Protostellar spin-down: a planetary lift?

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ABSTRACT
When they first appear in the HR diagram, young stars rotate at a mere 10% of their break-up velocity. They must have lost most of the angular momentum initially contained in the parental cloud, the so-called angular momentum problem. We investigate here a new mechanism by which large amounts of angular momentum might be shed from young stellar systems, thus yielding slowly rotating young stars. Assuming that planets promptly form in circumstellar disks and rapidly migrate close to the central star, we investigate how the tidal and magnetic interactions between the protostar, its close-in planet(s), and the inner circumstellar disk can efficiently remove angular momentum from the central object. We find that neither the tidal torque nor the variety of magnetic torques acting between the star and the embedded planet are able to counteract the spin up torques due to accretion and contraction. Indeed, the former are orders of magnitude weaker than the latter beyond the corotation radius and are thus unable to prevent the young star from spinning up. We conclude that star-planet interaction in the early phases of stellar evolution does not appear as a viable alternative to magnetic star-disk coupling to understand the origin of the low angular momentum content of young stars.

Key words: Stars: formation – Stars: rotation – Accretion, accretion disks – giant planet formation – Magnetohydrodynamics (MHD).

1 INTRODUCTION

It has long been known that the Sun and its siblings are characterized by quite modest rotational velocities, of order of a few km/s at most (e.g. Kraft 1970). This stands in sharp contrast to more massive stars that exhibit much higher spin rates all over their evolution. Schatzman (1962) first theorized that magnetized winds from solar-type stars would carry away a significant amount of angular momentum, thus braking the stars quite efficiently. Hence, regardless of their initial velocity, stars with outer convective envelopes would end up as slow rotators on the main sequence. This expectation was later confirmed by Skumanich (1972) who showed that the rotational velocity of solar-type stars appeared to steadily decrease on the main sequence, following the well-known $\Omega \propto t^{-1/2}$ relationship.

Extrapolating this relationship back in time to the early pre-main sequence (PMS), young stars were thus expected to be fast rotators at birth. More generally, as gravity dominates the late stages of protostellar gravitational collapse, newly-born stars were commonly thought to rotate close to break-up velocity. Surprisingly, the first measurements of spin rates for low-mass PMS stars, the so-called T Tauri stars (TTS), did not meet these expectations. On the contrary, young stars were found to have only moderate rotational velocities, on average about 10 times that of the Sun’s, i.e., a mere 10% of their break-up velocity (Vogel & Kuhfu[1981] Hartmann et al.[1986] Bouvier et al.[1986]). More than 30 years later, this aspect of the so-called “initial angular momentum problem” remains very much vivid.

Several physical processes have been proposed to account for the slow rotation rates of young solar-type stars. They all rely on the magnetic interaction between the young star and its circumstellar disk. At least 3 classes of models can be identified: X-winds (Shu et al.[1994]), accretion-powered stellar winds (Matt & Pudritz [2005]), and magnetospheric ejections (Zanni & Ferreira [2013]). All these models investigate how the angular momentum flux between the star and its surrounding is modified by the magnetospheric interaction with the inner accretion disk. A brief outline of these models and a discussion of their specific issues can be found in Bouvier et al.[2014]. While a combination of processes may eventually provide strong enough braking torques on young stars to account for their low spin rates, none have definitely proved to be efficient enough.

We investigate here an alternative process to account for the low angular momentum content of young stars. Specifically, we explore the flux of angular momentum being exchanged within a system consisting of a magnetic young stars surrounded by a close-in orbiting planet embedded in the inner circumstellar disk. We re-
view tidal and magnetic interactions between the protostar and a close-in planet to estimate whether spin angular momentum can be transferred from the central star to the planet’s orbital momentum and eventually from there to the disk by gravitational interaction, thus effectively spinning down the central star. Previous papers have extensively explored star-disk, star-planet, and planet-disk interactions, usually with the aim to investigate the orbital evolution of close-in planets (e.g. Zhang & Penev 2014; Laine et al. 2008; Laine & Lin 2012; Chang et al. 2010, 2012; Lai 2012). We build here from these earlier studies to explore the star-planet-disk interaction in the specific framework of the initial angular momentum problem. Indeed, while addressing the issue of halting planet migration near young stars through magnetic torques, Fleck (2008) mentions that such interactions may be at least partly responsible for the slow rotation rates of PMS stars. We will show here that, for realistic sets of stellar and planetary parameters, magnetic as well as tidal torques seem actually unable to prevent pre-main sequence stellar spin-up.

In Section 2 we describe the general idea developed here, summarize the parameters of the system, and express the requirement for an effective protostellar spin down. In Section 3 we summarize the accelerating torques on the central star, namely accretion and contraction, that tend to spin it up. In Section 4, we explore both tidal torques and magnetic torques acting between the young star and the inner planet and review their efficiency in removing angular momentum from the central object. We discuss the quantitative torque estimates and the limits of the model in Section 5, and highlight our conclusions in Section 6.

2 STAR – PLANET – INNER DISK INTERACTION (SPIDI): A GENERAL FRAMEWORK

The general idea that we develop here is to investigate whether a fraction of the spin angular momentum of the star can be transferred to the orbital angular momentum of a close-in planet, which in turn could lose its excess angular momentum to the disk through tidal interaction. In this way, the planet would act as a “lift” carrying angular momentum from the central star to the outer disk (cf. Fig. 1). We aim here at computing an equilibrium solution that would fulfill this requirement, and hence prevent the star from spinning up.

In the following subsections we compute the torques acting in a system consisting of a young star, its circumstellar disk, and a close-in orbiting planet. The parameters of the system are listed in Table 1. To set orders of magnitude, we compute the specific angular momentum of the star, \( J_\star \), and of the planet, \( J_{pl} \).

The specific spin angular momentum of the central star is

\[
J_\star = \frac{J}{M_\star} = \frac{k^2 R_\star^2 \Omega_\star}{M_\star},
\]

with \( k \approx 0.205 \) for a completely convective star, which yields \( J_\star = 5.8 \times 10^{16} \text{ cm}^2\text{s}^{-1} \). The specific orbital angular momentum of a non-eccentric planet orbiting at a distance \( a \) from the star is:

\[
J_{pl} = \frac{J_{pl}}{M_{pl}} = (G M_\star a)^{1/2}
\]

which is of order of a few \( 10^{18} \text{ cm}^2\text{s}^{-1} \) for an orbital semi-major axis of a few stellar radii. The specific angular momentum of the planetary orbit is 2 orders of magnitude larger than the specific angular momentum contained in the stellar spin. However, a Jupiter-mass planet orbiting at the corotation radius has nearly the same total angular momentum as the star,

\[
\frac{J_{pl}}{J_\star} = \frac{M_{pl} (G M_\star a)^{1/2}}{k^2 M_\star R_\star \Omega_\star} = \frac{M_{pl}}{k^2 M_\star} \left( \frac{\Omega_{br}}{\Omega_\star} \right)^{4/3} \approx 0.2,
\]

for \( a = r_{mig} = \Omega_\star^{-2/3} (G M_\star)^{1/3}, \) the break-up velocity \( \Omega_{br} = (G M_\star)^{1/2} R_\star^{3/2}, \) and the system parameters defined in Table 1.

3 SPIN-UP TORQUES: YOUNG STARS OUGHT TO ROTATE FAST

In this section, we describe torques that act to spin up young stars as they start their evolution on Hayashi tracks, namely the accretion and the contraction torques.

3.1 The accretion torque

A magnetic young star accreting from its circumstellar disk assumed to be in Keplerian rotation undergoes a spin-up torque expressed as

\[
T_{\text{acc}} = \dot{M}_{\text{acc}} a R_{\text{Kep}} (r_\text{f}) \Omega_{\text{Kep}} (r_\text{f}) = \dot{M}_{\text{acc}} (G M_\star r_\text{f})^{1/2},
\]

where \( \dot{M}_{\text{acc}} \) is the mass accretion rate onto the star, \( r_\text{f} \) is the inner disk truncation radius, \( \Omega_{\text{Kep}} \) the Keplerian velocity in the disk, and \( M_\star \) the stellar mass.

For the system parameters listed in Table 1, the alteration of the protostellar disk truncates the disk at a distance of

\[
r_\text{f} = 2R_\star^{17/2} M_{\text{acc}}^{-2/7} M_\star^{-1/7} R_\star^2 \approx 3.6 \times 10^{11} \text{ cm} = 5R_\star = 2.5R_\star.
\]
where \( B_\ast \) is the star’s magnetic field and \( R_\ast \) the stellar radius (Bessolaz et al. 2008).

Alternatively, assuming that the truncation radius is located close to the corotation radius in the disk
\[
t_r \approx r_{\text{co}} = \Omega_\ast^{-2/3}(GM_\ast)^{1/3} = 12R_\odot = 6R_\ast,
\]
with \( \Omega_\ast \) the star’s angular velocity.

For these values of the truncation radius, the accretion torque amounts to \( T_{\text{acc}} = 4.3\text{–}6.7 \times 10^{33} \text{ g cm}^2 \text{ s}^{-2} \). The spin up timescale of the central star due to accretion is
\[
\tau_{\text{acc}}^{\text{up}} = \frac{J_*}{T_{\text{acc}}} = \frac{k^2 M_\ast^2 R_\ast^6 \Omega_\ast}{M_{\text{acc}}(GM_\ast r_t)^{1/2}},
\]
with the stellar angular momentum \( J_* = 1.2 \times 10^{40} \text{ g cm}^2 \text{ s}^{-1} \). Using the system parameters listed in Table 1, the spin up timescale thus amounts to 7 Myr and reduces to 0.7 Myr for a protostellar mass accretion rate of \( 10^{-5} M_\odot \text{ yr}^{-1} \). Hence, as shown by Hartmann & Stauffer (1989), accretion from the circumstellar disk is expected to spin up the star to a significant fraction of its break-up velocity within a few Myr.

### 3.2 The contraction torque

As newly-born stars descend their Hayashi track, their radius decreases and they eventually develop a radiative core. Both effects yield a reduction of the stellar moment of inertia, and if angular momentum is conserved, the star spins up. Taking into account only radius contraction during the first few Myr, the fully convective star can be described as a \( n = 3/2 \) polytrope.

The potential energy of a polytrope of index \( n \), is given by
\[
E_{\text{pot}} = -\frac{3}{5-n} \frac{GM_\ast^2}{R_\ast},
\]
and the luminosity of a star undergoing homologous contraction is
\[
L_\ast = \frac{(3\gamma - 4)}{(3\gamma - 1)} \frac{dE_{\text{pot}}}{dt} = 4\pi R_\ast^2 \sigma T_{\text{eff}}^4.
\]

The torque that should be applied to prevent the star from spinning up as it contracts down on the pre-main sequence on a Kelvin-Helmholtz timescale is
\[
T_{KH} = \Omega_\ast \frac{dR_\ast}{dt} = 2k^2 M_\ast \Omega_\ast R_\ast \frac{dR_\ast}{dt},
\]
and combining equations 8 and 10 with \( n = 3/2 \) and \( \gamma = 5/3 \), one finally gets
\[
T_{KH} = \frac{14}{3} k^2 \Omega_\ast R_\ast^3 \frac{GM_\ast}{L_\ast}.
\]

With the reference parameters listed in Table 1 and assuming \( T_{\text{eff}} = 4000 \text{ K} \), one finds \( T_{KH} \approx 5 \times 10^{35} \text{ g cm}^2 \text{ s}^{-2} \). Hence, stellar contraction is equivalent to an accelerating torque whose magnitude is similar to that of the accretion torque. Both contraction and accretion will thus equally act to spin the young star up.

### 4 SPIN DOWN TORQUES: CAN THEY PREVENT FAST ROTATION?

In this section, we investigate the torque a planet orbiting outside the corotation radius would exert on the central star. Such a planet would extract angular momentum from star and migrate outwards. At the same time, the planet feels the tidal torque from the disk through Lindblad resonances and tends to migrate inwards. An equilibrium configuration could result if the torques acting on the planet balance outside the corotation radius, thus ensuring a continuous transfer of angular momentum from the star to the planet and from the planet to the disk (cf. Fig. 1). If the outwards net flux of angular momentum can counteract the accretion and contraction torques, the star will thus be prevented from spinning up. Whether such an equilibrium configuration can be reached, and whether it is able to effectively spin the star down, depends on the nature and strength of the star-planet-disk interaction. We first discuss tidal effects, and then turn to magnetic interactions.

### 4.1 Tidal torques

We investigate whether tidal torques are strong enough to allow the orbiting planet to extract spin angular momentum from the star and to pass it on to the outer disk. In order to spin the star down, the planet must be located beyond the corotation radius. The inward migration torque the disk-embedded planet experience must therefore be balanced by the star-planet tidal torque beyond the corotation radius for the lift to operate.

The migration torque exerted by the disk on the planet can be expressed as
\[
T_{\text{disk}} = \frac{dJ_{pl}}{dt} = \frac{1}{2} M_{pl}(GM_\ast)^{1/2} \frac{da}{dt} + \frac{J_{pl}}{\tau_{\text{acc}}},
\]
where \( \tau_{\text{acc}} \) is the planet’s migration timescale in the inner disk.

We may first verify that the magnitude of the migration torque outside the corotation radius would be able to compensate for the accretion and contraction torques. The condition can be written as
\[
|T_{\text{disk}}(a \gg r_{\text{co}})| \geq |T_{\text{acc}} + T_{KH}| \implies 2|T_{\text{acc}}|.
\]

Using the above expressions for the accretion and migration torques, and assuming the inner disk is located at the corotation radius (cf. Eq. 5), this translates to
\[
\frac{a}{r_{\text{co}}} \geq \left( \frac{2\tau_{\text{acc}} M_{\text{acc}}}{M_{pl}} \right)^{2/3}
\]
which, adopting parameters from Table 1 yields \( a \gg 4r_{\text{co}} \). Hence, if a giant planet can be hold off beyond the corotation radius, it may effectively extract enough angular momentum so as to compensate for the accretion and contraction torques, thus preventing the central star from spinning up.

Is the tidal effect between the star and the planet strong enough to prevent the planet from migrating inwards of the corotation radius? The tidal torque is given by
\[
T_{t} = \frac{9}{4} \left( \frac{M_{pl}}{M_\ast + M_\ast} \right) \Omega_{pl}(\Omega_{pl} - \Omega_\ast) \frac{R_\ast^3}{Q_\ast a^3},
\]
where the migration rate of the planet due to the tidal interaction with the star is given by:
\[
\frac{\left( \frac{da}{dt} \right)}{a} = \text{sign}(\Omega_\ast - \Omega_{pl}) \left( \frac{G}{aM_\ast} \right)^{1/2} \frac{R_\ast^{3/2}}{Q_\ast} M_{pl},
\]
where \( \Omega_{pl} \) is the planet orbital rotation rate, \( Q_\ast \) is the tidal dissipation factor (e.g. Zhang & Pencre 2014). The sign of the migration

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1. The factor of 2 difference between Eq. 15 above and Eq. 18 of Chang et al. (2010) arises from their use of the apsidal motion constant which amounts to half the tidal Love number used to estimate the tidal quality factor \( Q_\ast \) (cf. Madling & Lin 2002).
depends on whether the planet orbits within or beyond the corotation radius. The migration induced by the disk is always inwards
\[
\left(\frac{da}{dt}\right)_{\text{disk}} = -\frac{a}{\tau_{\text{mig}}}. \tag{17}
\]
In order to slow the star down, the planet must orbit beyond the corotation radius. Hence, the planet’s inward migration must be stopped before it reaches the corotation radius. We must then have
\[
\left(\frac{da}{dt}\right)_{\text{star}} > \left|\left(\frac{da}{dt}\right)_{\text{disk}}\right| \tag{18}
\]
for \(r > r_{\text{co}}\), all the way down to the corotation radius. Combining equations \(16\) and \(17\), we compute the critical semi-major axis at which the disk and stellar torques have the same amplitude on the planet (see also \cite{Lin1996}).
\[
a_{\text{crit}} = \left(\frac{9M_{\odot} T_{\text{disk}}}{2Q_s}\right)^{2/13} \left(\frac{G}{M_\star}\right)^{1/13} R_\star^{10/13}. \tag{19}
\]
At a semi-major axis larger than \(a_{\text{crit}}\), the planet will migrate inwards, while for distance smaller than \(a_{\text{crit}}\), the migration will be outwards. In order to ensure outwards migration at or beyond the corotation radius, as required to brake the central star, \(a_{\text{crit}}\) must thus to be larger than \(r_{\text{co}}\). Using parameters listed in Table \ref{tab:parameters} and \(Q_s = 10^3\), one finds \(a_{\text{crit}} = 2.4 \times 10^{11} \text{ cm} = 1.7R_\star = 0.3r_{\text{co}}\). The magnitude of the tidal torque thus appears unable to prevent the planet from migrating inward the corotation radius, at which point it will contribute to spin up the star. Indeed, the tidal torque decreases very steeply as distance as seen from Eq. \ref{eq:tidal_torque}. Its amplitude at the corotation radius is about 4 orders of magnitude weaker than required to balance the migration and accretion torques (cf. Fig. \ref{fig:planet_migration}), a result which is barely sensitive to the parameters of the system (cf. Eq. \ref{eq:crit}.

### 4.2 Magnetic torques

#### 4.2.1 Models and kind of magnetic interactions

In this section, we aim at estimating orders of magnitude of the magnetic torques exerted by the planet on the star. To do so, we model the planet and the star as objects with a given radial profile of electrical conductivities, possibly generating magnetic fields. Naturally, this also models magnetic interactions between a moon and its planet, such as Io and Jupiter. The following is thus valid for star-planet or moon-planet magnetic interactions.

In the simplest case considered by some authors \cite{Laine2008, Chang2010, Chang2012}, vacuum is assumed between the star and the planet. If the star and the planet both generate a dipolar field, the magnetic torque is simply given by the cross product of the magnetic moment with the magnetic field produced by the other dipole (which tends to align the magnetic moments). Torque can also come from dissipative effects. Typically, any time-variation of the magnetic field in an electrically conductive domain will generate eddy currents, leading to Joule dissipation and torques. This is the so-called Transverse Electric (TE) mode \cite{Laine2008}, and the associated torques are e.g. studied by \cite{Chang2012} in the case of a planet orbiting in a tilted stellar magnetic field, which generates eddy currents in the conducting planet.

Actually, the space between the star and the planet is rather filled by a stellar wind originating from the star, which can be roughly modeled as an electrically conducting fluid in motion. This has, in particular, two consequences: (i) the stellar magnetic field is adverted by the stellar wind, which is the so-called interplanetary magnetic field (IMF), and decays thus less rapidly than in vacuum, and (ii) waves and currents can exist between the star and the planet allowing new kinds of magnetic interactions. The former point leads to a stronger TE mode in the planet, due to the higher time-varying magnetic field advected by the stellar wind. The latter point requires to investigate the interaction between the planet and the surrounding magnetized flow of the stellar wind.

The star-planet interaction via the stellar wind can be of various kinds, depending of the planet’s velocity \(v_{\text{orb}}\) relative to the stellar wind (SW) local one \(v_{\text{sw}}\), the speed \(v_{\text{A}}\) of the fastest wave in the stellar wind (i.e. the so-called fast magnetosonic wave), and the speed \(v_{\text{A}}\) of the shear (or intermediate) Alfvén wave. First, if the (fast) Alfvén Mach number \(M_{\text{A,f}} = ||v_{\text{orb}} - v_{\text{sw}}||/v_{\text{A,f}}\) is larger than 1, the obstacle constituted by the planet generates a shock wave. In this so-called super-Alfvénic case, the flow is controlled upstream and disturbances are transmitted downstream. This is the case for Venus for instance, where the shock wave takes the usual form of a (detached) bow shock due to its bluff (i.e. non sharp-nosed) geometry. In the case where the planet generates its magnetic field, like the Earth, this changes the apparent radius of the obstacle (aka the magnetosphere radius) for the stellar wind, leading to a enhanced coupling. This case of magnetic interaction between the stellar wind and the planet is called magnetospheric interaction by \cite{Zarka2007}.

Second, we consider the sub-Alfvénic case \((M_{\text{A,f}} < 1)\), focusing first on the simple case where the planet is weakly magnetized or unmagnetized. In this case, the planet motion generates shear (or intermediate) Alfvén waves which propagate in the stellar wind and transport some energy from the planet to the star. The wave can then be reflected back, from the star to the planet, which leads to a planet-star interaction. Since the disturbance is only partially reflected, each subsequent reflection is of lower amplitude, and results in a lesser change to the current system \cite{Crary1997}. After several round-trip travel times, the system reaches a steady state, and a current loop is settled. In this case, the star-planet interaction can be modeled as a DC circuit \cite{Goldreich1969}, which is called the unipolar inductor model or the TM mode \cite{Lange2013}. In the other limiting case, the planet has moved away when the wave comes back and the planet’s interaction is thus decoupled from the star. This is the so-called Alfvén wings (or Alfvénic interactions) model (e.g. \cite{Saur2004}). These two models, sometimes presented as two different kinds of interaction, originates actually from the same phenomena and have been primarily applied to the Io-Jupiter interaction, but also to planets around magnetic dwarfs, ultra compact WD binaries, exoplanetary systems, etc.

In the regime \(M_{\text{A,f}} < 1\), we finally have the case where both the planet and the star generate a magnetic field. In the stellar system, the only example of such a case is the Ganymede-Jupiter interaction, which constitutes a textbook example of the expected plasma environment around close-in extrastellar planets \cite{Saur2014}. This kind of interaction is often termed the dipolar interaction (e.g. \cite{Zarka2007}).

#### 4.2.2 A generic torque formula

Focusing on dissipative torques, one can link the dissipated power \(P_d\) with the star-planet dissipative torque \(T\) by
\[
P_d = T||v_{\text{pl}} - v_{\text{sw}}||/\alpha. \tag{20}
\]
where $a$ is the star-planet distance, $v_{pl}$ is the planet velocity ($v_{pl} \approx \Omega_p a$) and $v^p$ is the advective velocity of the magnetic field at the planet orbit (i.e., the SW velocity $v^p = v^m$, or $v^p = \Omega_p a$ in absence of SW flow). Dimensional arguments allow to write any torque exerted on the planet by the star as

$$T = \epsilon a A_{eff}^2 P_{eff}^m,$$  \hspace{1cm} (21)

where $\epsilon$ is a dimensionless coefficient, $a$ is the star-planet distance, $A_{eff}$ is the typical surface area on which the star-planet magnetic coupling is effective, and $P_{eff}^m$ is the typical energy density of the coupling on the area $A_{eff}$ (which can be seen as the ram pressure of the stellar wind when a stellar wind is considered). Focusing only on the magnetic coupling, this formula reduces to

$$T = \epsilon a A_{eff} B_{eff}^2 = \epsilon a \pi R_{obs}^2 B_{eff}^2 / \mu_0,$$  \hspace{1cm} (22)

where $B_{eff}$ is a typical magnetic field value exerted on the area $A_{eff}$, $\mu_0$ is the magnetic permeability of the vacuum, and $R_{obs} = \sqrt{A_{eff}/\pi}$ is the typical radius of $A_{eff}$.

Equation (22) requires the knowledge of $\epsilon$, $B_{eff}$ and $A_{eff}$. The expression of $\epsilon$ depends on the magnetic interaction we consider, which is discussed in section 4.2.3. The calculation of magnetic field $B_{eff}$ depends on the medium between the planet and the star. If this is vacuum, then $B_{eff}$ is simply deduced from the Gauss coefficients of the stellar magnetic field. If a stellar wind exists, $B_{eff}$ is simply given by the model chosen for the stellar wind. Finally, $A_{eff}$ depends on the planet. In absence of any external conductive layer, $R_{obs} = R_{pl}$, which is thus a lower bound for $R_{obs}$. However, a planet has usually an ionosphere due to photoionization of the upper atmosphere (even without any own magnetic field), which leads to an induced magnetosphere above the ionosphere. This also means that the time-varying stellar magnetic field can be shielded by the ionosphere such that induced eddies currents in the planetary interior is small. Note that, as argued by Chang et al. (2012), atmosphere circulation may maintain this so-called exo-ionosphere on the permanent nightside of a tidally locked planet. In this case, the radius $R_{obs}$ to consider is the exo-ionospheric radius, which is typically $R_{ino} = 1.1 - 1.4 R_{pl}$ in the case of Io (Zarka 2007). Finally, the planet can also have a own magnetic field, either stored in its rocks, or generated by dynamo. In this case, the planet develops its own magnetosphere. The magnetopause radius $R_{MP}$ is then the apparent radius $R_{obs}$ to consider, which can be estimated by the balance between the total ram pressure $p_{ram}$ coming from the star and the planetary magnetic field $B_{pl}$:

$$R_{MP} = R_{pl} \left[ \frac{B_{pl}^2}{2 \mu_0 p_{ram}} \right]^{1/3},$$  \hspace{1cm} (23)

where $m$ depends on the planetary magnetic field geometry ($m = 3$ for a mainly dipolar magnetic field, $m = 4$ for a mainly quadrupolar magnetic, field, etc.), and where

$$p_{ram} = \frac{1}{2} \frac{m}{\sigma} v_{pl}^2 v^m + \frac{1}{2} \frac{B_{eff}^2}{\mu_0}. $$ \hspace{1cm} (24)

Finally, $R_{obs}$ is thus simply given by

$$R_{obs} = \max (R_{pl}, R_{ino}, R_{MP}). $$  \hspace{1cm} (25)

Particular forms of the generic formula (22) has already been considered in previous works. For instance, neglecting $v_{pl}$ in equation (20) allows to recover equation 9 of Zarka (2007), where $B_{eff} = B_\parallel$ is the IMF component perpendicular to the SW flow in the planet’s frame. Similarly, equation (9) of Chang et al. (2010) is recovered with equation (22), assuming no stellar wind and a dipolar stellar field, such that $B_{eff} = B_\parallel(R_\star/\alpha)^3$, and considering $R_{obs} = \sqrt{2} R_{pl}$.

As stated by Zarka (2007), this general expression is simply the fraction $\epsilon$ of the magnetic energy flux convec ted on the obstacle (aka the planet), and is expected to provide a correct order of magnitude whatever the interaction regime (unipolar or dipolar, super- or sub-Alfvenic as long as the obstacle conductivity is not vanishingly small. The physics of the interaction is now hidden in the coefficient $\epsilon$, which is the difficult estimate we have to obtain.

### 4.2.3 Application of the formula

In this work, we aim at studying if the star can be slowed down by planetary magnetic torque. Since the super-Alfvenic regime only allows disturbances to be transmitted downstream, this requires to consider sub-Alfvenic regime such that a torque can be exerted by the planet on the star. Given that the stellar wind accelerates when flowing away from the star, this imposes to consider orbital distance $a$ smaller than the so-called Alfven radius given by Bessolaz et al. (2008)

$$\frac{\epsilon}{\alpha} = \left( \frac{B^4 R^6 \epsilon^2}{2GM_\star M^2_{\alpha}} \right)^{1/7} = 12R_\star,$$ \hspace{1cm} (26)

distance at which the SW flow becomes super-Alfvenic. We have thus to estimate torques in four cases, the TE mode, the Alfven wings model), the dipolar interaction, and the unipolar inductor model.

**Torque due to the TE mode:** In this case, the torque is associated with the Joule dissipation in the interior of the planet, originating from the stellar magnetic field oscillating at the rate $\omega$. Owing to the diffusive nature of the problem, one can thus estimate the Ohmic dissipation by Campbell (1997, 1983; Laine et al. 2008; Chang et al. 2012):

$$P_{ohmic} = \epsilon \omega \frac{B_{eff}^2}{2\mu_0}, $$ \hspace{1cm} (27)

where $\epsilon$ is the volume where eddy currents take place. This is thus typically a thin shell of thickness $\delta$, which gives $\epsilon = 4\pi R^3 \delta^3$ if it takes place at the surface of an electrically conducting planet. Noting $\eta$ the typical planetary electrical diffusivity, $\delta = (2\eta/\omega)$ is then the skin depth for our magnetic induction problem.

Finally, considering that $\omega \sim \Omega_\star$, an upper bound of the torque is obtained for $\epsilon = 4\pi/3 \cdot R^3_\star$, and we will thus use equation (22) using $R_{obs} = R_{pl}$ and

$$\epsilon = 2 \frac{R_{pl}}{3 a}. $$ \hspace{1cm} (28)

**Alfven wings:** As shown by Drel et al. (1965), the radiation of two pure linear Alfven wings leads (when converted in S.I. units) to a torque given by formula (22), with the coefficient (see also Lat 2012)

$$\epsilon = 2 M_\alpha, $$ \hspace{1cm} (29)

Using the expression of the Alfven radius from Matt & Prudatz (2008)’s stellar wind models (their Eq.14) would result in twice as large an estimate.

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\[ \epsilon = \frac{2 M_A}{||M_A \hat{v}_{\text{rel}} + \hat{B}||} \]  

where the flow is perpendicular to the magnetic field ($\hat{v}_{\text{rel}} \cdot \hat{B} = 0$) leads then to (Zarka 2007)

\[ \epsilon = \frac{1}{\sqrt{1 + 1/M_A^2}}, \]  

for the Alfven wings interaction. Note that, in this case, \( R_{\text{obs}} \) is of the order \( R_{\text{obs}} \sim R_{\text{pl}} \), even in the presence of an exo-ionosphere. Note also that Alfven waves cannot be radiated when \( r > r_A \), and the Alfven wings interaction is thus only valid for \( r < r_A \).

Dipolar or magnetospheric interaction: As argued by Zarka (2007), the torques associated to these interactions can also be modeled by formula (22), using \( \epsilon = 0.1 - 1 \) (typically, \( \epsilon = 0.3 \) in the sub-Alfvenic Ganymede-Jupiter interaction). Note that \( \epsilon = 1 \) is in agreement with equation (31), which gives \( \epsilon \sim 1 \) for \( M_A \gg 1 \). The important feature of this kind of interaction is the presence of a own
planetary magnetic field, which can lead to large magnetic interactions (due to large $R_{\text{obs}}$).

Unipolar inductor model (DC circuit or TM mode): In this case, first studied by Goldreich & Lynden-Bell (1969), the Alfvén waves round trips between the star and the planet allow to reach a steady state, which can be modeled as a DC circuit. In the planetary frame, the electrical field $E = v \times B$ generated by the planetary velocity $v$ allows indeed to close a DC circuit through the surrounding electrically conductive medium. An electrical current is thus flowing through the electrical resistances of the planet, $R_p$, of the stellar footprint of the planet, $R_*$, and of the two flux tubes crossing the stellar wind, $2R_f$.

The difficult estimation of the three electrical resistances is performed here by following Laine & Lin (2012); Laine (2013), who neglect $R_f \ll R_p, R_*$, According to the formula obtained by Laine & Lin (2012); Laine (2013), we thus end up with

$$\epsilon = \frac{8 \mu_0}{\pi} \frac{\Omega_{\text{pl}} - \Omega_*}{(R_p + R_*)}$$

(32)

One can distinguish different regimes of closures of the DC circuit (Laine 2013). In the so-called favorable DC circuit closure regime, the Alfvén wave have the time to perform the round-trip planet-star-planet, which actually imposes here that the planet is on an orbit below a certain distance $r_\pi$. In our model, $r_\pi = 8 R_\odot$. Analytical estimates of $r_\pi$ are derived in Appendix A.

Note that the DC circuit is naturally opened for $r > r_\pi$. Note also that it exists an upper limit to the magnetic interaction torque generated using the unipolar inductor model (Lin 2012). Indeed, when the circuit resistance is too small, the large current flow severely twists the magnetic flux tube connecting the two binary components, leading to the breakdown of the circuit. According to Lai (2012), this breakdown appears when

$$\zeta = \frac{4 \mu_0}{\pi} \frac{\Omega_{\text{pl}} - \Omega_*}{R_p + R_*} = \frac{\epsilon}{2} > 1,$$

(33)

which limits $\epsilon$ to $\epsilon = 2$ (corresponding to $\zeta = 1$ in Lai 2012).

### 4.2.4 Results

In order to calculate the various torques estimates give above, a stellar wind model is required. In this work, we have considered two stellar wind models. One is adapted from Zarka (2007), as detailed in Appendix B, and the other one is described in Lovelace et al. (2008). Both give similar results as far as magnetic torques are concerned (cf. Appendix B). Using the stellar wind model adapted from Zarka (2007), the results are summarized in Figures 2 and compared with the tidal torque. Note that the magnetic torque associated with the DC circuit depends on the chosen planetary conductivity. It turns out that above $\sigma_{\text{pl}} = 10^3$ S/m (as used in Fig. 2), the resistance is very small ($\zeta \gg 1$), and the torque is thus maximum, given by $\epsilon = 2$.

Fig. 2 clearly illustrates the main result of this study. None of the planet-induced torques acting on the central star, be it tidal or magnetic, is able to balance the accretion torque beyond the corotation radius. Hence, at least with the parameters adopted here, the star-planet interaction cannot prevent the spin up of the central star as it accretes from its circumstellar disk and contracts down its Hayashi track. Indeed, even the most powerful torques, corresponding to the dipolar interaction and the unipolar inductor model, fail by several orders of magnitude to match the magnitude of the migration torque beyond the corotation radius. We have also compared these results to Strugarek et al. (2014) 2.5D numerical simulations and find that their parametric torque formulation predicts order of magnitude torques similar as the DC and dipolar cases investigated here (see Appendix C for a short account of their parametric torque formulation).

One may wonder why the magnetic coupling seems to be efficient enough in the similar study of Fleck (2008), in contrast to our results. This is partly due to the fact that he prescribes the cross-sectional area $A_{\text{eff}}$ of the magnetic flux tube linking a planet to its host star to be in the range $\alpha = 1 - 10\%$ of the stellar surface. Here, we define $A_{\text{eff}}$ self-consistently and show that it is of order of magnitude of $\pi R_{\text{obs}}^2$, where $R_{\text{obs}} \approx R_\odot$ when the planet is close from the star, and thus $\alpha = \pi R_{\text{obs}}^2/4 \pi R_\odot^2 \approx 0.2\%$ with our values. Also, Fleck (2008) assumed a stronger magnetic field (2 kG) and a larger stellar radius (3$R_\odot$) than we adopted here, which results in a much stronger torque (cf. his Eq. 14). Indeed, a 3 kG field associated to a 4 $R_\odot$ protostar would be required to obtain a torque comparable to the accretion and migration torques.

### 5 DISCUSSION

We have considered the various torques acting in a system consisting of a contracting pre-main sequence star still accreting from its circumstellar disk into which a newly-formed Jupiter-mass planet is embedded on a short-period orbit. We have shown that a balance between accelerating torques acting onto the star, due to accretion and contraction, and decelerating ones, due to the star-planet interaction, cannot be reached under the conditions we explored here. For all cases we investigated, the accretion torque exceeds any planetary torque acting on the central star by orders of magnitude.

The main limitation of the scenario outlined here lies in the adopted wind model. The model is an extrapolation from a solar-type wind properties, which is not necessarily adapted to the wind topology of young stars. Firstly, the topology of the magnetic field of young stars might be quite different from that of a mature solar-type stars (e.g. Donati & Landstreet 2009), although current measurements suggest that fully convective PMS stars at the start of their Hayashi tracks host strong dipolar fields, of order of a few kG (Gregory et al. 2012; Johnstone et al. 2014). Secondly, as shown by Zanni & Ferreira (2009), the structure of the stellar magnetosphere might be significantly impacted by its interaction with the circumstellar disk. In particular, an initially dipolar magnetic field may evolve into a much more complex and dynamical topology. Whether this would significantly modify the torques associated to the magnetic star-planet interaction is yet unclear.

Another issue of course is whether the framework proposed here is applicable to the vast majority of young stars that rotate slowly as they appear in the HR diagram. This would require that planet formation be not only a common occurrence, but also that it is fast enough to impact the earliest stages of stellar evolution, and that it is quite dynamic so as to frequently send massive planets on inner orbits. Whether all these conditions are met around protostars is yet unknown. The recent ALMA image of the numerous gaps in the disk of the protostar HL Tau suggests that multiple planet formation may indeed proceed quite rapidly after protostellar collapse. The significant fraction of hot Jupiters found to revolve around their host star on a non coplanar orbit further suggests that dynamical interaction between forming planets in the protostellar disk may be efficient to scatter massive planets close to their parent star (Fabrycky & Tremaine 2007; Triaud et al. 2010; Morton &
and unfavorable DC circuit close-up regimes (see Eq. [32]). Indeed, noting $t_0 = 2 R_\text{pl}/||v_\text{pl} - v^w||$ the advection time of the unperturbed plasma to cross the planet’s diameter, and $t_{\lambda,\text{FT}}$ the Alfvén wave travel time between the planet and the top of the footprint, the condition for favorable close-up regime reads then $t_0 \gg t_{\lambda,\text{FT}}$. Estimating $t_{\lambda,\text{FT}}$ by the time $t_{\lambda,\text{FT}0}$ it takes for the Alfvén wave to travel along the flux tube until the stellar chromosphere, we obtain the following asymptotic estimates:

$$r_c = R_* \left( \frac{3B_* R_{\text{pl}}}{R_* \Omega_*} \frac{4\pi V_*}{\mu_0 M_*} \right)^{1/4},$$

(A1)

for slow rotators ($\Omega_{\text{pl}} \ll \Omega_*$ at $r = r_c$), and

$$r_c = R_* \left( \frac{9B_* R_*^2 R_{\text{pl}}^3}{GM_* \mu_0 M_*^2} \right)^{1/5},$$

(A2)

for fast rotators ($\Omega_{\text{pl}} \gg \Omega_*$ at $r = r_c$).

**APPENDIX B: STELLAR WIND MODEL ADAPTED FROM ZARKA (2007)**

The star we consider being not very different from the Sun, we have chosen to slightly adapt the solar wind model proposed by Zarka (2007). First, let remind roughly the various usual scaling laws at play in the solar wind, before detailing the exact model used in this work. At a certain distance $r$ to the Sun (beyond a few Sun radius), the radial SW velocity can be considered as constant, and thus, mass and magnetic flux conservation give a SW density $N$ and a radial field $B$, decaying as $1/r^2$, whereas other conserved quantities (e.g. angular momentum) give an azimuthal velocity $v_\phi^\wedge$ and magnetic field $B_\phi$ which decays as $1/r$ (e.g. Weber & Davis [1967] Belcher & MacGregor [1976]). Close from the Sun, $B_r$ is rather dipolar and decays thus as $1/r^3$ (which leads to a decay in $1/r^2$ of $B_r$, see eq. [B2]).

The model proposed by Zarka (2007) for the Sun and the solar wind is (in spherical coordinates)

$$B_r = \frac{A}{(r/R_\star)^2} \left( 1 + \frac{f - 1}{(r/R_\star)^{3/2}} \right),$$

(B1)

$$B_\theta = B_r \frac{\Omega r}{||v_\text{pl} - v^w||},$$

(B2)

$$B_\phi = 0$$

(B3)

with the constants $4 \leq f \leq 9$ (here taken equal to $f = 6.5$) and $\lambda \approx 1.5 \times 10^{-4} \text{T}$, and

$$N_r (\text{cm}^{-3}) = \alpha_1 (a/R_\star)^{-15} + \alpha_2 (a/R_\star)^{-9/2} + \alpha_3 (a/R_\star)^{-2}$$

(B4)

with $\alpha_1 \approx 3 \times 10^6 \text{ cm}^{-3}$, $\alpha_2 \approx 4 \times 10^6 \text{ cm}^{-3}$, and $\alpha_3 \approx 2.3 \times 10^5 \text{ cm}^{-3}$, which gives the SW density $\rho = 1.1 m_p N_r$ (with the proton mass $m_p \approx 1.673 \times 10^{-27} \text{ kg}$). In this model, the solar wind velocity $v_\wedge$ can be described by

$$v_\wedge r = v_\wedge \left( 1 - \exp \left( \frac{r/R_\star - 1.45}{5} \right) \right)$$

(B5)

$$v_\wedge r = 0,$$

(B6)

$$v_\phi = 0.$$

(B7)

with $v_\wedge = 385 \text{ km/s}$. Note that the values given by this expression of $v_\wedge$ are naturally quite close from the ones derived from $\rho$ by mass conservation (i.e. using $r^2 \rho$ $v_\wedge$ = const).

We have adapted this model to our star by multiplying $N_r$ by $M_*^3/R_*^2$, the magnetic field by $B_\star$, and $v_\phi$ by $\sqrt{B_\star}/R_*$. 

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**APPENDIX A: ANALYTICAL ESTIMATES OF $R_*$ FOR THE UNIPOLAR INDUCTOR MODEL**

Based on the work of Laine (2013), we have been able to obtain analytical estimates of $r_c$, the transitional radius between favorable
all these quantities being expressed in solar units (which allows to recover the initial solar wind model of Zarka (2007), i.e. $R_0 = 6.96 \times 10^6$ m, $M_0 = 1.99 \times 10^{30}$ kg, $2\pi/\Omega_0 = 27$ d, $M_0^* = 1.6 \times 10^{-14} M_\odot$ yr$^{-1}$, and $B_0 = 10$ G (these two latter values are actually obtained as outputs from the solar wind model of Zarka (2007).

Note finally that theoretical models, such as the one of Weber & Davis (1967), rather advocates an azimuthal magnetic field given by $B_\phi = B_0 (v_{\phi,0} - \Omega_0 r)/v_{\phi,0}$, as well as non-zero azimuthal velocity, typically given by co-rotation ($v_{\phi,0} = \Omega_0 r$) for $r < r_A$ and $v_{\phi,0} = \Omega_0 r_A$ for $r > r_A$. We have checked that this does not change our conclusions, e.g., when using the stellar wind model of Lovelace et al. (2008). This is expected since, in our case, $v_{\phi,0}$ is at most of the order of $v_{\phi,0}$, or smaller, which would thus even further reduce the magnetic torques.

APPENDIX C: RESULTS OF STRUGAREK ET AL. (2014)

According to Strugarek et al. (2014), the torque applied by the planet to the star is given by (see their Eq. 16)

$$
\tau = K \tau_w \left( \frac{B_\phi + b}{B} \right)^p \cos \left( \frac{\theta_0 - \Theta}{s} \right) \left( \frac{a/R_*}{3} \right)^{-3},
$$

(C1)

where $\tau_w = M_0^* \Omega_0 r_A^2$, where $b$, $p$, $t$, $\Theta$ and $s$ are given by their Table 3, values obtained for $a/R_* = 3$, and where the last factor on the right hand side comes from their proposed scaling in $a^{-3}$ (see their Eq. 28). We have added a supplementary factor $K$ to remind that their values are all obtained for a planetary radius $R_p/R_* = 0.1$, and the extrapolation to other planetary radius requires thus a correction $K$. We propose for instance $K = (R_p/R_*)^2/0.1^2$, since eq. (22) gives a torque in $R_{\odot}^2$, and eq. (23) gives $R_{\odot} \sim R_p$.

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