Energy efficient motion control of the electric bus on route

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Abstract. At present, the urgent problem is the reduction of energy costs of urban motor transport. The article proposes a method of solving this problem by developing an energy-efficient law governing the movement of an electric bus along a city route. To solve this problem, an algorithm is developed based on the dynamic programming method. The proposed method allows you to take into account the constraints imposed on the phase coordinates, control action, as well as on the time of the route. In the course of solving the problem, the model of rectilinear motion of an electric bus on a horizontal reference surface is considered, taking into account the assumptions that allow it to be adapted for the implementation of the method. For the formation of a control action in the equations of motion dynamics, an algorithm for changing the traction / braking torque on the wheels of an electric bus is considered, depending on the magnitude of the control parameter and the speed of motion. An optimal phase trajectory was obtained on a selected section of the road for the prototype of an electric bus. The article presents the comparison of simulation results obtained with the optimal energy efficient control law with the results obtained by a test driver. The comparison proved feasibility of the energy efficient control law for the automobile city electric transport.

In the modern metropolis, a significant role in passenger transportation is played by land road transport. With an intensive increase in the city's population, urban boundaries are being expanded, the transport network is enlarged and the traffic schedule for road transport is consolidated. At present, the city is widely used by the city automobile electric transport - the electric bus. For this type of electric transport, the task of improving the energy efficiency of traffic on the urban route (which involves the presence of stops and a restriction on the time of travel between them) is especially important, since with a reduction in energy costs it will be possible to:

- increase the passenger capacity of the electric bus by rationally choosing the parameters of the energy storage device (the number of batteries) on board, and, consequently, reducing the curb weight;
- reducing the cost of the electric bus by reducing the required number of batteries to ensure the necessary power reserve;
- increase the power reserve of the electric bus with the same capacity of the battery.

The essence of the proposed method consists in software control of the speed of the electric bus on the way between stops for landing and disembarking passengers at a given time in order to minimize energy consumption during movement. In this case, the driver determines the beginning of the movement. The control system must provide the calculated speed of the electric bus as a function of the
traveled path, by means of the traction moment of the electric machine supplied to the wheels. The braking torque of the regenerative brake and the braking torque produced by the service braking system.

Thus, the task of an energy-efficient law governing the speed of an electric bus between stops on a city route should be formulated as an optimization problem. Determine the law of change in speed from the traversed path of the electric bus on the route when driving from a waypoint with phase coordinates \( v_1 = 0 \) to the waypoint \( v_2 = 0 \) during \( T = t_2 - t_1 \) with the minimization of the energy consumed by the motion determined by the objective function:

\[
J = \int_{t_1}^{t_2} (N_a - N_{rb} + N_b) dt,
\]

or

\[
J = \int_{s_1}^{s_2} \frac{(N_a - N_{rb} + N_b)}{v} ds
\]

where

- \( N_a \) – the capacity of the discharge of the accumulator, developed during the acceleration of the electric bus;
- \( N_{rb} \) – power charge of the drive with regenerative braking;
- \( N_b \) – power dissipated when using the service braking system.

To determine the energy-efficient control law, we describe the rectilinear motion of an electric bus on a horizontal reference surface as a motion of a rigid body. To do this, let's assume that the electric bus moves along the city route without slipping of its wheels, which means that \( r_{wi} \) (rolling radius of the i-wheel) \( \approx r_{w0} \) (rolling radius without sliding of the i-wheel) \( \approx r_{di} \) (dynamic radius of the i-wheel). In addition, we will assume that the rolling radii of all the wheels of an electric bus without slipping are equal. Then the equation of dynamics [1] electric bus will take the form:

\[
m\delta \frac{dv}{dt} = \frac{M_{el} \eta_{tr} u_{tr} - M_f - M_b}{r_{w0}} - P_w,
\]

where

- \( m \) – mass of the electric bus;
- \( \frac{dv}{dt} \), \( v \) – acceleration and speed of the electric bus;
- \( r_{w0} \) – rolling radius without sliding of the electric bus wheels;
- \( M_{el} \) – tractive / braking torque generated by the electric machine;
- \( M_f \) – moment of rolling resistance of an electric bus [1];
- \( M_b \) – reduced braking torque of the service braking system;
- \( P_w \) – force of aerodynamic resistance;
- \( \eta_{tr} \) – efficiency of the mechanical part of the transmission;
- \( u_{tr} \) – transmission ratio;
- \( \delta \) – rotational inertia coefficient.

\[
\delta = 1 + \frac{\sum_{i=1}^{I} J_{wi}}{mr_{w0}^2},
\]

where

- \( J_{wi} \) – the moment of inertia of the mechanical part of the transmission and the wheel reduced to the i-th wheel.

For further convenience, we write the equation of the dynamics of the electric bus in the coordinates path \( s - v \):

\[
m\delta \frac{dv}{ds} = \frac{1}{\nu} \left( \frac{M_{el} \eta_{tr} u_{tr} - M_f - M_b}{r_{w0}} - P_w \right),
\]

This simplified model of the motion of the electric bus as a rigid body is used to reduce computation time and increase the speed of the algorithm.
To determine the relationship $N_a$ and $N_b$ with $M_{el}$ and $v$ we apply the following sequence of power conversion in the rectilinear motion of the electric bus (figure 1):

\[
N_a \eta_{CP} \eta_{emp} = \sum_{i=3}^{4} M_{wi} \alpha_{wi} = \frac{v \cdot u_{tr}}{r_{w0}} M_{el}, \quad \text{traction mode}
\]

\[
N_b = \eta_{tr} \eta_{CT} \eta_{emr} \sum_{i=3}^{4} M_{wi} \alpha_{wi} = \eta_{CT} \eta_{emr} \frac{v \cdot u_{tr}}{r_{w0}} M_{el}, \quad \text{energy recuperation mode}
\]

where

- $M_{wi}$ – traction braking torque on the i-wheel created by the electric machine;
- $\frac{da_{wi}}{dt}$, $\alpha_{wi}$ – angular acceleration and angular velocity of the i-th wheel;
- $\eta_{CP}$ – efficiency of the electric power converter at dispersal and speed maintenance;
- $\eta_{CT}$ – efficiency of the electric power converter at regenerative braking;
- $\eta_{emr}$ – efficiency of the electric machine during acceleration and holding speed;
- $\eta_{emr}$ – efficiency of the electric machine with regenerative braking.

At the same time, the maximum power dissipated during braking of the electric bus by the service braking system is independent of the speed of movement and defined as:

\[
N_b = \sum_{i=1}^{4} M_{bi} \alpha_{wi} = \frac{v}{r_{w0}} M_{bi},
\]

where

- $M_{bi}$ – braking torque generated by the service braking system on the i-wheel.

Thus we have provided a possibility to determine the discharge charge power of the drive and the braking power by the working braking system and, consequently, to find the value of the minimized functional at each moment of time by solving the equations of dynamics.

Let us describe the law governing the change in the driving and braking moment of the electric bus. The driver, by pressing the accelerator pedal, determines the "desire" to move according to the calculated
traffic law. Proceeding from this, the control system must provide the desired speed of the electric bus, by means of the traction moment of the electric machine supplied to the wheels, the braking torque of the regenerative brake and the braking torque created by the service braking system. For convenience of calculations we combine $M_e$ and $M_b$ into one variable $M_h = M_e \eta_{tr} u_{tr} - M_b$ and determine its value depending on the magnitude of the control action $h$. To do this, it is necessary to integrate to determine the phase space, that is, the state space of the electric bus $v(s)$, in which it can be physically located under the condition that the boundary conditions are met. To do this, it is necessary to integrate the equation of motion of the electric bus while fixing the initial boundary conditions ($s_1, v_1 = 0$) and

$$
M_h = \begin{cases}
M_{em} u_{tr} \eta_{tr} (h/h_{em}), & \text{if } \frac{v \cdot u_{tr}}{r_w} < \frac{N_{em}}{M_{em}} \text{ and } v > 0 \\
\frac{N_{em}(h/h_{em})}{v/r_w} \eta_{tr}, & \text{if } \frac{v \cdot u_{tr}}{r_w} > \frac{N_{em}}{M_{em}} \text{ and } v < v_{em}, \text{ if } h > 0 \text{ and } h < h_{em} \\
\frac{M_{rec} u_{tr} (h/h_{rec})}{\eta_{tr}}, & \text{if } \frac{v \cdot u_{tr}}{r_w} < \frac{N_{rec}}{M_{rec}} \text{ and } v > 0 \\
\frac{N_{rec} (h/h_{rec})}{\eta_{tr} \cdot v/r_w}, & \text{if } \frac{v \cdot u_{tr}}{r_w} > \frac{N_{rec}}{M_{rec}} \text{ and } v < v_{rec}, \text{ if } h > -h_{rec} \text{ and } h < 0 \\
-M_{rec} u_{tr} \eta_{tr} (h/h_{rec}), & \text{if } \frac{v \cdot u_{tr}}{r_w} < \frac{N_{rec}}{M_{rec}} \text{ and } v > 0 \\
\frac{N_{rec} (h/h_{rec})}{\eta_{tr} \cdot v/r_w} + M_b \left( \frac{h + h_{rec}}{h - h_{rec}} \right), & \text{if } \frac{v \cdot u_{tr}}{r_w} > \frac{N_{rec}}{M_{rec}} \text{ and } v < v_{rec}, \text{ if } h < -h_{rec} \\
M_b \left( \frac{h + h_{rec}}{h - h_{rec}} \right), & \text{if } v > v_{rec}
\end{cases}
$$

where

- $M_{em}$ – the maximum tractive moment created by the electric machine in the traction mode;
- $M_{rec}$ – maximum braking torque generated by the electric machine in the energy recovery mode;
- $M_b$ – maximum braking torque produced by the service braking system;
- $N_{em}$ – maximum tractive power developed by the electric machine in traction mode;
- $N_{rec}$ – the maximum power developed by the electric machine in the mode of energy recovery;
- $v_{rec}$ – the maximum speed of the electric bus to which the electric machine can operate in traction mode;
- $v_{em}$ – the maximum speed of the electric bus to which the electric machine can operate in the energy recovery mode;
- $h_{em}$ – the amount of control at which the electric machine operates in the mode of maximum traction power;
- $h_{rec}$ – the control value at which the electric machine operates in the maximum recovery power mode;
- $h_s$ – the control value at which the electric machine operates in the maximum recovery power mode and the maximum braking torque of the service braking system is used.

Thus, a system of equations is obtained describing the driving traction / braking torque realized by the motion control system as a function of the speed of the electric bus $v$ and the magnitude of the control action $h$. value $h$ may vary from $-h_s$ to $h_{em}$, introducing into the equation of dynamics a limitation on the driving traction/braking torque.

The search for optimal control is carried out using Bellman’s dynamic programming method [2]. The algorithm for implementing the Bellman method is as follows. At the first stage, it is necessary to determine the phase space, that is, the state space of the electric bus $v(s)$, in which it can be physically located under the condition that the boundary conditions are met. To do this, it is necessary to integrate the equation of motion of the electric bus while fixing the initial boundary conditions ($s_1, v_1 = 0$) and...
feed the traction moment to the driving wheels according to the external characteristic of the electric motor, when moving from the initial state to the final state. The magnitude of the control will then be equal to $h_{em}$. After this, it is necessary to obtain solutions of the equation of motion for the fixed finite boundary conditions ($s_2, v_2 = 0$) and the braking torque the regenerative brake taken from its external characteristic, as well as the maximum torque of the service braking system $M_r$, moving from the final state to the initial state. The magnitude of the control will then be equal to $-h_t$. Then, if necessary, consider the restriction of the speed of the electric bus on the route in accordance with the traffic rules. Thus, curves are obtained that limit the space in which the electric bus can be located without violating the traffic rules, provided that the driving traction / braking torque when applying the permissible control action will ensure that the boundary conditions are met. A graphic illustration of the space obtained is given in figure 2.

![Figure 2. Phase space of electric bus states.](image)

The next stage is the discretization of the resulting space, that is, the partition of the phase surface onto the grid of the states considered. We divide the section of the path under consideration into $N$ points, and the allowable speed for $L$. Since for each point of the considered route the range of permissible speeds can be different, the number of points of the velocity partition at a given point of the path in question will be equal to $L_k$, where $k$ – the number of the point under consideration in the route under study. The calculation time and the accuracy of the obtained control law directly depend on the frequency of the partition.

Then it is necessary to compile the Bellman function for the system under consideration. For convenience, we will use a functional with a path dependence:

$$J = \int_{s_1}^{s_2} \frac{(N_a - N_{rb} + N_b)}{v} ds$$
Then in our case the Bellman function will have the form:

\[
Z_{N-k}(s(N-k), v(N-k, l)) = \min_{h(N-k,l) \in H} \left\{ N_a(s(N-k), v(N-k, l), h(N-k, l)) \\
- N_{rb}(s(N-k), v(N-k, l), h(N-k, l)) + N_b(s(N-k), v(N-k, l), h(N-k, l)) \\
+ \lambda \cdot \frac{(s(N-k+1) - s(N-k))}{0.5(v(N-k+1, p) + v(N-k, l))} \\
+ Z_{N-k+1}(s(N-k+1), v(N-k+1, p)) \right\},
\]

where

\( Z(s, v) \) – the minimum value of the Bellman function at the point of phase space with the length of the traversed route \( s \) and speed \( v \);

\( N \) – number of points of subdivision of the path under study;

\( H \) – area of admissible controls;

\( k \) – number of the point in question in the path under study \( k \in [1 ... N] \);

\( l \) – the number of the point in question in the array of permissible velocities for \( (N-k) \) point of the path \( l \in [1 ... L_{N-k}] \);

\( p \) – the number of the point in question in the array of permissible velocities for \( (N-k+1) \) point of the path \( p \in [1 ... L_{N-k+1}] \);

\( \lambda \) – indefinite Lagrange multiplier.

The value of the functional, at a point with phase coordinates \( k \), \( l \), is the minimum value of the sum of the value of the functional, at a point with phase coordinates \( k \), \( p \) and the amount of energy expended for the transition from the state \( k \), \( l \) to \( k \), \( p \), provided that the control applied to the system \( h \) be in the range of acceptable values. Thus, by calculating the value of the Bellman function at each point of the phase space, moving from the final position to the initial position, we obtain a two-dimensional array of minimum energy values that must be expended to move from the point of space to the finite point. The sequence of calculations occurs precisely from the end point to the initial one because the value of the function \( Z(s, v) \) at the end point of the route does not affect the very process of going to this point.

For the comfortable transportation of passengers, it is necessary to provide a restriction on the acceleration developed by the electric bus \( (1 \text{ m/s}^2) \). To do this, when calculating the values of the Bellman function, one must choose a control that is not only in the range of admissible values (from \(-h_{mr} \) to \( h_{em} \)), but also creates a rooting that does not exceed the permissible limits.

The next step is to write down the equation of state. Since the position of the electric bus on the phase surface depends on the previous state and control at this step, then:

\[
Z_k(s(k), v(k, l')) = \]
\[
= Z_k(s(k-1), v(k-1, p')) \\
+ N_a(s(k), v(k, l'), h(k, l')) - N_{rb}(s(k), v(k, l'), h(k, l')) \\
+ N_b(s(k), v(k, l'), h(k, l')) + \lambda \cdot \frac{(s(k) - s(k-1))}{0.5(v(k, l') + v(k-1, p'))}
\]

where

\( l' \) – the number of the point in question in the array of permissible velocities for \( k \) point of the path \( l' \in [1 ... L_k] \);

\( p' \) – the number of the point in question in the array of permissible velocities for \( (k-1) \) point of the path \( p' \in [1 ... L_{k-1}] \).
Now moving from the initial state to the final state, solving the equation of state, we obtain the optimal phase trajectory $s'(v')$ movement of the electric bus in the area under consideration, as well as control $h'$, providing the movement of the electric bus along the received trajectory.

In the case when the indeterminate Lagrange multiplier $\lambda$ is equal to 0, the method gives a trivial solution: to move with the minimum possible speed. In this case, the time to reach the final state tends to infinity. Thus, in order to ensure a given time of passage of the route, it is necessary to vary iteratively the value of the indeterminate Lagrange multiplier $\lambda$. Repeating the above actions.

Similar calculations can be made for various options for congestion of the electric bus, thus obtaining various traffic laws depending on the number of passengers in the cabin. In addition, if it is possible to solve the equation of state on board the electric bus, it is possible to find the optimal control law for moving from the current state of the electric bus to the end point. Thus, you can take into account, for example, the intervention of the driver to prevent a traffic accident or stop the electric bus at the traffic light [2].

In order to estimate the energy the developed method would save in comparison with the energy spent by the electric bus on the route now, simulation results with the use of the energy-efficient control law obtained by the method were compared with the results of the road tests.

As the test object, we used the electric bus developed by the "GAZ" group in cooperation with the Bauman Moscow State Technical University (figure 3).

![Electric bus built on the LiAZ 5292.30 chassis.](image)

**Figure 3.** Electric bus built on the LiAZ 5292.30 chassis.

The electric bus was built on the LiAZ 5292.30 chassis (figure 4). The curb weight of the electric bus is 13265 kg, the gross weight of the electric bus is 18000 kg.
The electric bus movement parameters registration for further analysis on a PC was made using a CAN bus through the MATLAB Simulink software package [[3]]. The following parameters were recorded:

- time of motion $t$, s;
- voltage on the motor $U$, V;
- current on the motor $I$, A;
- speed of the electric bus $v$, km/h (calculated by the controller with the help of a standard rotor speed sensor).

The traveled distance $s$ and the electrical energy $J$ spent for acceleration and recovered during braking of the electric bus were calculated by numerical integration of the recorded data using the following formulas:

$$s = \int_{t_1}^{t_2} v dt$$

$$J = \int_{t_1}^{t_2} U I dt.$$

In the course of the tests, the driver was asked to travel a measuring section of 100 m in length during 20 s so that a minimum amount of energy was used for movement.

The calculated optimal phase trajectory for the section of the route under consideration is presented in figure 5. The calculation was carried out taking into account the dependence of the efficiency of the electric machine on the rotational speed of the rotor and the perceived load.
Figure 5. The calculated optimal phase trajectory.

The optimal control law on the estimated measuring site is:
1. Acceleration (the route segment from 0 to 31 m in figure Figure 5);
2. Movement in the coasting mode (the route segment from 31 m to 79 m in figure Figure 5);
3. Braking with recuperative brake (the route segment from 79 m to 100 m in figure Figure 5, in the end, the electric bus is stopped by the mechanical brakes).

Due to the shortness of the section of the route in question, there is no speed maintenance mode. The calculated value of the spent energy is 238 kJ.

For the collection of the statistics and reducing the impact of a possible slope of the road the driver had to move 5 times uniformly in the forward and reverse directions. The number of measurements was chosen on the basis of the requirement for the width of the confidence interval \( \pm \sqrt{D} \) with probability value \( P = 95\% \). As a result, it was obtained that it is necessary to conduct at least 7 measurements [5].

Figure 6 presents the results of the road tests.
According to figure 6 it can be concluded that the driver cannot independently maintain a uniform motion even on a simple section of the route, which increases the average energy consumption for traffic along the measurement site.

After the experiment, a comparison was made between the calculated data reflecting the energy consumption of the electric bus when driving according to the obtained energy-efficient control law, with the results of driving the test driver. The results of the road test data processing are given in the table 1.

**Table 1. Results of the road test data processing.**

| Distance traveled s, m | Spent time t, s | Spent energy J, KJ |
|------------------------|-----------------|--------------------|
| 107,99                 | 19,71           | 343,88             |
| 103,83                 | 20,36           | 285,99             |
| 104,35                 | 20,81           | 252,44             |
| 103,69                 | 20,27           | 282,13             |
| 105,29                 | 22,00           | 244,24             |
| 104,22                 | 20,51           | 290,35             |
| 109,39                 | 19,61           | 394,47             |
| 105,32                 | 19,86           | 324,97             |
| 104,73                 | 21,94           | 265,05             |
| 104,84                 | 20,95           | 282,75             |

The result: $J = 296,63 \pm 32,71$ kJ

The results of the experiment confirmed that the proposed method makes it possible to store up to $(296,63$ kJ-238 kJ) / 296,63 kJ 100% = 19,7% of the energy when moving along the measuring site, due to the optimal choice of the timing of the switching of the driving regimes and providing a greater repeatability of the implementation of the law of motion.
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Conclusion
Comparison of the amount of energy spent on the movement obtained as a result of theoretical calculations with the energy expended when driving the electric bus by the test driver proved the possibility to save up to 19.7% of energy even on a simple part of the route. The received "gain" of energy is reached by an optimum choice of the moment of switching of modes of movement, and also due to the fact that the driver cannot maintain a uniform law of movement at repetition of the same site of a route. The obtained results prove the necessity of optimization of the laws of the automobile electric transport traffic between stops while driving along the city route, as well as the introduction of the traffic control system.

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