Memristance in human skin

Ø.G. Martinsen1,2, S. Grimnes1,2, C.A. Lütken1 and G.K. Johnsen1

1. Dept. of Physics, University of Oslo, Norway
2. Dept. of Biomed. and Clin. Eng., Oslo University Hospital, Rikshospitalet, Norway

E-mail: ogm@fys.uio.no

Abstract. The memristor is basically a resistor with memory, so that the resistance is dependent on the net amount of charge having passed through the device. It is the regarded the fourth fundamental component, in addition to the resistor, capacitor and inductor, that can be deduced from the four basic circuit variables; current, voltage, charge and magnetic flux. We show that memristors can be used for modelling electrical properties of human skin. In particular is electro-osmosis in human sweat ducts of memristive nature.

1. Introduction to memristance

The four basic circuit variables, current $i$, voltage $v$, charge $q$ and magnetic flux $\phi$, give six different, possible combinations. Five of them are the definitions of capacitance, inductance and resistance, the definition of electric current and Faraday’s law of induction. The last combination is the ratio of magnetic flux to electric charge, now called memristance [1]. However, the memristance is not necessarily linked to magnetic systems and can be regarded as the ratio between the time dependent voltage and current:

$$M \equiv \frac{d\phi}{dq} = \frac{v_{dt}}{i_{dt}} = \frac{v(t)}{i(t)} [\Omega]$$

The memristor can be thought of as a variable resistor where the resistance is dependent on the amount of charge having passed the device in a given direction. Predicted to exist on purely theoretical grounds many years ago [2], memristors did not gain acceptance as a new passive circuit element that complements the conventional ones (RLC) until a team from Hewlett-Packard recently announced that they had invented a nano-memristor that is fully compatible with conventional semi-conductor technology [3].

Figure 1: Applied potential (dashed line) and current response on ventral forearm [1,2].
2. Electroosmosis

The measured electrical current response of human skin shown in fig. 1 (reproduced from [4]), which was obtained using dry disc electrodes, was explained in [5] by electro-osmotic transport of fluid through pores in the skin, mostly the sweat ducts. In dry skin most of the ducts are empty or partially filled, but a conductive fluid film normally covers the inner wall of the duct, as shown in fig. 2.

When an electric field is applied the fluid column in the filled part of the duct is dragged along by the mobile part of the double layer near the wall. The amount of liquid dragged along is determined by the electric field strength $E$ and electro-kinetic (zeta-) potential $\zeta$ at the interface; the dielectric constant $\varepsilon$, resistivity $\rho$ and viscosity $\eta$ of the fluid; as well as the geometry of the duct [2]:

$$\frac{dV}{dt} = \alpha AE,$$  \hspace{1cm} (2)

where $dV$ is the volume of liquid transported in time $dt$ under the influence of the electric field $E$. $A = \pi r^2$ is the cross-sectional area of the duct, and $\alpha = \frac{\zeta \varepsilon}{4 \pi \eta}$.

![Figure 2: Schematic model of sweat duct in human epidermis.](image)

The average resistance in the filled part of the duct is significantly smaller than in the thin fluid film covering the wall of an otherwise empty duct. Because the location $w(t)$ of the fluid/gas interface (“water table”) is a function of the electrical history of the duct, this system has some memory and is hysteretic. The main feature of the mathematical model developed in [5] is that it attempts to take this into account, by coupling the change in conductivity to the motion of the fluid.

3. Memristive model

Our objective is to reexamine and refine this explanation, taking into account some effects that were neglected in previous work [6]. As a first step we include a kind of feedback effect: when the water table moves a large effective resistivity is traded for a smaller one, or vice versa. The resistance in the water column is $R_w(t) = \rho w(t)/A$, and the resistance in the film is $R_f(t) = \rho (D - w(t))/a$. $D$ is the total length of the duct and $a = 2\pi d$, with $d \ll r$ denoting the thickness of the film. The total resistance is then:

$$M = R_w + R_f = R(\tau + (1-\tau)x),$$  \hspace{1cm} (3)

with $R = \rho D/A$, $\tau = r/2d$ and $x = w/D$, whence $0 \leq x \leq 1$.

This type of immittance is logically and physically distinct from impedance and is called memristance [1,7,8]. (The analogue of admittance is called memductance.)

We argue that the memristive paradigm may be useful also in biophysics. In particular, we show that electro-osmosis in ducts can be usefully thought of as a memristive system [7]. This is a generalization of the memristor concept where the memristance is controlled by any number of additional state-variables (in our case just one, $x = x(t)$), which may also be coupled to each other in
complicated nonlinear ways. Thus, memristive systems should be rich enough to capture some of the nonlinearities that are ubiquitous in bioelectrics.

Consider first the “linear regime” where the water table remains at a finite distance from either end of the duct, $0 < x < 1$. Nonlinear effects at the ends where the liquid is injected and extracted are neglected here. Since the microscopic width $w(t)$ of the filled region is unobservable, we must relate it to macroscopic state-variables and device geometry that are easier to measure. Because the volume of fluid in our partially filled duct is $Adv$, we have $dV = Adv$, which combined with eq. (2) gives:

$$dx = \alpha \frac{E}{D} dt = \alpha \frac{\rho}{V} dq$$

(4)

Here $V = AD$ is the total volume of the duct, and we have assumed that $E$ is constant and that all the current $i(t) = dq/dt$ flows through the fluid column. Integrating eq. (4) gives $x(t) = x_0 + \alpha \rho q(t)/V$, which inserted in eq. (3) and using $\tau >> 1$, gives the charge controlled memristance of this system:

$$M(q) = \tau R(c_0 - \alpha \rho q/V),$$

(5)

where $0 < c_0 = 1 - x_0 < 1$ is determined by the initial position $w_0 = x_0 / D$ of the water table. Inserting this expression into $M(q)dq = v(t)dt$ and integrating we find the charge

$$q(t) = c_0 D^2 \left( \frac{1}{\rho R} - \frac{1}{\tau c_0 D^2} \left( \frac{2\alpha \rho \phi(t)}{\tau c_0 D^2} \right)^{1/2} \right)$$

(6)

as a function of the flux $\phi(t) = \int_0^t v(s)ds$. This gives the current:

$$i(t) = v(t) \left( c_0^2 - \frac{2\alpha \rho \phi(t)}{\tau D^2} \right)^{-1/2}$$

(7)

where $G_\tau = 1/\tau R$.

With a simple harmonic driving potential $v(t) = v_0 \sin(\omega t)$ this gives the $i$-$v$-characteristics shown in fig. 3 for three different frequencies. The signature of memristance is clearly visible at low frequencies: hysteresis anchored at the origin. At high frequency the memristance degenerates to ordinary resistance, giving the approximately linear characteristic seen in fig. 3.

![Figure 3](image1.png)

**Figure 3:** Parametric plot of the voltage and current of the “micro-memristive” model of skin capillaries described in the text, shown for three different frequencies. Inset: flux-charge characteristic.

![Figure 4](image2.png)

**Figure 4:** Input (thin line) and output (thick line) signals of the “micro-memristive” model of skin capillaries described in the text. The dashed line shows similar behavior in the model developed in ref.[5].
With this potential the flux is \( \phi(t) = (2v_0/\omega)\sin^2(\omega t/2) \), which gives the flux-charge characteristics shown in the inset in fig.3: it is single valued and frequency independent as it must be for a memristive system [1,7,8].

For suitable choices of parameters the current behaves as shown by the solid thick line in fig.4, when the driving potential is negative. For a limited range of parameters this is quite similar to the response found in [5], both experimentally and theoretically – compare the dashed line which shows similar behavior found in the model developed in [5] – thus demonstrating the viability of the memristive paradigm in this regime.

4. Discussion
Comparing the behavior of this memristive model with the data shown in fig. 1, we find that some of the main features agree, including the phase-shift, while others need more work. There are two main properties of the experimental response that remain to be explained: the increasing amplitude of the current with time, and the damping or “clipping” of the positive current.

We will in a future paper show that the increasing amplitude can be explained by “wetting” of the duct wall. As the fluid column oscillates it deposits more fluid on the liquid film coating the wall, thus decreasing the resistance in that part of the duct, at least temporarily. This wetting mechanism alone will only increase the current amplitude from the first to the second period of the applied potential. It will then reach a new, stable value. However, the lack of a reservoir of fluid at the top of the duct will lead to less transport of fluid away from the duct orifice than towards the orifice, and hence a net buildup of fluid in the duct over several cycles of the current. This explains the increasing amplitude with time demonstrated by Grimnes [5].

This other mechanism can be called “clipping”: when the potential repels the fluid film it eventually lets go of the surface electrode, and the current is cut. This gives rise to the characteristic spike in the positive current. The liquid then diffuses rapidly back to the electrode which reconnects the circuit, and the process repeats rapidly several times until the potential becomes insufficient to push the fluid away.

Hence, further improvements to the memristive model are possible. We have here also ignored all electrical activity in the stratum corneum itself. It is known [4] that this acts like a capacitive load coupled in parallel to the sweat capillaries, albeit a rather unusual one which is traditionally modeled using a device called a “constant phase element” (CPE) [4]. It is known that a CPE cannot be replaced by a finite number of conventional passive circuit elements (RLC). We shall return to this question elsewhere and argue that this activity in the epidermis is also memristive.

5. References
[1] Chua LO 1971 Memristor – the missing circuit element. IEEE Trans. Circuit Theory. CT-18(5):507-509
[2] Glasstone S 1946 Textbook on physical chemistry. Van Nostrand
[3] Strukov DB, Snider GS, Stewart DR, Williams RS (2008) The missing memristor found. Nature 453:80-83
[4] Grimnes S, Martinsen ØG 2008 Bioimpedance and bioelectricity basics. 2. ed. Academic Press
[5] Grimnes S 1983 Skin impedance and electro-osmosis in the human epidermis. Med. & Biol. Eng. & Comput 21:739-749
[6] Grimnes S, Lütken CA, Martinsen ØG 2009 Memristive properties of electro-osmosis in human sweat ducts. WC2009, IFMBE Proceedings. 25/VII: 696-698
[7] Chua LO, Kang SM 1976 Memristive devices and systems. Proc. IEEE 64:209-223
[8] Chua LO 2003 Nonlinear circuit foundations for nano-devices, Part1: the four-element torus. Proc. IEEE 91:1830-1859