The effect of primary loading on fatigue life of cylindrical roller bearings

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Abstract. Experimental and theoretical works shown that if a roller-raceway rolling contact is loaded in the elastic-plastic domain then, after the first rolling cycles, the material elastically shakedowns to a slightly modified axial profile and stable compressive residual stresses. Fatigue life tests carried out on four groups of NJ206 cylindrical roller bearings pointed out the superiority of the bearings groups that primary supported a few cycles of rolling loading in the elastic-plastic domain. An elastic-perfect plastic analysis was performed to reveal the role played by the roller’s crowning geometry, operating clearance, misalignment angle and operating loads on contact pressures distributions achieved between each roller and the corresponding inner and outer raceways. The modified reference rating lives, evaluated using the lamina technique, exposed a good agreement with the values provided by fatigue life tests.

1. Introduction

In any rolling bearing the working load is transmitted through concentrated contacts characterized by small areas and very high pressures. This loading has a cyclic character due to the inherent rolling. The cross-section of roller profile controls the pressure distribution in the contact area and radically affects the bearing’s basic dynamic load rating and rating lives, Ioannides et al. [1], Harris and Kotzalas [2], ISO 16281 [3]. Reviewing papers by Hoeprich [5], Reusner [6], de Mull et al. [7], Fujiwara et al. [8] revealed that the contact pressures have a tendency to be significantly higher around the profile’s discontinuities than those attained in the middle of the contact area. The edge loading effect is considerably amplified for one end and diminished at the other when the bearing operates with misalignment between the inner and outer raceways. Consequently, the primary target was to reduce the stresses in the roller-raceway contact by modifying the roller profile. The diversity of crowning profiles includes: the single straight line with chamfer ends, single circular arc or a combination of multiple circular arcs, cylindrical-crowned (ZB), and the logarithmic profile. When logarithmic profile [3-4] is used, the distribution of contact pressures might result axially uniform, but even this theoretical profile has an infinite drop at the end of the effective contact length and the edge effect manifests when the operating load or misalignment are large.

Around profile’s discontinuities the contact between roller’s generatrix and rings’ raceways is no longer a simple Hertz line contact but a more complicated three dimensional type contact. In fact, these local increases in the pressure distribution are able to overwhelm material’s yield limit and to cause plastic deformations, residual stresses and steel hardening. After few cycles the material elastically shakedowns to a slightly modified roller axial profile and a stable state of compressive residual stresses, [9 – 14]. Kapoor and Johnson [15], and Williams [16] revealed that the plastic...
deformations achieved in the first rolling cycles alter the initial contact geometry diminishing the contact pressure.

The non-linear dependency between the basic dynamic load rating $C_r$ of cylindrical roller bearings and internal design parameters is provided by the International Standard ISO 28 [17] and ISO 1281, [18] as follows:

$$C_r = b_m f_c (i L_{we} \cos(\alpha))^{7/9} Z^{3/4} D_{we}^{29/27}. \quad (1)$$

The equation for $f_c$ is derived in ISO 1281 [18] and incorporates the reduction factor $\lambda \nu$ for the edge effects:

$$f_c = 0.483 B_1 0.377 \lambda \nu \left( \frac{2}{(1+\gamma)} \right)^{29/27} \left( 1 + \left[ 1.04 \left( \frac{1+\gamma}{1+\nu} \right)^{143/108} \right]^{9/29} \right)^{-2/9} \quad (2)$$

where $\gamma = D_{we} \cos(\alpha) / D_{pw}$.

Values of $f_c$, as given in ISO 281, are calculated by substituting the reduction factor $\lambda \nu = 0.83$ into equation (2). Two essential notifications are stipulated in connection with the values for $f_c$:

i. they are the maximum values applicable only to roller bearings in which the contact stress is substantially uniform along the most heavily roller / raceway contact, and

ii. smaller values of $f_c$ than those given in ISO 281 should be used if an accentuated stress concentration is present in some part of the roller/raceway contact.

Based on a fully elastic approach, the International Standard ISO-16281 [3] introduced advanced calculation methods which enable the user to take into account the influence of bearing operating clearance and misalignment under general loading conditions. The ISO-16281 approach points out that the end increases in pressure distribution cause a severe diminishing of both the reference dynamic load rating and modified reference rating life.

2. Cylindrical roller bearings primary loaded in elastic-plastic domain

2.1. Rolling bearing and equipment for primary loading

The NJ206 cylindrical roller bearing was selected for this study. Bearing’s internal data were as follows: inner raceway diameter $F=38.5 \, \text{mm}$, roller diameter: $D_w=7.5 \, \text{mm}$, total roller length: $L_w=7.5 \, \text{mm}$, roller’s end chamfer: $R_{ch}=(0.3 - 0.5) \, \text{mm}$, number of cylindrical roller: $Z_w=12$, $C_2$ internal clearance: $S=(0 - 25) \, \mu\text{m}$, P0 precision class. The bearings rings and cylindrical rollers were manufactured from 100Cr6 through hardened rolling bearing steel with the traction ultimate strength $R_m = 2200 \, \text{MPa}$, and the traction yield limit $R_{P,0.2} = 1700 \, \text{MPa}$.

The equipment used to load the NJ206 bearing in the elastic-plastic domain assured small rotation speeds of both bearing rings during the gradual increasing of radial load to the required level. During the primary loading operation the NJ206 bearing was symmetrical positioned to avoid shaft misalignment and unequal deformations.

2.2. Experimental procedure and results

The target was to establish the new cylindrical roller profiles after bearing’s running a number of 200 inner ring rotations at high radial loads.

The radial loads were chosen to create, in the contact achieved between the most loaded roller and inner ring raceway, the following values of the Hertzian pressure: $p_{H0} \in \{3.5 \, \text{GPa}, 4 \, \text{GPa}, 4.5 \, \text{GPa}, 5 \, \text{GPa}\}$.

For each load level, the primary loading procedure were performed for two bearings. The results of measurements are summarized in figure 1.
2.3. Profile functions for cylindrical rollers subjected to elastic-plastic primary loadings.

Based on the experimental data presented in figure 1, particular profile functions $P(x)$ were obtained for each level of the primary loading force. The interpolations of the experimental data were accomplished using the model of logarithmic function revealed in standard ISO 16281 [3]:

$$P_k(x) = A_k \cdot D_{we} \cdot ln \left[ \frac{1}{1 - \left( \frac{2 \cdot x}{L_{we}} \right)^{0.6}} \right]$$

(3)

3. Fatigue life tests

The purpose of the fatigue life tests, carried out on NJ206 cylindrical roller bearings, was to verify experimentally the positive influence of primary loading on rating lives. Four groups of 24 bearings each group were durability tested in the same conditions, two groups without any primary loading and two groups with modified profiles of the rolling surfaces as result of a primary loading in the elastic-plastic domain.

3.1. Testing conditions

3.1.1. Testing machines. The fatigue life tests were performed on a battery of 12 testing machines. A special care has been taken to ensure the same testing conditions for all bearings to be tested. For this reason only two bearings of the group to be tested run simultaneously inside the testing machine. These bearings are positioned at the ends of the machine’s shaft while in the middle are two larger spherical roller bearings used to sustain the radial load created by a hydrostatic system. In this way the two cylindrical bearings support the same values for the radial load, misalignment angle, rotation speed, temperature of lubrication oil, temperatures of inner rings, outer rings and rollers, respectively.

3.1.2. Cylindrical roller bearings. The design parameter of NJ206 cylindrical roller bearings used in the durability tests were as presented in 2.1. All bearings were made of the same steel charge of 100Cr6 and using the same production series. The final heat treatment consisted of a hardening process (austenitization at 840 °C for 20 minutes, salt-bath quenching at 180 °C and tempering at 170 °C for 2 hours), the final hardness is 61-63 HRC.

Two groups of bearings had cylindrical rollers with straight line profile (SLP) terminated with chamfers, and the other two groups of bearings had the cylindrical rollers and raceways with modified profiles (PMP) as result of the primary loading operation.
The primary loading was performed in a number of 200 inner ring rotations with a radial force to attain, in the contact between the most loaded roller and inner raceway, a hertzian pressure $P_{H0} = 4.5$ GPa. The profile of cylindrical roller after the primary loading operation is presented in figure 2.

![Figure 2. Profile of cylindrical roller after the primary loading.](image)

Before the primary loading, the roughness of the active surfaces was: for inner and outer raceways $R_{a,r} = 0.14$ µm, and for cylindrical surface of roller $R_{a,w} = 0.1$ µm. After the primary loading operation the roughness diminishes to $R_{a,r} = 0.1$ µm for inner and outer raceways, and to $R_{a,w} = 0.08$ µm for cylindrical surface of roller.

3.1.3. Operating conditions. The fatigue life tests were carried out in the following specifications:
- radial load on a bearing $F_r = 8500$ N,
- misalignment due to shaft’s elastic bending $\psi = 1.6$ minutes,
- rotation speed of the inner ring $n_i=2880$ rev./min.,
- oil bath lubrication ($e_C = 0.6$),
- lubrication oil- ISO VG 46 ($\dot{v}_{40} = 46 \frac{mm^2}{s}$, $\dot{v}_{80} = 12 \frac{mm^2}{s}$),
- operating temperatures: inner ring $t_i = 65^\circ C$, outer ring $t_e = 45^\circ C$, ambient $t_a = 20^\circ C$,
- shaft tolerance – k6, housing tolerance - H7.

3.1.4. Testing method. In order to shorten the testing time, the accelerated durability test method [19] was used. To maintain a high confidence level, the fatigue life test of each group was performed until minimum 8 bearings of the group failed by rolling contact fatigue.

3.2. Results provided by the fatigue life testing program
3.2.1. Types of failures. All failures were caused by rolling contact fatigue, manifested practically complete as pitting on inner raceways, figure 3. For bearings with straight line profiles the failure manifested almost sole at the end of the contact area, figure 3a, that reveals the presence of a strong edge effect. The statistic of failures is revealed in table 1.
Table 1. Statistics of failures by rolling contact fatigue.

| Bearing group | Number of tested bearings | Number of failed bearings | Bearing's element failed |
|---------------|---------------------------|----------------------------|-------------------------|
|               |                           |                            | Inner raceway | Middle | Outer raceway | Roller |
| SLP 1         | 22                        | 11                         | 0            | 0      | 0            |
| SLP 2         | 26                        | 14                         | 1            | 0      | 0            |
| SLP 1+2       | 48                        | 25                         | 1            | 0      | 0            |
| PDP 1         | 24                        | 9                          | 7            | 0      | 0            |
| PDP 2         | 20                        | 8                          | 6            | 0      | 1            |
| PDP 1+2       | 44                        | 17                         | 13           | 0      | 1            |

Figure 3. Failures by rolling contact fatigue (pitting) on inner raceway
(a)- at the edge of raceway, (b)-at the middle of raceway.

For bearings which had sustained the primary loading, the large majority of failure locations appeared in the central zone of the inner raceways, figure 3b, that indicates a significant diminishing of the stress edge effect.

3.2.2. The distribution function and parameters estimation. The Weibull distribution function has been hypothesized as describing the distribution of the fatigue lives [20]. The estimation of parameters for the particular Weibull distribution function of each group was accomplished using two methods:

i. the graphical method in which the values $P(i)$ of the distribution function are plotted on Weibull probability papers (WPP), Johnson [21];

ii. the maximum likelihood method (MLM), [22, 23].

The summary of fatigue lives is presented in table 2.
Table 2. Results of fatigue life tests.

| Bearing Groupa | Hardness (roller/ring) (HRC) | Primary loading pressure (MPa) | Method of estimationb | Shape factor | L₁₀ rating (hours) | lives 90% confidence limits (hours) | Lₘ median lives (hours) |
|----------------|-------------------------------|-------------------------------|------------------------|--------------|-------------------|-------------------------------------|------------------------|
| SLP 1          | 61 / 61                       | WPP                           | 1.96                   | 18           | 11....30          | 50.3                                |                         |
|                |                               | ML                             | 1.76                   | 18.3         |                   | 58.8                                |                         |
| SLP 2          | 63 / 61                       | WPP                           | 1.73                   | 20.3         | 11....38          | 66.5                                |                         |
|                |                               | ML                             | 2.47                   | 23.5         |                   | 51.9                                |                         |
| PDP 1          | 61 / 61                       | 4500 WPP                      | 3.24                   | 112          | 85....168         | 202                                 |                         |
|                |                               | ML                             | 3.84                   | 118.5        |                   | 193                                 |                         |
| PDP 2          | 63 / 61                       | 4500 WPP                      | 1.49                   | 102          | 50....209         | 416                                 |                         |
|                |                               | ML                             | 0.92                   | 59.6         |                   | 762                                 |                         |

a SLP, straight line profiles; PDP, primary deformed profiles.
bWeibull probability paper; ML, maximum likelihood.

4. Rating lives

4.1. Basic rating life

The basic rating life represents the duration before which 10 percent of the bearings fail whereas 90 percent will exceed this duration. The basic rating life is obtained using the basic equation:

\[ L_{10}^b = \left( \frac{C_r}{P} \right)^p \]  (4)

where \( C_r \) is the basic dynamic load rated \( C_r \), \( P \) is the dynamic equivalent radial load, and the exponent has the value \( p=10/3 \) for line contact. For the case of bearings with unmodified line profiles, the value of the basic dynamic load rating \( C_r \) was diminished by the correction factor \( \lambda \nu=0.83 \), [17, 18].

For operating conditions used in fatigue life tests, the values for basic rating lives are:

- For bearings with no primary loadings, \( L_{10}^b = 9.88 \cdot 10^6 \text{ rev.} = 57.2 \text{ hours} \),
- For bearings which sustained the primary loading operation, \( L_{10}^b = 18.5 \cdot 10^6 \text{ rev.} = 107.1 \text{ hours} \).

4.2. Modified rating life

To evaluate the modified rating life of bearings, the International Standard ISO 281 uses the equation:

\[ L_{10m} = a_1 a_{ISO} \cdot L_{10} = a_1 \cdot a_{ISO} \cdot \left( \frac{C_r}{P} \right)^p \]  (5)

where for a reliability of 90% the reliability factor corresponds to unity, \( a_1 = 1 \).

The life modification factor \( a_{ISO} \) can be derived from equation:

\[ a_{ISO} = f \left( \frac{C_u C_w}{P}, \kappa \right) \]  (6)

where \( C_u \) is the fatigue load limit whereas the parameters \( e_c \) and \( \kappa \) take into consideration contamination and lubrication condition. Values of the fatigue load limit \( C_u \) can be determined by means of equations provided by ISO 281 as function of the bearing type, size, internal geometry, profile of rolling elements and raceways, manufacturing quality, material of rolling bearing. Operating conditions as: lubrication (type of lubricant, viscosity, bearing speed, bearing size, additives), environment (contamination level, seals), contaminant particles (hardness, and particle size, lubrication method, filtration), mounting, are considered by standard ISO 281 in selecting values for parameters \( e_c \) and \( \kappa \).

For mineral oil lubrication and bearing raceway surfaces machined with “good manufacturing quality”, the condition of lubricant separation is described by the viscosity ratio \( \kappa \), defined as the ratio of the actual kinematic viscosity \( \nu \) to the reference kinematic viscosity \( \nu_1 \):
\[ \kappa = \frac{V}{V_1} \]  

(7)

For a more detailed estimation of the \( \kappa \) value, e.g. for especially machined surfaces, \( \lambda \)-ratio is first evaluated as the ratio between the minimum thickness of the EHD oil film, \( h_{EHD\min} \) (formed between the most loaded roller and inner raceway) and the composed roughness of the involved surfaces:

\[ \lambda = \frac{h_{EHD\min}}{1.25 \cdot (Ra_{\text{raceway}}^2 + Ra_{\text{roller}}^2)^{1/2}} \]  

(8)

Further, the \( \kappa \) value is estimated with the equation:

\[ \kappa = \lambda^{1.3} \]  

(9)

For the operating conditions used in fatigue life tests, the values for modified rating lives are: 
\( L_{10m} = 6.93 \cdot 10^6 \) rev. = 40.1 hours, for the case of bearings with no primary loadings, and 
\( L_{10m} = 15.6 \cdot 10^6 \) rev. = 90.3 hours, for bearings which sustained the primary loading operation.

4.3. Modified reference rating life

Additional to the influencing parameters described by ISO 281 [17], the standard ISO 16281 [3] takes into account three more operating conditions: the misalignment, operating internal clearance and internal load distribution on rolling elements.

Experimental investigations [3, 35, 36] as well as theoretical and computer simulations [16, 37-41] shown that the presence of compressive residual stresses can have a positive effect on fatigue lives of rolling bearings. The mentioned standards do not consider the effect of residual stresses in the evaluations of rating lives.

4.4. Requirement for semi-analytical (SAM) methods

The slicing technique recommended in ISO 16281 [3] make possible to consider the profile modifications in the evaluation of the modified reference rating life of a roller bearing. However, the concentrated contact achieved between two finite length cylinders is no longer a hertzian contact. For these non-hertzian contacts there is not a general analytical solving method.

To find the pressures distribution on the common contact area, and further the stresses states developed in the stressed materials, numerical methods and corresponding computing codes have to be used. In cases where concentrated contacts are involved the Finite Element Method (FEM) proved to be computing time consuming when accurate results were needed. This drawback causes the FEM approach unaffordable when a large number of case studies are required.

Essentially, to express a semi-analytical method (SAM) the particular analytic equations developed in the theory of concentrated contacts [24-26] are converted into numerical formulations [27–34]. Using a today personal computer, a non-hertzian concentrated contact is solved thousands times faster by a SAM comparing with FEM.

Details on the SAM used are given in appendix A.

4.5. Computation algorithms

For computations presented in the paper, the uniformly spaced rectangular arrays were built with \( N_x = 256 \), \( N_y = 64 \). A single-loop conjugate gradient method (CGM) was used to solve the mentioned algebraic system of equations [29, 32 – 34]. To increase the efficiency of the numerical algorithm, a dedicated discrete fast Fourier transform routine (DC-FFT) for 3D contact problems was developed to solve the convolution products implicated in the calculation of the contact pressure, internal stress field and surface displacements [26, 31].
5. Computation of modified reference rating lives

The lamina technique, as presented in standard ISO-16281 [3], was used to evaluate the modified reference rating life. In this procedure the elastic pressures distributions developed in the contact areas of each loaded roller with both, the inner raceway \((i)\) and outer raceway \((e)\), have to be considered.

5.1. Internal load distribution

The method used to determine the internal load distribution took into account operating radial clearance and rollers’ tilt. The operating clearance considered both the different operating temperatures of inner and outer raceways and interference fits. For bearings which sustained a primary loading, the elastic-shak edown provided permanent deformations of inner raceways roughly one third of the value experienced by cylindrical rollers, while the permanent deformations of the outer raceways attained irrelevant values and were neglected.

The internal distributions of the applied radial load are presented in figure 4. The small dissimilarities between the two distributions are caused by changes of contact rigidities for rolling elements that sustained the primary loading operation.

5.2. Pressures distributions

The 3D and 2D pressures distributions developed by the internal loads \(Q_2\), \(Q_1\) and \(Q_{12}\) on contact areas between the corresponding rollers and inner raceway are exemplified in figure 5, figure 6 and figure 7 respectively. In each figure, the pressures distributions are comparatively illustrated for bearings with straight line profiles and bearings with profiles modified by the primary loading.

5.3. Modified reference rating lives

The operating conditions used in fatigue life tests were considered as the input data in computations for modified reference rating lives. The following values were obtained:

\[
L_{10\text{mr}} = 3.28 \cdot 10^6 \text{ rev.} = 19 \text{ hours}, \text{ for the case of bearings with no primary loadings, and}
\]

\[
L_{10\text{mr}} = 23.32 \cdot 10^6 \text{ rev.} = 135 \text{ hours}, \text{ for bearings which sustained the primary loading operation.}
\]

For bearings with straight line profiles as well as for bearings with modified profiles by primary loading the corresponding value of the modified reference rating life is well situated inside 90% confidence bands of the \(L_{10}\) rating life provided by the fatigue life tests, (table 3).
Figure 5. Pressures distributions along rollers 2 and 10
(a) - bearings with no primary loading, (b) - bearings with primary loading.
Figure 6. Pressures distributions along rollers 1 and 11:
(a) - bearings with no primary loading, (b) - bearings with primary loading.
Figure 7. Pressures distributions along the most loaded roller:
(a) - bearings with no primary loading, (b) - bearings with primary loading.
Table 3. Results of rating lives computations.

| Bearing Group | Rating lives (hours) | Life tests (hours) |
|---------------|----------------------|--------------------|
|               | Basic rating life $L_{10}$ | Modified rating life $L_{10m}$ | Modified reference rating life $L_{10m_r}$ | 90% life $L_{10}$ | 90% confidence limits |
| SLP           | 57.2                 | 40.1               | 19                  | 18            | 11.....30            |
| PDP           | 107.1                | 90.3               | 135                 | 112           | 85.....168           |
|               |                      |                    |                     | 102           | 50.....209           |

*SLP, straight line profiles; PDP, primary deformed profiles.

6. Conclusions
The profiles of rolling elements control the pressures distributions in the contact area and radically affects the bearing basic dynamic load rating and rating lives. A few number of loading cycles in elastic-plastic domain initiates an elastic shakedown phenomenon able to create a favorable change of the initial profiles of rolling elements. Fatigue life tests carried out on four groups of NJ206 cylindrical roller bearings pointed out the superiority of bearings groups that primary supported a few cycles of rolling loading in the elastic-plastic domain. An elastic-perfect plastic analysis was performed to reveal the role played by the roller’s crowning geometry, operating clearance, misalignment angle and operating loads on contact pressures distributions achieved between each roller and the corresponding inner and outer raceways. The modified reference rating lives, evaluated using the lamina technique, exposed a good agreement with the $L_{10}$ lives made available by durability tests.

Appendix A - Semi-analytical method to solve the elastic-perfect plastic concentrated contacts

A.1. Analytical formulation of SAM
The concentrated contact between two elastic bodies is a classical problem in the half-space theory of linear and isotropic elasticity; but formulae for stresses and deformations ready to use are available for some particular contact geometry only, the hertzian case being the most known, Solomon L [24], Johnson K L [25], Crețu S [26]. However, these classic analytical equations establish the basis for numerical formulation and, therefore, their derivation will be briefly presented in the following.

A hypothetical (virtual) rectangular contact area denoted $A_h$ is built on the common tangent plane, around the initial contact point, figure A1a. A Cartesian system $(x, y, z)$ is introduced, its $x-O-y$ plane being the common tangent plane and with its origin located at the left corner of the hypothetical rectangular area. The elastic deflection of each surface is measured in the direction of the corresponding

![Figure A1](image_url)
outer normal and is denoted by \( w_I(x, y) \) and \( w_{II}(x, y) \), respectively. The sum of the individual deflections at any generic point \((x, y)\) is defined as a composite deflection, denoted by \( w(x, y) \).

Three equations model the surface deformation:

1. **geometric equation of the elastic contact:**
   \[
g(x, y) = h(x, y) + w(x, y) - \delta_0 \tag{A.1}
   \]
   where: \( h(x,y) \) is initial separation between the surfaces when no load is applied, \( g(x,y) \) is the gap between the surfaces after deformation, \( \delta_0 \) is the rigid bodies approach.

2. **integral equation of the normal surface displacement:**
   \[
w(x, y) = \frac{1}{\pi \sqrt{E_I I_{II}}} \iint_{A_r} \frac{p(\xi, \eta)}{\sqrt{(x-\xi)^2+(y-\eta)^2}} \, d\xi \, d\eta \tag{A.2}
   \]
   where: \( E_I, E_{II}, v_I, v_{II} \) are Young’s modulus and Poisson’s ratio for the elastic materials, and \( p(x, y) \) is the interfacial contact pressure.

3. **load balance equation:**
   \[
   \iint_{A_r} p(x, y) \, dx \, dy = Q \tag{A.3}
   \]
   where \( Q \) is the applied normal force.

4. **constraint equations of non-penetration and non-adhesion:**
   \[
g(x, y) = 0, \quad (x, y) \in A_r, \quad p(x, y) > 0 \tag{A.4}
   \]
   \[
g(x, y) > 0, \quad (x, y) \notin A_r, \quad p(x, y) = 0 \tag{A.5}
   \]

5. **equation to simulate the elastic-perfect plastic constitutive law:**
   \[
p(x, y) \rightarrow_{yields}^\text{yield} p_L, \quad (x, y) \in A_r \tag{A.6}
   \]

### A.2. Numerical formulation of SAM

A uniformly spaced rectangular array is built on the hypothetical rectangular contact area with the grid sides parallel to the \( x \) and \( y \)-axes. The nodes of the grid are denoted by \((i, j)\), where indices \(i\) and \(j\) refer to the grid columns and rows, respectively. In the considered Cartesian system, the coordinates of the grid node \((i, j)\) are denoted by \((x_i, y_j)\) and are given by \(x_i = i \cdot \Delta x\), \(0 \leq i < N_x\) and \(y_j = j \cdot \Delta y\), \(0 \leq j < N_y\) where \(\Delta x\) and \(\Delta y\) are the grid spaces in the \(x\) and \(y\)-directions, respectively. The real pressure distribution is approximated by a virtual pressure distribution, figure 6b. The surface deformation is modelled by the following six linear algebraic equations:

1. **geometric equation of the elastic contact:**
   \[
g_{ij} = h_{ij} + w_{ij} - \delta_0 \tag{A.7}
   \]

2. **integral equation of the normal surface displacement:**
   \[
w_{ij} = \sum_{k=0}^{N_x-1} \sum_{l=0}^{N_y-1} (K_{i-k,j-l} * p_{kl}) \tag{A.8}
   \]

3. **load balance equation:**
   \[
   \Delta x \Delta y \sum_{i=0}^{N_x-1} \sum_{j=0}^{N_y-1} p_{ij} = Q \tag{A.9}
   \]

4. **constraint equations of non-penetration and non-adhesion:**
   \[
g_{ij} = 0, \quad p_{ij} > 0, \quad (i, j) \in A_r \tag{A.10}
   \]
   \[
g_{ij} > 0, \quad p_{ij} = 0, \quad (i, j) \notin A_r \tag{A.11}
   \]

5. **equation to simulate the elastic-perfect plastic constitutive law:**
As long as the pressures distribution is established, the components of the stress tensor are obtained by superposition, [24, 25]:

$$p_{ij} > p_l \rightarrow p_{ij} = p_l, \quad (i, j) \in A_r$$  \hspace{1cm} (A.12)

where the influence function $C_{ijkl}(x,y,z)$ describes the stress component $\sigma_{ij}(x,y,z)$ due to a unit pressure acting in patch $(k, l)$.

**Appendix B – The lamina technique**

**B.1. Pressure distributions**

The roller-raceway contacts of length $L_{wr}$ are divided into a number of $n_s$ laminae each of width $w$, and $L_{wr} = n_s \cdot w$. The internal load distribution, presented in 5.1 provides the normal loads $Q_j$ and further the contact stresses $q_{j,k}$ corresponding to the lamina $k$ of roller $j$.

The distribution of pressures $p_{Hij,k}$ and $p_{Hej,k}$ over the length of the $j$-roller is established using the pressure distribution provided by the semi-analytical method presented in Appendix 1.

**B.2. Rings loads for the basic dynamic load ratings**

For the inner ring and outer ring, the values $Q_{ci}$ and $Q_{ce}$ are calculated using the basic dynamic load rating, $C_r$, of the bearing, [3, 17]:

$$Q_{ci} = \frac{1}{k} \frac{C_r}{0.378 Z \alpha (\cos \alpha)^{1/9}} \left( 1 + \left[ \frac{1.038}{(1+\gamma)} \frac{143/108}{1+\gamma} \right]^{9/2} \right)^{2/9}$$  \hspace{1cm} (B.1)

$$Q_{ce} = \frac{1}{k} \frac{C_r}{0.364 Z \alpha (\cos \alpha)^{1/9}} \left( 1 + \left[ \frac{1.038}{(1+\gamma)} \frac{143/108}{1+\gamma} \right]^{-9/2} \right)^{2/9}$$  \hspace{1cm} (B.2)

For the case under study: $\alpha = 0$, $i = 1$ and $\gamma = D_w/D_{pw}$.

**B.3. Basic dynamic load rating of a bearing lamina**

For laminae of inner ring and outer ring, the basic dynamic load ratings are:

$$q_{ci} = Q_{ci} \left( \frac{1}{n_s} \right)^{7/9}$$  \hspace{1cm} (B.3)

$$q_{ce} = Q_{ce} \left( \frac{1}{n_s} \right)^{7/9}$$  \hspace{1cm} (B.4)

**B.4. Functions for stress concentrations stresses**

Functions of the stress concentration stresses over the roller length are further developed for the inner ring raceway $f_i[j,k]$, and outer ring raceway $f_e[j,k]$, (equations (58) and (59) from ISO 16281:2008, [3]):

$$f_i[j,k] = \left( \frac{p_{Hij,k}}{271} \right)^2 D_w (1-\gamma) \frac{L_{wr}}{n_s} / q_{j,k}$$  \hspace{1cm} (B.5)

$$f_e[j,k] = \left( \frac{p_{Hej,k}}{271} \right)^2 D_w (1+\gamma) \frac{L_{wr}}{n_s} / q_{j,k}$$  \hspace{1cm} (B.6)

The distribution of pressures $p_{Hij,k}$ and $p_{Hej,k}$ over the length of the $j$-roller is established using the pressure distribution provided by the semi-analytical method presented in Appendix A.

**B.5. Dynamic equivalent load on a lamina**
The dynamic equivalent load on a lamina \( k \) of the rotating inner ring is:

\[
q_{kei} = \left[ \frac{1}{Z_w} \sum_{j=1}^{Z_w} (f_i[j,k] \cdot q_{j,k})^4 \right]^{1/4}
\]  \hspace{1cm} (B.7)

The dynamic equivalent load on a lamina \( k \) of the stationary outer ring is:

\[
q_{kee} = \left[ \frac{1}{Z_w} \sum_{j=1}^{Z_w} (f_e[j,k] \cdot q_{j,k})^{4.5} \right]^{1/4.5}
\]  \hspace{1cm} (B.8)

B.6. Fatigue lives of roller-raceway contact laminae

Considering a fourth power load-life dependence for the line contact, the fatigue lives of roller-raceway contact laminae are given by:

\[
L_{ijk} = \left( \frac{q_{ci}}{q_{ijk}} \right)^4, \quad L_{ejk} = \left( \frac{q_{cej}}{q_{eqe}} \right)^{4.5}
\]  \hspace{1cm} (B.9)

B.7. Basic reference rating life

The basic reference rating life, \( L_{10r} \), is calculated using the basic life formula:

\[
L_{10r} = \left\{ \sum_{k=1}^{n_s} \left[ \left( \frac{q_{kei}}{q_{kei}} \right)^{-4.5} + \left( \frac{q_{kee}}{q_{kee}} \right)^{-4.5} \right] \right\}^{-8/9}
\]  \hspace{1cm} (B.10)

B.8. Dynamic equivalent reference load

For the entire cylindrical roller bearing the dynamic equivalent reference load, \( P_{ref,r} \), is:

\[
P_{ref,r} = \frac{C_r}{L_{10r}^{3/10}}
\]  \hspace{1cm} (B.11)

B.9. Dynamic equivalent load \( P_{ks} \) of the bearing lamina \( k \).

The value of \( P_{ks} \) is obtained using the equation (69) from ISO 16281:2008, [3]):

\[
P_{ks} = 0.323 \cdot Z_w \cdot \cos(\alpha) \cdot n_s \left\{ q_{kei}^{9/2} \left( 1 + \frac{0.323}{0.308} q_{ceq} q_{ke}^{9/2} \right) \right\} / \left[ 1 + \left( \frac{0.323}{0.308} q_{ceq} q_{ke}^{9/2} \right) \right]^{2/9}
\]  \hspace{1cm} (B.12)

B.10. Modified reference rating life, \( L_{nmr} \)

The modification factor \( a_{IS0k} \) calculated using equations (34) to (36) from ISO 281:2007, [17], are considered for each lamina, in the equation of the modified reference rating life of radial roller bearing:

\[
L_{nmr} = a_1 \left( \sum_{k=1}^{n_s} \left[ a_{IS0k} \left( \frac{C_r q_{ce}}{P_{ks}}, K \right)^{-9/8} \cdot \left( \frac{q_{kei}}{q_{kei}} \right)^{-4.5} + \left( \frac{q_{kee}}{q_{kee}} \right)^{-4.5} \right] \right)^{-8/9}
\]  \hspace{1cm} (B.13)

Nomenclature

\( A_k \) amplitude factor in equation for the profile function
\( a_{H} \) major half-axis of the hertzian contact area, in millimeters
\( a_{ISO} \) life modification factor, based on a system approach of life calculation
\( a_1 \) life modification factor for reliability
\( b_{H} \) minor half-axis of the hertzian contact area, in millimeters
\( b_m \) rating factor for contemporary hardened bearing steel, in accordance with good manufacturing practice
\( C_r \) basic dynamic radial load rating, in newtons
\( C_u \) fatigue load limit, in newtons
\( D_{pw} \) pitch diameter of roller set, in millimeters
\( D_{we} \) roller diameter, in millimeters
modulus of elasticity, in megapascals

subscript for outer ring

bearing radial load, in newtons

stress correction function for consideration of edge load
gap, in millimeters, between surfaces after deformation, at the point (x, y)
gap, in millimeters, between surfaces after deformation, at the grid point (i, j)
separation, in millimeters, between surfaces when no load is applied, at the grid point (i, j)
separation, in millimeters, between surfaces after deformation, at the point (x, y)
minimum film thickness in an elasto-hydrodynamic lubrication, in millimeters
subscript for inner ring

number of rows of rolling elements

influence coefficient

modified rating life, in million revolutions
modified reference rating life, in million revolutions
effective roller length, in millimeters
basic rating life, in million revolutions
basic reference rating life, in million revolutions
probability of failure
speed of rotation, in revolutions per minute
number of laminae
dynamic equivalent load, in newtons
dynamic equivalent load, in newtons, of a bearing lamina k
dynamic equivalent reference radial load, in newtons
profile function, in millimeters
contact stress, in megapascals, at the contact of outer ring and cylindrical roller
contact stress, in megapascals, at the contact of inner ring and cylindrical roller
pressure, in megapascals, at the grid point (i, j)
contact pressure, in megapascals, corresponding to elastic limit
pressure, in megapascals, at the point (x, y)
radial load on the roller j, in newtons
roller load, in newtons, for the basic dynamic load rating of outer ring
roller load, in newtons, for the basic dynamic load rating of inner ring
dynamic equivalent roller load, in newtons, on outer ring
dynamic equivalent roller load, in newtons, on inner ring
basic dynamic load rating, in newtons, of a bearing lamina at the outer ring contact
basic dynamic load rating, in newtons, of a bearing lamina at the inner ring contact
dynamic equivalent load, in newtons, of a bearing lamina at the outer ring contact
dynamic equivalent load, in newtons, of a bearing lamina at the inner ring contact
load, in newtons, on the lamina k of roller j
radial operating clearance, in millimeters
composite deflection, in millimeters, defined by sum of the individual deflections at a
generic point (x, y)
composite normal surface deflection, in millimeters, for the grid point (i,j)
number of cylindrical rollers
nominal contact angle, in degrees, of a bearing
auxiliary parameter, \( \gamma = \frac{D_{we} \cos \alpha}{D_{pw}} \)
total elastic deflection, in millimeters, of both contacts of a roller
elastic deflection, in millimeters, of the roller j
\( \delta_{j,k} \) elastic deflection, in millimeters, of the lamina \( k \) of the roller \( j \)
\( \delta_0 \) rigid bodies approach
\( \kappa \) viscosity ratio, \( \kappa = \nu / \nu_1 \)
\( \lambda \) film parameter, evaluated as the ratio between the minimum thickness and the composed roughness of the involved surfaces
\( \lambda \nu \) reduction factor for the consideration of stress concentrations and exponent variation
\( \sigma_{H} \) maximum pressure in a hertzian distribution
\( \nu \) actual kinematic viscosity at operating temperature, in mm\(^2\)/s
\( \nu_1 \) reference kinematic viscosity, required to obtain adequate lubrication, in mm\(^2\)/s
\( \nu_i \) Poisson’s ratio
\( \phi_j \) angular positions of the roller \( j \)
\( \psi \) total misalignment, in degrees, between the inner raceway and outer raceway
\( \psi_j \) total misalignment, in degrees, between the inner raceway and outer raceway in the plane of roller \( j \)

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