Phase improvement algorithm for NLFM waveform design to reduction of sidelobe level in autocorrelation function

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A phase improvement algorithm has been developed to design the non-linear frequency modulated (NLFM) signal for the four windows of Raised-Cosine, Taylor, Chebyshev, and Kaiser. The authors have already designed NLFM signal by stationary phase method (SPM). The simulation results for the peak sidelobe level of the autocorrelation function in the phase improvement algorithm reveal a significant average decrement of about 5 dB with respect to the stationary phase method. Moreover, to evaluate the efficiency of the phase improvement algorithm, the minimum error value for each iteration is calculated.

Introduction: The goal of pulse compression is to increase bandwidth and improve range resolution [1]. There are several methods for pulse compression. For example, coding methods such as Barker, Huffman, Zadoff-Chu and so on, are utilised in pulse compression [2], but due to the phase discontinuity and the signal amplitude variability (such as the Huffman codes), they result in a loss increment in the receiver (due to mismatching) [3]. The linear frequency modulation (LFM) method has received much attention since its phase continuity and the constant amplitude of the signal, but it suffers from relatively high sidelobes in autocorrelation function (ACF) [3].

The non-LFM (NLFM) method has been proposed to reduce the sidelobes level in ACF. In the NLFM method, the signal amplitude is constant and the frequency variations with respect to time are non-linear. Stationary phase concept (SPC) is commonly used in the NLFM method. SPC explains that power spectral density (PSD) in a frequency is relatively high if the related frequency variation is low with regard to time [3]. Using this method leads to noticeable sidelobe level decrement in ACF. In addition, it causes the main lobe width to increase slightly but negligible.

The phase improvement algorithm (PIA) is proposed here to be used after the stationary phase method. This method is designed based on the phase matching techniques. To start the algorithm, an appropriate value for the phase is used which comes from the stationary phase method. The algorithm is repeated several times in order to get closer to the optimal phase value where sidelobes level is significantly reduced compared to the stationary phase method.

The remaining of the paper is organised as follows: The following section outlines the proposed phase improvement algorithm. In the ‘Simulation and results’ section, the simulation results of the proposed algorithm are discussed and a comparison between SPC and the proposed method is made. Finally, the ‘Conclusion’ section concludes the Letter.

NLFM signal design with phase improvement algorithm: In the phase improvement algorithm, the goal is to find the desired signal phase in terms of the minimum error. To achieve this goal, first, a window is selected as the initial PSD. Then, the signal phase is obtained at each iteration by reducing the error of the algorithm. Due to the constant amplitude of the NLFM signal, the desired signal can be shown as follows:

\[ x(t) = A \exp(j\phi(t)), \quad |t| \leq T/2 \]

where \( \phi(t) \) is the signal phase and \( A \) is the signal amplitude, which is constant and \( T \) is the pulse width. If \( X(f) \) is the Fourier transform of \( x(t) \), so

\[ X(f) = \sqrt{\frac{T}{2\pi}} \int_{-T/2}^{T/2} x(t) \exp(-j2\pi ft) dt \]

(2)

Suppose \( |Y(f)| \) is the root of the initial PSD; therefore, the following equation is used to calculate the difference between the amplitude of \( |Y(f)| \) and \( |X(f)| \):

\[ \text{Error} = \int_{-B/2}^{B/2} (|Y(f)| - |X(f)|)^2 df \]

where \( B \) is the bandwidth of \( x(t) \). If the difference between the two complex numbers decreases, it can be concluded that their amplitude difference is also reduced; therefore, if the phase of \( X(f) \) is \( \theta(f) \), (3) is rewritten as shown in below to be applicable in the phase matching [4, 5]

\[ \text{Error} = \int_{-B/2}^{B/2} (|Y(f)| \exp(j\theta(f)) - |X(f)|)^2 df \]

(4)

The primary aim is to obtain \( X(f) \) in order to achieve \( x(t) \) by taking an inverse Fourier transform [6]. Consider \( y_m(f) = \int_{-B/2}^{B/2} |Y(f)| \exp(j\theta(f)) df \) and \( k = (K - 1)f/B \). Therefore, we can solve the integral in the discrete form

\[ \text{Error} = \sum_{k=0}^{K-1} |y_m(k) - X(k)|^2 \]

\[ = \sum_{k=0}^{K-1} |y_m(k) - X(k)|^2 \]

(5)

\[ X(k) = \sum_{n=0}^{K-1} y(n) \exp\left(-\frac{2\pi kn}{K}\right) \]

(6)

The symbol * in (5) indicates complex conjugation and \( N \) in (6) is the number of samples in the signal \( x(n) \). Now, consider \( y_m = [y_m(0) \ y_m(1) \ y_m(2) \cdots y_m(K-1)]^T \) and \( x = [x(0) \ x(1) \ x(2) \cdots x(N-1)]^T \). Thus, we have

\[ X(k) = \left[ 1 \exp(-\frac{2\pi k1}{K}) \cdots \exp(-\frac{2\pi (N-1)k}{K}) \right] \cdot x \]

(7)

Making use of \( \exp(j\theta) = \exp(-\frac{2\pi kn}{K}) \), the following equation for \( X(k) \) can be expressed in vector space:

\[ \left[ X(0) \right] = \left[ \begin{array}{c} y(0) \\ \vdots \\ y(K-1) \end{array} \right] = \left[ \begin{array}{c} w(0)_0 \\ \vdots \\ w(K-1)_0 \end{array} \right] \cdot [x(0) \cdots x(N-1)] \]

(8)

In (8), \( W_{K \times N} \) is the discrete Fourier transform (DFT) matrix. We now take a partial derivative of (5) with respect to the vector \( x \):

\[ \text{Error} = \sum_{k=0}^{K-1} |y_m - W_k W_k^H y_m| \]

\[ = y_m^H W_k y_m - y_m^H W_k x \cdot W_k^H y_m + x^H W_k^H W_k x \]

\[ \rightarrow \frac{\partial \text{Error}}{\partial x} = -2(W_k^H y_m - W_k^H W_k x) = 0 \]

(9)

\[ \rightarrow x = (W_k^H W_k)^{-1} W_k^H y_m \]

where \( W_k^H \) is the Hermitian (complex conjugate transpose) of \( W_k \); \( y_m \) are the vector observations or measurements, and \( x \) is a vector parameter [7], and the aim is to estimate \( x \) with the vector observations to have a minimum error. In other words, the answer given in (9) is the linear least square (LS) estimator of \( x \). Moreover, \( (W_k^H W_k)^{-1} W_k^H \) is called a pseudo inverse of \( W_k \). Since the matrix \( W_k \) has orthogonal columns, \( W_k^H W_k = \lambda I_N \) is always satisfied (\( I_N \) is the identity matrix of size \( N \)). It follows that:

\[ (W_k^H W_k)^{-1} = 1/(K) \lambda I_N \rightarrow \hat{x} = (1/K) W_k^H y_m \]

(10)

where \( \hat{x} \) is a linear estimator of the vector parameter \( x \). Since the amplitude of the desired signal is constant, \( \hat{x} \) is defined as a vector \( \hat{x} \) with normalised coefficients. We can processed to calculate the minimum error in below:

\[ \text{Error}_{\min} = ||y_m - W_k \hat{x}||^2 \]

(11)

where \( ||.|| \) is the \( L^2 \)-norm on vector space. The vector \( y_m \) is constructed by the phase values of \( \theta(k) \) which should be improved at each iteration; therefore, the algorithm is designed in a way that \( y_m \) and then \( \hat{x} \) are obtained proportional to \( \theta(k) \). Now, \( \hat{x} \) can be given for the \( r \)-th iteration as shown below:

\[ \hat{x}^{(r)} = \exp(j \text{phase}(W_k^{(r)})^{H} y_m) \]

(12)

Having determined \( \hat{x}^{(r)} \), we can calculate \( \theta \) in the iteration of the \( r \)-th by taking DFT of \( \hat{x}^{(r)} \)

\[ \theta^{(r)} = \text{phase}(W_k^{(r)}) \]

(13)
At the end of the $r$th iteration, designed signal is $y^{(r)}$, that can be multiplied by the constant coefficient $A$. We shall write the minimum error of the $r$th iteration as follows:

$$y^{(r)}_m = \begin{bmatrix} \lvert Y(0) \rvert \exp(i \theta^R(0)) \\ \lvert Y(1) \rvert \exp(i \theta^R(1)) \\ \vdots \\ \lvert Y(K-1) \rvert \exp(i \theta^R(K-1)) \end{bmatrix}$$

(14)

At the end of the $r$th iteration, designed signal is $y^{(r)}$, that can be multiplied by the constant coefficient $A$. We shall write the minimum error of the $r$th iteration as follows:

$$Error^{(r)}_{\text{Min}} = \lvert y^{(r-1)}_m - W^{(r)} \rvert^2$$

(15)

Noteworthy, to start the algorithm from an appropriate point, the obtained phase from the stationary phase concept is taken as the initial phase $\theta^R$, then, by the following algorithm, we try to get close to the optimal condition. The proposed algorithm is efficient as long as the minimum error value is reducing in each iteration. In the other words, the algorithm holds to be efficient when the following relation is true:

$$\Delta Error^{(r)}_{\text{Min}} = Error^{(r)}_{\text{Min}} - Error^{(r-1)}_{\text{Min}} < 0$$

(16)

As long as (16) is satisfied, the proposed phase improvement algorithm will be efficient. In the following section, the minimum error is discussed with regard to the number of iterations to analyse the algorithm efficiency.

**Simulation and results:** The phase improvement algorithm is performed for the four windows of Raised-Cosine, Taylor, Chebyshev, and Kaiser. Design parameters such as pulse width, bandwidth, and sampling rate are considered as 2.5 $\mu$s, 100 MHz, and 1 GHz, respectively. Fig. 1 illustrates the one-sided autocorrelation function of the designed signal using the phase improvement algorithm for the four windows of Raised-Cosine, Taylor, Chebyshev, and Kaiser.

| Windows | PSL with SPM (dB) [8] | PSL with PIA (dB) | Improvement (dB) |
|---------|-----------------------|------------------|------------------|
| Raised-Cosine | –33.34 | –37.67 | –4.33 |
| Taylor | –33.34 | –37.38 | –4.04 |
| Chebyshev | –31.77 | –36.50 | –4.73 |
| Kaiser | –30.98 | –36.98 | –6.00 |

Table 1 compares the results of the phase improvement algorithm for the four windows of Raised-Cosine, Taylor, Chebyshev, and Kaiser.

Table 1: PSL comparison for the phase improvement algorithm and stationary phase method

Fig. 2 Minimum error against the number of iterations for the four windows of Raised-Cosine, Taylor, Chebyshev, and Kaiser

**Conclusion:** Phase improvement algorithm demonstrates the reduction of the PSL about 5 dB with respect to the stationary phase method, which is a significant value. The phase improvement for the Kaiser window is higher than the other windows in the proposed algorithm. In fact, the proposed method comes after the stationary phase method for finding an optimal phase. In the design of NLFM signals due to the constant amplitude, the main concern is to find the optimal phase. According to the presented results, the minimum error in each iteration has been reduced indicating the superior efficiency of the proposed algorithm.

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One or more of the Figures in this Letter are available in colour online.
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