Turning the Cosmological Constant into Black Holes

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It is known that there is a quantum mechanical tunneling process which, through nucleation of a membrane, induces a transition between two de Sitter spaces, lowering the cosmological constant. It is shown in this paper that a different, new, membrane nucleation process exists, which, in addition to lowering the cosmological constant, leaves a black hole behind. Once a black hole is present, the relaxation of the cosmological constant may proceed via an analog of the old process, which decreases the black hole horizon area, or via the new process, which increases it.

Microscopic physics would appear to predict an enormously large value for the cosmological constant $\Lambda$ . However, the observed value appears to be very small and positive $\Lambda > 0$ . This makes it very appealing to consider $\Lambda$ as a variable that can evolve dynamically, and relax through quantum mechanical tunneling.

The simplest way of implementing this view is, in four spacetime dimensions, through the coupling to the gravitational field of a 3–form gauge potential $F_{\mu\nu\lambda\rho}$ . In addition to the gauge potential it is natural to bring in also its source, which is a two dimensional membrane that sweeps a three dimensional history. There exists then a quantum mechanical process – analogous to pair creation by an electric field in two spacetime dimensions – which nucleates a membrane and induces a transition between two de Sitter spacetimes, the final one having a value of the cosmological constant lower than the initial one $\Lambda > 0$ .

In the case of pair production, the energy of the pair is provided by the decrease in the energy of the electric field in the space between the pairs. Much in the same way, the energy of the membrane which is nucleated may be accounted for, in physical terms, by considering the reduction of the cosmological constant inside the bubble as a lowering of the vacuum energy.

Now, with the dynamical cosmological constant, one has a vast energy sea at one’s disposal. It is then natural to ask whether one could tap this reservoir to get from it more than just an expanding membrane, which escapes to infinity, and whose only effect is to lower the cosmological constant $\Lambda > 0$ . Could one perhaps transmute the cosmological background energy into “localized matter” $\Lambda > 0$ ?. And what localized matter can be more natural in the context of gravitation theory than a black hole $\Lambda > 0$ ?. The purpose of this note is to answer this question in the affirmative: One may also nucleate a membrane which contracts to form a black hole, leaving outside of the hole a lower cosmological constant.

To make the point in the most economical manner we will consider membrane nucleation around a pre–existent black hole of mass $M_\pm$ and will then study, in particular, the case $M_+ \to 0$ . This is useful because $M_\pm \neq 0$ is the generic situation and the limit $M_+ \to 0$ is somewhat singular, so that some important features become blurred. The case $M_\pm \neq 0$ was considered in [11] in the context of black hole thermodynamics. However, the new process presented in this paper, as well as some important features of the analog of the old process treated in [8], were not discussed there.

We consider an (Euclidean) spacetime element of the form

$$ds^2 = f^2(r)dt^2 + f^{-2}dr^2 + r^2(\,d\theta^2 + \sin^2 \theta d\phi^2) \, .$$

(1)

The antisymmetric field strength tensor takes the form

$$F_{\mu\nu\lambda\rho} = (dA)_{\mu\nu\lambda\rho} = E\sqrt{g} \epsilon_{\mu\nu\lambda\rho} \, .$$

(2)

The history of the membrane will divide the spacetime in two regions, one which will be called the interior, labeled by the suffix “−” and the other the exterior, labeled by the suffix “+”. The boundary in question may be described by the parametric equations

$$r = R(\tau) \quad t_\pm = T_\pm(\tau) \, ,$$

(3)

where $\tau$ is the proper time, so that its line element reads

$$ds^2 = d\tau^2 + R^2(\tau) (\,d\theta^2 + \sin^2 \theta d\phi^2) \, ,$$

(4)

with

$$1 = f_\pm^2(\,R(\tau)) \, \dot{R}_\pm^2 + f_\mp^{-2}(\,R(\tau)) \, \dot{R}^2 \,.$$

(5)
In the “+” and “−” regions the solution of the field equations read

\[ f_\pm^2 = 1 - \frac{2M_\pm}{r} - \frac{r^2}{l^2_\pm}, \quad (6) \]

\[ E_\pm^2 = \frac{1}{4\pi} \left( \frac{3}{l^2_\pm} - \lambda \right). \quad (7) \]

The actual cosmological constant \( \Lambda = 3/l^2 \) is thus obtained by adding \( \lambda \) (normally taken to be negative), coming from “the rest of physics” and not subject to change, and the contribution \( 4\pi E^2 \), which is subject to dynamical equations.

The discontinuities in the functions \( f^2 \) and \( E^2 \) across the membrane are given by

\[ f^2 T_- - f^2 T_+ = \mu R. \quad (8) \]
\[ E_+ - E_- = q. \quad (9) \]

Here \( \mu \) and \( q \) are the tension and charge on the membrane respectively (the action is given in (10) below). Eq. (8) follows from integrating Gauss’ law for the antisymmetric tensor across the membrane, whereas Eq. (9) represents the discontinuity in the extrinsic curvature of the membrane when it is regarded as embedded in either the “−” or the “+” spaces (12). In writing these equations the following conventions have been adopted, and will be maintained from here on: (i) The coordinate \( t \) increases anticlockwise around the cosmological horizon, (ii) the variable \( \tau \) increases when somebody traveling along the curve leaves the interior on his right side.

Equation (8) may also be interpreted as the first integral of the equation of motion for the membrane, which is thus obtained by differentiating it with respect to \( \tau \) (“equations of motion from field equations”). Hence, satisfying (8) amounts to solving all the equations of motion and, therefore, finding an extremum of the action.

We are interested in a solution of the equations of motion of the membrane, which is a closed orbit, that will be interpreted as an “instanton”, leading to a reduction of the cosmological constant accompanied by the production of a black hole.

To this end we examine the “radii of formation” of the membrane, namely, the values of \( R \) for which \( \dot{R} = 0 \). This is most efficiently analyzed if one uses the proper time condition (5) to rearrange (8) to read

\[ \Delta M = \frac{1}{2} \left( \alpha^2 - \mu^2 \right) R^3 - \mu R^2 f^2 T_+ , \quad (10a) \]
\[ \Delta M = \frac{1}{2} \left( \alpha^2 + \mu^2 \right) R^3 - \mu R^2 f^2 T_- , \quad (10b) \]

where

\[ \alpha^2 = \frac{1}{l^2_+} - \frac{1}{l^2_-} \]

The graph of \( \Delta M \) as a function of \( R \) when \( \dot{R} = 0 \) is given in Fig. 1. There are two branches which merge smoothly, corresponding to taking both signs of the square root of \( T_\pm \) in (10a) (much as \( x = \pm \sqrt{1 - y^2} \) gives the smooth circle \( x^2 + y^2 = 1 \)).

We see that for each \( \Delta M \) within the range shown in fig. 1, there are two roots, which correspond to the simultaneous creation of two membranes, which have opposite polarities, \( i.e. \), one has charge \( q \), the other \( -q \). In order for the solution to represent an extremum of the action associated with tunneling, these two roots must be the two turning points of a single, time symmetric, Euclidean orbit. Now, it may be verified by studying the orbits which solve (11), that this does not happen for the generic \( \Delta M \). However, there are two special “mass gaps” for which the orbit indeed closes. They correspond to the limiting cases when (i) the smaller membrane sits on the horizon of the initial black hole, (ii) the larger membrane sits on the final cosmological horizon. In the limiting case, \( M_+ = 0 \), discussed in (8) the smaller membrane has zero radius, disappearing from the problem. However, in the generic case, \( M_+ \neq 0 \), two membranes are formed simultaneously. The closed orbits and their properties are discussed in Fig. 2.

The joining of the Euclidean and Lorentzian sections is not devoid of subtleties, and will be discussed in detail in (13). The resulting processes may be described as follows. At a certain moment two membranes are created and the cosmological constant is lowered in the space in between them. For the old process, the first membrane materializes at the radius \( R_0 \) between the black hole and cosmological horizons, and proceeds to expand, “collapsing outwards”. At the same time, the second membrane materializes at a radius \( r_+ \), the value of \( r \) corresponding to the initial black hole horizon. With the creation of the membranes, the radius of the black hole horizon decreases \( (r_- < r_+) \), while
FIG. 1: Change in the black hole mass as a function of the turning point radius $R$. The closed curve is the graph of the function $\Delta M(R)$, obtained from Eq. (10) when $R = 0$. If the line $\Delta M = \text{constant}$ intersects the curve, it will in general do so at two points. The corresponding orbits will bounce between these two radii, but will not close upon themselves. The two special values of $\Delta M$ for which the orbit closes occur when the second root is $r_+$ (old process), or $r_-$ (new process). Here $r_+$ is the initial black hole horizon and $r_-$ is the final cosmological horizon. In each case we call the first root $R_0$. Also shown in the figure are the curves for which $\dot{T} = 0$, and the three regions corresponding to different signs for $\dot{T}$, as inferred from Eq. (10). These regions are crucial in determining the properties of the instantons (Fig. 2).

that of the cosmological horizon increases ($r_- > r_+ \)$. The second membrane collapses inwards beyond the new black hole horizon. Note that this second membrane can exist only if there is initially a black hole. The inside of the first membrane is the “interior” in this case.

For the new process the roles of the two horizons are interchanged. The first membrane materializes at the corresponding $R_0$ and proceeds to contract, collapsing inwards. At the same time, the second membrane materializes right on the final cosmological horizon. This time the black hole radius increases and the cosmological horizon radius decreases. The second membrane expands beyond the cosmological horizon. In this case the first membrane collapses onto the black hole, or forms one if initially there is none. Now the outside of the first membrane is the “interior”.

Note that with the existence of the new process the symmetry between cosmological and black hole horizons, which had been lost with the presence of the old process alone, is restored. The only remaining asymmetry is that one can consider the absence of an initial black hole, but one cannot allow, in this context, for the absence of an initial cosmological horizon.

The tunneling processes change both parameters $M$ and $l$ of the Schwarzschild–de Sitter solution. For a given change in the cosmological constant, the change in the mass is obtained by setting, in Eq. (10), $R$ equal to either the initial black hole horizon, $r_+$ (old process), or to the final cosmological horizon, $r_-$ (new process). This yields

$$\Delta M = \frac{1}{2}(\alpha^2 - \mu^2)r_+^3 \quad \text{(old process)},$$  
$$\Delta M = \frac{1}{2}(\alpha^2 + \mu^2)r_-^3 \quad \text{(new process)}.$$  

In particular, when $M_+ = 0$, Eq. (12) gives $M_- = 0$, in accordance with [8]. However, for the new process, one obtains for $M_-$ the value,

$$M_0 = \frac{1}{2} \left( \frac{\Lambda_+ - \Lambda_- + \mu^2}{\Lambda_+ + \mu^2} \right)^{3/2}$$  

for the mass of the black hole spontaneously formed out of de Sitter space, when the cosmological constant decreases from $\Lambda_+$ to $\Lambda_-$, through nucleation of a pair of membranes of tension $\mu$. 

FIG. 2: Old and new instantons for tunneling through membrane creation. The instantons are obtained by glueing, along the history of the membrane, pieces of two spacetimes with different values of the cosmological constant and the mass. In both, the old (a) and the new (b) processes, the space on the left corresponds to the initial value of the parameters, and the one on the right to the final value. In each case, the white regions in both spaces are deleted and the remaining gray regions are joined. In the old process the orbit starts at the black hole horizon, \( r_+ \), of the exterior region, which is a single point in \((r,t)\)-space. The orbit is therefore closed by construction when seen from the exterior. As seen from the interior, the orbit will close only if one defines the total amount of time elapsed \( T \) as being the period. This leads to identifying the endpoints without cutting out a piece of the orbit, and is possible because \( \dot{T} > 0 \) (Fig. 1). This procedure will, in general, generate a conical singularity at \( r_- \). Also, in order to avoid a conical singularity at the initial black hole horizon, which is no longer an horizon on the instanton geometry, one must properly adjust the period of \( T_+ \) around \( r_{++} \). In this way one will generically end up with two conical singularities, at \( r_- \) and at \( r_{++} \), (ordinary Euclidean de–Sitter spacetimes has at least one). The new process is characterized by the exchange of the roles played by the “+” and “-” spaces. Here the orbit starts at \( r_{--} \), and therefore it is closed by construction as seen from the exterior. Now it is necessary to fix the period of \( T_+ \) to close the orbit as seen from the exterior. One ends up again with two conical singularities. Note that, in this case, the old cosmological horizon is not in the instanton geometry and therefore there is no need to worry about a possible conical singularity there, which is just as well because the only adjustable period available, \( T_+ \), has already been used up to close the orbit. One ends up again with two conical singularities, one at \( r_+ \) the other at \( r_{--} \).

Having established the existence of the instantons, we pass to discuss the probability for each of the two processes. The general rule is that, in the semiclassical approximation, the probability is the exponential of the Euclidean action. However, since there are thermal effects at play due to the presence of event horizons, one must distinguish those from the underlying quantum mechanical probability for the process. This is equivalent to subtracting the action of the background.

When there is only one horizon at play, as it happens for the old process in the absence of an initial black hole, the identification of the thermal effects can be done neatly since there exists a complete action principle for the coupled system of the gravitational field, the antisymmetric field, and the membrane. The action, appropriate for keeping fixed the initial cosmological constant, \( \Lambda_+ \), is then equal to the sum of four contributions. They are the “bulk” gravitational and antisymmetric tensor hamiltonian actions, the membrane action (which includes couplings with the gravitational and antisymmetric tensor fields), and the surface term one fourth of the cosmological horizon area. The surface term, is the horizon entropy, and it stems from the thermal nature of the problem. The rest of the action may be thought of as yielding the quantum mechanical probability for the process itself. Furthermore, the on–shell value of the bulk Hamiltonian action turns out to be zero, and thus the action for the background is just one fourth of the...
Therefore, in the semi–classical approximation, the quantum mechanical probability for the process is given by the exponential of the membrane action alone.

\[ P = \exp[I_{\text{membrane}}], \]

(15)

with

\[ I_{\text{membrane}} = -\frac{\mu}{4\pi}V_3 + \frac{3}{8\pi}\alpha^2V_4. \]

(16)

Here \( V_3 \) is the 3–volume of the history of the membrane and \( V_4 \) is the 4–volume of its interior. The 4–volume term may be understood as arising from the standard minimal coupling through application of Stoke’s theorem. The coefficient \( 3\alpha^2/8\pi \) comes from writing

\[ \frac{3}{8\pi}\alpha^2 = \frac{3}{8\pi} \left( \frac{1}{l_+^2} - \frac{1}{l_-^2} \right) = \frac{1}{2}(E_+^2 - E_-^2) = (E_+ - E_-)\frac{1}{2}(E_+ + E_-) = qE_{av}, \]

where \( E_{av} = (E_+ + E_-)/2 \) is the field on the membrane.

When two horizons are present in the instanton, as it happens for the old process with an initial black hole or for the new process, always, there is no complete action principle. This is because one cannot include appropriate surface terms in the action which ensure that, on–shell, there is no conical singularity at either horizon (the time periods do not match). This problem is not a peculiarity of the membrane, or of the antisymmetric tensor. It is already present for the pure gravitational field. Thus, already the action for the background is not well defined. In physical terms this is a reflection of the fact that the black hole and cosmological horizons are not in thermal equilibrium [14]. However, in view of the previous discussion, it would seem justified to assume that Eq. (15) remains valid as the quantum mechanical probability for membrane nucleation for each of the two processes.

Since the main purpose of this communication is to point out that the new process exists, we will not dwell here on the actual evaluation of the semiclassical tunneling probability. We just point out that it will be necessary to appeal to approximations to make the problem tractable, because in all cases except for the tunneling between two de Sitter spaces discussed in [3], the geometry has a low degree of symmetry. For the old process with \( M_+ = M_- = 0 \), the instanton consists of two 4–spheres joined on a 3–sphere. However, in all other cases, for the new and the old processes, one has two \( S_2 \times S_2 \)’s joined on a \( S_1 \times S_2 \). Furthermore, the \( S_2 \)’s are just topological, not metrical spheres.

A fuller account of the results presented in this paper and their possible implications is now in preparation, and will be published elsewhere [13]. However, even at this early stage, it is hard to refrain from speculating that the low value of the cosmological constant might point to the presence of mass in the universe, in the form of black holes created by its relaxation.

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