Instanton Effects in the Decay of Scalar Mesons

C. Ritter, B. C. Metsch, C. R. Münz, H. R. Petry

Institut für Theoretische Kernphysik,
Universität Bonn, Nußallee 14-16, 53115 Bonn, Germany

(March 26, 2022)

Abstract

We show that instanton effects may play a crucial role in the decay of scalar mesons into two pseudoscalars. Particularly the branching ratios of two meson decays of the $f_0(1500)$, which is considered as a glue-ball candidate, are then compatible with an ordinary $q\bar{q}$-structure of this resonance and a small positive SU(3) mixing angle, close to a result recently calculated with the same instanton-induced force [1].
I. INTRODUCTION

Instanton effects seem to be outstandingly reflected in properties of scalar and pseudoscalar mesons. An interaction induced by instantons \[2,3\] solves naturally the $U_A(1)$ problem for the $\eta - \eta'$ masses in the pseudoscalar sector \[4,5\]. In the framework of a relativistic quark model, this interaction acting on scalar mesons has been suggested to explain quantitatively the unusual mass spectrum of scalar mesons roughly in terms of a low lying singlet and a higher lying octet of $q\bar{q}$-states \[1\]. Their peculiar strong decay pattern, however, has not yet been consistently described.

In particular the $f_0(1500)$ \[6\] was argued to have properties incompatible with a pure $q\bar{q}$ configuration and it was suggested to possess a large glue admixture \[4\]. One of the major arguments in favor of this interpretation is the decay phenomenology of the $f_0(1500)$: It is found to decay into $\pi\pi$ \[8\], $\eta\eta$ \[9\], $\eta\eta'$ \[10\] but not into $K\bar{K}$. The $q\bar{q}$ hypothesis cannot fit all of these branching ratios at any $SU(3)_f$ scalar mixing angle, when decaying through a conventional decay mechanism (see Fig.1 in \[7\]). Furthermore the full width $\Gamma(f'_0) = 116 \pm 17$ MeV is out of line with the scalar nonet: taking the widths $\Gamma(a_0) = 270 \pm 40$ MeV and $\Gamma(K^*_0) = 287 \pm 23$ MeV as a scale for the other members of the scalar nonet, a natural guess for the $f'_0$ is around 500 MeV. The $f_0(1500)$ thus does not naturally fit into the quarkonium nonet.

In this paper we investigate the contribution of an instanton induced six-quark-vertex to the strong decay of scalar mesons into pseudoscalars. We derive $SU(3)_f$ branching ratios for the scalar octet, in particular for the $f_0(1500)$, which allow for a region of the scalar mixing angle fulfilling all of the three experimental branching ratios. We also present results of a numerical calculation in the framework of a relativistic quark model \[4\].

II. THE INSTANTON-INDUCED SIX-QUARK-VERTEX

The flavor dependent effective quark interaction used here was computed by 't Hooft and others from instanton effects \[4,8,11\]. 't Hooft showed that an expansion of the (euclidian) action around the one instanton solution of the gauge fields assuming dominance of the zero modes of the fermion fields leads to an effective quark interaction not covered by perturbative gluon exchange. For three flavors this is a six-point quark vertex completely antisymmetric in flavor. After normal ordering this results in a contribution to the constituent quark masses, a two body interaction and a six quark term that can be written as \[3\]:

$$
\Delta L(3)(y) = \frac{27}{80} g^{(3)}_{\text{eff}} \{ \overline{\Psi}(y) \overline{\Psi}(y) \overline{\Psi}(y) [1 \cdot 1 + \gamma_5 \cdot \gamma_5 \cdot 1 + \gamma_5 \cdot 1 \cdot \gamma_5 + 1 \cdot \gamma_5 \cdot \gamma_5] P_1^F (2 P_{10}^C + 5 P_8^C) \Psi(y) \Psi(y) \Psi(y) \}.
$$

where $P_1^F$ is the projector onto a three-particle flavor singlet state, $P_{10}^C$ and $P_8^C$ are projectors onto the color decuplet and the color octet and $g^{(3)}_{\text{eff}}$ is an effective coupling constant \[5\]. The dots imply that the first Dirac operator acts on the first quark field and so forth. Because
of the special Dirac structure the adequate formulation for this Lagrangian is the Weyl representation for the Dirac spinors

$$\Psi(x) =: \begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix}$$ \hspace{1cm} (2)

with diagonal $\gamma_5$.

The antisymmetric part of the operator acting on the three fermion fields is given by the product of the antisymmetric flavor projector and a projector which is symmetric in spin and color. The only possible symmetric spin-color operators are realized by the combinations:

$$P_1 = P_S^4 \otimes P_C^{10}$$
$$P_2 = P_S^2 \otimes P_C^8$$ \hspace{1cm} (3)

with $P_S^4$ the projector on the spin quadruplet and $P_S^{Spin}$ the projector on the spin doublet. The interaction Lagrangian then can be written in terms of the Weyl spinors as:

$$\Delta L(3) = \frac{27}{20} g_{eff}^{(3)} \left\{ : \eta^\dagger \eta^\dagger \eta^\dagger P_F^F (2P_1 + 5P_2) \xi \xi \xi : \right\} + (\eta \leftrightarrow \xi).$$ \hspace{1cm} (4)

We want to calculate the contribution of (4) to the decay amplitude of one meson into two mesons. In the present framework mesons are described as bound $q\bar{q}$-states with a BS-Amplitude \[12\]

$$\chi_{JM_Z}(x, y) = \langle 0 | T\Psi(x)\overline{\Psi}(y) | P, JM_Z \rangle$$ \hspace{1cm} (5)

It can be shown (see below) that the six-quark-interaction (4) due to its pointlike nature only applies to spin zero states i.e. mesons with nonvanishing Bethe–Salpeter amplitudes at the origin proportional to $\gamma_5$ (for pseudoscalars) or 1 (for scalars). Thus the only relevant quantities for each meson are:

$$\langle 0 | T\xi(0)\eta^\dagger(0) | P \rangle \text{ and } \langle 0 | T\eta(0)\xi^\dagger(0) | P \rangle$$ \hspace{1cm} (6)

Moreover the parity transformation relates these quantities:

$$\langle 0 | T\xi(0)\eta^\dagger(0) | P \rangle = \pi \langle 0 | T\eta(0)\xi^\dagger(0) | P \rangle$$ \hspace{1cm} (7)

where $\pi$ is the parity of the meson. The amplitude given in (5) can be decomposed in a product of space, flavor, spin and color part:

$$\langle 0 | T\xi(0)\eta^\dagger(0) | P \rangle = R(0) \otimes F \otimes \Sigma \otimes C$$ \hspace{1cm} (8)

The full contraction of the six-point Green’s function

$$\mathcal{G}^{(6)} = i \int d^4 y \langle 0 \left| T \Psi_{\alpha_2} \overline{\Psi}_{\beta_3} \Psi_{\alpha_3} \overline{\Psi}_{\beta_2} \Psi_{\alpha_1} \overline{\Psi}_{\beta_1} (\Delta L(3)(y)) \right| 0 \rangle$$ \hspace{1cm} (9)
FIG. 1. Feynman diagram of the strong decay of a meson in two mesons a) in a conventional quark line diagram b) by 't Hooft interaction

according to Wick’s theorem leads to the transition matrix element in lowest order for the decay process:

$$\langle P_2 P_3 | \Delta \mathcal{L}(3) | P_1 \rangle = i \frac{81}{5} g_{\text{eff}}^{(3)} \text{tr}[6 \mathcal{P}_1^F \bigotimes_{j=1}^3 (R(0) \otimes \mathcal{F})_j] \text{tr}[(2 P_1 + 5 P_2) \bigotimes_{j=1}^3 (\Sigma_j \otimes \mathcal{C}_j)]$$

(10)

where

$$\text{tr}[\mathcal{O} \bigotimes_{j=1}^3 \chi^j] = \mathcal{O}^{\alpha_1 \alpha_2 \beta_1 \beta_2 \beta_3} \chi_{\alpha_1 \beta_1}^1 \chi_{\alpha_2 \beta_2}^2 \chi_{\alpha_3 \beta_3}^3$$

(11)

The second trace is non-vanishing only if all mesons have spin zero. It can easily be calculated, since the mesons are color singlet states, and yields a factor $\frac{160}{3 \sqrt{3}}$ (Color matrices are normalized by a factor $\frac{1}{\sqrt{3}}$). In the following section we will investigate the flavor dependence of (10).

III. DECAY PROPERTIES OF SCALAR MESONS IN THE SU(3) LIMIT

The explicit flavor dependence in the notation of equation (11) is:

$$6 \mathcal{P}_1^F (\mathcal{F}^1, \mathcal{F}^2, \mathcal{F}^3) := \mathcal{F}_{ii}^1 \mathcal{F}_{jj}^2 \mathcal{F}_{kk}^3 \epsilon^{ijk} \epsilon^{i'j'k'}$$

(12)

with $\mathcal{F}_{kk'}^j \ j = 1, 2, 3$ the flavor part of the meson amplitude $j$. With the Cayley-Hamilton theorem this can be written as:
The first term is recognized as the flavor dependence of the conventional quark line diagram (according to the OZI rule) of fig 1a). Defining \( \hat{F}_l \) as the traceless part of \( F_l \) we obtain:

\[
6P^F_1(F^1, F^2, F^3) = \text{tr}(F^1[F^2, F^3]) - (\text{tr}(F^1)\text{tr}(F^2) + \text{cycl}) + \text{tr}(F^1)\text{tr}(\hat{F}^2 \hat{F}^2) + \text{cycl}) + \text{tr}(\hat{F}^1 \text{tr}(F^2) \text{tr}(F^3)) \quad (13)
\]

The flavor dependence of the three-body interaction thus leads to a minimal violation of the OZI rule: only if \( \text{tr}(F^i) \) does not vanish, i.e. a flavor singlet participates, there is an additional contribution to the conventional decay mechanism given by the first term.

With a pseudoscalar mixing angle \( \Theta_{PS} = -17.3^\circ \) for the \( \eta\eta' \)-system [13] we find the following partial widths (normalized to \( \Gamma(a_0 \rightarrow K\bar{K}) = 1 \)) in the SU(3) limit:

\[
\begin{align*}
\Gamma(a_0 \rightarrow \pi\eta) &= 0.4 \\
\Gamma(K^* \rightarrow K\eta) &= 0.3 \\
\Gamma(K^* \rightarrow K\pi) &= 1.5
\end{align*}
\]

Let us now consider the decay properties of the \( f_0, f'_0 \) system. We parameterize the mixing of the singlet and octet \( f_0 \) as

\[
\begin{align*}
|f_0\rangle &= \sin(\Theta_S) |f_{0,8}\rangle + \cos(\Theta_S) |f_{0,1}\rangle \\
|f'_0\rangle &= \cos(\Theta_S) |f_{0,8}\rangle - \sin(\Theta_S) |f_{0,1}\rangle
\end{align*}
\]

The partial widths of the \( f'_0 \) are plotted in Fig.2 as a function of the scalar mixing angle \( \theta_S \). With an angle of \( \theta_S \approx 25^\circ \) we find the following partial widths (normalized as in (15)):

\[
\begin{align*}
\pi\pi : \eta\eta : \eta\eta' : K\bar{K} &= 1.45 : 0.32 : 0.18 : 0.03
\end{align*}
\]

This is in fair agreement with the observed partial widths of the \( f_0(1500) \) [7]:

\[
\begin{align*}
\pi\pi : \eta\eta : \eta\eta' : K\bar{K} &= 1.45 : 0.39 \pm 0.15 : 0.28 \pm 0.12 : < 0.15
\end{align*}
\]

if phase space and a form factor is divided out and the \( \pi\pi \)-width is normalized. It is interesting that in a relativistic quark model based on the same instanton induced quark-antiquark force [1] we also find a small positive mixing angle although the numerical value \( \theta \approx 6^\circ \) is somewhat smaller. Though in this SU(3) model the total width of the \( f'_0 \) is expected to be about as large as that of the \( K^* \) this is still too large to explain the remarkable small width of \( \Gamma(f'_0) = 116 \pm 17 \text{ MeV} \) [14,15].
IV. INSTANTON-INDUCED SCALAR WIDTHS IN A RELATIVISTIC QUARK MODEL

The transition matrix elements for a scalar to two pseudoscalars have also been calculated in the framework of a relativistic quark model based on the Salpeter equation, for details see [1]. There the mixing angle of the scalar states is found to be $\theta \approx 6^\circ$ [1]. As it stands, the instanton-induced interaction is point-like. We regularized the interaction by replacing the delta function by a normalized Gaussian function, which introduces a finite effective range [5]. The calculation uses all parameters as given in [1] and the calculated masses from this model. Our results for the partial widths are given in table I, where we have adjusted the six-quark coupling strength $g^{(3)}_{\text{eff}}$ to the process $K^* \rightarrow K\pi$. Again, the small $f_0(1500)$ total width cannot be accounted for. We infer that in future work we have to include also the conventional decay mechanism of fig. 1a). The interference between both terms may well solve the remaining problems of the scalar decay modes.

For the $K^*$ the Particle Data Group [16] lists one decay mode $K^* \rightarrow K\pi$ within $93 \pm 10\%$. Our calculated result ($K^* \rightarrow K\eta \approx 10\%$) is still compatible with this number. For the $f_0(980)$ there are two listed strong decay modes: $f_0(980) \rightarrow \pi\pi$ within $78.1 \pm 2.4\%$ and $f_0(980) \rightarrow K\bar{K}$ within $21.9 \pm 2.4\%$. Our results, 60% and 40% respectively are still in fair agreement.

In the following we compare the invariant couplings for the $f_0(1500)$, i.e. the branching ratios divided by phase space factors:

$$\pi\pi : \eta\eta' : K\bar{K} = 3 : 0.33 : 0.02 : 0.07 \quad (19)$$

The experimentally seen invariant couplings [17] are:
\[ \pi \pi : \eta \eta : K \bar{K} = 3 : 0.70 \pm 0.27 : 1.00 \pm 0.46 : < 0.36 \]  

The clear discrepancy (for the \( \eta \eta' \) channel) is mainly due to the small calculated scalar mixing angle \( \theta \approx 6^\circ \). The decay mechanism via the six-quark-vertex thus cannot explain the experimentally seen branching ratios alone. The conventional contribution has to be added coherently. It is interesting to note that the calculated invariant coupling

\[ \Gamma(f_0(1500) \rightarrow \pi \pi) : \Gamma(f_0(1500) \rightarrow \pi \pi(1300)) = 1.2 : 1 \]  

indicates that the \( \pi \pi'(1300) \)-channel is relevant for the decay of the \( f_0(1500) \).

**V. CONCLUSION**

In this paper an instanton induced six-point vertex has been worked out which contributes to the strong decay of scalar mesons into two pseudoscalars. Using a \( SU(3)_f \) mixing angle \( \theta \approx 25^\circ \) for the \( f_0 - f_0' \) system, the experimental branching ratios of the \( f_0(1500) \) can be explained in a \( SU(3)_f \) symmetric calculation in contrast to a mechanism via a conventional quark line diagram. The small total width of the \( f_0(1500) \), however, has not yet been explained in the \( SU(3) \) calculation nor in a relativistic quark model. It should be emphasized that none of the parameters was adjusted to the scalar spectrum, and some improvement might be obtained by changing slightly the strength of the instanton-induced force. In addition there is no logical argument which rules out the conventional decay mechanism. Actually the latter still should account for all other meson decays, since the contribution discussed here only works for (pseudo)scalars. Therefore in the future the interference between the conventional and the instanton-induced six-quark vertex has to be worked out quantitatively.
REFERENCES

[1] E. Klempt, B. C. Metsch, C. R. Münz, H. R. Petry, Phys. Lett. B361 (1995) 160.
[2] G. ’t Hooft, Phys. Rev. D14 (1976) 3432.
[3] M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, Nucl. Phys. B163 (1980) 46.
[4] W. H. Blask, U. Bohn, M. G. Huber, B. C. Metsch, H. R. Petry, Z. Phys. A337 (1990) 327.
[5] C. R. Münz, J. Resag, B. C. Metsch, H. R. Petry, Nucl. Phys. A578 (1994) 418.
[6] V. V. Anisovitch et al., Phys. Lett. B323 (1994) 333.
[7] C. Amsler, F. E. Close, Phys. Lett. B353 (1995) 385.
[8] D. Alde et al., Phys. Lett. B201 (1988) 160 and Refs. therein.
[9] S. Abatzis et al., Phys. Lett. B324 (1994) 509.
[10] D. V. Bugg et al., Phys. Lett. B353 (1995) 378.
[11] H. R. Petry, H. Hofestadt, S. Merk, K. Bleuler, Phys. Lett. B159, (1985) 363.
[12] J. Resag, C. R. Münz, B. C. Metsch, H. R. Petry, Nucl. Phys. A578 (1994) 397.
[13] C. Amsler et al., Phys. Lett. B294 451.
[14] C. Amsler et al., Phys. Lett. B342 (1995) 433.
[15] C. Amsler et al., Phys. Lett. B340 (1994) 259.
[16] Particle Data Group, Phys. Rev. D50 (1994) 1173.
[17] C. Amsler et al. (Crystal Barrel Collaboration), Phys. Lett. B355 (1995) 425.
TABLES

TABLE I. Calculated decay widths in MeV with Bethe-Salpeter amplitudes from \[1\] and instanton-induced six-quark-vertex

| Decay                  | Width (MeV) |
|------------------------|-------------|
| $a_0(1450) \to \pi\eta$ | 101         |
| $a_0(1450) \to KK$     | 153         |
| $K^*_0(1430) \to K\pi$ | 264         |
| $K^*_0(1430) \to K\eta$| 28          |
| $f_0(980) \to \pi\pi$  | 167         |
| $f_0(980) \to KK$      | 69          |
| $f'_0(1500) \to \pi\pi$| 304         |
| $f'_0(1500) \to KK$    | 5           |
| $f'_0(1500) \to \eta\eta$| 21         |