Analysis for active isolation of the equipment on flexible beam structure

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Abstract. This paper dealt with the active isolation of vibration. Many home appliance and vehicles in operation generate the vibration for several reasons. Due to the vibration propagated to the structure, it has a negative influence of the structure such as breakage and decreased reliability. As basic study on vibration isolation, the analytical model is as follows. The analytical model consists of the equipment as source of vibration and the flexible beam to be controlled. The equipment is assumed by rigid body and excited by a concentrated force as disturbance. And the equipment and the beam are connected to spring located at both ends the equipment. The actuators are located slightly away from spring. To minimize the vibration propagated to the beam structure through the equipment, the controller was designed by using the multi-channel Wiener filter after calculating the transfer path through the theoretical formula. The disturbance was given as harmonic components. The propagated vibration to the beam caused by the disturbance was controlled using a designed controller. Finally, the numerical results for the displacement obtained at each point of the beam structure with and without control were carried out and compared.

1. Introduction

When many home appliance and vehicles operate, the vibration occurs. Here, the vibration propagates through the structure, which has a negative influence on life of the structure \cite{1}. Therefore, it is necessary to study how to isolate the propagated vibration. To isolate the propagation vibration, there is a method of the passive damping and a method of redesigning the vibration transmission path of the structure \cite{2}. However, the method of the passive damping is only a control of high frequency band, and the redesign of the structure is costly. Thus, active vibration control (AVC) technique is used as a solution to these problems \cite{3}.

In this study, the control was implemented on a model in which the equipment with disturbance connected with spring on the flexible beam supporting both ends. The object to be controlled is a flexible beam. The controller was designed using a Wiener filter for isolate the vibration transmitted from the equipment. Using the designed filter, the displacement with and without control were carried out and compared.
2. System modelling

2.1. Analytical model

Figure 1 shows the analytical model. This model consists of the equipment as source of vibration and the flexible beam as receiver to be controlled. The equipment is assumed by rigid body and excited a concentrated force as disturbance. The equipment and flexible beam is connected to spring located at both ends of the equipment.

![Analytical model](image1)

Figure 1. Analytical model

Here, \( l \) and \( L \) are the length of receiver and source, respectively. And \( k \) is the spring constant. \( x_1, x_2 \) are the location of spring on receiver, \( f_k, f_{s1} \) are the controlled force.

2.2. Governing equation

The governing equation of analytical model can be obtained through the free body diagram. Figure 2 shows the free body diagram of analytical model.

![Free body diagram](image2)

Figure 2. Free body diagram of analytical model

Receiver was assumed the Euler-Bernoulli beam. The equation of motion of transverse vibration of a uniform cross section beam under concentrated force is as follows.

\[
EI \frac{\partial^4 w_i(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w_i(x,t)}{\partial t^2} = f(x,t)
\]  

(1)

Here, \( E \) is the elastic modulus, \( I \) is the moment of inertia of beam section, \( \rho \) is the density, \( A \) is cross section of beam and \( f \) is the external force per unit length.
In this case, the displacement of beam is expressed by the sum of the eigenmodes up to N-th order, as follows.

\[ w_i(x,t) = \sum_{n=1}^{N} d_n(\omega) \phi_n(x) e^{i\omega t} \]  

(2)

Here, \( d_n(\omega) \) is the n-th order modal displacement by frequency and \( \phi_n(x) \) is the n-th order eigenmode.

Using Fourier transform, Equation (2) in frequency domain can be expressed.

\[ w_i(x,\omega) = \sum_{n=1}^{N} d_n(\omega) \phi_n(x) \]  

(3)

In equation (3), if beam length is \( L \) and the boundary condition of beam is simply supported, normalized eigenmodes can be expressed.

\[ \phi_n(x) = \sqrt{2} \sin\left(\frac{n\pi x}{L}\right) \]  

(4)

The force on the receiver is assumed to be applied to the \( x_1 \) and \( x_2 \) concentrated loads. And it consists of force transmitted through the source and controlled force \( f_{h_1} \), \( f_{h_2} \). Therefore, the external force can be expressed.

\[ f(x,t) = \left[ k\left\{ w_2 - \frac{L}{2}\theta - w_i(x_1,t)\right\} - f_{h_1}\right] \delta(x-x_1) + \left[ k\left\{ w_2 + \frac{L}{2}\theta - w_i(x_2,t)\right\} - f_{h_1}\right] \delta(x-x_2) \]  

(5)

Here, \( w_i \) is the transverse degree of freedom of receiver, \( w_2 \) and \( \theta \) are the transverse and rotational degree of freedom, respectively.

Substituting equation (3) and equation (5) into equation (1), it can be expressed as follows.

\[ \omega_n^2 a_m - \omega_n^2 a_m = -\frac{k}{\rho A} \left\{ \sum_{n=1}^{N} d_n(\omega) \phi_n(x_1) \right\} \phi_m(x_1) - \frac{k}{\rho A} \left\{ \sum_{n=1}^{N} d_n(\omega) \phi_n(x_2) \right\} \phi_m(x_2) + \frac{k}{\rho A} \left\{ \phi_m(x_1) + \phi_m(x_2) \right\} W_2 - \frac{k}{\rho A} \left\{ \phi_m(x_1) - \phi_m(x_2) \right\} \Theta \]

\[ -\frac{1}{\rho A} F_s \phi_m(x_1) - \frac{1}{\rho A} F_s \phi_m(x_2) \]  

(6)

Here, \( m \) is the arbitrary order left by the mode orthogonality, \( W_2, \Theta, F_{h_1} \) and \( F_{h_2} \) are variables obtained by Fourier transforming \( w_2, \theta, f_{h_1} \) and \( f_{h_2} \).

The equation of motion of source can be expressed using Figure 2.

\[ m\ddot{w}_2 + 2kw_2 - k\left\{ w_i(x_1,t) + w_i(x_2,t)\right\} = f_{p_1} + f_{h_1} + f_{h_2} \]  

(7)

\[ \frac{ml^2}{12} \dddot{x} + \frac{k l^2}{2} \dddot{x} + \frac{kl}{2} \left\{ w_i(x_1,t) - w_i(x_2,t)\right\} = f_{x_1}x_{p_1} - f_{h_1} l + f_{h_2} \frac{l}{2} \]  

(8)

Here, equation (7) and equation (8) are transverse and rotational equation of motion, respectively.

Substituting equation (3) into equation (7) and equation (8), it can be expressed as follows.

\[ (-\omega^2 + 2k)W_2 - k\left\{ \sum_{n=1}^{N} d_n(\omega) \phi_n(x_1) + \sum_{n=1}^{N} d_n(\omega) \phi_n(x_2) \right\} = F_{p_1} + F_{h_1} + F_{h_2} \]  

(9)
\[
\left(-\omega^2 \frac{ml^2}{12} + \frac{kl^2}{2}\right)\Theta + \frac{kl}{2} \left(\sum_{i=1}^{N} d_i(\omega)\phi_i(x_i) - \sum_{i=1}^{N} d_i(\omega)\phi_i(x_i)\right) = F_p x_p - F_s \frac{l}{2} + F_s \frac{l}{2} \quad (10)
\]

3. Controller design

The controller was designed using the Wiener filter. The Wiener filter one of ways of obtaining the coefficient of fir filter, it calculates the coefficients to minimizes the mean square error between the estimated random process and the desired process.

In this study, the control was performed using SIMO (Single-input multiple-output) Wiener filter. A schematic diagram is shown in figure 3.

**Figure 3.** Schematic diagram using SIMO Wiener filter

Here, \( x \) is the input force, \( W \) is the controller (filter coefficient), \( H \) is the transfer function between force and displacement, \( d \) is the displacement before control, \( y \) is the displacement for control and \( e \) is the displacement after control.

The filter coefficient \( W \) can be obtained by minimizing the mean square of the \( e \).

\[
e^2(n) = \left[ d_1(n) - \left\{ y_{11}(n) + y_{21}(n) + \cdots + y_{M1}(n) \right\} \right]^2 + \cdots + \left[ d_L(n) - \left\{ y_{1L}(n) + y_{2L}(n) + \cdots + y_{ML}(n) \right\} \right]^2
\]

where,

\[
y_{ij} = x_i^T W_i = x_i^T H_i
\]

Here, \( M \) is the number of controlling force, \( L \) is the number of transducer to measure displacement. To minimize the \( e \), the equation (14) was used.

\[
\frac{\partial e^2(n)}{\partial W_1} = \frac{\partial e^2(n)}{\partial W_2} = \cdots = \frac{\partial e^2(n)}{\partial W_M} = 0
\]

Thus, filter coefficient \( W \) was calculated by the following equation (14).

\[
\begin{bmatrix}
W_1 \\
W_2 \\
\vdots \\
W_M
\end{bmatrix} = \begin{bmatrix}
\sum_{i=1}^{L} T_{1i,1} & \sum_{i=1}^{L} T_{1i,2} & \cdots & \sum_{i=1}^{L} T_{1i,M} \\
\sum_{i=1}^{L} T_{2i,1} & \sum_{i=1}^{L} T_{2i,2} & \cdots & \sum_{i=1}^{L} T_{2i,M} \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{i=1}^{L} T_{Mi,1} & \sum_{i=1}^{L} T_{Mi,2} & \cdots & \sum_{i=1}^{L} T_{Mi,M}
\end{bmatrix}^{-1} \begin{bmatrix}
\sum_{i=1}^{L} R_{1i,1} \\
\sum_{i=1}^{L} R_{1i,2} \\
\vdots \\
\sum_{i=1}^{L} R_{1i,M}
\end{bmatrix}
\]
where,

\[
T_{\text{ch}} = \begin{bmatrix}
R_{x_{\text{in}}x_{\text{in}}}(0) & R_{x_{\text{in}}x_{\text{in}}}(1) & \cdots & R_{x_{\text{in}}x_{\text{in}}}(J-1) \\
R_{x_{\text{in}}x_{\text{in}}}(1) & R_{x_{\text{in}}x_{\text{in}}}(0) & \cdots & R_{x_{\text{in}}x_{\text{in}}}(J-2) \\
\vdots & \vdots & \ddots & \vdots \\
R_{x_{\text{in}}x_{\text{in}}}(J-1) & R_{x_{\text{in}}x_{\text{in}}}(J-2) & \cdots & R_{x_{\text{in}}x_{\text{in}}}(0)
\end{bmatrix}
\]

Here, J is the filter length, T is the Toeplitz matrix, R is the cross correlation.

4. Analysis

4.1. Analysis condition

The specification and properties of analytical model are as follows.

| Parameter                  | Value | Unit   |
|----------------------------|-------|--------|
| Length                     | 1.0   | m      |
| Width                      | 8.6   | mm     |
| Thickness                  | 8.6   | mm     |
| Density                    | 7580  | Kg/m³  |
| Elastic modulus            | 2.07e11 | Pa    |

The length of equipment was assumed to be 0.4L. And the input force was assumed to be white noise at 0.2f on source. And location of spring on receiver \(x_{1}\), \(x_{2}\) were assumed to be 0.1L, 0.7L, respectively. The time interval for analysis was 0.1 seconds, and the record length was 60 seconds. And Wiener filter length is 2000.

4.2. Results

Figure 4 shows the displacement before and after at 0.3L on receiver. When the control is performed, it can be confirmed that the displacement is reduced compared the displacement before control. And Figure 5 shows the displacement after Fourier transform of result of Figure 4. It can be confirmed that the largest control effect occurs in the frequency band where the displacement is the largest.

![Figure 4](image1.png)  
**Figure 4.** Displacement with/without control (time domain)

![Figure 5](image2.png)  
**Figure 5.** Displacement with/without control (frequency domain)
5. Conclusion
The control analysis was carried out to isolate the vibration transmitted the equipment connected with flexible beam supported both ends. To design the controller, the SIMO Wiener filter was used.

As a result after analysis, it was confirmed that the displacement was reduced and the largest control effect is occurred in the frequency band where the displacement is the largest. Later, it will test on actual model by using the analytical results.

References
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