**Stress Ratio Method to Predict Fracture Pressure Gradient in Southern Iraqi Deep Wells**

Nagham Jasim Al – Ameri

Petroleum Engineering Dep. – College of Eng. – Baghdad University

**Abstract**

This research presents a method for calculating stress ratio to predict fracture pressure gradient. It also, describes a correlation and list ideas about this correlation.

Using the data collected from four wells, which are the deepest in southern Iraqi oil fields (3000 to 6000) m and belonged to four oil fields. These wells are passing through the following formations: Y, Su, G, N, Sa, Al, M, Ad, and B. A correlation method was applied to calculate fracture pressure gradient immediately in terms of both overburden and pore pressure gradient with an accurate results.

Based on the results of our previous research, the data were used to calculate and plot the effective stresses. Many equations relating horizontal effective stress and vertical effective stress are obtained for each well and used to calculate fracture pressure gradient. Similar equations are found for group of formations that calculate fracture pressure gradient and to find the most accurate correlation among them.
Introduction

Rock at depth is subjected to stresses resulting from the weight of overlying strata and from locked in stresses of tectonic origin (see fig. 1) [2]. When an opening is excavated in this rock, the stress field is locally disrupted and a new set of stresses are induced in the rock surrounding the opening. Knowledge of the magnitudes and directions of these insitu and induced stresses is an essential component of underground excavation design since, in many cases, the strength of the rock is exceeded and the resulting instability can have serious consequences on the behavior of the excavations.

Fig. (1) Stress elements (After Hubbert and Willis, 1957)[2].

The instantaneous shut in pressure (ISIP)[3] recorded during or after a fracturing job provides a good approximation to the minimum principal insitu total stress component $\sigma_{H_{\text{min}}}$. The vertical total stress $\sigma_V$ (normally the maximum principal insitu stress) can be derived by integration of the formation density, compensated (FDC) log. The intermediate principal stress, $\sigma_{H_{\text{max}}}$ can be assumed approximately
equal to the minimum principal stress ($\sigma_{\text{Hmin}} \leq \sigma_{\text{Hmax}} < \sigma_v$), but in the more general case, when $\sigma_{\text{Hmax}} > \sigma_{\text{Hmin}}$, the value of $\sigma_{\text{Hmax}}$ in principle can be derived from the formation breakdown pressure as measured at the start of a fracturing job. In practice, $\sigma_{\text{Hmax}}$ cannot be determined accurately because the breakdown value will be influenced by: hole geometry, hole integrity, mud-cake properties, and the extent to which the fluid penetrates in and pressurizes the pore space around the borehole. Also, hydraulic fracturing data are often lacking in many areas of the world. The only remaining way, then, to obtain information relating to insitu stresses is to analyze the results of formation integrity tests (leak-off test and casing seat test).

**Review of stress ratio analysis**

Using leak-off test (LOT) data, many authors have derived fracture gradient correlations to be used in planning drilling programs. All correlations methods begin with the fracture gradient equation as follows:

$$G_f = G_p + K (G_{ov} - G_p) \quad \ldots \ldots \ldots \ldots \ldots \ldots (1)$$

Where:

$G_f = \text{fracture pressure gradient, psi/ft.}$

$G_p = \text{pore pressure gradient, psi/ft.}$

$G_{ov} = \text{overburden pressure gradient, psi/ft.}$
The parameter \( K \) then is correlated with depth, \( D \), overburden gradient, \( G_{ov} \), or porosity, \( \Phi \) (for shale).

For gravitationally loaded rock mass in which no lateral strain was permitted during formation of the overlying strata, the value of \( K \) is independent of depth [2], and is given by:

\[
K = \left[ \frac{\nu}{1-\nu} \right] \quad \text{................................. (2)}
\]

Where \( \nu \) is the Poisson's ratio of the rock mass.

An expression for the minimum effective principal stress in term of overburden stress was made using Hooks law of elasticity [2]. The total fracture pressure, \( F_P \), that is required to keep open and extend a fracture is given by:

\[
F_P = \sigma_{(3)\text{eff}} + P_p \quad \text{................................. (3)}
\]

Where:

\( F_P \) = fracture pressure, psi.

\( P_p \) = pore pressure, psi.

\( \sigma_{(3)\text{eff}} \) = minimum effective stress \( \left( \frac{\sigma_{\text{eff}}}{\sigma_{\text{eff}}} = \frac{\nu}{1-\nu} \right) \).

Therefore, the stress ratio was expressed in term of Poisson's ratio \( \frac{\nu}{1-\nu} \), And with the assumption of a constant value of Poisson's
ratio (v=0.25, according to the authors), the stress ratio will be constant as a result [2].

The concept of a variable horizontal – to – vertical stress ratio was introduced as a function of depth, derived from the commonly equation [4]:

\[
F_p = K_i \left( \frac{\sigma_{(1)\text{eff}}}{D} \right) + P_p
\]

Where:

\( \sigma_{(1)\text{eff}} \) = greatest effective stress.

\( K_i \) = matrix stress coefficient for the depth at which the value of \( \sigma_{(1)\text{eff}} \) would be normal.

D = depth.

The authors used the available measured data, \( G_{p(mea)} \), \( G_{f(mea)} \) to calculate the matrix stress coefficient \( K_i \):

\[
K_i = \frac{G_{f\text{meas}} - G_{p\text{meas}}}{1 - G_{p\text{meas}}}
\]

With the assumption of constant overburden pressure gradient (\( G_{ov} = 1 \) psi/ft). Then \( K_i \) is plotted as a function of depth.

The stress ratio was correlated with depth but constant overburden pressure gradient was not, so the stress ratio became:

\[
K_i = \frac{G_{f\text{meas}} - G_{p\text{meas}}}{G_{ov} - G_{p\text{meas}}}
\]
A modification to Hubbert and Willis equation was made in which both overburden stress and Poisson's ratio are assumed to be variable [6]:

\[ F_p = \left( \frac{\nu}{1-\nu} \right) \sigma_{(1)eff} + P_p \] ..........................(7)

Where:

\[ \sigma_{(1)eff} = \sigma_v - P_p \]

Rearranging the above equation, the author plotted Poisson's ratio against depth using measured data.

\[ \frac{\nu}{1-\nu} = \left[ \frac{G_f - G_p}{\left( \frac{\sigma_v - P_p}{D} \right)} \right] \] ..........................(8)

Nevertheless, Eaton's procedure as stated by Breekels and Eekelen, 1982[3] is regarded to be unnecessary and somewhat dangerous complication because it might create the wrong impression that the effective stress ratio can be accurately determined by measuring Poisson's ratio \( \sigma \) on a core.

All the previous illustrated procedures are based on finding a correlation between horizontal to vertical effective stress ratio and depth. But soon it was concluded that this procedure generated a very poor correlations, which can be attributed to variation in the depth of the top of the abnormal pore pressure zone and the rate of change of
the pore pressure. To minimize these factors Brennan and Annis, 1984 [7] made a correlation between effective horizontal stress gradient versus effective vertical stress gradient. By this plot, the depth problem was eliminated and pore pressure effects minimized.

**Theory of proposed method**

Formation fracture gradient predictions have been given a considerable attention over the past years. The model developed by Hubbert and Willis, 1957 [2] has provided the foundation of the majority of the proposed method. In the proposed method, overburden pressure has been either assumed as 1.0 psi/ft. or more correctly, evaluated from velocity data, sonic logs, or density log. Pore pressure gradient has either been assumed as normal pressured (0.44 psi/ft) or, in the case of abnormal pressures, evaluated from resistivity (conductivity) logs, and sonic measurement. So, accurate calculation of pore and overburden pressure gradient can be obtained. The effective stress ratio ($K$) is usually developed as an empirical function of depth. The effective stress ratio is calculated using equation (6):

$$K_i = \frac{\text{effective horizontal stress gradient}}{\text{effective vertical stress gradient}} = \frac{EHSG}{EVSG} = \frac{G_k - G_p}{G_{ov} - G_p}. \hspace{1cm} \text{(9)}$$

A correlation between horizontal to vertical effective stress ratio and depth has been developed. But soon it was concluded that this procedure generated very poor correlation, which can be attributed to variation in the depth of abnormal pore pressure zone and the rate of
change of the pore pressure. To minimize these factors, EHSG versus EVSG was plotted. By this plot the depth problem was eliminated and pore pressure effects minimized.

**Results and conclusions of proposed method**

Entering the collected data for the studied wells in equation (9), EVSG and EHSG were calculated and drawn fig. (2-5) to find correlation among measured data and to obtain an equation that represent best fitting to that data.

An attempt was made to correlate the data for each well separately, see fig. (2-5) and obtain an equation to calculate fracture pressure gradient (FPG) from the beginning of abnormal pore pressure to the final depth. These equations are shown bellow, and the resulted FPG was illustrated in tables (1) to (4).

For well A; \[(G_f - G_p) = 1.08 (G_{ov} - G_p)^2 - 0.032(G_{ov} - G_p) + 0.108\] ..........(10)

For well B; \[(G_f - G_p) = 0.857 (G_{ov} - G_p)^2 - 1.361(G_{ov} - G_p) -0.047\] ..........(11)

For well C; \[(G_f - G_p) = 0.498 (G_{ov} - G_p)^2 + 0.404(G_{ov} - G_p) + 0.015\] ..........(12)

For well D; \[(G_f - G_p) = -0.521 (G_{ov} - G_p)^2 + 0.146(G_{ov} - G_p) + 0.084\] ..........(13)
Another attempt was made to divide the correlation due to pore pressure behavior: First, the correlation was developed for all studied wells from the beginning of abnormal pore pressure gradient until we reach the maximum value of it which was presented in $G$ (Salt-Anhydrate cycles) formation. This gives an equation to calculate FPG for all studied wells from the beginning of abnormal pore pressure till $G$ formation fig. (6), this equation is:

$$\left(G_f - G_p\right) = 1.211 \left(G_{ov} - G_p\right)^2 + 0.05 \left(G_{ov} - G_p\right) + 0.053 \quad \ldots \ldots \quad (14)$$

Second, the correlation was developed starting from $G$ formation (the maximum pore pressure gradient) and down ward to get an equation which calculates FPG for all studied wells from $G$ formation and down ward fig. (7), this equation is:

$$\left(G_f - G_p\right) = 0.353 \left(G_{ov} - G_p\right)^2 + 0.939 \left(G_{ov} - G_p\right) - 0.025 \quad \ldots \ldots \quad (15)$$

The above two equations (14) & (15) were used to calculate FPG using overburden pressure gradient (from bulk density log) and pore pressure gradient (using drilling and log data). The results were illustrated in Tables (1-4) for the studied wells.

Using the AAPE (absolute average percent error), the degree of accuracy for the two attempts for calculating FPG were illustrated in Table (5). The result AAPE for each attempt was compared with AAPE of the concluded best method of our previous research (Eaton's method). The results AAPE give a conclusion that the obtained
equations for each well separately was most accurate than equations obtained according to pore pressure behavior, and it was most accurate than Eaton's method (which was also applied for each well). This can be attributed to variation in G formation thickness for each well and due to missing formations in some wells, such as N formation and down. This appears from the closed values of AAPE (well equation) and AAPE (pore pressure behavior equations) for wells (A & C) where the same thickness of G formation is present with compared to other wells. While, in well (B) the formation Sa and down is present while it's not noticed in other wells, so its well equation gives the best AAPE than pore pressure behavior equation. In well (D), well equation gives best AAPE than pore pressure behavior equations due to missing in G formation and down ward.

As a conclusion, these correlations using the measured data give best AAPE among all other methods to estimate FPG.

![Graph](image)

**Fig. (2) Correlation of effective horizontal stress vs. effective vertical stress for well A.**
Fig. (3) Correlation of effective horizontal stress vs. effective vertical stress for well B.

Fig. (4) Correlation of effective horizontal stress vs. effective vertical stress for well C.
Fig. (5) Correlation of effective horizontal stress vs. effective vertical stress for well D.

\[ y = -0.521x^2 + 0.146x + 0.084 \]
\[ R^2 = 0.240 \]

Fig. (6) Correlation of effective horizontal stress vs. effective vertical stress for all wells (G formation and up).

\[ y = 1.211x^2 + 0.050x + 0.053 \]
\[ R^2 = 0.822 \]
Fig. (7) Correlation of effective horizontal stress vs. effective vertical stress for all wells (G formation and down).
Table (1) Results of fracture pressure gradient for well A.

| Depth, m | $G_f$ (meas.) | $G_f$ ($P_p$ behavior eq.) | $G_f$ (well eq.) | formation |
|----------|---------------|----------------------------|------------------|-----------|
| 3400     | 0.88765       | 0.91568                    | 0.891083         | R         |
| 3500     | 0.898         | 0.917007                   | 0.892191         | R         |
| 3600     | 0.9093        | 0.901884                   | 0.902187         | y         |
| 3700     | 0.92          | 0.905025                   | 0.913703         | y         |
| 3800     | 0.922         | 0.910951                   | 0.926711         | y         |
| 3900     | 0.93095       | 0.926509                   | 0.951633         | su        |
| 4000     | 0.9396        | 0.942612                   | 0.973232         | su        |
| 4100     | 0.9526        | 0.964894                   | 1.001787         | G         |
| 4200     | 0.955         | 0.991071                   | 1.032848         | G         |
| 4300     | 1.0067        | 1.0184                     | 1.064535         | G         |
| 4400     | 1.01755       | 1.023198                   | 1.033363         | N         |
| 4500     | 1.0176        | 1.028414                   | 0.987239         | N         |
| 4600     | 1.0176        | 1.030584                   | 0.988069         | N         |
| 4700     | 1.01755       | 1.032757                   | 0.988908         | N         |
| 4800     | 1.01755       | 1.032866                   | 1.005412         | N         |
| 4900     | 1.01755       | 1.032591                   | 1.00795          | sa        |
| 5000     | 1.0176        | 1.035136                   | 1.015475         | sa        |
| 5100     | 1.0176        | 1.038136                   | 1.018475         | Al- M     |
| 5200     | 1.0177        | 1.041036                   | 1.022524         | M-Ad      |
Table (2) Results of fracture pressure gradient for well B.

| Depth, m | $G_f$ (meas.) | $G_f$ ($P_p$ behavior eq.) | $G_f$ (well eq.) | formation |
|----------|---------------|----------------------------|------------------|-----------|
| 3600     | 0.90064       | 0.937571                   | 0.906324         | y         |
| 3700     | 0.9093        | 0.939992                   | 0.906933         | y         |
| 3800     | 0.93961       | 0.924127                   | 0.938149         | y         |
| 3900     | 0.9959        | 0.913565                   | 0.964676         | y         |
| 4000     | 1.00456       | 0.908666                   | 0.992768         | su        |
| 4100     | 1.00889       | 0.910575                   | 1.005681         | su        |
| 4200     | 1.01322       | 0.913419                   | 1.013974         | G         |
| 4300     | 1.0262        | 0.960308                   | 1.039572         | G         |
| 4400     | 1.03054       | 0.961109                   | 1.039265         | G         |
| 4500     | 1.03487       | 0.945163                   | 1.034972         | G         |
| 4600     | 1.037035      | 0.973766                   | 1.029629         | G         |
| 4700     | 1.039         | 0.979241                   | 1.028446         | G         |
| 4800     | 1.0392        | 1.02104                    | 1.030075         | N         |
| 4900     | 1.02621       | 1.029771                   | 1.031355         | N         |
| 5000     | 1.02621       | 1.032896                   | 1.032651         | N         |
| 5100     | 1.026         | 1.03963                    | 1.032729         | N         |
| 5200     | 1.02188       | 1.0402                     | 1.034303         | N         |
| 5300     | 1.0262        | 1.034991                   | 1.039354         | N         |
| 5400     | 1.0392        | 1.037117                   | 1.039088         | N         |
| 5500     | 1.04786       | 1.040891                   | 1.039794         | sa        |
| 5600     | 1.06085       | 1.037219                   | 1.036942         | Al- M     |
| 5700     | 1.045695      | 1.030841                   | 1.039708         | M-Ad      |
| 5800     | 1.0392        | 1.031408                   | 1.039885         | B         |
Table (3) Results of fracture pressure gradient for well C.

| Depth, m | $G_f$(meas.) | $G_f$(Pp behavior eq.) | $G_f$(well eq.) | formation |
|----------|--------------|-------------------------|-----------------|-----------|
| 3400     | 0.8387       | 0.856335                | 0.837842        | R         |
| 3500     | 0.866        | 0.856738                | 0.854988        | y         |
| 3600     | 0.8673       | 0.864477                | 0.866687        | y         |
| 3700     | 0.868        | 0.873089                | 0.878237        | y         |
| 3800     | 0.87033      | 0.886575                | 0.892494        | su        |
| 3900     | 0.90064      | 0.899254                | 0.904366        | su        |
| 4000     | 0.9526       | 0.918853                | 0.920971        | su        |
| 4100     | 0.957        | 0.946244                | 0.942342        | su        |
| 4200     | 0.96126      | 0.976794                | 0.964438        | G         |
| 4300     | 0.96126      | 0.977371                | 0.965507        | G         |
| 4400     | 0.96126      | 0.977839                | 0.966363        | G         |
| 4500     | 0.96126      | 0.977958                | 0.966579        | G         |
Table (4) Results of fracture pressure gradient for well D.

| Depth, m | $G_f$ (meas.) | $G_f$ ($P_p$ behavior eq.) | $G_f$ (well eq.) | formation |
|----------|---------------|----------------------------|------------------|-----------|
| 3500     | 0.71          | 0.807274                   | 0.720056         | R         |
| 3600     | 0.72          | 0.810215                   | 0.722254         | R         |
| 3700     | 0.73          | 0.813626                   | 0.728518         | R         |
| 3800     | 0.751         | 0.822993                   | 0.747841         | y         |
| 3900     | 0.773         | 0.830064                   | 0.760385         | su        |
| 4000     | 0.7803        | 0.838668                   | 0.776782         | su        |
| 4100     | 0.81025       | 0.8556                     | 0.820238         | su        |
| 4200     | 0.899         | 0.893166                   | 0.897101         | G         |

Table (5) AAPE for each well using the two correlations.

| Well | AAPE (well eq.) | AAPE ($P_p$ behavior eq.) | Eaton’s method | Thickness of G formation, m |
|------|----------------|---------------------------|----------------|-----------------------------|
| A    | 2.159%         | 1.5447%                   | 2.19%          | 300                         |
| B    | 0.803%         | 4.0086%                   | 3.4%           | 600                         |
| C    | 1.024%         | 1.46%                     | 3.6%           | 400                         |
| D    | 0.735%         | 8.547%                    | 3.8%           | 120                         |
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