Probing nonstandard bosonic interactions via $W$-boson pair production at lepton colliders

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Abstract

The process $e^+e^- \rightarrow W^+W^-$ provides a valuable laboratory to test the Standard Model (SM) and to search for new physics. The most general helicity amplitudes for this process require the introduction of nine form-factors which we calculate in the context of SU(2)×U(1) gauge-invariant extensions of the SM. The contributions of new physics are parametrized via an effective Lagrangian constructed from the light fields. Because the mechanism of electroweak symmetry-breaking remains an open problem we consider both the effective Lagrangian with a linearly realized Higgs sector, \emph{i.e.} with a light physical Higgs boson, and the effective Lagrangian which utilizes a nonlinear realization of the Higgs mechanism. The use of an effective Lagrangian allows one to calculate consistently nonstandard contributions to $e^+e^- \rightarrow W^+W^-$ amplitudes as well as the nonstandard contributions to other processes. We study the interplay of the low-energy and $Z$-pole measurements with mea-
surements via the processes $e^+e^- \rightarrow f\bar{f}$ and $e^+e^- \rightarrow W^+W^-$ at LEP II or a future linear $e^+e^-$ collider. Concrete relationships between operators of the linear and nonlinear realizations are presented where possible.

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I. INTRODUCTION

The Standard Model (SM) of electroweak interactions has been very successful when tested by experiments at and below the scale of the weak-boson masses. However, all available precision data concerns processes with four light external fermions only. There is very little data which directly reflects the couplings of electroweak bosons amongst themselves, and the symmetry-breaking sector remains wholly uninvestigated.

Studies of the process $e^+e^- \rightarrow W^+W^-$ will provide important data concerning both non-Abelian gauge-boson couplings and the Higgs sector. A convenient form-factor-based analysis of this process is indispossession if we wish to discuss the search for new physics effects in an efficient manner. The utility of this approach has been demonstrated in Ref. [1], where seven tensors, each with a scalar form-factor coefficient, were introduced to describe new physics in the s-channel. However, one should expect that realistic models may induce corrections in the t-channel and box graphs as well. Furthermore, it is desirable to have a framework which allows both standard radiative corrections [2] and nonstandard contributions to be straightforwardly combined. In Section II we demonstrate how a total of nine tensors may be used to obtain the most general amplitudes. While Section II concentrates on the kinematics of $e^+e^- \rightarrow W^+W^-$ amplitudes, Section III concentrates on the dynamical structure of each form factor at the one-loop level in the SM supplemented by small nonstandard contributions.

We then describe deviations from the SM via effective-Lagrangian techniques. In general one constructs an effective Lagrangian by adding to the SM Lagrangian terms which describe the new physics. These new terms will be constructed, subject to the various assumptions of the extended theory, from the fields of the SM and derivatives thereof. We will everywhere assume that the full theory is invariant under $SU(2)_L \times U(1)_Y$ spontaneously broken to $U(1)_{em}$. Furthermore, we will assume that the couplings of the new physics to the light fermions are suppressed, hence fermionic fields shall not be employed in the construction of effective operators. Because the existence or nonexistence of the Higgs boson has not yet been established, its inclusion or exclusion is open to debate. We therefore consider both scenarios by discussing the linear and the nonlinear realizations of the symmetry-breaking sector. Wherever possible we present our results in a fashion which facilitates comparison of the two scenarios.
In Section IV we present an effective Lagrangian with a linearly realized Higgs sector which may be written as the sum of the SM Lagrangian plus operators of energy-dimension greater than four. At the energy-dimension-six level we present a complete set of such operators, and we discuss the couplings affected by each operator. In Section V we describe the construction of the electroweak chiral Lagrangian. A complete set of operators through energy-dimension-four are presented. Each operator in the nonlinear representation is paired with its counterpart in the linear representation from which it may be obtained in the limit where the mass of the Higgs boson is taken to infinity. We discuss the electroweak gauge-boson couplings which are affected by each nonlinear operator.

In Section VI we show how a subset of the operators, in either realization of the symmetry-breaking sector, may be constrained by current data from the LEP/SLC and low-energy experiments. At low-energies three operators in the nonlinear representation are tightly constrained. Four operators in the linear representation may be constrained, albeit somewhat less stringently, by the low-energy data; the constraints on these four are very much improved through the study of $e^+e^- \rightarrow f\bar{f}$ at higher energies.

In Section VII we return to the process $e^+e^- \rightarrow W^+W^-$, for which we calculate the form-factors in the linear and in the nonlinear representation. In either representation seven operators contribute to $e^+e^- \rightarrow W^+W^-$. We also review the standard parameterization of the most general $WW\gamma/WWZ$ vertex.

In Section VIII we present a numerical study of the process $e^+e^- \rightarrow W^+W^-$ including nonstandard effects. In Section IX we discuss the numerical results and how they may be combined with constraints from the low-energy experiments, $Z$-pole data and further measurements of four-fermion observables at higher energies. Finally, in Section X we present our conclusions.

II. A FORM-FACTOR-BASED ANALYSIS OF $e^+e^- \rightarrow W^+W^-$

The process $e^-(k,\tau) + e^+(\bar{k},\bar{\tau}) \rightarrow W^-(p,\lambda) + W^+(\bar{p},\bar{\lambda})$ is depicted in Fig. 1. The four-momenta of the $e^-$, $e^+$, $W^-$ and $W^+$ are $k$, $\bar{k}$, $p$ and $\bar{p}$ respectively. The helicity of the $e^-$ ($e^+$) is given by $\frac{1}{2}\tau$ ($\frac{1}{2}\bar{\tau}$), and $\lambda$ ($\bar{\lambda}$) is the helicity of the $W^-$ ($W^+$). In the limit of massless electrons only $\tau = -\bar{\tau}$ amplitudes survive, and the most general amplitude for this process
FIG. 1. The process $e^-e^+ \rightarrow W^-W^+$ with momentum and helicity assignments. The momenta $k$ and $\bar{k}$ are incoming, but $p$ and $\bar{p}$ are outgoing. The arrows along the W-boson lines indicate the flow of negative electronic charge.

may be written as

$$M(k, \bar{k}; p, \bar{p}; \lambda, \bar{\lambda}) = \sum_{i=1}^{9} F_{i,\tau}(s, t) j_\mu(k, \bar{k}, \tau) T^{\mu\alpha\beta}_{i} \epsilon_\alpha(p, \lambda)^* \epsilon_\beta(\bar{p}, \bar{\lambda})^*, \quad (2.1)$$

where all dynamical information is contained in the scalar form-factors $F_{i,\tau}(s, t)$ with $s = (k + \bar{k})^2$ and $t = (k - p)^2$. The other factors in Eqn. (2.1) are of a purely kinematical nature; $\epsilon_\alpha(p, \lambda)^*$ and $\epsilon_\beta(\bar{p}, \bar{\lambda})^*$ are the polarization vectors for the $W^-$ and $W^+$ bosons respectively, and $j_\mu(k, \bar{k}, \tau)$, given by

$$j_\mu(k, \bar{k}, \tau) = \bar{v}(\bar{k}, -\tau) \gamma_\mu u(k, \tau), \quad (2.2)$$

is the fermion current for massless electrons.

The tensors $T^{\mu\alpha\beta}_{i}$ may be chosen as

$$T^{\mu\alpha\beta}_{1} = P^\mu q^\alpha q^\beta, \quad (2.3a)$$
$$T^{\mu\alpha\beta}_{2} = -\frac{1}{m_W^2} P^\mu q^\alpha q^\beta, \quad (2.3b)$$
$$T^{\mu\alpha\beta}_{3} = q^\alpha g^{\mu\beta} - q^\beta g^{\alpha\mu}, \quad (2.3c)$$
$$T^{\mu\alpha\beta}_{4} = i(q^\alpha g^{\mu\beta} + q^\beta g^{\alpha\mu}), \quad (2.3d)$$
$$T^{\mu\alpha\beta}_{5} = i \epsilon^{\mu\alpha\beta\rho} P_\rho, \quad (2.3e)$$
$$T^{\mu\alpha\beta}_{6} = -\epsilon^{\mu\alpha\beta\rho} q_\rho, \quad (2.3f)$$
$$T^{\mu\alpha\beta}_{7} = -\frac{1}{m_W^2} P^\mu \epsilon^{\alpha\beta\rho\sigma} q_\rho P_\sigma, \quad (2.3g)$$
$$T^{\mu\alpha\beta}_{8} = K^\beta g^{\alpha\mu} + K^\alpha g^{\mu\beta}, \quad (2.3h)$$
$$T^{\mu\alpha\beta}_{9} = \frac{i}{m_W^2} (K^\alpha \epsilon^{\beta\mu\rho\sigma} + K^\beta \epsilon^{\alpha\mu\rho\sigma}) q_\rho P_\sigma, \quad (2.3i)$$
where \( P = p - \bar{p}, q = k + \bar{k} = p + \bar{p}, K = k - \bar{k} \) and \( \epsilon_{0123} = -\epsilon^{0123} = +1 \). The properties of the associated form factors \( F_{i,\tau}(s, t) \) under the discrete transformations of charge conjugation \( (C) \), parity inversion \( (P) \) and the combined transformation \( CP \) are summarised in Table I.

|   | \( F_1 \) | \( F_2 \) | \( F_3 \) | \( F_4 \) | \( F_5 \) | \( F_6 \) | \( F_7 \) | \( F_8 \) | \( F_9 \) |
|---|---|---|---|---|---|---|---|---|---|
| \( C \) | + | + | + | - | - | + | + | + | - |
| \( P \) | + | + | + | + | - | - | - | + | - |
| \( CP \) | + | + | + | - | + | - | - | + | + |

**TABLE I.** The properties of the form factors \( F_{i,\tau}(s, t) \) under the discrete transformations \( C, P \) and \( CP \).

When working in the context of a particular model the calculation of the scalar form-factors, \( F_{i,\tau}(s, t) \), depends upon the dynamics particular to that model as well as the level of precision to which the calculation is performed. To the contrary, the kinematical aspects are completely general. Therefore, it is practical to choose a convenient frame and to tabulate

\[
J_\mu (k, \vec{k}, \tau) T^{\mu \alpha \beta}_i \epsilon_\alpha(p, \lambda)^* \epsilon_\beta(\bar{p}, \bar{\lambda})^* = \tau \sqrt{2s} \tilde{T}_{i,\tau}(k, \vec{k}; p, \bar{p}, \lambda, \bar{\lambda}) d_{J_0,\Delta \lambda}^{J_0}, \tag{2.4}
\]

for \( i = 1 \cdots 9 \). On the right-hand side of the equation an overall factor is extracted as well as the appropriate \( d \)-functions \([3,4]\), \( d_{\tau,\Delta \lambda}^{J_0} \), where \( \frac{1}{2} \tau \) is the electron helicity, \( \Delta \lambda = \lambda - \bar{\lambda} \) and \( J_0 \) is the angular momentum of the first partial wave which contributes. Those \( d \)-functions which are relevant to the current discussion are summarized in Table II.

| \( \tau \) | \( J_0 \) |
|---|---|
| 0 | \(-\tau \sqrt{\frac{1}{2} \sin \theta} \), \( d_{\tau,0}^1 = \frac{1}{2}(1 \pm \tau \cos \theta) \) |
| \( \pm 1 \) | \(-\tau \sqrt{\frac{1}{2} \sin \theta \cos \theta} \), \( d_{\tau,\pm 1}^1 = \frac{1}{2}(1 \pm \tau \cos \theta)(2 \cos \theta \mp \tau) \), \( d_{\tau,\pm 2}^2 = \pm \frac{1}{2}(1 \pm \tau \cos \theta) \sin \theta \) |

**TABLE II.** A list of the \( d \)-functions which are used in Eqn. (2.4), Table III and Table IV.

We choose the \( e^+e^- \)-collision center of momentum (CM) frame with the outgoing \( W^- \) boson momentum vectors along the \( z \)-axis. The angle \( \Theta \) is measured between the momentum vectors of the electron and the \( W^- \) boson. Then

\[
q^\mu = \sqrt{s} \left( 1, 0, 0, 0 \right), \tag{2.5a}
\]

\[
P^\mu = \sqrt{s} \left( 0, 0, 0, \beta \right), \tag{2.5b}
\]

\[
K^\mu = \sqrt{s} \left( 0, -\sin \Theta, 0, \cos \Theta \right), \tag{2.5c}
\]
and, in the notation of Ref. [5], the fermion current and the polarization vectors become

\[ j^\mu(k, \bar{k}, \tau) = \sqrt{s}(0, -\cos \Theta, -i\tau, -\sin \Theta) , \]  

(2.6a)

\[ \varepsilon^{\mu}(p, \pm)^* = \frac{1}{\sqrt{2}}(0, \mp 1, i, 0) , \]  

(2.6b)

\[ \varepsilon^{\mu}(p, 0)^* = \gamma(\beta, 0, 0, 1) , \]  

(2.6c)

\[ \varepsilon^{\mu}(\bar{p}, \pm)^* = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0) , \]  

(2.6d)

\[ \varepsilon^{\mu}(\bar{p}, 0)^* = \gamma(\beta, 0, 0, -1) , \]  

(2.6e)

with

\[ \beta = \sqrt{1 - m_W^2/E_W^2}, \quad \gamma = E_W/m_W, \quad E_W = \sqrt{s}/2 . \]  

(2.7)

The explicit form of the \( \hat{T}_{i,\tau} \) in this frame are summarised in Table III for \( i = 1, \cdots, 7 \), and in Table IV for \( i = 8, 9 \). Note that the results of these two tables are valid in any CM frame obtained from the frame of Eqn. (2.5) by a simple rotation.

### Table III

| \( \Delta \lambda \) | \( \lambda \bar{\lambda} \) | \( d^{d_{10}}_{\tau,\Delta \lambda} \) | \( \hat{T}_1 \) | \( \hat{T}_2 \) | \( \hat{T}_3 \) | \( \hat{T}_4 \) | \( \hat{T}_5 \) | \( \hat{T}_6 \) | \( \hat{T}_7 \) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 0              | 00             | \( d^{d_{10}}_{\tau,0} \) | \( -\gamma^2(1 + \beta^2) \) | \( 4\beta^3\gamma^4 \) | \( 2\gamma^2\beta \) | \( +i \) | \( +4i\beta^2\gamma^2 \) | \( -i \) | \( -4i\beta^2\gamma^2 \) |
| 0              | ++             | \( d^{d_{10}}_{\tau,0} \) | \( \beta \) | \( \beta \) | \( \beta \) | \( \beta \) | \( \beta \) | \( \beta \) | \( \beta \) |
| 0              | --             | \( d^{d_{10}}_{\tau,0} \) | \( \beta \) | \( \beta \) | \( \beta \) | \( \beta \) | \( \beta \) | \( \beta \) | \( \beta \) |
| +1             | +0             | \( d^{d_{10}}_{\tau,1} \) | \( \gamma\beta \) | \( -i\gamma\beta \) | \( +\gamma\beta^2 \) | \( +i\gamma \) | \( +i\gamma \) | \( +i\gamma \) | \( +i\gamma \) |
| +1             | 0-             | \( d^{d_{10}}_{\tau,1} \) | \( \gamma\beta \) | \( +i\gamma\beta \) | \( +\gamma\beta^2 \) | \( -i\gamma \) | \( -i\gamma \) | \( -i\gamma \) | \( -i\gamma \) |
| -1             | 0+             | \( d^{d_{10}}_{\tau,-1} \) | \( \gamma\beta \) | \( +i\gamma\beta \) | \( -\gamma\beta^2 \) | \( +i\gamma \) | \( +i\gamma \) | \( +i\gamma \) | \( +i\gamma \) |
| -1             | -0             | \( d^{d_{10}}_{\tau,-1} \) | \( \gamma\beta \) | \( -i\gamma\beta \) | \( -\gamma\beta^2 \) | \( -i\gamma \) | \( -i\gamma \) | \( -i\gamma \) | \( -i\gamma \) |

TABLE III. The \( \hat{T}_{i,\tau}(k, \bar{k}, \tau; p, \bar{p}, \lambda, \bar{\lambda}) \), \( i = 1 \cdots 7 \) evaluated in the CM frame. (For \( i = 8, 9 \) see Table IV.) In Eqn. (2.4) each \( \hat{T}_{i,\tau} \) is associated with a d-function; for each \( i \) the appropriate d-function is listed in the third column. In column 1, \( \Delta \lambda = \lambda - \bar{\lambda} \). Only nonzero results are presented. For \( i = 1 \cdots 7 \) there are no contributions to \( \Delta \lambda = 2 \) amplitudes.

Seven of the tensors, Eqn. (2.3a)-Eqn. (2.3g), follow the notation of Ref. [1], where the primary emphasis was the discussion of nonstandard \( WW\gamma \) and \( WWZ \) vertices which respect Lorentz invariance and electromagnetic gauge invariance, but not \( SU(2) \times U(1) \) gauge
TABLE IV. The $\hat{T}_i,\tau(k, p, \lambda, \bar{\lambda})$, $i = 8, 9$ evaluated in the CM frame. (For $i = 1 \cdots 7$ see Table II.) Each $\hat{T}_i,\tau$ is explicitly multiplied by the appropriate $d$-function or linear combination of $d$-functions. In column 1, $\Delta\lambda = \lambda - \bar{\lambda}$. Only nonzero results are entered in the table.

| $\Delta\lambda$ | $\lambda\bar{\lambda}$ | $\hat{T}_8 d_{\tau,\Delta\lambda}$ | $\hat{T}_9 d_{\tau,\Delta\lambda}$ |
|------------------|-----------------------|----------------------------------|----------------------------------|
| 0                | 00                    | $-\sqrt{\frac{2}{3}}d_{\tau,0}^2$ |                                   |
| 0                | ++                    | $-\sqrt{\frac{1}{3}}d_{\tau,0}^2$ | $-4\gamma^2\beta d_{\tau,0}^1$  |
| 0                | --                    | $-\sqrt{\frac{1}{3}}d_{\tau,0}^2$ | $-4\gamma^2\beta d_{\tau,0}^1$  |
| +1               | +0                    | $-\gamma d_{\tau,1}^2$           | $-2\gamma^3\beta(d_{\tau,1}^2 + \tau d_{\tau,1}^1)$ |
| +1               | 0−                    | $-\gamma d_{\tau,1}^2$           | $-2\gamma^3\beta(d_{\tau,1}^2 + \tau d_{\tau,1}^1)$ |
| −1               | 0+                    | $-\gamma d_{\tau,-1}^2$          | $2\gamma^3\beta(d_{\tau,-1}^2 - \tau d_{\tau,-1}^1)$ |
| −1               | −0                    | $-\gamma d_{\tau,-1}^2$          | $2\gamma^3\beta(d_{\tau,-1}^2 - \tau d_{\tau,-1}^1)$ |
| +2               | +−                    | $-\sqrt{2}d_{\tau,-2}^2$         | $-4\sqrt{2}\gamma^2\beta d_{\tau,-2}^2$ |
| −2               | −−                    | $-\sqrt{2}d_{\tau,-2}^2$         | $4\sqrt{2}\gamma^2\beta d_{\tau,-2}^2$ |

Invariance. Under these assumptions the most general $WWV$ vertex ($V = \gamma$ or $Z$) may be written

$$\Gamma_{\nu}^{\mu\alpha\beta}(q, p, p) = \sum_{i=1}^{7} f_i^{V}(s) T_i^{\mu\alpha\beta},$$  

where the $f_i^{V}$ are the form factors of Ref. [1]. The two tensors which are new, Eqn. (2.31)-Eqn. (2.34), are necessary to include all possible effects, including t-channel and box corrections.

III. CALCULATION OF THE FORM-FACTORS

In this section we write the scalar form-factors, $F_{i,\tau}(s, t)$, of Eqn. (2.1) in a form which is valid at the one-loop order for completely general corrections in the two- and three-point functions. For higher-order effects which include fermionic vertices or the self-energy corrections for fermions, only the SM corrections are explicitly included. We find

$$F_{i,\tau}(s, t) = \frac{1}{s} \left\{ \left[ Q \left[ \bar{e}^2(s) + \bar{e}^2 \Gamma_{1}^{1}(s) - i\bar{e}^2 \Delta_{\gamma\gamma}(s) \right] + I_3 \bar{e}^2 \Gamma_{1}^{1}(s) \right] f_i^{0} + Q \bar{e}^2 f_i^{1}(s) \right\}$$

8
\[
+ \frac{1}{s - m_Z^2 + i s \Gamma_Z \frac{\Gamma_2}{m_Z}} \left\{ \left( (I_3 - \hat{s}_Z^2) \hat{c}^2 [g_Z(s) + \hat{g}_Z \Gamma_1 \hat{c} Z(s) - i \hat{g}_Z \Delta Z Z(s)] + I_3 \hat{c}^4 \hat{g}_Z \Gamma_2 \hat{c} \right) f_i^{Z(0)} + (I_3 - \hat{s}_Z^2) \hat{c}^2 \hat{g}_Z f_i^{Z(1)}(s) - \hat{g}_Z^2 \left[ Q \hat{c}^2 f_i^{Z(0)} + (I_3 - \hat{s}_Z^2) Q \hat{c}^2 (s) \right] \left( s^2 (s) - s^2 + i \Delta Z (s) \right) \right\} \\
+ \frac{1}{2 t} I_3 \hat{g}_Z^2 \left( 1 + \Gamma_{ee'}(t) + \Gamma_{e''e''}(t) \right) f_i^{t(0)} + B_{i,\tau}(s, t),
\]

where the hatted couplings, \( \hat{c} = \hat{g}_s = \hat{g}_c = \hat{g}_Z \hat{c} s \), are the MS couplings with a short-hand notation \( \hat{s}_Z^2 = 1 - \hat{c}_Z^2 = \sin^2 \theta_W(\hat{\mu})_{\text{MS}} \). \( I_3 \) and \( Q \) refer to the SU(2)-isospin and electromagnetic-charge quantum numbers of the electron, i.e. \( Q = -1 \), and \( I_3 = -1/2 \) \( (I_3 = 0) \) for a left-handed (right-handed) electron. Gauge-boson propagator corrections are contained in the form factors \( \hat{e}^2(s), \hat{g}_Z^2(s), \hat{s}^2(s), \Delta_{\gamma\gamma}(s), \Delta_{\gamma Z}(s) \) and \( \Delta_{ZZ}(s); \Gamma_1(s) \) and \( \Gamma_2(s) \) contain corrections to the \( eeV \) vertex and \( e^{\pm} \) self-energy corrections \[6\].

In Eqn. \((3.1)\) we introduce additional form-factors through
\[
f_i^X(s) = f_i^{X(0)} + f_i^{X(1)}(s),
\]
where \( X = Z, \gamma, t \). At the tree level, \( f_i^\gamma(0) \) \( (f_i^Z(0)) \), for \( i = 1, \cdots, 7 \), corresponds to the SM contribution to the s-channel exchange of a photon (Z boson). See Fig. 2. The values of the \( f_i^V(0) \) may be obtained via the expansion of the tree-level WWV vertex according to

\[ \text{FIG. 2. The SM Feynman graphs for the process } e^- e^+ \rightarrow W^- W^+. \text{ Momentum and helicity assignments coincide with those of Fig. 1.} \]

Eqn. \((2.8)\). At higher orders there may be corrections directly to the WWV vertex, which are contained in \( f_i^V(1) \). Associated self-energy corrections for the external W bosons are also included in \( f_i^V(1) \).

In a similar fashion the t-channel contribution to the tree-level amplitude may be expanded to obtain \( f_i^t(0) \) for \( i = 1, \cdots, 9 \). We do not introduce an \( f_i^t(1) \) term. Such a term
would correspond to corrections to the $W\nu\nu$ vertex beyond the SM; as stated above, nonstandard couplings to the external fermions are not explicitly considered. The only nonstandard corrections which enter via the neutrino-exchange diagram are the $W$-boson self-energy corrections which are included in the $e^-\nu W^-$ vertex-correction factor, $\Gamma^{e\nu}$, and in the $e^+\nu W^+$ vertex-correction factor, $\Gamma^{e\nu}$.

The SM tree-level values for the $f_i^{X(0)}$ are shown in Table V. Refering to Table I, we see that $f_1^{\gamma(0)} = f_1^{Z(0)} = f_1^{t(0)} = 1$ all contribute to the $C$-even $P$-even form-factor $F_{1,\tau}(s, t)$. Similarly $f_3^{\gamma(0)} = f_3^{Z(0)} = f_3^{t(0)} = 2$ all contribute to the $C$-even $P$-even form-factor $F_{3,\tau}(s, t)$. Parity violation in the SM tree-level amplitudes enters through the $C$-odd $P$-odd form-factor $F_{5,\tau}(s, t)$; that parity violation appears only via the t-channel Feynman graph of Fig. 2 is apparent from the values $f_5^{\gamma(0)} = f_5^{Z(0)} = 0$, $f_5^{t(0)} = 1$. Finally, spin-greater-than-one contributions are manifest through the contribution of $f_8^{t(0)} = 1$ to the $C$-even $P$-even form-factor $F_{8,\tau}(s, t)$. The regular pattern that appears in Table V is extremely important, as will be discussed at the end of this section in the context of tree-level perturbative unitarity. While the SM employs neither the $C$-even $P$-even form-factor $F_{2,\tau}(s, t)$ nor the $C$-odd $P$-odd form-factor $F_{9,\tau}(s, t)$ at the tree level, at the one-loop level they attain nonzero values [2]. $F_{9,\tau}(s, t)$ is generated solely through box corrections.

The barred charges include the real parts of the gauge-boson two-point-functions [3]:

$$\bar{e}^2(q^2) = \hat{e}^2\left[1 - \text{Re}\Pi_{\tau,\gamma}(q^2)\right], \quad (3.3a)$$

$$\bar{s}^2(q^2) = \hat{s}^2\left[1 + \frac{\hat{c}}{s}\text{Re}\Pi_{\tau,\gamma}(q^2)\right], \quad (3.3b)$$

$$\bar{g}_2^2(q^2) = \hat{g}_Z^2\left[1 - \text{Re}\Pi_{\tau,Z}(q^2)\right], \quad (3.3c)$$

$$\bar{g}_W^2(q^2) = \hat{g}_W^2\left[1 - \text{Re}\Pi_{\tau,W}(q^2)\right]. \quad (3.3d)$$
While $e^2$, $s^2$ and $g_Z^2$ are employed explicitly in Eqn. (3.1), $g_W^2$ enters only through the $W$-boson wave-function-renormalization factor as discussed below. The $\Delta_{VV'}$, which are the imaginary parts of the two-point-function corrections, are given by

\[
\begin{align*}
\Delta_{\gamma\gamma}(q^2) &= \text{Im}\Pi^{\gamma\gamma}_{T,V}(q^2), \\
\Delta_{\gamma Z}(q^2) &= \hat{s}\hat{c}\text{Im}\Pi^{\gamma Z}_{T,V}(q^2), \\
\Delta_{ZZ}(q^2) &= \text{Im}\Pi^{ZZ}_{T,Z}(q^2) - \frac{\text{Im}\Pi^{ZZ}_T(m_Z^2)}{m_Z^2}.
\end{align*}
\]

Here

\[
\Pi^{AB}_{T,V}(q^2) = \frac{\Pi^{AB}_T(q^2) - \Pi^{AB}_T(m_V^2)}{q^2 - m_V^2},
\]

where $m_V$ denotes the physical mass of the gauge boson $V$. The subscript ‘$T$’ indicates the use of the transverse component of gauge-boson two-point-function; the longitudinal component makes no contribution when coupled to an external massless-fermion current. We employ the LEP convention for the $Z$-boson mass and running width \[7\] which accounts for the additional contribution to $\Delta_{ZZ}$ in Eqn. (3.4c). The pinch-term contributions \[8,9,10,11\] have been removed from the vertex-correction terms, \textit{i.e.} $\Gamma^{(1)}_2(s)$ (and also $f_1^V(s)$ and $f_3^V(s)$), but instead have been absorbed into the barred effective charges \[6\]. This standard procedure renders the effective charges gauge invariant and allows us to use them universally in both the four-fermion and $e^+e^- \rightarrow W^+W^-$ amplitudes.

The $\Gamma^{e}_1$ and $\Gamma^{e}_2$ terms contain the corrections to the $ee\gamma$ and $eeZ$ vertices as well as the associated self-energy corrections of the external electrons(positrons). The $e^-\nu W^- (e^+\nu W^+)$ vertex corrections are combined with the electron (positron) and $W^-$ ($W^+$) wave-function renormalization factors and one half (one half) of the internal neutrino self-energy corrections to produce finite form factors $\Gamma^{e\nu} (\Gamma^{e\nu})$. The final term in Eqn. (3.1), $B_{i,\tau}(s,t)$, includes all box-type corrections.

Finally, we conclude this section with a discussion of the $e^+e^- \rightarrow W^+W^-$ amplitudes at the tree level in the SM. Notice that, in Table \[11\] and Table \[15\], the contributions where either one or two $W$ bosons are longitudinally polarized grow at high energies as a power of the kinematical variable $\gamma = E_W/m_W$. If a perturbative description remains valid at high energies, then the tree-level unitarity of the amplitudes demands that these large contribu-
tions cancel among the various terms of Eqn. (2.1) and Eqn. (3.1). These cancellations are straightforwardly displayed in the current formalism.

Combining Eqns. (2.1), (3.1) and (2.4), then taking the limit \( \beta \to 1, \gamma \to \infty \), the leading contributions to the amplitudes may be expressed as

\[
M(\tau; \lambda, \bar{\lambda}) = \sum_{i=1}^{9} \tau \sqrt{2} \left\{ e^{2}Q f^{(0)}_{i} + (g^{2}I_{3} - e^{2}Q) f^{Z(0)}_{i} - \frac{\hat{g}^{2}I_{3}}{1 - \cos \Theta f^{(0)}_{i}} \right\} \hat{T}_{i} d^{0}_{\tau \Delta \lambda}. \tag{3.6}
\]

On the left-hand side momentum arguments have been suppressed for brevity. First, consider the amplitude for \( \lambda \bar{\lambda} = 00 \). From Table V we see that \( \hat{T}_{1}, \hat{T}_{3} \) and \( \hat{T}_{8} \) contribute to this amplitude. In Eqn. (3.6) it is immediately apparent that the nonzero terms proportional to \( \hat{e}^{2}Q \) vanish within each \( F_{i,\tau}(s, t) \) because \( f^{(0)}_{i} = f^{Z(0)}_{i} \) for both \( i = 1 \) and \( i = 3 \). For left-handed fermion currents the \( \hat{g}^{2}I_{3} \) terms also play a role. However, in this case the cancellations only take place upon summation over the various form-factors. Using \( d^{2}_{\tau,0} = \sqrt{3} \cos \Theta d^{1}_{\tau,0} \) for the treatment of the \( \hat{T}_{8,-} \) contribution one obtains

\[
M(-; 0, 0) \sim \sqrt{2} \hat{g}^{2}I_{3} \left\{ \frac{-2\gamma^{2}d^{1}_{-,-} + 4\gamma^{2}d^{1}_{+,0}}{1 - \cos \Theta} - \frac{\sqrt{4/3} \gamma^{2}d^{2}_{+,0}}{1 - \cos \Theta} \right\} \to \sqrt{2} \hat{g}^{2}I_{3} \left\{ \frac{2\gamma^{2}(1 - \cos \Theta)}{1 - \cos \Theta} - 2\gamma \right\} d^{1}_{-,-,0}, \tag{3.7}
\]

and we see that the terms proportional to \( \gamma^{2} \) cancel.

We may repeat this procedure for the +0 amplitude. The nonzero terms proportional to \( \hat{e}^{2}Q \) cancel within \( F_{3} \). However, for the terms proportional to \( \hat{g}^{2}I_{3} \), the cancellations only take place when the \( i = 3, 5 \) and 8 contributions are summed. Using \( d^{2}_{\tau,\pm 1} = (2 \cos \Theta \mp \tau) d^{1}_{\tau,\pm 1} \),

\[
M(-; +, 0) \sim \sqrt{2} \hat{g}^{2}I_{3} \left\{ \frac{2\gamma d^{1}_{+,-} + \gamma d^{2}_{-,-} - \gamma d^{2}_{-,-}}{1 - \cos \Theta} - 2\gamma d^{1}_{+,-} \right\} \to \sqrt{2} \hat{g}^{2}I_{3} \left\{ \frac{2\gamma(1 - \cos \Theta)}{1 - \cos \Theta} - 2\gamma \right\} d^{1}_{-,-}, \tag{3.8}
\]

and here the contributions proportional to \( \gamma \) cancel. We could repeat this analysis for the \( 0-, 0+ \) and \( -0 \) amplitudes, again to discover that the terms which grow with energy cancel.

There is an advantage of adopting the \( \overline{\text{MS}} \) couplings in the perturbative expansion of the form-factors. In brief, through a judicious choice of the renormalization scale for these couplings, the above tree-level unitarity cancellation straightforwardly prevails in the major part of the corrected amplitudes. In the numerical studies of subsequent sections we adopt
the renormalization conditions $\hat{e}^2 = \bar{e}^2(s)_{\text{SM}}$ and $\hat{s}^2 = \bar{s}^2(s)_{\text{SM}}$. The nonstandard corrections described by an effective Lagrangian appear to violate tree-level unitarity, but one should recall that the effective-Lagrangian description is valid only at energies below the threshold of the new physics. This will be revisited in Section VII.

IV. THE LINEAR REPRESENTATION

If the scale of new physics, $\Lambda$, is large compared to the vacuum expectation value (vev) of the Higgs field, $v \equiv (\sqrt{2}G_F)^{-1/2} = 246\text{GeV}$, then the effective Lagrangian may be expressed as the SM Lagrangian plus terms with energy dimension greater than four suppressed by inverse powers of $\Lambda$, i.e.

$$L_{\text{eff}} = L_{\text{SM}} + \sum_{n \geq 5} \sum_i f_i^{(n)} \frac{O_i^{(n)}}{\Lambda^{n-4}}. \quad (4.1)$$

The energy dimension of each operator is denoted by $n$, and the index $i$ sums over all operators of the given energy dimension. The coefficients $f_i^{(n)}$ are free parameters, though they may be determined explicitly once the full theory is known.

The higher-dimensional terms are constructed from the fields of the low-energy theory. In this section we assume that the low-energy theory, i.e. the SM, contains a light physical scalar Higgs particle which is the remnant of a complex Higgs-doublet field; the remaining three real fields of this doublet provide the longitudinal modes of the $W^\pm$ and $Z$ bosons. We will refer to the physics described by the effective Lagrangian of Eqn. (4.1) as the ‘light-Higgs scenario’ or the ‘linear realization of the symmetry-breaking sector’.

An exhaustive list of SU(2)×U(1)-gauge-invariant energy-dimension-five and -six operators has been compiled in Refs. [12]. As outlined in Section I, we exclude all operators which contain fermionic fields. Furthermore, we only consider operators which conserve CP. Upon restricting the analysis to operators not exceeding energy-dimension six we find that twelve operators form a basis set; all are dimension-six and separately conserve C and P. In the notation of Ref. [13,14] they are

$$O_{DW} = \text{Tr}\left([D_\mu, \hat{W}_{\nu \rho}] [D^\mu, \hat{W}^{\nu \rho}]\right), \quad (4.2a)$$

$$O_{DB} = -\frac{g'^2}{2} \left(\partial_\mu B_{\nu \rho} \right) \left(\partial^\mu B^{\nu \rho}\right), \quad (4.2b)$$
\[ O_{BW} = \Phi^\dagger B_{\mu\nu} \hat{W}^{\mu\nu} \Phi , \]  
\[ O_{\Phi,1} = \left[ \left( D_\mu \Phi \right)^\dagger \right] \left[ \Phi^\dagger \left( D^\mu \Phi \right) \right] , \]  
\[ O_{WWW} = \text{Tr} \left( \hat{W}_{\mu\nu} \hat{W}^{\rho\nu} \hat{W}^\rho_\mu \right) , \]  
\[ O_{WW} = \Phi^\dagger \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \Phi , \]  
\[ O_{BB} = \Phi^\dagger B_{\mu\nu} \hat{B}^{\mu\nu} \Phi , \]  
\[ O_{W} = \left( D_\mu \Phi \right)^\dagger \hat{W}^{\mu\nu} \left( D_\nu \Phi \right) , \]  
\[ O_{B} = \left( D_\mu \Phi \right)^\dagger \hat{B}^{\mu\nu} \left( D_\nu \Phi \right) , \]  
\[ O_{\Phi,2} = \frac{1}{2} \partial_\mu \left( \Phi^\dagger \Phi \right) \partial^\mu \left( \Phi^\dagger \Phi \right) , \]  
\[ O_{\Phi,3} = \frac{1}{3} \left( \Phi^\dagger \Phi \right)^3 , \]  
\[ O_{\Phi,4} = \left( \Phi^\dagger \Phi \right) \left[ \left( D_\mu \Phi \right)^\dagger \left( D^\mu \Phi \right) \right] . \]

The covariant derivative, \( D_\mu \), is given by

\[ D_\mu = \partial_\mu + igT^a W_\mu^a + ig'Y B_\mu , \]

where \( g \) is the SU(2) coupling with \( \text{Tr}(T^a T^b) = \frac{1}{2} \delta^{ab} \), \( g' \) is the U(1) coupling and \( Y \) is the hypercharge operator. For convenience when defining the normalizations of the individual operators we use the ‘hatted’ field strength tensors defined according to

\[ [ D_\mu, D_\nu ] = \hat{W}_{\mu\nu} + \hat{B}_{\mu\nu} , \]

hence

\[ \hat{W}_{\mu\nu} = igT^a W_\mu^a \quad \text{and} \quad \hat{B}_{\mu\nu} = ig'Y B_{\mu\nu} . \]

Combining the twelve operators of Eqn. (4.2) with Eqn. (4.1) completes the construction of the effective Lagrangian in the linear representation.

The calculation of the Feynman rules from Eqns. (4.1) and (4.2) is straightforward, though tedious. We do not present the Feynman rules, but in Table VI we indicate those vertices to which each operator contributes with an ‘X’ in the appropriate box. First, observe
that four of the operators, $O_{DW}$, $O_{DB}$, $O_{BW}$ and $O_{\Phi,1}$, contribute to gauge-boson two-point-functions at the tree level \[15\]. For this reason their respective coefficients, $f_{DW}$, $f_{DB}$, $f_{BW}$ and $f_{\Phi,1}$, are strongly constrained by LEP/SLC and low-energy data \[13,14,16\], and these constraints will be improved by the study of two-fermion final states at higher-energy lepton colliders \[16\]. (This will be discussed in greater detail in Section VI.) These four operators will contribute to the process $e^+ e^- \rightarrow W^+ W^-$ through corrections to the charge form-factors, $\bar{e}^2(q^2)$, $s^2(q^2)$, $\bar{g}_Z^2(q^2)$, and through the $W$-boson wave-function-renormalization factor.

The operators $O_{DW}$ and $O_{BW}$ also make a direct contribution to $WW\gamma$ and $WWZ$ vertices. Three additional operators contribute as well \[13,14\]. They are $O_{WWW}$, $O_W$ and $O_B$; their respective coefficients are $f_{WWW}$, $f_W$ and $f_B$.

Naively one would expect contributions to $WWV$ three-point-functions and to gauge-boson two-point-functions from the operators $O_{WW}$ and $O_{BB}$. However, their contributions may be completely absorbed by a redefinition of SM fields and gauge couplings,

$$
\left[ 1 + \frac{2m_W^2}{\Lambda^2} f_{WW} \right]^{1/2} W^{\mu\nu} \rightarrow W^{\mu\nu},
$$

$$
\left[ 1 + \frac{2m_W^2}{\Lambda^2} f_{WW} \right]^{-1/2} g \rightarrow g,
$$

$$
\left[ 1 + \frac{2m_Z^2}{\Lambda^2} s^2 f_{BB} \right]^{1/2} B^{\mu\nu} \rightarrow B^{\mu\nu},
$$

$$
\left[ 1 + \frac{2m_Z^2}{\Lambda^2} s^2 f_{BB} \right]^{-1/2} g' \rightarrow g',
$$

leading to a null contribution. For this reason an ‘O’ is used for these operators in Table VI.

Additionally $O_{\Phi,4}$ contributes to the $W$- and $Z$-mass terms, while $O_{\Phi,1}$ contributes to the $Z$-mass term only. Hence $O_{\Phi,1}$ violates the custodial symmetry \[17\], SU(2)$_c$, and the $T$ parameter \[18\] is explicitly dependent upon $f_{\Phi,1}$. On the other hand, the contributions from $O_{\Phi,4}$ exactly cancel in the calculation of $T$, hence it does not contribute to our analysis.

Notice that $O_{DW}$, $O_{WWW}$ and $O_W$ contribute to four-gauge-boson vertices, though none contribute to a $ZZZZ$ vertex. Furthermore, many of the operators (1.2) do contribute to processes which include Higgs bosons. For example, $O_{WWW}$ and $O_{BB}$ contribute to the $H\gamma\gamma$ vertex \[19\]. The operators $O_{\Phi,2}$, $O_{\Phi,3}$ and $O_{\Phi,4}$ are of concern only when discussing nonstandard Higgs-boson interactions.
TABLE VI. Energy-dimension-six operators in the linear representation of the Higgs mechanism. The contribution of an operator to a particular vertex is denoted by an ‘X’. In some cases an operator naively contributes to a vertex, yet that contribution does not lead to observable effects. In such cases the ‘X’ is replaced by an ‘O’.
V. THE NONLINEAR REALIZATION

The construction of the effective Lagrangian requires knowledge of the low-energy particle spectrum. The existence of a light Higgs boson has not been confirmed, and an intriguing possibility is that no such particle exists. The scale for the new physics is then set by the scale of electroweak symmetry breaking, \( v \). Typically

\[ \Lambda \sim 4\pi v . \]  

(5.1)

In general one should expect that the list of operators which contribute to the effective Lagrangian are related to those of the linear representation of Eqn. (4.1), but the operators which appear at leading order may be quite different than those enumerated in Eqn. (4.2). This may be seen by studying the nonlinear representation of the Higgs doublet field;

\[
\Phi(x) = \exp \left( \frac{i\chi^i(x)\tau^i}{v} \right) \left( \begin{array}{c} 0 \\ (v + H)/\sqrt{2} \end{array} \right),
\]

(5.2)

where \( \chi^i(x) \) are the Goldstone fields, \( H \) is the usual Higgs field and the \( \tau^i \) are the Pauli matrices. In the limit that the Higgs field is too massive to fluctuate the \( H \) term may be dropped. Then, in the unitary gauge,

\[
\Phi(x) = \frac{v}{\sqrt{2}} \left( \begin{array}{c} 0 \\ 1 \end{array} \right). 
\]

(5.3)

Therefore, if one starts with an operator of energy dimension \( n \) in the linear representation but removes \( m \) Higgs fields, \( H \), the residual operator may, by Eqn. (5.1), contain a coefficient proportional to \( v^m/\Lambda^n \sim 1/v^{n-m} \) for integers \( n \) and \( m \). Hence operators which appear at higher orders in the linear representation may appear at a reduced order in the nonlinear representation. Powers of \( 4\pi \) are absorbed into the numerical coefficients.

The full Lagrangian may be written as

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \mathcal{L}_i + \cdots. 
\]

(5.4)

In contrast to the linearly realized Lagrangian of Eqn. (4.1), the \( \mathcal{L}_{\text{SM}} \) term does not contain the physical Higgs field. We adopt the notation \([20][21][22]\).
\[ U \equiv \frac{\sqrt{2}}{v} (\Phi^c, \Phi) = \exp \left( \frac{2i\chi^i(x)\tau^i}{v} \right), \tag{5.5a} \]
\[ D_\mu U = \partial_\mu U + igT^a W^a_\mu U - ig' UT^3 B_\mu, \tag{5.5b} \]
\[ T \equiv 2UT^3U^\dagger, \tag{5.5c} \]
\[ V_\mu \equiv (D_\mu U)U^\dagger. \tag{5.5d} \]

Here \( \Phi^c = i\tau^2\Phi^* \) denotes the charge-conjugate Higgs doublet field, and \( T^a = \tau^a / 2 \) are the generators of the SU(2) algebra. In the unitary gauge these expressions become,

\[ U = 1, \tag{5.6a} \]
\[ D_\mu U = igT^a W^a_\mu - ig'T^3 B_\mu, \tag{5.6b} \]
\[ T = 2T^3, \tag{5.6c} \]
\[ V_\mu = D_\mu U. \tag{5.6d} \]

The custodial SU(2)_c symmetry which rotates \( \Phi \) and \( \Phi^c \) is broken by the hypercharge gauge interactions of Eqn. (5.5b) and the \( T \) term of Eqn. (5.5c).

We present a list of gauge-invariant chiral operators through energy-dimension four which conserve CP. There are twelve such operators given by [21,22]

\[ L'_1 = \frac{1}{4} \beta_1 v^2 \left[ \text{Tr} \left( TV_\mu \right) \right]^2, \tag{5.7a} \]
\[ L_1 = \frac{1}{2} \alpha_1 gg' \text{Tr} \left( B_{\mu\nu} TW^{\mu\nu} \right), \tag{5.7b} \]
\[ L_2 = \frac{i}{2} \alpha_2 g' B_{\mu\nu} \text{Tr} \left( T[V^\mu, V^\nu] \right), \tag{5.7c} \]
\[ L_3 = i\alpha_3 g \text{Tr} \left( W_{\mu\nu} [V^\mu, V^\nu] \right), \tag{5.7d} \]
\[ L_4 = \alpha_4 \left[ \text{Tr} \left( V_\mu V_\nu \right) \right]^2, \tag{5.7e} \]
\[ L_5 = \alpha_5 \left[ \text{Tr} \left( V_\mu V_\mu \right) \right]^2, \tag{5.7f} \]
\[ L_6 = \alpha_6 \text{Tr} \left( V_\mu V_\nu \right) \text{Tr} \left( TV_\mu \right) \text{Tr} \left( TV_\nu \right), \tag{5.7g} \]
\[ L_7 = \alpha_7 \text{Tr} \left( V_\mu V_\mu \right) \text{Tr} \left( TV_\nu \right) \text{Tr} \left( TV_\nu \right), \tag{5.7h} \]
\[ L_8 = \frac{1}{4} \alpha_8 g^2 \left[ \text{Tr} \left( TW_{\mu\nu} \right) \right]^2, \tag{5.7i} \]
\[ L_9 = \frac{i}{2} \alpha_9 g \text{Tr} \left( TW_{\mu\nu} \right) \text{Tr} \left( T[V^\mu, V^\nu] \right), \tag{5.7j} \]
\[ L_{10} = \frac{1}{2} \alpha_{10} \left[ \text{Tr} \left( TV_\mu \right) \text{Tr} \left( TV_\nu \right) \right]^2, \tag{5.7k} \]
\[ L_{11} = \alpha_{11} g \epsilon_{\mu\rho\sigma} \text{Tr} \left( TV_\mu \right) \text{Tr} \left( V_\nu W_{\rho\sigma} \right). \tag{5.7l} \]
The dimension-two operator $L'_1$ and the first ten dimension-four operators, $L_1$ through $L_{10}$, conserve both $C$ and $P$, whereas the last operator, $L_{11}$, is both $C$-odd and $P$-odd. We adopt the notation of Ref. [21] and Ref. [22].

In Table VII we indicate the vertices to which each operator contributes with an ‘X’. Additionally we present, with each chiral operator, its counterpart in the linear realization. In particular we may associate four of the chiral operators with energy-dimension-six operators of Section IV. Realizing that the $O_i$ depend explicitly upon the field $H$, but the $L_i$ do not, we may write

\begin{align}
L'_1 &= -\frac{4\beta_1}{v^2} O_{\Phi,1} , \\
L_1 &= \frac{4\alpha_1}{v^2} O_{BW} , \\
L_2 &= \frac{8\alpha_2}{v^2} O_B , \\
L_3 &= \frac{8\alpha_3}{v^2} O_W .
\end{align}

These operator identities give valid relations among matrix elements for processes that do not involve external Higgs particles. The linear-realization counterparts of the remaining chiral operators appear at energy-dimension eight, ten and twelve. These higher dimensional operators in the second column of Table VI are

\begin{align}
O^{(8)}_4 &= \left[ (D_\mu \Phi)^\dagger (D_\nu \Phi) + (D_\nu \Phi)^\dagger (D_\mu \Phi) \right]^2 , \\
O^{(8)}_5 &= \left[ (D_\mu \Phi)^\dagger (D^\mu \Phi) \right]^2 , \\
O^{(10)}_6 &= \left[ (D_\mu \Phi)^\dagger (D_\nu \Phi)^\dagger (D^\mu \Phi) \right] \left[ (D^\nu \Phi) \right] , \\
O^{(10)}_7 &= \left[ (D_\mu \Phi)^\dagger (D^\nu \Phi)^\dagger (D^\mu \Phi) \right] \left[ (D^\nu \Phi) \right] , \\
O^{(8)}_8 &= \left[ \Phi^\dagger W_{\mu\nu} \Phi \right]^2 , \\
O^{(8)}_9 &= \left[ \Phi^\dagger W_{\mu\nu} \Phi \right] \left[ (D^\mu \Phi)^\dagger (D^\nu \Phi) \right] , \\
O^{(12)}_{10} &= \left[ \Phi^\dagger (D_\mu \Phi) \right] \left[ \Phi^\dagger (D_\nu \Phi) \right] \left[ \Phi^\dagger (D^\rho \Phi) \right] , \\
O^{(8)}_{11} &= i \epsilon^{\mu\nu\rho\sigma} \left[ \Phi^\dagger (D_\mu \Phi) \right] \left[ \Phi^\dagger W_{\rho\sigma} (D_\nu \Phi) \right] + \text{h.c.}
\end{align}

The higher dimensionality of the associated operators in the linear realization indicates

---

1 Operators $L_1$ through $L_{10}$ were discussed in Ref. [21], but $L_{11}$ was added in Ref. [22].
\[ \mathcal{L}_{\text{chiral}} \]

| \( \mathcal{L} \) | \( \mathcal{O}_{\text{linear}} \) | MM | ZZ | AA | ZZM | ZAM | VMM | MMMM | ZZZM | VZZM | WZZ | WAA | ZZZZ |
|----------------|-----------------|-----|-----|-----|------|------|------|-------|-------|-------|------|------|-------|
| \( \mathcal{L}_1 = \frac{\beta_1 v^2}{2} \left[ \text{Tr}(TV_\mu) \right]^2 \) | \( -\frac{4\beta_1}{v} \mathcal{O}_8,1 \) | X | | | | | | | | | | | |
| \( \mathcal{L}_2 = \frac{i\alpha_3 g}{2} B_{\mu\nu} \text{Tr} \left( T[V_\mu, V_\nu] \right) \) | \( \frac{8\alpha_3}{v^2} \mathcal{O}_B \) | X | X | | | | | | | | | | |
| \( \mathcal{L}_3 = i\alpha_3 g \text{Tr} \left( W_{\mu\nu}[V_\mu, V_\nu] \right) \) | \( \frac{8\alpha_3}{v^2} \mathcal{O}_W \) | X | X | X | X | | | | | | | | |
| \( \mathcal{L}_4 = \alpha_4 \left[ \text{Tr}(V_\mu V_\nu) \right]^2 \) | \( \frac{4\alpha_4}{v^4} \mathcal{O}_4^{(8)} \) | X | X | | | | | | | | | | |
| \( \mathcal{L}_5 = \alpha_5 \left[ \text{Tr}(V_\mu V_\nu) \right]^2 \) | \( \frac{16\alpha_5}{v^4} \mathcal{O}_5^{(8)} \) | X | X | | | | | | | | | | |
| \( \mathcal{L}_6 = \alpha_6 \text{Tr}(V_\mu V_\nu) \text{Tr}(TV^\mu) \text{Tr}(TV_\nu) \) | \( -\frac{64\alpha_6}{v^8} \mathcal{O}_6^{(10)} \) | X | X | | | | | | | | | | |
| \( \mathcal{L}_7 = \alpha_7 \text{Tr}(V_\mu V_\nu) \text{Tr}(TV_\nu) \text{Tr}(TV_\nu) \) | \( -\frac{64\alpha_7}{v^8} \mathcal{O}_7^{(10)} \) | X | X | | | | | | | | | | |
| \( \mathcal{L}_8 = \frac{i\alpha_9 g^2}{2} \left[ \text{Tr}(TW_{\mu\nu}) \right]^2 \) | \( -\frac{4\alpha_9}{v^4} \mathcal{O}_8^{(8)} \) | X | X | X | X | X | | | | | | | |
| \( \mathcal{L}_9 = \frac{i\alpha_9 g}{2} \text{Tr}(TW_{\mu\nu}) \text{Tr}(T[V_\mu, V_\nu]) \) | \( -\frac{16\alpha_9}{v^4} \mathcal{O}_9^{(8)} \) | X | X | X | | | | | | | | | |
| \( \mathcal{L}_{10} = \frac{\alpha_10 g}{2} \left[ \text{Tr}(TV_\mu) \text{Tr}(TV_\nu) \right]^2 \) | \( \frac{128\alpha_{10}}{v^8} \mathcal{O}_{10}^{(12)} \) | X | | | | | | | | | | | |
| \( \mathcal{L}_{11} = \alpha_{11} g \epsilon_{\mu\nu\rho\sigma} \text{Tr}(TV_\mu) \text{Tr}(V_\nu W_{\rho\sigma}) \) | \( \frac{8\alpha_{11}}{v^4} \mathcal{O}_{11}^{(8)} \) | X | X | | | | | | | | | | |

**TABLE VII.** Column one lists operators in the nonlinear representation. The linear-representation counterparts appear in the second column. For the definitions of the operators \( \mathcal{O}_i^{(n)} \) the reader is referred to the text. An ‘X’ is used to indicate the the contribution of an individual operator to a particular vertex.
that the observation of effects arising from $\mathcal{L}_4$ through $\mathcal{L}_{11}$ are an indication of a strongly interacting Higgs sector.

Three of the operators, $\mathcal{L}'_1$, $\mathcal{L}_1$ and $\mathcal{L}_8$, contribute to gauge-boson two-point-functions. Like $\mathcal{O}_{8,1}$, $\mathcal{L}'_1$ contributes only to the $Z$-mass term but not to the $W$-mass term and leads to a violation of the custodial symmetry. Through contributions to the charge form-factors $\bar{e}^2(q^2)$, $\bar{s}^2(q^2)$ and $\bar{g}_Z^2(q^2)$ these three operators contribute to the process $e^+e^- \rightarrow W^+W^-$. None of the operators contributes to the $WW$ two-point-function, hence, in contrast to the linear realization, the non-SM operators do not contribute to the $W$-boson wave-function-renormalization factor, and the t-channel neutrino-exchange amplitudes are not modified.

In total six of the operators, $\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_3$, $\mathcal{L}_8$, $\mathcal{L}_9$ and $\mathcal{L}_{11}$, contribute directly to three-gauge-boson vertices, and a total of nine contribute to four-gauge-boson vertices. While operators of the linear representation contribute to the $WW\gamma\gamma$ vertex but not to a $ZZZZ$ vertex, precisely the opposite is realized in Table VII. And of course there are no Higgs-boson interactions in the nonlinear realization.

Finally, an alternative standard notation of Ref. [24] is related to our notation by

\begin{align}
\alpha_1 &= L_{10} , \\
\alpha_2 &= -\frac{1}{2}L_{9R} , \\
\alpha_3 &= -\frac{1}{2}L_{9L} .
\end{align}

Ref. [25] makes the estimate

\begin{align}
\alpha_1 &= L_{10} \approx -0.05
\end{align}

for one family of techniquarks and technileptons with chiral SU(8)$\times$SU(8) symmetry, and

\begin{align}
\alpha_1 &= L_{10} \approx -0.005
\end{align}

in the minimal model with one color-singlet technidoublet. Taking input from low-energy QCD [24,26],

\begin{align}
L_9 \approx -L_{10} , \quad \alpha_2 \approx \alpha_3 \approx 2\alpha_1 .
\end{align}

Considering the contributions from $N$ flavor doublets of heavy fermions $U$ and $D$ [26,27,28] Ref. [22] estimates
\[ \beta_1 = \frac{N}{24\pi^2} \frac{(\Delta m)^2}{v^2}, \]  
\[ \alpha_1 = -\frac{N}{96\pi^2} \approx -N \times 10^{-3}, \]  
\[ \alpha_2 = -\frac{N}{96\pi^2} \approx -N \times 10^{-3}, \]  
\[ \alpha_3 = -\frac{N}{96\pi^2} \left\{ 1 - \frac{2}{5} \delta^2 \right\} \approx -N \times 10^{-3}, \]  
\[ \alpha_8 = -\frac{N}{96\pi^2} \frac{16}{5} \delta^2, \]  
\[ \alpha_9 = -\frac{N}{96\pi^2} \frac{14}{5} \delta^2, \]  
\[ \alpha_{11} = -\frac{N}{96\pi^2} \delta, \]

where \( \Delta m = m_U - m_D, \) \( \delta = (m_U - m_D)/(m_U + m_D), \) and it has been assumed that the mass splitting is small compared to the masses \( m_U \) and \( m_D. \) Notice that \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) are approximately degenerate. Also, \( \beta_1 \) is suppressed relative to \( \alpha_1 \) by \( (\Delta m)^2/v^2 \) while \( \alpha_8 \) and \( \alpha_9 \) are suppressed by \( \delta^2. \) It is noteworthy that, while \( \alpha_{11} \) is also suppressed, the suppression factor is only one power of \( \delta. \) The above estimates will serve as useful benchmarks throughout the remainder of the paper.

VI. PROCESSES WITH FOUR EXTERNAL FERMIONS

A. The linear realization of the symmetry-breaking sector

In the linear realization four operators, \( O_{DW}, O_{DB}, O_{BW} \) and \( O_{\Phi,1}, \) have special significance [13,14] due to their contributions to low-energy processes involving four external light fermions. In short, they are the only operators from Eqns. (4.2) which are well constrained by the present data. This subset contributes to electroweak precision observables via their contributions to the transverse components of the gauge-boson propagators. If the one-particle-irreducible two-point-function is separated into SM and new-physics contributions according to \( \Pi = \Pi_{SM} + \Delta \Pi, \) then, in the notation of Ref. [13,14], we find

\[ \Delta \Pi_{T}^{QQ}(q^2) = 2\frac{q^2}{\Lambda^2} \left[ (f_{DW} + f_{DB})q^2 - f_{BW}\frac{v^2}{4} \right], \]  
\[ \Delta \Pi_{T}^{3Q}(q^2) = 2\frac{q^2}{\Lambda^2} \left[ f_{DW}q^2 - f_{BW}\frac{v^2}{8} \right], \]
\[
\Delta \Pi_T^{33}(q^2) = 2 \frac{q^2}{\Lambda^2} f_{DW} q^2 - \frac{v^2}{\Lambda^2} \left[ f_{\Phi,1} + f_{\Phi,4} \right] \frac{v^2}{8},
\]

\[
\Delta \Pi_T^{11}(q^2) = 2 \frac{q^2}{\Lambda^2} f_{DW} q^2 - \frac{v^2}{\Lambda^2} f_{\Phi,4} \frac{v^2}{8}.
\]

The two-point functions may also be expressed in a basis which refers to physical gauge bosons by

\[
\Pi_T^\gamma(q^2) = \epsilon^2 \Pi_T^{QQ}(q^2),
\]

\[
\Pi_T^{ZZ}(q^2) = \epsilon g_Z \left[ \Pi_T^{QQ}(q^2) - s^2 \Pi_T^{QQ}(q^2) \right],
\]

\[
\Pi_T^{QZ}(q^2) = g_Z \left[ \Pi_T^{QQ}(q^2) - 2 s^2 \Pi_T^{QQ}(q^2) + s \Pi_T^{QQ}(q^2) \right],
\]

\[
\Pi_T^{WW}(q^2) = g^2 \Pi_T^{11}(q^2).
\]

Either set of two-point functions may be employed, as convenience dictates. From Eqn. (6.1) follow the \(S, T\) and \(U\) parameters of Ref. [38] or some equivalent triplet of parameters [39]. In general we allow for an anomalous contribution to \(\alpha_{QED}(m_Z^2)\) [30]. Defining \(S, T\) and \(U\) according to Ref. [3],

\[
\Delta S \equiv 16\pi \Re \left[ \Delta \Pi_T^{33}(m_Z^2) - \Delta \Pi_T^{33}(0) \right] = -4\pi \frac{v^2}{\Lambda^2} f_{BW},
\]

\[
\Delta T \equiv \frac{4\sqrt{2} G_F}{\alpha} \Re \left[ \Delta \Pi_T^{33}(0) - \Delta \Pi_T^{11}(0) \right] = -\frac{1}{2\alpha \Lambda^2} f_{\Phi,1},
\]

\[
\Delta U \equiv 16\pi \Re \left[ \Delta \Pi_T^{33}(0) - \Delta \Pi_T^{11}(0) \right] = 32\pi \frac{m_Z^2 - m_W^2}{\Lambda^2} f_{DW},
\]

\[
\Delta \frac{1}{\alpha} \equiv 4\pi \Re \left[ \Delta \Pi_T^{QQ}(m_Z^2) - \Delta \Pi_T^{QQ}(0) \right] = 8\pi \frac{m_Z^2}{\Lambda^2} (f_{DW} + f_{DB}),
\]

where \(S = S_{SM} + \Delta S\), \(T = T_{SM} + \Delta T\), \(U = U_{SM} + \Delta U\), and

\[
\Pi_A^{AB}(q^2) = \frac{\Pi_A^{AB}(q^2) - \Pi_A^{AB}(m_V^2)}{q^2 - m_V^2}.
\]

Because the contributions of \(f_{\Phi,1}\) and \(f_{\Phi,4}\) to the two-point functions of Eqn. (6.1) are independent of \(q^2\), they may contribute only to \(T\). The \(f_{\Phi,4}\) contributions exactly cancel as expected. The charge form-factors of Ref. [6] follow directly:

\[
\frac{1}{\bar{g}_Z(0)} = \frac{1 + \delta \epsilon - \alpha T}{4\sqrt{2} G_F m_Z^2},
\]

\[
\bar{S}^2(m_Z^2) = \frac{1}{2} - \left[ \frac{1}{4} - \bar{S}^2(m_Z^2) \left( \frac{1}{\bar{g}_Z(0)} + \frac{S}{16\pi} \right) \right],
\]

\[
\frac{1}{\bar{g}_W(0)} = \frac{\bar{S}^2(m_Z^2)}{\bar{S}^2(m_Z^2)} - \frac{1}{16\pi} (S + U),
\]
where SM vertex and box corrections to the muon lifetime are incorporated in $\delta_G \approx 0.0055$. Additionally, the nontrivial $q^2$-dependence of the two-point-functions leads to a nonstandard running of the charge form-factors;

$$\Delta \left[ \frac{1}{\hat{e}^2(q^2)} - \frac{1}{4\alpha} \right] = \frac{2q^2}{\Lambda^2} (f_{DW} + f_{DB}), \quad (6.6a)$$

$$\Delta \left[ \frac{s^2(q^2)}{\hat{e}^2(q^2)} - \frac{\pi^2(m_Z^2)}{\hat{\tau}^2(m_Z^2)} \right] = \frac{2q^2 - m_Z^2}{\Lambda^2} f_{DW}, \quad (6.6b)$$

$$\Delta \left[ \frac{1}{\hat{g}_Z(q^2)} - \frac{1}{\hat{g}_Z(0)} \right] = \frac{2q^2}{\Lambda^2} (c^4 f_{DW} + s^4 f_{DB}), \quad (6.6c)$$

$$\Delta \left[ \frac{1}{\hat{g}_W(q^2)} - \frac{1}{\hat{g}_W(0)} \right] = \frac{2q^2}{\Lambda^2} f_{DW}. \quad (6.6d)$$

The combination of Eqn. (6.4) with Eqn. (5.6) leads to the convenient expressions

$$\Delta \underline{\sigma}(q^2) = -8\pi\alpha^2 \frac{q^2}{\Lambda^2} (f_{DW} + f_{DB}), \quad (6.7a)$$

$$\Delta \underline{g}_Z(q^2) = -2\hat{g}_Z^2 \frac{q^2}{\Lambda^2} \left( c^4 f_{DW} + s^4 f_{DB} \right) - \frac{1}{2} \hat{g}_Z^2 \frac{v^2}{\Lambda^2} f_{\Phi,1}, \quad (6.7b)$$

$$\Delta \underline{g}_W(q^2) = -\frac{s^2\hat{c}^2}{\hat{c}^2 - \hat{s}^2} \left[ 8\pi\alpha \frac{m_Z^2}{\Lambda^2} (f_{DW} + f_{DB}) + \frac{m_Z^2}{\Lambda^2} f_{BW} - \frac{1}{2} \frac{v^2}{\Lambda^2} f_{\Phi,1} \right]$$

$$+ 8\pi\alpha \frac{q^2 - m_Z^2}{\Lambda^2} \left( c^2 f_{DW} - s^2 f_{DB} \right), \quad (6.7c)$$

$$\Delta \underline{g}_W(q^2) = -8\pi\hat{g}_W^2 \frac{m_Z^2}{\Lambda^2} f_{DB} - \frac{1}{4} \frac{\hat{g}_W^2}{\Lambda^2} f_{BW} - \frac{1}{2} \frac{\hat{g}_W^2}{\Lambda^2} f_{DW}. \quad (6.7d)$$

The ‘hatted’ couplings are the $\overline{\text{MS}}$ couplings, and hence they satisfy the tree-level relationships $\hat{c} \equiv \hat{g}_s \equiv \hat{g}_Z \hat{s} \hat{c}$ and $\hat{c}^2 \equiv 4\pi\hat{\alpha}$. For numerical results concerning Z-pole observables we adopt the renormalization conditions of Ref. [30] and use $\hat{\alpha}(m_Z^2)_{\text{SM}} = 128.72$ and $\hat{s}^2(m_Z^2)_{\text{SM}} = 0.2312$.

We perform a $\chi^2$ analysis to constrain the corrections of Eqn. (5.4). We base our analysis on the results of the recent global analysis of Ref. [6,31]. In Ref. [31] the ‘barred’ charges are fit to the data with the following results. For measurements on the Z-pole,

$$\hat{g}_Z^2(m_Z^2) = 0.55557 - 0.00042 \frac{\alpha_s + 1.54\hat{\sigma}_b(m_Z^2) - 0.1065}{0.0038} \pm 0.00061, \quad (6.8a)$$

$$\hat{s}^2(m_Z^2) = 0.23065 + 0.00003 \frac{\alpha_s + 1.54\hat{\sigma}_b(m_Z^2) - 0.1065}{0.0038} \pm 0.00024, \quad (6.8b)$$

$$\rho_{\text{corr}} = 0.24. \quad (6.8c)$$

The parameter $\hat{\sigma}_b(m_Z^2)$, which is a function of $m_t$, accounts for corrections to the $Zb\overline{b}$ vertex,
for which adopt the SM values. Combining the W-boson mass measurement \(m_W = 80.356 \pm 0.125\text{GeV}\) with the input parameter \(G_F\), they find

\[
\bar{g}_W^2(0) = 0.4237 \pm 0.0013 .
\]  

(6.9)

And finally, from the low-energy data,

\[
\begin{align*}
\bar{g}_Z^2(0) &= 0.5441 \pm 0.0029 , \\
\bar{s}^2(0) &= 0.2362 \pm 0.0044 , \\
\rho_{\text{corr}} &= 0.70 .
\end{align*}
\]  

(6.10a, b, c)

For \(\alpha_s = 0.118\) we obtain the following constraints on \(f_{DW}, f_{DB}, f_{BW}\) and \(f_{\Phi,1}\):

\[
\begin{align*}
f_{DW} &= -0.32 + 0.0088 x_H - 0.55 x_t \pm 0.44 \\
f_{DB} &= -14 \pm 10 \\
f_{BW} &= 3.7 + 0.085 x_H \pm 2.4 \\
f_{\Phi,1} &= 0.30 - 0.028 x_H + 0.32 x_t \pm 0.16
\end{align*}
\]

(6.11)

where

\[
\begin{align*}
x_t &= \frac{m_t - 175\text{GeV}}{100\text{GeV}} , \\
x_H &= \ln \frac{m_H}{100\text{GeV}} ,
\end{align*}
\]  

(6.12)

and \(\Lambda = 1\text{TeV}\). The errors are at the one-sigma level. The parameterization of the central values is good to a few percent of the one-sigma errors in the range \(150\text{GeV} < m_t < 190\text{GeV}\) and \(60\text{GeV} < m_H < 800\text{GeV}\); the dependencies upon \(m_H\) and \(m_t\) arise from SM contributions only.

We note the very strong correlations among three of the parameters. This suggests that the data constrains one combination of the parameters particularly well. This should not be ignored since this most stringent constraint sets the present sensitivity limit for physics beyond the SM as parameterized by the effective Lagrangian of Eqn. (4.1). We diagonalize the covariance matrix and repeat the \(\chi^2\) analysis in the basis of eigenvectors to find this particular combination and its associated error with the following result:

\[
f_{\Phi,1} - 0.18 f_{BW} - 0.029 f_{DB} + 0.016 f_{DW} = 0.023 \pm 0.017 .
\]  

(6.13)
The implication of this measurement is that, barring accidental cancellations among the various parameters, the constraint on $f_{\Phi,1}$ is actually much more severe than one would expect from Eqn. (6.11).

This result may be explained by the dominance of the data from the $Z$-pole measurements. Comparing the errors associated with the charge form-factors of Eqn. (6.8), Eqn. (6.9) and Eqn. (6.10), it is clear that the measurements of $\bar{s}^2(m_Z^2)$ and $\bar{g}_Z^2(m_Z^2)$ are much more precise than the remaining measurements. Considering only these two measurements and including their associated correlation it is possible to predict which combination of parameters is best constrained, and that prediction is approximately Eqn. (6.13). Nevertheless, the low-energy neutral-current and the charged-current/$W$-boson-mass data play an important role in the fit.

B. The nonlinear realization of the symmetry-breaking sector

We may repeat the entire analysis for the chiral Lagrangian of Eqn. (5.4). The corrections to the two-point-functions are

$$
\Delta\Pi^{QQ}_T(q^2) = -q^2\left(2\alpha_1 + \alpha_8\right), \\
\Delta\Pi^{gQ}_T(q^2) = -q^2\left(\alpha_1 + \alpha_8\right), \\
\Delta\Pi^{33}_T(q^2) = \frac{1}{2}\beta_1 v^2 - q^2\alpha_8, \\
\Delta\Pi^{11}_T(q^2) = 0.
$$

A comparison of Eqns. (6.14) with Eqns. (6.1) reveals two important differences. The chiral Lagrangian leads, at the current level of calculation, to at most linear dependence of the two-point functions upon $q^2$. Also $\Delta\Pi^{11}_T(q^2)$ vanishes in Eqn. (6.14d). In analogy with Eqns. (6.3),

$$
\Delta S = -16\pi\alpha_1, \\
\Delta T = \frac{2}{\alpha}\beta_1, \\
\Delta U = -16\pi\alpha_8, \\
\Delta \frac{1}{\alpha} = 0.
$$
which agrees with Ref. [22]. The contributions to the charge form-factors may be calculated via Eqn. (6.5), but there is no additional contribution to the running of the charge form-factors in Eqn. (5.6). Here the analysis with the operators of the chiral Lagrangian through energy dimension four is equivalent to the standard $S, T, U$ analysis. In short, the results are

\[
\Delta \pi(q^2) = 0 , \tag{6.16a}
\]
\[
\Delta \bar{g}_Z^2(q^2) = 2 \hat{g}_Z^2 \beta_1 , \tag{6.16b}
\]
\[
\Delta \bar{\Sigma}^2(q^2) = - \frac{c^2 s^2}{c^2 - s^2} \left( 2 \beta_1 + \hat{g}_Z^2 \alpha_1 \right) , \tag{6.16c}
\]
\[
\Delta \bar{g}_W^2(q^2) = - \hat{g}_W^2 \frac{\Delta \Sigma^2 (m_Z^2)}{s^2} - \hat{g}^4 \left( \alpha_1 + \alpha_8 \right) . \tag{6.16d}
\]

A fit to the data as summarized by Eqn. (6.8), Eqn. (6.9) and Eqn. (6.10) produces the central values, the one-sigma errors and the correlation matrix which follow:

\[
\begin{align*}
\alpha_1 &= (4.3 - 4.8 \times 10^{-3}) \times 10^{-3} \\
\beta_1 &= (0.45 - 3.5 \times 10^{-3}) \times 10^{-3} \\
\alpha_8 &= (-0.71 + 9.1 \times 10^{-3}) \times 10^{-3}
\end{align*}
\]

\[
\begin{pmatrix}
1 & -0.87 & -0.12 \\
1 & 0.22 & \\
1 & & \\
\end{pmatrix}
\tag{6.17}
\]

For our reference values of $S, T$ and $U$ we use the SM values at $m_H = 1\text{TeV}$. Notice that the data favors a positive value of $\alpha_1$ while the estimates of Eqns. (5.11a), (5.11b) and (5.12b) all predict a negative value.

The two fits, (6.11) and (6.17), are not equivalent. The nontrivial running of the charge form-factors introduced by the energy-dimension-six operators $O_{DW}$ and $O_{DB}$ is a leading-order effect; similar effects will also be induced by dimension-six operators of the chiral Lagrangian [32] which are neglected in the present approximation. On the other hand, the contribution of $\alpha_8$ is equivalent to a dimension-eight effect in the linear realization, hence the contribution of its counterpart in the linear realization is expected to be suppressed. A partial comparison may be made only in the limit where $f_{DW} = f_{DB} = 0$ and $\alpha_8 = 0$, which corresponds to a fit in $\Delta S$ and $\Delta T$ with $\Delta U = 0$ [10].


C. Expectations for improved measurements

The study of four-fermion processes at higher energies will do little to further constrain the parameters of the chiral Lagrangian via contributions to the electroweak charge form-factors unless the precision of those high-energy experiments is competitive with the precision of LEP/SLC. The situation is markedly different when the symmetry breaking is linearly realized, and the anomalous running of the charge form-factors leads to enhanced sensitivity at higher center of mass energies. This enhanced sensitivity in turn implies improved constraints upon the contributing coefficients. At LEP II, with $\sqrt{s} = 175\text{GeV}$ and $\int \mathcal{L} dt = 500\text{pb}^{-1}$, the constraints may improve as

$$f_{DW} = -0.07 + 0.032 x_H - 0.67 x_t \pm 0.22 \begin{pmatrix} 1 & -0.490 & 0.211 & -0.182 \end{pmatrix}$$
$$f_{DB} = -0.3 + 0.13 x_H + 0.83 x_t \pm 1.9 \begin{pmatrix} 1 & -0.896 & -0.484 \end{pmatrix}$$
$$f_{BW} = 0.19 + 0.050 x_H \pm 0.46 \begin{pmatrix} 1 & 0.791 \end{pmatrix}$$
$$f_{\Phi,1} = 0.052 - 0.032 x_H + 0.34 x_t \pm 0.042 \begin{pmatrix} 1 \end{pmatrix}$$

(6.18)

We make the assumption that the measurement of the $W$-boson mass will improve to $\Delta m_W = 45\text{MeV}$ [33]. The corresponding improvement for the parameters of the chiral Lagrangian is much more modest. Only the $W$-boson mass measurement plays a role, reducing the error on $\alpha_8$ to $\delta \alpha_8 = \pm 3.3 \times 10^{-3}$ while increasing the correlation between $\alpha_8$ and $\beta_1$ to 0.50.

At a future linear collider with $\sqrt{s} = 500\text{GeV}$ and $\int \mathcal{L} dt = 50\text{fb}^{-1}$ we may expect

$$f_{DW} = -0.010 + 0.0089 x_H - 0.13 x_t \pm 0.055 \begin{pmatrix} 1 & 0.295 & -0.242 & -0.131 \end{pmatrix}$$
$$f_{DB} = 0.00 - 0.0070 x_H \pm 0.21 \begin{pmatrix} 1 & -0.340 & -0.140 \end{pmatrix}$$
$$f_{BW} = 0.06 + 0.097 x_H \pm 0.17 \begin{pmatrix} 1 & 0.904 \end{pmatrix}$$
$$f_{\Phi,1} = 0.037 - 0.028 x_H + 0.34 x_t \pm 0.025 \begin{pmatrix} 1 \end{pmatrix}$$

(6.19)

In this analysis we have also assumed that the error on the $W$-boson mass will be reduced to $\Delta m_W = \pm 20\text{MeV}$ by the TeV33 upgrade of the Fermilab Tevatron [34]. For the parameters of the chiral Lagrangian, the fit of Eqn. (6.17) is modified by a reduction of the error on $\alpha_8$ to $\delta \alpha_8 = \pm 2.3 \times 10^{-3}$ while the correlation between $\alpha_8$ and $\beta_1$ is increased to 0.72. In this case the improvement is from the precise measurement of the $W$-boson mass.

In Ref. [35] a scheme has been proposed which accounts for $Z$-pole measurements at LEP I when discussing new data at LEP II or a higher-energy linear $e^+e^-$ collider. Their “$Z$-peak
subtracted” scheme reduces the number of parameters required for these future experiments by using LEP I measurements as input parameters for the calculation of observables at higher energies; effectively they concentrate on \( f_{DW} \) and \( f_{DB} \), the operators which introduce a nonstandard running of the charge form-factors. The obvious advantage of their approach is a smaller parameter space which focuses on those parameters whose constraints should improve the most. However, with an exact calculation we obtain more stringent bounds, and we are able to take full advantage of the correlations among all four parameters; these correlations change dramatically at different scales. Because the details concerning our analyses are quite different, our results and theirs are not easily compared. However, we find rough agreement between their results and ours.

VII. THE PROCESS \( e^+e^- \rightarrow W^+W^- \)

Next we calculate the contributions of the effective Lagrangians of Section IV and Section V to the form factors of Eqn. (2.1) and Eqn. (3.1). Eventually it will be necessary to include both the complete SM radiative corrections and the effective-Lagrangian contributions in a combined analysis, but the scenario of immediate interest is where the nonstandard contributions are relatively large compared to the higher-order SM effects. The SM corrections have been considered by many authors \[23433738\].

When neglecting the SM loop-level corrections, Eqn. (3.1) may be simplified considerably. Because there are no corrections to fermionic vertices, the \( \Gamma_1^e \) and \( \Gamma_2^e \) terms vanish while the \( \Gamma^{e\nu} \) term becomes equivalent to the self-energy correction for an external \( W \) boson. In the effective Lagrangian the equivalent of a box correction is a contact term; there is no such contribution due to the exclusion of fermionic fields in the construction of effective operators. With these simplifications we may rewrite Eqn. (3.1) as

\[
F_{i,\tau}^{IB}(s, t) = \frac{Q}{s} \left\{ \bar{e}^2(s)_{SM} f^\gamma_i(0) + \Delta \bar{e}^2(s) f^\gamma_i(1)(s) \right\} \\
+ \frac{1}{s - m_Z^2 + i s \frac{m_Z^2}{m_Z}} \left\{ (I_3 - s^2 Q) \bar{e}^2 \left( \bar{g}_Z^2(s)_{SM} f_i^Z(0) + \Delta \bar{g}_Z^2(s) f_i^Z(0) + \hat{g}_Z^2 f_i^Z(1)(s) \right) \\
- \hat{g}_Z^2 \left( Q \bar{e}^2 f_i^Z(0) + (I_3 - s^2 Q) f_i^\gamma(0) \right) \Delta s^2(s) \right\} \\
+ \frac{I_3}{2t} \left( \bar{g}_W^2(m_W^2)_{SM} + \Delta \bar{g}_W^2(m_W^2) \right) f_i^t(0) . \tag{7.1}
\]
Notice that, through the $Z-\gamma$ mixing term, the $Z$-boson has acquired a coupling proportional to the charge of the electron, as discussed in Ref. [26]. Also note that Eqn. (7.1) remains valid when the SM corrections to the gauge-boson propagators are included.

A. The linear realization of the symmetry-breaking sector

The corrections to the charge form factors, $\Delta e^2(q^2)$, $\Delta s^2(q^2)$, $\Delta g^2_Z(q^2)$ and $\Delta g^2_W(q^2)$, may be found in Eqns. (5.1). The tree-level form factors $f_i^{X(0)}$ may be found in Table V. Hence, once we calculate the various $f_i^{X(1)}$ terms, the form factors of Eqn. (7.1) are completely determined. For the effective Lagrangian of Eqn. (4.1) we find

$$f_1^{\gamma(1)}(s) = -\hat{g}^2 \frac{s}{\Lambda^2} f_{DW} + \frac{3}{4} \hat{g}^2 \frac{s}{\Lambda^2} f_{WWW} ,$$

(7.2a)

$$f_2^{\gamma(1)}(s) = -6 \hat{g}^2 \frac{m^2_W}{\Lambda^2} f_{DW} + \frac{3}{2} \hat{g}^2 \frac{m^2_W}{\Lambda^2} f_{WWW} ,$$

(7.2b)

$$f_3^{\gamma(1)}(s) = 2 \hat{g}^2 \frac{2s - 3m^2_W}{\Lambda^2} f_{DW} + \frac{3}{2} \hat{g}^2 \frac{m^2_W}{\Lambda^2} f_{WWW} + \frac{1}{2} \frac{m^2_W}{\Lambda^2} (f_W - 2f_{BW} + f_B) ,$$

(7.2c)

for the $WW\gamma$ vertex, and

$$f_1^{Z(1)}(s) = f_1^{\gamma(1)}(s) + \frac{1}{2} \frac{m^2_Z}{\Lambda^2} f_W ,$$

(7.2d)

$$f_2^{Z(1)}(s) = f_2^{\gamma(1)}(s) ,$$

(7.2e)

$$f_3^{Z(1)}(s) = f_3^{\gamma(1)}(s) + \frac{1}{2} \frac{m^2_Z}{\Lambda^2} (f_W + 2f_{BW} - f_B) ,$$

(7.2f)

for the $WWZ$ vertex. Only nonzero results are reported. Notice that $f_1^{Z(1)}(s)$ differs from $f_1^{\gamma(1)}(s)$ by a term proportional to $f_W$; $f_2^{Z(1)}(s)$ and $f_2^{\gamma(1)}(s)$ are the same; the $f_W$, $f_{BW}$ and $f_B$ terms of $f_3^{\gamma(1)}(s)$ and $f_3^{Z(1)}(s)$ differ. Recall that the $W$-boson self-energy contributions for the external $W$-bosons are included in the $f_i^{X(1)}$ form factors.

Because the effective Lagrangian of Eqn. (4.1) is invariant under $U(1)_{em}$, the $g_1^\gamma(s)$ form-factor is required to assume its canonical value, $g_1^\gamma(0) = 1$, for on-shell photons. For readers who are unfamiliar with this standard notation $[\gamma], it will be reviewed later in this section. We may obtain the $g_1^\gamma(s)$ form-factor via

$$g_1^\gamma(s) = f_1^\gamma(s) - \frac{s}{2m^2_W} f_2^\gamma(s) .$$

(7.3)
However, care must be taken to account for direct corrections to the three-point vertex as well as self-energy corrections for the particles attached to each leg of the vertex. Motivated by the form of the first line of Eqn. (7.1) we define
\[ f_{\gamma}^{(\text{eff})}(s) = \left[ 1 + \frac{\Delta e^2(s)}{e^2} \right] f_{\gamma}^{(0)} + f_{\gamma}^{(1)}(s). \] (7.4)

Then, combining the above two equations,
\[ g_{\gamma}^{(s)} = 1 - \left\{ 2\frac{e^2}{\Lambda^2} \left( f_{DW} + f_{DB} \right) \right\} + \left\{ g^2 \frac{4m_W^2 - s}{\Lambda^2} f_{DW} + \frac{3}{4} g^2 \frac{s}{\Lambda^2} f_{WWW} \right\} - \left\{ 4g^2 \frac{m_W^2}{\Lambda^2} f_{DW} \right\} - \frac{s}{2m_W^2} \left\{ \frac{3}{2} g^2 \frac{m_W^2}{\Lambda^2} f_{WWW} - 6g^2 \frac{m_W^2}{\Lambda^2} f_{DW} \right\} \] (7.5a)
\[ = 1 + 2\frac{g^2}{\Lambda^2} \left( e^2 f_{DW} - s^2 f_{DB} \right) \]. (7.5b)

The first term on the right-hand side of Eqn. (7.5a) is the tree-level value of \( g_1, g_1^{(0)} = f_{\gamma}^{(0)} \). The second term is the contribution from the photon self-energy, given by \( \Delta e^2(s) \), obtainable from Eqn. (6.7a). The third term is the direct correction to the three-point vertex, and the fourth term arises from the wave-function renormalization factor for the external \( W \) bosons. Notice that the constant pieces in these third and fourth terms cancel, as required by \( U(1)_{\text{em}} \) gauge invariance. These first four terms comprise \( f_{\gamma}^{(\text{eff})}(s) \). The last term of Eqn. (7.5a) is \( f_{2\gamma}^{(\text{eff})}(s) \), which receives only a direct correction. Recall from Table V that \( f_{2\gamma}^{(0)} = 0 \). In the final result, displayed in Eqn. (7.5b), the correction term is proportional to \( s \), the square of the CM energy. Indeed \( g_{\gamma}^{(s)} \) does reduce to its canonical value, \( g_{\gamma}^{(0)} = 1 \), for on-shell photons.

However, some of the desirable properties of the SM \( e^+e^- \rightarrow W^+W^- \) amplitudes are not preserved in the amplitudes above. In the SM elegant cancellations between the various Feynman graphs insure that the full amplitudes are well behaved at high energies, and perturbative unitarity is satisfied. In particular, at high-energies the SM amplitudes behave like \( s^n \) where \( n \leq 0 \), and, for large \( \sqrt{s} \), the SM cross-sections decrease with increasing CM energy. To the contrary, the amplitudes of this section will, in some cases, behave as \( s^n \) where \( n > 0 \) leading to cross sections which do not decrease or even grow with increasing CM energy, violating tree-level perturbative unitarity at high energies. As we approach the scale of the new interactions described by our effective Lagrangian, higher order terms in the
expansion become increasingly important until the effective Lagrangian formalism breaks down.

An explicit calculation, like the calculation leading to Eqn. (3.7) and Eqn. (3.8), leads to the following high-energy limits for the amplitudes:

$$\mathcal{M}^{IB}(+; 0, 0) \rightarrow -\sqrt{2} e^2 \gamma \frac{m_Z^2}{\Lambda^2} \left\{ 4s^2 \hat{g}_Z^2 f_{DB} + f_B \right\} d_{+,0}, \quad (7.6a)$$

$$\mathcal{M}^{IB}(-; 0, 0) \rightarrow \sqrt{2} \gamma \frac{m_Z^2}{\Lambda^2} \left\{ 2\hat{c}_4^2 f_{DW} + \hat{s}^4 f_{DB} \right\} + \frac{1}{2} \hat{g}^2 \left( \hat{c}^2 f_W + \hat{s}^2 f_B \right) d_{-,0}, \quad (7.6b)$$

$$\mathcal{M}^{IB}(\pm; 0, 0) \rightarrow -\sqrt{2} e^2 \gamma \frac{m_Z^2}{\Lambda^2} \left\{ 4s^2 \hat{g}_Z^2 f_{DB} - \frac{1}{2} \left( f_W - 2f_{BW} - f_B \right) \right\} d_{+,\pm1}, \quad (7.6c)$$

$$\mathcal{M}^{IB}(+; 0, \mp) = \mathcal{M}^{IB}(+; \pm, 0), \quad (7.6d)$$

$$\mathcal{M}^{IB}(-; \pm, 0) \rightarrow \sqrt{2} \gamma \frac{m_Z^2}{\Lambda^2} \left\{ \hat{c}^2 \hat{g}_Z^4 \left( -\hat{c}^4 f_{DW} + 2\hat{s}^4 f_{DB} \right) + \frac{3}{4} \hat{c}^2 \hat{g}^4 f_{WWW} \right\} + \frac{1}{4} \left[ (2\hat{c}^2 - \hat{s}^2) f_W + 2\hat{s}^2 f_{BW} + \hat{s}^2 f_B \right] d_{-,\pm1}, \quad (7.6e)$$

$$\mathcal{M}^{IB}(-; 0, \mp) = \mathcal{M}^{IB}(-; \pm, 0), \quad (7.6f)$$

$$\mathcal{M}^{IB}(-; \pm, \pm) \rightarrow \sqrt{2} \hat{g}^4 \gamma \frac{2m_W^2}{\Lambda^2} \left\{ -6f_{DW} + \frac{3}{2} f_{WWW} \right\} d_{-,0}, \quad (7.6g)$$

where $\gamma = E_W/m_W$. The $\mathcal{M}^{IB}(+; \pm, \pm)$ and $\mathcal{M}^{IB}(\tau; \pm, \mp)$ amplitudes do not receive any contributions that grow with energy. Notice that $f_{\Phi,1}$ does not contribute to any of the above expressions. We have used the equivalence theorem at the qualitative level to verify the behavior of these high-energy approximations [39].

**B. The nonlinear realization of the symmetry-breaking sector**

We now repeat the discussion for the chiral Lagrangian. The corrections to the charge form factors, $\Delta \hat{c}^2(q^2)$, $\Delta \hat{s}^2(q^2)$, $\Delta \hat{g}_Z^2(q^2)$ and $\Delta \hat{g}_W^2(q^2)$ may be found in Eqns. (6.16). For the effective Lagrangian of Eqn. (5.4) the nonzero $f_i^{X(1)}(s)$ are

$$f_3^{\gamma(1)}(s) = \hat{g}^2 \left( -\alpha_1 + \alpha_2 + \alpha_3 - \alpha_8 + \alpha_9 \right), \quad (7.7a)$$

for the $WW\gamma$ vertex, and

$$f_1^{Z(1)}(s) = \hat{g}_Z^2 \alpha_3, \quad (7.7b)$$

$$f_3^{Z(1)}(s) = f_3^{\gamma(1)}(s) + \hat{g}_Z^2 (\alpha_1 - \alpha_2 + \alpha_3), \quad (7.7c)$$

$$f_5^{Z(1)}(s) = \hat{g}_Z^2 \alpha_{11}, \quad (7.7d)$$
for the $WWZ$ vertex. If $\Gamma^{(0)}_Z$ and $\Gamma^{(0)}_\gamma$ are the tree-level vertex functions and $\Gamma^{(1)}_Z$ and $\Gamma^{(1)}_\gamma$ are the one-loop vertex corrections, then we may define the ‘full’ vertex functions according to $\hat{g}_Z e^2 \Gamma^{(0)}_Z \to \Delta \hat{g}_Z(s) e^2 \Gamma^{(0)}_Z + \hat{g}_Z \Delta e^2(s) \Gamma^{(0)}_Z + \hat{g}_Z e^2 \Gamma^{(1)}_Z = \hat{g}_Z e^2 \Gamma^{(\text{full})}_Z$ and $\hat{e} \Gamma^{(0)}_\gamma \to \Delta \hat{e}(s) \Gamma^{(0)}_\gamma + \hat{e} \Gamma^{(1)}_\gamma = \hat{e} \Gamma^{(\text{full})}_\gamma$; we find that $\Gamma^{(\text{full})}_Z$ and $\Gamma^{(\text{full})}_\gamma$ calculated in this way agree with the results of Ref. [22]. Ref. [26] calculated the corrections to the $WW\gamma$ and $WWZ$ vertices for a small subset of the operators, but also discussed the $e^+ e^- \to W^+ W^-$ cross-section.

For the chiral Lagrangian the calculation of $g^1_1$ is trivial. There are no direct corrections from the $WW\gamma$ three-point vertex because $f^{Z(1)}_1(s) = f^{Z(1)}_2(s) = 0$. There is no correction from the $W$-boson wave-function-renormalization factor because $Z_W = 1$. Finally, $\Delta \hat{e}^2(s) = 0$. Therefore, $g^1_1 = 1$ respecting gauge invariance under $U(1)_{\text{em}}$.

We present the high-energy limit of the $e^+ e^- \to W^+ W^-$ amplitudes.

\begin{align}
\mathcal{M}^{\text{IB}}(++; 0, 0) &\to -2 \sqrt{2} \hat{e}^2 \gamma^2 \left\{ \hat{g}^2_Z \alpha_2 \right\} d^1_{+; 0} , \\
\mathcal{M}^{\text{IB}}(-; 0, 0) &\to \sqrt{2} \hat{g}^2 \alpha^2 \left\{ \hat{g}^2_Z \left( s^2 \alpha_2 + \hat{c}^2 \alpha_3 \right) + \hat{g}^2 \alpha_9 \right\} d^1_{-, 0} , \\
\mathcal{M}^{\text{IB}}(++; \pm, 0) &\to - \sqrt{2} \hat{c}^2 \gamma \left\{ \hat{g}^2_Z (\alpha_1 + \alpha_2 - \alpha_3 \mp \alpha_{11}) \right\} d^1_{+, \pm 1} , \\
\mathcal{M}^{\text{IB}}(++; 0, \mp) &\equiv \mathcal{M}^{\text{IB}}(++; \pm, 0) , \\
\mathcal{M}^{\text{IB}}(-; \pm, 0) &\to \sqrt{2} \hat{c}^2 \gamma \left\{ \frac{1}{2} \hat{g}^2_Z \left[ s^2 (\alpha_1 + \alpha_2 - \alpha_3 \mp \alpha_{11}) \right. \\
&\left. + \hat{c}^2 (2 \alpha_3 + \alpha_8 + \alpha_9 \pm \alpha_{11}) \right] \right\} d^1_{-, \pm 1} , \\
\mathcal{M}^{\text{IB}}(-; 0, \mp) &\equiv \mathcal{M}^{\text{IB}}(-; \pm, 0) .
\end{align}

The remaining amplitudes, $\mathcal{M}^{\text{IB}}(\tau; \pm, \pm)$ and $\mathcal{M}^{\text{IB}}(\tau; \pm, \mp)$, do not have any contributions that grow with energy. Like its counterpart $f_{\Phi,1}$ in the linear realization, $\beta_1$ makes no contribution which grows with energy. Eqn. (7.8) may be used to verify that Eqns. (7.6) and Eqns. (7.8) are consistent under the equivalence of $\mathcal{O}_{\Phi,1} \sim \mathcal{L}'_1$, $\mathcal{O}_{BW} \sim \mathcal{L}_1$, $\mathcal{O}_B \sim \mathcal{L}_2$ and $\mathcal{O}_W \sim \mathcal{L}_3$. Notice that $\alpha_1$ does not contribute to the high-energy behavior of the $\lambda \overline{\lambda} = 00$ amplitudes as was observed in Ref. [20]. On the other hand, we observe that $\alpha_1$ makes a contribution to the $\mathcal{M}^{\text{IB}}(\tau; \pm, 0)$ and $\mathcal{M}^{\text{IB}}(\tau; 0, \pm)$ amplitudes which is proportional to $\gamma$. 

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C. The ‘phenomenological Lagrangian’

To facilitate discussion and to make a connection with much of the standard literature, we also present a phenomenological Lagrangian of the most general $W W \gamma$ and $W W Z$ couplings which respects only U(1)$_{em}$ gauge invariance \[1\]. Retaining only those terms that are CP conserving,

$$L_{WW} = -ig_{WW} \left( g^{V}_1 (W_{\mu}^{\gamma} W_{-\nu}^{\gamma} - W_{\nu}^{\gamma} W_{-\mu}^{\gamma}) + \kappa W_{\mu}^{\gamma} W_{-\nu}^{\gamma} - \nu^\mu V_{-\nu}^{\gamma} \right) + \frac{\lambda}{m_W^2} W_{\mu}^{\gamma} W_{-\nu}^{\gamma} - ig_5^V \epsilon^{\mu\nu\sigma\rho} [W_{\mu}^{\gamma} (\partial_{\rho} W_{-\sigma}^{\gamma}) - (\partial_{\rho} W_{-\mu}^{\gamma}) V_{-\sigma}^{\gamma} ] V_{\sigma},$$  \(7.9\)

where the overall coupling constants are $\hat{g}_{WW} = \hat{e}$ and $\hat{g}_{WWZ} = \hat{g}_Z \hat{e}^2$. The field-strength tensors include only the Abelian parts, i.e. $W_{\mu\nu} = \partial_{\mu} W_{\nu}^{\gamma} - \partial_{\nu} W_{\mu}^{\gamma}$ and $V_{\mu\nu} = \partial_{\mu} V_{\nu}^{\gamma} - \partial_{\nu} V_{\mu}^{\gamma}$. The explicit relationships between the form factors $f^V_i$ and the effective Lagrangian of Eqn. \(7.9\) are given by

$$f^V_1(s) = g^{V}_1 + \frac{s}{2m_W^2} \lambda, \quad (7.10a)$$
$$f^V_2(s) = \lambda, \quad (7.10b)$$
$$f^V_3(s) = g^{V}_1 + \kappa + \lambda, \quad (7.10c)$$
$$f^V_5(s) = g_5^V. \quad (7.10d)$$

The $e^+ e^- \rightarrow W^+ W^-$ amplitudes with corrections from Eqn. \(7.9\) display the following high-energy behavior.

$$M^{IB}(+; 0, 0) \rightarrow -2\sqrt{2} e^2 \gamma^2 \left\{ \Delta \kappa - \Delta \kappa_Z \right\} d^1_{+,0}, \quad (7.11a)$$
$$M^{IB}(-; 0, 0) \rightarrow \sqrt{2} \tilde{g}^2 \gamma^2 \left\{ 2 \tilde{s}^2 \Delta \kappa + \left( \tilde{c}^2 - \tilde{s}^2 \right) \Delta \kappa_Z \right\} d^1_{-,0}, \quad (7.11b)$$
$$M^{IB}(+; \pm, 0) \rightarrow -\sqrt{2} e^2 \gamma \left\{ \left( \Delta \kappa + \lambda + \Delta g_1^\gamma \pm g_5^\gamma \right) - \left( \Delta \kappa_Z + \lambda \pm \Delta g_1^Z \pm g_5^Z \right) \right\} d^1_{+; \pm 1}, \quad (7.11c)$$
$$M^{IB}(+; 0, \mp) = M^{IB}(+; \pm, 0), \quad (7.11d)$$
$$M^{IB}(-; \pm, 0) \rightarrow \sqrt{2} \tilde{g}^2 \gamma \left\{ \tilde{s}^2 \left( \Delta \kappa + \lambda + \Delta g_1^\gamma \pm g_5^\gamma \right) + \frac{1}{2} \left( \tilde{c}^2 - \tilde{s}^2 \right) \left( \Delta \kappa_Z + \lambda \pm \Delta g_1^Z \pm g_5^Z \right) \right\} d^1_{-; \pm 1}, \quad (7.11e)$$
$$M^{IB}(-; 0, \mp) = M^{IB}(-; \pm, 0). \quad (7.11f)$$
\[ M^{IB}(\pm; \pm, \pm) = -2\sqrt{2}e^2\gamma^2 \left\{ \lambda_\gamma - \lambda_Z \right\} d^1_{+0}, \]  
\[ M^{IB}(-; \pm, \pm) = \sqrt{2}g^2\gamma^2 \left\{ 2\tilde{s}^2\lambda_\gamma + (\tilde{e}^2 - \tilde{s}^2)\lambda_Z \right\} d^1_{-0}. \]  

(7.11g)  
(7.11h)

The remaining amplitudes, \( M^{IB}(\tau; \pm, \mp) \), do not have any contributions that grow with energy. Here \( \Delta \kappa_V = \kappa_V - 1 \) and \( \Delta g^V_1 = g^V_1 - 1 \) for \( V = \gamma, Z \).

**VIII. NUMERICAL ANALYSIS**

In this section we shall determine the level to which the parameters of the effective Lagrangians (4.1), (5.4) and (7.9) may be measured/constrained through the study of \( W \)-boson pair production at LEP II and at a 500GeV linear collider. We are especially interested in comparing and contrasting the results which we obtain in the different realizations of the symmetry-breaking sector.

When analysing actual experimental data \( W \)-boson finite-width effects [37,38,40,41] and contributions from initial and final state radiation [41,42] are very important. However, as verified by Ref. [41], these contributions primarily lead to a shift in the measured quantities, but the sensitivity to non-standard couplings is minimally affected. Hence, we may justifiably use the simplified calculation of Section VII.

The calculation of the cross section for \( e^+e^- \rightarrow W^+W^- \), \( W^- \rightarrow f_1\bar{f}_2 \), \( W^+ \rightarrow f_3\bar{f}_4 \) requires, in general, the evaluation of an eight-dimensional integral. In Sec. II we introduced \( \Theta \), the angle between the momentum vectors of the \( W^- \) and the \( e^- \) as measured in the CM frame. The integration over the azimuthal angle of the \( W^- \)-boson momentum vectors, \( \Phi \), is trivial, and need not be considered explicitly. We do not consider transverse polarizations of the LEP II beams [39]. In the zero-width approximation for the decaying \( W \) bosons, two integrations are performed analytically, and a single event is characterized by five angles. Using the same notation as Ref. [41] we introduce the momentum vectors of \( f_1 \) and \( \bar{f}_2 \) as measured in the rest frame of the \( W^- \) as

\[ p_1^\mu = \frac{1}{2}\sqrt{s} \left( 1, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta \right), \]  
\[ p_2^\mu = \frac{1}{2}\sqrt{s} \left( 1, -\sin \theta \cos \phi, -\sin \theta \sin \phi, -\cos \theta \right). \]  

(8.1a)  
(8.1b)

For the momentum vectors of \( f_3 \) and \( \bar{f}_4 \) as measured in the rest frame of the \( W^+ \) we have
\[ p'_q = \frac{1}{2} \sqrt{s} \left( 1, \sin \theta \cos \varphi, \sin \theta \sin \varphi, -\cos \theta \right), \]  
\[ p''_q = \frac{1}{2} \sqrt{s} \left( 1, \sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta \right), \]

(8.2a)  
(8.2b)

The z-axis and the x–z plane are common to all three frames.

In practice, to perform a fit, we need to retain as much of the above angular information as possible. A straightforward approach to this problem is to compare the full five-fold differential cross-section calculated in the SM to that calculated with the nonstandard contributions. Suppose, for a moment, that individual events could be completely reconstructed. Suppose also that we divide each of the above angular variables into 10 bins. Then in total we have \(10^5\) bins to consider; at LEP II, where approximately \(8 \times 10^3\) \(W\)-pair events are expected, we can expect to have zero events in a very large number of these bins. Hence, we will use many fewer bins. In particular we employ four bins in each variable for \(4^5 = 1,024\) total bins, and we then perform a log-likelihood fit with Poisson statistics [3]. A similar strategy was employed by the authors of Ref. [44].

The most common final state, realized in 49% of the events [41], is where both \(W\) bosons decay hadronically, \(i.e.\) the \(jjjj\) final state. For many of these events it is possible to reconstruct the four-momenta of all four jets. The jets may then be paired such that each pair has the invariant mass of an on-shell \(W\)-boson. However, it is extremely difficult within each pair to determine which jet came from a quark and which came from an anti-quark. By tagging charm quarks it may be possible to make correct assignments of the jets in some fraction of the events. Color reconnection effects [33,45] may be important at LEP II where the \(W\) bosons are produced with a very small velocity, and hence their production and decay vertices are minimally displaced, and as a consequence their decay jets may interact between pairs. Following Ref. [41] we assume that there is an overall ambiguity in the assignment of the jets within each pair, and we also assume that we cannot determine the charges of the \(W\) bosons. In this respect our analysis is somewhat conservative. Again following Ref. [41], we assign an efficiency of 60% for the reconstruction of the \(jjjj\) final state.

Next we consider the final state where one \(W\) boson decays hadronically, and the other decays leptonically, \(i.e.\) the \(jjlv\) final state. The branching fraction is 14% for \(l = e\), 14% for \(l = \mu\) and 14% for \(l = \tau\). Due to difficulties in the reconstruction we will simply
ignore the $jj\tau\nu$ final state. For the reconstruction of the $jjl\nu$, $l = e, \mu$ final states, only one reconstruction ambiguity exists; it is difficult to correctly determine which of the jets is the quark jet and which is the anti-quark jet. Because the charge assignments of the $W$ bosons are determined by the measurement of the lepton charge, charm-quark tagging might be useful for assigning the jets in some portion of the events; for simplicity will we ignore this refinement. Following Ref. [11], we assign an efficiency of 95% for the reconstruction of the $jjl\nu$ final state.

Finally, there is the final state where both $W$ bosons decay leptonically, i.e. the $l\nu l'\nu'$ final state, which occurs 9% of the time. A portion of these events, where one or both of the leptons is a $\tau$ lepton, is difficult to reconstruct. The remaining events may be well reconstructed up to an overall two-fold ambiguity which is the result of having two neutrinos in the final state. We have chosen to neglect the $l\nu l'\nu'$ final state in this analysis, but it may be straightforwardly added to future analyses.

We make one kinematical cut, $|\cos \Theta| < 0.9$, and divide each of the five variables into four bins. In a more complete analysis we would need to add additional separation cuts on the final-state fermions, especially to allow for a jet-cone radius. These cuts are crudely included through the incorporation of the efficiencies.

For the one-sigma limits on the coefficients of the energy-dimension-six operators in the linear realization of the symmetry-breaking sector see Table VIII. We include results not

|    | $f_{DW}$ | $f_{DB}$ | $f_{BW}$ | $f_{\Phi,1}$ | $f_{WWW}$ | $f_{W}$ | $f_{B}$ |
|----|---------|---------|---------|-------------|-----------|--------|--------|
| LEP II | 2.1     | 12      | 1.5     | 0.19        | 10        | 7.1    | 46     |
| LC   | 0.063   | 0.39    | 0.32    | 0.045       | 0.23      | 0.10   | 0.25   |

TABLE VIII. One-sigma limits on the parameters of the linearly realized effective Lagrangian assuming $\Lambda = 1$TeV. In the first row are the constraints from LEP II at 175GeV with $\mathcal{L}^{\text{int}} = 500\text{pb}^{-1}$, and the second row contains results for a 500GeV future linear collider with $\mathcal{L}^{\text{int}} = 50\text{fb}^{-1}$. The one-sigma allowed region is approximately symmetric about zero.

only for LEP II with $\mathcal{L}^{\text{int}} = 500\text{pb}^{-1}$ at $\sqrt{s} = 175$GeV, but we also perform the analysis for a future linear collider with $\sqrt{s} = 500$GeV and $\mathcal{L}^{\text{int}} = 50\text{fb}^{-1}$. We repeat the analysis for the parameters of the effective Lagrangian in the scenario where symmetry breaking is realized.
nonlinearly. The results are presented in Table IX. The results for the analysis in the basis

|       | $\beta_1$ | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_8$ | $\alpha_9$ | $\alpha_{11}$ |
|-------|-----------|------------|------------|------------|------------|------------|--------------|
| LEP II| 0.0028    | 0.022      | 0.34       | 0.053      | 0.017      | 0.10       | 0.50         |
| LC    | 0.00064   | 0.0047     | 0.0018     | 0.00072    | 0.0022     | 0.00078    | 0.0045       |

TABLE IX. One-sigma limits on the parameters of the nonlinearly realized effective Lagrangian. In the first row are the constraints from LEP II at 175GeV with $\mathcal{L}^{\text{int}} = 500\text{pb}^{-1}$, and the second row contains results for a 500GeV future linear collider with $\mathcal{L}^{\text{int}} = 50\text{fb}^{-1}$. The one-sigma region is approximately symmetric about zero.

of Eqn. (7.9) appear in Table X. We report the one-sigma limits to two significant digits,

|       | $\Delta g_{\gamma}^1$ | $\Delta g_{\gamma}^2$ | $\Delta \kappa_\gamma$ | $\Delta \kappa_Z$ | $\lambda_\gamma$ | $\lambda_Z$ | $f_5^\gamma$ | $f_5^Z$ |
|-------|------------------------|------------------------|------------------------|-------------------|-----------------|-------------|------------|
| LEP II| 0.12                   | 0.073                  | 0.092                  | 0.067             | 0.11            | 0.068       | 0.48       | 0.28     |
| LC    | 0.0030                 | 0.0030                 | 0.00059                | 0.00077           | 0.0022          | 0.0017      | $^{+0.0036}_{-0.0023}$ | 0.0025 |

TABLE X. One-sigma limits on the couplings from the phenomenological effective Lagrangian of Eqn. (7.9). In the first row are the constraints from LEP II at 175GeV with $\mathcal{L}^{\text{int}} = 500\text{pb}^{-1}$, and the second row contains results for a 500GeV future linear collider with $\mathcal{L}^{\text{int}} = 50\text{fb}^{-1}$. The one-sigma region is approximately symmetric about zero except for $f_5^\gamma$.

even though the second digit is only approximate. In many cases we find that one-sigma region is not perfectly symmetric about zero, but generally the asymmetry is less than 10%.

We performed several cross checks of our results. First of all, for the LEP II constraints on $f_{WWW}$, $f_W$ and $f_B$, see Table VII, we were able to make some comparisons with the results of Ref. [41]; we found good agreement. For a few of the parameters in Table IX and Table X we were able to compare with the results of Refs. [44], again finding good agreement. Additionally we made several checks for the internal consistency of our results, for example, by using the relationships of Eqns. (5.8). Additional relationships connect some of the values in Table VII and Table X to those in Table X. Unfortunately the high-energy limits presented in Eqns. (7.6) for the light-Higgs scenario, in Eqns. (7.8) for the chiral Lagrangian and in Eqns. (7.11) for the phenomenological effective Lagrangian are not useful for explaining the improvement from LEP II to the linear collider. This is simply because
LEP II is much too close to the $W$-boson pair production threshold for the high-energy approximation to be useful.

IX. DISCUSSION

Compare the first row of Table VIII, obtained from studying $e^+e^- \rightarrow W^+W^-$, to the constraints in Eqn. (5.11), obtained from studying the low-energy and $Z$-pole data. The first observation we make is that, current constraints on $f_{DW}$ are sufficiently strong that we do not have sensitivity to this parameter at LEP II. On the other hand, the bounds on $f_{DB}$ and $f_{\Phi,1}$ here are only slightly weaker than current bounds. Hence, with the improvements to the analysis described in the previous section, perhaps the improved bounds on these two coefficients may become competitive. Finally, the new bound on $f_{BW}$ is an improvement over the current bound. The current data is not sufficient to rule out observable effects from the operator $O_{BW}$, contrary to some expectations [46]. However, next consider the constraints of Eqn. (6.18), obtained from studying $e^+e^- \rightarrow ff$ at LEP II. Immediately we see that, if any of the four coefficients $f_{DW}$, $f_{DB}$, $f_{BW}$ or $f_{\Phi,1}$ were to make an observable contribution at LEP II to $e^+e^- \rightarrow W^+W^-$, then there would be an even larger effect observed in the $e^+e^- \rightarrow ff$ channel. Hence, if measurements made on two-fermion final states are in good agreement with the SM, then we may disregard these four coefficients, and we are justified in considering only $f_{WWW}$, $f_W$ and $f_B$ when studying $W$-boson pair production. In this case we may employ the relations [14]

\begin{align}
g_1^Z(q^2) &= 1 + \frac{1}{2} \frac{m^2_W}{\Lambda^2} f_W, \\
\kappa_\gamma(q^2) &= 1 + \frac{1}{2} \frac{m^2_W}{\Lambda^2} (f_W + f_B), \\
\kappa_Z(q^2) &= 1 + \frac{1}{2} \frac{m^2_Z}{\Lambda^2} \left( \hat{c}^2 f_W - \hat{s}^2 f_B \right), \\
\lambda_\gamma(q^2) &= \lambda_Z(q^2) = \frac{3}{2} \frac{g^2 m^2_W}{\Lambda^2} f_{WWW}.
\end{align}

Out of the three parameters $g_1^Z$, $\kappa_\gamma$, and $\kappa_Z$, only two are independent. Notice that $g_1^Z = 1$. Similarly, only one of the $\lambda$ couplings is independent.

Next, consider the second row of Table VIII. We see that the bounds on all seven of the parameters improve at a 500GeV linear collider. In some cases, such as the constraints of
$f_{\Phi,1}$, we expect small improvements due to the high luminosity which more than compensates for the $1/s$ falloff in the cross-section. Approximately we might expect an improvement from statistics roughly of the order \(\sqrt{N_{LC}/N_{LEP \ II}} \approx \sqrt{\mathcal{L}_{LC}/\mathcal{L}_{LEP \ II}/s_{LC}/s_{LEP \ II}} \approx 3.5\); we see an improvement in the measurement of $f_{\Phi,1}$ by approximately a factor of 4. However, from studying Eqns. (7.6), we see that the $f_i$'s often appear multiplied by factors of $\gamma$ or $\gamma^2$, which, upon modification of the above argument, suggest an improvement due to statistics by a factor of 6 or 10 respectively. We see that these estimates tend to fail because the LEP II CM energy is too low for the high-energy approximations of the amplitudes to be useful.

Next we compare the linear-collider constraints on $f_{DW}$, $f_{DB}$, $f_{BW}$ and $f_{\Phi,1}$ from Table VIII with Eqn. (6.19), and we see that, if there is a signal from these coefficients in the $e^+e^- \rightarrow W^+W^-$ process, then there should be an even bigger signal in the $e^+e^- \rightarrow f\bar{f}$ channel. Hence, if we see no signal in the latter channel, then we can return to the three parameter fit in terms of $f_{WWW}$, $f_W$ and $f_B$, and Eqns. (9.1) may be employed.

Now we turn to the chiral Lagrangian. Recall that $\beta_1$, $\alpha_1$ and $\alpha_8$ are already constrained by the low-energy data through their contributions to the gauge-boson two-point-functions. One of these parameters, $\alpha_1$, also contributes directly to the $WW\gamma$ and $WWZ$ vertices. If we could justify neglecting these three parameters, then we can present a set of relations that parallels Eqns. (1.1):

\begin{align}
g_1^Z(q^2) &= 1 + g_2^Z \alpha_3, \tag{9.2a} \\
\kappa_{\gamma}(q^2) &= 1 + g_2^{\alpha} (\alpha_2 + \alpha_3 + \alpha_9), \tag{9.2b} \\
\kappa_{Z}(q^2) &= 1 + g_2^{\alpha} (-s^2 \alpha_2 + c^2 \alpha_3 + \bar{c}^2 \alpha_9), \tag{9.2c} \\
g_5^Z(q^2) &= \hat{g}_2^Z \alpha_{11}, \tag{9.2d} \\
\lambda_{\gamma}(q^2) &\approx \lambda_{Z}(q^2) \approx 0. \tag{9.2e}
\end{align}

These results agree with Ref. [22]. The numerical estimate of Eqn. (5.12f) suggests that $\alpha_9$ may be very small. If we neglect $\alpha_9$, then, upon using Eqns. (5.8), Eqns. (9.2a)-(9.2c) are equivalent to Eqns. (9.1a)-(9.1d); in general we must retain $\alpha_9$. The appearance of $g_5^Z$ at the leading order in the chiral Lagrangian has no leading-order counterpart in the light-Higgs scenario. By Eqn. (5.12g) we expect that $\alpha_{11}$, hence $g_5^Z$, may be small, again due to custodial
symmetry. However, its suppression is not so strong as for the other custodial-symmetry-violating couplings. Also, because it has no counterpart in the light-Higgs scenario, it is of special interest for discriminating between the two realizations of the symmetry-breaking sector. Finally, because the $\lambda$ couplings are inherently higher order in the chiral Lagrangian, Eqn. (9.2e) is obtained trivially at low energies.

Upon comparing the first row of Table IX with the $Z$-pole/low-energy constraints of Eqn. (6.17), we see that, at LEP II, we are justified in neglecting the contributions of $\beta_1$, $\alpha_1$ and $\alpha_8$, hence Eqns. (9.2) are valid. Also, if we believe the estimates of Eqns. (5.12), then we need to constrain the $\alpha$ parameters at the level of $10^{-3}$ before we can expect to see the effects of new physics. Clearly we do not yet have this type of sensitivity at LEP II.

Considering the second row of Table IX, we expect that the linear collider may be sensitive to new physics described by the chiral Lagrangian. However, the analysis now becomes more complicated. Linear-collider experiments may also be sensitive to $\alpha_8$ and marginally sensitive to $\alpha_1$. In stark contrast to the light-Higgs scenario, with the chiral Lagrangian we do not obtain additional constraints by studying $e^+e^- \rightarrow f\bar{f}$. Here the leading corrections to the gauge-boson propagators are independent of $q^2$, and hence there is no benefit from the higher CM energy. To the contrary, once we are away from the $Z$ pole, event rates are low and we are statistics limited. If, taking advantage of the high luminosity of the linear collider, we repeat the LEP experiments on the $Z$ pole, then, through the improved measurement of $\bar{s}^2(m_Z^2)$, it may be possible to improve the measurements of $\beta_1$, $\alpha_1$ and $\alpha_8$ directly. The measurement of the weak mixing angle may also be improved at the TeV33. However, the impact of these additional measurements is limited.

Finally, in Table X, we have presented constraints which treat corrections to three-gauge-boson vertices as independent from the two-point-function corrections. As we see from Eqn. (9.2d), we are justified in measuring $f_Z^5$ (which is equivalent to $g_5^Z$) separately from the rest. Eqn. (9.1d) implies that $\lambda_\gamma = \lambda_Z < 0.04$ at LEP II, and $\lambda_\gamma = \lambda_Z < 0.001$ at the linear collider; these results should be contrasted with the first-row and second-row results of Table X respectively. The other correlations described by Eqns. (9.1) and Eqns. (9.2) could also be explored in this way. However, at linear collider energies where the sensitivity to $WW\gamma$ and $WWZ$ couplings rivals the sensitivity to gauge-boson propagator corrections,
it is more sensible to abandon the analysis of Table \text{X} in favor of the analyses of Table \text{VIII} and Table \text{IX}.

\section*{X. CONCLUSIONS}

When we consider the effects of new physics described by an effective Lagrangian with the linearly realized symmetry-breaking sector, \textit{i.e.} the light Higgs scenario, then, at the leading order, the coefficients of seven operators contribute to $e^+e^- \rightarrow W^+W^-$ amplitudes. These coefficients are $f_{DW}$, $f_{DB}$, $f_{BW}$, $f_{\Phi,1}$, $f_{WWW}$, $f_W$ and $f_B$. The first four are already constrained via low-energy and $Z$-pole experiments, but the current constraints, in some cases, do not rule out observable contributions to $e^+e^- \rightarrow W^+W^-$ at LEP II. However, the constraints on these four coefficients may be strengthened by also studying $e^+e^- \rightarrow f\bar{f}$ processes at LEP II. In fact, the $e^+e^- \rightarrow f\bar{f}$ process is more sensitive to these four parameters than is the $e^+e^- \rightarrow W^+W^-$ process. Hence, if we fail to observe a signal for non-SM physics in the $e^+e^- \rightarrow f\bar{f}$ channel, then we can neglect these four coefficients when analysing $e^+e^- \rightarrow W^+W^-$. The analysis of $e^+e^- \rightarrow W^+W^-$ amplitudes then reduces to a three-parameter analysis in terms of $f_{WWW}$, $f_W$ and $f_B$. The analysis may be performed using the familiar parameters $\kappa_V$, $g_V^1$ and $\lambda_V$ with $V = \gamma, Z$ subject to the constraints of Eqn. (9.1). Essentially the same scenario occurs at the linear collider. We must use both the $W$-boson pair-production process as well as studies of two-fermion final states to separate the contributions of $f_{DW}$, $f_{DB}$, $f_{BW}$ and $f_{\Phi,1}$ from those of $f_{WWW}$, $f_W$ and $f_B$. 

When we consider the effects of new physics described by an effective Lagrangian with the symmetry-breaking realized nonlinearly, \textit{i.e.} the chiral Lagrangian which does not include a physical Higgs scalar boson, we must consider the contributions of $\beta_1$, $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_8$, $\alpha_9$ and $\alpha_{11}$. Three of these, $\beta_1$, $\alpha_1$ and $\alpha_8$, are already constrained by the low-energy and $Z$-pole data. Hence Eqns. (9.2) may be relevant. The inclusion of $\alpha_9$ makes these relations slightly more complicated than their light-Higgs-scenario counterparts, Eqns. (9.1). The parity violating coupling $\alpha_{11}$ also contributes, but certainly we can disentangle its effects by constructing some parity-violating observables. Because it has no leading-order parity-violating counterpart in the light-Higgs scenario, $\alpha_{11}$ is especially interesting. At the linear collider $e^+e^- \rightarrow W^+W^-$ amplitudes may be sensitive to all seven parameters, providing for
a rather complicated analysis.

We also presented an analysis where the most general contributions to the $WW\gamma$ and $WWZ$ vertices are assumed to be independent of the corrections to the gauge-boson two-point-functions. At LEP II this analysis is useful, especially for testing the relations of Eqns. (9.1). However, at a 500GeV linear collider, where the measurements of $WW\gamma$ and $WWZ$ couplings become competitive with measurements of gauge-boson propagator corrections, this approach may be less useful.

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