Effects of quantum deformation on the spin-1/2 Aharonov-Bohm problem

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Abstract

In this letter we study the Aharonov-Bohm problem for a spin-1/2 particle in the quantum deformed framework generated by the $\kappa$-Poincaré-Hopf algebra. We consider the nonrelativistic limit of the $\kappa$-deformed Dirac equation and use the spin-dependent term to impose an upper bound on the magnitude of the deformation parameter $\varepsilon$. By using the self-adjoint extension approach, we examine the scattering and bound state scenarios. After obtaining the scattering phase shift and the $S$-matrix, the bound states energies are obtained by analyzing the pole structure of the latter. Using a recently developed general regularization prescription [Phys. Rev. D. 85, 041701(R) (2012)], the self-adjoint extension parameter is determined in terms of the physics of the problem. For last, we analyze the problem of helicity conservation.

Keywords: $\kappa$-Poincaré-Hopf algebra, self-adjoint extension, Aharonov-Bohm, scattering, helicity

1. Introduction

Theory of quantum deformations based on the $\kappa$-Poincaré-Hopf algebra has been an alternative framework for studying relativistic and nonrelativistic quantum systems. The Hopf-algebraic description of $\kappa$-deformed Poincaré symmetries, with $\kappa$ a masslike fundamental deformation parameter, was introduced in \cite{1,2}. In this context, the space-like $\kappa$-deformed Minkowski spacetime is the more interesting among them because its phenomenological applications. Such $\kappa$-deformed Poincaré-Hopf algebra established in Refs. \cite{1,2} is defined by the following commutation relations

\begin{equation}
[\Pi_\nu, \Pi_\mu] = 0,
\end{equation}

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where $\varepsilon$ is defined by

$$
\varepsilon = \kappa^{-1} = \lim_{R \to \infty} (R \ln q),
$$

with $R$ being the de Sitter curvature and $q$ is a real deformation parameter, $\Pi_\mu = (\Pi_0, \Pi)$ are the $\kappa$-deformed generators for energy and momenta. Also, the $M_i$, $L_i$ represent the spatial rotations and deformed boosts generators, respectively.

The coalgebra and antipode for the $\kappa$-deformed Poincaré algebra was established in Ref. [7].

The physical properties of $\kappa$-deformed relativistic quantum systems can be accessed by solving the $\kappa$-deformed Dirac equation [3, 4, 8, 9]. The deformation parameter $\kappa$ can be usually interpreted as being the Planck mass $M_P$ [10]. The $\kappa$-deformation has implications for various properties of physical systems as for example, vacuum energy divergent [11], Landau levels [12], spin-1/2 Aharonov-Bohm (AB) interaction creating additional bound states [13], Dirac oscillator [14], Dirac-Coulomb problem [4] and constant magnetic interaction [15]. In Ref. [13] the spin-1/2 AB problem was solved for the first time in connection with the theory of quantum deformations. The AB problem [16] has been extensively studied in different contexts in recent years [17–24]. In this letter we study the scattering scenario of the model addressed in Ref. [13] where only the bound state problem was considered. We solve the problem by following the self-adjoint extension approach [25–27] and by using the general regularization prescription proposed in [20] we determine the self-adjoint extension parameter in terms of the physics of the problem. Such procedure allows discuss the problem of helicity conservation and, as an alternative approach, we obtain the bound states energy from the poles of $S$-matrix.

The plan of our Letter is the following. In Section 2 we introduce the $\kappa$-deformed Dirac equation to be solved and take its nonrelativistic limit in order to study the physical implications of $\kappa$-deformation in the spin-1/2 AB problem. A new contribution to the nonrelativistic Hamiltonian arises in this approach. These new term imply a direct correction on the anomalous magnetic moment term. We impose a upper bound on the magnitude of the deformation parameter $\varepsilon$. The Section 3 is devoted to study the $\kappa$-deformed Hamiltonian via self-adjoint extension approach and presented some important properties of the $\kappa$-deformed wave function. In Section 4 are addressed the scattering and bound states scenario within the framework of $\kappa$-deformed Schrödinger-Pauli equation. Expressions for the phase shift, $S$-matrix, and bound states are derived. We also derive a relation between the self-adjoint extension parameter and the physical parameters of the problem. For last, we make a detailed analysis of the helicity conservation problem in the present framework. A brief conclusion in outlined in Section 5.
2. \( \kappa \)-deformed Schrödinger-Pauli equation

In the minimal coupling prescription the (3+1)-dimensional \( \kappa \)-deformed Dirac equation supported by the algebra in Eq. (1) up to \( O(\varepsilon) \) order was derived in Ref. [13] (see also Refs. therein). We here analyze the (2+1)-dimensional \( \kappa \)-deformed Dirac equation, which follows from the decoupling of (3+1)-dimensional \( \kappa \)-deformed Dirac equation for the specialized case where \( \partial_3 = 0 \) and \( A_3 = 0 \), into two uncoupled two-component equations, such as implemented in Refs. [28–30]. This way, the planar \( \kappa \)-deformed Dirac equation (\( \hbar = c = 1 \)) is

\[
\hat{H}\psi = \left[ \beta \gamma \cdot \Pi + \beta M + \frac{\varepsilon}{2} (M \gamma \cdot \Pi + e s \sigma \cdot B) \right] \psi = E \psi, \tag{3}
\]

where \( \psi \) is a two-component spinor, \( \Pi = p - eA \) is the generalized momentum, and \( s \) is twice the spin value, with \( s = +1 \) for spin “up” and \( s = -1 \) for spin “down”. The \( \gamma \)-matrices in (2+1) are given in terms of the Pauli matrices

\[
\beta = \gamma_0 = \sigma_3, \quad \gamma_1 = i\sigma_2, \quad \gamma_2 = -i\sigma_1. \tag{4}
\]

Here few comments are in order. First, the \( \kappa \)-deformed Dirac equation is defined in the commutative spacetime and the corresponding \( \gamma \)-matrices are independent of the deformation parameter \( \kappa \) [31]. Second, it is important to observe that in Ref. [13] the authors only consider the negative value of the spin projection, here our approach considers a more general situation.

We shall now take the nonrelativistic limit of Eq. (3). Writing \( \psi = (\chi, \phi)^T \), where \( \chi \) and \( \phi \) are the “large” and “small” components of the spinor, and using \( E = M + \varepsilon \) with \( M \gg \varepsilon \), after expressing the lower component \( \phi \) in terms of the upper one, \( \chi \), we get the \( \kappa \)-deformed Schrödinger-Pauli equation for the large component

\[
\hat{H}\chi = E\chi, \tag{5}
\]

with

\[
H = \frac{1}{2M} \left[ \Pi_1^2 + \Pi_2^2 - (1 - M\varepsilon)e s B_3 \right], \tag{6}
\]

where it was assumed that \( \varepsilon^2 \gg 0 \). It can be seen from (6) that the magnetic moment has modified by a quantity proportional to the deformation parameter.

Another effect enclosed in Hamiltonian (6) is concerned with the anomalous magnetic moment of the electron. The electron magnetic moment is \( \mu = -e/2M \), with \( e \) the gyromagnetic factor. The anomalous magnetic moment of the electron is given by \( \mu = 2(1 + a) \), with \( a = \alpha/2\pi = 0.00115965218279 \) representing the deviation in relation to the usual case [32]. In this case, the magnetic interaction is \( \hat{H} = \mu(1+a)(\sigma \cdot B) \). In accordance with very precise measurements and quantum electrodynamics (QED) calculations [33], precision corrections to this factor are now evaluated at the level of 1 part in \( 10^{11} \), that is, \( \Delta a \leq 3 \times 10^{-11} \). In our case, the Hamiltonian (6) provides \( \kappa \)-tree-level contributions to the usual \( g = 2 \) gyromagnetic factor, which can not be larger than \( a = 0.00116 \) (the current experimental value for the anomalous magnetic moment). The total \( \kappa \)-deformed magnetic interaction in Eq. (6) is

\[
H_{\text{magn}} = (1 - M\varepsilon)s(\mu \cdot B). \tag{7}
\]
For the magnetic field along the $z$-axis and a spin-polarized configuration in the $z$-axis, this interaction assumes the form

$$ (1 - M\varepsilon) s\mu B_z, \quad (8) $$

with $M\varepsilon$ representing the $\kappa$-tree-level correction that should be smaller than 0.00116. Under such consideration, we obtain the following upper bound for $\varepsilon$:

$$ \varepsilon < 2.27 \times 10^{-9} \text{ (eV)}^{-1}, \quad (9) $$

where we have used $M = 5.11 \times 10^5 \text{eV}$.

We now pass to study the $\kappa$-deformed Schrödinger-Pauli equation in the AB background potential [16]. The vector potential of the AB interaction, in the Coulomb gauge, is

$$ A = -\frac{\alpha}{r} \hat{\phi}, \quad A_0 = 0, \quad (10) $$

where $\alpha = \Phi/\Phi_0$ is the flux parameter with $\Phi_0 = 2\pi/e$. The magnetic field is given in the usual way

$$ eB = e\nabla \times A = -\alpha \frac{\delta(r)}{r} \hat{z}. \quad (11) $$

So, the $\kappa$-deformed Schrödinger-Pauli equation can be written as

$$ \frac{1}{2M} \left[ H_0 + \eta \frac{\delta(r)}{r} \right] \chi = E\chi, \quad (12) $$

with

$$ H_0 = \left( \frac{1}{i} \nabla - eA \right)^2, \quad (13) $$

and

$$ \eta = (1 - M\varepsilon)\alpha s, \quad (14) $$

is the coupling constant of the $\delta(r)/r$ potential.

For the present system the total angular momentum operator in the $z$ direction,

$$ \hat{J}_3 = -i\partial_\phi + \frac{1}{2} \sigma_3, \quad (15) $$

commutes with the effective Hamiltonian. So, it is possible to express the eigenfunctions of the two dimensional Hamiltonian in terms of the eigenfunctions of $\hat{J}_3$. The eigenfunctions of this operator are

$$ \psi = \begin{pmatrix} \chi \\ \phi \end{pmatrix} = \begin{pmatrix} f_m(r) e^{i(m_j - 1/2)\phi} \\ g_m(r) e^{i(m_j + 1/2)\phi} \end{pmatrix}, \quad (16) $$

with $m_j = m + 1/2 = \pm 1/2, \pm 3/2, \ldots$, with $m \in \mathbb{Z}$. Inserting this into Eq. (12), we can extract the radial equation for $f_m(r)$ ($k^2 = 2ME$)

$$ \hbar f_m(r) = k^2 f_m(r), \quad (17) $$
where

\[ h = h_0 + \eta \frac{\delta(r)}{r}, \quad (18) \]

\[ h_0 = -\frac{d^2}{dr^2} - \frac{1}{r} \frac{d}{dr} + \frac{(m + \alpha)^2}{r^2}. \quad (19) \]

The Hamiltonian in Eq. (18) is singular at the origin. This problem can then be treated by the method of the self-adjoint extension \[27\], which we pass to discuss in the next Section.

3. Self-adjoint extension analysis

The operator \( h_0 \), with domain \( \mathcal{D}(h_0) \), is self-adjoint if \( h_0^\dagger = h_0 \) and \( \mathcal{D}(h_0^\dagger) = \mathcal{D}(h_0) \). For smooth functions, \( \xi \in C_0^\infty(\mathbb{R}^2) \) with \( \xi(0) = 0 \), we should have \( h\xi = h_0\xi \), and hence it is reasonable to interpret the Hamiltonian (18) as a self-adjoint extension of \( h_0|_{C_0^\infty(\mathbb{R}^2 \setminus \{0\})} \) \[34–36\]. In order to proceed to the self-adjoint extensions of (19), we decompose the Hilbert space \( \mathcal{H} = L^2(\mathbb{R}^2) \) with respect to the angular momentum \( H = H_r \otimes H_\phi \), where \( H_r = L^2(\mathbb{R}^+, dr) \) and \( H_\phi = L^2(S^1, d\phi) \), with \( S^1 \) denoting the unit sphere in \( \mathbb{R}^2 \). The operator \(-\partial^2/\partial\phi^2\) is essentially self-adjoint in \( L^2(S^1, d\phi) \) \[37\] and we obtain the operator \( h_0 \) in each angular momentum sector. Now, using the unitary operator \( U : L^2(\mathbb{R}^+, dr) \to L^2(\mathbb{R}^+ \setminus \{0\}, dr) \), given by \((U\xi)(r) = r^{1/2}\xi(r)\), the operator \( h_0 \) becomes

\[ \tilde{h}_0 = Uh_0U^{-1} = -\frac{d^2}{dr^2} - \left[(m + \alpha)^2 - \frac{1}{4}\right] \frac{1}{r^2}, \quad (20) \]

which is essentially self-adjoint for \( |m + \alpha| \geq 1 \), while for \( |m + \alpha| < 1 \) it admits a one-parameter family of self-adjoint extensions \( h_0,\lambda_m \), where \( \lambda_m \) is the self-adjoint extension parameter. To characterize this family we will use the approach in \[27\], which is based in a boundary conditions at the origin.

Following the approach in Refs. \[26, 27\], all the self-adjoint extensions \( h_0,\lambda_m \) of \( h_0 \) are parametrized by the boundary condition at the origin

\[ f_{0,\lambda_m} = \lambda_m f_{1,\lambda_m}, \quad (21) \]

with

\[ f_{0,\lambda_m} = \lim_{r \to 0^+} r^{|m + \alpha|} f_m(r), \quad (22) \]

\[ f_{1,\lambda_m} = \lim_{r \to 0^+} r^{-|m + \alpha|} \left[ f_m(r) - f_{0,\lambda_m} \frac{1}{r^{|m + \alpha|}} \right], \quad (23) \]

where \( \lambda_m \in \mathbb{R} \) is the self-adjoint extension parameter. The self-adjoint extension parameter \( \lambda_m \) has a physical interpretation, it represents the scattering length \[38\] of \( h_0,\lambda_m \) \[27\]. For \( \lambda_m = 0 \) we have the free Hamiltonian (without the \( \delta \) function) with regular wave functions at the origin, and for \( \lambda_m \neq 0 \) the boundary condition in Eq. (21) permit a \( r^{-|m + \alpha|} \) singularity in the wave functions at the origin.
4. Scattering and bound states analysis

The general solution for Eq. (17) in the $r \neq 0$ region can be written as

$$f_m(r) = a_m J_{|m+\alpha|}(kr) + b_m Y_{|m+\alpha|}(kr),$$

(24)

with $a_m$ and $b_m$ being constants and $J_\nu(z)$ and $Y_\nu(z)$ are the Bessel functions of first and second kind, respectively. Upon replacing $f_m(r)$ in the boundary condition (21), we obtain

$$\lambda_m a_m \nu k^{|m+\alpha|} = b_m \left[ \zeta k^{-|m+\alpha|} - \lambda_m (\beta k^{|m+\alpha|} + \zeta \nu k^{-|m+\alpha|} \lim_{r \to 0^+} r^{2-|m+\alpha|}) \right],$$

(25)

where

$$\nu = \frac{1}{2^{|m+\alpha|} \Gamma(1+|m+\alpha|)}, \quad \zeta = \frac{2^{|m+\alpha|} \Gamma(|m+\alpha|)}{\pi},$$

$$\beta = -\frac{\cos(\pi|m+\alpha|) \Gamma(-|m+\alpha|)}{\pi 2^{|m+\alpha|}}, \quad \nu = \frac{k^2}{4(1-|m+\alpha|)}.$$  

(26)

In Eq. (25), $\lim_{r \to 0^+} r^{2-|m+\alpha|}$ is divergent if $|m + \alpha| \geq 1$, hence $b_m$ must be zero. On the other hand, $\lim_{r \to 0^+} r^{2-|m+\alpha|}$ is finite for $|m + \alpha| < 1$, it means that there arises the contribution of the irregular solution $Y_{|m+\alpha|}(kr)$. Here, the presence of an irregular solution contributing to the wave function stems from the fact the Hamiltonian $\hat{h}$ is not a self-adjoint operator when $|m + \alpha| < 1$ (cf., Section 3), hence such irregular solution must be associated with a self-adjoint extension of the operator $\hat{h}_0$ 39, 40. Thus, for $|m + \alpha| < 1$, we have

$$\lambda_m a_m \nu k^{|m+\alpha|} = b_m (\zeta k^{-|m+\alpha|} - \lambda_m \beta k^{|m+\alpha|}),$$

(27)

and by substituting the values of $\nu$, $\zeta$ and $\beta$ into above expression we find

$$b_m = -\mu_m^\lambda a_m,$$  

(28)

where

$$\mu_m^\lambda = \frac{\lambda_m k^2 |m+\alpha| \Gamma(1-|m+\alpha|) \sin(\pi|m+\alpha|)}{B_k},$$

(29)

and

$$B_k = \lambda_m k^2 |m+\alpha| \Gamma(1-|m+\alpha|) \cos(\pi|m+\alpha|)$$

$$+ 4^{|m+\alpha|} \Gamma(1+|m+\alpha|).$$

(30)

Since a $\delta$ function is a very short range potential, it follows that the asymptotic behavior of $f_m(r)$ for $r \to \infty$ is given by 41

$$f_m(r) \sim \sqrt{\frac{2}{\pi k r}} \cos \left( kr - \frac{|m| \pi}{2} - \frac{\pi}{4} + \delta_m^\lambda (k; \alpha) \right),$$

(31)
where $\delta_{m}^{\lambda}(k, \alpha)$ is a scattering phase shift. The phase shift is a measure of the argument difference to the asymptotic behavior of the solution $J_{m}(kr)$ of the radial free equation which is regular at the origin. By using the asymptotic behavior of the Bessel functions \[42\] into Eq. (24) we obtain

$$f_{m}(r) \sim a_{m} \sqrt{\frac{2}{\pi kr}} \left[ \cos \left( kr - \frac{\pi|m+\alpha|}{2} - \frac{\pi}{4} \right) - \mu_{m}^{\lambda} \sin \left( kr - \frac{\pi|m+\alpha|}{2} - \frac{\pi}{4} \right) \right].$$

(32)

By comparing the above expression with Eq. (31), we found

$$\delta_{m}^{\lambda}(k, \alpha) = \Delta_{AB}^{m}(\alpha) + \theta_{\lambda m},$$

(33)

where

$$\Delta_{AB}^{m}(\alpha) = \pi \frac{|m| - |m + \alpha|}{2},$$

(34)

is the usual phase shift of the AB scattering and

$$\theta_{\lambda m} = \arctan(\mu_{m}^{\lambda}).$$

(35)

Therefore, the scattering operator $S_{\alpha,m}^{\lambda}$ (S-matrix) for the self-adjoint extension is

$$S_{\alpha,m}^{\lambda} = e^{2i\delta_{m}^{\lambda}(k, \alpha)} = e^{2i\Delta_{AB}^{m}(\alpha)} \left[ 1 + i\mu_{m}^{\lambda} \right] \left[ 1 - i\mu_{m}^{\lambda} \right].$$

(36)

Using Eq. (29), we have

$$S_{\alpha,m}^{\lambda} = e^{2i\Delta_{AB}^{m}(\alpha)} \times \frac{B_{k} + i\lambda_{m}k^{2|m+\alpha|}\Gamma(1 - |m + \alpha|)\sin(\pi|m + \alpha|)}{B_{k} - i\lambda_{m}k^{2|m+\alpha|}\Gamma(1 - |m + \alpha|)\sin(\pi|m + \alpha|)}. \quad (37)$$

Hence, for any value of the self-adjoint extension parameter $\lambda_{m}$, there is an additional scattering. If $\lambda_{m} = 0$, we achieve the corresponding result for the usual AB problem with Dirichlet boundary condition; in this case, we recover the expression for the scattering matrix found in Ref. [43], $S_{\alpha,m}^{\lambda} = e^{2i\Delta_{AB}^{m}(\alpha)}$.

If we make $\lambda_{m} = \infty$, we get $S_{\alpha,m}^{\lambda} = e^{2i\Delta_{AB}^{m}(\alpha) + 2\pi|m+\alpha|}$.

In accordance with the general theory of scattering, the poles of the S-matrix in the upper half of the complex plane \[44\] determine the positions of the bound states in the energy scale. These poles occur in the denominator of (37) with the replacement $k \rightarrow i\kappa$,

$$B_{i\kappa} + i\lambda_{m}(ik)^{2|m+\alpha|}\Gamma(1 - |m + \alpha|)\sin(\pi|m + \alpha|) = 0.$$  

(38)

Solving the above equation for $E$, we found the bound state energy

$$E = -\frac{2}{M} \left[ -\frac{1}{\lambda_{m}\Gamma(1 - |m + \alpha|)} \right]^{1/|m+\alpha|},$$

(39)
for $\lambda_m < 0$. Hence, the poles of the scattering matrix only occur for negative values of the self-adjoint extension parameter. In this latter case, the scattering operator can be expressed in terms of the bound state energy

$$S_{\alpha,m}^{\lambda_m} = e^{2i\Delta_{AB}^{\lambda_m}(\alpha)} \left[ \frac{e^{2i\pi|m+\alpha|} - (\kappa/k)^2|m+\alpha|}{1 - (\kappa/k)^2|m+\alpha|} \right].$$

(40)

The scattering amplitude $f_{\alpha}(k, \varphi)$ can be obtained using the standard methods of scattering theory, namely

$$f_{\alpha}(k, \varphi) = \frac{1}{\sqrt{2\pi ik}} \sum_{m=-\infty}^{\infty} \left( e^{2i\lambda_m m(k,\alpha)} - 1 \right) e^{im\varphi}$$

$$= \frac{1}{\sqrt{2\pi ik}} \sum_{m=-\infty}^{\infty} \left( e^{2i\Delta_{m}(\alpha)} \left[ \frac{1 + i\mu_{m}}{1 - i\mu_{m}} \right] - 1 \right) e^{im\varphi}.$$  

(41)

In the above equation we can see that the scattering amplitude differs from the usual AB scattering amplitude off a thin solenoid because it is energy dependent (cf., Eq. (29)). The only length scale in the nonrelativistic problem is set by $1/k$, so it follows that the scattering amplitude would be a function of the angle alone, multiplied by $1/k$ [45]. This statement is the manifestation of the helicity conservation [46]. So, one would to expect the commutator of the Hamiltonian with the helicity operator, $\hat{h} = \sum \cdot \Pi$, to be zero. However, when calculated, one finds that

$$[\hat{H}, \hat{h}] = e \varepsilon \begin{pmatrix} 0 & (\sigma \cdot B)(\sigma \cdot \Pi) \\ \sigma \cdot B)(\sigma \cdot \Pi) & 0 \end{pmatrix},$$  

(42)

which is nonzero for $\varepsilon \neq 0$. So, the inevitable failure of helicity conservation expressed in Eq. (41) follow directly from the deformation parameter $\varepsilon$ and it must be related with the self-adjoint extension parameter, because the scattering amplitude depend on $\lambda_m$. Indeed, as it was shown in [20] it is possible to find a relation between the self-adjoint extension parameter and the coupling constant $\eta$ in (14). By direct inspection we can claim that such relation is

$$\frac{1}{\lambda_m} = -\frac{1}{r_0^2|m+\alpha|} \left( \frac{\eta + |m + \alpha|}{\eta - |m + \alpha|} \right),$$

(43)

where $r_0$ is a very small radius smaller than the Compton wave length $\lambda_C$ of the electron [47], which comes from the regularization of the $\delta$ function (for detailed analysis see [48]). The above relation is only valid for $\lambda_m < 0$ (when we have scattering and bound states), consequently we have $|\eta| \geq |m + \alpha|$ and due to $|m + \alpha| < 1$ it is sufficient to consider $|\eta| \geq 1$ to guarantee $\lambda_m$ to be negative. A necessary condition for a $\delta$ function generates an attractive potential, which is able to support bound states, is that the coupling constant must be negative. Thus, the existence of bound states requires

$$\eta \leq -1.$$  

(44)
Also, it seems from the above equation and from (14) that we must have \( \alpha s < 0 \) and there is a minimum value for the magnetic flux \( \alpha \).

It is worthwhile observe that bound states and additional scattering still remain inclusive when \( \varepsilon = 0 \), i.e., no quantum deformation case, because the condition \( \lambda_m < 0 \) is satisfied, as it is evident from (43). It was shown in Refs. [20, 49].

Now, let us comeback to helicity conservation problem. In fact, the failure of helicity conservation expressed in Eq. (41), it stems from the fact that the \( \delta \) function singularity make the Hamiltonian and the helicity non self-adjoint operators [50–53], hence their commutation must be analyzed carefully by considering first the correspondent self-adjoint extensions and after that compute the commutation relation, as we explain below. By expressing the helicity operator in terms of the variables used in (16), we attain

\[
\hat{h} = \begin{pmatrix} 0 & -i \left( \partial_r + \frac{s|m + \alpha| + 1}{r} \right) \\ -i \left( \partial_r - \frac{s|m + \alpha|}{r} \right) & 0 \end{pmatrix}.
\]  

(45)

This operator suffers from the same disease as the Hamiltonian operator in the interval \( |m + \alpha| < 1 \), i.e., it is not self-adjoint [54, 55]. Despite that on a finite interval \([0, L]\), \( \hat{h} \) is a self-adjoint operator with domain in the functions satisfying \( \xi(L) = e^{i\theta} \xi(0) \), it does not admit a self-adjoint extension on the interval \([0, \infty)\) [56], and consequently it can be not conserved, thus the helicity conservation is broken due to the presence of the singularity at the origin [45, 51].

5. Conclusion

We have studied the AB problem within the framework of \( \kappa \)-deformed Schrödinger-Pauli equation. The new contribution to the Pauli’s term is used to impose a upper bound in the deformation parameter, \( \varepsilon < 2.27 \times 10^{-9} \ (eV)^{-1} \). It has been shown that there is an additional scattering for any value of the self-adjoint extension parameter and for negative values there is non-zero energy bound states. On the other hand, the scattering amplitude show a energy dependency, it stems from the fact that the helicity operator and the Hamiltonian do not to commute. These results could be compared with those obtained in Ref. [49] where a relation between the self-adjoint extension parameter and the gyromagnetic ratio \( g \) was obtained. The usual Schrödinger-Pauli equation with \( g = 2 \) is supersymmetric [57] and consequently it admits zero energy bound states [58]. However, in the \( \kappa \)-deformed Schrödinger-Pauli equation \( g \neq 2 \) and supersymmetry is broken, giving rise to non-zero energy bound states. Changes in the helicity in a magnetic field represent a measure of the departure of the gyromagnetic ratio of the electron or muon from the Dirac value of \( 2e/2M \) [46]. Hence, the helicity nonconservation is related to nonvanishing value of \( g - 2 \).
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