The mean free path of protons and neutrons in isospin-asymmetric nuclear matter

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Abstract

We calculate the mean free path of neutrons and protons in symmetric and asymmetric nuclear matter, based on microscopic in-medium nucleon-nucleon cross sections. The latter are obtained from calculations of the $G$-matrix including relativistic “Dirac” effects. The dependence of the mean free path on energy and isospin asymmetry is discussed. We conclude by suggesting possible ways our microscopic predictions might be helpful in conjunction with studies of rare isotopes.

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1 Introduction

Previously, we reported microscopic predictions of effective nucleon-nucleon (NN) cross sections in isospin symmetric and asymmetric nuclear matter [1]. In asymmetric matter, cross sections become isospin dependent beyond the usual and well-known differences between the basic $np$, $pp$, and $nn$ interactions. They depend upon the total density and the relative proton and neutron concentrations, which implies that the $pp$ and the $nn$ cases will in general be different from each other.

In-medium cross sections are a way to explore the effective NN interaction in a dense and isospin-asymmetric hadronic environment. This environment can be produced in the laboratory via energetic heavy ion collisions (HIC). Transport equations, such as the Boltzmann-Uehling-Uhlenbeck (BUU) equation, describe the evolution of a non-equilibrium gas of strongly interacting hadrons drifting in the presence of the mean field while undergoing two-body collisions. Thus HIC simulations require the knowledge of in-medium two-body cross sections as well as the mean field. In a microscopic approach, both are calculated self-consistently starting from the bare two-nucleon force.

Besides being a crucial part of the input for transport models, in-medium effective cross sections are important in their own right as they allow to establish an immediate connection with the nucleon mean free path, $\lambda$, one of the most fundamental properties characterizing the propagation of nucleons through matter. The mean free path enters the calculation of the nuclear transparency function. The latter is obviously related to the total reaction cross section of a nucleus, which can be used to extract nuclear r.m.s. radii within Glauber-type models [2]. Therefore, microscopic
in-medium isospin-dependent NN cross sections can ultimately help obtain information about the properties of exotic, neutron-rich nuclei. These studies are particularly timely due to the advent of radioactive beams, which allow to explore the unknown regions of proton/neutron rich unstable nuclei.

Applying our microscopic cross sections in calculations of the nucleon mean free path in symmetric and asymmetric nuclear matter is the focal point of this note. Recently, predictions of the mean free path have been obtained from the nucleon optical potential calculated in the relativistic impulse approximation, together with empirical NN scattering amplitudes and the relativistic mean field model [3]. Those were then used to extract in-medium cross sections. Our calculations are microscopic and proceed exactly in the opposite way, namely we obtain \( \lambda \) from the microscopically predicted cross sections. It will be interesting to see if some consistency can be found between the two sets of results.

In the next section, we recall the main aspects of the previously calculated cross sections, which, together with neutron and proton densities, completely determine the mean free path. We then present and discuss our results in Section III. Our conclusions and outlook are summarized in Section IV.

## 2 Effective cross sections and mean free path

Our cross sections are calculated from a \( G \)-matrix which includes all “conventional” medium effects as well as those associated with medium modifications of the nucleon Dirac wavefunction (DBHF effects). We choose the Bonn-B potential [4] as our model for the free-space two-nucleon force. The nuclear matter calculation of Ref. [5] provides, self-consistently with the nuclear equation of state, the single-proton/neutron potentials as well as their parametrizations in terms of effective masses. Those effective masses, together with the appropriate Pauli operator (depending on the type of nucleon involved), are then used in a separate calculation of the in-medium reaction matrix (or \( G \)-matrix) under the desired kinematical conditions. Coulomb effects are not included in the \( pp \) cross sections, which therefore differ from the \( nn \) ones entirely due to the proton and the neutron having different Fermi momenta. In Ref. [1] we found that the degree of sensitivity to the asymmetry in neutron and proton concentrations depends strongly on the region of the energy-density-asymmetry phase space under consideration, and can separate \( pp \) and \( nn \) scatterings under appropriate conditions of density and kinematics.

We recall that the neutron and proton Fermi momenta, \( k_F^n \) and \( k_F^p \), change with increasing neutron fraction according to the relations

\[
k_F^n = k_F (1 + \alpha)^{1/3}
\]

\[
k_F^p = k_F (1 - \alpha)^{1/3},
\]

where \( k_F \) is the average Fermi momentum, and \( \alpha = (\rho_n - \rho_p) / (\rho_n + \rho_p) \).

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Figure 1: Nucleon mean free path in symmetric nuclear matter as a function of the nucleon kinetic energy at an average Fermi momentum of $1.1 f m^{-1}$ (dash) and $1.4 f m^{-1}$ (solid).

In Ref. [1] we calculated the total cross section as

$$
\sigma(q_0, P_{tot}, \rho) = \int \frac{d\sigma}{d\Omega} Q(q_0, P_{tot}, \theta, \rho) d\Omega,
$$

where $\frac{d\sigma}{d\Omega}$ is given by the usual sum of amplitudes squared and phase space factors and $Q$ is the Pauli operator which prevents scattering into occupied states.

The momentum $q_0$ is the two-body c.m. frame momentum, and $P_{tot}$ is the total momentum of the two-nucleon system, which, in the present calculation, is taken to be equal to zero. This amounts to assuming that the c.m. frame of the nucleons and the nuclear matter rest frame coincide, a choice which is different from the one employed in Ref. [1] but consistent with how we calculated the cross sections in Ref. [6].

Notice that the Pauli operator in Eq. (3) is present in addition to Pauli blocking of the (virtual) intermediate states. The latter acts only on the intermediate states by cutting out part of the momentum spectrum during the integration of the scattering equation, whereas Eq. (3) prevents scattering into occupied final states. In the present case, because we are taking $P_{tot}$ to be equal to zero, and we are considering elastic scattering, the presence of $Q$ in Eq. (3) is equivalent to simply setting the cross section to zero whenever the momentum $q_0$ is below the Fermi level, since in such case the scattering is forbidden. In other words, our momenta and energies are defined relative to the bottom of the Fermi sea, and we have in mind a scenario where a nucleon is bound in a nucleus (or, more ideally, nuclear matter) through the mean field. If such nucleon is struck, (for instance, in a $(e, e')$ reaction), it may subsequently scatter from another nucleon.

It was shown by Negele and Yazaki [8] that the nucleon mean free path is related to the imaginary part of the dispersion relation through

$$
\lambda = \frac{1}{2k T},
$$

where $k$ is the Boltzmann constant and $T$ is the temperature.
Figure 2: Behaviour of the mean free path when the cross section is not suppressed by Pauli blocking of the final scattering state. The Fermi momentum is equal to $1.4 \text{fm}^{-1}$.

where $k_i^l$ is the imaginary part of the complex momentum. The mean free path can also be defined, for a proton, as

$$\lambda_p = \frac{1}{\rho_p \sigma_{pp} + \rho_n \sigma_{pn}},$$

with $\rho_n$ and $\rho_p$ the neutron and proton densities in asymmetric matter, equal to $\frac{(k_n^F)^3}{3\pi^2}$ and $\frac{(k_p^F)^3}{3\pi^2}$, respectively. (An analogous definition holds for the neutron.) The above expression represents the length of the unit volume in the phase space defined by the effective scattering area and the number of particles/volume [7]. Notice that we set the appropriate cross section to zero if the nucleon momentum is less than or equal to the Fermi momentum (of that particular nucleon type), since final states (as well as intermediate ones) are Pauli-blocked when calculating the mean free path for the reasons mentioned above. Equivalent considerations in Ref. [1] meant that the angular integration in Eq. (3) was restricted through a condition involving $P_{tot}$ as well.

3 Results and discussion

First, we show the mean free path of nucleons in symmetric matter as a function of the nucleon kinetic energy (calculated as $T = \sqrt{q_0^2 + m^2} - m$), see Fig. 1. The chosen densities correspond to Fermi momenta of $1.1 \text{fm}^{-1}$ and $1.4 \text{fm}^{-1}$ for the dashed and the solid curve, respectively. The density dependence is quite large. Again, in the present approximation, the cross section goes sharply to zero, and thus the mean
free path goes to infinity, for $q_0 \leq k_F$. Thus the lowest energy for which $\lambda$ is finite corresponds to the lowest momentum allowed by Pauli blocking of the final state. Table 1, together with Eq. (5), should facilitate the interpretation of the mean free path behaviour observed in Fig. 1. Reconnecting with the previous discussion which followed Eq. (3), we also show, see Fig. 2, the mean free path calculated without considerations of Pauli blocking of the final states. In this case, $\lambda$ becomes very small at low energy, due to the large values of the cross section in that region.

Back to Fig. 1, and focusing on the higher density first (solid line), we see the sharp drop from infinity at low energy, after which the mean free path slowly decreases with energy, due to the fact that the in-medium cross sections actually start to go up with energy at high densities, see Table 1. This feature, which may appear counterintuitive (being opposite to what is seen in free space), has been reported in other works as well \cite{3,9}. A similar behavior also sets in at the lower density (dashed line), but in that case the mean free path, after the sharp drop from infinity, rises with energy at first (corresponding to a reduction of the in-medium cross section). Notice that the tendency to rise with energy in dense matter appears more pronounced for scattering of identical nucleons, a behaviour which was traced to in-medium enhancement of some isospin-1 partial waves \cite{1}.

We now move to mean free path considerations in asymmetric matter. For scattering of like nucleons, the cross section is set equal to zero when $q_0 \leq k_{iF}^i$ ($i = n, p$), whereas for np scattering it is set to zero for $q_0 \leq k_F$, the average Fermi momentum. The corresponding behavior of the mean free path is shown in Fig. 3 for $\alpha=0.5$ and in Fig. 4 for a greater degree of asymmetry, $\alpha=0.8$. The large differences between the mean free path for protons and neutrons at the lowest energies are to be expected from what we stated above, namely the suppression of $pp$ and $nn$ cross sections is controlled by the (unequal) proton and neutron Fermi momenta. Proceeding from the lowest to the highest energies, the proton mean free path is infinity when both $pp$ and $np$ cross sections are Pauli blocked, followed by the small rise around 25 MeV, and then again the sharp drop when the $np$ cross section starts to contribute. Sim-
Table 1: \(pp\) and \(np\) total effective cross sections in symmetric matter calculated at two densities as a function of the kinetic energy.

| \(k_F (fm^{-1})\) | \(T(q_0)(MeV)\) | \(\sigma_{pp}(mb)\) | \(\sigma_{np}(mb)\) |
|-------------------|----------------|-----------------|-----------------|
| 1.1               | 5.31           | .0000           | .0000           |
|                   | 8.28           | .0000           | .0000           |
|                   | 11.91          | .0000           | .0000           |
|                   | 16.17          | .0000           | .0000           |
|                   | 21.06          | .0000           | .0000           |
|                   | 26.58          | 23.46           | 60.57           |
|                   | 32.71          | 18.00           | 34.39           |
|                   | 39.44          | 16.67           | 26.64           |
|                   | 46.76          | 16.41           | 23.14           |
|                   | 54.66          | 16.63           | 21.44           |
|                   | 63.11          | 17.08           | 20.63           |
|                   | 72.12          | 17.67           | 20.28           |
|                   | 81.65          | 18.31           | 20.17           |
|                   | 102.27         | 19.66           | 20.32           |
|                   | 124.83         | 21.03           | 20.71           |
|                   | 175.34         | 23.79           | 21.94           |
|                   | 232.22         | 26.66           | 23.83           |
|                   | 294.60         | 29.66           | 26.30           |
|                   | 361.68         | 32.63           | 29.08           |
| 1.4               | 5.31           | .0000           | .0000           |
|                   | 8.28           | .0000           | .0000           |
|                   | 11.91          | .0000           | .0000           |
|                   | 16.17          | .0000           | .0000           |
|                   | 21.06          | .0000           | .0000           |
|                   | 26.58          | .0000           | .0000           |
|                   | 32.71          | .0000           | .0000           |
|                   | 39.44          | .0000           | .0000           |
|                   | 46.76          | 13.70           | 17.26           |
|                   | 54.66          | 15.04           | 16.63           |
|                   | 63.11          | 16.31           | 16.77           |
|                   | 72.12          | 17.54           | 17.24           |
|                   | 81.65          | 18.71           | 17.85           |
|                   | 102.27         | 20.86           | 19.22           |
|                   | 124.83         | 22.80           | 20.61           |
|                   | 175.34         | 26.26           | 23.37           |
|                   | 232.22         | 29.47           | 26.28           |
|                   | 294.60         | 32.55           | 29.32           |
|                   | 361.68         | 35.40           | 32.34           |
ilar considerations explain the dashed curve, with the difference that the neutron Fermi momentum is the highest in this case. These effects are of course especially pronounced when $k^n_F$ is much larger than $k^p_F$, see Fig. 4.

In summary, strong variations with energy of the proton and neutron mean free path can be seen, as well as large differences between the two, in a rather narrow region around the Fermi “thresholds” for $pp$ and $nn$ scatterings. As energy increases, however, the mean free path becomes essentially insensitive to isospin asymmetry. This is in agreement with the conclusions of Ref. [3].

Finally, we show in Table 2 some of the in-medium cross sections used for the present calculations of the mean free path in asymmetric matter. The $pp$ and $nn$ cross sections are quite similar to each other except in the low energy region where one may be sizable while the other is still suppressed.

4 Conclusions and future prospects

We presented predictions of the mean free path for protons and neutrons in isospin symmetric or asymmetric matter based on microscopic predictions of in-medium cross sections. The mean free path in exotic matter is a fundamentally important quantity which finds applications in diverse areas including radiobiology.

As it appears reasonable, very low-energy protons and neutrons can have dramatically different propagation properties in strongly asymmetric matter. Our conclusion is that an experimental signature of sensitivity of in-medium scattering to isospin asymmetry may be sought by probing highly asymmetric matter with energies close to the proton and neutron Fermi surfaces. Otherwise, isospin asymmetry has only a very minor impact on the mean free path.

We recall that our baseline calculation of the cross sections is a microscopic one. The assumptions we made in this paper concerning kinematics and sharpness of the Pauli operator simply have the purpose to make the discussion more transparent and can be improved or removed depending on the specific needs of potential users and
Table 2: \( pp, nn, \) and \( np \) total effective cross sections in asymmetric matter under the same conditions as chosen in Fig. 3.

| \( T(q_0)(MeV) \) | \( \sigma_{pp}(mb) \) | \( \sigma_{nn}(mb) \) | \( \sigma_{np}(mb) \) |
|------------------|-----------------|-----------------|-----------------|
| 5.31             | 0.0000          | 0.0000          | 0.0000          |
| 8.28             | 0.0000          | 0.0000          | 0.0000          |
| 11.91            | 0.0000          | 0.0000          | 0.0000          |
| 16.17            | 0.0000          | 0.0000          | 0.0000          |
| 21.06            | 0.0000          | 0.0000          | 0.0000          |
| 26.58            | 15.51           | 0.0000          | 0.0000          |
| 32.71            | 13.80           | 0.0000          | 0.0000          |
| 35.33            | 13.82           | 0.0000          | 0.0000          |
| 38.05            | 14.00           | 0.0000          | 0.0000          |
| 40.86            | 14.27           | 0.0000          | 20.50           |
| 43.77            | 14.62           | 0.0000          | 18.32           |
| 46.76            | 15.01           | 0.0000          | 17.48           |
| 49.85            | 15.45           | 0.0000          | 17.04           |
| 53.03            | 15.90           | 12.52           | 16.83           |
| 56.30            | 16.37           | 13.54           | 16.76           |
| 59.66            | 16.85           | 14.29           | 16.78           |
| 63.11            | 17.33           | 14.92           | 16.87           |
| 81.65            | 19.70           | 17.51           | 17.90           |
| 102.27           | 21.85           | 19.68           | 19.22           |
| 124.83           | 23.79           | 21.61           | 20.57           |
| 175.34           | 27.20           | 25.09           | 23.29           |
| 232.22           | 30.34           | 28.37           | 26.16           |
| 294.60           | 33.32           | 31.53           | 29.19           |
| 361.68           | 36.06           | 34.50           | 32.21           |
the experimental conditions one may wish to simulate.

Through additional steps, which would involve the calculation of the nuclear transparency function (defined as the probability that at some impact parameter the projectile will pass through the target without interacting), the mean free path is closely related to the nuclear reaction cross section. Thus, analyses of reaction cross section data can ultimately shed light on the target density (a much needed information for nuclei with large neutron skin and, thus, hard-to-probe density distributions). On the other hand, a crucial input for the equations written above are the two-body cross sections, for which parametrizations of free-space NN cross sections are often adopted. This may not be reliable, and we suggest that keeping in touch with microscopic predictions can be of help when trying to constrain observables which depend on several (essentially unknown) degrees of freedom.

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