Resource Optimization for Time Slot-Split Noma in Full-Duplex Cooperative Communications

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Abstract. A time-slot-split non-orthogonal multiple access (TSS-NOMA) scheme for full-duplex (FD) relaying communication systems is proposed to improve frequency resource utilization. Aiming at the problem of low bandwidth efficiency when users pairing, TSS-NOMA divides the whole time slot into several parts to improve the matching process, where users can occupy all bandwidths in different time slots. TSS-NOMA decodes the information symbols of relay and re-encodes the messages through the new power allocation factors of successfully matched users. The mixed symbols are sent to increase the proportion of central users’ own signals received by themselves, thus improving the performance of the system. To maximize the minimum achievable rate of the system, TSS-NOMA optimizes the power allocation at the base station and the relay. Numerical results show that TSS-NOMA can achieve higher spectral and energy efficiency than some existing schemes.

1. Introduction
In wireless communication systems, non-orthogonal multiple access (NOMA) enables users to share the same time-frequency resources, which allows the system to achieve large capacity and low transmission delay, thus serving a large number of users [1-4]. Besides, cooperative technology greatly ameliorates the throughput and transmission rate of communication systems [5-6].

Recently, the combination of cooperative relaying system communication and NOMA (CRS-NOMA) becomes a hot research topic. The conception of CRS-NOMA was initially proposed in Ref. [7], which shows its strong advantages in outage performance. In Ref. [8], a new detection method is proposed to verify the better performance of CRS-NOMA with amplification and forwarding than that with decoding and forwarding. Besides, literatures [9-12] are devoted to the study of power allocation with effective algorithms, where Ref. [10] concludes that the system has better performance on the premise of intermediate signal to interference and noise ratio (SINR).

A drawback of the aforementioned schemes is that the performance of the system will decrease when the cell has a large number of NOMA users. As a result, we propose a time slot-split non-orthogonal multiple access (TSS-NOMA) scheme for FD cooperative communication systems to improve resource utilization in this paper. The contributions of this paper are as follows. We divided the whole communication time into two parts by exploiting slot-split, where the paired users communicate in NOMA mode. Then the paired users can utilize the entire bandwidth of the system in different slots. Provided that a small delay occurs between the cooperative and the direct link signals in TSS-NOMA, we derived the closed form solutions for outage probability by forwarding mixed signals instead of single signal, proving that TSS-NOMA can achieve higher performance.
Considering the power allocation problem from the perspective of maximizing the minimum achievable rate of the system, an algorithm is proposed to obtain the power allocation factors.

2. Description of Time Slot-Split Noma (TSS-NOMA)

As illustrated in figure 1, we consider a system consisting of a base station (BS), a relay node R and three NOMA users namely U1, U2 and U3, where U2 and U3 are almost the same distance to R. The channels are flat fading Rayleigh channels, and they satisfy \( h_q \sim CN(0,\lambda_q) \), where \( q = b, b_1, r_1, r_2, r_3, R \), and \( CN(0,\sigma^2) \) denotes the zero-mean complex Gaussian distribution with variance \( \sigma^2 \). The coefficients between BS and R (BS-R), BS and U1 (BS-U1), R and U1 (R-U1), R and U2 (R-U2), R and U3 (R-U3) are denoted as \( h_{b}, h_{b_1}, h_{r_1}, h_{r_2}, h_{r_3} \), respectively. Parameter \( h_R \) is the self-interference channel at relay. Also, we define \( \lambda_q = d_q^{-\nu} \), where \( d_q \) and \( \nu \) denote the distance between two related nodes and the path-loss exponent, respectively. The system noise is additive white noise, and it can be denoted as \( n_k \sim CN(0,\sigma_k^2) \) \( (k = r, 1, 2, 3, 11) \). Here \( \sigma_1^2, \sigma_2^2, \sigma_3^2 \) and \( \sigma_r^2 \) denote the noise variances at U1, U2, U3 and R, respectively. Particularly, parameter \( \sigma_{r_1}^2 \) denotes the noise variance of U1 after decoding its own signal by successive interference cancellation (SIC). U1 is the central user of the cell, it communicates directly with the BS because it is close to the BS. As edge users of the cell, U2 and U3 cannot constitute a direct link with the BS because of their long distances from the BS and the influence of obstacles. As a consequence, they must communicate with the BS through the assistance of the relay.

Considering several scenarios as described in figure 2, the power of U1, U2 and U3 is denoted as \( P_1, P_2, P_3 \), respectively, and \( B \) represents the bandwidth of the system. The orthogonal multiple access (OMA) scheme shows that users share the bandwidth resources equally, and signals from the BS and the relay are processed by maximal ratio combining (MRC). Furthermore, the NOMA-OMA scheme causes a certain waste of bandwidth resources because the unpaired users utilize OMA mode for communication. In TSS-NOMA, a complete communication time is divided into \( T_1 \) and \( T_2 \) where \( T_1 \) denotes the paired time of U1-U2, and \( T_2 \) denotes the paired time of U1-U3.

Considering the BS transmit power as \( P_B \), the overlapping coded signal in \( T_1 \) can be written as

\[
G[T_1] = \sqrt{\alpha_1 P_B} x_1[T_1] + \sqrt{\alpha_2 P_B} x_2[T_1]
\]  

Here, \( x_1[T_1] \) and \( x_2[T_1] \) represent messages which are sent to U1 and U2, respectively. Parameters \( \alpha_1 \) and \( \alpha_2 \) denote the power allocation factors of U1 and U2, where \( \alpha_1 + \alpha_2 = 1 \) and \( \alpha_1 < \alpha_2 \).

![Figure 1. System model of TSS-NOMA.](image)

![Figure 2. The Comparison of different communication schemes.](image)
Provided that the relay decodes the signals of U1 and U2 to obtain symbols $x_i[T_i]$ and $x_j[T_j]$ in $T_i$. The new mixed symbols are obtained by decoding with $\alpha_i$ and $\alpha_j$, where $\alpha_i$ and $\alpha_j$ denote the new power allocation factors of U1 and U2. Then the relay re-encodes $x_i[T_i]$ and $x_j[T_j]$ through the protocol of NOMA to generate a new mixed symbol to be broadcast as

$$
\sqrt{\alpha_i P_i x_i[T_i] - \tau} + \sqrt{\alpha_j P_j x_j[T_j] - \tau} \quad (\tau \geq 1)
$$

where $\tau \geq 1$ and $P_i$ indicate the processing delay and power at the relay, respectively. Also, U3 remains silent when U1 and U2 decode the signal in $T_i$ by SIC. The signal received by the relay can be expressed as

$$
y_i[T_i] = h_{i0} G[T_i] + h_{i1} \left( \sqrt{\alpha_i P_i x_i[T_i] - \tau} + \sqrt{\alpha_j P_j x_j[T_j] - \tau} \right) + n_i[T_i]
$$

where $k(0 \leq k \leq 1)$ represents the FD residual self-interference factor. In particular, $k=0$ implies perfect interference cancellation.

Next, we maximize the minimum achievable rate of the system by optimizing the power allocation at the BS and the relay. The power of the BS and the relay is denoted as $P_B$ and $P_r$, respectively. Moreover, $P_B$ and $P_r$ are optimally allocated when the total power $P$ is limited. The problem can be described as follows,

$$
(P_B, P_r) = \arg \max_{P_B, P_r} \left\{ \min \left\{ R_i(P_B, P_r), R_j(P_B, P_r) \right\} \right\}
$$

s.t. $P_B + P_r = P$

$$
0 < P_B < P_{B,max}
$$

$$
0 < P_r < P_{r,max}
$$

where $P_{B,max}$ and $P_{r,max}$ denote the maximum transmit power of the BS and the relay respectively.

Provided that the high quality of service (QOS) of the central user near the BS, then U1 has a high demand for the target rate. The corresponding optimization problem becomes maximizing the achievable rate of U2.

According to the maximization principle and the power allocation schemes in Refs. [12, 13], we consider

$$
\frac{\alpha_2 P_B |h_{i0}|^2}{\alpha_2 P_B |h_{i0}|^2 + k|P_i| |h_{i0}|^2 + \sigma_x^2} = \frac{\alpha_2 P_r |h_{i1}|^2}{\alpha_2 P_r |h_{i1}|^2 + \sigma_x^2}
$$

maximizing the achievable rate of U2.

Therefore, by substituting $P_B = P - P_r$ into equation (5), we obtain an equation which can be expressed as $AP^2 + BP + C = 0$, where $A$, $B$ and $C$ are respectively expressed as

$$
A = \left| h_{i2} \right|^2 \left( \alpha_2 k |h_{i0}|^2 - \alpha_i \alpha_j |h_{i0}|^2 + \alpha_i \alpha_j |h_{i0}|^2 \right)
$$

$$
B = \alpha_2 \left| h_{i2} \right|^2 \sigma_x^2 + \alpha_2 |h_{i0}|^2 \sigma_x^2 + \alpha_i \alpha_j \left| h_{i0} \right|^2 \left| h_{i2} \right|^2 P + \alpha_i \alpha_j \left| h_{i0} \right|^2 \left| h_{i2} \right|^2 P
$$

$$
C = -\alpha_2 \left| h_{i0} \right|^2 \sigma_x^2 P
$$

The expressions of $P_r$ which are greater than 0 can be acquired from the properties of parabolic. Hence, the power allocation factors of the BS and the relay are expressed as $P_r / P$ and $(1-P_r / P)$ respectively.
Since the two slots apply the identical principle, the processing manner is the same as the above steps when U1 and U3 are paired in $T_i$. U2 remains silent during this process.

3. Performance analysis of TSS-NOMA

Suppose that the target SINRs of decoding U1, U2 and U3 signals are $\gamma_{1,T} = 2^{R_1} - 1$, $\gamma_{2,T} = 2^{R_2} - 1$ and $\gamma_{3,T} = 2^{R_3} - 1$, where $R_1$, $R_2$ and $R_3$ denote the target rates when U1, U2 and U3 decode their own signals, respectively. The outage probability of U1 in $T_i$ can be expressed as $P^o_{out,1} = 1 - \xi_1 \xi_2 \xi_3 \xi_4$, where $\xi_1$ and $\xi_2$ are given by

$$
\xi_1 = P \left( \frac{\alpha_1 P_R |h_{10}|^2}{\alpha_1 P_R |h_{10}|^2 + kP_R |h_k|^2 + \sigma^2} \geq \gamma_{2,T} \right)
$$

$$
= \frac{\lambda_n P_R (1 - \alpha_1 - \alpha_1 \gamma_{2,T})}{\lambda_n P_R (1 - \alpha_1 - \alpha_1 \gamma_{2,T}) + \gamma_{2,T} kP_R \lambda_R} \exp \left( \frac{-\gamma_{2,T} \sigma^2}{\lambda_n P_R (1 - \alpha_1 - \alpha_1 \gamma_{2,T})} \right) \tag{9}
$$

$$
\xi_2 = P \left( \frac{\alpha_2 P_R |h_{10}|^2}{\alpha_1 P_R |h_{10}|^2 + \sigma^2} \geq \gamma_{1,T} \right) = \frac{\lambda_n \alpha_1 P_R}{kP_R \gamma_{1,T} \lambda_R + \lambda_n \alpha_1 P_R} \exp \left( \frac{-\sigma^2 \gamma_{1,T}}{\lambda_n (\alpha_2 P_R \gamma_{2,T} + \alpha_1 P_R \gamma_{2,T})} \right) \tag{10}
$$

Provided that relay to U1 at a high signal noise ratio (SNR) as $\frac{\alpha_1 P_R |h_{10}|^2}{\alpha_1 P_R |h_{10}|^2 + \sigma^2} = \frac{\alpha_1}{\alpha_1}$, then $\xi_3$ can be approximated as

$$
\xi_3 = P \left( \frac{\alpha_2 P_R |h_{10}|^2}{\alpha_2 P_R |h_{10}|^2 + \sigma^2} \geq \gamma_{1,T} \right) = \exp \left( \frac{-\gamma_{1,T} \sigma^2 - \alpha_2 \sigma^2}{\lambda_n (\alpha_2 P_R \gamma_{2,T} + \alpha_1 P_R \gamma_{2,T})} \right) \tag{11}
$$

According to the double integral, we can obtain the expression of $\xi_4$ by

$$
\xi_4 = P \left( \frac{\alpha_2 P_R |h_{10}|^2}{\alpha_2 P_R |h_{10}|^2 + \sigma^2} \geq \gamma_{1,T} \right) = \exp \left( \frac{-\gamma_{1,T} \sigma^2 - \alpha_2 \sigma^2}{\lambda_n (\alpha_2 P_R \gamma_{2,T} + \alpha_1 P_R \gamma_{2,T})} \right) \tag{12}
$$

Substituting $\xi_1$, $\xi_2$, $\xi_3$ and $\xi_4$ into $P^o_{out,1} = 1 - \xi_1 \xi_2 \xi_3 \xi_4$, the outage probability of U1 in $T_i$ can be expressed as

$$
P^o_{out,1} = 1 - \frac{\lambda_n P_R (1 - \alpha_1 - \alpha_1 \gamma_{2,T})}{\lambda_n P_R (1 - \alpha_1 - \alpha_1 \gamma_{2,T}) + \gamma_{2,T} kP_R \lambda_R} \frac{\lambda_n \alpha_1 P_R}{kP_R \gamma_{1,T} \lambda_R + \lambda_n \alpha_1 P_R} \exp \left( \frac{-\gamma_{2,T} \sigma^2}{\lambda_n P_R (1 - \alpha_1 - \alpha_1 \gamma_{2,T})} \right) \frac{\gamma_{1,T} \sigma^2}{\lambda_n (\alpha_2 P_R \gamma_{2,T} + \alpha_1 P_R \gamma_{2,T})} \exp \left( \frac{-\gamma_{1,T} \sigma^2 - \alpha_2 \sigma^2}{\lambda_n (\alpha_2 P_R \gamma_{2,T} + \alpha_1 P_R \gamma_{2,T})} \right) \tag{13}
$$

Also, the outage probability of U2 in $T_i$ is given by
\[ P_{\text{out}, 2} = 1 - \frac{\lambda_{\text{br}} P_B (1 - \alpha_1 - \gamma_{2, T})}{\lambda_{\text{br}} P_B (1 - \alpha_1 - \gamma_{2, T}) + \lambda_{\text{br}} \alpha_1 P_B k \rho_{\gamma_{1, T}} \gamma_{2, T} + \lambda_{\text{br}} \alpha_1 P_B} \exp \left( - \frac{\sigma_{\gamma_{1, T}}^2}{\lambda_{\text{br}} \alpha_1 P_B (1 - \alpha_1 - \gamma_{2, T})} - \frac{\gamma_{2, T} \sigma_{\gamma_{2, T}}^2}{\lambda_{\text{br}} \alpha_1 P_B (1 - \alpha_1 - \gamma_{2, T})} \right) \]  

(14)

Similarly, we can get the outage probabilities of U1 and U3 which are denoted as \( P_{\text{out}, 1} \) and \( P_{\text{out}, 3} \) in \( T_2 \). Then the outage probabilities in \( T_1 \) and \( T_2 \) are given by

\[ P_{\text{out}} = 1 - \left( 1 - P_{\text{out}, 1} \right) \left( 1 - P_{\text{out}, 2} \right) \]  

(15)

\[ P_{\text{out}} = 1 - \left( 1 - P_{\text{out}, 3} \right) \left( 1 - P_{\text{out}, 2} \right) \]  

(16)

Finally, the outage probability of the system can be calculated by

\[ P_{\text{out}} = 1 - \left( 1 - P_{\text{out}} \right) \left( 1 - P_{\text{out}} \right) \]  

(17)

4. Numerical Results

In this section, simulations are implemented to verify the performance of TSS-NOMA. We consider \( \lambda_1 = \lambda_2 = \lambda_3 = 0.8 \), \( \lambda_{\text{br}} = 1 \), \( \lambda_{\text{br}} = \lambda_R = 0.6 \) and \( \alpha_1 = 0.05 \), and assume the Monte Carlo simulation times are \( 10^7 \). The target SINRs of U1, U2 and U3 are \( \gamma_{1, T} = 1.5 \) and \( \gamma_{1, T} = 1 \), respectively.

We consider the scheme proposed in Ref. [10] as the comparison scheme to verify the analysis of outage probability, the outage probabilities of U1 and U2 with the change in SNR between two different schemes are demonstrated in figure 3. Different from the comparison scheme, the outage probability of U1 can be further decreased with the increase in SNR when it reaches a value in TSS-NOMA. As illustrated in figure 4, the outage probability as a function of the parameter \( \alpha \), where \( \alpha \) and \( 1 - \alpha \) denote power allocation factors at the BS and the relay respectively. Obviously, the curve of U3 has a slower trend because of its lower SINR than U2. Moreover, the outage probability of U1 first decreases, and it rises with the increase in \( \alpha \) when it is greater than a certain value. As such, the system achieves the best performance when the power allocation factor \( \alpha \) is about 0.25.
5. Conclusion
The paper proposes a TSS-NOMA scheme for FD cooperative communication systems to improve resource efficiency. It has been concluded that better system performance is achieved than other available schemes if users occupy the entire bandwidth. Also, the value of the power allocation factor has a significant impact on maximizing the minimum achievable rate of the system. Results indicate that, compared with other schemes mentioned in this paper, TSS-NOMA for FD cooperative communication system has better outage capacity, thus improving the performance of the system.

References
[1] Chen S Z, Ren B, Gao Q B, Kang S L, Sun S H and Niu K 2017 Pattern division multiple access-a novel nonorthogonal multiple access for fifth-generation radio networks IEEE Transactions on Vehicular Technology 66 3185-3196.
[2] Gau R H, Chiu H T, Liao C H and Wu C L 2018 Optimal power control for NOMA wireless networks with relays IEEE Wireless Communications Letters 7 22-25.
[3] Nguyen H S, Nguyen T S, Tin P T and Voznak M 2018 Outage performance of time switching energy harvesting wireless sensor networks deploying NOMA IEEE 20th International Conference on e-Health Networking pp 1-4.
[4] Kim B, Park Y and Hong D 2019 Partial non-orthogonal multiple access (P-NOMA) IEEE Wireless Communications Letters 8 1377-1380.
[5] Liu C H and Liang D C 2017 Towards accurate throughput analysis for dense heterogeneous networks with cooperative NOMA IEEE GLOBECOM pp 1-6.
[6] Sanada K and Mori K 2019 Throughput analysis for full duplex wireless local area networks with hidden nodes IEEE Annual Consumer Communications and Networking Conference (CCNC) pp 1-4.
[7] Ding Z G, Peng M G and Poor H V 2015 Cooperative non-orthogonal multiple access in 5G systems IEEE Communications Letters 19 1462-1465.
[8] Abbasi O, Ebrahimi A and Mokari N 2019 NOMA inspired cooperative relaying system using an AF relay IEEE Wireless Communications Letters 8 261-264.
[9] Ding J F, Cai J and Yi C Y 2019 An improved coalition game approach for MIMO-NOMA clustering integrating beamforming and power allocation IEEE Transactions on Vehicular Technology 68 1672-1687.
[10] Zhong C J and Zhang Z Y 2016 Non-orthogonal multiple access with cooperative full-duplex relaying IEEE Communications Letters 20 2478-2481.
[11] Yu D Y, Liu Y, Shen P and Zhang H L 2019 power allocation for self-coded distributed space-time codes in FD two-way relaying networks IEEE Transactions on Vehicular 68 12013-12024.
[12] Liu Y, Xu X W, Ma Y and Zhang H L 2018 Power allocation under global and individual power constraints for full-duplex relay networks IET Communications 12 317-325.
[13] Zamani M R, Eslami M and Khorramizadeh M 2018 Optimal sum-rate maximization in a NOMA system with channel estimation error Electrical Engineering (ICEE) pp 720-724.