The widths of quarkonia in quark gluon plasma

Yongjae Park*[1] Kyung-Ill Kim[1] Taesoo Song[1] and Su Houng Lee[1]

Institute of Physics and Applied Physics, Yonsei University, Seoul 120-749, Korea

Cheuk-Yin Wong[5]

Physics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37830 and

Department of Physics, University of Tennessee, Knoxville, TN 37996

Recent lattice calculations showed that heavy quarkonia will survive beyond the phase transition temperature, and will dissolve at different temperatures depending on the type of the quarkonium. In this work, we calculate the thermal width of a quarkonium at finite temperature before it dissolves into open heavy quarks. The input of the calculation are the parton quarkonium dissociation cross section to NLO in QCD, the quarkonium wave function in a temperature-dependent potential from lattice QCD, and a thermal distribution of partons with thermal masses. We find that for the $J/\psi$, the total thermal width above $1.4 T_c$ becomes larger than 100 to 250 MeV, depending on the order $10^{-2}$ to $10^{-1}$ mb at $1.4 T_c$. However, at similar temperatures, we find a much smaller thermal width and effective cross section for the $\Upsilon$.

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INTRODUCTION

Recently, a number of important progress have been made in the physics of $J/\psi$ suppression as a signature of quark gluon plasma that inevitably leads us to augment the original work by Matsui and Satz[1] with a more detailed study of heavy quark system at finite temperatures, before confronting the recent RHIC data[2], and in predicting results for LHC. Among these theoretical developments are the phenomenologically successful statistical model for $J/\psi$ production[3, 4, 5], based on a coalescence assumption near $T_c$, the recombination of charm pairs into $J/\psi$, and the recent lattice calculations, showing strong evidence that the heavy quarkonium will persist above $T_c$, and then perform a similar calculation for the bottomonium case.

The NLO calculation of $J/\psi$-parton dissociation calculations involves collinear divergence. When applying this elementary cross section to dissociation by hadrons, the collinear divergence is cured by mass factorization, which renormalizes the divergent part of the cross section into the parton distribution function of the hadron. Such complications disappear at finite temperatures, as the thermal masses of the partons automatically renders the divergence finite. The magnitude of the thermal mass as a function of temperature has been obtained previously by examining the equation of state[22]. In the region of $T_c$ to $2T_c$, the order of $300-400$ MeV for quarks and $400-600$ MeV for gluons. In this work, instead of following the detailed temperature dependence, we shall study results with a thermal mass of 400 and 600 MeV for both the quarks and gluons. As we will see, with an effective thermal mass of 400 MeV, we find that the effective thermal dissociation cross section above $1.4 T_c$ is larger than 250 MeV. The NLO calculation is proportional to the derivative of the momentum space wave function. We will use the Coulomb wave function, whose size have been fitted to reproduce the result obtained by
Wong\cite{10}, using a potential extracted from lattice gauge thermodynamical quantities.

In Section II, we will recapitulate the LO result. In Section III, we will discuss the NLO for $J/\psi$. The LO and NLO results for bottomonium are given in Section IV.

**LO RESULT**

The LO invariant matrix element for the $J/\psi$ dissociation cross section by gluon first obtained by Peskin\cite{18} and rederived by one of us\cite{23} using the Bethe-Salpeter equation is given as,

$$\mathcal{M}^{\mu\nu} = -g \sqrt{\frac{M_{\psi}}{N_c}} \left\{ k \cdot \frac{\partial \psi(p)}{\partial p} \gamma^\alpha + k_0 \frac{\partial \psi(p)}{\partial p^j} \gamma^j \right\} \bar{u}(p_1) \left[ \frac{1 + \gamma_0}{2} \gamma^i \frac{1 - \gamma_0}{2} T^a T^a \psi(p_2) \right]$$

(1)

Here, $\mu, \nu$ represents the polarization index of the $J/\psi$ and gluon respectively, and $k, k_1, p_2$ are the four momentum of the gluon, $c, \bar{c}$. The quantity $p$ is the relative three momentum between $c$ and $\bar{c}$, and $\psi(p)$ is the charmonium wave function. $M_{\psi}$ is the mass of a quarkonium, and $N_c$ is the number of color. Current conservation is easily shown to be satisfied, $k_0 M^{\mu\nu} = 0$. The energy conservation in the non-relativistic limit implies,

$$k_0 + m_{\psi} = 2m_c + \frac{p_1^2 + p_2^2}{2m_c},$$

(2)

from which, the counting scheme for both the LO\cite{18} and the NLO\cite{20} are given as follows,

$$|p_1| \sim |p_2| \sim |p| \sim O(m^2),$$

$$k_0 \sim |\vec{k}| \sim O(m^4).$$

(3)

The effective thermal width and cross section are obtained by folding the matrix element with the thermal parton distribution, $n(k_0)$,

$$\Gamma^{\text{eff}} = d_p \int \frac{d^3 k}{(2\pi)^3} n(k_0) v_{\text{rel}}(\sigma(k_0)),$$

$$\sigma^{\text{eff}} = \int \frac{d^3 k}{(2\pi)^3} n(k_0) \sigma(k_0) / \int \frac{d^3 k}{(2\pi)^3} n(k_0),$$

(4)

where $d_p$ is the parton degeneracy, which is taken to be 16 for the gluon in the LO calculation. Here, we will perform the calculation in the rest frame of $J/\psi$, so the relative velocity of $J/\psi$ and initial parton is $v_{\text{rel}} = |\vec{k}|/k_0$.

The effective thermal width or the effective cross section in Eq.(4), the cross section is integrated over the incoming energy, which effectively integrates over the absolute square of the derivative of the momentum space wave function. As a consequence, the results are sensitive to the size of the wave function only and not so much on its detailed functional form. Therefore, we will use a Coulomb wave function, whose Bohr radius is fitted to reproduce the rms radius obtained by one of us\cite{10} by solving the bound states in a temperature-dependent potential extracted from lattice gauge thermodynamical quantities.

For the binding energy, we use the values obtained in ref.\cite{10}. Table I summaries the $J/\psi$ binding energy, its rms radius, and its corresponding Bohr radius at finite temperature.

| $T/T_c$ | 1.13 | 1.18 | 1.25 | 1.40 | 1.60 | 1.65 |
|---------|------|------|------|------|------|------|
| $\epsilon_0$ (MeV) | 36.4 | 20.9 | 10.1 | 3.4 | 0.14 | 0.004 |
| $\sqrt{<r^2>}$ (fm) | 0.97 | 1.19 | 1.54 | 2.30 | 4.54 | 5.17 |
| $a_0$ (fm) | 0.56 | 0.69 | 0.89 | 1.35 | 2.62 | 2.99 |

TABLE I: The binding energy, the rms radius, and its corresponding Bhor radius of $J/\psi$ at finite temperature.
The width due to these contributions will be calculated in the next section.

\[ \Gamma(J/\psi) = \frac{1}{4\sqrt{q \cdot k_1}} \int d\sigma_3 |M|^2 \]  

(6)

where \( u^2 = (q+k_1)^2 \), \( m_{k_1} \) is the thermal mass of a parton, and \( d\sigma_3 \) is the 3-body phase space,

\[ d\sigma_3 = \frac{d^3k_2}{(2\pi)^3} \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \]

\[ \times (2\pi)^4 \delta^4(q + k_1 - k_2 - p_1 - p_2) \]  

(7)

Here, \( q, k_1, k_2, p_1 \) and \( p_2 \) are respectively the momentum of \( J/\psi \), incoming parton, outgoing parton, charm quark, and anti-charm quark.

In the \( J/\psi \) rest frame, the cross section can be written as,

\[ \sigma = \frac{1}{4\sqrt{q \cdot k_1}} \int \frac{d\alpha}{\alpha} \int_{\alpha}^{\beta} \frac{dw}{w} \] \[ \times \int_{\alpha}^{\beta} d\alpha' \frac{w^2}{16\pi^3 m_\Phi |k_1|} |M|^2, \]  

(8)

where \( p_\Delta^2 = (k_1 - k_2)^2 \), \( w^2 = (q + p_\Delta)^2 \). The integration range is

\[ \alpha = (m_{p_2} + m_{p_1})^2 = 4m_c^2, \]
\[ \beta = (u - m_{k_1})^2, \]
\[ \alpha' = -b - \sqrt{b^2 - ac}, \]
\[ \beta' = -b + \sqrt{b^2 - ac}, \]  

(9)

The variables appearing in \( \sigma \) can be expressed in terms of \( w^2, p_\Delta \) and \( u^2 \) as follows,

\[ q \cdot k_1 = \frac{(u^2 - m_{\Phi}^2 - m_{k_1}^2)}{2}, \]
\[ \frac{1}{|k_1|} = \sqrt{\left\{ (u^2 - m_{\Phi}^2 + m_{k_1}^2)/(2m_{\Phi}) \right\}^2 - u^2}, \]
\[ k_1 \cdot k_2 = \frac{(-p_\Delta^2 + 2m_{k_1}^2)}{2}, \]
\[ k_{10} = \sqrt{|k_1|^2 + m_{k_1}^2}, \]
\[ k_{20} = k_{10} - \frac{(u^2 - p_\Delta^2 - m_{\Phi}^2)/(2m_{\Phi})}{|p|} = \sqrt{m_c(k_{10} - k_{20} + m_{\Phi} - 2m_c).} \]  

(10)

**NLO RESULT**

The \( J/\psi \) dissociation cross section by partons in QCD at the NLO was performed by two of us [20]. The dissociation cross section can be divided into two parts; the dissociation by quarks and that by gluons (Fig. 4).

In both cases, the cross sections are given as,

\[ \sigma(u) = \frac{1}{4\sqrt{q \cdot k_1}} \int d\sigma_3 |M|^2 \]  

(6)

\[ \int d\sigma_3 = \int \frac{d^3k_2}{(2\pi)^3} \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \]

\[ \times (2\pi)^4 \delta^4(q + k_1 - k_2 - p_1 - p_2) \]  

(7)

Here, \( q, k_1, k_2, p_1 \) and \( p_2 \) are respectively the momentum of \( J/\psi \), incoming parton, outgoing parton, charm quark, and anti-charm quark.

In the \( J/\psi \) rest frame, the cross section can be written as,

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(8)

where \( p_\Delta^2 = (k_1 - k_2)^2 \), \( w^2 = (q + p_\Delta)^2 \). The integration range is

\[ \alpha = (m_{p_2} + m_{p_1})^2 = 4m_c^2, \]
\[ \beta = (u - m_{k_1})^2, \]
\[ \alpha' = -b - \sqrt{b^2 - ac}, \]
\[ \beta' = -b + \sqrt{b^2 - ac}, \]  

(9)

The variables appearing in \( \sigma \) can be expressed in terms of \( w^2, p_\Delta \) and \( u^2 \) as follows,

\[ q \cdot k_1 = \frac{(u^2 - m_{\Phi}^2 - m_{k_1}^2)}{2}, \]
\[ \frac{1}{|k_1|} = \sqrt{\left\{ (u^2 - m_{\Phi}^2 + m_{k_1}^2)/(2m_{\Phi}) \right\}^2 - u^2}, \]
\[ k_1 \cdot k_2 = \frac{(-p_\Delta^2 + 2m_{k_1}^2)}{2}, \]
\[ k_{10} = \sqrt{|k_1|^2 + m_{k_1}^2}, \]
\[ k_{20} = k_{10} - \frac{(u^2 - p_\Delta^2 - m_{\Phi}^2)/(2m_{\Phi})}{|p|} = \sqrt{m_c(k_{10} - k_{20} + m_{\Phi} - 2m_c).} \]  

(10)

**FIG. 4:** NLO diagrams induced by (a)quarks, (b)gluons.
We first consider the NLO effective thermal width induced by quark. Here, the quark degeneracy is 36 assuming 3 flavors. The invariant matrix element is given as\cite{20},

$$|M|^2 = \frac{4}{3} q^4 m^2 \left| \frac{\partial \psi(p)}{\partial p} \right|^2 \left( -\frac{1}{2} + \frac{k^2_{10} + k^2_{20}}{2k_1 \cdot k_2} \right). \quad (11)$$

Next, the gluon induced NLO calculation has the same degeneracy as the LO case, and the invariant matrix element is given as follows\cite{20},

$$|M|^2 = \frac{4}{3} q^4 m^2 \left( \frac{\partial \psi(p)}{\partial p} \right)^2 \left\{ -4 + \frac{k_1 \cdot k_2}{k_{10} k_{20}} + \frac{2k_{10}}{k_{20}} \frac{2k_{20}}{k_{10}} - \frac{k^2_{10}}{k_{20}} - \frac{k^2_{20}}{k_{10}} + \frac{2}{k_1 \cdot k_2} \times \left( \frac{k^2_{10} + k^2_{20}}{k_{10} k_{20}} \right)^2 - 2k^2_{10} - 2k^2_{20} + k_{10} k_{20} \right\}. \quad (12)$$

In the hadronic phase, the $1/(k_1 \cdot k_2)$ term, and the $1/k^2_{20}$ term give rise to collinear divergence, and soft divergence respectively. However, in QGP phase, the thermal mass of a parton $m_{k_1}$ in Eq.\,(10) plays the role of a cutoff. In contrast to the LO calculation, as can be seen from Fig.\,(a) for quark and from Fig.\,(b) for gluon, $\sigma(k_0)$ at the NLO does not vanish at large $k_0$. This is so because irrespective of how large the energies of the incoming quark or the gluon are, they can always radiate small energy gluon in the order of the binding energy to effectively dissociate J/ψ via the LO process. Hence, in Eq.\,(11), $\sigma$ has non trivial overlap with the maximum of $n(k_0)$ that increase with temperature, leading to the result shown in Fig.\,(a) and in Fig.\,(b) for quark and gluon induced NLO width respectively. One thing to note from Fig.\,(a) is that the elementary cross
section has a peak near threshold at $1.4 \, T_c$. Such peak structure only appears when the binding energy becomes very small and the corresponding momentum space wave function becomes highly peaked near zero momentum. When the incoming energy is small, these highly peaked region gives important contributions to the two dimensional phase space integral in Eq. (5). But when the incoming energy becomes large, the phase space for the peaked region becomes smaller and so does the total cross section. Such singular behavior as a function of the incoming energy disappears when the binding energy becomes large.

Here we take the thermal mass from 400 MeV to 600 MeV$^{22}$ within the temperature region of a few $T_c$. With those masses we obtained large thermal widths. Even with an upper limit thermal mass of 600 MeV, the width exceeds 100 MeV above $1.4 \, T_c$, where we have taken $T_c=170$ MeV. For example, if the thermal mass of partons is 600 MeV, and the produced $J/\psi$ remains at $1.4 \, T_c$ for 2 fm/c, its survival rate will be less than 40%.

As can be seen from Fig. 8(a) and Fig. 8(b), with a 600 MeV thermal mass at $1.4 \, T_c$, the effective dissociation cross section by a quark is about 1.0 mb and that by a gluon 1.5 mb. Hence, even though the $J/\psi$ might start forming at $1.6 \, T_c$, its effective width is very large and will not accumulate until the system cools down further.

**Υ DISSOCIATION**

Here, we present the result for the Υ case. The Υ wave function is less sensitive to changes in the temperature$^{16}$. In the LO calculation, while the trends in the temperature dependence of $\sigma$ is similar to that of the $J/\psi$, its variation in magnitudes is much smaller. Moreover, as can be seen in Table 11, the binding energy remains large. As a consequence, the overlap of $\sigma$ and the thermal distribution remains large even at low temperatures, and the effective thermal width slowly decreases with temperature as can be seen in Fig. 9.

In the NLO calculation, as shown in Fig. 10 and Fig. 11 the form of the cross section is similar to that of $J/\psi$. However, because the binding energy of Υ is still large at high temperature, there is no peak structure near threshold. Moreover, because Υ is more tightly bound and has a smaller dipole size than $J/\psi$, its corresponding $\sigma$ is also smaller. Therefore, as can be seen from Fig. 12(a) for the quark induced and Fig. 12(b) for the gluon induced width, the overall value of the width for Υ is smaller than that of $J/\psi$. With an upper limit of thermal mass of 600 MeV and at temperature of $1.65 \, T_c$, the sum of the LO and NLO thermal widths is less than 50 MeV. At this temperature, the effective dissociation cross section by a quark is less than 0.2 mb and by a gluon than 0.6 mb. Therefore, unlike the $J/\psi$ case, the Υ has a smaller thermal width and effective dissociation cross section, and will effectively start accumulating at higher temperatures. Fig. 13(b) is the sum of LO and NLO.
In this work, we have calculated the thermal widths of $J/\psi$ and $\Upsilon$ at finite temperature using the elementary parton-quarkonium dissociation cross section at NLO in QCD and assuming thermal partons with effective thermal mass. We find that for $J/\psi$ at $1.4 T_c$, the thermal width will be 100 to 250 MeV, which translates into an effective thermal cross section of several mb. However, the corresponding width and effective cross section for the $\Upsilon$ is much smaller. Recently Mocsy and Petreczky have also calculated the thermal widths of charmonium and bottomonium in QGP, assuming that the quarkonium and its constituents are in thermal equilibrium with the surrounding. The thermal width estimated by MP is similar to ours for the bottomonium but several times larger for the charmonium. The result for the charmonium by MP is obtained using a phenomenological formula obtained when the binding energy is much smaller than the temperature; hence further work has to be performed to understand the discrepancy.

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**TABLE II:** Binding energy and rms of the $\Upsilon$.

| $T/T_c$ | 1.13 | 1.18 | 1.25 | 1.65 | 1.95 |
|--------|------|------|------|------|------|
| $\epsilon_0$(MeV) | 313  | 247  | 203  | 150  | 111  | 86   |
| $\sqrt{\sigma^2}$(fm) | 0.294, 0.331, 0.366, 0.425, 0.494, 0.562 |
| $a_0$(fm) | 0.17, 0.19, 0.21, 0.246, 0.285, 0.324 |

**SUMMARY**

Because the contribution of LO is not negligible, the shape is quite different from the effective cross section due to quarks of Fig. 13(a).
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