The Nuclear Window to the Extragalactic Universe

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Abstract

We investigate two recent parameterizations of the galactic magnetic field with respect to their impact on cosmic nuclei traversing the field. We present a comprehensive study of the size of angular deflections, dispersion in the arrival probability distributions, multiplicity in the images of arrival on Earth, variance in field transparency, and influence of the turbulent field components. To remain restricted to ballistic deflections, a cosmic nucleus with energy $E$ and charge $Z$ should have a rigidity above $E/Z = 6$ EV. In view of the differences resulting from the two field parameterizations as a measure of current knowledge in the galactic field, this rigidity threshold may have to be increased. For a point source search with $E/Z \geq 60$ EV, field uncertainties increase the required signal events for discovery moderately for sources in the northern and southern regions, but substantially for sources near the galactic disk.

Keywords: astroparticle physics, magnetic fields, cosmic rays

1. Introduction

The origin of cosmic rays still remains an unanswered fundamental research question. Cosmic ray distributions of various aspects have been measured, most notably the steeply falling spectrum up to the ultra-high energy regime with cosmic ray energies even exceeding $E = 100$ EeV [1, 2].

For ultra-high energy cosmic rays, deflections in magnetic fields should diminish with increasing energy, such that directional correlations should lead to a straightforward identification of accelerating sites. However, even at the highest energies the arrival distributions of cosmic rays appear to be rather isotropic. Only hints for departures from isotropic distributions have been reported, e.g., a so-called hot spot [3], and a dipole signal [4]. At least with the apparent isotropy, limits on the density of extragalactic sources were derived which depend on the cosmic ray energy [5].

A recent determination of ultra-high energy cosmic ray composition from measurements of the shower depth in the atmosphere revealed contributions of heavy nuclei above $\sim 5$ EeV [6, 7]. This observation may explain the seemingly isotropic arrival distribution as deflections of nuclei in magnetic fields scale with their nuclear charges $Z$.

Obviously, when searching for cosmic ray sources, a key role is therefore attributed to magnetic fields. The galactic field in particular is strong enough to displace original arrival directions of protons with energy $E = 60$ EeV by several degrees from their original arrival directions outside the galaxy [8]. The displacement angles for nuclei even reach tens of degrees [9]. The knowledge on the extragalactic magnetic fields is much less certain, but is likely to be less important than the galactic field [10] and is not studied in this contribution.

To identify sources of cosmic rays, rather precise corrections for the propagation within the galactic magnetic field are needed which in turn can be used to constrain the field [11]. Beyond this, effects of lensing caused by the galactic field have been studied which influence the visibility of sources and the number of images appearing from a single source [12]. The influence of turbulent contributions to the galactic field has also been studied in the context of lensing [13] and nuclear deflections [14].

In previous directional correlation analyses of measured cosmic rays, only the overall magnitude of deflections was taken into account, e.g. [15], or
corrections for cosmic ray deflections were applied using analytic magnetic field expressions reflecting the spiral structure of our galaxy \[16\].

Recently, parameterizations of the galactic magnetic field have been developed which are based on numerous measurements of Faraday rotation \[17\] \[18\], and in addition polarized synchrotron radiation for the second reference. Based on directional characteristics and the field strength of the parameterizations, deflections of cosmic rays are predicted to depend strongly on their arrival direction, charge and energy. In the following we will refer to the regular field with the bisymmetric disk model of the first reference as the PT11 field parameterization, and to the regular field of the latter as the JF12 field parameterization, respectively.

Angular distributions of cosmic rays in these galactic field parameterizations have been studied before, e.g., with respect to general properties of the JF12 parameterization \[19\], specific source candidates \[20\], general properties of deflections and magnifications \[21\] \[22\], and to the potential of revealing correlations between cosmic rays and their sources \[23\].

In this work we investigate whether cosmic ray deflections in the galactic magnetic field can be reliably corrected for, given the current knowledge of the field. To simplify discussions of energy and nuclear dependencies we will define rigidity as the ratio of the cosmic ray energy and number \(Z\) of elementary charges \(e\)

\[
R = \frac{E}{Ze}. \tag{1}
\]

In our investigations we use galactic coordinates as our reference system, with longitude \(l\) and latitude \(b\). For a number of visualizations we use Cartesian coordinates alternatively with height \(z\) above the galactic plane, with the Earth being located at \((x_E, y_E, z_E) = (−8.5, 0, 0)\) kpc.

Based on the two field parameterizations PT11 and JF12 we initially discuss key distributions of cosmic ray deflection, dispersion effects in arrival distributions, directional variance in field transparency, and the influence of random field components. From the rigidity dependencies of these distributions, we recommend a minimum rigidity threshold above which cosmic ray deflection may be controlled in terms of probability distributions.

Furthermore, we take the different results of the two galactic field parameterizations as a measure of our current knowledge of the galactic field. We compare their cosmic ray angular deflections and study differences in the dispersion of arrival distributions. Finally, we study the practical consequences of galactic field corrections and their uncertainties by performing simulated point source searches and by quantifying the field impact in terms of discovery potential.

2. Field parameterizations

The two field parameterizations PT11 and JF12 each follow a different ansatz. Both take into account about 40,000 Faraday rotation measurements. The PT11 field has been fitted to two large sets of Faraday rotation measurements. The JF12 field has been adapted to several large sets of Faraday rotation measurements and to synchrotron polarization measurements, thereby increasing the information per analysed direction by two additional complementary measurements \[21\]. Both use the electron density model NE2001 \[24\] with an enlarged vertical scale for weighting the line-of-sight integrals of the magnetic field.

Fig. 1 shows the field strength as a function of the radial distance from the galactic center along the solar system line-of-sight and the distance perpendicular to the galactic plane. The fields exhibit different shapes and magnitudes; especially notable in Fig. 1 is the field extent of the JF12 parametrization above and below the galactic plane with non-negligible field strengths even at a distance of 10 kpc. The PT11 field (Fig. 1b), on the other hand, exhibits a rather concentrated halo field, which is centered around a distance of \(\sim 1.2\) kpc to the galactic plane.

When studying the magnitude of angular deflections of cosmic rays resulting from these parameterizations, we take the angle \(\beta\) between the incoming direction to the galaxy and the arrival direction on Earth as a measure of the directional change (Fig. 2).

To get a first impression of the different deflections resulting from the two field parameterizations we use backward tracking techniques of antiprotons through the galactic field. With this technique we obtain individual trajectories for matter particles entering from outside the galaxy and then following the reverse path. The method ensures that every trajectory leads to observation on Earth.

In Fig. 3 we show the magnitudes of the angular deflections \(\beta\) of cosmic rays with rigidity \(R = 60\) EV. The position in the map denotes the
initial direction on Earth in galactic coordinates for the backtracked antiprotons. The color code refers to the magnitude of angular deflections which reach up to $\beta = 28$ deg.

For the JF12 parameterization (Fig. 3a), deflections are largest near directions of the galactic center which is expected from the magnitude of the field shown in Fig. 1a. With the PT11 parameterization (Fig. 3b), deflections are largest in any direction near the galactic plane which is attributed to the strong disk field (Fig. 1b).

As expected, the differences in the field parameterizations relate directly to a different impact on cosmic ray deflections. In the following section we study a number of aspects related to the directional changes of cosmic rays when traversing the galactic field.
3. Impact of the galactic magnetic field on cosmic ray arrival

The goal of this section is to determine a kinematic regime where information on cosmic ray arrival directions can be obtained by transformation of probability distributions. For this purpose we first analyze cosmic ray angular deflections as a function of rigidity. As our primary criterion we require angular deflections to be below 90 deg in order to distinguish ballistic deflections from diffusive-type random walk.

Beyond this we investigate the dispersion of arrival probability distributions by the galactic field, and the splitting of arrival distributions into several images. Furthermore we show directional dependencies of the field transparency for cosmic matter and antimatter particles. We also study the influence of random components of the field which imposes uncertainties on the arrival directions.

3.1. Deflection angles

The two magnetic field parameterizations exhibit different field strengths above and below the galactic plane, and differ substantially in their field characteristics near the plane (Fig. 1). Therefore, we divide the sky into three regions of equal solid angles, and study angular deflections for each region separately. The boundaries of these regions are fixed at galactic latitudes of ±19.5 deg.

To ensure that every cosmic ray trajectory leads to observation on Earth we use the backward tracking method explained in the previous section. We use the term “northern region” to refer to antiparticles originating on Earth in the direction of positive longitudes, and “southern region”, while for latitudes in-between we use the term “disk region”.

In Fig. 1 we show the average directional change \(\langle \beta \rangle\) between the direction on Earth and the direction outside the galaxy as a function of the cosmic ray rigidity \(R\) using the JF12 field. The distribution was derived from 5 million simulated cosmic rays per rigidity interval. In Fig. 2 we show the corresponding spread \(\sigma_\beta\) in terms of standard deviations. For low rigidities \(R \sim 0.1 \text{ EV}\), cosmic ray confinement owing to the size of our galaxy and its magnetic field leads to large directional changes \(\langle \beta \rangle \sim 90\) deg and large average variations in \(\beta\) \((\sigma_\beta \sim 40\) deg).

At rigidity \(R = 6\) EV, the largest average deflection of \(\langle \beta \rangle \approx 50\) deg is found in the southern region (Fig. 3, downward-pointing triangles). The corresponding spread amounts to \(\sigma_\beta \approx 15\) deg (Fig. 3), such that for 95% of the cosmic rays the deflection angle remains below \(\beta = 90\) deg. The average deflection in the northern region is substantially smaller with only \(\langle \beta \rangle \approx 30\) deg, however, the spread of \(\sigma_\beta \approx 25\) deg is larger (upward-pointing triangles). Also here most of the cosmic ray deflections are below \(\beta = 90\) deg. A similar conclusion holds for the disk region (square symbols).

The PT11 parameterization exhibits very similar tendencies for rigidity \(R = 6\) EV as can be seen in Figs. 3b,d. In the disk region the deflections exceed those of the other regions for rigidities above \(R = 40\) EV (square symbols). Here the deflections also exceed that of the JF12 parameterization as was already visualized in Fig. 3 above.

Overall, at rigidities \(R > 6\) EV the deflections are consistently reduced and correspondingly enhance the control over cosmic ray deflection. In the following studies we will therefore use the rigidity of \(R = 6\) EV as a benchmark.

3.2. Dispersion

Cosmic rays originating from a point source may arrive slightly dispersed after their propagation through extragalactic fields. When traversing the galactic field the extent of the arrival distribution may even be enlarged.

In order to obtain information on the arrival direction and arrival probability on Earth of a cosmic ray that enters the galaxy in any direction, we use a lensing technique [25, 26]. The lenses consist of matrices based on the HEALPix format [27], where we divide the sphere into \(N_{\text{pix}} = 49,152\) equally sized pixels of approximately 1 deg in size. For each rigidity interval a separate matrix is produced by backtracking a set of \(N_{\text{pix}} \times 100\) antiparticles, which are distributed uniformly in each pixel. The matrices thus contain the probability of a cosmic ray entering the galaxy with rigidity \(R\) at pixel direction \((l_i, b_i)\) to be observed in pixel direction \((l'_j, b'_j)\). As defined above, \(l, l'\) refer to the galactic longitudes, and \(b, b'\) to the latitudes, respectively.

By design, the lenses project an extragalactic isotropic distribution onto an isotropic distribution on Earth. Note that some incoming directions have more simulated trajectories leading to observation on Earth, while other directions have less, such that there are directionally dependent variations in...
the transparency of the field. We normalize the lenses to ensure that the lenses return relative arrival probabilities, and that an isotropic cosmic ray flux is preserved. However, for cosmic rays arriving from individual sources the flux varies depending on the source directions which we will show below. The technical details of the lenses and their production are outlined in [26, 28]. The lenses used in this contribution were calculated with the CRPropa v3 program [29] using the PT11 and JF12 parameterizations.

In Fig. 5 we show examples of arrival directions on Earth for simulated cosmic rays with rigidity $R = 10$ EV together with their original source directions using the JF12 parameterization of the regular field. The incoming cosmic rays followed a Fisher probability distribution $f(\alpha, \kappa) = \kappa \exp (\kappa \cos \alpha)/(4\pi \sinh \kappa)$ with a Gaussian width of $\sigma = 1/\sqrt{R} = 3$ deg. Their arrival directions were calculated with the lensing techniques described above. The color code indicates the relation between sources and their cosmic rays. For all scenarios, the fraction of arriving cosmic rays is shown on the right side of the figure, normalized to the source with the highest arrival probability.

Different images of the cosmic rays appear depending on their incoming direction. For example, a source direction which coincides on average with the cosmic rays after traversing the galactic field is denoted by the green symbols. Only a widening of the directional distribution is observed. Another example is a source direction where the cosmic ray
distribution is displaced without a strong spread (purple symbols).

Examples of source directions where the cosmic rays are substantially deflected and exhibit a widespread distribution of arrival directions are denoted by the light blue and dark blue symbols. For some source directions, small variations in the cosmic ray incoming direction lead to largely different paths, and therefore to several distinct images of the arrival directions (red symbols).

To investigate dispersion effects in the galactic field we again choose initial cosmic rays to follow a Fisher probability distribution with a Gaussian width of 3 deg. In principle this value could be related to dispersion effects caused by extragalactic fields, which implies a dependency on cosmic ray rigidity. However, to ensure clarity of our galactic field investigations we will use a fixed Gaussian spread for all cosmic rays incoming to our galaxy throughout this work.

After the cosmic rays traversed the galactic field we quantify the direction and extent of the resulting arrival distribution by calculating around every HEALPix pixel \((l_j', b_j')\) with non-zero probability a circular curve which includes 50% of all arrival probabilities. We then use the radius \(r_{50}\) of the smallest circle as a measure of the extent of the probability distribution \([31]\).

In Fig. 6a we show the extent of the arrival probability distributions in terms of the smallest average radius \(\langle r_{50} \rangle\) as a function of cosmic ray rigidity \(R\). Again we show separately the three galactic regions defined above. As we start with extragalactic directions we use the term “northern region” to refer to initial directions with galactic latitudes above 19.5 deg, etc.

The projection of the incoming distribution causes a dispersion or a focusing effect, depending on the original cosmic ray direction and its rigidity \(R\). In the southern region (downward-pointing triangles) a focusing effect is visible for rigidities around \(R \sim 15\) EV. In contrast, the disk region (square symbols) exhibits dispersion effects up to the largest rigidities.

Below \(R = 1\) EV the average extent \(\langle r_{50} \rangle\) is large for both fields, such that it appears difficult to identify arrival directions on Earth from a given extragalactic direction. The JF12 field exhibits several pronounced features which appear to be specific to the JF12 field parametrization. Such effects are not visible in the PT11 parametrization where \(\langle r_{50} \rangle\) ap-

![Figure 5: Arrival distributions for cosmic rays with rigidity \(R = 10\) EV originating from arbitrarily chosen sources (star symbols), and the relative arrival probability in percent. Relations of cosmic rays with their sources and corresponding arrival probabilities are indicated by the color code (JF12).](image)

![Figure 6: Average extent \(\langle r_{50} \rangle\) of arrival probability distributions on Earth resulting from cosmic rays incoming to our galaxy with a fixed Gaussian width of 3 deg in three regions separated by galactic latitudes \(\pm 19.5\) deg as a function of rigidity \(R\), a) JF12, b) PT11. For the exact definition of \(r_{50}\) refer to the text.](image)
pears to increase continuously with reduced rigidity up to 70 deg.
Above $R = 6$ EV both parametrizations lead to similar results with a dispersion effect in the disk region (square symbols), and dispersion and focusing effects in the southern region (downward-pointing triangles). The largest extent of the probability region is found for the PT11 field corresponding to a dispersion of the initial probability distribution by a factor of 3 at $R = 6$ EV, which may still be acceptable for an analysis of cosmic ray arrival directions.

3.3. Multiple images

Extending the above study of dispersion we investigate multiple images arising in cosmic ray arrival distributions on Earth. Again cosmic rays from an incoming direction are Fisher-distributed with a Gaussian width of 3 deg. For some incoming directions, the small angular deviations within this distribution are sufficient to change the arrival direction on Earth substantially, leading to distinct maxima in the arrival distributions. Examples for such multiple images are denoted in Fig. 5 by the red symbols.

We count the number of arrival images by searching for connected areas of arbitrary shape in-between which the probability falls below a pre-defined threshold. For this we coarsen the HEALPix resolution in the arrival distribution from originally 1 deg to 4 deg and require each pixel to carry at least 20% of the pixel with the maximum arrival probability. The image multiplicity then arises from counting connected areas that are separated from one another by at least one pixel below the pre-defined threshold.

In Fig. 6 we show the average multiplicity $\langle n \rangle$ of arrival images as a function of cosmic ray rigidity $R$ in the three regions of the galactic sphere separated by galactic latitudes $\pm 19.5$ deg. Overall, the image multiplicity decreases with increasing rigidity. The multiplicity arising from the PT11 parametrization (Fig. 7b) appears to exceed that of the JF12 parameterization (Fig. 7a).

For directions in the northern and southern regions, typically 1 image of the arrival direction arises. In the disk region, however, multiple images appear even for cosmic rays with large rigidity. Here the image multiplicity is especially large for the PT11 parameterization (Fig. 7b, square symbols) which is related to the pronounced halo field visualized in Fig. 1b.

Multiple images reduce the predictive power of cosmic ray arrival directions and may require additional selection depending on the individual analysis. As criteria to reduce image multiplicity both cosmic ray rigidity and a selection of incoming directions away from the galactic disk region are relevant.

3.4. Field transparency

Related to the above study on dispersion we investigate extragalactic directions causing a relatively enhanced flux of cosmic rays on Earth [25], and directions for which the arrival probability disappears as no simulated trajectory leads to Earth.
These effects have a direct impact on the visibility of a source by cosmic ray messengers, and the luminosity required for observation on Earth. Examples of varying transparency of the field depending on the incoming directions are shown in Fig. 5.

To demonstrate the enhanced flux of a few extragalactic directions we show in Fig. 8a for incoming cosmic rays with rigidity $R = 6$ EV the probability $p$ of arriving on Earth coded in color. The lightly colored regions indicate incoming cosmic ray directions with a high probability of observation on Earth.

In order to quantify flux enhancement we organize the incoming directions (binned in $N_{\text{pix}} = 49,152$ pixels of 1 deg) according to their arrival probabilities $p_j$ on Earth and select the leading $k$ directions. These incoming directions cover a solid angular region of $\Omega = \left( \frac{k}{N_{\text{pix}}} \right) \cdot 4\pi$ and provide a relative flux contribution of $F(k) = \sum_{j=1}^{k} p_j / \sum_{j=1}^{N_{\text{pix}}} p_j$.

In Fig. 8b we show for the above example the observed flux $f$ of cosmic rays originating from the 1% incoming directions with the highest probabilities indicated in Fig. 8a, when assuming an isotropic extragalactic flux. We find that these few incoming directions cause a wide spread distribution and contribute $F = 30\%$ to the observed flux.

In a more general approach we show in Fig. 9 the relative flux $F$ of observed cosmic rays as a function of the solid angular region $\Omega/(4\pi)$ covered by the incoming directions with the highest arrival probabilities. At low rigidity $R = 6$ EV in the JF12 parameterization 95% of the cosmic ray flux on Earth is caused by about 50% of the extragalactic directions (Fig. 9a). With increasing rigidity all extragalactic directions contribute equally to the flux on Earth. The PT11 parameterization yields similar results as shown in Fig. 9b, however, with less inhomogeneity at low rigidity $R$.

In contrast, the flux from certain extragalactic directions is not only suppressed but can even disappear, as no path leads to observation on Earth. As an example we show in Fig. 10 example trajectories of cosmic antimatter with rigidity $R = 60$ EV from the same extragalactic direction traversing a thin slice of $\pm 500$ pc around the $x$-$z$-plane in the galactic coordinate system. The trajectories are expected to miss the solar environment (marked by the yellow star). To enable this demonstration for antimatter we produced a separate set of lenses by backtracking matter particles in the JF12 field.

In general, both matter and antimatter particles entering the galaxy exhibit directions with a negligible arrival probability on Earth. In Fig. 10 we show the fraction of the sky in terms of solid angles $\Omega_B$, where none of 100 simulated cosmic rays reached Earth as a function of their rigidity $R$. For antimatter trajectories in the JF12 field, invisible directions appear at small and large rigidities (dashed curve).

For matter particles of rigidity $R = 6$ EV traversing the JF12 field, the invisible sky fraction is about 20% and disappears above $R \sim 10$ EV (blue curve). In contrast, for the PT11 field the invisible fraction of the sky for matter particles appears to be generally small (red curve).

At cosmic ray rigidities of $R = 6$ EV, variations in the field transparency from specific extragalactic directions lead to relative suppression and enhancement effects which may eventually require corrections in individual arrival direction analyses. The variations are strongly reduced with increasing cos-
Figure 9: Integrated relative flux $F$ of observed cosmic rays originating from the incoming directions with the highest arrival probabilities covering $\Omega/(4\pi)$ of the sky, a) JF12, b) PT11.

3.5. Small-scale random field

Beyond the regular large-scale component, recent galactic magnetic field models also contain small-scale random structures which are motivated e.g. by supernovae \([32, 33, 34]\). Such local disturbances are expected to cause a randomly oriented field component which introduces uncertainties in the predicted arrival directions of extragalactic cosmic rays. We expect this impact to be small, since the direction of the random component changes on a scale that is substantially smaller than the gyroradius of cosmic rays constrained in the galaxy.

To investigate the influence of such random fields we compare two different realizations of the so-called striated and turbulent random components as described in \([33]\) with a coherence length of $\lambda = 60$ pc. Cosmic rays are then deflected in both the regular JF12 and the first random field realization, and in the regular JF12 and the second random field, respectively.

As we aim to investigate the influence of the random fields on arrival directions we use the lensing technique. To obtain the most probable arrival direction on Earth we use the same techniques described above when studying the dispersion of the probability distribution (section 3.2). We calculate the radius $r_{50}$ containing 50% of the arrival probabilities, and use the center of the pixel with the
Figure 11: a) Angular distance $\delta$ between two cosmic ray arrival directions (circular symbols) from an incoming direction (star symbol), and directional difference $\Psi$ of the deflections (tangential to the sphere) resulting from different magnetic field orientations. b) Angular distances $\delta$ between the arrival directions using two different realizations of the striated and turbulent random components in addition to the regular JF12 field parametrization as a function of rigidity $R$. The median is depicted by the curve.

As expected, the impact of the random field is significantly smaller compared to the deflections of the cosmic rays arising from the regular galactic field (Fig.4). However, the uncertainties generated by such random deflections are sufficiently sizable to consider optimization of the rigidity threshold for individual arrival direction analyses.

3.6. Summary of the field impact on cosmic ray arrival

In order to work in a phase space region in which cosmic ray deflections can be controlled at least in a probabilistic way a minimum cosmic ray rigidity of $R = 6$ EV is recommended. This value results from avoiding deflections leading to a bend of 90 deg which overlaps with cosmic ray diffusion. All other distributions presented above on the dispersion of arrival probability distributions and multiple images, variance in field transparency, and uncertainties due to random field components are in accordance with this minimal rigidity value.

For a typical large-scale analysis the $R = 6$ EV rigidity threshold may be sufficient. However, depending on individual analysis requirements, the above key distributions may help to determine whether the rigidity threshold needs to be adjusted to larger values, or whether restrictions to incoming directions aside the galactic disk region need to be introduced.

4. Influence of field uncertainties on cosmic ray arrival

For a typical point source search, the reliability of galactic field corrections are of utmost importance. As a first step we compare directly the cosmic ray deflections resulting from the two field parameterizations PT11 and JF12.

In order to exemplify the impact of these differences on point source searches we simulate a typical corresponding analysis. We determine the discovery potential when the true galactic field is known, and quantify the reduced discovery potential when taking into account the different deflections of the two field parameterizations.

4.1. Comparison of deflection angles

Uncertainties in the current knowledge of the galactic field can be obtained to some extent from the different arrival distributions of the two field parameterizations PT11 and JF12. Note that their

smallest radius $r_{50}$ as the expected arrival direction.

In Fig[11] we sketch the angular distance $\delta$ between the arrival directions resulting from the two random field realizations. Here the incoming cosmic ray direction to the galaxy is indicated by the star symbol, and the two alternative arrival directions are denoted by the circular symbols.

In Fig[11] we show the angular distances $\delta$ between the arrival directions on Earth using the two realizations of the random fields as a function of cosmic ray rigidity $R$. The red curve indicates the median values. For rigidity $R = 6$ EV the uncertainty in the arrival directions is below 10 deg for 50% of the cosmic rays, and rarely extends to more than 90 deg.
fields are not completely independent regarding the overlap in the measurements constraining their fits, and usage of similar electron density distributions. However, the two parameterizations follow different ansatzes and include disjoint measurements, such that a direct comparison of cosmic ray deflections at least gives an idea of limited knowledge in the magnetic field.

For cosmic rays originating from the identical extragalactic direction \((l, b)\) we investigate the different deflections resulting from the two field parameterizations. We use the above-mentioned lensing technique to deliver arrival probability distributions. As outlined in section 3.5 we calculate the most probable arrival direction on Earth by using the center of the pixel with the smallest radius \(r_{50}\) containing 50% of the arrival probabilities. This gives the expected arrival directions \((l', b')\) for the JF12 parametrization and for the PT11 parametrization, respectively.

We first study the different directions of cosmic ray deflections by their azimuthal angular distance \(\Psi\) which is measured tangentially to the sphere (see Fig.11a). These directional differences reflect different field orientations of the two parameterizations.

In Fig.12a we show the directional difference \(\Psi\) for cosmic ray rigidity \(R = 60\) EV for the three regions separated by galactic latitudes \(\pm 19.5\) deg. For the southern region almost all directions of the deflections are within 90 deg (red histogram). In the northern region most of the incoming directions show \(\Psi < 90\) deg (blue histogram). About 1/5 of the incoming directions exhibit \(\Psi > 90\) deg, i.e. here the directions are nearly opposite.

In the disk region the differences between the two fields are large as half of the incoming directions are within 90 deg, and the other half has directional differences above 90 deg (green histogram).

In Fig.12a we show the rigidity dependence of the directional difference \(\Psi\) for all regions where the curve represents the median values. For all rigidities above \(R = 6\) EV, the most likely differences in the directions of the deflections are below 10 deg, and for 50% of the cosmic rays the field directions are within 40 deg.

The second important aspect of the field differences is the absolute angular distance in the arrival directions resulting from the PT11 and JF12 parameterizations which are denoted by \(\delta\) (see Fig.11b).

In Fig.13a we show the angular distance \(\delta\) for cosmic rays with rigidity \(R = 60\) EV for the three regions separated by galactic latitudes \(\pm 19.5\) deg. For the northern and southern regions the angular distance between the two parametrizations is below \(\delta = 5\) deg for 3/4 of the incoming directions (blue, red histograms).

In the disk region, only 1/3 of the arrival directions show angular distances below \(\delta = 5\) deg, while the majority of incoming directions have larger angular distances up to \(\delta = 30\) deg.

In Fig.13b we show the angular distance \(\delta\) for all regions as a function of cosmic ray rigidity. The curve represents the median values. For cosmic rays with small rigidity \(R = 6\) EV, half of them result at an angular difference below 30 deg. However, there is a long tail towards large angular distances resulting from the two field parameterizations.

Although sizable differences in the directional
characteristics and magnitudes of the two field parameterizations exist, their influence is sufficiently reduced at large cosmic ray rigidity. For example, at $R = 60$ EV, the absolute deflection angles $\beta$ as well as the angular distances $\delta$ arising from the two fields become consistently small for most incoming directions. Further studies show that in the northern and southern regions the median ratio of the angular distance $\delta$ and the averaged deflection $(\beta(JF12) + \beta(PT11))/2$ remains approximately flat for rigidities $R > 10$ EV.

4.2. Comparison of arrival directions

In Fig. 14 we show example arrival distributions of cosmic rays with rigidity a) $R = 20$ EV, b) $R = 60$ EV originating from ten sources. The directions of the sources are denoted by the star symbols. The initial cosmic ray distributions followed the Fisher distribution with a Gaussian width of 3 deg.

Indicated by the dark (light) red regions are the 68% (95%) arrival probability distributions of the cosmic ray after deflections by the PT11 field. The blue regions give the corresponding arrival probability distributions from the JF12 field.

At cosmic ray rigidity $R = 20$ EV (Fig. 14a) at least half of the arrival probability distributions exhibit substantial overlap for the PT11 and JF12 fields. With increasing rigidity (Fig. 14b, $R = 60$ EV) the overlap increases as expected. The number of images is also reduced at larger rigidity.

This implies that, at large rigidities, the agreement of the two field parameterizations is sufficiently large to investigate the impact of the field uncertainties on a point source search which we present in the following section.

4.3. Simulated point source search

For our simulated search for origins of cosmic rays we study separately sources in the three regions of the galaxy (latitude $\pm 19.5$ deg). In each
region we repeatedly simulate ten sources and demand the cosmic rays to follow a Fisher probability distribution with a Gaussian width of 3 deg.

We also simulate isotropically distributed cosmic rays with full sky coverage as a background contribution. In the following we perform multiple analyses with sets of 500 cosmic rays for which we vary the contribution of signal cosmic rays, i.e. cosmic rays arriving from the ten sources, between signal fraction \( f_s = 0 \%) \) and \( f_s = 100 \% \).

To quantify the analysis sensitivity we use the log-likelihood function

\[
\ln L(a) = \sum_{i=1}^{N} \ln \left[ a P(R_i, l_i', b_i') + (1 - a) B \right].
\]

The sum refers to all simulated \( N = 500 \) cosmic rays. Parameter \( a \) denotes the anticipated fraction of signal cosmic rays from the sources when analyzing the data, and the isotropically distributed cosmic rays are assumed to contribute with \( (1 - a) \) correspondingly. The probabilities \( P(R, l', b') \) represent the anticipated arrival probability distributions for cosmic rays with rigidity \( R \) which originate from the sources and are expected to be observed in directions \( (l', b') \) on Earth. They were obtained using the lensing techniques. The background probability \( B \) corresponds to the inverse number of pixels for which we used the above \( N_{\text{pix}} = 49,152 \) pixels of approximately 1 deg.

As the test statistics we use the likelihood ratio

\[
t = 2 \ln \frac{L(a)}{L(a = 0)}
\]

which approximately follows a \( \chi^2 \) distribution with 1 degree of freedom [35]. For each anticipated signal fraction \( a \) we repeat the simulation of cosmic ray sets 1000 times and determine the average maximum \( t_{\text{max}} \). The significance by which isotropic arrival distributions can be excluded is then estimated by converting the integral

\[
\int_{t_{\text{max}}}^{\infty} \chi^2 dt
\]

to Gaussian standard deviations \( \sigma \).

In the analysis we use as the simulated scenario the JF12 arrival probability distributions \( P(R, l', b') \) to describe cosmic ray deflections. To obtain a benchmark for a best-case scenario, where the field and the cosmic ray rigidities are perfectly known, we first analyze cosmic rays with rigidity \( R = 20 \) EV by using the JF12 field, here representing the true field. Note that this scenario returns optimistic results as we neglect deflections in the small-scale random field and demand sources to be located in one of the three galactic regions exclusively.

In Fig 15k we show the significance \( \sigma \) as a function of the signal fraction \( f_s \) of the simulated sample. With perfect knowledge of the galactic field a signal fraction of \( f_s = 5\% \) is sufficient for a 5\( \sigma \) discovery (full curves).

To take into account uncertainties in the galactic field as encoded in the two different parameterizations, we then perform the analysis with the PT11 arrival probability distributions instead of the true JF12 probability distributions. As the results are slightly dependent on the exact directions of the sources, we repeat the analyses nine times in each region and present the average resulting values.

In Fig 15k we show the significance \( \sigma \) of a deviation from isotropic arrival distributions as a function of the average signal fraction \( f_s \), for the three regions (dashed curves). A signal fraction of \( f_s = 14\% \) is sufficient for a 5\( \sigma \) discovery for the northern and southern regions, and slightly larger for the disk region \( (f_s = 18\%) \). When compared to the above best-case scenario, the field uncertainties require the signal fraction for discovery to increase substantially by a factor of \( 3 - 4 \).

In Fig. 15b we present the required signal fraction \( f_s(5\sigma) \) for discovery as a function of cosmic ray rigidity \( R \). The full curves represent the required signal fraction for the benchmark scenario having perfect knowledge of the galactic magnetic field. The required signal fraction \( f_s(5\sigma) \) when including field uncertainties are shown by the dashed curves.

It is interesting to note that, although deflections are on average twice as large in the southern region compared to the northern region, their sensitivities are of similar value above rigidity \( R = 20 \) EV. In the disk region the different characteristics of the two parameterizations have a large impact on the point source search. Here the required signal fraction almost doubles irrespective of rigidity.

As expected, the sensitivity of the point source search improves consistently with increasing cosmic ray rigidity. Again comparing with the above best-case scenario, cosmic rays with \( R = 60 \) EV entering the galaxy in the northern or southern regions require a moderate 20\% increase in the required signal fraction for discovery owing to field uncertainties. In contrast, incoming directions at the disk region need a 2.5-fold larger signal fraction.

In view of our current knowledge of the galactic field, for analyses which aim at selected source
Figure 15: Simulated point source search with ten sources located in one of the three regions separated by galactic latitudes $\pm 19.5$ deg and sets of 500 cosmic rays with fraction $f_s$ originating from the sources and $(1 - f_s)$ from isotropic background with full sky coverage. a) Significances for deviation from isotropic arrival distributions at $R = 20$ EV using the true galactic field (full curve) for the anticipated arrival probability $P$ in eq. (2), and the other field alternatively (dashed curves). b) Required signal fraction for a $5\sigma$ deviation from isotropic arrival directions using the true field (full curve) compared to using the other field (dashed curves) as a function of rigidity $R$.

4.4. Summary of the influence of field uncertainties on cosmic ray arrival

Our direct comparisons of cosmic ray deflections in the two field parameterizations provide some information on the actual knowledge of the entire field map. For cosmic ray rigidity $R = 6$ EV, the angular distance after deflections in the two fields is within $\delta = 30$ deg for 50% of the directions incoming to our galaxy. This value is much smaller at $R = 60$ EV rigidity, where the differences are below $\delta = 5$ deg for 60% of the directions incoming to our galaxy.

To study the influence of uncertainties in the field on searches for cosmic ray origins, we analyzed simulated astrophysical scenarios using a log-likelihood method. The method includes the anticipated probability distributions for cosmic ray arrival after traversing the galactic field, and quantifies deviations from isotropic arrival distributions.

We estimated the influence of field uncertainties by using one field parameterization in the simulation of the scenario, and by applying the other field parameterization in the log-likelihood analysis. For cosmic rays with rigidity $R = 60$ EV arriving from sources within galactic latitudes $\pm 19.5$ deg (disk region) we found that the field uncertainties increase the required signal fraction for a $5\sigma$ discovery substantially by more than a factor of two. However, for sources in the northern and southern regions emitting cosmic rays with $R = 60$ EV, the field uncertainties are relatively small and increase the required signal fraction for a $5\sigma$ discovery by 20% only.

5. Conclusion

Corrections for deflections in the galactic magnetic field using the two parameterizations PT11 and JF12 can be meaningfully considered for cosmic ray rigidities above $R > 6$ EV. Above this rigidity, deflections can be distinguished from diffusive random walk. This has strong implications for analyses using cosmic ray data with mixed composition. For protons this rigidity corresponds to energies above $E = 6$ EeV. However, when analyzing, e.g., Neon nuclei with charge $Z = 10$, meaningful corrections can be performed for energies above $E = 60$ EeV only.

When quantifying uncertainties in the galactic field from comparisons of the two field parameterizations PT11 and JF12, the rigidity threshold needs to be raised substantially. Then both fields give similar predictions for cosmic ray deflections in the northern and southern regions with galactic latitudes $|l| > 19.5$ deg. In the disk region $|l| < 19.5$ deg, however, the differences in the predictions remain large. Consequently, in our simu-
lated search for cosmic ray origins the arising uncertainties are substantial for sources near the galactic disk, and may be considered acceptable for sources aside the disk emitting cosmic rays with rigidities $R \geq 20 \text{ EV}$.

Acknowledgments

We wish to thank very much G. Farrar for fruitful discussions, and G. Farrar, P. Tinyakov and M. Sutherland for valuable comments on the manuscript. This work is supported by the Ministerium für Wissenschaft und Forschung, Nordrhein-Westfalen, the Bundesministerium für Bildung und Forschung (BMBF), and the Helmholtz Alliance for Astroparticle Physics.

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