Limited Feedback Scheme for Device to Device Communications in 5G Cellular Networks with Reliability and Cellular Secrecy Outage Constraints

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Abstract—In this paper, we propose a device to device (D2D) communication scenario underlaying a cellular network where both D2D and cellular users (CUs) are discrete power-rate systems with limited feedback from the receivers. It is assumed that there exists an adversary which wants to eavesdrop on the information transmission from the base station (BS) to CUs. Since D2D communication shares the same spectrum with cellular network, cross interference must be considered. However, when secrecy capacity is considered, the interference caused by D2D communication can help to improve the secrecy communications by confusing the eavesdroppers. Since both systems share the same spectrum, cross interference must be considered. We formulate the proposed resource allocation into an optimization problem whose objective is to maximize the average transmission rate of D2D pair in the presence of the cellular communications under average transmission power constraint. For the cellular network, we require a minimum average achievable secrecy rate in the absence of D2D communication as well as a maximum secrecy outage probability in the presence of D2D communication which should be satisfied. Due to high complexity of convex optimization methods, to solve the proposed optimization problem, we apply Particle Swarm Optimization (PSO) which is an evolutionary approach. Moreover, we model and study the error in the feedback channel and the imperfectness of channel distribution information (CDI) using parametric and nonparametric methods. Finally, the impact of different system parameters on the performance of the proposed scheme is investigated through simulations. The performance of the proposed scheme is evaluated using numerical results for different scenarios.

Index Terms— Device to Device (D2D) communications, Limited Rate Feedback, Physical (PHY) layer security, Particle swarm optimization.

I. INTRODUCTION

A. Background and Motivation

The growth of the cellular networks and the number of users as well as the emergence of the new multimedia based services result in growing demands for high data rate and capacity which is beyond the capability of forth generation (4G) wireless networks. Recently, the fifth generation (5G) cellular network has triggered a great attention to provide high data rate and low latency services in a power and spectrally efficient manner. Introducing new applications like context-aware applications requires the direct communications of neighboring devices. In this context, device to device (D2D) communication has been considered as a promising technique for 5G wireless networks [1]–[3]. D2D communication operates as an underlay network to a cellular network [4]–[6] and enables reusing the cellular resources which increases the spectral efficiency and the system capacity. In D2D communications, two neighboring devices use the cellular bandwidth to communicate directly without the help of cellular base station (BS).

Although D2D communications can improve the spectral efficiency, it should provide access to licensed spectrum with a controlled interference to avoid the uncertainties of the cellular network performance. Therefore, interference management is a critical issue for D2D underlaying cellular networks without considering it, the effectiveness of D2D communication links will be deteriorated. In this sense, several papers have proposed mechanisms for interference mitigation and avoidance. To perform interference management in D2D underlaying cellular network, one approach is to consider cooperative communications. In this way, a D2D user equipped with multiple antennas acts as an in-band relay to a cellular link where the multi-antenna relay is able to help decoding messages, cancelling interference, and providing multiplexing gain in the network [7]–[9]. In [10], power control problem for the D2D users is investigated in order to optimize the energy efficiency of the user equipments (UEs) as well as to ensure that the quality of service (QoS) of D2D devices and UEs does not fall below the acceptable target. The problem of interference management through multi rate power control for D2D communications is studied in [11]. The transmission power levels of D2D users are optimized to maximize the cell throughput while preserving the signal-to-interference-plus-noise ratio (SINR) performance for the cellular user. In [12], authors guarantee the reliability of D2D links and mitigate the interference from the cellular link to the D2D receivers. A pricing framework has been suggested in [13] where BS protects itself by utilizing game theory approach.

To increase the security of wireless transmission, physical layer security has been developed based on information theoretic concepts [14]–[21]. From the physical layer point of view, the security is quantified by the secrecy rate which is defined as the difference of achievable rate between the legitimate receiver and the rate overheard by eavesdroppers [22]. In this sense, unlike the previous work on D2D underlaying cellular networks in which the focus is on the interference mitigation and avoidance, the interference works well when secrecy capacity of the cellular communication is taken into consideration [21]. In other words, it can be assumed that the D2D communication works as a friendly jammer and its interference is helpful for the secure cellular network to improve secrecy capacity. In practice, since the eavesdropper is a passive attacker, obtaining its channel state information (CSI) is impossible in many situations. In this case, the secrecy outage probability can be used as a security performance criterion.

The performance of previous works is based on the fact that the perfect CSI of all links is available. However, due to
the estimation errors and feedback delay, perfect CSI may not be available. In addition, the feedback channel has a limited capacity since transmitting unlimited feedback information between transmitters and receivers means passing a huge amount of bits for signaling. To tackle this issue, the limited feedback channel model can be employed. In the limited feedback channel, the space of channel gains is divided into a finite number of regions, and instead of channel gain values, the index of the fading region in which the actual channel gain lies is feedbacked [23]–[25]. In [25], the authors study the effect of the feedback information on the performance of the D2D underlaying cellular networks and develop user selection strategies based on limited feedback.

B. Contributions and Organization

In this paper, we study D2D communications in the presence of the cellular communications while there exists a malicious user which wants to eavesdrop the information transmitted from the BS to CU. We assume that the legitimate transmitters do not have the perfect values of the channel power gains and the knowledge about their respective direct channel power gains is obtained via their dedicated limited rate feedback channel. In other words, we assume that the space of the channel gains is divided into a finite number of regions. Then given the actual value of the channel gains, the receiver determines the index of the region in which the channel gain lies and feedbacks the index of that region to the corresponding receiver. Note that, the cellular system is superior to D2D communication and D2D pair uses the spectrum of cellular networks in an opportunistic manner. The concurrent transmission of cellular network and D2D pair, if exists, degrades the performance of both systems due to the cross interference between these two systems. Therefore, in this paper, we consider the performance of the cellular system in both the presence and the absence of D2D communication. Precisely, we require that the average transmission rate of cellular user in the absence of the D2D communication should be above a predefined threshold while its performance in the presence of the D2D communication, in terms of outage probability, satisfies a predefined threshold. Our objective is to maximize the average achievable data rate of the D2D pair in the presence of the cellular communication while individual constraints on the average transmission power of the cellular BS and the D2D pair should be satisfied. Due to non-convexity and nonlinearity of the proposed problem, to find the optimal solution of the problem, we use particle swarm optimization (PSO) method which is an evolutionary algorithm [26]–[30]. In reality, the feedback channels can be affected by the noise which makes the transmitter select an incorrect code-word from the designed code book. Therefore, in this paper, we consider the effect of error in the feedback channel on the performance of the proposed scheme by incorporating such error into the problem formulation. We further study the effect of channel distribution information (CDI) imperfection. Parametric and nonparametric methods are investigated in estimating the CDI of the channels. The contributions of this paper are as follows:

- We develop a mathematical model for the secure communication in D2D communication underlaying the cellular network in which the knowledge of transmitters about the CSIs is obtained via a limited rate feedback channel. In our model, we consider the cross interference between the cellular network and the D2D pair explicitly and formulate the resource allocation problem as an optimization framework.
- To solve this optimization problem and obtain its solutions which are the fading regions’ boundaries and transmission power levels, we use PSO algorithm which is an evolutionary algorithm.
- We further consider the effect of the noise in the feedback channel and incorporate it into our optimization problem. In this case, the error in the feedback channel would lead the transmitters to choose the incorrect code-words. We formulate the corresponding optimization problem and solve it using the PSO approach.
- We also consider the effect of the CDI imperfectness in our proposed scheme. In this case, the CDI’s parameters are not perfectly known and parametric and non-parametric approaches are used to estimate the CDI parameters. Finally, the performance of the proposed scheme in different scenarios is investigated via simulations.

The paper is organized as follows. System model is described in Section II. Limited rate feedback schemes are proposed in Section III. The limited rate feedback resource allocation problem is formulated and solved in Section IV. In Section V, practical considerations, i.e., noisy feedback channel and CDI estimation error, are investigated. Simulation and numerical results are provided in Section VI and finally conclusions are drawn in Section VII.

II. System Model

We consider a D2D communication scenario underlaying an existing cellular network. It is assumed the downlink transmission in the cellular network where the BS transmits information to a cellular user while at the same time; the existing D2D pair performs its own transmission on the same channel. Such a scenario can be interpreted as there are many cellular users in the network each of which is assigned to a channel over which the BS sends information to them. Assuming this assignment is performed based on some network parameters and is fixed, two cellular users exploit one of the available cellular channels to perform their information transmission directly. In this paper, we assume that this assignment is predefined. In addition to cellular user and D2D pair, we assume that there exists a malicious user which wants to eavesdrop on the information transmission of cellular network, i.e., from the BS to the CU. However, the malicious user does not eavesdrop on the D2D pair. Such assumption can be justified when the malicious user is not aware of the existence of D2D pair as such sharing can be performed opportunistically (i.e., D2D pair may or may not exist at any time) when the malicious user is not interested in D2D pair information, or when the D2D pair applies upper layer security measures, e.g., cryptography. In such case, the malicious user treats the signals from D2D pair as noise.
In this paper, we assume that the eavesdropper has complete knowledge about the instantaneous channel power gains and the CDI of the channels from BS to CU and from BS to itself. We further assume that the legitimate receivers, i.e., the cellular user and the RD2D, know the CDI of all channels and only the instantaneous channel power gains of their respective channels. We assume that the legitimate transmitters do not have perfect values of channel power gains and the knowledge of legitimate transmitters about their respective direct channel power gains is obtained via their respective limited rate feedback channels. In this case, the space of $h_{BC}$ is divided into a finite number of $M$ regions, i.e., $[0, \tilde{h}_{BC}(1))$, $[\tilde{h}_{BC}(1), \tilde{h}_{BC}(2))$, $\cdots$, $[\tilde{h}_{BC}(M-1), \tilde{h}_{BC}(M)]$ where $\tilde{h}_{BC}(M) = \infty$. Similarly, for D2D pair, the space of $h_{DD}$ is divided into a finite number of $N$ regions, i.e., $[0, \tilde{h}_{DD}(1))$, $[\tilde{h}_{DD}(1), \tilde{h}_{DD}(2))$, $\cdots$, $[\tilde{h}_{DD}(N-1), \tilde{h}_{DD}(N)]$ where $\tilde{h}_{DD}(N) = \infty$. The receiver, i.e., CU, measures the channel power gain $h_{BC}$ and feedbacks the index $m$ if $h_{BC}$ lies in the region $[\tilde{h}_{BC}(m), \tilde{h}_{BC}(m+1))$. Similarly, RD2D measures the channel power gain $h_{DD}$ and feedbacks the index $n$ if $h_{DD}$ lies in the region $[\tilde{h}_{DD}(n), \tilde{h}_{DD}(n+1))$. In this paper, we assume that the feedback links are confidential. This means that, the feedbacked index of the cellular network could not be overheard by D2D pair and that of D2D pair could not be overheard by the cellular network. In this case, the power-rate tuples will depend only on the corresponding feedbacked index, i.e., we have $(p_{BC}(m), r_{BC}(m), r_{S_{BC}}(m))$ for cellular communication which are the transmit power, the transmission rate and the secrecy rate at which BS transmits information to CU, respectively. Moreover, we consider $(p_{DD}(n), r_{DD}(n))$ for D2D communications which are the transmit power and the transmission rate at which TD2D transmits information to RD2D, respectively.

In fact, the proposed schemes operate in two phases. In the first phase (off-line phase), several parameters that are later used for resource allocation are computed. It is done before the communication established and based on the CDIs of the network’s links the optimum boundary regions and code-books are designed by the base station. At the end of this phase, all code-words are informed to users by BS. However, the channel partitioning structure is kept at the CU. In the second phase (on-line phase) that is employed during communication, the transmitters use the parameters obtained in off-line phase. In fact, in the on-line phase, the CU and D2D receiver measure the related CSIs and based on them find the related channel partition and boundary region. Then, they transmit back the index to the base station and D2D transmitter in order to select the corresponding code-word from the obtained code-book.

For the resource allocation, two approaches could be adopted. One is to consider the problem of pairing the D2D and cellular links as well as the designing limited feedback scheme jointly. In this way, the outcome of the resource allocation problem is which D2D link is paired with which cellular link as well as code-books (power allocation and boundary regions). However, this approach is much complex and would be computationally prohibitive. Another approach which could lower the complexity of the scheme is to consider the pairing problem and limited rate feedback design separately. In this case, one first solves the problem of pairing D2D and cellular link. Then, given this pairing result, the problem of designing a limited rate feedback scheme could be formulated and solved. In this paper, we assumed the second approach and assumed that the D2D and cellular links are paired and the pairing result is available based on which we design the limited rate feedback scheme. The pairing process could be performed based on network parameters as well some degrees of the required QoS level. For example, one could formulate a problem in which the aim is to pair the D2D and cellular links based on the average channel gains instead of instantaneous or long term channel considerations. Several authors have studied the problem of pairing D2D links with CUs for spectrum sharing and focus on the selection of the D2D link and CUs as a pair for better performance [36]–[38]. However, in our paper, we present the resource allocation in D2D underlaying cellular network and focus on devising a limited rate feedback model as well as power allocation problem encompassing different performance metrics. In this way, first, the D2D link and cellular communication link are scheduled based on the mean of channel power gain. In the next step, the resource allocation can be obtained based on the proposed scheme in this paper. To obtain the best optimum solution, they should be solved at the same time; however, it causes a high computational complexity. To reduce the complexity, they can be considered separately at the cost of a slight performance loss. However,
we can extend this approach for solving the problem at the same time as future works.

III. LIMITED RATE FEEDBACK SCHEMES

A. Capacity of Links

When the feedback links are confidential, the feedbacked indices cannot be heard by any party other than the eavesdropper. In this case, we assume that, the cellular network quantizes the main channel power gain, i.e., $h_{BC}$, independent of the index feedbacked by the RD2D. Knowing that the index $m$ is feedbacked by CU, from the designed code book $C_{BC}$, BS chooses transmit power level $p_{BC}(m)$ to send its information. In other word, it chooses the tuple $(p_{BC}(m), r_{BC}(m), r_{BC}^{S}(m))$ where $r_{BC}(m) = \log(1 + \tilde{h}_{BC}(m)p_{BC}(m))$ is the transmission rate over BS to CU. In this case, in the absence of D2D transmission, the capacity of the link between BS and CU and its corresponding secrecy capacity are, respectively, given by

$$C^C(m) = \log(1 + \tilde{h}_{BC}p_{BC}(m)),$$  \hspace{1cm} (1)

$$C^S_S(m) = \left[ \log(1 + \tilde{h}_{BC}p_{BC}(m)) - \log(1 + \tilde{h}_{BE}p_{BC}(m)) \right]^{+},$$  \hspace{1cm} (2)

while knowing indices $n$, the D2D pair chooses transmit power level $p_{DD}(n)$ to send its information. In other word, the D2D pair has chosen $(p_{DD}(n), r_{DD}(n))$ for concurrent transmission with cellular network and the capacity of the link between BS and CU and its corresponding secrecy capacity are, respectively, given by

$$C^C(m, n) = \log(1 + \tilde{h}_{BC}p_{BC}(m)),$$  \hspace{1cm} (3)

$$C^S_S(m, n) = \left[ \log(1 + \tilde{h}_{BC}p_{BC}(m)) - \log(1 + \tilde{h}_{BE}p_{BC}(m)) \right]^{+},$$  \hspace{1cm} (4)

where $\tilde{h}_{BC} = \frac{h_{BC}}{1 + h_{BC}p_{DD}(n)}$ and $\tilde{h}_{BE} = \frac{h_{BE}}{1 + h_{BE}p_{DD}(n)}$ are the effective channel gains between BS and CU and between BS and eavesdropper, respectively. Note that, the transmission capacity in (3) and the secrecy capacity in (4) can be achieved only if we have full knowledge of CSIs, i.e., the perfect values of $\tilde{h}_{BC}$ and $\tilde{h}_{BE}$.

On the other hand, given that the index $n$ is feedbacked by RD2D, from the designed code book $C_{DD}$, TD2D chooses the tuple $(p_{DD}(n), r_{DD}(n))$ where $r_{DD}(n) = \log(1 + \tilde{h}_{DD}(n)p_{DD}(n))$. In this case, in the absence of cellular transmission, the capacity of D2D link is given by

$$C^D(n) = \log(1 + \tilde{h}_{DD}p_{DD}(n)),$$  \hspace{1cm} (5)

while given that BS has chosen the tuple $(p_{BC}(m), r_{BC}(m), r_{BC}^{S}(m))$ for concurrent transmission with D2D pair, the capacity of the D2D link is given by

$$C^D(m, n) = \log(1 + \tilde{h}_{DD}p_{DD}(n)),$$  \hspace{1cm} (6)

where $\tilde{h}_{DD} = \frac{h_{DD}}{1 + h_{DD}p_{DD}(n)}$ is the effective channel gain between TD2D and RD2D. The perfect value of $\tilde{h}_{DD}$ is needed to achieve the transmission capacity in (6).

B. Outage Events

In our model, there are two types of outage, namely reliability outage which corresponds to the case where the transmission rate exceeds the channel capacity and secrecy outage whose definition depends on the availability of CSI at the transmitter. More precisely, consider the case where CU feedbacks the index $m$, i.e., BS chooses the tuple $(p_{BC}(m), r_{BC}(m), r_{BC}^{S}(m))$. In the absence of D2D transmission, reliability outage for cellular communication occurs if $r_{BC}(m) > C^C_S(m)$ where $C^C_S(m)$ is given by (1). This event corresponds to the case where $\tilde{h}_{BC}(m) > h_{BC}$ which never occurs. In addition, the secrecy outage occurs if $r_{BC}(m) > C^S_S(m)$ where $C^S_S(m)$ is given by (2). In this paper, however, we are interested in the outage event in the presence of D2D pair communication. In this case, reliability outage for cellular communication occurs if $r_{BC}(m) > C^C_S(m, n)$ where $C^C_S(m, n)$ is given by (3). This event corresponds to the case where $\tilde{h}_{BC}(m) > h_{BC}$ which is possible. However, as we assume that only the knowledge of direct channels is available, the secrecy outage does not correspond to the event $r_{BC}(m) > C^S_S(m, n)$ where $C^S_S(m, n)$ is given by (4). Note that, given that the indices $m$ and $n$ are feedbacked, the transmission rate is fixed to $r_{BC}(m) = \log(1 + \tilde{h}_{BC}(m)p_{BC}(m))$. In this case, any secrecy rate given by $r_{DD}(m) \leq \left( r_{BC}(m) - r_{DD}(m) \right)$ is achievable where $r_{DD}(m)$ is the maximum allowable equivocation rate of the eavesdropper. Now, assume for the feedbacked index $m$, the secrecy rate is fixed to $r_{BC}^{S}(m)$ and hence we have $r_{DD}(m) = \left( r_{BC}(m) - r_{BC}^{S}(m) \right)$. Therefore, given that D2D pair chooses $(p_{DD}(n), r_{DD}(n))$, the outage probability for cellular communication using tuple $(p_{BC}(m), r_{BC}(m), r_{BC}^{S}(m))$ is given by

$$P_{outage}^{BC}(m, n, p_{DD}(n), r_{DD}(n), r_{BC}(m), r_{BC}^{S}(m)) = 1 - P_{success}^{BC}(m, n, p_{DD}(n), r_{DD}(n), r_{BC}(m), r_{BC}^{S}(m)),$$  \hspace{1cm} (7)

where

$$P_{success}^{BC}(m, n, p_{DD}(n), r_{DD}(n), r_{BC}(m), r_{BC}^{S}(m)) = \Pr \left( r_{BC}(m) \leq C^C_S(m, n), C^S_S(m, n) \leq r_{BC}(m) - r_{BC}(m) \right),$$  \hspace{1cm} (8)

and we assumed that $h_{BC} \in \bar{h}_{BC}(m), \bar{h}_{BC}(m + 1)$ and $h_{DD} \in \bar{h}_{DD}(n), \bar{h}_{DD}(n + 1)$.

Using the above explanations and defining $R_{BC}^D = \frac{h_{BC}}{h_{BC} + h_{DD}}$ and $R_{DD}^D = \frac{h_{DD}}{h_{BC} + h_{DD}}$, the outage probability for cellular communication when it uses the tuple $(p_{BC}(m), r_{BC}(m), r_{BC}^{S}(m))$ is given by

$$P_{outage}^{BC}(m, n, p_{DD}(n), r_{BC}(m), r_{BC}^{S}(m)) = \frac{N - 1}{N} \sum_{n=1}^{N-1} \Pr \left( h_{DD} \in R_{DD}^D \right) R_{BC}^D \Pr \left( h_{BC} \in R_{BC}^D \right) P_{outage}^{BC}(m, n, p_{DD}(n), r_{BC}(m), r_{BC}^{S}(m)),$$  \hspace{1cm} (9)

and the outage probability of cellular link code book, i.e., $C_{BC}$, is given by

$$P_{outage}^{BC} = \sum_{m=1}^{M-1} \Pr \left( h_{BC} \in R_{BC}^D \right) P_{outage}^{BC}(m, n, p_{DD}(n), r_{BC}(m), r_{BC}^{S}(m)).$$  \hspace{1cm} (10)
Details on obtaining the above probabilities are deferred to Appendix A.

Similarly, for the D2D pair, assume that the transmitter chooses the pair \((p_{DD}(n), r_{DD}(n))\). In the absence of cellular transmission, reliability outage occurs if \(r_{DD}(n) > c_C(m)\) where \(c_C(m)\) is given by (5). This event corresponds to the case where \(h_{DD}(n) > h_{DD}\) which never occurs. On the other hand, in the presence of cellular communication, reliability outage for D2D communication occurs if \(r_{DD}(n) > \tilde{c}_D(m, n)\) where \(\tilde{c}_D(m, n)\) is given by (6). This event corresponds to the case where \(\tilde{h}_{DD}(n) > \tilde{h}_{DD}\) which is possible. Note that, as we assumed the malicious user is not interested in D2D communication, only reliable transmission is considered for D2D pair and no secrecy rate is defined.

### C. Transmit Powers and Achievable Rates

As we assumed, the transmitters only know the region number in which the channel power gains of direct channels lay. This means that it is impossible to know the value of effective channel gains when a concurrent transmission is running. Therefore, for cellular network the value of direct channel gain, i.e., \(h_{BC}\), and for D2D pair, the value of \(h_{DD}\) are quantized. Given that \(h_{BC}\) lies in the region \(R_{BC}^m\), BS chooses tuple \((p_{BC}(m), r_{BC}(m), r_S^{BC}(m))\). Note that, regardless of the channel power gains of other links, i.e., \(h_{BD}, h_{DD}, h_{BC}, h_{BE}\), and \(h_{DE}\), BS transmits with power level \(p_{BC}(m)\). Therefore, in this case, the average transmission power of BS only depends on \(h_{BC}\) and is given by

\[
\bar{P}_C = \sum_{m=1}^{M-1} \Pr(h_{BC} \in R_{BC}^m) p_{BC}(m),
\]

which is the same for both cases where D2D pair is not transmitting or concurrently transmits information. Similarly, given that \(h_{DD}\) lies in the region \(R_{DD}^n\), TD2D chooses the pair \((p_{DD}(n), r_{DD}(n))\). As we assumed that the feedback link of cellular network is confidential, the transmission power of D2D pair, i.e., \(p_{DD}(n)\), only depends on \(h_{DD}\). Therefore, in this case, the average transmission power of D2D pair is given by

\[
\bar{P}_D = \sum_{n=1}^{N-1} \Pr(h_{DD} \in R_{DD}^n) p_{DD}(n),
\]

which does not depend on whether BS is transmitting concurrently or not.

In addition, for cellular communication, the transmission is assumed successful if no outage occurs, i.e., we have both the reliable and secure communications. We define the average achievable secrecy rate for cellular communication as the adopted secrecy rate, i.e., \(r_S^{BC}(m)\), times the probability of success, i.e., no outage occurs, summed over all regions. When the D2D pair is absent, the average achievable secrecy rate is given by

\[
\bar{R}_S = \sum_{m=1}^{M-1} \Pr(h_{BC} \in R_{BC}^m, r_S^{BC}(m) \leq C_S(m)) r_S^{BC}(m),
\]

where \(C_S(m)\) is given by (2). Note that, it is not required to include the term \(r_{BC}(m) \leq C(m)\) in (3) because it is always satisfied. Please refer to Appendix B for more details on obtaining the probability terms in (13).

Since, we need only reliable transmission for D2D communication, the average achievable rate of D2D pair is defined as the adopted data rate, i.e., \(r_{DD}(n)\), times the probability of succeed, i.e., no outage occurs, summed over all region. The average transmission rate which is achievable by D2D pair in the presence of cellular communication is given by

\[
\bar{R}_D = \sum_{m=1}^{M-1} \sum_{n=1}^{N-1} \Pr(h_{BC} \in R_{BC}^m, h_{DD} \in R_{DD}^n, r_{DD}(n) \leq \tilde{c}_D(m, n)) r_{DD}(n),
\]

where \(\tilde{c}_D(m, n)\) is given by (6). Details on obtaining probability terms in (14) can be found in Appendix C.

### IV. Limited Rate Feedback Resource Allocation Problem

#### A. Problem Formulation

In this paper, we assume that D2D pair opportunistically uses the cellular network resources to maximize its average transmission rate. Note that, generally, D2D communication is underlay to cellular communication which means the later one is superior and should be protected against the side effects of concurrent transmission of D2D pair. There are several approaches to achieve this. One approach is to limit the amount of interference that D2D pair produces on the cellular receiver. Such approach can be seen exactly the same as the notion of interference temperature in cognitive radio networks [13]. However, note that this approach is effective when it is used in its instantaneous form (i.e., the exact amount of interference D2D pair produces) and not the averaged one (i.e., the average amount of interference D2D pair produces). However, since we only know the direct channel power gains using limited rate feedback, applying instantaneous interference constraint is not possible. Another approach is to maintain the average achievable rate of the cellular link above a predefined threshold [10], [11]. Moreover, the reliability of the cellular network is much of our concern, particularly, in the case that the resource is shared with D2D links. To this end, outage based approach is the next approach in which the outage probability for cellular communication is kept below a predefined threshold [11], [12].

In this paper, we combine the last two approaches. More precisely, our objective is to maximize the average achievable data rate for D2D pair in the presence of cellular communication, i.e., (14), while it is required to maintain a minimum amount of the average achievable data rate of cellular link in the absence of D2D communication, i.e., (13), and the outage probability for cellular communication in the presence of D2D communication, i.e., (10), is kept below a predefined threshold. In this way, we take into account the performance of cellular communication both in the absence and presence of D2D communication. Indeed, by doing so, we require that the average transmission rate of cellular link in the absence of D2D pair to stay above a predefined threshold while its performance in the presence of D2D communication, which is given by the outage probability, remains as satisfactory as is required. In addition, the average transmit power of the cellular link and D2D pair, which are, respectively, given by (11) and (12), should not exceed a predefined value. Mathematically,
defining \( A = \left\{ \mathbf{h}^{BC}, \mathbf{h}^{DD}, \mathbf{p}^{BC}, \mathbf{p}^{DD}, \mathbf{r}_S^{BC} \right\} \), we aim to solve the optimization problem which is given by
\[
\max_d \sum_{m=1}^{M-1} \sum_{n=1}^{N-1} \Pr \left( h^{BC} \in \mathcal{R}_m^{BC}, h^{DD} \in \mathcal{R}_n^{DD}, r^{DD}(n) \leq C^{DD}(m, n) \right),
\]
\[
\text{s.t.:} \sum_{m=1}^{M-1} \sum_{n=1}^{N-1} \Pr \left( h^{BC} \in \mathcal{R}_m^{BC}, r_S^{BC}(m) \leq C_S(m) \right) r_S^{BC}(m) \geq R^{BC}_{\min},
\]
\[
\sum_{m=1}^{M-1} \Pr \left( h^{BC} \in \mathcal{R}_m^{BC}, p^{BC}(m) \leq P^{BC}_{\max} \right),
\]
\[
\sum_{m=1}^{M-1} \Pr \left( h^{DD} \in \mathcal{R}_n^{DD}, p^{DD}(n) \leq P^{DD}_{\max} \right),
\]

This optimization problem is nonlinear and non-convex and it is hard to solve it, hence, we utilize the PSO method which has been used to solve highly non-linear mixed integer optimization problems in various research [27]–[30]. Since PSO is a computational intelligence-based technique and has global search ability, it can converge to the optimal solution and not largely affected by the size and non-linearity of the problem [28].

B. Particle Swarm Optimization Method

In this paper, to solve the optimization problem, we apply relatively new technique, PSO algorithm which is a computational intelligence-based technique. PSO is based on a moment of the swarm which searches to find the best optimal solution by updating generations [26], [30]. This method is not largely affected by the size and nonlinearity of the problem, and can converge to the optimal solution. In PSO algorithm, all particles which are the potential moves, move towards its optimum value. For each iteration all the particles in this swarm are updated by its position and velocity for optimization ability and based on them the aim function for the system is evaluated. PSO starts with the random initialization of swarm of particles in the search space. Then, by adjusting the path of each particle to its own best location and the best particle of the swarm at each step, the global best solution is found. The path of each particle in the search space is adjusted by its velocity, according to moving experience of that particle and other particles in the search space.

In this paper, we consider different particles for each variable, i.e., \( A = \left\{ \mathbf{h}^{BC}, \mathbf{h}^{DD}, \mathbf{p}^{BC}, \mathbf{p}^{DD}, \mathbf{r}_S^{BC} \right\} \), which denote a solution of the problem. The PSO algorithm consists of \( A_i \) as the vector of \( i \)-th particle in \( d \)-dimension, i.e., for \( \{ \mathbf{h}^{BC}, \mathbf{p}^{BC}, \mathbf{r}_S^{BC} \} \), \( d \) is equal to \( M-1 \) and for \( \{ \mathbf{h}^{DD}, \mathbf{p}^{DD} \} \), \( d \) is equal to \( N-1 \) [30].

The position and the velocity of the \( i \)-th particle in the \( d \) dimensional search space can be shown as \( X_i = [x_{i,1}, x_{i,2}, \ldots, x_{i,d}]^T \) and \( V_i = [v_{i,1}, v_{i,2}, \ldots, v_{i,d}]^T \), respectively. A best position of each particle is denoted by \( \text{pbest}_i \) and \( \text{gbest} \), the best particle at time \( t \) in the entire swarm. The new velocity of each particle can be obtained as follows [30]:
\[
v_{i,j}(t + 1) = w v_{i,j}(t) + c_1 p_{i,j} - x_{i,j}(t) + c_2 r_2 (p_g - x_{i,j}(t)), \quad j = 1, \ldots, d,
\]
where \( c_1 \) and \( c_2 \) are constants called acceleration coefficients, \( w \) is the inertia factor, \( r_1 \) and \( r_2 \) are two independent random numbers uniformly distributed in \([0, 1]\). Thus, the position of each particle is updated in each step as follows:
\[
x_{i,j}(t + 1) = x_{i,j}(t) + v_{i,j}(t + 1).
\]

The standard form of PSO uses (16) to calculate the new velocity of each particle based on its previous velocity and the distance of its current position from both its best position and global best position. To control search of particles outside the search space \([X_{i,min}, X_{i,max}]\), we can limit the value of \( V_i \) to the range \([V_{min}, V_{max}]\) and according to (17), each particle moves to a new position. The process is repeated until a stopping criterion is satisfied. This algorithm is summarized in Table I [26].

V. PRACTICAL CONSIDERATION

A. CDI Estimation Error

The most practical assumption made in this paper is that the instantaneous channel power gains of the eavesdropper’s links, i.e., \( h^{BE} \) and \( h^{DE} \), are not available which is mostly due to the fact that the eavesdropper is passive and hence acquiring its channel power gains are not possible. Generally, the CDI of a channel depends on the environmental property of the communication channel. If the propagation environment is known, one can assume that the channel CDIs, including those of the eavesdropper, are available. The statistical property of the signal propagation in the coverage area of the network can be easily obtained as the legitimate users are present and can be involved in finding the required statistical properties. Since for small geographical areas, a unified distribution can be applied to all channels\(^1\), we can have the CDIs of eavesdropper’s links at hand.

Due to the availability of limited statistical data, the distribution function is hard to drive and cannot be fit into the known ones, e.g., Rayleigh distribution. In such cases, schemes developed based on the availability of the perfect CDI may exhibit performance worse than that expected. Therefore, the imperfection of CDIs should be taken into account. Generally, such consideration can be performed by assuming that the true distribution differs from the nominal distribution by the value known as Kullback–Leibler distance [39] and incorporate such inaccuracy into problem formulation [40]–[42]. We investigate imperfect CDI through two parametric and nonparametric methods.

\(^1\)This assumption is reasonable when the size of the area under investigation is small which is the case for nowadays cellular networks specially for small cells.
Initializaiton:

Step 1) Max It: Iteration number of PSO algorithm
nPop: Number of particles of PSO algorithm
For each variable of $A = \{-h_{BC}, -h_{DD}, p_{BC}, p_{DD}, r_s\}$:

\[
X_i: \text{Position of one particle, } i = 1, 2, \ldots, n Pop
\]

\[
V_i: \text{Velocity of one particle, } i = 1, 2, \ldots, n Pop
\]

Step 2) Evaluate (15a)-(15e) as a cost for all particles, named $cost_i$:

- Set $pbest_i = X_i$ and $pbest.cost_i = cost_i$
- Set $gbest$ and $gbest.cost$ value equal to the value of the best initial particle.

For $t = 1, 2, \ldots$, Max It
For $i = 1, 2, \ldots, n Pop$

Use (16), (17) to update the velocity and position of particles

Evaluate (15a)-(15e):

If $cost_i > pbest.cost_i$:

- $pbest_i = X_i$ and $pbest.cost_i = cost_i$.
- If $pbest.cost_i > gbest$:

\[
gbest = pbest_i \text{ and } gbest.cost = pbest.cost_i.
\]

end
end

where $L$ is the number of observation vectors. Moreover, each $x_i = x_{i1}, \ldots, x_{ij}, i = 1, \ldots, L$ is a sequence of $j$ data in the vector $x_i$. The kernel density estimate of $f(x)$ given by

\[
\hat{f}_{KDE}(x) = \frac{1}{L} \sum_{i=1}^{L} k_3(x, x_i),
\]

where $k_3(x, x_i)$ is the kernel function which commonly is a Gaussian kernel:

\[
k_3(x, x_i) = \frac{1}{\sqrt{2\pi}\delta} \exp\left(-\frac{\|x - x_i\|^2}{2\delta^2}\right),
\]

where $\delta$ is the smoothing parameter and referred to as the bandwidth. It is set to the median distance of a training point $x_i$ to its nearest neighbor.

2.2. Robust Kernel Density Estimation: In practice, the channel gain samples might include contaminated data, referred to as outlier data, which makes it necessary to use robust density estimation methods such as robust KDE (RKDE). In the presence of the contaminated samples, RKDE can give robustness to contamination of the training sequence and estimate the density. Contaminated data consists of realizations from both a nominal or clean distribution in addition to outlying or anomalous measurements. In an increasing number of applications, data arises from high dimensional or high-throughput systems where the nominal distribution itself may be quite complex and not amenable to parametric modelling. The RKDE has the following form:

\[
\hat{f}_{RKDE}(x) = \sum_{i=1}^{L} \omega_i k_3(x, x_i),
\]

where $k_3(x, x_i)$ is a kernel function and $\omega_i$ are nonnegative weights that sum to one. The RKDE can be implemented based on the iteratively reweighed least square (IRWLS) [44] algorithm in which the main goal is to find the optimal value of $\omega_i$.

B. Noisy Feedback Channel

So far, we assumed that the feedback channels are error free meaning that the received index is the same as the feedbacked one. However, in reality, the feedback channel could be affected by the noise which makes transmitter to select an incorrect code word from the designed code book. Note that, designing limited rate feedback systems with incorporating feedback error is complicated, especially for our scheme with two interfering links. In this paper, to consider the feedback error, we utilize the scheme which is commonly used in the literature [30], [45]–[48]. We consider the memoryless feedback channel which characterized by index transition probabilities $\rho_{m,n'}^C(m, m' = 0, \ldots, M - 1)$ for cellular link which is the probability of receiving index $m$ in BS given the index $m'$ was sent by CU, and $\rho_{n,m'}^D(n, n' = 0, \ldots, N - 1)$ for D2D pair which is the probability of receiving index $n$ in TD2D given the index $n'$ was sent by RD2D. It is assumed $b_M = \log_2(M)$ bits feedback for cellular link and $b_N = \log_2(N)$ bits feedback for D2D pair. Let $m_1 n_1 \cdots m_b m_1' n_1' \cdots m_b' n_b'$ and $n_1 n_2 \cdots n_b$ and $n_1' n_2' \cdots n_b'$ indicate the binary display of indices $m, m', n,$ and $n'$, respectively. We assume that the cellular and D2D pair’s feedback channel can be considered as, respectively, $b_M$ and $b_N$ independent use of binary symmetric channel (BSC) to sent each of the feedback bits presented.
in binary representations of cellular link and D2D pair’s feedbacked indices. Let \( q^C \) and \( q^D \) represent the cross over probabilities of the feedback channels of cellular link and D2D pair, respectively. The index transition probabilities of the feedback channels of cellular link and D2D pair can be obtained, respectively, by

\[
\rho^C_{m,m'} = (q^C)_m (1 - q^C)_m (1 - d_{m,m'}),
\]

\[
\rho^D_{n,n'} = (q^D)_n (1 - q^D)_n (1 - d_{n,n'}),
\]

where \( d_{m,m'} \) and \( d_{n,n'} \) denote the Hamming distances between, indices \( m \) and \( m' \) and indices \( n \) and \( n' \), respectively [45]–[47].

With the above definitions and assumptions, the average transmission powers in (11) and (12), average transmission data rates in (13) and (14), and the outage probabilities in (7), (8), (9), and (10) should be manipulated to incorporate the effect of noisy feedback channel. Note that, choosing the transmit power level from a code book only depends on the corresponding channel region index which is feedbacked by the respective transmitter. This mean that, in (11), we should only consider the noise effect of the feedback channel of cellular link, and in (12), we should only consider the noise effect of the feedback channel of D2D pair. Therefore, the average transmit powers of cellular link and D2D pair, when noisy channel feedback is assumed, are given, respectively, by

\[
\bar{p}^C = \sum_{m=1}^{M-1} \sum_{m'=1}^{M-1} p_{m,m'} \Pr \left( h^{BC} \in \mathcal{R}_{m,m}' \right) p^{BC}(m),
\]

\[
\bar{p}^D = \sum_{n=1}^{N-1} \sum_{n'=1}^{N-1} p_{n,n'} \Pr \left( h^{DD} \in \mathcal{R}_{n,n}' \right) p^{DD}(n).
\]

For the average transmission data rate in (13), we assumed the D2D pair is absent, hence, it is not affected by the noise in feedback channel of D2D pair. Therefore, the average transmission data rate can be written as

\[
\bar{R}^C = \sum_{m=1}^{M-1} \sum_{m'=1}^{M-1} \Pr \left( h^{BC} \in \mathcal{R}_{m,m}' \right) r^{BC}(m),
\]

\[
, C^{BE}(m) \leq r^{S}(m) \right) r^{SC}(m),
\]

where \( r^{S}(m) = r^{BC}(m) - r^{BC}(m) \) and \( C^{BE}(m) = \log(1 + h^{BE} p^{BC}(m)) \) where we note that actually, the value of \( C^{BE}(m) \) does not depend on \( m' \). In (26), \( m' \rightarrow m \) is the event that the feedbacked index \( m' \) is received as \( m \). Here, we highlight that, in (26), \( C^{C}(m/m') \) means that its value is given by (1) with \( h^{BC} \in \mathcal{R}_{m,m}' \) and \( p^{BC}(m) \). Note that, in contrast to (13), we must include \( r^{BC}(m) \leq C^{C}(m/m') \) in (26) because reliability outage can occur when the feedback is noisy.

From (1), we know that the event \( r^{BC}(m) \leq C^{C}(m/m') \) occurs when \( m \leq m' \). Therefore, (26) can be rewritten as follows:

\[
\bar{R}^{C} = \sum_{m=1}^{M-1} \sum_{m'=1}^{M-1} p_{m,m'} \Pr \left( h^{BC} \in \mathcal{R}_{m,m}' \right) r^{BC}(m).
\]

The remaining steps are similar to those in obtaining probability terms in (13) in Appendix B, and hence omitted. However, as we assumed in (14) that both the cellular link and D2D pair transmit simultaneously, the cross effect of noisy feedback channel should be considered. In other words, given that the transmitted index \( m' \) was received as \( m \) by BS and the transmitted index \( m' \) was received as \( n \) by transmitter of D2D pair, the average data rate of D2D pair in the presence of cellular communication with noisy feedback channels is given by

\[
\bar{R}^D = \sum_{m=1}^{M-1} \sum_{n=1}^{N-1} \Pr \left( h^{BC} \in \mathcal{R}_{m,m}', h^{DD} \in \mathcal{R}_{n,n}' \right) r^{DD}(n).
\]

Note that, in (28), the value of \( \hat{C}^{D}(m,n/m') \) does not depend on \( m' \). In addition, the effect of noise in feedback channel of cellular link on the value of \( \hat{C}^{D}(m,n/m') \) appears through the choice of transmit power level \( p^{BC}(m) \) which affects the value of the effective channel gain \( \hat{h}^{DD} = \frac{1}{1 + h^{BD} p^{BD}(n)} \) with \( h^{DD} \in \mathcal{R}_{n,n}' \). Obtaining probability terms in (28) is similar to obtaining probability terms in (14) in Appendix C, and hence omitted.

Like (28), for the outage probabilities in (7), (8), (9), and (10), we should consider the cross effect of noisy feedback channels. If we consider the noisy feedback channel effect in the outage probability of cellular communication in the presence of D2D pair, we observe that given the feedback indices \( m' \) and \( n' \) were received by the corresponding receiver as \( m \) and \( n \), respectively, BS uses the code word \( (p^{BC}(m), r^{BC}(m), r^{BC}(m)) \) while we have \( h^{BC} \in \mathcal{R}_{m,n}' \) and the transmitter of D2D pair uses the code word \( p^{DD}(n), r^{DD}(n) \) while we have \( h^{DD} \in \mathcal{R}_{n,n}' \). In this case, the outage probability is given by

\[
P_{\text{outage}}(m/m', n) = \frac{P_{\text{BC}}(m/m', n)}{p^{BC}(m), r^{BC}(m), r^{BC}(m), p^{DD}(n), r^{DD}(n)} = 1 - P_{\text{success}}(m/m', n) p^{BC}(m), r^{BC}(m), r^{BC}(m), p^{DD}(n), r^{DD}(n),
\]

where

\[
P_{\text{success}}(m/m', n) = \Pr \left( \hat{r}^{BC}(m) \leq \hat{C}(m/m', n), \hat{C}^{BE}(m, n) \leq r^{BC}(m) \right),
\]

\[
\hat{C}(m/m', n) \text{ is given by (3) with } \hat{h}^{BC} = \frac{P_{\text{outage}}(m/m', n)}{1 + h^{BD} p^{BD}(n)} \text{ and } h^{BC} \in \mathcal{R}_{m,n}', \text{ and } \hat{C}^{BE}(m, n) = \log \left( 1 + \hat{h}^{BE} P^{BC}(m) \right) \text{ with } \hat{h}^{BE} = \frac{1}{1 + h^{BD} p^{BD}(n)}. \text{ To obtain (30) one can follow the similar steps as those for (8) in Appendix A.}
\]

Using the above explanations, the outage probability for cellular communication when it uses the tuple \( (p^{BC}(m), r^{BC}(m), r^{BC}(m)) \) and under noisy feedback channel model, is given by

\[
P_{\text{outage}}(m/m', n) = \sum_{n=1}^{N-1} \sum_{n'=1}^{N-1} \Pr \left( h^{DD} \in \mathcal{R}_{n,n}' \right) \times P_{\text{outage}}(m/m', n) p^{BC}(m), r^{BC}(m), r^{BC}(m), p^{DD}(n), r^{DD}(n),
\]

and the outage probability of cellular link code book, i.e., \( C^{BC} \), is given by
To take the noisy feedback channel model into consideration, in the optimization problem (15a), we must use (24), (25), (26), (28), and (32).

VI. SIMULATION RESULTS

In this section, numerical results are presented to evaluate the performance of the proposed limited feedback scheme in a D2D communication through the simulations under various system parameters. The channel gain is an exponential random variable with the probability density function (PDF) given by

\[ f(h) = \frac{1}{\sigma} \exp\left(-\frac{h}{\sigma}\right), \]

where \(\sigma\) can be used to model the average channel gain as \(\sigma = s\left(\frac{d_0}{d}\right)^\gamma\) where \(d\) is the distance between the transmitter and the receiver, \(d_0\) is the reference distance, \(\gamma\) is the amplitude path-loss exponent, and \(s\) characterizes the shadowing effect. The users are assumed to be uniformly distributed in a cell of radius 100 m. The small-scale channel fading is assumed to be Rayleigh distributed. The path-loss exponent is equal to 4, and the shadowing effect follows a log-normal distribution, i.e., \(10\log_{10}(s) \sim N(0, 8\text{dB})\). System parameters are equal to \(P_{\text{max}}^BC = 10\text{ dB}, P_{\text{max}}^C = 5\text{ dB}, P_{\text{max}}^D = 0.1, P_{\text{Smin}}^C = 0.1\) bps/Hz, \(q^C = q^D = 0.25\). We set the coefficients \(c_1 = c_2 = 1.496\) and \(w = 0.729\) for PSO algorithm and simulated for 1000 iterations.

A. Convergence

Fig. 2 shows the convergence of the algorithm. For the limited feedback scheme, we consider 1, 2, and 3 bits to display the results clearly. To demonstrate the performance of the proposed system, the results are obtained for non-noisy and noisy limited-feedback schemes for both the perfect and imperfect CDI. The PSO method, generally, does not guarantee to achieve global optimum for n-dimensional functions. It is difficult to prove and show mathematically that PSO can guarantee global optima in our problem. However, we have used different searches to show the reliability of the PSO in Fig. 3. As it is shown, with different random initialization of swarm of particles in the different part of problem space, all of the solutions converge to the same point.

B. The Effect of the System Parameters

In Fig. 4 and Fig. 5, the D2D average rate is plotted versus the maximum transmit power of D2D user \(P_{\text{Dmax}}\) and the different number of BC and D2D feedback bits \((M, N)\). In Fig. 4, the D2D average rate is studied for perfect CDI and parametric CDI estimation method as well as noisy feedback. Obviously with increasing \(P_{\text{Dmax}}\), the average rate of D2D increases due to increasing the feasibility set of the resource allocation problem with the relaxation of constraint on the transmit power of D2D user. As we can see, some curves are flattened when the D2D power constraint is increased. This is because the cellular rate constraint becomes the dominant factor in the optimization problem and D2D rate cannot increase with increasing the transmit power. To study the effect of percent error of imperfect CDI, in Fig. 5 the D2D
network increases, the operation of D2D communication is obviously, when the minimum secrecy rate of the cellular communication can withstand slight secrecy outage probability, i.e., the quality of the feedback link degrades, we see the decline in the rate of D2D.

As it is seen in all figures, the increasing number of feedback bits results in the improvement of the D2D performance, and the average rate increases. Also, the results demonstrate that the performances of the limited-feedback scheme without noise have the better performance in comparison with the noisy case.

C. CDI Estimation Error

To check out the effect of CDI estimation on the performance of the system, $\Delta r$ is defined as the percent of the difference between the average rate of D2D obtained based on the perfect and estimated CDI.

Fig. 9 demonstrates $\Delta r$ as a function of the total number of users for different numbers of the nominal data where the number of outlier data is set to $\kappa = 10$. As the figure shows, for small $L$ both KDE and RKDE methods perform very poor. As $L$ grows, the performance of both methods improves, and for $L = 200$, the average rate obtained based on RKDE is very close to that of the perfect CDI case.

In Fig. 10, $\Delta r$ is plotted versus the number of feedback bits for the different number of outlier data $\kappa$ where the number of nominal data is set to $L = 200$. As it is seen, the value of $\Delta r$ is close to zero for RKDE method with $\kappa = 10$. As $\kappa$ grows, $\Delta r$ increases implying the divergence from the actual pdf. It is also observed that the value of $\Delta r$ for KDE is far away from zero and the performance degrades faster compared to that of RKDE as $\kappa$ grows.

VII. Conclusions

In this paper, we studied a limited-feedback radio resource allocation problem for the D2D communication scenario underlying an existing cellular network with the objective of maximizing the D2D average rate subject to average users transmit power limitations, the average secrecy rate and outage probability threshold for the cellular network. Through the PSO algorithm, the appropriate code book for the channel partitioning was designed. In addition, we solved the problem when the feedback channel is noisy. To investigate the effect of the CDI imperfectness on the performance, we applied both the parametric and non-parametric methods. Using simulations, we studied the impact of the system parameters, such as the maximum allowable transmit power of D2D user, the number of feedback bits, and the minimum secrecy rate of cellular network, on the achievable rate of D2D. As it was shown, by more feedback bits, better D2D performance can be achieved.

**Appendix A**

**Finding Outage Probability in (7)**

To compute the outage probability in (7), we should compute the success probability in (8). Note that we have $h_{BC} \in R_{BC} = [\tilde{h}_{BC}(m), \tilde{h}_{BC}(m + 1)]$ and $h_{DD} \in R_{DD} = [\tilde{h}_{DD}(n), \tilde{h}_{DD}(n + 1)]$. The success probability can be written as follows.
Note that in the above equations, we have implicitly assumed that $p^{BC}_m$, $p^{DD}(n)$, and $\kappa$ are independent and hence the product term in (A.1) follows.

To compute (A.1), we need to find PDFs of $\hat{h}^{BC}$ and $\hat{h}^{BE}$. The CDF of $\hat{h}^{BC}$ can be computed as follows:

$$ F_{\hat{h}^{BC}}(x) = \Pr \left( \hat{h}^{BC} \leq x | h^{BC} \in \mathcal{R}_{m}^{BC}, h^{DD} \in \mathcal{R}_{n}^{DD} \right) $$

$$ = \Pr \left( \frac{\hat{h}^{BC}}{1 + h^{BC}p^{DD}(n)} \leq x | h^{BC} \in \mathcal{R}_{m}^{BC}, h^{DD} \in \mathcal{R}_{n}^{DD} \right) $$

$$ = \Pr \left( h^{BC} \leq (1 + h^{BC}p^{DD}(n))x | h^{BC} \in \mathcal{R}_{m}^{BC}, h^{DD} \in \mathcal{R}_{n}^{DD} \right) $$

$$ = \begin{cases} \Pr \left( \hat{h}^{BC} \leq x | h^{BC} \in \mathcal{R}_{m}^{BC}, h^{DD} \in \mathcal{R}_{n}^{DD} \right) & \text{if } n = 0, \\ \Pr \left( \frac{\hat{h}^{BC}}{p^{BC}_{m,n}} \leq \hat{h}^{DC} | h^{BC} \in \mathcal{R}_{m}^{BC}, h^{DD} \in \mathcal{R}_{n}^{DD} \right) & \text{if } n \neq 0. \end{cases} \quad (A.2) $$

In (A.2), for the cases $n = 0$ and $n \neq 0$, respectively, we have the followings:

$$ F_{\hat{h}^{BC}}^{m,n}(x) = \Pr \left( \hat{h}^{BC} \leq x, h^{BC} \in \mathcal{R}_{m}^{BC} \right) $$

$$ = \frac{1}{\Pr (h^{BC} \in \mathcal{R}_{m}^{BC})} \times \int_{\mathcal{R}_{m}^{BC}} f_{h^{BC}}(h^{BC}) dh^{BC}, \text{ if } \bar{h}^{BC}(m) \leq x \leq \bar{h}^{BC}(m+1), \quad (A.3) $$

The CDF of $\hat{h}^{BE}$ is given by

$$ F_{\hat{h}^{BE}}(x) = 1 - \frac{\hat{h}^{BE}}{\hat{h}^{BE} + h^{DD}p^{DD}(n)x} \exp \left(-\frac{x}{\hat{h}^{BE}}\right), \quad \text{if } 0 \leq x < \infty. \quad (A.5) $$

Finally, we will have the following:

$$ P_{succe}^{outage} = \sum_{n=1}^{N-1} \sum_{m=1}^{M-1} \Pr \left( h^{DD} \in [\bar{h}^{DD}(n), \bar{h}^{DD}(n+1)) \right) \times \Pr \left( h^{BC} \in [\hat{h}^{BC}(m), h^{BC}(m+1)] \right) \times P_{succe}^{outage}(m,n), \quad (A.6) $$

**APPENDIX B**

**FINDING SUCCESS PROBABILITY IN (13)**

To obtain the success probability in (13), i.e., $\Pr \left( h^{BC} \in \mathcal{R}_{m}^{BC}, r^{BC}_{m,n} \leq C_{S}^{BC}(m) \right)$, we first define new random variables $x_{m} = 1 + h^{BC}p^{BC}(m)$ and $y_{m} = 1 + h^{BE}p^{BC}(m)$ with

Fig. 9. The percent of difference between the average rate of D2D based on perfect and estimated CDI, $\Delta d/r$, vs. number of feedback bits, for KDE and RKDE, and different number of outlier data, $L$ and $\kappa = 10$.

Fig. 10. The percent of difference between the average rate of D2D based on perfect and estimated CDI, $\Delta d/r$, vs. number of feedback bits, for KDE and RKDE, and different number of outlier data, $\kappa$ and $L = 200$. 

To compute (A.1), we need to find PDFs of $\hat{h}^{BC}$ and $\hat{h}^{BE}$. The CDF of $\hat{h}^{BC}$ can be computed as follows:
respective distributions \( f_{x_m}(x_m) = \frac{1}{p^m} \mathcal{P} \mathcal{H}(x_m-1) \) and \( f_{y_m}(y_m) = \frac{1}{p^n} \mathcal{P} \mathcal{H}(y_m-1) \). Therefore, we have

\[
\Pr \left( r_S^{BC}(m) \leq C^S(m), h^{BC} \in \mathcal{R}_m^{BC} \right) = \int_{1+h^{BC}(m+1)}^{1+h^{BC}(m+1)} f_{y_m}(y_m) f_{x_m}(x_m) dy_m dx_m.
\]

\[\text{(B.1)}\]

**APPENDIX C**

**Finding Success Probability in (14)**

To obtain the success probability in (14), i.e., \( \Pr \left( h^{BC} \in \mathcal{R}_m^{BC}, h^{DD} \in \mathcal{R}_n^{DD}, r^{DD}(n) \leq \hat{C}^D(n) \right) \), first, we introduce the cumulative distribution function of \( h \), which is equal to \( F_h(x) = 1 - e^{-\frac{x}{\kappa}} \) and

\[
G_m^h = F_h^m(h(m+1)) - F_h^m(h(m)) = e^{-\frac{h(m)}{\kappa}} - e^{-\frac{h(m+1)}{\kappa}},
\]

which is the probability that \( h \) falls into the region of \([h(m), h(m+1)]\). Therefore, for \( h^{BC} \) we have the following:

\[
G_m^{h^{BC}} = F_h^{h^{BC}}(h^{BC}(m+1)) - F_h^{h^{BC}}(h^{BC}(m)) = e^{-\frac{h^{BC}(m+1)}{\kappa}} - e^{-\frac{h^{BC}(m)}{\kappa}}.
\]

\[\text{(C.2)}\]

Then, we have the following:

\[
\Pr \left( h^{BC} \in \mathcal{R}_m^{BC}, h^{DD} \in \mathcal{R}_n^{DD}, r^{DD}(n) \leq \hat{C}^D(n) \right) = G_m^{h^{BC}} \Pr \left( h^{DD} \in \mathcal{R}_n^{DD}, r^{DD}(n) \leq \hat{C}^D(n) \right),
\]

\[\text{(C.3)}\]

and given that \( \hat{h}^{DD} = \frac{h^{DD}}{1+h^{DD}p^{DD}(m)} \), we will have the following:

\[
\Pr \left( h^{DD}(n) \leq \hat{h}^{DD}, h^{DD} \in \mathcal{R}_n^{DD} \right) = \Pr \left( 0 \leq h^{DD} \leq \frac{h^{DD}(n) - 1}{h^{BC}(m)} - h^{DD} \in \mathcal{R}_n^{DD} \right).
\]

\[
\Pr \left( h^{DD}(n+1) \right) \left[ 1 - e^{-\frac{h^{DD}(n+1)}{\kappa p^{BC}(m)}} \right] \frac{1}{\hat{h}^{DD}} e^{-\frac{h^{DD}(n)}{\kappa}} dh^{DD}.
\]

\[\text{(C.4)}\]

Finally, the success probability in (14) is equal to:

\[
\Pr \left( h^{BC} \in \mathcal{R}_m^{BC}, h^{DD} \in \mathcal{R}_n^{DD}, r^{DD}(n) \leq \hat{C}^D(n) \right) = G_m^{h^{BC}} \int_{\hat{h}^{DD}(n)}^{\hat{h}^{DD}(n+1)} \left[ 1 - e^{-\frac{h^{DD}(n+1)}{\kappa p^{BC}(m)}} \right] \frac{1}{\hat{h}^{DD}} e^{-\frac{h^{DD}(n)}{\kappa}} dh^{DD}.
\]

\[\text{(C.5)}\]
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