Vortex liquids and vortex quantum Hall states in trapped rotating Bose gases

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Abstract. We discuss the feasibility of quantum Hall states of vortices in trapped low-density two-dimensional Bose gases with large particle interactions. For interaction strengths larger than a critical dimensionless 2D coupling constant $g_c \approx 0.6$, upon increasing the rotation frequency, the system is shown to spatially separate into vortex lattice and melted vortex lattice (vortex liquid) phases. At a first critical frequency, the lattice melts completely, and strongly correlated vortex and particle quantum Hall liquids coexist in inner respectively outer regions of the gas cloud. Finally, at a second critical frequency, the vortex liquid disappears and the strongly correlated particle quantum Hall state fills the whole sample.

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1. Introduction

There has been enormous interest in the last few years in the properties of trapped Bose gases set under rotation, both theoretically and experimentally (a small selection of recent contributions is [1] [2] [3] [4] [5] [6] [7]). Of particular interest is the behavior of (effectively) two-dimensional gases in highly correlated fractional quantum Hall states (FQHE) [8]. These states lend themselves for an exploration of physics beyond the mean-field picture, where the role of the strong magnetic field in the FQHE of ultrapure 2D electron gases is taken over by a fast rotation of the gas.

The formation of FQHE states in a rapidly rotating, harmonically trapped dilute Bose gas, such that the Landau level mixing due to interactions can be neglected, was studied in Ref. [5]. It was found that, after the vortex lattice melts, at a critical filling factor of the rotational Landau levels the system of particles enters a highly correlated quantum ground state. The many-body wave function corresponding to this state was numerically shown to have large overlap with a bosonic version of the many-body quantum Hall ground state introduced by Laughlin for electrons [8].

Instead of a quantum Hall fluid of particles, one can equally well consider the vortices in the fluid to be the elementary objects, forming themselves a highly correlated quantum fluid. Previously, a quantum Hall fluid of vortices has been invoked to explain certain features of Hall conductance and magnetization experiments in high-$T_c$ superconductors [9] [10] [11] [12] [13]. In the present paper, we propose to investigate vortex liquid phases in a rotating, spatially inhomogeneous, harmonically trapped dilute Bose gas. It is shown, in particular, that there exists a critical dimensionless 2D interaction strength, $g = g_c \approx 0.6$, above which several phases can coexist (we set $\hbar = m = 1$) [14]. In the limit of weak interaction, for $g < g_c$, and after the vortex lattice has melted, the particles are confined to the lowest rotational Landau level and, for low enough temperatures, the system makes a transition into the particle Laughlin state. For $g > g_c$, on the other hand, a transition into a vortex quantum Hall state can occur. Upon partial lattice melting, the system separates into a low-density phase in which the vortex lattice has melted, and an inner high-density region, in which a vortex lattice resides. At a critical rotation velocity, the lattice melts completely, and the inner region consists of a vortex quantum liquid, while the outer shell enters a highly correlated particle quantum Hall state. Finally, at a second critical velocity, the vortex liquid phase disappears and the particle quantum Hall state fills the whole sample.

Below, we first introduce the basic vortex Hamiltonian of superfluid hydrodynamics in the rotating frame, upon which our discussion of the various vortex states will be built. We then give in section III the criterion for the melting of the vortex lattice into a vortex quantum liquid for a general rotating, uncharged superfluid. In section IV, this criterion is specialized to harmonically trapped Bose gases, where it is demonstrated how the phase separation described above is obtained from the inhomogeneous density profile of the gas cloud. In section V, we describe possible experimental means to verify the existence of the vortex quantum liquid. Finally, the conclusions section contains a discussion of how the zero temperature limitation, used in the following sections, can be lifted at practically all temperatures for which the vortex quantum liquid can be observed.
2. Vortex Hamiltonian and Plasma Analogy

Consider a two-dimensional (2D) Bose gas at temperatures well below the Kosterlitz-Thouless transition temperature $T_{KT} = \pi \rho / 2$, where $\rho$ is the 2D particle density. In this superfluid state, phase fluctuations are provided by phonons and bound vortex-antivortex pairs. Vorticity is a good quantum number, and the vortices are “fundamental” objects. At sufficiently low temperatures, we can neglect the dissipation which arises from scattering of thermal excitations on vortices, so that their dynamics is conservative. The situation considered then corresponds to the Magnus force dominated limit of vortex motion in very clean superconductors \[9, 15\]. The vortex motion is governed by a Hamiltonian corresponding to one of point particles, with a charge equal to the quantum of circulation $\kappa = 2\pi$, interacting with electromagnetic fields. For a large collection of $N_v \gg 1$ vortices (cf. Fig. 1), in a frame rotating with angular velocity $\Omega$, the vortex Hamiltonian reads \[16, 17\]:

$$H_v = \sum_{i=1}^{N_v} \left( \frac{P_i - \kappa A_i}{2m_v} \right)^2 - \Omega (X_i \times P_i)_z - \frac{\rho \kappa^2}{2\pi} \sum_{i<j} \ln \left| \frac{X_i - X_j}{\alpha_c} \right|.$$  \hspace{1cm} (1)

We emphasize that the hydrodynamic vortex Hamiltonian above does in its validity not depend on the (microscopic) form of the many-body wave function; it describes the system accurately if the matter we deal with is a superfluid. The Hamiltonian is also valid if the density varies like in the (harmonically) trapped case of currently experimentally realized Bose-Einstein condensates, as long as this variation takes place on scales much larger than the intervortex distance. This is the case here, because we will consider below large vortex densities, with intervortex separations of order a few times the coherence length.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{vortex_hamiltonian}
\caption{A large collection of quantized vortices, each with mass $m_v$ and circulation $\kappa = 2\pi$, in a rotating gas of radial size $R$ and density $\rho = \rho(r)$. The vortices $i$ are moving in the rotating frame at velocities $|X_i| \ll c_s$, at a distance $|X_i|$ from the center of rotation. This system is described by the Hamiltonian (1).}
\end{figure}
The first term in \((1)\) is the kinetic energy of vortices, the second stems from a \(-\mathbf{\Omega} \cdot \mathbf{L}\) coupling, the Coriolis force on vortices, and the last term is the repulsive logarithmic Coulomb interaction of point vortices. In the symmetric gauge, the vector potential \(A_i\), responsible for the vortex velocity part of the Magnus force on the vortex (the analog of the Lorentz force), takes the form

\[ A_i = \frac{1}{2} \rho \kappa \left( x_i, y_i \right). \]

This vector potential corresponds to motion of a charged particle in a “magnetic field” perpendicular to the plane, of magnitude \(\rho\). The canonical vortex momentum of the vortex \(i\) is consequently given by the sum of the kinetic and field momenta,

\[ \mathbf{P}_i = m_v (\dot{\mathbf{X}}_i + \mathbf{\Omega} \times \mathbf{X}_i) + \kappa A_i. \]  

The vortex mass is due to the density disturbance a moving vortex causes in the system. For a single central vortex, it is given by

\[ m_v = \left[ \frac{\kappa^2}{4\pi g} \right] \ln \left[ \frac{R}{a_c} \right] = \left[ \frac{\kappa^2}{4\pi g} \right] \ln \left[ \frac{R}{\xi_0} \right], \]

where the core size \(a_c = O(\xi_0)\) is of order the bulk coherence length \(\xi_0 = 1/\sqrt{2\rho g}\), and \(c_s = \sqrt{\frac{g}{\rho}}\) the speed of sound \(19\). The interaction strength parameter \(g\) is related to the compressibility \(\chi\) of the fluid by

\[ \chi = \frac{1}{g \rho^2} = \frac{1}{\rho c_s^2}. \]

We will take below the limit of \(m_v \to 0\), so that the precise value of \(m_v\), in particular its modification in the presence of a dense vortex lattice, is not of importance to us, but only that \(m_v\) has a finite value. The latter fact is important for the quantization of the vortex position and momentum degrees of freedom in a straightforward canonical manner, because for \(m_v = 0\) exactly, vortex phase and configuration space coincide \(20\), that is, vortex momenta become (gauge dependent) functions of the vortex coordinates alone.

The ground state of the Hamiltonian \((1)\) can be analyzed by setting all the vortex velocities to zero (in the rotating reference frame) and finding the vortex configuration by minimizing the energy:

\[ H_v = \frac{1}{2} ho \kappa \mathbf{\Omega} \sum_{i=1}^{N_v} X_i^2 - \frac{\rho \kappa^2}{2\pi} \sum_{i<j} \ln \left| \frac{X_i - X_j}{a_c} \right|. \]  

We neglected the centrifugal potential \(-\frac{1}{2} m_v \mathbf{\Omega}^2 \sum_{i=1}^{N_v} X_i^2\) in the above expression, which is justified by taking the small mass limit \(m_v \to 0\) in \((1)\). In a plasma analogy, the first term in the Hamiltonian \(H_v\) provides the neutralizing background to the potential created by the vortices. In the rotating frame the system is identical to a system of charges interacting by 2D Coulomb repulsion in a neutralizing background (the total circulation being zero as seen in the rotating frame), such that the ground state of the system is given by the spatially homogeneous vortex density

\[ n_v = 2\Omega/\kappa = \Omega/\pi. \]  

3. Vortex Lattice Melting

The uniform vortex density solution is only a first approximation for the ground state properties of the rotating superfluid. Since the vortices repel each other, the energy of the liquid can be lowered further by forming highly correlated states. At relatively low vortex density, the vortices form a triangular lattice \(24\). The excitations above this vortex lattice ground state in a rotating dilute Bose-Einstein condensate have been the subject of recent experimental and theoretical studies \(21, 22\). They represent Tkachenko waves \(23\), modified by the interaction of the vortex lattice vibrations with sound \(22\).

According to the electrodynamical analogy, the Abrikosov-Tkachenko vortex lattice may equivalently be understood as the (bosonic) Wigner crystal of vortices,
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analogous to the (fermionic) Wigner crystals formed by electrons \[25, 26, 27\]. On the other hand, for large vortex densities \(n_v\), the lattice melts, at zero temperature due to quantum fluctuations, and the vortices form a highly correlated quantum liquid. Indeed, long-range logarithmic interactions, experienced by our (bosonic) vortices, have already been shown by Laughlin in his seminal paper on the (fermionic) FQHE to yield excellent overlap with incompressible quantum Hall states (cf. \[8\], Table I; see Ref. \[28\] for the present bosonic case).

In the limit of small vortex mass \(m_v \to 0\), we have large quantum fluctuations of the vortices around their equilibrium lattice sites, as well as confinement of the vortices to the lowest Landau level of the Hamiltonian (1). The dimensionless parameter

\[ \nu_v = \frac{\Omega}{\pi \rho} = \left( \frac{d}{\ell_v} \right)^2, \tag{4} \]

where \(d \equiv \rho^{-1/2}\) is the interparticle distance and \(\ell_v = \sqrt{\pi / \Omega}\) the mean distance between vortices, then controls the relative strength of quantum fluctuations and therefore the (zero temperature) melting transition into the quantum Hall state. The above parameter is the vortex filling fraction corresponding to the ratio of the number of vortices, associated with \(n_v = \Omega / \pi\), and the number of “flux quanta” associated with the “magnetic” field of strength \(\rho\), which is just the number of particles itself. The filling factor \(\nu_v\) of the vortex rotational Landau levels is just the inverse of the filling of particle rotational Landau levels, \(\nu = 1 / \nu_v\). The Laughlin wave function for the vortices reads, in the usual complex notation for the vortex coordinates \(Z_i = X_i + iY_i\),

\[ (\Psi_v)_{\text{Laughlin}} = \prod_{i>j} (Z_i - Z_j)^{1/\nu_v} \exp \left[ -\frac{\pi \rho}{2} \sum_i |Z_i|^2 \right], \tag{5} \]

where \(1 / \nu_v\) is even.

The Hamiltonian (1) has been studied in the context of vortex matter phase transitions in layered high-\(T_c\) superconductors [9, 10, 11, 13], where the possibility of a melting transition for the vortex lattice is due to the low Cooper-pair superfluid density \(\rho\) in these materials [9]. In the simplest approach to melting of matter ordered in crystals, one assumes a Lindemann criterion [29], which simply states that the departure of the vortices from their lattice positions should approximately equal their separation \(\ell_v\). In more refined treatments [11], one takes the vortex lattice and Laughlin state wave functions for the vortices, and evaluates their corresponding energies, equating them at the transition point. In the limit of small vortex mass, the zero temperature transition point from the vortex lattice to the Laughlin state is, using either method, Lindemann or “exact” evaluation of correlation energies, consistently within a factor of order unity found to be at \[5, 6, 11\]

\[ \nu_v = (\nu_v)_m \approx 0.1 \quad \text{(melting filling factor)}. \tag{6} \]

At absolute zero, for \(\nu_v < (\nu_v)_m\), the system is in the vortex lattice phase, and for \(\nu_v > (\nu_v)_m\), it is a vortex quantum liquid, i.e., a “melted” vortex crystal. For small interactions \(g \ll 1\), the resulting state was shown to high accuracy to be a quantum Hall state of particles \[5, 6\]. In the next section, we argue that for sufficiently large interactions \(g\), the resulting quantum Hall liquid is, conversely, one of vortices as the elementary constituents of the liquid.
4. Vortex Liquid States in Harmonic Traps

4.1. Coexistence of Solid and Liquid Phases

An isotropic, harmonically trapped 2D gas possesses, in the Thomas-Fermi limit, an inhomogeneous density profile of the form

$$\rho(r) = \rho_0 \left(1 - \frac{r^2}{R^2}\right).$$  

(7)

Here,

$$\rho_0 = \left(\frac{N}{\pi g}\right)^{1/2} (\omega_\perp - \Omega)^{1/2}, \quad R^2 = 2 \sqrt{\frac{gN/\pi}{\omega_\perp - \Omega^2}}$$  

(8)

are the central density and the squared Thomas-Fermi radius of the condensate cloud, respectively. In contrast to an essentially homogeneous system like a 2D superconductor, the melting condition then becomes a local criterion due to its dependence on the local density. While the density is a locally dependent quantity, it is important to recognize for the ensuing analysis that the observed vortex arrays possess a striking regularity [1, 2]. Even though the particle density is inhomogeneous, the vortex density is to a very good approximation the constant $n_v = \Omega/\pi$ [30, 31, 32].

To reach the vortex quantum Hall state in the melted phase, in preference to a particle quantum Hall state, the following two conditions have to be fulfilled. Firstly, for the lowest particle Landau level description to break down, large particle interactions have to mix the first two rotational particle Landau levels [4], such that $\omega_\perp + \Omega \approx 2\Omega < g\rho$. Second, for melting to occur the rotation rate has to be larger than $\pi \rho(p(n_v))$. These two conditions give

$$\pi \rho(p(n_v)) < \Omega < g\rho/2.$$  

(9)

We conclude that, for a finite $\Omega$–region to exist in which the vortex quantum Hall state is preferable over other states, $g > g_c$ needs to be fulfilled, where

$$g_c = 2\pi(p(n_v)) \approx 0.6.$$  

(10)

In addition, our small mass assumption can only be justified if the Coriolis force on vortices is (much) smaller than the vortex velocity part of the Magnus force, i.e., when $2m_v\Omega \ll pK$. This leads to $\Omega < g\rho/\ln[R/a_c]$ which is for large systems more stringent than the Landau level mixing criterion $\Omega < g\rho/2$ by a logarithmic factor $\ln[R/a_c]/2$. However, for the purpose of the present discussion, in the mesoscopic trapped gases, the difference between the two criteria can be neglected.

We distinguish the following cases, cf. Fig. A. At sufficiently low angular velocity, $\Omega < \Omega_{c1}$, with

$$\Delta \Omega_{c1} = \omega_\perp - \Omega_{c1} = \frac{g\omega_\perp}{2\pi N[(n_v)_m]^2},$$  

(11)

the system separates into three distinct phases, cf. Fig. B (a) (it is understood that all expressions for $\Delta \Omega$ are valid in the large $N$ limit, i.e., for $\Delta \Omega/\omega_\perp \ll 1$). The center of the trap is occupied by the vortex lattice (region I), surrounded by two vortex liquid
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Figure 2. Different vortex phases in a harmonically trapped gas for various regions of rotation velocity. (a) The region I signifies the vortex lattice region; the region II the melted lattice (vortex liquid), and in the very-low-density outer region IIIa the particles are in their lowest rotational Landau level. (b) For increasing Ω, the inner “solid” vortex lattice phase disappears completely at Ω\(_{c1}\) given by Eq. (11), and particle and vortex Laughlin state coexist in outer (IIIb) respectively inner (II) regions of the gas. Finally, at Ω > Ω_{c2}, given by Eq. (17), the vortex quantum Hall state disappears, and the particle quantum Hall state prevails throughout the sample.

phases, in which the vortex lattice has melted. Phase II represents the vortex liquid, and is defined by Eq. (9). The inner and outer radii of the shell are given by

\[
\frac{x_{\text{min}}^2}{R^2} = 1 - \frac{1}{g_c \sqrt{N/g \pi}} \frac{2\Omega}{\sqrt{\omega^2 - \Omega^2}},
\]

and the area of the shell \(\Delta A\), relative to \(\pi R^2\), therefore is

\[
\frac{\Delta A}{\pi R^2} = \frac{2\Omega}{\sqrt{(gN/\pi)(\omega^2 - \Omega^2)}} \left( \frac{g}{g_c} - 1 \right).
\]

The number of vortices in the shell is

\[
(N_v)_{\text{shell}} = \frac{\Omega^2}{\omega^2 - \Omega^2} \left( \frac{g}{g_c} - 1 \right).
\]

For the width of the vortex liquid shell to be a significant fraction of \(R\) in the limit \(N \to \infty\), such that \((N_v)_{\text{shell}} \propto N\), \(\Delta \Omega\) has to be a significant fraction of \(\Delta \Omega_{c1}\). Furthermore, the density must be approximately homogeneous for the “magnetic field” \(\rho\) to be approximately constant and one well-defined vortex Laughlin state to exist.

The two conditions that the region II phases in Fig. 2 be sufficiently thin for a constant density approximation to apply, and \((N_v)_{\text{shell}} \propto N\), therefore require that the coupling \(g\) is sufficiently close to its critical value \(g_c\). The outermost shell IIIa shown in Fig. 2 (a) is filled by particles confined to their lowest rotational Landau level.

The maximum number of vortices in the vortex liquid shell is reached at \(\Omega = \Omega_{c1}\). For given \(N\) and \(g\), this maximum number has the value

\[
\max [(N_v)_{\text{shell}}] = \frac{Ng_c}{\pi} \left( 1 - \frac{g}{g_c} \right).
\]
If the rotation velocity grows and reaches $\Omega_{c1}$, the vortex lattice completely melts, see Fig. 2(b). The gas cloud in the region II remains superfluid. Further increase of $\Omega$ leads to the contraction of region II and eventual disappearance of the vortex liquid phase at $\Omega = \Omega_{c2}$, where

$$\Delta \Omega_{c2} = \omega_\perp - \Omega_{c2} = \frac{2\pi \omega_\perp}{gN}.$$  

(17)

For $\Omega_{c1} \leq \Omega \leq \Omega_{c2}$, the number of vortices in the inner melted area II is given by the expression

$$N_{v,\text{shell}} = 2\Omega \left( \frac{gN/\pi}{\omega_\perp^2 - \Omega^2} \right)^{1/2} \left( 1 - \frac{2\sqrt{\pi\Omega}}{\sqrt{gN(\omega_\perp^2 - \Omega^2)}} \right),$$  

(18)

which vanishes at $\Omega = \Omega_{c2}$ and coincides with (16) at $\Omega = \Omega_{c1}$.

The strongly correlated fractional quantum Hall states of particles in a harmonic trap require that the detuning $\Delta \Omega_{\text{particle}}/\omega_\perp = 1 - \Omega_{\text{particle}}/\omega_\perp \approx \nu^2 g/N$ [33]. On the other hand, provided the inequality $g \gtrsim g_c$ is fulfilled, the vortex quantum Hall states occurs in an outer shell already at rotation frequencies $\Omega < \Omega_{c1} \sim \Omega_{\text{particle}}$ smaller than the rotation frequencies needed for the particle quantum Hall states. Thus, even if only an outer shell of the sample is filled by the vortex quantum Hall state, the number of vortices can be still sufficiently large to observe the characteristics of the vortex Laughlin state to be discussed in the next subsection.

4.2. Detection

We now come to discuss experimental procedures to verify that the state of the vortex system is either crystallized or a quantum fluid. We assume in the following that the part of the sample under consideration is occupied either by the vortex lattice or the vortex Laughlin phase.

The melting transition can, for example, be detected by letting the gas expand, and then investigating the resulting spatial configuration of vortices (cf. Fig. 3). The primary difference which reveals itself is that vortex liquids present a disordered structure compared to the regular crystal. The information on the degree of order is encoded in the (assumed isotropic) pair-correlation function $g(r)$. This information is obtained from pictures like the ones shown in Fig. 3 by counting the number of vortices inside a shell, of radial size the intervortex spacing, at a distance $r$ from a given (central) vortex. For a crystalline ordered structure, there are oscillations in the pair-correlation function around its asymptotic value, whereas these oscillations vanish for the liquid. If the number of vortices is large so that the counting statistics can be made accurate enough to extract the small-distance behavior $r \to 0$ of the pair-correlation function, one should find $g(r) \propto r^{2/\nu_v}$ for the vortex Laughlin liquid. An alternative method to detect the transition to a melted quantum Hall state was proposed in [35], by directly verifying a relation connecting condensate fraction and density profile. One may also conceive of using Fourier-space techniques in situ, and analyze Bragg diffraction patterns by scattering light off a collection of vortices. Finally, we mention that a very recent experiment has indeed observed, adding a quartic trapping potential term to achieve $\Omega \simeq \omega_\perp$, a transition from a vortex lattice to a disordered structure [36].
5. Conclusions

The preceding analysis was carried out at absolute zero. In fact, the limitation to zero temperature can be relaxed. The temperature dependence of the friction force on the vortices can be estimated using the result of Reference [37]: the 2D friction coefficient \( \eta \sim T^4/v_c^6 \). Magnus force domination corresponds to \( \rho \gg \eta \), which leads to \( T \ll \rho g^{3/4} \), provided \( T \ll \rho g \). On the other hand, since quantum Hall states are incompressible, excitations of the vortex liquid are separated from the ground state, for finite \( m_v \), by a gap of order of the Landau level spacing \( T_{\text{Landau}} = \kappa \rho / m_v \sim \rho g \ln[R/a_c] \). In the case of large \( g \), the gap is dominated by the interaction between the vortices, and \( T_{\text{int}} \sim \rho \kappa^2 \ln[R/a_c] \). Therefore, if \( T \ll T_{\text{Landau}} \) respectively \( T \ll T_{\text{int}} \), excitations are frozen out and the state of the system in thermal equilibrium essentially coincides with the Laughlin state. For the interaction strength under consideration here, the temperatures \( T_{\text{Landau}} \) and \( T_{\text{int}} \) are both comparable with the Kosterlitz-Thouless temperature \( T_{c}^{\text{KT}} = \pi \rho / 2 \). The vortex Laughlin state thus forms practically for any temperature well below the transition temperature \( T_{c}^{\text{KT}} \) to superfluid behavior.

To realize the vortex quantum Hall liquid, at the low particle densities required, moderately strong particle interactions are necessary, \( g \gtrsim 0.6 \), both to accommodate a sufficiently large number of vortices into the rotating sample and to drive the system into the vortex quantum Hall state. The ultimate limit on the maximal value of the particle coupling constant \( g \) in a rapidly rotating, inhomogeneous 2D low-density Bose gas is not known. However, we may conclude from the study of Bose gases with large scattering lengths in Reference [40] that \( g \sim O(1) \) should be possible before the system enters a solid, uncondensed phase.

Experimental confirmation of the vortex quantum Hall state provides a
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verification of the Hamiltonian [39][41], which stems from superfluid hydrodynamics and the consequent quantization procedure for vortices as massive, “charged” particles. The present investigation should, then, help to shed light on the debate about the physical reality of vortex matter quantum phase transitions, whose occurrence has been extensively discussed for the high-$T_c$ superconductors. It provides an analogous phenomenon in a pure, uncomplicated system, for which the force fields on the vortices are exactly known.

Finally, the present investigation raises the interesting question if vortices can also exist if only a very small number of particles per vortex, of order $5 \cdot \cdot \cdot 10$, is present. In an abstract sense, a quantized vortex is a singular hole in the order parameter manifold with a certain residue. The hole effectively acts on the particles, making them spin around the hole in a manner prescribed by the quantization of circulation condition. The question which naturally poses itself, then, is if the vortex vacuum can also exist even in the limit of no particles being present. Formulated differently, one may ask for the lower limit of particle density, if any, for which the entity quantized vortex still remains a well-defined physical object.

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