Flavor without Flavor Symmetry

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Abstract

Non-trivial patterns of quark and lepton masses and mixings can arise without there being any underlying flavor symmetry that distinguishes among the three families. Two realistic examples are given.
There are two outstanding features of the quark and lepton spectrum. First, the masses of the fermions of each kind exhibit an interfamily hierarchy: $m_3 \gg m_2 \gg m_1$. Second, these hierarchies appear to be nearly aligned, at least in the case of the quarks, in the sense that $u_3$ is aligned with $d_3$, $u_2$ with $d_2$, and $u_1$ with $d_1$. That is to say, the mixing angles are very small. There is also some evidence that the lepton hierarchies are aligned, namely the fact that the lepton mixing angle $U_{e3}$ is very small. But the large mixing angle $U_{\mu 3}$ seen in atmospheric neutrino oscillations shows that the alignment is not as good for the leptons as for the quarks.

Attempts to explain the quark and lepton masses are usually based on either grand unification or flavor symmetry, or a combination of the two approaches. The way that flavor symmetry would explain the parallel hierarchies of the fermion masses is quite straightforward. If flavor symmetry distinguished the different families from each other, then the masses of the fermions of different families would typically arise at different order in flavor symmetry breaking. Moreover, the mixing of families would also be a flavor-symmetry-breaking effect. Consequently, if family-symmetry-breaking effects were small, one might expect both a hierarchical pattern of masses and small mixing angles, as observed.

Turning to grand unification, one finds that grand unified gauge symmetries are able to explain small mixing angles in a completely different way, which is most readily understood by considering the minimal $SO(10)$ model. In minimal $SO(10)$ all four Dirac mass matrices, those of the neutrinos, up quarks, down quarks, and charged leptons (which matrices we denote henceforth as $N$, $U$, $D$, $L$) are exactly proportional: $N = U \propto D = L$. Obviously this means that the hierarchies in minimal $SO(10)$ are exactly aligned, and the CKM angles vanish. In realistic unified models the relation between the mass matrices is more complicated, but for models based on $SO(10)$ and some related groups the CKM angles do tend to be small.

The question arises whether grand unification can also explain the other main feature of the quark and lepton spectrum — the interfamily mass hierarchies — without flavor symmetry. If so, then it becomes attractive to dispense with flavor symmetry altogether. By flavor symmetry, we mean here any symmetry that distinguishes among the three families.

If there is no flavor symmetry, then how could one explain that some families have much larger mass than others? A simple possibility was suggested quite a long time ago [1] [2], namely that the different families may mix
differently with superheavy fermions in real representations of the unified group. The idea can be illustrated very simply. Suppose that we consider an $SO(10)$ model \[2\] in which, in addition to the three families $16_i$, $i = 1, 2, 3$, there is a “vectorlike” family-antifamily pair $16 + \overline{16}$. We can imagine a $Z_2$ parity under which the ordinary families are odd and the vectorlike ones are even. We do not consider this a flavor symmetry because it does not distinguish among the three families. Suppose further that there are Higgs in the vector and adjoint representations of $SO(10)$, $10_H$ and $45_H$. If these are also odd under the $Z_2$, then only the following types of renormalizable Yukawa coupling are allowed:

$$W_{Yukawa} = M(\overline{16}16) + a_i(\overline{16}16_i)45_H + b_i(1616_i)10_H.$$  \hspace{1cm} (1)

The mass $M$ is of the GUT scale. The interesting point to note is that even though no symmetry has been imposed that distinguishes among the families, two of the light families get mass and one does not. The reason for this is that the two Yukawa coupling vectors, $a_i$ and $b_i$, span only a two dimensional subspace of the full three dimensional family space. To be more concrete, one can without any loss of generality choose the axes in family space so that $a_i = a(0, 0, 1)$ and $b_i = b(0, \sin \theta, \cos \theta)$. It is then clear that the first family has no Yukawa couplings at all and remains exactly massless. The other families get mass through mixing, as follows. The first two terms of Eq. (1) lead to a superheavy fermion mass term that can be written as $\overline{16}(M16 + a(45_H)16_3)$. One sees that the superheavy spinor is a linear combination of the $16$ and $16_3$. The linear combination orthogonal to this is light and in fact is just the third family. In other words, the $16$ without an index actually contains in part the light fermions of the third family. Consequently, the third term in Eq. (1) generates weak-scale mass terms of the form $b(10_H)16_3(\cos \theta 16_3 + \sin \theta 16_2)$. That is, it generates 23, 32, and 33 elements of the light fermion mass matrices.

To summarize what is going on, the light fermions are only able to obtain mass through mixing with superheavy fermions. But not all the three families are able to mix in this way, since there are not enough superheavy fermions for them all to mix with. Thus, perforce, an interfamily mass hierarchy results even though all three families have exactly the same quantum numbers.

Let us examine the structure we have just described in more detail. One may write the VEV of the adjoint Higgs field as $\langle 45_H \rangle = \Omega T$, where $T$ is a
generator of \(SO(10)\). Then, integrating out the vectorlike fields \(\mathbf{16} + \mathbf{16}\), as shown in Fig. 1, one obtains the effective operator

\[
W_{\text{eff}} \cong \left[ a_i \langle 45_H \rangle \mathbf{16}_i \right] \left[ b_j \langle 10_H \rangle \mathbf{16}_j \right] / M
\]

\[
= (a_3 T \cdot \mathbf{16}_3) (b_3 \mathbf{16}_3 + b_2 \mathbf{16}_2) \langle \Omega / M \rangle (\Omega / M)
\]

\[
\propto T_{(16_3)} \mathbf{16}_3 (\cos \theta \mathbf{16}_3 + \sin \theta \mathbf{16}_2).
\]

Consider now fermions of type \(f\), where \(f\) can be (left-handed) up-type quarks, down-type quarks, charged leptons, or neutrinos. The left-handed anti-fermions are denoted \(f^c\). There arises straightforwardly from the previous equation the following effective three-by-three mass matrix for the light fermions of type \(f\):

\[
f^c_i M_{ij} f_j \cong M_f \left( f^c_1, f^c_2, f^c_3 \right) \left( \begin{array}{ccc}
0 & 0 & 0 \\
0 & \sin \theta T_f & \sin \theta T_{f^c} \\
0 & \cos \theta (T_f + T_{f^c}) & 0
\end{array} \right) \left( \begin{array}{c}
f_1 \\
f_2 \\
f_3
\end{array} \right).
\]

(3)

The factor \(M_f\) has one value, which we shall call \(M_U\), for up quarks and neutrinos, and another value, which we shall call \(M_D\), for down quarks and charged leptons. The symbols \(T_f\) and \(T_{f^c}\) stand for the charges of the fields \(f\) and \(f^c\) under the \(SO(10)\) generator \(T\).

There are several interesting features of this structure that have been pointed out in earlier papers \[2\] \[3\]. Not only is the first family singled out as massless despite there being no flavor symmetry that distinguishes one family from another, but the structure naturally accommodates a mass hierarchy between the second and third families as well. For example, if we assume that \(T_f \sim T_{f^c}\) and that \(\sin \theta\) is somewhat small, then \(m_{f_2}/m_{f_3} \sim \frac{1}{4} \tan^2 \theta \ll 1\).

Another interesting feature of this structure is that it naturally explains why the minimal \(SU(5)\) relation \(m_b \cong m_\tau\) works well while the corresponding relation for the second family, \(m_s \cong m_\mu\), does not. The reason has to do with the way the \(SO(10)\) generators appear in Eq. (3). Since the same Higgs doublet \(H_d\) in \(10_H\) couples to both \(d^c d\) and \(\ell_+ \ell_-\), it follows that \(T_{d^c} + T_d = T_{\ell^+} + T_{\ell^-} = -T_{H_d}\). Consequently, the 33 elements of the mass matrices for the down quarks and charged leptons are approximately equal. However, the 23 and 32 elements are different for the two matrices since \(T_d\) and \(T_{d^c}\) are not equal to \(T_{\ell^-}\) and \(T_{\ell^+}\).
Finally, the structure incorporates automatically a sort of Fritzschian form \[ \frac{m_c}{m_t} \] for the heavier families, with a “texture zero” in the 22 element. This relates the smallness of $V_{cb}$ to the smallness of $\frac{m_c}{m_t}$ and $\frac{m_s}{m_b}$.

It is remarkable that the one effective Yukawa term given in Eq. (2) goes so far toward providing a satisfactory framework for describing the masses and mixings of the fermions of the second and third families. In several earlier papers attempts were made to construct models of the quark and lepton masses and mixings on the basis of exactly this term. The first attempt was \[ \frac{m_s}{m_b}, \frac{m_c}{m_t}, \text{and } V_{cb} \] that had to be fit using two parameters, namely $\theta$ and a parameter specifying the $SO(10)$ generator $T$. (Since $T$ must commute with the Standard Model group, it must be a linear combination of weak hypercharge and the generator $X$ in $SU(5) \times U(1)_X$, or equivalently a linear combination of $B-L$ and $I_{3R}$. Thus, only a single parameter is needed to specify $T$.) It turned out that at least one quantity got very badly fit for all choices of parameters. In the same paper, a better fit was sought by extending the model in the obvious way to the group $E_6$. One more parameter was thereby introduced, since in $E_6$ two parameters are needed to specify the generator $T$. It seems almost prophetic in light of recent results that the best fit obtained in the $E_6$ model had very small $m_s$ and a large mixing between $\mu_L$ and $\tau_L$, i.e. a large contribution to the leptonic mixing $U_{\mu 3}$. Unfortunately, the values of $m_s$ obtained were too small even compared to the recent lattice results, and the value of the $\mu \tau$ mixing angle was not large enough to account for the atmospheric neutrino oscillations.

In \[ \frac{m_s}{m_b}, \frac{m_c}{m_t}, \text{and } V_{cb} \] a realistic model was obtained by introducing structures that went beyond Eq. (2) to account for the heavy two families. It was assumed that $T = (I_{3R}) + \epsilon (B-L)$, $\epsilon \ll 1$, which gave an appealingly simple explanation of the Georgi-Jarlskog relation; however, the ratio $m_c/m_t$ remained a problem, since with this choice of $T$, the matrices in Eq. (3) give the minimal $SO(10)$ result $m_c/m_t \approx m_s/m_b$. A way of suppressing $m_c/m_t$ was found in \[ \frac{m_s}{m_b}, \frac{m_c}{m_t}, \text{and } V_{cb} \] that, although elegant, was somewhat involved.

In \[ \frac{m_s}{m_b}, \frac{m_c}{m_t}, \text{and } V_{cb} \] a very successful model of quark and lepton models was constructed that also included the operator in Eq. (2). (For a similar model see \[ \frac{m_s}{m_b}, \frac{m_c}{m_t}, \text{and } V_{cb} \].) However, to give a satisfactory account of the heavy two families, two other operators in addition to that in Eq. (2) were needed. Nevertheless,
this did not lead to a loss of predictivity for two reasons. First, the generator $T$ was fixed to be exactly $B - L$ in order to solve the doublet-triplet splitting problem via the Dimopoulos-Wilczek mechanism [8], thus reducing the number of parameters by one. Second, with $T = B - L$, the 33 elements in Eq. (3) vanish, since $(B - L)_f + (B - L)_{f^c} = 0$, making the parameter $\theta$ irrelevant. (This necessitated, of course, that a different operator be introduced to generate the 33 elements.) The model of [9] and [10] was extremely predictive and simple in structure. One of its great successes was that it naturally accounted for the largeness of the atmospheric neutrino mixing angle. However, though very simple, it made an enormous sacrifice from the point of view of the idea we are exploring in the present paper: it was based on a flavor symmetry, i.e. a symmetry that distinguished the three families from each other.

To summarize the past efforts, one can say that no completely satisfactory model exists that succeeds in explaining the flavor structure of the quarks and leptons without a flavor symmetry. In this paper we shall pursue this goal again. We present two models. Both incorporate the operator in Eq. (2). The first model is similar in spirit to the models of [2] and [3]. It is rather simple and has a single prediction, namely the mass of the strange quark, which comes out smaller than the Georgi-Jarlskog prediction and more in line with the recent lattice estimates. The second model is very close to the model of [5] and [6], but is obtained without recourse to flavor symmetry.

Model 1. A realistic variant of the models of [5] and [6] can be constructed in a simple fashion. Consider an $SO(10)$ model with the following Yukawa superpotential:

$$W_{\text{Yukawa}} = M(\overline{16}16) + a_i(\overline{16}16_i)1_H$$

$$+ M'(\overline{16}'16') + b_i(\overline{16}'16_i)45_H$$

$$+ c(1616)10_{up}^H + d(16'16)10_{down}^H. \quad (4)$$

The vector Higgs fields $10_{up}^H$ and $10_{down}^H$ are supposed, respectively, to obtain VEVs in their $Y/2 = +1/2$ and $Y/2 = -1/2$ components. Thus, the former gives mass only to up quarks and neutrinos, while the latter gives mass to down quarks and charged leptons.
The structure that emerges from these terms can be understood readily. As before, we can write \( \langle 45_H \rangle = \Omega T \), where \( T \) is an \( SO(10) \) generator, which we choose to parametrize as \( T = 2I_{3R} + 3d(B - L) \). (The parameter called \( d \) here is the same up to a normalization as the parameter called \( \epsilon \) in [3].) Without loss of generality a basis in family space can be chosen so that the Yukawa coupling vectors take the form \( a_i = a(0, 0, 1) \) and \( b_i = b(0, \sin \theta, \cos \theta) \). The combination of the two terms involving the \( 16 \), namely the term with \( M \) and the term with \( 1_H \), cause the fields \( 16 \) and \( 16_3 \) to mix with each other. That means that the field \( 16 \) is not purely superheavy, but also contains an admixture of the fields of the third family. Similarly, the two terms involving the \( 16' \), namely the term with \( M' \) and the term with \( 45_H \), cause the \( 16' \) and the linear combination \( b_i 16_i = b(\cos \theta 16_3 + \sin \theta 16_2) \) to mix with each other. Thus, the \( 16' \) is also not purely superheavy, but contains an admixture of the fields of the second and third families in a proportion that depends on the angle \( \theta \).

Given these facts, one sees that the term \( (1616)10_H^{up} \) contributes only to the 33 element of the up quark mass matrix \( U \). Similarly, the term \( (16'16)10_H^{down} \) contributes to the 23, 32, and 33 elements of the mass matrix of the down quarks, \( D \), and the mass matrix of the charged leptons, \( L \). At this stage, then, one has that the charm quark is massless. It, like the fermions of the first family, is supposed to get mass from other smaller terms. This is not unreasonable, in light of the fact that \( m_c/m_t \) is an order of magnitude smaller than \( m_s/m_b \) and \( m_\mu/m_\tau \).

From the foregoing one can write down the following expressions for the mass matrices \( D \) and \( L \):

\[
D = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & (1 - d) \sin \theta \\
0 & d \sin \theta & \cos \theta
\end{pmatrix} \quad \text{and} \quad L = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & (1 + 3d) \sin \theta \\
0 & -3d \sin \theta & \cos \theta
\end{pmatrix}
\]

In this model, two parameters, \( d \) and \( \theta \) are available to predict three dimensionless quantities, \( V_{cb}, m_\mu/m_\tau \), and \( m_s/m_b \). There is therefore one prediction, which can be taken to be for the strange quark mass. For brevity, let us define \( t \equiv \tan \theta, v \equiv V_{cb}, \) and \( 3\ell \equiv m_\mu/m_\tau \). Then, to the leading two orders in small quantities one can write
\begin{align*}
v \equiv V_{cb} &= \frac{dt}{1 + [(1 - d)^2 - d^2]t^2}. \quad (6) \\
\ell \equiv \frac{1}{3} \frac{m_\mu}{m_\tau} &= \frac{d(1 + 3d)t^2}{1 + [(1 + 3d)^2 + (3d)^2]t^2}. \quad (7) \\
m_s/m_b &= \frac{d(1 - d)t^2}{1 + [(1 - d)^2 + d^2]t^2}. \quad (8)
\end{align*}

Eq. (6) can be inverted to give \( d = \frac{v}{t} \left( \frac{1 + t^2}{1 + 2vt} \right) \). Substituting this into Eq. (7), one finds that the expressions simplify to yield a quadratic equation for the parameter \( t = \tan \theta \) in terms of the experimentally known \( v \) and \( \ell \):

\begin{equation}
0 = t^2[5v(1 - \frac{34}{5}\ell)] + t[1 - 10\ell] - \frac{\ell}{v + 18\ell v - 3v]. \quad (9)
\end{equation}

Using \( v = 0.035 \) and \( \ell = 1/50.4 \) (these are evaluated at the GUT scale), one finds that

\begin{equation}
t \equiv \tan \theta = 0.536, \quad d = 0.081. \quad (10)
\end{equation}

Note that the angle \( \theta \) is not particularly small. This means that the Yukawa vectors \( a_i \) and \( b_i \) are not “unnaturally” aligned in family space. This is consistent with the philosophy that there is no family symmetry. The small parameter in this model is really \( d \). As is evident from Eqs. \( (6) \) – \( (8) \), it is the smallness of \( d \) that accounts for the mass hierarchy between the second and third families and for the smallness of the mixing between them. That is, remarkably, it is not a flavor symmetry that produces these “flavor” features, but the pattern of \( SO(10) \) breaking. Note also that in the limit \( d \to 0 \), the Georgi-Jarlskog relation \( m_s/m_b = \frac{1}{2} m_\mu/m_\tau \) becomes exact, as can be seen from Eqs. \( (7) \) and \( (8) \). In fact, this is how the Georgi-Jarlskog relation was obtained in the model of [3]. However, since the parameter \( d \) is significantly different from zero, there is a significant deviation from the exact Georgi-Jarlskog prediction for \( m_s \). One finds that

\begin{equation}
m_s/m_b \cong 0.866(m_s/m_b)_{GJ} \cong 1/58.2. \quad (11)
\end{equation}

This is the value at the unification scale. It translates into a strange quark mass of about 137 MeV at 1 GeV, or about 100 MeV at 2 GeV, which is in the range given by recent lattice calculations.
Model 2. The second model is a realization of the model of [5] and [6] constructed without use of flavor symmetries. Again based on $SO(10)$ it has the following Yukawa superpotential terms:

$$W_{Yukawa} = M(\overline{16} 16) + a_i(\overline{16} 16_i) 1_H$$

$$+ M'(1010) + b_i(10 16_i) 16_H$$

$$+ c(16 16) 10 + d(1610) 16'_H$$

$$+ g_i(16 16_i) 10 45_H 1_H / M_G^2.$$  \hfill (12)

Here, the spinor Higgs field $16_H$ is supposed to acquire a VEV in the $SU(5)$-singlet direction, i.e. one that commutes with the Standard Model group, whereas $16'_H$ is supposed to acquire a VEV in the Weak-doublet direction [9]. As in the model of [5] and [6], the adjoint Higgs field $45_H$ is supposed to acquire a VEV in the $B - L$ direction, as needed to solve the doublet-triplet splitting problem by the Dimopoulos-Wilczek mechanism. This set of terms can be shown to be the most general that is consistent with a $Z_3$ symmetry under which all the quark and lepton multiplets transform trivially except for the three ordinary families $16_i$, which all transform non-trivially and in the same way.

The form of the mass matrices that result from these Yukawa terms can be determined by the same kind of reasoning that was used above. As before, we can choose our axes to make $a_i = a(0, 0, 1)$ and $b_i = b(0, \sin \theta, \cos \theta)$. The coefficient $c_i$ in the higher-dimension operator will then in general have three non-zero components. It is easy to see that integrating out the $\overline{16}$ leads to a mixing of the $16$ and $16_3$, as discussed before. Similarly, integrating out the superheavy fermions in the $10$ leads to a mixing of the $SU(5)$ 5’s in $10$ and in $b_i 16_i = b(\cos \theta 16_3 + \sin \theta 16_2)$. The term with coefficient $c$ then gives 33 entries to all the Dirac mass matrices $N$, $U$, $D$, and $L$. These contributions are denoted “1” in the matrices shown below. The term with coefficient $d$ gives contributions only to the matrices $D$ and $L$. This can be seen by looking at the $SU(5)$ decomposition of this term: $d[10(16) 5(10)] 5(16'_H)$. This reduces to a term proportional to $10_3(\cos \theta 5_3 + \sin \theta 5_2) 5_H$. These contributions are denoted by “$\sigma$” in the matrices below. Note that these contributions are “lopsided”, giving only a contribution to $D_{23}$ but not $D_{32}$ and to $L_{32}$ but not $L_{23}$. Finally, the higher-dimension operator with coefficient $g_i$ gives terms

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proportional to $16_3(g_316_3 + g_216_2 + g_116_1)$, which are denoted by “$\epsilon_i$” in the matrices below. Altogether, then, the matrices have the form:

$$U = \begin{pmatrix} 0 & 0 & -\epsilon_1/3 \\ 0 & 0 & -\epsilon_2/3 \\ \epsilon_1/3 & \epsilon_2/3 & 1 \end{pmatrix} M_U, \quad D = \begin{pmatrix} 0 & 0 & -\epsilon_1/3 \\ 0 & 0 & \sigma s_\theta - \epsilon_2/3 \\ \epsilon_1/3 & \epsilon_2/3 & 1 + \sigma c_\theta \end{pmatrix} M_D,$$

$$L \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon \\ -\epsilon_1 & \sigma s_\theta - \epsilon_2 & 1 + \sigma c_\theta \end{pmatrix} M'_D$$

(13)

where $s_\theta \equiv \sin \theta$, $c_\theta \equiv \cos \theta$. By rotations in the 1-2 planes these can be brought to the forms:

$$U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon/3 \\ 0 & \epsilon/3 & 1 \end{pmatrix} M_U, \quad D \cong \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma s_\theta - \epsilon_2/3 \\ 0 & \epsilon/3 & 1 + \sigma c_\theta \end{pmatrix} M_D,$$

$$L \cong \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon \\ -\epsilon_1 & \sigma s_\theta - \epsilon_2 & 1 + \sigma c_\theta \end{pmatrix} M'_D$$

(14)

where $\epsilon \equiv \sqrt{\epsilon_1^2 + \epsilon_2^2}$. These matrices go over to those of the model of [5] in the limit that $\cos \theta \to 0$ and $\epsilon_1/\epsilon_2 \to 0$. As far as the fits to the quark masses and mixings and the charged lepton masses are concerned, the parameter $\epsilon_1$ makes very little difference. (It does, however, make a contribution to the neutrino mixing parameter $U_{e3}$.) The presence of the $\cos \theta$ and $\sin \theta$ in these matrices is important, on the other hand, since it introduces an additional parameter compared to the model of [5]. There it was found that an excellent fit was obtained with $\sigma \cong 1.7$ and $\epsilon \cong 0.14$. Here, because of the additional parameter $\theta$ a slightly better fit is possible. We find the best fit to be $\sigma \cong 1.6$, $\epsilon \cong 0.15$, and $\cos \theta \cong 0.13$.

Essentially, then, the model is the same as that in [5], although slightly less predictive. It has the same important feature of highly “lopsided” mass matrices $D$ and $L$, i.e. $D_{32} \ll D_{23} \sim 1$ and $L_{23} \ll L_{32} \sim 1$. This, as emphasized in [5], gives a natural explanation of why the atmospheric neutrino
mixing \( U_{\mu 3} \) is so large. Other important features are the natural explanation of the Georgi-Jarlskog factor and of the fact that \( m_c/m_t \ll m_s/m_b \). The reader is referred to \cite{5} for further details.

The interesting thing is that we have succeeded in taking a highly successful model of quark and lepton masses that exists in the literature and that was constructed by means of flavor symmetries which distinguish among the three families, and constructing a model that is virtually the same without making any use of flavor symmetries of that kind.

What we have shown is that in the context of grand unification it is possible to construct interesting and predictive models which reproduce the important features of the quark and lepton mass spectrum, while at the same time treating all three families on exactly the same footing, that is, giving them the same quantum numbers. In other words, one can have flavor without flavor symmetry.
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Fig. 1. The diagram that leads to the effective operator given in Eq. (2).