An expanding 4D universe in a 5D Kaluza-Klein cosmology with higher dimensional matter

F. Darabi*
Department of Physics, Azarbaijan University of Tarbiat Moallem, 53714-161, Tabriz, Iran.
Research Institute for Astronomy and Astrophysics of Maragha, 55134-441, Maragha, Iran.

December 3, 2009

Abstract

In the framework of Kaluza-Klein theory, we investigate a (4+1)-dimensional universe consisting of a (4+1) dimensional Robertson-Walker type metric coupled with a (4+1) dimensional energy-momentum tensor. The matter part consists of an energy density together with a pressure subject to 4D part of the (4+1) dimensional energy-momentum tensor. The dark part consists of just a dark pressure \( \bar{p} \), corresponding to the extra-dimension endowed by a scalar field, with no element of dark energy. It is shown that the reduced Einstein field equations are free of 4D pressure and are just affected by an effective pressure produced by the 4D energy density and dark pressure. It is then proposed that the expansion of the universe may be controlled by the equation of state in higher dimension rather than four dimensions. This may account for the emergence of unexpected current acceleration in the middle of matter dominant era.

PACS: 95.36.+x; 98.80.-k; 04.50.Cd
Key words: Dark pressure; inflation; accelerating universe; non compact Kaluza-Klein cosmology.

*e-mail: f.darabi@azaruniv.edu
1 Introduction

The recent distance measurements from the light-curves of several hundred type Ia supernovae [1, 2] and independently from observations of the cosmic microwave background (CMB) by the WMAP satellite [3] and other CMB experiments [4, 5] suggest strongly that our universe is currently undergoing a period of acceleration. This accelerating expansion is generally believed to be driven by an energy source called dark energy. The question of dark energy and the accelerating universe has therefore been the focus of a large amount of activities in recent years. Dark energy and the accelerating universe have been discussed extensively from various point of views over the past few years [6, 7, 8]. In principle, a natural candidate for dark energy could be a small positive cosmological constant. One approach in this direction is to employ what is known as modified gravity where an arbitrary function of the Ricci scalar is added to the Einstein-Hilbert action. It has been shown that such a modification may account for the late time acceleration and the initial inflationary period in the evolution of the universe [9, 10]. Alternative approaches have also been pursued, a few example of which can be found in [11, 12, 13]. These schemes aim to improve the quintessence approach overcoming the problem of scalar field potential, generating a dynamical source for dark energy as an intrinsic feature. The goal would be to obtain a comprehensive model capable of linking the picture of the early universe to the one observed today, that is, a model derived from some effective theory of quantum gravity which, through an inflationary period would result in today accelerated Friedmann expansion driven by some $\Omega_\Lambda$-term. However, the mechanism responsible for this acceleration is not well understood and many authors introduce a mysterious cosmic fluid, the so called dark energy, to explain this effect [14].

Since in a variety of inflationary models scalar fields have been used in describing the transition from the quasi-exponential expansion of the early universe to a power law expansion, it is natural to try to understand the present acceleration of the universe by constructing models where the matter responsible for such behavior is also represented by a scalar field. Such models are worked out, for example, in Ref [15].

Bellini et al, on the other hand, have published extensively on the evolution of the universe from noncompact vacuum Kaluza-Klein theory [16]. They used the essence of STM (Space-Time-Matter) theory and developed a 5D mechanism to explain, by a single scalar field, the evolution of the universe including inflationary expansion and the present day observed accelerated expansion. The STM theory is proposed by Wesson and his collaborators, which is designed to explain the origin of matter in terms of the geometry of the bulk space in which our 4D world is embedded, for reviews see [17]. More precisely, in STM theory, our world is a hypersurface embedded in a five-dimensional Ricci-flat $(R_{AB} = 0)$ manifold where all the matter in our world can be thought of as being manifestations of the geometrical properties of the higher dimensional space according to $G_{\alpha\beta} = 8\pi G T_{\alpha\beta}$, provided an appropriate definition is made for the energy-momentum tensor of matter in terms of the extra part of the geometry. Physically, the picture behind this interpretation is that curvature in $(4 + 1)$ space induces effective properties of matter in $(3 + 1)$ space-time. The fact that such an embedding can be done is supported by Campbell’s theorem [18] which states that any analytical solution of the Einstein field equations in $N$ dimensions can be locally embedded in a Ricci-flat manifold in $(N + 1)$ dimensions. Since the matter is induced from the extra dimension, this theory is
also called the induced matter theory. The sort of cosmologies stemming from STM theory is studied in [19] [20] [21].

Another higher dimensional work has already been done with a multi-dimensional compact Kaluza-Klein cosmological model in which the scale factor of the compact space evolves as an inverse power of the radius of the observable universe [22]. The Friedmann-Robertson-Walker equations of standard four-dimensional cosmology were obtained where the pressure in the $4D$ universe was an effective pressure, expressed in terms of the components of the higher dimensional energy-momentum tensor, capable of being negative to explain the acceleration of our present universe.

In this work, motivated by the work done in the compact model [22] and interested in its non-compact version, a $5D$ non-compact Kaluza-Klein cosmological model is introduced which is not Ricci flat, but is extended to couple with a higher dimensional energy momentum tensor. In the present non-compact model, it is shown that a dark pressure along the higher dimensional sector together with the $4D$ energy density may induce an effective pressure in four dimensional universe so that the reduced field equations on $4D$ universe are free of $4D$ pressure and are just affected by the effective pressure. The main point of this paper is to show the possibility that perhaps $4D$ pressure does not directly control the dynamics of the universe, rather the cosmological eras including inflation, deceleration and current acceleration are just happened due to either the evolution in equation of state along higher dimension or an interplay between equations of state in $4D$ universe and along higher dimension. Moreover, it is appealing to consider the current acceleration of the universe as a result of a new phase which is started recently along extra dimension. In this way, the emergence of unexpected acceleration in the middle of matter dominated era is easily justified because in this model the real dynamics of the universe is controlled not by $4D$ physics but through the higher dimensional physics. In other words, the unexpected emergence of current acceleration may be related to a higher dimensional effect which is hidden for $4D$ observers.

2 The Model

We start with the $5D$ line element

$$dS^2 = g_{AB}dx^A dx^B,$$

in which $A$ and $B$ run over both the space-time coordinates $\alpha, \beta$ and one non compact extra dimension indicated by $4$. The space-time part of the metric $g_{\alpha\beta} = g_{\alpha\beta}(x^\alpha)$ is assumed to define the Robertson-Walker line element

$$ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right),$$

where $k$ takes the values $+1, 0, -1$ according to a close, flat or open universe, respectively. We also take the followings

$$g_{4\alpha} = 0, \quad g_{44} = \epsilon \Phi^2(x^\alpha),$$

where $\epsilon^2 = 1$ and the signature of the higher dimensional part of the metric is left general. This choice has been made because any fully covariant $5D$ theory has five coordinate degrees of freedom which can lead to considerable algebraic simplification, without loss of generality.
The extra dimensional independence of the scalar field and the 4D metric may pose an ambiguity between the compact or non-compactness of the present model. In the compact theory the cylindrical condition is imposed to account for this independence and justify the non-observability of extra dimension. In non-compact theory, however, one may account for this independence within a different framework in which the non-observability of extra dimensions is justified in a different way. Therefore, the answer to the question of whether a higher dimensional model in which all variables are independent of extra dimension is compact or non-compact depends on the way by which the model justifies the non-observability of extra dimensions. In the present model we aim to follow the non-compact model where a reasonable justification will be given for the non-observability of extra dimension.

Unlike the noncompact vacuum Kaluza-Klein theory, we will assume the fully covariant 5D non-vacuum Einstein equation

\[ G_{AB} = 8\pi GT_{AB}, \] (3)

where \( G_{AB} \) and \( T_{AB} \) are the 5D Einstein tensor and energy-momentum tensor, respectively. Note that the 5D gravitational constant has been fixed to be the same value as the 4D one.

In the following we use the geometric reduction from 5D to 4D as appeared in [23]

\[ \hat{R}_{\alpha\beta} = R_{\alpha\beta} + \partial_4 \Gamma^4_{\alpha\lambda} + \partial_\lambda \Gamma^\lambda_{\alpha\beta} - \partial_\beta \Gamma^4_{\alpha\lambda} + \Gamma^\lambda_{\alpha\beta} \Gamma^\lambda_{\alpha\lambda} + \Gamma^\lambda_{\alpha\beta} \Gamma^\lambda_{\beta\lambda} - \Gamma^\lambda_{\alpha\lambda} \Gamma^\lambda_{\beta\lambda} - \Gamma^\lambda_{\alpha\beta} \Gamma^\lambda_{\beta\lambda} - \Gamma^\lambda_{\alpha\lambda} \Gamma^\lambda_{\beta\lambda}, \] (4)

where \( \hat{\cdot} \) denotes the 4D part of the 5D quantities. Evaluating the Christoffel symbols for the metric \( g_{AB} \) gives

\[ \hat{R}_{\alpha\beta} = R_{\alpha\beta} - \nabla_\alpha \nabla_\beta \Phi. \] (5)

In the same way we obtain

\[ R_{44} = -\epsilon \Phi \Box \Phi. \] (6)

We now construct the space-time components of the Einstein tensor

\[ G_{AB} = R_{AB} - \frac{1}{2} g_{AB} R_{(5)}. \]

In so doing, we first obtain the 5D Ricci scalar \( R_{(5)} \) as

\[ R_{(5)} = g^{AB} R_{AB} = g^{\alpha\beta} \hat{R}_{\alpha\beta} + g^{44} R_{44} = g^{\alpha\beta} (R_{\alpha\beta} - \frac{\nabla_\alpha \nabla_\beta \Phi}{\Phi}) + \frac{\epsilon}{\Phi^2} (-\epsilon \Phi \Box \Phi) = R - \frac{2}{\Phi} \Box \Phi, \] (7)

where the \( \alpha 4 \) terms vanish and \( R \) is the 4D Ricci scalar. The space-time components of the Einstein tensor is written \( \hat{G}_{\alpha\beta} = \hat{R}_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R_{(5)}. \) Substituting \( \hat{R}_{\alpha\beta} \) and \( R_{(5)} \) into the space-time components of the Einstein tensor gives

\[ \hat{G}_{\alpha\beta} = G_{\alpha\beta} + \frac{1}{\Phi} (g_{\alpha\beta} \Box \Phi - \nabla_\alpha \nabla_\beta \Phi). \] (8)

\[ ^1 \text{In compact Kaluza-Klein theory one may define a 5D gravitational constant } G^{(5)} \text{ which is reduced to 4D one as } G = \frac{G^{(5)}}{\int dy}, \text{ where } \int dy \text{ is the volume of extra compact dimension. In non-compact theory, however, we do not require a 5D gravitational constant because there is no finite volume of extra dimension, and assuming the gravitational constant as } G \text{ will result in the correct 4D Einstein equations.} \]
In the same way, the 4-4 component is written \( G_{44} = R_{44} - \frac{1}{2}g_{44}R_{(5)} \), and substituting \( R_{44}, \ R_{(5)} \) into this component of the Einstein tensor gives

\[
G_{44} = -\frac{1}{2} \epsilon R \Phi^2. \tag{9}
\]

We now consider the 5D energy-momentum tensor without specifying its nature or origin. The form of energy-momentum tensor is dictated by Einstein’s equations and by the symmetries of the metric \( (2) \). Therefore, we may assume a perfect fluid with nonvanishing elements

\[
T_{\alpha\beta} = (\rho + p)u_\alpha u_\beta - pg_{\alpha\beta}, \tag{10}
\]

\[
T_{44} = -\bar{p}g_{44} = -\epsilon\bar{p}\Phi^2, \tag{11}
\]

where \( \rho \) and \( p \) are the conventional density and pressure of perfect fluid in the 4D standard cosmology and \( \bar{p} \) acts as a pressure living along the higher dimensional sector. Notice that the perfect fluid is isotropic on the 3D geometry and anisotropic regarding the 5th dimension.

The field equations \( (3) \) are to be viewed as constraints on the simultaneous geometric and physical choices of \( G_{AB} \) and \( T_{AB} \) components, respectively.

Substituting the energy-momentum components \( (10), (11) \) in front of the 4D and extra dimensional part of Einstein tensors \( (8) \) and \( (9) \), respectively, we obtain the field equations

\[
G_{\alpha\beta} = 8\pi G[(\rho + p)u_\alpha u_\beta - pg_{\alpha\beta}] + \frac{1}{\Phi} [\nabla_\alpha \nabla_\beta \Phi - \Box \Phi g_{\alpha\beta}], \tag{12}
\]

and

\[
R = 16\pi G\bar{p}. \tag{13}
\]

By evaluating the \( g^{\alpha\beta} \) trace of Eq.\((12)\) and combining with Eq.\((13)\) we obtain

\[
\Box \Phi = \frac{1}{3}(8\pi G(\rho - 3p) + 16\pi G\bar{p})\Phi. \tag{14}
\]

This equation infers the following scalar field potential

\[
V(\Phi) = -\frac{1}{6}(8\pi G(\rho - 3p) + 16\pi G\bar{p})\Phi^2, \tag{15}
\]

whose minimum occurs at \( \Phi = 0 \), for which the equations \( (12) \) reduce to describe a usual 4D FRW universe filled with ordinary matter \( \rho \) and \( p \). In other words, our conventional 4D

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2Since this model is supposed to describe the radiation and matter dominated eras, it seems that this fifth dimensional pressure component would be a dark property of conventional matter, including standard model fields (see refs in [24] for some derivations of the matter contribution in Kaluza-Klein cosmology).

3The same choice has been made in [22] with the components of the higher dimensional energy-momentum tensor as \( T_{ij} = \text{diag}[-\rho, p, p, p, p_d, ... , p_d] \) where \( \rho, p \) and \( p_d \) are the density, pressure on the 3D geometry and pressure along the extra dimensions, respectively.

4The \( \alpha 4 \) components of Einstein equation \( (3) \) result in

\[
R_{\alpha 4} = 0,
\]

which is an identity with no useful information.
universe corresponds to the vacuum state of the scalar field \( \Phi \). From Eq. (14), one may infer the following replacements for a nonvanishing \( \Phi \)

\[
\frac{1}{\Phi} \Box \Phi = \frac{1}{3} (8\pi G (\rho - 3p) + 16\pi G \bar{\rho}),
\]

(16)

\[
\frac{1}{\Phi} \nabla_\alpha \nabla_\beta \Phi = \frac{1}{3} (8\pi G (\rho - 3p) + 16\pi G \bar{\rho}) u_\alpha u_\beta.
\]

(17)

Putting the above replacements into Eq. (12) leads to

\[
G_{\alpha\beta} = 8\pi G [ (\rho + \bar{\rho}) u_\alpha u_\beta - \bar{\rho} g_{\alpha\beta}],
\]

(18)

where

\[
\bar{\rho} = \frac{1}{3}(\rho + 2\bar{p}).
\]

(19)

This energy-momentum tensor effectively describes a perfect fluid with density \( \rho \) and pressure \( \bar{p} \). The four dimensional field equations lead to Friedmann equation

\[
3 \frac{\dot{a}^2 + k}{a^2} = 8\pi G \rho,
\]

(20)

and

\[
2a \ddot{a} + a^2 + \frac{k}{a^2} = -8\pi G \bar{p}.
\]

(21)

Differentiating (20) and combining with (21) we obtain the conservation equation

\[
\frac{d}{dt} (\rho a^3) + \bar{p} \frac{d}{dt} (a^3) = 0.
\]

(22)

The equations (20) and (21) can be used to derive the acceleration equation

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3\bar{p}) = -\frac{8\pi G}{3} (\rho + \bar{p}).
\]

(23)

The acceleration or deceleration of the universe depends on the negative or positive values of the quantity \( (\rho + \bar{p}) \).

From extra dimensional equation (13) ( or 4-dimensional Eqs. (19), (20) and (21) ) we obtain

\[
-\frac{6(k + \dot{a}^2 + \ddot{a}a)}{a^2} = 16\pi G \bar{p}.
\]

(24)

Using power law behaviors for the scale factor and dark pressure as \( a(t) = a_0 t^\alpha \) and \( \bar{p}(t) = \bar{p}_0 t^\beta \) in the above equation, provided \( k = 0 \) in agreement with observational constraints, we obtain \( \beta = -2 \).

Based on homogeneity and isotropy of the 4D universe we may assume the scalar field to be just a function of time, then the scalar field equation (14) reads as the following form

\[
\ddot{\Phi} + 3\frac{\dot{a}}{a} \dot{\Phi} - \frac{8\pi G}{3} ((\rho - 3p) + 2\bar{p}) \Phi = 0.
\]

(25)
Assuming \( \Phi(t) = \Phi_0 t^\gamma \) and \( \rho(t) = \rho_0 t^\delta \) (\( \rho_0 > 0 \)) together with the equations of state for matter pressure \( p = \omega \rho \) and dark pressure \( \bar{p} = \Omega \rho \) we continue to calculate the required parameters for inflation, deceleration and then acceleration of the universe. In doing so, we rewrite the acceleration equation (23), scalar field equation (25) and conservation equation (22), respectively, in which the above assumptions are included as

\[
\alpha(\alpha - 1) + \frac{8\pi G}{3} \rho_0 (1 + \Omega) = 0, \tag{26}
\]

\[
\gamma(\gamma - 1) + 3\alpha\gamma - \frac{8\pi G}{3} \rho_0((1 - 3\omega) + 2\Omega) = 0, \tag{27}
\]

\[
2\rho_0[(2 + \Omega)\alpha - 1] = 0, \tag{28}
\]

where \( \delta = -2 \) has been used due to the consistency with the power law behavior \( t^{3\alpha - 3} \) in the conservation equation. The demand for acceleration \( \ddot{a} > 0 \) through Eq. (23) with the assumptions \( \rho(t) = \rho_0 t^\delta \) and \( \bar{p} = \Omega \rho \), requires \( \rho_0(1 + \Omega) < 0 \) or \( \Omega < -1 \) which accounts for a negative dark pressure. This negative domain of \( \Omega \) leads through the conservation equation (22) to \( \alpha > 1 \) which indicates an accelerating universe as expected. On the other hand, using Friedmann equation we obtain \( \alpha = \frac{1}{2 + \Omega} \) which together with the condition \( \alpha > 1 \) requires that \( -2 < \Omega < -1 \). Now, one may recognize two options as follows.

The first option is to attribute an intrinsic evolution to the parameter \( \Omega \) along the higher dimension so that it can produce the 4D expansion evolution in agreement with standard model including early inflation and subsequent deceleration, and also current acceleration of the universe. Ignoring the phenomenology of the evolution of the parameter \( \Omega \), we may require

\[
\left\{ \begin{array}{ll}
\Omega \gtrsim -2 & \text{for inflation} \\
\Omega > -1 & \text{for deceleration} \\
\Omega \lesssim -1 & \text{for acceleration}. \end{array} \right. \tag{29}
\]

The first case corresponds to highly accelerated universe due to a large \( \alpha \gg 1 \). This can be relevant for the inflationary era if one equate the power law with exponential behavior. The second case corresponds to a deceleration \( \alpha < 1 \), and the third case represents an small acceleration \( \alpha \gtrsim 1 \). In this option, there is no specific relation between the physical phase along extra dimension, namely \( \Omega \), and the ones defined in 4D universe by \( \omega \).

The second option is to assume a typical relation between the parameters \( \Omega \) and \( \omega \) as \( \Omega = f(\omega) \) so that

\[
\left\{ \begin{array}{ll}
\Omega \gtrsim -2 & \text{for } \omega = -1 \\
\Omega > -1 & \text{for } \omega = \frac{1}{3} \\
\Omega \lesssim -1 & \text{for } \omega = 0. \end{array} \right. \tag{30}
\]

The case \( \omega = -1 \) corresponds to the early universe and shows a very high acceleration due to \( \alpha \gg 1 \). The case \( \omega = \frac{1}{3} \) corresponds to the radiation dominant era and shows a deceleration \( \alpha < 1 \). Finally, the case \( \omega = 0 \) corresponds to the matter dominant era and shows an small acceleration \( \alpha \gtrsim 1 \) in agreement with observations.

\[5\text{As we discussed earlier, the fifth dimensional pressure component could be a dark property of conventional matter }\rho, \text{ through a dark parameter }\Omega, \text{ according to } \bar{p} = \Omega \rho.\]
Conclusion

A (4 + 1)-dimensional universe consisting of a (4 + 1) dimensional metric of Robertson-Walker type coupled with a (4 + 1) dimensional energy-momentum tensor in the framework of non-compact Kaluza-Klein theory is studied. In the matter part, there is energy density $\rho$ together with pressure $p$ subject to 4D part of the (4 + 1) dimensional energy-momentum tensor, and a dark pressure $\bar{p}$ corresponding to the extra-dimensional part endowed by a scalar field. A particular (anisotropic) equation of state in 5D is used for the purpose of realizing the 4D expansion in agreement with observations. This is done by introducing two parameters $\omega$ and $\Omega$ which may be either independent or related as $\Omega = f(\omega)$. The physics of $\omega$ is well known but that of the parameter $\Omega$ needs more careful investigation based on effective higher dimensional theories like string theory or Brane theory. The reduced 4D and extra-dimensional components of 5D Einstein equations together with different equations of state for pressure $p$ and dark pressure $\bar{p}$ may lead to a 4D universe which represents early inflation, subsequent deceleration and current acceleration. In other words, all eras of cosmic expansion may be explained by a single simple mechanism.

The important point of the present model is that the reduced Einstein field equations are free of 4D pressure and are just affected by an effective pressure produced by the 4D energy density and dark pressure along the extra dimension. This provides an opportunity to consider the expansion of the universe as a higher dimensional effect and so justify the unexpected current acceleration in the middle of matter dominant era, along this line of thought. Moreover, there is no longer “coincidence problem” within this model. This is because, in the present model there is no element of “dark energy” at all and we have just one energy density $\rho$ associated with ordinary matter. So, there is no notion of coincidental domination of dark energy over matter densities to trigger the acceleration at the present status of the universe. In fact, a dark pressure with different negative values along the 5th dimension by itself may produce expanding universe including inflation, deceleration and acceleration without involving with the coincidence problem. These stages of the 4D universe may occur as well because of negative, positive and zero values of the four dimensional pressure, respectively, which leads to a competition between energy density $\rho$ and dark pressure $\bar{p}$ in the acceleration equation \cite{23}. For the same reason that there is no element of dark energy in this model, the apparent phantom like equation of state for dark pressure $\Omega < -1$ is free of serious problems like unbounded from below dark energy or vacuum instability \cite{25}.

The above results are independent of the signature $\epsilon$ by which the higher dimension takes part in the 5D metric. Moreover, the role played by the scalar field along the 5th coordinate in the 5D metric is very impressed by the role of scale factor over the 4D universe. At early universe during the inflationary era the scalar field is highly suppressed and the 5th coordinate is basically ignored in 5D line element. At radiation dominant era the scalar field is much less suppressed and the 5th coordinate becomes considerable in 5D line element. Finally, at matter dominant era the scalar field and its possible fluctuations starts to be super-suppressed and the observable effect of 5th coordinate becomes vanishing in 5D line element at $t \approx 10^{17}$ Sec, leaving an effective 4D universe in agreement with observations.

A clear similarity is seen between the results of our 5D non compact model and that of multi-dimensional compact one \cite{22}. Both of these models predict an effective 4D pressure,
expressed in terms of the components of the higher dimensional energy-momentum tensor, capable of being negative to explain the acceleration of our present universe. Moreover, both higher dimensional metrics dynamically evolves towards an effective four-dimensional one.

Acknowledgment

This work has been financially supported by the Research Institute for Astronomy and Astrophysics of Maragha (RIAAM).

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