Lepton mass matrices from a quark-lepton analogy

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ABSTRACT

We study the implications of having a similarity between quark and lepton mixing in the Dirac sector of the Standard Model plus the right-handed neutrino. This enable us to describe all masses and mixings in the Dirac sector in terms of only five parameters: three mass scales $m_b$, $m_{\tau}$ and $m_t$, one parameter $\lambda$ describing all the mixings, and a CP violating phase in the quark sector. Then, from experimental data on neutrino masses and mixings, we extract the heavy neutrino mass matrix. The approach considered, although does not contradict Grand Unified Theories, can help to find other theoretical models of fermion masses.

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The Super-Kamiokande Collaboration has confirmed the oscillation of atmospheric neutrinos [1]. This evidence, as well as the indications of oscillation of solar neutrinos to solve the solar neutrino problem [2], leads to a finite mass of neutrinos. However, in the Minimal Standard Model (MSM), neutrino is massless. In fact, with only the left-handed neutrino $\nu_L$ we cannot get a Dirac mass, and with only the Higgs doublet we cannot get a Majorana mass for $\nu_L$. Adding a right-handed neutrino $\nu_R$ allows to build a Dirac mass term of neutrino, in analogy with the other Dirac masses of fermions in the theory. But neutrino mass is very small if compared with the other fermion masses. The see-saw mechanism [3] explains this feature, giving a large Majorana mass to $\nu_R$. This mass is not constrained and in fact defines a new scale.

Regarding the mixings, recent data imply large mixing between second and third lepton family [4], while the mixing between first and second lepton family may be large or small [4]. On the contrary, in the quark sector all mixings are small [5]. The see-saw mechanism helps to understand also these features: the Dirac sector of the theory may give similar quark and lepton mixings, while the effect of the Majorana sector can enhance the effective lepton mixing [6].

Motivated by that, we construct explicitly a quark-lepton analogy in the Dirac sector, suggested by ref.[7], and derive all fermion mass matrices, using experimental limits on neutrino masses and mixings [8]. A very simple scheme comes out. This scheme has some resemblance with the Grand Unified Theory (GUT) scheme [9] (see also [10]), but we work it out within a more general approach, where the MSM is enlarged to include the right-handed neutrino only.

Now, we first summarize the experimental data on lepton mixing, then we briefly describe the formalism of the see-saw mechanism, and finally explore the
consequences of a quark-lepton analogy in the Dirac mixing of the theory.

Weak and mass eigenstates of neutrino are connected by the relation

$$\nu_\alpha = U_{\alpha i} \nu_i$$

(1)

where $U$ is a unitary matrix, $\alpha = e, \mu, \tau$, and $i = 1, 2, 3$. According to ref. we write $U$ as

$$U = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}c_{23} & c_{12}c_{23} & s_{23} \\ s_{12}s_{23} & -c_{12}c_{23} & c_{23} \end{pmatrix}.$$  

(2)

(We put zero in position 1-3, although it is only constrained to be much less than one.) The experimental data on oscillation of atmospheric and solar neutrinos lead to three possible numerical forms for $U$. These correspond to the three solutions of the solar neutrino problem, namely small mixing or large mixing MSW, and vacuum oscillations. The small mixing solution is now preferred over the large mixing one, and we refer to this solution and the vacuum oscillations, although the same calculation can be done for the large mixing MSW. Choosing the central values of $s_{12}$ and $s_{23}$ of ref., we obtain, for small mixing MSW,

$$U = \begin{pmatrix} 1 & 0.04 & 0 \\ -0.032 & 0.80 & 0.60 \\ 0.024 & -0.60 & 0.80 \end{pmatrix}.$$  

(3)

and, for vacuum oscillations,

$$U = \begin{pmatrix} 0.80 & 0.60 & 0 \\ -0.474 & 0.632 & 0.61 \\ 0.366 & -0.488 & 0.79 \end{pmatrix}.$$  

(4)

In the following we need also the values of light neutrino masses. From the experimental limits on $\Delta m^2_{32}$, $\Delta m^2_{21}$ we take, for example,

$$m_{\nu_1} = 2.2 \times 10^{-4}, \ m_{\nu_2} = 2.8 \times 10^{-3}, \ m_{\nu_3} = 3.6 \times 10^{-2} \ \text{(eV)}$$  

(5)
and
\[ m_{\nu_1} = 3.0 \times 10^{-9}, \quad m_{\nu_2} = 1.0 \times 10^{-5}, \quad m_{\nu_3} = 3.6 \times 10^{-2} \text{ (eV)}, \tag{6} \]
for small mixing MSW and vacuum oscillations, respectively. We assume a hierarchical pattern \( m_{\nu_1} \ll m_{\nu_2} \ll m_{\nu_3} \), which gives \( m_{\nu_3} / m_{\nu_2} = m_{\nu_2} / m_{\nu_1} \), which gives \( m_{\nu_1} \).

Now we briefly explain the formalism of the Standard Model plus \( \nu_R \) with the see-saw mechanism. The part of the Lagrangian we are interested in is
\[ \overline{e}_L M_e e_R + \overline{\nu}_L M_\nu \nu_R + g \overline{\nu}_L e_L W + \overline{\nu}_L M'_R \nu_R \tag{7} \]
where \( M_e \) and \( M_\nu \) are the Dirac mass matrices of charged leptons and neutrinos respectively, and \( M'_R \) is the Majorana mass matrix of right-handed neutrinos. We assume the elements of \( M'_R \) much greater than those of \( M_\nu \). When we diagonalize the Dirac mass matrices we have (renaming the fermion fields)
\[ \overline{e}_L D_e e_R + \overline{\nu}_L D_\nu \nu_R + g \overline{\nu}_L V_D e_L W + \overline{\nu}_L M'_R \nu_R \tag{8} \]
where \( D_e \) and \( D_\nu \) are diagonal and \( V_D \) is the analogous of \( V_{\text{CKM}} \) in the sense that it rises from the diagonalization of the Dirac lepton sector. The see-saw mechanism leads to the effective Lagrangian
\[ \overline{e}_L D_e e_R + \overline{\nu}_L M_L \nu_R + g \overline{\nu}_L V_D e_L W + \overline{\nu}_L M'_R \nu_R \tag{9} \]
with the Majorana mass matrix of left-handed neutrinos
\[ M_L = D_\nu M_{R}^{-1} D_\nu. \tag{10} \]
Now, if we diagonalize also \( M_L \), we obtain
\[ \overline{e}_L D_e e_R + \overline{\nu}_L D_L \nu_R + g \overline{\nu}_L V_{\nu} e_L W + \overline{\nu}_L M_R \nu_R \tag{11} \]
where the unitary matrix $V_s$ specifies the effect of the see-saw mechanism on lepton mixing that is the effect of the Majorana mass matrix of the right-handed neutrino $\nu_R$. The product

$$V_{lep} = V_s V_D$$

is the lepton mixing matrix that appears in the charged current interaction, and is related to the neutrino mixing matrix $U$ by

$$V_{lep} = U^+.$$  \hspace{1cm} (13)

As for the quark sector, we have now

$$V_D = V_\nu^+ V_e,$$  \hspace{1cm} (14)

which is the analogous of $V_{CKM} = V_u^+ V_d$, while

$$V_s M_L V_s^+ = D_L$$

(15)

where $D_L = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$ gives the masses of light neutrinos.

In the quark sector we can always choose $M_u$ diagonal and $M_d$ with three zeros in certain positions \cite{7}. In the same way, in the Dirac lepton sector, we can always choose $M_\nu$ diagonal and $M_e$ with three zeros in the same positions (then $M_R' = M_R$). For the quark case, using one of these bases, in ref.\cite{7}, a very simple description of the down quark mass matrix $M_d$ and the $V_{CKM}$ matrix was inferred:

$$M_d \simeq \begin{pmatrix} 0 & \sqrt{m_d m_s} & 0 \\ \sqrt{m_d m_s} & m_s & m_s \\ 0 & m_b/\sqrt{5} & 2m_b/\sqrt{5} \end{pmatrix}$$

(16)

that yields

$$V_{us} \simeq \sqrt{\frac{m_d}{m_s}}, \quad V_{cb} \simeq \frac{3}{\sqrt{5}} \frac{m_s}{m_b}, \quad V_{ub} \simeq \frac{1}{\sqrt{5}} \frac{\sqrt{m_d m_s}}{m_b}.$$  \hspace{1cm} (17)
Such a basis shows, in a single matrix, \( M_d \), the hierarchy of both down quark masses and flavor mixings. As also charged lepton masses are hierarchical, one can imagine a similar structure of mixing in the Dirac lepton sector and in the quark sector. Then, we set

\[
V_D \simeq \begin{pmatrix}
1 - \frac{m_e}{m_{\mu}} & \sqrt{\frac{m_e}{m_{\mu}}} & \frac{1}{\sqrt{5}} \sqrt{\frac{m_e m_{\mu}}{m_r}} \\
-\sqrt{\frac{m_e}{m_{\mu}}} & 1 - \frac{m_e}{m_{\mu}} & \frac{3}{\sqrt{5}} \frac{m_e}{m_{\tau}} \\
\frac{2}{\sqrt{5}} \sqrt{\frac{m_e m_{\mu}}{m_r}} & \frac{3}{\sqrt{5}} \frac{m_e}{m_{\tau}} & 1
\end{pmatrix}
\]  
(18)

and hence, in analogy with ref. [7], we can write

\[
M_e \simeq \begin{pmatrix}
0 & \sqrt{m_e m_{\mu}} & 0 \\
\frac{m_{\mu}}{m_{\mu}} & m_{\mu} & \frac{m_{\mu}}{m_{\mu}} \\
0 & \frac{m_{\mu}}{m_{\mu}} & \frac{2 m_{\mu}}{\sqrt{5}}
\end{pmatrix}
\]  
(19)

(we set the CP violating phase in the leptonic sector equal to zero), while \( M_\nu \) is diagonal. Using quark and lepton masses (at the scale \( M_Z \)) as calculated in ref. [10], one can check from Eqns.(16),(19) that

\[
M_d \simeq m_b \begin{pmatrix}
0 & \lambda^3 & 0 \\
\lambda^3 & \lambda^2 & \lambda^2 \\
0 & \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}}
\end{pmatrix},
\]  
(20)

\[
M_e \simeq m_{\tau} \begin{pmatrix}
0 & \frac{1}{2} \lambda^3 & 0 \\
\frac{1}{2} \lambda^3 & \frac{1}{2} \lambda^2 & \frac{2}{\sqrt{5}} \lambda^2 \\
0 & \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}}
\end{pmatrix},
\]  
(21)

with \( \lambda \simeq \sqrt{m_d/m_s} \simeq \sqrt{m_s/m_b} \). This means that, on the basis of ref. [7] (both \( M_u \) and \( M_\nu \) are diagonal), if we assume Eqn.(18), then we obtain a simple description of both \( M_d \) and \( M_e \) in terms of three real parameters plus one phase in \( M_d \) (not reported). Actually we have

\[
V_{D12} \simeq \frac{1}{3} \lambda,
\]  
(22)

\[
V_{D23} \simeq \frac{9}{2 \sqrt{5}} \lambda^2 \simeq 2 \lambda^2,
\]  
(23)
\[ V_{D13} \simeq \frac{1}{2\sqrt{5}} \lambda^3 \simeq \lambda^4. \]  

(24)

Now we proceed towards the determination of Majorana mass matrices. From Eqn.(12) we have

\[ V_s = V_{lep} V_D^+ \]  

(25)

from which, by Eqn.(13), we can calculate \( V_s \). Then, from Eqn.(15) we get

\[ M_L = V_s^+ D_L V_s, \]  

(26)

and from Eqn.(10) we have also

\[ M_R = D_\nu M_L^{-1} D_\nu. \]  

(27)

As \( M_d \) is almost proportional to \( M_e \) by the factor \( m_b/m_\tau \), we can assume

\[ D_\nu \simeq \frac{m_\tau}{m_b} D_u, \]  

(28)

which gives the Dirac masses of neutrinos \( m_1 = 0.00136, m_2 = 0.394, m_3 = 105 \) (GeV). In GUT’s the ratio \( m_b/m_\tau \) is related to renormalization group evolution of Yukawa couplings from the intermediate scale, rather than the unification scale \([17]\), to the weak scale. We have also \( \sqrt{m_u/m_t} \simeq m_e/m_t \simeq \lambda^4 \) \([18]\), thus

\[ M_u = D_u \simeq m_t \text{diag}(\lambda^8, \lambda^4, 1), \]  

(29)

and then all Dirac masses and mixings are expressed in terms of four real parameters, that is \( m_b, m_\tau, m_t, \) and \( \lambda \). By Eqn.(27) we can now calculate the heavy neutrino mass matrix \( M_R \). We yield (in GeV) for the two cases of small mixing MSW and vacuum oscillations, respectively:

\[ M_R = \begin{pmatrix}
8.3 \times 10^6 & -2.2 \times 10^8 & 1.6 \times 10^{10} \\
-2.2 \times 10^8 & 3.9 \times 10^{10} & -7.2 \times 10^{12} \\
1.6 \times 10^{10} & -7.2 \times 10^{12} & 1.9 \times 10^{15}
\end{pmatrix}, \]  

(30)
with eigenvalues

\[ M_1 = 6.0 \times 10^6, M_2 = 1.2 \times 10^{10}, M_3 = 1.9 \times 10^{15}, \quad (31) \]

and

\[ M_R = \begin{pmatrix}
3.6 \times 10^{11} & -6.8 \times 10^{13} & 1.5 \times 10^{16} \\
-6.8 \times 10^{13} & 1.3 \times 10^{16} & -2.8 \times 10^{18} \\
1.5 \times 10^{16} & -2.8 \times 10^{18} & 6.0 \times 10^{20}
\end{pmatrix}, \quad (32) \]

with eigenvalues

\[ M_1 = 2.6 \times 10^7, M_2 = 1.9 \times 10^{11}, M_3 = 6.0 \times 10^{20}. \quad (33) \]

The highest mass scale appearing is near the grand unification scale, in the former case, while it is near the super unification scale (the Planck mass), in the latter. We also observe a huge hierarchy. However, \( M_R \) depends on values of light neutrino masses, which are not well established. Therefore, if the origin of the right-handed neutrino mass is at some intermediate scale, then, from Eqns. (30), (32), we find that the small mixing MSW solution is favoured.

Now we compare our scheme with the one in ref. [3], based on a GUT (for example \( SO(10) \)). There, mass matrices are symmetric:

\[ M_d \simeq m_b \begin{pmatrix}
0 & \lambda^3 & 0 \\
\lambda^3 & \lambda^2 & \lambda^2 \\
0 & \lambda^2 & 1
\end{pmatrix}, \quad (34) \]

\[ M_e \simeq m_\tau \begin{pmatrix}
0 & \lambda^3 & 0 \\
\lambda^3 & -3\lambda^2 & \lambda^2 \\
0 & \lambda^2 & 1
\end{pmatrix}, \quad (35) \]

\[ M_u \simeq m_t \begin{pmatrix}
0 & 0 & \lambda^4 \\
0 & \lambda^4 & 0 \\
\lambda^4 & 0 & 1
\end{pmatrix}, \quad (36) \]

\[ M_\nu \simeq \frac{m_\tau}{m_b} M_u, \quad (37) \]
and lead to

\[ V_{D12} \simeq \frac{1}{3} \lambda, \quad (38) \]

\[ V_{D23} \simeq \lambda^2, \quad (39) \]

\[ V_{D13} \simeq \lambda^4. \quad (40) \]

The dependence of mixings on \( \lambda \) is similar to our scheme, but Eqns.(32-35) rely on some *ad hoc* couplings of Higgs bosons to fermions (for example, in \( SO(10) \), the coefficient \(-3\) should come from a \( \bf{126} \) which couples only to the second generation \[19\], while the other couplings come from a \( \bf{10} \)). We argue that Eqns. (20),(21), with \( M_u \) and \( M_\nu \) diagonal, should approximately be the expression of Eqns.(34)-(37) on our basis. Only \( V_{23} \) in Eqn.(23) is twice the value in Eqn.(39).

A further remark can be done: if we change the numerical coefficients in \( V_D \) (Eqn.(18)), leaving the same dependence on mass ratios, the result of a few-parameter description of mass and mixing does not change. This suggests also an intriguing possibility, that is \( V_D = V_{CKM} \), which differs from GUT mixings for \( V_{D12} \simeq \lambda \), and should imply a CP violating phase in the leptonic sector too. A better experimental knowledge of fermion masses and mixings, as well as theoretical speculations \[20\], can decide among different patterns of lepton mass and mixing in the Dirac sector. Using our basis should help this task.

In conclusion, from experimental data on neutrino mass and mixing, and a quark-lepton analogy, we obtained all lepton mass matrices. The ideas developed here, although could be in agreement with GUT’s, can also be useful to study some other models of fermion masses and mixings.

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References

[1] Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 81, 1562 (1998)

[2] R. Davis, D. Harmer, and K. Hoffman, Phys. Rev. Lett. 20, 1205 (1968)
   V. Gribov and B. Pontecorvo, Phys. Lett. B 28, 493 (1969)

[3] M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, eds. P. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979)
   T. Yanagida, Proceedings of the Workshop on Unified Theory and Barion Number in the Universe, eds. O. Sawada and A. Sugamoto (KEK, 1979)

[4] N. Hata and P. Langacker, Phys. Rev. D 56, 6107 (1997)

[5] Particle Data Group, C. Caso et al., Eur. Phys. J. C 3, 1 (1998), p. 103
   L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983)

[6] A.Yu. Smirnov, Phys. Rev. D 48, 3264 (1993)

[7] D. Falcone and F. Tramontano, [hep-ph/9806496], to be published in Phys. Rev. D

[8] S.M. Bilenky and C. Giunti, [hep-ph/9802201]
   M. Narayan, G. Rajasekaran, and S.U. Sankar, Phys. Rev. D 58, 031301 (1998)

[9] M. Bando, T. Kugo, and K. Yoshioka, Phys. Rev. Lett. 80, 3004 (1998)

[10] P. Ramond, R.G. Roberts, and G.G. Ross, Nucl. Phys. B 406, 19 (1993)
[11] S.P. Mikheyev and A. Yu. Smirnov, Nuovo Cimento C 9, 17 (1986); Sov. J. Nucl. Phys. 42, 913 (1985)
   L. Wolfenstein, Phys. Rev. D 17, 2369 (1978)

[12] G.L. Fogli, E. Lisi, and D. Montanino, Astropart. Phys. 9, 119 (1998)

[13] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963)
   M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973)

[14] A.Yu. Smirnov, Nucl. Phys. B 466, 25 (1996)

[15] D. Falcone, O. Pisanti, and L. Rosa, Phys. Rev. D 57, 195 (1998)
   R. Haussling and F. Scheck, Phys. Rev. D 57, 6656 (1998)

[16] H. Fusaoka and Y. Koide, Phys. Rev. D 57, 3986 (1998)

[17] N.N. Singh and S.B. Singh, Eur. Phys. J. C 5, 363 (1998)
   H. Arason et al., Phys. Rev. Lett. 67, 2933 (1991)

[18] W.-S. Hou and G.-G. Wong, Phys. Rev. D 52, 5269 (1995)

[19] H. Georgi and C. Jarlskog, Phys. Lett. B 86, 297 (1979)

[20] N. Irges, S. Lavignac, and P. Ramond, Phys. Rev. D 58, 035003 (1998)