TOP QUARK TOTAL AND DIFFERENTIAL CROSS SECTIONS
AT NNLO AND NNLL∗

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I present recent NNLO-NNLL results for top quark hadroproduction at the Tevatron.
The total cross section as well as transverse momentum and rapidity distributions are shown.

1. Introduction

The top quark production cross section at the Tevatron receives contributions from the threshold region where there are potentially large corrections. Near threshold for the production of the $t\bar{t}$ pair, there is restricted phase space for real gluon emission. This results in an incomplete cancellation of infrared divergences between real and virtual graphs which manifests itself in large logarithmic corrections.

At $n$th order in $\alpha_s$, these corrections are of the form $[\ln^k(s_4/m^2)]/s_4$, $k \leq 2n - 1$, with $m$ the top mass and $s_4 \equiv s + t_1 + u_1$, $s_4 \to 0$ at threshold. They can be formally resummed in moment space at next-to-leading logarithmic (NLL) or higher accuracy to all orders in perturbation theory,1–7 but a prescription is required in practice to obtain numerical results. In this talk, I present the next-to-next-to-leading order (NNLO) threshold corrections for $t\bar{t}$ production at next-to-next-to-leading logarithmic (NNLL) accuracy.8 These follow from the two-loop expansion of the resummed cross section without need of a prescription.

The factorized top quark production cross section can be written as a convolution of parton distributions $\phi$ with the perturbative cross section $\tilde{\sigma}$: $\sigma_{b_1 b_2 \to t\bar{t}} = \sum_f \phi_{f_i/b_1} \otimes \phi_{f_j/b_2} \otimes \tilde{\sigma}_{f_if_j \to t\bar{t}}$. The main partonic processes involved are $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$. By taking moments, the above expression for the cross section simplifies to $\tilde{\sigma}_{f_if_j \to t\bar{t}}(N) = \tilde{\phi}_{f_i/f}(N) \tilde{\phi}_{f_j/f}(N) \tilde{\sigma}_{f_if_j \to t\bar{t}}(N)$ where the moments are defined by $\tilde{\sigma}(N) = \int (ds_4/s) e^{-Ns_4/m^2} \tilde{\sigma}(s_4)$, etc., with $N$ the moment variable. Under moments, $[(\ln s_4/m^2)]/s_4 \to \ln^2 N$, and our goal becomes to resum logarithms of $N$.

We may refactorize the cross section1,3,5 as $\tilde{\sigma}_{f_if_j \to t\bar{t}}(N) = \tilde{\psi}_{f_i/f}(N) \tilde{\psi}_{f_j/f}(N) \times H_{IJ} \tilde{S}_{JI}(m/(N\mu_F))$, where the $\psi$ are center-of-mass parton distributions, $H$ is

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the hard-scattering matrix (N-independent), and $S$ is the soft-gluon function that describes noncollinear soft gluon emission. $H$ and $S$ are matrices in the space of color exchanges and depend on the partonic process.

Solving for the perturbative cross section $\hat{\sigma}$, we have $\hat{\sigma}(N) = (\psi/\phi)^2 \text{Tr}[H \hat{S}]$. After resumming the $N$-dependence in the exact NLO cross section $\psi/N$, we obtain the resummed top quark cross section in moment space at NLL accuracy:

$$\hat{\sigma}_{f_1, f_2 \to t\bar{t}}(N) = \exp \left[ E^{(f_1)}(N_i, \mu_F) + E^{(f_2)}(N_j, \mu_F) \right] \exp \left[ 4 \int_{\mu_R}^{\mu_F} \frac{d\mu'}{\mu'} \beta(\alpha_s(\mu'^2)) \right] \times \hat{S}(1) \mathcal{P} \exp \left[ \int_{\mu}^{\mu_F} \frac{d\mu'}{\mu'} \Gamma^i_S(\alpha_s(\mu'^2)) \right],$$

\[ \text{(1)} \]

where the incoming parton $N$-dependence is in $E^{(f)}$, $\gamma_i$ is the anomalous dimension of the field $\psi_i$, and $\Gamma^i_S$ is the process-dependent soft anomalous dimension matrix which has been calculated explicitly at one-loop.\(^1\)

The resummed cross section is then expanded to NNLO\(^8\) and even higher orders,\(^9\) thus avoiding prescription dependence and unphysical terms.\(^9\) After matching with the exact NLO cross section\(^10\) one can determine the next-to-next-to-leading logarithms and thus obtain the NNLO-NLL corrections to top quark production. Similar expansions have already been presented for electroweak-boson,\(^11\) direct photon,\(^12\) and jet\(^13\) hadroproduction.

2. NNLO-NLL corrections

We now expand the resummed cross section in the $\msbar$ scheme to two-loops in single-particle inclusive kinematics.\(^8\) The NLO threshold corrections agree with exact NLO results.\(^10\)

The NNLO-NLL threshold corrections in the $q\bar{q}$ channel are

$$\sigma_{q\bar{q} \to t\bar{t}}^{\msbar(2)}(s_4, m^2, t_1, u_1) = \sigma_{q\bar{q} \to t\bar{t}}^B \left( \frac{\alpha_s(\mu^2_R)}{\pi} \right)^2 \left[ 8C_F \left( \frac{\ln^3(s_4/m^2)}{s_4} \right) + \left[ \ln^2(s_4/m^2) \right] s_4 \right] + \left[ \ln(s_4/m^2) \right] s_4 \left[ C_F \left[ -\beta_0 + 12 \left( \text{Re} \Gamma''_{22} - C_F + C_F \ln \left( \frac{m^2}{t_1u_1} \right) - C_F \ln \left( \frac{\mu^2_F}{m^2} \right) \right) \right] \right]$$

$$\times \left[ 4 \left[ \text{Re} \Gamma'_{22} - C_F - C_F \ln \left( \frac{t_1u_1}{sm^2} \right) - C_F \ln \left( \frac{\mu^2_F}{m^2} \right) \right] \right]$$

$$+ 4\Gamma'_{12} \Gamma''_{12} - \beta_0 \left[ \text{Re} \Gamma_{22} - C_F - C_F \ln \left( \frac{t_1u_1}{sm^2} \right) - C_F \ln \left( \frac{\mu^2_F}{m^2} \right) \right]$$

$$+ 2C_F K - 16\zeta_2 C_F^2 \left] + 4C_F \frac{\alpha_s(\mu^2_R)}{\pi} \frac{(1) q\bar{q} S+V}{\msbar} \left[ \frac{\ln(s_4/m^2)}{s_4} \right] \right] + \mathcal{O} \left( \frac{1}{s_4} \right),$$

\[ \text{(2)} \]

where $\sigma^{(1) q\bar{q} S+V}$ denotes the soft plus virtual $\delta(s_4)$ terms in the NLO cross section.\(^10\)
Top quark cross section at the Tevatron

Fig. 1. Top quark production at the Tevatron: total cross section.

Note that $\hat{\sigma}^{\overline{MS}}(2)$ actually stands for any relevant double differential cross section, such as $d^2\sigma/(dt_1du_1)$ or $d^2\sigma/(dp_t^2dy)$, with appropriate expressions for the Born term $\sigma^B$. Analogous results have been derived for the $gg$ channel.8

In Fig. 1 we show the top quark cross section at the Tevatron as a function of its mass. We use the CTEQ5M14 parton densities. We note a dramatic decrease of the scale dependence when we include the NNLO-NNLL corrections. The NNLO-NNLL cross section is 6.3 pb versus 5.2 pb at NLO, an enhancement of over 20%, at $\mu = m$. Good agreement is observed with recent results from CDF and D0. At Run II, with $\sqrt{s} = 2.0$ TeV, we predict a NNLO-NNLL cross section of 8.8 pb versus 7.1 at NLO with $\mu = m$.

The top quark transverse momentum distribution is shown in Fig. 2. We note an overall enhancement at NNLO with little change of shape. Similar conclusions are also reached for the rapidity distribution.

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Fig. 2. Top quark production at the Tevatron: transverse momentum distribution.

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