Numerical study of edge crack interaction with inclusion and void of an isotropic plate under different loadings by extended finite element method

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Abstract: In the present work Extended Finite Element Method (XFEM) is utilized for interaction study of cracked isotropic plate under different loading conditions. XFEM is a powerful tool to handle fracture mechanics problems where, extra enrichment functions for the crack faces and tip are get utilized. In this present work main focus is interaction study of edge cracked isotropic plate with inclusion and void under tensile, shear, combined (tensile and shear) and exponential loading. Here, MATLAB environment is used to evaluate mixed mode stress intensity factor (MMSIF) of edge cracked isotropic plate when interacted with inclusion and void.

1. Introduction
In many practical applications in spite of using advanced production and operational methods it is very difficult to make defect free component. Materials during casting, drilling, grinding etc. can have various unavoidable defects just as crack, inclusion, void etc. which degrade the material and finally affect the life. So, interaction study of cracked isotropic plate with inclusion and void under different loading conditions for higher safety reliability.

Structures are always subjected to various types of inplane loadings such as tensile, shear, combine (combination of tensile and shear) and exponential. Under the action of such loadings crack propagation behavior is abnormal. Hence the evaluation of crack behavior under such loading conditions is one of the areas of research for higher safety and reliability. The interaction between a crack and inclusion in the material is very important for understanding the fracture behavior and improving the fracture resistance. The crack tip in various engineering materials is affected by the geometry and the stiffness of near second phase inclusions and/or voids. The density and geometry of inclusion, the stiffness ratio between inclusion and voids and materials play a major role to enhance or reduce the stress at the crack tip. Crack-inclusion, interaction studies have great importance to understand the near crack-tip field for the fracture behavior of the material. In this direction Sih [1] proposed energy density based fracture mechanics, where “energy-density factor” ($S$) is measured similar to stress intensity factor ($K$). The fracture toughness of the material is measured from critical value of $S$ and this proposed theory is also well suited for various mixed-mode crack problems.
Chienet et al [2] applied the concept of singularity at the crack tip with conventional FEM. In this present study SIF is calculated for edge cracked plate and compared the accuracy of these results. Sukumar and Prevost [3] presented an analysis of the 2-D crack of an isotropic and biomaterial plate by XFEM. In this present approach discontinuity is modelled by utilizing partition of unity in the standard finite element package. Li and Chen [4] studied the crack inclusion study based on transformation toughening theory for the prediction of stress intensity factors of cracked plate under loading. Sukumar et al [5] utilized XFEM with level set function for modelling discontinuities like hole and inclusion. The local enrichment functions are also incorporated for better accuracy of results. Limtrakarn and Dechaumphai [6] presented crack inclusion study of polycarbonate by adaptive FEM. In the present study SIF $K_I$ and $K_{II}$ are calculated and the accuracy of the present results is compared with results from photoelastic technique. Zhang et al. [7] Studied crack inclusion, interaction by distributing dislocation method with Gauss–Chebyshev quadrature and the result obtained from the present study is compared with the result of XFEM. Sharma [8] implemented XFEM based fracture analysis of a cracked plate with other discontinuities. In this present study, the interaction integral method is used to calculate SIF and confirmed that SIF at the crack tip is reduced due to interaction of hard inclusion. Jiang et al [9] presented XFEM based fracture analysis of a plate with multiple discontinuities and the crack path is observed when interacted with hard and soft inclusions. Lal et al [10] presented stochastic fracture response of edge cracked composite beam under different loadings. In this present study randomness of various parameters is considered to check the severity of individual fracture parameter. The result obtained is well compared with other method in the literature. Khatri and Lal [11] investigated stochastic fracture analysis of the behavior of a hole emerging crack of a plate under in-plane loadings by XFEM. It is observed that the crack length of shorter length is also very sensitive because of very high stress concentration at the crack tip.

From the above literature review, it is observed that researchers are showing great interest in XFEM based fracture mechanics with the interaction of crack and inclusions/voids. In this present work main focus is numerical investigation of various discontinuities like crack, voids, soft and hard inclusions in an isotropic plate with an edge crack through implementing XFEM under different loading (Tensile, shear, combined and exponential).

2. Mathematical Formulation

Modelling of crack growth in finite element method is time-consuming and here it’s necessary to update meshing at every step of crack growth. In XFEM remeshing is not required during crack growth and discontinuities are modelled by enrichment functions. Displacement vector in XFEM framework is given below.

$$u^e(x) = \sum N_i(x)u_i + \sum N_i(x)H_i\alpha_i + \sum \Phi_i^a(x)b_i^{a1} + \sum \Phi_i^b(x)b_i^{a2} + \sum N_i(x)\chi(x)c_i$$ (1)

Where, $u_i$, $\alpha_i$, $b_i^{a1}$, $b_i^{a2}$ and $c_i$ are the conventional degrees of freedom (dofs), and extra dofs is added for crack face, crack tip and for inclusion/void. A body is considered whose area is denoted by $\Omega$ and its outer boundary $\Gamma$ containing a crack and inclusion indicated by $\Gamma_c$, as in Figure 1(a). The body is experiencing uniform volume/ body loads $b$, and the surface load or forces are applied at the boundary $\Gamma_f$. The parameter $\bar{u}$ is the displacement and $\bar{f}$ is the tractions. It is assumed that a surface with crack and inclusion is traction free. Figure 1 (b) represents an isotropic plate with edge crack and inclusion/void under tensile, shear and exponential loadings.

Here, a set of nodes for FEM mesh, enrichment due to fully cut the elements by crack, enrichment for the elements at the crack tip, and enrichment for the inclusion are denoted by $n_e$, $n_c$, $n_t$ and $n_h$ respectively. The total dofs is represented as
\[ \text{dofs} = \text{size}(n) + \text{size}(n_c) + \text{size}(n_t) + \text{size}(n_h) \]

Where, \( n = n_c + n_t \)

The parameters \( H(x) \) and \( \chi(x) \) denote the Heaviside function for the enrichment of crack and inclusion/void which values are +1 or -1. Crack tips asymptotic functions are represented by \( \Phi_{\alpha}^{1} \) and \( \Phi_{\alpha}^{2} \). The displacement function or asymptotic function near the crack tip is represented as:

\[
\Phi_{\alpha}^{1} = \sqrt{r} \sin \left( \frac{\theta}{2} \right), \Phi_{\alpha}^{2} = \sqrt{r} \cos \left( \frac{\theta}{2} \right), \Phi_{\alpha}^{3} = \sqrt{r} \sin \theta \cos \left( \frac{\theta}{2} \right), \Phi_{\alpha}^{4} = \sqrt{r} \sin \theta \cos \left( \frac{\theta}{2} \right)
\] (2a)

**Figure 1.** (a) An arbitrary body with edge crack and inclusion/void, subjected to traction \( \mathbf{t} \) and displacement \( \mathbf{u} \) (b) Edge cracked plate with inclusion under different loading conditions.

The relationship between MMSIF and J-integral for mixed-mode problems in 2D can be represented as,

\[
J = \frac{K_{I}^{2} + K_{II}^{2}}{E_{\text{eff}}} \quad \text{Where,} \quad E_{\text{eff}} = \begin{cases} E & \text{for plane stress} \\ \frac{E}{1-\nu^2} & \text{for plane strain} \end{cases}
\]

(3)

The \( J \) integral for the body with crack is given in equation (4), where \( \Gamma \) area inside contour \( W \) is the strain energy density, \( \delta_{ij} \) is Kronecker delta, \( n_j \) is the outward unit normal to \( \Gamma \) for \( j \)th component, \( \sigma_{ij} \) is stress tensor and \( \mathbf{u}_i \) is displacement field vector. A crack body is denoted by two states and equation 5 represents combination of two states.

\[
J = \int_{\Gamma} \left( W \delta_{ij} - \sigma_{ij} \frac{\partial \mathbf{u}_i}{\partial x_j} \right) n_j d\Gamma
\]

(4)

\[
J^{(1,2)} = \frac{1}{2} \left( \sigma_{ij}^{(1)} + \sigma_{ij}^{(2)} \right) (k_{ij}^{(1)} + k_{ij}^{(2)} \delta_{ij} - \left( \sigma_{ij}^{(1)} + \sigma_{ij}^{(2)} \right) \frac{\partial (\sigma_{ij}^{(1)} + \sigma_{ij}^{(2)})}{\partial x_j} n_j d\Gamma
\]

(5)

On further solving, get

\[
J^{(1,2)} = J^{(1)} + J^{(2)} + \frac{2}{E_{\text{eff}}}(K_{I}^{(1)}K_{I}^{(2)} + K_{II}^{(1)}K_{II}^{(2)})
\]

(6)

Now, from Equations. (3) and (6) get,

\[
J^{(1,2)} = \frac{2}{E_{\text{eff}}}(K_{I}^{(1)}K_{I}^{(2)} + K_{II}^{(1)}K_{II}^{(2)})
\]

(7)
Where, $I_1$ (Mode I) and $I_2$ (Mode II) are interaction integrals. The SIF in XFEM for two states $K_{I_1}^{(1)}$ and $K_{I_2}^{(1)}$ are evaluating by putting $K_{I_1}^{(2)} = 1$ and $K_{I_2}^{(2)} = 0$ and $K_{II}^{(2)} = 1$ in Equation 7, we get

\[
K_{I_1}^{(2)} = \frac{M_{I_1}^{(2)}}{2} E_{eff}, \quad K_{I_2}^{(2)} = \frac{M_{I_2}^{(2)}}{2} E_{eff}
\]

(8)

3. Result and discussion.

An edge crack isotropic plate is considered for the validation study. The geometry of an edge crack isotropic plate under uniaxial tensile loading is shown in Figure 2(a). The geometry dimension of this plate is taken 72 mm x 36 mm (L x W), crack length ratio ($a/W$) = 0.3, 0.4, 0.5, 0.6 (10.8 ≤ $a$ ≤ 21.6), polycarbonate material (PSI) is taken with E = 2.50 GPa, $\nu = 0.38$, tensile stress $\sigma = 1.1$ MPa as taken from [6].

For validation purpose, a standard single edge crack isotropic plate under uniaxial tensile loading is considered. By applying XFEM Mode-1 $K_1$ is determine for $a/W = 0.3, 0.4, 0.5, 0.6$ and compare with analytical result. Here, 2(a) show isotropic plate with edge crack and inclusion/void under tensile loading and Analytically SIF can be calculated as:

\[
K_1 = \sigma \sqrt{(\pi a)} f(\alpha), \text{ Where, } \alpha = \frac{a}{W}
\]

(9)

\[
f(\alpha) = 1.12 - 0.23 \alpha + 10.55\alpha^2 - 21.72 \alpha^3 + 30.39\alpha^4
\]

(10)

A validation graph 2(b) shows, results of present work are very close to the analytical result. Some cases are solved by using this XFEM mixed-mode, numerical solution for an edge crack isotropic plate with inclusion/void under different loadings.

Figure 2. (a) geometry of an edge crack isotropic plate with a single inclusion/void under different loading (b) Validation graph for $K_1$ with respect to $a/W$

The geometry dimensions are taken 36 mm × 72 mm (W × L), crack length ratio $a/W = 0.2, 0.3, 0.4, 0.5, 0.6$ (7.2 ≤ $a$ ≤ 21.6), polycarbonate material (PSI) with E = 2.50 GPa, $\nu = 0.38$. A circular void and hard inclusion is considered with radius 6 mm. The position of a void/inclusion is taken at L/4 and W/4 for all cases as shown in Figure 1(b). The soft inclusion is of Teflon with $E_{soft}/E_{PSI}$ ratio = 0.24 and hard inclusion taken of AZ61 with $E_{hard}/E_{PSI}$ ratio = 18, tensile stress $\sigma = 1.1$ MPa [6] is considered. The shear stress $\tau = 1.1$ MPa considered and in combine stress tensile and shear both are...
subjected on the plate. The exponential loading is considered with \( p = \sigma x e^{cx} \), \( x(0 \leq x \leq W) \) and \( c = 0.02778 \).

**Figure 3.** MMSIF \( K_I \) for an edge crack isotropic plate with a single inclusion/void under different loadings

**Figure 4.** MMSIF \( K_{II} \) for an edge crack isotropic plate with a single inclusion/void under different loadings

The same meshing and elements are taken as above validation problem. MMSIF \( K_I \) and \( K_{II} \) are determined by XFEM method for various crack length ratios \( a/W = 0.2, 0.3, 0.4, 0.5, 0.6 \) as shown in Figure 3 and Figure 4. It is observed that MMSIF \( K_I \) and \( K_{II} \) increase by increasing \( a/W \) under different loadings. It is found that for void MMSIF \( K_I \) is increased up to 10% in exponential loading, around 3 times under shear loading and around 4 times under combine loading as compare to tensile loading. For soft inclusion, the MMSIF \( K_I \) is of the same nature and decreases near to 15%as compared to void under different loading. For hard inclusion, the MMSIF \( K_I \) is ofthe same nature and decreases near to 22% as compared to soft inclusion under different loading, which is due to the strength of the plate increase with the interaction with hard inclusion whereas decreases with the interaction with soft inclusion and void. The SIF \( K_{II} \) is nearly zero for tensile and exponential loadings whereas for shear and combined loadings are higher. It is concluded that maximum SIF \( K_I \) is found for void and
minimum in hard inclusion. So the plate strength is decreased in XFEM fracture behavior for void and it is increasing in XFEM fracture behavior for hard inclusion.

4. Conclusions

In the present work mixed mode SIF isotropic edge cracked plate with various discontinuities like crack, voids, soft and hard inclusions numerical are evaluated using XFEM under different loadings. The following observations are found during this numerical investigation study:

- With the increase of crack length MMSIF increases for voids and/or inclusions under different loadings.
- An isotropic plate with discontinuities like edge crack, void and inclusion are found more sensitive under shear and compound loading.
- The fracture behavior of the plate with a crack and voids/inclusions at near to crack tip is more sensitive.

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