Symmetry between time and space in a quantum field

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Abstract. Despite nature’s preference for symmetries, the treatment of time and space in quantum theory is not symmetrical. To restore the symmetry, we introduce an additional degree of freedom, allowing matter to vibrate in the temporal direction. We find that a system with matter vibrating in time has the same basic properties of a zero-spin bosonic field. In addition, the internal time of this system can be represented by a self-adjoint operator despite the fact that the Hamiltonian is bounded from below. The properties of a zero-spin bosonic field can be reconciled from a field with vibrations of matter in time.

1. Introduction

The dynamical nature of spacetime, as treated in general relativity, is a direct conflict with the non-dynamical way in which time and space are treated separately in quantum physics [1,2]. For example, the quantum theory postulates an independent time parameter with respect to which the dynamics unfold. There is nothing dynamical about time in the theory. On the other hand, spacetime is mingled together as unity in general relativity. Space and time are to be treated on the same footing. Therefore, how time is treated in quantum theory and general relativity is conceptually quite different.

The use of operators lies at the heart of quantum physics. Physical observables such as position, momentum and energy are represented by operators in the formulation of quantum theory. However, any attempt to treat time as an operator will encounter the problem suggested by Pauli [3]. Based on his reasoning, a time operator canonically conjugates to a semi-bounded or discrete Hamiltonian, cannot be treated as a self-adjoint operator that has a spectrum spanning the entire real line. In order to circumvent this difficulty, it has been advocated in many studies [4–9] that a time operator should be generalized, by allowing not only self-adjoint operators, but also by the use of non-self-adjoint operators, e.g. positive operator valued measures. Apart from these efforts, many different approaches have been proposed, intending to resolve the dynamical nature of time in quantum physics [10–18]. The asymmetrical way in which time and space are treated in quantum theory has been a continuing source of inspiration for the quest of a time operator.

In refs. [19–21], we demonstrate a possibility that the amplitude of a matter wave can be taken as a 4-vector, with vibrations in time and space. The vibration of matter in time is an additional degree of freedom introduced to restore the symmetry between time and space in a matter field. The real scalar field describing this system obeys the Klein-Gordon equation and Schrödinger equation. The energy in this system must be quantized under the constraint that a particle’s mass is on shell. In addition, the internal time of this system can be represented by a self-adjoint operator. The spectrum of this operator spans the entire real line without
contradicting Pauli’s theorem. The properties of a bosonic field can be reconciled from this field with vibrations of matter in space and time.

In this paper, we consider an alternate approach that matter can have temporal vibrations, but without the correlated spatial vibrations as proposed in the aforementioned references. We show that the same properties of the bosonic field can be attained. Under this assumption, the temporal vibration is a ‘pure’ intrinsic property of matter. It has effects on the rate of change for the particle’s intrinsic properties. However, there is no variation in speed for a propagating particle as we have proposed previously. Since both approaches can give us the same properties of a bosonic field, we have to rely on future experiments to determine which approach is the preference of nature. This will be discussed further in the last section.

2. Harmonic oscillator in time

Nature in general has a preference for symmetry [22, 23]. If matter can have oscillation in the spatial directions, is it possible that matter can also have oscillation in the temporal direction? More importantly, can the properties of this temporal oscillator have something to do with our real physical world? In this paper, we will study a system containing oscillation of matter in time. This oscillation in time is an additional degree of freedom introduced, to restore the symmetry of time and space in a matter field. In the following analysis, we will work in the natural units whereby $c = \hbar = 1$.

Let us assume that an oscillator in time is observed at the origin of the $x$ coordinates in a flat space-time background coordinate system $(t, x)$,

$$t_f = t - T_0 \sin(\omega_0 t).$$

(1)

Time, in this background coordinate system, is the ‘external time’ [24–27] as measured by clocks that are not coupled to the oscillator under investigation. The coordinate system adopted here is observed in an inertial frame by an observer stationary with respect to and very far from the oscillator, i.e. far enough that the effects of the oscillator are negligible. As we shall note, the external time is an independent variable in the equations of motion and a parameter used as adopted in quantum theory. There is nothing dynamical about its nature. We will use this background time as a reference for measuring the oscillation that takes place.

In analogous to the amplitude $X$ for a classical oscillator with oscillation in the spatial directions, we define the proper time amplitude $T_0$ as the maximum difference between the ‘internal time’ $t_f$ of matter in the oscillator and the ‘external time’ $t$. If the matter in the oscillator carries a clock measuring its internal time, this internal clock will be running at a varying rate, relative to the inertial observer’s clock,

$$\frac{\partial t_f}{\partial t} = 1 - \omega_0 T_0 \cos(\omega_0 t).$$

(2)

Therefore, the internal clock of matter will measure a different time $t_f$ relative to the external time $t$. The internal time is a function of the external time $t$ and a dynamical variable for the system.

The oscillator is assumed to have no oscillation in the spatial directions; the temporal oscillation is an intrinsic property of matter. As we shall recall, the total potential energy and kinetic energy of an oscillator with amplitude $X$ in the classical theory is $E = m\omega_0^2|X|^2/2$. Analogous to the classical system, we make an ansatz that the total energy of the oscillator in time is,

$$E = m\omega_0^2 T_0^2.$$ 

(3)

Since the oscillation in time is an intrinsic property of matter, we expect that the energy resulted from the temporal oscillation of mass $m$ is also an intrinsic property of matter.
As shown in Eq. (3), there are no force fields and charges included in the energy equation for the oscillator in time. The oscillator involves only the oscillation of matter with mass $m$. As a result, the only energy present in this system is the internal energy of matter. Here, we will consider that the energy which arises from the oscillation in time is the internal energy of mass $m$, i.e. $E = m$.

The internal mass-energy of mass $m$ must be on shell. Eq. (3) then becomes,

$$\omega_0^2 T_0^2 = 1.$$  \hspace{1cm} (4)

This implies that an oscillator in time can only have one unique proper time amplitude which we will call it $T_0$, i.e.

$$T_0 = 1/\omega_0,$$  \hspace{1cm} (5)

Unlike the case for an oscillator in the classical theory, the energy of an oscillator in time must be quantized under the constraint that mass is on shell\(^1\). Therefore, only the energy of the integer number of temporal oscillators can be observed.

The energy must be quantized in a system with oscillations of matter in time. This is a property we will expect for a quantum field. In fact, if this temporal oscillator has something to do with quantum physics, we can, in theory, define an annihilation operator,

$$a = \omega_0 T_0,$$  \hspace{1cm} (6)

and a creation operator,

$$a^\dagger = \omega_0 T_0^\dagger,$$  \hspace{1cm} (7)

such that the number operator for a multiple oscillators system is,

$$N = \omega_0^2 T_0^\dagger T_0 = a^\dagger a.$$  \hspace{1cm} (8)

These operators are postulated based on the concepts developed for a bosonic field; they are different from those defined for the quantum oscillator with oscillation in the spatial directions. The state $|n\rangle$ of the temporal oscillator shall be a state with $n$ particles and not the state which corresponds to a particle with energy level $E_n = \omega(n + 1/2)$. In the following sections, we will show that a field with temporal oscillators has the same properties of a bosonic field. Since we are considering the energy of a quantum particle, $\omega_0$ can be taken as the de Broglie’s frequency for the mass-energy of a particle.

### 3. Matter field with vibrations in time

Let us consider a real scalar field in a cubic box with volume $V$ that can have multiple particles with mass $m$ vibrating in time,

$$\varphi(\vec{x}) = \sum_k (2V\omega)^{-1/2} (\omega_0 T_{0k} e^{-i\vec{k} \cdot \vec{x}} + \omega_0 T_{0k}^* e^{i\vec{k} \cdot \vec{x}}),$$  \hspace{1cm} (9)

which satisfies the Klein-Gordon equation,

$$\partial_a \partial^a \varphi(\vec{x}) + \omega_0^2 \varphi(\vec{x}) = 0.$$  \hspace{1cm} (10)

\(^1\) Note that by making the ansatz as stated, Eq. (3) has the appearance of the Einstein’s equation after we rewrite it as $E = mc^2 \omega_0^2 T_0^2 = mc^2$. In relativity, the speed of light $c$ can be interpreted as the time component of the 4-velocity for a rest mass. Here, $c\omega_0 T_0 = c$ is the maximum oscillating time rate of the harmonic oscillator, i.e. $\max |c\partial t_f/\partial t - c| = \max |c\omega_0 T_0 \cos(\omega_0 t)| = c$ from Eqs. (2) and (4). With the approach we have taken in this paper, the Einstein’s equation can be conceived as the energy equation of a temporal oscillator.
Periodic boundary conditions are to be imposed at the box walls. \( T_{0k} \) is a complex proper time amplitude analogous to the one defined in the previous section. It is measured in the rest frame of the observed particle with \( \omega_0^2 T_{0k}^* T_{0k} \) as a Lorentz scalar.

Before we quantize this field, we will first consider its properties as a classical field. In other words, \( \varphi(\vec{x}) \) and \( T_{0k} \) are to be taken as functions and not operators, for a beginning. The Lagrangian density and Hamiltonian density for this real scalar field are,

\[
\mathcal{L} = \frac{1}{2}[(\partial_\nu \varphi)^2 - (\nabla \varphi)^2 - \omega_0^2 \varphi^2],
\]

\[
\mathcal{H} = \frac{1}{2}[(\partial_\nu \varphi)^2 + (\nabla \varphi)^2 + \omega_0^2 \varphi^2].
\]

The conjugate momentum of \( \varphi(\vec{x}) \) is,

\[
\pi(\vec{x}) = \frac{\partial \mathcal{L}}{\partial [\partial_\nu \varphi(\vec{x})]} = \partial_\nu \varphi(\vec{x}) = -i \sum_k \omega_0 \sqrt{\frac{\omega}{2V}} [T_{0k} e^{-i\vec{k} \cdot \vec{x}} - T_{0k}^* e^{i\vec{k} \cdot \vec{x}}].
\]

From Eq. (9), the real scalar field \( \varphi(x) \) is the superposition of plane waves,

\[
\varphi_{nk}^+(\vec{x}) = \frac{\omega_0 T_{0k}}{\sqrt{V}} e^{-i\vec{k} \cdot \vec{x}},
\]

and their conjugates

\[
\varphi_{nk}^-(\vec{x}) = \frac{\omega_0 T_{0k}^*}{\sqrt{V}} e^{i\vec{k} \cdot \vec{x}},
\]

such that

\[
\varphi(\vec{x}) = \frac{1}{\sqrt{2}} \sum_k [\varphi_{nk}^+(\vec{x}) + \varphi_{nk}^-(\vec{x})].
\]

Based on Eq. (12), the Hamiltonian density for the plane wave \( \varphi_{nk}^+(\vec{x}) \) is,

\[
\mathcal{H}_{nk}^\pm = \frac{1}{2}[(\partial_\nu \varphi_{nk}^\pm)^* (\partial_\nu \varphi_{nk}^\pm) + (\nabla \varphi_{nk}^\pm)^* \cdot (\nabla \varphi_{nk}^\pm) + \omega_0^2 (\varphi_{nk}^\pm)^* (\varphi_{nk}^\pm)] = \frac{m \omega_0^2 T_{nk}^* T_{nk}}{V},
\]

where

\[
T_{nk} = \sqrt{\frac{\omega}{\omega_0}} T_{0k}.
\]

This result is obtained after we put the first de Broglie’s frequency as \( \omega_0 = m \) in Eq. (17).

\( \varphi_{nk}^+(\vec{x}) \) is a plane wave with vibrations of matter in time. From Eq. (17), the energy contained inside volume \( V \) is \( E = m \omega_0^2 T_{nk}^* T_{nk} \). Analogous to Eq. (3), \( E \) is the energy of an oscillating system with amplitude \( T_{nk} \). To describe these vibrations, let us define a temporal vibration plane wave \( \zeta^+_{nk}(\vec{x})) \) and its conjugate \( \zeta^-_{nk}(\vec{x}) \) as,

\[
\zeta^+_{nk}(\vec{x}) = -iT_{nk} e^{-i\vec{k} \cdot \vec{x}},
\]

\[
\zeta^-_{nk}(\vec{x}) = iT_{nk}^* e^{i\vec{k} \cdot \vec{x}}.
\]

Both of these plane waves have the same magnitude of amplitude \( |T_{nk}| \). They are analogous to the classical waves, except that the vibrations are in the temporal direction and not in the spatial directions.

A temporal vibration field \( \zeta(\vec{x}) \) can be written as a superposition of the plane waves \( \zeta^+_{nk}(\vec{x}) \) and their conjugates \( \zeta^-_{nk}(\vec{x}) \), i.e.

\[
\zeta(\vec{x}) = \frac{1}{\sqrt{2}} \sum_k [\zeta^+_{nk}(\vec{x}) + \zeta^-_{nk}(\vec{x})] = \sum_k -\frac{i}{\sqrt{2}} [T_{nk} e^{-i\vec{k} \cdot \vec{x}} - T_{nk}^* e^{i\vec{k} \cdot \vec{x}}].
\]
From Eqs. (13) and (21), $\zeta_t(\vec{x})$ can be expressed in terms of the conjugate momentum $\pi(\vec{x})$:

$$\zeta_t(\vec{x}) = \pi(\vec{x}) = \frac{\omega_0}{\bar{\rho}_m/2} \frac{\partial_0 \varphi(\vec{x})}{\omega_0^{1/2}},$$

(22)

where

$$\bar{\rho}_m = \frac{m}{V},$$

(23)

is a constant mass density of the system. Similarly, we can express the temporal vibration plane wave $\zeta_{nk}(\vec{x})$ in terms of the plane wave $\varphi_{nk}(\vec{x})$:

$$\zeta_{nk}(\vec{x}) = \frac{\partial_0 \varphi_{nk}(\vec{x})}{\omega_0^{1/2}}.$$

(24)

As shown, $\varphi(\vec{x})$ is a real scalar field that has matter vibrating in time. These vibrations are described by the temporal vibration field $\zeta_t(\vec{x})$ that can be obtained from Eq. (22).

4. Particle with vibration in time

Let us consider a plane wave $\varphi_{n0}(\vec{x})$ with $\omega = \omega_0$ and $|k| = 0$:

$$\varphi_{n0}(\vec{x}) = \sqrt{\omega_0/V} T_0 e^{-i\omega_0 t}.$$

(25)

From Eq. (17), its Hamiltonian density is,

$$\mathcal{H}_{n0} = \frac{m \omega_0^2 T_0^* T_0}{V}.$$

(26)

The energy contained inside volume $V$ is $E = m \omega_0^2 T_0^* T_0$. As discussed in Section 2, this is the energy of a simple harmonic oscillating system in proper time. A particle observed in this plane wave has a unique proper time amplitude of $\bar{\varphi_{n0}} = 1/\omega_0$. Only the integer number of particles are observable. As a result, the energy in this plane wave must be quantized. The system as a whole shall be treated as a quantum field.

From Eq. (24), the temporal vibration plane wave $\zeta_{n0t}(\vec{x})$ corresponding to $\varphi_{n0}(\vec{x})$ is:

$$\zeta_{n0t}(\vec{x}) = -i T_0 e^{-i\omega_0 t}.$$

(27)

We will define the temporal vibrations of matter as the real component of this complex plane wave. Analogous to Eq. (1), the internal time observed is,

$$t_{0f}^t(\vec{x}) = t + \text{Re}[\zeta_{n0t}(\vec{x})] = t - T_0 \sin(\omega_0 t).$$

(28)

For a plane wave with $T_0 = 1/\omega_0$, the Hamiltonian density from Eq. (26) is $\mathcal{H}_{n0} = \omega_0/V$. This is equivalent to one particle with energy $\omega_0$ in volume $V$. The plane wave is normalized and the particle observed has an amplitude of $\bar{\varphi_{n0}} = 1/\omega_0$. The internal time of the particle observed is,

$$t_{0f}^t = t - \frac{\sin(\omega_0 t)}{\omega_0}.$$

(29)

This is the same internal time of the oscillator discussed in Section 2. The particle observed in this plane wave is at rest relative to the cubic box.
The internal time rate relative to the external time for this particle is,

\[ \frac{\partial \hat{t}_{nf}^+}{\partial t} = 1 - \cos(\omega_0 t). \]  (30)

The particle will appear to travel along a time-like geodesic when averaged over many cycles. In addition, the internal time rate is bounded between 0 and 2. The particle will never travel backward in time. However, in order to detect the particle’s oscillation in time, the inertial observer’s clock has to be sensitive enough to detect the high frequency and small amplitude of the vibration. Its accuracy shall be dictated by the time-energy uncertainty relation [28–32].

Comparing Eqs. (14) and (15), the plane wave \( \varphi^+_{n0}(\vec{x}) \) with a particle traveling forward in time is mathematically equivalent to the plane wave \( \varphi^-_{n0}(\vec{x}) \) with a particle traveling backward in time - time reversal symmetry [33]. The oscillator observed in \( \varphi^-_{n0}(\vec{x}) \) is an anti-particle. Its internal time and internal time rate are,

\[ \hat{t}_{nf}^- = -t + \frac{\sin(\omega_0 t)}{\omega_0}, \]  (31)

\[ \frac{\partial \hat{t}_{nf}^-}{\partial t} = -1 + \cos(\omega_0 t). \]  (32)

Therefore, the internal time rate of this anti-particle is bounded between 0 and -2. Its average rate is -1. The anti-particle will appear to travel along a time-like geodesic when averaged over many cycles.

Next, let us consider the plane wave \( \varphi^+_{nk}(\vec{x}) \) from Eq. (14) and its corresponding temporal vibration plane wave \( \zeta^+_{nk}(\vec{x}) \) from Eq. (19). For a normalized plane wave with amplitude \( |T_{nk}| = 1/\omega_0 \), the temporal vibration plane wave \( \zeta^+_{nk}(\vec{x}) \) is,

\[ \zeta^+_{nk}(\vec{x}) = -i \sqrt{\frac{\omega}{\omega_0}} e^{-i\vec{k} \cdot \vec{x}}. \]  (33)

Its Hamiltonian density from Eq. (17) is,

\[ \mathcal{H}^+_{nk} = \frac{m\omega_0^2 T^*_{nk} T_{nk}}{V} = \frac{\omega}{V}. \]  (34)

This is equivalent to one particle with energy \( \omega \) and momentum \( \vec{k} \) in a volume \( V \). The particle observed will be traveling with a velocity \( \vec{v} = \vec{k}/\omega \).

The internal time observed in the normalized temporal vibration plane wave \( \zeta^+_{nk}(\vec{x}) \) is,

\[ t_{nf}^+(t) = t + \text{Re}[\zeta^+_{nk}(\vec{x})] = t - \sqrt{\frac{\omega}{\omega_0}} \sin(\omega_0 t - k \cdot \vec{x}). \]  (35)

Assuming a particle is first observed at the origin of the \( \vec{x} \) coordinates at \( t = 0 \), the internal time of this particle’s clock traveling along a trajectory \( \vec{x} = \vec{v} t \) is,

\[ t_{nf}^+(t) = t - T_n \sin(\omega_0 t), \]  (36)

where

\[ T_n = \sqrt{\frac{\omega}{\omega_0}}, \]  (37)
\[ \omega_p = \frac{\omega_0^2}{\omega}. \]  

(38)

Note that \( \omega_p \) is the angular frequency of a moving particle and not the angular frequency \( \omega \) of the plane wave.

To illustrate the magnitude of this temporal oscillation, let us consider the frequency and amplitude of a neutrino with an assumed mass of \( m = 0.07 \text{eV} \) \( (\omega_0 = 1.06 \times 10^{14} \text{s}^{-1} \) and \( |T_0| = 9.4 \times 10^{-15} \text{s}) \) [34],

\[ E = 1 \text{MeV} \Rightarrow |\hat{T}_n| = 3.5 \times 10^{-11} \text{s}, \quad \omega_p = 7.4 \times 10^6 \text{s}^{-1}, \]  

(39)

\[ E = 1 \text{Gev} \Rightarrow |\hat{T}_n| = 1.1 \times 10^{-9} \text{s}, \quad \omega_p = 7.4 \times 10^3 \text{s}^{-1}. \]  

(40)

Therefore, a neutrino traveling at a higher speed will have a lower frequency and larger amplitude of oscillation, which may facilitate the detection of these effects. Owing to its extremely light weight, a neutrino can be projected to a higher speed easier than a heavier particle. The study of neutrino in high energy experiments may provide useful information for our future investigations.

5. Internal time in a bosonic field

The energy in the plane wave \( \varphi^\pm_{nk} \) is quantized under the constraint that mass is on shell. As the superposition of these plane waves, the energy in the real scalar field \( \varphi(\vec{x}) \) must also be quantized. A particle observed in this real scalar field can only have an unique amplitude \( |\hat{T}_0| = 1/\omega_0 \). As a result, \( \varphi(\vec{x}) \) has no classical analogy. It must be treated as a quantum field. Following the same concepts developed in quantum theory, the transition of the field with temporal vibrations to a quantum field can be done via canonical quantization. In other words, \( \varphi(\vec{x}) \) and \( T_{0k} \) shall be treated as operators.

Analogous to the creation and annihilation operators defined in Section 2 for the harmonic oscillator in proper time, we can also adopt a set of similar operators for the real scalar field \( \varphi(\vec{x}) \), i.e.

\[ a_k = \omega_0 T_{0k}, \]  

(41)

\[ a_k^\dagger = \omega_0 T_{0k}^\dagger. \]  

(42)

The operators \( a_k, a_k^\dagger, T_{0k} \) and \( T_{0k}^\dagger \) shall satisfy the commutation relations,

\[ [a_k, a_{k'}^\dagger] = \delta_{kk'}, \]  

(43)

\[ [a_k, a_{k'}] = [a_{k}^\dagger, a_{k'}^\dagger] = 0, \]  

(44)

\[ [T_{0k}, T_{0k'}^\dagger] = \frac{\delta_{kk'}}{\omega_0^2}, \]  

(45)

\[ [T_{0k}, T_{0k'}] = [T_{0k}^\dagger, T_{0k'}^\dagger] = 0. \]  

(46)

As we shall note, \( \varphi(\vec{x}) \) is the same bosonic field from quantum theory after we rewrite Eq. (9) in terms of the creation operator \( a_k^\dagger \) and the annihilation operator \( a_k \), i.e.

\[ \varphi(\vec{x}) = \sum_k (2\omega V)^{-1/2}[a_k e^{-i\vec{k}\cdot\vec{x}} + a_k^\dagger e^{i\vec{k}\cdot\vec{x}}]. \]  

(47)

In fact, we can express other operators in a bosonic field by rewriting them in terms of \( T_{0k} \) and \( T_{0k}^\dagger \). For example, the conjugate momentum field is,

\[ \pi(\vec{x}) = -i \sum_k \sqrt{\frac{\omega}{2V}} [\omega_0 T_{0k} e^{-i\vec{k}\cdot\vec{x}} - \omega_0 T_{0k}^\dagger e^{i\vec{k}\cdot\vec{x}}]. \]  

(48)
The particle number operator is
\[ N_k = \omega_0^2 T_{0k}^\dagger T_{0k}, \] (49)
and the Hamiltonian is,
\[ H = \sum_k \omega(\omega_0^2 T_{0k}^\dagger T_{0k} + \frac{1}{2}). \] (50)

The real scalar field with vibrations of matter in time has the same basic properties of a zero-spin bosonic field.

The temporal vibration field operator \( \zeta_t(\vec{x}) \) is,
\[ \zeta_t(\vec{x}) = \sum_k -\frac{i}{\sqrt{2}}[T_{nk} e^{-ik\cdot\vec{x}} - T_{nk}^\dagger e^{ik\cdot\vec{x}}], \] (51)
where the temporal vibration amplitude \( T_{nk} \) and its Hermitian conjugate \( T_{nk}^\dagger \) shall satisfy commutation relations,
\[ [T_{nk}, T_{nk}^\dagger] = \frac{\omega}{\omega_0}\delta_{kk'}, \] (52)
\[ [T_{nk}, T_{nk'}] = [T_{nk}^\dagger, T_{nk'}^\dagger] = 0. \] (53)

From Eq. (22), \( \zeta_t(\vec{x}) \) has a linear expression in terms of \( \pi(\vec{x}) \). We can also use \( \pi(\vec{x}) \) to describe the temporal vibrations. Since \( \pi(\vec{x}) \) is a self-adjoint operator, \( \zeta_t(\vec{x}) \) is also a self-adjoint operator. \( \varphi(\vec{x}) \) and \( \zeta_t(\vec{x}) \) shall satisfy the equal-time commutation relations:
\[ [\varphi(t, \vec{x}), \zeta_t(t', \vec{x})] = \frac{\delta(x - x')}{\omega_0 \rho_m^{1/2}}, \] (54)
\[ [\zeta_t(t, \vec{x}), \zeta_t(t', \vec{x'})] = 0. \] (55)

\( \zeta_t(\vec{x}) \) is a field that describes the temporal vibrations of matter in the system. From quantum field theory, \( \pi(\vec{x}) \) forms a conjugate pair with \( \varphi(\vec{x}) \) and not with the Hamiltonian. Based on our discussions above, a similar argument can also be made for \( \zeta_t(\vec{x}) \). There is no commutation relation with the semi-bounded Hamiltonian which restricts the spectrum of the temporal vibration operator \( \zeta_t(\vec{x}) \) to be bounded. The spectrum of this operator spans the entire real line.

The internal time in the bosonic field is,
\[ t_f(t, \vec{x}) = t + \zeta_t(t, \vec{x}). \] (56)
Since external time \( t \) is a parameter and \( \zeta_t(t, \vec{x}) \) is a self-adjoint operator, the internal time \( t_f(t, \vec{x}) \) is also a self-adjoint operator which shall satisfy the equal time commutation relations,
\[ [\varphi(t, \vec{x}), t_f(t', \vec{x'})] = \frac{\delta(x - x')}{\omega_0 \rho_m^{1/2}}, \] (57)
\[ [t_f(t, \vec{x}), t_f(t', \vec{x'})] = 0. \] (58)

The spectrum of the internal time operator \( t_f(t, \vec{x}) \) also spans the entire real line.
6. Conclusions and discussions

In refs. [19–21], we demonstrate a possibility that the amplitude of a matter wave can be taken as a 4-vector with vibrations in time and space. The properties of a bosonic field can be attained.

In this approach, a plane wave with vibrations of matter in proper time is the same as described by Eq. (27) in this paper. Under a Lorentz transformation, the proper time amplitude \( T_0 \) of this plane wave transforms as the 0-component of a 4-vector, i.e. \( (T_0, 0, 0, 0) \rightarrow (T, X) \), where \( X \) is the amplitude for the vibrations in the spatial directions. The plane wave therefore has vibrations of matter in both the spatial and temporal directions when observed in another reference frame.

A particle observed will have a zigzag motion in the spatial direction of the propagation. As the superposition of the plane waves which were discussed, a bosonic field has the vibrations of matter in both the spatial and temporal directions. To illustrate the magnitude of the spatial vibration, we will consider the same examples in Section 4 for a neutrino,

\[
E = 1\text{Mev} \Rightarrow |T_n| = 3.5 \times 10^{-11}s, \quad |X_n| = 1.07\text{cm}, \quad \omega_p = 7.4 \times 10^6\text{s}^{-1},
\]

\[
E = 1\text{Gev} \Rightarrow |T_n| = 1.1 \times 10^{-9}s, \quad |X_n| = 33.7\text{cm}, \quad \omega_p = 7.4 \times 10^9\text{s}^{-1}.
\]

The spatial vibration amplitude \( \dot{X}_n = k/\sqrt{\omega^2 \omega} \) is derived from the results presented in ref. [21].

In this paper, we consider another possibility that a matter wave can have vibrations in time but without the correlated spatial vibrations. The properties of a bosonic field can also be obtained. In this approach, the temporal vibration is a 'pure' intrinsic property of matter. It dictates the rate of change of the particle’s intrinsic properties, e.g. decay rate of an unstable particle. However, the matter observed does not have the spatial vibrations discussed above.

In addition, the temporal vibrations can be described either by the conjugate momentum field \( \pi(\vec{x}) \) or the temporal vibration field \( \zeta_t(\vec{x}) \). Being the conjugate momentum of the real scalar field \( \varphi(\vec{x}) \), it is equally feasible to formulate the properties of the bosonic field by the use of \( \pi(\vec{x}) \). Similarly, we can develop the same Lorentz covariant formulations based on the temporal vibration field \( \zeta_t(\vec{x}) \). All these results can be obtained without the need to define another field with spatial vibrations of matter as we have shown in this paper. By defining only the temporal vibration field \( \zeta_t(\vec{x}) \), it is sufficient to derive a relativistic theory for the bosonic field. Note that in this field, with vibrations of matter in time only, the internal time of an observed propagating particle is vibrating, relative to the external time. However, there is no zigzag motion in the spatial direction of the propagation.

The temporal vibration is an additional degree of freedom introduced. Reasonable assumptions have been made in our studies about the properties of this vibration. By comparing Eqs. (39), (40), (59) and (60), we can see that the temporal vibration amplitude and frequency are the same with or without the correlated spatial vibrations. The two different approaches have the same predicted results for the vibrations of matter in time. The only difference is whether there are the correlated spatial vibrations. Since both approaches are theoretically consistent and can reconcile the same properties of a bosonic field, we will have to rely on future experiments to determine which one is the nature's true preference.

References

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