Quantum transfer functions, weak nonlocality and relativity

by

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Abstract

The method of transfer functions is developed as a tool for studying Bell inequalities, alternative quantum theories and the associated physical properties of quantum systems. Non-negative probabilities for transfer functions result in Bell-type inequalities. The method is used to show that all realistic Lorentz-invariant quantum theories, which give unique results and have no preferred frame, can be ruled out on the grounds that they lead to weak backward causality.
1. Introduction

In John Bell’s study of the general properties of realistic alternative quantum theories [B14], he showed that quantum systems have properties that cannot be simulated by classical systems (References to Bell’s book ‘Speakable and Unspeakable in Quantum Mechanics’ [1] will be denoted by [Bp], where p is a page number). This discovery led to modern quantum technology and a new branch of quantum physics. In order to treat the relativistic version of these theories, we need to follow chains of causality. For this purpose, section 2 develops the general theory of transfer functions that can be applied to both classical and quantum systems. The transfer functions link output events like the results of quantum measurements to input events like the preparation of quantum systems.

Section 3 describes the causal relations of classical and quantum systems in terms of the properties of transfer functions and the location of the input and output events in spacetime. Section 4 shows how the transfer function theory is used to obtain inequalities of the Bell type, and suggests the use of linear programming. Section 5 then uses the transfer function theory to put some very strong constraints on realistic relativistic quantum theories.

2 Transfer functions for classical and quantum systems

A quantum experiment consists of a quantum system linked to classical systems through preparation events $i_A, i_B, \ldots$, which we shall call inputs, and measurement events $j_A, j_B, \ldots$, which we call outputs. The input $i = (i_A, i_B, \ldots)$ (in the singular) represents all the input events, and the output $j = (j_A, j_B, \ldots)$ all the output events. Classical systems can similarly be prepared by input events and measured by output events.

We do not consider as an input or output any event which is is the same for all runs of an experiment, for example the production of a system with total spin zero in a Bell-inequality experiment: inputs and outputs are variables of an experiment, not constants. The same event could be both an input and an output. Input and outputs occupy restricted regions of space and time, and there is no reason why all the inputs should take place before all the outputs. Spacetime relations are discussed in the next section.

For classical systems without noise, and for some special quantum experiments, the output $j$ is determined uniquely by the input $i$, and the relation between them is given by a transfer function $F$ such that

$$j = F(i).$$ (1)
If there is a finite number of possible inputs and outputs, then there is a finite number of possible transfer functions. For continuous variations, including continuous preparation and measurement of quantum systems, there is an infinite number, the mathematics is more subtle, but the physical principles are the same, so we restrict our treatment to the finite number.

Suppose for simplicity that the system has only two inputs and two outputs. The transfer function then has two components

\[ F = (F_A, F_B) \quad \text{such that} \quad j_A = F_A(i_A, i_B), \quad j_B = F_B(i_A, i_B). \]  

(2)

Sometimes the input and output \( i_A, j_A \) in spacetime region A are far from the input and output \( i_B, j_B \) in spacetime region B, and causal relations can be expressed in terms of signals between the two regions, as follows:

\[ F_A(i_A i_B) = F_A(i_A), \quad F_B(i_A i_B) = F_B(i_B) \quad \text{(No signals between A and B)} \]  

(3a)

\[ F_A(i_A i_B) = F_A(i_A), \quad F_B(i_A i_B) \neq F_B(i_B) \quad \text{(Signal from A to B only)} \]  

(3b)

\[ F_A(i_A i_B) \neq F_A(i_A), \quad F_B(i_A i_B) = F_B(i_B) \quad \text{(Signal from B to A only)} \]  

(3c)

\[ F_A(i_A i_B) \neq F_A(i_A), \quad F_B(i_A i_B) \neq F_B(i_B) \quad \text{(Signals both ways)} \]  

(3d)

where, for example, the inequality in (3b) means that \( j_B \) depends explicitly on \( i_A \). Those transfer functions that allow signals are called signaling (transfer) functions. Those that do not are null. The signals are constrained by the conditions of special relativity, depending on whether the intervals between the inputs and outputs are spacelike or timelike.

More generally, classical systems are noisy, and the results of quantum measurements are not determined. The noise in a classical system can be represented by a background variable \( \lambda \), which usually represents unknown effects, typically microscopic fluctuations, which influence the output. The variable \( \lambda \) can be discrete or continuous, a point in a function space, or any other space for which a probability distribution \( \Pr(\lambda) \) can be defined [B15]. An example is an electrical circuit with one or more resistors, where the background variable is a combination of some of the internal variables of the resistors, whose values are unknown. The output depends on these background variables, which produce noise in the resistors. Bell has given some beautiful examples of background variables from everyday life [B83,B105,B139,B147,B152].

Suppose that for a given value of \( \lambda \), the transfer function \( F_\lambda \) is determined, so that we can express the dependence of the output on the input in terms of transfer functions \( F_\lambda \) such that

\[ j = F_\lambda(i). \]  

(4)
This is just another way of saying the \( j \) depends only on the background variable \( \lambda \) and the input \( i \), and is uniquely determined by them. There are background variables for which the output is not uniquely determined, but we will suppose that they are supplemented if necessary by further background variables to make the output unique for a given \( i \) and \( \lambda \).

Given the probability \( \Pr(\lambda) \) for the background variable, the probability of the output \( j \), given the input \( i \), is

\[
\Pr(j/i) = \int d\lambda \Pr(\lambda)\delta(j,F_\lambda(i)),
\]

where as usual the \( \delta \)-function is unity when its arguments are equal and zero otherwise. For a resistor, \( \Pr(\lambda) \) is just the appropriate Boltzmann distribution.

For a particular system, two values of the background variable \( \lambda \) which have the same transfer function \( F_\lambda \) are completely equivalent. For a given input they result in the same output. Only the function \( F_\lambda \) matters. So all the information that is needed about the probability distribution \( \Pr(\lambda) \) is contained in the probability distribution

\[
\Pr(F) = \int d\lambda \Pr(\lambda)\delta(F,F_\lambda).
\]

For this system, the transfer functions can be treated as if they were background variables. The conditional probability for finding the output \( j \), when the input is \( i \), is then given by

\[
\Pr(j/i) = \sum_F \Pr(F)\delta(j,F(i)).
\]

The causal relations for a noisy system with background variables can be expressed in terms of signals just as they were for deterministic systems in (3). If the probability \( \Pr(F) \) of a signalling transfer function is not zero, then there are corresponding signals for the noisy system, but unless the probability is unity, the channel capacity is less.

For the classical dynamics of an electrical circuit with resistors, those internal freedoms of the resistors which produce noise in the circuit are background variables, which are hidden, because we cannot see the motion of the ‘classical electrons’. Unlike some background variables of quantum mechanics, there is no physical principle that prevents the hidden variables of the classical circuit from becoming visible.

The same analysis applies to a quantum system prepared in a pure state, with a very important difference: the background variables are now hidden variables in principle, and there is no guarantee that they have the properties of classical background variables. When a system does not have these properties, the dependence
of the output on the input must have a quantum component. A quantum system can do things that a classical system cannot, which is the basis of technologies like quantum cryptography and quantum computation. If the system is in a black box, an experimenter can tell, just by controlling the input and looking at the output, that the output is linked to the input by a quantum system. This is why Bell’s study of hidden variables has had such important consequences.

Equation (7) for a Bell experiment, together with the condition of non-negative $\Pr(F)$ for all $F$, requires $\Pr(j/i)$ to satisfy inequalities, including the Bell inequality. The method can be used to find inequalities of the Bell type. The study of the constraints on $P(j/i)$ for a general quantum system becomes a problem in linear programming.

3. Spacetime and causality for classical and quantum systems

Special relativity puts strong constraints on causality in spacetime.

According to classical special relativity, causal influences such as signals operate at a velocity less than or equal to the velocity of light, so that an input event influences only output events in or on its forward light cone, and an output is only influenced by events in or on its backward light cone. They cannot operate over spacelike intervals, nor can they operate backwards in time. There is only forward causality.

Nevertheless, there can be correlations between systems with spacelike separation, due to background variables that originate in the region of spacetime common to their backward light cones [2, B55]. Causality which acts over spacelike intervals is generally called nonlocal causality, or simply nonlocality. It does not exist for relativistic classical systems. Correlation between events with spacelike separation is not sufficient evidence for nonlocality, since it can be due to common background variables.

Causality which acts backwards over timelike intervals means that the future influences the present, and the present influences the past. This is backward causality. This term will only be used when there is timelike separation between cause and effect.

These relations can be expressed in terms of inputs and outputs. Suppose that the input events or inputs $i_A, i_B, \ldots$ and output events or outputs $j_A, j_B, \ldots$ are all so confined in spacetime that the interval between any pair of such events is well-defined as spacelike or timelike. The special case of null intervals is excluded, as it is not needed for our purposes. Since no signal can propagate faster than the velocity of light, an output $j_Y$ depends only on those $i_X$ that lie in its backward light cone, so that any variation of the other $i_X$ makes no difference to $j_Y$. This condition on the transfer function $F$ is a condition of forward causality, which applies to all
those deterministic classical and quantum systems for which the input uniquely
determines the output.

The background variable $\lambda$ of noisy classical systems may be considered as an
additional input, so it does not affect the causality relations between the inputs and
outputs. Each of the transfer functions $F_\lambda$ satisfies the same causality conditions as
the transfer function $F$ of a deterministic system. There is only forward causality.

The same applies to the transfer functions $F_\lambda$ of some quantum systems, but not
to all. According to quantum mechanics, there are experiments for which the $F_\lambda$
with hidden variables $\lambda$ do not satisfy the conditions of forward causality: causality
is nonlocal. These include experiments designed to test the violation of Bell’s
inequalities. Experimental evidence has been overwhelmingly in favour of quantum
mechanics, and also tends to favour nonlocal causality through violation of inequal-
ities, although there are still some loopholes that need to be closed [3]. Nonlocal
causality, conditional on hidden background variables, is called weak nonlocality,
and the corresponding signals are weak nonlocal signals. Weak signals require the
hidden variables to be known at the receiver, and since the variables are hidden, the
signals are hypothetical. Nevertheless, the example of Bell experiments shows that
the presence or absence of weak signals can tell us something about the properties
of quantum systems. Because the background variables are hidden, weak nonlocal
signals cannot be used to send signals faster than the velocity of light. However,
systems with weak nonlocal signals have observable properties that cannot be sim-
ulated by certain other systems, such as any system whose inputs and outputs are
linked by classical variables only.

There are is absolutely no experimental evidence for backward causality of any
kind in classical or quantum systems, even weak backward causality. We show
in section 5, that without weak backward causality, certain relativistic realistic
quantum theories are impossible.

4. Inequalities of the Bell type

Einstein-Podolsky-Rosen-Bohm experiments, and Bell experiments which test Bell’s
or Clauser-Holt-Horne-Shimony (CHHS) [B86] inequalities, are typical quantum
nonlocality thought experiments. In one run of an experiment, two spin-half par-
ticles with total spin zero are ejected in opposite directions from a central source.
This stage of the preparation is not included in the input, as it happens for every
run of the experiment, and the inputs in $i$ are the variable inputs, not the constant
inputs. Before the particles are detected, they pass into Stern-Gerlach magnetic
fields. The classical settings of the orientations of the magnetic fields are the input
events. The measurements, which include the classical recording of one of the two
spin directions parallel or antiparallel to the field, are the output events. Most real
experiments have been based on photon polarization, but the principles are the same.

There are two inputs and two outputs, given by

$$i = (i_A i_B), \quad j = (j_A j_B),$$

where one pair of inputs and outputs is confined to a spacetime region A and the other is confined to a spacetime region B. The output $j_A$ is in the forward light cone of $i_A$ and similarly for $j_B$ and $i_B$. The input preparation at A must be separated by a spacelike interval from the output measurement at B and vice versa. This is a considerable experimental challenge.

For local hidden variables, it is not possible to send even weak signals between A and B, so there are only transfer functions of type (3a), and the other 3 types of transfer function have probability zero. So the only transfer functions that have nonzero probability have the form

$$F = (F_A, F_B), \quad \text{where} \quad j_A = F_A(i_A), \quad j_B = F_B(i_B).$$

The conditional probability for the outcome $j_A, j_B$ given inputs $i_A, i_B$ is then

$$\Pr(j_A, j_B/i_A, i_B) = \sum_{F_A, F_B} \Pr(F_A, F_B) \delta(j_A, F_A(i_A)) \delta(j_B, F_B(i_B)).$$

Using $(F_A, F_B)$ as a hidden variable $\lambda$ in equations (6) and (10) results in Bell’s [B16], equation (2) for independent determination of the results at A and B, given $\lambda$.

The spin of a particle is denoted $+$ when it is in the same direction as the field and $-$ when it is in the opposite direction. The output $j = (j_A j_B)$ then consists of one of four pairs of spin components, $(++)$, $(+-)$, $(-+)$ and $(--)$, where the first sign represents the output at A and the second sign the output at B. We will adopt a convention whereby the angles at A and B are measured from zeros that point in opposite directions, so that, for total spin zero, when the angles are equal, the measured signs of the spin components are the same, not opposite.

Consider first an experiment for which each of the magnetic field orientations at A and at B has only two possible angles, $\theta_i$ with $i = 1, 2$, the same pair of angles at A and B, with the above convention. Then the function $F_A$ can be labelled by a table of its output values ($\pm$) for $i = 1$ and $i = 2$, and similarly for $F_B$. Thus if $j_A = +$ for $i_A = 1$ and $j_A = -$ for $i_A = 2$, we can denote $F_A$ by $F_A = [+-]$. The function $F$ can then be labelled by a table of 4 values, those for A first, and those for B second, eg $[+-, +]$. For general functions $F$ we would have $2^4 = 16$ output
values. But since we are assuming local hidden variables, \( F \) has the form (9), and only 2 arguments each are needed to specify \( F_A \) and \( F_B \), so we need only 4 output values to specify \( F \).

The number of independent probabilities \( \Pr(F) = \Pr(F_A, F_B) \) is then severely restricted by the fact that for opposite input orientations of the magnetic fields at \( A \) and \( B \), corresponding to \( \theta_A = \theta_B \) and \( i_A = i_B \), the output signs \( j_A, j_B \) must be the same. So when they are different the probability of \( F \) is zero. Reversing the direction of all spins does not affect the probabilities. Thus all \( \Pr(F) \) are zero except

\[
P_1 = \Pr([++, ++]) = \Pr([- -, - -]), \\
P_2 = \Pr([+-, + -]) = \Pr([-+, -+]),
\]

where the right-hand equalities follow by symmetry. Notice that the two strings of signs, for \( A \) and for \( B \), are the same.

There are no contradictions here: two directions can be handled with local hidden variables. But now consider 3 directions \( \theta_1, \theta_2, \theta_3 \), as in a Bell inequality. The values of \( i_A \) and \( i_B \) are 1,2,3, and there are four independent probabilities, given by

\[
P_0 = \Pr([+++, +++]) = \Pr([- --, -- -]), \\
P_1 = \Pr([-+++, --+]) = \Pr([-+-, +--]), \\
P_2 = \Pr([+-+, ++-]) = \Pr([-+-, -+-]), \\
P_3 = \Pr([++-, +++,]) = \Pr([- ++, - - -]).
\]

From these probabilities, we can obtain the known conditional probabilities for experiments in which \( i_A, i_B \) with \( i = 1, 2, 3 \) correspond to the three orientations \( \theta_i \) of the fields. \( \Pr(j_A j_B / i_A i_B) \) is given by adding together the probabilities for which \( j_A \) appears in position \( i_A \) in the first sequence of signs, and \( j_B \) appears in position \( i_B \) in the second sequence:

\[
\Pr(+/+/11) = \Pr(--/11) = P_0 + P_1 + P_2 + P_3 = 1/2 \quad \text{and cyclic,} \quad (13a) \\
\Pr(+/+/23) = \Pr(--/23) = P_0 + P_1 \quad \text{and cyclic,} \quad (13b) \\
\Pr(+/+/31) = \Pr(--/31) = P_2 + P_3 \quad \text{and cyclic,} \quad (13c)
\]

where the cyclic permutations permute 1,2 and 3 but not 0.

The equations can be solved for the \( P_k \), all of which must be nonnegative. Their solutions, with resultant conditions on \( \Pr(+/+/i_A i_B), \Pr(--/i_A i_B) \) are

\[
2P_0 = \Pr(+/+/23) + \Pr(+/+/31) + \Pr(+/+/12) - \frac{1}{2} \geq 0 \quad \text{and cyclic,} \quad (14a)
\]
\[ 2P_1 = \frac{1}{2} + \Pr(\text{+} + /23) - \Pr(\text{+} + /31) - \Pr(\text{+} + /12) \geq 0 \quad \text{and cyclic, (14b)} \]

\[ 2P_1 = \Pr(\text{+} - /31) + \Pr(\text{+} - /12) - \Pr(\text{+} - /23) \geq 0 \quad \text{and cyclic. (14c)} \]

The inequalities (14a,b) are always satisfied, but (14c) gives the Bell inequalities [B18], his equation (15). Bell uses the same origin for the angles at A and B, so his opposite signs are the same signs here, and his expectation values of products are

\[ P(b, c) = 4\Pr(\text{+} - /23) - 1 \quad \text{and cyclic. (15)} \]

The condition of weak nonlocality and the conditions that the transfer function probabilities should all be non-negative, and that sums over them are all less than or equal to 1, provide a systematic method for obtaining inequalities of the Bell type.

Since quantum mechanics violates the Bell inequalities, it is weakly nonlocal, and there are signalling transfer functions \( F_s \) with nonzero probabilities \( \Pr(F_s) \).

5. **Relativistic hidden variables**

It has long been known that it is difficult to reconcile special relativity and quantum theories that are based on background variables [B169-172], including the de Broglie-Bohm pilot wave theory [4,5], and theories based on spontaneous localization or quantum state diffusion [6-13]. Local ‘beables’, in the sense of Bell [B52] appear to be incompatible with special relativity. The difficulty is to reconcile the weak nonlocality of measurement processes with a theory that provides a unique Lorentz-invariant result for a measurement when there is no preferred frame, as discussed in [4].

By taking Einstein’s old simultaneity thought experiment using light signals [14], and replacing his central source and two receivers by a Bell experiment, the following shows that any relativistic hidden variable theory with unique results and no preferred frame results in weak backward causality, and can be rejected for this reason. If we exclude backward causality, then relativistic hidden variable quantum theories of measurement must depend on environmental or cosmological influences, which define a special frame.

Here is the argument. It does not require weak locality. Because of the Bell inequalities, it must allow weak nonlocal signals.

A Bell inequality experiment on spin one-half particles has a source \( S_1 \) and two receivers \( A_1 \) and \( B_1 \) at equal distances from \( S_1 \), all at rest in the same frame, number 1. This experiment is symmetrical with respect to interchange of \( A_1 \) and \( B_1 \). Bell’s theorem shows that, given the hidden variable \( \lambda_1 \), the result of the
experiment at B₁ is dependent on the angle of measurement at A₁, as in (3b), or the result of the experiment at A₁ is dependent on the angle of the measurement at B₁, as in (3c), or both. Now because of symmetry with respect to interchange of A and B, it must be both. There are weak signals in both directions, with the same probabilities.

Let $F_\text{s}$ with nonzero probability $\Pr(F_\text{s})$ be one of the signalling transfer functions. Then for $j = F_\text{s}(i)$, there must be at least one value of $i_B$, defining the orientation at B, and two values $i_A = 1, 2$ which then result in different signs for $j_B$. Otherwise there could be no weak signal. If we fix the value of $i_B$ and allow $i_A$ to have those two values, $F_\text{s}$ defines a one-bit weak signal, which is sent with probability $\Pr(F_\text{s})$ in the Bell experiment.

Now choose two such Bell experiments which are in the same line, but are at rest in different Lorentz frames with a nonzero relative velocity parallel to the line of the experiments. Let the second frame, with its source and two receivers, be labelled by 2. We use $\lambda_2$ to represent the hidden variable that affects the result of experiment 2. As in Einstein’s experiment, A₁ is in the past light cone of A₂ but B₁ is in the future light cone of B₂. It is first assumed that the results of the experiments are not correlated through the hidden variables, so that

$$\Pr(\lambda_1, \lambda_2) = \Pr(\lambda_1)\Pr(\lambda_2).$$

(16)

Later we will drop this assumption.

There is a weak channel from B₁ to A₁ and another from A₂ to B₂.

Set up the apparatus with a classical signal from A₁ to A₂ such that results of the measurements of A₁ determine the orientation of the measurement at A₂. For example, we could set up the apparatus at A so that if the result of the measurement at A₁ is +, then the orientation of A₂ is given by $i_A = 1$, and if the result of the measurement is −, then the orientation of A₂ is given by $i_A = 2$.

Given $\lambda_1$ and $\lambda_2$, the orientation of B₁ affects the result of measurement A₁, which determines the orientation of A₂, which affects the result of measurement B₂. Thus the orientation of B₁ affects the sign of the spin at B₂. This means that weak causality goes backwards in time: there is weak backward causality.

It would indeed be remarkable if the assumption (16) were false, so that the correlation of $\lambda_1$ and $\lambda_2$ produced a correlation between the outputs $j$ for the two Bell experiments. Then two Bell experiments set up with the spacetime configuration of Einstein’s simultaneity thought experiment, but otherwise unconnected, would have correlated outputs, which could be checked experimentally. This is not inconsistent with some alternative quantum theories, like primary state diffusion [12,13].
However, our conclusion does not depend on (16).

From section 2, the only way that the correlation of $\lambda_1$ and $\lambda_2$ could affect the results of the experiments is through the transfer functions, and since it is the signalling functions that produce the backward causality, they would have to be correlated:

$$\Pr(F_{s1}, F_{s2}) \neq \Pr(F_{s1})\Pr(F_{s2}).$$

(17)

This could be achieved if the probability $\Pr(F_{s1}, F_{s2})$ of the two signalling functions occurring together was zero. In that case a signal from B1 to A1 and from A2 to B2 would never occur together, so there would be no backward causality. But in that case the total probability of either $F_{s1}$ or $F_{s2}$ would have to be the sum of the probabilities, which are equal, so that, since no probability can be greater than 1:

$$\Pr(F_{s1} \text{ or } F_{s2}) = 2\Pr(F_{s1}) \leq 1.$$  

(18)

Since $\Pr(F_{s1})$ is small, this inequality can be satisfied. But the probability $\Pr(F_{s1})$ cannot be zero, since there is weak nonlocality. Suppose that $N$ is the smallest integer for which

$$\Pr(F_{s1}) > \frac{1}{2N + 1}$$

(19)

Now replace the two Bell experiments by $2N+1$ of them, in which each pair has the spacetime overlap property of the original two. This can be achieved by choosing the preparation events at points which are distance $L$ apart in some frame:

$$\left(t_k, x_k, y_k, z_k\right) = (\mp L \sinh k\phi, \pm L \cosh k\phi, 0, 0) \quad (k = -N, \ldots, N),$$

(20)

where the frames correspond to boosts of $v = \tanh k\phi$ relative to a standard Lorentz frame, with $c = 1$. The measurement events take place after a time interval $\tau$ small compared with $L \sinh \phi$ in the appropriate frame.

If there is to be no weak signal for any pair, then none of the weak signals can occur together, so

$$(2N + 1)\Pr(F_{s1}) \leq 1,$$

(21)

which contradicts the inequality (19). So at least two of the Bell experiments have a finite probability of their signalling functions occurring together, resulting in backward causality. So it is not possible to avoid backward causality even by allowing the hidden variables of the Bell experiments to be correlated.
6. Conclusions

Transfer functions and their probabilities are a powerful tool for studying hidden variable theories and the associated physical properties of quantum systems. Inequalities of the Bell type follow from the condition that all transfer functions have positive or zero probabilities, which suggests the use of the methods of linear programming. Transfer function analysis also shows that an important type of relativistic Lorentz-invariant hidden variable theory can be ruled out on the grounds that it leads to backward causality.

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