Trans-Planckian quantum corrections and inflationary vacuum fluctuations of non-minimally coupled scalar fields

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In the present paper we discuss how trans-Planckian physics affects the inflationary vacuum fluctuation or the primordial density perturbation and leaves a specific imprint on the cosmic microwave background (CMB). The inflationary trans-Planckian problem has been under debate and these is a variety of conclusions in literature. Here we consider this problem by using the two-point correlation function \( \langle \delta \phi^2 \rangle \) of the non-minimally coupled scalar fields and constructing the effective potential in curved spacetime. We point out that in the UV regime the quantum fluctuation does not drastically change even in de Sitter spacetime and the trans-Planckian corrections can be embedded in effective potential. Thus, the UV sensitivity on the primordial density perturbation is sufficiently sequestered and we only receive a tiny trans-Planckian imprint on the CMB in contrast with previous suggestions. However, the trans-Planckian corrections give a strong impact on the fine-tuning problems of the inflaton potential and also inflationary vacuum fluctuation. We simply obtain a inflationary conjecture \( \Lambda_{UV} \ll H/g^{1/2} \) where \( g \) is the interaction coupling at the UV scale \( \Lambda_{UV} \).

I. INTRODUCTION

Recently an intriguing possibility has been discussed in the literature [1–32] where inflation can provide important clues about ultraviolet (UV) physics or trans-Planckian physics. In the standard paradigm of the inflation, quantum fluctuations are assumed not to be modified up to the infinitely short length. However, it is not realistic because new physics is to be expected below the Planck scale and this simple assumption could not be correct. There are so many discussions about how the UV physics or trans-Planckian physics modify the inflationary fluctuation and leave a specific imprint on the cosmic microwave background (CMB). The inflationary trans-Planckian problem has been under debate and a variety of the literature have reached a range of conclusions.

In the literature [14–17], the trans-Planckian problem has been discussed from the viewpoint of the choice of the initial vacuum. Taking the initial vacuum as \( \alpha \)-vacua defined at the finite time [15, 33, 34], the Bogoliubov coefficients \( \alpha_k \) and \( \beta_k \) are constrained by the following relation

\[
\beta_k = \frac{i e^{-2i k \eta_0}}{2 k \eta_0} + i \alpha_k, \tag{1}
\]

where \( \eta_0 \) is the conformal initial time and \( k \) is the wave mode. Note that the Bunch-Davies vacuum is restored in the infinite past (\( \eta_0 \to -\infty \), \( \alpha_k = 1 \) and \( \beta_k = 0 \)). Initial condition should be imposed when the wavelength crosses to some fundamental length scale. Therefore, the initial condition could be imposed at the \( k \)-dependent initial time \( \eta_0 = -\Lambda_{UV}/Hk \) [14] where \( \Lambda_{UV} \) is the UV cut-off scale and \( H \) is the Hubble constant during inflation. In this condition, the quantum fluctuation \( \langle \delta \phi^2 \rangle \) can be written as follows:

\[
\langle \delta \phi^2 \rangle = \int d^3k |\delta \phi_k (\eta, x)|^2 = \int \frac{dk}{k} P_{\delta \phi} (\eta, k), \tag{2}
\]

where \( P_{\delta \phi} (\eta, k) \) is the power spectrum of the quantum fluctuation given by

\[
P_{\delta \phi} (\eta, k) = \left( \frac{H}{2 \pi} \right)^2 \left( 1 - \frac{H}{\Lambda_{UV}} \sin \left( \frac{2 \Lambda_{UV}}{H} \right) \right), \tag{3}
\]
which shows a optimistic size of the UV corrections as \( \mathcal{O}(H/\Lambda_{UV}) \) and provides a window towards physics beyond the UV cut-off scale, e.g. the Planck scale or the string scale. On the other hand, in the literature [7, 8] based on the effective field theory, the inflationary quantum fluctuation can be given as follows:

\[
\langle \delta \phi^2 \rangle \bigg|_{k \approx H} = H^2 + c_1 H^2 (H^2/\Lambda_{UV}^2) + c_2 H^2 (H^2/\Lambda_{UV}^2)^2 + c_3 H^2 (H^2/\Lambda_{UV}^2)^3 + \cdots ,
\]

where the coefficients \( c_i \) are determined by the cut-off scale \( \Lambda_{UV} \). The above estimates suggest that the contributions from UV physics can not be larger than \( \mathcal{O}(H^2/\Lambda_{UV}^2) \) and have been criticized as being too pessimistic. At least there are two competing estimates of the UV corrections to the inflationary quantum fluctuation or the CMB power spectrum in the literature and no consensus has been reached.

In this paper we discuss how the UV or trans-Planckian corrections modify the inflationary vacuum fluctuation based on the standard analysis of the quantum field theory (QFT) in curved spacetime. Here we deal with this problem by using the two-point correlation function \( \langle \delta \phi^2 \rangle \) of the non-minimally coupled scalar fields which can be strictly calculated under the QFT in curved spacetime. As a consequence, we clearly show that the correction of the UV or trans-Planckian physics can be embedded in effective potential, and the UV sensitivity on the primordial density perturbation is sufficiently sequestered and we only receive a tiny imprint on the CMB correction of the UV or trans-Planckian physics can be a window towards the high-scale physics.

II. THE RENORMALIZATION OF QUANTUM FLUCTUATION

The quantum fluctuation necessarily causes a problem about renormalization. As well-known facts in QFT, the two-point correlation function \( \langle \delta \phi^2 \rangle \) which express the quantum fluctuation have the UV (quadratic and logarithmic) divergences and therefore some regularization or renormalization methods are required. In flat spacetime the quantum fluctuation with the divergences can be eliminated by the bare parameters of the Lagrangian through standard renormalization technique. But in curved spacetime [38] due to the quantum particle creations the representation of the quantum fluctuation and the renormalization has some ambiguity which complicates the trans-Planckian problem [1–32]. In this section, let us consider carefully the renormalization of the quantum fluctuation in de-Sitter spacetime using the adiabatic regularization [45–53] which is a powerful method to remove the divergences. We clearly show that the inflationary fluctuation or quantum particle creation are sequestered from the cut-off sensitivity of the UV or trans-Planckian physics under reasonable assumptions.

Through this paper, we consider a spatially flat Friedmann-Lemaitre-Robertson-Walker (FLRW) spacetime,

\[
ds^2 := -dt^2 + a^2(t) \delta_{ij} dx^i dx^j ,
\]

where \( a(t) \) is the scale factor and the scalar curvature is give as \( R = 6(\dot{a}/a)^2 + 6(\ddot{a}/a) = 6(a''/a^3) \) where \( \eta \) is the conformal time and defined by \( d\eta = dt/a \). The scale factor becomes \( a(t) = e^{H t} \) and the scalar curvature is expressed as \( R = 12H^2 \) in de Sitter spacetime. For simplicity, we consider two nonminimal coupled scalar fields with the interaction coupling \( g \). The bare (unrenormalized) Lagrangian is defined by,

\[
\mathcal{L}[\phi, S] := \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} \left( m_\phi^2 + \xi_\phi R \right) \phi^2 + \frac{\lambda}{4} \phi^4 + \frac{1}{2} g^{\mu\nu} \partial_\mu S \partial_\nu S + \frac{1}{2} \left( m_S^2 + \xi_S R \right) S^2 + \frac{g}{2} \phi^2 S^2 ,
\]

where the inflaton field is \( \phi \), the massive scalar field is \( S \) with \( m_S \simeq \Lambda_{UV} \gg m_\phi \) and \( \xi_\phi, \xi_S \) are the nonminimal curvature couplings. The Klein-Gordon equations for these two scalar field are given as follows:

\[
\Box \phi + m_\phi^2 \phi + \xi_\phi R \phi + \lambda \phi^3 + g S^2 \phi = 0 ,
\]

\[
\Box S + m_S^2 S + \xi_S RS + g \phi^2 S = 0 ,
\]

\footnote{In principle, these problems should be discussed in the framework of quantum gravity (QG) theory. However, owing essentially to the non-renormalizable and non-unitary properties, we do not have even a consistent theory for QG yet [35–37], and furthermore, it is well-known that the primordial density perturbation can be successfully described by a semiclassical approach to gravity. In this sense, our goal will be sufficiently achieved by simply adopting QFT in curved spacetime [38–44] which is a completely self-consistent theory.}
where \(\Box = g^{\mu \nu} \nabla_\mu \nabla_\nu = 1/\sqrt{-g} \partial_\mu (\sqrt{-g} \partial^\mu)\) express generally covariant d’Alembertian operator.

Next we treat the scalar fields \(\phi, S\) as the field operators acting on the ground states and these scalar fields \(\phi, S\) are decomposed into the classical parts and the quantum parts: \(\phi = \bar{\phi} (\eta, x) + \delta \phi (\eta, x)\), \(S = \bar{S} (\eta, x) + \delta S (\eta, x)\) and we assume \(\langle 0 | \delta \phi (\eta, x) | 0 \rangle = \langle 0 | \delta S (\eta, x) | 0 \rangle = 0\). By introducing the renormalized parameters and the counterterms to be \(m^2_\phi = m^2_\phi (\mu) + \delta m^2_\phi, \xi_\phi = \xi_\phi (\mu) + \delta \xi_\phi, \lambda = \lambda (\mu) + \delta \lambda\) and \(g = g (\mu) + \delta g\), the one-loop Klein-Gordon equations of the inflaton field are given by

\[
\begin{align*}
\Box \phi + (m^2_\phi (\mu) + \delta m^2_\phi) \phi &+ (\xi_\phi (\mu) + \delta \xi_\phi) \Box \phi + 3 (\lambda (\mu) + \delta \lambda) \langle \delta \phi^2 \rangle \phi + (\lambda (\mu) + \delta \lambda) \phi^3 + (g (\mu) + \delta g) \langle \delta S^2 \rangle \phi = 0, \\
\Box m^2_\phi + (\xi_\phi (\mu) + \delta \xi_\phi) R \phi + 3 \lambda (\mu) \phi^2 + (g (\mu) + \delta g) S^2 \delta \phi = 0,
\end{align*}
\]

where the first equation shows the dynamics of the average inflaton field whereas the second equation shows the quantum fluctuation of the inflaton field. The quantum field \(\delta \phi\) can be decomposed into each \(k\) modes by

\[
\delta \phi (\eta, x) = \int d^3k \left( a_k \delta \phi_k (\eta, x) + a^*_k \delta \phi^*_k (\eta, x) \right),
\]

where the creation and annihilation operators of \(\delta \phi_k\) are required to satisfy the standard commutation relations \([a_k, a_l] = [a^*_k, a^*_l] = 0\) and \([a_k, a^*_l] = \delta (k - k')\). The in-vacuum state \(|0\rangle\) is defined by \(a_k |0\rangle = 0\) and corresponds to the initial conditions of the mode functions of \(\delta \phi_k\). The quantum fluctuation \(\langle \delta \phi^2 \rangle\) of the inflaton field can be written by

\[
\langle 0 | \delta \phi^2 | 0 \rangle = \int d^3k |\delta \phi_k (\eta, x)|^2 = \frac{1}{2 \pi^2 a^2 (\eta)} \int_0^\infty dk k^2 |\delta \chi_k (\eta)|^2,
\]

where we introduce the rescaled mode functions \(\delta \chi_k (\eta)\) as \(\delta \phi_k (\eta, x) = e^{ik \cdot x} \delta \chi_k (\eta) / (2\pi)^{3/2} a (\eta)\). From Eq. (9), the Klein-Gordon equation for the quantum rescaled field \(\delta \chi\) is given by

\[
\delta \chi''_k (\eta) + \Omega^2_k (\eta) \delta \chi_k (\eta) = 0,
\]

where:

\[
\Omega^2_k (\eta) = k^2 + a^2 (\eta) \left( m^2_\phi + 3 \lambda \phi^2 + (\xi - 1/6) R \right).
\]

Now, we rewrite the rescaled mode function \(\delta \chi (\eta)\) by the Bogoliubov coefficients \(\alpha_k (\eta), \beta_k (\eta)\) as

\[
\delta \chi_k (\eta) = \frac{1}{\sqrt{2 \Omega_k (\eta)}} \left\{ \alpha_k (\eta) \delta \chi_k + \beta_k (\eta) \delta \chi^*_k \right\},
\]

where \(\alpha_k (\eta), \beta_k (\eta)\) satisfy the Wronskian condition: \(|\alpha_k (\eta)|^2 - |\beta_k (\eta)|^2 = 1\). The initial conditions for \(\alpha_k (\eta_0), \beta_k (\eta_0)\) are equivalent to the choice of the in-vacuum state. From Eq. (13) the quantum fluctuation \(\langle \delta \phi^2 \rangle\) can be given by

\[
\langle \delta \phi^2 \rangle = \frac{1}{4 \pi^2 a^2 (\eta)} \int_0^\infty dk k^2 \Omega^{-1}_k \left\{ 1 + 2 |\beta_k|^2 + \alpha_k \beta^*_k \delta \phi^2_k + \alpha^*_k \beta_k \delta \phi^*_2 \right\}.
\]

For convenience, we introduce the following quantities \(n_k = |\beta_k|^2\) and \(z_k = \alpha_k \beta^*_k \delta \phi^2_k\) where \(n_k = |\beta_k (\eta)|^2\) can be interpreted as the particle number density created in curved spacetime. By using \(n_k\) and \(z_k\), we obtain the following expression of the quantum fluctuation of the inflaton field as

\[
\langle \delta \phi^2 \rangle = \langle \delta \phi^2 \rangle^{(q)} + \langle \delta \phi^2 \rangle^{(c)},
\]

where:

\[
\langle \delta \phi^2 \rangle^{(q)} = \frac{1}{4 \pi^2 a^2 (\eta)} \int_0^\infty dk k^2 \Omega^{-1}_k \left\{ 1 + 2 |\beta_k|^2 + \alpha_k \beta^*_k \delta \phi^2_k + \alpha^*_k \beta_k \delta \phi^*_2 \right\},
\]

and

\[
\langle \delta \phi^2 \rangle^{(c)} = \frac{1}{4 \pi^2 a^2 (\eta)} \int_0^\infty dk k^2 \Omega^{-1}_k \{ 2n_k + 2 \text{Re} z_k \},
\]

where \(\langle \delta \phi^2 \rangle^{(c)}\) can be regarded as the classic field fluctuations and expresses finite particle creations in curved spacetime [38], whereas \(\langle \delta \phi^2 \rangle^{(q)}\) obviously diverges as with the flat spacetime. Thus, we regularize the divergences
of \( \langle \delta \phi^2 \rangle^{(q)} \) by the cut-off or dimensional regularization, and cancel them by the counterterms of the couplings. By using the dimensional regularization, we obtain the following expression,

\[
\langle \delta \phi^2 \rangle^{(q)} = \frac{M^2(\phi)}{16\pi^2} \left[ \ln \left( \frac{M^2(\phi)}{\mu^2} \right) - N_\epsilon - \frac{3}{2} \right],
\]

where:

\[
M^2(\phi) = m_\phi^2(\mu) + g(\mu) S^2 + 3\lambda(\mu) \phi^2 + (\xi_\phi(\mu) - 1/6) R,
\]

where \( N_\epsilon = -1/\epsilon - \log 4\pi - \gamma \) which is offset by the coupling counterterms, \( \gamma \) is the Euler-Mascheroni constant and \( \mu \) is the renormalization parameter. From Eq. (16) the quantum fluctuation in de-Sitter spacetime can be written as

\[
\langle \delta \phi^2 \rangle = \langle \delta \phi^2 \rangle^{(q)} + \langle \delta \phi^2 \rangle^{(c)} = \frac{M^2(\phi)}{16\pi^2} \left[ \ln \left( \frac{M^2(\phi)}{\mu^2} \right) - N_\epsilon - \frac{3}{2} \right] + \langle \delta \phi^2 \rangle^{(c)}.
\]

By using Eq. (18) the one-loop effective potential in de-Sitter spacetime can be given as follows [53, 54]:

\[
V_{\text{eff}}(\phi) = \frac{1}{2} m_\phi^2(\mu) \phi^2 + \frac{1}{2} \xi_\phi(\mu) R \phi^2 + \frac{\lambda(\mu)}{4} \phi^4 + \frac{3\lambda(\mu)}{2} \langle \delta \phi^2 \rangle^{(c)} \phi^2 + \frac{M^4(\phi)}{64\pi^2} \left[ \ln \left( \frac{M^2(\phi)}{\mu^2} \right) - \frac{3}{2} \right] + O \left( \left\{ \langle \delta \phi^2 \rangle^{(c)} \right\}^2 \right),
\]

where the effective potential is closely related with the energy density \( \rho \) and pressure \( p \) to be

\[
\rho = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \nabla \phi^2 + V_{\text{eff}}(\phi),
\]

\[
p = \frac{1}{2} \dot{\phi}^2 - \frac{1}{6} \nabla \phi^2 - V_{\text{eff}}(\phi).
\]

Thus, the UV divergences can be eliminated by the renormalization parameters and the radiative corrections of the UV or trans-Planckian physics are sequestered form the cosmological observations as with the flat spacetime. On the other hand, primordial density perturbations originate from \( \langle \delta \phi^2 \rangle^{(c)} \) corresponding to the quantum particle creations and one is troubled with whether \( \langle \delta \phi^2 \rangle^{(c)} \) has the sensitivity of the UV or trans-Planckian physics.

However, the sensitivity of the cut-off scale is also sufficiently sequestered from the inflationary fluctuations. Let us discuss the issues using the adiabatic regularization method. The adiabatic regularization is a powerful method to calculate the vacuum fluctuation or the energy density from the quantum particle creation in curved spacetime, and proceed the regularization through subtracting \( \langle \delta \phi^2 \rangle^{(q)} \) from \( \langle \delta \phi^2 \rangle \). Using the method, the vacuum fluctuation for the inflaton field can be given by

\[
\langle \delta \phi^2 \rangle^{(c)} = \int \frac{dk}{k} P_{\delta \phi}(\eta, k)
= \langle \delta \phi^2 \rangle - \langle \delta \phi^2 \rangle^{(q)} = \frac{1}{4\pi^2 a^2(\eta)} \int_0^\infty dk k^2 \Omega_k^{-1} \{ 2n_k + 2 \text{Re} z_k \}
= \frac{1}{4\pi^2 a^2(\eta)} \left[ \int_0^\infty dk k^2 \delta \chi_k^2 - \int_0^\infty dk k^2 \Omega_k^{-1} \right],
\]

where we must choose an appropriate initial vacuum and determine the mode function of \( \delta \chi(\eta) \). In the discussion from now on, let us consider the massive non-minimally coupled case (for the details, see Ref.[51–53]) where the mode function \( \delta \chi_k(\eta) \) is given by

\[
\delta \chi_k(\eta) = \sqrt{\frac{\pi}{4}} \eta^{1/2} \left\{ \alpha_k H_\nu^{(2)}(k \eta) + \beta_k H_\nu^{(1)}(k \eta) \right\},
\]

with

\[
\nu \equiv \sqrt{\frac{9}{4} - \frac{M^2(\phi)}{H^2}} \simeq \frac{3}{2} - \frac{M^2(\phi)}{3H^2},
\]
Thus, we can obtain the following expression of the mode function,

\[ \alpha_k = \frac{1}{2i} \sqrt{\frac{\pi k \eta_0}{2}} \left( \left( -i + \frac{H}{2k} \right) H_\nu^{(1)}(k \eta_0) - H_\nu^{(1)\prime}(k \eta_0) \right) e^{ik/H}, \tag{24} \]

\[ \beta_k = -\frac{1}{2i} \sqrt{\frac{\pi k \eta_0}{2}} \left( \left( -i + \frac{H}{2k} \right) H_\nu^{(2)}(k \eta_0) - H_\nu^{(2)\prime}(k \eta_0) \right) e^{ik/H}. \tag{25} \]

From Eq. (21) the vacuum fluctuations are given as follows:

\[ \langle \delta \phi^2 \rangle^{(c)} = \lim_{\lambda \to \infty} \frac{1}{4\pi^2 C(\eta)} \left[ \int_0^\Lambda 2k^2 |\delta \chi_k|^2 dk - \int_{\sqrt{2}/|\eta|}^\Lambda dk k^2 \Omega_k^{-1} \right] \]

\[ = \frac{\eta^2 H^2}{2\pi^2} \int_0^H k^2 |\delta \chi_k|^2 dk + \frac{\eta^2 H^2}{2\pi^2} \int_{\sqrt{2}/|\eta|}^H k^2 |\delta \chi_k|^2 dk. \tag{26} \]

The divergence parts exactly cancel as follows,

\[ \lim_{\lambda \to \infty} \frac{1}{4\pi^2 C(\eta)} \left[ \int_{\sqrt{2}/|\eta|}^\Lambda 2k^2 |\delta \chi_k|^2 dk - \int_{\sqrt{2}/|\eta|}^\Lambda dk k^2 \Omega_k^{-1} \right], \tag{27} \]

where we take the adiabatic mode cut-off as \( k > \sqrt{2 - M^2/(\phi/|H^2/|\eta| \simeq \sqrt{2}/|\eta| \simeq \sqrt{2a}H} \). Beyond the mode cut-off the mode function does not drastically change against the evolution of the universe. In this sense the mode cut-off can be recognized as the UV cut-off of the inflationary quantum fluctuations and therefore the corrections of the trans-Planckian physics are sequestered as long as the Hubble parameter is smaller than the Planck scale.

By using the formula of the Hankel functions

\[ H_\nu^{(1,2)}(k \eta_0) = H_\nu^{(1,2)}(k \eta_0) - \frac{\nu}{k \eta_0} H_\nu^{(1,2)}(k \eta_0), \tag{28} \]

and the Bessel function of the first kind defined by \( J_\nu = (H_\nu^{(1)} + H_\nu^{(2)})/2 \), we can obtain the expression

\[ |\alpha_k - \beta_k| = \sqrt{\frac{\pi k}{2H}} \left| J_{\nu-1}(k \eta_0) + \left( i - \frac{H}{2k} + \frac{\nu H}{k} \right) J_{\nu}(k \eta_0) \right|. \tag{29} \]

For small \( k \) modes, the the Bessel function and the Hankel function asymptotically behave as

\[ J_\nu(k \eta_0) \simeq \frac{1}{\Gamma(\nu + 1)} \left( \frac{k \eta_0}{2} \right)^\nu, \tag{30} \]

\[ H_\nu^{(2)}(k \eta_0) \simeq -H_\nu^{(1)}(k \eta_0) \simeq \frac{i}{\pi} \Gamma(\nu) \left( \frac{k \eta_0}{2} \right)^{-\nu}. \tag{31} \]

Thus, we can obtain the following expression of the mode function,

\[ |\delta \chi_k|^2 \simeq \frac{\pi}{4} |\eta| |\alpha_k - \beta_k|^2 |H_\nu^{(2)}(k \eta)|^2 \simeq \frac{2}{9k} (H |\eta|^2)^{1-2\nu} \quad (0 \leq k \leq H). \tag{32} \]

For large \( k \) modes, we approximate the Bogoliubov coefficients to be \( \alpha_k \simeq 1 \) and \( \beta_k \simeq 0 \) and evaluate the mode function as

\[ \delta \chi_k(\eta) \simeq \sqrt{\frac{\pi}{4}} \eta^{1/2} H_\nu^{(2)}(k \eta). \tag{33} \]

Thus, we can get the following expression

\[ |\delta \chi_k|^2 \simeq \frac{|\eta|}{16} \left( \frac{k |\eta|}{2} \right)^{-2\nu} \quad (H \leq k \leq \sqrt{2}/|\eta|). \tag{34} \]
From Eq. (32) and Eq. (34), the vacuum fluctuations are written as
\[
\langle \delta \phi^2 \rangle^{(c)} \simeq \frac{(H |\eta|)^{3-2\nu}}{9\pi^2} \int_0^H k dk + \frac{H^2 |\eta|^{3-2\nu}}{4\pi^2 \cdot 2^{3-2\nu}} \int_H^{\sqrt{2}/|\eta|} k^{2-2\nu} dk \\
\simeq \frac{H^2}{18\pi^2 e^{-2m^2/\nu}} \left[ \frac{3H^4}{8\pi^2 M^2 (\phi)} \left( 1 - e^{-2m^2/\nu} \right) \right].
\]  
(35)

For late cosmic-time \( (N_{\text{tot}} = Ht \gg H^2/M^2 (\phi) ) \), the vacuum fluctuations \( \langle \delta \phi^2 \rangle^{(c)} \) in de Sitter background are approximately written as \(^2\)
\[
\langle \delta \phi^2 \rangle^{(c)} \simeq \frac{3H^4}{8\pi^2 M^2 (\phi)}, \quad (M (\phi) \ll H).
\]  
(37)

which modify the effective potential of Eq. (19) in de Sitter spacetime and provides primordial density perturbations. Now, we found out that the UV divergences are safely sequestered and the sensitivity of the trans-Planckian physics is negligible. This conclusion is consistent with the approach of the effective field theories \(^7, 8\) but the quantum effects of the UV or trans-Planckian physics only changes the effective potential through the radiative corrections. Note that the effective mass \( M (\phi) \) of the inflaton should be smaller than the Hubble scale if not the inflationary vacuum fluctuations are strongly suppressed as \( \langle \delta \phi^2 \rangle^{(c)} \to 0 \) for \( M (\phi) \gg H \) \(^34\), and therefore, the radiative corrections of the UV or trans-Planckian physics to the inflaton mass must be hardly small and this fact is consistent with the fine-tuning of the inflaton potential. Precisely, however, \( \langle \delta \phi^2 \rangle^{(c)} \) has the mode dependence and might leave a tiny trans-Planckian imprint on the CMB at the short distance.

### III. THE UV COMPLETION AND INFLATIONARY QUANTUM FLUCTUATION

Next, let us consider the UV corrections of the massive scalar field \( S \) to the inflationary vacuum fluctuation and construct the effective potential \( V_{\text{eff}} (\phi) \) in curved spacetime. By using \( \langle \delta \phi^2 \rangle \) and \( \langle \delta S^2 \rangle \) we obtain the one-loop effective potential in de-Sitter spacetime as follows:
\[
V_{\text{eff}} (\phi) = \frac{1}{2} m_\phi^2 (\mu) \phi^2 + \frac{1}{2} \xi_\phi (\mu) R \phi^2 + \frac{\lambda (\mu)}{4} \phi^4 + \frac{3\lambda (\mu)}{2} \langle \delta \phi^2 \rangle^{(c)} \phi^2 + \mathcal{O} \left( \left\{ \langle \delta \phi^2 \rangle^{(c)} \right\}^2 \right) \\
+ \frac{M^4 (\phi)}{64\pi^2} \left[ \ln \left( \frac{M^2 (\phi)}{\mu^2} \right) - \frac{3}{2} \right] + \frac{g (\mu)}{2} \phi^2 S^2 + \frac{M^4 (S)}{64\pi^2} \left[ \ln \left( \frac{M^2 (S)}{\mu^2} \right) - \frac{3}{2} \right] + \cdots,
\]  
(38)

with
\[
M^2 (S) = m_S^2 (\mu) + g (\mu) \phi^2 + \left( \xi_S (\mu) - 1/6 \right) R,
\]

where \( \langle \delta S^2 \rangle^{(c)} \) is sufficiently suppressed and can be negligible. By using Eq. (38), we can read off the \( \mu \) dependence of these couplings and the one-loop \( \beta \) function of \( m_\phi^2 \) can be given as follows:
\[
\beta_{m_\phi^2} = \mu \frac{\partial}{\partial \mu} m_\phi^2 = \frac{6\lambda m_\phi^2 + 2gm_S^2}{(4\pi)^2} + \cdots.
\]  
(39)

Now we can simply understand that the UV corrections of the massive scalar field \( S \) drastically changes the effective potential of the inflaton and the fine-tuning matters appear to realize the inflation. These issues can be

\(^2\) In this case, the power spectrum on super-horizon scale \( (|k\eta| \ll 1) \) can be approximately written by \(^55\)
\[
P_{\delta \phi} (\eta, k) = \left( \frac{H}{2\pi} \right)^2 \left( \frac{k}{aH} \right)^{3-2\nu}
\]  
(36)

where the inflationary vacuum fluctuation with non-vanishing mass \( (M (\phi) \ll H) \) has a tiny \( k \)-dependence, i.e scale invariance.
interpreted by the inflationary vacuum fluctuation of Eq. (37). To improve the expression of Eq. (37) we shifts the inflaton field to be
\[ \phi^2 \rightarrow \phi^2 + \langle \delta \phi^2 \rangle_{\text{eff}} = \phi^2 + \frac{M^2(\phi)}{16\pi^2} \left[ \ln \left( \frac{M^2(\phi)}{\mu^2} \right) - N_e - \frac{3}{2} \right] + \langle \delta \phi^2 \rangle_{\text{eff}} \],
which is reasonable to include quantum backreactions in perturbative contexts and we can get the following expression of the inflationary vacuum fluctuation
\[ \langle \delta \phi^2 \rangle_{\text{eff}} \simeq \frac{3H^4}{8\pi^2M_{\text{eff}}^2(\phi)}, \quad (M_{\text{eff}}(\phi) \ll H), \]
where \( M^2(\phi) \) on Eq. (37) is replaced by \( M_{\text{eff}}(\phi) \) including quantum radiative corrections as follows:
\[ M_{\text{eff}}^2(\phi) = m^2_\phi(\mu) + g(\mu) S^2 + 3\lambda(\mu) \phi^2 + (\xi_\phi(\mu) - 1/6) R 
+ \frac{3\lambda(\mu) M^2(\phi)}{16\pi^2} \left[ \ln \left( \frac{M^2(\phi)}{\mu^2} \right) - \frac{3}{2} \right] + g(\mu) M^2(S) \frac{2}{16\pi^2} \left[ \ln \left( \frac{M^2(S)}{\mu^2} \right) - \frac{3}{2} \right] + \cdots. \]
In \( m_S \simeq \Lambda_{\text{UV}} \gg m_\phi \), the quantum radiative corrections are approximately order of the UV scale as \( M_{\text{eff}}(\phi) \simeq g^{1/2}m_S \simeq g^{1/2}\Lambda_{\text{UV}} \). Therefore, the inflationary vacuum fluctuation can be simplified as follows:
\[ \langle \delta \phi^2 \rangle_{\text{eff}} \simeq \frac{3H^4}{8\pi^2(m_\phi^2 + g\Lambda_{\text{UV}}^2)}, \quad \left( m_\phi + g^{1/2}\Lambda_{\text{UV}} \ll H \right), \]
where the UV corrections are of order \( \mathcal{O}(1) \) rather than \( \mathcal{O}(H/\Lambda_{\text{UV}}) \) or \( \mathcal{O}(H^2/\Lambda_{\text{UV}}^2) \) in comparison with \( m_\phi \), and therefore, the inflationary vacuum fluctuation or the primordial CMB perturbation are naively depend on the UV contributions. In this sence, for the very massive case \( (m_\phi + g^{1/2}\Lambda_{\text{UV}} \gg H) \), the slow-roll condition of the inflation violates or the inflationary fluctuation breaks the scale invariance of the spectrum of CMB perturbations and are sufficiently suppressed (see e.g. Ref.[56–58]). Therefore, we can impose a tight constraint on the UV physics as follows:
\[ \Lambda_{\text{UV}} \ll H/g^{1/2}. \]

Now, we can obtain an upper bound of the interaction coupling to be \( g \ll \mathcal{O}(10^{-10}) \) if we assume the Planck scale cut-off as \( \Lambda_{\text{UV}} \simeq M_{\text{Pl}} \simeq 10^{18} \text{ GeV} \) and take the current upper value of the Hubble parameter \( H \simeq 10^{13} \text{ GeV} \) [59, 60]. Note that in the same way we obtain the constraint of the existence of the coherent fields \( \phi, S \) as \( \phi, S \ll H/g \) from Eq. (42). From these points, we found out that the inflaton sector should be decoupled with the high energy physics like the Planck or string physics. From this standpoint, the Starobinsky inflation [61] or the Higgs inflation [62, 63] are attractive due to the non-requirement for the new physics in comparison with any other inflation models [66–69], and furthermore, matched with the current constraints of the CMB observations. These things from Eq. (42) are consistent with the fine-tuning problems of the inflaton potential, and therefore, if we neglect the fine-tuning for inflation the quantum effects of the UV or trans-Planckian physics on the primordial density perturbation is sufficiently sequestered. The reason for this is that in the UV regime the quantum fluctuation does not drastically change described by Eq. (27) and we only receive a tiny imprint on the CMB at the short distance rather than \( \mathcal{O}(H/\Lambda_{\text{UV}}) \) or \( \mathcal{O}(H^2/\Lambda_{\text{UV}}^2) \) as previously discussed. However, the trans-Planckian corrections give non-negligible impact on the inflaton potential and the inflationary vacuum fluctuation as the fine-tuning issues of the inflation itself.

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3 From the viewpoint of the quantum radiative corrections on the curved spacetime, there is a strong correspondence between the Starobinsky inflation \((a_1 R^2)\) and the Higgs inflation \((\xi_H H^4)\). In fact, the one-loop \( \beta \)-function of the gravitational coupling \( a_1 \) can be written as follows [44, 64, 65]:
\[ \beta_{a_1} = \mu \frac{\partial}{\partial \mu} a_1 = \frac{(\xi_H - 1/6)^2}{2(4\pi)^2}, \]
where \( a_1 \approx 10^9 \) for the Starobinsky inflation or \( \xi_H \approx 10^4 \) for the Higgs inflation are compatible with the latest Planck data.
IV. CONCLUSION AND SUMMARY

In this paper, we have discussed how the UV or trans-Planckian physics affect the inflationary vacuum fluctuation and the primordial density perturbation from the rigid perspective of the QFT in curved spacetime. Here we have dealt with this problem by using the two-point correlation function $\langle \delta \phi^2 \rangle$ of the non-minimally coupled scalar fields and constructing the effective potential in curved spacetime. Clearly we have shown that in the UV regime the quantum fluctuation does not drastically change as described by Eq. (27) and the UV or trans-Planckian corrections can be embedded in the effective potential, and the UV sensitivity on the primordial density perturbation is sufficiently sequestered. However, the trans-Planckian corrections give a strong impact on the fine-tuning problems of the inflation itself and we have obtained a inflationary conjecture which can be a window towards the UV or trans-Planckian physics.

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