Digital Object Identifier 10.1109/ACCESS.2020.3042698

Connected Ecological Cruise Control Strategy Considering Multi-Intersection Traffic Flow

CHUNMING LI, TAO ZHANG, (Graduate Student Member, IEEE), XIAOXIA SUN, AND NING ZHAO
China North Vehicle Research Institute, Beijing 100072, China
Corresponding author: Tao Zhang (ztao1208@126.com)

This work was supported by the National Natural Science Foundation of China under Grant 51705480.

ABSTRACT This paper constructs the signal light model of intersection considering dynamic traffic flow, and proposes an ecological cruise control strategy using the pseudo-spectral method to optimize the energy consumption of electric connected and automated vehicles when passing through multiple intersections. Firstly, a method for a dynamic effective red light duration model considering the average queue effect and the real-time flow fluctuation effect is proposed. Secondly, a recursive and traversal search algorithm is used to identify different combinations of traffic schemes to reduce the complexity of solving global optimization problems. Finally, considering the constraints of dynamic effective red light duration, the pseudo-spectral method is proposed which can be used in the traffic energy-saving planning of single intersection and multiple intersections. Simulation results demonstrate that compared with dynamic programming benchmark algorithm, the energy-saving effect of this control strategy is much close to the global optimum, and the calculation efficiency is far better. Besides, compared with the intelligent driver model which used as the energy consumption benchmark, this control strategy can produce a better energy-saving speed reference trajectory.

INDEX TERMS Dynamic effective red light duration, traffic flow, queue effect, pseudo-spectral method.

I. INTRODUCTION

Connected and automated vehicle (CAV) technology is revolutionizing the automotive industry. In particular, CAV has the ability of self-driving, path/velocity planning, vehicle-to-infrastructure (V2I), and vehicle-to-vehicle (V2V) communications. In this way, vehicles can be connected to the intelligent transportation system, exchange and share massive data of current traffic status and vehicle operating data [1]. These data can be used to improve the overall vehicular performance, in terms of mobility, safety, and fuel economy [2], [3].

By using V2I or V2V communication, traffic jams or unnecessary stop-and-go behavior and emergency acceleration behavior at signalized intersections in the main road are reduced to shrink energy consumption, which is the important research contents of driving in current urban road ecology [4], [5]. In general, the main idea of eco-driving for electric vehicle (EV) on urban roads is to find the optimal speed trajectory for specific journey conditions from the perspective of reducing battery energy consumption [6]. When the signal phase and timing (SPaT) information is not considered, the eco-driving control scenario is simpler on urban roads. In [7], the model predictive control (MPC) based approach was used for acceleration and speed control of an EV to obtain the optimal energy-saving speed profile. Dynamic programming (DP) was further applied to an EV in [8] for eco-driving control. Both of them demonstrated that a properly designed speed profile is able to significantly improve the vehicle energy efficiency. Barth and H. Yang used average road speed statistics to provide drivers with suggested speeds to reduce fuel consumption levels [9], [10]. However, the energy-saving driving strategy on urban roads should also consider the constraints of road SPaT information to increase the practical application scope of the strategy.

SPaT information is critical in urban driving environment and greatly increases the complexity of eco-driving problems. Assuming the SPaT information is available, the optimal eco-departing problem was solved at signalized intersections in [11]. Similarly, the sophisticated on-board driver assistance was developed to calculate the optimal energy-saving speed profile with deterministic traffic signals [4].
Some existing researches have considered the problem of optimal energy management for scenario of multiple intersections. For example, Ref. [12] used a DP algorithm, and Ref. [13] used the MPC cruise algorithm. Both of them hoped to maximize the probability of encountering a green light signal when passing through multiple intersections to reduce unnecessary energy consumption. Ref [14] considered the arrival time at each intersection of traffic flow and a two-stage solution method was constructed to find the optimal speed trajectory with lowest fuel consumption for multiple intersections. In [4], by considering the constraints of traffic signal position and SPaT information, the vehicle was forced to pass the green light time window to optimize the optimal driving speed. In [15], the optimization problem with continuous constraints of multiple time windows was established and solved by a preliminary velocity velocity pruning algorithm. In addition, Ref [16] proposed a simple solution to ensure the vehicle pass through multiple intersections smoothly with constant speed.

However, most of the above articles optimize operation regarding motor and gear by considering SPaT, speed limit, etc., and use traffic arrival rate and energy consumption rate constraints to plan control strategies. These methods treat the green light duration as a complete transit time window, which is ideal for reflecting the actual driving situation. For this problem, Sun Chao [17] proposed a robust and optimal eco-driving strategy. The green light time of uncertainty was equivalent to the Poisson distribution function, which was used as a random variable in the DP algorithm to describe the feasibility of passing through the signalized intersection. The shortcoming of the article is that the uncertainty of the signal light was assumed to be a probability problem, ignoring the actual signal delay caused by the traffic queues effect. In [18], based on the queuing time of the traffic flow, the optimal vehicle speed within a specific distance upstream and downstream of the intersection was calculated.

It should be noticed that the effective red-light duration (ERD) was affected not only by the average queue delay but also by the delay caused by the dynamic fluctuation of the real-time traffic flow. However, when dealing with the problem of energy saving optimization for electric CAVs passing through multiple intersections, there are still relatively few studies that consider queuing effect and real-time traffic fluctuation effect simultaneously. In actual traffic scenarios, traffic flow determines the average speed of the road. The arrival time of the vehicle at the previous intersection determines the arrival time combination of the next intersection. The queuing effect causes the actual effective transit time window of the vehicle passing through each intersection to be less than the full green signal time provided by SPaT. Different queued entry time points and queue lengths result in dynamic changes of the effective green light duration which is actually available for planning.

Considering the above factors, we define the concept of dynamic effective red light duration (DERD), including three parts: base red light time, average queue effect time and real-time flow fluctuation effect time. Accurate modeling and estimation of DERD are the basis of energy-saving optimization for all SPaT constraints. Inspired by the above problems, this paper hopes to find a control framework and optimization algorithm suitable for dealing with energy-saving traffic at continuous multiple intersections under the constraints of DERD, so as to realize energy-saving planning of electric CAVs.

Some other energy-saving strategies, such as Particle Swarm Optimization (PSO) [19], Equivalent Consumption Minimization Strategy (ECMS) [20], MPC [21] and reinforcement learning (RL) [22], realize the power distribution by minimizing the value of the objective function while satisfying system constraints. But these methods are typically based on specific powertrain and off-line cycle driving conditions to optimize energy efficiency, the optimization results obtained are not globally optimal solutions.

DP and Pontryagin’s minimum principle (PMP) [23], are two widely used approaches for obtaining the globally optimal solution in energy management problem. The globally optimal solution yielded through the DP method was generally used as a benchmark to evaluate the controller [24]. However, this method involves heavy computational load and will suffer from the so-called “curse of dimensionality”, when there is a relatively high number of state and control grid variables. The PMP method is used to formulate the analytical necessary condition equation, which defines the mathematical relationship among the state variables, the Hamiltonian, and co-states. When dealing with nonlinearities and complex constraints, this method is usually incapable of obtaining optimal solutions in an efficient manner [25].

As a typical direct method belonging to nonlinear programming (NLP)solution, the pseudo-spectral method (PM) has been increasingly used for numerical solving of global optimal control problems for various dynamic systems [26], [27]. The PM uses the orthogonal matching point to discretize the continuous optimal control problem, and approximates the state and control variables through global interpolation polynomial, thus transforming the problem into an NLP problem. At the same time, the PM is easy to deal with the segmentation problem, which can transform the continuous multiple intersections traffic problem into a continuous single-signal intersection speed planning problem. By splicing the constraints of multi-segment planning, the energy management problem of continuous flow intersections is solved with the goal of the lowest global energy consumption. The PM is more attractive due to the merits of higher accuracy, lower sensitivity to the initial value and faster convergence [28]. They are potentially more efficient than DP methods in terms of computation because they exploit well developed NLP codes, and the energy allocation problem under dynamic traffic can be reformulated more flexibly.

Because of the queuing effect of traffic flow, the actual passable green time window is shorter than the real green time window, which poses great challenge for improving vehicles’ energy efficiency when passing through
multiple intersections. Inspired by the above, this paper proposes an ecological cruise control (Eco-CC) strategy for electric CAV which is suitable for energy-saving driving at multiple intersections.

The main contributions include:

1) Dynamic effective red-light duration (DERD) is proposed to describe the dynamic transit time affected by traffic flow at continuous signal intersections, including base red light time, average queue effect time and real-time flow fluctuation effect time.

2) Based on the signal light model considering DERD, the recursive and traversal search algorithm is applied to obtain all possible combinations of passing green light windows in advance. Then, by using the computer’s multi-threaded computing technology, multiple sets of traffic plans can be optimized at the same time, which greatly reduces the difficulty of solving global optimization problems.

3) The pseudo-spectral method (PM) algorithm is proposed to optimize the eco-driving strategy under DERD constraints. In addition, the secondary determination in the local adaptation stage is used to perform lane change for improving the safety of the vehicle.

![FIGURE 1. The ecological cruise control framework of CAVs.](image)

### II. DEFINITION AND PROBLEM FORMULATION

#### A. THE CONNECTED ECOLOGICAL CRUISE CONTROL FRAMEWORK

The proposed Eco-CC framework of CAVs on an urban road with signal lights and traffic limits is shown in Fig. 1. The host vehicle obtains traffic information through V2I, including short-term traffic statistics, SPaT information, maps. The system divides the energy-saving optimization problem of continuous flow multiple intersection into several determined transit time schemes through a recursive and traversal search algorithm. Then, according to the vehicle longitudinal motion dynamics, energy consumption model and dynamic traffic information, energy-saving optimization are implemented for each time combination scheme. After determining the travel time and distance of different combinations, the on-board system selects the best travel plan. Finally, the control signal is loaded directly into the vehicle’s longitudinal control system.

#### B. ELECTRIC VEHICLE AND ENERGY CONSUMPTION MODEL

The nonlinear longitudinal motion of a vehicle at time $t$ can generally be governed by the state update equation

$$\dot{X}(t) = f(X(t), u(t))$$

where $X(t) = [s(t), v(t), e(t)]^T$ denotes the state vector of traveled distance, speed, and energy consumption, and $u(t)$ is the control input. A one-dimensional vehicle motion based on Newton’s second law is shown in (2), wherein the tire slip is ignored and the vehicle is considered as a mass point. The vehicle acceleration is proportional to the difference between the traction force $F_{\text{trac}}$ and the driving resistance forces.

$$\dot{V} = \frac{F_{\text{trac}} - (Mgfr \cos \alpha + \frac{1}{2} \rho A_f d v^2 + Mg \sin \alpha)}{M}$$

where $M$ is the equivalent mass of the vehicle, $f_r$ is the rolling resistance coefficient, $\rho$ is the air density, $A_f$ is the frontal area, $C_d$ is the aerodynamic drag coefficient, and $\alpha$ is the slope angle of the rode.

The speed and traction force have a significant influence on the vehicle’s energy consumption, and various approaches have been proposed for modeling the energy consumption of EVs [29], [30]. The traction force depends on the equivalent mass and control input as $F_{\text{trac}} = Mu(t)$. When cruising under constant conditions, the energy consumption is equal to the resistance power [31]. It can be approximated through the curve-fit process with measurement data as

$$P_{\text{cruise}} = b_3 v^3 + b_2 v^2 + b_1 v + b_0.$$  

The acceleration and deceleration, only considering the regenerative energy zone in the hybrid brake system can be approximated by a similar curve-fitting process with measurement data using a polynomial of the control input as

$$\dot{p} = p(v, u) = f_u(p_{\text{trac}}/M) + p_{\text{cruise}} = (a_2 u^2 + a_1 u + a_0)(uv) + (b_3 v^3 + b_2 v^2 + b_1 v + b_0)$$

where $p_{\text{trac}}$ is the tractive power, $a_i$ and $b_i$ are the weight coefficients. This model is capable of representing the regenerative braking effect when $u(t) < 0$ for the full-range speed and control input limits. In addition, we adapt the first-order internal resistance battery model for study. This kind of fitting method is commonly used in industrial application due to its simplicity. It incorporates not only the driving resistance power but also some latent power, for example the power caused by mechanical loss.

#### C. TRADITIONAL TRAFFIC SIGNAL MODEL

The traffic signal at an intersection is a spatial-temporal system in the optimal eco-driving control problem.
Assuming that the traffic lights timing is available in advance through V2I communication and does not change during the course of driving. An example of a straight route with three upcoming traffic lights is presented in Fig. 2. The green lines represent time intervals when the corresponding traffic light is green. The black lines represent the possible driving signal combination window at different driving speeds.

**Fig. 3. Traffic dynamics of the vehicle at an intersection.**

**D. MODIFIED TRAFFIC SIGNAL MODEL**

If one vehicle arrives at the intersection when the traffic signal indication is green, it passes through the intersection without any delay, while it has to wait at the stop bar for a green indication when the traffic signal indication is red. In the case of actual traffic flow, taking into account the queue effect of the queue length at the intersection, even if the traffic light turns green, the host vehicle needs to wait for the queue to pass through the intersection. The red signal indication generates a shock wave upstream of the intersection, so the actual effective transit time for the host vehicle is less than the green time of the traffic light.

Fig. 3 shows the movements of a series of vehicles approaching and passing an intersection with traffic waves. \( t_r \) and \( t_s \) are the time that the traffic signal turns to a red indication and a green indication, respectively, according to \( T_i \) and \( T_i^G \). The black solid line indicates the preceding vehicle, and the blue solid line indicates the host vehicle. Without the Eco-CC algorithm planning, the host vehicle follows the traffic flow at a constant speed and approaches the intersection, and then slows down, stops, accelerates, etc. For EVs, sudden braking, parking start, and low speed also consume more energy. Therefore, it is important to reduce the unnecessary acceleration and deceleration behavior to improve energy efficiency, as the red dotted line shown in Fig. 3. The premise of planning needs to consider not only the timing of traffic lights but also the queuing phenomenon caused by real-time traffic.

With the green and red indications, traffic flow generates various shock waves and rarefaction waves, which lead to significant variations in vehicular movements. The application of wave behavior to traffic study was first presented independently by Lighthill and Whitham in 1955 and Richards in 1956 [32], [33]. Shockwave structure derived from Lighthill–Whitham–Richards (LWR) traffic flow theory is widely applied to describe queue dynamics and performance measures of a signalized intersection. These two classic works also provide a preliminary attempt to answer how the theory might be utilized to describe the traffic state at road junctions. Assume that \( \{ K(x, t), v(x, t), Q(x, t) \} \) represent the traffic density, average speed, and flow at location \( x \) and time \( t \), respectively, then the LWR model can be described as follows:

\[
\frac{dK(x, t)}{dt} + \frac{dQ(x, t)}{dx} = 0 \quad (5)
\]

The above model assumes a relationship between flow and density, i.e., \( K(x, t) = Q(K(x, t)) \). Fig. 4 illustrates a general fundamental diagram, and the average flow \( Q \) is a concave function of the density \( K \).

Assume that the average flow entering the intersection is \( Q(x, t) \), and the traffic state upstream of the intersection is \( x \), as shown in Fig. 3. With a flow state \( x \), the shockwaves among different states can be graphically demonstrated in Fig. 4. When the signal turns red, no vehicles can proceed through the intersection. At this point, the traffic flow queue causes the maximum road density \( K_1 \), and the upstream state becomes 0. The queueing effect produces a shock wave...
propagating upstream of the intersection, and the speed of the shock wave is determined by the flow and density at states $x$ and $\bar{1}$. Once the traffic signal turns green, the intersection starts to discharge vehicles at the saturation flow rate $Q_2$. As a result, a rarefaction wave is formed to release the queue upstream of the intersection, and the downstream state becomes $\bar{2}$, and the speed of the shock wave is determined by the flow and density at states $\bar{1}$ and $\bar{2}$.

$W_{x1}$ denotes the shockwave between state $x$ and $1$, and $W_{12}$ is the rarefaction wave between state $\bar{1}$ and $\bar{2}$. Throughout this article, $W$ represents the wave speed and can be calculated as:

$$W_{x1} = \frac{|Q_x - Q_1|}{K_x - K_1} \quad (6)$$

$$W_{12} = \frac{|Q_1 - Q_2|}{K_1 - K_2} \quad (7)$$

At time $t$, the Eco-CC vehicle enters the control segment with speed $v_x = \frac{Q_x}{K_x}$ and the distance from the signal light is $d_i$. When the host vehicle follows the tail of the traffic flow and stops at the signal light, the tail location $d_0$ of the host vehicle can be estimated according to the formulas $(6)$, $(7)$ and the triangular geometric relationship shown in and Fig. 3.

$$d_0 = \begin{cases} \frac{W_{x1}}{v_x} \cdot \frac{d - v_x(t_r - t)}{v_x + W_{x1}(t_g - t)}, & \forall t \in [t_r - \frac{d}{v_x}, t_g + \frac{W_{x1}(t_g - t)}{W_{x1} + W_{12}}] \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

The queuing effect leads to increased waiting time $\Delta t'_c = d_0/W_{12}$ and $\Delta t''_c = d_0/v_x$, and the parking waiting time of the host vehicle is

$$t_{x,0} = t_g - t + \Delta t'_c - (d - d_0)/v_x \quad (9)$$

Therefore, the new ERD due to the queuing effect is approximately equal to

$$T_i,\text{ERD} = \Delta t_c + \Delta t'_c + \Delta t''_c \quad (10)$$

where $\Delta t_c = T_i - T_i^G$. Fig. 5 exhibits the concept of ERD and DERD.

Due to fluctuation effect of the traffic flow, the above mentioned ERD parameters obtained based on the average traffic flow need to be modified in the actual application to improve the accuracy of the traffic signal model. Therefore, it is necessary to add the change time based on the real-time traffic flow fluctuation effect to the ERD. Further, in order to analyze the the changing fluctuation characteristics of the real-time traffic flow fluctuation effect, we specifically counted the traffic flow information in a 900-meter-long signal road section in Beijing was collected.

Fig. 6 shows the traffic flow under different time-scales. It demonstrates that the larger the time-scale, the smoother the statistical results. When the time-scale is small, the curve exhibits periodic fluctuations. Therefore, the average flow cannot fully reflect the dynamic changes in short-term traffic.

Due to the truncation of the signal light, the real-time traffic flow fluctuates periodically and the changes in the adjacent cycle are approximately the same. When the host vehicle is at the peak of the flow, the queuing effect is aggravated. When the host vehicle is in the flow trough, the queuing effect is relieved. Considering the real-time traffic and average traffic, the real-time flow fluctuation effect time $\alpha$ is defined based on the remaining green light time as follows:

$$\alpha = (T_i^G - \Delta t'_c - \Delta t''_c)\eta = (T_i^G - \Delta t'_c - \Delta t''_c)\frac{Q_x^* - Q_x}{Q_{peak}^* - Q_x} \quad (11)$$

where $Q_x^*$ and $Q_{peak}^*$ represent the instantaneous flow and short-term peak flow obtained by the traffic observer respectively. $Q_x$ is the statistical average flow value based on big data. $Q_{peak}^*$ is the predicted real-time traffic at the cycle time $\Delta t_c + \Delta t'_c$, and $Q_{peak}^*$ is the peak traffic flow in the adjacent period. $\eta$ is the proportional coefficient. The parameter
\[ T_{i,\text{DERD}} = \Delta t_i + (\Delta t_i' + \Delta t_i'') + \alpha \]

\[
T_{i,\text{DERD}} = \begin{cases} 
T_{i,\text{ERD}} + \alpha, & Q_{i}^a > Q_i \\
T_{i,\text{ERD}}, & Q_{i}^a \leq Q_i
\end{cases}
\]

(12)

It needs to be mentioned here that the model does not reduce the estimated average effective red light time so, ensuring that the vehicle passes through the intersection as much as possible at the end of the green light signal, which avoids abrupt acceleration and ensures driving comfort. Therefore, in the above model, when \( Q_{i}^a \leq Q_i \), we take \( \eta = 0 \).

According to the average flow and instant flow information of each traffic section predicted, the modified traffic signal model is obtained (13), as shown at the bottom of the page, where \( i, j \) indicates the \( i \)th intersection and the \( j \)th signal light cycle.

### III. ECO-CC ALGORITHM AND STRATEGY

In this section, the Eco-CC strategy is developed to adapt to real-time changing traffic scenarios. The framework and working mechanism are described as follows:

1) According to the driver’s setting demand (taking the lowest energy consumption as an example), combined with the road speed limit and the traffic SPaT, the system uses a recursive and traversal search algorithm to search for all possible time window combinations of the upcoming intersection and obtain corresponding time window limits.

2) Based on the pseudo-spectral method and combined with the advantages of computer multithreaded computation, all combinations can be optimized at the same time to select the best speed trajectory. The most appropriate action sequence is selected and actively applied to the longitudinal control of the host vehicle. Compared with traditional sequential optimization, parallel optimization in form of time window combination can greatly reduce the computational burden and shorten the calculation time.

3) Combined with advanced driver assistant system, the host vehicle can detect the distance from the preceding vehicle in real-time. In case of external interference, the distance is adjusted to ensure safe driving. When the interference disappears, the Eco-CC algorithm re-plans the optimal driving curve.

### A. SEARCH WINDOWS BASED ON A RECURSIVE AND TRAVERSAL ALGORITHM

Since the optimization problem consists of multiple traffic segments, different vehicle speeds result in different combinations of time window constraints. We perform a recursive and traversal search by applying algorithm in Table 1 to every intersection one by one starting from the closest one. That means, we get the passable time window for the first intersection, then based on which, the passable time window for the second intersection and so on can be obtained iteratively. Finally, we pick up the best solution out of all combinations of passable time windows. In Table 1, \( i \) represents the index of the intersection. By increasing \( i \), traversing multiple intersections can be realized. \( [t_{i,j}^{\text{min}}, t_{i,j}^{\text{max}}] \) indicates the time window of the \( j \)th signal light that the vehicle reaches the \( i \)th intersection from the current time. \([t_{\text{fast}}, t_{\text{slow}}]\) indicates two extreme time constraints when the vehicle reaches the \( i \)th intersection. The time window constraints for the \( i \)th intersection are used as inputs in the crossing window search algorithm for the \((i+1)\)th intersection. \( V_{\text{top}} \) is the maximum allowed speed of the road. \( V_{\text{max}} \) is the smaller one between \( V_{\text{top}} \) and the average traffic flow \( V_{\text{flow}} \) on the interval. The minimum speed limit \( V_{\text{min}} \) also needs to be defined.

\[
s_j(t) = \begin{cases} 
1, & T_i^0 + (j - 1)T_i + T_{i,\text{DERD}} + T_i^G < t \leq T_i^0 + jT_i + T_i^G \\
0, & T_i^0 + jT_i + T_i^G < t \leq T_i^0 + jT_i + T_{i,\text{DERD}} + T_i^G
\end{cases}
\]

(13)
B. OPTIMAL PROBLEM BASED ON PM

Since different combinations of arrival time windows have been determined, the optimization target of a certain traffic scheme is to minimize the energy consumption in the determined arrival time window \([t_0, t_f]\). Therefore, the cost function can be defined as (19), shown at the bottom of the page, where the function \(f\) represents the system dynamics in formulas (1)–(3). The values of variables \(n_{mot}, T_{mot}, t_i\) and \(v_i\) are constrained within a feasible range.

The commonly used pseudo-spectral methods are Gauss pseudospectral method, Radau pseudo-spectral method and Legendre pseudo-spectral method [34]. Compared with the first two methods, Legendre pseudo-spectral method has advantages in computational convenience [35]. The optimal control problem corresponding to the eco-driving strategy can be transformed into the NLP problem with the state and control variables discretized by Legendre-Gauss-Lobatto (LGL) coordination points. The specific conversion steps are as follows:

Step1: Transformation of the time domain. The time interval is transformed from \([t_0, t_f]\) to \([-1, 1]\) via the affine transformation,

\[
\tau = \frac{2t - t_f - t_0}{t_f - t_0}, \quad \tau \in [-1, 1]
\]  

(20)

Step2: Discretization. Disperse the state and control variables at LGL collocations to form \(N + 1\) discrete state variables \(X = \{X_0, X_1, \ldots, X_N\}\) and \(U = \{U_0, U_1, \ldots, U_N\}\), where \(X_i = x(t_i)\) and \(U_i = u(t_i)\). The PM only optimizes the state \(X_i\) and the control \(U_i\) at the discrete point, and the actual continuous state \(x(\tau)\) and the control \(u(\tau)\) are approximated by the Lagrange interpolation polynomial, ie

\[
x(\tau) = \sum_{i=0}^{N} L_i(\tau)X_i
\]

\[
u(\tau) = \sum_{i=0}^{N} L_i(\tau)U_i
\]

(21)

Step3: Transformation of state equation. The differential operation of the state can be transformed into a differential operation on the interpolation basis function, ie

\[
\dot{x}(t_k) \approx \hat{X}(t_k) = \sum_{i=0}^{N} \hat{L}_i(t_k)X_i = \sum_{i=0}^{N} D_{ki}X_i
\]

(22)

where \(k = 0, 1, 2, \ldots, N, D_{ki}\) is an \((N + 1) \times (N + 1)\) differential matrix, represents the differential value of each Lagrange basis function at each LGL point. From this, the equation of state constraint can be transformed into the equality constraint of the \(N + 1\) group at the LGL collocation point.

\[
\sum_{i=0}^{N} D_{ki}X_i - \frac{t_f - t_0}{2}f(X_k, U_k, t_k) = 0
\]

(23)

Step4: Function transformation. The integral term in the performance function can be calculated by the Gauss-Lobatto integration method, ie

\[
\int_{t_0}^{t_f} \dot{e} dt = \frac{t_f - t_0}{2} \sum_{i=0}^{N} w_i E(X_k, U_k, \tau)
\]

(24)

where \(w_i\) is the weight coefficient in the Gauss integration.

Step5: Problem conversion. Through the above steps, the original control problem can be transformed into the NLP problem with the control variable and the state variable at the distribution point as the variables to be optimized,

\[
\min_{u} J = \frac{t_f - t_0}{2} \sum_{i=0}^{N} w_i E(X_i, U_i, t_i)
\]

\[\text{s.t.,} \quad \left\| \frac{\partial \phi(X_0, X_N, \tau_0, \tau_N)}{\partial \tau} \right\|_{\infty} \leq \xi
\]

\[
\left\| C_1(X_k, U_k, t_k) \right\|_{\infty} \leq \xi
\]

\[
C_2(X_k, U_k, t_k) \leq 0
\]

(26)

where \(k, i = 0, 1, 2, \ldots, N\). \(\xi\) is the amount of relaxation of the equality constraint. The transformed NLP belongs to the high-dimensional sparse problem, which can be solved by a mature solver [36], such as SNOPT, IPOPT, etc.
IV. RESULTS AND DISCUSSION

A. INTELLIGENT DRIVER MODEL

The intelligent driver model (IDM) is originally developed by Treiber [37] based on the desired distance or speed limit. This paper uses IDM as the benchmark for comparison with the optimal eco-driving model. Besides the distance \( s \) to the leading vehicle and the actual speed \( v \), the IDM also takes into account the speed difference \( v_1 - v_{lead} \) to the leading vehicle. Acceleration at each time step is computed by comparing the desired distance with the current distance between the host vehicle and the vehicle in front.

\[
a_{IDM} = a_{max} \left[ 1 - \left( \frac{v}{V} \right)^\delta - \left( \frac{d_{des}}{s} \right)^2 \right] \\
d_{des} = s_0 + vT + \frac{v \Delta v}{2 \sqrt{a_{max} a_c}}
\]

where \( d_{des} \) is the desired distance, \( a_{max} \) is the maximal vehicle acceleration ability, and \( a_c \) is the preferred deceleration for comfort, \( v_{lead} \) is the speed of leading vehicle.

In simulation, to make our constructed driver model closer to the reality, we collected the real-world data when the vehicle drives towards the intersection and finally stops at the intersection when the traffic light turns red. The collected data is used to simulate the leading vehicle of the queue in order to formulate the tracking calibration of the IDM. The parameters of the IDM adopted in this paper is shown in TABLE 2.

B. OPTIMIZATION RESULTS OF SINGLE INTERSECTION

Firstly, the simulation is performed at a single signal light intersection under specific green light duration. The distance between the host vehicle and the signal intersection is \( d_1 = 900m \), speed limits \( V_{top} = 65km/h \) and \( V_{min} = 25km/h \). The green and red (include yellow signal time) signal durations are both set to 40s and the green signal time has passed 12s (That means \( t_0 = 12s \)). Moreover, the traffic property of the current road segment is set as follow: \( Q_1(t) = 1000 \text{ veh/h}, K_1(t) = 16 \text{ veh/km}, Q_2(t) = 1600 \text{ veh/h}, K_2(t) = 20 \text{ veh/km}, K_1(t) = 12 \text{ veh/km} \) and set \( \eta = 0 \). For IDM, it is assumed that the vehicle mode is traveling at a constant speed \( v_x \) to the tail of the vehicle queue, and then the corresponding action is performed following Eq (10). Conversely, the vehicle performing Eco-CC is controlled by the system recommended actions. To validate the reliability of PM algorithm, DP is applied to solve the same problem.

Fig. 7 compares the distance and speed trajectories between IDM and proposed method in the single intersection simulation. As indicated by the black line in the figure, it can be seen that the IDM follows the traffic flow to the intersection and stops until the queue moves again. The green dotted line in Fig. 7(a) represents the impassable time caused by the queue effect, which causes the ERD to become longer. As can be seen from Fig. 7, the PM algorithm can obtain optimization results close to the DP algorithm, and ensure that the vehicle is able to cruise to the intersection and catch the tail of the queue just when it is released. Fig. 7(b) shows the actual speed of the vehicle. Compared with IDM, the eco-driving vehicle (with PM or DP) decreases the speed and avoids the stoping behavior at the intersection. In addition, it accelerates when it is close to the intersection so as to pass the intersection before the light turns red.

Fig. 8 shows the power requirements of the vehicle in different drive modes and the corresponding positive motor torque operating area. Fig. 8 (a) shows that the two algorithms
have similar power requirements. In phase A, the vehicle decelerates and the kinetic energy is mainly absorbed by the battery. In phase B, the vehicle performs free-slipping driving in a low power consumption state. In phase C, the vehicle maintains a constant speed and the power demand fluctuates within a certain range. In this phase, the power demand of eco-driving vehicles changes more frequently, which can be understood as a frequent Pulse and Glide state in a small range. In addition, although the dynamic motor characteristics under PM do not fully match that of DP, the driving states of the two are similar.

The motor operation areas of PM and DP are shown in Fig. 8 (b). Although the motor operates in similar area in both methods, the operating points of PM algorithm distribute in a wider range, which is related to the time step of the algorithm. It must be mentioned that, unlike DP, the hp-adaptive mesh refinement technique [38] is adopted in our PM algorithm. The number of mesh intervals of the trajectory and the Lagrange polynomials order is adjusted adaptively to keep a balance between the convergence rate and solving accuracy. The time step of DP algorithm is fixed, and the PM algorithm fits the state variables by freely matching points, so that a better fitting effect can be obtained, thus causing the control variables to be more dispersed in the local range.

Table 3 lists the calculation time and the energy consumption for the two methods. The energy consumption for IDM is 0.1195kWh. For DP, when the state variables are discretized into 21 × 21 grids, the solving process takes 1077s; when the variables are discretized into 81 × 81 grids, the solving process takes 9687s; when the variables are discretized into 91 × 91 grids, the solving process takes 12867s. The energy consumption for these three different discrete density levels are 0.1125kWh, 0.1092kWh and 0.1062kWh, respectively. By contrast, PM takes 305s and the energy consumption is 0.1073kWh. The energy consumption obtained by PM is very close to that of DP with 91 × 91 grids. Notably, PM needs less calculation time than DP especially in the case of a problem with a larger number of state variable grids. In addition, we find that when the time window of the optimal control problem is reduced to 10s, the PM calculation duration will be less than 1s. It means that the PM method has great potential to be used online.

In the actual application of the algorithm, when the difference between real-time local traffic flow and average flow is large, Fig. 9 shows different driving trajectory curves based on different η thresholds. For η = 0, the best trajectory passes through the intersection immediately after approaching the ERD. As η increases, the passing time gradually increases as the solution becomes more cautious to delayed ERD, and the time is gradually constrained to DERD. As real-time traffic gradually increases to peak traffic, the vehicle’s pass time is closer to the end of the green light.

The black and red solid lines in Fig. 10 show the energy consumption of the two driving modes at different entry time $t_0$. In the above scenario, due to the limitation of DERD, the earlier the host vehicle enters a single signal cycle, the longer the queuing time it needs to wait. It needs to be highlighted in Fig.10 that IDM is more efficient than the PM-optimal trajectory for $t_0 < 4s$. This is because in Fig.10, we assume that the duration of red light is 30s, so for $t_0 = 0s$, it represents that it will take 30s from current moment to the moment when the light turns green. Therefore, when $t_0 = 0s$, to ensure the vehicle pass through the intersection without stopping, the PM-optimal trajectory will decrease the vehicle’s speed due to the long duration the red light will last, thus making the motor work in low-speed area with low-efficiency, which deteriorates the energy-saving performance of the proposed method. Therefore, for $t_0 = 0s$, the energy consumption of the PM-optimal trajectory is higher than that
of IDM, where the driver drives at high speed at first and then stops at the intersection. It is also the same for other values falling in range $t_0 < 4s$. As $t_0$ increases, the duration of the red light will decrease. Therefore, the speed optimized by the proposed method will increase, which makes the working points of the motor shift from low-efficiency to high-efficiency area, so the energy consumption of the proposed method will decrease and its superiority will gradually demonstrate.

Despite above disadvantage of the proposed algorithm, the possibility for the scenario where the energy consumption of PM-optimal trajectory is higher than that of IDM is small and this is for the single intersection scenario. For multiple intersections scenario, this possibility will further decrease because the proposed method will optimize the speed trajectory from a holistic view while IDM will waste more energy on re-starting moments. Therefore, our proposed method generally has a better performance than IDM in most cases.

### C. OPTIMIZATION RESULTS OF MULTI-INTERSECTION

This section establishes a simulation of a three-intersection road. The settings of the roads, the signal phasing and timing plan can be found in Table 4. Since the vehicles on each road come from the three directions of the intersection, the road traffic density information is different. In the simulation, assuming the initial vehicle speed $v_0 = 50km/h$, the minimum road traffic speed is $V_{\text{min}} = 25km/h$, the maximum speed limit is $V_{\text{max}} = 65km/h$.

According to the road configuration at different stages, the speed of the traffic flow can be calculated. Combined with the road limit, eight kinds of traffic signal combinations can be obtained, as shown in Fig. 11. Considering the impact of the queuing effect, the recursive and traversal search algorithm filters out 7 possible cross-windows for the driver. Among them, Path #7 is not available because it is impossible for the vehicle to pass smoothly from the signal periods 5 to 7 considering the traffic speed limit.

With $\eta = 0$, Fig. 12 yields the travel distance and speed curves (three color lines) of path#3 using Eco-CC strategy planning. The effective green transit time window obtained by the solution is $[t_{\min}^{\text{g},1}, t_{\max}^{\text{g},1}] = [57.3s, 70s]$, $[t_{\min}^{\text{g},2}, t_{\max}^{\text{g},2}] = [153.6s, 170s]$, and $[t_{\min}^{\text{g},3}, t_{\max}^{\text{g},3}] = [239.1s, 260s]$. After passing the first intersection, the speed is reduced to ensure that the vehicle can pass smoothly at the cyclic signal lights 4 at the second intersection. After that, the vehicle speed accelerated to a certain level and stably traveled until it passed the third intersection (signal 7). For the CAV under PM, due to the queuing effect of traffic flow, the vehicle will encounter different queuing lengths when it arrives at each intersection, resulting in the actual effective green light window time of Eco-CC vehicles being reduced by $\Delta t_{c,i,j}^{\prime} + \Delta t_{c,i,j}^{\prime\prime}$.

The black line in Fig. 12 is the distance and speed curve for IDM driving. Assume that IDM uses the maximum road speed limit. Due to the faster speed, IDM reached the third intersection earlier than Eco-CC driving mode. On the other hand, unplanned driving leads to fierce deceleration and parking of IDM at every intersection, and the final energy consumption is 0.3382kW.h.

### TABLE 4. Multi-intersection traffic information.

| Parameter            | Values | Values | Values |
|----------------------|--------|--------|--------|
| traffic light number | 1      | 2      | 3      |
| distance, m          | 900    | 1700   | 2600   |
| initial phase $T_i^p$, s | 40     | 20     | 35     |
| cycle $T_i/T_i^G$, s  | 60/30  | 60/30  | 65/35  |
| traffic $Q_x$, veh/h  | 1000   | 850    | 900    |
| traffic $K_x$, veh/km | 16     | 16     | 15     |
| traffic $Q_x$, veh/h  | 1400   | 1400   | 1400   |
| traffic $K_x$, veh/km | 20     | 22     | 24     |
| traffic $K_1$, veh/km | 12     | 12     | 11     |
TABLE 5. Result of the different method under multi-intersection (path#3).

| Driving method | Energy consumption | Computation duration (s) |
|----------------|--------------------|--------------------------|
| IDM            | 0.3382kW.h         | <1                       |
| PM             | 0.3185kW.h         | 1230                     |
| DP             | 0.3165kW.h (states=91x91;control=81;Time step=0.1s) | 31246 |

For the above scenarios and conditions, DP is applied to solve the same problem, and the comparison result is described below. Table 5 lists the computation durations and the energy consumption for the three methods. The energy consumption for IDM is 0.3382kWh. For the DP method, when the state variables are discretized into 91 × 91 grids, the solving process takes 31246s, and the energy consumption is 0.3165kWh. By contrast, PM takes 1230s and the energy consumption is 0.3185kWh. Although the better energy-saving performance of DP can be obtained, a lot of calculation time is sacrificed. In contrast, PM can get good results in a relatively small amount of time. To sum up, for the optimization of multiple intersections under DRED, PM still shows the characteristics of high efficiency and energy saving.

FIGURE 13. The cost function for available crossing windows, \( \eta = 0 \).

Further, using the same conditions as the above mentioned, we apply the PM algorithm to every possible path separately. The cost functions associated with the possible path windows for the Eco-CC mode under PM are presented in Fig. 13. The simulation results demonstrate that path #1, path #3 and path #5 based on PM-optimized use less energy consumption than IDM driving. Among them, path #1 and IDM driving have experienced the same signal cycle, but the speed trajectory optimized by PM produces less energy consumption. The comprehensive comparison shows that path #5 depicts the optimal crossing sequence, which means that the fastest traffic path #1 is not the most energy-efficient driving mode. Observing other energy consumption results, it can be found that the energy consumption of the paths #4, #6, and #8 is relatively large. This is because the average driving speed is low due to long driving time, and the motor cannot be in an efficient and energy-saving working area. From the above results, it can be known that the Eco-CC algorithm proposed in this paper can quickly optimize the path energy consumption under the constraints of different DERD combinations and select the best driving scheme. When the on-board resources meet the needs of efficient computing, the proposed framework provides a solution for online optimization control of energy-saving driving on real urban roads.

In all optimization results, the most energy-efficient speed trajectory may not be the most time-saving trajectory. This is because the driving speed limit, road length, and vehicle characteristics have a great impact on the final optimization results. In the future research, weight coefficients can be introduced to include the travel time in the cost function, so the optimization result will try to make a compromise between energy efficiency and travel time.

FIGURE 14. Different travel trajectory curves at multi-signal intersection.

For the path#3, Fig. 14 shows the different travel trajectory curves based on different \( \eta \) values when passing through three intersections. In order to facilitate the observation of the trajectory curve, it is assumed that the parameters \( \eta \) of the three intersections are identical. The results show that as \( \eta \) increases, the vehicle’s trajectory gradually moves backward, ensuring that the vehicle avoids the peak of the traffic queue as much as possible. Combined with the results of Figure 10, it can be seen that the ERD based on the average traffic information can ensure a smooth passage of the CA V to a certain extent. Undeniably, there is a certain difference between real dynamic traffic and average traffic information. Hence, the proposed DERD scheme based on average traffic and dynamic traffic flow can provide a safer driving space for the CA V.

V. CONCLUSION

The traffic light model is constructed by considering the basic red light time, the average queuing effect time and the real-time traffic fluctuation effect time to improve the authenticity of the model and ensure that connected and automated vehicles can smoothly pass through intersections with queues interfere. The pseudo-spectral method was applied to solve the optimal energy saving problem for electric vehicles under the constraints of dynamic effective red light
duration. In this process, the recursive and traversal search clarifies the time constraints under different traffic combinations, reducing the complexity of the global optimization problem. The optimal control problem is transformed into a finite-dimensional nonlinear programming problem by using the Legendre-Gauss-Lobatto pseudo-spectral scheme. Both of these efforts are conducive to improving the efficiency of solving control strategies. Compared with the intelligent driver model, the electric vehicle under the pseudo-spectral method reduces energy consumption and avoids the parking behavior at the intersection. The effectiveness of the pseudo-spectral method was demonstrated through the comparison with dynamic programming. It is found that the pseudo-spectral method provides slightly accurate results close to dynamic programming at less computation cost. In summary, the pseudo-spectral method is a promising candidate for optimal energy-saving driving of electric vehicles under dynamic traffic flow constraints on urban roads and can be used to promote the popularization of eco-driving systems in actual driving.

Actually, a better and more feasible framework for the ecological driving is that in the upper-level, when given the start position and end position, a route will be planned considering the holistic traffic situation. In the lower-level, the driver will follow the planned route and drive the vehicle according to the advised speed trajectory. Thus, in the lower-level, only the traffic information in the planned route direction will be necessary. In this research, we focus on the lower-level design. So we only consider the vehicle pass through the intersection in straight. If the vehicle needs to turn left or turn right, only the green wave in the direction of left or right needs to be incorporated into the model. Our proposed method is still useful.

Due to the limited computing resources at present, the control algorithm can not be transplanted to the vehicle for real-time application, which is also the shortcoming of this paper. After a large number of scene optimization analyses, we found that the speed curve after PM optimization presented regular changes, which can be used to formulate the on-line ecological driving strategy table and conduct real vehicle verification. In addition, there are a lot of microscopic traffic models like full velocity difference model (FVDM), optimal velocity mode (OVM) [39], or Wiedemann model, which can be used to compare with our proposed model. Because IDM is the most commonly used vehicle-following model in current researches, it is used as the benchmark to evaluate the effectiveness of our proposed method. Due to limited time and resources, we didn't analyze more models like FVD/OVM or Wiedemann model. All above mentioned contents may be the next step of our ongoing work.

REFERENCES

[1] T. Zhang, Y. Zou, X. Zhang, N. Guo, and W. Wang, “Data-driven based cruise control of connected and automated vehicles under cyber-physical system framework,” IEEE Trans. Intell. Transp. Syst., early access, May 12, 2020, doi: 10.1109/TITS.2020.2991223.

[2] T. Zhang, Y. Zou, X. Zhang, N. Guo, and W. Wang, “A cruise control method for connected vehicle systems considering side vehicles merging behavior,” IEEE Access, vol. 7, pp. 6922–6936, 2019.

[3] Y. J. Zhang, A. Malikopoulos, and C. G. Cassandras, “Optimal control and coordination of connected and automated vehicles at urban traffic intersections,” in Proc. Amer. Control Conf. (ACC), Jul. 2016, pp. 6227–6232.

[4] V. A. Butakov and P. Ioannou, “Personalized driver assistance for signalized intersections using V2I communication,” IEEE Trans. Intell. Transp. Syst., vol. 17, no. 7, pp. 1910–1919, Jul. 2016.

[5] K. Katsaros, R. Kernchen, M. Dianati, D. Rieck, and C. Zinoviou, “Application of vehicular communications for improving the efficiency of traffic in urban areas,” Wireless Commun. Mobile Comput., vol. 11, no. 12, pp. 1657–1667, 2011.

[6] P. Guo, H. Chen, Q. Liu, and B. Gao, “A computationally efficient and hierarchical control strategy for velocity optimization of on-road vehicles,” IEEE Trans. Syst., Man, Cybern. Syst., vol. 49, no. 1, pp. 31–41, Jan. 2019.

[7] S. A. Sajadi-Alamdari, H. Voos, and M. Darouach, “Nonlinear model predictive extended eco-cruise control for battery electric vehicles,” in Proc. Mediterranean Conf. Control Automat. (MED), Aug. 2016, pp. 467–472, doi: 10.1109/MED.2016.7535929.

[8] W. Dih, A. Chasse, P. Moulin, A. Scarrettura, and G. Corde, “Optimal energy management for an electric vehicle in eco-driving applications,” Control Eng. Pract., vol. 29, pp. 299–307, Aug. 2014.

[9] M. Barth and K. Borbionsomnis, “Energy and emissions impacts of a freeway-based dynamic eco-driving system,” Transp. Res. Part D, Transp. Environ., vol. 14, no. 6, pp. 400–410, Aug. 2009.

[10] H. Yang and W.-L. Jin, “A control theoretic formulation of green driving strategies based on inter-vehicle communications,” Transp. Res. Part C, Emerg. Technol., vol. 41, pp. 48–60, Apr. 2014.

[11] H. Suzuki and Y. Marumo, “Safety evaluation of green light optimal speed advisory (GLOSA) system in real-world signalized intersection,” J. Robot. Mechatronics, vol. 32, no. 3, pp. 598–604, 2020.

[12] S. Mandava, K. Borbionsomnis, and M. Barth, “Arterial velocity planning based on traffic signal information under light traffic conditions,” in Proc. 12th Int. IEEE Conf. Intell. Transp. Syst., Oct. 2009, pp. 1–6.

[13] B. Asadi and A. Vahidi, “Predictive cruise control: Utilizing upcoming traffic signal information for improving fuel economy and reducing trip time,” IEEE Trans. Control Syst. Technol., vol. 19, no. 3, pp. 707–714, May 2011.

[14] E. Ozatay, U. Ozguner, D. Filev, and J. Michelinii, “Analytical and numerical solutions for energy minimization of road vehicles with the existence of multiple traffic lights,” in Proc. 52nd IEEE Conf. Decis. Control, Dec. 2013, pp. 7137–7142.

[15] G. Thomas and P. G. Voulgaris, “Fuel minimization of a moving vehicle in suburban traffic,” in Proc. Amer. Control Conf., Jun. 2013, pp. 4009–4014.

[16] G. De Nunzio, C. C. de Wit, P. Moulin, and D. Di Domenico, “Eco-driving in urban traffic networks using traffic signals information,” Int. J. Robust Nonlinear Control, vol. 26, no. 6, pp. 1307–1324, Apr. 2016.

[17] C. Sun, X. Shen, and S. Moura, “Robust optimal ECO-driving control with uncertain traffic signal timing,” in Proc. Ann. Amer. Control Conf. (ACC), Jun. 2018, pp. 5548–5553.

[18] H. Yang, H. Rakha, and M. V. Ala, “Eco-cooperative adaptive cruise control at signalized intersections considering queue effects,” IEEE Trans. Intell. Transp. Syst., vol. 18, no. 6, pp. 1575–1585, Jun. 2017.

[19] C. Yin, S. Wang, C. Yu, J. Li, and S. Zhang, “Fuzzy optimization of energy management for power split hybrid electric vehicle based on particle swarm optimization algorithm,” Adv. Mech. Eng., vol. 11, no. 2, Feb. 2019, Art. no. 168781401983079.

[20] Y. Zeng, J. Sheng, and M. Li, “Adaptive real-time energy management strategy for plug-in hybrid electric vehicle based on simplified-ECMS and a novel driving pattern recognition method,” Math. Problems Eng., vol. 2018, pp. 1–12, Oct. 2018.

[21] X. Li, L. Han, H. Liu, W. Wang, and C. Xiang, “Real-time optimal energy management strategy for a dual-mode power-split hybrid electric vehicle based on an explicit model predictive control algorithm,” Energy, vol. 172, pp. 1161–1178, Apr. 2019.

[22] Y. Wu, H. Tan, J. Peng, H. Zhang, and H. He, “Deep reinforcement learning of energy management with continuous control strategy and traffic information for a series-parallel plug-in hybrid electric bus,” Appl. Energy, vol. 247, pp. 454–466, Aug. 2019.
[23] M. R. Amini, X. Gong, Y. Feng, H. Wang, I. Kolmanovsky, and J. Sun, “Sequential optimization of speed, thermal load, and power split in connected HEVs,” in Proc. Amer. Control Conf. (ACC), Philadelphia, PA, USA, Jul. 2019, pp. 1–7.

[24] N. Guo, J. Shen, R. Xiao, W. Yan, and Z. Chen, “Energy management for plug-in hybrid electric vehicles considering optimal engine ON/OFF control and fast state-of-charge trajectory planning,” Energy, vol. 163, pp. 457–474, Nov. 2018.

[25] R. F. Hartl, S. P. Sethi, and R. G. Vickson, “A survey of the maximum principles for optimal control problems with state constraints,” in Proc. Soc. Ind. Appl. Math., 1995, vol. 37, no. 2, pp. 181–218.

[26] S. Xu, K. Deng, S. E. Li, S. Li, and B. Cheng, “Legendre pseudospectral computation of optimal speed profiles for vehicle eco-driving system,” in Proc. IEEE Intell. Vehicles Symp., Jun. 2014, pp. 1103–1108.

[27] S. Wei, Y. Zou, F. Sun, and O. Christopher, “A pseudospectral method for solving optimal control problem of a hybrid tracked vehicle,” Appl. Energy, vol. 194, pp. 588–595, May 2017.

[28] F. Fahroo and I. M. Ross, “Advances in pseudospectral methods for optimal control,” in Proc. AIAA Guid., Navigat. Control Conf. Exhib., Honolulu, HI, USA, Aug. 2008, pp. 18–21.

[29] T. Wang, C. G. Cassandras, and S. Pourazarm, “Optimal motion control for energy-aware electric vehicles,” Control Eng. Pract., vol. 38, pp. 37–45, May 2015.

[30] S. Yang, C. Deng, T. Tang, and Y. Qian, “Electric vehicle’s energy consumption of car-following models,” Nonlinear Dyn., vol. 71, nos. 1–2, pp. 323–329, Jan. 2013.

[31] M. A. S. Kamal, M. Mukai, J. Murata, and T. Kawabe, “Ecological vehicle control on roads with up-down slopes,” IEEE Trans. Intell. Transp. Syst., vol. 12, no. 3, pp. 783–794, Sep. 2011.

[32] M. J. Lighthill and G. B. Whitham, “On kinematic waves II. A theory of traffic flow on long crowded roads,” Proc. Roy. Soc. London. Ser. A, Math. Phys. Sci., vol. 229, no. 1178, pp. 317–345, 1955.

[33] P. I. Richards, “Shock waves on the highway,” Oper. Res., vol. 4, no. 1, pp. 42–52, Feb. 1956.

[34] G. Huntington, D. Benson, and A. Rao, “A comparison of accuracy and computational efficiency of three pseudospectral methods,” in Proc. AIAA Guid., Navigat. Control Conf. Exhib., Aug. 2007, p. 6405.

[35] I. M. Ross and M. Karpenko, “A review of pseudospectral optimal control: From theory to flight,” Annu. Rev. Control, vol. 36, no. 2, pp. 182–197, Dec. 2012.

[36] P. E. Gill, W. Murray, and M. A. Saunders, “SNOPT: An SQP algorithm for large-scale constrained optimization,” SIAM Rev., vol. 47, no. 1, pp. 99–131, 2005.

[37] A. Kesting, M. Treiber, and D. Helbing, “Enhanced intelligent driver model to assess the impact of driving strategies on traffic capacity,” Phil. Trans. Roy. Soc. A: Math. Phys. Eng. Sci., vol. 368, no. 1928, pp. 4585–4605, Oct. 2010.

[38] C. L. Darby, W. W. Hager, and A. V. Rao, “An hp-adaptive pseudospectral method for solving optimal control problems,” Optim. Control Appl. Methods, vol. 32, no. 4, pp. 476–502, Jul. 2011.

[39] Y. Li et al., “Complexity and applicability analysis among OVM, GFM and FVDM models,” J. Southeast Univ., vol. 31, no. 3, pp. 424–426, 2015.

CHUNMING LI received the Ph.D. degree from the Beijing Institute of Technology, China, in 2011. He is currently the Head Technology Principal of the China North Vehicle Research Institute. His current research interests include the vehicle overall performance matching, power management, and control strategy optimization.

TAO ZHANG (Graduate Student Member, IEEE) received the M.S. degree from the Beijing University of Technology, China, in 2015, and the Ph.D. degree from the Beijing Institute of Technology, China, in 2020. He currently works as an Assistant Research Fellow with the China North Vehicle Research Institute. His current research interests include the hardware design of vehicle controller, vehicle dynamics control, hybrid energy management, energy efficient driving assistance systems, and reinforcement learning.

XIAOXIA SUN received the Ph.D. degree from the Beijing Institute of Technology, China, in 2011. She is currently a Researcher with the China North Vehicle Research Institute. Her current research interests include the vehicle power management, thermal management, and control strategy optimization.

NING ZHAO received the M.S. degree from the Beijing University of Technology, China, in 2020. He currently works as a Junior Research Fellow with the China North Vehicle Research Institute. His current research interests include the hardware design of vehicle controller, hybrid energy management, and machine learning.