The discovery of the series of minima in the transverse resistivity in the quantum Hall effect as distinguished from the integer quantum Hall effect which gives the value of $e^2/h$ and the fractional value, $(1/3)e^2/h$, deserves recognition by the American Physical Society. The priorities in performing the experimental work as well as in theoretical understanding of the series of minima in the quantum Hall effect are pointed out. It is found that the sequence in which the discoveries are made, as recorded by the APS, is incorrect. Similarly, the assignment of credits for the discovery of the series, by the APS is found to be incorrect. Therefore, the discovery well deserves the award but others could have been given.

1 Introduction

In 1980 von Klitzing identified that $e^2/h$ can be measured from the plateau in the Hall resistivity. The values measured by this method are found to be correct. Since accurate measurement of the fundamental constants can be of interest, von Klitzing was awarded the Nobel prize by the Royal Swedish Academy of Sciences in 1985. In 1982, Tsui, Stomer and Gossard found that the plateau occurs at $(1/3)e^2/h$ in addition to the one at $e^2/h$ and the physics of the problem giving rise to fractional effective charge, is different from that of the integer, $e^2/h$. Therefore, another Nobel prize was awarded to Tsui and Stormer in 1998. In the first publication, it was thought that the “fine structure” is important and the numerical values of the electron charge, Planck’s constant and that of the velocity of light were correct. However, later on, it was realized that the problem has nothing to do with the atomic fine structure. Then what is the value of $e^2/h$ is due to? The experiment of Tsui and Stormer showing $(1/3)e$ is all the more puzzling but Robert Laughlin wrote down the wave function for a quasiparticle of charge $(1/3)e$ so he shared half of the Nobel prize of 1998. No one tried to explain the origin of either $e^2/h$ or $(1/3)e^2/h$. If we know the wave function of quasiparticles of charge $(1/3)e$, does it mean that we can explain the Hall effect? Usually, the Hall effect gives the concentration of carriers but the wave function is
not used to find the Hall resistivity. So it does not help to know the wave
function and the experiment remains unexplained.

In 1987, a publication appeared by Willett, Eisensein, Stormer, Tsui,
Gossard and English [1]. Let us take only the first author of this paper,
Willett, but what is more important is that earlier to this paper, the frac-
tions reported were isolated. The authors wanted to report a new fraction
every time, a new plateau was found. So there were lots of fractions in the
literature. The paper of Willett et al changed the discovery from $e^2/h$ to a
full series of fractions. Thus the credit of the discovery of the series is as-
signed to Willett. The experiments are done at the temperature of a few mK
so that only a few laboratories in the world can do. Therefore, most likely,
the assignment of the discovery of series of fractions to Willett is correct.
In 1999, Willett, West and Pfeiffer showed [2] that the experimental data
is symmetric about $\nu = 1/2$. Again, taking only the first author, we select
Willett.

The OEB prize was awarded to B.I. Halperin in 1982 and to P. A. Lee in
1991. Therefore, we can safely ignore Halperin and Lee from the credits to
be given to Halperin, Lee and Read. However, Halperin, Lee and Read [3] is
an extension of Jain [4]. Halperin et al used the series $\nu = p/(2p + 1)$ which
is the same as one of Jain’s. Therefore, there is no need to expand on Read
any further.

The motion of an electron in a curved path produces a magnetic field
normal to the plane of the path. This is, of course, well known principle of
making a magnet. What Jain has said is that even number of flux quanta,
$\phi_o = hc/e$, are attached to the electron so that the magnetic field produced
by electron with flux quanta attached is $B \pm 2n\phi_o$ where $n$ is the number of
electrons per unit area. The sign depends on the alignment of flux quanta
with respect to the external magnetic field. Apparently, this kind of flux
attachment gives the correct series of fractions which are the same as those
experimentally found by Willett.

Thus we have selected Jain, Read and Willett (in alphabetical order)
to receive the prestigious Oliver E. Buckley (OEB) prize of the American
Physical Society in March 2002.
2 The Discovery.

The value of transverse and longitudinal resistivities, $\rho_{xx}$ and $\rho_{xy}$ in the Hall effect of a single interface of GaAs/AlGaAs have been measured by Willett et al at a temperature of 150 mK and at high fields at 85 mK. One of the very good measurements is shown in Fig.1. This graph shows the fraction $2/5$, $3/7$, $5/11$ and $6/13$ symmetrically located on the right hand side of $1/2$. The values on the left hand side are $2/3$, $3/5$, $4/7$, $5/9$, $6/11$ and $7/13$. The resistivity is written as

$$\rho_{xy} = \frac{h}{\nu e^2}$$  \hspace{1cm} (1)

and $\nu$ determines the fractions as given above. It can be interpreted as the fractional charge,

$$e_{\text{eff}} = \nu e .$$  \hspace{1cm} (2)

The symmetry around $\nu = 1/2$ has been emphasized again in a later work where it is pointed out that Jain’s formula is correct.

Jain suggested that flux-quanta attached to the electron explains the quantum Hall effect. It produces the series of charges, $\nu = p/(2p \pm 1)$ and then the magnetic field becomes $B^* = B - 2n\phi_o$. Jain’s series of effective charges is correct and agrees with the experimental data of Willett. The effective magnetic field is a result of having the correct series of charges. The expression for the effective charge is constructed from even number plus or minus one, with a number in the numerator. This construction leads to an effective magnetic field. The charges are, 1, 2, or 3 divided by an odd number, and this is what is found in the experimental data. Fixing the charges, fixes the field as $B - 2n\phi_o$ and hence even number of flux quanta are attached to the electron. If somehow, the field expression can be proved to be correct, then the model can be accepted.

Assuming that the field expression can be found to be correct in the future, we can justify the award of OEB to Jain, Read and Willett.

3 Is the field correct?

Let us subject the field to a few tests.

(a) Flux quantization.
Usually the flux is quantized as,

$$AB = n \phi_o$$  \hspace{1cm} (3)

with $n/A = n_o$, the number per unit area, $\phi_o = \hbar c/e$ and $B$ is the field. So, what is the difference between flux quantization above and Jain’s formula,

$$B^* = B - 2n\phi_o \hspace{1cm} \text{(Jain)}$$  \hspace{1cm} (4)

The Jain’s formula for the field is inconsistent with the flux quantization. If, it is a new discovery, then it need be consistent with flux quantization.

(b) Even feature.

The field required by the Jain’s formula is $B^* = B \pm 2n\phi_o$. If even number of flux quanta are attached, there should be features at

$$B - 2n\phi_o, \ B + 2n\phi_o, \ B - 4n\phi_o, \ B + 4n\phi_o, \ etc$$  \hspace{1cm} (5)

when a Hall resistivity is plotted against field, there should occur some feature at the above fields but no such feature is present in the experimental data.

(c) Nuclear magnetic resonance.

Let us do the NMR to measure the magnetic field. This is a well known method to measure the magnetic field. We take some odd nuclei such as $^1H$ (proton, i.e., hydrogen in water). The resonance occurs when,

$$\omega = \gamma H$$  \hspace{1cm} (6)

where $\gamma$ is the nuclear gyromagnetic ratio. We can use the nuclear g-factor, $g_N$ and the nuclear magneton, $\mu_N$ to write the above expression as,

$$g_N \mu_N H = \hbar \omega .$$  \hspace{1cm} (7)

We can change $\omega$ from a r.f. oscillator to detect the resonance so that if we know $H$, we can determine $g_N$ and of course, if we know $g_N$, we can determine the field. This is usually a continuous field. Now if CF theory is correct the Ga nuclei will not see $H$ but they will see $B - 2n\phi_o$. The NMR experiment near a Hall plateau has actually been performed but such fields with flux attached have not been found. Some of the Ga nuclei may see $B$ and some others see $B^*$, then NMR will be split. No such splitting has been seen.
(d) **Electron spin resonance.** The electron spin resonance occurs at the resonance frequency determined by,
\[ g\mu_B H = h\nu. \]  
(8)

If such fields as \( B - 2n\phi_0 \) are present, the electrons will see them as,
\[ g\mu_B(B - 2n\phi_0) = h\nu. \]  
(9)

Many ESR experiments have been done but such fields with flux attached have never been found.

(e) **Biot and Savart’s law.** The field is proportional to the current. Therefore, additional flux can not be added to \( B \). Therefore, the flux attachment violates the elementary electrodynamics.

In view of the above five experimental data, the idea of “flux attached to electrons” should be dropped. Jain back calculated the field from the quantum Hall effect data and such a field is not found. Recent experiments performed by Spielman et al in Caltech require a boson so that the composite fermion (CF) model will not explain the data.

4 **Statistics.**

The consistency demands that Jain’s quasiparticles should be “composite fermions”. There is no way for them to become “bosons” because of the even number of flux quanta attachment. The odd number of flux quanta attached shall be “composite bosons”. However, it has been reported that the composite fermions become mixtures of bosons and fermions. Therefore, model of “composite fermions” is internally inconsistent.

Therefore, CF model is incorrect. It creates a really helpless situation. We are left with no theory at all for the fine experimental work of Bell Laboratories. Let us go back to the wave function of Laughlin. What the wave function does is to create quasiparticles of fractional charge by introducing “incompressibility”. However, if GaAs is compressed, the charge leaks and the fractionally charged particles disappear. Therefore, Laughlin’s theory is not relevant to the experimental data on quantum Hall effect.
5 Special handling.

The APS is observing special handling of manuscripts so that only papers arriving from a closed group of authors are published and others are rejected. Therefore, only new type of interpretation is sought for quantum Hall effect and more conventional interpretations are not considered. When it comes to awarding OEB prize, the APS members living outside U.S.A. should also be considered but that means that there are two issues, to publish the papers is one problem and to award prizes is another. If the manuscripts of only a certain group of authors are published, the prize can still go to articles published in non-APS journals. That is very strange, the best articles are published in the APS journals and yet the prize should go to articles published in non-APS journals.

6 The correct series.

In the section 2 above, we pointed out that the series which gives the effective fractional charge, i.e. \( \nu = p/2p + 1 \) is correct. Indeed, the series \( p/2p + 1 \) is correct but it was found by Shrivastava [5] at least three years before Jain. It is based on \( l \) and \( s \) values so that the effective fractional charge comes from the modifications of the Bohr mangeton. The symmetry about \( \nu = 1/2 \) found by Willett is also present in Shrivastava’s paper. If that is the case, then why \( g \)-values were not considered by the APS? The APS authors would have liked to discover the correct series from the \( g \) values but such values were very high and did not agree with the data. Therefore, they were ignored by most of the authors. What is the meaning of \( g \) values? You can get \( g \)-values by comparing the Zeeman energy, \( g\mu_BH.S \) with the klystron frequency which can be tuned upto resonance. In semiconductors there is a band gap so that when \( g\mu_BH = gap \), a different type of \( g \) value emerges. Actually, Shrivastava considered half the \( g \) value, many values of \( l \) and \( s \), including negative \( s \). The entire data from PRL and PRB was considered and in all cases Shrivastava’s theory is found to be correct. It is amazing that Shrivastava has a quasiparticle of zero charge and spin \( \frac{1}{2} \), with a new phenomenon called “superresistivity”. Laughlin has a wave function of charge \( 1/3 \). In Shrivastava’s paper \( 1/3 \) comes from a certain combination of \( l \) and \( s \). Laughlin talks about spinons of spin \( 1/2 \) and zero charge. In
Shrivastava’s formula zero charge gives infinite resistivity and spin 1/2, which are not due to Laughlin.

It may be added that quantization of resistivity at $h/e^2$ is not due to atomic fine structure, it is due to flux quantization, and the quantization at fractional charge is not due to quasiparticles of fractional charge and fluxes are not attached to electrons. The fractional quantization occurs due to combinations of $l$ and $s$.

The correct theory of the quantum Hall effect which explains the experimental data is given in a recently published book [6].

7 Conclusions.

There is no doubt that the fine experimental work of Willett is different from that of integer quantized Hall effect for which von Klitzing was awarded the Nobel prize in 1985. The experimental identification of series of effective charges is also different from that of fractional plateaus found by Tsui and Stormer which was also awarded the Noble prize of 1998. It is thought that Read’s papers are developments of original work done by others. Jain’s back calculation of fields from the experimental data is surely not correct and the claim made that flux quanta are attached to the electrons is not justified and the award of the OEB prize relies on possible future developments. Shrivastava’s work published several years before Jain, is pointed out as the correct interpretation of the quantum Hall effect.

Keshav Shrivastava obtained his Ph.D. from the Indian Institute of Technology in 1966. He worked in the UC Santa Barbara, Univ. Houston, Univ. Nottingham, the State University, Utrecht, etc. He has published 170 papers in the last 36 years. He is the author of Superconductivity: Elementary Topics, World Scientific, 2000.

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**Fig.1:** The integer quantized Hall effect and the fractional quantized Hall effect have disappeared and a “series quantized Hall effect has appeared when data is recorded properly. The experimental data of Willett et al is shown. The series on the right hand side of 1/2 as well as that on the left hand side are the same as in ref. 5.
This figure "OEB.gif" is available in "gif" format from:

http://arxiv.org/ps/cond-mat/0207391v1