Planck Scale to Hubble Scale

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Abstract

Within the context of the usual semi classical investigation of Planck scale Schwarchild Black Holes, as in Quantum Gravity, and later attempts at a full Quantum Mechanical description in terms of a Kerr-Newman metric including the spinorial behaviour, we attempt to present a formulation that extends from the Planck scale to the Hubble scale. In the process the so called large number coincidences as also the hitherto inexplicable relation between the pion mass and the Hubble Constant, pointed out by Weinberg, turn out to be natural consequences in a consistent description.

1 Introduction

Recently a description of Fermions was given in terms of the Kerr-Newman metric, and a background Zero Point Field with a cut off at the Compton wavelength scale[1, 2, 3, 4]. Indeed as is well known the Kerr-Newman metric describes the gravitational and electromagnetic field of an electron including the anomalous gyro magnetic ratio[5]. On the other hand the Zero Point Field with a Compton wavelength cut off not only avoids divergences, but it leads to the energy of a typical ementary particle[2, 6, 7]. It was shown that it is thus possible to think of particle formation from the Zero Point Field, the particles themselves being Kerr-Newman type Black Holes, but with the Quantum Mechanical input that the Zitterbewegung effects within the Compton wavelength remove a naked singularity.

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This scheme then leads to a cosmology that not only predicts puzzling empirical relations and is consistent, but in fact predicts an ever expanding universe with a cosmological constant \([1, 8, 9]\). Indeed it is quite remarkable that latest observations by different teams of observers confirm all this \([10, 11, 12]\).

At the other end of length scales, it can also be shown that the usual quark picture can be recovered from the above description \([13]\).

\section{The Planck Scale to Hubble Scale}

The above formulation works for Fermions. In Quantum Gravity however, we deal with Schwarzchild Black Holes, without the all important spin half. This is a semi classical domain brought out by the fact that for the Planck mass \(m_P \sim 10^{-5} \text{gms}\), we have

\[
\frac{Gm_P}{c^2} \sim \frac{\hbar}{mc}
\]

the left side is the classical Schwarzchild radius while the right side gives the Quantum Mechanical Compton wavelength. From (1) it follows

\[
\frac{Gm_P^2}{e^2} \sim 1
\]

which shows that all the energy at this scale is gravitational.

For a typical elementary particle, the pion for example we would have instead of (2), the well known relation,

\[
\frac{Gm^2}{e^2} \sim 10^{-40}
\]

which shows that at these scales it is electromagnetism that predominates.

We will now throw further light on the fact that at the Planck scale it is gravitation alone that manifests itself. Indeed Rosen \([14]\) has pointed out that one could use a Schrodinger equation with a gravitational interaction to deduce a mini universe, namely the Planck particle. The Schrodinger equation for a self gravitating particle has also been considered \([15]\), from a different point of view. We merely quote the main results.

The energy of such a particle is given by

\[
\frac{Gm^2}{L} \sim \frac{2m^5G^2}{\hbar^2}
\]
where
\[ L = \frac{\hbar^2}{2m^3G} \] (5)
(4) and (5) bring out the characteristic of the Planck particles and also the difference with elementary particles, as we will now see.
We first observe that for a Planck mass, (4) gives, self consistently,
\[ \text{Energy} = m_Pc^2, \]
while (5) gives,
\[ L = 10^{-33}\text{cms}, \]
as required.
However, the situation for pions is different: They are parts of the universe and do not constitute a mini universe. Indeed, if there are \( N \) pions in the universe, then the total gravitational energy is given by, from (4),
\[ \frac{NGm^2}{L} \] (6)
As this equals \( mc^2 \), we get back as can easily be verified, equation (3), or equivalently we deduce the pion mass!
Indeed given the pion mass, one can verify from (3) that \( L = 10^{28}\text{cms} \) which is the radius of the universe, \( R \). Remembering that \( R \approx \frac{c}{H} \), (3) in fact gives the empirical and otherwise inexplicable Weinberg formula\[16],
\[ m = \left( \frac{\hbar^2H}{Gc} \right)^{1/3} \] (7)
where \( m \) is the pion mass and \( H \) is the Hubble Constant.
Again gravitation dominates at large scales and the universe itself shows up as a Black Hole: Not only is \( R \approx \frac{GM}{c^2} \), where \( M \) is the mass of the universe, as for the Schwarzchild Black Hole, but also it can be shown that the age of the universe equals the proper time for travel from the centre to the edge as in the case of a Black Hole\[9].

3 The Stochastic Universe

The cosmological scheme referred to above, also deduces the various so called Dirac large number ”coincidences”, while at the same time deducing the mysterious and poorly understood relation (7). What makes (7) remarkable is
the correlation between an elementary particle and a cosmological constant, called the micro-macro nexus in [1].

The following suggestion was made to explain all this in reference [13]: If we consider the $N$ particles of the universe, as a statistical collection, then the typical uncertainty length $l$ is given by

$$l = \frac{R}{\sqrt{N}}$$

where $R$ is the dimension of the system, in this case the radius of the universe. As is well known, and this is one of Dirac’s large number coincidences, $l$ in (8) is given by the Compton wavelength of the typical elementary particle, namely the pion.

This brings us right back to the Compton wavelength and the Kerr-Newman description referred to above which also leads to the various fundamental interactions as detailed in references [1, 2, 4] and [13].

Let us now consider the stochastic picture in a little greater detail. As is well known, it is possible to obtain the Schrodinger equation from the theory of Brownian motion [17, 18, 19, 20]. In this case the motion of particles whose position is given by $x(t)$ is subject to random stochastic corrections $\Delta x(t)$. The Brownian character is expressed by the fact that the average of $\Delta x(t)$ in a time $\Delta t$ vanishes, that is,

$$< \Delta x > = 0$$

whereas the random change is given by

$$|\Delta x| = \sqrt{< \Delta x^2 >} \approx \nu \sqrt{\Delta t}$$

where the diffusion constant is given by

$$\nu = \frac{\hbar}{m}$$

and is related to the correlation length or mean free path by the equation

$$\nu = lv$$

(9)

We can then go on to the usual derivation of the Fokker-Planck equations. In Nelson’s derivation referred to above, the Schrodinger wave function, as in the De Broglie-Bohm theory (cf.ref.[2]), is decomposed as

$$\psi = \sqrt{\rho} e^{iS/\hbar}$$
The Schrödinger equation itself decomposes into two equations:

\[ v = \frac{1}{m} \nabla S \]

\[ \frac{\partial S}{\partial t} = -\frac{1}{2m} (\partial S)^2 + V + \frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \]

\( v \) is identified with the velocity, but there is a new term,

\[ V \equiv V_{\text{quantum}} = \frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \quad (10) \]

It is with this term, which is absent in the classical Hamilton-Jacobi theory, that Quantum Mechanics diverges from classical theory—this term also contains the reduced Planck Constant \( \hbar \). As Smolin notes\[19\], "...any attempt to explain Quantum Mechanics as arising from a probabilistic description of some more fundamental level of dynamics must come down to an explanation of this term'.

We would like to point out that all these equations, in fact, come up quite naturally in the above Kerr-Newman type Black Holes formulation (cf. ref.\[2\] for details). To put it simply, with \( v = c \), equation (9) above gives for the correlation length, the Compton wavelength, which is the radius of a relativistic vortex, or more accurately the Kerr-Newman Black Hole in this context, while the equation (10) gives the rest energy of the particle. This could be deduced independently in the relativistic case (cf. ref.\[2\]).

What all this means is that, as pointed out in\[1\] the assembly of \( N \) particles leads to a minimum uncertainty length, namely the Compton wavelength, and similarly uncertainty within the Compton time. In other words we have to consider minimum space time intervals: One cannot go to arbitrarily small space time intervals or points. Indeed in Quantum Mechanics space time points are very debatable concepts, though they have been used invariably (cf. ref.\[21\] and references therein).

In the above light, as in Nelson’s formulation, we come across what may be called Stochastic Holism—each particle is governed by the probability distribution of the entire assembly. There is a two-way feedback. Indeed as pointed out, in references\[1\] and\[21\] the supposedly mysterious equation (7) symbolises this holistic aspect. So also equation (8) and other large number coincidences including (3) become perfectly meaningful in this light.
The underpinning mechanism is an ambient ZPF. At the Compton wavelength scales, as pointed out above, this shows up as particles. Indeed at the length scale $l$ the energy density of the background ZPF is given by \[ \frac{\hbar c}{l^4} \] so that the energy within the Compton wavelength scale $l$ volume $\sim mc^2$, the rest energy of the particle as required.

It is again this ZPF which throws up a cosmological constant at large scales\[1, 9, 23\].

4 Discussion

We now make a few observations:

i) It must be remarked that what have hither to been considered to be two different fluctuations viz., the Quantum fluctuations and the Statistical fluctuations have really been unified into a single consideration.

ii) The $V$ term in equation (10) which characterises Quantum Mechanics now shows up as the rest energy of particles, the correlation between the Brownian processes being within the Compton wavelength correlation length which characterises the particle.

iii) Once we have a formulation of particles within the Compton wavelength as Kerr-Newman type Black Hole, then we not only get a unified picture of electromagnetism and gravitation\[4\] but also, as pointed out in the introduction the strong interactions and a quark picture emerge (cf. ref.\[4\] and \[13\] for details).

iv) Incidentally it may be pointed out that from much in the spirit of the foregoing considerations one could obtain the Planck Constant in terms of the fluctuations in the particle number $N$ of the universe. The fluctuation in the mass of a typical elementary particle like the pion due to the fluctuation of the particle number is given by

$$ \frac{G\sqrt{Nm^2}}{c^2R} $$

So we have

$$ (\Delta mc^2)T = \frac{G\sqrt{Nm^2}}{R}T = \frac{G\sqrt{Nm^2}}{c} $$

(11)
as \( cT = R \). It can be easily seen that the right side of (11) equals \( \hbar \)! That is we have

\[
\hbar \approx \frac{G\sqrt{N}m^2}{c},
\]

The equation (12) expresses the Planck constant in terms of non Quantum Mechanical quantities.

v) The considerations of Section 2 indicate why the Planck particles at the lowest scale and pions at the elementary particle scale are fundamental. Thus equation (4) immediately leads to the Planck mass and the Planck length for a single particle universe, whereas equation (8) for a \( N \) particle universe leads to the pion mass and the radius of the universe. This can also be seen in the light of the background ZPF. For a single particle universe, equating the gravitational energy, given by the left side of (4) with the ZPF energy in a volume \( \sim L^3 \), where now \( L \) is the Planck length, viz., \( \hbar c/L \), we get back (4) or equivalently the Planck mass.

However equating the corresponding energy for an elementary particle given by (4) with a similar ZPF energy and using (8), we recover this time the mass of the pion, which thus shows up as a natural choice for a typical elementary particle.

vi) In the light of the comments in Section 3 the Quantum non locality is no longer mysterious [21, 24].

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