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X(3872) diagnostics with decays to $D\bar{D}\gamma$

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Abstract

It is pointed out that in the decays $X(3872) \to D\bar{D}\gamma$ the possible ‘molecular’
component of the $X(3872)$ should give rise essentially only to a $D^0\bar{D}^0\gamma$ final state
with a predictable spectrum, and should yield virtually no contribution to decays into
$D^+D^-\gamma$. The latter final state with charged $D$ mesons should however arise from
the radiative decays of the short-distance core of the $X(3872)$ through the transition
$X(3872) \to \psi(3770)\gamma$. Thus an observation of the radiative decays of $X(3872)$ to pairs
of neutral and charged $D$ mesons would provide an insight into the internal dynamics
of the $X(3872)$ resonance.
1 Introduction

The narrow resonance \(X(3872)\) observed through its decay channel \(X \to \pi^+ \pi^- J/\psi\) in the \(B \to X K\) decays\(^{[1, 2]}\) and in inclusive production in \(p\bar{p}\) annihilation\(^{[3, 4]}\) attracts a great theoretical and experimental interest. The peculiarity of this resonance is that its mass is within approximately \(0.6 \pm 1.0\) MeV from the \(D^0 \bar{D}^{*0}\) threshold, which strongly suggests that its internal composition can be significantly contributed by a ‘molecular’ state made of the charmed mesons. An existence of such states of heavy hadrons was suggested\(^{[5]}\) and discussed\(^{[6, 7, 8]}\) long ago, and this idea was revived by the observation of \(X(3872)\)\(^{[9, 10, 11, 12]}\). The interpretation of \(X(3872)\) as largely a ‘molecular’ state is further boosted by the observation\(^{[13]}\) of the decay \(X \to \pi^+ \pi^- \pi^0 J/\psi\), with a rate similar to that of the discovery mode \(X \to \pi^+ \pi^- J/\psi\). The co-existence of these two decay modes implies that the isospin is badly broken in the resonance \(X(3872)\), which would be impossible if it were just another charmonium state. Furthermore, the observation\(^{[13]}\) of the decay \(X \to \gamma J/\psi\) implies positive C parity of \(X\), and a detailed study\(^{[14]}\) of the decay \(X(3872) \to \pi^+ \pi^- J/\psi\) most strongly favors the identification of its quantum numbers as \(J^{PC} = 1^{++}\). Such quantum numbers as well as a substantial isospin violation agree well\(^{[18]}\) with that a molecular C-even S-wave state of neutral charmed mesons: \(D^0 \bar{D}^{*0} + D^{*0} \bar{D}^0\) makes up a significant part of the wave function of the \(X(3872)\) resonance.

There is however no reason to expect that there is only the molecular component in the wave function that determines all of the properties of the \(X(3872)\) boson. Rather one should consider the wave function in terms of a general Fock decomposition:

\[
\psi_X = a_0 \psi_0 + \sum_i a_i \psi_i ,
\]

where \(\psi_0\) is the state of the neutral \(D\) mesons \((D^0 \bar{D}^{*0} + D^{*0} \bar{D}^0)/\sqrt{2}\), while \(\psi_i\) refer to ‘other’ hadronic states. Due to the extreme proximity of the mass of \(X\) to the \(D^0 \bar{D}^{*0}\) threshold, the \(\psi_0\) part should be dominant at long distances. Indeed, assuming that the mass of \(X\) is below the threshold by the binding energy \(w\): \(m_{D^0} + m_{D^{*0}} - M_X = w\), the spatial extent of the \(\psi_0\) is determined as \((m_D w)^{-1/2} \approx 5\) fm \((1\) MeV/\(w)^{1/2}\), and \(\psi_0\) thus describes the ‘peripheral’ part of the wave function, in fact beyond the range of strong interaction. On the other hand, the ‘other’ states in the sum in the Fock decomposition\(^{[11]}\) are localized at shorter distances and constitute the ‘core’ of the \(X(3872)\) wave function. In other terms one may think of this picture as that of a mixing in \(X(3872)\) of the molecular component \(D^0 \bar{D}^{*0} + D^{*0} \bar{D}^0\) with
‘other’ states, such as e.g. a ‘pure’ $c\bar{c}$ charmonium, which then has to be in a $^3P_1$ state, also favored by the heavy quark spin selection rule\cite{18}.

Since the internal composition of the $X(3872)$ can be quite different at different distances, one or the other part of the Fock decomposition (1) may be important in specific processes. It appears that the pionic transitions from $X(3872)$ to $J/\psi$ are determined by a long distance dynamics, where the $D^0\bar{D}^\ast + D^\ast 0 \bar{D}^0$ component dominates, so that the isospin states are mixed, and the $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$ transitions have approximately the same strength. The production of $X$ however is determined by short distances, and proceeds through the core component\cite{18}, which is approximately an isospin singlet, as evidenced\cite{15} by a comparable relative rate of the decays $B^+ \rightarrow X K^+$ and $B^0 \rightarrow X K^0$, while contribution of the molecular $D^0 D^\ast 0 + D^\ast 0 \bar{D}^0$ would correspond to a strong suppression\cite{16, 17} of the $B^0 \rightarrow X K^0$ decay in comparison with $B^+ \rightarrow X K^+$.

The purpose of the present paper is to point out that both the peripheral molecular component of the $X(3872)$ and, to some extent, its core can be studied in the decays $X(3872) \rightarrow D\bar{D}\gamma$. It has been previously noted\cite{11} that the rates of the decays $X \rightarrow D^0 \bar{D}^0\pi^0$ and $X \rightarrow D^0\bar{D}^0\gamma$ are sensitive to the coordinate wave function of the molecular component due to the interference between the decays of $D^0$ and $D^\ast 0$ mesons in a state of definite $C$ parity. Here the decays of $X$ into $D^0\bar{D}^0\gamma$ as well as into $D^+D^-\gamma$ are considered in more detail, including the effects of re-scattering $D^0\bar{D}^0 \leftrightarrow D^+D^-$ and of the charmonium resonance $\psi(3770)$ strongly coupled to the $D\bar{D}$ channel. As a result it will be argued here that at the photon energy $\omega$ near that in the decay $D^\ast 0 \rightarrow D^0\gamma$, $\omega_0 \approx 137 \text{ MeV}$, the contribution of the molecular component of $X(3872)$ should dominate with the radiative decay going almost exclusively into the channel with neutral mesons: $D^0\bar{D}^0\gamma$, and the underlying process being the decays $D^\ast 0 \rightarrow D^0\gamma$ and $\bar{D}^\ast 0 \rightarrow \bar{D}^0\gamma$. At a somewhat lower photon energy $\omega \approx 100 \text{ MeV}$ the final state with charged mesons, $D^+D^-\gamma$, should also appear due to the underlying process of radiative transition from a charmonium ‘core’ component of $X(3872)$ to $\psi(3770)\gamma$. Thus a measurement of the photon spectra in the radiative decays of $X$ into $D^0\bar{D}^0\gamma$ and $D^+D^-\gamma$ would provide an information on both the peripheral molecular component and the ‘core’ of the $X(3872)$.

The rest of the paper is organized as follows. In Sec.2 the wave function of the $D\bar{D}^\ast + D^\ast \bar{D}$ mesons within the $X(3872)$ resonance is discussed as well as the possible states making up the ‘core’. In Sec.3 the contribution of the peripheral meson component to the decay $X \rightarrow D^0\bar{D}^0\gamma$ is calculated, and in Sec.4 it is argued that the peripheral contribution to the
decay \( X \rightarrow D^+D^-\gamma \) is very small, both due to the direct decays \( D^{*\pm} \rightarrow D^{\pm}\gamma \) as well as due to the rescattering \( D^0\bar{D}^0 \rightarrow D^+D^- \). In Sec.5 the radiative transition \( X(3872) \rightarrow \psi(3770)\gamma \) as proceeding due to a charmonium ‘core’ of the \( X \) resonance is considered, which results in the decay \( X \rightarrow D\bar{D}\gamma \) with both the neutral and the charged mesons. Finally, Sec.6 contains a discussion and concluding remarks.

2 States in the Fock decomposition of \( X(3872) \)

The extreme proximity of the mass of \( X(3872) \) to the \( D^{*0}\bar{D}^{*0} \) guarantees that the mesons in the \( D^0\bar{D}^{*0} + D^{*0}\bar{D}^0 \) component move freely beyond the range of the strong interaction, where their wave function in the coordinate space is given by

\[
\psi_n(r) = c\frac{\exp(-\kappa_n r)}{r}, \tag{2}
\]

where \( \kappa_n \) is determined by the binding energy \( w \) and the reduced mass \( m_r \approx 966 \text{ MeV} \) in the \( D^0\bar{D}^{*0} \) system as \( \kappa_n = \sqrt{2 m_r w} \). The normalization coefficient \( c \) determines the statistical weight of the \( D^0\bar{D}^{*0} + D^{*0}\bar{D}^0 \) component in \( X(3872) \), and its definition is correlated with that of the coefficient \( a_0 \) in the Fock decomposition \( (1) \). We resolve this ambiguity in the definition by requiring that the coordinate wave function of the neutral meson pair be normalized to one, so that the statistical weight of the state \( (D^0\bar{D}^{*0} + D^{*0}\bar{D}^0)/\sqrt{2} \) is given as \( |a_0|^2 \). If the wave function of the form \( (2) \) is used down to \( r = 0 \), this requirement corresponds to \( c = \sqrt{\kappa_n/(2\pi)} \).

The dominance of the \( D^0\bar{D}^{*0} + D^{*0}\bar{D}^0 \) at long distances translates into a substantial isospin violation in the processes determined by the ‘peripheral’ dynamics, examples of which are apparently the observed decays \( X \rightarrow \pi^+\pi^- J/\psi \) and \( X \rightarrow \pi^+\pi^-\pi^0 J/\psi \). It is quite likely however that this isospin-breaking behavior is only a result of the ‘accidentally’ large mass difference \( \delta \approx 8 \text{ MeV} \) between \( D^+D^- \) and \( D^{*0}\bar{D}^{*0} \). Therefore it is natural to expect that at shorter distances within the range of the strong interaction the isospin symmetry is restored and at those distances the wave function of \( X(3872) \) is dominated by \( I = 0 \). In this picture the wave function of a \( D^+D^- + D^-D^+ \) state within the region beyond the range of the strong interaction can be found from eq.\( (2) \) by requiring that at short distances the pairs of charged and neutral mesons combine into an \( I = 0 \) state\(^1\), so that the wave function of the

\(^1\)The effects of the Coulomb interaction between the charged mesons are neglected here.
charged meson pair has the form
\[ \phi_c(r) = c \frac{\exp(-\kappa_c r)}{r}, \]
where \( \kappa_c = \sqrt{2m_r (\delta + w)} \approx 125 \text{ MeV} \). It should be noticed that both the neutral (eq.2) and the charged (eq.3) meson wave function have the same normalization factor \( c \) (determined by \( \kappa_n \)) and differ only in the exponential power. This simple picture allows one to estimate the relative statistical weight of the charged and neutral \( D \) meson components in the \( X(3872) \):
\[ \lambda \equiv \frac{| \langle X | D^+ D^{*-} + D^- D^{*-} \rangle |^2}{| \langle X | D^0 \bar{D}^{*0} + D^{*0} \bar{D}^0 \rangle |^2} = \frac{\kappa_n}{\kappa_c}. \]

Clearly, the wave functions in eq.(2) and eq.(3) cannot be applied at short distances in the region of strong interaction, where the mesons overlap with each other and cannot be considered as individual particles. In order to take into account this behavior an ‘ultraviolet’ cutoff should be introduced. One widely used method for introducing such cutoff is to consider the meson wave functions only down to a finite distance \( r_0 \), at which distance the boundary condition of the state being that with \( I = 0 \) is imposed (a discussion of a similar situation can be found e.g. in Ref.\[19\]). An alternative, somewhat more gradual cutoff, described by parameter \( \Lambda \), can be introduced\[17\] by subtracting from the wave functions (2) and (3) an expression \( c e^{-\Lambda r}/r \). It should be noticed, that such regularization also results in a modification of the normalization coefficient \( c \), which for the gradual cutoff takes the form
\[ c = \sqrt{\frac{\kappa_n}{2\pi}} \frac{\sqrt{\Lambda (\Lambda + \kappa_n)}}{\Lambda - \kappa_n}. \]
One can also readily see, that an introduction of any such cutoff eliminates relatively more of the charged meson wave function than of the neutral one, thus reducing the estimate of the relative statistical weight as compared to that in eq.(4), so that eq.(4) gives in fact the upper bound for the ratio.

The structure of the ‘core’ of \( X(3872) \) in the region of strong interaction can be quite complicated. As mentioned above, it is natural to expect that the states in the ‘core’ are dominantly isotopic scalars, which however still leaves the possibility that the essential states contain light-quark pairs and/or gluons in addition to the \( c\bar{c} \) charm quark pair. However the simplest configuration with just a charmonium \( ^3P_1 \) state is also allowed by the quantum numbers, and in what follows only the contribution of the charmonium ‘core’ to the discussed...
properties is considered. In other words, the states in the Fock decomposition taken into account in this paper are the charm meson pairs at the ‘peripheral’ distances and a $J^{PC} = 1^{++}$ charmonium $c\bar{c}$ pair in the ‘core’. It can be mentioned that such simplified structure corresponds to the approach based on channel mixing between the charmed meson pairs and the states of charmonium.

3 Peripheral contribution to the decay $X(3872) \rightarrow D\bar{D}\gamma$

The dominant peripheral contribution to the decay $X(3872) \rightarrow D^0\bar{D}^0\gamma$ arises from the radiative decays of individual vector mesons: $D^{*0} \rightarrow D^0\gamma$ and $D^{*0} \rightarrow D^0\gamma$, while the peripheral contribution to the decay $X(3872) \rightarrow D^+D^-\gamma$ originates dominantly from the decays $D^{*\pm} \rightarrow D^{\pm}\gamma$. As will be argued, the rate for the latter process should be expected considerably smaller than for the radiative decay with neutral mesons. It will be also argued that the effects of a rescattering $D^0\bar{D}^0 \leftrightarrow D^+D^-$ should be very small in the kinematical region, where the peripheral contribution dominates. In this section the rescattering effects are completely neglected.

The amplitude of the radiative decay can be written as

$$A(D^* \rightarrow D\gamma) = \mu \epsilon_{ij} \epsilon_i k_j a_k,$$

where $\vec{a}$ and $\vec{\epsilon}$ are the polarization amplitudes for the photon and the vector meson, $\vec{k}$ is the momentum of the photon, and the non-relativistic normalization for the heavy-meson states is assumed, so that the decay rate is expressed in terms of the parameter $\mu$ as

$$\Gamma(D^* \rightarrow D\gamma) = \frac{|\mu|^2 \omega_0^3}{3\pi}.$$

The widths of the decays $D^{*0} \rightarrow D^0\gamma$ and $D^{*\pm} \rightarrow D^{\pm}\gamma$ can be readily deduced from the data: $\Gamma(D^{*\pm} \rightarrow D^{\pm}\gamma) = 1.5 \pm 0.5$ KeV, $\Gamma(D^{*0} \rightarrow D^0\gamma) = 26 \pm 6$ KeV. One can see from these numbers that the transition magnetic moment $\mu$ for the charged meson decay is substantially smaller than that for the neutral meson radiative decay: $|\mu_c|^2 \ll |\mu_n|^2$.

If the final-state interaction between the pseudoscalar mesons is neglected, the amplitude of the decay $X(3872) \rightarrow D\bar{D}\gamma$ due to the peripheral $D\bar{D}^* + D^*\bar{D}$ component can be written as

$$A_0(X \rightarrow D\bar{D}\gamma) = \frac{\mu a_0}{\sqrt{2}} \epsilon_{ijk} \epsilon_i k_j a_k \left[ \phi \left( \frac{\vec{k}}{2} + \vec{p} \right) - \phi \left( \frac{\vec{k}}{2} - \vec{p} \right) \right].$$
Here $a_0$ is the weight amplitude in eq. (11) for the ‘molecular’ state in $X(3872)$, $\vec{p} = (\vec{p}_D - \vec{p}_{\bar{D}})/2$ is the momentum of the $D$ meson in the c.m. frame of the $D\bar{D}$ system, and $\phi(q)$ is the wave function of the peripheral meson component in the momentum space. Clearly, one should use the neutral-meson wave function $\phi_n$ and the corresponding transition magnetic moment $\mu_n$ for the decay $X \to D^0\bar{D}^0\gamma$ and the charged-meson one wave function $\phi_c$ and $\mu_c$ for the decay $X \to D^+D^-\gamma$. The two terms in the square braces in eq. (8) correspond respectively to the processes $D^* \to D\gamma$ and $\bar{D}^* \to \bar{D}\gamma$, and the relative minus sign is due to the opposite C parity of the photon and of the initial state of the mesons.

The differential decay rate found from eq. (8) can be written in terms of the radiative width of $D^*$ meson (7) as

$$
d\Gamma(X \to D\bar{D}\gamma) = \Gamma(D^* \to D\gamma) a_0^2 \left( \frac{\omega}{\omega_0} \right)^3 \left[ \phi\left(\frac{\vec{k}}{2} + \vec{p}\right) - \phi\left(\frac{\vec{k}}{2} - \vec{p}\right) \right]^2 \frac{d^3p}{(2\pi)^3},
$$

where $\omega_0$ stands for the photon energy in the corresponding decay $D^* \to D\gamma$ ($\omega_0 = 137$ MeV in the decay $D^{*0} \to D^0\gamma$ and $\omega_0 = 136$ MeV in the radiative decay of $D^{*\pm}$), the photon energy $\omega$ and the momentum $\vec{p}$ in eq. (9) are related by the energy conservation relation

$$
\omega + \frac{\omega^2}{4m_D} + \frac{\vec{p}^2}{m_D} = \Delta
$$

with $\Delta = M_X - 2m_D$.

The photon spectrum given by eq. (9) will be further discussed in some detail. It is clear on general grounds that the dominant contribution to the total rate comes from the region where $\omega$ is very close to $\omega_0$. Such values of $\omega$ are kinematically allowed for the $X \to D^0\bar{D}^0\gamma$ decay and are forbidden for the decay $X \to D^+D^-\gamma$. Thus for the former decay the total rate can be approximated by setting $\omega = \omega_0$ in eq. (9), which results in

$$
\Gamma(X \to D^0D^0\gamma) = \Gamma(D^{*0} \to D^0\gamma) a_0^2 \left[ 1 - \int \phi_n\left(\frac{\vec{k}}{2} + \vec{p}\right) \phi_n\left(\frac{\vec{k}}{2} - \vec{p}\right) \frac{d^3p}{(2\pi)^3} \right]
$$

$$
= \Gamma(D^{*0} \to D^0\gamma) a_0^2 \left[ 1 - 4\pi \int \phi^2_n(r) \frac{\sin kr}{kr} r^2 dr \right].
$$

In this formula the normalization condition and the absence of an angular dependence of an $S$ state wave function are explicitly taken into account.

With the unregularized free-motion wave functions (2) and (3) one readily finds the contribution of the peripheral meson component to the total rate of the discussed decays
\[ \Gamma(X \to D^0\bar{D}^0\gamma) = \Gamma(D^{*0} \to D^0\gamma) a_0^2 \left[ 1 - \frac{2\kappa_n}{\omega_0} \arctan \frac{\omega_0}{2\kappa_n} \right]. \quad (12) \]

The regularization at short distances in fact enhances the total rate of the dominant decay \( X \to D^0\bar{D}^0\gamma \) when the rate is expressed in terms of \( a_0^2 \). Indeed, the integral weight in the negative term in the interference factor in eq. (11) is the largest at short distances. Thus, eliminating by regularization a short-distance part of the meson wave function reduces this negative term and enhances the rate. In particular for the gradual cutoff regularization the expression for the total decay rate takes the form

\[
\Gamma(X \to D^0\bar{D}^0\gamma) = \Gamma(D^{*0} \to D^0\gamma) a_0^2 \times 
\left[ 1 - \frac{2\kappa_n}{\omega_0} \frac{\Lambda(\Lambda + \kappa_n)}{(\Lambda - \kappa_n)^2} \left( \arctan \frac{\omega_0}{2\kappa_n} + \arctan \frac{\omega_0}{2\Lambda} - 2 \arctan \frac{\omega_0}{\Lambda + \kappa_n} \right) \right]. \quad (13)
\]

At \( \omega_0 = 137 \text{ MeV} \) and \( \kappa_n \approx 44 \text{ MeV} \) (corresponding to the binding energy \( w = 1 \text{ MeV} \)) the numerical value of the expression in the square braces, the interference factor, varies between 0.36 at \( \Lambda \to \infty \) and 0.61 at \( \Lambda = 200 \text{ MeV} \).

\[
\frac{d\Gamma}{d\omega}
\]

Figure 1: The photon spectrum in the decay \( X(3872) \to D^0\bar{D}^0\gamma \) for \( \kappa_n = 44 \text{ MeV} \) at \( \Lambda = 200 \text{ MeV} \) (dashed) and at \( \Lambda \to \infty \) (solid). The vertical scale is in arbitrary units.

\[\text{\footnotesize \( ^2 \)This equation corrects the corresponding formula of Ref. [11], which erroneously contains an extra factor of two.}\]
The photon spectrum in the decay $X \rightarrow D^0 \bar{D}^0 \gamma$ can be found from eq.(9) and eq.(10) using the regularized form of the wave function in the momentum space,

$$\phi_c(\vec{q}) = 4\pi c \left( \frac{1}{q^2 + \kappa_n^2} - \frac{1}{q^2 + \Lambda^2} \right).$$

(14)

The final expression is found after approximating the solution to eq.(10) as $p \approx \sqrt{m_D(\omega_1 - \omega)}$ with $\omega_1 \approx 140$ MeV being the maximal kinematically allowed photon energy in the decay and after performing the integration over the angle between the photon momentum and $\vec{p}$. At finite regularization parameter $\Lambda$ the resulting expression is quite lengthy. However the effect of a finite $\Lambda$ in the essential part of the spectrum, i.e. near $\omega = \omega_0$ to a very good accuracy reduces to an overall rescaling, according to the previously mentioned enhancement of the decay rate for a regularized wave function. This behavior is illustrated in Fig.1.

In the limit of no regularization, i.e. at $\Lambda \rightarrow \infty$, the expression for the spectrum takes a more concise form:

$$\frac{d\Gamma}{d\omega}(X \rightarrow D^0 \bar{D}^0 \gamma) = \Gamma(D^{*0} \rightarrow D^0 \gamma) a_0^2 \omega_0^3 \frac{2 m_D \kappa_n}{\pi} p f(p^2 + \omega^2/4 + \kappa_n^2, p \omega),$$

(15)

where

$$f(x, y) = \frac{1}{x^2 - y^2} - \frac{1}{xy} \arctanh \frac{y}{x},$$

(16)

and the momentum $p$ is related to $\omega$ by the energy conservation relation (10). These formulas allow to illustrate the dependence of the spectrum on the mass gap $w$ between the $X(3872)$ and the $D^0 \bar{D}^{*0}$ threshold, as shown in Fig.2 for two representative values of $w$: $w \approx 1$ MeV ($\kappa_n = 44$) and $w \approx 0.3$ MeV ($\kappa_n = 24$ MeV). One can readily see the expected behavior: with decreasing $w$ the spectrum becomes more narrowly peaked around $\omega_0$ and is enhanced due to diminishing destructive interference.

4 Peripheral contribution to the decay $X \rightarrow D^+ D^- \gamma$ and the effects of final-state interaction

For the decay $X \rightarrow D^+ D^- \gamma$ the maximal allowed energy of the photon is $\omega_2 \approx 131$ MeV, which is below the photon energy $\omega_0 \approx 136$ MeV in the decay $D^{*\pm} \rightarrow D^{\pm} \gamma$, so that the peak in the spectrum, corresponding to a free-meson decay is kinematically inaccessible. Thus
the peripheral contribution to this decay has to be calculated by integrating the differential decay rate. The latter rate can be estimated from a formula similar to eq. (15):

\[
\frac{d\Gamma}{d\omega}(X \rightarrow D^+ D^- \gamma) = \Gamma(D^{*\pm} \rightarrow D^{\pm} \gamma) a_0^2 \lambda \frac{\omega^3}{\omega_0^3} \frac{2 m_D \kappa_c}{\pi} p f(p^2 + \omega^2/4 + \kappa_c^2, p\omega),
\]

and the total rate found from this expression is very small, corresponding to the bound (on the peripheral contribution to the rate) at all values of \(\omega\) below 1 MeV:

\[
\left| \frac{\Gamma(X \rightarrow D^+ D^- \gamma)}{\Gamma(X \rightarrow D^0 D^{*0} \gamma)} \right|_{\text{peripheral}} < 10^{-3}. \tag{18}
\]

Such strong suppression of the decay \(X \rightarrow D^+ D^- \gamma\) is a result of three suppression factors: the small rate of \(D^{*\pm} \rightarrow D^{\pm} \gamma\), the small statistical weight of the charged meson pair in the \(X(3872)\) relative to that of the neutral pair (eq. (15)), and the stronger destructive interference in emission of photon by the charged meson pair in the \(X(3872)\), the latter suppression factor being combined with the discussed kinematical restriction on the photon energy, which further suppresses the decay rate.

The effects of the rescattering of the \(D\) mesons in the final state were completely neglected in the previous estimates. It can be argued that these effects could give rise only to a very
small rate of the decay $X \to D^+ D^- \gamma$ and have very little impact on the photon spectrum in the decay $X \to D^0 \bar{D}^0 \gamma$ arising from the peripheral component of the $X(3872)$.

Indeed, the $D$ meson pair produced in the discussed decay is $C$ odd and therefore the relative angular momentum of the meson is also odd. It can be noted that the decay rate is dominated by the kinematical region, where the relative momentum $\vec{p}$ of the $D$ mesons is small, so that the main contribution to the rate comes from the states where the $D$ meson pair is in the lowest partial wave with odd angular momentum, i.e. in the $P$ wave. One can readily verify that in the previous estimates of the peripheral contribution to the rate of the decay $X \to D^0 \bar{D}^0 \gamma$ at $w < 1$ MeV more than 93% of the rate is associated with the production of the meson pair in the $P$ wave. In the kinematical region of this decay with the photon energy $\omega$ above the maximal energy $\omega_2$ for the decay $X \to D^+ D^- \gamma$ only the elastic scattering between $D^0$ and $\bar{D}^0$ is possible, so that only the phases of the partial-wave production amplitudes can be modified by small scattering phases, but not the differential or the total rate of the decay. At $\omega < \omega_2$ the $D$ meson pair is above the threshold for $D^+ D^-$ and the charged meson pair can in principle be produced by the rescattering $D^0 \bar{D}^0 \to D^+ D^-$. However in this region the amplitude of the decay $X \to D^0 \bar{D}^0 \gamma$ is already small, and the rescattering effect is further suppressed by the small $P$-wave factor $p^3_\pm$ with $p_\pm$ being the relative momentum of the $D^+ D^-$ pair. The rescattering of the $D$ mesons is kinematically suppressed near the threshold below the $\psi(3770)$ resonance, and its effect can be rather conservatively estimated as resulting in the rate of the charged meson pair production $\Gamma(X \to D^+ D^- \gamma)$ being less than about 1% of $\Gamma(X \to D^0 \bar{D}^0 \gamma)$. Although this bound is considerably larger than the estimate in eq.(18) without the rescattering, it is still quite small in absolute terms. The mixing between the pairs of neutral and charged $D$ mesons becomes of order one in the $\psi(3770)$ resonance, which corresponds to $\omega \approx 100$ MeV. However in this kinematical region the estimated in eq.(15) peripheral contribution is already very small.

\section{The ‘core’ decay $X(3872) \to \psi(3770) \gamma$}

At the $\psi(3770)$ resonance the c.m. momentum of the $D$ mesons is quite large: $p_\pm \approx 242$ MeV and $p_0 \approx 275$ MeV, so that not only the estimate of the peripheral contribution to the decay $X \to D \bar{D} \gamma$ is small, but it also becomes rather unreliable, since the amplitude in this kinematical region is sensitive to the short-distance ‘core’ of the $X(3872)$. In the picture
discussed above, where this ‘core’ is a $^3P_1$ charmonium, it is natural to describe the radiative decays as due a charmonium radiative transition $^3P_1 \to \psi(3770)\gamma$ with subsequent decay $\psi(3770) \to D\bar{D}$. Clearly, this mechanism should give the relative yield of the charged and neutral pairs as is measured in the decay of the $\psi(3770)$ resonance[22]:

$$\frac{\Gamma(X \to D^+D^-)}{\Gamma(X \to D^0D^0\gamma)}_{\text{core}} = \frac{\Gamma[\psi(3770) \to D^+D^-]}{\Gamma[\psi(3770) \to D^0D^0]} = 0.776 \pm 0.024^{+0.014}_{-0.006}. \quad (19)$$

The estimate of the absolute decay rate provided by this mechanism obviously depends on two factors: the amplitude of the transition $^3P_1 \to \psi(3770)\gamma$ and the statistical weight $|a_{cc\bar{c}}|^2$ of the $^3P_1$ charmonium in the wave function of the $X(3872)$ resonance. Needless to mention that any estimate of each of these factors is necessarily quite approximate. Perhaps the best way of estimating the amplitude of the radiative transition $^3P_1 \to \psi(3770)\gamma$ is to use the recent data[23] on a similar transition $\psi(3770) \to \chi_{c1}\gamma$:

$$\Gamma[\psi(3770) \to \chi_{c1}\gamma] = 75 \pm 14 \pm 13 \text{ KeV }, \quad (20)$$

and to rescale the rate as $\omega^3$, assuming that the wave function of the $^3P_1$ ‘core’ charmonium state in $X(3872)$ is similar to that in the $\chi_{c1}$ resonance. Such estimate gives

$$\Gamma[X(3870) \to \psi(3770)\gamma] \approx |a_{cc\bar{c}}|^2 \times 5 \text{ KeV }. \quad (21)$$

In the approximation where the core of $X(3872)$ is a $^3P_1$ charmonium similar to $\chi_{c1}$ the amplitude $a_{cc\bar{c}}$ can be estimated from the rate of production of the $X$ resonance in $B$ decays relative to $\chi_{c1}$, since in $B$ decays $X(3872)$ is dominantly produced through its core[18, 17], rather than through the peripheral meson component. In particular, it is known[21] that $\mathcal{B}(B \to \chi_{c1}K) \approx \mathcal{B}(B \to \psi'K)$, and also that

$$\frac{\mathcal{B}(B^+ \to K^+X) \mathcal{B}(X \to \pi^+\pi^-J/\psi)}{\mathcal{B}(B^+ \to K^+\psi') \mathcal{B}(\psi' \to \pi^+\pi^-J/\psi)} = 0.063 \pm 0.014. \quad (22)$$

Therefore one can estimate

$$|a_{cc\bar{c}}|^2 \approx 0.06 \frac{\mathcal{B}(\psi' \to \pi^+\pi^-J/\psi)}{\mathcal{B}(X \to \pi^+\pi^-J/\psi)} \approx \frac{0.02}{\mathcal{B}(X \to \pi^+\pi^-J/\psi)}. \quad (23)$$

The recent experimental lower limit[21] on the branching fraction, $\mathcal{B}(X \to \pi^+\pi^-J/\psi) > 4.2\%$, then implies $|a_{cc\bar{c}}|^2 \lesssim 0.5$, with a more realistic value, perhaps, being in the neighborhood of 0.1.
6 Discussion

The estimates presented here illustrate that a study of the decays $X(3872) \rightarrow D\bar{D}\gamma$, if experimentally feasible, may provide important information about the internal structure of the $X$ resonance. In particular, it may be quite helpful in finding out the relative weight of the charmonium state and the molecular one in the wave function describing the composition of $X(3872)$. Recently the dilemma of whether this resonance is just another charmonium level or a ‘molecule’ was discussed in detail in Ref.\[^{17}\]. It is likely however that we will not have to resolve it in one way or the other, and that the $X$ resonance is a part of both. In this picture the two competing components contribute to different properties of the $X$: soft processes such as the decays of the type $X \rightarrow \pi^+ \pi^- J/\psi$ and $X \rightarrow \pi^+ \pi^- \pi^0 J/\psi$ proceed due to the peripheral molecular part with a strong isospin violation, while the hard processes such as production of $X(3872)$ in $B$ decays or in hadronic collisions are dominantly due to the charmonium ‘core’. In this picture the $X$ resonance should behave as an isoscalar in the hard processes. This implies in particular, that one should expect\[^{18, 25}\] approximately equal rates of the decays $B^+ \rightarrow XK^+$ and $B^0 \rightarrow XK^0$, which expectation is in agreement with the most recent data\[^{15}\]. Furthermore, if the core of $X(3872)$ is similar to the $^3P_1$ charmonium state $\chi_{c1}$, this similarity can be tested in other production processes where the yield of the $\chi_{c1}$ is known, e.g. in the decays $B \rightarrow XK^*$ vs. the decays $B \rightarrow \chi_{c1}K^*$.

The decays $X \rightarrow D\bar{D}\gamma$ have the advantage that they receive contribution from both the peripheral component and the core in different parts of the photon spectrum. The peripheral contribution gives rise to essentially only the decay with neutral mesons in the final state, $X \rightarrow D^0\bar{D}^0\gamma$, with the spectrum peaking near the photon energy $\omega_0 \approx 137$ MeV as in the free $D^{*0}$ radiative decay. The charmonium core contribution however is peaked near the photon energy $\omega \approx 100$ MeV, corresponding to the transition $X \rightarrow \psi(3770)\gamma$. In the latter peak the ratio of the yield of the pairs of charged and neutral mesons is determined by the decay properties of the $\psi(3770)$ resonance (eq.\(^{(19)}\)). The ratio of the rates under those peaks, if measured, would allow to quantitatively estimate, using the equations \(^{(12)},\ (^{13}),\) and \(^{(21)},\) the statistical weights $|a_0|^2$ and $|a_{cc}|^2$, while the shape of the peak in the spectrum near $\omega_0$ may provide further information on the details of the wave function of the ‘molecular’ component.
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