Research Article

On Quantum Differential Subordination Related with Certain Family of Analytic Functions

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Recently, there is a rapid increase of research in the area of Quantum calculus (known as \(q\)-calculus) due to its widespread applications in many areas of study, such as geometric functions theory. To this end, using the concept of \(q\)-conic domains of Janowski type as well as \(q\)-calculus, new subclasses of analytic functions are introduced. This family of functions extends the notion of \(\alpha\)-convex and quasi-convex functions. Furthermore, a coefficient inequality, sufficiency criteria, and covering results for these novel classes are derived. Besides, some remarkable consequences of our investigation are highlighted.

1. Introduction

Recently, there is a rapid increase in the area of Quantum calculus (known as \(q\)-calculus) due to its widespread applications in many areas of study such as geometry functions theory (GFT), combinatorial analysis, Lie theory, mechanical engineering, cosmology, and statistics. The concept of \(q\)-integral was first introduced and studied by Jackson et al. [6] at the beginning of the twentieth century.

The development of the concept of \(q\)-calculus in GFT had its history from the work of Ismail et al. [5], where the notion of \(q\)-starlike functions was extensively studied. As such, many subclasses of univalent functions correlated with \(q\)-calculus have been on increase (see [1, 14, 17, 20, 23–25, 27, 28]). In recent times, various family of \(q\)- extension of starlike functions, which are connected to Janowski functions in the open unit disc \(U\), were initiated and examined from many different viewpoints and perspective (see [17, 20, 28]).

In an attempt to generalize the notion of uniformly closed-to-convex functions considered by Goodman [3], Kanas and Wisniowska [8–10, 13] and Kanas and Srivastava [12] introduced the conic domain \(\Omega_m(\alpha, \gamma, \lambda)\) and studied the classes \(m = \text{UCV}\) and \(m = \text{UST}\) of \(m\)-uniformly convex and starlike functions. Furthermore, Noor and Malik [22], using the concept of Janowski class, extended the domain \(\Omega_m(\alpha, \gamma, \lambda)\), \(-1 \leq \lambda < \gamma \leq 1\). In the latest article by Mahmood et al. [17], the importance of \(q\)-calculus was used to improve the Noor–Malik conic domains to \(\Omega_{q,m}(\gamma, \lambda)\).

Using this domain, they examined the coefficient inequalities associated with the class \(m = \text{UST}_q(\gamma, \lambda)\) of \(q\)-uniformly starlike functions. Afterward, the same coefficient problems were also explored for the classes \(m = \text{UCV}_q(\gamma, \lambda)\), \(m = \text{UKV}_q(\gamma, \lambda)\), \(m = \text{UCV}_q^*(\gamma, \lambda)\) of \(m\)-uniformly \(q\)-convex, close-to-convex and quasi-convex functions by Naeem et al. [20].

Motivated by these recent articles [15, 17, 20, 28], our aim is to introduce the novel classes \(m = \text{UM}_{q}(\alpha, \gamma, \lambda)\) and \(m = \text{UQ}_{q}(\alpha, \gamma, \lambda)\) consisting of \(m\)-uniformly \(q\)-alpha convex and quasi-convex functions. We study the coefficient inequalities associated with these classes and some other related properties. Some relevant consequences of our results which were studied in previous work show the significance of our investigation.

2. Materials and Methods

Now, we give some useful preliminaries which are necessary for our study.
Let $\mathcal{A}$ be the class of normalized analytic functions $f(z)$ in $U = \{z: |z| < 1\}$ with
\[ f(z) = c + a_2z^2 + a_3z^3 + \cdots. \]  
(1)

Let $S, C, V, ST, QV,$ and $KV$ be the subclasses of $\mathcal{A}$ consisting functions that are univalent, convex, starlike, quasi-convex, and close-to-convex functions, respectively. The function $f(z)$ of form (1) is subordinate to the analytic function $g(z)$ (written as $f(z) \prec g(z)$) of the form
\[ g(z) = c + b_2z^2 + b_3z^3 + \cdots, \]  
(2)
if there exists a Schwarz function $w(z)$ in $U$ such that
\[ f(z) = g(w(z)), \quad z \in U. \]  
(3)

Let $-1 \leq \lambda < \gamma \leq 1$. Then, class $P(\gamma, \lambda)$ (see [7]) of function $p(z)$ satisfies the subordination condition
\[ p(z) \prec \frac{1 + \gamma z}{1 + \lambda z}, \quad z \in U, \]  
(4)
or equivalently,
\[ p(z) = \frac{(1 + \gamma)h(z) + (1 - \gamma)}{(1 + \lambda)h(z) + (1 - \lambda)}, \quad z \in U, \]  
(5)
where $h(z)$ is the class of functions with positive real part. For $\gamma = 1 - 2\beta, 0 \leq \beta < 1$, and $\lambda = -1$, the class $P(\gamma, \lambda)$ reduces to the class $P(\beta)$, the class of functions whose real part is greater than $\beta$.

The conic domains $\Omega_m(\gamma, \lambda) (m \geq 0)$ of Janowski type introduced by Noor and Malik [22] are defined as follows:

\[ \Omega_m(\gamma, \lambda) = \left\{ u + iv: \left[ \left( \lambda^2 - 1 \right)\left( u^2 + v^2 \right) - 2\left( \gamma \lambda - 1 \right)u + \left( y^2 - 1 \right) \right]^2 > m^2 \left[ -2\left( \lambda + 1 \right)\left( u^2 + v^2 \right) + 2\left( \gamma + \lambda + 2 \right)u - 2\left( y + 1 \right)^2 + 4\left( \gamma - 1 \right)v^2 \right] \right\}. \]  
(6)

Geometrical interpretation of $\Omega_m(\gamma, \lambda)$ and its effect on $\Omega_m$ were also demonstrated in [22]. The class $m = P(\gamma, \lambda)$ represents the class of all functions that maps $U$ onto $\Omega_m(\gamma, \lambda)$. Equivalently, a function $p(z)$ belongs to $m = P(\gamma, \lambda)$ if and only if
\[ p_m(z) = \frac{1 + cz}{1 - cz}, \quad m = 0, \]
\[ \frac{1 + 2}{\pi} \left( \log \frac{1 + \sqrt{c}}{1 - \sqrt{c}} \right)^2, \quad m = 1, \]
\[ 1 + \frac{2}{1 - m} \sin^2 \left[ \frac{\pi}{2} \arccos m \right] \arctan \sqrt{c}, \quad 0 < m < 1, \]
\[ 1 + \frac{m}{1 - m} \sin \left[ \frac{\pi}{2R(t)} \right] \int_0^{\mu(\sqrt{z})/\sqrt{T}} \frac{1}{\sqrt{1 - x^2} \sqrt{1 - (tx)^2}} \, dx \right] + \frac{1}{m^2 - 1}, \quad m > 1, \]  
(8)

where $u(z) = ((c - \sqrt{t})/(1 - \sqrt{c})), t \in (0, 1), c \in U$ and $t$ is chosen such that $m = \cosh((\pi R(t))/((4R(t))^n))$. $R(t)$ is Legendre’s complete elliptic integral of the first kind, and $R'(t)$ is the complementary integral of $R(t)$.

Definition 1 (see [2]). Let $q \in (0, 1)$. Then, the $q$-number $[n]_q$ is given as
\[ [n]_q = \begin{cases} \frac{1 - q^n}{1 - q}, & n \in \mathbb{C}, \\ \sum_{r=0}^{n-1} q^r = 1 + q + q^2 + \cdots + q^{n-1}, & n \in \mathbb{N}, \text{ as } q \to 1^- \end{cases} \]  
(9)
and the $q$-derivative of a complex valued function $f(z)$ in $U$ is given by

$$D_q f(z) = \begin{cases} 
\frac{f(qz) - f(z)}{(q-1)z}, & z \neq 0, \\
f'(0), & z = 0, \\
f'(z), & z \rightarrow 1^-.
\end{cases}$$

From the above explanation, it is easy to see that, for $f(z)$ given by (1),

$$D_q f(z) = 1 + \sum_{n=2}^{\infty} [n]_q a_n z^n. \tag{11}$$

**Definition 2** (see [28]). An analytic function $p(z)$ in $U$ belongs to $P_q(y, \lambda)$ if and only if the condition

$$p(z) = \frac{(O_1 y + O_3) h(z) - (\lambda - 1) O_1}{(O_1 \lambda + O_3) h(z) - (\lambda - 1) O_1}, \tag{12}$$

is satisfied, where $O_1 = 1 + q, O_3 = 3 - q$ and $h \in P.$

**Definition 3** (see [17]). An analytic function $p(z)$ in $U$ belongs to $m - P_q(y, \lambda)$ if and only if the subordination condition

$$p(z) < \frac{(O_1 y + O_3) p_m(z) - (\lambda - 1) O_1}{(O_1 \lambda + O_3) p_m(z) - (\lambda - 1) O_1}, \tag{13}$$

is satisfied, where $p_m(z) = (8).$ Equivalently, $p \in m - P_q(y, \lambda)$ if and only if $p(z)$ conformally maps $U$ onto the domain $\Omega_{q,m}(y, \lambda)$ defined by

$$\Omega_{q,m}(y, \lambda) = \{u + iv: \Omega_1^2 \left[ (\lambda - 1)(\lambda O_1 + O_3)(u^2 + v^2) - (\lambda - 1)(y O_1 + O_3) \right] + (\lambda - 1)(\lambda O_1 + O_3) u + (\lambda - 1)(y O_1 + O_3)]^2 > 4m^2 \left[ (\lambda O_1 + O_3)(u^2 + v^2) + (\lambda + \lambda) O_1 + 2O_3) u - (y O_1 + O_3) \right]^2 + (\lambda - 1) O_1^2 O_1 v^2 \}, \tag{16} \quad m \geq 0, 0 < q < 1.$$

We note that

(i) $m - P_q(y, \lambda) \subset P(\beta),$ where $\beta$ is given by

$$\beta = \frac{4m + (1 - y) O_1}{4m + (1 - y) O_1} \tag{17}.$$ 

(ii) As $q \rightarrow 1^-,$ the class $m - P_q(y, \lambda)$ becomes the class $m - P(y, \lambda)$ and $\Omega_{q,m}(y, \lambda) \equiv \Omega_m(y, \lambda) [22].$

(iii) When $q \rightarrow 1^- \quad \text{and} \quad y = 1, \lambda = 1,$ the class $m - P_q(y, \lambda)$ reduces to the class $P(p_m)$ and $\Omega_{q,m}(y, \lambda) \equiv \Omega_m \{10].$

**Definition 4** (see [20]). Let $g(z)$ of form (2) be in $A$ and $\alpha \geq 0, m \in [0, \infty).$ Then, $g \in m - \text{UCV}_q (y, \lambda)$ if and only if

$$D_q \left( \frac{c D_q g(z)}{D_q g(z)} \right) \frac{(O_1 y + O_3) p_m(z) - (\lambda - 1) O_1}{(O_1 \lambda + O_3) p_m(z) - (\lambda - 1) O_1} \tag{18}$$

where $p_m(z) = (8).$

Inspired by the above recent mentioned work, we announce the following novel classes of analytic functions.

**Definition 5.** Let $f(z) \in A, \alpha \geq 0, m \in [0, \infty).$ Then, $f \in m - \text{UM}_q (a, y, \lambda)$ if and only if

$$\text{Re} \left( \frac{(\lambda - 1) O_1 J_q(a, f; z) - (\lambda - 1) O_1}{(\lambda O_1 + O_3) J_q(a, f; z) - (\lambda O_1 + O_3)} \right) > m \left( \frac{(\lambda - 1) O_1 J_q(a, f; z) - (\lambda - 1) O_1}{(\lambda O_1 + O_3) J_q(a, f; z) - (\lambda O_1 + O_3)} \right) - 1. \tag{19}$$

Equivalently, $f \in m - \text{UM}_q (a, y, \lambda)$ if and only if

$$J_q(a, f; z) = (1 - a) \frac{c D_q f(z)}{f(z)} + a \frac{c D_q f(z)}{D_q f(z)} \frac{(O_1 y + O_3) p_m(z) - (\lambda - 1) O_1}{(O_1 \lambda + O_3) p_m(z) - (\lambda - 1) O_1} \tag{20}.$$
Definition 6. Let \( f(z) \in \mathcal{S} \), \( \alpha \geq 0 \), \( m \in [0, \infty) \). Then, \( f \in m - UQ_q(\alpha, \gamma, \lambda) \) if and only if there exists an analytic function \( g(z) \in m - UCV_q(\gamma, \lambda) \) such that

\[
\Re\left( \frac{(\lambda - 1)O_1J_q(a, f; c) - (\gamma - 1)O_1}{(\lambda O_1 + O_3)J_q(a, f; c) - (\gamma O_1 + O_3)} \right) > m \quad \text{(21)}
\]

Equivalently, \( f \in m - UQ_q(\alpha, \gamma, \lambda) \) if and only if

\[
J_q(a, f; c) = (1 - \alpha)\frac{D_qf(c)}{D_qg(c)} + \alpha \frac{D_q(cD_qf(c))}{D_qg(c)} \quad \text{and} \quad \frac{(O_1\gamma + O_3)p_m(c) - (\gamma - 1)O_1}{(O_1\lambda + O_3)p_m(c) - (\lambda - 1)O_1} > m \quad \text{(22)}
\]

We note the following special cases:

(i) When \( q \to 1^- \), the classes \( m - UM_q(\alpha, \gamma, \lambda) \) and \( m - UQ_q(\alpha, \gamma, \lambda) \) reduce to the classes \( m - UM(\alpha, \gamma, \lambda) \) [21] and \( m - UQ(\alpha, \gamma, \lambda) \) [18], respectively.

(ii) When \( q \to 1^- \) and \( \alpha = 0 \), the classes \( m - UM_q(\alpha, \gamma, \lambda) \) and \( m - UQ_q(\alpha, \gamma, \lambda) \) cut down to the classes \( m - UST(\gamma, \lambda) \) [22] and \( m - UKV(\gamma, \lambda) \) [16].

(iii) When \( q \to 1^- \) and \( \alpha = 1 \), the classes \( m - UM_q(\alpha, \gamma, \lambda) \) and \( m - UQ_q(\alpha, \gamma, \lambda) \) scale down to the classes \( m - UCV(\gamma, \lambda) \) [22] and \( m - UCV^*(\gamma, \lambda) \) [16].

(iv) When \( \alpha = 0 \) and \( \alpha = 1 \), the classes \( m - UM_q(\alpha, \gamma, \lambda) \) and \( m - UQ_q(\alpha, \gamma, \lambda) \) diminish, respectively, to those classes of functions considered in [17, 20].

(v) When \( \alpha = 0 = m \) in Definition 2, the class \( m - UM_q(\alpha, \gamma, \lambda) \) becomes the class \( q \)-starlike functions \( ST_q(\gamma, \lambda) \) of Janowski type recently explored by Srivastava et al. [28].

To effectively establish our findings, the following set of lemmas is required.

### 3. A Set of Lemmas

**Lemma 1** (see [4]). Let \( m \geq 0 \) and \( p_m(z) \) given by (8) be of the form \( p_m(z) = 1 + \ell_1 z + \ell_2 z^2 + \cdots \). Then,

\[
|c_2 - \delta c_1^2| \leq \begin{cases} 
-4\delta + 2, & \delta \leq 0, \\
2, & 0 \leq \delta \leq 1, \\
4\delta + 2, & \delta \geq 1,
\end{cases}
\]

**Lemma 2** (see [17]). If \( P(z) = 1 + c_1 z + c_2 z^2 + \cdots \in P \), then, for any real number \( \delta \),

\[
|c_2 - \delta c_1^2| \leq \begin{cases} 
-4\delta + 2, & \delta \leq 0, \\
2, & 0 \leq \delta \leq 1, \\
4\delta + 2, & \delta \geq 1,
\end{cases}
\]
when \( \delta < 0 \) or \( \delta > 1 \), the equality holds if and only if \( p(\zeta) = (1 + \zeta)/(1 - \zeta) \) or one of its rotations. If \( 0 < \delta < 1 \), then the equality holds if and only if \( p(\zeta) = (1 + \zeta)/(1 - \zeta^2) \) or one of its rotations. If \( \delta = 0 \), then the equality holds if and only if
\[
p(\zeta) = \left( \frac{1 + \psi}{2} \right) \frac{1 + \zeta}{1 - \zeta} + \left( \frac{1 - \psi}{2} \right) \frac{1 - \zeta}{1 + \zeta}, \quad 0 \leq \psi \leq 1, \tag{26}
\]
or one of its rotations. If \( \delta = 1 \), then the equality holds if and only if \( p(\zeta) \) is the reciprocal of one of the functions such that equality holds for the case \( \delta = 0 \). Although the above upper bound is sharp, when \( 0 < \delta < 1 \) it can be improved as follows:
\[
|c_2 - \delta c_1| + |c_1^2| \leq 2, \quad \left( 0 < \delta \leq \frac{1}{2} \right), \tag{27}
\]
\[
|c_2 - \delta c_1^2| + (1 - \delta)|c_1|^2 \leq 2, \quad \left( \frac{1}{2} < \delta \leq 1 \right).
\]

**Lemma 3** (see [20]). Let \( g(\zeta) \) be of form (2) and \( g \in m - \text{UCV}_q(\gamma, \lambda) \). Then,
\[
|a_n| \leq \frac{1}{|n|} \prod_{i=0}^{n-2} \left( |\varrho_i \Omega_1 (\gamma - \lambda) - 4q[i]_q| \right), \tag{28}
\]
where \( \varrho_1 \) is defined by (23).

### 4. Results and Discussion

We now turn our attention to the main results of this article.

#### 4.1. Sufficient Conditions

**Theorem 1.** A function \( f(\zeta) \) of form (1) belongs to the class \( m - \text{UM}_q(\alpha, \gamma, \lambda) \) if it satisfies the condition
\[
\phi_n(\alpha, m, q, \gamma, \lambda) < |\lambda - \gamma| \Omega_1, \tag{29}
\]
where

\[
\phi_n(\alpha, m, q, \gamma, \lambda) = \sum_{n=2}^{\infty} \left[ 4(m + 1) \left( [n]_q - 1 \right) \left( 1 + a \left( [n]_q - 1 \right) \right) + \left( \lambda \Omega_1 + \Omega_2 \right)(2 [n]_q - \lambda \Omega_1 + \Omega_2) \right] |a_n| + \left( \lambda \Omega_1 + \Omega_2 \right) \left( n + 1 - i \right) |a_n|.
\]

**Proof.** Suppose condition (29) holds. Then, we need to prove that

\[
m \left| (\lambda - 1) \Omega_1 f_q(\alpha, f; \zeta) - (\gamma - 1) \Omega_1 \right| - \text{Re} \left( \frac{(\lambda - 1) \Omega_1 f_q(\alpha, f; \zeta) - (\gamma - 1) \Omega_1}{(\lambda \Omega_1 + \Omega_2) f_q(\alpha, f; \zeta) - (\gamma \Omega_1 + \Omega_3) - 1} \right) < 1.
\]

Therefore,

\[
m \left| (\lambda - 1) \Omega_1 f_q(\alpha, f; \zeta) - (\gamma - 1) \Omega_1 \right| - \text{Re} \left( \frac{(\lambda - 1) \Omega_1 f_q(\alpha, f; \zeta) - (\gamma - 1) \Omega_1}{(\lambda \Omega_1 + \Omega_2) f_q(\alpha, f; \zeta) - (\gamma \Omega_1 + \Omega_3) - 1} \right)
\]

\[
\leq (m + 1) \left| (\lambda - 1) \Omega_1 f_q(\alpha, f; \zeta) - (\gamma - 1) \Omega_1 \right| - \text{Re} \left( \frac{(\lambda - 1) \Omega_1 f_q(\alpha, f; \zeta) - (\gamma - 1) \Omega_1}{(\lambda \Omega_1 + \Omega_2) f_q(\alpha, f; \zeta) - (\gamma \Omega_1 + \Omega_3) - 1} \right)
\]

\[
= 4(m + 1) \left| (1 - \alpha) cD_q f(\zeta) D_q f(\zeta) + a f(\zeta) D_q f(\zeta) - f(\zeta) D_q f(\zeta) \right|
\]
We have
\[ cD_q f (c)D_q f (c) = \left( \sum_{n=0}^{\infty} [n]_q a_n c^n \right) \left( \sum_{n=0}^{\infty} [n]_q a_n c^{n-1} \right), \quad a_0 = 0, a_1 = [0]_q = [1]_q = 1 \]
\[ = \sum_{n=0}^{\infty} \left( \sum_{i=0}^{n} [i]_q [n-i]_q a_i a_{n-i} \right) c^{n-1} \]
\[ = \zeta + \sum_{n=1}^{\infty} \left( \sum_{i=1}^{n} [i]_q [n-1-i]_q a_i a_{n-1-i} \right) c^n \]
\[ = \zeta + \sum_{n=1}^{\infty} \left( 2 [n]_q a_n + \sum_{i=1}^{n-1} [i]_q [n-1-i]_q a_i a_{n-1-i} \right) c^n. \]

Similarly,
\[ D_q (cD_q f (c)) = \zeta + \sum_{n=1}^{\infty} \left( \left( [n]_q^2 + 1 \right) a_n + \sum_{i=2}^{n-1} \left[ n+1-i \right]_q a_i a_{n-i} \right) c^n, \] (33)
\[ f (c)D_q f (c) = \zeta + \sum_{n=1}^{\infty} \left( \left( [n]_q + 1 \right) a_n + \sum_{i=2}^{n-1} \left[ n+1-i \right]_q a_i a_{n-i} \right) c^n. \] (34)

From inequality (32) and equations (33) and (34), we arrive at
\[ m \left| \frac{(\lambda - 1)O_1 f_q (\alpha, f; c) - (\gamma - 1)O_1}{(\lambda O_1 + O_3) f_q (\alpha, f; c) - (\gamma O_1 + O_3)} - 1 \right| \leq 4 (m + 1) \left\{ \sum_{n=2}^{\infty} \left[ [n+1]_q - [n]_q - (\alpha([n]_q - 1]) a_n + \sum_{i=2}^{n-1} \left( [i]_q - 1 \right) + a \left( [n+1-i]_q - [i]_q \right) \right] \right. \]
\[ \left. \times \left( \lambda - \gamma \right) O_1 \zeta + \sum_{n=2}^{\infty} \left( \left( \lambda O_1 + O_3 \right) \left( 2 [n]_q + a \left( [n]_q - 1 \right) \right) \right) \right. \]
\[ - \left( \gamma O_1 + O_3 \right) \left( [n+1]_q + 1 \right) a_n + \sum_{i=2}^{n-1} \left( \left( \lambda O_1 + O_3 \right) \left( [i]_q + a \left( [n+1-i]_q - [i]_q \right) \right) \right) \]
\[ - \left( \gamma O_1 + O_3 \right) \left( [n+1-i]_q - [i]_q \right) \right) \left[ \left. \lambda - \gamma \right) O_1 - \sum_{n=2}^{\infty} \left( \left( \lambda O_1 + O_3 \right) \left( 2 [n]_q + a \left( [n]_q - 1 \right) \right) \right) \right. \]
\[ - \left( \gamma O_1 + O_3 \right) \left( [n+1]_q + 1 \right) a_n + \sum_{i=2}^{n-1} \left( \left( \lambda O_1 + O_3 \right) \left( [i]_q + a \left( [n+1-i]_q - [i]_q \right) \right) \right) \left. \right\}^{-1}. \] (36)
The last inequality is bounded by 1 if (29) is satisfied. This completes the proof.

As $q \to 1^-$ in Theorem 1, we are led to Theorem 1 in [21].

\[ \phi_n(a, m, y, \lambda) = \sum_{n=2}^{\infty} \left[ 2(m+1)(n-1)(1+\alpha(n-1)) + \lambda(n+1)(2n+\alpha(n-1)^2) \right] - \frac{(y+1)(n+1)|a_n|}{2} + \sum_{n=2}^{\infty} \sum_{i=2}^{n-1} \left[ 2(m+1)(i-1) + \alpha(n+1-2i) \right] (n+1-i)|a_{n-i}| \]  

(38)

Setting $\alpha = 0$ in Theorem 1, we obtain a variance version of the result presented in [17].

\[ \sum_{n=2}^{\infty} \left[ 4q(m+1)(n-1)_q + \left( \lambda O_1 + O_3 \right)[n]_q - (yO_1 + O_3) \right] |a_n| < |\lambda - y|O_1, \]  

(39)

then $f \in m - UST_q(\gamma, \lambda)$.

If we choose $\alpha = 0$ and allow $q \to 1^-$ in the above theorem, our investigation comes down to Theorem 1 in [22].

**Corollary 3.** A function $f(\zeta)$ demonstrated by (1) is in the class $m - UST(\gamma, \lambda)$ if it satisfies the condition

\[ \sum_{n=2}^{\infty} \left[ 2(m+1)(n-1) + |(\lambda + 1)n - (\gamma + 1)|a_n| \right] |a_n| < |\lambda - y|, \]  

(40)

**Corollary 4** (see [11]). If $f(\zeta) \in \mathcal{A}$ satisfies the condition

\[ \sum_{n=2}^{\infty} (m(n-1)+n)|a_n| < 1, \]  

(41)

then $f \in m - UST_{q-1}:(0, 1, -1) = m - UST$.

**Theorem 2.** Let $f \in \mathcal{A}$. Then, $f \in m - UQ_q(a, \gamma, \lambda)$ if inequality (42) is satisfied,

\[ \sum_{n=2}^{\infty} \prod_{i=0}^{\infty} \left( 4(m+1) + (yO_1 + O_3) \right) \frac{a_{n+i} \left( \gamma - \lambda \right) - 4q[i]_q}{4q[i+1]_q} \]  

(42)

where $\partial_1$ is given by (23).

**Proof.** Assuming (42) holds, then it suffices to establish that

\[ m \left| (\lambda - 1)O_1 \mathcal{F}_q(a, f; \zeta) - (y - 1)O_1 \right| \left( \lambda O_1 + O_3 \right) \mathcal{F}_q(a, f; \zeta) - (yO_1 + O_3) - 1 \]  

(43)
Following the same process in the proof of Theorem 1, we have

\[
m \left\lfloor \frac{(\lambda - 1)O_1 f_q(\alpha, f; \zeta) - (\gamma - 1)O_1}{(\lambda O_1 + O_3) f_q(\alpha, f; \zeta) - (\gamma O_1 + O_3)} - 1 \right\rfloor - \Re \left( \frac{(\lambda - 1)O_1 f_q(\alpha, f; \zeta) - (\gamma - 1)O_1}{(\lambda O_1 + O_3) f_q(\alpha, f; \zeta) - (\gamma O_1 + O_3)} - 1 \right)
\]

\[
\leq 4(m + 1) \left\lfloor \lfloor 1 - a \rfloor D_q f_q(\zeta) + a D_q \left( c D_q f_q(\zeta) - D_q g_q(\zeta) \right) \right\rfloor \left( \lambda O_1 + O_3 \right) \left( \lfloor 1 - a \rfloor D_q f_q(\zeta) + a D_q \left( c D_q f_q(\zeta) - D_q g_q(\zeta) \right) \right) - (\gamma O_1 + O_3) D_q g_q(\zeta)
\]

(\text{where } g \in m - \text{UCV}_q(\gamma, \lambda))

\[
\leq \frac{4(m + 1) \left( \sum_{n=2}^{\infty} \left( 1 + a \left( \lfloor n \rfloor_{q} - 1 \right) \right) \lfloor n \rfloor_{q} |a_n| + \lfloor n \rfloor_{q} |b_n| \right)}{\lfloor \lambda - \gamma \rfloor O_1 - \sum_{n=2}^{\infty} \left( \lambda O_1 + O_3 \right) \left( 1 + a \left( \lfloor n \rfloor_{q} - 1 \right) \right) \lfloor n \rfloor_{q} - (\gamma O_1 + O_3) \sum_{n=2}^{\infty} \lfloor n \rfloor_{q} |b_n|}
\]

\[
\leq \frac{4(m + 1) \left( \sum_{n=2}^{\infty} \left( 1 + a \left( \lfloor n \rfloor_{q} - 1 \right) \right) \lfloor n \rfloor_{q} |a_n| + \sum_{n=2}^{\infty} \prod_{r=0}^{n-2} \left( \left( \left( \beta_1 O_1 (\gamma - \lambda) - 4q[I]_{q} \right) / 4q[I + 1]_{q} \right) \right) \right)}{\sum_{n=2}^{\infty} \prod_{r=0}^{n-2} \left( 4(m + 1) + (\gamma O_1 + O_3) \right) \lfloor n \rfloor_{q} |a_n| < |\lambda - \gamma| O_1}
\]

\[
\sum_{n=2}^{\infty} \prod_{r=0}^{n-2} \left( 4(m + 1) + (\gamma O_1 + O_3) \right) \lfloor n \rfloor_{q} |a_n| < |\lambda - \gamma| O_1.
\]

\[
\text{(44)}
\]

where we have used Lemma 3. Thus, the last inequality is bounded by 1 if (42) is satisfied. Hence, we complete the proof.

As \( q \rightarrow 1^- \) in Theorem 2, we obtain the similar result proved in [18].

Corollary 5. Let \( f \in \mathcal{A} \). Then, \( f \in m - \text{UQ}(\alpha, \gamma, \lambda) \) if

\[
\sum_{n=2}^{\infty} \prod_{r=0}^{n-2} \left( 2(m + 1) + (\gamma + 1) \right) \left( \beta_1 O_1 (\gamma - \lambda) - 2\lambda \right) / 2(i + 1)
\]

\[
+ \sum_{n=2}^{\infty} \left( 2(m + 1) + (\gamma + 1) \right) (1 + a(n-1))n|a_n| < |\lambda - \gamma|,
\]

holds.

For \( \alpha = 0 \) in Theorem 2, we get the result established by Naeem et al. [20].

Corollary 6. Let \( f \in \mathcal{A} \). Then, \( f \in m - \text{UKV}_q(\gamma, \lambda) \) if inequality (46) is satisfied,

\[
\text{(45)}
\]

For \( \alpha = 0 \) and \( \alpha = 1 \) as \( q \rightarrow 1^- \) in Theorem 2, we obtain the following results established in [16].

Corollary 7. Let \( f \in \mathcal{A} \). Then, \( f \in m - \text{UKV}(\gamma, \lambda) \) if

\[
\sum_{n=2}^{\infty} \prod_{r=0}^{n-2} \left( 2(m + 1) + (\gamma + 1) \right) \left( \beta_1 O_1 (\gamma - \lambda) - 2\lambda \right) / 2(i + 1)
\]

\[
+ \sum_{n=2}^{\infty} \left( 2(m + 1) + (\gamma + 1) \right) n|a_n| < |\lambda - \gamma|
\]

is satisfied.

Corollary 8. Let \( f \in \mathcal{A} \). Then, \( f \in m - \text{UKV}_q(\gamma, \lambda) \) if
4.2. Fekete Szegő Inequality

\[ \sum_{n=2}^{\infty} \prod_{m=1}^{n-2} \frac{(2(m+1)+(\gamma+1))|\vartheta_1\vartheta_3(y-\gamma)-2\lambda|}{2(s+1)} + \sum_{n=2}^{\infty} (2(m+1)+(\lambda+1))n^2|a_n|<|\lambda-\gamma|, \]

is satisfied.

Theorem 3. Let \( f \in m-UM_q(\alpha,\gamma,\lambda) \). Then, for any real number \( \delta \), we have

\[ |a_n-\delta a_n|^2 \leq \frac{(\gamma-\lambda)|\vartheta_1\vartheta_3|}{8q(1+aq[2]_q)} \times \begin{cases} 
\Lambda_1(\alpha, m, q, \gamma, \lambda), & \delta < \rho_1, \\
2, & \rho_1 \leq \delta \leq \rho_2, \\
\Lambda_2(\alpha, m, q, \gamma, \lambda), & \delta > \rho_2,
\end{cases} \]

where

\[
\begin{align*}
\rho_1 & = \frac{1}{[2]_q(1+aq[2]_q)} \left[ \frac{8q(1+aq)^2}{(\gamma-\lambda)\vartheta_1\vartheta_3} \left( \frac{\vartheta_2 - \vartheta_1}{2\vartheta_1} - \frac{(\lambda\vartheta_1 + \vartheta_3)\vartheta_1}{8} \right) + (1+aq[2]_q+1) \right], \\
\rho_2 & = \frac{1}{[2]_q^2(1+aq[2]_q^2)} \left[ \frac{8q(1+aq)^2}{(\gamma-\lambda)\vartheta_1\vartheta_3} \left( \frac{\vartheta_2 - \vartheta_1}{2\vartheta_1} - \frac{(\lambda\vartheta_1 + \vartheta_3)\vartheta_1}{8} \right) + (1+aq[2]_q+1) \right], \\
\Lambda_1(\alpha, m, q, \gamma, \lambda) & = \frac{4\vartheta_2 - (\lambda\vartheta_1 + \vartheta_3)\vartheta_1^2}{2\vartheta_1} + \left( \frac{1+aq[2]_q+1}{\delta[2]_q} - \delta[2]_q \right) (\gamma-\lambda)\vartheta_1\vartheta_3, \\
\Lambda_2(\alpha, m, q, \gamma, \lambda) & = \frac{2q(1+aq)^2}{4(\lambda\vartheta_1 + \vartheta_3)\vartheta_1^2}, \\
\end{align*}
\]

and \( \vartheta_1 \) and \( \vartheta_2 \) are given by (23) and (24). This result cannot be improved. Then, \( f(\zeta) \in m-UM_q(\alpha,\gamma,\lambda) \) implies there exists a Schwarz function \( w(\zeta) \) such that

\[ J_q(\alpha,f;\zeta) = \frac{(\lambda\vartheta_1 + \vartheta_3)\vartheta_1^2 + (\gamma-\lambda)\vartheta_1\vartheta_3}{\vartheta_1^2} - (\lambda\vartheta_1 + \vartheta_3)\vartheta_1^2 \]

Using the representation for \( p_m(\zeta) \) in Lemma 3 and the relationship between \( w(\zeta) \) and \( p(\zeta) = 1 + c_1\zeta + c_2\zeta^2 + \cdots \in P \), we can write

\[ w(\zeta) = \frac{p(\zeta) - 1}{p(\zeta) + 1} = \frac{c_1}{2} \zeta + \frac{c_2 - c_1^2\zeta^2}{2} + \cdots, \]

\[ p_m(\zeta) = 1 + \frac{c_1\vartheta_1}{2} \zeta + \left( \frac{2c_2 - c_1^2}{4} \right) \zeta^2 + \cdots \]

Thus,

\[ J_q(\alpha,f;\zeta) = \frac{4 + ((\lambda\vartheta_1 + \vartheta_3)\vartheta_1^2)/2)\zeta + ((\gamma-\lambda)\vartheta_1\vartheta_3)/4}{4 + ((\lambda\vartheta_1 + \vartheta_3)\vartheta_1^2)/2)\zeta + ((\gamma-\lambda)\vartheta_1\vartheta_3)/4} \]

\[ = 1 + \frac{(\gamma-\lambda)\vartheta_1\vartheta_3}{8} \zeta + \left( \frac{\gamma-\lambda}{16} \right) \vartheta_1(\vartheta_2 - c_1^2)\vartheta_1 + c_1^2\vartheta_2 \]

\[ = \frac{(\gamma-\lambda)(\lambda\vartheta_1 + \vartheta_3)c_1^2\vartheta_1^2}{64} \zeta^2 + \cdots. \]
However,

\[
J_q(\alpha, f; \varsigma) = (1 - \alpha) \frac{cD_q f(c)}{f(c)} + \alpha \frac{D_q(cD_q f(c))}{D_q f(c)}
\]

\[= 1 + q(1 + aq)\varsigma + q\left(\left[12q(1 + aq[2]_q)a_3 - \left(1 + aq[2]_q + 1\right)\right]a_3\right)^2 + \ldots.
\]

On comparing the coefficients of \(\varsigma\) and \(\varsigma^2\) of (53) and (54), we obtain

\[
a_2 = \frac{(y - \lambda)c_1O_1\mathcal{E}_1}{8q(1 + aq)},
\]

\[
a_3 = \frac{1}{q[2]_q(1 + aq[2]_q)} \left\{ \left(\frac{y - \lambda}{16}\right)O_1\left(2c_2 - c_1^2\right)\mathcal{E}_1 + c_1^2\mathcal{E}_1 \right\} - \frac{(y - \lambda)(\Lambda O_1 + O_3)c_1^2O_1\mathcal{E}_1^2}{64}
\]

\[+ \frac{(1 + aq[2]_q + 1)}{q[2]_q(1 + aq[2]_q)}a_2^2
\]

Now, for a real number \(\delta\), we have

\[
|a_3 - \deltaa_3^2| = \frac{1}{q[2]_q(1 + aq[2]_q)} \left(\frac{y - \lambda}{8}\right)O_1 \left[\mathcal{E}_1 c_2 + \left(\frac{\mathcal{E}_2 - \mathcal{E}_1}{2} - \frac{(\Lambda O_1 + O_3)c_1^2}{8}\right)\mathcal{E}_1\right]
\]

\[+ (y - \lambda)O_1\mathcal{E}_1^2\left(\frac{y - B}{8}\right)O_1 \left[\left(1 + aq[2]_q + 1\right) - \delta[2]_q(1 + aq[2]_q)\right]
\]

\[\frac{8q^2(1 + aq)^2[2]_q(1 + aq[2]_q)}{\left(\left(1 + aq[2]_q + 1\right) - \delta[2]_q(1 + aq[2]_q)\right)(y - \lambda)O_1\mathcal{E}_1\right]c_2\mathcal{E}_1^2}
\]

\[= \frac{(y - \lambda)O_1\mathcal{E}_1}{8q[2]_q(1 + aq[2]_q)} \left[\mathcal{E}_1 - \frac{\mathcal{E}_2}{2\mathcal{E}_1} + \frac{(\Lambda O_1 + O_3)c_1}{8}\right]
\]

\[\left(\left(1 + aq[2]_q + 1\right) - \delta[2]_q(1 + aq[2]_q)\right)(y - \lambda)O_1\mathcal{E}_1\right]c_2\mathcal{E}_1^2}
\]

\[= \frac{(y - \lambda)O_1\mathcal{E}_1}{8q[2]_q(1 + aq[2]_q)} |\mathcal{E}_2 - \beta\mathcal{E}_1|,
\]

where

\[
\beta = \frac{\mathcal{E}_1 - \mathcal{E}_2 + (\Lambda O_1 + O_3)c_1}{2\mathcal{E}_1} - \frac{\left(\left(1 + aq[2]_q + 1\right) - \delta[2]_q(1 + aq[2]_q)\right)(y - \lambda)O_1\mathcal{E}_1}{8q(1 + aq)^2}.
\]

Hence, the result follows from Lemma 2.

If \(\alpha = 0\) and \(0 < m < 1\), then Theorem 3 reduces to Theorem 10 in [17].

Corollary 9. Let \(0 < m < 1\) and \(f \in \text{UST}_q(y, \lambda)\) be of form (1). Then, for any real number \(\delta\), we have
Lemma 1. The proof is straightforward from Theorem 3 and

\[
|a_2 - \delta a_2^2| \leq \frac{(y - \lambda)^2}{4q(1 - m^2)} \times \begin{cases} 
\Lambda_1(m, q, \gamma, \lambda), & \delta < \lambda_1, \\
2, & \lambda_1 \leq \delta \leq \lambda_2, \\
\Lambda_2(m, q, \gamma, \lambda), & \delta > \lambda_2,
\end{cases}
\]

where

\[
\Lambda_1(m, q, \gamma, \lambda) = \frac{4q(1 - m^2)}{(y - \lambda)(1 + q)^2} \left( \frac{T^2 - 1}{6} - \frac{T^2(\lambda(1 + q) + 3 - q)}{4(1 - m^2)} \right) + \frac{1}{1 + q},
\]

\[
\Lambda_2(m, q, \gamma, \lambda) = \frac{4q(1 - m^2)}{(y - \lambda)(1 + q)^2} \left( \frac{T^2 + 5}{6} - \frac{T^2(\lambda(1 + q) + 3 - q)}{4(1 - m^2)} \right) + \frac{1}{1 + q},
\]

\[
\lambda_1 = \frac{4q(1 - m^2)}{(y - \lambda)(1 + q)^2} \left( \frac{T^2 - 1}{6} - \frac{T^2(\lambda(1 + q) + 3 - q)}{4(1 - m^2)} \right) + \frac{1}{1 + q},
\]

\[
\lambda_2 = \frac{4q(1 - m^2)}{(y - \lambda)(1 + q)^2} \left( \frac{T^2 + 5}{6} - \frac{T^2(\lambda(1 + q) + 3 - q)}{4(1 - m^2)} \right) + \frac{1}{1 + q},
\]

The result is sharp.

Proof. The proof is straightforward from Theorem 3 and

\[
\Lambda_1(m, q, \gamma, \lambda) = \frac{2(2 + T^2)}{3} - \frac{T^2(\lambda(1 + q) + 3 - q)}{(1 - m^2)} + \frac{T^2(\delta(1 + q))/(1 + q)(y - \lambda)}{q(1 - m^2)}.\]

\[
\Lambda_2(m, q, \gamma, \lambda) = \frac{T^2(\lambda(1 + q) + 3 - q)}{(1 - m^2)} - \frac{2(2 + T^2)}{3} + \frac{T^2(\delta(1 + q) - 1)/(1 + q)(y - \lambda)}{q(1 - m^2)}.
\]

where

\[
\nu_1 = \frac{1}{1 + q} - \frac{q\pi^2}{(y - \lambda)(1 + q)^2} \left( \frac{1}{6} + \frac{\lambda(1 + q) + 3 - q}{\pi^2} \right),
\]

where

Corollary 10. Let m = 1, and f(z) of form (1) belongs to

\[
|a_3 - \delta a_3^2| \leq \frac{(y - \lambda)}{q\pi^2} \times \begin{cases} 
\Lambda_1(q, \gamma, \lambda), & \delta < \nu_1, \\
2, & \nu_1 \leq \delta \leq \nu_2, \\
\Lambda_2(q, \gamma, \lambda), & \delta > \nu_2,
\end{cases}
\]

where

\[
\nu_2 = \frac{1}{1 + q} - \frac{q\pi^2}{(y - \lambda)(1 + q)^2} \left( \frac{1}{6} + \frac{\lambda(1 + q) + 3 - q}{\pi^2} \right).
\]

\[
\Lambda_1(q, \gamma, \lambda) = \frac{4\lambda(1 + q) + 3 - q}{\pi^2} - \frac{4}{3} \frac{1}{\pi^2} + \frac{(1 - \delta(1 + q))/(1 + q)(y - \lambda)}{q\pi^2},
\]

\[
\Lambda_2(m, q, \gamma, \lambda) = \frac{4\lambda(1 + q) + 3 - q}{\pi^2} - \frac{4}{3} \frac{\delta(1 + q) - 1/(1 + q)(y - \lambda)}{q\pi^2}.
\]
The result is sharp. Theorem 3 becomes Theorem 1 in [28] when \( \alpha = m = 0 \).

\[
|a_1 - \delta a_2|^2 \leq \left( \frac{\sqrt{y - \lambda}}{4q} \right)^2 \times \begin{cases} 
(y - \lambda) + (y - 2\lambda - 1)q + (1 - \lambda)q^2 - \delta(y - \lambda)(1 + q)^2, & \delta < \sigma_1, \\
(\lambda - y) + (2\lambda - y + 1)q + (\lambda - 1)q^2 + \delta(y - \lambda)(1 + q)^2, & \delta > \sigma_2,
\end{cases}
\]

where

\[
\sigma_1 = \frac{1}{1 + q} \left( 1 - \frac{q(\lambda + 1 + q + 3 - q)}{(y - \lambda)(1 + q)} \right),
\]

\[
\sigma_2 = \sigma_1 + \frac{4q}{(y - \lambda)(1 + q)^2}.
\]

4.3. Covering Theorems

**Theorem 4.** The range of every univalent functions \( f \in m - UM_q(\alpha, \gamma, \lambda) \) contains the disc:

\[
|c| < \frac{2q(1 + \alpha q)}{8q(1 + \alpha q) + (y - \lambda)\sigma_1|\sigma_1|}.
\]

Proof. From the proof of Theorem 3, we can see that

\[
|a_2| \leq \frac{(y - \lambda)\sigma_1|\sigma_1|}{4q(1 + \alpha q)}
\]

Since the Koebe one-quarter theorem asserted that each omitted value \( w \) of the univalent function \( f(c) \) of form (1) satisfies

\[
|w| \geq \frac{1}{2 + |d_2|} \geq \frac{4q(1 + \alpha q)}{8q(1 + \alpha q) + (y - \lambda)\sigma_1|\sigma_1|},
\]

\[
(1 - \alpha)D_q f(c) + aD_q(cD_q f(c)) = \left( \frac{O_1\gamma + O_2}{O_1\lambda + O_3} \right) p_m(w(c)) - (y - 1)\sigma_1|\sigma_1| D_q y(c),
\]

which in turn implies

\[
1 + [2]_q(1 + \alpha q)a_2 \zeta + \cdots = 1 + \left( \gamma_1 + [2]_q b_2 \right) \zeta \cdots.
\]

It is easy to see that

\[
b_2 = \frac{\gamma_1}{q[2]_q},
\]

\[
a_2 = \frac{\gamma_1}{q(1 + \alpha q)}.
\]

**Corollary 11.** Let \( f(c) \) of the series representation (1) be in \( ST_q(\gamma, \lambda) \). Then, we have the sharp inequality:

\[
|c| < \frac{2q}{4q + (y - \lambda)\sigma_1}.
\]

Thus, we have the result.

**Corollary 12.** The range of every univalent function \( f \in ST_q(\gamma, \lambda) \) contains the disc:

\[
|c| < \frac{2q}{4q + (\lambda - 1)\sigma_1}.
\]

**Theorem 5.** The range of every univalent function \( f \in m - UQ_q(\alpha, \gamma, \lambda) \) contains the same disc given by (66).

Proof. Let \( w(c) \) be a Schwarz function. We note first in Theorem 3 that

\[
\frac{(O_1\gamma + O_2)}{(O_1\lambda + O_3)} p_m(w(c)) - (y - 1)\sigma_1|\sigma_1| = 1 + \gamma_1\zeta + \gamma_2\zeta^2 + \cdots,
\]

where

\[
\gamma_1 = \frac{(y - \lambda)\sigma_1|\sigma_1|}{8},
\]

\[
\gamma_2 = \left( \frac{y - \lambda}{16} \right) O_1 (2c_1 - c_1^2) \sigma_1 + c_1^2 \sigma_2 - \frac{(y - \lambda)(\lambda O_1 + O_3)c_1 \sigma_1 \sigma_2^2}{64}
\]

Since \( f \in m - UQ_q(\alpha, \gamma, \lambda) \), then for some

\[
g(c) = \zeta + b_2 \zeta^2 + b_3 \zeta^3 + \cdots \in m - UCV_q(\gamma, \lambda),
\]

we have

\[
\left( 1 - \alpha \right) D_q f(c) + a D_q (c D_q f(c)) = \left( \frac{O_1\gamma + O_2}{O_1\lambda + O_3} \right) p_m(w(c)) - (y - 1)\sigma_1|\sigma_1| D_q y(c),
\]

Therefore, comparing the coefficients of \( c \) of (73) and applying (74), we obtain

\[
a_2 = \frac{\gamma_1}{q(1 + \alpha q)}.
\]
Now, proceeding the same way as in the proof of Theorem 4, we have the required result.

5. Conclusion

Using the concept of $q$-calculus, we have introduced some new subclasses of analytic functions in the unit disc related to Janowski class of functions. In addition, sufficient conditions, Fekete–Szegő inequality as well as covering results for functions belonging to these new classes were established. Consequently, many remarkable special cases of our findings which were studied in the previous work were obtained [19, 26].

Data Availability

No data were used to support the findings of the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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