Kernel-based online learning for real-time voltage control in distribution networks

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Abstract: This paper presents a new data-driven voltage control approach for distribution networks based on kernel methods. Voltage control becomes more and more challenging due to the increased penetration of Distributed Generation (DG), bidirectional power flow and faster voltage dynamics. State-of-art strategies for voltage control rely on physics model-based Optimal Power Flow (OPF) solutions, which can be implemented in a centralized or distributed manner. Nevertheless, such strategies require a detailed model of the network, and often lack scalability due to the large number of nodes and the limited communication infrastructure in distribution networks. In order to achieve real-time voltage control in distribution networks of meshed and radial topology, this paper presents a data-driven approach, which relies on local or regional measurements and does not require accurate models of the grid or an advanced communication infrastructure. Specifically, the proposed data-driven approach uses functional stochastic gradient descent in Reproducing Kernel Hilbert Spaces (RKHSs), to learn the control strategies for Distributed Generation (DG) units in real-time that lead to near-optimal operation costs, while maintaining adequate voltage profiles in the network and alleviating congestions for time-varying load and generation conditions.

1 Introduction

Traditionally, distribution networks assumed passive consumer behavior and unidirectional power flow from the substation to the end-consumers. Hence, voltage control was achieved by actuating reactive power compensators like shunt reactances and capacitor banks [1], whose set-points were updated at a slow time scale.

The high penetration of Distributed Generation (DG), has shaped a new reality with more challenging voltage dynamics, as well as, bidirectional power flow and line congestions, which demands for changes in the operation practice of Distribution System Operators (DSOs). A growing trend is to utilize controllable inverter-interfaced DGs to provide controllability at higher time resolutions, where the active and reactive power set-points are obtained by solving the Optimal Power Flow (OPF) problem [1, 2].

The OPF problem is a mathematical optimization problem that relies on system-wide sensor information, detailed modelling of the network topology, line impedances and controlled equipment, and aims at minimizing a network operational objective (e.g. total operation costs or active power losses), subject to a set of safety constraints such as voltage limits, power flow, line limits, etc.

Due to the non-linear nature of power flow equations, the OPF problem corresponds to a Non-linear Programming (NLP) problem, which is computationally expensive to solve. In order to control the distribution system voltage dynamics, fast computation and communication of the OPF solutions are required.

A popular method for reducing the computation times of such OPFs is convex relaxations via Semidefinite programming (SDP) [3, 4], and Second Order Cone Programming (SOCP) [5]. Although exact for radial network topologies, SDP relaxations often fail to achieve a rank-1 solution for meshed topologies [6]. According to [7], it is possible to extend SDP relaxations to weakly meshed networks by considering a penalized convex relaxation, neglecting the network losses and effectively limiting the feasible injection region. Nevertheless, the behaviour of such penalized convex relaxation is highly sensitive to the penalty parameter and feasible solutions are not always guaranteed.

SOCP relaxations exploit the sparsity of the network to reduce the computation time compared to SDP relaxations. However, they suffer from an ill-fate when solving OPF problems for meshed and weakly meshed network topologies [5]. The reason behind is that the so-called cycle-condition (angle differences sum up to zero over each cycle), as well as, the conic relaxation must both be exact to ensure the feasibility of the SOCP solution in such topologies. For both SDP and SOCP relaxations a detailed model of the network, and communication between a central coordinator and all nodes is required.

In recent years, numerous works have aimed at further reducing the computation times by solving the OPF in a distributed fashion (i.e. D-OPF) [8–11], by using linear and convex approximations. Hereby, most works focus on radial network topologies [8, 10–12]. In addition, such D-OPF solutions rely on global communication between all the buses or with a central coordinator, which yields at expensive deployments and long delays. Very recently, D-OPF based on local voltage measurements and communications from its neighbours in the network has been proposed [12].

In practice, the large number of nodes and the limited communication infrastructure hinder the implementation of real-time OPF-based voltage control settings. Hence, approaches based on local or regional measurements are now considered a viable alternative.

Leveraging recent advances in the fields of machine learning and signal processing, data-driven approaches have been recently proposed for tackling the voltage control problem in distribution networks, in a decentralized manner and by exploiting the knowledge extracted from local or regional measurements. In [13–16], data-driven approaches have been proposed to learn control strategies for each controlled asset, using data from off-line OPF calculations to train the respective local models (one for each controlled asset), under a variety of operating conditions that are either historically observed or synthetically created using simulations [16].

In this work, we focus on an online data-driven approach for real-time voltage control of distribution networks; specifically, we propose the Kernel-based Decentralized Online Learning (K-DOL) algorithm, where each inverter-interfaced controlled asset learns a local control strategy, as local (or regional) observations become available. Compared to the works in [13–16], K-DOL is valid for meshed, as well as, radial network topologies, and does not require
off-line OPF calculations to learn control strategies for each controlled asset. Moreover, K-DOL has several advantages over centralized and distributed OPF solutions, as it does not require a detailed model of the network or a fully deployed communication infrastructure to infer solutions that contribute to the voltage control goal.

In addition, kernel methods enable the usage of linear structures to represent non-linearities, e.g. the non-linearities of the power system, by using an expansion of kernel functions and weights. Furthermore, thanks to the Representer Theorem, it is possible to transform a search over an infinite space into one over a set of weights and data samples; this means that in order to search a given control strategy, one can simply search for a combination of weights and previously observed features in a decentralized manner.

Our theoretical contribution is the formulation of the real-time voltage control problem as a functional stochastic gradient descent in RKHSs (Reproducing Kernel Hilbert Spaces). Our practical contribution is an algorithm for very efficient control of DGs, based using sparse RKHS representations to learn strategies of limited complexity, faster than centralized and distributed OPF approaches.

The remainder of this paper is organized as follows. Section 2 explains the main characteristics of kernel methods and discusses their implementation in the context of power systems. Section 3 describes the proposed data-driven approach for real-time voltage control in distribution networks. Section 4 describes the case study used for validation. Section 5 discusses the simulation results and Section 6 concludes the paper.

2 Kernel methods

In machine learning, kernel methods map data into a (higher-dimensional feature space) vector space, where the relationships among the data are linear; thanks to this mapping, it is possible to apply linear algorithms to learn an underlying function in that space, in a computationally efficient manner [17].

Let \( \mathcal{X} \subset \mathbb{R} \) denote the original input space of the data, then using a non-linear function \( f(\cdot, \cdot) \), the input data is mapped into a feature space where the non-linear pattern or relation now appears linear. Hereby, \( f : x \in \mathcal{X} \rightarrow f(x) = x(\cdot, \cdot) \in \mathcal{H} \) is the corresponding mapping and \( \kappa : \mathcal{X} \times \mathcal{X} \rightarrow \mathbb{R} \) is a Mercer kernel [18], which is a continuous, symmetric, and positive-definite function.

A Mercer kernel that is applied to pairs of input vectors, is interpreted as an inner product in a higher-dimensional feature space i.e. RKHS \( \mathcal{H} \), hence allowing inner products in the feature space to be computed without making direct reference to feature vectors. This idea, known as the kernel trick, allows the use of linear algorithms to learn non-linear functions e.g. strategies for controlled assets.

Kernel methods have been widely used in off-line learning settings, where a large set of historical observations is available to learn an underlying function e.g. in regression or classification tasks. Typical implementations are based on Support Vector Machines (SVM). In decentralized online learning settings, kernel methods have been recently proven successful for a general case of regression over networks, which promotes sparse representations [19].

In the context of power systems, off-line kernel methods were successfully applied to learn strategies for controlled assets in a centralized [14–16] or decentralized fashion [12]. However, off-line OPF calculations are necessary to train the learning models for each controlled asset, and applications are typically restricted to radial networks topologies.

Online learning with kernels has not yet being proposed to learn control strategies for controlled assets. The main advantage of online learning is that is possible to progressively update the underlying control strategy while adapting to evolving network conditions, whereas in offline learning, large training sets are required, and the optimality might substantially degrade for network conditions that were not part of the training set. Furthermore, online learning generally performs faster than offline learning in both training and execution.

In the following section, we formulate the real-time voltage control problem in distribution networks, as a functional stochastic gradient descent in RKHSs, while using notions from [19, 20], to learn strategies of limited complexity for each controlled asset.

3 Real-time voltage control formulation

The OPF-based voltage control problem consists in determining the control setpoints e.g. active and reactive power injections, for a set of controlled assets \( \mathcal{F} \), that minimize a system-level objective, while maintaining the voltage and other operational quantities in the prescribed limits. The system-level objective, in this case, is to minimize the network operation costs.

In K-DOL, we consider a distribution network of arbitrary topology, where each node with a controlled asset, e.g. inverter-interfaced DG unit or a controllable load, is denoted by \( i \). Each controlled asset \( i \in \mathcal{F} \) observes a local data sequence as realizations \( (x_i, y_i) \) from random pair \( (x_i, y_i) \in \mathcal{X} \times \mathcal{Y} \) and seeks to learn a common globally optimal strategy \( f^* \in \mathcal{H} \), which corresponds to a set of optimal injections for the controlled assets and belongs to the Hilbert space \( \mathcal{H} \). Hereby, \( x_i \) is the nodal voltage measurement at the controlled asset (magnitude \( v_i \) and angle \( \theta_i \)) and \( y_i \) correspond to measurements of a load node \( j \) (e.g. voltages and line currents) belonging to the neighbourhood of \( i \), i.e. \( j \in n_i \), which are considered visible for each controlled asset \( i \).

\[
f^* = \arg \min_{f \in \mathcal{H}} \sum_{i \in \mathcal{F}} \left( \| E_{y_i}(f(f_i^*(x_i), y_i)) \| + \frac{1}{2} \| f_i^* \|_{\mathcal{H}} \right)
\]

In this setting, the optimal strategy is learned by each controlled asset locally i.e. \( f_i \), with the locally available data; therefore \( f_i \) is near-optimal as compared to \( f^* \), which is the set of optimal strategies calculated in a centralized manner i.e. by solving the \( \lambda \)-regularized minimization problem defined in (1), with all data gathered in the distribution network. To overcome this limitation, restricted communication among controlled assets can be allowed; specifically, a consensus constraint on the strategies can be imposed between neighbours, as proposed by Koppel et al. in [19] in the context of regression over networks with kernels. Nevertheless, by forcing all control strategies to be very similar, we may degrade the local controller performance, ignoring the diversity available at each controlled asset's data stream.

The control strategies aggregated over the network are stacked in \( f(\cdot) = [f_1(\cdot), \ldots, f_n(\cdot)] \) that yield vectors of length \( \mathcal{F} \) when evaluated at local random vectors \( f(x) = [f_1(x_1), \ldots, f_n(x_N)] \in \{ \mathbb{R} \}^n \), \( f_i \) in the hypothesized function class \( \mathcal{H} \) admits an expansion in terms of kernel evaluations only over training examples.

\[
f_i(x_i) = \sum_{n=1}^M a_i \kappa(x_i, x_n)
\]

where \( a_i = [a_i, \ldots, a_M]^T \in \mathbb{R}^M \) denotes a set of weights and the upper index \( M \) is the model order.

The Gram or kernel matrix is defined as \( \mathbf{K} \in \mathbb{R}^{n \times n} \), with entries given by the kernel evaluations between \( x_i, x_n \), as \( K_{x_i, x_n} = \kappa(x_i, x_n) \). The vector of kernel evaluations is denoted as \( \mathbf{k}(\cdot) = [\kappa(x_1, \ldots, \kappa(x_M, \cdot))] \). The training points associated with the kernel matrix are defined as \( x_i = [x_1, \ldots, x_M]^{T} \).

The one we have selected, which is also the most popular and the universal approximator, is the Gaussian kernel (RBF) [21], described in (3), where \( \xi \) is the kernel bandwidth.

\[
\kappa(x_i, x_n) = \exp \left( -\frac{\| x_i - x_n \|^2}{2\xi^2} \right)
\]

The kernel bandwidth \( \xi \), is the distance one has to cover in the input space for the kernel function value to change significantly [22].
3.1 Clustering the distribution network in different neighbourhoods

In order to define the neighbourhood $n_i$ and voltage control area for each controlled asset $i$, we cluster the distribution network in different neighbourhoods, one for each controlled asset, according to an electrical distance measure. Hereby, the load nodes electrically closer to a controlled asset form its neighbourhood and respective voltage control area.

Several electrical distance measures can be used for clustering a network into a set of neighbourhoods for a variety of power-system operations and control tasks [23–25]. In this work, we calculate the Power Distribution Factor (PTDF) matrix for each node to build a power transfer distance measure [24], which is then used for clustering the network.

The PTDF matrix, described in (4), measures the shift in the active power flow in a line that connects node $k$ to node $l$, due to an active power transfer between two buses $m$ and $n$, where $m$ is the source node and $n$ is the receiver node. Hereby, $x_{kl}$ is reactance of the line connecting node $k$ to node $l$ and $X_{in}$ is the entry at the kth row and the mth column of the bus reactance matrix.

$$PTDF_{kl,mn} = \frac{1}{x_{kl}}(X_{in} - X_{kn}) - (X_{in} - X_{mn})$$

The PTDF matrix is calculated for each node as the source node. Thereafter, a power transfer distance measure $PT_{trans}$ is calculated as in (5), which provides the total shift in active power flows across all lines in the network, for a unit active power injection at $m$ and withdrawal at $n$, i.e.

$$PT_{trans} = \sum_{k\in \mathcal{K}} |PTDF|_{P_{n} + 1, P_{o} - 1}$$

Each load node is then assigned to the controlled asset for which the smallest $PT_{trans}$ value is obtained.

For a broader description on different electrical distance measures, the reader is referred to [24].

3.2 Cost functional

To map each control strategy to a scalar cost, the following cost function $\psi_i$ is introduced:

$$\psi_i(f) = \sum_{i \in \mathcal{Y}} \left[ E_{x,y} [\ell_i(f_i(x), y)] + \frac{\lambda}{2} \| f_i \|_H^2 \right]$$

where $\ell_i(f_i(x), y)$ is the operation cost for each controlled asset; $\ell_i(f_i(x), y)$ is averaged over all possible observations to define the statistical cost $E_{x,y} [\ell_i(f_i(x), y)]$. To penalize the complexity of $f_i$, we use the Tikhonov regularizer (second term in (6)), where $\| \cdot \|_H$ is the square-Hilbert-norm.

From (6), a local cost functional is defined as in (7). Observe that $\psi_i(f) = \sum \psi_i(x)$. An stochastic approximation of the cost functional $\psi_i(f)$ is utilized, which is denoted by $\tilde{\psi}_i(f(x), y)$ evaluated at the realization ($x$, $y$) of the stacked random pair ($x$, $y$):

$$\tilde{\psi}_i(f(x), y) = \sum_{i \in \mathcal{Y}} \left[ \ell_i(f_i(x), y) + \frac{\lambda}{2} \| f_i \|_H^2 \right]$$

Similarly, the local instantaneous cost functional is defined as $\tilde{\psi}_i(f_i(x), y)$, for $i \in \mathcal{Y}$. For voltage control in distribution networks, we define the local loss $\ell_i(f_i(x), y)$ as the operation cost of each controlled asset $i$, namely $C_i(f_i(x))$, which is typically defined as a quadratic function of the power injections. Besides, $\ell_i(f_i(x), y)$ includes a term for penalizing the strategies that may cause voltage violations at any node belonging to the neighbourhood of $i$, where $\mu_i$ is the voltage penalty factor.

Assuming that the line current measurements of adjacent nodes and the slack bus power injections are available to each controlled asset $i$, we can readily account for violations in the neighbouring line ratings and slack bus power limits, as described in (9). Hereby, $\mu_{jl}$, $\mu_{pn}$, and $\mu_{o}$ are the respective penalty factors. $C_i(f_i(x))$ correspond to the active and reactive power costs.

We use the function $\phi = \max \{x, 0\}^2$ as in [26], which takes a positive value when the limit of the variable in question is violated and zero when it is inside the prescribed boundary; this is further illustrated for $v_j$ in (11). Alternative approaches for tackling the constraints visible to each controlled asset include, decentralized primal-dual for online distributed optimization in RKHS, such as the one proposed in [27, 28] for learning a regression function in heterogenous networks, as well as, Lagrange multipliers in RKHS as exploited in [29] for motion control.

$$\ell_i(f_i(x), y) = C_i(f_i(x)) + \mu_{jl} \sum_{j=1}^n \left( \phi(v_j(f_i(x), y)) - v_j^{(x)} \right) + \sum_{i=1}^m \left( \phi(v_i(f_i(x), y) - v_i^{(x)}) \right)$$

for $i \in \mathcal{Y}$

Note that variables such as neighbour voltages and line currents, are defined as local dependent variables, which are functions of the local control strategies, i.e. $v_j(f_i(x))$. Each $f_i(x)$ is continuously differentiable with respect to $f_i$. We assume that these functions are continuously differentiable with respect to $f_i$.

$$\phi(v_j(f_i(x), y) - v_j^{(x)}) = \begin{cases} (v_j - v_j^{(x)})^2, & \text{if } v_j > v_j^{(x)} \\ 0, & \text{if } v_j \leq v_j^{(x)} \end{cases}$$

3.3 Functional stochastic gradient descent

The next step in K-DOL is to compute the functional stochastic gradient of the approximated cost functional $\psi_i(f_i(x), y)$, i.e. $\nabla_{f_i} \psi_i(f_i(x), y)$. To accomplish this, the gradient of the local loss $\ell_i(f_i(x), y)$ needs to be determined (see (11)). $\nabla_{f_i} \psi_i(f_i(x), y)$ is used to steer the online learning algorithm and for updating the control strategies at each controllable asset when new observations become available.

For the first term in (11), we will use the shorthand notation $\ell_i(f_i(x), y) := (\partial_0 f_i(x), y, \partial_0 f_i(x))$. Using functional derivative properties, the second term in (11) is equivalent to a kernel evaluation at the current observation, as expressed in (12).

$$\nabla_{f_i} \psi_i(f_i(x), y) := \frac{\partial \ell_i(f_i(x), y)}{\partial f_i} \left( \partial_{0} f_i(x), y, \partial_0 f_i(x) \right)$$

$$\frac{\partial \ell_i(f_i(x), y)}{\partial f_i} = \frac{\partial f_i(x)}{\partial f_i} \left( \partial_{0} f_i(x), y, \partial_0 f_i(x) \right)$$

With this in mind, the functional stochastic gradient step for the $\lambda$-regularized minimization problem in (1) is defined in (13), where $\eta_t$ is the learning rate.

$$f_{i,t+1} = (1 - \eta_t) f_i - \eta_t \ell_i(f_i(x), y) \phi_k(x_i, y)$$

If the model order for each controlled asset is unlimited, the functional update in (13) can be performed by updating the
dictionary and the set of weights as expressed in (14) and (15). However, if the goal is to learn control strategies of limited complexities, \( f_{i+1} \) can be projected onto subspace \( \mathcal{H}_{D_{i+1}} \subseteq \mathcal{H} \), where \( \mathcal{H}_{D_{i+1}} \) consists only of functions that can be represented using the dictionary of features extracted from past data observations. This is achieved by greedy compression of the dictionary of features \( D_{i+1} \), where we use, specifically, the Kernel Orthogonal Matching Pursuit (KOMP) method \([19, 20]\). In KOMP, the compressed dictionary \( D_{i+1} \) is formed by selecting a subset of columns from \( D_{i+1} \) that best approximate \( f_{i+1} \), in terms of Hilbert norm error, with error tolerance \( \epsilon_t \). Thereafter, the weights are revised based on the compressed dictionary, which results in a sparse approximation of the original function \( f_{i+1} = f_{i+1} \), with a lower model order. Fig. 1 illustrates the algorithm for the K-DOL approach.

\[
\tilde{D}_{i+1} = [D_{i}, x_i] \tag{14}
\]

\[
[a_{i+1}]_u = \begin{cases} 
(1 - \eta_t)[a_i]_u & \text{if } 0 \leq u \leq t - 1 \\
-\eta_t f'(f_i(x_i), y_i) & \text{if } u = t 
\end{cases} \tag{15}
\]

For updating the control strategies, \( f'(f_i(x_i), y_i) \) must be analytically derived. This is achieved by determining the partial derivative of (9) with respect to the active and reactive power injections of each controlled asset, i.e., \( f_i = [P_i, Q_i] \). For the operation cost term, which usually takes a quadratic form, the sensitivities of slack power injections to controlled assets injections, \( \partial P_i/\partial v_i, \partial Q_i/\partial v_i \), which are obtained by computing the partial derivatives of \( v_i \) with respect to the local active and reactive power injections.

Besides \( \partial v_i/\partial P_i, \partial v_i/\partial Q_i, \partial v_i/\partial P_{\text{slack}}, \partial v_i/\partial Q_{\text{slack}} \), we require the sensitivities of slack power injections to controlled assets injections \( \partial P_{\text{slack}}/\partial P_i, \partial P_{\text{slack}}/\partial Q_i, \partial Q_{\text{slack}}/\partial P_i, \partial Q_{\text{slack}}/\partial Q_i \). To achieve this, we perform a multi-variable linear regression based on historical data, which uses power flow scenarios for multiple demand and generation scenarios. Hereby, we use the models (17)–(20) to compute the corresponding sensitivities with respect to \( P_i \); the sensitivities with respect to \( Q_i \) are calculated analogously.

\[
\frac{\partial v_i}{\partial P_i} = a_i P_i + b_i Q_i + c_i Q_{\text{slack}} + d_i P_{\text{slack}} \tag{17}
\]

\[
\frac{\partial v_i}{\partial Q_i} = b_i P_i + v_i P_i + d_i P_{\text{slack}} \tag{18}
\]

\[
\frac{\partial P_{\text{slack}}}{\partial P_i} = a_{\text{slack}} P_i + b_{\text{slack}} Q_i + c_{\text{slack}} P_i + d_{\text{slack}} Q_i \tag{19}
\]

\[
\frac{\partial Q_{\text{slack}}}{\partial Q_i} = a_{\text{slack}} Q_i + b_{\text{slack}} Q_i + c_{\text{slack}} Q_i + d_{\text{slack}} Q_i \tag{20}
\]

which correspond to gradient terms penalizing violations in voltages’ limits, line limits and slack bus power limits. The traditional method for obtaining the required sensitivities is based on the Jacobian matrix inversion. This method is accurate; however, the process of updating and inverting the Jacobian matrix is computationally expensive and lacks scalability \([30]\); hence, it is not suitable for real-time computations.

Furthermore, calculating the Jacobian matrix and its inverse requires full system monitoring and robust communication infrastructure, which is often not available in distribution systems. For these reasons, we prefer a data-driven method to learn the sensitivities from historical data, which is discussed in the next section.

3.4 Data-driven sensitivities computation

Several authors have focused on determining the voltage to active and reactive power sensitivities for online voltage control applications \([30–33]\). In \([31]\), the sensitivities are obtained using a perturb-and-observe method via dividing the voltage by the power perturbation.

In \([30]\), the sensitivities are calculated relying the equivalent resistances and reactances between the source substation node and the node where the power source is located; this approach requires accurate parameter knowledge and is suitable only for radial network topologies.

Similarly, in \([32]\) the sensitivities are estimated using measurements of voltage magnitudes and power injections, while relying on a line parameter estimator and a topology estimator, which assumes a radial topology.

In \([33]\), the voltage to power sensitivities are calculated using a surface fitting technique to produce an estimate the local voltage, which is a high-dimension function of the controlled asset’s injections, i.e., \( v_i = f(P_i, Q_i) \). This approach does not require accurate knowledge of the line parameters and is valid for meshed and radial network topologies; nevertheless, a large set of power flow scenarios for different generations and demand conditions is necessary to produce an accurate estimate of the sensitivities \( \partial v_i/\partial P_i, \partial v_i/\partial Q_i \), which are obtained by computing the partial derivatives of \( v_i \) with respect to the local active and reactive power injections.
this elaboration does not constitute a contribution of this paper but is rather an enabler of a decentralized implementation for the K-DOL approach.

4 Case study

The K-DOL approach described in the previous section is evaluated for voltage control of a distribution network with meshed topology (see Fig. 2), which is the 12.7 kV IEEE 69 bus network with 5 tie lines, (considered closed) and a rated apparent power of 10 MVA.

The test system data is available in MATPOWER. Data for tie lines are taken from [34].

Four DG units, considered PQ controlled, are added to the network at the nodes 11, 49, 61 and 65. To account for volatility in the generation profiles, the DG units are modelled as two PV utility scale generators and two wind generators, with injection limits as shown in Table 1, and operation costs as in Table 2 [35].

![Fig. 2 Modified IEEE 69 bus network topology - PTDF clusters. Controlled assets’ locations (green circles) (Image)](image)

Solar and wind energy profiles were obtained from [36, 37], with a granularity of 15 and 10 min respectively, have been in interpolated to have values on a per-min basis.

The slack bus active power lower limit is set to zero to avoid reverse power flow at the slack bus; this condition can be modified in case it is desired to provide active power towards the transmission system, and if the protections are configured accordingly. For the voltage limits, we assume the range [0.95, 1.05] as normal operation conditions for distribution networks.

The set of neighbours that belong to each controlled asset (n) is defined using the PTDF (Power Transfer Distribution Factors) as electrical distance measure [23]. Within each neighbourhood, we assume different time-varying load profiles; namely, Industrial (I), Commercial (C) and Residential (R). The data is taken from [38].

The proposed algorithm is implemented in MATLAB in an Intel core i7-7500U with 2.70 GHz processor and 16 GB or RAM. For an efficient real-time execution, we use the threshold η = 10−4 to promote sparsification of the learned control strategies. The online learning rate is η = 1. We employ the Gaussian kernel with bandwidth is ξ = 0.13.

The control strategies are computed independently when new data samples become available at each controlled asset; we assume a sampling frequency of 1 min; however, higher sampling frequencies are possible.

Prior to testing the K-DOL approach, the data-driven sensitivities were estimated using the linear regression models presented in Section 3.4. To train the models, a total number of 50,625 scenarios were considered, combining 15 non-simultaneous active power levels for the controlled assets, 15 reactive power levels for the controlled assets, and 15 demand levels for the load nodes.

5 Results

To test the performance of the proposed control approach, we consider two scenarios. In the first scenario (S1), normal operating conditions, for time-varying load profiles for different types of consumers are considered. We compare the solution of K-DOL with the solution obtained with a centralized OPF formulation. In the second scenario (S2), we evaluate the voltage control and congestion alleviation capabilities of the K-DOL approach. The metrics selected to evaluate the performance of the proposed controller are suboptimality gap and average computation time per iteration.

5.1 Scenario 1 (S1). minimization of operation costs

In this scenario we compare the K-DOL approach with the centralized OPF. The centralized OPF is solved using MATPOWER Interior Point Solver. The K-DOL approach is implemented for every controlled asset as shown in Fig. 1.

In Fig. 3, the slack bus active and reactive power injections are displayed in the top and middle graphs. The results show that the slack power injections resulting from the K-DOL implementation, are very close to the injections computed by the centralized OPF.

Furthermore, the total operation costs for the K-DOL and the centralized OPF are presented in bottom graph of Fig. 3. The K-DOL results are near-optimal with a small average suboptimality gap of 8.05% and a maximum deviation of 17.71%.

Note that, compared to the centralized OPF, the K-DOL approach does not require access to all measurement data; instead, each controlled asset needs only the measurements of the nodes

| Table 1 | Generators’ location and injection limits |
| --- | --- | --- | --- |
| Node | Type | Act. power limits | React. power limits |
| slack | brown coal | [0, 5] | [−5, 5] |
| 11 | PV utility | [0, 0.825] | [−0.1725, 0.1725] |
| 49 | PV utility | [0, 0.125] | [−0.2410, 0.2410] |
| 61 | wind | [0, 2.000] | [−0.4465, 0.4465] |
| 65 | wind | [0, 1.000] | [−0.1590, 0.1590] |

| Table 2 | Operation costs |
| --- | --- | --- | --- | --- | --- |
| Node | Fuel costs | Variable costs | Fixed costs |
| --- | --- | --- | --- | --- | --- |
| slack | €, MW | €, MVAr | €, MW | €, MVAr | €, MW | €, MVAr |
| 1.8 | 0.558 | 1.55 | 0.07 | 0.07 |
| 11 | 0 | 0 | 0 | 0.04 | 0.04 |
| 49 | 0 | 0 | 0 | 0.04 | 0.04 |
| 61 | 0 | 0 | 1.55 | 0.057 | 0.057 |
| 65 | 0 | 0 | 1.55 | 0.057 | 0.057 |

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belonging its neighbourhood and the slack bus. Furthermore, topology and network parameters are not required.

In Fig. 4, we present the injections of the controlled assets for both the K-DOL and the centralized OPF approach. The results show that the K-DOL approach are very close to the solutions provided with the centralized OPF approach. There are small discrepancies in the reactive power injections of node 61 and 65. Both controlled assets are wind generators and share the same reactive power costs coefficients; hence, such variations do not affect the total operation costs incurred in the network.

The voltage profile of the network is shown in Fig. 5 across a full week for the K-DOL approach. All node voltages (dashed lines in green colour) are kept in the prescribed limits (solid lines in black colour) during operation. We do not plot the voltages for the centralized OPF, as these are very similar to the voltages obtained with the K-DOL approach.

5.2 Scenario 2 (S2). Voltage control and congestion alleviation capabilities

To test the voltage control capability of the K-DOL approach, the voltage limits are tightened to [1.005, 0.95] to create network conditions in which voltage violations would occur. The sensitivities models are the same as in the previous section.

As shown in Fig. 6, for line 65-27, the proposed approach adequately manages congestions, by curtailing the injections of the asset in node 65 to avoid violations in the line thermal limits.

Next, we test the congestion alleviation capability of the proposed approach. Specifically, we check if the thermal loading limits are satisfied.

As shown in Fig. 7, for line 65-27, the proposed approach adequately manages congestions, by curtailing the injections of the asset in node 65 to avoid violations in the line thermal limits.
The performance metrics for S1 and S2 are summarized in Table 3.

5.3 Discussion

The results obtained show the capability of K-DOL for real-time voltage control, as well as congestion alleviation. Moreover, the communication requirement is less stringent than for the centralized OPF approach, since only intra-neighbourhood communication is necessary. Such requirement can be further reduced, by considering only the most representative load node for each neighbourhood in terms of the voltage magnitude i.e. pilot node, instead of the complete set. The concept of pilot nodes has been leveraged since decades for secondary voltage control in transmission systems.

Compared to other data-driven control approaches for distribution networks, K-DOL does not require off-line OPF calculations and is suited for networks of both radial and meshed topologies. Nevertheless, the proposed approach requires knowledge of the voltage to active and reactive power sensitivities, as well as, slack power injections to controlled assets injections sensitivities. An interesting question that arises is to which extent erroneous sensitivities estimation could produce inaccurate control strategies, particularly when topology changes occur. For such cases, a periodic retraining procedure for the sensitivities could be necessary so that the learned control strategies remain valid.

Furthermore, the impact of clustering on the network in different neighbourhoods might play an important role in the sub-optimality of the learned control strategies; especially, when the loads and controlled assets are concentrated in different areas. In the results obtained so far, the controlled assets communicate with neighbouring nodes; thus, their injections may trigger the demand of the nodes nearby. In the case we observe one extra node belonging to the neighbourhood of other controlled asset, the control actions will respond to the high or low demand in that other area.

6 Conclusion

We propose here a data-driven approach i.e. K-DOL for real-time voltage control in distribution networks based on kernel methods. We formulate an online algorithm for learning the control strategies of each asset, based on functional stochastic gradient descent in RKHSs. The proposed algorithm allows to learn strategies of limited complexity, yielding a very efficient control of inverter-interfaces DGs. Compared to other data-driven control approaches for inverter-interfaced DGs in distribution networks, K-DOL is suited for networks of both radial and meshed topologies and does not require off-line OPF calculations.

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Table 3 Performance metrics

| Metric | Scenario 1 (S1) | Scenario 2 (S2) |
|--------|----------------|----------------|
| Sub-opt. gap, p.u. | 0.0812 | 0.0935 |
| Vol. tightening time, s | 6.5969 | 2.9156 |

The performance metrics for S1 and S2 are summarized in Table 3.
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