Evolution of stellar orbits around merging massive black hole binary

Liu, Bin; Lai, Dong

Published in:
Monthly Notices of the Royal Astronomical Society

DOI:
10.1093/mnras/stac1200

Publication date:
2022

Document version
Publisher's PDF, also known as Version of record

Document license:
CC BY

Citation for published version (APA):
Liu, B., & Lai, D. (2022). Evolution of stellar orbits around merging massive black hole binary. Monthly Notices of the Royal Astronomical Society, 513(3), 4657-4668. https://doi.org/10.1093/mnras/stac1200
Evolution of stellar orbits around merging massive black hole binary

Bin Liu\textsuperscript{1,*} and Dong Lai\textsuperscript{2}

\textsuperscript{1}Niels Bohr International Academy, Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen, Denmark
\textsuperscript{2}Cornell Center for Astrophysics and Planetary Science, Department of Astronomy, Cornell University, Ithaca, NY 14853, USA

Received 2022 April 26. Accepted 2022 April 4; in original form 2022 January 24

ABSTRACT

We study the long-term orbital evolution of stars around a merging massive or supermassive black hole binary (BHB), taking into account the general relativistic effect induced by the black hole (BH) spin. When the BH spin is significant compared to and misaligned with the binary orbital angular momentum, the orbital axis ($\hat{I}$) of the circumbinary star can undergo significant evolution during the binary orbital decay driven by gravitational radiation. Including the spin effect of the primary (more massive) BH, we find that starting from nearly coplanar orbital orientations, the orbital axes $\hat{I}$ of circumbinary stars preferentially evolve towards the spin direction after the merger of the BHB, regardless of the initial BH spin orientation. Such alignment phenomenon, i.e. small final misalignment angle between $\hat{I}$ and the spin axis of the remnant BH $\hat{S}$, can be understood analytically using the principle of adiabatic invariance. For the BHBs with extremely mass ratio ($m_2/m_1 \lesssim 0.01$), $\hat{I}$ may experience more complicated evolution as adiabatic invariance breaks down, but the trend of alignment still works reasonably well when the initial binary spin–orbit angle is relatively small. Our result suggests that the correlation between the orientations of stellar orbits and the spin axis of the central BH could provide a potential signature of the merger history of the massive BH.

Key words: black hole physics – gravitational waves – binaries: general – stars: black holes – stars: kinematics and dynamics.

1 INTRODUCTION

Massive black hole binaries (BHBs), with orbital separations $\lesssim 10$ pc, are natural products of galaxy mergers (e.g. Begelman et al. 1980; Milosavljević & Merritt 2001; Escala et al. 2005; Milosavljević & Phinney 2005; Dotti et al. 2007; Mayer et al. 2007; Cuadra et al. 2009; Chapon et al. 2013; Fragione 2022). Significant observational efforts have been devoted to searching for such binaries, and a number of candidate systems have been detected using various techniques (e.g. Sillanpää et al. 1988; Komossa et al. 2003, 2008; Rodriguez et al. 2006; Bianchi et al. 2008; Bogdanović et al. 2009; Boroson & Lauer 2009; Comerford et al. 2009, 2018; Dotti et al. 2009; Green et al. 2010; Deane et al. 2014; Liu et al. 2014; Bansal et al. 2017; De Rosa et al. 2019). These massive BHBs are likely surrounded by stars (or compact objects) associated with the merging galaxies. Alternatively, the stars could form in a circumbinary disc or be captured by the disc from a nuclear star cluster (e.g. Tagawa et al. 2020, 2021). For sufficiently small orbit separations, the massive binary black holes (BHs) experience orbital decay and eventually merge, producing low-frequency gravitational waves (GWs). How would the orbits of the circumbinary stars change?

The secular gravitational interaction between a central binary and a surrounding object dictates the long-term evolution of the system. For a hierarchical triple (with the semimajor axis $a_{\text{out}}$ of the outer orbit much larger than that of the inner orbit $a_{\text{in}}$), the secular evolution equations for arbitrary orbital eccentricities and orientations can be derived using expansion in $a_{\text{in}}/a_{\text{out}}$ [see Ford et al. (2000)] for the equations to the octupole order; more compact equations in the vector form can be found in Liu et al. (2015b) and Petrovich (2015). Such systems may exhibit excitations/oscillations in eccentricities and inclinations in both the inner and outer orbits (e.g. the well-known Lidov–Kozai effect; von Zeipel 1910; Kozai 1962; Lidov 1962; Naoz 2016). In general, the evolution can be highly irregular when the octupole effects are significant. If the outer body has a negligible mass compared to the inner binary, the dynamics of the outer body becomes simpler and analytical results can be obtained (e.g. Farago & Laskar 2010; Li et al. 2014). In particular, the inner eccentric binary can drive significant inclination evolution of the outer orbit (e.g. Zanazzi & Lai 2018) and produce orbit flipping from extreme eccentricity excitation (e.g. Naoz et al. 2017). Vinson & Chiang (2018) carried out a systematic study of the (secular) restricted three-body problem by expanding the potential to the hexadecapolar order (see also Gallardo et al. 2012) and identified various secular resonances.

In this paper, we study the secular evolution of stellar orbits around an inner massive BHB undergoing GW-induced orbital decay. We are particularly interested in the case of inner massive BHBs with relatively small mass ratios, such that the spin of the primary BH may play an important role. To the Newtonian leading order, the (inner) massive BHB makes the (outer) stellar orbit precess around the inner binary. However, when the BH spin is significant compared to the (inner) binary orbital angular momentum, the inner orbit axis undergoes Lens–Thirring (LT) precession around the BH spin axis. Therefore, the angular momentum axis of the stellar orbit can also be affected by the LT precession in an indirect way. In several recent studies (Liu et al. 2019; Liu & Lai 2020, 2022), we have shown that the general relativity (GR) effects induced by a spinning tertiary supermassive black hole (SMBH) play an important role in the evolution of an inner stellar mass binary. Here, we extend our

* E-mail: liusaky@gmail.com
previous studies to the 'inverse' secular problem, in which the tertiary is essentially a test mass. By evolving the inner massive BHB until merger, we seek to identify the correlation (or signature) between the distribution of the surrounding stellar orbits and the final spin orientation of the BHB merger remnant.

This paper is organized as follows. In Section 2, we review the essential GR effects in the 'BHB + outer test particle' system and present the secular equations in post-Newtonian (PN) theory. In Section 3, we identify different dynamical behaviours of the outer orbit for different parameters of the system. We perform analytical calculations of the final spin-orbit misalignment angles using the principle of adiabatic invariance. In Sections 4 and 5, we explore the final configurations of the stellar orbits at different distances from the central BHB, considering a range of mass ratios of BHB, coplanar/inclined initial orientations, and eccentricities of the stellar orbits. We summarize our main results in Section 6.

2 EVOLUTION EQUATIONS

We first review the secular dynamics of massless particles around a massive binary. Consider a BHB with semimajor axis $a_{in}$, eccentricity vector $e_{in}$, total mass $m_{12} = m_1 + m_2$ (where $m_1$ and $m_2$ are the individual masses), and reduced mass $\mu_{in} = m_1 m_2 / m_{12}$. The outer test particle moves around the BHB with semimajor axis $a_{out}$ and eccentricity $e_{out}$. The orbital angular momenta of two orbits are $L_{in} = \hat{L}_{in} = \mu_{in} \sqrt{G m_{12} a_{in}(1 - e_{in}^2)} \hat{J}_{in}$ and $L_{out} = \hat{L}_{out} L_{out}$ (see Fig. 1).

Throughout the paper, for convenience of notation, we will frequently omit the subscript 'out' for the outer orbit. The evolution of the system is governed by the double-averaged (averaging over both the inner and outer orbital periods) secular equations of motion.

For the inner binary, we set $e_{in} = 0$. The primary BH ($m_1$) in the binary has spin $S_1 = S_1 \hat{S} = (\chi, G m_1^2 / c) \hat{S}_1$, where $\chi \leq 1$ is the Kerr parameter. Throughout the paper, we assume $S_2 \ll S_1$, thus neglecting the dynamical effect of the spin of the low-mass secondary ($m_2$); this approximation allows some of the dynamical spin-orbit behaviours to be understood analytically (see Section 3). However, all the equations shown below are valid for arbitrary mass ratio of the inner binary. The angular momentum $L_{in}$ evolves according to

$$\frac{dL_{in}}{dt} = \frac{dL_{in}}{dt}_{GW} + \frac{dL_{in}}{dt}_{L_{out}S},$$

where the two terms represent dissipation due to GW emission and the spin–orbit coupling, respectively. Gravitational radiation draws energy and angular momentum from the BH orbit, with (e.g. Peters 1964)

$$\frac{dL_{in}}{dt}_{GW} = -\frac{32 G^3 \mu_{in} m_{12}^2}{5 c^5 a_{in}^6} L_{in}.$$

For reference, the merger time due to GW radiation of a binary with the initial semimajor axis $a_{in}$ is given by

$$T_{in} = \frac{5 c^2 a_{in}^2}{256 G^3 m_{12}^2 q} (1 + q)^2 \left( \frac{1 + q^2}{q} \right)^{12} \left( \frac{a_{in}}{224 \text{ au}} \right)^4 \text{yr},$$

where we have introduced the mass ratio $q = m_2 / m_1$. Spin–orbit coupling (1.5 PN effect) induces mutual precession of $\hat{L}_{in}$ around $\hat{S}_1$ (e.g. Barker & O’Connell 1975):

$$\frac{dL_{in}}{dt}_{L_{out}S} = \Omega_{L_{out}S} \hat{S}_1 \times L_{in},$$

where

$$\Omega_{L_{out}S} = \frac{GS_1(4 + 3q/m_1)}{2 c^2 a_{in}^3}.$$ (5)

The spin vector $S_1$ follows

$$\frac{dS_1}{dt} = \Omega_{L_{out}S} \hat{S}_1 \times S_1,$$

with

$$\Omega_{L_{out}S} = \frac{L_{in}}{S_1} = \frac{3G \mu_{in} (m_2 + \mu_{in}/3)}{2 c^2 a_{in}^3}.$$ (7)

where $n_{in} = (G m_{12} / a_{in}^2)^{1/2}$ is the mean motion of the inner binary.

The time evolution equations of the outer angular momentum axis $\hat{l}$ and eccentricity $e$ vectors are given by

$$\frac{dl}{dt} = \frac{dl}{dt}^{(N)} + \frac{dl}{dt}^{(GR)} + \frac{dl}{dt}^{L_{out}S},$$

and

$$\frac{de}{dt} = \frac{de}{dt}^{(N)} + \frac{de}{dt}^{(GR)} + \frac{de}{dt}^{L_{out}S} + \frac{de}{dt}^{GR} + \frac{de}{dt}^{L_{out}S}.$$ (9)

The precession of $\hat{l}$ around $\hat{L}_{in}$ includes the Newtonian and GR components. The Newtonian precession can be described in the quadruple order

$$\frac{dl}{dt}^{(N)} = -\Omega_{L_{out}S} \hat{l} \times \hat{l},$$

where $\Omega_{L_{out}S} = \frac{3 \mu_{in} a_{in}^2}{4 m_{12} c^2 (1 - e^2)^2}$,

$$\Omega_{L_{out}S} = \frac{3 \mu_{in} a_{in}^2}{4 m_{12} c^2 (1 - e^2)^2}.$$ (11)

where $n = (G m_{12} / a_{in}^2)^{1/2}$. Note that since $e_{in} = 0$, the high-order Newtonian perturbation acting on the outer orbit can be ignored. Similarly,

$$\frac{de}{dt}^{(N)} = -\Omega_{L_{out}S} \left( \hat{l} \times \hat{l} \times e - \left[ \frac{1}{2} - \frac{5}{2} (\hat{l} \cdot \hat{e})^2 \right] \hat{l} \times e \right).$$ (12)

The GR components are given by (e.g. Liu et al. 2019; Liu & Lai 2020)

$$\frac{dl}{dt}^{(GR)} = \Omega_{L_{out}S} \hat{l} \times l,$$

$$\frac{de}{dt}^{(GR)} = \Omega_{L_{out}S} \hat{l} \times e - 3 \Omega_{L_{out}S} (\hat{l} \cdot e) \hat{l} \times e,$$ (14)
with
\[ \Omega_{\text{out}}^{(GR)} = \frac{2Gm_{12}}{c^2a_{\text{in}}^3} \sqrt{\frac{a_{\text{in}}}{a_{\text{in}}^3(1-e^2)^2}}. \] (15)

GR (1-PN correction) introduces pericentre precession of the outer binary,
\[ \frac{d}{dt} | \Omega_{\text{GR, out}} \hat{i} \times e, \] (16)

where
\[ \Omega_{\text{GR, out}} = \frac{3n}{c^2d(1-e^2)^3/2}. \] (17)

Finally, the spin–orbit coupling also induces the precession of \( \hat{i} \) around \( S_1 \):
\[ \frac{d}{dt} | \Omega_{\text{Lout}} \hat{s}_1 \times \hat{i}, \] (18)
\[ \frac{d}{dt} | \Omega_{\text{Lout}} \hat{s}_1 \times e - 3\Omega_{\text{Lout}}(\hat{s}_1 \times \hat{e} \times \hat{e}), \] (19)

where
\[ \Omega_{\text{Lout}} = \frac{2GJ}{c^2a_{\text{in}}^3(1-e^2)^3/2}. \] (20)

By comparing equations (15) and (20), we find that \( \Omega_{\text{Lout}} / \Omega_{\text{out}}^{(GR)} = S_1 / L_{\text{in}}(\text{at } e_{\text{in}} = 0) \).

3 ANALYTICAL RESULTS

3.1 Different types of \( \hat{i} \) behaviours

To develop an analytical understanding of the dynamics, we assume the outer test particle has a circular orbit. If we define \( J = J_1 J_2 \), equation (4) gives
\[ \frac{d}{dt} | \Omega_{\text{in}} J \times \hat{i}_{\text{in}}, \] (21)

with
\[ \Omega_{\text{in}} = \frac{GJ}{S_1(4 + 3m_2/m_1)}. \] (22)

Combining equations (10), (13), and (18), we find that the orbital axis \( \hat{i} \) of the test particle evolves according to
\[ \frac{d}{dt} | -\Omega_{\text{in}}(\hat{i}_{\text{in}} \cdot \hat{j}) \hat{i}_{\text{in}} \times J + \Omega_{\text{out}}^{(GR)} J \times \hat{i}, \] (23)

where
\[ \Omega_{\text{out}}^{(GR)} J = \frac{\Omega_{\text{out}}^{(GR)} J}{L_{\text{in}}}. \] (24)

In the absence of GW dissipation, \( \hat{i}_{\text{in}} \) rotates around \( \hat{j} \) at a constant rate, \( \Omega_{\text{in}} \), so it is useful to consider the evolution of \( \hat{i} \) in the frame corotating with \( \hat{i}_{\text{in}} \). Combining equations (21) and (23), we have
\[ \frac{d}{dt} | -\frac{1}{2} \Omega_{\text{out}}^{(GR)}(\hat{i}_{\text{in}} \cdot \hat{j}) \hat{i}_{\text{in}} \times J = \Omega_{\text{out}}^{(GR)}(\hat{i} \cdot J) \hat{i}. \] (25)

The corresponding Hamiltonian can be given by
\[ \mathcal{H} = -\frac{1}{2} \Omega_{\text{out}}^{(GR)}(\hat{i}_{\text{in}} \cdot \hat{j})^2 + (\Omega_{\text{out}}^{(GR)} - \Omega_{\text{in}})(\hat{i} \cdot J). \] (26)

We define the dimensionless Hamiltonian
\[ \tilde{\mathcal{H}} = \frac{\mathcal{H}}{\Omega_{\text{out}}^{(GR)}} = -\frac{1}{2} (\hat{i} \cdot J)^2 + \left( \Omega_{\text{out}}^{(GR)} - \Omega_{\text{in}} \right) (\hat{i} \cdot J). \] (27)

Note that compared to the Newtonian precession \( \Omega_{\text{out}}^{(N)} \), the GR precession \( \Omega_{\text{out}}^{(GR)} \) of \( \hat{i} \) is only important near the merger of the inner binary. We thus ignore the \( \lambda \) term in our analytical analysis. Depending on the value of \( \eta \), we expect three possible \( \hat{i} \) behaviours:

(i) For \( \eta \ll 1 \), \( \hat{i} \) closely follows \( \hat{i}_{\text{in}} \), maintaining an approximately constant \( \hat{i} = \cos^{-1}(\hat{i}_{\text{in}} \cdot \hat{i}) \).

(ii) For \( \eta \gg 1 \), \( \hat{i} \) effectively precesses around \( \hat{i} \) with approximately constant \( \hat{i} = \cos^{-1}(\hat{i} \cdot \hat{j}) \).

(iii) When \( \eta \sim 1 \), a resonance behaviour of \( \hat{i} \) may occur, and large oscillation in \( \hat{i} \) can be generated.

Fig. 2 presents the parameter space indicating how the dynamical behaviour of \( \hat{i} \) can change during the merger of the inner BHB. We set the primary component of the BHB to be \( m_1 = 4 \times 10^6 M_\odot \), and vary the mass of the secondary component \( (m_2) \) and the semimajor axis of the BHB \( (a_{\text{out}}) \). The contours of constant \( \eta \) are evaluated for the closest stable test particle orbits around the binary (Holman & Wiegert 1999):
\[ a_{\text{out},c} = (1.6 + 5.1e - 2.22e^2 + 4.12\mu_e - 4.27e\mu_e - 5.09\mu_e^2 + 4.61e^2\mu_e^2)a_{\text{in}}. \] (28)

where \( \mu_e = m_2/m_{12} \). We see that for a given \( m_2 \), as \( a_{\text{in}} \) decreases, \( \eta \) increases and the outer orbit may experience three types of dynamical behaviours successively.

To study such behaviours, we set up a coordinate system with \( \hat{z} = \hat{i}_{\text{in}} \) and \( \hat{y} \) sin \( \alpha \equiv \hat{i}_{\text{in}} \times \hat{j} \), and let \( \hat{i} = \sin \hat{\theta} (\cos \phi \hat{x} + \sin \phi \hat{y}) + \cos \hat{\theta} \hat{z} \), where \( \alpha \) is the angle between \( \hat{i}_{\text{in}} \) and \( \hat{j} \) (see panel a of Fig. 3). Equation (27) becomes (neglecting the \( \lambda \) term)
\[ \frac{d}{dt} \hat{i} = -\frac{1}{2} \left( \cos \hat{\theta} \hat{i} \times J + \sin \hat{\theta} \sin \phi \hat{j} \cos \phi \right). \] (29)

Alternatively, we can also set up a coordinate system with \( \hat{z} = \hat{j} \), as shown in the panel (A) of Fig. 3. In this case, we have
\[ \frac{d}{dt} \hat{i} = -\frac{1}{2} \left( \cos \hat{\theta} \hat{i} \times J + \sin \hat{\theta} \sin \phi \hat{j} \cos \phi \right). \] (30)
Figure 3. Different types of $\hat{l}$ behaviours for three values of $a_{in}$ (as labelled), representing the different stages of the orbital decay of the inner BHB. Panels (a) and (A) show the coordinate system used to describe the triple system, where $z$-axis is aligned with $\hat{l}_{in}$ and $\hat{J}$, respectively. Panels (b)–(d) and (B)–(D) show the phase-space portraits with two different sets of canonical variables ($\cos I - \psi$ and $\cos \theta - \psi$). The system parameters studied here are $m_1 = 4 \times 10^6 M_\odot$, $m_2 = 10^5 M_\odot$, $a_{out} = 1000$ au, and $e_{in} = e_{out} = 0$. The solid lines shown in the panels (b)–(d) and (B)–(D) are contours of constant $\mathcal{H}$ (see equations 30 and 31), where we keep a constant $\theta_{S,in} = 90^\circ$ ($\theta_{S,in}$ is the angle between $\hat{l}_{in}$ and $\hat{S}_1$).
In the presence of GW dissipation, when the rate of change of $\Omega_{\text{eff}}$ is much smaller than $|\Omega_{\text{eff}}|$, i.e.
\[
\frac{\dot{\Omega}_{\text{eff}}}{|\Omega_{\text{eff}}|} \ll |\Omega_{\text{eff}}|.
\] (34)

$\Omega_{\text{eff}}$ becomes a slowly changing vector, and the angle between $\Omega_{\text{eff}}$ and $\hat{l}$ is expected to be an adiabatic invariant, i.e.
\[
\theta_{\text{eff, out}} \simeq \text{constant} \quad \text{(adiabatic invariant).}
\] (35)

After the inner binary has decayed, we have $|\Omega_{\text{out}}^{(N)}| \ll |\Omega_{\text{in}}|$, and $\Omega_{\text{eff}} \simeq \Omega_{\text{in}} \hat{J}$. Therefore,
\[
\theta_{\text{out}}^{0} \simeq \theta_{\text{eff, out}}^{0} = \theta_{0}^{0}.
\] (36)

To obtain $\theta_{\text{eff, out}}^{0}$, we note that the orientation of the initial $\Omega_{\text{eff}}$ is determined by both $\hat{l}_{\text{in}}$ and $\hat{J}$. For the outer orbits with $|\Omega_{\text{out}}^{(N)}| \gg |\Omega_{\text{in}}|$ (generally corresponding to the systems with small $\alpha_{\text{in}}$), we have $\Omega_{\text{eff}} \simeq -\Omega_{\text{in}} \hat{J}$. As a result, the final spin–orbit misalignment angle is equal to the initial inclination angle between $\hat{l}_{\text{in}}$ and $\hat{l}$, i.e.
\[
\theta_{\text{out}}^{0} \simeq \theta_{\text{eff, out}}^{0} \simeq \theta_{0}^{0}.
\] For the example shown in Fig. 4, we see that the adiabatic criterion (equation 34) is satisfied and the adiabatic invariant $\theta_{\text{eff, out}}$ is almost a constant. Since $\hat{l}$ and $\hat{l}_{\text{in}}$ are initially aligned, $\theta_{0}^{0} = 0$, the final spin–orbit misalignment angle $\theta_{\text{out}}^{0} = 0$.

For the distant outer orbits, we have $|\Omega_{\text{out}}^{(N)}| \ll |\Omega_{\text{in}}|$, and $\Omega_{\text{eff}} \simeq -\Omega_{\text{in}} \hat{J}$. Therefore, we expect that $\theta_{\text{out}}^{0} \simeq \theta_{\text{eff, out}}^{0} \simeq \theta_{0}^{0}$, where $\theta_{0}$ is the angle between $\hat{l}$ and $\hat{J}$ at the initial moment. For the specific configuration with $\theta_{0} = 0$, we have $\theta_{\text{out}}^{0} \simeq \alpha_{0}$, where $\alpha_{0}$ is the initial angle between $\hat{l}$ and $\hat{J}$.

4 RESULTS FOR INITIALLY COPLANAR OUTER ORBITS

4.1 Fiducial case: $m_{2} = 10^{6} M_{\odot}$

We now study the evolution of the outer orbits with different radius ($a_{\text{out}}$) as the inner BHB decays. We consider the initially coplanar case with $l_{0} = 0^\circ$ and $m_{2} = 10^{5} M_{\odot}$ in this section.
Figure 5. The variation of the values of $\eta$ and $\lambda$ as the inner BHB decays. The inner BHB has masses $m_1 = 4 \times 10^6 \, M_\odot$ and $m_2 = 10^5 \, M_\odot$. The results (purple and orange lines) are obtained by using equation (28) with $\theta_{\text{S, in}} = 0^\circ$, where the solid, dashed, and dash-dotted lines are for the given $a_{\text{out}}$ as labelled; the minimum $a_{\text{out}}$ is evaluated by equation (29). We also show the ratio of $S_1/L_{\text{in}}$ as a function of $a_{\text{in}}$.

Fig. 5 shows $\lambda^{-1}$ and $\eta^{-1}$ (see equation 28) as a function of $a_{\text{in}}$ for a given $a_{\text{out}}$. The values of $\eta$ are obtained by setting $\theta_{\text{S, in}} = 0^\circ$. We find that the nodal precession induced by GR ($\Omega_{\text{GR}}$) is always weaker than the Newtonian one ($\Omega_{\text{N}}$), until the inner BHB has become sufficiently compact. On the other hand, when the BHB is wide, the systems, especially for the close test particle orbits (e.g. $a_{\text{out}} = 810$ au), are in the ‘$\eta \gg 1$’ regime, in which the Newtonian precession of $\hat{\mathbf{l}}$ around $\hat{\mathbf{l}}_{\text{in}}$ is much stronger than the precession of $\hat{\mathbf{l}}_{\text{in}}$ around $\hat{\mathbf{l}}$. This implies that the direction of $\Omega_{\text{eff}}$ is approximately parallel to $\hat{\mathbf{l}}_{\text{in}}$ and $\theta_{\text{S, eff}} \approx \theta_{\text{S, in}}$. However, if the test particle is further away from the central BHB (i.e. $a_{\text{out}} > 3000$ au), $\eta$ is close to unity and the orientation of $\Omega_{\text{eff}}$ is determined by both $\hat{\mathbf{l}}_{\text{in}}$ and $\hat{\mathbf{l}}$.

In Fig. 6, panel (A) shows the final spin–orbit angles $\theta_{\text{S, out}}$ for a series of test particle orbits with different separations, for several values of $\theta_{\text{S, in}}$. We obtain the numerical results (dots) by integrating equations (1), (6), (8), and (9) and the analytical results based on equation (36). We find that the analytical predictions (dashed lines) agree well with the numerical results. For the close test particle orbits, the final angular momentum $\hat{\mathbf{l}}$ always points in the direction of the spin $\mathbf{S}_1$, i.e. $\theta_{\text{S, out}} \approx \theta_{\text{S, in}} = 0^\circ$, regardless of the initial spin orientation. This is because $\Omega_{\text{eff}} \propto \hat{\mathbf{l}}_{\text{in}}$ for the orbits with $a_{\text{out}} \leq 3000$ au (as shown in Fig. 5). On the other hand, for $a_{\text{out}} \geq 3000$ au, the final value $\theta_{\text{S, out}}$ is only determined by $\theta_{\text{S, out}}^0$ and $\theta_{\text{S, out}} \approx \theta_{\text{S, in}}$ (the angle between $\hat{\mathbf{l}}$ and $\hat{\mathbf{l}}_{\text{in}}$) as $a_{\text{out}} \gtrsim 10^4$ au. Since the initial orientation of $\hat{\mathbf{l}}$ depends on $\theta_{\text{S, in}}$, we see that the angles $\theta_{\text{S, out}}$ corresponding to different $\theta_{\text{S, in}}$ differ at large $a_{\text{out}}$.

Panels (b)–(d) of Fig. 6 show the dependence of $\theta_{\text{S, out}}^i$ on $\theta_{\text{S, in}}$ for three values of $a_{\text{out}}$. We find that the analytical results are in excellent agreement with the numerical calculations.

4.2 $m_2 = 10^4 \, M_\odot$ and $m_2 = 10^5 \, M_\odot$

If $m_2$ becomes lighter, in order to have BHB merging within the Hubble time-scale, $a_{\text{out}}$ should be smaller (as shown in Fig. 5). The initial systems may be close to or even already in the ‘$\eta \sim 1$’ regime, indicating that the angular momentum of the close test particle orbit $\hat{\mathbf{l}}$ may experience more complicated evolution at the early stage of the merger of the inner BHB.

Figure 6. Panel (a) shows the final spin–orbit misalignment angles $\theta_{\text{S, out}}^i$ as a function of $a_{\text{out}}$, for different initial spin orientations (as labelled). The system parameters follow the example shown in Fig. 5. The stability criterion is given by equation (29). All the dots are the numerical results obtained by integrating equations (1), (6), (8), and (9). The dashed lines are the analytical results based on equation (33). Panels (c) and (d) show the final angles $\theta_{\text{S, out}}^i$ as a function of a full range of $\cos \theta_{\text{S, in}}^i$, with three values of $a_{\text{out}}$. Again, the dots and the dashed lines are obtained numerically and analytically, respectively.
Fig. 7. Same as Fig. 5, except for $m_2 = 10^4 M_\odot$ (left-hand panel) and $m_2 = 10^3 M_\odot$ (right-hand panel).

Fig. 8. Same as Fig. 6, except for $m_2 = 10^4 M_\odot$.

Fig. 7 shows how $\lambda$ and $\eta$ change as $a_{in}$ decreases when $m_2 = 10^4 M_\odot$ (left-hand panel) and $m_2 = 10^3 M_\odot$ (right-hand panel). Here, since $S_1 \gg L_{in}$, the orientation of $\hat{J}$ is dominated by $\hat{S}_1$.

Fig. 8 shows the final angle $\theta_{S,\text{out}}'$ as a function of $a_{out}$ for a range of $\theta_{S,\text{in}}'$ values. Compared to the results shown in Fig. 6, the analytical predictions are only valid for the small $\theta_{S,\text{in}}'$ or the distant outer orbits (see also the panel d); for the test particle orbit with small $a_{out}$, the analytical results break down when $\theta_{S,\text{in}}' \gtrsim 90^\circ$ (see also panels b and c).

Fig. 9 shows two evolution examples for a system with small $a_{out}$. We identify two main reasons for the discrepancy between the analytical and numerical results for $\theta_{S,\text{out}}'$: (i) The time of entry into ‘$\eta \sim 1$’ regime. The systems with small $m_2$ tend to have a relatively large $\eta$ ($\lesssim 1$), thus will enter the ‘$\eta \sim 1$’ regime earlier. The inclination angle $l$ shown in Fig. 9 has a chance to be excited (left-hand panel) or experience oscillations (right-hand panel) at earlier times compared to the example shown in Fig. 4. Note that the exact value of $\eta$ depends on the choice of $\theta_{S,\text{in}}$ (see Fig. 2). (ii) Crossing 90° in $l$. For the BHB with small mass ratio, the direction of $\hat{J}$ is dominated by the spin vector $\hat{S}_1$ instead of $\hat{I}_{in}$ (see Fig. 7). Thus, for a given $\theta_{S,\text{in}}'$, the angle between $\hat{I}_{in}$ and $\hat{J}$ (i.e. $\alpha$) is larger than the one for a BHB with comparable masses (e.g. Fig. 5). The large $\alpha$ value may easily induce large inclinations ($l \gtrsim 90^\circ$) due to the precession of $\hat{I}_{in}$ around $\hat{J}$ as the system reach the ‘$\eta \sim 1$’ regime. Therefore, the crossing through 90° in $l$ may occur and induces significant oscillations in $|A|$ and $|\Omega_{\text{eff}}|$, breaking the adiabaticity condition.

Fig. 10 shows the results for $m_2 = 10^3 M_\odot$. Similar to Fig. 8, we find that the analytical results are in agreement with the numerical
calculations except when $a_{\text{out}}$ is small ($a_{\text{out}} \lesssim 400$ au) and $\theta_{S, \text{in}}^0$ is large ($\theta_{S, \text{in}}^0 \gtrsim 90^\circ$).

Different from the case of $m_2 = 10^5 M_\odot$, the system with $m_2 = 10^3 M_\odot$ has $\eta \simeq 1$ at the initial time, which means it will pass through the 'η ~ 1' regime much earlier. We see in Fig. 11 that the inclination angle $I$ undergoes small amplitude oscillations in the early stage, which is a result of the precession of $I_\text{in}$ around $\hat{J}$. After the excitation, $I$ keeps oscillating for a long time until the inner BHB merges.

5 NUMERICAL RESULTS FOR MISALIGNED AND ECCENTRIC OUTER ORBITS

5.1 Initially inclined $\hat{l}$

We now consider the general case in which $\hat{l}$ is not aligned with $\hat{l}_\text{in}$ initially, focusing on systems with $m_1 = 4 \times 10^6 M_\odot$ and $m_2 = 10^5 M_\odot$.

Fig. 12 shows our results when the initial $\hat{l}$ is inclined to $\hat{l}_\text{in}$ by $20^\circ$. We find that the analytical results for $\theta_{S, \text{out}}^0$ agree well with the numerical results. In addition, we see that three lines from different initial phase angles converge into a single line at small $a_{\text{out}}$. This is because in this case, $\Omega_{\text{eff}} \simeq -\Omega_{\text{in}}(\hat{l}_\text{in} \cdot \hat{l})|l_\text{in}|$, and $\theta_{S, \text{out}}^0$ is only determined by $I_0$ instead of $\varphi$. If $a_{\text{out}}$ is sufficient large, $\Omega_{\text{eff}} \simeq -\Omega_{\text{in}} \hat{J}$ and $\theta_{S, \text{out}}^0 = \theta_0$, which depends on the initial phase angle. As seen in the panel (A) of Fig. 3, the minimum and maximum values of $\theta_0$ can be achieved when $\varphi = 0^\circ$, $180^\circ$, respectively. Therefore, the range of $\theta_{S, \text{out}}^0$ can be well characterized for the distant test particle orbits.

To determine the final orientation of a stellar disc with finite 'thickness', we consider a range of initially inclined $\hat{l}$ with misalignment angle $\theta_{\text{out}, \text{in}} \in (0^\circ, 20^\circ)$ ($\theta_{\text{out}, \text{in}}$ is the angle between $\hat{l}$ and $z$-axis, i.e. initial $l_a$) at each $a_{\text{out}}$. For each $l_a$, we consider a random phase $\varphi$ from 0 to $2\pi$. The results are shown in Fig. 13. A wide range of $\theta_{S, \text{out}}^0$ are produced for a given $a_{\text{out}}$. 

![Figure 9](image_url) Figure 9. Similar to Fig. 4, but the system parameters here are $m_1 = 4 \times 10^6 M_\odot$, $m_2 = 10^4 M_\odot$, $a_{\text{in}} = 265$ au, $a_{\text{out}} = 430$ au, $e_{\text{in}} = e_{\text{out}} = 0$, and $I_0 = 0^\circ$. We consider two values of $\theta_{S, \text{in}}$ (as labelled) in the left-hand and right-hand panels.

![Figure 10](image_url) Figure 10. Same as Fig. 6, except for $m_2 = 10^3 M_\odot$. 

To characterize the role of the initial spin orientation, we perform the similar calculations with $\theta_{S,\text{in}} = 90^\circ$. The results are shown in Fig. 14. Compared to Fig. 13, the distribution of $\theta_{S,\text{out}}$ is widened, but all $\theta_{S,\text{out}}$ values are within $40^\circ$.

### 5.2 Eccentric outer orbits

Here, we consider how the results are changed when the outer orbits have finite eccentricities.

Fig. 15 presents the results from the fiducial example (see Fig. 6) but with $e = 0.9$. Since the outer eccentricity $e$ only appears in the expression for $\Omega_{\text{out}}$, we carry out the analytical calculations by using equation (11) with $e \neq 0$. We find that the numerical results and the analytical calculations are still in good agreement.

Note that here we do not consider the mutual interactions between different outer orbits. For the realistic system, the adjacent eccentric outer orbits could experience orbital crossings. However, we expect that the results for $\theta_{S,\text{out}}^i$ remain largely valid.

### 6 DISCUSSION AND CONCLUSION

In this paper, we have studied the secular dynamics of stars (modelled as test particles) around a merging massive/supermassive BHB, taking into account the GR effect induced by the rotating BH in the inner binary. We focus on the circular BHB with relatively small mass ratio, so that we only need to include the spin of the (more massive) primary BH. Our goal is to determine the final orbital orientations of the outer (circumbinary) stellar orbits relative to the spin axis of the merger remnant, assuming that the initial stellar orbital axes are approximately aligned with the BHB orbital axis.

The evolution of the angular momentum vector of the stellar orbit ($\hat{l}$) is determined by the competition between the precession of the BHB axis $\hat{l}_m$ around the primary spin axis $\hat{s}_1$ and the precession of $\hat{l}$ around $\hat{l}_m$. During the orbital decay of the BHB, the ratio of the two precession rates can change from $\leq 1$ to $\geq 1$, leading to a significant change in the orientation of $\hat{l}$. The final direction of $\hat{l}$ carries the imprint of the spin of the remnant BH ($\hat{s}_1$). Our main findings are as follows:

(i) For central BHBs with modest mass ratio ($m_2/m_1 \sim 0.1$), there is a quasi-alignment phenomenon for the evolution of the outer stellar orbits. Namely, starting with nearly coplanar outer orbits (i.e. $\hat{l} \parallel \hat{l}_m$), the orbital axis $\hat{l}$ of the circumbinary star will preferentially evolve towards the spin direction after the merger of inner BHB, regardless of the initial spin–orbit misalignment angle of the BHB (see Fig. 6). This alignment is particularly strong for close stellar orbits. Such trend of alignment, where the final spin–orbit misalignment angle ($\theta_{S,\text{out}}^i$) is small, can be understood analytically based on the principle
of adiabatic invariance (equation 35). Also, our analytical analysis can be applied to inclined and eccentric outer orbits (Figs 13–15).

(ii) When the mass ratio of the BHB is more extreme (i.e. $m_2/m_1 \lesssim 0.01$), the angular momentum axis of the outer stellar orbit can experience complicated evolution in general. The adiabaticity condition in the analytical calculation may break down and the evolution of the stellar orbits can only be resolved numerically by using the full secular equations of motion. Nevertheless, the alignment effect still works reasonably well when the initial spin–orbit misalignment angle is small (i.e. $\theta_{S,\text{in}}^0 \lesssim 90^\circ$; see Figs 8 and 10).
There are several caveats in our study:

(i) We have neglected the effect due to the secondary spin in the central BHB. This is reasonable if the secondary spin $S_2$ is negligible compared to $S_1$ (e.g. when the mass ratio $m_2/m_1$ is relatively small or when $\chi_2 \ll \chi_1$). For comparable-mass BHBs, the final spin axis of the merger remnant is approximately aligned with the pre-merger orbital axis; thus, we expect the circumbinary stellar orbital axis to be aligned with the final BH spin (assuming $\hat{I}$ is initially aligned with the binary axis).

(ii) We have not considered the merger kick acting on the remnant BH, which may change the orientation of the stellar orbit relative to the final BH spin axis. For the BHB studied in our paper ($m_1 = 4 \times 10^6 M_\odot$ and $m_2 = 10^5 M_\odot$, with mass ratio 0.025), assuming the primary BH has the maximum spin with isotropic orientation, the kick velocity ($V_{\text{kick}}$) on the merger remnant evaluated using the fitting formula of Lousto et al. (2010) is less than $\sim 40 \text{ km s}^{-1}$. Compared to the orbital velocity ($V_{\text{orb}}$) of the stellar orbits studied here ($a_{\text{out}} \leq 10^5 \text{ au}$), we always have $V_{\text{orb}} \gg V_{\text{kick}}$. Thus, the kick effect is negligible. However, for BHBs with higher mass ratios, the merger kick could play an important role, especially for the distant stellar orbits with $V_{\text{orb}} \gtrsim V_{\text{kick}}$. In this case, the post-kick orbital orientation can be modified (e.g. Liu & Lai 2021), and the final spin–orbit misalignment angle must be evaluated based on the corrected orientation of $\hat{I}$.

(iii) We have only considered BHBs in circular orbit in this paper. When the BHB has a finite eccentricity, the outer stellar orbit can also gain modest eccentricity through octupole-order secular interactions (e.g. Liu et al. 2105a, b). The finite eccentricity may influence the orbital inclination evolution indirectly.

Our result suggests that the relative orientation between the spin of a central massive/supermassive BH and the surrounding stellar orbits might provide a probe of the merger history of the BH. In particular, the Galactic Centre hosts a population of young massive stars (e.g. Ghez et al. 1998, 2008; Genzel et al. 2000; Merritt 2013; Alexander 2017). If the SMBH, Sagittarius A*, has experienced a previous merger with an intermediate-mass BH, it could have left some imprints on the nearby S-star orbits. It has been suggested that the orbital distribution of S-stars could put constraints on the Sagittarius A* spin (e.g. Levin & Beloborodov 2003; Fragione & Loeb 2020). Therefore, the precise measurements of the S-star orbits (including the orbital orientations) and the spin axis of central BH would be highly desirable.

ACKNOWLEDGEMENTS

BL thanks Johan Samsing, Daniel D’Orazio, and Adrian Hamers for useful discussion. DL was supported in part by NSF grants AST-1715246 and AST-2107796. This project has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie Actions grant agreement no. 847523 ‘INTERACTIONS’.

DATA AVAILABILITY

The simulation data underlying this article will be shared on reasonable request to the corresponding author.

REFERENCES

Alexander T., 2017, ARA&A, 55, 17
Bansal K., Taylor G. B., Peck A. B., Zavala R. T., Romani R. W., 2017, ApJ, 843, 14
Barker B. M., O’Connell R. F., 1975, Phys. Rev. D, 12, 329
Beloborodov M. C., Blandford R. D., Rees M. J., 1980, Nature, 287, 307
Bianchi S., Chiaberge M., Piconcelli E., Guainazzi M., Matt G., 2008, MNRAS, 386, 105
Bogdanović T., Eracleous M., Sigurdsson S., 2009, ApJ, 697, 288
Boroson T. A., Lauer T. R., 2009, Nature, 458, 53
Chapon D., Mayer L., Teyssier R., 2013, MNRAS, 429, 3114
