C 39, 1 (2005).
[18] M. Diehl, T. Feldmann, H. W. Huang and P. Kroll, Phys. Rev. D 67, 037502 (2003).
[19] M. Diehl, Eur. Phys. J. C 19, 485 (2001).
[20] H. Rollnik and P. Stichel, In *E.Paul Et Al., Elementary Particle Physics.*Springer Tracts Vol.79*, Berlin 1976.
[21] S. Alekhin, J. Blumlein and S. Moch, Phys. Rev. D 86, 054009 (2012).
[22] T. Kitagaki et al., Phys. Rev. D 28, 436 (1983).
[23] D. de Florian, R. Sassot, M. Stratmann and W. Vogelsang, Phys. Rev. D 80, 034030 (2009).
[24] M. Burkardt, Int. J. Mod. Phys. A 18, 173 (2003).
[25] D. J. Hamilton et al. [Jefferson Lab Hall A Collaboration], Phys. Rev. Lett. 94, 242001 (2005).

[26] C. Fanelli et al., Phys. Rev. Lett. 115, no. 15, 152001 (2015).

[27] S. K. Dagaonkar, P. Jain and J. P. Ralston, Eur. Phys. J. C 74, no. 8, 3000 (2014).

[28] S. D. Drell and T. M. Yan, Phys. Rev. Lett. 24, 181 (1970).

[29] S. J. Brodsky, M. Burkardt and I. Schmidt, Nucl. Phys. B 441, 197 (1995).

[30] G. A. Miller, Phys. Rev. C 69, 052201 (2004).

[31] A. Danagoulian et al. [Hall A Collaboration], Phys. Rev. Lett. 98, 152001 (2007).

[32] D. Babusci, G. Giordano, A. I. L’vov, G. Matone and A. M. Nathan, Phys. Rev. C 58, 1013 (1998).

[33] D. J. Quinn, J. P. Rutherford, M. A. Shupe, D. Sherden, R. Siemann and C. K. Sinclair, Phys. Rev. D 20, 1553 (1979).

[34] N. Kivel and M. Vanderhaeghen, Eur. Phys. J. C 75, no. 10, 483 (2015).

[35] S. Dagaonkar, arXiv:1611.00147 [hep-ph].

[36] C. Bourrely, J. Soffer and E. Leader, Phys. Rept. 59, 95 (1980).