Longitudinal fluid dynamics for ultrarelativistic heavy–ion collisions

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Abstract

We develop a 1+1 dimensional hydrodynamical model for central heavy–ion collisions at ultrarelativistic energies. Deviations from Bjorken’s scaling are taken into account by implementing finite–size profiles for the initial energy density. The calculated rapidity distributions of pions, kaons and antiprotons in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV are compared with experimental data of the BRAHMS Collaboration. The sensitivity of the results to the choice of the equation of state, the parameters of initial state and the freeze–out conditions is investigated. Experimental constraints on the total energy of produced particles are used to reduce the number of model parameters. The best fits of experimental data are obtained for soft equations of state and Gaussian–like initial profiles of the energy density. It is found that initial energy densities required for fitting experimental data decrease with increasing critical temperature of the phase transition.

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I. INTRODUCTION

High–energy heavy–ion collisions provide a unique tool for studying properties of hot and dense strongly interacting matter in the laboratory. The theoretical description of such collisions is often done within the framework of a hydrodynamic approach. This approach opens the possibility to study the sensitivity of collision dynamics and secondary particle distributions to the equation of state (EOS) of the produced matter. The two most famous realizations of this approach, which differ by the initial conditions, have been proposed by Landau [1] (full stopping) and Bjorken [2] (partial transparency). In recent decades many versions of the hydrodynamic model were developed ranging from simplified 1+1 [3, 4, 5, 6, 7, 8] and 2+1 dimensional models [6, 9, 10, 11, 12, 13] of the Landau or Bjorken type to more sophisticated 3+1 dimensional models [14, 15, 16, 17, 18, 19]. One should also mention the multi–fluid models [20, 21, 22, 23, 24, 25, 26] which consider the whole collision process including the nuclear interpenetration stage. Recent theoretical investigations show that fluid–dynamical models give a very good description of many observables at the SPS and RHIC bombarding energies (see e.g. Ref. [27]).

The 2+1 dimensional hydrodynamical models have been successfully applied [9, 10, 11, 12, 13] to describe the $p_T$ distributions of mesons and their elliptic flow at midrapidity. These models assume a boost–invariant expansion [2] of matter in the longitudinal (beam) direction and, therefore, cannot explain experimental data in a broad rapidity region, where strong deviations from the scaling regime have been observed. More realistic 3+1 dimensional fluid–dynamical simulations have been already performed for heavy–ion collisions at SPS and RHIC energies. But as a rule, the authors of these models do not study the sensitivity of the results to the choice of initial and final (freeze–out) stages. On the other hand, it is not clear at present, which initial conditions, Landau–like [1] or Bjorken–like [2], are more appropriate for ultrarelativistic collisions.

Our main goal in this paper is to see how well the fluid–dynamical approach can describe the RHIC data on $\pi$, $K$, $\bar{p}$ distributions over a broad rapidity interval, reported recently by the BRAHMS Collaboration [28, 29]. Within our approach we explicitly impose a constraint on the total energy of the produced particles which follows from these data. For our study we apply a simplified version of the hydrodynamical model, dealing only with the longitudinal dynamics of the fluid. This approach has as its limiting cases the Landau and Bjorken
models. We investigate the sensitivity of the hadron rapidity spectra to the fluid’s equation of state, to the choice of initial state and freeze–out conditions. Modification of these spectra due to the feeding from resonance decays is also analyzed. Special attention is paid to possible manifestations of the deconfinement phase transition. In particular, we compare the dynamical evolution of the fluid with and without the phase transition.

II. FORMULATION OF THE MODEL

A. Dynamical equations

Below we study the evolution of highly excited, and possibly deconfined, strongly interacting matter produced in ultrarelativistic heavy–ion collisions. It is assumed that after a certain thermalization stage this evolution can be described by the ideal relativistic hydrodynamics. The energy–momentum tensor is written in a standard form

$$ T^{\mu\nu} = (\epsilon + P) U^\mu U^\nu - P g^{\mu\nu}, $$

where $\epsilon$, $P$ and $U^\mu$ are the rest–frame energy density, pressure and the collective 4–velocity of the fluid.

We consider central collisions of equal nuclei disregarding the effects of transverse collective expansion. It is convenient to parametrize $U^\mu$ in terms of the longitudinal flow rapidity $Y$ as $U^\mu = (\cosh Y, 0, \sinh Y)^\mu$. All calculations are performed using the light–cone variables, namely, the proper time $\tau$ and the space–time rapidity $\eta$, defined as

$$ \tau = \sqrt{t^2 - z^2}, \quad \eta = \tanh^{-1} \left( \frac{z}{t} \right) = \frac{1}{2} \ln \frac{t + z}{t - z}. $$

In these coordinates, the equations of relativistic hydrodynamics, $\partial_\nu T^{\mu\nu} = 0$, for an ideal baryon–free fluid take the following form

$$ \left( \tau \frac{\partial}{\partial \tau} + \tanh(Y - \eta) \frac{\partial}{\partial \eta} \right) \epsilon + (\epsilon + P) \left( \tanh(Y - \eta) \tau \frac{\partial}{\partial \tau} + \frac{\partial}{\partial \eta} \right) Y = 0, $$

$$ (\epsilon + P) \left( \tau \frac{\partial}{\partial \tau} + \tanh(Y - \eta) \frac{\partial}{\partial \eta} \right) Y + \left( \tanh(Y - \eta) \tau \frac{\partial}{\partial \tau} + \frac{\partial}{\partial \eta} \right) P = 0. $$

To solve Eqs. (3)–(4), one needs to specify the EOS, $P = P(\epsilon)$, and the initial profiles $\epsilon(\tau_0, \eta)$ and $Y(\tau_0, \eta)$ at a time $\tau = \tau_0$ when the fluid may be considered as thermodynamically equilibrated.
Following Ref. [17], we choose the initial conditions for a finite-size fluid, generalizing the Bjorken scaling conditions:

\[ Y(\tau_0, \eta) = \eta, \quad \epsilon(\tau_0, \eta) = \epsilon_0 \exp \left[ - \frac{(|\eta| - \eta_0)^2}{2\sigma^2} \Theta(|\eta| - \eta_0) \right] , \]

where \( \Theta(x) \equiv (1 + \text{sgn} x)/2 \). The particular choice \( \eta_0 = 0 \) corresponds to the pure Gaussian profile of the energy density. At small \( \sigma \) such a profile can be similar to the Landau initial condition [54]. On the other hand, when \( \sigma \) or \( \eta_0 \) tends to infinity, one gets the limiting case of the Bjorken scaling solution. Below we adopt the value \( \tau_0 = 1 \text{ fm}/c \).

The numerical solution of Eqs. (3)–(4) is obtained by using the relativistic version [31] of the flux-corrected transport algorithm [32].

B. Equation of state

As well known, a deconfinement phase transition is predicted by quantum chromodynamics (QCD). This phase transition is implemented through a bag-like EOS in the parametrization suggested in Ref. [13]. This EOS consists of three parts, denoted below by indices \( H, M \) and \( Q \) corresponding, respectively, to the hadronic, ”mixed” and quark–gluon phases. In the case of equilibrated baryon–free matter the pressure \( P \), energy density \( \epsilon \) and entropy density \( s \) may be regarded as functions of the temperature only. The hadronic phase consists of pions, kaons, meson resonances and baryon–antibaryon pairs. It corresponds to the domain of low energy densities, \( \epsilon < \epsilon_H \), and temperatures, \( T < T_H \). The sound velocity, \( c_s = \sqrt{dP/d\epsilon} \), is assumed to be constant \( (c_s = c_H) \) in this phase:

\[ P = c_H^2 \epsilon, \quad T = T_H \left( \frac{\epsilon}{\epsilon_H} \right) \frac{c_H^2}{1 + c_H^2} \quad (\epsilon < \epsilon_H). \]

The mixed phase corresponds to intermediate energy densities, from \( \epsilon_H \) up to \( \epsilon_Q \). The following parametrization is used for this region:

\[ P = c_M^2 \epsilon - (1 + c_M^2) B_M, \quad T = T_H \left( \frac{\epsilon - B_M}{\epsilon_H - B_M} \right) \frac{c_M^2}{1 + c_M^2} \quad (\epsilon_H < \epsilon < \epsilon_Q). \]

Here \( B_M \) is the bag constant, determined from the condition of continuity of \( P(\epsilon) \) at \( \epsilon = \epsilon_H \). Due to the small sound velocity \( c_M \) (see Table II), both pressure and temperature increase.
only weakly with $\epsilon$ in the mixed phase region. The third, quark–gluon plasma region of the EOS corresponds to energy densities above $\epsilon_Q$:

$$P = c_Q^2 \epsilon - (1 + c_Q^2) B_Q, \quad T = T_Q \left( \frac{\epsilon - B_Q}{\epsilon_Q - B_Q} \right) \frac{c_Q^2}{1 + c_Q^2} \quad (\epsilon > \epsilon_Q).$$

(8)

Here $B_Q$ is the bag constant in the deconfined phase. The corresponding formulae for the entropy density are obtained from the thermodynamic relation $s = (\epsilon + P)/T$. We use the sound velocities $c_H^2, c_M^2, c_Q^2$ close to those used in Refs. [9, 13].

**TABLE I: Parameters of EOSs with the deconfinement phase transition.**

|        | $\epsilon_H$ (GeV/fm$^3$) | $\epsilon_Q$ (GeV/fm$^3$) | $c_H^2$ | $c_M^2$ | $c_Q^2$ | $T_H$ (MeV) | $T_Q$ (MeV) | $B_M$ (MeV/fm$^3$) | $B_Q$ (MeV/fm$^3$) |
|--------|------------------------|--------------------------|---------|---------|---------|-------------|-------------|-----------------|-----------------|
| EOS–I  | 0.45                   | 1.65                     | 0.15    | 0.02    | 1/3     | 165         | 169         | -57.4          | 344             |
| EOS–II | 0.79                   | 2.90                     | 0.15    | 0.02    | 1/3     | 190         | 195         | -101           | 605             |

The parameters $T_H$ and $T_Q$ define the boundaries of a mixed phase region separating the hadronic and quark–gluon phases. The critical temperature $T_c$ as defined by lattice calculations should lie between $T_H$ and $T_Q$, i.e. $T_c \simeq (T_H+T_Q)/2$. Earlier lattice calculations (see e.g. Ref. [33]) predicted the values $T_c = (170 \pm 10)$ MeV for the baryon–free two–flavor QCD matter. However, a noticeably larger value $T_c = (192 \pm 11)$ MeV was reported recently in Ref. [34]. To probe sensitivity to the actual position of the phase transition, we consider two EOSs with different $T_H$ and $T_Q$ (see Table I). The EOS–I corresponds to $T_H = 165$ MeV and the parameters $\epsilon_H, \epsilon_Q$ used in the parametrization LH12 of Ref. [13]. In the EOS–II we choose $T_H = 190$ MeV and scale $\epsilon_H, \epsilon_Q$ to get the same values of $\epsilon/T^4$ as a function of $T/T_H$ [55]. Finally, the parameters $B_Q, T_Q$ are found from the continuity conditions for $P$ and $T$.

Unless stated otherwise, these EOSs are used in the calculations presented in this paper. For comparison, we have performed also calculations with several purely hadronic EOSs. In this case we extend Eq. (6) to energy densities $\epsilon > \epsilon_H$ with the same $\epsilon_H, T_H$ as in Table I but choosing different $c_H^2 = 0.15$ and 1/3. In Fig. 1 we compare the EOS–I and EOS–II as well as two purely hadronic EOSs with constant sound velocities $c_s = c_H$. One can see that the mixed phase region in the EOS–II occupies larger interval of energy densities, i.e.
FIG. 1: Comparison of different EOSs used in this paper. The solid (dashed–dotted) line is calculated using Eqs. [6]–[8] with parameters given by the set EOS–I (II) in Table I. The dashed and dotted lines correspond to the hadronic EOSs with constant $c_s^2 = 0.15$ and $1/3$, respectively.

This EOS has a larger latent heat, $\epsilon_Q - \epsilon_H$, as compared to the EOS–I. By this reason, the life–times of the mixed phase will be longer for the EOS–II, assuming the same initial state.

C. Total energy and entropy

Using the equations of fluid dynamics one can show that the total energy and entropy of the fluid can be expressed as [7]

$$E = \int d\sigma [\epsilon T^0 + \epsilon \cosh Y \cosh (Y - \eta) + P \sinh Y \sinh (Y - \eta)],$$  

$$S = \int d\sigma \tau^0 = S_0 \int d\eta s \cosh (Y - \eta),$$

where $\epsilon$ is the energy density, $P$ is the pressure, $c_s^2$ is the sound speed squared. The variables $\tau^0$, $Y$, and $s$ are related to the pressure and entropy through the equations of state.
where $S_\perp$ is the transverse area of the fluid. The right-hand sides of Eqs. (9)–(10) give the values of the energy and entropy at $\tau = \tau_0$. Equations (9) and (10) can be considered as sum rules for the total energy and entropy of the produced particles.

Below we use Eq. (9) to constrain possible values of the parameters characterizing the initial state. This is possible since the total energy of produced particles can be estimated from experimental data. Indeed, the value of the total energy loss, $\Delta E = 73 \pm 6$ GeV per nucleon, has been obtained from the the net baryon rapidity distribution in most central Au+Au collisions \cite{28}. This gives the estimate of the total energy of secondaries in the considered reaction:

$$E = N_{\text{part}} \Delta E \simeq 26.1 \text{ TeV},$$

where $N_{\text{part}} \simeq 357$ is the mean number of participating nucleons. Substituting the parametrization (5) into Eq. (9) and taking the value of $E$ from Eq. (11), one gets the relation between the parameters $\epsilon_0, \eta_0, \sigma$.

We have checked that our numerical code conserves the total energy $E$ and entropy $S$ at any hypersurface $\sigma_\mu$ lying above the initial hyperbola $\tau = \tau_0$, on the level better than 1% up to very long times, $\tau \sim 10^3 \text{ fm}/c$.

D. Particle spectra at freeze–out

The momentum spectra of secondary hadrons are calculated by applying the standard Cooper–Frye formula \cite{35}, assuming that particles are emitted without further rescatterings from the elements $d\sigma_\mu$ of the freeze–out hypersurface $\tau = \tau_F(\eta)$. Then, the invariant momentum distribution for each particle species is given by the expression

$$E \frac{d^3 N}{d^3 p} = \frac{d^3 N}{dy d^2 p_T} = \frac{g}{(2\pi)^3} \int d\sigma_\mu p^\mu \left\{ \exp \left( \frac{p_\nu U_\nu^F - \mu_F}{T_F} \right) \pm 1 \right\}^{-1},$$

where $p^\mu$ is the 4–momentum of the particle, $y$ and $p_T$ are, respectively, its longitudinal rapidity and transverse momentum, $g$ denotes the particle’s statistical weight. The subscript $F$ in the collective 4–velocity $U^\mu$, temperature $T$ and chemical potential $\mu$ implies that these quantities are taken on the freeze–out hypersurface \cite{56}. The plus or minus sign in the right-hand side of Eq. (12) correspond to fermions or bosons, respectively.

As has been already stated, the effects of transverse expansion are disregarded in our approach. Due to this reason, we cannot describe realistically the $p_T$ spectra of pro-
duced hadrons, and analyze below only the rapidity spectra. For a cylindrical fireball with transverse cross section $S_\perp$ expanding only in the longitudinal direction, one can write $d\sigma^\mu = S_\perp (dz, 0, dt)^\mu$. Using Eq. (2) one arrives at the following relation

$$d\sigma_\mu p^\mu = S_\perp m_T \left\{ \tau_F(\eta) \cosh(y - \eta) - \tau_F'(\eta) \sinh(y - \eta) \right\} d\eta.$$  (13)

Here $m_T$ is the particle’s transverse mass defined as $m_T = \sqrt{m^2 + p_T^2}$, where $m$ is the corresponding vacuum mass. In the same approximation one can also write the expression

$$p_\nu U^\nu_F = m_T \cosh(y - Y_F(\eta)),$$  (14)

where $Y_F(\eta) = Y(\tau_F(\eta), \eta)$. An explicit expression for particle spectra at freeze–out is obtained after substituting (13)–(14) into Eq. (12) and integrating over $\eta$ from $-\infty$ to $+\infty$.

Note that Bjorken’s model [2] corresponds to $Y_F = \eta$ and $\tau_F, T_F$ independent of $\eta$. As can be seen from Eqs. (12)–(14), the rapidity distributions of all particles should be flat in this case.

We adopt a freeze–out criterion, assuming that a given fluid element decouples from the rest of the fluid when its temperature decreases below a certain value $T_F$. For finite–size initial conditions, $T(\tau_0, \eta) \to 0$ at $|\eta| \to \infty$, so that the fluid elements at large $|\eta|$ have temperatures below $T_F$ from the very beginning, i.e. at $\tau = \tau_0$. We treat these elements as decoupled instantaneously ($\tau_F = \tau_0$) and use in Eq. (12) the initial values of $Y$ and $T$ instead of $Y_F$ and $T_F$. Direct calculation shows, that such elements contribute only little to the tails of the rapidity distributions. The value of $T_F$ is considered as an adjustable model parameter which is found from the best fit to experimental data.

E. Feeding from resonance decays

In calculating particle spectra one should take into account not only directly produced particles but also feeding from resonance decays. Below we assume that the freeze–out temperatures for directly produced particles and corresponding resonances are the same. One of the most important contributions to the pion yield is given by $\rho(770)$–mesons. The spectrum of $\pi^+$–mesons originating from these decays is calculated by using the expression [36]

$$E_\pi \frac{d^3 N_{\rho \to \pi^+}}{d^3 p} = \frac{1}{3\pi} \int_2^{\infty} \frac{d m_R w(m_R)}{\sqrt{m_R^2 - 4m_\pi^2}} \int d^3 p_R \frac{d^3 N_R}{d^3 p_R} \delta \left( \frac{pp_R}{m_R} - \frac{m_R}{2} \right),$$  (15)

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where the first integration corresponds to averaging over the mass spectrum of $\rho$–mesons, $p_R$ and $p$ are, respectively, the 4–momenta of the $\rho$–resonance and of the secondary pion. The normalization coefficient in Eq. (15) takes into account that the number of $\pi^+$–mesons produced in $\rho$–decays equals $2/3$ of the total multiplicity of $\rho$–mesons. The freeze–out momentum spectrum of $\rho$–mesons, $d^3N_R/d^3p_R$, is calculated using Eqs. (12)–(14) with $m = m_R$, $g = g_\rho = 9$. We use the parametrization of the $\rho$–meson mass distribution, $w(m_R)$, suggested in Ref. [36].

The feeding of the pion yields from other meson and baryon resonances ($R = \eta, \omega, K^*, \Delta \ldots$) is obtained in the zero–width approximation, assuming that the contribution of the resonance $R$ is proportional to its equilibrium density $n_R(T_F)$, multiplied by a factor $d_R$, the average number of $\pi^+$ mesons produced in this resonance decay ($d_\rho = 2/3, d_\eta = 0.65 \ldots$). The details of $n_R$ and $d_R$ calculations can be found in Ref. [37]. We have checked for several resonances with two–body decays (e.g. for $R = K^*$) that such a procedure yields a very good accuracy. As a result, we get the following formula for the total resonance contribution to the spectrum of $\pi^+$ mesons:

$$\sum_R \frac{d^3N_{R\rightarrow\pi^+}}{dyd^2p_T} = \alpha \frac{d^3N_{\rho\rightarrow\pi^+}}{dyd^2p_T},$$

(16)

where the enhancement factor $\alpha$ is defined as follows

$$\alpha = \sum_R \frac{d_R n_R(T_F)}{d_\rho n_\rho(T_F)}.$$

(17)

We include meson (baryon and antibaryon) resonances with masses up to 1.3 (1.65) GeV and widths $\Gamma < 150$ MeV. The statistical weights, masses and branching ratios of these resonances are taken from Ref. [38]. The factor $\alpha$ decreases gradually with decreasing freeze–out temperature: $\alpha = 2.8, 2.4, 2.3$ for $T_F = 165, 130, 100$ MeV, respectively.

When calculating the kaon spectra we explicitly include feeding from decays of $K^*(892)$ (in the zero–width approximation). Higher resonances ($R = \phi, K_1 \ldots$) are taken into account by applying the same procedure as for pions. In this case the enhancement factor changes from 1.5 to 1.2 when $T_F$ goes from 165 to 100 MeV.
III. RESULTS

A. Best fits of rapidity spectra

Below we show the results for rapidity distributions of $\pi^-$ and $K^-$ mesons as well as antiprotons produced in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. In all calculations we use the fireball radius $R = 6.5$ fm and $S_\perp = \pi R^2 \simeq 133$ fm$^2$. The results are compared with data of the BRAHMS Collaboration \cite{28,29} for 5% most central collisions.

We have considered different profiles of the initial energy density, ranging from the Gaussian–like ($\eta_0 = 0$) to the table–like ($\sigma = 0$). We found that in the case of EOS–I it is not possible to reproduce the BRAHMS data on the pion and kaon rapidity spectra in Au+Au collisions by choosing either too small ($\epsilon_0 \lesssim 5$ GeV/fm$^3$) or too large ($\epsilon_0 \gtrsim 15$ GeV/fm$^3$) initial energy densities. For such $\epsilon_0$ values the pion and kaon yields can not be reproduced with any $T_F$. It is also found that the quality of fits is noticeably reduced for initial energy density profiles with sharp edges, corresponding to $\sigma < 1$. As follows from the constraint (11), such profiles should have either very large $\epsilon_0$ or a wide plateau $-\eta_0 < \eta < \eta_0$. This would lead to more flat rapidity distributions of pions and kaons as compared to the BRAHMS data.

A few parameter sets which give good fits with the EOS–I are listed in Table II. All three sets from Table II give very similar rapidity distributions for both pions and kaons. In these calculations we choose various $\epsilon_0$ and $\sigma$ and determine $\eta_0$ from the total energy constraint (11). It is interesting that the initial states A–C have approximately the same total entropy $S \simeq 3 \times 10^4$. This, in fact, should follow from the correct description of

| set | $\epsilon_0$ (GeV/fm$^3$) | $\sigma$ | $\eta_0$ | $T_0$ (MeV) | $E_1$ (TeV) | $E_3$ (TeV) | $E/S$ (GeV) |
|-----|----------------|-------|--------|-------------|------------|------------|-------------|
| A   | 10            | 1.74  | 0      | 279         | 1.53       | 9.25       | 0.89        |
| B   | 9             | 1.50  | 0.62   | 271         | 1.54       | 9.59       | 0.86        |
| C   | 8             | 1.30  | 1.14   | 263         | 1.49       | 9.55       | 0.86        |
total pion and kaon multiplicities. As one can see from the last column of Table II, the corresponding $E/S$-ratios fall into a narrow interval 0.86 – 0.89 GeV. This observation is similar to the result of Ref. [39] that the observed ratio of the rest frame energy to the multiplicity of produced hadrons is constant as a function of the bombarding energy.

To check the sensitivity to the parameters of the phase transition, we also calculate the pion and kaon rapidity distributions for the EOS–II. It is found that with the same initial energy profiles as for the EOS–I it is not possible to reproduce the observed spectra at any freeze-out temperature. In particular, the predicted kaon yield is strongly overestimated at 100 MeV $< T_F < T_H = 190$ MeV. Nevertheless, the BRAHMS data can be well reproduced with the EOS–II too when taking smaller initial energy densities as compared with the EOS–I. Fits of approximately same quality are obtained for $\epsilon_0 \simeq 5$ GeV/fm$^3$. As before, in choosing the initial conditions we apply the constraint (II) for the total energy of produced particles. Similarly to the case of the EOS–I, the data are better reproduced for initial profiles with small $\eta_0 \lesssim 1$.

Figures 2–3 show the model results for pion and kaon rapidity distributions obtained for the EOS–I and EOS–II. These results correspond to Gaussian initial profiles with $\eta_0 = 0$. For both EOSs we choose the parameter $\epsilon_0$ to obtain the best fit of the BRAHMS data. Although the overall fits are very similar for both EOSs, the rapidity spectra obtained with the EOS–II are slightly broader than those with the EOS–I. In the same figures we demonstrate sensitivity to the choice of the freeze-out temperature. The best fits of the pion spectrum for EOS–I and EOS–II are achieved with $T_F \simeq 130$ MeV (see Fig. 2). On the other hand, the kaon spectrum can be well reproduced only by assuming that kaons decouple at the very beginning of the hadronic stage, i.e. at $T_F \simeq T_H = 165 (190)$ MeV for EOS–I (II). The contribution of resonance decays turns out to be rather significant, especially in the central rapidity region, where it amounts to about 35% (45%) of the total pion (kaon) yield.

According to Fig. 2 larger yields of secondary pions are predicted for smaller freeze-out temperatures. A much weaker sensitivity to $T_F$ is found for kaons (see Fig. 3). This difference can be explained by the large difference between the pion and kaon masses. Indeed, in the case of direct pions, a good approximation at $T_F > 100$ MeV is to replace the transverse mass $m_T$ in Eqs. (12)–(13) by the pion transverse momentum $p_T$. Neglecting the second term in the right-hand side of Eq. (13), one can show that the rapidity distribution of pions at $y = 0$ is proportional to $\xi = \tau_F(\eta) \cosh \eta \cdot T_F^3 / \cosh^3 Y_F(\eta)$ integrated over all $\eta$. For a rough
FIG. 2: Rapidity distribution of $\pi^+$–mesons in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Left panel shows results of hydrodynamical calculations for the EOS–I and initial conditions (5) with the parameters $\epsilon_0 = 10$ GeV/fm$^3$, $\eta_0 = 0$, $\sigma = 1.74$ (set A from Table II). Right panel corresponds to the EOS–II and the parameters $\epsilon_0 = 5$ GeV/fm$^3$, $\eta_0 = 0$, $\sigma = 2.02$. Solid, dashed and dashed–dotted curves are calculated for different values of the freeze–out temperature $T_F$. The dotted lines show contributions of resonance decays in the case $T_F = 130$ MeV. Experimental data are taken from Ref. [29].

FIG. 3: Same as Fig. 2 but for $K^+$ rapidity distributions.

estimate, one can use the Bjorken relations $Y_F = \eta$, $s_F \tau_F = s_0 \tau_0$, where $s_F$ is the entropy
density at $T = T_F$. Using Eq. (6) one gets $\tau_F \propto s_F^{-1} \propto T_F^{-1/c_H^2}$ and therefore, $\xi \propto T_F^{3-1/c_H^2}$. This shows that for $c_H^2 < 1/3$ the pion yield grows with decreasing $T_F$. Qualitatively, one can say that at low enough $c_H$ the increase of the spatial volume at freeze–out compensates for the decrease of the pion occupation numbers at smaller $T_F$. This effect is somewhat reduced because of a decreasing resonance contribution at smaller temperatures. It is obvious that for kaons this effect should be much weaker due to the presence of the activation exponent $\exp (-m_K/T_F)$. In fact, a numerical calculation for the same EOS and initial state shows that the kaon yield changes nonmonotonically: first it slightly increases when temperature goes down but then it starts to decrease at lower $T_F$.

To study sensitivity of particle spectra to the presence of the phase transition, we have performed calculations with purely hadronic EOSs. In this case we use the same initial conditions as before and apply Eq. (6) for all stages of the reaction, including high density states. Our analysis shows that for soft hadronic EOS with $c_H^2 \simeq 0.1 - 0.2$ it is possible to reproduce the observed pion and kaon data with approximately the same fit quality as in the calculations with the quark–gluon phase transition. Furthermore, the corresponding freeze–out temperatures do not change significantly. However, we could not achieve satisfactory fits for the ”hard” hadronic EOS with $c_H^2 \geq 1/3$. This is demonstrated in Fig. 4 where we compare calculations for two EOSs with and without phase transition. In both calculations $c_H^2 = 1/3$. In the first case with we use Eqs. (6)–(8) with the same $T_H$, $\epsilon_H$ and $\epsilon_Q$ as for the EOS–I, but choose $c_H^2 = 1/3$. The hadronic EOS is obtained by extending Eq. (6) to all energy densities. We have found that calculations with $c_H^2 = 1/3$ require much higher initial energy densities as compared to the EOS with $c_H^2 = 0.15$. One can see that this hadronic EOS predicts a too wide pion rapidity distribution. The same conclusion is valid for kaons. The reason is that the higher pressure gives a stronger push to the matter in forward and backward directions. From these findings we conclude that a certain degree of softening of the EOS is required to reproduce the pion and kaon rapidity distributions.

It turns out that our model can also reproduce reasonably well the antiproton rapidity spectra measured by the BRAHMS Collaboration [28]. Figure 5 shows the antiproton rapidity distributions, calculated for the EOS–I and the parameter set A. In this case we explicitly take into account the contribution of the $\Delta(1232) \to \pi p$ decays, ignoring the width of $\Delta$–isobars. Contributions of higher antibaryon resonances are taken into account in a similar way as for pions and kaons. The resonance contribution is about 55% at $T_F = 165$ MeV.
FIG. 4: Rapidity distributions of $\pi^+$-mesons in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV calculated for the initial condition with the parameters $\epsilon_0 = 50$ GeV/fm$^3$, $\eta_0 = 0$, $\sigma = 0.96$. Solid line corresponds to the EOS with the phase transition (see the text). The dashed line is calculated the purely hadronic EOS. In both cases $c_H^2 = 1/3$. Experimental data are taken from Ref. [29].

One should consider these results as an upper bound for the antiproton yield. A more realistic model should include effects of the nonzero baryon chemical potential which will certainly reduce the antibaryon yield. The thermal model analysis of RHIC data, performed in Refs. [40, 41], gives rather low values for the baryon chemical potentials, $\mu_F \sim 30$ MeV, at midrapidity. This will suppress the antiproton yield by a factor $\sim \exp \left( -\frac{\mu_F}{T_F} \right) \sim 0.8$.

B. Rapidity distribution of total energy

We have calculated additionally the rapidity distribution of the total energy of secondary particles, $dE/dy$, in order to check the energy balance in the considered reaction. In this calculation we take into account not only direct pions and kaons (charged and neutral), but also heavier mesons and $B\bar{B}$ pairs (the same set of resonances as in the calculation of pion
FIG. 5: Rapidity distributions of antiprotons in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Shown are results of hydrodynamical calculations for the EOS–I and the parameter set A. All results are obtained assuming $\mu_F = 0$. Experimental data are taken from Ref. [28].

and kaon spectra). The contribution of heavy mesons and $B\bar{B}$ pairs was calculated in the zero–width approximation at the temperature $T_F = 165$ MeV. By integrating $dE/dy$, we have determined $E_1$ and $E_3$, the total energies of secondaries within the rapidity intervals $|y| < 1$ and $|y| < 3$, respectively. The BRAHMS Collaboration estimated $E_{1,3}$ from the rapidity distributions of charged pions, kaons, protons and antiprotons in most central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The values $E_1 \simeq 1.5$ TeV, $E_3 \simeq 9$ TeV have been reported in Ref. [42]. From Table III one can see that these values are well reproduced by the model.

Based on the above analysis we conclude that within the hydrodynamical model the BRAHMS data can be well described with the EOS–I and EOS–II and the parameters of the initial state ($\tau_0 = 1$ fm/$c$) $\sigma \simeq 1.5 - 2$, $\eta_0 \lesssim 1$. The maximal initial energy density, $\epsilon_0$, is sensitive to the critical temperature of the phase transition. For the EOS–I ($T_c \simeq 167$ MeV) we get the estimate $\epsilon_0 \simeq 9 \pm 1$ GeV/fm$^3$ while for the EOS–II ($T_c \simeq 192$ MeV) the required
values of \( \epsilon_0 \) are lower by about a factor of two.

These profiles are intermediate between the Landau and Bjorken limits. It is worth noting that the observed pion rapidity distribution can be well approximated by the Gaussian with the width \( \sigma_{\text{exp}} \simeq 2.3 \) \cite{29}. According to the Landau model, the width of the distribution is given by the expression \cite{43}

\[
\sigma_{\text{Lan}}^2 \simeq \frac{8}{3} \frac{c_s^2}{1 - c_s^4} \ln \frac{\sqrt{s_{NN}}}{2m_N},
\]

where \( m_N \) is the nucleon mass. For \( c_s^2 = 1/3 \) this gives \( \sigma_{\text{Lan}} \simeq 2.16 \), the value often quoted in the literature (see e.g. \cite{29}). On the other hand, for \( c_s^2 = 0.15 \) (which is preferable within our model) the width is only 1.38 i.e. noticeably smaller than observed by the BRAHMS Collaboration. This shows that deviations from the simple Landau model are rather significant.

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C. Dynamical evolution of matter

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FIG. 6: Time dependence of temperature as a function of \( \eta \) calculated for the parameter set A (only the forward hemisphere is shown). Left and right panels correspond, respectively, to the EOS–I and the hadronic EOS \( P = c_H^2 \epsilon \) with \( c_H^2 = 0.15 \).

Finally, after we have determined the initial conditions which lead to a reasonable description of the observed rapidity spectra, we can use the strength of the hydrodynamical
model to follow the dynamical evolution of matter. Below we present the results for two EOSs, with and without the phase transition, for the parameter set A. Figures 6–7 show profiles of the temperature and the collective rapidity at different proper times \( \tau \). The main difference is that in the case of phase transition the model predicts appearance of a flat shoulder in \( T(\eta) \) and local minima in \( Y(\eta) \) which are clearly seen at \( \tau \lesssim 10 \text{ fm}/c \). This is a consequence of the mixed phase which has a life time \( \Delta \tau \sim 10 \text{ fm}/c \). According to Figs. 6–7 the ”memory” of the quark–gluon phase is practically washed out at \( \tau \gtrsim 30 \text{ fm}/c \). As one can see from Fig. 7, at such times deviations from the Bjorken scaling \( (Y = \eta) \) do not exceed 5%.

Figure 8 shows the matter isotherms in the \( \eta – \tau \) plane. One can clearly see that the initial stage of the evolution, when matter is in the quark–gluon phase, lasts only for a very short time, of about 5 fm/c. The region of the mixed phase is crossed in less than 10 fm/c. This clearly shows that the slowing down of expansion associated with the ”soft point” of the EOS plays no role, when the initial state lies much higher in energy density than the phase transition region. In this situation the system spends the longest time in the hadronic phase and the late evolution is not sensitive to the phase transition. The freeze-out at \( T_F = 130 \text{ MeV} \) requires an expansion time of about 60 fm/c at \( \eta = 0 \). This is certainly a very long time which is apparently in contradiction with experimental findings. Indeed,
FIG. 8: Isotherms in the $\eta - \tau$ plane calculated for the parameter set A. Left and right panel corresponds to the same EOSs as in Fig. 6. Shaded region indicates the mixed phase.

The interferometric measurements \[^{[44]}\] show much shorter times of hadron emission, of the order of 10 fm/$c$. This discrepancy can not be removed by considering other EOS or initial conditions. A considerable reduction of the freeze–out times can be achieved by including the effects of transverse expansion and chemical nonequilibrium \[^{[17]}\]. However, this will not change essentially the dynamics of the early stage ($\tau \lesssim 10$ fm/$c$) when expansion is predominantly one–dimensional. A more radical solution of the ”short time puzzle” could be an explosive decomposition of the quark–gluon plasma, proposed in Ref. \[^{[45]}\]. This may happen at very early times, right after crossing the critical temperature line, when the plasma pressure becomes rather small. We shall consider this possibility in a forthcoming publication.

**IV. SUMMARY AND DISCUSSION**

In this paper we have generalized Bjorken’s scaling hydrodynamics for finite–size profiles of energy density in pseudorapidity space. The hydrodynamical equations were solved numerically in $\tau - \eta$ coordinates starting from the initial time $\tau_0 = 1$ fm/$c$ until the freeze–out stage. The sensitivity of the final particle distributions to the initial conditions, the freeze–
out temperature and the EOS has been investigated. A comparison of \( \pi, K, p \) rapidity spectra with the BRAHMS data for central Au+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV has been made. Best agreement with these data is obtained for initial states with nearly Gaussian profiles of the energy density. In choosing the initial conditions we impose the constraint on the total energy of produced particles. It is found that the maximum energy density of the initial state, \( \epsilon_0 \), is sensitive to the parameters of a possible deconfinement transition. The BRAHMS data are well reproduced with \( \epsilon_0 \) of about 10 (5) GeV/fm\(^3\) for the critical temperature \( T_c \sim 165 (190) \) MeV. The only unsatisfactory aspect of these calculations is the prediction of a very long freeze–out times, \( \sim 50 \) fm/c for pions.

We would like to comment on several points.

It is clear that our 1+1 dimensional model can not be valid at late stages of heavy–ion collisions, and the transverse flow effects should be included into a more realistic approach. On the other hand, the above–mentioned 2+1 dimensional models [9, 10, 11, 12, 13], which assume Bjorken scaling in the beam direction, are apparently not accurate too, even for the slice around \( \eta = 0 \). Indeed, in contrast to the Bjorken model, our calculations show that total entropy in different pseudorapidity intervals does not stay constant during the expansion. Due to the pressure gradients along the beam axis and corresponding fluid’s acceleration, the entropy is transferred from central pseudorapidity bins to the periphery. For instance, the entropy in the central bin \(|\eta| < 1\) drops by about 15% during the evolution. Therefore, only full 3D models can provide a more reliable description.

It is interesting to note that the viscosity terms, omitted in this paper, should lead to the opposite effects, namely to slower cooling and smaller acceleration of the fluid [46, 47]. Therefore, to describe the observed data, we would need somewhat broader initial energy density profiles and accordingly lower \( \epsilon_0 \) values. In principle, our simple model can be used for a more quantitative study of these effects.

We have performed calculations with the initial time \( \tau_0 = 1 \) fm/c. Of course, one can start the hydrodynamical evolution from an earlier time, i.e. assuming smaller \( \tau_0 \). In this case one should choose accordingly higher initial energy densities. But \( \tau_0 \) cannot be taken too small, since at very early times the energy is most likely stored in strong chromofields [48]. The quark–gluon plasma is produced as a result of the decay of these fields (see e.g. Ref. 49 and references therein). Estimates show that the characteristic decay times are in the range \( 0.3 - 1.0 \) fm/c. At earlier times the system will contain both the fields as well as produced
partons, and the evolution equations will be more complicated, see e.g. Refs. [50, 51].

It is obvious that the Cooper–Frye scenario of the freeze-out process, applied in this paper is too simplified. This was demonstrated e.g. in Ref. [52]. One should also have in mind that the freeze–out temperatures obtained in our model will be modified by the effects of transverse expansion and chemical nonequilibrium. Attempts to achieve a more realistic description of the freeze–out stage have been recently made in Refs. [10, 13] where a transport model was applied to describe evolution of the hadronic phase. In this approach the solution of fluid–dynamical equations is used to obtain initial conditions for transport calculations at later stages of a heavy–ion collision. We are planning to use a similar approach in the future.

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[53] Units with ħ = c = 1 are used throughout the paper.
[54] Within the Landau model [1] σ ∝ γ−1 and ε0 ∝ γ2 where γ is the c.m. Lorentz–factor of colliding nuclei.
[55] This is achieved by choosing the same εi/T_H^4 (i = H, Q) for these two EOSs.
[56] Below we assume that the chemical and thermal freeze–out hypersurfaces coincide. In this case μF = 0 for baryon–free matter.
[57] According to the Landau model [1], the multiplicity of produced particles is proportional to the total entropy.
[58] We have checked that at fixed T_F and the same initial conditions the freeze–out times predicted for the EOS–II are noticeably longer than for the EOS–I.
[59] We did not try to achieve a perfect fit of these data, bearing in mind that their systematic errors are quite big, about 15% in the rapidity region |y| > 1.3 [29].