Four-Tier Diagnostic Test Method to Identify Conceptual Understanding in Calculus

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Abstract. The study aims to identify the ability to understand concepts with four-tier diagnostics test method. Every student has a different ability to understand mathematical concept. It is influenced by various factors such as intelligence, emotional conditions, environment, etc. This research uses the qualitative method with study case strategy. The data obtained from the four-tier diagnostic test on the calculus chapter. The subjects in this research are XIIth grade students. There are four stages of work in a four-tier diagnostic test, 1) question, 2) level of confidence in the answers, 3) reason, and 4) level of confidence in the reason. The result of this research show that most students have low ability to understand concepts. Most students are not sure of the answers to calculus problems given. Students also do not give the correct reason for the answer.

1. Introduction

Mathematics is a very important science in human life [1-3]. The importance of learning mathematics cannot be separated from its role in various aspects of life. In addition, by learning mathematics someone will get used to thinking systematically, scientifically, using logic, critically, and can increase their creativity [4-6]. Mathematics is important as a tool, as a science (for scientists), as a former of attitude, and as a guide to the mindset [7]. Considering the importance of mathematics in everyday life, mathematics needs to be understood and mastered by all levels of society including school students as the next generation,

The role of mathematics for students in the life to come, is none other than to improve students' abilities and intelligence in various aspects of life. Not only practicing counting and applying mathematical logic, but also students are also required to be able to think about how life takes place by utilizing mathematics. The usefulness of mathematics that is applied in society is very much and even the activities that arise are always related to mathematics. Thus, if someone learns mathematics, he will learn to connect mathematics. Mathematics consists of various topics that are interrelated to one another [8,9]. The linkage is not only between topics in mathematics, but there are also links between mathematics with other disciplines and with everyday life, hereinafter referred to as mathematical connections.

A survey conducted by the Program for International Student Assessment (PISA) in 2009 found that 69% of Indonesian students were only able to recognize the theme of the problem, but were unable to find a connection between the theme of the problem and the knowledge they had [10]. The
connection intended in this case is the connection between the theme of the problem with all existing knowledge. Deep thinking about the importance of mathematics will make students practice and try to be able to master and understand mathematics. Therefore, in mastering mathematics, students must be able to solve mathematical problems with a coherent, systematic, and correct settlement process. But often we find problems that are not directly addressed in the final solution. Students must go through several stages before the final step and then re-ascertain whether the answer is appropriate or is there something lacking in the completion. This certainly requires a thought process, because thinking is an attempt to solve the problem at hand, and of course the thought process requires an ability.

Calculus is a branch of mathematics focused on a series of limits, functions, derivatives, integrals, and infinity. This subject is one of the main parts of mathematics. Calculus has two main branches, namely differential calculus and integral calculus which are interconnected through the basic theorem of calculus. Differential Calculus is a branch of calculus in mathematics which also studies how the value of a function will change according to changes in its input value. The main topic of learning differential calculus is derivatives. The reaction rate of a chemical reaction is also a derivative. Integral Calculus is a mathematical analysis of the technique of finding expressions and evaluating integral functions, especially for the calculation of area, length, curvature, volume, and numbers and solving simple differential equations.

The basic idea of formative assessment is that classroom assessment with immediate feedback to students during instruction has a significant positive effect on student learning [11,12]. Formative assessment can be highly effective in fostering student learning output. The effectiveness depends on several features of assessment, individual characteristics, and contextual factors [13]. Significant effects for formative assessment have been reported in the domains of reading and mathematics [14]. One possible solution to this problem is concepts (Concept Tests) which have been discussed in the field of mathematics education. These concepts are high-level multiple-choice questions that focus on the key concepts of a lesson [15]. Each item is made up of a four-tier multiple-choice question. The first-tier question is a content question; the following one or two require the students to give one or more reason for their first choice. Such diagnostic tests with three- or four-tier multiple-choice items have already been developed in different scientific domains [16,17]. There are some practical advantages of concepts and multi-tier diagnostic tests. First, students re easy to administer; second, teachers can use them to assess pre-knowledge as well as the learning process during instruction; third, little technical knowledge is required to develop and administer multiple-tier diagnostic assessments in learning management systems [18]. Based on the description, the researcher aims to identify the ability of students of class XII IPA MAN Magelang in understanding derivative and integral material with the four-tier method.

2. Method
This research uses qualitative methods with case studies strategy. Qualitative method is a method that aims to find out the phenomena experienced by the subject with the description with words and sentences [19]. The Case study is an empirical inquiry that purposed to investigates a contemporary phenomenon within its real-life context [20]. The problem in this study is the ability to understand concepts XIIth Grade students on Calculus Chapter by Four-Tier Diagnostic Test. The research was held at the high school in Magelang in August 2019. This research uses a four-tier diagnostic test method. Where the approach is to calculate the correct number, false, confident, and unsure of the student's answer to each question. Data were analyzed with Miles and Huberman steps of analyzing involve data reduction, data display, and verification [21]. The data in the form of the results of students and then analyzed to find out the ability to understand concepts by the Four-Tier Diagnostic Test.
3. Result and Discussion

The Sub-material in this study consisted of derived concepts, minimum and maximum value concepts, and integral concepts. Based on the results of the study, obtained information that the research subjects understand the concept of the tunings, but for the sub material that other subjects research has not understood well. The research Instrument uses tests with four-tier diagnostic test methods. The understanding of the student concept is identified based on the student confidence level in answering the four-tier diagnostic test instruments at the second and four-tier while identifying concepts of understanding can be identified through Instrument in step three. The results of the study are explained below.

Table 1. Student conception category based on answers to the four-tier diagnostic test

| Number | Category                  | Option | Confidence | Reason | Confidence |
|--------|---------------------------|--------|------------|--------|------------|
|        |                           |        | Level      |        | Level      |
| 1      | Misconception             | False  | Sure       | False  | Sure       |
| 2      | Don’t                     | False  | Sure       | False  | Not sure   |
| 3      | Understand the concept    | False  | Not sure   | False  | Sure       |
| 4      |                           | False  | Not sure   | False  | Not sure   |
| 5      | Understand the concept    | True   | Sure       | True   | Sure       |
| 6      | Concept                   | True   | Sure       | True   | Not sure   |
| 7      |                           | True   | Not sure   | True   | Sure       |
| 8      |                           | True   | Not sure   | True   | Not sure   |
| 9      |                           | True   | Sure       | False  | Sure       |
| 10     |                           | True   | Sure       | False  | Not sure   |
| 11     | Understand some           | True   | Not sure   | False  | Sure       |
| 12     |                           | True   | Not sure   | False  | Not sure   |
| 13     |                           | False  | Sure       | True   | Sure       |
| 14     |                           | False  | Sure       | True   | Not sure   |
| 15     |                           | False  | Not sure   | True   | Sure       |
| 16     |                           | False  | Not sure   | True   | Not sure   |
| 17     | Cannot be encoded         |        |            |        | If one, two, three or all of them are not filled |

Tabel 2. Student response analysis by the four-tier diagnostic test method

| Number | Category                                      | Total Answer |
|--------|-----------------------------------------------|--------------|
| 1.     | Misconception                                 | 2            |
| 2.     | Don’t Understand the concept                  | 49           |
| 3.     | Understand the concept                        | 38           |
| 4.     | Understand some                               | 21           |
1. Unknown \( f(x) = \frac{4x-1}{3x-4} \) and \( f'(x) \) is the first derivative of \( f(x) \). Value \( f'(1) = \ldots \)
   
   A. -24
   B. -13
   C. 13
   D. \( \frac{13}{5} \)
   E. 24

   Are you sure with the answer...
   
   A. Yes
   B. Not

   The reason why you choose the answer is ...
   
   A. Supposing \( f(x) = \frac{u(x)}{v(x)} \), then \( u'(x) = 3 \ v'(x) = 4 \)
   B. Supposing \( f(x) = \frac{u(x)}{v(x)} \), then \( u'(x) = 4 \ v'(x) = 3 \)
   C. Supposing \( f(x) = \frac{u(x)}{v(x)} \), then \( u'(x) = 4x \ v'(x) = 3x \)
   D. Supposing \( f(x) = \frac{u(x)}{v(x)} \), then \( u'(x) = 3-4 \ v'(x) = 4-1 \)

   Are you sure?
   
   A. Yes
   B. Not

**Figure 1. Question 1**

Question 1 read,” It is known that \( f(x) = \frac{4x-1}{3x-4} \) and \( f'(x) \) are first derivative of \( f(x) \). Value \( f'(1) \)”

,”“Are you sure about the answer”,“The reason you chose the answer is”,” Are you sure ?” Based on the research data from table 1 in question number 1, it is obtained that as many as 12 subjects answered questions and reasons appropriately. Also, the subject believes in the answers and the reasons chosen to resolve the matter. There are also 4 subjects answered correctly and the reason is wrong but the subject is confident in the answer. One subject exists that answers properly with no confidence but the reason is right with confidence and there is 1 subject answered correctly with confidence but the wrong reason is not sure. All the subjects were included in the concept of understanding. While 4 subjects belong to the category partially understood with the wrong answer is a true reason and answered confidently.
2. Known function \( f(x) = 2x^3 + 3x^2 - 12x + 6 \) for \(-3 \leq x \leq 2\). The following statement is true . . .
   A. The function \( f(x) \) reaches its maximum when \( x = -1 \).
   B. The function \( f(x) \) reaches a minimum when \( x = -2 \).
   C. The function \( f(x) \) reaches a minimum when \( x = -1 \).
   D. The minimum value of the function \( f(x) \) is -2.
   E. The maximum value of the function \( f(x) \) adalah -2.

Are you sure about the answer . . .
   A. Yes
   B. Not

The reason why you choose the answer is . . .
   A. Diagram of the function value sign \( f'(x) = 6x^2 + 6x - 12 \) in each \( x \) value as

   followers

   B. Diagram of the function value sign \( f'(x) = 6x^2 + 6x - 12 \) in each \( x \) value as

   followers

   C. Diagram of the function value sign \( f'(x) = 6x^2 + 6x - 12 \) in each \( x \) value as

   followers

   D. Diagram of the function value sign \( f'(x) = 6x^2 + 6x - 12 \) in each \( x \) value as

   followers

Are you sure ?
   A. Yes
   B. Not

**Figure 2. Question 2**

Based on the research data from table 1 in question number 2, obtained as much information 6 subjects included in the category Misskonsepsi with 1the subject answered wrong with the reason but sure and 5 subjects that answered the wrong answer but sure and why Wrong not sure. While 12 subjects come in not understand the concept by answering the wrong that followed the lack of a. There are then 3 subjects in the category of partial understanding by answering the wrong questions but having the right reason and only 1 subject that comes in the category of concepts.
3. Given the function \( g(x) = \frac{1}{3} x^3 - A^2 x + 7 \), \( A \) contains. If \( f(x) - g(2x + 1) \) and the derivative at \(-\frac{3}{2} < x < \frac{1}{2}\) the minimum relative value of \( g \) is . . .

A. \( \frac{4}{3} \)
B. \( \frac{5}{3} \)
C. 2
D. \( \frac{7}{3} \)

Are you sure about the answer . . .

A. Yes
B. No

The reason why you choose the answer is . . .

A. The graph of function \( f(x) \) rises at intervals \(-\frac{3}{2} \leq x \leq \frac{1}{2}\), then \( f \) down at \( (x \leq \frac{3}{2} \text{ or } x \geq \frac{1}{2}) \)
B. The graph of function \( f(x) \) rises at intervals \(-\frac{3}{2} < x < \frac{1}{2}\), then \( f \) down at \( (x \leq \frac{3}{2} \text{ or } x \geq \frac{1}{2}) \)
C. The graph of function \( f(x) \) Dropped at intervals \(-\frac{3}{2} < x < \frac{1}{2}\), then \( f \) rises on \( (x \leq -\frac{3}{2} \text{ or } x \geq \frac{1}{2}) \)
D. The graph of function \( f(x) \) Dropped at intervals \(-\frac{3}{2} < x < \frac{1}{2}\), then \( f \) rises on \( (x < -\frac{3}{2} \text{ or } x > \frac{1}{2}) \)

Are you sure ?

A. Yes
B. No

**Figure 3. Question 3**

Based on the research data from table 1 in question number 3, obtained as many as 7 subjects in the category are not conceptualized with the wrong answer followed by the unequal in answering the problem. Also, there are 13 subjects included in the concept of understanding with as many as 3 subjects answered questions and reasons appropriately. Besides Tu 1 The subject answered the question correctly and was sure but in his true, but not sure. 6 subjects answered questions and reasons correctly but not sure. Besides, 1 subject answered the question correctly and was convinced, but the reason was wrong and convinced. While the other 2 subjects answered the question correctly and were convinced, but with the wrong reason and not sure. Two subjects belong to the partially understood category, with the correct answer but not sure.
4. The results of \( \int 2x \sqrt{4-x} \, dx \) is . . .
   A. \(-\frac{4}{15} (3x + 8)(4 - x)^{\frac{3}{2}} + C\)
   B. \(-\frac{2}{15} (3x + 8)(4 - x)^{\frac{3}{2}} + C\)
   C. \(-\frac{4}{15} (3x - 8)(4 - x)^{\frac{3}{2}} + C\)
   D. \(\frac{2}{15} (3x - 8)(4 - x)^{\frac{3}{2}} + C\)

Are you sure about the answer . . .
   A. Yes
   B. No

The reason why you choose the answer is . . .
   A. Because \( \int 2x \sqrt{4-x} \, dx \) using partial integrals
   B. Because \( \int 2x \sqrt{4-x} \, dx \) using an unnecessarily integral
   C. Because \( \int 2x \sqrt{4-x} \, dx \) using an unnatural integral
   D. Because \( \int 2x \sqrt{4-x} \, dx \) using integral substitutions

Are you sure?
   A. Yes
   B. No

**Figure 4. Question 4**

Based on the research data from table 1 in Question No. 4, obtained as many as 7 subjects entered into an unconceptualized category. The 3 subjects answered the question wrong but were convinced because of being wrong and unsure. Besides, 4 subjects answered questions and reasons wrong and not sure. 6 subjects go into the concept of conceptualized with 1 subject answering questions and reasons correctly and confidently. Besides, there are 3 subjects answered questions and reasons correctly but not sure. 1 The subject answered the question correctly and was convinced but with the wrong reason and convinced. There are also 1 subjects answered correctly and confidently, but the reason is wrong and not sure. Also, 9 subjects enter into a partial category with 1 subject answering the correct question but not sure and answer the wrong reason but sure. In addition, 7 subjects answer the question correctly but are unsure and answer the wrong reason and are unsure. 1 subjects answer the question wrong and are unsure and then answer the reason correctly but not sure.
5. The results of \( \int \frac{x+2}{\sqrt{x^2+4x-5}} \, dx \) is ....
   A. \( \sqrt{x^2} + 4x - 5 + C \)
   B. \( 2 \sqrt{x^2} + 4x - 5 + C \)
   C. \( 4 \sqrt{x^2} + 4x - 5 + C \)
   D. \( 6 \sqrt{x^2} + 4x - 5 + C \)

Are you sure about the answer . . .
   C. Yes
   D. Not

The reason why you choose the answer is . . .
   A. \( 2 \, du = (x+2) \, dx \)
   B. \( \frac{du}{x+2} = \frac{1}{2} \, dx \)
   C. \( \frac{1}{2} \, du = (x+2) \, dx \)
   D. \( du = 2 \, dx \)

Are you sure?
   C. Yes
   D. Not

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**Figure 5. Question 5**

Based on the research data from Table 1 in Question No. 5, obtained as many as 1 subject entered into category miss concepts by answering the question and the reason is wrong but has a level of confidence in answering. There are 18 subjects in the concept of conceptualized with 1 subject answered the wrong matter and unsure and answered the wrong reason but sure. Also, 17 subjects answered questions and reasons wrong and not sure. 3 subjects enter into a partial category with 2 subjects answering the correct question but not sure and answering the wrong reason and not sure and 1 subject answered the wrong problem and was unsure and answered the reason properly and confidently.

4. **Conclusion**

The ability to understand the mathematical concept of the XII MAN calculus class in Magelang can be divided into 2 misconception cases, 49 answers cases that indicated the student don't understand the Concept, 38 answers cases that indicated the students understand the concept answers, and 21 answers cases that indicated the students understand some answers.

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