A Unified Approximation Framework for Deep Neural Networks

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Abstract

Deep neural networks (DNNs) have achieved significant success in a variety of real world applications. However, tons of parameters in the networks restrict the efficiency of neural networks due to the large model size and the intensive computation. To address this issue, various compression and acceleration techniques have been investigated, among which low-rank filters and sparse filters are heavily studied. In this paper we propose a unified framework to compress the convolutional neural networks by combining these two strategies, while taking the nonlinear activation into consideration. The filter of a layer is approximated by the sum of a sparse component and a low-rank component, both of which are in favor of model compression. Especially, we constrain the sparse component to be structured sparse which facilitates acceleration. The performance of the network is retained by minimizing the reconstruction error of the feature maps after activation of each layer, using the alternating direction method of multipliers (ADMM). The experimental results show that our proposed approach can compress VGG-16 and AlexNet by over $4 \times$. In addition, $2.2 \times$ and $1.1 \times$ speedup are achieved on VGG-16 and AlexNet, respectively, at a cost of less increase on error rate.

1 Introduction

As the neural network goes deeper, the representation ability of neural network keeps improving, leading to great improvement on the performance. However, the model size and the computation cost of a neural network are also increasing due to the huge amount of weights learned, which results in low throughput in inference stage and limits the deployment of the models. For example, embedded devices may lack of enough storage and computation power. Recognition in surveillance video may require real-time response. Meanwhile, deep neural networks are demonstrated to be over-parametrized [1], which motivates researchers to explore practical approaches to make the deep models compact.

Compacting the deep models involves removing the redundancy and seeking for simplified structures. Low-rankness and sparse connection are the most commonly applied assumptions when compacting a model. Sparse connection can be realized by pruning a pre-trained network, which is the most straightforward approach. A hard thresholding approach is proposed in [2], which achieves high sparsity by removing the weights with less importance. Structured sparse connections can be learned by imposing group sparse regularizations during the training [3,4]. Structured sparse also favors computation acceleration. In additional to sparse connections, low-rank is another desired structure for model compression and acceleration. The weight matrices in the neural networks with lower-rank can be further decomposed into smaller matrices, which reduces the amount of weights as well as the computation cost. An approach of low-rank approximation with a rank selection strategy is proposed in [5] for fast inference. Tensor decomposition is applied in [6], in which the weight tensor in fully-connected layer is approximated by a series of kernels. Similar to sparse connection networks, the low-rank filters can also be learned by imposing regularizations [7,8]. An intuitive extension is to consider both low-rank structure and sparse structure simultaneously. In [9], a layer in the pre-trained neural network is decomposed into a low-rank component and a sparse component by a fast greedy algorithm.
Performing approximation or pruning to a pre-trained network may inevitably result in performance degradation. In order to retain the accuracy of a pre-trained model, some approaches aim to minimize the reconstruction error of the feature maps in each layer, which depends on solving an optimization problem with specified constraints on the rank or sparsity of the filters. The reconstruction error is measured between the linear response in original network and approximated one [9, 10]. Since the non-linearity such as Rectified Linear Units (ReLU) [11] follows the linear filters in most modern neural networks, only the error of positive response is accumulated and the error of negative response is omitted, which makes the accuracy more dependent on to positive response reconstruction. In [5], a method for reconstructing non-linear response is proposed.

In this work, we propose to compact the convolutional layers with two components, including a structured sparse component and a low-rank component. We retain the accuracy by approximating the nonlinear response after activation. The problem is formulated as minimizing the reconstruction error of the response after the ReLU. Two constraints are added in the problem to impose low-rank or sparsity on each component. Both constraints of this optimization problem are not easy to tackle since they are non-convex. To overcome this, we apply convex relaxation to those constraints. The relaxed problem is solved by alternating direction method of multipliers (ADMM) [12].

The proposed method is evaluated on VGG-16 [13] and AlexNet [14]. For VGG-16 on CIFAR-10, we achieve 22.5% compression rate and 2.2× speedup with only 0.4% increase on error rate. In order to tackle the problem of accumulative error in the deep models, we use the asymmetric data to approximate deeper layers, which can improve the accuracy by more than 1%. For AlexNet on ImageNet, we achieve 20.6% compression rate and 1.1× speedup with only 1.3% increase on top-5 error rate.

2 Related Work

Neural network sparsification Despite the appealing performance of the deep neural network, it has been demonstrated that there are much redundancy. Therefore, sparsifying a neural network is a intuitive method to eliminate the redundancy while preserving the accuracy. The sparse weights in a network can be stored with compressed representation like compressed sparse row (CSR) format, thus a model can be compressed. A three-stage pipeline is proposed in [2]. The weights which are below a threshold are considered as less important and are discarded. Then retraining is performed to restore the accuracy. The approach used in [2] results in highly sparse networks. However, the sparse pattern is non-structured which cannot bring speedup during inference due to the poor weight locality. Structured sparse networks are studied in [3, 4, 15] A structured sparse learning algorithm is proposed in [3, 4], which enables to learn a network with structured sparsity by applying group sparse regularizations. Since structured sparsity leads to zero-columns and zero-rows in the lowered matrices, [3] further proposes to reduce the dimension of lowered matrices by removing these zero-columns and zero-rows, which reduces computation cost and results in inference speedup. A channel pruning method is proposed in [10], which can be considered as a special case of structured sparsity. Differently, channel pruning is performed on a pre-trained model rather than training the model from scratch. It is formulated as \(l_0\)-norm problem which minimizes the reconstruction error, trying to find the “informative” channels of the feature map and the corresponding weights. In this paper, we are more interested in exploring structured sparsity since it can not only be used for compressing the networks but also achieve practical speedup in modern hardware platform.

Low-rank approximation In addition to sparsifying a network, low-rank approximation is another kind of approach which can be applied for both network compression and acceleration. In modern convolutional neural networks (CNNs) structure, filters are usually a 4-D tensor. Some tensor decomposition techniques are leveraged for acceleration and compression. A straightforward idea is to replace the 4-D tensor with two consecutive filters with lower-rank [16]. In addition, other kinds of tensor decomposition can also be applied [6, 17]. In [6], fully-connected layers are converted to the Tensor Train format, resulting in compression by a huge factor. CP-decomposition of the filter tensors is proposed in [18], while Tucker decomposition is used in [6]. The low-rank approximation is achieved by performing tensor decomposition on the pre-trained network in these works. In order to conduct low-rank approximation efficiently, a new
method for training neural networks with low-rank filters is proposed in [7,8,19]. Accounting such low-rank assumptions during training results in DNNs with fast inference time [7, 19] and much more compact models [8].

Most of those low-rank approximation-based methods for a pre-trained model like [16,20] consider reconstructing the response of linear block of a network, while ignoring the following non-linear activation function like ReLU [11]. A method for low-rank approximation of non-linear response in convolutional networks is proposed in [5], which achieves good speedup in various modern CNNs.

It should be stressed that some previous work aims to train a network with both group-sparse and low-rank regularizations [8,21]. Considering that filters tends to be both low-rank and sparse, a layer in a pre-trained DNN is approximated by the sum of a non-structured sparse component and a low-rank component for compression in [9]. Similar to [16], it relies on reconstructing the linear response of a layer to constrain accuracy loss. In our approach, we are minimizing the error of the reconstructed feature maps in each layer after non-linearity, which is generated by a structured sparse filter and a low-rank filter. Therefore, the model compacted by our approach not only can be compressed to smaller size but also shows speedup on inference time.

### 3 Mathematical Formulation

In this section, we introduce our mathematical formulation for simultaneous sparse and low-rank layer decomposition, while taking non-linearity into account. To this end, we propose to formulate the problem into a unified optimization model. In the following context, we focus on CNNs whose inference time bottleneck lies in computation of convolutional layers, and whose most weights gather in the fully-connected layers.

In a fully-connected layer of a CNN, the output feature map can be computed as

\[
Y = WX, \tag{1}
\]

where \(X \in \mathbb{R}^m\) and \(Y \in \mathbb{R}^n\) represent the input feature vector and output response, respectively. \(W \in \mathbb{R}^{m \times n}\) denotes the weight matrix. For a convolutional layer with a filter \(W \in \mathbb{R}^{n \times c \times h \times w}\) with a bias of size \(n\)-by-1, where \(h\) and \(w\) are spatial size. \(c\) is the number of input channels and \(n\) is the number of filters. The filter can be reshaped to a matrix with size \(n\)-by-\((hwc + 1)\). The input \(X\) is lowered to a matrix such that each \(h \times w \times c\) volume involved in a convolution forms a column. Then the convolution operation can also be represented as Equation (1).

The information loss is inevitable when we approximate the original filters by low-rank or sparse filters, which may cause performance degradation. In order to compress the network and accelerate the computation, we perform low-rank approximation and network sparsification simultaneously. The output feature maps in a layer is generated by the sum of these two filters. The network structure is shown in Figure 1.

In order to preserve the performance, we aim at minimizing the reconstruction error of the response in
each layer after activation, using two components. The problem is formulated as follows:

$$\min_{A,B} \sum_{i=1}^{N} \|Y_i - r((A + B)X_i)\|_F,$$

s.t. $\|A\|_0 \leq S,$

$$\text{rank}(B) \leq L.$$  

Here $Y_i$ and $X_i$ represent the output feature map and the input feature map, respectively. Structured sparse component $A$ and low-rank component $B$ are two weight matrices we are looking for. $N$ is the total number of samples used for approximation. $\|\cdot\|_F$ is Frobenius norm. $r(\cdot)$ is the activation function in the network, which is ReLU(·) [11] in this paper. $S$ and $L$ are user-defined target sparsity level and target rank for the filters.

4 Optimization

4.1 Problem Relaxation

Solving Problem (2) directly involves both $l_0$ minimization and rank minimization, which is NP-hard. To tackle this, we apply convex relaxation to the constraints. The rank constraint on $B$ is relaxed by nuclear norm of $B$, which is the sum of the singular values. As we discussed above, structured sparse patterns can be easily made use of to computation acceleration. Therefore, here we relax $l_0$ constraint by $l_2,1$ norm such that the zero elements in $A$ appear column-wise. The problem (2) is reformulated as

$$\min_{A,B} \sum_{i=1}^{N} \|Y_i - r((A + B)X_i)\|_F^2 + \lambda_1 \|A\|_{2,1} + \lambda_2 \|B\|_*,$$

where $\|\cdot\|_{2,1}$ is $l_2,1$ norm and $\|\cdot\|_*$ is nuclear norm. $\lambda_1$ and $\lambda_2$ are Lagrange multipliers. The problem (3) is a convex problem. To solve this problem efficiently, we apply alternating direction method of multipliers (ADMM), which is a well suited method that is widely used in large-scale problems arising in statistics [12]. Especially, the optimal solution of the sub-problems involving $l_2,1$-norm and nuclear norm can be obtained in closed-form as in subspace learning [22] and the singular value thresholding (SVT) operator [23], respectively. By introducing an auxiliary variable $M$, the problem (3) can be written as

$$\min_{A,B,M} \sum_{i=1}^{N} \|Y_i - r(MX_i)\|_F^2 + \lambda_1 \|A\|_{2,1} + \lambda_2 \|B\|_*,$$

s.t. $A + B = M.$

Then the augmented Lagrangian function is

$$L_t(A, B, M, \Lambda) = \sum_{i=1}^{N} \|Y_i - r(MX_i)\|_F^2 + \lambda_1 \|A\|_{2,1} + \lambda_2 \|B\|_* +$$

$$\langle \Lambda, A + B - M \rangle + \frac{t}{2} \|A + B - M\|_F^2,$$

where $t > 0$ is the penalty parameter and $\Lambda$ is Lagrange multiplier. $\langle \cdot, \cdot \rangle$ represents the inner product operator.
4.2 Variables Update

ADMM solves the problem iteratively. The variables are alternatively updated in each iteration. To update $A, B, M$ in iteration $k + 1$, we consider the following three sub-problems.

$$\min_A \lambda_1 \|A\|_{2,1} + \frac{t}{2} \left\|A + B_k - M_k + \frac{\Lambda_k}{t}\right\|_F^2,$$

(6)

$$\min_B \lambda_2 \|B\|_* + \frac{t}{2} \left\|B + A_{k+1} - M_k + \frac{\Lambda_k}{t}\right\|_F^2,$$

(7)

$$\min_M \sum_{i=1}^N \|Y_i - r(MX_i)\|_F^2 + \langle \Lambda_k, A_{k+1} + B_{k+1} - M\rangle + \frac{t}{2} \|A_{k+1} + B_{k+1} - M\|_F^2.$$  

(8)

All these three problems are proximal mapping problems. For Problem (6), the optimal solution is given by

$$A_{k+1} = \text{prox}_{\frac{\lambda_1}{t} \|\cdot\|_{2,1}} \left(M_k - B_k - \frac{\Lambda_k}{t}\right).$$  

(9)

The explicit representation of Equation (9) can be derived based on [22]. Let $C = M_k - B_k - \frac{\Lambda_k}{t}$, then the column $i$ in $A_{k+1}$ is given as

$$[A_{k+1}]_{i} = \begin{cases} \|C\|_2 - \frac{\lambda_1}{t} C_{i,i}, & \text{if } \|C\|_2 > \frac{\lambda_1}{t}; \\ 0, & \text{otherwise.} \end{cases}$$  

(10)

For Problem (7), the optimal solution is given by

$$B_{k+1} = \text{prox}_{\frac{\lambda_2}{t} \|\cdot\|_*} \left(M_k - A_{k+1} - \frac{\Lambda_k}{t}\right).$$  

(11)

The explicit representation of Equation (11) can be obtained based on SVT operator [23]. Let $D = M_k - A_{k+1} - \frac{\Lambda_k}{t}$. We perform singular value decomposition on $D$ such that $D = U\Sigma V$, where $\Sigma = \text{diag}\{\sigma_i\}_{i \leq s}$ and $\sigma_i$ is the $i$-th largest singular value. Then $B_{k+1}$ is given by

$$B_{k+1} = U D_{\frac{\lambda_2}{t}} (\Sigma) V, \quad \text{where } D_{\frac{\lambda_2}{t}} (\Sigma) = \text{diag}\{\{\sigma_i - \frac{\lambda_2}{t}\}_+\}. $$  

(12)

For Problem (8), it is non-trivial to derive the closed-form of the optimal solution of the sub-problem with respect to $M$ since $r(\cdot)$ is a piecewise linear function. However, it can be noted that the function is continuous and convex. Then we can approaching the optimal solution of $M$ iteratively by applying gradient-based method. The overall optimization procedure is summarized in Algorithm 1.

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**Algorithm 1** ADMM for solving Problem (4)

**Require**: Feature maps $Y_i, X_i, i = 1 \cdots N$, given $\lambda_1, \lambda_2$.

**Ensure**: Structured sparse matrix $A$ and low-rank matrix $B$.

1. Initialize $k \leftarrow 0, \Lambda_0, A_0, B_0, M_0$, error tolerance $\epsilon, t$;
2. while not converged do
3. Update $A_{k+1}$ by Equation (10);
4. Update $B_{k+1}$ by Equation (12);
5. Update $M_{k+1}$ by iteratively solving Problem (8) with gradient-based method;
6. Update Lagrange multiplier by $\Lambda_{k+1} \leftarrow \Lambda_k + t(A_{k+1} + B_{k+1} - M_{k+1})$;
7. $k \leftarrow k + 1$;
8. end while
9. return $A_k$ and $B_k$;
Experimental Results

All implementations and experiments are conducted on Caffe [24] using CIFAR-10 [25] and ILSVRC-2012 [26]. We evaluate the proposed method on modern CNNs such as VGG-16 [13] and AlexNet [14]. The compression ratio and speedup ratio are reported as well as the accuracy of the models after approximation.

5.1 Experiments on VGG-16 with CIFAR-10

VGG-16 network is a convolutional neural network consisting of 13 convolution layers and 3 fully-connected layers. We test our approach with experiments on CIFAR-10 dataset [25] which consists of 50K training images and 10K test images. We first train a VGG-16 network on CIFAR-10 dataset from scratch as baseline, which has an error rate of 7.92%. Then we collect the input and output feature maps of each layer as the samples. In our experiment, 1000 samples are collected from the images in training set.

We replace the convolution filters by the approximated low-rank and structured sparse components. The experiments are conducted on Intel Xeon E7-4830 CPU. The approximation is performed for each layer sequentially, and the first convolutional layer is not approximated. The sparse component $A$ can be stored in CSR format for size reduction. The low-rank component can be further decomposed into two smaller filters with rank $k$. The compression ratio provided by $B$ is $k(m+n)/mn$, where $mn$ is the size of original filter. The values are shown as $CR(B)(\%)$ in Table 1. For fully-connected layers, we only use sparse component for approximation. In our experiment, we can achieve over 90% sparsity for component $A$ for all approximated layers. Therefore, the compression ratio of component $A$ are quite small. Moreover, we constrain the sparse component $A$ to be structured sparse which can be made use of to accelerate the computation [3]. After the approximation, we achieve $2.2 \times$ speedup on inference and $4.44 \times$ compression rate. Without fine-tuning, the classification accuracy drop is less than 2%. In order to restore the accuracy of the compressed model, it can be fine-tuned with the training set. We retrained the compressed network for 5 epochs, the error rate increase can be further improved from 1.9% to 0.4%.

If a shallow layer is approximated, the approximation error may be accumulated when deeper layers are approximated. In order to handle this issue, we take the ‘asymmetric’ strategy used in [27]. We approximate the layers from shallow to deep. When approximating a deep layer, use the response produced by all previous layers instead of the non-approximate response as the input feature map $X_i$. Figure 2 shows the comparison of classification error increase. We can observe that with more layers being approximated, the performance becomes worse for both strategies. However, the asymmetric version loses less accuracy.

We further compare the performance between reconstructing non-linear response and reconstructing linear response. We perform the comparison on a single layer each time, while the remaining layers are kept unchanged. In Figure 3, we plot the relation between the compression ratio and the accuracy degradation of two approaches of different layers. The performance is evaluated by the accuracy drop compared with original
Table 1: Results of VGG-16 on CIFAR-10

| Layer     | CR(A)(%) | CR(B)(%) | CR(A + B)(%) | Accuracy drop(%) |
|-----------|----------|----------|--------------|------------------|
| conv1-1   | 0        | 100      | 100          | 0                |
| conv1-2   | 5.38     | 52.07    | 57.45        | 0.86             |
| conv2-1   | 5.39     | 38.18    | 43.58        | 0.22             |
| conv2-2   | 2.17     | 28.64    | 60.33        | 0.66             |
| conv3-1   | 2.77     | 47.73    | 50.51        | 0.23             |
| conv3-2   | 4.04     | 54.25    | 58.29        | 0.46             |
| conv3-3   | 10.04    | 59.02    | 69.07        | 0.46             |
| conv4-1   | 2.04     | 15.99    | 18.04        | 0.01             |
| conv4-2   | 2.02     | 22.35    | 24.37        | 0.01             |
| conv4-3   | 4.02     | 16.27    | 24.37        | 0.01             |
| conv5-1   | 2.02     | 9.77     | 11.78        | 0.01             |
| conv5-2   | 2.02     | 8.68     | 10.70        | 0.01             |
| conv5-3   | 2.02     | 6.72     | 8.75         | 0.01             |
| fc1       | 44.19    | 0        | 44.19        | 0.02             |
| fc2       | 36.20    | 0        | 36.20        | 0.01             |
| fc3       | 24.02    | 0        | 24.02        | 0.00             |

Overall compression rate 22.5% (4.44 ×)
Overall speedup 2.2 ×
Error rate increase 1.9%
Error rate increase after FT 0.4%

Figure 3: Comparison of reconstructing linear response and non-linear response: (a) layer conv2-1; (b) layer conv3-1.

model. We take two convolutional layers in two different stages of the VGG-16, including conv2-1 and conv3-1. Figure 3 shows that under the same compression rate, reconstructing non-linear response achieves lower accuracy drop than reconstructing linear response, which verified the advantage of reconstructing the non-linear response. In Figure 4, we visualize the sparse filter and low-rank filter after the approximation of layer conv3-1. The B has rank 126 and it can be further decomposed by \( B = U V \), where both \( U \) and \( V \) have rank 126.

5.2 Experiments on AlexNet with ILSVRC-2012

In this part we show the experimental results on AlexNet for ImageNet classification. AlexNet has five convolutional layers and three fully-connected layers. We evaluate the top-5 accuracy with single-view. In our implementation, the top-5 error rate of the baseline AlexNet on ILSVRC-2012 dataset is 22.83%. For ILSVRC-2012, we collect 1500 images in the training set and collect their response for approximation. Images are resized with 256 pixels on the shorter side. The testing image is on the center crop of 224 × 224 pixels. The first convolutional layer is not approximated. The second layer is not compressed yet in our experiment since it is hard to achieve good trade off between compression rate and accuracy without
Figure 4: Approximated filters of conv3-1. Low-rank filter $B$ with rank 126 is decomposed into $UV$, both of which have rank 126. (a) Sparse filter $A$; (b) Low-rank filter $U$; (c) Low-rank filter $V$.

Table 2: Results of AlexNet on ILSVRC-2012

| Layer | $CR(A)$(%) | $CR(B)$(%) | $CR(A + B)$(%) | Top-5 accuracy drop (%) |
|-------|------------|------------|-----------------|------------------------|
| conv1 | 0          | 100        | 100             | 0                      |
| conv2 | 0          | 100        | 100             | 0                      |
| conv3 | 2.06       | 78.29      | 80.35           | 4.6                    |
| conv4 | 5.53       | 50.35      | 55.88           | 3.1                    |
| conv5 | 4.04       | 43.40      | 47.44           | 1.0                    |
| fc1   | 14.08      | 0          | 14.08           | 0.9                    |
| fc2   | 28.32      | 0          | 28.32           | 0.8                    |
| fc3   | 28.29      | 0          | 28.29           | 0.8                    |

Overall compression rate 20.6% (4.85×)
Overall speedup 1.1×
Accuracy drop 8.9%
Accuracy drop after FT 1.3%

retraining. The results of the compression rate and accuracy drop are shown in Table 2. Without fine-tuning, the network is compressed by nearly 5× at a cost of 8.9% top-5 error rate increase, and the top-5 error rate increase is reduced to 1.3% after fine-tuning for 10 epochs. In addition, 1.1× speedup is achieved in inference runtime on CPU platform.

6 Conclusion

In this paper, we propose a unified approximation model for deep neural networks with low-rank approximation and network sparsification. Different from other approaches, we reconstruct the non-linear response with two components. ADMM is applied to solve the problem. The effectiveness is verified on VGG-16 and AlexNet. By sacrificing less accuracy, both networks are compressed by over 4×. What’s more, thanks to the structured sparsity imposed during approximation, 2.2× and 1.1× speedup in inference runtime are achieved on VGG-16 and AlexNet, respectively.

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