An Amplifying RIS Architecture with a Single Power Amplifier: Energy Efficiency and Error Performance Analysis

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Abstract—Reconfigurable intelligent surfaces (RISs) have recently attracted the attention of the community as a potential candidate for next generation of wireless communication networks. Various studies have been carried out on this technology, which allows the control of the signal propagation environment. However, when an RIS is used in its inherently passive structure, it appears to be only a supportive technology for communications, while suffering from a multiplicative path loss. Therefore, researchers have lately begun to focus on RIS hardware designs with minimal active elements to further boost the benefits of this technology. In this paper, we present a simple hardware architecture for RISs including a single variable gain amplifier for reflection amplification to confront the multiplicative path loss. The end-to-end signal model for communication systems assisted with the proposed amplifying RIS design is presented, together with an analysis for the capacity maximization and the theoretical bit error probability performance, which is verified by computer simulations. In addition, the advantages of the proposed amplifying RIS design compared to its passive counterpart is presented, together with an analysis for the capacity maximization and the theoretical bit error probability performance, which is verified by computer simulations.

Index Terms—Reconfigurable intelligent surface, energy efficiency, performance analysis, reflection amplification.

I. INTRODUCTION

THE number of individuals trying to keep up with the emerging technologies is increasing day by day. This situation affects the socio-cultural environment of people or even the economy of countries, and especially affects communication networks. Recent generations of wireless communication have been developed to meet the high data demand arising from these technological developments [1]. For these reasons, the fact that the existing communication systems will start to become insufficient in the future, as it was before, will make the development of the next generation of the wireless communication, 6G and beyond, inevitable. This new generation should improve many important factors such as data rate, security, consistency, and mobility, much more than its predecessor. In order to meet such a huge future demand for these factors, numerous studies are being carried out towards 6G wireless networks. As a result of these studies, the use of advanced technologies such as extreme multiple-input multiple-output (MIMO) systems with power efficient front-ends [2], as well as terahertz and millimeter wave communications might play an important role in supplying these demands.

Reconfigurable intelligent surface (RIS)-empowered communication has become one of the most popular developments among these 6G technologies in recent years [3]–[6]. Supporting very high wireless channel capacity, expanding the signal coverage, reducing the bulkiness of the multiple antenna systems as well as the energy consumption, low-cost implementation and mitigating several negative effects of the wireless channel, such as multipath fading and the Doppler effect can be given as some of the reasons why RISs are being investigated thoroughly [7]–[9].

Differently from the massive antenna arrays that include Radio Frequency (RF) chains attached to the antennas, RIS includes only a certain number of reflecting elements that reflect the incoming signals with a specific phase. This can be done by changing the electromagnetic properties of its reflecting elements, such as their reflection coefficient, with the help of a software-defined controller, in that way, the wireless propagation environment becomes configurable. The potential of RI-is stems from the fact that they perform these operations, while being almost passive, because tiny reflecting elements are used instead of power-hungry phase shifters. Thus, energy efficient and low cost setups can be realized at very small sizes with the help of RISs.

Numerous studies have been conducted by combining RISs with many research fields. Assisting a relay system with an RIS [10], RIS-based index modulation [11], RIS-enabled reflection modulation [12], implementation of non-orthogonal multiple access (NOMA) by using RISs [13], [14], and channel modeling in the presence of an RIS [15]–[17] can be given as examples. These studies have achieved a great success with the help of RISs by avoiding or abiding some of the negative effects of them, such as the double pathloss problem [18], [19], and lossy reflection through reflecting elements. For this reason, passive RISs only perform efficiently when they are positioned close to terminals and cannot go beyond being a supportive technology for the communication system. To overcome these unfavourable properties, some studies have been conducted by combining active elements with RISs. In [20], a single receive RF chain that includes a low-noise amplifier...
In this paper, we propose an amplifying RIS design that incorporates a single PA, enabling reflection amplification. According to our design, the impinging signal in a portion of the RIS, after getting phase configured, is fed to the PA which feeds it to the remaining portion of the RIS that reflects it with controllable phase configuration. The amplifying design works solely in the RF domain similar to the waveguide-based approach in [22], unlike full-duplex multi-antenna decode-and-forward (DF) relays, which perform down-conversion and baseband processing [10]. Therefore, although our design looks more similar to a full-duplex multi-antenna amplify-and-forward (AF) relay, they have key differences. Relays usually perform linear processing techniques, such as maximum ratio combining (MRC), and realize power allocation optimization algorithms. Furthermore, they include bulky phase-shifter networks for transmit beamforming, and are subject to loopback self-interference [25]. Our design is simpler and does not require complex algorithms, and prevents loopback self-interference via spatial separation in the form of back to back placement of the different RIS portions. Note that simple self-interference cancellation techniques in the RF domain can be applied [27]. We note that a similar concept was considered in [28], assuming that the received signal is amplified and fed back as a leakage to the input, but without a sufficient mathematical framework and a detailed modeling of the system operation. On the other hand, we propose a unique amplifying and active RIS scheme with a solid mathematical framework, and provide the system model for it. Moreover, we analyze the theoretical bit error probability (BEP), capacity, and energy efficiency (EE) of the proposed design and compare it with a benchmark passive RIS structure. The proposed design includes interesting trade-offs. For instance, amplifying the signal further results in better system performance, but it also requires more power consumption, as well as greater maximum output power for the amplifier. Also, the advantage of the proposed scheme compared to the other schemes is that the number of active devices is not increasing with respect to the number of reflecting elements. In that way, the EE performance of the system can be boosted considerably. The contributions of this study are emphasized as follows:

- We propose a new and energy-efficient amplifying RIS design. The end-to-end system model of the proposed amplifying RIS design is presented.
- We examine the achievable rate of the system under different configurations.
- Theoretical BEP is obtained under different conditions, and also verified through computer simulations.
- An optimization problem is addressed to maximize the system capacity. EE analyses are performed for different key metrics.
- We make comprehensive comparisons of the proposed active model with a passive RIS setup.

The rest of the paper is organized as follows. In Section II, we present the end-to-end system model of the proposed amplifying RIS design. Section III presents the performance analysis of the system by investigating the BEP with a theoretical approach. In Section IV, the EE analysis is performed by considering characteristics of the PAs. In Section V, we exhibit our numerical results to evaluate the system performance. Finally, the paper is concluded in Section VI.

II. SYSTEM MODEL

In this section, we propose an amplifying RIS-assisted system model. Beside this, a conventional passive RIS system model is given as the benchmark to the proposed scheme.

A. Amplifying RIS-assisted System

In this subsection, the considered single-input single-output (SISO) amplifying RIS-assisted system model is introduced. In this scenario, there are two passive RISs, denoted as RIS$_1$ and RIS$_2$, that are connected with a PA, and both have $N$ number of reflecting elements while assuming there is no direct link between the transmitter (Tx) and the receiver (Rx) as shown in Fig. 1. All of the reflecting elements of RIS$_1$ receive signals from the Tx and these signals are combined by RIS$_1$ to be received as a single signal. After that, the signal is amplified by the PA by taking into account that there is also some additional noise to be amplified, which is due to all of the reflecting elements of the RIS$_1$. Also, we consider the signal power at the output of the amplifier is limited. The system
passes the resulting amplified signal through the RIS2, such that the reflecting elements of the RIS2 share and transmit the amplified signal. One should consider that there might be leakage from the signal transmitted by RIS2 to the signal received by RIS1. However, this issue can be resolved by properly positioning the RISs, since an RIS is a one-sided surface. In our design, RIS1 and RIS2 are placed back-to-back and separately as shown in Fig. 1 so that the signals directed by RIS2 do not contaminate the signal captured by RIS1.

In light of these, the received complex baseband signal can be expressed as

\[
y = \sqrt{\frac{G}{N}} (\phi^T h \sqrt{P} s + \sqrt{F} n_{\text{tot}}) \theta^T g + n_{\text{rx}} \tag{1}
\]

where \( s \) and \( y \) correspond to the transmitted and received signals, respectively. \( P_t \) is the transmit power at the Tx and \( N \) is the number of reflecting elements. \( \phi \in \mathbb{C}^{N \times 1} \) and \( \theta \in \mathbb{C}^{N \times 1} \) are the reflecting element phase shift vectors of the RIS1 and RIS2, respectively, where \( \phi = [\phi_1 \phi_2 \cdots \phi_N]^T \) and \( \theta = [\theta_1 \theta_2 \cdots \theta_N]^T \). \( G \) and \( F \) are the gain and the noise figure of the amplifier, \( n_{\text{rx}} \) is the noise sample at the Rx, and \( n_{\text{tot}} \) is the total amount of noise at the input of the amplifier. \( h \in \mathbb{C}^{N \times 1} \) is the channel between the Tx and RIS1 and \( g \in \mathbb{C}^{N \times 1} \) stands for the collected signal received by the RIS1. The signal is multiplied by \( \sqrt{G} \) because of the amplification. Besides, it is divided by \( \sqrt{N} \) because of the phase shift network of the RIS2 that is a kind of power-division circuit, so the total power is distributed among all of the reflecting elements of the RIS2. After that, the amplified and distributed signal is steered to the Rx with the help of the RIS2. A similar scenario is also valid for the \( n_{\text{tot}} \). After multiplying it with the noise figure of the amplifier, it is also amplified, and distributed among the RIS2. Here, the noise at the RIS2 is ignored, where the noise at the RIS1 is not because it is subjected to amplification.

The channels exhibit either Rayleigh or Rician fading depending on the distance as the probability of having a line-of-sight (LOS) component, \( p_{\text{LOS}} \), is also a function of the distance. The channel coefficients are modeled as

\[
h_i = \sqrt{\frac{1}{\lambda^h}} \left( \sqrt{\frac{K_i}{K_1+1}} h_i^L + \sqrt{\frac{1}{K_1+1}} h_i^{NL} \right) \tag{2}
\]

\[
g_i = \sqrt{\frac{1}{\lambda^g}} \left( \sqrt{\frac{K_i}{K_2+1}} g_i^L + \sqrt{\frac{1}{K_2+1}} g_i^{NL} \right) \tag{3}
\]

where \( K_1, K_2, \lambda^h \) and \( \lambda^g \) denote the Rician factors for Tx-RIS1, and RIS2-Rx links, and the pathloss components for the channels \( h \) and \( g \), respectively. Here, \( h_i^L \) and \( g_i^L \) represent the LOS components, and \( h_i^{NL} \) and \( g_i^{NL} \) represent the non-line-of-sight (NLOS) components and are modeled by Rayleigh fading, \( h_i^{NL}, g_i^{NL} \sim \mathcal{CN}(0,1) \) for \( i = 1, \ldots, N \), where \( \mathcal{CN}(0,1) \) stands for the complex Normal distribution with zero mean and unit variance. If the channels include an NLOS component, which mostly refer to Rayleigh fading, we consider \( K_1 = K_2 = 0 \). Furthermore, the pathloss components \( \lambda^h \) and \( \lambda^g \) depend on having NLOS or LOS communication, and calculated as follows by considering 3GPP study on channel models for frequencies from 0.5 to 100 GHz [29]:

\[
\lambda_{\text{LOS}}[\text{dB}] = 32.4 + 17.3 \log_{10}(d_t) + 20 \log_{10}(f_c) \tag{4}
\]

\[
\lambda_{\text{NLOS}}[\text{dB}] = \max (\lambda_{\text{LOS}}, 32.4 + 31.9 \log_{10}(d_t) + 20 \log_{10}(f_c)) \tag{5}
\]

where \( d_t \in \{d_1, d_2\} \) depends on the channel. Moreover, \( d_1 = \sqrt{d_v^2 + d_h^2} \) is the distance between the Tx and RIS1, \( d_2 = \sqrt{d_v^2 + (d - d_h)^2} \) is the distance between the RIS2 and Rx, \( d_v \) and \( d_h \) is the vertical and horizontal distance between the Tx and the RISs, and \( d \) is the distance between the Tx and Rx in meters, respectively, as shown in Fig. 1. Also, \( p_{\text{LOS}} \) is given as

\[
p_{\text{LOS}} = \begin{cases} 
1 & \text{if } d_n \leq 5 \\
ed^{-\frac{(d_n-5)^2}{10}} & \text{if } 5 < d_n \leq 49 \\
0.54e^{-\frac{(d_n-49)^2}{10}} & \text{if } 49 < d_n.
\end{cases} \tag{6}
\]

To determine the signal-to-noise ratio (SNR) of the amplifying RIS-assisted system, (1) is expanded as

\[
y = \sqrt{\frac{G P_t}{N}} \phi^T h \left( \begin{bmatrix} \theta^T g \\ \sqrt{N} \theta^T g \end{bmatrix} s + \sqrt{\frac{G F}{N}} \theta^T g \right) n_{\text{tot}} + n_{\text{rx}} \tag{7}
\]

so we obtain

\[
\gamma_{\text{act}} = \frac{P_t \left| \sqrt{\frac{G}{N}} (\phi^T h) (\theta^T g) \right|^2}{\sqrt{\frac{G F}{N} \theta^T g} \sigma^2 + \sigma^2_{\text{rx}}} \tag{8}
\]

where \( \gamma_{\text{act}} \) is the instantaneous received SNR of the amplifying RIS-assisted system and the achievable rate of the system is expressed as \( R_{\text{act}} = \log_2 (1 + \gamma_{\text{act}}) \).

### B. Passive RIS-assisted System

This section is based on a passive RIS-assisted model shown in Fig. 2. In this scenario, only one RIS is used and it contains \( 2N \) reflecting elements such that it can be considered as a fair benchmark to the amplifying RIS-assisted model. The signal model of the passive system is specified as

\[
y = g_p^T \psi_p \sqrt{P_t} s + n_{\text{rx}} \tag{9}
\]
where \( \Psi \in \mathbb{C}^{2N \times 2N} \) is the reflecting element phase shift matrix of the RIS and \( \Psi = \text{diag} \left( [\psi_1 \psi_2 \cdots \psi_{2N}] \right) \). \( \mathbf{b}_p \in \mathbb{C}^{2N \times 1} \) is the channel between the Tx and RIS, while \( \mathbf{g}_p \in \mathbb{C}^{2N \times 1} \) stands for the channel between the RIS and Rx. They follow either Rician or Rayleigh fading depending on the distance similar to the channels in the active model. Here, SNR of the passive model \( \gamma_{pas} \) is determined as

\[
\gamma_{pas} = \frac{P_t |\mathbf{g}_p^T \Psi \mathbf{h}_p|^2}{\sigma_{rx}^2} \tag{10}
\]

where the achievable rate for the passive model is \( R_{pas} = \log_2 (1 + \gamma_{pas}) \).

### III. Performance Analysis

In this section, we analyze the maximization of system capacity and the distribution of the received SNR for the amplifying RIS-assisted system, and present our theoretical BEP calculations accordingly. In order to maximize the system capacity, the received SNR for the amplifying RIS-assisted system should be maximized by finding optimum values of the phase shift vectors of RIS1, RIS2 and the gain of the amplifier \( G \). The corresponding optimization problem is formulated as follows:

\[
\gamma_{act} = \max_{G, \phi, \theta} \frac{P_t}{N} \frac{G}{2} \left( \phi^T \mathbf{h} (\theta^T \mathbf{g}) \right)^2 \tag{11}
\]

subject to:

- \( |\phi_i| = |\theta_i| = 1, \quad i \in \{1, 2, \ldots, N\} \)
- \( GP_t |\phi^T \mathbf{h}|^2 \leq P_{\text{max}} \)
- \( G \leq G_{\text{max}} \)

where \( G_{\text{max}} \) stands for the maximum gain of the amplifier. Here, the optimal value of \( G \) depends on the phase shift matrix of RIS1 as well. Therefore, the optimal solution to the phases of the reflecting elements should be determined first. The optimal solutions to reflecting element phases for RIS1 and RIS2 are obtained respectively as:

\[
\phi_i = e^{-j \chi_i} \tag{12}
\]

\[
\theta_i = e^{-j \phi_i} \tag{13}
\]

where \( \chi \) denotes the phase of a complex term. Next, we should specify the value for an optimal \( G \) by fixing the optimal value of \( \phi \) as in (12). It is mentioned earlier that there is a maximum power level for the amplified signal. Therefore, an optimal gain value should be determined so that the power of the input signal to the amplifier, \( P_{in} = P_t \left( \sum_{i=1}^{N} |h_i|^2 \right) \), should not exceed the maximum output power when it is amplified. Thus, the equation below should be satisfied to obtain the optimal value of \( G \):

\[
\overline{G}_{\text{opt}} P_t \left( \sum_{i=1}^{N} |h_i|^2 \right)^2 = P_{\text{max}} \tag{14}
\]

where \( \overline{G}_{\text{opt}} \) and \( P_{\text{max}} \) stand for the optimal gain value without the limitation of \( G_{\text{max}} \) and the maximum output power of the amplifier, respectively. Considering that \( \overline{G}_{\text{opt}} \) can exceed \( G_{\text{max}} \), we define the optimal gain as:

\[
G_{\text{opt}} = \min \left( \overline{G}_{\text{opt}}, G_{\text{max}} \right) \tag{15}
\]

By substituting \( G_{\text{opt}} \) in (15) into (8) for \( G \) and arranging the reflecting element phases of \( \phi \) and \( \theta \) as in (12) and (13), respectively, the maximized received SNR for the amplifying RIS-assisted system can be written as

\[
\gamma_{act} = \frac{P_t P_{\text{max}} F}{N} A^2 B^2 \tag{16}
\]

where \( A = \sum_{i=1}^{N} |h_i|^2 \) and \( B = \sum_{i=1}^{N} |g_i|^2 \). Here, one can easily observe that the maximized received SNR increases proportional to \( N, P_t \), and \( P_{\text{max}} \). However, there are also other constraints that can limit the effect of the RIS size and the Tx power, such as \( P_{\text{max}} \) and \( G_{\text{max}} \). The effects of these constraints are further investigated in Section V. Moreover, the amplifying RIS model experiences multiplicative path loss just like the passive RIS since (16) includes the multiplication of \( A \) and \( B \). Nevertheless, the gain factor significantly compensates this double path loss effect.

Probability analysis of the maximized SNR in (16) gives us useful insights. The terms \( A \) and \( B \) converge to Gaussian distributed random variables due to the Central Limit Theorem (CLT) for sufficiently large \( N \), where \( |h_i| \) and \( |g_i| \) are Rayleigh or Rician distributed random variables. Thus, \( A^2 \) and \( B^2 \) follow non-central chi-square distribution with one degree of freedom. The denominator and numerator of the SNR term in (16) are correlated and follow Gamma distribution due to the combination of weighted non-central chi-square random variables. The received SNR term in (15) becomes the ratio of two Gamma random variables. The received SNR term in (15) includes the multiplication of \( A \) and \( B \). Nevertheless, the gain factor significantly compensates this double path loss effect.

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specifications such as $N$, $P_{\text{max}}$, and $P_t$. In Fig. 3 the standard deviation of the distribution increases as the number of RIS element increases.

Theoretical symbol error probability (SEP) is calculated by using the moment generation function (MGF) of the Gamma distribution as follows [34]:

$$M_{Y_{\text{act}}} (s) = (1 - \nu s)^{-k}, \text{ for } s < \frac{1}{\nu}$$  \hspace{1cm} (17)

where $k$ and $\nu$ represents the shape and scale parameters for fitted Gamma distribution and provided in Table I for different system specifications. The average SEP for $M$-ary phase-shift keying ($M$-PSK) signaling by using the MGF in (17) given as follows

$$P_s = \frac{1}{\pi} \int_0^{\pi} M_{\nu} \left(\frac{-\sin^2(\nu/2)}{\sin^2(\delta)}\right) d\delta,$$  \hspace{1cm} (18)

We can numerically calculate the SEP in (18) by using the Gamma distribution parameters and obtain BEP as $P_e \approx P_s/\log_2 M$. Accordingly, the BEP for binary phase-shift keying (BPSK) simplifies to

$$P_e = \frac{1}{\pi} \int_0^{\pi/2} \left(1 + \frac{\nu}{\sin^2(\delta)}\right)^{-k} d\delta.$$  \hspace{1cm} (19)

TABLE I

| $P_{\text{max}}$ (dBm) | $P_t$ (dBm): | -10 | -5 | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
|------------------------|-------------|-----|----|---|---|----|----|----|----|----|
| 64                     | $k$         | 44.8922 | 44.7180 | 44.7905 | 44.7109 | 44.8358 | 44.8963 | 48.0934 | 58.6428 | 58.7049 |
|                        | $\nu$       | 0.000405 | 0.001287 | 0.004063 | 0.012868 | 0.040595 | 0.128166 | 0.375527 | 0.648473 | 0.634983 |
| 256                    | $k$         | 178.8281 | 178.8481 | 179.1008 | 178.2996 | 233.7027 | 234.8395 | 234.5761 | 233.9432 | 234.3112 |
|                        | $\nu$       | 0.006486 | 0.020508 | 0.064762 | 0.205720 | 0.330019 | 0.328426 | 0.328834 | 0.329725 | 0.329212 |
| 64                     | $k$         | 44.8284 | 44.7199 | 44.7593 | 44.8439 | 44.8633 | 44.7232 | 44.7599 | 44.7586 | 47.9274 |
|                        | $\nu$       | 0.000406 | 0.001287 | 0.004066 | 0.012868 | 0.040595 | 0.128166 | 0.375257 | 0.588695 | 0.585764 |
| 20                     | $k$         | 178.2811 | 178.9068 | 178.4629 | 178.8242 | 178.8931 | 178.6688 | 234.2556 | 234.8709 | 233.5700 |
|                        | $\nu$       | 0.006505 | 0.020503 | 0.064994 | 0.205099 | 0.328377 | 0.328377 | 0.329275 | 0.329212 |

IV. TOTAL POWER CONSUMPTION MODEL AND ENERGY EFFICIENCY ANALYSIS

In this section, we present power consumption models for the proposed amplifying and passive RIS designs, and we analyze and compare the EE for both systems. The main power consuming elements can be listed as transmitter, receiver, amplifiers and RIS elements. There are two PAs in the system, one at the Tx and one between the RISs. We assume ideal PAs whose power efficiency is given as [35]

$$\frac{P_{\text{out}}}{P_{\text{amp}}} = \eta_{\text{max}} \left(\frac{P_{\text{out}}}{P_{\text{max}}}\right)^{\varepsilon}$$  \hspace{1cm} (20)

where $P_{\text{amp}}$ and $P_{\text{out}}$ corresponds to power consumed by the amplifier and the output power of the amplifier, respectively. The maximum output power of the amplifier is set to $P_{\text{max}}$ to ensure that it operates in the linear region. Here, $\eta_{\text{max}} \in (0, 1]$ is the maximum efficiency of the amplifier and $\varepsilon$ is a parameter that depends on the amplifier class. We assume $\varepsilon = 0.5$ for more accurate modeling as in [35]. The power consumed by the PA can be obtained by reorganizing (20) as

$$P_{\text{amp}} = \frac{1}{\eta_{\text{max}}} \sqrt{P_{\text{out}} P_{\text{max}}}.$$  \hspace{1cm} (21)

The phase shift of each RIS element is arranged by programmable electronic circuits that consume power as well. The power consumption of the RIS depends on the phase resolution of RIS elements [37] and modeled as

$$P_{\text{RIS}} = N P_n(b)$$  \hspace{1cm} (22)

where $P_n(b)$ is the power consumption of each RIS element which is a function of bit-resolution. We consider 6-bit phase resolution for each RIS element, which consumes 7.8 mW power according to [4]. The total power consumption of the amplifying RIS-assisted system is expressed as:

$$P_{\text{tot}}^{\text{act}} = \alpha P_t + P_{\text{Tx}} + P_{\text{Rx}} + N P_n(b) + \beta \sqrt{P_{\text{out}} P_{\text{max}}}$$  \hspace{1cm} (23)

where $\alpha = \omega_{\text{max}}^{-1}$ and $\beta = \eta_{\text{max}}^{-1}$ with $\omega_{\text{max}}$ and $\eta_{\text{max}}$ represent the maximum efficiency of the transmit PA and the PA between the RISs, respectively. We assume that $P_t = P_{\text{max}}$ for the transmit PA. Here, $P_{\text{Tx}}$ and $P_{\text{Rx}}$ are the hardware dissipated static powers at Tx and Rx, respectively and therefore they are constant and does not depend on the system parameters. The values for the power consumption model parameters are given

1Interested readers can produce arbitrary results as well by using the source code provided: https://github.com/recepakiftasci/Amplifying-RIS
Table II: Power Consumption Model Parameters

| Parameter | Value    |
|-----------|----------|
| $\alpha$  | 1.2      |
| $\beta$  | 1.2      |
| $P_n(b)$  | 7.8 mW   |
| $P_{tx}$  | 9 dBW    |
| $P_{rx}$  | 10 dBm   |

in Table II as stated in [4]. Likewise, the power consumption of the passive RIS-assisted system can be written as follows only by omitting the power consumed by the PA between the RISs:

$$P_{tot}^{out} = \alpha P_t + P_{tx} + P_{rx} + NP_n(b)$$  \hfill (24)

The bit-per-joule energy efficiency ($\eta_{EE}^b$) of a system can be expressed as

$$\eta_{EE}^b = \frac{BW \cdot R_i}{P_{tot}^d}$$  \hfill (25)

where $BW$ is the communication bandwidth, $R_i$ is the achievable rate, and $P_{tot}^d$ is the total consumed power by the system where $i \in \{act, pas\}$. The amplifying RIS-assisted system consumes more power but provides higher capacity in return. On the other hand, passive RIS-assisted system is less power consuming and provides less capacity due to its fully passive nature. The EE analysis is an important criterion determining which system performs better in terms of energy consumption.

Achievable rates of both designs for varying $d_h$ with several $N$ values are presented in Fig. 5. For this figure, the lines

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Fig. 4. BER results for (a) $P_{max} = 10$ dBm and (b) $P_{max} = 20$ dBm.

Fig. 5. Achievable rates of the system for (a) $d_h = 50$ m and (b) $d_h = 100$ m.

In this study, we compute EE for different systems through computer simulations without particularly focusing on the optimization of the EE.

V. NUMERICAL RESULTS

In this section, we provide numerical results for both the amplifying and passive RIS models to evaluate and compare the performances of them under several configurations. Achievable rate, BER, power consumption, and EE computer simulations have been presented and discussed in detail. We consider the systems as shown in Figs. 1 and 2 with parameters $d_v = 5$ m, $d_r = 5$ m, $d = 50$ m, $P_t = 30$ dBm, $P_{max} = 30$ dBm, $f_c = 28$ GHz, $BW = 180$ kHz, $F = 5$ dB, $K_1 = K_2 = 5$, $N = 128$, $n_{rx} = n_{tot} = -100$ dBm, and $G_{max} = 30$ dB, unless specified otherwise. Achievable rate and energy efficiency simulations are performed with $10^6$ iterations.

For the BER analysis, two different setups with $P_{max} = 10$ dBm and $P_{max} = 20$ dBm are considered. Fig. 4(a) shows the BER performance when $P_{max} = 10$ dBm, while Fig. 4(b) exhibits the results for $P_{max} = 20$ dBm. Here, the corresponding theoretical values are given, as well. From Fig. 4(a), one can easily observe that an error floor occurs after $P_t = 25$ dBm for $N = 32$ case. Besides, an error floor is not observed in Fig. 4(b). This is because of the limitation of $P_{out}$ when the input signal becomes too strong such that the amplifier cannot stay in the linear region if it continues to enhance the signal with the current gain. In this case, when $P_{out}$ is fixed to $P_{max}$, further increment in $P_t$ does not lead a better error performance after a certain point, and thus we observe an error floor. On the other hand, in Figs. 4(a) and (b) we do not observe any error floor when $P_{out}$ is smaller than $P_{max}$. Furthermore, the better BER performance is achieved with larger $N$ and higher $P_t$ in both cases as long as $P_{out}$ does not reach $P_{max}$. Based on these investigations, we can conclude that it may be beneficial to keep $P_{max}$ high enough to prevent any error floor. In addition, it should be emphasized that using a larger RIS can further strengthen the received signal and an error floor appears for lower $P_t$ values due to the limitation of $P_{out}$.
RIS can overcome the common problem that generally affects system performance unpleasingly in passive RIS designs. This problem is the necessity of placing the RIS close to the Rx or Tx. As seen in the Fig. 5(a), the worst achievable rate is obtained when the RIS is placed in the middle of the Rx and Tx for the passive RIS. This gets even worse in the Fig. 5(b) because the Tx and Rx are further apart. The amplifying RIS design significantly reduces the multiplicative path loss effect by amplifying the combined signal by RIS, as it uses a PA between the two RISs. Using the amplifying RIS instead of the passive one can compensate this performance drop, even enhance the performance further.

Similar to passive RIS scenarios, the RIS size has a considerable impact on achievable rate also for the active RISs. Fig. 6 illustrates the achievable rates of the amplifying RIS design where Figs. 6(a) and (b) stand for $P_T = 20$ dBm and $P_T = 10$ dBm, respectively. Here, the numbers for each marker signify the $G_{opt}$ for the corresponding $N$. Here, $G_{opt}$ begins to decrease after a point for all of the cases except for $P_{max} = 20$ dBm in Fig. 6(b). Differently from the previous case, the reason behind the occurrence of the break points is the bigger $N$ values, such that $P_{in}$ increases if more reflecting elements are used and again limits $G_{opt}$. In Fig. 6(a), $P_{out}$ reaches $P_{max}$ for large $N$ and amplifier cannot operate with $G_{max}$ even if $P_{max} = 20$ dBm. However, when $P_t$ is reduced to 10 dBm as in Fig. 6(b), we see that $P_{out}$ cannot reach $P_{max} = 20$ dBm even for large $N$ and the amplifier can boost the signal with $G_{max}$.

Tx power and RIS size cause similar effects on the system performance because both of them enhances $P_{in}$. Looking at Figs. 6(a) and (b), one can prove that keeping $P_T$ high does not always have an extra constructive effect on the system capacity by observing the fact that the achievable rates become identical for the same $P_{max}$ after a certain point as clearly demonstrated in Fig. 7 as well. In Fig. 7, the achievable rates for $P_T = 20$ dBm and $P_T = 10$ dBm are the same after $N = 200$ when $P_{max} = 20$ dBm. A similar scene is also valid for $P_{max} = 10$ dBm, where the achievable rates are identical after $N = 60$. These occur when $P_{out} = P_{max}$. Since $P_{max} = 10$ dBm can be reached by lower $P_{in}$ values, the intersection point occurs

![Achievable rates of the amplifying RIS-assisted system for different $P_T$ and $P_{max}$.](image)
at lower $N$ compared to the case $P_{\text{max}} = 20$ dBm. On the other hand, a considerable performance difference is examined where $N$ is smaller than 100 and 50 for $P_{\text{max}} = 20$ dBm and $P_{\text{max}} = 10$ dBm, respectively. These observations indicate that the system performance is not enhanced with $P_t$ after the break points, on the other hand, it can be boosted slightly by using larger $N$. This can be explained by the fact that there are two RISs at both sides of the amplifier. Increasing $N$ for RIS$_1$ can enhance $P_{\text{in}}$ just as $P_t$, however, $P_{\text{out}}$ will be the same if it is already at $P_{\text{max}}$, so this increment in $N$ for RIS$_1$ does not affect the achievable rate. Despite this, if more reflecting elements are used for RIS$_2$, slightly higher achievable rates are achieved although the total power is the same such that it is distributed among those reflecting elements. The reason of this advance is the beamforming gain which increases with $N$.

Fig. 8(a) shows the EE values for varying $N$ and different $P_{\text{max}}$ values where Fig. 8(b) stands for the corresponding achievable rates and $P_{\text{tot}}$ values. In Fig. 8(a), the EE values start to decline after some points because the achievable rates are almost at a constant level for $P_{\text{max}} = 20$ dBm and $P_{\text{max}} = 10$ dBm while $P_{\text{tot}}$ is still increasing as seen in the Fig. 8(b). When $P_{\text{max}} = 50$ dBm, a worse EE performance is obtained as the increase in power consumption has a greater effect than the increase in the achievable rate. Unless $P_{\text{max}}$ is too high, the total power consumption of the active design is nearly the same as the passive RIS. Consequently, the active design performs better in terms of EE.

EE is investigated for varying $P_t$ and different $P_{\text{max}}$ values in Fig. 9(a). Achievable rates and $P_{\text{tot}}$ values are also given in Fig. 9(b) for the same scenario. For $P_{\text{max}} = 20$ dBm and $P_{\text{max}} = 10$ dBm, although the amplifying RIS does not consume high amount of power, the EE begins to decrease after some points. The reason is that increasing $P_t$ after these points does not affect the achievable rate, but increases $P_{\text{tot}}$. For the case of $P_{\text{max}} = 50$ dBm, as we do not reach $P_{\text{max}}$, both the power consumption of the PA at the base station and the PA between the RISs increases. Accordingly, we observe that the EE starts to decrease after a certain point even if the achievable rate keeps increasing.

Finally, Figs. 10(a) and (b) represent the EE, achievable rate, and $P_{\text{tot}}$ for varying $P_t$ and different $P_{\text{max}}$ values. By contrast with the Figs. 8 and 9, the break points occur when the amplifier cannot amplify the signal more than $G_{\text{max}} = 30$ dB. Thus, increasing the $P_{\text{max}}$ only leads to higher power consumption as explained in (21) while it does not have a positive effect on the achievable rate. This causes the EE to
In this paper, we have proposed an amplifying RIS design that utilizes a single PA. The differences and advantages of amplifying RIS design have been clearly pointed out compared to conventional passive RIS design. We have presented the signal model and optimized the amplifier gain, the phase response of RIS\(_1\) and RIS\(_2\) to maximize the achievable rate. Ultimately, active RIS-assisted systems provides much more communication capacity and better error rate performance than the passive RIS-assisted systems. In addition, we have the flexibility to place the amplifying RIS anywhere in between Tx and Rx as it greatly reduces the effect of double pathloss, unlike passive RIS that should be placed closer to Tx and Rx.

![Energy efficiency and achievable rates for different P\(_t\) values](image)

Fig. 10. (a) Energy efficiencies for different P\(_t\) values with P\(_{\text{max}}\) and (b) corresponding achievable rates and P\(_{\text{tot}}\) values

show an interesting downward trend.

In the view of these observations, one can conclude that the EE is affected by many parameters such as P\(_t\), P\(_{\text{max}}\), N, and G\(_{\text{max}}\). Increasing these parameters to have a higher achievable rate is not always leads to a more energy efficient system. To reach a system with a maximum EE, all of these parameters should be jointly optimized, which requires advanced optimization techniques and might be a topic for the future works.

VI. CONCLUSION

In this paper, we have proposed an amplifying RIS design that utilizes a single PA. The differences and advantages of amplifying RIS design have been clearly pointed out compared to conventional passive RIS design. We have presented the signal model and optimized the amplifier gain, the phase response of RIS\(_1\) and RIS\(_2\) to maximize the achievable rate. Ultimately, active RIS-assisted systems provides much more communication capacity and better error rate performance than the passive RIS-assisted systems. In addition, we have the flexibility to place the amplifying RIS anywhere in between Tx and Rx as it greatly reduces the effect of double pathloss, unlike passive RIS that should be placed closer to Tx and Rx.

On top of that, active RIS-assisted systems are more energy efficient, although consuming more power. This study provides a new design for the active-RIS architectures by deploying a single active component. Future works may include joint uplink and downlink communication, the optimization and extension of this system to MIMO systems and real-world experimental results.

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