Research Article

MHD Analysis of Couple Stress Hybrid Nanofluid Free Stream over a Spinning Darcy-Forchheimer Porous Disc under the Effect of Thermal Radiation

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This article develops the semianalytical analysis of couple stress hybrid nanofluid free stream past a rotating disc by applying the magnetic flux effects and radiation of thermal energy. The analysis of such kind of mixed convective flow is most important due to numerous industrial applications such as electronic devices, atomic reactors, central solar energy equipment, and heat transferring devices. The impact of variable permeability is also considered in the study. The permeability of the disc obeys the Darcy-Forchheimer model. The hybrid nanofluid is composed of water, titanium dioxide, and aluminum oxide. The set of governing equations in the PDE form are transformed to couple ODEs by applying similarity transformations. The ODE set are solved by applying the technique of HAM. The graphs of impacts of numerous physical parameters over momentum, energy, and concentration profiles are drawn in computer-based application Mathematica 11.0.1. In the sundry physical parameters, the porosity parameter, Reynolds number, inertial parameter, Prandtl number, Schmidt number, couple stress, and quotient of rotational momentum to elongating rate are included. During the analysis, it is found that the momentum profile of the couple stress hybrid nanofluid enhances with local inertial parameter, couple stress parameter, porosity parameter, and Reynolds number but declines for the growth in Hartmann number. Heat transfer rate enhances for radiation parameter but decreases in variable for temperature, thermal stratification parameter, thermophoresis parameter, and Brownian parameter.

1. Introduction

The era in which we are living the use of machinery and modern technology is too much like in civil, mechanical, and engineering. It is fact that production of heat is associated with machinery. The extra production of heat causes lower efficiency of the machinery and more consumption of energy. To reduce such production of useless heat in the running machinery, we used such technologies and approaches in which the use of such fluids is very common like water, mineral, organic (ethylene glycol, refrigerants, and tri-ethylene-glycol), bio, and herbal oils. The uses of such fluids are in cement, lather, garments, tiles, and rock cutting industries. Such kind of fluids was used in scientific purposes like hydroelectric, thermal electric, and nuclear power stations. But the uses of such common fluids have some limitations, because these common fluids used as a coolant has low heat capacity. Metals (gold, silver, and copper), metal oxides (for example, aluminum oxide (Al₂O₃), iron oxide (Fe₂O₃), silver oxide (Ag₂O), zirconia (ZrO₂), titania (TiO₂), alloy of metals, metal nitrides (AlN, SiN, and SiN), and metal carbides (SiC) have high heat capacity as compared to the common fluids; thus,
by the amalgamation of common fluids and metals, metal oxides and alloy of metals enhance the heat capacity of the common fluids. This concept was presented by the pioneer scientist Choi and Eastman [1]. At the beginning of invention of such new kind of suspension, the particle size in base fluids was in such a range that they were sediment after some time and its efficiency was not up to mark. So, after this, the researched recommended that the particle size should be smaller to enhance the heat capacity of the suspension and reduces the chances of sedimentation of the particles in the base fluids. When magnitude of metal particle inside the common liquid is in the range 10^-10^, then, the efficiency of such suspension is better. The solid particles are called nanoparticles, and common fluids in the suspensions are known as base fluids. Therefore, this new kind of suspension is called nanofluid. For the better performance of these nanofluids, the researchers add another kind of solid powder of nano-sized particles. In the means due to hybridization of nanoparticles in the nanoliquid, the nanofluid heat capacity enhances. This is the next generation of the nanofluid. The size of the nanoparticles in the nanofluid or in the hybridnaofluid should be so small to flow through the porous medium [2]. Madesh et al. [3] studied the experimental flow of heat in the hybrid nanofluid of Cu-TiO_2 across a tubular shaped container; the nanoparticles volume fraction during the experiment was 0.1% to 2%. Senthilaraja et al. [4] studied experimentally the thermal conductivity of hybrid nanofluid composed of Cu-Al_2O_3/H_2O, during the experiment the nanoparticle volume fraction was (0.05, 0.1, and 0.2%) and temperature variation was 20°C to 60°C. Bhosale and Borse [5] experimentally showed that the hybrid nanofluid is better significantly than conventional fluid in critical heat flux (CHF) by using a Ni-Cr wire in pool boiling. He et al. [6] prepared a new kind of hybrid nanofluid composed of SWCNTs (single-wall carbon nanotubes) and CuSNPs (copper sulfide nanoparticles) into the base fluid PSS (styrene sulfonate) and analyze this new kind of hybrid nanofluid for production of electricity. Esfe et al. [7] experimentally analyzed the impact of nanoparticle percentage in the base fluid over the thermal conductivity of the hybrid nanofluid. Syam et al. [8] demonstrated experimentally the impact of MHD on the thermal conductivity and viscosity of hybrid nanofluid of ND-Co_3O_4 (ND: nanodiamond). The shape of the nanoparticles like spherical, cylindrical, brick, platelet, or blade shape affects the thermal capacity of the nanofluid. The nanofluid emerged as a favorable heat transferring agent from last few decades of years due to contempt of inconsistency in the literature of the nanofluid and lacking of understanding of the heat transfer mechanisms. With regard to the extension of nanofluid study, the researchers invent a new kind of fluid by adding various kinds of nanoparticles in mixture/composite form in the conventional fluid; this type of fluid is identified by a hybrid nanofluid. As compared to the nanofluid, the hybrid nanofluid is the good heat transferring agent due to less disadvantages, enhanced thermal characteristics, and synergistic effect of nanoparticles of multikind of nanoparticles versus single kind of nanoparticle and less pressure drop characteristic. For the industrial use production of nanofluid, hybrid nanofluid is the great challenge for researchers due to selection of suitable nanomaterial which is affordable in cost and fulfills the required needs, long-term stability, and simple in production [9]. Haidera et al. [10] have investigated the stream of hybrid nanofluid across the medium obeying the Darcy-Forchheimer model. They solve this flow by the shooting method by keeping the slip condition of momentum as well as applying thermal radiation and stratification. Acharya et al. [11] have explored the results related to the MHD stream of nanofluid over a spinning disc under the inspirations of Hall current and thermal radiation. They solve the problem through the RK-4 approach and then compare their results with a shooting technique. Farooq et al. [12] have inspected the entropy production in the hybrid nanofluid over a stretched rotating circular disc. They investigated the influence of transportation and viscous effects. TD, MG, and Brinkman models are applied to entropy generation to find the effective thermal conductivity. Islam et al. [13] have scrutinized the flow of the Maxwell nanofluid on an elongating cylinder. For the best analysis, they apply the heat in the form of radiation; the fluid is also heated up due to joule heating. Zubair et al. [14] have explored the results of nanofluid flow inside a couple of spinning discs with Cattaneo-Cristove heat flux; the discs are porous following the Darcy-Forchheimer model. They also analyze the entropy production during the flow of the nanofluid. Khan et al. [15] have explored the stream of couple stress fluid past a stretched surface. This flow on the stretched sheet is due to the periodic oscillations. Hybrid nanofluid with mixed convection boundary layer stream has a lot of practical applications like in electronic devices as a coolant, solar receivers, nuclear reactors, and heat exchangers [16]. Due to these practical applications of mixed convection boundary layer stream, this is mandatory to deeply study its benefits and welfare. Hayat et al. [17] have scrutinized analytically the mixed convection boundary layer stream of the nanofluid past a stretched cylinder. Turkylmazoglu [18] has examined the heat transfer in the flow of mixed convection nanofluid past a moving flat plate. Haroun et al. [19] have investigated the nanofluid stream over a stretching/shrinking belt with convective boundary condition and solved by applying spectral relaxation method. Mirmosoumi et al. [20] have investigated the nanofluid flow inside a cylindrical tube with convective boundary condition. Mahday [21] explored the numerical results about the heat transfer in the mixed convective boundary layer flow over a vertically standing stretched sheet due to variable thermal conductivity and viscosity. Vassamati and Rao [22] investigated the laminar flow of mixed convective boundary under MHD past over the nonlinear stretched surface. Sumalatha and Shanker [23] studied the impact of Soret and Dufour effects on the mixed convective boundary layer stream under MHD past a nonuniform heated plate. Kumar et al. [24] explored the nanofluid flow in a vertical channel with robin boundary constrains. Shehzad et al. [25] investigated the viscous fluid flow by keeping interference of
viscous dissipation and joule heating. Malvandi et al. [26] have considered the mixed convective flow of Al₃O₃ and water nano-fluid annulus under the impact of the magnetic field. Selimefeghili et al. [27] have reported the result of their studies about the mixed convective flow of the nano-fluid made up of SiO₂ and H₂O over a spinning cylinder. Hayat and Nadeem [28] studied the transfer of heat through the hybrid nano-fluid composed of Ag, CuO solid nanoparticles, and water; the impact of chemical reaction in the hybrid nano-fluid is also encountered. Rashad et al. [29] have explored the outcomes of the hybrid nano-fluid composed of Cu, Al, O₂, and water; natural convective flow under the impact of magnetic field; and joule heating. Usman et al. [30] investigated the impact of nonlinear thermal radiation over a time-dependent variable conductive hybrid nano-fluid past a spinning plate. Mansour et al. [31] analyze the entropy generation in the MHD convective boundary flow inside a square-shaped cavity with heat source/sink. Sheikholeslami et al. [32] investigated the MHD flow of the hybrid nano-fluid inside a round-shaped cavity with two heaters. Kumar and Sarkar [33] investigated the transfer of heat and drop of pressure of heat sink experimentally and present results numerically. Boulahia et al. [34] analyzed the entropy production ina convective boundary flow of the hybrid nano-fluid inside a closed figure, wavy-shaped heat source placed at the center. Urmı et al. [35] explore experimentally the viscosity and thermal conductivity of hybrid nano-fluid TiO₂, A l₁O₃, and ethylene glycol is used as the base fluid. The flow of convective fluid across the porous medium have a lot of practical applications like biomedical engineering, ecofriendly engineering, petroleum engineering, civil engineering, and agrarian sciences. By means of porous medium studies, we can calculate underground water pollution by toxic fluids, collect more and more data about gas and petroleum at underground reservoirs, water seepage in ground during drainage and irrigation, biosensors, to study the infrastructure of concrete construction, as well as in petrochemicals. For the study of these kinds of flows in the porous medium, we adopt Darcy’s law to build a mathematical model for the flow. Darcy’s law is applicable to the fluid when the Reynolds number does not exceeds unity, but as the Reynolds number beats unity the pressure drops in the fluid due to inertial effects. Therefore, Forchheimer [36] introduce a nonlinear term in his work to calculate the inertial effects to the flow in permeable medium. Muskat [37] also reported their results about the Forchheimer term. Seddeek [38] investigated the mass and heat transfer through the mixed convective boundary hybrid nano-fluid past an isothermal upstanding flat sheet. Sadiq and Hayat [39] studied the 2D MHD flow of the Maxwell nano-fluid past a linearly elongating belt. Bakar et al. [40] analyzed the convective boundary layer flow near a stagnation point with slip condition past a Darcy-Forchheimer porous surface. Umavathi et al. [41] studied the natural convective stream inside an upstanding porous medium-filled rectangular duct. Hayat et al. [42] investigated the heat transfer in SWCNTs and MWCNTs over the Darcy-Forchheimer porous disc. Shah et al. [43] explored the 3D micropolar flow past a spinning Darcy-Forchheimer porous disc. Sajjadi et al. [44] analyzed MRT, LBM, and MHD natural convective boundary flow across a porous medium. Muhammad et al. [45] demonstrated the flow of a viscous nano-fluid past a curved sheet stretched nonlinearly with the Darcy-Forchheimer model. Sadiq et al. [46] illustrated impacts of slip conditions (thermal and velocity) over the heat transfer in a nano-fluid past a Darcy-Forchheimer porous spinning disc. Alghamdi et al. [47] examined the transfer of heat in the Casson biphasic nano-fluid through the Darcy-Brinkman porous medium. Khan et al. [48] explored the transfer of heat and mass in a nonaxisymmetric flow over a linearly stretched sheet. Rasool et al. [49] investigated numerically the MHD Darcy-Forchheimer flow of nano-fluid over a nonlinear stretched sheet. Waqas et al. [50] investigated numerically the Darcy-Forchheimer flow of couple stress bioconvective nano-fluid past an inclined porous stretched cylinder.

Now, researchers take interest to analyze the nano-fluid flow across the medium having variable permeability and porosity. By keeping in mind the above study, we examined the hybrid couple stress nano-fluid across the medium having variable permeability and porosity under the impact of MHD. In this study, the rotation of the medium having variable permeability and porosity was also encountered.

2. Mathematical Formulation

We assumed the 3D stream of hybrid nano-fluid past a rotating circular disc. The angular momentum of the disc is represented by \( \Omega \). The disc is made up of a porous medium obeying the Darcy-Forchheimer model. The permeability and porosity of the circular disc are variable. This flow of the couple stress hybrid nano-fluid is in a steady state. The fluid is electrically conducted and heated up by applying thermal radiation which is also affected by the magnetic field. The electrical conductivity of the hybrid nano-fluid is symbolized by \( \sigma_{\text{hf}} \). The flow of the couple stress fluid past a rotating disc is represented geometrically as below.

Here, \((u, v, w)\) are the momentum components taken in the direction of \((r, \varphi, z)\). The effect of concentration is also encountered in the study. The principal equations of the assumed stream are as follows:

\[
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0,
\]

\[
\frac{u \partial u}{\partial r} + \frac{v^2}{r} + \frac{w \partial u}{\partial z} = \nu_{\text{hf}} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{\rho_{\text{hf}} \sigma_{\text{hf}}}{K(z)} u - \frac{\rho_{\text{hf}}}{\sqrt{K(z)}} \frac{\partial^2 u}{\partial z^4} - \frac{C_v \epsilon(z) u}{\sqrt{K(z)}} u \sqrt{u^2 + v^2},
\]
The boundary conditions for the problem are

\[
\begin{align*}
    u & \to u_n = K(z) + \frac{\partial u}{\partial r} + \frac{\partial u}{\partial z} = u_n \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{\partial^2 u}{\partial z^2} \\
    - \frac{\partial^2 u}{\partial r^2} & \quad (z = 0) \\
    \left. \frac{\partial u}{\partial r} \right|_{r = r_0} & \quad (r = r_0) \\
    \left. \frac{\partial u}{\partial z} \right|_{z = z_0} & \quad (z = z_0)
\end{align*}
\]

The variable permeability and porosity of the disc are given in the mathematical form as

\[
\begin{align*}
    K(z) & = K_0 \left(1 + \frac{d e^{-z i r}}{z}ight), \\
    \varepsilon(z) & = \varepsilon_0 \left(1 + \frac{d e^{-z i r}}{z}ight).
\end{align*}
\]

The boundary conditions for the problem are

\[
\begin{align*}
    u & \to u_0 = L_0 \frac{\partial u}{\partial z}, \quad v = r \Omega L_0 \frac{\partial u}{\partial z}, \quad w = 0, \quad T = T_0, \quad C = C_0, \quad \text{at} \quad z = 0, \\
    u & \to 0, \quad v \to 0, \\
    r & \to r_0, \quad C \to C_0, \quad \text{at} \quad z \to \infty.
\end{align*}
\]

The similarity transformation selected for the problem to transform the governing PDEs given in equation (1) to (6) to ODEs is

\[
\begin{align*}
    u & = r \Omega f'(\xi), \quad v = r \Omega g(\xi), \quad w = -\sqrt{2 \Omega} u f' (\xi), \\
    \theta(\xi) & = \frac{T - T_0}{T_w - T_0}, \quad \phi(\xi) = \frac{C - C_0}{C_w - C_0}, \quad \text{Here} \quad \xi = \sqrt{\frac{2 \Omega}{u_f}}.
\end{align*}
\]

The theoretical mathematical model for the hybrid nanofluid [12] applies to governing equations (2) to (6):

\[
\begin{align*}
    \frac{\partial v}{\partial r} + \frac{uv^2}{r} + w \frac{\partial u}{\partial z} = & \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{\partial^2 u}{\partial z^2} \\
    \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = & \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} - \frac{\partial^2 w}{\partial z^2} \\
    \frac{\partial T}{\partial r} + \frac{w}{r} \frac{\partial T}{\partial z} = & \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} - \frac{\partial^2 T}{\partial z^2} \\
    u & \to 0, \quad v \to 0, \\
    r & \to r_0, \quad C \to C_0, \quad \text{at} \quad z \to \infty.
\end{align*}
\]

\[
\begin{align*}
    \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} = & \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{\partial^2 u}{\partial z^2} \\
    \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = & \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} - \frac{\partial^2 w}{\partial z^2} \\
    \frac{\partial T}{\partial r} + \frac{w}{r} \frac{\partial T}{\partial z} = & \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} - \frac{\partial^2 T}{\partial z^2}.
\end{align*}
\]

\[
\begin{align*}
    \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} = & \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{\partial^2 u}{\partial z^2} \\
    \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = & \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} - \frac{\partial^2 w}{\partial z^2} \\
    \frac{\partial T}{\partial r} + \frac{w}{r} \frac{\partial T}{\partial z} = & \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} - \frac{\partial^2 T}{\partial z^2}.
\end{align*}
\]

\[
\begin{align*}
    \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} = & \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{\partial^2 u}{\partial z^2} \\
    \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = & \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} - \frac{\partial^2 w}{\partial z^2} \\
    \frac{\partial T}{\partial r} + \frac{w}{r} \frac{\partial T}{\partial z} = & \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} - \frac{\partial^2 T}{\partial z^2}.
\end{align*}
\]

\[
\begin{align*}
    \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} = & \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{\partial^2 u}{\partial z^2} \\
    \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = & \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} - \frac{\partial^2 w}{\partial z^2} \\
    \frac{\partial T}{\partial r} + \frac{w}{r} \frac{\partial T}{\partial z} = & \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} - \frac{\partial^2 T}{\partial z^2}.
\end{align*}
\]

\[
\begin{align*}
    \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} = & \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{\partial^2 u}{\partial z^2} \\
    \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = & \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} - \frac{\partial^2 w}{\partial z^2} \\
    \frac{\partial T}{\partial r} + \frac{w}{r} \frac{\partial T}{\partial z} = & \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} - \frac{\partial^2 T}{\partial z^2}.
\end{align*}
\]
shows porosity parameter, \( K = \eta_0 b / \rho u_f \) symbolizes the parameter of couple stress, Re \( (= U_x r / u_f) \) represents local Reynolds number, \( Fr = C_g \rho_0^2 r^2 f / \sqrt{K_w} \) expresses local inertial parameter with \( K_w \) permeability and \( \kappa_0 \) porosity, \( \gamma = \sqrt{\alpha / \pi r^2} (r / K_w)^{1/2} \) is the parameter where \( \gamma \) represents \( \gamma \) to \( \gamma \) and \( \gamma \) to \( \gamma \) of thermophysical properties of titanium dioxide \( \gamma \) and \( \gamma \) and \( \gamma \) of thermophysical properties of titanium dioxide \( \gamma \).

3. Thermophysical Properties

Thermophysical properties are the properties of the material which is considered very important in the transfer of heat. The thermophysical properties of a substance comprise specific heat capacity \( (\rho c_p) \), thermal conductivity \( \kappa \), coefficient of liner and volumetric expansion \( \alpha_l \) and \( \beta_v \), heat of vaporization \( \Delta H_{\text{vap}} \), and heat of combustion \( \Delta H_{\text{comb}} \). Some of the thermophysical properties of titanium dioxide TiO2, aluminum oxide Al2O3, and water H2O related to our problem are given in the table given below.

4. Engineer Interest Quantities

In the modern era, the use of mechanical and electrical equipment is much more; the generation of heat and friction must be associated with running machineries. The production of useless heat and friction reduces the efficiency of equipment. For the reduction of friction and transfer of useless heat, nanofluids are used in the equipment and machineries. For the flow analysis of couple stress hybrid nanofluid through permeable medium, the two terms skin friction and Nusselt number are more important. Skin friction is important to study the friction among the fluid and solid parts of the equipment, and the Nusselt number is calculated to examine the heat transfer rate through fluids. These two terms are explained below.

4.1. Skin Friction. The skin friction for the current phenomenon is defined mathematically in the nondimensional differential form as follows:

\[
(\text{Re})^{1/2} C_f = \frac{1}{(1 - \phi_1 - \phi_2) f''(0)}, \quad (\text{Re})^{1/2} C_g = \frac{1}{(1 - \phi_1 - \phi_2) g''(0)}.
\]

4.2. Local Nusselt Constant. The Nusselt number for the current problem is defined mathematically in the nondimensional differential form as follows:

\[
\frac{1}{2} (\text{Re})^{1/2} \text{Nu} = -\frac{k_{\text{fluid}}}{\kappa_f} + \frac{4}{3} \text{Rd} \left( \frac{1}{\partial_w + s_i} + 1 \right)^3 \theta'(0).
\]

5. Solution by HAM

The semianalytic technique HAM is applied to our modeled problem given in equations (12)–(15) by utilizing the boundary conditions in equation (16). The schematic representation of the HAM technique is as follows.

The linear operators \( L_f, L_g, L_{\phi} \), and \( L_{\phi} \) for the HAM are demarcated as

\[
L_f \left( f \right) = f', \quad L_g \left( g \right) = g', \quad L_{\phi} \left( \phi \right) = \phi', \quad L_{\phi} \left( \phi \right) = \phi'.
\]

The nonlinear operators \( N_f, N_g, N_{\phi} \), and \( N_{\phi} \) for the HAM are defined as follows:

\[
N_f \left[ f \left( \xi ; p \right), g \left( \xi ; p \right) \right] = \frac{1}{\left( 1 - \phi_1 - \phi_2 \right) \left( 1 - \phi_1 - \phi_2 + \left( \rho_1 / \rho_f \right) \phi_1 + \left( \rho_2 / \rho_f \right) \phi_2 \right)} \left( \begin{array}{c} \text{Re} f' \left( \xi ; p \right) - \frac{1}{\partial_w + s_i} f' \left( \xi ; p \right) \\ \text{Re} g' \left( \xi ; p \right) - \frac{1}{\partial_w + s_i} g' \left( \xi ; p \right) \end{array} \right).
\]

\[
N_g \left[ f \left( \xi ; p \right), g \left( \xi ; p \right) \right] = \frac{1}{\left( 1 - \phi_1 - \phi_2 \right) \left( 1 - \phi_1 - \phi_2 + \left( \rho_1 / \rho_f \right) \phi_1 + \left( \rho_2 / \rho_f \right) \phi_2 \right)} \left( \begin{array}{c} \text{Re} f' \left( \xi ; p \right) - \frac{1}{\partial_w + s_i} f' \left( \xi ; p \right) \\ \text{Re} g' \left( \xi ; p \right) - \frac{1}{\partial_w + s_i} g' \left( \xi ; p \right) \end{array} \right).
\]

\[
N_{\phi} \left[ f \left( \xi ; p \right), g \left( \xi ; p \right), \phi \left( \xi ; p \right) \right] = \frac{1}{\left( 1 - \phi_1 - \phi_2 \right) \left( 1 - \phi_1 - \phi_2 + \left( \rho_1 / \rho_f \right) \phi_1 + \left( \rho_2 / \rho_f \right) \phi_2 \right)} \left( \begin{array}{c} \text{Re} f' \left( \xi ; p \right) - \frac{1}{\partial_w + s_i} f' \left( \xi ; p \right) \\ \text{Re} g' \left( \xi ; p \right) - \frac{1}{\partial_w + s_i} g' \left( \xi ; p \right) \end{array} \right).
\]
The 0th order solution is as follows:

\[
(1-p)L_j \left[ \tilde{f}(\xi; p) - \tilde{f}_0 \right] = phN_j \left[ \tilde{f}(\xi; p), \tilde{g}(\xi; p) \right],
\]

\[
(1-p)L_g \left[ \tilde{g}(\xi; p) - \tilde{g}_0 \right] = phN_g \left[ \tilde{f}(\xi; p), \tilde{g}(\xi; p) \right],
\]

\[
(1-p)L_\theta \left[ \tilde{\theta}(\xi; p) - \tilde{\theta}_0 \right] = phN_\theta \left[ \tilde{f}(\xi; p), \tilde{g}(\xi; p); \tilde{\phi}(\xi; p) \right],
\]

\[
(1-p)L_\phi \left[ \tilde{\phi}(\xi; p) - \tilde{\phi}_0 \right] = phN_\phi \left[ \tilde{f}(\xi; p), \tilde{g}(\xi; p); \tilde{\phi}(\xi; p) \right].
\]

The conditions at the boundary point for the current phenomenon are transformed to nondimensional form as follows:

\[
\tilde{f}(\xi; p) \bigg|_{\xi=0} = 0, \quad \frac{\partial \tilde{f}(\xi; p)}{\partial \xi} \bigg|_{\xi=0} = \frac{\gamma}{\Delta} \frac{\partial \tilde{f}(\xi; p)}{\partial \xi} \bigg|_{\xi=0},
\]

\[
= 1 - \xi_0, \quad \tilde{\phi}(\xi; p) \bigg|_{\xi=0} = 1,
\]

\[
\tilde{f}(\xi; p) \bigg|_{\xi=\infty} = 0, \quad \tilde{g}(\xi; p) \bigg|_{\xi=\infty} = 0, \quad \tilde{\theta}(\xi; p) \bigg|_{\xi=\infty} = 0, \quad \tilde{\phi}(\xi; p) \bigg|_{\xi=\infty} = 0.
\]

(28)

In the above system, \( \tilde{p} \) represents the embedding parameter lies in the range \( 0 \leq \tilde{p} \leq 1 \). To regular the solution convergence for the problem, we choose the parameters \( h_f, h_g, h_\theta, \) and \( h_\phi \) in a best range. At the extreme limits of the embedding parameter \( \tilde{p} \) that is \( p = 0 \) and \( p = 1 \), the problem solution reduces to the expressions as

\[
\tilde{f}(\xi; 0) = \tilde{f}_0(\xi), \quad \tilde{g}(\xi; 0) = \tilde{g}_0(\xi), \quad \tilde{\theta}(\xi; 0) = \tilde{\theta}_0(\xi), \quad \tilde{\phi}(\xi; 0) = \tilde{\phi}_0(\xi),
\]

(29)

Expand the functions \( \tilde{f}(\xi; p), \tilde{g}(\xi; p), \tilde{\theta}(\xi; p), \) and \( \tilde{\phi}(\xi; p) \) by using the Taylor's series.

\[
f(\xi; p) = f_0(\xi) + \sum_{j=1}^{\infty} f_j(\xi)p^j, \quad g(\xi; p) = g_0(\xi) + \sum_{j=1}^{\infty} g_j(\xi)p^j,
\]

\[
\theta(\xi; p) = \theta_0(\xi) + \sum_{j=1}^{\infty} \theta_j(\xi)p^j, \quad \phi(\xi; p) = \phi_0(\xi) + \sum_{j=1}^{\infty} \phi_j(\xi)p^j.
\]

(30)

Where

\[
f_j(\xi) = \frac{1}{j!} \frac{\partial^j}{\partial \xi^j} f(\xi; p) \bigg|_{p=0}, \quad g_j(\xi) = \frac{1}{j!} \frac{\partial^j}{\partial \xi^j} g(\xi; p) \bigg|_{p=0},
\]

\[
\theta_j(\xi) = \frac{1}{j!} \frac{\partial^j}{\partial \xi^j} \theta(\xi; p) \bigg|_{p=0}, \quad \phi_j(\xi) = \frac{1}{j!} \frac{\partial^j}{\partial \xi^j} \phi(\xi; p) \bigg|_{p=0}.
\]

(31)

The \( j^{th} \) order solution for the current problem is

\[
L_j \left[ \tilde{f}_j(\xi) - \chi_j \tilde{f}_{j-1}(\xi) \right] = h_j \tilde{R}_f^j(\xi),
\]

\[
L_g \left[ \tilde{g}_j(\xi) - \chi_j \tilde{g}_{j-1}(\xi) \right] = h_g \tilde{R}_g^j(\xi),
\]

\[
L_\theta \left[ \tilde{\theta}_j(\xi) - \chi_j \tilde{\theta}_{j-1}(\xi) \right] = h_\theta \tilde{R}_\theta^j(\xi),
\]

\[
L_\phi \left[ \tilde{\phi}_j(\xi) - \chi_j \tilde{\phi}_{j-1}(\xi) \right] = h_\phi \tilde{R}_\phi^j(\xi).
\]

(32)

Here,

\[
\tilde{R}_f^j(\xi) = \frac{1}{(1 - \phi_1 - \phi_2)} \left( \frac{1}{1 - \phi_1 - \phi_2 + \left( \frac{\rho_f}{\rho_p} \right) \phi_1 + \left( \frac{\rho_f}{\rho_p} \right) \phi_1} \right)
\]

\[
\cdot \left( 2 \sum_{r=0}^{j-1} \tilde{g}_{j-1-r} \tilde{f}_{r}' + \frac{1}{2 \lambda \text{Re}} \frac{1 + \frac{d^* e^{-\xi}}{1 + d e^{-\xi}}} \right)
\]

\[
- F_r \left( \frac{1 + \frac{d^* e^{-\xi}}{1 + d e^{-\xi}}} \right) \left( \sum_{r=0}^{j-1} \tilde{g}_{j-1-r} \tilde{g}_{r}' + \frac{1}{2} \sum_{r=0}^{j-1} \tilde{f}_{j-1-r} \tilde{f}_{r}' \right)
\]

\[
- \sum_{r=0}^{j-1} \tilde{f}_{j-1-r} \tilde{f}_r + 2 \sum_{r=0}^{j-1} \tilde{f}_{j-1-r} \tilde{f}_r ' - \frac{\sigma_{\text{had}}/j_f}{\rho_{\text{had}}/\rho_f} M \tilde{g}_r
\]

\[
+ 4 \frac{\rho_f}{\rho_{\text{had}}} K_f \tilde{f}_r',
\]

\[
\tilde{R}_g^j(\xi) = \frac{1}{(1 - \phi_1 - \phi_2)} \left( \frac{1}{1 - \phi_1 - \phi_2 + \left( \frac{\rho_f}{\rho_p} \right) \phi_1 + \left( \frac{\rho_f}{\rho_p} \right) \phi_1} \right)
\]

\[
\cdot \left( \frac{1 + \frac{d^* e^{-\xi}}{1 + d e^{-\xi}}} \right) \left( \sum_{r=0}^{j-1} \tilde{g}_{j-1-r} \tilde{g}_{r}' + \frac{1}{2} \sum_{r=0}^{j-1} \tilde{f}_{j-1-r} \tilde{f}_{r}' \right)
\]

\[
- \sum_{r=0}^{j-1} \tilde{f}_{j-1-r} \tilde{f}_r + 2 \sum_{r=0}^{j-1} \tilde{f}_{j-1-r} \tilde{f}_r ' - \frac{\sigma_{\text{had}}/j_f}{\rho_{\text{had}}/\rho_f} M \tilde{g}_r
\]

\[
+ 4 \frac{\rho_f}{\rho_{\text{had}}} K_f \tilde{f}_r',
\]

\[
\tilde{R}_\theta^j(\xi) = \tilde{\phi}_r ' - \frac{\sigma_{\text{had}}/j_f}{\rho_{\text{had}}/\rho_f} M \tilde{g}_r
\]

\[
+ \frac{\rho_f}{\rho_{\text{had}}} K_f \tilde{f}_r '.
\]

(33)

(34)

(35)

(36)
6. Result and Discussion

Figure 1 shows the physical flow of the couple stress fluid past a rotatory disc. Figures 2–6 show the momentum profile $f(\eta)$ for various physical parameters, while Figures 7–9 demonstrate the impact of diverse physical parameters over the momentum profile $g(\eta)$. Figures 10–13 are the representation of concentration against different physical parameters. The implications of novel physical parameters on the energy profile of the couple stress nanofluid are represented in Figures 14–20. Figure 2 indicates the effect of local inertial parameter $Fr$ on the momentum profile $f(\eta)$. Figure 2 indicates that the momentum profile decreases by enhancing the local inertial parameter $Fr$. Physically the increase in inertial parameter the increase in resistance in medium means decrease in the permeability of the medium as a result the decrease in momentum of the fluid. Figure 3 displays the impact of couple stress parameter $K$ over the momentum profile $f(\eta)$. This Figure 3 demonstrates that momentum profile boosts with the escalation of couple stress parameter. Physically the increase in couple stress parameter means the decrease in viscosity of the fluid so it is obvious that non-viscous fluid moves rapidly. Figure 4 portrays the relation of porosity parameter $\lambda$ versus momentum profile $f(\eta)$ of the couple stress nanofluid. Figure 4 indicates that there is direct relation of porosity parameter $\lambda$ with the momentum profile $f(\eta)$ of the couple stress nanofluid. Physically, it is understood that the fluid moves fast in a porous medium with more permeability. Figure 5 displays the impression of Hartmann number $M$ over the momentum profile $f(\eta)$. From Figure 5, we noted that as we enhance the Hartmann number the momentum profile declines. Physically, the increase in Hartmann number creates Lorentz force, which provides the opposing force to the flow as a result retardation in the flow. Another very novel physical parameter in the motion of nanofluid is the Reynolds number $Re$; the impact of this Reynolds number on the momentum profile $f(\eta)$ of the couple stress nanofluid is portrayed in Figure 6. From this Figure 6, it is observed that as the Reynolds number $Re$ increases, the momentum profile $f(\eta)$ upsurges.

The impact of local inertial parameter $Fr$ on the momentum profile $g(\eta)$ is portrayed in Figure 7. From the same figure, it is noted that momentum profile $g(\eta)$ declines with escalation of inertial parameter $Fr$. The impact of couple stress parameter $K$ over the momentum profile $g(\eta)$ is demonstrated in Figure 8. From Figure 8, it is distinguished that the momentum profile $g(\eta)$ declines by the enhancing of couple stress parameter. Figure 9 portrays the relation of porosity parameter $\lambda$ versus momentum profile $g(\eta)$ of the couple stress nanofluid. Figure 9 indicates that there is direct relation of porosity parameter $\lambda$ with the momentum profile $g(\eta)$ of the couple stress nanofluid.

Figure 10 indicates the effect of Brownian parameter $Nb$ over the concentration profile $\phi(\eta)$ of the couple stress nanofluid past a rotting disc. From Figure 10, it is noted that the concentration profile $\phi(\eta)$ of the couple stress nanofluid declines with the escalation of the Brownian parameter $Nb$. Figure 11 demonstrates the impact of thermophoresis parameter $Nt$ over the concentration profile $\phi(\eta)$ of the couple stress nanofluid. From Figure 11, we observed that as the thermophoresis parameter enhances the concentration profile of the nanofluid enhances. From Figure 12, it is noted that the concentration $\phi(\eta)$ of the nanoparticles becomes low where the Reynolds number $Re$ is high during the flow. Figure 13 portrayed the influence of Schmidt number $Sc$ over the concentration profile $\phi(\eta)$ of the nanofluid. From Figure 13, it is obvious that the concentration $\phi(\eta)$ of the couple stress nanofluid declines with the escalation of the Schmidt number $Sc$.

Figure 14 is the sketch of energy profile $\theta(\eta)$ against local inertial parameter $Fr$. From Figure 14, it is very clear that the energy profile $\theta(\eta)$ of the couple stress nanofluid enhances as we increase the inertial parameter $Fr$ of the medium. Physically, the increase in inertial parameter means the decrease in permeability of the medium which reduces the pace of the fluid provides more time to fluid in contact with hot wall which enhances the temperature profile of the couple stress hybrid nanofluid. Figure 15 demonstrates the effect of couple stress parameter $K$ over the energy profile $\theta(\eta)$ of the flowing couple stress nanofluid. It is very clear from Figure 15 that the energy profile $\theta(\eta)$ drops as the couple stress feature of the fluid boosts. Figure 16 is the sketch of energy profile $\theta(\eta)$ of the couple stress nanofluid against the Brownian parameter $Nb$. Figure 17 demonstrates the effect of thermophoresis parameter $Nt$ over the energy profile $\theta(\eta)$ of the nanofluid. It is obvious from Figure 17 that...
\[M = 0.2, Fr = 0.2, \\
Re = 10, \lambda = 0.5,  \\
\Phi 1 = 12\% \text{ and } \Phi 2 = 13\%\]

\[\eta = 0.2, \lambda = 0.4,  \\
\lambda = 0.6, \lambda = 0.8 \]

\[\eta = 0.2, \lambda = 0.4,  \\
\lambda = 0.6, \lambda = 0.8 \]

\[\eta = 0.2, \lambda = 0.4,  \\
\lambda = 0.6, \lambda = 0.8 \]

**Figure 3:** Influence of couple stress parameter \(K\) over momentum profile \(f(\eta)\).

**Figure 4:** Influence of porosity parameter \(\lambda\) over momentum profile \(f(\eta)\).

**Figure 5:** Influence of Hartmann number \(M\) over momentum profile \(f(\eta)\).
\[ M = 0.3, \ K = 0.17, \ Fr = 0.35, \ \lambda = 0.5, \ \Phi_1 = 12\% \ \text{and} \ \Phi_2 = 13\% \]

**Figure 6:** Impact of Reynolds number Re over momentum profile \( f(\eta) \).

\[ \eta \]

- \( \text{Re} = 2 \)
- \( \text{Re} = 3 \)
- \( \text{Re} = 4 \)
- \( \text{Re} = 5 \)

\[ \eta \]

**Figure 7:** Impact of local inertial parameter Fr over momentum profile \( g(\eta) \).

\[ \eta \]

- \( \text{Fr} = 0.2 \)
- \( \text{Fr} = 0.3 \)
- \( \text{Fr} = 0.4 \)
- \( \text{Fr} = 0.5 \)

**Figure 8:** Impact of couple stress parameter \( K \) over momentum profile \( g(\eta) \).

\[ \eta \]

- \( K = 1.1 \)
- \( K = 1.4 \)
- \( K = 1.6 \)
- \( K = 1.8 \)
$M = 0.2, Fr = 0.35, K = 3, Re = 10, \Phi_1 = 12\%$ and $\Phi_2 = 13\%$

Figure 9: Impact of porosity parameter $\lambda$ over momentum profile $g(\eta)$.

$M = 2, K = 1.5, Pr = 0.4, Sc = 0.27, Nt = 0.3, Fr = 0.4, \Phi_1 = 12\%, \Phi_2 = 13\%$

Figure 10: Impact of Brownian motion parameter $Nb$ over concentration profile $\phi(\eta)$.

$M = 2, K = 1.5, Pr = 0.4, Sc = 0.25, Fr = 0.4, Nb = 0.35, \Phi_1 = 12\%$ and $\Phi_2 = 13\%$

Figure 11: Impact of thermophoresis parameter $Nt$ on the concentration profile $f(\eta)$.  

$Nt = 0.2$ $Nt = 0.3$ $Nt = 0.4$ $Nt = 0.5$
Figure 12: Impact of Prandtl number Pr on the concentration profile $\phi(\eta)$.

Figure 13: Impact of Schmidt number Sc on the concentration profile $\phi(\eta)$.

Figure 14: Influence of local inertial parameter Fr over energy profile $\theta(\eta)$. 

\[ M = 2, K = 1.5, Fr = 0.4, Sc = 0.25, \\
Nt = 0.3, Nb = 0.35, \\
\Phi_1 = 12\%, \Phi_2 = 13\% \]
Figure 15: Impact of couple stress parameter $K$ on the energy profile $\theta(\eta)$.

Figure 16: Impact of Brownian motion parameter $Nb$ over energy profile $\theta(\eta)$.

Figure 17: Impact of thermophoresis parameter $Nt$ over energy profile $\theta(\eta)$. 
\( M = 2, K = 1.5, Pr = 0.4, Sc = 0.25, \)
\( Nt = 0.3, Nb = 0.35, Fr = 0.4, \Phi_2 = 13\% \)

\( \Phi_1 = 09\% \)
\( \Phi_1 = 11\% \)
\( \Phi_1 = 10\% \)
\( \Phi_1 = 12\% \)

**Figure 18:** Impact of nanoparticle fraction \( \Phi_1 \) over energy profile \( \theta(\eta) \).

\( M = 2, K = 1.5, \)
\( Pr = 0.4, Sc = 0.25, \)
\( Nt = 0.3, Nb = 0.35, \Phi_1 = 12\%, \Phi_2 = 13\% \)

\( \Phi_2 = 09\% \)
\( \Phi_2 = 11\% \)
\( \Phi_2 = 10\% \)
\( \Phi_2 = 12\% \)

**Figure 19:** Influence of nanoparticle fraction \( \Phi_2 \) over energy profile \( \theta(\eta) \).

\( M = 2, K = 1.5, Fr = 0.4, Sc = 0.25, \)
\( Nt = 0.3, Nb = 0.35, \Phi_1 = 12\%, \Phi_2 = 13\% \)

\( Pr = 0.2 \)
\( Pr = 0.6 \)
\( Pr = 0.4 \)
\( Pr = 0.8 \)

**Figure 20:** Effect of Prandtl number \( Pr \) over momentum profile \( f(\eta) \).
the energy profile $\theta(\eta)$ boosts as the thermophoresis parameter $N_t$ upsurges. Figure 18 is the plot of energy profile $\theta(\eta)$ of the flowing fluid against the volume fraction $\phi_1$ of the titanium oxide TiO$_2$. From Figure 18, it is very clear that the energy profile $\theta(\eta)$ enhances with the escalation of nanoparticles volume fraction $\phi_1$ (9%, 10%, 11%, and 12%) of the titanium oxide TiO$_2$ in the base fluid (H$_2$O). Figure 19 indicates that as we increase the volume fraction $\phi_1$ (9%, 10%, 11%, and 12%) of the nanoparticles, aluminum oxide Al$_2$O$_3$ enhancing the energy profile $\theta(\eta)$ of the couple stress nanofluid declines. Figure 20 indicates the relation of energy profile $\theta(\eta)$ with the Prandtl number $Pr$. From Figure 20, it is noted that the energy profile enhances as the Prandtl number $Pr$ enhances.

Figures 21 and 22 show the combine impact of local inertial parameter Fr and couple stress parameter $K$ over the skin friction $f''(0)$ and $g'(0)$. From Figures 21 and 22, we see clearly that with the increase in inertial parameter Fr the decrease in permeability increase the pressure drop due to this the velocity of the nanofluid decreases this generates the reduction in the skin friction $Re^{1/2}C_f$ and $Re^{1/2}C_g$. Also, we see from Figures 21 and 22 that with the

![Figure 21](image1.png)  
**Figure 21:** Effect of local inertia parameter Fr and couple stress parameter $K$ on skin friction $f''(0)$.

![Figure 22](image2.png)  
**Figure 22:** Impact of local inertia parameter Fr and couple stress parameter $K$ on skin friction $g'(0)$.

| Physical properties                      | Base fluid H$_2$O | TiO$_2$ | Solid nanoparticles Al$_2$O$_3$ |
|------------------------------------------|-------------------|---------|-------------------------------|
| Density $\rho$ (kg/m$^3$)                | 997.1             | 4230    | 4000                          |
| Thermal conductivity $\kappa$ (W/mK)     | 0.613             | 8.4     | 40                            |
| Specific heat capacity $c_p$ (J/kgK)     | 4179              | 692     | 773                           |
| Electric conductivity $\sigma$ (Sm$^{-1}$) | $5.5 \times 10^{-6}$ | $2.38 \times 10^6$ | $5.96 \times 10^7$ |

Table 1: Thermophysical properties of H$_2$O, titanium oxide, and aluminum oxide.
Table 2: Impact of some physical parameters over skin friction coefficient Re$^{1/2}C_f$ and Re$^{1/2}C_g$ when $R = 0.3$, $\gamma^* = 0.3$, $\gamma = 0.3$ when $\phi_1 = 12\%$ and $\phi_2 = 13\%$.

| Pr | M  | $K$ | Re | $\lambda$ | Re$^{1/2}C_f$ | Re$^{1/2}C_g$ |
|----|----|----|----|---------|-------------|-------------|
| 0.5 | 0.45 | 0.35 | 0.25 | 0.5 | 0.60245520 | 0.7633975 |
| 0.6 | | | | | 0.3533196 | 0.4935312 |
| 0.7 | | | | | 0.08601030 | 0.2042981 |
| 0.5 | | | | | 0.0835144 | 0.2017363 |
| 0.6 | | | | | 0.0785259 | 0.1966159 |
| 0.7 | | | | | 0.0735418 | 0.1914999 |
| 0.5 | | | | | 0.2390841 | 0.3674829 |
| 0.6 | | | | | 0.3472050 | 0.4823447 |
| 0.7 | | | | | 0.4601849 | 0.620653 |
| 0.5 | | | | | 0.1058902 | 0.2247019 |
| 0.6 | | | | | 0.1092103 | 0.2281093 |
| 0.7 | | | | | 0.1115829 | 0.2305443 |
| 0.5 | | | | | 0.0860103 | 0.2042981 |
| 0.6 | | | | | 0.0926293 | 0.2110917 |
| 0.7 | | | | | 0.0973618 | 0.2159489 |

increase in couple stress parameter the velocity field enhances due to which the skin friction Re$^{1/2}C_g$ enhances. Table 1 shows the thermophysical properties of the base fluid (water H$_2$O) and solid nanoparticles (TiO$_2$, Al$_2$O$_3$). Also, Table 2 displays the impact of various physical parameters over the skin friction coefficient when the nanoparticle volume fraction in the base fluid is $\phi_1 = 12\%$(TiO$_2$) and $\phi_2 = 13\%$(Al$_2$O$_3$). Table 3 displays the impact of various physical parameters over the heat transfer rate through the couple stress hybrid nanofluid when there is 12% volume fraction of the titanium oxide (TiO$_2$) nanoparticles and 13% volume fraction of the aluminum oxide (Al$_2$O$_3$) nanoparticles in 75% water (H$_2$O).

7. Main Findings

(i) Velocity $f(\eta)$ enhances with the increase in local inertial parameter Pr, couple stress parameter $K$, porosity parameter $\lambda$, and Reynolds number Re but declines by the escalation in Hartmann number $M$. Also, the velocity $g(\eta)$ decreases with the increase in local inertial parameter Pr and porosity parameter $\lambda$ but decreases with an escalation in couple stress parameter $K$.

(ii) Concentration profile $\phi(\eta)$ decreases with the increase in Nb, Pr, and Sc but enhances with $M$.

(iii) Temperature profile $\theta(\eta)$ decreases with the increase in Nb, $\phi_2$, and $K$ but enhances with $Nt$, Nb, $\phi_1$, and Pr.

(iv) Skin friction enhances with the increase in local inertial parameter Pr, couple stress parameter $K$, Reynolds number Re, and porosity parameter $\lambda$ but decreases for Hartmann number $M$. Also, the heat transfer rate Re$^{1/2}Nu$ enhances for radiation parameter Rd but decreases in $\theta^\ast_w$ (variable for temperature), $s_t$ (the thermal stratification parameter), thermophoresis parameter $Nt$, and Brownian parameter $Nb$.

Abbreviations

| Abbreviation | Description |
|--------------|-------------|
| RK | Runge-Kutta |
| TD | Tiwari and Das |
| MG | Maxwell-Garnett |
| HAM | Homotopy analysis method |
| PDEs | Partial differential equations |
| ODEs | Ordinary differential equation |
| MRT | Multirelaxation time |
| LBM | Lattice Boltzmann method |

Nomenclature

$u, v, w$: Velocity components (m)
$(r, \psi, z)$: Coordinate direction
$\varepsilon(z)$: Variable porosity function
$K(z)$: Variable permeability function
$\sigma$: Electric conductivity (S/m)
$\sigma^*$: Stefan-Boltzmann constant (Wm$^{-2}$K$^{-4}$)
$\beta_0$: Magnetic field (Tesla)
$\rho$: Density (kg/m$^3$)
$\eta^\ast_0$: Material constant for couple stress parameter
$C_\beta$: Drag coefficient
$k$: Mean absorption coefficient
$D_b$: Brownian diffusion coefficient (m$^2$/s)
$D_T$: Thermophoresis diffusion coefficient
$\tau$: Ratio between heat capacity of solid particles to heat capacity of base fluid
$\Omega$: System angular velocity (rev/min)
$L$: Velocity slip coefficient
$\varepsilon_{w0}$: Porosity proportionality constant
$K_{w0}$: Permeability proportionality constant
$T_w$: Thermal stratification at the disc (K)
$T_{w0}$: Thermal stratification at the free stream (K)
$T_0$: Reference temperature (K)
$\lambda$: Porosity parameter
$Re$: Reynolds number
$d$: Variable permeability
$d^*$: Variable porosity
$Fr$: Local inertial parameter
$M$: Hartman number
$K$: Couple stress parameter
$R$: Radiation parameter
$Pr$: Prandtl number
$Nt$: Thermophoresis parameter
$Nb$: Brownian motion parameter
$Sc$: Schmidt number
$\gamma$: Parameter
$\kappa$: Thermal conductivity (Wm$^{-1}$K$^{-1}$)
$c_p$: Specific heat capacity (J/K)
$\eta$: Material constant for couple stress parameter
$\alpha_{ahnf}$: Thermal diffusivity of hybrid nanofluid
$\theta_w$: Variable for temperature
$\Theta_{w0}$: Thermal stratification parameter

Subscript

hnf: Hybrid nanofluid
$f$: Base fluid
$0$: Reference point
1: Related to first solid nanoparticles
2: Related to second solid nanoparticles
$w$: At the wall level
$\infty$: At the free stream level.

Data Availability

Complete data available inside the manuscript.

Conflicts of Interest

The authors declare that they have no competing interest.

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