Research Article
Hamilton-Connected Mycielski Graphs *

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Jarnicki, Myrvold, Saltzman, and Wagon conjectured that if $G$ is Hamilton-connected and not $K_2$, then its Mycielski graph $\mu(G)$ is Hamilton-connected. In this paper, we confirm that the conjecture is true for three families of graphs: the graphs $G$ with $\delta(G) > |V(G)|/2$, generalized Petersen graphs $GP(n, 2)$ and $GP(n, 3)$, and the cubes $G^3$. In addition, if $G$ is pancyclic, then $\mu(G)$ is pancyclic.

1. Introduction

All graphs considered in this paper are simple and finite. For notations and terminologies not defined here, we refer to Bondy and Murty [1]. A spanning cycle (path) of a graph is called Hamilton cycle (Hamilton path). A graph which contains a Hamiltonian path between every two vertices of $G$ is called Hamilton-connected (HC). Mycielski [2] proved that the chromatic numbers of triangle-free graphs can be arbitrarily large by introducing a graph transformation as follows. For a graph $G$ on vertices $V = \{v_1, v_2, \ldots, v_n\}$, its Mycielski graph, denoted by $\mu(G)$, is the graph on vertices $X \cup Y \cup \{z\} = \{x_1, x_2, \ldots, x_n\} \cup \{y_1, y_2, \ldots, y_n\} \cup \{z\}$ with edges $yz_i$ for all $i$ and edges $x_ix_j, y_ix_j, x_iy_j$ for all edges $v_iv_j$ in $G$. In recent years, a number of papers are devoted to various properties of Mycielski graphs, such as Hamilton-connectedness, Hamiltonicity [3–7], total chromatic number [8, 9], circular chromatic number [10–14], and connectivity [15, 16]. Fisher et al. [4] obtained the following results.

Theorem 1 (see Fisher et al. [4]). The following results hold for a graph $G$:

(1) If $G$ is Hamiltonian, then $\mu(G)$ is Hamiltonian
(2) If $G$ is not connected, then $\mu(G)$ is not Hamiltonian
(3) If $G$ has at least two pendant vertices, then $\mu(G)$ is not Hamiltonian

Cheng, Wang, and Liu studied Hamiltonicity and Hamilton-connectedness in Mycielski graphs of bipartite graphs.

Theorem 2 (see Cheng et al. [3]). For a bipartite graph $G$, the following are true:

(1) If $\mu(G)$ is Hamiltonian, then $G$ is balanced
(2) If $\mu(G)$ is Hamiltonian, then $G$ has a Hamilton path

In 2017, Jarnicki et al. [17] established the following results for $\mu(G)$ being Hamilton-connected or not.

Theorem 3 (see Jarnicki et al. [17]). The following results hold for a graph $G$:

(1) If $G$ is an odd cycle, then $\mu(G)$ is Hamilton-connected
(2) If $G$ is a Hamilton-connected graph with order odd, then $\mu(G)$ is Hamilton-connected
(3) If $G$ is an even cycle, then $\mu(G)$ is not Hamilton-connected

They posed the following conjecture.

Conjecture 1 (see Jarnicki et al. [17]). If $G$ is Hamilton-connected and not $K_2$, then $\mu(G)$ is Hamilton-connected.

In this paper, we confirm that the conjecture is true for three families of graphs: the graphs $G$ with $\delta(G) > |V(G)|/2$,
generalized Petersen graphs $GP(n, 2)$ and $GP(n, 3)$, and the cubes $G^3$. In addition, if $G$ is pancyclic, then $\mu(G)$ is pancyclic.

2. Mycielski Factor

Let $G$ be a connected graph of order $n$ even, and $v_1 \in V(G)$. We call a connected spanning subgraph of $G$ to be a Mycielski factor starting at $v_1$ if it consists of an even number of odd cycles $C_1, \ldots, C_s$ (possibly $s = 0$) and an even cycle $C_{2s+1}$ with the chord (possibly empty), joined by $2s$ edges $e_1, \ldots, e_{s}$, where $e_i = \{v_{2i+1}, v_{2i+2}\}$ for each $i \in \{1, \ldots, 2s\}$ such that $v_{2i}v_{i+1} \in V(C_i)$ for each $i \in \{1, \ldots, 2s+1\}$, and the chord joins $v_{2s+1}$ and a vertex at distance even on $C_{2s+1}$.

Lemma 1. Assume that a graph $G$ is Hamilton-connected. If, for any $v \in V(G)$, there exists a Mycielski factor starting at $v$, then $\mu(G)$ is Hamilton-connected.

Proof. As in the assumption, let $G$ be HC. Trivially, $G$ has a Hamilton cycle. By Theorem 3 (2), $\mu(G)$ is HC if the order of $G$ is odd. So, it remains to tackle the case when the order is even. Let $V(G) = \{v_1, \ldots, v_{2n}\}$, where $n \geq 2$. Recall that $V(\mu(G)) = X \cup Y \cup \{z\}$. Take any two vertices $A, B \in V(\mu(G))$. We consider five cases in terms of the location of $A$ and $B$ in $X, Y$, and $z$.

$\Box$

Case 1. $A \in X$ and $B \in X$.

Without loss of generality, let $A = x_1$ and $B = x_{2n}$. Since $G$ is HC, there exists a Hamilton path $P$ connecting $v_1$ and $v_2$ in $G$. We shall find a Hamilton path of $\mu(G)$ depending on $P$ as follows. Zigzag up from $x_1$ until $y_{2n}$ is reached. Then, jump via $z$ to $y_1$, and zigzag right until $x_{2n}$ is reached. Formally, it is

$$x_1 - y_2 - x_3 - \cdots - y_{2n} - z - y_1 - x_2 - \cdots - x_{2n},$$

(1)

as shown in Figure 1.

Case 2. $A \in Y$ and $B \in Y$.

Without loss of generality, let $A = y_1$ and $B = y_{2n}$. Since $G$ is HC, there exists a Hamilton path $P$ connecting $v_1$ and $v_2$ in $G$. Thus, there exists a neighbor, say $v_2$, of $v_1$, Zigzag up from $y_1$ to $x_2$ and then back to $x_1$, zigzag up to $x_{2n-1}$ and then up to $x_{2n}$, and zigzag left to $y_1$ and then up to $z$ and $y_{2n}$, as shown in Figure 2. Formally,

$$y_1 - x_2 - x_1 - y_2 - \cdots - x_{2n-1} - x_2 - y_{2n-1} - x_{2n} - y_{2n-2} - \cdots - y_3 - z - y_{2n},$$

(2)

Case 3. $A \in X$ and $B \in Y$.

Without loss of generality, let $A = x_1$ and $B = y_{2n}$. Since $G$ is HC, there exists a Hamilton path $P$ connecting $v_1$ and $v_2$ in $G$. We are able to find a Hamilton path joining $A$ and $B$: zigzag up from $x_1$ to $x_{2n-1}$ and then up to $x_{2n}$, and zigzag left to $y_1$, and then reach $z$ and $y_{2n}$, as shown in Figure 3. Formally, it is

$$x_1 - y_2 - x_3 - \cdots - x_{2n-1} - x_{2n} - y_{2n-1} - x_{2n} - \cdots - y_3 - z - y_{2n}.$$  

(3)

Case 4. $A \in Y$ and $B = z$.

Without loss of generality, let $A = y_1$. Since $G$ is HC, $G$ has a Hamilton cycle $C$. Label the vertices of $C$ as $v_1, v_2, \ldots, v_{2n}$. We are able to find a Hamilton path joining $A$ and $B$: zigzag from $y_1$ to $x_2$ and then go to $x_1$, and then zigzag right to $y_2$ and finish at $z$, as shown in Figure 4. Formally, it is

$$y_1 - x_2 - y_3 - \cdots - x_{2n} - x_1 - y_2 - \cdots - y_{2n} - z.$$  

(4)

Case 5. $A \in X$ and $B = z$.

Without loss of generality, let $A = x_1$. Let $H$ be a Mycielski factor of $G$ starting at $v_1$, which consists of an even number of odd cycles $C_1, \ldots, C_s$ (possibly $s = 0$) and an even cycle $C_{2s+1}$ with the chord (possibly empty), joined by $2s$ edges $e_1, \ldots, e_{s}$, where $e_i = \{v_{2i+1}, v_{2i+2}\}$ for each $i \in \{1, \ldots, 2s\}$ such that $v_{2i+1}v_{2i+2} \in V(C_i)$ for each $i \in \{1, \ldots, 2s+1\}$, and the chord joins $v_{2s+1}$ and a vertex at distance even on $C_{2s+1}$.

3. Hamiltonian Connectedness

Theorem 4. Assume that $G$ is a Hamilton-connected graph of order $n \geq 3$. If $\delta(G) \geq (n/2) + 1$, then $\mu(G)$ is Hamilton-connected.

Proof. Let $v$ be a vertex of $G$. We consider a Hamilton cycle $C$ of $G$. Let $u$ be a neighbor of $v$ on $C$. Since $d(u) \geq (n/2) + 1$, it has a neighbor at distance even on $C$. By Lemma 1, $\mu(G)$ is HC.

The $k$th power of a graph $G$, denoted by $G^k$, is a graph with the same vertex set as $G$ in which two vertices are adjacent if and only if their distance in $G$ is at most $k$. Thus,
Theorem 5 (see Karaganis [18]). The cube $G_3$ of every connected graph $G$ of order $n \geq 3$ is Hamilton-connected.

Theorem 6. For any connected graph $G$ of order $n \geq 3$, $\mu(G^3)$ is Hamilton-connected.

Proof. By Theorem 5, $G^3$ is HC for $G$. Since $\mu(H^3)$ is a spanning subgraph of $\mu(G^3)$ for any spanning graph $H$ of $G$, to show $\mu(G^3)$ is HC, it suffices to show that $\mu(T^3)$ is HC for any tree $T$ of order $n \geq 3$. Since $T^3$ is HC, by Theorem 3 (1), we may assume that $n$ is even. By Lemma 1, it remains to show that $T^3$ has a Mycielski factor starting from each vertex $v \in V(T)$.

Let $w$ be a neighbor of $v$ in $T$, and let $T_v$ and $T_w$ be the components of $T - vw$ containing $v$ and $w$, respectively. Let $n_v$ and $n_w$ be the order of $T_v$ and $T_w$, respectively. Let $v'$ be a neighbor of $v$ in $T_v$ and let $w'$ be a neighbor of $w$ in $T_w$.

Case 1: both $n_v$ and $n_w$ are at least 3.

Subcase 1.1: both $n_v$ and $n_w$ are odd.

By Theorem 5, both $T_v^3$ and $T_w^3$ are HC. Let $C_v$ and $C_w$ be Hamilton cycles of $T_v$ and $T_w$, respectively. One can see that $C_v \cup C_w + v'w'$ is a Mycielski factor of $T^3$ starting at $v$.

Subcase 1.2: both $n_v$ and $n_w$ are even.

By the induction hypothesis, $T_v^3$ has a Hamilton path $P_{v,v'}$ joining $v$ and $v'$, and $T_w^3$ has a Hamilton path $P_{w,w'}$ joining $w$ and $w'$. One can see that $P_{v,v'} \cup P_{w,w'} + vw + v'w'$ is a Mycielski factor of $T^3$ starting at $v$.

Case 2: $\min\{n_v, n_w\} \leq 2$.

Subcase 2.1: $\min\{n_v, n_w\} = n_v = 2$.

Since $n_v + n_w = n$ is an even number at least 3, $n_w$ is an odd number at least 3. By Theorem 5, let $C_{vw}$ be a Hamilton cycle of $T^3_w$ containing $v$. It is easy to see that $C_{vw} + v + v'$ is a Mycielski factor of $T^3$ starting at $v$.

Subcase 2.1.1: $\min\{n_v, n_w\} = n_v = 1$.

Since $n_v + n_w = n$ is an even number at least 3, $n_w$ is an odd number at least 3. By Theorem 5, let $C_{vw}$ be a Hamilton cycle of $T^3_w$ containing $w$. It is easy to see that $C_{vw} + v + v'$ is a Mycielski factor of $T^3$ starting at $v$.

Subcase 2.2.2: $\min\{n_v, n_w\} = n_w = 2$.

If $\min\{n_v, n_w\} = n_w = 2$, then $n = 4$. Trivially, $T^3 = K_4$ has a Mycielski factor starting at $v$.

Subcase 2.2.1: $\min\{n_v, n_w\} = n_v = 2$.

If $\min\{n_v, n_w\} = n_v = 2$, then $n = 4$. By Theorem 5, let $P_{wv}$ be a Hamilton path of $T^3_w$. It can be seen that $P_{wv} + v + v'$ is a Mycielski factor starting at $v$.

Theorem 7 (see Alspach and Liu [21]). The generalized Petersen graph $GP(n,2)$ with $n \geq 6$ is Hamilton-connected if and only if $n \equiv 1, 2, 3 (\text{mod} 6)$.

Theorem 8. If $GP(n,2)$ is Hamilton-connected for $n \geq 6$, then $\mu(GP(n,2))$ is Hamilton-connected.

Proof. In view of Lemma 1, it suffices to show that $GP(n,2)$ has a Mycielski factor starting at any $v \in V(GP(n,2))$. We consider two cases:

Case 1: $n \equiv 1$ or $3 (\text{mod} 6)$.

Since $n$ is odd, $GP(n,2)$ is vertex-transitive, and we may assume that $v = u_1$, without loss of generality. Let $C_1$ and $C_2$ be the outer cycle and inner cycle of $GP(n,2)$. Let $v$ be a vertex of $GP(n,2)$. It is clear that $C_1 \cup C_2 + u_1v_2$ is a Mycielski factor of $GP(n,2)$ starting at $v$.

Case 2: $n \equiv 2 (\text{mod} 6)$.

By the symmetry, it suffices to tackle two possibilities according to the location of $v$ in $GP(n,2)$: $v$ lies on the outer cycle or inner cycle of $GP(n,2)$. Without loss of generality, let $v = u_1$ or $v = v_1$.

First, for the case when $n = 8$, we can find a Mycielski factor $F_n$ of $GP(n,2)$ as follows:
obtain a Mycielski factor

\[ C \]
get

where

\[ C \]

illustrated in Figures 6, 8, and 9. For the case when the outer cycle and for the case that

\[ \mu \]

Case 1:

Mycielski factor starting at

\[ n \]

is odd, at least 7. Let \( u \) be a vertex of \( G(n, 3) \). In view of Lemma 1, it suffices to show that \( G(n, 3) \) has a Mycielski factor starting at \( v \). We consider two cases:

Case 1: \( n \equiv 1 \) or \( 5 \) (mod 6).

Since \( n \) is odd, \( G(n, 3) \) is vertex-transitive; by the symmetry, we may assume that \( v = u_1 \), without loss of generality. Let \( C_1 \) and \( C_2 \) be the outer cycle and inner cycle of \( G(n, 3) \). It is clear that \( C_1 \cup C_2 + u_2 v_2 \) is a Mycielski factor of \( G(n, 3) \) starting at \( v \).

Case 2: \( n \equiv 3 \) (mod 6).

By the symmetry, it suffices to tackle two possibilities according to the location of \( v \) in \( G(n, 3) \): \( v \) lies on the outer cycle or inner cycle of \( G(n, 3) \). Without loss of generality, let \( v = u_1 \) or \( v = v_1 \).

First, for the case when \( n = 9 \), we can find a Mycielski factor \( F_9 \) of \( G(n, 3) \) starting at \( v \) as follows:

\[
F_9 = \begin{cases} 
C_9 + uv_7 & \text{if } v = u_1, \\
C_9 + u_1 u_9 & \text{if } v = v_1,
\end{cases}
\]

where \( C_9 \) is a Hamilton cycle of \( G(n, 3) \) as its subgraph, \( C_9 \) contains a cycle of length \( 9 \), by inserting 12 new vertices to \( C_9 \) as illustrated in Figures 10–12.

For the case when \( n \geq 15 \), by inserting 12 new vertices to \( C_9 \) of \( F_9 \), we get \( C_{15} \) as illustrated in Figures 10–12. For the case when \( n \geq 21 \), by inserting 12 new vertices to \( F_{n-6} \) with type A insertion, we obtain a Mycielski factor \( F_n \) of \( G(n, 3) \) starting at \( v \).

4. Pancyclicity

In this section, we show that if a graph \( G \) is pancyclic, then \( \mu(G) \) is also pancyclic.

Theorem 11. If \( G \) is pancyclic, then \( \mu(G) \) is pancyclic.

Proof. Let \( G \) be a pancyclic graph of order \( n \). Since \( \mu(G) \) contains \( G \) as its subgraph, \( \mu(G) \) contains a cycle of length \( l \) for each \( l \in \{3, \ldots, n\} \).

Now, we find a cycle of length \( n + 1 \) in \( \mu(G) \). Take a cycle \( C \) of length \( n - 2 \) in \( G \). Without loss of generality, let \( P = v_1 v_2, \ldots, v_{n-2} \) be a path resulting from \( C \) deleting an edge. It can be seen that \( x_1 x_2 \ldots x_{n-2} y_1 y_{n-2} x_1 \) is a cycle of length \( n + 1 \), as illustrated in Figure 13. In a similar way, one can find a cycle of length \( n + 2 \) in \( \mu(G) \) in terms of a cycle of length \( n - 1 \) in \( G \).

Next, we will find a cycle of length \( n + k \) in \( \mu(G) \) for each \( k \in \{3, \ldots, n + 1\} \). Take a Hamilton cycle \( C \). Without loss of
generality, let $C = v_1 v_2, \ldots, v_n v_1$ in $G$. We consider two cases according to the parity of $k$:

Case 1: $k$ is odd.

One can find a cycle of length $n+k$ in $\mu(G)$, as shown in Figure 14. Formally, it is

$$x_1 - y_2 - x_3 - \cdots - y_{k-1} - z - y_1 - \cdots - x_{k-1} - x_k - \cdots - x_n - x_1. \quad (10)$$

Case 2: $k$ is even.

Zigzag up from $x_1$ to $x_{k-1}$ and left to $x_{k-2}$, then zigzag left to $y_1, z, y_{k-1}$, and $x_k$, and go right to $x_n$ and back to $x_1$, as shown in Figure 15. Formally, it is

$$x_1 - \cdots - x_{k-2} - x_{k-1} - y_{k-2} - \cdots - y_1 - z - y_{k-1} - x_k - \cdots - x_n - x_1. \quad (11)$$
Figure 10: $F_9$ from $u_1$ to $v_1$ in $GP(9, 3)$.

Figure 11: Type $B$ insertion.

Figure 12: $F_{15}$ obtained from $F_9$ by type $B$ insertion in $GP(9, 3)$.

Figure 13: Finding a cycle $C_{n+1}$ in $\mu(G)$ from a cycle $C_{n-2}$ in $G$.

Figure 14: A cycle of length $n + k$ in $\mu(G)$ if $k$ is odd.

Figure 15: A cycle of length $n + k$ in $\mu(G)$ if $k$ is even. □
5. Conclusion

In this paper, we introduce the notion of the Mycielski factor of a graph. If a graph $G$ has a Mycielski factor starting at $v$ for any $v \in V(G)$, then $\mu(G)$ is Hamilton-connected. Applying this result, we are able to show that if a graph $G$ belongs to three (well-defined) families of graphs, then $\mu(G)$ is Hamilton-connected. However, the full conjecture of Jar- nicki, Myrvold, Saltzman, and Wagon is not yet solved. We also prove that if $G$ is pancyclic, then $\mu(G)$ is pancyclic.

One of the reviewers proposed the following two interesting problems.

Zhong et al. [7] showed that the line graph of the generalized Petersen graph $GP(n,k)$ is always Hamilton-connected. Is it easy to show that the Mycielski graph of $L(GP(n,k))$ is Hamilton-connected?

It is known that the line graph of a Hamilton-connected graph $G$ is also Hamilton-connected. Is $\mu(L(G))$ Hamilton-connected if $L(G)$ is Hamilton-connected? [22].

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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