Active Colloids with Position-Dependent Rotational Diffusivity

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The non-thermal nature of self-propelling colloids offers new insights into non-equilibrium physics. The central mathematical model to describe their trajectories is active Brownian motion, where a particle moves with a constant speed, while randomly changing direction due to rotational diffusion. While several feedback strategies exist to achieve position-dependent velocity, the possibility of spatial and temporal control over rotational diffusion, which is inherently dictated by thermal fluctuations, remains untapped. Here, we decouple rotational diffusion from thermal noise. Using external magnetic fields and discrete-time feedback loops, we tune the rotational diffusivity of active colloids above and below its thermal value at will and explore a rich range of phenomena including anomalous diffusion, directed transport, and localization. These findings add a new dimension to the control of active matter, with implications for a broad range of disciplines, from optimal transport to smart materials.

INTRODUCTION

The behavior of self-propelling colloidal particles sheds light on far-from-equilibrium physics and offers tantalizing options to perform tasks beyond the reach of other micro- and nanoscale systems [1]. Many of these functions are inspired by the striking similarity that synthetic active matter exhibits with living systems such as motile bacteria. This analogy therefore also provides an ideal opportunity to understand the motion and (self-)organization of living systems through synthetic models [2]. The fundamental mathematical model for active colloids is given by active Brownian motion [1]. A microscopic particle of radius $R$ in a fluid of viscosity $\eta$ moves with constant speed $v$, while its orientation undergoes thermal rotational diffusion with a coefficient $D_B = (k_BT)/(8\pi\eta R^3)$, with $k_BT$ being the characteristic thermal energy at absolute temperature $T$ and $k_B$ the Boltzmann constant. This minimal model has been successfully employed to describe a wealth of phenomena, from the motion of active particles in complex structures [3] to the optimization of search strategies [4].

Recently, there has been a growing interest in pushing the control of synthetic active matter beyond the standard active Brownian particle (ABP) model to sustain a wealth of complex behavior, including directed transport and pattern formation. A key to these phenomena is the spatio-temporal modulation of particle velocity and fluctuation spectra. A position-dependent translational diffusion coefficient has been proposed as a fundamental biological mechanism leading to anomalous diffusion and localization of biomolecules in cellular membranes [5], while a position-dependent velocity allows controlling the regions explored by artificial active particles as well as their interactions [6,9]. Beyond their fundamental interest, these mechanisms can also be exploited in a range of applications, such as environmental remediation and targeted drug delivery [1]. Experimentally, control of the translational diffusion or the velocity of the active particles as a function of their position [3,10] or interactions [11,12] has already been shown.

However, an unexplored avenue to engineer active Brownian motion exists through the targeting of rotational dynamics. Biological swimmers, such as chemotactic bacteria [13], already follow this route to climb up or down chemical gradients in order to reach food sources or escape harmful chemicals by adjusting their reorientation frequency (tumbling rate). A change of tumbling rate can be straightforwardly mapped into an effective increase of rotational diffusivity [14]. Nonetheless, translational and rotational fluctuations are coupled via the thermal bath and their lower bound is dictated by thermal fluctuations.

Here, we decouple the spectrum of rotational fluctuations from the thermal bath and experimentally control the rotational diffusion of an active Brownian particle by using magnetic fields and a discrete-time feedback loop. This strategy allows us to control the effective rotational temperature above and below the actual temperature. Depending on the space-time modulation of the rotational diffusion and on the sampling/update time in the feedback loop, a broad range of exotic phenomena emerges, from anomalous diffusion and non-Gaussian statistics, to directed transport and localization. We support our results with numerical simulations, which also indicate new directions for future developments.

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FIG. 1. Experimental control of rotational diffusion. (a) Schematic of the experimental setup. A silica particle, half-coated with a magnetized Ni cap and immersed in water between two transparent electrodes, is subjected to a homogeneous AC electric field and actuated by induced-charge electro-phoresis (the yellow arrows depict the local electro-kinetic flows). A uniform magnetic field \( \mathbf{B} \) (blue arrow), produced by four coils, controls the orientation of the particle in the plane, and the motion is observed using an upright optical microscope. The inset shows an optical micrograph of a 4 \( \mu \)m particle, with the Ni cap in black. The white arrow shows the propulsion direction, at an angle \( \theta \), following the alignment of the cap’s magnetic moment to the imposed magnetic field. Scale bar: 5 \( \mu \)m. (b) Imposed orientation angle vs time for three values of \( D_R \). The colored bands delimit a \( 2\pi \) range. Data are shifted along y for clarity. (c) Probability distributions of measured angular displacements \( G(\Delta \theta, \Delta t) \) for different \( D_R \) (dashed lines: magnetic field; solid lines: ABPs) with the same colors as in (b). The grey points show \( G(\Delta \theta, \Delta t) \) for the thermal \( D_{th} \) at room temperature, with the field off (grey line: Gaussian fit, see Supplementary Table 1). (d) Trajectory of an ABP with \( v = 5.5 \, \mu \text{m s}^{-1} \) (dashed lines) and where \( D_R \) is varied over time as shown in the inset. (e) MSD of an ABP with \( v = 5.5 \, \mu \text{m s}^{-1} \) for different imposed \( D_R \): 0.1 (diamonds), 1 (triangles), 5 (circles) and 10 (squares) \( \text{rad}^2 \text{s}^{-1} \), with same colors as in (d). (f) Persistence length \( (L_P) \) of ABPs as a function of rotational relaxation time \( D_R^{-1} \) for different values of \( v \): 8.2 (squares), 6.7 (triangles) and 2.7 (circles) \( \mu \text{m s}^{-1} \). The inset shows \( L_P \) as a function of \( v \) for \( D_R^{-1} = 0.014 \, \text{rad}^2 \text{s}^{-1} \) (grey squares) and for imposed values of 0.07 and 0.144 \( \text{rad}^2 \text{s}^{-1} \) (black triangles and squares, respectively).

RESULTS

Controlling rotational dynamics

Our model active Brownian particles (ABPs) self-propel thanks to induced-charge electro-phoresis [15–17]. They consist of 4 \( \mu \)m-diameter silica colloids, half-coated with 120 nm of magnetized nickel so that the propulsion direction is aligned with the magnetic moment. We place the particles in a liquid cell made of two planar transparent electrodes vertically separated by a gap \( h = 120 \, \mu \text{m} \) and record their position and cap orientation at a frame rate of 10 fps by video microscopy (see Materials and Methods for more details). When the electrodes are connected to a 1 kHz AC voltage source, the colloids swim on the bottom substrate due to locally unbalanced electro-kinetic flows, as schematically depicted by the yellow arrows in Fig. [1]a. The swimming velocity \( v \) goes as \( v \propto (V_{pp}/h)^2 \), where \( V_{pp} \) is the peak-to-peak voltage, set between 1 and 10 V.

The orientation angle \( \theta \) relative to the lab reference frame (see microscopy image in Fig. [1]b) is controlled by two pairs of independent Helmholtz coils that generate a spatially uniform in-plane magnetic field, whose direction can be freely controlled (Fig. [1]h, Supplementary Fig. S1, and Movie S1). In contrast to previous applications of magnetic fields to remote-control active colloids [18–22], we randomize the direction of the magnetic field to supply the particles with an externally controlled rotational diffusivity, which is decoupled from the thermal bath and from the propulsion scheme (Supplementary Movie S2). We change the field orientation at 1 kHz extracting random angular displacements from a zero-mean Gaussian distribution whose variance is \( \sigma^2 = 2D_R\delta t \), where \( \delta t \) is the time between reorientation events and \( D_R \) is the imposed rotational diffusivity. Figure [1] shows the magnetic field’s angle as a function of time for three different values of \( D_R \). The field direction randomly and freely rotates over the entire \( 2\pi \) range from slightly slower up to several orders of magnitude faster than the thermal rota-
tional diffusion $D_R^{th}$ of 0.014 rad$^2$ s$^{-1}$ (rotational cooling is also shown in SI for 2 $\mu$m-diameter colloids). The corresponding probability distributions of angular displacements $G(\Delta \theta, \Delta t)$, where $\Delta t = 100$ ms is the time between consecutive frames, show that the field’s and the particle’s orientations correspond to identical Gaussian distributions (Fig. 1).

By varying $D_R$ in a step-wise manner, we can gradually increase the propensity of the ABPs to move along straight paths (Fig. 1 and Supplementary Fig. S2 ally increase the propensity of the ABPs to move along.

Figure 2e illustrates the probability distributions of angular displacements due to ballistic segments. For $D_R^{th} \sim \Delta t \leq (D_R^{th})^{-1}$, the distribution is predominantly Gaussian, with small deviations due to ballistic segments. For $D_R^{th} \leq \Delta t \leq (D_R^{th})^{-1}$, large non-Gaussian tails develop, reminiscent of the distribution of step sizes in Lévy flights [24], although here the variance of $G(\Delta \theta/\sqrt{\Delta t}, \Delta t)$ is always finite with an upper bound of the order of $v^2 \Delta t^2$. The Gaussian behavior is recovered in the limit of long times, i.e. $\Delta t \gg (D_R^{th})^{-1}$, where the spatial inhomogeneity imposed by the checkerboard pattern averages out over large length scales, thus allowing the central limit theorem to hold in the limit of long times. The non-Gaussian parameter, $\alpha$, defined as a re-scaled excess kurtosis of the distribution of the $\alpha = \frac{(\Delta x^4)}{5(\Delta x^2)^2} - \frac{3}{5}$, has a maximum at $\Delta t \approx 10$ s (Fig. 2), which marks the transition from a super-diffusive scaling $\sim t^2$, with $a \sim 1.6$, to a diffusive regime, with $a \sim 1$ (Fig. 2).

The observed trend of the non-Gaussian parameter is in stark contrast to the behaviour of ABPs with constant rotational diffusivity, where $\alpha$ is zero on either short or long timescales, and negative otherwise [25]. In our system, the degree of non-Gaussianity depends on the timescale $L/v$, that is the time an ABP takes to cross low-$D_R$ regions, and on the feedback sampling time $\tau$. At a given $\tau$ (e.g. $\tau = 400$ ms as in the experiments), $L/v$ dictates both the maximum value of $\alpha$ and the lag time at which it is attained (see Figure 3a). For small $L/v$, the length scale of the spatial variation in $D_R$ is smaller than any persistence length and the ABP swims in a homogeneous environment with a constant effective rotational diffusivity, where $\alpha$ is constant between consecutive sampling times ($\tau = 400$ ms), and in the Langevin dynamics simulations by letting the rotational friction vary according to a zero-order hold (ZOH) model as described in the Methods section (see Fig. 2).

We apply this scheme to study the effect of two-dimensional periodic changes of rotational diffusivity by letting $D_R$ vary according to checkerboard patterns comprising square regions of size $L$ of alternating high and low random orientation (Figure 2b), over a wide range of timescales, velocities, and region sizes. In particular, we set $D_R$ such that particle motion is predominantly diffusive in one region and ballistic in the other, and that the overall dynamics is largely unaffected by the ballistic-to-diffusive transition undergone by the ABPs within the independent regions (see Methods and SI). Figure 2c shows experimental (black) and numerical (red) trajectories of an ABP swimming at $v = 3.5$ $\mu$m s$^{-1}$ through squares of $L = 32$ $\mu$m (Supplementary Movie S5 and Fig. S7).

Non-Gaussianity and anomalous diffusion

Imposing this spatial modulation of $D_R$ leads to anomalous diffusion, revealing unique dynamical features. Figure 2f illustrates the probability distribu-
transitions can be distinguished in the particle’s MSDs at $L/v$ and $3L/v$ lag times (yellow band in Fig. 3f). As $\tau$ grows, we observe non-monotonic variations in the time dependence of $\alpha$ and the MSDs of the ABP (Fig. 3a-f). Up to $\tau \sim L/v \sim 9\ s$, an increase in $\tau$ leads to higher values of $\alpha$ and the gradual disappearance of the peaks at lag times that are odd multiples of $\sim L/v$ (Fig. 3c). Consequently, the MSDs on timescales $\sim L/v$ decrease and the subdiffusive scaling gradually vanishes (see Figure 3d). For $\tau \gtrsim L/v$, $\alpha$ decreases again with no detectable peaks. In this regime, the MSDs increase across all timescales (Fig. 3).

**Localization**

While a position-dependent rotational dynamics alone cannot sustain pattern formation [20], the introduction of discrete-time feedback leads to the localization of ABPs in regions of high rotational diffusion [27], following again a non-trivial response to changes of $\tau$ (Fig. 3a-d). For instantaneous updates ($\tau = 0$), the steady-state particle distribution is homogeneous and there are no traces of underlying diffusivity patterns or localization (Fig. 3a). As $\tau$ becomes non-zero, the position-dependent steady-state particle density $\rho(x, y)$ starts to deviate from the uniform value $\rho_{eq}$ reaching a maximum localization for a critical $\tau$ (Fig. 3a-c), before becoming smooth again for greater sampling times (Fig. 3f). We quantify the degree of departure from the homogeneous distribution as the fraction of ABPs residing in high-$D_R$ regions, i.e. $\eta = N_L/(N_L + N_H)$ (Fig. 3), where $N_L$ and $N_H$ are the number density of ballistic and diffusive ABPs, respectively.

The maximum value of $\eta = \eta^{\text{max}}$ as a function of $\tau$ can be estimated with a simple transport argument based on the dynamic asymmetry of particles crossing different regions. Ballistic ABPs crossing into a diffusive region keep traveling up to lengths $\sim vt$ before updating, and particles diffusing into a ballistic region keep diffusing up to lengths $\sim \sqrt{D_R/\tau}$. This imbalance causes $\eta$ to be maximized at the $\tau$ that maximizes the penetration length into diffusive regions $L^*$; the deeper ABPs penetrate into diffusive regions, the longer it takes them to escape. Because at equilibrium the net flux between regions must be zero, we can write:

$$v_{in}N_L = v_{out}N_H,$$

where $v_{in} = v$ is the average velocity with which ballistic ABPs enter diffusive regions and $v_{out} = L^*/\Delta t^* - \Delta t^*$ is the time at which the MSD of an ABP in a diffusive region is equal to $L^*/\Delta t^* - \Delta t^*$; is the average velocity with which ABPs escape from diffusive regions. Since in our system $\Delta t^*D_R^H \gg 1$ and translational diffusion plays a negligible role.
role relative to propulsion, we obtain

\[ L^* = \sqrt{2D^*_R t^*} \]  

which in turn leads to a simple estimation of \( \eta_{\text{max}} \) as (more details in the SI):

\[ \eta_{\text{max}} \simeq \frac{1}{1 + (2/D_R^H)(v/L^*)}. \]  

Rescaling \( \eta_{\text{H}} \) by \( \eta_{\text{max}} \) and normalizing \( \tau \) by \( L/v \) causes all curves in Fig. 3 to collapse onto a single master curve (Fig. 3f), with \( L^* \) roughly equal to \( \sqrt{2L/4} \), i.e., the average between half of the squares diagonal and 0. Figure 3g shows that, using the optimum \( \tau \), a steady-state particle localization is rapidly reached and sustained as a function of time. Instead, larger sampling times lead to strong oscillations in \( \eta \) (Figure 3h-i) with period \( \tau \) during which the particle distributions tend to relax towards the uniform distribution.

**DISCUSSION AND CONCLUSIONS**

Our data show that spatial and temporal control over rotational diffusivity leads to the emergence of anomalous diffusion and of localization for ABPs. Concerning the first aspect, we in particular observe subdiffusion and positive values of non-Gaussianity \( \alpha \). These traits might suggest an analogy to the dynamics found in proximity of the colloidal glass transition, where neighbors cause transient confinement of the particles’ motion, a phenomenon referred to as caging [28]. In our system, however, the origin of caging is purely dynamical given that there is no physical confinement. Up to lag times on the order of \( L/v \), where \( L/v \gg D_R^{H-1} \), the total MSD is mostly determined by the ABPs traveling within ballistic regions (the space explored by ABPs in the diffusive regions over the same time scale, i.e., \( v^2/D_R^H \Delta t + v^2 \Delta t^2 \approx v^2 \Delta t^2 \), is in fact negligible). However, on timescales \( \gtrsim L/v \), ABPs travelling in a ballistic fashion within low-\( D_R \) regions eventually cross into high-\( D_R \) ones and have a finite probability to diffuse back and cross the low-\( D_R \) region in the opposite direction (Supplementary Movie 6). These deflection events translate into virtual or null displacements over timescales up to \( \sim 2L/v \), that is the time that a ballistic ABP takes to cross a low-\( D_R \) square and travel back. The MSD only grows diffusively for those particles that cross a low-\( D_R \) region an odd number of times and remain in a high-\( D_R \) region, which identifies the boundary of the subdiffusive regime at \( L/v \) and \( 3L/v \) (Fig. 3a,c).
FIG. 4. Emergence of localization. (a–d) Normalized particle density distributions $\rho(x,y)/\rho_{eq}$ of simulated ABPs subject to position-dependent rotational dynamics for different values of $\tau$ ($\rho_{eq}$ is the homogeneous distribution in the case of constant $D_R$). The simulation parameters are $v = 3.5\,\mu m\,s^{-1}$, $L = 32\,\mu m$, $D_H^R = 10\,rad^2\,s^{-1}$ and $D_L^R = 0.01\,rad^2\,s^{-1}$. The distributions are obtained by binning the positions of $2.5\times10^4$ particles for a simulated time of 900 s, after letting the particles move from their initially uniform-random-distributed positions for 100 s. Periodic boundary conditions are enforced over a square simulation box of size $10L \times 10L$. (e) Evolution of the degree of departure from the homogeneous distribution with $\tau$ for different values of $L/v$, as quantified in terms of the time-averaged fraction of particles residing in high-$D_R$ regions $\eta$. (f) Evolution of the latter quantity with the rescaled sampling period $\tau L^2 v$ after normalization by $\eta_{\text{max}}$, as defined in Eq. 3. (g–i) Evolution in time of $\eta$ for different values of $\tau$.

Higher-order contributions are washed out with the exception of a small shoulder at $\Delta t \sim 5L/v$. This dynamical caging becomes less prominent with decreasing $L/v$ ratios as the difference between diffusive and ballistic displacements diminishes. The second aspect, particle localization, is inherently connected to the introduction of short discrete feedback times, which generate a state-dependent asymmetry in their motion. In fact, the distance that ABPs can travel into a region before updating their rotational dynamics depends on the region where they come from; in particular, ballistic ABPs penetrate into diffusive regions up to lengths $\sim v\tau$ before adjusting their rotational dynamics. The striking consequence of this asymmetry is the localization of ABPs in the regions of high rotational diffusivity (Figs. 4c and 4g). ABPs thus spend more time in diffusive regions, where the MSD grows at a slower rate (as $t^2$ rather than as $t$), leading to an overall slowing down of the dynamics and to an increase of $\alpha$ (Fig. 3f from black to green). Conversely, longer sampling times can prevent the ABPs from sensing the periodicity of the environment. This crossover can be viewed in terms of the Nyquist-Shannon’s theorem. When $\tau \sim L/v$, the frequency at which the ABPs sample their environment is comparable to the highest frequency at which the ABPs can cross a region of the checkerboard pattern. This means that for $\tau > L/v$ the ABPs cannot sense changes of $D_R$ happening on a length scale $L$. At all times, the ABPs will retain a given $D_R$ for a period equal to the sampling period. Since there is no memory of the path, this fact leads to two different ABP populations moving in an either ballistic or diffusive fashion depending on their respective initial positions, which are updated every $\tau$. In this regime, $\alpha$ remains negative or close to zero (Fig. 3f from dark to light green) and no steady-state localization can be sustained (Figs. 4d), as highlighted by the large oscillations of $\eta$ in Fig. 4g–i.

Looking towards the future, our findings open up new interesting avenues to direct the dynamics and organization of ABPs. Local control over the rotational dynamics offers an alternative means to control the persistence of active trajectories at a given velocity, which can be harnessed to optimize the navigation of ABPs in complex environments. The introduction of periodic modulations in the rotational dynamics of ABPs also defines a new framework to study a variety of anomalous diffusion phenomena, beyond the cases presented here, with interesting analogies with glassy dynamics. Since ABPs enable directed transport and pattern formation, even in the absence of particle interactions and external forces, introducing various forms of feedback, communication and information flows.

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defines new opportunities. Borrowing ideas and tools from signal processing and control systems, we envisage the engineering of more complex dynamical responses. We could, for instance, adapt ideas developed for nuclear detectors (dead-time analysis) or robotic systems (feed-forward responses or negative delays) to devise new signal reconstruction strategies between discrete sampling events for ABPs, or design higher-order or integral responses to mimic the way in which biological microswimmers sense and adapt to chemical signals [37].

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AUTHOR CONTRIBUTIONS

Author contributions are defined based on the CRediT (Contributor Roles Taxonomy) and listed alphabetically. Conceptualization: FG, GV, IB, LA, LI, MAFR. Formal analysis: FG, LA. Funding acquisition: LI. Investigation: FG, LA, MAFR, MR. Methodology: FG, IB, LA, LI, MAFR. Project administration: LI. Software: FG, GV, LA, MAFR. Supervision: LI. Validation: FG, LA, MAFR. Visualization: FG, LA, LI, MAFR. Writing original draft: FG, LA, LI, MAFR. Writing review and editing: FG, GV, IB, LA, LI, MAFR.

ADDITIONAL INFORMATION

Supplementary Information is available for this paper. Correspondence and requests for materials should be addressed to lucio.isa@mat.ethz.ch.

DATA_AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

METHODS

Janus particle fabrication

In order to fabricate the active Janus particles, silica colloids with 4.28 μm diameter (5% w/v, microParticles GmbH, Germany) are diluted to 1:6 in MilliQ water and spread on a glass slide, previously made hydrophilic by a 2-minute air plasma treatment. Upon drying of the suspension, a close-packed particle monolayer is formed. The monolayer is then sputter-coated with 120 nm of nickel (Safematic CCU-010, Switzerland) to create the Janus surface. After coating, the glass slide is placed overnight above a neodymium magnet (50 × 50 × 12.5 mm³, 1.2 T) to align the magnetic moments of all particles in the direction of the compositional asymmetry. The particles are retrieved from the glass slide by pipetting and withdrawing a droplet of water on top of the monolayer. An identical procedure is followed for 2 μm silica particles, for which data are shown in the SI.

Cell preparation

The transparent electrodes are fabricated from 24 mm × 24 mm No. 0 coverglasses (85 – 115 μm-thick, Menzel Gläser, Germany) covered with 3 nm of chromium and 10 nm of Au deposited by metal evaporation (Evatec BAK501 LL, Switzerland), followed by 10 nm of SiO₂ deposited by plasma enhanced chemical vapor deposition (STS Multiplex CVD, UK) to minimize the adhesion of particles to the substrate. A water droplet containing the particles is deposited on the bottom electrode within the 9 mm-circular opening of a 0.12 mm-thick sealing spacer (Grace Bio-Labs SecureSeal, USA). The top and bottom electrodes are connected to a signal generator (National Instruments Agilent 3352X, USA) that applies the AC electric field, with a fixed frequency of 1 kHz and varying the VPP voltage between 1 and 10 V. For 5 V, the applied field is 42 V mm⁻¹.

Experiments and data analysis

The magnetic moment of the Janus particles is confined to the electrode plane and freely rotated within it using a custom-built setup with two pairs of independent Helmholtz coils [38]. The magnetic field is constant within a few percent over a 1 mm² area in the center of the cell and the maximum applicable magnetic field is 65 mT. In order to impose an effective rotational diffusivity to the particles, the magnetic field angle at step \( n + 1 \) (\( \theta_{n+1} \)) is obtained by adding to \( \theta_n \) a random angular displacement \( \Delta \theta \), which is given by Equation (4), where \( D_R \) is the target rotational diffusivity, \( \Delta t \) is the time step (1 ms in our experiments), and \( \eta \) is a random number sampled from a normal Gaussian distribution.

\[
\theta_{t+1} = \theta_t + \sqrt{2D_R \Delta t \eta}
\]  

(4)

The Janus particles are imaged with a home-built bright-field microscope in transmission and image sequences are taken with a sCMOS camera (Andor Zyla) at 10 fps with a 512 × 512 pixels² field of view. The image series to measure the thermal and imposed rotational diffusivity are acquired using a 50× objective (Thorlabs). The positions of the center of the JPs and of the metal
cap are located using a customized software written in Matlab. Then, a vector connecting both centers is used to determine the orientation of the particle at each frame for different $D_R$, from which the angular displacement distribution in Fig. 1b was calculated. The image series of ABPs actuated by the magnetic and the AC electric fields, are acquired with a 10s objective (Thorlabs). In this case, only the particle center of mass is located, and all the dynamical information is extracted from the final particle trajectory.

For the experiments with a spatial dependence of rotational diffusivity, single particles are located in real time by a custom LabView software. In particular, series of 1024 $\times$ 1024 pixels$^2$ images are recorded at 2.5 fps. The output coordinates from Labview are the input for a Matlab code that updates the particle $D_R$ based on a predefined landscape. For the data presented in the main text, the field of view was divided into checkerboard patterns with 5×5, 10×10 and 20×20 squares, respectively, with alternating regions of $D_R^H = 10$ rad$^2$ s$^{-1}$ and $D_R^L = 0.01$ rad$^2$ s$^{-1}$. $D_R$ is updated every 400 ms based on the particle coordinates.

The particle thermal translational ($D_T$) and rotational ($D_R$) diffusion coefficients at room temperature (24°$^\circ$C) are extracted from their 2D trajectories with no magnetic or electric fields applied. The thermal $D_T$ and $D_R$ in proximity of the substrate are extracted from the slope of the mean square displacement and the mean angular square displacement at short times and are measured to be 0.055 $\pm$ 0.002 m$^2$ s$^{-1}$ and 0.014 $\pm$ 0.01 rad$^2$ s$^{-1}$, respectively.

**Numerical simulations**

We simulate the dynamics of the ABPs by solving the equations of motion:

\[
\begin{align*}
    m\ddot{x} &= f_x(\theta) - \gamma_x + \sqrt{2kT}\eta_x(t) \\
    m\ddot{y} &= f_y(\theta) - \gamma_y + \sqrt{2kT}\eta_y(t) \\
    I\ddot{\theta} &= \gamma_R(\mathbf{r}, \tau)\dot{\theta} + \sqrt{2kT}\gamma_R(\mathbf{r}, \tau)\eta_\theta(t)
\end{align*}
\]

where $m$ and $I$ are the mass and the moment of inertia of the colloid, respectively, $f_x(\theta)$ and $f_y(\theta)$ are the $x$ and $y$ components of the active force acting on the colloid, $\gamma$ is the friction coefficient associated with translational motion, $\gamma_R(\mathbf{r}, \tau)$ is the state-dependent friction coefficient associated with rotational motion, and $\eta_x(t)$, $\eta_y(t)$, and $\eta_\theta(t)$ are uncorrelated random numbers satisfying:

\[
\langle \eta_x \rangle = \langle \eta_y \rangle = \langle \eta_\theta \rangle = 0; \quad \langle \eta_x^2 \rangle = \langle \eta_y^2 \rangle = \langle \eta_\theta^2 \rangle = 1
\]  

The active forces $f_x(\theta)$ and $f_y(\theta)$ are set equal to $\gamma v \cos(\theta)$ and $\gamma v \sin(\theta)$, respectively, such that in the absence of thermal noise and in the limit of long times the particles move at a constant velocity equal to $v$. We solved Equation 5 in the underdamped limit through a Verlet-type integration scheme proposed by Gronbech-Jensen and Farago (GJF) using the Itô convention [39]. Although Equation 5 could also be solved in the overdamped limit, this approach allowed us to achieve a faster convergence to the homogeneous distribution for $\tau = 0$ using a relatively small integration step $dt$: 0.001 s.

In particular, in the numerical simulations we control $D_R$ by letting the rotational friction $\gamma_R(\mathbf{r}) = k_BT/D_R(\mathbf{r})$ vary as a function of the ABP’s position $\mathbf{r} = [x(t), y(t)]$ according to a checkerboard pattern as follows:

\[
\gamma_R(\mathbf{r}) = \gamma_R^H - \frac{\gamma_R^L}{2} \left[ \text{sgn} \left[ \sin \left( \frac{\pi x}{L} \right) \sin \left( \frac{\pi y}{L} \right) \right] + \gamma_R^L \right]
\]

where:

\[
\text{sgn}(x) = \begin{cases} 
1, & x \geq 0 \\
-1, & x < 0 
\end{cases}
\]

and $\gamma_R^H$ and $\gamma_R^L$ correspond to the regions of high and low $D_R$, respectively.

For the implementation of the discrete time-feedback loop using the ZOH model, we update $\gamma_R(\mathbf{r})$ every $t = n\tau$, where $n$ is the number of samples. Since the rotational diffusivity is a physical quantity that must be continuous in time, we reconstruct it from discrete-time inputs using the following function:

\[
\gamma_R(\mathbf{r}, \tau) = \sum_{n=0}^{\infty} \gamma_R[n] \Pi(t - n\tau),
\]

where $\gamma_R[n]$ is $\gamma_R(\mathbf{r})$ evaluated at $\mathbf{r} (t = n\tau)$,

\[
\gamma_R[n] = \int_{-\infty}^{+\infty} \gamma_R(\mathbf{r}) \delta(t - n\tau) dt,
\]

and $\Pi$ is a rectangular function defined as

\[
\Pi(t) = \begin{cases} 
1, & 0 \leq t < \tau \\
0, & \text{otherwise} 
\end{cases}
\]

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