Appointed-Time Prescribed Performance Fault Tolerant Control for QUAV With Actuator Fault and Model Parameters Uncertainties

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ABSTRACT
In this paper, an appointed-time prescribed performance fault-tolerant control (ATPPFTC) method for quadrotor unmanned aerial vehicle (QUAV) system is studied in the presence of model parameters uncertainties, actuator faults and external disturbances. For the position loop and the attitude loop, the proposed method overcomes the model parameters uncertainties caused by the changes of rotary inertia, the mass and aerodynamic damping coefficients. Meanwhile, unknown actuator faults and unknown external disturbances are investigated by using adaptive method. Moreover, a new appointed-time prescribed performance function and coordinate transformation are introduced to impose performance characteristics on the tracking errors. Based on the Lyapunov stability theory, an adaptive dynamic surface control method is proposed to ensure that the position and attitude tracking errors of the closed-loop system can achieve the appointed-time convergence and prescribed transient and steady-state performance. Incidentally, the convergence speed and convergence time of the system tracking error can be set flexibly. Finally, the simulation verifies the effectiveness and superiority of the proposed control scheme from both qualitative and quantitative aspects.

INDEX TERMS
Quadrotor unmanned aerial vehicle (QUAV), appointed-time prescribed performance function (ATPPF), actuator fault, external disturbance.

I. INTRODUCTION
Nonlinear control systems are widely used in many fields. In addition to the general engineering system, it is also of great significance in the control of UAV. UAV system is a typical underactuated nonlinear coupling system. UAV has strong maneuverability and hovering ability. Due to the characteristics of low production cost and convenient operation, it has been widely used in many fields and has broad development prospects. At present, there are sliding mode control [1], [2], [3], backstepping control [4], [5], adaptive control [6] and other methods for the stability control of QUAV.

The QUAV is vulnerable to unknown external disturbance. Therefore, it is a more practical consideration. In [7], a tracking flight control scheme is proposed based on a disturbance observer for a quadrotor with external disturbances, but the parameter uncertainties in QUAV systems did not been considered. In [8], an attitude system of a QUAV with disturbance and gyroscopic effect was developed, however, the moment of inertia was not considered, and only the yaw direction which is most commonly executed in the QUAV was discussed in this paper. In [9], a new robust nonlinear adaptive controller was investigated subjected to disturbances. It is proved that the estimator is more insensitive to noise. However, the fault tolerant control problem of the quadrotor actuators and sensors has not been considered.

These actuators of quadrotors may expose to measurement noise, under the joint influence of component loss, temperature and humidity change, external disturbance and other factors, the probability of actuator fault in the actual system is increasing, actuator fault will not only deteriorate the control
performance, but also make the system difficult to stabilize. Therefore, the possible faults should be considered in the actual control, and the fault-tolerant controller should be designed to improve the reliability and safety of the system. In [10], the problem of asymptotic tracking control for nonlinear systems with actuator fault was investigated. In [11], a robust fault estimation observer was designed for a class of linear sampled data systems with external disturbances, but the boundary of external disturbances was known, moreover, the active fault-tolerant control method was not adopted. In [12], a novel adaptive fault-tolerant control was introduced to mitigate the constant or time-varying actuator faults to deal with its pneumatic complexity and some actuator faults. However, the disturbance problem is not considered.

The above-mentioned researches focused more on the fault-tolerant control of the systems. When the system fails, it is equally important to pay attention to its transient and steady-state performance. The prescribed performance control can make the convergence speed and maximum overshoot of the tracking error of the system set within the preset performance requirements, and ensure to meet the prescribed transient and steady-state performance requirements. It is of great significance to improve the performance of the QUAV system. In [13], an adaptive prescribed performance control scheme was proposed for a QUAV with actuator fault, but the unknown dynamic parameters and time-varying loads were not considered, and the prescribed performance function needed to be inverted. In [14], a novel model-free saturated prescribed performance reinforcement learning framework was proposed in the presence of the model uncertainties, nonlinearities and external disturbances. However, the problem of actuator fault was not considered. The finite time prescribed performance control strategy for the trajectory tracking problem of QUAVs with time-vary disturbance, model uncertainty and constrain of the output error was studied in [15]. However, the external disturbance and dynamic parameters were known, and the prescribed performance was controlled in a limited time, and the prescribed performance still needed to be inverted. In [16], a robust finite time control method for position and attitude was proposed, however the system tracking error cannot converge in the appointed-time and reach the preset transient performance and steady-state performance, and the system parameters were known. In [17], a trajectory tracking control problem of a quadrotor with unknown dynamics and disturbances was studied, but the kinetic parameters were known and actuator fault was also not considered. In [18], an adaptive finite-time extended state observer was designed for quadrotor attitude control in the presence of wind disturbances and actuator faults, however only the actuator fault-tolerant control of attitude subsystem was studied, and the influence of actuator fault in position system on the control system of QUAV was also very important, moreover, the prescribed performance control at the appointed-time was not considered. The convergence time of finite time control method depends on the initial conditions [19], [20], [21], and the convergence time cannot be estimated in advance. In [22], two fixed-time fault-tolerant control schemes were developed to address the problem of attitude stabilization of a quadrotor unmanned aerial vehicle with external disturbances and actuator faults, nevertheless, the bound of external disturbance was known, and the controller was designed only for the attitude system. In [23], an appointed-time prescribed performance control was designed for the position of the QUAV. But the attitude loop was controlled in a finite time, and the problem of actuator fault was not considered.

According to the above analysis, inspired by the above mentioned control strategies, an appointed-time prescribed performance fault-tolerant control (ATPPFTC) is proposed in this paper. The main contributions are as follows: (1) In order to make the QUAV with actuator fault asymptotically stable under the disturbance of unknown air flow, unknown air resistance and model parameters uncertainties, the adaptive method is used to overcome the limitation of knowing system parameters, air resistance, external disturbances and actuator fault. (2) Combined with the new prescribed performance function, the coordinate transformation function acts on the tracking error, which ensures that the tracking error of the closed-loop system meets the prescribed transient and steady-state performance constraints within an appointed-time. And the convergence speed can be adjusted flexibly. (3) The complex inverse process in the existing prescribed performance control [13], [14], [15] has been avoided.

This paper is divided into four sections. In Section II, the system model, performance function and necessary preliminaries are given. In Section III, the new appointed-time prescribed performance fault tolerant controllers are presented for outer loop position subsystem and inner loop attitude subsystem based on dynamic surface control scheme, finally, in order to validate the effectiveness and superiority of the presented controllers, the corresponding simulation results are illustrated in Section IV, the corresponding errors indicators are also given in the simulation section to better illustrate the quantitative of the proposed controller, and the conclusions are given in the last section.

II. UNCERTAIN DYNAMIC MODEL AND SYSTEM DESCRIPTION

A. UNCERTAIN SYSTEM MODEL

Assumption 1: In order to facilitate the establishment of the model, the following assumptions are made for the QUAV [24]:

- Design is symmetrical.
- QUAV body is rigid.
- Propellers are rigid.
- The flexibility of the blade is relatively small and can be neglected.

Consider the ground coordinate system and body coordinate system, as shown in Fig 1. Inertial coordinate system $O_e (OX_eY_eZ_e)$ is stationary relative to the earth’s surface, body coordinate system $O_b (OX_bY_bZ_b)$ is firmly connected...
with the aircraft, and the origin $O$ is the center of gravity of the aircraft. The distance between each motor and the center of mass is $L_i, i = 1, 2, 3, 4$ is the lift of a single rotor. $\phi, \theta, \psi$ are the positions and attitude angles of the QUAV.

For the inertial coordinate system $O_E$ and the body coordinate system $O_b$, the transformation relationship or rotation matrix between them is:

$$
R_{b-E} = \begin{bmatrix}
    c_\phi c_\psi & -c_\phi s_\psi + s_\phi c_\psi & s_\phi c_\psi + c_\phi s_\psi \\
    s_\phi c_\psi & c_\phi c_\psi + s_\phi s_\psi & -s_\phi c_\psi + c_\phi s_\psi \\
    -s_\psi & c_\psi & s_\phi \end{bmatrix}
$$

where $c$ and $s$ are abbreviations for functions cos and sin, respectively.

According to $F = ma$, it has:

$$
\begin{bmatrix}
    0 \\
    0 \\
    f_1 + f_2 + f_3 + f_4
\end{bmatrix}
- \begin{bmatrix}
    0 \\
    0 \\
    mg
\end{bmatrix}
+ \begin{bmatrix}
    d_x \\
    d_y \\
    d_z
\end{bmatrix}
- \begin{bmatrix}
    k_1 \dot{x} \\
    k_2 \dot{y} \\
    k_3 \dot{z}
\end{bmatrix}
= m \begin{bmatrix}
    a_x \\
    a_y \\
    a_z
\end{bmatrix}
$$

where $x, y, z$ denotes the position. $a_x, a_y, a_z$ is the acceleration of $x, y, z$ respectively, $m$ is the mass of the QUAV, $k_1, k_2, k_3$ is the aerodynamic damping coefficient of the system, $d_x, d_y, d_z$ represent for the lumped disturbances, and the force $f_i, i = 1, 2, 3, 4$ are lifts generated by the four motors, $g$ is gravitation acceleration.

Using Newton Euler equation, the inner loop attitude subsystem dynamic equation of the quadrotor is obtained as follows:

$$
\dot{\mathbf{W}} + W \times I W = M_i + M_p - M_k + M_d
$$

where $I = diag(I_x, I_y, I_z)$, matrix $I_{xx}, I_{xy}, I_{xz}$ represents the moment of inertia about the lower $x, y, z$ axis. $W = W_\phi, W_\theta, W_\psi$ is the angular velocity of roll, pitch, and yaw with respect to the body-fixed frame. $M_p$ is the gyroscopic torque. $M_p = J_p [\dot{\psi}, 0, 0]^T \Omega$, $J_p$ is the moment of inertia of each rotor. $\Omega$ is the angular velocity, indicating the propeller speed margin. $M_i, i = \phi, \theta, \psi$ is the torque provided by the rotors with respect to the body-fixed frame, $M_k = diag(k_4, k_5, k_6), k_i, i = 4, 5, 6$ is the aerodynamic damping coefficient of the system. $M_d = [d_\phi, d_\psi, d_\theta]^T \delta, \hat{\delta} = [\phi, \theta, \psi]^T$ $d_\phi, d_\psi, d_\theta$ represent for the lumped disturbances.

Consider the influence of unknown time-varying load, external disturbance and air resistance on the system, equation (2) (3) can be written as follows:

$$
\begin{align*}
\dot{x} &= \frac{1}{m_\phi} (c_\phi s_\psi c_\phi + s_\phi s_\psi) F_z + \frac{1}{m_x} d_x - \frac{1}{m_x} k_1 \dot{x} \\
\dot{y} &= \frac{1}{m_\psi} (c_\phi s_\psi s_\psi + s_\phi c_\psi) F_x + \frac{1}{m_y} d_y - \frac{1}{m_y} k_2 \dot{y} \\
\dot{z} &= \frac{1}{m_\psi} c_\phi c_\theta c_\psi + s_\phi s_\psi F_z + \frac{1}{m_z} d_z - \frac{1}{m_z} k_3 \dot{z}
\end{align*}
$$

where $m_\phi, m_\psi, m_\theta, m_x, m_y, m_z$ are the gyroscopic terms of Eq. (4a) and Eq. (4b) can be expressed as the following outer loop position subsystem (5a) and inner loop attitude subsystem (5b):

$$
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= \eta_{mx} u_x + \eta_{mx} d_x - \eta_{mx} k_1 \dot{x} \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= \eta_{my} u_y + \eta_{my} d_y - \eta_{my} k_2 \dot{y} \\
\dot{x}_5 &= x_6, \\
\dot{x}_6 &= \eta_{mx} u_z + \eta_{mx} d_z - \eta_{mx} k_3 \dot{z}
\end{align*}
$$

where parameters $\eta_{mx} = 1/m_x, \eta_{my} = 1/m_y, \eta_{mz} = 1/m_z, \eta_{m_\phi} = 1/I_{xx}, \eta_{m_\theta} = 1/I_{yy}, \eta_{m_\psi} = 1/I_{zz}, \eta_{m_\phi} = [I_{yy} - I_{yx}] \eta_{m_\psi}, u_\phi = M_\phi, u_\theta = M_\theta, u_\psi = M_\psi, u_\psi = (c_\phi s_\phi c_\psi + s_\phi c_\psi) F_y, u_\phi = (c_\phi s_\phi s_\psi + s_\phi c_\psi) F_x, u_\psi = c_\phi c_\phi F_z$.

Assumption 2: Suppose parameters $\eta_{m_\phi}, \eta_{m_\psi}, \eta_{m_\theta}, \eta_{m_\phi}$ and $\eta_{m_\psi}$ are unknown, and their lower bound parameters $\eta_{m_\phi}^{\text{min}}, \eta_{m_\psi}^{\text{min}}, \eta_{m_\theta}^{\text{min}}, \eta_{m_\phi}^{\text{min}}$ and $\eta_{m_\psi}^{\text{min}}$ are also unknown and are greater than zero. Aerodynamic drag coefficient $k_i, i = 1, 2, 3, 4, 5, 6$ are also unknown and are greater than zero.

Assumption 3: Suppose these conditions satisfy max $\{\eta_{m_\phi} d_\phi, \eta_{m_\psi} d_\psi, \eta_{m_\theta} d_\theta\} \leq T_{m_\phi}, \max \{|\eta_{m_\phi} d_\phi|, |\eta_{m_\phi} d_\psi|\} \leq T_{m_\psi}$, where upper parameters $T_{m_\phi}, T_{m_\psi}$ and $T_{m_\psi}$ are unknown, meanwhile, suppose these
in the upper parameters $u_1$ and $\psi$. The actuator faults existing in the position loop and attitude loop are comprehensively considered, in [18], only the actuator fault-tolerant control of inner loop attitude subsystem is studied.

Remark 2: The fault of actuator often endangers the control strategy of the whole system, so as to effectively solve the trajectory tracking problem of QUAV in case of actuator fault, the portion of the control action $u_{af}$ is considered to be bounded and unknown. And, the bounds of these unknown parameters are unknown and estimated by the adaptive method here. The actuator faults existing in the position loop and attitude loop are comprehensively considered, in [18], only the actuator fault-tolerant control of inner loop attitude subsystem is studied.

C. APPORTED-TIME PRESCRIBED PERFORMANCE FUNCTION

In order to meet the requirements of tracking error performance constraints, a novel positive and decreasing smooth function $F_{bi}$ is designed to meet the transient and steady-state performance, as described below:

$$F_{bi} = \begin{cases} 
\ln(\xi_0 (t_e - t)^n + 1) + \xi_\infty, & 0 < t < t_e \\
\xi_\infty, & t \geq t_e
\end{cases} \quad (7)$$

where $\xi_0 = \frac{e^{(-c-n-1)}}{(c-\eta-n)}, t, n, \xi_\infty, t_e$ are positive design parameters, $t_0$ represents initial value of time, $\ln(\cdot)$ represents natural logarithmic function.

From the appointed-time performance function (ATPPF) specified above, it is easy to see that the performance function $F_{bi}$ is positive and decreasing and satisfies $F_{bi}(0) = t > 0$. $F_{bi}(t_e) = \xi_\infty > 0$. Initial value of the performance function $F_{bi}$ can be adjusted by adjusting parameters $t$, and the steady-state value can be adjusted by adjusting parameter $\xi_\infty$. The convergence rate can be changed by adjusting parameter $n$.

If define tracking errors $e_i, i = 1, 3, 5, 7, 9, 11$ meet the following relationship,

$$-d_2 F_{bi} < e_i < d_1 F_{bi}, \quad i = 1, 3, 5, 7, 9, 11 \quad (8)$$

where tunable parameters of prescribed performance requirement $d_1$ and $d_2$ are positive constants. $d_1 F_{bi}$ and $-d_2 F_{bi}$ are the upper and lower bounds of the prescribed performance that the tracking error needs to meet. To sum up, by selecting appropriate parameters $t, n, \xi_\infty, t_e$, and tunable parameters $d_1$ and $d_2$, the appropriate selection performance described by (8) can be impose performance characteristics on the tracking error, as shown in Fig.2. The performance function used in this paper can be set by setting parameter $t_e$ to control converge time, so that the constraint error can converge in the range of $(-d_2 \xi_\infty, d_1 \xi_\infty)$ within $t_e$ time.

Remark 3: According to the characteristics of traditional prescribed performance function (PPF) $F_{bi} = \xi_0 e^{-at} + \xi_\infty$, it is easy to obtain that when the selected parameter $a = a_0$, the convergence speed and convergence time of the PPF are fixed. When we choose different values $a_1$ and $a_2$ respectively, the convergence time of the PPF will decrease or increase. If we want to avoid a large initial control demand, we need to reduce the initial convergence speed, which can only be met by selecting a smaller value $a_2$, which will
increase the steady-state convergence time and make the convergence time at the appointed-time impossible to achieve. To explain the characteristics of PPF, Fig.3 is given to show the dynamic process of PPF with different \( q \). The transient convergence rate of ATPPF described by (7) can also be increased or decreased by selecting \( n = 3 \) or \( n = 2 \). However, no matter what value of \( n \) we choose, it can be convergent at the appointed-time \( t_a = 5 \). To explain the characteristics of ATPPF, Fig.4 is given to show the dynamic process of ATPPF with different \( n \) values. Based on the above analysis, it is easy to see that the proposed ATPPF can adjust convergence time and convergence speed flexibly, which is more suitable for practical application than the researches [13], [15], [25]. Compared with the work [25], the prescribed performance function used in this paper avoids inversion and simplifies the calculation process, and the convergence time can be setting by designing parameter \( t_a \), which do not depend on initial tracking errors. In [26], it considered finite time, which cannot flexibly set the convergence time, and the introduction of prescribed performance requires inverse calculations.

In order to make the tracking error meet the constraints of prescribed performance, the following coordinate transformation is performed on the tracking error as used in [23] and [27]:

\[
z_i = \tan \left( \frac{\pi e_i}{2d_1F_{bi}} \right) q_i + \tan \left( \frac{\pi e_i}{2d_2F_{bi}} \right) (1 - q_i) \tag{9}
\]

where \( q_i \) satisfy:

\[
q_i = \begin{cases} 
1, & e_i \geq 0, \ i = 1, 3, 5, 7, 9, 11 \\
0, & e_i < 0, \ i = 1, 3, 5, 7, 9, 11
\end{cases}
\tag{10}
\]

It can be seen from equation (9), when \( e_i(0) \in (-d_2F_{bi}, d_1F_{bi}) \), as long as \( z_i \to 0 \), then \( e_i \to 0 \), and \( e_i \in (-d_2F_{bi}, d_1F_{bi}) \). Therefore, as long as the controller is designed to ensure \( z_i \to 0 \), then the tracking error \( e_i \) can be guaranteed to be asymptotically stable and meet the constraints (8).

From equation (9), further derivation of \( z_i \) is obtained:

\[
\dot{z}_i = F_{bi}A_{Fi}e_i - \dot{F}_{bi}A_{Fi}e_i
\tag{11}
\]

where

\[
A_{Fi} = \sec^2 \left( \frac{\pi e_i}{2d_1F_{bi}} \right) \left( \frac{\pi}{2d_1F_{bi}} \right) \cdot \frac{1}{F_{bi}^2} q_i + \sec^2 \left( \frac{\pi e_i}{2d_2F_{bi}} \right) \left( \frac{\pi}{2d_2F_{bi}} \right) \cdot \frac{1}{F_{bi}^2} (1 - q_i),
\]

\[i = 1, 3, 5, 7, 9, 11\]

**III. DOUBLE LOOPS APPOINTED-TIME PRESCRIBED PERFORMANCE FAULT TOLERANT CONTROL**

**A. POSITION SUBSYSTEM CONTROLLER DESIGN**

In this section, a fault-tolerant controller with appointed-time prescribed performance is designed for the outer loop position subsystem. Combined with the dynamic surface technology, the adaptive method is used to adaptively estimate the external disturbance, unknown aerodynamic damping coefficient, actuator fault parameters and unknown dynamic parameters.

Firstly, the tracking error variables are defined as follows:

\[
e_1 = x_1 - x_{1d}, \quad e_2 = x_2 - \alpha_1, \quad e_3 = x_3 - x_{3d}, \quad e_4 = x_4 - \alpha_3, \quad e_5 = x_5 - x_{5d}, \quad e_6 = x_6 - \alpha_5,
\tag{12}
\]

where, \( x_{1d}, x_{3d}, x_{5d} \) are given reference signals, \( \alpha_i, i = 1, 3, 5 \) are virtual control inputs.

Define boundary layer error:

\[
y_i = a_{i-1} - \alpha_{i-1}^* \quad (i = 2, 4, 6)
\tag{13}
\]

where, \( \alpha_{i-1}^*, i = 1, 3, 5 \) are ideal virtual control inputs.

Select the Lyapunov function in the following form:

\[
V_1 = z_i^2/2
\tag{14}
\]

Then, the derivative of \( V_1 \) can be written as follows:

\[
\dot{V}_1 = z_i \dot{z}_i = z_i F_{bi}A_{Fi} \dot{e}_i - z_i \dot{F}_{bi}A_{Fi} e_i
\]

\[
= z_i F_{bi}A_{Fi} \left( e_2 + y_2 + \alpha_1^* - \dot{x}_{1d} \right) - z_i F_{bi}A_{Fi} e_i
\]

\[
\leq z_i F_{bi}A_{Fi} \left( e_2 + \frac{1}{2} z_i F_{bi}A_{Fi} + \alpha_1^* - \dot{x}_{1d} \right)
\]

\[
+ \frac{1}{2} y_2^2 - z_i \dot{F}_{bi}A_{Fi} e_i
\tag{15}
\]

The ideal virtual control input is designed as:

\[
\alpha_1^* = -\frac{c_1}{F_{bi}A_{Fi}} z_i + \dot{x}_{1d} - \frac{1}{2} z_i F_{bi}A_{Fi} + \frac{\dot{F}_{bi}A_{Fi}}{F_{bi}} e_i
\]

where design parameter \( c_1 \) is greater than 0.
Substitute the ideal virtual control input $\alpha_1^*$ into equation (15) to obtain
\begin{equation}
\dot{V}_1 \leq z_1 F_{b1} A_{F1} e_2 - c_1 z_1^2 + \frac{1}{2} y_1^2 
\end{equation}
(16)

Next, Lyapunov function $V_2$ is selected as follows:
\begin{equation}
V_2 = V_1 + \frac{1}{2} \bar{e}_1^2 + \frac{1}{2} \gamma_1^2 
\end{equation}
(17)

The derivative of $V_2$ is obtained as follows:
\begin{equation}
\dot{V}_2 \leq z_1 F_{b1} A_{F1} e_2 - c_1 z_1^2 + \frac{1}{2} \gamma_1^2 + e_2 \bar{e}_2 + y_2 \gamma_2 
\end{equation}
(18)

Further, define the first-order filter:
\begin{equation}
\lambda_2 \bar{u}_1 + \alpha_1 = \alpha_1^* 
\end{equation}
(19)

where $\lambda_2 > 0$ is a time constant.

Considering that the following inequality holds:
\begin{equation}
y_2 \gamma_2 = y_2 \left( -\frac{1}{\lambda_2} y_2 - \alpha_1^* \right) \leq \left( -\frac{1}{\lambda_2^2} + \frac{\mu_2^2}{\gamma_1^2} \right) \gamma_2^2 + \frac{1}{2} \gamma_1^2, \mu_2 \geq |\alpha_1^*| 
\end{equation}
(20)

where $\gamma_2 > 0$.

Then substitute equation (6) and (20) into equation (18) to obtain:
\begin{equation}
\dot{V}_2 \leq z_1 F_{b1} A_{F1} e_2 - c_1 z_1^2 + \left( -\frac{1}{\lambda_2^2} + \frac{\mu_2^2}{\gamma_1^2} \right) \gamma_2^2 + \frac{1}{2} \gamma_1^2 + e_2 \left( \eta_{mx} \bar{x}_u + u_{sf} \right) + \eta_{mx} d_{\bar{x}} - \eta_{nm} k_1 x_2 - \bar{u}_1 
\end{equation}
(21)

Design outer loop position subsystem controller:
\begin{equation}
u_{sa} = -\frac{e_2 \bar{u}_2^2 \bar{d}_{\bar{x}}}{\sqrt{e_2 \bar{u}_2^2 \bar{d}_{\bar{x}}^2 + \frac{\lambda_2^2}{\gamma_1^2}}} 
\end{equation}
(22)

where, design parameter $c_2 > 0$, $D_{\hat{x}}$ is the estimation of $D_{x}$, $\hat{G}_{\hat{x}}$ is the estimation of $G_{\hat{x}}$, $d_{\bar{x}} = \frac{1}{\eta_{mx}}$, $\bar{d}_{\bar{x}}$ is the estimation of $1/\eta_{mx}$, $\chi_x = 1/\tau_x$, $\bar{x}_u = \chi_x - \bar{x}_u$, $\bar{u}_{sf} = u_{sf} - \bar{u}_{sf}$, they are the estimation errors of the actuator efficiency factor and the portion of the control action in actuator fault.

According to lemma 1 and controller (19), equation (18) can be rewritten as follows:
\begin{equation}
\dot{V}_2 \leq -c_1 z_1^2 + z_1 F_{b1} A_{F1} e_2 - \frac{e_2 \bar{u}_2^2 \bar{d}_{\bar{x}}^2}{\sqrt{e_2 \bar{u}_2^2 \bar{d}_{\bar{x}}^2 + \frac{\lambda_2^2}{\gamma_1^2}}} + e_2 \bar{u}_2 + e_2 \bar{d}_{\bar{x}} + e_2 \bar{G}_x \bar{x}_2 - e_2 \bar{u}_1 + \left( -\frac{1}{\lambda_2^2} + \frac{\mu_2^2}{\gamma_1^2} \right) \gamma_2^2 + \frac{1}{2} \gamma_1^2 
\end{equation}
(23)

where $\bar{d}_{\bar{x}} = d_{\bar{x}} - \bar{d}_{\bar{x}}$, $\bar{D}_{\bar{x}} = D_{\bar{x}} - \bar{D}_{\bar{x}}$, $\bar{G}_{\hat{x}} = G_{\hat{x}} - \bar{G}_{\hat{x}}$, $\bar{u}_{sf} = u_{sf} - \bar{u}_{sf}$.

The total Lyapunov function $V_x$ is selected as:
\begin{equation}
V_x = V_2 + \frac{1}{2r_1} \bar{\eta}_{mx} \bar{d}_{\bar{x}}^2 + \frac{1}{2r_2} \bar{\tilde{D}}_{\bar{x}} + \frac{1}{2r_3} \bar{\tilde{G}}_x^2 + \frac{1}{2r_4} \bar{\tilde{\chi}}_x^2 + \frac{1}{2r_5} \bar{\tilde{u}}_{sf}^2 
\end{equation}
(24)

where, design parameter $r_1 > 0$, $r_2 > 0$, $r_3 > 0$, $r_4 > 0$, $r_5 > 0$.

The design adaptive laws are:
\begin{equation}
\tilde{d}_{\bar{x}} = r_1 e_2 \bar{u}_{sf} - \tilde{\delta}_1 r_1 \tilde{d}_{\bar{x}} 
\end{equation}
(25)

\begin{equation}
\tilde{D}_{\bar{x}} = r_2 |e_2| - \delta_2 r_2 \tilde{D}_{\bar{x}} 
\end{equation}
(26)

\begin{equation}
\tilde{G}_{\hat{x}} = r_3 |e_2| - \delta_3 r_3 \tilde{G}_{\hat{x}} 
\end{equation}
(27)

\begin{equation}
\tilde{\chi}_x = r_4 \left( e_2 \bar{u}_{sf} - \tilde{\chi}_x \right) 
\end{equation}
(28)

\begin{equation}
\tilde{u}_{sf} = r_5 \left( |e_2| - \delta \tilde{u}_{sf} \right) 
\end{equation}
(29)

where, design parameter $\delta_1 > 0$, $\delta_2 > 0$, $\delta_3 > 0$, $\delta_5 > 0$.

The derivative of $V_x$ is obtained as follows:
\begin{equation}
\dot{V}_x \leq -c_1 z_1^2 + z_1 F_{b1} A_{F1} e_2 + \tau_x \bar{\eta}_{mx} e_2 \bar{u}_2 \bar{d}_{\bar{x}} + \tau_x \bar{\eta}_{mx} \tilde{e}_1 + e_2 \tau_x \tilde{\chi}_x (\bar{u}_1 - c_2 e_2 - \bar{D}_{\bar{x}} \tilde{\chi}_x - e_2 \bar{u}_1) 
\end{equation}
(22)

where, design parameter $c_2 > 0$, $\bar{D}_{\bar{x}}$ is the estimation of $D_{\bar{x}}$, $\bar{G}_{\hat{x}}$ is the estimation of $G_{\hat{x}}$, $d_{\bar{x}} = \frac{1}{\eta_{mx}}$, $\bar{d}_{\bar{x}}$ is the estimation of $1/\eta_{mx}$, $\chi_x = 1/\tau_x$, $\bar{x}_u = \chi_x - \bar{x}_u$, $\bar{u}_{sf} = u_{sf} - \bar{u}_{sf}$, they are the estimation errors of the actuator efficiency factor and the portion of the control action in actuator fault.
\[-\frac{\tau_x}{2} x_2^2 - \frac{1}{2} \delta \tilde{u}^2_F + \tau_x \tilde{\eta}_{max} \varepsilon_1 + \frac{\delta_1}{2} \tau_x \tilde{\eta}_{max} \tilde{d}_{max}^2 + \frac{\delta_2}{2} D_x^2 + \frac{\delta_3}{2} G_x^2 + \frac{\tau_x}{2} \lambda_x^2 + \frac{1}{2} \delta \tilde{u}^2_F + \left( \frac{1}{\delta_2} - \frac{1}{\mu_2} \right) y_2^2 + \frac{1}{4} \gamma_2^2 \leq -\chi_1 V_x + \Sigma_x \]  

where,
\[
\chi_1 = \min \left\{ 2c_1, 2c_2, \delta_1 r_1, \delta_2 r_2, \delta_3 r_3, r_4, r_5, \right\} 
\]

\[
\Sigma_x = \frac{\delta_1}{2} \tau_x \tilde{\eta}_{max} \tilde{d}_{max}^2 + \frac{\delta_2}{2} D_x^2 + \frac{\delta_3}{2} G_x^2 + \frac{\tau_x}{2} \lambda_x^2 + \frac{1}{2} \tilde{u}^2_F + \frac{1}{4} \gamma_2^2 + \tau_x \tilde{\eta}_{max} \varepsilon_1
\]

By the same design process, controllers \(u_y\) and \(u_z\) are designed.

\[
u_{ya} = -\frac{e_6 \tilde{u}_z^2 \tilde{d}_{\eta_{my}}}{\sqrt{e_6^2 \tilde{u}_z^2 \tilde{d}_{\eta_{my}} + e_5^2}}
\]

\[
u_y = -\tilde{y}_y \tilde{x}_3 - c_4 e_4 - \tilde{D}_y \text{sign} (e_4) - |x_4| \hat{G}_y \text{sign} (e_4)
\]

\[
u_z = -\tilde{z}_y \tilde{x}_3 - c_4 e_4 - \tilde{D}_y \text{sign} (e_4) - |x_4| \hat{G}_y \text{sign} (e_4)
\]

\[
\nu_{za} = -\frac{e_6 \tilde{u}_z^2 \tilde{d}_{\eta_{my}}}{\sqrt{e_6^2 \tilde{u}_z^2 \tilde{d}_{\eta_{my}} + e_5^2}}
\]

\[
u_z = -\tilde{z}_z \tilde{y}_3 - c_6 e_6 - \tilde{D}_z \text{sign} (e_6) - |x_6| \hat{G}_z \text{sign} (e_6)
\]

\[
u_e = -\tilde{z}_z \tilde{y}_3 - c_6 e_6 - \tilde{D}_z \text{sign} (e_6) - |x_6| \hat{G}_z \text{sign} (e_6)
\]

\[
u_{ea} = -\frac{e_6 \tilde{u}_z^2 \tilde{d}_{\eta_{my}}}{\sqrt{e_6^2 \tilde{u}_z^2 \tilde{d}_{\eta_{my}} + e_5^2}}
\]

B. ATTITUDE SUBSYSTEM CONTROLLER DESIGN

In this section, the inner loop attitude subsystem controller is designed. Firstly, the two subsystems are connected with each other through the attitude extraction algorithm. The relationship is as follows:

\[
u_x = (c \psi \theta \phi \psi + s \phi \psi) F_z, \nu_y = (c \phi \theta \psi \psi - s \phi \psi) F_z
\]

\[
u_z = c \phi \rho \theta \psi \phi F_z\]

The reference input of the attitude angle of the inner ring can be obtained:

\[x_{7d} = \phi_{sd} = \arcsin \frac{u_x \sin \psi_x - u_y \cos \psi_x}{\sqrt{u_x^2 + u_y^2 + u_z^2}}\]

\[x_{9d} = \theta_{sd} = \arctan \left( \frac{u_z \cos \psi_x + u_y \sin \psi_x}{u_x} \right)\]

The tracking error of the defined attitude is as follows:

\[e_7 = x_7 - x_{7d}, \quad e_9 = x_9 - x_{9d}, \quad e_{11} = x_{11} - x_{11d},\]

\[e_8 = x_8 - \alpha_7, \quad e_{10} = x_{10} - \alpha_9, \quad e_{12} = x_{12} - \alpha_{11}\]

where \(\alpha_7, \alpha_9, \alpha_{11}\) are virtual control inputs, \(x_{7d}, x_{9d}, x_{11d}\) are the reference signals of attitude angle.

Boundary layer error is defined as follows:

\[y_i = \alpha_{i-1} - \alpha^*_i \quad (i = 8, 10, 12)\]

where \(\alpha^*_8, \alpha^*_9, \alpha^*_{11}\) are ideal virtual control inputs.

Lyapunov function \(V_\phi\) is selected as follows:

\[V_\phi = V_{\phi 1} + V_{\phi 2} + V_{\phi 3}\]

where \(V_{\phi 1} = \frac{1}{2} \tilde{\tau}_1^2, V_{\phi 2} = \frac{1}{2} \tilde{\tau}_2^2 + \frac{1}{2} \tilde{\tau}_8^2, V_{\phi 3} = \frac{1}{2} \tilde{\tau}_{16} \tilde{\eta}_{mob} \tilde{\phi}_{mob}^2 + \frac{1}{2} \tilde{\tau}_{17} \tilde{\phi}_{mob}^2 + \frac{1}{2} \tilde{\tau}_{18} \tilde{\phi}_{mob}^2 + \frac{1}{2} \tilde{\tau}_{19} \tilde{\phi}_{mob}^2 + \frac{1}{2} \tilde{\tau}_{20} \tilde{\phi}_{mob}^2 \]

The design parameters \(r_{16} > 0, r_{17} > 0, r_{18} > 0, r_{19} > 0, r_{20} > 0, d_{\eta_{mob}} = d_{\phi_{mob}} - d_{\eta_{mob}} - d_{\phi_{mob}}\)

where \(\eta_{mob}, \phi_{mob}, \tilde{n}_{mob}, \tilde{\eta}_{mob}, \tilde{n}_{mob}\) are the estimation of \(\eta_{mob}, \phi_{mob}\) and \(d_{\eta_{mob}}, d_{\phi_{mob}}\).
Substituting the derivative of $V_{\phi_1}$ into equation (49), one get:

$$
\dot{V}_{\phi_1} = z_7 \zeta_7 = z_7 F_{b7} A F_7 e_7 = z_7 \dot{F}_{b7} A F_7 e_7
$$

where, design parameters $c_7 > 0$. Let $\zeta_7$ through the first-order filter

$$
\lambda_8 \dot{\zeta}_7 + \zeta_7 = \zeta_7 A F_7 e_7
$$

where time constant $\lambda_8 > 0$.

Substituting the derivative of $V_{\phi_1} + V_{\phi_2}$ into Equation (49), and substitute $\zeta_7$ into the derivative to get:

$$
\dot{V}_{\phi_1} + \dot{V}_{\phi_2} = z_7 F_{b7} A F_7 \left( e_8 + \frac{1}{2} z_7 F_{b7} A F_7 + \zeta_7 A F_7 e_7 \right)
$$

where $\mu_8 \geq \zeta_7$, $\gamma_8 > 0$.

Design inner loop attitude subsystem controller:

$$
\dot{u}_\theta = -x_10 J_\phi \Omega_r / \tau_\phi - \frac{e_{\hat{\theta} \Phi} \phi^2}{\hat{\Phi}_m^2 \phi_m^2 + \phi^2} + \frac{\hat{\Phi}_m^2 \phi^2}{\phi_m^2 \phi_m^2 + \phi^2}
$$

where design parameters $\epsilon > 0$, $\epsilon > 0$.

According to lemma 1 and controller (52), equation (51) can be rewritten as follows:

$$
\dot{V}_{\phi_1} + \dot{V}_{\phi_2} \leq z_7 F_{b7} A F_7 e_8 - c_7 z_7 + e_8 [x_10 T_m + \eta_m \phi + x_10 J_\phi \hat{\phi} + \hat{\phi} - k_4 x_8 - \dot{\zeta}_7]
$$

where $\dot{V}_{\phi_1} + \dot{V}_{\phi_2}$ is the derivative of Lyapunov function $V_{\phi}$ is obtained, and substitute the equations (54)-(59) into the derivative of $V_{\phi}$ to obtain:

$$
\dot{V}_{\phi} = \dot{V}_{\phi_1} + \dot{V}_{\phi_2} \leq z_7 F_{b7} A F_7 e_8 - c_7 z_7 + e_8 [x_10 T_m + \eta_m \phi + x_10 J_\phi \hat{\phi} + \hat{\phi} - k_4 x_8 - \dot{\zeta}_7]
$$

where design parameters $\epsilon > 0$, $\epsilon > 0$.

According to lemma 1 and controller (52), equation (51) can be rewritten as follows:

$$
\dot{V}_{\phi_1} + \dot{V}_{\phi_2} \leq z_7 F_{b7} A F_7 e_8 - c_7 z_7 + e_8 [x_10 T_m + \eta_m \phi + x_10 J_\phi \hat{\phi} + \hat{\phi} - k_4 x_8 - \dot{\zeta}_7]
$$

where design parameters $\epsilon > 0$, $\epsilon > 0$.

According to lemma 1 and controller (52), equation (51) can be rewritten as follows:

$$
\dot{V}_{\phi_1} + \dot{V}_{\phi_2} \leq z_7 F_{b7} A F_7 e_8 - c_7 z_7 + e_8 [x_10 T_m + \eta_m \phi + x_10 J_\phi \hat{\phi} + \hat{\phi} - k_4 x_8 - \dot{\zeta}_7]
$$

where design parameters $\epsilon > 0$, $\epsilon > 0$.
where

\[ \chi_\phi = \min \{2c_7, 2c_8, \delta_{16}r_{16}, \delta_{17}r_{17}, \delta_{18}r_{18}, \}
\]

\[ r_{19}, r_{20}, 2 \left( \frac{1}{\chi_8} - \frac{1}{2} - \frac{\mu_2^2}{\gamma_8^2} \right) \}, \]

\[ \Sigma_\phi = \tau_0 n_{mph} e_7 + \frac{\delta_{16}}{2} \tau_0 n_{mph} d_{mph}^2 + \frac{\delta_{17}}{2} \tilde{T}_{mph}^2
\]

\[ + \frac{\delta_{18}}{2} \tilde{T}_{mph}^2 + \frac{\tau_0}{2} \chi_\phi + \frac{1}{4} \eta^2 F \]

By the same process, controllers \( u_\theta \) and \( u_\psi \) are designed:

\[ u_\theta = -x_8 J_p \Omega_r - e_10^2 \tilde{d}_{mph} \sqrt{e_10^2 \tilde{d}_{mph}^2 + e_{11}^2} \]

\[ \tilde{u}_\theta = -\tilde{x}_\theta [\tilde{u}_\theta - c_{10} e_{10} - i |x_8|x_{12} + 1] \tilde{T}_{mph} \text{sign} (e_{10}) \]

\[ - |x_10| \tilde{N}_{mph} \text{sign} (e_{10}) - z_9 F_{b9} A_9 - \text{sign} (e_{10}) \tilde{u}_\theta F \]

\[ (60) \]

\[ u_\psi = -x_8 J_p \phi_r - (e_10^2 \tilde{d}_{mph}^2 + e_{11}^2) \]

\[ \tilde{u}_\psi = -\tilde{x}_\psi [\tilde{u}_\psi - c_{12} e_{12} - i |x_8|x_10 + 1] \tilde{T}_{mph} \text{sign} (e_{12}) \]

\[ - |x_12| \tilde{N}_{mph} \text{sign} (e_{12}) - z_9 F_{b9} A_9 - \text{sign} (e_{12}) \tilde{u}_\psi F \]

\[ (61) \]

where, design parameters \( e_g, e_{11}, e_{12} > 0, c_{10}, c_{12} > 0, \tilde{T}_{mph} \) is the estimation of \( T_{mph} \), \( \tilde{N}_{mph} \) is the estimation of \( N_{mph} \), \( d_{mph} \) is the estimation of \( d_{mph}^2 = 1/\tilde{N}_{mph} \), \( \tilde{u}_\theta \) is the estimation of \( u_\theta F \), \( \tilde{T}_{mph} \) is the estimation of \( T_{mph} \), \( \tilde{N}_{mph} \) is the estimation of \( N_{mph} \), \( d_{mph} \) is the estimation of \( d_{mph}^2 = 1/\tilde{N}_{mph} \), \( \tilde{u}_\psi \) is the estimation of \( u_\psi F \).

The ideal virtual control input \( a_\theta^a \) and \( a_\psi^a \) are designed as:

\[ a_\theta^a = -\frac{c_0}{F_{b9}A_9} z_9 + \tilde{x}_\theta - \frac{1}{2} \tilde{z}_9 F_{b9} A_9 + F_{b9} e_9 \]

\[ a_\psi^a = -\frac{c_{11}}{F_{b11} A_{11}} z_9 + \tilde{x}_\psi - \frac{1}{2} \tilde{z}_{11} F_{b11} A_{11} + F_{b11} e_{11} \]

\[ (62) \]

\[ (63) \]

where, design parameters \( c_0 > 0, c_{11} > 0 \). Let \( a_\theta^a \) and \( a_\psi^a \) through the first-order filter \( \tilde{\lambda}_{10} \tilde{a}_\theta + \alpha_a = a_\theta^a \) and \( \lambda_{12} \tilde{a}_{11} + \alpha_1 = a_\psi^a \), where \( \lambda_{10} > 0, \lambda_{12} > 0 \) are time constants.

In addition, the adaptive laws can be designed as:

\[ \dot{d}_{mph} = r_{21} e_{10} \tilde{u}_\theta - r_{21} \tilde{d}_{mph} \]

\[ (64) \]

\[ \dot{T}_{mph} = r_{22} (|x_8|x_{12} + 1) e_{10} - r_{22} \tilde{d}_{mph} \]

\[ \dot{N}_{mph} = r_{23} (|x_8|e_{10} - r_{23} \tilde{N}_{mph} \]

\[ \dot{\chi}_\theta = r_{24} (e_{10} \tilde{u}_\theta - \chi_\theta) \]

\[ \dot{u}_\theta = r_{23} (|e_8| - \chi_\theta) \tilde{u}_\theta F \]

\[ \dot{\theta}_\psi = r_{26} e_{12} \tilde{u}_\psi - r_{26} \tilde{d}_{mph} \]

\[ \dot{m}_\psi = r_{27} (|x_8|x_{12} + 1) e_{12} - r_{27} \tilde{m}_\psi \]

\[ \dot{\tilde{N}}_{mph} = r_{28} (|x_2|x_{12} - r_{28} \tilde{N}_{mph} \]

\[ \dot{\tilde{\psi}} = r_{29} (e_{12} \tilde{u}_\psi - \tilde{\psi}) \]

\[ \dot{u}_\psi = r_{30} (|e| - \chi_\theta) \tilde{u}_\psi F \]

where design parameters \( r_i > 0, \chi_i > 0, i = 21, 22, 23, 24, 25, 26, 27, 28, 29, 30. \)

Remark 4: The realization of the prescribed performance is independent of the selection of these controller design parameters, as long as the design parameters meet the requirements in the design process (such as \( t, n, \xi_{\infty}, \gamma_c > 0 \)) and these design parameters \( c_1, c_2, c_3, r_1, r_2, r_3, r_4, r_5, r_2, \lambda_2 > 0 \), and should be appropriately selected to ensure that \( \Sigma_\chi \) is small enough or \( \chi_1 \) is large enough. In the simulation, these design parameters should be properly selected to ensure that the simulation can be carried out normally.

To sum up, the following conclusions can be drawn from the controller design for the QUAV system:

Theorem 1: For the outer loop position subsystem (5a) and inner loop attitude subsystem (5b) of the QUAV with actuator faults described by (6), if the initial conditions \( e_1 (0) \in (-d_2 F_{b1} (0), d_1 F_{b1} (0)) \) hold, the controllers can be designed as (22)(31)(32), (52)(60) and (61), and the corresponding adaptive laws can be obtained by (25)-(29), (35)-(44), (54)- (58) and (64)-(73). It can ensure that the tracking errors of the outer loop position subsystem and inner loop attitude subsystem are uniformly ultimately boundedness, and meet the appointed-time prescribed performance constraints in the whole dynamic process, that is, \( -d_2 F_{b1} < e_1 (t) < d_1 F_{b1} \), \( i = 1, 3, 5, 7, 9, 11, \) for \( \forall t > 0. \)

Based on Theorems 1, the overall control structure of the QUAV can be displayed as Fig. 5.
IV. NUMERICAL SIMULATIONS

A. SIMULATIONS WITH ATPPFCT

In order to verify the effectiveness of the adaptive appointed-time prescribed performance fault-tolerant control method proposed in Theorem 1, numerical simulation is carried out in this section.

During the simulation, the QUAV system model parameters are set as follows: $p_1 = 0.1 \sin (10t)$, $\eta_{\text{max}} = \eta_{\text{ny}} = \eta_{\text{mc}} = 0.67 + p_1$ (kg), $\eta_{\text{mb}} = 33 + 5p_1$ (kg · m$^2$), $\eta_{\text{mb}} = 33 + 5p_1$ (kg · m$^2$), $\eta_{\text{mp}} = 25 + 5p_1$ (kg · m$^2$), the moment of inertia parameters are: $I_z = I_y = 1 / (33 + 5p_1)$ (kg · m$^2$), $I_z = 1 / (25 + 5p_1)$ (kg · m$^2$), the acceleration of gravity is: $g = 9.8m/s^2$, the external disturbance signal and aerodynamic damping coefficient are set as follows: $\hat{d}_x = \hat{d}_v = \hat{d}_z = 0.08 \sin (5t)$, $\hat{d}_\phi = \hat{d}_\theta = \hat{d}_\psi = 0.08 \sin (5t)$, $k_1 = k_2 = k_3 = 0.08 \sin (5t)$ (kg/s), $k_4 = k_5 = k_6 = 0.08 \sin (5t)$ (kg/rad). In addition, the reference input signal: $[x_d, y_d, z_d, \psi_{\text{d}}]^T = (0.5 \sin (t), 0.5 \cos (t), 0.1t, 0)^T$. The initial states of the position and attitude of the QUAV are set as follows: $[1.5, 1.5, 1.5]^T$ and $[1.5, 1.5, 1.5]^T$.

The parameters of the prescribed performance function at the appointed-time are: $d_1 = 1.5$, $d_2 = 1$, $d_3 = 2.5$, $\xi_{\infty} = 0.05$, $t_0 = 0$, $t_e = 5$, $n = 5$. The controllers proposed by Theorem 1 are used and the design parameters of the controllers and adaptive laws are selected as follows:

$c_1 = c_2 = 90$, $c_3 = c_4 = c_5 = 90$, $c_6 = c_7 = 90$, $c_9 = 90$, $c_{10} = c_{11} = c_{12} = 90$, $r_1 = 0.01$, $r_2 = 0.01$, $r_3 = 0.01$, $r_4 = 0.01$, $r_5 = 0.01$, $r_6 = 0.01$, $r_7 = 0.01$, $r_8 = 0.01$, $r_9 = 0.01$, $r_{10} = 0.01$, $r_{11} = r_{12} = 0.01$, $r_{13} = 0.01$, $r_{14} = r_{15} = r_{16} = r_{17} = r_{18} = 0.01$, $r_{19} = r_{20} = r_{21} = r_{22} = r_{23} = 0.01$, $r_{25} = r_{27} = r_{28} = 0.01$, $r_{29} = r_{30} = 0.01$, $\delta_1 = 0.01$, $\delta_2 = 0.01$, $\delta_3 = 0.08$, $\delta_5 = 0.01$, $\delta_6 = 0.01$, $\delta_7 = 0.01$, $\delta_8 = 0.01$, $\delta_{10} = 0.01$, $\delta_{11} = 0.01$, $\delta_{12} = 0.01$, $\delta_{13} = \delta_{15} = 0.01$, $\delta_{25} = \delta_{27} = 0.01$, $\delta_{16} = 0.01$, $\delta_{20} = 0.01$, $\varepsilon_3 = \varepsilon_5 = \varepsilon_7 = 0.01$, $\varepsilon_9 = 0.01$, $\varepsilon_{11} = 0.01$, $\lambda_2 = 0.1$, $\lambda_4 = 0.1$, $\lambda_6 = 0.1$, $\lambda_8 = \lambda_{10} = 0.1$, $\lambda_{12} = 0.5$

The simulation results are shown in Fig.6-Fig.10. Among them, Fig.6 shows the three-dimensional tracking trajectory. The curve with the pentagram as the starting point is the expected trajectory, and the curve with the asterisk as the starting point is the actual trajectory of the QUAV. Fig 7 and Fig 8 give the tracking trajectory of the position and attitude of the QUAV system, respectively. From Fig.6-8, it can see that the position and attitude of the QUAV system can track the given signals quickly. In order to observe the tracking transient and steady-state performance clearly, Figure 9 and Figure 10 are given to show tracking errors of the position subsystem and attitude subsystem, respectively. Obviously, the tracking errors of the position loop and the attitude loop are convergent with time $t_e = 3$, and satisfy the appointed-time prescribed transient and steady-state performance constraints in the whole dynamic process.

Moreover, in order to show the system with our proposed controller can achieve convergence with arbitrary appointed-time, the parameter of ATPPF $t_e$ is selected a small value
as $t_e = 2$. The other design parameters are selected to be the same as above-mentioned. The tracking errors of outer loop position subsystem and inner loop attitude subsystem are shown in Fig.11-Fig.12. In order to be clearer, the curve is also partially enlarged. When the parameter of ATPPF $t_e$ is selected as $t_e = 2$, the convergence time is shortened and the tracking error still meets prescribed transient and steady-state performance, and satisfy uniformly ultimately boundedness throughout the whole dynamic process.

### B. COMPARISON SIMULATIONS

In order to clearly show that the actuator faults can lead to system performance deterioration and the proposed method in Theorem 1 can effectively deal with actuator faults while the general controller (controller design process without considering actuator faults) cannot, the comparison simulations are done with the proposed controller (considering actuator faults) and with the general controller (without considering actuator faults), respectively. And, the comparison results are shown in Fig.13-Fig.18. The curves are also partially enlarged to see clearly. From the comparison figures, it is easy to see that the trajectory cannot be well tracked expected trajectory when the system with actuator faults is controlled by the general controller. That is, the actuator faults can affect the performance of the system. However, the proposed controller can ensure that the states of the system track the expected trajectory excellently, although the actuator faults exist.

In addition, in order to clearly show the effectiveness and superiority of the proposed method in Theorem 1 (with ATPPFTC), the comparison simulations are made with ATPPFTC, with traditional PPFTC and without ATPPFTC, respectively. And, the simulation results are shown in Fig.19-Fig. 24. It is easy to see that the tracking performance is poor without ATPPFTC, and the whole dynamic process
of tracking errors cannot satisfy the transient and steady-state performance. In addition, it also can obtain that the tracking errors can meet the prescribed transient and steady-state performance by using the controllers with ATPPFTC or with traditional PPFTC. However, the appointed-time convergence can be satisfied with ATPPFTC, which is not accomplished with traditional PPFTC. Moreover, as can be seen from Figure 19-24, the tracking error using PPFTC exceeds the upper and lower performance boundaries of ATPPFTC,
while the tracking error using ATPPFTC is always within the prescribed boundary. Furthermore, to clearly illustrate the superiority of the designed controller, the compared quantitative results are also given in Table 1 and Table 2. For position errors and attitude errors, three performance indicators for three different controllers are shown in Table 1 and Table 2, respectively, where \( \sigma_e \) is standard deviation, \( V_e \) is variance, \( M_e \) is maximum error.

From Table 1 and Table 2, it can be seen that all the tracking performance indicators of ATPPFTC are smaller than the other two controllers, indicating that the performance indicators of the ATPPFTC controller in the event of a fault are significantly better than the other two controllers. Hence, the performance of the ATPPFTC control strategy is significantly better than the other two control strategies (PPFTC and without ATPPFTC).

V. CONCLUSION

In this paper, a new adaptive appointed-time prescribed performance fault-tolerant dynamic surface control scheme is proposed for the uncertain QUAV system with model parameters uncertainties, external disturbances and actuator faults, which ensures that the tracking errors of the closed-loop system are uniformly ultimately boundedness, and satisfy the appointed-time convergence and the requirements of prescribed transient and steady-state performance. The corresponding errors indicators also illustrates the superiority of the designed controller. The influence of unknown external disturbance, model parameters uncertainties and actuator faults are solved simultaneously. It overcomes the limitations of accurately knowing the mass or model parameters in existing works. Moreover, the convergence time can be setting by designing parameter \( t_e \), do not depend on initial tracking errors, which is convenient and effective than the researches on fix-time control. In addition, based on dynamic surface control scheme, the introduction of new appointed-time prescribed performance function (ATPPF) and coordinate transformation simplify the design of the controller effectively.

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REFERENCES

[1] C.-C. Hua, K. Wang, J.-N. Chen, and X. You, “Tracking differentiator and extended state observer-based nonsingular fast terminal sliding mode attitude control for a quadrotor,” Nonlinear Dyn., vol. 94, no. 1, pp. 343–354, Oct. 2018.
[2] C. Hua, J. Chen, and X. Guan, “Fractional-order sliding mode control of uncertain QUAVs with time-varying state constraints,” Nonlinear Dyn., vol. 95, no. 2, pp. 1347–1360, Jan. 2019.
[3] H. Ramirez-Rodriguez, V. Parra-Vega, A. Sanchez-Orta, and O. Garcia-Salazar, “Robust backstepping control based on integral sliding modes for tracking of quadrotors,” J. Intell. Robot. Syst., vol. 73, no. 4, pp. 51–66, 2014.
[4] L. X. Xu, H. J. Ma, D. Guo, A. H. Xie, and D. L. Song, “Backstepping sliding-mode and cascade active disturbance rejection control for a quadrotor UAV,” IEEE/ASME Trans. Mechatronics, vol. 25, no. 6, pp. 2743–2753, Dec. 2020.
[5] K. Eliker and W. Zhang, “Finite-time adaptive integral backstepping fast terminal sliding mode control application on quadrotor UAV,” Int. J. Control, Autom. Syst., vol. 18, no. 2, pp. 415–430, Feb. 2020.
[6] T. Chingozha and O. Nyandoro, “Adaptive sliding backstepping control of quadrotor UAV attitude,” IFAC Proc. Volumes, vol. 47, no. 3, pp. 11043–11048, 2014.
[7] M. Chen, S. Xiong, and Q. Wu, “Tracking flight control of quadrotor based on disturbance observer,” IEEE Trans. Syst., Man, Cybern., Syst., vol. 51, no. 3, pp. 1414–1423, Mar. 2021.
[8] H. Bi, G. Qi, and X. Li, “Characteristic analyses, experimental testing and control for attitude system of QUAV under disturbance,” Appl. Math. Model., vol. 100, pp. 77–91, Dec. 2021.
[9] M. Labbadi and M. Cherkaoui, “Robust adaptive nonsingular fast terminal sliding-mode tracking control for an uncertain quadrotor UAV subjected to disturbances,” ISA Trans., vol. 99, pp. 290–304, Apr. 2020.
[10] Z. Ma, H. Kang, and H. Ma, “Adaptive output-feedback asymptotic tracking control for a class of nonlinear systems with actuator failure,” J. Franklin Inst., vol. 359, no. 5, pp. 1881–1898, Mar. 2022.
[11] J. Yan, “Robust actuator fault estimation for a class of linear sampled-data systems with disturbance,” Control Theory Appl., Jun. 2022.
[12] W. Fan, B. Xu, Y. Zhang, S. Tang, and C. Xiang, “Adaptive fault-tolerant control of a novel ducted-fan aerial robot against partial actuator failure,” Aerosp. Sci. Technol., vol. 122, Mar. 2022, Art. no. 107371.
[13] G. Zhao, R. Gao, and J. Chen, “Adaptive prescribed performance control of quadrotor with unknown actuator fault,” J. Control Decis., vol. 36, no. 9, pp. 2103–2112, 2021.
[14] O. Elhaki and K. Shojaei, “A novel model-free robust saturated reinforcement learning-based controller for quadrotors guaranteeing prescribed transient and steady state performance,” *Aerosp. Sci. Technol.*, vol. 119, Dec. 2021, Art. no. 107128.

[15] Z. Hu, C. Hua, and L. Zhang, “Finite time prescribed performance control of quadrotor UAVs with time varying disturbances,” *J. Control Decis.*, vol. 37, no. 12, pp. 3216–3222, 2022.

[16] Z. Hou, P. Lu, and Z. Tu, “Nonsingular terminal sliding mode control for a quadrotor UAV with a total rotor failure,” *Aerosp. Sci. Technol.*, vol. 98, Mar. 2020, Art. no. 105716.

[17] N. Wang, Q. Deng, G. Xie, and X. Pan, “Hybrid finite-time trajectory tracking control of a quadrotor,” *ISA Trans.*, vol. 90, pp. 278–286, Jul. 2019.

[18] P. Tang, D. Lin, D. Zheng, S. Fan, and J. Ye, “Observer based finite-time fault tolerant quadrotor attitude control with actuator faults,” *Aerosp. Sci. Technol.*, vol. 104, Sep. 2020, Art. no. 105968.

[19] K. Liu, R. Wang, X. Wang, and X. Wang, “Anti-saturation adaptive finite-time neural network based fault-tolerant tracking control for a quadrotor UAV with external disturbances,” *Aerosp. Sci. Technol.*, vol. 115, Aug. 2021, Art. no. 106790.

[20] A. Arif, H. Wang, Z. Liu, H. Castañeda, and Y. Wang, “Adaptive visual servo control law for finite-time tracking to land quadrotor on moving platform using virtual reticle algorithm,” *Robot. Auto. Syst.*, vol. 141, Jul. 2021, Art. no. 103764.

[21] F. Wang, H. Gao, K. Wang, C. Zhou, and C. Hua, “Disturbance observer-based finite-time control design for a quadrotor UAV with external disturbance,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 57, no. 2, pp. 834–847, Dec. 2020.

[22] W. Gong, B. Li, Y. Yang, H. Ban, and B. Xiao, “Fixed-time integral-type sliding mode control for the quadrotor UAV attitude stabilization under actuator failures,” *Aerosp. Sci. Technol.*, vol. 95, Dec. 2019, Art. no. 105444.

[23] X. Wu, W. Zheng, X. Zhou, and S. Shao, “Adaptive dynamic surface and sliding mode tracking control for uncertain QUAV with time-varying load and appointed-time prescribed performance,” *J. Franklin Inst.*, vol. 358, no. 8, pp. 4178–4208, May 2021.

[24] A. Freddi, A. Lanzon, and S. Longhi, “A feedback linearization approach to fault tolerance in quadrotor vehicles,” *IFAC Proc. Volumes*, vol. 44, no. 1, pp. 5413–5418, 2011.

[25] C. Hua, J. Chen, and X. Guan, “Adaptive prescribed performance control of QUAVs with unknown time-varying payload and wind gust disturbance,” *J. Franklin Inst.*, vol. 355, no. 4, pp. 6323–6338, 2018.

[26] Z. Yu, Y. Zhang, Z. Liu, Y. Qu, C.-Y. Su, and B. Jiang, “Decentralized finite-time adaptive fault-tolerant synchronization tracking control for multiple UAVs with prescribed performance,” *J. Franklin Inst.*, vol. 357, no. 16, pp. 11830–11862, Nov. 2020.

[27] X. Wu, X. Wu, X. Luo, and X. Guan, “Indirect adaptive neural network dynamic surface control for non-linear time-delay systems with prescribed performance and unknown dead-zone input,” *IET Control Theory Appl.*, vol. 12, no. 14, pp. 1895–2001, 2018.

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