Abstract

We examine the possibility of extracting $\mathcal{CP}$-violating terms in the decay $K^0 \rightarrow \pi^0 e^+ e^-$ by studying the time evolution of a $K^0$ beam. We focus on the interference region and search for clear effects. We find that experiments which average over the electron and positron momenta can detect $\mathcal{CP}$ violation as an oscillation in the decay rate. The branching ratio is $(3 - 5) \times 10^{-12}$ and direct $\mathcal{CP}$ violation dominates over a wide range of the parameters.

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1 Introduction

One property of the standard model which is still under active consideration is the origin of $\mathcal{CP}$ violation. Up to now $\mathcal{CP}$-odd contributions have been observed only in the decays of $K^0$ mesons [1]. In the decay one distinguishes two types of $\mathcal{CP}$ violation effects: direct $\mathcal{CP}$ violation occurring in the amplitudes (described by $\epsilon'$) and $\mathcal{CP}$-asymmetric terms in the mass matrix which is called indirect (described by $\epsilon$). The value of $\epsilon$ is precisely known ($|\epsilon| = 2.258 \times 10^{-3}$), but there are still uncertainties concerning the value of $\epsilon'$. The CERN experiment NA31 found [2]

$$Re\left(\frac{\epsilon'}{\epsilon}\right) = (2.3 \pm 0.7) \times 10^{-3}$$

(1)

while the measurement of the FERMILAB experiment E731 is [3]

$$Re\left(\frac{\epsilon'}{\epsilon}\right) = (0.74 \pm 0.59) \times 10^{-3}$$

(2)

which is still consistent with the predictions of the superweak theory. Thus it is still interesting to investigate other processes in order to find if direct $\mathcal{CP}$ violation is different from zero and providing another crucial test of the standard model. Examples for such processes are B meson and rare $K^0$ meson decays, which are actively investigated both theoretically and experimentally. A promising decay channel is $K^0 \rightarrow \pi^0 e^+ e^-$, where $\mathcal{CP}$ violation may be relatively large. The specific decay $K_L \rightarrow \pi^0 e^+ e^-$ has been studied extensively and its status was recently reviewed [4, 5].

We study the time development of this decay channel starting with a pure $K^0$ beam and pose the question if one could identify a $\mathcal{CP}$-violating signal in the interference region. In particular, we are interested in a signature of direct $\mathcal{CP}$ violation which we present in this article. We will show that an experiment which studies the time development of $K^0$ decays and averages over the momenta of electron and positron is sensitive to $\mathcal{CP}$-violating terms. The new effect appears in the interference region of the $K_S$ and $K_L$ component and manifests itself as a time oscillation. The effect follows from general symmetry considerations as is explained in the next section. In addition, we present an estimate for the magnitude of the effect. This experiment is especially suited for laboratories with intense K beams like Brookhaven [6].

Our paper is organized as follows. In section 2 we derive the formula describing the time evolution of a pure $K^0$ state and classify the different contributions. In section 3 we discuss the calculations available for the amplitudes and their dependence on the parameters. Furthermore we give the range of parameters, which is used later on in the numerical analysis. In section 4 we present the numerical results for the time evolution of the $K^0$ state with special emphasis on $\mathcal{CP}$ violation in the interference region. Finally, the reader who is interested on the experimental possibility can study section 2 and the conclusions in section 4.
2 Classification of the various amplitudes

The time evolution of a pure $K^0$ state is given in terms of the time development of the physical states $K_L$ and $K_S$ as follows

$$|K^0(t)\rangle = \frac{1}{\sqrt{2}} \left[ e^{-iX_L t} (|K_2\rangle + \epsilon |K_1\rangle) + e^{-iX_S t} (|K_1\rangle + \epsilon |K_2\rangle) \right].$$

The decay proceeds through two intermediate states, $K^0 \to \pi^0 \gamma$ and $K^0 \to \pi^0 \gamma \gamma$, with the single or two photons converting into electron positron pairs. The decaying kaon has spin 0, and angular momentum conservation demands the intermediate state $\pi^0 \gamma$ to be in a $p$-wave. It follows, then, that the $CP$ eigenvalue of $\pi^0 \gamma$ is $(-1)^s(+1,(-1)^l=1) = +1$. Thus $K_1$ can decay through the $\pi^0 \gamma$ channel in terms of $CP$-conserving parts, whereas $K_2$ decays through the $CP$-violating parts of the Hamiltonian. We denote these amplitudes as

$$A_1 = \langle \pi^0 e^+ e^- | \mathcal{H} \gamma | K_1\rangle; \quad CP\text{-conserving (It gives indirect CP violation through } \epsilon), (4)$$

$$B = \langle \pi^0 e^+ e^- | \mathcal{H} \gamma | K_2\rangle; \quad CP\text{-violating (direct). (5)}$$

These would be the only two amplitudes if there were no higher order terms. In fact the decay of a $K^0$ can also proceed through the intermediate state $\pi^0 \gamma \gamma$, which is higher order in the electromagnetic coupling. This contribution to the decay width is not a priori negligible, because, as we will show, it has to be compared with $CP$-violating terms which are suppressed. To be specific, the intermediate state of a pion and two photons has many partial waves so that both $CP = +1$ and $CP = -1$ states are allowed. Thus the decay

$$K_2 \to \pi^0 \gamma \gamma \to \pi^0 e^+ e^-$$

is $CP$-conserving with the final state odd under the $CP$ transformation. In fact the decay $K_2 \to \pi^0 \gamma \gamma$ has already been observed. We define the relevant amplitude as

$$A_2 = \langle \pi^0 e^+ e^- | \mathcal{H} \gamma\gamma | K_2\rangle; \quad CP\text{-conserving. (7)}$$

The decay of a pure $K^0$ beam has the general form

$$\langle \pi^0 e^+ e^- | \mathcal{H} | K^0\rangle (t) = \frac{1}{\sqrt{2}} \left\{ e^{-iX_L t} \bar{\Psi}(k_-) \Phi_K \left[ (B^+ + \epsilon A_1^+ + A_2^+) + B^- \gamma_5 \right] v(k_+) \right. + e^{-iX_S t} \bar{\Psi}(k_-) \Phi_K \left[ A_1^+ + \epsilon (B^+ + A_2^+) + \epsilon B^- \gamma_5 \right] v(k_+) \right\}$$

where the $+, -$ indicate that the spinors are written out explicitly, with $A_1^+, A_2^+, B^+$ being vector amplitudes and $B^-$ being the axial-vector. For explicit definitions see equations (12), (17) and (24).

The term $\epsilon (B + A_2)$ is small in comparison to $A_1$ on several reasons:

i) $B$ is small, being $CP$-violating,
ii) $A_2$ is small, being higher order in electromagnetism

iii) these two small terms are multiplied by the small parameter $\epsilon$.

Neglecting $\epsilon(B + A_2)$, the $K_S$ decays are $\mathcal{CP}$-conserving. The $K_L$ decays contain $A_2$, which is $\mathcal{CP}$-conserving, and the $\mathcal{CP}$-violating amplitudes $B$ and $\epsilon A_1$. The amplitude $B$ represents direct $\mathcal{CP}$ violation, whereas the violation in $\epsilon A_1$ arises through the mass matrix. Since both terms are very likely suppressed, it becomes necessary to consider the $A_2$ term, as mentioned above.

Next, we compute the time evolution of the decays.

$$
\frac{d\Gamma}{dsd\Delta}(t) = \frac{1}{512 \pi^2 m_K^3} \left\{ e^{-\Gamma_{LT} t} \left( |B^+ + A_2^+ + \epsilon A_1^+|^2 + |B^-|^2 \right) + e^{-\Gamma_{ST} t} |A_1^+|^2 
+ e^{-\frac{\Gamma_{LT} + \Gamma_{ST}}{2} t} 2 \text{Re} \left( e^{-i \Delta m_K t} (B^+ + A_2^+ + \epsilon A_1^+) A_1^{+\ast} \right) \right\}
$$

where

$$
\lambda(s, m_K^2, m_\pi^2) = s^2 + m_K^4 + m_\pi^4 - 2 s m_K^2 - 2 s m_\pi^2 - 2 m_K^2 m_\pi^2 \quad \text{and} \quad \Delta = (p_K - k_-)^2 - (p_K - k_+)^2.
$$

This expression shows three time intervals: decays for $K_S^-$, $K_L$ mesons and an interference region. The first two show the typical exponential behavior for the decays, and the interference has an oscillatory term as well. We point out an important property of the interference term. The $A_2$ amplitude is odd under the $\mathcal{CP}$ transformation and thus anti-symmetric under the exchange of the electron and positron energies or momenta. This is born out by explicit calculation, with equation (24) being linear in $\Delta$. The $A_1^+$ amplitude is even under exchange of the electron and positron momenta. Therefore the term $A_2^+ A_1^{+\ast}$ drops out in an experiment which symmetrizes over the electrons and positrons. The remaining interference terms in equation (9) are $\mathcal{CP}$-odd. Thus the presence of an oscillation in the interference region is a clear indication of $\mathcal{CP}$ violation. In the remaining article we estimate each of the amplitudes, we calculate the magnitude of the effect and demonstrate it with formulas and several figures.

### 3 Estimates for the Amplitudes

#### 3.1 The direct $\mathcal{CP}$-violating amplitude $B$

Among the amplitudes, $B$ is the best known in the standard model. As discussed by several authors, the $B$-amplitude is calculated according to Fig. 1 by means of an effective Hamiltonian for $\Delta S = 1$ semileptonic transitions derived by using the operator-product-expansion [7–10]. One starts at a high energy scale, where the interaction is point-like, and
scales down to low energies by means of the renormalization group equation. The procedure also includes electromagnetic and strong effects contained in the Wilson coefficients. Since the $B$-amplitude is $\mathcal{CP}$-violating, it involves the imaginary parts of the Wilson coefficients, and the dominant terms are \[11\]

$$\text{Im}(C_7) \, O_7 \quad \text{and} \quad \text{Im}(C_8) \, O_8$$

with

$$O_7 = (\bar{s}_L \gamma_\mu d_L) (\bar{e} \, \gamma^\mu \, e), \quad \text{and} \quad O_8 = (\bar{s}_L \gamma_\mu d_L) (\bar{e} \, \gamma^\mu \gamma_5 \, e).$$

These Wilson coefficients receive their main contribution from energy scales between $m_t$ and $m_c$ where perturbative QCD is more reliable. The reduced matrix elements of the operators involve quark currents between hadronic states and can be related to $K_{l3}$ decays through an isospin rotation \[11\]. Neglecting the mass of the electron, the final form of the amplitude is

$$B = \pi(k_-) \psi_K \left[ B^+ + B^- \gamma_5 \right] \nu(k_+)$$

with

$$B^+ = \frac{i G_F}{\sqrt{2}} V_{ud} V_{us}^* \alpha \, 2\sqrt{2} f_+(s) \text{Im}(C_7), \quad B^- = \frac{i G_F}{\sqrt{2}} V_{ud} V_{us}^* \alpha \, 2\sqrt{2} f_+(s) \text{Im}(C_8).$$

Following the Kobayashi Maskawa parametrization for the quark mixing matrix we use for the coefficients the values \[10\]

$$\text{Im}(C_7) = -\frac{1}{V_{ud} V_{us}^*} \text{Im}(V_{td} V_{ts}^*) \, 0.74 \quad \text{and} \quad \text{Im}(C_8) = -\frac{1}{V_{ud} V_{us}^*} \text{Im}(V_{td} V_{ts}^*) \, (-0.70)$$

with $m_t = 170$ GeV. The main uncertainty comes from the factor

$$\text{Im}(\lambda_t) = \text{Im}(V_{td} V_{ts}^*) = -s_1 s_2 s_3 c_2 \sin\delta$$

for which we will allow the range $(1.0 - 2.0) \times 10^{-4}$. The branching ratio from the $B$-amplitude alone is given by the formula

$$\frac{d\Gamma}{ds d\Delta} = \frac{1}{512 \pi^3 m_K^3} \left[ |B^+|^2 + |B^-|^2 \right] \frac{1}{2} \left[ \lambda(s, m_{K}^2, m_{\pi}^2) - \Delta^2 \right].$$

Varying the parameters, we obtain the range

$$\text{BR}(K_L \to \pi^0 e^+ e^-)_{\text{direct}} = (2.4 - 9.7) \times 10^{-12}.$$

Later on we will use the $B$-amplitude as given above with the corresponding ranges of the parameters in order to study the development of a pure $K^0$ beam.
3.2 The $\mathcal{CP}$-Conserving Amplitude $A_1$

The amplitude $A_1$ has the same diagrams (see Fig. [4]) as the amplitude $B$. But the approach of using the same effective Hamiltonian as in section 3.1 involves the real parts of the Wilson coefficients with large contributions from regions far below $m_e$, where perturbative QCD is not reliable. For this reason one does not use the QCD effective Hamiltonian, but resorts to other low energy methods like chiral perturbation theory. We define $A_1$ through the equation

$$A_1 = <\pi^0 e^+ e^- | \mathcal{H}_\gamma | K_1 > = \overline{\nu}(k_-) A^+_1 \gamma \nu_K v(k_+),$$

(17)

and $A^+_1$ is given by

$$A^+_1 = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* g_8 \frac{\alpha}{\pi} 2 \left[ w_+ + \frac{1}{6} \ln \frac{m^2}{m^2_K} + 2 \phi(s) \right]$$

with the loop-function $\phi(s)$

$$\phi(s) = -\frac{4m^2_K}{3s} + \frac{5}{18} + \frac{1}{3} \left( \frac{4m^2_K}{s} - 1 \right)^2 \arctan \left( \frac{1}{\sqrt{4m^2_K/s}} - 1 \right)$$

(18)

as calculated in [12]. Most of the factors here have standard definitions except for $g_8$, which is the coupling constant of the octet of pseudoscalar mesons and $\omega_+$, which is a dimensionless coupling constant. Both have to be determined experimentally. From the decay $K \rightarrow \pi\pi$ it was found $g_8 = 5.1$. \omega_+ was determined from a $\chi^2$-analysis of the spectrum for the decay $K^+ \rightarrow \pi^+ e^+ e^-$, based on a calculation of the spectrum in $\chi$PT [12], including the same set of parameters. From their data set the BNL E777 group [13] derived a value of

$$w_+ = 0.89^{+0.24}_{-0.14}. $$

(19)

The decay-width from the $A_1$ amplitude reads

$$\frac{d\Gamma}{dsd\Delta} = \frac{1}{512 \pi^3 m^3_K} |A^+_1|^2 \frac{1}{2} \left[ \lambda(s, m^2_K, m^2_\pi) - \Delta^2 \right].$$

(20)

This yields a branching ratio for the decay

$$BR(K_L \rightarrow \pi^0 e^+ e^-) = 1.71 \times 10^{-15} - 1.14 \times 10^{-12}$$

(21)

if the decay occurs only through the $\mathcal{CP}$-violating piece of the $K^0-K^0$ mass-matrix.

In the same way as $B$, the $A_1$-amplitude with the corresponding range of parameters will be used as an input for the time development of a pure $K^0$ beam.
3.3 The $CP$-conserving amplitude $A_2$

We have already mentioned that we should include the $A_2$ amplitude even though it is of $O(\alpha^2)$. Since the decay $K^0 \to \pi^0 e^+ e^-$ has not yet been observed it was suggested to study the intermediate decay $K_L \to \pi^0 \gamma \gamma$, which has recently been observed, with the branching ratio \[ BR(K_L \to \pi^0 \gamma \gamma) = (1.7 \pm 0.3) \times 10^{-7}. \] (22)

Starting from these studies, one should couple the two photons to the final electron-positron pair. This is useful but not very direct, because some amplitudes contributing to $K_L \to \gamma \gamma$ are suppressed in the $K_2 \to \pi^0 e^+ e^-$ amplitude, being proportional to $m_e$ and their contribution being negligible. As will become clear later on, an amplitude which gives a significant contribution to the $m_{\gamma\gamma}$-distribution gives a very small contribution to the semileptonic decay.

The amplitude for the decay $K_L \to \pi^0 \gamma \gamma$ has been estimated by two different methods. One method uses a two-component model developed by Sehgal and collaborators [15, 16]. The two contributions included are: ($\alpha$) a diagram with a charged pion loop to which the photons are attached and ($\beta$) vector meson intermediate states. The second method applies chiral perturbation theory [12]. The two approaches differ in several respects, but for the amplitude which is dominant in our investigation they agree. This comes about as follows: the decay $K_L \to \pi^0 \gamma \gamma$ has several amplitudes, but only one of them is significant for $A_2$. It is fortunate that estimates of this amplitude give similar results in the two methods. We describe the results of the two-component model [17, 18].

The amplitude for $K_L(p_K) \to \pi^0(p_\pi)\gamma(k)\gamma(k')$ has the general structure

\[
M = \epsilon^\mu \epsilon'^\nu \left[ F \left( k'_\mu k'_\nu - g_{\mu\nu} k' \cdot k \right) + G \left( g_{\mu\nu} k \cdot p_K k' \cdot p_K + p_K \mu p_K \nu k' \cdot k - p_K \mu k' \cdot p_K - k'_\mu p_K \nu k \cdot p_K \right) \right].
\] (23)

The pion-loop diagrams contribute only through the $F$ amplitude, whose contribution to $A_2$ is proportional to $m_e$ and thus small. This follows by considering the general structure of the loop integral and the tensor structure of the term that multiplies the $F$ amplitude in equation (23). The vector meson pole diagram contributes to the amplitude $A_2^+(s, \Delta)$ defined by

\[
A_2 = A(K_2 \to \pi^0 e^+ e^-)_{2\gamma} = \mp(k_-) A_2^+ \varphi_K v(k_+). \] (24)

Conservation of $CP$ demands that $A_2^+(s, \Delta)$ is an odd function of $\Delta$. Defining $\beta = \sqrt{1 - \frac{4m_e^2}{s}}$, the absorptive part of $A_2^+(s, \Delta)$ is

\[
\text{Im } A_2^+ = \frac{\alpha}{16} \frac{G_{\text{eff}}}{m_V^2} \frac{\Delta}{\beta} \left[ \frac{2}{3} + \frac{2}{\beta^2} - \left( \frac{1}{\beta^2} - \beta^2 \right) \frac{1}{\beta} \ln \frac{1 + \beta}{1 - \beta} \right],
\] (25)

\[
= \frac{\alpha}{16} \frac{G_{\text{eff}}}{m_V^2} \frac{8}{3} \Delta \quad \text{when } \beta \to 1.
\]
We notice that $\text{Im} A^+_2$ is an odd function of $\Delta$ and in addition the limit $\beta \to 1$ is justified for the decay $K_2 \to \pi^0 e^+ e^-$. In this limit, equation (24) is in agreement with formula (28) of [19]. The formula above has only one coupling $G_{\text{eff}}$ which is chosen in such a way that it reproduces the branching ratio in equation (22).

The dispersive part is calculated with the help of a dispersion relation [18]

$$\text{Re} A^+_2 = \frac{1}{\pi} \frac{\Lambda^2 = m^2_\rho}{s_{\text{min}}(\Delta)} \int \frac{\text{Im} A^+_2(\Delta)}{z-s} \, dz$$

where the lower limit is given by

$$s_{\text{min}}(\Delta) = 4 m^2_e \left[1 - \frac{\Delta^2}{((m_K + m_\pi)^2 - 4m^2_e) ((m_K - m_\pi)^2 - 4m^2_e)} \right]^{-1}$$

and the upper limit is determined by the heaviest particle considered, this being the $\rho$ meson. The differential decay-width reads

$$\frac{d\Gamma}{dsd\Delta} = \frac{1}{512\pi^3 m^3_K} |A^+_2|^2 \frac{1}{2} \left[ \lambda(s, m^2_K, m^2_\pi) - \Delta^2 \right].$$

Inserting the numerical values we obtain a branching ratio dominated by the vector meson coupling constant

$$\text{Br}(K_L \to \pi^0 e^+ e^-)_{2\gamma} = 4.61 \times 10^{-12} \left( \frac{G_{\text{eff}} m^2_K}{0.25 \times 10^{-7}} \right)^2 (1 + \rho)$$

$$= 4.15 \times 10^{-12}$$

with $\rho = \Gamma_{\text{disp}}/\Gamma_{\text{abs}} = 1.5$

and $G_{\text{eff}} m^2_K = 0.15 \times 10^{-7}$.

The second method for calculating the decay $K_L \to \pi^0 \gamma \gamma$ is chiral perturbation theory. The authors include effects of the order $p^4$ [20] and $p^6$ [21] in the momentum expansion of $\chi$PT, as well as vector mesons [22]. This enables them to reproduce the observed decay rate and spectrum. In this approach there are again two effective coupling constants which have to be fixed experimentally. Here the vector meson coupling constant was chosen in such a way so that the measured decay rate is reproduced.

The calculation of the two-photon-exchange contribution to the decay $K_L \to \pi^0 e^+ e^-$ on the basis of the $\chi$PT prediction is analogous. The branching ratio achieved in this manner is dominated by the vector meson intermediate state and reads

$$\text{Br}(K_L \to \pi^0 e^+ e^-)_{2\gamma} = 1.8 \times 10^{-12}(1 + \rho)$$

$$= 4.5 \times 10^{-12}$$

with the same $\rho$. In the numerical analysis of section 4 we can choose any of the two calculations for $K_2 \to \pi^0 e^+ e^-|_{2\gamma}$, because the term relevant for our purpose is practically the same. In addition, we shall demonstrate that in experiments which average over the $e^+$ and $e^-$ momenta, this $A^+_2$ amplitude drops out in the interference region.
4 Numerical results

With the amplitudes developed and the general equation (9) we calculate the time evolution for the decay $K^0 \rightarrow \pi^0 e^+ e^-$. As already discussed, there are three regions of physical interest.

1) The $K_S$-region, where the $CP$-conserving amplitude contributes to the decay width. A measurement in this region will determine parameters of the $K_S$-decay like $\omega_+$. 

2) The $K_L$-decay region, which has been studied in several articles interested in $CP$ phenomena (see the review [4, 5]). The relevant amplitude in this case is $|B + A_2 + \epsilon A_1|^2$, which gives several terms. The interference term $Re(A_2 B^+ + \epsilon A_1 A_2)$ is odd in $\Delta$ and one could define an asymmetry in $\Delta$ in order to extract this term. The sum of the absolute values squared is even in $\Delta$, and we will need precise measurements of the amplitudes in order to observe an excess of events.

3) More interesting is the interference region occurring in the time interval $(8 - 9) \cdot \tau_{K_S}$. This term has an oscillatory behavior. The term $A_2 A_1^+$ is a linear function in $\Delta$ and drops out when we average over the electron positron pair. The remaining terms $B A_1^+$ and $\epsilon |A_1|^2$ are both $CP$-violating. Thus the appearance of an oscillation in the interference region gives evidence for $CP$ violation.

We have studied this phenomena numerically and show the effect in several figures. Fig. (2) and Fig. (3) show the branching ratio as a function of time for two different time scales. We note that an oscillation is evident. We use $\omega_+ = 0.89$ and three values for $Im \lambda_t = 1.0 \times 10^{-4}, 1.5 \times 10^{-4}$ and $2.0 \times 10^{-4}$. We plotted the same curves in figures (4) and (5) where the $CP$-violating terms $B A_1^+$ and $\epsilon |A_1|^2$ are set equal to zero. We noticed that the curves to be compared are different. In the interference region there is a clear oscillation and for very long times the curves which contain the $CP$ amplitudes lie above the curve without the $CP$-violating terms. For the latter region experiments studying $K_L$-decays need a precise measurement of magnitudes in order to establish a signal. The branching ratio is in the range $(3-5) \times 10^{-12}$. In contrast to this situation, the oscillation in the interference region is unambiguous. A comparison of the magnitudes of the contributing terms shows that $B A_1^+$ dominates over $\epsilon |A_1|^2$. Only if we choose $\omega_+$ at the upper bound ($\omega_+ = 1.13$) the two contributions are of comparable size. But the direct $CP$-violating term is still larger by $(6 - 113)\%$ for $Im(\lambda_t) = (1.0 - 2.0) \times 10^{-4}$.

We conclude that an experiment searching for a branching ratio down to $10^{-12}$ and sensitive to the time development of the decay can observe $CP$ violation as an oscillation in the interference region. The experiment does not require a measurement of the $e^+ e^-$ energy asymmetry.
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Figure Captions

Fig. 1 Short distance contributions to the decay $K \to \pi^0 e^+e^-$

Fig. 2 Time development of the partial branching ratio $\Gamma(K^0 \to \pi^0 e^+e^-)/\Gamma(K_L \to all)$ for the time interval $(6 - 20) \cdot \tau_{K_S}$.

Fig. 3 Same as in fig. 2 for a larger time interval.

Fig. 4 The same curves as in fig. 2 together with the modified decay rate in which the CP-violating terms are set to zero.

Fig. 5 The same curves as in fig. 3 together with the modified decay rate when the CP-violating terms are set to zero.
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Figure 1: Short distance contributions to the decay $K \rightarrow \pi^0 e^+ e^-$
Fig. 2

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Graph showing the partial branching ratio over time for different values of \( \text{Im} \lambda_t \).}
\end{figure}
Figure 2: Time development of the decay $\Gamma(K^0 \to \pi^0 e^+ e^-) / \Gamma(K^0 \to all)$ for the time interval $(6 - 20) \tau_{K_S}$
Figure 3: Same as in fig. 2 for a longer time interval.
Figure 4: The same curves as in fig. 2 together with the decay when the $CP$-violating terms are set to zero.
Figure 5: The same curves as in fig. 3 together with the decay when the $\mathcal{CP}$-violating terms are set to zero.
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