LIPSCHITZ SPACES AND CALDERÓN-ZYGMUND OPERATORS ASSOCIATED TO NON–DOUBLING MEASURES

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Abstract. In the setting of $\mathbb{R}^d$ with an $n$–dimensional measure $\mu$, we give several characterizations of Lipschitz spaces in terms of mean oscillations involving $\mu$. We also show that Lipschitz spaces are preserved by those Calderón-Zygmund operators $T$ associated to the measure $\mu$ for which $T(1)$ is the Lipschitz class 0.

1. Introduction

In the sequel $\mathbb{R}^d$ will denote the euclidean space of vectors $x = (x_1, \ldots, x_d)$ with $d$ real components and $\mu$ a Radon $n$–dimensional measure on $\mathbb{R}^d$. We say that $\mu$ is an $n$–dimensional measure, $n \in \mathbb{R}$, $0 < n \leq d$, if it satisfies the following growth condition: There is a constant $C$ such that

\begin{equation}
\mu(B(x, r)) \leq Cr^n
\end{equation}

for every ball $B(x, r)$ with center $x \in \mathbb{R}^d$ and radius $r > 0$. This allows, in particular, non-doubling measures.

The study of Calderón-Zygmund operators associated to an $n$–dimensional measure was carried out, in the Lebesgue spaces, by Nazarov, Treil and Volberg (see [NTV1, NTV3]) and also by Tolsa (see [To1, To2]). Further results, dealing with $BMO$ and $H^1$ and providing boundedness criteria in the spirit of the $T(1)$ or $T(b)$ theorems, were obtained as well (see [NTV3, MN0, To3])

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Supported in part by DGES, Spain, under grant BFM2001-0189. It is a pleasure for the second author, to thank the members of the Mathematics Department of Universidad Autónoma de Madrid for their friendly hospitality.
In [GG] we have also studied the theory of fractional integral operators associated to an \( n \)-dimensional measure \( \mu \) on Lebesgue spaces and Lipschitz spaces.

The present paper is devoted to further study the Lipschitz spaces associated to an \( n \)-dimensional measure \( \mu \) on \( \mathbb{R}^d \) and Calderón-Zygmund operators associated to \( \mu \) on these spaces.

In section 2, we obtain several characterizations in terms of mean oscillations of functions in the Lipschitz spaces associated to \( \mu \). Then these characterizations are used in section 3 to establish the boundedness of Calderón-Zygmund operators.

From now on, all balls that we consider will be centered at points in the support of \( \mu \).

2. Characterization of Lipschitz spaces

In order to prove the main theorem of this section we will need the following known definition and lemma (see [163]):

**Definition 2.1.** Let \( \beta \) be a fixed constant. A ball \( B \) is called \( \beta \)-doubling if

\[
\mu(2B) \leq \beta \mu(B).
\]

**Lemma 2.2.** Let \( f \in L^1_{\text{loc}}(\mu) \). If \( \beta > 2^d \), then, for almost every \( x \) with respect to \( \mu \), there exists a sequence of \( \beta \)-doubling balls \( B_j = B(x, r_j) \) with \( r_j \to 0 \), such that

\[
\lim_{j \to \infty} \frac{1}{\mu(B_j)} \int_{B_j} f(y) \, d\mu(y) = f(x).
\]

**Proof.** We will show that for almost every \( x \) with respect to \( \mu \) there is a \( \beta \)-doubling ball centered at \( x \) with radius as small as we wish. This fact, combined with the differentiation theorem, completes the proof of the lemma.

We know that for almost every \( x \) with respect to \( \mu \)

\[
\lim_{r \to 0} \frac{\mu(B(x, r))}{r^d} > 0
\]

(differentiation of \( \mu \) with respect to Lebesgue measure, see [163]). Now for \( x \) satisfying (2.1) take \( B = B(x, r) \) and assume that none of the balls \( 2^{-k}B \), \( k \geq 1 \), is \( \beta \)-doubling. Then it easy to see that \( \mu(B) > \beta^k \mu(2^{-k}B) \) for all \( k \geq 1 \). Therefore

\[
\frac{\mu(2^{-k}B)}{(2^{-k}r)^d} < \left( \frac{2^d}{\beta} \right)^k \frac{\mu(B)}{r^d}.
\]
Note that, since $\beta > 2d$, the right hand side tends to zero for $k \to \infty$, which is a contradiction.

Now we can state and prove the main result of this section.

**Theorem 2.3.** For a function $f \in L^1_{\text{loc}}(\mu)$, the conditions I, II, and III below, are equivalent

(I) There exist some constant $C_1$ and a collection of numbers $f_B$, one for each ball $B$, such that these two properties hold: For any ball $B$ with radius $r$

\[
\frac{1}{\mu(2B)} \int_B |f(x) - f_B| \, d\mu(x) \leq C_1 r^\alpha,
\]

and for any ball $U$ such that $B \subset U$ and radius $(U) \leq 2r$.

\[
|f_B - f_U| \leq C_1 r^\alpha,
\]

(II) There is a constant $C_2$ such that

\[
|f(x) - f(y)| \leq C_2 |x - y|^\alpha
\]

for $\mu$—almost every $x$ and $y$ in the support of $\mu$.

(III) For any given $p$, $1 \leq p \leq \infty$, there is a constant $C(p)$, such that for every ball $B$ of radius $r$, we have

\[
\left( \frac{1}{\mu(B)} \int_B |f(x) - m_B(f)|^p \, d\mu(x) \right)^{1/p} \leq C(p) r^\alpha,
\]

where $m_B(f) = \frac{1}{\mu(B)} \int_B f(y) \, d\mu(y)$ and also for any ball $U$ such that $B \subset U$ and radius $(U) \leq 2r$.

\[
|m_B(f) - m_U(f)| \leq C(p) r^\alpha
\]

In addition, the quantities: $\inf C_1$, $\inf C_2$, and $\inf C(p)$ with a fixed $p$ are equivalent.

**Proof.** (I) $\Rightarrow$ (II). Consider $x$ as in the Lemma and let $B_j = B(x, r_j)$, $j \geq 1$, a sequence of $\beta$—doubling balls with $r_j \to 0$. We will show first that (2.2) implies

\[
\lim_{j \to \infty} f_{B_j} = f(x).
\]

It suffices to observe that

\[
|m_{B_j}(f) - f_{B_j}| \leq \frac{1}{\mu(B_j)} \int_{B_j} |f(y) - f_{B_j}| \, d\mu(y)
\]

\[
\leq \frac{\mu(2B_j)}{\mu(B_j)} \frac{1}{\mu(2B_j)} \int_{B_j} |f(y) - f_{B_j}| \, d\mu(y) \leq \beta C_1 r_j^\alpha.
\]
Next, let \( x \) and \( y \) be two points as in the lemma. Take \( B = B(x,r) \) any ball with \( r \leq |x - y| \) and let \( U = B(x,2|x - y|) \). Now define \( B_k = B(x,2^k r) \), for \( 0 \leq k \leq \bar{k} \), where \( \bar{k} \) is the first integer such that \( 2^k r \geq |x - y| \). Then

\[
|f_B - f_U| \leq \sum_{k=0}^{\bar{k}-1} |f_{B_k} - f_{B_{k+1}}| + |f_{B_{\bar{k}}} - f_U| \\
\leq C_1 \sum_{k=0}^{\bar{k}} \left( 2^k r \right)^\alpha \leq C' C_1 |x - y|^\alpha ,
\]

where \( C' \) is independent of \( x \) and \( B \).

A similar argument can be made for the point \( y \) with any ball \( B' = B(y,s) \) such that \( s \leq |x - y| \) and \( V = B(y,3|x - y|) \). Therefore

\[
|f_B - f_{B'}| \leq |f_B - f_U| + |f_U - f_V| + |f_V - f_{B'}| \leq C'' C_1 |x - y|^\alpha .
\]

Finally, take two sequences of \( \beta \)-doubling balls \( B_j = B(x,r_j) \) and \( B'_j = B(y,s_j) \) with \( r_j \to 0 \) and \( s_j \to 0 \). We have

\[
|f(x) - f(y)| = \lim_{j \to \infty} \left| f_{B_j} - f_{B'_j} \right| \leq C'' C_1 |x - y|^\alpha .
\]

\((II) \Rightarrow (III)\). It is immediate. Note also that \((II) \Rightarrow (I)\) is immediate as well.

\((III) \Rightarrow (I)\). Define first \( f_B = m_B(f) \). Then (2.3) is exactly (2.6). To prove (2.2), we write

\[
\frac{1}{\mu(2B)} \int_B |f(y) - f_B| \, d\mu(y) \leq \frac{1}{\mu(2B)} \left( \int_B |f(y) - f_B|^p \right)^{1/p} \mu(B)^{1/p'} \\
\leq \frac{\mu(B)}{\mu(2B)} \left( \frac{1}{\mu(B)} \int_B |f(y) - f_B|^p \, d\mu(y) \right)^{1/p} \leq C(p) r'^\alpha .
\]

This concludes the proof of the theorem.

\(\square\)

Remark 2.4. Theorem 2.3 is also true if the number 2 in condition (I) is replaced by any fixed \( \rho > 1 \). In that case, the proof uses \( (\rho, \beta) \)-doubling balls, that is, balls satisfying \( \mu(\rho B) \leq \beta \mu(B) \). However this extension is not needed in our paper.

The idea of combining the mean oscillation condition with an extra condition as in (I) originates in the work of Tolsa [To3] on regular BMO, whereas the introduction of the \( \rho \) factor in (I) comes from [NTV3].
Definition 2.5. We shall call Lipschitz function of order $\alpha$ with respect to $\mu$ to a function, or rather the corresponding Lebesgue class in $L^1_{\text{loc}}(\mu)$, which satisfies any, and hence all, of the conditions of theorem 2.3.

The linear space of all Lipschitz functions of order $\alpha$, with respect to $\mu$, modulo constants, becomes, with the norm $\inf C_2$ of Theorem 2.3, a Banach space, which we shall call $\text{Lip}(\alpha, \mu)$.

Remark 2.6. It is easy to see that $\text{Lip}(\alpha, \mu)$ coincides with the space of Lipschitz functions of order $\alpha$ on the support of $\mu$.

Note that by the extension theorem of Banach (see [B] or [Mi]), any Lipschitz function of order $\alpha$ with respect to $\mu$ has an extension to $\mathbb{R}^d$ that is a Lipschitz function of order $\alpha$ with an equivalent norm.

Remark 2.7. For $0 < \alpha \leq 1$, a telescoping argument like the one used in the proof of (I) $\Rightarrow$ (II) in Theorem 2.3 shows that (2.3) is equivalent to

$$|f_B - f_U| \leq C'_1 \text{radius}(U)^\alpha$$

for any two balls $B \subset U$. Furthermore (2.7) is also equivalent to

$$|f_B - f_U| \leq C''_1 K_{B,U} \text{radius}(U)^\alpha,$$

for any two balls $B \subset U$, where $K_{B,U}$ is the constant introduced by X. Tolsa in [To3], given by

$$K_{B,U} = 1 + \sum_{j=1}^{N_{B,U}} \frac{\mu(2^j B)}{(2^j B)^\alpha},$$

with $N_{B,U}$ equal to the first integer $k$ such $2^k \text{radius}(B) \geq \text{radius}(U)$. Indeed (2.8) for comparable balls, that is, for $\text{radius}(U) \leq 2 \text{radius}(B)$, reduces to (2.3) because, in this case, $K_{B,U}$ is controlled by an absolute constant.

Note that (2.2) and (2.8) make sense also for $\alpha = 0$ and the space defined by them is the space $\text{RBMO}(\mu)$ of X. Tolsa (see [To3]). Therefore, the spaces $\text{Lip}(\alpha, \mu)$, $0 < \alpha \leq 1$ can be seen as members of a family containing also $\text{RBMO}(\mu)$.

3. Calderón-Zygmund operators

Definition 3.1. An “$n$-dimensional” standard kernel $K$ on $\mathbb{R}^d$ will be a function $K \in L^1_{\text{loc}}(\mathbb{R}^d \times \mathbb{R}^d \setminus \{(x, y) : x = y\}, \mu \times \mu)$, satisfying the following two conditions for some constants $C > 0$ and $\varepsilon \in [0, 1]$:
\((i)\) \( |K(x, y)| \leq \frac{C}{|x - y|^n} \), for all \( x \neq y \) and
\[(ii)\] \( |K(x, y) - K(x', y)| \leq C \frac{|x - x'|^\varepsilon}{|x - y|^{n+\varepsilon}} \) \(\text{for } |x - y| \geq 2 |x - x'|.\)

When we want to be precise about \( \varepsilon \), we shall say that \( K \) is an \( n \)-dimensional, \( \varepsilon \)-regular, standard kernel.

**Definition 3.2.** By a Calderón-Zygmund operator on \( \mathbb{R}^d \) with respect to \( \mu \), we shall mean a linear operator \( T \), bounded on \( L^2(\mu) \), which is associated to an \( n \)-dimensional standard kernel \( K \) in the sense that for every \( f \in L^2(\mu) \), with compact support
\[
T f(x) = \int_{\mathbb{R}^d} K(x, y) f(y) \, d\mu(y), \quad \text{for } \mu\text{-almost every } x \in \mathbb{R}^d \setminus \text{supp } f.
\]

**Remark 3.3.** We have not required \( \varepsilon \)-regularity of \( K \) in the second variable, as is usually done, because we do not need it for proving Theorem 3.6.

The definition of a Calderón-Zygmund operator \( T \) can be extended to Lipschitz functions in the following way:

**Definition 3.4.** Assume that \( T \) is a Calderón-Zygmund operator associated to \( \mu \) having an \( \varepsilon \)-regular kernel. First, for each ball \( B \), with center \( x_B \), \( x \in B \) and \( f \in \text{Lip}(\alpha, \mu) \), \( 0 < \alpha < \varepsilon \), we define
\[
(3.1) \quad T_B(f)(x) = T(f \chi_{2B})(x) + \int_{\mathbb{R}^d \setminus 2B} (K(x, z) - K(x_B, z)) f(z) \, d\mu(z)
\]

Now \( T(f) \) is defined as the class of all functions \( g \) on \( \mathbb{R}^d \) such that for any ball \( B \) there is a constant \( c_B \) satisfying
\[
g(x) = T_B(f)(x) + c_B \quad \mu\text{-almost everywhere on } B.
\]

**Remark 3.5.** In order to see that \( T(f) \) is well defined, it suffices to realize that if \( B \subset V \) are balls, then \( T_B(f) - T_V(f) \) is constant \( \mu\)-a.e. over \( B \). Then, considering an increasing sequence of balls whose union is \( \mathbb{R}^d \), we can see that there is a function \( g \) such that \( g - T_B(f) \) is constant \( \mu\)-a.e. on \( B \) for each ball \( B \) in the sequence. Consider now any ball \( B \). Since this ball \( B \) is contained in some ball of the sequence, by our previous observation, \( g - T_B(f) \) is constant \( \mu\)-a.e. on \( B \).

**Theorem 3.6.** Let \( T \) be a Calderón-Zygmund operator associated to \( \mu \) having an \( \varepsilon \)-regular kernel and \( 0 < \alpha < \varepsilon \leq 1 \). Then \( T \) is bounded on \( \text{Lip}(\alpha, \mu) \) if and only if \( T(1) = 0 \) in \( \text{Lip}(\alpha, \mu) \).
Proof. The fact that $T(1) = 0$ in $\mathcal{L}_{\text{lip}}(\alpha, \mu)$ is a necessary condition follows immediately from the fact that $\|1\|_{\mathcal{L}_{\text{lip}}(\alpha, \mu)} = 0$.

To prove that this condition is sufficient, take $f \in \mathcal{L}_{\text{lip}}(\alpha, \mu)$ and two balls $B \subset U$ with $\text{radius}(U) \leq 2r$, where $r$ is the radius of $B$. We want to show that there are two constants $a_B$ and $a_U$ such that

$$\tag{3.2} \frac{1}{\mu(2B)} \int_B |T_U(f)(x) - a_B| \, d\mu(x) \leq C \|f\|_{\mathcal{L}_{\text{lip}}(\alpha, \mu)} r^\alpha$$

and

$$\tag{3.3} |a_B - a_U| \leq C \|f\|_{\mathcal{L}_{\text{lip}}(\alpha, \mu)} r^\alpha$$

with $C$ independent of $B, U$ and $f$.

Since $T(f)(x) = T_U(f)(x) + c_U$ on $U$, where $c_U$ is a constant, then (3.2) and (3.3) follow for $T(f)$ with constants $a'_B = a_B + c_U$ and $a'_U = a_U + c_U$.

By the hypothesis $T(1) = 0$, we can assume that $\int_{2U} f \, d\mu = 0$. Let’s now define

$$a_B = \frac{1}{\mu(2B)} \int_B T_U(f)(x) \, d\mu(x)$$

and, similarly $a_U$ with $U$ instead of $B$. Observe that

$$\frac{1}{\mu(2B)} \int_B |T_U(f)(x) - a_B| \, d\mu(x) \leq \frac{1}{\mu(2B)} \int_B |T_U(f)(x)| \, d\mu(x) + \frac{1}{\mu(2B)} \int_B |a_B| \, d\mu(x) \leq 2 \frac{1}{\mu(2B)} \int_B |T_U(f)(x)| \, d\mu(x),$$

since $\frac{\mu(B)}{\mu(2B)} \leq 1$. On the other hand

$$|a_B - a_U| \leq \frac{1}{\mu(2B)} \int_B |T_U(f)(x)| \, d\mu(x) + \frac{1}{\mu(2U)} \int_U |T_U(f)(x)| \, d\mu(x).$$

Now we want to estimate the quantity

$$\tag{3.4} \frac{1}{\mu(2B)} \int_B |T_U(f)(x)| \, d\mu(x),$$

where $B$ can also be $U$. To do that we write

$$T_U(f)(x) = T(f \chi_{2B})(x) + T(f \chi_{2U \setminus 2B})(x)$$

$$+ \int_{\mathbb{R}^d \setminus 2U} (k(x, z) - k(x_U, z)) \, f(z) \, d\mu(z).$$
We then have
\[
\frac{1}{\mu(2B)} \int_B |T_U(f)(x)| \, d\mu(x) \\
\leq \frac{1}{\mu(2B)} \int_B |T(f \chi_{2B})(x)| \, d\mu(x) + \frac{1}{\mu(2B)} \int_B |T(f \chi_{2U \setminus 2B})(x)| \, d\mu(x) \\
+ \frac{1}{\mu(2B)} \int_{\mathbb{R}^d \setminus 2U} (k(x, z) - k(x_U, z)) f(z) \, d\mu(z) \, d\mu(x) = A_1 + A_2 + A_3.
\]

To estimate $A_1$, we use that $T$ is bounded in $L^2(\mu)$ and the fact that
\[
|m_{2B}(f)| \leq C \|f\|_{\mathcal{L}_{\text{Lip}}(\alpha, \mu)} r^\alpha,
\]
which follows from
\[
|m_{2B}(f) - m_{2U}(f)| \leq C \|f\|_{\mathcal{L}_{\text{Lip}}(\alpha, \mu)} r^\alpha \text{ and } m_{2U}(f) = 0.
\]
We have
\[
\frac{1}{\mu(2B)} \int_B |T(f \chi_{2B})(x)| \, d\mu(x) \leq \left( \int_B |T(f \chi_{2B})(x)|^2 \, d\mu(x) \right)^{1/2} \frac{\mu(B)^{1/2}}{\mu(2B)} \\
\leq C \left( \int |f \chi_{2B}|^2 \, d\mu \right)^{1/2} \frac{\mu(B)^{1/2}}{\mu(2B)} \\
\leq C \left( \frac{1}{\mu(2B)} \int_{2B} |f(z) - m_{2B}(f)|^2 \, d\mu(z) \right)^{1/2} \\
+ C \left( \frac{1}{\mu(2B)} \int_{2B} |m_{2B}(f)|^2 \, d\mu(z) \right)^{1/2} \leq C \|f\|_{\mathcal{L}_{\text{Lip}}(\alpha, \mu)} r^\alpha.
\]

For $A_2$, note first that in case $B = U$, $A_2$ doesn’t appear. Otherwise, since $x \notin \text{supp} (f \chi_{2U \setminus 2B})$ and $\mu(2U) \leq C(4r)^n$, we have
\[
\frac{1}{\mu(2B)} \int_B \int_{2U \setminus 2B} |k(x, z)||f(z)| \, d\mu(z) \, d\mu(x) \\
\leq \frac{1}{\mu(2B)} \int_B C \int_{2U} |f(z)| \, d\mu(z) \, d\mu(x) \\
\leq C \frac{\mu(2U)}{r^n} \frac{1}{\mu(2B)} \int_{2U} |f(z) - m_{2U}(f)| \, d\mu(z) \leq C \|f\|_{\mathcal{L}_{\text{Lip}}(\alpha, \mu)} r^\alpha.
\]

Finally for $A_3$ using the $\epsilon-$regularity of the kernel, we have
\[
A_3 \leq \frac{1}{\mu(2B)} \int_B \int_{\mathbb{R}^d \setminus 2U} \frac{|x - x_U|^\epsilon}{|x_U - z|^{n+\epsilon}} |f(z)| \, d\mu(z) \, d\mu(x) \\
\leq C \frac{\mu(B)}{\mu(2B)} \|f\|_{\mathcal{L}_{\text{Lip}}(\alpha, \mu)} r^\alpha \leq C \|f\|_{\mathcal{L}_{\text{Lip}}(\alpha, \mu)} r^\alpha.
\]
The inequalities above are justified by the fact that, for $z \notin 2U$

$$|f(z)| = |f(z) - m_{2U}(f)| \leq \frac{1}{\mu(2U)} \int_{2U} |f(z) - f(y)| \, d\mu(y) \leq (2|z - x_U|)^\alpha \|f\|_{Lip(\alpha, \mu)}$$

and also, that

$$\int_{\mathbb{R}^d \setminus 2U} \frac{1}{|z - x_U|^{n+\epsilon-\alpha}} \, d\mu(z) \leq C r^{\alpha - \epsilon}$$

(see [GG]). This concludes the proof of the theorem.

□

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