Electromagnetic Higgs production

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ABSTRACT: The cross section for central diffractive Higgs production is calculated, for the LHC range of energies. The graphs for the possible mechanisms for Higgs production, through pomeron fusion and photon fusions are calculated for all possibilities allowed by the standard model. The cross section for central diffractive Higgs production through pomeron fusion, must be multiplied by a factor for the survival probability, to isolate the Higgs signal and reduce the background. Due to the small value of the survival probability \((4 \times 10^{-3})\), the cross sections for central diffractive Higgs production, in the two cases for pomeron fusion and photon fusion, are competitive.

KEYWORDS: Higgs boson, BFKL pomeron, diffractive production, LHC, electromagnetic Higgs production, photon photon fusion, pomeron pomeron fusion, survival probability, standard model.

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1. Introduction

The most promising process for observation of the Higgs boson at the LHC is central diffractive production of the Higgs, with large rapidity gaps (LRG) between the Higgs and the two emerging protons, after scattering. Namely,

\[ p + p \rightarrow p + [\text{LRG}] + H + [\text{LRG}] + p \]  

(1.1)

The large rapidity gaps either side of the Higgs reduces the background, so that the Higgs signal will be easier to isolate in central exclusive production. Hence, this process gives the best experimental signature.
for detecting the Higgs at the LHC, and is very interesting for experiments in the search for the Higgs boson.

In this paper, two mechanisms are compared for central exclusive Higgs production; (1) $\gamma\gamma$ fusion, namely $pp \rightarrow \gamma\gamma \rightarrow H$ and (2) pomeron exchange, namely $pp \rightarrow PP \rightarrow H$, where $P$ denotes a pomeron. For $\gamma\gamma$ fusion, there is no hard re-scattering of the photons to fill up the rapidity gaps, so that large rapidity gaps are automatically present. The motivation for considering $PP \rightarrow H$, is the observation that for gluon gluon fusion, namely the process $gg \rightarrow H$ the colour flow induces many secondary parton showers which fill up the rapidity gaps. Instead the process $PP \rightarrow H$ is a colour singlet exchange, where the colour flow is screened, and the large rapidity gaps are preserved.

However in the case of $PP$ fusion, there are hard re-scattering corrections, giving additional inelastic scattering, which will give emission filling up the rapidity gaps (see ref. [1]). To guarantee the presence of large rapidity gaps after scattering in central exclusive production, in the case of $PP$ fusion, one has to multiply by the survival probability. This is the probability that large rapidity gaps, between the Higgs boson and the emerging protons, will be present after scattering.

The motive of this paper, was driven by the potentially small value for the survival probability, namely $< |S^2| > = 4 \times 10^{-3}$, calculated in ref. [1] for central exclusive Higgs production via $PP$ fusion. This is an order of magnitude less than previous estimates, namely $< |S^2| > = 0.02$ for LHC energies (see ref. [2]). Hence, previous calculations in ref. [2] of the cross section for central exclusive Higgs production via $PP$ fusion, which included a factor for the survival probability of $< |S^2| > = 0.02$, gave for the exclusive cross section $\sigma_{PP}^{exc} (pp \rightarrow p + H + P) = 3 \text{fb}$. Since the survival probability is predicted in ref. [1] to be an order of magnitude less, it follows that the cross section $\sigma_{PP}^{exc} (pp \rightarrow p + H + P)$ for central exclusive Higgs production will also be an order of magnitude smaller. It also follows that this cross section will be competitive with the cross section for central exclusive Higgs production via $\gamma\gamma$ fusion, which is predicted in ref. [1] to be $\sigma_{\gamma\gamma}^{exc} (p + p \rightarrow p + H + p) = 0.1 \text{fb}$.

To illustrate that for the process of Eq. (1.1), the cross sections will be competitive for $\gamma\gamma$ fusion and $PP$ fusion, it is instructive to consider the diagrams for these processes. The notation used for the couplings in the standard model are the following.
where the mass of the Higgs boson, is derived from the vacuum expectation value of the Higgs SU(2) weak isodoublet, which is 

\[
\begin{pmatrix}
\tilde{H} \\
\tilde{H}
\end{pmatrix}
\]

= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix},
\]

where \( v = \sqrt{-\mu^2 / \lambda} \), and \( \mu \) and \( \lambda \) are parameters in the Higgs potential, which is introduced into the standard model in the spontaneous symmetry breaking of \( SU_L(2) \times U_Y(1) \to U_{EM}(1) \), which is responsible for giving rise to the W and Z boson masses. In the case for \( \gamma\gamma \) fusion shown in Fig. 3, there are four vertices proportional to \( \alpha_{em} \) coupling the photons to the two protons and the photons either side of the subprocess for \( \gamma\gamma \to H \). A factor of \( g_w^2 = 4\sqrt{2}G_F m_w^2 \) should also be included, to account for the weak coupling of the Higgs to the sub-process, depicted by the shaded area in Fig. 3. So it is expected that the cross section will be proportional to

\[
\sigma_{\gamma\gamma}^{exc} ( p + p \to p + H + P) \propto 4\sqrt{2}G_F m_w^2 \alpha_{em}^4 = 0.6 \text{ fb}
\]

where units are defined as \( 1 \text{ GeV}^{-2} = 0.3893 \text{ mb} \). In central exclusive Higgs production for the case for \( \gamma\gamma \) fusion, all the couplings of the photons shown in Fig. 3 are known constants, namely they are proportional to \( \alpha_{em} \). Hence, the cross section for this diagram can be calculated exactly. On the other hand, when considering central exclusive Higgs production for the case for \( IP\bar{P} \) fusion shown in Fig. 1, the couplings of the gluons are not constants. In Fig. 1 there are four gluon couplings with the protons, giving a contribution to the cross section proportional to \( \alpha_s^4 (Q^2) \), where \( Q^2 \) is the momentum transferred along the pomeron, and it is assumed that \( \alpha_s (Q^2) \sim 0.2 \). There are also two gluon couplings with the subprocess for \( IP\bar{P} \to H \), giving a contribution to the cross section proportional to \( \alpha_s^2 (M_H^2) \). Taking the mass of the Higgs to be \( M_H \sim 100 \text{ GeV} \), then it is expected that \( \alpha_s (M_H^2) \sim 0.12 \). Also in the case of \( IP\bar{P} \) fusion, a factor of \( g_w^2 = 4\sqrt{2}G_F m_w^2 \) should also be included, to account for the weak coupling of the Higgs to the sub-process shown in Fig. 1. For \( IP\bar{P} \) fusion, this subprocess is the quark triangle subprocess shown in Fig. 2. Since the gluon itself couples weakly to the Higgs boson, only the contribution of the quark triangle subprocess is taken into account. The amplitude for the quark triangle subprocess, is

| coupling constant | expression | value ( GeV / c^2 ) |
|-------------------|------------|----------------------|
| \( \alpha_{em} \) | \( \frac{e^2}{4\pi} \) | \( \approx \frac{1}{137} \) |
| \( G_F \) | \( \frac{\sqrt{2}g_w^2}{8m_w^2} \) | \( 1.17 \times 10^{-5} \) |
| \( m_w \) | \( \frac{1}{2}v g_w \) | 80 |
| \( M_H \) | \( \sqrt{2\lambda v^2} \) | 120 |
| \( \alpha_s (M_H^2) \) | \( g_w^2 (M_H^2) / 4\pi \) | 0.12 |
derived in section 3.1 for the electromagnetic case, where $\alpha_s$ replaces $\alpha_{em}$. After multiplying by a factor for the survival probability, which includes the survival probability $<|S^2_{\text{hard}}|>$ $= 4 \times 10^{-3}$ which takes into account hard re-scattering of the pomeron, and $<|S^2_{\text{soft}}|>$ $= 5 \times 10^{-2}$ which takes into account soft re-scattering of the pomeron, then one obtains for the exclusive cross section for central exclusive Higgs production, in the case of $IPIP$ fusion the value

$$\sigma_{IPIP}^{\text{exc}}(p + p \rightarrow p + H + p) \propto 4 \sqrt{2} G_F m_w^2 \alpha_s^2 (M_H^2) \alpha_s^4 (Q^2) <|S^2_{\text{hard}}|> <|S^2_{\text{soft}}|> = 0.9 \text{ fb} \quad (1.3)$$

Comparing the estimates of Eq. (1.2) and Eq. (1.3), it is expected that $\sigma_{\gamma\gamma}^{\text{exc}}(p + p \rightarrow p + H + p)$ and $\sigma_{IPIP}^{\text{exc}}(p + p \rightarrow p + H + p)$ will be competitive. This is the motivation for this paper which re-examines $\sigma_{\gamma\gamma}^{\text{exc}}(p + p \rightarrow p + H + p)$ for electromagnetic Higgs production.

This paper is organised in the following way. In section 2, the details of the calculation of the cross section for central exclusive Higgs production in the case for $IPIP$ fusion is explained in detail. The cross section which is obtained, is multiplied by the factor for the survival probability in ref. [1], to give the exclusive cross section. In section 3, the cross section for central exclusive Higgs production, in the case of $\gamma\gamma$ fusion is calculated. The mechanism $\gamma\gamma \rightarrow H$ proceeds via the fermion triangle and Boson loop sub-processes illustrated in Fig. 4 and Fig. 5. The total cross section for central exclusive Higgs production, in the case of $\gamma\gamma$ fusion is calculated by taking the sum over all the contributions for these sub-processes for the $\gamma\gamma \rightarrow H$ mechanism. Finally, in the conclusion the results found in section 2 and section 3 are compared.

All calculations in this paper, are based on the Feynman rules for the standard electro-weak theory, given in Fig. 6 which is to be found in the appendix.

2. Double diffractive Higgs production at the LHC

In this section, an explicit expression is derived for the Born amplitude for double diffractive Higgs production (see Fig. 1). The Higgs couples weakly to the gluon, so the main contribution comes from the quark triangle subprocess (see Fig. 2), and all six flavours of quarks are taken into account. The second $t$ channel gluon in Fig. 1 is included. This is because of the large rapidity (LRG) gap between the Higgs and the proton, which demands that in the $t$ channel, one has colorless exchange. Indeed, if the LRG wasn’t present between the protons, then the Higgs could simply be produced from gluon gluon fusion in a single channel. However, the colour flow induced by a single channel exchange process, could produce many secondary particles. These secondary particles could fill up the LRG. To screen the colour flow, it is necessary to exchange a second $t$ channel gluon. At lowest order in $\alpha_s$, this gluon couples only to the incoming quark lines. The Born amplitude for double diffractive Higgs production by gluon exchange, is given by the expression $[3, 4]$.
where $\vec{k}_1 \cdot \vec{k}_2 = \frac{M_H^2}{2}$ has been used. In Eq. (2.1), the Weizsäcker - Williams approach, explained in ref. [5] has been used. The factor $A\vec{k}_1 \cdot \vec{k}_2$ is the amplitude for the quark triangle subprocess of Fig. 2, where $A$ takes the value, [6, 7, 8, 9]

$$A = \frac{2}{3} \left( \frac{-\alpha_s (M_H^2) (\sqrt{2G_F})^{1/2}}{\pi} \right)$$

(2.2)

The Born amplitude shown in Fig. 1 is extended to proton scattering instead of just quark scattering here. The typical momentum transferred, $t_1 = (q_1 - Q)^2$ and $t_2 = (q_2 - Q)^2$, are rather small and of the order of $\frac{1}{b}$, where $b$ is the slope of the gluon - proton form factor, and can be estimated to be $b = 5.5 GeV^2$. Therefore, one takes into account the proton couplings to the gluon ladder (see Fig. 2), by including the proton form factors in the gaussian form

$$\exp \left( -\frac{1}{2} bt_1 - \frac{1}{2} bt_2 \right) \text{ where } t_1 = (q_1 - Q)^2 \text{ and } t_2 = (q_2 - Q)^2$$

(2.3)
Figure 2: Quark triangle subprocess for Higgs production in gluon gluon fusion

to describe the dependence on the transferred momentum. If the momentum transfer is small, it can be assumed that $k_1 \sim k_2 \sim Q$. Therefore, with the definition of Eq. (2.3), $t_1 \to 0$ and $t_2 \to 0$. Hence, the proton form factors $e^{-\frac{1}{2}bt_1}$ and $e^{-\frac{1}{2}bt_2}$ tend to 1, and can be taken outside the integral, and the Born amplitude behaves as

$$
\frac{2}{9} A e^{-\frac{1}{2}bt_1} e^{-\frac{1}{2}bt_2} \int \frac{d^2Q}{Q^2} 8\alpha_s \left( Q^2 \right)
$$

(2.4)

To consider the exclusive process only, with the condition of the LRG, bremsstrahlung gluons must be suppressed. The bremsstrahlung gluons are shown by the dashed lines in Fig. 1, which are suppressed by multiplying by the Sudakov form factor

$$
F_s = e^{-S(k_1^2, E_1^2)}
$$

(2.5)

$F_s$ is the probability not to emit bremsstrahlung gluons. $S$ is the mean multiplicity of Bremsstrahlung gluons given as

$$
S \left( k_1^2, E_1^2 \right) = \int_{k_1^2}^{E_1^2} \frac{dp_1^2}{p_1^2} \int_{p_1^2}^{\infty} \frac{d\omega}{\omega} \frac{3\alpha_s \left( p_1^2 \right)}{\pi} \left( \ln \left( \frac{E_1^2}{k_1^2} \right) \right)^2
$$

(2.6)

Secondly, evolution of BFKL ladder gluons between the two channels (see Fig. 1), must be taken into account. For proton scattering instead of quark scattering, the naive coupling of the gluons to the external
quarks must be replaced by a coupling of the gluons to the external proton lines. To include both of these modifications, the naive gluon density for quarks is replaced by the density for protons by the following substitution

\[ \frac{4\alpha_s (Q^2)}{3\pi} \to f(x, Q^2) \]  

where \( f(x, k^2) \) is the un-integrated gluon density of the proton. After including the Sudakov form factor of Eq. (2.1), and the gluon density function of Eq. (2.7), the amplitude in Eq. (2.1) becomes

\[
M_{\P\P} (p+p \to p+H+p) = A_{\pi}^3 s \int \frac{dQ^2}{Q^2_\perp} e^{-S(k^2_\perp, E^2)} f(x_1, Q^2) f(x_2, Q^2)
\]  

where for the gluon densities \( f(x_{1,2}) = 2 (Q^2) \gamma_{1,2} e^{\omega(\gamma_{1,2}) \ln \left( \frac{1}{x_{1,2}} \right)} \)

where \( \gamma_{1,2} \) are the anomalous dimensions and the numerical coefficient 2 in Eq. (2.9) can be taken from MRST-2002-NLO parameterizations. Using Eq. (2.9), the integration over \( Q_\perp \) and over \( \gamma_1 \) and \( \gamma_2 \) can be evaluated. It turns out that the integrand of Eq. (2.8) has a saddle point given by \( \ln Q^2_\perp = \ln \frac{M_H^2}{4\alpha_s} + \frac{2\pi}{3\alpha_s} (\gamma_1 + \gamma_2 - 1) \), and the essential values of \( \gamma_1 \) and \( \gamma_2 \) are close to \( \frac{1}{2} \). Hence, the typical \( Q_\perp \) is rather large, and depends on the mass of the Higgs. After integrating over \( \gamma_1 \) and \( \gamma_2 \) and \( q_\perp \), the final result for the amplitude \( M_{\P\P} (p+p \to p+H+p) \) is derived in the appendix (see section A-1), and the final result is given in Eq. (A-1-16) as

\[
M_{\P\P} (p+p \to p+H+p) = -2A_{\pi}^4 s \left( \frac{4\pi^2}{3\alpha_s} \right) \exp \left( -\frac{\left( \ln \frac{M_H^2}{4\alpha_s} \right)^2}{\omega(\frac{1}{2}) \ln s_1} \left( \frac{1}{2} - \frac{\pi}{\alpha_s} \omega(\frac{1}{2}) \ln s_1 \right) \right)
\]  

Now that the amplitude is known, \( \sigma_{\P\P} (p+p \to p+H+p) \) for central Higgs production can be calculated for the case of \( \P\P \) fusion. To derive the cross section for exclusive central Higgs production, one has to multiply by a factor which takes into account the survival probability for large rapidity gaps \( < |S^2_{\text{hard}}| > = 0.004 \), to suppress hard re-scattering. Therefore, the cross section \( \sigma^\text{exc}_{\P\P} (p+p \to p+H+p) \), for central exclusive Higgs production, without any further hard re-scattering for the case of \( \P\P \) fusion, takes the value

\[
< |S^2_{\text{hard}}| > \sigma_{\P\P} (p+p \to p+H+p) = \sigma^\text{exc}_{\P\P} (p+p \to p+H+p) = 0.47 \text{fb}
\]
3. Electromagnetic Higgs production at the LHC

In the case of central exclusive Higgs production for the case of $\gamma\gamma$ fusion, shown in Fig. 3, there is no hard re-scattering to take into account, and all the couplings are known precisely. The shaded area in Fig. 3 depicts the subprocess for the mechanism $\gamma\gamma \rightarrow H$. The possible mechanisms are illustrated in Fig. 4 and Fig. 5, and their contributions to the amplitude are calculated in this section. Gauge invariance requires that the contribution of a subprocess for the mechanism $\gamma\gamma \rightarrow H$, takes the form

$$2\gamma \rightarrow H,$$

where $A$ is a constant, depending on the particular subprocess. It turns out that for the case of when the subprocess is the fermion triangle shown in Fig. 4, summed over all six flavours of quarks, and all three lepton flavours, then the expression for the amplitude takes the form of Eq. (3.1). However in the case of when the subprocess is one of the Boson loops shown in Fig. 5, the expression for the amplitude is not of the form of Eq. (3.1). The correct statement is that the sum over all the amplitudes for the sub-processes shown in Fig. 5, gives a gauge invariant expression of the form of Eq. (3.1).

The amplitude for the diagram of Fig. 3, where the process $\gamma\gamma \rightarrow H$ proceeds via the sum over the fermion triangle shown in Fig. 4, and Boson loops shown in Fig. 5, is given by

$$M_{\gamma\gamma} (p+p \rightarrow p+H+p) = -\frac{4\pi\alpha_{em}}{q_1 q_2} 4p^\mu_2 p^\nu_1 \left( A^f_{\mu\nu} + A^b_{\mu\nu} \right)$$  (3.2)

where $A^f_{\mu\nu} = A_f \left( q^\mu_1 q^\nu_2 - \frac{M^2_H}{2} g^\mu\nu \right)$ denotes the amplitude of the quark/anti-quark triangle subprocess, summed over all six quark flavours/anti-quark flavours ($q, \bar{q} = u/\bar{u}, d/\bar{d}, s/\bar{s}, c/\bar{c}, t/\bar{t}, b/\bar{b}$) and all
three lepton/anti-lepton flavours \((L^\pm = e^\pm, \mu^\pm, \tau^\pm)\), shown in Fig. 4, and \(A_{\mu\nu}^b = A_b \left( q_1^\mu q_2^\nu - \frac{M^2_H}{s} g^{\mu\nu} \right)\) denotes the amplitude of the sum over all the Boson loop sub-processes shown in Fig. 4. At this point, the Weizsacker - Williams formula is used, as explained in refs. [3, 5, 4]. In this approach, the substitution \(p_1^\mu p_2^\nu A_{\mu\nu} = -\frac{2s}{M^2_H} q_1^\mu q_2^\nu A_{\mu\nu} \) is used. In the notation used in this paper, \(q_1^\mu\) and \(q_2^\mu\) denotes two dimensional vectors, in the plane transverse to the direction of the momenta of the two incoming protons \(p_1^\mu\) and \(p_2^\nu\). Hence, the amplitude of Eq. (3.2) can be written as

\[
M_{\gamma\gamma} (p+p \rightarrow p+H+p) = -\frac{4\pi \alpha_{em}}{q_1^2 q_2^2} \frac{2s}{M^2_H} 4q_2^\mu q_1^- (A_f^\mu + A_b^\mu) \tag{3.3}
\]

In order to calculate the cross section, one has to integrate the squared amplitude over all the transverse momenta \(q_{1\perp}\) and \(q_{2\perp}\). For central exclusive Higgs production in the case of \(\gamma\gamma\) fusion, it is required that the lower limits of integration are \((q_{1\perp}^{min})^2, (q_{2\perp}^{min})^2 = m_p^2 \frac{M^2_H}{s}\), where \(m_p\) is the proton mass, which is assumed in this paper to be 1 GeV. The upper limits of integration, are taken from the electromagnetic form factors for the proton, namely \(G_p (q^2) = \frac{1}{2(1+ \frac{4\pi}{m^2})}\), from which the upper limits of the integration are derived to be \((q_{1\perp}^{max})^2, (q_{2\perp}^{max})^2 = 0.72\).

\[
\sigma_{\gamma\gamma} (p+p \rightarrow p+H+p) = \frac{\alpha_{em}^2}{32\pi} \ln \frac{s}{m^2} (A_f + A_b)^2 \ln^2 \left( \frac{0.72}{m_p^2 \sqrt{\frac{M^2_H}{s}}} \right) \tag{3.4}
\]

where \(y = \ln \frac{s}{m^2}\) is the rapidity gap between the two incoming protons in Fig. 4, and \(m\) is the proton mass, assumed to be 1 GeV. This calculation is for central exclusive Higgs production at the LHC, where it is expected that \(\sqrt{s} = 14000\) GeV, which gives for the value of the rapidity gap \(y = 19\).

### 3.1 The fermion triangle subprocess for Higgs production in \(\gamma\gamma\) fusion

Central exclusive Higgs production for the case of \(\gamma\gamma\) fusion, can proceed through the subprocess \(\gamma\gamma \rightarrow \) fermion triangle \(\rightarrow H\) shown in Fig. 4, where the fermions include the six flavours of quarks and anti quarks \((u, d, s, c, t, b)\) and the three lepton and anti lepton flavours \((e, \mu, \tau)\). A derivation of the amplitude for the fermion triangle, can be found in refs. [6, 9, 10] for the case where the mass of the fermion in the triangle is much larger than the Higgs mass. In this section, the amplitude of the fermion triangle is derived by taking the sum over all the fermions which could contribute, including the six quark flavours and the three lepton flavours. The \(H \rightarrow f f\) vertex coupling, is proportional to the mass \(m_f\) of the fermion at the vertex, so that the sub-process amplitude of the fermion triangle will be proportional to the mass of the fermion in the triangle. The fermion masses are assumed to take the following values listed in the table below.
| fermion | mass (GeV/c$^2$) |
|---------|------------------|
| quarks  |                  |
| u       | $3 \times 10^{-3}$ |
| d       | $6 \times 10^{-3}$ |
| s       | 1.3              |
| c       | 0.1              |
| t       | 175              |
| b       | 4.3              |
| fermions|                  |
| e       | $5.11 \times 10^{-4}$ |
| $\mu$   | 0.106            |
| $\tau$  | 1.7771           |

Hence, these values indicate that the most significant contribution to the amplitude will come from the top quark triangle.

Figure 4: Fermion triangle subprocess for Higgs production through $\gamma\gamma$ fusion

To derive explicitly the amplitude of the subprocess Fig. 4, the labeling of momenta shown in the
where \( m_f \) denotes the mass of the fermion which forms the triangle, which could be one of the quark flavours or one of the lepton flavours. Then the amplitude for the fermion triangle subprocess, summed over all possibilities of quark and anti-quark flavours, and lepton and anti-lepton flavours takes the form

\[
A_q^{\mu\nu} = \sum_f 4\pi\alpha_{em} \left( \sqrt{2}G_F \right)^{\frac{1}{2}} m_f \int \frac{d^4l}{(2\pi)^d} \left( \frac{I_f^{\mu\nu}}{D_1D_2D_3} \right) \quad \text{where} \quad \sum_f = \sum_q + \sum_{L=e,\mu,\tau}
\]

where \( \sum_q \) denotes the sum over all six quark flavours \( q = u, d, s, c, t, b \) and \( \sum_{L=e,\mu,\tau} \) denotes the sum over all three lepton flavours \( L = e, \mu, \tau \). On the RHS of Eq. (3.1.2), \( d \) is the space-time dimension, and at the end of the calculation, \( d \to 4 \) is imposed. The reason for not specifying this in the beginning, is because it will be necessary to use dimensional regularisation to cancel divergences, which requires integration over \( d + \epsilon \) dimensions, in the limit that \( d \to 4 \) and \( \epsilon \to 0 \). Using the notation shown in the diagram of Fig. 4 for the flow of momenta in the quark and anti-quark triangles, the trace term \( I_f^{\mu\nu} \) is given by

\[
I_f^{\mu\nu} = Tr \left( \gamma^x \left( l_x + q_2^x + m_f \right) \gamma^\nu \gamma^\sigma \left( l_\sigma + m_f \gamma^\nu \gamma^\tau \left( l_\tau - q_1^\tau + m_f \right) \right) \right)
+ Tr \left( \gamma^x \left( -l_x + q_1^x + m_f \right) \gamma^\nu \gamma^\sigma \left( -l_\sigma + m_f \gamma^\nu \gamma^\tau \left( -l_\tau - q_2^\tau + m_f \right) \right) \right)
= 8m_f \left( q_1^\mu q_2^\nu + 4l^\mu l^\nu + 2 \left( l_1^\mu q_1^\nu - l_2^\mu q_2^\nu \right) - g^{\mu\nu} \left( \vec{q}_1 \cdot \vec{q}_2 + l^2 - m_f^2 \right) \right)
\]

where \( \vec{q}_1 \) and \( \vec{q}_2 \) denotes four dimensional vectors. The first line on the RHS of Eq. (3.1.3) corresponds to the contribution given by the triangle formed by the fermions, (i.e. quarks \( q = (u, d, s, c, t, b) \) and negatively charged leptons \( L^- = (e^-, \mu^-, \tau^-) \), and the second line corresponds to the contribution given by the triangle formed by the anti-fermions (i.e. anti-quarks \( \bar{q} = (\bar{u}, \bar{d}, \bar{s}, \bar{c}, \bar{t}, \bar{b}) \) and positively charged leptons \( L^+ = (e^+, \mu^+, \tau^+) \)). Introducing Feynman parametres to rewrite the quotient \((D_1D_2D_3)^{-1}\) in a more convenient form, Eq. (3.1.2) simplifies to

\[
A_f^{\mu\nu} = \sum_f 4\pi\alpha_{em} \left( G_F \sqrt{2} \right)^{\frac{1}{2}} \int \frac{d^4\tilde{l}}{(2\pi)^d} \int_0^1 dx \int_0^{1-x} dy \frac{8m_f I_f^{\mu\nu}}{(\tilde{l}^2 - \Delta_f(x,y))^3}
\]

where \( \Delta_f = \left( \frac{m_f^2 - M_H^2}{x} \right) \) and \( \tilde{l}^\mu = l^\mu - x q_1^\nu + y q_2^\nu \)

Note in the last step, it was assumed that \( q_1^2 \ll q_2^2 \ll M_H^2, m_f^2 \) so that these terms can be ignored. From the kinematics shown in the diagram of Fig. 3 and Fig. 4, it is clear that \( 2\vec{q}_1 \cdot \vec{q}_2 = M_H^2 \). The trace
term $I^\mu\nu_f$ was given in Eq. (3.1.3) in terms of the unknown momentum $l^\mu$ in the fermion triangle in Fig. 4. In terms of the new variable $\tilde{l}^\mu$, the trace term takes the form

$$I^\mu\nu_f = m_f \left( q_1^\mu q_2^\nu + 4\tilde{l}^\mu \tilde{l}^\nu - 4q_1^\mu q_2^\nu x y - g^\mu\nu \left( \tilde{q}_1 \cdot \tilde{q}_2 (1 - 2x y) - m_f^2 + \tilde{l}^2 \right) \right)$$

(3.1.5)

The details of the integration over the momentum $\tilde{l}$ on the RHS of Eq. (3.1.4) are given in section A-3 of the appendix, (see Eq. (A-3-1) - Eq. (A-3-4)). Here, dimensional regularisation is used, a technique where one integrates over $d + \epsilon$ dimensions, and afterwards $d \to 4$ and $\epsilon \to 0$. This removes non gauge invariant terms, in the numerator of the integrand on the RHS of Eq. (3.1.4). In this way, one obtains the following gauge invariant result for the RHS of Eq. (3.1.4):

$$A^\mu\nu_f = -\frac{2\alpha_{em} G_F^\frac{1}{2} 2^{\frac{1}{2}}}{\pi} \left( q_1^\mu q_2^\nu - \frac{M_H^2}{2} g^\mu\nu \right) \sum_I I_f$$

(3.1.6)

where

$$\sum_I I_f = \sum_{f=u, d, s, c, t, b, e, \mu, \tau} m_f^2 \int_0^1 dx \int_0^{1-x} dy \frac{1 - 4xy}{\Delta_f(x, y)}$$

(3.1.7)

$$\Delta_f(x, y) = m_f^2 - M_H^2 x y$$

The integral $I_f$ is evaluated in section A-3 of the appendix (see Eq. (A-3-5) - Eq. (A-3-13)). It turns out that, due to the dependence of the factor for the $H \to f f$ vertex coupling on the mass of the fermion $m_f$, that the only fermion triangle which gives a significant contribution to the amplitude is for the case where $m_f \gg M_H$. From the table, this is true only for the top quark $m_t$. Hence, it turns out that the only fermion triangle that is necessary to take into, is the top quark triangle, and the contributions from the rest of the triangle sub-processes formed by the rest of the quarks, and the leptons can be neglected. Using this result, the amplitude for the contribution of the fermion triangle has the expression

$$A^\mu\nu_f = A_f \left( q_1^\mu q_2^\nu - g^\mu\nu (\tilde{q}_1 \cdot \tilde{q}_2) \right) \quad \text{where} \quad A_f = -\frac{2 G_F^{\frac{3}{2}} 2^{\frac{1}{2}} \alpha_{em}}{3\pi}$$

(3.1.8)

3.2 Boson loop sub-processes for Higgs production in $\gamma\gamma$ fusion

For central exclusive Higgs production, the Higgs can also be produced through the subprocess $\gamma\gamma \to$ boson loop $\to H$, where the possible boson loops are shown in Fig. 5 ( taken from ref. [7]). $H^-$ in Fig. 5 is an un-physical, charged Higgs boson, and $\phi^\pm$ is a Fadeev-Popov ghost. The formalism for calculating the amplitude of each diagram in Fig. 5, is similar to the approach used to calculate the amplitude of the fermion triangle above, in section 3.1. Similarly here, after integration over the unknown momentum $l$ in the loop of each diagram in Fig. 5, and after integration over Feynman parametres, the expression for each diagram takes the general form

$$- 12 -$$
\[ A^\mu_\nu = \frac{\alpha em 2^\frac{1}{2} G^2_F}{2\pi} m_w^2 \left( B \Gamma \left( 2 - \frac{d}{2} \right) g^{\mu\nu} + C \frac{q_1^\nu q_2^\mu}{m_w^2} + D \frac{M_H^2}{2 m_w^2} g^{\mu\nu} \right) \]  

(3.2.1)

where \( d \) is the dimension of space-time. Terms proportional to \( q_1^2 \) and \( q_2^2 \) were assumed to vanish, and from the kinematics shown in Fig. 3, it was assumed that \( \vec{q}_1 \cdot \vec{q}_2 = \frac{M_H^2}{2} \). In the limit that \( d \to 4 \), the term proportional to \( \Gamma \left( 2 - \frac{d}{2} \right) \) on the RHS of Eq. (3.2.1) tends to infinity. However, when one sums over the contributions to the amplitude given by all the boson sub-processes shown in Fig. 3, these divergencies cancel exactly (see table below). One requires also, that this sum over all the amplitudes for the boson loop sub-processes shown in Fig. 3, satisfies the condition

\[
\sum C = -\sum D
\]  

(3.2.2)

such that the amplitude for the sum is gauge invariant. In ref. [7], this sum was taken and the result was an almost gauge invariant expression, since terms proportional to \( \frac{m_w^2}{M_H^2} \) and higher were neglected. In the calculation which lead to the results in this paper, the result gives an exactly gauge invariant expression, after using dimensional regularisation to remove terms which do not satisfy the gauge invariance condition.

| graph         | \( B \)   | \( C \)   | \( D \)   |
|---------------|-----------|-----------|-----------|
| a + crossed   | 3 \( (d-1) \) | -4        | 5         |
| b + crossed   | -2 \( (d-1) \) | 0         | \( 2 \frac{m_w^2}{M_H^2} \) |
| c + d + crossed | -\frac{1}{7} \( (d-1) \) | -4        | 2         |
| e + crossed   | 0         | 0         | \(-\frac{m_w^2}{M_H^2}\) |
| f + crossed   | 0         | 0         | 1         |
| g + h + crossed | 1        | 0         | 0         |
| i + crossed   | -2        | 0         | 0         |
| 2j + crossed  | -\frac{1}{7} | 0         | \(-\frac{m_w^2}{M_H^2}\) |
| sum           | \( \frac{1}{7} d - 2 \) | -8        | 8         |

Hence, plugging the results shown in the table for the coefficients into Eq. (3.2.1), the result of taking the sum over the amplitudes for the sub-processes shown in Fig. 3 gives a gauge invariant expression, and in the limit that \( d \to 4 \), the divergencies cancel exactly, such that the expression of Eq. (3.2.1) reduces to

\[ A^\mu_\nu = A_b \left( q_1^\nu q_2^\mu - \frac{M_H^2}{2} g^{\mu\nu} \right) \]  

where \( A_b = 4 \frac{\alpha em 2^\frac{1}{2} G^2_F}{\pi} \)  

(3.2.3)
Figure 5: Boson loop subprocess for Higgs production in $\gamma\gamma$ fusion

It should be noted from the results for the amplitude of the subprocess of Fig. 5 (a), the subprocess $\gamma\gamma \rightarrow W$ triangle $\rightarrow H$, interferes destructively with the subprocess $\gamma\gamma \rightarrow$ fermion triangle $\rightarrow H$ shown in Fig. 4.
3.3 The cross section for central exclusive Higgs production through $\gamma\gamma$ fusion

Now that the amplitudes for the sub-processes $\gamma\gamma \rightarrow$ fermion triangle $\rightarrow H$ and $\gamma\gamma \rightarrow$ boson loop $\rightarrow H$ have been calculated, the results can be plugged into Eq. (3.4) to derive the cross section, for central exclusive Higgs production through $\gamma\gamma$ fusion. The result, taking into account all possible sub-processes shown in Fig. 4 and Fig. 5 is found to be

$$\sigma^{\text{exc}}_{\gamma\gamma} \left( p+p \rightarrow p+H+p \right) = 0.1 \text{ fb}$$

(3.3.1)

4. Conclusion

The results of this paper are summarized in the table below. $\sigma^{\text{exc}}$ is the exclusive cross section, which includes multiplication by a factor for the survival probability, for central exclusive Higgs production. The results are given for the mechanisms $pp \rightarrow \gamma\gamma \rightarrow H$ and $pp \rightarrow IP\overline{P} \rightarrow H$. Note that in these results, the cross section for central exclusive Higgs production in the case of $\gamma\gamma$ fusion, is multiplied by a factor for the survival probability of 1. This is because in the case of photon exchange, there is no hard re-scattering to suppress, and the large rapidity gaps between the Higgs and the two emerging protons are automatically present.

| process | $<|S^2|>$ | $\sigma^{\text{exc}}$ (fb) |
|---------|---------|-----------------|
| $\overline{IP}P$ | 0.023 | 2.7 |
| $IP\overline{P}$ | 0.004 | 0.47 |
| $\gamma\gamma$ | 1 | 0.1 |

The results show that, taking the survival probability to be 0.02, which is the value used in ref. [2], then the result for $\sigma^{\text{exc}}_{IP\overline{P}}$ for central exclusive Higgs production at the LHC, almost agrees with the prediction of ref. [4], (which was 3 fb). However, if the survival probability is an order of magnitude smaller as predicted in ref. [1], then $\sigma^{\text{exc}}_{IP\overline{P}}$ will be an order of magnitude smaller and, it becomes competitive with $\sigma^{\text{exc}}_{\gamma\gamma}$ for central exclusive Higgs production at the LHC.

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A. Appendix

A-1 Evaluation of the integral over the anomalous dimensions $\gamma_1$ and $\gamma_2$ of the momentum $Q^2$ in the gluon density function

The Born amplitude was calculated in Eq. (2.8), in terms of the gluon density as a function of the anomalous dimensions $\gamma_1$ and $\gamma_2$, for the two gluon ladders in Fig. 1. One now needs to integrate over $\gamma_1$ and $\gamma_2$, and also over the momentum in the t-channel gluon, namely $Q$. Altogether the necessary integrations take the form

$$M_{\mathcal{F} \mathcal{F}} (p+p \rightarrow p+H+p) = A \pi^3 s \int d\gamma_1 d\gamma_2 \int \frac{dQ^2}{Q_\perp^2} \ e^{-S(k_\perp^2, E_\perp^2)} f(x_1, Q^2) f(x_2, Q^2)$$  \hspace{1cm} (A-1-1)

Firstly, the integral over $Q_\perp$ is evaluated using the steepest descent technique. The gluon density is given by

$$f(Q^2, x_1, x_2) = 2(Q^2)^{\gamma_1, \gamma_2} e^{\omega(\gamma_1, \gamma_2) \ln \frac{s_1, s_2}{s_0}}$$  \hspace{1cm} (A-1-2)

where $\ln \left(\frac{1}{x_{1,2}}\right) \sim \ln \left(\frac{s_1, s_2}{s_0}\right)$, where $s_0 \sim 1$GeV. This comes from the BFKL ladder gluon exchange (see Fig. 1), while the coefficient was taken from MRST - NLO - 2002 data (see Ref.[?]). $\omega(\gamma_1, \gamma_2)$ is the BFKL kernel defined as

$$\omega(\gamma_1, \gamma_2) = \bar{\alpha}_s \chi(\gamma_1, \gamma_2) = \bar{\alpha}_s (\psi(1) - \psi(\gamma_1, \gamma_2) - \psi(1 - \gamma_1, \gamma_2))$$  \hspace{1cm} (A-1-3)

where $\psi(f)$ is the digamma function and $\psi(f) = \frac{d\Gamma(f)}{df}$. In Eq. (A-1-2), $S(k_\perp^2, E_\perp^2)$ is the Sudakov form factor with the typical value $\Re \left[ \Im \left( \frac{E_\perp^2}{Q_\perp^2} \right) \right]$, in the notation that $E_\perp = \frac{M_H}{2}$. Using this substitution Eq. (A-1-1) then becomes

$$M_{\mathcal{F} \mathcal{F}} (p+p \rightarrow p+H+p) = 4A \pi^3 s \int_{-\infty}^\infty d\gamma_1 d\gamma_2 \int \frac{dQ^2}{Q_\perp^2} \ e^{-S(k_\perp^2, E_\perp^2)} f(x_1, Q^2) f(x_2, Q^2) \times \exp \left( -\phi(Q_\perp^2) \right) \ \exp \left( \omega(\gamma_1) \ln \frac{s_1}{s_0} + \omega(\gamma_2) \ln \frac{s_2}{s_0} \right)$$  \hspace{1cm} (A-1-4)

where

$$\phi(Q_\perp^2) = \frac{3\alpha_s}{4\pi} \left( \ln \left( \frac{E_\perp^2}{Q_\perp^2} \right) \right)^2 - (\gamma_1 + \gamma_2 - 1) \ln Q_\perp^2$$  \hspace{1cm} (A-1-5)

Differentiating the right hand side of Eq. (A-1-5) with respect to $\ln Q_\perp^2$, one sees that $\phi$ has a saddle point at $\ln Q_\perp^2 = \ln \frac{M_H^2}{\pi s_0} + \frac{2\alpha_s}{\bar{\alpha}_s} (\gamma_1 + \gamma_2 - 1)$. Hence, changing the integration variable to $u = \ln Q_\perp^2$, and expanding $\phi$ around the saddle point, Eq. (A-1-4) can be written as

\[ \]
\[ M_{IPP} (p+p \to p+H+p) = 4A\pi^3 s e^{-\phi(u_0)} \int d\gamma_1 d\gamma_2 \int du e^{\left(-\frac{1}{2} (u-u_0)^2 \frac{d^2 \phi(u_0)}{du^2}\right)} e^{\left(\omega(\gamma_1) \ln \frac{s_1}{s_0} + \omega(\gamma_2) \ln \frac{s_2}{s_0}\right)} } \]

where \[ u_0 = \ln \frac{M^2}{4} + \frac{2\pi}{3\alpha_s} (\gamma_1 + \gamma_2 - 1) \] (A-1-7)

Now the right hand side of Eq. (A-1-6) has reduced to a Gaussian integral over \( u \), which can be evaluated by the steepest descent technique, to give the expression

\[ M_{IPP} (p+p \to p+H+p) = 4A\pi^3 s \left( \frac{4}{3\alpha_s} \right)^{\frac{1}{2}} \int d\gamma_1 d\gamma_2 \exp \left( (\gamma_1 + \gamma_2 - 1) \left( \frac{\pi}{3\alpha_s} (\gamma_1 + \gamma_2 - 1) + \ln \frac{M^2}{4} \right) \right) \]

\[ \times \exp \left( \omega(\gamma_1) \ln \frac{s_1}{s_0} + \omega(\gamma_2) \ln \frac{s_2}{s_0} \right) \] (A-1-8)

The BFKL function \( \omega(\gamma) \) has a saddle point at \( \gamma = \frac{1}{2} \). Near to this point \( \omega(\gamma) \) can be written as

\[ \omega(\gamma_1, \gamma_2) = \omega \left( \frac{1}{2} \right) + \frac{1}{2} \left( \gamma_1 - \frac{1}{2} \right)^2 \omega'' \left( \frac{1}{2} \right) \] (A-1-9)

Hence using Eq. (A-1-9), Eq. (A-1-8) can be reduced to

\[ M_{IPP} (p+p \to p+H+p) = 4A\pi^3 s \left( \frac{4}{3\alpha_s} \right)^{\frac{1}{2}} \int d\gamma_1 d\gamma_2 \exp \left( f(\gamma_1, \gamma_2) \right) \] (A-1-10)

where the function \( f(\gamma_1, \gamma_2) \) has the form

\[ f(\gamma_1, \gamma_2) = \omega \left( \frac{1}{2} \right) \ln \frac{s_1 s_2}{s_0^2} + \frac{1}{2} \left( \gamma_1 - \frac{1}{2} \right)^2 \omega'' \left( \frac{1}{2} \right) \ln \frac{s_1}{s_0} + \frac{1}{2} \left( \gamma_2 - \frac{1}{2} \right)^2 \omega'' \left( \frac{1}{2} \right) \ln \frac{s_2}{s_0} \]

\[ + (\gamma_1 + \gamma_2 - 1) \left( \frac{\pi}{3\alpha_s} (\gamma_1 + \gamma_2 - 1) + \ln \frac{M^2}{4} \right) \] (A-1-11)

This function has a saddle point with respect to \( \gamma_1 \) given by

\[ \gamma_1^s = -\frac{2\pi}{3\alpha_s} (\gamma_2 - 1) - \ln \frac{M^2}{4} + \frac{1}{2} \omega'' \left( \frac{1}{2} \right) \ln \frac{s_1}{s_0} \]

\[ - \left( \frac{2\pi}{3\alpha_s} + \omega'' \left( \frac{1}{2} \right) \ln \frac{s_1}{s_0} \right) \] (A-1-12)

Hence, expanding \( f(\gamma_1, \gamma_2) \) around \( \gamma_1^s \), the integration over \( \gamma_1 \) is evaluated using the steepest descent technique to give the expression
\[ M_{\pi^p p} (p+p \rightarrow p+H+p) = 2A\pi s \left( \frac{4\pi}{3\alpha_s} \right)^{\frac{3}{2}} \int d\gamma_2 \frac{\exp \left( f (\gamma_1^{sp}, \gamma_2) \right)}{\sqrt{-\frac{1}{2} \left( \frac{2\pi}{3\alpha_s} + \ln \frac{M_f^2}{4} + \omega'' \left( \frac{1}{2} \right) \ln \frac{s_1}{s_0} \right)}} \]  

(A-1-13)

Now the function \( f (\gamma_1^{sp}, \gamma_2) \) has a saddle point with respect to \( \gamma_2 \) given by

\[ \gamma_2^{sp} = \frac{1}{2} - \frac{\ln \frac{M_f^2}{4} \omega'' \left( \frac{1}{2} \right) \ln \frac{s_1}{s_0}}{\frac{2\pi}{3\alpha_s} \omega'' \left( \frac{1}{2} \right) \ln \frac{s_1}{s_0} + \omega'' \left( \frac{1}{2} \right) \ln \frac{s_1}{s_0}} \sim \frac{1}{2} - \frac{\ln \frac{M_f^2}{4} \omega'' \left( \frac{1}{2} \right) \ln \frac{s_1}{s_0}}{\left( 1 + \frac{4\pi}{3\alpha_s} \right) \omega'' \left( \frac{1}{2} \right) \ln \frac{s_1}{s_0}} \]  

(A-1-14)

Here in the second line it is assumed that \( s_1 \sim s_2 \). For large \( s_1 \) and \( s_2 \), \( \gamma_2^{sp} \) is approximately \( \frac{1}{2} \). Using the same method as above, expanding \( f (\gamma_1^{sp}, \gamma_2) \) around \( \gamma_2^{sp} \), the integration over \( \gamma_2 \) is evaluated using the steepest descent technique for Eq. (A-1-13), to give the result

\[ M_{\pi^p p} (p+p \rightarrow p+H+p) = A\pi s \left( \frac{4\pi^2}{3\alpha_s} \right)^{\frac{3}{2}} \frac{-2 \exp \left( f (\gamma_1^{sp}, \gamma_2^{sp} \sim \frac{1}{2}) \right)}{\left( \frac{2\pi}{3\alpha_s} + \ln \frac{M_f^2}{4} + \omega'' \left( \frac{1}{2} \right) \ln \frac{s_1}{s_0} \right)} \]  

(A-1-15)

where

\[ f (\gamma_1^{sp}, \gamma_2^{sp}) = -\left( \frac{\ln \frac{M_f^2}{4}}{\omega'' \left( \frac{1}{2} \right) \ln \frac{s_1}{s_0}} \right)^2 \left( \frac{1}{2} - \frac{\frac{\pi}{3\alpha_s}}{\omega'' \left( \frac{1}{2} \right) \ln \frac{s_1}{s_0}} \right) \]  

(A-1-16)
A-2 Feynman rules for the standard electro-weak theory

\[
\begin{align*}
H(q) & \rightarrow H(p) = (1/2)i g_w(p+q)_\mu \\
H(q) & \rightarrow W^\mu w = (1/2)i g_w(p-q)_\mu \\
H(q) & \rightarrow W^\mu v = i g_w m_w g_{\mu\nu} \\
H(q) & \rightarrow H = (-1/2)i g_w/2 m_w M_H^2
\end{align*}
\]

Figure 6:

\[
\begin{align*}
H(q) & \rightarrow (1/2)i g_w m_w \\
\phi^{+/−} & = (1/2)i g_w m_w \\
\phi^{+−} & = (1/2)i g_w m_w \\
\phi^{−+} & = (1/2)i g_w m_w \\
\phi^{−−} & = (1/2)i g_w m_w
\end{align*}
\]

Figure 7: Feynman rules in the standard electroweak theory

A-3 Evaluation of the integral over the momentum in the fermion triangle loop

The amplitude for the fermion triangle subprocess, summed over all quark flavours \( q = ( u, d, s, c, t, b ) \)
and lepton flavours \( L = (e, \mu, \tau) \), for the mechanism \( \gamma\gamma \) fermion triangle \( \rightarrow H \) was found in equation Eq. 3.1.4 to take the form

\[
A_\mu^\nu = 2 \sum_f 4\pi\alpha_{em} \left( G_F^2 \frac{1}{2} \right)^{\frac{1}{2}} \int_0^1 \frac{d\tilde{l}}{(2\pi)^d} \int_0^{1-x} \frac{dy}{y^{\frac{1}{2}}} \frac{8m_0^2 \left( q_1^\mu q_2^\nu - \frac{M^2}{2} g^{\mu\nu} \right) \left( 1 - 2xy - \frac{M^2}{2} g^{\mu\nu} + m_f^2 g^{\mu\nu} \right)} \left( \tilde{l}^2 - \Delta_f (x, y) \right)^3
\]

+ 2 \sum_f 4\pi\alpha_{em} \left( G_F^2 \frac{1}{2} \right)^{\frac{1}{2}} \int_0^1 \frac{d\tilde{l}}{(2\pi)^d} \int_0^{1-x} \frac{dy}{y^{\frac{1}{2}}} \frac{8m_0^2 \left( 4\tilde{l}^\mu \tilde{l}^\nu - \tilde{l}^2 g^{\mu\nu} \right)} \left( \tilde{l}^2 - \Delta_f (x, y) \right)^3
\]

where \( \Delta_f = m_f^2 - \frac{1}{2} H_{0} \)

\[
\Delta_f = m_f^2 - \frac{1}{2} H_{0} \quad \text{and} \quad \sum_f = \sum_q + \sum_{L=e,\mu,\tau}
\]

where \( \sum_q \) denotes the sum over all six quark flavours \( q = u, d, s, c, t, b \) and \( \sum_{L=e,\mu,\tau} \) denotes the sum over all three lepton flavours \( L = e, \mu, \tau \).

From the numerator of the integrand, on the RHS of Eq. (A-3-1), one can see that there is a gauge invariant term \( \left( q_1^\mu q_2^\nu - \frac{M^2}{2} g^{\mu\nu} \right) (1 - 4xy) \), and the numerator in the integrand of the second line on the RHS gives a vanishing contribution to the integration over \( \tilde{l} \), for \( d \rightarrow 4 \). However one is still left with the terms \( -2xy \frac{M^2}{2} g^{\mu\nu} + m_f^2 g^{\mu\nu} \) in the numerator of the integrand, which are certainly not gauge invariant. However, conveniently this non gauge invariant piece is exactly equal to \( \Delta_f (x, y) \) introduced in Eq. (3.1.2). To deal with this non gauge invariant piece, it is useful to use dimensional regularisation, when integrating over the \( \tilde{l}^2 \) term in the numerator of the integrand on the second line. In this approach, one initially integrates over \( d + \epsilon \) dimensions in the limit that \( \epsilon \rightarrow 0 \) and \( d ightarrow 4 \). In this way the non gauge invariant terms disappear. Hence, evaluating the integral over \( \tilde{l} \) on the RHS of Eq. (A-3-1) gives

\[
A_\mu^\nu = 2 \sum_f 4\pi\alpha_{em} G_F^2 \left( \frac{1}{2} \right)^{\frac{1}{2}} \frac{(-1)^3 \Gamma \left( 3 - \frac{d}{2} \right)}{\Gamma (3) (4\pi)^{\frac{d}{2}}} \int_0^1 \frac{dx}{x} \int_0^{1-x} \frac{dy}{y^{\frac{1}{2}}} \frac{8m_0^2 \left( q_1^\mu q_2^\nu - \frac{M^2}{2} g^{\mu\nu} \right) (1 - 4xy) + \Delta_f (x, y) g^{\mu\nu}} {\Delta_f (x, y)}
\]

+ 2 \lim_{\epsilon \rightarrow 0} \sum_f 4\pi\alpha_{em} G_F^2 \left( \frac{1}{2} \right)^{\frac{1}{2}} \frac{(-1)^{3+1} \Gamma \left( 3 - \frac{d+\epsilon}{2} - 1 \right)}{2\Gamma (3) (4\pi)^{\frac{d+\epsilon}{2}}} \int_0^1 \frac{dx}{x} \int_0^{1-x} \frac{dy}{y^{\frac{1}{2}}} \frac{8m_0^2 \left( 4\tilde{l}^\mu \tilde{l}^\nu - \tilde{l}^2 g^{\mu\nu} - (d + \epsilon) g^{\mu\nu} \right)} {\Delta_f (x, y)}
\]

In the limit that \( \epsilon \rightarrow 0 \) and \( d \rightarrow 4 \), the gamma function \( \Gamma \left( 3 - \frac{d+\epsilon}{2} - 1 \right) \rightarrow \frac{2}{\epsilon} \) and the RHS of Eq. (A-3-2) reduces to

\[
A_\mu^\nu = -2 \sum_f 4\pi\alpha_{em} G_F^2 \left( \frac{1}{2} \right)^{\frac{1}{2}} \frac{1}{(4\pi)^2} \int_0^1 \frac{dx}{x} \int_0^{1-x} \frac{dy}{y^{\frac{1}{2}}} \frac{8m_0^2 \left( q_1^\mu q_2^\nu - \frac{M^2}{2} g^{\mu\nu} \right) (1 - 4xy) + \Delta_f (x, y) g^{\mu\nu}} {\Delta_f (x, y)}
\]

+ 2 \sum_f 4\pi\alpha_{em} G_F^2 \left( \frac{1}{2} \right)^{\frac{1}{2}} \frac{1}{\epsilon (4\pi)^2} \int_0^1 \frac{dx}{x} \int_0^{1-x} \frac{dy}{y^{\frac{1}{2}}} 8m_0^2 \epsilon g^{\mu\nu}
\]

(A-3-3)
Thus, after canceling $\epsilon$ in the numerator and the denominator in the second line on the RHS of Eq. (A-3-3), the second line exactly cancels the non gauge invariant part of the integrand on the first line. Hence, one is left with the purely gauge invariant expression

$$A_{\mu \nu}^{ij} = -2 \frac{\alpha_{em} G_F^2 \gamma^2}{\pi} \left( q_1^\mu q_2^\nu - \frac{M_H^2}{2} g^{\mu \nu} \right) \sum_f I_f$$ \hspace{1cm} (A-3-4)

where $I_f$ is the only remaining integral to evaluate, which takes the form

$$\sum_f I_f = \sum_{f=u, d, s, c, t, b, e, \mu, \tau} m_f^2 \int_0^1 dx \int_0^{1-x} dy \frac{1 - 4xy}{\Delta_f(x, y)} \quad \text{where} \quad \Delta_f(x, y) = m_f^2 - M_H^2 xy$$ \hspace{1cm} (A-3-5)

To evaluate this integral, there are two cases to consider, namely (1) when $m_f^2 \gg M_H^2$, which is true for the top quark when $m_t = 175$ GeV, and (2) when $m_f^2 \ll M_H^2$, which is true for all the rest of the fermions listed in the table. Therefore, $\sum_f I_f$ can be separated into two parts, namely

$$\sum_f I_f = I_t + \sum_{f \neq t} I_f$$ \hspace{1cm} (A-3-6)

For case (1), where $m_f = m_t \gg M_H$, one can write $I_t$ in a more convenient way as

$$I_t = \frac{m_t^2}{M_H^2} \int_0^1 dx \int_0^{1-x} dy \frac{1 - 4xy}{\frac{m_f^2}{M_H^2} - xy} = \frac{m_t^2}{M_H^2} \int_0^1 \left( 4 + \frac{m_f^2}{M_H^2} \right) dx \int_0^{1-x} \frac{1 - 4xy}{\frac{m_f^2}{M_H^2} - xy}$$

\[= \frac{m_t^2}{M_H^2} \left( 2 + \left(1 - 4 \frac{m_t^2}{M_H^2} \right) \int_0^1 dx \ln \left( 1 - \frac{M_H^2}{m_t^2} x (1 - x) \right) \right) \]

\[= \frac{m_t^2}{M_H^2} \left( 2 + \left(1 - 4 \frac{m_t^2}{M_H^2} \right) \int_0^1 dx \left( \frac{M_H^2}{m_t^2} (1 - x) + \frac{1}{2} \left( \frac{M_H^2}{m_t^2} \right)^2 x (1 - x)^2 + \frac{1}{3} \left( \frac{M_H^2}{m_t^2} \right)^3 x^2 (1 - x)^3 + \ldots \right) \right) \]

(A-3-7)

where in the last step, the logarithm was expanded in a Taylor series around $x = 0$. Evaluating the integral over $x$, and since it is assumed that $M_H^2 \ll m_t^2$, retaining terms no smaller than $\left( \frac{M_H^2}{m_t^2} \right)$, the RHS of Eq. (A-3-7) becomes

$$I_t = \frac{1}{3} \left( 1 + \frac{7}{120} \frac{M_H^2}{m_t^2} + \ldots \right)$$ \hspace{1cm} (A-3-8)
For case (2), where \( m_f \ll M_H \) which includes all the fermions in the table except for the top quark, there are two possible regions of integration, namely (I) when \( M_H^2 xy > m_f^2 \) and (II) when \( M_H^2 xy < m_f^2 \).

In the region where \( M_H^2 xy > m_f^2 \), the RHS of Eq. (A-3-5) reduces to

\[
\sum_{f \neq t} I_f \text{ region (I)} = - \sum_{f \neq t} \frac{m_f^2}{M_H^2} \int_{\frac{m_f^2}{M_H^2}}^{1} dx \int_{\frac{m_f^2}{M_H^2}}^{1-x} dy \frac{1 - 4xy}{xy} 
\]

\[
= - \sum_{f \neq t} \frac{m_f^2}{M_H^2} \int_{\frac{m_f^2}{M_H^2}}^{1} dx \ln \left( \frac{M_H^2}{m_f^2} x (1-x) \right) + 4 \sum_{f \neq t} \frac{m_f^2}{M_H^2} \int_{\frac{m_f^2}{M_H^2}}^{1} \left( 1 - x - \frac{m_f^2}{M_H^2} x \right) 
\]

\[
= - \sum_{f \neq t} \frac{m_f^2}{2M_H^2} \ln^2 \frac{M_H^2}{m_f^2} + \frac{m_f^2}{M_H^2} \text{polylog} (2, x = 1) - \sum_{f \neq t} \frac{m_f^2}{M_H^2} \text{polylog} \left( 2, x = \frac{m_f^2}{M_H^2} \right) 
\]

\[+ 2 \sum_{f \neq t} \left( \frac{m_f^2}{M_H^2} + 4 \left( \frac{m_f^2}{M_H^2} \right)^2 \ln \frac{m_f^2}{M_H^2} \right) \]  

(A-3-9)

In the region (II) where \( M_H^2 xy < m_f^2 \), the RHS of Eq. (A-3-5) reduces to

\[
\sum_{f \neq t} I_f \text{ region (II)} = \sum_{f \neq t} \frac{m_f^2}{M_H^2} \int_{0}^{\frac{m_f^2}{M_H^2}} dx \int_{\frac{m_f^2}{M_H^2}}^{m_f^2} dy \frac{(1 - 4xy)}{m_f^2} = \sum_{f \neq t} \int_{\frac{m_f^2}{M_H^2}}^{1} dx \frac{m_f^2}{M_H^2} \frac{1}{x} \left( 1 - 2 \frac{m_f^2}{M_H^2} \right) 
\]

\[
= \sum_{f \neq t} \frac{m_f^2}{M_H^2} \left( 1 - 2 \frac{m_f^2}{M_H^2} \right) \ln \frac{M_H^2}{m_f^2} 
\]

(A-3-10)

Hence, adding the contributions of Eq. (A-3-9) and Eq. (A-3-10) for the contributions of region (I) and region (II) of the integral, gives the result

\[
\sum_{f \neq t} I_f = \sum_{f \neq t} \frac{m_f^2}{M_H^2} \left( 1 - 6 \frac{m_f^2}{M_H^2} - \frac{1}{2} \ln \frac{M_H^2}{m_f^2} \right) \ln \frac{M_H^2}{m_f^2} 
\]

\[
+ \sum_{f \neq t} \frac{m_f^2}{M_H^2} \left( \text{polylog} (2, x = 1) - \text{polylog} \left( 2, x = \frac{m_f^2}{M_H^2} \right) + 2 \right) 
\]

\[
\approx \sum_{f \neq t} \frac{1}{2} \frac{m_f^2}{M_H^2} \ln^2 \frac{M_H^2}{m_f^2} \quad \text{for} \ m_f \ll M_H 
\]

(A-3-11)

From Eq. (A-3-12) and Eq. (A-3-8), the result for the evaluation of the integral \( I_f \) has its main contribution from the top quark triangle, such that
\[
\sum_f I_f \approx I_t, = \frac{1}{3} \text{ for } m_t \gg M_H
\]  
(A-3-13)

Plugging this result into Eq. (A-3-4) gives the final expression for the amplitude of the fermion triangle subprocess shown in Fig. 4, for the sum over all quark \( q = (u, d, s, c, t, b) \) contributions and lepton contributions \( L = e, \mu, \tau \) as

\[
A_{f}^{\mu\nu} = A_f \left( q_1^\mu q_2^\nu - \frac{M_H^2}{2} g^{\mu\nu} \right) \text{ where } A_f = \frac{-2 \alpha_{em} G_F^2 2^4}{3 \pi}
\]  
(A-3-14)

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