Research on the position and attitude control method of the spatial arc for a five-axis hybrid tapping robot

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Abstract. Automatic rubber harvesting can not only reduce the cost of latex production, but also stabilize the output and improve the production efficiency. In order to lay the foundation for the movement of the robot around the tapping trajectory and meet the performance requirements of the robot in the tapping operation, this paper studies the position and attitude interpolation algorithm of the spatial arc for a five-degree-of-freedom hybrid tapping robot to realize the synchronous motion of the spatial arc pose control. In the verification test, the tip of the pen installed at the end of the robot draws a coherent clear arc in the space. The end vector is perpendicular to the normal vector of the arc trajectory and keeps pointing to the center of the trajectory at the same time, which verifies the correctness of the position and attitude control method.

1. Introduction
Natural rubber is an important national strategic material in modern society. The output of rubber in major rubber producing countries has declined due to the large shortage of rubber workers caused by COVID-19.[11] However, there is still a large demand for natural rubber around the world in 2020.[2] Traditional rubber harvesting is mainly obtained through manual tapping, which the labor cost invested accounts for about 70% of the total cost.[3] Realizing the automation of its tapping can reduce labor cost input, improve production efficiency, stabilize rubber output, and promote the development of natural rubber industry.[4, 5]

Agricultural robots are widely used in the harvesting of agricultural products at present.[6-9] The task of the tapping robot is to achieve stable continuous tapping and ensure that the tapping surface is flat. Zhang Chunlong[10], Zhang Weimin[11], Qiu Jihong[12], etc. studied the tapping method based on the robot arm respectively and realized the tapping function. However, the cork structure on the surface of the rubber tree bark, will cause the load on the tool increase suddenly. In order to ensure the accuracy of the local bark tapping parameters, the performance can be met by using a hybrid robot in tapping.

The end tool of the tapping robot has to not only meet agronomic demands during its movement, but also the tool attitude has to be synchronized with the tool position. The surface of the trunk is regarded as a cylindrical surface because the latex tube in the rubber bark forms a 2-7° included angle with the trunk axis. The tapping trajectory can be regarded as a circular helix curve from top left to bottom right. Circular helix can be decomposed into a straight line along the axial direction and a circular arc along the radial direction, according to the geometric definition of cylindrical helix. Therefore, the spiral trajectory can be decomposed into a straight trajectory and an arc trajectory to study separately. This
paper takes a five-axis hybrid robot as the object to study the arc position and attitude control method, and lays the foundation for the tapping robot to realize the tapping operation.

2. Position and attitude control method of the spatial arc

According to the agronomic demands of tapping, in order to reduce the damage to the cambium during the tapping process, the robot has to keep the tool vector perpendicular to the trunk axis (That is, the tool vector is perpendicular to the normal vector of the trajectory) as Figure 1.

(a) Physical diagram of rubber cutting robot        (b) Diagram of the tool position and attitude in tapping

Figure 1. Five-axis hybrid rubber tapping robot

2.1. Spatial arc position interpolation algorithm

Spatial arc parameters include position information and geometric information. In the Cartesian coordinate, position includes start coordinates \( P_{s_{xyz}} \), end coordinates \( P_{e_{xyz}} \), center coordinates \( P_{c_{xyz}} \) of the arc, and spatial arc unit normal vector \( r_{xyz} \). Geometric includes radius \( R \) and rotation angle \( \beta \). In practical applications, the starting position \( P_{s} \), the ending position \( P_{e} \) and the radius \( R \) of the arc are easy to obtain more accurately and directly.

In this paper, the rotation angle \( \beta \) is equally divided into several radians \( \Delta \beta \), the corresponding points are obtained on the arc. The coordinates of the corresponding points are described relative to the coordinate system established with the center of the circle as the origin, which ensure each point on the spatial arc and realize the position interpolation through the rotation transformation with the base coordinate \( \{0\} \). In the interpolation process, the position of each interpolation point are on the spiral curve. There is no approximate error, and higher interpolation accuracy can be achieved.

In the working space of the robot, the arc starts from \( P_{s} \) and rotates around the unit vector \( r \) with the radius \( R \) to the end point \( P_{e} \). Assuming that the center of the arc is \( P_{c_{xyz}} \), according to the characteristics of arcs \( P_{s}P_{e} \perp r \) and \( P_{e}P_{e} \perp r \), we can get

\[
P_{s}P_{e} \cdot r = (x_{e} - x_{s}) \cdot r_{x} + (y_{e} - y_{s}) \cdot r_{y} + (z_{e} - z_{s}) \cdot r_{z} = 0
\]

\[
P_{e}P_{e} \cdot r = (x_{e} - x_{c}) \cdot r_{x} + (y_{e} - y_{c}) \cdot r_{y} + (z_{e} - z_{c}) \cdot r_{z} = 0
\]

Both \( |P_{s}P_{e}| \) and \( |P_{e}P_{e}| \) are the arc radius long.

\[
\left[ (x_{e} - x_{s})^2 + (y_{e} - y_{s})^2 + (z_{e} - z_{s})^2 \right]^{1/2} = R
\]

\[
\left[ (x_{e} - x_{c})^2 + (y_{e} - y_{c})^2 + (z_{e} - z_{c})^2 \right]^{1/2} = R
\]

Combine (1),(2) and (3) to get arc center that passes \( P_{s} \) and \( P_{e} \). When \( |P_{s}P_{e}| = 2R \), the unique center \( P_{c} \) can be determined, as shown in Figure 2(a). When \( |P_{s}P_{e}| < 2R \), two circle centers \( P_{c_{1}} \) and \( P_{c_{2}} \) are obtained, as shown in Figure 2(b).
Judge the direction of the normal vector \( \mathbf{n} = \mathbf{P}_{P} \times \mathbf{P}_{P_c} \) perpendicular to \( \mathbf{P}_{P} \) and \( \mathbf{P}_{P_c} \). The center of the target circle makes the normal vector \( \mathbf{n} \) the same direction as the unit normal vector \( \mathbf{r} \).

The rotation angle \( \beta \) is

\[
\cos \langle \mathbf{P}_{P}, \mathbf{P}_{P_c}, \mathbf{P}_{P} \rangle = \frac{\mathbf{P}_{P} \cdot \mathbf{P}_{P_c}}{|\mathbf{P}_{P}||\mathbf{P}_{P_c}|} = \frac{(x_c - x_s)(x_c - x_s) + (y_c - y_s)(y_c - y_s) + (z_c - z_s)(z_c - z_s)}{R^2}
\]

(4)

Establish a spatial arc coordinate \( \{C\} \), with the center point \( P_c \) as the origin, the vector \( r \) as the axis \( X_c \), and the vector in the same direction as vector \( P_c P_c \) as the axis \( Z_c \). The conversion relationship between the coordinate \( \{C\} \) and the base coordinate \( \{0\} \) is derived as Figure 3.

The unit vectors of coordinate \( \{C\} \) refer to coordinate \( \{0\} \) as

\[
\begin{align*}
\mathbf{x}_c &= (x_c, y_c, z_c) \\
\mathbf{y}_c &= \frac{1}{R}(x_c - x_s, y_c - y_s, z_c - z_s) \\
\mathbf{z}_c &= \mathbf{y}_c \times \mathbf{x}_c
\end{align*}
\]

(5)

Assuming that the conversion relationship from the coordinate \( \{C\} \) to the base coordinate \( \{0\} \) is \( \alpha_{CT} \).
\[
\begin{bmatrix}
    n_x & o_x & a_x & P_x \\
    n_y & o_y & a_y & P_y \\
    n_z & o_z & a_z & P_z \\
    0 & 0 & 0 & 1
\end{bmatrix} = \begin{bmatrix}
    r_x & r_y & r_z \\
    0 & 0 & 0
\end{bmatrix} = \begin{bmatrix}
    \frac{r_x}{R} & \frac{r_y}{R} & \frac{r_z}{R} \\
    0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
    (y_e - y_i) - (x_e - x_i) & (z_e - z_i) \\
    (z_e - z_i) & (x_e - x_i)
\end{bmatrix} = \begin{bmatrix}
    \frac{x_e - x_i}{R} & \frac{y_e - y_i}{R} & \frac{z_e - z_i}{R} \\
    0 & 0 & 0
\end{bmatrix} \\
(6)
\]

Calculate the \( i \) interpolation points \( P_i(x_i, y_i, z_i) \) \( (i=1,2,...,n_\beta) \) in the coordinate \( \{ C \} \) to simplify the calculation process, Regardless of the tool vector provisionally.

![Figure 4. Schematic diagram of spatial arc position interpolation](image)

The segmentation accuracy of the arc is \( \Delta L \), that is, within one period, the arc length from \( P_{i-1} \) to \( P_i \) is \( \Delta L \), and the corresponding rotation angle is \( \Delta \beta \) as Figure 4.

\[
\Delta \beta = \frac{\Delta L \cos \gamma}{R} \quad (7)
\]

The interpolation number \( n_\beta \) is:

\[
n_\beta = \left[ \frac{\beta}{\Delta \beta} \right] \quad (8)
\]

The \( i \) interpolation point coordinates \( P_i \) in the coordinate \( \{ C \} \) are expressed as

\[
\begin{align*}
x_i &= 0 \\
y_i &= R \sin \beta_i \\
z_i &= -R \cos \beta_i
\end{align*} \quad (\beta_i = i \Delta \beta, i = 1,2,...,n_\beta) \quad (9)
\]

The interpolation point coordinates \( P_i \) in the coordinate \( \{ 0 \} \) can be calculated referring to the conversion relationship \( ^0cT \), and then the spiral arc position interpolation operation can be completed.

\[
\begin{bmatrix}
x_i \\
y_i \\
z_i
\end{bmatrix} = ^0cT \begin{bmatrix}
x_{i,t} \\
y_{i,t} \\
z_{i,t}
\end{bmatrix} \\
(10)
\]

2.2. Spatial arc attitude interpolation algorithm
The attitude interpolation is realized by describing the transformation of the rotation matrix between each two interpolation points. On the trajectory of the robot motion as spatial arc, the tool vector at the beginning and destination of the arc was described, then the rotation angle of the two rotation matrices based on the interpolation accuracy was divided, and the direction vector of the interpolation point was confirmed to completed the attitude interpolation.
At the starting of the spatial arc, the tool vector \( a \) is parallel to the space plane of the arc, and points to the arc rotation axis \( X_c \). The rotation matrix of the tool vector at the point \( P_s \) is a third-order identity matrix.

In the interpolation process, \( 0_j R \) is the rotation matrix which represents the coordinate \( \{j\} \) of the \( i \) interpolation point on the arc to the base coordinate, and \( j+1 \) \( 0 R \) is the rotation matrix which represents the coordinate \( \{j+1\} \) of the \( i+1 \) interpolation point on the arc to the base coordinate. The rotation matrix from the coordinate \( \{j\} \) to the coordinate \( \{j+1\} \) is:

\[
\begin{bmatrix}
    a_{11} & a_{12} & a_{13} \\
    a_{21} & a_{22} & a_{23} \\
    a_{31} & a_{32} & a_{33}
\end{bmatrix}
\]

Regardless the translation transformation between the coordinate systems temporarily, the rotation matrix from the coordinate \( \{j\} \) to the coordinate \( \{j+1\} \) can also be regarded as the \( \{j\} \) rotating around the unit vector \( \omega = \omega_i \hat{i} + \omega_j \hat{j} + \omega_k \hat{k} \) which starting from the origin by the angle \( \alpha \) and coinciding with the \( \{j+1\} \).

During the movement, the Z axis of each point coordinate system on the arc keeps pointing to the axis \( X_c \) which the arc rotated. And \( \{j\} \) rotates around the axis \( X_c \) by the angle \( \Delta \beta \) and coincide with \( \{j+1\} \). The attitude transformation matrix from the coordinate \( \{S\} \) to the coordinate \( \{j\} \) of interpolation point is:

\[
R(\omega, \beta) = \begin{bmatrix}
    \omega_i^2 (1 - \cos \beta) + \cos \beta & \omega_i \omega_j (1 - \cos \beta) - \omega_i \sin \beta & \omega_i \omega_k (1 - \cos \beta) + \omega_i \sin \beta \\
    \omega_j \omega_i (1 - \cos \beta) + \omega_j \sin \beta & \omega_j^2 (1 - \cos \beta) + \cos \beta & \omega_j \omega_k (1 - \cos \beta) - \omega_j \sin \beta \\
    \omega_k \omega_i (1 - \cos \beta) - \omega_k \sin \beta & \omega_k \omega_j (1 - \cos \beta) + \omega_k \sin \beta & \omega_k^2 (1 - \cos \beta) + \cos \beta
\end{bmatrix}
\]

\[
\beta = i \Delta \beta (i = 1, 2, \ldots, n_p)
\]

\[
R(\omega, \beta) = \begin{bmatrix}
    \omega_i^2 (1 - \cos \beta) + \cos \beta & \omega_i \omega_j (1 - \cos \beta) - \omega_i \sin \beta & \omega_i \omega_k (1 - \cos \beta) + \omega_i \sin \beta \\
    \omega_j \omega_i (1 - \cos \beta) + \omega_j \sin \beta & \omega_j^2 (1 - \cos \beta) + \cos \beta & \omega_j \omega_k (1 - \cos \beta) - \omega_j \sin \beta \\
    \omega_k \omega_i (1 - \cos \beta) - \omega_k \sin \beta & \omega_k \omega_j (1 - \cos \beta) + \omega_k \sin \beta & \omega_k^2 (1 - \cos \beta) + \cos \beta
\end{bmatrix}
\]

\[
\beta = i \Delta \beta (i = 1, 2, \ldots, n_p)
\]

In equation (12), \( n_p \) is the time of interpolations in arc position interpolation. When the \( i \) interpolation, the tool attitude matrix based on the coordinate \( \{C\} \) is

\[
\begin{bmatrix}
    \epsilon_{n_{ij}} & \epsilon_{o_{ij}} & \epsilon_{a_{ij}} \\
    \epsilon_{n_{ij}} & \epsilon_{o_{ij}} & \epsilon_{a_{ij}} \\
    \epsilon_{n_{ij}} & \epsilon_{o_{ij}} & \epsilon_{a_{ij}}
\end{bmatrix} = \begin{bmatrix}
    \epsilon_{n_{ij}} & \epsilon_{o_{ij}} & \epsilon_{a_{ij}} \\
    \epsilon_{n_{ij}} & \epsilon_{o_{ij}} & \epsilon_{a_{ij}} \\
    \epsilon_{n_{ij}} & \epsilon_{o_{ij}} & \epsilon_{a_{ij}}
\end{bmatrix} R(\omega, \beta)
\]

Through the inverse kinematic, the tool attitude matrix can solve the joint variables of each step. The joint variables can be used to obtain the joint variable increments, and then the joint motors are controlled to transform the attitude.

According to equation (10) and equation (13), the position matrix and attitude rotation matrix of interpolation points are constructed into a fourth-order homogeneous rotation matrix, and transform it to a new rotation matrix based on the base coordinate \( \{0\} \). Let the tool position and pose matrix be equal to the new rotation matrix. Give out each joint variable through the inverse kinematics model, control
the joint motion, and drive the end actuator to change the position and attitude according to the track planning and complete a spatial arc movement with the position and attitude changes of the tool. In summary, the position and attitude interpolation expression from \( P_s \) to \( P_e \) is (19).

\[
\begin{bmatrix}
    n_x & o_x & a_x & p_x \\
    n_y & o_y & a_y & p_y \\
    n_z & o_z & a_z & p_z \\
    0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    e_x \\
    e_y \\
    e_z \\
    0
\end{bmatrix}
= e^{T} \begin{bmatrix}
    e_{n_x} \\
    e_{n_y} \\
    e_{n_z} \\
    0
\end{bmatrix}
= \begin{bmatrix}
    e_{a_x} \\
    e_{a_y} \\
    e_{a_z} \\
    0
\end{bmatrix}
= \begin{bmatrix}
    P_x \\
    P_y \\
    P_z \\
    p_z
\end{bmatrix} (i = 1, 2 \ldots n_β)
\] (14)

2.3. Motion control of the robot

The 3D structure of the five-axis hybrid robot is shown in Figure 6(a). The robot is assembled by a boom, a jib and an end actuator. The boom is a 2SPU+U two-degree-of-freedom parallel mechanism, the jib is a rotating link, and the end actuator is a two-degree-of-freedom series mechanism. These three parts are connected in series to form a five-degree-of-freedom hybrid mechanism. In the 2SPU+U type parallel part, the two electric cylinders are drive branches, the boom is a restrained branch, and the drive branches only provide driving force for the parallel mechanism. Therefore, the Kinematics analysis of this robot can be equivalent a 5R series mechanism with 5 revolute joints as figure 6(b).

![3D structure diagram](image)

(a) 3D structure diagram

![Equivalent structure diagram](image)

(b) Equivalent structure diagram

Figure 6. Five-axis hybrid robot structure diagram

The control system of the five-degree-of-freedom hybrid robot uses DSP as the motion controller. Set up a kinematic model of the 5R mechanism by the D-H method, obtained the transform relationship \( i^{-1}T(i = 1 \ldots 6) \) between two adjacent link coordinate systems, builded the forward kinematics equation of the hybrid robot. Set up the inverse kinematics model by using the algebra law of the equivalent structure and the geometric method of the parallel structure part. During movement, the motion controller calls the interpolation program to calculate the center coordinates, rotation angle and arc length according to the starting point coordinate, the ending point coordinate, the unit normal vector and the radius. And the motion controller calculates the times of interpolation \( n_β \), the coordinate of the \( i \) interpolation point \( P_i(x_i, y_i, z_i) (i = 1, 2 \ldots n_β) \), and the pose matrix of the point according to the arc mapping the radian division accuracy \( Δβ \). Through the inverse kinematics model, calculate each joint variable and joint variable increment. The interpolation cycle \( T_c \) is set and the motor driving program is called to make the motor run. In the motor drive control, it is needed to input the joint variable increment \( ΔX \) and the interpolation period \( T \), judge the rotation direction of the motor and calculate the number of pulses. the motion control of the robot is realized based on the motion position increment command to drive the motor to rotate according to the CAN bus program.

3. Spatial arc motion control test

The test object is the five-degree-of-freedom hybrid manipulator, the CAN bus communication strategy is used, the hardware platform is built with TMS320F28335 as motion controller, and the software
development is completed under the CCS6.1.1 integrated environment. A spatial arc is given as the robot motion trajectory, start point (1460, -100, 900), end point (1460, 100, 900), arc unit normal vector \( r = (0,0,1) \), radius \( R = 100 \text{mm} \), a pen is installed at the end instead of the tool, the spatial arc motion control method is verified.

Line 1 is a circle around the cylinder. The trajectory of the robot is parallel to line 1, and is the same normal vector with line 1 as a space arc. During the movement, the pose of the tool points to the center of the spatial arc as shown in Figure 7. The correctness of the above motion control method is verified.

![Figure 7. Robot position and attitude diagram of space arc motion process](image)

### 4. Conclusion
(1) In order to realize the automatic tapping of natural rubber, the spatial arc motion control method of a five-axis hybrid tapping robot is proposed. In the Cartesian coordinate, The position and attitude of the space arc are synchronously interpolated at the end of the rubber cutting robot to realize the motion control of the spatial arc.

(2) In the verification test, the tool attitude of the robot keeps pointing to the center of the spatial arc trajectory all the time. The correctness of the motion control method is verified.

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