Questioning the wobbling interpretation of low-spin bands in γ-soft nuclei within the interacting boson-fermion model

K. Nomura1,∗ and C. M. Petrache2
1Department of Physics, Faculty of Science, University of Zagreb, HR-10000, Croatia
2Université Paris-Saclay, CNRS/IN2P3, IJCLab, 91405 Orsay, France
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An alternative interpretation of the recently reported low-lying excited bands in γ-soft odd-mass nuclei as wobbling bands is presented in terms of the interacting boson-fermion model. The model Hamiltonian is determined based on the mean-field calculations with the nuclear energy density functionals. The predicted mixing ratios of the ∆I = 1 electric quadrupole to magnetic dipole transition rates between yrast bands and those yrare bands previously interpreted as wobbling bands in 135Pr, 133La, 127Xe, and 105Pd are consistently smaller in magnitude than the experimental values on which the wobbling interpretation is based. These calculated mixing ratios indicate predominant magnetic character in agreement with the new experimental data. The earlier wobbling assignments are severely questioned.

I. INTRODUCTION

Ground-state shape of most nonspherical nuclear systems is characterized by axially-symmetric quadrupole deformation [1]. The axial symmetry, i.e., invariance under rotation about the symmetry axis of the intrinsic frame, is, however, broken in many nuclei. The nonaxial nuclear shapes as well as the resulting triaxially deformed rotors are a prominent feature of nuclear structure. A fingerprint of the rigid triaxiality is wobbling motion [1], a collective mode in which the principal axis of a triaxial rotor corresponding to the largest moment of inertia oscillates about the space-fixed angular momentum. The phenomenon has attracted much attention in nuclear physics, and is also recognized in finite many-body microscopic and macroscopic systems in general.

The wobbling motion in nuclei can be identified experimentally through the observation of rotational bands that are connected to each other by predominant ∆I = 1 electric quadrupole (E2) transitions, because the collective oscillation of the entire nuclear charge is involved. Traditionally, excited bands that manifest features of wobbling motion have been identified in high-spin bands of the odd-mass Lu and Ta nuclei in the mass A ≈ 160 region [2–7]. More recent experiments have shown new evidence for wobbling bands in odd-mass nuclei in several other mass regions, observed in the low-spin regime, e.g., in 135Pr [8, 9], 133La [10], 105Pd [11], 127Xe [12], 187Au [13], and 183Au [14], as well as at medium spins in 130Ba [15] and 136Nd [16]. In comparison to the high-spin wobbling bands of strongly-deformed triaxial nuclei in the mass A ≈ 160 region, the new experiments have proposed the occurrence of low-spin wobbling motion in normal-deformed γ-soft nuclei, which are characterized by a collective potential that is soft in nonaxial deformation and has small quadrupole deformation. The search for new regions of wobbling motion expands the frontier of nuclear collective motion, but should be accompanied by increasing experimental rigour.

In fact, it is of crucial importance to critically assess the reported experimental evidence for the wobbling bands. The wobbling interpretation requires connecting transitions with predominant electric character, which can be established by extracting mixing ratios δ(E2/M1) with magnitudes larger than 1, which signifies predominance of the electric over the magnetic components. Actually, new experiments that involve angular distribution combined with linear polarization measurements on excited bands in 187Au [17] and 135Pr [18] showed that the interband transition between the proposed wobbling bands and the yrast bands in these nuclei are predominantly magnetic. The wobbling interpretation of the newly found bands has been mostly based on a particle-rotor picture, in which the configuration that embodies the wobbling motion is explicitly considered within the intrinsic frame of reference [8–11, 18, 19]. On the other hand, it would be useful to give an alternative theoretical interpretation of the proposed low-spin wobbling bands if the character of the connecting transitions is not predominantly electric, as required by the collective wobbling motion.

In this paper, we shall consider the recently proposed low-spin wobbling bands of γ-soft odd-mass nuclei 135Pr, 133La, 127Xe, and 105Pd, within the interacting boson-fermion model (IBFM) [20, 21], with the Hamiltonian determined by the constrained mean-field calculations that are based upon the nuclear energy density functional (EDF) [22–25]. The aim of this work is to provide an alternative interpretation of the observed non-yrast bands of the above odd-mass nuclei. Our calculation reproduces the new data on 135Pr [18] and the old data on 105Pd [26], but is in contradiction with those experimental data on which the wobbling interpretation is based. Here we mainly focus on the excitation spectra and electromagnetic transition properties of these non-yrast bands, that are obtained from the diagonalization of the IBFM Hamiltonian in the laboratory frame of ref-

∗ knomura@phy.hr
II. THEORETICAL PROCEDURE

In even-even nuclei, to a good approximation, nucleons are coupled pairwise and the presence of such pairs play an important role in nuclear dynamics, determining basic parameters of vibrational and rotational spectra. In odd-mass nuclear systems, one has to consider explicitly the unpaired nucleon and treat the collective and noncollective (single-particle) degrees of freedom on the same footing [34]. A major assumption in the present work is that the low-lying states of an even-even nucleus is described by the interacting boson model (IBM) [35], consisting of the monopole $s$ (with spin and parity $L = 0^+$) and quadrupole $d$ ($L = 2^+$) bosons, which represent the collective $S$ and $D$ pairs of valence nucleons, respectively.

The low-energy structure of a given odd-mass nucleus is determined by the interaction between an odd fermion and the even-even boson (IBM) core. Specifically, $^{135}$Pr ($^{134}$La) is a system composed of even-even core $^{134}$Ce ($^{132}$Ba) plus an odd proton particle, while $^{127}$Xe ($^{105}$Pd) is composed of the $^{128}$Xe ($^{106}$Pd) even-even core coupled to the odd neutron hole. For all these odd-mass nuclei, the fermion space corresponds to the proton $Z$ or neutron $N = 50 – 82$ major oscillator shell, hence only the orbital $1h_{11/2}$ is considered to describe negative-parity states. In general, the IBM Hamiltonian is given by

$$
\hat{H} = \hat{H}_B + \hat{H}_F + \hat{V}_{BF},
$$

where $\hat{H}_B$ stands for the IBM Hamiltonian for an even-even core, $\hat{H}_F$ is the single-nucleon Hamiltonian, and $\hat{V}_{BF}$ represents the boson-fermion interaction.

In the next step we build the IBM Hamiltonian $\hat{H}_B$. In this study, we employ the proton-neutron IBM (IBM-2) [41]. The IBM-2 comprises the proton $s$, and $d$ bosons, and the neutron $s\nu$ and $d\nu$ bosons, which represent the collective monopole and quadrupole proton-proton and neutron-neutron pairs, respectively. For the IBM Hamiltonian $\hat{H}_B$ we adopt the form

$$
\hat{H}_B = \epsilon_d (\hat{n}_{d\nu} + \hat{n}_{d\nu}) + \kappa \hat{Q}_\pi \cdot \hat{Q}_\nu,
$$

where in the first term $\hat{n}_{d\rho} = d^\dagger_\rho d_\rho$ ($\rho = \pi$ or $\nu$), represents the number operator for the $d_\rho$ bosons, with $\epsilon_d$ the single $d$-boson energy relative to the $s$-boson one, and $\hat{Q}_\rho$ are the collective $s$-boson operators.
so that the fermionic and bosonic PESs become similar to those of [45]. In other words, the IBM parameters are calibrated by mapping the fermionic PES onto the expectations, however, comparison between the fermionic and bosonic deformations can be related to each other. No phenomenological adjustment to experiment is made in this procedure. We also note that the IBM Hamiltonian (2) is a rather specific form of the most general IBM-2 Hamiltonian. A more accurate theoretical description of the relevant spectroscopic properties of both even-even and odd-mass nuclei might require the inclusion of additional terms to $\hat{H}_B$, in particular, the so-called Majorana terms, which could play an important role in calculations of $M1$ properties. The Majorana terms, however, do not add independent contributions to the energy surface, unless the proton and neutron deformations are taken to be different [44]. Under the present assumption that the deformations for the proton and neutron boson systems are equal to each other, the strength parameters of these terms cannot be determined only by the comparison of the fermionic and bosonic PESs that are given in terms of the $\beta$ and $\gamma$ degrees of freedom only.

In Fig. 2 we show the mapped (bosonic) PESs for the even-even core nuclei $^{134}$Ce, $^{132}$Ba, $^{128}$Xe, and $^{106}$Pd, which can be compared with the original fermionic PESs in Fig. 1. In general, the bosonic PES appears to be flat especially in the region far from the global minimum. This reflects the fact that the IBM is built on the valence space of collective nucleon pairs in one major shell, while the mean-field model involves all nucleons. The bosonic PESs for $^{134}$Ce, $^{132}$Ba, and $^{128}$Xe exhibit a minimum on the prolate axis $\gamma = 0^\circ$, while a shallow triaxial minimum at $\gamma \approx 20^\circ$ is suggested in the corresponding fermionic PESs. This discrepancy could be corrected by including a higher-order term in the IBM Hamiltonian [46]. However, the triaxial minimum in the fermionic PES is so shallow that the discrepancy is expected to have a minor influence on the low-lying spectra of odd-mass nuclei.

Having fixed the boson-core Hamiltonian, we introduce the unpaired nucleon degree of freedom. The single-nucleon Hamiltonian in Eq. (1) reads

$$\hat{H}_F = -\epsilon_{j_r} \sqrt{2j_{r}} + 1 (a_{j_r}^\dagger \times \tilde{a}_{j_r}) |0\rangle \equiv \epsilon_{j_r} \hat{n}_{j_r},$$

where $\epsilon_{j_r}$ is single-particle energy for the odd proton $(\rho = \pi)$ or neutron $(\rho = \nu)$ in orbital $j_r$, $a_{j_r}^\dagger$ denotes the operator that annihilate (create) a single nucleon with $\tilde{a}_{j_r, m_\rho} = (-1)^{j_r-m_\rho} a_{j_r, -m_\rho}$, and $\hat{n}_{j_r}$ stands for the fermion number operator.

The interaction term $\hat{V}_{BF}$ for the coupling between the odd nucleon with the angular momentum $j_r$ and the boson core has the form

$$\hat{V}_{BF} = \Gamma_{j_r} \tilde{Q}_{\rho'} \cdot (a^\dagger_{j_r} \times \tilde{a}_{j_r})^{(2)} + \Lambda_{j_r} \left[ (s^\dagger_{j_r} \times \tilde{d}_{\rho'})^{(2)} \cdot \left( (d^\dagger_{j_r} \times \tilde{a}_{j_r})^{(j_r)} + (a^\dagger_{j_r} \times \tilde{s}_{j_r})^{(j_r)} \right)^{(2)} : + \hat{H}_{ec} \right]$$

$$+ \Lambda_0 \hat{n}_{d_{\rho'}} \hat{n}_{j_r},$$

where $\rho' \neq \rho$. The first, second, and third terms are the quadrupole dynamical, exchange, and monopole interactions, respectively. The $j$-dependent parameters $\Gamma_{j_r}$ and

FIG. 2. Same as Fig. 1, but for the bosonic PESs.
\[ \Lambda_{j_\rho} \] are given by [21, 47]

\[
\Gamma_{j_\rho} = \Gamma_0 (u_{j_\rho}^2 - v_{j_\rho}^2) Q_{j_\rho j_\rho}
\]

(6a)

\[
\Lambda_{j_\rho} = \Lambda_0 \left[ -4u_{j_\rho}^2 v_{j_\rho}^2 Q_{j_\rho j_\rho}^2 \sqrt{\frac{10}{N_\rho (2j_\rho + 1)}} \right]
\]

(6b)

\[ Q_{j_\rho j_\rho} \] stands for the matrix element of the spherical harmonic in the single-particle basis, i.e., \[ Q_{j_\rho j_\rho} = (\frac{1}{2} L_{j_\rho}) Y^{(2)} || L_{j_\rho} \parallel j_\rho \], and \[ u_{j_\rho}^2 + v_{j_\rho}^2 = 1 \] is satisfied. The dots : ( : : ) in Eq. (5) denotes normal ordering. Based on the microscopic considerations in terms of the generalized seniority scheme [47], it is assumed that both the dynamical and exchange terms are dominated by the interaction between unlike particles, i.e., between the odd proton (neutron) and the neutron (proton) bosons. The exchange term takes into account the fact that the bosons are made of nucleon pairs. For the monopole term, the interaction between like-particles, i.e., between the odd proton (neutron) and the proton (neutron) bosons, is considered. The specific boson-fermion interaction of the form (5), which is based on the generalized seniority, has been frequently used in a number of phenomenological IBM calculations including the spectroscopic studies of strongly deformed nuclei [21, 47, 48].

The building blocks of \( \hat{H}_B \) and \( \hat{V}_{BF} \) are spherical single-particle energy \( \epsilon_j \) and occupation probability \( v_{j_\rho}^2 \) of the odd fermion with \( j = 11/2 \), which are computed by the same mean-field calculations constrained to zero deformation [49]. The three coupling constants \( \Gamma_0, \Lambda_0, \) and \( \Lambda_0 \), defined in Eqs. (5) and (6), are fitted to reproduce to a reasonable accuracy the low-lying excitation energies of each odd-mass nucleus. Table I lists the adopted values of the parameters of \( \hat{H}_B \) (2), the occupation probability \( v_{j_\rho}^2 \) of the odd particle, and the fitted coupling constants of \( \hat{V}_{BF} \).

The IBFM Hamiltonian thus constructed is diagonalized in the basis \( || L_{\pi}, L_{\nu} (I); j:j \parallel I \rangle \) [50, where \( L_{\pi} \) (\( L_{\nu} \)) and \( L \) are the angular momentum of proton (neutron) boson system, and the total angular momentum of the even-even boson core, respectively. \( I \) stands for the total angular momentum of the coupled boson-fermion system.

### III. RESULTS AND DISCUSSIONS

#### A. Excitation spectra

Now we turn to the discussion about the spectroscopic properties of low-spin bands in odd-mass nuclei, which are produced by the diagonalization of the IBFM Hamiltonian (1) with the parameters obtained by the aforementioned procedure. In what follows, the band built on the \( 11/2^- \) state, the first, and the second excited bands resulting from the IBFM calculations are denoted by Yrast, B1, and B2, respectively.

In Fig. 3 the predicted five lowest-energy negative-parity bands of \( ^{135}\text{Pr} \) are shown. The theoretical Yrast, B1, and B2 bands consist of the states that exhibit dominant \( \Delta I = 2 \) inband \( E2 \) transitions. The comparison between the experimental and theoretical energy spectra demonstrates that the IBFM describes well the observed \([8, 18]\) low-lying bands in the odd-mass nucleus \( ^{135}\text{Pr} \).

For \( ^{135}\text{Pr} \), the experiment performed by Matta et al.
[8] showed that beside the $I = 11/2^-$ yrast band, the first excited band based on the $13/2^-$ state can be identified as unfavored signature partner (SP) band, and the second excited band built on the $17/2^-$ state can be assigned as an one-phonon wobbling band (denoted by “Wobb” in Fig. 3). In the IBFM, band B1 with bandhead $9/2^-$ corresponds to the proposed SP band, and band B2 including the $13/2^-, 17/2^- \ldots$ states is the theoretical counterpart of the proposed wobbling band.

As for $^{135}$Pr, two additional negative-parity bands have been considered in the new measurement by Lv et al. [18]: the first one comprising the $19/2^-, 23/2^-$, and $27/2^-$ states (band 4 in Ref. [18] and Fig. 3), and the second one comprising the $23/2^+, 27/2^+$, and $31/2^-$ states (band 5). The IBFM predicts additional two bands built on the $19/2^-$ and $23/2^+$ states, which are in agreement with the newly identified bands 4 and 5 in Ref. [18]. Our calculation shows that each of the two bands is composed by strong inband $\Delta I = 2E2$ transitions. In addition, the predicted interband $B(E2; 23/2^- \rightarrow 19/2^-)$ transition is an order of magnitude weaker than the inband $B(E2; 23/2^- \rightarrow 19/2^-)$ transition. This result supports the finding in the new measurement of Ref. [18], but is in contradiction to the earlier experiment of Ref. [9], in which the $19/2^-, 23/2^+$, and $27/2^-$ states were grouped into a single band and interpreted as two-phonon wobbling band. The new bands proposed in Ref. [18] do not follow the $I(I+1)$ energy dependence in a rotor picture, but appear to be rather vibrational, a fingerprint of the rotational softness in this mass region.

Figures 4, 5, and 6 show the calculated and experimental three lowest-lying negative-parity bands of the odd-mass nuclei $^{133}$La, $^{127}$Xe, and $^{105}$Pd. In general, the IBFM description of the observed low-lying band structure is satisfactory, except for the fact that the spins of the predicted bandhead states of some bands disagree with the experimental ones. The observed bandhead energies of the first (B1) and second (B2) excited bands are reproduced well by the IBFM.

For $^{133}$La, besides the $11/2^-$ ground-state band, the experiment by Biswas et al. [10] identified the first excited band based on the $13/2^-$ state as wobbling band. In Fig. 4, we associate the IBFM band B2 with bandhead $13/2^-$, with the proposed wobbling band, according to the facts (i) that the calculated energy levels are in a better agreement with the experimental ones than band B1, (ii) that the spin of the bandhead state $I = 13/2$ is correctly reproduced, and (iii) that, as shown later, the calculated $B(E2)_{out}/B(E2)_{in}$ and $B(M1)_{out}/B(E2)_{in}$ ratios are in a better agreement with data [10] than those for band B1.

The lowest three observed bands for $^{127}$Xe [12] are the lowest band (band 2) built on the $9/2^-$ state, the first ex-
cited band (band 1) built on the \( I = 11/2^- \) state, and the second excited band (band 3) built on the \( 13/2^- \) state, which was assigned to be wobbling band \([12]\). Band B2 in the IBFM consisting of the \( 9/2^+, 13/2^+, 17/2^+, 21/2^+, 25/2^- \) states is the theoretical counterpart of band 3. The IBFM yields band B1 with the bandhead \( 9/2^- \) corresponding to the ground state, consistently with the observed band 2.

For \(^{105}\text{Pd}\), Timár et al. \([11]\) interpreted the first excited \( \Delta I = 2 \) band (band B), which is built on the \( 13/2^- \) state as wobbling band. This is the first evidence for a low-spin wobbling band in the mass \( A \approx 100 \) region. The present IBFM calculation yields the equivalent \( \Delta I = 2 \) band (B1) consisting of the \( 9/2^- , 13/2^- , \ldots \) states.

**B. \( E2/M1 \) mixing ratio**

The \( E2 \) to \( M1 \) mixing ratio \( \delta \) is a criterion for the wobbling interpretation, and is calculated by using the formula \([51, 52]\)

\[
\delta = 0.835 \times E_\gamma \frac{\langle I_f \| \hat{T}^{(E2)} \| I_i \rangle}{\langle I_f \| \hat{T}^{(M1)} \| I_i \rangle},
\]

(7)

where \( E_\gamma = E_{I_f} - E_{I_i} \), with the resultant excitation energies of the initial \( I_i \) and final \( I_f \) states, and \( \langle I_f \| \hat{T}^{(E2)} \| I_i \rangle \) and \( \langle I_f \| \hat{T}^{(M1)} \| I_i \rangle \) represent reduced matrix elements of the \( E2 \) and \( M1 \) transition operators, respectively. The \( E2 \) operator \( \hat{T}^{(E2)} \) here takes the form \([21]\):

\[
\hat{T}^{(E2)} = \hat{T}_B^{(E2)} + \hat{T}_F^{(E2)},
\]

(8)

where

\[
\hat{T}_B^{(E2)} = e_\pi^B \hat{Q}_\pi + e_\nu^B \hat{Q}_\nu,
\]

(9)

and

\[
\hat{T}_F^{(E2)} = -e_\nu \frac{1}{\sqrt{5}} (u_{j\nu}^2 - v_{j\nu}^2) \hat{Q}_{j\nu} (a_{j\nu}^+ \times \hat{a}_{j\nu})^{(2)}
\]

(10)

are the bosonic and fermionic parts of the \( E2 \) operators, respectively. Note that \( \hat{Q}_\rho \) has been defined in (2). We assume that the effective \( E2 \) charges for proton \( e_\pi^B \) and neutron \( e_\nu^B \) bosons are equal to each other, \( e_\pi^B = e_\nu^B = e^B \), and fix \( e^B \) so that the experimental \( B(E2; 2^-_1 \rightarrow 0^+_1) \) rate of each even-even core nucleus is reproduced. For the fermion part, standard effective charge \( e^\nu = 1.5 \) (0.5) \( eB \) is adopted for the odd proton (neutron). The \( M1 \) transition operator \( \hat{T}^{(M1)} \) reads:

\[
\hat{T}^{(M1)} = \hat{T}_B^{(M1)} + \hat{T}_F^{(M1)}
\]

(11)

where

\[
\hat{T}_B^{(M1)} = \sqrt{\frac{3}{4\pi}} (g_\pi^B \hat{L}_\pi + g_\nu^B \hat{L}_\nu),
\]

(12)

\[
\hat{T}_F^{(M1)} = - \frac{1}{\sqrt{3\pi}} \langle j \| g_{1l} + g_{s} \| j \rangle \langle a_{j\nu}^{+} \times \hat{a}_{j\nu} \rangle^{(1)}
\]

(13)

are the boson and fermion parts of \( \hat{T}^{(M1)} \), respectively.

The effective gyromagnetic \((g_z)\)-factors for the proton \( g_\pi^B \) and neutron

\[
\text{FIG. 7. The } E2 \text{ to } M1 \text{ mixing ratios } \delta \text{ for the } \Delta I = 1 \text{ yrare to }\]

\( \text{yrast transitions in the odd-mass nuclei } ^{135}\text{Pr}, ^{133}\text{La}, ^{127}\text{Xe}, \text{ and } ^{105}\text{Pd, for which wobbling bands were previously suggested. The experimental values are taken from Refs. } [8, 10–12, 18, 26]. \text{ The shaded area in each panel indicates } |\delta| < 1. \text{ The notations of the experimental bands in each panel follow the ones used in the above references. The IBFM } \delta \text{ values for those bands that were previously assigned to be wobbling bands are represented by solid symbols.} \)
and neutron \( g_{v}^{B} \) bosons are chosen to be close to the empirical values \([53, 54]\) that satisfy \( g_{v}^{B} \approx 1.0 \mu_{N} \), and \( g_{v}^{B} \approx 0.1 \mu_{N} \). For the odd proton (neutron) \( g \)-factors, the standard Schmidt values \( g_{i} = 1.0 \mu_{N} \) and \( g_{i} = 5.58 \mu_{N} \) \((g_{i} = 0 \mu_{N} \) and \( g_{i} = -3.82 \mu_{N} \) are used, with \( g_{i} \) quenched by 30\% with respect to the free value. The adopted values of the boson effective \( E_{2} \) charge \( e_{B}^{2} \), \( g \)-factors for proton \( g_{B}^{P} \) and neutron \( g_{B}^{N} \) are found in Table I.

We show in Fig. 7 the calculated \( \delta(E2/M1) \) ratios for the \( \Delta I = 1 \) transitions between the yrast and yrare bands. The predicted \( \delta \) values for \( ^{135}\text{Pr} \) shown in Figs. 7(a) and 7(b), are close to zero for both the B1 \( \rightarrow \) Yrast and B2 \( \rightarrow \) Yrast transitions. The experimental absolute values \( |\delta| \) for the Wobb \( \rightarrow \) Yrast transitions \([8]\) increase with spin \( I \). The updated data of Lv \textit{et al.} \([18]\), however, provide smaller mixing ratios for the same band (band 3) at \( I = 17/2 \) and 21/2, which agree with the present calculations.

The computed mixing ratios for \( ^{133}\text{La} \), shown in Figs. 7(c) and 7(d), are generally small, \( |\delta| < 1 \). The measured \( \delta \) ratios for the wobbling (Wobb) band, which are the basis of the wobbling interpretation of the \( 13/2^{+} \) band, are depicted in Fig. 7(d). The corresponding IBFM \( \delta \) values for the B2 \( \rightarrow \) Yrast transitions, plotted also in Fig. 7(d), are much smaller in magnitude than these experimental values, but are rather close to those obtained with the quasiparticle-plus-triaxial-rotor (QTR) model \([10]\), giving more weight to a non-wobbling description of the band. We note that the interpretation of the experimental \( 13/2^{+} \) band as wobbling band \([10]\) has been recently questioned in Ref. \([55]\), with respect to the reported \( E2 \) dominance of the transitions connecting the proposed wobbling and normal bands.

The calculated \( \delta \) for the B1 \( \rightarrow \) Yrast transitions for \( ^{127}\text{Xe} \) shown in Fig. 7(e) are small, except for the one at \( I = 13/2 \). As seen in Fig. 7(f), the absolute \( \delta \) ratios for the B2 \( \rightarrow \) Yrast transitions are predicted to be smaller in magnitude than the experimental counterparts for \( I \geq 21/2 \). However, the predicted ratios are unusually large \( |\delta| \approx 5 \) for \( I < 21/2 \). An earlier in-beam spectroscopic study by Urban \textit{et al.} \([56]\) gave the mixing ratio \( \delta = -1.7^{+0.4}_{-0.6} \), or \( -0.45 \pm 0.12 \) for the \( E_{2} \) value of 483 keV (or \( 13/2^{+} \) \( \rightarrow \) \( 11/2^{+} \) ) decay for \( ^{127}\text{Xe} \), the absolute value of which is considerably overestimated by the present calculation. As we show later, the too large \( |\delta| \) is here obtained because the calculated \( M1 \) matrix elements for \( ^{127}\text{Xe} \), especially the one for the \( 13/2^{+} \) \( \rightarrow \) \( 11/2^{+} \) transition, are negligibly small. For \( ^{125}\text{Xe} \), the \( 1h_{11/2} \) single-neutron orbital is nearly half filled, \( v_{11/2}^{2} \approx 0.5 \) (see Table I), in which case the contributions from both the dynamical and exchange terms of \( V_{\text{HP}} \) are rather sensitive to the choice of their strength parameters. The chosen set of the strength parameters might have yield substantial amount of configuration mixing in the lower spin states, thus resulting in the too small \( M1 \) matrix elements.

The absolute values of the calculated \( \delta \) for both bands B1 and B2 for \( ^{109}\text{Pd} \) shown in Figs. 7(g) and 7(h), are all less than one except for the lowest \( I = 13/2 \) state of band B2. The calculated \( \delta \) values for band B1 are by a factor of two to three smaller than the measured values for the proposed wobbling band B \([11]\). In contrast, the mixing ratios obtained by Rickey \textit{et al.} \([26]\), also shown in Fig. 7(h), agree with the calculated values at \( I = 17/2 \) and 21/2. In this case we are again faced with contradicting experimental values, like in the case of \( ^{135}\text{Pr} \), which keeps open the question of the real nature of the band.

C. \( B(E2) \) and \( B(M1) \) transitions

In Fig. 8 we show the predicted \( B(E2)_{\text{out}}/B(E2)_{\text{in}} \) \((B(M1)_{\text{out}}/B(E2)_{\text{in}})\) ratios of the interband \( \Delta I = 1 \) \( E2 \) (\( M1 \)) to the inband \( \Delta I = 2 \) \( E2 \) transitions for the bands B1 and B2 for \( ^{135}\text{Pr} \). We see from Fig. 8(b) that, as compared to the B1 \( \rightarrow \) Yrast \( E2 \) transitions, the predicted B2 \( \rightarrow \) Yrast \( E2 \) transitions are generally weak. The calculated \( B(E2)_{\text{out}}/B(E2)_{\text{in}} \) ratios for band B2 are much smaller than the experimental data reported by Matta \textit{et al.} \([8]\), but are closer to the new data of Lv \textit{et al.} \([18]\) at \( I = 21/2 \). The calculated \( B(M1)_{\text{out}}/B(E2)_{\text{in}} \) ratios for band B2, shown in Fig. 8(d), are much larger than the data reported in \([8]\), but are in a better agreement with the new value at \( I = 21/2 \) \([18]\).

The ratios \( B(E2)_{\text{out}}/B(E2)_{\text{in}} \) and \( B(M1)_{\text{out}}/B(E2)_{\text{in}} \) for \( ^{133}\text{La} \) are shown in Fig. 9. The predicted \( B(E2)_{\text{out}}/B(E2)_{\text{in}} \) ratios for the B2 \( \rightarrow \) Yrast transitions...
agree with the experimental values reported in Ref. [10] (Fig. 9(b)), in which the first excited band (denoted as “Wobb” in Figs. 4 and 9) has been interpreted as wobbling band, as well as with the QTR model calculations [10]. The $B(M1)_{out}/B(E2)_{in}$ ratios for the $B2 \rightarrow$ Yrast transition are calculated to be much larger than the measured values of the Wobb $\rightarrow$ Yrast transitions (Fig. 9(d)), but rather agree with the QTR results [10], which also overestimated the data.

As for $^{127}$Xe, the calculated $B(E2)_{out}/B(E2)_{in}$, shown in Fig. 10(b), are much smaller than the experimental ones for band 3, which was assigned to be wobbling band [12]. The experimental $B(E2)_{out}/B(E2)_{in}$ ratios indicate strong $E2$ transitions from band 3 to the yrast band (band 1 in Fig. 5), especially for the spin $I \geq 21/2$, while the errors are also large. In Fig. 10(d), the predicted $B(M1)_{out}/B(E2)_{in}$ ratios for the band $B2$ at $I = 17/2$ and $21/2$ are considerably smaller than the experimental values for band 3, which was identified as wobbling band. We recall that the too large $\delta$ values are calculated at these spins (see Fig. 7(f)).

The $B(E2)_{out}/B(E2)_{in}$ ratios for both the $B1 \rightarrow$ Yrast and $B2 \rightarrow$ Yrast transitions in $^{105}$Pd shown in Fig. 11 are relatively small, in comparison to the measured values for band $B$ [11], which was interpreted as the wobbling band. Generally, the $B(M1)_{out}/B(E2)_{in}$ ratios are calculated to be larger than the data. Thus we can confirm the $M1$ dominance of the predicted transitions between yrare and yrast bands for $^{105}$Pd.

For the sake of completeness, the calculated values for the mixing ratios $\delta$, the $B(M1)_{out}/B(E2)_{in}$, the $B(E2)_{out}/B(E2)_{in}$ ratios of the lowest two excited bands, and the corresponding experimental values based on the wobbling interpretation of the yrare bands [8, 10–12], and those from the new measurement for $^{135}$Pr [18] and the old experimental $\delta$ values for $^{105}$Pd [26] are listed in Table II.
connecting the low-lying yrare bands to the yrast bands in $^{135}\text{Pr}$, $^{133}\text{La}$, $^{127}\text{Xe}$, and $^{105}\text{Pd}$. In all cases, the boson and fermion matrix elements, for both bands B1 and B2, and for all transitions from the theoretical bands B1 and B2 to inband B(E2; \text{I} \rightarrow \text{I} - 2)_\text{in} transition rates, connecting the low-lying yrare bands to the yrast bands in $^{135}\text{Pr}$, $^{133}\text{La}$, $^{127}\text{Xe}$, and $^{105}\text{Pd}$. The boson contribution to the matrix elements to those of band B2 (B1) is opposite in sign to the boson one. As for band B2 of $^{105}\text{Pd}$, which is here associated with the proposed wobbling band [8], both the boson and fermion parts make small in magnitude, but coherent contributions to the E2 and M1 matrix elements. Band B1 (B2) of $^{133}\text{La}$ appears to show similar patterns of the boson and fermion contributions to the matrix elements to those of band B2 (B1) of $^{135}\text{Pr}$. We note that band B2 of $^{133}\text{La}$ is here considered to be the theoretical counterpart of the proposed wobbling band [11] (see Fig. 4), while both bands B1 and B2 in the same nucleus are shown to be close in energy (see Fig. 4) and have similar electromagnetic properties (see Figs. 7(c,d) and 9). There is essentially no fermion contribution to the E2, as well as M1, matrix elements of both bands B1 and B2 of $^{127}\text{Xe}$. For this nucleus, the calculated M1 matrix elements at $\text{I} = 13/2$ are especially small, and this corroborates the too large $\delta$ mixing ratios at lower spins (see Figs. 7(e) and 7(f)). For band B1 of $^{105}\text{Pd}$, here associated with the wobbling band [11], the boson and fermion parts make coherent contributions to the matrix elements (see Figs. 12(d) and 12(h)). This systematic trend is similar to the one observed for band B2 of $^{135}\text{Pr}$, which is also associated with the wobbling band in the present study. The boson contribution to band B2 of $^{105}\text{Pd}$ becomes increasingly larger for higher spin.

### Table II. Comparisons between the calculated and experimental $\delta(E2/M1)$ mixing ratios of the \(\Delta I = 1\) interband transitions, and the ratios of the interband $B(M1; \text{I} \rightarrow \text{I} - 1)_\text{out}$ and $B(E2; \text{I} \rightarrow \text{I} - 1)_\text{out}$ to inband $B(E2; \text{I} \rightarrow \text{I} - 2)_\text{in}$ transition rates, connecting the low-lying yrare bands to the yrast bands in $^{135}\text{Pr}$, $^{133}\text{La}$, $^{127}\text{Xe}$, and $^{105}\text{Pd}$.

| Nucleus | $E_\text{c}$ (keV) | $\delta$ | \(B(M1)\text{out}/B(E2)_\text{in}\) | \(B(E2)\text{out}/B(E2)_\text{in}\) |
|---------|-----------------|----------|-------------------------------|--------------------------|
|         | EXP  | IBFM | EXP  | IBFM | EXP  | IBFM |
| $^{135}\text{Pr}$ [8] | 747.0 | 17/2$^+_1$ | -1.24 ± 0.13 | -0.646 | 0.164 ± 0.014 | 0.425 | 0.854 ± 0.032 | 0.387 |
|        | 812.8 | 21/2$^+_1$ | -1.54 ± 0.09 | -0.368 | 0.035 ± 0.028 | 0.771 | 0.506 ± 0.025 | 0.946 |
| $^{133}\text{La}$ [10] | 618 | 13/2$^+_1$ | -1.48 ± 0.32 | -1.167 | 0.107 ± 0.035 | 0.232 | 1.126 ± 0.140 | 0.683 |
| $^{127}\text{Xe}$ [12] | 483 | 13/2$^+_1$ | -2.1 ± 0.04 | -3.699 | 0.085 | 1.924 |
|        | 639 | 21/2$^+_1$ | -2.2 ± 0.1 | -4.901 | 0.138 ± 0.012 | 0.309 | 2.352 ± 0.565 | 1.874 |
|        | 735 | 21/2$^+_1$ | -2.4 ± 0.1 | -1.801 | 0.098 ± 0.005 | 0.375 | 1.500 ± 0.172 | 0.645 |
|        | 800 | 25/2$^+_1$ | -2.9 ± 0.1 | -1.117 | 0.071 ± 0.031 | 0.050 | 1.346 ± 0.879 | 0.864 |
|        | 884 | 29/2$^+_1$ | -3.1 ± 0.1 | -0.355 | 0.052 ± 0.044 | 0.310 | 0.922 ± 0.895 | 0.011 |
|        | 651 | 13/2$^+_1$ | +0.15 ± 0.05 | +1.698 | 0.180 ± 0.004 | 0.022 | 0.014 ± 0.009 | 0.329 |
|        | 876 | 17/2$^+_1$ | +0.26 ± 0.10 | +0.857 | 0.053 ± 0.002 | 0.462 | 0.007 ± 0.005 | 0.005 |
| $^{105}\text{Pd}$ [11] | 991 | 17/2$^+_1$ | +1.8 ± 0.5 | +0.727 | 0.162 ± 0.097 | 0.316 | 0.66 ± 0.18 | 0.389 |
|        | 1034 | 21/2$^+_1$ | +2.3 ± 0.3 | +0.817 | 0.089 ± 0.026 | 0.166 | 0.60 ± 0.09 | 0.236 |
|        | 994 | 25/2$^+_1$ | +2.7 ± 0.6 | +0.851 | 0.249 ± 0.057 | 0.101 | 0.34 ± 0.07 | 0.182 |
| $^{105}\text{Pd}$ [26] | 991 | 17/2$^+_1$ | +0.46 ± 0.10 | +0.727 | 0.316 | 0.389 |
|        | 1034 | 21/2$^+_1$ | +0.62 ± 0.18 | +0.817 | 0.166 | 0.236 |
|        | 994 | 25/2$^+_1$ | +1.5 ± 1.0 | +0.851 | 0.101 | 0.182 |
predicted an oblate minimum for calculations using the Skyrme [60] and Gogny [61] forces exist near the ground state [59]. Previous mean-field calculations often characterized by oblate and prolate shapes that co-
exist near the ground state. In addition, the even-even core nuclei generally exhibit pronounced $\gamma$ softness characteristic for nonaxial nuclei. The calculated $E2$ to $M1$ mixing ratios $\delta$ for the $\Delta I = 1$ transitions between the yrare and yrast bands in the considered nuclei are consistently small, $|\delta| < 1$. These mixing ratios indicate the $M1$ dominance of the transitions connecting the yrare bands in question to the yrast bands, which is in contradiction with the wobbling interpretation, and are in agreement with the updated experimental mixing ratios for $^{135}$Pr [18] and the old data for $^{105}$Pd [26]. This work sheds new light upon the excited low-lying bands in $\gamma$-soft nuclei, questioning their wobbling interpretation.

E. The gold nuclei

Finally, we make a remark on the recently proposed wobbling bands in the heavier nuclei, i.e., $^{187}$Au [13] and $^{183}$Au [14]. Empirically, their low-lying negative-parity states are understood as the proton $\pi h_{9/2}$ orbital coupled with a prolate-deformed core. Within the standard IBFM, the above nuclei would be described by the coupling between the odd proton $\pi h_{11/2}$ orbital and the even-even Hg core. Earlier phenomenological IBFM calculations [57, 58] considered the $\pi h_{9/2}$ intruder orbital coupled to the even-even Pt core in order to describe the $I = 9/2^-$ ground state bands of odd-mass Au nuclei. In principle, both the $\pi h_{11/2}$ and $\pi h_{9/2}$ orbitals could be simultaneously included in our model but, due to the huge energy difference between their spherical single-particle levels across the proton $Z = 82$ major shell gap, contribution from the latter is expected to be negligible. In addition, the even-even core nuclei $^{188}$Hg and $^{184}$Hg are often characterized by oblate and prolate shapes that coexist near the ground state [59]. Previous mean-field calculations using the Skyrme [60] and Gogny [61] forces predicted an oblate minimum for $^{188}$Hg and a pronounced prolate-oblate shape coexistence for $^{184}$Hg. In both cases, the PESs were shown to be far from $\gamma$ soft, in contrast to the nuclei considered here (see Fig. 1). These facts indicate a potential difficulty in the treatment of the $\pi h_{11/2}$ orbital for $^{187}$Au and $^{183}$Au within our model calculations.

IV. CONCLUDING REMARKS

In summary, an alternative interpretation of the recently reported low-spin wobbling bands in the odd-mass nuclei $^{135}$Pr, $^{133}$La, $^{127}$Xe, and $^{105}$Pd has been presented through IBFM calculations. The bosonic Hamiltonian for the even-even core nuclei, and the essential building blocks of the boson-fermion interaction have been determined by using the constrained mean-field approach based on a given nuclear EDF. The PESs for the even-even core nuclei, obtained from the mean-field calculations with representative classes of the universal EDF, generally exhibit pronounced $\gamma$ softness characteristic for nonaxial nuclei. The calculated $E2$ to $M1$ mixing ratios $\delta$ for the $\Delta I = 1$ transitions between the yrare and yrast bands in the considered nuclei are consistently small, $|\delta| < 1$. These mixing ratios indicate the $M1$ dominance of the transitions connecting the yrare bands in question to the yrast bands, which is in contradiction with the wobbling interpretation, and are in agreement with the updated experimental mixing ratios for $^{135}$Pr [18] and the old data for $^{105}$Pd [26]. This work sheds new light upon the excited low-lying bands in $\gamma$-soft nuclei, questioning their wobbling interpretation.
FIG. 13. The same as Fig. 12, but for the transitions from bands B2 to yrast bands. The theoretical bands B2 for $^{135}$Pr, $^{133}$La, and $^{127}$Xe are here associated with the proposed wobbling bands in Refs. [8, 10, 12], respectively.

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