Calculation of Dissociation Temperature of Nucleon Using Gaussian Expansion Method

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The first study of the dissociation temperature of nucleon in hot QCD medium in the framework of constituent quark model is presented. The temperature-dependent potential energy of the three quark system, as taking as the internal energy of the system are obtained from the free energy of the system, and the temperature-dependent free energy is derived based on Debye-Hückel theory. The lattice QCD results of the free energy for heavy three-quark system are employed and extended to the light three-quark system. The Schrödinger equation for nucleon is solved with the help of Gaussian expansion method and the dissociation temperature of the nucleon is determined according to the temperature dependence of binding energy and radii. Comparing with the dissociation temperature of \( J/ψ \), the dissociation temperature of nucleon is higher. So, nucleon is more difficult to melt than Charmonium.

\textbf{Keywords:} Nucleon; Dissociation temperature; Gaussian Expansion Method.

I. INTRODUCTION

The relativistic heavy-ion collider experiments show that human beings may have produced quark-gluon plasma (QGP) in the laboratory [1]. It is generally believed that some quark bound states can survive in QGP. In thermal Quantum chromodynamics (QCD), the thermal properties of QGP can be determined by studying the behavior of these quark bound states in hot medium. In 1986, Satz pointed out that the suppression of \( J/ψ \) could be a signature of QGP formation in the relativistic heavy ion collisions [2]. Since then, many people have systematically studied the melting of Charmonium and Bottomonium. But the dissociation temperature of nucleon, the lightest baryon, has not been studied systematically. This is due to the difficulties of solving three-body system and obtaining the temperature dependent quark-quark interaction potential within the nucleon.

QCD is the fundamental theory of strong interaction. It works well in the perturbative region but it does not work in the non-perturbative region. It is difficult for us to use QCD to study the thermal properties of quark bound states directly. In this case, people have to use the model [3–6] to study the dissociation of quarkonium states. In high temperatures and density, the interaction between quarks is screened [7] and the binding energy will be decreased. As a result, the nucleon will start to melt when the binding energy become low enough. The melting of nucleon can be solved by Schrödinger equation of three body. For the calculation, we need the interacting potentials among quarks of the nucleon in the hot medium, which is temperature-dependent. Unfortunately, the potentials are not yet well understood up to now. The free energy of a static heavy three-quark system \( F_{qqq}(r,T) \) can be calculated in lattice QCD and the internal energy can be obtained by using thermodynamic relation. In the present approach, the needed potential is assumed to be the internal energy, i.e. \( V = F + sT \) with \( s \) being the entropy density \( s = -\frac{\partial F}{\partial T} \). The temperature-dependent form of \( F_{qq}(r,T) \) can be constructed based on Debye-Hückel theory [8], as having been done in Ref. [7]. Then, we can determine the \( T \)-dependent parameter in the free energy by fitting it with the lattice data. According to the relation between \( F_{qq}(r,T) \) and \( F_{qqq}(r,T) \), we can obtain the free energy of heavy three-quark system. We assume the conclusions are applicable to light quark system due to the flavor independence of the strong interaction. After constructing the potential of nucleon system at finite temperature, we can obtain the temperature dependence of binding energies and radii by solving the corresponding Schrödinger equation. The dissociation temperature is the point where the binding energy decreases to zero. The Gaussian expansion method (GEM), which is an efficient and powerful method in few-body system [9], is employed to calculate dissociation temperature of nucleon in this paper.

This paper is organized as follows. In Sec.II, we show the rationality of GEM on studying the melting of Charmonium and Bottomonium by comparing our results with others. In Sec.III, we construct the potential of nucleon and apply GEM to solve corresponding Schrödinger equation. In Sec.IV, we show the results at quenched and 2-flavor QCD, respectively. Sec.V contains summary and conclusion.
II. THE RESULTS ON DISSOCIATION TEMPERATURE OF QUARKONIUM

Before studying the dissociation of nucleon, we test the rationality of GEM on studying dissociation temperatures of quarkonium by comparing our results with others. To compare with Satz’s results, the potential of quarkonium at finite temperature we use is the same as Satz’s work [4]. The results on dissociation temperature of Charmonium, Bottomonium in Ref. [4] and our calculation results are listed in Table I, Table II, which show our results are consistent with that in Ref. [4]. So GEM can give accurate results on the dissociation temperature of quarkonium. Giving accurate binding energy and wave function [9] makes GEM very suitable for studying dissociation temperature of quark bound states (more detail can be found in Appendix A). In the following, we will use this method to calculate the dissociation temperature of nucleon.

III. FORMALISM

A. Constituent Quark Model

The constituent quark model is a non-relativistic quark model [10]. In the constituent quark model, baryons are formed by three constituent quarks, which are confined by a confining potential and interact with each other [11]. The potential of baryon can be described by a sum of the potential of corresponding two-quark system. In Kaczmarek’s work [12], it has been calculated in lattice QCD that the potential of diquark system is about half of that of corresponding quark-antiquark system, i.e. \( V_{qq} = \frac{1}{2} V_{q\bar{q}} \). The simplest and most frequently used potential for a \( qq \) system is the Cornell potential [7],

\[
V_{qq}(r) = -\frac{\alpha}{r} + \sigma r
\]

where \( \alpha \) is the coupling constant, and \( \sigma \) is the string tension. In the present work, we neglect the spin-dependent part of potential here. Thus our Hamiltonian is written as

\[
H = \sum_{i=1}^{3} \left( m_i + \frac{p_i^2}{2m_i} \right) - T_{cm} + \sum_{1 \leq i < j}^{3} \frac{1}{2} V(r_{ij})
\]

\[
V(r_{ij}) = \sigma r_{ij} - \frac{\alpha}{r_{ij}}
\]

where \( m_i \) is the constituent quark mass of the \( i \)-th quark, and \( T_{cm} \) is the kinetic energy of center-of-mass frame (cm). \( r_{ij} = r_i - r_j \) is the relative coordinate between \( i \)-th quark and \( j \)-th quark. In this model, the mass of light quark (u and d quark) we use is 300 MeV. The parameters of Cornell potential we use are: \( \alpha = 1.4, \sqrt{\sigma} = 0.131 \) GeV. Solving the corresponding Schrödinger equation, \( H\Psi_{\text{total}} = E_m\Psi_{\text{total}} \), with GEM, we can get the mass \( E_m \) and corresponding wave function of nucleon \( \Psi_{\text{total}} \). We define the radii of nucleon as

\[
R = \frac{1}{3} \sum_{i=1}^{3} \sqrt{\langle r_i^2 \rangle}
\]

with

\[
\langle r_i^2 \rangle = \int \Psi_{\text{total}}^* r_i^2 \Psi_{\text{total}} d\tau
\]

where \( r_i \) is the distance between the center of nucleon and \( i \)-th quark. Using the calculated wave function, we can calculate the radii of nucleon. The calculating mass and radii of nucleon are 939 MeV and 0.8393 fm, respectively. While the corresponding experimental data are about 939 MeV and 0.841 fm. We can see this model gives a good estimation of the properties of nucleon even if the spin-dependent part is neglected. So it is reasonable for us to use this potential model to study the dissociation of nucleon. Of course, we need notice that the spin-dependent part plays an important role in the baryon spectrum.

B. Wave Function

Here, we solve the Schrödinger equation with GEM. In this method, three sets of Jacobi coordinates (Fig. 1) are introduced to express three-quark wave function. The Jacobi coordinates in each channel \( c(c = 1, 2, 3) \) are defined as

\[
r_i = x_j - x_k
\]
The potential of nucleon at zero temperature has been discussed above and its parameters have been determined by fitting the properties of nucleon. To determine the dissociation temperature of nucleon, we need the potential in hot medium, i.e. $V_{qqq}(r, T)$ (the index $q$ represents u or d quark). Here, we assume that the potential is just the internal energy

$$V_{qqq}(r, T) = U_{qqq}(r, T) = F_{qqq}(r, T) + sT$$

where $s$ is the entropy density $s = -\partial F_{qqq}/\partial T$. In Refs. [13–15], Kaczmarek's works show that the color singlet free energies of the heavy three-quark system ($F_{qqq}^{1}$) can be described by the sum of antitriplet free energies of the corresponding diquark system ($F_{qq}^{1}$) plus self energy contributions when the temperature is above $T_{c}$. It can be expressed as

$$F_{qqq}(P, T) \approx \sum_{i<j}F_{qq}^{1}(R_{ij}, T) - 3F_{qq}(T)$$

where $P = \sum_{i<j}R_{ij}$ and the self energy $F_{qq}(T) = \frac{1}{2}F_{qq}^{1}(\infty, T)$. In Ref. [12], O.Kaczmarek's work suggests a simple relation between free energies of anti-triplet $qq$ states and color singlet $q\bar{q}$

$$F_{q\bar{q}}(r, T) \approx 2(F_{qq}^{1}(r, T) - F_{qq}(T))$$

The form of $F_{q\bar{q}}^{1}$ can be obtained based on studies of screening in Debye-Hückel theory. It can be written as [7]

$$F_{q\bar{q}}^{1}(r, T) = -\frac{\alpha}{r} \left[ e^{-\mu r} + \mu r \right] + \frac{\sigma}{\mu} \left[ \frac{\Gamma(1/4)}{23/4\Gamma(3/4)} - \frac{\sqrt{\mu r}}{23/4\Gamma(3/4)} K_{1/4}((\mu r)^2 + \kappa(\mu r)^4) \right]$$

where screening mass $\mu$ and the parameter $\kappa$ are temperature-dependent, and $K_{1/4}[x]$ is the modified Bessel function. We can determine the T-dependent $\mu$ and $\kappa$ by fitting $F_{q\bar{q}}^{1}(r, T)$ to the lattice result obtained in quenched [16] and 2-flavor [17] QCD. At $r = \infty$, the free energy $F_{q\bar{q}}^{1}(T)$ is written as

$$F_{q\bar{q}}^{1}(T) = \frac{\sigma}{\mu(T)} \frac{\Gamma(1/4)}{23/4\Gamma(3/4)} - \alpha \mu(T)$$

Thus, the form of $\mu(T)$ is given as function of $F(T)$

$$\mu(T) = \left( \sqrt{F_{q\bar{q}}^{1}(T)^2 + 4\sigma \alpha \frac{\Gamma(1/4)}{23/4\Gamma(3/4)} - F_{q\bar{q}}^{1}(T)} \right) / 2\alpha$$

Once we obtain the temperature dependence of $\mu(T)$, we fit Eq. (17) to the lattice data to obtain $\kappa(T)$. The results for $\mu(T)$ and $\kappa(T)$ are shown in Fig. 2 and Fig. 3.
3, respectively. In Fig. 4, we show our fit curves (solid lines) together with the lattice results. We can see that the resulting $F_{qq}(r,T)$ fits the lattice results quite well for all $r$ and in a broad range of temperatures from $T_c$ to $4T_c$ in the two cases. For higher temperature, the resulting $F_{qq}(r,T)$ cannot be fitted quite well to the lattice results in quenched QCD. There can be higher order corrections to Poisson equation [7].

FIG. 2. Results for $\mu(T)$ in quenched (upper figure) and 2-flavor (lower figure) QCD.

To obtain the binding energies of nucleon, we define an effective potential as

$$\tilde{V}_{qqq}(R,T) = V_{qqq}(R,T) - V_{qqq}(\infty, T) \quad (20)$$

Combining Eqs.(14-16,20), we get a relation between effective potential and free energies of $q\bar{q}$

$$\tilde{V}_{qqq}(R,T) = \sum_{i<j} \frac{1}{2} (\tilde{F}_{qq}^{1}(R_{ij},T) - T \frac{\partial \tilde{F}_{qq}^{1}(R_{ij},T)}{\partial T}) \quad (21)$$

where

$$\tilde{F}_{qq}^{1}(R_{ij},T) = F_{qq}^{1}(R_{ij},T) - F_{qq}^{1}(\infty, T) \quad (22)$$

Replacing the potential term, $\sum_{i<j} \frac{1}{2} V(r_{ij})$, in Eq. (2) with this effective potential $\tilde{V}_{qqq}(R,T)$, we can get a new Hamiltonian for nucleon at finite temperature written as

$$H_{new} = \sum_{i=1}^{3} \frac{p_{i}^{2}}{2m_{i}} - T_{CM} + \tilde{V}_{qqq}(R,T) \quad (23)$$

FIG. 3. Results for $\kappa(T)$ in quenched (upper figure) and 2-flavor (lower figure) QCD.

FIG. 4. Results for free energy ($F_{qq}$) in quenched (upper figure) and 2-flavor (lower figure) QCD.
Solving corresponding Schrödinger equation,

\[ H_{new} \Psi_{J/M}^{JM} = \epsilon(T) \Psi_{J/M}^{JM}, \]

with GEM, we can get the binding energies \( \Delta E(T)(= -\epsilon(T)) \) and corresponding wave function at finite temperature. Using the wave function, we can calculate the T-dependent radii according to Eq. (4).

**IV. NUMERICAL RESULTS**

In Fig. 5, we show the resulting binding energies behaviour for nucleon in quenched and 2-flavor QCD. We can see there is little difference between the two lines. When they vanish, the nucleon no longer exists. So \( \Delta E(T) = 0 \) determines the dissociation temperature. From Fig. 5, we get the dissociation temperature in quenched and 2-flavor QCD are about \( 3.0T_c \) and \( 3.3T_c \), respectively; in Fig. 6, we show the corresponding nucleonic radii. The dissociation temperature determined from Fig. 6 is consistent with that determined from Fig. 5. It is seen that the divergence of the radii defines quite well the different dissociation points in the two cases. The resulting dissociation temperatures have a little difference between the two cases.

![FIG. 5. T-dependent of binding energy in quenched and 2-flavor QCD, respectively.](image)

**V. SUMMARY AND CONCLUSION**

The free energies of quark-antiquark system we construct based on Debye-Hückel theory at finite temperature fit the lattice results quite well from \( T_c \) to \( 4T_c \), but not well for higher temperature. According to Kaczmarek’s works, we can get a relation between color singlet free energy of heavy \( qqq \) system \( F_{qqq}^1 \) and color singlet free energy of heavy \( q\bar{q} \) system \( F_{q\bar{q}}^1 \), written as \( F_{qqq}^1 \approx \sum_{i<j} \frac{1}{2} F_{q\bar{q}}^1 \). The dissociation temperature of nucleon in quenched and 2-flavor QCD we calculate are about \( 3.0T_c \) and \( 3.3T_c \), respectively. There are a little difference between the two results. Comparing with \( J/\psi \), the dissociation temperature of nucleon is higher. So, nucleon is more difficult to melt than charmonium. For the potential, we neglect the spin-dependent part in this work which may has some effects to the resulting dissociation temperature. The effects arising from spin-dependent part deserve further studies.

![FIG. 6. T-dependent of radii. in quenched and 2-flavor QCD, respectively](image)

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[1] J. M. Torres-Rincon, B. Sintes, and J. Aichelin, Physical Review C 91, 065206 (2015).

[2] T. Matsui and H. Satz, Physics Letters B 178, 416 (1986).
Appendix A: The Calculation on Dissociation Temperature of Quarkonium

We have given our results on dissociation temperature of quarkonium in Sec.II. Here we present the calculation in detail. For quarkonium, the potential we use is the Cornell potential mentioned above. The Hamiltonian for quarkonium is written as

\[ H = \frac{p^2}{2\mu_{12}} + V_{qq}(r) \quad (A1) \]

where \( p = p_2 - p_1 \) and \( \mu_{12} \) is the reduced mass. The parameters taken from Ref.[4] are: \( m_c = 1.25 \) GeV, \( \sqrt{s} = 0.445 \) GeV, \( \alpha = \pi/12 \) and \( m_b = 4.65 \) GeV. Then we can construct the corresponding free energies at finite temperature based on Debye-Hückel theory as having done above. To compare with H.Satz’s work, we neglect the term \( \kappa (\mu r)^4 \) in Eq. (17). We can obtain the T-dependent parameter \( \mu(T) \) by fitting Eq. (17) to lattice data calculated in two-flavour QCD[17]. According to the potential model mentioned above, we can construct the potential of quarkonium at finite temperature. After obtaining the potential of quarkonium, we can get the Schrödinger equation

\[ \left( -\frac{1}{2\mu_{12}} \nabla^2 + V_{qq}(r,T) - V_{qq}(\infty,T) \right) \psi_i(r,T) = \epsilon_i(T) \psi_i(r,T) \quad (A2) \]

where the index \( i \) represents a quarkonium state and \( \Delta E_i(T)(= -\epsilon_i(T)) \) is the binding energy at temperature \( T \). According to Ref. [9], we expand the total wave function in terms of a set of basis functions as

\[ \psi_{lm} = \sum_{n=1}^{n_{\text{max}}} c_n \phi_{nlm} \quad (A3) \]

with

\[ \phi_{nlm}(r) = \phi_{nl}(r)Y_{lm}(\hat{r}) \quad (A4) \]

\[ \phi_{nl}(r) = N_{nl} r^l e^{-\nu_n r^2} \quad (A5) \]

where the \( N_{nl} \) is the normalization constant. And Rayleigh-Ritz variational principle leads to a generalized matrix eigenvalue problem,

\[ \sum_{n'=1}^{n_{\text{max}}} (H_{nn'} - EN_{nn'}) c_{n'l} = 0 \quad (A6) \]

Therefore, we can obtain the eigenvalues and corresponding wave functions of both ground state and excited states. We define the radii for quarkonium as:

\[ \langle r_i \rangle = \int \psi_i^* r \psi_i d\tau \quad (A7) \]

Then we can use the calculating wave function to calculate the temperature-dependent radii. In Fig. 7 and Fig. 8, we show the resulting binding energy behaviour for the different charmonium states and bottomonium states, respectively. We show the T-dependence of bound state radii for each states in Fig. 9 and Fig. 10. According to these figures, we can determine the dissociation temperature.
FIG. 7. Binning energy of $J/\psi(1S)$, $\chi_c(1P)$, and $\psi'(2S)$ dependent to $T$.

FIG. 8. Binning energy of $\Upsilon(1S)$, $\chi_b(1P)$, $\Upsilon(2S)$, $\chi_b(2P)$, and $\Upsilon(3S)$ dependent to $T$. 
FIG. 9. Bound state radii of $J/\psi(1S)$, $\chi_c(1P)$, and $\psi'(2S)$ dependent to $T$

FIG. 10. Bound state radii of $\Upsilon(1S)$, $\chi_b(1P)$, $\Upsilon(2S)$, $\chi_b(2P)$, and $\Upsilon(3S)$ dependent to $T$