Study of MHD stability limits of reactor-scale spherical tokamak

A Yu Chirkov\textsuperscript{1,3}, J E Lopez\textsuperscript{2}, E A Orozco\textsuperscript{2}, D E Fedyunin\textsuperscript{1}

\textsuperscript{1} Bauman Moscow State Technical University, Vtoraya Baumanskaya 5/1, Moscow, 105005 Russia
\textsuperscript{2} Universidad Industrial de Santander, Bucaramanga, Santander, 105005 Colombia
\textsuperscript{3} E-mail: chirkov@bmstu.ru

Abstract. The MHD equilibria in low aspect ratio tokamak plasma are studied. The parameters correspond to the regimes of a system of relatively large sizes, which can be considered as the concept of a reactor (or source of fusion neutrons) on deuterium fuel. The efficiency of such a system substantially depends on the limitations associated with the conditions of MHD equilibrium and stability.

1. Introduction

A spherical tokamak with deuterium fuel (without external tritium) can potentially be a neutron source for a fusion–fission hybrid [1]. The parameters of such a tokamak, in principle, correspond to a moderate extrapolation of technologies adopted in the ITER project. The possibility of regimes with plasma power gain $Q \sim 0.5$ was shown at aspect ratio $A \sim 2$, magnetic fields on the axis of the plasma column $B_0 \sim 4$ T, and a minor radius $a \sim 2$ m [1]. For a fusion reactor with $Q \sim 10$, the requirements for the parameters of the magnetic system significantly exceed today's level of technological capabilities [2]. The present study is aimed at substantiating the limiting parameters of a spherical tokamak corresponding to regimes with $Q \sim 0.5$–1.

We mainly focus on the parameters of thermonuclear systems with a spherical tokamak, proposed for the realization of a neutron source based on deuterium plasma (without external tritium) for hybrid fusion–fission reactors. Integral models developed for tokamaks [3–5] and other systems [6–8] with advanced fusion fuel cycles (D–D, D–\textsuperscript{3}He) are also used for evaluations.

To substantiate the concept of a reactor, one of the primary tasks is MHD simulation of an equilibrium configuration. The selection of the main parameters takes into account the known limitations for tokamaks depending on the aspect ratio [9–12] and conceptual designs of power plants with spherical tokamaks [13].

One of the urgent tasks is to study the conditions of MHD stability of the plasma in the enhanced confinement mode. In particular, the tokamak parameters are limited by the development of edge localized modes (ELMs). Studies of regimes that mitigate these phenomena require the establishment of stability limits based on geometric factors and corresponding field values. Note that recent experiments on the DIII-D tokamak have shown a significant improvement in confinement in the case of reverse D-shape plasma [14].

In present study, we consider the properties of the equilibrium configurations. A numerical study of the plasma dynamics of a spherical tokamak for reactor conditions using the MHD code [15]...
is presented. Equilibrium is considered on the basis of the Grad–Shafranov equation. The perturbation analysis makes it possible to determine the limits of MHD stability and the limiting values of the parameters for reactor regimes.

2. Reactor parameters and limitations

In the Table, parameters of the following fusion systems based on low-aspect tokamaks are presented: deuterium fuelled fusion neutron source for fusion fission hybrid with no external tritium and tritium breeding [1]; the same system with tritium breeding in the plasma for external consumption of the part of the produced tritium [1]; D–³He reactor [2]; low-scale demonstrator calculated using previously developed models [1, 2, 4–6].

Tokamak parameters are limited by the conditions of MHD stability. In particular, the plasma temperature is limited by the ratio $\beta$ of the plasma pressure to the magnetic pressure. It was established [9–12] that the maximum elongation of the plasma cross section depends on the aspect ratio as follows

$$ k = 1.082 + 2.747/A , $$

and the maximum value of the Troyon coefficient (normalized beta) is associated with aspect ratio and elongation:

$$ \beta_N = (1.37/A + 1.60/\sqrt{A} + 1.26)/\sqrt{k} . $$

In terms of placement of coils and internal tokamak elements, the distance from the axis of the tore to the boundary of the plasma column should be not less then $r_0 \sim 2$ m. It follows the minimum value of the minor radius (radius of the plasma column)

$$ a = r_0/(A - 1) . $$

Magnetic induction is inversely proportional to the distance from the axis of the tore. Its value is limited by the value of $B_m$ on the surface of the superconductor of the inner part of the core of the magnetic system ($B_m = 12.5$ T, for today’s superconductor technology). Therefore, the maximum magnetic field on the axis of the plasma column is

$$ B_0 = \frac{B_m r_0}{Aa} . $$

Figure 1 illustrates the above limitations. The limiting parameters of a tokamak with $A \sim 2$ are as follows: $a \sim 2$ m, $B_0 \sim 4$ T, $k \sim 2.5$, $\beta_N \sim 5$, $\beta \sim 0.25$. These parameters basically correspond to tokamaks in the Table.

| Table. Parameters of fusion systems based on low-aspect tokamaks |
|---------------------------------------------------------------|
| Fusion fuel cycle | D–D fuelled hybrid | D–D hybrid with tritium breeding [1] | D–³He reactor [2] | Low-scale demonstrator |
| Minor radius $a$, m | 2.0 | 2.0 | 2.0 | 0.67 |
| Aspect ratio $A$ | 2.0 | 2.0 | 1.7 | 2.2 |
| Elongation $k$ / Triangularity $\delta$ | 2.5 / 0.35 | 2.5 / 0.35 | 2.8 / 0.5 | 1.8 / 0.35 |
| Normalized beta $\beta_N$ / Beta $\beta$ | 4.5 / 0.25 | 3.0 / 0.16 | 5.0 / 0.5 | 4 / 0.12 |
| Magnetic field on the axis $B_{ls}$, T | 4.0 | 4.5 | 5.5 | 2.0 |
| Plasma current $I_p$, MA | 42 | 48 | 110 | 3.4 |
| Plasma temperature $T$, keV | 15 | 12.5 | 44 | 8 |
| Energy confinement time $\tau_E$, s | 7.0 | 5.7 | 9.4 | 0.07 |
| Plasma power gain $Q$ | 0.7 | 0.25 | 10 | 0.004 | 0.11 |
| Fusion power $P_{ fus }$, MW | 250 | 110 | 2200 | 0.042 | 9.7 |
The study of the processes that occur in thermonuclear fusion devices are usually approached by various plasma models at different levels of approximation, depending essentially on the phenomenon to be analyzed. The magnetohydrodynamic (MHD) model is the most suitable when study macro instabilities. In order to simulate the high confinement phase of the reactor, it was decided to model the plasma under the resistive MHD equations [15].

**3. Modeling**

When studying the equilibrium conditions in fusion device, the first search is instances the topology of the magnetic field for plasma control, together with the parameters such as pressure, current density, etc. The first basic consideration is to treat plasma in static and stationary equilibrium, that is, the speed of fluid element of is null, \( V = 0 \), at the same as the temporal derivatives of the other physical variables are zeros. For two-dimensional geometry it is possible to find the steady state equilibrium through the Grad–Shafranov equation. This equation is deduced from the system of equations for a toroidal geometry under the condition of axial symmetry, where the equilibrium is represented in terms of the poloidal magnetic flux \( \psi \).

The geometry of toroidal systems is simplified by considering symmetry on the toroidal axis. Thus, the simulations are performed on the pole plane of the tokamak, which for simplicity is limited to a rectangular region that includes the open and closed magnetic field lines of the system. With these ideas in mind it is appropriate to work on the cylindrical coordinate system \((r, z)\), where the radial coordinate coincides with the axis of the major radius of the toroid, the angular coordinate with the toroidal angle and the \( z \) axis represents the vertical distance of the system from the radial axis. Due to the symmetry, any function that describes the state of the plasma is entirely described by the variables that represent the poloidal plane \((r, z)\).

In the study of the MHD equilibria in magnetic fusion reactors, it is essential to provide a solution to the Grad–Shafranov equation. Numerical techniques take on great relevance in this regard. Typical models are based on iterative schemes. In this work, two solvers were developed using the finite difference scheme under the successive over-relaxation method (SOR), capable of solving this problem. A solver is based on the plasma free boundary problem, using a homogeneous rectangular mesh. The second is developed, to which a non-equidistant mesh is fitted.

Figure 2 shows the results of calculating the equilibrium configuration of a spherical tokamak reactor, the parameters of which correspond to the system based on D–D fusion fuel.
To analyze the plasma dynamics, the equilibrium conditions obtained through the numerical solution of the Grad–Shafranov equation are disturbed. To do this, a code based on finite differences and Runge–Kutta algorithms were developed that solve the resistive MHD equations in the linear regime. These simulations show a strong disposition of the disturbance in the outer edge zone of the plasma. It was achieved or determined that as the disturbance evolves, some poloidal modes become more visible around the magnetic axis.

The dynamic evolution of the disturbances presents a damping effect, achieving establish that the disturbances at about 60 periods of Alfvén times in are undetectable. In these simulations it is not possible to determine the cause that generates this damping, but based on results of previous investigations and simulation reports, it seems to indicate that the energy dissipation is generated by inappropriate viscosity values. From this point of view, the problem is left open for future research.

The strong plasma damping anomalous behavior cannot to be understood, it is relevant to highlight the excitation of several poloidal modes near the edge of the plasma, this because it is typical that in the high-confinement region, these modes are excited in said zone, generating instabilities of outer edge, being precursors of the ELMs. It is clear that damping has a greater effect than the possible development of these instabilities, however, is an indicator that the simulations in their structure are correct.

4. Conclusions
In this work, the MHD equilibria in plasma was studied in the first instance from axially symmetrical to toroidal geometry, which is characteristic of fusion devices like tokamak. The results obtained are able to predict in great approximation the equilibrium stationary regimes of the spherical tokamak reactor (or neutron source for fusion–fission hybrid), suitable for analyzing the parameters of such a system. Details in the geometry of the plasma column differ slightly from the characteristic D-shape.
MHD equilibrium allows to make the initial estimates necessary to develop the concept of a thermonuclear system based on a spherical tokamak with an alternative fuel cycle. Such a system, in particular, could potentially be the basis of a fusion–fission hybrid system in which only deuterium is a fusion fuel, and no needs for tritium production.

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