Symmetry breaking of quantum droplets in a dual-core trap

Bin Liu\textsuperscript{1}, Hua-Feng Zhang\textsuperscript{2}, Rong-Xuan Zhong\textsuperscript{1}, Xi-Liang Zhang\textsuperscript{1}, Xi-Zhou Qin\textsuperscript{1}, Chunjing Huang\textsuperscript{1}, and Yong-Yao Li\textsuperscript{1,3}\textsuperscript{*} and Boris A. Malomed\textsuperscript{3}

\textsuperscript{1}School of Physics and Optoelectronic Engineering, Foshan University, Foshan 528000, China
\textsuperscript{2}School of Physics and Optoelectronic Engineering, Yangtze University, Jingzhou 434023, China and
\textsuperscript{3}Department of Physical Electronics, School of Electrical Engineering, Faculty of Engineering, and Center for Light-Matter Interaction, Tel Aviv University, Tel Aviv 69978, Israel

We consider the dynamical model of a binary bosonic gas trapped in a symmetric dual-core cigar-shaped potential. The setting is modeled by a system of linearly-coupled one-dimensional Gross-Pitaevskii equations with the cubic self-repulsive terms and quadratic attractive ones, which represent the Lee-Huang-Yang corrections to the mean-field theory in this geometry. The main subject is spontaneous symmetry breaking (SSB) of quantum droplets (QDs), followed by restoration of the symmetry, with respect to the symmetric parallel-coupled trapping cores, following the increase of the QD’s total norm. The SSB transition and inverse symmetry-restoring one form a bifurcation loop, whose shape in concave at small values of the inter-core coupling constant, $\kappa$, and convex at larger $\kappa$. The loop does not exist above a critical value of $\kappa$. At very large values of the norm, QDs do not break their symmetry, featuring a flat-top shape. Some results are obtained in an analytical form, including an exact front solution connecting constant zero and finite values of the wave function. Collisions between moving QDs are considered too, demonstrating a trend to merger into breathers.

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I. INTRODUCTION

Recently, a new type of self-bound quantum liquid states, in the form of three-dimensional (3D) droplets, was created experimentally in dipolar bosonic gases of dysprosium \textsuperscript{1} and erbium \textsuperscript{2}, as well as in mixtures of two atomic states of \textsuperscript{39}K with contact interactions \textsuperscript{3}, following the theoretical proposal elaborated in Refs. \textsuperscript{4} and \textsuperscript{5}. These quantum droplets (QDs) are formed by the balance of attractive forces, which drive the collapse of the quantum gases in the mean-field approximation, and the repulsive force induced by quantum fluctuations around the mean-field states, which is represented by the quartic Lee-Huang-Yang (LHY) corrections \textsuperscript{6} to the respective Gross-Pitaevskii equations (GPEs) with the usual cubic terms. In the dysprosium and erbium gases, the attractive force is generated by the dipole-dipole interactions \textsuperscript{6,7,8,9}, while in the binary mixture it is provided by the inter-component attraction, which can be made stronger than the intra-component repulsion by means of the Feshbach parameter space \textsuperscript{10,11,12}.

QDs are made of extremely dilute quantum fluids \textsuperscript{13}. The droplets may be considered as soliton-like objects, with the unique property of stability in 2D and 3D geometries, where usual nonlinear models give rise to solitons that are subject to strong instabilities \textsuperscript{16,17} (an exception is provided by pairs of GPEs with spin-orbit-coupling (SOC) terms, which predict absolutely stable 2D solitons, i.e., the system’s ground states \textsuperscript{18,19}, and metastable 3D ones \textsuperscript{20}). Accordingly, stable QDs offer potential use in various applications, such as matter-wave interferometry \textsuperscript{21,22} and manipulations of quantum information \textsuperscript{24}. Furthermore, it was recently predicted that 2D QDs (whose effective nonlinearity is different from the above-mentioned quartic form, amounting to cubic terms multiplied by a logarithmic factor \textsuperscript{5}) with embedded vorticity $S = 1, 2, 3, ...$ may be stable too, up to $S = 5$ \textsuperscript{25}. A related result is the stability of 2D QDs of the mixed-mode type (mixing vortical and zero-vorticity constituents), formed by the SOC effect \textsuperscript{26}. Full 3D QDs with embedded vorticity $S = 1$ and 2 have also been predicted to have stability domains in the respective parameter space \textsuperscript{27}.

One of fundamental aspects of the soliton phenomenology is spontaneous symmetry breaking (SSB) of self-trapped modes in symmetric two-component systems. In particular, the SSB of optical solitons was considered in various settings \textsuperscript{28,29,30}. Applications of this effect, such as design of power-switch devices based on soliton light propagation in fibers, were proposed \textsuperscript{31,32}. In Bose-Einstein condensates (BECs), SSB of matter-wave solitons has been considered in many configurations \textsuperscript{33,34,35,36,37,38}, but not, as yet, for QDs. In this work, we address effectively one-dimensional QDs in the binary bosonic gas loaded in a symmetric double-core cigar-shaped potential. Unlike the usual SSB mechanism for matter-wave solitons, which is induced by mean-field interactions, the SSB of QDs in this system is driven by the interplay of the mean-field and LHY terms.

The rest of the paper is structured as follows. The model is introduced in Sec. II, where some analytical results are presented too, such as an exact solution for

\textsuperscript{*}Electronic address: yongyaoli@gmail.com
a front interpolating between zero and a constant wave function. Basic numerical results for the SSB of QDs are reported in Sec. III, which, in addition, includes some approximate analytical results related to the numerical ones. Collisions of two-component QDs are addressed in Sec. IV. The paper is concluded by Sec. V.

II. THE MODEL

The system under the consideration is sketched in Fig. 1. QDs, which are formed in the binary bosonic gas with wave functions $\Phi_\pm$ of the two components, are trapped in the nearly-1D symmetric double-core potential. Assuming, as it is usually done, that the wave-function components of the two species are equal in each core, i.e., $(\Phi_+)_n = (\Phi_-)_n = \Psi_n$, with $n = 1, 2$ being the core’s number, the system of linearly-coupled GPEs, including the LHY terms, are written in the scaled form as [14]:

$$\begin{align*}
&i\partial_t \Psi_1 = -\frac{1}{2}\partial_{xx} \Psi_1 + g |\Psi_1|^2 \Psi_1 - |\Psi_1| \Psi_1 - \kappa \Psi_1,
&i\partial_t \Psi_2 = -\frac{1}{2}\partial_{xx} \Psi_2 + g |\Psi_2|^2 \Psi_2 - |\Psi_2| \Psi_2 - \kappa \Psi_2,
\end{align*}$$

where $g \sim (g_+ + \sqrt{g_+g_-})/\sqrt{g_+g_-} > 0$ is the effective coefficient of the cubic repulsion [14] ($g_+,-,$ and $g_+,-,$ are, respectively, strengths of the self- and cross-interaction of the $\Phi_+$ and $\Phi_-$ components), and $\kappa > 0$ is the hopping rate which couples the parallel cores. By means of additional rescaling, we fix $g \equiv 1$, making $\kappa$ the single control parameter in Eq. (1). The competition of the self-repulsive cubic and attractive quadratic terms in Eq. (1) determines the formation of QDs in this setting [14]. Previously, a dual-core model with the competition of cubic self-attraction and quintic repulsion in each core was introduced in optics [33].

The total norm of the wave function, which is a dynamical invariant in the model, being proportional to the total number of atoms in the dual-core system, is

$$N = N_1 + N_2 \equiv \int_{-\infty}^{+\infty} dx \left( |\Psi_1|^2 + |\Psi_2|^2 \right).$$

Also conserved are the system’s Hamiltonian and total momentum:

$$\begin{align*}
H &= \int_{-\infty}^{+\infty} dx \sum_{n=1,2} \left[ \frac{1}{2} |\partial_x (\Psi_n)|^2 + \frac{1}{2} |\Psi_n|^4 - \frac{2}{3} |\Psi_n|^3 \right] - \kappa (\Psi_1 \Psi_2^* + \text{c.c.).}
\end{align*}$$

$$P = i \int_{-\infty}^{+\infty} dx \sum_{n=1,2} \Psi_n \partial_x (\Psi_n^*),$$

where both * and c.c. stand for the complex conjugation.

Stationary QDs with chemical potential $\mu$ are sought for as solution to Eq. (1) in the form of

$$\{\Psi_1, \Psi_2\} = \{\psi_1, \psi_2\} e^{-i\mu t},$$

with real stationary wave functions $\psi_1$ and $\psi_2$ obeying equations (the prime stands for $d/dx$)

$$\begin{align*}
\mu \psi_1 &= -\frac{1}{2} \psi_1'' + \psi_1^3 - \psi_1^2 - \kappa \psi_2,
\mu \psi_2 &= -\frac{1}{2} \psi_2'' + \psi_2^3 - \psi_2^2 - \kappa \psi_1.
\end{align*}$$

Symmetric soliton solutions of Eq. (5), with $\psi_1 = \psi_2$ and the chemical potential taking values

$$-2/9 < \mu + \kappa < 0,$$

have the known form [14]:

$$\psi_{1,2} \approx \frac{-3(\mu + \kappa)}{1 + \sqrt{1 + (9/2)(\mu + \kappa)} \cosh \left(\sqrt{2(\mu + \kappa)} x\right)} \equiv \psi_{\text{symm}}(x).$$

In the limit of $(\mu + \kappa) \to -0$, they take the bell-shaped form,

$$\psi_{1,2} \approx \frac{-3(\mu + \kappa)}{2 \cosh^2 \left(\sqrt{-(\mu + \kappa)/2} x\right)}.$$

In the opposite limit of

$$\mu + \kappa \to -2/9$$

[see Eq. (6)], the soliton features an extended flat-top shape, with a nearly constant wave function,

$$\psi_{1,2} \approx 2/3,$$

of size

$$L \approx (3/2) \ln \left( (\mu + \kappa + 9/2)^{-1} \right).$$

This flat-top wave form is bounded by two fronts, which are represented by exact solutions of Eq. (5), available precisely at $\mu + \kappa = -2/9$:

$$\psi_{1,2} \approx \frac{2/3}{1 + \exp \left[ \pm \left(2/3\right)(x - x_0)\right]},$$

($x_0$ is an arbitrary shift of the coordinate), each interpolating between $\psi_{1,2} = 0$ and $\psi_{1,2} = 2/3$, cf. Eq. (10). The energy of the front pattern, calculated as per Eq. (2), is

$$H_{\text{front}} = 8/81.$$
A similar exact front solution of the GPE with the cubic-quintic nonlinearity is known too [46].

The SSB point is determined by the condition that the linearization of Eq. (5) around the symmetric solution produces a critical antisymmetric eigenmode with the zero eigenvalue, \( \delta \psi_{1,2} = \pm \delta \psi_0 \), which satisfies the linear equation [43, 47]

\[
(\mu - \kappa) \delta \psi_0 = \left[ -\frac{1}{2} \frac{d^2}{dx^2} + 3\psi_0^2(x) + 2\psi_{symm}(x) \right] \delta \psi_0.
\] (14)

In this work, we obtained numerical asymmetric solutions of Eq. (5) by means of the finite-difference method. As concerns Eq. (14) with \( \kappa = 0 \) has an obvious exact solution, which, however, is a spatially antisymmetric (odd) one,

\[
\delta \psi_0(x; \kappa = 0) = \frac{\partial}{\partial \kappa} [\psi_{symm}(x; \kappa = 0)].
\] (15)

This fact implies that asymmetric solutions cannot branch off from the symmetric ones at \( \kappa = 0 \), keeping the spatial parity.

The linear-stability analysis for the stationary states was performed by adding small perturbations to solution [4]:

\[
\Psi_1(x, t) = \left[ \psi_1 + \varepsilon w_1 e^{iGt} + \varepsilon \psi_1^* e^{-iGt} \right] e^{-i\mu t},
\]

\[
\Psi_2(x, t) = \left[ \psi_2 + \varepsilon w_2 e^{iGt} + \varepsilon \psi_2^* e^{-iGt} \right] e^{-i\mu t},
\] (16)

where \( \varepsilon \) is a real infinitesimal amplitude of the perturbation with eigenfunctions \( \psi_1, \psi_2, \psi_1 \) and \( \psi_2 \). As usual, the existence of an imaginary part in a perturbation eigenfrequency, \( G \), implies an instability. The substitution of expression (1D) in Eq. (4) and subsequent linearization leads to the eigenvalue problem in the matrix form,

\[
\begin{pmatrix}
\hat{L}_1 & -\kappa & 0 \\
-\kappa & \hat{L}_2 & 0 \\
0 & -\kappa & \hat{L}_4
\end{pmatrix}
\begin{pmatrix}
\psi_1 \\
\psi_2 \\
\psi_1
\end{pmatrix} = -\kappa
\begin{pmatrix}
\psi_1 \\
\psi_2 \\
\psi_1
\end{pmatrix},
\] (17)

with operators

\[
\hat{L}_1 = -\frac{1}{2} \frac{\partial^2}{\partial x^2} - \mu + 2|\psi_1|^2 - \frac{3}{2} |\psi_1|,
\]

\[
\hat{L}_2 = -\frac{1}{2} \frac{\partial^2}{\partial x^2} - \mu + 2|\psi_2|^2 - \frac{3}{2} |\psi_2|,
\]

\[
\hat{L}_3 = \psi_1^2 - \frac{\psi_1^2}{2 |\psi_1|},
\]

\[
\hat{L}_4 = \psi_2^2 - \frac{\psi_2^2}{2 |\psi_2|}.
\] (18)

The linear eigenvalue problem based on Eq. (17) can be solved by means of the finite-difference method.

### III. Symmetric and Asymmetric Quantum Droplets

#### A. Generic numerical results

Solutions for symmetric and asymmetric QDs (defined so with respect to the coupled symmetric cores) were produced with the help of the imaginary-time-integration method [48, 49], applied to Eq. (1). Figure 2 displays typical examples of stable and unstable QDs with different values of norm \( N \). Similar to the situation in the single-core model [see Eqs. (2) and (11)], QDs in the present system feature spatial density profiles of two different types: bell-shaped and flat-top ones, for relatively small and large values of \( N \), as can be seen in Figs. 2(a1)-(c1), (e1) and (d1), respectively.

Examples of stable symmetric and asymmetric QDs can be seen, respectively, in Figs. 2(a1), (b1), (d1) and (c1), (e1). The asymmetry is characterized by parameter

\[
\delta = \left| \frac{N_1 - N_2}{N_1 + N_2} \right|.
\] (19)

Accordingly, the SSB is characterized by dependences of \( \delta \) on \( N \) and \( \kappa \).

First, in Fig. 3 we produce \( \delta(N) \) curves for different values of \( \kappa \). Due to the competition between the quadratic self-attraction and cubic repulsion they take the form of bifurcation loops (somewhat similar to those in the cubic-quintic model [33]), which exist at \( \kappa \leq \kappa_{\text{max}} \approx 0.0592 \). With the increase of \( N \), the \( \delta(N) \) curves first show the SSB bifurcation of the supercritical (forward) type, driven by the quadratic self-attraction, which is followed by a reverse symmetry-restoring bifurcation of the subcritical (backward) type, which occurs when the cubic repulsion becomes a dominant nonlinear term. The latter bifurcation lends the loop a concave shape, at \( \kappa < \kappa_0 \approx 0.044 < \kappa_{\text{max}} \). In the interval of \( \kappa_0 < \kappa < \kappa_{\text{max}} \), the symmetry-restoring bifurcation is supercritical, making the loop a convex figure, which shrinks at \( \kappa \to \kappa_{\text{max}} \) and disappears at \( \kappa = \kappa_{\text{max}} \). It is relevant to mention that the bifurcations of the subcritical and supercritical types are tantamount to phase transitions of the first and second kinds, respectively (see, e.g., Refs. [33, 36] and references therein), thus predicting the possibilities of these phase transition in the QDs trapped in the dual-core potential.

The bifurcation loops are chiefly built of the QDs of the sech type, corresponding to relatively small and moderate values of the norm, while the flat-top modes are found for large values of \( N \), at which the SSB is, in most cases, suppressed by the strong self-repulsive nonlinearity.

Results pertaining to the bifurcation loops are collected in Fig. 4. Namely, Fig. 4(a) displays values \( N_D \)
FIG. 2: (Color online) Profiles of $|\psi_1(x)|$ and $|\psi_2(x)|$ components of the QDs, and the exact solution given by Eq. (7), are shown by red solid, black solid, and blue dashed curves, respectively, for different values of the total norm, $N = 0.3, 1, 20,$ and $4$, severally, in panels (a1)-(e1). These examples of self-trapped modes correspond to points a, b, c, and e, which are marked in Figs. 3(a,c). The perturbation eigenvalues for the corresponding symmetric [in (a1,b1,d1)] and asymmetric [in (c1,e1)] QDs, and direct simulations of the perturbed evolution of their $\Psi_1$ component, are displayed, respectively, in panels (a2)-(e2) and (a3)-(e3). Parameters of Eq. (1) are $\kappa = 0.05$ in panels (a3)-(d3) and $\kappa = 0.03$ in panels (e3); the amplitude of small random perturbations in Eq. (16) is $\varepsilon = 0.01$.

and $N_R$ of the total norm at the direct- and reverse-bifurcation points, which merge at $\kappa = \kappa_{\text{max}}$, and Fig. 4(b) plots the largest value of asymmetry $\delta_{\text{max}}(\kappa)$, vs. the coupling constant. Figure 4(a) also includes a plot (the blue line) showing the largest value $N_{\text{max}}$ of $N$ attained in the concave loops, in the case of $\kappa < \kappa_0$. Obviously, $N_{\text{max}} = N_R$ at $\kappa_0 < \kappa < \kappa_{\text{max}}$.

B. Analytical results for the weakly-coupled system

Approximate analytical results can be obtained in the limit of small $\kappa$ and, accordingly, small $N_D$. In this case, Eq. (17) with $\psi_{\text{symm}}$ approximated according to Eq. (8) can be solved exactly, using the well-known result from quantum mechanics, similar to how this was done, in another context, in Refs. [43, 47]:

$$\mu \approx -(13/5)\kappa, \quad N_D \approx 192 (\kappa/5)^{3/2}. \quad (20)$$
Maximum values of the asymmetry parameter, $N$, occur for different values of the linear-coupling parameter, $\kappa$: (a) $\kappa = 0.03$; (b) $\kappa = 0.04$; (c) $\kappa = 0.05$; (d) $\kappa = 0.059$. Red, blue dotted, and black curves represent symmetric stable, symmetric unstable, and asymmetric stable states, respectively.

In Fig. 4(a), the approximate dependence given by the second equation in (20) is plotted by the dotted black line, showing that it fits well to the numerically found dependence at $\kappa \lesssim 0.04$.

Further, both $N_{\text{max}}$ and $N_R$ diverge in the limit of $\kappa \to 0$, when one component in the asymmetric state (e.g., $\psi_2$) is vanishing, its amplitude in the flat-top states (which correspond to large $N$) being

$$\psi_2 \approx 3\kappa,$$

as it follows from Eqs. (5), (11), and (13), while a correction to the amplitude of the larger component is determined by the conservation of the total norm:

$$\psi_1 \approx 2/3 - (27/4)\kappa^2.$$  \hspace{1cm} (22)

At $\kappa \to 0$, the value of $N_{\text{max}}$ can be estimated, taking into regard that the energy of the flat-top symmetric soliton is larger than its single-component counterpart, with the same total norm, by the amount equal to the front’s energy (13), as the single-component state includes only two fronts, unlike four ones in the two-component state, and the energy (13) actually pertains to the double front in the two-component symmetric state. Actually, this is an energy barrier which supports the existence of the asymmetric soliton. On the other hand, the weak linear coupling between the components in the flat-top symmetric state of length $L$ corresponds to the negative energy, which is

$$H_{\text{coupling}} \approx -(8/9)\kappa L,$$  \hspace{1cm} (23)

according to Eq. (2). The asymmetric state ceases to exist, by a jump [like in Fig. 3(a)] under condition $H_{\text{front}} + H_{\text{coupling}} < 0$, i.e., at $L > L_{\text{max}} \approx (9\kappa)^{-1}$. Eventually, the respective prediction for the largest norm, up to which the asymmetric states exist at $\kappa \to 0$, is

$$N_{\text{max}} \approx 2(2/3)^2 L_{\text{max}} \approx 8/(81\kappa).$$  \hspace{1cm} (24)

Further, in the same limit of $\kappa \to 0$, value $N_R$ at the reverse-bifurcation point also diverges, because, as shown above [see Eq. (13)], the SSB cannot take place in the form of an asymmetric branch stemming from a symmetric one at $\kappa = 0$ and some finite value of $N$. A rough estimate for the divergence can be derived by noting that large size $L$ of the symmetric QD is associated with a shift of the eigenvalue in Eq. (13), $\kappa \sim L^{-2}$, hence the respective norm is estimated as

$$N_R \approx 2(3/2)^2 L \sim \kappa^{-1/2},$$  \hspace{1cm} (25)

cf. Eq. (24).

The dotted black line, dash-dotted blue line, and short-dashed red line show the analytical approximations given by Eqs. (20), (23) and (24) for $N_D$, $N_{\text{max}}$, and $N_R$, respectively (the curve representing $N_R$ is drawn with a fitting parameter). It is seen that the analytically predicted values $N_D$ and $N_R$ show good agreement with their numerical counterparts. The prediction given by Eq. (24) is less accurate, in comparison with the numerical findings, but, still, it is qualitatively correct.

IV. COLLISIONS OF TWO-COMPONENT DROPLETS

Once stable solitons are available in the Galilean-invariant system (1), it is relevant to explore collisions between them. In the framework of the usual dual-core
FIG. 5: (Color online) (a) The value of the kick, \(k_c\), which is a boundary between the merger and passage of colliding QDs, launched as per Eq. (26) with \(D = 64\), versus the total norm, \(N\). Typical examples of the density plots of the colliding droplets: (b1) \(N = 0.3, k = 0.03\); (b2) \(N = 0.3, k = 0.065\); (c1) \(N = 1, k = 0.09\); (c2) \(N = 1, k = 0.2\); (d1) \(N = 15, k = 0.03\); (d2) \(N = 15, k = 0.15\). In this figure, \(\kappa = 0.05\) is fixed. The colliding QDs are symmetric in (b1,b2) and asymmetric in (c1,c2).

We simulated the collisions, solving Eq. (1) with initial conditions

\[
\Psi_{1,2}(x, t = 0) = \psi_{1,2}(x + D) e^{ikx} + \psi_{1,2}(x - D) e^{-ikx+\varphi},
\]

where \(\psi_{1,2}\) represent the stationary shape of two-component QDs, \(\pm k\) is a kick, which sets two initial droplets, separated by distance \(2D\), in motion with velocities also equal to \(\pm k\), and \(\varphi\) is the initial phase difference between them.

The simulations demonstrate a trend to inelastic outcomes of the collisions between the solitons in the in-phase configuration, i.e., with \(\varphi = 0\) in Eq. (26). When the QDs are of the sech type, they merge at relatively small values of \(k\), and collide quasi-elastically (passing through each other) at large \(k\). A boundary value, \(k_c\), which separates the inelastic and elastic collisions is displayed in Fig. 5(a), as a function of \(N\), for \(\kappa = 0.05\). In particular, it demonstrates that \(k_c\) is smaller for symmetric sech-shaped QDs than for asymmetric ones of the same sech type. This difference is explained by the fact that the nonlinear interaction between larger components in the asymmetric state is stronger than in the symmetric one, hence larger kinetic energy is necessary to overcome the interaction and let the colliding QDs pass through each other.

Figures 5(b1,c1) and (b2,c2) show typical collision pictures for \(k < k_c\) and \(k > k_c\), respectively. The examples displayed in panels (b1,b2) and (c1,c2) correspond, respectively, to the red dot and black triangle marks in panel (a). These pictures demonstrate that, when the sech-shaped QDs pass through each other at \(k > k_c\), the collisions essentially perturb them. In particular, symmetric QDs emerge from the collision with excited intrinsic oscillations and velocities different from the original ones. In addition, colliding asymmetric QDs of the sech type generate an extra oscillating localized pulse (breather) with zero velocity.

We have also considered cross-symmetric collisions between two asymmetric QDs, i.e., with opposite placements of the larger and smaller components with respect to the two cores, as shown in Fig. 5(a)-(b). In this case, strongly inelastic, quasi-elastic, and completely elastic outcomes are observed too.

As the sech-shaped QDs carry over into flat-top ones with the increase of \(N\), the newly generated quiescent breather grows larger, and eventually absorbs almost all the norm of the colliding QDs, see an example in Fig. 5(d1,d2) (a similar outcome of collisions of single-component QDs was reported in Ref. [14]). Actually, this is a different mechanism of the merger of colliding QDs, cf. panels...
kicks being play, severally, strongly inelastic and quasi-elastic collisions
between in-phase solitons with norms $N$ = 1, the respective kicks being $k_a = 0.05$ and $k_b = 0.20$. Panels (c) and (d) show completely elastic collisions between out-of-phase QDs, i.e., ones with $\varphi = \pi$ in Eq. (26), for $N = 1$, $k = 0.05$ and $N = 15$, $k = 0.03$, respectively. In this figure, $\kappa = 0.05$ is fixed.

(d1) and (d2) in Fig. 5 which correspond to the blue rhombic marks in Fig. 5(a). We find that about 91% of the total norm is absorbed by the quiescent breather in Fig. 5(d2).

Lastly, also similar to the results reported in Ref. [14] for the single-component model, completely elastic collisions occur between the two-component QDs with opposite signs, i.e., $\varphi = \pi$ in Eq. (26), as shown in Fig. c,d, and is observed in other cases too.

V. CONCLUSION

The objective of this work is to study the SSB (spontaneous symmetry breaking) of effectively one-dimensional QDs (quantum droplets) created in the binary bosonic gas loaded in the dual-core trapping potential. The matter-wave dynamics in this system is governed by the linearly-coupled GPEs (Gross-Pitaevskii equations) with the cubic repulsive and quadratic attractive nonlinear terms, the latter ones represented the LHY (Lee-Huang-Yang) correction to the mean-field approximation. QDs in this system feature sech density profiles for smaller values of total norm, $N$, and flat-top profiles for larger $N$. The SSB bifurcation takes place with the increase of $N$, while the QDs keep the sech shape. Further increase of $N$ leads to the restoration of the symmetry via the reverse bifurcation, hence the flat-top QDs, which realize large values of $N$, are symmetric, in most cases. The resulting bifurcations loops are concave and convex in the cases of small and larger values of the inter-core coupling constant, and vanish when it exceeds a critical value. Some results have been obtained in the analytical form – in particular, the exact solution was produced for a front separating zero and finite constant values of the wave function, in the flat-top states.

Collisions between two-component QDs have been considered too. Unless the colliding in-phase QDs move very fast, they tend to demonstrate inelastic interactions, leading to their merger into breathers.

An interesting extension of the present analysis is to perform it for the two-dimensional dual-core system, where the effective nonlinear terms in the GPE is different, $\sim |\Psi|^2 \ln (|\Psi|^2)$. In that case, it will be possible to study the SSB not only in fundamental two-component QDs, but also in ones with embedded vorticity, cf. Refs. [8, 51].

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