On Learning and Testing of Counterfactual Fairness through Data Preprocessing

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ABSTRACT

Machine learning has become more important in real-life decision-making but people are concerned about the ethical problems it may bring when used improperly. Recent work brings the discussion of machine learning fairness into the causal framework and elaborates on the concept of Counterfactual Fairness. In this article, we develop the Fair Learning through dAta Preprocessing (FLAP) algorithm to learn counterfactually fair decisions from biased training data and formalize the conditions where different data preprocessing procedures should be used to guarantee counterfactual fairness. We also show that Counterfactual Fairness is equivalent to the conditional independence of the decisions and the sensitive attributes given the processed nonsensitive attributes, which enables us to detect discrimination in the original decision using the processed data. The performance of our algorithm is illustrated using simulated data and real-world applications. Supplementary materials for this article are available online.

1. Introduction

The rapid popularization of machine learning methods and the growing availability of personal data have enabled decision-makers from various fields such as graduate admission (Waters and Miikkulainen 2014), hiring (Ajunwa, Scheidegger, and Venkatasubramanian 2016), credit scoring (Thomas 2009), and criminal justice (Brennan, Dieterich, and Ehret 2009) to make data-driven decisions efficiently. However, the community and the authorities have also raised concern that these automatically learned decisions may inherit the historical bias and discrimination from the training data and would cause serious ethical problems when used in practice (Dwoskin 2015; Nature Editorial 2016; Angwin and Larson 2016; Executive Office of the President et al. 2016).

Consider a training dataset $D$ consisting of sensitive attributes $S$ such as gender and race, nonsensitive attributes $A$ and decisions $Y$. If the historical decisions $Y$ are not fair across the sensitive groups, a powerful machine learning algorithm will capture this pattern of bias and yield learned decisions $\hat{Y}$ that mimic the preference of the historical decision-maker, and it is often the case that the more discriminative an algorithm is, the more discriminatory it might be.

While researchers agree that methods should be developed to learn fair decisions, opinions vary on the quantitative definition of fairness. In general, researchers use either the observational or counterfactual approaches to formalize the concept of fairness. The observational approaches often describe fairness with metrics of the observable data and predicted decisions (Hardt, Price, and Srebro 2016; Chouldechova 2017; Yeom and Tschantz 2018). For example, Demographic Parity (DP) or Group Fairness (Zemel et al. 2013; Khademi et al. 2019) considers the learned decision $\hat{Y}$ to be fair if it has the same distribution for different sensitive groups, that is, $P(\hat{Y}|S = s) = P(\hat{Y}|S = s')$. The Individual Fairness (IF) definition (Dwork et al. 2012) views fairness as treating similar individuals similarly, which means the distance between $\hat{Y}(s_i, a_i)$ and $\hat{Y}(s_j, a_j)$ should be small if individuals $i$ and $j$ are similar.

The other branch of fairness and/or discrimination definitions are built upon the causal framework of Pearl (2009a), such as direct/indirect discrimination (Zhang, Wu, and Wu 2017; Nabi and Shpitser 2018), path-specific effect (Wu et al. 2019), counterfactual error rate (Zhang and Bareinboim 2018a) and counterfactual fairness (Kusner et al. 2017; Wang, Sridhar, and Blei 2019; Wu, Zhang, and Wu 2019). These definitions often involve the notion of counterfactuals, which means what the attributes or decision would be if an individual were in a different sensitive group. With the help of the potential outcome concept, the measuring of fairness is no longer restricted to the observable quantities (Kilbertus et al. 2017; Zhang and Bareinboim 2018b). For example, the Equal Opportunity (EO) definition (Wang, Sridhar, and Blei 2019) has the same idea as IF but it can directly compare the actual and counterfactual decisions of the same individual instead of the actual decisions of two similar individuals. The Counterfactual Fairness (CF) definition (Kusner et al. 2017) or equivalently, the Affirmative Action (AA) definition (Wang, Sridhar, and Blei 2019) goes one step further than EO and derives the counterfactual decisions from the counterfactual nonsensitive attributes. It first asks what the nonsensitive attributes $A$ would be had $S$ been different and then compare the counterfactual decisions of an individual given her/his counterfactual nonsensitive attributes. For example, a female student with a test score of 85 and a male...
student with the same score should have the same probability of being admitted under the Equal Opportunity definition. When considering the Counterfactual Fairness definition, we first imagine a counterfactual world where the female student was treated as a boy since she was born. There she received the same educational resources as her male siblings in her family and finally reached a score of 95. We would then use 95 as her counterfactual nonsensitive attribute and conclude that she should have a higher probability of being admitted than her male competitor. The fact that her score is 85 in the real world is due to the historical disadvantage of limited education resources and the Equal Opportunity definition will ignore this kind of unfairness. We adopt CF as our definition of fairness and it is formally described in Section 2. We believe causal reasoning is the key to fair decisions as DeDeo (2014) pointed out that even the most successful algorithms would fail to make fair judgments due to the lack of causal reasoning ability.

For the observational definitions, fair decisions can be learned by solving optimization problems, either adding the fairness condition as a constraint (Dwork et al. 2012) or directly optimize the fairness metric as an objective function (Zemel et al. 2013; Edwards and Storkey 2015; Zhang, Lemoine, and Mitchell 2018; Zhao et al. 2019). When using the counterfactual definitions, however, those optimization methods cannot be directly applied since the counterfactuals are unobservable. Thus, an approximation of the causal model or the counterfactuals is often needed. In the FairLearning algorithm proposed by Kusner et al. (2017), the unobserved parts of the graphical causal model are sampled using the Markov chain Monte Carlo method. Then they use only the nondescendants of $S$ to learn the decision, which ensures CF but will have a low prediction accuracy. In Wang, Sridhar, and Blei (2019), the counterfactual of $A$ had $S$ been $s'$ is imputed as the sum of the counterfactual group mean $E(A|S = s')$ and the residuals from the original group $A - E(A|S = s)$. As we discuss later, this approach would only work when a strong assumption of the relationship between $A$ and $S$ is satisfied.

1.1. Contributions

We develop the Fair Learning through dAta Preprocessing (FLAP) algorithm to learn counterfactually fair decisions from biased training data. While current literature is vague about the assumptions needed for their algorithms to achieve fairness, we formalize the weak and strong conditions where different data preprocessing procedures should be used to guarantee CF and prove the results under the causal framework of Pearl (2009a). We show that our algorithm can predict fairer decisions with similar accuracy when compared with other counterfactual fair learning algorithms using three simulated datasets and three real-world applications, including the loan approval data from a fintech company, the adult income data, and the COMPAS recidivism data.

On the other hand, the processed data also enable us to detect discrimination in the original decision. We prove that CF is equivalent to the conditional independence of the decisions and the sensitive attributes given the processed nonsensitive attributes under certain conditions. Therefore, any well-established conditional independence tests can be used to test CF with the processed data. To our knowledge, it is the first time that a formal statistical test for CF is proposed. We illustrate the idea using the Conditional Distance Correlation test (Wang et al. 2015) in our simulation and test the fairness of the decisions in the loan approval data using a parametric test.

2. Causal Model and Counterfactual Fairness

For the discussion below, we consider the sensitive attributes $S \in S$ to be categorical, which is a reasonable restriction for the commonly discussed sensitive information such as race and gender. The nonsensitive attributes $A \in A \subseteq \mathbb{R}^d$, and the decision $Y$ is binary as admit or not in graduate admission, hire or not in the hiring process, approve or not in loan assessment.

To bring the discussion of fairness into the framework of causal inference, we begin by constructing the Structural Causal Model (SCM) for the data. As described in Pearl (2009b), an SCM $M$ consists of a set of exogenous variables $U$, a set of endogenous variables $V$, and $F$, a set of functions that assign value to each endogenous variable given its parents in $V$ and the exogenous variables $U$. In our case (Figure 1), we consider $V = \{S, A, Y, \hat{Y}\}$, where $(S, A, Y)$ are the observed data and $\hat{Y}$ is the prediction of $Y$ we made based on $S$ and $A$. The only exogenous variable affecting $\hat{Y}$ is a Uniform(0, 1) random variable $U_{\hat{Y}}$ so that we can conveniently express the value of $\hat{Y}$ with a structural equation. We assume that $U_{S}$, $U_{A}$, and $U_{Y}$, which are the exogenous variables that affect $S$, $A$, and $Y$, respectively, are independent of each other. The structural equations on the right side of Figure 1 are described with the functions in $F$, one for each component in $V$. Here we express $f_{\hat{Y}}$ as an indicator function so that $\hat{Y}$ is a Bernoulli random variable that takes value one with probability $\rho(S, A)$. In general, $\rho(s, a)$ could be any function that maps $S \times A$ to $[0, 1]$, but we are more interested in such functions that will result in a fair decision, more details of which will be discussed in Section 3. It can be seen that the subset of exogenous variables $\{U_{S}, U_{A}, U_{Y}\}$ characterize everything we should know about a unit. Any two units with the same realization will have the same behavior and result irrespective of the different identities in their environments.
One important assumption included in our SCM is that there is no common cause of $S$ and $A$. In most cases where the sensitive attributes are biologically determined, this is a reasonable assumption. For example, it is hard to think of a cause of sex that will also affect nonsensitive attributes like education time and income.

Here we give a simplified loan approval model as a running example to help understand the SCM we considered.

**Definition 1.** Counterfactual Fairness. Given a new pair of attributes $(s^*, a^*)$, a (predicted) decision $Y$ is counterfactually fair if for any $s^* \in S$,

$$Y_{c}(U) | (S = s^*, A = a^*) \overset{d}{=} Y_{c}(U) | (S = s^*, A = a^*)$$

In other words, the conditional distribution of the counterfactual result should not depend on the sensitive attributes. It should be noted that there are two cases in evaluating the conditional counterfactuals $Y_{c}(U) | (S, A)$. The first stage is updating the conditional distribution of $U$ given $(S, A)$, and the second stage is deriving the conditional distribution of the counterfactuals from the structural equations of $M_t$ and the conditional distribution of $U$. Thus, $Y_{c}(U) | (S, A)$ depends on $S$ directly and on $(S, A)$ indirectly through $U$. Take the decision $Y$ from Example 1, if $s^* = 0$, then $U_S | (S = s^*, A = a^*)$ is from Uniform$(0, 1)$ and $U_A | (S = s^*, A = a^*)$ is a constant $(\log(a^*/c1) - c2)/c3$, but $U_Y | (S = s^*, A = a^*)$ is still a Uniform$(0, 1)$ random variable since $U_Y$ is independent of $S$ and $A$ from the SCM. In the second stage, $Y_{1}(U) | (S = 0, A = a^*)$ would be equal in distribution to

$$f_y(1, f_A(1, U_A), U_Y) | (S = 0, A = a^*)$$

and $Y_{0}(U) | (S = 0, A = a^*) \overset{d}{=} 1(U_Y < \expit(\beta_0 + \beta_a a^* + a^* (1 - \sigma_a) c2 + \beta_s))$. Thus, the bank’s decision $Y$ would be counterfactually fair if $\sigma_a = 1$, $\lambda_a = 0$ and $\lambda_s = 0$.

**3. Preprocessing, Learning, and Testing**

Our data preprocessing methods try to remove from $A$ the sensitive information carried by $S$ while retaining the information from $U_A$. In the following, we first present our preprocessing methods and then give an intuition for why they would work.

Define a preprocessing procedure $P^{D}(s, a) : S \times A \rightarrow A^\prime$ to be a function that maps attributes $(s, a)$ to the processed attributes $a^\prime$ given the training data $D$. Here we consider two such procedures. Denote $P_n(S = s)$ as the empirical p.m.f. of $S$ and $E_n(A|S = s)$ as the empirical conditional mean of $A$ given $S$ learned from data $D$.

**Definition 2 (Orthogonalization).** An orthogonalization procedure $P^{D}_{O}$ is a preprocessing procedure such that

$$P^{D}_{O}(s, a^*) = \sum_s \hat{a}(s)P_n(S = s),$$

where $\hat{a}(s) = a^* - E_n(A|S = s^*) + E_n(A|S = s), \forall s \in S$.

Denote $P_{hs}(x) = P_n(A_j \leq x|S = s)$ as the empirical marginal cumulative distribution function (CDF) of the $j$th element of the nonsensitive attributes given the sensitive attribute $S = s$. Define its inverse as

$$P_{hs}^{-1}(z) = \inf\{x : P_n(A_j \leq x|S = s) \geq z\}.$$  (1)
Definition 3 (Marginal Distribution Mapping). A marginal distribution mapping \( \mathcal{P}_D^O \) is a pre-processing procedure such that

\[
\mathcal{P}_D^O(s^*, a^*) = \sum_s \hat{a}(s) \mathbb{P}_n(S = s),
\]

where the \( j \)th element of \( \hat{a}(s) \) is \( \hat{a}(s)_j = \hat{F}_{j-1}(\hat{F}_{j^*}(|a^*|)_j) \) for \( j = 1, \ldots, d \).

These two pre-processing procedures are motivated by the notion of counterfactuals. Both \( \mathcal{P}_D^O(s^*, a^*) \) and \( \mathcal{P}_D^O(s^*, a^*) \) are weighted average of the counterfactual nonsensitive attributes had \( S \) been \( s \) (under assumptions we introduce in Section 3.1) with the weights being the sample proportion of \( S = s \). Therefore, the influence of \( S \) is averaged out while the information from \( U_A \) is preserved in the counterfactuals.

Let \( \mathcal{P}, \mathcal{P}_O, \) and \( \mathcal{P}_M \) denote the population level pre-processing procedure corresponding to \( \mathcal{P}_D^O, \mathcal{P}_D^O, \) and \( \mathcal{P}_D^O \), respectively. It is easy to see that \( \mathcal{P}_D^O(s^*, a^*) = a^* - \mathbb{E}_n(A|S = s^*) + \mathbb{E}_n(A) \) is a one-to-one function of \( a^* \) for any fixed \( s^* \), and the same property holds for \( \mathcal{P}_M(s^*, a^*) \). For the Marginal Distribution Mapping procedure, note the \( j \)th element of \( \mathcal{P}_M(s^*, a^*) \) is

\[
[\mathcal{P}_M(s^*, a^*)]_j = \sum_s F_{j-1}(F_{j^*}(|a^*|)) \mathbb{P}(S = s),
\]

where \( F_{j^*} \) is the marginal CDF of the \( j \)th element of \( A \) given \( S = s \) and \( F_{j-1} \) is defined similarly to (1) but replacing \( \mathbb{P}_n \) with \( \mathbb{P} \). It can be seen that if \( A_j \) is a discrete variable, then \( F_{j^*}(F_{j^*}(x)) \) is strictly increasing for \( s = s^* \); and if \( A_j \) is a continuous variable, then \( F_{j^*}(F_{j^*}(x)) \) may not be strictly increasing when \( F_{j^*}(x) \) is constant on some interval of \( x \). Therefore, \( \mathcal{P}_M(s^*, a^*) \) is only a one-to-one function of \( a^* \) for any fixed \( s^* \) when the marginal CDF of each continuous element in \( A \) given \( S = s^* \) is strictly increasing.

3.1. Fair Learning Algorithm

Besides pre-processing procedures, the choice of learners also plays an important role in fairness learning. A Fairness-Through-Unawareness (FTU) predictor \( f_{FTU}(a) \) only uses the nonsensitive attributes \( A \) to predict the conditional mean of \( Y \). A Machine Learning predictor \( f_{ML}(s, a) \) uses both the sensitive and nonsensitive attributes to predict \( \mathbb{E}(Y|S, A) \). An Averaged Machine Learning (AML) predictor \( f_{AML}(a) = \sum_s f_{AML}(s, a) \mathbb{P}_n(S = s)ds \). Note that we still need to train the ML predictor to obtain the AML predictor, but it only needs the nonsensitive attributes as its input when making a prediction since the sensitive attributes are averaged out. Algorithm 1 could use any learner \( f \in \{f : A \rightarrow [0, 1]\} \) to learn the decisions from the processed data, and we would consider the FTU and AML learners in our numerical studies.

Algorithm 1 tries to achieve counterfactually fair learning with two steps. It first removes the indirect effect of sensitive information in nonsensitive attributes through data preprocessing, then it removes the direct effect of sensitive attributes by restricting the learner to the class \( \{f : A \rightarrow [0, 1]\} \), which forces the learner to either not use the sensitive attributes or averaging out the effect of them. Therefore, the final output \( \hat{Y} \) would not be dependent on the sensitive information.

### Algorithm 1: Fair Learning through dAta Preprocessing (FLAP)

**Input:** Training data \( D \), preprocessing procedure \( \mathcal{P}_D \), learner \( f \), test attributes \( (s, a) \)

for \( (s_i, a_i, y_i) \) in \( D \) do

\( a'_i = \mathcal{P}_D(s_i, a_i) \)

end for

Create the processed data \( D' = \{(s_i, a'_i, y_i)\}_{i=1}^n \)

Learn predictor \( f \) from \( D' \)

Calculate \( a' = \mathcal{P}_D(s, a) \)

Draw \( \hat{Y} \) from Bernoulli(\( f(a') \))

**Output:** \( \hat{Y} \)

Apart from the structural assumptions made in Figure 1, extra conditions of the structural equation \( f_A(s, u_A) \) must be satisfied for the preprocessing method to work.

**Condition 1 (Strong nonsensitive).** The partial derivative \( \frac{\partial}{\partial u_A} f_A(s, u_A) \) does not involve \( s \).

**Condition 2 (Weak nonsensitive).** The sign of \( \frac{\partial}{\partial u_A} f_A(s, u_A) \) does not change with \( s \) for all \( u_A \) and all \( j = 1, \ldots, d \).

These two conditions describe the relationship between the sensitive and nonsensitive attributes. **Condition 2** is weaker than **Condition 1**. For example, an additive model \( f_A(s, u_A) = \beta_0 + \beta_1 s + \beta_2 u_A \) satisfies both conditions, while in our running example, \( \frac{\partial}{\partial u_A} f_A(s, u_A) = c_1 c_3 \sigma^2_A \exp\{c_2 + \lambda_1 s + c_3 \sigma^2_A u_A\} > 0 \) for \( s = 0, 1 \). So it meets **Condition 2** but not **Condition 1**. We prove in the following theorem that these conditions, together with the SCM, are sufficient for **Algorithm 1** to generate counterfactually fair decisions.

**Theorem 1.** Given an SCM \( M = (U, V, F) \) with structural equations defined in Figure 1 and let \( \hat{Y} \) be the output from **Algorithm 1**, that is, \( Y \leq f(\mathcal{P}_D(S, A)) \). As the data size \( |D| \) goes to infinity:

1. If the procedure \( \mathcal{P}_O \) is adopted, \( \hat{Y} \) is counterfactually fair under **Condition 1**.
2. If the procedure \( \mathcal{P}_M \) is adopted, \( \hat{Y} \) is counterfactually fair under **Condition 2**.

The proof of **Theorem 1** is given in the Appendix, supplementary materials. The intuition is that the FLAP algorithm learns the decision from processed data only, and the processed data contain no sensitive information since the pre-processing procedure can remove \( A \)'s dependence on \( S \) under the nonsensitive condition.

**Theorem 1** identifies the conditions for achieving counterfactually fair decisions under a certain SCM. However, the functional form of the structural equations of the SCM is often unidentifiable given the observational data, that is, two sets of structural equations with different functional forms could generate the data with the same joint distribution. For example, let \( h \) be a one-to-one function and \( U'_A = h(U_A) \). Since we cannot identify if \( U_A \) or \( U'_A \) are the true exogenous variables, the structural equation of \( A \) can be either \( f_A(S, U_A) \) or \( f'_A(S, U'_A) = f_A(S, h(U_A)) \)
Since we do not have any restrictions on the functional forms of the structural equations in the SCM, this kind of reparameterization of $U_A$ to $U'_A$ and $f_A \to f'_A$ does not break any assumptions of the SCM. Therefore, the effectiveness of the FLAP algorithm only requires there exists at least one set of $U$ and $F$ that satisfies one of the nonsensitive conditions.

This could be used to further relax the conditions for applying the FLAP algorithm to observational data. For illustration, consider the case where $A$ is a continuous random variable, let $U'_A$ be a uniform random variable and $f'_A$ be the inverse conditional CDF of $A$ given $S$. Condition 2 will hold true if $f'_A$, or equivalently, the conditional CDF of $A$ given $S$ is strictly increasing since the partial derivative of the inverse of a strictly increasing CDF is always positive. Then we can apply the Marginal Distribution Mapping procedure to learn counterfactually fair decisions. It is easy to check that this reparameterization trick also works for discrete random variables and random vectors where each element is either continuous or discrete. The only exception where Condition 2 does not hold even with reparameterization is when $A$ contains mixtures of continuous and discrete random variables. One example is test score where the distribution below the maximum score is continuous but there is also a positive probability of getting 100. In this sense, we can learn counterfactually fair decisions for most common types of nonsensitive attributes using the Marginal Distribution Mapping preprocessing procedure.

### 3.2. Test for Counterfactual Fairness

Data preprocessing not only allows us to learn a counterfactually fair decision but also enables us to test if the decisions made in the original data are fair. When Condition 1 holds, we can use the data processed by the orthogonalization procedure to test fairness. When the strong condition does not hold but Condition 2 is satisfied, we need an extra condition to use the marginal distribution mapping procedure for fairness testing.

**Condition 3.** The conditional marginal CDF $F_Y(s|x)$ is strictly increasing for all such $j$ that $A_j$ is continuous and all $s \in S$.

In other words, each nonsensitive attribute $A_j$ should be either a discrete random variable or a continuous one with nonzero density on $\mathbb{R}$. This condition ensures that $P_M(s^*,a^*)$ is a one-to-one function as discussed earlier. With these conditions, we can establish the equivalence between CF and the conditional independence of decision and sensitive information given the processed nonsensitive information.

**Theorem 2.** Consider the original decision $Y$:

1. Under Condition 1, $Y$ is counterfactually fair if and only if $Y \perp S | P_D(S,A)$.
2. Under Conditions 2 and 3, $Y$ is counterfactually fair if and only if $Y \perp S | P_M(S,A)$.

The proof of Theorem 2 is given in the Appendix, supplementary materials. Theorem 2 allows us to test CF using any well-established conditional independence test. In practice, given a decision dataset $D = (s_i,a_i,y_i)_{i=1}^n$, we can obtain the empirical processed nonsensitive attributes $P^D(s,a)$ and test if $Y \perp S | P^D(S,A)$. If the $p$-value of the test is small enough for us to reject the conditional independence hypothesis, then the original decision is probably biased and algorithms such as FLAP should be used to learn fair decisions.

### 4. Numerical Studies

In this section, we compare the decisions made by different algorithms in terms of fairness and accuracy using simulated and real data, and also investigate the empirical performance of the fairness test using simulated data with small sample sizes. We consider three cases for generating the simulation data. The first one is Example 1 and the second one is a multivariate extension of it.

**Example 2.** The bank now collects the race $S$, education year $E$ and annual income $A$ information from loan applicants. There are three possible race groups $S = \{0,1,2\}$ and $S = 1\{U_S > 0.76\} + 1\{U_S < 0.92\}$, meaning that a random applicant could be from the majority race group (0) with probability 0.76, or from the minority group 1 or 2 with probability 0.16 or 0.08. Let $U_E$ be a standard normal random variable and $\mu_E = \lambda_{e0} + 1\{S = 1\} \lambda_{e1} + 1\{S = 2\} \lambda_{e2}$, the education year is $E = \max(0, \mu_E + 0.4\mu_E U_E)$. Let $\mu_A = \log(\lambda_{a0} + 1\{S = 1\} \lambda_{a1} + 1\{S = 2\} \lambda_{a2})$, the annual income is $A = \exp(\mu_A + 0.4\mu_E U_E + 0.1 U_A)$. The decision of the bank is modeled as

$$Y = 1\{U_Y < \expit(\beta_0 + 1\{S = 1\} \beta_1 + 1\{S = 2\} \beta_2 + \beta_A A + \beta_E E)\}.$$ 

Here $\lambda_{e0}, \lambda_{e1},$ and $\lambda_{e2}$ decide the mean education year of the three race groups. $\lambda_{a0}, \lambda_{a1},$ and $\lambda_{a2}$ decide the median annual income. The annual income and the education year are positively correlated through $U_E$. $\beta_1$ and $\beta_2$ characterize the direct effect of the race information while the $\beta$’s indicate the indirect effect together with $\beta_A$ and $\beta_E$. In this example, neither of Conditions 1 and 2 holds if $\beta_2$ and $\lambda_{a1}$ and/or $\lambda_{e2}$ are not zero due to the maximum operator in $f_E$. Even if $\lambda_{a1} = \lambda_{e2} = 0$, only the weaker Condition 2 will hold due to the same reason for Example 1.

The third example is a replica of the admission example constructed by Wang, Sridhar, and Blei (2019).

**Example 3.** The admission committee of a university collects the gender $G$ and test score $T$ information from applicants. The gender is simulated from $S = 1\{U_S < 0.5\}$, where $S = 1$ for male and $S = 0$ for female. Let $U_T \sim \text{Uniform}(0,1)$ and we generate the test score as $T = \min(\max(0,\lambda S + U_T), 1)$. The decision of the committee is

$$Y = 1\{U_Y < \expit(\beta_0 + \beta_1 T + \beta_S S)\}.$$ 

It is worth noting that Example 3 also does not satisfy either of Conditions 1 and 2 due to the cutoff in the test score. There will be a positive probability (let $\lambda$ be exact) of seeing male students with scores equal to 1 if $\lambda > 0$. Check that

$$\frac{\partial}{\partial u_T} f_T(s,u_T) = \begin{cases} 1, & 0 < u_T < 1 - \lambda s \\ 0, & 1 - \lambda s < u_T < 1 \end{cases}$$

and we can see that its sign does change with $s$ for any fixed $u_T$. Therefore, neither of the proposed preprocessing methods can achieve CF in theory.
4.1. Fairness Evaluation

We compare our FLAP algorithm with

1. ML: the machine learning method using both sensitive and nonsensitive attributes without preprocessing, which is a logistic regression of $Y$ on $S$ and $A$;
2. FTU: the Fairness-Through-Unawareness method which fits a logistic model of $Y$ on nonsensitive attributes $A$ alone without preprocessing;
3. FL: the FairLearning algorithm in Kusner et al. (2017);
4. AA: the Affirmative Action algorithm in Wang, Sridhar, and Blei (2019).

The FTU method addresses fairness problems using the most intuitive but often ineffective way—excluding sensitive attributes from the learning process. It cannot achieve CF if $S$ has a causal effect on $A$ because the decision learned from $A$ and $Y$ is affected by $S$ indirectly. Our FLAP method is mainly compared against FL and AA methods which are designed to achieve CF under the assumption of the SCM.

All these methods can output a predicted score $p$ given the training data $D$ and test attributes $(s, a)$, denoted $p(s, a; D)$ and draw the random decision $Y$ from Bernoulli($p(s, a; D)$). For ML method, $p(s, a; D) = f_{ML}(s, a)$; for FTU method, that is $f_{FTU}(a)$. We denote the predicted scores of the FairLearning and AA algorithms as $f_{FL}(s, a; D)$ and $f_{AA}(s, a; D)$, respectively. For our FLAP method, we use the marginal distribution mapping procedure and try both the ML and the FTU learners described in Section 3 and name the methods as FLAP-1 and FLAP-2. Their predicted scores are $f_{FLAP}(P_M^D(s, a))$ and $f_{FTU}(P_M^D(s, a))$, respectively. We use the test accuracy to measure the prediction performance and consider two metrics for measuring the counterfactual fairness. The CF-metric is defined as

$$\max_{r,s \in S} \frac{1}{N} \sum_{i=1}^N \left| p(r, a^*_M(r, s_i, a_i); D) - p(t, a^*_M(t, s_i, a_i); D) \right|,$$

where $N$ is the size of the test set and $a^*_M(s, a^*)$ is defined as $a_M$ in Definition 3. Note that the CF-metric should be zero when decisions are CF under Condition 2. This metric is different from the AA-metric proposed by Wang, Sridhar, and Blei (2019) in two folds. First, it allows us to consider more than two sensitive groups by taking the maximum of the pairwise difference of predicted scores, but it reduces to the AA-metric for two sensitive groups. Second, we use the marginal distribution mapping method to compute the counterfactual nonsensitive attributes $a^*_M(s, a^*)$ had the unit been in a different sensitive group $s$. This ensures that all the derived counterfactual attributes are within the range of observed attribute values. In comparison, Wang, Sridhar, and Blei (2019) use the orthogonalization method to compute the counterfactual attributes and thus a female student having test score 0.98 would have a counterfactual score of 1.48 had she been a male if the male mean test score is 0.5 higher than female. This out-of-range counterfactual score is unreasonable and problematic when being used as the input of the score prediction function $p$.

We should note that the CF-metric is a good representative of CF when the weak nonsensitive condition is met. When it is not satisfied, however, a CF decision is not guaranteed to have zero CF-metric. In absence of the nonsensitive conditions, the counterfactual nonsensitive attributes are unidentifiable, and thus Wu, Zhang, and Wu (2019) propose to assess CF using the lower and upper counterfactual fairness bounds, which evaluate to

$$\min_{i, (s', a') \in D, d \neq s_i} \left( p(s', a'; D) - p(s_i, a_i; D) \right),$$

$$\max_{i, (s', a') \in D, d \neq s_i} \left( p(s', a'; D) - p(s_i, a_i; D) \right),$$

under our SCM. Though requiring no assumption on the functional form of $f_A$, these bounds are often too wide to be used for comparing different methods. It can be seen that if the range of the prediction probabilities $p$ is $[0, 1]$ for each sensitive group $s$, then the bounds are always $(-1, 1)$. In order for the metric to be more informative, we propose the CF-bound defined as follows.

$$\max_{i, (s', a') \in D, d \neq s_i} \left[ \hat{p}(s', a'; D) - p(s_i, a_i; D) \right],$$

where $\hat{p}(s', a'; D)$ is the average of predicted scores $p(s', a'; D)$ for a sample of $a'$ randomly selected from the set $\{ a : |r_j([a])| \leq \delta_{ns}, j = 1, \ldots, d \}$. The prespecified parameter $\delta$ determines the sampling range of the nonsensitive attributes. If we set $\delta = 1$, meaning that $a'$ is sampled from all nonsensitive attributes seen in $D_s$, then the CF-bound performs like the maximum absolute value of the bounds defined in Wu, Zhang, and Wu (2019) and it is also not very informative. On the other hand, setting $\delta = 0$ will only make sense when the rank of each element of the counterfactual nonsensitive attributes $a'$ in the $s'$ sensitive group is the same as the rank of each element of $a_i$ in the $s_i$ group, which is an assumption that may not hold in real world applications. Therefore, $\delta$ should be chosen from $(0, 1)$ to tell apart different methods and weaken the assumption needed for $f_A$. We use $\delta = 0.05$ for the discussions below, and a comparison of the CF-bound results for different $\delta$ is shown in Section 5.

In Figure 2(b), we set $\beta_0 = -1$, $\beta_2 = 2$, $\beta_1 = 1$ and increase $\lambda$ from 0 to 0.8 to see how the mean difference of test scores affects fairness.

For Example 1, we choose $c_1 = 0.01$, $c_2 = 4$, $c_3 = 0.2$, fix $\beta_0 = -1$, $\beta_2 = 2$, $\beta_1 = 1$, and $\lambda = 0.5$ while increase $\sigma_\alpha$ from 1 to 2.8 to see how the difference in the variation of the nonsensitive attribute between sensitive groups affects fairness. As shown in Figure 2(a), the AA algorithm which essentially uses the orthogonalization method cannot achieve CF since Condition 1 is not met. However, both FLAP algorithms’ CF-metrics are zero when using the marginal distribution mapping preprocessing. The CF-bounds also show that the FLAP methods are the fairest among the methods we consider.

Wang, Sridhar, and Blei (2019) showed that the AA algorithm can achieve zero AA-metric in Example 3, but it does not satisfy either of the nonsensitive conditions for achieving CF. In Figure 2(b), we fix $\beta_0 = -1$, $\beta_2 = 2$, $\beta_1 = 1$ and increase $\lambda$ from 0 to 0.8. It can be seen that all algorithms we consider cannot achieve CF, but the FLAP-1 algorithm still has the lowest CF-metric and CF-bound. There is no significant difference between the accuracy of the FL, AA, and FLAP algorithms in all examples. In general, we expect fairer predictions to have
lower accuracy since they correct the discriminatory bias of the original decisions.

In Example 2, we choose $\beta_0 = -1$, $\beta_1 = \beta_2 = 0$, $\beta_3 = 1$, $\beta_e = 2$, $\lambda_{e0} = 1.07$, $\lambda_{e1} = 0.58$. In Figure 3(a), we change $(\lambda_{i1}, \lambda_{i2})$ while fix $\lambda_{e1} = 0$ and $\lambda_{e2} = 0$ to see how the mean difference of income affect fairness. The results are telling the same story as Figure 2(a): since only the weaker nonsensitive condition is met, the AA-algorithm cannot achieve CF but the FLAP algorithms with marginal distribution mapping procedure can.

In Figure 3(b), we change $(\lambda_{e1}, \lambda_{e2})$ while fix $\lambda_{e1} = 0$ and $\lambda_{e2} = 0$ to see how the mean difference of education affect fairness. The results are similar to those of Figure 2(b) where all algorithms we consider cannot achieve CF but the FLAP algorithms still have the lowest CF-metric and CF-bound.

We conduct additional simulation studies to examine the performance of the FLAP algorithm when the given structural equations violates the nonsensitive conditions. The results show that the FLAP algorithm is effective even when the given structural equations violates the nonsensitive conditions.

### 4.2. Fairness Test

The Conditional Distance Correlation (CDC) test (Wang et al. 2015) is a well-established nonparametric test for conditional independence. We use it here to illustrate the performance of the fairness test with the three simulated examples. For each example, we use different combinations of parameters to obtain simulated datasets with different fairness levels, which are measured by the CF-metric. A CDC test with a significance level of 0.05 is then conducted to test if $Y \perp S \mid \mathcal{P}^D(S, A)$ for each dataset. The simulation-test process is repeated 1000 times for each combination of parameters to estimate the power of the test, namely the probability of rejecting the null hypothesis that the decisions are counterfactually fair. The results are summarized in Figure 4.
When the decisions are generated fair, which are shown as the points with CF-metrics equal to zero, the Type I error rate is around 0.05 for all examples. The power of the test grows as we make the decisions more unfair, or increase the sample size.

5. Real Data Analysis

We apply our methods to a loan application dataset from a fintech company, the adult income dataset from UCI Machine Learning Repository\(^2\) and the COMPAS recidivism data from ProPublica\(^3\) (Angwin and Larson 2016).

In the loan application case, the fintech lender aims to provide short-term credit to young salaried professionals by using their mobile and social footprints to determine their creditworthiness even when a credit history may not be available. To get a loan, a customer has to download the lending app, submit all the requisite details and documentation, and give permission to the lender to gather additional information from her/his smartphone, such as the number of apps, number of calls, and SMSs, and number of contacts and social connections. We obtained data from the lending firm for all loans granted from February 2016 to November 2018. The decisions \(Y\) are whether or not the lender approves the loan applications. The attributes are applicants’ gender, age, salary, and other information collected from their smartphones. Both gender and age are regarded as sensitive information here and we find that the decisions are made in favor of the senior and female applicants. Since we can only deal with categorical sensitive attributes, we divide the applicants into two age groups by the lower quartile of the age distribution and create a categorical variable \(S \in \{0, 1, 2, 3\}\) to denote the group of the applicants: female younger than 28; male younger than 28; female older than 28; and male older than 28. The effective sample size after removing missing values is 203,656.

Nonparametric conditional independence tests will not be efficient for this real case due to the large sample size. Therefore, we test the conditional independence of \(Y\) and \(S\) given \(P_M\) (\(S, A\))

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\(^2\)https://archive.ics.uci.edu/ml/machine-learning-databases/adult/
\(^3\)https://github.com/propublica/compas-analysis
by fitting a simple logistic model for \( Y \) with \( S \) and \( P_{\text{MD}}(S,A) \) as the explanatory variables and testing if the coefficient of \( S \) is significantly different from zero. The \( p \)-value of the \( F \)-test is almost zero and indicates that the decisions are unfair for applicants in different groups. When other attributes are fixed to their means, the predicted approval probabilities of the four groups from the logistic model are 0.924 (young female), 0.899 (young male), 0.948 (senior female), and 0.946 (senior male), also indicating that the decisions are most in favor of the senior and female applicants.

We randomly split the data into a training set of 193,656 samples and a test set of 10,000 samples 50 times. The training datasets are used to learn the decisions with different algorithms and the test datasets are used to evaluate the CF-metric, CF-bound, and accuracy. The average and standard error of the 50 evaluation results are summarized in Table 1. Since the nonsensitive condition may not be satisfied in this real world application, the CF-bound may be a better indicator of CF than the CF-metric when they disagree with each other. While both the FLAP algorithms using the marginal distribution mapping preprocessing procedure have lower CF-metrics and CF-bounds compared with other algorithms, the FLAP algorithm using the AML learner (FLAP-1) is fairer than the one using the FTU-learner (FLAP-2) as shown by the CF-bound. Their test accuracy is slightly lower than the ML method. Note that in real-world applications, fairer decisions may not have lower accuracy as expected in the simulation studies because we do not have access to all the variables possessed by the original decision-maker. When the original decisions depend on additional information, the FLAP and other fair learning methods may yield predictions closer to or further away from the original decisions, and thus leading to lower or higher accuracy.

Table 2 shows the effect of \( \delta \) on the CF-bound. In general, higher \( \delta \) means a broader range of \( a' \)’s are considered as the possible counterfactual nonsensitive attributes \( a' \) had \( S \) been \( s' \), and thus the bound will become higher. When \( \delta \) is close to 1, the metrics become similar to each other and make it difficult to tell the best method. As shown from the table, the results for different \( \delta \)'s are mostly consistent with each other.

The COMPAS (Correctional Offender Management Profiling for Alternative Sanctions) recidivism data contains the demographic data such as sex, age, race, and record data such as priors count, juvenile felonies count, and juvenile misdemeanors count of over 10,000 criminal defendants in Broward County, Florida. The task is to predict whether they will re-offend in two years. According to ProPublica, “Black defendants were often predicted to be at a higher risk of recidivism than they actually were.” Here we treat sex and race as sensitive information and try to predict recidivism in a counterfactually fair manner. We only
Table 2. Comparison of the CF-bound of decision making algorithms on the loan application data using different range parameter \( \delta \).

|          | ML  | FTU | FL  | AA  |
|----------|-----|-----|-----|-----|
| \( \delta = 0 \) | 0.1251 | 0.1454 | 0.1644 | 0.1652 |
| \( \delta = 0.025 \) | 0.1401 | 0.1610 | 0.1709 | 0.1709 |
| \( \delta = 0.05 \) | 0.1354 | 0.1564 | 0.1675 | 0.1675 |
| \( \delta = 0.1 \) | 0.1346 | 0.1558 | 0.1649 | 0.1649 |
| \( \delta = 1 \) | 0.1313 | 0.1558 | 0.1649 | 0.1649 |

Table 3. Comparison of the CF-metric, CF-bound, and test accuracy of decision making algorithms on the COMPAS data.

|          | ML (0.0230) | FTU (0.0125) | FL (0.0125) | AA (0.0125) |
|----------|-------------|--------------|-------------|-------------|
| CF-metric | 0.2320(0.2320) | 0.1422(0.0060) | 0.0050(0.0014) | 0.0054(0.0016) |
| CF-bound | 0.6284(0.3529) | 0.5845(0.2611) | 0.4830(0.2911) | 0.4868(0.0724) |
| accuracy | 0.5726(0.0025) | 0.5708(0.0025) | 0.5590(0.0026) | 0.5602(0.0027) |
| FLAP-1(O) | 0.0964 | 0.1218 | 0.1320 | 0.1465 |
| FLAP-2(O) | 0.0975 | 0.1225 | 0.1327 | 0.1471 |
| FLAP-1(M) | 0.0501 | 0.1060 | 0.1217 | 0.1380 |
| FLAP-2(M) | 0.0512 | 0.1067 | 0.1227 | 0.1390 |

NOTE: The metrics are averaged from 50 random training/test data split, and standard errors are shown in parenthesis.

Table 4. Comparison of the CF-metric, CF-bound, and test accuracy of decision making algorithms on the adult income data.

|          | ML  | FTU | FL  | AA  |
|----------|-----|-----|-----|-----|
| CF-metric | 0.2779 | 0.2338 | 0.0228 | 0.0268 |
| CF-bound | 0.9152 | 0.8421 | 0.7166 | 0.7656 |
| accuracy | 0.7612 | 0.7604 | 0.7594 | 0.7644 |
| FLAP-1(O) | 0.0280 | 0.0228 | 0.0020 | 0.0022 |
| FLAP-2(O) | 0.7357 | 0.7151 | 0.7721 | 0.7470 |
| FLAP-1(M) | 0.7548 | 0.7594 | 0.7570 | 0.7599 |

use the data for Caucasian, Hispanic, and African-American individuals due to the small sample sizes of other races. The sensitive attribute \( S \) is a combination of the two sex categories and three race categories. Since all nonsensitive attributes are either continuous or discrete, the data meet our assumptions for conducting preprocessing and evaluation. We randomly split the data into a training set of 5090 samples and a test set of 1697 samples 50 times to conduct the analysis. The average results and standard errors are shown in Table 3.

We use the adult income data to predict whether an individual’s income is higher than $50K with information including sex, race, age, workclass, education, occupation, marital-status, capital gain and loss. Sex and race are regarded as sensitive information. We combine the two sex categories and five race categories (White, Asian-Pac-Islander, Amer-Indian-Eskimo, Black, Other) into a sensitive attribute \( S \) with 10 categories. We should note that this dataset does not satisfy the nonsensitive assumption even under reparameterization because one of its nonsensitive attribute “working hour per week” is a continuous variable capped at 40. The training set with 32,561 samples and the test set with 16,281 samples are provided by the UCI website. We directly report the evaluation results on the test set in Table 4 so others can better compare their results with ours.

Here we still use \( \delta = 0.05 \) for calculating the CF-bound. For the COMPAS data, both the CF-bound and CF-metric show that the FLAP methods using the marginal distribution mapping preprocessing procedure are fairer than other fair learning algorithms. Since the adult income data does not satisfy the nonsensitive conditions, the CF-bound is very high for all methods and it is likely that none of the methods achieves CF. However, the CF-bound metric still favors the FLAP-2 algorithm with the orthogonalization procedure. The accuracy of all fair learning algorithms is comparable to the ML method for both datasets.

We conduct additional analyses showing the effect of \( \delta \) on the CF-bound for the adult income data and the COMPAS data. The results are given in the Appendix, supplementary materials.

6. Discussion

We propose two data preprocessing procedures and the FLAP algorithm to make counterfactually fair decisions. The algorithm is general enough so that any learning methods from logistic regression to neural networks can be used, and counterfactual fairness is guaranteed regardless of the learning methods. The orthogonalization procedure is faster and ensures counterfactually fair decisions when the strong nonsensitive condition is met. The marginal distribution mapping procedure is more complex but guarantees fairness under the weaker nonsensitive condition, which is satisfied by most common types of nonsensitive attributes after reparameterization. Even when the nonsensitive attributes contains mixtures of continuous and discrete variables, the FLAP method is still fairer than other methods we considered as shown in our data analysis.

We also prove the equivalence between counterfactual fairness and the conditional independence of decisions and sensitive attributes given the processed nonsensitive attributes under the nonsensitive assumptions. We illustrate that the CDC test is reliable for testing counterfactual fairness when the sample size is small. When the size gets bigger, however, we need a more efficient testing method for the fairness test.

Supplementary Materials

The supplementary materials provide proofs of Theorems 1 and 2, and additional simulation and real data analysis results.

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