Metal cutting of tooth gears by a method of a nonlinear generating process of a profile

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Abstract. In this article, the perspective methods of milling of tooth gears by disk cutters with the nonlinear coordination between feeding movements are presented. Besides, the algorithm of the contour calculation of sophisticated reconciled movements of a tool and a gear blank is described.

1. Introduction

Metal cutting of gear wheels of a various profile including an involute one is carried out by special pitch tools: hobbing cutters, gear shape cutters, chasers and other pitch tools. Such surfaces are shaped by two coordinated movements of shaping at a constant ratio of a feeding rotation of a work piece and gear shape cutters, chasers tools, the rotational main movement of a cutter and a feeding rotation of a work piece for hobbing cutters. Thus the shape of a tool generating flat varies at the profile form change of a gear wheel tooth contour, a module, an angle of involute profile. It demands the application of a big spectrum of complicated, expensive and special tools \cite{1, 2}. Therefore, these methods are low efficient in small-batch and unit productions, and the application of tools running based on a method of copying, does not ensure high accuracy and quality of a surface \cite{3-5}.

For the purpose of the efficiency increase of the cutter life use, it is necessary to develop a system of control and cutter recovery taking into account the mentioned factors of dispersion. For this, the development of a simulator for the process of adequate cutter life use, taking into consideration the factors emphasized, is necessary \cite{8, 9}.

2. Materials and methods

During the development of the simulator of cutter life use, taking into account the dispersion factors mentioned, the following models of tool wear were used: a fan model, an accumulation model, a complex model and a breakout pattern. To solve the problem identified, we used the methods of probability theory and those of mathematical statistics.

3. Metal cutting of gear wheels with three nonlinear movements of forming

Shaping of working surfaces of teeth is carried out by a disk cutter of a trapezoidal (conical) profile, herewith, each working side of a gear tooth profile is executed by the rectilinear corresponding side of a tool profile, and a combined, slot profile of the gear wheel coupling with working sites of a work piece profile is carried out by its cylindrical surface.
Profile processing of each slot is made by three simultaneous forming movements which are coordinated nonlinearly and lie in one plain of shaping and the principal rotation of cutter \( \omega_{zo} \) (Figure 1). One of them, a uniform rotation of gear wheel \( \omega_{x1} \), and the other two, \( S_x \) and \( S_y \), will agree with rotational \( \omega_{x1} \) so that during processing of profiles sites, rectilinear generators roll along a work surface. In this manner, the rectilinear generator of a flared section of a cutter should be sequentially tangent to each point of a processed profile. Ratios between separate movements constantly vary depending on the position of the top of the cutting edge and the magnitude of its slippage regarding the cutting surface.

![Figure 1. The diagram of a nonlinear rolling of a gear wheel slot with a disk cutter of a trapezoidal profile with three sophisticated and coordinated forming movements.](image)

The designed method of processing of difficult surfaces allows increasing precision owing to profiles generating process of a work piece by rectilinear generators of a tool; besides, it is multipurpose since involute wheels processing of a various module and wheels with non-standard profile angles can be manufactured by cutters of one standard size.

### 4. Sophisticated trajectory installation and coordinated feedings movements during processing of gear wheels of a non-evolvent profile

The method of a nonlinear rolling is applicable for wheels of a non-evolvent profile, for example, for Novikov’s meshing wheels or a cycloidal profile [10].

The main task for the implementation of the described method by any function is establishment of the functional correlation between separate movements of shaping that is reduced to the establishment of tool top migrations on the shaft, \( Z_i \Delta S_x \), and \( Y_i \Delta S_y \) in step-by-step increments depending on a rotation angle of a work piece \( \varphi \) in the direction of angle feeding \( \omega_{x1} \). For this purpose, the cutter should sequentially touch the working surface of the tooth at each point of the profile [9]. That is, the processing side of the tool profile at each point should be tangent to the processed side of the profile of a work piece (Figure 2).

The following designations were used: 1, 1’ – consecutive positions of the side of a tool profile; 2, 2’ – consecutive positions of the rolled side of the work piece profile; \( A_iA' \) – an initial point of the contact of the tool with the processed surface in two consecutive positions of a work piece; \( A_iA' \) – correspondingly subsequently contact point; \( B_iB' \) – two consequent positions of the tool top during migration of the tool along shafts \( Z \) and \( Y \) without rolling; \( B' \) – the position of the tool top in the following engagement point with a work piece during the tool migration simultaneously on shafts \( Z \) and \( Y \) during rolling.
In the course of nonlinear rolling, moment positions of tooth profile 1 are defined by an angle of its tilting to shaft $O_1Z$ in coordinate system $O_1YZ$ during certain moments of time or an angle of a conical site of cutting profile $\alpha/2$. Positions of the tool working profile vary in coordinate system $OYZ$ under the following law:

$$\varphi = \omega t. \quad (1)$$

The cutter profile should be tangent to all consecutive positions of the surface profile described by function $y = f(z)$ in the same coordinate system. During the turning movement of the tooth profile to angle $\varphi$, the cutter profile will be displaced from point $A(z_1, y_1)$ to point $A'(z'_1, y'_1)$; it means during processing with slippage $p = 0$ (the tooth top of the cutter does not change its position in processing relatively the cutting edge), profile top $B$ transfers on shaft $Z$ from point $A_i$ to point $A'_i(B')$.

For rolling provision without a slippage ($p = 1$) (the tooth top of the cutter changes its position in processing relating to the cutting edge) the profile top is displaced from a point $B$ to the point $B''$ so that the length of the profile $B'B''$ was equal to the curvilinear site of the processed profile $A_iA_j$ or $A_iA_2$. Therefore, the initial position of the profile top in shaft direction $O_1Y$ makes interchange $\Delta S_1$ in shaft direction $O_1Z = \Delta S_2$.

Thus, the problem of calculation of the coordinates of the work piece migration during processing is reduced to the definition of the first derived functions describing the tooth profile, which defines the position of the tangents drawn at an angle to shaft $O_1Z$, in two positions of functions $y = f(z)$, different from angle of rotation $\varphi$. This problem is solved quite easily for the initial profile position at interval $A_iA_2$, and it is difficult enough for the following position of functions at interval $A_iA_2$.

A tangent angle is set with regard to the function of the tooth profile, drawn to some point $A_2$ (with coordinates $z_2, y_2$) belonging to the profile in its home position. The tangent analysis is conducted in...
a similar way with z-coordinate value \( z_2 \), there can be some equations, concerning the given function. The tangent angle of inclination at point \( A_2 \) is the following:

\[
K = \arctg\left( \frac{df}{dz} \right)z_2 . \tag{2}
\]

A new position of point \( A_2 \) is set (point \( A_2' \) with coordinates \( \left( z_2', \ y_2' \right) \), in which the tangent will occupy tool profile position 1; in other words, the tangent position turned at angle \( \varphi \), will occupy a position at the angle to shaft \( O_1Z \):

\[
\Delta \varphi = 90^\circ - \frac{\alpha}{2} - K . \tag{3}
\]

As point \( A_2' \) turns at angle \( \varphi \) towards rotation centre \( O_1 \), its new coordinates are recorded with the function of the coordinated transformation:

\[
\begin{align*}
z_2' &= -y_2 \cos \varphi + z_2 \cos \varphi \\
y_2' &= y_2 \cos \varphi + z_2 \cos \varphi 
\end{align*} \tag{4}
\]

Coordinates of point \( A_2 \) are set, through which the tangent parallel to the position of tool profile 1 and makes an angle \( \left( 90^\circ - \frac{\alpha}{2} \right) \) with the shaft:

\[
\frac{df}{dz} \bigg|_{z_1} = \tan \left( 90^\circ - \frac{\alpha}{2} \right) . \tag{5}
\]

Substituting discovered value \( z_1 \) in the equation of the profile function, coordinates of tangent point \( A_2 \) are set during profile turning movement at angle \( \varphi \).

Migration of the tool top is set from point \( B' \) to point \( B \) without offset of the tooth top concerning cutting edge \( (p = 1) \):

\[
\begin{align*}
\Delta S_{z1} &= z_1 - z_2' , \\
\Delta S_{y1} &= y_1 - y_2' 
\end{align*} \tag{6}
\]

Migration of the tool top is set from point \( B_1 \) to point \( B' \), taking into account pure rolling (curve \( AB \) can be substituted with the sufficient degree of accuracy by a chord).

With regard to pure rolling \( (p = 1) \), the following holds:

\[
\begin{align*}
\Delta S_z &= \Delta S_{z1} + \sqrt{\left( z_2 - z_1 \right)^2 + \left( y_2 - y_1 \right)^2} \frac{\sin \alpha}{2} \\
\Delta S_y &= \Delta S_{y1} + \sqrt{\left( z_2 - z_1 \right)^2 + \left( y_2 - y_1 \right)^2} \frac{\cos \alpha}{2} 
\end{align*} \tag{7}
\]

During the rolling action with slippage \( (p \neq 1) \), the following is true:

\[
\begin{align*}
\Delta S_z &= \Delta S_{z1} + p \sqrt{\left( z_2 - z_1 \right)^2 + \left( y_2 - y_1 \right)^2} \frac{\sin \alpha}{2} \\
\Delta S_y &= \Delta S_{y1} + p \sqrt{\left( z_2 - z_1 \right)^2 + \left( y_2 - y_1 \right)^2} \frac{\cos \alpha}{2} 
\end{align*} \tag{8}
\]
5. Trajectory establishment of sophisticated and coordinated movements of feedings during teeth processing of the evolvent profile

Setting of the functional correlation between separate forming movements for tooth gears of the evolvent profile begins with setting of the profile angle of cutter $\alpha/2$ (Figure 3).

The profile angle is defined by the following formula:

$$\alpha/2 = \eta_0 + \gamma_{of} = \frac{ob \cdot 180^\circ}{r_e \cdot \pi} + \gamma_{of},$$

where $ob$ is an arc site on the basic circle of a wheel, which is selected with regard to the possibility of wheel processing of several various modules by a cutter of one standard size and makes $(0.3 - 0.4)E$, where $E$ is the width of a slot on the arc of basic radius $r_e$:

$$r_e = \left(\frac{mz \cdot \cos \alpha_o}{2}\right)$$

where $\alpha_o$ is a standard or an accepted angle of linkage, $\gamma_{of}$ is the angle of the evolvent at point $f$:

$$\gamma_{of} = \left(\frac{g \alpha_o \cdot 180^\circ}{\pi} - \alpha_{of}\right);$$

$$\alpha_{of} = \arccos \frac{r_e}{r_f}$$

$r_f$ defines the junction point of a working site of the tooth profile with a slot:

$$r_f = 0.5mz - m.$$
\[
\alpha_{oa} = \arccos \left( \frac{r_a}{r_o} \right),
\]
where \( \gamma_{oa} \) is the angle of the evolvent at point \( f \):
\[
\gamma_{oa} = \left( \left( \frac{5\alpha_{oa} \cdot 180^\circ}{\pi} \right) - \alpha_{oa} \right),
\]
where \( r_o \) is the radius of teeth tops:
\[
r_o = 0.5mz + m.
\]

An angle of rotation is set between two subsequent reference points \( \Delta \gamma \) and the absolute angle of rotation \( n \) of the point of tooth profile \( \alpha_n \), for the provision of the condition of the contingence of point \( \alpha_n \) (the first point is point \( \alpha \)):
\[
\Delta \gamma = \left( \alpha_{oa} - \alpha_{of} \right) / N;
\]
\[
\alpha_n = \alpha_{oa} - \Delta \gamma n.
\]
Radii of all consecutive points of the contingence from \( \alpha \) to \( f \) are set. For point \( \alpha_n \) with radius \( r_n \), the following is true:
\[
r_n = r_o / \cos \left( \alpha_{oa} - \Delta \gamma n \right).
\]

Coordinates of point \( n \) of the evolvent profile are determined by formulas
\[
y_n = r_n \cos \left( \eta_o + \tan \frac{\alpha_n \cdot 180^\circ}{\pi} - \alpha_n \right),
\]
\[
z_n = r_n \sin \left( \eta_o + \tan \frac{\alpha_n \cdot 180^\circ}{\pi} - \alpha_n \right).
\]

Since the absolute angle of rotation of each point of profile is \( y_n = \varphi \), (Figure 2), new coordinates of points of the profile during the contingence moments are defined by formulas (4) of coordinated transformations:
\[
y_n' = y_n \cos \Delta \gamma n + z_n \sin \Delta \gamma n;
\]
\[
z_n' = -y_n \sin \Delta \gamma n + z_n \cos \Delta \gamma n.
\]

Calculation of sophisticated and coordinated tool migrations taking into account profile rolling is made by formulas (6) and (7).

Processing of the right working part of the tooth profile is completed at point \( f \) and begins at point \( a \); for the left working part of the profile, it begins at point \( f_1 \) and completes at the point.

6. Trajectory establishment of feeding movements during slot processing of tooth gears of the evolvent profile
Slot processing is conducted after processing of the working site of the profile from the tooth top (point \( a \) with radius \( r_o \)) to the conjugated site at the tooth fillet (point \( f \) with radius \( r_f \)) (Figure 4).

For this purpose, the motion on shaft \( Z \) is set for the cutter on shaft \( Z \) at the value of a difference between doubled intercept \( F \) and intercept \( S_1 \), and the wheel is simultaneously turned at radius \( r_f \) at the value of a difference of intercepts \( E \) and \( 2F \), with working feeding which equals the sum of rates of these motions. Such motions ensure the contingence with the opposite side of the tool profile of the working site of the opposite tooth profile at point \( f_1 \), that is at the junction point of the working site of the tooth profile with a slot.

Thus, the tool and the work piece occupy a new position (Figure 4, it is shown by a dotted line). From point \( f_1 \) to point \( a_1 \), the opposite profile of the working site of the tooth is processed by the profile processing algorithm at the site, but in the opposite direction.
7. Conclusions
Methods of nonlinear rolling of tooth gears have been developed and presented. These methods allow processing of gears with a working surface of various forms of a profile by a disk cutter of a trapezoidal profile. The narrow range of machines with NC allows enhancing the accuracy and productivity of processing under conditions of serial and individual manufacture.

Algorithms of trajectories calculation of sophisticated and coordinated movements of the workpiece and the tool for processing of wheels with the various forms of the tooth profile are presented.

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