Proton emission from the deformed odd-odd nuclei near drip line

M Patial*, P Arumugam¹, A K Jain¹, E Maglione² and L S Ferreira³
¹Department of Physics, Indian Institute of Technology Roorkee, Roorkee - 247667, India.
²Dipartimento di Fisica e Astronomia “G. Galilei”, Via Marzolo 8, I-35131 Padova, Italy and Istituto Nazionale di Fisica Nucleare, Padova, Italy.
³Centro de Física das Interacções Fundamentais, and Departamento de Física, Instituto Superior Técnico, Avenida Rovisco Pais, P1049-001 Lisbon, Portugal.
E-mail: *monikapatial@gmail.com

Abstract. Proton emission from odd-odd nuclei is studied within the two quasiparticle plus rotor model which includes the non-adiabatic effects and the residual interaction between valence proton and neutron. Justification of the formalism is discussed through corroboration of our results with the experimental spectrum of $^{180}$Ta. Exact calculations are performed to get the proton emission half-lives. Our results for the proton emitter $^{130}$Eu leads to the assignment of spin and parity $J^\pi = 1^+$ for the ground state. The role of Coriolis and residual neutron-proton interactions on the proton emission half-lives and their interplay are also discussed.

1. Introduction
Proton decay in the exotic nuclei near proton drip-line mostly takes place from the continuum of energy level spectrum which has single particle character due to Fermi level being closer to the continuum. We can study the physics of these nuclei through the decay width which provides relevant information about the structure and we can infer the details of amplitudes of wave functions of the decaying state. Hence the proton emission decay widths, complemented by the spectra built on the decaying states, is a perfect tool to study the structure and decay of nuclei right on the proton drip line. In a theoretical description within the adiabatic limit, it is assumed that decaying proton moves in the single-particle Nilsson resonances with the unbound core plus neutron system. It has been clearly demonstrated [1] in the case of odd-even nuclei that it is important to consider the nonadiabatic effects, which take care of the rotational excitations of the daughter nucleus, through the Coriolis interaction in a particle plus rotor model approach. A proper treatment of the pairing residual interaction [1] provides a more complete and consistent description of proton emission in agreement with the experimental data. The extension of this model to triaxial nuclei confirmed strong triaxial deformations in $^{145}$Tm [2] and $^{141}$Ho [3]. These studies show how powerful the study of proton radioactivity can be for the knowledge of nuclear structure.

A proper theoretical framework has been formulated recently [4, 5] to study the structure of odd-odd nuclei which is least explored. An exciting feature that can be studied in these nuclei is the interaction between the odd proton and the odd neutron [6]. Though this residual neutron-proton ($np$) interaction could be weak, in combination with the zero point rotational energy, it leads to several possible combinations of spin and parity for the ground state of the nucleus.
Here we present our results for the spectrum of $^{180}$Ta and for the proton emission half-life of $^{139}$Eu. We focus on the role of the Coriolis and the residual $np$ interactions and their interplay while describing the rotational states and the decay widths.

2. The nonadiabatic quasiparticle approach

In this approach, we consider a two quasiparticle plus rotor model [4, 6] based on the mean field of Woods-Saxon potential with deformed spin-orbital potential. The presence of residual pairing interactions are considered by the transformation of single-particle energy to quasiparticle energy through the relation $\tilde{\epsilon}_k = \sqrt{\Delta^2 + (\epsilon_k - \lambda)^2}$, where $\lambda$ is the Fermi energy, $\epsilon_k$ is the single-particle energy and $\Delta$ is the pairing gap chosen as $12/\sqrt{A}$. The complete Hamiltonian for the odd-odd nucleus can be expressed as sum of intrinsic part and rotational part where latter consists of sum of various terms including the Coriolis interaction between valence particles and collective rotation, the particle-particle coupling and the recoil term. We have introduced by a suitable constant value, the residual $np$ interaction (GM splitting [7]) and the Newby shift [8]. We consider a variable moment of inertia (VMI) defined as $\mathcal{I}$.

\[
\mathcal{I} = \mathcal{I}_0 + b I(I+1),
\]

where $b$ is the VMI parameter and the constant $\mathcal{I}_0$, is fixed such that $\frac{b^2}{2(2I+1)} = \frac{E^{2+}}{6}$, where the $E^{2+}$ is the first excited $2^+$ energy of the core. The symmetrized wave function for the parent nucleus takes the form as shown below:

\[
\Psi^{I,M} = \sum_{K_p,K_n,K_T=K_n \pm K_p} \sqrt{\frac{2I+1}{16\pi^2}} a^{I}_{K_T,K_n,K_p} \{ D^{I}_{M,K_T} T^{K_p,K_n}_{K_T} + (-1)^{I+K_T} D^{I}_{M,-K_T} R_{I} T^{K_p,K_n}_{K_T} \}. \tag{1}
\]

$a^{I}_{K_T,K_n,K_p}$ is the mixing coefficient obtained after diagonalising the total Hamiltonian matrix for the parent nucleus. Similarly, the mixing coefficient, $c^{I}_{K_n}$, for the daughter nucleus is obtained by solving the Hamiltonian for the odd-odd system. The decay width is expressed in terms of the overlap between the initial (parent nucleus) and final (daughter and proton) states, where the parent is considered to be a two quasiparticle plus rotor system. In this way the partial decay width can be written as

\[
\Gamma^{I,J_d}_{I_p,J_p} = \frac{\hbar^2}{\mu} \sum_{K_n,K_p(K_T=K_n \pm K_p),I} a^{I}_{K_T,K_n,K_p} c^{I}_{K_p} \times \sqrt{\frac{2I_d + 1}{2I+1}} \langle I_d, K_n, j_p, K_p | I, K_T \rangle u_{K_p} N^{K_p}_{I_p,j_p}^2, \tag{2}
\]

where $N^{K_p}_{I_p,j_p}$ represents the asymptotic normalization factor, $u_{K_p}$ is the spectroscopic factor calculated from BCS approach and hence $u_{K_p}^2$ represents the probability of unoccupancy of proton level in daughter nucleus. The CG coefficients define angular momentum coupling between daughter and emitted proton. The total decay width is obtained by taking sum of partial decay widths from every possible combination of the angular momentum values of proton ($I_p$ and $j_p$ values) and hence

\[
\Gamma^{I,J_d} = \sum_{j_p=|I-I_d|}^{I+I_d} \Gamma^{I,J_d}_{I_p,j_p}. \tag{3}
\]
Figure 1. (a) Comparison between the theoretical and experimental [9] level spectrum of the ground state band of $^{180}$Ta. (b) Calculated odd-even staggering pattern in the ground state band ($K^\pi_t = 1^+$) of $^{180}$Ta. Solid black circles represent the experimental data and the open circles represent the calculated values. The parameters used for the calculations are also presented.

Figure 2. Calculated half-lives for proton emission from the $K^\pi_s = 2^+$ state of $^{130}$Eu plotted as a contour with the $x$ and $y$ axes representing the Coriolis attenuation coefficient and the residual $np$ interaction.
3. Results
We present the calculated energy levels for $^{180}$Ta in Fig. 1, for the band built on the ground state. From Fig. 1(a), it is clear that we have a very good agreement with the experimental results even for very high spins. It has to be noted that the parameters involved in the calculations such as the Coriolis attenuation coefficient ($\rho$) and VMI parameter ($b$) are tuned to achieve a good fit for the spectrum. The odd-even (Newby) shift in $K=0$ band can be transmitted further to high-$K$ bands through Coriolis mixing and such a shift can occur in the ground state ($K^\pi = 1^+$) band of $^{180}$Ta. Introducing a Newby shift leads to a better agreement with the experimental values as shown in the odd-even staggering presented in Fig. 1(b). The Newby shift is tuned to reproduce the first excited state ($2^+$) accurately and the staggering pattern grossly. This leads to the best fit at 40 keV. With our results for $^{180}$Ta we demonstrate that our formalism could explain the measured spectrum very well while the Coriolis and residual $np$ interactions are taken in account.

Our results for the highly deformed proton emitter $^{130}$Eu are presented in Ref. [4], where it is shown that the decay widths corresponding to both the probable ground states $I^\pi = 1^+, 2^+$ are close to the experimental value. The $I^\pi = 1^+$ state is a triplet state which is favoured by GM rule as lower in energy than the singlet state. Hence $I^\pi = 1^+$ is assigned to be the ground state of $^{130}$Eu from which the proton is emitted and this conclusion does not depend on the attenuation of the Coriolis interaction. In Fig. 2, we present the half-lives for the decay from the $2^+$ state where we can identify the effect of interplay between the Coriolis and residual $np$ interactions on the proton emission half-lives [5]. For example, at very low Coriolis attenuation $\rho \sim 0.0$, the halflives are almost independent of $V_{np}$ and for very high $\rho$, the halflives increase for lower $V_{np}$ but start decreasing for higher $V_{np}$. We note that $V_{np}$ affects only the singlet state because it can mix through Coriolis interaction with a level of same spin built on the triplet state. If such a singlet state or other low-lying state which is sensitive to $V_{np}$, happens to be an isomeric state, it would be interesting to study proton emission from such states.

4. Conclusion
A proper formalism to study the low lying states of odd-odd nuclei is justified with our results for $^{180}$Ta and has been successfully extended to explain the proton emission from $^{130}$Eu. We propose that it would be interesting to study proton emission from isomeric states because the corresponding decay widths could be quite sensitive to the residual $np$ interaction.

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