Multi-objective Reliability-based Design Optimization for Climbing drive System of Stair-climbing Wheelchair

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Abstract. The 2K-H planetary gear reducer is an important driving part in the climbing system of the stair-climbing wheelchair. Based on the conventional mechanical design parameters, the mathematical model of the reliability optimization of the planetary gear reducer is established by using the reliability theory and the computer optimization method. Seeking optimal design parameters by the optimization algorithm of Matlab software. On the basis of satisfying the reliability constraint, the purpose of reducing the volume of the reducer and increasing the transmission efficiency is realized. It provides a theoretical basis for further optimization of structural design.

1. Introduction

The stairs and steps in life are inconvenient for the disabled and the elderly to travel. Under this premise, in order to improve the elderly and disabled travel convenience, the development of stair climbing function wheelchair has great significance. Star wheel stair climbing wheelchair has been widely concerned in the research of stair climbing wheelchair because of its advantages, such as convenient driving, small turning radius and little damage to steps. [1] The driving system of the staircase wheelchair is an important part of the device. Based on the characteristics of small planetary gears, small mass, compact structure, large carrying capacity, high transmission efficiency and arbitrary angle of stopping or turning, we designed a 2K-H planetary gear reducer as the climbing transmission system [2].

The traditional design method of planetary gear reducer is: the designer carries on the design research based on the data, finding the reference book and combining the design experience with the traditional design formula. The reducer designed by this traditional design method can fulfill the requirements even though the strength can fulfill the requirements, but the reliability requirements are not taken into account. Moreover, the structural size margin is often too large, lead to excessive waste material. Therefore, considering the requirements of reliability, the design of structural size optimization has a certain practical significance [3].

Reliability-based design and optimization are all modern design methods. The purpose of reliability-based design is to improve product reliability, and it is an important means of product quality assurance and safety improvement. The mathematical model based on the concept of reliability design can effectively improve the safety of the product. The optimization design is based on the
mathematical optimization method, but the mathematical model established does not necessarily take into account the requirements of the reliability, so it can not accurately reflect the actual situation. In order to make a more reasonable design plan of the climbing transmission system, this paper designs it by combining the reliability design method with the optimization design [4,5].

2. Reliability-based design optimization formulation

The schematic diagram of the designed 2K-H planetary gear reducer is shown in Figure 1. The mechanical relation of each part in the transmission system of the double stage 2K-H gear reducer is very complicated, and all the variables are dynamic. Therefore, the design assumes that the input load obeys the normal distribution function, The other parameters are treated as static quantity [6]. These hypothesis fulfill the design requirement of the planetary reducer, so the reliability optimization design of the reducer is carried out on the basis of this hypothesis.

According to the requirements of reference [7], The reliability of a gear requiring high reliability should be greater than or equal to 0.99. Because of the special application range of the staircase wheelchair, it requires high safety and high reliability. Therefore, it is required that the gear strength reliability of the transmission system is greater than or equal to 0.99. In this condition, the drive system is required to have the smallest volume and the highest transmission efficiency. The main parameters affecting the drive system's volume and transmission efficiency are the modulus m, the number of teeth z, the tooth width coefficient \( \varphi_d \). In the design of the transmission system, the two gear drives take the same modulus, so the design variables are as follows:

\[
\begin{bmatrix}
z_1 \\
z_2 \\
z_3 \\
z_4 \\
m
\end{bmatrix}
= [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T
\]

The minimum volume of the gear drive system is the objective function, and the volume expression is as follows:

\[
V = \varphi_d B m (n_p z_1 + z_2 + n_p z_3 + z_4)
\]

Where B is Gear thickness; \( z_1 \) is tooth number of gear 1; \( z_2 \) is tooth number of gear 2; \( z_3 \) is tooth number of gear 3; \( z_4 \) is tooth number of gear 4; \( m \) is Gear modulus; \( n_p \) is The number of planetary gear(\( n_p = 3 \)); \( \varphi_d \) is Coefficient of tooth width.

Because the planetary gear reducer is used for large torque transmission, and the DC motor torque margin for climbing the stairs is very small. Therefore, improving the transmission efficiency and saving energy is another pursuit in the design, so the improvement of transmission efficiency should be considered as a design optimization goal. According to the research result of document [8], The transmission efficiency of planetary gear mechanism depends on the transmission ratio, and the transmission ratio depends on the number of teeth of planetary mechanism, so its efficiency value can be obtained from the following formula:

\[
\eta_e = \frac{1}{1 + (1 - \eta_e^s)}
\]

Where \( \eta_e \) is the efficiency of planetary gear reducer; \( i_x \) is Drive ratio of reducer; \( \eta_e^s \) - The single transmission efficiency of gears; \( i_x = \frac{z_2}{z_1 z_3 z_4} \) [9]; \( \eta_e^s \approx 0.97 \).

The design variables were substituted into the formula (1), the formula (2) obtained that:

\[
F_1(x) = x_6 x_5 (3 x_1 + x_2 + 3 x_3 + x_4)
\]

\[
F_2(x) = \frac{x_1 x_4}{x_1 x_4 - x_2 x_3 - 1}
\]

In order to optimize the calculation, considering the different importance of each objective function in design, weights can be used to form an integrated objective function [10]:

\[
F(x) = aF_1(x) \times \beta / F_2(x)
\]
According to the importance of the selected target in the design, the size of the weighted values of \( \alpha \) and \( \beta \) is chosen appropriately, and then the multi-objective optimization design is realized.

![2K-H planetary gear reducer mechanism diagram](image)

**Figure 1 2K-H planetary gear reducer mechanism diagram**

### 3. Constraints

#### 3.1 Reliability constraints of contact fatigue strength of tooth surface

In the process of the design of the reducer, the same modulus is adopted for the two stage gear, and the stress load of gear 3 and gear 4 is slightly larger than that of gear 1 and gear 2. Therefore, when the strength reliability is constrained, the strength reliability of the gear 3 and gear 4 meets the requirements, which can satisfy the design requirements. The mathematical model of normal distribution is easy to deal with, and it is more safe than other probability distributions. Therefore, the stress and strength of the gear can be assumed to be normal distribution. The expression of reliability is:

\[
\zeta_H = \frac{\sigma_{H\text{lim}} - \bar{\sigma}_H}{\sqrt{s^2_{\sigma_{H\text{lim}}} + s^2_{\sigma_H}}} \tag{6}
\]

Where \( \sigma_{H\text{lim}} \) is the average contact fatigue limit; \( \bar{\sigma}_H \) is the average contact stress; \( s_{\sigma_{H\text{lim}}} \) is the standard deviation of contact fatigue limit; \( s_{\sigma_H} \) is the standard deviation of contact stress.

1) The mean and standard deviation of the contact stresses

\[
\sigma_H = Z_H Z_E Z_\varepsilon Z_\beta \left( \frac{K_F K_A K_V K_{H\beta} K_{Ha}}{db} \right)^{\frac{1}{2}} \tag{7}
\]

Where \( Z_H \) is node regional coefficient; \( Z_E \) is elastic influence coefficient; \( Z_\varepsilon \) is contact ratio factor; \( Z_\beta \) is helix angle coefficient; \( K_A \) is working condition coefficient; \( K_V \) is dynamic load coefficient; \( K_{H\beta} \) is coefficient of calculation of load distribution between contact strength teeth; \( K_{Ha} \) is coefficient of contact strength to calculate the distribution of tooth load; \( u \) is Tooth number ratio of gear; \( F_t \) is tangential force. By the \( T = \frac{9.55 \times 10^6 P}{n} \), \( F_t = \frac{T}{d} = \frac{2 \times 9.55 \times 10^6 P}{dn} \), \( d = mz; \beta = \phi_d d \).

Fetching coefficient \( K=1 \), the contact stress of the deformed tooth surface is as follows:

\[
\sigma_H = Z_H Z_E Z_\varepsilon Z_{\beta} \left( \frac{2 \times 9.55 \times 10^6 P}{m z \phi_d n} \cdot \frac{u^{\pm 1}}{u} K_A K_V K_{H\beta} K_{Ha} \right)^{\frac{1}{2}} \tag{8}
\]

The mean, coefficient of variation and standard deviation of contact stress are as follows:

\[
\bar{\sigma}_H = Z_H Z_E Z_\varepsilon Z_{\beta} \left( \frac{2 \times 9.55 \times 10^6 P}{m z \phi_d n} \cdot \frac{u^{\pm 1}}{u} K_A k_{K_V} K_{H\beta} K_{Ha} \right)^{\frac{1}{2}} \tag{9}
\]

The \( \bar{\sigma}_H \) and \( Z_E \) in the formula are the corresponding average values.

\[
V_{\sigma_H} = \sqrt{V_{Z_E}^2 + \frac{1}{4} \left( V_{K_F}^2 + V_{K_A}^2 + V_{K_V}^2 + V_{K_{H\beta}}^2 + V_{K_{Ha}}^2 + V_{K_V} V_{K_{H\beta}} + V_{K_{Ha}} V_{K_{H\beta}} \right)} \tag{10}
\]
The $\sigma_{H}$, $V_{Zg}$ in the formula is the coefficient of variation of the corresponding value; and $s_{\sigma_{H}} = V_{\sigma_{H}}\bar{\sigma}_{H}$. 

2) The mean and standard deviation of contact strength

According to international GB3480-97, the contact fatigue limit of gear is as follows:

$$\sigma_{flimj} = \sigma_{flimj} Z_{n} Z_{f} Z_{R} Z_{x} Z_{y}$$

Where $Z_{n}$ is life coefficient; $Z_{f}$ is roughness coefficient; $Z_{v}$ is velocity coefficient; $Z_{x}$ is calculation coefficient; $Z_{R}$ is life coefficient; $\sigma_{flimj}$ is contact fatigue limit of gear.

The mean, coefficient of variation and standard deviation of the contact fatigue limit are as follows:

$$\bar{\sigma}_{flimj} = \bar{\sigma}_{flimj} Z_{n} Z_{f} Z_{R} Z_{x} Z_{y}$$

$$s_{\sigma_{flimj}} = s_{\sigma_{flimj}} Z_{n} Z_{f} Z_{R} Z_{x} Z_{y}$$

If the reliability of the design is $R_{H}$ and the corresponding reliability index $Z_{R_{H}}$, the value is checked by the normal distribution table. Then the constraint function is:

$$G_{s}(X) = Z_{R_{H}} - \frac{\bar{\sigma}_{flimj} - \bar{\sigma}_{H}}{s_{\sigma_{flimj}} + s_{\sigma_{H}}} \leq 0$$

### 3.2 Reliability constraints of tooth root bending fatigue strength

The bending limit and bending stress of the gear are all normal distribution, and the reliability index is derived from the joint equation:

$$z_{F} = \frac{\bar{\sigma}_{flimj} - \bar{\sigma}_{F}}{\sqrt{s_{\sigma_{flimj}}^{2} + s_{\sigma_{F}}^{2}}} \quad (i = 1, 2)$$

Where $\bar{\sigma}_{flimj}$ is mean value of bending fatigue limit; $\bar{\sigma}_{F}$ is mean value of bending stress; $s_{\sigma_{flimj}}$ is standard deviation of bending fatigue limit; $s_{\sigma_{F}}$ is standard deviation of bending stress.

1) The mean and standard deviation of the bending stress of the root of the tooth. The tooth root fracture is the limit state in the calculation of the bending fatigue strength of the gear. The bending stress is:

$$\sigma_{F} = \frac{K_{F} F_{t}}{m z^{2} \phi_{d} n} Y_{F} Y_{sa} Y_{v} K_{A} K_{V} K_{p} K_{F} K_{Fa}$$

Where $F_{t}$ is tangential force on standard pitch circle of end face; $b$ is tooth width; $K_{A}$ is use coefficient; $K_{V}$ is velocity load coefficient; $K_{p}$ is tooth load distribution coefficient calculated by bending strength; $K_{Fa}$ is the load distribution coefficient between the teeth calculated by the bending strength; $Y_{sa}$ is the stress correction coefficient of the load acting on the top of the tooth; $Y_{v}$ is the coincidence coefficient of the calculation of the bending strength; $Y_{\beta}$ is the helical angle coefficient of the calculation of the bending strength.

When the coefficient $K=1$ is taken, the distortion bending stress of the root of the tooth is as follows:

$$\sigma_{F} = \frac{2x9.55x10^{6}xP}{m z^{2} \phi_{d} n} Y_{F} Y_{sa} Y_{v} K_{A} K_{V} K_{p} K_{F} K_{Fa}$$

The mean, variation coefficient and standard deviation of the bending stress are as follows:

$$\bar{\sigma}_{F} = \frac{2x9.55x10^{6}xP}{m z^{2} \phi_{d} n} Y_{F} Y_{sa} Y_{v} K_{A} K_{V} K_{p} K_{F} K_{Fa}$$

The $\bar{\sigma}_{F}$ and $K_{F}$ in the formula are the corresponding average values.
\[ V_{\sigma_F} = \sqrt{V_p^2 + V_{K_v}^2 + V_{K_{Fa}}^2 + V_{K_{F\beta}}^2 + V_{K_v} V_{K_{Fa}} + V_{K_{F\beta}}} \]  \hspace{1cm} (19)

The \( V_{\sigma_h}, V_{Z_k}^2 \) and so on are the coefficient of variation of the corresponding values. And \( s_{\sigma_F} = V_{\sigma_F} \sigma_F \).

2) The mean and standard deviation of the bending fatigue strength of the root of the teeth

According to the design requirements of planetary gear and the tooth number is selected in the range of 17~80, the bending fatigue limit of the tooth root is:

\[ \sigma_{\text{lim}} = \sigma_{\text{lim}} Y_{ST} V_{NT} Y_{\text{rel}} Y_{R\text{rel}} Y_X \]  \hspace{1cm} (20)

Where \( Y_{ST} \) is stress correction coefficient of experimental gear; \( Y_{NT} \) is the life coefficient of the calculation of the bending strength; \( Y_{\text{rel}} \) is relative root rounded angle sensitivity coefficient; \( Y_{R\text{rel}} \) is the surface condition coefficient of the relative root rounded corners; \( Y_X \) is calculation coefficient; \( \sigma_{\text{lim}} \) is the bending fatigue limit of a gear.

The mean, coefficient of variation and standard deviation of the bending fatigue limit are:

\[ \bar{\sigma}_{\text{lim}} = \bar{\sigma}_{\text{lim}} Y_{ST} \bar{V}_{NT} \bar{Y}_{\text{rel}} \bar{Y}_{R\text{rel}} \bar{Y}_X \]  \hspace{1cm} (21)

The following formulas are derived from the reference data: \( V_{\sigma_{\text{lim}}} = \sqrt{V_{\text{lim}}^2 + V_{Y_{NT}}^2 \sigma_{\text{lim}} = V_{\sigma_{\text{lim}}} \bar{\sigma}_{\text{lim}} \).

The reliability of the bending fatigue strength required by the design is \( R_F \), and the corresponding reliability index is \( Z_{R_F} \). Then there is a constraint function: \( Z_{R_F} - Z_F \leq 0 \). That is:

\[ G_2(X) = Z_{R_F} - \frac{\bar{\sigma}_{\text{lim}} - \bar{\sigma}_F}{\sqrt{\bar{\sigma}_{\text{lim}}^2 + s_{\sigma_{\text{lim}}}} \leq 0} \]  \hspace{1cm} (22)

### 3.3 Modulus constraint

The modulus of the transmission gear is generally selected within the range of 2 ~ 8mm, and there is a constraint function:

\[ G_3(X) = 2 - x_5 \leq 0 \]  \hspace{1cm} (23)
\[ G_4(X) = x_5 - 8 \leq 0 \]  \hspace{1cm} (24)

### 3.4 Tooth number constraint

In the transmission form, the closed type soft tooth gear transmission is adopted. Considering the radial size limit of the transmission device, the tooth number is selected in the range of 17~80, and the constraint function is:

\[ G_5(X) = 17 - x_1 \leq 0 \]  \hspace{1cm} (25)
\[ G_6(X) = x_1 - 80 \leq 0 \]  \hspace{1cm} (26)
\[ G_7(X) = 17 - x_2 \leq 0 \]  \hspace{1cm} (27)
\[ G_8(X) = x_2 - 80 \leq 0 \]  \hspace{1cm} (28)
\[ G_9(X) = 17 - x_3 \leq 0 \]  \hspace{1cm} (29)
\[ G_{10}(X) = x_3 - 80 \leq 0 \]  \hspace{1cm} (30)
\[ G_{11}(X) = 17 - x_4 \leq 0 \]  \hspace{1cm} (31)
\[ G_{12}(X) = x_4 - 80 \leq 0 \]  \hspace{1cm} (32)

### 3.5 Tooth width coefficient constraint

According to the design requirements of planetary gear: \( 0.3 \leq \varphi_d \leq 1.4 \), then there is a constraint function:

\[ G_{13}(X) = 0.3 - x_6 \leq 0 \]  \hspace{1cm} (33)
\[ G_{14}(X) = x_6 - 1.4 \leq 0 \]  \hspace{1cm} (34)

### 3.6 Concentric condition constraint [11]
\[ G_{15}(X) = (x_2 - x_3) - (x_1 - x_4) = 0 \] (35)

3.7 Assembly condition constraint
\[ G_{16}(X) = \left( x_2 + x_4 \right) / n_p - \text{INT}(\text{integer}) = 0 \] (36)

In summary, the mathematical model can be expressed as:
\[
X = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T;
\]
\[
\min F(X) = \alpha F_1(x) + \beta / F_2(x);
\]
\[
s.t. G_i(X) \leq 0 \quad (i = 1, 2, \ldots, 16).
\]

4. Calculation example and result discussion

4.1 Calculation examples
As shown in Figure 1, the driving device for the climbing and climbing system of the stairs is 2K-H planetary gear reducer. After climbing power calculation, the transmission power is 76.6W, the speed of gear is 4r/min, according to the selected motor parameters, the transmission ratio is i=11. For the purpose of improving the stability, anti-shock and vibration capability of planetary gear system, make \( n_p = 3 \). Gear materials are used for surface hardening, the hardness of the tooth surface is 48 ~ 50HRC, gear accuracy rating is 8 levels.

4.2. Results and discussion
The MATLAB optimization toolbox contains a series of optimization algorithm functions, which can solve the linear and nonlinear minimum values conveniently and quickly, and solve the engineering problems of nonlinear systems, such as equation solving, curve fitting, two time programming. The reliability design optimization of the planetary gear reducer includes multiple constraints and multiple nonlinear constraints, which belong to nonlinear constrained optimization design problems. Therefore, the Fmincon function is used to optimize the design. The method of function call can be referred to [12]. The optimization results of Multi-objective reliability design for planetary gear reducer are shown in Table 1.

| Table 1 Comparison of the optimization results with the original design results |
|-------------------------------|----------------|----------------|
| \( z_1 \) | 23 | 20 |
| \( z_2 \) | 70 | 63 |
| \( z_3 \) | 20 | 17 |
| \( z_4 \) | 67 | 60 |
| \( m \) | 2 | 2 |
| \( \varphi_d \) | 0.3 | 0.3 |
| \( V \) | 159.6 | 140.4 |
| \( \eta_e \) | 0.77 | 0.78 |

From the calculation results of Table 1, compared with the conventional design scheme, the reliability optimization design can reduce the overall volume of the reducer by 12.03% while the reliability is 0.99, while the transmission efficiency is increased by 1.28%.

5. Conclusion
In this paper, reliability design optimization method is applied to the reliability design optimization of climbing wheelchair climbing drive system, by using the mathematical optimization of the Fmincon function in the Optimization Toolbox in MATLAB software, in the condition of ensuring the reliability of climbing drive system of climbing wheelchair is more than 0.99, It reduces the weight and volume of the whole reducer to a large extent, thus reducing the cost of production and the weight of the whole machine, and improving the efficiency of the drive. The economic benefit and safety guarantee of the
staircase wheelchair device are further promoted.

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