Dual Description for SUSY $SO(N)$ Gauge Theory with a Symmetric Tensor

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Abstract

We consider $\mathcal{N} = 1$ supersymmetric $SO(N)$ gauge theory with a symmetric traceless tensor. This theory saturates ’t Hooft matching conditions at the origin of the moduli space. This naively suggests a confining phase, but Brodie, Cho, and Intriligator have conjectured that the origin of the moduli space is in a Non-Abelian Coulomb phase. We construct a dual description by the deconfinement method, and also show that the theory indeed has an infrared fixed point for certain values of $N$. This result supports their argument.

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Our understanding of the non-perturbative dynamics in supersymmetric (SUSY) gauge theories has made remarkable progress during the past several years. In particular, $\mathcal{N} = 1$ SUSY gauge theory has rich low energy behaviors e.g. the runaway superpotential (no vacuum), some kinds of confining phases, (infrared) free magnetic phase, Non-Abelian Coulomb phase, (infrared) free electric phases and various applications to phenomenology e.g. models of dynamical SUSY breaking, models of composite quarks and leptons. In this paper, we discuss $\mathcal{N} = 1$ SUSY $SO(N)$ gauge theory with a symmetric traceless tensor, which is one of the theories with an Affine quantum moduli space classified by Dotti and Manohar \cite{1}, and its low energy behavior has been investigated by Brodie, Cho, and Intriligator \cite{2}. In order to make our discussion clear, we briefly review the argument of Brodie, Cho, and Intriligator.

Matter content and symmetries of the model are displayed in Table 1\footnote{Throughout this paper, we use the Young tableau to denote the representation of the superfields. $\mathbf{1}$, $\mathbf{\bar{1}}$, $\mathbf{1}$ stand for vector, adjoint (anti-symmetric), symmetric (and traceless) representations under $SO$ group, respectively.}. There is no $SO(N)$ $U(1)_R$ tree level superpotential. Here $U(1)_R$ is an anomaly free global symmetry. The 1-loop beta function coefficient is $b_0 = 2(N-4)$,\footnote{The following Dynkin indices are adopted: $\mu(\mathbf{1}) = 2$, $\mu(\mathbf{\bar{1}}) = 2N - 4$, $\mu(\mathbf{1}) = 2N + 4$.} so the theory is asymptotically free for $N \geq 5$. The classical moduli space is parameterized in terms of the diagonal vacuum expectation values (VEVs) of $S$, which is of $(N - 1)$ complex dimensions.

At the quantum level, the superpotential of the form

$$W_{\text{dyn}} = C \left[ \frac{S^{2N+4}}{\Lambda^{2N-8}} \right]^{1/4}$$

\label{1}

can appear, which is determined by holomorphy and symmetries. Here $\Lambda$ denotes the dynamical scale of $SO(N)$ theory, and $C$ is a constant. In the weak coupling limit $\langle S \rangle / \Lambda \to \infty$, Eq. (1) diverges, and cannot reproduce the classical moduli space. Therefore, $C$ must vanish. This part of the moduli space is referred to as the “Higgs branch”.

This classical moduli space can also be parameterized by VEVs of the gauge invariant composite operators\footnote{det$S$ and Tr$S^n (n \geq N+1)$ can be expressed by $O_n (n = 2, \cdots, N)$, in other words, these operators are not linearly independent.}

$$O_n = \text{Tr} S^n \quad (n = 2, \cdots, N).$$

\label{2}

Table 1: The field content of the original theory
the $SO(N)$ model is in the confining phase at the origin. However, Brodie, Cho, and Intriligator have discussed that this confining picture at the origin is misleading because of the following three arguments.

1. Free electric subspaces exist and intersect at the origin. This implies that the massless spectrum at the origin cannot simply consist of the confining moduli $O_n$.

2. In the presence of the mass term for $S$, the moduli space must have another confining branch to be consistent with Witten index argument, where the non-perturbative superpotential is generated, while no superpotential exists on the Higgs branch.

3. A nontrivial phase and branch structure must arise when the original $SO(N)$ model is perturbed with a general tree level superpotential.

From these arguments, they concluded that the $SO(N)$ model’s moduli space origin is in a non-Abelian Coulomb phase. If this is the case, it is natural to ask whether the dual description exists. However, explicit dual description has not yet been found so far. The purpose of this paper is to construct the dual description of $SO(N)$ model and show that it has a nontrivial infrared fixed point.

Let us recall the “deconfinement” method introduced by Berkooz [3] in order to construct the dual description of the $SO(N)$ gauge theory with a symmetric, traceless tensor. This method has been applied to the theories in which a two-index tensor field is included, and no tree level superpotential exists. According to this method, the new strong gauge dynamics is introduced, and the two-index tensor field is regarded as a composite field (meson) by the strong gauge dynamics, namely,

$$X_{ab} \rightarrow g^{\alpha\beta} F_{\alpha a} F_{\beta b}. \quad (3)$$

Here $X$ denotes a composite superfield, $F$ is an elementary superfield charged under both the original gauge group and the new strong gauge group, and $g$ is an invariant metric of the strong gauge group. Greek letters are indices of the new strong gauge group, while Roman letters are those of the original gauge group. For instance, the symmetric tensor, the antisymmetric tensor, the adjoint representation of $SU$ gauge group correspond to mesons of the strong $SO$, $Sp$, $SU$ gauge group, respectively. The advantage of this method is that a “deconfined” theory has only defining representations, therefore one can use a well-known duality to derive a new duality.

We apply here this method to the symmetric traceless tensor of $SO(N)$ gauge group. Note that a symmetric tensor of $SO(N)$ is not irreducible, so there always appears a singlet under $SO(N)$, which is a trace part of the symmetric tensor.

The matter content and symmetry of the deconfined theory is given in Table 2. The tree level superpotential is

$$W = y z p + z^2 s. \quad (4)$$
Table 2: The field content of the deconfined theory

\[ SO(N+5) \text{ gauge theory with } (N+1) \text{ flavors has a branch in which the dynamically generated superpotential vanishes } [4], \]

\[ W_{\text{dyn}} = 0. \quad (5) \]

One can easily verify that the above deconfined theory is reduced to the original theory at the low energy. Consider the case \( \Lambda_{SO(N)} \ll \Lambda_{SO(N+5)} \), where \( \Lambda_{SO(N),SO(N+5)} \) is the dynamical scale of \( SO(N), SO(N+5) \) gauge theory, respectively. We know that \( SO(N+5) \) gauge theory with \( (N+1) \) flavors is confining [4], and the effective fields are mesons \( y^2, yz, z^2 \). As can be seen in the superpotential, \( yz, p, z^2, s \) become massive at \( \Lambda_{SO(N+5)} \). After integrating them out, we see that only \( y^2 \) is massless and the superpotential vanishes. Thus, the original theory is recovered [4].

Taking a dual of \( SO(N) \) gauge theory with \( (N+6) \) flavors [4], we obtain the following theory given in Table 3. Here the fields in parentheses stand for the elementary \( SO(10) \) gauge singlet meson fields.

Table 3: The field content of the dual theory

\[ \tilde{W} = (yp)z + z^2 s + (y^2)\tilde{y}^2 + (yp)\tilde{y}\tilde{p} + (p^2)\tilde{p}^2 \quad (6) \]

\[ ^6\text{More precisely, } y^2 \text{ includes a singlet under } SO(N) \text{ as noted before. This will be integrated out by adding the mass term.} \]

\[ ^7\text{For simplicity, we set the scale dependent coefficients of the last three terms to be of order one.} \]
\[ \begin{array}{|c|c|c|c|} \hline & \text{SO}(10) & \text{SO}(N+5) & \text{U}(1)_R \\ \hline \hat{y} & 0 & 0 & \frac{N+2}{2(5N+6)} \\ \hat{\rho} & 1 & 0 & \frac{N}{N+2} \\ s & 1 & 1 & \frac{N+2}{12(N+1)} \\ (y^2) & 1 & \Box \oplus 1 & \frac{N+2}{N+2} \\ (p^2) & 1 & 1 & \frac{N+2}{N+2} \\ \hline \end{array} \]

Table 4: The field content of the resulting dual theory

Since \((yp), z\) are massive, integrating them out by the equations of motion

\[ \begin{align*}
0 &= \frac{\partial W}{\partial (yp)} = z + \hat{y}\hat{\rho} \rightarrow z = -\hat{y}\hat{\rho}, \\
0 &= \frac{\partial W}{\partial z} = (yp) + 2zs \rightarrow (yp) = 2s\hat{y}\hat{\rho},
\end{align*} \]

we obtain the effective superpotential of the dual theory,

\[ \tilde{W}_{\text{eff}} = y^2\hat{\rho}^2s + (y^2)\hat{y}^2 + (p^2)\hat{p}^2. \]

The field content of the resulting dual theory is given in Table 4. Note here that while

\begin{table}[h]
\centering
\begin{tabular}{c|c|c|c|}
\text{original} & \text{deconfined} & \text{dual} \\
\hline
\text{tr} S^n & \text{tr}(y^2)^n & \text{tr}(y^2)^n \\
\text{(}n = 2, \cdots, N\text{)} & \text{(}n = 2, \cdots, N\text{)} & \text{(}n = 2, \cdots, N\text{)} \\
\text{S_{singlet}} & (y^2)_{\text{singlet}} & (y^2)_{\text{singlet}} \\
\hline
\end{tabular}
\caption{The operator mapping}
\end{table}

\((y^2)\) in deconfined theory denotes a composite meson, \((y^2)\) in the dual denotes a gauge singlet elementary meson. As mentioned earlier, there always exists a trace part of the symmetric tensor, so we add its mass term to the superpotential

\[ \delta W = mS^2_{\text{singlet}}, \]

and integrates it out. Then, the low energy effective theory becomes \(SO(N)\) gauge theory with a symmetric traceless tensor, which we would like to consider. This deformation corresponds to

\[ \delta \tilde{W} = m(y^2_{\text{singlet}})^2 \]

in the dual description. Using the equation of motion for \((y^2)_{\text{singlet}}\)

\[ 0 = \frac{\partial \tilde{W}}{\partial (y^2)_{\text{singlet}}} = \hat{y}^2 + 2m(y^2)_{\text{singlet}}, \]
we obtain the following effective superpotential

$$\tilde{W}_{\text{eff}} = \tilde{y}^2 \tilde{p}^2 s - \frac{1}{2m} \tilde{y}^4 + (\tilde{p}^2)\tilde{p}^2 + (\tilde{y}^2)\tilde{y}^2. \quad (13)$$

Now, let us check the consistency of the duality. The ’t Hooft anomaly matching conditions are trivially satisfied since we use the deconfined method which guarantees the anomaly matching. The mapping of the gauge invariant operators which describes the moduli space is also trivial as depicted in Table 5. This mapping is consistent with the global symmetry $U(1)_R$.

Next, we consider various flat direction deformations. First, consider the direction $\langle y^2 \rangle \neq 0$, namely,

$$\langle y \rangle = \begin{pmatrix} y_1 \\ \vdots \\ \vdots \\ y_N \end{pmatrix}, \quad (14)$$

where $\langle y \rangle$ is a $(N + 5) \times N$ matrix and $y_i (i = 1, \cdots, N)$ are constants. To simplify the analysis, let us suppose that $y_1 \neq 0$ and $y_i (i = 2, \cdots, N) = 0$. In the deconfined theory, the following symmetry breaking occur

$$SO(N) + (N + 6) \rightarrow SO(N - 1) + (N + 5), \quad (15)$$
$$SO(N + 5) + (N + 1) \rightarrow SO(N + 4) + N. \quad (16)$$

Since one component of $z$ and $p$ become massive from the coupling in the superpotential, integrating them out by the equations of motion, we obtain the effective superpotential

$$W_{\text{eff}} = y' z' p' + z'^2 s + m(y_{\text{single}}')^2. \quad (17)$$

$y'$, $z'$ and $p'$ are transformed as $y'(\square, \square)$, $z'(1, \square)$ and $p'(\square, 1)$ under $SO(N - 1) \times SO(N + 4)$. On the other hand, the corresponding direction in the dual is

$$\langle \langle (y^2) \rangle \rangle = \begin{pmatrix} y_1^2 \\ 0 \\ \vdots \\ 0 \end{pmatrix}. \quad (18)$$

Along this direction, the symmetry breaking goes as follows

$$SO(10) + (N + 6) \rightarrow SO(10) + (N + 5), \quad (19)$$
$$SO(N + 5) + \square + 10 \rightarrow SO(N + 4) + \square + 10. \quad (20)$$

Since one component of $\tilde{y}$ becomes massive due to the coupling in the superpotential, integrating them out, we obtain the effective superpotential

$$\tilde{W}_{\text{eff}} = \tilde{y}^2 \tilde{p}^2 s + (y^2)'\tilde{y}^2 + (\tilde{p}^2)\tilde{p}^2 - \frac{1}{2m} \tilde{y}^4. \quad (21)$$
The fields with dash are transformed as $\tilde{y}’(\square, \square), (y^2)(1, \square \oplus 1)$ under $SO(10) \times SO(N + 4)$. The above result is consistent simply because $N$ is replaced by $N - 1$. In fact, taking a dual of $SO(N - 1) + (N + 5) \square$ in the deconfined theory, we obtain $SO(N - 1) \times SO(N + 4)$ as the dual gauge group. We will arrive at the following theory as in Table 6. The dual superpotential is

$$W = \tilde{y}’ z’ p’ + z^2 s + m(y^2)^2_{\text{singlet}}.$$  \hspace{1cm} (22)

Integrating the massive fields $(y'p'), z'$ and $(y^2)_{\text{singlet}}$ by using their equations of motion, one can obtain the same superpotential as that in Eq. (21).

It is straightforward to extend the above result to the more general case where the VEV of $y$ takes the form as

$$\langle y \rangle = \begin{pmatrix} y_1 \\ \vdots \\ y_l \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad (l = 1, \cdots, N).$$  \hspace{1cm} (23)

In this case, the symmetry breaking is

$$SO(N) + (N + 6) \square \rightarrow SO(N - l) + (N + 6 - l) \square,$$  \hspace{1cm} (24)

$$SO(N + 5) + (N + 1) \square \rightarrow SO(N + 5 - l) + (N + 1 - l) \square,$$  \hspace{1cm} (25)

and $l$ components of $z$ and $p$ become massive, so after integrating them out, we find the effective superpotential of the form

$$W = y' z' p' + z^2 s + m(y^2)^2_{\text{singlet}}.$$  \hspace{1cm} (26)
The representations of $y'$, $z'$ and $p'$ under $SO(N-l) \times SO(N+5-l)$ are $y'(\square,\square)$, $z'(1,\square)$ and $p'(\square, 1)$, respectively. In the dual, the corresponding direction is

$$\langle (y^2) \rangle = \begin{pmatrix} y_1^2 \\ \vdots \\ y_l^2 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad (l = 1, \cdots, N), \quad (27)$$

and the symmetry breaking along this direction is

$$SO(10) + (N + 6) \square \rightarrow SO(10) + (N + 6 - l) \square, \quad (28)$$

$$SO(N + 5) + \square + 10 \square \rightarrow SO(N + 5 - l) + \square + 10 \square. \quad (29)$$

Since $l$ components of $\tilde{y}$ are massive, after integrating them out, we can find the effective dual superpotential of the form

$$\tilde{W}_{\text{eff}} = \tilde{y}^2 \tilde{p}^2 s + (y^2)\tilde{y}^2 + (p^2)\tilde{p}^2. \quad (30)$$

This result is also consistent. One can easily show explicitly that taking a dual of $SO(N-l) + (N + 6 - l) \square$ in the deconfined theory, one obtains the same deformed dual theory as seen in the above simple case.

Next, we consider the other flat direction deformations $\langle p \rangle \neq 0$. In the deconfined theory, the following symmetry breaking occur:

$$SO(N) + (N + 6) \square \rightarrow SO(N - 1) + (N + 5) \square, \quad (31)$$

$$SO(N + 5) + (N + 1) \square \rightarrow SO(N + 5) + (N - 1) \square \quad (32)$$

The low energy deformed deconfined theory is given in Table 7, and the effective superpotential is

$$W_{\text{eff}} = m(y^2)^2_{\text{singlet}}. \quad (33)$$

In the dual, the direction under consideration corresponds to $\langle (p^2) \rangle \neq 0$. Then, the low energy deformed dual theory is displayed in Table 8, and the dual superpotential is

$$\tilde{W}_{\text{eff}} = (y^2)\tilde{y}^2 - \frac{1}{2m}\tilde{y}^4. \quad (34)$$
This resulting theory is also consistent under the deformation along \( \langle p \rangle \neq 0 \). In fact, taking a dual of \( SO(N - 1) + (N + 5) \) and integrating out the massive modes, we can easily derive the dual in Table 8 and the superpotential (34).

Furthermore, we can check the consistency under the mass term deformation. Adding the mass term \( \delta W = \frac{1}{2} m' p^2 \) to the superpotential in the deconfined theory, and integrating out \( p \), we can derive the effective theory: \( SO(N) \) gauge theory with \( (N + 5) \) vectors and \( SO(N + 5) \) gauge theory with \( (N + 1) \) vectors and a singlet \( s \). The effective superpotential takes the form

\[
W_{\text{eff}} = -\frac{1}{2m'} (yz)^2 + z^2s + m(y^2)_{\text{singlet}}.
\]

On the dual side, this deformation corresponds to adding the term \( \tilde{\delta}W = \frac{1}{2} m' (p^2) \) to the dual superpotential. The equation of motion for \( (p^2) \) forces \( \tilde{p} \) to develop a VEV. This leads to break \( SO(10) \) to \( SO(9) \), then we arrive at the following effective theories:

\[
SO(9) \text{ gauge theory with } (N + 5) \text{ vectors and } SO(N + 5) \text{ gauge theory with a symmetric tensor and 10 vectors and a singlet } s, \text{ a gauge singlet meson } (y^2).
\]

The effective dual superpotential becomes

\[
\tilde{W}_{\text{eff}} = -\frac{1}{2} m' \tilde{y}_0^2 s + (y^2) \tilde{y}^2 + (y^2) \tilde{y}_0^2
- \frac{1}{2m} (y^2)^2_{\text{singlet}} - \frac{1}{2m} (\tilde{y}_0^2)^2_{\text{singlet}} - \frac{1}{m} (y^2)_{\text{singlet}} (\tilde{y}_0^2)_{\text{singlet}},
\]

where \( \tilde{y}_0 \) stands for a field transformed as \((1, \Box)\) under \( SO(9) \times SO(N + 5) \), which should be identified with the field \( z \) in the deconfined theory. By rescaling the fields appropriately, one can see that the above result is consistent.

Although we have constructed the dual description for \( SO(N) \) SUSY gauge theory with a symmetric traceless tensor by deconfinement technique, it is not so trivial to see that the theory under consideration has a non-trivial infrared fixed point at the origin of the moduli space since the gauge groups are products. Following Terning’s argument [5], we would like to show explicitly that the theory has indeed the infrared fixed point.

We note that one can analyze the theory for an arbitrary ratio of the two dynamical scales \( \Lambda_1, \Lambda_2 \) thanks to holomorphy [6], where \( \Lambda_1, \Lambda_2 \) are the scales of \( SO(10) \) gauge theory with \( (N + 6) \) flavors, \( SO(N + 5) \) gauge theory with a symmetric traceless tensor and ten vector flavors, respectively. Furthermore, there is no phase transition when the

### Table 8: The deformed dual theory for \( \langle p \rangle \)

| \( \tilde{y} \) | \( \Box 
| \hline s | 1 \Box 
| (y^2) | 1 \Box \oplus 1 


ratio is varied. There are three cases to be considered. For \( N < 4 \), \( SO(N + 5) \) theory is asymptotically non-free and \( SO(10) \) theory is asymptotically free. This implies \( \Lambda_1 \ll \Lambda_2 \) and if we renormalize the gauge coupling of \( SO(N + 5) \) theory \( g \) at the scale \( \Lambda_1 \), then \( g(\mu \sim \Lambda_1) \ll 1 \). For \( 4 < N \leq 18 \), \( SO(N + 5) \) theory becomes asymptotic free and the limit \( \Lambda_1 \gg \Lambda_2 \) corresponds to weak coupling of \( SO(N + 5) \) theory. For \( N > 18 \), \( SO(10) \) becomes asymptotically non-free, which implies \( \Lambda_1 \gg \Lambda_2 \) and \( g(\mu \sim \Lambda_1) \ll 1 \). For \( N = 4 \), since the gauge coupling \( g \) does not run, we can take an arbitrary small coupling. In any cases, the gauge coupling \( g \rightarrow 0 \) as the ratio \( \Lambda_1/\Lambda_2 \rightarrow 0 \) or \( \infty \), so we can perform the perturbative analysis for \( g \).

Let us first consider the zero-th order case in \( g \), i.e. \( SO(N + 5) \) dynamics is turned off. The dimensions of the gauge invariant operators have to satisfy the following constraints to be in unitary representations of the superconformal algebra [7]:

\[
\begin{align*}
D(\tilde{y}^2) &= 2 + 2\gamma_{\tilde{y}}(g = 0) \geq 1, \\
D(\tilde{y}\tilde{p}) &= 2 + \gamma_{\tilde{y}}(g = 0) + \gamma_{\tilde{p}}(g = 0) \geq 1, \\
D(\tilde{y}^10) &= 10 + 10\gamma_{\tilde{y}}(g = 0) \geq 1, \\
D(\tilde{y}\tilde{p}^9) &= 10 + 9\gamma_{\tilde{y}}(g = 0) + \gamma_{\tilde{p}}(g = 0) \geq 1, \\
D((y^2)) &= 1 + \gamma(y^2)(g = 0) \geq 1,
\end{align*}
\]

(37)

where \( \gamma_{\phi} \) is the anomalous dimension of the field \( \phi \), and the bound is saturated for free fields. We note that the first term in the dual superpotential (13) is a product of three gauge invariant operators. Thus, these are irrelevant because they can be relevant only if the dimensions of these gauge invariants are one, which means that these operators are free. The fields \( s \) interacts only through the first term which is irrelevant, so these are free fields and their anomalous dimensions vanish. Therefore the equalities (37) cannot be saturated.

In order to obtain more relations among the anomalous dimensions, we use the exact \( \beta \) function for the \( SO(10) \) coupling \( g_1 \) [8]

\[
\beta(g_1) = -\frac{g_1^3}{16\pi^2} \times 3 \times 8 - (N + 5)(1 - \gamma_{\tilde{y}}(g = 0)) - (1 - \gamma_{\tilde{p}}(g = 0)),
\]

(38)

and at the fixed point

\[
0 = 18 - N + (N + 5)\gamma_{\tilde{y}}(g = 0) + \gamma_{\tilde{p}}(g = 0).
\]

(39)

The second and the last term in Eq. (13) are relevant operators with R-charge 2 for \( g = 0 \), so the following conditions have to be satisfied,

\[
\begin{align*}
D(\tilde{y}^4) &= 4 + 4\gamma_{\tilde{y}}(g = 0) = 3, \\
D((y^2)\tilde{y}^2) &= 3 + \gamma(y^2)(g = 0) + 2\gamma_{\tilde{y}}(g = 0) = 3.
\end{align*}
\]

(40)

On the other hand, \( \tilde{y}^2, \tilde{p}^2 \) and \( s \) are gauge invariant operators for arbitrary \( g \), and the corresponding constraint for the dimensions are

\[
D(\tilde{y}^2) = 2 + 2\gamma_{\tilde{y}}(g) \geq 1,
\]

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\[ D(p^2) = 2 + 2\gamma_{\tilde{\eta}}(g) \geq 1, \]
\[ D(s) = 2 + 2\gamma_s(g) \geq 1. \]

The second and the fourth inequalities of (37) and the first one of (40) lead to
\[ \gamma_{\tilde{\eta}}(g = 0) > -\frac{3}{4}, \]
therefore one can derive the bound for \( N \) using the conditions (39), the first inequality of (40) and (42):
\[ N > \frac{64}{5}. \]

The above result implies that \( SO(10) \) theory has an infrared fixed point if \( N \) is in the range of (43). Next, we would like to show that \( SO(N + 5) \) theory also has an infrared fixed point.

The exact beta function for \( g \) is
\[ \beta(g) = -\frac{g^3}{16\pi^2} \frac{3(N + 3) - 10(1 - \gamma_{\tilde{\eta}}(g)) - (N + 7)(1 - \gamma_{(y^2)}(g))}{1 - (N + 3)\frac{g^2}{8\pi^2}}, \]
where we assume that \( \gamma_{\tilde{\eta}} \) and \( \gamma_{(y^2)} \) can be expanded in \( g \) perturbatively as follows
\[ \gamma_{\tilde{\eta}} = -\frac{g^2}{8\pi^2} \frac{N + 4}{2} + \mathcal{O}(g^4), \]
\[ \gamma_{(y^2)} = -\frac{g^2}{8\pi^2}(N + 5) + \mathcal{O}(g^4). \]

If the 1-loop beta function coefficient is negative but the 2-loop one is positive, then the infrared fixed point will exist [1]:
\[ \beta_0 = -(2N - 8) - 10\gamma_{\tilde{\eta}}(g = 0) - (N + 7)\gamma_{(y^2)}(g = 0) < 0 \]
\[ \Leftrightarrow N > \frac{14}{5}, \]
\[ \beta_1 = 5N + 20 + N^2 + 12N + 35 - (2N^2 - 2N - 24) - 10(N + 3)\gamma_{\tilde{\eta}}(g = 0) - (N^2 + 10N + 21)\gamma_{(y^2)}(g = 0) > 0 \]
\[ \Leftrightarrow -3 \leq N \leq 14, \]
where \( \beta_0, \beta_1 \) denote 1- or 2-loop beta function coefficients. Taking into account the conditions (43), (47), we find \( N = 13, 14 \).

In summary, we have discussed a \( \mathcal{N} = 1 \) SUSY \( SO(N) \) gauge theory with a symmetric traceless tensor. This theory saturates 't Hooft matching conditions among the fundamental fields and the gauge invariant composites at the origin of the moduli space. This naively suggests a confining phase but Brodie, Cho, and Intriligator have conjectured that the origin of the moduli space is in a Non-Abelian Coulomb phase. If this is the case, it is natural to ask whether the dual description exists. In
In this paper, we have constructed its dual by the deconfinement technique. Since this approach leads to the product gauge groups, it is not so trivial that the theory has a non-trivial infrared fixed point. Following [3], we have shown that the theory has indeed a non-trivial fixed point at the origin of the moduli space for $N = 13, 14$. Thus, this result supports the argument of Brodie, Cho, and Intriligator. Of course, the dual description is not necessarily unique, we may be able to find other dualities by further explorations.

We hope this work will provide a useful guide to analyzing the theory where 't Hooft anomaly matching appears to be coincidental.

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