Collective effects of multi-scatterer on coherent propagation of photon in a two-dimensional network

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We study the collective phenomenon in the scattering of a single photon by one or two layers of two-level atoms. By modeling the photon dispersion with a two-dimensional coupled cavity array (2D CCA), we analytically derive the scattering probability of a single photon. We find that the translational symmetry of the atomic distribution leads to many important effects in the single-photon scattering. In the case with one layer of atoms, the atomic collective Lamb shift is related to the photonic density of states (DOS) of a 1D CCA, rather than the photonic DOS of a 2D CCA. As a result, the photon is effectively not scattered by the atoms when the incident momentum of the photon takes some special values. In the case with two layers of atoms, an inter-layer effective coupling appears and induces an electromagnetic-induced-transparency-like phenomenon. Our work provides a new scheme of analyzing photon coherent transport in 2D and may help to understand the recent experiments about the high energy photon scattering by the layered nuclei material.

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I. INTRODUCTION

For the purpose of controlling the transport and the scattering of a single photon in a full quantum fashion, much progress has been made in confined atom-photon hybrid systems [1, 2]. It is shown that the transport of a single photon can be coherently manipulated by the interaction between the photon and the doped natural or artificial atoms in low-dimensional quantum networks [3-6]. The corresponding physical implementations could be realized in several different ways, such as: defected photonic crystals [7, 8] and superconducting transmission line resonators [9, 10]. So far, many investigations have been done on one-dimensional (1D) atom-photon hybrid system [11-13]. For example, people find that with a tunable two-level atom inside one of the cavities, the transmission and reflection of a single photon can be well controlled in 1D coupled cavity array (CCA) [6]. In order to fabricate the integrated all-optical on-chip devices, it is also necessary to study the control of the photon transport in two-dimensional (2D) structures [14, 15]. Nevertheless, to our best knowledge, this kind of investigation is still lack.

In this paper we begin to study the control of a single-photon transport in a 2D CCA with atoms. The 2D CCA is a good candidate for the 2D quantum networks. It has been experimentally realized in defected photonic crystals [14, 15]. The photon localization was observed in such a system with disorder [16]. Many authors proposed that the 2D CCA with atoms can be used on quantum information [17] and the quantum simulation of many-body physics, such as the superfluid-Mott transition [18] and quantum Hall effect [19-22], etc.

We investigate the scattering of a single photon in the 2D CCA, with two-level atoms which are periodically located in one or two rows of the cavities. It is pointed out that, the research for such a problem is not only helpful for the development of control technique for photon transport in a 2D quantum network, but also important for the investigations of collective effects of periodically located atoms on the single-photon scattering. Since Dicke’s initial work on super-radiance [23], many authors have studied the influence of the collective behaviors of atoms on the spontaneous radiation process [24-32]. Most of these researches focused on the cases where the atoms are randomly distributed [28, 29], or confined in a region which is much smaller than the cube of the photonic wave length [30], or the systems with finite atoms [31, 32]. Here we consider the scattering process of a single photon, and focus on the effects of infinite number of periodically distributed atoms, with the distance between two nearest atoms comparable with the photonic wave length. Our research is also closely related to the recent experiments of the single X-ray photon scattering on the layered nuclear material, where the 57Fe nuclei are periodically distributed [33, 34].

For the 2D CCA system, we analytically solve the single-photon scattering problem, and obtain a clear understanding of the physics behind the collective effects. We find that when the atoms are located in one row of the cavities in the 2D CCA, due to the translational symmetry of the atomic distribution, the atomic collective energy shift induced by the photon-atom interaction (i.e., collective Lamb shift) [35] is related to the photonic density of states (DOS) of a 1D CCA, rather than the photonic DOS of a 2D CCA. Furthermore, the collective Lamb shift diverges when the incident momentum of the photon takes some special values. In these cases the pho-
ton cannot be scattered by the atoms. When the atoms are located in two rows of the cavities, as a result of the periodical structure, only two atomic collective states are coupled to the photonic states. Effective coupling between these two collective states, which is described by the non-diagonal elements of the self-energy matrix, can be induced by the photon-atom interaction. We can thus obtain two dressed states with different effective energies, which are the eigen-states of the self-energy matrix. The maximum of the scattering probability appears when the incident photon is resonant with one of these two states. Similar as the atomic susceptibility in the system with electromagnetic induced transparency (EIT), the scattering probability has a double-peak behavior as a function of photon-atom detuning.

The rest of the paper is organized as follows. In Sec. II we investigate the single-photon scattering in a 2D CCA with one layer of atoms, and discuss the collective shift of the atomic energy. In Sec. III we consider the case with two layers of atoms, and illustrate the EIT-like behavior of the scattering probability. A brief conclusion is given in Sec. IV. Some details of our calculation are presented in the appendix.

II. SINGLE-PHOTON SCATTERING WITH ONE LAYER OF ATOMS

A. System and Hamiltonian

We consider a 2D array of identical single-mode cavities as shown in Fig. 1. We further assume that the photons can hop between neighbor cavities. Then the cavity array is described by a tight-binding model as

\[
H_c = \sum_{x,y=-\infty}^{+\infty} \omega_c a(x,y)^\dagger a(x,y) - \xi [a(x+1,y)^\dagger a(x,y) + a(x,y+1)^\dagger a(x,y) + \text{h.c.}] ,
\]

(1)

Here \( \omega_c \) is the frequency of the photons in the cavities, \( \xi \) is the hopping intensity or the inter-cavity coupling strength in both \( x \) and \( y \) directions, \( a(x,y) \) and \( a(x,y)^\dagger \) are the annihilation and creation operators of the photon in the cavity at position \( (x,y) \), respectively. Here and after we set \( \hbar = 1 \). When there is a single photon propagating in the system, the eigenstate \( |\vec{k}\rangle \) of \( H_c \) takes the form of 2D plane wave with momentum \( \vec{k} = (k_x, k_y) \) as

\[
|\vec{k}\rangle = \frac{1}{2\pi} \sum_{x,y=-\infty}^{+\infty} e^{i(k_x x + k_y y)} a(x,y)^\dagger |\text{vac}\rangle
\]

(2)

with \( |\text{vac}\rangle \) the vacuum state of all the cavities. The single-photon dispersive relation of the 2D cavity array

\[
\epsilon_{\vec{k}} \equiv \epsilon(k_x, k_y) = \omega_c - 2\xi (\cos k_x + \cos k_y)
\]

(3)

is naturally obtained by the stationary Schrödinger equation \( H_c |\vec{k}\rangle = \epsilon_{\vec{k}} |\vec{k}\rangle \).

B. Single-photon scattering state and T-matrix

Now we calculate the scattering probability of a single photon scattered by the two-level atoms in our system. To this end, we first derive the single-photon scattering state and the on-shell element of the T-matrix in this subsection. With the help of these results, we will obtain the single-photon scattering probability in the next subsection.

Figure 1: (Color online) The single-photon scattering by one layer of atoms in a 2D CCA. The two-level atoms are confined in one row of the array with period \( d \) (in the figure we show the case with \( d = 3 \)). During the scattering process, the incident photon (solid line) can be scattered in several different outgoing directions (waved lines).

To explore the scattering character of the incident photon on a collection of identical atoms with geometrical configuration, we embed a two-level atom in every \( d \) cavities along the \( y \)-axis, and the \( j \)-th atom is in the cavity at \((0, dj)\). The free Hamiltonian of these atoms is

\[
H_a = \omega_a \sum_j |e\rangle_j \langle e| + h.c.
\]

(4)

with \( \omega_a \) the energy-level spacing of the two-level atom and \( |e\rangle_j (|g\rangle_j) \) the excited (ground) state of the \( j \)-th atom. The atom-photon coupling is described by the Jaynes-Cummings Hamiltonian

\[
V = \sum_j \Omega a(0, dj) |e\rangle_j \langle g| + h.c.
\]

(5)

with \( \Omega \) the coupling strength.

The setup studied in this paper can be physically implemented by placing the two-level atoms in the defected cavities of the 2D optical crystals [14]. Under the condition \( (k_x, k_y) \sim \pi/2 \), the energy \( \epsilon_{\vec{k}} \) becomes a linear function of the momentum \( \vec{k} \) as \( \epsilon_{\vec{k}} \approx \omega_c + 2\xi (k_x + k_y) \). Therefore, in this region our model can also characterize the scattering of photons on the array of atoms in the free space, e.g., the scattering process in the recent experiments with X-ray photon scattered by the nuclei [33, 34].
The scattering state $|\Psi^{(+)}\rangle$ is given by the Lippmann-Schwinger equation

$$|\Psi^{(+)}\rangle = |\tilde{k}\rangle |\tilde{g}\rangle + \frac{1}{\epsilon_{\tilde{k}} - (H_a + H_c) + i0^+} V |\Psi^{(+)}\rangle$$

(6)

with $|\tilde{g}\rangle \equiv \prod_j |g_j\rangle$ the collective ground state of all atoms. It is clear that in our system the total excitation number $\sum_{x,y=-\infty}^{+\infty} a^\dagger_{(x,y)} a_{(x,y)} + \sum_j |e_j\rangle \langle e_j|$ is conserved. Thus, in the subspace with one excitation we can expand the stationary eigenstate as

$$|\psi^{(+)}\rangle = |\phi\rangle |\tilde{g}\rangle + \sum_j \beta_j |\text{vac}\rangle |\tilde{e}_j\rangle.$$  

(7)

Here $|\phi\rangle$ is a single-photon state of the cavity modes. The state $|\tilde{e}_j\rangle$ is defined as $|\tilde{e}_j\rangle \equiv |e_j\rangle \otimes \prod_{i \neq j} |g_i\rangle$ and represents the state with only the $j$-th atom excited. It follows from Eqs. (6) and (7) that $|\phi\rangle$ and $\beta_j$ satisfy

$$|\phi\rangle = |\tilde{k}\rangle + \sum_j \frac{\Omega^j \beta_j}{\epsilon_{\tilde{k}} - H_c + i0^+ a^\dagger_{(0,dj)}} |\text{vac}\rangle,$$

(8)

$$\beta_j = \frac{\Omega}{\epsilon_{\tilde{k}} - \omega_\alpha} a_{(0,dj)} |\phi\rangle.$$  

(9)

Eqs. (8) and (9) can be analytically solved by the following approach. First, we notice that our system is invariant under the translation for $d$ cavities along the $y$-axis. Namely, the total Hamiltonian $H_a + H_c + V$ of our system is commutative with the translation operator $D$, which is defined as $D a_{(x,y)} D^\dagger = a_{(x,y+d)}$, $D |\tilde{e}_j\rangle D^\dagger = |\tilde{e}_j+1\rangle (\tilde{e}_j+1)$ and $D |\tilde{g}\rangle D^\dagger = |\tilde{g}\rangle (\tilde{g})\rangle$. As a result of this symmetry, the scattering state $|\Psi^{(+)}\rangle$ in Eq. (6) satisfies $D|\Psi^{(+)}\rangle = \exp(i k_x d) |\Psi^{(+)}\rangle$. Therefore, we can conclude that the coefficient $\beta_j$ takes the form

$$\beta_j = 2 \beta e^{i k_y d j}.$$  

(10)

Second, substituting Eq. (10) into Eqs. (8)(9), we obtain the expression for the $j$-independent coefficient $\beta$:

$$\beta = \frac{\Omega}{2 \pi \epsilon_{\tilde{k}} - \omega_\alpha - \Sigma(\tilde{k})},$$

(11)

where the self-energy $\Sigma(\tilde{k})$ is given by

$$\Sigma(\tilde{k}) = \sum_{l=0}^{d-1} \Sigma_l(\tilde{k}).$$

(12)

Here the function $\Sigma_l(\tilde{k})$ is defined as

$$\Sigma_l(\tilde{k}) = \frac{\Omega^2}{2\pi d} \int_{-\pi}^{\pi} dq_y \frac{1}{\epsilon_{\tilde{k}} - \epsilon_{(l, p_l(k_y))} + i0^+}$$

(13)

with

$$p_l(k_y) \equiv (k_y + \pi + \frac{2\pi |l|}{d}) \text{mod}[2\pi] - \pi.$$  

(14)

To obtain Eq. (13) we have also used the formula

$$\sum_{j=-\infty}^{+\infty} e^{-i(q_y - k_y) dj} = \frac{2\pi}{d} \sum_{l=0}^{d-1} \delta[q_y - p_l(k_y)],$$

(15)

for $q_y \in [-\pi, \pi]$. It is pointed out that, in our system the self-energy $\Sigma(\tilde{k})$ takes finite value (except for some special momentums which will be discussed later) and thus the renormalization technique is not required. This is due to the fact that the single-photon energy $\epsilon_c$ has a finite upper limit $\omega_c + 2\epsilon_c$.

Furthermore, we can treat the integral in $\Sigma_l(\tilde{k})$ analytically and get the result

$$\Sigma_l(\tilde{k}) = \begin{cases} -i \Omega |\phi|^2 \left[2d\xi \sqrt{1 - A_l^2}\right]^{-1}, & |A_l| < 1, \\
-\text{sign}(A_l) \Omega |\phi|^2 \left[2d\xi \sqrt{A_l^2 - 1}\right]^{-1}, & |A_l| > 1, \end{cases}$$

(16)

with $A_l$ defined as

$$A_l \equiv \cos k_x + \cos k_y - \cos[p_l(k_y)].$$

(17)

Substituting the result in Eq. (16) into Eqs. (12), we finally obtain the analytical expressions of the state $|\phi\rangle$, the coefficient $\beta_j$, and the scattering state $|\Psi^{(+)}\rangle$.

With the analytical expression of the scattering state, we can calculate the on-shell element $t(\vec{k}' \leftarrow \vec{k})$ of the $T$-matrix. According to the scattering theory \cite{37}, $t(\vec{k}' \leftarrow \vec{k})$ is defined as $t(\vec{k}' \leftarrow \vec{k}) = \langle \tilde{g}\rangle (\tilde{k}'\rangle |\Psi^{(+)}\rangle$. The straightforward calculation yields

$$t(\vec{k}' \leftarrow \vec{k}) = u_1(\vec{k}) \sum_{l=0}^{d-1} \delta(k_x' - p_l(k_y)), $$

(18)

where $\vec{k}' = (k_x', k_y')$ and the function $u_1(\vec{k})$ is defined as

$$u_1(\vec{k}) = \frac{|\Omega|^2}{2\pi d \Delta - 2\xi (\cos k_x + \cos k_y) - \Sigma(\vec{k})}.$$  

(19)

Here the photon-atom detuning $\Delta$ is defined as

$$\Delta = \omega_c - \omega_\alpha.$$  

(20)

Due to the delta functions in Eq. (18), the $y$-component $k_y'$ of the outgoing momentum can only take $d$ possible values. This is also the result of the translation symmetry along the $y$-axis in our system.

C. Single-photon scattering probability

Using the above results of the $T$-matrix element, we can calculate the single-photon scattering probability. To this end, we consider the scattering of a single-photon
wave packet on the atoms. In the scattering process, the incident wave packet of the photon can be expressed as

\[ |\Phi^{(in)}\rangle = \int d\vec{k}' |\vec{k}'\rangle |\vec{g}\rangle. \]  

(21)

Here \( \phi^{(in)}(\vec{k}) \) is single-photon wave function in the momentum representation and satisfies \( \int d\vec{k}|\phi^{(in)}(\vec{k})|^2 = 1 \). We further assume \( \phi^{(in)}(\vec{k}) \) sharply peaks at a specific momentum \( \vec{k}_0 = (k_{0x}, k_{0y}) \). According to the scattering theory, when the scattering process is completed, the single-photon state can be expressed as

\[ |\Phi^{(out)}\rangle = \int d\vec{k}' |\vec{k}'\rangle |\vec{g}\rangle \langle \vec{g}|S|\Phi^{(in)}\rangle \]  

(22)

\[ \equiv \int d\vec{k}' \phi^{(out)}(\vec{k}')(\vec{k}) |\vec{g}\rangle \]  

(23)

in the interaction picture. Here the \( S \)-matrix satisfies

\[ \langle \vec{g}|(\vec{k}')S|\vec{k}\rangle |\vec{g}\rangle = \delta(\vec{k}' - \vec{k}) - 2\pi i \delta(\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}'}) \tau(\vec{k}' \leftarrow \vec{k}). \]  

(24)

Substituting Eqs. (18, 24) into Eqs. (22, 23), it is easy to find that after the scattering process, the incident wave packet splits into \((2d-1)\) different ones (see Appendix). Namely, the output wave function in Eq. (25) is given by

\[ \phi^{(out)}(\vec{k}) = \sum_{l=-d}^{d-1} \phi_l^{(out)}(\vec{k}). \]  

(25)

Here the \( l \)-th wave packet \( \phi_l^{(out)}(\vec{k}) \) sharply peaks at a momentum \( \vec{k}_l = (k_{lx}, p_l(k_{dy})) \) with \( \varepsilon_{\vec{k}_l} = \varepsilon_{\vec{k}_0} \) and \( \text{sign}(k_{lx}) = \text{sign}(l) \) (see Appendix).

Then the probability for the photon being scattered to the \( l \)-th \((l \neq 0)\) outgoing momentum \( \vec{k}_l \) is \( P_l = \int d\vec{k}' |\phi_l^{(out)}(\vec{k}')|^2 \). As shown in the Appendix, \( P_l \) can be expressed as

\[ P_l = \frac{|u_l(\vec{k}_0)|^2}{4\xi^2 |\sin k_{0x} \sin k_{lx}|}. \]  

(26)

Therefore, for an incident photon with central momentum \( \vec{k}_0 \), the scattering probability \( R_l(\vec{k}_0) \) is

\[ R_l(\vec{k}_0) = \sum_{l=-d}^{d-1} P_l = 2 \sum_{l=1}^{d-1} P_l + P_0, \]  

(27)

where we have used the fact \( P_1 = P_{-1} \).

**D. Behavior of the scattering probability**

Now we discuss the behavior of the scattering probability \( R_l(\vec{k}) \) for an incident photon with central momentum \( \vec{k} \). According to Eqs. (19) and (20), it is clear that for fixed value of \( \vec{k} \), \( R_l(\vec{k}) \) is a Lorentz function of the photon-atom detuning \( \Delta \), and takes the maximum value under the condition \( \Delta = 2k(\cos k_{lx} + \cos k_{vy}) + \text{Re}[\Sigma(\vec{k})] \). In Fig. 2, we illustrate \( R_l(\vec{k}) \) with different periods of atoms. The Lorentz-shape of the \( R_l(\vec{k}) \) is clearly shown.

The single-peak behavior of \( R_l(\vec{k}) \) can be explained by the following simple picture. Owing to the periodic structure of our system in the \( y \)-direction, in the single-photon scattering process the incident state \( |\vec{k}\rangle \) of the photon is coupled to the atomic spin-wave state

\[ |S_{k_y}\rangle = \sum_j e^{ik_y j} |\vec{e}_j\rangle. \]  

(28)

As a result of this coupling, the effective energy of state \( |S_{k_y}\rangle \) is shifted from the bare value \( \omega_n \) to \( \omega_n + \text{Re}[\Sigma(\vec{k})] \). Thus, \( \text{Re}[\Sigma(\vec{k})] \) can be considered as the collective Lamb shift of the atomic state \( |S_{k_y}\rangle \). The scattering probability \( R_l(\vec{k}) \) takes the maximum value when the energy \( \varepsilon_{\vec{k}} \) of the incident photon equals to the shifted energy \( \omega_n + \text{Re}[\Sigma(\vec{k})] \) of the spin-wave state. The cooperative effect of the atomic ensemble is reflected by the atomic-density-dependence of this collective Lamb shift \( \text{Re}[\Sigma(\vec{k})] \) and the width \( \text{Im}[\Sigma(\vec{k})] \) of the peak of \( R_l(\vec{k}) \) [27, 28, 30].

Now we consider the effect of the translational symmetry of the atomic distribution on the collective Lamb shift. Due to this symmetry, the atomic spin-wave state \( |S_{k_y}\rangle \) is only coupled to the photonic states in the subspace \( \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus ... \oplus \mathcal{H}_{d-1} \) during the scattering process. Here \( \mathcal{H}_i \) is the subspace spanned by the states \( |\vec{k}_i\rangle = |k_{lx}, k_{ly}\rangle \), with \( k_{lx} \in (-\pi, \pi) \) and \( k_{ly} \) taking a fixed value \( p_l(k_{dy}) \). It is apparent that each space \( \mathcal{H}_i \) is isomorphic to the state space of a photon propagating in...
a 1D CCA, rather than the one of a photon propagating in the 2D CCA. In the expression (12) of the self-energy, the term $\Sigma_l(\vec{k})$ is essentially contributed by the coupling between $|S_{k_0}\rangle$ and the states in $\mathcal{H}_l$. With straightforward calculation, we can re-write $\Sigma_l(\vec{k})$ defined in Eq. (13) as

$$\Sigma_l(\vec{k}) = \frac{|\Omega|^2}{2\pi d} \int_{a_l}^{b_l} dE \rho(E) \frac{\epsilon_\vec{k} - E + i0^+}{\epsilon_\vec{k} - E + i0^+}$$

$$= -i \frac{|\Omega|^2}{2d} \rho(\epsilon_\vec{k}) + \frac{|\Omega|^2}{2\pi d} \int_{a_l}^{b_l} dx \frac{\rho(E)}{\epsilon_\vec{k} - E},$$

(29)

where P means the principle-value integral, $a_l = \epsilon_{0,p_f(k_0)}$ and $b_l = \epsilon_{x,p_f(k_0)}$ are the lower and upper bounds of the energy $\epsilon_\vec{k}$ of the states in $\mathcal{H}_l$, respectively. In Eq. (29) $\rho(x)$ is the density of states in $\mathcal{H}_l$ and can be expressed as $\rho(x) = dk_{\text{eff}}/dE$, with $k_{\text{eff}}$ related to $E$ via the relation $E = \epsilon_{\vec{k}} = \omega_c - 2\xi [\cos k_{x_0} + \cos p_{y_0}]$. It is apparent that $\rho(x)$ is nothing but the density of states of a single photon in a 1D CCA.

Usually, the self-energy $\Sigma_l(\vec{k})$ is convergent for a system with bounded energy spectrum. However, in the present problem, when the energy $\epsilon_\vec{k}$ of the incident photon is just at the boundaries of the energy spectrum of the states in $\mathcal{H}_l$, i.e., the condition $\epsilon_\vec{k} = a_l$ or $\epsilon_\vec{k} = b_l$ is satisfied), the principle-value integral in Eq. (29) diverges. This observation is verified by Eq. (15) which shows that $\Sigma_l(\vec{k}) = \infty$ when $|A_l| = \pm 1$, i.e., $\epsilon_\vec{k} = a_l$ or $b_l$.

Furthermore, according to Eqs. (19,26,27), when the self-energy diverges we have $R_l(\vec{k}) = 0$, i.e., the photon is not scattered by the atoms. In Fig. 3 we plot the scattering probability $R_l(\vec{k})$ and the self-energy $\Sigma(\vec{k})$ as functions of $\cos k_{x_0}$ for fixed values of $\Delta$ and $k_y$. It is clearly shown that $\Sigma(\vec{k}) = \infty$ at the points where $R_l(\vec{k}) = 0$.

In the end of this section, we would like to point out that, although $\Sigma_l(\vec{k})$ given in Eq. (16) is proportional to the atomic density $d^{-1}$, the total self-energy $\Sigma(\vec{k})$ is not a simple linear function of $d^{-1}$, because $\Sigma(\vec{k}) = \sum_{l=0}^{d-1} \Sigma_l(\vec{k})$ is the summation of $d$ terms. Therefore, the dependence of the collective Lamb shift $\text{Re}[\Sigma(\vec{k})]$ on atomic density is rather complicated. For instance, in the cases shown in Fig. 2, the collective Lamb shift increases when $d^{-1}$ is decreased from 1 to 1/3. We emphasize that, the complicated relation between $\text{Re}[\Sigma(\vec{k})]$ and $d^{-1}$ is caused by the expressions of $\Sigma(\vec{k})$ and $\Sigma_l(\vec{k})$, and thus is essentially a result of the translational symmetry of the atomic distribution in our system.

III. SINGLE-PHOTON SCATTERING WITH TWO LAYERS OF ATOMS

In the above section we have studied the single-photon scattering on one layer of atoms in a 2D cavity array.
We show that the single-photon scattering probability takes the maximum value when the incident photon is resonant with the shifted atomic energy \( \omega_a + \Re[\Sigma(\vec{k})] \), and thus has a single peak as a function of the photon-atom detuning \( \Delta \). In this section, we consider the single-photon scattering in the 2D cavity array with two layers of atoms located in the cavities at \((x_1, d_j)\) and \((x_2, d_j)\) with \( j = 0, \pm 1, \pm 2, \ldots \) (Fig. 4). We will show that as a function of \( \Delta \), the scattering probability has two peaks rather than a single one. The double-peak behavior is due to the photon-induced effective coupling between atoms in different layers, and can be considered as an EIT-like phenomenon.

In the presence of two layers of atoms, the atom-photon interaction reads

\[
V = \sum_{s=1,2} \sum_{j=-\infty}^{\infty} \Omega_s a_{s(x_j, d_j)} |e_j^{(s)}\rangle \langle g| + h.c.
\]  

(30)

with \( |g(e)\rangle^{(s)}_j \) the ground (excited) state of the \( j \)-th atom in the \( s \)-th layer. The scattering state \( |\Psi(+)\rangle \) can be written as

\[
|\Psi(+)\rangle = |\phi\rangle \langle g| + \sum_{s=1,2} \sum_{j=-\infty}^{\infty} \beta_j^{(s)} |\text{vac}\rangle |e_j^{(s)}\rangle.
\]  

(31)

Here \( |g\rangle \equiv \Pi_j |g\rangle^{(1)}_j |g\rangle^{(2)}_j \) is the collective ground state of all atoms, and \( |e_j^{(s)}\rangle \) \((s = 1, 2)\) defined as \( |e_j^{(s)}\rangle \equiv |e_j^{(s)}\rangle \prod_{i\neq j} |g\rangle^{(1)}_i |g\rangle^{(2)}_i \prod_n |g\rangle_n^{(3-s)} \) denotes the state in which only the \( j \)-th atom in the \( s \)-th layer is excited. Moreover, the translation invariance along the \( y \)-axis leads to the result \( \beta_j^{(s)} = \beta^{(s)} \exp[i(k_xx_j + k_y d_j)] \). Substituting this result into the Lippmann-Schwinger equation, we find that the \( j \)-independent coefficients \( \beta^{(1,2)} \) have similar expressions with the parameter \( \beta \) in Eq. (11), and can be written as

\[
\left( \begin{array}{c} \beta^{(1)} \\ \beta^{(2)} \end{array} \right) = \frac{1}{2\pi} \frac{1}{\omega_0 - \omega_a - \Sigma(\vec{k})} \left( \begin{array}{c} \Omega_1 \\ \Omega_2 \end{array} \right),
\]  

(32)

Here the self-energy \( \Sigma(\vec{k}) \) is now a \( 2 \times 2 \) matrix

\[
\Sigma(\vec{k}) = \left[ \begin{array}{cc} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{array} \right]
\]  

(33)

with elements \( \Sigma_{ij} \) \((i, j = 1, 2)\) given by

\[
\Sigma_{ij} = \frac{\Omega_i \Omega_j^*}{2\pi d} \sum_{l=e}^{d-1} \int_{\pi}^{\pi} dq \exp[-i(k_x - q_x)(x_j - x_i)]
\]  

\[
\times \epsilon_{\vec{k}} - \epsilon_{[q_x, p_y(k_y)]} + i0^+.
\]  

(34)

It is clear that Eq. (32) can be solved straightforwardly and we have

\[
\beta^{(s)} = \frac{\Omega_s}{2\pi (\Sigma_+ - \Sigma_-)} \left( \frac{\Sigma_- - J_s}{\Delta_{\vec{k}} - \Sigma_+} - \frac{\Sigma_+ - J_s}{\Delta_{\vec{k}} - \Sigma_-} \right)
\]  

(35)

for \( s = 1, 2 \). Here \( \Delta_{\vec{k}} = \omega_\vec{k} - \omega_a \) and \( J_s \) is defined as

\[
J_s = \Sigma_{(3-s), (3-s)} - \frac{\Omega_{(3-s)}}{\Omega_s} \Sigma_{s, (3-s)}.
\]  

(36)

In Eq. (35), \( \Sigma_{\pm}(\vec{k}) \) is the eigenvalue of matrix \( \Sigma(\vec{k}) \) and takes the form

\[
\Sigma_{\pm} = \frac{1}{2} \left[ \Sigma_{11} + \Sigma_{22} \pm \sqrt{(\Sigma_{11} - \Sigma_{22})^2 + 4\Sigma_{12}^2} \right].
\]  

(37)

With these results, we can derive the expressions of the scattering state \( |\Psi(+)\rangle \), and the on-shell element \( t(\vec{k'} \leftarrow \vec{k}) \) of the \( T \)-matrix is

\[
t(\vec{k'} \leftarrow \vec{k}) = u_{11}(\vec{k}) \sum_l \delta [k_{1y} - p_1(k_{1y})].
\]  

(38)

Now the coefficient function \( u_{11}(\vec{k}) \) is given by

\[
u_{11}(\vec{k}) = \frac{1}{2\pi d} \sum_{s=1,2} e^{-i(k_{1x} - k_x)d_y} \Omega_s^* \beta^{(s)}.
\]  

(39)

Similar to the above section, for an incident photon with central momentum \( \vec{k} \), the scattering probability \( R_{11}(\vec{k}) \) can be expressed in terms of the \( T \)-matrix element, which is given by

\[
R_{11}(\vec{k}) = 2 \sum_{l=1}^{d} \left| \frac{u_{11}(\vec{k})}{4k_x^2 \sin k_x \sin k_{1x}} \right|^2
\]  

(40)

with \( k_{1x} \) determined by the equation \( \epsilon_{\vec{k}} = \epsilon_{[k_{1x}, p_y(k_y)]} \).

Now we investigate the behavior of the scattering probability \( R_{11}(\vec{k}) \) with respect to the bare detuning \( \Delta \). With Eqs. (35) and (39), we find that \( R_{11}(\vec{k}) \) takes local maximum values when the condition \( \Re[\Delta_\pm - \Sigma_{\pm}(\vec{k})] = 0 \) is satisfied. Namely, unlike the single-peak behavior shown in Eq. (11) and Fig. 2 for the single-layer case, in the current system \( R_{11}(\vec{k}) \) has two peaks around the positions

\[
\Delta_{\pm} \equiv \Re[\Sigma_{\pm}] + 2\xi (\cos k_x + \cos k_y).
\]  

(41)
Figure 5: (Color online) The single-photon scattering probability $R_{11}(\vec{k})$ for the two-layer case (solid blue line). The peak behavior of $R_{11}(\vec{k})$ at $\Delta = \Delta_\pm$ is illustrated for the cases with $\Omega_1 = 7$, $\Omega_2 = 5$, $(k_x, k_y) = (\pi/8, \pi/4)$ and $x_2 - x_1 = 7$ (a), $x_2 - x_1 = 8$ (b), as well as the cases with $\Omega_1 = \Omega_2 = 5$ and $(k_x, k_y) = (\pi/8, \pi/4)$, $x_2 - x_1 = 15$ (c), $(k_x, k_y) = (\pi/8, \pi/4)$, $x_2 - x_1 = 14$ (d), $(k_x, k_y) = (\pi/8, \pi/3)$, $x_2 - x_1 = 12$ (e) and $(k_x, k_y) = (\pi/8, \pi/3)$, $x_2 - x_1 = 8$ (f). Here we choose $\xi = 1$ and $d = 3$. As a comparison, for each parameter $(\xi, k_x, k_y, \Omega_1, \Omega_2)$ we also plot the scattering probability $R_1(\vec{k})$ for the corresponding single-layer case with $\Omega = \Omega_2$ (dashed red line).

Furthermore, in the two-layer case, the cooperative effect of the atoms is reflected in the dependence of the collective Lamb shifts $\text{Re}[\Sigma_\pm(\vec{k})]$, the peak widths $\text{Im}[\Sigma_\pm(\vec{k})]$ and the distance between the two peaks $\text{Re}[\Sigma_+ - \Sigma_-](\vec{k})$ on the atomic density $d^{-1}$.

This observation is verified by our exact numerical calculation for Eq. (31). In Fig. 5 we illustrate $R_{11}(\vec{k})$ for the two-layer case (with the comparisons to the one-layer case). The double-peak behavior of $R_{11}(\vec{k})$ at $\Delta = \Delta_\pm$ is clearly shown in Figs. (a-f). It is pointed out that, such a behavior essentially has the same physical mechanism as that of the atomic susceptibility in an EIT system, where the two internal states of the $\Lambda$-type atom are dressed with the control laser beam, and form two dressed states with different energies. Then the atomic susceptibility takes local maximum value when the incident photon is resonant with one of these two states. In our problem, the incident state $|\bar{k}\rangle$ of the photon is coupled to two quasi spin-wave states

$$|S_{k_y}^{(1,2)}\rangle \equiv \sum_j e^{i k_y d_j} |E_j^{(1,2)}\rangle$$

with respect to the excitations of the atoms in the 1st and 2nd layers, respectively. These two states have the same bare energy $\omega_n$. Nevertheless, due to the atom-photon
coupling, \(|S_{k_y}^{(1)}\) and \(|S_{k_y}^{(2)}\) are effectively coupled with each other via the non-diagonal elements \(\Sigma_{12}\) and \(\Sigma_{21}\) of the self-energy matrix \(\Sigma(\vec{k})\). As a result of this effective coupling, \(|S_{k_y}^{(1)}\) and \(|S_{k_y}^{(2)}\) form two dressed states with different effective energies \(\omega_u + \Sigma_{\pm}\).

Figure 6: (a): In the two-layer case, the two quasi spin-wave states \(|S_{k_y}^{(1)}\) and \(|S_{k_y}^{(2)}\), which have the same bare energy \(\omega_u\), are coupled to the continuous spectrum of the photonic states. Due to this atom-photon coupling, \(|S_{k_y}^{(1)}\) and \(|S_{k_y}^{(2)}\) are effectively coupled with each other by the non-diagonal terms \(\Sigma_{12}\) and \(\Sigma_{21}\) of the self-energy matrix \(\Sigma(\vec{k})\). (b): As a result of this effective coupling, \(|S_{k_y}^{(1)}\) and \(|S_{k_y}^{(2)}\) form two dressed states with different effective energies \(\omega_u + \Sigma_{\pm}\).

result, when the incident momentum takes some special finite values, the collective Lamb shift diverges and the photon is effectively not scattered by the atoms. In the two-layer case, the periodicity of our system renders only two collective atomic states to couple with the photon. An EIT-like double-peak behavior appears when the effective coupling eliminates the degeneracy between them. Our approach and results will be helpful for the study of the 2D photonic quantum devices and X-ray quantum optics from the perspective of scattering.

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Appendix: single-photon outgoing wave packets and probability \(P_l\)

In this appendix we derive the outgoing wave function of a single photon scattered by the atoms, and prove Eq. (20) for the probability \(P_l\). Substituting Eqs. (18,24) into Eqs. (22,23), it is easy to find that after the scattering process, the incident wave packet splits into \((2d+1)\) different ones. Namely, the out-going wave function in Eq. (23) is given by

\[
\phi^{(out)}(\vec{k}) = \sum_{l=-d}^{d} \phi_l^{(out)}(\vec{k}).
\]

Here the function \(\phi_l^{(out)}(\vec{k})\) is defined as

\[
\phi_0^{(out)}(\vec{k}) = \frac{u(\vec{k})\phi^{(in)}(\vec{k})}{2\xi |\sin k_x|},
\]

for \(l = 0\) and

\[
\phi_l^{(out)}(\vec{k}) = \frac{u(\vec{k})\phi^{(in)}(\vec{k})}{2\xi |\sin f_{lx}(\vec{k})|}\delta_{\text{sign}(k_x),\text{sign}(l)}
\]

with the function \(f_l(\vec{k}) = [f_{lx}(\vec{k}), f_{ly}(\vec{k})]\) defined as

\[
f_{lx} (\vec{k}) = \arccos \left\{ \cos k_x + \cos k_y - \cos \left[ f_{ly} (\vec{k}) \right] \right\},
\]

\[
f_{ly} (\vec{k}) = (k_y + \pi + \frac{2\pi|l|}{d}) \text{mod}[2\pi] - \pi
\]

and the Kronecker symbol \(\delta_{i,j}\) satisfies \(\delta_{i,j} = 1\) for \(i = j\) and \(\delta_{i,j} = 0\) for \(i \neq j\). In Eqs. (43) and (44), we have \(u(\vec{k}) = u_1(\vec{k})\) for the case with one layer of atoms.
and \( u(\vec{k}) = u_{11}(\vec{k}) \) for the two-layer case. Then it is easy to see that, when the incident wave packet \( \phi^{(\text{in})}(\vec{k}) \) sharply peaks at a specific momentum \( \vec{k}_0 = (k_{0x}, k_{0y}) \), the out-put wave packet \( \phi^{(\text{out})}(\vec{k}) \) sharply peaks at the momentum \( \vec{k}_l = (k_{lx}, p_l(k_{0y})) \) which satisfies \( \varepsilon_{k_l} = \varepsilon_{k_0} \) and \( \text{sign}(k_{lx}) = \text{sign}(l) \).

Now we calculate the probability \( P_l = \int |\phi^{(\text{out})}(\vec{k})|^2 d\vec{k} \) for the cases with \( l \neq 0 \). Apparently, we have

\[
P_l = \int \left| \frac{u[f_l(\vec{k})] \phi^{(\text{in})}[f_l(\vec{k})]}{2\xi \sin f_{lx}(\vec{k})} \delta_{\text{sign}(k_x), \text{sign}(l)} \right|^2 d\vec{k}.
\] (48)

We define \( g_{lx} = f_{lx}(\vec{k}) \) and \( g_{ly} = f_{ly}(\vec{k}) \). Then using the fact that \( \phi^{(\text{in})}(\vec{k}) \) sharply peaks at a specific momentum \( \vec{k}_0 \) and the relations

\[ dk_x = \frac{\sin g_{lx}}{\sin k_{lx}} \, dg_{lx}, \quad dk_y = dg_{ly}, \] (49)

we have

\[
P_l \approx \int \left| \frac{u[f_l(\vec{k})] \phi^{(\text{in})}[f_l(\vec{k})]}{2\xi \sin f_{lx}(\vec{k})} \right|^2 d\vec{k} \approx \frac{|u(\vec{k}_0)|^2}{4\xi^2 |\sin k_{0x} \sin k_{lx}|} \int |\phi^{(\text{in})}(g_{lx}, g_{ly})|^2 d g_{lx} d g_{ly}.
\] (50)

Then we have proved Eq. \( (26) \).

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