Efficient Linearized Model of Pressurizer System in Pressurized Water Reactors for Control Purposes

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Abstract. The pressurizer has an important role to ensure the protected operation of pressurized water reactor (PWR) by keeping the reactor coolant system pressure among allowed tolerances. In this paper, a non-equilibrium two-region nonlinear pressurizer (PZR) model was linearized using Taylor technique to introduce the linear models of pressurizer transients for the controller design purposes. Taylor series expansion of functions around an equilibrium point was calculated. The same assumptions of the reference model are considered in linearized model. The linearized model consists of five states; rate of changes for: mass of steam & water in the PZR, mass of water in the PZR, steam temperature in PZR, and water temperature in PZR. The nonlinear and linear PZR models was implemented in MATLAB/Simulink. The linearized model of PZR is verified. The performance of linearized model of PZR and nonlinear model is consistent with small and acceptable errors. The closed-loop of the linearized model of PZR pressure and water level is implemented and investigated using standard Proportion-Integration-Differentiation (PID) controllers. Simulation results and the evaluation performance indicate that the proposed linear model of PZR is efficient, and it can be used for control purposes.

1. Introduction

The development of new energy policies become a great demand taking into account renewable energy sources and nuclear power that provides energetic security for years. The principle of NPPs operation is based on the process of changing nuclear energy as a result of nuclear fission. PWR is a type of nuclear reactor used to the generate electricity. It quickly grew to become the most widely used reactor in NPPs; with 301 in operation around the world as of 2019 [1]. The PZR has two main responsibilities; the alignment of volume changes of the coolant and preserving the required pressure that can be disturbed by the temperature changes resulted due to the fluctuations occur in power. A pressure airbag is produced using electric heaters to compensate changes of coolant volume and prevents boiling of coolant. Since 1960, great efforts are made to develop high-fidelity mathematical models of the PWR-PZR based on the basic conservation laws of mass, energy and momentum for various kinds of research purposes. These models can be generally, divided into the equilibrium [2] and non-equilibrium pressurizer models. Baek et al. [3] proposed a complicated, non-equilibrium three-region PZR model. It further divides the water region of pressurizer into main water and surge water regions that consists of nearly all the important thermal-hydraulics processes taking place in the three regions. Pini et al. [4] proposed
a non-equilibrium control-oriented model for the PZR dynamics, including a two regions triple-volume sub-model that divides the steam and water regions in PZR into a moving-boundary control volume filled with saturated steam and water and two control volumes with sub-cooled water. Compared with the two- and three-region models, it not solely takes the effects of water droplets and vapor bubbles in the continuous water and vapor regions into consideration, however also uses distinct pressures and temperatures for the upper vapor and lower water regions [5]. Zhong et al [6] proposed multi-region model improved by Wang et al [7], which is capable of calculating the pressure and temperature distributions in PZR with satisfactory accuracy. The PZR water level control acts like a compensator of water loss in the primary coolant (PC) to maintain a steady PZR water level. Mathematical models of water level in the PZR available in the literature are linear input-output transfer function models or complex models that provide a full description of the dynamics of the phenomena occurring in the PZR [8, 9]. The literatures on the development of the linearized or transfer function models of the PZR that used for the conventional controller design are very limited. For this reason, the present study focuses on the development of a linearized model of the PZR for control purposes. The commonly used systems for controlling the PZR water level and pressure in NPPs with VVER (water-water energetic reactor) PWR are P, PI, or PID controllers. All the introduced solutions have a common feature- PID algorithm with different approaches either to tune or to synthesis the PID controller [9-12].

The paper is organized as follows. Presentation of the nonlinear mathematical model for PZR system shown in section 2. Linearization model and transfer functions of PZR system model shown in section 3. The nonlinear model of PZR and the validation and verification of linearized model of PZR shown in section 4. In addition, the implementation and simulation results of pressure and water level of PZR shown in section 4. Finally, conclusions presented in section 5.

### Nomenclature

| Parameter                  | Symbol | Value       | Unit    |
|----------------------------|--------|-------------|---------|
| Reactor                    |        |             |         |
| Reactor power              | Wr     | 13.654×10⁸  | W       |
| Primary circuit            |        |             |         |
| Overall mass in the primary circuit (state) | Mₚc     | 2 × 10⁵      | Kg      |
| Water average temperature (input) | Tₚc    | 281.13      | °C      |
| Inlet mass flow rate (input) | mₚin   | 1.4222      | kg/s    |
| Outlet mass flow rate (disturbance) | mₚout  | 2.11        | kg/s    |
| Hot leg water temperature (input) | Tₚc,HL | 296.13      | °C      |
| Cold leg water temperature | Tₚc,cl | 266.13      | °C      |
| Inlet temperature (disturbance) | Tₚc,i  | 258.85      | °C      |
| Specific heat at 282°C     | cₚ,pc  | 5355        | J/kg/K  |
| Heat transfer coefficient  | kₜ,sg  | 9.5296 × 10⁶ | W/K     |
| Heat loss                  | Wₚc.loss | 2.996 × 10⁷  | W       |
| Water nominal volume       | Vₚc,0  | 242         | m³      |
| Water nominal mass         | Mₚc,0  | 2 × 10⁵      | kg      |
| Differences Tₚc,HL – Tₚc = Tₚc – Tₚc,cl | ∆      | 15          | °C      |
| Pressurizer                 |        |             |         |
| Water temperature (state)  | Tₚc,par | 326.57      | °C      |
| Steam temperature (state)  | Tₛ,pzr | 326.57      | °C      |
| Heating power              | Pₚc,heat | 168        | kW      |
| Water level (output)       | lₚc    | 4.8         | M       |
| Pressure (output)          | pₚc    | 1.23 × 10⁴  | kPa     |
| Water specific heat at 325°C | cₚ,pzr | 6873.1        | J/kg/K  |
| Heat loss                  | Wₚc.loss | 1.6823×10⁵  | W       |
| Water mass (state)         | Mₚc,pzr | 19400       | Kg      |
| Steam mass (state)         | Mₛ,pzr | 10000       | Kg      |
Vessel cross section $A_{pzr}$ 4.52 m$^2$
Vessel volume $V_{pzr,vessel}$ 44 m$^3$
Saturated vapor pressure $p^*, T (T)$ 28884.78 kPa
Coefficients for quadratic approximation $c_0$ 28884.78 kPa
$c_1$ 258.01 kPa/°C
$c_2$ 0.63455 kPa/°C$^2$
Water density $\rho(T)$
Coefficients for quadratic approximation $c_{\rho,0}$ 581.2 kg/m$^3$
$c_{\rho,1}$ 2.98 kg/m$^3$/°C
$c_{\rho,2}$ 0.00848 kg/m$^3$/°C$^2$

2. Nonlinear model for pressurizer unit
In this section, a non-equilibrium two-region PZR model is described as in [8, 9]. This model is based on neglecting the assumptions of mechanical work effect, mass change due to phase transition and pressure dependence of the thermo-dynamical variables. In the two-region model water (liquid) at the lower portion is considered as control volume (a) and the steam (vapor) at the upper portion as control volume (b). Each control volume has uniform thermodynamic properties and temperature. Typical layout of the PZR in PWR and its two-region structure with important thermodynamic variables is shown in figure 1.

![Figure 1](image-url)  
*Figure 1. The location of the PZR in pressurized water reactor and its two-region structure.*

2.1. Mathematical Description
The PZR is divided into two balance volumes (see figure 1) over which mass and energy balances are constructed. The mass flows of the feed water and surge to and from the cold leg are negligible compared to the main flow. These justify that the energy transfer attached to the surges is neglected. The rate change of (water mass inside PZR, steam mass inside PZR, and water mass in primary circuit) based on mass conservation law are estimated in equations (1), (2), and (3) [8, 9].

$$\frac{dM_{w,PZR}}{dt} = m_{PZR} - m_{x,PZR}$$  \hspace{1cm} (1)

$$\frac{dM_{s,PZR}}{dt} = m_{x,PZR}$$  \hspace{1cm} (2)

$$\frac{dM_{w,pc}}{dt} = m_{in} - m_{out} - m_{PZR}$$  \hspace{1cm} (3)

A good description for the rate of the PZR steam temperature is deduced as illustrated in equation (4).
$$\frac{dT_{s,PZR}}{dt} = \frac{1}{c_{t,s,M_{s,PZR}}} \left[ -C_{pp,s} \frac{dP_{s,PZR}}{dt} M_{s,PZR} - h_{s,PZR} m_{s,PZR} + V_{s,PZR} \frac{dP}{dt} + h_{w,PZR} m_{s,PZR} + K_{PZR} (T_{w,PZR} - T_{s,PZR}) - P_{s,loss} \right]$$

(4)

In addition, the rate of PZR water temperature \( \frac{dT_{w,PZR}}{dt} \), extracted as in equation (5)

$$\frac{dT_{w,PZR}}{dt} = \frac{1}{C_{t,w,M_{w,PZR}}} \left[ -C_{pp,w} \frac{dP_{w,PZR}}{dt} M_{w,PZR} - h_{w,PZR} (m_{PZR} - m_{s,PZR}) + h_{w,HL} m_{PZR} - h_{s} m_{s,PZR} + V_{w,PZR} \frac{dP}{dt} + K_{PZR} (T_{w,PZR} - T_{s,PZR}) - P_{s,loss} + P_{w,heat} \right]$$

(5)

The rate of mass change in PZR is caused by temperature change of the primary circuit coolant, the feed and surge water. Mass balance equations for each of the balance volumes are represented as in equations (1) to (3) based on the mass conservation law. Equation (1) is modified by neglecting the value of mass flow rate of steam compared to mass flow rate to PZR in the mass balance equation, and so steam mass flow rate formed as in equation (6).

$$m_{s,PZR} = -m_{PZR} \frac{P(T_{s,PZR})}{P(T_{w,PZR})}$$

(6)

Now, going to represent the mass flow rate of the pressurizer using equations (7).

$$m_{PZR} = \left( \frac{m_{in} - 31 m_{out}}{M_{pc}} \right) V_{pc} V_{pc,c_{1}} (m_{in} T_{pc}) + V_{pc} c_{1} \left( W_{loss,pc} - W_{R} + 6 K_{T,SG} (T_{PC} - T_{SG}) + \right)$$

$$\begin{align*}
& V_{pc} c_{2} (m_{in} - m_{out}) - 2 V_{pc} c_{2} m_{in} T_{pc} + 2 V_{pc} c_{2} (m_{in} - m_{out}) T_{PC} + \\
& 2 V_{pc} c_{2} (W_{loss,pc} + 6 K_{T,SG} (T_{PC} - T_{SG}) - W_{R} - 30 m_{out} c_{pc}) \right) \frac{M_{pc}}{c_{pc} M_{pc}} * T_{PC} + \right)
& \frac{2 V_{pc} c_{2} (W_{loss,pc} + 6 K_{T,SG} (T_{PC} - T_{SG}) - W_{R} - 30 m_{out} c_{pc}) \right) \frac{M_{pc}}{c_{pc} M_{pc}} * T_{PC,HL} 
\end{align*}$$

(7)

3. Linearization of pressurizer system model

A non-linear dynamic model mentioned before that describes the dynamics of the PZR used. PZR model consists of the rate change of masses (water in the PC, water in the PZR, and steam in the PZR) and the energy balance (energy conservation) contained in the PZR. For the purposes of designing the control system, there is a need for linear model [13].

- Vectors of nonlinear functions: \( \dot{X} = f(X, u, v, t) \) and \( Y = g(X, u, v, t) \).
- State variables: \( X(t) = [M_{pc}, M_{w,PZR}, M_{s,PZR}, T_{s,PZR}, T_{w,PZR}] \).
- Inputs vector: \( u(t) = [m_{in}, T_{PC}, T_{PC,HL}] \).
- Disturbance vector: \( v(t) = [m_{out}, T_{PC}] \).
- Output vector: \( y(t) = [l_{PZR}, P_{PZR}] \).

In order to linearize non-linear dynamics of PZR, Taylor series expansion of functions around an equilibrium point was calculated [8, 9]. The equilibrium point [14] \( \dot{X}_{eq} = f(X_{eq}, u_{eq}, v_{eq}, t) \). Where \( X_{eq} = [M_{pc,eq}, M_{w,PZR,eq}, M_{s,PZR,eq}, T_{s,PZR,eq}, T_{w,PZR,eq}] \), \( u_{eq} = [m_{ineq}, T_{Pceq}, T_{PC,leq}] \), \( v_{eq} = [m_{outeq}, T_{PC,leq}] \). By expanding in Taylor series and by neglecting the high order terms. Finally, linear state-space model of PZR model derived in equations (1), (2), (3), (4), and (5) describe the linearized model which consists of five states; rate of change of (mass of steam in the PZR, mass of water in the PZR, mass of water in the PC, steam temperature in PZR, water temperature in PZR). This after derivation and mathematical treatment concluded in equations (8), (9), (10), (11), and (12). The Rate of change of mass of steam in the PZR is presented in equation 8.
\[
\frac{dM_{w,PC}}{dt} \text{ linearized} = m_{in} - m_{out} - m_{PZR,linearized}
\]

(9)

\[
\frac{dM_{w,PZR}}{dt} \text{ linearized} = \left(1 - \frac{\mathcal{P}(T_{s,PZR})}{\mathcal{P}(T_{w,PZR})}\right) m_{PZR,linearized}
\]

(10)

\[
\frac{dT_{w,PZR}}{dt} \text{ linearized} = -\frac{\psi m_{PZR,eq} + K_{PZR}T_{s,PZR,eq} + P_{loss} - K_{PR}T_{w,PZR,eq}}{C_{p,w}M_{w,PZR,eq}^2} M_{w,PZR} + \frac{K_{PZR}}{C_{p,w}M_{w,PZR,eq}} T_{w,PZR} - \frac{K_{PZR}}{C_{p,w}M_{w,PZR,eq}} T_{S,PZR} + \frac{\psi}{C_{p,w}M_{w,PZR,eq}} m_{PZR,linearized}
\]

(11)

\[
\frac{dT_{s,PZR}}{dt} = \frac{\phi m_{PZR,eq} + K_{PZR}T_{s,PZR,eq} + P_{loss} - K_{PR}T_{w,PZR,eq}}{C_{p,s}M_{s,PZR,eq}^2} M_{s,PZR} + \frac{K_{PZR}}{C_{p,s}M_{s,PZR,eq}} T_{w,PZR} - \frac{K_{PZR}}{C_{p,s}M_{s,PZR,eq}} T_{S,PZR} + \frac{\phi}{C_{p,s}M_{s,PZR,eq}} m_{PZR,linearized}
\]

(12)

Where; the mass flow rate of water of pressurizer is given as:

\[
m_{PZR,linearized} = \left[\begin{array}{c}
2V^o_{pc}c_{z,1}(m_{in} - m_{out}) - 2V^o_{pc}c_{z,2}m_{in}T_{PC,eq} + 2V^o_{pc}c_{z,2}T_{PC,HL,eq} + 4V^o_{pc}c_{z,2}(m_{in} - m_{out})T_{PC,eq} + \\
24V^o_{pc}c_{z,2}K_{LSG} + 6V^o_{pc}c_{z,1}K_{LSG} + 2V^o_{pc}c_{z,2}(W_{loss,pc} - W_R - 6K_{LSG}T_{SG,eq} - 30m_{out}(\epsilon P_C))T_{PC,eq} + \\
M_{PC}^2 + \\
-2V^o_{pc}c_{z,1}T_{PC,HL,eq}m_{out} + 2V^o_{pc}c_{z,2}m_{in}T_{PC,eq} + 2V^o_{pc}c_{z,2}T_{PC,eq} + 2V^o_{pc}c_{z,2}m_{out} + 2V^o_{pc}c_{z,2}T_{PC,eq}T_{PC,HL,eq} + \\
M_{PC}^2 + \\
-2V^o_{pc}c_{z,1}T_{PC,HL,eq}m_{out} + 2V^o_{pc}c_{z,2}m_{in}T_{PC,eq} + 2V^o_{pc}c_{z,2}T_{PC,eq} + 2V^o_{pc}c_{z,2}m_{out} + 2V^o_{pc}c_{z,2}T_{PC,eq}T_{PC,HL,eq} + \\
M_{PC}^2 + \\
-2V^o_{pc}c_{z,1}T_{PC,eq}m_{out} - 2V^o_{pc}c_{z,2}T_{PC,eq} + 2V^o_{pc}c_{z,2}T_{PC,eq} + 2V^o_{pc}c_{z,1}T_{PC,eq} + 2V^o_{pc}c_{z,1}T_{PC,eq}
\end{array}\right] M_{PC} + \\
\frac{-V^o_{pc}c_{z,1}m_{in}m_{out}}{M_{PC}^2} T_{PC,eq} + \left[1 + \frac{2V^o_{pc}c_{z,2}T_{PC,eq} + 2V^o_{pc}c_{z,1}T_{PC,eq}m_{out}}{M_{PC}^2}\right] m_{in} + \frac{2V^o_{pc}c_{z,2}T_{PC,eq} + 2V^o_{pc}c_{z,1}m_{in}}{M_{PC}^2} \times \\
T_{PC,HL} + \left[31 + \frac{-60V^o_{pc}c_{z,2}T_{PC,eq} - 2V^o_{pc}c_{z,2}T_{PC,eq} + 2V^o_{pc}c_{z,1}T_{PC,eq} + 2V^o_{pc}c_{z,1}T_{PC,eq} + 2V^o_{pc}c_{z,1}m_{out}}{M_{PC}^2}\right] m_{out}
\]

\[
\phi = \frac{\mathcal{P}(T_{s,PZR})}{\mathcal{P}(T_{w,PZR}) - \mathcal{P}(T_{w,PZR})}(h_{w,HL} - h_{s,PZR}), \quad \psi = \frac{\mathcal{P}(T_{s,PZR})}{\mathcal{P}(T_{s,PZR}) - \mathcal{P}(T_{w,PZR})}(h_{w,HL} - h_{s,PZR})
\]

The linearized model outputs are the water level and pressure in the PZR given by equations (13) and (14):

\[
l_{PZR} = \frac{V_{PZR}}{A_{PZR}} + l_0 = \frac{1}{A_{PZR}} \left[\frac{M_{PC}}{\mathcal{P}(T_{PC})} - V^o_{pc}\right] + l_0 = \frac{1}{A_{PZR}(c_{z,0} + c_{z,1}T_{PC,eq} + c_{z,2}T^2_{PC,eq})} M_{PC} - \\
\]

\[
l_{PZR} = \frac{V_{PZR}}{A_{PZR}} + l_0 = \frac{1}{A_{PZR}} \left[\frac{M_{PC}}{\mathcal{P}(T_{PC})} - V^o_{pc}\right] + l_0 = \frac{1}{A_{PZR}(c_{z,0} + c_{z,1}T_{PC,eq} + c_{z,2}T^2_{PC,eq})} M_{PC} - \\
\]

5
\[
\frac{(c_v+2c_2 T_{PC\text{eq}})}{A_{PZR}(c_v+ct_{PC\text{eq}}+c_v t_{PC\text{eq}})^2} T_{PC}
\]

(13)

\[
p_{PZR} = p^T (T_{S, PZR}) = (-a_1 + 2a_2 T_{S,PZR,eq}) T_{S, PZR}
\]

(14)

State space form for linear model that given in equations from (8) to (14) is organized and written in matrix form as shown in equation (15) and equation (16).

\[
\begin{bmatrix}
\frac{dM_{PC}}{dt} \\
\frac{dM_{w, PZR}}{dt} \\
\frac{dM_{s, PZR}}{dt} \\
\frac{dP_{PZR}}{dt}
\end{bmatrix} =
\begin{bmatrix}
a_{11} & 0 & 0 & 0 & 0 & a_{12} & b_{11} & b_{12} & b_{13} \\
a_{21} & 0 & 0 & 0 & 0 & a_{22} & b_{21} & b_{22} & b_{23} \\
a_{31} & 0 & 0 & 0 & 0 & a_{32} & b_{31} & b_{32} & b_{33} \\
a_{41} & a_{42} & 0 & a_{44} & a_{45} & a_{54} & b_{41} & b_{42} & b_{43} \\
an_{51} & 0 & a_{53} & a_{54} & a_{55} & a_{56} & b_{51} & b_{52} & b_{53}
\end{bmatrix}
\begin{bmatrix}
M_{PC} \\
M_{w, PZR} \\
M_{s, PZR} \\
T_{P_{PZR}} \\
T_{S, PZR}
\end{bmatrix} +
\begin{bmatrix}
d_{11} & d_{12} & d_{13} & d_{21} & d_{22} & d_{23} & d_{31} & d_{32} & d_{33} & d_{41} & d_{42} & d_{43} & d_{51} & d_{52} & d_{53}
\end{bmatrix}
\begin{bmatrix}
m_{in} \\
T_{P_{PC,HL}} \\
T_{PC,HL}
\end{bmatrix}
\]

(15)

\[
L_{PZR} = 
\begin{bmatrix}
P_{PZR}
\end{bmatrix}
\begin{bmatrix}
M_{PC} \\
M_{w, PZR} \\
M_{s, PZR} \\
T_{P_{PZR}} \\
T_{S, PZR}
\end{bmatrix} +
\begin{bmatrix}
d_{11} & d_{12} & d_{13} & d_{21} & d_{22} & d_{23} & d_{31} & d_{32} & d_{33} & d_{41} & d_{42} & d_{43} & d_{51} & d_{52} & d_{53}
\end{bmatrix}
\begin{bmatrix}
m_{in} \\
T_{P_{PC,HL}} \\
T_{PC,HL}
\end{bmatrix}
\]

(16)

To generate the parameters of Matrices A, B, C, D, and E in equations (15) and (16), the transfer functions of \( I_{PZR} \) with the three inputs \( (m_{in}, T_{PC}, T_{PC,HL}) \) are presented in \( G_{11}, G_{12}, G_{13} \) deduced in equation (17) with pole at \((-2.43 * 10^5)\). In addition, the transfer functions of PZR with the three inputs \( (m_{in}, T_{PC}, T_{PC,HL}) \) are presented in \( G_{21}, G_{22}, \) and \( G_{23} \) with poles at \((0, 0, -2.43 * 10^5, -0.0002)\) using equations (18), (19), and (20) respectively.

\[
G_{11} = \frac{0.0003}{S + 2.43*10^5}, G_{12} = \frac{0.0001 S + 27.87}{S + 2.43*10^5}, \text{and } G_{13} = \frac{7.64 * 10^{-7}}{S + 2.43*10^5}
\]

(17)

\[
G_{21} = \frac{-3.76 * 10^{-6} S^4 + 16.68 S^3 - 2298 S^2 - 0.359 S - 1.784 * 10^{-19}}{S^5 + 2.43 * 10^5 S^4 + 37.96 S^3}
\]

(18)

\[
G_{22} = \frac{-6.075 * 10^{-6} S^4 + 0.0008 S^3 + 1.3 * 10^{-7} S^2 - 9.338 * 10^{-18} S - 2.866 * 10^{-33}}{S^5 + 2.43 * 10^5 S^4 + 37.96 S^3}
\]

(19)

\[
G_{23} = \frac{-5.76 * 10^{-10} S^4 + 7.937 * 10^{-9} S^3 + 1.24 * 10^{-11} S^2 - 8.86 * 10^{-22} S + 4.33 * 10^{-41}}{S^5 + 2.43 * 10^5 S^4 + 37.96 S^3}
\]

(20)

4. Results and discussion

4.1. Transient response of PZR non-linear model

In this section, the implementation and simulation of the nonlinear and linear models for the PZR system are carried out using MATLAB package. The data used in our simulation are given in \([8, 9]\). The transient response of PZR model for irregular four inputs of PZR is simulated. The inlet and outlet mass flow rate of PC hypothesis signal of PZR is assumed in irregular variation as shown at figure 2. Hot Leg temperature and average temperature of the PC are shown at figure 3. In the simulation of irregular changes, the variations in masses flow rate and temperature changes in the input of the PZR are similar.
The simulation results for pressure variations, water level, water temperature, and steam temperature of PZR model for irregular input variations that described in figures 2 and 3 are shown in figures 4, 5, 6 and 7 respectively. Since the pressure inside the PZR shown at figure 4. The water level of PZR shown at figure 5. The water and steam temperature of PZR of nonlinear model shown at figures 6 and 7 respectively.

4.2. Validation of PZR linearized model
The nonlinear model of PZR is linearized with the operating points for control purposes. The state space form of PZR in this work is developed from the linearized model with some assumptions and simplifications, their accuracy and performance are tested and verified against the nonlinear PZR model during inlet flow rate, average temperature and hot-leg temperature of PC transients. For these transients, the PZR operates at steady-state condition with the inlet flow rate keeping at 2 kg/s along 1000s. At the same time, the average temperature and hot-leg temperature of PC keeping at 280 ºC and 300 ºC respectively. In addition, the outlet flow rate keeping at 1 kg/s along 1000s and the inlet temperature of PC keeping at 258.57 ºC. The dynamic responses of pressure and water level of PZR is
obtained using the state space model which are compared and tested with the presented nonlinear model in figures 8 and 9 respectively. The simulation of validation results indicate the performance of the linearized model of PZR is mostly agree well with those of the nonlinear model with small and acceptable errors, demonstrating the accuracy and feasibility of the developed state space for controller design.

4.3. Controller performance assessment and discussion
The block diagram of open loop PZR linearized model used for evaluation of the controller response is shown in figure 10. Figures 11, 13, 15, 17, 19 and 21 shows the impulse response of PZR water level and pressure with inlet flow rate, average temperature, and hot leg temperature of primary circuit respectively with their associated bode plots that are shown at figures 12, 14, 16, 18, 20 and 22 respectively.
Figure 13. Impulse response of PZR level with average temperature of PC.

Figure 14. Bode diagram of PZR level with average temperature of PC.

Figure 15. Impulse response of PZR level with hot leg temperature of PC.

Figure 16. Bode diagram of PZR level with hot leg temperature of PC.

Figure 17. Impulse response of PZR Pressure with inlet flow rate of PC.

Figure 18. Bode diagram of PZR pressure with inlet flow rate of PC.

Figure 19. Impulse response of PZR Pressure with average temperature of PC.

Figure 20. Bode diagram of PZR pressure with average temperature of PC.
Figure 21. Impulse response of PZR pressure with hot leg temperature of PC.

Figure 22. Bode diagram of PZR pressure with hot leg temperature of PC.

The block diagram of the PZR with the PID control of pressure and water level is shown in figure (23). The PID controllers for level and pressure are used to analysis and confirm the performance of proposed linear model of PZR as shown in figures 24 and 25. With the gains of PID controller derivative, integrator, and differentiator respectively are $K_p = 0$, $K_i = 1.89 \times 10^3$, $K_d = 0$ for level and $K_p = 5.39$, $K_i = 0.014$, $K_d = 398.75$ for pressure. It is clear that the transient time of water level is 5s to reach its set point at 6.2 m, on the other hand the pressure stated at 12000 kPa with settling time about 650s.

Figure 23. Block diagram of pressurizer with the controls of pressure and water level.

Figure 24. PID response of water level of PZR.

Figure 25. PID response of pressure of PZR.
5. Conclusion

In this paper, a non-equilibrium two-region pressurizer model for PWR was developed first based on the basic conservation laws of mass and energy for the steam and water in pressurizer. Then using Taylor series expanding, the linearization model derived and the state space model represented. Five states of PZR are developed; also, the transfer functions of pressurizer extracted. The linearized model is verified with the nonlinear model of pressurizer. Based on the developed transfer function models, the closed-loop pressurizer pressure and water level control systems including conventional PID controllers designed. The results show that the effectiveness of the proposed linear model of PZR in control purposes.

6. References

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