Shot noise fluctuations in disordered graphene nanoribbons near the Dirac point

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Abstract
Random fluctuations of the shot-noise power in disordered graphene nanoribbons are studied. In particular, we calculate the distribution of the shot noise of nanoribbons with zigzag and armchair edge terminations. We show that the shot noise statistics is different for each type of these two graphene structures, which is a consequence of presence of different electron localizations: while in zigzag nanoribbons electronic edge states are Anderson localized, in armchair nanoribbons edge states are absent, but electrons are anomalously localized. Our analytical results are verified by tight binding numerical simulations with random hopping elements, i.e., off diagonal disorder, which preserves the symmetry of the graphene sublattices.

Keywords: Shot noise power, graphene disordered nanoribbons, electron localization,

1. Introduction
It is widely recognized that time dependent current fluctuations due to the discreteness of the electrical charges—shot noise power—provide further physical information of an electronic system than other transport quantities such as the conductance. For instance, the shot noise takes into account the Pauli principle and it can reveal electron correlations. Markus Böttiker and collaborators recognized the importance of the shot noise for the understanding of the problem of quantum electron transport and made major contributions to this topic.

The shot-noise power can be studied by using a scattering approach to quantum transport. Within this framework, Böttiker found that the shot-noise power spectrum \(P\), in the zero frequency limit and zero temperature, can be written as

\[
P = 2eVG_0 \sum_{n=1}^{N} T_n (1 - T_n),
\]

where \(G_0(= 2e^2/h)\) is the conductance quantum, \(V\) the applied voltage, while the \(T_n\)'s are the transmission eigenvalues of the Hermitean matrix \(tt^\dagger\), \(t\) being the \(N \times N\) transmission matrix. If there were no correlations among electrons, the shot noise is given by the Poisson value \(P_P = 2eVG_0 \sum T_n\). The shot noise power has been extensively studied both experimentally and theoretically in small electronic devices such as quantum wires and quantum dots, in which quantum coherence is preserved. The literature on this topic is very extensive, we thus refer the reader to the review articles Refs. [3, 4].

In general, electron correlations reduce the shot noise respect to the case of fully uncorrelated electrons. The Fano factor \(F\) measures that suppression of the shot noise and it is defined by the ratio \(F = \langle P \rangle / \langle P_P \rangle\), where the brackets indicate energy or ensemble average. Using random matrix theory to quantum transport, it has been predicted that the Fano factor takes the value \(1/3\) for disordered quantum wires in the diffusive regime limit [5, 6], while for ballistic chaotic quantum dots \(F = 1/4\), in the limit of large number of channels supported by the leads attached to the dots [7, 8]. Thus, universal values of the shot noise suppression have been predicted for both transport regimes.

With respect to graphene, several electronic properties have been intensively studied since its
Figure 1: Disordered graphene nanoribbons (shaded areas) of length \( L \) with zigzag (a) and armchair (b) edges with perfect graphene leads (non-shaded areas) attached. The width \( W \) of the nanoribbons has been fixed: \( W = 11a/\sqrt{3} \) and \( 5a \) for zigzag and armchair nanoribbons (\( a \) being the lattice constant \( a \approx 2.46 \text{Å} \), respectively. With those widths, the attached perfect leads have gapless metallic band structures.

discovery in 2004 and the shot noise is not an exception (for a review, see [8] and [10]). For pristine graphene structures whose lengths are shorter than their widths, it has been predicted a 1/3 shot-noise suppression, at the Dirac point [11]. This suppression value coincides with the Fano factor value for disordered normal-metal wires in the diffusive regime.

Although pristine graphene structures show many interesting electronic properties, in real graphene-based devices those properties may be affected by the presence of disorder [11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. In general, different sources of disorder can be present in graphene such as ripples, vacancies, adatoms, or distortions of the lattice produced by interactions with the substrate [21, 22], however, even suspended graphene structures are not free of defects [23, 24].

In particular, effects of the presence of disorder on the shot noise in graphene sheets have been experimentally and theoretically investigated [10, 23, 24, 25, 26, 27]. It has been found that the Fano factor is affected by the strength of the disorder, as well as the length-to-width ratio of the graphene sample. Most of those studies, however, have been concentrated on wide geometries and models of disorder that break the chiral symmetry of the graphene sublattices.

Here, we are interested in the properties of the shot noise at the Dirac point in finite disordered graphene samples, i.e., disordered nanoribbons, in which the edge terminations as well as the symmetry of the graphene sublattices play a crucial role in the electronic properties. We thus investigate the random fluctuations of the shot noise power for the two different edge terminations: zigzag and armchair, Fig. 1. For both terminations, we consider the so-called off-diagonal disorder (random hopping connecting the two graphene sublattices) in order to preserve the chiral symmetry of the graphene sublattices. This kind of short-range disorder might model distortions like ripples in the graphene lattice. Actually, experimental evidence of short range disorder in graphene has been observed [23, 24, 25]. Therefore, here we are interested in studying the effects of the nanoribbon edge terminations and the presence of disorder on the statistical properties of the shot noise power.

The most interesting properties of graphene are found at low energies, i.e., near the Dirac point where a linear dispersion relation holds and an analogy with relativistic massless particles has attracted much attention. Also, the lattice symmetry of graphene or chiral symmetry, resulting in a symmetric energy spectrum around the Fermi energy plays an important role in the description of the electronic properties; hence, we are interested in calculating statistical properties of the shot noise near the Dirac point. We recall that in clean graphene nanoribbons the band structure is determined by the edge termination and the width of the nanoribbons. Here we consider disordered nanoribbons with attached perfect leads whose band structure is metallic. In the next section we introduce a statistical model to describe the random fluctuations of the shot noise power, Eq. (1).

2. Statistical Model

The statistical properties of the shot noise power in disordered graphene nanoribbons can be analyzed through the statistical properties of the transmission eigenvalues \( T_n \), as we can see from Eq. (1). A well established theoretical framework to study the transmission statistics of disordered systems is the one-parameter scaling approach to localization [33] and random matrix theory [34]. Our statistical analysis of the shot noise is based on that scaling approach to Anderson localization and a recent extension to the case of anomalous localization [35, 36]. In general, the manner in which electrons are localized determines the statistical properties of quantum transport. Therefore, the statistics of the transmission, or conductance, for standard (Anderson) and anomalous localizations is different. For
instance, the average of the conductance of disordered systems in the presence of Anderson localization decays exponentially with the system length, whereas the conductance average has a power-law decay with the length for disorder systems with anomalous localization. Both kind of localizations have been investigated in disordered zigzag and armchair nanoribbons.[37]

On the one hand, near the Dirac point, only a single transmission eigenvalue contributes to the electronic transport. Therefore, since we are interested in the shot noise near the Dirac point, we only need to consider one transmission eigenvalue and the shot-noise power in Eq. (1) is reduced to 

$$P = 2eVG_0T(1 - T).$$

For convenience, we introduced the dimensionless shot-noise power $S$ defined as:

$$S = \frac{1}{2eVG_0}P = T(1 - T).$$

Thus, along this work we will be interested in describing the statistical properties of the shot noise power $S$, as given by Eq. (2).

On the other hand, within the scaling approach to localization and random matrix theory, the distribution of the transmission $P(T)$ is given by the solution of the so-called Mel’nikov’s equation, which is an evolution equation of $P(T)$ with the length of the disordered system.[38, 39, 40] The exact solution of the Mel’nikov’s equation is known in terms of quadratures. To simplify the calculations and provide analytical expressions for shot noise distribution, we find convenient to consider an approximation to the exact solution:[41]

$$P_s(T) = C \sqrt{\frac{\text{acosh}(1/\sqrt{T})}{T^3}} e^{-s^{-1}\text{acosh}^2(1/\sqrt{T})},$$

where $C$ is a normalization constant and $s$ is the ratio $L/l$, $L$ being the length of the system and $l$, the mean free path. The above distribution, Eq. (3), has been verified in a number of numerical simulations for any practical value of the disorder strength (measured by the value of $s$) as well as in microwave experiments.[42] Additionally, the value of the parameter $s$ can be obtained from the numerical or experimental data through the linear dependence of the average $(-\ln T)$ with $L$: $(-\ln T) = L/l(\approx s)$.

We point out that the distribution given in Eq. (3) is appropriate for systems in which the presence of disorder leads to an exponential localization of electron wavefunctions (Anderson localization) with the distance $r$: $|\psi| \sim e^{-\gamma r}$, $\gamma$ being a constant. This exponential decay has been experimentally and theoretically studied in several different disordered systems.[43]

The presence of disorder, however, can lead to a different electron localization, or anomalous localization, in relation to the above-mentioned standard exponential decay. For example, in one-dimensional disordered systems, at the center of the band, the wavefunction decays as $|\psi| \sim e^{-\gamma x^\alpha}$ with $\alpha = 1/2$.[44, 45] For disordered armchair nanoribbons, it has been also shown that electrons are anomalously localized ($\alpha = 0.69$).[37]

Recently, it has been proposed a generalization of the single scaling approach to describe the transmission through disordered systems with anomalous localization.[39] In this case, the distribution of the transmission is given in terms of long-tailed probability density functions or Lévy-type distributions $q_\alpha(x)$, where $\alpha$ is the power-tail exponent of the density function, i.e., for large $x$, $q_\alpha(x) \sim 1/x^{1+\alpha}$. The model is summarized by the following equation which gives the distribution of the transmission in terms of two quantities: the average $\langle \ln T \rangle$ and $\alpha$:

$$P_{\xi,\alpha}(T) = \int_0^\infty P_s(T)q_\alpha(z)dz,$$

where we have defined $\xi = (-\ln T)$. $P_{\xi,\alpha,\xi,\alpha}(T)$ is given by Eq. (3) with $s$ replaced by the function $\xi(\alpha, \xi, z) = \xi/(2z^\alpha I_\alpha)$, where $I_\alpha = 1/2 \int_0^\infty z^{-\alpha}q_\alpha(z)dz$. The density function $q_\alpha(z)$ is part of a family of probability densities commonly known as $\alpha$-stable distributions. We remark that only two quantities ($\xi$ and $\alpha$) determine the distribution of the transmission. Also, we point out that Eq. (4) predicts a nonlinear behavior of the logarithm of the transmission with the length $L$: $(\ln T) \propto L^\alpha$, which is in contrast to the linear behavior expected for the standard Anderson localization. The model summarized in Eq. (4) has been applied recently to describe the transmission of microwaves in disordered waveguides.[46]

In the following two sections, we will apply Eqs. (3) and (4) to calculate the distribution of the shot noise power $S$, Eq. (2), in zigzag and armchair nanoribbons, respectively. The theoretical predictions will be compared with numerical simulations using a standard tight-binding Hamiltonian model:

$$H = \sum_{<i,j>} t_{i,j}(c_i^\dagger c_j + c_j^\dagger c_i),$$

(5)
where \( i \) and \( j \) are nearest neighbors and \( c_i^{\dagger} (c_i) \) is the creation (annihilation) operator for spinless fermions, while the hopping elements \( t_{i,j} \) between the two graphene sublattices are randomly obtained from the distribution \( p(t) = 1/\Delta t \) with \( \exp(-w/2) \leq t \leq \exp(w/2) \), \( w \) being the strength of the disorder which is fixed to 1. This short-range disorder models random distortions in graphene sheets without breaking the symmetry of the graphene lattice. The numerical simulations were performed near the Dirac point: \( \Delta = 10^{-6} \) (in units of the hopping energy of the perfect leads) for zigzag and armchair nanoribbons of widths \( W = 11 a/\sqrt{3} \) and \( W = 9 a \) (\( a \) being the lattice constant \( a \approx 2.46 \AA \), respectively. The transmission eigenvalues in Eq. (2) were calculated by using a recursive Green’s function and, for all cases shown in this work, the shot noise statistics were collected from an ensemble of \( 2 \times 10^4 \) disorder realizations.

### 3. Zigzag Nanoribbons: Shot Noise via Edge States

Firstly, we consider the case of disordered zigzag nanoribbons, Fig. 1(a). In pristine zigzag nanoribbons, it is well known the existence of edge states which are perfectly transmitted. In the presence of disorder, those states remain at the border of the nanoribbons, but they are (Anderson) localized near the Fermi energy, those states remain at the border of the nanoribbons, but they are (Anderson) localized near the Fermi energy.

In the present case, however, the average indicated by Eq. (4) to calculate the distribution of the shot-noise power given by \( P(S) = \langle \delta (S - T(1 - T)) \rangle \). Thus, using Eqs. (2) and (3), it is straightforward but lengthy to perform the average indicated with brackets. The final expression for the distribution of the shot noise power is

\[
P_s(S) = \frac{C}{\sqrt{1 + 4S}} \times \left\{ \begin{array}{l}
\frac{\text{acosh}(1/\sqrt{S_+})}{S_+^{3/2}} e^{-s^{-1} \text{acosh}^2(1/\sqrt{S_+})} \\
\frac{\text{acosh}(1/\sqrt{S_-})}{S_-^{3/2}} e^{-s^{-1} \text{acosh}^2(1/\sqrt{S_-})}
\end{array} \right\},
\]

where \( C \) is a normalization constant and \( S_{\pm} = (1 \pm \sqrt{1 + 4S})/2 \). We notice that the statistical properties of the shot noise are determined by the single parameter \( s \), which can be extracted from experimental or numerical data, as we have pointed out.

In order to verify the above result, we compare the theoretical distribution, Eq. (6), with the tight binding numerical simulations, described previously. In Fig. 2, the numerical distributions (histograms) and \( P_s(S) \) (solid line) are compared for two different strengths of disorder, measured by the parameter \( s \). The value of \( s \) is obtained from the numerical data and it is plugged into Eq. (6). Figs. 2(a) and 2(b) correspond to zigzag nanoribbons which are characterized by the average dimensionless conductance \( \langle G \rangle = 0.46 \) and \( \langle G \rangle = 0.04 \), respectively. The Fano factor values for those cases are \( F = 0.37 \) for Fig. 2(a), while \( F = 0.6 \) for the case shown in Fig. 2(b). We can observe a good agreement between the model [Eq. (6)] and the numerical simulations.

### 4. Armchair Nanoribbons: Shot Noise via Anomalously Localized States

We now consider the case of nanoribbons with armchair terminations, Fig. 1(b). As we have mentioned, evidence of the presence of anomalous localization has been found when the hopping elements of the tight binding Hamiltonian [Eq. (5)] have random fluctuations, which may model random distortions in the graphene lattice. As in the case of zigzag nanoribbons, near the Fermi energy, only a single channel contributes to the electronic transport and the shot noise distribution can be obtained by performing the average \( \langle \delta (S - T(1 - T)) \rangle \). In the present case, however, the average indicated with brackets is performed with the transmission distribution given by Eq. (4). The \( \alpha \) parameter of \( q_{\alpha}(z) \) in Eq. (4) that characterize the anomalous localization of our disordered armchair nanoribbons has been numerically obtained \((\alpha = 0.69) \) in [37]. This value of \( \alpha \) does not depend on the length, width, and strength of the disorder, according to the numerical simulations. On the other hand, unfortunately, there is no a close analytical expression for the Lévy-type distribution \( q_{\alpha}(z) \) with \( \alpha = 0.69 \); thus, we express the shot noise distribution for armchair nanoribbons in terms of quadratures:

\[
P_{\xi, \alpha}(S) = \int \left[ P_{\xi}(S+) + P_{\xi}(S-) \right] q_{\alpha}(z) dz,
\]

where \( P_{\xi}(S) \) is given by Eq. (6) with \( s \) replaced by \( \bar{s} = \xi/(2z^\alpha I_\alpha) \). As we have defined previously:

\[4\]
Figure 2: Distribution of the shot noise power for disordered zigzag nanoribbons. Panels (a) and (b) show the numerical (histograms) and theoretical distribution (solid line) for disordered nanoribbons with dimensionless conductance average $\langle G \rangle = 0.46$ ($s = 1.03$) and $\langle G \rangle = 0.043$ ($s = 6.21$), respectively. The value of the Fano factor is 0.37 for panel (a), while $F = 0.6$ for panel (b). A good agreement is seen between theory and numerical simulations.

\[ \xi = \langle -\ln T \rangle, \quad I_0 = 1/2 \int_0^\infty z^{-\alpha} q_0 \, dz, \quad \text{and} \quad S_\pm = (1 \pm \sqrt{1 + 45})/2. \]

We now compare our expression in Eq. (7) with numerical simulations of disordered armchair nanoribbons. In Fig. 3 we show the numerical (histogram) and theoretical (solid line) shot noise distributions for two different values of $\xi$. The values of $\xi$ are extracted from the numerical data and they are plugged into Eq. (7). We may also characterized the nanoribbons by the average dimensionless conductance: in Fig. 3(a), $\langle G \rangle = 0.46$, while in Fig. 3(b) $\langle G \rangle = 0.15$. The values of the Fano factor of these two cases are $F = 0.27$ for panel (a), while $F = 0.35$ for panel (b). We can observe that our model describe correctly the trend of the shot noise distributions. As in the previous case of disordered zigzag nanoribbons, we point out that no parameters have been adjusted in our theoretical results.

Finally, we would like to contrast the statistics of the shot noise power for both type nanoribbon terminations; hence, in Fig. 4 we show the shot noise distributions for both armchair (dashed line) and zigzag (solid line) nanoribbons. For both cases, we have chosen nanoribbons with the same average of the dimensionless conductance ($\langle G \rangle = 0.46$), shown in Figs. 2(a) and 3(a). For this value of $\langle G \rangle$, the Fano factor values are: $F = 0.37$ for zigzag nanoribbons, while $F = 0.27$ for armchair nanoribbons. From Fig. 4 the main differences in the distributions are seen at small values of the shot noise. We can also observe that the probability of having small values of the shot noise is larger in armchair nanoribbons than in zigzag nanoribbons. This can be explained by the larger fluctuations in the transmission eigenvalues in armchair nanoribbons, as a consequence of the presence of anomalous localization, which lead to large fluctuations of the shot noise power. Thus, Fig. 4 shows an example of the effects of the edge termination on the statistics of the shot noise power.

5. Summary and Conclusions

We have investigated the statistics of the random fluctuations of the shot noise power in disordered graphene nanoribbons with zigzag and armchair edge terminations, near the Dirac point. Within a scattering approach developed by Büttiker and collaborators, the shot noise can be written in terms of the transmission eigenvalues. We thus apply a...
Figure 3: Shot noise distribution for disordered armchair nanoribbons. Panels (a) and (b) show the numerical (histograms) and theoretical distribution (solid line) for disordered nanoribbons with transmission average \( \langle G \rangle = 0.46 \) \((-\ln G) = 1.37\) and \( \langle G \rangle = 0.15 \) \((-\ln G) = 6.37\) for panels (a) and (b), respectively. The value of the Fano factor is 0.27 for panel (a), while \( F = 0.35\) for panel (b). We can observe that the theoretical predictions reproduce the trend of the numerical distributions.

Figure 4: Shot noise distributions for zigzag (solid line) and armchair (dashed line) nanoribbons with the same average dimensionless conductance \( \langle G \rangle = 0.46 \). Stronger transmission fluctuations due to the presence of anomalous localization in armchair nanoribbons increase the probability of smaller values of the shot noise power, in relation to the zigzag nanoribbons.

random matrix theory of quantum transport to calculated the complete distribution of the shot noise power for both type of nanoribbon edge terminations.

Our theoretical model predicts different statistical properties of the shot noise for zigzag and armchair nanoribbons. Those differences come from the fact that electronic edge states in zigzag nanoribbons are exponentially localized in space, i.e., electrons are (Anderson) localized, while in armchair nanoribbons, electrons are anomalously localized or delocalized, in relation to the case of Anderson localization. Anomalous localization produces stronger random fluctuations of the transmission which lead to a different statistics of the shot noise.

We point out that while for zigzag nanoribbons the shot noise distribution depends on a single parameter, for armchair nanoribbons the shot noise distribution is determined by two parameters. Those parameters are not free in our model in the sense that their values are obtained from the numerical simulations, or experimental data, and they are used as an input in the analytical expressions. Our theoretical predictions have been verified by numerical simulations of graphene nanoribbons with off-diagonal disorder, which preserve the chiral symmetry of the graphene sublattices. We have found that the value of the Fano factor is not universal but it depends on the strength of the disorder, which is in contrast to the known universal value 1/3 for short and wide
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References

[1] G. B. Lesovik, Pis’ma Zh. Eksp. Teor. Fiz. 49 (1989), 515. [JETP Lett. 49 (1989) 594].
[2] M. Büttiker, Phys. Rev. Lett. 65 (1990) 2901.
[3] M. J. M. de Jong and C. W. J. Beenakker, Shot noise in Mesoscopic systems, in “Mesoscopic Electron Transport,” edited by L. L. Sohn, L. P. Kouwenhoven, and G. Schoen, NATO ASI Series Vol. 345 (Kluwer Academic Publishers, Dordrecht, 1997) 225.
[4] Ya. M. Blanter and M. Büttiker, Phys. Reports, 336 (2000) 1.
[5] C. W. J. Beenakker and M. Büttiker, Phys. Rev. B 46 (1992) 1889.
[6] K. E. Nagaev, Phys. Lett. A 169 (1992) 103.
[7] R. A. Jalabert, J.-L. Pichard, C.W.J. Beenakker, Europhys. Lett. 27 (1994) 255.
[8] For finite number of channels see: V. A. Gopar, S. Rotter, H. Schomerus, Phys. Rev. B 73 (2006) 155308; E. N. Bulgakov, V. A. Gopar, P. A. Mello, I. Rotter, Phys. Rev. B 73 (2006) 155302; D. V. Savin and H.-J. Sommers, Phys. Rev. B 73 (2006) 081307(R); P. Braun, S. Heusler, S. Müller, and Fritz Haake, J. Phys. A: Math. Gen. 39 (2006) L159.
[9] A. Castro Neto, F. Guinea, N. Peres, K. Novoselov, and A. Geim, Rev. Mod. Phys. 81, 109 (2009).
[10] S. Das Sarma, Shaffique Adam, E. Hwang, and Enrico Rossi, Rev. Mod. Phys. 83 (2011) 407.
[11] J. Tworzydło, B. Trauzettel, M. Titov, A. Rycerz, and C. W. J. Beenakker, Phys. Rev. Lett. 96 (2006) 246802.
[12] A. R. Mucciolo and C. H. Lewenkopf, J. Phys.: Condens Matter 22 (2010) 274201.
[13] D. A. Areshkin, D. Gunlycke, and C. T. White, Nano Lett. 7 (2007) 204.
[14] J. H. Bardarson, J. Tworzydło, P. W. Brouwer, and C. W. J. Beenakker, Phys. Rev. Lett. 99 (2007) 106801.
[15] S.-J. Xiong and Y. Xiong, Phys. Rev. B 76 (2007) 214204.
[16] E. Louis, J. A. Vergés, F. Guinea, and G. Chiape, Phys. Rev. B 75 (2007) 085440.
[17] M. Evaldsson, I. V. Zozoulenko, H. Xu, and T. Heinzel, Phys. Rev. B 78 (2008) 161407(R).
[18] F. Libisch, S. Rotter, and J. Burgdörfer, New J. Phys. 14 (2012) 123006.
[19] K. Wakabayashi, Y. Takane, and M. Sigrist, Phys. Rev. Lett. 99 (2007) 666601.
[20] I. Kleftogiannis and I. Amanatidis, Eur. Phys. J. B 87 (2014) 16.
[21] P. Gallagher, K. Todd, and D. Goldhaber-Gordon, Phys. Rev. B 81 (2010) 115409.
[22] M. Y. Han, J. C. Brant, and P. Kim, Phys. Rev. Lett. 104 (2010) 056801.
[23] J. C. Meyer, A. K. Geim, M. I. Katsnelson K. S. Novoselov, T. J. Booth, and S. Roth, Nature Mater. 10 (2011) 282.
[24] J. Xue, J. Sanchez-Yamagishi, D. Bulmash, P. Jacquod, A. Deshpande, K. Watanabe, T. Taniguchi, P. Jarillo-Herrero, B. J. LeRoy, Nature Mat. 10 (2011) 282.
[25] R. Danneau, F. Wu, M. F. Craciun, S. Russo, M. Y. Tomi, J. Salzlehtoh, A. M. Morpurgo, and P. J. Hakonen Phys. Rev. Lett. 100 (2008) 196802.
[26] L. DiCarlo, J. R. Williams, Yiming Zhang, D. T. McClure, and C. M. Marcus Phys. Rev. Lett. 100 (2008) 146805.
[27] P. San-Jose, E. Prada, and D. S. Golubev, Phys. Rev. B 76, (2007) 195445.
[28] C. H. Lewenkopf, E. R. Mucciolo, and A. H. Castro Neto, Phys. Rev. B 77 (2008) 081410(R).
[29] R. L. Dragomirova, D. A. Areshkin, and B. K. Nikolici, Phys. Rev. B 79 (2009) 2414401(R).
[30] Y.-W. Tan, Y. Zhang, K. Bolotin, Y. Zhao, S. Adam, E.-H. Hwang, S. Das Sarma, H.-L. Stormer, and P. Kim, Phys. Rev. Lett. 99, (2007) 246803.
[31] C. Jang, S. Adam, J.-H. Chen, E. D. Williams, S. Das Sarma, and M. S. Fuhrer, Phys. Rev. Lett. 101 (2008) 146805.
[32] J. M. Zeuner, M. C. Rechtsman, S. Nolte, A. Szameit, arXiv:1304.6911.
[33] E. Abrahams, P. W. Anderson, D. C.Licciardello, and T. Ramakrishnan, Phys. Rev. Lett. 42 (1979) 673.
[34] Pier A. Mello and Naredra Kumar, Quantum Transport in Mesoscopic Systems, (Oxford University Press, Oxford, 2004).
[35] F. Falceto and V. A. Gopar, Europhys. Lett. 92 (2010) 57014.
[36] I. Amanatidis, I. Kleftogiannis, F. Falceto, V. A. Gopar, Phys. Rev. B 85 (2012) 245450.
[37] I. Kleftogiannis, I. Amanatidis, V. A. Gopar, Phys. Rev. B 88 (2013) 205414.
[38] O. N. Dorokhov, Pis’ma Zh. Eksp. Teor. Fiz. 36 (1982) 259 [JETP Lett. 36 (1980) 318].
[39] V. I. Mel’nikov, Pis’ma Zh. Eksp. Teor. Fiz. 34 (1981) 5698.
[40] A. A. Abrikosov, Solid State Commun. 37 (1981) 997.
[41] V. A. Gopar and R. A. Molina, Phys. Rev. B 81 (2010) 115415.
[42] A. Pea, A. Girschik, F. Libisch, S. Rotter, and A. A. Chabanov, Nature Commun. 5 (2014) 3488.
[43] Ad Lagendijk, Bart van Tiggelen, and Diederik S. Wiersma, Phys. Today 62 (2009) 24, and references therein.
[44] C. M. Soukoulis and E. N. Economou, Phys. Rev. B 24 (1981) 5698.
[45] M. Inui, S. A. Trugman, Elihu Abrahams Phys. Rev. B 49, 3190 (1994).
[46] A.A. Fernández-Marín, J.A. Méndez-Bermúdez, J. Carbonell, F. Cervera, J. Sánchez-Dehesa, and V.A. Gopar
Phys. Rev. Lett. 113 (2014) 233901.