Bottomonium masses, decay rates and scalar charge radii

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Abstract

The masses of bottomonium s and p-states, decay constants, leptonic as well as radiative decay widths are computed in the framework of extended harmonic confinement model without any additional parameters.

1 Bottomonia masses from ERHM

The mass of a hadron having p number of quarks in ERHM can be obtained as

\[ M_N(q_1q_2\ldots) = \sum_{i=1}^{p} \epsilon_N(q_i,p)_{\text{conf}} + \sum_{i<j=1}^{p} \epsilon_N(q_iq_j)_{\text{coul}} + \sum_{i<j=1}^{p} \epsilon_{JN}^f(q_i,q_j)_{\text{SD}} \quad (1) \]

where the first sum is the total confined energies of the constituting quarks of the hadron, the second sum corresponds to the residual colour coulomb

\[ \epsilon_{JN}^f(q_i,q_j)_{\text{coul}} \]
interaction energy between the confined quarks and the third sum is due to spin dependent interaction.

The intrinsic energy of the quarks in a mesonic system is given by

$$\epsilon_N(q_i, 2)_{\text{conf}} = \sqrt{(2N + 3)\Omega_N(q_i) + M_i^2 - 3M_0(q_i)/(M_1 + M_2)} \quad (2)$$

Here $M_{i=1,2}$ represent the masses of the quark and the antiquark constituting the meson. The coulombic part of the energy is computed using the residual coulomb potential given by

$$V_{\text{coul}}(q_i, q_j) = k\alpha_s(\mu)/\omega_n r,$$

where $\omega_n$ represents the state dependent colour dielectric “coefficient”.

We construct the wave functions for bottomonium by retaining the nature of single particle wave function but with a two particle size parameter $\Omega_N(q_i, q_j)$ instead of $\Omega_N(q_i)$.

Coulomb energy is computed perturbatively using the confinement basis with two particle size parameter defined above for different states as

$$\epsilon_{N}(q_i, q_j)_{\text{coul}} = \langle N | V_{\text{coul}} | N \rangle.$$

The fitted parameters to obtain experimental ground state mass are $m_b = 4637$ MeV, $k = 0.19252$ and the confinement parameter $A = 2166$ MeV$^{3/2}$.

From the center of weight masses, the pseudoscalar and vector mesonic masses are computed by incorporating the residual two body chromomagnetic interaction through the spin-dependent term of the confined one gluon exchange propagator perturbatively as

$$\epsilon_{N}(q_i, q_j)_{\text{S.D.}} = \langle N | V_{\text{SD}} | N \rangle.$$ We consider the two body spin-hyperfine interaction of the residual (effective) confined one gluon exchange potential (COGEP).

The computed masses in comparison with experimental and other theoretical model results are given in Table.

2 Decay properties and scalar charge radii

We employ radial wave functions to compute the hadronic as well as radiative decay widths of the vector and pseudoscalar mesons of $b\bar{b}$ system based on the treatment of perturbative QCD. The standard Van - Royen - Weisskopf formula has been used without radiative correction term for computing leptonic decay widths. The computed leptonic decay widths are tabulated in Table along with other theoretical as well as experimental values.

The Van Royen - Weisskopf formula used for the meson decay constants is obtained in the two-component spinor limit. $f_P$ and $f_V$ are related to the ground state radial wave function $R_{1S}(0)$ at the origin, by the VR-W formula modified for the colour as

$$f_{P/V}^2 = (3/\pi M_{P/V})|R_{1S}(0)|^2,$$ where $M_{P/V}$
Table 1: Masses (in MeV/c^2) of the bottomonium system

| State       | Present | 9) | 11 | 12 | 13 | 14 |
|-------------|---------|----|----|----|----|----|
| η_b(1S_0)   | 9425    | 9300 ± 23 | 9457 | 9414 | 9421 | 9400 | 9300 |
| η_b(2S_0)   | 10012   | – | 10018 | 9999 | 10004 | 9993 | 9974 |
| η_b(3S_0)   | 10319   | – | 10380 | 10345 | 10350 | 10328 | 10333 |
| η_b(4S_0)   | 10572   | – | 10721 | 10623 | 10632 | – | – |
| η_b(5S_0)   | 10752   | – | 11059 | – | – | – | – |
| Υ(1S_1)     | 9461    | 9460 | 9460 | 9460 | 9460 | 9460 |
| Υ(2S_1)     | 10027   | 10023 | 10023 | 10024 | 10023 | 10023 |
| Υ(3S_1)     | 10329   | 10355 | 10385 | 10364 | 10366 | 10355 | 10381 |
| Υ(4S_1)     | 10574   | 10579 | 10727 | 10643 | – | – | 10787 |
| Υ(5S_1)     | 10753   | 10865 | 11065 | – | – | – | 11278 |
| χ_b0(1P_0)  | 9839    | 9859 | 9894 | 9861 | 9860 | 9863 | 9865 |
| χ_b1(1P_1)  | 9873    | 9893 | 9941 | 9891 | 9892 | 9892 | 9895 |
| χ_b2(1P_2)  | 9941    | 9912 | 9983 | 9912 | 9910 | 9913 | 9919 |
| h_b1(1P_1)  | 9907    | – | 9955 | 9900 | 9900 | 9901 | 9894 |
| χ_b0(2P_0)  | 10197   | 10232 | 10234 | 10231 | 10231 | 10234 | 10238 |
| χ_b1(2P_1)  | 10207   | 10255 | 10283 | 10255 | 10258 | 10255 | 10264 |
| χ_b2(2P_2)  | 10227   | 10268 | 10326 | 10272 | 10271 | 10268 | 10283 |
| h_b2(2P_1)  | 10217   | – | 10296 | 10262 | 10263 | 10261 | 10260 |

p = perturbative and np = nonperturbative computations in Tables 1 & 2

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Table 2: Leptonic decay widths (in keV) of $\Upsilon(n^3S_1)$

| State   | Present | [9] | [11] | [12]_p | [12]_np | [15] | [16] |
|---------|---------|-----|------|--------|---------|------|------|
| $\Upsilon(1^3S_1)$ | 1.320 1.340 ± 0.018 – 5.30 1.73 1.45 ± 0.07 | 1.314 |
| $\Upsilon(2^3S_1)$ | 0.628 0.612 ± 0.011 0.426 2.95 1.04 0.52 ± 0.02 | 0.576 |
| $\Upsilon(3^3S_1)$ | 0.263 0.443 ± 0.008 0.356 2.17 0.81 0.35 ± 0.02 | 0.476 |
| $\Upsilon(4^3S_1)$ | 0.104 0.272 ± 0.029 0.335 1.67 0.72 – 0.248 |
| $\Upsilon(5^3S_1)$ | 0.040 0.310 ± 0.070 0.311 – – – 0.310 |

Table 3: Radiative M1 transitions of bottomonia (eV)

| Transition | Present | [12] | [13] | [16] | [17] | [18] |
|------------|---------|------|------|------|------|------|
| $1^3S_1 \rightarrow 1^1S_0$ | 2.242 (36) 4.0 5.8 (60) 9.2 7.7 (59) 8.95 |
| $2^3S_1 \rightarrow 2^1S_0$ | 0.145 (15) 0.5 1.40 (33) 0.6 0.53 (25) 1.51 |
| $3^3S_1 \rightarrow 3^1S_0$ | 0.041 (10) – 0.80 (27) – 0.13 (16) 0.826 |

The values in the parentheses are the energy of emitted photons in MeV.

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