Finite-Time Gradient Descent-Based Adaptive Neural Network Finite-Time Control Design for Attitude Tracking of a 3-DOF Helicopter

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Abstract—This paper investigates a novel finite-time gradient descent-based adaptive neural network finite-time control strategy for the attitude tracking of a 3-DOF lab helicopter platform subject to composite disturbances. First, the radial basis function neural network (RBFNN) is applied to estimate lumped disturbances, where the weights, centers and widths of the RBFNN are trained online via finite-time gradient descent algorithm. Then, a finite-time backstepping control scheme is constructed to fulfill the tracking control of the elevation and pitch angles. In addition, a hybrid finite-time differentiator (HFTD) is introduced for approximating the intermediate control signal and its derivative to avoid the problem of "explosion of complexity" in the traditional backstepping design protocol. Moreover, the errors caused by the HFTD can be attenuated by the combination of compensation signals. With the aid of the stability theorem, it is proved that the closed-loop system is semi-globally uniformly ultimately boundedness in finite time. Finally, a comparison result is provided to illustrate the effectiveness and advantages of the designed control strategy.

Index Terms—Neural network (NN), finite-time backstepping control, finite-time gradient descent, 3-DOF helicopter platform

I. INTRODUCTION

In the past few years, the laboratory 3-DOF helicopter (Fig. 1) has attracted considerable research interests owing to the similar dynamics with the real one and facilitating to implement various control approaches [1]. In recent years, numerous nonlinear and intelligent control schemes have been established to work out the attitude tracking problem of the helicopter platform [1]–[7].

In [1], a nonlinear robust control method, augmented with UDE terms to approximate compound disturbances, was presented to achieve the semi-global asymptotic stability of a table-mount helicopter. The authors in [2] developed a NN-based fault-tolerant control strategy for the helicopter platform under actuator faults, and the NN was applied to estimate the composite disturbance. By integrating the interval type-2 fuzzy logic into SMC, the developed control protocol in [3] fulfilled the exponential convergence of helicopter platform tracking error. The authors in [4] addressed the attitude tracking problem of an experimental helicopter with unknown time delay by utilizing an adaptive NN control approach, where NN was served as the uncertainty compensator. In [6], combined with RBFNNs to estimate lumped disturbances, an adaptive backstepping control framework was constructed for the attitude control of a 3-DOF helicopter, where all parameters of the RBFNNs are trained online by means of the gradient descent algorithm. In [7], a command filter was introduced into the NN-based backstepping control framework to settle the complexity explosion problem in the design of the 3-DOF helicopter. Nevertheless, all the above-mentioned control strategies are asymptotic stability or uniformly ultimately boundedness, while the finite-time control strategy is more applicable for the attitude tracking control of the 3-DOF helicopter.

Recently, finite-time backstepping control has sparked extensive attention owing to the combination of the merits of conventional backstepping control and finite-time control scheme [8]. Due to the universal approximation capability of NNs and FLSs under certain conditions, the adaptive neural and fuzzy control methods [9]–[16] are introduced into the finite-time backstepping control to cope with the uncertain nonlinearities existing in systems. However, the above-mentioned direct adaptive NN control approaches only consider the adaptation of weights, while the centers and widths of the RBFNN are set empirically and remain fixed during the entire control process, which may diminish the
control quality [17].

Motivated by the aforementioned discussion, a novel finite-time gradient descent-based adaptive NN finite-time control strategy is developed and applied to the attitude tracking control of a 3-DOF helicopter platform with composite disturbances in this work. First, a RBFNN is employed to approximate composite disturbances, where all the parameters of the RBFNN embracing the weights, centers and widths are trained online via finite-time gradient descent algorithm [18], [19]. Then, inspired by [16], a finite-time backstepping control strategy is designed in Section III. The comparative simulation experiment is implemented in Section V. Section II presents the system description and preliminaries.

A. System description

The compound disturbances $d_1(x)$ and $d_2(x)$ denote compound disturbances. The definitions and values of other variables can be seen in [21].

The control goal of this paper is to develop a finite-time gradient descent-based adaptive NN finite-time control strategy such that the tracking errors of the elevation angle and pitch angle can converge to an adequately small domain around the origin in finite time.

Assumption 1: The compound disturbances $d_1(x), d_2(x)$ are smooth and bounded.

Assumption 2: The reference signals $x_{1r}(t), x_{3r}(t)$ and their first derivative are known, smooth as well as bounded.

B. Preliminaries

Lemma 1 ([22]): For $y_j \in \mathbb{R}, j = 1, 2, \cdots, n, 0 < \gamma \leq 1$

$$\left(\sum_{j=1}^{n} |y_j|\right)^{\gamma} \leq \sum_{j=1}^{n} |y_j|^{\gamma} \leq n^{1-\gamma} \left(\sum_{j=1}^{n} |y_j|\right)^{\gamma} \tag{2}$$

Lemma 2 ([23]): For any $\chi_1, \chi_2 \in \mathbb{R}$, one has:

$$|\chi_1| c_1 |\chi_2| c_2 \leq \frac{c_1}{c_1+c_2} |\chi_1| c_1+c_2 + \frac{c_2}{c_1+c_2} |\chi_2| c_1+c_2 \tag{3}$$

where $c_1 > 0, c_2 > 0$ are constants.

To approximate the compound disturbances in the 3-DOF helicopter platform, the following lemma is required.

Lemma 3 ([24]): For any continuous function $d(x)$ defined on a compact set $\Omega$, there exist $\bar{\sigma} > 0$ and a RBFNN $W^T Q(x)$ satisfying

$$d(x) = W^T Q(x) + \sigma(x), |\sigma(x)| \leq \bar{\sigma} \tag{4}$$

where $x = [x_1, x_2, \cdots, x_N]^T$ and $W = [w_1, w_2, \cdots, w_M]^T$ stand for the input vector and weight vector of the RBFNN, respectively. $Q(x) = [q_1(x), q_2(x)\cdots, q_M(x)]^T$ represents the basis function vector, where the basis function $q_i(x)$ is defined as

$$q_i(x) = \exp\left(-\frac{||x - \mu_i||^2}{\delta_i^2}\right) \tag{5}$$

with $\mu_i = [\mu_{i1}, \mu_{i2}, \cdots, \mu_{iN}]^T$ and $\delta_i$ being the center and width vectors of the basis function, respectively.

III. Main Results

In this section, the design process of the control law for the elevation angle tracking will be displayed in detail.

A. Control Design

Consider the elevation channel of the system (1):

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{L_a}{J_a} \ddot{u}_1 - \frac{g}{J_a} mL \cos(x_1) + d_1(x) \tag{6}$$

Then, the tracking errors can be defined as follows:

$$e_1 = x_1 - x_{1r}$$

$$e_2 = x_2 - x_{1c} \tag{7}$$
where \( x_{1,c} \) is the approximation of intermediate control input \( \alpha_m \) by utilizing HFTD, which is formulated as [20].

\[
\begin{align*}
\dot{x}_{1,c} &= x_{2,c} \\
\dot{\varepsilon}^2 x_{2,c} &= \alpha_{m} \bigl( x_{1,c} - \alpha_{m} \bigr) - \alpha_{m} \text{sign}(x_{1,c} - \alpha_{m})^T \\
&\quad - b_{0} x_{2,c} - b_{1} \text{sign}(x_{2,c})^T
\end{align*}
\]

(8)

According to lemma 6 in [25], we have

\[
\begin{align*}
x_{1,c} - \alpha_{m} (t) &= O \bigl( \varepsilon^{n_{r}r_{2}} \bigr),
\quad \dot{x}_{2,c} - \dot{\alpha}_{m} (t) = O \bigl( \varepsilon^{n_{r}r_{2}^{-1}} \bigr) \quad (t > t_1)
\end{align*}
\]

(9)

To compensate the error generated by HFTD, the error compensation signals \( \xi_1, \xi_2 \) are constructed as follows [16]

\[
\begin{align*}
\dot{\xi}_1 &= -k_1 \xi_1 + \xi_2 + (x_{1,c} - \alpha_{m}) - m_1 \xi_1^h \\
\dot{\xi}_2 &= -k_2 \xi_2 - \xi_1 - m_2 \xi_2^h
\end{align*}
\]

(10)

where \( k_1 > 0, k_2 > 0, m_1 > 0, m_2 > 0 \) are the appropriate design parameters. \( h = h_2/h_1 < 1 \), \( h_1, h_2 \) are positive odd numbers.

Define the compensated tracking errors \( z_1, z_2 \) as the following form

\[
\begin{align*}
z_1 &= e_1 - \xi_1 \\
z_2 &= e_2 - \xi_2
\end{align*}
\]

(11)

Then, intermediate control input \( \alpha_m \) and the controller \( \bar{u}_1 \) are developed as

\[
\begin{align*}
\alpha_{m} &= -k_1 e_1 + \dot{x}_r - m_1 z_1^h \\
\bar{u}_1 &= \frac{J_a}{L_a} \left( -k_2 \dot{e}_2 - e_1 + x_{2,c} + \frac{g}{J_a} m L_a \text{cos}(x_1) \right) \quad (t > t_1)
\end{align*}
\]

(12)

where \( m_1 > 0, m_2 > 0 \) are the appropriate design parameters and \( W^T Q (x) \) denotes a RBFNN to estimate the composite disturbance \( d_1(x) \).

To train the parameters of the RBFNN, we construct the NN approximation error as follows

\[
E = \dot{z}_{2,c} + k_2 z_2 + z_1 + m_2 \dot{z}_2^h - n_2 \xi_2^h
\]

(13)

where \( \dot{z}_{2,c} \) is the approximation of \( \dot{z}_2 \) by adopting HFTD.

Design the cost function as \( Y = 0.5 E^2 \), then in terms of finite-time gradient descent algorithm [18], [19], the iterative equations of the weights \( w_i \), centers \( \mu_{i,j} \) and widths \( \delta_i \) can be given as

\[
\begin{align*}
\Delta w_i (k) &= E q_i \\
w_i (k + 1) &= w_i (k) + \lambda \text{sign}(\Delta w_i (k))^p \\
\Delta \mu_{i,j} (k) &= E w_i q_i \frac{(x_j - \mu_{i,j} (k))}{\delta_j^2 (k)} \\
\mu_{i,j} (k + 1) &= \mu_{i,j} (k) + \lambda \text{sign}(\Delta \mu_{i,j} (k))^p \\
\Delta \delta_i (k) &= E w_i q_i \frac{\| x_j - \mu_{i,j} (k) \|^2}{\delta_j^2 (k)} \\
\delta_i (k + 1) &= \delta_i (k) + \lambda \text{sign}(\Delta \delta_i (k))^p
\end{align*}
\]

(14)

where \( \delta \in (0, 1) \) denotes the learning rate, \( i = 1, 2, \cdots, M, j = 1, 2, \cdots, N \) and \( 0 \leq p < 1 \).

### B. Stability Analysis

Theorem 1: Consider the system (6) subject to Assumptions 1 and 2. If the HFTD is adopted as (8), the error compensation signals are constructed as (10), the intermediate control input is designed as (12), and the iterative equations of the parameters of the RBFNN are developed as (14), then the designed control laws \( \bar{u}_i \) can guarantee that the closed-loop system is semi-globally uniformly ultimately bounded in finite time, while the attitude tracking errors can converge to an adequately small domain around the origin in finite time.

Proof: By substituting (9), (10), (12) into (13) yields

\[
\begin{align*}
E &= \dot{z}_{2,c} + k_2 z_2 + z_1 + m_2 \dot{z}_2^h - n_2 \xi_2^h \\
&= \dot{z}_2 - O \bigl( \varepsilon^{n_{r}r_{2}^{-1}} \bigr) + k_2 z_2 + z_1 + m_2 \dot{z}_2^h - n_2 \xi_2^h \\
&= d_1 - W^T Q (x) - O \bigl( \varepsilon^{n_{r}r_{2}^{-1}} \bigr)
\end{align*}
\]

In terms of lemma 3 and finite-time gradient descent algorithm, there exists \( t_2 > 0 \) when \( t > t_2 \), one has

\[
\begin{align*}
d_1 - O \bigl( \varepsilon^{n_{r}r_{2}^{-1}} \bigr) &= W^T Q (x) + \sigma, \quad |\sigma| \leq \bar{\sigma}
\end{align*}
\]

(16)

Consider the following Lyapunov function

\[
V = \frac{1}{2} \sum_{i=1}^{2} (z_i^2 + \xi_i^2)
\]

(17)

Taking the time derivative of \( V \), we have

\[
\dot{V} \leq - \left[ k_1 z_1^2 + \left( k_2 - \frac{1}{2} \right) z_2^2 + \left( k_1 - \frac{1}{2} \right) \xi_1^2 + k_2 \xi_2^2 \right] \\
- \sum_{i=1}^{2} \left( m_i - \frac{n_i}{1+h} \right) z_{i+1}^2 + \sum_{i=1}^{2} \frac{n_i}{1+h} \xi_{i+1}^2 \right] + \eta_3
\]

(18)

where \( \eta_3 = 0.5 O \bigl( \varepsilon^{n_{r}r_{2}^{-1}} \bigr) + 0.5 (\bar{\sigma} + O \bigl( \varepsilon^{n_{r}r_{2}^{-1}} \bigr)^2 \bigr) \).

According to lemma 1, (19) can be further rewritten as

\[
\begin{align*}
\dot{V} (x) &\leq -\eta_1 V (x) - \eta_2 V (x)^{\frac{1+p}{2}} + \eta_3
\end{align*}
\]

(20)

where

\[
\begin{align*}
\eta_1 &= \min \left\{ (2k_1 - 1), (2k_2 - 1) \right\} \\
\eta_2 &= \min \left\{ \left( m_i - \frac{n_i}{1+h} \right)^{\frac{1+p}{2}}, \left( \frac{n_i}{1+h} \right)^{\frac{1+p}{2}} \right\}
\end{align*}
\]

(21)

By adopting lemma 5 in [25], the closed-loop system is semi-globally uniformly ultimately bounded in finite time. By designing the appropriate parameters, \( z_i, \xi_i \) can converge
to adequately small domains around the origin in finite time \( T \), which are given as follows

\[
|z| \leq \min \left\{ \frac{2\eta_3}{(1 - \kappa) \eta_1}, \sqrt{2 \left( \frac{\eta_3}{(1 - \kappa) \eta_2} \right)^{\frac{2}{1+\kappa}}} \right\}
\]

\[
|\xi| \leq \min \left\{ \frac{2\eta_3}{(1 - \kappa) \eta_1}, \sqrt{2 \left( \frac{\eta_3}{(1 - \kappa) \eta_2} \right)^{\frac{2}{1+\kappa}}} \right\}
\]

(22)

where \( \kappa \in (0, 1) \).

For \( t \geq T \), the tracking error will stay in

\[
|e_1| \leq |z| + |\xi|
\]

\[
\leq \min \left\{ \frac{2\eta_3}{(1 - \kappa) \eta_1}, \sqrt{2 \left( \frac{\eta_3}{(1 - \kappa) \eta_2} \right)^{\frac{2}{1+\kappa}}} \right\}
\]

(23)

The proof is completed.

IV. SIMULATION RESULTS

This section conducts a contrastive simulation experiment on the tracking control of a table-mount experimental helicopter using output feedback. The initial condition of the elevation angle is \( x_1(0) = -24^\circ \) and the compound disturbance is selected as \( d_1 = \sin(2t) \).

The target signal is given by

\[
x_{1r}(t) = \frac{\pi}{18} \sin(0.3\pi t - \frac{\pi}{2})
\]

(24)

Design the parameters of the constructed control strategy as \( k_1 = 1, k_2 = 2, h = 0.6, n_1 = n_2 = 0.5, n_3 = 0.5, a_0 = 5, a_1 = 0.5, \varepsilon = 0.01, b_0 = 2, b_1 = 0.5, r_1 = r_2 = 0.5, p = 0.6, M = 5, \lambda = 0.005 \), and the adaptive NN backstepping control method in [6] is applied as the comparison approach. The comparison simulation results are shown in Figs. 2-3 where Fig. 2 depicts the curves of the attitude tracking error by adopting the developed control strategy as well as the NN backstepping control method, and Fig. 3 displays the curves of control input.

It is apparent from the simulation results that our developed control scheme can guarantee that the attitude tracking error converges to an adequately small domain around the origin in finite time. Furthermore, it can be observed that compared with the approach in [6], the control scheme developed by us possesses the features of faster convergence speed coupled with higher tracking accuracy.

V. CONCLUSION

In this work, a new finite-time gradient descent-based adaptive neural network finite-time control scheme has been presented to deal with the tracking problem of a 3-DOF helicopter platform with composite disturbances. A RBFNN is introduced to approximate composite disturbances, where all the parameters of the RBFNN are trained online by utilizing finite-time gradient descent algorithm. By integrating HFTD and error compensation system into the finite-time backstepping control framework, the problem of complexity explosion is addressed, and the influence of filter error is alleviated. The theoretical analysis illustrates that the closed-loop system is semi-globally uniformly ultimately boundedness in finite time. The effectiveness and advantages of the designed control strategy are well demonstrated by the comparative simulation.

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