Plastic flow, voltage bursts, and vortex avalanches in superconductors

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We use large-scale parallel simulations to compute the motion of superconducting magnetic vortices during avalanches triggered by small field increases. We find that experimentally observable voltage bursts correspond to pulsing vortex movement along branched channels or winding chains, and relate vortex flow images to features of statistical distributions. As pin density is increased, a crossover occurs from interstitial motion in narrow easy-flow winding channels with typical avalanche sizes, to pin-to-pin motion in broad channels, characterized by a very broad distribution of sizes. Our results are consistent with recent experiments.

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Recent studies of systems characterized by avalanche dynamics, where energy dissipation occurs in sudden bursts of collective activity, have revealed many interesting dynamical behaviors. Granular assemblies \textsuperscript{[1]}, magnetic domains \textsuperscript{[2]}, fluid flow \textsuperscript{[3]}, and flux lines in type-II superconductors \textsuperscript{[4,5]} all display avalanches, but produce broad distributions of avalanche sizes only under certain conditions, indicating a lack of universal response to small perturbations. Probing the \textit{microscopic} origins of different avalanche behaviors by experimentally recording the microscopic motion over time scales long enough to statistically characterize the system is difficult. In contrast, numerical simulations provide both precise control of microscopic parameters and dynamical information, making them a useful tool. Early avalanche simulations used simple discrete models \textsuperscript{[6]}, but recent advances in parallel processing allow the use of more realistic continuous dynamic models (MD) models. In this paper, we present extensive parallel MD simulations of flux-gradient-driven superconducting vortex avalanches. By varying the pinning density \( n_p \) and maximum pinning strength \( f_p \), we obtain avalanche distributions in good agreement with recent experiments \textsuperscript{[7,8]}, and quantify how the density and strength of pinning sites affects the breadth of these distributions. Characteristic avalanche sizes and lifetimes appear at low pin densities when narrow easy-flow winding channels of interstitial vortices form. At higher pin densities, pin-to-pin transport through vortex chains occurs, and the avalanche size and lifetime distributions remain very broad. We find that there is no universal distribution valid for all pinning parameters.

Flux penetrates a type-II superconductor in the form of discrete quantized vortices. As an external field is slowly increased, a metastable gradient in vortex density, termed the Bean state \textsuperscript{[9]}, forms as vortices are driven into the sample by their mutual repulsion and are held back by defects in the material \textsuperscript{[10,11]}. To model this system, we simulate an infinite slab with a magnetic field \( \mathbf{H} = H\hat{z} \) applied parallel to the surface so that there are no demagnetization effects. The rigid vortices and straight columnar pins we consider are all parallel to \( \hat{z} \), so we can obtain all relevant dynamical information from a transverse two-dimensional slice, in the \( x-y \) plane, of the three-dimensional slab. The flux lines evolve according to a \( T = 0 \) MD algorithm. The vortex-vortex repulsion, given by the modified Bessel function \( K_1(r/\lambda) \), is cut off beyond \( r = 6\lambda \), where \( \lambda \) is the penetration depth, so that each vortex interacts with up to 100 neighbors, and important \textit{collective} effects, neglected in simulations with shorter interaction ranges, are observed. Each \( 24\lambda \times 26\lambda \) sample contains up to 3700 attractive parabolic pins of radius \( \xi_p = 0.15\lambda \) with pinning densities \( n_p = 0.96/\lambda^2 \), \( n_p = 2.40/\lambda^2 \), or \( n_p = 5.93/\lambda^2 \), and pinning strengths uniformly distributed over the range \( f_p^{\text{max}} / 5 \) to \( f_p^{\text{max}} \), where \( f_p^{\text{max}} = 0.3 f_0 \), \( 1.0 f_0 \), or \( 3.0 f_0 \). All forces are given in units of \( f_0 = \Phi_0^2/(8\pi^2\lambda^2) \) and lengths in units of \( \lambda \).

The total force on vortex \( i \) is given by \( \mathbf{f}_i = \mathbf{f}_i^{vv} + \mathbf{f}_i^{pp} = \eta\mathbf{v}_i \), where the force on vortex \( i \) from the vortex \( v \) is \( \mathbf{f}_i^{vv} = \sum_{j=1}^{N_v} f_0 K_1(|\mathbf{r}_i - \mathbf{r}_j|/\lambda)\hat{r}_{ij} \), and the force from pinning sites is \( \mathbf{f}_i^{pp} = \sum_{k=1}^{N_p} (f_p/\xi_p)|\mathbf{r}_i - \mathbf{r}_k^{(p)}|/\lambda\Theta((\xi_p - |\mathbf{r}_i - \mathbf{r}_k^{(p)})/\lambda)\hat{f}_{ik} \). Here, \( \Theta \) is the Heaviside step function, \( \mathbf{r}_i (\mathbf{v}_i) \) is the location (velocity) of vortex \( i \), \( \mathbf{r}_k^{(p)} \) is the location of pinning site \( k \), there are \( N_p \) pinning sites and \( N_v \) vortices, \( \mathbf{v}_i = (\mathbf{r}_i - \mathbf{r}_j)/|\mathbf{r}_i - \mathbf{r}_j| \), \( \hat{f}_{ik} = (\mathbf{r}_i - \mathbf{r}_k^{(p)})/|\mathbf{r}_i - \mathbf{r}_k^{(p)}| \), and we take \( \eta = 1 \). Using roughly \( 10^4 \) hours on an IBM SP parallel computer, we recorded more than \( 10^4 \) avalanches for each of five combinations of \( n_p \) and \( f_p \). In addition to the results described in this paper, we obtained distributions of vortex velocities and displacements as well as analytical results supporting our observations \textsuperscript{[7,8]}. Further simulation details appear in \textsuperscript{[10,11]}. A slowly increasing external field is modeled by adding a single vortex to an unpinned region along the sample edge whenever the system reaches mechanical equilibrium \textsuperscript{[12]}. The majority of these small field increases
result in only slight shifts in vortex positions, but occasionally one or more vortices will become depinned, producing an avalanche. We find that avalanche disturbances propagate as an uneven pulse, as seen in Fig. 1 where arrows indicate instantaneous vortex velocities. Events with longer lifetimes often contain more than one pulse of motion (i.e., multiple oscillations in the total avalanche velocity) [14].

By imaging individual avalanches in our samples, we find that a chain of vortices is displaced in a typical event. Each vortex in the chain is depinned, moves a short distance, and comes to rest in a nearby pinning site. Vortices outside the chain transmit stress by shifting very slightly inside pinning sites, but are not depinned. In Fig. 2, filled circles represent the initial positions of vortices that were depinned during the avalanche, while small crosses mark the remaining vortices that remained pinned. Easily visible chains of moving vortices extend down the flux gradient in the $x$–direction, winding slightly in the $y$–direction. Chain size varies from event to event: in some cases a chain spans the sample, as shown in Fig. 2, while in other events the chain contains only three or four vortices. In each case, although the disturbance may cross the sample, an individual vortex does not. Thus, the time span of a typical avalanche is much shorter than the time a single vortex takes to traverse the sample.

When the pinning density is high, chains of moving vortices are equally likely to form anywhere in the sample. As the pinning density is lowered, vortices move only through narrow well-defined interstitial winding channels in which vortices are weakly held in place only by the repulsion of other vortices which sit in pinning sites. To identify the cumulative pattern of flow channels for different pinning parameters, in Fig. 3 we plot vortex trajectories with lines over the course of many avalanches. A concentration of trajectory lines indicates a heavily-travelled region of the sample. In Fig. 3(a) we show a small portion of a sample with low pinning density. We see that all motion occurs through narrow easy-flow interstitial channels. In these channels, mobile interstitial vortices move plastically [15] around their strongly pinned neighbors in a manner similar to that recently imaged experimentally [6]. This sample also contains vortices oscillating in interstitial “magnetic traps.” As the pin density increases, the amount of interstitial pinning decreases and the number of channels increases, as in Fig. 3(b), until at high pin densities [Fig. 3(c)], there is no interstitial flow and the vortices move only from pin to pin. Here, where no easy-flow channel exists, avalanches are spread evenly throughout the sample.

We determine how the microscopic pinning parameters affect avalanche size by finding the total avalanche lifetime $\tau$ for each event. Since vortices typically hop from pin to pin in samples with high pinning density, a natural unit of time is the interval $t_h$ a vortex spends hopping between pinning sites, and so we use scaled lifetimes $\tau^* = \tau/t_h$. To find $t_h$, we assume that
The average distance between pinning sites, and that the vortex speed $v_c$ is proportional to the depinning force, $v_c = f_i/\eta$, where $f_i \approx -f_p$. This gives $t_h = d_p/v_c \approx \eta f_p^{-1} f_p^{-1/2}$. In Pbln, for example, $\eta \sim 0.3$ G-m/Ω, so using $f_p = 3.0f_0$, $n_p = 5.93/\lambda^2$, and $\lambda \sim 65$ nm gives $t_h \sim 150$ ns. The plot of $P(\tau^*)$ in Fig. 4(a) for samples with dense pinning shows that in each case the distribution is very broad and can be written as $P(\tau^*) \sim (\tau^*)^{-1.4}$ over a range of $\tau^*$. If the pinning density $n_p$ is reduced, the form of $P(\tau^*)$ changes noticeably, as in Fig. 4(b). Here, we find an enhanced probability for avalanches with a characteristic value as a result of the appearance of easy-flow interstitial channels. Many avalanches in these samples consist of a single sample-spanning pulse of motion through one of these channels. The estimated pulse lifetime produced by a straight channel is $\tau_{\text{est}} \sim t_h N_h$, where $N_h$ is the number of vortices in the channel. Since $N_h \sim L_x \sqrt{n_v}$, where $L_x$ is the sample length, we find $\tau_{\text{est}} = \eta L_x f_p^{-1} \sqrt{n_v/n_p}$, which gives a characteristic value of $\tau^* = \tau_{\text{est}}/t_h \approx L_x \sqrt{n_v} = 26\lambda \sqrt{1.5/\lambda^2} \approx 30$. This value, indicated by an arrow in Fig. 4(b), agrees well with the peak in the distribution of $\tau^*$.

For all pin densities, only a small fraction of the vortices move significantly while the rest shift in pinning sites, as seen by considering the distance $d_i$ each vortex is displaced in an avalanche. Vortices that hop from pin to pin create a peak in $P(d_i)$, marked with arrows in Fig. 3(c–d), at $d_i \approx d_p = n_p^{-1/2}$. For those vortices that remain pinned and accumulate stress in the vortex lattice, $d_i < d_p$, we can approximate the distribution by $P(d_i) \sim d_i^\rho$, where $\rho \sim 1.4$ for all samples except $\rho \sim 1.2$ for $n_p = 0.96/\lambda^2$, $f_p \max = 3.0f_0$, and $\rho \sim 0.9$ for $n_p = 5.93/\lambda^2$, $f_p \max = 0.3f_0$. An analytical argument [11], sketched here, predicts a similar $\rho$ for all samples since the distribution is generated only by pinned vortices and is not affected by the presence of easy-flow channels. The addition of a vortex to the sample creates a small additional force $f$ on an arbitrary vortex located a small distance $r$ away ($r < \lambda$), displacing this vortex a distance $\delta u(r) = (f/\eta)\delta t \sim K_1(r/\lambda) \sim 1/r$. Since there are $\delta N(r) = 2\pi\eta r_0\lambda$ vortices a distance $r$ from the added vortex, we find $\rho = -d\ln \delta N/d\ln \delta u = -(1/r)(-1/r)^{-1} = 1$, in general agreement with our computed values of $\rho$, $\rho \sim 0.9 - 1.4$. 

![Figure 3](image1.png)  
![Figure 4](image2.png)
Pinning density $n_p$ has a significant impact on the number of vortices $N_t$ that move a distance $d_t$ larger than the pin diameter $2a_p$. We plot $P(N_t)$ in Fig. 4(e–f). When $n_p$ is low, $n_p \lesssim 2.4/\lambda^2$, vortices move only in narrow easy-flow winding channels, producing a characteristic value of $N_a$. $N_a \approx N_b$, marked with an arrow in Fig. 4(f). The strongly pinned vortices around the interstitial channel cannot be depinned, so large values of $N_a$ are not observed. At high pinning densities, as in Fig. 4(e) where $n_p = 5.93/\lambda^2$, we can approximate the form of $P(N_a)$ for small $N_a$ as $P(N_a) \sim N_a^{-\beta}$. As the pin strength decreases, $\beta$ decreases slightly: $\beta \sim 1.4$ for $f_p^{\text{max}} = 3.0f_0$, $\beta \sim 1.0$ for $f_p^{\text{max}} = 1.0f_0$, and $\beta \sim 0.9$ for $f_p^{\text{max}} = 0.3f_0$. This is in contrast to the lifetime distributions which were not affected by changing pinning strength. Smaller values of $\beta$ indicate a relative increase in the frequency of large avalanches compared to small ones. The trend in $\beta$ indicates the importance of avalanche width in determining $N_a$. In samples with strong pinning, avalanche width is suppressed since strongly pinned vortices on either side of the moving chain are not depinned. As the pinning weakens, wider avalanches occur when weakly pinned vortices adjacent to a slowly moving chain depin and join the motion. Thus, avalanches with similar scaled lifetimes are likely to be wider in samples with weak pinning than in samples with strong pinning.

Altering the pinning parameters affects the number of vortices $N_f$ that exit the sample during an avalanche, as shown in the plot of $P(N_f)$ in Fig. 4(g–h). If we approximate $P(N_f)$ for low $N_f$ by the form $P(N_f) \sim N_f^{-\alpha}$, we find that all samples with high pinning density, $n_p = 5.93/\lambda^2$, have $\alpha \sim 2.4$. As $n_p$ decreases, $\alpha$ increases: $\alpha \sim 3.4$ for $n_p = 2.40/\lambda^2$ and $\alpha \sim 4.4$ for $n_p = 0.96/\lambda^2$. When all vortex motion occurs in an easy-flow interstitial channel, $\alpha$ increases since the channel does not build up enough stress to allow events with large $N_f$ to occur. Smaller events continually relieve the accumulated stress instead. For example, with the small number of flux paths in Fig. 4(b), events with large $N_f$ are rare, and $\alpha \sim 3.4$. Fig. 4(a) corresponds to the extreme case of a sample with only one channel, for which $n_p = 0.96/\lambda^2$ and $\alpha \sim 4.4$. In samples with high pin density, even after the stress in one vortex path has been depleted by a large avalanche, other areas still contain enough stress to remain active in large and small events while the depleted regions build up stress again. This leads to a greater likelihood of large events and correspondingly smaller values of $\alpha$, $\alpha \lesssim 2.4$. Distributions similar to this high pin density case have been obtained experimentally in [1], where values of $\alpha$ ranging from 1.4 to 2.2 are observed. The pinning density from grain boundaries in the experimental sample is very high, $n_p \sim 100/\lambda^2$, so it is reasonable that the $\alpha$ values observed in [1] are similar to the $\alpha$ values produced by our most densely pinned samples. Broad distributions with $\alpha$’s of 1.7 to 2.2 were also observed in [2], in good agreement with both [1] and our results. In addition, [2] finds a regime where avalanches of a characteristic size occur, offering an experimental example in which samples with a lower density of weaker pinning sites produce narrower distributions, as we also observe.

We have quantitatively shown how pinning determines the nature of vortex avalanches. By using large-scale MD simulations, we image pulses of motion in chain-like disturbances through the sample. The presence or absence of distinct channels for flow leads to a crossover from broad distributions of avalanche size to characteristic sizes. Pinning strength causes a transition between strongly plastic flow to mildly plastic, “semi-elastic” flow. Lowering the pinning density causes a transition from broad distributions to distributions with characteristic sizes.

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[15] Plastic motion is also discussed in R.D. Merithew et al., Phys. Rev. Lett. 77, 3197 (1996), and references therein. Note that in our work, vortices are flux-gradient-driven, with no artificial “uniform force” applied to them.