Firefly Algorithm: Recent Advances and Applications

Xin-She Yang  
School of Science and Technology,  
Middlesex University, The Burroughs, London NW4 4BT, UK.  

Xingshi He  
School of Science, Xi’an Polytechnic University,  
No. 19 Jinhua South Road, Xi’an 710048, P. R. China.

Abstract

Nature-inspired metaheuristic algorithms, especially those based on swarm intelligence, have attracted much attention in the last ten years. Firefly algorithm appeared in about five years ago, its literature has expanded dramatically with diverse applications. In this paper, we will briefly review the fundamentals of firefly algorithm together with a selection of recent publications. Then, we discuss the optimality associated with balancing exploration and exploitation, which is essential for all metaheuristic algorithms. By comparing with intermittent search strategy, we conclude that metaheuristics such as firefly algorithm are better than the optimal intermittent search strategy. We also analyse algorithms and their implications for higher-dimensional optimization problems.

Keywords: Algorithms; bat algorithm; cuckoo search; firefly algorithm; metaheuristic; nature-inspired algorithms.

Reference to this paper should be made as follows: Xin-She Yang and Xingshi He, (2013). ‘Firefly Algorithm: Recent Advances and Applications’, Int. J. Swarm Intelligence, Vol. 1, No. 1, pp. 36–50. DOI: 10.1504/IJSI.2013.055801

1 Introduction

Metaheuristic algorithms form an important part of contemporary global optimisation algorithms, computational intelligence and soft computing. These algorithms are usually nature-inspired with multiple interacting agents. A subset of metaheuristics are often referred to as swarm intelligence (SI) based algorithms, and these SI-based algorithms have been developed by mimicking the so-called swarm intelligence characteristics of biological agents such as birds, fish, humans and others. For example, particle swarm optimisation was based on the swarming behaviour of birds and fish [24], while the firefly algorithm was based on the flashing pattern of tropical fireflies [32, 33] and cuckoo search algorithm was inspired by the brood parasitism of some cuckoo species [37].

In the last two decades, more than a dozen new algorithms such as particle swarm optimisation, differential evolution, bat algorithm, firefly algorithm and cuckoo search have appeared and they have shown great potential in solving tough engineering optimisation problems [32, 5, 16, 27, 34, 35, 17]. Among these new algorithms, it has been shown that firefly algorithm is very efficient in dealing with multimodal, global optimisation problems.

In this paper, we will first outline the fundamentals of firefly algorithm (FA), and then review the latest developments concerning FA and its variants. We also highlight the reasons...
why FA is so efficient. Furthermore, as the balance of exploration and exploitation is important to all metaheuristic algorithms, we will then discuss the optimality related to search landscape and algorithms. Using the intermittent search strategy and numerical experiments, we show that firefly algorithm is significantly more efficient than intermittent search strategy.

2 Firefly Algorithm and Complexity

2.1 Firefly Algorithm

Firefly Algorithm (FA) was first developed by Xin-She Yang in late 2007 and 2008 at Cambridge University [32, 33], which was based on the flashing patterns and behaviour of fireflies. In essence, FA uses the following three idealized rules:

- Fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex.
- The attractiveness is proportional to the brightness, and they both decrease as their distance increases. Thus for any two flashing fireflies, the less brighter one will move towards the brighter one. If there is no brighter one than a particular firefly, it will move randomly.
- The brightness of a firefly is determined by the landscape of the objective function.

As a firefly’s attractiveness is proportional to the light intensity seen by adjacent fireflies, we can now define the variation of attractiveness $\beta$ with the distance $r$ by

$$\beta = \beta_0 e^{-\gamma r^2},$$  \hspace{1cm} (1)

where $\beta_0$ is the attractiveness at $r = 0$.

The movement of a firefly $i$ is attracted to another more attractive (brighter) firefly $j$ is determined by

$$x_{t+1}^i = x_t^i + \beta_0 e^{-\gamma r^2_{ij}} (x_t^j - x_t^i) + \alpha_t \epsilon^i_t,$$  \hspace{1cm} (2)

where the second term is due to the attraction. The third term is randomization with $\alpha_t$ being the randomization parameter, and $\epsilon^i_t$ is a vector of random numbers drawn from a Gaussian distribution or uniform distribution at time $t$. If $\beta_0 = 0$, it becomes a simple random walk. On the other hand, if $\gamma = 0$, it reduces to a variant of particle swarm optimisation [32]. Furthermore, the randomization $\epsilon^i_t$ can easily be extended to other distributions such as Lévy flights [32]. A demo version of firefly algorithm implementation by Xin-She Yang, without Lévy flights for simplicity, can be found at Mathworks file exchange web site.

2.2 Parameter Settings

As $\alpha_t$ essentially control the randomness (or, to some extent, the diversity of solutions), we can tune this parameter during iterations so that it can vary with the iteration counter $t$. So a good way to express $\alpha_t$ is to use

$$\alpha_t = \alpha_0 \delta^t, \hspace{1cm} 0 < \delta < 1,$$  \hspace{1cm} (3)

[1] http://www.mathworks.com/matlabcentral/fileexchange/29693-firefly-algorithm
where \( \alpha_0 \) is the initial randomness scaling factor, and \( \delta \) is essentially a cooling factor. For most applications, we can use \( \delta = 0.95 \) to 0.97 [32].

Regarding the initial \( \alpha_0 \), simulations show that FA will be more efficient if \( \alpha_0 \) is associated with the scalings of design variables. Let \( L \) be the average scale of the problem of interest, we can set \( \alpha_0 = 0.01L \) initially. The factor 0.01 comes from the fact that random walks requires a number of steps to reach the target while balancing the local exploitation without jumping too far in a few steps [33, 34].

The parameter \( \beta \) controls the attractiveness, and parametric studies suggest that \( \beta_0 = 1 \) can be used for most applications. However, \( \gamma \) should be also related to the scaling \( L \). In general, we can set \( \gamma = 1/\sqrt{L} \). If the scaling variations are not significant, then we can set \( \gamma = O(1) \).

For most applications, we can use the population size \( n \) = 15 to 100, though the best range is \( n = 25 \) to 40 [32, 33].

2.3 Algorithm Complexity

Almost all metaheuristic algorithms are simple in terms of complexity, and thus they are easy to implement. Firefly algorithm has two inner loops when going through the population \( n \), and one outer loop for iteration \( t \). So the complexity at the extreme case is \( O(n^2 t) \). As \( n \) is small (typically, \( n = 40 \)), and \( t \) is large (say, \( t = 5000 \)), the computation cost is relatively inexpensive because the algorithm complexity is linear in terms of \( t \). The main computational cost will be in the evaluations of objective functions, especially for external black-box type objectives. This latter case is also true for all metaheuristic algorithms. After all, for all optimisation problems, the most computationally extensive part is objective evaluations.

If \( n \) is relatively large, it is possible to use one inner loop by ranking the attractiveness or brightness of all fireflies using sorting algorithms. In this case, the algorithm complexity of firefly algorithm will be \( O(nt \log(n)) \).

2.4 Firefly Algorithm in Applications

Firefly algorithm has attracted much attention and has been applied to many applications [3, 10, 19, 30, 36, 20, 21]. Horng et al. demonstrated that firefly-based algorithm used least computation time for digital image compression [20, 21], while Banati and Bajaj used firefly algorithm for feature selection and showed that firefly algorithm produced consistent and better performance in terms of time and optimality than other algorithms [4].

In the engineering design problems, Gandomi et al. [17] and Azad and Azad [2] confirmed that firefly algorithm can efficiently solve highly nonlinear, multimodal design problems. Basu and Mahanti [6] as well as Chatterjee et al. have applied FA in antenna design optimisation and showed that FA can outperform artificial bee colony (ABC) algorithm [10]. In addition, Zaman and Matin have also found that FA can outperform PSO and obtained global best results [43].

Sayadi et al. developed a discrete version of FA which can efficiently solve NP-hard scheduling problems [30], while a detailed analysis has demonstrated the efficiency of FA over a wide range of test problems, including multiojective load dispatch problems [3, 33, 39]. Furthermore, FA can also solve scheduling and travelling salesman problem in a promising way [26, 23, 42].
Classifications and clustering are another important area of applications of FA with excellent performance [31, 28]. For example, Senthilnath et al. provided an extensive performance study by compared FA with 11 different algorithms and concluded that firefly algorithm can be efficiently used for clustering [31]. In most cases, firefly algorithm outperform all other 11 algorithms. In addition, firefly algorithm has also been applied to train neural networks [25].

For optimisation in dynamic environments, FA can also be very efficient as shown by Farahani et al. [13, 14] and Abshouri et al. [1].

2.5 Variants of Firefly Algorithm

For discrete problems and combinatorial optimisation, discrete versions of firefly algorithm have been developed with superior performance [30, 19, 23, 15, 12], which can be used for travelling-salesman problems, graph colouring and other applications.

In addition, extension of firefly algorithm to multiobjective optimisation has also been investigated [3, 41].

A few studies show that chaos can enhance the performance of firefly algorithm [11, 38], while other studies have attempted to hybridize FA with other algorithms to enhance their performance [18, 21, 22, 29].

3 Why Firefly Algorithm is So Efficient?

As the literature about firefly algorithm expands and new variants have emerged, all pointed out the firefly algorithm can outperform many other algorithms. Now we may ask naturally “Why is it so efficient?”. To answer this question, let us briefly analyze the firefly algorithm itself.

FA is swarm-intelligence-based, so it has the similar advantages that other swarm-intelligence-based algorithms have. In fact, a simple analysis of parameters suggests that some PSO variants such as Accelerated PSO [40] are a special case of firefly algorithm when $\gamma = 0$ [32].

However, FA has two major advantages over other algorithms: automatical subdivision and the ability of dealing with multimodality. First, FA is based on attraction and attractiveness decreases with distance. This leads to the fact that the whole population can automatically subdivide into subgroups, and each group can swarm around each mode or local optimum. Among all these modes, the best global solution can be found. Second, this subdivision allows the fireflies to be able to find all optima simultaneously if the population size is sufficiently higher than the number of modes. Mathematically, $1/\sqrt{\gamma}$ controls the average distance of a group of fireflies that can be seen by adjacent groups. Therefore, a whole population can subdivide into subgroups with a given, average distance. In the extreme case when $\gamma = 0$, the whole population will not subdivide. This automatic subdivision ability makes it particularly suitable for highly nonlinear, multimodal optimisation problems.

In addition, the parameters in FA can be tuned to control the randomness as iterations proceed, so that convergence can also be sped up by tuning these parameters. These above advantages makes it flexible to deal with continuous problems, clustering and classifications, and combinatorial optimisation as well.

As an example, let use use two functions to demonstrate the computational cost saved by FA. For details, please see the more extensive studies by Yang [33]. For De Jong’s function
Figure 1: Four global maxima at \((\pm 1/2, \pm 1/2)\).

with \(d = 256\) dimensions

\[
f(x) = \sum_{i=1}^{256} x_i^2.
\]

(4)

Genetic algorithms required \(25412 \pm 1237\) evaluations to get an accuracy of \(10^{-5}\) of the optimal solution, while PSO needed \(17040 \pm 1123\) evaluations. For FA, we achieved the same accuracy by \(5657 \pm 730\). This save about 78% and 67% computational cost, compared to GA and PSO, respectively.

For Yang’s forest function

\[
f(x) = \left(\sum_{i=1}^{d} |x_i|\right) \exp\left[-\sum_{i=1}^{d} \sin(x_i^2)\right], \quad -2\pi \leq x_i \leq 2\pi,
\]

(5)

GA required \(37079 \pm 8920\) with a success rate of 88% for \(d = 16\), and PSO required \(19725 \pm 3204\) with a success rate of 98%. FA obtained a 100% success rate with just \(5152 \pm 2493\). Compared with GA and PSO, FA saved about 86% and 74%, respectively, of overall computational efforts.

As an example for automatic subdivision, we now use the FA to find the global maxima of the following function

\[
f(x) = \left(\sum_{i=1}^{d} |x_i|\right) \exp\left(-\sum_{i=1}^{d} x_i^2\right),
\]

(6)

with the domain \(-10 \leq x_i \leq 10\) for all \((i = 1, 2, ..., d)\) where \(d\) is the number of dimensions. This function has multiple global optima. In the case of \(d = 2\), we have 4 equal maxima \(f_* = 1/\sqrt{e} \approx 0.6065\) at \((1/2, 1/2)\), \((1/2, -1/2)\), \((-1/2, 1/2)\) and \((-1/2, -1/2)\) and a unique global minimum at \((0, 0)\).

In the 2D case, we have a four-peak function are shown in Fig. 1 and these global maxima can be found using the implemented Firefly Algorithms after about 500 function
evaluations. This corresponds to 25 fireflies evolving for 20 generations or iterations. The initial locations of 25 fireflies and their final locations after 20 iterations are shown in Fig. 2. We can see that FA can indeed automatically subdivide its population into subgroups.

4 Search Optimality

4.1 Intensification versus Diversification

The main components of any metaheuristic algorithms are: intensification and diversification, or exploitation and exploration [9, 39]. Diversification means to generate diverse solutions so as to explore the search space on the global scale, while intensification means to focus on the search in a local region by exploiting the information that a current good solution is found in this region. This is in combination with the selection of the best solutions.

Exploration in metaheuristics can be achieved often by the use of randomization [9, 32, 33], which enables an algorithm to have the ability to jump out of any local optimum so as to explore the search globally. Randomization can also be used for local search around the current best if steps are limited to a local region. When the steps are large, randomization can explore the search space on a global scale. Fine-tuning the right amount of randomness and balancing local search and global search are crucially important in controlling the performance of any metaheuristic algorithm.

Exploitation is the use of local knowledge of the search and solutions found so far so that new search moves can concentrate on the local regions or neighborhood where the optimality may be close; however, this local optimum may not be the global optimality. Exploitation tends to use strong local information such as gradients, the shape of the mode such as convexity, and the history of the search process. A classic technique is the so-called hill-climbing which uses the local gradients or derivatives intensively.

Empirical knowledge from observations and simulations of the convergence behaviour of common optimisation algorithms suggests that exploitation tends to increase the speed of convergence, while exploration tends to decrease the convergence rate of the algorithm.
On the other hand, too much exploration increases the probability of finding the global optimality, while strong exploitation tends to make the algorithm being trapped in a local optimum. Therefore, there is a fine balance between the right amount of exploration and the right degree of exploitation. Despite its importance, there is no practical guideline for this balance.

4.2 Landscape-Based Optimality or Algorithm Optimality?

It is worth pointing out that the balance for exploration and exploitation was often discussed in the context of optimisation algorithms; however, this can be more often related to the problem type of interest, and thus such balance is problem-specific, depending on the actual landscape of the objective function. Consequently, there may be no universality at all. Therefore, we may have to distinguish landscape-dependent optimality for exploration and exploitation, and algorithm-based optimality. The former focuses on landscape, while the latter focuses on algorithms.

Landscape-based optimality focuses on the information of the search landscape and it is hoped that a better (or even best) approach/algorithm may find the optimal solutions with the minimum effort (time, number of evaluations), while algorithm-based approach treats objective functions as a black-box type and tries to use partly the available information during iterations to work out the best ways to find optimal solutions. As to which approach is better, the answer may depend on one’s viewpoint and focus. In any case, a good combination of both approaches may be needed to reach certain optimality.

4.3 Intermittent Search Strategy

Even there is no guideline in practice for landscape-based optimality, some preliminary work on the very limited cases exists in the literature and may provide some insight into the possible choice of parameters so as to balance these components. In fact, intermittent search strategy is a landscape-based search strategy [7].

Intermittent search strategies concern an iterative strategy consisting of a slow phase and a fast phase [7, 8]. Here the slow phase is the detection phase by slowing down and intensive, static local search techniques, while the fast phase is the search without detection and can be considered as an exploration technique. For example, the static target detection with a small region of radius $a$ in a much larger region $b$ where $a \ll b$ can be modelled as a slow diffusive process in terms of random walks with a diffusion coefficient $D$.

Let $\tau_a$ and $\tau_b$ be the mean times spent in intensive detection stage and the time spent in the exploration stage, respectively, in the 2D case. The diffusive search process is governed by the mean first-passage time satisfying the following equations [8]

$$D \nabla^2 r_1 + \frac{1}{2\pi \tau_a} \int_0^{2\pi} [t_2(r) - t_1(r)] d\theta + 1 = 0,$$

$$u \cdot \nabla r_2(t_2(r)) - \frac{1}{\tau_b} [r_2(t_2(r)) - t_1(r)] + 1 = 0,$$

where $t_2$ and $t_1$ are mean first-passage times during the search process, starting from slow and fast stages, respectively, and $u$ is the mean search speed.

After some lengthy mathematical analysis, the optimal balance of these two stages can be estimated as

$$r_{\text{optimal}} = \frac{\tau_a}{\tau_b} \approx \frac{D}{a^2 \left[2 - \frac{1}{\ln(b/a)}\right]^2}.$$
Assuming that the search steps have a uniform velocity $u$ at each step on average, the minimum times required for each phase can be estimated as

$$
\tau_a^{\min} \approx \frac{D}{2u^2} \frac{\ln^2(b/a)}{[2 \ln(b/a) - 1]},
$$

and

$$
\tau_b^{\min} \approx \frac{a}{u} \sqrt{\ln(b/a) - \frac{1}{2}}.
$$

(11)

When $u \to \infty$, these relationships lead to the above optimal ratio of two stages. It is worth pointing out that the above result is only valid for 2D cases, and there is no general results for higher dimensions, except in some special 3D cases [7]. Now let us use this limited results to help choose the possible values of algorithm-dependent parameters in firefly algorithm [32, 33], as an example.

For higher-dimensional problems, no result exists. One possible extension is to use extrapolation to get an estimate. Based on the results on 2D and 3D cases [8], we can estimate that for any $d$-dimensional cases $d \geq 3$

$$
\frac{\tau_1}{\tau_2^d} \sim O \left( \frac{D}{a^2} \right), \quad \tau_m \sim O \left( \frac{b}{u} \left( \frac{b}{a} \right)^{d-1} \right),
$$

(12)

where $\tau_m$ the mean search time or average number of iterations. This extension may not be good news for higher dimensional problems, as the mean number of function evaluations to find optimal solutions can increase exponentially as the dimensions increase. However, in practice, we do not need to find the guaranteed global optimality, we may be satisfied with suboptimality, and sometimes we may be ‘lucky’ to find such global optimality even with a limited/fixated number of iterations. This may indicate there is a huge gap between theoretical understanding and the observations as well as run-time behaviour in practice. More studies are highly needed to address these important issues.

5 Numerical Experiments

5.1 Landscape-Based Optimality: A 2D Example

If we use the 2D simple, isotropic random walks for local exploration to demonstrate landscape-based optimality, then we have

$$
D \approx \frac{s^2}{2},
$$

(13)

where $s$ is the step length with a jump during a unit time interval or each iteration step. From equation (9), the optimal ratio of exploitation and exploration in a special case of $b \approx 10a$ becomes

$$
\frac{\tau_a}{\tau_b} \approx 0.2.
$$

(14)

In case of $b/a \to \infty$, we have $\tau_a/\tau_b^2 \approx 1/8$, which implies that more times should spend on the exploration stage. It is worth pointing out that the naive guess of 50-50 probability in each stage is not the best choice. More efforts should focus on the exploration so that the best solutions found by the algorithm can be globally optimal with possibly the least computing effort. However, this case may be implicitly linked to the implicit assumptions
Table 1: Variations of $Q$ and its effect on the solution quality.

| $Q$  | 0.4 | 0.3 | 0.2 | 0.1 | 0.05 |
|------|-----|-----|-----|-----|------|
| $f_{\text{min}}$ | 9.4e-11 | 1.2e-12 | 2.9e-14 | 8.1e-12 | 9.2e-11 |

that the optimal solutions or search targets are multimodal. Obviously, for a unimodal problem, once we know its modality, we should focus more on the exploitation to get quick convergence.

In the case studies to be described below, we have used the firefly algorithm to find the optimal solutions to the benchmarks. If set $\tau_0 = 1$ as the reference timescale, then we found that the optimal ratio is between 0.15 to 0.24, which are roughly close to the above theoretical result.

### 5.2 Standing-Wave Function

Let us first use a multimodal test function to see how to find the fine balance between exploration and exploitation in an algorithm for a given task. A standing-wave test function can be a good example [33, 34]

$$\begin{align*}
f(x) &= 1 + \left\{ \exp\left[-\sum_{i=1}^{d} \left(\frac{x_i}{\beta}\right)^{10}\right] - 2 \exp\left[-\sum_{i=1}^{d} (x_i - \pi)^2\right] \right\} \cdot \prod_{i=1}^{d} \cos^2 x_i, \\
&= f(x) = 1 + \left\{ \exp\left[-\sum_{i=1}^{d} \left(\frac{x_i}{\beta}\right)^{10}\right] - 2 \exp\left[-\sum_{i=1}^{d} (x_i - \pi)^2\right] \right\} \cdot \prod_{i=1}^{d} \cos^2 x_i,
\end{align*}$$

which is multimodal with many local peaks and valleys. It has a unique global minimum at $f_{\text{min}} = 0$ at $(\pi, \pi, ..., \pi)$ in the domain $-20 \leq x_i \leq 20$ where $i = 1, 2, ..., d$ and $\beta = 15$. In this case, we can estimate that $R = 20$ and $a \approx \pi/2$, this means that $R/a \approx 12.7$, and we have in the case of $d = 2$

$$
p_e \approx \tau_{\text{optimal}} \approx \frac{1}{2\pi - 1/\ln(R/a)} \approx 0.19.
$$

This indicate that the algorithm should spend 80% of its computational effort on global explorative search, and 20% of its effort on local intensive search.

For the firefly algorithm, we have used $n = 15$ and 1000 iterations. We have calculated the fraction of iterations/function evaluations for exploitation to exploration. That is, $Q = \text{exploitation}/\text{exploration}$, thus $Q$ may affect the quality of solutions. A set of 25 numerical experiments have been carried out for each value of $Q$ and the results are summarized in Table 1.

This table clearly shows that $Q \approx 0.2$ provides the optimal balance of local exploitation and global exploration, which is consistent with the theoretical estimation.

Though there is no direct analytical results for higher dimensions, we can expect that more emphasis on global exploration is also true for higher dimensional optimisation problems. Let us study this test function for various higher dimensions.

### 5.3 Comparison for Higher Dimensions

As the dimensions increase, we usually expect the number of iterations of finding the global optimality should increase. In terms of mean search time/iterations, Bénichou et al.’s
intermittent search theory suggests that \([7, 8]\)

\[
\tau_m \bigg|_{(d=1)} = \frac{2b}{u} \sqrt[3]{\frac{b}{3a}},
\]

(17)

\[
\tau_m \bigg|_{(d=2)} = \frac{2b^2}{au} \sqrt{\ln(\frac{b}{a})},
\]

(18)

\[
\tau_m \bigg|_{(d=3)} = \frac{2.2b}{u} \left(\frac{b}{a}\right)^2.
\]

(19)

For higher dimensions, we can only estimate the main trend based on the intermittent search strategy. That is,

\[
\frac{\tau_1}{\tau_2} \sim O\left(\frac{D}{a^2}\right), \quad \tau_m \sim O\left(\frac{b}{u} \left(\frac{b}{a}\right)^{d-1}\right),
\]

(20)

which means that number of iterations may increase exponentially with the dimension \(d\). It is worth pointing out that the optimal ratio between the two stage should be independent of the dimensions. In other words, once we find the optimal balance between exploration and exploitation, we can use the algorithm for any high dimensions.

Now let us use firefly algorithm to carry out search in higher dimensions for the above standing wave function and compare its performance with the implication of intermittent search strategy. For the case of \(b = 20\), \(a = \pi / 2\) and \(u = 1\), Fig. 3 shows the comparison of the numbers of iterations suggested by intermittent search strategy and the actual numbers of iterations using firefly algorithm to obtain the globally optimal solution with a tolerance or accuracy of 5 decimal places. It can be seen clearly that the number of iterations needed by the intermittent search strategy increases exponentially versus the number of dimensions, while the actual number of iterations used in the algorithm only increases slightly, seemingly weakly a low-order polynomial. This suggests that firefly algorithm is very efficient and requires far fewer (and often many orders lower) number of function evaluations.
6 Conclusions

Nature-inspired metaheuristic algorithms have gained popularity, which is partly due to their ability of dealing with nonlinear global optimisation problems. We have reviewed the fundamentals of firefly algorithm, the latest developments with diverse applications. As the time of writing, a quick Google search suggests that there are about 323 papers on firefly algorithms from 2008. This review can only cover a fraction of the literature. There is no doubt that firefly algorithm will be applied in solving more challenging problems in the near future, and its literature will continue to expand.

On the other hand, we have also highlighted the importance of exploitation and exploration and their effect on the efficiency of an algorithm. Then, we use the intermittent search strategy theory as a preliminary basis for analyzing these key components and ways to find the possibly optimal settings for algorithm-dependent parameters.

With such insight, we have used the firefly algorithm to find this optimal balance, and confirmed that firefly algorithm can indeed provide a good balance of exploitation and exploration. We have also shown that firefly algorithm requires far fewer function evaluations. However, the huge differences between intermittent search theory and the behaviour of metaheuristics in practice also suggest there is still a huge gap between our understanding of algorithms and the actual behaviour of metaheuristics. More studies in metaheuristics are highly needed.

It is worth pointing out that there are two types of optimality here. One optimality concerns that for a given algorithm what best types of problems it can solve. This is relatively easy to answer because in principle we can test an algorithm by a wide range of problems and then select the best type of the problems the algorithm of interest can solve. On other hand, the other optimality concerns that for a given problem what best algorithm is to find the solutions efficiently. In principle, we can compare a set of algorithms to solve the same optimisation problem and hope to find the best algorithm(s). In reality, there may be no such algorithm at all, and all test algorithms may not perform well. Search for new algorithms may take substantial research efforts.

The theoretical understanding of metaheuristics is still lacking behind. In fact, there is a huge gap between theory and applications. Though theory lags behind, applications in contrast are very diverse and active with thousands of papers appearing each year. Furthermore, there is another huge gap between small-scale problems and large-scale problems. As most published studies have focused on small, toy problems, there is no guarantee that the methodology that works well for such toy problems will work for large-scale problems. All these issues still remain unresolved both in theory and in practice.

As further research topics, most metaheuristic algorithms require good modifications so as to solve combinatorial optimisation properly. Though with great interest and many extensive studies, more studies are highly needed in the area of combinatorial optimisation using metaheuristic algorithms. In addition, most current metaheuristic research has focused on small scale problems, it will be extremely useful if further research can focus on large-scale real-world applications.

References

[1] A. A. Abshouri, M. R. Meybodi and A. Bakhtiary, New firefly algorithm based on multiswarm and learning automata in dynamic environments, Third Int. Conference on
Signal Processing Systems (ICSPS2011), Aug 27-28, Yantai, China, pp. 73-77 (2011).

[2] Sina K. Azad, Saeid K. Azad, Optimum Design of Structures Using an Improved Firefly Algorithm, *International Journal of Optimisation in Civil Engineering*, 1(2), 327-340(2011).

[3] Apostolopoulos T. and Vlachos A., (2011). Application of the Firefly Algorithm for Solving the Economic Emissions Load Dispatch Problem, *International Journal of Combinatorics*, Volume 2011, Article ID 523806. [http://www.hindawi.com/journals/ijct/2011/523806.html](http://www.hindawi.com/journals/ijct/2011/523806.html)

[4] H. Banati and M. Bajaj, Firefly based feature selection approach, *Int. J. Computer Science Issues*, 8(2), 473-480 (2011).

[5] J. C. Bansal and K. Deep, Optimisation of directional overcurrent relay times by particle swarm optimisation, in: Swarm Intelligence Symposium (SIS 2008), IEEE Publication, pp. 1-7, (2008).

[6] B. Basu and G. K. Mahanti, Firefly and artificial bees colony algorithm for synthesis of scanned and broadside linear array antenna, *Progress in Electromagnetic Research B.*, 32, 169-190 (2011).

[7] O. Bénichou, C. Loverdo, M. Moreau and R. Voituriez, Two-dimensional intermittent search processes: An alternative to Lévy flight strategies, *Phys. Rev.*, E74, 020102(R), (2006).

[8] O. Bénichou, C. Loverdo, M. Moreau, and R. Voituriez, Intermittent search strategies, *Review of Modern Physics*, 83, 81-129 (2011).

[9] C. Blum and A. Roli, Metaheuristics in combinatorial optimisation: Overview and conceptual comparision, *ACM Comput. Surv.*, Vol. 35, 268-308 (2003).

[10] A. Chatterjee, G. K. Mahanti, and A. Chatterjee, Design of a fully digital controlled reconfigurable switched beam concomcentric ring array antenna using firefly and particle swarm optimisation algorithm, *Progress in Electromagnetic Research B.*, 36, 113-131 (2012).

[11] L. dos Santos Coelho, D. L. de Andrade Bernert, V. C. Mariani, a chaotic firefly algorithm applied to reliability-redundancy optimisation, in: *2011 IEEE Congress on Evolutionary Computation (CEC’11)*, pp. 517-521 (2011).

[12] K. Durkota, Implementation of a discrete firefly algorithm for the QAP problem within the sage framework, BSc thesis, Czech Technical University, (2011).

[13] S. M. Farahani, A. A. Abshouri, B. Nasiri and M. R. Meybodi, A Gaussian firefly algorithm, *Int. J. Machine Learning and Computing*, 1(5), 448-453 (2011).

[14] S. M. Farahani, B. Nasiri and M. R. Meybodi, A multiswarm based firefly algorithm in dynamic environments, in: *Third Int. Conference on Signal Processing Systems (IC-SPS2011)*, Aug 27-28, Yantai, China, pp. 68-72 (2011).
[15] I. Fister Jr, I. Fister, J. Brest, X. S. Yang, Memetic firefly algorithm for combinatorial optimisation, in: *Bioinspired Optimisation Methods and Their Applications (BIOMA2012)* edited by B. Filipiˇ c and J. ˇSilc, 24-25 May 2012, Bohinj, Slovenia, pp. 75-86 (2012).

[16] C. A. Floudas and P. M. Pardalos, *Encyclopedia of Optimisation*, 2nd Edition, Springer (2009).

[17] A. H. Gandomi, X. S. Yang, and A. H. Alavi, Cuckoo search algorithm: a metaheuristic approach to solve structural optimisation problems, *Engineering with Computers*, 27, article DOI 10.1007/s00366-011-0241-y, (2011).

[18] G. Giannakouris, V. Vassiliadis and G. Doumas, Experimental study on a hybrid nature-inspired algorithm for financial portfolio optimisation, SETN 2010, Lecture Notes in Artificial Intelligence (LNAI 6040), pp. 101-111 (2010).

[19] T. Hassanzadeh, H. Vojodi and A. M. E. Moghadam, An image segmentation approach based on maximum variance intra-cluster method and firefly algorithm, in: *Proc. of 7th Int. Conf. on Natural Computation (ICNC2011)*, pp. 1817-1821 (2011).

[20] M.-H. Horng, Y.-X. Lee, M.-C. Lee and R.-J. Liou, Firefly metaheuristic algorithm for training the radial basis function network for data classification and disease diagnosis, in: *Theory and New Applications of Swarm Intelligence* (Edited by R. Parpinelli and H. S. Lopes), pp. 115-132 (2012).

[21] M.-H. Horng, Vector quantization using the firefly algorithm for image compression, *Expert Systems with Applications*, 39, pp. 1078-1091 (2012).

[22] M.-H. Horng and R.-J. Liou, Multilevel minimum cross entropy threshold selection based on the firefly algorithm, *Expert Systems with Applications*, 38, pp. 14805-14811 (2011).

[23] G. K. Jati and S. Suyanto, Evolutionary discrete firefly algorithm for travelling salesman problem, ICAIS2011, Lecture Notes in Artificial Intelligence (LNAI 6943), pp.393-403 (2011).

[24] J. Kennedy and R. Eberhart, Particle swarm optimisation, in: *Proc. of the IEEE Int. Conf. on Neural Networks*, Piscataway, NJ, pp. 1942-1948 (1995).

[25] S. Nandy, P. P. Sarkar, A. Das, Analysis of nature-inspired firefly algorithm based back-propagation neural network training, *Int. J. Computer Applications*, 43(22), 8-16 (2012).

[26] S. Palit, S. Sinha, M. Molla, A. Khanra, M. Kule, A cryptanalytic attack on the knapsack cryptosystem using binary Firefly algorithm, in: *2nd Int. Conference on Computer and Communication Technology (ICCCT)*, 15-17 Sept 2011, India, pp. 428-432 (2011).

[27] R. S. Parpinelli and H. S. Lopes, New inspirations in swarm intelligence: a survey, *Int. J. Bio-Inspired Computation*, 3, 1-16 (2011).

[28] A. Rajini, V. K. David, A hybrid metaheuristic algorithm for classification using micro array data, *Int. J. Scientific & Engineering Research*, 3(2), 1-9 (2012).
[29] B. Rampriya, K. Mahadevan and S. Kannan, Unit commitment in deregulated power system using Lagrangian firefly algorithm, *Proc. of IEEE Int. Conf. on Communication Control and Computing Technologies (ICCCCT2010)*, pp. 389-393 (2010).

[30] Sayadi M. K., Ramezanian R. and Ghaffari-Nasab N., (2010). A discrete firefly meta-heuristic with local search for makespan minimization in permutation flow shop scheduling problems, *Int. J. of Industrial Engineering Computations*, 1, 1–10.

[31] J. Senthilnath, S. N. Omkar, V. Mani, Clustering using firefly algorithm: performance study, *Swarm and Evolutionary Computation*, 1(3), 164-171 (2011).

[32] X. S. Yang, *Nature-Inspired Metaheuristic Algorithms*, Luniver Press, UK, (2008).

[33] X. S. Yang, Firefly algorithms for multimodal optimisation, *Proc. 5th Symposium on Stochastic Algorithms, Foundations and Applications*, (Eds. O. Watanabe and T. Zeugmann), Lecture Notes in Computer Science, 5792: 169-178 (2009).

[34] X. S. Yang, *Engineering Optimisation: An Introduction with Metaheuristic Applications*, John Wiley and Sons, USA (2010).

[35] X. S. Yang, A new metaheuristic bat-inspired algorithm, in: *Nature Inspired Cooperative Strategies for Optimisation* (NISCO 2010) (Eds. J. R. Gonzalez et al.), Springer, SCI Vol. 284, 65-74 (2010).

[36] X. S. Yang, Firefly algorithm, stochastic test functions and design optimisation, *Int. J. Bio-Inspired Computation*, 2(2), 78-84 (2010).

[37] X. S. Yang and S. Deb, Cuckoo search via Lévy flights, *Proceedings of World Congress on Nature & Biologically Inspired Computing* (NaBIC 2009, India), IEEE Publications, USA, pp. 210-214 (2009).

[38] X. S. Yang, Chaos-enhanced firefly algorithm with automatic parameter tuning, *Int. J. Swarm Intelligence Research*, 2(4), pp. 1-11 (2011).

[39] X. S. Yang, Swarm-based metaheuristic algorithms and no-free-lunch theorems, in: *Theory and New Applications of Swarm Intelligence* (Eds. R. Parpinelli and H. S. Lopes), Intech Open Science, pp. 1-16 (2012).

[40] X. S. Yang, S. Deb and S. Fong, (2011). Accelerated particle swarm optimization and support vector machine for business optimization and applications, *Networked Digital Technologies (NDT’2011)*, Communications in Computer and Information Science, Vol. 136, Part I, pp. 53-66.

[41] X. S. Yang, Multiobjective firefly algorithm for continuous optimization, *Engineering with Computers*, Online Fist, DOI: 10.1007/s00366-012-0254-1 (2012).

[42] A. Yousif, A. H. Abdullah, S. M. Nor, A. A. abdelaziz, Scheduling jobs on grid computing using firefly algorithm, *J. Theoretical and Applied Information Technology*, 33(2), 155-164 (2011).

[43] M. A. Zaman and M. A. Matin, Nonuniformly spaced linear antenna array design using firefly algorithm, *Int. J. Microwave Science and Technology*, Vol. 2012, Article ID: 256759, (8 pages), 2012. doi:10.1155/2012/256759