One-loop effective action in $\mathcal{N} = 2$ supersymmetric massive Yang-Mills theory

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Abstract

We consider the $\mathcal{N} = 2$ supersymmetric theory of the massive Yang-Mills field formulated in the $\mathcal{N} = 2$ harmonic superspace. The various gauge-invariant forms of writing the mass term in the action (in particular, using the Stueckelberg superfield), which result in dual formulations of the theory, are presented. We develop a gauge-invariant and explicitly supersymmetric scheme of the loop off-shell expansion of the superfield effective action. In the framework of this scheme, we calculate gauge-invariant and explicitly $\mathcal{N} = 2$ supersymmetric one-loop counterterms including new counterterms depending on the Stueckelberg superfield. Component structure of one of these counterterms is analyzed.

1 Introduction

Study of quantum aspects of massive non-Abelian Yang-Mills theories has long history (see, e.g., [1]). It is very well known that to understand particle phenomenology, the massive degrees of freedom of vector bosons must be taken into account. However the mass terms in vector field Lagrangian violates a gauge invariance of massless theory.

Several different mechanisms for generating vector field mass are currently known that are compatible with the gauge invariance. Their common feature it is increasing the number of physical degrees of freedom in comparison with the massless theory. Of course, the main commonly accepted paradigm of the standard model is the mechanism of spontaneous symmetry breaking in which additional physical degrees of freedom are due to scalar Higgs

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fields. However for phenomenological aims is useful to consider other mechanisms for generating the boson and fermion masses in gauge theories such that no additional physical fields appear in the Lagrangian.

The most popular alternative to the Higgs mechanism is the model based on the massive Yang-Mills theory. In such a model, the gauge invariance is attained by introducing the real pseudoscalar auxiliary Stueckelberg field [2], which corresponds to coupling the Yang-Mills field to a gauge nonlinear sigma model. In the unitary gauge, this field is absorbed by the longitudinal component of the massive vector field (see [3] for a comprehensive review and reference list; we mention [4] among the numerous latest publications).

In addition to models with the Stueckelberg fields, non-Abelian vector-tensor gauge theories with topological constraints can be considered [5]. All such theories are classically equivalent to non-Abelian theories of massive vector fields with the Stueckelberg fields. The same degrees of freedom can be described by either of the two dual representations. Depending on a problem context, one of the formulations can be more convenient, and both formulations and their interrelations are therefore worth studying. We mention that such models appear naturally in the low-energy limit of the superstring theory and also in the context of supergravity in higher dimensions. For example, degrees of freedom of the massive skew-symmetric tensor field related to the mechanism of natural spontaneous supersymmetry breaking appear naturally in the recently found compactifications of the type-II superstring on the Calabi-Yau manifolds in the presence of nontrivial flows of the 3-form (see, e.g., [7]). This revived the interest in a more detailed study of massive $\mathcal{N} = 1$ and $\mathcal{N} = 2$ tensor multiplets and their relations to scalar and vector multiplets. We note that such the relations play an important role in the mechanism of anomaly cancellation in superstring models.

The construction of the $\mathcal{N} = 1$ supersymmetric massive tensor multiplet as the version dual to the massive vector multiplet has long been known (see, e.g., [8]). In the $\mathcal{N} = 2$ supersymmetry, the strength of the skew-symmetric tensor fields is contained in the tensor multiplet $G^{++}$ defined on the analytic subspace of the harmonic $\mathcal{N} = 2$ superspace restricted by constraints [9], [10]. The action contains only $G^{++}$ in the case of massless tensor multiplet, but if the skew-symmetric tensor acquires mass, then the gauge invariance results in the relation between Stueckelberg fields and the vector multiplet [11]. Studying the quantum properties of dual realizations of the same supersymmetry representation is especially interesting.

Here, we consider the quantum properties of the $\mathcal{N} = 2$ massive Yang-Mills field theory with the Stueckelberg fields. This model is a direct $\mathcal{N} = 2$ supersymmetrization of the $\mathcal{N} = 0$ nonsupersymmetric massive Yang-Mills theory in the Kunimasa-Goto formalism [12]. Several aspects of this problem were already considered in [13], where it was found that the theory is finite in the second order in the dimensionless Yang-Mills coupling constant $g^2$ and that the massive term is not renormalized, but the theory then becomes nonrenormalizable in the sector containing the dimension full coupling constant $m^2 g^2$. It was concluded from this that the theory is finite in all orders of the loop expansion in the vector multiplet sector.

But we note that already on the classical level, the action of the $\mathcal{N} = 2$ massive Yang-Mills theory has the form of an infinite series containing all orders of the vector multiplet potential $V^{++}$ in the framework of the harmonic superspace formalism [9]. Moreover, the sigma-

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The quantum equivalence of such different dual formulations is a more delicate problem requiring a separate investigation for each concrete case (see, e.g., [6]).
model Lagrangian of the Stueckelberg superfield is itself highly nonlinear. To analyze the quantum properties of the theory in a gauge invariant way even on the one-loop level, we therefore cannot restrict ourselves to considering only the simplest diagrams resulting in gauge-noninvariant counterterms and must instead sum over all one-loop diagrams with all possible external legs for the effective action. In the framework of the standard noncovariant diagram technique, this problem seems to be very difficult technically, if not impossible.

Here, to construct the effective action, we use the formulation of the $\mathcal{N} = 2$ supersymmetric Yang-Mills field theory and the corresponding Stueckelberg formalism in the harmonic superspace [13] and the background field method [14], which allows effectively summing all the diagrams with the increasing number of insertions of the external lines. Our conclusions are ideologically close to the results in [16], [17], where the problem of constructing off-mass-shell invariant counterterms for the nonsupersymmetric ($\mathcal{N} = 0$) massive Yang-Mills theory was solved. To preserve the gauge invariance at all calculation stages, we use the invariant perturbation theory developed in models of principal chiral fields long ago [18].

The paper is organized as follows. In Sec. 2, we describe the formulation of the $\mathcal{N} = 2$ supersymmetric theory of the massive Yang-Mills field in the harmonic superspace taking into account the Stueckelberg superfield. Excluding nonphysical degrees of freedom, we obtain an explicitly gauge-invariant nonlocal expression for the mass term in the Lagrangian. It is expected that the dual relation to the theory of $\mathcal{N} = 2$ massive tensor multiplet [11] becomes more transparent just in this formulation. In Sec. 3, we discuss the procedure for constructing the effective action based on the $\mathcal{N} = 2$ supersymmetric background field method and indicate the special features of using this method in the theory under consideration. Sec. 4 is devoted to calculating the one-loop divergences in the effective action. There, we first present gauge-invariant and explicitly $\mathcal{N} = 2$ supersymmetric counterterms depending on the Stueckelberg superfield. In Sec. 5, we discuss the derivation of the component structure of the bosonic sector of one of these counterterms.

## 2 The $\mathcal{N} = 2$ supersymmetric theory of the massive Yang-Mills field in the harmonic superspace

The formulation of the $\mathcal{N} = 2$ supersymmetric field theories in terms of unrestricted superfields defined on an analytic subspace of the harmonic superspace [9], [10] turns out to be exceptionally useful for investigating the quantum effects (see, e.g., [14], [15]). The concept of the harmonic $\mathcal{N} = 2$ superfield was introduced in [9]; it consists in enhancing the standard $\mathcal{N} = 2$ superspace with the coordinates $z^M = (x^m, \theta_i^\alpha, \bar{\theta}_i^{\dot{\alpha}})(i = 1, 2)$, by adding the spherical harmonics $u_i^\pm$ parameterizing the two-dimensional sphere $S^2 = SU(2)/U(1)$:

$$u_i^+ u_i^- = 1, \overline{u_i^+} = u_i^-.$$  

The main advantage of using the harmonic superspace is that unconstrained superfields of matter hypermultiplets and those of the vector Yang-Mills field multiplet are defined in the analytic subspace with the coordinates $\zeta_M = (x^m_A, \theta^+ \alpha, \overline{\theta}^+_{\dot{\alpha}}, u_i^\pm)$ in which the so-called analytic basis is closed under the transformations of the $\mathcal{N} = 2$ supersymmetry:

$$x^m_A = x^m - i \theta^+ \sigma^m \overline{\theta}^- - i \theta^- \sigma^m \theta^+, \quad \theta^\pm = u_i^\pm \theta^i, \quad \overline{\theta}^\pm = \overline{u_i^\pm} \overline{\theta}^i.$$


The $\mathcal{N} = 2$ vector multiplet with a finite number of physical and auxiliary component fields but with an infinite number of gauge degrees of freedom is described by a real analytic superfield $V^{++} = V_a^{++} T_a$ taking values in the Lie algebra of the gauge group. The hypermultiplets $\omega$ and $q^+$ containing off-shell an infinite number of auxiliary fields and transforming on a representation $R$ of the gauge group are determined by the analytic superfields $\omega(\zeta)$, $q^+(\zeta)$, and their conjugate $\tilde{q}^+(\zeta)$ (see [10] for the definition of the generalized conjugation as a composition of the standard conjugation and the antipodal mapping on the two-sphere). The scalar component fields $\omega(x_A)$ and $\omega^{(ij)}(x_A)$ of the $\omega$-multiplet, which are the respective isoscalar and isotriplet of the $SU(2)$ group of internal isomorphisms of the supersymmetry algebra, and the doublet of the Weyl fermions $\psi^i$, $\tilde{\psi}^{i\dot{a}}$ appear as lower components in the expansion of $\omega(\zeta)$ in powers of $\theta^+, \tilde{\theta}^+ + u_1^+$. Other $\mathcal{N} = 2$ matter multiplets with a finite number of auxiliary fields are described by analytic superfields subject to proper harmonic constraints. The vector $\mathcal{N} = 2$ potential $V^{++}$ satisfies the reality constraint with respect to the generalized conjugation $V^{++} = V^{++}$ and transforms as $\delta V^{++} = -D^{++} \lambda$ under the gauge transformations, where $\lambda$ is an arbitrary real analytical superfield and $D^{++}$ is the covariant harmonic derivative in the analytic basis

$$D^{++} = D^{++} + iV^{++} = e^{ib(z,u)} D^{++} e^{-ib(z,u)},$$

$$D^{++} = u_1^{+i} \frac{\partial}{\partial u_1^{-i}} - 2i\theta^+ \sigma^a \tilde{\theta}^+ \frac{\partial}{\partial x_A^m} + \theta^+ \alpha \frac{\partial}{\partial \theta^{-\alpha}} + \tilde{\theta}^{+\dot{a}} \frac{\partial}{\partial \tilde{\theta}^{-\dot{a}}}$$

and $b(z,u)$ is the so-called gauge bridge. This gauge freedom allows eliminating an infinite number of auxiliary fields by choosing the Wess-Zumino gauge in which the analytic superfield $V^{++}$ contains a finite number of physical and auxiliary fields. As shown in [9], [10], all the geometric characteristics, such as the field strength, can be expressed in terms of a unique unrestricted potential $V^{++}(\zeta, u)$.

We are not going to discuss the $\mathcal{N} = 2$ supersymmetric Yang-Mills field theory (see [10]) in detail and only present the action of the non-Abelian vector multiplet. On-shell this multiplet consists of the following component fields: the vector field $A_m(x)$, the complex scalar field $M(x) + i N(x)$, the Majorana isodoublet of spinors $\lambda^i_\alpha(x)$ and $\bar{\lambda}^{i\dot{a}}(x)$ and the triplet of auxiliary fields $F^{ij}(x)$. Off-shell, this multiplet is given by the superfield potential $V^{++}$ taking values in the Lie algebra of the gauge group. The corresponding action is:

$$S = \frac{1}{2g^2} \text{tr} \sum_{n=2}^\infty \frac{(-i)^n}{n} \int d^2 z d^2 u_1 \cdots d^2 u_n \frac{V^{++}(z, u_1) \cdots V^{++}(z, u_n)}{(u_1^+ u_2^+ \cdots u_n^+ u_1^+)} = -\frac{1}{2g^2} \text{tr} \int d^8 z W^2,$$

where we use the harmonic distributions $\frac{1}{u_1^+ u_2^+}$ or, in other words, the Green’s functions on the sphere $G^{-1,-1}(u_1, u_2)$, which satisfy the equation $\partial^{++} G^{-1,-1}(u_1, u_2) = \delta^{(1,-1)}(u_1, u_2)$. The rules of differentiation with respect to harmonics and integration over harmonics were defined in the pioneering papers [9], [10].

The harmonic-independent chiral superfield strength $W = -\frac{1}{4} (\bar{D}^+) V^{--}$ is determined in terms of the nonanalytic superfield

$$V^{--}(z,u) = \int du' e^{i b(z,u)} e^{-i b(z,u')} V^{++}(z,u') \left( u^+ u'^+ \right)^2,$$
satisfying the zero-curvature equation:

\[ D^{++}V^{--} - D^{--}V^{++} + i[V^{++}, V^{--}] = 0. \]  \(2\)

In the \(\lambda\)-basis, action \(\Pi\) is invariant under the gauge transformations

\[ gV^{++} = e^{i\lambda}(V^{++} - iD^{++})e^{-i\lambda}, \quad V^{++} = V^{++}_a T_a, \]  \(3\)

where \(T_a\) are the generators of the gauge group given by the formulas:

\[ [T_a, T_b] = if^{abc}T_c, \quad \text{tr}(T_a T_b) = \delta_{ab}, \]

and the superfield gauge parameter \(\lambda(\zeta, u)\) is a real analytic superfield.

We consider a construction of the gauge-invariant expression for the mass term in the superfield action. For this, we use the known Kunimasa-Goto formalism \[12\], \[3\], developed for describing gauge field masses in the \(\mathcal{N} = 0\) Yang-Mills theory. In the \(\mathcal{N} = 2\) superfield description, this formalism requires introducing the additional Goldstone \(\omega\)-hypermultiplet in the adjoint representation of the gauge group. The corresponding mass term in the action is given as follows:

\[ S_m = -\frac{m^2}{2g^2} \text{tr} \int d\zeta d(-\bar{\zeta}) \{\Omega^{-1}(V^{++} - iD^{++})\Omega\}^2, \]  \(4\)

where \(\Omega = \Omega(\omega) = e^{-i\omega}\). In such a form of writing\(^2\) the mass term is explicitly invariant under the simultaneous transformations \(\Pi\) and the transformations

\[ g\Omega = e^{i\lambda}\Omega, \]  \(5\)

where \(g\Omega = g\Omega\) and \(g\) is the gauge group element. We note that mass term \(\Pi\) is also invariant under the global right transformations \(g_R\): \((V^{++})^{g_R} = V^{++}, \Omega^{g_R} = \Omega g_R\). Because action \(\Pi\) of the \(\mathcal{N} = 2\) supersymmetric massless Yang-Mills field theory is gauge invariant, the substitution \(V^{++} \rightarrow \Omega V^{++}\) does not change the structure of action \(\Pi\).

It is interesting to present another, explicitly gauge-invariant but nonlocal form of the mass term expressed in terms of the superfield strength \(W\). We consider the action

\[ S[V^{++}, \omega] = -\frac{1}{2g^2} \text{tr} \int d\theta W^2 - \frac{m^2}{2g^2} \text{tr} \int d\zeta^{(-4)} du \{\gamma^{++} + \nabla^{++}\}^2, \]  \(6\)

where

\[ \gamma^{++} = V^{++} - L^{++}, \]  \(7\)

\[ L^{++} = i(D^{++}\Omega)\Omega^{-1} = \int_0^1 d\tau e^{-i\tau\omega} D^{++}\omega e^{i\tau\omega} = \int_0^1 d\tau D^{++}\omega_a R_{ab}(\tau\omega)T_b. \]

\(^2\)Another useful form of writing \(\Pi\) is

\[ S_m = -\frac{m^2}{2g^2} \text{tr} \int d\zeta^{(-4)} du (V^{++} - i\Omega^{-1}(D^{++}\Omega + i[V^{++}, \Omega]))^2, \]

in which the specific structure of the gauge sigma model is more transparent.
(we define the isotopic matrix $R_{ab}(\omega)$ below; see \[16\]), and write the equation of motion that follows from this action:

$$\frac{1}{4}(D^+)^2W + m^2\mathcal{V}^{++} = 0. \quad (8)$$

Because $W$ is independent of harmonics, the consistency condition for this equation is the equation of motion of the $\omega$-multiplet:

$$D^{++}\mathcal{V}^{++} = 0. \quad (9)$$

As is clear from definition \[7\], the component content of the gauge-covariant ($g\mathcal{V}^{++} = g\mathcal{V}^{++}g^{-1}$) potential $\mathcal{V}^{++}$ is determined by complicated nonpolynomial combinations composed from physical components of the vector multiplet in the Wess-Zumino gauge and from the components of the $\omega$-multiplet. But because of constraint \[9\], we can parameterize the component decomposition on the mass shell:

$$\mathcal{V}^{++}(\zeta, u) = f^{++}(x_A) + (\theta^+)^2\bar{\varphi}(x_A) + (\bar{\theta}^+)^2\varphi(x_A) + 2i(\theta^+\sigma^m\bar{\theta}^+)(A_m(x_A) + \partial_m f^{+-}(x_A)) \quad (10)$$

$$+ 2[\theta^+\psi^i - \bar{\theta}^+\bar{\psi}^{\dot{i}}]u_i^+ + 2i[(\theta^+)^2\theta^+\alpha\partial_{\alpha}\bar{\psi}^{\dot{i}} + (\bar{\theta}^+)^2\bar{\theta}^+\bar{\alpha}\psi^i]u_i^- - (\theta^+)^2(\bar{\theta}^+)\Box f^{-\gamma}(x_A).$$

The mass term for the vector component of the superfield $\mathcal{V}^{++}$ then has a form similar to the form of the mass term in the Stueckelberg formalism for the nonsupersymmetric massive Yang-Mills field theory \[17\]:

$$S_m = -\frac{m^2}{2g^2}\text{tr} \int d^4x du(A_m - L_m)^2,$$

where the Cartan form $L_m$ on the group is determined in terms of the isoscalar and isotriplet physical components $\omega$, $\omega^{(ij)}$ of the $\omega$-supermultiplet. Deriving an explicit component expression for $S_m$ is technically difficult because we must integrate over harmonics in each term of the infinite series for the Cartan form $L_m = e^{-if^{ij}u_i^+u_j^-}\partial_m e^{if^{ij}u_i^+u_j^-}$, where $f^{ij} = \omega^{(ij)} + \omega^{ij}$. But the analogy with the corresponding nonsupersymmetric theory is nevertheless transparent.

We can treat Eq. (8) as a constraint imposed on the $\omega$-multiplet, which can also be resolved perturbatively in the non-Abelian theory, but whose solution has an especially simple form in the Abelian case:

$$\omega(\zeta_1, u_1) = \int d\zeta_2 d_u_2 G_0^{(0,0)}(1|2)D_2^{++}V^{++}(2), \quad (11)$$

where

$$G_0^{(0,0)}(1|2) = -\frac{1}{\Box}(D_+^+)^4(D_+^+)^4\delta^{12}(1|2)\frac{u_1^+u_2^-}{(u_1^+u_2^-)^3} \quad (12)$$

is the Green’s function of the omega-multiplet \[9\], \[10\]. Acting with the operator $D^{++}$ on the both sides of (11), we obtain:

$$D_1^{++}\omega(\zeta_1, u_1) = -\int d\zeta_2 d_u_2 \frac{1}{\Box}(D_1^+)^4(D_2^+)^4\delta^{12}(1|2)\mathcal{V}^{++}(2)\left\{\frac{1}{(u_1^+u_2^-)^2} \right\}$$
\begin{align*}
+ \frac{1}{2} (D^-_2)^2 \delta^{(2,-2)}(u_2, u_1) &= \int d\zeta_2^{(-4)} d\zeta_2^{(-4)} du_2 \{ \Pi^{(2,2)}_\Gamma(1|2) + \delta^{(2,2)}_A (1|2) \} V^{++}(2),
\end{align*}

where \( \Pi_\Gamma \) is an analytic distribution with the properties of the projection operator \([10]\). Now excluding the gauge degrees of freedom of \( \omega \) from \([6]\), after a chain of transformations for the mass term, we obtain:

\[
S_m = -\frac{m^2}{2g^2} \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} du_2 V^{++}(1) \Pi^{(2,2)}_\Gamma(1|2) V^{++}(2) = \frac{m^2}{2g^2} \int d^8 z W \frac{1}{\square} W. \tag{13}
\]

As a result, the gauge-invariant form of the mass term can be completely formulated in terms of the field strength \( W \). In a non-Abelian case, we can obviously also find the \( \omega \)-multiplet perturbatively as a polynomial in powers of \( V^{++} \). This expansion is definitely a nonlocal expression, but we can localize it by introducing the proper tensor multiplets, \([19]\), where the nonlocal gauge-invariant functional related to \( A^2 \) contains information about the topological structure of the theory vacuum with a nonvanishing mean of the operator \([20]\) and the references therein for a detailed description of the infrared dynamics of the \( \mathcal{N} = 0 \) Yang-Mills theory.

Our further goal here is to determine the effective action and to analyze the structure of one-loop divergences in the theory with action \([6]\).

### 3 The background field formalism

A gauge-invariant loop expansion of the effective action in supersymmetric theories is given on the base of the superfield background field method (see, e.g., \([8]\) for \( \mathcal{N} = 1 \) theories and \([13]\) for \( \mathcal{N} = 2 \) theories). In the background field formalism, we subsequently perform the background-quantum splitting of all the fields, fix the gauge degrees of freedom of quantum fields, and integrate only over quantum fields in path integral. The contribution to the effective action in a given loop order then comes from a finite number of terms in expansion of action in the integrand in quantum fields.

\(^3\)We write the action for the massive tensor multiplet in the harmonic superspace \([11]\) in the form

\[
S = \frac{1}{2} \int d\zeta^{-4} (G^{++})^2 + \frac{1}{2} m \{ \int d^8 z W \psi + c.c. \} + \frac{1}{2} \int d^8 z W^2,
\]

where \( G^{++}(z, u) \) is the real analytic superfield satisfying the equation \( D^{++} G^{++} = 0 \). This equation can be resolved in terms of the harmonic-independent unconstrained chiral superfield \( \psi(z) \) and its conjugate in the form \( G^{++}(z, u) = \frac{1}{8} (D^{++})^2 \psi(z) + \frac{1}{8} (\bar{D}^{++})^2 \bar{\psi}(z) \). This superfield remains invariant under the gauge transformations

\[
\delta \psi = i \Lambda, \quad \bar{D}_a \Lambda = 0, \quad D^{\alpha i} D^{\bar{j} \bar{a}} \Lambda = \bar{D}_a \bar{D}^{\bar{j} \bar{a}} \Lambda.
\]

We choose the gauge-fixing function in the form \( F^{++} = \frac{1}{8} (D^{++})^2 \psi(z) - \frac{1}{8} (\bar{D}^{++})^2 \bar{\psi}(z) \). Integrating in the generating functional over the prepotentials \( \psi \) and \( \bar{\psi} \),

\[
Z = \int D\psi D\bar{\psi} (\text{Det} \square) e^{\frac{i}{\hbar} \int d\zeta^{-4} ((G^{++})^2 + (F^{++})^2) + \frac{i}{2} m \int d^8 z W \psi + c.c.} = e^{\frac{i}{\hbar} \int d^8 z W \frac{1}{\square} W}
\]

we obtain the nonlocal mass term for vector potential \([13]\).
We consider the theory of the fields \( V^{++} \) and \( \omega \) with action (6). In the \( \mathcal{N} = 2 \) sector of the vector supermultiplet \( V^{++} \), we split \( V^{++} \rightarrow V^{++} + g\omega^{++} \) and repeat all the steps as in the massless theory \([14]\). In the sector of the \( \omega \)-multiplet with a nonlinear chiral Lagrangian, we must construct the expansion following the perturbation theory in terms of parameterization-independent invariant quantities \([18]\). The main principle of the background-quantum splitting of fields taking values in the group is a nonlinear rule for the group addition of elements of the Lie group \( \Omega(\omega) \) and \( \Omega(\chi) \) determined by the relation \([18]\):

\[
\Omega(\omega \oplus \chi) = \Omega(\omega)\Omega\left(\frac{m}{g}\chi\right).
\]  

Under such a rule \( \Omega(\omega \oplus \chi) \) is an element in the same space as \( \Omega(\omega) \) and has the same group transformation law as \( \Omega(\omega) \).

We define the background-quantum splitting of the superfield \( \omega \) into the background superfield \( \omega \) and the quantum superfield \( \chi \) according to rule (14). It is easy to demonstrate that the background \( \omega \)-fields transform as in (5), while the quantum fields \( \chi \) are merely invariant. Under such a splitting of fields into background and quantum fields, both the Lagrangian and all the terms of the Taylor expansion in quantum fields are invariant under both the local and global transformation groups. Consequently, all the obtained counterterms are automatically invariant under classical gauge and global transformations.

For the one-loop calculation, it is sufficient to expand the Lagrangian up to terms of the second order in the quantum fields \( v^{++} \) and \( \chi \):

\[
S^{(2)} = \frac{1}{2} \int d^{12}z du_1 du_2 \frac{v^{++}_{a}(1)v^{++}_{a}(2)}{(u_1 u_2)^2} - \frac{1}{2} \int d\zeta(-4) du \{m^2(v^{++}_{a})^2 - 2mD^{++}_a\chi_bR_{ab}v^{++}_b + D^{++}_a\chi_bD^{++}_b\chi_a + f_{abc}D^{++}_a\chi_bR_{cd}V^{++}_d\},
\]

where the isotopic matrix \( R_{ab}(\omega) \) is determined by the equality \( \Omega^T_a\Omega^{-1} = R_{ab}T_b \). As in the \( \mathcal{N} = 0 \) case \([17]\), it has the properties:

\[
R_{ae}R_{be} = \delta_{ab}, \quad D^{++}_aR_{ab} = -R_{ae}f_{bec}L^{++}_c, \quad f_{abc} = R_{ad}R_{be}R_{cg}f_{dgg}.
\]

To (15), we must also add the term fixing the gauge in the sector of the quantum vector superfield, which can be conveniently chosen in the background gauge-invariant form,

\[
F^{(+4)} = D^{++}v^{++},
\]

and the action of the Faddeev-Popov and Nielsen-Kallosh ghosts. Here, we follow the approach developed in \([14]\). Following the Faddeev-Popov procedure, to fix the gauge in the functional integral \( Z = N \int \mathcal{D}v^{++}e^{iS} \), we must insert unity in the form \( 1 = \Delta_{FP}\delta(F^{(+4)} - f^{(+4)}) \), where the Faddeev-Popov determinant is \( \Delta_{FP}[v^{++},V^{++}] = \text{Det}(D^{++}(D^{++} + iv^{++})) \). Further, we must insert unity in the form

\[
1 = \Delta_{NK} \int \mathcal{D}f^{(+4)} \exp\left\{ \frac{i}{2\alpha} \text{tr} \int d^{12}z du_1 du_2 f^{(+4)}(u_1 u_2)^3 f^{(+4)} \right\},
\]

into the functional integral, where \( \alpha \) is the gauge parameter, which for convenience we set equal to \( \alpha = -1 \) in what follows, \( \Delta_{NK}[V^{++}] \) is the Nielsen-Kallosh determinant, and
\[ f^{(+4)} = e^{-ib}f^{(+4)}e^{ib} \] is a gauge-invariant function taking values in the Lie algebra of the gauge group. We note that the Nielsen-Kallosh determinant depends on the background superfield, which indicates the presence of the third Nielsen-Kallosh ghost. The details of calculation of this determinant \( \Delta_{NK}[V^{++}] = \text{Det}^{-1/2}(\mathcal{D}^{++})^2 \text{Det}^{1/2} \bar{\Box}_{(4,0)} \) are given in [13]. Here \( \bar{\Box} \) is the covariant-analytic D’Alembertian transforming analytic superfields again into analytic superfields [14]:

\[
\Box = -\frac{1}{2}(\mathcal{D}^{+})^4(\mathcal{D}^{-})^2 = \frac{1}{2} \mathcal{D}^{\dot{a}\dot{\alpha}} \mathcal{D}_{\dot{a}\dot{\alpha}} + \frac{i}{2} (\mathcal{D}^{+a} W) \mathcal{D}_{-a} + \frac{i}{2} (\bar{\mathcal{D}}^+ \bar{W}) \bar{\mathcal{D}}^{-}\dot{\alpha} \\
+ \frac{1}{2} \{W, \bar{W}\} - \frac{i}{4} (\mathcal{D}^{+} \mathcal{D}^{+} \bar{W}) \mathcal{D}^{-} + \frac{i}{8} [\mathcal{D}^{+}, \mathcal{D}^{-}] W .
\]

The final result for the Lagrangian determining one-loop quantum corrections to the effective action in the vector multiplet sector is

\[
S_2 + S_{GF} = -\frac{1}{2} \text{tr} \int d\zeta(-4) du v^{++}(\Box + m^2) v^{++}.
\]

The ghost action is:

\[
S_{\text{ghost}} = \text{tr} \int d\zeta(-4) u du (\mathcal{D}^{++})^2 c + \frac{1}{2} \int d\zeta(-4) d\phi (\mathcal{D}^{++})^2 \phi + \text{tr} \int d\zeta(-4) d\rho^{(+4)} \bar{\Box}_{(4,0)} \sigma. \tag{20}
\]

where \( b \) and \( c \) are the anticommuting superfield Faddeev-Popov ghosts, \( \rho^{(+4)} \) and \( \sigma \) are the anticommuting Nielsen-Kallosh ghosts, and \( \phi \) are additional commuting ghosts taking values in the Lie algebra of the gauge group.

It is convenient to write the superfield action for the quantum field \( \chi \) and for its interaction with the quantum field \( v^{++} \) in the matrix form:

\[
S_{\text{SYM}}^{(2)} + S_m^{(2)} = -\frac{1}{2} \int d\zeta(-4) (v^{++}, \chi_a) \left( \begin{array}{cc} \Box_{ab} + m^2 & -m R_{ba} D^{++} \\ m D^{++} R_{ab} & -\nabla^{++} D^{++} \end{array} \right) \begin{pmatrix} v^{++}_b \\ \chi_b \end{pmatrix} , \tag{21}
\]

where \( D^{++} \) is the standard harmonic derivative [10] and

\[
\nabla^{++} \chi_a = D^{++} \chi_a + f_{abc} \chi_b R_{ca} V^{++}_d \tag{22}
\]

is the long derivative in the \( \lambda \)-basis in which \( \Omega V^{++}_a T_a \Omega^{-1} \) plays the role of the (gauge-invariant) connection. Because this field is analytic, we have the standard constraints

\[
[\nabla^{++}, \nabla^{++}_{\dot{\alpha}}, \dot{\alpha}] = 0.
\]

For uniformity here and hereafter, we use the notation \( \nabla^{++}_{\dot{\alpha}, \dot{\alpha}} = D^{++}_{\dot{\alpha}, \dot{\alpha}} \). Other commutation relations, for instance,

\[
[\nabla^{++}, \nabla^{-}] = D^0, \quad [\nabla^{\pm \mp}, \nabla^{\pm}_{\dot{\alpha}, \dot{\alpha}}] = \nabla^{\mp}_{\dot{\alpha}, \dot{\alpha}}, \quad \{\nabla^{+}_{\dot{\alpha}}, \nabla^{-}_{\dot{\alpha}}\} = 2i \nabla_{\dot{\alpha} \dot{\alpha}},
\]

\[
\{\nabla^{+}_{\alpha}, \nabla^{-}_{\dot{\alpha}}\} = -\{\nabla^{+}_{\dot{\alpha}}, \nabla^{-}_{\alpha}\} = 2i \nabla_{\dot{\alpha} \alpha}, \quad \{\nabla^{+}_{\alpha}, \nabla^{-}_{\beta}\} = -2i \varepsilon_{\alpha \beta \dot{\alpha}} \bar{W}, \quad \{\nabla^{+}_{\dot{\alpha}}, \nabla^{-}_{\dot{\beta}}\} = 2i \varepsilon_{\dot{\alpha} \dot{\beta}} \bar{W},
\]

exactly replicate the commutation relations for the covariant derivatives \( D^{\pm}_{\dot{\alpha}, \dot{\alpha}} \) and \( D^{\mp, 0} \) [14] and determine \( \nabla^{-}_{\alpha, \dot{\alpha}} \) and the chiral superfield of the harmonically independent strength \( \mathcal{W}[V^{++}] = -\frac{1}{4}(\nabla^{+})^2 \mathcal{V}^{--} \) and \( \bar{\mathcal{W}}[\mathcal{V}^{++}] = -\frac{1}{4}(\nabla^{+})^2 \mathcal{V}^{--} \) for the gauge invariant potential \( \Omega \mathcal{V}^{++} \Omega^{-1} \).

9
4 One-loop divergences

We now analyze one-loop divergences in the theory under consideration. The effective action is the sum of action (20) of quantum superfields of ghosts and action (21) of the quantum superfields $v^{++}$ and $\chi$:

$$\Gamma^{(1)}[V^{++},\omega] = \Gamma^{(1)}_1[V^{++}] + \Gamma^{(1)}_2[V^{++},\chi].$$

(24)

where $\Gamma^{(1)}_1[V^{++}]$ is the ghost contribution to the effective action and $\Gamma^{(1)}_2[V^{++},\chi]$ is the contribution from the superfields $v^{++}$ and $\chi$. Here, $\chi^{++}$ is given by expression (7). We note that the whole dependence of the effective action on the Stueckelberg superfield $\omega$ is contained in $V^{++}(7)$. Actions (20) and (21) completely determine the structure of a perturbative expansion needed for calculating the one-loop effective action of the massive $\mathcal{N} = 2$ supersymmetric Yang-Mills field theory in an explicitly supersymmetric and gauge-invariant form. Further, we are interested only in the structure of divergences of the considered theory. For this, we use the dimensional regularization (see [8]) about using dimensional regularization in superfield theories) and the minimal subtraction scheme.

The ghost contribution to the one-loop effective action depends only on the potential $V^{++}$ and coincides completely with the corresponding contribution in the standard massless $\mathcal{N} = 2$ supersymmetric Yang-Mills field theory [14]:

$$i\Gamma^{(1)}_1[V^{++}] = \text{Tr} \ln(D^{++})^2 - \frac{1}{2} \text{Tr} \ln(D^{++})^2 + \frac{1}{2} \text{Tr} \ln \hat{\square}_{(4,0)}.$$

(25)

We can therefore directly use the results in [13], [15] to calculate the divergent part of the effective action:

$$\Gamma^{(1)}_{1,\text{div}}[V^{++}] = -\frac{C_2}{32\pi^2\varepsilon} \text{tr} \int d^8z W^2,$$

(26)

where $C_2$ is the quadratic Casimir operator of the gauge group and $\varepsilon$ is the dimensional regularization parameter.

New contributions to divergences correspond to one-loop corrections to the effective action related to the quantum fields $v^{++}$ and $\chi$ running along the loop and to their mixing. To calculate the functional determinant of the matrix operator in action (21), it is convenient to reduce the matrix to the diagonal form and write it as:

$$\begin{pmatrix} 1 & mRD^{++} \frac{1}{\nabla^{++}D^{++}} \\ 0 & \frac{1}{\nabla^{++}D^{++}} \end{pmatrix} \begin{pmatrix} \hat{\square} + m^2 - m^2RD^{++} \frac{1}{\nabla^{++}D^{++}} D^{++}R & 0 \\ 0 & -\nabla^{++}D^{++} \end{pmatrix} \begin{pmatrix} 1 & mD^{++}R \\ 0 & 1 \end{pmatrix}.$$

(27)

All the contributions to the effective action are then ensured by the diagonal elements of the matrix:

$$\begin{pmatrix} \hat{\square} + m^2\Pi^T & 0 \\ 0 & -\nabla^{++}D^{++} \end{pmatrix}.$$

(28)

where we use the notation $\Pi^T$ for the covariant-analytic distribution [13] with the projection operator properties. It is well known that the operator $\hat{\square} + m^2\Pi^T$ and also $\hat{\square}_{(4,0)}$ do not
contribute to the holomorphic part of the effective action \[14], \[15\]. All possible contributions to the one-loop counterterm are then due to the known ghost contribution \((26)\) and due to the contribution
\[
\Gamma^{(1)}_{V^{++}} = \frac{i}{2} \text{Tr} \ln(\nabla^{++} D^{++})
\]
(29)
of the quantum superfields \(\chi\) coming from the lower-right block of matrix \((28)\). To use the known calculation tools \[14], \[15\] for \(\Gamma^{(1)}_{V^{++}}\), we reduce the differential operator in \((29)\) to the form
\[
(\nabla^{++})^2 + U^{(+4)},
\]
(30)
where
\[
U^{(+4)}_{ab} = \frac{1}{2} f_{abc} D^{++} V^{++} c + \frac{1}{4} f_{ace} f_{bde} V^{++} d.
\]
(31)
Obviously, the exact Green’s function for the operator \((\nabla^{++})^2\) coincides with \((12)\) after the replacement \(D^{++} \rightarrow \nabla^{++}\) and \(W \rightarrow W\). The Green’s function for the omega-multiplet in the external field \(U^{(+4)}\) is determined by the equation
\[
[(\nabla^{++})^2 + U^{(+4)}] G^{(0,0)}_{U} (1|2) = \delta^{(4,0)}_{A} (1|2).
\]
(32)
We define the analytic superfield kernel by the law
\[
Q^{(4,0)} (1|2) = \delta^{(4,0)}_{A} (1|2) + U^{(+4)}_{1} G^{(0,0)}_{U} (1|2)
\]
(33)
where the Green’s function \(G^{(0,0)}_{U} (1|2)\) in the external field \(V^{++}\) satisfies the equation
\[
(\nabla^{++})^2 G^{(0,0)}_{U} (1|2) = \delta^{(4,0)}_{A} (1|2).
\]
This kernel contains all the external field effects. The effective action is then determined as
\[
\Gamma[V^{++}] = \frac{i}{2} \text{Tr} \ln((\nabla^{++})^2 + U^{(+4)}) = \frac{i}{2} \text{Tr} \ln(\nabla^{++})^2 + \frac{i}{2} \text{Tr} \ln Q^{(4,0)}.
\]
(34)
The first term in the right-hand side of this equation exactly coincides with the known one-loop contribution of the hypermultiplet in an external field \[15\]. This easily follows from comparing this contribution with expression \((26)\). The only difference is that we have the opposite sign and replace the superfield strength \(W[V^{++}]\) with \(W[V^{++}] = -\frac{1}{4} (\nabla)^2 V^{--}\). We can therefore write the contribution to the divergent part of the effective action additional to \((26)\) without calculations:
\[
\Gamma^{(1)}_{\text{div}}[V^{++}] = \frac{C_2}{32 \pi^2 \varepsilon} \text{tr} \int d^8 z W^2.
\]
(35)
On the diagram level, the expansion of the second term in the right-hand side of equality \((34)\) in a power series in interactions of fields inside the loop with the external insertions \(U^{(+4)}\) and with the propagator in the external field \(V^{++}\) is
\[
\Gamma[U^{(+4)}] = \sum_{n=1}^{\infty} \Gamma_n[U^{(+4)}],
\]
(36)
where the $n$th term of this series is described by the supergraph with $n$ external lines $U^{(+4)}$. Functional (34) therefore contains the complete information about one-loop contributions with an arbitrary number of external lines $\mathcal{V}^{++}$.

The first term in (29) in the expansion of $\Gamma[\mathcal{V}^{++}]$ in a power series in $U^{(+4)}$ vanishes because it contains the harmonic product, which becomes zero in the limit of coinciding arguments, $u_1 u_2 |_{u_1 = u_2} = 0$. The effective action in the second order is

$$2\Gamma^{(1)}[\mathcal{V}^{++}] = -\frac{i}{4} \text{tr} \int d\xi_1 (-4) d\xi_2 (-4) du_1 du_2 U^{(+4)}(1) U^{(+4)}(2) \frac{1}{\Box_1} (\nabla_1^+)^4 (\nabla_2^+)^4 \delta^{12}(1|2) \frac{u_1^+ u_2^-}{(u_1^+ u_2^-)^3} \times \frac{1}{\Box_2} (\nabla_2^+)^4 (\nabla_2^+)^4 \delta^{12}(2|1) \frac{u_2^- u_1^-}{(u_2^- u_1^-)^3}.$$  

Reconstructing the total Grassmann integration measure, we remove one of the delta functions and use the identity $\langle D_1^+ \rangle^4 \langle D_2^+ \rangle^4 \delta^8(\theta - \theta')|_{\theta = \theta'} = (u_1^+ u_2^+)^4$. Further, after standard transformations, the divergent contribution becomes

$$2\Gamma^{(1)}_{\text{div}}[\mathcal{V}^{++}] = \frac{1}{(8\pi)^2 \varepsilon} \int d^2 z du_1 du_2 U^{(+4)}(z, u_1) U^{(+4)}(z, u_2) \frac{(u_1^- u_2^-)^2}{(u_1^+ u_2^+)^2}. \quad (37)$$

Subsequent terms in the expansion of (29) give finite contributions to the effective action.

Relation (37) is the main result in this paper. It is a new superfield counterterm in the $N = 2$ supersymmetric massive Yang-Mills field theory in the Stueckelberg formalism depending on the background superfield $\omega$. Obviously, this functional does not contain on-shell harmonic singularities. This follows because the nonzero contribution to the integral over odd variables must contain the maximum power of the Grassmann coordinates. But $\langle (\theta_1^+)^2 (\theta_1^-)^2 (\theta_2^+)^2 (\theta_2^-)^2 \rangle = (u_1^+ u_2^+)^4 (\bar{\theta}^4 \theta)^4$. Among many terms arising in its component form, functional (37) contains nonstandard contact four-vector interactions and terms necessary for their supersymmetrization. For example, for the gauge group $SU(2)$, these interactions are $a^i_1 a^j_2 a^j_m a^i_n$, where $a^i_m = A^i_m - L^i_m$ is the vector component of the $SU(2)$-superfield $\mathcal{V}^{++}$ given by (7). For the gauge group $SU(3)$, the corresponding interactions are $\hat{\delta}^i_6 (a^i_n a^a_m)^2 + a^a_m a^a_n a^b_m b^b_n - d_{a b c d e} a^a_m a^b_n c^c_m d^d_n$. Precisely this counterterm arises as an obstruction to the renormalizability of the standard nonsupersymmetric massive Yang-Mills field theory [16], [17]. Such deviations from the standard model result in interesting phenomenological consequences, for instance, for the processes of the creation of $W^+ W^-$ and $W Z$ vector bosons (see [21] and the references therein). We note that in contrast to the nonsupersymmetric case [17], the mass term in the $N = 2$ supersymmetric massive Yang-Mills field theory is not renormalized.

All counterterms (26), (35), and (37) are gauge-invariant, however the counterterms (35) and (37) do not reproduce the form of the initial Lagrangian. Similar to the massive $N = 0$ nonsupersymmetric Yang-Mills field theory, the massive $N = 2$ supersymmetric Yang-Mills field theory can then be regarded only as an effective low-energy theory. In other words, action (33) is not the most general $N = 2$ supersymmetric functional compatible with the local left and global right gauge symmetries of the theory, and for the theory to be renormalized in the modern sense (see, e.g., [22]) in the next order of the derivative expansion of the effective action, we must therefore include new vertices induced by functionals (35) and (37).

\footnote{In the minimal subtraction scheme, counterterms differ only in signs from divergences (26), (35), and (37).}
5 The component structure of $\mathcal{N} = 2$ superfield functional (35)

A separate interesting problem is to study features of the component expansion of the $\mathcal{N} = 2$ superfield strength $\mathcal{W}$ constructed on the base of the gauge-invariant potential $\Omega V^{++}\Omega^{-1}$, which includes an additional degree of freedom in the vector multiplet due to the Stueckelberg superfield $\omega$. Obviously, this gauge-invariant superfield differs from the covariant chiral superfield strength constructed using the potential $V^{++}$ only by the $\Omega$ operators, which disappears under the trace.

As a rule, the $\mathcal{N} = 2$ superfield formalism is most useful for description of interacting off-shell supermultiplets. Passing to component fields nevertheless requires excluding an infinite number of auxiliary fields, which is a rather difficult technical problem. Finding the component structure of counterterm (26) is easy; its component form coincides with that of the classical action of the $\mathcal{N} = 2$ supersymmetric Yang-Mills field theory [9], [10]. But the component form of (35) and (37) needs a special investigation because potential (7) transforms as $g V^{++} = e^{i\lambda} V^{++} e^{-i\lambda}$ in contrast to the transformation law for $V^{++}$, this transformation law does not contain the term with the derivative $D^{++}\lambda$. The superfield $V^{++}$ therefore contains nonremovable longitudinal degrees of freedom in the vector field sector. In particular, this makes imposing the Wess-Zumino gauge impossible for the field $V^{++}$. The problem of finding the component form of superfield functional (35) therefore implies the component decomposition of $\mathcal{W}$ without imposing a gauge condition on $V^{++}$. A general solution of this problem is still missing from the literature. Particular aspects of the component structure of the massive vector supermultiplet without fixing a supergauge were studied in [23] for the Abelian case and for the non-Abelian case in the first order in the coupling constant expansion.

In this section, we describe the procedure for finding the component form of superfield functional (35) in the bosonic sector, in which the component content of the superfield $V^{++}$ necessary for writing (35) and (37) in terms of physical fields actually coincides with the component structure of the superfield $V^{++}$ in the Wess-Zumino gauge, but each component is endowed with an infinite tower of interactions with the longitudinal degrees of freedom, related to the component fields of the $\omega$-multiplet.

A convenient way to find the component content of the strength superfield for nonstandard theories [24] is based on solving the harmonic zero-curvature equation [9], [10] for a nonanalytic potential $V^{--}$:

$$D^{++} V^{--} - D^{--} V^{++} + i[V^{++}, V^{--}] = 0.$$  (38)

Because $V^{++} = V^{++} - L^{++}$ and $V^{++}$ is subject to the standard gauge transformations, we can impose the Wess-Zumino gauge on $V^{++}$. As the result, gauge-invariant analytic potential (7) takes on form (11):

$$V^{++}(\zeta) = f^{++}(x_A) + (\theta^+)^2 \varphi(x_A) + (\bar{\theta}^+)^2 \bar{\varphi}(x_A) + 2i(\theta^+ \sigma^m \bar{\theta}^+) (A_m(x_A) + \partial_m f^{+-}(x_A))$$

$$- (\theta^+)^2 (\bar{\theta}^+)^2 \Box f^{--}(x_A) + \text{fermions},$$

where $\zeta = (x^A, \theta^{+\alpha}, \bar{\theta}^{+\dot{\alpha}}, u^+_i)$ and we preserve the notation for the component form of the potential in the Wess-Zumino gauge. We can then remember that each of the components
of $V^{++}_{WZ}$ is endowed with an infinite series in powers of the interaction with components of the $\omega$-multiplet $\omega(\zeta) = \omega(x_A) + \omega^{(ij)}(x_A)u_i^+u_j^-$. Because the superfield strength

$$W = -\frac{1}{4}(\bar{D}^+)^2 \mathcal{V}$$

(40)
is a harmonic-independent $N = 2$ chiral superfield, it is convenient to seek a solution of (38) in the chiral-analytic coordinates $Z_c = (z_c, \theta^{\pm\bar{\alpha}})$, where

$$z_c = (x^m_L, \theta^{\pm\alpha}), \quad x^m_L = x^m_A + 2i\theta^-\sigma^m\bar{\theta}^+.$$

In this basis, each of the components of potential (39) decomposes as:

$$f(x_A) = f(x_L) - 2i\theta^-\sigma^m\bar{\theta}^+\partial_m f(x_L) + (\theta^-)^2(\bar{\theta}^+)^2 \Box f(x_L).$$

Following [24], it is convenient to represent the decomposition of $\mathcal{V}^{--}$ in these coordinates in the form

$$\mathcal{V}^{--}(Z_c, u) = v^{--}(x_L, \theta^\pm, u) + \bar{\theta}^{+\alpha}v^{(-3)\alpha} + \bar{\theta}^{-\bar{\alpha}}v^{-\bar{\alpha}} + (\theta^-)^2\mathcal{A} + (\bar{\theta}^-\bar{\theta}^-)\varphi^{--} + \bar{\theta}^-\bar{\theta}^+\varphi^{--} + (\bar{\theta}^+)^2 v^{(-4)} + (\theta^-)^2\bar{\theta}^{-\bar{\alpha}}\tau^{-\bar{\alpha}} + (\bar{\theta}^+)^2\bar{\theta}^{+\alpha}\tau^{+\alpha} + (\bar{\theta}^+)^2(\bar{\theta}^-)^2 \tau^{-\bar{\alpha}}.$$

We note that strength (40) depends only on the fields $\mathcal{A}$, $\tau^{-\bar{\alpha}}$, and $\tau^{-\bar{\alpha}}$, but not on all the chiral superfields in this decomposition. However, as shown in [24], only the chiral superfield

$$\mathcal{A} = A_1 + (\theta^-)^2A_4^{++} + (\theta^-\bar{\theta}^+)A_5 + \theta^{-\bar{\alpha}}\theta^{+\bar{\alpha}}A_{6\alpha\bar{\beta}} + (\theta^+)^2A_7^{--} + (\theta^-)^2(\theta^+)^2A_{10} + \text{fermions}$$

actually determines the component structure of superfield functional (35), which has the form:

$$S_{bos} = \frac{1}{4}\text{tr} \int d^4x_L du \{2A_1A_{10} + 2A_4^{++}A_7^{--} - \frac{1}{2}A_5^2 - \frac{1}{4}A_6^2\}$$

(41)
in the bosonic sector.

The equations that determine the components of the superfield $\mathcal{A}$ are constructed as coefficients in expansion of (38) in $\theta^\pm, \bar{\theta}^{\pm}$:

$$d^{++}A_1 = 0, \quad d^{++}A_4^{++} = 0, \quad d^{++}A_5 + 4A_4^{++} = 0, \quad d^{++}A_{6\alpha\bar{\beta}} = 0$$

$$d^{++}A_7^{--} + 2A_5 + [\bar{\phi}, A_1] = 0, \quad d^{++}A_{10} + [\bar{\phi}, A_4^{++}] = 0,$$

(42)

where we introduce the notation

$$d^{++} = \partial^{++} + i[f^{++}, ...].$$

The general solution of this set of harmonic equations can be written in terms of Green’s functions for the operator $\partial^{++}$ [10]. For example, the solution of the first equation in chain (42) is:

$$A_1(u) = \varphi - i \int du_1 \frac{u_1^+u_1^-}{u_1^+u_1^-} [f^{++}(u_1), A_1(u_1)]$$

(43)
\begin{equation}
\sum_{n=0}^{\infty} (-i)^n \int du_1 ... du_n \frac{u^+ u_1^- ... u_{n-1}^+ u_n^-}{u^+ u_1^+ ... u_{n-1}^- u_n^+} \left[ f^{++}(u_1), ..., [f^{++}(u_n), \varphi] \right] = e^{ib} \varphi,
\end{equation}

where \( \varphi \) is a particular solution of the homogeneous equation and \( e^{ib} \) is a nonanalytic superfield called the bridge [9], [10] for the field \( f^{++} = -ie^{ib} \partial^{++} e^{-ib} \). This solution can be constructed iteratively using the Taylor series in \( f^{++} \) as in [9].

Each component of \( A \) is therefore determined by an infinite series in powers of interactions of the standard components of the superfield \( W \) with the field \( f^{++} \):

\begin{align}
A_{i+}^{++} &= e^{ib} f^{++}, & A_{i-}^{+-} &= 4e^{ib} f^{--}, & A_{i+}^{0\beta} &= e^{ib} F^{\alpha\beta} \\
A_5 &= e^{ib} (-4 \Box f^{+-} + \frac{1}{2} [\varphi, \bar{\varphi}]), & A_{10} &= e^{ib} (-\Box \varphi + \frac{1}{8} [\bar{\varphi}, [\bar{\varphi}, \varphi]]).
\end{align}

Here, the component fields \( f^{ij}, \varphi, \bar{\varphi} \) and \( F^{\alpha\beta} \) are determined by decomposition (39) and the action of the matrix operator \( e^{ib} \) is given by relation (43).

Our analysis therefore proves that a formal solution for the components of superfield strength (40) exists in a nonpolynomial form and that in addition to the standard action in the Wess-Zumino gauge modified by interaction with the \( \omega \)-multiplet components, action (41) contains fourth powers of the space-time derivatives of \( \omega, \omega^{(ij)} \) components of the \( \omega \)-multiplet. Finding a more detailed component form of expression (41) is a very complicated technical problem although in principle such a form can be found using the above procedure. We also note that in the sector of the vector multiplet components (i.e., when we switch off the dependence on the omega-multiplet components), divergences (26) and (35) cancel.

6 Conclusion

We present the main results of the paper.

1. We considered the \( N = 2 \) supersymmetric massive Yang-Mills field theory whose action depends on the \( N = 2 \) gauge superfield \( V^{++} \) and on the hypermultiplet Stueckelberg superfield \( \omega \). We proposed the various dual-equivalent formulations of this theory differing by form gauge-invariant mass term in the superfield action.

2. We developed a background field method that allows obtaining the loop expansion of the effective action in an explicitly gauge-invariant and \( N = 2 \) supersymmetric form. We demonstrated that the contribution of the Stueckelberg superfield \( \omega \) to the effective action can be formulated in terms of the superfield \( V^{++} \) given by (7), which is a special gauge-invariant combination of the background superfields \( V^{++} \) and \( \omega \).

3. We studied the structure of one-loop divergences in the theory under consideration. We obtained explicitly gauge-invariant and \( N = 2 \) supersymmetric expressions for one-loop divergences (26), (35) and (37). The expression (37) is a new gauge-invariant and \( N = 2 \) supersymmetric functional constructed from the superfields \( V^{++} \) and \( \omega \). The appearance of this functional as an expression determining one-loop divergences results in the (multiplicative) nonrenormalizability of the theory. This functional can be treated as an \( N = 2 \) supersymmetrization of the covariant counterterm in the nonsupersymmetric Yang-Mills field theory [16], [17]. But in contrast to the nonsupersymmetric case, the mass term in the massive \( N = 2 \) supersymmetric Yang-Mills field theory is not renormalized. We gave the
complete analysis of the $\mathcal{N} = 2$ superfield structure of the one-loop divergences in the considered theory. The case when the interaction of the vector multiplet with the Stueckelberg multiplet is switched off or, in other words, the zero-mass case, partly verifies the obtained results. In this limit, we have a single divergence due to one-loop ghost contributions (26), which determines the known value of the beta-function of the pure $\mathcal{N} = 2$ supersymmetric Yang-Mills field theory [15].

4. We considered the component structure of the counterterm of form (35) in the bosonic sector. Because the gauge transformation for $V^{++}$ given by (7) does not contain the derivative of the gauge parameter, we cannot impose the Wess-Zumino gauge on $V^{++}$, which is standardly used when passing from the superfield description of the vector multiplet to its component description. We therefore encountered the problem of finding the component form of superfield functional (35). A procedure for solving this problem in the bosonic sector is proposed (see (11) and (14)).

We briefly discuss the prospect for further studying the massive $\mathcal{N} = 2$ supersymmetric Yang-Mills field theory. As is known, the massless $\mathcal{N} = 2$ supersymmetric Yang-Mills theory is finite beyond the one-loop approximation (see, e.g., [14]). The problem of divergences of the considered massive theory in higher loops remains open. Finding finite contributions to the one-loop effective action and studying the effective action in the presence of the interaction between the massive gauge $\mathcal{N} = 2$ superfield and the matter hypermultiplets is therefore interesting. In our opinion, the problem of the quantum equivalence of the massive $\mathcal{N} = 2$ supersymmetric Yang-Mills field theory in the Stueckelberg formalism and the $\mathcal{N} = 2$ supersymmetric non-Abelian vector-tensor model, which (as was shown) are dual on the classical level, is especially interesting.

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