Strange matter and its stability in presence of magnetic field

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Abstract

We study the effect of a magnetic field on the interacting quark matter and apply to strange star. We consider the low temperature approximation to strange matter. We find that the interacting strange quark matter is more stable compare to free quark gas in presence of strong external magnetic field with zero and finite temperature. We then calculate strange star structure parameters such as mass and radius and find that the strange star is more compact for interacting quark matter than free quark matter in presence of strong magnetic field.

Subject Headings: dense matter- elementary particles - stars: neutron - stars: quark- stars: magnetic field: general

It has been argued that there is a strong magnetic field in the vicinity of astrophysical objects such as neutron stars, white dwarfs and supernovae. Particularly, the magnetic field is very high
in the surface of the neutron stars. The estimation of the magnetic field strength at the surface of neutron stars is done in theoretical models of pulsar emission, (Ruderman 1972) the accretion flow in the binary X-ray sources (Ghosh and Lamb 1978) and observation of features in the spectra of pulsating X-ray sources e.g., cyclotron lines (Trumper, J. et al. 1978, Wheaton et al. 1979, Gruber et al. 1980 and Mihara et al. 1990). It has been observed from a sample of more than 400 pulsars, the surface magnetic field strength is in the range $2 \times 10^{10} \text{G} \leq H \leq 2 \times 10^{13} \text{G}$.

Very recently, two different physical mechanism leading to an amplification of some initial magnetic field in a collapsing star have been proposed by several authors (Duncan and Thompson 1992, Thompson and Duncan 1993 and Bisnovatyi-Kogan 1993). In the first scenario (Bisnovatyi-Kogan 1993), the field amplification leads to the formation of an additional toroidal field from the poloidal one by twisting of the field lines due to rapid differential rotation of proto-neutron stars. The induced toroidal magnetic field could be as large as $H \sim 10^{15} - 10^{17} \text{G}$, after the first twenty seconds of the life of a new-born neutron star. The second mechanism (Duncan and Thompson 1992, Thompson and Duncan 1993) is a dynamo action in a differentially rotating and convective young neutron star, which is responsible for the strengthening of some initial dipole field up to values of $H \sim 10^{12} - 3 \times 10^{13} \text{G}$ or to $H \sim 10^{14} - 10^{15} \text{G}$. The former value is due to the convective episodes arose during the main-sequence and later value is so, if the dipole field is generated after the collapse. So, the new born neutron stars have values of magnetic field as strong as $H \sim 10^{14} - 10^{16} \text{G}$, or even more. In the interior of neutron star, it probably reaches $\sim 10^{18} \text{G}$. Therefore, it is advisable to study the effect of strong magnetic field and corresponding quantum corrections on compact neutron stars.

There are strong reasons for believing that the hadrons are composed of quarks, and the idea of quark stars has already been existed for about twenty years. If the neutron matter density at the core of neutron stars exceeds a few times normal nuclear density a deconfining phase transition to quark matter may take place. As a consequence, a normal neutron star will be
converted to a hybrid star with an infinite cluster of quark matter core and a crust of neutron matter. In 1984, Witten suggested that strange matter, e.g., quark matter with strangeness per baryon of order unity, may be the true ground state (Witten 1984). The properties of strange matter at zero pressure and zero temperature were subsequently examined and it was found that the strange matter can indeed be stable for a wide range of parameters in the strong interaction calculations (Farhi and Jaffe 1984). Therefore, at the core, the strange quarks will be produced through the weak decays of light quarks (u and d quarks) and ultimately a chemical equilibrium will be established among the participants. Since, the strange matter is energetically favorable over neutron matter, there is a possibility that whole star may be converted to a strange star.

In the present paper, we study the effect of strong magnetic field on strange quark matter including interaction up to one order of strong coupling constant. Throughout the calculation, as a first approximation, we have assumed the existence of constant uniform magnetic field $H \approx 10^{16}$ G in the strange star. The interior of the quark star has high conductivity and has therefore zero electric field and uniform current density. The magnetic field $H \propto \frac{I}{R^2} \propto J \sim \text{constant}$, where $I$ is the total current, $J$ is current density and $R$ is the radius of the spherical quark star. Taking into account the low - temperature corrections in the thermodynamic potentials, we investigate the stability of strange matter for finite temperature, finite chemical potentials and finite magnetic field. At the end, we apply strange quark matter equation of state to structure equations of a relativistic spherical static star to calculate the structure parameters of strange quark stars.

For a constant magnetic field along the z-axis ($\vec{A} = (Hy, 0, 0)$), the single energy eigenvalue is given by (Landau and Lifshitz 1965)

$$\varepsilon_{p,n,s} = \sqrt{p_i^2 + m_i^2 + q_i H (2n + s + 1)} ,$$

where $n=0, 1, 2, ..., $ being the principal quantum numbers for allowed Landau levels, $s = \pm 1$ refers to spin up(+) and down(−) and $p_i$ is the component of particle(species i) momentum along
the field direction. Setting $2n + s + 1 = 2\nu$, where $\nu = 0, 1, 2...$, we can rewrite the single particle energy eigenvalue in the following form

$$\varepsilon_i = \sqrt{p_i^2 + m_i^2 + 2\nu_q_i H}.$$  \hfill (2)

Now, it is very easy to see that $\nu = 0$ state is singly degenerate, whereas, all other states with $\nu \neq 0$ are doubly degenerate. Thus we set $b_\nu = 2 - \delta_{\nu0}$. For $\nu = 0$, $b_\nu = 1$, which is the lowest Landau level. Since the temperature $T << \mu_i$ at the core of quark star, the presence of anti-particles can be ignored. Now instead of infinity the upper limit of $\nu$ sum can be obtained from the following relation

$$p_{Fi}^2 = \mu_i^2 - m_i^2 - 2\nu_q_i H \geq 0,$$  \hfill (3)

where $p_{Fi}$ is the Fermi momentum of the species $i$, which gives

$$\nu \leq \frac{\mu_i^2 - m_i^2}{2q_i H} = \nu_{max}^{(i)} \text{ (nearest integer)}. \hfill (4)$$

Therefore, the upper limit is not necessarily same for all the components. As is well known, the energy of a charged particle changes significantly in the quantum limit if the magnetic field strength is equal to or greater than some critical value $H^{(c)} = m_i^2 c^3/(q_i \hbar)$ (in G), where $m_i$ and $q_i$ are respectively the mass and the absolute value of charge of particle $i$, $\hbar$ and $c$ are the reduced Planck constant and velocity of light respectively, both of which along with Boltzman constant $k_B$ are taken to be unity in our choice of units. For an electron of mass 0.5 MeV, this critical field as mentioned above is $\sim 4.4 \times 10^{13}$G, whereas for a light quark of current mass 5 MeV, this particular value becomes $\sim 4.4 \times 10^{15}$G.

Then the thermodynamic potential in presence of strong magnetic field $H(> H^{(c)}$, critical value discussed later) is given by

$$\Omega_i = -\frac{g_i q_i H T}{4\pi^2} \int d\varepsilon_i \sum_{\nu} \nu_{max}^{(i)} b_\nu \frac{dp_i}{d\varepsilon_i} \ln[1 + \exp(\mu_i - \varepsilon_i)/T], \hfill (5)$$

where $g_i$ is the degeneracy of the species $i$. 
Integrating by parts and substituting

\[ p_i = \pm \sqrt{\varepsilon_i^2 - m_i^2 - 2\nu q_i H} \],

for all \( T \), one finds

\[ \Omega_i = -\frac{g_i q_i H}{4\pi^2} \int d\varepsilon_i \sum_{\nu} b_{\nu} \frac{2\sqrt{\varepsilon_i^2 - m_i^2 - 2\nu q_i H}}{e^{\varepsilon_i - \mu_i}/T + 1} \]

where the sum over \( \nu \) is restricted by the condition \( \varepsilon_i > \sqrt{m_i^2 + 2\nu q_i H} \) and the factor 2 takes into account the freedom of taking either sign in eq(6). For \( T = 0 \), therefore, approximate the Fermi distribution by a step function and interchange the order of the summation over \( \nu \) and integration over \( \varepsilon_i \),

\[ \Omega_i^1 = -\frac{g_i q_i H}{2\pi^2} \sum_{\nu} \left[ \mu_i \sqrt{\mu_i^2 - m_i^2 - 2\nu q_i H} \right] 
- (m_i^2 + 2\nu q_i H) \ln \frac{\mu_i + \sqrt{\mu_i^2 - m_i^2 - 2\nu q_i H}}{\sqrt{m_i^2 + 2\nu q_i H}} \]

which is first order thermodynamic potential.

Considering interaction upto one order of strong coupling constant \( \alpha_c \) (Freedman and McLerran 1978, Alcock et al. 1986 and Haensel et al. 1986), the second order thermodynamic potential in presence of strong magnetic field for zero temperature is given by

\[ \Omega_i^2 = -\frac{g_i q_i H}{4\pi^2} \frac{8\alpha_c}{\pi} \sum_{\nu} b_{\nu} \left( \mu_i^2 - m_i^2 - 2\nu q_i H \right) \ln \frac{\mu_i^2 - m_i^2 - 2\nu q_i H}{q_i H} \]

The low-temperature corrections to the first order and second order thermodynamic potentials have been derived in Ref. (Sahu 1995) and these are:

\[ \Omega_i^1 = \]
\[-\frac{g_i q_i H}{4\pi^2} \sum_{\nu=0} \left[ \mu_i \sqrt{\mu_i^2 - m_i^2 - 2q_i H \nu} - (m_i^2 + 2q_i H \nu) \ln \frac{\mu_i + \sqrt{\mu_i^2 - m_i^2 - 2q_i H \nu}}{\sqrt{m_i^2 + 2q_i H \nu}} \right] 
+ \frac{T^2 \mu_i^2}{6} \left( \mu_i^2 - m_i^2 - 2q_i H \nu \right)^{1/2} \]
\[ \Omega_i^2 = \]
\[-\frac{g_i q_i H}{4\pi^2} \sum_{\nu=0}^{\nu_{\text{max}}} b_{\nu} \frac{8\alpha_e}{\pi} \left\{ \left( \mu_i^2 - m_i^2 - 2q_i H \nu \right) \ln \frac{\mu_i^2 - m_i^2 - 2q_i H \nu}{q_i H} \right\} \]
\[+ \frac{T^2 \pi^2}{6} \left[ 1 + \frac{2\mu_i^2}{\mu_i^2 - m_i^2 - 2q_i H \nu} + \ln \frac{\mu_i^2 - m_i^2 - 2q_i H \nu}{q_i H} \right] \bigg\} . \tag{11} \]

This expansion is valid for \( \frac{T}{\mu_i - \sqrt{m_i^2 + 2q_i H \nu}} \ll 1 \). This means that the distance from the edge of any Landau level \( \varepsilon_{\nu}(p_z) = \sqrt{m_i^2 + 2q_i H \nu} \) to the Fermi surface \( \mu_i \) is much greater than the temperature, or in otherwords, the Landau level with \( \nu = \nu_{\text{max}} \) should be partially filled, i.e., its edge cannot concide with the Fermi surface.

So, the total thermodynamic potential is

\[ \Omega_i(T, H, \mu_i) = \Omega_i^1(T, H, \mu_i) + \Omega_i^2(T, H, \mu_i). \tag{12} \]

In our study, we assume that strange quark matter is charge neutral and also chemical equilibrium, then

\[ \mu_d = \mu_s = \mu = \mu_u + \mu_e, \tag{13} \]

and charge neutrality conditions gives

\[ 2n_u - n_d - n_s - 3n_e = 0. \tag{14} \]

The baryon number density of the system is given by

\[ n_B = \frac{1}{3} (n_u + n_d + n_s). \tag{15} \]

Using above eqs(13, 14, 15) one can solve numerically for the chemical potentials of all the flavors and electron. Having the expression for the total thermodynamic potential, we may move forward to calculate the number density \( -\partial \Omega_i / \partial \mu_i \) and the magnetization \( -\partial \Omega_i / \partial H \) of the
species $i$ ($u$, $d$, $s$, $e$) (Sahu 1995). Using Eq. (12) and definition of number density, one has the following expression:

\[ n_i(H, \mu, T) = \]

\[ \frac{g_i q_i H}{2\pi^2} \sum_{\nu=0}^{\nu} b_{\nu} \sqrt{\mu_i^2 - m_i^2 - 2q_i H \nu} \left\{ 1 - \frac{T^2 \pi^2}{12} \frac{m_i^2 + 2q_i H \nu}{(\mu_i^2 - m_i^2 - 2q_i H \nu)^2} \right\} + \frac{8\alpha_c}{\pi} \mu_i \left( \frac{\mu_i^2 - m_i^2 - 2q_i H \nu}{q_i H} \right) \ln \left( \frac{\mu_i^2 - m_i^2 - 2q_i H \nu}{q_i H} \right) \]

\[ - \frac{T^2 \pi^2}{6} \mu_i \left( \frac{3(m_i^2 + 2q_i H \nu) - \mu_i^2}{(\mu_i^2 - m_i^2 - 2q_i H \nu)^{5/2}} \right) \]  \tag{16}

Now, for $T = 0$, we have the number density of the species $i$:

\[ n_i(H, \mu, T = 0) = \]

\[ \frac{g_i q_i H}{2\pi^2} \sum_{\nu=0}^{\nu} b_{\nu} \left\{ \sqrt{\mu_i^2 - m_i^2 - 2q_i H \nu} + \frac{8\alpha_c}{\pi} \mu_i \left( 1 + \ln \frac{\mu_i^2 - m_i^2 - 2q_i H \nu}{q_i H} \right) \right\} \]  \tag{17}

In figure 1, we have plotted the number density of $u$ quarks as a function of magnetic field for fixed chemical potential ($\mu_u = 20$ MeV). The $u$ quark density is showing an oscillating behavior as consecutive Landau levels are passing the Fermi level. The number density is low for case (a), where $\alpha_c = 0.0$ and the number density increases with increase in $\alpha_c = 0.01$ for case (b) and $\alpha_c = 0.05$ for case (c) respectively. Thus, we noticed that the number density is high for strongly interacting quarks.

In the weak magnetic field limit ($H_0 \ll (\mu_i^2 - m_i^2)$), one may reduce the number density as follows:

\[ n_i(H_0 \ll (\mu_i^2 - m_i^2), \mu, T = 0) \approx \frac{g_i}{4\pi^2} \left\{ \frac{2}{3}(\mu_i^2 - m_i^2)^{3/2} \right\} \]
where, we can substitute in the limit $2q_i H \nu = \theta$

$$\sum_{\nu=0} \left[ \frac{\mu_i^2 - m_i^2}{2q_i H \nu} \right] = \int_0^\theta \left[ \frac{\mu_i^2 - m_i^2}{2q_i H} \right] d \nu = \lim_{H \to 0} \frac{1}{2q_i H} \int_0^{(\mu_i^2 - m_i^2)} d \theta$$

The ratio of $u$ quark density, $n_u(H_0 \ll (\mu_u^2 - m_u^2))/n_u(H)$ as function of chemical potentials for fixed magnetic field, curve (a) $H \approx 5 \times 10^{15}$ G and curve (b) $H \approx 10^{16}$ G, is shown in figure 2. We choose the weak field limit, $H_0$ to be $4.4 \times 10^{13}$ G. As the chemical potential increase, the ratio decreases and reaches to unity for both curves (a) and (b). We noticed that the magnetic field has significant contribution to the quark densities and hence to the total thermodynamic potential. For illustration purpose, we choose here the $u$ quark density.

From the definition of magnetization $M = -\frac{\partial \Omega}{\partial H}$ where $M(H, \mu, T) = M(H) + \tilde{M}(H, \mu, T)$, $M(H)$ is the vacuum magnetization, one has

$$\tilde{M}(H, \mu, T = 0) = \frac{g q_i}{4\pi^2} \sum_{\nu=0} b_{\nu} \left\{ \mu_i \sqrt{\mu_i^2 - m_i^2 - 2q_i H \nu} - (m_i^2 + 4q_i H \nu) \ln \left( \frac{\mu_i + \sqrt{\mu_i^2 - m_i^2 - 2q_i H \nu}}{\sqrt{m_i^2 + 2q_i H \nu}} \right) \right\} + \frac{8\alpha_c}{\pi} \left[ (\mu_i^2 - m_i^2 - 4q_i H \nu) \ln \frac{\mu_i^2 - m_i^2 - 2q_i H \nu}{q_i H} - (\mu_i^2 - m_i^2) \right].$$

One may calculate the magnetic susceptibility $\chi$, which is defined as $(\partial M/\partial H)$.

Figure 3 shows the magnetization as a function of magnetic field for fixed chemical potential ($\mu_u = 20$ MeV). We have not considered the vacuum magnetization in this figure, because the vacuum contribution is small (Sahu 1995). We noticed that the quark gas exhibits the de Hass - van Alphen effect. We considered the strong coupling constant in curves (a) $\alpha_c = 0.0$, (b) $\alpha_c = 0.01$ and (c) $\alpha_c = 0.05$ respectively for illustration purpose. The magnetization is high at
small value magnetic field and approaches to case (a) (e.g., free quark gas) for large magnetic field. Thus the de Hass - van Alphen effect amplifies with increase in strength of the strong coupling constant.

The total energy density and the external pressure of the strange quark matter is given respectively by

\[ \varepsilon = \sum_i \Omega_i + B + \sum_i n_i \mu_i - T \left( \frac{\partial \Omega_i}{\partial T} \right) \mu_i \]

\[ p = - \sum_i \Omega_i - B, \] (21)

where \( i = u, d, s, e \). The last term is the total energy density is entropy of the system, which is non zero and is given by

\[ - T \left( \frac{\partial \Omega_i}{\partial T} \right) \mu_i \]

\[ = \frac{g_i q_i H}{4 \pi^2} \sum_{\nu=0}^1 b_\nu \frac{T^2 \pi^2}{3} \left\{ \frac{\mu_i}{(\mu_i^2 - m_i^2 - 2q_i H \nu)^{1/2}} \right\} \]

\[ + \frac{8\alpha_c}{\pi} \left[ 1 + \frac{2\mu_i^2}{\mu_i^2 - m_i^2 - 2q_i H \nu} + \ln \frac{\mu_i^2 - m_i^2 - 2q_i H \nu}{q_i H} \right]. \] (22)

Here, we have considered the conventional bag model for the sake of simplicity in presence of magnetic field. We are assuming that the self interacting quarks are moving within the system and as usual the current masses of both u and d quarks are extremely small, e.g., 5 MeV each, whereas, for s-quark the current quark mass is to be taken 150 MeV. We choose the bag pressure \( B \) to be 56 MeV \( fm^{-3} \) and the strong coupling constant \( \alpha_c < 1 \). Also, we set the magnetic field to be \( H \approx 10^{16} \) G in our calculations. Since we choose the magnetic field along the z-axis, it follow from energy-stress tensor that the constant uniform magnetic field \( (H = 10^{16} G) \) contribute to the pressure and the energy density by \( \frac{H^2}{8\pi} \), which is much smaller than the bag pressure, e.g., 0.0025 \( << 56 MeV \) \( fm^{-3} \).
The variation of energy per baryon with baryon number density is shown in figure 4. The curve (a) is for free quark gas, $\alpha_c = 0.0$ at zero temperature and curve (b) is for interacting quark gas with interaction strength $\alpha_c = 0.1$ at zero temperature, whereas, the curve (c) is for $\alpha_c = 0.1$ and temperature $T = 50$ MeV. For all the cases, the magnetic field strength is $\approx 10^{16}$ G. We concluded from the figure that the interaction strength and finite temperature make the strange quark matter energetically more stable compare to the zero temperature and zero interaction strength.

Next, we have shown the variation of pressure with energy density in figure 5. These are the equation of states of strange quark matter. The curve (a) is for free quark gas ($\alpha_c = 0.$) at zero temperature and curve (b) is for interacting quark gas ($\alpha_c = 0.1$) at zero temperature. From this figure, we noticed that the interaction strength makes the quark equation of state soft. Also, we have seen that the softness increases with increase in interaction strength.

The mass and radius for nonrotating strange quark stars are obtained by integrating the structure equations of a relativistic spherical static star composed of a perfect fluid which is derived from Einstein equation. These equations are given in Ref. (Datta et al. 1992, Sahu, Basu and Datta 1993 and Ghosh and Sahu 1993), hence, we are not reproducing here. For a given equation of state, and given central density, the structure equations are integrated numerically with the boundary conditions $m(r = 0) = 0$, to give $R$ and $M$. The radius $R$ is defined by the point where $p \simeq 0$. The total gravitational mass $M$, moment of inertia $I$, surface red shift $z$ and the period $P_0$ corresponding to fundamental frequency $\Omega_0$ are then given by $M = m(R)$, $I = I(R)$, $z = (1-2GM/Re^2)^{-1/2} - 1$ and $P_0 = \frac{2\pi}{\Omega_0}$ respectively, where $\Omega_0 = (\frac{3GM}{4R^3})^{1/2}$ (Cutler, Lindblom and Splinter 1990). These are correspond to maximum mass of stable star that can have, presented in Table 1. Figure 6 shows the variation of mass with central density for two equation of states as illustrated in Fig. 5. We noticed from this figure and table that with increase in interaction strength, the maximum mass and the corresponding radius decrease.
from 1.83 to 1.64 solar mass and from 10.4 km to 9.58 km and therefore, the star becomes more compact. Similarly, the values for surface red shift, moment of inertia and fundamental period decrease. Also, we have noticed that with increase in magnetic field strength, the density also increases and therefore, the star becomes more compact (Chakrabarty and Goyal 1994, Chakrabarty and Sahu 1995 and Chakrabarty 1995).

In conclusion, we concluded that the presence of strong magnetic field in interacting strange quark matter reduces the mass and radius of strange star. That is the star becomes more compact. With inclusion of interaction strength of quarks at finite temperature and at zero temperature in presence of strong magnetic field, the strange quark matter is more stable compare to free quark gas at zero temperature.

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Table 1: The radius \((R)\), mass \((M)\), surface red shift \((z)\), moment of inertia \((I)\) and period of fundamental frequency \((P_0)\) of strange stars versus central density \(\epsilon_c\) for two cases; (a): \(\alpha_c = 0.0\) and (b): \(\alpha_c = 0.1\). For both the cases \(H \approx 10^{16} \text{ G}\). These values are correspond to maximum mass of stable strange star.

| \(\epsilon_c\) | \(R\) | \(M/M_\odot\) | \(z\) | \(I\) | \(P_0\) | Cases |
|----------------|------|---------------|------|------|-------|-------|
| \(g \text{ cm}^{-3}\) | \text{km} | \(g \text{ cm}^{-2}\) | \text{ms} | |
| 2.0 \times 10^{15} | 10.37 | 1.83 | 0.44 | 1.85 \times 10^{45} | 0.49 | (a) |
| 2.5 \times 10^{15} | 9.58 | 1.64 | 0.42 | 1.38 \times 10^{45} | 0.48 | (b) |
Figure 1: The number density of $u$ quarks as a function of magnetic field for fixed chemical potential ($\mu_u = 20$ MeV). Where Case (a) is for $\alpha_c = 0$, case (b) is for $\alpha_c = 0.01$ and case (c) is for $\alpha_c = 0.05$ respectively.

Figure 2: The ratio of $u$ quark density, $n_u(H_0 \ll (\mu_u^2 - m_u^2))/n_u(H)$ as function of chemical potentials for fixed magnetic field, curve (a) $H \approx 5 \times 10^{15}$ G and curve (b) $H \approx 10^{16}$ G. The choice of the weak field limit, $H_0$ is $4.4 \times 10^{13}$ G.

Figure 3: The magnetization as a function of magnetic field for fixed chemical potential ($\mu_u = 20$ MeV). The curves (a) is for $\alpha_c = 0.0$, (b) is for $\alpha_c = 0.01$ and (c)$\alpha_c = 0.05$ respectively.

Figure 4: The variation of energy per baryon with baryon number density with constant magnetic field $H \approx 10^{16}$. The curve (a) is for free quark gas, $\alpha_c = 0.0$ at zero temperature and curve (b) is for interacting quark gas ($\alpha_c = 0.1$) at zero temperature and curve (c) is for $\alpha_c = 0.1$ and temperature $T = 50$ MeV.

Figure 5: Pressure and energy density ($\epsilon$) curves for case (a) $\alpha_c = 0.0$ and case (b) $\alpha_c = 0.1$ with constant magnetic field $H \approx 10^{16}$.

Figure 6: Mass and central density ($\epsilon_c$) curves for two cases as mentioned in figure 5.