Phenomenological implications of the anomalous baryon current in the Standard Model are discussed, in particular neutrino-photon interactions at finite baryon density. A pedagogical derivation of the baryon current anomaly is given.

1 Introduction

The baryon current in the Standard Model is not conserved in the presence of electroweak gauge fields. Although classically we have

$$\partial_\mu J^\mu = \partial_\mu \left( \frac{1}{3} \sum_q \bar{q} \gamma^\mu q \right) = 0,$$

the baryon current divergence acquires quantum corrections when gauge fields are coupled differently to left- and right-handed quarks. For the Standard Model electroweak gauge fields, we have

$$\partial_\mu J^\mu = -\frac{1}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} \left( g_2^2 F_{\mu\nu}^a F_{\rho\sigma}^a - g_1^2 F_{\mu\nu}^Y F_{\rho\sigma}^Y \right) \neq 0,$$

where $F_{\mu\nu}^a = \partial_\mu W_\mu^a - \partial_\nu W_\mu^a + g_2 \epsilon^{abc} W_\mu^b W_\nu^c$ is the covariant $SU(2)_{L}$ field strength and $F_{\mu\nu}^Y$ is the weak hypercharge field strength. This curious fact may have profound cosmological implications through the generation of baryon number at the electroweak phase transition.

As discussed in Refs. and reviewed in this talk, nonconservation of baryon number is connected to novel effects that can be observed in laboratory experiments, and that may have interesting astrophysical implications. This report begins with a theoretical review by analyzing the baryon number anomaly in analogy to the perhaps more familiar axial anomaly. Turning to
phenomenology, some observable consequences in neutrino scattering experiments are described, and several other directions to explore are mentioned.

2 Theoretical excursion

2.1 The axial current anomaly and $\pi^0 \rightarrow \gamma\gamma$

A famous implication of gauge anomalies is the necessity for a nonzero $\pi^0 \rightarrow \gamma\gamma$ amplitude due to the nonconservation of the iso-triplet axial-vector quark current,

$$J_5^\mu = \frac{1}{2}(\bar{u}\gamma^\mu\gamma_5u - \bar{d}\gamma^\mu\gamma_5d).$$

(3)

In the presence of electromagnetism we have

$$\partial_\mu J_5^\mu = \frac{e^2}{32\pi^2}\epsilon_{\mu\rho\sigma}F_{\mu\rho}F_{\rho\sigma}.$$  

(4)

If low-energy QCD is described by a theory of mesons, a nonzero $\pi^0 \rightarrow \gamma\gamma$ amplitude is necessary in order to reproduce this result.

Let us recall how this works explicitly, by considering the object:

$$\int d^4x e^{-iq\cdot x}\langle \gamma(p)\gamma(k)\mid J_5^\mu(x)\mid 0 \rangle \equiv \left[ B_5(q,\mu) \right] \times \epsilon^*_\rho(p)\epsilon^*_\sigma(k)(2\pi)^4\delta^4(p + k - q),$$

(5)

first at the quark level, and then at the meson level. The field $B_5$ denotes a background field coupled to $J_5^\mu$, and $A$ is the photon. At the quark level, after a proper definition of the relevant triangle diagram that ensures vector current conservation, a standard calculation shows that in lowest order perturbation theory,

$$iq_\mu \left[ \begin{array}{c} B_5 \\ A \end{array} \right] = \frac{e^2}{4\pi^2}\epsilon^{\nu\rho\alpha\beta}p_\alpha k_\beta,$$

(6)

consistent with (4). How is this result reproduced in terms of the low-energy effective action where the quarks are replaced by mesons? First, there is no gauge invariant operator connecting $B_5$ and two photons directly, so that

$$B_5 \rightleftharpoons A = 0.$$  

(7)

A nonzero contribution is however obtained from the pion pole (consider the limit of vanishing quark masses),

$$B_5 \rightleftharpoons A = -iC_1q^\mu \times \frac{i}{q^2} \times (-iC_2)\frac{e^2}{4\pi^2}\epsilon^{\nu\rho\alpha\beta}p_\alpha k_\beta.$$  

(8)
Here $C_1$ denotes the strength of the $\pi$ coupling to the axial current, and $C_2$ is the strength of the pion-photon vertex. From the chiral lagrangian with Wess-Zumino-Witten term, we necessarily have $C_1 = f_\pi$, $C_2 = 1/f_\pi$. Contracting (8) with $iq_\mu$ reproduces (9) and hence (4). Phrased differently, if low-energy QCD is described by an effective theory of pions, then the process $\pi^0 \rightarrow \gamma\gamma$ occurs with a fixed strength.

### 2.2 The baryon current anomaly

The anomalous baryon current can be treated in close analogy to the anomalous axial-vector current above. We must however pay close attention to which currents are conserved, since in the present case it is no longer true that “vector currents are conserved, axial-vector currents are anomalous,” as the usual intuition suggests. Suppose that we introduce a background field $B_\mu$ coupled to baryon number. Then the baryon current is defined by varying the action with respect to $B_\mu$:

$$J^\mu = \frac{\delta S}{\delta B_\mu},$$

and its divergence is read off from

$$\delta S = -\int d^4x \epsilon(x) \partial_\mu J^\mu,$$

where $\delta B_\mu = \partial_\mu \epsilon$. Thus the problem of calculating the anomalous divergence of the baryon current is reduced to the introduction of $B_\mu$.

However, we must not be too naive in introducing $B_\mu$; otherwise we may start with a gauge invariant theory, but end up with a non-gauge-invariant (i.e., nonsensical) theory. Varying the nonsensical theory would not give the correct symmetry current and its divergence. To be explicit, let us return to the example of the axial-vector current for a single fermion, and suppose that we add the perturbation

$$\bar{\psi}(i\partial + A)\psi \rightarrow \bar{\psi}(i\partial + A + B_5\gamma_5)\psi.$$  

Then the theory naively remains invariant under electromagnetic gauge transformations,

$$\psi \rightarrow e^{i\epsilon} \psi, \quad A_\mu \rightarrow A_\mu + \partial_\mu \epsilon, \quad B_5 \rightarrow B_5.$$  

However, due to the effects of anomalies the theory is in fact not gauge invariant. For a sensible theory, we must add at the same time as the perturbation (11), a counterterm:

$$\bar{\psi}(i\partial + A)\psi \rightarrow \bar{\psi}(i\partial + A + B_5\gamma_5)\psi + \mathcal{L}_{ct}(A, B_5),$$  

where explicitly,

$$\mathcal{L}_{ct}(A, B_5) = \frac{1}{6\pi^2} \epsilon^{\mu\nu\rho\sigma} B_\mu A_\nu \partial_\rho A_\sigma.$$  

The results (4) and (6) have an implicit dependence on the choice of counterterm. In particular, the “Bardeen” form of the counterterm, of which (14) is an example, is employed to conserve vector currents in the presence of arbitrary background fields.

When nonvectorlike currents are gauged, a different counterterm must be employed. For the general case the explicit counterterm is given in Ref. Let us consider the baryon current for

\(^{a}\) That is, due to the effects of the fermion measure, in path integral language.

\(^{b}\) The dependence can be made explicit by performing the calculation with Weyl fermions.
a single standard model generation, and for simplicity restrict attention to the neutral gauge bosons \( A \) and \( Z \). The Bardeen counterterm is then

\[
\mathcal{L}_{\text{Bardeen}} = \frac{eg^2}{24\pi^2 \cos \theta_W} \epsilon^{\mu\nu\rho\sigma} (B_\mu Z_\nu \partial_\rho A_\sigma + A_\mu Z_\nu \partial_\rho B_\sigma),
\]

whereas the full counterterm is

\[
\mathcal{L}_{\text{ct}} = \frac{eg^2}{24\pi^2 \cos \theta_W} \epsilon^{\mu\nu\rho\sigma} (-2B_\mu Z_\nu \partial_\rho A_\sigma + A_\mu Z_\nu \partial_\rho B_\sigma).
\]

If we now write

\[
S = [S + S_{\text{Bardeen}} - S_{\text{ct}}] + S_{\text{ct}} - S_{\text{Bardeen}},
\]

then the variation \((10)\) vanishes for the bracketed combination in \((17)\), and from the remainder we can read off immediately using \((15)\) and \((16)\):

\[
\partial_\mu J^\mu = -\frac{eg^2}{8\pi^2 \cos \theta_W} \epsilon^{\mu\nu\rho\sigma} \partial_\mu A_\nu \partial_\rho Z_\sigma,
\]

yielding the result \((2)\).

In the language of chiral lagrangians, the new counterterm has the novel property that it leaves residual “pseudo-Chern Simons” terms in the action, i.e., terms involving the epsilon tensor, but having no pion fields. Such terms are subtracted if the Bardeen counterterm is used instead, since it can be shown that \(\mathcal{L}_{\text{Bardeen}} = -\mathcal{L}(\pi = 0)\). Equivalently, the counterterm requires a different boundary condition for “integrating the anomaly” to obtain the anomalous part of the chiral lagrangian\(^9\); this is again related to the statement that \(\mathcal{L}(\pi = 0) \neq 0\).

2.3 Vector mesons

The preceding discussion shows how to incorporate spin-1 background fields into the chiral lagrangian without upsetting gauge invariance in the fundamental gauge fields. In particular, the \(SU(2)_L \times U(1)_Y\) gauge anomaly cancellation between quark and lepton sectors is not upset when background fields are coupled to the quark flavor symmetries. With these background field probes in place, it is straightforward to derive properly defined (covariant) currents and the associated anomalous divergences, by an appropriate variation of the action.

The relevance of the background field discussion for vector mesons is twofold. First, since physical spin-1 mesons such as \(\rho\) and \(\omega\) behave mathematically like these background fields, we have found the “slots” which these fields fit into when constructing our chiral lagrangian. Second, and relatedly, once we know that the vector mesons inhabit these slots, we find new physical effects related to the quark level anomalies, e.g. to the baryon current anomaly. These effects can be observed experimentally. For example, at the level of vector meson dominance, new effects will be described by the interaction

\[
\mathcal{L} = \frac{eg^2}{8\pi^2 \cos \theta_W} \epsilon^{\mu\nu\rho\sigma} \omega_\mu Z_\nu \partial_\rho A_\sigma.
\]

This is in the same spirit as using \(\pi^0 \rightarrow \gamma\gamma\) as a probe of the axial current anomaly.

The theoretical description can be refined at low energy by integrating out the vector mesons; the vector dominance assumption then translates into a prediction for the coefficients of certain \(1/m_\omega^2\) suppressed operators.
3 Phenomenology

To see that the vector mesons are indeed described as part of the WZW term structure, we can verify that the same coupling strength is observed in accessible decay modes, such as $\omega \rightarrow 3\pi$ and $\omega \rightarrow \pi\gamma$. As depicted in the Fig. 1, these are all parts of the baryon current, expressed in terms of the fields, including nuclear sources, in the low-energy chiral lagrangian:

$$J^\mu = \bar{N} \gamma^\mu N + \frac{1}{4\pi^2} e^{\mu\nu\rho\sigma} \left( -\frac{2i}{f_\pi^2} \partial_\nu \pi^0 \partial_\rho \pi^0 - \frac{e}{f_\pi} \partial_\nu \pi^0 \partial_\rho A_\sigma + \frac{e g_2}{2 \cos \theta_W} Z_\nu \partial_\rho A_\sigma + \ldots \right). \quad (20)$$

For example,

$$\Gamma(\omega \rightarrow \pi\gamma) \approx \frac{3\alpha E^-_\gamma}{64\pi^3 f_\pi^2} \left( \frac{2}{3} g_\omega \right)^2 \approx 0.76 \text{ MeV} \left( \frac{2}{3} g_\omega \right)^2. \quad (21)$$

Similarly, $\frac{2}{3} g_\omega \approx 6$ is obtained for $\omega \rightarrow 3\pi$, including the $\omega \rightarrow \rho\pi \rightarrow 3\pi$ contributions. A consistent, although somewhat uncertain, value of the $\omega$ coupling to the baryon current is also obtained for the first diagram in Fig. 1 using one-meson exchange models of the force between nucleons, and isolating the isoscalar $J^{P}=1^{-}$ channel. The effective coupling is expected to be somewhat larger in this case, since “$\omega$” is actually representing a tower of resonances.

We wish to access the final diagram in Fig. 1, i.e., the pure gauge field part of the baryon current, that is most directly related to the baryon current anomaly. We expect $\omega$ to couple to this part of the current with the same strength as the other parts. Now, if the $Z$ mass were small, the Standard Model would predict a decay mode,

$$\Gamma(\omega \rightarrow Z\gamma) = \frac{3\alpha E^-_\gamma}{256\pi^3 m_Z^2 \cos^2 \theta_W} \frac{g_2^2}{g^2} \left( \frac{2}{3} g_\omega \right)^2 \left( 1 + \frac{m_Z^2}{m_\omega^2} \right). \quad (22)$$

Of course, the decay $\omega \rightarrow Z\gamma$ is not physically allowed. Nevertheless, processes involving virtual $Z^*$ are allowed, and can lead to interesting effects. Since there will be a weak suppression, we should focus on situations in which the $Z$ is “useful”, e.g., processes involving neutrinos or parity violation. We can also make the $\omega$ “useful”, e.g., by utilizing its strong coupling to baryon number to look for enhanced rates when scattering off nuclei, rather than searching for the tiny branching fraction $\omega \rightarrow \gamma\nu\bar{\nu}$. As depicted in Fig. 2, this is in analogy to probing the $\pi^0\gamma\gamma$ coupling via the Primakoff effect, where one of the photons couples coherently to the electric charge of the nucleus.
3.1 Neutrino scattering

The interaction (19) will induce neutrino-photon interactions in the presence of baryon number. For example, single photons are produced in neutrino-nucleus scattering, as depicted in Fig. 3. In the approximation where the nuclear interactions are described by one-meson exchange, there will be competing contributions from virtual $\pi^0$ and $\rho^0$ exchange. However, $\pi^0$ exchange is suppressed by the accidental smallness of $1 - 4 \sin^2 \theta_W$ in the Standard Model, and the $\rho^0$ exchange diagram is suppressed in amplitude by $\sim (1 + 1 - 1)^2/(1 + 1 + 1)^2 = 1/9$ relative to $\omega$, due to the fact that $\omega$ is isoscalar, whereas $\omega$ is isotriplet; this can be thought of as a coherence effect at the nucleon level. Further enhancement of the $\omega$ exchange due to coherence over adjacent nucleons can occur in kinematics where small enough momentum is exchanged with the nucleus.

![Figure 3: Photon production in neutrino scattering in the presence of baryon number.](image)

Neglecting effects such as coherence, form factors and recoil, the cross section for the process depicted in Fig. 4 for scattering off an isolated nucleon is

$$\sigma \approx \frac{\alpha g_{\omega}^4 G_F^2}{480 \pi^6 m_\omega^4} E_\nu^6 \approx 2.2 \times 10^{-41} (E_\nu/1 \text{ GeV})^6 (g_\omega/10)^4 \text{ cm}^2.$$  \hspace{1cm} (23)

The photon energy distribution in this approximation is

$$\frac{d\sigma}{dE_\gamma} \propto E_\gamma^3 (E_\nu - E_\gamma)^2,$$  \hspace{1cm} (24)

and the angular distributions is flat,

$$\frac{d\sigma}{d\cos \theta_\gamma} \propto \text{constant}.$$  \hspace{1cm} (25)

Form factors will suppress the cross section at large momentum exchange, pulling the angular distribution forward. As an illustration, the photon energy and angular distribution for a

![Figure 4: Photon energy distribution (left figure) and angular distribution (right figure), including nuclear recoil and $\omega$(780) form factor, for 700 MeV neutrino incident on stationary nucleon (arbitrary normalization).](image)
700 MeV neutrino incident on an isolated nucleon, including nuclear recoil and the form factor induced by $\omega(780)$ exchange, is depicted in Fig. 4. A more detailed analysis of single photon events will be presented elsewhere.\(^\text{[5]}\)

![Diagram of neutrino interaction](image1)

In the absence of large coherent enhancements, e.g. for scattering on small nuclei, we should ideally use relatively large incident neutrino energies, in order to overcome the mass of $\omega$. Also, if it is not possible to distinguish photon showers from electron showers, a pure $\nu_\mu$ beam should be used in order to avoid a background from charged-current scatters, $\nu_e+n\rightarrow e^-+p$. In fact, these requirements have overlap with experiments looking for $\nu_e$ appearance in a $\nu_\mu$ beam. For example, MiniBooNE\(^\text{[13]}\) and (in the future) T2K\(^\text{[14]}\) have $\nu_\mu$ beams with energy spectra of order several hundreds of MeV, but primarily $\lesssim 1$ GeV, largely within the range of a chiral lagrangian description. Single photons that are mistaken for electrons are a background to $\nu_e$ appearance searches, as depicted in Fig. 5. It is interesting that an excess of events observed by MiniBooNE is in the same order of magnitude as predicted by (23), and has similar characteristics to the distributions in Fig. 4. Experiments with higher energy neutrinos are also of interest, but pass beyond a simple chiral lagrangian description.

### 3.2 Neutrino pair production

![Diagram of neutrino pair production](image2)

Similar interactions can give rise to photon conversion into neutrino pairs in the presence of baryon number, as depicted in Fig. 6. A nonnegligible contribution to neutron star cooling via this mechanism was computed in Ref. 3. Similar effects will occur in the hot and dense environment of a supernova core.

### 3.3 Parity violation

Besides neutrino interactions, we can use the $Z$ to mediate parity violation. The interaction (19) will give rise to potentially interesting effects in various parity-violating observables. These will be investigated elsewhere.\(^\text{[5]}\)

### 4 Summary

This report began with a pedagogical derivation of the baryon current anomaly in the Standard Model. The counterterm structure in this derivation is interesting because it requires residual
“pseudo-Chern-Simons” terms in the action when background vector fields are coupled to the quark flavor symmetries. This exercise is significant for phenomenology because the same framework can be used to describe vector meson interactions in vector dominance approximation. The resulting extension of the QCD chiral lagrangian provides a useful guide to new effects, such as “baryon-catalyzed” neutrino-photon interactions and parity violation. Other applications of the formalism that have not been discussed here include a description of “natural parity violating” QCD vector meson decays, such as $f_1 \to \rho \gamma$. It is also interesting to relate this framework to five-dimensional descriptions of QCD\cite{5}, both as a means of constraining “AdS-QCD” models, and potentially using such models to predict undetermined constants appearing in the chiral lagrangian.

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\* That is, odd under the parity transformation for which scalars and vectors are even, and pseudoscalars and axial-vectors are odd.