Queue theory for triangular and weibull arrival distribution models (case study of Banyumanik toll)

Sugito¹, Rita Rahmawati², Jenesia Kusuma Wardhani³
¹,²,³ Statistics Department FSM Diponegoro University

E-mail: sugitozafi@undip.ac.id

Abstract. Queuing is one of the most common phenomena in daily life. Queued also happens on highway during busy time. The Electronic Toll Collection (ETC) was the new system of the Banyumanik toll gate which operates in 2014. Before ETC, Banyumanik toll gate users got regular service (regular toll gate) by paying in cash only. The ETC benefits more than regular service, but automatic toll gate (ETC) users are still few compared to regular toll gate users. To know the effectiveness of service substance, this paper used analysis of queuing system. The research was conducted at Toll Gate Banyumanik with the implementation time on 26-28 December 2016 for Ungaran-Semarang direction, and 29-31 December 2016 for Semarang-Ungaran direction. In one day, observation was done for 11 hours. That was at 07.00 a.m. until 06.00 p.m. There are 4 models of queues at Banyumanik toll gate. Here the four models will be used on the number of arrival and service time. Based on the simulation with Arena, the result showed that queue model regular toll gate in Üngaran-Semarang direction is (Tria/G/3):(GD/∞/∞) and the queue model for automatic toll gate is (G/G/3):(GD/∞/∞). While the queue model for the direction of Semarang-Üngaran regular toll gate is (G/G/3):(GD/∞/∞) and the queue model of automatic toll gate is (Weib/G/3):(GD/∞/∞).

1. Introduction
Queuing is one of the phenomena that often occur in everyday life. The situation also occurs in toll roads to be exact at the toll gate, especially during busy times. In this paper the distribution of arrivals will be determined by using data on the number of services whereas the distribution of services will be determined using data on the service time.

PT. Trans Marga Central Java is a subsidiary of PT. Jasa Marga (Persero) Tbk which is established to carry out the Semarang-Solo Toll Road Concession. The Electronic Toll Collection (ETC) system uses the e-toll card which was the new system of the Banyumanik toll gate which began operating in 2014. Previously, Banyumanik toll gateway users only got regular service. They’re paying the toll fee in cash at the toll gate and took longer to complete the transaction. The advantages of ETC are much higher than those of regular systems. When this research is done, there are 2 types of substations used in Banyumanik toll, which is regular and automatic (ETC). And in facts was show that e-toll card users are fewer than regular toll collection users.

In this study there are 12 substations at the toll gate of Banyumanik, among which 6 substations serve the toll collection system on a regular basis and 6 substations serve the electronic toll collection system (ETC). In this research, the queuing system model will be analyzed further to obtain the characteristics of vehicle arrival and toll service, in order to obtain the queuing model and system performance measurement in automatic toll booths and regular substations at the toll gate Banyumanik.
2. Literature Review

Research on queues on tolls has been done by Nugraha [5] under the title "Determination of Vehicle Queue System Model at Banyumanik Toll Gate". The toll substation that operated at that time was 6 substations with 3 regular substations for each, the direction of Ungaran-Semarang and Semarang-Ungaran. The result of the research is the queuing model according to the condition of the service facility at the toll gate Banyumanik both for the direction of Ungaran-Semarang and the direction of Semarang-Ungaran is the model (G / G / c):(GD / ∞ / ∞) [5].

While in this study, there are 12 substations at the toll gate Banyumanik, which were 6 substations serving the system of toll collection on a regular basis and 6 substations serving the electronic toll collection (ETC) system. The general queuing system model will be analyzed further to obtain the characteristics of vehicle arrival and toll service characteristic characteristics, so that the queue model and the system performance measure in automatic toll booths and regular toll booths at toll gate of Banyumanik.

2.1. Queue Theory

The queue theory was proposed and developed by Agner Kraup Erlang from Denmark, in 1910. The discovery occurred when Erlang as an engineer worked on the Copenhagen Telephone Company faced with the classic problem of determining how many sets were needed to provide received telephone service to how many telephone operators which is required to handle a certain volume on the call. According to Siagian [7], the queue is a waiting line from customers requiring service from one or more servants (service facility).

Taha [9] said that the basic elements of the queuing model depend on several important factors, these are arrival distribution, service distribution, service facility, service discipline, queue size (number of costumer in the queue) and calling sources. About Forms of Service Facility, Subagyo [8] said commonly in a queue system, there are four basic structures of queuing models, these are single channel single phase, single channel multiple phase, multiple channel single phase, and multiple channel multiple phase. While according to Kakiay [3] there are four commonly used service disciplines, First Come First Served (FCFS), Last Come First Served (LCFS), Services in Random Order (SIRO), and Priority Service (PS).

2.2. Notation

In grouping different queuing models a notation called Kendall Notation is used. Kendal notation is often used because it is an efficient tool for identifying not only queuing models but also assumptions that must be met [8]. The standard notation used is (a/b/c):(d/e/f), with:

- a = arrival distribution
- b = time distribution of service or departure
- c = number of servants in parallel (where c = 1, 2, 3, ...)
- d = service discipline, such as FCFS, LCFS, SIRO, or PS
- e = the maximum number allowed in the system
- f = number of customers who want to enter the system as a source

2.3. Steady-State Size of Performance

Queues with combined arrivals and start departures under transient conditions and gradually reach steady state after considerable time has elapsed, provided that system parameters allow reaching steady state [9]. Steady state are the conditions when \( \lambda < \mu \) so that \( \rho = \frac{\lambda}{\mu} < 1 \), with \( \lambda \) is the average number of customers arriving per unit of time and \( \mu \) is the average number of services per unit of time. If \( \lambda > \mu \), the number of queues will continue to grow so that if this happens there is something wrong with the queue system [6].
3. Research Methods

According to Gross and Haris [2], in general in the queue process it is assumed that the time between arrival and service time follows the exponential distribution, or equal to the average arrival and average service following the Poisson distribution.

3.1. Distribution Test of Kolmogorov-Smirnov

The Kolmogorov-Smirnov test is designed to test continuous data alignment [1]. In the Kolmogorov-Smirnov test it allows testing of a null hypothesis stating that two samples have been drawn from the same or identical population. The assumption in the Kolmogorov-Smirnov test is that the data consist of free observation results $X_1, X_2, ..., X_n$, which are a random sample of size $n$ of an unknown distribution function and represented by $F(x)$. Kolmogorov-Smirnov test steps are following:

a. Determining the hypothesis
   $H_0$: The sample distribution follows the specified distribution
   $H_1$: The sample distribution does not follow the specified distribution

b. Determining the level of significance
   The significance level used is $\alpha = 5\%$

c. Test Statistics
   \[ D = \text{Sup } | S(x) - F_0(x) | \]
   With are
   $S(x)$ : the cumulative probability function calculated from the sample data
   $F_0(x)$ : the hypothesized distribution function (cumulative probability function)

d. Test Criteria
   Reject $H_0$ at significance level $\alpha$ when $D > D^*(1-\alpha)$. The $D^*(1-\alpha)$ is the critical value obtained from the Kolmogorov-Smirnov table.

3.2. Queue Models

There are two queuing models that used in this paper.

3.2.1. Queue Model (M/M/c):(GD/$\infty$/ $\infty$). Taha [9] said, on the model (M/M/c):(GD/$\infty$/ $\infty$), the customer arrives at a constant rate, not more than $c$ customers can be served at the same time, and the service rate per service is constant. The probability for $n$ customers can be written as follows:

\[
p_n = \begin{cases} 
\frac{\lambda^n}{n!\mu^n} p_0, & 0 \leq n \leq c \\
\frac{\lambda^n}{c!c^{n-c} \mu^n} p_0, & n > c 
\end{cases}
\]

\[
p_0 = \left(\sum_{n=0}^{c} \frac{\rho^n}{m^n} + \frac{\rho^c}{c! (1-\frac{\rho}{c})} \right)^{-1}, \quad \frac{\rho}{c} < 1
\]

The formula for finding performance measures on the queue model (M/M/c):(GD/$\infty$/ $\infty$) is:

\[
L_q = \frac{\rho^{c+1}}{(c-1)! (c-\rho)^2} p_0 = \left( \frac{c \rho}{(c-\rho)^2} \right) p_0
\]

\[
L_s = L_q + \rho
\]

\[
W_q = \frac{L_q}{\lambda}
\]

\[
W_s = W_q + \frac{1}{\mu}
\]

3.2.2. Queue Model (G/G/c):(GD/$\infty$/ $\infty$). Gross and Harris [2] said, the queue model (G/G/c):(GD/$\infty$/ $\infty$) is a queue model with general distributed arrival pattern and general service pattern with $c$ number
of service facilities, \(c = 1, 2, 3, \ldots\). The queuing discipline used in this model is generalized as FCFS (First Come First Service), the maximum capacity allowed in the system is infinite, and has an unlimited calling source. The formula for finding performance measures in the model \((G/G/c):(GD/\infty/\infty)\) is:

\[
L_q = \frac{\rho^{c+1}}{(c-1)(c-\rho)^2}p_0 \frac{\mu^2 \nu(t) + \nu(t') \lambda^2}{2} = L_q M/M/c \frac{\mu^2 \nu(t) + \nu(t') \lambda^2}{2}
\]

where
\(\nu(t)\) is variance of service time
\(\nu(t')\) is variance of time between arrivals

\[
L_s = L_q + \rho
\]

\[
W_q = \frac{L_q}{\lambda}
\]

\[
W_s = W_q + \frac{1}{\mu}
\]

### 3.3. Triangular Distribution

Triangular Distribution is a continuous distribution with minimum and maximum fixed values and frequent events (mode). The most likely values lie between the minimum and maximum values form a triangular distribution [4]. Triangular distribution density function with minimum parameter values (a), mode (m), and maximum value (b) are:

\[
f(x) = \begin{cases} 
\frac{2(x-a)}{(m-a)(b-a)}, & a \leq x \leq m \\
\frac{2(b-x)}{(b-m)(b-a)}, & m \leq x \leq b \\
0, & \text{others}
\end{cases}
\]

The mean and variance for the Triangular distribution is:

\[
\mu = \frac{a + m + b}{3} \quad \sigma^2 = \frac{a^2 + m^2 + b^2 - ma - ab - mb}{18}
\]

### 3.4. Weibull Distribution

Weibull distribution is often used in handling issues such as reliability and age test [10]. The Weibull distribution density function with parameters \(\alpha\) and \(\beta\) is:

\[
f(t) = \alpha \beta t^{\beta-1}e^{-\alpha t^\beta}, \quad t > 0
\]

where
\(\alpha > 0\) and \(\beta > 0\).

While the mean and variance in the Weibull distribution is:

\[
\mu = \alpha^{-1/\beta} \Gamma \left(1 + \frac{1}{\beta}\right) \quad \sigma^2 = \alpha^{-1/\beta} \left\{ \Gamma \left(1 + \frac{2}{\beta}\right) - \left[ \Gamma \left(1 + \frac{1}{\beta}\right) \right]^2 \right\}
\]

### 3.5. System Simulation

Simulation is the duplication or abstraction of real life problems into mathematical models. In this case is usually done simplification, so the solution with mathematical models can be done [8]. Troubleshooting with simulations is usually done using a computer, because many things are too
complicated if calculated manually. One of the software that can be used to simulate the queue is Arena.

3.6. Stages of Data Analysis
The research was conducted at Banyumanik toll gate in December 2016. The software used is XNote Stopwatch, Microsoft Excel, SPSS 22, WinQSB, and Arena. Observed data are vehicle arrival time, service time of vehicle starts, and service time of vehicle ends. The data obtained are then organized based on the specified intervals. Stages of research and data analysis are as follows:
1. Doing research directly to get data of arrival amount and service time data in specified time unit.
2. The data obtained must meet steady-state conditions. If steady-state conditions have not been met then the repair service system can be done by adding a substation service tailored to the situation and conditions at the toll gate.
3. Conducting the distribution alignment test to determine the distribution of arrival number and service quantity by using Kolmogorov-Smirnov test. In this case, if the hypothesis is accepted, it can be concluded that the Poisson distributed data, if the hypothesis is rejected then the data is considered to follow the general distribution.
4. Determine the characteristics and model of the queue system accordingly.
5. General arrival and service distributions are tested to determine their distribution through output in the Arena. Then do the simulation with Arena.
6. Test the distribution alignment that has been obtained from the Arena
7. Determine the size of system performance, Lq, Ls, Wq, and Ws.
8. Creating results and discussion obtained from the size of system performance, so as to obtain an optimal model.

4. Result and Discussion
The results of the analysis and discussion on this research are as follows:

4.1. Overview of Queue System
Semarang-Ungaran toll road is a highway connecting Semarang City with Ungaran area, which is part of the Semarang-Solo toll road series. The toll gate of Banyumanik is located in Banyumanik area. When this study was conducted there are 12 substations at the toll gate Banyumanik, including six substations service for the direction Ungaran-Semarang and six relay stations for the direction of Semarang-Ungaran. In each direction there are three substations serving the toll collection system in cash (regular) and three substations serving the electronic toll collection (ETC) system.

4.2. Descriptive Analysis of Vehicle Traffic
The total number of vehicles per day for the direction of Ungaran-Semarang and Semarang-Ungaran direction from 07.00 a.m. until 06.00 p.m. can be seen in Table 1.

| Day     | Ungaran-Semarang | Day     | Semarang-Ungaran |
|---------|------------------|---------|------------------|
|         | Regular | Automatic |         | Regular | Automatic |
| Monday  | 9064     | 3939     | Thursday | 9864     | 4145     |
| Tuesday | 8351     | 3086     | Friday   | 9747     | 4440     |
| Wednesday | 8434    | 2923     | Saturday | 11044    | 5699     |

4.3. Steady State Size Analysis of Service System Performance
The value of utility service level for each substation is less than 1 (see Table 2) which means the average arrival time of the vehicle does not exceed the average service time given, so it can be said that the queue system meets steady-state conditions, with the average number of arrivals not exceeding
the number of services then all vehicles that come can be served by substations that exist. This means the service system is good and can be calculated queue system performance.

| Data                | Substation                                           | P         |
|---------------------|------------------------------------------------------|-----------|
| Amount – Time       | Ungaran-Semarang Regular Substation                  | 0.27193863|
|                     | Ungaran-Semarang Automatic Toll Substation           | 0.084002  |
|                     | Semarang-Ungaran Regular Substation                  | 0.34456848|
|                     | Semarang-Ungaran Automatic Toll Substation           | 0.11139769|

4.4. Goodness of Fit

In the goodness of fit (distribution test) will be determined the assumption that the number of arrivals and the number of service vehicles at the toll gate Banyumanik follow the Poisson or Exponential distribution. From the match fit test obtained the test statistic value as Table 3. From the table it can be concluded that the queue model in automatic toll booths and regular toll booths at the Banyumanik toll gate is (G/G/3):(GD/∞/∞).

| Substation   | Data                | Sig.   | Decision          |
|--------------|---------------------|--------|-------------------|
| Ungaran-Semarang Regular | The number of arrivals | 0.000  | H0 is rejected    |
|              | Service time        | 0.000  | H0 is rejected    |
| Ungaran-Semarang Automatic | The number of arrivals | 0.000  | H0 is rejected    |
|              | Service time        | 0.000  | H0 is rejected    |
| Semarang-Ungaran Regular | The number of arrivals | 0.002  | H0 is rejected    |
|              | Service time        | 0.003  | H0 is rejected    |
| Semarang-Ungaran Automatic | The number of arrivals | 0.000  | H0 is rejected    |
|              | Service time        | 0.000  | H0 is rejected    |

4.5. Queue System Model

Before calculating the system performance measure, firstly testing the distribution of the number of arrivals and the number of service vehicles to determine the actual distribution based on Arena output.

| Substation   | Data                | Distribution | Sig.   | Decision          |
|--------------|---------------------|--------------|--------|-------------------|
| Ungaran-Semarang Regular | The number of arrivals | Triangular  | 0.341  | H0 is accepted    |
|              | Service time        | Lognormal    | <0.005 | H0 is rejected    |
| Ungaran-Semarang Automatic | The number of arrivals | Weibull     | 0.0294 | H0 is rejected    |
|              | Service time        | Exponential  | <0.005 | H0 is rejected    |
| Semarang-Ungaran Regular | The number of arrivals | Normal      | <0.005 | H0 is rejected    |
|              | Service time        | Lognormal    | <0.005 | H0 is rejected    |
| Semarang-Ungaran Automatic | The number of arrivals | Weibull     | 0.368  | H0 is accepted    |
|              | Service time        | Weibull      | <0.005 | H0 is rejected    |

Based on Table 4, the final model for queue model at Banyumanik toll gate for the direction of Ungaran-Semarang regular substation is (Tria/G/3):(GD/∞/∞) and queuing model
for automatic toll substation is \((G/G/3):(GD/\infty/\infty)\). While the queue model for the direction of Semarang-Ungaran regular substation is \((G/G/3):(GD/\infty/\infty)\) and the queue model in automatic toll substation is \((\text{Weib}/G/3):(GD/\infty/\infty)\).

4.6. System Performance Measures

Based on the output obtained by using WINQSB software, it is known that the queue system performance measure is presented in the following table:

| Substation        | \(c\) | \(\Lambda\) | \(\mu\) | \(L_s\)  | \(L_q\)  | \(W_s\) | \(W_q\) | \(P_0\)  |
|-------------------|-------|-------------|---------|---------|---------|---------|---------|---------|
| Ungaran-Semarang  | 3     | 195.826     | 240.027 | 73.044  | 72.228  | 0.373   | 0.369   | 0.43995 |
| Regular           |       |             |         |         |         |         |         |         |
| Automatic         | 3     | 75.364      | 299.025 | 0.743   | 0.491   | 0.0099  | 0.0065  | 0.77718 |
| Semarang-Ungaran  | 3     | 232.235     | 224.662 | 155.691 | 154.657 | 0.6704  | 0.666   | 0.35101 |
| Regular           |       |             |         |         |         |         |         |         |
| Automatic         | 3     | 108.212     | 323.801 | 2.6813  | 2.3471  | 0.0248  | 0.0217  | 0.7158  |

From Table 5, it is shown that for the direct Ungaran-Semarang, regular substation, the number of service substations provided to serve the vehicle \((c)\) is 3 with an average vehicle arrival rate every 15 minutes \((\Lambda)\) is 195.826. Average service rate of vehicles \((\mu)\) is 240.027. The estimated number of vehicles in the system \((L_s)\) and in each queue \((L_q)\) are 73.044 and 72.228, while the expected wait time in the system \((W_s)\) and in the queues \((W_q)\) are 0.373 and 0.369 respectively. Services probability of empty time (idle) is 0.43995. For other substations it can be interpreted in the same way.

5. Conclusion

The final model for the queue model at the toll gate of Banyumanik with the arrival and service time data for the direction of Ungaran-Semarang regular substation is \((\text{Tria}/G/3):(GD/\infty/\infty)\) and the queue model for automatic toll substation is \((G/G/3):(GD/\infty/\infty)\) and the queue model in automatic toll substation is \((\text{Weib}/G/3):(GD/\infty/\infty)\).

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