Ratio of Photoproduction Rates of $J/\psi$ and $\psi(2S)$

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Abstract

There are different approaches for diffractive photoproduction of charmonia. Recently, a new approach is proposed, in which charm quarks are taken as heavy quarks and the nonperturbative effect related to charmonia can be handled with nonrelativistic QCD. The interaction between the $c\bar{c}$ pair and the initial hadron is through exchange of soft gluons. The exchange of soft gluons can be studied with heavy quark effective theory and an expansion in the inverse of charm quark mass $m_c$ can be employed. In this approach a simple formula for the S-matrix can be derived by neglecting higher orders in $m_c^{-1}$ and relativistic corrections related to charmonia. The S-matrix is related to the usual gluon distribution $g(x)$ at small $x$. This result is different than those from other approaches. Confronting experiment the result is not in agreement with experimental measurement because large errors from higher order in $m_c^{-1}$ and from relativistic corrections. Nevertheless the ratio of cross sections of $J/\psi$ and $\psi(2S)$ can be predicted more precisely than cross-sections. In this letter we show that the ratio predicted in this approach with an estimation of relativistic corrections is in good agreement with the recent measurement at HERA.

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Diffractive photoproduction of a charmonium has been studied extensively. Experimentally diffractive photoproduction of $J/\psi$ and $\psi(2S)$ has been measured with increasing precision at HERA[^1], while there are different approaches for the process[^4]-[^9]. Recently H1 has studied diffractive photoproduction of $\psi(2S)$ and obtained the ratio[^3]

$$R = \frac{\sigma(\psi(2S))}{\sigma(J/\psi)} = 0.166 \pm 0.007\text{(stat.)} \pm 0.008\text{(sys.)} \pm 0.007\text{(BR)},$$

which is in agreement with an earlier measurement by H1[^2]. The measured ratio has a weak dependence on the invariant mass $s$ of the initial states. Although this ratio and the dependence on $s$ can be explained theoretically, e.g., with the approaches presented in[^6]-[^7], but there are always certain free parameters in these approaches. In this letter we will show that the ratio and the weak $s$-dependence can be well explained with the soft gluon approach proposed in[^9], in which relativistic correction for charmonia is taken into account and predictions are made without free parameters.

In previous approaches[^5]-[^8] the nonperturbative properties of charmonium is described by light-cone wave-functions, which are defined with operators classified by twists. For diffractive photoproduction only a leading-twist wave-function is used. It is then questionable if contributions from higher-twist wave-functions are suppressed or not, and also for consistency of the twist-expansion the $c$-quark should be taken as a light quark and its mass $m_c$ should be neglected. However, taking $m_c = 0$ will cause some problems in these approaches and hence $m_c$ is not neglected. Because the leading-twist wave-function is unknown, either a parameterization is introduced or it is related to the nonrelativistic wave-function of charmonia. By solving Schrödinger equation with a suitable potential, the nonrelativistic wave-function, then the leading-twist wave-function is obtained through a Lorentz boost. It is clear that certain free parameters are introduced for determining the leading-twist wave-function. For the interaction between the initial hadron and the $c\bar{c}$-pair one usually uses the small-size color dipole approximation, in which some modification is also introduced, e.g., in[^5]. Hence the color dipole approximation also contains some parameters, which need to be determined from other sources.

A new approach[^9], which is different than the above discussed, is used to study diffractive photoproduction of $J/\psi$. In this approach the $c$-quark is taken as a heavy quark, hence one can use nonrelativistic QCD(NRQCD) to describe the nonperturbative property of charmonia[^10] by noting the fact that the $c$-quark in $J/\psi$ in its rest-frame moves with a small velocity $v$. A systematic expansion in $v$ can be employed. At leading order of $v$ the chamonium can be taken as a bound state of a $c\bar{c}$ pair. The interaction between the $c\bar{c}$ pair and the initial hadron is through exchange of gluons. These gluons are soft in the diffractive region. Because the $c$-quark is taken as a heavy quark, the heavy quark effective theory(HQET)^[11] can be used to study the exchange of soft gluons and an expansion in $m_c^{-1}$ for the exchange can be performed systematically. This approach was first used to study the decay $J/\psi \rightarrow e^+e^- + \pi + \pi$ where the two pions are soft[^12]. Keeping only leading orders, results for the $S$-matrix element are obtained and they are different than those in[^1][^5]. The reason for this is discussed in detail in[^9].

Considering the process

$$\gamma + p \rightarrow p + J/\psi$$  \hspace{1cm} (2)
in the diffractive region, the differential cross-section in the approach in [9] is given by:

\[
\frac{d\sigma}{dt}\big|_{t \to 0} = \frac{4Q_c^2\alpha_em}{9}\frac{\langle J/\psi|O_1(3S_1)|J/\psi\rangle \cdot \sum |T_R|^2 \cdot \left\{1 + O\left(\frac{\Lambda_{QCD}}{m_c}\right) + O(v^2)\right\}}{s^2m_c^5}.
\]

where \(\sum\) is the summation over spin of the final hadron \(h\) and the spin average of the initial hadron. \(t\) is the squared momentum transfer between protons. The matrix element \(\langle J/\psi|O_1(3S_1)|J/\psi\rangle\) is defined with NRQCD fields [10] and is related to the leptonic decay width

\[
\Gamma(J/\psi \rightarrow e^+e^-) = \frac{\alpha^2_em}{6m^2_c} \cdot \langle J/\psi|O_1(3S_1)|J/\psi\rangle \cdot \left\{1 + O(v^2)\right\}.
\]

The nonperturbative effect of the initial proton is represented by \(T_R\) and \(T_R\) is defined by a matrix element of field strength operators, which are separated in the moving direction of \(J/\psi\) in the space-time. In the case \(s \gg m_c\), the contribution from leading-twist operator is dominant and \(T_R\) can be related to the usual gluon distribution \(g(x)\) by:

\[
T_R|_{t \to 0} \approx -i\pi^2m_c\alpha_s^2(m_c)g(x_c) \left[1 - i\tan\left(\frac{1}{2}\alpha\pi\right)\right],
\]

with

\[
x_c = \frac{2m^2_c}{s}.
\]

The term with \(\tan(\frac{1}{2}\alpha\pi)\) is obtained by assuming \(xg(x) \to x^{-\alpha}\) for \(x \to 0\). In Eq.(5) we have set the renormalization scale \(\mu\) to be \(m_c\). The total cross section is obtained by:

\[
\frac{d\sigma(J/\psi)}{dt} = \frac{d\sigma(J/\psi)}{dt}\big|_{t \to 0} \cdot \exp^{-b_{J/\psi}|t|},
\]

where the slope parameter \(b\) is measured in experiment for \(J/\psi\) and for \(\psi(2S)\).

It should be noted that at the orders we consider \(J/\psi\) has the same helicity as that of the initial photon and the final proton also has the same helicity as that of the initial one. In Eq.(3) and (4) we also give the orders of theoretical errors. One of errors is from the expansion in \(m_c^{-1}\) for the exchange of soft gluons, which is at the order of \(\lambda_{QCD}/m_c\). The parameter \(\lambda_{QCD}\) is the typical scale of nonperturbative QCD and is at order of several hundreds MeV. Another is the relativistic correction at order of \(v^2\), because we only used leading order in the expansion in \(v\) for \(J/\psi\). These errors are likely very large and result in that the predicted cross-section of \(J/\psi\) is not in agreement with experiment. For \(\Upsilon\), these errors are significantly smaller than those for \(J/\psi\), and the predicted cross-section of \(\Upsilon\) agrees with experiment fairly well [11]. Although these errors are large for charmonia, but some of them is cancelled in the ratio defined in Eq.(1), e.g., the error at order of \(\lambda_{QCD}/m_c\). Using the above results we obtain the prediction:

\[
R = \frac{b_{J/\psi}}{b_{\psi(2S)}} \cdot \frac{\Gamma(\psi(2S) \rightarrow e^+e^-)}{\Gamma(J/\psi \rightarrow e^+e^-)} \cdot \left\{1 + O(v^2)\right\}.
\]

In this ratio some corrections at higher orders of \(\alpha_s\) are also cancelled. Using the experimentally measured slope parameters \(b_{J/\psi} = (4.99 \pm 0.13 \pm 0.39)\) GeV and \(b_{\psi(2S)} = (4.31 \pm 0.57 \pm 0.46)\) GeV, and the experimental data for the leptonic decay widths we obtain:

\[
R \approx 0.47.
\]

2
Figure 1: Prediction of Eq.(12) for the ratio R, where the curves are drawn with different parameterizations of the gluon distribution. Experimental data are from [3].

This is almost three times of the experimentally measured. With the prediction the ratio in Eq.(1) should have no s-dependence. Experimentally the dependence is observed and is parameterized by $R \propto s^\delta$ with $\delta = 0.24 \pm 0.17$ [3]. This is also in conflict with our prediction.

It is clear that the relativistic correction plays an important role for predicting the ratio. The correction is unknown and requires to be analyzed. The analysis for this correction will be in general complicated. Nevertheless, in the framework of NRQCD [10] the differential cross-section can be written as:

$$\left. \frac{d\sigma(J/\psi)}{dt} \right|_{t \rightarrow 0} = A_1 \cdot \langle J/\psi | O_1(3S_1) | J/\psi \rangle + A_2 \cdot \langle J/\psi | P_1(3S_1) | J/\psi \rangle + \mathcal{O}(v^4),$$

(10)

where $A_1$ can be read from Eq.(3) and $A_2$ is unknown. The relativistic correction is characterized by the matrix element $\langle J/\psi | P_1(3S_1) | J/\psi \rangle$, whose definition can be found in [11]. With equation of motion it can be shown [13]:

$$\langle J/\psi | P_1(3S_1) | J/\psi \rangle = m_c(M_{J/\psi} - 2m_c) \cdot \langle J/\psi | O_1(3S_1) | J/\psi \rangle \cdot \left\{ 1 + \mathcal{O}(v^2) \right\}. \quad (11)$$

Replacing $J/\psi$ with $\psi(2S)$ one obtains the relation for matrix elements of $\psi(2S)$. It should be noted that the pole mass $m_c$ is not well determined yet, it lies in the range from 1.3GeV to 1.7GeV. Based on the relation in Eq.(11) an estimation of relativistic corrections can be made if one takes $m_c$ is free parameter and sets $2m_c = M_{J/\psi}$ for $J/\psi$ and $2m_c = M_{\psi(2S)}$ for $\psi(2S)$ respectively, then the error due to relativistic correction at order $v^2$ is absent formally and the correction is taken into account. With this setting it has been shown that one could reduce the violation of the famous 14% in the case of radiative decays into $\eta'$ of $J/\psi$ and $\psi(2S)$ [14]. However, it should
be noted that this should be regarded as a phenomenological estimation, a detailed analysis and a precise determination of $m_c$ is needed to study the correction in a consistent way.

Using the setting we obtain the ratio as:

$$ R = \frac{b_{J/\psi}}{b_{\psi(2S)}} \cdot \frac{\Gamma(\psi(2S) \rightarrow e^+e^-)}{\Gamma(J/\psi \rightarrow e^+e^-)} \cdot \frac{\alpha_s(\frac{1}{2}M_{\psi(2S)})}{\alpha_s(\frac{1}{2}M_{J/\psi})} \cdot \left( \frac{M_{J/\psi}}{M_{\psi(2S)}} \right)^5 \cdot \frac{x_2 g(x_2)}{x_1 g(x_1)^2}, \quad (12) $$

with

$$ x_2 = \frac{M_{\psi(2S)}^2}{2s}, \quad x_1 = \frac{M_{J/\psi}^2}{2s}. \quad (13) $$

In Eq.(12) we have neglected the terms with $\tan(\frac{1}{2}\alpha \pi)$. This term has little effect on the ratio. With available gluon distributions\cite{15, 16, 17} we obtain numerical results given in Fig.1, where the experimental data in\cite{3} is also drawn.

From Fig.1 we can see that the prediction based on Eq.(12) agrees with experimental data very well. Also, the weak $s$-dependence is predicted in which $R$ increases with increasing $s$. This agreement shows clearly how important the relativistic correction for charmonia is. From Eq.(12) one can see that the most important factor is $(M_{J/\psi}/M_{\psi(2S)})^5 = 0.419$ for reducing the predicted $R$-value in Eq.(9). Hence, the relativistic correction in $R$ can be larger than 50%. It should be noted that by taking $2m_c = M_{J/\psi}$ for $J/\psi$ and $2m_c = M_{\psi(2S)}$ for $\psi(2S)$ respectively the uncertainty from $O(\lambda_{QCD}/m_c)$ in Eq.(3) will be not cancelled in the ratio defined in Eq.(1), but it appears in the ratio at order of $\lambda_{QCD}(M_{\psi(2S)} - M_{J/\psi})/(M_{J/\psi} M_{\psi(2S)}) \approx 0.03$ by taking $\lambda_{QCD} = 500\text{MeV}$, hence the uncertainty is essentially smaller in the ratio than that in the differential cross-sections.

For $\Upsilon$-systems, if we neglect the relativistic correction and correction from higher orders of $m_b^{-1}$, we obtain the ratio

$$ \frac{\sigma(\Upsilon(2S))}{\sigma(\Upsilon(1S))} = \frac{\Gamma(\Upsilon(2S) \rightarrow e^+e^-)}{\Gamma(\Upsilon(1S) \rightarrow e^+e^-)} \approx 0.39, \quad (14) $$

where we assume the same slope parameters for $\Upsilon(1S)$ and $\Upsilon(2S)$. If we take the relativistic correction in analogy to Eq.(12) into account, i.e., by taking $2m_b = M_{\Upsilon(1S)}$ for $\Upsilon(1S)$ and $2m_b = M_{\Upsilon(2S)}$ for $\Upsilon(2S)$ respectively, we obtain numerical results for the ratio with available gluon distributions\cite{15, 16, 17}. The results are given in Fig.2. From Fig.2 we can see that the predicted ratio has a significant deviation from the one given in Eq.(14). Again, the most important factor here for reducing the value given in Eq.(14) is $(M_{\Upsilon(1S)}/M_{\Upsilon(2S)})^5 = 0.75$. This indicates that the relativistic correction in the ratio is also significant and this significance may be beyond the expectation based on that the $b$-quark moves inside a $\Upsilon$-system with a velocity $v^2 \sim 0.1$. The reason for the significance in the ratio may be explained by the following: The pole mass $m_b$ is known more precisely than $m_c$. A lattice determination gives $m_b = 5.0 \pm 0.2\text{GeV}$\cite{18}. With this value and experimental data of masses of $\Upsilon$-systems, one knows that the matrix element of relativistic correction for $\Upsilon(1S)$ in Eq.(10) is negative, while for $\Upsilon(2S)$ it is positive, based on the similar relation given in Eq.(11) for the case of $J/\psi$. It results in that the relativistic correction in the ratio is the sum of absolute values of the both corrections. Hence, the correction is significant. Experimentally, the ratio is not measured. If it is measured, then our prediction given in Fig.2 can be tested.
Figure 2: The Predicted production ratio for Υ-systems, where the curves are drawn with different parameterizations of the gluon distribution.

To summarize: In the soft gluon approach for diffractive photoproduction of $1^-\bar{1}^+$ charmonium we take $c$-quark as a heavy quark, this allows one to use NRQCD to describe nonperturbative properties of the quarkonium and to use HQET to handle the exchange of soft gluons between the $c\bar{c}$-pair and the initial proton. In this letter we use this approach to predict the ratio of the cross sections of $J/\psi$ and $\psi(2S)$. It is shown that the prediction by adding relativistic correction is in good agreement with the recently measured ratio. This indicates that the relativistic correction is very important for charmonia. Finally, it should be noted that our estimation for relativistic correction is a phenomenological estimation based on theoretical results, a detailed analysis and an precise determination of $m_c$ is needed to study the correction in a consistent way.

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