Magnetic quantization of electronic states in \( d \)-wave superconductors

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We derive a general quasiclassical approach for long–range magnetic–field quantization effects in superconductors. The method is applied to superclean \( d \)-wave superconductors in the mixed state. We study the delocalized states with energies \( \epsilon \gg \Delta_0 \sqrt{H/H_c2} \). We find that the energy spectrum consists of narrow energy bands whose centers are located at the Landau levels calculated in absence of the vortex potential. We show that transitions between the states belonging to the different Landau levels give rise to resonances in the a.c. quasiparticle conductivity and in the a.c. vortex friction.

I. INTRODUCTION

The unusual behavior of thermodynamic and transport properties of \( d \)-wave superconductors as functions of magnetic field is being a subject of extensive experimental and theoretical studies. This behavior is attributed to nontrivial energy dependence of the electronic density of states \( \mathcal{N}(\omega) \), and to specific kinetic processes which are very sensitive to fine details of electronic states brought about by the presence of vortices \( v \). There exists, however, a conceptual controversy about the structure of electronic states in \( d \)-wave superconductors in the mixed state. One of the views is that the states below the maximum gap \( \Delta_0 \) have a discrete spectrum due to Andreev reflections; some states are localized within vortex cores \( v \) while others are quantized at longer distances \( \sim \xi \) as a particle moving along a curved trajectory in a magnetic field hits the gap for a current momentum direction such that \( \epsilon = \Delta_p \). Other authors advocate that, instead of the magnetic quantization, energy bands should appear in a periodic vortex potential due to the vortex lattice \( v \).

In the present paper, we develop a general quasiclassical approach for calculating the long–range magnetic–field quantization effects in superconductors in the regime where the wave–length of electrons is much shorter than the coherence length \( \xi \) such that \( p_F \xi \gg 1 \). The proposed method is applied to superclean \( d \)-wave superconductors in the mixed state in the low field limit, \( H \ll H_c2 \). We demonstrate that quantization effects are in fact a compromise between the two abovementioned extremes. In the first part of the paper (Sections I - IV), we show that the influence of a magnetic field on delocalized excitations in a superconductor cannot be reduced to simply an action of an effective vortex lattice potential. The effect of magnetic field is rather two-fold: (i) It creates vortices and thus provides a periodic potential for electronic excitations. (ii) It also affects the long range motion of quasiparticles in a manner similar to that in the normal state. The latter long range effects are less pronounced for low energy excitations. The spectrum of excitations with energies \( \epsilon \sim \Delta_0 \sqrt{H/H_c2} \), however, is mostly determined by the long range motion and exhibits magnetic quantization.

We study the delocalized states with energies \( \epsilon \gg \Delta_0 \sqrt{H/H_c2} \) and calculate their energy spectrum. We find that the spectrum indeed consists of energy bands as it should be in a periodic potential. However, in the quasiclassical limit, the bands are rather narrow; their centers are located at the Landau levels calculated in Refs. 11, 12.

In the second part, Sections V - VI, we consider effects of the energy spectrum on the vortex dynamics and on the quasiparticle conductivity. We show that both the vortex friction for oscillating vortices and the a.c. quasiparticle conductivity for fixed (pinned) vortices display resonances at transitions between the states belonging to different Landau levels.

II. LONG–RANGE EFFECTS OF THE MAGNETIC FIELD

We start with the conventional Bogoliubov-de Gennes equations

\[
\left( \hat{p} - \frac{e}{c} \mathbf{A} \right)^2 - p_F^2 \right) u + 2m\Delta_p v = 2meu, \\
\left( \hat{p} + \frac{e}{c} \mathbf{A} \right)^2 - p_F^2 \right) v - 2m\Delta_p^* u = -2mev
\]

(1)

where \( \hat{p} = -i\nabla \) is the canonical momentum operator. Equations (1) have the particle–hole symmetry such that \( u \rightarrow v^*, v \rightarrow -u \) under complex conjugation and \( \epsilon \rightarrow -\epsilon \). For a vortex array, the order–parameter phase is a multiple–valued function defined through

\[
\text{curl} \nabla \chi = \sum_i 2\pi \delta(\mathbf{r} - \mathbf{r}_i).
\]

As a result
such that, on average, $\nabla \chi \approx eHr/c$ for large $r$.

Consider a quasiparticle in a magnetic field in the presence of the vortex lattice for energies ranging from the above the gap to infinity. If the particle mean free path is longer than the Larmor radius, i.e., $\omega_c \tau \gg 1$ where $\omega_c$ is the cyclotron frequency, such particle can travel away from each vortex up to distances of the order of the Larmor radius $r_L = v_F/\omega_c$. This brings new features to Eqs. (4). Assume for a moment that $\Delta = 0$. The wave function $u$ describes then a particle with the kinetic momentum $P_+ = p - (e/c)A$ and an energy $\epsilon = P_+^2/2m - E_F$ while $v$ describes a hole with the kinetic momentum $P_- = p + (e/c)A$ and an energy $\epsilon = E_F - P_-^2/2m$. A particle and a hole which start propagation from the same point will then move in different directions and along different trajectories which transform one into another under the transformation $H \rightarrow -H$. For a finite order parameter, the wave function is a linear combination of a particle and a hole. It is not convenient, however, to use such a combination at distances where the trajectories of a particle and a hole go far apart, i.e., when the vector potential is no longer small compared to the Fermi momentum $p_F$.

Eq. (4) shows that the phase of $u$ differs from that of $v$ by the order parameter phase $\chi$. To construct a proper basis, one needs to bring the phases of $u$ and $v$ in correspondence with each other. We note that the usual transformation

$$
\begin{pmatrix}
u \\
v
\end{pmatrix} = \begin{pmatrix}
e^{ix/2}u \\
e^{-ix/2}v
\end{pmatrix}
$$

is not convenient when considering a particle which can move at distances much larger than the size of one unit cell. The problem is that the new functions $\tilde{u}$ and $\tilde{v}$ have extra phase factors $\pm x/2$ as compared to the initial functions $u$ and $v$, respectively. These phases increase with the distance resulting thus in a shift in the action $A \rightarrow A \pm x/2$. The latter and is equivalent to a shift in the momentum $\mathbf{p} = \nabla A \rightarrow \mathbf{p} \pm \nabla \chi/2$. This transformation is not dangerous if the particle is bound to distances of the order of one intervortex distance because the phase gradient is limited $|\nabla \chi| \ll p_F$. However, for a vortex array, the phase gradient increases with distance and can reach values comparable with $p_F$. It means that components of the new momentum can not be integrals of motion (i.e., they change along the trajectory) even in absence of the vortex potential associated with the superconducting velocity and spatial variations of the order parameter magnitude.

To avoid these complications we use another transformation which also removes the coordinate dependence of the order parameter phase. The results, of course, should be independent of the choice of the transformation due to the gauge invariance. Following Refs. [11] and [14] we put in Eq. (4)

$$\begin{align*}
u = \tilde{u}, & \quad v = \exp(-ix)\tilde{v}.
\end{align*}
$$

This is a single-valued transformation. We obtain

$$
\begin{align*}
\left[\hat{P}_+^2 - p_F^2\right] \tilde{u} + 2me^{-ix}\Delta \hat{p}_+ \tilde{v} &= 2me\tilde{u}, \\
\left[\hat{P}_+ - 2mv_s\right]^2 - p_F^2 \tilde{v} - 2meix\Delta \hat{p}_+ \tilde{u} &= -2me\tilde{v}
\end{align*}
$$

where $\hat{P}_+ = \mathbf{p} - \frac{e}{c}\mathbf{A}$ is the operator of the particle kinetic momentum, and

$$
\hat{P}'_+ = \mathbf{p} - \nabla \chi/2 = \hat{P}_+ - mv_s.
$$

The superconducting velocity is

$$
2mv_s = \nabla \chi - \frac{2e}{c}\mathbf{A}.
$$

In Eqs. (4), (5) we use that, for a general pairing symmetry, $\Delta \propto \mathbf{u}^\ast \mathbf{v}$ depends actually on $\mathbf{p}' = (\mathbf{p}_u + \mathbf{p}_v)/2$ where $\mathbf{p}_u,v$ are the canonical momentum operators which act on the Bogoliubov wave functions $u$ and $v$, respectively. The term $-\nabla \chi/2$ appears in the order parameter together with the canonical momentum $\mathbf{p}$ because only one half of the momentum operator in $\Delta \mathbf{p}'$ acts on each of the wave functions $u$ or $v$.

The transformation of Eq. (4) is “$u$-like” and brings the phase of $v$ in correspondence with the phase of $u$. The resulting equations are not symmetric with respect to $u$ and $v$: the term $\mathbf{v}_s$ is present in the second equation together with $\mathbf{P}$ while it does not appear in the first equation. Let us perform one more transformation

$$
\begin{pmatrix}
u \\
v
\end{pmatrix} = \begin{pmatrix}
U \\
V
\end{pmatrix} e^{ix/2}
$$

where $\nabla \chi_v = 2mv_s$ such that

$$
curl \nabla \chi_v = \sum_i 2\pi\delta(r - r_i) - \frac{2e}{c}\mathbf{H}
$$

and $\chi_v = \chi - \chi_A$ where

$$
\chi_A = \frac{2e}{c} \int_{r_0}^{r} \left[\mathbf{H} \times \mathbf{r}'\right] dr'.
$$

The “phase” $\chi_v$ is not single valued within each unit cell, it depends on the particular path of integration. However, it is single valued on average, i.e., on a scale much larger than the intervortex distance since

$$
\int \nabla \nabla \chi_v d^2r = 0.
$$

It also implies that $\chi_s$ does not have large terms increasing with distance. The transformation Eq. (4) is thus not dangerous. The total transformation Eqs. (4,5) has the form

\begin{align*}
\left[\hat{P}_+^2 - p_F^2\right] \tilde{u} + 2me^{-ix}\Delta \hat{p}_+ \tilde{v} &= 2me\tilde{u}, \\
\left[\hat{P}_+ - 2mv_s\right]^2 - p_F^2 \tilde{v} - 2meix\Delta \hat{p}_+ \tilde{u} &= -2me\tilde{v}
\end{align*}
With this transformation we finally obtain
\[
\left( \hat{\mathbf{P}}_+ - m \mathbf{v}_s \right)^2 - p_F^2 \right] U + 2m \hat{\Delta} \hat{\mathbf{P}}_+ V = 2mcU ,
\]
\[
\left( \hat{\mathbf{P}}_+ + m \mathbf{v}_s \right)^2 - p_F^2 \right] V - 2m \hat{\Delta} \hat{\mathbf{P}}_+ U = -2meV
\] (9)

where
\[
\hat{\Delta} \hat{\mathbf{P}}_+ = e^{-i\chi} \Delta \mathbf{A} = e^{i\chi} \Delta \mathbf{A}.
\]

As distinct from Eq. (11), a particle and a hole determined by Eq. (8) move along the same trajectory though, of course, in different directions.

One can transform these equations further by putting
\[
\hat{\Psi} = \begin{pmatrix} U \\ V \end{pmatrix} = \exp \left( i \int \mathbf{p} \cdot d\mathbf{r} \right) \hat{\phi} \ ; \ \hat{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}
\] (10)

where
\[
\mathbf{p} - \frac{e}{c} \mathbf{A} = p_F^2.
\] (11)

If \( \text{div} \mathbf{A} = 0 \) we have
\[
\begin{align*}
\mathbf{P}_+ (-i\nabla + m \mathbf{v}_s) \phi_1 + m \hat{\Delta} \mathbf{P}_+ \phi_2 &= m\phi_1 , \\
\mathbf{P}_+ (-i\nabla - m \mathbf{v}_s) \phi_2 - m \hat{\Delta} \mathbf{P}_+ \phi_1 &= -m\phi_2 .
\end{align*}
\] (12)

Another equation can be obtained using the transformation
\[
\begin{align*}
u = e^{i\chi} e^{-i\chi_s/2} U &= \exp \left( i\chi/2 + i\chi_A/2 \right) U , \\
v &= e^{-i\chi_s/2} V = \exp \left( -i\chi/2 + i\chi_A/2 \right) V.
\end{align*}
\] (13)

We get
\[
\begin{align*}
\left[ \hat{\mathbf{P}}_+ - m \mathbf{v}_s \right]^2 - p_F^2 \right] U + 2m \hat{\Delta} \hat{\mathbf{P}}_+ V &= 2mcU , \\
\left[ \hat{\mathbf{P}}_+ + m \mathbf{v}_s \right]^2 - p_F^2 \right] V - 2m \hat{\Delta} \hat{\mathbf{P}}_+ U &= -2meV
\end{align*}
\] (14)

where \( \hat{\mathbf{P}}_+ = \hat{\mathbf{p}} + (e/c) \mathbf{A} \) is the “hole” kinetic momentum. The transformation Eq. (14) is “\( v \)-like,” it brings the phase of \( u \) in correspondence with that of \( v \). Using Eq. (11) we can transform Eq. (12) to its quasiclassical version which is Eq. (12) where \( \hat{\mathbf{P}}_+ \) is substituted with \( \mathbf{P}_+ \) under the condition \( \left| \mathbf{P}_+ \right|^2 = p_F^2 \). Eq. (12) and its \( v \)-like analogue possess the particle–hole symmetry. Under transformation
\[
\mathbf{p} \rightarrow -\mathbf{p}, \ v \rightarrow -v; \ \phi_1 \rightarrow \phi_2^\ast, \ \phi_2 \rightarrow -\phi_1^\ast
\]
they go one into another. Moreover, each set of equations has the particle–hole symmetry separately for a given position on the trajectory if the kinetic momenta

\[
\mathbf{P}_\bot = \mathbf{p} \mp (e/c) \mathbf{A}
\]
are reversed for a fixed position of the particle. Due to Eq. (11) \( \mathbf{p} \mp (e/c) \mathbf{A} = (q \cos \alpha, q \sin \alpha) \), where \( \alpha \) is the local direction of the momentum. The reversal corresponds to \( \alpha \rightarrow \pi + \alpha \).

We take the \( z \) axis along the magnetic field and define the quasiclassical particle-like trajectory in Eq. (12) by
\[
\frac{dx}{dy} = \frac{p_x - (e/c)A_y}{p_y - (e/c)A_x}.
\] (15)

When the magnetic filed penetration length is much longer than the distance between vortices, \( \lambda_L \gg a_0 \), the magnetic field can be considered homogeneous. With \( \mathbf{A} \) taken in the Landau gauge
\[
\mathbf{A} = (-H_y, 0, 0)
\] (16)

the trajectory is a circle
\[
(x - x_0)^2 + (y + c \ell_H/eH)^2 = (p_\bot c/eH)^2
\] (17)

where \( p_\bot^2 = p_x^2 - p_y^2 \). The local direction of the kinetic momentum is \( p_x + eH_y/e = p_x \sin \alpha, p_y = p_y \cos \alpha \). The distance along the trajectory is \( ds = r_L d\alpha \) where the Larmor radius is \( r_L = p_\bot / m \omega c \).

Eq. (12) has a simple physical meaning. It is the quasiclassical version of the usual Bogoliubov–de Gennes equation for vortex state modified to take into account long range effects of magnetic field. Eq. (12) can be written in terms of the particle trajectory Eq. (13). We have from Eq. (12)
\[
\begin{align*}
v_\perp \left( -i \frac{\partial}{\partial s} + m \mathbf{v}_t \right) \phi_1 + \Delta (\alpha) \phi_2 &= \epsilon \phi_1 , \\
v_\perp \left( -i \frac{\partial}{\partial s} - m \mathbf{v}_t \right) \phi_2 - \Delta (\alpha) \phi_1 &= -\epsilon \phi_2.
\end{align*}
\] (18)

Here \( v_\perp = p_\perp / m \), and \( v_t \) is the projection of \( \mathbf{v}_t \) on the local direction of the trajectory. \( \Delta (\alpha) \) and \( v_t \) are functions of coordinates \( x(s), y(s) \), and of the angle \( \alpha(s) \) taken at the trajectory. Eqs. (18) look exactly as the usual Bogoliubov–de Gennes equations.

### III. ELECTRONIC STATES IN ZERO LATTICE POTENTIAL

For \( d \)-wave superconductors, we take the order parameter in the form \( \Delta \mathbf{p} = \Delta_0 (2p_x p_y) / (p_x^2 + p_y^2) \) so that \( \Delta_{p-(e/c)\mathbf{A}} = \Delta_0 \sin(2\alpha) \). Consider first the limit \( v_s = 0 \) and \( \Delta_0 = \text{const} \). Eqs. (13) become
\[
\begin{align*}
-i\omega c \frac{\partial \phi_1}{\partial \alpha} + \Delta_0 \sin(2\alpha) \phi_2 &= \epsilon \phi_1 , \\
i\omega c \frac{\partial \phi_2}{\partial \alpha} + \Delta_0 \sin(2\alpha) \phi_1 &= -\epsilon \phi_2.
\end{align*}
\]

With
\[ \phi = \hat{C} \exp \{ i f(\alpha) \} \]
we obtain
\[ f(\alpha) = \pm \int \frac{d\alpha}{\omega_c} \sqrt{\epsilon^2 - \Delta_0^2 \sin^2(2\alpha)} . \]

The quantization rule also includes the integral over the momentum \( p \) defined by Eqs. (11, 13). We have
\[ \oint p \, d\alpha \pm \oint \frac{d\alpha}{\omega_c} \sqrt{\epsilon^2 - \Delta_0^2 \sin^2(2\alpha)} = 2\pi n . \tag{19} \]

The quasi-classical approximation holds for \( n \gg 1 \). The \( \pm \) signs distinguish between particles and holes. As it was already mentioned, a particle (with the plus sign in Eq. (13)) and a hole (with the minus sign) move along the same trajectory but in the opposite directions. The phase \( \chi_v \) which was introduced in Eqs. (6, 8) gives a contribution to the action of the order of \( 2\pi \) because it is limited from above by an increment of the order of circulation around one vortex unit cell; it can thus be neglected for large \( n \).

A. Sub-gap states

In the range \( |\epsilon| < \Delta_0 \), the turning points correspond to vanishing of the square root at \( \alpha = \pm \alpha_c \) where \( \sin(2\alpha_c) = |\epsilon|/\Delta_0 \). We have
\[ \frac{4}{\omega_c} \int_0^{\alpha_c} d\alpha \sqrt{\epsilon^2 - \Delta_0^2 \sin^2(2\alpha)} = 2\pi n . \tag{20} \]
where \( n > 0 \). The first integral in Eq. (19) disappears because the turning points of the momentum \( p \) are not reached: the particle cannot go far along the trajectory Eq. (13) and remains localized on a given trajectory at distances \( s \sim r_L(\epsilon/\Delta_0) \) smaller than the Larmor radius \( r_L \). Note also that the contribution from \( \chi_v \) vanishes identically because the particle after being Andreev reflected transforms into a hole which returns to the starting point along the same trajectory. Using the substitution \( \sin x = (\Delta_0/\epsilon) \sin(2\alpha) \) we find
\[ \int_0^{\alpha_c} d\alpha \sqrt{\epsilon^2 - \Delta_0^2 \sin^2(2\alpha)} = \frac{\Delta_0}{2} \left[ E \left( \frac{\epsilon}{\Delta_0} \right) - \left( 1 - \frac{\epsilon^2}{\Delta_0^2} \right) K \left( \frac{\epsilon}{\Delta_0} \right) \right] \]
where \( K(k) \) and \( E(k) \) are the full elliptic integrals of the first and second kind, respectively. Applying the Bohr–Sommerfeld quantization rule Eq. (19) we obtain
\[ \frac{2\Delta_0}{\omega_c} \left[ E \left( \frac{\epsilon}{\Delta_0} \right) - \left( 1 - \frac{\epsilon^2}{\Delta_0^2} \right) K \left( \frac{\epsilon}{\Delta_0} \right) \right] = 2\pi n . \tag{21} \]

These states are degenerate with the same degree as in the normal state: for each \( n \), there are \( \Phi/2\Phi_0 = N_e/2 \) states for particles and \( N_e/2 \) states for holes, where \( \Phi \) is the total magnetic flux through the superconductor, and \( N_e \) is the total number of vortices.

Consider \( \epsilon \ll \Delta_0 \), expanding in small \( k \)
\[ E(k) = \frac{\pi}{2} \left( 1 - \frac{k^2}{4} \right) , \quad K(k) = \frac{\pi}{2} \left( 1 + \frac{k^2}{4} \right) \]
we find from Eq. (21)
\[ \epsilon_n = \pm \sqrt{4\Delta_0 \omega_c n} . \tag{22} \]
Eq. (22) agrees with the result of Refs. 10, 11.

B. Extended states

If \( |\epsilon| > \Delta_0 \), we get for the Landau gauge Eq. (10)
\[ p_x = \text{const and} \]
\[ \oint p \, dy = \oint p_y \, dy = 2 \int_{y_1}^{y_2} \sqrt{p_x^2 - (p_x + eHy/c)^2} \, dy = \pi c p_x^2 / eH \]

The turning points \( y_{1,2} \) correspond to the values of Larmor radius where \( p_x + eHy_{1,2}/c = \pm p_x \). The corresponding trajectory is a closed circle where \( \alpha \) varies by \( 2\pi \). The second integral in Eq. (13) gives
\[ \int_0^{2\pi} \frac{d\alpha}{\omega_c} \sqrt{\epsilon^2 - \Delta_0^2 \sin^2(2\alpha)} = 4e \left( \frac{\Delta_0}{\epsilon} \right) . \tag{23} \]

The quantization rule (19) yields
\[ \pm \frac{2e}{\pi} \frac{\Delta_0}{\epsilon_n} = \omega_c n + \frac{p_x^2}{2m} - E_F . \tag{24} \]
For an \( s \)-wave superconductor we get, in particular,
\[ \pm \sqrt{\epsilon_n^2 - \Delta_0^2} = \omega_c n + \frac{p_x^2}{2m} - E_F . \tag{25} \]

IV. EFFECTS OF THE PERIODIC POTENTIAL

A. Bloch functions

At low magnetic fields \( H \ll H_c2 \), one can consider that the particle trajectory always passes far from cores. The oscillating part of the order parameter comes mostly from the superconducting velocity. The corresponding Doppler energy \( \eta = p_x v_t \) is of the order of \( \Delta_0 \sqrt{H/H_c2} \). This periodic potential can split the energy spectrum into bands. Eqs. (9, 14) or the quasiclassical version Eq. (12) are invariant under the magnetic translations by periods of the regular vortex lattice. Consider the particle-like equations (9) or (12). The particle-like operator of magnetic translations in a homogeneous field is [1]
\[ \hat{T}(\mathbf{R}_l) = \exp \left[ -i\mathbf{R}_l \left( \hat{\mathbf{p}} + \frac{e}{c} \mathbf{A} \right) \right] \]  

(26)

where \( \hat{\mathbf{p}} = -i\nabla \) is the canonical momentum and \( \mathbf{R}_l \) is a vector of the vortex lattice. Its zero-field version corresponds to a shift

\[ \hat{T}_0(\mathbf{R}_l) f(\mathbf{r}) = \exp \left[ -i\mathbf{R}_l \mathbf{p} \right] f(\mathbf{r}) = f(\mathbf{r} - \mathbf{R}_l) . \]

The operator \( \hat{T}(\mathbf{R}_l) \) commutes with the Hamiltonian because \( \mathbf{v}_s \) and \( \Delta \) are periodic in the vortex lattice and the commutator

\[ \left[ \left( \hat{\mathbf{p}} + \frac{e}{c} \mathbf{A} \right)_i , \left( \hat{\mathbf{p}} - \frac{e}{c} \mathbf{A} \right)_j \right] = 0 . \]

Since \( P_+ \) does not change under the action of the operator Eq. (26), magnetic translations for functions \( \hat{\phi} \) in Eq. (13) are equivalent to usual translations \( \hat{T}_0(\mathbf{R}_l) \) in space for a fixed kinetic momentum of the particle.

When deriving these expressions we have used the periodicity of \( \mathbf{v}_s \) and the fact that the trajectory depends on \( y \) only through \( y + c p_x / e H \). The turning point \( y_1 \) is thus shifted by \( lb_0 \) when \( p_x \) is shifted by \( -eHb_0/c \).

The functions Eq. (27) can be used to construct two independent basis functions

\[ \hat{\Phi}^+_n(k_x, k_y; x, y) = \sum_l e^{ik_y 2lb_0} \hat{T}_y(2lb_0) \hat{\Psi}_n(k_x; x, y) , \]

(30)

\[ \hat{\Phi}^-_n(k_x, k_y; x, y) = \sum_l e^{ik_y (2l+1)b_0} \times \hat{T}_y((2l+1)b_0) \hat{\Psi}_n(k_x; x, y) \]

(31)

with even and odd translations, respectively. Starting from Eq. (30) we replace \( p_x \) with \( k_x \). The functions \( \hat{\Phi}^\pm \) belong to the same energy. The wave vector \( k_y \) has an arbitrary value, we shall establish it later. The generic translation is \( 2b_0 \) which is the size of the magnetic unit cell along the \( y \) axis. The magnetic unit cell contains two vortices because the superconducting magnetic flux quantum correspond to one half of the \( 2\pi \) phase circulation of a single–particle wave function. The functions Eqs. (30, 31) have the Bloch form

\[ \hat{T}_x(la_0) \hat{\Phi}^\pm(k_x, k_y) = (\pm 1)^l e^{-ik_x la_0} \hat{\Phi}^\pm(k_x, k_y) , \]

\[ \hat{T}_y(2mb_0) \hat{\Phi}^\pm(k_x, k_y) = e^{-i k_y 2mb_0} \hat{\Phi}^\pm(k_x, k_y) . \]

(32)

We omit the coordinates \( x, y \) in the arguments of \( \hat{\Phi}^\pm \) for brevity. The functions \( \hat{\Phi}^\pm \) transform into each other under odd translations

\[ \hat{T}_y((2m+1)b_0) \hat{\Phi}^\pm(k_x, k_y) = e^{-ik_y(2m+1)b_0} \hat{\Phi}^\mp(k_x, k_y) . \]

(33)

Since the magnetic translation \( \hat{T}_y(lb_0) \) commutes with the Hamiltonian, the energy is degenerate with respect to \( k_y \). This degeneracy is spurious, however. To see this, consider the transformations Eqs. (32, 33). For \( l = 1 \), the transformed function in Eq. (32) is periodic in \( k_x \) with the period \( 2\pi/a_0 \). This period corresponds to the shift of the center of orbit \( y_0 = c k_x/eH \) by one size of the magnetic unit cell \( 2b_0 \). Obviously, the transformation Eq. (33) should also have the same symmetry. For one magnetic unit cell, a shift by \( 2b_0 \) (i.e., for \( m = 1 \)) along the \( y \) axis should combine with one period along the \( x \) axis. The period in \( k_y \) is \( \pi/b_0 \); it should thus correspond to the shift of the coordinate \( x_0 \) by \( a_0 \). This fixes

\[ k_y = eHx_0/c . \]

(35)

The energy depends on the position of the trajectory within the vortex unit cell through the Doppler energy \( \eta \). The energy \( \epsilon(k_x, k_y) \) has a band structure due to periodicity of \( \eta \); it is periodic with the periods \( eHb_0/c = \pi/a_0 \) and \( eHb_0/c = \pi/b_0 \) in \( k_x \) and \( k_y \), respectively, which correspond to shifts of the center of orbit by one vortex unit cell vector.

**B. Spectrum**

Consider energies \( \epsilon \gg \Delta_0 \sqrt{H/H_{c2}} \). Applying the quasiclassical approximation to Eq. (18) we find

\[ \hat{\phi} = \hat{C} \exp \left[ \pm i A(s) \right] \]

(36)

where the action is

\[ A(s) = \int_{s_1}^{s} \sqrt{\left( \epsilon - \bar{\eta} \right)^2 - \Delta_0^2 \sin^2 2(s - s_1)/\nu_\perp} . \]

(37)
The quasiclassical approximation is justified because the wave vector \( \partial A / \partial s \sim e / v_F \) is much larger than the inverse characteristic scale \( 1 / a_0 \) of variation of the potential \( \eta \) for \( \epsilon \gg \Delta_0 \sqrt{H / H_{c2}} \). The function \( \eta = P_2 v_1 \) is taken at the trajectory which is a part of a circle specified by the coordinates of its center \( x_0 \) and \( y_0 = -c p_x / e H \); they determine the position of the trajectory within the vortex unit cell.

For energies \( \Delta_0 \sqrt{H / H_{c2}} \ll \epsilon < \Delta_0 \), quasiparticle trajectory is extended over distances of the order of \( r_L (\epsilon / \Delta_0) \). The quantization rule defines the energy
\[
\int_{s_1}^{s_2} \sqrt{(\epsilon - \eta)^2 - \Delta_0^2 \sin^2(2\alpha)} ds / v_L = \pi n .
\]
Here \( s_1 \) and \( s_2 \) are the turning points. Expanding in small \( \eta \ll \epsilon \) we find
\[
m \int_{y_1}^{y_2} \sqrt{\epsilon^2 - \Delta_0^2 \sin^2(2\alpha)} dy / p_y - m \int_{y_1}^{y_2} \eta(x, y) \epsilon dy / p_y = \pi n .
\]
Here \( \eta(x, y) = (k_x + e H y / c) v_{sx} + p_y v_{sy} \) while \( y_1 \) and \( y_2 \) are the turning points which correspond to vanishing of the square root: \( k_x + e H y_{1,2} / c = p_\perp \sin(2\alpha_0) \). The energy \( \epsilon_n \) is a function of \( k_x \) and \( x_0 \) which determine the location of the particle trajectory with respect to vortices. The energy is thus periodic in \( k_x \) with the period \( e H b_0 / c \) and in \( x_0 \) with a period \( a_0 \) when the center is shifted by one period of the vortex lattice.

The \( \eta \) term under the second integral in Eq. (33) oscillates rapidly over the range of integration and mostly averages out. The remaining contribution determines the variations of energy with \( k_x \) and \( x_0 \) and can be estimated as follows. For example, variation of action for \( \epsilon \ll \Delta_0 \) due to a change in energy \( \delta \epsilon \) is
\[
\delta A \sim (\delta \epsilon / v_F)(\epsilon / \Delta_0) r_L \sim (\epsilon \delta \epsilon) / (\Delta_0 \omega_c) .
\]

Variation of action due to a shift of the center of orbit by a distance of the order of the lattice period is \( \delta A \sim (a_0 / v_F) \eta \sim 1 \). The corresponding energy variation is thus \( \delta \epsilon \sim \Delta_0 \omega_c / \epsilon \). Since \( x_0 \) is coupled to \( k_y \) through Eq. (33) the energy can be written as
\[
\epsilon_n (k_x, k_y) = \sqrt{4 \Delta_0 \omega_c [n + \eta_0 (k_x, k_y)]}
\]
where \( \eta_0 \sim 1 \) can depend on energy. The energy Eq. (40) has a band structure; the bandwidth is of the order of the distance between the Landau levels. It is small as compared to the energy itself. It is clear that the spectrum for energies \( \epsilon \gtrsim \Delta_0 \) can also be obtained from Eqs. (22), (24) and (25) through the substitution \( n \rightarrow n + \eta_0 (k_x, k_y) \).

One can check that, for given \( k_x \) and \( k_y \), the quasiparticle states with different principle quantum numbers indeed concentrate near the levels determined by Eq. (22) if \( \epsilon \gg \Delta_0 \sqrt{H / H_{c2}} \). This is because the contribution to the action from the oscillating potential picked up on the distance of the order of the size of the unit cell \( a_0 \) is \( p_F a_0 / v_F \sim 1 \). The discrete structure of the levels Eq. (22) would be preserved if the contribution to the action from the oscillating potential changes by an amount much less than unity for transitions between the neighboring levels. For an energy \( \epsilon \), the distance between the neighboring levels is \( \delta \epsilon \sim \Delta_0 \omega_c / \epsilon \). This corresponds to a change in the length of the trajectory by
\[
\delta s_c \sim v_F \delta \epsilon / \omega_c \sim \delta \epsilon / \epsilon .
\]
The variation in the length is much smaller that the intervortex distance \( a_0 \sim \xi \sqrt{H_{c2}/H} \) if \( \epsilon \gg \Delta_0 \sqrt{H / H_{c2}} \), and the action changes by a quantity much less than 1. It shows that the distance between the levels with different \( n \) is indeed determined by Eq. (22).

The situation changes for smaller energies \( \epsilon \lesssim \Delta_0 \sqrt{H / H_{c2}} \). The centers of bands will deviate strongly from positions determined by Eq. (22) due to a considerable contribution from the periodic vortex potential to the turning points in Eq. (33). Moreover, the applicability of the quasiclassical approximation, i.e., of Eq. (28) itself is violated; the potential \( \eta \) is strong enough to cause large deformations of the energy spectrum. As was shown in Ref. 8 some states can even become effectively localized near the vortex cores.

V. INDUCED TRANSITIONS BETWEEN THE LANDAU LEVELS

Vortex motion induces transitions between the quasiparticle states. The transitions between low-energy core states with \( \epsilon \ll \Delta_0 \sqrt{H / H_{c2}} \) were considered in Ref. 1. It was shown that the vortex core states determine the vortex response to d.c. and a.c. electric fields. For temperatures \( T < \sqrt{H / H_{c2}} \) extended states dominate. It was found in Ref. 8 that the vortex response is determined by what was called “collective modes” which are associated with the electron states outside the vortex cores. In this Section we demonstrate that these collective modes are nothing but transitions between the electronic states Eq. (10) specified by the same quasimomentum but by different principal quantum numbers \( n \). We start with noting that the transition matrix elements are proportional to \( \langle \Phi_n (k_j) | \nabla \hat{H}_1 | \Phi_m (k_j) \rangle \) where the Hamiltonian \( \hat{H}_1 \) is composed of \( \Delta \hat{p} \) and \( \eta \) while \( k \) is the quasimomentum. \( \hat{H}_1 \) is periodic with the period of the vortex lattice thus the transitions are possible between the quasimomenta which differ by vectors of the reciprocal lattice. Since the band energy is periodic in the quasimomentum with the periods of the reciprocal lattice, the energy difference for these transitions corresponds to the energy difference for states with the same quasimomentum but with different quantum numbers \( n \). For \( \eta_0 \ll n \) the transition
energy is just the distance between the Landau levels: 
\[ \delta \epsilon_n (k_x, k_y) = \delta \epsilon_n \] determined by Eqs. (21, 24) or (25).
For low energies in a \( d \)-wave superconductor, one has
\[ \delta \epsilon (k_x, k_y) = 2 \Delta_0 \omega_c / \epsilon_n \] in accordance with Eq. (22).
Consider transitions between the levels which are ex-
cited by oscillating the vortices in more detail. We use the
microscopic kinetic-equation approach which has been
applied earlier for \( s \)-wave superconductors in Ref. 13.
The kinetic equations for the distribution functions \( f_1 \) and \( f_2 \) have the form

\[ \left\{ \begin{align*}
\left[ e (v_F \cdot E) g_+ + \frac{1}{2} \left( f_+ \frac{\partial \Delta_p}{\partial t} + f_+^\dagger \frac{\partial \Delta_p^*}{\partial t} \right) \right] \frac{\partial f_1}{\partial \epsilon} &+ \left( v_F \cdot \nabla \right) (g_+ f_2) + g_- \frac{\partial f_1}{\partial \epsilon} \\
+ \left[ e \left[ v_F \times H \right] g_- - \frac{1}{2} \left( f_- \nabla \Delta_p^* + f_-^\dagger \nabla \Delta_p \right) \right] \frac{\partial f_1}{\partial p} &+ \frac{1}{2} \left( f_- \frac{\partial \Delta_p^*}{\partial p} + f_-^\dagger \frac{\partial \Delta_p}{\partial p} \right) \cdot \nabla f_1 = J
\end{align*} \right. \] (41)

and
\[ g_- (v_F \cdot \nabla) f_1 = 0. \] (42)

Here
\[ \hat{g}^{R(A)} = \left( \begin{array}{cc}
g^{R(A)} & f^{R(A)} \\
-f^{R(A)} & -g^{R(A)}
\end{array} \right) \]
are the retarded (advanced) quasiclassical Green functions, and \( \hat{g}_- = \left( \hat{g}^R - \hat{g}^A \right) / 2. \)
For an extended state with an energy \( \epsilon > \Delta_p \), the
particle trajectory crosses many vortex unit cells at various
distances from vortices. Since the distribution function
\( f_1 \) is constant along the trajectory according to Eq. (42),
it should be also independent of the impact parameter
(i.e., of the distance from the trajectory to the vortex).
We thus look for a distribution function \( f_1 \) which is
independent of coordinates. One can then omit the last term
in the l.h.s. of Eq. (41). Let us average Eq. (41) over an
area which contains many vortex unit cells but has a size
small compared with the Landor radius, \( a_0 \ll r \ll r_L \).
Since \( r \ll r_L \) the momentum \( p \) is still an integral of
motion. We have (compare with Ref. 22)

\[ \frac{1}{2} \text{Tr} \int_{S_0} g_- \frac{\partial f_1}{\partial t} \, d^2 r - \frac{1}{2} \text{Tr} \int_{S_0} d^2 r \hat{g}_- \left( \nabla \hat{H} \right) \cdot \frac{\partial f_1}{\partial \epsilon} - \int_{S_0} J d^2 r \\
= \frac{1}{2} \text{Tr} \int_{S_0} d^2 r \hat{g}_- \left( v_L \cdot \nabla \hat{H} \right) \cdot \frac{\partial f_1^{(0)}}{\partial \epsilon}. \]

Here \( \text{Tr} \) is the trace in the Nambu space, \( S_0 = \Phi_0 / B \) is
the area of the vortex unit cell,
\[ J = -\frac{1}{\tau} \left[ \langle f_1 g_- \rangle - \langle f_1 g_- \rangle \right] g_- \\
- \left( \langle f_1^\dagger \rangle - \langle f_1 f_1^\dagger \rangle \right) f_- + \left( \langle f_1 \rangle - \langle f_1 f_1^\dagger \rangle \right) f_1. \]
Using the identity
\[ \frac{1}{2} \text{Tr} \int_{S_0} d^2 r \left[ \left( \nabla \hat{H} \right) \hat{g}_- \right] = \pi \left[ \mathbf{z} \times v_\perp \right] \]
derived in Ref. 19 we find
\[ \left. \frac{1}{2} \text{Tr} \int_{S_0} d^2 r \left[ \left( \nabla \hat{H} \right) \hat{g}_- \right] = \pi \left[ \mathbf{z} \times v_\perp \right] \right|_{\mathbf{z} = \mathbf{z}_0} \]
and
\[ g_- (v_F \cdot \nabla) f_1 = 0. \] (42)

We shall concentrate on energies \( \epsilon \gg \Delta \sqrt{H/H_c} \). In
the leading approximation
\[ g_- = \frac{\epsilon}{\sqrt{\epsilon^2 - \Delta^2}} \Theta \left[ \epsilon^2 - \Delta^2 (\alpha) \right], \]
\[ f_- = \frac{\Delta (\alpha)}{\sqrt{\epsilon^2 - \Delta^2 (\alpha)}} \Theta \left[ \epsilon^2 - \Delta^2 (\alpha) \right]. \]
We have
\[ \langle f_1 \rangle = \langle f_1 g_- \rangle = 0; \quad \langle f_1 f_- \rangle = \langle f_1 f_1^\dagger \rangle = 0. \]
For a \( d \)-wave superconductor also \( \langle f_- \rangle = \langle f_1 f_1^\dagger \rangle = 0. \)

In the collision integral, the main contribution for
\( \epsilon \gg \Delta \sqrt{H/H_c} \) comes from the delocalized states. In-
deed, including contributions from the bound states in
the core \[ \hat{H}_n \] with energies \( E_n (b) \) we would have
\[ \int_{S_0} J d^2 r \approx -S_0 \left[ \sum_n \frac{p_{\perp} \omega_c}{\tau_n} \int \delta (\epsilon - E_n) \, db + \langle g_- \rangle \right] f_1 \]
where \( b \) is the impact parameter. The first term in square
brackets comes from the core states. Since \( \tau_n \sim \tau \) and
\( b \sim \xi \sqrt{H/H_c} \), the core contribution is of the order
of \( \tau^{-1} \sqrt{H/H_c} \). The delocalized states, however, give
\( \langle \epsilon / \Delta_n \rangle \tau^{-1} \) which is much larger than the first term.
Neglecting the core contribution we find
\[ J = -\frac{1}{\tau} \langle g_- \rangle g_- f_1. \]

Let us put
\[ f_1 = -\frac{\partial f_1^{(0)}}{\partial \epsilon} \left[ (\mathbf{u} \times \mathbf{p}_\perp) \cdot \mathbf{z} \right] \gamma_0 + (\mathbf{u} \cdot \mathbf{p}_\perp) \gamma_H \] (44)
The functions \( \gamma_{0,H} \) satisfy the following set of equations
\[ \frac{\partial \gamma_{\alpha}}{\partial \alpha} - \gamma_{H} - V(\alpha) \gamma_{O} + 1 = 0 \]
\[ \frac{\partial \gamma_{H}}{\partial \alpha} + \gamma_{O} - V(\alpha) \gamma_{H} = 0 \] (45)

which is derived from Eq. (43). Here

\[ V(\alpha) = \frac{-i(\omega + (g_{-})/\tau)}{\omega_{c}}. \] (46)

The general solution of Eqs. (45) can be obtained by putting \( W_{\pm} = \gamma_{H} \pm i\gamma_{O} \). We have

\[ \frac{\partial W_{\pm}}{\partial \alpha} + iW_{\pm} - V(\alpha) W_{\pm} \pm i = 0. \]

The solution is

\[ W_{\pm} = \left[ C_{\pm} + i \int_{0}^{\alpha} e^{\mp i\alpha' - F(\alpha')} d\alpha' \right] e^{\pm i\alpha + F(\alpha)} \] (47)

where

\[ F(\alpha) = \int_{0}^{\alpha} V(\alpha') d\alpha'. \]

The constant \( C_{\pm} \) is found from the condition of periodicity \( W(\alpha) = W(\alpha + \pi/2) \)

\[ C_{\pm} = \exp \left\{ F(\pi/2) \right\} \int_{0}^{\pi/2} \exp \left\{ \mp i\alpha - F(\alpha) \right\} d\alpha \]
\[ - 1 - \exp \left\{ \mp i\pi/2 + F(\pi/2) \right\}. \] (48)

In the limit \( \tau \to \infty \), the solution

\[ \gamma_{H} = (W_{+} + W_{-})/2; \quad \gamma_{O} = (W_{+} - W_{-})/2i \]

has poles when

\[ F(\pi/2) = \frac{\pi i}{2}(1 + 2M) \] (49)

where \( M \) is an integer. If \( \epsilon > \Delta_{0} \), one obtains resonances at

\[ \frac{\omega}{\omega_{c}} \int_{0}^{2\pi} \frac{\epsilon}{\sqrt{\epsilon^{2} - \Delta^{2}(\alpha)}} d\alpha = 2\pi (1 + 2M). \]

The lowest frequency \( M = 0 \) exactly corresponds to the condition

\[ \omega = (d\epsilon_{n}/dn) \]

where \( d\epsilon_{n}/dn \) is the distance between the Landau levels determined by Eq. (49).

Note that, for an s-wave superconductor, Eqs. (45) has the form

\[ \gamma_{H} + V\gamma_{O} = 1 \]
\[ \gamma_{O} - V\gamma_{H} = 0 \] (50)

where

\[ \frac{\omega}{\omega_{c}} \int_{0}^{2\pi} \frac{\epsilon}{\sqrt{\epsilon^{2} - \Delta^{2}(\alpha)}} d\alpha = 2\pi (1 + 2M). \]

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where

\[ \omega = (d\epsilon_{n}/dn) \]

with

\[ J = -\frac{1}{\tau} (f_{1} - f_{1}) \Theta \left[ \epsilon^{2} - \Delta_{0}^{2} \right]. \]

One has from Eq. (50)

\[ \gamma_{H} = \frac{1}{1 + V^{2}}, \quad \gamma_{O} = \frac{V}{1 + V^{2}}. \]

The resonances appear when \( \omega_{c}\tau \gg 1 \); the poles correspond to \( V = \pm i \) so that

\[ \omega = \omega_{c} \sqrt{\epsilon^{2} - \Delta_{0}^{2}} = \frac{d\epsilon_{n}}{dn} \]

where \( \epsilon_{n} \) is determined by Eq. (22).

### A. Low energies

For energies \( \epsilon < \Delta_{0} \), the resonance condition Eq. (49) is not just the distance between the Landau levels determined by Eq. (21). One has from Eq. (21)

\[ \frac{d\epsilon_{n}}{dn} \int_{-\alpha_{c}}^{\alpha_{c}} \frac{\epsilon}{\sqrt{\epsilon^{2} - \Delta^{2}(\alpha)}} d\alpha = \pi \omega_{c} \]

where \( \Delta(\alpha_{c}) = \epsilon \). At the same time, Eq. (40) gives the lowest resonant frequency

\[ \frac{\omega}{\omega_{c}} N \int_{-\alpha_{c}}^{\alpha_{c}} \frac{\epsilon}{\sqrt{\epsilon^{2} - \Delta^{2}(\alpha)}} d\alpha = 2\pi \]

where \( N \) is the number of gap nodes (\( N = 4 \) for a d-wave superconductor). We see that the resonance occurs at

\[ N\omega = 2 \frac{d\epsilon_{n}}{dn}. \] (51)

When the vortex oscillates, all \( N \) nodes participate in exciting quasiparticles which accounts for the factor \( N \) in the l.h.s. of Eq. (51). This is similar to the process of multi-photon absorption. The factor 2 in the r.h.s. is explained by noting that states with momentum directions \( \alpha \) and \( \alpha + \pi \) are simultaneously excited.

Solution of Eqs. (47, 48) for \( \Delta_{0} \sqrt{H/H_{c2}} \ll \epsilon \ll \Delta_{0} \) was obtained in Ref. [1]. For the main region of angles, \( |\alpha| > \alpha_{c} = \epsilon/2\Delta_{0} \), it is

\[ \gamma_{O} = A \cos \alpha + B \sin \alpha \]
\[ \gamma_{H} = 1 - A \sin \alpha + B \cos \alpha \] (52)

with

\[ A = \frac{e^{\lambda} \sinh \lambda}{2 \sinh^{2} \lambda + 1}, \quad B = \frac{e^{-\lambda} \sinh \lambda}{2 \sinh^{2} \lambda + 1}. \] (53)
Here we use that $F(\pi/2 - \alpha) = 2\lambda - F(\alpha)$ where
\[ \lambda = F(\alpha_c) ; F(\pi/2) = 2\lambda. \]
One has
\[ \lambda = \frac{-i\omega + 1/\tau_{eff} \pi |\epsilon|}{\omega_c} \]
where $1/\tau_{eff} = |\epsilon|/\Delta_0 \tau$ since $\langle g_\tau \rangle = |\epsilon|/\Delta_0$. Note that a $\tau$-approximation was used in Ref. [1] for the collision integral. To get the present expression for $\lambda$ from that obtained in Ref. [3] one has to replace $1/\tau$ with $1/\tau_{eff}$.

For $\tau \to \infty$, the response Eqs. (52), (53) has poles at $\omega = (2M + 1) \pi/4$, i.e., for
\[ \omega = (2M + 1)E_0(\epsilon); \quad E_0(\epsilon) = \Delta_0 \omega_c / |\epsilon|. \] (54)
We have for $M = 0$
\[ \omega = \frac{1}{2} \frac{\partial m_0}{\partial n} \]
where $\epsilon_n$ is determined by Eq. (28). This condition agrees with Eq. (53).

These resonances were first predicted in Ref. [3]. Note the different numerical factor in Eq. (54) as compared to Ref. [3]; this is because a simplified version of $V(\alpha)$ has been used in Ref. [3]. The main effect of resonances is that vortices experience a considerable friction force Eq. (53) even in a superclean case $\omega \tau_{eff} \gg 1$.

**B. Vortex friction**

A vortex moving with a velocity $v_L$ experiences a force from the environment
\[ \mathbf{F}_{env} = -Dv_L - D'[v_L \times \mathbf{z}]. \] (55)
According to Ref. [1], the delocalized states contribute to the friction constant
\[ D_{del} = \pi N \left\langle \int_{\gamma_0} \frac{df^{(0)}(\epsilon)}{2} \right\rangle, \] (56)
where $\langle \cdots \rangle_\alpha$ is an average over $d\alpha$. The factor $D'$ is determined by the same expression where $\gamma_0$ is replaced with $\gamma_H$.

The presence of resonances makes the dissipative constant $D_{del}$ finite even in the superclean limit $\omega_c \tau \to \infty$. Indeed, for an $s$-wave case,
\[ \gamma_0 = \frac{\pi E}{2} \left[ \delta(\omega - E) + \delta(\omega + E) \right] \]
where $E = \omega_c \sqrt{1 - \Delta_0^2/\epsilon^2}$. The friction constant becomes
\[ D_{del} = \pi^2 N \Delta_0 \frac{\omega^2/\omega_c^2}{(1 - \omega^2/\omega_c^2)^{3/2}} \frac{df^{(0)}(\epsilon_0)}{d\epsilon} \] (57)
where $\epsilon_0 = \Delta_0 / \sqrt{1 - \omega^2/\omega_c^2}$. A more detailed discussion of the resonant vortex friction for a $d$-wave superconductor at low temperatures can be found in Ref. [4].

**VI. QUASIPARTICLE CONDUCTIVITY**

Consider the a.c. quasiparticle conductivity which can be observed if vortices are pinned. The distribution function can be found from Eqs. (11, 12). We are looking again for the distribution function $f_1$ which is independent of coordinates. One has
\[ \epsilon \left( v_F \cdot \mathbf{E} \right) g_0 - \frac{\partial f^{(0)}}{\partial \epsilon} + \left( v_F \cdot \nabla \right) (g_0 f_2) + \frac{\partial f_1}{\partial t} \]
\[ + \left[ \frac{\epsilon}{c} \left( v_F \times \mathbf{H} \right) g_0 - \frac{1}{2} \left( f_- \nabla \Delta_p^* + f_+ \nabla \Delta_p \right) \right] \cdot \frac{\partial f_1}{\partial \mathbf{p}} = J. \]

We omit the time derivatives of $\Delta$ because vortices do not move. After averaging over the vortex lattice we get
\[ \pi \left[ \mathbf{z} \times v_L \right] \cdot \frac{\partial f_1}{\partial \mathbf{p}} - \frac{\partial f^{(0)}}{\partial \epsilon} \int_{S_0} g_- d^2r + \int_{S_0} J d^2r \]
\[ = \epsilon \left( v_F \cdot \mathbf{E} \right) \frac{\partial f^{(0)}}{\partial \epsilon} \int_{S_0} g_- d^2r. \]

For the distribution function in the form
\[ f_1 = -\frac{\partial f^{(0)}}{\partial \epsilon} \left[ (\mathbf{E} \cdot \mathbf{p} \times \mathbf{z}) \gamma_0 - (\mathbf{E} \times \mathbf{p} \times \mathbf{z}) \gamma_H \right] \] (58)
we obtain
\[ \frac{\partial \gamma_0}{\partial \alpha} - \gamma_H - V(\alpha) \gamma_0 = -\frac{e}{m\omega_c} g_-, \]
\[ \frac{\partial \gamma_H}{\partial \alpha} + \gamma_0 - V(\alpha) \gamma_H = 0. \] (59)

The solution is
\[ W_\pm = \left[ \pm \frac{i\epsilon}{m\omega_c} \int_0^\alpha g_- e^{i\alpha' - F(\alpha')} d\alpha' + C_\pm \right] e^{\pm i\alpha + F(\alpha)}. \] (60)

The periodicity condition $W(0) = W(\pi/2)$ gives for $\epsilon \ll \Delta_0$
\[ C_\pm = \frac{e}{m} \frac{1}{-i\omega + \langle g_\tau \rangle / \tau} \left[ \sinh \lambda \cos \lambda + i \left( 1 - \frac{\cosh \lambda}{\cosh 2\lambda} \right) \right] \] (61)
where $\lambda = F(\alpha_c)$.

The quasiparticle current is
\[ j^{(qp)} = -\nu(0) e \int v_F g_- f_1 d\Omega = \sigma^{(qp)} E + \sigma_H^{(qp)} [E \times \mathbf{z}] \]
where
\[ \sigma_{\alpha H}^{(qp)} = \frac{\nu(0)e}{2} \int v_{\perp} p g(\tilde{\gamma} - \gamma_0, H) \frac{d\Omega}{4\pi} \frac{d\tilde{f}^{(0)}}{d\epsilon} d\epsilon. \] (62)
Calculating the integral over d\alpha in Eq. (62) we find
\[ \langle \tilde{\gamma}_O g_\alpha \rangle = \pi e \frac{\langle g_- \rangle^2}{4m\omega_c} \left( \lambda - \frac{\tan \lambda}{\tan^2 \lambda + 1} \right), \] (63)
\[ \langle \tilde{\gamma}_H g_\alpha \rangle = \pi e \frac{\langle g_- \rangle^2}{4m\omega_c} \frac{\tan^2 \lambda}{\tan^2 \lambda + 1}, \] (64)
and
\[ \lambda = \frac{\pi}{4} \langle g_- \rangle - i\omega + \langle g_- \rangle / \tau \omega_c^{-1}. \]
Since \lambda is independent of the momentum directions, the quasiparticle conductivity becomes
\[ \sigma_{\alpha}^{(qp)} = Ne \int \langle g_- \rangle \frac{d\tilde{f}^{(0)}}{d\epsilon} \frac{d\epsilon}{2}, \] (65)
and the same expression for \sigma_{H}^{(qp)} where \langle g_- \rangle is replaced with \langle g_- \tilde{\gamma}_H \rangle.
Consider first the superclean limit \omega_c \tau_{eff} \gg T/T_c, such that Re \lambda \ll 1. For \omega \tau_{eff} \gg 1, where \tau_{eff} \sim (T_c/T) \tau the response Eq. (63) has resonances at \lambda = (2M + 1)\pi/4 which is again the condition of Eq. (54):
\[ \langle g_- \tilde{\gamma}_H \rangle = \frac{4e}{\pi m\omega_c} \sum_M \frac{\langle g_- \rangle^2 E_0(\epsilon)}{(2M + 1)^2} \delta[\omega - (2M + 1)E_0(\epsilon)] + \frac{ie \langle g_- \rangle}{m\omega}. \]
The dissipative part of the quasiparticle conductivity becomes
\[ \text{Re} \sigma_{\alpha}^{(qp)} = \frac{2Ne^2\omega_c^2\Delta_0}{\pi mT} \sum_{M=0}^\infty \cosh^{-2} \left[ \frac{\Delta_0 \omega_c(2M + 1)}{2T |\omega|} \right]. \]
It is
\[ \text{Re} \sigma_{\alpha}^{(qp)} = \frac{2Ne^2\omega_c}{\pi m\omega_c^2} \]
for \omega \gg E_g where \( E_g = \Delta_0 \omega_c / 2T \), but decreases exponentially for smaller \omega \ll E_g. The dissipative part for \omega \ll E_g is mostly due to \tau. Since \lambda \ll 1 one has in this limit
\[ \text{Re} \sigma_{\alpha}^{(qp)} = \frac{\pi Ne^2}{3m\omega_c} \int \frac{d\tilde{f}^{(0)}}{d\epsilon} \frac{d\epsilon}{2} \frac{\langle g_- \rangle}{2} \frac{\lambda}{\tau \omega_c^2} = \frac{5\pi^7 T^4}{24m^2w^2\tau^2\Delta_0^4}. \]
On the moderately clean side, such that \omega_c \tau_{eff} \ll T/T_c one has Re \lambda \gg 1. The conductivity has a Drude form
\[ \sigma_{\alpha}^{(qp)} = \frac{Ne^2}{m} \int \frac{d\tilde{f}^{(0)}}{d\epsilon} \frac{d\epsilon}{2} \left[ -i\omega + \frac{\langle g_- \rangle}{\tau} \right]. \]
The Hall conductivity does not contain contributions from poles because the resonances in Eq. (64) with \( M > 0 \) cancel those with \( M < 0 \). For \omega \ll \langle g_- \rangle / \tau one has
\[ \langle \tilde{\gamma}_H g_\alpha \rangle = \frac{e\tau}{m} \frac{\tan^2 \omega_c}{\tan^2 \omega_c + 1}. \]
where \( w(\epsilon) = \pi \langle g_- \rangle / 4 \omega_c \tau \). The conductivity is
\[ \sigma_{H}^{(qp)} = \frac{Ne^2\omega_c}{m} \int_0^\infty \frac{d\tilde{f}^{(0)}}{d\epsilon} \frac{\tan^2 \omega_c}{\tan^2 \omega_c + 1} \frac{d\epsilon}{\tau}. \]
If \( T \ll \Delta_0 \omega_c \tau \) one has \( \epsilon \sim T \) and \( w \ll 1 \). In this limit
\[ \sigma_{H}^{(qp)} = \frac{Ne^2\omega_c}{m} \int_0^\infty \frac{d\tilde{f}^{(0)}}{d\epsilon} \frac{\tan^2 \omega_c}{\tan^2 \omega_c + 1} \frac{d\epsilon}{\tau}. \]
If \( T \gg \Delta_0 \omega_c \tau \) and \( \omega_c \tau \ll 1 \) the integral is determined by \( w \sim 1 \) and \( \epsilon \sim \Delta_0 \omega_c \tau \ll T \). We have
\[ \sigma_{H}^{(qp)} = \frac{0.39Ne^2\omega_c^2\Delta_0}{Tm}. \]

VII. CONCLUSIONS

We discuss and analyze the “Landau levels” vs “energy bands” opposition concerning the structure of the excitation spectrum in the mixed state of superconductors, and in particular, of d-wave superconductors. We find that the actual picture of quantization is an interplay between the two limiting images of the energy spectrum. Our analysis shows that the influence of the magnetic field on delocalized excitations in a superconductor can not be reduced to a mere action of the effective vortex lattice potential. In fact, magnetic field has a two-fold effect: on one hand, it creates vortices and thus provides a periodic potential for excitations, on the other hand, it also affects the long range motion of quasiparticles in a manner similar to that in normal metals. For low energy excitations, the long range effects are less pronounced. However, excitations with energies \( \epsilon > \Delta_0 \sqrt{H/H_c} \) mostly show the long range quantization. The energy spectrum consists of “Landau levels” which are split into bands by the periodic vortex potential. In the quasiclassical approximation \( p_T \xi \gg 1 \), the bandwidth is of the order of the distance between the Landau levels; it is small compared to the energy itself.

An a.c. electric field induces transitions between the states belonging to different Landau levels. Using the microscopic kinetic equations we demonstrate that these transitions can be seen as an increase in the vortex friction and/or in the quasiparticle conductivity due to a resonant absorption at frequencies corresponding to the energy differences between the Landau levels.
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