On sub-Alfven MHD flows with acceleration in coaxial channels

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Abstract. Magnetohydrodynamic (MHD) flows in coaxial channels of plasma accelerators with a longitudinal magnetic field are considered. Sub-Alfven MHD flows with acceleration transonic to the slow magnetosonic speed are studied. Sub-Alfven MHD flows are related to the case when the plasma flow velocity doesn’t exceed the Alfven speed corresponding to the longitudinal field along the channel. A narrow coaxial nozzle channel with a constant lower bound is considered. The non-stationary and stationary mathematical MHD models are considered in the quasi-one-dimensional approximation. The corresponding MHD problems are solved numerically. Features of the relaxation process and characteristics of the transonic sub-Alfven flow type with acceleration in the channel are studied. The influence of the longitudinal magnetic field on the steady-state plasma flows are researched. The mechanism of energy transformations is described.

1. Introduction
Mathematical modeling and numerical simulation are a key element in physical process researches and new technique development. Plasma acceleration is an object of interest due to its diverse technical applications. Plasma accelerators based on this effect are used to solve a number of problems facing humanity. In the transport industry, these devices are a component of electric thrusters installed on aerospace and aviation techniques. A spacecraft or an aircraft equipped with a plasma accelerator could achieve values of speed significantly exceeding those in their gas dynamic analogs. In the nuclear energy sector, high energy plasma fluxes formed in a plasma accelerator output could have parameters necessary for the thermonuclear fusion process activation and could be injected into the currently developed magnetic traps.

Depending on a principle of acceleration and plasma parameters, there are various technical realizations of plasma accelerators. In the present paper, plasma accelerators based on plasma acceleration in crossed electric and magnetic fields [1, 2] are considered. In these devices connected to an external source of electric energy, the interaction of the current flowing in plasma with a generated own transverse magnetic field results in plasma acceleration towards the channel output (figure 1).
The quasi-steady-state high-current plasma accelerator (QSPA) with record-high values of the output plasma flow velocity is a prime example of accelerators based on this principle [3]. The development and realization of QSPA were successfully completed because of, inter alia, the mathematical modeling and numerical simulation of plasma fluxes flowing in channels of such accelerators. In these devices, plasma can be considered as a continuous medium which behavior is described in terms of magnetic hydrodynamics (MHD) (see, e.g. [4])

The above-mentioned scheme of the plasma accelerator under consideration can be modified by including in it a longitudinal magnetic field induced by external current-carrying conductors. According to the theory [5], MHD flows in the channels are subdivided into two classes of MHD flows in this case: super-Alfven in which plasma velocity \( v \) exceeds the Alfven speed \( C_A \) corresponding to the longitudinal field and sub-Alfven with the opposite relation. In turn, each of the classes is subdivided into flow types depending on relation of the velocity and one of the magnetosonic speeds – fast magnetosonic \( C_f \) or slow magnetosonic \( C_s \). The classification of MHD flows in channels [6] is presented in figure 2.

The classification was obtained for a narrow nozzle channel with a constant lower bound, but its basic consequences are applied to MHD flows in a channel of arbitrary geometry. Accelerating super-Alfven transonic relative to the fast magnetosonic speed \( C_f \) (type 3 on figure 2) and sub-Alfven transonic relative to the slow magnetosonic speed \( C_s \) (type 8) are the main object of interest in the theory of plasma accelerators.
The relevant researches were connected with the study of the super-Alfven accelerating flow type in the transverse magnetic field [7] and in the presence of the longitudinal magnetic field [8, 9].

Features of relaxation process and characteristics of the Alfven type (type 5) in which plasma velocity coincides with the Alfven speed in each point of a channel with a constant lower bound and the near-Alfven types (types 4 and 6) are presented in [10]. The transition of the plasma velocity through the Alfven speed in combined flows with super-Alfven and sub-Alfven regions simultaneously existing in a channel of arbitrary geometry is studied in [11].

Following the previous researches, the present paper is focused on the sub-Alfven transonic (relative to the slow magnetosonic speed) flow (type 8 on figure 2).

2. Mathematical model
In the present paper, the object of modeling is plasma fluxes flowing in a coaxial channel in the presence of a longitudinal magnetic field (figure 3). The nozzle channel is formed by two coaxial electrodes connected to the external source of electric current. The axisymmetric plasma flow considered as a continuous conducting medium with the known thermodynamic parameters enters the channel and completes the formed electric circuit. In this system, the plasma is accelerated by the Ampere force $\mathbf{F}_\parallel$ of interaction between the radial electric current $\mathbf{j}$ flowing in it and the transverse magnetic field $\mathbf{H}_\perp$ generated by the current flowing in the central electrode. The longitudinal magnetic field $\mathbf{H}_\parallel$ additionally induced in the system influence on the plasma flow causing, in particular, its rotation in the channel transverse direction by the force $\mathbf{F}_\perp$.

![Figure 3. Scheme of coaxial plasma accelerator with longitudinal magnetic field in cylindrical coordinate system $(r, \varphi, z)$. Cross-section by the plane $\varphi=\text{const}$.](image_url)

Main regularities in parameters and characteristics of the plasma flows under consideration can be obtained in terms of the one-dimensional MHD model in the quasi-one-dimensional approximation (see, e.g. [6]). This approach is applied to narrow channels in which distance between electrodes is much less than its length or thin tubes between close flow trajectories. In this case, the researched parameters are averaged over a channel cross-section and depend only on the longitudinal spatial coordinate $z$.

In the present paper, a narrow nozzle-type coaxial channel with a constant lower bound is considered (figure 4). Its geometry is determined only by its ring cross-section $S(z)$. The non-stationary axisymmetric ($\partial / \partial \varphi = 0$) MHD flows in such channels in the quasi-one-dimensional approximation are described by the following system of equations:
\[
\begin{align*}
\frac{\partial}{\partial t} \rho S + \frac{\partial}{\partial z} \rho v_z S &= 0 \\
\frac{\partial}{\partial t} \rho v_z S + \frac{\partial}{\partial z} \left( \rho v_z^2 + \frac{H_z^2}{2} \right) S &= \left( p + \frac{H_\rho^2}{2} \right) S' \\
\frac{\partial}{\partial t} \rho \varphi S + \frac{\partial}{\partial z} \left( \rho v_\varphi v_\varphi - H_\rho H_z \right) S &= 0 \\
\frac{\partial}{\partial t} \rho \varphi S + \frac{\partial}{\partial z} \left( \rho v_\varphi v_\varphi + p \frac{\partial}{\partial z} v_z \right) S &= 0 \\
\frac{\partial}{\partial t} H_\rho S + \frac{\partial}{\partial z} \left( H_\rho v_\varphi - H_\varphi v_\rho \right) S &= 0 \\
\frac{\partial}{\partial t} H_z S &= 0, \quad \frac{\partial}{\partial z} H_z S = 0,
\end{align*}
\]

where \( p = (\gamma - 1) \rho c^2 \).

The last two equations of the system (1) imply that the longitudinal magnetic flux \( H_z S \) is constant in time and space, and it can be considered as given. In addition, an initial-boundary value problem about a sub-Alfven transonic MHD flow with acceleration in a narrow nozzle channel with a constant lower bound requires to set four boundary conditions [6]: three at the channel input and one at the channel output.

At the channel input, the role of boundary conditions is played by the known parameters of the accelerator and the plasma entering the channel. Plasma enters the channel with defined thermodynamic parameters in the absence of rotation. The total electric current in the circuit is also known. Turning to the dimensionless variables composed of the aforementioned dimension parameters of the model, these conditions are:

\[
\rho = 1, \quad v_\varphi = 0, \quad H_\varphi = 1
\]

The considered flow type is characterized by the transition of the plasma flux through the slow magnetosonic speed \( C_s \) at the thinnest channel cross-section, and the plasma velocity exceeds it at the channel output. Hence, the boundary condition on the right of the calculation area can be defined as follows:

\[
v_z = C_s + \delta,
\]

where \( \delta \) as well as the magnetic flux \( H_z S \) is a parameter of the considered mathematical model. The value of \( \delta > 0 \) indirectly defines the energy of the plasma flowing into the channel.

Initial conditions of the problem should only provide initial plasma acceleration and therefore can be quite arbitrary.

The MHD problem under consideration with equations (1) and boundary conditions (2), (3) is solved numerically by using the flux-corrected transport (FCT) algorithm with the Boris-Book correction [11]. Its steady-state solutions are obtained in the process of relaxation in time.

Part of the computation results presented in the paper is obtained in the stationary quasi-one-dimensional model based on the first integrals (the laws of conservation) of the system (1). In this case, the computation algorithm is reduced to numerical solving of a single algebraic equation for the plasma density followed from the first integrals. In more detail, the previous researches contain the description of the aforementioned calculation method and its implementation in relation to the research of super-Alfven transonic accelerating [7, 9] and trans-Alfven non-accelerating [11] MHD flows in narrow channels of curvilinear geometry.
3. Computation results

In this section, the results of numerical simulation of sub-Alfven transonic MHD flows with acceleration (type 8 on figure 2) in narrow coaxial channels are provided.

The coaxial nozzle narrow channel which has a constant lower bound with the following cross-section is considered:

\[ S(z) = 2(z - 0.5)^2 + 0.5 \]

In the non-stationary initial-boundary value problem, the chosen value of the longitudinal magnetic flux \( H_z S = 0.9 \) corresponds to a sufficiently strong longitudinal magnetic field. As mentioned before, the parameter \( \delta \) indirectly defines the energy and temperature of the input plasma, which are arbitrary in the considered problem statement, and is chosen \( \delta = 0.345 \).

Relaxation process of the MHD flow of this type is presented in figure 5. The initial distributions \((t = 0)\) of the plasma longitudinal velocity \( u_z \) (figure 5a) and azimuthal magnetic field \( H_\phi \) (figure 5b) are changed in time during numerical solving of the initial-boundary value problem until the steady-state regime of the flow \((t \sim 15)\) is obtained in the calculations.

![Figure 5a. Relaxation of longitudinal plasma velocity.](image)

![Figure 5b. Relaxation of azimuthal magnetic field.](image)

At the first stage of the relaxation \((0 < t < 1)\), the portrait of the sub-Alfven plasma acceleration along the channel is similar to the super-Alfven one, except to its output section where the boundary condition (3) on the output plasma velocity takes effect. The presented distributions of the plasma parameters attest to the shock wave propagating to the channel enter during the relaxation process. Over time, the influence of the right boundary condition extends to the entire length of the channel, and the distributions become smooth.

The steady-state regime \((t \sim 15)\) is characterized by the plasma acceleration along the channel axis. The velocity distribution isn’t monotonic and has a local maximum at the output section of the channel. In the steady-state regime of the flow, the longitudinal plasma velocity \( u_z \) doesn’t exceed Alfven speed \( C_{A_{z\infty}} \) corresponding to the longitudinal field \( H_z \) along the channel length and transits through the slow magnetosonic speed \( C_{s_{\infty}} \) at its center (figure 6).

During the steady-state plasma motion in the channel, generation of an electric current in the flow takes place: the output value of the azimuthal magnetic field intensity \( H_\phi \) is greater than the input one. The decrease of the plasma density \( \rho \) corresponds to the thermal energy consumption (figure 7). In addition, the flow also rotates in the transverse channel direction with the velocity \( \nu_\phi \).
Summarizing the aforementioned observations, in sub-Alfven transonic MHD flows with acceleration, the accumulated thermal energy of an input plasma transforms into its electromagnetic and kinetic energies of acceleration and rotation. In the super-Alfven transonic accelerating type, on the contrary, electromagnetic energy of a plasma flux isn’t generated and also turns into energy of its motion (see, e.g. [9]).

![Figure 6](image6.png)

**Figure 6.** Relation between longitudinal plasma velocity and characteristic magnetic speeds in steady-state sub-Alfven transonic flow type with acceleration.

![Figure 7](image7.png)

**Figure 7.** Steady-state distributions of plasma flow parameters.

The final part of the calculations was connected with the study of the longitudinal magnetic field influence on the stationary flows (figure 8). The longitudinal flux increase leads to more intensive thermal energy consumption (a plasma density at the channel output decreases) and growth of acceleration characteristics of the channel (the difference between output and input plasma velocities increases). This is another distinction of the sub-Alfven accelerating mechanism from its super-Alfven analog: in super-Alfven flows, the longitudinal field reduces acceleration properties of the device (see, e.g. [9]). The azimuthal magnetic field of the flow and its rotation aren’t directly proportional to the longitudinal field intensity. There is a critical value of the longitudinal flux in which behavior of these parameters is qualitatively changed. The increasing electric current generation corresponding to the increase of the azimuthal field intensity is replaced by its following decrease. This observation is also applied to the flow rotation.

Finally, it is important to note that the sub-Alfven transonic flow type with acceleration exists under any longitudinal magnetic flux \( H_z S > 0 \).
4. Conclusions

The results of the numerical researches of sub-Alfven transonic accelerating flows in coaxial channels of plasma accelerators with a longitudinal magnetic field are presented. The non-stationary MHD model based on initial-boundary value problems formulated in the quasi-one-dimensional approximation for narrow channels with a constant lower bound and its stationary form are considered.

The following results are highlighted as the most significant:

- The features of the relaxation process and characteristics of the considered flow type are defined;
- The mechanism of energy transformation taking place in the flow is described;
- The influence of the longitudinal field on the steady-state flow parameters is determined.
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