Impact of the nuclear mass uncertainties on the $r$ process

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(Dated: February 5, 2019)

Abstract
Based on a simple site-independent approach, we attempt to reproduce the solar $r$-process abundance with four nuclear mass models, and investigate the impact of the nuclear mass uncertainties on the $r$ process. In this paper, we first analyze the reliability of an adopted empirical formula for $\beta$-decay half-lives which is a key ingredient for the $r$ process. Then we apply four different mass tables to study the $r$-process nucleosynthesis together with the calculated $\beta$-decay half-lives, and the existing $\beta$-decay data from FRDM+QRPA is also considered for comparison. The numerical results show that the main features of the solar $r$-process pattern and the locations of the abundance peaks can be reproduced well via the $r$-process simulations. Moreover, we also find that the mass uncertainties can significantly affect the derived astrophysical conditions for the $r$-process site, and resulting in a remarkable impact on the $r$ process.

PACS numbers: 21.10.Dr, 21.60.Jz, 26.30.Hj

Keywords: nuclear mass; $\beta$-decay half-life; $r$-process nucleosynthesis; solar $r$-process abundance

I. INTRODUCTION

It has long been thought that the rapid neutron capture process ($r$-process) is responsible for the synthesis of half of the heavy elements beyond iron [1, 2]. In the $r$ process, the iron group or the nuclei up to $A \approx 90$ are regarded as seed nuclei, and then capture the neutrons in time scales shorter than $\beta$ decay. In this way the corresponding pathway of $r$-process involves many neutron-rich nuclei which are far away from the $\beta$ stability, even close to the neutron drip-line. Unfortunately only few of these very exotic nuclei can be produced in current or even next-generation rare isotope beam facilities. As a result, calculations based on theoretical models of the reaction chain are essential to understand the origin of the heaviest nuclei in the universe and the $r$-process nucleosynthesis.

As to the $r$-process nucleosynthesis, one of the underlying difficulties is the astrophysical sites, which have not yet been unambiguously identified up to now. However, there is common agreement that the $r$-process occurs on the premise of extreme neutron densities, and then runs through very neutron-rich nuclei far-off the valley of stability [3]. So far many studies have been performed for the candidate sites [4–15] (for examples, the "neutron star (NS) merger" [7, 8], the "prompt explosion" from a low mass SN [9, 10], and the "collapsar" from a massive progenitor [11, 12]). Nevertheless each of them still faces severe problems and cannot be asked to explain the production of the $r$-process nuclei observed in nature, and leading to a lack of consensus at the present time.

The other puzzle is the nondeterminacy of the nuclear properties which are essential for the $r$-process and can not be gained using the current experimental techniques. In particular, mass predictions for neutron-rich nuclei have a vital influence on the relevant nuclear reactions in $r$-process, such as the photodisintegration,
the neutron-capture, the fission probabilities, and the $\beta$-decay rates. In view of the importance of nuclear mass in the $r$ process, many related theoretical calculations were performed in the past decades. One conventional method is the local mass relations, which have a high precision of prediction for nearby nuclei, such as the Garvey-Kelson relations [16], residual proton-neutron interactions [17, 18], Coulomb-energy displacement [19, 20], and systematics of $\alpha$-decay energies [21]. The other method depends on the global mass models. These mass models are generally regarded to have a better extrapolation for nuclei far from the known region, for examples, the macroscopic-microscopic finite-range droplet model (FRDM) [22], the Weizs"acker-Skyrme (WS) model [23], the microscopic Hartree-Fock-Bogoliubov (HFB) theory with a Skyrme force [24] and the relativistic mean-field (RMF) model [25–27]. Compared to the former mass relations, the accuracies of these global mass models are worse. Therefore, several methods have been introduced to improve their accuracies, such as the CLEAN image reconstruction technique [28], the radial basis function approach [29–33], and the neural network approach [34–36].

After Burbidge’s systematic introduction to the $r$ process for the first time [37], due to the lag of experimental and theoretical development, only the phenomenological nuclear droplet mass formula [38] could be considered for the $r$-process calculations in a long period. Fortunately by now, the theoretical study of nuclear properties has made a great progress which results in lots of various $r$-process calculations [39–43].

In addition to the mass, the $\beta$-decay half-life plays key role in estimating the $r$-process abundances, which determines the time-scale for the matter flow from the seeds to the heavy nuclei. In fact, the evaluation of $\beta$-decay rates for the waiting-point nuclei is one of the important issues of the nucleosynthesis through the $r$ process. As we known, the famous Fermi theory [44] in the 1930’s is commonly regarded as the starting point of the theoretical investigations for the nuclear $\beta$-decay. Nowadays, as the extremes, there are mainly two types of microscopic approaches for the large-scale calculations of nuclear $\beta$-decay half-lives: the shell model and the proton-neutron quasiparticle random-phase approximation (QRPA). Due to extremely large configuration spaces, the shell-model calculations can not be applied to the heavy nuclei far from the magic numbers. In contrast, the proton-neutron QRPA are feasible for the arbitrary heavy systems [45–47].

Thus far, the nuclear $\beta$-decay calculations have already been carried out using the QRPA based on the FRDM [48], the extended Thomas-Fermi plus Strutinsky integral (ETFSl) model [49], the Skyrme Hartree-Fock-Bogoliubov (SHFB) model [45], the density functional of Fayan (DF) [50], and the covariant density functionals [51–53]. Recently, Zhang et al. [54] presented a new exponential law for calculating $\beta$-decay half-lives of nuclei far from the stability line. In 2017, Zhou et al. [55] proposed an empirical formula for $\beta$-decay half-lives via investigating systematically the variation of $\beta$-decay half-lives with the decay energy $Q$ and nucleon number $(Z, N)$ based on experimental data. Here both shell and pairing effects on $\beta$-decay half-lives versus the nucleon number $(Z, N)$ are taken into account.

In this paper, based on Zhou’s empirical formula for $\beta$-decay half-lives, we consider four nuclear mass models to provide the theoretical $Q$-values for $\beta$-decay half-lives. In order to testify the precision of this formula for reliably predicting $\beta$-decay half-lives in this paper, comparisons about the root-mean-square (rms) deviation between the calculated $\beta$-decay half-lives and the experimental data [56], together with the existing $\beta$-decay properties from FRDM+QRPA [48], are performed. Then with the calculated $\beta$-decay half-lives, four nuclear mass models are applied to investigate the impact of theoretical uncertainty of unknown masses in the $r$-process calculation. The numerical results show that the acquired astrophysical conditions for the $r$-process are significantly different with each other and very sensitive to the adopted mass models, and the impact of mass uncertainties on the solar $r$-process abundances is very important to completely understand the $r$-process.

This paper is organized as follows. In section II, the calculating formula for $\beta$-decay half-lives and the introduction to a site-independent $r$-process approach are briefly described. In section III, four nuclear mass models are considered to provide the $Q$-values for $\beta$-decay half-lives with the empirical formula. Then they are subsequently applied to reproduce the solar $r$-process abundances. In addition, the numerical
results of the simulation results as well as some detailed discussions are given, including some comparisons between different mass models with (without) the calculated $\beta$-decay half-lives. Finally, a concise summary is delivered in section IV.

II. THE THEORETICAL METHOD

A. The employed formula for the $\beta$ decay half-lives

Recently, with corrections to both pairing and shell effects, Zhou et al. [55] proposed a formula to calculate the $\beta^-$-decay half-lives of the neutron-rich nuclei far from the stability line,

$$\ln t_{1/2} = a_6 + (\alpha^2 Z^2 - 5 - a_7 \frac{N - Z}{A}) \ln(Q - a_8 \delta) + a_9 \alpha^2 Z^2 + \frac{1}{3} \alpha^2 Z^2 \ln(A) - \alpha Z \pi + S(Z, N),$$

(1)

in which the relationship among $\beta^-$-decay half-lives, $Q$-values and nucleon numbers $(Z, N)$ is detailed shown. In this equation, $\alpha$ is the fine structure constant with value $\frac{1}{137}$. The most important term $(\alpha^2 Z^2 - 5 - a_7 \frac{N - Z}{A}) \ln(Q - a_8 \delta)$ has a dominant effect. The accuracy of $Q_{\beta}$-values is an strong influence on this formula, and $Q_{\beta}$-values can be calculated using mass data as $Q_{\beta} = M_p - M_d$, $M_p$ and $M_d$ are the masses of parent and daughter nuclei, respectively. $\delta$ denotes the even-odd staggering caused by pairing effects on $\beta^-$-decay half-lives versus $Q$-values, and can be described by

$$\delta = (-1)^N + (-1)^Z.$$

It is pointed out that since the mass table [23] already contains the pairing energy, the pairing effects on $Q$-values versus neutron number have already been embodied in the calculation of $Q$-values.

As for the shell correction $S(Z, N)$, its contributions only appear near the nucleon magic-numbers, and can be written as

$$S(Z, N) = a_1 e^{-((Z-20)^2 + (N-28)^2)/12} + a_2 e^{-((Z-38)^2 + (N-50)^2)/43} + a_3 e^{-((Z-50)^2 + (N-82)^2)/13} + a_4 e^{-((Z-58)^2 + (N-82)^2)/24} + a_5 e^{-((Z-70)^2 + (N-110)^2)/244}. $$

(2)

In the equation (1), all the nine parameters in Eq.(1) can be obtained explicitly through a least-squares fit to the available experimental data of $\beta^-$-decay half-lives [56]. In addition, as we known, the ratio of calculated value $T_{1/2}^{\text{theo}}$ to experimental value $T_{1/2}^{\text{expt.}}$ of $\beta$-decay half-lives can reflect the ability of reproducing experiment data and the extrapolation capacity via this analytical formula. So we defined the rms deviation of the decimal logarithm of the $\beta$-decay half-life, in this paper, as

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left[ \log_{10}(T_{1/2}^{\text{theo}}) - \log_{10}(T_{1/2}^{\text{expt.}}) \right]^2}. $$

(2)

$N$ is the number of nuclei used for evaluation of the rms deviation. The acquired experimental $\beta^-$-decay half-lives are taken from Ref. [56].

B. The site-independent $r$-process approach

The $r$ process not only depends on various inputs of nuclear properties, but is also affected by multifarious astrophysical conditions: the entropy and temperature of the explosion environment, the electron-to-baryon
number ratio \((Y_e)\), and the neutrino processes, etc. It is unsatisfactory that the astrophysical sites for \(r\)-process nucleosynthesis have still not been unambiguously stated. Even many candidate sites have been tried to proposed and supernovae appears to be well suited as the \(r\)-process site \([51]\). However, up to now there has been no conclusion as the correct astrophysical model. In this paper, we employed a site-independent approach (for recent reviews, see, e.g., Refs. \([39, 41, 58, 59]\)), in which the solar \(r\)-process abundances \([60]\) have been used to constrain the astrophysical conditions, to calculate the \(r\)-process abundance with different nuclear physics inputs.

In this approach, it is supposed that neutron sources irradiate the seed nuclei over a time scale \(\tau\) in a high-temperature environment \((T \sim 1 \text{ GK})\). It is particularly important that the neutron sources have high and continuous neutron densities \(n_n\) ranging from \(10^{20}\) to \(10^{28} \text{ cm}^{-3}\). Because of the very high neutron densities, the \(\beta\) decays will be largely overwhelmed by the competing neutron captures, leading to the equilibrium \((n, \gamma) \leftrightarrow (\gamma, n)\) for every element. The abundance ratio of two isotopes on a time scale \(\tau\) can be written as

\[
\frac{Y(Z, A+1)}{Y(Z, A)} = n_n \left( \frac{h^2}{2\pi m_\mu \kappa T} \right)^{3/2} \frac{G(Z, A+1)}{2G(Z, A)} \left( \frac{A+1}{A} \right)^{3/2} \exp \left[ \frac{S_n(Z, A+1)}{\kappa T} \right].
\]

In the above equation, the symbols \(Y(Z, A)\), \(S_n\), \(G(Z, A)\) denotes the abundance of the nuclide \((Z, A)\), the one-neutron separation energy and the partition function of nuclide \((Z, A)\), and \(h, \kappa, \text{ and } m_\mu\) are the Planck constant, Boltzmann constant, and atomic mass unit, respectively. In addition, suppose \(Y(Z, A+1)/Y(Z, A)\) is close to 1 at the highest isotopic abundance for each element, and all other quantities are fixed to be constant, the average neutron-separation energy \(\overline{S}_n\) will be the same for all the nuclides with the highest abundance for each isotopic chain. In this method, \(\overline{S}_n\) is defined as

\[
\overline{S}_n \approx \kappa T \log \left[ \frac{2}{n_n} \left( \frac{2\pi m_\mu \kappa T}{h^2} \right)^{3/2} \right] = T_\beta \{2.79 + 0.198 \log(\frac{10^{20}}{n_n}) + \frac{3}{2} \log(T_\beta)\},
\]

where \(T_\beta\) correspond to the temperature in \(10^9\) K.

While we ignore the fission reaction, the matter flow in \(r\) process is governed by \(\beta\) decays. The corresponding abundance can be shown using a group of differential equations:

\[
\frac{dY(Z, A)}{dt} = Y(Z-1) \sum_A P(Z-1, A) \lambda^{Z-1, A}_\beta - Y(Z) \sum_A P(Z, A) \lambda^{Z, A}_\beta,
\]

here \(Y(Z) = \sum_A Y(Z, A) = \sum_A P(Z, A)Y(Z)\) is the total abundance for each isotopic chain \((P(Z, A)\) is the isotopic abundance distribution\). \(\lambda^{Z, A}_\beta\) is the total decay rate of the nuclide \((Z, A)\) via the delayed neutron emission or the \(\beta\) decay. Then after the neutrons freeze out, all the isotopes will go to the corresponding stable elementary substance via the \(\beta\) decays, for which the abundance can be obtained based on the two equations (3) and (5).

III. RESULTS AND DISCUSSIONS

A. The reliability of the systematic formula

In the \(r\) process, there are two reasons why \(\beta\)-decay lifetimes are regarded as the critical inputs. The first one is that they set the timescale for heavy element production if the equilibrium \((n, \gamma) \leftrightarrow (\gamma, n)\) occurs. Another is that \(\beta\)-decay lifetimes help to shape the final pattern as the path moves back to stability. In this paper, the needed \(\beta\)-decay lifetimes will be calculated by using the empirical formula \([55]\). However, it is doubtful to what extent the predictive ability of the adopted formula in this paper can achieve. According to the formula expressed in Eq.(1), one can see half-life calculations depend somewhat on the values of the
TABLE I: Comparisons between the rms deviations of the experimental $\beta^-$-decay half-lives and the calculated data from four nuclear mass tables, together with the deviations arising from FRDM+QRPA and the Eq.(1) itself. The first column includes the symbols for the corresponding nuclear mass tables, the extractive $\beta^-$-decay properties from Ref. [48] (FRDM+QRPA) and the equation 1 itself (EQ.(1)). The first row contains the adopted regions of experimental $\beta^-$-decay half-life values. See text for more details.

|        | $T \leq 1\text{ s}$ | $T \leq 1000\text{ s}$ |
|--------|---------------------|-------------------------|
| FRDM   | 0.381               | 0.598                   |
| RMF    | 0.433               | 0.730                   |
| HFB-31 | 0.375               | 0.651                   |
| WS4    | 0.316               | 0.529                   |
| Eq.(1) | 0.364               | 0.584                   |
| FRDM+QRPA | 0.391           | 0.597                   |

parameter $Q$, then reliable theoretical predictions of the nuclear mass, bringing about precise $Q$-values, are essential to study $\beta$ transitions when these nuclear masses can not be given experimentally.

In this subsection, four different mass tables, i.e., the FRDM model [22], the latest version of WS (WS4) model [23], the recent version of the HFB model (HFB-31) [61], and the RMF model with TMA effective interaction [25], are considered to demonstrate the predictive ability of the systematic formula [55]. The model deviation of binding energy $B$ with respect to the experimental data can be characterized by the rms deviation $\sigma_{rms} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (B_{theo} - B_{expt})^2}$, and the corresponding numerical numbers for the FRDM, WS4, HFB-13, and RMF with respect to experimentally determined values [62] are 0.677, 0.302, 0.584 and 2.258 MeV, respectively. The other model deviations of $\log_{10}(T_{1/2}^{theo} / T_{1/2}^{expt})$ for these four mass models and the Eq.(1) itself are list in Tab. I, and for comparison we also take into account the rms deviations between the existing $\beta$-decay properties taken from Ref. [48] (FRDM+QRPA) and recent experiment data [56]. By the way, according to the updated information of Audi et al. [56], there are totally about 1101 nuclei for all possible $\beta^-$ transitions with well-defined half-lives ($T$). The eligible data set adopted in this paper contains 824 nuclei with their $Z$, $N$ and half-life values being in the selected regions of $Z \geq 8$, $N \geq 8$ and $T \leq 1000$ second (s), respectively. Furthermore, for the sake of illustration, two categories of comparisons are performed according to the regions of adopted experimental $\beta^-$-decay half-life values $T$, i.e., $T \leq 1\text{s}$ and $T \leq 1000\text{s}$.

As can be seen in Tab. I in the first category of $T \leq 1\text{ s}$ (including 381 nuclei), the WS4 model [23], combined with the employed formula [55], gives the smallest rms values of 0.316, which means the overall ratios between calculated $\beta^-$-decay half-life values and experimental ones are about a little more than twice. For FRDM [22], HFB-31 [61] and RMF [25], the corresponding rms deviations are 0.381, 0.375 and 0.433, respectively, less than the rms deviation of 0.391 deduced from traditional FRDM+QRPA [48] except for RMF [25]. The rms deviation obtained using the empirical formula [55] and the experimental $Q$-values [62] is 0.364, also lower than that of FRDM+QRPA [48]. All of these mean that in this category of $T \leq 1\text{s}$, the best estimations of $\beta^-$-decay half-lives come from WS4 [23] while the worst one from RMF [25]. For FRDM [22], HFB-31 and the Eq.(1) itself, anyone of them has a higher precision of prediction for $\beta^-$-decay half-lives than FRDM+QRPA [48]. In another category of $T \leq 1000\text{s}$ (including 824 nuclei), one can see that WS4 [23] still provides the optimal outcome with the $\sigma$-values being only 0.529. The following one is 0.584 deduced from the Eq.(1) itself [55] and the experimental $Q$-values [62]. FRDM+QRPA [48] wins third place with the corresponding value being 0.597. While considering FRDM [22], HFB-31 [61] and RMF [24], the calculated rms deviations are 0.598, 0.651 and 0.730, respectively, all higher than that of FRDM+QRPA [48]. These mean that the similar conclusions for both WS4 model [23] and the Eq.(1) itself [55] can be attained in comparison with FRDM+QRPA [48]. But for FRDM [22] and HFB-31 [61], the opposite is true. That is
with the empirical formula [55], WS4 [23] brings the best accuracy of $\beta^-$-decay half-life prediction, followed by the combination of the formula itself [55] and the experimental $Q$-values [62]. But the accuracies resulted from both FRDM [22] and HFB-31 [61] are slightly inferior than that from FRDM+QRPA [48]. As is shown, RMF [25] is still last.

Based on above, one can see that the numerical results of the rms deviation deduced from the mass models or the empirical formula itself proposed in Eq.(1) are very different with each other in the two categories. However, it’s worth noting that these rms values of calculated $\beta^-$-decay half-lives deviating from experiment data [56] are generally within the same order of magnitude as that resulted from FRDM+QRPA [48]. That is to say, compared with the existing $\beta^-$-decay properties [48], our evaluations of $\beta^-$-decay half-lives using the employed formula and the four mass models agree well with the experiment values [56] to some extent. So it is believable that the adopted systematic formula, combining with the mass models, for reliably predicting $\beta^-$-decay half-lives is effective, and the corresponding precisions in this work are appropriate.

![FIG. 1: (Color online) Decimal logarithms of $T_{1/2}^{\text{theo}}/T_{1/2}^{\text{exp}}$ for 824 nuclei derived from WS4 model, FRDM+QRPA and the experimental data. Panel (a) corresponds to the results for $\beta^-$-decay half-lives taken from FRDM+QRPA, panel (b) denotes the case for the calculated $\beta^-$-decay half-lives using the WS4 model and the systematic formula proposed in Eq. (1), respectively. The pairs of gray solid parallel lines denote the magic numbers.](image)

For further insight of the extrapolating capacity of the systematic formula, we calculated the $\beta^-$-decay half-lives for whole 824 nuclei, and here the WS4 model is taken as an example. Comparisons of logarithms of the ratios between the experimental half-life values [56], the calculated ones and the existing $\beta^-$-decay properties from FRDM+QRPA [48] are performed, respectively, and are drawn in Fig.1. It can be seen that the calculated $\beta^-$-decay half-lives show a better agreement with the experimental ones except the regions near magic numbers. The reason may be that, as proposed in Ref. [52], for extrapolation to nuclei far from $\beta^-$-stability line, enormous differences in tendency are evident between the results with this formula and those from the exponential law [54]. As a result, The correction of both the shell and pairing effects on $\beta^-$-decay half-lives for the nuclei near the magic numbers in this work is not perfect. However, compared to the case of FRDM+QRPA, the effect of systematic correction is notable, all of which provide strong support for the reliability of the systematic formula and its usefulness for eliminating the discrepancy.
B. The influence of mass uncertainties on the \( r \)-process

As we known, the evaluation of mass formulae and \( \beta \)-decay rates, particularly at the waiting point nuclei, is one of the important issues of the nucleosynthesis through the \( r \) process. In this section, four nuclear mass tables proposed above will be considered to reproduce the solar \( r \)-process abundance. The required \( \beta \)-decay half-lives will be obtained using the empirical formula and the \( Q \)-values from the mass tables themselves and, to have a comparison, the existing \( \beta \)-decay properties from FRDM+QRPA are considered as well.

Similar to the method used in Refs. [39, 41, 58, 59], 16 components with neutron densities in the range of \( 10^{20} \) to \( 3 \times 10^{27} \) cm\(^{-3}\) are applied in our calculations. Then a temperature \( T = 1.5 \) GK is chosen, for which we suppose the corresponding weight \( \omega \) and the irradiation time \( \tau \) follow an exponential dependence on neutron density \( n_n \),

\[ \omega(n_n) = d \times n_n^a, \quad \tau(n_n) = b \times n_n^c, \]  

the four parameters \( a, b, c \) and \( d \) can be obtained by fitting the solar \( r \)-process abundances with a least-square fit. Furthermore, it is also assumed that the longest neutron irradiation time should be longer than 0.5 s but shorter than 20 s.

![FIG. 2: The configuration of sixteen \( r \)-process components that recur the \( r \)-process abundances with different mass inputs. The neutron density \( n_n \) is in units of cm\(^{-3}\). As a function of the neutron density \( n_n \), we display the weighting factor \( \omega(n_n) \) and the neutron irradiation time \( \tau(n_n) \) in the upper and lower panel, respectively. The total weighting factor has been normalized to 100.](image)

In order to investigate the impact of theoretical uncertainties of unknown masses in the \( r \)-process calculations, we take our best simulation using the four mass models, together with the calculated \( \beta \)-decay half-lives. The astrophysical conditions resulted from the various mass inputs are shown in Fig. 2. As it is shown, in the upper panel, the weighting factors for FRDM are almost completely overlaid by those for WS4, and so are RMF and HFB31 models. Moreover, the astrophysical conditions obtained from the former two mass-model simulations require a smaller weighting factor for low neutron density than those from the latter ones, while the case is reverse for high neutron density. On the other hand, in the lower panel, the astrophysical conditions found using the FRDM and WS4 mass inputs favor a relatively constant neutron
irradiation time for different neutron densities, and for the HFB-31 case, the time becomes longer, up to 4 s, as the neutron density increases. The longest neutron irradiation time comes from the simulation using the RMF model, which requires component durations of as long as 10 s. As a result, it is clear that the acquired astrophysical conditions for the $r$-process are significantly different with each other and very sensitive to the adopted mass models.

The solar $r$-process abundances calculated by using different mass models and the calculated $\beta$-decay half-lives are displayed in Fig. 3, and the results obtained with the existing $\beta$-decay properties from FRDM+QRPA are also plotted for comparison. From Fig. 3, one can see, the results of the $r$-process abundance calculations with various nuclear mass models differ from each other, however, all of them bring about an abundance underproduction at $A \sim 120$, which has traditionally been put down to the overestimated strength of the $N = 82$ shell closure\cite{39, 59} in the theoretical nuclear physics model. Compared to the results with the existing $\beta$-decay properties from FRDM+QRPA, the $r$-process simulations with our calculated $\beta$-decay half-lives agree better with the observation for all nuclear mass models, particularly for RMF, the results with the existing $\beta$-decay properties from FRDM+QRPA lead to a significant underestimation of the isotopic abundances between the $A \sim 130$ peak and the $A \sim 195$ peak, then with our calculated $\beta$-decay half-lives, the abundance trough is largely filled in, as shown in Fig. 3(c). Moreover, it also happened for
HFB-31, but to a lesser degree (see Fig. 3(b)). As to nuclear mass input, one can see, the best agreement with the solar abundance pattern comes from WS4 models, followed by FRDM and HFB-31, the worst one is produced by RMF, particularly between the $A \sim 130$ peak and the $A \sim 195$ peak.

IV. SUMMARY

In summary, we first adopt an empirical formula proposed by Zhou et al. [55] for calculating $\beta$-decay half-lives with four nuclear mass tables as well as the experimental $Q$-values [62]. Then we compare the calculated results with the experimental data, together with the existing $\beta$-decay properties from FRDM+QRPA, to demonstrate the precision of the formula for reliably predicting $\beta$-decay half-lives. It is shown that the adopted formula is reliable and useful for eliminating the discrepancy.

Subsequently, in order to investigate the impact of theoretical uncertainties of unknown masses in $r$-process calculations, we apply four nuclear mass models to reproduce the main features of the solar $r$-process pattern and the locations of the abundance peaks based on the site-independent $r$-process approach. The required $\beta$-decay half-lives are calculated using the empirical formula with the theoretic $Q$-values come from these mass tables and, for the sake of comparison, also include the existing $\beta$-decay properties taken from FRDM+QRPA. As is shown above, compared with the results using the existing $\beta$-decay properties, the $r$-process simulations with the calculated data can lead to a better agreement with the observation for all four mass tables, particularly for RMF, the significant underestimation of the isotopic abundances can be largely corrected between the $A \sim 130$ peak and the $A \sim 195$ peak. Furthermore, we have also compared the results of $r$-process simulations obtained from different mass models. One can see that the deduced astrophysical conditions for the $r$-process are significantly different and change along with the used mass model. The $r$-process abundances obtained from WS4 agree best with the observed data, followed by FRDM and HFB-31, the worst one is produced by RMF, all of which provide strong support for a remarkable impact of mass uncertainties on the $r$-process nucleosynthesis.

V. ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China (Grant No.11505001), and the Key Research Foundation of Education Ministry of Anhui Province of China (Grant No. KJ2015A041).

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