Experimental and numerical study of transverse flux shaking in MgB$_2$ superconductors

J Luzuriaga$^1$, A Badía-Majós$^2$, G Nieva$^1$, J L Giordano$^3$, C López$^4$, A Serquis$^1$ and G Serrano$^1$

$^1$Centro Atómico Bariloche, CNEA, Inst. Balseiro, UNC, Argentina
$^2$Dep. de Física de la Materia Condensada–ICMA, Universidad de Zaragoza–CSIC, Spain
$^3$Departamento de Ciencias de la Ingeniería, Universidad de Talca, Chile
$^4$Dep. Matemáticas, Universidad de Alcalá de Henares, Spain

E-mail: luzuriag@cab.cnea.gov.ar

Abstract. Magnetization measurements in the mixed state of thick strips of carbon nanotube doped MgB$_2$ in crossed fields configurations are reported, together with numerical simulations performed with a geometry equivalent to the sample shape. The samples were subjected to magnetic field components along mutually perpendicular directions, an oscillatory field in one direction and a remanent magnetization in the perpendicular direction. The magnetic response along the oscillatory field and the magnetic relaxation perpendicular to it are observed and simulated using the critical state theory. A remarkable quantitative agreement between the experiment and the theory was obtained.

1. Introduction

The application of a transverse ac field has been found to induce relaxation of the non equilibrium perpendicular magnetization as has been recently reported in high-$T_c$ superconducting samples.$[1, 2]$

From the theoretical point of view, the critical state theory seems to capture the essential physics behind the experimental facts. Of special note are the so-called shaking effects on the flux lines. They have been predicted, either for thin strips within a purely analytical approach,$[3, 4, 5]$ or for arbitrary section strips in numerical studies.$[6]$ Remarkably, the irreversible component $M_{irr}$ perpendicular to the oscillating magnetic field relaxes in the complete absence of thermal activation effects. However, it was also shown$[6]$ that relaxation may stop in some metastable state or eventually lead to a null remanent magnetization depending on the actual experimental conditions.

We study the shaking effect on both components of $M$ for MgB$_2$ samples. In our experiment a magnetic field is ramped with a typical period of around an hour, and the changes are large enough to put the sample in the critical state. Therefore this shaking effect is different from that reported in experiments of the type performed by Pasquini et al.$[7]$ which use smaller and faster flux shaking that can change pinning of the flux lines. The practical absence of anisotropy and the high pinning forces, make MgB$_2$ a good candidate for measuring the connection between the phenomenological critical state approach and flux shaking effects. We have matched the geometry of the measured sample and the simulations, and also paid particular attention to the
magnetic field dependence of the critical current. In this way, all the parameters which enter into the calculation are fixed, and we are therefore able to test the validity of the approximations. We obtain full agreement between the theoretical prediction and experiments at a quantitative level. A single phenomenological relation \( J_{c\perp}(B) \), i.e. the perpendicular (field dependent) critical current density has been used. Such quantity is obtained from a detailed analysis of the one-dimensional \( M(H) \) response [8]. This gives confidence in the applicability of the critical state model[6], and proves that the essential physics is captured by it.

2. Experimental details
For the experiments a polycrystalline \( \text{MgB}_2 \) sample doped with a 10at-% concentration of single walled carbon nanotubes (SWCNT) was used. The \( T_c \sim 36 \text{ K} \) [9] and its dimensions were \( 0.31 \times 0.66 \times 3.0 \text{ mm} \). The longest dimension (3mm) was set parallel to the rotation axis used for establishing the crossed field configuration. The smallest dimension, (0.31 mm), is matched with the y-axis, and the remaining (0.66 mm) with the x-axis.

We have used a custom built probe[10] with a rotating sample holder, inside a Quantum Design SQUID magnetometer which allows rotation around an axis perpendicular to the magnetic field direction. The crossed field configuration was achieved experimentally by placing the sample so that the y-axis was parallel to the magnetic field direction. Then, it was cooled in this position down to 10 K in zero field. Subsequently, a field \( H_y \) was applied and then removed, producing a remanent magnetic moment along the y-axis. With no magnetic field present so as to avoid magnetic induction effects, the sample was rotated 90° placing the x-axis parallel to the field direction. Following this, \( H_x \) was cycled several times.

Taking advantage of the SQUID’s capability for simultaneously recording the parallel and perpendicular components of magnetization with respect to the magnet’s field, we measured the vector \((M_x(t), M_y(t))\) while oscillating the applied field \( H_x \). All the experiments reported were performed at 10 K. We have checked that at this temperatures flux creep effects are negligible, i.e. \( \Delta M/M < 0.01 \) in a typical measurement period of five hours.

3. Experimental and Numerical results
In Fig. 1 we show measurements of \( M_y \) as a function of time compared with the results of the simulations, both in linear and logarithmic scales.

Experimental time scales are small with regard to flux creep and long compared with transients associated with flux changes. The initial field amplitude for the trapped magnetic field was \( \mu_0 H_{y,a} = 0.4 \text{ T} \) while \( H_{x,a} \) has been swept up to maximum values \( \mu_0 H_{x,a} = 0.2; 0.4; 0.6; 0.8 \) and 1.2 T.

Two regimes are visible, as predicted in Ref. [6], depending on the amplitude of the cycling field \( H_{x,a} \). From the graphs, it is clear that the measured \( M_y \) tends to a finite saturation value for low values of the amplitude \( H_{x,a} \) and to zero for values of \( H_{x,a} \) above the penetration field \( H_{px} \). The excellent agreement between the simulations and the measurements can be clearly seen. It should be remarked that the measurement of low values of \( M_y \) is subject to larger experimental errors due to some unshielded induction of the oscillating field along the x-axis. This is apparent in particular in the logarithmic plots, where the small discrepancies are enlarged by the scale.

In Fig. 2 we show the measurements of \( M_x \) which follows the cycle imposed by the field, together with the corresponding numerical results, showing also quantitative agreement. The magnetization along x is cyclic after the first complete cycle in \( H_x \). It should be noticed that \( M_x \) is always in a metastable state during the cycle so that the sample as whole is not in thermodynamic equilibrium, even when the value of \( M_y \) is close to zero (or to the reversible equilibrium value). In experiments such as those of Beidenkopf et al[11] which use perpendicular fields and aim at measuring a thermodynamic state this fact could perhaps be relevant, especially if the dimension \( d \) cannot be neglected.
Figure 1. Plots of $M_y(t)$ against time both in logarithmic and linear scales. Symbols: measurements, full lines: model. Notice the two regimes, for $H < H_{px}$ a finite magnetization remains, for $H > H_{px}$ the magnetization collapses to zero. The curves, from top to bottom correspond to $\mu_0 H_{x,a} = 0.2; 0.4; 0.6; 0.8$ and 1.2 T. The insets show the magnetic flux lines penetrating the sample: (1) just after inducing the remanent state for $\mu_0 H_{y,a} = 0.4$ T and rotating the sample, (2) subsequent to the first half-cycle of $H_x$ for an amplitude $\mu_0 H_{x,a} = 0.4$ T and (3) the same for $\mu_0 H_{x,a} = 0.8$ T.

Figure 2. Magnetization along the x-axis $M_x(t)$ as a function of time. Symbols: measurements, full lines: model. The curves correspond to $\mu_0 H_x = 0.2; 0.4 ; 0.6; 0.8$ and 1.2 T. For higher values of $H_x$ the response of $M_x$ also shows a greater amplitude.

As a final experimental issue, we stress that the quantitative analysis of the sample’s response ($M_x(t), M_y(t)$) has required a careful determination of the critical current density dependence $J_{c\perp}(B)$. In the low field region problems may arise when extracting $J_{c\perp}(B)$ from the width of the hysteresis loops $\Delta M(H)\, [12]$ so we used a thin disk from the same batch of material. From $\Delta M$ obtained by measuring with the magnetic field along the axis of the disk, we have estimated a zero field critical current $J_{c0} = 6.0 \times 10^5 A/cm^2$ which is in agreement with other measurements in this compound.[9] From the same magnetization loop, we have found that for fields below 3 T, one can use the following fit $J_{c\perp}(B) = \frac{J_{c0}}{1+B/B_0}$ with $B_0 = 1.25$ T.

We have also done some preliminary measurements in which the shielding currents run mainly
Figure 3. Comparison of the remanent magnetization in two different configurations of the oscillating field. The remanent magnetization $m_Y$ of 0.4 T is applied in the same way, but the oscillating field is along the sample’s long axis (black open circles) or along the shorter side of the parallelepiped (blue closed circles, the same data as in Fig. 2).

parallel to the oscillating field in order to test the model in this different condition. We used the same sample, but placed in the holder so that the rotation axis is along the $x$ direction of the sample. The remanent magnetization is established in the same way as before, in the $y$ direction of the sample, but now (after rotation) the oscillating field is parallel to $z$, the longer dimension of the sample. Consideration of the geometry shows that most of the shielding current associated with $M_y$ is now parallel to the oscillating field. Results are shown in Fig. 3 (open circles) compared to the earlier measurements (closed circles). The relaxation is slightly slower when the shielding currents run parallel to the oscillating field, as may be expected from the lower Lorentz force in this configuration. Numerical simulations are being performed to compare quantitatively with the measurements.

3.1. Theoretical modeling
With a high accuracy, predictions of the macroscopic response of type-II superconducting samples, i.e. average magnetic moment vs. applied magnetic field, are typically done in terms of the usual critical state theory. In this approach, flux quanta are treated by the phenomenological relation $|J \times B| \leq F_p$ between the volume pinning force and the average values of the current density and magnetic flux density. Thus, a maximum value for the pinning force is equivalent to a critical value in the component of the current density, perpendicular to the local magnetic induction ($J_\perp \leq J_{c,\perp}$). Finally, corresponding to a large dissipation when flux is eventually unpinned, one may treat the response of the sample by the Maxwell equations for the average fields, plus the condition $J_\perp = J_{c,\perp}$ in a magnetoquasistationary approach. In physical terms, the dissipation related time constant may be neglected, and evolution is linked to the external excitation process by Ampère’s and Faraday’s laws.

The application of the above ideas to flux shaking experiments for non-idealized geometry is not simple, and has been done either under ad hoc simplifying hypotheses for extreme geometries or by using numerical minimization techniques. In our case, variational solutions of Faraday’s law when the sample undergoes a magnetic process as shown in Fig. 3 have been obtained under the mutual inductance approach (eq.(2) in Ref. [6]).

In the experiments we measure the decay of a remanent magnetization, while in Ref. [6] the field penetration was calculated so new calculations have been performed relying on the same physical principles and similar numerical methods. Because the field-independence of $J_{c,\perp}$ may not be assumed here, a local field dependence of the critical current $I_{c,\perp}(B_i)$ has been used. Such refinement was needed for a more realistic approach to the experimental facts. As explained above, $J_{c,\perp}(B)$ was derived from a specifically designed experiment.
4. Discussion
The results of our calculations are shown in Figs. 1 and 2, in which the comparison between theory and experiment is displayed. The general trends are those described in Ref. [6], i.e. the remanent magnetization along the y-axis relaxes toward a constant $M_{y,\infty}$, by means of the ac oscillation of $H_x(t)$. The actual value of $M_{y,\infty}$ depends on the amplitude of the applied ac field $H_{x,a}$ (going to zero as $H_{x,a}$ increases). Finally, the relaxation process takes place in a step-like descent, modulated by the ac cycle. However, some peculiarities have been observed in the experiment that may be clearly ascribed to the field dependence of the critical current. To be specific, small bumps are visible in the step-descent of $M_y$ that cannot be obtained from the field independent $J_{c\perp}$ approximation. Additionally, in the present case, the $M_x(t)$ response does not reach a saturation as predicted in the conditions of Ref. [6].

In conclusion, transverse flux experiments in MgB$_2$ samples have revealed a number of features that provide new insights in the problem of applying mutually perpendicular field components to type-II superconductors.

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