Hard scattering cross sections at LHC in the Glauber approach: from \( pp \) to \( pA \) and \( AA \) collisions

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Abstract
The scaling rules of the invariant yields and cross sections for hard scattering processes in proton-nucleus \((pA)\) and nucleus-nucleus \((AB)\) reactions at LHC energies relative to those of nucleon-nucleon \(NN\) (isospin averaged \(pp\)) collisions are reviewed within the Glauber geometrical formalism. The number of binary \(NN\) inelastic collisions for different centrality classes in p+Pb and Pb+Pb collisions at \(\sqrt{s_{NN}}=8.8\) TeV and 5.5 TeV respectively, as obtained from a Glauber Monte Carlo, are also given.

0.1 Proton-nucleus \((pA)\) collisions

0.11 Glauber formalism
The inelastic cross-section of a \(p + A\) reaction, \(\sigma_{pA}\), can be derived in the eikonal limit (straight line trajectories of colliding nucleons) from the corresponding inelastic nucleon-nucleon \(NN\) cross-section, \(\sigma_{NN}(s)\) at the center-of-mass energy \(\sqrt{s}\), and the geometry of the \(pA\) collision simply determined by the impact parameter \(b\) of the reaction. In the Glauber multiple collision model [1], such a cross-section reads

\[
\sigma_{pA} = \int d^2 b \left[ 1 - e^{-\sigma_{NN}(s)T_A(b)} \right],
\]

where \(T_A(b)\) is the nuclear thickness function (or nuclear profile function) of the nucleus \(A\) at impact parameter \(b\):

\[
T_A(b) = \int dz \rho_A(b, z).
\]

\(T_A(b)\) gives the number of nucleons in the nucleus \(A\) per unit area along a direction \(z\) separated from the center of the nucleus by an impact parameter \(b\). The nuclear density, \(\rho_A(b, z)\), is usually parametrized by a Woods-Saxon distribution with nuclear radius \(R_A = 1.19 \cdot A^{1/3} - 1.61 \cdot A^{-1/3}\) fm and surface thickness \(a = 0.54\) fm as given by the experimental data [2] and normalized so that

\[
\int d^2 b T_A(b) = A.
\]

0.12 Hard scattering cross-sections
Though Eq. (1) is a general expression for the total inelastic cross-section, it can be applied to an inclusive \(p + A \rightarrow h + X\) process of production of particle \(h\). When one considers hard scattering processes, the corresponding cross-section \(\sigma_{hard}^{NN}\) is small and one can expand Eq. (1) in orders of \(\sigma_{NN}T_A(b)\) and then, to first approximation

\[
\sigma_{pA}^{hard} \approx \int d^2 b \sigma_{NN}^{hard} T_A(b)
\]

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0.13 “Minimum bias” hard scattering cross-sections

Integrating Eq. (4) over impact parameter, and using (3), one gets the minimum bias (MB) cross-section for a given hard process in \( pA \) collisions relative to the same cross-section in \( pp \) (or \( NN \)) collisions:

\[
(\sigma_{\text{hard}}^{pA})_{\text{MB}} = A \cdot \sigma_{\text{hard}}^{NN}
\]  

(5)

From this expression it is easy to see that the corresponding minimum bias multiplicity (invariant yield per nuclear reaction: \( N_{\text{hard}}^{NN,pA} = \sigma_{\text{hard}}^{NN,pA}/\sigma_{\text{geo}}^{pA} \)) for a given hard-process in a \( pA \) collision compared to that of a \( pp \) collision is

\[
\langle N_{\text{hard}}^{pA} \rangle_{\text{MB}} = A \cdot \sigma_{\text{hard}}^{NN} \cdot \frac{A}{\sigma_{\text{geo}}^{pA}} \cdot \sigma_{\text{hard}}^{NN},
\]  

(6)

where \( \sigma_{\text{geo}}^{pA} \) is the geometrical \( pA \) cross-section given, in its most general form, by Eq. (1). The average nuclear thickness function for minimum bias reactions [making use of Eq. (3)] reads:

\[
\langle T_A \rangle_{\text{MB}} = \frac{\int d^2 b T_A}{\int d^2 b} = \frac{A}{\pi R_A^2} = \frac{A}{\sigma_{\text{geo}}^{pA}}.
\]  

(7)

Thus, for a \( p+Pb \) \((A(Pb) = 208)\) collision at LHC energies \( \sqrt{s_{\text{NN}}} = 8.8 \text{ TeV} \) with

\[
\sigma_{\text{NN}} \approx 77 \text{ mb} \text{ [3]}, \text{ and } \sigma_{\text{geo}}^{pPb} \approx 2162 \text{ mb} \text{ [6]},
\]  

(8)

(9)

one obtains: \( \langle N_{\text{hard}}^{pPb} \rangle_{\text{MB}} \approx 7.4 \cdot N_{\text{hard}}^{NN} \), and the average nuclear thickness function amounts to \( \langle T_{Pb} \rangle_{\text{MB}} = 0.096 \text{ mb}^{-1} = 0.96 \text{ fm}^{-2} \).

0.2 Nucleus-nucleus (AB) collisions

0.21 Glauber formalism

As in the proton-nucleus case, the inclusive inelastic cross-section \( \sigma_{AB} \) for a collision of nuclei \( A \) and \( B \) is given in the multiple-scattering Glauber approximation by:

\[
\sigma_{AB} = \int d^2 b \left[ 1 - e^{-\sigma_{\text{NN}}(s) T_{AB}(b)} \right],
\]  

(10)

where now \( T_{AB}(b) \) is the nuclear overlap function of the nuclei \( A \) and \( B \) separated by impact parameter \( b \). \( T_{AB}(b) \) can be written as a convolution of the corresponding thickness functions of \( A \) and \( B \) over the element of overlapping area \( d^2 \vec{s}(\vec{s} = (x, y)) \) is a 2-D vector in the transverse plane, and \( \vec{b} \) is the impact parameter between the centers of the nuclei):

\[
T_{AB}(b) = \int d^2 \vec{s} T_A(\vec{s}) \cdot T_B(|\vec{b} - \vec{s}|).
\]  

(11)

\( T_{AB}(b) \) is normalized so that integrating over all impact parameters one gets:

\[
\int d^2 b T_{AB}(b) = A B.
\]  

(12)
0.22 Hard scattering cross-sections

As in the \( pA \) case, for hard processes of the type \( A + B \rightarrow h + X \), Eq. (10), can be approximated by:

\[
\sigma_{AB}^{\text{hard}} \approx \int d^2b \, \sigma_{NN}^{\text{hard}} T_{AB}(b).
\]  

(13)

0.23 “Minimum bias” hard scattering cross-sections and yields

Integrating Eq. (13) over impact parameter and using (12), one gets the minimum bias (\( MB \)) cross-section for a given hard process in \( AB \) collisions relative to the corresponding \( pp \) cross-section:

\[
\langle \sigma_{AB}^{\text{hard}} \rangle_{MB} = A \cdot B \cdot \sigma_{NN}^{\text{hard}} 
\]  

(14)

Again the corresponding minimum bias multiplicity (invariant yield per nuclear reaction: \( N_{hard,AB}^{\text{MB}} = \sigma_{NN,AB}^{\text{hard}} / \sigma_{NN,AB}^{\text{geo}} \)) for a given hard-process in a \( AB \) collision compared to that of a \( pp \) collision is

\[
\langle N_{AB}^{\text{hard}} \rangle_{MB} = A \cdot B \cdot \frac{\sigma_{NN}^{\text{hard}}}{\sigma_{AB}^{\text{geo}}} \cdot N_{NN}^{\text{hard}} 
\]  

(15)

where \( \sigma_{AB}^{\text{geo}} \) is the geometrical \( AB \) cross-section given, in its most general form, by Eq. (10). The average nuclear overlap function for minimum bias reactions [making use of Eq. (12)] reads now:

\[
\langle T_{AB} \rangle_{MB} \equiv \frac{\int d^2b \, T_{AB}(b)}{\int d^2b} = \frac{A \cdot B}{\pi (R_A + R_B)^2} = \frac{AB}{\sigma_{AB}^{\text{geo}}},
\]  

(16)

Thus, for a Pb+Pb \( (A^2(Pb) = 43264) \) collision at LHC energies \( \sqrt{s_{NN}} = 5.5 \) TeV with

\[
\sigma_{NN} \approx 72 \text{ mb} \,[3], \text{ and}
\sigma_{PbPb}^{\text{geo}} \approx 7745 \text{ mb} \,[7],
\]

one gets: \( \langle N_{PbPb}^{\text{hard}} \rangle_{MB} \approx 400 \cdot N_{NN}^{\text{hard}}, \) and the average nuclear overlap function amounts to \( \langle T_{PbPb} \rangle_{MB} = 5.58 \text{ mb}^{-1} = 55.8 \text{ fm}^{-2} \).

0.24 Binary collision scaling

For a given impact parameter \( b \) the average hard scattering yield can be obtained by multiplying each nucleon in nucleus \( A \) against the density it sees along the \( z \) direction in nucleus \( B \), then integrated over all of nucleus \( A \), i.e.

\[
\langle N_{AB}^{\text{hard}} \rangle(b) = \sigma_{NN}^{\text{hard}} \int d^2\vec{s} \int \rho_A(\vec{s}, z') \int \rho_B(\vec{b} - \vec{s}, z'') \, d\vec{z}' \, dz'' \equiv \sigma_{NN}^{\text{hard}} \cdot T_{AB}(b) 
\]  

(19)

where we have made use of expressions \( (2) \) and \( (11) \). In the same way, one can obtain a useful expression for the probability of an inelastic \( NN \) collision or, equivalently, for the average number of binary inelastic collisions, \( \langle N_{\text{coll}} \rangle \), in a nucleus-nucleus reaction with impact parameter \( b \):

\[
\langle N_{\text{coll}} \rangle(b) = \sigma_{NN} \cdot T_{AB}(b)
\]  

(20)

From this last expression one can see that the nuclear overlap function, \( T_{AB}(b) = N_{\text{coll}}(b)/\sigma_{NN} \) \([\text{mb}^{-1}]\), can be thought as the luminosity (reaction rate per unit of cross-section) per \( AB \) collision at a
given impact parameter. As an example, the average number of binary collisions in minimum bias Pb+Pb reactions at LHC ($\sigma_{NN} = 72 \text{ mb} = 7.2 \text{ fm}^2$) is:

$$\langle N_{\text{coll}} \rangle_{\text{MB}} = 7.2 \text{ fm}^2 \cdot 55.9 \text{ fm}^{-2} = 400.$$  \hfill (21)

From (19) and (20), we get so-called “binary (or point-like) scaling” formula for the hard scattering yields in heavy-ion reactions:

$$\langle N_{\text{hard}}^{AB} \rangle (b) \approx \langle N_{\text{coll}} \rangle (b) \cdot N_{\text{hard}}^{NN}$$ \hfill (22)

0.25 Hard scattering yields and cross-sections in a given centrality class

Equation (13) gives the reaction cross-section for a given hard process in $AB$ collisions at a given impact parameter $b$ as a function of the corresponding reaction cross-section in $pp$ collisions. Very usually, however, in nucleus-nucleus collisions we are interested in calculating such a reaction cross-section for a given centrality class, $(\sigma_{AB}^{\text{hard}})_{C_1 - C_2}$, where the centrality selection $C_1 - C_2$ corresponds to integrating Eq. (13) between impact parameters $b_1$ and $b_2$. It is useful, in this case, to define two parameters [8, 9]:

- The fraction of the total cross-section for hard processes occurring at impact parameters $b_1 < b < b_2$ ($d^2b = 2\pi bdb$):

$$f_{\text{hard}}(b_1 < b < b_2) = \frac{2\pi}{AB} \int_{b_1}^{b_2} bdb T_{AB}(b).$$ \hfill (23)

- The fraction of the geometric cross-section with impact parameter $b_1 < b < b_2$:

$$f_{\text{geo}}(b_1 < b < b_2) = \left[ \frac{2\pi}{\int_{b_1}^{b_2} bdb \left( 1 - e^{-\sigma_{NN}T_{AB}(b)} \right)} \right] / \sigma_{AB}^{\text{geo}},$$ \hfill (24)

[f_{\text{geo}} simply corresponds to a $0.75 \text{ (e.g. 0.1)}$ factor for the $X\%$ ($10\%$) centrality.]

Hard scattering production is more enhanced for increasingly central reactions (with larger number on $N_{\text{coll}}$) as compared to the total reaction cross-section (which includes “soft”, - scaling with the number of participant nucleons $N_{\text{part}}$ - , as well as “hard” contributions). The growth with $b$ of the geometric cross-section is slower than that of the hard component. For this reason, the behaviour of $f_{\text{hard}}$ and $f_{\text{geo}}$ as a function of $b$, although similar in shape is not the same (see [8]): $f_{\text{hard}} \approx 1$ for $b = 2R_A$, but $f_{\text{geo}} \approx 0.75$ for $b = 2R_A$.

Similarly to (16), we can obtain now the nuclear overlap function for any given centrality class $C_1 - C_2$:

$$\langle T_{AB} \rangle_{C_1 - C_2} = \frac{\int_{b_1}^{b_2} d^2b T_{AB}(b)}{\int_{b_1}^{b_2} d^2b} = \frac{A \cdot B \cdot f_{\text{hard}}}{\sigma_{AB}^{\text{geo}} f_{\text{geo}}}$$ \hfill (25)

The number of hard processes per nuclear collision for reactions with impact parameter $b_1 < b < b_2$ is given by

$$\langle N_{\text{hard}}^{AB} \rangle_{C_1 - C_2} = \frac{\sigma_{AB}^{\text{hard}}(b_1 < b < b_2)}{\sigma_{AB}^{\text{geo}}(b_1 < b < b_2)} = \frac{A \cdot B \cdot \sigma_{NN}^{\text{hard}}}{\sigma_{AB}^{\text{geo}} f_{\text{geo}}},$$ \hfill (26)

which we could have just obtained directly from (19) and (25). From (15) and (26) it is easy to see that:

$$\langle N_{\text{hard}}^{AB} \rangle_{C_1 - C_2} = \langle N_{\text{hard}}^{AB} \rangle_{\text{MB}} \cdot \frac{f_{\text{hard}}}{f_{\text{geo}}}$$ \hfill (27)
Finally, the cross-section for hard processes produced in the centrality class $C_1 - C_2$ (corresponding to a fraction $f_{geo}$ of the reaction cross-section, $(\sigma_{AB}^{geo})_{C_1 - C_2}$) is:

$$(\sigma_{AB}^{hard})_{C_1 - C_2} = A \cdot B \cdot f_{hard} \cdot \sigma_{NN}^{hard}$$  \hspace{1cm} (28)$$

Figure 1, extracted from [8], plots the (top) fraction of the hard cross-section, $f_{hard}(0 < b < b_2)$ (labeled in the plot as $f$), as a function of the top fraction of the total geometrical cross-section, $f_{geo}(0 < b < b_1)$, for several nucleus-nucleus reactions.

Fig. 1: Figure 4 of ref. [8]. Fraction of the hard cross-section, $f \equiv f_{hard}(0 < b < b_2)$, vs. the fraction of the total geometrical cross-section, $f_{geo}(0 < b < b_1)$, for several heavy-ion collisions (from left to right): 197$^+$197, 110$^+$197, 63$^+$193, 27$^+$197, and 16$^+$197.

As a practical application of Eq. (28) and the results of Fig. 1, the hard-scattering cross-sections in Pb+Pb for the top 0-10% ($f_{hard} = 0.41$ for $f_{geo} = 0.1$ from the practically equivalent Au+Au system of figure 1) and 0-20% ($f_{hard} = 0.664$ for $f_{geo} = 0.2$) central collisions relate to the $pp$ cross-section, in the absence of nuclear effects, respectively as:

$$(\sigma_{AB}^{hard})_{0-10\%} = (208)^2 \cdot 0.41 \cdot \sigma_{NN}^{hard} \approx 1.7 \cdot 10^4 \cdot \sigma_{NN}^{hard}$$  \hspace{1cm} (29)$$

$$(\sigma_{AB}^{hard})_{0-20\%} = (208)^2 \cdot 0.664 \cdot \sigma_{NN}^{hard} \approx 2.9 \cdot 10^4 \cdot \sigma_{NN}^{hard}$$  \hspace{1cm} (30)$$

A straightforward way to compute the invariant yield for a given hard process in a given centrality class of a nucleus-nucleus collision from the corresponding yield in $pp$ collisions consists in determining, via a Glauber MC calculation, the average number of inelastic $NN$ collisions corresponding to that centrality class via

$$\langle N_{coll} \rangle_{C_1 - C_2} = \langle T_{AB} \rangle_{C_1 - C_2} \cdot \sigma_{NN} ,$$  \hspace{1cm} (31)$$

and then use this value in the “binary-scaling” formula.
\[ \langle N_{AB}^{\text{hard}} \rangle_{C_1-C_2} = \langle N_{\text{coll}} \rangle_{C_1-C_2} \cdot \langle \sigma_{AB}^{\text{hard}} \rangle_{C_1-C_2}, \quad \text{or} \]
\[ \langle \sigma_{AB}^{\text{hard}} \rangle_{C_1-C_2} = \langle T_{AB} \rangle_{C_1-C_2} \cdot (\langle \sigma_{AB}^{\text{geo}} \rangle_{C_1-C_2} \cdot \sigma_{NN}^{\text{hard}}) \]  

or (32)

\[ \sigma_{\text{hard}}^{\text{AB}}_{C_1-C_2} = \langle T_{AB} \rangle_{C_1-C_2} \cdot (\langle \sigma_{AB}^{\text{geo}} \rangle_{C_1-C_2} \cdot \sigma_{NN}^{\text{hard}}) \]

The same two formulae above apply to \( pA \) collisions (of course substituting \( AB \) by \( pA \) and computing \( N_{\text{coll}} \) from \( T_A \) instead of from \( T_{AB} \)).

Finally, to obtain the experimental rates, \( \langle N_{AB}^{\text{hard}} \rangle_{C_1-C_2} \), actually measured in a given centrality bin one needs to take into account the expected integrated luminosity \( L_{\text{int}} \) [mb\(^{-1}\)] as follows:

\[ \langle N_{AB}^{\text{hard}} \rangle_{C_1-C_2} = L_{\text{int}} \cdot (\sigma_{AB}^{\text{hard}})_{C_1-C_2} \]  

or (33)

0.3 Hard scattering yields and cross-sections for \( pPb \) and \( PbPb \) collisions at LHC

As a practical application of the Glauber approach described here, in Table I the values of \( \langle N_{\text{coll}} \rangle \) and \( \langle T_{pPb} \rangle \) are quoted for different centrality classes obtained from a Monte Carlo calculation \[10\] for \( pPb \) (\( \sqrt{s_{NN}} = 8.8 \text{ TeV} \) and \( \sigma_{NN} = 77 \text{ mb} \)) and \( PbPb \) (\( \sqrt{s_{NN}} = 5.5 \text{ TeV} \), \( \sigma_{NN} = 72 \text{ mb} \)) collisions (Woods-Saxon Pb density parametrization with \( R_A = 6.78 \text{ fm} \) and \( a = 0.54 \text{ fm} \)).

Table 1: Number of inelastic \( NN \) collisions, \( \langle N_{\text{coll}} \rangle \), and nuclear thickness \( \langle T_{pPb} \rangle \) or overlap \( \langle T_{PbPb} \rangle \) function per centrality class, in \( pPb \) (\( \sqrt{s_{NN}} = 8.8 \text{ TeV} \), \( \sigma_{NN} = 77 \text{ mb} \)) and \( PbPb \) collisions at LHC (\( \sqrt{s_{NN}} = 5.5 \text{ TeV} \), \( \sigma_{NN} = 72 \text{ mb} \)) obtained with the Glauber Monte Carlo code of ref. \[10\]. The errors in \( \langle N_{\text{coll}} \rangle \), not shown, are of the same order as the current uncertainty in the value of the nucleon-nucleon inelastic cross section, \( \sigma_{NN} \), at LHC energies (\~{}10%).

| Centrality \((C_1-C_2)\) | \( \langle N_{\text{coll}} \rangle \) | \( \langle T_{pPb} \rangle \) [mb\(^{-1}\)] | \( \langle N_{\text{coll}} \rangle \) | \( \langle T_{PbPb} \rangle \) [mb\(^{-1}\)] |
|-------------------------|-----------------|-----------------|-----------------|-----------------|
| 0-5%                    | 15.7            | 0.203           | 1876.0          | 26.0            |
| 0-10%                   | 15.3            | 0.198           | 1670.2          | 23.2            |
| 10-20%                  | 13.8            | 0.179           | 1019.5          | 14.2            |
| 20-30%                  | 12.0            | 0.155           | 612.4           | 8.50            |
| 30-40%                  | 9.9             | 0.128           | 351.8           | 4.89            |
| 40-50%                  | 7.8             | 0.101           | 188.0           | 2.61            |
| 50-60%                  | 5.6             | 7.27\cdot10^{-2}| 92.9            | 1.29            |
| 60-70%                  | 3.8             | 4.93\cdot10^{-2}| 41.4            | 5.75\cdot10^{-1}|
| 70-80%                  | 2.6             | 3.37\cdot10^{-2}| 16.8            | 2.33\cdot10^{-1}|
| 80-90%                  | 1.7             | 2.20\cdot10^{-2}| 6.7             | 9.31\cdot10^{-2}|
| 90-100%                 | 1.2             | 1.55\cdot10^{-2}| 2.7             | 3.75\cdot10^{-2}|
| min. bias               | 7.4             | 9.61\cdot10^{-2}| 400.0           | 5.58            |

Using \[32\], \[33\] and Table I, we can now easily get the scaling factors of the cross-sections and yields from \( pp \) to, e.g., central (0-10%), minimum bias, and semi-peripheral (60-80%), from the combined average 60-70% and 70-80%) \( pPb \) (8.8 TeV) and \( PbPb \) (5.5 TeV) collisions:
For p+Pb collisions ($\sigma_{pPb}^{geo} = 2162$ mb):

\[
\langle N_{pPb}^{hard} \rangle_{0-10\%} = 15.3 \cdot N_{NN}^{hard} , \quad \langle \sigma_{pPb}^{hard} \rangle_{0-10\%} = 0.198 \cdot 0.1 \cdot 2162 \cdot \sigma_{NN}^{hard} \approx 450 \cdot \sigma_{NN}^{hard} \tag{35}
\]

\[
\langle N_{pPb}^{hard} \rangle_{60-80\%} = 3.2 \cdot N_{NN}^{hard} , \quad \langle \sigma_{pPb}^{hard} \rangle_{60-80\%} = 0.042 \cdot 0.2 \cdot 2162 \cdot \sigma_{NN}^{hard} \approx 18 \cdot \sigma_{NN}^{hard} \tag{36}
\]

\[
\langle N_{pPb}^{hard} \rangle_{MB} = 7.4 \cdot N_{NN}^{hard} , \quad \langle \sigma_{pPb}^{hard} \rangle_{MB} = 0.096 \cdot 2162 \cdot \sigma_{NN}^{hard} \approx 2 \cdot 10^2 \cdot \sigma_{NN}^{hard} \tag{37}
\]

For Pb+Pb collisions ($\sigma_{PbPb}^{geo} = 7745$ mb):

\[
\langle N_{PbPb}^{hard} \rangle_{0-10\%} = 1670 \cdot N_{NN}^{hard} , \quad \langle \sigma_{PbPb}^{hard} \rangle_{0-10\%} = 23.2 \cdot 0.1 \cdot 7745 \cdot \sigma_{NN}^{hard} \approx 1.6 \cdot 10^4 \cdot \sigma_{NN}^{hard} \tag{38}
\]

\[
\langle N_{PbPb}^{hard} \rangle_{60-80\%} = 29.1 \cdot N_{NN}^{hard} , \quad \langle \sigma_{PbPb}^{hard} \rangle_{60-80\%} = 0.4 \cdot 0.2 \cdot 7745 \cdot \sigma_{NN}^{hard} \approx 6.2 \cdot 10^2 \cdot \sigma_{NN}^{hard} \tag{39}
\]

\[
\langle N_{PbPb}^{hard} \rangle_{MB} = 400 \cdot N_{NN}^{hard} , \quad \langle \sigma_{PbPb}^{hard} \rangle_{MB} = 5.58 \cdot 7745 \cdot \sigma_{NN}^{hard} \approx 4.3 \cdot 10^4 \cdot \sigma_{NN}^{hard} \tag{40}
\]

### 0.4 Nuclear effects in pA and AB collisions

Eqs. (41) and (43) for the hard scattering cross-sections in pA and AB collisions have been derived within an eikonal framework which only takes into account the geometric aspects of the reactions. Any differences of the experimentally measured $\sigma_{pA, AB}^{hard}$ with respect to these expressions indicate “de facto” the existence of “nuclear effects” (such as e.g. “shadowing”, “Cronin enhancement”, or “parton energy loss”) not accounted for by the Glauber formalism. Indeed, in the multiple-scattering Glauber model each nucleon-nucleon collision is treated incoherently and thus, unaffected by any other scattering taking place before (initial-state) or after (final-state effects) it.

If the Glauber approximation holds, from (41) and (43) one would expect a $\propto A^1$, and $\propto A^2$ growth of the hard processes cross-section with system size respectively. Equivalently, since $N_{NN,AB}^{hard} = \sigma_{NN,AB}^{hard} / \sigma_{NN,AA}^{geo}$ and $\sigma_{NN}^{geo} \sim R_A^2$ with $R_A \sim A^{1/3}$, one would expect a growth of the number of hard process as $\propto A^{1/3}$, $\propto A^{4/3}$ for pA, AA collisions respectively. Experimentally, in minimum bias pA and AB collisions, it has been found that the production cross-sections for hard processes actually grow as:

\[
(\sigma_{pA}^{hard})_{MB} = A^\alpha \cdot \sigma_{NN}^{hard} , \quad \text{and} \quad (\sigma_{AB}^{hard})_{MB} = (AB)^\alpha \cdot \sigma_{NN}^{hard} , \quad \text{with } \alpha \neq 1 \tag{41}
\]

More precisely, in high-pT processes in pA and heavy-ion collisions at SPS energies one founds $\alpha > 1$ (due to initial-state pT broadening or “Cronin enhancement”); whereas $\alpha < 1$ at RHIC energies (“high-pT suppression”). Theoretically, one can still make predictions on the hard probe yields in pA, AB collisions using the pQCD factorization machinery for the pp cross-section complemented with the Glauber formalism while modifying effectively the nuclear PDFs and parton fragmentation functions to take into account any initial- and/or final- state nuclear medium effect.

### 0.5 Summary of useful formulae

Finally, let us summarize a few useful formulae derived here to determine the hard-scattering invariant yields, cross-sections, or experimental rates, from pp to pA and AB collisions for centrality bin $C_1 - C_2$ (corresponding to a nuclear thickness $T_A$ or nuclear overlap function $T_{AB}$ and to an average number of NN inelastic collisions $\langle N_{coll} \rangle$):

\[
\frac{d^2 N_{pA,AB}^{hard}}{dpT dy} \big|_{C_1-C_2} = \langle T_{A,AB} \rangle_{C_1-C_2} \cdot \frac{d^2 \sigma_{pp}^{hard}}{dpT dy} \tag{42}
\]
\[
\frac{(d^2\sigma_{h\text{ard}}^{pA,AB})_{C_1-C_2}}{dp_T dy} = \langle T_{A,AB} \rangle_{C_1-C_2} \cdot \sigma_{\text{geo}}^{pA,AB}_{C_1-C_2} \cdot \frac{d^2\sigma_{pp}^{\text{hard}}}{dp_T dy} \tag{43}
\]
\[
\frac{(d^2N_{h\text{ard}}^{pA,AB})_{C_1-C_2}}{dp_T dy} = \mathcal{L}_{\text{int}} \cdot \langle T_{A,AB} \rangle_{C_1-C_2} \cdot \sigma_{\text{geo}}^{pA,AB}_{C_1-C_2} \cdot \frac{d^2N_{pp}^{\text{hard}}}{dp_T dy} \tag{44}
\]
\[
\frac{(d^2\sigma_{h\text{ard}}^{pA,AB})_{C_1-C_2}}{dp_T dy} = \langle N_{\text{coll}} \rangle_{C_1-C_2} \cdot \frac{d^2N_{pp}^{\text{hard}}}{dp_T dy} \tag{45}
\]
\[
\frac{(d^2\sigma_{h\text{ard}}^{pA,AB})_{C_1-C_2}}{dp_T dy} = \langle N_{\text{coll}} \rangle_{C_1-C_2} \cdot \sigma_{\text{geo}}^{pA,AB}_{C_1-C_2} \cdot \frac{d^2N_{pp}^{\text{hard}}}{dp_T dy} \tag{46}
\]
\[
\frac{(d^2\sigma_{h\text{ard}}^{pA,AB})_{C_1-C_2}}{dp_T dy} = \mathcal{L}_{\text{int}} \cdot \langle N_{\text{coll}} \rangle_{C_1-C_2} \cdot \sigma_{\text{geo}}^{pA,AB}_{C_1-C_2} \cdot \frac{d^2N_{pp}^{\text{hard}}}{dp_T dy} \tag{47}
\]

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