A Remark on the Continuous Subsolution Problem for the Complex Monge-Ampère Equation

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Abstract
We prove that if the modulus of continuity of a plurisubharmonic subsolution satisfies a Dini-type condition then the Dirichlet problem for the complex Monge-Ampère equation has the continuous solution. The modulus of continuity of the solution also given if the right hand side is locally dominated by capacity.

Keywords Dirichlet problem · Complex Monge-Ampère equation · Weak solutions · Subsolution problem

Mathematics Subject Classification (2010) 32W20 · 32U40

1 Introduction

In this note, we consider the Dirichlet problem for the complex Monge-Ampère equation in a strictly pseudoconvex domain $\Omega \subset \mathbb{C}^n$. Let $\psi$ be a continuous function on the boundary of $\Omega$. We look for the solution to the equation:

$$u \in PSH(\Omega) \cap C^0(\bar{\Omega}),$$

$$(dd^c u)^n = d\mu,$$

$$u = \psi \quad \text{on } \partial \Omega.$$

(1.1)
Here, \( PSH \) stands for plurisubharmonic functions, and \( d^c = i(\bar{\partial} - \partial) \). It was shown in [9] and [10] that for the measures satisfying certain bound in terms of the Bedford-Taylor capacity [4], the Dirichlet problem has a (unique) solution. The precise statement is as follows.

Let \( h : \mathbb{R}_+ \to (0, \infty) \) be an increasing function such that

\[
\int_1^\infty \frac{1}{x[h(x)]^{\frac{1}{n}}} dx < +\infty.
\]

We call such a function admissible. If \( h \) is admissible, then so is \( Ah \) for any number \( A > 0 \). Define

\[
F_h(x) = \frac{x}{h(x^{\frac{1}{n}})}.
\]

Suppose that for such a function \( F_h(x) \) a Borel measure \( \mu \) satisfies

\[
\int_E d\mu \leq F_h(\text{cap}(E))
\]

for any Borel set \( E \subset \Omega \). Then, by [9] the Dirichlet problem (1.1) has a solution.

This statement is useful as long as we can verify the condition (1.2). In particular, if \( \mu \) has density with respect to the Lebesgue measure in \( L^p \), \( p > 1 \) then this bound is satisfied [9]. By the recent results in [12, 13] if \( \mu \) is bounded by the Monge-Ampère measure of a Hölder continuous plurisubharmonic function \( \varphi \)

\[
\mu \leq (dd^c \varphi)^n
\]

in \( \Omega \), then (1.2) holds for a specific \( h \), and consequently, the Dirichlet problem (1.1) is solvable with Hölder continuous solution. The main result in this paper says that we can considerably weaken the assumption on \( \varphi \) and still get a continuous solution of the equation.

Let \( \varphi \in PSH(\Omega) \cap C^0(\overline{\Omega}) \), \( \varphi = 0 \) on \( \partial\Omega \). Assume that its modulus of continuity satisfies the Dini type condition

\[
\int_0^1 \frac{[\sigma(t)]^\frac{1}{2}}{t|\log t|} dt < +\infty.
\]

(1.3)

If the measure \( \mu \) satisfies \( \mu \leq (dd^c \varphi)^n \) in \( \Omega \), then the Dirichlet problem (1.1) admits a unique solution.

Theorem 1.1

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Let us mention in this context that it is still an open problem if a continuous subsolution \( \varphi \) implies the solvability of (1.1).

The modulus of continuity of the solution to the Dirichlet problem (1.1) was obtained in [3] for \( \mu = f dV_{2n} \) with \( f(x) \) being continuous on \( \overline{\Omega} \). We also wish to study this problem for the measures which satisfy the inequality (1.2). For simplicity, we restrict ourselves to measures belonging to \( H(\alpha, \Omega) \). In other words, we take the function \( h(x) = Cx^n\alpha \) for positive constants \( C, \alpha > 0 \) in the inequality (1.2).

We introduce the following notion, which generalizes the one in [8]. Consider a continuous increasing function \( F_0 : [0, \infty) \to [0, \infty) \) with \( F(0) = 0 \).
Definition 1.2 The measure $\mu$ is called uniformly locally dominated by capacity with respect to $F_0$ if for every cube $I(z, r) = I \subset B_I := B(z, 2r) \subset \subset \Omega$ and for every set $E \subset I$,

$$\mu(E) \leq \mu(I) F_0 (\text{cap}(E, B_I)).$$

(1.4)

According to [1], the Lebesgue measure $dV_{2n}$ satisfies this property with $F_0 = C_\alpha \exp(-\alpha/x^{-1/n})$ for every $0 < \alpha < 2n$. The case $F_0(x) = Cx$ was considered in [8]. We refer the reader to [5] for more examples of measures satisfying this property. Here is our second result.

Theorem 1.3 Assume $\mu \in \mathcal{H}(\alpha, \Omega)$ with compact support and satisfying the condition (1.4) for some $F_0$. Then, the modulus of continuity of the solution $u$ of the Dirichlet problem (1.1) satisfies for $0 < \delta < R_0$ and $2R_0 = \text{dist}(\text{supp} \mu, \partial \Omega) > 0$,

$$\varpi(\delta; u, \Omega) \leq \varpi(\delta; \psi, \partial \Omega) + C \left[ \left( \frac{\log R_0}{\delta} \right)^{-\frac{1}{2}} + F_0 \left( \frac{C_0}{[\log(R_0/\delta)]^{1/2}} \right)^{\alpha_1} \right],$$

where the constants $C, \alpha_1$ depend only on $\alpha, \mu, \Omega$.

2 Preliminaries

Here, we gather some basic facts from pluripotential theory taken from [4], and used in the sequel. Given a compact set $K$ in a domain $\Omega \subset \mathbb{C}^n$, its relative extremal function $u_K$ is given by

$$u_K = \sup \{ u \in PSH(\Omega) : u < 0, u \leq -1 \text{ on } K \}.$$

Its upper semicontinuous regularization $u^*_K$ is plurisubharmonic. When $u_K$ is continuous, we call $K$ a regular set. It is easy to see that the $\epsilon$-envelope

$$K_\epsilon = \{ z : \text{dist}(z, K) \leq \epsilon \}$$

of a compact set $K$ is regular, and thus any compact set can be approximated from above by regular compact sets.

The relative capacity of a compact set $K$ with respect to $\Omega$ (now usually called the Bedford-Taylor capacity) is defined by the formula

$$\text{cap}(K, \Omega) = \sup \left\{ \int_K (dd^c u)^n : u \in PSH(\Omega), -1 \leq u \leq 0 \right\},$$

and by [4], can be expressed as

$$\text{cap}(E, \Omega) = \int_K (dd^c u^*_K)^n.$$

We say that a positive Borel measure $\mu$ belongs to $\mathcal{H}(\alpha, \Omega)$, $\alpha > 0$, if there exists a uniform constant $C > 0$ such that for every compact set $E \subset \Omega$,

$$\mu(E) \leq C \left[ \text{cap}(E, \Omega) \right]^{1+\alpha}.$$
3 Proof of Theorem 1.1

In this section, we shall prove Theorem 1.1. We need the following lemma. The proof of this lemma is based on a similar idea as the one in [11, Lemma 3.1] where the complex Hessian equation is considered. The difference is that we have much stronger volume-capacity inequality for the Monge-Ampère equation.

Lemma 3.1 Assume the measure $\mu$ is compactly supported. Fix $0 < \alpha < 2n$ and $\tau = \alpha/(2n+1)$. There exists a uniform constant $C$ such that for every compact set $K \subset \Omega$,

$$
\mu(K) \leq C \left\{ \varpi \left( \exp \left( \frac{-\tau}{2[\text{cap}(K)]^{\frac{1}{n}}} \right) \right) + \exp \left( \frac{2n\tau - \alpha}{2[\text{cap}(K)]^{\frac{1}{n}}} \right) \right\} \cdot \text{cap}(K), \tag{3.1}
$$

where $\text{cap}(K) := \text{cap}(K, \Omega)$.

**Proof** Fix a compact subset $K \subset \subset \Omega$. Without loss of generality, we may assume that $K$ is regular. Denote by $\varphi_\varepsilon$ the standard regularization of $\varphi$ in the terminology of [10]. We choose $\varepsilon > 0$ so small that

$$
\text{supp } \mu \subset \Omega'' \subset \subset \Omega' \subset \subset \Omega,
$$

where $\Omega_\varepsilon = \{ z \in \Omega : \text{dist}(z, \partial \Omega) > \varepsilon \}$. Since for every $K \subset \Omega''$ we have

$$
C_0 \text{cap}(K, \Omega) \leq \text{cap}(K, \Omega') \leq C_0^{-1} \text{cap}(K, \Omega)
$$

(for a constant $C_0$ depending only on $\Omega, \Omega'$) in what follows we shall write $\text{cap}(K)$ for either one of these capacities. We have

$$
0 \leq \varphi_\varepsilon - \varphi \leq \varpi(\varepsilon) := \delta \quad \text{on } \Omega'.
$$

Let $u_K$ be the relative extremal function of $K$ with respect to $\Omega'$. Consider the set $K' = \{ 3\delta u_K + \varphi_\varepsilon < \varphi - 2\delta \}$. Then,

$$
K \subset K' \subset \left\{ u_K < -\frac{1}{2} \right\} \subset \Omega'. \tag{3.2}
$$

Hence, by the comparison principle [4],

$$
\text{cap}(K') \leq 2^n \text{cap}(K). \tag{3.3}
$$

Note that

$$
dd^c \varphi_\varepsilon \leq \frac{C}{\varepsilon^2} \, dd^c |z|^2, \quad \| \varphi_\varepsilon + u_K \|_\infty =: M \leq \| \varphi \|_\infty + 1. \tag{3.4}
$$

The comparison principle, the bounds (3.4), and the volume-capacity inequality from [1, Theorem A] (in the last inequality below) give the following:

$$
\int_{K'} (dd^c \varphi)^n \leq \int_{K'} (dd^c (3\delta u_K + \varphi_\varepsilon))^n
\leq 3\delta \int_{K'} [dd^c (u_K + \varphi_\varepsilon)]^n + \int_{K'} (dd^c \varphi_\varepsilon)^n
\leq 3\delta M^n \text{cap}(K') + C(\alpha) \varepsilon^{-2n} \exp \left( \frac{-\alpha}{[\text{cap}(K')]^{\frac{1}{n}}} \right) \text{cap}(K').
$$

Choose

$$
\varepsilon = \exp \left( \frac{-\tau}{[\text{cap}(K')]^{\frac{1}{n}}} \right)
$$
(we assume that $\varepsilon$ is so small that it satisfies (3.2), otherwise the inequality (3.1) holds true by increasing the constant) and plug in the formula for $\delta$ to get that

$$\mu(K) \leq \int_{K'} (dd^c \varphi)^n$$

$$\leq 3 M^n \sigma \left( \exp \left( \frac{-\tau}{[\text{cap}(K')]^{\frac{1}{n}}} \right) \right) \cdot \text{cap}(K') + C \exp \left( \frac{2n\tau - \alpha}{[\text{cap}(K')]^{\frac{1}{n}}} \right).$$

This combined with (3.3) gives the desired inequality. \hfill \Box

We are ready to finish the proof of the theorem. It follows from Lemma 3.1 that

$$h(x) = \frac{1}{C \sigma(\exp(-\tau x))}$$

is a function which satisfies (1.2) for the measure $\mu$ once we have

$$\int_1^{\infty} \frac{1}{x[h(x)]^{\frac{1}{n}}} \, dx < +\infty.$$  

By changing the variable $s = 1/x$, and then $t = e^{-\tau/s}$, this is equivalent to

$$\int_0^{e^{-\tau}} \frac{[\sigma(t)]^{\frac{1}{n}}}{t[\log t]} \, dt < +\infty.$$  

The last inequality is guaranteed by (1.3). Thus, our assumption on the modulus of continuity $\sigma(t)$ implies that $h$ is admissible in the case of $\mu$ with compact support. Then, by [10, Theorem 5.9] the Dirichlet problem (1.1) has a unique solution.

To deal with the general case, consider the exhaustion of $\Omega$ by compact sets $E_j = \{ \varphi \leq -1/j \}$ and define $\mu_j$ to be the restriction of $\mu$ to $E_j$. Denote by $u_j$ the solution of (1.1) with $\mu$ replaced by $\mu_j$. By the comparison principle

$$u_j + \max(\varphi, -1/j) \leq u \leq u_j,$$

and so the sequence $u_j$ tends to $u = \lim u_j$ uniformly and the continuity of $u$ follows. The proof is complete.

4 The Modulus of Continuity of Solutions

In this section, we study the modulus of continuity of the solution of the Dirichlet problem with the right hand side in the class $H(\alpha, \Omega)$ under the additional condition that a given measure is locally dominated by capacity.

In what follows we need [8, Lemma 2] whose proof is based on the lemma due to Alexander and Taylor [2, Lemma 3.3]. For the reader’s convenience, we give the proofs. The latter can be simplified by using the Błocki inequality [6].

Lemma 4.1 Let $B' = \{ |z - z_0| < r \} \subset \subset B = \{ |z - z_0| < R \}$ be two concentric balls centered at $z_0$ in $\mathbb{C}^n$. Let $u \in PSH(B) \cap L^\infty(B)$ with $u < 0$. There is a constant $C = C(n, \frac{R}{r})$ independent of $u$ such that

$$\int_{B'} (dd^c u)^n \leq C |u(z_0)| \sup_{z \in B} |u(z)|^{n-1}.$$  

In particular, if $R/r = 3$ then the constant $C$ depends only on $n$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Example of a modulus of continuity}
\end{figure}
Proof Without loss of generality, we may assume $z_0 = 0$. Set $\rho := (r + R)/2$ and \( B(\rho) = \{|z - z_0| < \rho\} \). We use the Blocki inequality [6] for $v(z) = |z|^2 - \rho^2$ and $\beta := dd^c v = dd^c |z|^2$, to get

$$
\int_{B'} (dd^c u)^n \leq \frac{1}{(\rho^2 - r^2)^{n-1}} \int_{B(\rho)} |v|^{n-1} (dd^c u)^n \leq \frac{(n - 1)! |u|^{n-1}}{(\rho^2 - r^2)^{n-1}} \int_{B(\rho)} dd^c u \wedge \beta^{n-1}.
$$

By Jensen’s formula

$$
u(0) + N(\rho) = \frac{1}{\sigma_{2n-1}} \int_{|\xi| = 1} u(\rho \xi) d\sigma(\xi),$$

where $\sigma_{2n-1}$ is the area of the unit sphere,

$$N(\rho) = \int_0^\rho \frac{n(t)}{t^{2n-1}} dt$$

and

$$n(t) = \frac{1}{\sigma_{2n-1}} \int_{|z| \leq t} \Delta u(z) dV_{2n}(z) = a_n \int_{|\xi| \leq t} dd^c u \wedge \beta^{n-1}.$$

Since $n(t)/t^{2n-2}$ is increasing, we have

$$N(R) \geq \int_0^R \frac{n(t)}{t^{2n-1}} dt \geq \frac{n(\rho)}{\rho^{2n-2}} \log(R/\rho).$$

From $u < 0$, it follows that $N(R) < -u(0)$. Hence,

$$\int_{B(\rho)} dd^c u \wedge \beta^{n-1} \leq \frac{n(\rho)}{a_n} \leq \frac{N(R)\rho^{n-2}}{\log(R/\rho)} \leq \frac{\rho^{2n-2} |u(0)|}{\log(R/\rho)}.$$

Combining the above inequalities, we get the desired estimate with the constant

$$C = \frac{(n - 1)! \rho^{2n-2}}{(\rho^2 - r^2)^{n-1} \log(R/\rho)}.$$

If $R = 3r$, then $C$ is also independent of $r$. \(\square\)

Lemma 4.2 Denote for $\rho \geq 0$, $B_\rho = \{|z - z_0| < e^\rho R_0\}$. Given $z_0 \in \Omega$ and two numbers $R > 1$, $R_0 > 0$ such that $B_M \subset \subset \Omega$, and given $v \in PSH(\Omega)$ such that $-1 < v < 0$, denote by $E$ the set

$$E = E(\delta) = \left\{ z \in B_0 : (1 - \delta)v \leq \sup_{B_0} v \right\},$$

where $\delta \in (0, 1)$. Then, there exists $C_0$ depending only on $n$ such that

$$\text{cap}(E, B_2) \leq \frac{C_0}{R\delta}.$$

Proof From the logarithmic convexity of the function $r \leftrightarrow \sup_{|z-z_0|<r} v(z)$ it follows that for $z \in B_R \setminus B_0$ and $a_0 := \sup_{B_0} v$ we have

$$v(z) \leq a_0 \left( 1 - \frac{1}{R} \log \frac{|z - z_0|}{R_0} \right).$$
Hence,

\[ a := \sup_{B_2} v \leq a_0 \left( 1 - \frac{2}{R_0} \right). \]

Let \( u = u_{E, B_2} \) the relative extremal function of \( E \) with respect to \( B_2 \). One has

\[ \frac{v - a}{a - a_0/(1 - \delta)} \leq u. \]

So, for some \( z_1 \in B_0 \), we have

\[ u(z_1) \geq \frac{a_0 - a}{a - a_0/(1 - \delta)} \leq \frac{2(\delta - 1)}{(M - 2)\delta + 2}. \]

Note that \( E \subset \{ |z - z_1| < 2R_0 \} \subset B_2 \). Therefore, Lemma 4.1 gives

\[ \text{cap}(E, B_2) = \int_{|z-z_1|<2R_0} (dd^c u)^n \leq C_0 \| u \|^{n-1}_{B_2} |u(z_1)| \leq \frac{C_0}{R\delta}. \]

This is the desired inequality.

Let us proceed with the proof of Theorem 1.3. Since \( \mu \in \mathcal{H}(\alpha, \Omega) \), according to [9] and [10, Theorem 5.9] we can solve the Dirichlet problem (1.1) to obtain a unique continuous solution \( u \). Define for \( \delta > 0 \) small

\[ \Omega_\delta := \{ z \in \Omega : \text{dist}(z, \partial \Omega) > \delta \}; \]

and for \( z \in \Omega_\delta \) set

\[ u_\delta(z) := \sup_{|\zeta| \leq \delta} u(z + \zeta). \]

Thanks to the arguments in [12, Lemma 2.11] it is easy to see that there exists \( \delta_0 > 0 \) such that

\[ u_\delta(z) \leq u(z) + \varpi(\delta; \psi, \partial \Omega) \quad (4.1) \]

for every \( z \in \partial \Omega_\delta \) and \( 0 < \delta < \delta_0 \). Here, we used the result of Bedford and Taylor [3, Theorem 6.2] (with minor modifications) to extend \( \psi \) plurisubharmonically onto \( \Omega \) so that its modulus of continuity on \( \bar{\Omega} \) is controlled by the one on the boundary. Therefore, for a suitable extension of \( u_\delta \) to \( \Omega \), using the stability estimate for measure in \( \mathcal{H}(\alpha, \Omega) \) as in [7, Theorem 1.1] (see also [12, Proposition 2.10]), we get

**Lemma 4.3** There are uniform constants \( C, \alpha_1 \) depending only on \( \Omega, \alpha, \mu \) such that

\[ \sup_{\Omega_\delta} (u_\delta - u) \leq \varpi(\delta; \psi, \partial \Omega) + C \left( \int_{\Omega_\delta} (u_\delta - u) d\mu \right)^{\alpha_1} \]

for every \( 0 < \delta < \delta_0 \).

Thanks to this lemma, we know that the right hand side tends to zero as \( \delta \) decreases to zero. We shall use the property “locally dominated by capacity” to obtain a quantitative bound via Lemma 4.2.

Let us denote the support of \( \mu \) by \( K \). Since \( \| u \|_\infty \) is controlled by a constant \( C = C(\alpha, \Omega, \mu) \), without loss of generality, we may assume that

\[ -1 \leq u \leq 0. \]

Then for every \( 0 < \varepsilon < 1 \)

\[ \int_{\Omega_\delta} (u_\delta - u) d\mu \leq \varepsilon \mu(\Omega) + \int_{\{ u < u_\delta - \varepsilon \} \cap K} d\mu. \quad (4.2) \]
We shall now estimate the second term on the right hand side. We may assume that $\Omega \subset \subset [0, 1]^{2n}$. Let us write $z = (x^1, \ldots, x^{2n}) \in \mathbb{C}^n : -r \leq x^i - x^i_0 < r, \forall i = 1, \ldots, 2n$.

Then, by the assumption, $\mu$ satisfies for every cube $I(z, r) =: I \subset \subset B(z, 2r) \subset \subset \Omega$ and for every set $E \subset I$, the inequality $\mu(E) \leq \mu(I(z, r)) F_0 \left( \cap (E(\varepsilon, u, B_I), B_I) \right)$, (4.3)

where $F_0 : [0, \infty] \to [0, \infty]$ is an increasing continuous function and $F_0(0) = 0$.

Consider the semi-open cube decomposition of $\Omega \subset \subset I_0 := [0, 1)^{2n} \subset \mathbb{R}^{2n}$ into $3^{2n}$ congruent cubes of diameter $3^{-s} = 2\delta$, where $s \in \mathbb{N}$. Then

\[
\{u < u_\delta - \varepsilon\} \cap I_s \subset \left\{ z \in B_{I_s} : u < \sup_{B_{I_s}} u - \varepsilon \right\},
\]

(4.4)

where $I_s = I(z_s, \delta)$ and $B_{I_s} = B(z_s, 2\delta)$ for some $z_s \in I_0$. Hence,

\[
\int_{\{u < u_\delta - \varepsilon\}} d\mu \leq \sum_{I_s \cap K \neq \emptyset} \int_{\{u < u_\delta - \varepsilon\} \cap I_s} d\mu.
\]

Using (4.3), (4.4), and then applying Lemma 4.2 for $r = 2\delta$ and $R = 2R_0$, we have for $B_s := B(z_s, 4\delta)$ corresponding to each cube $I_s$

\[
\int_{\{u < u_\delta - \varepsilon\} \cap I_s} d\mu \leq \mu(I_s) F_0 \left( \cap (B(\varepsilon, u, I_s), I_s) \right) \leq \mu(I_s) F_0 \left( \frac{C_0}{\varepsilon \log(R_0/\delta)} \right),
\]

(4.5)

where $2R_0 = \text{dist}(K, \partial \Omega)$. Therefore, combining the above inequalities, we get that

\[
\int_{\{u < u_\delta - \varepsilon\}} d\mu \leq \mu(\Omega) F_0 \left( \frac{C_0}{\varepsilon \log(R_0/\delta)} \right).
\]

We conclude from this and Lemma 4.3 that

\[
\omega(\delta; u, \tilde{\Omega}) \leq \sup_{\Omega_s} (u_\delta - u) \leq \sigma(\delta; \psi, \partial \Omega) + C \left[ \varepsilon + F_0 \left( \frac{C_0}{\varepsilon \log(R_0/\delta)} \right) \right]^\alpha_1.
\]

If we choose $\varepsilon = (\log R_0/\delta)^{-1/2}$, then Theorem 1.3 follows.

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