Competition and dual users in complex contagion processes

Byungjoon Min1,2 & Maxi San Miguel2

We study the competition of two spreading entities, for example innovations, in complex contagion processes in complex networks. We develop an analytical framework and examine the role of dual users, i.e. agents using both technologies. Searching for the spreading transition of the new innovation and the extinction transition of a preexisting one, we identify different phases depending on network mean degree, prevalence of preexisting technology, and thresholds of the contagion process. Competition with the preexisting technology effectively suppresses the spread of the new innovation, but it also allows for phases of coexistence. The existence of dual users largely modifies the transient dynamics creating new phases that promote the spread of a new innovation and extinction of a preexisting one. It enables the global spread of the new innovation even if the old one has the first-mover advantage.

The spread of rumors, fads, innovations or new technologies in a global scale occurs more frequently than before due to the intensive use of new information and communications technologies1,2. In order to model and understand the mechanisms of spreading phenomena3–11, quantitative approaches to contagion processes have become important. A pioneer work about diffusion of collective behavior4 and its variant adapted for the spread on complex networks5 proposed a model of “complex contagion”12–15 incorporating a threshold mechanism. This is often called the threshold model in which each agent requires a sufficiently large fraction of neighbors that have already adopted a spreading new technology to adopt it. Therefore, neighbors affect collectively (group interaction) the probability of adoption rather than independently as in the “simple contagion” of well known epidemic models16.

Research on spreading by complex contagion sheds light on fundamental aspects of collective phenomena, nonlinearity in diffusion, cascading dynamics, and first order phase transitions in complex networks. It has focused on aspects such as seed effect17, clustering18, modularity6,19–21, multiplexity22,23, multi-stage contagion24, and interaction with simple contagion19 or coordination processes25.

Models of complex contagion typically assume that there exists no preexisting technology that competes with a new one, so that the new technology spreads in a system of susceptible agents2,5,6. However, it is common that a preexisting technology is used by some fraction of the agents in the system. Therefore, the spread of a new competing technology involves the decline, coexistence or extinction of the actual predominant technology27–29.

Competition between ideas or products30–33 is an essential ingredient in the modeling of contagion process. While there exist several studies of simple contagion with cooperative epidemics34–36, competing epidemics37,38, and interacting epidemics on multi-layer networks39–41, complex contagion with interactions among multiple spreading entities has not been, so far, addressed in detail.

A most clear example of competition in technology spreading is that of new social networking services in top of a preexisting technology27,28. A new service competes with preexisting ones in the market share. The initial technology can take first-mover advantage such as preemption, technological leadership, and switching cost to other ones, and thus the spread of the new technology is effectively hindered. Therefore, one might expect that when an existing product occupies a dominant position in the market, the global spread of a newly launched product of similar characteristics is hardly successful. However, it is not rarely observed that a late technology successfully spreads into population, such as the success of new online social networking services28. These observations call for an understanding of the competition dynamics in complex contagion processes. In this paper we address this question considering the spreading of a new technology competing with a preexistent one. An important fact in this context is that some agents use multiple technologies at the same time. For instance, it is reported that 52% of people use multiple different social networking services because different neighbors use different software29. We thus explore the role of the agents using at the same time the old and new technologies. The consideration of these “dual users” is also motivated by analogy with bilingual agents on problems of language.

1Department of Physics, Chungbuk National University, Cheongju, Chungbuk, 28644, Korea. 2IFISC, Instituto de Física Interdisciplinar y Sistemas Complejos (CSIC-UIB), Campus Universitat Illes Balears, E-07122, Palma de Mallorca, Spain. Correspondence and requests for materials should be addressed to B.M. (email: bmin@chungbuk.ac.kr) or M.S.M. (email: maxi@ifisc.uib-csic.es)
In all three models, changes to state number of neighbors of \( A \) in the transition from an intermediate state \( AB \) to state \( B \) is in state \( A \) exclusive model, \( A \) and \( B \) compete and additional requirements exist for a user of \( A \) to change its state adopting \( B \). To be specific, if \( k_{iB}^j/k^i \geq \theta_B \), node \( i \) in state \( S \) adopts \( B \), as the independent model. However, if node \( i \) is in state \( A \), it changes to state \( B \) only if the fraction of \( B \) neighbors of \( i \) is larger than \( \theta_B \) and at the same time the fraction of \( A \) neighbors is smaller than \( \theta_A \) \( k_{iB}^j/k^i \geq \theta_B \) and \( k_{iA}^j/k^i < \theta_A \). (iii) In the compatible model, we introduce dual users in an intermediate state \( AB \) in the transition from \( A \) to \( B \) modeling agents that use both technologies \( A \) and \( B \). Nodes in the susceptible state \( S \) change to state \( B \) as in the independent or exclusive model. However, a node \( i \) in state \( A \) becomes a dual user \( AB \) by adopting \( B \) when \( (k_{iB}^j + k_{iAB}^j)/k^i \geq \theta_B \). A dual user \( i \) (\( AB \) state) changes into \( B \) state by discarding \( A \), when the fraction of neighbors in \( A \) is less than the threshold for \( A \) \( (k_{iA}^j/k^i < \theta_A) \). In all three models, dynamics proceeds from the initial state until a final frozen configuration is reached which depends on parameters such as threshold parameters \( \theta_A, \theta_B \), initial fraction \( \rho_A, \rho_B \), and network structure as well.

Theory

The general question addressed is about the conditions for the spreading of \( B \), but in particular we ask about the possible coexistence of \( A \) and \( B \) and the role of dual users in the spreading of \( B \). To compute the final fraction of nodes in each state we provide a general theory of complex contagion in which each node can be in \( \{S, A, B, AB\} \) independent, \{Fig. 1(a)\}, exclusive \{Fig. 1(b)\}, and compatible \{Fig. 1(c)\} models. (i) In the independent model, the preexisting technology \( A \) and the new technology \( B \) are independent and do not interact with each other. If a fraction of neighbors of node \( i \) in state \( B \) is larger than a threshold \( \theta_B (k_{iB}^j/k^i \geq \theta_B) \) where \( k^i \) is the number of neighbors of \( i \) in \( j \) state and \( k^j \) is the total number of neighbors of node \( i \) (degree of node \( i \)), node \( i \) changes to state \( B \) state regardless of the initial state of node \( i \), either susceptible \( S \) or \( A \). Therefore, the independent model is essentially the same as the original threshold model for the spreading of a single technology \( \theta \). (ii) In the exclusive model, \( A \) and \( B \) compete and additional requirements exist for a user of \( A \) to change its state adopting \( B \). To be specific, if \( k_{iB}^j/k^i \geq \theta_B \), node \( i \) in state \( S \) adopts \( B \), as the independent model. However, if node \( i \) is in state \( A \), it changes to state \( B \) only if the fraction of \( B \) neighbors of \( i \) is larger than \( \theta_B \) and at the same time the fraction of \( A \) neighbors is smaller than \( \theta_A \) \( k_{iA}^j/k^i \geq \theta_B \) and \( k_{iA}^j/k^i < \theta_A \). (iii) In the compatible model, we introduce dual users in an intermediate state \( AB \) in the transition from \( A \) to \( B \) modeling agents that use both technologies \( A \) and \( B \). Nodes in the susceptible state \( S \) change to state \( B \) as in the independent or exclusive model. However, a node \( i \) in state \( A \) becomes a dual user \( AB \) by adopting \( B \) when \( (k_{iB}^j + k_{iAB}^j)/k^i \geq \theta_B \). A dual user \( i \) (\( AB \) state) changes into \( B \) state by discarding \( A \), when the fraction of neighbors in \( A \) is less than the threshold for \( A \) \( (k_{iA}^j/k^i < \theta_A) \). In all three models, dynamics proceeds from the initial state until a final frozen configuration is reached which depends on parameters such as threshold parameters \( \theta_A, \theta_B \), initial fraction \( \rho_A, \rho_B \), and network structure as well.

![Diagram of models](image-url)

**Figure 1.** Schematic illustration of (a) independent, (b) exclusive, and (c) compatible models. Open circle, triangle, square, and filled circle respectively represent susceptible \( S \), \( A \), \( B \), and dual user \( AB \).
Assuming locally-tree like structures, we can compute k^P k(\theta, \rho) = \kappa \rho 0.22, 0.16, 0.19, and 0.22. The local peak of NOI accurately indicates the structure and produces very good agreement with numerical simulations for sparse tree-like graphs.

Next, we define the excess degree of a node for a given state x as \kappa_x. Considering g(x, y, z) where a node is connected to a node in state z, we can define the vector of excess degree for each state as \kappa_z = \{k_1, k_2, ..., k_z = 1, ..., k_n\}. Assuming locally-tree like structures, we can compute g(x, y, z) using the self-consistency equations:

\[
q_x(y) = \sum_{w=1}^{n} [\rho_w g(w, x, y) - \rho_x g(x, w, y)].
\]

(3)

where \eta(x, y, z) is a threshold function for the transition from state x to y for a node connected to a node in state z and \eta(\kappa_z) = \sum_{n=1}^{\kappa_z} q_n(x)^n \eta(x, y, z),

(4)

where \eta(x, y, z) is a threshold function for the transition from state x to y for a node connected to a node in state z and \eta(\kappa_z) = \sum_{n=1}^{\kappa_z} q_n(x)^n \eta(x, y, z),

(4)

The transition between different phases can be accurately identified by a local maximum in the number of iteration (NOI) of the recursion equations (Eq. 4), reflecting critical slowing down at the transition point. As the initial fraction of nodes in state A, \rho_A, decreases we can observe two transitions in R_A and R_B: a transition to extinction of A and a transition to spreading of B. For the compatible model, the transitions observed for R_A and R_B coincide when \theta = 0.22 (single peak of NOI), but they appear at different values of \rho_B when \theta < 0.2.
allowing for a coexistence phase $A + B$. Note also that these transitions are discontinuous for $\theta > 0.2$ ($\theta = 0.22$), but become continuous for $\theta < 0.2$. For the exclusive model, a coexistence phase ($A + B$) appears consistently for all $\theta_B$. The spreading transition for $B$ is also discontinuous here for $\theta_B > 0.2$ and continuous for $\theta_B < 0.2$.

Proceeding along these lines we can construct from our analytical solution phase diagrams as a function of different parameters: Fig. 3 shows a plot density of $R_B$ for the exclusive and compatible models in the plane $\theta_B - \rho_A$ with different transition lines (see Methods for their calculation). Varying $\theta_B$, the transition lines in the phase diagram are discontinuous rather than a continuous curve because the threshold of our model is based on the ratio to the degree. Specifically, we can observe sharp transitions at the point where $\theta_B = n/k$ with $n$ being an integer. Note that such discontinuity is also found in the prototypical threshold model. Phase $A$ always exists for $\theta_B > 1/4$ larger than the critical value of the independent model ($\theta_B = 1/4$) since $\theta_A$ is less than $1/4$. For $\theta_B < 1/4$ phase $A$ occurs also in both models when $\rho_A$ is above the critical value for spreading of $B$ in the compatible model. On the other hand, also for $\theta_B < 1/4$, phase $B$ occurs in both models when $\rho_B$ is below the transition line for spreading of $B$ in the exclusive model. In between these two regimes we find three domains of parameters in which results are different for the two models. These domains give evidence of the important role of dual users $AB$ in favoring the spread of $B$. In region I we find phase $A$ in the exclusive model and phase $B$ in the compatible model: This is an extreme effect of dual users leading to the extinction of $A$ and spreading of $B$. In region II we find phase $A + B$ in the exclusive model and phase $B$ in the compatible model: Dual users destroy coexistence in favor of full spreading of $B$. In region III we find phase $A$ in the exclusive model and phase $A + B$ in the compatible model: Dual users spread $B$ allowing for coexistence. In this coexistence regime, there is a large proportion $\rho_{AB}$ of dual users in the final state, while in regions I and II dual users play an important transient role in the spreading of $B$, but essentially disappear in the final state. In summary, dual users behave as an intermediate step in the transition from $A$ to $B$, mitigating the first-mover advantages of $A$.

We next look at network effects that confirm the important role of dual users in the spreading of $B$ (Fig. 4). We consider a point in region II of Fig. 3 and examine the effect of varying the average degree $\langle k \rangle$ of the ER network.
The compatible model shows three transitions when increasing \( \langle k \rangle \). Since networks do not form a globally connected component below the percolation threshold \( \langle k \rangle \approx 1 \), spreading of \( B \) does not occur when \( \langle k \rangle < 1 \). A first continuous spreading transition for \( B \) leads from phase \( A \) to the coexistence phase \( A + B \). A second continuous extinction transition for \( A \) occurs around \( \langle k \rangle \approx 4 \) leading from phase \( A + B \) to phase \( B \). A final transition from \( B \) to \( A \) occurs for the high connectivity regime \( \langle k \rangle > 10 \) where initial fraction of \( B \) does not exceed the threshold \( \theta_B \), so that the spread of \( B \) disappears abruptly. For the exclusive model, phase \( B \) between \( A + B \) and \( A \) no longer exists and \( R_B \) > 0.5 for all values of \( \langle k \rangle \), indicating a strong preemptive advantage of \( A \). Comparing the two models we find again regions I and II with the same characteristics than in Fig. 3 where dual users play a dominant role in the spreading of \( B \). Again this transient dynamics affects with only a significant number of dual users in the final state \( R_{AB} \) in the coexistence phase \( A + B \) of the compatible model.

Dependence of our results on \( \theta_B \) is shown in Fig. 4(c,d) where results for \( R_B \) are shown in the \( \langle k \rangle - \theta_B \) plane. The range of parameters in which \( B \) spreads in the exclusive model is smaller than in the independent model due to the hindering effect of a preexisting technology \( A \), but this effect is smaller for the compatible model. More importantly, when \( B \) spreads, the final fraction \( R_B \) is much larger in the compatible model than in the exclusive model. Note that phase \( B \) in Fig. 4(a) is within the \( R_B \) transition line of the compatible model [Fig. 4(c)], while phase \( A + B \) in Fig. 4(b) is within the \( R_B \) transition line of the exclusive model [Fig. 4(d)]. Results for the full range of values of \( \langle k \rangle \), \( \rho_B \), \( \theta_B \) and \( \theta_A \) are further shown in the SI. We find that our main conclusions are consistently valid for a broad range of parameters.

Discussion

In summary, we have discussed models of complex contagion for the spreading of a new innovation \( B \) competing with an established one \( A \). We obtain perfect agreement between theory and simulation. Competition, as compared with the independent model, is shown to have a hindering effect on the spreading of the new technology. However, competition can create a gap between the extinction and spreading transitions that allows for a coexistence phase. It can also change the nature of the spreading transition from first to second order. Moreover, the existence of dual users is found to be a key mechanism to promote the spreading of \( B \) in a system dominated by \( A \), allowing to overcome the hindering effect of tightly clustered of agents using \( A \). Dual users create new phases in which they either lead to extinction of \( A \) in favor of spreading of \( B \), or destroy coexistence in favor of spreading of \( B \) or create coexistence by \( B \) spreading. The role of dual users provides a plausible explanation to reconcile two seemingly contradictory factors: the first-mover advantage meaning that the prevalence of \( A \) potentially blocks the spread of \( B \), and the success of late-mover frequently observed in reality. Practically, our study suggests a strategic way of spreading a new technology in a competitive environment by offering it as a good compatible alternative instead of promoting changing to the new technology. Further studies may be needed to examine the role of dual-users in simple contagion models, opinion dynamics, and coevolutionary dynamics, to name a few.

Methods

Theory of compatible model. We implement the developed general theory of complex contagion into the compatible model. The final fraction of state \( A \), \( B \), \( AB \), and \( S \) are defined as \( R_A \), \( R_B \), \( R_{AB} \), and \( R_S \), respectively. We further define the initial fraction of \( A \), \( B \), \( AB \), and \( S \) as \( \rho_A \), \( \rho_B \), \( \rho_{AB} \) and \( \rho_S \). These variables are normalized as \( R_A + R_B + R_{AB} + R_S = 1 \) and \( \rho_A + \rho_B + \rho_{AB} + \rho_S = 1 \). Note that \( \rho_{AB} = 0 \) since the coexisting state cannot exist initially. Then the main purpose of the theory is to calculate the final fraction of nodes in each state for a given initial condition. Given the initial fraction of each state, the final fraction of each state for the compatible model is given by

\[
R_A = \rho_A [1 - f(A, AB) - f(A, B)],
\]

\[
R_B = \rho_B [1 - f(B, AB) - f(B, A)],
\]

\[
R_{AB} = \rho_{AB} [1 - f(AB, AB) - f(AB, A) - f(AB, B)],
\]

\[
R_S = \rho_S [1 - f(S, AB) - f(S, A) - f(S, B)].
\]

Figure 4. Theory (line) and numerical results (symbol) for the final fraction of \( A \), \( B \), \( AB \) nodes, and NOI for (a) compatible and (b) exclusive models on ER networks \( N = 10^5 \) as a function of \( \langle k \rangle \) with \( \theta_A = 0.2 \), \( \theta_B = 0.16 \), \( \rho_A = 0.6 \), and \( \rho_B = 0.01 \). The phases \( (A, B) \) and \( (A + B) \) and transitions between them are identified by the peak of NOI. \( R_B \) for (c) compatible and (d) exclusive models in the \( \langle k \rangle - \theta_B \) plane, with \( \theta_A = 0.2 \), \( \rho_A = 0.6 \), and \( \rho_B = 0.01 \). Transition lines are indicated as in Fig. 3.
\[ R_B = \rho_B + \rho_S f(S, B) + \rho_A f(A, B), \]
\[ R_{AB} = \rho_A f(A, AB), \]

where \( f(x, y) \) represents the transition probability from state \( x \) to \( y \). Then, the transition probability \( f(x, y) \) on a locally-tree like network can be calculated as

\[ f(x, y) = \sum_k P(k) T_k(x) q_B(x) q_A(x) f_{BA}(x) q_A(x) q_B(x) \eta(x, y). \]

Here, \( k \) represents the degree of a node for a given state \( x \), and the degree distribution of underlying networks, the vector of degree for each state is \( T_k(x) = \frac{k^i_{BA} k^i_{AB} k^i_{SA}}{k^i_{SA} + k^i_{AB} + k^i_{BA}} \), and multinomial distribution of them is \( T_k(x) = \frac{k^i_{BA} k^i_{AB} k^i_{SA}}{k^i_{SA} + k^i_{AB} + k^i_{BA}} \). \( q_B(y) \) is the probability that a node, connected to a node in state \( x \), is in state \( y \). And, threshold \( \eta(x, y) \) is given by the rule of adoption for a pair of state \( (x, y) \). Specifically

\[ \eta(S, B) = H(\frac{k^i_{BA} + k^i_{AB} - \theta_S}{k}), \eta(A, B) = H(\frac{k^i_{BA} - \theta_B}{k}), \eta(A, AB) = H(\frac{k^i_{BA} - \theta_B}{k}), \eta(A, AB, S) = H(\frac{k^i_{BA} - \theta_B}{k})H(\frac{k^i_{SA} + k^i_{AB} - \theta_S}{k}). \]

A set of variables \( q_B(y) \) can be obtained by introducing the transition probability \( g(x, y, z) \) from state \( x \) to \( y \) for a node connected to a node in state \( z \):

\[ q_B(z) = \rho_B [1 - g(A, AB, z) - g(A, B, z)], \]
\[ g(x, y, z) = \sum_{z'} \frac{k^{i_{z,z'}}}{z} T_{z'} q_B(x) q_A(x) f_{BA}(x) q_A(x) q_B(x) \eta(x, y). \]

where \( k^i \) is the excess degree of state \( i \), and \( \eta(x, y, z) \) is a threshold function for the transition from state \( x \) to \( y \) for a node connected to a node in state \( z \). For our compatible model is given by \( \eta(S, B, S) = \eta(S, B, A) = H(\frac{k^i_{BA} - \theta_B}{k}), \eta(A, B, S) = H(\frac{k^i_{BA} - \theta_B}{k} - \theta_S), \eta(A, B, A) = H(\frac{k^i_{BA} - \theta_B}{k})H(\frac{k^i_{SA} + k^i_{AB} - \theta_S}{k}). \)

Theory of exclusive model. For the exclusive model, since coexisting \( AB \) state is not allowed, the final fraction of each state is given by

\[ R_A = \rho_A [1 - f(A, B)], \]
\[ R_B = \rho_B + \rho_S f(S, B) + \rho_A f(A, B). \]

Similarly, the variables \( q_A(y) \) are

\[ q_A(z) = \rho_A [1 - g(A, B, z)], \]
\[ q_B(z) = \rho_B + \rho_S g(S, B, z) + \rho_A g(A, B, z). \]

where \( z \in [S, A] \). The transition probability \( f(x, y) \) and \( g(x, y, z) \) can be computed by using the same equations for compatible model (Eqs. 8 and 12) but with different \( \eta(x, y) \) and \( \eta(x, y, z) \). To be specific, for the exclusive model, \( \eta(S, B) = H(\frac{k^i_{BA}}{k} - \theta_B), \eta(A, B) = H(\frac{k^i_{BA}}{k} - \theta_B)H(\frac{k^i_{BA}}{k}), \eta(A, S) = \eta(S, B, A) = H(\frac{k^i_{BA} + k^i_{SA} - \theta_B}{k} - \frac{k^i_{SA}}{k}), \eta(A, B, S) = H(\frac{k^i_{BA}}{k} - \theta_B). \)

Theory of independent model. For the independent model, nodes in \( A \) state do not interact with nodes in \( B \) state and thus the final fraction of nodes is given by

\[ R_A = \rho_A [1 - f(A, B)], \]
\[ R_B = \rho_B + \rho_S f(S, B) + \rho_A f(A, B). \]

And \( q_A(y) \) are

\[ q_A(z) = \rho_A [1 - g(A, B, z)]. \]
\[ q_g(z) = \rho_B + \rho_g(S, B, z) + \rho_g(A, B, z), \quad (20) \]

where \( z \in \{S, A\} \). The thresholds for the independent model are simply given by \( \eta(S, B) = \eta(A, B) = H(B - \theta_B) \)

and \( \eta(S, B, A) = \eta(A, B) = H(B - \theta_B) \).

References

1. Geroski, P. A. Models of technology diffusion. Research policy 29, 603–625 (2000).
2. Karsai, M., Íniguez, G., Karsai, M. & Kertész, J. Complex contagion process in spreading of online innovation. J. R. Soc. Interface 11, 20140694 (2014).
3. Schellings, T. C. Hockey helmets, concealed weapons, and daylight saving: A study of binary choices with externalities. J. Confl. Resolut. 17, 381 (1973).
4. Granovetter, M. Threshold models of collective behavior. Am. J. Sociol. 83, 1420 (1978).
5. Watts, D. J. A simple model of global cascades on random networks. Proc. Natl. Acad. Sci. 99, 5766–5771 (2002).
6. Centola, D. The spread of behavior in an online social network experiment. Science 329, 1194 (2010).
7. Gai, P. & Kapadia, S. Contagion in financial networks. Proc. R. Soc. A 466, 2401 (2010).
8. Dodds, P. S. & Watts, D. J. Universal behavior in a generalized model of contagion. Phys. Rev. Lett. 92, 218701 (2004).
9. Christakis, N. A. & Fowler, J. H. Social contagion theory: examining dynamics social networks and human behavior. Statist. Med. 32, 356 (2013).
10. Singh, P., Sreenivasan, S., Szymanski, B. K. & Korniss, G. Threshold-limited spreading in social networks with multiple initiators. Sci. Rep. 3, 2330 (2013).
11. Ruan, Z., Íniguez, G., Karsai, M. & Kertész, J. Kinetics of social contagion. Phys. Rev. Lett. 115, 218702 (2015).
12. Centola, D., Eguíluz, V. M. & Macy, M. W. Cascade dynamics of complex propagation. Physica A 374, 449 (2007).
13. Lugo, H. & San Miguel, M. Learning and coordinating in a multilayer network. Sci. Rep. 7, 046117 (2018).
14. Bailey, N. The mathematical theory of infectious diseases and its applications. 2nd ed. (Griffon, London, 1975).
15. Gleson, J. P. & Cahalane, D. J. Seed size strongly affects cascades on random networks. Phys. Rev. E 75, 056103 (2007).
16. Melnik, S., Hackett, A., Porter, M. A., Mucha, P. J. & Gleson, J. P. The unreasonable effectiveness of tree-based theory for networks with clustering. Phys. Rev. E 83, 036112 (2011).
17. Galstyan, A. & Cohen, P. Cascading dynamics in modular networks. Phys. Rev. E 75, 036109 (2007).
18. Gleson, J. P. Cascades on correlated and modular random networks. Phys. Rev. E 77, 046117 (2008).
19. Nematzadeh, A., Ferrara, E., Flammini, A. & Ahn, Y.-Y. Optimal network modulation for information diffusion. Phys. Rev. Lett. 113, 088701 (2014).
20. Brummitt, C. D., Lee, K.-M. & Goh, K.-I. Multiplexity-facilitated cascades in networks. Phys. Rev. E 85, 045102(R) (2012).
21. Lee, K.-M., Brummitt, C. D. & Goh, K.-I. Threshold cascades with response heterogeneity in multiplex networks. Phys. Rev. E 90, 062816 (2014).
22. Melnik, S., Ward, J. A., Gleeson, J. P. & Porter, M. A. Multi-stage complex contagion. Chaos 23, 013124 (2013).
23. Czaplicka, A., Toral, R. & San Miguel, M. Competition of simple and complex adoption on interdependent networks. Phys. Rev. E 94, 062301 (2016).
24. Lugo, H. & San Miguel, M. Learning and coordinating in a multilayer network. Sci. Rep. 5, 7776 (2015).
25. https://ondeviceresearch.com/blog/messenger-wars-how-facebook-climbed-number-one/.
26. http://www.pewinternet.org/2015/01/09/social-media-update-2014/.
27. https://www.techinasia.com/cyworld-vs-facebook/.
28. http://www.pewinternet.org/2015/01/09/social-media-update-2014/.
29. Kocis, G. & Kun, F. Competition of information channels in the spreading of innovations. Phys. Rev. E 84, 026111 (2011).
30. Newman, M. E. J. Threshold effects for two pathogens spreading on a network. Phys. Rev. Lett. 95, 108701 (2005).
31. Ahn, Y.-Y., Jeong, H., Masuda, N. & Noh, J. D. Epidemic dynamics of two species of interacting particles on scale-free networks. Phys. Rev. E 74, 066113 (2006).
32. Karrer, B. & Newman, M. E. J. Competing epidemics on complex networks. Phys. Rev. E 84, 036106 (2011).
33. Chen, L., Ghanbarnejad, F., Cai, W. & Grassberger, P. Outbreaks of coinfections: the critical role of cooperativity. EPL 104, 50011 (2013).
34. Cai, W., Chen, L., Ghanbarnejad, F. & Grassberger, P. Avalanche outbreaks emerging in cooperative contagion. Nat. Phys. 11, 936–940 (2015).
35. Choi, W., Lee, D. & Kahng, B. Critical behavior of a two-step contagion model with multiple seeds. Phys. Rev. E 95, 062115 (2017).
36. Weng, L., Flammini, A., Vespignani, A. & Menczer, F. Competition among memes in a world with limited attention. Sci. Rep. 2, 355 (2012).
37. Sahneh, F. D. & Scoglio, C. Competitive epidemic spreading over arbitrary multilayer networks. Phys. Rev. E 89, 062817 (2014).
38. Granell, C., Götzsche, S. & Arenas, A. Dynamical interplay between awareness and epidemic spreading in multiplex networks. Phys. Rev. Lett. 111, 128703 (2013).
39. Min, B. & Goh, K.-I. Multiple resource demands and viability in multiplex networks. Phys. Rev. E 89, 040802(R) (2014).
40. Granell, C., Karsai, M., Goh, K.-I. & Kertész, J. Dynamics of interacting diseases. New J. Phys. 16, 041005 (2014).
41. Castelló, X., Eguíluz, V. M. & San Miguel, M. Ordering dynamics with two non-excluding opinions: bilingualism in language competition. New J. Phys. 8, 306 (2006).
42. Carro, A., Toral, R. & San Miguel, M. Coupled dynamics of node and link states in complex networks: A model for language competition. New J. Phys. 18, 113056 (2016).
43. Amato, R., Kouvaris, N., San Miguel, M. & Díaz-Guilera, A. Opinion competition dynamics on multiplex networks. New J. Phys. 19, 123019 (2017).
44. Newman, M. E. J., Strogatz, S. H. & Watts, D. J. Random graphs with arbitrary degree distributions and their applications. Phys. Rev. E 64, 026118 (2001).

Acknowledgements

We acknowledge financial support from Agencia Estatal de Investigacion (AEI, Spain) and Fondo Europeo de Desarrollo Regional under Project ES0TECoS Grant No. FIS2015-63628-C2-2-R (AEI/FEDER,UE) and from Agencia Estatal de Investigacion (AEI, Spain) through the Maria de Maeztu Program for Units of Excellence in R&D (MDM-2017-0711). This work was supported by the research grant of the Chungbuk National University in 2018.
Author Contributions
All authors conceived the study, B.M. performed the research, all authors analyzed the data, and wrote and reviewed the paper.

Additional Information
Supplementary information accompanies this paper at https://doi.org/10.1038/s41598-018-32643-4.

Competing Interests: The authors declare no competing interests.

Publisher's note: Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

© The Author(s) 2018