RELATIVISTIC BONDI–HOYLE–LYTTLETON ACCRETION ONTO A ROTATING BLACK HOLE: DENSITY GRADIENTS

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ABSTRACT

In this work, for the first time, we present a numerical study of Bondi–Hoyle accretion with density gradients in the fully relativistic regime. In this context, we consider the accretion onto a Kerr black hole (BH) of a supersonic ideal gas with density gradients perpendicular to the relative motion. The parameters of interest in this study are the Mach number, \( \mathcal{M} \), the spin of the BH, \( a \), and the density-gradient parameter of the gas, \( \epsilon_p \). We show that, unlike in the Newtonian case, all of the studied cases, especially those with a density gradient, approach a stationary flow pattern. To illustrate that the system reaches a steady state, we calculate the mass and angular momentum accretion rates on a spherical surface almost located at the event horizon. In the particular case of \( \mathcal{M} = 1 \), \( \epsilon_p = 0.5 \), and BH spin \( a = 0.5 \), we observe a disk-like configuration surrounding the BH. Finally, we present the gas morphology and some of its properties.

Key words: accretion, accretion disks – black hole physics – hydrodynamics – methods: numerical – relativistic processes

1. INTRODUCTION

1.1. Accretion Onto Astrophysical Black Holes

Stellar-Mass Black Holes (sBHs). Astrophysical black holes (BHs) come with a variety of masses and spins. sBH candidates with \( 4 M_\odot \lesssim M_{BH} \lesssim 30 M_\odot \) have been observed (see, e.g., Table 1 in Moreno Méndez 2013 and references therein). The masses of BHs can be obtained through two main channels. The first channel is through the collapse of the progenitor star and the capture of fallback mass. The second channel is through mass transfer from a companion star. Of course, other channels are possible; for example, the BH could be the result of the merger of two compact objects (NS–NS, BH–NS, WD–WD, NS–WD, BH–WD, etc.), a compact object and a main-sequence star, or through a common-envelope (CE) phase; the last two mechanisms could produce a Thorne–Żytkow object that eventually forms a BH (Thorne & Zytkow 1975).

In addition to the masses of sBHs, it has also been possible to obtain their spins. These may reveal important features of the BHs histories. Some of these spins have been estimated through three different methods and seem to be distributed between \( a_c \sim -0.2 \) and \( \gtrsim 0.98 \). All of these methods depend on observing the phenomena of an accretion disk surrounding the sBH, and thus a donor star is necessary. One method for measuring sBH spins is by fitting the X-ray continuum (see McClintock et al. 2015, for a review). A second method is through X-ray reflection spectroscopy (of the Fe K-\( \alpha \) line; see Reynolds et al. 2015, for a review). The third method is based on quasi-periodic oscillations (QPOs) from the precession of the accretion disk (see, e.g., Axelsson et al. 2005).

Stellar-mass BHs may acquire their spins through a variety of mechanisms (Lee et al. 2002; Yoon & Langer 2005; Wooley & Heger 2006). In most known BHs in low-mass X-ray binaries (LMXBs), the spins may be the result of pre-BH formation processes, either accretion through mass transfer (Moreno Méndez et al. 2011) or tidal-synchronization after a post-Case-C-mass-transfer-and-CE episode (Brown et al. 2007; Moreno Méndez 2014); this mechanism may even lead to the production of gamma-ray burst and hypernova events (e.g., Brown et al. 2008; Moreno Méndez et al. 2014; but see also Fragos & McClintock 2014).

For BHs in high-mass X-ray binaries (HMXBs), the spin estimates seem to pile on the high end, i.e., they vary between 0.84 and 0.98. Stellar evolution in binaries has trouble explaining these spins, as do core-collapse mechanisms (E. Moreno Méndez & M. Cantiello 2015, in preparation); thus, Moreno Méndez et al. (2008) and Moreno Méndez (2011) have suggested that wind-driven mass transfer with hypercritical accretion may be necessary to explain these binaries.

Intermediate-Mass Black Holes (IMBHs). IMBHs are the logical intermediary between sBHs and supermassive black holes (SMBHs). If the latter are produced by accretion onto the former or from mergers of massive stars or sBHs, then IMBHs should be abundant. On the other hand, if they are only produced from the core collapse of Population III stars, then they may have already turned into SMBHs. Indeed, they are excellent candidates to explain observed ultra-luminous X-ray (ULX) sources. Nonetheless, many ULXs have been identified as LMXBs (Bachetti et al. 2014) and HMXBs (Liu et al. 2013; Motch et al. 2014). However, there seems to be a good candidate IMBH in a ULX source where, using high-frequency QPOs (with 3:2 ratio), Pasham et al. (2014) used the inverse-scaling of sBH as well as a relativistic precession model to determine a mass of \( M_{BH} \approx 400 M_\odot \).

Super-Massive Black Holes (SMBHs). Explaining the mass of SMBHs is currently one of the most interesting problems in astrophysics. A standard approach to the problem assumes that these holes are the result of accretion onto IMBH seeds. This
1.2. Bondi–Hoyle–Lyttleton Accretion

Bondi–Hoyle–Lyttleton (BHL) accretion deals with the evolution of a homogeneously distributed gas moving uniformly toward a central compact object (Hoyle & Lyttleton 1939; Bondi & Hoyle 1944). Depending on whether or not the velocity of the gas is supersonic, a shock cone will or will not form. This process shows interesting properties when considered within the Newtonian and relativistic regimes, which have been explored based on several numerical studies. In the classical regime, which is ruled by Newtonian gravity, the most important subjects are the consequences on the morphology of the wind and the supersonic shocks that develop. A summary of results, under Newtonian gravity, can be found in Edgar (2004) and Foglizzo et al. (2005).

Unlike in the Newtonian regime, the relativistic approach allows the study of BHL accretion in regions where the gravitational field is strong. Some studies in this direction have been carried out. The first study, performed by Petrich et al. (1989), investigated the different accretion patterns developed by the relativistic gas during accretion onto a BH. Later, considering axial and equatorial symmetries, Font & Ibáñez (1998a, 1998b) and Font et al. (1998, 1999) reviewed the results obtained by Petrich et al. (1989) using more accurate methods. In an astrophysical context and using equatorial symmetry, Dömmez et al. (2011) showed that the shock-cone vibrations can be associated with the sources of QPOs. They also found a flip-flop type of unstable oscillation in the shock cone. However, it was later shown by Cruz-Osorio et al. (2012) that the flip-flop oscillation of the shock cone depends on the coordinates used to describe the rotating BH, specifically, it was found that the flip-flop oscillation does not appear when Kerr–Schild coordinates are used to describe the rotating BH. More recently, considering axially symmetric fluxes, Lora-Clavijo & Guzmán (2013b) studied the shock-cone oscillations as a potential source of low- and high-frequency QPOs. The ultrarelativistic BHL accretion onto a rotating BH was recently reported considering axisymmetric fluxes (Penner 2013) and equatorial symmetry (Cruz-Osorio et al. 2013). More realistic scenarios introduce astrophysically relevant ingredients like magnetic fields (Penner 2011), radiative terms (Zanotti et al. 2011), and full general relativity in the context of supermassive BH binary mergers (Farris et al. 2010).

This work uses penetrating coordinates (Kerr–Schild), which offer the possibility of placing the excision inside the event horizon of the BH. It is well known that what happens inside a BH, stays inside the BH, and thus this further avoids the implementation of boundary conditions there.

In the Newtonian regime, the BHL problem is attended by the assumption that the accretor is point-like and the size of the accretor is determined in terms of the accretion radius. In our case, we are doing the exact opposite; this implies that if we want to resolve the accretor, the Schwarzschild radius must be similar in order of magnitude to the accretion radius, and thus the velocity of the wind must be high. This is so that the ratio of the accretion radius (as well as that of the compact object) can be small enough compared to the exterior numerical domain radius (the ratio must be at least an order of magnitude) so that no numerical artifacts affect the simulations (Cruz-Osorio et al. 2012; Font et al. 1998). Hence, this allows us to obtain good resolution and a reasonable computational timescale. The accretion radius is defined to help decide when a particle falls into the compact object. Such a scale is determined by the...
density that hits the BH. The velocities of the flows hitting the BH are $0.1c$, $0.2c$, $0.3c$, $0.4c$, and $0.5c$.

Next, we list a set of astrophysical situations where our model would become a good fit with the proper scaling.

### 2.1. Stellar-mass BHs

Relativistic winds and shockwaves. An astrophysical scenario where our models may apply for stellar-mass BHs is during a hypernova in a binary system. Suppose we have a short-orbital-period binary ($P \lesssim 0.4$ days) with a massive WR star ($M_{\text{WR}} > 10 M_\odot$) and a ~ $10 M_\odot$ BH (e.g., Cyg X-3 may be a system in a similar situation). The WR tidally synchronizes and rotates rapidly. Such a star will collapse into a compact object and will likely launch a GRB/HN explosion (see, e.g., Moreno Méndez et al. 2011). If ~ $5 M_\odot$ of material collapse, the resulting BH could have $\alpha_s \gtrsim 0.8$ and the available energy would be $\gtrsim 10^{54}$ erg (or 1000 Bethes; 1 Bethe = 1 $B = 10^{51}$ erg). If ~ $5 M_\odot$ of material are expelled with a fraction (say 10% to 50%) of this energy, then their average velocity would be between 0.1$c$ and 0.7$c$. At the time of the explosion, the orbital separation is about 3–4 $R_\odot$. Thus, the density of the ejecta at the orbit of the BH could be as large as one-fifth of its density inside the star if the expansion is mostly equatorial or a hundredth if the expansion is spherically symmetric. This translates to densities as high as $10^9$ g cm$^{-3}$ if material from the C core is expelled. For a 10$M_\odot$ BH, the density is $\rho_{\text{BH}} \gtrsim 10^6$ g cm$^{-3}$. Thus, our model deals with $\rho_{\text{BH}} \sim 10^4$ g cm$^{-3}$, which is 1 to 2 orders of magnitude above what this model provides. Hence, our simulations provide a good qualitative description of what should occur for such a scenario.

### 2.2. IMBHs

IMBHs have a density range from $\rho_{\text{BH}} \sim 10^5$ g cm$^{-3}$ ($M_{\text{BH}} = 10^5 M_\odot$) to $\rho_{\text{BH}} \sim 10^7$ g cm$^{-3}$ ($M_{\text{BH}} = 10^9 M_\odot$). Thus, given that our simulations are bound to $\rho_{\text{BH}} \sim 10^{12}$, we have a wind density regime that goes from $10^5$ g cm$^{-3}$ to $10^3$ g cm$^{-3}$.

A scenario similar to the previous one would be ideal. That is, an IMBH in the mass range of $10^5 M_\odot \lesssim M_{\text{BH}} \lesssim 10^6 M_\odot$ and rotating rapidly. Such an orbital separation, the density of the ejecta should be around $5 \times 10^{-5} - 10^{-2}$ g cm$^{-3}$ at $10 R_\odot$, and $10^{-3} - 5 \times 10^{-2}$ g cm$^{-3}$ at 50 $R_\odot$. Thus, these models are well within the range of our numerical calculations.

### 2.3. SMBHs

SMBHs ($10^6 M_\odot$ to $10^9 M_\odot$), although there is recent evidence for an SMBH as large as $M_{\text{BH}} \gtrsim 10^4 M_\odot$ (Lópe-Cruz et al. 2014a) have a density range that goes from $10^6$ to $10^{-1}$ g cm$^{-3}$. The wind densities in our numerical simulations are thus confined between $10^{-7}$ and $10^{-12}$ g cm$^{-3}$.

For example, one possible scenario where our models may be applied for SMBHs is a SMBH binary where one of the BHs has relativistic jets pointing along the orbital plane. As the
second BH steps into or out of the jet of the first BH, it is submersed in a stream of material with a density gradient. It is conceivable that the last model discussed for IMBHs (a nearby GRB pointing toward the BH) could similarly apply to SMBHs but at somewhat larger distances.

3. RELATIVISTIC HYDRODYNAMIC EQUATIONS AND NUMERICAL METHODS

3.1. Relativistic Hydrodynamics Equations

In order to solve numerically the relativistic Euler equations, we use the 3 + 1 decomposition of spacetime, in which the spacetime is foliated with a set of non-intersecting space-like hypersurfaces \( \Sigma_t \) (see e.g., Alcubierre 2008; Baumgarte & Shapiro 2010; Rezzolla & Zanotti 2013). The spacetime is described with the line element

\[
 ds^2 = -\alpha^2 dt^2 + \tilde{\gamma}_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt),
\]

where \( \alpha \) is the lapse function, \( \beta^i \) are the shift vector components, and \( \tilde{\gamma}_{ij} \) are the components of the induced three-metric that relates proper distances on the spatial hypersurfaces.

The background spacetime corresponds to a rotating BH in Kerr–Schild coordinates which allow us to place the inner boundary of the computational domain inside the event horizon. A discussion of the advantage on the use of these coordinates can be found in Cruz-Osorio et al. (2012). Once the geometrical elements of the spacetime background are known, it is necessary to track the evolution of the fluid, for which we write down the general relativistic Euler equations. For a generic spacetime, these can be derived from the local conservation of the stress-energy tensor,

\[
 \nabla_{\mu}(T^{\mu\nu}) = 0, \quad (2)
\]

and the local conservation of the rest mass density

\[
 \nabla_{\mu}(\rho u^\mu) = 0, \quad (3)
\]

where \( \rho \) is the proper rest mass density, \( u^{\mu} \) is the four-velocity of the fluid, and \( \nabla_{\mu} \) is the covariant derivative consistent with the four-metric \( g_{\mu\nu} \) of the spacetime (1).

We assume that the matter field in the above equations is that of a perfect fluid with a stress-energy tensor

\[
 T^{\mu\nu} = \rho u^\mu u^\nu + pg^{\mu\nu}, \quad (4)
\]

where \( \rho \) is the pressure, \( g_{\mu\nu} \) are the components of the four-metric, and \( h \) is the relativistic specific enthalpy given by \( h = 1 + \epsilon + p/\rho \), where \( \epsilon \) is the rest-frame specific internal energy density of the fluid.

It is well known that Euler’s equations develop discontinuities in the hydrodynamical (HD) variables even if smooth initial data are considered. Thus, one may solve hydrodynamics equations using finite volume methods as long as the system of equations is written in a flux balance law form, which in turn requires the definition of conservative variables.

In order to obtain the general relativistic Euler equations as a set of flux balance laws (Banyuls et al. 1997; Font et al. 2000), we project the local conservation equations along the space-like hypersurfaces and the normal direction to such hypersurfaces. A straightforward calculation yields the set of equations in the desired form:

\[
 \frac{1}{\sqrt{-g}} \left[ \partial_i (\sqrt{\tilde{g}} q^i) + \partial_j (\sqrt{-g} f^{(i)}(q)) \right] = s(q), \quad (5)
\]

where \( g \) is the determinant of the four-metric (1), \( q \) is a vector of conservative variables, \( f^{(i)}(q) \) are the fluxes along each spatial direction, and \( s(q) \) is a source vector. These last quantities are given by

\[
 q = [D_v M_f, \tau]^T = [\rho \Gamma, \rho \phi \Gamma^2 v_j, \rho \phi \Gamma^2 - \rho \Gamma^2]^T, \quad (6)
\]

\[
 f^{(i)}(q) = \left[ \left( v^i - \beta^i \alpha \right) D_v, \left( v^i - \beta^i \alpha \right) M_f \right] \\
 + \delta^i_j p, \left[ v^i - \beta^i \alpha \right] \tau + \tau p \right]^T, \quad (7)
\]

\[
 s(q) = [0, T^{\alpha\mu}\partial_{\nu}\alpha - \alpha T^{\mu\nu}\Gamma^{ij}_{\mu\nu}]^T. \quad (8)
\]

In these expressions, \( \tilde{g} \) is the determinant of the spatial metric, \( \Gamma^{ij}_{\mu\nu} \) are the Christoffel symbols, and \( v^i \) is the 3-velocity measured by a Eulerian observer and defined in terms of the spatial part of the 4-velocity \( u^i \) as \( v^i = u^i/\Gamma + \beta^i/\alpha \), where \( \Gamma \) is the Lorentz factor given by \( \Gamma = 1/\sqrt{1 - \tilde{\gamma}_{ij} v^i v^j} \).

It is still necessary to close the system of Equations (5), for which an equation of state relating \( p = p(\rho, \epsilon) \) is used. We choose the gas to obey an ideal gas equation of state:

\[
 p = \rho c_s^2 (\gamma - 1), \quad (9)
\]

where \( \gamma \) is the adiabatic index or the ratio of specific heats. Something that stands out is that the relativistic sound velocity \( c_s \) for an ideal equation of state can be written as \( c_s^2 = p\gamma(\gamma - 1)/(\rho\gamma - \rho(\gamma - 1)) \), where its asymptotic value or its maximum permitted value is \( c_{s,\text{max}} = \sqrt{\gamma - 1} \). Thus, the choice of our initial values is restricted to this condition.

3.2. Numerical Methods

Gas Evolution. The general relativistic Euler system of Equation (5) is solved in time using the method of lines, which uses a third-order total variation diminishing (TVD) Runge–Kutta time integrator (Shu & Osher 1988). These are discretized using a finite volume approximation together with high-resolution shock capturing methods. Specifically, we use the HLLE (Harten et al. 1983; Einfeldt 1988) approximate Riemann solvers in combination with the minmod linear piecewise reconstructor. The numerical fluxes and sources in Equation (5) depend both on the conservative and the primitive variables \( w = (\rho \phi, \tau, p) \). Then, in order to express primitive in terms of conservative variables, we use a Newton–Raphson algorithm for each time step within the evolution scheme.

Boundary Conditions. We numerically study the relativistic gas on the equatorial plane, in the domain \([r_{\text{exc}}, r_{\text{max}}] \times [0, 2\pi] \) with resolution \( (\Delta r, \Delta \phi) = (0.158, 0.05) \) for all cases. We choose the interior boundary \( r_{\text{exc}} \) to be inside the BH horizon, where we apply a numerical excision (Seidel & Suen 1992), i.e., we apply a cutoff inside the event horizon, which is possible due to the Kerr–Schild coordinates used by CAFE. The exterior boundary \( r_{\text{max}} \) is split into two halves: one in
which the gas enters the domain where we apply inflow boundary conditions, and a second half where the gas leaves the domain and we apply outflow boundary conditions. In addition, in the angular domain, we use periodic boundary conditions.

Initial data. As initial data, we consider a wind that fills the whole domain, moving on the equatorial plane along the x direction with a non-constant density profile. We characterize the initial velocity field $v'$ in terms of the asymptotic initial velocity $v_\infty$, as in Cruz-Osorio et al. (2012) and Font et al. (1999), where the relation $v^2 = v_1 v' = v_\infty^2$ is satisfied. Using Kerr–Schild coordinates, the explicit expressions for the velocity vector field $v'$ are given by

$$v^r = H_1 v_\infty \cos \phi + H_2 v_\infty \sin \phi,$$

$$v^\phi = -H_3 v_\infty \sin \phi + H_4 v_\infty \cos \phi,$$

where $H_i (i = 1, 2, 3, 4)$ represent functions associated with the components of the three metric:

$$H_1 = \frac{1}{\gamma_{rr}},$$

$$H_2 = \frac{H_3 H_4 \gamma_{\phi\phi} + H_1 H_3 \gamma_{rr}}{H_1 \gamma_{rr} + H_4 \gamma_{\phi\phi}},$$

$$H_3 = \frac{H_2 \gamma_{rr} + H_4 \gamma_{\phi\phi}}{\sqrt{(\gamma_{rr} \gamma_{\phi\phi} - \gamma_{\phi\phi}^2)(H_1^2 \gamma_{rr} + H_2^2 \gamma_{\phi\phi} + 2 H_1 H_4 \gamma_{rr})}},$$

$$H_4 = \frac{2 \gamma_{\phi\phi}}{\sqrt{\gamma_{rr} \gamma_{\phi\phi}}}.\quad (15)$$

The profiles of the components of the field velocity can be seen in Figure 1.

Following Ruffert’s work (Ruffert & Anzer 1995; Ruffert 1997, 1999), the initial density gradient is chosen in such a way that it is perpendicular to the gas motion and is also proposed as a hyperbolic function in order to serve as a cutoff at large distances for large density gradients. The density distribution, in polar coordinates, is given by the following expression:

$$\rho_{\infty} = \rho_0 \left[ 1 - \frac{1}{2} \tanh \left( 2 \epsilon_{\rho} \frac{(r \sin \phi + a \cos \phi)}{r_a} \right) \right],\quad (16)$$

where $\rho_0$ is a constant density value, $a = J/M$ is the spin of the BH, $\epsilon_{\rho}$ is the parameter specifying the magnitude of the density gradient, and $r_a$ is the accretion radius, which is defined in terms of the asymptotic values of the sound speed $c_{\infty}$ as in Petrich et al. (1989):

$$r_a = \frac{M}{v_\infty + c_{\infty}}.\quad (17)$$

Figure 2 illustrates the density profile for two values of $\epsilon_{\rho}$.

Once we fix the value of $c_{\infty}$ and assume the gradient density profile (16), the pressure can be found from the equation of state as $p_{\infty} = c_{\infty}^2 \rho_{\infty}/(\gamma - c_{\infty}^2 \gamma)$, where $\gamma = \gamma/(\gamma - 1)$. In order to avoid negative and zero values on the pressure, the condition $c_{\infty} < \sqrt{\gamma - 1}$ has to be satisfied.

Now, $v_\infty$ and $c_{\infty}$ are two useful parameters which define the relativistic Mach number at infinity, $\mathcal{M}_\infty = v_\infty/c_{\infty}$, and $\mathcal{M}_\infty = \Gamma_{\infty} \mathcal{M}_\infty$. Here, $\Gamma_{\infty}$ is the Lorentz factor calculated with the speed of sound and $\mathcal{M}_\infty$ is the asymptotic Newtonian Mach number used to parametrize the initial configurations. The initial data is parametrized using the Mach number given in Table 1.

We use CAFE (Lora-Clavijo et al. 2015), a fully three-dimensional relativistic-MHD code. Although the MHD in CAFE is written for a Minkowsky spacetime (it does not use curved spacetime), the HD routine does utilize the equatorial (Cruz-Osorio et al. 2012, 2013) and axial (Lora-Clavijo & Guzmán 2013b) symmetries in order to allow simulations using fixed curved spacetimes. CAFE also solves the Einstein field equations coupled with relativistic HD in spherical symmetry (Guzman et al. 2012; Lora-Clavijo et al. 2015).
The numerical methods available in CAFE include several reconstructors. For instance, MINMOD and MC linear piecewise methods; for higher reconstructors, CAFE uses the PPM parabolic method and the WENO5 polynomial method. All of these reconstructors are used in combination with an HLLE approximate Riemann solver flux formula.

Regarding relativistic MHD, in order to preserve magnetic field divergence, CAFE uses flux constraint-transport and divergence cleaning methods. Numerical tests of the numerical implementation in CAFE can be found for the relativistic HD in Lora-Clavijo et al. (2013a) and Lora-Clavijo & Guzmán (2013b), and for relativistic MHD in Lora-Clavijo et al. (2015).

### 3.3. Diagnostics

In order to diagnose the amount of mass and angular momentum accreted by the Kerr BH, we implement a detector located as close as possible to the (outer) event horizon at

\[ r_e = M + \sqrt{M^2 - a^2}. \]

This implies that we define a sphere where we compute said scalars. The following formulae represent both quantities, respectively:

\[ M = \int_0^{2\pi} \alpha \sqrt{\gamma} D(v^r - \beta^r/\alpha) d\phi, \]  \hspace{1cm} (18)

\[ \dot{\rho}^\phi = -\int_0^{2\pi} \alpha \sqrt{\gamma} T^\phi r d\phi + \int_0^{2\pi} \int_{r_{\text{exc}}}^{r_{\text{det}}} S^\phi dr d\phi, \]  \hspace{1cm} (19)

where \( S^\phi \) stands for the \( \phi \) component (\( j = \phi \)) of the source term of Equation (8), \( r_{\text{exc}} \) is the excision radius, and \( r_{\text{det}} \) is the radius where the detector lies.
Figure 3. This figure shows the evolution of the mass accretion rates (the mass accretion rate is rescaled by $10^{-10}$). The mass accretion rate is measured near the event horizon of the BH (whose radius is a function of the spin parameter). Each model is run at least until steady state is achieved. We show the cases considering various values of the Mach number $\mathcal{M} = 2, 3, 4, 5$ (columns) and different values for the BH spin parameter $a = 0.3, 0.5, 0.7, 0.9$ (rows). In each figure, we consider three values of the density gradient $\epsilon_\rho = 0, 0.2, 0.5$. We find that the mass accretion rate increases as the Mach number decreases. In contrast, we can see that the density gradient has little influence on the accretion rate. We remind the reader that the time is in units of $M$ with $1 M_e \equiv 4.93 \times 10^{-6}$ s.
4. RESULTS

We have studied the parameter space for mass and angular-momentum accretion rates. Our parameters are the Mach number, $\mathcal{M}$ (the columns in our plot array in Figures 3 and 4), the spin of the BH, $a$ (the rows in our plot arrays in Figures 3 and 4) and the density-gradient parameter of the gas, $\epsilon_{\rho}$.
represented by the different color lines in our plot array in Figures 3 and 4).

All the images in Figure 5 are snapshots of the system in steady state or the stationary regime. These figures illustrate the BH rotating counter-clockwise for positive values of $a$ and with the flow moving from left to right. That is, they correspond to timescales where mass and angular momentum accretion rates have stabilized, and thus the curves (in Figures 3 and 4) have plateaued.

Our results, as seen in the first column (for $\epsilon_\rho = 0$) of Figure 5, correctly reproduce the expected properties of the shock cone when there exists no density gradient in the medium where the BH moves. That is, we observe the formation of a symmetric shock cone whose width depends on the Mach number; as the Mach number increases, the shock-cone angle decreases (Font & Ibáñez 1998a, 1998b; Font et al. 1998, 1999; Cruz-Osorio et al. 2012; Lora-Clavijo & Guzmán 2013b).

4.1. Morphology

We present, for the first time, the two-dimensional (2D) morphology of the relativistic BHL accretion onto a BH considering density gradients (Figure 5). It is worth mentioning that in order to illustrate the general morphology of the system, we present only the case of $\mathcal{M} = 5$; however, we have covered all of the configurations presented in this paper. This first attempt is performed in slab symmetry, which is the first of a series of steps toward more realistic 3D simulations. The final objective would be to consider a non-fixed background in full nonlinear GR while studying the gravitational wave signature of this configuration. The color gradient represents the

**Figure 5.** Morphology of the rest mass density at stationary state; we can observe that the shock cones are dragged due to the density gradients. We show models with Mach number $\mathcal{M} = 5$. Different rows, from top to bottom, show results for $a = 0, 0.5, 0.9$. The columns, from left to right, show density-gradient parameters $\epsilon_\rho = 0, 0.2, 0.5$. The contour line range is $[-12.6, -10]$ and the space between one line and another is 0.1 in each plot. We remind the reader that the length is in units of $M$ with $1 M_\odot = 1.48 \times 10^5$ cm.
logarithm of the gas density in geometrical units normalized to the BH mass. The contour lines simply emphasize the density gradient. These figures dramatically illustrate the effect that the density gradient has on the shock cone once steady state is achieved.

From the first column of Figure 5, we observe (as in Font et al. 1999, where zero density gradient is considered) that as \(a\) increases, so does the induced angular momentum in the wind. Now, as the density gradient increases, we observe the most notable feature in these figures, i.e., the shock cone is further pushed toward the lower density region.

4.2. Mass and Angular Momentum Accretion Rates

From the plots on Figures 3 and 4, we observe that, as expected, the mass accretion rate decreases as the Mach number, \(M_{\infty}\), increases. Unlike the Newtonian case where no trend between the mass accretion rate fluctuations and the density gradient was discerned (Ruffert 1999), our results exhibit a stationary state rather quickly and show that as \(\epsilon_{\rho}\) increases, the mass accretion rate slightly decreases. The faster the fluid moves with respect to the BH, the faster the steady states settle in. The mass accretion rate is, mostly, independent of the spin parameter of the BH. The reason for the steady-state settling rather quickly may be due to the fact that our simulations are performed in slab symmetry as opposed to 3D. It could also be that we are dealing with relativistic flows. We confirm that the accretion rates do decrease slightly when the density gradient increases. Besides, when the BH spin increases, the effects of the wind density gradient on the mass accretion rate become slightly stronger.

We further explore cases where the accretor has angular momentum. As \(M_{\infty}\) decreases, the difference between the mass accretion rate, for different \(\epsilon_{\rho}\), becomes larger. This is not noticeable, however, in the case where \(M_{\infty} = 2\) because for lower velocities, the steady state settles in much later (note that the timescale is \(\sim 3\) times longer and it is still not fully achieved; this would be consistent with the Newtonian results in Ruffert 1999).

The behavior of the angular momentum accretion rate (Figure 4), instead, clearly shows that for higher wind velocities (increasing \(M_{\infty}\)), steady state is reached much more quickly. For \(M_{\infty} = 4\) and \(M_{\infty} = 5\), steady state is achieved after about \(t = 200\) and \(t = 100\), respectively, regardless of BH spin or density gradient. Instead, looking at \(M_{\infty} = 3\) and, in particular, at \(M_{\infty} = 2\), it is clear that a steady state is not fully reached in \(t = 600\) for the former and not even in \(t = 2000\) for the latter. This trend is more noticeable for a lower density gradient and/or higher BH spin. In other words, it appears that increasing the density gradient helps to stabilize the angular momentum accretion rate at an earlier time. We also observe that the angular momentum rates extered on the wind decrease further and further to negative values as the BH spin, \(a\), increases. It can also be seen that higher Mach number has the effect of slightly increasing the angular momentum accretion rate at steady state for a large density gradient (\(\epsilon_{\rho} = 0.5\)). For a lower density gradient (\(\epsilon_{\rho} = 0.2\)), this trend is not evident. When no density gradient exists (\(\epsilon_{\rho} = 0\)), no correlation seems to exist.

We have also produced a set of simulations where the spin of the BH counter-rotates, \(a < 0\). The first case has \(a = -0.9\), \(\gamma = 5/3\) and \(M_{\infty} = 0.3\); the second case has \(a = -0.9\), \(\gamma = 5/3\) and \(M_{\infty} = 0.4\); finally, the third case has \(a = -0.99\), \(\gamma = 5/3\) and \(M_{\infty} = 0.4\). In all three cases, we performed simulations for the same three values of the density gradient, i.e., \(\epsilon_{\rho} = 0\), 0.2, and 0.5. As opposed to the cases described above, here the angular momentum of the accreted material is parallel to that of the BH, and thus it is expected that mass accretion will tend to increase the spin of the BH over the long run.

As can be observed from the simulations for low Mach number (see Figure 7), a disk-like structure forms. The velocity field has a positive radial coordinate, and thus little or no accretion occurs.

5. DISCUSSION AND CONCLUSIONS

We have performed a parameter study of BHL accretion for a relativistic wind with a density gradient onto Schwarzschild and Kerr BHs with different spin parameters. We discuss our results in the following paragraphs.

Comparing Figures 3, 4, and 6, it is interesting to note that the mass accretion rate, unlike the angular momentum rate (see first column plots in Figure 4), does not depend strongly on the wind density gradient regardless of Mach number or BH spin. As can be observed from our Figure 3, observing the plots from left to right, as the Mach number increases the mass accretion rate decreases, which agrees with previous studies as well as theory. It is also important to note that as the Mach number increases, the steady state is obtained much more rapidly. This is further implied by the results in Ruffert (1999), where only Newtonian velocities are achieved and the plots show much more variability in both mass and angular momentum accretion.

From the plots in Figure 4, we find that the wind may be accreted with low or even negative values of angular
momentum with respect to the spin of the BH. This implies that were a significant amount of material to be accreted (over long periods of time), the spin of the BH could be lowered significantly or even reversed in those cases where the angular momentum has negative values. On the other end of the BH spin spectrum, from Figure 6, it is clear that the spin of a BH can increase over time if the spin and the wind density gradient are properly aligned.

From the morphology plots (Figures 5), we observe a new signature, i.e., the Mach cones wrap around the BH, even with low or nil spin, due to the density gradient of the winds.

Probably due to the fact that our runs do stabilize rather quickly, we observe that an accretion disk starts to form for \( \gamma = 5/3 \), low Mach number, high BH spin, and high density gradient (\( \epsilon_\rho = 0.5 \); see Figure 7).

As noted above, in the simulations where a disk forms, the velocity field has a positive radial component, which would imply that no accretion occurs. However, the fluid modeled by our code has no account of radiative losses. Thus, in a more realistic simulation, it is highly likely that energy would be radiated away and angular momentum would be transported outwards within the disk, allowing for substantial accretion onto the BH.

5.1. Astrophysical Scenarios and Applications

From Figure 3, we can observe that for a \(~10 M_\odot\) BH, steady-state accretion can be achieved after a few milliseconds. For IMBHs the timescales necessary for reaching steady-state accretion go from seconds to hours. Meanwhile, for SMBHs said timescales go from hours to years.

In the hypernova-explosion scenario for sBHs and IMBHs, accretion will probably last a few minutes, and thus little mass or angular momentum may be accreted.
For the scenario where a SMBH crosses through the jet of a companion SMBH, substantial mass may be accreted if the orbit and the supply of material to the BH producing the jet are stable. However, the effects of the density gradient modifying the spin of the BH may be cancelled out if the axis of the jet is on the orbital plane. This occurs because the density increases as the BH enters the cone of the jet, however, as the BH exits the jet, the opposite effect takes place, thus cancelling most of the effect. If the axis of the jet is slightly off the orbital plane, then there is a component of the density gradient that does not get cancelled out and a BH spin can be built up over many orbital periods. If two jets, as opposed to one, exist and they are symmetric, then the effect of the density gradient on the BH spin will also be cancelled out. Were the jet to precess (so that the axis of the jet is sometimes above the orbital plane and sometimes below) on a timescale much larger than the orbital period, spin reversal could be observed on the BH that cuts the jet. Perhaps more important is the fact that these processes are accreting mass but, more likely than not, no angular momentum, and thus the spin, of the BH will decrease (as the spin $a \propto J/M^2$).

The reader may ask how likely it is that the jet of a SMBH may hit another one. There are a few examples where binary SMBHs have been observed (e.g., Valtonen et al. 2008, 2012; Fabbiano et al. 2011; Graham et al. 2015). The most relevant parameters to estimate the likelihood of such an event, we suspect, are the following: first, the jet must have a large angle, otherwise chances are very small. Second, it is suspected that these binaries are the product of a merger of two galaxies, and hence it is likely that if the spin of the SMBHs are similar to those of their hosts (e.g., Barausse 2012) and they collide at random angles, the SMBHs spins will also be random. Third, the impact parameter of the collision should, preferably, be small such that the BHs do not have to travel far within the newly merged galaxy, otherwise they may accrete substantial amounts of matter (see, e.g., Armitage & Natarajan 2002; Dotti et al. 2009) which will have a preferred angular momentum (that of the host galaxy) and may reorient the axes toward that of the reshaped host (Dotti et al. 2009), thus preventing them from hitting each other with their jets. The masses of the SMBHs will be important as well, as the spin of a more massive BH will be less affected by accretion as it drifts inwards to meet the companion BH (Barausse 2012).

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