THE MAGNETOHYDRODYNAMICAL MODEL OF KILOHERTZ QUASI-PERIODIC OSCILLATIONS IN NEUTRON STAR LOW-MASS X-RAY BINARIES (II)

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ABSTRACT

We study the kilohertz quasi-periodic oscillations (kHz QPOs) in neutron star low-mass X-ray binaries (LMXBs) with a new magnetohydrodynamics (MHD) model, in which the compressed magnetosphere is considered. The previous MHD model is reexamined and the relation between the frequencies of the kHz QPOs and the accretion rate in LMXBs is obtained. Our result agrees with the observations of six sources (4U 0614+09, 4U 1636–53, 4U 1608–52, 4U 1915–15, 4U 1728–34, and XTE 1807–294) with measured spins. In this model, the kHz QPOs originate from the MHD waves in the compressed magnetosphere. The single kHz QPOs and twin kHz QPOs are produced in two different parts of the accretion disk and the boundary is close to the corotation radius. The lower QPO frequency in a frequency–accretion rate diagram is cut off at a low accretion rate and the twin kHz QPOs encounter a top ceiling at a high accretion rate due to the restriction of the innermost stable circular orbit.

Key words: accretion, accretion disks – magnetohydrodynamics (MHD) – stars: neutron – X-rays: binaries

Online-only material: color figures

1. INTRODUCTION

The fastest variability, i.e., the high frequency quasi-periodic oscillations (QPOs), has been observed in both neutron star low-mass X-ray binaries (NS-LMXBs) and black hole LMXBs, which gives us an important channel to understand the physics of the accretion process in the accretion disks of LMXBs. Specifically, it gives us a clue to find the parameters of compact stars, e.g., their equation of state (van der Klis 2006). In NS-LMXBs, the frequencies of the high frequency QPOs approach the Kepler frequency of the matter at the surface of the NS and often exceed 1000 Hz, referred to as kilohertz (kHz) QPOs. The kHz QPOs always appear as broad peaks in their Fourier power spectra and are often discovered in pairs; they are labeled as upper QPOs ($\nu_{\text{upper}}$) and lower QPOs ($\nu_{\text{lower}}$) according to their frequencies. The kHz QPOs change with the source states and the X-ray count rates (Méndez et al. 2003). Barret et al. (2005, 2006) discovered that the lower QPO frequency from 4U 1636–53 in the frequency–count rate diagram has a maximum value (around 920 Hz), above which frequency no QPO was detected for any count rate; they named this maximum value as the ceiling frequency. In this work, we simply refer to this maximum value as the ceiling frequency.

Many models have been suggested to explain the physics of kHz QPOs in NS-LMXBs (see, e.g., van der klis 2006; Shi & Li 2009). Lin et al. (2011) discussed the frequency relationship of kHz QPOs for 4U 1636–53 and Sco X–1 by comparing the observations to some theoretical models. Generally, the models of the kHz QPOs mainly include several types: beat-frequency models (Miller et al. 1998; Cui 2000; Campana 2000; Lamb & Miller 2001, 2004); rotation, precession, and epicyclic frequency models (Stella & Vietri 1998; Romanova & Kulkarni 2009; Bachetti et al. 2010); disk-oscillation and resonance models (Osherovich & Titarchuk 1999; Abramowicz & Kluźniak 2001; Abramowicz et al. 2003; Kato 2001, 2004; Urbanec et al. 2010); and wave models (Zhang 2004; Li & Zhang 2005; Rezania & Samson 2005; Shi & Li 2009, 2010). The above classification is not strict, because some factors overlap in those studies. In most of the models for NS-LMXBs, the characteristic radius is very important because the changing QPO frequencies are highly dependent on the radius in those models.

Zhang (2004) considered the Alfvén waves as the source of the kHz QPOs of the accreting X-ray binaries. Rezania & Samson (2005) also discussed the magnetohydrodynamics (MHD) turbulence effect coming from the accretion process when the plasma hits the magnetosphere, i.e., the excited resonant shear Alfvén waves in a region of enhanced density gradients from a collision in the magnetosphere led to the kHz QPOs. Shi & Li (2009) have considered the two MHD oscillation modes in NS magnetospheres including the effect of gravity and the rotation of an NS as the source of the twin kHz QPOs. A linear relation between the frequencies of the upper QPOs and the lower QPOs was obtained and the model fitted well the observation of the change of the upper kHz QPO frequencies and the frequency difference with lower kHz QPOs frequencies.

In this work, we reexamine the previous MHD model (Shi & Li 2009) and find the relation between the kHz QPOs and the accretion rate in NS-LMXBs, based on the interpretation of the MHD waves by Shi & Li (2009). Unlike the result of others, we find that the Alfvén-like transverse waves only exist under...
special conditions. We start in Section 2 with the new MHD model and provide the solutions of the dispersion equation for the MHD wave frequencies. Then, we compare our results with observations in Section 3. Finally, we present our discussion in Section 4 and summarize our results in Section 5.

2. THE MHD MODEL

In this section, we consider that the steady standard disk of Shakura & Sunyaev (1973) in an NS-LMXB is truncated by the stellar magnetosphere, and the plasma is accreted to the polar cap along the magnetic field (e.g., Ghosh et al. 1977; Elsner & Lamb 1977). As shown in Figure 1, in the accretion process, the plasma hits the magnetic field lines and compresses the primary polar magnetic field, which might lead to a deformation of the magnetic field and some instability (Elsner & Lamb 1977). The MHD waves produced at the magnetosphere radius from a small perturbation lead to the kHz QPOs.

2.1. The MHD Waves

We start from a balance of gravity, barometric pressure, and magnetic pressure for the plasma in the border between the magnetosphere and the steady thin accretion disk rotating around the NS; this provides us with a definition of the magnetosphere radius of the LMXBs. The balance equation can be expressed as follows:

\[
\frac{\partial \rho_0}{\partial t} + \rho_0 \mathbf{u}_0 \cdot \nabla \mathbf{u}_0 = - \nabla P + \frac{1}{\mu} (\nabla \times \mathbf{B}) \times \mathbf{B}_0
\]

\[
+ 2 \rho_0 \mathbf{u}_0 \times \mathbf{\Omega} + \rho_0 \mathbf{\Omega} \times \mathbf{r}_0 - \rho_0 (\mathbf{\Omega} \cdot \mathbf{r}_0) \mathbf{\Omega} - \frac{G M}{r_0^3} \mathbf{r}_0 ,
\]

where \( \mathbf{u} \) is the plasma velocity, \( \mu \) is the vacuum magnetic conductivity, \( P \) is the barometric pressure, \( \mathbf{B} \) is the magnetic field, \( \mathbf{r} \) is the displacement from the NS, \( \rho \) is the plasma density, \( G \) is the gravitational constant, \( M \) is the mass of the NS, and \( \mathbf{\Omega} \) is the angular velocity of the NS, respectively. The subscript “0” denotes variables in the equilibrium state and the bold italic expresses vectors. This is very different from Shi & Li (2009) in that now we consider that the magnetic field and the density of the plasma in the magnetosphere are not uniform, but we do consider the same initial conditions of Shi & Li (2009) that the balance is steady at first, i.e., \( \partial (\rho_0 / \partial t) = 0, \partial P / \partial t = 0, \partial \mathbf{B}_0 / \partial t = 0, \) and \( \partial \mathbf{\Omega} / \partial t = 0 \). We consider the plasma to be an ideal conductor, so the vacuum electroconductivity \( \sigma \rightarrow \infty \), and then we can obtain the equation according to the Faraday principle of electromagnet induction as follows:

\[
\frac{\partial \mathbf{B}_0}{\partial t} = (\mathbf{B}_0 \cdot \nabla) \mathbf{u}_0 - (\mathbf{u}_0 \cdot \nabla) \mathbf{B}_0 - (\nabla \cdot \mathbf{u}_0) \mathbf{B}_0 .
\]

The continuity and the adiabatic condition are always used in the accretion process and they can be expressed as follows:

\[
\frac{\partial \rho_0}{\partial t} + \nabla \cdot (\rho_0 \mathbf{u}_0) = 0 ,
\]

\[
P_0 \rho_0^{-\gamma} = \text{const},
\]

where \( \gamma \) is the adiabatic index. The plasma at the magnetosphere radius is always disturbed by strong excitation (Rezania & Samson 2005), so now we discuss the MHD waves that are produced from a small disturbance. The disturbed MHD equations are written as

\[
\frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = - \nabla P + \mathbf{J} \times \mathbf{B} + 2 \rho_0 \mathbf{u} \times \mathbf{\Omega}
\]

\[
+ \rho \mathbf{\Omega} \times (\mathbf{r} \times \mathbf{\Omega}) - \rho \frac{G M \mathbf{r}}{r^3},
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla) \mathbf{u} - (\mathbf{u} \cdot \nabla) \mathbf{B} - (\nabla \cdot \mathbf{u}) \mathbf{B},
\]

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 ,
\]

\[
P \rho^{-\gamma} = \text{const},
\]

where \( \mathbf{u} = \mathbf{u}_0 + \mathbf{u}_s, \mathbf{B} = \mathbf{B}_0 + \mathbf{B}_s, \mathbf{r} = \mathbf{r}_0 + \mathbf{r}_s, \rho = \rho_0 + \rho_s, P = P_0 + P_s, \) and the subscript “s” denotes the variation of a physical quantity due to the perturbation, except in the sound velocity (c_s) below. We consider that a small perturbation leads to the MHD waves, and so \( u_s \ll |\mathbf{\Omega} \times \mathbf{r}|, B_s \ll B_0, r_s \ll r_0, \rho_s \ll \rho_0, \) and \( P_s \ll P_0 \). Combining Equations (1)–(8), we can obtain the equations concerning the variation of the perturbed physical quantities in the first-order approximation in the corotation reference frame (so \( \mathbf{u}_0 = 0 \)).

\[
\rho_0 \frac{\partial \mathbf{u}_s}{\partial t} = - \nabla P_s + \frac{1}{\mu} \left[ (\nabla \times \mathbf{B}_0) \times \mathbf{B}_s + (\nabla \times \mathbf{B}_s) \times \mathbf{B}_0 \right]
\]

\[
+ 2 \rho_0 \mathbf{u}_s \times \mathbf{\Omega} + \rho_0 \mathbf{\Omega} \times \mathbf{r}_s + \rho_0 \mathbf{\Omega} \times \mathbf{r}_s - \rho_0 (\mathbf{\Omega} \cdot \mathbf{r}_s) \mathbf{\Omega} - \rho_0 \frac{G M \mathbf{r}_s}{r_0^3} ,
\]

where \( \mathbf{u}_s \) is the plasma velocity, \( r_s \) is the displacement from the NS, \( \Omega \) is the angular velocity of the NS, \( P_s \) is the sound pressure in the first-order approximation, then we can obtain the equation according to the Faraday principle of electromagnet induction as follows:

\[
\frac{\partial \mathbf{B}_s}{\partial t} = (\mathbf{B}_0 \cdot \nabla) \mathbf{u}_s - (\mathbf{u}_s \cdot \nabla) \mathbf{B}_0 - (\nabla \cdot \mathbf{u}_s) \mathbf{B}_0 .
\]

The continuity and the adiabatic condition are always used in the accretion process and they can be expressed as follows:

\[
\frac{\partial \rho_s}{\partial t} = - \nabla \rho_s - \rho_s \nabla \cdot \mathbf{u}_s ,
\]

\[
P_s = \frac{\gamma P_0}{\rho_0} \rho_s ,
\]
Unlike Equation (13) in Shi & Li (2009), Equation (9) includes the previously neglected term \(-3\Omega r_0 \cdot r/s/0\) that comes from \((r_0 + r/s/0 + r/s)^3 - (r_0/r_0)^3\), and sometimes \((3\Omega r_0 \cdot r/s/0/r_0)^3\), so in addition, we also consider the nonuniform distribution of the density and magnetic field for the plasma in the magnetosphere in those equations.

Differentiating Equation (9) and substituting Equation (12) into it, we obtain
\[
\frac{\partial^2 u_s}{\partial t^2} = -\frac{\partial}{\partial t} \left( \nabla \frac{\rho_0}{\rho} \rho_0 \right) + \frac{\partial}{\partial t} \left[ (\nabla \times B_0) \times B_0 \right] + \left( \nabla \times B_0 \right) \times B_0 + 2\rho_0 \frac{\partial}{\partial t} (u_s \times \Omega) + \rho_0 \nabla^2 u_s
\]
\[
+ \frac{\partial}{\partial t} \rho_0 \nabla^2 r_0 - \rho_0 \frac{GM}{r_0^3} \left[ \nabla - \frac{3r_0 \cdot u/s}{r_0^2} \right].
\]  

Now, we discuss the physical process in the rectangular coordinate system \((x-x,y)\) in the corotation reference frame (see Figure 1). We assume that (1) the accretion disk does not warp, i.e., the compressed magnetic field lines are normal to the disk in the equatorial plane, and the magnetic field and the density are only functions of the longitudinal displacement \(r_0\) after being compressed, i.e., \(B_0 = (0, B_0(r_0)), \rho_0 = \rho_0(r_0), \Omega = (0, 0, \Omega)\); (2) the MHD waves propagate along the magnetic field lines or they would easily dissipate in the disk (Shi & Li 2009), i.e., the wave vector can be expressed as \(k = (0, k, k)\) (\(k\) is the wavenumber). The balance equation of the plasma at the magnetosphere radius, Equation (1), can be simplified as
\[
0 = -\nabla P_0 + \frac{1}{\mu_0} \left( \nabla \times B_0 \right) \times B_0 + \rho_0 \nabla^2 r_0 - \rho_0 \frac{GM}{r_0^3} r_0,
\]
and the further simplified form is
\[
\Omega^2 r_0 - \frac{GM}{r_0} = \left( \frac{c_s^2}{\rho_0} \frac{\partial \rho_0}{\partial r} + \frac{1}{\mu_0 \rho_0} B_0 \frac{\partial B_0}{\partial r} \right) |_{r=r_0},
\]

where \(c_s = \sqrt{\gamma P_0/\rho_0}\) is the sound velocity.

After substituting Equations (10)–(12) and Equation (15) into Equation (13), we can obtain
\[
\frac{\partial^2 u_s}{\partial t^2} \simeq \frac{\rho_0}{\rho_0} \left[ \nabla \rho_0 \times (\nabla \times u_s) + (\nabla \rho_0 \cdot \nabla) u_s \right]
+ (u_s \cdot \nabla) (\nabla \rho_0) + \rho_0 \nabla (\nabla \times u_s) + \rho_0 \nabla (\nabla \times u_s)
+ \frac{1}{\mu_0} \left( \nabla B_0 \times (\mathbf{e} \cdot \nabla) u_s \right) + B_0 \nabla \times (\mathbf{e} \cdot \nabla) u_s
- (\nabla \times u_s) \times B_0 - (\nabla \times u_s) \nabla \times B_0 - \nabla \times [(u_s \cdot \nabla) B_0] \times B_0 + \frac{1}{\mu_0} \left( \nabla \times B_0 \right) \times (B_0 \cdot \nabla) u_s
- \frac{1}{\mu_0} \left( \nabla \times B_0 \right) \times (\nabla \cdot u_s) B_0 - \frac{1}{\mu_0} \left( \nabla \times B_0 \right) \times (u_s \cdot \nabla) B_0
\]
\[
+ \frac{GM}{r_0} \left[ u_s - \frac{3r_0 \cdot u/s}{r_0^2} + 2\rho_0 \frac{\partial}{\partial t} (u_s \times \Omega) \right]
- \frac{GM}{r_0} \nabla \rho_0 \cdot \nabla \left( \frac{\rho_0}{\rho_0} \right) \Omega^2 r_0 - (\Omega \cdot r_0) \Omega
\]
\[
- \frac{GM}{r_0} r_0 - \nabla \left( \frac{\rho_0}{\rho_0} \right) \Omega^2 r_0 - (\Omega \cdot r_0) \Omega.
\]

where \(e\) is the unit vector with the same direction as \(B_0\). We then carry out a Fourier transformation \((f \rightarrow f e^{ikr-i\omega t})\) for Equation (16) and simplify the result, and obtain the following dispersion equations:
\[
(-\omega^2 - \Omega^2 - 2\omega_k^2 + k^2 V_A^2 + m) u_{sx} = -i k \left( \frac{\rho_0}{\rho_0} B_0 \frac{\partial B_0}{\partial x} u_{sz} \right)
+ i k (\gamma - 1) \frac{c_s^2}{\rho_0} \frac{\partial \rho_0}{\partial x} u_{sz} - i 2\omega \Omega u_{sy},
\]
\[
(-\omega^2 - \Omega^2 + \omega_k^2 + k^2 V_A^2) u_{sy} = -i 2\omega \Omega u_{sy},
\]
\[
(-\omega^2 + \omega_k^2 + k^2 c_s^2) u_{sz} = \left( \frac{c_s^2}{\rho_0} \frac{\partial \rho_0}{\partial x} + \frac{1}{\mu_0 \rho_0} B_0 \frac{\partial B_0}{\partial x} \right) i k u_{sx},
\]
\[
m = \left[ \frac{\partial}{\partial r} \left( \frac{c_s^2}{\rho_0} \frac{\partial \rho_0}{\partial r} + \frac{1}{\mu_0 \rho_0} B_0 \frac{\partial B_0}{\partial r} \right) \right] |_{r=r_0},
\]

where \(V_A = \sqrt{B_0^2/\mu_0 \rho_0}\) is the Alfvén velocity and \(\omega_k = \sqrt{GM/r_0}\) is the Kepler angular frequency. If the magnetic field and the plasma are uniform, \((\partial B_0/\partial x) = (\partial B_0/\partial r) = 0\) and \((\partial \rho_0/\partial y)/(\partial \rho_0/\partial r) = 0\) (so \(m = 0\)), then this magnetosphere radius is also the corotation radius of the NS from Equation (15), and the frequencies of the MHD Alfvén-like waves are
\[
\omega^2 = k^2 V_A^2 + \frac{1}{2} \Omega^2 + \frac{1}{2} \sqrt{\Omega^4 + 16 k^2 V_A^2 \Omega^2}.
\]

Since the coefficients of the highest-order derivatives in Equation (16) are not small, we cannot make expansions for small coefficients or assume that these coefficients are equal to zero. Therefore, our approximation is a classical method when the total energy is greater than the potential energy in all space, compared with the WKB approximation method.

In the case of \(m \neq 0\), several solutions are obtained from Equations (17)–(20).

1. If \(u_{sx} = 0, u_{sy} = 0, u_{sz} \neq 0\), then it is a sound-like wave solution, \(\omega^2 = \omega_k^2 + k^2 c_s^2\), on the condition that \(\gamma c_s^2 (\partial \rho_0/\partial x) = (1/\mu_0) B_0 (\partial B_0/\partial x \times \mathbf{e}) + c_s^2 (\partial \rho_0/\partial x \times \mathbf{e})\).
2. If \(u_{sx} = 0, u_{sy} = 0, u_{sz} \neq 0\), then it is an Alfvén-like wave solution, \(\omega^2 = \omega_k^2 + k^2 V_A^2 - \Omega^2\), on the condition that the Coriolis force is neglected.
3. If \(u_{sx} \neq 0, u_{sy} \neq 0, u_{sz} = 0\), i.e., the magnetosphere radius is also the corotation radius, then we can obtain the MHD waves as \(\omega^2 = k^2 V_A^2 + (1/2) \Omega^2 \pm (1/2) \sqrt{\Omega^4 + 16 k^2 V_A^2 \Omega^2}\), which revert to Equation (21).
4. If \(u_{sx} \neq 0, u_{sy} \neq 0, u_{sz} \neq 0\), then we can obtain the dispersion equation from Equations (17)–(20) as follows:
\[
(-\omega^2 - \Omega^2 - 2\omega_k^2 + k^2 V_A^2 + m) (-\omega^2 - \Omega^2 + \omega_k^2 + k^2 V_A^2)
\]
\[
\times (-\omega^2 + \omega_k^2 + k^2 c_s^2) - \Omega^2 \omega^2 (-\omega^2 + \omega_k^2 + k^2 c_s^2)
\]
\[
= k^2 r_0^2 (2\omega_k^2 - 2\omega_k c_s^2) (-\omega^2 - \Omega^2 + \omega_k^2 + k^2 V_A^2)
\]
\[
- (\omega^2 - \omega_k^2) (-\omega^2 - \Omega^2 + \omega_k^2 + k^2 V_A^2)
\]
\[
\times c_s^2 \frac{r_0}{\rho_0} \frac{\partial \rho_0}{\partial r} \bigg|_{r=r_0}.
\]
The above solutions of the dispersion equations are special solutions under special conditions except for those of Equation (22), and so only the solutions of Equation (22) are ordinary and we consider them to be the source of the kHz QPOs in NS-LMXBs. Because there is not a concise analytic solution for Equation (22), we will discuss the numerical solutions below.

2.2. Magnetosphere Radii

The magnetosphere radius is the characteristic radius of the kHz QPOs in this model. Since Lamb et al. (1973) provided a definition of the magnetosphere radius in the globular accretion process onto a compact star, many authors have explored the outer boundary of the magnetospheres of pulsars (e.g., Cui 1997) and obtained not identical but analogous conclusions. The magnetosphere radius obtained by Lamb et al. (1973) is written as

$$r_m \approx 2.29 \times 10^6 \mu_{26}^{-4/7} M_{16}^{-2/7} m_{1,4\odot}^{-1/7} \text{cm},$$  \hspace{0.5cm} (23)

where \(r_m\) is the magnetosphere radius in the accretion process; \(\mu\) is the magnetic moment of the star; \(M\) is the accretion rate; \(m\) is the mass of the NS; and the subscripts “26,” “16,” and “1,4\odot” express the quantities in units of \(10^{26}\) G cm\(^3\), \(10^{16}\) g s\(^{-1}\), and 1.4 times the mass of the Sun, respectively.

McCray & Lamb (1976) considered that the magnetic pressure could resist the falling spherical plasma layer and they found a magnetosphere radius ranging from \(1.3 \times 10^8\) cm to \(7 \times 10^8\) cm. Elsner & Lamb (1977) continued to discuss the spherical accretion process and they considered that the central star rotates sufficiently slowly. In addition, many results for the magnetosphere radius close to \(10^8\) cm were calculated if the star rotates sufficiently slowly. In addition, many results for the magnetosphere radius close to \(10^8\) cm were calculated if the star rotates sufficiently slowly. In addition, many results for the magnetosphere radius close to \(10^8\) cm were calculated if the star rotates sufficiently slowly.

Long et al. (2005) considered disk accretion onto the magnetized star with a dipole magnetic field and using simulations found that the magnetosphere radius is half that from Elsner & Lamb (1977). Recently, Kulkarni & Romanova (2013) performed three-dimensional MHD simulations of magnetospheric accretion at a quasi-equilibrium state, in which the gravitational, centrifugal, and pressure gradient forces are in balance. Then, a different dependence of the magnetosphere radius on the accretion rate of the LMXBs, the magnetic field, and the radius of the NS is found as

$$r_m \approx 2.50 \times 10^6 \mu_{26}^{2/5} M_{16}^{-1/5} m_{1,4\odot}^{-1/10} R_8^{3/10} \text{cm},$$  \hspace{0.5cm} (24)

where \(R\) denotes the radius of the NS in units of \(10^6\) cm. The more gradually changing trend with \(\mu\) and \(M\) comes from the compression of the magnetosphere by the disk matter, which leads to the non-dipole magnetic field of the external magnetosphere.

We also consider that a central NS with a dipole magnetic field, whose mass is \(m\), accretes plasma from the standard thin disk, in which the density of the plasma is

$$\rho = 3.1 \times 10^{-8} \alpha^{-7/10} M_{15}^{11/20} m_{1,4\odot}^{-5/8} R_8^{-15/8} f^{11/5} \text{g cm}^{-3},$$  \hspace{0.5cm} (25)

where \(f = (1 - \sqrt{R/r})^{1/4}\), and \(\alpha\) is the viscosity parameter (Frank et al. 2002).

We can obtain the barometric pressure as \(P = \rho KT/m_p\) and \(T\) is the temperature in the standard thin disk, where \(K\) is the Boltzmann constant and \(m_p\) is the mass of the proton. We consider the balance of the plasma by magnetic pressure, barometric pressure, and collision, and the magnetosphere radius can be estimated according to \((B^2/8\pi) \simeq P + \rho u^2\) (Elsner & Lamb 1977; Romanova et al. 2002). Due to \(P \gg \rho u^2\) in the standard thin disk when the radius ranges from \(10^6\) cm to \(10^{10}\) cm, the balance condition can be simplified as \(P \simeq (B^2/8\pi)\). The magnetosphere radius can be estimated when the dipole magnetic field is adopted:

$$r_m \simeq 6.64 \times 10^5 \alpha^{-4/15} \mu_{26}^{16/27} M_{16}^{-34/135} m_{1,4\odot}^{-7/27} f^{136/135} \text{cm}.$$  \hspace{0.5cm} (26)

We have also made the strict computation of the magnetosphere radius using the equation \(|\nabla(B^2/8\pi)| \simeq |\nabla P|\), the result of which is very close to the above magnetosphere radius.

If we select the characteristic value, i.e., \(m = 1.4 M_{\odot}\), \(R = 10^6\) cm, and \(\alpha = 0.1\) (King et al. 2007), then the entire magnetosphere radius can be simplified. The magnetosphere radius in the spherical accretion process can be simplified from Equation (23) as

$$r_{ms} \simeq 2.29 \times 10^6 \mu_{26}^{-2/7} B_{s8}^{4/7} \text{cm},$$  \hspace{0.5cm} (27)

where \(B_{s8}\) denotes the magnetic flux density at the surface of the NS in units of \(10^8\) G. After the parameters are substituted into Equation (26), the magnetosphere radius is simplified as

$$r_{mA} \simeq 3.60 \times 10^5 \mu_{26}^{-34/135} B_{s8}^{16/27} f^{-136/135} \text{cm},$$  \hspace{0.5cm} (28)

which we refer to as “model A” when the radius \(r_{mA}\) is used. We also simplify Equation (24) as

$$r_{mB} \simeq 2.50 \times 10^6 \mu_{26}^{-1/5} B_{s8}^{2/5} \text{cm},$$  \hspace{0.5cm} (29)

which we refer to as “model B” when the compressed magnetosphere radius \((r_{mB})\) is considered.

As shown in Figure 2, the magnetosphere radius in the spherical accretion process \((r_{ms})\) is much larger and covers a larger range than the others. The magnetosphere radius in model A approaches the radius of an NS when \(M\) is high enough or the magnetic field at the surface of the NS is weak enough. The compressed magnetosphere radius \((r_{mB})\) changes slower than the other two radii \((r_{ms}\) and \(r_{mA})\), which reveals the clear differences between the compressed magnetic field and the dipolar magnetic field. In this study, we only discuss the accretion process of the plasma in the accretion disks of LMXBs, so the magnetosphere radius of Equation (27) for spherical accretion will not be used hereafter. The innermost stable circular orbit (ISCO) is the smallest stable circular orbit for the accretion plasma and the plasma will fall into the central NS after it passes through the orbit due to a dynamical instability for circular geodesics in general relativity. The ISCO of the accretion plasma around an NS with the mass \((1.4 M_{\odot})\) is marked as the dashed lines in Figure 2 according to its radius \((66 m)\). Inside ISCO, the assumption of the standard disk model breaks down, so we assume that the disk is truncated at ISCO at high \(M\).

2.3. General Solutions in Two Kinds of Magnetic Configurations

We suppose that the MHD waves at the magnetosphere radius are the origin of the kHz QPOs, therefore numerical solutions...
of Equation (22) should be obtained according to the different parameters, $B_\ast$, $M_\ast$, and $k$, and for the characteristic value of NS-LMXBs, i.e., $m = 1.4 M_\odot$, $R = 10^6$ cm, and $\sigma = 0.1$ (King et al. 2007). In the two models (i.e., model A and model B), two types of magnetic field configurations are adopted, distinguished by whether the magnetic field is compressed or not.

In model A, the standard $\alpha$-disk comes into contact with the dipolar magnetic field at the magnetosphere radius and an equilibrium state is reached. We substitute the balance equation, $(c_s^2/\rho_0)(\partial \rho_\ast/\partial r) = \Omega^2 r_\odot - (GM/r_\odot^3) - (1/\mu_\ast \rho_\ast) B_\odot (\partial B_\odot/\partial r)$ (i.e., Equation (15)), into Equation (22) and obtain the equation for when the dipolar magnetic field is considered:

$$\left(\frac{f}{\omega^2} - \frac{\Omega^2}{2} + \frac{\mathcal{V}_A^2}{k} + m\right)\left(\frac{f}{\omega^2} - \frac{\Omega^2}{2} + \frac{\mathcal{V}_A^2}{k} + \frac{2}{k} \mathcal{V}_A^2\right)$$

$$\times \left(-\frac{f}{\omega^2} + \frac{\mathcal{V}_A^2}{k} + \frac{2}{k} \mathcal{V}_A^2\right) - 4\Omega^2 \left(-\frac{f}{\omega^2} + \frac{\mathcal{V}_A^2}{k} + \frac{2}{k} \mathcal{V}_A^2\right)$$

$$= (1 - \gamma) k^2 r_\odot^2 \left(\frac{\mathcal{V}_A^2 - \mathcal{V}_A^2}{k} - \frac{2}{k} \mathcal{V}_A^2\right)$$

$$- 3\gamma k^2 \mathcal{V}_A^2 \left(-\frac{f}{\omega^2} - \frac{\mathcal{V}_A^2}{k} + \frac{2}{k} \mathcal{V}_A^2\right).$$

(30)

In Equation (30), the Kepler angular frequency, Alfvén velocity, and sound velocity can be obtained as follows:

$$\omega_k = 13628.4 \frac{c_s}{f_{A, 6}} \text{ Hz, } V_A = 2.56 \times 10^7 B_{8, 6} m_{16}^{-11/40} f_{A, 6}^{-3/16} \text{ cm s}^{-1}, \text{ and } c_s = 1.29 \times 10^8 M_{16}^{1/20} r_{A, 6}^{-5/8} f_{A, 6}^{1/5} \text{ cm s}^{-1},$$

where $f_A = (1 - \sqrt{1/f_{A, 6}^{1/4}})^{1/4}.

In model B, the standard $\alpha$-disk compresses the magnetic field until an equilibrium state is reached and the relevant physical quantities for Equation (22) are obtained as follows:

$$\omega_k = 3447.74 M_{16}^{3/10} B_{8, 5}^{-3/5}, \text{ and } c_s = 4.10 \times 10^7 B_{8, 5}^{-3/20} M_{16}^{3/40} f_B^{1/5} \text{ cm s}^{-1},$$

where $f_B = (1 - 0.63 B_{8, 5}^{1/5} M_{16}^{1/10})^{1/4}$. Because the dipole magnetic field is compressed within the magnetic sphere ($\sigma \rightarrow \infty$), we adopt the approximation for the magnetic field structure that all of the magnetic field lines, which are compressed between the magnetosphere radius ($r_{mB}$) and the magnetosphere radius of the uncompressed magnetic field ($r_{mA}$), are uniform. During the compressing process, the magnetic flux remains unchanged, and so the magnetic field can be estimated from the following equation:

$$\frac{1}{r_{mB}} \int_{r_{mB}}^{r_{mA}} B_p^2 2\pi r dr = \frac{1}{r_{mA}} \int_{r_{mA}}^{r_{mB}} B_p^2 2\pi r dr,$$

(31)

where $B_0$ is the magnetic flux density of the compressed magnetic field and $B_p$ is that of the uncompressed dipole magnetic field in the equatorial plane. The Alfvén velocity can be obtained as

$$V_A = 1.51 \times 10^9 B_{8, 6}^{11/8} M_{16}^{-37/80}$$

$$\times \left(r_{mB, 6}^2 - r_{mA, 6}^2\right)^{1/2} r_{mA, 6}^{-11/10} f_B^{-11/10} \text{ cm s}^{-1}.

(32)

In order to derive the general solutions, we should derive the secondary derivative of the density and the magnetic field (i.e., $m$ in Equation (20)) in models A and B. We assume that the condition on the balance of the plasma (i.e., Equation (15)) is extended to the vicinity of the magnetosphere radius, and so $m \approx -\partial (\Omega^2 - GM/r^3)/\partial r = -\frac{\Omega^2}{2} - 2\omega_k^2$. In addition, we use the distribution of the density of the $\alpha$-disk ($\partial \rho/\partial r$) in Equation (22) in model B, or the distribution of the dipolar magnetic field ($\partial B/\partial r$) in Equation (30) in model A. Finally, we obtain the general numerical solutions after the two kinds of magnetosphere radii in Equations (28) and (29) are substituted into Equations (22) and (30), respectively (see Figure 3).

As shown in Figure 3, we find that there are both twin solutions (the upper solution $v_u$ and the lower solution $v_l$) and single solutions in the two models; the solutions within $r_{t1}$ can be considered as twin solutions and the solutions outside $r_{t1}$ are single solutions (see the left panel of Figure 3). In the left panel, the four characteristics of the solutions of Equation (30) in model A are listed as follows. (1) The solutions are divided into two parts, i.e., the left solutions corresponding to low $M$ and the right ones corresponding to high $M$. (2) A transition accretion rate corresponding to the transition radius ($r_{t1}$) can be found by the vertical dash-dotted line; the transition radius is the border between the twin solutions and the single solution for an accretion rate and it is near the corotation radius. (3) In the right part, there are only twin solutions for an accretion rate. (4) $v_u$ and $v_l$ increase with $M$ until they reach their ceiling frequencies.
at $v_a = 1508$ Hz and $v_1 = 1083$ Hz, due to the restriction of ISCO.

In the right panel of Figure 3, the general real solutions of Equation (22) are obtained and their five characteristics are listed as follows. (1) The single solution described by the bold solid line is also the upper solution and supposed to be the origin of the single kHz QPOs. (2) $v_a$ and $v_1$ increase with $M$ until they reach their ceiling frequencies at $v_a = 1862$ Hz and $v_1 = 1521$ Hz when the accretion disk is truncated by ISCO. (3) There is a transition point (corresponding to a transition radius $r_{t2}$) that splits the solutions into the twin solutions and the single solution; the lower real solutions will disappear when $M$ is lower than the transition accretion rate. (4) The turning point of the single solutions (corresponding to a transition radius $r_t$) separates the decreasing trend from the increasing trend of $v_a$ and indicates the changing of the key factor dominating the balance of the plasma. (5) All the two transition radii ($r_t$ and $r_{t2}$) are close to the corotation radius at which the Kepler rotation frequency of the NS.

The changing trend of the single solutions in model B is different from that in model A; the main reason for this can be found in Figure 4 in model B and Equation (22) as follows. (1) The Kepler angular frequency is very high ($\omega_k \simeq 50 \sim 110kv_A$ and $\omega_k \simeq 100 \sim 210kv_A^2$) and it is the key factor in determining the solutions of Equation (22). (2) With the increase of $M$, $r_m$ decreases and the other variables ($\omega_k$, $k \ast V_A$, $k \ast c_s$, and $(\rho_0/\rho_0)(\partial \rho_0/\partial r)\mid_{r=r_m}$) in Equation (22) increase. (3) The expression $l_2 = -(\Omega^2 - \omega_k^2)k^2c_s^2(r_0/\rho_0)(\partial \rho_0/\partial r)\mid_{r=r_m}$ is smaller than $l_1 = k^2r_0^2(\Omega^2 - \omega_k^2)$, except for a small section near the turning point (see panel (E) of Figure 4), and so Equation (22) can be simplified as

$$\omega^2 + 2\Omega^2 + 4\omega_k^2(-\omega^2 - \Omega^2 + \omega_k^2)(-\omega^2 + \omega_k^2) + 4\Omega^2\omega^2 \rightarrow (-\omega^2 + \omega_k^2) \simeq k^2r_0^2(\Omega^2 - \omega_k^2)^2(\omega^2 + \Omega^2 - \omega_k^2).$$ (33)

(4) In Equation (33), it can be inferred that the expression $l_1$ dominates the changing trend because all of the other expressions maintain the changing trend with increasing $M$. The changing trend of $l_1$ is shown in panel (F) of Figure 4 and its transition accretion rate is also close to that of the single solution.

We can draw the conclusion that the frequencies of the solutions of Equation (22) will change with the increase of the accretion rate in the same trend as $l_1$.

The negative factor $(1 - \gamma)$ in Equation (30) in model A from the balance Equation (15) leads to a different changing trend of the frequencies of the single solutions with the accretion rate. The configuration of the magnetic field is related to the distribution of the density by the balance equation. Finally, we can conclude that the differences of the distribution of the magnetic field and the distribution of the density lead to the different changing trends of the frequencies of the solutions in models A and B.

3. COMPARISON WITH OBSERVATIONS

3.1. Twin kHz QPOs

We take several steps to select suitable parameters, such as the wavenumber and the surface magnetic field of NSs, to match the observations.

1. We choose the initial value of the magnetic flux density (e.g., $B_s = 10^8$ G) and the accretion rate (e.g.,

\[ B = 5 \times 10^8 \, \text{G}; \quad \Omega = 415.0 \times 2\pi \, \text{Hz}; \quad k = 1.705 \times 10^{-6} \, \text{cm}^{-1}; \]

\[ \Omega = 415.0 \times 2\pi \, \text{Hz}; \]

\[ B = 0.178 \times 10^8 \, \text{G}; \]

\[ k = 1.2 \times 10^{-5} \, \text{cm}^{-1}; \]

\[ \Omega = 110 \times 2\pi \, \text{Hz}; \]

\[ B = 5 \times 10^8 \, \text{G}; \]

\[ \Omega = 415.0 \times 2\pi \, \text{Hz}; \]

\[ B = 0.178 \times 10^8 \, \text{G}; \]

\[ k = 1.2 \times 10^{-5} \, \text{cm}^{-1}; \]

\[ \Omega = 110 \times 2\pi \, \text{Hz}; \]

\[ B = 5 \times 10^8 \, \text{G}; \]

\[ \Omega = 415.0 \times 2\pi \, \text{Hz}; \]

\[ B = 0.178 \times 10^8 \, \text{G}; \]

\[ k = 1.2 \times 10^{-5} \, \text{cm}^{-1}; \]
\[ \dot{M} = 10^{16} \text{ g s}^{-1}, \]

and then substitute them into \( r_{\text{mb}} \) or \( r_{\text{nA}} \), and the detailed expressions for \( \omega_k, c_s, V_A, \) and \( m \).

2. The solutions of Equation (22) or (30) can be obtained when different wavenumbers, \( k \), are considered.

3. We select the value of \( k \) for which the model predicted frequency difference (\( \Delta \nu = \nu_u - \nu_l \)) is closest to the discovered average value of \( \Delta \nu = \nu_{\text{upper}} - \nu_{\text{lower}} \) (such as 322 Hz for 4U 0614+09).

4. The solutions of Equation (22) or (30) can again be obtained with the above \( k \), the same accretion rate (e.g., \( \dot{M} = 10^{16} \text{ g s}^{-1} \)), and different values of \( B_* \).

5. We select the value of \( B_* \) for which the twin solutions are closest to the observed frequencies of the twin kHz QPOs.

6. After substituting the above \( k, B_* \), and different values of \( \dot{M} \) into Equation (22) or (30), we can obtain new solutions.
7. We change the value of $B_0$ slightly and then repeat step (6) until the solutions match the frequencies of most observed twin kHz QPOs.

8. We change the value of $k$ slightly and repeat step (6) until the solutions match the frequencies of most observed twin kHz QPOs.

9. We repeat steps (7) and (8) above in turn until the numerical solutions match the maximum number of the observed twin kHz QPOs.

10. With the last $k$ and $B_0$, we can obtain both $v_u$ and $v_l$, changing with $M$ in the two models.

11. Using all of the parameters determined above (and listed in Table 1), we compute $M$ by requiring that $v_{lower} = v_l$.

12. For each $M$, we numerically calculate $v_u$.

13. Finally, in Figure 5, we plot the numerically calculated and observed twin kHz QPOs as functions of $M$.

Because the analytic solutions are too complex and their expressions are too long to be used, we just choose the “best” parameters that match the most data as much as possible by checking by eye. As shown in Table 1, most of the wavenumbers ($k$) are close to $1 \times 10^{-6}$ cm$^{-1}$ in Table 1. Rezania & Samson (2005) discussed the fact that the wavelength of the MHD wave in LMXBs was on the order of the magnitude of the radius of NS.

The data points describe the observational data and the solid lines come from our numerical solutions in Figure 5. In the left panels of Figure 5, we consider the dipole magnetic field and use the magnetosphere radius in model A from Equation (28). In a1, b1, c1, suitable values of $M$ for several groups of observed QPOs are not found with model A, and thus those QPOs are not plotted. The right panels describe the result when a compressed magnetic field is considered and the magnetosphere radius from Equation (29) is used.

In the left panels of Figure 5, we could not find a group of parameters for 4U 0614+09, 4U 1608–52, and 4U 1636–53 in order to match the observations and we can only adopt not one but two different $k$ ($k_{a1}$ and $k_{a2}$) for 4U 1636–53 for the best result to match the observations including all of the twin kHz QPOs. The relation between $v_{upper}$ and $M$ is reproduced well in the right panels of Figure 5 and the result from model B is much better than that from model A.

In the right panels of Figure 5, our numerical solutions in 4U 1608–52 and 4U 1636–53 deviate from the observations slightly in the tails of the curves, perhaps due to ISCO. Barret et al. (2005, 2006) discovered the ceiling of the lower QPO frequency in 4U 1636–53 and 4U 1608–52 in a frequency–count rate diagram. As shown in Figure 5, however, the ceiling of $v_{upper}$ in 4U 1636–53 and 4U 1608–52 seems to be clearer than $v_{lower}$. This is different from Barret et al. (2005), mainly because when plotting our results in the figure, $v_l = v_{lower}$ is required to determine $M$ and so deviations can only exist in $v_{upper}$. Then, the magnetosphere radius can be determined with Equation (29).

We can estimate the masses of the two NSs in 4U 1636–53 and 4U 1608–52 if the magnetosphere radius corresponding to the ceiling frequency is ISCO (see Figure 3), i.e., $M_{NS} = r_2/r_1 \times 1.4 M_{⊙}$, where $r_1$ is the ISCO of an NS with 1.4 $M_{⊙}$ and $r_2 = r_c$ in Equation (29) when the QPO frequency reaches the ceiling frequency. Then, the ceiling frequencies and their errors can be found as follows (see Figure 6).

1. The histogram of the lower (or upper) QPO frequencies is plotted with a bin size of 20 Hz (slightly larger than the error (about 17 Hz) of the observational data).
2. We look for a peak in the histogram near the high frequency end.
3. We pick all of the QPO frequencies if their absolute differences between the peak frequency are less than 40 Hz.
4. The average of those QPO frequencies is considered to be the ceiling frequency and their rms is the error of the ceiling frequency.

With a ceiling frequency of either $v_{upper}$ or $v_{lower}$, we can obtain $M$ from the numerical solution for model B. Then, with $B_0$ (see Table 1) determined from the fitting (see Section 3.1 for details), $r_2$ can be determined. In Figure 7, we show the relations between the ceiling frequencies as functions of $M_{NS}$.

For 4U 1636–53 we have $v_{upper, ceiling} = 1181 \pm 13$ Hz and $v_{lower, ceiling} = 902 \pm 17$ Hz, corresponding to $M_{NS} = 1.90 \pm 0.01 M_{⊙}$ and $1.85 \pm 0.02 M_{⊙}$, respectively. In comparison, $M_{NS} = 2.02 \pm 0.12 M_{⊙}$ was obtained by Kaaret et al. (1997) by assuming the maximum $v_{upper} = 1171$ Hz as the Kepler frequency at ISCO in a Kerr spacetime. Similarly, in 4U 1608–52, $M_{NS} = 2.03 \pm 0.02 M_{⊙}$ or $1.98 \pm 0.03 M_{⊙}$ for $v_{upper, ceiling} = 1042 \pm 15$ Hz or $v_{lower, ceiling} = 772 \pm 14$ Hz, respectively.

3.2. Single kHz QPOs

The corresponding $M$ for single kHz QPOs cannot be identified using the above method for the twin kHz QPOs and can be considered as our model prediction in the right panels of Figure 5. In the left panels of Figure 5, the single numerical solutions in model A are too small to match the observed single kHz QPOs so we will discuss the single kHz QPOs only in model B.

Generally, we use a Lorentzian function to describe the finite-width peak in the power spectrum, i.e., QPOs, and the QPOs are

| Sources      | $\nu$ (Hz) | $k_{a1}$ (10$^{-6}$ cm$^{-1}$) | $k_{a2}$ (10$^{-6}$ cm$^{-1}$) | $B_{a1}$ (10$^{8}$ G) | $k_{l}$ (10$^{-6}$ cm$^{-1}$) | $B_{l}$ (10$^{8}$ G) | $r_{co}$ (10$^6$ cm) | $r_{l2}$ (10$^6$ cm) |
|-------------|-----------|-------------------------------|-------------------------------|-----------------------|-----------------------------|---------------------|---------------------|---------------------|
| 4U 0614+09  | 415       | 1.71                          | ~                             | 5                     | 1.20                        | 0.18                | 3.00                | 3.01                |
| 4U 1728–34  | 363       | 1.52                          | ~                             | 7                     | 1.15                        | 0.80                | 3.28                | 3.29                |
| XTE 1807–294| 190.6     | 1.03                          | ~                             | 20                    | 1.13                        | 0.53                | 5.04                | 5.08                |
| 4U 1915–05  | 270       | 1.35                          | ~                             | 11                    | 1.33                        | 0.27                | 4.00                | 4.01                |
| 4U 1608–52  | 619       | 0.05                          | ~                             | 6                     | 1.10                        | 0.53                | 2.30                | 2.31                |
| 4U 1636–53  | 581       | 0.36                          | 1.97                          | 6                     | 1.10                        | 0.46                | 2.40                | 2.41                |

This table lists the parameters used in our numerical computation for two different kinds of magnetosphere radii: the spin $\nu = (\Omega/2\pi)$, the selected parameters ($k_{a1}$, $k_{a2}$, and $B_{a1}$) for model A, the selected parameters ($k_{l}$, $B_{l}$ and $r_{co}$) for model B, and the corotation radius $r_{co}$ in the corresponding sources respectively.
Figure 5. Relations between the frequencies of the twin kHz QPOs, $\nu_{\text{upper}}$, $\nu_{\text{lower}}$, and accretion rate $\dot{M}$ for the six sources (for the measured data of 4U 0614+09: van Straaten et al. 2000; van Straaten et al. 2002; 4U 1636–53: Altamirano et al. 2008; Di Salvo et al. 2003; Jonker et al. 2002; Wijnands et al. 1997; 4U 1608–52: van Straaten et al. 2003; 4U 1915–05: Boirin et al. 2000; 4U 1728–34: van Straaten et al. 2002; Migliari et al. 2003; Di Salvo et al. 2001; Jonker et al. 2000; Strohmayer et al. 1996; XTE 1807–294: Linares et al. 2005; Zhang et al. 2006). Left: The magnetosphere radius from Equation (28) is used and the dipolar magnetic field is considered in model A. Right: The compressed magnetic field is considered and we use the magnetosphere radius (Equation (29)) which is obtained from the simulation of Kulkarni & Romanova (2013) in model B.

(A color version of this figure is available in the online journal.)
Figure 6. Histograms of the twin kHz QPOs in 4U 1636−53 and 4U 1608−52 (for the measured data: same with the caption of Figure 5).
(A color version of this figure is available in the online journal.)

Figure 7. Relation between the ceiling frequency of the twin KHz QPOs and the estimated mass of the NS in 4U 1636−53 and 4U 1608−52.
(A color version of this figure is available in the online journal.)
confirmed only if the qualify factor is larger than two. As shown in Figure 8, the single kHz QPOs with low count rate outside the box are related to a low qualify factor and the other of the twin kHz QPOs may also be related to a low qualify factor. The other kHz QPOs with very low qualify factors may be missed and some reported single kHz QPOs may be one of the twin kHz QPOs.

Barret et al. (2006) believed that the origin of $\nu_{\text{upper}}$ was different from that of $\nu_{\text{lower}}$ due to the different changing trends of their qualify factors. It is also possible that the qualify factors are related to the different origins of the twin kHz QPOs and single kHz QPOs. As shown in the right panel of Figure 8 for 4U 1728–34, three reported “single” kHz QPOs clustered around twin kHz QPOs probably belong to the twin kHz QPOs, but with one of the twin kHz QPOs missed from detection due to a possibly lower signal-to-noise ratio. On the other hand, the reported single kHz QPOs in the box of each of the two panels of Figure 8 are located quite distinctively separated from all of the others, and are thus considered to be true single kHz QPOs; their frequencies decrease with the increase of the count rate (probably an indicator of accretion rate), as predicted by model B shown in Figure 5. The real single kHz QPOs in the other sources are identified by the same method as above and the unselected single kHz QPOs listed in the references in the caption of Figure 5 are omitted from Figure 5.

4. DISCUSSION

In this study, we only consider that the rotation axis of an NS coincides with the magnetic axis, i.e., the inclination angle is zero. Méndez et al. (2001) concluded that the properties of kHz QPOs are determined only by the mass accretion rate through the disk. Lamb et al. (2009) considered that the magnetic inclination is likely to be very small for accretion-powered millisecond pulsars. Romanova & Kulkarni (2009) found that the moving spots in the magnetic boundary layer regime might produce QPO features in some cases. Further simulation by Kulkarni & Romanova (2013) showed that the magnetosphere radius mainly depends on the accretion rate in Equation (24), but not on the misalignment angle of the dipole magnetic field (see also Figure 4 in Kulkarni & Romanova 2013). It seems that there is little influence on the kHz QPOs from the magnetic inclination because the central frequencies of kHz QPOs in our model are mainly determined by the magnetosphere radius.

There are some complex wave frequency solutions from Equations (22) and (30) such that the oscillations also grow or decrease in amplitude when the real solutions also exist for the same parameters; however, the corresponding periods are too long to be observed. The accretion rates of the other complex wave frequency solutions are too small and the solutions are obtained in an unstable accretion region where the magnetosphere radius is much larger than the corotation radius and the propeller effect will begin to dominate. We have therefore omitted them and selected only the real solutions, which are considered to be the source of the kHz QPOs in NS-LMXBs. The magnetosphere radius is always regarded as the termination radius of the accretion onto the NS and it probably determines the position of the boundary layer. Due to the effect of the centrifugal force, it is often discussed whether the magnetosphere radius should be restricted within the corotation radius (Pringle & Rees 1972; Spruit & Taam 1993; Rappaport et al. 2004), which would lead to an accretion process with instability when the magnetosphere radius is more than the corotation radius due to the propeller mechanism. After substituting the compressed magnetosphere radius into our kHz QPOs model, the kHz QPOs can be divided into two parts, i.e., the single kHz QPOs and the twin kHz QPOs in Figure 5. Because the result for the magnetosphere radius from Kulkarni & Romanova (2013) better matches the observation, we will only discuss the result for model B below. According to the result for the new magnetosphere radius, we find that the transition radius ($r_t$) is very close to the corotation radius (see Table 1) and so the twin kHz QPOs may originate from the steady accretion process and the single kHz QPOs may mainly originate from the unstable accretion process; the latter may be responsible for the low quality factor of the true single kHz QPOs shown in the box of right panel of Figure 8.

With the decrease of the accretion rate, the frequencies of the single kHz QPOs increase outside $r_t$ and will exceed the ceiling of the twin kHz QPOs in model B when the accretion rate of LMXBs is very low (about $5 \times 10^{10}$ g s$^{-1}$ for the parameters
in the right panel of Figure 3); however, the expected high frequency cannot be detected due to the following reasons. (1) An NS-LMXB in such a very low accretion rate cannot be detected with a sufficiently high signal-to-noise ratio. (2) We can confirm kHz QPOs only when the fluctuation of the signal is small enough and the signal-to-noise ratio is high enough. (3) The magnetosphere radius was not simulated by Kulkarni & Romanova (2013) when the accretion rate is very low and it may be considered as an uncompressed one, i.e., the modes of the kHz QPOs in the compressed magnetic field may be converted to those in the dipolar magnetic field.

In our model, the surface magnetic field of the NS is considered to be invariant and so the accretion rate of the LMXBs that determines the magnetosphere radius is the key parameter. In Figures 3 and 5, the accretion rate is not an observed result because it is determined by our selection from comparing the central frequencies of the observational lower kHz QPOs to our lower numerical solution. Generally, the energy spectrum from each observation needs to be fitted to calculate the corresponding physical accretion rate. However, the quality of the available data from RXTE is not good enough to do that. This is why we compare \( v_{\text{upper}} \) in the twin kHz QPOs with the observation by means of a model-derived accretion rate, and why the detailed relation between the frequencies of kHz QPOs and the measured accretion rate needs to be tested with future observations from better X-ray instruments.

5. SUMMARY

In this study, we reexamined the MHD model of kHz QPOs, and the relation between the frequencies of the kHz QPOs and the accretion rate is predicted in the two models. In model A, the magnetic field of an NS keeps its dipolar topology and the accretion disk is compressed due to the magnetic pressure of its dipolar field. In model B, the accretion disk keeps the standard \( \alpha \)-disk and the magnetosphere of an NS is compressed due to the gas pressure of the standard disk. We find that the results of model B match the observations much better. Our main results are summarized as follows.

1. The Alfvén-like wave and the sound-like wave at the transition radius is

\[ \tau_1, \tau_1, \text{ and } \tau_2 \]

as shown in Figure 3. All of the observed ceiling frequencies of each NS originate from the magnetosphere radius truncated at the ISCO of the NS.

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