Proposals for Calculation of Bucking Coefficient for Concrete-Filled Steel Tube Columns

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Abstract: This paper demonstrates that the methodology currently standardized in Russia to factor in the flexibility of reinforced concrete components under extra-central compression produce results that satisfactorily match the experimental values; however, that only holds for the components with a flexibility of $\lambda=40÷60$. Given the complex stress state of the concrete core and the steel shell as well as due to the concrete-filled steel tube columns being prone to deformation, this method cannot be used to reliably calculate their load capacity. The literature review has revealed many researchers' suggestions to factor in the flexibility of concrete-filled steel tubes by means of the buckling coefficient that reduces the limit value of longitudinal force a short compressed element can take. We have analyzed the methods currently standardized in Europe and China as well as more advanced methods proposed by Chinese scientists. Calculating by these methods led to the results that excessively deviated from experimental values. By statistically analyzing a large volume of own and third-party research data as well as the data obtained by non-linear deformation computing, we have derived a new formula to determine the bucking coefficient depending on the relative flexibility.

1. Introduction

The longer are compressed components, the more the critical load applied with a specified eccentricity will depend not only on the strength of materials and the size and shape of cross-section, but also on the flexibility of structures calculated.

The problem of factoring in such flexibility is rather complex. Calculating the load capacity of concrete-filled steel tube columns is specific due to a number of factors. Those are mostly due to the complex stress state of the concrete core and the steel shell. As of today, both Russian [1-7] and non-Russian [8-15] researchers have studied the problem.

Paper [16] demonstrates that non-linear deformation models maximize the accuracy of calculating such structures while factoring in their flexibility. However, such calculations are very complex and require the development of special software. To make engineering calculations for feasibility studies, simpler methods are preferable that can quickly return sufficiently reliable results.

2. Finding the sustainable strength

In accordance with Russia's current SP 63.13330 standard, when reinforced-concrete structures are exposed to extra-central compression with eccentricities in excess of random values, flexibility should
be factored by multiplying the initial eccentricity by the coefficient $\eta$ that depends on the ratio of the acting force $N$ and the critical Euler force $N_{cr}(1 \leq \eta \leq 2.5)$. This method is based on the plastic hinge method that specialists have found to be erroneous [17]. The method does return results that fit the experimental values; however, that only holds for components with a flexibility around $\lambda_{eff} = 40 \div 60$.

The method returns up to 60% lower load capacity values for low-flexibility structures; for more flexible structures, it returns an up to 40% higher-than-actual load capacity value.

That calculations by this method do not match experimental values is mostly due to using the averaged bending stiffness of structures when finding the value of longitudinal bend. The actual bending stiffness value will differ for different normal sections along the height of a compressed component; for the assumed initial concrete and steel properties, it will largely depend on the corresponding material elasticity factors. Therefore, the height, or the calculation length of a compressed component significantly affects the value of its averaged stiffness. This is not factored in by the standardized calculation methods.

Due to the proneness to deformations, normal CSTC sections will at different heights have a greater range of possible concrete and steel elasticity factor values. Therefore, the SP 63.13330 method cannot be used to calculate their load capacity while factoring in the flexibility.

Many researchers suggest that the flexibility of CSTC be factored in by reducing the maximum value of longitudinal force a short compressed element can take, which is done multiplying the same value by the bucking coefficient $\phi$ [2,4,8,13]. For such calculations, the sustainable strength condition can be written as

$$N \leq \phi N_0$$  \hspace{1cm} (1)

where $N_0$ is the strength of the normal section of a short component under central compression; $\phi$ is the bucking coefficient.

Let us analyze the most well-known and substantiation proposals on how to find the coefficient $\phi$ for round cross-section CSTC. One of the earliest such proposals was made in [2] where the coefficient $\phi$ is the function of reduced flexibility $\lambda_{eff}$ and reduced eccentricity $meff$.

By comparing the values calculation by this method against the experimental data, we find that those do not match satisfactorily. This is mostly due to the wrong theoretical assumptions made when deriving formulae to evaluate the load capacity of flexible CSTC. In particular, buckled concrete and steel shells were analyzed as if in a state of yield whereas in reality, buckled concrete and steel shells still have their strength properties non-actualized. This is why for practical calculations, the bucking coefficient is mostly found by means of empirical dependencies.

These dependencies are proposed in the papers of L.I. Storozhenko’s research school. Paper [4] contains a formulate for calculating $\phi$; the formula was derived by statistical analysis of experimental data. This formula is limited in application not only in terms of the column concrete strength, but also in terms of the calculated column length and the initial eccentricity of the compressive force; as a result, the application of the formula is limited. We believe it is erroneous to propose a limitation to calculated length when unrelated to the cross-section parameters.

As for non-Russian researchers, the most notable are the standards and papers by Chinese scholars. The current design standards of the PCR contain a simplified method for calculating $\phi$

$$\phi = 1 - 0.115 \left( \frac{l_0}{d} - 4 \right)^{+} \hspace{1cm} (2)$$

Calculations show that when using the formula (2), the bucking coefficient is somewhat lower-than-actual resulting in excessive CSTC load capacity values.

The authors [18] suggest that the bucking coefficient be found according to the effective flexibility $\lambda_{eff}$. A positive side of this approach to calculating compressed CST components consists in the
differentiated determination of the bucking coefficient depending on the range of their flexibility. Indeed, the effect of flexibility is negligible for shorter CSTC. On the other hand, for more flexible components the load capacity whereof is related to the second-order bucking, the strength properties of concrete and steel have ever lesser effect on their resistance.

A serious drawback here is that the flexibility $\lambda$ is considerable undervalued. As it seems, there is an error in this dependency whereas the use of empirical coefficients fails to make the formula versatile.

The main drawback of this method is that the bucking coefficient value found by using the formulae above is considerably different from experimental values we [19] and other researchers [20] obtained.

In one of the today’s most relevant Chinese papers on the problem [21], the bucking coefficient is found by a method similar to that of the European standards. To factor in the flexibility of CSTC, Eurocode suggests the EN 1993-1-1 method designed for steel structures. This method is based on plotting the dependency of the coefficient $\phi$ on the relative stressed component flexibility $\bar{\lambda}$. In case of CSTC, it is suggested that $\bar{\lambda}$ be calculated by the formula:

$$\bar{\lambda} = \frac{l}{\pi \sqrt{E_b I_b + E_p I_p + E_s I_s}}$$

(3)

where $E_b$, $E_p$, $E_s$, $I_b$, $I_p$, $I_s$ – are the elasticity moduli and the inertia moments of the concrete core, the steel pipe, and the longitudinal reinforcement.

The formula for calculating the bucking coefficient is therefore written as follows:

$$\phi = \frac{l}{2\bar{\lambda}} \left( \Phi - \sqrt{\Phi^2 - 4\bar{\lambda}^2} \right)$$

(4)

where

$$\Phi = \frac{\bar{\lambda}^2}{2} + 0.16\bar{\lambda} + 1$$

(5)

These dependencies are not validated experimentally [19]. As seen in the Figure, for CSTC samples with the rod flexibility $\lambda_{eff}$ with a $\lambda_{eff} = 35 \div 75$ cross-section applied to the concrete, the formulae return a higher-than-actual bucking coefficient value. This is why this method has to be adjusted, too.

**Figure 1.** Theoretical and experimental dependencies of the bucking coefficient on the reduced flexibility.
3. Results
From the materials analyzed above, we conclude that as of today, there are no acceptable dependencies that can be reliably used to find the CSTC buckling coefficient.

To obtain a more reliable dependency to calculate $\phi$, we have statistically analyzed a large sample of experimental data and also reviewed the calculations based on the deformation model. As a result, we obtained a rather simple formula that can be written as follows:

$$\phi = \frac{1}{1 + 0.42\lambda}$$ (6)

This formula is recommendable to use for calculating CSTC of the flexibility of $\lambda \leq 2.5$.

In case of prolonged load, the buckling coefficient $\phi$ should be multiplied by the coefficient $\gamma_l$ assumed depending on the flexibility of the calculated component:

- $\gamma_l = 1$ for flexibility $\lambda_{eff} \leq \lambda_0$;
- $\gamma_l = 0.8$ for flexibility $\lambda_{eff} = 80$.

For interim flexibility values, the coefficient $\gamma_l$ should be assumed based on linear interpolation.

If the eccentricity of applying the compressive force $N$ exceeds the random value whilst the condition $e_0 / d \leq 0.25$ holds, first-approximation calculations of the sustainable strength of the CSTC can be made by the breaking stress method. The calculations are down to verifying the condition

$$N \leq m_e\phi N_0$$ (7)

where $m_e$ is the coefficient that factors in the effect the initial eccentricity of applying the compressive force has on the sustainable CSTC strength [22].

4. Conclusion
The excessive literature review of both Russian and non-Russian researchers' papers has shown that there are currently no acceptable dependencies that could be reliably used to find the buckling coefficient of concrete-filled steel tube columns while factoring in the resistance of such structures. Results obtained by Chinese researchers seem appropriate yet they are far from perfection, too.

The authors have statistically analyzed a large volume of their own and third-party data and reviewed the calculations based on the deformation model to have obtained a rather simple dependency of the buckling coefficient on the relative flexibility. Using this dependency enables one to approximately calculate the load capacity of concrete-filled steel tube components of varying flexibility while under extra-central compression; the breaking stress method accuracy thereby is acceptable.

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