Methods for photoelastic determination of the stress intensity factors

N V Korihin
Peter the Great St. Petersburg Polytechnic University, St. Petersburg, Russia
E-mail: korihin_nv@spbstu.ru

Abstract. Methodologies for the stress intensity factors (SIF) photoelastic determination are described. Practical recommendations are given for correct SIF determination.

1. Introduction
Despite the development of computer technology, experimental methods have not lost their relevance since the accuracy of numerical methods depends on the idealization of the structure and the correct setting of boundary conditions, including the values of stress intensity factors (SIF).

There are a significant number of photoelastic methods for determining SIF [1]. Therefore, it was decided to conduct a study in order to establish the most accurate methods.

2. Method
When calculating the strength of structures and parts of machines with technological and operational damages such as cracks the most reliable are methods of linear fracture mechanics, based on the stress intensity factors (SIF). For constructions and complex geometry parts obtaining SIF values in an analytical way is associated with significant mathematical difficulties; therefore, numerical and experimental methods are used. Among numerical methods, the finite element method has found the greatest application, but in determining the SIF its accuracy depends on the art of splitting and the idealization of the structure and the proper setting of boundary conditions. Therefore, it is most reliable to use experimental methods and, in particular, the method of photoelasticity.

The work of Irwin [1] is mainstream for developing methods photoelastic SIF determination.

According to the classification of stress fields at the top [2, 3], if the picture of isochromatic bands is symmetric with respect to the crack plane, photoelastic methods of determining the SIF $K_I$ are used. When determining $K_I$, according to photoelasticity data, the most well-known and proven in both domestic and foreign practice are the methods of Bradley and Kobayashi, Shroedl and Smith, Radner, Iokimidis and Theokaris, Shilov and Dolgopolov, Pearson and Ruiz, Etheridge and Daley.

In cases when the picture of isochromatic bands at the vertices of the cracks under study is asymmetric with respect to the crack plane, there is a crack of normal separation and transverse shear, the stress field here is described by the SIF type $K_I$ and $K_{II}$, and in such cases photoelastic separation methods $K_I$ and $K_{II}$ are used.

Since all existing and photoelastic methods for determining the SIF that are considered here are based on asymptotic representations of the stress field at the crack tip, it is advisable to examine possible areas around the crack tip to find out where these singular stresses are. According to [4], there
are three regions around the crack tip. The first area is the area near the top of the crack. In this area, the asymptotic controls are not suitable, since the material of the models in this area does not behave elastically, and the tip of the crack during “freezing” of deformations, although slightly, is dulled. The second is a singular region, where the stresses are described by asymptotic expressions, and the effect of changing the shape of the crack tip is small. The third is a remote area. The singular stresses here are of the same order as the stresses that will arise without a crack, and the proximity of the surfaces strongly influences their value.

To correctly find the singular region, it is necessary to graphically construct the dependence of the difference between the principal stresses \( \sigma_1 - \sigma_2 \) or the optical path \( \delta \) on the value \( r^{1/2} \) – the polar radius from the poles at the crack tip. In this case, only those data that lie on a straight line can be taken in the calculation of the SIF. Departure from linearity shows that the asymptotic Irwin equations are invalid.

To verify the accuracy and complexity of each individual photoelastic SIF method, infinite plates with central through cracks located perpendicularly \( (K_1) \) and at an angle \( (K_\perp) \) to the action of uniformly tensile load were investigated. The plates were made of epoxy resin ED-16MTGFA. The method of “freezing” deformations was used, since the polymers used as model materials in the highly elastic state are perfectly elastic, while their creep at room temperature makes it very difficult to study stresses at the tip of the crack.

Measurements of optical quantities were performed on a coordinate-synchronous polarimeter KSP-7.

Experimental values \( K_1 \), as well as \( K_1 \) and \( K_\perp \), obtained in this work, are given in Table 1 and 2, respectively, where a comparison of these values with known calculated SIF values is also given. As can be seen from the tables, all the presented methods are sufficient for the practice of engineering calculations accuracy.

To select the method of determination \( K_1 \) in the study of spatial structures and parts with cracks, the tops of which are located near stress concentrators or near free surfaces, an infinite thick slab, with a non-through central crack, a cylindrical sample with an annular crack, and a sample with a crack loaded according to the three-point bend pattern were tested.

### Table 1. Comparison of photoelastic determination methods \( K_1 \).

| № | Author          | The equation                                                                                                                                                                                                 | \( K_1 \) deviation, % |
|---|-----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------|
| 1 | Irwin           | \( K_1 = \frac{\delta \sqrt{2 \pi r_m}}{C d \sin \theta_m} \left[ 1 + \left( \frac{2}{3tg \theta_m} \right)^2 \right]^{1/2} \left( 1 + \frac{2tg \frac{3\theta_m}{2}}{3tg \theta_m} \right) \)                                                                                       | 3,45 3,0 |
| 2 | Bradley,        | \( f_{1,2} = \sin^2 \theta_m + 2\delta \left( \frac{2r_{m2}}{l} \right)^2 \sin \theta_{m1} \sin \frac{3\theta_m}{2} + 2r_{m2} \delta^2 \)                                                                                                                                 | 3,2 -4,4 |
|    | Kobayashi       | \( K_1 = \sqrt{2 \pi r_m} \left( \frac{2r_{m1}}{l} \right)^2 \sin^2 \theta_m + 2\delta \left( \frac{2r_{m2}}{l} \right) \sin \theta_{m1} \sin \frac{3\theta_m}{2} + 2r_{m2} \delta^2 \)                                                                 |            |
| 3 | Shroedl,        | \( K_1 = \frac{2 \sqrt{\pi r_m}}{2 \sqrt{r_{\max}}} \left( \frac{2r_{\max}}{l} \right)^2 \sin^2 \theta_m + 2\delta \left( \frac{2r_{m2}}{l} \right) \sin \theta_{m1} \sin \frac{3\theta_m}{2} + 2r_{m2} \delta^2 \)                                                                 | 3,52 5,1 |

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\[
K_i = \frac{A}{r^{1+2}} + B + \sum_{N=1}^{N} C_N \cdot r^N
\]

Smith

\[
K_i = \frac{2(2\pi)^{3/2}d}{\sin \theta};
\]

\[
\tau_m^2 = A^2 + d^2 + 2At \sin^3 \frac{3\theta}{2}; A \to 0;
\]

\[
K_i = \frac{\pm \sqrt{2 \pi \tau_m}}{\cos \frac{\theta}{2}};
\]

Iokimidis, Theokaris

\[
\tau_m \cos \theta = |A \cos^3 \frac{3\theta}{2}; A \neq 0|
\]

Etheridge, Daily

\[
K_i = \frac{\delta}{Cd} \sqrt{2 \pi m f(\theta_m)}
\]

Redner

\[
K_i = \frac{\delta}{Cd} \sqrt{\frac{\pi b}{\cos \theta}}
\]

Redner

\[
K_i = \frac{2\delta}{Cd} \sqrt{2 \pi}
\]

Shilov, Dolgopolov

\[
K_i = \frac{\delta}{Cd} \sqrt{\frac{2 \pi}{\cos \theta}}
\]

Pearson, Ruiz

\[
K_i = \frac{\sigma f}{\sqrt{\pi d}}
\]

The estimated value \(K_i = 3,35 \text{ kg/cm}^{3/2}\)

**Table 2. Comparison of experimental photoelastic calculation methods** \(K_i\) and \(K_{II}\)

| №  | Author | The equation | \(K_i\) | \(K_{II}\) | deviation \(K_i\) \% | deviation \(K_{II}\) \% |
|----|--------|--------------|------|------|-----------------|-----------------|
| 1  | Cheng  | \(\tau_{\text{max}}^2 = \frac{1}{8\pi r} \left[K_i^2 \sin^2 \theta + 2K_iK_{II} \sin 2\theta + K_{II}^2 (4 - 3 \sin^2 \theta)\right]\) | 1,246 | 0,84 | -6,3 | 10,2 |
| 2  | Gduots, Teokaris | \(\left(\frac{K_{II}}{K_i}\right)^2 - 4 \frac{K_{II} \cdot \cot 2\theta - 1}{3} = 0\) | 1,239 | 0,843 | -6,8 | 9,9 |
| 3  | Kobayashi | \(\tau_{\text{max}} \mid_{\theta=90^\circ} = \left(K_i^2 + K_{II}^2\right)^{1/2}\) | 1,515 | 0,617 | 13,9 | -19,6 |
\[
f(K_I, K_{II}, \sigma_{\alpha}) = \\
= \frac{1}{2\alpha} \left[ (K_I \sin \theta + 2K_{II} \cos \theta)^2 + (K_{II} \sin \theta)^2 \right] + \\
\quad \quad + \frac{2\sigma_{\alpha}}{\sqrt{2\alpha}} \sin \theta \left[ K_I \sin \theta (1 + 2 \cos \theta) \right] + \\
\quad \quad + K_{II} \left( 1 + 2 \cos^2 \theta + \cos \theta \right) + \sigma_{\alpha}^2 - \tau_{\max}^2 \\
\]

Sanford, Daily

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\[
K_I^2 \sin^2 \theta + 2K_IK_{II} \sin 2\theta + K_{II}^2 \left( 1 + 3 \cos^2 \theta \right) = \\
2r \left( \frac{\delta}{C_d} \right)^2 - 2\frac{\delta}{C_d} \sigma_0 \sin 2\varphi - \sigma_0^2 \\
\]

Grilitsky, Sorokaty, Dumanskiy

5

1,394 0,818 4,8 6,6

Theoretical value \( K_I = 1,33 \text{ kg/cm}^{3/2} \), \( K_{II} = 0,767 \text{ kg/cm}^{3/2} \)

3. Results and discussions

Experiments have shown that when modeling problems of fracture mechanics, it is necessary to carefully select the magnitude of the external load. Thus, by setting a high load with an obligatory condition of crack stability in the course of the experiment, we can obtain, on the one hand, reliable recording of optical effects, and on the other hand, the nonlinearity zone between stresses and strains at the crack tip can so narrow the size of the singular region that obtaining reliable SIF values are problematic and sometimes impossible. By setting a low level of external load, you can increase the size of the singular region, but there will be difficulties associated with measuring optical effects. The literary analysis and carrying out of the experiment allow us to recommend the external load for the ED-16MTGFA material to be set to such a value that the nominal stresses in the crack area of the model are within 0.1 ... 0.15 N/ mm².

You can evaluate the effect of external and internal surfaces, and the stress concentrators on the stress field at the crack tip can be entered in the asymptotic expression for stresses \( \sigma_x \) for the nonsingular term \( \sigma_{\alpha} \), which is a uniform stress at a certain distance from the crack. Therefore, when solving applied problems of fracture mechanics using three-dimensional photoelasticity, preference should be given to the methods described in [5, 6].

The experience of studying the details of power equipment with cracks — the shell of a nuclear reactor, the pressure manifold, the threaded joint of the main connector of the shell of a nuclear reactor, the rotor of the turbogenerator, the welded joint with a fillet weld — showed the following. To determine the \( K_I \) most effective methods described in lines 1, 9 and 10 of table 1, in view of the fact that the measurement of the physical quantities required for the calculation is carried out at a small distance from the crack tip, where the picture of the strips has the appearance that contributes to their more accurate measurement.

4. Conclusion

For determining \( K_I \) and \( K_{II} \), the most convenient methods are shown in lines 4 and 5 of the table 2.

Comparison of various experimental methods for determining \( K_I \), \( K_{II} \) (table 2) showed their practical equivalence, provided sufficiently accurate measurements and high-quality optical picture. The author believes that from the point of view of sensitivity to measurement errors and the quality of
the picture of the bands, the most stable is the multipoint method, since it allows minimizing the error resulting from measurement errors $r$ and $\theta$.

To reduce the errors of the first type, it is necessary to manufacture models with cracks of given sizes, located in the places strictly required for study and having a minimum radius of curvature at the top. Therefore, it is advisable to apply cracks to the model using the method proposed in [7, 8].

When determining the SIF of three types, it is necessary to use the method proposed in [9]. In [10], examples of modeling problems of fracture mechanics are described in detail using the techniques described in this paper as applied to problems of power engineering.

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