Radiatively Induced Neutrino Mass Matrix in a SUSY GUT Model with $R$-Parity Violation

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In order to evade the proton decay which appears when the Zee model is embedded into a SUSY GUT scenario with $R$-parity violation, a new idea based on a discrete $Z_2$ symmetry is proposed. Under the symmetry $Z_2$, the quark and lepton mass matrices are tightly constrained. The admissible form of the radiatively-induced neutrino mass matrix is investigated.

1. Introduction

The Zee model [1] is one of promising models of neutrino mass generation mechanism, because the model has only 3 free parameters and it can naturally lead to a large neutrino mixing [2], especially, to a bimaximal mixing [3]. However, the original Zee model is not on a framework of a grand unification theory (GUT), and moreover, it is recently pointed out [4] that the predicted value of $\sin^2 2\theta_{\text{solar}}$ must be satisfied the relation $\sin^2 2\theta_{\text{solar}} > 0.99$ for $\Delta m^2_{\text{solar}}/\Delta m^2_{\text{atm}} \sim 10^{-2}$, i.e.,

$$\sin^2 2\theta_{\text{solar}} \geq 1 - \frac{1}{16} \left( \frac{\Delta m^2_{\text{solar}}}{\Delta m^2_{\text{atm}}} \right)^2. \quad (1.1)$$

The conclusion cannot be loosened even if we take the renormalization group equation (RGE) effects into consideration.

The simple ways to evade the constraint (1.1) may be as follows: One is to consider [5] that the Yukawa vertices of the charged leptons can couple to both scalars $\phi_1$ and $\phi_2$. Another one [6] is to introduce a single right-handed neutrino $\nu_R$ and a second singlet Zee scalar $S^+$. Also, a model with a new doubly charged scalar $k^{++}$ is interesting because the two loop effects in such a model can give non-negligible contributions to the neutrino masses [7]. As another attractive model, there is an idea [8] that in an $R$-parity violating supersymmetric (SUSY) model we identify the Zee scalar $h^+$ as the slepton $\tilde{e}_R$. Then, we can obtain additional contributions from the down-quark loop diagrams to the neutrino masses, so that such a model can be free from the constraint (1.1).

However, these models have not been embedded into a GUT scenario. As an extended Zee model based on a GUT scenario, there is, for example, the Haba-Matsuda-Tanimoto model [9]. They have regarded the Zee scalar $h^+$ as a member of the messenger field $M_{10} + M'_{10}$ of SUSY-breaking on the basis of an SU(5) SUSY GUT. However, their model cannot escape from the constraint (1.1) because the radiative masses are only induced by the charged lepton loop diagrams.

In the present paper, we will investigate an extended Zee model which is based on a framework of a SUSY GUT with $R$-parity violation, and which is free from the severe constraint (1.1). Usually, it is accepted that SUSY models with $R$-parity violation are incompatible with a GUT scenario, because the $R$-parity violating interactions induce the proton decay. In order to suppress the proton decay due to the $R$-parity violating terms, we will introduce a discrete $Z_2$ symmetry.

2. How to evade the proton decay

We identify the Zee scalar $h^+$ as the slepton $\tilde{e}_R$ which is a member of SU(5) 10-plet sfermions
Then, the Zee interactions correspond to the following $R$-parity violating interactions
\[
\lambda_{ij}^{k}(\bar{\psi}_{10}^{A})(\psi_{10}^{B})k_{AB} = \frac{1}{\sqrt{2}} k_{ij} \left\{ \varepsilon_{\alpha,\beta} \left( \bar{d}_{R}^{i}_{\alpha} \bar{c}_{R}^{j}_{\beta} \eta_{10}^{j} \right) \right\} ,
\]
\[
= \frac{1}{\sqrt{2}} k_{ij} \left\{ \varepsilon_{\alpha,\beta} \left( \bar{d}_{R}^{i}_{\alpha} \bar{c}_{R}^{j}_{\beta} \eta_{10}^{j} \right) \right\} ,
\]
\[
-[(\bar{\psi}_{L}^{i})(\nu_{L})_{j} - (\bar{\psi}_{R}^{i})(e_{L})_{j}](e_{R})_{k} ,
\]
\[
-[\bar{\psi}_{L}^{i}(d_{R}^{j})_{\alpha} - (\bar{d}_{R}^{i})_{\alpha}(e_{L})_{j}](\bar{u}_{L})_{k} ,
\]
\[
+[(\bar{\psi}_{L}^{i})(d_{R}^{j})_{\alpha} - (\bar{d}_{R}^{i})_{\alpha}(e_{L})_{j}](\bar{u}_{L})_{k} \right\} ,
\]
where $\psi^{c} \equiv C\bar{\psi}^{T}$ and the indices $(i,j,\cdots)$, $(A,B,\cdots)$ and $(\alpha,\beta,\cdots)$ are family-, SU(5)$_{GUT}$- and SU(3)$_{color}$-indices, respectively. However, in GUT models, if the interactions (2.1) exist, the following $R$-parity violating interactions will also exist:
\[
\lambda_{ij}^{k}(\bar{\psi}_{10}^{A})(\psi_{10}^{B})k_{AB} = \frac{1}{\sqrt{2}} k_{ij} \left\{ \varepsilon_{\alpha,\beta} \left( \bar{d}_{R}^{i}_{\alpha} \bar{c}_{R}^{j}_{\beta} \eta_{10}^{j} \right) \right\} ,
\]
\[
-[(\bar{\psi}_{L}^{i})(\nu_{L})_{j} - (\bar{\psi}_{R}^{i})(e_{L})_{j}](e_{R})_{k} ,
\]
\[
-[\bar{\psi}_{L}^{i}(d_{R}^{j})_{\alpha} - (\bar{d}_{R}^{i})_{\alpha}(e_{L})_{j}](\bar{u}_{L})_{k} ,
\]
\[
+[(\bar{\psi}_{L}^{i})(d_{R}^{j})_{\alpha} - (\bar{d}_{R}^{i})_{\alpha}(e_{L})_{j}](\bar{u}_{L})_{k} \right\} ,
\]
which contribute to the proton decay through the intermediate state $\bar{d}_{R}$.

In order to forbid the contribution of the interactions (2.2) to the proton decay, for example, we can assume that the $R$-parity violating interactions occur only when the field $\psi_{10}$ of the third family is related, i.e., we assume the interactions
\[
\lambda_{ij}^{k}(\bar{\psi}_{10}^{A})(\psi_{10}^{B})k_{AB} = \frac{1}{\sqrt{2}} k_{ij} \left\{ \varepsilon_{\alpha,\beta} \left( \bar{d}_{R}^{i}_{\alpha} \bar{c}_{R}^{j}_{\beta} \eta_{10}^{j} \right) \right\} ,
\]
instead of the interaction (2.2). Then, the terms $\lambda_{ij}^{k}(\bar{\psi}_{10})(\bar{d}_{R}^{i})(\bar{c}_{R}^{j})(u_{R}^{c})_{j}$ cannot contribute to the proton decay. In order to realize the constraints
\[
\lambda_{12}^{k} = \lambda_{23}^{k} = \lambda_{31}^{k} = 0 \quad \text{for} \quad k = 1,2 ,
\]
we introduce a discrete symmetry $Z_{2}$, which exactly holds at every energy scale, as follows:
\[
(\bar{\psi}_{L})_{i} \rightarrow \eta_{i}(\bar{\psi}_{L})_{i} , \quad (\bar{\psi}_{R}^{i})_{i} \rightarrow \eta_{i}(\bar{\psi}_{R}^{i})_{i} ,
\]
\[
(\psi_{10})_{i} \rightarrow \xi_{i}(\psi_{10})_{i} , \quad (\psi_{10})_{i} \rightarrow \xi_{i}(\psi_{10})_{i} ,
\]
where $\eta_{i}$ and $\xi_{i}$ take
\[
\eta = (+1,+1,+1) , \quad \xi = (+1,+1,+1) ,
\]
under the $Z_{2}$ symmetry. Then, the $Z_{2}$ invariance leads to the constraints (2.4).

However, if the RGE causes a mixing between the first and third families, the interactions (2.3) can again contribute to the proton decay. If we assume that 5 and $\bar{5}$ Higgs fields $H_{u}$ and $H_{d}$ transform as
\[
H_{u} \rightarrow +H_{u} , \quad H_{d} \rightarrow +H_{d} ,
\]
under the $Z_{2}$ symmetry, the up-quark mass matrix $M_{u}$ is given by the form
\[
M_{u} = \begin{pmatrix}
c_{u} & d_{u} & 0 \\
0 & d_{u} & b_{u} \\
0 & 0 & a_{u}
\end{pmatrix} .
\]
This guarantees that the top quark $u_{3}$ in the $R$-parity violating terms (2.3) does not mix with the other components ($u_{1}$ and $u_{2}$) even if we take the RGE effects into consideration, so that the interactions (2.3) cannot contribute to the proton decay at any energy scales.

On the other hand, the down-quark mass matrix $M_{d}$ and the charged lepton mass matrix $M_{e}$, which are generated by the Higgs scalar $H_{d}$, have the form
\[
M_{d} = M_{e}^{T} = \begin{pmatrix}
c_{1} & c_{2} & c_{3} \\
0 & b_{1} & b_{2} \\
0 & 0 & a_{1}
\end{pmatrix} .
\]
The mass matrix form (2.9) cannot explain the observed masses and mixings. In order to give reasonable masses and mixings of the quarks and charged leptons, we must consider additional SU(5) 45-plet Higgs scalars, which do not contribute to the up-quark mass matrix because $\bar{\psi}_{10}^{c} M_{u}^{c} \psi_{10} \in (10 \times 10)_{symmetric}$. Then, we obtain the down-fermion mass matrices
\[
M_{d} = \begin{pmatrix}
c_{1} & c_{2} & c_{3} \\
b_{1} & b_{2} & b_{3} \\
a_{1} + a_{1} & a_{2} + a_{2} & a_{3} + a_{3}
\end{pmatrix} ,
\]
(2.10)
\[
M_{e}^{T} = \begin{pmatrix}
-3c_{1} & -3c_{2} & -3c_{3} \\
-3b_{1} & -3b_{2} & -3b_{3} \\
a_{1} - 3a_{1} & a_{2} - 3a_{2} & a_{3} - 3a_{3}
\end{pmatrix} ,
\]
(2.11)
where $a'_i$ and $(b'_i, c'_i)$ denote contributions from the 45-plet Higgs scalars $H^{(+)}_{45}$ and $H^{(-)}_{45}$ which transform $H^{(+)}_{45} \to +H^{(+)}_{45}$ and $H^{(-)}_{45} \to -H^{(-)}_{45}$ under the symmetry $Z_2$, respectively. Note that if we consider either $H^{(+)}_{45}$ or $H^{(-)}_{45}$, we cannot still give realistic down-fermion masses. For example, if we have $H_5$ ($H_u$ and $H_d$) and $H^{(-)}_{45}$, we obtain an unwelcome relation

$$m^2_l + m^2_\mu + m^2_e - (m^2_\nu + m^2_\tau + m^2_d)$$

$$= 8 \sum (|b'_i|^2 + |c'_i|^2) > 0, \quad (2.12)$$

from the trace of $M^U_1 M^L_2 - M^d M^d_1$. Therefore, in order to obtain realistic down-fermion mass matrices, we need, at least, the three types of the Higgs scalars $H_5$, $H^{(+)}_{45}$ and $H^{(-)}_{45}$.

However, such additional Higgs scalars $H^{(+)}_{45}$ cause another problem. One is a problem of the flavor changing neutral currents (FCNC). This problem is a common subject to overcome not only in the present model but also in most GUT models. The conventional mass matrix models based on GUT scenario cannot give realistic mass matrices without assuming Higgs scalars more than two. For this problem, we optimistically consider that only one component of the linear combinations among those Higgs scalars survives at the low energy scale $\mu = \Lambda_L$ ($\Lambda_L$ is the electroweak energy scale), while other components are decoupled at $\mu < \Lambda_X$ ($\Lambda_X$ is a unification scale).

Another problem is that the 45 Higgs scalars can have vacuum expectation values (VEV) at the electroweak energy scale $\Lambda_L$, so that the $Z_2$ symmetry is broken at $\mu = \Lambda_L$. Therefore, the proton decay may occur through higher order Feynman diagrams. In the conventional GUT models, it is still a current topic whether the colored components of the SU(5) 5-plet Higgs scalar can become sufficiently heavy or not to suppress the proton decay. We again optimistically assume that the colored components of the 45-plet Higgs scalars are sufficiently heavy to suppress the proton decay, i.e., that such effects will be suppressed by a factor $(\Lambda_L/\Lambda_X)^2$.

Figure 1. Radiatively induced neutrino mass through the down-quark loop. The vertices A, B, C and D are given by $(U^d R \lambda U^c_L)_{ij} (U^d_L)_{3n}$, $(U^d L M^d R)_{kl}$, $(\tilde{U}^d L \lambda U^c_L)_{mi} (U^d L)_{3k}$, and $(U^d L \lambda M^d R)_{kl}$, respectively.

3. Radiatively induced neutrino masses

We define fields $u_i$, $d_i$ and $e_i$ as those corresponding to mass eigenstates, i.e.,

$$H_{\text{mass}} = \begin{pmatrix} u_L \end{pmatrix}^T M_u \begin{pmatrix} u_R \end{pmatrix} + \begin{pmatrix} d_L \end{pmatrix}^T M_d \begin{pmatrix} d_R \end{pmatrix}$$

$$+ \begin{pmatrix} e_L \end{pmatrix}^T M_e \begin{pmatrix} e_R \end{pmatrix} + \text{h.c.} \quad , \quad (3.1)$$

and fields $\nu L_\ell$ as partners of the mass eigenstates $\nu_{L\ell}$, i.e., $\nu_{L\ell} = (\nu_{L\ell}, \nu_{L\ell})$. We define the neutrino mass matrix $M_\nu$ as

$$H_{\nu \text{ mass}} = \begin{pmatrix} \nu_L \end{pmatrix}^T M_\nu \begin{pmatrix} \nu_L \end{pmatrix} \quad . \quad (3.2)$$

Therefore, a unitary matrix $U^\nu_L$ which is defined by

$$U^{\nu T}_L M_\nu U^\nu_L = D_\nu \equiv \text{diag}(m^\nu_1, m^\nu_2, m^\nu_3) \quad , \quad (3.3)$$

is identified as the Maki-Nakagawa-Sakata-Pontecorvo [11] neutrino mixing matrix $U_{MNSP} = U^\nu_L$.

In addition to the $R$-parity violating terms (2.1) and (2.2) [(2.3)], we assume SUSY breaking terms $\tilde{\psi}_i^{\nu} \psi_i H^\nu_d$ (and $\tilde{\psi}_i^{s} \psi_i H^s_d$). For the moment, we do not consider $\tilde{\psi}_i^{\nu} H^\nu_d$ mixing as in the original Zee model. (The $H^\nu_d$ contribution to the neutrino mass matrix will be discussed in the end of this section.) Then, the neutrino masses are radiatively generated. In Fig. [1], we illustrate the Feynman diagram for the case with the down-quark loop. The amplitude is proportional to the
coefficient
\[
(U_R^d)^2 \lambda U_L^e)_{ij} (\tilde{U}_L^d)_{3\lambda} \cdot (U_L^d M_d U_R^d)_{kl}
\]
\[
(\tilde{U}_R^d \lambda^T U_L^e)_{mi} (U_L^d M_d U_R^d)_{2\lambda} \cdot (U_L^d m^2_{dR})_{nm}
\]
\[
= (\tilde{m}_d^2 \lambda U_L^e)_{3\lambda} (M_d \lambda U_R^e)_{3\lambda},
\]
where \((\tilde{m}_d^2)_{ij}\) are coefficients of \((\tilde{d}_L^i),(\tilde{d}_R^j)\), and \((\lambda)_{ij} = \lambda_{ij}^L\). Similarly, we obtain the contributions from the charged lepton loops. Therefore, the radiatively induced neutrino mass matrix \(M_\nu\) is given by the following form
\[
(M_\nu)_{ij} = (f_i^e g_j^e + f_j^e g_i^e) K_e + (f_i^d g_j^d + f_j^d g_i^d) K_d,
\]
where \(K_f (f = e, d)\) are common factors independent of the families, and
\[
f_i^e = (M_\nu^T \lambda U_L^e)_{3\lambda}, \quad g_i^e = (\tilde{m}_e^2 \lambda U_L^e)_{3\lambda},
\]
\[
f_i^d = (M_d \lambda U_R^e)_{3\lambda}, \quad g_i^d = (\tilde{m}_d^2 \lambda U_R^e)_{3\lambda}.
\]
On the other hand, the contributions due to the \(H_d^+ - \tilde{e}^+\) mixing are as follows. There are no contributions from bilinear terms \(H_u(5)|H_d(5) + \sum_i \tilde{\psi}(5)\), because we can always eliminate the contributions from \(H_u(5)\bar{\tilde{\psi}}(5)\) by re-definition of the scalar \(H_d(5)\). Also, there are no contributions from \(H_d(5)H_d(5)\bar{\tilde{\psi}}(10)\), because \(\bar{\tilde{\psi}}(10)\) is anti-symmetric in SU(5) indices. However, the terms \(H_d(5)H_d(45)\bar{\tilde{\psi}}(10)\) can cause the \(H_d^+ - \tilde{e}^+\) mixing. These contributions affect only to the charged lepton loop diagrams.

The final result which includes the \(H_d^+\) contributions is given by
\[
M_{ij} = (f_i^e g_j^e + f_j^e g_i^e) K_e + (f_i^d g_j^d + f_j^d g_i^d) K_d + F_{ij} K_e,
\]
where
\[
F_{ij} = (U_L^e)^T \lambda M_e M_e^{-1} U_L^e)_{ij} + (i \leftrightarrow j),
\]
and \(M_e\) denotes the contributions due to
\[
\sum_{i \epsilon \nu_L H_u^\perp \lambda H_d^\perp} \sum_{i \in \nu_L} H_u^\perp H_d^\perp \lambda (H_d(5)\bar{\tilde{\psi}}(10)\), \quad \text{and} \quad (H_d(5)\bar{\tilde{\psi}}(10)\).
\]
We have already assumed that only one component of the linear combinations among those Higgs scalars survives at the low energy scale. Then, we can regard \(M_e\) as \(M_e = M_e\), so that we obtain \(F_{ij}\) as follows:
\[
F_{ij} = (\lambda D_e^\nu)_{ij} + (i \leftrightarrow j) = \lambda_{ij} (m_j^2 - m_i^2),
\]
where \(\lambda = U_L^e \lambda U_R^e\) is antisymmetric tensor as well as \(\lambda\), and \(m_i^+ = (m_1^d, m_2^d, m_3^d)\). Note that, in this case, the third term in (3.7) has the same mass matrix form as that in the original Zee model.

4. Phenomenology

In the SUSY GUT scenario, there are many origins of the neutrino mass generations. For example, the sneutrinos \(\tilde{\nu}_L\) can have the VEV, and thereby, the neutrinos \(\nu_\nu\) acquire their masses (for example, see Ref.[11]). Although we cannot rule out a possibility that the observed neutrino masses can be understood from such compound origins, in the present paper, we do not take such the point of view, because the observed neutrino masses and mixings appear to be rather simple and characteristic. We simply assume that the radiative masses are only dominated even if there are other origins of the neutrino mass generations.

However, even under such the assumption, we still have many parameters as shown in the expression (3.7). For simplicity, we assume that the contributions from the charged lepton loop are dominated compared with those from the down-quark loop, i.e., \(K_e \gg K_d\), which corresponds to the case \(m^2(d_L) - m^2(d_R) \gg m^2(\tilde{e}_L) - m^2(\tilde{e}_R)\). (We can give a similar discussion for the case \(K_d \gg K_e\). In the present paper, we do not discuss which case is reasonable.) Furthermore, we neglect the third term (\(H_d^+\) contributions) in (3.7). Then, the neutrino mass matrix (3.5) becomes a simple form
\[
(M_\nu)_{ij} = m_0 (f_i g_j + f_j g_i).
\]
Hereafter, for convenience, we will normalize \(f_i\) and \(g_i\) as
\[
|f_1| + |f_2| + |f_3| = 1, \quad |g_1|^2 + |g_2|^2 + |g_3|^2 = 1.
\]
In the most SUSY models, it is taken that the form of \(\tilde{m}_e^2 (f = e, d)\) is proportional to the fermion mass matrix \(M_f\). Then, the coefficients \(g_i\) are proportional to \(f_i\), so that the mass matrix (4.1) becomes \((M_\nu)_{ij} = 2m_0 f_i f_j\), which is a rank one matrix. Therefore, we rule out the case with \(\tilde{m}_e^2 \propto M_f\).
For convenience, hereafter, we assume that $f_i$ and $g_i$ ($i = 1, 2, 3$) are real. The mass eigenvalues and mixing matrix elements for the neutrino mass matrix (4.1) are given as follows:

$$
m_{1}' = (1 + \varepsilon)m_0, \quad m_{2}' = -(1 - \varepsilon)m_0, \quad m_{3}' = 0, \quad (4.3)
$$

$$
U_{11} = \frac{1}{\sqrt{2}} \frac{f_1 + g_1}{\sqrt{1 + \varepsilon}}, \quad U_{12} = \frac{1}{\sqrt{2}} \frac{f_1 - g_1}{\sqrt{1 - \varepsilon}}, \quad U_{13} = -\varepsilon_{ijk} \frac{g_k}{\sqrt{1 - \varepsilon^2}}, \quad (4.4)
$$

where

$$
\varepsilon = f_1 g_1 + f_2 g_2 + f_3 g_3. \quad (4.5)
$$

As seen in (4.3), the mass level pattern of the present model shows the inverse hierarchy as well as that of the Zee model. From (4.3), we obtain

$$
\Delta m^2_{21} \equiv (m_{2}')^2 - (m_{1}')^2 = -4\varepsilon m_0^2, \\
\Delta m^2_{32} \equiv (m_{3}')^2 - (m_{2}')^2 = -(1 - \varepsilon^2)m_0^2, \\
R \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{32}} = \frac{4\varepsilon}{(1 - \varepsilon^2)^2}, \quad (4.6)
$$

For a small $R$, the mixing parameters $\sin^2 2\theta_{\text{solar}}$, $\sin^2 2\theta_{\text{atm}}$ and $U_{13}^2$ are given by

$$
\sin^2 2\theta_{\text{solar}} \equiv 4U_{11}^2 U_{12}^2 = \frac{1}{1 - \varepsilon^2}(f_1^2 - g_1^2)^2, \quad (4.7)
$$

$$
\sin^2 2\theta_{\text{atm}} \equiv 4U_{23}^2 U_{33}^2 = \frac{1}{1 - \varepsilon^2}(f_1^2 + g_1^2 - 2\varepsilon f_1 g_1)^2, \quad (4.8)
$$

Let us demonstrate that the mass matrix (4.1) has reasonable parameters for the observed neutrino data. The atmospheric neutrino data \cite{12} suggests a large $\nu_{\mu}$-$\nu_{\tau}$ mixing, i.e., $\sin^2 2\theta_{23} \simeq 1$. It is known that the nearly maximal mixing is derived from the neutrino mass matrix $M_\nu$ with $2 \leftrightarrow 3$ symmetry \cite{13}. Therefore, we take

$$
f_1 = s_\alpha, \quad f_2 = f_3 = \frac{1}{\sqrt{2}}c_\alpha, \quad g_1 = c_\beta, \quad g_2 = g_3 = -\frac{1}{\sqrt{2}}s_\beta, \quad (4.9)
$$

where $c_\alpha = \cos \alpha$, $s_\alpha = \sin \alpha$ and so on. Then, the parameterization (4.11) gives

$$
\varepsilon = \sin(\alpha - \beta), \quad (4.10)
$$

$$
\sin^2 2\theta_{\text{solar}} = \cos^2(\alpha + \beta), \quad (4.11)
$$

$$
\sin^2 2\theta_{\text{atm}} = 1, \quad (4.12)
$$

$$
U_{13}^2 = 0. \quad (4.13)
$$

We assume that the values of $\alpha$ and $\beta$ are highly close each other, i.e., $\sin(\alpha - \beta) \sim 10^{-2}$. The result (4.13) with $\alpha \simeq \beta$ is free from the constraint (1.1) in the original Zee model, so that we can fit the value of $\sin^2 2\theta_{\text{solar}}$ with the observed value \cite{14} $\sin^2 2\theta_{\text{solar}} \sim 0.8$ from the solar neutrino data by adjusting the parameter $\alpha \simeq \beta$. Of course, the parameterization (4.11) is taken only from the phenomenological point of view, so that the results (4.13)-(4.15) are not theoretical consequences in the present model.

From the recent atmospheric and solar neutrino data \cite{12,14} $R \simeq (4.5 \times 10^{-5}\text{ eV}^2)/(2.5 \times 10^{-3}\text{ eV}^2) = 1.8 \times 10^{-2}$, we estimate $\varepsilon = 4.5 \times 10^{-3}$, and $m_0 \simeq m_1 \simeq |m_3| \simeq \sqrt{\Delta m^2_{32}} = 0.050\text{ eV}$. The effective neutrino mass $\langle m_\nu \rangle$ from the neutrinoless double beta decay experiment is given by $\langle m_\nu \rangle = (M_\nu)_{11} = 2m_0 s_\alpha s_\beta \simeq m_0 \sqrt{1 - \sin^2 2\theta_{\text{solar}}} \simeq 2.2 \times 10^{-2}\text{ eV}$, where we have used the observed value \cite{14} $\sin^2 2\theta_{\text{solar}} \simeq 0.8$.

5. Conclusion

In conclusion, we have proposed a neutrino mass matrix model based on a SUSY GUT model where only top quark takes $R$-parity violating interactions and the $Z_2$ symmetry plays an essential role, so that the proton decay due to the $R$-parity interactions can be evaded safely.

The general form of the radiatively induced neutrino mass matrix in the present model is given by the expression (3.7). Since the form (3.7) has too many parameters, we have investigated phenomenology for a simplified case (4.1). Although we have investigated the case $\varepsilon \simeq 0$ [where $\varepsilon$ is defined by (4.5)], the case $1 - \varepsilon \simeq 0$ is also possible. However, the purpose of the present paper
is to give the general form of the radiatively induced neutrino mass matrix (3.7) under the discrete symmetry $Z_2$. A systematic study of the mass matrix (3.7) will be given elsewhere.

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