Cover Note

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The results presented in this paper are preliminary. A new version that discusses an updated methodology, and shows new simulation and hardware results on a different robotic platform, can be found here: https://arxiv.org/abs/2211.10270
Meta Learning MPC using Finite-Dimensional Gaussian Process Approximations

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Abstract

Data availability has dramatically increased in recent years, driving model-based control methods to exploit learning techniques for improving the system description, and thus control performance. Two key factors that hinder the practical applicability of learning methods in control are their high computational complexity and limited generalization capabilities to unseen conditions. Meta-learning is a powerful tool that enables efficient learning across a finite set of related tasks, easing adaptation to new unseen tasks. This paper makes use of a meta-learning approach for adaptive model predictive control, by learning a system model that leverages data from previous related tasks, while enabling fast fine-tuning to the current task during closed-loop operation. The dynamics is modeled via Gaussian process regression and, building on the Karhunen-Loève expansion, can be approximately reformulated as a finite linear combination of kernel eigenfunctions. Using data collected over a set of tasks, the eigenfunction hyperparameters are optimized in a meta-training phase by maximizing a variational bound for the log-marginal likelihood. During meta-testing, the eigenfunctions are fixed, so that only the linear parameters are adapted to the new unseen task in an online adaptive fashion via Bayesian linear regression, providing a simple and efficient inference scheme. Simulation results are provided for autonomous racing with miniature race cars adapting to unseen road conditions.

1 Introduction

Data-driven model-based control techniques have gained increasing attention, most commonly focusing on improving system knowledge during operation. While this enhances control performance, data-driven control methods come with important challenges [13], particularly regarding the trade-off between model complexity and computational requirements. Gaussian process (GP) regression, e.g., offers appealing properties for modeling complex interactions, but scales poorly, and can be potentially intractable for real-time operation. For this reason, existing work [15], [24], [22], [16], [7], [19] reverted to the use of sparse GP approaches [34], [26] or techniques for approximating GPs along the control horizon [27]. These methods are typically based on collecting data for a single task, where the learning algorithm improves the model based on online data, under the assumption that the task will not change. This approach naturally limits adaptation to changing conditions, for instance in the case of a car driving on dry and wet asphalt, or a robotic manipulator tracking a given
reference trajectory that changes from task to task, which is often addressed by means of heuristic
data selection procedures. Meta-learning [9] systematically addresses this issue with the goal of
transferring acquired knowledge to unseen tasks with minimal new data.

In this work, we exploit the strength of meta-learning for enabling data-efficient adaptive model
predictive control (MPC), making use of GP regression for modeling the system dynamics. The
idea is to leverage the Karhunen-Loève expansion [25] and approximate the GP as a finite linear
combination of basis functions. In a meta-learning fashion, the associated hyperparameters are meta-
trained over a set of related tasks making use of a variational bound for the log-marginal likelihood,
which is typically used for training GPs [30]. During meta-testing, the basis functions are fixed
and only the linear parameters are adapted online to a new task via Bayesian linear regression [2].
We analyse and validate the proposed method in simulation on two different control problems. The
first is the mountain-car problem [32], where the tasks are specified as different slopes. In the
second example, we consider the control of a miniature race car along a track under changing road
conditions.

The contributions of this paper are twofold:

- We develop a meta-learning approach for adaptive MPC, providing both generalization
capabilities to unseen conditions, and a low complexity inference scheme where at meta-
testing only the linear parameters are adapted via Bayesian linear regression.
- We provide a meta-training formulation that extends to eigenfunction approximations
whenever these are not readily available in closed-form, such that further hyperparameters
can be optimized, e.g. the input locations for a subset of regressors.

2 Related Work

Meta-learning, also known as learning to learn, was first proposed as a model-agnostic procedure [9]
that enables to construct a strong prior across tasks, which can then be transferred to new unseen
tasks. Given the versatility of the approach, it has naturally been extended, and reformulated, in the
context of reinforcement learning (RL), particularly for model-free approaches [36], [37], [1]. These,
however, still need large amounts of data to achieve acceptable learning results, which can be imprac-
tical for real-world systems, thus motivating a shift towards model-based approaches [40]. In [6],
the authors propose a meta-learning approach for neural network dynamics models to be adapted online
in an RL framework. The main idea is to define a task as a collection of past time-steps, used to meta-
learn a model that can be easily adapted to future time-steps. Other model-based approaches were
proposed in [31] and [11]. In [31], the dynamics are modeled as a latent variable GP, interpreting
meta-learning as hierarchical Bayes [10]. While we make use of a similar variational approach for
enabling meta-training, our problem is formulated to provide exact probability updates for the task
parameters by using Bayesian linear regression, and by employing a linear finite-dimensional GP
approximation. The approach developed in [11] is based on Bayesian last-layer models [12], where
the features consist in neural networks for which the weights are meta-learned, and only the last
layer of linear parameters is adapted to new unseen tasks. This scheme enables efficient online adap-
tration, similarly to the approach employed in this paper, with the difference that the meta-training
phase is formulated as an optimization problem over the posterior predictive distribution for finding
a good prior over the linear parameters. In contrast, we optimize over a variational bound of the log-
marginal likelihood for learning kernel hyperparameters and, potentially, additional basis function
hyperparameters when closed form eigenfunctions are not available.

Another related area of research considers multi-task GPs, e.g. [38], [18], where in particular the
first makes use of finite-dimensional approximations, but, similarly to [12], seeks for a good prior
over the linear parameters using an expectation-maximization approach. The second is also based
on the concept of linear combinations of basis functions, but rather in the context of neural networks
and restricted to the case of radial basis functions. An interesting generalization of the variational
inference procedure proposed in this paper, and similarly in [31], can be found in [3], where a
generic multi-task objective function for deep GPs is formulated as a sum of task specific losses
and shared regularizers. In our case, while the employed GP has a shallow structure, the obtained
variational bound also contains the terms highlighted in [3]: the task-specific losses are expressed
as log-likelihood terms and the shared regularizers are in the form of the KL divergence between
the variational distributions associated to each task and the prior over all tasks. In the context of
Markov decision processes (MDPs), a related approach is developed in [8], which is based on [33], and makes use of GP linear mixture models.

Extensive prior work on online adaptation at test time has been developed in the area of learning-based control and robotics, in particular on the idea of fine-tuning global models with a learned residual [5], [14], [21], [4] or based on learning local models [35], [20] - which can be interpreted as particular instances of tasks. These methods, however, typically do not use the training time for easing the adaptation at test time as, on the contrary, is done in a meta-learning approach.

3 Preliminaries

We consider measurement models of the form

\[ y = f(x) + \epsilon, \]

where \( f : \mathcal{X} \to \mathbb{R} \) is an unknown function mapping from a compact space \( \mathcal{X} \in \mathbb{R}^d \), with input locations \( x \) distributed as \( x \sim \mu(\mathcal{X}) \), where \( \mu \) is a non-degenerate probability measure on \( \mathcal{X} \), and \( \epsilon \sim \mathcal{N}(0, \sigma^2) \) is i.i.d. additive measurement noise. The mapping is approximated as a GP, described by a covariance function of the form \( k : \mathcal{X} \times \mathcal{X} \to \mathbb{R} \) as

\[ f \sim GP(0, k(x, x')). \]

(1)

As discussed in [30], GPs of the form (1) can be expressed as linear combinations of possibly infinite basis functions, and can, therefore, be interpreted as Bayesian linear regression. One possible set of basis functions are the eigenfunctions, which allow for reformulating the associated GP \( f \) via the Karhunen-Loève expansion [25]:

\[ f(x) = \sum_{i=1}^{\infty} \phi_i(x) \alpha_i, \quad \alpha_i \sim \mathcal{N}(0, \lambda_i). \]

(2)

This expansion can be truncated to a finite linear combination of eigenfunctions, by selecting \( E \) principal components, or in other words, choosing the best \( E \)-dimensional subspace approximation for \( f \) in (2), see e.g. [39],

\[ \hat{f}(x) = \sum_{i=1}^{E} \phi_i(x) \alpha_i, \quad \alpha_i \sim \mathcal{N}(0, \lambda_i). \]

(3)

Obtaining eigenfunctions under a known input distribution is not an easy task, although there exist important known closed-form solutions, e.g., for a Gaussian input distribution and Gaussian kernel expansion using Hermite polynomials [25]. In practice, it is possible to construct basis functions using, for instance, a subset of regressors or the Nyström method [10], or random features [28] in the case of stationary kernels.

4 Method

Data-efficiency and competitive computation times are key factors for the success of a learning-based controller, and to enable real-time closed-loop operation. The presented approach allows for exploiting data collected from a range of related tasks to quickly adapt online to a new task by learning shared information in a meta-training phase and re-using it during meta-testing. This is achieved by making use of the finite-dimensional GP approximation [3], so that basis functions \( \phi_i(\cdot) \) are shared while only the linear parameters \( \alpha_i \) are specialized to each task. The meta-training phase optimizes the basis function hyperparameters, which are then fixed during meta-testing, and the linear parameters \( \alpha_i \) are adapted via Bayesian linear regression, rendering the approach real-time feasible. In the next sections, we will first state the problem formulation and then introduce the overall meta-learning procedure for learning-based MPC. A summary of the method is given in Algorithm[1].

4.1 Problem formulation

We consider dynamic systems of the form

\[ x_{k+1} = g(x_k, u_k) + w_k, \]

(4)
where \( x_k \in \mathbb{R}^{n_x} \) is the state variable, \( u_k \in \mathbb{R}^{n_u} \) is the input variable, \( w_k \sim \mathcal{N}(0, \sigma_w^2 I) \) is i.i.d. additive process noise, and \( q \) is the (partially) unknown dynamics of the system.

The system dynamics can be reformulated as

\[
y_k = f(x_k, u_k) + w_k,
\]

where \( y_k \) can represent, for instance, state differences \( y_k = x_{k+1} - x_k \), or incorporate known dynamics of the system, so that the mapping \( f \) represents an unknown residual. This allows for approximating each dimension of \( f \) as an independent zero-mean GP (1), which corresponds to modeling (4) as a collection of GPs with a particular mean function. For the remaining part of the section we will refer to one dimension of the system, but multiple dimensions can be constructed analogously.

4.2 Meta-training

Task data is collected in sets \( D_m = \{(x_i, y_i)\}_{i=1}^{N_m} \) where \( N_m \) is the number of input and output data for each task \( m \), and the overall dataset formed by \( M \) different tasks is defined as \( D = \{D_m\}_{m=1}^{M} \).

A naïve approach would be to train a single GP on each \( D_m \), specializing the fit to each task. However, this results in an inefficient use of the available information when the tasks share similarities, described as a distribution \( p(m) \) from which each task \( m \) is sampled [9]. A more data-efficient approach, which facilitates generalization to a new unseen task, is to train the GP to leverage task similarities and tailor for the re-use of information. We use the approximation in (5), and obtain a posterior model description for each task data set \( D_m \):

\[
\hat{f}(x)\mid D_m = \sum_{i=1}^{E} \phi_i(x) \alpha_i^m,
\]

where \( \alpha^m = [\alpha_1^m, \ldots, \alpha_E^m]^T \sim \mathcal{N}(\mu^m, \Sigma^m) \), with mean and variance computed via Bayesian linear regression. The main idea is that the basis functions are the same for all \( m \), leaving \( \alpha^m \) as the only term characterizing a specific task. Finding the best linear approximation (6), such that a finite number of basis functions is shared among tasks, can be cast as an optimization problem over the basis function hyperparameters. For each \( m \), we define a variational distribution \( q^m(f) \) such that \( \hat{f}(x)\mid D_m \sim q^m(f) \), i.e.:

\[
q^m(f) \sim \mathcal{N}(\mu^m_q, \Sigma^m_q), \quad \mu^m_q = \Phi \mu^m, \quad \Sigma^m_q = \Phi \Sigma^m \Phi^T,
\]

where \( \Phi = [\phi_1(\cdot), \ldots, \phi_E(\cdot)] \). We construct a variational bound for the marginal log-likelihood of the GP modeling (5), building on the classical approach for optimizing kernel hyperparameters [30], and ensuring that the approximate linear model (6) is as close as possible to the true underlying GP (5).

We consider the following model likelihood

\[
p(Y, f|X, \theta) = p(Y|f, X, \theta)p(f),
\]

where \( X \) and \( Y \) are the overall collection of input and output data, such that \( \mathcal{D} = \{X, Y\} \), the likelihood \( p(y|f, X, \theta) = \mathcal{N}(f|\mu_w^2) \) is Gaussian, and the prior \( p(f) \) over the latent variable \( f \) is a zero-mean GP (1). Parameter \( \theta \) represents the set of basis function hyperparameters that should be optimized. For instance, in the case of a subset of regressors, each basis function is defined by the kernel function evaluated at some input location. Typically, the set of input locations can be chosen a priori as a subset of training data, or test data, but in our formulation, these are considered as hyperparameters in \( \theta \), together with the kernel hyperparameters. The model likelihood (8) is used to construct the following known variational bound (2):

\[
\log p(Y|X, \theta) \geq \mathbb{E}_q \log \frac{p(Y, f|X, \theta)}{q(f)},
\]

where \( q(f) \) represents the joint variational distribution \( q(f) = \prod_{m=1}^{M} q^m(f) \). The obtained bound, which is typically referred to as evidence lower bound (ELBO), is maximized in order to approximate the log-marginal likelihood, while obtaining a variational distribution that approximates the
true posterior $p(f|Y, X, \theta)$. The ELBO [5] can be shown to decompose into

$$
\mathbb{E}_q \log \frac{p(Y, f|X, \theta)}{q(f)} = \mathbb{E}_q \log \frac{p(Y|X, f, \theta)p(f)}{q(f)} \\
= \mathbb{E}_q \log p(Y|X, f, \theta) + \mathbb{E}_q \log \frac{p(f)}{q(f)} \\
= \mathbb{E}_q \log \prod_{m=1}^M \prod_{i=1}^{N_m} p(y_i|x_i, f, \theta) + \mathbb{E}_q \log \prod_{m=1}^M \frac{p^m(f)}{q^m(f)} \\
= \sum_{m=1}^M \left[ \sum_{i=1}^{N_m} \mathbb{E}_q \log p(y_i|x_i, f, \theta) + \mathbb{E}_q \log \frac{p^m(f)}{q^m(f)} \right],
$$

where $p^m(f)$ is equal to the zero-mean GP prior, with covariance function evaluated at the points $\{x_i\}_{i=1}^{N_m}$. The structure of the obtained loss function, similar to [31], contains log-likelihood terms specific to each task and KL divergence terms, which share the same prior distribution, regularizing the variational distribution associated to each task. The expected log-likelihood term can be computed by generating samples from $q^m(f)$, which is a Gaussian distribution itself (7), and evaluating the likelihood $p(y_i|x_i, f, \theta) = N(f, \sigma_y^2)$ at samples of $f$. Furthermore, an analytic form of the KL divergence between two Gaussian distributions is available in terms of mean and variance information. The overall minimization (see Algorithm 1) can be carried out via gradient descent methods.

### 4.3 Meta-testing

During closed-loop control, the basis functions $\phi_i(\cdot)$ learned during meta-training are fixed while the linear parameters $\alpha_i$ are adapted online as new data is collected during operation via Bayesian linear regression. The adaptive MPC then makes use of the current parameter mean (and potentially also variance) information in the prediction horizon, which is updated at every time-step. We consider a typical MPC formulation in which a control input sequence $U = \{u_0, \ldots, u_{N-1}\}$ is minimized with respect to a cost function $J(U, x_0) = \mathbb{E}_w[\sum_{i=0}^N l_k(x_k, u_k)]$, where $N$ is the length of the control horizon, and $l_k : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}$ is a potentially time-varying stage cost function. The optimization problem is subject to the model dynamics and state or input constraints. At each time-step, only the first input of the optimal sequence is applied to the system, i.e. the MPC controller is implemented in a receding-horizon fashion.

**Algorithm 1 Meta Learning MPC**

1. Collect task data $D$ using a nominal controller for tasks $m \sim p(m)$ for meta-training. Fix number of basis functions $E$.
2. Meta-training (offline):
   3. Set the basis functions $\phi_i(\cdot)$, and initialize hyperparameters $\theta$.
   4. Optimize $\theta$ for all tasks in $D$:
      $$\min_{\theta} - \sum_{m=1}^M \left[ \sum_{i=1}^{N_m} \mathbb{E}_q \log p(y_i|x_i, f, \theta) + \mathbb{E}_q \log \frac{p^m(f)}{q^m(f)} \right]$$
3. Meta-testing (online):
   6. Fix hyperparameters $\theta$ from meta-training.
   7. Initialize prior mean and variance of $\alpha^0 = [\alpha^0_1, \ldots, \alpha^0_E]$.
   8. for $t \leftarrow 1$ to $T$ do
      9. Solve MPC problem using current residual model $f^t = \Phi \alpha^t$.
      10. Apply input and obtain new measurement $y^t$.
      11. Update $\alpha^t$ via Bayesian linear regression.
   12. end for
5 Results

In the following, we first present an illustrative example of the well-known mountain-car problem, which is used for describing the mechanisms of the proposed meta-learning approach. As a second example, we consider the control of an autonomous miniature race car, where the goal is to enable adaptation to unseen road conditions via the proposed meta-learning MPC approach. The details of the examples and the implementations are provided in the supplementary.

5.1 Mountain-car problem

The mountain-car problem is typically used as a benchmark for RL, and considers the problem of driving from a valley past the top of a hill. The dynamics are given by [32]:

\[
x_{k+1} = \begin{bmatrix} p_{k+1} \\ v_{k+1} \end{bmatrix} = \begin{bmatrix} p_k + T_s v_k \\ v_k - T_s \cos(3p_k) \theta_1 + T_s u_k \theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ w_k \end{bmatrix},
\]

where \( p \) is the position of the car, \( v \) is the velocity, \( u \) corresponds to the acceleration and the additive disturbance is \( w_k \sim \mathcal{N}(0, 0.001^2) \). The sampling time is \( T_s = 0.2 \text{s} \) and the actuator gain is set as \( \theta_2 = 0.3 \). The residual dynamics to be learned is given by \( y_k = v_{k+1} - v_k - T_s u_k \theta_2 = -T_s \cos(3p_k) \theta_1 + w_k \), and is therefore linear in the parameter \( \theta_1 \). The basis function can be directly defined as \( \phi(p) = -T_s \cos(3p) \), and the parameter determining each task is \( \theta_1 \). Varying parameter \( \theta_1 \) corresponds to changing the slope of the mountain, and in turn defines a task. The training trajectories, depicted in Figure 1a, are generated by a nominal MPC controller with perfect knowledge of the overall dynamics, with \( \theta_1 \in D^{\text{train}}_\theta = \{0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6\} \). The test tasks, depicted in Figure 1b, are similarly generated, with \( \theta_1 \in D^{\text{test}}_{\theta_1} = \{0.65, 0.9, 1.3\} \), and are used as ground truth for comparisons.

We first consider the case of a known basis function \( \tilde{\phi}(p) \), and with hyperparameter represented by the cosine frequency \( \sigma \). Figure 1c shows the associated (negative) ELBO [9], which displays two clear minima at \( \sigma = \pm 3 \), which is expected given symmetry of the cosine.

![Figure 1: Meta-training results for known basis function of the residual.](image)

We then consider \( \tilde{\phi}(p) \) unknown, and carry out the proposed meta-learning MPC scheme. The basis functions chosen for this example are a subset of regressors, which consist in evaluating the kernel function at a finite number of input locations \( E \), so that \( \tilde{\phi}_i(\cdot) = K(\vec{p}^i, \cdot), \ i = 1, \ldots, E \), where \( E = 4 \). The considered kernel is the squared-exponential, so that the overall basis function hyperparameters are \( \theta = \{\lambda, \sigma_w^2, \sigma^2, \{\vec{p}^i\}_{i=1}^E\} \), corresponding to lengthscale, output noise, scaling factor and input locations respectively. In this example, we focus on investigating learning quality rather than the control performance, showing in Figure 2 the residual dynamics learned using previously collected meta-testing data, which is considered as ground truth. Closed-loop adaptation is further investigated in the proposed miniature race car example.
Figure 2: Comparison between ground truth and predicted residual dynamics, for which the mean is highlighted in bold. The shaded area around the mean is bounded by $\pm 2\sigma$ confidence intervals: (left) $\theta_1 = 0.65$, (center) $\theta_1 = 0.9$, (right) $\theta_1 = 1.3$.

5.2 Miniature race car

In this example, we consider the problem of controlling a miniature race car modeled by a kinematic bicycle model [29], to race on a track with different road conditions. Front and rear tire lateral forces, $F_f$ and $F_r$, are modeled using a simplified Pacejka model [23]

$$
F_r = D_r \sin(C_r \arctan(B_r s_r)) \\
F_f = D_f \sin(C_f \arctan(B_f s_f)),
$$

(11)

where $s_r$ and $s_f$ are the slip angles, which depend on the current velocity of the car. The Pacejka coefficients $B_f, C_f, D_f, B_r, C_r,$ and $D_r$ are usually identified from data, can vary over time, and depend on specific road conditions, e.g., driving on a dry or wet surface. In this example, we use meta-learning to learn online the Pacejka front and rear tire models leveraging previous tasks, by approximating (11) as $F_i = \Phi(s_i^{\alpha}, i = r, f$. We collect meta-training data for 7 tasks with different tire models, illustrated in Figure 3, where every data set consists of 200 points.

![Figure 3: Pacejka models for the 7 different meta-training tasks.](image)

We feed the training data to the proposed meta-training algorithm using again a subset of regressors, with $E = 14$. After the meta-training phase, meta-testing is performed in closed-loop using an adaptive formulation of the model predictive contouring controller (MPCC) [17], where the vector of parameters $\alpha = [\alpha_1, \ldots, \alpha_E]$ is adapted online via Bayesian linear regression at each time step, making use of the mean sequential update\footnote{In the case of Gaussian distributions, the posterior mean update obtained via Bayesian linear regression corresponds to the maximum a posteriori (MAP) estimate [2]. Furthermore, it can be shown that the MAP estimate corresponds to a maximum likelihood estimate with Ridge regularization, which justifies the regularizing term in (12).} i.e.,

$$
\mu_{\alpha_{k+1}} = \mu_{\alpha_k} + \eta \left( (y_k - \mu_{\alpha_k}^T \Phi(x_k)) \Phi(x_k) + \sigma_w^2 \mu_{\alpha_k} \right),
$$

(12)

where $\eta = 0.0005$ is the learning rate, and $\sigma_w = 0.02$ is the noise standard deviation.

The meta-testing results are shown in Figure 5a and in Figure 4. We compare our proposed meta-learning MPC (MMPC) against a variation, which we refer to as MMPC-GP, which specifies the variational distribution $q^m(f)$ as an exact GP, and extracts the basis functions a-posteriori, thus
optimizing only the kernel hyperparameters during meta-training. The approaches are compared against Alpaca [11], [12], where the basis functions are designed as neural networks, adapting the last linear layer online. The network weights are meta-trained together with the prior mean and variance over the last layer, by optimizing the posterior predictive distribution. The performance of all methods is comparably good already after the first lap, shown for one noise realization in Figure 4 which is quantified in terms of the cumulative root mean squared error (RMSE) with respect to a ground truth MPCC in Figure 5a. The neural network for Alpaca was chosen trading-off the model complexity against computational requirements for enabling closed-loop control. While the RMSE for Alpaca is only slightly higher, it has significantly more parameters to be tuned: in this example, considering weights and biases of the neural network, these are around \( \sim 100 \) times more than for the proposed approach. MMPC achieves lower RMSE while reducing the number of parameters, which becomes essential for embedded platforms with limited computation power and memory storage. The difference in performance w.r.t. MMPC-GP is minimal and likely due to the lack of input location optimization during meta-training. Furthermore, while this does not pose a real constraint given that meta-training is performed offline, the hyperparameter optimization using basis functions, rather than exact GPs, is consistently faster while providing the same performance in this example. Finally, we tested MMPC with changing grip conditions, where the grip decreases by about 36% in the (orange) second half of the track. In Figure 5b, the resulting trajectory shows fast adaptation to the new grip, and a performance that is very close to the result of the ground truth MPCC controller which has perfect knowledge of the changing grip.

![Figure 4: Counter-clockwise trajectory using a ground truth model predictive contouring controller (dashed red line) compared to MMPC (4a), MMPC-GP (4b), and Alpaca (4c).](image)

![Figure 5: (a) Comparison of cumulative RMSE for 1000 noise realizations along the first lap. (b) Counter-clockwise trajectory using a ground truth MPCC (- - -) compared to MMPC (- - -).](image)

### 6 Conclusion

We presented a meta-learning approach for data-efficient adaptive MPC, making use of linear GP approximations via basis functions. We provide both the ability to generalize by extracting shared information from related task data, and an efficient inference scheme by applying Bayesian linear regression to tailor the linear combination of basis functions online to the task. Results are shown in simulation examples for the mountain-car problem and for a miniature race car adapting to changing road conditions. The presented performance, both in terms of low cumulative errors with respect to a ground truth trajectory, and in terms of computational tractability, motivates future experimental work in this direction.
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