Auto-oscillations in complex plasmas

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Abstract. Experimental results on an auto-oscillatory pattern observed in a complex plasma are presented. The experiments are performed with an argon plasma, which is produced under microgravity conditions using a capacitively coupled rf discharge at low power and gas pressure. The observed intense wave activity in the complex plasma cloud correlates well with the low-frequency modulation of the discharge voltage and current and is initiated by periodic void contractions. Particle migrations forced by the waves are of long-range repulsive and attractive character.

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1. Introduction

The ability to self-sustain oscillations is typical for auto-oscillation systems with inertial self-excitation such as the Helmholtz resonator, well known in acoustics [1].

Hydrodynamic (or hydrodynamic-like) systems provide other examples of oscillatory patterns fed by streaming in the system: a flow-acoustic resonance [2], hydrothermal [3] and plastic deformation flows [4], mobile dunes [5], surface tension auto-oscillations [6], oscillating domains in planar discharges [7], self-excited dust density waves [8] and many others.

The usual assumption is that auto-oscillations are maintained by a sufficiently powerful instability allowing recirculation (hysteresis) in phase space [9, 10].

In this paper, we discuss self-sustained oscillatory and wave patterns observed in complex plasmas with the PK-3 Plus setup on board the ISS [11]. These active experiments (including as an item wave excitations [12]) have been performed recently in a particularly wide range of plasma parameters [11].

Complex plasmas are low-pressure low-temperature plasmas containing microparticles. These particles can be visualized individually with a laser beam, the light of which is scattered by the particles and then recorded with a CCD camera. Under microgravity conditions in experiments on board the ISS [11] or in parabolic flight experiments [13], the recorded particle clouds are essentially three-dimensional structures that are more or less homogeneous, albeit commonly containing a void—a ‘visibly empty region’ free of particles—at the center [14, 15].

By providing investigations of microparticle migrations at the atomistic level, such experiments may help us to understand the fundamental nature of inanimate auto-oscillations and the intrinsic dynamics of highly dissipative nonlinear structures. In particular, we will show that for fluid systems a range of processes may occur (at the microscopic level), which all play a role in the self-organization and finally lead to a simple auto-oscillatory pattern for the system as a whole.
Stable regular auto-oscillations have been observed for the first time in the experiments under microgravity conditions. It is still a challenging open question to explain the physics of this phenomenon. The intention of the paper is twofold:

- detailed analysis of available experimental data;
- focus attention on the physical mechanism of auto-oscillations.

The paper is organized as follows: in section 2, the observation results are collected. This includes the experimental conditions (section 2.1), the dynamical patterns shown by the particle vibrations (section 2.2), their correlation with measured low-frequency oscillatory components of the voltage and the current of the feeding circuitry (section 2.3), and the parameters of the complex plasma (section 2.4). The capability of a complex plasma to generate auto-oscillations (in this respect complex plasmas resemble many other dynamical systems) is discussed in section 3 and physical aspects of auto-oscillations in complex plasmas in section 4. We propose a simple electromechanical model description. To ensure that our simple scenario is valid, we check the local consistency of the time and space scales (section 4.1), the charge balance (section 4.2), the momentum and the energy balance (4.3), and possible trigger (section 4.4) and driving (section 4.5) mechanisms for oscillations. In section 5, the results are briefly summarized.

2. Observation results

2.1. Experimental conditions

The PK-3 Plus chamber, a parallel plate capacitively coupled rf discharge, is symmetrically driven by two electrodes, which have a diameter of 6 cm and are separated by 3 cm (measured voltage asymmetry does not exceed \(\simeq 2\%\); all measured electrical values shown below are arithmetic means). The electrodes are surrounded by a grounded ring of 9 cm diameter and 1.5 cm width. (More technical details of the setup can be found in [11].) Particle vibrations were recorded at a rate of 50 fps and a spatial resolution of 45.05 \(\mu\)m pixel\(^{-1}\) (49.6 \(\mu\)m pixel\(^{-1}\)) in the vertical (horizontal) direction. The sample rate of the low-frequency electrical measurements was 10 Hz at the stage of stable vibrations.

The experiment we address here was performed in argon at a pressure of 9 Pa and was arranged in two stages (figure 1). In the first stage, the discharge was ignited with a peak-to-peak voltage of 37 V at an applied (rms) power of 0.181 W (the discharge power factor for the entire circuitry was estimated as about 45–60%). Melamine-formaldehyde particles with a diameter of \((9.2 \pm 1\%) \mu\)m and a mass density of 1\,51 g cm\(^{-3}\) were inserted into the chamber. They formed a cloud stretched horizontally (the aspect ratio width/height \(\equiv D/H \approx 64 \text{ mm}/15 \text{ mm}\)) with a visually pulsating elliptically shaped void. In the maximal ‘stretched’ phase the void is \(\sim 7 \times 3 \text{ mm}^2\). This discharge regime allows us to observe stable oscillations at a frequency of 3–15 Hz. The estimated gas damping rate, attenuating particle motion, was \(\gamma_{\text{damp}} = 10.7 \text{ s}^{-1}\). Unlike [16]–[18] no contaminating (sputtering) components affected the discharge.

This is one of the main differences of this experiment compared to [17]: the particles are injected, not grown in a plasma, and the particle size is larger (at least one order of magnitude).

The experiment started out with a stochastically stabilized complex plasma \((t = 0 \text{ s in figure 1})\; \text{for details of stochastic stabilization see [11]}\). At \(t = 20 \text{ s}\), the external stabilization was turned off, and after \(\Delta t \simeq 1 \text{ s}\) delay the auto-oscillations appeared. Next at \(t = 31 \text{ s}\) the
plasma was stimulated by a series of six short-time voltage pulses produced by the function generator. The pulses, with a negative amplitude of $-50$ V, were applied to the bottom electrode through a $20 \text{k}\Omega$ feed resistor. This produces short-time dc voltage shifts of a few volts, which leads to shock compression of the particle cloud. After this the system was observed to freely oscillate for $\approx 83$ s without external forcing.

After $\approx 150$ s the applied power was lowered to 0.12 W, and another form of the heartbeat instability with its almost irregular large-amplitude void constrictions started. The stable oscillation phase we address here would seem to be a completely different phenomenon. Details of the unstable (heart-beat) phase will be published elsewhere.

2.2. The global dynamical pattern and oscillons

Figure 2 visualizes the dynamical activity. Surprisingly the ‘shaking’ divides the particle cloud into two counter-moving parts, which form a stagnation zone with nearly zero particle velocity at the interface. Outside the stagnation zone the particles are seen to move along the axis at nearly constant velocity (in this particular half-cycle) as if they were attracted by the void. Apparently, in this phase the void behaves like a negatively charged probe, and long-range attraction is due to the ion collection effect [19]. In the next half-cycle, as the void tends to close, the particles are repelled from the center.

Simple estimates based on particle behavior analysis show a rather weak plasma charge oscillation with maximal decompensation of the order of $\delta n/n = 0.5\text{–}1\%$ inside the stagnation zone.

The fluctuation spectrum of the oscillating particle cloud is shown in figure 3. It consists of discrete lines, indicating regular nonlinear vibration, and the dust fluctuations with a continuous spectrum $\omega \propto k$ at a mean speed of $6.4 \pm 0.3 \text{ mm s}^{-1}$. Waves in the continuum are identified as dust acoustic waves.

The periodic auto-oscillation pattern formed by particle horizontal vibrations is shown in figure 4. For depicting the oscillation pattern, we follow a simple procedure proposed in [8].
Figure 2. Ten superimposed images shifted by one period in time are shown to reveal the global dynamical pattern of the complex plasma. The main elements are: a void (the dark elliptic-shaped area to the right, at this active open stage), a quasi-spherical halo (highlighted by the dash-dotted line) with concentric waves spreading around the vibrating void, two horizontal counter-rotating global vortices with angular velocity ≃ 0.2 s\(^{-1}\) and horizontal radius ≃ 7.5 mm, and an edge ‘buffer’ zone to the left. The boundaries of the vortices form a ‘waveguide’ for oscillons in-between (oscillons are identified in the middle as a few brighter vertical stripes). For the given half-cycle the dominant particle motion is as indicated by the arrows. The stagnation zone with nearly zero particle velocity is located at \(x \sim 5\) mm (see also figure 7). The semi-transparent arrows indicate general particle drifts. The dashed lines cross at the position of the void center. The illuminating laser sheet FWHM is about 80 µm.

From each image of the recorded sequence a narrow slab of size 35 × 5 mm\(^2\) centered over the void’s vertical position is extracted. Then the slab is ‘compressed’ into a line by adding up the pixel intensities perpendicular to its longer side. The result is plotted (as a periodgram) for every frame as shown in figure 4, forming a \((x, t)\) map. The brighter regions of this map correspond to higher particle densities. The darkest regions are particle-free, which means that the void is maximally open. The fundamental oscillation frequency shown by the cloud is

\[
f_{\text{osc}} = 2.81 \pm 0.03 \text{ Hz.} \tag{1}\]

The lines of the spectrum shown in figure 3 form a sequence of harmonics of fundamental frequency \(f_n = n f_{\text{osc}}, n = 1, 2, 3, \ldots\). All harmonics up to the fourth order can be properly identified.

2.3. Correlation between particle vibrations and electrical signals

Figure 5 shows the correlation between the vertical cloud oscillations and all electrical signals. There is no correlation with the applied power. To depict the pattern of vertical oscillations the
Figure 3. Spectral characteristics of the auto-oscillating complex plasma. The spectral energy is shared between two components: (a) a sound-like continuum (dust acoustic waves) and (b) discrete spectral lines. The two bottom panels show the spatial distribution (c) of the spectral intensity of the mean particle velocity fluctuations (I) and (d) the phase variation (φ) at the fundamental frequency. Both parameters are distributed inhomogeneously along the cloud. The spectrum (a) was calculated by making use of over 2700 particle trajectories. The spectrum (b) of the density fluctuations was obtained locally (5.5 mm < x < 6.5 mm) inside the stagnation zone. Note that the dashed line in (a) with the slope $C_{DAW} = 6.4$ mm s$^{-1}$ fits well to the DAW spectrum.

The aforementioned procedure is carried out over a narrow vertical slab of size $2 \times 20$ mm$^2$. The result is shown in figure 5(a) as a superposition of consecutive periods.

A comparison with the oscillatory components of the effective voltage and current (figures 5(b) and (c)) shows an almost anti-phase behavior that is typical of pattern-creating gas discharges (see e.g. [20]). The mean components of these signals (corrected for measurement offset errors) are

$$\langle I \rangle = 3.24 \pm 0.03 \text{ mA}, \quad \langle U \rangle = 12.70 \pm 0.02 \text{ V}. \quad (2)$$

From this we obtain an average ohmic discharge resistance of $\approx 4$ kΩ. (Without particles the plasma resistance was $\approx 3.4$ kΩ. This value agrees well with that estimated from [21].)
Figure 4. Periodogram showing horizontal oscillations of the cloud. The brighter spikes in the middle (indicating enhanced particle density) are oscillons (see also figure 2) slowly propagating towards the outer edge of the cloud (to the left) at an approximately constant speed of 0.4 mm s\(^{-1}\) away from the pulsating void (the horizontal periodic dark stripes to the right). The dashed line indicates the position of the void center. The life-time of the oscillons is \(\simeq 20\) s, i.e. about 200 damping times. Oscillons are ‘fed’ by the oscillatory energy. The faster edge wave-ridges (see [12]) are also clearly seen at \(x > 25\) mm.

Fitting a VI-curve (figure 6) obtained from the data shown in figure 5(b) and (c) yields (to the main order) a constant negative differential conductivity (NDC; see [22])

\[-dI/dU = 0.41 \pm 0.07\ \text{mA V}^{-1}.\]

(3)

Note that sharp current minima (voltage maxima) are apparently linked to the vertical cloud compression, whereas the void opening relates to a weak rise in the current. All this suggests an association with the variation of the discharge capacitance—an ‘electromechanical effect’. In this sense, the complex plasma resembles a varicap, a device whose capacitance varies under the applied voltage.

Based on this analogy and using an equivalent circuit model [21], one can show that the expected variation of the rms current is of the order

\[
\frac{\delta I}{\langle I \rangle} \approx -\eta \frac{C_{\text{cloud}}}{C}, \quad \eta = \frac{\omega^2 \tau^2}{1 + \omega^2 \tau^2}, \quad \tau = RC,
\]

(4)

where \(R\) is the discharge resistance, \(C\) the discharge capacitance, and \(C_{\text{cloud}}\) the cloud capacitance. Assuming \(\omega \tau \simeq 1\) we estimate \(C \simeq 3\) pF. Hence, the observed oscillation amplitude would be explained if \(C_{\text{cloud}} \simeq 0.4\) pF, which is quite reasonable.

The maximal volume of the void considered as an oblate (disc-shaped) spheroid is

\[V = \frac{4}{3} \pi ca^2 \simeq 0.077\ \text{cm}^3,\]

(5)

where \(a \simeq 0.35\) cm, and \(c \simeq 0.15\) cm are the corresponding semi-axes.
Figure 5. (a) Vertical oscillations of the cloud, (b) the (rms) rf current, (c) rf voltage and (d) applied (forward) rf power versus reduced time ($f_{osc}t$). Two ‘reconstructed’ oscillation periods are shown. Reconstruction, based on superposition of the data points taken over 20 consecutive periods (phase 3 in figure 1), demonstrates clearly local quasi-periodicity of the discharge electrical signals (periodicity time is limited by slowly drifting oscillons). The lack of any periodicity in the applied power signal is also evident. (The forward power is held constant by a servo control loop inside the rf generator.)

The geometric capacitance of such a spheroid is

$$C = \frac{\sqrt{a^2 - c^2}}{\arccos(c/a)} \simeq 0.3 \text{ pF.}$$

(6)

This agrees surprisingly well with the above estimate derived from current variation.

2.4. Plasma parameters

From probe measurements [23] we estimate the plasma parameters as $n_e \sim 10^8 \text{ cm}^{-3}$, $T_e \sim 2$–$3 \text{ eV}$. The interparticle separation averaged over the entire cloud area (figure 2) is $\langle \Delta \rangle \simeq 230 \mu \text{m}$. The highest compression occurs at the spikes (figure 4), where the interparticle separation is $\Delta_{\text{min}} = 173 \pm 16 \mu \text{m}$. Outside the spikes compression is less; the interparticle separation is $\langle \Delta \rangle = 300$–$350 \mu \text{m}$. At the kinetic level we see that particles are first accelerated...
to high speeds \((v_{\text{max}} = 18.9 \pm 0.4 \text{ mm s}^{-1})\) and then they are decelerated, forming spikes—oscillons, which are clearly seen in figure 2 as vertically elongated constrictions.

The process of periodic capture and release of particles by oscillons is quite similar to that realized inside the dust density wave fronts \([8]\). Following \([8]\) and using the parameters listed above, we calculated the particle charge \(Z_d \approx 8900\). From \([25]\) it follows \(Z_d \sim 9000\).

Thus taking \(Z_d \approx 9000\), we can estimate now all other dynamical parameters. For instance, the dust sound speed is \(C_{\text{DAW}} \sim 6–7 \text{ mm s}^{-1}\), and the dust plasma frequency is \(f_{\text{dust}} = \omega_{\text{dust}}/2\pi \approx 14–20 \text{ Hz}\).

The fact that the estimated value of \(C_{\text{DAW}}\) agrees well with the measured value (see section 2.2) is worth noting. It indicates that the plasma parameters shown above are properly identified.

It is remarkable also from another viewpoint. In the general case the dust acoustic speed depends equally strongly on the particle charge and plasma density \([24]\). Hence, it is difficult to identify the dominant parameter—either the electron temperature or density—controlling the dynamics. In dense particle clouds if the Havnes parameter \(H = Z_d n_d/n_e\) is large, \(H > 1\) (exactly the case we deal with), the situation might be different. The dust acoustic speed \(C_{\text{DAW}} \propto \sqrt{Z_d(1 + H^{-1})^{-1}}\) is still strongly dependent on the particle charge but only weakly on the electron density. In our case, we estimated \(H \approx 2\); thus a \(\pm 50\%\) variation in the value of \(H\) would give only \(\approx 10\%\) variation in \(C_{\text{DAW}}\). The particle charge is roughly proportional to the electron temperature. Therefore the value of the dust acoustic speed can be a good measure for the electron temperature.

Finally, let us emphasize again that the values of both parameters \(Z_d\) and \(C_{\text{DAW}}\) are obtained from observations: charge—from the dynamics of oscillons, dust acoustic speed—directly from the fluctuation spectrum.
3. Complex plasma as a dissipative system capable of auto-oscillations

The auto-oscillating cloud of the microparticles is one of the most intriguing phenomena detected in experiments with complex plasmas: the complex plasma gives rise to self-excited macroscopic motions—it sets the paradigm of a dissipative system capable of auto-oscillations. (This stable auto-oscillating complex plasma should not be confused with cyclic spatiotemporal microparticle generations \[16\], and the heart-beat instability \[17\] studied in detail in dust-forming plasmas). Depending on discharge conditions and plasma parameters, the complex plasma could be kept stable, or excited externally into an oscillatory state, which even in the presence of damping remains autonomically excited.

We associate low-frequency current and voltage self-pulsations, and the accompanying particle oscillations in complex plasmas, with the negative differential conductivity. In this sense, the complex plasma exhibits properties similar to some types of photoconductors \[22\], semiconductors \[26\], semi-metals \[27\], ferroelectric liquid-crystalline films \[28\], carbon nanotubes \[29\], nanocrystalline heterostructures \[30\] and other microelectromechanical systems \[31, 32\].

The nonlinear dissipative compact formations in the patterns seen in complex plasmas remotely resemble oscillons. These standing undulations can be produced on the free surface of a liquid (so-called Faraday's waves \[33\]), a granular medium (in this case localized excitations can self-organize with possible assembly into 'molecular' and 'crystalline' structures \[34\]), or nonlinear electrostatic oscillations on a plasma boundary \[35\]. Oscillons 'feed' from external shaking of the system, and dissipation seems to be inherent for their existence, likewise in our case. There remain still many open questions in explaining the physical mechanism of oscillons.

Streaming ions may act as an activator of the instability in the complex plasma (see e.g. \[8\]). The formation of a void itself is explained frequently by the counteraction of the confinement force and the ion drag force \[18, 36\]. Void vibrations in an energized complex plasma are believed to be due to the heart-beat instability, the free energy of which may arise from streaming ions \[17\]. There are a number of direct observations of the interaction of streaming ions and dust particles \[36\]–\[38\].

Since straightforward measurements of flowing ions in complex plasmas are not possible to perform without perturbing the particle cloud structure \[39\], evidence might be extricated from the direct observations of particle vibrations. There are two possible options:

1. elucidate the long-range character of particle vibrations;
2. decode the patterns of the secondary wavefronts.

Fortunately, both options are realizable as has been proven by the given experiments.

4. Physical aspects of auto-oscillations in complex plasmas

The observations of an auto-oscillating fluid-like system at the individual (atomistic) particle level have revealed a number of different effects/processes \[40\]–\[42\]:

1. Complex plasmas are thermodynamically ‘open’ systems—energy has to be constantly provided to maintain the plasma against recombination;
2. Complex plasmas are (weakly) dissipative due to neutral friction;
3. Complex plasmas exhibit natural frequencies associated with global modes, dust acoustic wave generation, periodic flow patterns, dust plasma frequency, etc as well as characteristic frequencies due to particle confinement;

4. Self-sustained oscillations, ‘feeding’ on the free energy in the system (ultimately the external supply provided by the rf power) and driven by an instability (e.g. caused by ion–particle drift), are therefore not entirely unsuspected;

5. The ‘kinetic details’, colliding particle flows, generation of waves, oscillons, etc presumably all play the role of optimizing energy dissipation and transporting the oscillatory energy out to the edge of the particle cloud.

The surprising (global energy) result appears to be the agreement of our measurements with a simple electromechanical model description, which does not concern itself with the details and speed (rate) of energy transport (dissipation) by the various means at disposal. This suggests that the system automatically ‘adjusts’ these paths of dissipation according to the minimum action principle—without auto-oscillations free energy would build up and the system would have to expand; when it does, energy is released at a rate that could be equal to or larger than that at which it is supplied. External confinement prohibits continuous expansion as a means of equilibration; hence an oscillatory solution would appear to be the most reasonable. Such a scenario requires that the potential energy of the central region be a significant part of the available local energy—the system is clearly not overdamped. That the process occurs in the spatial regime where ion drag and electrostatic forces approximately balance strongly suggests that the free energy of the ion drift is the source of the instability required.

To assure that this scenario is valid, we must do a charge balance, a momentum balance, and an energy balance involving systematic kinetic energy, potential energy, thermal energy and energy carried away by dust acoustic waves and oscillations. A simple estimate, by restricting the considerations to only the main processes, actually is not difficult to get.

4.1. Time and space scales of the momentum transfer

The local consistency of the timescales, for instance, immediately follows from a comparison of the DAW speed (see section 2.4) and the velocity of the void boundary vibrations (see section 2.1). At each half-cycle the void shrinks in size by a factor of \( k \simeq 1.5 \). The variation of the horizontal (maximal) void semi-axis is of the order of \( \delta x = a(1 - k^{-1}) \simeq 0.12 \) cm, and the mean boundary velocity is then

\[
\langle c \rangle = \frac{\Delta x}{\Delta t} \approx 2f_{\text{osc}} \delta x \simeq 6.6 \text{ mm s}^{-1}.
\]

This is consistent with the above estimated \( C_{\text{DAW}} \sim 6-7 \text{ mm s}^{-1} \) and close to that obtained from the fluctuation spectrum of figure 3. The variation of the void volume, assuming that both axes shorten in the same proportion, can be estimated as

\[
\frac{\delta V}{V} = 1 - k^{-3} \simeq 0.7, \quad \delta V \simeq 0.054 \text{ cm}^3.
\]

On average the rate of volumetric variation is a bit less than the estimate (7) predicts:

\[
\langle c_{\text{vol}} \rangle \approx 2f_{\text{osc}} \langle R \rangle (1 - k^{-1}) \simeq 5 \text{ mm s}^{-1}
\]

but is still of the same order. Here, \( \langle R \rangle = \sqrt{\frac{3}{4\pi}} V \simeq 0.26 \) cm is the averaged size of the maximal open void.
Propagating periodic density modulations with a wavelength of 1–2 mm are clearly seen, e.g. in figure 5(a). At these scales the DAWs are dominantly responsible for the particle cloud elasticity.

4.2. A charge exchange

Every half-cycle the volume (8) should be cleared out of approximately

\[ \delta N_d = \langle n_d \rangle \delta V \simeq 1060 \]  

particles. Here, \( \langle n_d \rangle = \left( \frac{\Delta}{\pi} \right)^3 \simeq 2 \times 10^4 \text{ cm}^{-3} \) is the dust cloud density (see section 2.4). The total number of electrons stuck to these particles is \( Z_d \delta N_d \). This number must be a match for the number of elementary charges that are exchanged while the ‘void-capacitor’ is recharging. According to section 2.2, this number can be estimated as

\[ \delta N_{\text{el}, \text{ch}} \approx \frac{R \langle I \rangle}{\epsilon} \left( \frac{\delta C_{\text{cloud}}}{C_{\text{cloud}}} + \frac{\delta I}{\langle I \rangle} \right). \]  

Numerically these two amounts are close, \( Z_d \delta N_d \approx \delta N_{\text{el}, \text{ch}} \simeq 10^7 \). This is not surprising if the charge balance is fulfilled.

4.3. Momentum and energy balance

In order to examine the global behavior, we investigate the conservation of the horizontal momentum. The momentum of the oscillating particles averaged over the entire oscillation period \( \Delta t \) is naturally equal to zero, \( \langle \delta V_x \rangle_{\text{period}} = 0 \). Hence, also the impulse of the forces acting at the void interface is zero:

\[ \oint_{\Delta t} \delta F_x \, dt = 0. \]  

We separate the acting forces into the ‘electrostatic’ ones \( \delta F_{\text{es}} \), and the ‘ion drag’ forces \( \delta F_{\text{id}} \). Simplifying, we can assume that these counteracting forces play a role dominantly only in one of the consecutive half-cycles. Then the relationship (12) can be approximately rewritten as

\[ (\langle \delta F_{\text{es}} \rangle - \langle \delta F_{\text{id}} \rangle) \Delta t / 2 = 0. \]  

The force \( \delta F_{\text{id}} \) is due to the variation of the ion drift (compared to the global ambipolar ion drift component always directed outwards, i.e. towards the discharge chamber walls, see figure 2). For our parameter set (see section 2.4) it can be estimated from the relationship (see equation (11) of [43]; the relationship is valid in the limiting case of small ion drift velocities):

\[ \delta F_{\text{id}} \simeq 0.02 M_d g \frac{\delta u_i}{v_{T_i}}, \]  

if \( M_T = \frac{\delta u_i}{v_{T_i}} \), the ‘thermal Mach number’ of the drift velocity fluctuation is known. To get a sense of how big \( M_T \) is, we note that far away from the void boundary where the particles move at an approximately constant speed \( |v| \simeq 4–5 \text{ mm s}^{-1} \) (figure 7), the force (14) should be compensated for by the friction force \( \delta F_{\text{fr}} \equiv M_T \gamma \text{damp} \delta v \). This balance yields the magnitude of the ion drift velocity fluctuation:

\[ \frac{\delta u_i}{v_{T_i}} = 0.2–0.3, \]  

which is quite reasonable.
Figure 7. Time–space map of the horizontal velocity of the particles. Particle positions are identified, their velocities are calculated in consecutive frames, and shown color-coded, using a narrow horizontal slab $1.2 \times 30$ mm$^2$ crossing the void center. The void is clearly seen to the right as the darker region with (conventionally) zero velocity. It should not be confused with the stagnation zone located at $x \sim 5$ mm on the first and last quarters of the period, and at the middle of the period. The enhanced counter streaming advancing and retreating particle fluxes are also clearly seen there. Note that outside this region at $x > 7$ mm the incoming and outgoing particle fluxes are formed practically simultaneously at all distances, indicating the presence of the ‘latent’ ion drift activating motion of the particles. To reduce the stochastic noise, the data points are taken and smoothed over 20 consecutive periods (the same as in figure 5).

Next, after introducing the electrostatic force by the relationship

$$\delta F_{es} = Z_d e E, \quad E \approx -\frac{\delta \varphi}{\delta x},$$

the force balance (13) allows us to estimate:

$$\delta \varphi \approx 0.03 \text{ V}.$$  \hspace{1cm} (17)

This result is in fairly good agreement with that measured in [36].

Finally, the energy balance written in the form:

$$W_{\text{ext}} = \Delta W + 2\langle W_{\text{kin}} \rangle_{\text{damp}} \Delta t, \quad W = W_{\text{pot}} + W_{\text{kin}},$$

yields the work $W_{\text{ext}}$ of the external forces needed to compensate for frictional losses. Here $W_{\text{pot}} = Z_d e \delta N_d \delta \varphi$ is the electrostatic potential energy, $W_{\text{kin}}$ is the kinetic energy, and $\Delta W$ is the variation of the total energy during the time of an oscillation $\Delta t$.

In the beginning of every expansion cycle the particles first accelerate, acquiring kinetic energy, and then decelerate. Let us adopt a simple spherical model of the void of size $a$. To consider the last stage, we put for the particles’ speed a distribution

$$v = v_m \frac{a - r}{a}, \quad 0 < r \leq a,$$

where $v_m$ is the maximal particle velocity; particles stop at $r = a$. Initially the particle density is constant, $n_d = \delta N_d / V, \quad V = \frac{4}{3} \pi a^3$; next, at expansion it is $\sim \delta (r - a)$. The maximal kinetic
energy ‘to clear the void’ is then

\[ W_{\text{kin}} = \int_{0}^{a} \frac{1}{2} M_{d} n_{d} v^{2} 4\pi r^{2} dr = \frac{1}{20} M_{d} v_{m}^{2} \delta N_{d}. \]  

(20)

At \( v_{m} \sim 5 \text{ mm s}^{-1} \), we have \( W_{\text{kin}} \sim 5 \text{ keV} \), that is, roughly 5 eV per particle.

The directed kinetic energy dissipation rate in every expansion–contraction cycle is \( \dot{W}_{\text{kin}} = 2W_{\text{kin}} f_{\text{osc}} \sim 30 \text{ keV s}^{-1} \). This is an elastic, ‘recoverable’, part of dissipation: The kinetic energy is returned to the particles by work done by potential forces. The average inelastic (frictional) energy loss is defined by the relationship

\[ \langle \dot{W} \rangle_{\text{fr}} = 2\gamma_{\text{damp}} \langle W_{\text{kin}} \rangle, \]  

(21)

and is of the same order as \( \dot{W}_{\text{kin}} \).

In contrast, the oscillon-driving mechanism must be much more powerful because the energy dissipation rate through outgoing oscillons is higher, of the order of \( \langle \dot{W} \rangle_{\text{oscillon}} \sim 100–200 \text{ keV s}^{-1} \). This can easily be verified making use of the parameters listed above (see sections 2.2 and 2.4) and noting that the fully developed oscillon is a circular belt-shaped wave of horizontal width \( \approx 0.5–1 \text{ mm} \) and transverse cross-section \( S_{\text{oscillon}} = (2\pi R \times H)_{\text{oscillon}} \approx 5–7 \text{ cm}^{2} \).

4.4. A trigger mechanism for inner oscillations

According to the general theory of nonlinear oscillations, the auto-oscillation patterns arise in dynamical systems with feedback. In order to clarify the mechanism of feedback in the complex plasma we experimented with, it is necessary to have an idea of the distribution and evolution of internal and external fields controlling particle motion. The observed behavior may be due to a different (collective) physical process that needs a different treatment. In the experimental studies discussed above such detailed measurements could not be made and for that reason the creation of a rigorous model is difficult. There is one common point, however. A possible indication of such a feedback is the ‘intrinsic non-Hamiltonianity’ of complex plasma dynamics [44, 45]. There are two aspects worth discussing: This type of feedback based on charge gradients has been used in [45] to explain self-excitations of some simple systems—a few oscillating particles, and chains of charged particles. It was mentioned that the complexity of the system is not an obstacle for auto-oscillations. Furthermore, often space- and time-dependent particle charges are considered as an inherent attribute of many complex plasmas, see e.g. [46] and references therein.

Another possibility for particles to gain energy originates from the classical tunneling effect [25]. Moving particles may slide through a cloud of similar particles, elastically deforming the cloud as they slide through it. For instance, at expansion as the central particles are pushed away outwards, forming a void, they are forced to penetrate into the rest of the particle cloud, which is still immobile. When a particle passes through the layers of a cloud, its charge changes, helping penetration. For particles of the same size possible charge variation through the ‘charge sharing process’ is \( (\delta Z/Z)_{\text{max}} = 1/4 \) on maximum [25]. In our case, the charge variation needed to compensate for frictional losses (21) is 3–4 times less than this limit:

\[ \frac{\delta Z_{d}}{Z_{d}} = \frac{\langle \dot{W} \rangle_{\text{fr}}}{Z_{d} |e| \delta \varphi \delta N_{d} f_{\text{osc}}} \sim 0.07 < \left( \frac{\delta Z}{Z} \right)_{\text{max}}, \]  

(22)

so that this is a promising way to gain the energy.
4.5. A driving mechanism for outer oscillations

In the dynamics of auto-oscillations, not only short time and space scales of DAWs are important, but also the global ones of the size of the cloud. At distances further from the void, starting from \( \sim 5–7 \) mm and approximately up to the edge of the cloud, the particles are seen to move as if they acquire momentum at all positions simultaneously (see figure 7). Apparently the ‘wave exciting agent’ (that is, the plasma ions), sharing the momentum with particles, must be much lighter compared to the particles themselves. At these distances the ion collection effect appears to be more important (see section 2.2). This highlights once again the crucial role of the ion drift as the source of the instability.

Undoubtedly the oscillations in the outer part of the cloud are induced by fluctuating central charges (figure 2). But what is the physics explaining why these oscillations form such a kind of ‘supersonic’ pattern?

The explanation of ‘simultaneous’ excitation is indeed simple: the particles behave this way because they are forced to by the breathing mode \([12]\). Compared to the dust acoustic mode, this one is a fast-phase mode:

\[
C_{bm} = f_{osc} \Delta L = 70–80 \text{ mm s}^{-1} \gg C_{DAW} = 6–7 \text{ mm s}^{-1}.
\]

(Here \( \Delta L \approx D/2 = 25–30 \) mm is approximately half the size of the cloud.) This naturally explains the observed simultaneous excitation: the cloud disturbance appears to be transmitted supersonically with respect to the speed of the dust acoustic waves.

Next, an important issue is to explain the value of the frequency \((1)\) involved in auto-oscillations. This frequency is difficult to calculate rigorously. However, there are several possible ways to estimate it: it is well known that the breathing mode is determined primarily by the confinement conditions, that is, by the strength of the electric field caging the cloud. As a first step, assuming \([11]\) that the radial voltage drop \( \delta U_r \sim U \) (see \((2)\)), we estimate the radial component of the electric field strength, and then the confinement parameter as \((M_d \) is the mass of the dust particle)

\[
f_{conf} \sim \frac{1}{\pi D} \sqrt{\frac{2Z_d|e|\delta U_r}{M_d}} \simeq 1.4 \text{ Hz},
\]

this explaining the breathing mode oscillation frequency. (According to \([47]\), for the breathing mode it should be \( f_{bm} \simeq 2f_{conf} \simeq 2.8 \) Hz in good agreement with the experimentally observed value \((1)\)).

It is worth noting that a more detailed consideration based on the assumptions of the ambipolar model \([48]\) confirms \((23)\). It follows that

\[
f_{conf} \sim \frac{1}{\pi D} \sqrt{\frac{2Z_d|e|\delta U_{amb}}{M_d}}, \quad \delta U_{amb} \simeq \left(\frac{1}{2}D\right)^2 \text{div} E_{amb} \simeq \xi^2 \frac{T_e}{|e|}.
\]

Here \( \xi \simeq 2.405 \) is the first root of the zeroth-order Bessel function, \( J_0(\xi) = 0 \). Numerically \( \delta U_{amb} \approx U \), and this frequency estimate agrees well with \((23)\).

Note also that a reasonable estimate of the frequency can be obtained by considering the behavior of the particles at the very edge of the cloud. The key point is that there, at the edge, the quasi-equilibria are stable because of the balance between confinement and repulsion:

\[
E_{conf} = \delta E^{(1)}_{\text{rep}} n_{\text{eff}},
\]

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Figure 8. Averaged velocities $v^I_I$, $v^I_{II}$, $v^II_I$, and $v^II_{II}$ for inner (I) and outer (II) particles, oscillating to the left of the void boundary (see figure 2). $x^I$ and $x^{II}$ are the ‘centres of mass’ of the inner ($2.5 \text{ mm} < x < 7.5 \text{ mm}$) and outer ($7.5 \text{ mm} < x < 25 \text{ mm}$) particles. In both cases the width of the horizontal slab (centered at the height of the void center) was $\Delta y = 4.4 \text{ mm}$. The inner and outer oscillations are evidently auto-correlated, forming a type I attractor (as classified in [45]).

\[
\delta E^{(1)}_{\text{rep}} \approx \frac{Z_d|e|}{\langle \Delta \rangle^2} \left( 1 + \frac{\langle \Delta \rangle}{\lambda_{\text{scr}}} \right) \exp \left( -\frac{\langle \Delta \rangle}{\lambda_{\text{scr}}} \right),
\]

\[
\delta U_{\text{conf}} \sim \frac{1}{2} DE_{\text{conf}},
\]

where $\lambda_{\text{scr}}$ is the screening length, $\delta E^{(1)}_{\text{rep}}$ is the strength of repulsion (per particle), and $\delta E^{(1)}_{\text{rep}} n_{\text{eff}}$ is the total repulsion exerted on a given edge particle by the effective nearest neighbors $n_{\text{eff}}$, pushing the particle out of the cloud. From elementary calculations it follows that $\delta U_{\text{conf}}/n_{\text{eff}} \simeq 2–3 \text{ V}$. Hence, to obtain the expected $\delta U_{\text{conf}} \approx U \approx 13 \text{ V}$, one needs 5–6 associated neighbors. This number has to be reduced roughly by a factor 2 when the ion drag force is included in the balance.

The time–space pattern formed by the oscillating particles has a rather complicated structure (see figure 2 and 7). To simplify the analysis, we divided the cloud’s cross-section by two (not equal) subparts: the inner (I) ($2.5 \text{ mm} < x < 7.5 \text{ mm}$) and the outer (II) ($7.5 \text{ mm} < x < 25 \text{ mm}$) regions (in both cases the width of the horizontal slab was chosen as specified in the caption to figure 8). We explored the particle oscillations in these regions separately and found a surprisingly clear auto-correlation as shown in figure 8. For the particles oscillating closer to
the void, the mean kinetic energy is $\langle W_{\text{kin}}^I \rangle = 5.1 \pm 0.3 \text{eV}$, which agrees very well with that predicted by the model (20), whereas $\langle W_{\text{kin}}^{II} \rangle = 15.3 \pm 0.6 \text{eV}$ for the particles inside the outer region.

5. Conclusion

The origin of the proposed electromechanical effect could be due to the cloud stretching, multiplication or selective harmonic amplification of coupled oscillations of the particles and the electrical circuitry feeding the discharge. In the case studied here, it is a self-sustained low-frequency resonant oscillator. We conclude that the self-excitation leading to the regular repeatable constrictions of the void in the particle cloud is due to the free energy in plasma ions drifting relative to the microparticles.

Note finally that intense shaking due to the auto-oscillating breathing mode is the necessary condition of the oscillation excitations observed in our wave patterns. Hereby, the complex plasma sets a new ‘kinetically resolved’ paradigm to a variety of ‘vibro-excited’ oscillons in many other media [33]–[35].

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