EFFECTIVE FIELD THEORY OF THE SINGLE-NUCLEON SECTOR

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We address the issue of a consistent power counting scheme in manifestly Lorentz-invariant baryon chiral perturbation theory. We discuss the inclusion of vector mesons in the calculation of the nucleon electromagnetic form factors. We comment on the chiral expansion of the nucleon mass to order $O(q^6)$.

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1. Introduction

Effective field theory (EFT) is a powerful tool for describing the strong interactions at low energies. Starting point is the chiral SU($N_L \times SU(N_R)$) symmetry of QCD in the limit of $N$ massless quarks and its spontaneous breakdown to SU($N_V$) in the ground state. Instead of solving QCD in terms of quarks and gluons, its low-energy physics (of the mesonic sector) is described using the most general Lagrangian containing the Goldstone bosons as effective degrees of freedom. Physical quantities are calculated in terms of an expansion in $p/\Lambda$, where $p$ stands for momenta or masses that are smaller than a certain momentum scale $\Lambda$ (see, e.g., Refs. [8,9] for an introduction). In the following we will outline some recent developments in devising a renormalization scheme leading to a simple and consistent power counting for the renormalized diagrams of a manifestly Lorentz-invariant approach to baryon chiral perturbation theory.

2. Renormalization and Power Counting

The standard effective Lagrangian relevant to the single-nucleon sector consists of the sum of the purely mesonic and $\pi N$ Lagrangians, respectively,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_\pi + \mathcal{L}_{\pi N} = \mathcal{L}_2 + \mathcal{L}_4 + \cdots + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \cdots$$
which are organized in a derivative and quark-mass expansion. The aim is to devise a renormalization procedure generating, after renormalization, the following power counting: a loop integration in $n$ dimensions counts as $q^n$, pion and fermion propagators count as $q^{-2}$ and $q^{-1}$, respectively, vertices derived from $\mathcal{L}_{2k}$ and $\mathcal{L}_{\pi N}^{(k)}$ count as $q^{2k}$ and $q^k$, respectively. Here, $q$ generically denotes a small expansion parameter such as, e.g., the pion mass.

Several methods have been suggested to obtain a consistent power counting in a manifestly Lorentz-invariant approach. As an illustration consider the integral

$$H(p^2, m^2; n) = \int \frac{d^nk}{(2\pi)^n} \frac{i}{[(k - p)^2 - m^2 + i0^+][k^2 + i0^+]},$$

where $\Delta = (p^2 - m^2)/m^2 = \mathcal{O}(q)$ is a small quantity. In the infrared (IR) regularization of Becher and Leutwyler $^1$ one makes use of the Feynman parametrization

$$\frac{1}{ab} = \int_0^1 \frac{dz}{az + b(1 - z)^2},$$

with $a = (k - p)^2 - m^2 + i0^+$ and $b = k^2 + i0^+$. The resulting integral over the Feynman parameter $z$ is then rewritten as

$$\int_0^1 dz \cdots = \int_0^\infty dz \cdots - \int_1^\infty dz \cdots,$$

where the first, so-called infrared (singular) integral satisfies the power counting, while the remainder violates power counting but turns out to be regular and can thus be absorbed in counterterms. The central idea of the extended on-mass-shell (EOMS) scheme $^{12,13}$ consists of performing additional subtractions beyond the MS scheme. In Ref. $^{14}$ the IR regularization of Becher and Leutwyler was reformulated in a form analogous to the EOMS renormalization scheme. Within this (new) formulation the subtraction terms are found by expanding the integrands of loop integrals in powers of small parameters (small masses and Lorentz-invariant combinations of external momenta and large masses) and subsequently exchanging the order of integration and summation. The new formulation of IR regularization can be applied to diagrams with an arbitrary number of propagators with various masses (e.g., resonances) and/or diagrams with several fermion lines as well as to multi-loop diagrams.

3. Applications

3.1. Nucleon Form Factors

It has been known for some time that ChPT results at $\mathcal{O}(q^4)$ only provide a decent description of the electromagnetic Sachs form factors $G_E$ and $G_M$ up to $Q^2 = 0.1$ GeV$^2$ and do not generate sufficient curvature for larger values of $Q^2$. To improve these results higher-order contributions have to be included. This can be achieved by performing a full calculation at $\mathcal{O}(q^5)$ which would also include the
analysis of two-loop diagrams. Another possibility is to include additional degrees of freedom, through which some of the higher-order contributions are re-summed. Both the reformulated IR regularization and the EOMS scheme allow for a consistent inclusion of vector mesons which already a long time ago were established to play an important role in the description of the nucleon form factors. Figure 1 shows the results for the electric and magnetic Sachs form factors in the EOMS scheme (solid lines) and the infrared renormalization (dashed lines)\(^{17}\). A consistent inclusion of vector mesons clearly improves the quality of the description. Similarly, the inclusion of the axial-vector meson \(a_1(1260)\) results in an improved description of the experimental data for the axial form factor\(^{18}\).

Fig. 1. The Sachs form factors of the nucleon in manifestly Lorentz-invariant chiral perturbation theory at \(\mathcal{O}(q^4)\) including vector mesons as explicit degrees of freedom. Full lines: results in the extended on-mass-shell scheme; dashed lines: results in infrared regularization.

### 3.2. Chiral Expansion of the Nucleon Mass to Order \(\mathcal{O}(q^6)\)

Using the reformulated infrared regularization\(^{14}\) we have calculated the nucleon mass up to and including order \(\mathcal{O}(q^6)\) in the chiral expansion\(^{19,20}\):

\[
m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln \frac{M}{\mu} + k_4 M^4 + k_5 M^5 \ln \frac{M}{\mu} + k_6 M^5 + k_7 M^6 \ln \frac{M}{\mu} + k_8 M^6 \ln \frac{M}{\mu} + k_9 M^6.
\]

In Eq. (1), \(m\) denotes the nucleon mass in the chiral limit, \(M^2\) is the leading term in the chiral expansion of the square of the pion mass, \(\mu\) is the renormalization scale; all the coefficients \(k_i\) have been determined in terms of infrared renormalized parameters. Our results for the renormalization-scheme-independent terms agree with the heavy-baryon ChPT results of Ref.\(^{21}\).

The numerical contributions from higher-order terms cannot be calculated so far since, starting with \(k_4\), most expressions in Eq. (1) contain unknown low-energy coupling constants (LECs) from the Lagrangians of order \(\mathcal{O}(q^4)\) and higher. The coefficient \(k_5\) is free of higher-order LECs. Figure 2 shows the pion mass dependence...
of the term $k_5 M^5 \ln(M/m_N)$ (solid line) in comparison with the term $k_2 M^3$ (dashed line) for $M < 400$ MeV. For $M \approx 360$ MeV the $k_5$ term is as large as the $k_2$ term.

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