New holographic dark energy and the modified Bekenstein-Hawking entropy

Stefano Viaggiu,
Dipartimento di Matematica, Università di Roma “Tor Vergata”,
Via della Ricerca Scientifica, 1, I-00133 Roma, Italy.
E-mail: viaggiu@axp.mat.uniroma2.it

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Abstract

We show that in the standard derivation of the holographic dark energy some conceptual issues are present. In particular, the formula used in the literature to avoid black holes is not suitable in expanding universes. A more suitable expression for a holographic motivated dark energy must contain the energy density of the remaining matter content of the universe. However, we show that under some reasonable hypothesis, we can obtain a new physically motivated expression for the holographic dark energy. By considering an appropriate time-dependence of the saturation-level parameter, a de Sitter phase arises. Moreover, by adopting an argument similar to the original Bekenstein one, our approach justifies a correction of the Bekenstein-Hawking entropy for non-static isotropic expanding universes. Finally, we write down the equation of state of a black hole embedded in Friedmann spacetimes.

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1 Introduction

A lot of observations during the past decade (see [1] [2] [3]) strongly support a present day accelerating universe. In the standard ΛCDM cosmological model, this acceleration is caused by the the presence of the so-called dark energy expressed in terms of a cosmological constant Λ representing about 70% of the present universe matter-energy. However, the physical origin of
this constant is still obscure. Moreover, the reasons why $\Lambda$ is so small and
begins to dominate only recently still remains mysterious.

Many attempts are present in the literature [4, 5, 6, 7, 8], based on
quintessence models, k-essence models, phantom models to cite some of
them, to alleviate the issues of the standard cosmological paradigm. Per-
haps the more intriguing alternative is given by the holographic prnciple
(see for example [9, 10, 11, 12]). By following this principle, one can obtain
an expression for the dark energy density depending on some unspecified
size of the universe that can justify the smallness of the cosmological con-
stant. Thanks to its relevance in the field of dark energy models, we analyze
critically the standard derivation leading to the upper holographic limit for
the energy density. In this context, the aim of this paper is to show the
weakness of the standard derivation together with a possible solution to al-
leviate these issues and its physical consequences. In section 2 we analyze
the standard derivation together with our criticisms. In section 3 we attempt
to amend the standard derivation by using exact theorems of general rela-
tivity concerning black hole formation. In section 4 we study a modification
of the Bekenstein-Hawking entropy for black holes embedded in Friedmann
spacetimes. Finally, section 5 collects some conclusions.

2 Criticisms of the standard derivation

In [13] has been suggested that the limit set by the black hole formation can
be applied in quantum field theory to justify a short distance cut-off. This
idea has been analyzed in the context of dark energy models. In particular,
denoting with $\rho_\Lambda$ the quantum zero-point energy density, the total amount
of mass-energy in a region of size $L$ cannot exceed the mass of a black hole
of the same size:

$$\rho_\Lambda = k \frac{3c^2}{8\pi G L^2},$$

(1)

where the real constant $k \in (0, 1]$ denotes the saturation level: for $k > 1$
black hole arises. This crude estimation presents several issues.

First of all, the derivation of (1) makes use of the well known condition
for black holes suitable for static spherically symmetric spacetimes. In a
static context, $L$ represents a radial radius in spherical coordinates, not a
proper distance, while in the holographic context $L$ is a proper distance. In
this regard, some justification should be done for the fact that condition (1)
works when proper distances are used.

A second criticism is that condition (1) holds for a spherical black hole.

2
General relativity itself cannot account for a maximal allowed value for the energy-density by using black hole physics only. This can be obtained when quantum mechanics arguments are taken into account [14, 15]. Moreover, its value depends on the shape of the collapsing object. For a spherically symmetric body immersed in a minkowskian spacetime, the bound (1) is correct. But for a general shape, this value can change. In any case, if we consider Friedmann universes, sphericity is an exact symmetry for the spacetime and then seems to be a natural choice.

A third criticism concerns the interpretation of condition (1). This represents the maximal mass-energy density excess within a certain volume allowed to avoid a black hole. Thus we have a given unperturbed background (minkowskian for (1)) and a perturbation representing the collapsing matter. The Schwarzschild solution represents an exact spherically symmetric perturbation of minkowskian spacetime. Hence, when we apply the condition (1) to Friedmann universes, the second member of (1) must represent the energy density excess of finite extension with respect to a given unperturbed configuration, otherwise the naive use of the formula (1) is not suitable and can also lead to wrong results. To this purpose, consider a Friedmann flat background filled with ordinary dust dark matter $\rho_m$ and a spherical perturbation given by a time constant energy density, i.e. $\rho_\Lambda = k c^2 \Lambda / (8 \pi G)$. Condition (1) for the holographic cosmological constant it gives:

$$\Lambda = \frac{k}{L^2}. \tag{2}$$

Since the cosmological constant fills the whole universe, then one can in principle perform the limit $L \to \infty$ and then obtain $\Lambda = 0$ in order to avoid black holes. But, as well known, in the standard cosmological model the presence of a small but finite cosmological constant does not lead to black hole formation. As a consequence, we cannot take, for example, the standard cosmological concordance $\Lambda$CDM model and then apply merely condition (1). Condition (1) must thus be interpreted as an energy density excess perturbation of finite extension with respect to a given background, i.e. in our case a cosmological background not a minkowskian one.

Finally, the black hole formation also depends on the dynamics of the collapsing matter. Hence, it is natural to suppose that in an expanding non-static universe the Hubble flow plays an important role for the formation of black holes. This is certainly true if we look for black holes formation in a Friedmann context. In fact, there exist exact theorems in general relativity [16, 17, 18, 19] giving sufficient conditions for the non formation of trapped surfaces for exact spherical perturbations. As well known from the Hawking
theorems [20], the presence of trapped surfaces is related to the formation of the black hole singularity. In the next section, with the help of these theorems, we alleviate the weakness depicted above.

3 Holographic principle from suitable black holes theorems

To our purposes, it is sufficient to consider the theorem shown in [17, 19] for open Friedmann flat cosmologies.

Consider, in a flat Friedmann cosmology with a background energy-density $\rho_m$, a spherical surface of proper radius $L$ and proper area $A$ and a perturbation of proper mass $\delta M > 0$ within $A$. Suppose that the current matter perturbation $\delta J$ is vanishing on the boundary of $A$, if

$$\frac{\delta M G}{c^2} < \frac{L}{2} + A \sqrt{G \frac{\rho_m}{6\pi c^2}},$$

then $A$ is not trapped. Note that $L, A$ are respectively the proper length and the proper area with respect to the perturbed configuration, i.e. backreaction is taken into account. Moreover, the upper bound for the mass-excess depends on the energy-density of the background on which the perturbation acts.

In the following, we make the reasonable assumption (radial inhomogeneities are not considered) that the mass-excess within $A$ is spatially constant, i.e. $\delta M = \delta \rho(t) 4/3\pi L^3$. Under this assumption, by following the same reasonings leading to (1), from condition (3) we obtain the new holographic energy given by:

$$\rho_\Lambda = \frac{k}{G} \left[ 3 \frac{c^2}{8\pi L^2} + 3 \frac{L c}{L c} \sqrt{\frac{G \rho_m}{6\pi}} \right].$$

Note that the new holographic dark energy (4) depends on the matter content of the background that is supposed, for our purposes, to be filled with dark matter. This is in agreement with the fact that theorems at our disposal relate the proper mass-excess to the matter content just present in the universe. Hence, any physically reasonable dark energy expression motivated by the holographic principle must be a function of the background energy density. In practice, the maximum energy-density mass excess must depend on the matter-energy present in the universe and on its dynamics. In a Friedmann context, it is natural that the Hubble flow $H$ makes more difficult the black holes formation and as a consequence an higher value for
the energy-density excess is expected. Otherwise, the resulting dark-energy expression is conceptually wrong in light of black hole formation theorems. In fact, note that the term involving $\rho_m$ in (4) can as well be of the same order of the first term and depends, thanks to the Friedmann equations, on the Hubble flow $H$. This confirms that, according to physical intuition, in the presence of an expanding universe it is more difficult to build black holes.

The expression (4) can be put into Einstein equations together with the density $\rho_m$ and then solve them with a suitable expression for $L$ and $k$. To this purpose, thanks to (4), for the running cosmological constant we have:

$$\Lambda(t) = k \left[ \frac{3}{L^2} + \frac{24\pi}{cL} \sqrt{\frac{G\rho_m}{6\pi}} \right].$$

(5)

According to the reasonings of the section 2, we cannot naively take the limit $L \to \infty$ in (5). In fact, in a given gedanken localizing experiment, reasonable limits must be imposed to the size of the localizing object, i.e. causality cannot be violated. To this purpose, expression (5) can be written in terms of the density parameters:

$$\Omega_\Lambda(t) = k \left[ \frac{c^2}{H^2L^2} + \frac{2c}{cL} \sqrt{\frac{\Omega_m(t)}{HL}} \right],$$

(6)

together with the Friedmann equation $\Omega_m(t) + \Omega_\Lambda(t) = 1$.

After calculating expression (6) at the present time $t_0$ and solving with respect to $L_0H_0$ we have:

$$L_0H_0 = \frac{c}{\Omega_\Lambda_0} \left[ k\sqrt{\Omega_{m0}} + \sqrt{k^2\Omega_{m0} + k\Omega_\Lambda_0} \right].$$

(7)

From a physical point of view, a value for $k$ such that $k \simeq 1$ seems most desirable, but obviously it depends on the physical motivation that inhibits the saturation with the maximum allowed value for $\delta\rho$.

Moreover, we expect that the size $L$ is of the order of the particle horizon or less and not of the future event horizon. This is because a reasonable limit for an ideal experiment on large scales involving black holes is dictated by causality. The future event horizon struggles with causality. These choices are in agreement with the idea that dark energy has an origin similar to the one leading to Hawking or Unruh effects, where the presence of horizons play an important role. For a numerical example we set $k = 1$ and the concordance values $\Omega_\Lambda_0 \simeq 0.68, \Omega_{m0} \simeq 0.32$. As a result we obtain $L_0 \sim$
2.4c/H_0, being c/H_0 the Hubble radius. Remember that for the particle horizon of the ΛCDM model we have L_0 \sim 3.3c/H_0. Note that without the fundamental term involving \( \mathcal{P}_m \), i.e. by using practically the expression (1), we obtain, for \( k = 1 \), \( L_0 \sim 1.2c/H_0 \).

A viable model for holographic dark energy must also contain a phase, after the recombination era, where dark energy is negligible with respect to dark matter. At this epoch, by denoting with \( L \) the particle horizon, we have \( L \simeq 3c/H \). Moreover, by considering soon after recombination \( \Omega_m \simeq 0.96 \) and \( \Omega_\Lambda \simeq 0.04 \), from (7) we obtain \( k \simeq 0.065 \). As a consequence, by setting the saturation-level parameter \( k \) as a truly time constant near to the limit \( k \simeq 1 \), it is not possible to obtain an early matter dominated era, which is obviously in contradiction with cosmological data. This is a typical problem plaguing holographic models. The way in which in the literature one attempts to alleviate this problem is to consider ad hoc interactions between dark matter and dark energy. Our approach strongly suggests that a physically viable expression for \( \rho_\Lambda \), motivated by holography, must depend on the other kinds (and their dynamics) of matter-energy density present in the universe in a form dictated by (4). To alleviate this problem, the most simple assumption is to consider a time-dependent \( k(t) \) reaching the saturation value only asymptotically. In particular, it may be supposed that the emission mechanism to create 'holographic' dark energy is similar to the one of a black body radiation emitted at some radius of the order of (or less) the particle horizon. In this regard, \( k(t) \) can be seen as a kind of efficiency parameter: a low \( k \) indicates an inefficient emission, while a value near to the saturation value indicates that the emission is efficient as the one of a perfect black body radiation, i.e. dark energy is an ideal holographic radiation. In fact, it is physically reasonable that the efficiency of the emission process of the holographic dark energy depends on the presence of other kinds of matter: the process becomes more and more efficient as \( \mathcal{P}_m \) decreases. If we take for \( L \) instead of the particle horizon the Hubble one, then by setting again \( \Omega_\Lambda_0 \simeq 0.68, \Omega_{m0} \simeq 0.32 \), we have \( k_0 \simeq 0.32 \). We stress again that a time-running \( k \) is certainly more justified in our context with respect to the old one given by (11). This idea is not exotic in cosmology. In fact, during the radiation dominated era, before nucleosynthesis, the radiation was in a black body configuration, as firstly conjectured by Gamow.

To be more quantitative, we are tempted to write:

\[
\rho_\Lambda = k \frac{4\sigma_\Lambda}{c^3} T^4 = k \frac{c^2}{G} \left[ \frac{3}{8\pi L^2} + \frac{3}{cL} \sqrt{\frac{G\mathcal{P}_m}{6\pi}} \right],
\]

(8)
In any case, independently on the correctness of the formula (8), the interpretation of the parameter $k$ as a time-dependent efficiency parameter remains a viable possibility. In this view, the saturation value $k = 1$ would correspond to an 'ideal' holographic emitter.

Hence, if we consider $L$ of the order of the particle horizon (or less, the Hubble radius), the variability of $k$ with time is essential to regain agreement with real astrophysical data. In alternative, we can use for $L$ the expression of the future event horizon with a constant $k$ but, as stated above, it is plagued by causality problems that have not yet been addressed.

Concerning the Hubble radius proposal, as well known [10], by using the Hubble radius for $L$ with the old expression (1) and a constant $k$, we cannot obtain dark energy. However, note that the arguments of [10] do not apply in our new holographic paradigm because $k$ is a time function and hence $\rho_\Lambda$ is not merely proportional to $H^2$ and thus does not scale as a dust-like solution. Nevertheless, when $L = c/H$, the dark density (8) assumes the expression

$$\Omega_\Lambda(t) = k(t) \left[1 + 2\sqrt{\Omega_m(t)}\right]. \quad (9)$$

Since $\Omega_m(t) + \Omega_\Lambda(t) = 1$, from (9) we get:

$$k(t) = \frac{1 - \Omega_m}{1 + 2\sqrt{\Omega_m}}. \quad (10)$$

The expression (10) has the right behaviour for an emissivity parameter: at the recombination $k \simeq 0$ and a late times $k \to 1$. At the present time $k_0 \simeq \Omega_{m0} \simeq 0.32$, that could be an interesting coincidence. Recently, in [21] has been shown that the Hubble radius has an unexpected link concerning the value of the dark energy, although in apparently different context with respect to holography. Note that this nice feature cannot hold for $L$ given by particle horizon, future event horizon or inverse square Ricci radius. Moreover, by using the Friedmann equations with the expression (11) we have:

$$\varpi_m = \frac{4H^2(1 - k)^2}{G\left[k\sqrt{\frac{32\pi}{3}} + \sqrt{\frac{32\pi}{3}(k^2 - k + 1)}\right]^2}. \quad (11)$$

By inserting expression (11) in (4) we see that, for a running $k$, $\rho_\Lambda$ cannot be a dust component. This happens only for $k$ constant. Hence our model introduces a well defined link between the dark sectors of the universe driven by the emissivity parameter. By solving equation (10) for $\Omega_m(t)$ in terms of $k(t)$, we can obtain the empirical expression for $k(t)$ from the distance-redshift relation on the light cone. This can be calculated also for the other
cases, i.e. particle horizon, inverse Ricci square, agegraphic holography etc. Note that the use of the Hubble radius is consistent with the inflationary period. Here $\Omega_m = 0$ and the Hubble radius is constant, i.e. $L = c/H_I$, where $H_I$ is the inflationary potential. In this context, $k(t) < 1$ in order to assures the Planck-level measured non-gaussianity.

Concerning the far future limit $t \to \infty$, it is easy to see that, under suitable conditions on $k(t)$, also with $L$ the Hubble radius, we can have a late time de Sitter phase. To show this, we write the equation for $\rho_\Lambda$:

$$\rho_{\Lambda, t} + 3H\rho_{\Lambda}(1 + \gamma_{\Lambda}) = 0,$$

(12)

where $\gamma_{\Lambda}$ is the equation of state parameter for the holographic dark energy. As an example, we look for solutions such that for $t \to \infty$, $H(t)$ approaches a constant value $H_s \sim c\sqrt{\Lambda}/3$. To this purpose, thanks to (11), note that for $t \to \infty$, $\sqrt{\rho_m} \sim H(1 - k)$. As a result, since $k \to 1$, we have that in this asymptotic limit $H\sqrt{\rho_m} \sim H^2(1 - k) = o(H^2)$. Under the conditions above, the first term in (4) is dominant with respect to the one involving $\rho_m$. Finally, by putting the first term of (4) in (12), the desired asymptotic behaviour for $k(t)$ such that $H \to H_s$ and $\gamma_{\Lambda} \to -1$ is

$$k(t) = \left(\frac{H_s}{H}\right)^2 + o(1).$$

(13)

We stress that the behaviour (13) for $k(t)$ must hold only asymptotically for $t \to \infty$. Nevertheless, note that also by taking $k(t) \sim 1/H(t)^2$ after recombination, from expression (11), we see that the dark-energy density (4) looks like a constant with added a monotonically decreasing term. Summarizing, a time-dependent $k$ can account for an early and a later de Sitter phase together with a dark matter dominated era also by using the mistreated Hubble radius in the holographic context.

In any case, we stress again that the crude estimation (1), although can be justified in a more rigorous way, is not suitable in a Friedmann cosmological context. As clearly shows expression (6), the term involving $\rho_m$ is at least at present time, of the same order (but greater) of the old usual holographic term and therefore cannot be neglected.

We leave in a further paper the study of the efficiency parameter in terms of the astrophysical data by using for $L$ the particle horizon and Hubble radius expressions.
4 Bekenstein-Hawking entropy in expanding universes

As a first consideration of this section, note that the reasoning leading to the expression (1) can be formulated in terms of the well known Bekenstein-Hawking entropy, initially proposed for black holes in a vacuum asymptotically flat spacetime. Hence, a first interesting consequence of our result is that expression (4) suggests a correction term caused by the degrees of freedom that are due to the non-static nature of Friedmann spacetimes. Denoting with $k_B$ the Boltzmann constant, with $L_P$ the Planck length and with $\rho_m$ the matter-energy content of the universe and adopting the original Bekenstein argument [22], we have:

$$S_{BH} = \frac{k_B A}{4L_P^2} + \frac{k_B A^3}{cL_P^3} \sqrt{G \rho_m / 6}.$$  \hspace{1cm} (14)

The correction term takes into account the degrees of freedom of a non-static (flat) Friedmann universe. By using theorems present in [16, 19], the expression (14) can also be generalized to a Friedmann universe with negative curvature: in this case another positive term arises proportional to $\sim H^2 A^2$.

As well known, the identification of the surface $A$ with the analogue of the event horizon in the Schwarzschild case in expanding universes is not (see for example [23]) a simple task. However, the theorem (4) leaves undetermined the nature of the enclosing surface $A$: it can be as well a light-like surface that can be identified, for example, with the apparent horizon of the black hole. These issues, although important, play no role at this stage of our formulation of the black hole thermodynamics in expanding universes.

In a static universe, the added term is vanishing. Moreover, if all the energy-densities present in the universe $\sum_i \rho_i$ satisfy the Friedmann equation $H^2 = 8/3\pi G \sum_i \rho_i$ (i.e. we have a spherical black hole embedded in a Friedmann expanding universe, see [23, 24]), then expression (14) becomes

$$S_{BH} = \frac{k_B A}{4L_P^2} \left( 1 + \frac{H}{c} \sqrt{\frac{A}{\pi}} \right).$$ \hspace{1cm} (15)

Note that, by taking for $A$ the Hubble area, the entropy (15) becomes 3 times the usual expression. According to holography, an higher admissible upper bound for the density implies an higher upper bound for the entropy. Generally, the correction term is negligible when $A << c^2/H^2$. When the dimensions of an object become comparable with the Hubble radius, this
correction cannot be neglected. We expect that the added term will be relevant also at the early stage of the universe, near the Planck era.

It is interesting to study the fate of (15) near the big bang. The first term is always vanishing at $t = 0$. Concerning the added term, if $H \sim 1/t$, then for a power law cosmologies with $a(t) \sim t^\alpha$, for $\alpha \in (0, 1/3)$, $S_{BH} \to \infty$. Conversely, for a spacetime with $\alpha > 1/3$, $S_{BH} \to 0$. Interestingly enough, for stiff matter, i.e. $\alpha = 1/3$, then entropy reaches at the big bang a finite non vanishing limit.

As a further remark, we analyze the expressions (14) and (15) from the point of view of the first law of thermodynamics. To this purpose, note that in (14) the term $A^{3/2}$ can be written also as $V$. Since we are in a spherically symmetric context, in (14) we have firstly explicitly written this term as a function of the proper area. However, in light to first law of thermodynamics, could be more natural and useful to express the added term as a function of the proper volume of the apparent horizon of the black hole. With this choice, we have:

$$S_{BH} = \frac{k_B A}{4 L_P^2} + \frac{3k_B}{2c L_P} V H. \tag{16}$$

By differentiating (16) we get

$$dS_{BH} = \frac{k_B}{4 L_P^2} dA + \frac{3k_B}{2c L_P^2} V dH + \frac{3k_B}{2c L_P^2} H dV. \tag{17}$$

The first two terms in the right side of (17) can be interpreted as representing $1/T$ times the internal energy of the black hole. In particular, the one involving $dH$ can be seen as the increases ($dH \geq 0$) of the internal energy due to the expansion of the universe caused by the presence of some unspecified kind of matter. The term proportional to $dV$ can be seen as a work term due to the Hubble flow. In particular, we can write:

$$\frac{P}{T} = \frac{3k_B H}{2c L_P^2}, \tag{18}$$

where $P$ denotes the pressure. For a universe with negative curvature, a further positive term proportional to $H^2 A^2$ also appears in (18). Unfortunately, we have not been able to obtain a convincing expression for the temperature $T$ from general considerations. In fact, also for an isotropic expanding universe, we have not at our disposal an universally accepted analogous of the surface gravity parameter for a black hole in an asymptotically flat spacetime. To do this, an explicit black hole solution in expanding universes can
be useful. To this purpose, we stress again that all the distances, areas
must be calculated from the exact solution representing a spherically sym-
matic black hole embedded in an expanding Friedmann universe. However,
by denoting with $R_H$ the Hubble radius, we can write formula (18) in the
following form:

$$PR_H L_P^2 = \frac{3}{2} k_B T.$$  \hfill (19)

For an ideal gas we have $PV = N k_B T$, where $N$ is the particles number.
It is interesting that in the formula (19) explicitly appears the apparent
horizon. After multiplying both members of (19) for the proper volume $V$
of the apparent horizon of the black hole, we have:

$$PV = \left( \frac{3V}{2R_H L_P^2} \right) k_B T.$$

(20)

Suppose now to decompose the proper volume $V(t)$ of the horizon in $n(t)$
elementary spherical cells of fixed proper radius $L_P$, i.e. $V = 4/3 \pi n(t)L_P^3$.
Then expression (20) becomes:

$$PV = \left( \frac{2\pi}{R_H} \right) n(t) K_B T.$$

(21)

The expression (21) generalizes the usual equation of state suitable for ideal
gases in the context of black hole thermodynamics in Friedmann flat expanding
spacetimes. Formula (21) can have a nice physical interpretation. When
the universe is cold, as at present time, many quantum degrees of freedom
are frozen: this is described by the low ratio $L_P/R_H$ at present time. But
when the universe has been hot, more and more degrees of freedom have
been excited ($L_P/R_H \sim 1$). At the Planck epoch, when $R_H \sim L_P$, we have
$PV \sim nK_B T$, with $2\pi$ a geometric factor due to the sphericity of the black
hole (there is not reason to put a sphere into a rectangular box). A similar
phenomenon happens for the ordinary statistical mechanics. Moreover,
note that $n(t)$ is an integer and so also $R_H/L_P = N_P(t)$, provided that the
Planck length is the minimum distance available in the real world. Hence
the equation (21) can be written in the expressive form:

$$PV = \left( \frac{2\pi}{N_P} \right) K_B T,$$

(22)
a kind of Bohr-Sommerfield quantization rule for the black hole equation of
state. We do not speculate further on this formula.
The presence of a volume term in \((17)\) could struggle with the holographic principle. However, note that the work term \(PdV\) of usual thermodynamics arises thanks to a volume dependence of the entropy, and is unavoidable. Otherwise, no work would be associated to the expansion of \(A\), that seems rather unlikely.

As a further remark, note that all the reasonings of this section are a consequence of the theorem leading to \((1)\) and thus do not depend on the interpretations concerning the nature of the holographic dark energy.

As a final consideration, it should also be noticed that the theorem \((4)\) remains valid if we substitute the energy-density \(\rho_m\) with a constant energy-density, i.e. in a de Sitter expanding universe. In such a case, we have again the formula \((16)\), but with \(H(t)\) the constant de Sitter value \(H = \sqrt{\Lambda/3}\). In this case, the term involving \(dH\) in \((17)\) is vanishing, and as a result the internal energy of a black hole in a de Sitter universe is left unchanged with respect to the asymptotically flat case.

5 Conclusions

In this paper, we have discussed the usual calculations supporting the holographic dark energy expression. The usual derivation is plagued by the use of expressions for black holes formation that are not suitable in a cosmological context. In fact, expression \((11)\) used in the literature is suitable for the black holes formation caused by a spherically symmetric static configuration in the vacuum. Unfortunately, condition \((11)\) cannot work in a cosmological context. In an expanding universe, as well known from theorems of general relativity, the formation of trapped surfaces depends on the matter content of the universe and on its dynamics. In particular, the Hubble flow \(H\) plays an important role making black hole formation more difficult. In this context, by using the theorems present in \([17, 19]\), we can obtain a new expression for the holographic dark energy. In order to have a conceptually and mathematically correct expression for an holographic motivated dark energy, it is essential its dependence on the background density, for example dark matter after recombination. A choice for \(L\) based on the future event horizon is plagued by serious yet not resolved causality problems. Moreover, in these models the far future limit \(t \to \infty\) is rather problematic and can lead to a spacetime singularity.

We have tested our new holographic dark energy by requiring that at present time the proper length \(L\) be of the order of the particle horizon or the Hubble radius and that soon after recombination the dark matter dom-
inated the dark energy. The only way to satisfy these requirements is to take the parameter $k$ depending on the cosmic time $t$. In this regard, the parameter $k(t)$ can be seen as measuring the efficiency of the holographic dark emission. Also by using for $H$ the Hubble radius expression and for $k$ a suitable time-dependent function, we can account for an early and late times de Sitter phase together with a dark matter dominated era after recombination. Remember that in a flat Friedmann spacetime, the apparent horizon is nothing else that the Hubble radius.

It is important to note that with our new view of the holographic dark energy, the time variability of $k$ can be more easily justified.

It is worth to note that also with the new holographic dark energy (4), an interaction terms can be considered in the right hand side of equation (12) (and obviously also for $\rho_m$).

As a consequence of our approach, we are in the position to generalize the Bekenstein-Hawking entropy in an expanding universe. This allows us to write the equation of state of a black hole (equation (21)) together with a possible physical motivation. Further investigations are needed to a more deep physical understanding.

As a final hint of this paper, we can conclude with the claim that all holographic motivated reasonings in a cosmological context must take into account, from the onset, the dynamical nature of the spacetime due to the Hubble flow.

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