Brane Dynamics and Four-Dimensional Quantum Field Theory \(^a\)

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We review the relation between the classical dynamics of the M-fivebrane and the quantum low energy effective action for \(N = 2\) Yang-Mills theories. We also discuss some outstanding issues in this correspondence.

1 Introduction

In recent years there have been some remarkable and surprising advances in non-perturbative gauge theory arising from the study of branes in string theory and M-theory. While there have been many interesting developments, here we will only review the precise connection between the classical dynamics of the M-fivebrane and four-dimensional \(N = 2\) quantum Yang-Mills theories. Let us begin by reviewing some of Witten’s analysis of type IIA brane configurations.\(^1\)

A similar role for the M-fivebrane also appeared in\(^2\).

We start by considering type IIA string theory. We place two parallel NS-fivebranes in the \(x^0, x^1, x^3, \ldots, x^5\) plane separated along the \(x^6\) direction by a distance \(\Delta x^6\). These two NS-fivebranes will preserve sixteen of the thirty-two spacetime supersymmetries. Next we introduce \(N_c\) parallel D-fourbranes in the \(x^0, x^1, x^2, x^3, x^6\) plane. These D-fourbranes stretch between the two NS-fivebranes and reduce the number of preserved supersymmetries to eight.

At weak string coupling the NS-fivebranes are heavy and their motion can be ignored. The low energy fluctuations of this system are then described by the D-fourbranes. As is well known the low energy dynamics of \(N_c\) parallel D-fourbranes is given by a five-dimensional \(U(N_c)\) gauge theory with sixteen supersymmetries. The presence of the NS-fivebranes has two effects. Firstly they reduce the number of preserved supersymmetries to eight. Secondly, since the \(x^6\) direction of the D-fourbrane is finite in extent, at low energy their worldvolume is four-dimensional. An overall \(U(1)\) factor of the gauge group \(U(N_c)\) is trivial and simply describes the centre of mass motion of the D-fourbranes so we may ignore it. Thus the low energy, weak coupling description

\(^a\)Talk given by P.C. West at the Trieste Conference on Superfivebranes and Physics in 5+1 Dimensions, April 1998.
of this configuration is given by four-dimensional $N = 2$ $SU(N_c)$ gauge theory.

One can also consider adding $N_f$ semi-infinite D-fourbranes to this configuration. These intersect the left or right NS-fivebrane at one end but extend to infinity at the other. Since they are infinitely heavy as compared to the finite D-fourbranes their motion is suppressed. However in the D-brane picture there are stretched open strings with one end on a semi-infinite D-fourbrane and the other on a finite D-fourbrane. These strings give rise to $N_f$ massive hyper-multiplets in the (anti-) fundamental representation of $SU(N_c)$ in the four-dimensional gauge theory. Their bare mass given by the length of the open strings, which is the distance between the finite and semi-infinite D-fourbranes.

What Witten noticed was that there is an elegant strong coupling description of this configuration in M-theory. Increasing the string coupling lifts us up to eleven dimensions and introduces another coordinate $x^{10}$ which is periodic with period $2\pi R$. Furthermore one can go to strong coupling keeping the curvatures small, so that supergravity is a good approximation, yet also keeping the Yang-Mills coupling constant fixed. The NS-fivebranes simply lift to M-fivebranes. The D-fourbranes also lift to M-fivebranes, only wrapped on the $x^{10}$ dimension. Thus in eleven dimensions the entire configuration appears as intersecting M-fivebranes. An important realisation is that this configuration can be viewed as a single M-fivebrane wrapped on a two-dimensional manifold, embedded the four-dimensional space with coordinates $x^4, x^5, x^6, x^{10}$.

The condition that an M-fivebrane wrapped around a manifold breaks only of half the supersymmetry, leaving eight unbroken supersymmetries, is that the manifold is complex, i.e. it must be a Riemann surface. It is helpful then to introduce the complex notation

$$ s = (x^6 + ix^{10})/R, \quad t = e^{-s}, \quad z = x^4 + ix^5. \quad (1) $$

Thus the supersymmetric intersecting M-fivebrane configuration can be realised by any holomorphic embedding $F(t, z) = 0$ of its worldvolume into the $x^4, x^5, x^6, x^{10}$ dimensions of eleven-dimensional spacetime.

Let us return to the configuration in question. At a given point $z$ there should be two solutions for $s$, corresponding to the two NS-fivebranes. Therefore we take $F$ to be second order in $t$

$$ A(z)t^2 - 2B(z)t + C(z) = 0. \quad (2) $$

We expect that as $z \to \infty$, we are on either the left or right NS-fivebrane so that $t \to \infty, 0$ respectively. If $A(z)$ or $C(z)$ has a zero at any finite value of $z$, so that $t \to \infty, 0$ there, this can be interpreted as a semi-infinite D-fourbrane ending on the left or right NS-fivebrane respectively. Let us just consider semi-infinite D-fourbranes ending on the right NS-fivebrane. In this case we may
set $A = 1$ by rescaling $t$. For $N_f$ semi-infinite D-fourbranes we need $N_f$ zeros of $C(z)$. Thus $C(z)$ must take the form
\[ C(z) = \Lambda \prod_{a=1}^{N_f} (z - m_a), \tag{3} \]
where $\Lambda$ is a constant and $m_a$ are the positions of the semi-infinite D-fourbranes, i.e. their bare masses. For $N_f = 0$ we simply take $C = \Lambda$. Finally we need to determine $B(z)$. For a fixed $s$ we need there to be $N_c$ solutions for $z$, corresponding to the $N_c$ finite D-fourbranes. Thus $B(z)$ must take the form
\[ B(z) = \prod_{i=1}^{N_c} (z - e_i), \tag{4} \]
note that we can set the coefficient of $z^{N_c}$ to one by rescaling $z$. For large $z$ the $e_i$ then appear as the positions of the $N_c$ finite D-fourbranes. Since we have frozen out the centre of mass motion we set $\sum_{i=1}^{N_c} e_i = 0$, which can also be achieved by redefining $z$. With these conditions imposed ones see that $s(z)$ defines precisely the Seiberg-Witten curve
\[ y^2 = \left( \prod_{i=1}^{N_c} (z - e_i) \right)^2 - \Lambda \prod_{a=1}^{N_f} (z - m_a), \tag{5} \]
where $y = t - B$.

In summary then this brane configuration has a weak coupling description as four-dimensional $N = 2$ $SU(N_c)$ Yang-Mills theory with $N_f$ hypermultiplets in the fundamental representation and a strong coupling description as an M-fivebrane wrapped around the correct Seiberg-Witten curve. Thus from the brane configuration we can identify the Riemann surfaces that are known to be associated with the exact quantum low energy effective action of four-dimensional $N = 2$ Yang-Mills theory. This analysis also suggests why the scalar modes in the Seiberg-Witten solution correspond to moduli of a Riemann surface, since the zero modes of the M-fivebrane are just the Riemann surface moduli. In addition Witten was able to derive the appropriate curves for many new classes of Yang-Mills theories which were previously unknown.

This remarkable correspondence left open the question as to whether or not the classical M-fivebrane could predict the precise perturbative and instanton corrections of the Yang-Mills Theory and not just the Seiberg-Witten curve. In other words, a knowledge of the elliptic curve alone is not enough to compare with the four-dimensional Yang-Mills quantum field theory. One must also know how to calculate the low energy effective action from the M-fivebrane dynamics, including all the instanton corrections.
2 Brane Dead

Since the paper \cite{1} first appeared there have been several discussions of how to construct the low energy effective action from M-theory but many papers present a seriously flawed argument, apparently based on misinterpretations of comments in \cite{1}. It would be invidious to reference all of these articles here, however the following argument, which has appeared in a substantial review article, illustrates many of these issues.

The M-fivebrane worldvolume theory has a self-dual three-form $H$. The argument states that the action therefore contains the standard kinetic term for $p$-form fields

$$S_{\text{SYM}} = \int d^6x H^2. \quad (6)$$

To obtain the effective action one must decompose $H$ in a basis of non-trivial one-forms $\Lambda_i$ of the Riemann surface $\Sigma$ (the Seiberg-Witten curve), with genus $N_c - 1, I = 1, \ldots, N_c - 1$,

$$H = \sum_{I=1}^{N_c-1} F_I \wedge \Lambda_I + *F_I \wedge *\Lambda_I. \quad (7)$$

Substituting eq. (7) into eq. (6) leads to

$$S_{\text{SYM}} = \int d^4x (\text{Im} \tau_{IJ}) F_I \wedge *F_J + (\text{Re} \tau_{IJ}) F_I \wedge F_J, \quad (8)$$

with $\tau$ the period matrix of $\Sigma$

$$\text{Im} \tau_{IJ} = \int_\Sigma \Lambda_I \wedge *\Lambda_J + c.c., \quad (9)$$

$$\text{Re} \tau_{IJ} = \int_\Sigma \Lambda_I \wedge \Lambda_J + c.c. \quad (10)$$

It should not take the alert reader much time to realise the following errors in the above argument. Firstly, and most seriously, since the ansatz in eq. (7) ensures $H$ is self-dual the action eq. (6) 

vanishes identically. This can be easily seen since $H^2 = H \wedge *H = H \wedge H$. Now $H$ is an odd form so $H \wedge H = 0$. Therefore this argument predicts that the Seiberg-Witten effective action vanishes identically.

To avoid this problem some articles reference the paper \cite{5} where a first order six-dimensional action is given for a self-dual three form which is non-zero. However, this action is not coupled to scalars so that the resulting
four-dimensional action contains a constant period matrix $\tau_{ij}$. Therefore this method leads only to the classical low energy effective action with no quantum corrections. Even if one ignored this problem and let the Riemann surface moduli $e_i$ become spacetime dependent, one could not arrive at the correct low energy effective action because the relations between the moduli $e_i$ and the Yang-Mills scalar fields $a_I$ are still unknown.

Given that this argument starts with zero one might wonder how its advocates obtain a non-trivial answer. In fact the expressions eq. 8 and eq. 10 are incorrect. To see this one only needs to consider the case of genus one ($N_c=2$) and let us choose our basis of one forms so that $\Lambda_1 = \Lambda$ is a holomorphic one form, $\ast\Lambda = i\Lambda$ and $\Lambda_2$ is an anti-holomorphic one-form, $\bar{\Lambda}$, $\ast\bar{\Lambda} = -i\bar{\Lambda}$. Then one finds that the two equations 8 and 10 are the same (up to a constant). Furthermore no redefinition will alter this since, in complex notation, it is clear that the only independent integrand one could write down is $\Lambda \wedge \bar{\Lambda}$ and this is purely imaginary, i.e. only $\text{Im} \tau$ has a simple integral formula.

### 3 The Fivebrane Equations of Motion

Let us now discuss in detail the worldvolume theory of the M-fivebrane. It has a six-dimensional $(2,0)$ tensor multiplet of massless fields on its worldvolume. The component fields of this supermultiplet are five real scalars $X^{a'}$, a gauge field $B_{mn}$ whose field strength satisfies a modified self-duality condition and sixteen spinors $\Theta^i_\beta$. The scalars are the coordinates transverse to the fivebrane and correspond to the breaking of eleven-dimensional translation invariance by the presence of the fivebrane. The sixteen spinors correspond to the breaking of half of the thirty-two component supersymmetry of M-theory. The classical equations of motion of the fivebrane in the absence of fermions and background fields are

$$G^{mn} \nabla_m \nabla_n X^{a'} = 0, \quad (11)$$

and

$$G^{mn} \nabla_m H_{npq} = 0. \quad (12)$$

where the worldvolume indices are $m, n, p = 0, 1, \ldots, 5$ and the world tangent indices $a, b, c = 0, 1, \ldots, 5$. The transverse indices are $a', b' = 6, 7, 8, 9, 10$. The usual induced metric and vielbien for a $p$-brane is given, in static gauge, by

$$g_{mn} = \eta_{mn} + \partial_m X^{a'} \partial_n X^{b'} \delta_{a'b'} = e_m^a \eta_{ab} e_n^b. \quad (13)$$
The covariant derivative $\nabla$ is defined as the Levi-Civita connection with respect to the metric $g_{mn}$. The inverse metric $G^{mn}$ which also occurs is related to $g^{mn}$ by the equation

$$G^{mn} = (e^{-1})^m{}_c g^{ca} m_a d (e^{-1})^m{}_b ,$$

(14)

where the matrix $m$ is given by

$$m_a^b = \delta_a^b - 2 h_{acd} h^{bcd} .$$

(15)

The field $H_{abc}$ is an anti-symmetric three-form and is the curl of $B_{ab}$. However it satisfies a non-linear self-duality constraint. To construct $H_{abc}$ we start from the three-form $h_{abc}$ which is self-dual;

$$h_{abc} = \frac{1}{3!} \epsilon_{abcdef} h^{def} ,$$

(16)

but it is not the curl of a three form gauge field. The field $H_{mnp}$ is then obtained as

$$H_{mnp} = \epsilon_m{}^a e_n{}^b e_p{}^c (m^{-1})_c{}^d h_{abcd} .$$

(17)

Clearly, the self-duality condition on $h_{abd}$ transforms into a condition on $H_{mnp}$ and vice-versa for the Bianchi identify $dH = 0$.

4 Soliton Dynamics

In this section we wish to provide a complete derivation of the low energy effective action for the wrapped M-fivebrane. We will review the discussion given in which treats the vector as well as scalar modes and we refer the interested reader there for more details. It is possible to derive the Seiberg-Witten action relatively simply by considering the scalar modes alone and using $N = 2$ supersymmetry to complete the action from only its scalar part.

We have chosen to discuss the complete analysis here because the purely scalar argument is blind to many subtle and interesting features of the M-fivebrane, notably how the Abelian three-form can reproduce the low energy effective action for non-Abelian vector fields. In addition the argument presented below can be generalised to cases with less supersymmetry. In these cases there is no a priori relation between the vector and scalar dynamics. We also note that the construction can be performed in a manifestly $N = 2$ supersymmetric form which perhaps best highlights the underlying geometry.

Our approach is to view the intersecting M-fivebrane configuration as a threebrane soliton on a single M-fivebrane worldvolume. Viewed in this way the the soliton is a purely scalar field configuration of the worldvolume theory and the Bogomol’nyi condition is just the Cauchy-Riemann equation for $s(z)$. 
From this point of view we can obtain the low energy effective equations by expanding the equations of motion to second order in derivatives $\partial_\mu$, $\mu = 0, 1, 2, 3$ and field strengths $H_{mnp}$ around the threebrane background. To this order $h_{mnp} = H_{mnp}$ so that the field strength $H_{mnp}$ is self-dual but with respect to the induced metric $g_{mn}$. This implies that the ansatz in eq. 3 is incorrect as there are additional terms in $H_{mnp}$. In particular there is a non-zero contribution to $H_{\mu\nu\lambda}$. Taking this into account and expanding the equations of motion for two scalars (i.e. $X^6$ and $X^{10}$) leads to the expressions

$$E \equiv \eta^{\mu\nu} \partial_\mu \partial_\nu s - \partial_\zeta \left[ (\partial_\zeta s \partial^2 s) \bar{s} \right] / (1 + |s|^2) - 16 (1 + |s|^2)^2 H_{\mu\nu\zeta} H^{\mu\nu} \bar{z} \partial \partial s = 0 ,$$

(18)

and

$$E_\nu \equiv \partial_\nu H_{\mu\nu\zeta} - \partial_\zeta \left[ (\partial_\zeta s \partial^2 s) H_{\mu\nu\zeta} - (\partial_\zeta s \partial^2 \bar{s}) H^{\mu\nu} \bar{z} \right] / (1 + |s|^2) H_{\mu\nu\zeta} = 0 .$$

(19)

In these expressions we have assumed that the threebrane soliton defined by the Seiberg-Witten curve $s(z)$ is dynamical due to its moduli $e_i$ becoming $x^\mu$-dependent.

Before reducing these equations to four-dimensions we need an ansatz for the three-form components $H_{\mu\nu\zeta}$. Let us restrict our attention here to the simplest case of $N_c = 2, N_f = 0$, although the analysis can be generalised by considering the appropriate curve. Explicitly, eq. 3 leads to the curve

$$s = - \ln \left[ z^2 - u \pm \sqrt{(z^2 - u)^2 - \Lambda} \right] ,$$

(20)

where $u = e_1 e_2$ is the only moduli since $e_1 + e_2 = 0$. Solving the self-duality condition requires $H_{mnp}$ to take the form

$$H_{\mu\nu\zeta} = \kappa F_{\mu\nu} ,$$

(21)

where $F_{\mu\nu} = F_{\mu\nu} + i \ast F_{\mu\nu}$ and $\kappa$ is undetermined. The closure of $H_{mnp}$ requires that $\kappa$ is holomorphic. We therefore write $\kappa = \kappa_0(x) \lambda_z$ where $\lambda_z dz = ds/du dz$ is the holomorphic one-form of the Riemann surface $\Sigma$. We are now free to choose $\kappa_0$ and we do this to ensure that $F_{mn}$ is a closed two-form. We will see below that this in turn fixes

$$\kappa = \left( \frac{da}{du} \right)^{-1} \lambda_z .$$

(22)

We have also introduced the periods

$$a = \int_A s dz , \quad a_D = \int_B s dz ,$$

(23)
where $A$ and $B$ are a basis of one-cycles of $\Sigma$, i.e. $sdz$ is identified with the Seiberg-Witten differential. Note that the factor $(da/du)^{-1}$ normalises the period of the form $\kappa dz$ to be one around the $A$-cycle. This reveals another error in the argument in section two as the choice of $\Lambda$ in eq. 7 will not lead to the Seiberg-Witten solution.

Finally to reduce these equations to four dimensions we project them over a complete set of one-forms of $\Sigma$

$$
0 = \int_\Sigma Edz \wedge \bar{\lambda} = \int_\Sigma E_\nu dz \wedge \bar{\lambda} = R^2 \Lambda \int_\Sigma \partial_z \left( \frac{\bar{s}^2 \partial_{\bar{s}}}{1 + R^2 \Lambda \partial_{\bar{s}} \partial_s \bar{s}} \right) dz \wedge \bar{\lambda} = 0,
$$

$$
0 = \int_\Sigma E_\mu dz \wedge \bar{\lambda} = \int_\Sigma E_\mu dz \wedge \bar{\lambda} = R^2 \Lambda \int_\Sigma \partial_{\bar{s}} \left( \frac{\bar{s}^2 \partial_s}{1 + R^2 \Lambda \partial_s \partial_{\bar{s}} \bar{s}} \right) dz \wedge \bar{\lambda} = -\left( \frac{\partial_s}{\partial \bar{s}} \right) \frac{3}{\partial \bar{\tau}} d\bar{\tau},
$$

(24)

Here we encounter integrals over $\Sigma$ labelled by $I, J$ and $K$ and given below. While it is straightforward to evaluate $I$ using the Riemann bilinear relation the $J$ and $K$ integrals require a more sophisticated analysis. This was done indirectly in 8 and directly in 9 using properties of modular forms resulting in

$$
I \equiv \int_\Sigma \lambda \wedge \bar{\lambda} = \frac{da_D \partial_{\bar{s}} d\bar{a}}{\partial_{\bar{s}} d\bar{a}} - \frac{da \partial_{\bar{a}} d\bar{a}}{\partial_{\bar{a}} d\bar{a}},
$$

$$
J \equiv R^2 \Lambda \int_\Sigma \partial_z \left( \frac{\bar{s}^2 \partial_{s}}{1 + R^2 \Lambda \partial_s \partial_{\bar{s}} \bar{s}} \right) dz \wedge \bar{\lambda} = 0,
$$

$$
K \equiv R^2 \Lambda \int_\Sigma \partial_{\bar{s}} \left( \frac{s^2 \partial_s}{1 + R^2 \Lambda \partial_s \partial_{\bar{s}} s} \right) dz \wedge \bar{\lambda} = -\left( \frac{\partial_s}{\partial \bar{a}} \right) \frac{3}{\partial \bar{\tau}} d\bar{\tau},
$$

(25)

where $\tau = da_D/da$. With these integrals we can now evaluate the four-dimensional equations of motion

$$
0 = \partial^\nu \partial_\mu a(\tau - \bar{\tau}) + \partial^\nu a \partial_\mu a \frac{d\tau}{da} + 16 \bar{F}_{\mu
u} \bar{F}^{\mu\nu} \frac{d\bar{\tau}}{d\bar{a}},
$$

$$
0 = \partial^\nu F_{\mu\nu}(\tau - \bar{\tau}) + F_{\mu\nu} \partial^\nu u \frac{d\tau}{du} - \bar{F}_{\mu\nu} \partial^\nu \bar{u} \frac{d\bar{\tau}}{d\bar{u}}.
$$

8
Note that real part of the second equation is just the Bianchi identity for $F_{mn}$.

Finally we see that these equations may be derived from the Seiberg-Witten effective action

$$S_{SW} = \int d^4 x \ \text{Im} (\tau \partial_{\mu} a \partial^\mu \bar{a} + 16 \tau F_{\mu \nu} F^{\mu \nu}) .$$

(27)

5 Discussion

In this review we have shown that by using brane dynamics one can obtain not only qualitative features of quantum $N = 2$ gauge theories such as the Seiberg-Witten curve but also the precise details of the low energy effective action, including instanton corrections. The example that we presented above also sets out a general method which can be applied to configurations with less supersymmetry. One first identifies a solitonic solution of the M-fivebrane and then the low energy dynamics can be obtained from the M-fivebrane equations of motion. The low energy effective action will contain scalar zero modes from the moduli of the soliton and vector zero modes arising from the three-form $H_{mnp}$ and the brane topology.

Note that this construction is more subtle than the direct relation between the geometry of the brane and the $\beta$-function that has been suggested. For example the one-loop $\beta$-function coefficient can be recovered from the bending of the branes. However this interpretation has difficulties since if one identifies the coupling constant with the distance between the two NS-fivebranes this gives a function $\tau(z)$ rather than $\tau(u)$. In addition for $SU(N_c)$ gauge theories there are in fact $\frac{1}{2} N_c (N_c - 1)$ coupling constants and these cannot all be identified with the distance between the two NS-fivebranes.

It is of course natural to see if M-theory can produce other details of the Yang-Mills quantum field theory. For example it was pointed out in reference that the M-fivebrane also predicts an infinite number of higher derivative terms which are not holomorphic. Although these terms are complicated one can easily see that they depend upon the radius of the eleventh dimension and do not seem to agree with the terms obtained from quantum field theory. One can also consider the M-fivebrane prediction for low energy monopole dynamics to see if this leads to the correct form for the monopole moduli space metric.

In closing let us mention some unsolved issues which warrant further study. There are other formulations of the M-fivebrane dynamics, which in addition admit an action, and therefore it is natural to see if they can also reconstruct

\[ \text{We are grateful to V. Khoze for this point.} \]
the Seiberg-Witten effective action. However there is one immediate difficulty with using an action. Namely, since there is no a priori distinction between $A$- and $B$-cycles, one expects that the construction will be modular invariant. On the other hand, the Seiberg-Witten action is not invariant under the $SL(2, \mathbb{Z})$ modular group, even though its equations of motion are.

Lastly there are in fact discrepancies between the instanton coefficients predicted by the Seiberg-Witten curves and those obtained by explicit calculations using instanton calculus for the cases $N_f = 2N_c$. Perhaps a more detailed consideration using M-theory will lead to alternative forms for these curves.

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