Boundary element analysis of dynamic interaction of anisotropic elastic half-space and structure under impact load

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Abstract. In this paper, a direct boundary element formulation for numerical modelling of three-dimensional dynamic soil-structure interaction is briefly described. Formulation is applicable to problems involving isotropic and anisotropic elastic materials. It is based on the Laplace domain regularized boundary integral equations for the displacements under assumption of vanishing initial conditions and zero body forces. Mixed boundary elements are employed for the spatial discretization of the boundary integral equations. Subdomain technique is used to tackle problems involving piecewise homogeneous material properties. The described formulation is employed for the boundary element analysis of dynamic soil-structure interaction problem involving a structure which contains a cavity with a complex geometrical form. Soil is represented by a transversely isotropic elastic half-space. Effect of material anisotropy of half-space on the transient response of the embedded structure is assessed. It is observed that the shear modulus anisotropy have substantial impact on the displacements response. Influence of the Young’s modulus in the plane of isotropy is lesser but still pronounced.

1. Introduction

Consideration of the dynamic interaction between the supporting soil and the embedded structure is essential for ensuring safety, reliability and integrity of complex structures, buildings and foundations subjected to dynamic loadings like blast or impact. Naturally occurring soils often exhibit anisotropic behavior. Representing soil in practical problems as an isotropic medium can lead to inadequate or unsatisfactory results. Boundary Element Method (BEM) is well-known as an efficient and accurate numerical method for modelling transient wave propagation in linearly elastic solids and media, but the number of works on the BEM applied to dynamic problems which involves anisotropic elastic soils is quite low [1-8]. Primarily it is attributed to the difficulties associated with numerical implementation of the dynamic fundamental solutions of anisotropic elasticity, since the numerical integration of double integrals has to be performed for each pair of field and source points.

In this work we briefly describe a direct boundary element formulation for numerical analysis of three-dimensional dynamic soil-structure interaction problems and apply it to a problem which involves a protective cover structure of a nuclear heating plant (nuclear plant which produce thermal energy for district heating) partially embedded into a transversely isotropic soil. Effect of anisotropic characteristics of the soil on transient response of the structure is studied.

2. Boundary element formulation
The Laplace domain equations of motion for a homogeneous linearly elastic solid $\Omega \in \mathbb{R}^3$ bounded by a surface $\Gamma = \partial \Omega$ assuming zero initial conditions and absence of the body forces are

$$\bar{\sigma}_{ij}(\mathbf{x},s) - \rho s^2 \bar{\nu}_{ij}(\mathbf{x},s) = 0, \quad \mathbf{x} \in \Omega, \quad i, j = 1,3,$$

(1)

where $\bar{\sigma}_{ij}$ is the stress tensor, $\rho$ is the mass density, $s$ is the Laplace transform parameter and $\bar{\nu}_{ij}$ are the displacements.

The symmetric linear strain tensor $\bar{\varepsilon}_{ij}$ is defined from the strain-displacement relationship

$$\bar{\varepsilon}_{ij}(\mathbf{x},s) = \frac{1}{2} \left( \bar{\nu}_{ij}(\mathbf{x},s) + \bar{\nu}_{ji}(\mathbf{x},s) \right).$$

(2)

The stress tensor and the strain tensor are linearly related through the generalized Hooke’s law via the fourth-order elasticity tensor $C_{ijkl}$ as follows

$$\bar{\sigma}_{ij}(\mathbf{x},s) = C_{ijkl} \bar{\varepsilon}_{kl}(\mathbf{x},s), \quad k, l = 1,3.$$

(3)

Combining equations (1)-(3) and symmetry of stress tensor the linear field equations are obtained

$$C_{ijkl} \bar{\nu}_{kl}(\mathbf{x},s) - \rho s^2 \bar{\nu}_{ij}(\mathbf{x},s) = 0, \quad \mathbf{x} \in \Omega$$

(4)

Problem is supplemented with an appropriate boundary conditions

$$\bar{\nu}_{i}(\mathbf{x},s) = \bar{\nu}_{i}'(\mathbf{x},s), \quad \mathbf{x} \in \Gamma_u,$$

(5)

$$\bar{t}_{i}(\mathbf{x},s) = \bar{\sigma}_{ik}(\mathbf{x},s)n_{k}(\mathbf{x}) = \bar{t}_{i}'(\mathbf{x},s), \quad \mathbf{x} \in \Gamma_t,$$

(6)

where $n_{k}$ denote components of the outward unit normal vector to the boundary $\Gamma$ at point $\mathbf{x}$.

The Laplace domain regularized direct boundary integral equations (BIEs) for displacements corresponding to the differential equations (4) can be expressed as follows

$$\int_{\Gamma_u} \left[ \bar{u}_{i}(\mathbf{y},s)h_{ik}(\mathbf{y},\mathbf{x},s) - \bar{u}_{i}(\mathbf{x},s)h_{ik}^s(\mathbf{y},\mathbf{x}) - \bar{t}_{i}(\mathbf{y},s)\bar{s}_{ik}(\mathbf{y},\mathbf{x},s) \right] d\Gamma(\mathbf{y}) = 0, \quad \mathbf{x} \in \Gamma,$$

(7)

where $\mathbf{x}$ is the field point and $\mathbf{y}$ is source point. $\bar{s}_{ik}$ and $h_{ik}$ are three-dimensional Laplace domain displacement and traction dynamic full-space fundamental solutions, respectively, $h_{ik}^s$ is the singular term of $\bar{h}_{ik}$.

In the present formulation, so-called mixed boundary elements [9] are employed for the spatial discretization of BIEs (7). The geometry of a mixed quadrangular boundary element is interpolated using quadratic shape functions. Linear variation of displacements and a constant value of tractions are assumed over each boundary element. For the problems with anisotropic materials, fundamental solutions are expressed in terms of finite double integrals [10, 11]. Special attention is paid to the numerical evaluation of these integrals due to the highly oscillatory nature of the integrand for large values of the imaginary part of the complex frequency $s$.

Applying nodal collocation procedure, carrying out necessary integrations and taking into account prescribed boundary conditions we obtain a complex-valued system of linear algebraic equations for a given value of the Laplace transform parameter $s$. For the problems involving piecewise homogeneous material properties (e.g. soil-structure interaction) we employ a subdomain technique: boundary element procedure is repeated for each subdomain in which the material properties are constant and the coupling constraints follow from the conditions of equilibrium of tractions and continuity of the displacements at the interfaces between the subdomains.
3. Numerical analysis
The described formulation is employed to conduct the boundary element analysis of complex soil-structure interaction problem involving a protective cover structure of a nuclear heating plant partially embedded into a soil which is represented by a homogeneous anisotropic elastic half-space. Symmetrical part (for \( x_z \geq 0 \)) of the considered problem’s geometrical configuration is depicted in figure 1. Protective cover structure is represented by an isotropic elastic rectangular parallelepiped with dimensions \( L \times L \times H \) and it contains a cavity with a complex geometrical form as shown in detail in figure 2. The values of constants indicated in figure 2 are given as follows: \( H = 34.5 \text{ m} \), \( L = 15.6 \text{ m} \), \( H_e = 14.455 \text{ m} \), \( l_1 = 9.6 \text{ m} \), \( l_2 = 11.6 \text{ m} \), \( h_1 = 2 \text{ m} \), \( h_2 = 9.3 \text{ m} \), \( h_3 = 4.215 \text{ m} \), \( h_4 = 4.01 \text{ m} \), \( h_5 = 8.445 \text{ m} \), \( h_6 = 2 \text{ m} \), \( h_7 = 2.2 \text{ m} \), \( h_8 = 0.33 \text{ m} \), \( r_1 = 3.6 \text{ m} \), \( r_2 = 3.8 \text{ m} \), \( r_3 = 4.29 \text{ m} \), \( r_4 = 5.8 \text{ m} \) and point \( O(0,0,0) \) is the origin of a rectangular Cartesian coordinate system \( \{O; x_1, x_2, x_3\} \). On the area specified by relations \( 0 \text{ m} \leq x_1 \leq 7.8 \text{ m} \), \( 0 \text{ m} \leq x_2 \leq 7.8 \text{ m} \), \( x_3 = 20.045 \text{ m} \), the cover structure is subjected to an impact loading \( t_3(t) = -t_3 \cdot H(t), t_3 = 1 \cdot 10^7 \text{ Pa}, H(t) \) is the Heaviside step function. The half-space and the rest of the protective cover structure are considered to be free of tractions. The elastic properties of the protective cover structure are as follows: \( E_{\text{struct}} = 3 \cdot 10^3 \text{ Pa}, \nu_{\text{struct}} = 0.2, \rho_{\text{struct}} = 2000 \text{ kg/m}^3 \).

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**Figure 1.** Configuration of the soil-structure interaction problem.

**Figure 2.** Geometry of protective cover structure.
To assess the effect of material anisotropy of the half-space on the transient response of the embedded structure we employ one artificial isotropic and several transversely isotropic materials. Density of all these fictitious materials is \( \rho = 2000 \) kg/m\(^3\). Young’s modulus of isotropic material is \( E_{iso} = 2 \cdot 10^{10} \) Pa and Poisson’s ratio is \( \nu_{iso} = 0.25 \). Planes of isotropy of transversely isotropic materials are parallel to \( x_1 - x_2 \) plane and \( x_3 \)-axis is assumed to be the axis of rotational material symmetry. Non-zero elastic moduli in Voigt notation \( c_{ij} \) for a transversely isotropic soil can be expressed through the five independent engineering elastic constants \( E \), \( E', \nu \), \( \nu' \) and \( G' \) by

\[
c_{11} = c_{22} = \frac{E(1-\nu'\nu)}{(1+\nu)(1-2\nu'\nu)} , \quad c_{12} = c_{21} = \frac{E(\nu+\nu'\nu)}{(1+\nu)(1-2\nu'\nu)} , \quad c_{44} = c_{55} = G' , \quad (8)
\]

\[
c_{13} = c_{23} = c_{31} = c_{32} = \frac{Ev'}{1-2\nu'\nu}, \quad c_{33} = \frac{E'(1-\nu')}{1-2\nu'\nu}, \quad c_{66} = \frac{E}{2(1+\nu)} , \quad \eta = \frac{E'}{E}, \quad (9)
\]

where \( E \) and \( \nu \) are the Young’s modulus and the Poisson’s ratio in the \( x_1 - x_2 \) symmetry plane; \( E' \) and \( G' \) is the Young’s modulus and the shear modulus in the \( x_3 \)-direction; \( \nu' \) is the Poisson’s ratio characterizing the lateral strain response in the \( x_1 - x_2 \) plane to a stress applied in the \( x_3 \)-direction.

For the problem under consideration we chose \( E' = 2 \cdot 10^{10} \) Pa, \( \nu = \nu' = 0.25 \). Properties of all materials involved in analysis are summarized in Table 1.

| Material        | \( \frac{E}{E'} \) | \( \frac{G'}{E'} \) | \( c_{11} \) | \( c_{12} \) | \( c_{13} \) | \( c_{33} \) | \( c_{44} \) | \( c_{66} \) |
|-----------------|---------------------|---------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Material 1      | 1                   | 0.1                 | 24           | 8            | 8            | 24           | 2            | 8            |
| Material 2      | 1                   | 0.2                 | 24           | 8            | 8            | 24           | 4            | 8            |
| Material 3      | 1                   | 0.3                 | 24           | 8            | 8            | 24           | 6            | 8            |
| Material 4 (isotropic) | 1               | 0.4                 | 24           | 8            | 8            | 24           | 8            | 8            |
| Material 5      | 2                   | 0.4                 | 56           | 24           | 20           | 30           | 8            | 16           |
| Material 6      | 3                   | 0.4                 | 104          | 56           | 40           | 40           | 8            | 24           |
| Material 7      | 5                   | 0.4                 | 440          | 360          | 200          | 120          | 8            | 40           |

Figures 3 and 4, 5 and 6 demonstrate transient responses of displacements \( u_s(t) \) and \( u_v(t) \) at the points \( A(4.8,0,18.045) \)m and \( B(3.8,0,-4.01) \)m located on the surface of the cavity inside the protective cover structure, respectively. Even though point \( A \) is located far above the surface of the half-space and very near the loading area, material parameters of the transversely isotropic half-space have noticeable influence on the displacements response. Much more pronounced impact on the response of the structure is evident from figures 5 and 6. In all cases, increasing ratio \( G'/E' \) and increasing ratio \( E/E' \) differently affects absolute values of the maximal displacements. Comparing to the solution for the isotropic half-space, shear modulus anisotropy have substantial influence on the vertical displacements \( u_v(t) \) and lesser on the displacements \( u_s(t) \). In contrast, the influence of the Young’s modulus in the plane of isotropy is more notable for \( u_s(t) \) than for \( u_v(t) \), but altogether less than effect of \( E/E' \). In addition, for a static formulation of the problem, figure 7 displays changes in distribution of displacements \( u_v \) over a part of the cavity located in an embedded section of a structure. Obtained results convincingly illustrate the necessity to take into account material anisotropy of the embedding medium when solving static and dynamic soil-structure interaction problems.
Figure 3. Displacements $u_x(t)$ at point A for (a) materials 1-4, (b) materials 4-7.

Figure 4. Displacements $u_y(t)$ at point A for (a) materials 1-4, (b) materials 4-7.

Figure 5. Displacements $u_z(t)$ at point B for (a) materials 1-4, (b) materials 4-7.
Figure 6. Displacements $u_3(t)$ at point B for (a) materials 1-4, (b) materials 4-7.

Figure 7. Distribution of displacements $u_3$ for (a) material 1, (b) material 4.

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