Monitoring Constraints and Metaconstraints with Temporal Logics on Finite Traces

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Abstract. Runtime monitoring is one of the central tasks in the area of operational decision support for business process management. In particular, it helps process executors to check on-the-fly whether a running process instance satisfies business constraints of interest, providing an immediate feedback when deviations occur. We study runtime monitoring of properties expressed in LTL on finite traces (LTLf), and in its extension LDLf. LDLf is a powerful logic that captures all monadic second order logic on finite traces, and that is obtained by combining regular expressions with LTLf, adopting the syntax of propositional dynamic logic (PDL). Interestingly, in spite of its greater expressivity, LDLf has exactly the same computational complexity of LTLf. We show that LDLf is able to declaratively express, in the logic itself, not only the constraints to be monitored, but also the de-facto standard RV-LTL monitors. On the one hand, this enables us to directly employ the standard characterization of LDLf based on finite-state automata to monitor constraints in a fine-grained way. On the other hand, it provides the basis for declaratively expressing sophisticated metaconstraints that predicate on the monitoring state of other constraints, and to check them by relying on standard logical services instead of ad-hoc algorithms. In addition, we devise a direct translation of LDLf formulae into nondeterministic finite-state automata, avoiding to detour to Büchi automata or alternating automata. We then report on how this approach has been effectively implemented using Java to manipulate LDLf formulae and their corresponding monitors, and the well-known ProM process mining suite as underlying operational decision support infrastructure.

1 Introduction

Runtime monitoring is one of the central tasks to provide operational decision support to running business processes [1]. While traditional process mining techniques analyze event data of already completed process instances, operational
support lifts process mining to running, live process executions, providing an online feedback that can be used to influence the future continuations of such executions. In this setting, the goal of monitoring is to check on-the-fly whether a running process instance complies with business constraints and rules of interest, promptly detecting deviations [12]. Such indicators can, in turn, be used to compute different monitoring metrics, obtaining a succinct summary about the degree of compliance of a running process instance.

In order to provide provably correct runtime monitoring techniques with a well-defined semantics and a solid formal background, monitoring is typically rooted into the field of formal verification, the branch of formal methods aimed at checking whether a system meets some property of interest. Being the system dynamic, properties are typically expressed by making use of temporal logics, that is, modal logics whose modal operators predicate about the evolution of the system along time.

Among all the temporal logics used in verification, Linear-time Temporal Logic (LTL) is particularly suited for monitoring, as an actual system execution is indeed a linear sequence of events. Since the LTL semantics is given in terms of infinite traces, LTL monitors analyze the trace of interest by considering it as the prefix of an infinite trace that will continue forever [2]. However, this hypothesis falls short in several contexts, where the usual assumption is that each trace produced by the system is in fact finite. This is often the case in Business Process Management (BPM), where each process instance is expected to eventually reach one of the foreseen ending states of the process [23]. In this setting, a monitored trace has to be considered as the prefix of an unknown, but still finitely long, trace. To handle this type of setting, finite-trace variants of LTL have been introduced. In this work, we consider in particular the logic LTL\textsubscript{f} (LTL on finite traces), investigated in detail in [8], and at the basis of one of the main declarative process modeling approaches: DECLARE [23][21][14].

Following [14], monitoring in LTL\textsubscript{f} amounts to check whether the current execution belongs to the set of admissible prefixes for the traces of a given LTL\textsubscript{f} formula \( \varphi \). To achieve such a task, \( \varphi \) is usually first translated into a corresponding finite-state automaton that exactly recognizes all and only those finite traces that satisfy \( \varphi \). Despite the presence of previous operational decision support techniques to monitoring LTL\textsubscript{f} constraints over finite traces [14][16], two main challenges have not yet been tackled in a systematic way.

First of all, several alternative semantics have been proposed to make LTL suitable for runtime verification, such as the de-facto standard RV monitor conditions [2], which interpret LTL formulae using four distinct truth values that account at once for the current trace and its possible future continuations. Specifically, in the RV-LTL framework, a formula is associated to a corresponding RV state, which may witness: (i) permanent violation (the formula is currently violated, and the violation cannot be repaired anymore); (ii) temporary violation (the formula is currently violated but it is possible to continue the execution in a way that makes the formula satisfied); (iii) permanent satisfaction (the formula is currently satisfied and it will stay satisfied no matter how the execution
continues; \( iv \) temporary satisfaction (the formula is currently satisfied but may become violated in the future). The main issue is that no comprehensive, formal framework based on finite-state automata is available to handle such RV states. On the one hand, this is because runtime verification for temporal logics typically focus on the infinite-trace setting [2], with the consequence that the corresponding automata-theoretic techniques detour to Büchi automata for building and using the monitors. On the other hand, the incorporation of such an RV semantics in a finite-trace setting has only been tackled so far with ad-hoc techniques. This is in particular the case of [14], which operationally proposes to “color” automata to support the different RV states, but it does not come with an underlying formal counterpart justifying the correctness of the approach.

A second, fundamental challenge is the incorporation of advanced forms of monitoring, going beyond what can be expressed with LTL\(_f\). In particular, contemporary monitoring approaches do not systematically account for metaconstraints that predicate on the RV state of other constraints. This is especially important in a monitoring setting, where it is often of interest to consider certain constraints only when specific circumstances arise, such as when other constraints become violated. For example, metaconstraints provide the basis for monitoring compensation constraints, which can be considered as the temporal version of so-called contrary-to-duty obligations [26] in normative reasoning, that is, obligations that are put in place only when other obligations have not been fulfilled. While this feature is considered to be a fundamental compliance monitoring functionality [12], it is still an open challenge, without any systematic approach able to support it at the level of the constraint specification language.

In this article, we attack these two challenges by proposing a formal and operational framework for the monitoring of properties expressed in LTL\(_f\) and in its extension LDL\(_f\) [8]. LDL\(_f\) is a powerful logic that completely captures Monadic Second-Order Logic on finite traces, in turn, expressively equivalent to the language of regular expressions. LDL\(_f\) does so by combining regular expressions with LTL\(_f\), adopting the syntax of propositional dynamic logic (PDL). Interestingly, in spite of its greater expressivity, LDL\(_f\) has exactly the same computational complexity of LTL\(_f\). At the same time, it provides a balanced integration between the expressiveness of regular expressions, and the declarativeness of LTL\(_f\).

Our first, technical contribution is the formal development, accompanied by a proof-of-concept implementation, of an automata-theoretic framework for monitoring LTL\(_f\) and LDL\(_f\) constraints using the four truth values of the RV approach. We do this in two steps. In the first step, we devise a direct translation of LDL\(_f\) (and hence of LTL\(_f\)) formulae into nondeterministic automata, which avoid the usual detour to Büchi automata. The technique is grounded on alternating automata (AFW), but it actually avoids their introduction all together: in fact, the technique directly produces a standard non-deterministic finite-state automaton (NFA), which can then be manipulated using conventional automata techniques (such as determinization and minimization). In the second step, we show that LDL\(_f\) is able to capture, in the logic itself, special formulae that capture all RV monitoring conditions. More specifically, given an arbitrary LDL\(_f\) formula \( \varphi \), we
show how to construct, for each RV monitor condition, another $\text{LDL}_f$ formula that characterizes all and only the traces culminating in a time point where $\varphi$ is associated to that RV state. By studying the so-obtained four $\text{LDL}_f$ special formulae, we then describe how to construct a single automaton that, given a trace, outputs the RV state associated to $\varphi$ by that trace. This, in turn, provides for the first time a proof of correctness of the “colored automata” approach proposed in [14].

We exploit this meta-level ability of $\text{LDL}_f$ in our second, major contribution, which shows how to use the logic to capture metaconstraints, and how to monitor them by relying on usual logical services instead of ad-hoc algorithms. Metaconstraints provide a well-founded, declarative basis to specify and monitor constraints depending on the monitoring state of other constraints. To concretely show the flexibility and sophistication of our approach, we introduce and study three interesting classes of metaconstraints. The first class is about contextualizing a constraint, by expressing that it has to be enforced only in those time points where another constraint is in a given RV state. The second class deals with two forms of the aforementioned compensation constraints, which capture that a compensating constraint has to be monitored when another constraint becomes permanently violated. The third and last class targets the interesting case of conflicting constraints, that is, constraints that, depending on the circumstances, may contradict each other. In particular, we show how to express a preference on which constraint should be satisfied when a contradiction arises.

In the final part of the paper, we report on how our monitoring framework has been concretely implemented, and exposed as an operational decision support plug-in within PROM, one of the most widely adopted infrastructures for process mining.

This article is a largely extended version of the conference paper in [7]. In relation with [7], we expand all technical parts, including here full proofs of the obtained results and a completely novel part on the construction of “colored automata” for monitoring. In addition, we provide here a much more detailed account on metaconstraints, introducing three metaconstraint classes that have not yet been investigated in prior work. We also report here on the complete implementation of our monitoring framework.

The rest of the article is structured as follows. In Section 2, we introduce syntax and semantics of $\text{LDL}_f$ and $\text{LTL}_f$. In Section 3, we then show how an $\text{LDL}_f/\text{LTL}_f$ formula can be translated into a corresponding NFA that accepts all and only the traces that satisfy the formula. In Section 4, we show how $\text{LDL}_f$ is able to capture the RV states in the logic itself, and employ the automata-theoretic approach developed in Section 3 to construct RV monitors for $\text{LDL}_f/\text{LTL}_f$ formulae. In Section 5, we discuss how the resulting framework can be applied in the context of the DECLARE constraint-based process modeling approach. In Section 6, we turn to metaconstraints, introducing the three interesting metaconstraint classes of contextualization, compensation, and preference in case of

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http://www.promtools.org/
conflict. The implementation of our monitoring framework in Java and PROM is reported in Section 7. Conclusion follows.

2 Linear Temporal Logics on Finite Traces

In this work, we adopt the standard LTL and its extension LDL, interpreted on finite traces. LTL on finite traces, called LTL\(_f\) [8], has exactly the same syntax as LTL on infinite traces [25]. Namely, given a set of \(\mathcal{P}\) of propositional symbols, LTL\(_f\) formulae are obtained through the following:

\[
\varphi ::= \phi \mid \neg \varphi \mid \varphi_1 \land \varphi_2 \mid \varphi_1 \lor \varphi_2 \mid O \varphi \mid \bullet \varphi \mid \varphi_1 U \varphi_2 \mid \varphi_1 R \varphi_2
\]

where \(\phi\) is a propositional formula over \(\mathcal{P}\), \(O\) is the next operator, \(\bullet\) is the weak next operator, for which we have the equivalence \(\bullet \varphi \equiv \neg O \neg \varphi\) (notice that in the finite trace case \(\neg O \neg \varphi \neq \varphi\)), \(U\) is the until operator and \(R\) is release operator, for which we have the equivalence \(\varphi_2 R \varphi_2 \equiv \neg (\neg \varphi_2 U \neg \varphi_2)\). In addition, we have common abbreviations. For example, eventually \(\Diamond \varphi\) abbreviates \(true U \varphi\); and always \(\square \varphi\) abbreviates \(false R \varphi\) or equivalently \(\neg \Diamond \neg \varphi\).

Notice that, for convenience and without loss of generality, we allow negation only in propositional formulae, i.e., we essentially assume the temporal formulae to be in negation normal form (NNF). An arbitrary temporal formula can be put in NNF in linear time.

The semantics of LTL\(_f\) is given in terms of finite traces denoting finite, possibly empty, sequences \(\pi = \pi_0, \ldots, \pi_n\) of elements from the alphabet \(2^\mathcal{P}\), containing all possible propositional interpretations of the propositional symbols in \(\mathcal{P}\). We denote the length of the trace \(\pi\) as \(\text{length}(\pi) = n + 1\). We denote as \(\pi(i) = \pi_i\) the \(i\)-th step in the trace. If the trace is shorter and does not include an \(i\)-th step, \(\pi(i)\) is undefined. We denote by \(\pi(i,j) = \pi_i, \pi_{i+1}, \ldots, \pi_{j-1}\) the segment of the trace \(\pi\) starting at the \(i\)-th step and ending at the \(j\)-th step (excluded). If \(j > \text{length}(\pi)\) then \(\pi(i,j) = \pi(i, \text{length}(\pi))\). For every \(j \leq i\), we have \(\pi(i,j) = \epsilon\), i.e., the empty trace. Notice that here, differently form [8], we allow the empty trace \(\epsilon\) as in [3]. This is convenient for composing monitors, as it will become clear later on in the article. Given a finite trace \(\pi\), we inductively define when an LTL\(_f\) formula \(\varphi\) is true at a step \(i\) written \(\pi, i \models \varphi\), as follows (we include abbreviations for convenience):

- \(\pi, i \models \phi\) iff \(0 \leq i \leq \text{length}(\pi)\) and \(\pi(i) \models \phi\) (\(\phi\) propositional);
- \(\pi, i \models \neg \varphi\) iff \(\pi, i \not\models \varphi\);
- \(\pi, i \models \varphi_1 \land \varphi_2\) iff \(\pi, i \models \varphi_1\) and \(\pi, i \models \varphi_2\);
- \(\pi, i \models \varphi_1 \lor \varphi_2\) iff \(\pi, i \models \varphi_1\) or \(\pi, i \models \varphi_2\);
- \(\pi, i \models O \varphi\) iff \(0 \leq i < \text{length}(\pi) - 1\) and \(\pi, i+1 \models \varphi\);
- \(\pi, i \models \bullet \varphi\) iff \(0 \leq i < \text{length}(\pi) - 1\) implies \(\pi, i+1 \models \varphi\);
- \(\pi, i \models \Diamond \varphi\) iff for some \(j\) s.t. \(0 \leq i \leq j < \text{length}(\pi)\), we have \(\pi, j \models \varphi\);
- \(\pi, i \models \Box \varphi\) iff for all \(j\) s.t. \(0 \leq i \leq j < \text{length}(\pi)\), we have \(\pi, j \models \varphi\);
- \(\pi, i \models \varphi_1 U \varphi_2\) iff for some \(j\) s.t. \(1 \leq i \leq j < \text{length}(\pi)\), we have \(\pi, j \models \varphi_2\), and for all \(k, i \leq k < j\), we have \(\pi, k \models \varphi_1\);
\( \pi, i \models \varphi_1 R \varphi_2 \) iff for all \( j \) s.t. \( 0 \leq i \leq j < \text{length}(\pi) \), either we have \( \pi, j \models \varphi_2 \) or for some \( k, i \leq k < j \), we have \( \pi, k \models \varphi_1 \).

Observe that for \( i \geq \text{length}(\pi) \), hence e.g., for \( \pi = \epsilon \) we get:

- \( \pi, i \not\models \varphi \) (\( \varphi \) propositional);
- \( \pi, i \models \neg \varphi \) iff \( \pi, i \not\models \varphi \);
- \( \pi, i \models \varphi_1 \land \varphi_2 \) iff \( \pi, i \models \varphi_1 \) and \( \pi, i \models \varphi_2 \);
- \( \pi, i \models \varphi_1 \lor \varphi_2 \) iff \( \pi, i \models \varphi_1 \) or \( \pi, i \models \varphi_2 \);
- \( \pi, i \not\models \Diamond \varphi \);
- \( \pi, i \models \Box \varphi \);
- \( \pi, i \not\models \varphi_1 U \varphi_2 \);
- \( \pi, i \models \varphi_1 R \varphi_2 \).

It is known that LTL\(_f\) is as expressive as First-Order Logic over finite traces, so strictly less expressive than regular expressions, which, in turn, are as expressive as Monadic Second-Order logic over finite traces. On the other hand, regular expressions are a too low level formalism for expressing temporal specifications, since, for example, they miss a direct construct for negation and for conjunction [8].

To be as expressive as regular expressions and, at the same time, convenient as a temporal logic, in [8] Linear Dynamic Logic of Finite Traces, or LDL\(_f\), has been proposed. This logic is as natural as LTL\(_f\), but with the full expressive power of Monadic Second-Order logic over finite traces. LDL\(_f\) is obtained by merging LTL\(_f\) with regular expressions through the syntax of the well-know logic of programs PDL, Propositional Dynamic Logic [10,11], but adopting a semantics based on finite traces. LDL\(_f\) is an adaptation of LDL introduced in [27], which, like LTL, is interpreted over infinite traces.

Formally, LDL\(_f\) formulae are built as follows:

\[
\varphi ::= \ttt \mid \fff \mid \neg \varphi \mid \varphi_1 \land \varphi_2 \mid \varphi_1 \lor \varphi_2 \mid \langle \rho \rangle \varphi \mid [\rho] \varphi
\]

\[
\rho ::= \phi \mid \rho? \mid p_1 + p_2 \mid \rho_1; \rho_2 \mid \rho^*
\]

where \( \ttt \) and \( \fff \) denote respectively the true and the false LDL\(_f\) formula (not to be confused with the propositional formula \( \text{true} \) and \( \text{false} \)); \( \phi \) denotes propositional formulae over \( \mathcal{P} \); \( \rho \) denotes path expressions, which are regular expressions over propositional formulae \( \phi \) over \( \mathcal{P} \) with the addition of the test construct \( \rho? \) typical of PDL and are used to insert into the execution path checks for satisfaction of additional LDL\(_f\) formulae; and \( \varphi \) stand for LDL\(_f\) formulae built by applying boolean connectives and the modal operators \( \langle \rho \rangle \varphi \) and \( [\rho] \varphi \). These two operators are linked by the following equivalence \( [\rho] \varphi \equiv \neg \langle \rho \rangle \neg \varphi \).

Intuitively, \( \langle \rho \rangle \varphi \) states that, from the current step in the trace, there exists an execution satisfying the regular expression \( \rho \) such that its last step satisfies \( \varphi \). While \( [\rho] \varphi \) states that, from the current step, all executions satisfying the regular expression \( \rho \) are such that their last step satisfies \( \varphi \).
Notice that $\text{LDL}_f$, as defined above, does not include propositional formulae $\phi$ as $\text{LDL}_f$ formulae, but only as path expressions. However, they can be immediately introduced as abbreviations: $\phi \equiv \langle \phi \rangle tt$. For example, to say that eventually proposition $a$ holds, instead of writing $(\text{true}^*a)$, we can write $(\text{true}^*; a) tt$. This is analogous to what happens in (extensions with regular expressions of) XPath, a well-known formalism developed for navigating XML documents and graph databases [6,17,4]. We may keep $\phi$ as $\text{LDL}_f$ formulae for convenience, however, we have to be careful of the difference we get if we apply negation to propositional formula $\phi$ or to $\langle \phi \rangle tt$. In the first case, we get $\neg \phi$, which is equivalent to $\langle \neg \phi \rangle tt$. In the second case, we get $[\phi] ff$, which is equivalent to $[\text{true}?] ff \lor \langle \neg \phi \rangle tt$, which says that either the trace is empty or $\phi$ holds in the current state. We drop the use of $\phi$ to avoid this ambiguity.

It is also convenient to introduce the following abbreviations specific for dealing with the finiteness of the traces: $\text{end} = [\text{true}] ff$, which denotes that the trace has been completed (the current instant is out of the range of the trace, or the remaining fragment of the trace is empty); and $\text{last} = (\text{true}) end$, which denotes the last step of the trace.

As for $\text{LTL}_f$, the semantics of $\text{LDL}_f$ is given in terms of finite traces denoting a finite, possibly empty, sequence of consecutive steps in the trace, i.e., finite words $\pi$ over the alphabet of $2^P$, containing all possible propositional interpretations of the propositional symbols in $P$. The semantics of $\text{LDL}_f$ is given in the following. An $\text{LDL}_f$ formula $\varphi$ is true at a step $i$, in symbols $\pi, i \models \varphi$, if:

- $\pi, i \models \text{tt}$;
- $\pi, i \not\models \text{ff}$;
- $\pi, i \models \neg \varphi$ iff $\pi, i \not\models \varphi$;
- $\pi, i \models \varphi_1 \land \varphi_2$ iff $\pi, i \models \varphi_1$ and $\pi, i \models \varphi_2$;
- $\pi, i \models \varphi_1 \lor \varphi_2$ iff $\pi, i \models \varphi_1$ or $\pi, i \models \varphi_2$;
- $\pi, i \models [\rho]\varphi$ iff for some $j$ s.t. $i \leq j$, we have $\pi(i, j) \in L(\rho)$ and $\pi, j \models \varphi$;
- $\pi, i \models [\rho]\varphi$ iff for all $j$ s.t. $i \leq j$; and $\pi(i, j) \in L(\rho)$, we have $\pi, j \models \varphi$.

The relation $\pi(i, j) \in L(\rho)$ is defined inductively as follows:

- $\pi(i, j) \in L(\phi)$ if $j = i + 1$ and $0 \leq i \leq \text{length}(\pi)$ and $\pi(i) \models \phi$ (\phi propositional);
- $\pi(i, j) \in L(\text{ff})$ if $j = i$ and $\pi, i \models \varphi$;
- $\pi(i, j) \in L(\rho_1 + \rho_2)$ if $\pi(i, j) \in L(\rho_1)$ or $\pi(i, j) \in L(\rho_2)$;
- $\pi(i, j) \in L(\rho_1; \rho_2)$ if exists $k$ s.t. $\pi(i, k) \in L(\rho_1)$ and $\pi(k, j) \in L(\rho_2)$;
- $\pi(i, j) \in L(\rho^*)$ if $j = i$ or exists $k$ s.t. $\pi(i, k) \in L(\rho)$ and $\pi(k, j) \in L(\rho^*)$.

Note that if $i \geq \text{length}(\pi)$, hence, e.g., for $\pi = \epsilon$, the above definitions still apply; though, $\langle \phi \rangle \varphi$ (\phi prop.) and $\langle \psi \rangle \varphi$ become trivially false. As usual, we write $\pi \models \varphi$ as a shortcut for $\pi, 0 \models \varphi$. 
It easy to encode $\text{LTL}_f$ into $\text{LDL}_f$: we can define a translation function $\text{tr}$ defined by induction of the $\text{LTL}_f$ formula as follows:

\[
\begin{align*}
\text{tr}(\phi) &= \langle \phi \rangle \text{tt} \quad (\phi \text{ propositional}) \\
\text{tr}(\neg \phi) &= \neg \text{tr}(\phi) \\
\text{tr}(\varphi_1 \land \varphi_2) &= \text{tr}(\varphi_1) \land \text{tr}(\varphi_2) \\
\text{tr}(\varphi_1 \lor \varphi_2) &= \text{tr}(\varphi_1) \lor \text{tr}(\varphi_2) \\
\text{tr}(\diamond \varphi) &= \langle \text{true} \rangle(\text{tr}(\varphi) \land \text{end}) \\
\text{tr}(\bullet \varphi) &= \text{tr}(\neg (\diamond \neg \varphi))) \\
\text{tr}(\Box \varphi) &= \langle \text{true}^* \rangle(\text{tr}(\varphi) \land \text{end}) \\
\text{tr}(\varphi_1 \mathcal{U} \varphi_2) &= \langle (\text{tr}(\varphi_1) ?; \text{true})^* \rangle(\text{tr}(\varphi_2) \land \text{end}) \\
\text{tr}(\varphi_1 \mathcal{R} \varphi_2) &= \text{tr}(\neg (\neg \varphi_1 \mathcal{U} \neg \varphi_2))
\end{align*}
\]

where $\text{nff}(\psi)$ is the function that transform $\psi$ by pushing negation inside until it is just used in front of atomic propositions. It is also easy to encode regular expressions, used as a specification formalism for traces into $\text{LDL}_f$: $\rho$ translates to $\langle \rho \rangle \text{end}$.

We say that a trace satisfies an $\text{LTL}_f/\text{LDL}_f$ formula $\varphi$, written $\pi \models \varphi$, if $\pi, 0 \models \varphi$. Note that if $\pi$ is the empty trace, and hence 0 is out of range, still the notion of $\pi, 0 \models \varphi$ is well defined. Also sometimes we denote by $L(\varphi)$ the set of traces that satisfy $\varphi$. i.e., $L(\varphi) = \{ \pi \mid \pi \models \varphi \}$.

### 3 \text{LDL}_f \text{ Automaton}

We can associate with each $\text{LDL}_f$ formula $\varphi$ an (exponential) NFA $A(\varphi)$ that accepts exactly the traces that make $\varphi$ true. Here, we provide a simple direct algorithm for computing the NFA corresponding to an $\text{LDL}_f$ formula. The correctness of the algorithm is based on the fact that (i) we can associate each $\text{LDL}_f$ formula $\varphi$ with a polynomial alternating automaton on words (AFW) that accepts exactly those traces that make $\varphi$ true [9], and (ii) every AFW can be transformed into an NFA, see, e.g., [9]. However, to formulate the algorithm, we do not need these notions, but we can work directly on the $\text{LDL}_f$ formula. Then, we define an auxiliary function $\delta$ as in Figure 1 which takes an $\text{LDL}_f$ formula $\psi$ (in negation normal form) and a propositional interpretation $\Pi$ for $\mathcal{P}$, or a special symbol $\epsilon$, and returns a positive boolean formula whose atoms are (quoted) sub-formulae of $\psi$. Note that for defining $\delta$ we make use of extra symbols of the form $T(p^r)\varphi$ and $F[p^r]\varphi$, for handling formulae $\langle p^r \rangle \varphi$ and $[p^r] \varphi$. Such extra symbols act in $\delta$ as they were additional states excepts that during the recursive computation of $\delta$ they disappear, either because evaluated to true or false or because they are syntactically replaced by $\langle p \rangle \varphi$ and $[p] \varphi$, respectively, when a new state is returned. For the latter, we use an auxiliary function $E(\varphi)$, which takes as input a formula $\varphi$ with these extra symbols $T_\psi$ and $F_\psi$ used as additional atomic propositions, and recursively substitutes in it all their occurrences with the formula $\psi$ itself. Notice also that for $\phi$ propositional, $\delta(\phi, \Pi) = \delta(\langle \phi \rangle \text{tt}, \Pi)$, as a consequence of the equivalence $\phi \equiv \langle \phi \rangle \text{tt}$.
\[ \delta(tt, \Pi) = true \]
\[ \delta(ff, \Pi) = false \]
\[ \delta(\phi, \Pi) = \delta(\langle \phi \rangle tt, \Pi) \quad (\phi \text{ propos.}) \]
\[ \delta(\phi_1 \land \phi_2, \Pi) = \delta(\phi_1, \Pi) \land \delta(\phi_2, \Pi) \]
\[ \delta(\phi_1 \lor \phi_2, \Pi) = \delta(\phi_1, \Pi) \lor \delta(\phi_2, \Pi) \]
\[ \delta(\langle \phi \rangle \phi, \Pi) = \begin{cases} E(\phi) & \text{if } \Pi \models \phi \\ false & \text{if } \Pi \not\models \phi \end{cases} \quad (\phi \text{ propos.}) \]
\[ \delta(\phi_1 \land \phi_2, \Pi) = \delta(\phi_1, \Pi) \land \delta(\phi_2, \Pi) \]
\[ \delta(\phi_1 \lor \phi_2, \Pi) = \delta(\phi_1, \Pi) \lor \delta(\phi_2, \Pi) \]
\[ \delta(\langle \phi \rangle \phi, \Pi) = \begin{cases} E(\phi) & \text{if } \Pi \models \phi \\ false & \text{if } \Pi \not\models \phi \end{cases} \quad (\phi \text{ propos.}) \]
\[ \delta(\langle \psi \rangle \phi, \Pi) = \delta(\psi, \Pi) \land \delta(\phi, \Pi) \]
\[ \delta(\langle \rho \rangle \phi, \Pi) = \delta(\langle \rho \rangle \langle \rho \rangle \phi, \Pi) \]
\[ \delta(\langle \rho \rangle \phi, \Pi) = \delta(\rho, \Pi) \lor \delta(\langle \rho \rangle F_{\langle \rho \rangle} \phi, \Pi) \]
\[ \delta(\langle \rho \rangle \phi, \Pi) = \begin{cases} E(\phi) & \text{if } \Pi \models \phi \\ false & \text{if } \Pi \not\models \phi \end{cases} \quad (\phi \text{ propos.}) \]
\[ \delta(\langle \rho \rangle \phi, \Pi) = \delta(\rho, \Pi) \land \delta(\langle \rho \rangle T_{\langle \rho \rangle} \phi, \Pi) \]
\[ \delta(\rho \phi, \Pi) = false \]
\[ \delta(\rho \phi, \Pi) = true \]

Fig. 1: Definition of \( \delta \), where \( E(\phi) \) recursively replaces in \( \phi \) all occurrences of atoms of the form \( T_{\psi} \) and \( F_{\psi} \) by \( \psi \).

The auxiliary function \( \delta(\phi, \epsilon) \), i.e., in the case the (remaining fragment of the) trace is empty, is defined exactly as in Figure 1 except for the following base cases:

\[ \delta(\langle \phi \rangle \phi, \epsilon) = false \quad (\phi \text{ propositional}) \]
\[ \delta(\langle \phi \rangle \phi, \epsilon) = true \quad (\phi \text{ propositional}) \]

Note that \( \delta(\phi, \epsilon) \) is always either \( true \) or \( false \).

Using the auxiliary function \( \delta \), we can build the NFA \( A(\phi) \) of an LDL\(_f\) formula \( \phi \) in a forward fashion as described in Figure 1, where: states of \( A(\phi) \) are sets of atoms (recall that each atom is quoted \( \phi \) sub-formulae) to be interpreted as a conjunction; the empty conjunction \( \emptyset \) stands for \( true \); \( \Pi \) is a propositional interpretation and \( q' \) is a set of (quoted) sub-formulae of \( \phi \) that denotes a minimal interpretation such that \( q' \models \Pi \models \phi \). Note that we do not need to get all \( q \) such that \( q' \models \Pi \models \phi \), but only the minimal ones. In addition, trivially we have \( (\emptyset, a, \emptyset) \in \rho \) for every \( a \in \Sigma \).
The algorithm \texttt{LDL}_f2\texttt{NFA} terminates in at most an exponential number of steps, and generates a set of states \( S \) whose size is at most exponential in the size of \( \varphi \). We observe that the algorithm \texttt{LDL}_f2\texttt{NFA} implicitly constructs the \texttt{AFW} for \( \varphi \), and transforms it into a corresponding \texttt{NFA}. In particular, given an \texttt{LDL}_f formula \( \varphi \), its sub-formulae are the states of the \texttt{AFW}, with initial state the formula itself, and no final states. The auxiliary function \( \delta \) grounded on the sub-formulae of \( \varphi \) becomes the transition function of such an \texttt{AFW}. This directly leads to the following result.

**Theorem 1** ([8]). Let \( \varphi \) be an \texttt{LDL}_f formula and \( A(\varphi) \) the \texttt{NFA} obtained by applying the algorithm \texttt{LDL}_f2\texttt{NFA} to \( \varphi \). Then \( \pi \models \varphi \) iff \( \pi \in L(A(\varphi)) \) for every finite trace \( \pi \).

We can check the satisfiability of an \texttt{LDL}_f formula \( \varphi \) by checking whether its corresponding \texttt{NFA} \( A(\varphi) \) is nonempty. The same applies for validity and logical implication, which are linearly reducible to satisfiability. It is easy to see that \( A(\varphi) \) can be built on-the-fly, and hence we can check non-emptiness in \texttt{PSPACE} in the size of \( \varphi \). Considering that it is known that satisfiability in \texttt{LDL}_f is \texttt{PSPACE}-hard, we can conclude that the proposed construction is optimal with respect to the computational complexity for satisfiability (see [8] for details).

### 4 Runtime Monitoring

From a high-level perspective, the monitoring problem amounts to observe an evolving system execution and report the violation or satisfaction of properties of interest at the earliest possible time. As the system progresses, its execution trace increases, and, at each step, the monitor checks whether the trace seen so far conforms to the properties, by considering that the execution can still continue.
This evolving aspect has a significant impact on the monitoring output: at each step, indeed, the outcome may have a degree of uncertainty due to the fact that future executions are yet unknown.

Several variants of monitoring semantics have been proposed (see [2] for a survey). In this work, we adopt the semantics in [14], which is essentially the finite-trace variant of the RV semantics in [2]. Interestingly, in our finite-trace setting the RV semantics can be elegantly defined, since both trace prefixes and their continuations are finite.

Given an \( \text{LTL}_f/\text{LDL}_f \) formula \( \varphi \), and a current trace \( \pi \), the monitor returns one among the following four RV states:

- **temptrue**, meaning that \( \pi \) temporarily satisfies \( \varphi \), i.e., it satisfies \( \varphi \), but there is at least one possible continuation of \( \pi \) that violates \( \varphi \);
- **tempfalse**, meaning that \( \pi \) temporarily violates \( \varphi \), i.e., \( \varphi \) is not satisfied by \( \varphi \), but there is at least one possible continuation of \( \pi \) that does so;
- **permtrue**, meaning that \( \pi \) permanently satisfies \( \varphi \), i.e., \( \varphi \) is satisfied by \( \pi \) and it will always be, no matter how \( \pi \) is extended;
- **permfalse**, meaning that \( \pi \) permanently violates \( \varphi \), i.e., \( \varphi \) is not satisfied by \( \pi \) and it will never be, no matter how \( \pi \) is extended.

Formally, let \( \varphi \) be an \( \text{LDL}_f/\text{LTL}_f \) formula, and let \( \pi \) be a trace. Then, we define whether \( \varphi \) is in RV state \( s \in \{\text{temptrue}, \text{tempfalse}, \text{true}, \text{false}\} \) (written \( \lbrack \varphi = \text{temptrue} \rbrack \)) on trace \( \pi \) as follows:

- \( \pi \mid = \lbrack \varphi = \text{temptrue} \rbrack \) if \( \pi \mid = \varphi \) and there exists a trace \( \pi' \) such that \( \pi \pi' \neq \varphi \), where \( \pi \pi' \) denotes the trace obtained by concatenating \( \pi \) with \( \pi' \);
- \( \pi \mid = \lbrack \varphi = \text{tempfalse} \rbrack \) if \( \pi \mid \neq \varphi \) and there exists a trace \( \pi' \) such that \( \pi \pi' \mid = \varphi \);
- \( \pi \mid = \lbrack \varphi = \text{permtrue} \rbrack \) if \( \pi \mid = \varphi \) and for every trace \( \pi' \), we have \( \pi \pi' \mid = \varphi \);
- \( \pi \mid = \lbrack \varphi = \text{permfalse} \rbrack \) if \( \pi \mid \neq \varphi \) and for every trace \( \pi' \), we have \( \pi \pi' \mid \neq \varphi \).

By inspecting the definition of RV states, it is straightforward to see that a formula \( \varphi \) is in one and only one RV state on a trace \( \pi \).

The RV states **temptrue** and **tempfalse** are not definitive: they may change into any other RV state as the system progresses. This reflects the general unpredictability of how a system execution unfolds. Conversely, the RV states **permtrue** and **permfalse** are stable since, once outputted, they will not change anymore. Observe that a stable RV state can be reached in two different situations: (i) when the system execution terminates; (ii) when the formula that is being monitored can be fully evaluated by observing a partial trace only. The first case is indeed trivial, as when the execution ends, there are no possible future evolutions and hence it is enough to evaluate the finite (and now complete) trace seen so far according to the \( \text{LDL}_f \) semantics. In the second case, instead, it is irrelevant whether the systems continues its execution or not, since some \( \text{LDL}_f \) properties, such as eventualities or safety properties, can be fully evaluated as soon as something happens, e.g., when the eventuality is verified or the safety requirement is violated. Notice also that, when a stable state is returned by the monitor, the monitoring analysis can be stopped.

From a more theoretical viewpoint, given an \( \text{LDL}_f \) property \( \varphi \), the monitor looks at the trace seen so far, assesses if it is a prefix of a full trace not yet
completed, and categorizes it according to its potential for satisfying or violating \( \varphi \) in the future. We call a prefix \emph{possibly good} for an \LTLf\ formula \( \varphi \), if there exists an extension of it that satisfies \( \varphi \). More precisely, given an \LTLf\ formula \( \varphi \), we define the set of \emph{possibly good prefixes for} \( L(\varphi) \) as the set:

\[
L_{\text{poss,good}}(\varphi) = \{ \pi \mid \text{there exists } \pi' \text{ such that } \pi \pi' \in L(\varphi) \}. \tag{1}
\]

Prefixes for which every possible extension satisfies \( \varphi \) are instead called \emph{necessarily good}. More precisely, given an \LTLf\ formula \( \varphi \), we define the set of \emph{necessarily good prefixes for} \( L(\varphi) \) as the set:

\[
L_{\text{nec,good}}(\varphi) = \{ \pi \mid \text{for every } \pi' \text{ such that } \pi \pi' \in L(\varphi) \}. \tag{2}
\]

The set of \emph{necessarily bad prefixes} \( L_{\text{nec,bad}}(\varphi) \) can be defined analogously as:

\[
L_{\text{nec,bad}}(\varphi) = \{ \pi \mid \text{for every } \pi' \text{ such that } \pi \pi' \not\in L(\varphi) \}. \tag{3}
\]

Observe that the necessarily bad prefixes for \( \varphi \) are the necessarily good prefixes for \( \neg \varphi \), i.e., \( L_{\text{nec,bad}}(\varphi) = L_{\text{nec,good}}(\neg \varphi) \).

Such language-theoretic notions allow us to capture all the RV states defined before. More precisely, it is immediate to show the following.

\textbf{Proposition 1.} Let \( \varphi \) be an \LTLf\ formula and \( \pi \) a trace. Then:

\begin{itemize}
  \item \( \pi \models [\varphi = \text{temp\_true}] \iff \pi \in L(\varphi) \setminus L_{\text{nec,good}}(\varphi) \);
  \item \( \pi \models [\varphi = \text{temp\_false}] \iff \pi \in L(\neg \varphi) \setminus L_{\text{nec,bad}}(\varphi) \);
  \item \( \pi \models [\varphi = \text{perm\_true}] \iff \pi \in L_{\text{nec,good}}(\varphi) \);
  \item \( \pi \models [\varphi = \text{perm\_false}] \iff \pi \in L_{\text{nec,bad}}(\varphi) \).
\end{itemize}

We close this section by exploiting the above language-theoretic notions to better understand the relationships that hold over the various kinds of prefixes. We start by observing that the set of all finite words over the alphabet \( 2^\mathbb{N} \) is the union of the language of \( \varphi \) and its complement \( L(\varphi) \cup L(\neg \varphi) = (2^\mathbb{N})^* \). Also, any language and its complement are disjoint \( L(\varphi) \cap L(\neg \varphi) = \emptyset \). Since from the definition of possibly good prefixes we have \( L(\varphi) \subseteq L_{\text{poss,good}}(\varphi) \) and \( L(\neg \varphi) \subseteq L_{\text{poss,good}}(\neg \varphi) \), we also have that \( L_{\text{poss,good}}(\varphi) \cup L_{\text{poss,good}}(\neg \varphi) = (2^\mathbb{N})^* \). Also, from this definition, it is easy to see that \( L_{\text{poss,good}}(\varphi) \cap L_{\text{poss,good}}(\neg \varphi) \) corresponds to:

\[
\{ \pi \mid \text{there exists } \pi' \text{ such that } \pi \pi' \in L(\varphi) \text{ and there exists } \pi'' \text{ such that } \pi \pi'' \in L(\neg \varphi) \}
\]

meaning that the set of possibly good prefixes for \( \varphi \) and the set of possibly good prefixes for \( \neg \varphi \) do intersect, and in such intersection there are paths that can be extended to satisfy \( \varphi \), but can also be extended to satisfy \( \neg \varphi \). It is also easy to see that \( L(\varphi) = L_{\text{poss,good}}(\varphi) \setminus L(\neg \varphi) \).

Turning to necessarily good prefixes and necessarily bad prefixes, it is easy to see that \( L_{\text{nec,good}}(\varphi) = L_{\text{poss,good}}(\varphi) \setminus L_{\text{poss,good}}(\neg \varphi) \), that \( L_{\text{nec,bad}}(\varphi) = L_{\text{poss,good}}(\neg \varphi) \setminus L_{\text{poss,good}}(\varphi) \), and also that \( L_{\text{nec,good}}(\varphi) \subseteq L(\varphi) \) and \( L_{\text{nec,bad}}(\varphi) \subseteq L(\neg \varphi) \).
Interestingly, necessarily good, necessarily bad, and possibly good prefixes partition all finite traces. In fact, by directly applying the definitions of necessarily good, necessarily bad, possibly good prefixes of $L(\varphi)$ and $L(\neg \varphi)$, we obtain the following.

**Proposition 2.** The set of all traces $(2^P)^*$ can be partitioned into

\[
L_{\text{nec-good}}(\varphi) \quad L_{\text{poss-good}}(\varphi) \cap L_{\text{poss-good}}(\neg \varphi) \quad L_{\text{nec-bad}}(\varphi)
\]

such that

\[
L_{\text{nec-good}}(\varphi) \cup (L_{\text{poss-good}}(\varphi) \cap L_{\text{poss-good}}(\neg \varphi)) \cup L_{\text{nec-bad}}(\varphi) = (2^P)^* \\
L_{\text{nec-good}}(\varphi) \cap (L_{\text{poss-good}}(\varphi) \cap L_{\text{poss-good}}(\neg \varphi)) \cap L_{\text{nec-bad}}(\varphi) = \emptyset.
\]

### 4.1 Monitoring $\text{LDL}_f$ Formulae

As pointed out in the previous section, the core issue in monitoring is prefix recognition. $\text{LTL}_f$ is not expressive enough to talk about prefixes of its own formulae. Roughly speaking, given an $\text{LTL}_f$ formula, the language of its possibly good prefixes cannot be in general described as an $\text{LTL}_f$ formula. For such a reason, building a monitor usually requires direct manipulation of the automaton for the formula.

$\text{LDL}_f$, instead, can capture any nondeterministic automaton as a formula, and it has the capability of expressing properties on prefixes. We can exploit such an extra expressivity to capture the monitoring condition in a direct and elegant way. We start by showing how to construct formulae representing (the language of) prefixes of other formulae, and then we show how to use them in the context of monitoring.

Technically, given an $\text{LDL}_f$ formula $\varphi$, it is possible to express the language $L_{\text{poss-good}}(\varphi)$ with an $\text{LDL}_f$ formula $\varphi'$. Such a formula is obtained in two steps.

**Lemma 1.** Given an $\text{LDL}_f$ formula $\varphi$, there exists a regular expression $\text{pref}_\varphi$ such that $L(\text{pref}_\varphi) = L_{\text{poss-good}}(\varphi)$.

**Proof.** The proof is constructive. We build the NFA $A(\varphi)$ for $\varphi$. We then build a new NFA $A_{\text{poss-good}}(\varphi)$ by taking $A(\varphi)$ and setting as final states all states from which we can reach a final state of $A(\varphi)$. The so-obtained NFA $A_{\text{poss-good}}(\varphi)$ is such that $L(A_{\text{poss-good}}(\varphi)) = L_{\text{poss-good}}(\varphi)$. Since NFA are exactly as expressive as regular expressions, we can translate $A_{\text{poss-good}}(\varphi)$ to the corresponding regular expression $\text{pref}_\varphi$.

Since $\text{LDL}_f$ is as expressive as regular expressions (cf. [3]), we can translate $\text{pref}_\varphi$ into an equivalent $\text{LDL}_f$ formula.

**Theorem 2.** Given an $\text{LDL}_f$ formula $\varphi$,

\[
\pi \in L_{\text{poss-good}}(\varphi) \text{ iff } \pi \models (\text{pref}_\varphi)\text{end} \\
\pi \in L_{\text{nec-good}}(\varphi) \text{ iff } \pi \models (\text{pref}_\varphi)\text{end} \land \neg(\text{pref}_\neg \varphi)\text{end}
\]
Proof. Any regular expression $\rho$, and hence any regular language, can be captured in LDL$_f$ as $\langle \rho \rangle_{end}$. Specifically, the language $L_{\text{pos_good}}(\varphi)$ is captured by $\langle \text{perm}_\varphi \rangle_{end}$, and the language $L_{\text{acc_good}}(\varphi)$, which is equivalent to $L_{\text{pos_good}}(\varphi) \setminus L_{\text{pos_good}}(\neg \varphi)$, is captured by $\langle \text{pref}_\varphi \rangle_{end} \land \neg \langle \text{pref}_{\neg \varphi} \rangle_{end}$.

In other words, given an LDL$_f$ formula $\varphi$, formula $\varphi' = \langle \text{pref}_\varphi \rangle_{end}$ is an LDL$_f$ formula such that $L(\varphi') = L_{\text{pos_good}}(\varphi)$. Similarly for $L_{\text{acc_good}}(\varphi)$.

Exploiting this result, and the results in Proposition 1, we reduce the evaluation of RV states to the standard evaluation of LDL$_f$ formulae over a (partial) trace. Formally:

**Theorem 3.** Let $\pi$ be a trace. The following equivalences hold:
- $\pi \models [\varphi = \text{temp_true}]$ iff $\pi \models \varphi \land \langle \text{perm}_{\neg \varphi} \rangle_{end}$;
- $\pi \models [\varphi = \text{temp_false}]$ iff $\pi \models \neg \varphi \land \langle \text{pref}_\varphi \rangle_{end}$;
- $\pi \models [\varphi = \text{perm_true}]$ iff $\pi \models \langle \text{pref}_\varphi \rangle_{end} \land \neg \langle \text{pref}_{\neg \varphi} \rangle_{end}$;
- $\pi \models [\varphi = \text{perm_false}]$ iff $\pi \models \langle \text{perm}_{\neg \varphi} \rangle_{end} \land \neg \langle \text{pref}_\varphi \rangle_{end}$.

**Proof.** The theorem follows directly from Proposition 1 and Theorem 2.

This result provides an actual procedure to return the RV state of an LDL$_f$ formula $\varphi$: we build four automata, one for each of the four formulae above, and then follow the evolution of the trace $\pi$ simultaneously on each one of them. Since Proposition 2 proves that the languages of the four automata are a partition for the set of all languages over $(2^P)^*$, we are guaranteed that, at each step, one and only one automaton is in a final state, namely, one and only one truth value is returned as output of the monitoring procedure.

**Example 1.** Figure 3a shows the graphical representation of the automaton for formula $\Phi := O(a \rightarrow (\bullet b))$, where $s_0$ is the initial state and final states are double-circled. Moreover, for the sake of readability, labels on edges are logical formulae, a shortcut for every interpretation satisfying that formula, e.g., the edge labeled with $\neg a$ from state $s_1$ to $s_2$ is a shortcut for $(s_1, \neg a, b, s_2), (s_1, \neg a, \neg b, s_2) \in \rho$. Formula $\Phi$ intuitively requires that, in the next step, if $a$ is performed then either the trace ends (for the semantics of the weak next operator $\bullet$) or, if it continues, then it is forced to continue by performing $b$. Figures 3b – 3e represent the four automata for monitoring the different RV truth values. More specifically:
- The automaton in Figure 3b is used to check whether $\pi \models [\varphi = \text{temp_true}]$.
  Indeed, its final state is $s_2$, which corresponds to the subset of the final states in the original automaton from which some non-final state (in this case $s_3$) can still be reached.
- The automaton in Figure 3c is used to check whether $\pi \models [\varphi = \text{temp_false}]$.
  Indeed, its final states are $s_0$ and $s_1$, which correspond to the subset of the non-final states in the original automaton from which some final state (in this case $s_3$) can still be reached.
- The automaton in Figure 3d is used to check whether $\pi \models [\varphi = \text{perm_true}]$.
  Indeed, its final state is $s_3$, which corresponds to the subset of the final states in the original automaton from which no non-final state can ever be reached.
The automaton in Figure 3e is used to check whether \( \pi \models \lbrack \varphi = \text{perm\_false} \rbrack \). Indeed, its final state is \( s_4 \), which corresponds to the subset of the final states in the original automaton from which no final state can ever be reached.

In the next section, we prove that using four automata is indeed redundant, and that monitoring can be performed by making use of just a single automaton that retains, at once, all the necessary monitoring information.
4.2 Monitoring using Colored Automata

We now show that we can merge the four automata for monitoring the four RV truth values into a single automaton with "colored" states. The idea is grounded on the intuition that the four automata in the previous section "have the same shape", and only differ in determining which states are final. It is hence possible to build one automaton only and then mark its states with four different colors, each corresponding to the final states of a specific formula in Theorem 3, hence each representing one among the four RV truth values. The intention of using a single automaton for runtime verification is not novel [14], but here, for the first time, we provide a formal justification of its correctness.

As a first step, we formally define the notion of shape equivalence to capture the intuition that two automata have the same "shape", i.e., they have corresponding states and transitions, but possibly differ in their final states.

Formally, let $A_1 = (2^P, S^1, s^1_0, g^1, S^1_f)$ and $A_2 = (2^P, S^2, s^2_0, g^2, S^2_f)$ be two NFAs defined over a set $P$ of propositional symbols. We say that $A_1$ and $A_2$ are shape equivalent, written $A_1 \sim A_2$, if there exists a bijection $h : S^1 \rightarrow S^2$ such that:

1. $h(s^1_0) = s^2_0$;
2. for each $(s^1_1, \Pi, s^2_1) \in g^1$, $(h(s^1_1), \Pi, h(s^2_1)) \in g^2$; and
3. for each $(s^2_1, \Pi, s^2_2) \in g^2$, $(h^{-1}(s^2_1), \Pi, h^{-1}(s^2_2)) \in g^1$.

We write $A_1 \sim h \sim A_2$ to explicitly indicate the bijection $h$ from $A_1$ to $A_2$ that induces their shape equivalence.

It is easy to see that bijection $h$ preserves the initial states (condition (1)) and transitions (conditions (2) and (3)), but does not require a correspondence between final states.

**Lemma 2.** Shape equivalence $\sim$ is indeed an equivalence relation.

**Proof.** Reflexivity: the identity function trivially satisfies (1)-(3) above. Symmetry: let $A \sim h \sim A$. Given that $h$ is a bijection, then $A \sim h^{-1} \sim A$. Transitivity: Let $A_1 \sim h \sim A_2$ and $A_2 \sim g \sim A_3$. Then $A_1 \sim h \circ g \sim A_3$, where $h \circ g$ is the composition of $h$ and $g$.

Hence, $\sim$ induce (equivalence) classes of automata with the same shape. Automata for the basic formulae in Theorem 3 belong to the same class.

**Lemma 3.** For each LDL$_f$ formula $\varphi$, $A(\varphi)$, $A(\neg \varphi)$, $A((\text{pref}_{\varphi}) \text{end})$ and $A((\text{pref}_{\neg \varphi}) \text{end})$ are in the same equivalence class by $\sim$.

**Proof.** From automata theory, $A(\neg \varphi)$ can be obtained from $A(\varphi)$ by switching the final states with the non-final ones. Hence, the identity $i : S^\varphi \rightarrow S^{\neg \varphi}$ is such that $A(\varphi) \sim i \sim A(\neg \varphi)$. Moreover, $A(\varphi) \sim A((\text{pref}_{\varphi}) \text{end}) \sim A((\text{pref}_{\neg \varphi}) \text{end})$ as $A((\text{pref}_{\varphi}) \text{end})$, respectively, $A((\text{pref}_{\neg \varphi}) \text{end})$, can be obtained from $A(\varphi)$, respectively, $A(\neg \varphi)$, by setting as final states all states from which there exists a non-zero length path to a final state of $A(\varphi)$, respectively, $A(\neg \varphi)$, as explained in the proof of Lemma 1. Hence, again, the identity relation $i$ is such that $A(\varphi) \sim i \sim A((\text{pref}_{\varphi}) \text{end}) \sim A((\text{pref}_{\neg \varphi}) \text{end})$. 
As the last step for proving that automata for the four formulae in Theorem 3 are in the same class, we show that the formula conjunction does not alter shape equivalence, in the following precise sense.

**Theorem 4.** Let \( \varphi_1, \varphi_2, \psi_1 \) and \( \psi_2 \) be LDL\(_f\) formulae so that \( A(\varphi_1) \sim A(\psi_1) \) and \( A(\varphi_2) \sim A(\psi_2) \). Then \( A(\varphi_1 \land \varphi_2) \sim A(\psi_1 \land \psi_2) \).

**Proof.** From the semantics of LDL\(_f\) and Theorem 1 it follows that \( A(\varphi_1 \land \varphi_2) \equiv A(\varphi_1) \cap A(\varphi_2) \). Recall that states of \( A(\varphi_1) \cap A(\varphi_2) \) are ordered pairs \( (s^{\varphi_1}, s^{\varphi_2}) \) in \( S^{\varphi_1} \times S^{\varphi_2} \). Let \( h_1 \) and \( h_2 \) be bijections such that \( A(\varphi_1) \xrightarrow{h_1} A(\psi_1) \) and \( A(\varphi_2) \xrightarrow{h_2} A(\psi_2) \). We use \( h_1 \) and \( h_2 \) to construct a new bijection \( h : S^{\varphi_1} \times S^{\varphi_2} \to S^{\psi_1} \times S^{\psi_2} \) such that \( h(s^{\varphi_1}, s^{\varphi_2}) = (h_1(s^{\varphi_1}), h_2(s^{\varphi_2})) \). We show that \( h \) satisfies criteria (1)-(3) of shape equivalence, hence inducing \( A(\varphi_1 \land \varphi_2) \xrightarrow{h} A(\psi_1 \land \psi_2) \). The starting state \( s_0^{\varphi_1 \land \varphi_2} \) of \( A(\varphi_1 \land \varphi_2) \) corresponds to \( (s_0^{\varphi_1}, s_0^{\varphi_2}) \) by definition of \( A(\varphi_1) \cap A(\varphi_2) \). At the same time, \( s_0^{\psi_1 \land \psi_2} = (h_1(s_0^{\varphi_1}), h_2(s_0^{\varphi_2})) = (s_0^{\psi_1}, s_0^{\psi_2}) \) by definition of \( h \), which proves (1). Now, consider a transition \( ((s_1^{\varphi_1}, s_1^{\varphi_2}), \Pi, (s_2^{\varphi_1}, s_2^{\varphi_2})) \) in \( g^{\varphi_1 \land \varphi_2} \). By construction, this means that there exist transitions \( (s_1^{\varphi_1}, \Pi, s_1^{\varphi_2}) \in g^{\varphi_1} \) and \( (s_2^{\varphi_1}, \Pi, s_2^{\varphi_2}) \in g^{\varphi_2} \). Since \( A(\varphi_1) \xrightarrow{h_1} A(\psi_1) \) and \( A(\varphi_2) \xrightarrow{h_2} A(\psi_2) \), we have that \( (h_1(s_1^{\varphi_1}), \Pi, h_1(s_1^{\varphi_2})) \in g^{\psi_1} \) and \( (h_2(s_2^{\varphi_2}), \Pi, h_2(s_2^{\varphi_2})) \in g^{\psi_2} \). It follows that \( ((h_1(s_1^{\varphi_1}), h_2(s_1^{\varphi_2})), \Pi, (h_1(s_2^{\varphi_1}), h_2(s_2^{\varphi_2}))) \) in \( g^{\psi_1 \land \psi_2} \), which proves (2). Condition (3) is proved analogously with \( h^{-1} \).

**Corollary 1.** Given an LDL\(_f\) formula \( \varphi \), automata \( A(\varphi \land \langle \text{pref}_\varphi \rangle \text{end}) \), \( A(\neg \varphi \land \langle \text{pref}_\varphi \rangle \text{end}) \), \( A(\langle \text{pref}_\varphi \rangle \text{end} \land \neg \langle \text{pref}_\varphi \rangle \text{end}) \) and \( A(\langle \text{pref}_\varphi \rangle \text{end} \land \neg \langle \text{pref}_\varphi \rangle \text{end}) \) are in the same equivalence class by \( \sim \).

This result tells that the automata of the formulae used to capture the RV states of an LDL\(_f\) formula of interest, as captured by Theorem 3, are identical modulo final states. In addition, by definition of the four LDL\(_f\) formulae, we directly get that each state is marked as final by one and only one of such
This, in turn, allows us to merge all the four automata together into a single automaton, provided that we recall, for each state in the automaton, which of the four formula marks it as final (which corresponds to declare to which of the four RV states it corresponds). In practice, we can simply build the automaton $A(\varphi)$ for $\varphi$, and “color” each state in the automaton according to its corresponding RV state. This can be realized with the following, direct procedure. We first build $A(\varphi) = (2^P, S, s_0, \varphi, S_f)$, with $S_f$ the set of its final states, and, for each $s \in S$, we compute the set $\text{Reach}(s)$ of states reachable from $s$. Then:

- if (i) $s \in S_f$, (ii) $\text{Reach}(s) \not\subseteq S_f$, and (iii) $\text{Reach}(s) \cap \{s_f\} \neq \emptyset$, then we mark $s$ as temp\textunderscore true;
- if (i) $s \not\in S_f$, (ii) $\text{Reach}(s) \not\subseteq (S \setminus S_f)$, and (iii) $\text{Reach}(s) \cap S_f \neq \emptyset$, then we mark $s$ as temp\textunderscore false;
- if (i) $s \in S_f$ and (ii) $\text{Reach}(s) \subseteq S_f$, then we mark $s$ as perm\textunderscore true;
- if (i) $s \not\in S_f$ and (ii) $\text{Reach}(s) \subseteq (S \setminus S_f)$, then we mark $s$ as perm\textunderscore false.

It is easy to see that the four bullets above match the four ones of Theorem 3. The soundness of the marking immediately follows from the definitions and results in the previous section.

Example 2. Figure 4 depicts the colored automaton for the formula in Example 1 and Figure 3a. States $s_0$ and $s_1$, which were final in the automaton for $\pi \models \llbracket \varphi = \text{temp\textunderscore false} \rrbracket$, are indeed marked as temp\textunderscore true (orange, dashed line); state $s_2$, which was final in the automaton for $\pi \models \llbracket \varphi = \text{temp\textunderscore true} \rrbracket$, is marked as temp\textunderscore true (blue, solid thin line); state $s_3$, which was final in the automaton for $\pi \models \llbracket \varphi = \text{perm\textunderscore true} \rrbracket$, is marked with perm\textunderscore true (green, solid thick line); and, lastly, state $s_4$, which was final in the automaton for $\pi \models \llbracket \varphi = \text{perm\textunderscore false} \rrbracket$, is marked with perm\textunderscore false (red, dotted line).

Upon building the colored automaton, we can determinize it and keep it untrimmed, that is, including trap states from which no final state can be reached. In this way, every symbol from the entire supporting set $P$ is accepted by the automaton in each of its states. This becomes then our monitor.

We conclude by noticing that the presented solution is very flexible, as the reachability analysis can be performed on-the-fly: indeed, this is the procedure we actually implemented in our runtime verification tool, as explained in Section 7.

5 Monitoring Declare Constraints

We now ground our monitoring approach to the case of declare monitoring. declare\footnote{http://www.win.tue.nl/declare/} is a language and framework for the declarative, constraint-based modeling of processes and services. A thorough treatment of constraint-based processes can be found in \cite{2219}. As a modeling language, DECLARE takes a complementary approach to that of classical, imperative process modeling. In
imperative process modeling, all allowed control flows among tasks must be explicitly represented, and execution traces not falling within this set are implicitly considered as forbidden. Instead of this procedural and “closed” approach, DECLARE has a declarative, “open” flavor: the agents responsible for the process execution can freely choose in which order to perform the involved tasks, provided that the resulting execution trace satisfies the business constraints of interest. This is the reason why, alongside traditional control-flow constraints such as sequence (called in DECLARE chain succession), DECLARE supports a variety of more refined constraints that impose loose temporal orderings, and/or that explicitly account for negative information, i.e., the explicit prohibition of task execution.

Given a set $\mathcal{P}$ of tasks, a DECLARE model $\mathcal{M}$ is a set $\mathcal{C}$ of LTL$_f$ (and hence LDL$_f$) constraints over $\mathcal{P}$. A finite trace over $\mathcal{P}$ complies with $\mathcal{M}$, if it satisfies all constraints in $\mathcal{C}$. Among all possible LTL$_f$ constraints, some specific patterns have been singled out as particularly meaningful for expressing DECLARE processes, taking inspiration from [9]. Such patterns are grouped into four families:

- **existence** (unary) constraints, stating that the target task must/cannot be executed (for an indicated amount of times);
- **choice** (binary) constraints, accounting for alternative tasks;
- **relation** (binary) constraints, connecting a source task to a target task and expressing that, whenever the source task is executed, then the target task must also be executed (possibly with additional temporal conditions);
- **negation** (binary) constraints, capturing that whenever the source task is executed, then the target task is prohibited (possibly with additional temporal conditions).

Table 1, at the end of this document, summarizes some of these patterns. See [21] for the full list of patterns.

**Example 3.** Consider a fragment of a ticket booking process, whose DECLARE representation is shown in Figure 5. The process fragment consists of four tasks and four constraints, but in spite of its simplicity clearly illustrates the main features of declarative, constraint-based process modeling.

Specifically, each process instance is focused on the management of a specific registration to a booking event made by an interested customer. For simplicity, we assume that the type of registration is selected upon instantiating the process, and is therefore not explicitly captured as a set of tasks within the process itself. The process fragment then consists of four tasks:

- **accept regulation** is the task used to accept the regulation of the booking company for the specific type of registration the customer is interested in;
- **pay registration** is the task used to pay for the registration;
- **get ticket** is the task used to physically withdraw the ticket containing the registration details;
- **cancel registration** is the task used to abort the instance of the registration process.

The execution of the aforementioned tasks is subject to the following behavioral constraints. First of all, within an instance of the booking process, a
customer may pay for the registration at most once. This is captured in DECLARE by constraining the pay registration task with an absence 2 constraint.

After executing the payment, the customer must eventually get the corresponding ticket. On the other hand, the ticket can be obtained only after having performed the payment. This is captured in DECLARE by constraining pay registration and get ticket with a response constraint going from the first task to the second, and with a precedence constraint going from the second task to the first. This specific combination is called succession; it is graphically depicted by combining the graphical notation of the two constraints, and logically corresponds to their conjunction.

When a payment is executed, the customer must accept the regulation of the registration. There is no particular temporal order required for accepting the registration: upon the payment, if the regulation has been already accepted, then no further steps are required; otherwise, the customer is expected to accept the regulation afterwards. This is captured in DECLARE by connecting pay registration to accept regulation by means of a responded existence constraint.

Finally, a customer may always decide to cancel the registration, with the only constraint that the cancelation is incompatible with the possibility of getting the registration ticket. This means that, when the ticket is withdrawn, no cancelation is accepted anymore, and, on the other hand, once the registration is canceled, no ticket will be issued anymore. This is captured in DECLARE by relating get ticket to cancel registration through a not coexistence constraint.

Several, logic-based techniques have been proposed to support end-users in defining, checking, and enacting DECLARE models [24][22][15][19][21]. More recently, the \( \text{LTL}_f \) characterization of DECLARE, together with its operational automata-theoretic counterpart, have been exploited to provide advanced monitoring and runtime verification facilities [14][10]. In particular, monitoring DECLARE models amounts to:

- Track the evolution of a single DECLARE constraint against an evolving trace, providing a fine-grained feedback on how the truth value of the constraint evolves, when tasks are performed. This is done by adopting the RV semantics for \( \text{LTL}_f \). Specifically, in [14], the evolution of DECLARE constraints through the different RV states is tackled using the ad-hoc “colored automaton” construction technique that we have formally justified in Section 4.2.
• Track the compliance of an evolving trace to the entire DECLARE model, by considering all its constraints together. This is done by constructing the colored automaton for the conjunction of all constraints in the model. Monitoring the evolving trace against such a “global” automaton is crucial for inferring complex violations that cannot be ascribed to the interaction of the current trace with a single constraint in the model, but arise due to the interplay between the trace and multiple constraints at once. Such violations emerge due to conflicting constraints, i.e., constraints that, in the current circumstances, contradict each other and consequently cannot be all satisfied anymore [16]. By considering all constraints together, the presence of this kind of conflict can be detected immediately, without waiting for the later moment when an explicit violation of one of the single constraints involved in the conflict eventually arises. This important feature has been classified as early detection of violations in a reference monitoring survey [12].

Monitoring DECLARE Constraints with LDF. Since LDF includes LTLf, DECLARE constraints can be directly encoded in LDF using their standard formalization [23,21]. Thanks to the translation into NFAs discussed in Section 3 (and, if needed, their determinization into corresponding DFAs), the automaton obtained from the LTLf encoding of a constraint can then be used to check whether a (partial) finite trace satisfies that constraint or not. This is not very effective, as the approach does not support the detection of fine-grained truth values, as the four RV ones.

By exploiting Theorem 3, however, we can reuse the same technique, this time supporting all RV truth values. In fact, by formalizing the good prefixes of each DECLARE pattern, we can immediately construct the four LDF formulae that embed the different RV truth values, and check the current trace over each of the corresponding automata. Table 1 reports the good prefix characterization of some of the DECLARE patterns; it can be seamlessly extended to all other patterns as well.

Pragmatically, we can even go a step further, and employ colored automata by following the technique discussed in Section 4.2. More specifically, given a DECLARE model $M$, we proceed as follows:

• For every constraint $c \in M$, we derive its LTLf formula $\varphi_c$, and construct its corresponding deterministic colored automaton $A(c)$. This colored automaton, acts as local monitor for its constraint $c$. This can be used to track the RV state of $c$ as tasks are executed.

• We build the LTLf formula $\Phi_M$ standing for the conjunction of the LTLf formulae encoding all constraints in $M$, and construct its corresponding deterministic colored automaton $A(M)$. This colored automaton acts as global monitor for the entire DECLARE model $M$. This can be used to track the overall RV state of $M$ as tasks are executed, and early detect violations arising from conflicting constraints.

• When the monitoring of a process execution starts, the initial state of each local monitor, as well as that of the global monitor, are outputted.
Whenever an event witnessing the execution of a task is tracked, it is delivered to each local monitor and to the global monitor. The new, current state of each monitor is then computed and outputted based on the current state and on the received task name.

When the process execution is completed (i.e., not further events are expected to occur), the final state of each monitor is outputted, depending on whether its colored automaton is in an accepting state or not. In particular, if upon completion the colored state of the monitor is perm_true or temp_true, then the trace is judged as compliant; if, instead, upon completion the colored state of the monitor is perm_false or temp_false, then the trace is judged as non-compliant.

The global monitor can be inquired to obtain additional information about how the monitored trace interacts with the constraints. For example, when the current state of the global monitor is temp_false or temp_true, retrieving the names of tasks whose execution leads to a perm_false state is useful to return which tasks are currently forbidden by the model. This information is irrelevant when the monitor is in a perm_true or perm_false state: by definition, in the first case no task is forbidden, whereas in the latter all tasks are.

Example 4. Figure 6 depicts the result computed by monitoring the DECLARE model introduced in Example 3 and shown in Figure 5 against a trace where a registration is paid, the corresponding regulation is accepted, and then the registration is canceled.
When monitoring starts, all local monitors are in state temp_true, and so is the global monitor. Task get ticket is forbidden, since according to the precedence constraint connecting that task to pay registration (i.e., formula \(\neg (\text{get}U\text{pay}) \lor \neg \Diamond \text{pay} \)) a previous execution of pay registration is needed. When the payment is executed:

- the local monitor for the responded existence constraint linking pay registration to accept regulation (i.e., formula \(\Diamond \text{pay} \rightarrow \Diamond \text{acc} \)) moves to temp_false, because it requires acceptance of the regulation (which has not been done yet);
- the local monitor for the precedence constraint linking get ticket to pay registration (i.e., formula \((\neg \text{get}U\text{pay}) \lor \neg \Diamond \text{pay} \)) moves to perm_true, enabling once and for all the possibility of executing get ticket;
- the local monitor for the response constraint linking pay registration to get ticket (i.e., formula \(\Box (\text{pay} \rightarrow \Diamond \Diamond \text{get}) \)) moves to temp_false, because its satisfaction now demands a consequent execution of the get ticket task.

The global monitor also moves to temp_false, since there are two tasks that must be executed to satisfy the responded existence and response constraints, and it is indeed possible to execute them without violating other constraints. At the same time, further payments are now forbidden, due to the absence 2 constraint attached to the pay registration task (i.e., formula \(\neg \Diamond (\text{pay} \land \Box \Diamond \text{pay}) \)).

The consequent execution of accept regulation turns the state of the responded existence constraint linking pay registration to accept regulation (i.e., formula \(\Diamond \text{pay} \rightarrow \Diamond \text{acc} \)) to perm_true: since the regulation has now been accepted, the constraint is satisfied and will stay so no matter how the execution is continued.

The most interesting transition is the one triggered by the consequent execution of the cancel registration task. While this event does not trigger any state change in the local monitors, it actually induces a transition of the global monitor to the permanent, perm_false RV state. In fact, no continuation of the trace will be able to satisfy all constraints of the considered model. More specifically, the sequence of events received so far induces a so-called conflict [16] for the response constraint linking pay registration to get ticket (i.e., formula \(\Box (\text{pay} \rightarrow \Diamond \Diamond \text{get}) \)), and the not coexistence constraint relating get ticket and cancel registration (i.e., formula \(\neg (\Diamond \text{get} \land \Diamond \text{cancel}) \)). In fact, the response constraint requires a future execution of the get ticket task, which is however forbidden by the not coexistence constraint. Consequently, no continuation of the current trace will satisfy both constraints at once.

Since no further task execution actually happens, the trace is finally declared to be complete, with no execution of the get ticket task. This has the effect of respectively moving the response and not coexistence constraints to perm_false and perm_true. Also the absence 2 constraint on payment becomes perm_true, witnessing that no double payment occurred in the trace.

6 Modeling and monitoring metaconstraints

In Section [4] we have demonstrated that \(\text{LDL}_f\) has the ability of expressing formulae that capture the RV state of other formulae. This can be interpreted as
the ability of \( \text{LDL}_f \) to express meta-level properties of \( \text{LDL}_f \) constraints within the logic itself. Such properties, which we call \textit{metaconstraints}, can, in turn, be themselves monitored using the automata-theoretic approach described in Section 4.

In this section we elaborate on this observation, discussing how metaconstraints can be built, and illustrating interesting metaconstraint patterns.

6.1 Modeling Metaconstraints

Theorem 3 shows that, for an arbitrary \( \text{LDL}_f \) formula \( \varphi \), four \( \text{LDL}_f \) formulae can be automatically constructed to express whether \( \varphi \) is in one of the four RV states. Consequently, given \( s \in \{ \text{temp}_\text{true}, \text{temp}_\text{false}, \text{true}, \text{false} \} \), and an \( \text{LTL}_f/\text{LDL}_f \) formula \( \varphi \), we can consider formulae of the form \( J \varphi = s K \) as special atoms of the logic itself.

Such special atoms are used to check whether a trace brings \( \varphi \) in state \( s \). However, they cannot be used to explicitly characterize which are the paths that lead \( \varphi \) to RV state \( s \), i.e., that make formula \( \llbracket \varphi = s \rrbracket \) true. Such paths can be readily obtained by constructing the regular expression for language \( L(\llbracket \varphi = s \rrbracket) \), which we denote as \( re_{\llbracket \varphi = s \rrbracket} \). For example, \( re_{\llbracket \varphi = \text{perm}_\text{false} \rrbracket} = L(\langle \text{pref}_\neg \varphi \rangle \text{end} \land \neg \langle \text{pref}_\varphi \rangle \text{end}) \) describes all paths culminating in a permanent violation of \( \varphi \).

With these notions at hand, we can build \( \text{LTL}_f/\text{LDL}_f \) metaconstraints as standard \( \text{LTL}_f/\text{LDL}_f \) formulae that include:

- formulae of the form \( \llbracket \varphi = s \rrbracket \) as atoms;
- formulae of the form \( re_{\llbracket \varphi = s \rrbracket} \) as path expressions.

A metaconstraint is then translated back into a standard \( \text{LDL}_f \) formula by replacing each sub-formula of the form \( \llbracket \varphi = s \rrbracket \) with its corresponding \( \text{LDL}_f \) formula according to Theorem 3 and each sub-formula of the form \( re_{\llbracket \varphi = s \rrbracket} \) with its corresponding regular expression. A direct (non-optimized) way to calculate the regular expression for \( re_{\llbracket \varphi = s \rrbracket} \) is to construct the automaton for \( \llbracket \varphi = s \rrbracket \), and then to fold this automaton back into a regular expression (using standard techniques).

6.2 Some Relevant Metaconstraint Patterns

We present three types of metaconstraints, demonstrating the sophistication and versatility of the resulting framework.

\textit{Contextualizing constraints}. This type of metaconstraint is used to express that a constraint must hold while another constraint is in some RV state. The latter constraint, together with the specified state, consequently provides a monitoring context for the former, contextualized constraint.

Let us specifically consider the case of a contextualized absence, where, given a task \( a \), the contextualized constraint has the form \( \Box \neg a \), and the context is provided by an arbitrary constraint \( \varphi \) being in a given RV state \( s \). This is formalized as:

\[
[re_{\llbracket \varphi = s \rrbracket}](\neg a \lor \text{end})
\] (4)
where end denotes the end of the trace, as defined in Section 2; this is needed since, in $\mathcal{DL}_f$, $\neg a$ expresses that some task different than $a$ is executed, while we also want to accept the case where no task is performed at all (and the trace completes). The idea of formula (4) is to relativize the unrestricted $\square$ operator to all and only those paths leading to RV state $s$ for $\varphi$, which are, in turn, characterized by the regular expression $re[\varphi=s]$.

A monitor for formula (4) returns temp_true either when $\varphi$ is not in state $s$, but may evolve into such a state, or when $\varphi$ is in state $s$. In the latter situation, by inspecting the monitor one can see that task $a$ if forbidden; this also means that upon the execution of $a$, the monitor evolves into perm_false. Finally, the monitor returns perm_true if $\varphi$ is not in state $s$, and cannot enter into state $s$ in the future, no matter how the trace is continued.

**Example 5.** Consider the constraint model in Figure 5. We now want to express that it is not possible to get the ticket after the payment is done, until the regulation is accepted (if it was accepted before, no restriction applies). This can be seen as a contextualized absence constraint forbidding get ticket when the responded existence that links pay registration to accept regulation (i.e., formula $\Diamond pay \rightarrow \Diamond acc$) is temporarily violated. Formally, to encode this, we instantiate formula (4) into:

$$re[\{\Diamond pay \rightarrow \Diamond acc\}=temp\false](\neg get \lor end)$$

which, in turn, expands into:

$$[\neg pay^*; pay; \neg acc^*](\neg get \lor end)$$

**Compensation constraints.** In general terms, compensation refers to a behavior that has to be enforced when the current execution reaches an unexpected/undesired state. In our setting, the undesired state triggering a compensation is the permanent violation of a property that captures a desired behavior, which, in turn, triggers the fact that another formula, capturing the compensating behavior, has to be satisfied. We call the first formula the default constraint, and the second formula its compensating constraint.

Let us consider the general case of a default $\mathcal{DL}_f$ constraint $\varphi$, and a compensating $\mathcal{DL}_f$ constraint $\psi$. By noticing that once a trace permanently violates a constraint, then every possible continuation still permanently violates that constraint, we capture the compensation of $\varphi$ by $\psi$ as:

$$[\varphi = perm\false] \rightarrow \psi$$

(5)

The intuitive interpretation of formula (5) is that either $\varphi$ never enters into the $perm\false$ RV state, or $\psi$ holds. No requirement is placed regarding when $\psi$ should be monitored in case $\varphi$ gets permanently violated. In fact, the overall compensation formula (5) gets temporarily/permanently satisfied even when the compensating constraint $\psi$ is temporarily/permanently satisfied before the moment when the default constraint $\varphi$ gets permanently violated. This may sound
counterintuitive, as it is usually intended that the compensating behavior has to be exhibited as a reaction to the violation. We can capture this intuition by turning formula 5 into the following reactive compensation formula:

$$\| \varphi = \text{perm\_false} \| \rightarrow \langle \text{re}_{\varphi = \text{perm\_false}} \rangle \psi$$

This formula imposes that, in case of a permanent violation of $\varphi$, the compensating constraint $\psi$ must hold after $\varphi$ has become permanently violated.

Assuming that $\varphi$ can be potentially violated (which is the reason why we want to express a compensation), a monitor for formula (6) starts by emitting $\text{temp\_true}$. As soon as the monitored execution is so that $\varphi$ cannot be permanently violated anymore, the monitor switches to $\text{perm\_true}$. If instead the monitored execution leads to permanently violate $\varphi$, from the moment of the violation onwards, the evolution of the monitor follows that of $\psi$.

**Example 6.** Consider the **not** coexistence constraint in Figure 5. We now want to model that, whenever this constraint is permanently violated, that is, whenever a ticket is retrieved and the registration is canceled, then a return ticket (return for short) task must be executed. This has to occur in reaction to the permanent violation. Hence, we rely on template (6) and instantiate it into:

$$\| \neg(\Diamond \text{get} \land \Diamond \text{cancel}) = \text{perm\_false} \| \rightarrow \langle \text{re}_{\neg(\Diamond \text{get} \land \Diamond \text{cancel}) = \text{false}} \rangle \Diamond \text{return}$$

This formula is equivalent to

$$(\Diamond \text{get} \land \Diamond \text{cancel}) \rightarrow \langle \text{re}_{\neg(\Diamond \text{get} \land \Diamond \text{cancel})} \rangle \Diamond \text{return}$$

which, in turn, becomes

$$\langle \Diamond \text{get} \land \Diamond \text{cancel} \rangle \rightarrow \left( (o^*; \text{get}; (\neg\text{cancel})^*; \text{cancel}; \text{true}^*) + (o^*; \text{cancel}; \neg\text{get}^*; \text{get}; \text{true}^*) \right) \Diamond \text{return}$$

where $o$ is a shortcut notation for any task different than get and cancel.

**Constraint priority for conflict resolution.** Thanks to the fact that RV states take into considerations all possible future evolution of a monitored execution, our framework handles the subtle situation where the execution reaches a state of affairs in which the conjunction of two constraints is permanently violated, while none of the two is so if considered in isolation. This situation of conflict has been already recalled in Section 5 in the case of DECLARE. A situation of conflict involving two constraints $\varphi$ and $\psi$ witnesses that even though none of $\varphi$ and $\psi$ is permanently violated, they contradict each other, and hence in every possible future course of execution at least one of them will eventually become permanently violated. In such a state of affairs, it may become relevant to specify which ones of the two constraints has priority over the other, that is, which one should be preferably satisfied.

Formally, a trace culminates in a conflict for two $\text{LDL}_f$ constraints $\varphi$ and $\psi$ if it satisfies the following metaconstraint:

$$\| \{\varphi \land \psi\} = \text{perm\_false} \| \land \neg\| \varphi = \text{perm\_false} \| \land \neg\| \psi = \text{perm\_false} \|$$

(7)
Specifically, assuming that \( \varphi \) and \( \psi \) can potentially enter into a conflict, a monitor for formula (7) proceeds as follows:

- Initially, the monitor outputs \( \text{temp\_false} \), witnessing that no conflict has been seen so far, but it may actually occur in the future.
- From this initial situation, the monitor can evolve in one of the following two ways:
  - the monitor turns to \( \text{perm\_false} \), witnessing that from this moment on neither of the two constraints will ever be violated anymore, irrespectively of how the trace continues;
  - the monitor turns to \( \text{temp\_true} \), whenever the monitored execution indeed culminates in a conflict – this witnesses that a conflict is currently in place.
- From the latter situation witnessing the presence of a conflict, the monitor evolves then to \( \text{perm\_false} \) when one of the two constraints indeed becomes permanently violated; this witnesses that the conflict is not anymore in place, due to the fact that now the permanent violation can actually be ascribed to one of the two constraints taken in isolation from the other.

Using this monitor, we can identify all points in the trace where a conflict is in place by simply checking when the monitor returns \( \text{temp\_true} \).

Notice that the monitor never outputs \( \text{perm\_true} \), since a conflicting situation will always eventually permanently violate \( \varphi \) or \( \psi \), in turn, permanently violating (7). In addition, the notion of conflict defined in formula (7) is inherently “non-monotonic”, as it ceases to exist as soon as one of the two involved constraints becomes permanently violated alone. This is the reason why we cannot directly employ formula (7) as a basis to define which constraint we prefer over the other when a conflict arises. To declare that \( \varphi \) is preferred over \( \psi \), we then relax formula (7) by simply considering the violation of the composite constraint \( \varphi \land \psi \), which may occur due to a conflict or due to the permanent violation of one of the two constraints \( \varphi \) and \( \psi \). We then create a formula expressing that whenever the composite constraint is violated, then we want to satisfy the preferred constraint \( \varphi \):

\[
\langle \mathcal{R} \{ \varphi \land \psi \} = \text{perm\_false} \rangle \text{tt} \rightarrow \varphi
\]

This pattern can be generalized to conflicts involving \( n \) formulae, using their proper maximal subsets as building blocks. In the typical situation where a permanent violation of \( \varphi \land \psi \) does not manifest itself at the beginning of the trace, but may indeed occur in the future, a monitor for (8) starts by emitting \( \text{temp\_true} \). When the composite constraint \( \varphi \land \psi \) becomes permanently violated (either because of a conflict, or because of a permanent violation of one of its components), formula \( \mathcal{J} \{ \varphi \land \psi \} = \text{perm\_false} \) turns to \( \text{perm\_true} \), and the monitor consequently switches to observe the evolution of \( \varphi \) (that is, of the head of the implication in (8)).

**Example 7.** Consider again Figure 5, and in particular the response and not coexistence constraints respectively linking pay registration to get ticket, and get ticket to cancel registration, which we compactly refer to as \( \psi_r \) and \( \psi_{nc} \). These two constraints conflict when a registration is paid and canceled, but the ticket is not retrieved (this would indeed lead to a permanent violation of \( \varphi_{nc} \) alone).
Let $o$ denote any task that is different from pay, get, and cancel. The traces that culminate in a conflict for $\psi_r$ and $\varphi_{nc}$ are those that satisfy the regular expression:

$$ (o^*; \text{pay}; (o + \text{pay})^*; \text{cancel}; (\neg \text{get})^*) + (o^*; \text{cancel}; (o + \text{cancel})^*; \text{pay}; (\neg \text{get})^*) \quad (9) $$

Recall that, as specified in Section 2, testing whether a trace satisfies this regular expression can be done by encoding it in LDL as:

$$ \langle (o^*; \text{pay}; (o + \text{pay})^*; \text{cancel}; (\neg \text{get})^*) + (o^*; \text{cancel}; (o + \text{cancel})^*; \text{pay}; (\neg \text{get})^*) \rangle \text{end} \quad (10) $$

We want now express that we prefer the not coexistence constraint over the response one, i.e., that, upon cancelation, the ticket should not be retrieved even if the payment has been done. To this end, we first notice that, for an evolving trace, the composite constraint $\psi_r \land \varphi_{nc}$ is permanently violated either when $\varphi_{nc}$ is so, or when a conflict arise. The first situation arises when the trace contains both cancel and get (in whatever order), whereas the second arises when the trace contains both cancel and pay (in whatever order). Consequently, we have that $re_{[(\varphi_{nc} \land \psi_r) = \text{perm_false}]}$ corresponds to the regular expression:

$$ (o^*; \text{pay}; (\neg \text{cancel})^*; \text{cancel}; (\text{true})^*) + (o^*; \text{get}; (\neg \text{cancel})^*; \text{cancel}; (\text{true})^*) + (o^*; \text{cancel}; (\text{cancel} + o)^*; (\text{get} + \text{pay}); (\text{true})^*) $$

We then use this regular expression together with $\varphi_{nc}$ to instantiate formula (8) as follows:

$$ \left\langle (o^*; \text{pay}; (\neg \text{cancel})^*; \text{cancel}; (\text{true})^*) + (o^*; \text{get}; (\neg \text{cancel})^*; \text{cancel}; (\text{true})^*) + (o^*; \text{cancel}; (\text{cancel} + o)^*; (\text{get} + \text{pay}); (\text{true})^*) \right\rangle \text{tt} \rightarrow \neg (\Diamond \text{get} \land \Diamond \text{cancel}) $$

We conclude by showing the evolution of the monitors for the metaconstraints discussed in the various examples of this section.

Example 8. Figure 7 reports the result computed by the monitors for the metaconstraints discussed in Examples 5, 6, and 7 on a sample trace. When the payment occurs, the contextual absence constraint forbids to get tickets. The prohibition is then permanently removed upon the consequent acceptance of the regulation, which ensures that the selected context will never appear again.

The execution of the third step, consisting in the cancelation of the order, induces a conflict for $\neg (\Diamond \text{get} \land \Diamond \text{cancel})$ and $\Box (\text{pay} \rightarrow \Diamond \Box \text{get})$, since they respectively forbid and require to eventually get the ticket. The monitor for the conflict metaconstraint witnesses this by switching to temp_true. The preference stays instead temp_true, but while up to this point it was emitting temp_true because no conflict had occurred yet, it now emits temp_true because this is the current RV state of the preferred, not coexistence constraint.

The execution of the get ticket task induces a permanent violation for constraint $\neg (\Diamond \text{get} \land \Diamond \text{cancel})$, which, in turn, triggers a number of effects:
Monitoring Metaconstraints with Temporal Logics

begin
  do
    pay
    do
      acc
      do
        cancel
        do
          get
          do
            return
            do
              complete

3 \rightarrow 3\text{acc}
temp: true
perm: true

¬(3\text{get} \land 3\text{cancel})
temp: true
perm: false

Contextual absence: get task forbidden while \(3\text{pay} \rightarrow 3\text{acc}\) is \text{temp}\_false.

RV state
perm: true

Reactive compensation: permanent violation of \(¬(3\text{get} \land 3\text{cancel})\) compensated by a consequent \(3\text{return}\).

RV state
perm: true

Conflict: presence of a conflict for \(¬(3\text{get} \land 3\text{cancel})\) and \(3\text{pay} \rightarrow 3\text{get}\).

RV state

Preference: preference of \(¬(3\text{get} \land 3\text{cancel})\) over \(3\text{pay} \rightarrow 3\text{get}\).

RV state

Fig. 7: Result computed by monitoring the metaconstraints in Examples 5, 6, and 7 against the trace \(3\text{pay} \cdot 3\text{acc} \cdot 3\text{cancel} \cdot 3\text{get} \cdot 3\text{return}\); for readability, we also report the evolution of the monitors for the constraints mentioned by the metaconstraints.

Since the preference metaconstraint is now following the evolution of the preferred constraint \(¬(3\text{get} \land 3\text{cancel})\), it also moves to perm\_false.

The conflict is not present anymore and will never be encountered again, given that one of its two constraints is permanently violated on its own. Thus, the monitor for the conflict metaconstraint turns to perm\_false.

The reactive compensation is triggered by the permanent violation of \(¬(3\text{get} \land 3\text{cancel})\), and asserts that, from now on, the compensating constraint \(3\text{return}\) must be satisfied; since the ticket is yet to be returned, the metaconstraint turns to temp\_false.

The execution of the last step, consisting in returning the ticket, has the effect of permanently satisfying the compensation metaconstraint, which was indeed waiting for this task to occur.

7 Implementation

The entire approach has been implemented as an operational decision support (OS) provider for the PROM 6 process mining framework called LDL Monitor. PROM 6 provides a generic OS environment [28,15] that supports the

[http://www.promtools.org/prom6/]
interaction between an external workflow management system at runtime (producing events) and ProM. In Section 7.1 we will sketch some relevant aspects of the general architecture of the OS backbone implemented inside ProM 6. In Section 7.2 we ground the discussion to the specific case of the LDL Monitor, discussing the skeleton of our compliance verification OS Provider. The data exchanged between the LDL Monitor client and provider is illustrated in Section 7.3. In Section 7.4 we describe the implemented LDL Monitor client. At the back-end of the LDL Monitor, there is a software module specifically dedicated to the construction and manipulation of Nfas from \( \text{ldl/ltl} \) formulae (detailed in Section 7.5), concretely implementing the technique presented in Section 3. This software is called FLLOAT, which stands for “From \( \text{ltl/ldl} \) To AuTomata”, its code is open source and publicly available at https://github.com/RiccardoDeMasellis/FLLOAT.

7.1 General Architecture

The ProM OS architecture (shown in Figure 8) relies on the well-known client-server paradigm [13]. More specifically, the ProM OS Service manages the interaction with running process instances and acts as a mediator between them and the registered specific OS providers. Sessions are created and handled by the OS Service to maintain the state of the interaction with each running client. To establish a stateful connection with the OS Service, the client creates a session handle for each managed running process instance, by providing host and port of the OS Service. When the client sends a first query related to one of such running instances to the OS Service, it specifies information related to the initialization of the connection (such as reference models, configuration parameters, etc.) and to the type of the queries that will be asked during the execution. This latter information will be used by the OS Service to select, among the registered active providers, the ones that can answer the received query. The session handle takes care of the interaction with the service from the client point of view, hiding the connection details and managing the information passing in a lazy way. The interaction between the handle and the service takes place over a TCP/IP connection.
7.2 LDL Monitor Skeleton

In the LDL Monitor, the interaction between a client and the OS Service mainly consists of two aspects. First of all, before starting the runtime compliance verification task, the client sends to the OS Service the LDL$_f$ reference model to be used. This model is then placed inside the session by the OS service. The reference model is a set of LDL$_f$ constraints represented as strings. The client can also set further information and properties. For example, each constraint in the LDL$_f$ reference model can be associated to a specific weight, that can be then exploited to compute metrics and indicators that measure the degree of adherence of the running instance to the reference model. Secondly, during the execution, the client sends queries about the current monitoring status for one of the managed process instances. The session handle augments these queries with the partial execution trace containing the evolution that has taken place for the process instance after the last request. The OS Service handles a query by first storing the events received from the client, and then invoking the LDL Monitor provider. The LDL Monitor provider recognizes whether it is being invoked for the first time with respect to that process instance. If this is the case, it takes care of translating the reference model onto the underlying formal representation. The provider then returns a fresh result to the client, exploiting a reasoning component for the actual result’s computation. The reasoning component behind the provider is described in Section 7.5. After each query, the generated result is sent back to the OS Service, which possibly combines it with the results produced by other relevant providers, finally sending the global response back to the client.

7.3 Exchanged Data and Business Constraints States

We now discuss the data exchanged by the LDL Monitor client and provider. The partial execution traces sent by the client to the OS use the XES format (www.xes-standard.org/) for event data. XES is an extensible XML-based standard recently adopted by the IEEE task force on process mining. The response produced by the LDL Monitor provider is composed of two parts. The first part contains the temporal information related to the evolution of each monitored business constraint from the beginning of the trace up to now. At each time point, a constraint can be in one state, which models whether it is currently: (permanently) satisfied, i.e., the current execution trace complies with the constraint; possibly satisfied, i.e., the current execution trace is compliant with the constraint, but it is possible to violate it in the future; (permanently)
violated, i.e., the process instance is not compliant with the constraint; possibly violated, i.e., the current execution trace is not compliant with the constraint, but it is possible to satisfy it by generating some sequence of events. This state-based evolution is encapsulated in a fluent model which obeys to the schema sketched in Figure 9. A fluent model aggregates fluents groups, containing sets of correlated fluents. Each fluent models a multi-state property that changes over time. In our setting, fluent names refer to the constraints of the reference model. The fact that the constraint was in a certain state along a (maximal) time interval is modeled by associating a closed MVI (Maximal Validity Interval) to that state. MVIs are characterized by their starting and ending timestamps. Current states are associate to open MVIs, which have an initial fixed timestamp but an end that will be bounded to a currently unknown future value.

7.4 LDL Monitor Client

We have developed two LDL MONITOR clients, in order to deal with different settings: (a) replay of a process instance starting from a complete event log, and (b) acquisition of events from an information system. The first client is mainly used for testing and experimentation. The second client requires a connection to some information system, e.g., a workflow management system. The two clients

Fig. 10: Screenshot of one of the LDL MONITOR clients.
differ on how the user is going to provide the stream of events, but they both include an interface with a graphical representation of the obtained fluent model, showing the evolution of constraints and also reporting the trend of the compliance indicator. Figure 10 shows the interface running with the example in Figure 7.

7.5 Reasoning Component

The reasoning component of the LDL Monitor, FLLOAT ("From LTLf/LDLf To AuTomata", [https://github.com/RiccardoDeMasellis/FLLOAT](https://github.com/RiccardoDeMasellis/FLLOAT)) implements the logics of the runtime verification by building the automaton of the reference model LDLf constraints with the algorithms presented in Section 3.

Fig. 11: UML-like diagram of the backend main components.

The FFLOAT code has been implemented in the Java language and exploits the inheritance features of object oriented languages. It is made up by several conceptual modules and makes use of external libraries as shown by the UML-like diagram in Figure 11, where the main java classes are depicted by the usual rectangles, their surrounding boxes represent the conceptual modules they belong to and dashed arrows show the dependencies. In what follows we will address each conceptual module separately.

**Formulae** It contains classes and methods to represent and manipulate logical formulae. Classes in this module have a complex hierarchy, as formulae are characterized by several independent aspects: the language (LDLf, LTLf, REf); the structure (atomic, unary, binary) and the temporal characterization (local
or temporal). Since Java does not allow multiple inheritance, such a hierarchy has been reproduced by a suitable use of subclasses and interfaces. Besides, formulae are implemented with an inductive structure, i.e., formulae have as instance variable a formula if unary, or two if binary, which allows us to elegantly implement all recursive functions for their manipulation. Each formula implements the interface Formula, which is then extended by six interfaces, each representing a specific characteristic: Temporal if the formula contains a temporal operator; Local if it does not; BooleanOp if its main operator is a boolean operator; Atomic if it is atomic, i.e., if it is (propositional) true (abstract class TrueLocal), (propositional) false (abstract class FalseLocal) or a propositional variable LocalVar; Unary if its main operator is a unary operator and lastly Binary if it is instead binary. Besides, BooleanOp is extended by interfaces representing the usual boolean operators, such as Not, which also extends Unary, and And, which also extends Binary, and so on for the other boolean connectives. We remark that it is necessary to express such different characteristics by means of interfaces, as each formula is indeed a combination of those. At a lower level of abstraction, we have three main type of formulae: LDLf, LTLf and REf each of which is again an interface extending Formula. Here we provide a detailed description of the structure of LDLf formulae only, but the same ideas also hold for LTLf and REf formulae. Figure 12 provides a UML-like class diagram for LDLf formulae, where dashed boxes represent interfaces, simple boxes are abstract classes; boxes with bold text are classes and arrows mean both extends or implements, depending on whether the extending/implementing entity is an interface or a (abstract) class. LDLf extends Formula, and is ex-

Fig. 12: UML-like diagram of the classes for LDLf formulae.
tended by interface LDLfTemp (which also extends Temporal); LDLfBooleanOp (which also extends BooleanOp) and LDLfLocal (which also extends Local). Moreover, LDLf is implemented by abstract classes LDLfUnary and LDLfBinary, which also extend Unary and Binary, respectively. An atomic LDLf formula is clearly local, and, as presented in Section 2, can be LDLfLocalTrue, i.e., the propositional true (hence extending TrueLocal and implementing LDLfLocal) or, analogously, LDLfLocalFalse or LDLfLocalVar. Other local LDLf formulae are boolean combinations of other local formulae, hence they all implement interface LDLfBooleanOpLocal, such as LDLfLocalNot (which also implements Not) and LDLfLocalAnd (which also implements And). LDLfBooleanOpTemp formulae have an analogous structure. The other LDLf temporal formulae are the atomic LDLfTt and LDLfTf formulae and formulae with a temporal operator, thus extending abstract class LDLfTempOpTemp, that is, LDLfBox and LDLfDiamond formulae.

Although such a hierarchical structure may seem cumbersome, when dealing with a relevant number of classes it is essential to keep the code modular. As an example, let us consider the negation normal form (NNF) of a formula: regardless if a formula is a LDLfLocalAnd, LDLfTempAnd, LTLfLocalAnd or LTLfTempAnd, the logic for transforming an And formula in NNF is the same. Indeed, non-static method nnf() has been implemented as a default method in the And interface, and it is hence inherited by all implementing classes. Similar considerations also hold for method getSig(), which returns the set of propositional variables appearing in the formula, as it can be defined as a default method in interfaces Unary and Binary. Another notable example is the delta() method, which implements the delta function in Figure 1, which, being the same for all LDLfLocal formulae, has been implemented as a default method in interface LDLfLocal. Also, methods returning sub-formulae, such as getNested, getLeft and getRight are defined in the abstract classes LDLfUnary and LDLfBinary.

Automaton construction The main functionality FLLOAT provides is the automaton generation for an LTLf/LDLf formula \( \varphi \) given as input, which is implemented in the static method ldlf2Automaton of class AutomatonUtils. The whole procedure works as follows. First, the formula is parsed and an LTLf or LDLf object is created. This is achieved by classes that are previously and automatically generated by ANTLR\footnote{http://www.antlr.org} starting from grammar files. Then, if the input formula is LDLf, then it is translated in negation normal form (nnf() method) and the algorithm for the automata generation is called. Conversely, if it is LTLf, it must be first converted to LDLf by method toLDLf() implementing the translation explained in Section 2. Once an LDLf formula \( \varphi \) in negation normal form has been obtained, the automaton is generated with Algorithm 2. The method ldlf2Automaton consists of two nested cycles, the outer on states in \( S \) yet to be analyzed and the inner on interpretation for \( P \) or the empty trace (line 8 of Algorithm 2): at each iteration \( \delta(s, \Theta) \) is computed, where \( s \in S \) and \( \Theta \in 2^P \cup \epsilon \), possibly generating new states \( q' \) to be added to the set of states to be analyzed at the next iteration.
along with the respective transitions (line 8–9 of Algorithm 2). Since function \( \delta \) is recursively defined on the structure of LDL\(_f\) formulae, it is implemented by the recursive non-static \( \text{delta} \) method of LDL\(_f\) class, exploiting the java inheritance features. The Tweety library\(^9\) is used to compute the models, i.e., \( q' \) states, of the formula \( \bigwedge_{\psi \in q} \delta(\psi, \Theta) \) in line 8 of Algorithm 2. The procedure starts by analyzing \( \varphi \), the only state in \( S \), and ends when all states have been analyzed and no others have been generated in the meanwhile. The data structures for automata are defined in the jautomata library\(^{10}\) which also provides methods for automata manipulation, such as union, intersection, trimming and determinization.

**Runtime Verification** The runtime verification functionalities are provided by the \texttt{ExecutableAutomaton} class. An executable automaton is essentially a deterministic automaton (every NFA can be determinized) with a reference to the current state. When an executable automaton is created from an automaton, the current state is set to the initial state (by construction there is always a unique initial state). The idea is to navigate the automaton and return \( \text{RV} \) truth values while events are executed. Recalling the results presented in Section 4.2, each automaton state represents a \( \text{RV} \) truth value. Hence, an operative way to implement a \( \text{RV} \) monitor is to analyze one-by-one the occurring events and to perform the corresponding transitions on the automaton of the constraints. Each time a state change is triggered by a transition leading to state \( s \), we calculate \( \text{Reach}(s) \) and return the corresponding truth value. In our implementation when a new event is executed, the non-static method \text{step}, taking \( \Theta \in 2^P \cup \{\varepsilon\} \) as input, is called, updating the current state by traversing the corresponding \( \Theta \) transition. Method \text{currentRVTruthValue} computes \( \text{Reach} \) for the current state and returns one among \( \text{perm\_true}, \text{perm\_false}, \text{temp\_true}, \text{temp\_false} \) as explained at the end of Section 4.2 thus effectively implementing the \( \text{RV} \) semantics.

### 8 Conclusions

In this article, we have brought forward a foundational and practical approach to formalize and monitor linear temporal constraints and metaconstraints, under the assumption that the traces generated by the system under study are \textit{finite}. This is, e.g., the typical case in the context of business process management and service-oriented architectures, where each execution of a business process or service invocation leads from a starting state to a completion state in a possibly unbounded, yet finite, number of steps.

The main novelty of our approach is to adopt a more powerful specification logic, that is, LDL\(_f\) (which corresponds to Monadic Second-Order Logic over finite traces), instead of the typical choice of LTL\(_f\) (which corresponds to First-Order Logic over finite traces). Like in the case of LTL\(_f\), also LDL\(_f\) comes with

\(^9\) http://tweetyproject.org
\(^{10}\) https://github.com/abailly/jautomata
an automata-theoretic characterization that employs standard finite-state automata. Differently from \( \text{LTL}_f \), though, \( \text{LDL}_f \) can declaratively express, within the logic, not only constraints that predicate on the dynamics of task executions, but also constraints that predicate on the monitoring state of other constraints.

The approach has been fully implemented as an independent library to specify \( \text{LDL}_f/\text{LTL}_f \) formulae as well as obtain and manipulate their corresponding automata, which is then invoked by a process monitoring infrastructure that has been developed within the state-of-the-art ProM process mining framework.

As a next step, we intend to incorporate other monitoring perspectives, such as the data perspective taking into consideration the data carried by the monitored events. This setting is reminiscent of stream query languages and event calculi. For example, the logic-based Event Calculus has been applied to process monitoring against data-aware extensions of the DECLARE language in [20], also considering some specific forms of compensation [5]. However, all these approaches are only meant to query and reason over (a portion of) the events collected so far in a trace, and not to reason upon its possible future continuations, as we do in our approach. Genuine investigation is then required towards understanding under which conditions it is possible to lift the automata-based techniques presented here to the case where events are equipped with a data payload and constraints are expressed in (fragments of) first-order temporal logics over finite traces.

References

1. van der Aalst, W.M.P.: Process Mining - Data Science in Action, Second Edition. Springer (2016). https://doi.org/10.1007/978-3-662-49851-4
2. Bauer, A., Leucker, M., Schallhart, C.: Comparing LTL Semantics for Runtime Verification. Logic and Computation (2010)
3. Brafman, R.I., De Giacomo, G., Patrizi, F.: Ltlf/ldlf non-markovian rewards. In: AAAI. pp. 1771–1778. AAAI Press (2018)
4. Calvanese, D., De Giacomo, G., Lenzerini, M., Vardi, M.Y.: An automata-theoretic approach to Regular XPath. LNCS, vol. 5708, pp. 18–35. Springer (2009)
5. Chesani, F., Mello, P., Montali, M., Torroni, P.: Verification of choreographies during execution using the reactive event calculus. In: 5th Int. Workshop on Web Services and Formal Methods (WS-FM). LNCS, vol. 5387. Springer (2008)
6. Clark, J., DeRose, S.: XML Path Language (XPath) version 1.0. W3C Recommendation, World Wide Web Consortium (Nov 1999)
7. De Giacomo, G., De Masellis, R., Grasso, M., Maggi, F.M., Montali, M.: Monitoring business metaconstraints based on LTL and LDL for finite traces. LNCS, vol. 8659. Springer (2014)
8. De Giacomo, G., Vardi, M.Y.: Linear temporal logic and linear dynamic logic on finite traces. In: Proc. of the 23rd Int. Joint Conf. on Artificial Intelligence (IJCAI). AAAI (2013)
9. Dwyer, M.B., Avrunin, G.S., Corbett, J.C.: Patterns in property specifications for finite-state verification. In: Boehm, B.W., Garlan, D., Kramer, J. (eds.) Proc. of the 1999 International Conf. on Software Engineering (ICSE). ACM Press (1999)
10. Fischer, M.J., Ladner, R.E.: Propositional dynamic logic of regular programs. Journal of Computer and System Science 18 (1979)
11. Harel, D., Kozen, D., Tiuryn, J.: Dynamic Logic. MIT Press (2000)
12. Ly, L.T., Maggi, F.M., Montali, M., Rinderle-Ma, S., van der Aalst, W.M.P.: A framework for the systematic comparison and evaluation of compliance monitoring approaches. In: Proc. of the 17th IEEE Int. Enterprise Distributed Object Computing Conf. (EDOC). IEEE (2013)
13. Maggi, F.M., Montali, M., van der Aalst, W.M.P.: An operational decision support framework for monitoring business constraints. In: Fundamental Approaches to Software Engineering - 15th International Conference, FASE 2012, Held as Part of the European Joint Conferences on Theory and Practice of Software, ETAPS 2012, Tallinn, Estonia, March 24 - April 1, 2012. Proceedings. pp. 146–162 (2012)
14. Maggi, F.M., Montali, M., Westergaard, M., van der Aalst, W.M.P.: Monitoring business constraints with linear temporal logic: An approach based on colored automata. In: Proc. of the 9th Int. Conf. on Business Process Management (BPM). LNCS, vol. 6896. Springer (2011)
15. Maggi, F.M., Westergaard, M.: Designing software for operational decision support through coloured petri nets. Enterprise IS 11(5), 576–596 (2017)
16. Marx, M.: XPath with conditional axis relations. LNCS, vol. 2992, pp. 477–494. Springer (2004)
17. Montali, M.: Specification and Verification of Declarative Open Interaction Models: a Logic-Based Approach. Ph.D. thesis, Department of Electronics, Computer Science and Telecommunications Engineering, University of Bologna (2009), [http://amsdottorato.cib.unibo.it/1829/](http://amsdottorato.cib.unibo.it/1829/)
18. Montali, M.: Specification and Verification of Declarative Open Interaction Models: a Logic-Based Approach, LNBIP, vol. 56. Springer (2010)
19. Montali, M., Maggi, F.M., Chesani, F., Mello, P., van der Aalst, W.M.P.: Monitoring business constraints with the event calculus. ACM Trans. on Intelligent Systems and Technology 5(1) (2013)
20. Montali, M., Pesic, M., van der Aalst, W.M.P., Chesani, F., Mello, P., Storari, S.: Declarative specification and verification of service choreographies. ACM Trans. on the Web 4(1) (2010)
21. Pesic, M.: Constraint-Based Workflow Management Systems: Shifting Controls to Users. Ph.D. thesis, Beta Research School for Operations Management and Logistics, Eindhoven (2008)
22. Pesic, M., van der Aalst, W.M.P.: A declarative approach for flexible business processes management. In: Proc. of the BPM 2006 Workshops. LNCS, vol. 4103. Springer (2006)
23. Prakken, H., Sergot, M.J.: Contrary-to-duty obligations. Studia Logica 57(1) (1996)
27. Vardi, M.: The rise and fall of linear time logic (2011), http://www.cs.rice.edu/~vardi/papers/gandalf11-myv.pdf 2nd Int’l Symp. on Games, Automata, Logics and Formal Verification

28. Westergaard, M., Maggi, F.: Modelling and Verification of a Protocol for Operational Support using Coloured Petri Nets. In: Proc. of ATPN (2011)
Table 1: Some DECLARE constraints, including graphical representation, $\text{LTL}_f$ encoding, good prefix characterization, and colored automaton. In the table: $o$ is a shortcut notation for other task, that is, any task that is not explicitly mentioned by the constraint; given a task $a$, $\neg a$ is a shortcut notation for any task different from $a$.

| CONSTRAINT | $\text{LTL}_f$ | pref | COLORED AUTOMATON |
|------------|-----------------|------|-------------------|
| 1.°        | $\Diamond a$    | true $^*$ | ![Diagram 1](image1) |
| absence    | $\neg \Diamond a$ | $o^*$ | ![Diagram 2](image2) |
| absence 2  | $\neg \Diamond (a \land \Box \Diamond a)$ | $o^* + (o^*; a; o^*)$ | ![Diagram 3](image3) |
| choice     | $\Diamond (a \lor b)$ | true $^*$ | ![Diagram 4](image4) |
| responded  | $a \rightarrow \Diamond b$ | true $^*$ | ![Diagram 5](image5) |
| response   | $\Box (a \rightarrow \Diamond b)$ | true $^*$ | ![Diagram 6](image6) |
| precedence | $\neg b \lor a \lor \neg \Diamond b$ | $(-b)^* + (o^*; a; true^*)$ | ![Diagram 7](image7) |
| not        | $\neg (\Diamond a \land \Diamond b)$ | $(a + o)^* + (b + o)^*$ | ![Diagram 8](image8) |