Angular Momentum Conservation Law for Randall-Sundrum Models *

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Abstract

In Randall-Sundrum models, by the use of general Noether theorem, the covariant angular momentum conservation law is obtained with the respect to the local Lorentz transformations. The angular momentum current has also superpotential and is therefore identically conserved. The space-like components $J_{ij}$ of the angular momentum for Randall-Sundrum models are zero. But the component $J_{04}$ is infinite.

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I. INTRODUCTION

Conservation laws of energy-momentum and angular momentum have been of fundamental interest in gravitational physics [1]. Using the vierbein representation of general relativity, Duan (one of the present authors) et al obtained a general covariant conservation law of energy-momentum in 3+1 dimension which overcomes the difficulties of other expressions [2]. This conservation law gives the correct quadrupole radiation formula of energy which is in good agreement with the analysis of the gravitational damping for the pulsar PSR1916-13 [3]. Also, from the same point of view, Duan and Feng [4] proposed a covariant conservation law of angular momentum in four-dimensional Riemann space-time which does not suffer from the flaws of the others [5, 6, 7].

Recently, there has been considerable activity in the study of models that involve new extra dimensions. The possible existence of such dimensions got strong motivation from theories that try to incorporate gravity and gauge interactions in a unique scheme, in a reliable manner. The idea dates back to the 1920’s, to the works of Kaluza and Klein [8, 9] who tried to unify electromagnetism with Einstein gravity by assuming that the photon originates from the fifth component of the metric. In the course of the last several years, there has been active interest in the brane world scenarios [10, 11, 12, 13, 14, 15, 16, 17, 18, 19] and fermionic zero modes in Large dimensions [20, 21, 22]. The pioneering work was done by Randall and Sundrum [14, 15]. In their works, they present the so called Randall-Sundrum models [14, 15, 23, 24] for warped backgrounds, with compact or even infinite extra dimensions. The RSI scenario provides a way to solve the hierarchy problem, and the RSII scenario gives Newton’s law of gravity on the brane of positive tension embedded in an infinite extra dimension.

As in the (3+1)-dimensional case, we should have some conservation laws in order to understand high dimensional gravity well. In our previous work [25], we obtained the general covariant conservation laws of energy-momentum in (4+1)-dimensional Randall-Sundrum models. The purpose of the present paper is to study the general covariant conservation law of angular momentum for Randall-Sundrum models via the vierbein representation. General relativity without vierbein is like a boat without a jib—without these vital ingredients the going is slow and progress inhibited. Consequently, vierbein has grown to be an indispensable tool in many aspects of general relativity. More important, it is relevant to the physical observability [26]. Based on the Einsteins observable time and space interval, we take the local point of view that any measurement in physics is performed in the local flat reference system whose existence is guaranteed by the equivalence principle, i.e. an observable object must carry the indices of the internal space. Thus, we draw the support from vierbein not only for mathematical reasons, but also because of physical measurement consideration.

This paper is arranged as follows. In section III, we give a general description of the scheme for establishing general covariant conservation laws in general relativity. In section III, we first give a simple review of the Randall-Sundrum models. Then use local $SO(1, 4)$
transformations and the scheme in section II to obtain a covariant conservation law of angular momentum for Randall-Sundrum models. Finally, we calculate the angular momentum of the bulk for Randall-Sundrum solution by superpotential. Section IV is devoted to some remarks and discussions.

II. GENERAL CONSERVATION LAWS IN GENERAL RELATIVITY

The conservation law is one of the important problems in gravitational theory. It is due to the invariance of the action corresponding to some transformations. In order to study the covariant angular momentum conservation law of more complicated systems, it is necessary to discuss conservation laws by Noether theorem in the general case [2, 27, 28, 29, 30, 31]. Suppose that the space-time manifold $\mathcal{M}$ is of dimension $n = 1 + d$ and the Lagrangian density is in the first order formalism, i.e.

$$ I = \int_{\mathcal{M}} d^n x L(\phi^A, \partial_{\mu} \phi^A), $$

where $\phi^A$ denotes the general fields. If the action is invariant under the infinitesimal transformations

$$ x'^\mu = x^\mu + \delta x^\mu, $$

$$ \phi'^A(x') = \phi^A(x) + \delta \phi^A(x), $$

and $\delta \phi^A$ vanishes on the boundary of $\mathcal{M}$, $\partial \mathcal{M}$, then following relation holds [2, 27, 32]

$$ \partial_{\mu} (L \delta x^\mu + \frac{\partial L}{\partial \partial_{\mu} \phi^A} \delta \phi^A) + [L]_{\phi^A} \delta \phi^A = 0, $$

where $[L]_{\phi^A}$ is

$$ [L]_{\phi^A} = \frac{\partial L}{\partial \phi^A} - \partial_{\mu} \frac{\partial L}{\partial \partial_{\mu} \phi^A}, $$

and $\delta \phi^A$ is the Lie derivative of $\phi^A$

$$ \delta \phi^A = \phi'^A(x) - \phi^A(x) = \delta \phi^A(x) - \partial_{\mu} \phi^A \delta x^\mu. $$

If $L$ is the total Lagrangian density of the system, the field equation of $\phi^A$ is just $[L]_{\phi^A} = 0$. Hence from Eq. (4), we can obtain the conservation equation corresponding to transformations (2) and (3)

$$ \partial_{\mu} (L \delta x^\mu + \frac{\partial L}{\partial \partial_{\mu} \phi^A} \delta \phi^A) = 0. $$

This is just the conservation law in general case. It is important to recognize that if $L$ is not the total Lagrangian density of the system, e.g. the gravitational part $L_g$, then so long as the action of $L_g$ remains invariant under transformations (2) and (3), Eq. (4) is still valid yet Eq. (7) is no longer admissible because of $[L_g]_{\phi^A} \neq 0$. 

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In a gravitational theory with the vierbein as elementary fields, we can separate \( \phi^A \) as \( \phi^A = (e^\mu_a, \psi^B) \), where \( e^\mu_a \) is the vierbein field and \( \psi^B \) is an arbitrary tensor under general coordinate transformations. When \( \psi^B \) is \( \psi^{\mu_1 \mu_2 \cdots \mu_k} \), we can always scalarize it by

\[
\psi^{a_1 a_2 \cdots a_k} = e^{a_1}_{\mu_1} e^{a_2}_{\mu_2} \cdots e^{a_k}_{\mu_k} \psi^{\mu_1 \mu_2 \cdots \mu_k},
\]

so we can take \( \psi^B \) as a scalar field under general coordinate transformations. In later discussion we can simplify the equations by such a choice.

III. COVARIANT ANGULAR MOMENTUM CONSERVATION LAW FOR RANDALL-SUNDRUM MODELS

In this section, we first give a brief introduction of the Randall-Sundrum background. Then, with these foundations above, we use local SO(1,4) transformations to obtain the covariant angular momentum conservation law for Randall-Sundrum models.

A. Randall-Sundrum background

Let us consider the following setup. A five dimensional spacetime with an orbifolded fifth dimension of radius \( r \) and coordinate \( y \) which takes values in the interval \([0, \pi r]\). Consider two branes at the fixed (end) points \( y = 0, \pi r \); with tensions \( \tau \) and \( -\tau \) respectively. The brane at \( y = 0 \) (\( y = \pi r \)) is usually called the hidden (visible) or Planck (SM) brane. We will also assign to the bulk a negative cosmological constant \( -\Lambda \). Here we shall assume that all parameters are of the order the Planck scale.

The classical action describing the above setup is given by

\[
S = S_g + S_h + S_v, \tag{8}
\]

here

\[
S_g = \int d^4x dy \sqrt{g} \left( \frac{1}{2k_s^2} R + \Lambda \right) \tag{9}
\]

gives the bulk contribution, whereas the visible and hidden brane parts are given by

\[
S_{v,h} = \pm \tau \int d^4x \sqrt{-g_{v,h}}, \tag{10}
\]

where \( g_{v,h} \) stands for the induced metric at the visible and hidden branes, respectively. And \( 2k_s^2 = 8\pi G_s = M_5^{-3} \). Five dimensional Einstein equations for the given action are

\[
G_{\mu\nu} = - k_s^2 \Lambda g_{\mu\nu} + k_s^2 \tau \left[ \frac{-g_v}{g} \delta^\nu_\mu \delta^\rho_\nu g_{\mu\rho} \delta(y) \right. \\
- k_s^2 \tau \left[ \frac{-g_v}{g} \delta^\nu_\mu \delta^\rho_\nu g_{\mu\rho} \delta(y - \pi r) \right], \tag{11}
\]
where the Einstein tensor $G_{\mu \nu} = R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R$ as usual, Greek indices without bar $\mu, \nu = 0, \cdots, 4$ and the others with bar $\bar{\mu}, \bar{\nu} = 0, \cdots, 3$. The solution that gives a flat induced metric on the branes is

$$ds^2 = g_{\mu \nu} dx^\mu dx^\nu = e^{-2k|y|} \eta_{\bar{\mu} \bar{\nu}} dx^{\bar{\mu}} dx^{\bar{\nu}} - dy^2,$$

in which $x^\mu$ are coordinates for the familiar four dimensions, $k$ is a scale of order of the Planck length

$$k^2 = \frac{k^2 \Lambda}{6} = \frac{\Lambda}{6M_3^3}, \quad \Lambda = \frac{\tau^2}{6M_3^3}.$$}

The effective Planck scale in the theory is given by

$$M_P^2 = \frac{M_3^3}{k} \left( 1 - e^{-2k\pi r} \right).$$

Notice that for large $r$, the exponential piece becomes negligible, and above expression has the familiar form given in ADD models for one extra dimension of (effective) size $R_{ADD} = 1/k$:

$$M_P^2 = M_4^2 + n^2 R_{ADD}^n.$$}

**B. Covariant conservation law of angular momentum**

It is well known that in deriving the general covariant conservation law of energy-momentum in general relativity, the general displacement transformations, which is a generalization of the displacement transformations in the Minkowski space-time, was used. In the local Lorentz reference frame, the general displacement transformations take the same form as that in the Minkowski space-time. This implies that general covariant conservation laws are corresponding to the invariant of the action under local transformations. We may conjecture that since the conservation law for angular momentum in special relativity corresponds to the invariance of the action under the Lorentz transformations, the general covariant conservation law of angular momentum in general relativity may be obtained by means of the local Lorentz invariance.

The Lagrangian density for Randall-Sundrum background can be written as

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_h + \mathcal{L}_v + \mathcal{L}_m,$$

where $\mathcal{L}_g$ is the Lagrangian density of gravity which gives the bulk contribution, whereas $\mathcal{L}_h$ and $\mathcal{L}_v$ to the hidden and visible brane parts, respectively, and $\mathcal{L}_m$ denotes the matter part

$$\mathcal{L}_g = \sqrt{g} \left( \frac{1}{2k^2} R + \Lambda \right),$$

$$\mathcal{L}_h = \tau \sqrt{-g_h} \delta(y - \pi r),$$

$$\mathcal{L}_v = -\tau \sqrt{-g_v} \delta(y),$$

$$\mathcal{L}_m = \mathcal{L}_m(\phi^A, D_\mu \phi^A).$$
The matter fields $\phi^A$ belong to some representation of $SO(1,4)$ whose generators are $I_{ab}(a,b = 0,1,2,3,4)$ and $I_{ab} = -I_{ba}$. $D_\mu$ is the local $SO(1,4)$ gauge covariant derivative operator of $\phi^A$

$$D_\mu \phi^A = \partial_\mu \phi^A - \frac{1}{2} \omega_{\mu ab}(I^{ab})^A_B \phi^B. \quad (21)$$

Under the local $SO(1,4)$ gauge transformations

$$e^a_\mu(x) \to e'^a_\mu(x) = \Lambda^a_b(x)e^b_\mu(x), \quad \eta_{ab}\Lambda^a_c\Lambda^b_d = \eta_{cd}. \quad (22)$$

$\phi^A$ transforms as

$$\phi^A(x) \to \phi'^A(x) = D(\Lambda(x))^A_B \phi^B(x). \quad (23)$$

The infinitesimal $SO(1,4)$ rotations can be linearized as follows

$$\Lambda^a_b(x) = \delta^a_b + \alpha^a_b(x), \quad \alpha_{ab} = -\alpha_{ba}. \quad (24)$$

$D(\Lambda)$ can be linearized as

$$[D(\Lambda)]^A_B = \delta^A_B + \frac{1}{2}(I^{ab})^A_B \alpha^{ab}. \quad (25)$$

Thus the variation of $\phi^A$ is

$$\delta \phi^A = \phi'^A - \phi^A = \frac{1}{2}(I^{ab})^A_B \alpha^{ab} \phi^B. \quad (26)$$

As in the (3+1)-dimensional case, we have the following decomposition

$$L = L_\omega + L_\Delta + L_b + L_m, \quad (27)$$

where

$$L_\omega = \frac{1}{2k^2}(\omega_a \omega^a - \omega_{abc}\omega^{cba})\sqrt{g}, \quad (28)$$

$$L_\Delta = -\frac{1}{2k^2} \partial_\mu(\sqrt{g} e^{a\mu} \partial_\nu e^{\nu}_a - \sqrt{g} e^{\nu}_a \partial_\nu e^{a\mu}), \quad (29)$$

$$L_b = \Lambda \sqrt{g} - \tau \sqrt{-g} \delta(y) + \tau \sqrt{-g} \delta(y - \pi r), \quad (30)$$

$$\omega_{abc} = \frac{1}{2}(\Omega_{abc} - \Omega_{bca} + \Omega_{cab}), \quad \Omega_{abc} = e^c_\nu e_b^a(\partial_\mu e_{cv} - \partial_\nu e_{cm}),$$

$$\omega_a = \eta^{bc}\omega_{bac} = \omega^c_{ac}. \quad (31)$$

We choose vierbein $e^a_\mu$ and the matter field $\phi^A$ as independent variables. Since the coordinates $x^\mu$ do not transform under the local Lorentz transformations, $\delta x^\mu = 0$, from Eq. (6), it can be proved that in this case, $\delta_0 \to \delta$. It is required that $L_m$ is invariant under (22) and $L_\omega$, $L_\Delta$ and $L_s$ are invariant obviously. So under the local Lorentz transformations (22) $L$ is invariant. In the light of the discussion in section II, we would like to take the relation

$$\frac{\partial}{\partial x^\mu} \left( \frac{\partial L}{\partial \partial_\mu e^\nu_a} \delta e^\nu_a + \frac{\partial L}{\partial \partial_\mu \phi^A} \delta \phi^A \right) + [L]_{e^\nu_a} \delta e^\nu_a + [L]_{\phi^A} \delta \phi^A = 0, \quad (31)$$
where $[\mathcal{L}]_{e^a}$ and $[\mathcal{L}]_{\phi^A}$ are the Euler expressions defined as

$$[\mathcal{L}]_{e^a} = \frac{\partial \mathcal{L}}{\partial e^\nu_a} - \partial^\mu \frac{\partial \mathcal{L}}{\partial \partial^\mu e^\nu_a},$$

$$[\mathcal{L}]_{\phi^A} = \frac{\partial \mathcal{L}}{\partial \phi^A} - \partial^\mu \frac{\partial \mathcal{L}}{\partial \partial^\mu \phi^A}.$$

Using the Einstein equation $[\mathcal{L}]_{e^a} = 0$ and the equation of motion of matter $[\mathcal{L}]_{\phi^A} = 0$, we get following equation by (31)

$$\partial^\mu \left( \frac{\partial \mathcal{L}}{\partial \mu e^\nu_a} \delta e^\nu_a + \frac{\partial \mathcal{L}}{\partial \mu e^\nu_a e^\nu_b} \delta e^\nu_b + \frac{1}{2} \frac{\partial \mathcal{L}}{\partial \mu \phi^A} (I_{ab})^A_B \delta \phi^B \right) = 0,$$

(32)

where we have used the fact that only $\mathcal{L}_m$ contains the matter field $\phi^A$.

We introduce $j^\mu_{ab}$

$$\sqrt{g} j^\mu_{ab} = \left( \frac{\partial \mathcal{L}_\omega}{\partial \partial^\mu e^\nu_a} e^\nu_b + \frac{\partial \mathcal{L}_\Delta}{\partial \partial^\mu e^\nu_a} e^\nu_b + \frac{\partial \mathcal{L}_m}{\partial \partial^\mu e^\nu_a} e^\nu_b + \frac{1}{2} \frac{\partial \mathcal{L}_m}{\partial \partial^\mu \phi^A} (I_{ab})^A_B \delta \phi^B \right),$$

(33)

then (32) can be rewritten as

$$\partial^\mu \left( \sqrt{g} j^\mu_{ab} \alpha^{ab} \right) + \partial^\mu \left( \frac{\partial \mathcal{L}_\Delta}{\partial \partial^\mu e^\nu_a} e^\nu_b \alpha_{ab} \right) = 0.$$  

(34)

From (29) one can get easily that

$$\frac{\partial \mathcal{L}_\Delta}{\partial \partial^\lambda e^\nu_a} e^\nu_b \alpha_{ab} = - \frac{1}{2k^2} \alpha^{ab} \partial^\mu (\sqrt{g} V^{\mu\lambda}_{ab}),$$

(35)

where

$$V^{\mu\lambda}_{ab} = e^\mu_a e^\lambda_b - e^\mu_b e^\lambda_a.$$  

(36)

Substituting (36) into (34), we obtain

$$\partial^\mu (\sqrt{g} j^\mu_{ab}) \alpha^{ab} + \left[ \sqrt{g} j^\mu_{ab} - \frac{1}{2k^2} \partial^\nu (\sqrt{g} V^{\nu\mu}_{ab}) \right] \partial^\mu \alpha^{ab} = 0.$$  

(37)

Since $\alpha^{ab}$ and $\partial^\mu \alpha^{ab}$ are independent of each other, we must have

$$\partial^\mu (\sqrt{g} j^\mu_{ab}) = 0,$$

(38)

$$j^\mu_{ab} = \frac{1}{2k^2} \nabla_\nu V^{\nu\mu}_{ab}.$$  

(39)

From (38) and (39), it can be concluded that $j^\mu_{ab}$ is conserved identically. As usual, we call $V^{\nu\mu}_{ab}$ superpotential. Since the current $j^\mu_{ab}$ is derived from the local Lorentz invariance of the total Lagrangian, it can be interpreted as the total angular momentum tensor density of the
gravity-matter system, and it contains the spin density of the matter field: \( (\partial L_m)/(\partial e^\mu_a) \). From the above discussion, we see that not only the current \( j^\mu_a \), but also the superpotential \( V^\mu\nu_{ab} \) does not have any terms relevant to the visible and hidden branes, both of them are only determined by the vierbein.

For a globally hyperbolic manifold \( M \), there exist a series of Cauchy surfaces \( \Sigma_t \) foliating \( M \). We choose a submanifold \( D \) of \( M \) joining any two Cauchy surfaces \( \Sigma_{t_1} \) and \( \Sigma_{t_2} \) so the boundary \( \partial D \) of \( D \) consists of six parts \( \Sigma_{t_1}, \Sigma_{t_2}, B_v, B_h, A_1 \) and \( A_2 \), in which \( \Sigma_{t_1} \) and \( \Sigma_{t_2} \) are Cauchy surfaces, \( B_v \) and \( B_h \) are the visible and hidden branes, respectively, \( A_1 \) and \( A_2 \) are at the spatial infinity of the two branes. For the solution (12), we can obtain the following vierbein

\[
e^a_\mu = (e^{-k|y|}\delta^a_\mu, \delta_4^\mu) \quad (\bar{a} = 0, 1, 2, 3)
\]

(40)

So, the following relations are tenable

\[
(\partial_\mu e_{\alpha\nu} - \partial_\nu e_{\alpha\mu})|_{B_v,h} = 0, \quad (\partial_\mu e_{\alpha\nu} - \partial_\nu e_{\alpha\mu})|_{A_1} = (\partial_\mu e_{\alpha\nu} - \partial_\nu e_{\alpha\mu})|_{A_2}.
\]

(41)

Since

\[
\sqrt{g} \ V^\mu\nu_{ab} = \frac{1}{2} e^{\mu\nu\lambda\rho} \epsilon_{abcd} e_\lambda e_\rho,
\]

we have

\[
\partial_\lambda (\sqrt{g} \ V^{\lambda\mu}_{ab})|_{B_v,h} = 0, \quad \partial_\lambda (\sqrt{g} \ V^{\lambda\mu}_{ab})|_{A_1} - \partial_\lambda (\sqrt{g} \ V^{\lambda\mu}_{ab})|_{A_2} = 0.
\]

(43)

Thus, by the use of the following identity

\[
\int_D (\nabla_\mu j^\mu_{ab}) \sqrt{g} \, dx = 0,
\]

(45)

we can get the total conservative angular momentum from (38) and (39)

\[
J_{ab} = \int_{\Sigma_t} j^\mu_{ab} \sqrt{g} \ d\Sigma_\mu = \frac{1}{2k_s^2} \int_{\partial \Sigma_t} \sqrt{g} \ V^\mu\nu_{ab} \, d\sigma_{\mu\nu},
\]

(46)

where \( \sqrt{g} \ d\Sigma_\mu \) is the covariant surface element of \( \Sigma_t \), \( d\Sigma_\mu = \frac{1}{4!} \epsilon_{\mu\nu\lambda\rho\beta} dx^\nu \wedge dx^\lambda \wedge dx^\rho \wedge dx^\beta \), \( d\sigma_{\mu\nu} = \frac{1}{3!} \epsilon_{\mu\nu\lambda\rho\beta} dx^\lambda \wedge dx^\rho \wedge dx^\beta \). Because \( dx^0 = dt = 0 \) on the Cauchy surface \( \Sigma_t \), the expression (46) can be rewritten as following

\[
J_{ab} = \int_{\Sigma_t} j^0_{ab} \sqrt{g} \, dx^1 dx^2 dx^3 dy.
\]

(47)

The calculation result of \( J_{ab} \) is

\[
J_{04} = \frac{1}{2k_s^2} V_{3D} (1 - e^{-k\pi r}),
\]

(48)

\[
J_{0i} = 0, \quad (i = 1, 2, 3)
\]

(49)

\[
J_{ij} = 0, \quad (i, j = 1, 2, 3, 4)
\]

(50)
where $V_{3D}$ stands for the volume of usual three dimensional space
\[ V_{3D} = \int d^3x. \] (51)
So the space-like components the angular momentum are zero. But the non-space-like component $J_{04}$ of it is infinite, this is caused by the gravity on the warped extra dimension. When the radius $r$ of the extra dimension is taken the limit $r \to \infty$, $J_{04} = \frac{1}{2k^2} V_{3D}$.

IV. DISCUSSIONS

To summarize, by the use of general Noether theorem, we have obtained the conservation law of angular momentum for the Randall-Sundrum models with the respect to local $SO(1, 4)$ transformations. This conservation law is a covariant theory with respect to the generalized coordinate transformations, but the angular momentum tensor is not covariant under the local Lorentz transformation which, due to the equivalent principle, is reasonable to require.

The angular momentum current has also superpotential and is therefore identically conserved. The conservative angular momentum current and the corresponding superpotential for the Randall-Sundrum models are the same with those in (3+1)- and (2+1)-dimensional Einstein theories, the Lagrangian density $\mathcal{L}_{h,v}$ corresponding to the hidden and visible brane parts do not play a role in the conservation law. Both angular momentum current and the superpotential are determined only by vierbein field.

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