Intelligent Reflecting Surface Enhanced Wideband MIMO-OFDM Communications: From Practical Model to Reflection Optimization

Hongyu Li, Student Member, IEEE, Wenhao Cai, Yang Liu, Member, IEEE, Ming Li, Senior Member, IEEE, and Qian Liu, Member, IEEE

Abstract—Intelligent reflecting surface (IRS) is envisioned as a revolutionary technology for future wireless communication systems since it can intelligently change radio environment and integrate it into wireless communication optimization. However, most recent investigation utilized an ideal IRS reflection model, which is impractical and can cause significant performance degradation in realistic wideband systems. In this work, we first study the amplitude-frequency-phase relationship of reflected signals and present a simplified practical IRS reflection model for wideband signals. Then, an IRS enhanced wideband multi-user multi-input single-output orthogonal frequency division multiplexing (MU-MISO-OFDM) system is investigated. We aim to jointly design the transmit beamformer and IRS reflection to maximize the average sum-rate over all subcarriers. With the aid of the relationship between sum-rate maximization and mean square error (MSE) minimization, the original problem is equivalently transformed into a multi-block/variable problem, which can be solved by classic block coordinate descent (BCD) method. Complexity and convergence for both cases are analyzed or illustrated. Simulation results demonstrate that the proposed algorithm can offer significant average sum-rate enhancement compared to that achieved using the ideal reflection model, which confirms the importance of the use of the practical model for the design of wideband systems.

Index terms—Intelligent reflecting surface (IRS), multi-user multi-input single-output (MU-MISO), orthogonal frequency division multiplexing (OFDM), beamforming.

I. INTRODUCTION

The continuous popularizing of intelligent devices and the rapid development of emerging wireless services have caused the exponential increase of the demand for wireless network traffic. This motivates the research on key enabling technologies, such as massive multi-input multi-output (MIMO), ultra-dense network, and the use of millimeter wave (mmWave) bands [1]-[3], for the fifth-generation (5G) and beyond networks. However, the above technologies still inevitably face challenges mainly due to high cost and power consumption when employing multiple antennas, cells (base stations (BSs)), and/or hardware components (e.g., radio frequency (RF) chains). Therefore, researchers have never stopped their efforts to seek spectral- and energy-efficiency (SE/EE) solutions to accommodate the demanding data rate and diverse quality of service (QoS) requirements for future wireless communications.

In the current paradigm of wireless communication optimization, the radio environment and wireless propagation medium remain an uncontrollable factor, which cannot be included in the optimization formulations. Thus, channel fading effect due to the randomness in the radio environment is generally a major challenge for the maximization of EE/SE performance of wireless communications. Recently, an innovative concept of intelligent reflecting surface (IRS) has been introduced in the wireless communication research community as a revolutionary technology, which can realize controllable radio environment and combat stochastic wireless propagation medium [4]-[12].

The IRS consists of a large number of nearly passive elements with ultra-low power consumption. Particularly, each element of IRS is composed of configurable electromagnetic (EM) internals, which are capable of controlling the phase shift and amplitude of the incident EM wave in a programmable manner. Adaptively adjusting elements of IRS can collaboratively achieve reflection beamforming and shape the propagation environment suitable for wireless communications. The channel/beamforming gain can be effectively improved and the communication quality can be enhanced. Free of containing radio frequency (RF) chains, large-scale IRS can be deployed in different communication situations with lower power consumption and cost. Therefore, IRS is envisioned to revolutionize the current communication optimization paradigm by integrating the smart radio environment and expected to play an important role in future wireless communications.

Attracted by the sheer advantage of IRS, the investigation of IRS for improving the performance of various wireless communication systems is a thriving research area in the last few years. A majority of recent research efforts have been devoted to the IRS designs with focus on power allocation and/or beamformer for both single-user systems [13]-[15] and multi-user systems [16]-[20] using different metrics, e.g., power minimization [14], [19], max-min fairness [19], [20], SE maximization [13]-[15], [18], and EE maximization [16]. In some recent works [21]-[23], practical IRS implementation with finite/low-resolution phase shifts are considered. In order to further highlight the flexibility of the IRS employment, many researchers also studied the coordination of multiple IRSs [24]-[26]. Moreover, IRS technique has also been employed in other applications, e.g., physical layer security [27]-
are summarized as follows: Our main contributions a simplified practical reflection model of IRS and take it into MISO-OFDM communication system. Specifically, we present

The aforementioned work assumes that IRS have an ideal model with perfect signal reflection, i.e. each element has constant magnitude, variable phase shift, and the same response for wideband signals. The design of IRS with such an ideal reflection model can be easily implemented using classical optimization tools, e.g. semidefinite relaxation (SDR), manifold optimization, majorization minimization (MM), etc. However, it is extremely difficult to implement an IRS having such an ideal reflection model due to the hardware circuit limitation. Therefore, it is important and necessary to analyze the response characteristic of a practical IRS and establish an accurate and practical IRS reflection model. The authors in have illustrated the fundamental relationship between reflection amplitude and phase shift under a narrowband scenario and demonstrated the performance enhancement with their proposed practical model compared to that with the ideal one. When expanding to wideband communications, unfortunately, the above two-dimensional amplitude-phase relationship cannot accurately describe the response of the practical IRS, which will vary with the frequencies of incident signals. In our previous work, we have analyzed this issue and established a three-dimensional amplitude-frequency-phase relationship to precisely describe the response of practical IRS in wideband systems. Nevertheless, this practical model is so complicated that it will cause great difficulties in the IRS reflection design. This motivates us to further simplify the practical IRS model in order to facilitate the reflection design without significant accuracy loss.

In this paper, we consider an IRS-enhanced wideband MU-MISO-OFDM communication system. Specifically, we present a simplified practical reflection model of IRS and take it into consideration for the reflection design. Our main contributions are summarized as follows:

- We re-analyze the characteristic of IRS elements, i.e. phase and amplitude variations of IRS elements when responding to signals with different frequencies. Based on our previous work, we present a leaner practical model of IRS reflection, which is applicable to the designs of typical communication scenarios.
- Then, we aim to jointly design the beamformer and the reflection of IRS to achieve maximum average sum-rate over all subcarriers. Based on the equivalence between sum-rate maximization and mean square error (MSE) minimization, the problem is converted to a multi-block/variable optimization, which can be solved by the classical block coordinate descent (BCD) method.
- Finally, we evaluate our proposed design. We analyze the complexity and illustrate the convergence. Moreover, the performance of the proposed algorithm is validated by extensive simulation studies, which confirm the effectiveness of the design with the practical model compared to that with the ideal one.

**Notations:** Boldface lower-case and upper-case letters indicate column vectors and matrices, respectively. \( \mathbb{C} \) and \( \mathbb{R}^+ \) denote the set of complex and positive real numbers, respectively. \( (\cdot)^*, (\cdot)^T, (\cdot)^H, \) and \((\cdot)^{-1}\) denote the conjugate, transpose, conjugate-transpose operations, and inversion, respectively. \( \mathbb{E}\{\cdot\} \) represents statistical expectation. \( \mathbb{R}\{\cdot\} \) denotes the real part of a complex number. \( \mathbf{1}_L \) indicates an \( L \times L \) identity matrix. \( \|\mathbf{A}\|_F \) denotes the Frobenius norm of matrix \( \mathbf{A} \). \( \|a\|_2 \) denotes the \( \ell_2 \) norm of vector \( a \). \( \otimes \) denotes the Kronecker product. \( \text{blkdiag}(\cdot) \) denotes a block matrix such that the main-diagonal blocks are matrices and all off-diagonal blocks are zero matrices. Finally, \( \mathbf{A}(i,\cdot), \mathbf{A}(\cdot,j), \) and \( \mathbf{A}(i,j) \) denote the \( i \)-th row, the \( j \)-th column, and the \( (i,j) \)-th element of matrix \( \mathbf{A} \), respectively. \( a(i) \) denotes the \( i \)-th element of vector \( a \).

**II. Practical IRS Modeling**

The hardware construction of IRS is usually based on the printed circuit board (PCB) with uniformly distributed reflecting elements on a planar surface. A typical IRS generally consists of three layers: \( i \) an outer layer with a large number of metal elements printed on the PCB dielectric substrate; \( ii \) a copper plate to avoid the leakage of signal energy; \( iii \) a control circuit board for IRS control [4]. A semiconductor device, such as the positive-intrinsic-negative (PIN) diode, is embedded into each metal element in the outer layer to tune the reflecting response, e.g. phase shift and amplitude. The response of each reflecting element can be equivalently modeled as a parallel resonance circuit as shown in Fig. 1. Thus, the impedance of an IRS element for the signal of frequency \( f \) can be written as

\[
Z(C, f) = \frac{j2\pi f L_1 (j2\pi f L_2 + \frac{1}{j2\pi f C} + R)}{j2\pi f L_1 + j2\pi f L_2 + \frac{1}{j2\pi f C} + R},
\]

where \( L_1, L_2, C, \) and \( R \) denote the metal plate inductance, outer layer inductance, effective capacitance, and the loss resistance, respectively. The reflection coefficient of each IRS element, denoted as \( \phi \), is fundamentally the ratio of the power.
of the reflected signal to that of the incident one, which is therefore given by
\[
\phi = \frac{Z(C, f) - Z_0}{Z(C, f) + Z_0},
\]
where \(Z_0\) denotes the free space impedance. Here, we should emphasize that the reflection of the IRS element is a function of \(C\) and \(f\). When each element is controlled by selecting an appropriate capacitance \(C\), the response of each element is also associated with the frequency of the incident signals. Our previous work [42] has demonstrated that the same IRS element actually exhibits different responses (i.e. different amplitudes and phase shifts) to signals with different frequencies, which is referred to as dual phase- and amplitude-squint effect in this paper. Fig. 2(a) illustrates an example of the amplitude and phase shift variations of an IRS element as a function of frequency. Let us name the phase shift \(\theta\) for signal of central carrier frequency \(f_c\) as the basic phase shift (BPS) for clear and concise description. We can observe from Fig. 2(a) that, if we change the BPS \(\theta\), the phase shifts and amplitudes for other frequencies will be quite different, which illustrates the severe beam deviations due to the dual phase- and amplitude-squint. It is worth noting that this kind of dual phase- and amplitude-squint is an intrinsic phenomenon depending on the practical IRS circuit implementation, which cannot be simply ignored in realistic IRS-enhanced wideband systems. Therefore, it is necessary to consider the phase- and amplitude-squint into account by developing an accurate reflection model of each IRS element, which is crucial for the following joint beamforming and reflecting design.

In [42], we have established an accurate three-dimensional amplitude-phase-frequency model to describe the dual phase- and amplitude-squint. Unfortunately, this model is so complicated that it may significantly increase the difficulty and complexity of IRS design. To effectively simplify this model while maintaining its accuracy, we consider a more practical wideband situation that the relative bandwidth, i.e. the ratio of bandwidth and the carrier frequency \(B/f_c\), is less than 5\%. Take the case that the carrier frequency \(f_c = 2.4\) GHz and bandwidth \(B = 100\) MHz as example. It can be observed from

\[
\begin{align*}
\phi &= \frac{Z(C, f) - Z_0}{Z(C, f) + Z_0},
\end{align*}
\]

\[
\begin{array}{c|c|c|c|c|c}
\hline
i & 1 & 2 & 3 & 4 & 5 \\
\hline
a_i & 0.06 & 11.27 & 10.88 & 89.64 & 26.11 \\
b_i & 0.02 & 0.008996 & 0.9799 & 0.01268 & 0.9796 \\
c_i & 0.5736 & -1.897 & -1.471 & 0.2899 & 1.673 \\
\hline
\end{array}
\]
Algorithm to jointly design the transmit beamforming and IRS reflecting.

III. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider a wideband MU-MISO-OFDM system with \(N\) subcarriers, as shown in Fig. 3. The BS employs \(N_t\) antennas to communicate with \(K\) single-antenna users. This wireless transmission is assisted by an IRS of \(M\) passive elements. Denote \(\mathcal{N} = \{1, \ldots, N\}\), \(\mathcal{N}_t = \{1, \ldots, N_t\}\), \(\mathcal{K} = \{1, \ldots, K\}\), and \(\mathcal{M} = \{1, \ldots, M\}\) as the set of the indices of subcarriers, transmit antennas, users, and elements of the IRS, respectively. The phase shifts of IRS elements are individually adjusted via a controller. In this paper, exact and instantaneous channel state information (CSI) is assumed to be available at the BS, which can be obtained via the efficient channel estimation approaches proposed by the recent works [36], [43], [44]. Next, we will describe the communication process in detail.

Transmitter: Let \(s_i \triangleq [s_{i1}, \ldots, s_{iK}]^T \in \mathbb{C}^K\) be the transmit symbols for all users associated with the \(i\)-th subcarrier with \(\mathbb{E}\{s_i s_i^H\} = \mathbf{I}_K\), \(\forall i \in \mathcal{N}\). The vector \(s_i\) is first digitally precoded by a precoder matrix \(\mathbf{W}_i = [\mathbf{w}_{i1}, \ldots, \mathbf{w}_{iK}] \in \mathbb{C}^{N_t \times K}, \forall i \in \mathcal{N}\), in the frequency domain and then converted to the time domain by the inverse discrete Fourier transform (IDFT), which yields the overall time-domain signal \(\tilde{s}\) as

\[
\tilde{s} = (\mathbf{F}^H \otimes \mathbf{I}_{N_t}) \mathbf{W} s_i
\]

where \(\mathbf{F} \in \mathbb{C}^{N \times N}\) is the normalized discrete Fourier transform (DFT) matrix and is defined as \(\mathbf{F}(m, n) \triangleq \frac{1}{\sqrt{N}} e^{-j2\pi(m-1)(n-1)/N}, \forall m, n \in \mathcal{N}\). The overall precoding matrix \(\mathbf{W}\) is given by \(\mathbf{W} \triangleq \text{blkdiag}(\mathbf{W}_1, \ldots, \mathbf{W}_N)\), and the overall transmit symbol vector \(s\) can be written as \(s \triangleq [s_1^T, \ldots, s_N^T]^T\). After adding the cyclic prefix (CP) of size \(N_{cp}\), the signal is up-converted to the RF domain via \(N_t\) RF chains.

Channel: In the considered wideband MU-MISO-OFDM system, the wideband channel from the BS to user \(k\) is modeled by a \(D\)-tap \((D \leq N_{cp})\) finite-duration impulse response \(\{\mathbf{h}_{k,0}^d, \ldots, \mathbf{h}_{k,D-1}^d\}\), where \(\mathbf{h}_{k,d}^d \in \mathbb{C}^{N_t}, d \in \mathcal{D} \triangleq \{0, \ldots, D-1\}, \forall k \in \mathcal{K}\), is the impulse response corresponding to the \(d\)-th delay tap. Similarly, the wideband channel from the BS to the IRS is given by \(\{\mathbf{G}_0, \ldots, \mathbf{G}_{D-1}\}\) with \(\mathbf{G}_d \in \mathbb{C}^{M \times N_t}, \forall d \in \mathcal{D}\). The wideband channel from the IRS to user \(k\) is given by \(\{\mathbf{h}_{k,0}^d, \ldots, \mathbf{h}_{k,D-1}^d\}\) with \(\mathbf{h}_{k,d}^d \in \mathbb{C}^M, \forall d \in \mathcal{D}, \forall k \in \mathcal{K}\).

Receiver: After propagating through the wideband channels of both the BS-user link and the BS-IRS-user link, the signal \(\tilde{s}\) is corrupted by additive Gaussian white noise (AGWN). Down-converting to the baseband and removing the CP, we obtain the time-domain received signal for user \(k\) given as follows

\[
\mathbf{y}_k = (\mathbf{H}_k^d + \mathbf{H}_k^r \Phi \mathbf{G})(\mathbf{F}^H \otimes \mathbf{I}_{N_t}) \mathbf{W} s_i + \mathbf{n}_k, \forall k,
\]

where the block cyclic channel matrix \(\mathbf{H}_k^d \in \mathbb{C}^{N_t \times N_t}\) of the BS-user link is defined as

\[
\mathbf{H}_k^d = \begin{bmatrix}
(\mathbf{h}_{k,0}^d)^H & 0_{N_t}^T & \cdots & (\mathbf{h}_{k,1}^d)^H \\
\vdots & \vdots & \ddots & \vdots \\
0_{N_t}^T & (\mathbf{h}_{k,D-1}^d)^H & \cdots & (\mathbf{h}_{k,D-1}^d)^H \\
0_{N_t}^T & \cdots & 0_{N_t}^T & (\mathbf{h}_{k,0}^d)^H
\end{bmatrix},
\]

\(\forall k \in \mathcal{K}\). Similarly, we define \(\mathbf{G}_d^H, \ldots, \mathbf{G}_{D-1}^H, 0_{N_t \times M}, \ldots, 0_{N_t \times M}\) as the first block column of the block cyclic channel matrix \(\mathbf{G} \in \mathbb{C}^{MN_t \times N_t}\) of the BS-IRS link and \(\mathbf{h}_{k,0}^d, \ldots, \mathbf{h}_{k,D-1}^d, 0_M, \ldots, 0_M\) as the first block column of the block cyclic channel matrix \(\mathbf{H}_k^r \in \mathbb{C}^{NM_t \times M}\) of the IRS-user link. The reflection matrix \(\Phi\) of IRS is defined as \(\Phi = \text{blkdiag}(\Phi_1, \ldots, \Phi_N)\), where \(\Phi_i \triangleq \text{diag}(\phi_{i1}, \ldots, \phi_{iM})\), \(\forall i \in \mathcal{N}\). Here, \(\phi_{im}\) denotes the reflection coefficient of the \(m\)-th IRS element for the \(i\)-th subcarrier. Different from the ideal model that each element exhibits the same reflection coefficient for different frequencies (i.e., \(|\phi_{im}| = 1\), and \(\angle \phi_{i,im} = \ldots = \angle \phi_{in,im}, \forall i \in \mathcal{N}, \forall m \in \mathcal{M}\)), we adopt the practical model presented in the previous section. In particular, the amplitude and phase shift of \(\phi_{im}\) actually vary with...
Here $H$ is the BPS $\theta_m$, and follow the relationship given in (3), i.e.

$$|\phi_{i,m}| = F(\theta_m, f_i), \quad \angle\phi_{i,m} = G(\theta_m, f_i), \quad \forall \theta_m \in \mathcal{N}, \forall i \in \mathcal{M}. \quad \tilde{n}_k \in \mathcal{CN}(0, \sigma^2).$$

$k$ is the AGWN. After DFT, the received signal in the frequency domain can be written as

$$y_k = F(\tilde{H}_k, \tilde{H}_k, \tilde{G})(F^H \otimes I_N),$$

$$= H_k W s + n_k, \quad \forall k, \quad (7)$$

where $n_k \triangleq F\tilde{n}_k, \forall k \in \mathcal{K}$. The equivalent frequency-domain channel $H_k$ for user $k$ is given by (5) on the top of this page, where (a) holds by introducing two column permutation square matrices $\Gamma_1$ and $\Gamma_2$ with $\Gamma_1^T \Gamma_1 = I_N^M$, $\Gamma_2^T \Gamma_2 = I_{N^M}$, which convert a block cyclic matrix to several cyclic matrices arranged in rows [45]. In this way, the block cyclic channels are rearranged as a sequence of cyclic matrices. Specifically, (b) holds by defining cyclic channel matrices $\tilde{H}_{k,m}, \tilde{G}_{m,n} \in \mathcal{C}^{N \times N}$, and $\tilde{H}_{k,m}^d, \tilde{G}_{m,n} \in \mathcal{C}^{N \times N}$, $\forall m \in \mathcal{M}, \forall n \in \mathcal{N}$, as

$$\tilde{H}_{k,m}^d(\cdot, i) \triangleq \tilde{H}_{k,m}^d(\cdot, n + (i - 1)N),$$

$$\tilde{G}_{m,n}(p, q) \triangleq \tilde{G}(m + (p - 1)N, n + (q - 1)N),$$

$$\tilde{H}_{k,m}(\cdot, i) \triangleq \tilde{H}_{k,m}^d(\cdot, m + (i - 1)M), \quad \forall i, \forall k, \forall n \in \mathcal{N}. \quad (8)$$

Additionally, the rearranged reflection matrix $\tilde{G}$ is given by $\tilde{G} \triangleq \text{blkdiag}(\tilde{G}_1, \ldots, \tilde{G}_M)$, where $\tilde{G}_m \triangleq \text{diag}(\phi_{i,m}, \ldots, \phi_{N,m}), \forall m \in \mathcal{M}$. Then (c) holds since the DFT matrix can diagonalize the cyclic matrix. Here $A_{k,m}, A_{k,m}^\tau, A_{k,m}^\tau \in \mathcal{M}$ are diagonal matrices whose diagonal elements are the corresponding eigenvalues of $\tilde{H}_{k,m}^d$. $\tilde{H}_{k,m}$, and $\tilde{G}_{m,n}$, respectively. Finally, (d) holds by defining frequency-domain channels $\tilde{h}_{k,i}^d \in \mathcal{C}^{N_i}$, $\tilde{h}_{k,i} \in \mathcal{C}^{M}$, and $G_i \in \mathcal{C}^{M \times N_i}, \forall k \in \mathcal{K}, \forall i \in \mathcal{N}$, as

$$\tilde{h}_{k,i}^d(n) \triangleq (A_{k,m}^\tau(n, i))^*, \quad \tilde{h}_{k,i}(m) \triangleq (A_{k,m}^\tau(m, i))^*,$$

$$G_i(m, n) \triangleq \Xi_{m,n}(i, i), \quad \forall m \in \mathcal{M}, \forall n \in \mathcal{N}_i. \quad (9)$$

Substituting (5g) into (7), we can obtain the received signal on the $i$-th subcarrier for user $k$ as

$$y_{k,i} = (h_{k,i}^d)^H + (h_{k,i}^d)^H \Phi \Gamma_1 \Gamma_1^T |W_s| + n_{k,i}$$

$$= (h_{k,i}^d)^H + (h_{k,i}^d)^H \Phi \Gamma_1 \Gamma_1^T |W_p, i| s_{k,i} + n_{k,i}$$

$$\sum_{p=1, p \neq k}^K (h_{k,i}^d)^H \Phi \Gamma_1 |W_p, i| s_{k,i} + n_{k,i}$$

$$\sum_{p=1, p \neq k}^K (h_{k,i}^d)^H \Phi \Gamma_1 |W_p, i| s_{k,i} + n_{k,i}$$

$$\sum_{p=1, p \neq k}^K (h_{k,i}^d)^H \Phi \Gamma_1 |W_p, i| s_{k,i} + n_{k,i}$$

where $n_{k,i}$ denotes the $i$-th element of $n_k$.

### IV. Joint Transmit Beamformer and IRS Reflection Design

#### A. Problem Formulation

With the received signal given in (10), the signal-to-interference-plus-noise ratio (SINR) on the $i$-th subcarrier for user $k$ can be calculated as

$$\gamma_{k,i} = \frac{|(h_{k,i}^d)^H + (h_{k,i}^d)^H \Phi \tilde{G}_{i}|^2}{\sum_{p \neq k} |(h_{k,i}^d)^H + (h_{k,i}^d)^H \Phi \tilde{G}_{i}|^2 + \sigma^2}, \forall k, \forall i. \quad (11)$$

In this paper, our goal is to jointly design the transmit beamformer $W$ and the BPS matrix $\Theta \triangleq \text{diag}(\theta_1, \ldots, \theta_M)$, which essentially control the IRS reflection of wideband signals, to maximize the average sum-rate for the MU-MISO-OFDM system, subject to the constraints of the phase shift matrix and the transmit power constraint. Therefore, the joint transmit beamformer and IRS reflection design problem can be formulated as

$$\max_{W, \Theta} \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{K} \log_2(1 + \gamma_{k,i}) \quad (12a)$$

s.t. $|\phi_{i,m}| = F(\theta_m, f_i), \forall i, m$, \quad (12b)

$\angle\phi_{i,m} = G(\theta_m, f_i), \forall i, m$, \quad (12c)

$\theta_m \in [-\pi, \pi], \forall m$, \quad (12d)

$$\sum_{i=1}^{N} \|W_i\|^2 \leq P, \quad (12e)$$

where $P$ is the total transmit power.

Problem (12) is difficult to solve due to the complex form of the objective and the non-convex constraints of the BPS.
matrix. Additionally, it is worth noting that the amplitude and phase shift of each IRS element will change with different frequencies when considering practical IRS responses for wideband signals. In other words, we focus on the design of BPS matrix \( \Theta \), but the response of practical IRS for signals with different subcarriers varies, i.e., reflection matrix \( \Phi_i \), \( \forall i \in \mathcal{N} \), are different at each subcarrier. This fact will further perplex the problem. To deal with these issues, in the next section, we attempt to first transform problem (12) into a more tractable multi-variable/block optimization and then iteratively cope with each block.

B. Problem Reformulation

To tackle the difficulty rising from the \( \sum \log(\cdot) \) function and the fractional form of “SINRs” in problem (12), we first reformulate the original sum-rate maximization problem as a modified MSE minimization problem [46]. Let us first define the modified MSE function for user \( k \) on the \( i \)-th subcarrier as

\[
\text{MSE}_{k,i} = \mathbb{E}\left\{ (\varpi_{k,i}^* y_{k,i} - s_{k,i}) (\varpi_{k,i} y_{k,i} - s_{k,i})^* \right\}
\]

\[
= \sum_{p=1}^{K} |\varpi_{k,i}^* (h_{k,i}^d)^H + (h_{k,i}^r)^H \Phi_i G_i | w_{p,i}|^2
\]

\[
- 2\Re\{ \varpi_{k,i}^* (h_{k,i}^d)^H (h_{k,i}^r)^H \Phi_i G_i | w_{k,i} \} + |\varpi_{k,i}|^2 \sigma^2 + 1, \forall k, \forall i,
\]

where \( \varpi_{k,i} \in \mathbb{C}, \forall k \in \mathcal{K}, \forall i \in \mathcal{N} \), are auxiliary variables. By introducing weighting parameters \( \rho_{k,i}, \in \mathbb{R}^+ \), \( \forall k \in \mathcal{K}, \forall i \in \mathcal{N} \), problem (12) can be equivalently transformed into the following form [46]:

\[
\max_{\mathbf{w},\Theta,\rho,\varpi} \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{K} (\log_2 \rho_{k,i} - \rho_{k,i} \text{MSE}_{k,i} + 1) \quad (14a)
\]

s.t. \quad (12b)-(12e),

(14b)

where \( \rho \) and \( \varpi \) denote the sets of variables \( \rho_{k,i} \) and \( \varpi_{k,i} \), \( \forall k \in \mathcal{K}, \forall i \in \mathcal{N} \), respectively. Now, the newly formulated problem (14) is more tractable than the original problem after removing the complex fractional term (i.e., SINRs) from the \( \log(\cdot) \) function. In particular, problem (14) is a typical multi-variable/block problem, which can be solved using classical block coordinate descent (BCD) iterative algorithms [47]. In the following subsection, we will decompose problem (14) into four block optimizations and discuss the solution for each block in details.

C. Block Update

1) Weighting parameter \( \rho \): Fixing beamformers \( \mathbf{W}_i, \forall i \in \mathcal{N} \), the BPS matrix \( \Theta \), and auxiliary variables \( \varpi_{k,i}, \forall k \in \mathcal{K}, \forall i \in \mathcal{N} \), the sub-problem with respect to the weighting parameter \( \rho_{k,i} \) is given by

\[
\max_{\rho_{k,i}} \log_2 \rho_{k,i} - \rho_{k,i} \text{MSE}_{k,i}, \forall k, \forall i,
\]

and the optimal solution can be easily obtained by checking the first-order optimality condition of problem (15), i.e.,

\[
\rho_{k,i} = \text{MSE}_{k,i}^{-1} = 1 + \gamma_{k,i}, \forall k, \forall i. \quad (16)
\]

2) Auxiliary variable \( \varpi \): When the beamformers \( \mathbf{W}_i, \forall i \in \mathcal{N} \), the BPS matrix \( \Theta \), and weighting parameters \( \rho_{k,i}, \forall k \in \mathcal{K}, \forall i \in \mathcal{N} \), are all fixed, the sub-problem with respect to the auxiliary variable \( \varpi_{k,i} \) can be formulated as

\[
\min_{\varpi_{k,i}} \rho_{k,i} \text{MSE}_{k,i}, \forall k, \forall i, \quad (17)
\]

which is an unconstrained convex problem. Thus, problem (17) can be solved by setting the partial derivative of the objective in (17) with respect to \( \varpi_{k,i} \) to zero, which yields the optimal value of \( \varpi_{k,i} \) as

\[
\varpi_{k,i}^* = \frac{[(h_{k,i}^d)^H + (h_{k,i}^r)^H \Phi_i G_i | w_{k,i}]^2}{\sum_{p=1}^{K} [(h_{k,i}^d)^H + (h_{k,i}^r)^H \Phi_i G_i | w_{p,i}]^2 + \sigma^2}, \forall k, \forall i. \quad (18)
\]

3) Beamformer \( \mathbf{W} \): With weighting parameters \( \rho_{k,i} \), auxiliary variables \( \varpi_{k,i}, \forall k \in \mathcal{K}, \forall i \in \mathcal{N} \), and the BPS matrix \( \Theta \) given, the sub-problem with respect to the beamformer \( \mathbf{W}_i, \forall i \in \mathcal{N} \), can be written as

\[
\min_{\mathbf{w}} \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{K} \rho_{k,i} \left( \sum_{p=1}^{K} |(h_{k,i}^d)^H + (h_{k,i}^r)^H \Phi_i G_i | w_{p,i}|^2 - 2\Re\{ (h_{k,i}^d)^H (h_{k,i}^r)^H \Phi_i G_i | w_{k,i} \} \right)
\]

\[
to \min_{\mathbf{w}} \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{K} \left( \sum_{p=1}^{K} |h_{k,i}^d|^2 |w_{p,i}|^2 - 2\rho_{k,i} \Re\{ h_{k,i}^d w_{k,i} \} \right)
\]

s.t. \quad \sum_{i=1}^{N} \| \mathbf{w}_i \|_F^2 \leq P. \quad (19b)

For convenience, we define the equivalent channel \( h_{k,i} \equiv (\varpi_{k,i}^* (h_{k,i}^d)^H + (h_{k,i}^r)^H \Phi_i G_i))^H, \forall k \in \mathcal{K}, \forall i \in \mathcal{N} \). Then, problem (19) can be concisely rewritten as

\[
\min_{\mathbf{w}} \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{K} \left( \sum_{p=1}^{K} |h_{k,i}^d|^2 |w_{p,i}|^2 - 2\rho_{k,i} \Re\{ h_{k,i}^d w_{k,i} \} \right)
\]

s.t. \quad \sum_{i=1}^{N} \| \mathbf{w}_i \|_F^2 \leq P. \quad (20b)

Since the objective and constraint of problem (20) are all convex, this problem can be optimally solved using the classic Lagrange multiplier optimization. To be specific, by introducing a multiplier \( \mu \geq 0 \) corresponding to the power constraint (20b), problem (20) can be transformed into an unconstrained Lagrangian optimization:

\[
\min_{\mathbf{w}, \mu} \sum_{i=1}^{N} \sum_{k=1}^{K} \left( \sum_{p=1}^{K} |h_{k,i}^d|^2 |w_{p,i}|^2 - 2\rho_{k,i} \Re\{ h_{k,i}^d w_{k,i} \} \right) + \mu \left( \sum_{i=1}^{N} \| \mathbf{w}_i \|_F^2 - P \right) \quad (21a)
\]

\[
= \min_{\mathbf{w}, \mu} \sum_{i=1}^{N} \sum_{k=1}^{K} \left( \sum_{p=1}^{K} \rho_{k,i} |h_{k,i}^d|^2 |w_{p,i}|^2 - 2\rho_{k,i} \Re\{ h_{k,i}^d w_{k,i} \} \right) + \mu \left( \sum_{i=1}^{N} \| \mathbf{w}_i \|_F^2 - P \right) - \mu P. \quad (21b)
\]

Similar to the solution of the previous two blocks, this unconstrained convex problem can be solved by checking
the first-order optimality condition, which yields the optimal beamforming vector as

$$w^*_{k,i} = \left( \sum_{p=1}^{K} \rho_{k,i} h_{p,i} h_{p,i}^H + \mu I_{N_k} \right)^{-1} \rho_{k,i} h_{k,i}^H \forall k, \forall i,$$ (22)

where the optimal multiplier $\mu$ is associated with the total power constraint and can be easily determined using a bisection search over the set $S_{\mu} \triangleq \{ \mu \geq 0 | \sum_{i=1}^{N} ||w^*_i||_F^2 \leq P \}$.

4) BPS matrix $\Theta$: Given weighting parameters $\rho_{k,i}$, auxiliary variables $\varpi_{k,i}$, and beamformers $W_i, \forall i \in \mathcal{N}, \forall k \in \mathcal{K}$, the sub-problem with respect to the BPS matrix $\Theta$ can be presented as

$$\min_{\Theta} \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{K} \rho_{k,i} \left( \sum_{p=1}^{K} ||w^*_{k,i}||_F^2 \right)^2$$

$$- 2\Re\{\varpi_{k,i}(h_{k,i}^d H + (h_{k,i}^r)^H G_i)w_{k,i}^r\} \right)$$

s.t. (12b)-(12d).

By defining $\phi_i \triangleq [\phi_1, \ldots, \phi_M]^T$, $\varpi_{k,i} \triangleq (h_{k,i}^d)^H w_{k,i}$, and $v_{k,p,i} \triangleq (h_{k,i}^d)^H \text{diag}(G_i)w_{k,i}$, problem (23) can be concisely rearranged as

$$\min_{\Theta} \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{K} \rho_{k,i} \left( \sum_{p=1}^{K} ||v_{k,i}||_F^2 \right)^2$$

$$- 2\Re\{\varpi_{k,i}(h_{k,i}^d H + (h_{k,i}^r)^H G_i)w_{k,i}^r\} \right)$$

s.t. (12b)-(12d),

where

$$A_i \triangleq \sum_{k=1}^{K} \rho_{k,i} ||v_{k,i}||_F^2 \sum_{p=1}^{K} v_{k,p,i}^H v_{k,p,i}^H, \forall i,$$ (25a)

$$b_i \triangleq \sum_{k=1}^{K} \rho_{k,i} \left( \varpi_{k,i} v_{k,i}^r - ||w_{k,i}||_F^2 \sum_{p=1}^{K} v_{k,p,i}^H h_{k,p,i}^d \right), \forall i.$$ (25b)

Problem (24) is still difficult to solve since the BPS matrix $\Theta$ to be optimized is embedded into a summation of $N$ complicated functions. To simplify the design, one feasible solution is to decompose the joint optimization of the entire matrix $\Theta$ into sub-problems, each of which deals with only one entry of $\Theta$ while fixing others. This alternative update of $\Theta$ is conducted iteratively until the objective value converges.

Towards this end, we first split the objective (24b) as

$$\frac{1}{N} \sum_{i=1}^{N} (\phi_i^H A_i \phi_i - 2\Re\{\phi_i^H b_i\})$$

$$= \frac{1}{N} \sum_{i=1}^{N} \sum_{m=1}^{M} \left( \sum_{n=1}^{M} A_i(m,n)\phi_i^* m, m - 2\Re\{\phi_i^* m, b_i(m)\} \right)$$

$$= \frac{1}{N} \sum_{i=1}^{N} \sum_{m=1}^{M} \left( \sum_{n \neq m} (A_i(m,n)\phi_i^* m, m + A_i(n,m)\phi_i^* m, n) \right)$$

$$+ A_i(m,m)|\phi_i^* m, m|^2 - 2\Re\{\phi_i^* m, b_i(m)\}$$

and (a) holds since $A_i = A_i^H, \forall i \in \mathcal{N}$. Then, the sub-problem with respect to the $m$-th BPS element $\theta_m$ while fixing other elements can be formulated as

$$\min_{\theta_m} \sum_{i=1}^{N} \left( 2\Re\{\sum_{n \neq m} (A_i(m,n)\phi_i^* m, n - b_i(m)) \phi_i^* m, m \right)$$

$$+ A_i(m,m)|\phi_i^* m, m|^2 \right)$$

s.t. (12b)-(12d). (27a)

We further define $\chi_{i,m} \triangleq \sum_{n \neq m} A_i(m,n)\phi_i^* n - b_i(m), \forall i \in \mathcal{N}, \forall m \in \mathcal{M}$, and substitute the constraints (12b), (12c) into the objective (27a). Then, sub-problem (27) can be reformulated as

$$\min_{\theta_m} \sum_{i=1}^{N} \left( 2\Re\{\sum_{n \neq m} (A_i(m,n)\phi_i^* m, n - b_i(m)) \phi_i^* m, m \right)$$

$$+ A_i(m,m)|\phi_i^* m, m|^2 \right)$$

s.t. $\theta_m \in [-\pi, \pi].$ (28a)

The objective of problem (28) is a summation of $N$ complicated functions involving both trigonometric and quadratic terms, which is difficult to deal with. The computational complexity will be quite high when the numbers of IRS elements and/or subcarriers become large, which is the case for practical communication systems. To reduce the calculation complexity, we propose to further divide the whole bandwidth into $N_s$ sub-bands, each of which comprises $S = N/N_s$ subcarriers. By approximating each sub-band as a “narrowband” channel which has identical reflection coefficient configuration, problem (28) can be further simplified as the optimization of a summation of much smaller number of functions, i.e.

$$\min_{\theta_m} g(\theta_m)$$

s.t. $\theta_m \in [-\pi, \pi].$ (29a)

where the objective $g(\theta_m)$ is defined as

$$g(\theta_m) = \sum_{i=1}^{N_s} \left( 2\Re\{\sum_{n \neq m} (A_i(m,n)\phi_i^* m, n - b_i(m)) \phi_i^* m, m \right)$$

$$+ A_i(m,m)|\phi_i^* m, m|^2 \right)$$

$$+ \chi_{i,m} F^2(\theta_m, f_i)$$

(30)
with \( f_{s,i} \triangleq f_c + (i - \frac{\pi}{\sqrt{2}}) \frac{\beta}{N_s}, \chi_{i,m} \triangleq \frac{1}{S} \sum_{j=1}^{S} (i-j)A_{s+j}(m,m), \forall i = 1, \ldots, N_s. \)

Unfortunately, the above problem is still difficult to solve since we cannot easily calculate the derivative of the objective and obtain the close-form solution. To tackle this difficulty, we first try to explore the characteristic of the objective (30) with the aid of numerical experiments. After numerous simulations (more than 5000 times), we find that objective (30) has only one minimum point within the range \([\pi, \pi]\).

More concretely, objective (30) behaves like a kind of smooth double-peak-trough curve, whose minimum is achieved either at the minimum point or at two border points. Some of examples are shown in Fig. 4. Motivated by this finding, we propose a three-phase one-dimensional search method to efficiently find optimal solutions, which is summarized as follows:

**Phase 1:** Narrow the search range by a success-failure method: Initialize a starting point \( \theta_0 \) as well as a step size \( h > 0 \). If \( g(\theta_0 + h) < g(\theta_0) \), enlarge the step size and search forward until the objective rises; otherwise, search reversely until the objective rises.

**Phase 2:** Find the minimum point \( \bar{\theta} \) by a golden section method: Successively section the search range which includes the minimum point in the golden ratio until reaching a predefined threshold.

**Phase 3:** Determine the minimum value: Compare the values of \( g(\bar{\theta}), g(-\pi) \), and \( g(\pi) \) to determine the minimal value as well as its corresponding phase shift.

The detail of the three-phase search algorithm is summarized in Algorithm 1. Furthermore, red points marked in Fig. 4 are search results, which illustrate the accuracy of the proposed algorithm.

In realistic applications, the IRS is usually realized by finite- or even low-resolution phase shifters to effectively reduce the hardware consumption. Therefore, we also consider the case that the BPS \( \theta_m \) for IRS has discrete phases controlled by \( b \) bits, which are uniformly spaced within the range \([\pi, \pi]\), i.e.

\[
\theta_m \in \mathcal{F} \triangleq \{2\pi \frac{m}{2^b} | i = 0, 1, \ldots, 2^b\}, \forall m.
\]

In this case, the IRS design sub-problem is given by

\[
\begin{aligned}
\min_{\theta_m} \sum_{i=1}^{N} (2|\chi_{i,m}|F(\theta_m, f_i)\cos(\angle \chi_{i,m} - G(\theta_m, f_i))) \\
+ A_i(m,m)F^2(\theta_m, f_i))
\end{aligned}
\]

s.t. \( \theta_m \in \mathcal{F}. \) (32a)

Similarly, we simplify this problem by dividing the whole bandwidth into several sub-bands, which yields the following problem:

\[
\begin{aligned}
\min_{\theta_m} g(\theta_m) \\
\text{s.t.} \quad \theta_m \in \mathcal{F}.
\end{aligned}
\]

5) **Summary:** Having approaches to solve the above four sub-problems with respect to \( p_{k,i}, \varpi_{k,i}, w_{k,i}, \forall k \in K, \forall i \in N, \) and \( \Theta, \) the overall procedure for the joint beamformer and IRS design is finally straightforward. Given appropriate initial values of \( w_{k,i}, \forall k \in K, \forall i \in N, \) and \( \Theta, \) we iteratively update the above four blocks alternatively order until convergence. The proposed joint beamformer and IRS design algorithm is therefore summarized in Algorithm 2.

**D. Complexity Analysis**

In this subsection, we provide an analysis of the complexity for the proposed joint beamformer and IRS design algorithm. In each iteration, updating the weighting parameter \( p \) has a complexity of \( \mathcal{O}(NK^2N_iM^2) \) approximately; updating the auxiliary variable \( w \) requires \( \mathcal{O}(NK(K + 1)N_iM^2) \) operations; updating beamformer \( W \) requires about \( \mathcal{O}(IN_iNK^2(3M^2 + N_i^2)) \) operations, where the parameter \( I_1 \) denotes the iterations of bisection search. Finally, the order of complexity for updating BPS matrix \( \Theta \) for continuous phases is about \( \mathcal{O}((5MNI_i + M^3)NK^2 + I_2N_iM(I_3 + I_4)) \), where \( I_3 \) and \( I_4 \) denotes the iterations for success-failure
Algorithm 1 Three-Phase One-Dimensional Search

Input: $f_{\lambda,i}, \lambda_{i,m} \in \mathcal{N}, \forall i \in \mathcal{N}$.
Output: $\theta_m^\ast$.

1: **Phase 1: Success-failure method**
   2: Initialize $\theta_0, h > 0. \theta_1 = \theta_0, \theta_2 = \theta_1 + h.$
   3: if $g(\theta_2) < g(\theta_1)$ then
   4: $\theta_3 = \theta_2 + h.$
   5: if $g(\theta_2) \leq g(\theta_3)$ then
   6: Obtain the narrowed range $[\theta_1, \theta_3]$ as $\theta_1 = \min\{\theta_1, \theta_3\}, \theta_3 = \max\{\theta_1, \theta_3\}$, and stop.
   7: else
   8: $h = 2h, \theta_1 = \theta_2, \theta_2 = \theta_3, \theta_3 = \theta_2 + h.$
   9: Goto step 5.
10: **end if**
11: else
12: $h = -h, \theta_3 = \theta_1, \theta_1 = \theta_2, \theta_2 = \theta_3, \theta_3 = \theta_2 + h.$
13: Goto step 5.
14: **end if**
15: **Phase 2: Golden section method**
16: Set $\overline{\theta} = \theta_1 + 0.382(\theta_1 - \theta_1), \overline{\theta} = \theta_1 + 0.618(\theta_1 - \theta_1), \varepsilon$.
17: while $\theta_2 - \theta_1 > \varepsilon$ do
18: if $g(\bar{\theta}) \leq g(\bar{\theta})$ then
19: $\theta_3 = \bar{\theta}, \overline{\theta} = \bar{\theta}, \bar{\theta} = \theta_1 + 0.382(\theta_1 - \theta_1).$
20: else
21: $\theta_1 = \bar{\theta}, \overline{\theta} = \bar{\theta}, \bar{\theta} = \theta_1 + 0.618(\theta_1 - \theta_1).$
22: **end if**
23: end while
24: Obtain $\theta_m^\ast = (\theta_1 + \theta_2)/2.$
25: **Phase 3: Determine $\theta_m^\ast$**
26: if $g(\pi) \leq g(\theta_m^\ast)$ and $g(\pi) \leq g(\pi)$ then
27: $\theta_m^\ast = \pi.$
28: else if $g(\pi) \leq g(\theta_m^\ast)$ and $g(\pi) \leq g(\pi)$ then
29: $\theta_m^\ast = \pi.$
30: **end if**
31: Return $\theta_m^\ast.$

Algorithm 2 Joint Transmit Beamformer and IRS Reflection Design

Input: $h_{d,k,i}, h_{f,i}, G_{k}, \forall k \in \mathcal{K}, \forall i \in \mathcal{N}, P, B.$
Output: $w_{k,i}^\ast, \forall k \in \mathcal{K}, \forall i \in \mathcal{N}, \Theta^\ast.$

1: Initialize $w_{k,i}, \forall k \in \mathcal{K}, \forall i \in \mathcal{N}, \Theta.$
2: while no convergence of objective (14a) do
3: Update $\rho_{k,i}, \forall k \in \mathcal{K}, \forall i \in \mathcal{N}$ by (16).
4: Update $w_{k,i}, \forall k \in \mathcal{K}, \forall i \in \mathcal{N}$ by (18).
5: Update $w_{k,i}, \forall k \in \mathcal{K}, \forall i \in \mathcal{N}$ by (22).
6: while no convergence of $\Theta$ do
7: for $m = 1 : M$ do
8: Update $\theta_m$ by Algorithm 1 for continuous phases or by an exhaustive search for low-resolution phases.
9: **end for**
10: **end while**
11: **end while**
12: Return $w_{k,i}^\ast, \forall k \in \mathcal{K}, \forall i \in \mathcal{N}, \Theta^\ast.$

method and golden section method, respectively, and that for discrete phases is $O((5MN_t + M^3)NK^2 + I_2N_sM^2)$, where parameters $I_2$ and $I_3$ denote the numbers of iterations for calculating $\Theta$. Therefore, the total complexity of the proposed algorithm is given by

$$C_c = O(I_c(NK^2N_tM_2^2 + NK(K + 1)N_tM_2$$

$$+ I_1NN_tK(3M^2 + N_t^2) + (5MN_c + M^3)NK^2$$

$$+ I_2N_sM(I_3 + I_4)))$$

$$\approx O(I_c(NK^2M_3^3 + 3I_1NN_tKMK^2 + I_2N_sM(I_3 + I_4)))$$

$$C_d = O(I_d(NK^2N_tM_2^2 + NK(K + 1)N_tM_2$$

$$+ I_1NN_tK(3M^2 + N_t^2) + (5MN_t + M^3)NK^2$$

$$+ I_2N_sM^2))$$

$$\approx O(I_d(NK^2M_3^3 + 3I_1NN_tKMK^2 + I_2N_sMLN_sM^2))$$

where (a) holds under assumptions $M \gg N_t, M \gg K$. Parameters $I_c$ (for continuous phases) and $I_d$ (for discrete phases) are the numbers of iterations for Algorithm 2. Simulation results in the next section show that, under different settings, the proposed algorithm for both continuous and discrete scenarios can converge within limited iterations, which demonstrates the efficiency of the proposed algorithm.

V. SIMULATION RESULTS
A. Simulation Settings

In this section, we present simulation results to demonstrate the performance of the IRS-enhanced wideband MU-MISO-OFDM system by showing the average sum-rate of the proposed joint beamformer and IRS design. In the considered IRS-enhanced MU-MISO-OFDM system, we assume the number of subcarriers is $N = 64$. The number of taps is set as $D = 16$ with half non-zero taps modeled as circularly symmetric complex Gaussian (CSCG) random values. The CP length is set to be $N_{CP} = 16$. The carrier frequency and bandwidth is given by $f_c = 2.4$GHz and $B = 100$MHz, respectively. The signal attenuation is set as $\varphi_0 = 30$ dB at a reference distance 1 m for all channels. The path loss exponent of the BS-IRS channel, the IRS-user channel, and the BS-user channel is set as $\varepsilon_{BI} = 2.8, \varepsilon_{BU} = 2.5$, and $\varepsilon_{BU} = 3.7$, respectively. The noise power at each user is set as $\sigma^2 = -70$ dBm.

In the following simulation results, we assume a three-dimensional (3D) coordinate system is considered as shown in Fig. 5, where a uniform linear array (ULA) with antenna spacing $d_{A} = 0.3$ m at the BS and a uniform planar array (UPA) with element-spacing $d_{l} = 0.03$ m at the IRS and are located in y-z plane and x-y plane, respectively. The distance between the reference antenna of the BS and the reference element of the IRS is given by $d_{BI}$. $K$ users are randomly located in x-z plane with the same distance $d_{IU} = 3$ m as well as random phase $\varphi_k$ between the reference element of the IRS and the $k$-th user. Based on the relative position given in Fig. 5, the distances between the $(p, q)$-th IRS element and the $k$-th user $d_{IU}^{p,q,k},$ the $n$-th antenna and the $k$-th user $a_{BI}^{n,k},$ as
well as the $n$-th antenna and the $(p, q)$-th IRS element $d_{BI}^{n,p,q}$, are given by
\[
\begin{align*}
d_{IU}^{n,p,q} &= \sqrt{(pd_1 - d_{IU} \cos \varphi_k)^2 + q^2d_1^2 + d_{IU}^2 \sin^2 \varphi_k}, \\
d_{BU}^{n,k} &= \sqrt{(d_{BI} - d_{IU} \sin \varphi_k)^2 + n^2d_A^2 + d_{BI}^2 \cos^2 \varphi_k}, \\
d_{BU}^{n,p,q} &= \sqrt{(qd_1 - nd_A)^2 + p^2d_1^2 + d_{BI}^2}, \\
\forall n &\in \mathcal{N}, \forall p, q = 1, \ldots, M, \forall k \in \mathcal{K}.
\end{align*}
\] (35)

Then the fading component for the BS-IRS link, the BS-User link, and the IRS-User link is given by
\[
\begin{align*}
\xi_{BI}^{n,p,q} &= \sqrt{\xi_{IU}}^{n,p,q} - \varepsilon_{BI}, \\
\xi_{IU}^{n,k} &= \sqrt{\xi_{BU}}^{n,k} - \varepsilon_{IU}, \\
\forall n &\in \mathcal{N}, \forall p, q, \forall k.
\end{align*}
\] (36)

Thus, the channels for three links are given by
\[
\begin{align*}
\hat{h}_{k,i}(m) &= \xi_{IU}^{n,p,q}\hat{h}_{k,i}^{n,k}(m), \\
\hat{G}_i(m, n) &= \xi_{BI}^{n,p,q}\hat{G}_i^{n,k}(m, n), \\
\forall n &\in \mathcal{N}, \forall p, q, \forall k, \forall i \in \mathcal{N}, \forall m = (p - 1)M + q.
\end{align*}
\] (37)

**B. System Performance**

We start with presenting the convergence of the proposed joint beamformer and IRS design by plotting the average sum-rate versus the number of iterations in Fig. 6. Simulation results illustrate that the proposed algorithm can converge within 30 iterations when using continuous phase shifters and within 20 iterations when using low-resolution phase shifters to realize the IRS. When the numbers of antennas and IRS elements increase, the proposed algorithm can still converge within limited iterations. Next in Fig. 7, we plot the average sum-rate as a function of the resolution $b$ (LowRes) of each IRS element. Fig. 7 shows that $b = 4$ is a sufficiently precise resolution level and the performance improvement is marginal when $b$ is larger than 4. Moreover, considering the both results of the convergence speed as illustrated in Fig. 6 and the influence of resolution $b$ as shown in Fig. 7, it is more practical and efficient to realize the IRS using low-resolution phase shifters in realistic systems.

Fig. 8 shows the average sum-rate among all subcarriers versus the transmit power $P$ with the proposed algorithm for
the cases of using continuous and low-resolution (i.e. $b = 1, 2$-bit) phase shifters with different settings (e.g. number of antennas and/or IRS elements). For fair comparison, we also plot the average sum-rate for the following schemes:

- The average sum-rate designed by our proposed simplified IRS model in this paper and testified by the same IRS model, which is marked as “w/ IRS, Proposed”.
- The average sum-rate designed by the ideal IRS model in [38] but testified by the proposed IRS model, which is marked as “w/ IRS, Ideal”.
- Lower bound I: The system with an IRS whose BPSs are randomly selected within the range $[-\pi, \pi]$ and calculated by the proposed IRS model, which is marked as “w/ IRS, Random”.
- Lower bound II: The system with direct link only, which is marked as “w/o IRS”.

It can be observed from Fig. 8 that the proposed algorithm can achieve significantly better performance compared with two lower bounds for all transmit power ranges, which illustrates the advantages of employing IRS in wireless communications. Moreover, the proposed algorithm also outperform the “w/ IRS, Ideal” scheme, which demonstrates the importance of precisely modeling the reflection characteristics of the practical IRS.

To illustrate the advantage of employing IRS in enhancing wideband wireless communications, in Fig. 9 we plot the average sum-rate versus different numbers of IRS elements $M$. A similar conclusion can be drawn from Fig. 9 that the proposed algorithm can always achieve better performance compared with its competitors. Moreover, with the number of IRS elements growing, the performance gap between the “w/ IRS” scheme and the “w/o IRS” one is becoming larger.

Finally, the average sum-rate as a function of the number of transmit antennas is illustrated in Fig. 10. A similar conclusion can be obtained from the above simulation results. More
importantly, the performance gap between the proposed “w/ IRS. Proposed” scheme and the “w/ IRS. Ideal” scheme becomes smaller with the increasing number of the transmit antennas. This trend can be explained as follows: when the number of transmit antennas grows, the channel gain achieved by the direct link will gradually dominate the effective gain of the entire channels, which, to some extent, weakens the influence of the IRS.

VI. CONCLUSIONS

In this paper, we first simplified the practical IRS model and validated the accuracy of the proposed model based on numerical simulations. With the simplified practical model, we considered the problem of joint beamformer and IRS design with both continuous and low-resolution phase shifters to maximize the average sum-rate of a wideband MU-MISO-OFDM system. We proposed a sub-optimal iterative algorithm with the aid of the equivalence between sum-rate maximization and MSE minimization. Simulation results demonstrated the significance of modeling the imperfect response characteristics of IRS reflecting elements and its associated configuration design. With the tremendous difference between the ideal reflection model and the practical reflection model, there are many issues worthy to be studied and investigated, such as IRS deployment, resource allocation, user scheduling, etc.

REFERENCES

[1] A. L. Swindlehurst, E. Ayanoglu, P. Heydari, and F. Capolino, “Millimeter-wave massive MIMO: The next wireless revolution?” IEEE Commun. Mag., vol. 52, no. 9, pp. 56-62, Sept. 2014.

[2] S. Zhang, Q. Wu, S. Xu, and G. Y. Li, “Fundamental green tradeoffs: Progress, challenges, and impacts on 5G networks,” IEEE Commun. Surveys & Tutorials, vol. 19, no. 1, pp. 33-56, First Quarter 2017.

[3] Q. Wu, G. Y. Li, W. Chen, D. W. K. Ng, and R. Schober, “An overview of sustainable green 5G networks,” IEEE Wireless Commun., vol. 24, no. 4, pp. 72-80, Aug. 2017.

[4] Q. Wu and R. Zhang, “Towards smart and reconfigurable environment: Intelligent reflecting surface aided wireless network,” IEEE Commun. Mag., vol. 58, no. 1, pp. 106-112, Jan. 2020.

[5] C. Liaskos, S. Nie, A. Tsolliarioud, A. Pitsillides, S. Ioannidis, and I. Akylidiz, “A new wireless communication paradigm through software-controlled metasurfaces,” IEEE Commun. Mag., vol. 56, no. 9, pp. 162-169, Sep. 2018.

[6] M. Di Renzo, A. Zappone, M. Debbah, M.-S. Alouini, C. Yuen, J. de Rosny, and S. Tretiyakov, “Smart radio environments empowered by reconfigurable intelligent surfaces: How it works, state of research, and road ahead,” June 2020. [Online]. Available: https://arxiv.org/abs/2004.09352

[7] E. Bjornson, O. Özdogan, and E. G. Larsson, “Intelligent reflecting surface vs. decode-and-forward: How large scales are needed to beat relaying?” IEEE Wireless Commun. Lett., vol. 9, no. 2, pp. 244-248, Feb. 2019.

[8] S. Gong, et al., “Towards smart radio environment for wireless communications via intelligent reflecting surface: A comprehensive survey,” IEEE Commun. Surveys & Tutorials, to appear.

[9] K-K. Wong, K-F. Tong, Z. Chu, and Y. Zhang, “A vision to smart radio environment: Surface wave communication superhighways,” May 2020. [Online]. Available: https://arxiv.org/abs/2005.14062

[10] Q. Wu, S. Zhang, B. Zheng, C. You, and R. Zhang, “Intelligent reflecting surface aided wireless communications: A tutorial,” July 2020. [Online]. Available: https://arxiv.org/abs/2007.02759

[11] C. Huang, S. Hu, G. C. Alexandropoulos, A. Zappone, C. Yuen, R. Zhang, M. D. Renzo, M. Debbah, “Holographic MIMO surfaces for 6G wireless networks: Opportunities, challenges, and trends,” IEEE Wireless Commun. Magazine, to appear.

[12] Y. Liu, X. Liu, X. Mu, T. Hou, J. Xu, Z. Qin, M. D. Renzo, and N. Al-Dhahir, “Reconfigurable intelligent surfaces: Principles and opportunities,” July 2020. [Online]. Available: https://arxiv.org/abs/2007.03435

[13] X. Yu, D. Xu, and R. Scholhar, “MISO wireless communication systems via intelligent reflecting surface,” in Proc. IEEE Int. Conf. Commun. China (ICCC), Changchun, China, Dec. 2019.

[14] Q. Wu and R. Zhang, “Intelligent reflecting surface enhanced wireless network via joint active and passive beamforming,” IEEE Trans. Wireless Commun., vol. 18, no. 11, pp. 5394-5409, Nov. 2019.

[15] J. Han, W. Tang, S. Jin, G. Wu, and X. Ma, “Large intelligent surface-assisted wireless communication exploiting statistical CSI,” IEEE Trans. Veh. Technol., vol. 68, no. 8, pp. 8238-8242, Aug. 2019.

[16] C. Huang, A. Zappone, G. C. Alexandropoulos, M. Debbah, and C. Yuen, “Reconfigurable intelligent surfaces for energy efficiency in wireless communication,” IEEE Trans. Wireless Commun., to appear.

[17] C. Huang, R. Mo, and C. Yuen, “Reconfigurable intelligent surface assisted multiuser MISO systems exploiting deep reinforcement learning,” IEEE J. Sel. Areas Commun. (JSAC), to appear.

[18] H. Guo, Y.-C. Liang, J. Chen, and E. G. Larsson, “Weighted sum-rate optimization for intelligent reflecting surface enhanced wireless networks,” in Proc. IEEE Global Commun. Conf. (GLOBECOM), Waikoloa, HI, Dec. 2019.

[19] J. Zhao, “Optimizations with intelligent reflecting surfaces (IRSs) in 6G wireless networks: Power control, quality of service, max-min fair beamforming forunicast, broadcast, and multicast with multi-antenna mobile users and multiple IRSs,” Aug. 2019. [Online]. Available: https://arXiv.org/abs/1908.03965

[20] Y. Liu, J. Zhao, M. Li, and Q. Wu, “Intelligent reflecting surface aided MISO uplink communication network: Feasibility and SINR optimization,” July 2020. [Online]. Available: https://arxiv.org/abs/2007.01482

[21] Y. Di, H. Zhang, L. Song, Y. Li, Z. Han, and H. V. Poor, “Hybrid beamforming for reconfigurable intelligent surface based multi-user communications: Achievable rates with limited discrete phase shifts,” IEEE J. Sel. Areas Commun., to appear.

[22] Q. Wu and R. Zhang, “Beamforming optimization for wireless network aided by intelligent reflecting surface reflecting surface with discrete phase shifts,” IEEE Trans. Commun., vol. 68, no. 3, pp. 1838-1851, Mar. 2020.

[23] J. Xu, W. Xu, and A. L. Swindlehurst, “Discrete phase shift design for practical large intelligent communication,” in Proc. IEEE Pacific Rim Conf. on Commun., Computers and Signal Process. (PACRIM), Victoria, Canada, Aug. 2019.

[24] J. He, K. Yu, and Y. Shi, “Coordinated passive beamforming for distributed intelligent reflecting surfaces network,” Feb. 2020. [Online]. Available: https://arxiv.org/abs/2002.05915

[25] Z. Li, M. Hua, Q. Wang, and Q. Song, “Weighted sum-rate maximization for multi-IRS aided cooperative transmission,” IEEE Wireless Commun. Lett., to appear.

[26] D-W. Yue, H. H. Nguyen, and Y. Sun “MmWave doubly-massive-MIMO communications enhanced with an intelligent reflecting surface”, Mar. 2020. [Online]. Available: https://arxiv.org/abs/2003.00282

[27] M. Cui, G. Zhang, and R. Zhang, “Secure wireless communication via intelligent reflecting surface,” IEEE Wireless Commun. Lett., vol. 8, no. 6, pp. 1410-1414, Oct. 2019.

[28] X. Guan, Q. Wu, and R. Zhang, “Intelligent reflecting surface assisted secrecy communication: Is artificial noise helpful or not?” IEEE Wireless Commun. Lett., to appear.

[29] D. Xu, X. Yu, Y. Sun, D. W. K. Ng, and R. Schober, “Resource allocation for secure IRS-assisted multiuser MISO systems,” in Proc. IEEE Global Commun. Conf. (GLOBECOM), Waikoloa, HI, Dec. 2019.

[30] L. Zhang, C. Pan, Y. Wang, H. Ren, K. Wang, and A. Nallanathan, “Robust beamforming design for intelligent reflecting surface aided cognitive radio systems with imperfect cascaded CSI,” Apr. 2020. [Online]. Available: https://arxiv.org/abs/2004.04095

[31] D. Xu, X. Yu, and R. Schober, “Resource allocation for intelligent reflecting surface-assisted cognitive radio networks,” Jan. 2020. [Online]. Available: https://arxiv.org/abs/2001.11729

[32] X. Guan, Q. Wu, and R. Zhang, “Joint power control and passive beamforming in IRS-assisted spectrum sharing,” IEEE Commun. Lett., to appear.

[33] A. Khaled and E. Basar, “Reconfigurable intelligent surface empowered MIMO systems,” Apr. 2020. [Online]. Available: https://arxiv.org/abs/2004.02238

[34] E. Basar, “Reconfigurable intelligent surface-based index modulation: A new beyond MIMO paradigm for 6G,” IEEE Trans. Commun., to appear.

[35] Y. Yang, B. Zheng, S. Zhang, and R. Zhang, “Intelligent reflecting surface meets OFDM: Protocol design and rate maximization,” Nov. 2019. [Online]. Available: https://arxiv.org/abs/1906.09956
[36] B. Zheng and R. Zhang, “Intelligent reflecting surface-enhanced OFDM: Channel estimation and reflection optimization,” IEEE Wireless Commun. Lett., vol. 9, no. 4, pp. 518-522, Apr. 2020.

[37] T. Bai, C. Pan, H. Ren, Y. Deng, M. Elkashlan, and A. Nallanathan, “Resource allocation for intelligent reflecting surface aided wireless powered mobile edge computing in OFDM systems,” Mar. 2020. [Online]. Available: https://arxiv.org/abs/2003.05511

[38] H. Li, R. Liu, M. Li, Q. Liu, and X. Li, “IRS-enhanced wideband MU-MISO-OFDM communication systems,” in Proc. IEEE Wireless Commun. Networking Conf. (WCNC), Seoul, South Korea, May 2020.

[39] H. Rajagopalan and Y. Rahmat-Samii, “Loss quantification for microstrip reflectarray: Issue of high fields and currents,” in Proc. IEEE Antennas and Propag. Society Int. Symposium, San Diego, CA, July 2008.

[40] W. Tang et al., “MIMO transmission through reconfigurable intelligent surface: System design, analysis, and implementation,” Dec. 2019, [Online]. Available: https://arxiv.org/abs/1912.09955

[41] S. Abeywickrama, R. Zhang, Q. Wu, and C. Yuen, “Intelligent reflecting surface: Practical phase shift model and beamforming optimization,” IEEE Trans. Commun., to appear.

[42] W. Cai, H. Li, M. Li, and Q. Liu, “Practical modeling and beamforming for intelligent reflecting surface aided wideband systems,” IEEE Commun. Lett., vol. 24, no. 7, pp. 1568-1571, July 2020.

[43] A. Taha, M. Alrabeiah, and A. Alkhateeb, “Enabling large intelligent surfaces with compressive sensing and deep learning,” Apr. 2019. [Online]. Available: https://arxiv.org/abs/1904.10136

[44] B. Zheng, C. You, and R. Zhang, “Intelligent reflecting surface assisted multi-user OFDMA: Channel estimation and training design,” Mar. 2020. [Online]. Available: https://arxiv.org/abs/2003.00648

[45] Y. Kwon, J. Chung, and Y. Sung, “Hybrid beamformer design for mmWave wideband multi-user MIMO-OFDM systems,” in Proc. IEEE Int. Workshop on Signal Process. Advances in Wireless Commun. (SPAWC), Sapporo, Japan, July 2017.

[46] Q. Shi, M. Razaviyayn, Z. Q. Luo, and C. He, “An iteratively weighted MMSE approach to distributed sum-utility maximization for a MIMO interfering broadcast channel,” IEEE Trans. Signal Process., vol. 59, no. 9, pp. 4331-4340, Sept. 2011.

[47] D. Bertsekas, Nonlinear Programming, 2nd ed. Belmont, MA, USA: Athena Scientific, 1999.