Account of the force resistance in the calculation of reinforced concrete structures

A V Borovskih¹, V P Gorbachevskii¹,² and L A Pachomova¹

¹ Moscow State University of Civil Engineering (National Research University), Yaroslavskoye Shosse, 26, Moscow, 129337, Russia

Abstract. The aggressiveness of the environment of their placement has a direct impact on reinforced concrete structures. This work is aimed to present the method for constructing fundamentally modified equations of the force resistance of concrete and reinforcing steel. The results of this work present the method for the constructing quasi-linear equations of the force resistance of concrete and reinforcing steel for use in the calculation of reinforced concrete structures. The sequence for producing a quasi – linear equation of the force resistance of concrete and reinforcing steel, as well as the diagram of the stress – total relative deformations is presented.

1. Key words
Force resistance, force resistance of the concrete, force resistance of the fittings, stress-strain diagram, deformation chart, forces loss.

2. Introduction
The operated reinforced concrete structures besides their own weight and technological loads perceive loads caused by interaction with the environment of their placement (including the environment). The aggressiveness of the environment of their placement, which is the dominant set of the physical and chemical factors, has a direct impact on reinforced concrete structures operating in the exploitative stage, at the same time with temperature, humidity, barometric pressure, in some cases. The existing methods of the estimation the influence of the aggressive environment on the force resistance is carried out by the introduction of empirical coefficients «working conditions», taking into account «...factors that don't have an acceptable analytical description». However, this approach doesn’t exhaust the aspects of the deformation of materials and structures operated in the aggressive environments.

Among the factors of aggressive influence, the most common are the chemical effects of the placement environment, causing corrosion damage to the components of reinforced concrete (concrete, reinforcement), and the conditions of their joint work (including adhesion of reinforcement and concrete).

The corrosion damages of the reinforced concrete structures reduce their bearing capacity (strength, stability) and crack resistance, change their dynamic characteristics [1-6].

The level of the corrosion damage at the time of observation depends on the combination of initial qualities of concrete and characteristics of the chemically aggressive environment placement: firstly, the type, class and specificity of the technological limits of manufacture, formation, mainly from the initial data of the structural features of concrete and, secondly, the concentration, chemical aggressiveness, humidity and temperature of the environment.
At the same time, the structure of concrete and, consequently, its permeability, determining the origin and depth of corrosion damage, depends on the level and duration of the force action. In particular, it is experimentally established that under compression under the influence of the different levels of loading – from zero values to half of the lengthy strength – the concrete structure is compacted, the deformation modulus is reduced, and the permeability decreases; at higher stress levels, with their growth cracks in the concrete increase, unite in the main, the modulus of deformation decreases, and the permeability increases. The changes in permeability cause differences in the depth of the corrosion damage of the concrete body.

The cause-and-effect relationship of the listed factors of the influence of level of loading and corrosion influences is classified on temporary signs: on instant and on lagging. At the same time, the force deformation of concrete is characterized by both instantaneous and delayed manifestations, and the accumulation of corrosion damage – only delayed.

This work is aimed to present the method for constructing fundamentally modified equations of the force resistance of concrete and reinforcing steel.

3. Methods
To achieve this aim the method of computational analysis was used.

4. Results
As the equation of the force resistance of concrete at a homogeneous stress state the entry (1) is accepted:

$$
\varepsilon(t, t_0) = \varepsilon_{im}(t) = [\varepsilon_{im}(t) + \varepsilon_{ac}(t, t_0)],
$$

with:

$$
\varepsilon_{im}(t) = \frac{S_{im}(t)}{E_{im}(t)} + S_c(t)c(t, t),
$$

$$
\varepsilon_{ac}(t, t_0) = -\int_{t_0}^{t} S_c(t) \frac{ds(t, t_0)}{dt} d\tau,
$$

where \(\varepsilon(t, t_0)\) – total relative deformation obtained by summing partial relative deformations;

\(\varepsilon_{im}(t)\) – immediate relative deformation that monitors stresses, including elasto – instantaneous strain and the strain is fast – starting creep;

\(\varepsilon_{ac}(t, t_0)\) – mode creep deformations accumulated over a period of time \((t, t_0)\);

\(E_{im}(t)\) – elastic – instantaneous deformation modulus;

\(\varepsilon_{fr}(\tau, t_0)\) – fast-accumulating creep;

\(S_{im}(t)\) – the stress function for elasto – instantaneous strain;

\(S_c(t)\) – stress function for creep strain (both fast-creep and time-accumulated);

\(t, \tau, t_0\) – start time of the observations, current time, end of the observation.

In this case, the stress functions in the linear formulation:

$$
S(\sigma) = \sigma,
$$

In the nonlinear formulation on the proposal of P.I. Vasiliev [2]:

$$
S(\sigma) = \sigma(t) \left(1 + \frac{\sigma(t)}{R(t)}\right)^m
$$

or at the Graff's suggestion [2]:

$$
\varepsilon(t) = \bar{a}\sigma^b,
$$

$$
S(t) = \bar{a}\sigma^{b-1},
$$

(5)
\[
\sigma = \left( \frac{1}{\alpha} \right)^{1/b} \varepsilon^{1/b},
\]

The calculation formulas for calculating \(a\) and \(b\) is presented below. Currently, there are experimentally established values of the parameters \(V\) and \(m\) for instantaneous deformations and creep deformations [2]:

\[
S_{im}(t) = \sigma(t) \left( 1 + V_{im} \left[ \frac{\sigma(t)}{R(t)} \right]^m \right),
\]

\[
S_c(t) = \sigma(t) \left( 1 + V_c \left[ \frac{\sigma(t)}{R(t)} \right]^m \right),
\]

Table 1 shows the nonlinearity parameters expressed in terms of the prismatic strength of concrete \(R\) and the tensile strength \(R\sigma_t\).

| The type of deformation | Stage 1 Designation | Stage 2 Compression | Stage 3 Stretching |
|-------------------------|---------------------|---------------------|-------------------|
| Instant creep           | \(V_{im}\)          | 32.5 \(R^{-1}\)     | 0.3+0.37 \(R\sigma_t\), |
|                         | \(M_{im}\)          | 5.7–0.05\(R^{-1}\)  | 0.8+0.23 \(R\sigma_t\), |
| Creep                   | \(V_c\)             | 45.0 \(R^{-1}\)     | 1.5               |
|                         | \(M_c\)             | 5.0–0.07\(R^{-1}\)  | 1.0               |

The quantitative differences of the same nonlinearity parameters for instantaneous (elastic) deformations and for creep deformations complicate or exclude the application of the equation of force resistance (1) and (2) for many tasks of calculation of the reinforced concrete structures.

Using the proposal of Yu. Rabotnov, implemented by S. Bondarenko [2], it was possible to overcome these difficulties by replacing equation (1) with the quasi-linear equation (9):

\[
\varepsilon(t, t_0) = \frac{S[\sigma(t)]}{E_{np}(t,t_0)} = \frac{\sigma[1+V(\frac{\sigma(t)}{R(t)})^m]}{E_{np}(t,t_0)}
\]

or (10) and (11)

\[
\varepsilon(t, t_0) = \frac{\sigma(t)}{E_{np}(t,t_0)},
\]

\[
\varepsilon_{np}(t, t_0) = \frac{E_{np}(t,t_0)}{1+V(\frac{\sigma(t)}{R(t)})^m},
\]

where \(S\) – generalized uniform stress function of the quasilinear equation:

\[
S[\sigma(t)] = \sigma \left( 1 + V \left[ \frac{\sigma(t)}{R(t)} \right]^m \right);
\]

\(E_{np}(t,t_0)\) – linear temporary modulus of the concrete deformation:

\[
E_{np}(t, t_0) = \frac{E_{im}(t)}{1+E_{im}(t)\cdot C(t,t_0)}
\]

The nonlinearity parameters \(V\) and \(m\) for the quasilinear equation are calculated from the condition of equality of the total relative deformation calculated by (4) and (5) at two fixed points of the stress level \(\gamma \frac{G}{R}\) at \(\gamma = 1\) and \(\gamma = 0.7\), according to the formulas (14) and (15).

\[
V = \frac{(1+V_{im}) \frac{1}{E_{im}(t)} + (1+V_c) C(t,t_0)}{V_{im}(t) + C(t,t_0)}
\]
\[
\bar{m} = \frac{1}{ln\gamma} \ln \left( \frac{\frac{1}{E_{im}(t)} \frac{1}{C(t,t_0)}}{\frac{1}{E_{im}(t)} + \frac{1}{C(t,t_0)}} \right) \quad \text{(15)}
\]

Similarly are \(\bar{a}\) and \(b\) for the record (5):

\[
a(t,t_0) = \frac{(1+V)R^{1-b}}{E_{np}(t,t_0)}
\]

\[
b = 1 + \frac{1}{ln\gamma} \ln \frac{\gamma+V\gamma^n}{1+V} \quad \text{(16)}
\]

It is established that reinforcing steels under loading are deformed nonlinearly. At the same time it is accepted that at operational levels of loading the nonlinearity of deformation of reinforcing steel can be neglected and deformation of steel is considered proportional to stresses. At the same time, when steel is deformed, except for the so-called soft low-carbon steels at a temperature below 300°C, creep (T.N. Kripp) is manifested and, therefore, after loading and under conditions of the tightness of deformations, the stresses in them caused by the load relax.

In this regard, for reinforcing steels, the record of the equation of force resistance in the form of T.N. Hooke (17) is used.

\[
E_s(t) = \frac{\sigma(t)}{\varepsilon(t)}
\]

\[
E_s(t) = \frac{\sigma_{im}(t)}{1+\frac{\sigma_{im}(t)}{C(t,t_0)}} \quad \text{(17)}
\]

where \(E_s\) is the modulus of instantaneous deformation; \(C(t,t_0)\) is a measure of simple creep.

The quasilinear equation (10) allows to construct the diagram – total relative deformations of concrete (in the part of the ascending branch OT and T) (Fig.1).

In diagrams I and II (Fig.1) the initial strain modules have the common modular line 1-1t; points 2 and 2t mean the reached stress level \(\sigma\) and \(\sigma_t\), \(\varepsilon\) and \(\varepsilon_t\) – the corresponding total relative strain, points T and Tt – the end of the loading branch (T and Tt – the point of "no return"), the descending branches of the diagram T-K, Tt-TK fix the possible stress levels, and \(\varepsilon_T\) and \(\varepsilon_{Tt}\) – correspond to the point of relative strain, \(\varepsilon_T\) and \(\varepsilon_{Tt}\) – the reversible part of the strain during unloading, \(\varepsilon_{nr}\) and \(\varepsilon_{tnr}\) are irreversible deformation points.

The unloading lines 23 and 2t3t according to the Engesser –Yasinsky sign are parallel to the initial modular line 1-1t, figures 023 and 2t3t are the hysteresis loop areas equal to the deformation energy losses (according to A.N. Dinnik – energy intensity) for one loading cycle (\(\sigma - \varepsilon\)).
Figure 1. Diagram of the stress – total relative deformations: diagram I-under compression, diagram II-under tension (in square II, the notations have an index $i$).

The equations of the force resistance of type (1), in which the nonlinearity of different partial deformations are different, don’t allow to construct the specified diagram.

It follows from (10) that the coefficient of the reversibility of deformation can be obtained by the following formula:

$$K_r = \frac{\varepsilon_r}{\varepsilon} = \frac{1}{1 + \frac{\nu}{E} \frac{\varepsilon}{m}}$$

(18)

That is, the coefficient of the reversibility decreases with increasing stresses. The coefficient of the irreversibility of deformation can be found as follows:

$$K_{nr} = \frac{\varepsilon_{nr}}{\varepsilon} = \frac{\varepsilon - \varepsilon_r}{\varepsilon} = 1 - K_r$$

(19)

Formula (20) allows you to calculate the area of the hysteresis loop:

$$\Delta \tilde{w} = \tilde{w} - \tilde{w}$$

(20)

where:

$$\tilde{w} = \int_0^\varepsilon \sigma \, d\varepsilon = \left( \frac{E_{np}}{1 + \nu} \right)^{\frac{1}{2}} b \frac{R^{1/b}}{1 + \sigma} \varepsilon^{1/b}$$

(21)

$$\tilde{w} = \int_0^\varepsilon \sigma \, d\varepsilon = \frac{1}{2} \left( \frac{E_{np}}{1 + \nu} \right) \varepsilon^2$$

(22)

5. Discussion
The results of this work present the method for constructing quasi-linear equations of force resistance of concrete and reinforcing steel for use in the calculation of reinforced concrete structures.

6. Conclusions
The procedure for obtaining the quasi–linear equation of the force resistance of concrete and reinforcing steel, as well as the diagram of the stress – total relative deformations is presented.
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