Prediction of Engineering Settlement and Deformation Based on Grey Theory Model

Jinsong Tian*
School of Science, Anhui Agricultural University, Hefei, Anhui 230036, China
*Corresponding author’s e-mail: anhuiceliang@sina.com

Abstract: Research purposes: the traditional methods of building deformation data processing were based on regression statistical analysis and time series analysis etc. These prediction methods need larger prediction sample data. Relatively large amounts of errors were caused by a small amount of data. Research methods and procedures: the paper uses the grey theory in operational research to solve these problems. The grey theory model only needs less sample observation data to predict the characteristics and the catastrophic & abnormal values. Prediction model of vertical displacement (settlement) deformation is established based on GM (1, 1) model. Compare the errors between the model value and the measured value and form fitting graphs of monitoring points. Research conclusion: the application results show that the grey theory model is suitable for data processing and prediction analysis of engineering building settlement and obtain the high prediction accuracy.

1. Introduction
Influenced by the internal and external forces, engineering buildings will deform in both horizontal and vertical directions. Regardless of horizontal displacement, inclination or vertical displacement (settlement), when the deformation value exceeds a certain limit, the safety of the building will be negatively affected. Therefore, for high-rise buildings, it is of great significance to conduct regular deformation monitoring during construction and operation. Settlement monitoring is critical to the deformation monitoring of construction engineering [1]. Therefore, using mathematical methods to find or predict the spatial distribution of deformation and its temporal regularity and summarizing the relationships among deformation quantity and various internal and external factors for accurate early warning are important works in construction engineering[1]. Generally, a predictive model is a model of factors, and there is always a direct or indirect relationship among the factors. If the behavior of the system is predicted according to the change of factors, due to the fact that there are many other factors in the factors, the prediction would be quite complicated [1-5].

The grey system theory is the prediction of grey system, it concerns the theory of grey system analysis, modeling, prediction, decision making and control. With its requirement of less modeling information, convenient calculation, and high modeling precision, the grey prediction model has been widely applied to various fields of production. It is an effective tool for dealing with small sample prediction problems. Based on operational research, the grey prediction model can establish mathematical models and make predictions with a small amount of incomplete information[6]. According to the grey system theory, the system state equation described by the GM (1,1) model provides a description of the uncertainty correlation between the system's main behavior and other behavioral factors. As a single factor prediction, on the basis of the similarity of the developmental
trends, it analyzes the dynamic association analysis between system behavior and other behavioral factors[1].

In this investigation, the observation data of the main load-bearing wall of the garbage chamber of a main plant of an energy-saving power plant were used to establish a settlement prediction model. The statistical analysis of the discrete data of settlement monitoring was carried out and the gross error was eliminated, so that the sequence data in a certain time period is obtained as the original sequence of the later grey model modeling. Subsequently, the modeling data was used by the GM(1,1) model of the grey system. The model fitting value was compared with the measured value for detailed analysis and accuracy rating, and the elevation value of the settlement monitoring point in a certain period of time in the future was predicted. The results show that the model can be used to extrapolate the subsequent elevation values according to a small number of measured settlement monitoring points. The grey theory model is suitable for the prediction and analysis of building settlement deformation.

2. Grey system modeling principle and accuracy analysis model

2.1 Modeling Principle

The initial observation data sequence is \( x^{(0)} = \{x^{(0)}(1), x^{(0)}(2) \cdots x^{(0)}(n)\} \). After the accumulation of the observation data, a new data sequence \( x^{(1)} = \{x^{(1)}(1), x^{(1)}(2) \cdots x^{(1)}(n)\} \) is obtained, wherein \( x^{(1)}(i) = \sum_{j=1}^{i} x^{(0)}(j) \).

Assuming that the first-order differential equation is satisfied[6]:

\[
\frac{dx^{(1)}}{dt} + ax^{(1)} = u
\]

In the equation, \( a \) and \( u \) are constants. According to the grey system theory, \( a \) is called the development grey number, and \( u \) denotes the internal control grey number, which is the constant input to the system. When the equation satisfies the initial condition \( t = t_0 \), the solutions of \( x^{(1)} = x^{(1)}(t_0) \) can be expressed as[6]:

\[
x^{(1)}(t) = \left[ x^{(1)}(t_0) - \frac{u}{a} \right] e^{-at(t-t_0)} + \frac{u}{a}
\]

\[
x^{(1)}(k+1) = \left[ x^{(1)}(1) - \frac{u}{a} \right] e^{-ak} + \frac{u}{a}
\]

The method to build a grey model includes accumulating sequences and estimating the constant \( a \) and \( u \) by least squares. Since \( x^{(1)}(1) \) is the initial value, \( x^{(1)}(2) \), \( x^{(1)}(3) \) are substituted into Equation 1, and the differential combination is replaced by the difference to collect samples at equal intervals, therefore we can obtain [6]:

\[
\frac{\Delta x^{(1)}(2)}{\Delta t} = \Delta x^{(1)}(2) = x^{(1)}(2) - x^{(1)}(1) = x^{(0)}(2)
\]

In the same way, we can obtain:

\[
\frac{\Delta x^{(1)}(3)}{\Delta t} = \Delta x^{(1)}(3) = x^{(1)}(3) - x^{(1)}(2) = x^{(0)}(3)
\]

\[
\frac{\Delta x^{(1)}(n)}{\Delta t} = \Delta x^{(1)}(n) = x^{(1)}(n) - x^{(1)}(n-1) = x^{(0)}(n)
\]

According to Equation 1:
\[
\begin{align*}
\begin{bmatrix} x^{(0)}(2) + ax^{(1)}(2) \\ x^{(0)}(3) + ax^{(1)}(3) \\ \vdots \\ x^{(0)}(n) + ax^{(1)}(n) \end{bmatrix} &= u 
\end{align*}
\]

\[ (7) \]

\[ ax^{(1)}(i) \] is moved to the right and written in a vector form:

\[
\begin{align*}
\begin{bmatrix} x^{(0)}(2) = u - ax^{(1)}(2) = \left(1, -x^{(1)}(2)\right) \\ x^{(0)}(3) = u - ax^{(1)}(3) = \left(1, -x^{(1)}(3)\right) \\ \vdots \\ x^{(0)}(n) = u - ax^{(1)}(n) = \left(1, -x^{(1)}(n)\right) \end{bmatrix}
\end{align*}
\]

\[ (8) \]

Since \( \frac{\Delta x^{(1)}(i)}{\Delta t} \) relates to two time values of the accumulated column \( x^{(i)} \), it is more appropriate to replace \( x^{(i)}(i) \) with the average value of the two time values, which means replaced \( x^{(i)}(i) \) with \( \frac{1}{2} \left[ x^{(i)}(i) + x^{(i)}(i-1) \right] \), and Equation 8 can be rewritten in a matrix form:

\[
\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} = \begin{bmatrix} 1, -\frac{1}{2}x^{(1)}(2) + x^{(1)}(1) \\ 1, -\frac{1}{2}x^{(1)}(3) + x^{(1)}(2) \\ \vdots \\ 1, -\frac{1}{2}x^{(1)}(n) + x^{(1)}(n-1) \end{bmatrix}
\]

\[ (9) \]

Assuming that \( y = \left( x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n) \right)^T \),

\[
B = \begin{bmatrix} 1, -\frac{1}{2}x^{(1)}(2) + x^{(1)}(1) \\ 1, -\frac{1}{2}x^{(1)}(3) + x^{(1)}(2) \\ \vdots \\ 1, -\frac{1}{2}x^{(1)}(n) + x^{(1)}(n-1) \end{bmatrix}, \quad U = \begin{bmatrix} u \\ a \end{bmatrix}
\]

\[ (10) \]

Abbreviated the assumption as \( y = BU \), the least squares estimation is performed to calculate the constant \( a \) and \( u \):

\[
\hat{U} = \begin{bmatrix} \hat{u} \\ \hat{a} \end{bmatrix} = (B^T B)^{-1} B^T y
\]

\[ (11) \]
Substituting the estimated values of $a$ and $\hat{a}$ into Equation 4, the time response function equation of the future predicted value can be expressed as:

$$\hat{x}^{(i)}(k+1) = \left[ x^{(i)}(1) - \frac{\hat{a}}{a} \right] e^{-\hat{a}k} + \frac{\hat{a}}{a}$$

When $k = 1, 2, \cdots, n - 1$, the model fitting value $\hat{x}^{(i)}(k+1)$ of the original sequence $x^{(0)}$ can be obtained; when $k \geq n$, the predicted value $\hat{x}^{(0)}(k+1)$ of the original sequence $x^{(0)}$ can be obtained [6].

2.2 Precision Analysis Model

The main purpose of modeling is to predict. In order to improve the prediction accuracy, it is prior to ensure a higher filtering accuracy. There are three kinds of evaluation methods for the degree of model fitting, which are residual size test, correlation test and posterior variance test. The posterior variance test method is commonly applied to the grey system theory model for the accuracy test[6]. The relevant calculation model can be expressed as follows:

(1) Residual test: calculating the residual and relative residual separately

The residual is the difference between the original data sequence and the calculated value of the model $E(k) = x^0(k) - \hat{x}^{(0)}(k)$, $k = 2, 3, \cdots, n$. Subsequently, the relative residual can be calculated based on the residual, which is $e(k) = (x^0(k) - \hat{x}^{(0)}(k))/x^0(k)$, $k = 2, 3, \cdots, n$.

(2) Posterior variance test

According to the posterior variance test method, the variance of the original data sequence $x^{(0)}$ is $S_1 = \sqrt{\frac{1}{n} \sum_{k=1}^{n} (x^0(k) - \bar{x})^2}$, and then the variance of the residual is calculated according to the residual, which is $S_2 = \sqrt{\frac{1}{n-1} \sum_{k=1}^{n} (E(k) - \bar{E})^2}$. The posterior variance ratio $C = \frac{S_2}{S_1}$ is then calculated according to the variance and residual variance of the original data sequence. With the mathematical statistics knowledge, the small error probability is taken as $p = \{ |E(k) - \bar{E}| < 0.6745S_1 \}$.

According to the literature [6], the prediction accuracy level is shown in Table 1:

| Prediction accuracy levels | P value | C value |
|---------------------------|--------|--------|
| Good                      | $P>0.95$ | $C<0.35$ |
| Qualified                 | $P>0.80$ | $C<0.45$ |
| Nearly qualified          | $P>0.70$ | $C<0.50$ |
| Below standard            | $P\leq0.70$ | $C\geq0.65$ |

There are two requirements. First, the original data is required to be non-negative. Otherwise, the accumulation will be offset, and the data sequence will not be incremented or decremented with time. Second, if a negative value is included in the original data sequence of the actual problem, a constant
can be added to the original data sequence, so that, the values of the original data sequence are positive.

3. Case Analysis

3.1 Project Overview

The project is located in the energy-saving power plant area of a city's circular economy demonstration park. The main plant, chimney, trestle, comprehensive building, office building, pump house, anaerobic tank, garbage storage main load-bearing wall, etc. were monitored for settlement, and these buildings had been put into use for more than 4 years.

According to the relevant regulations, design drawings and the requirements of the owner, the first year monitoring cycle is four times in a continuous monitoring period every three months. The post-monitoring period and frequency are increased or decreased according to the requirements of owner, the settlement rate of the monitoring points as well as the stability of the settlement of the structure. According to the requirements of the Engineering Measurement Specification, the settlement monitoring after the capping of the building (or in the operational phase) is measured once every three months and observed for one year. If the average settlement rate of the last two observation periods is less than 0.02 mm/day, the overall stability is considered to be stable. If the average settlement at each point is less than 0.02 mm/day, the observation can be stopped. Otherwise, it should continue to be observed every 3 months until the building (structure) is stable. In this paper, the settlement monitoring data of the main load-bearing wall of the garbage chamber are selected as the original data sequence. A total of three monitoring points had been set up on the north main load-bearing wall, numbered J61-J63. From December 2016 to December 31, 2017, a total of 6 observations were conducted, followed by another 4 observations. The elevation values measured in the first 6 times of the monitoring points are shown in Table 2.

| Monitoring point number | 1st measured value | 2nd measured value | 3rd measured value |
|-------------------------|--------------------|--------------------|--------------------|
| J61                     | 20.73322           | 20.73313           | 20.73220           |
| J62                     | 20.75556           | 20.75521           | 20.75545           |
| J63                     | 20.71316           | 20.71334           | 20.71325           |

| Monitoring point number | 4th measured value | 5th measured value | 6th measured value |
|-------------------------|--------------------|--------------------|--------------------|
| J61                     | 20.73197           | 20.73175           | 20.73162           |
| J62                     | 20.75534           | 20.75548           | 20.75577           |
| J63                     | 20.71318           | 20.71317           | 20.71309           |

3.2 Building a model

3.2.1 Model fitting values

According to the above-mentioned modeling principle and modeling steps, with the established BY matrix and the measured values of the three monitoring points, the estimated values of the constants $a$ and $u$ corresponding to each monitoring point are obtained by calculating $(B^TB)^{-1}$, and the prediction time response function corresponding to each monitoring point is formed. The prediction time response function is used to calculate the model fitting value $\hat{x}^{(1)}(i)$, and then the model calculation value $\hat{x}^{(0)}(i)$ and the residual, relative residual and average residual are calculated.
according to the post-reduction calculation. The calculation results are shown in Table 3, Table 4 and Table 5.

| Table 3 J61 point model value |
|-----------------------------|
| Model fitting value | Measured value | Residual | Relative error | Average relative error | Number of observations |
|-------------------|---------------|---------|----------------|------------------------|------------------------|
| 20.73283 | 20.73313 | 0.30200 | 0.00145 | | 2 |
| 20.73248 | 20.73220 | -0.28100 | -0.001355 | | 3 |
| 20.73213 | 20.73197 | -0.16400 | -0.000791 | 0.00039 | 4 |
| 20.73179 | 20.73175 | -0.03560 | -0.000178 | | 5 |
| 20.73144 | 20.73162 | 0.18000 | 0.00086 | | 6 |

Note: In the table, the unit of calculated values and measured values of the model are all m, and the unit of the residual is mm.

| Table 4 J62 point model value |
|-----------------------------|
| Model fitting value | Measured value | Residual | Relative error | Average relative error | Number of observations |
|-------------------|---------------|---------|----------------|------------------------|------------------------|
| 20.75522 | 20.75521 | -0.01000 | -0.000048 | | 2 |
| 20.75533 | 20.75545 | 0.12000 | 0.00055 | | 3 |
| 20.75545 | 20.75534 | -0.11000 | -0.00053 | 0.000395 | 4 |
| 20.75556 | 20.75548 | -0.08453 | -0.00041 | | 5 |
| 20.75568 | 20.75577 | 0.09000 | 0.000434 | | 6 |

Note: In the table, the unit of calculated values and measured values of the model are all m, and the unit of the residual is mm.

| Table 5 J63 point model value |
|-----------------------------|
| Model fitting value | Measured value | Residual | Relative error | Average relative error | Number of observations |
|-------------------|---------------|---------|----------------|------------------------|------------------------|
| 20.713322 | 20.71334 | -0.018 | -0.00008777 | 0.000077 | 2 |
| 20.713264 | 20.71325 | 0.014 | 0.000068 | | 3 |
Note: In the table, the unit of calculated values and measured values of the model are all m, and the unit of the residual is mm.

It can be seen from Table 3, Table 4 and Table 5 that the residuals between the model fitting values and the measured values of the three monitoring points are small. By comparison with the measured values, we can find that the minimum residual value of the J61 point is 0.035mm, and its maximum value is 0.302mm, while the minimum value of J62 point residual is 0.01mm, and its maximum value is 0.12mm, the minimum residual value of J63 point is 0.00mm, and the maximum value is 0.026mm. The model fitting values and the measured values of the three monitoring points are now input into the EXCEL software to form charts of the model fitting value and the measured value, as shown in Figures 1-3. It can be seen from the figures that the fitting value of the model obtained by this model is highly in accordance with the measured value, and the applicability and feasibility of the model are verified, which indicates that the model can be applied to data processing and data prediction of settlement monitoring.
3.2.2 Accuracy test and the predicted value

The above-mentioned calculations were carried out to analyze and compare the calculated value of the model form the perspective of the residuals and other aspects. In order to further verify the prediction accuracy and applicability of the model, the accuracy of the fitting values of the three monitoring point models was calculated and analyzed, and the following four model values were predicted. According to the prediction accuracy calculation model mentioned in Section 1.2, the calculation process can be summarized as follows:

(1) The variance of point J61 is 0.6350mm; the mean of the residuals is 0.00028mm; the ratio of the posterior variance is $C = 0.3381$. Small error probability $p = \left\{ \left| E(k) - \bar{E} \right| < 0.6745S_1 = 0.4283 \text{mm} \right\}$. According to the residual and residual mean values, it is easy to conclude that $\left| E(k) - \bar{E} \right|$ satisfies the above-mentioned small error probability condition, thus the small error probability $p < 0.95$. According to the classification principle in Table 1, it can be concluded that the prediction accuracy level is "good", and the time response equation (prediction equation) of the J61 point can be used in the settlement prediction of the J61 point. Subsequent observations on J61 point will be carried out 4 times. The model is used for prediction and the predicted values are 20.73109m, 20.73074m, 20.73040m, 20.73005m, respectively.

(2) The variance of point J62 is $S_1 = 0.1746 \text{mm}$; the variance of the residual is $S_2 = 0.0716 \text{mm}$; the posterior variance ratio is $C = 0.4100$; the small error probability is $p = \left\{ \left| E(k) - \bar{E} \right| < 0.6745S_1 = 0.1178 \text{mm} \right\}$. It is easy to determine that most of the $\left| E(k) - \bar{E} \right|$ satisfied the small error probability conditions based on the residual and residual mean values, thus the small error probability $p > 0.8$. According to the level comparison table, it can be determined that the prediction accuracy level is "qualified". It can be determined that the time response equation (prediction equation) of point J62 can be used in the settlement prediction of point J62, but the level is "qualified". Subsequent observations of J62 point were also performed 4 times, and the model was used for prediction. The predicted values are 20.75579m, 20.75591m, 20.75602m, 20.75614m, respectively.

(3) The variance of point J63 is $S_1 = 0.0787 \text{mm}$; the mean of the residuals is $\bar{E} = 0.0000 \text{mm}$; the variance of the residuals is $S_2 = 0.0183 \text{mm}$; the posterior variance ratio $C = 0.2325$; the small error probability $p = \left\{ \left| E(k) - \bar{E} \right| < 0.6745S_1 = 0.0531 \text{mm} \right\}$. According to the residual and residual...
mean values calculated above, it is easy to determine that all of the \( \left| E(k) - \bar{E} \right| \) satisfy the small error probability condition, thus the small error probability \( p > 0.95 \). According to the level comparison table, it can be concluded that the model prediction accuracy level is "good". It can be determined that the time response equation (prediction equation) of point J63 can be used in the settlement prediction of point J63. Subsequent observations on J63 point were carried out 4 times. Using this model for prediction, the predicted values are 20.71303m, 20.71297m, 20.71291m, 20.71286m, respectively. The statistics of the prediction accuracy level are shown in Table 6.

3.2.3 Analysis of results
According to the above calculation and analysis, the errors in the prediction model residuals are 0.21mm, 0.07mm, and 0.01mm, respectively. The errors of the measured data sequence are 0.63mm, 0.17mm, and 0.07mm, respectively. Its prediction accuracy is high. It can be seen from Figure 1 to Figure 3 that when the measured values are fitted to the calculated values of the model, the degree of fitting is good, and the precision is high, indicating that the model can determine the deformation trend of the settlement monitoring project. Through the correlation analysis of \( P \) value and \( C \) value, it was found that the model prediction level accuracy is high, and the prediction level of the two monitoring points reaches the “good” level, while the prediction level of one monitoring point reaches the “qualified” level, indicating that the model can be used to extrapolate the subsequent elevation value according to the pre-tested elevation value of the settlement monitoring point. Besides, the fitting curve can be used to analyze the settlement results[7-8]. The grey theory model is suitable for the prediction and analysis of settlement deformation.

Table 6 Measured values, model value, and forecast level statistics

| Number | 1       | 2       | 3       | 4       | 5       | 6       | P value | C value | degree |
|--------|---------|---------|---------|---------|---------|---------|---------|---------|--------|
| J61    | 20.73322| 20.73313| 20.73220| 20.73197| 20.73175| 20.73162| >0.95   | 0.3381  | Good   |
| Residual | 0.000   | 0.300   | -0.280  | -0.160  | -0.040  | 0.180   |         |         |        |
| J62    | 20.75556| 20.75521| 20.75545| 20.75534| 20.75548| 20.75577| >0.80   | 0.4100  | Qualified |
| Residual | 0.000   | -0.010  | 0.120   | -0.110  | -0.080  | 0.090   |         |         |        |
| J63    | 20.71316| 20.71334| 20.71325| 20.71318| 20.71317| 20.71309| >0.95   | 0.2325  | Good   |
| Residual | 0.000   | -0.018  | 0.014   | 0.026   | -0.022  | 0.000   |         |         |        |

Note: In the table, the unit of calculated values and measured values of the model are all m, and the unit of the residual is mm.

4. Conclusions and prospects
Based on the grey theory model GM(1,1) model, the settlement monitoring data of the main load-bearing wall of the garbage chamber was used as the original data sequence, and the measured values are calculated and analyzed. The example shows that the observed original sequence data can be is fitted by the model and the fitting accuracy is high, which indicates that the model can be used to
describe the settlement variation law of the building. There are many factors influences building settlement. The case data selected in the paper is the main load-bearing wall of the main plant, which is the key part connecting the garbage chamber and the boiler house. The second floor of the main plant is the platform that carrying urban garbage (for energy-saving power generation), and the design load of the load-bearing wall is 15000 kg. Therefore, the selected case has certain representativeness. When using this method to predict building settlement, mathematical models can be built with a small amount of incomplete information and then make predictions. At the same time, it should be noted that the prediction time interval of the grey theory model should not be too long, otherwise it will affect the accuracy of the prediction. The more the number of subsequent predictions, the lower the prediction accuracy will be, thus it is necessary to establish a new time response equation (predictive model) in time according to the new observation data, so as to maintain high prediction accuracy.

Acknowledgment
Foundation Items: Project supported by the Natural Science Foundation of the Anhui Higher Education Institutions of China (Grant No KJ2013Z082) and the Teaching Research Foundation of Anhui Agriculture University (Grant No 201537XM37).

Received: Date: 29th August, 2018/Accepted: date: 30th September, 2018

References
[1] Yue, J.P. (2014) Deformation Monitoring Technology and Application. National Defense Industry University Publishing, Beijing.
[2] Jia, M.J. (2016) Research on Building Subsidence Prediction Based on Gray Model. Geomatics & Spatial Information Technology, 39(1):44-46.
[3] Liu, N. (2011) Bridge Deformation Monitoring and Forecasting Based on Time Series Analysis. Science of Surveying and Mapping, 36(6):46-48.
[4] Zhao, L.M. (2017) A Model of Settlement Prediction of Open-pit Mine Slope Based on Time series analysis. Engineering of Surveying and Mapping, 26(9):46-50.
[5] Yang, W. (2014) Building Subsidence Monitoring Analysis Based on Piecewise Linear Interpolation and Grey System Combination Model. Scientific and Technological Information, 6(6):224-226.
[6] Jiao, B.C. (2008) The Thought and Method of Operations Research and Application. Peking University Publishing, Beijing.
[7] Zhang, Z.L. (2013) Engineering Surveying. WuHan University Publishing, WuHan.
[8] Chen, Y.Q. (2016) Engineering Surveying. Surveying and Mapping Publishing, Beijing.