Detection Performance of Single Pulse Photon Accumulation with Typical Emission Waveform

A-Hui Hou1,2,a, Yi-Hua Hu1,2,b and Nan-Xiang Zhao1,2,c
1State Key Laboratory of Pulsed Power Laser Technology, National University of Defense Technology, Hefei 230037, China
2Key Laboratory of Electronic Restriction Technology Anhui, National University of Defense Technology, Hefei 230037, China
ahou_a_hui068@163.com, bskl_hyh@163.com, csouthfly@163.com

Abstract. Based on the theory of photon statistics, the cumulative detection probability model of mono-pulse echo is established. The influences of rectangular, Gaussian and triangular pulses on the photon detection performance are simulated, and it is analyzed the detection performances of three waveforms with different pulse widths. The results showed that the detection performance of rectangular pulse was slightly better than that of triangular pulse, and the detection performance of the Gaussian pulse was the worst. The variation of the pulse width of the three waveforms within a certain range had no effect on the detection performance.

1. Introduction
Photon counting laser radar is widely used in space detection, remote sensing imaging, aerial reconnaissance and other fields with high sensitivity, high detection range and low system energy consumption. Photon counting laser radars often use Geiger-mode single photon detection. In order to accurately describe the detection performance of the system, two parameters of detection probability (PD) and false alarm probability (Pf) are generally selected. In 2003, S. Johnson et al. [1] first proposed the use of discrete intervals to study the detection performance of single-photon detectors, which laid the foundation for the study of the detection performance of subsequent single-photon detectors. In 2003, D.G. Fouche et al. [2] studied the expression of detection and false-alarm probability of single-photon detection Lidar by means of multi-pulse cumulative detection and mono-pulse direct detection. In 2005, H Markus [3] analyzed the detection and false alarm probability expressions of photon detection mode, and studied the detection performance of multi-pulse cumulative detection. In 2011, Ying Guo [4] studied the multi-pulse cumulative detection of photon counting three-dimensional imaging laser radar, showing the method of multi-cumulative detection distinguishes echo photon and dark count. In 2012, Lu Xu et al. [5] studied the single-pulse cumulative detection performance under long dead time, indicating that the detection probability was significantly improved when the cumulative detection reached a certain number of times. In 2013, Hanjun Luo [6] studied the mono-pulse cumulative detection theory of Q-switched pulses, and analyzed the influence of pulse width on the detection probability. In 2017, Hanjun L et al. [7] studied the influence of turbulence on the detection performance of single-photon detection Lidar by means of mono-pulse accumulation, and the results showed mono-pulse accumulation could improve the detection probability. It can be found the detection efficiency of single photon detection has always been a hot issue. Recently,
research has been inclined to use multi-pulse cumulative detection to improve detection efficiency, and there are few studies on single pulse cumulative detection, especially based on the basic theoretical level of single-pulse cumulative detection.

In this paper, the photon number distribution of photon counting Lidar echo was studied. Based on the statistical distribution of echo photons, the principle of mono-pulse cumulative detection was analyzed. The single pulse cumulative detection performance of rectangular, Gaussian and triangular pulse echoes was compared. The effects of pulse width on detection performance under three waveforms were studied.

2. Single Pulse Photon Accumulation Detection Model

2.1 Signal Photon Echo Distribution

The emitted laser power is \( P_t \), the distance between the laser radar and the target is \( R \), and the target is Lambert. According to the Lidar equation, the wide pulse laser echo energy and photon numbers received within \( t_1 - t_2 \) are \[ N_{s(t_1-t_2)} = \frac{E_{t_1-t_2}}{\eta q} = \rho \tau^2 \eta_p \eta_t \eta_q \cos \theta \frac{A_r a}{\pi R^2} \frac{1}{\hbar \nu} \int_{t_1}^{t_2} P_t dt \] and \[ E_{t_1-t_2} = \int_{t_1}^{t_2} P_t dt \] (1)

Wherein, \( N_{s(t_1-t_2)} \) is average signal photon number of detector response within \( t_1 - t_2 \); \( E_{t_1-t_2} \) is laser echo energy; \( \eta_q \) is quantum efficiency of detector; \( h \nu \) is single photon energy; \( \rho \) is target emissivity; \( \tau \) is the atmospheric transmittance; \( \eta_p \) and \( \eta_q \) are the optical efficiencies of transmitting and receiving system; \( \theta \) is the angle between the optical axis and target; \( A_r \) is the effective receiving area; \( A_r \) and \( A_l \) are the projected areas of laser beam in cross-sectional and at the target.

The noise mainly includes the background noise and the dark count of the detector, which are independent of each other. The total noise photon number of the system is the sum of the two.

\[ N_n = N_b + N_d \] (2)

Wherein, \( N_d \) is the dark count of the detector, \( N_b \) is the background noise entering the receiving optical system within a unit time;

In order to improve the detection accuracy and working efficiency of the detector, this paper only measures within a certain period, which is called distance gate. Because the photon counting needs to consider the sampling time, the distance gate is divided into several time intervals \[ \text{(also called time bin). The discussion and research below in this paper are based on the above settings.} \]

2.2 Photon Count Detection Probability Model

In the Lidar detection system, it is generally believed that the detection probability is the probability of photons of the real target detected in the target time interval. In addition to this interval, the probability in other time intervals of detecting the number of photons caused by background or other noises is false alarm probability, also known as false alarm rate.

According to optical statistical theory, the photon number of noise and signal echo detected by the detector obeys Poisson distribution \[ \text{[2][10]. Based on the Poisson distribution, the probability of } m \text{ event being detected within } t_1 - t_2 \text{ is} \]

\[ P(m; M) = \frac{1}{m!} [M(t_1, t_2)]^m \exp[-M(t_1, t_2)] \] and \[ M(t_1, t_2) = \int_{t_1}^{t_2} R_t dt \] (3)
Wherein, \( M(t_1, t_2) \) is the average number of primary photons within \( t_1 - t_2 \); \( R_r \) is the rate function of the primary photoelectron.

The probabilities that no photons are excited and of generating a detectable pulse at a single time interval are respectively

\[
P(0, M) = \exp\left[ -M(t_1, t_2) \right]; \quad \overline{P} = 1 - \exp\left[ -M(t_1, t_2) \right]
\] (4)

### 2.3 Single Pulse Cumulative Detection Probability Model

When the echo information occupies multiple time intervals, the single pulse cumulative detection model can obtain higher detection probability and lower false-alarm rate. The model is studied when the dead time is zero or short, that is, the influence of dead time is not considered. The key point of mono-pulse cumulative detection is how to stack multiple detection \([6][8]\).

For photon counting Lidar target detection, the detection probability in the \( i-th \) interval is

\[
P(i) = \exp\left[ -\int_0^{(i-1)\tau_{\text{死}}} R_r dt \right] \cdot \left[ 1 - \exp\left[ \int_{(i-1)\tau_{\text{死}}}^{i\tau_{\text{死}}} R_r dt \right] \right]
\]

\[
= \exp\left[ -\sum_{j=1}^{i-1} N(j) \right] \cdot \left[ 1 - \exp\left[ -N(i) \right] \right]
\] (5)

Wherein, \( \tau_{\text{bin}} \) is the time interval; \( N(i) \) is the detectable photon number in the \( i-th \) interval, i.e.

\[
N(i) = N_s(i) + N_n(i) \quad \text{and} \quad N_s(i) = \eta_q \frac{E(i)}{h\nu} = \eta_q \int_{(i-1)\tau_{\text{死}}}^{i\tau_{\text{死}}} P_d dt / h\nu, N_n(i) = N_n \cdot \tau_{\text{bin}} = n_b.
\]

Combined with the above equation, we can get

\[
P(i) = \begin{cases} 
\exp\left[ -(i-1)n_{b_T} \right] \left[ 1 - \exp\left( -n_{b_T} \right) \right] & 1 \leq i < T \\
\exp\left[ -(i-1)n_{b_T} \right] \prod_{j=T}^{i-1} P\left[ 0, N_s(j) \right] \left[ 1 - P\left[ 0, N_s(i) \right] \right] & T \leq i \leq T + r \\
\exp\left[ -(i-1)n_{b_T} \right] \prod_{j=T+1}^{i+r} P\left[ 0, N_s(j) \right] \left[ 1 - \exp\left( -n_{b_T} \right) \right] & T + r < i \leq b
\end{cases}
\] (6)

Wherein, \( T = \tau_0 / \tau_{\text{bin}}, r = \tau / \tau_{\text{bin}}, b = T_g / \tau_{\text{bin}}, \tau_0 \) is the time interval between the echo signal and the beginning of the distance gate, \( \tau \) is the emitted laser pulse width, \( T_g \) is the distance gate width.

In order to reduce false-alarm rate, the threshold of the number of photons that can be detected in a single time interval is set as \( N_{\text{th}} \), that is, when the number of input photons is lower than \( N_{\text{th}} \), it is considered as noise and not accumulated. Therefore, the cumulative lower limit and upper limit are respectively set as \( \text{Th1} \) and \( \text{Th2} \).

Therefore, the detection probability and false-alarm probability of mono-pulse cumulative detection are respectively \([11]\)

\[
P_d = \sum_{i=\text{Th1}}^{\text{Th2}} P(i) \quad \text{and} \quad P_f = 1 - P_d - \exp\left[ -\sum_{j=T+1}^{T+r} N_s(j) - N_n \cdot T_g \right]
\] (7)

Wherein, the second term of the formula (7) means the probability that the signal and noise are present at the same time, but the detector has no response.
3. Simulation of Detection Performance

3.1 Different Waveform Detection Performance

Studying the influence of waveforms on detection probability is helpful to determine what kind of modulation at the transmitter can make the best detection performance. Suppose the emission pulse occupies multiple time intervals; the dead time of the single-photon detector is ignored, that is, as long as there is a signal pulse exceeding the threshold, the single-photon detector will produce a response. Based on the single-pulse cumulative detection probability model, the detection and false alarm rates of three waveforms are calculated, and the best waveform is obtained by comparison.

Reference [12] points that the mono-pulse energy of laser pulse is the integral of its power function over time. Suppose the energy and peak power of three pulses are equal. The single pulse energy is \(0.2 \mu J\), the pulse width of the rectangular and triangular pulse is \(10 \text{ns}\), the half-height width of the Gaussian is \(8 \text{ns}\), the divergence angle is \(1 \text{mrad}\), \(\lambda = 1064 \text{nm}\), \(\rho = 0.5\), \(R = 100 \text{km}\), \(\eta_s = 0.75\), \(\eta_a = 0.35\), \(N_d = 200 \text{kHz}\). Required \(P_f \leq 5\%\), \(P_d \geq 90\%\) the maximum allowable number of noise photons in time bin is calculated as \(2 \times 10^{-3}\), so \(N_{th} = 0.02\).

![Graphs showing detection performance for different waveforms](image)

The simulation results are shown in Fig.1, (a) (c) (e) represents the photon number distribution received by the detector, which is the sum of signal and noise photons; (b) (d) (f) represents the cumulative number of photons that can cause response within the pulse width.

The detection and false alarm rate of the three waveforms were calculated as shown in Table 1.

| Waveform    | Rectangular | Gaussian | Triangular |
|-------------|-------------|----------|------------|
| Detection Probability /% | 91.78       | 91.37    | 91.60      |
| False-alarm Rate /%       | 1.60        | 1.77     | 1.78       |
Table 1 showed that the probabilities of single pulse detection of three waveforms were greater than 90%, and false-alarm rates were less than 2%. The detection probability was: rectangular > triangular > Gaussian; the false-alarm probability was: triangular > Gaussian > rectangular. The single pulse energy of the three waveforms is equal, that is, the maximum cumulative number of detected photons is equal. The cumulative photon number of the response caused by the pulse width of three waveforms was compared: rectangular = triangular > Gaussian, and the detection probability of the Gaussian pulse was the lowest, so we can get the conclusion: the probability of detection is related to the cumulative number of detected photons causing the response in the pulse. The durations of different waveform pulses are different, resulting in different cumulative times: Gaussian = triangular > rectangular, and found that the rectangular pulse had the lowest false-alarm probability. It is concluded as the cumulative time increases, the detection probability increases, and the noise is superimposed, which increases the false-alarm probability to some extent. The (a) (c) (e) revealed only few low-signals time interval in the Gaussian pulse were filtered out when the threshold was 0.02, and the cumulative detected photons of the three pulses were basically the same, besides the detection probabilities were similar. So we can get the conclusion: three kinds of pulse detection performances are no difference in nature under low threshold.

In summary: 1) the detection probability is related to the cumulative number of detected photons in response to the pulse; 2) the increase of cumulative times increases the false-alarm probability to some extent; 3) there is no essential difference in the detection performance of the three pulses when the threshold is low; 4) In general, rectangular pulse has the best detection performance, followed by triangular pulse and Gaussian pulse.

### 3.2 Different Pulse Width Detection Performance

In 3.1, the detection performances of different waveforms were compared and analyzed. In order to further compare the three waveforms, the influences of the pulse width of the three waveforms on the detection performance were studied in this section. Set the energy of single pulse with different pulse widths to be the same, the dark count rate to be 200 kHz, ignore the background noise, and the threshold to be the same as above. The simulation results are shown in Fig. 2.

The abscissa is the pulse width, and the value is 0–50 ns. In order to facilitate the analysis of the detection performance when the pulse width is smaller than time bin, during 0–1 ns the step is 0.2 ns, and the remaining steps are 1 ns.

Fig.2 showed the detection and false-alarm probability remained unchanged when widths of three waveforms were smaller than a time interval. When the width was wider than time bin, the detection probability decreased with the increase of pulse width. Within 0–50 ns, the detection probabilities of rectangular, Gaussian and triangular pulse reduced 0.73%, 8.10%, 5.35%; the false alarm probabilities increased 0.74%, 2.75%, 2.28%. In order to analyze the changes of data in more detail, some other data are listed in Table 2.
Fig. 2 Detection performances with rectangular, Gaussian and triangular Pulse Width
(a)(b) detection probability and false-alarm probability with pulse width of three waveforms

Table 2 Comparison of partial pulse width detection performance of rectangular, Gaussian and triangular pulse

| waveform      | Pulse width/ns | 1   | 5   | 10  | 20  | 30  | 40  | 50  |
|---------------|---------------|-----|-----|-----|-----|-----|-----|-----|
|               | cumulative-detection times | 1   | 5   | 10  | 20  | 30  | 40  | 50  |
| Rectangular   |               | 2.5158 | 2.5158 | 2.5158 | 2.5158 | 2.5158 | 2.5158 | 2.5158 |
|               | cumulative-detection photons |       |       |       |       |       |       |       |
| Gaussian      |               | 2.5158 | 2.5001 | 2.4331 | 2.3207 | 2.2270 | 2.0959 | 1.9222 |
|               | cumulative-detection times | 1   | 13  | 21  | 35  | 47  | 55  | 59  |
|               | cumulative-detection photons |       |       |       |       |       |       |       |
| Triangular    |               | 2.5158 | 2.5158 | 2.4403 | 2.3592 | 2.2705 | 2.1333 |       |
|               | cumulative-detection times | 1   | 9   | 19  | 33  | 45  | 55  | 61  |
|               | cumulative-detection photons |       |       |       |       |       |       |       |

Table 2 showed the cumulative number of detections of three waveforms increased with the increase of pulse width; the cumulative number of detected photons of rectangular pulse was constant; that of Gaussian pulse decreased 0.5936; that of triangular pulse decreased 0.3824.

For the reason of analysis, the rectangular pulse width increases, but the number of photons in each time interval is still higher than the threshold, the cumulative number of detected photons of the rectangular pulse remains unchanged, so the detection probability of rectangular pulse remains basically unchanged; False alarm probability increases with the increase of detection probability and cumulative times. Therefore, it is concluded that different pulse widths of rectangular pulses have little effect on the detection performance. The Gaussian and triangular pulse widths increase, and the number of photons in each time interval gradually decrease. Particularly, the numbers of photons at both ends of the Gaussian and triangular pulse are lower than the threshold, so that the cumulative numbers of detected photons gradually reduce, the detection probabilities decreases. The cumulative-detection times increases and the false-alarms probabilities of Gaussian and triangular pulse increase. When the pulse width was 0–50ns, the pulse width of Gaussian and triangular pulse had a great influence on the detection performance and cannot be ignored. However, when the Gaussian pulse width was 0–10ns and the triangular pulse width was 0–20ns, the detection probability and the false-alarm probability of Gaussian and triangular pulse changed around 1%, and the influences of the pulse width on the detection performance can be ignored. Therefore, it is concluded that when the Gaussian pulse width is smaller than 10ns and the Triangle pulse width is smaller than 20ns, the influences of different pulse widths on the detection performance are negligible. Comparing the three waveforms, the variation of Gaussian pulse width has the greatest impact on the detection performance, while the rectangular pulse width has no effect on the detection performance basically. The reason is analyzed: rectangular pulse has the highest utilization rate of energy, triangle pulse has the second, Gaussian pulse has the lowest. There are many low energy distributions at both ends of the triangle and Gaussian pulse, which cannot generate response. As the pulse width increases,
the energy is more dispersed, the energy utilization rate is further lowered, and the detection performance is further worse.

In conclusion: 1) when the pulse width is smaller than time bin, the detection performances of different pulse widths are basically equal; 2) the Gaussian pulse width has the greatest influence on the detection performance of single-photon detector; 3) when pulse is less than 10ns, the influence of different pulse widths of the three waveforms on detection performance can be ignored.

4. Conclusion
This paper mainly studies the basic theoretical level problems based on single pulse cumulative detection. The content includes the detection performance of different waveforms based on single pulse cumulative detection, and that of three waveforms with different pulse widths. The results showed the detection performance of rectangular pulse was better than that of triangular pulse, and the Gaussian pulse was the worst. However, due to the small difference in the number of photons detected by the three waveforms, the detection performances were not much different, and the detection probability was around 91.5% and false alarm probability was within 2%. The Gaussian width had the greatest influence on the detection performance, the detection probability reduced 8.1% within 0–50ns, and the false-alarm probability increased 2.75%. The pulse width of the triangular pulse had a slightly smaller influence, the detection probability reduced 5.35% and the false-alarm probability increased 2.28%, and the influence of the pulse width of the rectangular pulse was negligible. When the pulse width is within 0–10ns, the influence of the pulse width variation of the three waveforms on the detection performance is negligible.

Acknowledgements
This work was supported by the National Natural Science Foundation of China (Grant No.61871389) and the Research Plan Project of the National University of Defense Technology (Grant No.ZK18-01-02).

References
[1] Johnson S , Gatt P , Nichols T:Proceedings of SPIE - The International Society for Optical Engineering(2003).
[2] Fouche D G:Applied Optics (2003)
[3] Markus H:Applied Optics (2005).
[4] GUO Ying: University of Chinese Academy of Science (2011).
[5] HOU Libing: University of Chinese Academy of Science (2013)
[6] XU Wei, ZHANG Yu, ZHANG Yong, et al:Chinese Journal of Lasers(2012).
[7] Luo Hanjun:Huazhong University of Science and Technology(2013).
[8] Hanjun L , Zhengbiao O , Qiang L , et al.: (2017).
[9] Dai Yongjiang: Publishing House of Electronics Industry (2010).
[10] Johnson S , Gatt P , Nichols T: Proceedings of SPIE - The International Society for Optical Engineering (2003).
[11] Oh M S , Kong H J , Kim T H , et al.: Journal of the Optical Society of America A (2011).
[12] Zhou Bingzhen. : National Defense Industry Press (2014).