Cosmology in Nonlinear Born-Infeld Scalar Field Theory With Negative Potentials

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Abstract

The cosmological evolution in Nonlinear Born-Infeld (hereafter NLBI) scalar field theory with negative potentials was investigated. The cosmological solutions in some important evolutive epoches were obtained. The different evolitional behaviors between NLBI and linear (canonical) scalar field theory have been presented. A notable characteristic is that NLBI scalar field behaves as ordinary matter nearly the singularity while the linear scalar field behaves as "stiff" matter. We find that in order to accommodate current observational accelerating expanding universe the value of potential parameters $|m|$ and $|V_0|$ must have an upper bound. We compare different cosmological evolutions for different potential parameters $m, V_0$.

Keywords: Negative Potential; Born-Infeld field; Dark Energy; Cyclic Model.

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1 Introduction

The role of rolling homogeneous scalar field has been widely discussed in the various epoch for a variety of purposes\cite{1}. Recently, with the surprising discovery of an accelerating expansive and spatially flat universe, the scalar field has gained another newly discussion as a candidate for dark energy. It can drive current accelerating expansion while its energy density can fill in the universe as "missing matter density". The most popular models with scalar field may be the linear scalar field model\cite{2-6}, a canonical scalar field described by the lagrangian $L = \frac{1}{2} \dot{\phi}^2 - V(\phi)$\cite{2-6}, the K-essence model\cite{7-22} a scalar field with a non-canonical kinetic energy terms and the "phantom" model\cite{23-52} a scalar field with the negative kinetic energy terms\cite{23-52}. The potentials in these models are chosen non-negative to avoid negative potential energy density. It is shown that the expanding universe with non-negative potentials have a common property that they will expand for ever, though the evolutionsal behavior of future universe has significant differences corresponding to different potentials. However, research shows that negative potentials can also lead to a viable cosmology\cite{53-56}. Moreover, the universes with negative potentials are entirely different with the universes with non-negative potentials. They can trigger our flat universe from expansion ($H > 0$) to contraction ($H < 0$), which will never occur in standard FRW model\cite{previous oscillatory model only appears in a close universe in standard FRW model}. Hence negative potentials are used to propose the "cyclic universe" model. In this cyclic scenario, when the scalar field rolls to a minimum of its effective potential with $V(\phi) < 0$, the universe will stop expanding and contract to a singularity eventually. Additionally, negative potentials also appeared in supergravity theory and in brane cosmology. It is theoretically important to continue investigating the cosmological features in other models where the effective potential $V(\phi)$ may become negative for some values of the field $\phi$.

Nonlinear Born-Infeld scalar field theory is firstly proposed by W.Heisenberg in order to describe the process of meson multiple production connected with strong field regime\cite{57-59} and then is discussed in cosmology\cite{60-64}. It shows that the lagrangian density of this NLBI scalar field posses some interesting characteristics\cite{65-66}. In Ref\cite{65}, the author showed that a singular horizon exists for a large class of solution in which the scalar field is finite. Naked singularities with everywhere well-behaved scalar field in another class of solution have also been found in Ref\cite{65}. Lately the quantum cosmology with the NLBI scalar field has been considered\cite{67}. In the extreme limits of small and large cosmological scale factor the wave function of the universe was found by applying the methods developed by Vilenkin, Hartle and Hawking. The result has suggested a non-zero positive cosmological constant with largest probability, which is consistent with current observational data. The classical wormhole solution and wormhole wavefunction with the NLBI scalar field has been obtained in Ref\cite{68}. The phantom cosmology based on NLBI scalar field with a special potential had been considered in Ref\cite{69-70}. The results show that the universe will evolve to a de-sitter like attractor regime in the future and the phantom NLBI scalar field can survive till today without interfering with
the nucleosynthesis of the standard model. Very recently, with the analysis to Gold supernova data, we show that maybe the NLBI scalar field model is superior to conventional quintessence model[71]. Furthermore it is showed that in another analogous NLBI theories with the lagrangian \( p(\phi, X) = \alpha^2(\sqrt{1 + \frac{2X}{\alpha^2}} - 1) - \frac{1}{2}m^2\phi^2 \) (where \( X = \frac{1}{2}\dot{\phi}^2 \)) the contribution of the gravitational waves to the CMB fluctuations can be substantially larger than that naively expected in simple inflationary models, which make the prospects for future detection much more promising[72-73]. It is also showed that with the same lagrangian, one can send information from inside a black hole[74]. In Ref[75-76], authors consider a non-Abelian Einstein-Born-Infeld-dilaton theory, where they concern a non-abelian vector field which couples to the dilaton and then describe a dark energy mechanism in a cosmological framework.

The key idea of NLBI scalar field theory is that the conventional quintessence scalar field can not describe the reality correctly in the case of strong field. The lagrangian of conventional quintessence model (here we also call it linear scalar field):

\[
L = \frac{1}{2}\dot{\phi}^2 - V(\phi)
\]  

should be substituted by the lagrangian of NLBI scalar field

\[
L_{NLBI} = \frac{1}{\eta}[1 - \sqrt{1 - \eta\dot{\phi}^2}] - V(\phi)
\]  

which can recover to conventional case when \( \dot{\phi} \to 0 \). In fact, the lagrangian of NLBI scalar field (Eq.2) implies that there exists a maximum constant value \( \frac{1}{\sqrt{\eta}} \) for field velocity \( \dot{\phi} \), which is very analogous to the universal constant velocity \( c \). It means that \( \dot{\phi} \) never reaches infinity while in linear scalar field model there are no such constraint.

In this paper, we combine the two ideas (negative potentials and NLBI scalar field theory) and consider the cosmology based on the NLBI scalar field with negative potentials. We think it may be very interesting and meaningful to see what will happen in this case. The paper is organized as follows: In section 2 we will describe theoretical model in NLBI scalar field theory and consider several basic regimes which are possible to happen in NLBI scalar field: the potential energy dominated regime, the kinetic energy dominated regime and the transient regime that the universe switches from expansion to contraction. In section 3, we investigate the different cosmological evolution in different cases and plot corresponding evolutive behaviors in detail. For the potential \( V(\phi) = \frac{1}{2}m^2\phi^2 + V_0(V_0 < 0) \) We consider the universe evolution with different slope \( m \) and different potential well \( V_0 \). The cases that \( V_0 > 0 \) and \( V_0 = 0 \) are also presented to compare with the case \( V_0 < 0 \), moreover we compare the different evolution between NLBI scalar field and linear scalar field. In section 4 we mention the cyclic model and consensus model. Conclusion and summary is also presented in section 4.

2 Theoretical Model in NLBI Scalar Field theory

We consider the behavior of the NLBI scalar field in the Friedmann universe with the spatially flat FRW metric \( ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \). The energy density and pressure density
for NLBI scalar field are:

\[ P_{NLBI} = \frac{1}{\eta} \left[ 1 - \sqrt{1 - \eta \dot{\phi}^2} \right] - V(\phi) \] (3)

\[ \rho_{NLBI} = \frac{1}{\sqrt{1 - \eta \dot{\phi}^2}} - \frac{1}{\eta} + V(\phi) \] (4)

Corresponding Friedmann equation is

\[ H^2 = \frac{1}{3M_p^2} (\rho_{NLBI} + \rho) = \frac{1}{3M_p^2} \left[ \frac{1}{\eta \sqrt{1 - \eta \dot{\phi}^2}} - \frac{1}{\eta} + V(\phi) + \rho \right] \] (5)

\( \rho \) is the energy density of a matter with baryotropic equation of state \( p = \alpha \rho \), where \( \alpha \) is a constant. For nonrelativistic matter \( \alpha = 0 \), for radiation \( \alpha = \frac{1}{3} \). The evolution equations with Hubble parameter \( H \) are:

\[ \dot{H} = -\frac{1}{2M_p^2} (\rho_{NLBI} + \rho + p_{NLBI} + p) = -\frac{1}{2M_p^2} \left[ \frac{\dot{\phi}^2}{\sqrt{1 - \eta \dot{\phi}^2}} + (1 + \alpha) \rho \right] \] (6)

\[ \dot{\rho} = -3H (\rho_{NLBI} + \rho_{\alpha}) \] (7)

\[ \ddot{\phi} + (1 - \eta \dot{\phi}^2) [3H \dot{\phi} + \frac{dV(\phi)}{d\phi} (1 - \eta \dot{\phi}^2)^{1/2}] = 0 \] (8)

When \( \dot{\phi} \rightarrow 0 \), ignoring the higher-order term of \( \dot{\phi} \) Eq.(8) will recover to quintessence model:

\[ \ddot{\phi} + 3H \dot{\phi} + \frac{dV(\phi)}{d\phi} = 0 \] (9)

where potential \( V(\phi) = \frac{1}{2} m^2 \phi^2 + V_0 \) (\( V_0 < 0 \)). Eq.(8) also tells us that when \( \dot{\phi} \) increase to its maximum \( \frac{1}{\sqrt{\eta \eta}} \), \( \dot{\phi} \) will decrease to zero and this prevents \( \dot{\phi} \) from increasing continuously.

We will use a system of units in which \( M_p = (8\pi G)^{-1/2} = \eta = 1 \) for convenience.

It is very difficult to obtain the general exact solution for the Eqs.(6-8). However we are able to obtain the solutions in some special regimes:

**A. The potential regime: Energy density dominated by \( V(\phi) \)**

In this case \( \dot{\phi}^2 / 2, \rho_{\alpha} << V(\phi) \) and \( |\dot{\phi}| << |3H \dot{\phi}| \). We will find that in this case the result for NLBI scalar field is similar to the one shown in Ref.[77]. It corresponds to the vacuum-like equation of state:

\[ p = -V(\phi) = -\rho \] (10)

The equations for \( a \) and \( \phi \) in this regime have the following form:

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{m^2 \phi^2}{6} + \frac{V_0}{3} \] (11)

\[ 3H \dot{\phi} + m^2 \phi (1 - \dot{\phi}^2)^{1/2} = 0 \] (12)

When \( m^2 \phi^2 >> |V_0| \), we can obtain the solutions for \( \phi \) and \( a \):

\[ \phi(t) = \phi_0 - \sqrt{\frac{2}{2m^2 + 3}} mt \] (13)
\[a(t) = a_0 e^{\sqrt{\frac{2m^2+3}{4t}}(\phi_0^2-\phi^2(t))}\] (14)

Where \(\phi_0\) and \(a_0\) are different integral constant. These solution are very analogous to the results in Ref[77-79]. It has been argued that these solutions describe inflationary universe. However if we consider positive potential \(V(\phi) = \frac{1}{2}m^2\phi^2 + V_0 (V_0 > 0)\) and \(V_0 >> \frac{m^2\phi^2}{2}\). From Eq.(12), we can get

\[\frac{d\phi^2}{dt} = -\frac{2m^2\phi^2}{\sqrt{m^4\phi^2 + 3V_0}}\] (15)

Integrating Eq.(15), we get the solutions for \(\phi\) and \(a\):

\[t + \frac{\sqrt{m^4\phi^2 + 3V_0}}{m^2} - \sqrt{3V_0}\arctanh\sqrt{\frac{m^4\phi^2 + 3V_0}{3V_0}} + C_1 = 0\] (16)

\[a(t) = a_0 e^\sqrt{\frac{V_0}{3t}}\] (17)

where \(C_1\) is the integral constant. For linear scalar field, the corresponding equation is

\[\frac{d\phi^2}{dt} = -\frac{2m^2\phi^2}{\sqrt{3V_0}}\] (18)

and the solutions are

\[\phi = \phi_0 e^{-m^2t/\sqrt{3V_0}}, a(t) = a_0 e^{\sqrt{\frac{V_0}{3t}}}\] (19)

Eqs.(16,17) show the evolution of universe when \(\phi\) is in the bottom of potential while Eqs.(13,14) describe the universe when \(\phi\) is far from the bottom. This two regime are both the accelerating expansion, so it can be considered as a simple version to describe an eternally self-reproducing inflationary universe, as well as the present stage of accelerating expansion. Eq.(19) shows that \(\phi\) will roll down the potential and settle on the bottom of the potential permanently to mimic the de-sitter accelerating expansion. From Eqs.(15,18), we can also know, due to the nonlinear effect, the attenuation of NLBI scalar field is slower than linear scalar field. However if \(V_0 < 0\), the later time accelerating expansion will never occur. The evolution of universe will be completely different with the case \(V_0 > 0\)

B. The kinetic Regime:Energy Density Dominated by Kinetic Energy

This regime is very important because in this case the nonlinear effects are distinct. This regime corresponds to strong field \(\dot{\phi}\). When energy density dominated by kinetic energy, we can neglect \(V(\phi)\) and \(\rho_\alpha\). Then the Eqs.(5,8) become:

\[H^2 = \frac{1}{3} \left( \frac{1}{\sqrt{1 - \dot{\phi}^2}} - 1 \right)\] (20)

\[\ddot{\phi} + 3H\dot{\phi}(1 - \dot{\phi}^2) = 0\] (21)

we have the solution

\[\dot{\phi} = \pm \sqrt{\frac{1}{1 + a^6}}\] (22)
Since in the kinetic energy dominated epoch, $\dot{\phi}$ is close to 1, from Eq.(22) we know that the scalar factor $a(t)$ will be very small. Eq.(22) describes an expanding universe from a singularity or a contracting universe towards a singularity. Form Eqs.(20,22), we can obtain

$$\rho \sim H^2 = \frac{\sqrt{1+a^6}}{a^3} - 1 \sim \frac{1}{a^3} + \frac{1}{2}a^3 - 1 \sim \frac{1}{a^3}$$

The solutions can be written as follows for small scale factor and strong field:

$$a(t) \sim t^{2/3}$$

$$\dot{\phi} = \pm \sqrt{\frac{1}{1+a^6}} = \pm \sqrt{\frac{1}{1+t^4}} \approx \pm (1 - \frac{1}{2}t^4)$$

$$\phi = \phi_0 \pm (t - t_0) + \frac{1}{10}(t - t_0)^5$$

We can represent the kinetic energy term and the potential energy term as: $E_k \sim H^2 \sim \frac{1}{t^2}$ and $E_p = V(\phi) \sim \phi^2$. Therefore we can conclude that: Firstly, if the solution describes an expanding universe from a singularity, $E_k$ drops down rapidly while $E_p$ changes slowly. Therefore the regime with energy density dominated by potential $V(\phi)$ will appear when the evolution is far from the singularity. Secondly, if the solution describes a contracting universe towards a singularity, $E_p$ grows slowly while $E_k$ diverges as $t^{-2}$, ($t$ is the time remaining before the big crunch singularity), and in this case $E_k$ will dominate the universe. Therefore we can conclude that the kinetic energy $E_k$ will always dominate the universe in the vicinity of the singularity.

Here we should point out the different results between NLBI scalar field and linear scalar field. The solution for $a(t)$ and $\phi(t)$ of linear scalar field are given in[77]: $a(t) \sim t^{1/3}, \phi = \phi_0 \pm \sqrt{\frac{2}{3}}\ln t_0$, $\dot{\phi}^2 = \frac{1}{a^2}, \dot{\phi} \sim \frac{1}{a^2}$ near the singularity while the linear scalar field behaves like ”stiff” matter($a(t) \sim t^{1/3}, \rho \sim \frac{1}{a^6}$). This may lead to some interesting cosmological implies. However we should point out that if the evolution happens in the presence of other fields or other source of matter whose density energy behaves as $\frac{1}{a^n}(n > 3)$, then our NLBI scalar field will not dominate the universe near the singularity. In this case the evolution presented here will be modified by the influence of other fields near the singularity.

### C. The transient regime: Switch from expansion to contraction

After analyzing two special regimes, we now pay attention to another important regime: the transient regime that the universe begin to contraction from a expanding phase. Before numerically studying this process, we try to get some solutions by some simple approximation. Since we study the very vicinity where Hubble parameter vanishes($H \sim 0$), we can neglect the term $3H\dot{\phi}(1 - \dot{\phi}^2)$ and rewrite Eq.(8) as:

$$\ddot{\phi} + m^2\rho(1 - \dot{\phi}^2)^{3/2} = 0$$

Furthermore, the value of $\dot{\phi}$ will be also very small in this case. Taking the first order approximation of $(1 - \dot{\phi}^2)^{3/2}$ in Eq.(27), we get:

$$\ddot{\phi} + m^2\phi(1 - \frac{3}{2}\dot{\phi}^2) = 0$$

(28)
Integrating Eq.(28), we obtain:
\[ \dot{\phi}^2 = 2 - e^{\frac{3}{2}m^2\phi^2} \]
(29)

In Ref[77], it is argued that only if \(|V_0| \sim 10^{-120}\) the time that universe begins to collapse can be greater than the age of present universe. In the transient regime, \(\frac{3}{2}m^2\phi^2 \sim |V_0| \sim 10^{-120}\), so the term \(e^{\frac{3}{2}m^2\phi^2}\) can be taken as \(1 + \frac{3}{2}m^2\phi^2\). Eq.(29) can be written as
\[ 3\dot{\phi}^2 + \frac{3}{2}m^2\phi^2 = 1 \]
(30)

The solution of Eq.(30) is
\[ \phi \propto \cos \sqrt{\frac{3}{2}}mt \]
(31)

Up to now, by some reasonable approximation, we know that in the vicinity where the universe evolves to contraction from expansion, the field \(\phi\) will experience a simple oscillatory motion \(\phi \propto \cos \sqrt{\frac{3}{2}}mt\) (Fig.2). With respect to the equation in Ref[77], where \(\phi \propto \cos mt\), it is the nonlinear effect that makes the different evolution.

Next we will investigate what happens during the transient regime (where the sign of \(\dot{a}\) changes). We also try to obtain the analytical solution by some reasonable approximations

First of all, we represent \(V(\phi) = \frac{1}{2}m^2\phi^2 - |V_0|\) in the form \(V(\phi) = \frac{1}{2}m^2(\phi^2 - \phi_0^2)\) for convenience. we assume that the field \(\phi\) begins the oscillation at \(t = 0\), moving with zero initial velocity from a point \(\phi_1 \approx \phi_0\). The initial energy density of the field is \(\Delta V = V(\phi_1) = \frac{1}{2}m^2(\phi_1^2 - \phi_0^2) < |V(0)|\). We will evaluate the turning point moment \(t_c\) when \(H \sim 0\)(i.e, \(\dot{a} \sim 0\)).

Using the same method in Ref[77], we will consider the series expansion of the Hubble parameter around the beginning of this process

\[ H(t) = H_1 + H_1^{(1)}t + \frac{1}{2!}H_1^{(2)}t^2 + \frac{1}{3!}H_1^{(3)}t^3 + \cdots \]
(32)

where \(H_1\) and \(H_1^{(n)}\) are taken the value of \(H\) and \(H^{(n)} = \frac{d^nH}{dt^n}\) at \(t = 0\). From Eq.(6) we have \(\dot{H} = -\frac{\dot{\phi}^2}{2\sqrt{1 - \phi^2}}\) (here we ignore the presence of baryotropic matter \(\rho_b\)), then we find that \(H_1^{(1)} = H_1^{(2)} = 0\) for vanishing initial velocity \(\dot{\phi} = 0\) at \(t = 0\). The first non-vanishing coefficient \(H_1^{(3)} = -\dot{\phi}^2(1 - \dot{\phi}^2)^{-\frac{1}{2}} = -V''(1 - \dot{\phi}^2)^{5/2} = -V'' = m^4\phi^2(0)\). Including the terms up to \(t^3\) in Eq.(32), we get:
\[ t_c = \left(\frac{12V(\phi_1)}{V''(\phi_1)}\right)^{1/6} = m^{-1}\left(\frac{12\Delta V}{m^2\phi_0}\right)^{1/6} \]
(33)

This means that the Hubble parameter vanishes at the time \(t_c\), where
\[ \phi_c = \phi_0 - \left(\frac{3\Delta V}{16m^2\phi_0}\right)^{1/3} \]
(34)

Here the result of \(t_c\)(Eq.(33)) is the same as Ref[77], this is because we set the initial velocity \(\dot{\phi}(0) = 0\). When \(\Delta V < m^2\phi_0^4\), this results imply that the turn occurs in the vicinity of the point \(\phi_0\) where the potential becomes negative. In the short time when universe begins to contract.
from expansion, we can also study the subsequent evolution of \( \phi(t) \) and \( a(t) \). We therefore take \( a = 1 \) during this time, and \( \phi(t) = \phi_1 \cos \frac{\sqrt{2}}{2} mt \). The potential can be expressed as:

\[
V(\phi) = \Delta V - \frac{m^2 \phi_1^2}{2} \sin^2 \frac{\sqrt{2}}{2} mt
\]  

(35)

The acceleration of the universe is given by

\[
\ddot{a} \simeq \frac{\Delta V}{3} - \frac{1}{3} m^2 \phi_1^2 \sin^2 \frac{\sqrt{2}}{2} mt - \frac{1}{16} m^4 \phi_1^4 \sin^4 \frac{\sqrt{2}}{2} mt
\]  

(36)

The initial value of \( \dot{a} \) equals \( \dot{a} = \pm a \sqrt{\Delta V/3} = \pm \sqrt{\Delta V/3} \), this yields

\[
\dot{a} = \pm \sqrt{\frac{\Delta V}{3}} + \frac{\Delta V}{6} t - \frac{1}{6} m^2 \phi_1^2 t + \frac{\sqrt{2}}{12} m \phi_1^2 \sin(\sqrt{2} mt)
\]

\[
-\frac{3}{128} m^4 \phi_1^4 t + \frac{\sqrt{2}}{64} m^3 \phi_1^4 \sin(2\sqrt{2} mt) - \frac{\sqrt{2}}{512} m^3 \phi_1^4 \sin(2\sqrt{2} mt)
\]  

(37)

Where the sign “+” denotes an expanding universe at the beginning of the oscillation. In this case the universe will stop its expansion at \( \phi = \phi_c \) and then collapse to singularity. “−” denotes a collapsing universe at the beginning of the oscillation. In this time the universe will continue collapsing to singularity (see Fig.1). We numerically plot the evolution of the scale factor \( a \) and scalar field \( \phi \) (Fig.1 and Fig.2) at the time when universe switches from expansion to contraction.

![Fig1](image1.png) Fig1. The evolution of scalar factor when \( H \sim 0 \). Solid line describes an expanding universe at the beginning of the oscillation. Clearly it stops its expansion and collapses to singularity. Dash line describes a collapsing universe, it continues collapsing to singularity.

![Fig2](image2.png) Fig2. The evolution of \( \phi \) and \( \dot{\phi} \) when \( H \sim 0 \). Solid line is the evolution of \( \phi \), dash line is the evolution of \( \dot{\phi} \). From the figure we know that the approximate solution Eq.(31) is reliable. We set \( \phi_0 = 0.001 \), the initial value of \( \phi(0) = 0.001000001 \), \( m = 0.4 \), \( \dot{\phi}(0) = 0 \), \( a(0) = 1 \).

3 The Cosmological Evolution in NLBI Scalar Field and Linear Scalar Field Theory

In this section, we will investigate the cosmological behaviors using numerical approach and plot the results in details. For the potential \( \frac{1}{2} m^2 \phi^2 + V_0 (V_0 < 0) \), we will consider the different cos-
mological evolutions with different potential wells (i.e., different negative $V_0$ value) and different potential slope (i.e., different $m$ value). We will also plot the different behaviors between NLBI scalar field and linear scalar field theory. The different evolutions in case of $V_0 > 0$, $V_0 = 0$, $V_0 < 0$ are also studied. In fact, the evolutions of these three cases describe the common feature of three class of potentials: positive potentials, non-negative potentials and negative potentials.

**Case 1. Same slope $m$ but different potential well $V_0$**

![Fig3](image1.png)

Fig3. The three potentials with same slope $m (= 0.1)$ but different potential well $V_0$. $V_0 = -2.0 \times 10^{-4}$ for solid line, $V_0 = -4.5 \times 10^{-4}$ for dash line and $V_0 = -8.0 \times 10^{-4}$ for dot line.

![Fig4](image2.png)

Fig4. The evolution of scale factor $a(t)$ with the three potentials. The value of $V_0$ is the same as Fig.3 and the initial condition is $\phi(0) = 6, \dot{\phi}(0) = 0$

From Fig.4, it shows that the universe can undergo an accelerating expansion, then a decelerating expansion and ultimately contract to singularity. Though the potential well are different, the universe have nearly same evolution at the beginning. It also shows that the deeper the potential well is, the shorter the age of universe is. Since we now live in a expanding universe where the large scale structure had formed, there must exist a upper bound for the value of potential well $|V_0|$. At the time when universe begins to contract from an expansion, the scalar field $\phi$ begins to oscillate and ultimately moves to $-\infty$ (Fig.5). The velocity of field $|\dot{\phi}|$ will reach its maximum value 1 finally.
Fig5. The evolution of scalar field $\phi$ with the three potentials. The initial condition and the value of parameters are the same as Fig4.

Case 2. Same potential well deep $V_0$ but different slope $m$

Fig7. The three potentials with same potential well $V_0 (= -0.0018)$ but different slope $m$. $m = 0.1$ for solid line, $m = 0.2$ for dash line and $m = 0.3$ for dot line.

Fig8. The evolution of scale factor $a(t)$ with the three potentials. The value of parameters are the same as Fig7. The initial condition is $\phi(0) = 8$, $\dot{\phi}(0) = 0$. 

Fig6. The evolution of field velocity $\dot{\phi}$ with the three potentials. The initial condition and the value of parameters are the same as Fig4.
Fig 9. The evolution of scalar field \( \phi \) with the three potential. The initial condition and the value of parameters are the same as Fig. 8.

The result presented in Fig. 8 shows that the steeper the potential slope is, the shorter the universe age is. Therefore there also exist a upper bound for the value of slope \( m \) to accommodate an accelerating expansive universe with the large scale structure formed. At the time when universe begins to contract from an expansion, the scalar field \( \phi \) begins to oscillate and ultimate moves to \( \pm \infty \) (Fig. 9). The velocity of field \( |\dot{\phi}| \) will reach its maximum value 1 finally.

**Case 3. Comparison between Linear Scalar Field and NLBI Scalar Field**

In NLBI scalar field theory, the linear scalar field theory is considered only correctly in weak field regime and will be not valid in strong field regime. It is necessary to investigate their different cosmological evolution in this two theories with the same parameter value and initial conditions. The results are plotted in Figs. 11-14.

For the same parameter value and initial condition, Figs. 11-14 show that this difference is quite noticeable. The different evolution of scale factor (Fig. 12) leads to different expansive rate \( H_i \) at time \( i \) (Fig. 11): the value of Hubble parameter in NLBI scalar field theory is larger than the value in linear scalar field.
Fig 11. The evolution of Hubble parameter $H$ with respect to $t$. Solid line for NLBI scalar field, dot line for linear scalar field. The value of parameters is chosen for $m = 1, V_0 = 0.02$. The initial condition is $\phi(0) = 6, \dot{\phi}(0) = 0$.

Fig 12. The evolution of scale factor $a$ with respect to $t$. Solid line for NLBI scalar field theory, dot line for linear scalar field theory. The value of parameters and the initial condition is the same as Fig.11.

Fig 13. The evolution of scalar field $\phi$ with respect to $t$. Solid line for NLBI scalar field theory, dot line for linear scalar field theory. The value of parameters and the initial condition is the same as Fig.11.

Fig 14. The evolution of scalar field $\dot{\phi}$ with respect to $t$. Solid line for NLBI scalar field theory, dot line for linear scalar field theory. The value of parameters and the initial condition is the same as Fig.11.

**Case 4. Comparison between positive, non-negative and negative potentials**

Now we have known that whether the potential can evolve to negative value is very important to the destiny of universe. A spatially flat universe with a negative potential may eventually collapses, which is not the same as in the general textbook. We plot the different cosmological evolution with the potential parameter $V_0 > 0, V_0 = 0, V_0 < 0$ in NLBI scalar field theory. The
results are plotted in Figs.15-17.

Fig15. The evolution of scale factor $a$ with respect to $t$. Solid line for $V_0 = -0.02$, dash line for $V_0 = 0$, dot line for $V_0 = 0.02$. $m = 1$ and the initial condition is $\phi(0) = 6$, $\dot{\phi}(0) = 0$.

Fig16. The evolution of scalar field $\phi$ with respect to $t$. The value of parameter and the initial condition is the same as Fig15.

Furthermore, we plot the evolution of the density parameter $\Omega$ when $V_0 > 0, V_0 = 0$ and $V_0 < 0$ (Fig.18). The starting point is chosen at the equipartition epoch, at which $\Omega_{Mi} = \Omega_{ri} = 0.5$. We should emphasize that in fact we plot the evolution of three cases when $V_0 = 4.5 \times 10^{-6}$, $V_0 = 0$, $V_0 = -4.5 \times 10^{-6}$ in Fig.18, but we can not find any difference from Fig.18. It shows that up to now there are no difference in the evolution of the density parameter $\Omega$ for small value of $|V_0|$, though the future of the universe is dramatically different for those three cases (see Fig.15).
4 Negative potentials and Cyclic Universe

Till now we have studied the evolution of the universe and classified new possibilities which appear in NLBI scalar field theories with negative potentials. It is very interesting to investigate its cosmological evolution and its differences with linear scalar field. It is true that the universe with negative potentials will end with a crunch and never expand again. They are born in a singularity and end in a singularity.

Recently, P.J. Steinhardt and N. Turok proposed a version of cyclic scenario[80-82]. It is based on the idea that we live on one of two branes whose separation can be parametrized by a scalar field $\phi$. It is assumed that one can describe the brane interaction by an effective 4D theory with the effective potential $V(\phi)$ having a negative minimum. According to brane-view, the potential $V(\phi)$ is the inter-brane potential caused non-perturbative virtual exchange of membranes between the boundaries. The interbrane force is what causes the branes to repeatedly collide and bounce. Consequently, the scale factor bounces and begins to expand. It is assumed that the potential $V(\phi)$ equals an extremely small value($\sim 10^{-120}$) at large $\phi$, and therefore the universe experiences a stage of extremely low-scale inflation associated with present stage of accelerating expansion. In the cyclic universe scenario the perturbations responsible for the formation of the structure of the universe are produced during the contracting regime of precious cycle. After the new cycle creates from singularity, the universe will experience radiation and matter domination, a low-scale inflation(i.e, dark energy domination) and contract again. Obviously there is no inflation in the very early universe. The author claimed that the cyclic model is able to reproduce all of the successful predictions of the consensus model (inflationary+Big Bang cosmology) with the same exquisite detail.
However, later, G.Felder, A.Frolov, L.Kofman and A.Linde\cite{77} investigate the cyclic universe and show that there are some problems that need to be resolved in order to realize a cyclic regime in this scenario. They propose several modifications of this scenario and conclude that the best way to improve it is to add a usual stage of inflation after the singularity and use the inflationary stage to generate perturbations in the standard way. In fact the inflationary mechanism is not the alternative to big bang model, it can be accommodated into big bang model instead. So in order to solve some problems appeared in cyclic model we can also involve the inflationary mechanism, just as G.Felder, A.Frolov, L.Kofman and A.Linde have suggested in Ref\cite{77}. In this cyclic model we will find that all the cosmological detail in the consensus model will appear. Very recently\cite{83}, P.J.Steinhardt and N.Turok show that the cyclic model can naturally incorporate a dynamical mechanism that automatically relaxes the value of the cosmological constant. It can explain why the cosmological constant is small and positive, as observed today.

5 Conclusion and Summary

The main goal of this paper is to perform a general investigation of the NLBI scalar field cosmology with negative potentials. The cosmological solutions in different regime have obtained through some approximate approach. The results obtained in NLBI scalar field theory are quite different with that obtained in linear scalar field theory. A notable characteristic is that NLBI scalar field behaves as ordinary matter nearly the singularity while the linear scalar field behaviors as "stiff" matter. We also find that, due to the nonlinear effect, the oscillatory motion of $\phi$ in the vicinity when the universe evolve to contraction from expansion is different to linear scalar field. Moreover, the value of Hubble parameter $H_i$ at time $i$ in NLBI scalar field theory is large than the one in linear scalar field theory. With the investigation of evolution with different value of $m$ and $V_0$, we find that in order to accommodate an accelerating expansive universe in which the large scale structure had formed, the value of $m$ and $|V_0|$ must have a upper bound. Finally we review the negative potentials and the new cyclic model.

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