I. INTRODUCTION

The amount of squeezing that can be generated is of interest in many potential applications, ranging from high resolution spectroscopy\(^1\) to gravity wave detection\(^2\). In these, the reduced quantum fluctuations (squeezing) in one quadrature of a light field are used to obtain a better signal to noise ratio than that given by the standard quantum limit. In this paper I consider a commonly suggested scheme for generating squeezed light, the use of a parametric amplifier, where a high frequency “pump” field is down converted into half-frequency sub-harmonic fields (often called the “signal” and “idler”). In calculations for an idealised lossless parametric amplifier\(^1, 7, 8, 9\), and for the more realistic non-equilibrium calculations at the threshold of parametric amplification\(^1, 7, 8, 9\), the squeezed quadrature variance in the non-equilibrium calculations at the threshold of parametric amplifier\(^1, 7, 8, 9\), the squeezed quadrature variance in the sub-harmonic field(s) is found to scale as \(N^{-1/2}\), where \(N\) is the number of photons in the pump field. Crouch and Braunstein\(^1\) suggested on intuitive grounds that for the lossless parametric oscillator, the “phase uncertainty” in the pump field is a lower bound on the phase uncertainty in the sub-harmonic field. Since the phase variance in the initial coherent state in the pump is of the order of \(N^{-1}\), they inferred an approximate lower bound for the variance of the squeezed quadrature which varies as \(N^{-1/2}\).

This result of \(N^{-1/2}\) scaling in the variance of the squeezed quadrature implies a \(N^{1/2}\) scaling in the unsqueezed fluctuations. Consequently, the efficiency of energy transfer from the \(N\)-photon pump field to the signal field scales with \(N^{-1/2}\), since the unsqueezed fluctuations contain most of the energy. Kinsler, Fernee and Drummond\(^1\) conjectured that this bound on efficiency is universal when a coherent field and a phase-invariant Hamiltonian combine to form a squeezed vacuum. The reason for this is straightforward. We should not be able to infer more information about the pump phase than its inherent variance of \(N^{-1}\), its standard deviation divided by its amplitude. This is precisely the phase uncertainty of an ideal squeezed vacuum that has been generated with a relative efficiency of \(N^{-1/2}\).

In the applications mentioned above, there are often different strategies which involve utilising increased coherent power levels rather than squeezed light to obtain greater precision \(^3\). If the production of squeezed light itself requires large coherent power inputs, then it may be advantageous to use a strategy involving only an increased coherent power input – assuming the equipment can tolerate the higher powers. As a result, the power input required to produce a known degree of squeezing is a significant factor.

II. PHASE RESOLUTION

Squeezed light is often suggested as a way to obtain an improved phase reference. To see if this is practical, we need to consider the amount of squeezing produced and the efficiency of its production, as well as defining some measure of how good this phase reference is. To this end I define a phenomenological measure of the phase information available from an optical state: the phase resolution \(S\). For example, with a coherent state, for a larger field amplitude \(\alpha\) we get a better defined phase – this is because the uncertainty in the position of the amplitude is always the same (the standard quantum limit of 1), but \(|\alpha|\), the distance from the origin, has increased. For \(|\alpha| \gg 1\), the phase resolution is

\[
S_{\text{coh}} = \frac{\text{distance from origin}}{\text{uncertainty in position}} = \frac{|\alpha|}{1} = N^{1/2}, \quad (2.1)
\]

since there are \(N\) photons in a coherent state with amplitude \(\alpha\). This is the reciprocal of the phase variance. Note that Freyberger and Schleich have considered a variety of definitions of phase in a more comprehensive fashion\(^6\). A quantum mechanical approach such as that introduced by Pegg and Barnett\(^1\) could also be used, but quantum phase effects are unlikely to be

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important in the large photon number limit considered here.

For more general states, such as squeezed states, we need a more general definition. We can get this by defining two quadratures of the optical state. The first quadrature \( \hat{Y} \) is the “distance” quadrature, oriented along the line between the origin and the expectation value of the amplitude. For a coherent state, \( \langle \hat{Y} \rangle = \sqrt{\langle \hat{Y}^+ \hat{Y} \rangle} = n^{1/2} \). The second is the “uncertainty” quadrature \( \hat{X} \), which is oriented orthogonal to \( \hat{Y} \). The variance of \( \hat{X} \) is the usual \( \langle \Delta \hat{X}^2 \rangle = \langle \hat{X}^2 \rangle - \langle \hat{X} \rangle^2 \). Using this approach, we can define a phase resolution of

\[
S = \left[ \frac{\langle \hat{Y}^+ \hat{Y} \rangle}{\Delta \hat{X}^2} \right]^{1/2} ,
\]

which is large for a well-defined phase. The situation is shown pictorially in Fig. 1, which corresponds to a phase space picture derived from the Wigner representation. This definition also works for squeezed vacuums, which have a zero average field but a non-zero intensity that increases with the degree of squeezing. Consequently, the greater the squeezing, the better defined (to within \( \pi \)) is the orientation of the squeezed state.

III. THE PARAMETRIC OSCILLATOR

Optical parametric oscillators are one of the most interesting and well characterized devices in nonlinear quantum optics. Novel discoveries made with them include demonstrations of large amounts of squeezing, significant quantum intensity correlations, and a quadrature correlation measurement that provided the first experimental demonstration of the original EPR paradox. In addition, other work has presented solutions for the at-threshold behaviour of the degenerate and non degenerate systems, when they are far from equilibrium and dominated by quantum mechanical effects. Practical applications include their use as highly efficient and tunable frequency converters.

The degenerate parametric oscillator is an idealised interferometer, which is resonant at two frequencies, the sub-harmonic (or signal) frequency \( \omega_1 \), and the pump field frequency \( \omega_2 = 2\omega_1 \). Both fields are damped due to cavity losses. It is externally driven by a laser field tuned to the pump frequency. Pump photons are down converted to pairs of resonant sub-harmonic photons due to a \( \chi^{(2)} \) nonlinearity present inside the cavity; and the reverse process where a pair of sub-harmonic photons combine to form a pump photon also occurs. The non degenerate parametric oscillator is similar, except that there are two lower frequency fields, called the signal and idler. Pump photon are down converted into a signal–idler pair of photons, and as before, the reverse process also occurs.

From eqn (2.2), the phase resolution for the sub-harmonic (or signal and idler) fields for the degenerate \((i = 2)\) and non degenerate \((i = 3)\) parametric oscillators are denoted \( S_2 \) and \( S_3 \) respectively, are

\[
S_i = \left[ \frac{\langle \hat{Y}_{i}^+ \hat{Y}_{i} \rangle}{\langle \Delta \hat{X}_{i}^2 \rangle} \right]^{1/2} .
\]

If \( \hat{a} \) is the annihilation operator for the sub-harmonic field mode of the degenerate parametric oscillator, the “distance” quadrature is \( \hat{Y}_2 = \hat{a}_1 + \hat{a}_1^\dagger \). The “uncertainty” quadrature \( \hat{X}_2 \) is defined as \( \hat{X}_2 = -i(\hat{a}_2 - \hat{a}_2^\dagger) \). Similarly, in the non degenerate system, if \( \hat{a}_2 \) and \( \hat{a}_3 \) are annihilation operators for the signal and idler modes, the “distance” quadrature is a combination of \( \hat{a}_2 \) and \( \hat{a}_3 \), being \( \hat{Y}_3 = \hat{a}_+ = i(\hat{a}_2^\dagger - \hat{a}_3) \). The uncertainty quadrature \( \hat{X}_3 \) is defined as \( \hat{X}_3 = -i(\hat{a}_2^\dagger - \hat{a}_3^\dagger) \). These definitions are equivalent to those used in [1].

Consider a parametric oscillator with perfect mirrors. At \( t = 0 \) it has a coherent state in the pump mode, and vacuum(s) in the sub-harmonic mode(s). As the system evolves in time, the energy oscillates between the modes; and at a certain time \( t_{eq} \), the squeezing in the sub-harmonic mode(s) is maximised. Figure 3 shows the numerically calculated optimum values of the phase resolution for this case as a function of the number of
FIG. 2: Phase resolution of the squeezed state inside a lossless parametric oscillator. The data is from the simulations presented in [1, 3]. The graphs show the phase resolutions for the squeezed vacuum state produced by a degenerate (×) and non degenerate (□) system.

photons N initially in the pump mode. The phase resolution scales as $N^{1/2}$ for both types of parametric oscillator. This is just as expected because the (squeezed) uncertainty $\langle \Delta \hat{X}_i^2 \rangle$ scales as $N^{-1/2}$, and the (intensity) distance $\langle \hat{Y}_i^\dagger \hat{Y}_i \rangle$ scales as $N^{1/2}$ (and $[N^{1/2}/N^{-1/2}]^{1/2} = N^{1/2}$). Clearly this phase resolution scales with $N$ in exactly same fashion as it would for the coherent state used as the initial condition.

Defining a phase resolution for the non equilibrium case of a driven (and lossy) parametric oscillator involves a little more thought, as we need to measure the output fields, not the internal cavity fields. The output squeezing spectrum $\tilde{V}_{i-out}(\omega)$ is the non equilibrium counterpart to the uncertainty quadrature $\hat{X}_i$. So what is the counterpart to the distance quadrature $\hat{Y}_i$? The two candidates are either the steady state output intensity $\tilde{W}_{i-out}$, and the intensity spectrum $\tilde{W}_{i-out}(\omega)$. The latter choice gives a consistent definition, that of the phase resolution $\tilde{S}_{i-out}(\omega)$ of a particular frequency component. For the degenerate ($i = 2$) and non degenerate ($i = 3$) cases this is

$$
\tilde{S}_{i-out}(\omega) = \left[ \frac{\tilde{W}_{i-out}(\omega)}{\tilde{V}_{i-out}(\omega)} \right]^{1/2},
$$

where the spectra of the (squeezed) uncertainty and (unsqueezed) distance quadratures are

$$
\tilde{V}_{i-out}(\omega) = \frac{4\pi\gamma}{T} \left\langle \hat{X}_i^\dagger(\omega), \hat{X}_i(\omega) : \right\rangle + 1,
$$

$$
\tilde{W}_{i-out}(\omega) = \frac{4\pi\gamma}{T} \left\langle \hat{Y}_i^\dagger(\omega)\hat{Y}_i(\omega) : \right\rangle + 1.
$$

The tilde is used to denote the Fourier transformed quadratures, and the colons(;) denote normal ordering. Figure 3 shows the numerical calculations of $\tilde{S}_{i-out}(\omega)$, worked out near threshold since this is where the best squeezing is obtained. These scale as $N^{1/2}$, exactly as would the same quantity calculated for a simple coherently driven cavity containing $N$ photons.

For a squeezed vacuum, the intensity spectrum $\tilde{W}_{i-out}(\omega)$ in eqn (3.2) could be replaced by the spectral variance in the unsqueezed quadrature. This would give

FIG. 3: Phase resolution of the output from a driven parametric oscillator. The data is from the simulations presented in [2, 3] and are at threshold in order to get the best possible squeezing. The graphs show the phase resolutions for the squeezed vacuum state produced by a degenerate (×) and non degenerate (□) system.
a definition entirely in terms of the quadratures, which might seem better than the one used above. However, the zero frequency part of the unsqueezed spectra \( \tilde{W}_{\text{out}}(0) \) scales not as \( N^{1/2} \), but as \( N^3 \). This quadrature-only definition would lead to a phase resolution that scaled as \( N^{3/4} = [N/N^{-1/2}]^{1/2} \). At first sight this has exceeded that of the coherent driving field which is the input to the system. This apparent gain is due to the slowing down in the critical fluctuations that occur in the unsqueezed quadrature near threshold. These fluctuations vary over a time that is \( N^{1/2} \) longer than those in the squeezed quadrature, and as a result a phase resolution based on this quadrature-only definition has a time averaging built into it. The gain occurs at the expense of ignoring what is going on in the other frequency components of the output light. When an integration over all frequency components is added, the \( N^{1/2} \) scaling would be recovered.

IV. MAKING SQUEEZED COHERENT LIGHT

It is possible to combine the squeezed vacuum with a coherent field to produce a squeezed field with a coherent amplitude by using a beam-splitter or interferometer. Here I consider whether this can improve the phase resolution given the \( N^{-1/2} \) squeezing efficiency limitation.

A schematic diagram of a beam-splitter is given in Fig. 4(a). The two input modes are \( \hat{a}_1 \) and \( \hat{a}_2 \), and these are coupled by the beam-splitter to the output modes \( \hat{b}_1 \) and \( \hat{b}_2 \). The coefficients of transmission and reflection of the beam-splitter are given by \( t_i, r_i \). The equations coupling the two sets of modes are

\[
\begin{align*}
\hat{b}_1 &= \exp(i\Delta)[t_1\hat{a}_1 + \exp(iv)r_2\hat{a}_2] \\
\hat{b}_2 &= \exp(i\Delta)[t_2\hat{a}_2 - \exp(iv)r_1\hat{a}_1].
\end{align*}
\]

(4.1)

Here \( \Delta \) is an overall phase shift, and \( \psi \) is a relative phase shift. The input states to this beam-splitter will be a coherent state \( |\alpha\rangle \) in \( \hat{a}_1 \), and a squeezed vacuum \( |0, se^{i\theta}\rangle \) in \( \hat{a}_2 \), and we want to analyze the squeezed coherent light in the \( \hat{b}_1 \) output mode. The uncertainty quadrature \( \Delta^{2} \) has a variance given by

\[
\langle \Delta^{2} \rangle = 1 + 2r_{2}^{2}\sinh^2(s)[\sinh^2(s) - \cosh^2(s)]
\]

(4.2)

The optimum amount of squeezing occurs for \( 2\Delta + 2\psi + \theta = 0 \), and is

\[
\langle \Delta^{2} \rangle = 1 - r_{2}^{2}[1 - \exp(-2s)]
\]

(4.3)

The distance quadrature \( \hat{Y}_{BS} = \hat{b}_1 \), so

\[
\langle \hat{Y}_{BS}^{\dagger}\hat{Y}_{BS} \rangle = \langle \hat{b}_1^{\dagger}\hat{b}_1 \rangle = t_{1}^{2}|\alpha|^{2} + r_{2}^{2}\sinh^2(s)
\]

(4.4)

This results in an optimum “phase resolution” in the output beam of

\[
S_{BS} = \left[ \frac{\langle \hat{Y}_{BS}^{\dagger}\hat{Y}_{BS} \rangle}{\langle \Delta^{2} \rangle} \right]^{1/2} = \left[ \frac{t_{1}^{2}|\alpha|^{2} + r_{2}^{2}\sinh^2(s)}{1 - r_{2}^{2}[1 - \exp(-2s)]} \right]^{1/2}
\]

(4.5)

This is the best possible phase resolution that can be obtained in the output for a coherent state in one input, and a squeezed vacuum (with squeezing parameter \( s \)) in the other. It depends on both the reflectivity \( r_{2} \) and transmissivity \( t_{1} \) of the beam-splitter, which control the mixing proportions of the squeezed and coherent fields. Note that the corresponding expression that can be derived for the other output mode (\( \hat{b}_2 \)) is equivalent to this, the only difference being in the optimum choice of phases.

An interferometer can also be used to give the squeezed vacuum some coherent amplitude. Figure 4(b) shows the layout schematically. The equations relating the input mode operators \( \hat{a}_1 \) and \( \hat{a}_2 \) to the output mode operators \( \hat{c}_1 \) and \( \hat{c}_2 \) are...
\[
\hat{c}_1 = \exp(i\Phi) \left[-i\exp(-i\psi)\hat{a}_1\sin(\phi/2) + \hat{a}_2\cos(\phi/2)\right], \\
\hat{c}_2 = \exp(i\Phi) \left[\hat{a}_1\cos(\phi/2) - i\exp(-i\psi)\hat{a}_2\sin(\phi/2)\right].
\]

For the same input fields that were used for the beam-splitter example above, the output uncertainty quadra-
tic for a coherent state in one input, and a squeezed state \(\hat{c}_1 + \hat{c}_1^\dagger\) has a variance of

\[
\langle \Delta \hat{X}_I^2 \rangle = 1 - \left[1 - \exp(-2s)\right] \cos^2(\phi/2). \tag{4.7}
\]

The distance quadrature \(\hat{Y}_I = \hat{c}_1\), so

\[
\langle \Delta \hat{X}_I^2 \rangle = \langle \hat{c}_1^\dagger \hat{c}_1 \rangle = |\alpha|^2 \sin^2(\phi/2) + \sinh^2(s)\cos^2(\phi/2). \tag{4.8}
\]

The phase resolution in output mode \(\hat{c}_1\) of the interferometer is

\[
S_{IN}^2 = \left[\frac{\langle \Delta \hat{Y}_I^2 \rangle}{\langle \Delta \hat{X}_I^2 \rangle}\right]^{1/2} \tag{4.9} = \left[\frac{|\alpha|^2 \sin^2(\phi/2) + \sinh^2(s)\cos^2(\phi/2)}{1 - \left[1 - \exp(-2s)\right] \cos^2(\phi/2)}\right]^{1/2}. \tag{4.10}
\]

This is the best possible phase resolution that can be obtained for a coherent state in one input, and a squeezed vacuum (with squeezing parameter \(s\)) entering the other. It depends on the relative phase shift \(\phi\) between the two arms of the interferometer. The relative phase controls the proportions of the mixing between the squeezed and coherent fields.

**V. SQUEEZING EFFICIENCY**

To work out the overall energy efficiency of producing a light field with a better phase resolution by mixing squeezed light with a coherent field, it is necessary to consider how the two fields are created and mixed. A simple theoretical scheme to create squeezed coherent light is shown in Fig. 5. An initial pump coherent field \((1)\) is passed through a parametric oscillator to generate the squeezed vacuum \((2)\). If the initial coherent field has \(N = |\alpha|^2\) photons, the squeezed state contains \(N_{sq} = (N/2)^{1/2}\) photons. For large \(N\), the unconverted part of the coherent field exiting the oscillator is largely unaffected, and this is then down-converted \((3)\) into \(2N\lambda\) signal frequency photons, where \(\lambda\) is an efficiency factor. This down conversion could be achieved using a parametric amplifier. The two fields at the sub-harmonic frequency are now mixed by either the beam-splitter or the interferometer at \((4)\). The inputs to this mixer are a coherent state with \(2N\lambda\) photons, and a squeezed vacuum containing \(N_{sq}\) photons.

![Diagram of squeezing efficiency](image)

FIG. 5: Scheme to create squeezed light with a coherent amplitude from an initial coherent input \((1)\). The input is fed into a parametric oscillator, producing a squeezed vacuum \((2)\). The unconverted part of the coherent input is then down converted using another parametric system \((3)\). The two signal frequency fields are then mixed at \((4)\) by either a beam-splitter or an interferometer. The resulting output is a squeezed state with coherent amplitude \((5)\).

The number of photons in an ideal squeezed vacuum is \(N_{sq} = \sinh^2(s)\), so in the limit of large squeezing \(|s| \gg 0\), and with \(N^{1/2} \gg 1\), we have \(s \approx \frac{1}{4} \ln(N/2)\), which is now substituted into the equations for the phase resolution. For the ideal case of a lossless beam-splitter with \(t_1^2 = 1 - r_2^2\), the phase resolution from eqn \((4.8)\) becomes

\[
S_{BS} = 2N \left[\frac{\lambda (1 - r_2^2) + r_2^2 N^{-1/2} \sqrt{8}}{1 - r_2^2 + r_2^2 N^{-1/2} \sqrt{8}}\right]^{1/2}. \tag{5.1}
\]

For large \(N\) and small \(r_2\) this can be approximated to

\[
S_{BS} \approx N^{1/2} (2\lambda + 4r_2^2)^{1/2} \approx (2N\lambda)^{1/2}, \tag{5.2}
\]

since \(N^{-1/2} \ll 1\). The scaling of \(S_{BS}\) remains the same, although for \(r_2 \approx 1\) the coefficient changes to become \(S_{BS} \approx (2N)^{1/2}\) since this is the limit corresponding to giving the squeezed vacuum no coherent amplitude at all. A similar analysis with the same approximations can be done for the interferometer. Using equation \((4.10)\), the result is

\[
S_{IN} = 2N \left[\frac{\sqrt{8} \lambda \tan^2(\phi/2) + N^{-1/2}}{\sqrt{8} \lambda \tan^2(\phi/2) + N^{-1/2}}\right]^{1/2}. \tag{5.3}
\]

Just as for the beam splitter, for \(|\phi - \pi/2| \ll 1\) the first terms in the numerator and denominator dominate, so that

\[
S_{IN} \approx (2N\lambda)^{1/2}. \tag{5.4}
\]

and for \(\phi \ll 1/N\) the second terms dominate to give \(S_{IN} \approx (2N)^{1/2}\). However, as for the beam-splitter with
FIG. 6: Graphs of the phase "signal to noise" ratio for (a) a beam splitter and (b) an interferometer, as a function of input photon number and (a) reflectivity or (b) phase difference. The coherent squeezed light is generated using the scheme shown in the previous figure, and the coherent down conversion efficiency is 50% ($\lambda = 0.50$).

$r_2 \approx 1$, the $\phi \ll 1/N$ region corresponds to a field with negligible coherent amplitude, so again no improvement in the scaling is achieved.

The results are shown on Fig. 6, which shows the phase resolution defined in eqn (3.1) for a squeezed vacuum with $N^{1/2}$ photons mixed with a coherent state with $N$ photons. Figure 6(a) shows the results from equation (4.5) for a lossless beam-splitter – the $r_2$ axis goes from $r_2 = 0$ (passing only the coherent input), to $r_2 = 1$ (passing only the squeezed input). Figure 6(b) shows the results for the interferometer from equation (4.10). The phase axis $\phi$ varies from $\phi = 0$, when only the squeezed input is passed into the output mode $\hat{b}_1$; through to $\phi = \pi$, when only the coherent input is passed.

VI. CONCLUSION

For both the beam-splitter and the interferometer the scaling of the phase resolution is identical, regardless of the relative mixing proportions ($r_2$ or $\phi$). In these terms, no benefit is obtained by mixing with a coherent signal. The reason is easy to see, since letting part of the coherent field into the output also lets through an equal proportion of its vacuum fluctuations. This means the accuracy with which the phase of the output of the system can be measured scales in the same way as the accuracy with which the input can be measured. This result confirms the conjectures of Kinsler, Fernee, and Drummond suggesting the existence of a bound on "phase information" rates in systems with a phase-invariant Hamiltonian. This is one step beyond a consideration of a constant phase in a system, since it involved non equilibrium processes.

Xiao et al [19] constructed a Mach-Zehnder interferometer using squeezed light at the vacuum port. They reported an improved measurement precision for the in-phase quadrature of the squeezed light, but did not consider any type of phase measurement. However, the efficiency limit discussed here can just as easily be applied to schemes involving quadrature measurements or fringe visibility. For example, a gravity wave interferometer would be set up to measure the small changes in length of the two arms caused by a passing gravity wave. In this case, the relative phases of the light from the two arms would not be not directly measured – but the movement of the interference fringes would be.

Caves [6] has also considered the use of squeezed light in interferometers. He found that partially squeezing the field going into the vacuum port improved the sensitivity of the interferometer by reducing the vacuum fluctuations added to the measurement. However, the large number of photons in a highly squeezed input causes increased uncertainty from the radiation pressure as individual photons reflect off the mirrors. As a result, the trade-off between reduced quantum noise and increased radiation pressure means that the optimum sensitivity is obtained for a medium value of the squeezing, at a lower input power. Caves’s result did not beat the standard quantum limit for the noise in the interferometer, and so the benefit from his squeezed light scheme was that it operated at a lower power. Above I showed that if the total available power is limited, then the maximum amount of squeezing possible is also limited, whereas Caves assumed that any desired amount of squeezing is available at the vacuum input port. Adding the squeezing efficiency constraint to the Caves interferometer puts an even lower limit on the power reduction possible with such a squeezed light scheme.

Subsequent schemes have shown that the standard quantum limit can be beaten by either careful choice of the phase of the squeezed light [20, 21, 22, 23], or by putting a non linear Kerr medium in the arms of the interferometer [24]. Taking into account the efficiency consideration would not interfere with the lower noise in these schemes, but it does put a limit on the squeezing available for a given maximum power. This will in turn limit by how much the standard quantum limit can be beaten. For example, Pace et al [25] constructed a model that included mirror noise by describing the mir-
rors as harmonic oscillators coupled to the light field. They found that by optimising the phase of the squeezed light the standard quantum limit could be beaten. The optimum intensity sensitivity was roughly proportional to $N^{-1}$, the reciprocal of the number of photons in the squeezed vacuum. However, as the squeezing is increased, the detector sensitivity was found to be limited by the noise caused by the mirrors so that beyond a certain point more squeezing had no effect. This type of result had also been obtained by Jaekel and Reynaud [22], and Luis and Sanchez-Soto [23].

If the mirror noise due to the radiation pressure is arbitrarily ignored, and the optimum input power is used, Pace et al's [25] results show that the minimum detectable gravitational wave amplitude $h_{\text{min}}$ is proportional to $e^{-s}$. If $P_{\text{opt}}$ is the optimum power, which is the same as that for a non squeezed light scheme, and $s$ is the squeezing parameter of the squeezed vacuum; then given the efficiency constraint of $N^{-1/2}$ for the squeezed vacuum generated by the parametric oscillator, this means that $h_{\text{min}}$ is proportional to $P_{\text{opt}}^{1/4}$. However, it is still possible to use squeezing to compensate for a lower, and no longer optimum, power. This is because the decrease in sensitivity due to the characteristics of the light does not matter if the measurements are still limited by the background noise caused by the mirrors.

In summary, these results suggest that when using squeezed light generated with passive time-independent parametric devices, the chief advantage is not one of overall efficiency or improved sensitivity. Instead, the advantage is one of lower power levels. Given a fixed incident coherent pump power, the limits to the phase resolution available from passive sources of squeezed radiation ensure that we cannot get improved information capacity relative to the pump itself, nor is there any improvement in phase resolution at a given pump photon number.

**Acknowledgments**

The author would like to thank P.D. Drummond and the Department of Physics at the University of Queensland. The paper has also gone through revisions while I worked at the University of Sheffield and the University of Leeds, and an earlier version was at one stage submitted to Optics Communications but never published there.

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