Could the cosmic acceleration be transient? A cosmographic evaluation

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Abstract
A possible slowing down of the cosmic expansion is investigated through a cosmographic approach. By expanding the luminosity distance to fourth order and fitting the SN Ia data from the most recent compilations (Union, Constitution and Union 2), the marginal likelihood distributions for the deceleration parameter today suggest a recent reduction of the cosmic acceleration and indicate that there is a considerable probability for \( q_0 > 0 \). Also in contrast to the prediction of the \( \Lambda \)CDM model, the cosmographic \( q(z) \) reconstruction permits a cosmic expansion history where the cosmic acceleration could already have peaked and be presently slowing down, which would imply that the recent accelerated expansion of the universe is a transient phenomenon. It is also shown that to describe a transient acceleration the luminosity distance needs to be expanded at least to fourth order. The present cosmographic results depend neither on the validity of general relativity nor on the matter–energy contents of the universe.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Independent measurements by various groups have suggested that the recent expansion of the universe is speeding up and not slowing down, as believed for many decades [1]. In other words, by virtue of some unknown mechanism, the cosmic expansion underwent a ‘dynamic phase transition’ whose main effect was to change the sign of the universal deceleration parameter \( q(z) \) at low redshifts. Together with other complementary observations, such results are strongly indicating that we live in a flat, accelerating universe composed of \( \sim 1/3 \)
of matter (baryonic + dark) and \(\sim 2/3\) of an exotic component with large negative pressure, usually named dark energy (see [2] for some reviews).

However, what seems more certain is that the universe was accelerating in the recent past, but that its current state of acceleration is less well determined [3]. Some authors, using a dynamical ansatz for the dark energy equation of state, have suggested that the cosmic acceleration has already peaked and that we are currently witnessing its slowing down [4]. This behavior was also suggested by other works combining different methods and data [5].

The possibility of a transient cosmic acceleration appears in several theoretical scenarios [6] and is also theoretically interesting because eternally accelerating universes (like \(\Lambda\)CDM) are endowed with a cosmological event horizon which prevents the construction of a conventional \(S\)-matrix describing particle interactions. Indeed, such a difficulty has been pointed out as a severe theoretical problem for any eternally accelerating universe [7, 8]. Naturally, whether the acceleration of the universe is slowing down, the overall dynamic behavior in the future may be profoundly different from the one predicted by the \(\Lambda\)CDM evolution.

Here, we investigate the question related to a possible slowing down of the recent accelerating stage of the universe by using a distinct method. In order to access the present value of the deceleration parameter we consider the so-called cosmographic approach [9–16] instead of assuming the validity of a specific gravitational theory together with a given dynamical dark energy model. As happens with kinematic models [3, 14, 17–24], the basic advantage of the cosmographic approach over dynamic models is that the former depends neither on the validity of general relativity nor on the matter-energy contents of the universe. In addition, to verify the dependence of the method on a particular set of supernova data, we repeat our analysis by considering three different samples, namely, the Union [25], Constitution [26], and Union 2 [27] compilations.

### 2. Cosmographic approach

Following a traditional approach, the late time cosmic expansion can be expanded as [10, 28]

\[
a(t) = 1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2 (t - t_0)^2 + \frac{1}{3!} j_0 H_0^3 (t - t_0)^3 + \frac{1}{4!} s_0 H_0^4 (t - t_0)^4 + \mathcal{O}[(t - t_0)^5],
\]

(1)

where \(H_0, q_0, j_0\) and \(s_0\) are the present day values of the Hubble, deceleration, jerk and snap parameters, respectively. Similarly, the luminosity distance can also be expanded, yielding an extended version of the Hubble law

\[
d_L(z) = \frac{cz}{H_0} \left[ z + \frac{1}{2} (1 - q_0) z^2 - \frac{1}{6} (1 - q_0 - 3 q_0^2 + j_0) z^3 
    + \frac{1}{24} (2 - 2 q_0 - 15 q_0^2 - 15 q_0^3 + 10 q_0 j_0 + 5 j_0 + s_0) z^4 
    + \mathcal{O}(z^5) \right],
\]

(2)

where a flat universe was assumed, as is done throughout this work. As appears, the truncation of the above expression is endowed with some convergence issues [12], which is a source of concern since the SN Ia samples do have events at \(z \gtrsim 1\). One way to circumvent this problem of convergence at high redshift is to limit the supernova sample to events of lower redshift. Another approach is to parametrize cosmological distances by the so-called \(y\)-redshift [12], defined by

\[
y = \frac{\lambda_0 - \lambda_y}{\lambda_0},
\]

(3)

\[
y = \frac{z}{1 + z}.
\]
Using the y-redshift we can expand the luminosity distance as

\[
d_L(y) = \frac{c}{H_0} \left[ y + \frac{1}{2}(3 - q_0)y^2 + \frac{1}{6}(11 - 5q_0 + 3q_0^2 - j_0)y^3 \\
+ \frac{1}{24}(50 - 26q_0 + 21q_0^2 - 15q_0^3 + 10q_0j_0 - 7j_0 + s_0)y^4 \right] + O(y^5).
\]

This parametrization has the nice property that its convergence radius comprises the full range of past cosmological events.

In what follows, in order to fit the SN Ia data compiled by the Union (307 events), Constitution (397 events) and Union 2 (557 events) samples, we consider (i) the fourth order expansion of the luminosity distance in redshift, which we will label \(d_L^{(4)}(z)\), that is, the truncation of (2), and (ii) the fourth order expansion of the luminosity distance in y-redshift, \(d_L^{(4)}(y)\), the truncation of (4). For the redshift expansion fit we use sub-samples with \(z < z_{\text{cut}}\), labeling the expansions \(d_L^{(4)}(z < z_{\text{cut}})\), which for \(z_{\text{cut}} = 0.5\) yields 157 events (51%) for the Union, 247 events (62%) for the Constitution, and 402 events (72%) for the Union 2 compilation.

We construct the likelihood as \(L = \exp[-\chi^2/2]\), where

\[
\chi^2 = \sum_i \left( \frac{|\mu_B(i; M_B, \alpha, \beta) - \mu_B(i; H_0, q_0, j_0, s_0)|^2}{\sigma^2_\mu(i)} \right),
\]

where \(i\) runs through the SN Ia events, \(\mu = 5 \log_{10}[d_L/(1\text{Mpc})] + 25\) is the predicted distance modulus in a given model, and \(\sigma_\mu\) is the total uncertainty on the distance moduli. In the likelihood analysis we analytically marginalize over [21, 29] the nuisance parameter containing the Hubble parameter (\(H_0\)) and the supernova absolute magnitude (\(M_B\)), and numerically maximize for two remaining nuisance parameters related to the supernova light curve fitting (\(\alpha\) and \(\beta\)). Both cosmographic and Constitution sets use the SALT method values for \(m_B^{\text{max}}\), \(s\) (stretch) and \(c\) (color) were used: \(\mu_B = m_B^{\text{max}} - M_B + \alpha(s - 1) - \beta c\). Union 2 uses SALT2 light curve fitting: \(\mu_B = m_B^{\text{max}} - M_B + \alpha x_1 - \beta c\), where \(x_1\) is a free parameter that measures the deviation from the average decline rate.

3. Current deceleration probability

Since we are mainly interested in the value of the deceleration parameter today, we show in figure 1 the marginal likelihoods for \(q_0\) (marginalization over \(j_0\) and \(s_0\)). Both cosmographic parametrizations, \(d_L^{(4)}(z < 0.5)^\dagger\) and \(d_L^{(4)}(y)\), are statistically compatible and have similar dispersion, even though the \(z < 0.5\) implies a considerably smaller number of events in the \(d_L^{(4)}(z < 0.5)\) analysis in relation to the full SN Ia sets used in the \(d_L^{(4)}(y)\) analysis. This effect was already noticed before [13], and that may be indicating that the y-redshift expansion is not optimal to maximize the Fisher information.

The basic distinction feature between the Union and Constitution sets comes from 90 events at low redshift (\(z < 0.5\)). They have the effect of shifting the \(q_0\) likelihoods to higher values (figure 1), thereby increasing the probability of a decelerating universe (table 1). Union 2 has an even higher percentage of low redshift events and the deceleration likelihood derived from \(d_L^{(4)}(z < 0.5)\) is further shifted to higher values of \(q_0\). This suggests that the low-\(z\) SN Ia events are pointing to a higher \(q_0\) than the full sample. Such a trend could help to explain why Shafieloo et al [4] found a slowing down of the cosmic acceleration when analyzing the \(\dagger\) The choice of \(z_{\text{cut}} = 0.5\) here is meant to be conservative in terms of possible convergence problems in the \(d_L^{(4)}(z)\) expansion.
Figure 1. Marginal likelihood for the deceleration parameter today. The top panel uses the Union SN Ia compilation, the central panel uses the Constitution, and the lower panel uses the Union 2. In the upper-left corner of each panel is indicated the number of events in each data set. Curves are scaled to max\[L(q_0)\] = 1.

Table 1. Probability of a positive deceleration today, \(P(q_0 > 0)\).

| Fit\set | Union | Constitution | Union 2 |
|---------|-------|--------------|---------|
| \(dL^{(4)}(z)\) \(z > 1\) | 0.006 | 0.097 | 0.008 |
| \(dL^{(4)}(z < 0.9)\) | – | – | 0.082 |
| \(dL^{(4)}(z < 0.7)\) | – | – | 0.40 |
| \(dL^{(4)}(z < 0.5)\) | 0.08 | 0.27 | 0.55 |
| \(dL^{(4)}(z < 0.3)\) | – | – | 0.57 |
| \(dL^{(4)}(y)\) | 0.15 | 0.65 | 0.33 |

*The use of \(z > 1\) SNe in the \(dL^{(4)}(z)\) parametrization incurs more severe convergence problems; therefore, these results must be taken with greater caution."

Constitution data set, but probably would find (i) a weaker result had they used the first Union compilation and (ii) maybe an even stronger result had they used Union 2.

To further explore the suggestion that low-\(z\) SN Ia events may be pointing to higher \(q_0\) values, we examine the Union 2 data setup to various redshift cuts, \(z_{\text{cut}}\), with the \(dL^{(4)}(z < z_{\text{cut}})\) parametrization. This point of view is reinforced by the results shown in figure 2 and table 1. The more recent the SN Ia events, the higher is the peak and average deceleration values.
today and the larger is the probability for positive $q_0$. Note that the higher deceleration probability with lower redshift cutoff is due not only to a higher peak and average deceleration parameter, but also to a higher variance in the distribution, consequence of looser constraining of the parameter space. The use of various redshift cuts in the $d_L^{(4)}(y)$ parametrization also corroborates these findings (numerical results not shown).

In table 1, by using $d_L^{(4)}(z < z_{\text{cut}})$ and $d_L^{(4)}(y)$, we quantify the probability for the universe to be slowing down today for the three SN Ia samples. It has been estimated by computing the integrated marginal likelihood for $q_0 > 0$:

$$P(q_0 > 0) = \frac{\int_{q_0}^{\infty} L(q_0) \, dq_0}{\int_{-\infty}^{\infty} L(q_0) \, dq_0}. \tag{6}$$

When just $z < 0.5$ events are fitted, then the Constitution and Union 2 sets indicate a considerable probability of a slowing down universe. This probability is also large (and even dominant in the Constitution case) when the samples are fitted by $d_L^{(4)}(y)$. Using a fifth-order expansion, John [9, 16] also found a significant probability of a positive $q_0$.

It is not totally surprising that both parametrizations, $d_L^{(4)}(z < z_{\text{cut}})$ and $d_L^{(4)}(y)$, lead to distributions for $q_0$ that are not identical. Previous works (e.g. [14]) showed that best-fit values are model (parametrization) dependent, even for models with the same good fit.

Because the $d_L^{(4)}(z < 0.5)$ and $d_L^{(4)}(y)$ fits yield broad marginal likelihoods that are nearly symmetrical and peaked around $q_0 = 0$, most notably with the Constitution and Union 2 compilations, $P(q_0 > 0)$ is very sensitive to the point of maximum likelihood, which is not a robust feature of those fits in relation to the various data sets. This explains the variability of the results shown in table 1 in relation to the different SN Ia sets.

As a byproduct, we also obtain the marginal likelihood distributions for the jerk and snap parameters today (figure 3) from the Union 2 data. The $d_L^{(4)}(z < 0.5)$ parametrization yields somewhat tighter constraints than $d_L^{(4)}(y)$, even though it uses less events. Our results for the $y$-redshift parametrization are also in general agreement with what [15] found using the first Union data set. For reference, the flat $\Lambda$CDM model has $q_0 = \frac{1}{2} \Omega_m - 1$, $j_0 = 1$ and
\[ s_0 = 1 - \frac{9}{2} \Omega_m. \] Therefore, this model has a single free parameter, for which the best fit to the Union 2 data gives \( q_0 = -0.565 \pm 0.032. \)

### 4. Cosmographic deceleration history reconstruction

Independently of the cosmic deceleration today, it is of interest to know if there is another change in the sign of the cosmic acceleration after a prior transition from a decelerated to an accelerated phase at moderate redshifts \((z < 0.5).\) From the power series expansion of the scale factor (1) one can also express the deceleration parameter as a power series in time. This time variable can be written as a power series in redshift or \(y\)-redshift, yielding respectively

\[
q(z) = q_0 + \left(-q_0 - 2q_0^2 + j_0\right)z + \frac{1}{2} \left(2q_0 + 8q_0^2 + 8q_0^3 - 7q_0j_0 - 4j_0 - s_0\right)z^2 + \mathcal{O}(z^3),
\]

and

\[
q(y) = q_0 + \left(-q_0 - 2q_0^2 + j_0\right)y + \frac{1}{2} \left(4q_0 + 8q_0^2 - 7q_0j_0 - 2j_0 - s_0\right)y^2 + \mathcal{O}(y^3).
\]

Equivalently, if the truncation of (1) is made at third order, then the resulting expression for \(q(z)\) and \(q(y)\) is linear. Several works \([14, 18, 20, 21, 24]\) were based on the \(q(z)\) parametrization for the deceleration, which does not allow a transient cosmic acceleration. The derivation of a power expansion for the deceleration from the scale factor expansion establishes a natural link with the kinematic studies based on a \(q(z)\) expansion in power series. Cunha\([32]\) employed the \(q(z) = q_0 + q_1z\) parametrization and found a suggestion of transient acceleration (see \([22]\) for another formulation of a quadratic kinematic model). The quadratic form of (7) and (8) allows for a decelerated past, a transition to an accelerated phase, a point of maximum acceleration, then a slowing down of the acceleration and a transition to a recent or future decelerating phase. Similar behavior of transient acceleration is predicted or allowed by several dynamic models \([4, 8]\). In contrast, the \(\Lambda\)CDM model predicts a monotonic deceleration history connecting its asymptotic limits in the past and future, \(q(z \to \infty) = 0.5\) and \(q(z \to -1) = -1.\)

Even though (7) and (8) are expansions of same order in their respective variables, the second is a more precise expression since the order of its error \(\mathcal{O}(y^3) = \mathcal{O}(z^3/[1 + z]^3)\) is always lower than the order of the error for the first, \(\mathcal{O}(z^3),\) in the past.

We use the cosmographic parameters \((q_0, j_0\) and \(s_0)\) obtained in each parametrization with the Union 2 sample to reconstruct \(q(z)\) using (8) and (3), which is shown in figure 4. The confidence regions shown in the figure are drawn from all \(q(z)\) curves allowed
Figure 4. Deceleration history reconstruction. The gray area and the darker area inside it delimit the 1σ confidence region for the $q(z)$ reconstruction obtained from various Union 2 SN Ia analyses. Top panel: $d_L^4(z < 0.3)$ (gray), $d_L^4(z < 0.7)$ (darker internal area) and flat $\Lambda$CDM model (green thinner area). Bottom panel: $d_L^4(y)$ with no extra conditions (gray) and with the condition that the deceleration parameter must be positive at $z = 2$ inside the 1σ confidence region (darker internal area).

by the parameter values such that $\mathcal{L}(q_0, j_0, s_0)/\mathcal{L}_\text{max}$ is less than a threshold [30]. The top panel shows the deceleration reconstruction for the $d_L^4(z < z_{\text{cut}})$ parametrization for $z_{\text{cut}} = 0.3$ and $z_{\text{cut}} = 0.7$. The first constrains very loosely $q(z)$, not even showing a phase of accelerated cosmic expansion with confidence above 1σ. The higher cut redshift allows a tighter constraining, being maximum for $z \sim 0.1$.

The $q(z)$ reconstruction obtained from the $d_L^4(y)$ parametrization is shown in the bottom panel of figure 4. It defines a clear phase of accelerated expansion around $z \sim 0.2$, but suggests it is slowing down around $z \sim 0.1$, where the $q(z)$ reconstruction is maximally constrained.

For comparison, figure 4 also plots the results for a flat $\Lambda$CDM model, showing that under this parametrization all redshifts have equally well-determined deceleration. As we know that the SN Ia data are limited to a redshift range, it is clear that the preponderant factor in this $\Lambda$CDM result is the model itself, not the data. We also note that, even though the comparison of the cosmographic parametrizations with the $\Lambda$CDM model is not the objective of this work, all have very similar goodness of fit as measured by the reduced $\chi^2$.

All cosmographic parametrizations contain transient acceleration solutions in their 1σ confidence regions with a considerable positive deceleration probability today and in the near future, if we assume that the cosmographic parametrization has some predictive power. However, due to the large uncertainties in the $q(z)$ reconstructions at large redshifts or in a projection into the future ($z < 0$) from the cosmographic parametrizations of the Union 2 data alone, it is not possible to affirm if there is a transition from a phase of accelerated expansion to a present or future phase of deceleration.
An improvement in the containment of the allowed $q(z)$ could be achieved by imposing external conditions to the cosmographic analysis, for example that in the remote past the matter dominated universe should be decelerated. As an exploratory investigation, we implement this external restriction to the analysis in the bottom panel of figure 4 by imposing that $q(z = 2) > 0$ at the 1σ confidence level. As a result, the allowed $q(z)$ is further constrained, only leaving transient acceleration solutions.

The strict transient behavior obtained with the external $q(z = 2) > 0$ condition may be a consequence of the fourth-order cosmographic parametrization for the luminosity distance, as this implies a quadratic $q(z)$ which may be forced to have two zeros (two transition redshifts). Every parametrization implies a bias, and to test if this is the case for the transient solutions in the approach adopted, one would need to go to higher order in the cosmographic expansion. However, models with a higher number of parameters would need more data to meaningfully constrain the parameter space.

5. Conclusion

To address the main question of this work, we should focus on the results for the Union 2 compilation, since it is currently the largest and most updated. Both kinds of cosmographic parametrization, $d_L^{(4)}(z < z_{cut})$ and $d_L^{(4)}(y)$, allow the circumvention of convergence issues related to the power expansion, broadly agree and (i) suggest that lower redshift SN events point to lower cosmic acceleration, and (ii) indicate that there is a considerable probability for the universe to be decelerating today. The slowing down of the cosmic acceleration and even a transient acceleration scenario is allowed in all cosmographic parametrizations.

The possibility that those results may be due to bias induced by the form of the parametrizations and data analysis artifacts may be examined inside the cosmographic approach by going to higher order in the scale factor expansion and through the use of simulated data inside some conveniently chosen fiducial cosmological models. That in itself would constitute an interesting follow-up study that could be useful to improve the understanding of the limitations of the observational data sets and of the analysis methods.

What may be causing a possible slowing down of cosmic acceleration and transient cannot be answered with a basis only in a cosmographic approach. We recall that in the accepted scenario (inflation followed by radiation, dark matter and dark energy dominated stages), the universe already had two accelerating stages and what is being discussed is whether the present supernova data are compatible with a new transition to a decelerating phase. Some examples of dynamical scenarios discussed in the literature include quintessence models in which the equation of state parameter becomes positive in the future, and some D-branes inspired scenarios, among others [6, 8]. Naturally, we cannot exclude the possibility that such a slowing down might be provoked by some exotic kinematic (or dynamic effect) in the presence of inhomogeneities [31].

In conclusion, we used fourth-order expansions of the luminosity distance to describe the recent cosmic expansion as probed by SN Ia observations. We showed that one needs to go at least to fourth order in the expansion of the luminosity distance to be able to describe a transient acceleration. Other works [13, 15], using smaller data sets than some of those used here, indicate that going to higher orders does not add meaningful statistical information. Under a cosmographic analysis of the most recent SN Ia compilations, we have found a suggestion of cosmic acceleration slowing down and the existence of a considerable probability in the

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2 The choice of $z = 2$ is arbitrary, but meant to be a conservative compromise. A larger redshift would imply a lower constraint on the fitting, and a with a smaller redshift, the constraint would be stronger, but then we would be less certain about the reality of the $q(z) > 0$ condition.
relevant parameter space that the universe is already in a decelerating expansion regime. This result is in great contrast to the standard ΛCDM model, which predicts that the cosmic expansion is accelerating and must remain so forever with an ever increasing acceleration.

The predictability of the cosmographic parametrizations is considerably inferior to the ΛCDM model and other dynamical models, but that seems to be the price for avoiding the usual assumptions about the physical contents of the universe and the underlying gravity theory. It should also be stressed that the price for avoiding convergence issues for the expansion in powers of $z$ has severe consequences on model predictability, but this could be overcome by a much larger SN Ia sample (which is foreseen in future surveys), by the use of other kinds of data and priors, and maybe by a smarter (optimal) cosmographic modeling.

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