E2/M1 Mixing Ratio of $\Delta \to N\gamma$ and Hyperon Resonance Radiative Decay

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Abstract

We compute the leading contribution to the E2/M1 mixing ratio of the decay $\Delta \to N\gamma$ in heavy baryon chiral perturbation theory. We find the mixing ratio to be $4\% \lesssim |\delta_{E2/M1}| \lesssim 11\%$, much larger than estimates based on the quark model and other hadronic models. We also compute the mixing ratio for the radiative decay of the hyperon resonances. The decays $\Sigma^{*+} \to \Sigma^{+}\gamma$ and $\Xi^{*-} \to \Xi^{-}\gamma$ provide a particularly sensitive probe of deviations from heavy baryon spin-flavour SU(6).

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The ratio of electric quadrupole radiation (E2) to magnetic dipole radiation (M1) in the decay $\Delta \rightarrow N\gamma$ is of great interest as it provides valuable information on the ground state structure of the lowest lying baryons. In the simplest constituent quark model this radiative transition is purely M1 arising from the spin flip of a single constituent quark. However, the hyperfine interaction between constituent quarks induces admixtures of non-zero orbital angular momentum states into the hadronic ground states. Such configurations give rise to non-vanishing E2 matrix elements. We will denote the ratio of reduced matrix elements for E2 radiation to M1 radiation by $\delta_{E2/M1}$. Pion–photoproduction has been used to arrive at an experimental determination of this mixing ratio, but only after model dependent analyses have been performed [1]–[4]. These analyses would indicate that $-3\% < \delta_{E2/M1} < +4\%$, where only local counterterms have been used in forming $\delta_{E2/M1}$, forcing it to be real and model dependent. Recent lattice computations [5] constrain the mixing ratio to lie in the interval $-5\% < \delta_{E2/M1} < 11\%$. Since lattice calculations treat the $\Delta$ as a stable particle, this ratio is also real. The nonrelativistic constituent quark model predicts $\delta_{E2/M1}$ to be very small on the order of $-0.4\%$, and to be essentially zero when relativistic wavefunctions are used [6]. However, it has been suggested that contributions from the pion cloud are likely to dominate this mixing ratio [7]. In this work we will compute the leading, model independent contribution to $\delta_{E2/M1}$ in heavy baryon chiral perturbation theory (HBChPT).

Unlike the M1 amplitude, which is dominated by short distance physics through a local counterterm and kaon loops, the E2 amplitude is dominated by long distance pion loops which are enhanced by factors of $\log(m_{\pi}^2/\Lambda^2)$ over naive estimates for the size of the E2 counterterm. Such chiral logs can be unambiguously computed in HBChPT, and as these are formally the leading contribution to the E2 amplitude we will use them to predict the mixing ratios $\delta_{E2/M1}$. We will not review here the formalism for chiral perturbation theory when the baryons are treated as heavy, the reader should refer to [8] (for a review see [9]).

In an earlier work [10], we studied the electromagnetic branching fractions of the decuplet of baryons using HBChPT, and also the strong interaction couplings at one–loop. One interesting outcome was the striking similarity in the relative strengths of the strong couplings extracted from measured widths, and the relationships predicted by heavy baryon spin–flavour SU(6) [11]–[14]. This would seems to indicate some manifestation of a higher symmetry group than the SU(3) of QCD. To investigate this further, we can compare the
HBChPT predictions for $\delta_{E2/M1}$, using the independent values of the strong couplings from SU(3), to predictions when the SU(6) relations between the couplings are imposed.

There is one dimension five counterterm that contributes to the radiative decay of the baryon decuplet of the form

$$\mathcal{L}^{M1}_v = i\Theta_1 \frac{e}{\Lambda^\chi} \overline{B}_v S_\nu^\mu Q T_\nu^\nu F_\mu^\nu,$$

where $Q$ is the electromagnetic charge matrix,

$$Q = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix},$$

(to lowest order in meson fields), and $\Theta_1$ is an unknown coefficient that was determined in [10] by fitting to the branching ratio for $\Delta \rightarrow N\gamma$. This local counterterm contributes only to the M1 transition as can easily be seen by going to the baryon rest frame, where $\mu, \nu$ are spacelike only. The $\pi$ and $K$ loops contribute at order $1/\Lambda^2$. Consequently, the rates for the SU(3) allowed transitions are dominated by the local counterterm while the SU(3) forbidden transitions are dominated by the loops. There is a dimension six counterterm that contributes to the E2 amplitude of the form

$$\mathcal{L}^{E2}_v = i\Theta_2 \frac{e}{\Lambda^2} \overline{B}_v (S_\nu^\mu Q T_\nu^\nu + S_\nu^\nu Q T_\mu^\mu) v^\alpha \partial_\mu F_\alpha^\nu,$$

whose coefficient $\Theta_2$ is also unknown. However, as mentioned earlier, this local counterterm is formally subleading to the contribution from the $\pi$ loops which are enhanced by a factor of $\log(m^2_\pi/\Lambda^2)$ over the naive estimate of $\Theta_2$, so we will neglect $\Theta_2$ for the purposes of this work. The matrix element for radiative decays can be written as [10]

$$\mathcal{M} = X \overline{B} S \cdot k T \cdot \epsilon_\gamma \quad + \quad Y \overline{B} S \cdot \epsilon_\gamma k \cdot T,$$

where $X$ and $Y$ include contributions from both one-loop graphs and the M1 local counterterm. The coefficients $X$ and $Y$, which were computed in [10], are functions of the mass of the $\pi$ or $K$, the mass splitting between the initial and intermediate state baryon, and the energy of the emitted photon. They are given by

$$X = -ie \left[ Q_{TB} \frac{\Theta_1}{\Lambda^\chi} + \frac{1}{4\pi^2 f^2} \left( \beta_{TBB} I_2 - \beta_{TTB} \frac{I_2}{3} - \frac{2}{3} I_1 \right) \right],$$

$$Y = -ie \left[ -Q_{TB} \frac{\Theta_1}{\Lambda^\chi} + \frac{1}{4\pi^2 f^2} \left( \beta_{TBB} I_1 - \beta_{TTB} \frac{I_1}{3} - \frac{2}{3} I_2 \right) \right].$$
where $Q_{TB}$, $\beta_{TBB}$ and $\beta_{TTB}$ are Clebsch–Gordan coefficients that can be found in [10], and $f$ is the meson decay constant; $f_\pi$ for $\pi$'s and $f_K$ for $K$'s. The integrals $I_1$ and $I_2$ are given by

$$I_1 = \int_0^1 dx x \Gamma(\epsilon) I(-\epsilon, \omega_\gamma x - \Delta m, M^2),$$

$$I_2 = \int_0^1 dx (x - 1) \Gamma(\epsilon) I(-\epsilon, \omega_\gamma x - \Delta m, M^2),$$

where $\omega_\gamma$ is the energy of the photon emitted during the decay, $\Delta m$ is the mass difference between the initial decuplet baryon and the intermediate state baryon, and $M$ is the mass of either the $\pi$ or $K$ in the loop. An SU(3) symmetric $\overline{\text{MS}}$ subtraction scheme is used to define the finite parts of $I_1$ and $I_2$, and we use

$$\Gamma(\epsilon) I(-\epsilon, b, c) = b \left[ \frac{\log(c/\Lambda^2)}{2} - 2 \right] - \sqrt{b^2 - c} \log\left( \frac{b - \sqrt{b^2 - c + i\epsilon}}{b + \sqrt{b^2 - c + i\epsilon}} \right),$$

where only finite pieces are shown.

The $E2$ and $M1$ amplitudes can be found from the symmetric and antisymmetric combinations of $X$ and $Y$ respectively,

$$A_{E2} = \frac{\sqrt{5}}{2} (X + Y),$$

$$A_{M1} = \frac{1}{2} (X - Y),$$

from which we find that the total width for the decay is

$$\Gamma(T \to B\gamma) = \frac{\omega_\gamma^3}{12\pi} \left[ |A_{E2}|^2 + |A_{M1}|^2 \right].$$

We note that the overall sign of $A_{E2}$ cannot be determined uniquely, and that we have chosen the sign defined in Eq. (8). The mixing ratio, $\delta_{E2/M1}$, is defined as

$$\delta_{E2/M1} = \frac{1}{\sqrt{3}} \frac{A_{E2}}{A_{M1}},$$

using the common convention for this quantity. The rates computed in [10] using (9) agree, within errors, with the lattice computations of [3]. However, these rates are dominated by the $M1$ amplitude, where a true test of these methods is in the $E2$ component.

The $M1$ counterterm is determined by the rate for $\Delta \to N\gamma$ and consequently depends on the choice of strong couplings constants $F$, $D$, $C$ and $\mathcal{H}$. These couplings and their
associated uncertainties have been found from axial current matrix elements amongst the octet baryons, from the mass splitting of the lowest lying baryons and baryon resonances, from nonleptonic hyperon decays, and from the strong decay of baryon resonances. In order to find the predicted value for each mixing ratio we randomly chose values for the constants $F$, $D$, $C$ and $H$ from the intervals $0 < D < 0.7$, $0.3 < F < 0.5$, $-1.3 < C < -1.1$ and $-2.8 < H < -1.6$. As there is an uncertainty in the radiative width for $\Delta \rightarrow N\gamma$, we also randomly chose a value for the radiative width from the allowed $2\sigma$ region. With these values we solved for the $M_1$ counterterm $\Theta_1$, and then computed the mixing ratio $\delta_{E2/M_1}$. We studied a sample of 6000 points in the allowed region, and the results are shown in fig. 1 and fig. 2 as the gray region.

It is interesting to see what the constraints imposed by a heavy baryon spin-flavour $SU(6)$ would have on our predictions. Such a symmetry relates all the lowest order strong coupling constants, $F = \frac{2}{3}D$, $C = -2D$ and $H = -3D$. We chose a value of $D$ from the interval $0.5 < D < 0.7$ and then predicted each mixing ratio, the results of which appear as the dark lines in fig. 1 and dark points in fig. 2. Note that the SU(3) forbidden decays are independent of the choice of the coupling constant $D$, since it divides out in the ratio, yielding sharp predictions based on heavy baryon spin–flavour $SU(6)$ for these transitions, depending only on baryon masses, meson masses, and photon energy.

Typical values of the ratio for the SU(3) forbidden decays ($\Sigma^{*+} \rightarrow \Sigma^{+}\gamma$ and $\Xi^{*-} \rightarrow \Xi^{-}\gamma$) are large and might be measured at CEBAF. It has also been suggested that polarized hyperon beams at FNAL could be used. Again, these SU(3) suppressed decay modes are important in that they are dominated by loops and do not depend on the counterterms $\Theta_1$ and $\Theta_2$ (the same U–spin argument that forbids a contribution from $\Theta_1$ also forbids a contribution from $\Theta_2$). We do not show $\delta_{E2/M_1}$ for the decay $\Sigma^{*0} \rightarrow \Sigma^{0}\gamma$. Since the one-loop contribution to its imaginary part vanishes in the limit of exact isospin symmetry (from exact cancellations between contributing intermediate states), this mixing ratio is very sensitive to isospin breaking and reliable predictions for this particular mode cannot be made at this order.

An important observation can be made from fig. 1 and fig. 2. The decays $\Sigma^{*+} \rightarrow \Sigma^{+}\gamma$ and $\Xi^{*-} \rightarrow \Xi^{-}\gamma$ are particularly sensitive to deviations from heavy baryon spin–flavour SU(6). This symmetry predicts that $\text{Re}(\delta_{E2/M_1})$ for the transition $\Sigma^{*+} \rightarrow \Sigma^{+}\gamma$ is very much smaller than $\text{Im}(\delta_{E2/M_1})$ while the possible range of values determined from the experimental uncertainties in the couplings constants without imposing the SU(6) relations is large. A similar tendency is seen for the $\Sigma^{*-} \rightarrow \Sigma^{-}\gamma$ result. For both this transition
and for $\Xi^{*-} \rightarrow \Xi^- \gamma$, the SU(6) prediction of $\delta_{E2/M1}$ is very restrictive, and at values large enough to be more experimentally accessible than some of the regions allowed without imposing this symmetry.

We should comment on the presence of an imaginary part in $\delta_{E2/M1}$. This arises from final–state interactions where the intermediate state is an octet baryon and a pion; a strong decay mode of the decuplet baryon. As mentioned, most pion–photoproduction analyses attempt to extract the contribution to $\delta_{E2/M1}$ arising from local counterterms only\[1\]–\[4\]. Such an extraction will necessarily yield a $\delta_{E2/M1}$ which is real, and relegates the pionic dressing of the vertex to be a background contribution. A comparison of the value of $\delta_{E2/M1}$ extracted for the bare vertices to the value of $\delta_{E2/M1}$ extracted for pion dressed vertices has been made in \[16\], using the model of ref. \[4\]. However, it is clear that the value of $\delta_{E2/M1}$ extracted from the dressed vertex is the only quantity with any physical significance; the ratio of bare vertices will be renormalization scheme dependent in chiral perturbation theory, and highly model–dependent in any quark or bag model. The lattice calculations \[5\], on the other hand, do dress the vertex to some extent, and do not make an unphysical separation into local counterterm and loop contributions. However, by treating the $\Delta$ and the hyperon resonances as stable particles they do not include the final state interactions which lead to an imaginary contribution to the ratio.

There is also the question of $\Theta_2$, which was neglected in our computation. Formally its contribution to $\delta_{E2/M1}$ is subdominant to the contribution from pion loops, which are enhanced by a chiral log. However, we should note that our counterterm $\Theta_1$ (in the $\overline{MS}$ subtraction scheme) is approximately three times larger than we would naively expect \[10\], and if nature were to be particularly unkind, $\Theta_2$ could also be larger than our naive estimate. If this is in fact true then we would find disagreement between our predictions and experimentally determined values of $\delta_{E2/M1}$ for the SU(3) allowed transitions. However, by measuring $\delta_{E2/M1}$ for one of the allowed transitions the counterterm $\Theta_2$ could be fixed, enabling us to predict the mixing ratio of the other SU(3) allowed decays. It is important to stress that this does not affect in any way our prediction for the SU(3) forbidden decays, which do not receive contributions from either counterterm.

The allowed region for each mixing ratio, shown in fig. \[1\] and fig. \[2\], was determined from the coupling constants $F, D, C$ and $H$, and associated uncertainties, and from the radiative width of $\Delta \rightarrow N \gamma$ and its uncertainty. It is important to realise that there are other uncertainties in these predictions arising from terms higher order in the chiral expansion that are not shown in fig. \[1\] and fig. \[2\]. An example is the two-loop contribution
to the E2 matrix element. These will generically be $\sim 25\%$ of the lowest order result in addition to the uncertainties shown in fig. 1 and fig. 2. One of these uncertainties arising from higher order terms is exhibited in the choice of the $\omega_\gamma$ used in evaluating the loop integrals and the formula for the overall decay rate. In the infinite baryon mass limit the photon energy is simply equal to the mass difference between the initial and final state baryon. Corrections to this relation, however, occur at order $1/M_B$ and are hence higher order in the chiral expansion. In reality, these are about a 10% correction but become potentially important when $\omega_\gamma$ is raised to a high power.

In conclusion, we have used heavy baryon chiral perturbation theory to compute the leading contribution to the ratio of electric quadrupole to magnetic dipole radiation, $\delta_{E2/M1}$, in the radiative decay of baryon resonances. The magnetic dipole transition is dominated by the local counterterm and from nonanalytic $m_s^{1/2}$ contributions from loop graphs involving kaons. On the other hand, the electric quadrupole transition is dominated by pion loop graphs contributing terms of order $\log(m^2_\pi/\Lambda^2)$ and thus by calculable long-distance physics. These long distance effects are formally larger than the naive estimate of the local counterterm for the E2 amplitude and so our result is formally the dominant contribution. We find that the mixing ratio for $\Delta \rightarrow N\gamma$ is larger than previously estimated in hadronic models such as the constituent quark model and also from model dependent extractions from pion photoproduction, but is consistent in magnitude with the latest results from the lattice. Since we have used a systematic, consistent field theoretic approach to the calculation and we have been able to find the leading, formally dominant contribution, we have some confidence in our result.

We have also shown that an experimental determination of $\delta_{E2/M1}$ for $\Sigma^{*+} \rightarrow \Sigma^+\gamma$ and $\Xi^{*-} \rightarrow \Xi^-\gamma$ would provide stringent constraints on deviations from heavy baryon spin-flavour SU(6). We feel that these are very exciting predictions and strongly urge experimentalists to test them.

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Figure Captions

Fig. 1. The E2/M1 ratio ($\delta = \delta_{E2/M1}$) for the SU(3) allowed radiative decays of the decuplet. The gray region corresponds to the 6000 points randomly chosen only from the uncertainties in the couplings $F$, $D$, $C$, $H$, and the width for $\Delta \rightarrow N\gamma$. The dark lines are the predictions of heavy baryon SU(6), chosen from the uncertainty in $D$, and still including the uncertainty in the $\Delta \rightarrow N\gamma$ width. We have assumed the E2 counterterm to be small compared to the contribution from pion loops.

Fig. 2. The E2/M1 ratio ($\delta = \delta_{E2/M1}$) for the SU(3) suppressed radiative decays of the decuplet. The gray region corresponds to the 6000 points randomly chosen from the uncertainties in the couplings $F,D,C$, and $H$. The dark points are the predictions of heavy baryon SU(6). Note that these SU(6) predictions are independent of the choice for $D$, and independent of the $\Delta \rightarrow N\gamma$ decay width. (These decays do not receive a contribution from either local counterterm $\Theta_1$ or $\Theta_2$ because of the lowest order U-spin invariance of electromagnetic interactions.)
