Use of the chord method for analyzing workspaces of a parallel structure mechanism

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Abstract: The size and shape of the mechanism working area are one of the most important criteria of its performance. Known approaches to the analysis of the workspace, such as the geometric method or the discretization method, have a number of features that limit their effective application. This paper considers the optimization chord method for determining the working areas of various types on the example of a planar parallel mechanism. The simulation results for maximum, constant orientation and dextrous workspaces are presented.

1. Introduction
The parallel mechanisms are the closed-loop mechanisms, the output link of which is connected to the base with several kinematic chains. Such mechanisms have a number of advantages compared with traditional serial manipulators: high load capacity, good rigidity, and increased accuracy. This allows them to find applications in various areas of human life [1, 2].

However, these mechanisms have a number of disadvantages. One of the most significant drawbacks is the small size of the working area compared to the manipulators of a serial structure. This fact is due to the presence of additional design restrictions imposed by the kinematic chains of the mechanism, for example, such as permissible angles and displacements in the passive joints and requirements of the links’ non-intersection.

The dimensions and the shape of the workspace are one of the most important parameters of the mechanism, and the successful performance depends largely on the workspace knowledge since the trajectories of the mechanism should not go beyond it. The structural features of parallel mechanisms often do not allow an obvious answer about the shape of the working area. Various methods are used to evaluate it [3].

One of the most famous approaches is a geometric one [4, 5]. This method is based on the construction of the surfaces, that can be achieved by the mechanism joints and links, and the analysis of the intersections of these surfaces. This approach is quite intuitive, but it becomes difficult to apply for mechanisms with a complex structure and taking into account a large number of construction limitations.

Another widely used method is based on the workspace discretization [6–8]. In this case, space is divided into many points, for each of which the restrictions are checked. This approach allows one to deal with almost any constraints for mechanisms with a complex structure, but accurate results require a large number of points for analysis. This leads to an increase in the calculation time, and this is the main drawback of the method.
This paper presents an approach to the analysis of the various types of workspaces for the parallel structure mechanism, based on the optimization chord method. This method is an effective way to determine the boundaries of the working area dealing with various constraints. The method was originally designed for planar mechanisms, however, it can be adapted for the analysis of the spatial ones by sequential application for slices along any of the coordinate axes. A description of the chord method with examples of its application is presented in detail in [9–11] and is not the main aim of the work.

The purpose of this paper is to show how this method can be applied for a new mechanism, which structure can be defined as a parallel-serial one. The paper is organized as follows. First, the description of the mechanism is presented and the workspace analysis problem. Next, the different types of workspaces are calculated for the mechanism. Finally, the discussion section summarizes the results and gives several recommendations.

2. Mechanism description

The 3D model of the considered mechanism and its working prototype are presented in figure 1.

![Mechanism 3D model (a) and its prototype (b).](image)

**Figure 1.** Mechanism 3D model (a) and its prototype (b).

This mechanism has a parallel-serial structure. The planar parallel mechanism with three degrees of freedom is placed on a plate 1 (figure 1, a) which can move along a coordinate axis $Z$, perpendicular to the plane of this planar mechanism. Though the parallel mechanism has three degrees of freedom, it has four equal kinematic chains, each with rotational joints 2 on both ends and a linear drive 3 between them. A redundant kinematic chain is added to improve mechanism rigidity and decrease the number of its singular configurations. One more movable link 5 is placed on a top platform 4 of the planar parallel mechanism. This link comprises an instrument and has a rotational degree of freedom which axis 6 lies in the plane of a parallel mechanism. Thus, the mechanism has five degrees of freedom in total.

This mechanism is designed to perform operations of selective laser sintering, and its output link (instrument) is a laser beam unit. The prototype (figure 1, b) has an output link model represented by a steel shaft with the same weight and dimensions as a real instrument.
As the introduction said, a workspace is one of the most important characteristics of the mechanism. For the given mechanism, it is easy to find the workspace dimension along the $Z$ direction, because it is fully defined by the limits of the corresponding linear guides. The main difficulty is to evaluate the working area of the planar parallel mechanism, and the next section presents this procedure.

3. Workspace analysis of the planar parallel mechanism

Figure 2 shows a kinematic scheme of the planar parallel mechanism. For convenience, the instrument is fixed in a vertical position, and it is considered that the instrument and the platform form a single solid body. Other instrument orientations will be addressed in the next section.

![Kinematic scheme of the planar parallel mechanism.](image)

One can distinguish the following design constraints for the mechanism:
- permissible ranges of the kinematic chains’ lengths $q_1 – q_4$ (figure 2);
- permissible rotation angles in the base joints $A_1 – A_4$;
- permissible rotation angles in the platform joints $B_1 – B_4$.

Though the real prototype has the points $B_1, B_2$ and $B_3, B_4$ being coincident, here we consider the general case. The mechanism parameters according to the prototype are presented in Table 1.

| Table 1. The geometric parameters of the planar parallel mechanism. |
|---------------------------------------------------------------|
| Parameter description                                | Value                     |
| Coordinates $(X, Y)$ of the points $A_1 – A_4$, mm          | $(-500, 212), (-250, 92), (250, 92), (500, 112)$ |
| Platform length $(B_1B_4$ or $B_2B_3)$, mm                  | 298                       |
| Instrument length $(CD)$, mm                              | 225                       |
| Permissible ranges for $q_1 – q_4$, mm                     | 538 – 838                 |
| Permissible angles in $A_1 – A_4$, deg                     | 25 – 155                  |
| Permissible angles in $B_1 – B_4$, deg                     | 7 – 173                   |

In the current work we will focus on the three following types of the workspaces [12]:
1) a maximum workspace – a set of positions which the output link can place without any restrictions on its orientation;
2) a constant orientation workspace – a set of positions which the output link can place for one fixed orientation;
3) a dextrous workspace – a set of positions which the output link can place for any orientation in the given range.

An optimization chord method [9–11] allows finding the boundaries for all these types of working areas. Corresponding MATLAB programs were written, and the next few figures present the obtained results. The mechanism scheme in an arbitrary position also presents on these plots for better clearness.

First, the maximum workspace was obtained. In this case, there were no restrictions on the output link orientation angle, i.e. on the angle $\phi$ (figure 2). Figure 3 demonstrates the boundary of the working area. The pink arrows indicate output link orientation on the workspace boundary.

![Figure 3. A maximum workspace.](image)

The next figure shows the constant orientation workspace for $\phi = 0$.

![Figure 4. A constant orientation workspace ($\phi = 0$).](image)
Finally, the dextrous workspaces for three ranges of $\phi$: $[-5^\circ, +5^\circ]$, $[-15^\circ, +15^\circ]$, and $[-30^\circ, +30^\circ]$, are presented on figure 5.

![Figure 5](image.png)

**Figure 5.** Dextrous workspaces for different ranges of the angle $\phi$: red for $[-5^\circ, +5^\circ]$, blue for $[-15^\circ, +15^\circ]$, and green for $[-30^\circ, +30^\circ]$.

4. **Discussion on results**

As we can see, the optimization chord technique allows building different types of workspaces. The accuracy of the results depends on the chosen chord length, i.e. the distance between adjacent points on the boundary. In the considered examples, the chord length was equal to 30 mm, except for the case of a dextrous workspace with $\phi$ in a range $[-30^\circ, +30^\circ]$, where the chord length was decreased to 10 mm to obtain more precise results (figure 5, green line).

Figures above also demonstrate that the constant orientation workspace and the dextrous working areas are smaller than the maximum one. Even with constant zero orientation, the workspace (figure 4) is significantly smaller than one with no limitations on the rotations (figure 3). The greater the required range for the angle $\phi$, the smaller the workspace (figure 5).

Though the proposed method is a fast and effective way to find the workspace boundary, it also has some limitations. One can see on the figures above that there is some strange behavior of the boundary at the corner points (for example, the left corner of the workspace in figure 3). Such points called bifurcation points [11] and should be concerned separately. The chord method also includes several optimization procedures, which are non-convex [13] in a general case. This results in a local solution, which sometimes can be not the correct one, and can even get stuck in this solution. The dense points at left and right boundary corners on figures 4 and 5 are examples of this situation (they are also the bifurcation points).

At the beginning of the 3rd section, there was an assumption that the instrument had a vertical orientation. One can also deal with other orientations. Changing the instrument orientation will lead to a change in the length of $CD$ (figure 2). The shape of the resulting working area will not change, but the whole workspace will move up or down.

In conclusion, this paper presented an effective approach for workspace evaluation of the real robot prototype. The results of this work were successfully used in the experiments and were applied in the robot control system at the trajectory planning stage. The future work and the potential possibilities are to use the presented optimization chord method for the optimal dimensional synthesis with respect to the given workspaces. An example of such a technique can be found in [14].
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