Super-resonant transport of topological surface states subjected to in-plane magnetic fields

Song-Bo Zhang,1, * Chang-An Li,1 Francisco Peña-Benitez,2,3 Piotr Surówka,2,3,4 Roderich Moessner,2,3, Laurens W. Molenkamp,5,6,3,7 and Björn Trauzettel1,3

1 Institut für Theoretische Physik und Astrophysik, Universität Würzburg, 97074 Würzburg, Germany
2 Max-Planck-Institut für Physik komplexer Systeme, Nöthnitzer Strasse 38, 01187 Dresden, Germany
3 Würzburg-Dresden Cluster of Excellence ct.qmat, Germany
4 Department of Theoretical Physics, Wrocław University of Science and Technology, 50-370 Wrocław, Poland
5 Physikalisches Institut (EP3), Universität Würzburg, Am Hubland, 97074 Würzburg, Germany
6 Institute for Topological Insulators, Universität Würzburg, Am Hubland, 97074 Würzburg, Germany
7 Max Planck Institute for Chemical Physics of Solids, Dresden D-01187, Germany

(Dated: April 28, 2021)

Magnetic oscillations of Dirac surface states of topological insulators are typically expected to be associated with the formation of Landau levels or the Aharonov-Bohm effect. We instead study the conductance of Dirac surface states subjected to an in-plane magnetic field in presence of a barrier potential. Strikingly, we find that, in the case of large barrier potentials, the surface states exhibit pronounced oscillations in the conductance when varying the magnetic field, in the absence of Landau levels or the Aharonov-Bohm effect. These novel magnetic oscillations are attributed to the emergence of super-resonant transport by tuning the magnetic field, in which many propagating modes cross the barrier with perfect transmission. In case of small and moderate barrier potentials, we identify a positive magnetoconductance due to the increase of the Fermi surface by tilting the surface Dirac cone. Moreover, we show that for weak magnetic fields, the conductance displays a shifted sinusoidal dependence on the field direction with period $\pi$ and phase shift determined by the tilting direction with respect to the field direction. Our predictions can be applied to various topological insulators, such as HgTe and Bi$_2$Se$_3$, and provide important insights into exploring and understanding exotic magnetotransport properties of topological surface states.

Introduction.—Topological insulators host gapless surface states which stem from nontrivial bulk topology [1–3]. These surface states can be modeled by a single Dirac cone. Over the last two decades, topological insulators have been discovered in numerous materials [4–12] including HgTe [13], Bi$_{1-x}$Sb$_x$ [14] and Bi$_2$Se$_3$ [15–18]. Magnetotransport of Dirac surface states has been an active research topic [19–51], theoretically and experimentally, since the discovery of topological insulators. It provides vital features, which include particularly magnetic oscillations, to detect and characterize Dirac surface states. Magnetic oscillations are usually associated with the formation of Landau levels or the Aharonov-Bohm effect [19–24, 33–38]. Thus, a fundamentally intriguing question is whether magnetic oscillations of topological surface states can appear in the absence of Landau levels or the Aharonov-Bohm effect. In slab geometries of topological insulators, in-plane magnetic oscillations as a function of field strength are often times observed [44, 52, 53]. To the best of our knowledge, a convincing explanation of these oscillations is still lacking.

Notably, in typical topological insulators, electron-hole symmetry in the energy spectrum of surface states is broken by the presence of higher-order momentum corrections [54–56]. To fully understand the transport properties of surface states in realistic systems, the consideration of this electron-hole asymmetry is important. Interestingly, the interplay of electron-hole asymmetry and in-plane magnetic fields tilts the surface states at low energies [50].

In this Letter, we study the conductance of topological surface states in presence of a barrier potential and external in-plane magnetic fields, taking into account the electron-hole asymmetry of the energy spectrum. We find that for small and moderate barrier potentials (comparable to the Fermi energy), the surface states exhibit a positive magnetoconductance due to the increase of the Fermi surface by the tilting in any direction. Remarkably, for larger barrier potentials, super-resonant transport of surface states appear by tuning the magnetic field, which enable many surface propagating modes to tunnel through the barrier.

---

Fig. 1. Schematic of surface states (cyan and magenta) of a topological insulator (gray) with a barrier potential $V_0$ extending over a length of $L$. An in-plane magnetic field $B$ (blue arrows) is applied to the system.
through the barrier without backscattering. This super-
resonant transport results in pronounced oscillations in
the conductance as strength or direction of the magnetic
field are varied. Moreover, we show that for moderate
magnetic fields, the conductance is a sinusoidal function
of field direction with period $\pi$ and phase shift depen-
dent on the angle between tilting and field directions.
Our predictions are applicable to a variety of topological
insulators including HgTe and Bi$_2$Se$_3$ as we argue below.

Effective Hamiltonian of surface states.—The states of
a topological insulator on a surface can be described by a
single Dirac cone [15, 54, 55]

$$H(\mathbf{k}) = m\mathbf{k}^2 + v(k_x s_y - k_y s_x),$$

(1)

where $\mathbf{k} = (k_x, k_y)$ are momenta in the vicinity of the
$\Gamma$ point, $v$ is the Fermi velocity, $s_x$ and $s_y$ are Pauli
matrices acting on spin space. Moreover, in certain topo-
logical insulators, for instance, HgTe with zinc-blende crys-
tal structure, bulk inversion symmetry is broken, leading
to extra terms $H_{\text{BLA}} = v_y(k_x s_x + k_y s_y) + \gamma k_x k_y$ [57].
Note that we have included the quadratic terms in mo-
momentum, $mn^2$ and $\gamma k_x k_y$, which preserve time-reversal
symmetry. These terms are often ignored in previous
studies for simplicity [59]. However, they break electron-
hole symmetry in the energy spectrum and can lead to
interesting physics as we show below.

Applying an in-plane magnetic field $\mathbf{B} = B(\cos \theta, \sin \theta)$
introduces a Zeeman term $H_Z = g_\mu B \mathbf{B} \cdot \mathbf{s}/2$, where $g$
is the $g$-factor, $\mu_B$ is the Bohr magneton, $B$ and $\theta$
denote the strength and direction of the magnetic field,
respectively. The Zeeman term not only shifts the Dirac
cone away from the $\Gamma$ point in momentum space but also
tilts the Dirac cone [50]. Considering that $v_\parallel$ is typi-
cally much smaller than $v$ [58], we can find the position
shift of the Dirac point as $\mathbf{k}_s = k_\parallel (-\sin \theta, \cos \theta)$ with
$k_\parallel = g_\mu B \mathbf{B} \cdot \mathbf{s}/2v$. Near the Dirac point, the effective model
for surface states can be written for [57]

$$\mathcal{H}(\mathbf{k}) = v(\tilde{k}_x s_y - \tilde{k}_y s_x) + t_x \tilde{k}_x + t_y \tilde{k}_y,$$

(2)

where $\mathbf{k} = \mathbf{k} - \mathbf{k}_s$ and the tilting vector $\mathbf{t} \equiv (t_x, t_y)$ is
given by

$$\mathbf{t} = k_\parallel (\gamma \cos \theta - 2m \sin \theta, 2m \cos \theta - \gamma \sin \theta).$$

(3)
The eigen-energies are tilted as $E_{\pm}(\mathbf{k}) = \mathbf{t} \cdot \mathbf{k} \pm v|\mathbf{k}|$. The tilting
strength $|\mathbf{t}|$ is proportional to the field strength and the tilting direction is controllable by the field di-
rection. We focus on the realistic case with small tilting
$|\mathbf{t}| < |v|$. throughout.

Transmission probability.—We consider the surface
states with a barrier potential $V_0$ extending over a length of
$L$ in $x$-direction, as sketched in Fig. 1. The in-plane
magnetic field is applied to the whole system. This setup
can be described by

$$\mathcal{H}_{\text{tot}} = \mathcal{H}(-i\partial_x) - E_F + V(x)$$

(4)

with $E_F$ the Fermi energy and the local electronic po-
tential $V(x) = V_0$ for $|x| \leq L/2$ and 0 otherwise [60]. $V_0$
can be created by local gating [61]. It can be positive or
negative. For simplicity, we assume the system to be
large in $y$-direction such that the transverse momentum
$k_y$ is conserved.

To study the transport properties of the system, we
employ the scattering approach. In each region, we find
two eigenstates for given energy $E$ and momentum $k_y$.
In the regions away from the barrier, their wavefunctions
can be written as

$$\psi_{\pm}(x, y) = e^{i\tilde{k}_y y}e^{i\tilde{k}_x x} (e^{i\theta_\pm}, -1)^T / N_{\pm},$$

(5)

where $e^{i\theta_\pm} = v(\tilde{k}_x + i\tilde{k}_y)/(E_{k_y} - t_x \tilde{k}_x)$, $E_{k_y} = E + E_F -
t_y \tilde{k}_y$, $N_{\pm} = \sqrt{1 + |e^{i\theta_\pm}|^2}$, and the wavenumbers $\tilde{k}_\pm$ in
$x$-direction are given by

$$\tilde{k}_\pm = [-t_x E_{k_y} \pm v \sqrt{E_{k_y}^2 - (v^2 - t_y^2)\tilde{k}_y^2}]/(v^2 - t_y^2).$$

(6)

In the barrier region, the wavefunctions have the same
form as Eq. (5) but with $E_{k_y}$ replaced by $E_B^{k_y} = E_{k_y} - V_0$.
Correspondingly, we use superscript $B$ to indicate the
angles $\theta_\pm^B$ and wavenumbers $\tilde{k}_\pm^B$ inside the barrier region.

The scattering state of injecting an electron from the
one lead to the junction can be expanded in terms of the
basis wavefunctions, Eq. (5). Matching the wavefunction
of the scattering state at the interfaces, we derive the
transmission coefficient as

$$t_{k_y} = e^{-ik_y L}e^{i(\tilde{k}_x^B + \tilde{k}_y^B)L} (e^{i\theta_+^B} - e^{i\theta_-^B})(e^{i\theta_+^B} - e^{i\theta_-^B})/Z(T)$$

where $Z = e^{i\tilde{k}_x^B L}(e^{i\theta_+^B} - e^{i\theta_-^B})(e^{i\theta_+^B} - e^{i\theta_-^B})
- e^{i\tilde{k}_y^B L}(e^{i\theta_+^B} - e^{i\theta_-^B})$. The transmission probability is then
given by $T_{k_y} = |t_{k_y}|^2$. More details of the derivation
are presented in the Supplemental Material (SM) [57].

For the incident modes with $k_y = 0$, we always have
$\theta_+ = \theta_-^B = \pm \pi/2$ and hence $T_{k_y = 0} = 1$. This per-
fected transmission results from spin conservation and is
related to Klein tunneling [62]. Notably, without tilting, the
results are independent of the magnetic field. This
indicates that a simple position shift of the Dirac cone in
momentum space does not change the transport prop-
erties of surface states.

Positive magnetoconductance.—With the transmission
probability, the (differential) conductance $G$ (per unit
length) at zero temperature and zero bias voltage can be
evaluated as

$$G = \frac{e^2}{h} \int \frac{d\tilde{k}_y}{2\pi} T_{k_y} (E = 0),$$

(8)

where the sum runs over all modes distinguished by $\tilde{k}_y$.

We first look at the case of small and moderate barrier
potentials, i.e., $|V_0| \lesssim |E_F|$, as shown in Fig. 2. Notably, $G$
increases as we increase the tilting strength in any di-
rection. Recalling that the tilting strength grows linearly
ductance. These oscillations can be understood as the function of the tilting strength in three respectively. (b) Number \( N_k \) of propagating modes (in units of \( k_F/\pi \)) as a function of \( t_x \) for \( t_y = 0 \), \( t_y = t \) (magenta) for \( E_F = 100e/L \) and \( V_0 = 0 \) (solid), \( 0.3E_F \) (dotted), and \( E_F \) (broken), respectively. (b) Number \( N_k \) of propagating modes (in units of \( k_F/\pi \)) as a function of \( t_x \) for \( t_y = 0 \), \( t_y = t \) (magenta) for \( E_F = 100e/L \) and \( V_0 = 0 \) (solid), \( 0.3E_F \) (dotted), and \( E_F \) (broken), respectively. Inset: Fermi surfaces for \( t = (0.5, 0)eV \), \((0, 0.5)eV\), and \((0.5, 0.5)eV\).

with increasing the magnetic field \( B \), this indicates a positive magnetococonductance. For small barrier potentials, \( |V_0| \ll |E_F| \), \( G \) increases monotonically with increasing \( B \). A larger barrier potential suppresses \( G \) and induces slight oscillations. However, \( G \) increases overall, with increasing \( B \) [Fig. 2(a)]. These oscillations are closely related to the super-resonant transport of tilted surface electrons, which we discuss later.

The positive magnetococonductance can be attributed to the enhanced Fermi surface of tilted surface states. To understand this, it is instructive to consider the zero-barrier limit \( V_0 = 0 \). In this limit, all propagating modes transmit through the junction without reflection. Thus, the conductance is simply given by the number \( N_k \) of propagating modes, i.e., \( G = (e^2/h)N_k \). \( N_k \) is determined by the size of the Fermi surface in \( k_y \)-direction, as illustrated in the inset of Fig. 2(b). Tilting the surface Dirac cone in any direction enlarges the Fermi surface and hence the number of propagating modes. As shown by the circles in Fig. 2(b), we calculate \( N_k \) numerically as a function of the tilting strength in three different directions as considered in Fig. 2(a). Evidently, this dependence nicely agrees with the magnetococonductance (solid curves). When the tilting occurs in \( x \)- or \( y \)-direction, we can find \( N_k \) analytically from the tilted spectrum. Namely, \( N_k = |E_F|/(\pi \sqrt{v^2 - t_x^2}) \) for tilting in \( x \)-direction and \( N_k = |vE_F|/|v(\sqrt{v^2 - t_y^2})| \) for tilting in \( y \)-direction.

**Super-resonant transport and conductance oscillations.**—Now, we consider larger barrier potentials, \( |V_0| > |E_F| \), and analyze the magnetic oscillations of the conductance. These oscillations can be understood as the emergence of super-resonant (transport) regimes of surface states, where many propagating modes perfectly transmit through the barrier at the same magnetic field (i.e. the same tilting). To make this clearer and simplify the analysis, we first focus on the large barrier limit, \( |V_0| \gg |E_F| \). In this limit, we can approximate \( \theta^B \approx \pm \pi/2 \) in Eq. (7) and simplify

\[
T_{k_y} = \frac{1 - \cos(\theta_+ - \theta_-)}{1 - \sin \theta_+ \sin \theta_- - \cos(\tilde{k}_y^2 - k_y^2)L} \cos \theta_+ \cos \theta_-.
\]

From this expression [more generally Eq. (7)], we find that the barrier becomes transparent for the mode with index \( \tilde{k}_y \) when the resonance condition, \( \left| \tilde{k}_y^2 - k_y^2 \right|L/2 = 0 \), is fulfilled. Using the expressions for \( \tilde{k}_y^2 \) in Eq. (6), the resonance condition reads explicitly

\[
(V_0 - t_y\tilde{k}_y)^2 - (v^2 - t_x^2)\tilde{k}_y^2 = \frac{n\pi(v^2 - t_x^2)(vL)}{h}^2 \quad (10)
\]

with \( n \) an integer. This means that an electron acquires a phase shift \( 2n\pi \) in one round trip between the interfaces [63].

When the tilting is in junction (i.e. \( x \)-) direction, we find the solutions of \( t_x \) to Eq. (10) as

\[
t_x \approx \pm \sqrt{v^2 - |vV_0|/\pi n} \quad (11)
\]

for integers \( n > |V_0|L/\pi v \). Strikingly, these solutions are independent of the mode index \( \tilde{k}_y \). This indicates the super-resonant regimes where all propagating modes with different \( \tilde{k}_y \) exhibit perfect transmission. As a result, we observe resonance peaks in \( T_{k_y} \) and thus the maximal conductance \( G_{\text{max}} = (e^2/h)N_k \) at \( t_x \propto B \) determined by Eq. (11). Moreover, we find that at \( t_x = \pm \sqrt{v^2 - |vV_0|/\pi (n + 1/2)} \), all modes have the lowest transmission probabilities given by \( T_{k_y} = 1 - (v^2 - t_x^2)/E_F^2 \) [57]. Summing over all modes, we obtain the minimal conductance as \( G_{\text{min}} = (2e^2/3h)N_k \). Therefore, we observe pronounced oscillations of \( G \) with magnitude \( \Delta G_{\text{osc}} \) as large as one third of the maximal conductance:

\[
\Delta G_{\text{osc}} = G_{\text{max}}/3. \quad (12)
\]

Interestingly, the values of \( G_{\text{max}}, G_{\text{min}} \) and \( \Delta G_{\text{osc}} \) (in units of \( N_k \)) are universal and independent of the potential \( V_0 \) and length \( L \) of the barrier [64]. Considering the increase of \( N_k \), when strengthening \( B \), \( \Delta G_{\text{osc}} \) increases. In contrast, according to Eq. (11), the separations between the conductance peaks depend strongly on the product \( V_0L \), whereas they are insensitive to \( E_F \). Moreover, they decrease with increasing \( B \). All these results are in accordance with our numerical results displayed in Fig. 3(a), (b) and (g).

When the tilting is in transverse (i.e. \( y \)-) direction, the solutions to Eq. (10) are given by \( t_y = V_0/\tilde{k}_y - v\sqrt{1 + (\pi n/\tilde{k}_y L)^2} \). For \( n \) close to \( n_c \equiv \lfloor |V_0L/\pi v| \rfloor \), the greatest integer less than \( |V_0L/\pi v| \), we find that most of the propagating modes exhibit a resonance condition at

\[
t_y \approx \pm(|V_0|L - \pi nv^2)/|E_F|L. \quad (13)
\]
Hence, in this tilting direction, we also observe the resonance peaks and pronounced oscillations in the conductance [Fig. 3(d)]. In contrast to the case with the tilting in junction direction, the separations between the conductance peaks are sensitive not only to \( L \) and \( V_0 \) individually but also to \( E_F \). Moreover, Eq. (13) indicates that the separations between the conductance peaks are almost constant with respect to \( B \). However, as increasing \( B \), the resonance positions for different propagating modes become more extended [Fig. 3(c) and (h)]. Consequently, the magnitude of oscillations is strongest for small \( B \) but suppressed for large \( B \).

For the general case with the tilting direction deviating from \( x \)- and \( y \)-directions, \( t_x = \varepsilon t_y \) with \( \varepsilon \neq 0 \), we can also observe magnetoconductance oscillations [Fig. 3(e) and (f)]. These oscillations can be similarly attributed to the super-resonant transport of surface states as varying \( B \). However, they are less regular, compared to the two special cases discussed above. The oscillations are aperiodic in \( B \) and the positions of the peaks become hard to predict in general.

Note that although we focus on the large barrier limit in the above analysis, the conductance oscillations remain pronounced even when the barrier potential is of the same order as the Fermi energy, \( |V_0| \lesssim |E_F| \), see Fig. 2(a) and Sec. VI in the SM [57].

Dependence on field direction.—As we have discussed before, the conductance \( G \) depends on the tilting direction which, in turn, is determined periodically by the field direction \( \theta \), according to Eq. (3). Therefore, \( G \) depends periodically on \( \theta \). This field-direction dependence stems from two origins: (i) the anisotropic Fermi surface and (ii) the barrier transparency for conducting channels. In Fig. 4, we calculate numerically \( G \) as a function of \( \theta \). Several interesting features can be observed.

First, \( G \) has a period of \( \pi \) in \( \theta \). For small field strengths...
$B < B_c$, $G(\theta)$ displays approximately a sinusoidal dependence, $G(\theta) - G_0 \propto \sin[2(\theta - \theta_0)]$, where $G_0$ is a ($\theta$-independent) constant and $B_c$ corresponds to the field strength at which the first conductance peak is located [65]. If the tilting direction is parallel ($m = 0$) or perpendicular ($\gamma = 0$) to the field direction, the phase shift becomes $\theta_0 = 0$ or $\pi/2$. However, if the tilting direction is neither parallel nor perpendicular to the field direction ($m\gamma \neq 0$), then $\theta_0$ is different from 0 and $\pi/2$ [Fig. 4(b)]. Second, if we increase the field strength $B$, the dependence on $\theta$ becomes more pronounced [Fig. 4(a)]. This shows that the anisotropy of surface states is enhanced by increasing $B$ via the tilting effect. Third, for stronger field strengths $B > B_c$, $G$ oscillates with a number of peaks and valleys in one period $\theta \in [0, \pi]$ (blue curve). These dense oscillations with respect to $\theta$ can also be related to the super-resonant transport of surface states analyzed before. It is interesting to note that clear field-direction dependence in the resistance of topological surface states has been observed recently [44, 52].

**Conclusion and discussion.**—We have identified a positive magnetoconductance of topological surface states, which stems from the increase of Fermi surface by applying in-plane magnetic fields. We have unveiled the super-resonant transport of the surface states by tuning the magnetic field, which enables many propagating modes to transmit a barrier potential without backscattering. This super-resonant transport results in pronounced oscillations in the magnetoconductance.

We note that the appearance of the positive magnetoconductance and conductance oscillations can be directly attributed to the deformation of the surface Dirac cone by in-plane magnetic fields. In this work, the crucial role of deforming is played by tilting the Dirac cone via the Zeeman effect. Particularly, the anomalous conductance oscillations arising from the super-resonant transport of surface states are essentially different from conventional magnetic oscillations, which typically stem from the formation of Landau levels or the Aharonov-Bohm effect.

Our predictions can be implemented in various candidate materials including HgTe and Bi$_2$Se$_3$ where in-plane magnetic fields have been successfully applied to surface states [27, 44, 52, 66–69]. Consider HgTe with parameters $v = 256$ meV·nm, $m = 108$ meV·nm$^2$, $\gamma = -64$ meV·nm$^2$, $g = 20$ [57], $L = 2\mu m$, and $V_0 = 40$ meV [75]. We could observe magnetic oscillations for tilting $t_{c} > v\sqrt{1-V_0L/[\pi v(n_c+1/2)]} \approx 0.017v$ and thus for magnetic fields $B > 2t_{c}/(gj\mu B/\sqrt{4m^2+\gamma^2}) \approx 8.4$ T. For Bi$_2$Se$_3$ with $v = 330$ meV·nm, $m = 237$ meV·nm$^2$, $\gamma = 0$, $g = 19.4$, $L = 2\mu m$, and $V_0 = 150$ meV [57], we could observe the oscillations for $B > 3.6$ T [80].

We thank Abu Aravindnath, Mohamed AbdelGhany, Wouter Beugeling, Hartmut Blumhann, Charles Gould, and Benedikt Mayer for valuable discussion. This work was supported by the DFG (SPP1666, SFB1170 “ToCoTronics”, and SFB1143 (project-id 247310070)), the Würzburg-Dresden Cluster of Excellence ct.qmat (EXC2147, project-id 390858490), and the Elitenetzwerk Bayern Graduate School on “Topological Insulators”. P.S. is also supported by the DFG through the Leibniz Program and the National Science Centre Sonata Bis grant 2019/34/E/ST3/00405.

* songbo.zhang@physik.uni-wuerzburg.de

[1] M. Z. Hasan and C. L. Kane, “Colloquium : Topological insulators”, Rev. Mod. Phys. 82, 3045 (2010).
[2] X. L. Qi and S. C. Zhang, “Topological insulators and superconductors”, Rev. Mod. Phys. 83, 1057 (2011).
[3] S.-Q. Shen, Topological Insulators: Dirac Equation in Condensed Matters (Springer, Berlin, 2012).
[4] L. Fu and C. L. Kane, “Topological insulators with inversion symmetry”, Phys. Rev. B 76, 045302 (2007).
[5] T. Sato, K. Segawa, H. Guo, K. Sugawara, S. Souma, T. Takahashi, and Y. Ando, “Direct Evidence for the Dirac-Cone Topological Surface States in the Ternary Chalcogenide TlBiSe$_2$”, Phys. Rev. Lett. 105, 136802 (2010).
[6] M. Z. Hasan and J. E. Moore, “Three-dimensional topological insulators”, Annu. Rev. Condens. Matter Phys. 2, 55 (2011).
[7] Y. Tanaka, Z. Ren, T. Sato, K. Nakayama, S. Souma, T. Takahashi, K. Segawa, and Y. Ando, “Experimental realization of a topological crystalline insulator in SnTe”, Nat. Phys. 8, 800 (2012).
[8] S.-Y. Xu, C. Liu, N. Alidoust, M. Neupane, D. Qian, I. Belopolski, et al., “Observation of a topological crystalline insulator phase and topological phase transition in Pb$_3$Sn$_2$Te”, Nat. Commun. 3, 1192 (2012).
[9] T. H. Hsieh, H. Lin, J. Liu, W. Duan, A. Bansil, and L. Fu, “Topological crystalline insulators in the SnTe material class”, Nat. Commun. 3, 982.
[10] P. Dziawa, B. J. Kowalski, K. Dybko, R. Buczko, A. Szczepakow, M. Szot, et al., “Topological crystalline insulator states in Pb$_{1-x}$Sn$_x$Te}, Nat. Mater. 11, 1023.
[11] B. Yan and S. C. Zhang, “Topological materials”, Rep. Prog. Phys. 75, 096501 (2012).
[12] Y. Ando, “Topological insulator materials”, J. Phys. Soc. Jpn. 82, 102001 (2013).
[13] C. Brüne, C. X. Liu, E. G. Novik, E. M. Hankiewicz, H. Buhmann, Y. L. Chen, X. L. Qi, Z. X. Shen, S. C. Zhang, and L. W. Molenkamp, “Quantum Hall effect from the topological surface states of strained bulk HgTe”, Phys. Rev. Lett. 106, 126803 (2011).
[14] D. Hsieh, D. Qian, L. Wray, Y. Xia, Y. S. Hor, R. J. Cava, and M. Z. Hasan, “A topological Dirac insulator in a quantum spin Hall phase”, Nature 452, 970 (2008).
[15] H. Zhang, C. X. Liu, X. L. Qi, X. Dai, Z. Fang, and S. C. Zhang, “Topological insulators in Bi$_2$Se$_3$, Bi$_2$Te$_3$ and Sb$_2$Te$_3$ with a single Dirac cone on the surface”, Nat. Phys. 5, 438 (2009).
[16] Y. L. Chen, J. G. Analytis, J. H. Chu, Z. K. Liu, S. K. Mo, X. L. Qi, et al., “Experimental Realization of a Three-Dimensional Topological Insulator, Bi$_2$Te$_3$”, Science 325, 178 (2009).
[17] D. Hsieh, Y. Xia, D. Qian, L. Wray, F. Meier, J. H. Dil, et al., “Observation of Time-Reversal-Protected Single-
Dirac-Cone Topological-Insulator States in Bi$_2$Te$_3$ and Sb$_2$Te$_3$, Phys. Rev. Lett. 103, 146401 (2009).

[18] Y. Xia, D. Qian, D. Hsieh, L. Wray, A. Pal, H. Lin, et al., “Observation of a large-gap topological-insulator class with a single Dirac cone on the surface”, Nat. Phys. 5, 398 (2009).

[19] A. A. Taskin and Y. Ando, “Quantum oscillations in a topological insulator Bi$_{1-x}$Sbx”, Phys. Rev. B 80, 055303 (2009).

[20] D. X. Qu, Y. S. Hor, J. Xiong, R. J. Cava, and N. P. Ong, “Quantum Oscillations and Hall Anomaly of Surface States in the Topological Insulator Bi$_2$Te$_3$”, Science 329, 821 (2010).

[21] J. G. Analytis, R. D. McDonald, S. C. Riggs, J. H. Chu, G. S. Boebinger, and I. R. Fisher, “Two-dimensional surface state in the quantum limit of a topological insulator”, Nat. Phys. 6, 960 (2010).

[22] P. Cheng, C. Song, T. Zhang, Y. Zhang, Y. Wang, J. F. Jia, et al., “Landau quantization of topological surface states in Bi$_2$Se$_3$”, Phys. Rev. Lett. 105, 076801 (2010).

[23] Z. Ren, A. A. Taskin, S. Sasaki, K. Segawa, and Y. Ando, “Large bulk resistivity and surface quantum oscillations in the topological insulator Bi$_2$Te$_3$Se”, Phys. Rev. B 82, 241306 (2010).

[24] A. A. Taskin, S. Sasaki, K. Segawa, and Y. Ando, “Manifestation of Topological Protection in Transport Properties of Epitaxial Bi$_2$Se$_3$ Thin Films”, Phys. Rev. Lett. 109, 066803 (2012).

[25] J. Chen, H. J. Qin, F. Yang, J. Liu, T. Guan, F. M. Qu, et al., “Gate-voltage control of chemical potential and weak antilocalization in Bi$_2$Se$_3$”, Phys. Rev. Lett. 105, 176702 (2010).

[26] H.-Z. Lu, J. Shi, and S.-Q. Shen, “Competition between weak localization and antilocalization in topological surface states”, Phys. Rev. Lett. 107, 076801 (2011).

[27] H.-T. He, G. Wang, T. Zhang, I. K. Sou, G. K. L. Wong, J.-N. Wang, H.-Z. Lu, S.-Q. Shen, and F.-C. Zhang, “Impurity effect on weak antilocalization in the topological insulator Bi$_2$Te$_3$”, Phys. Rev. Lett. 106, 166805 (2011).

[28] G. Tkachov and E. M. Hankiewicz, “Weak antilocalization in light quantum wells and topological surface states: Massive versus massless Dirac fermions”, Phys. Rev. B 84, 035444 (2011).

[29] I. Garate and L. Glazman, “Weak localization and antilocalization in topological insulator thin films with coherent bulk-surface coupling”, Phys. Rev. B 86, 035422 (2012).

[30] J. H. Bardarson and J. E. Moore, “Quantum interference and Aharonov-Bohm oscillations in topological insulators”, Rep. Prog. Phys. 76, 056501 (2013).

[31] M. Sitte, A. Rosch, E. Altman, and L. Fritz, “Topological insulators in magnetic fields: Quantum Hall effect and edge channels with a nonquantized $\theta$ term”, Phys. Rev. Lett. 108, 126807 (2012).

[32] X. Wang, Y. Du, S. Dou, and C. Zhang, “Room Temperature Giant and Linear Magnetoresistance in Topological Insulator Bi$_2$Te$_3$ Nanosheets”, Phys. Rev. Lett. 108, 266806 (2012).

[33] H. Peng, K. Lai, D. Kong, S. Meister, Y. Chen, X. L. Qi, S. C. Zhang, Z. X. Shen, and Y. Cui, “Aharonov-bohm interference in topological insulator nanoribbons”, Nat. Mater. 9, 225 (2010).

[34] Y. Zhang and A. Vishwanath, “Anomalous aharonov-bohm conductance oscillations from topological insulator surface states”, Phys. Rev. Lett. 105, 206601 (2010).

[35] J. H. Bardarson, P. W. Brouwer, and J. E. Moore, “Aharonov-bohm oscillations in disordered topological insulator nanowires”, Phys. Rev. Lett. 105, 156803 (2010).

[36] Y. Xu, I. Miotkowski, C. Liu, J. Tian, H. Nam, N. Alidoust, J. Hu, C. K. Shih, M. Z. Hasan, and Y. P. Chen, “Observation of topological surface state quantum Hall effect in an intrinsic three-dimensional topological insulator”, Nat. Phys. 10, 956 (2014).

[37] R. Yoshihi, A. Tsukazaki, Y. Kozuka, J. Falson, K. Takahashi, J. Checkelsky, N. Nagaosa, M. Kawasaki, and Y. Tokura, “Quantum Hall effect on top and bottom surface states of topological insulator (Bi$_{1-x}$Sbx)$_2$Te$_3$ films”, Nat. Commun. 6, 6627 (2015).

[38] Y. S. Fu, M. Kauamura, K. Igarashi, H. Takagi, T. Hanaguri, and T. Sasa-gawa, “Imaging the two-component nature of Dirac-Landau levels in the topological surface state of Bi$_2$Se$_3$”, Nat. Phys. 10, 815 (2014).

[39] R. Ilan, F. de Juan, and J. E. Moore, “Spin-Based Mach-Zehnder Interferometer in Topological Insulator $p$–$n$ Junctions”, Phys. Rev. Lett. 115, 096802 (2015).

[40] S.-B. Zhang, H.-Z. Lu, and S.-Q. Shen, “Edge states and integer quantum Hall effect in topological insulator thin films”, Sci. Rep. 5, 13277 (2015).

[41] Y. Xu, I. Miotkowski, and Y. P. Chen, “Quantum transport of two-species Dirac fermions in dual-gated three-dimensional topological insulators”, Nat. Commun. 7, 11434 (2016).

[42] N. H. Tu, Y. Tanabe, Y. Satake, K. K. Huyhn, and K. Tanigaki, “In-plane topological pn junction in the three-dimensional topological insulator Bi$_2$Sb$_2$Te$_3$, Nat. Commun. 7, 13763 (2016).

[43] P. Burset, B. Lu, G. Tkachov, Y. Tanaka, E. M. Hankiewicz, and B. Trauzettel, “Superconducting proximity effect in three-dimensional topological insulators in the presence of a magnetic field”, Phys. Rev. B 92, 205424 (2015).

[44] A. Taskin, H. F. Legg, F. Yang, S. Sasaki, Y. Kanai, K. Matsumoto, A. Rosch, and Y. Ando, “Planar Hall effect from the surface of topological insulators”, Nat. Commun. 8, 1340 (2017).

[45] V. Dziom, A. Shuvaev, A. Pimenov, G. Astakhov, C. Ames, K. Bendias, et al., “Observation of the universal magneto-electric effect in a 3D topological insulator”, Nat. Commun. 8, 15197 (2017).

[46] P. He, S. S.-L. Zhang, D. Zhu, Y. Liu, Y. Wang, J. Yu, G. Vignale, and H. Yang, “Bilinear magneto-electric resistance as a probe of three-dimensional spin texture in topological surface states”, Nat. Phys. 14, 495 (2018).

[47] S.-B. Zhang and B. Trauzettel, “Perfect Crossed Andreev Reflection in Dirac Hybrid Junctions in the Quantum Hall Regime”, Phys. Rev. Lett. 122, 257701 (2019).

[48] A. Assouline, C. Feuillet-Palma, N. Bergeal, T. Zhang, A. Mottaghizadeh, A. Zimmers, et al., “Spin-orbit induced phase-shift in Bi$_2$Se$_3$ Josephson junctions”, Nat. Commun. 10, 126 (2019).

[49] H. Wu, P. Zhang, P. Deng, Q. Lan, P. Han, S. A. Razavi, et al., “Room-Temperature Spin-Orbit Torque from Topological Surface States”, Phys. Rev. Lett. 123, 207205 (2019).

[50] S.-H. Zheng, H.-J. Duan, J.-K. Wang, J.-Y. Li, M.-X. Deng, and R.-Q. Wang, “Origin of planar Hall effect on the surface of topological insulators: Tilt of Dirac cone
by an in-plane magnetic field’, Phys. Rev. B 101, 041408 (2020).

[51] A. Dyrdał, J. Barnaś, and A. Fert, ‘Spin-Momentum-Locking Inhomogeneities as a Source of Bilinear Magnetoresistance in Topological Insulators’, Phys. Rev. Lett. 124, 046802 (2020).

[52] A. Sulavč, M. Zeng, S.-Q. Shen, S. K. Cho, W. G. Zhu, Y. P. Feng, S. V. Eremeev, Y. Kawazoe, L. Shen, and L. Wang, ‘Electrically Tunable In-Plane Anisotropic Magnetoresistance in Topological Insulator BiSbTeSe₂ Nanodevices’, Nano Lett. 15, 2061 (2015).

[53] Unpublished results from the Molenkamp lab.

[54] W.-Y. Shan, H.-Z. Lu, and S.-Q. Shen, ‘Effective continuous model for surface states and thin films of three-dimensional topological insulators’, New J. Phys. 12, 043048 (2010).

[55] C.-X. Liu, X.-L. Qi, H. Zhang, X. Dai, Z. Fang, and S.-C. Zhang, ‘Model Hamiltonian for topological insulators’, Phys. Rev. B 82, 045122 (2010).

[56] A. Jost, M. Bendias, J. Böttcher, E. Hankiewicz, C. Brüne, H. Buhmann, et al., ‘Electron-hole asymmetry of the topological surface states in strained HgTe’, Proc. Nat. Acad. Sci. 114, 3831 (2017).

[57] See the Supplemental Material including Ref. [54, 62, 71–74] for details.

[58] We note, however, that the main results discussed below are not restricted to this condition.

[59] In a more elaborate model, we may include the warping effect of surface states. However, using realistic parameters for Bi-based topological insulators, we find that the influence of the warping effect on our main results is ignorable [57].

[60] The local potential creates a Fermi-surface mismatch at the interfaces. This allows us to properly use the scattering approach to solve the transport problem, as discussed in Ref. [70].

[61] A. Banerjee, A. Sundaresh, S. Biswas, R. Ganesan, D. Sen, and P. S. Anil Kumar, ‘Topological insulator n-p-n junctions in a magnetic field’, Nanoscale 11, 5317 (2019).

[62] M. I. Katsnelson, K. S. Novoselov, and A. K. Geim, ‘Chiral tunnelling and the Klein paradox in graphene’, Nat. Phys. 2, 620 (2006).

[63] We may look at the system as a Fabry–Pérot resonator between two interfaces. An electron can circulate in the resonator. In one round trip, the electron acquires a phase shift \((k_F^+ - k_F^-) L\).

[64] Note that this is only true when \(|V_0| \gg |E_F|\). When this condition is softened, the value \(\Delta G_{osc}/G_{max}\) becomes smaller.

[65] Using Eqs. (11) and (13), \(B_c\) could be estimated as \(\min(2/|\gamma|, 2\pi v_e F/m E_F L)\sqrt{v_F/(g\mu_B)}\) with \(F = \sqrt{v_e^2 - V_0^2 L/(\pi(n_e + 1))}\).

[66] S. Wiedmann, A. Jost, B. Fauquère, J. van Dijk, M. J. Meijer, T. Khouri, et al., ‘Anisotropic and strong negative magnetoresistance in the three-dimensional topological insulator BiSbTeS₂’, Phys. Rev. B 94, 081302 (2016).

[67] S. Hart, H. Ren, M. Kosowsky, G. Ben-Shach, P. Leubner, C. Brüne, H. Buhmann, L. W. Molenkamp, B. I. Halperin, and A. Yacoby, ‘Controlled finite momentum pairing and spatially varying order parameter in proximitized HgTe quantum wells’, Nat. Phys. 13, 87 (2017).

[68] D. Rakhmilevich, F. Wang, W. Zhao, M. H. W. Chan, J. S. Moolerda, C. Liu, and C.-Z. Chang, ‘Unconventional planar Hall effect in exchange-coupled topological insulator–ferromagnetic insulator heterostructures’, Phys. Rev. B 98, 094404 (2018).

[69] B. Wu, X.-C. Pan, W. Wu, F. Fei, B. Chen, Q. Liu, H. Bu, L. Cao, F. Song, and B. Wang, ‘Oscillating planar Hall response in bulk crystal of topological insulator Sn doped Bi₁₋₀.₉Sb₁₉TeS₂’, Appl. Phys. Lett. 113, 011902 (2018).

[70] D. Breunig, S.-B. Zhang, B. Trauzettel, and T. M. Klapwijk, ‘Directional electron-filtering at a superconductor-semiconductor interface’, Phys. Rev. B 103, 165414 (2021).

[71] E. G. Novik, A. Pfeuffer-Jeschke, T. Jungwirth, V. Latusek, C. R. Becker, G. Landwehr, H. Buhmann, and L. W. Molenkamp, ‘Band structure of semimagnetic Hg₁₋₀.₉Mn₀.₉Te quantum wells’, Phys. Rev. B 72, 035321 (2005).

[72] M. Cardona, N. E. Christensen, and G. Fasol, ‘Terms linear in \(k\) in the band structure of zinc-blende-type semiconductors’, Phys. Rev. Lett. 56, 2831 (1986).

[73] S.-B. Zhang, H.-Z. Lu, and S.-Q. Shen, ‘Linear magnetococonductivity in an intrinsic topological Weyl semimetal’, New J. Phys. 18, 053039 (2016).

[74] V. H. Nguyen and J.-C. Charlier, ‘Klein tunneling and electron optics in Dirac–Weyl fermion systems with tilted energy dispersion’, Phys. Rev. B 97, 235113 (2018).

[75] In HgTe, the phase coherence length can be larger than 5 \(\mu m\) at temperature \(T = 40\) mK [76], and the bulk energy gap can be larger than 50 meV [77, 78]. To the best of our knowledge, the in-plane g-factor of topological surface states in HgTe is still not known experimentally. However, we notice that in a related 2D system, the quantum Hall phase of HgTe quantum wells, the 2D electron gas has an in-plane g-factor of 20.5 [79]. We may expect a similar g-factor for the surface states.

[76] J. Ziegler, R. Kozlovsky, C. Gorini, M. H. Liu, S. Weishäupl, F. Maier, et al., ‘Probing spin helical surface states in topological HgTe nanowires’, Phys. Rev. B 97, 035157 (2018).

[77] S.-C. Wu, B. Yan, and C. Felser, ‘Ab initio study of topological surface states of strained HgTe’, Europhys. Lett. 107, 57006 (2014).

[78] T. Rauch, S. Achilles, J. Henk, and I. Mertig, ‘Spin Chirality Tuning and Topological Semimetals in Strained HgTe\(_{2−x}\)’, Phys. Rev. Lett. 114, 236805 (2015).

[79] M. König, H. Buhmann, L. W. Molenkamp, T. Hughes, C.-X. Liu, X.-L. Qi, and S.-C. Zhang, ‘The Quantum Spin Hall Effect: Theory and Experiment’, J. Phys. Soc. Jpn. 77, 031007 (2008).

[80] For larger parameters \(V_0 L, v, g \or 4m^2 + \gamma^2\), we may expect the oscillations to appear in smaller magnetic fields.