Abstract

We present a detailed and complete calculation of the doubly charged Higgsino contribution to the decays $\mu \to e\gamma$ and $\mu \to 3e$ and to the anomalous magnetic moment of the muon $\Delta a_\mu$ within the left-right supersymmetric model. We include the mixing of the scalar partners of the left and right handed leptons, and show that it leads to a strong enhancement of the decay modes in certain scenarios. We find that the contribution of the doubly charged Higgsino can be close to the known experimental values and are reachable by future experiments.
1 Introduction

The quest for a supersymmetric grand unified theory is plagued by lack of direct signals which would distinguish such a theory from supersymmetry in general. Supersymmetry, in particular the Minimal Supersymmetric Model (MSSM) \([1]\), can be probed experimentally through the high energy production of superpartners. However, the MSSM, while filling in some of the theoretical gaps of the Standard Model, fails to explain other phenomena such as the weak mixing angle, the small mass (or masslessness) of the known neutrinos, the origin of CP violation, to quote a few. Extended gauge structures such as grand unified theories, introduced to provide an elegant framework for unification of forces \([2]\), would connect the standard model with more fundamental structures such as superstrings, and also resolve the puzzles of the electroweak theory.

Phenomenologically, grand unified theories would either predict relationships between otherwise independent parameters of the standard model, or new interactions (i.e. interactions forbidden or highly suppressed in the standard model).

Among supersymmetric grand unified theories, SO(10) \([3]\) and SU(5) \([4]\) have received significant attention. In this article we shall study a model based on the left-right symmetric extension of the MSSM, based on \(SU(2)_L \times SU(2)_R \times U(1)_{B-L}\). Its attraction is that on one hand the Left-Right Supersymmetric Model (LRSUSY) is an extension of the Minimal Supersymmetric Standard Model based on left-right symmetry and on the other hand it could be viewed as a low-energy realization of certain SUSY GUTs, such as SO(10).

LRSUSY shares some of the attractive properties of the MSSM, like providing a natural solution for the gauge hierarchy problem. LRSUSY also suppresses naturally rapid proton decay, by disallowing terms in the Lagrangian that explicitly violate either baryon or lepton numbers \([5]\). It gauges the only quantum number left ungauged, \(B-L\). The LRSUSY model also shares some of the attractive features of the original left-right symmetric model \([6]\), such as providing a possible explanation for the smallness of the neutrino mass and for the origin of parity violation. Recently, this model has received a lot of attention, because it could offer a framework for solving both the strong and the weak CP problem \([7]\). Furthermore it has the very attractive feature of automatically conserving R-parity.

So far, there is no experimental evidence for the right-handed interactions predicted by the \(SU(2)_L \times SU(2)_R \times U(1)_{B-L}\) theory, let alone by supersymmetry. Yet the foundation of
LRSUSY has so many attractive features that the model deserves some experimental and theoretical investigation. The next generation of linear colliders will provide an excellent opportunity for such a study. The theoretical and experimental challenge lies in finding distinctive features for the left-right supersymmetric model, which allow it to be distinguished from both the SUSY version of the Standard Model and from the non-supersymmetric version of the left-right theory \([8]\). Lepton-flavor violation decays are just the right type of such phenomena. The LRSUSY model provides a natural framework for large lepton flavor-violating effects through two mechanisms: on one hand it can give rise to a leptonic decay width of the \(Z\)-boson through both left-handed and right-handed scalar lepton mixing \([9]\), on the other hand it contains lepton-flavor-blind higgsinos which couple to leptons only and enhance lepton-flavor violation.

In a recent paper \([10]\) we analysed the lepton flavor-violating decay \(\mu \to e\gamma\), whose signature can be reliably calculated and, by the structure of the model, showed to be greatly enhanced over the MSSM, and already accessible at the current experimental accuracy.

The contribution of the doubly charged Higgs boson to lepton-flavor violation decays were considered in \([11]\) and quite strong constraints to its mass and its Yukawa coupling to the leptons were presented. Here we will study a new source of enhancement coming from the existence, in the supersymmetric sector, of a doubly-charged higgsino. Since the measurement of \(\mu \to e\gamma\) has a very precise bound \(B(\mu \to e\gamma) < 4.9 \times 10^{-11}\) and \(B(\mu \to 3e) < 1.0 \times 10^{-12}\) \([12]\), we are able to obtain values close to the experimental bound and restrict some of the parameters in the theory.

A similar contribution from the doubly-charged higgsino (with different mass parameters) appears in the calculation of another accurately measured quantity, the anomalous magnetic moment of the muon (AMMM) \(a_\mu = (g-2)/2\). Its experimental value is \(a_\mu^{\exp} = 1165922(9) \times 10^{-9}\) \([12]\). The measured deviation of the AMMM lies within a range of \(-2 \times 10^{-8} \leq \Delta a_\mu^{\exp} \leq 2.6 \times 10^{-8}\) \([13]\)

Experiment E821, under way at the Brookhaven National Laboratory (BNL), is designed to improve the existent data on the AMMM by a factor of 10–20. When completed, it would be possible to test deviations from the 1-loop predictions of the Minimal Supersymmetric Standard Model, believed to arise most likely from supersymmetric contributions. For completeness, we also investigate the effect of the contribution of the doubly-charged higgsino to the AMMM. We then restrict the parameter space of the left-right supersymmetric model
by combining these effects.

Our paper is organized as follows: in section 2 we give a brief description of the model, followed by the numerical analysis of the decay $\mu \rightarrow e\gamma$ in section 3 and the AMMM in section 4. Our conclusions are reached in section 5. In addition, Appendices A and B will present the detailed analytical calculations.

2 The Left-Right Supersymmetric Model

The $LRSUSY$ model, based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, has matter doublets for both left- and right- handed fermions and the corresponding left- and right-handed scalar partners (sleptons and squarks). In the gauge sector, corresponding to $SU(2)_L$ and $SU(2)_R$, there are triplet gauge bosons $(W^+,-,W^0)_L$, $(W^+,-,W^0)_R$ and a singlet gauge boson $V$ corresponding to $U(1)_{B-L}$, together with their superpartners. The Higgs sector of this model consists of two Higgs bi-doublets, $\Phi_u(\frac{1}{2}, \frac{1}{2}, 0)$ and $\Phi_d(\frac{1}{2}, \frac{1}{2}, 0)$, which are required to give masses to both the up and down quarks. In addition, the spontaneous symmetry breaking of the group $SU(2)_R \times U(1)_{B-L}$ to the hypercharge symmetry group $U(1)_Y$ is accomplished by introducing the Higgs triplet fields $\Delta_L(1, 0, 2)$ and $\Delta_R(0, 1, 2)$. The choice of the triplets (versus four doublets) is preferred because with this choice a large Majorana mass can be generated for the right-handed neutrino and a small one for the left-handed neutrino. In addition to the triplets $\Delta_L,R$, the model must contain two additional triplets $\delta_L(1, 0, -2)$ and $\delta_R(0, 1, -2)$, with quantum number $B-L = -2$ to insure cancellation of the anomalies that would otherwise occur in the fermionic sector. Given their strange quantum numbers, the $\delta_L$ and $\delta_R$ do not couple to any of the particles in the theory so their contribution is negligible for any phenomenological studies.

As in the standard model, in order to preserve $U(1)_{EM}$ gauge invariance, only the neutral Higgs fields acquire non-zero vacuum expectation values ($VEV’s$). These values are:

$$\langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}, \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \text{ and } \langle \Phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa'Ie^{i\omega} \end{pmatrix}.$$  

$\langle \Phi \rangle$ causes the mixing of $W_L$ and $W_R$ bosons with $CP$-violating phase $\omega$. In order to simplify, we will take the $VEV’s$ of the Higgs fields as: $\langle \Delta_L \rangle = 0$ and

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \langle \Phi_u \rangle = \begin{pmatrix} \kappa_u & 0 \\ 0 & 0 \end{pmatrix} \text{ and } \langle \Phi_d \rangle = \begin{pmatrix} 0 & 0 \\ 0 & \kappa_d \end{pmatrix}.$$
Choosing $v_L = \kappa' = 0$ satisfies the more loosely required hierarchy $v_R \gg \max(\kappa, \kappa') \gg v_L$ and also the required cancellation of flavor-changing neutral currents. The Higgs fields acquire non-zero $VEV$’s to break both parity and $SU(2)_R$. In the first stage of breaking the right-handed gauge bosons, $W_R$ and $Z_R$ acquire masses proportional to $v_R$ and become much heavier than the usual (left-handed) neutral gauge bosons $W_L$ and $Z_L$, which pick up masses proportional to $\kappa_u$ and $\kappa_d$ at the second stage of breaking. [5]

The supersymmetric sector of the model, while preserving left-right symmetry, has four singly-charged charginos (corresponding to $\tilde{\lambda}_L, \tilde{\lambda}_R, \tilde{\phi}_u$, and $\tilde{\phi}_d$), in addition to $\tilde{\Delta}^-_L$, $\tilde{\Delta}^-_R$, $\tilde{\delta}^-_L$, and $\tilde{\delta}^-_R$. The model also has eleven neutralinos, corresponding to $\tilde{\lambda}_Z, \tilde{\lambda}_{Z'}, \tilde{\lambda}_V, \tilde{\phi}^0_{1u}, \tilde{\phi}^0_{2u}, \tilde{\phi}^0_{1d}, \tilde{\phi}^0_{2d}, \tilde{\Delta}_L^0, \tilde{\Delta}_R^0, \tilde{\delta}_L^0$, and $\tilde{\delta}_R^0$. It has been shown that in the scalar sector, the left-triplet $\Delta_L$ couplings can be neglected in phenomenological analyses of muon and tau decays [14]. Although $\Delta_L$ is not necessary for symmetry breaking [15], it is introduced only for preserving left-right symmetry, we will not neglect the couplings of $\Delta_L$ in the fermionic sector, which leads to important consequences as shown later.

The doubly charged $\Delta^{--}_R$ is however very important: it carries quantum number $B - L$ of 2 and couples only to leptons, therefore breaking lepton-quark universality. It and its supersymmetric partner could, as will be seen in the next section, play an important role in flavor-violating leptonic decays.

In the scalar matter sector, the $LRSUSY$ contains two left-handed and two right-handed scalar fermions as partners of the ordinary leptons and quarks, which themselves come in left- and right-handed doublets. In general the left- and right-handed scalar leptons will mix together. Some of the effects of this mixings, such as enhancement of the AMMM, have been discussed elsewhere [3]. Only global lepton-family-number violation would prevent $\tilde{e}$, $\tilde{\mu}$ and $\tilde{\tau}$ to mix arbitrarily. Permitting this mixing to occur, we could expect small effects to occur in the non-supersymmetric sector, such as radiative muon or tau decays, in addition to other nonstandard effects such as massive neutrino oscillations and violation of lepton number itself. But, in general, allowing for the mixing, we have six charged-scalar lepton states (involving 15 real angles and 10 complex phases) and six scalar neutrinos (also involving 15 real angles and 10 complex phases). In order to reduce the (large) number of parameters we shall assume in what folows that only two generations of scalar leptons (the lightest) mix significantly [4].

[1] However in the appendix [4], we will present general analytical expressions obtained by including the
The mixings are as follows: $\tilde{\mu}_{L,R}$ and $\tilde{e}_{L,R}$ with angle $\theta_{L,R}$; $\tilde{\nu}_{\mu,L,R}$ and $\tilde{\nu}_{e,L,R}$ with angle $\alpha_{L,R}$; so that, for example:

\[
\tilde{l}_{L1} = \tilde{\mu}_{L}\cos\theta_{L} + \tilde{e}_{L}\sin\theta_{L},
\]

\[
\tilde{l}_{L2} = -\tilde{\mu}_{L}\sin\theta_{L} + \tilde{e}_{L}\cos\theta_{L},
\]

and similar for $\tilde{\nu}_{L1,2}$ and $\tilde{\nu}_{R1,2}$.

Furthermore we include the mixing of the scalar partners to the left and right handed leptons; that is, we take $\tilde{e}_{L} = \cos\theta_{e}\tilde{e}_{1} - \sin\theta_{e}\tilde{e}_{2}$ and $\tilde{e}_{R} = \sin\theta_{e}\tilde{e}_{1} + \cos\theta_{e}\tilde{e}_{2}$, and similar for the other generations, where we express the mixings by the matrix $\tilde{K}_{a}^{m}$ in Appendix A.

The physical mass eigenstates are then given by eq.(1-2).

Next we consider the implications of the above-mixing in the $LRSUSY$ in lepton-flavor violating decay $\mu \rightarrow e\gamma$.

### 3 The Decay $\mu \rightarrow e\gamma$

To obtain the contributions of the doubly charged higgsino to the decay rate of $\mu \rightarrow e\gamma$ we have to calculate the diagrams of fig.(1).

The detailed calculations are shown in the Appendix A. The matrix element given in eq.(14) leads to the following result:

\[
iM = \frac{eh_{LR}^{2}m_{\mu}}{4\pi^{2}m_{\Delta}}p_{\mu}i\sigma_{\mu\nu}\epsilon^{\nu}(c_{\theta_{L}}s_{\theta_{L}}(B_{e}^{L} - B_{\mu}^{L})P_{R} + c_{\theta_{R}}s_{\theta_{R}}(B_{e}^{R} - B_{\mu}^{R})P_{L})u_{\mu}\epsilon_{q}^{\nu}
\]

\[
B_{i}^{L} = c_{\theta_{L}}B_{i1} + s_{\theta_{L}}B_{i2} + \frac{m_{\Delta}}{m_{\mu}}c_{\theta_{L}}s_{\theta_{L}}(c_{\theta_{R}}B_{i1}^{L} - s_{\theta_{R}}B_{i2}^{L})
\]

\[
B_{i}^{R} = s_{\theta_{L}}B_{i1} + c_{\theta_{L}}B_{i2} + \frac{m_{\Delta}}{m_{\mu}}c_{\theta_{L}}s_{\theta_{L}}(c_{\theta_{R}}B_{i1}^{R} - s_{\theta_{R}}B_{i2}^{R})
\]

\[
B_{i,m}^{L} = e_{\Delta}[-B_{G}(x_{i,m}^{L}) + B_{F}(x_{i,m}^{L})] + e_{i}B_{F}(x_{i,m}^{L})
\]

\[
B_{i,m}^{R} = e_{\Delta}B_{G}(x_{i,m}^{L}) + e_{\mu}B_{F}(x_{i,m}^{L})
\]

In order to not to have too many parameters, we assume that $h_{LR}$ of different generations are all of the same order. We will however keep the difference of the couplings within the mixing of all generations via a supersymmetric version of the Kobayashi-Maskawa matrix.
Figure 1: The penguin diagrams with scalar leptons and doubly charged Higgsinos within the loop. In the left-right model without SUSY the same diagrams occur with $\Delta \leftrightarrow l$ and $\tilde{l} \leftrightarrow \Delta_{L,R}$

first and second generation in the mixing angles $c_{\theta_{L,R}}$, eq. (1-2). $c_{\theta_{l}}$ is the mixing angle of the scalar partners of the left and right handed leptons. Note that if we would include the mixing of all 3 generations the expression is far more complicated, in particular for the Higgsino mass term. Since $m_\Delta \gg m_l$ the mixing of the scalar leptons cannot be neglected. If we exclude the contribution of the left handed Higgsino then $B_{L}$ and, as can be easily seen from eq. (14) (all $\tilde{K}_{m1}^a$ set to be 0), the term proportional to the Higgsino mass are absent[3].

A matrix element of the form $iM = e\pi p q(u_L P_L + u_R P_R)_{L,R}^{q} u_p$ leads to a decay rate of $\Gamma(\mu \rightarrow e\gamma) = m_\mu^3(a_L^2 + a_R^2)/4$ and $\Gamma(\mu \rightarrow 3e) = \frac{1}{4 \pi}(a_L^2 + a_R^2)m_\mu^3[\log \frac{m_\mu^2}{m_\mu^2} - \frac{11}{4}]$. With $\Gamma(\mu \rightarrow \nu e\gamma) = \frac{\alpha^2}{16\pi} m_\mu^5 = \frac{\alpha^2}{384\pi s_W} \left(\frac{m_\mu}{m_W}\right)^4 m_\mu$, where $s_W$ is the Weinberg angle $\sin \theta_W$, we obtain the following branching ratio for eq. (3):

$$B(\mu \rightarrow e\gamma) = \frac{3s_W^4 h_{\Delta_{L,R}}^4}{8\pi^3} \left(\frac{m_W}{m_\Delta}\right)^4 \left[c_{\theta_L}^2 s_{\theta_L}^2 (B_L^L - B_L^R)^2 + c_{\theta_R}^2 s_{\theta_R}^2 (B_R^L - B_R^R)^2\right]$$

$^2$ The calculation of the diagrams with the doubly charged left and right Higgs bosons and leptons within the loop, which occur in the left-right model without SUSY are very similar ($e_\Delta \leftrightarrow e_{L,R}^c = +1$, $e_\Delta \leftrightarrow e_{L,R}$ and $x_{\Delta_{L,R}}^4 \leftrightarrow x_{\Delta_{L,R}}^4 = m_{\Delta_{L,R}}^2/m_l^2$). Limits of the couplings and the masses of the doubly charged Higgs boson have been given elsewhere and will not presented here[11].
Similiar for $B(\mu \rightarrow 3e)$. It is $\frac{\Gamma(\mu \rightarrow 3e)}{\Gamma(\mu \rightarrow e\gamma)} = \frac{g^2}{3\pi} \log \frac{m_{\mu}^2}{m_{\tau}^2} - \frac{11}{4}$.

If we neglect the mixing of the scalar partners of the left and right handed leptons, that is, setting $\sin \theta_l = 0$ eq. (1) leads to:

$$B(\mu \rightarrow e\gamma) = \frac{3s_W^4 h_{LR}^4}{8\pi^3} \frac{m_W}{m_\Delta}^4 \left[ c_{\theta_L}^2 s_{\theta_L}^2 (B_{e_1} - B_{\mu_1})^2 + c_{\theta_R}^2 s_{\theta_R}^2 (B_{e_2} - B_{\mu_2})^2 \right]$$ \hspace{1cm} (5)

The finite result depends strongly on the mass differences of the first and second generation. If we take as, required by left-right symmetry $\theta_L \equiv \theta_R \equiv \theta$ and $B_{l_i} = B_{l_2}$ with $\Delta m_i^2 = \Delta m_l^2$ for different generations and $m_\Delta \sim m_i$, a first estimate is given by:

$$B(\mu \rightarrow e\gamma) = \frac{3s_W^4 h_{LR}^4}{4\pi^3} \frac{m_W}{m_\Delta}^4 \left[ c_{\theta_L}^2 s_{\theta_L}^2 \left( \frac{7}{120} \right)^2 \left( \frac{m_\mu}{m_\Delta} \right)^4 \right]$$ \hspace{1cm} (6)

Eq. (6) leads to $B(\mu \rightarrow e\gamma) \approx 2.97 \times 10^{-16} h_{LR}^4 c_{\theta_L}^2 s_{\theta_L}^2$ for $m_\Delta = 100$ GeV and thus far smaller than the experimental limit.

However the situation is changed when the mixing of the scalar partners of the right and left handed leptons is included. Since $m_l \ll m_\tilde{l}$ the mixing angle is given by $\sin \theta_l \sim m_l(A_l - \mu \tan \beta)/m_\tilde{l}^2 \sim m_l/m_\tilde{l}$ with $(A_l - \mu \tan \beta) \sim m_l$, where $A_l$ is the trilinear scalar interaction, $\mu$ the mixing mass terms of the Higgs bosons and $\tan \beta = v_2/v_1$ the ratio of their vacuum expectation values (vev's). This leads to $B_{l_i}^L \approx B_{l_1} + \frac{m_\Delta m_{1\mu}}{m_\tilde{l}} (B_{l_1} - B_{l_2})$ where we assumed that after left-right symmetry breaking the relations $c_{\theta_L}/c_{\theta_R} \sim s_{\theta_L}/s_{\theta_R} \sim 1$ still hold.

If the left-right symmetry is conserved we have $\tilde{B}_{l_1} = \tilde{B}_{l_2}$ and eq. (3) is recovered. However after breaking, the mass difference between the scalar partners of the left handed and right handed leptons is much larger than the mass difference between the generations. We expect $\frac{m_\Delta^2 - m_{1\mu}^2}{m_\tilde{l}^2} \sim 10^{-2} - 10^{-1}$ \cite{4, 19}. That is $B_{l_i}^{L,R} - B_{l_{12}}^{L,R} \sim \frac{m_\Delta}{m_\tilde{l}} (\tilde{B}_{\mu_2} - \tilde{B}_{\mu_1}) \sim \frac{5}{12} (10^{-2} - 10^{-1})$ with $m_\Delta \sim m_l$. With these values eq. (3) leads to $B(\mu \rightarrow e\gamma) \approx 1.25 \times (10^{-6} - 10^{-4}) h_{L,R}^4 c_{\theta_L}^2 s_{\theta_L}^2$ for $m_\Delta = 100$ GeV and thus far above the known experimental limit of $4.9 \times 10^{-11}$, leading to the constraint $h_{L,R}^2 < (10^{-9} - 10^{-8}) m_\Delta^2$ (GeV) when maximal electron-muon mixing is assumed.
4 The Anomalous Magnetic Moment of the Muon

The calculations to the AMMM are similar as to the $\mu \to e\gamma$ decay. From eq.(15) we obtain the following result:

$$\Delta a_\mu = -\frac{2h^2_{LR}}{(4\pi)^2} \left(\frac{m_\mu}{m_\Delta}\right)^2 \left\{ B^L_\mu + B^R_\mu \right\}$$ (7)

$B^L_\mu$ and $B^R_\mu$ as given in eq.(3). Again $B^L_\mu$ is absent when we exclude the contribution of the left handed higgsino.

Eq.(7) leads to

$$\Delta a_\mu = -\frac{2h^2_{LR}}{(4\pi)^2} \left(\frac{m_\mu}{m_\Delta}\right)^2 (B_{\tilde{\mu}_1} + B_{\tilde{\mu}_2})$$ (8)

Eq.(8) leads to $\Delta a_\mu \approx -3.49 \times 10^{-9}h^2_{LR}$ for $m_\Delta \sim m_\mu = 100$ GeV, thus already very close to the experimental lower limit of $\Delta a_\mu^{\exp}$, leading to the constraint $h^2_{LR} < 5 \cdot 10^{-4}m_\Delta^2$ (GeV).

5 Conclusions

We presented the results of the contribution of the doubly charged Higgsino to the decays $\mu \to e\gamma$ and $\mu \to 3e$ and to the anomalous magnetic moment of the muon $\Delta a_\mu$. It was shown that when mixing of the scalar partners of the left and right handed leptons are neglected the contribution to the decay modes is far below the experimental limits. However for the anomalous magnetic moment it is within the reach of the future BNL experiments. When mixing of the scalar leptons is included we obtain a constrain for the lepton-scalar lepton-doubly charged Higgsino coupling $h_{LR}$ in the order of less than $10^{-4}m_\Delta$ (GeV) if maximal electron-muon mixing is assumed (not to be confused with the upper mixing of the scalar leptons). The effect of this mixing on $\Delta a_\mu$ is negligible.

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A Appendix: The matrix elements

In this Appendix we present the final results of the calculations of the Feynman diagrams as shown in Fig.1 with flavor non diagonal couplings of the doubly charged Higgsino to the left and right handed leptons and their scalar partners. We define the following matrix:

\[ iM = +\frac{e\hbar L R}{(4\pi)^2}(M_S + M_F + M_{SE}) \]  

(9)

where \( M_S \) is the matrix element for the two scalar leptons in the loop, \( M_F \) the one for the two doubly charged Higgsinos in the loop and \( M_{SE} \) the one for the self energy diagrams. The calculations of the diagrams are very similar to those presented in the Appendix A of [10]; where we refer the interested reader to for more details. Here we only present the finite results for each diagram.

\[ iM_F = \int_0^1 d\alpha_1 e_1 \left\{ \overline{\nu}_\mu (q^2\gamma_\mu - \gamma q_\mu)(T_{11}^{\alpha L} P_L + T_{22}^{\alpha R} P_R) u_p, \frac{1}{6}(2 - 3\alpha_1 + \alpha_1^2) \right\} \frac{\epsilon_{\mu}^\alpha}{D_{SE}} \]  

(10)

\[ iM_S = \int_0^1 d\alpha_1 e_1 \left\{ \overline{\nu}_\mu (q^2\gamma_\mu - \gamma q_\mu)(T_{11}^{\alpha L} P_L + T_{22}^{\alpha R} P_R) u_p, \frac{1}{6}\alpha_1^2 \right\} \frac{\epsilon_{\mu}^\alpha}{D_{SE}} \]  

(11)

\[ iM_{SE} = \int_0^1 d\alpha_1 \left[ -e_\nu \left\{ u_p, (1 - \alpha_1) m_\Delta (T_{11}^{\alpha L} P_L + T_{12}^{\alpha R} P_R) \right\} \right] \frac{\epsilon_{\mu}^\alpha}{D_{SE}} \]  

(12)

\[ D_{SE} = m_\Delta^2 - (m_\Delta^2 - m_{i_{am}}^2) \alpha_1 \]

\[ T_{11}^{\alpha L} := K_{ae,\mu}^L K_{aq}^L K_{m_1}^2 \quad T_{22}^{\alpha R} := K_{ae,\mu}^R K_{aq}^R K_{m_2}^2 \]

\[ T_{12}^{\alpha L} := K_{ae,\mu}^L K_{aq}^L K_{m_1}^a K_{m_2}^a \quad T_{12}^{\alpha R} := K_{ae,\mu}^R K_{aq}^R K_{m_1}^a K_{m_2}^a \]

where \( e_\Delta = -2, e_\ell = +1 \) and \( e_\mu = -1 \). We have to sum over the generation indices \( a = 1 - 3 \) and over the eigenstates of the scalar partners of the left and right handed leptons \( m = 1, 2, 3 \). The divergencies cancel after the summation of all diagrams \( e_\Delta + e_\ell - e_\mu \equiv 0 \). Due to this equation many more terms like terms proportional to \( m_\Delta \alpha_1 (1 - \alpha_1)(p_1 + p_2) \) drop out after summation and are not explicitely written down in eqs.([10][12]). Furthermore
we made use of eqs.(A.5)-(A.8) in [16] and eqs.(A.6)+(A.7) in [17]. After summation of eqs.(10-12) the final matrix element is given by:

\[ iM = + \frac{e^2 L_R}{(4\pi)^2} \frac{1}{m^2} \left\{ \overline{\nu}_{p_2}(q^2 \gamma_\mu - \not{q} q_\mu)(T_{11}^L P_L + T_{22}^R P_R) u_{p_1} \right\} \]

The final functions after Feynman integration are shown in Appendix B. Since we are not interested in CP violation we can skip the * in \[ T_{kl}^{L,R} \]. Furthermore for a photon on mass shell we have \[ q^2 = 0 \] and due to gauge invariance \[ \epsilon^* q \epsilon q = 0 \].

Neglecting the electron mass the matrix element for the decay \( \mu \to e \gamma \) is then given by:

\[ iM = + \frac{e^2 L_R}{(4\pi)^2} \frac{1}{m^2} \left\{ \overline{\nu}_{p_2} i \sigma_{\mu \nu} q^\nu \left\{ m_\mu [K_{a1}^{L} K_{a2}^{L} \tilde{K}_{m1}^{a2} P_R + K_{a1}^{R} K_{a2}^{R} \tilde{K}_{m2}^{a2} P_L] \left\{ e_\Delta [-B_G(x_{\Delta}^l) + B_F(x_{\Delta}^l)] + \epsilon_i B_F(x_{\Delta}^l) \right\} + m_\Delta \tilde{K}_{m1}^{a} \tilde{K}_{m2}^{a} (K_{a1}^{R} K_{a2}^{L} P_R + K_{a1}^{L} K_{a2}^{R} P_L) [e_\Delta \tilde{B}_G(x_{\Delta}^l) + \epsilon_i \tilde{B}_F(x_{\Delta}^l)] \right\} u_{p_1} \epsilon^* \mu \right\} \]

After summation over the mass eigenstates of the scalar leptons and all the generations making use of the unitarity of the leptonic KM matrix eq.(14) leads to the branching ratio \( B(\mu \to e \gamma) \) as given in eq.(3).

The ANMM can be extracted from \( V_\mu = \frac{ie}{2m_\mu} F(q^2) \overline{\nu}_{p_2} i \sigma_{\mu \nu} q^\nu u_{p_1} \) at \( q^2 = 0 \). We obtain the result:

\[ F(q^2) |_{q^2=0} = \Delta a_\mu = - \frac{2e^2}{(4\pi)^2} \frac{m_\mu}{m_\Delta} \left\{ \left[ |K_{a2}^{L} |^2 \tilde{K}_{m1}^{a2} + |K_{a2}^{R} |^2 \tilde{K}_{m2}^{a2} \right] \left\{ e_\Delta [-B_G(x_{\Delta}^l) + B_F(x_{\Delta}^l)] + \epsilon_i B_F(x_{\Delta}^l) \right\} \right\} \]

We could also have skipped the \( L \) and \( R \) indices (that is \( \theta_L = \theta_R \)) since the difference between the left and right diagonalizing matrices lies in a relative minus sign of the SUSY phase, see eq.(4) in [18]. This would simplify the results strongly giving us an overall factor of \( K_{ae,\mu} K_{a\mu} \) and \( T_{12}^L = \pm T_{12}^R \). For the moment however we will keep \( \theta_L \) different from \( \theta_R \).
As above after summation over all mass eigenstates and generations of the leptons eq.(15) leads to the result as given in eq.(7).

B Appendix: The final functions

\[ A_G(x_\tilde{l}_{am}) = \frac{1}{6} \int_0^1 d\alpha_1 [2 - 3\alpha_1^2 - 3\alpha_1(1 - \alpha_1)]/D_{\tilde{l}_{am}} \]
\[ = \frac{1}{(1 - x_\tilde{l}_{am})^2/2} \{1 - x_\tilde{l}_{am} + \frac{1}{3}(1 + 2x_\tilde{l}_{am}) \log(x_\tilde{l}_{am})\} \quad (16) \]

\[ A_F(x_\tilde{l}_{am}) = \frac{1}{6} \int_0^1 d\alpha_1 \alpha_1^3 / D_{\tilde{l}_{am}} \]
\[ = \frac{1}{(1 - x_\tilde{l}_{am})^4} \{1 - x_\tilde{l}_{am} + 2x_\tilde{l}_{am} \log(x_\tilde{l}_{am})\} \quad (17) \]

\[ B_G(x_\tilde{l}_{am}) = \frac{1}{2} \int_0^1 d\alpha_1 (1 - \alpha_1)/(1 - \alpha_1) / D_{\tilde{l}_{am}} \]
\[ = \frac{1}{(1 - x_\tilde{l}_{am})^3} \{1 - x_\tilde{l}_{am} + 2x_\tilde{l}_{am} \log(x_\tilde{l}_{am})\} \quad (18) \]

\[ B_F(x_\tilde{l}_{am}) = \frac{1}{2} \int_0^1 d\alpha_1 \alpha_1^3(1 - \alpha_1)/(1 - \alpha_1) / D_{\tilde{l}_{am}} \]
\[ = \frac{1}{(1 - x_\tilde{l}_{am})^4} \{2 + 3x_\tilde{l}_{am} - 6x_\tilde{l}_{am}^2 + x_\tilde{l}_{am}^3 + 6x_\tilde{l}_{am} \log(x_\tilde{l}_{am})\} \quad (19) \]

\[ \tilde{B}_G(x_\tilde{l}_{am}) = \int_0^1 d\alpha_1 (1 - \alpha_1)/(1 - \alpha_1) / D_{\tilde{l}_{am}} \]
\[ = \frac{1}{(1 - x_\tilde{l}_{am})^2} \{1 - x_\tilde{l}_{am} + x_\tilde{l}_{am} \log(x_\tilde{l}_{am})\} \quad (20) \]

\[ \tilde{B}_F(x_\tilde{l}_{am}) = 2B_G(x_\tilde{l}_{am}) \]
\[ D_{\tilde{l}_{am}} = 1 - (1 - x_\tilde{l}_{am})\alpha_1 \]
\[ x_\tilde{l}_{am} = \frac{m^2_\tilde{l}_{am}}{m^2_\Delta} \]

We have the following relations between the eqs(B.3-B.6) and the eqs.(4.10), (4.11),(4.17) and (4.21) of [3]. \[ \frac{m^2_\mu}{m^2_\Delta} \tilde{B}_F = G'/6, \frac{m^2_\mu}{m^2_\Delta} \tilde{B}_F = G/2, \frac{m^2_\mu}{m^2_\Delta} (B_G - \tilde{B}_F) = F'/12 \] and \[ \frac{m^2_\mu}{m^2_\Delta} (\tilde{B}_G - \tilde{B}_F) = F/2. \] Furthermore we have \[ \frac{m^2_\mu}{m^2_\Delta} G = (2G' + F')/3. \]
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