Anomalous U(1),
Gauge-Mediated Supersymmetry Breaking and
Higgs as Pseudo-Goldstone Bosons

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Abstract

We study the breaking of supersymmetry in models with anomalous U(1). These models are simple to construct and contain natural candidates for being the messengers of gauge-mediated supersymmetry breaking. When some of the ordinary matter fields transform under the anomalous U(1), we find a hybrid scenario in which the U(1) and the gauge interactions mediate the breaking of supersymmetry. This leads to a hierarchy of soft masses between the charged and neutral fields and provides a solution to the $\mu$-problem. Among these models, we present a scenario in which the Higgs arises as a pseudo-Goldstone boson. This scenario naturally allows for values of the $\mu$-term and the scalar soft masses larger than the weak scale.
1 Introduction

Anomalous U(1) allow for simple models of supersymmetry breaking\footnote{In a different context the possibility of gauge-mediation with anomalous U(1) was considered in\cite{ref3}.}. The presence of a gravitational anomaly for the U(1), $\text{Tr} Q \neq 0$ ($Q$ being the U(1)-generator), is necessary for the generation of a Fayet-Iliopoulos (FI) term and the triggering of supersymmetry breaking. In effective field theories arising from string, an anomalous U(1) is often present. Interestingly enough, one finds that these theories always contain fields that transform simultaneously under the anomalous U(1) and the standard model (SM) group. This is a necessary condition to cancel the anomaly by the Green-Schwarz mechanism\cite{ref3}.

This allows for two possibilities to transmit the breaking of supersymmetry to the SM fields:

$(i)$ The quiral superfields of the SM (quarks, leptons and Higgs) are neutral under the anomalous U(1). In this case, there should be extra matter fields (to cancel the mixed anomalies) that transform nontrivially under the U(1). These fields can act as messengers and communicate the breaking of supersymmetry to the SM fields at a higher loop level. (In ref.\cite{ref1} this effect was the main source for the gaugino masses). This will correspond to ordinary models of gauge-mediated supersymmetry breaking\cite{ref3,ref4}.

$(ii)$ There are SM fields transforming under the anomalous U(1). These models will present a hierarchy of the soft masses; fields charged under the U(1) will get tree-level scalar masses, while neutral ones and gauginos will get soft masses at higher loop orders. We will see that the tree-level masses are renormalization-scale invariant and are therefore (almost) not modified by radiative corrections. Consequently, they are not affected by the physics at high energies. Models of this type have already been considered in refs.\cite{ref1,ref3,ref4}. In particular, in ref.\cite{ref1} we proposed a model with the first and second generations of quarks and lepton charged under the anomalous U(1) as a solution to the flavor and CP supersymmetric problem.

Here we will present models of the type $(i)$ and $(ii)$, and compare the different pattern of soft masses that they generate. We will consider models with the Higgs transforming under the anomalous U(1) and show that this case allows for a new solution to the $\mu$-problem. Furthermore, these models can easily implement scenarios in which one of the Higgs arises as a pseudo-Goldstone boson (PGB) associated with the breaking of an accidental symmetry of the Higgs potential\cite{ref3}. We will study this scenario and show that it leads naturally to values of $\mu$ and soft masses larger than the weak scale. The origin of this hierarchy ($\mu, m_{H,Q} > m_Z$) can be understood from symmetry principles\cite{ref3}. The weak scale $m_Z$ is determined by the PGB mass that, due to the accidental symmetry of the Higgs potential, is smaller than the other mass parameters (soft supersymmetry breaking masses) of the Higgs potential. This accidental symmetry is not preserved beyond the tree-level approximation and then one does not expect that the hierarchy will be stable under radiative corrections. We find, however, that, due to a partial cancellation between the one-loop corrections, $\mu$ is maintained much larger than $m_Z$. In this scenario the scalar superpartners and Higgsinos will be heavier than expected and not accessible at LEPII.

In section 2 we will give the conditions for supersymmetry breaking in theories with anom-
lous U(1). In section 3 we will show that the scalar soft masses are renormalization-scale invariant and will study its consequences. In section 4 we will present a model with gauge-mediated supersymmetry breaking and compare it with models in which the breaking of supersymmetry is also communicated by the anomalous U(1). In section 5 we show how the $\mu$-term can arise in these models. We also present a model in which one of the Higgs is a PGB and study how large the $\mu$-term can be. Section 6 is devoted to conclusions. We include an appendix in which we show an explicit model with dynamical supersymmetry breaking.

2 Supersymmetry breaking with an anomalous U(1)

Let us consider a U(1) theory with $\text{Tr} Q \neq 0$ and therefore that it is anomalous. At the one-loop level, this results into the appearance of a tadpole $D$-term, the Fayet-Iliopoulos (FI) term, $\xi$. This tadpole is quadratically divergent and thus $\xi \sim \frac{1}{16\pi^2} \Lambda^2 \text{Tr} Q$ where $\Lambda$ is the cut-off of the theory. In string theories the generated FI-term can be calculated with a stringy regularization and is given by \[ \xi = \varepsilon M_P^2, \] (1)

where

$$\varepsilon = \frac{g^2 \text{Tr} Q}{192\pi^2} \left( \frac{\sqrt{2} M_{st}}{g M_P} \right)^2, \tag{2}$$

where $M_{st} = 1/\sqrt{\alpha'}$ is the string scale. In the weakly-coupled heterotic string, $M_{st}$ is related to the reduced Planck scale $M_P \approx 2 \times 10^{18}$ GeV by $M_{st} = g M_P / \sqrt{2}$ and therefore we have $\varepsilon \sim 10^{-3}$ for $\text{Tr} Q = O(1)$.

Let us also consider that exists a pair of chiral superfields $\phi$ and $\bar{\phi}$ with charges equal to $-1$ and $q_{\bar{\phi}}$ respectively under the anomalous U(1). The $D$-term contribution of the U(1) to the effective potential takes the form

$$\frac{g^2}{2} D^2 = \frac{g^2}{2} \left( q_{\bar{\phi}} |\bar{\phi}|^2 - |\phi|^2 + \xi \right)^2. \tag{3}$$

If eq. (3) is the only term in the potential, the vacuum expectation value (VEV) of $\phi$ adjusts to compensate $\xi$ (that we assume to be positive), and supersymmetry will not be broken. Nevertheless, as pointed out by Fayet long time ago [11], supersymmetry can be spontaneously broken if the field $\phi$ has a nonzero mass term in the superpotential:

$$W = m \phi \bar{\phi}. \tag{4}$$

Notice that $m$ carries U(1)-charge equal to $(1 - q_{\bar{\phi}})$ that can be different from zero if $m$ arises, as we will see, from some nonperturbative dynamics. In this case $m$ should have a nontrivial dilaton dependence

$$m \sim e^{-S_{192\pi^2} \text{Tr} Q (1 - q_{\bar{\phi}}) M_P}, \tag{5}$$

in the normalization in which the dilaton shifts under a U(1)-transformation as $S \rightarrow S + i \alpha \frac{\text{Tr} Q}{192\pi^2}$ and $\phi \rightarrow e^{i\alpha} \phi$. 

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For the moment, let us consider $m$ as a parameter whose value is much smaller than $M_P$. As we will show, the mass term $m$ has to lay $\sim 1$ TeV to generate realistic masses. Minimizing the potential derived from eqs. (3) and (4), we obtain that the VEVs of the scalar components are

$$\langle \bar{\phi} \rangle = 0, \quad \langle \phi \rangle^2 = \xi - \frac{m^2}{g^2},$$

and the VEVs of the $F$- and $D$-components are given by

$$\langle F_{\bar{\phi}} \rangle = m\sqrt{\xi - \frac{m^2}{g^2}}, \quad \langle F_{\phi} \rangle = 0, \quad \langle D \rangle = \frac{m^2}{g^2}.$$  

A important feature of these models is that supersymmetry is broken by nonzero VEVs of the two auxiliary fields, $F$ and $D$.

Embedding the above model in a supergravity theory will not restore supersymmetry as it was proven in ref. [1]; the VEVs of eqs. (6) and (7) are just shifted by the gravity corrections. The only noticeable effect is a tadpole for $\bar{\phi}$ generated from the bilinear soft-term $\approx \frac{m^3}{M_P}$ where $m_{3/2} \approx \langle F_{\bar{\phi}} \rangle / M_P$ is the gravitino mass. Thus, $\bar{\phi}$ gets a nonzero VEV given by

$$\langle \bar{\phi} \rangle \simeq \frac{m_{3/2}}{m} \langle \phi \rangle \simeq \sqrt{\varepsilon} \langle \phi \rangle.$$  

The fact that the VEV of $\bar{\phi}$ is smaller than that of $\phi$ (since $\varepsilon \ll 1$) will have important phenomenological consequences.

Let us comment on the origin of the small mass-parameter $m$. It can be generated dynamically [1, 2]. For example, it can arising from a field condensation, $m = \langle \Phi \bar{\Phi} \rangle / M_P$, where $\Phi$ and $\bar{\Phi}$ are some fields that transform under a strongly interacting gauge group [12]. In section 4 and the appendix we show some examples. If $m$ is dynamical, the minimization of the full potential is slightly different from the case above in which we took $m$ as a frozen parameter. We find (see appendix) that supersymmetry is always broken, although the VEV of $\bar{\phi}$ cannot be exactly determined since it depends on unknown nonperturbative effects. We, however, estimate that $\langle \bar{\phi} \rangle$ takes a value between $\sqrt{\varepsilon} \langle \phi \rangle$ and $\langle \phi \rangle$.

Alternatively, one can consider that $m$ arises from a higher dimensional operator suppressed by powers of the Planck scale. For example, a superpotential term such as

$$W = \frac{\phi^{12} \bar{\phi}}{M_P^{10}},$$

leads to an effective $m$ of order

$$m \simeq \frac{\langle \phi \rangle^{11}}{M_P^5} \sim \varepsilon^5 \sqrt{\xi} \sim 1 \text{ TeV},$$

for $\varepsilon \approx 10^{-3}$. Notice that this option does not need for nonperturbative dynamics to generate a scale smaller than $M_P$. This is possible due to the fact that the FI-term is generated at the one-loop level and this introduces a new scale in the theory $\sqrt{\xi} \ll M_P$. Although possible, we do not find the latter approach very appealing since it requires a very peculiar charge assignment $q_{\bar{\phi}} = 12$. 


3 Implications for the scalar soft masses

Any scalar field in the theory charged under the anomalous U(1) receive soft supersymmetry breaking masses from the VEV of the \( D \)-term:

\[
m^2_{Q_i} = q_i m^2,
\]

where \( q_i \) is the U(1)-charge of \( Q_i \). The pattern of soft masses of eq. (11) has very interesting consequences. First, we have scalar mass degeneracy for fields with equal charges. Second, fields with trilinear couplings \( hQ_1Q_2Q_3 \) in the superpotential has \( q_1 + q_2 + q_3 = 0 \), and then their soft masses will fulfill the following sum rule:

\[
m^2_{Q_1} + m^2_{Q_2} + m^2_{Q_3} = 0.
\]

The renormalization group equations (RGE) of the soft masses is given by

\[
\frac{dm^2_{Q_i}}{dt} = \frac{h^2}{8\pi^2}(m^2_{Q_1} + m^2_{Q_2} + m^2_{Q_3} + A^2) - \frac{2C^a_i\alpha_a}{\pi} m^2_{\lambda_a},
\]

where \( C^a_i \) is the quadratic Casimir of the \( Q_i \) representation of the group \( G_a \), \( A \) is the trilinear soft term and \( m_{\lambda_a} \) are the gaugino masses. Therefore, the sum rule (12) implies that the soft masses are RG invariant up to corrections of \( \mathcal{O}(m^2_{\lambda_a}, A^2) \). These corrections, however, are small since gaugino masses and trilinears do not receive contributions from the \( D \)-terms; they will be generated from gravity contributions as in ref. [1], or at a higher loop-order as we will see in the next section.

The above has important phenomenological implications. If we assume that the first and second families are equally charged under the U(1), then the squarks and sleptons will be degenerate at the scale \( \sqrt{\xi} \). The running from the scale \( \sqrt{\xi} \) to lower scales will not modify the scalar masses, since, as we said, they are renormalization-scale invariant for \( m_Q \gg m_\lambda, A \). Therefore, even if at a high energy all the Yukawa coupling are of \( \mathcal{O}(1) \) as in certain theories of flavor [13], the squarks will keep their degeneracy at lower energies. This property can avoid the supersymmetric flavor problem [13].

Of course, this will be true up to gravitational corrections. The squark masses receive gravity contributions that, for nonminimal Kähler potential, are nonuniversal [14]. These contributions are of the order of the gravitino mass

\[
m^2_{3/2} \simeq \frac{(F_\phi)^2}{M^2_P} \simeq \varepsilon m^2.
\]

Therefore, gravity effects to eq. (11) are smaller than 1\% for \( \varepsilon \ll 10^{-2} \).

\(^2\) We are assuming that \( \text{Tr}[QY] = 0 \) where \( Y \) is the hypercharge generator [1].
4 Gauge-mediated supersymmetry breaking

Let us consider that the ordinary SM fields are neutral under the anomalous U(1). In this case, to cancel the anomalies by the Green-Schwarz mechanism \cite{3}, the theory must contain extra matter fields transforming under the anomalous U(1). Let us consider that these extra matter fields, $\Psi$ and $\bar{\Psi}$, are vector-like under the SM group but chiral under the anomalous U(1) ($q_\Psi + q_{\bar{\Psi}} \neq 0$). If $n = -(q_\Psi + q_{\bar{\Psi}})/q_\phi$ is positive, then $\Psi \bar{\Psi}$ will couple to $\phi$\footnote{Equivalently, if $q_\Psi + q_{\bar{\Psi}}$ is positive, then $\Psi \bar{\Psi}$ will couple to $\phi$. The effects of this coupling will be similar to those discussed below.}.

$$W = \frac{\phi^n}{M_p^n} \Psi \bar{\Psi}.$$ (15)

This coupling will not change the vacuum of eqs. (3) and (4); the supersymmetry breaking vacuum, however, will be now a local minimum. The scalar-component VEV of $\phi$ will give a supersymmetric mass to $\Psi$ and $\bar{\Psi}$, while the $F$-component VEV of $\phi$ will induce a mass splitting inside the scalar components of $\Psi$ and $\bar{\Psi}$. Therefore, $\Psi$ and $\bar{\Psi}$ can act as messengers and transmit the supersymmetry breaking to the SM fields. Like ordinary models with gauge-mediated supersymmetry breaking, the gaugino masses will be given at the one-loop level \cite{4}

$$m_{\lambda_a} = \frac{\alpha_a}{4\pi} n S_\Psi \frac{\langle F_\phi \rangle}{\langle \phi \rangle},$$ (16)

where $S_\Psi^a$ is the Dynkin index of the $\Psi$-representation of the gauge group $G_a$. Scalar masses will arise at the two-loop level

$$m_{Q_i}^2 = 2C_i^a n^2 S_\Phi \left( \frac{\alpha_a}{4\pi} \right)^2 \frac{\langle F_\phi \rangle^2}{\langle \phi \rangle^2}.$$ (17)

This contribution is universal for fields with equal quantum numbers; therefore it does not induce flavor-violating interactions. Trilinear and bilinear soft terms are generated at the two-loop level, $A \simeq \frac{\alpha}{4\pi} m_\lambda$, unless there is a messenger-matter mixing that will induce them at the one-loop level \cite{15}. Eqs. (16) and (17) depend on the ratio

$$\frac{\langle F_\phi \rangle}{\langle \phi \rangle} = \frac{\langle \phi \rangle}{\langle \phi \rangle} m.$$ (18)

We will therefore consider two separate scenarios:

**Scenario (a):** The VEV of $\phi$ and $\bar{\phi}$ are of the same order and we have

$$\frac{\langle F_\phi \rangle}{\langle \phi \rangle} \simeq m.$$ (19)

**Scenario (b):** The VEV of $\phi$ is smaller than the VEV of $\phi$ as in eq. (8). We then have

$$\frac{\langle F_\phi \rangle}{\langle \phi \rangle} \simeq \sqrt{\frac{1}{\varepsilon}} m.$$ (20)
Scenario (b) arises when \( m \) is considered a spurion field with a frozen VEV or when \( m \) is effectively generated from a higher dimensional operator such as eq. (9). When \( m \) is induced dynamically by a field condensation (as in the appendix), we can have both scenarios (see section 2).

Scenario (a) and (b) lead to different pattern of soft masses, as it is shown in table 1. To have \( m_Q \sim 100 \text{ GeV} \), we need \( m \sim 100 \text{ TeV} \) and \( \sim 10 \text{ TeV} \) respectively for the scenario (a) and (b). There are also, however, gravity contributions to the scalar masses that can be comparable to the gauge contributions depending on the scenario –see table 1. From eqs. (14) and (17), we have using eqs. (19) and (20) respectively

\[
\frac{m_{3/2}^2}{m_Q^2} \simeq \begin{cases} 
10^4 \varepsilon & \text{scenario (a)}, \\
10^4 \varepsilon^2 & \text{scenario (b)}. 
\end{cases}
\]  

To avoid flavor problems coming from gravity contributions, the ratio of eq. (21) has to be smaller than \( \sim 10^{-2} \). We see that only for the the scenario (b) this can be satisfied for values of \( \varepsilon \sim 10^{-3} \). Of course, for string theories with \( M_{st} \ll g M_P \) we see from eq. (2) that both scenarios can accommodate \( \frac{m_{3/2}^2}{m_Q^2} \lesssim 10^{-2} \).

### 4.1 A Complete model of dynamical supersymmetry breaking

Here we present a model of gauge-mediation that, at the same time, addresses the problem of the generation of the small scale \( m \). The model has the minimal gauge group and field content necessary to break supersymmetry dynamically and transmit it by gauge interactions to the SM fields [19, 20].

Take the group \( \text{SU}(N) \times \text{U}(1) \times \text{SU}(5)_{SM} \), where the \( \text{SU}(N) \) becomes strong at some scale \( \Lambda \ll M_P \), the \( \text{U}(1) \) is anomalous and the \( \text{SU}(5)_{SM} \) includes the SM group. Add to the SM fields, the following field content:

\[
\begin{align*}
\phi & \quad (1, -1, 1), \\
\Psi & \quad (N, \frac{1}{2}, 5), \\
\bar{\Psi} & \quad (\bar{N}, \frac{1}{2}, \bar{5}),
\end{align*}
\]

with the classical superpotential

\[
W = \lambda \phi \bar{\Psi} \Psi.
\]  

This simple model leads to dynamical supersymmetry breaking [1] with \( \Psi \) and \( \bar{\Psi} \) playing the role of condensates and messengers. To show that, let us ignore, for the moment, the \( D \)-term of the anomalous \( \text{U}(1) \). Classically the vacuum manifold (moduli space) has a flat direction parameterized by the expectation value of \( \phi \). Along this branch \( \Psi \) and \( \bar{\Psi} \) are massive, \( M_\Psi = \lambda \phi \), and can be integrated out. For \( \phi \geq \Lambda \) the low-energy theory is a pure super-Yang-Mills theory

\[4\] As far as supersymmetry breaking is concerned, this example is similar to that of ref. [2].
with a massless chiral superfield $\phi$ (and, of course, all the SM states). Gaugino condensation in the strongly coupled group, $\langle \lambda \lambda \rangle$, induces the nonperturbative superpotential \[ W = \langle \lambda \lambda \rangle = N\Lambda_L^3, \tag{26} \]

where $\Lambda_L$ is the low-energy scale of the SU(N) theory. This scale is $\phi$-dependent; the explicit dependence can be found by the one-loop matching of the gauge coupling in the low and the high energy theories at the scale $\phi$ \cite{20}. By doing this we arrive to the following effective low-energy superpotential:

\[ W_{\text{eff}} = N \left( \Lambda^5 \phi^5 \Lambda^{3N-5} \right)^{\frac{1}{N}}. \tag{27} \]

The superpotential (27) leads to spontaneous supersymmetry breaking if $\phi$ gets a nonzero VEV. This is the case as soon as the anomalous $D$-term is switched-on \cite{5}. The pattern of the induced soft masses is like that in the scenario (a).

The superpotential (27) can also be derived by adding the instanton-generated superpotential (for $N > 5$) \cite{21}

\[ W = (N - 5) \left( \frac{\Lambda^{3N-5}}{\det M} \right)^{\frac{1}{N}}, \tag{28} \]

and solving the equations of motion for the mesons $M_i^j = \tilde{\Psi}^j \Psi_i$ (see appendix). For $N = 5$ the same can be done but using the quantum modified constraint \cite{12}

\[ W = \mathcal{A} \left( \text{Det}M - B\bar{B} - \Lambda^{10} \right), \tag{29} \]

where $B = \text{Det} \Psi$ and $\bar{B} = \text{Det} \bar{\Psi}$ are baryons. Note that if all the other states charged under the SU(5)$_{\text{SM}}$ are neutral under the U(1), then anomaly cancellation by the Green-Schwarz mechanism \cite{4} requires $N = 5$. This case ($N = 5$) shares some similarity with the models of ref. \cite{17}. The only difference resides in the mechanism that stabilizes $\phi$ at some large value. In the models of ref. \cite{17} this stabilization was achieved by the Kähler renormalization as in the Witten “inverse hierarchy” model \cite{22}. Due to the logarithmic nature of this effects the stabilization is only possible for $N = 5$ (when the superpotential is linear). In our case $\langle \phi \rangle \gg \Lambda$ is imposed by the U(1) $D$-term and supersymmetry can be broken for arbitrary $N \geq 5$ as well.

The above conclusions can be trivially generalized for arbitrary higher dimensional couplings

\[ W = \frac{\lambda \phi^n}{M_p^{n-1}} \bar{\Psi} \Psi. \tag{30} \]

This will lead to the change $\phi \rightarrow \phi^n/M_p^{n-1}$ in the effective superpotential (27).

The question of whether the gauge contribution to the scalar soft masses dominate over the gravity contribution is directly linked to the value of $\xi$. This is the subject on which we will not

\footnote{In effective field theories arising from strings the anomalous U(1) is also broken by the dilaton VEV and the eaten-up component is a combination of $\phi$ and the dilaton; the remaining combination is a light field. So strictly speaking, the question becomes linked to the story of dilaton stabilization, which we will not analyze here; we are assuming that the dilaton is stabilized by some dynamics which \textit{per se} preserves supersymmetry.}
speculate much here. Just note that, \textit{a priori}, the FI-term can be generated in the low-energy limit of some anomaly-free theory, after integrating out the heavy modes, provided that the light ones have $\text{Tr}Q \neq 0$. Obviously, such a situation is impossible within a four-dimensional field theory with unbroken supersymmetry, since the massive chiral superfields appear in pairs with opposite charges and do not contribute to $\text{Tr}Q$. Nevertheless, the situation is different if the theory changes dimensionality below the integration scale, \textit{e.g.}, if the four-dimensional chiral theory with $\text{Tr}Q \neq 0$ is obtained by compactifying some higher odd-dimensional one. Since the original theory is anomaly free, the anomaly in the daughter chiral theory must be cancelled by an effective Green-Schwarz term \cite{3}; the quadratic divergence of the one-loop diagram that induces the $D$-term must cancel out for momenta larger than the inverse of the radius of compactification $R^{-1}$ (the localization width in the extra dimension(s)). The resulting FI-term then must be controlled by the radius $R^{-1}$ and can be small if $R$ is large. In particular, in the M-theory picture \cite{23}, where the scales are more flexible, the value of the FI-term may be smaller \cite{6}.

4.2 **Hybrid models of supersymmetry breaking**

If some of the SM fields are charged under the U(1), the supersymmetry breaking will be transmitted by the U(1) and the gauge interactions. Scalar masses (of the U(1)-charged fields) will arise from eq. (11) while gaugino masses from eq. (16). We can see that there is a hierarchy of soft masses, $m_{Q_i} \gg m_\lambda$. For the scenario (a), the scalar masses are two orders of magnitude larger than the gaugino masses. If only the first and second families are charged under the U(1), this hierarchy can aminorate the flavor and CP supersymmetric problem \cite{1}.

If also the third family and the Higgs are charged, we have in the scenario (a) that either we take $m \approx 100$ GeV and the gaugino masses are too small (see table \cite{4}), or we take $m = \mathcal{O}(10$ TeV) and the weak scale (that is related to the Higgs soft mass) is too large. Thus, a large degree of fine tuning is required in this case. In the scenario (b), this fine tuning problem is aminorited since the scalar masses of the fields charged under the U(1) are only slightly larger than the gaugino masses (depending on the value of $\varepsilon$ as can be seen in table \cite{4}). It is also possible to have $m^2_H > m^2_Z$ in a natural way if the SM Higgs arise as PGBs from the breaking of some symmetry realized at high energies. As will see in the next section, this can be easily implemented in models with anomalous U(1).

\footnote{We thank Michel Peskin for a valuable discussion on this issue.}
5 A solution to the $\mu$-problem and Higgs as pseudo-Goldstone bosons

Let us suppose that the SM Higgs $H$ and $\tilde{H}$ have the same $U(1)$-charges as $\phi$ and $\bar{\phi}$ respectively. In this case the superpotential (31) can be extended to

$$ W = m(\phi\bar{\phi} + \lambda HH). $$

This will not alter the minimization procedure of section 2 and supersymmetry will be broken. From eq. (31) one has a supersymmetric mass for the Higgs $\mu = \lambda m$ that results to be of the right order of magnitude (i.e., of the order of the soft masses). This mechanism for generating the $\mu$-term has different origin from that in ref. [24]. There, the $\mu$-term is generated when supersymmetry is broken. Here, we start with a $\mu$-term before the breaking of supersymmetry, and it is this term the responsible for the generation of the soft masses.

Considering the case that $m$ is a frozen parameter (scenario (b)), we have that the Higgs mass matrix (prior electroweak breaking) is given by

$$ M^2_{HH} = \begin{pmatrix} \bar{m}^2_H & -B\mu \\ -B\mu & m^2_H \end{pmatrix}, $$

where up to $O(m^2_3/2)$,

$$ \bar{m}^2_H \equiv \mu^2 + m^2_H = \lambda^2 m^2 - m^2, $$

$$ m^2_H \equiv \mu^2 + m^2_H = \lambda^2 m^2 + q_m^2, $$

$$ B = 0. $$

Let us now consider the limit $\lambda = 1$. In this case one finds that the scalar $H$ is a massless state. This is in fact expected since in the limit $\lambda = 1$, the superpotential (31) and the $D$-term of the anomalous $U(1)$ posses a global $SU(3)_L$ symmetry with $\bar{\phi}$, $\bar{\phi}$ and the Higgs transforming as triplets, $(\phi, H) \in 3$ and $(\bar{\phi}, \bar{H}) \in \bar{3}$. When $\phi$ and $\bar{\phi}$ get a VEV, the $SU(3)_L$ symmetry is broken down to the $SU(2)_L$ symmetry of the SM. There is a Goldstone boson associated with this breaking that transforms as a doublet under the $SU(2)_L$: this is the Higgs field $H$. Of course, if the field $H$ is a true Goldstone boson, it will be massless to all orders and the electroweak symmetry will not be broken. We will assume that the global $SU(3)_L$ symmetry is broken by the gauge interactions and Yukawa couplings; it is just an accidental symmetry of the superpotential (31). In this case $H$ is just a pseudo-Goldstone boson (PGB) and will gain mass at higher loop orders. Models with Higgs as PGBs have been extensively considered in the literature to solve the doublet-triplet splitting problem in grand unification theories (GUTs) [7]. In these GUTs the global $SU(3)_L$ is embedded in a global $SU(6)$.

The above scenario suggests that the mass of $H$ can be smaller than $m$, and consequently $m_Z < m = \mu$. We want to analyze this point with more detail. The minimization condition of the SM Higgs potential is given by

$$ \frac{m_Z^2}{2} = \frac{\bar{m}^2_H - m^2_H \tan^2 \beta}{\tan^2 \beta - 1}, $$

where $m^2_H = \lambda^2 m^2 - m^2$. This will not alter the minimization procedure of section 2 and supersymmetry will be broken.
where
\[ \sin 2\beta = \frac{2B\mu}{m_H^2 + \bar{m}_H^2}. \] (35)

For \( q_\phi \neq -1 \), we have from eqs. (33) that \( B\mu \ll \bar{m}_H^2 \) and we are naturally in the large \( \tan \beta \) regime. Notice that this is different from models with gravity-mediated supersymmetry breaking where one finds [25] that Higgs as PGBs usually leads to \( \tan \beta \) close to one. In the large \( \tan \beta \) region, eq. (34) gives
\[ m_Z^2 \simeq -2\bar{m}_H^2, \] (36)
that relates the weak scale to the PGB mass. At the scale \( \sqrt{\xi} \), we already showed that \( \bar{m}_H \) is zero at tree-level independently of the value of \( \mu \). Nevertheless, it will receive radiative corrections. The largest corrections arises from the scale evolution of \( \bar{m}_H \) from \( \sqrt{\xi} \) to \( m_Z \). Up to corrections of \( O(m_\lambda, A, m_{3/2}) \) (that are small in these models), we have from section 3 that the soft mass of \( H, m_H \), is not modify; therefore only the \( \mu \)-parameter will receive large corrections (a similar scenario is obtained in ref. [26] but in a different context). The \( \mu \)-parameter, being a supersymmetric parameter, will be only renormalized by the wave-function renormalization constants of \( H \) and \( H^* \), i.e., \( \mu \rightarrow Z_H Z_{H^*} \mu \). Nevertheless, it is interesting to notice that \( Z_H Z_{H^*} \) are slightly modified for values of \( m_t \) close to the experimental values. This is due to a partial cancellation between the (positive) gauge contribution and the (negative) top contribution. Explicitly, one finds at the scale \( m \):
\[ \bar{m}_H^2 = \left[ f_1^{3\alpha_1} f_2^{\alpha_2} \sqrt{1 - \frac{m_t^2}{m_{FP}^2}} - 1 \right] m^2 + O(m_{3/2}^2, m_\lambda^2), \] (37)
where
\[ f_a = 1 + b_a \frac{\alpha_a(\sqrt{\xi})}{4\pi} \ln \frac{\xi}{m^2}, \quad b_{1.2} = (33/5, 1), \] (38)
\( \alpha_a(\sqrt{\xi}) \) is the gauge coupling at the scale \( \sqrt{\xi} \); \( m_t \) and \( m_{FP} \) are respectively the running top mass and the infrared fixed-point value for the top mass \( m_t \) at the scale \( m \). We have neglected the bottom contribution that could be important if \( \tan \beta \) is very large. We see that for
\[ m_t^2 = (1 - f_1^{3\alpha_1} f_2^{\alpha_2}) m_{FP}^2 \simeq (168 \text{ GeV})^2 \sin^2 \beta, \] (39)
the dominant contribution to \( \bar{m}_H^2 \) (the first term of eq. (37)) cancels and the value of \( \bar{m}_H^2 \) remains of \( O(m_{3/2}^2, m_\lambda^2) \). This suggests that for values of \( m_t \) close to that of eq. (39), we can have \( \mu = m \gg \bar{m}_H \sim m_Z \sim m_\lambda \). Experimentally, we have [28] \( m_t^{\text{pole}} = 175.6 \pm 5.5 \text{ GeV} \) that implies a running mass \( m_t(m_t) = 167.1 \pm 5.2 \text{ GeV} \); therefore the value of eq. (39) lays inside the experimental window.

In order to have the right electroweak breaking without fine tuning, we must require that the value of \( |\bar{m}_H^2| \) coming from eq. (37), i.e., \( |\bar{m}_H^2| \lesssim m_Z^2 \). This

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7 It is defined as \( m_{FP}^2(t) = \frac{16\pi^2 m_\chi}{3 g^2(\sqrt{t})} \sin^2 \beta \), where the functions \( E(t) \) and \( F(t) \) are given in ref. [26].

8 In fact, one could determine, using eq. (36), the exact value of \( \mu \). For this purpose, however, one must include the gravitino and gaugino contributions to the low-energy parameter \( \bar{m}_H^2 \); one should also consider the one-loop effective potential instead of the tree-level one. Here we are only interested in obtaining a rough estimate of the upper bound on \( m_t \).
puts an upper bound on $\mu$. In fig. 1 we plot this bound as a function of $m_t^{\text{pole}}$ and for $\tan \beta = 10$. We have made the following approximations: We have taken $\sqrt{\xi} = M_{\text{GUT}} \approx 2 \times 10^{16}$ GeV and evaluated $\tilde{m}_H^2$ at the scale $\approx 200$ GeV where $m_{\tilde{F}_P} \approx 196 \times \sin \beta$ GeV. We have not included the effects on $\tilde{m}_H^2$ arising from the gaugino, trilinears or gravitino; these effects are subdominant and very model dependent. From fig. 1 we see that large values of $\mu$ can be natural for certain values of the top mass. Note that large values of $m$ also lead to heavy stop and sbottom. This is because the sum rule (12) implies $m_{Q_3}^2 + m_{U_3}^2 = -m_H^2 = m^2$ up to small radiative corrections (we are assuming that the Yukawa coupling of the top arises from the tree-level term $h_t H Q_3 U_3$). Hence, the only light superparticles in these models are the gauginos and gravitino.

If $m$ in eq. (31) is dynamical, the only difference from the case above is that $B\mu$ can be of the order of $m^2$ (see appendix) and one can have smaller values of $\tan \beta$. An upper bound on $\mu$, similar to that of fig. 1, can be also obtained in this case. A complete analysis of the small $\tan \beta$ regime will be presented elsewhere.

![Figure 1: Upper bound on $\mu$ arising from $|\tilde{m}_H^2| \leq m_Z^2$, as a function of $m_t^{\text{pole}}$ and for $\tan \beta = 10$.](image)
6 Conclusions

The superparticle mass spectrum and its future experimental test depends on the mechanism responsible for supersymmetry breaking. It is then important to study the different alternatives that we know to break supersymmetry and analyze its phenomenological consequences.

Here we have shown that models with anomalous U(1) allow for different possibilities to transmit the breaking of supersymmetry to the SM fields. We have shown that:

• If the SM fields are neutral under the anomalous U(1), then the model contain extra matter fields that act as messenger of the breaking of supersymmetry. In this case, the pattern of soft masses are like that in models of gauge-mediated supersymmetry breaking. Gravity contribution can be important in these models, depending on the value of the FI-term –see eq. (21).

• When the SM fields are charged under the anomalous U(1), the soft masses present a hierarchy of values (see table 1). For \( \varepsilon \sim 10^{-3} \), we have:

\[
\text{Scenario(a)} : \quad m_{Q_i} \gg m_{3/2} \gg m_\lambda, \quad (40)
\]

\[
\text{Scenario(b)} : \quad m_{Q_i} \simeq m_\lambda > m_{3/2}, \quad (41)
\]

where the scenarios (a) and (b) are defined in section 4.

• The soft masses of the fields charged under the anomalous U(1) are (approximately) RG invariant. This implies that, even that supersymmetry is communicated to those fields at high energies, the soft masses are not modified by the physics at the ultraviolet. This provides a way to avoid the effect of ref. [13].

• When the Higgs is charged under the anomalous U(1), the \( \mu \)-parameter is present in the theory with the right order of magnitude.

• One of the Higgs of the SM can arise as a PGB. In this case, its mass (before electroweak breaking) is smaller than the other soft masses since it is generated at the one-loop order. As a consequence the weak scale is smaller than the \( \mu \)-parameter and the supersymmetry breaking mass \( m \) (see fig. 1).

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**Appendix**

In this appendix we analyze the vacuum of a theory where the mass $m$ is generated dynamically. Consider a SU($N$) gauge group with $N_f$ flavors, $\Phi_i$ and $\bar{\Phi}_j$ with $i, j = 1, ..., N_f < N$. The gauge interactions becomes strong at some scale $\Lambda$, and the quantum vacuum is described by the mesons $M^i_j \equiv \bar{\Phi}_j \Phi_i$. Let us assume that $M^i_j$ transforms under the anomalous U(1) with charge $(1 - q_{\phi})$. In this theory the superpotential is given by

$$W = \frac{\text{Tr} M}{M_P} \phi \bar{\phi} + (N - N_f) \left( \frac{\Lambda^{3N-N_f}}{\text{Det} M} \right) \frac{1}{N-N_f},$$

where the second term is generated nonperturbatively [21, 19]. The $F$-terms are given by

$$F_{\phi} = \frac{\phi \text{Tr} M}{M_P},$$
$$F_{\bar{\phi}} = \frac{\bar{\phi} \text{Tr} M}{M_P},$$
$$F_{M^i_j} = \frac{\phi \bar{\phi}}{M_P} \delta^i_j - (M^{-1})^j_i \left( \frac{\Lambda^{3N-N_f}}{\text{Det} M} \right) \frac{1}{N-N_f}.$$ 

Due to the $D$-term of the anomalous U(1) [eq. (3)], $\phi$ is forced to get a VEV and therefore the $F$-terms of eqs. (43)-(45) cannot all be zero, i.e., supersymmetry is broken. The value of $m$ is given by

$$m = \frac{\langle \text{Tr} M \rangle}{M_P},$$

where $\langle M^i_j \rangle$, for $\Lambda \ll M_P$, is given by

$$\langle M^i_j \rangle = \delta^i_j \left[ \Lambda^{3N-N_f} \left( \frac{\langle \phi \bar{\phi} \rangle}{M_P} \right)^{\frac{N_f-N}{N}} \right].$$

Notice that since $\Lambda \ll \langle \phi \bar{\phi} \rangle / M_P \sim \sqrt{\xi}$, one has $\langle M^i_j \rangle \ll \Lambda^2$ and the condensation takes place in the strong regime. In this regime we do not know the dependence of the Kähler on $M$ and therefore we cannot calculate the exact value of the ratio $\langle \bar{\phi} \rangle / \langle \phi \rangle$ that depend on the Kähler metric $K_{MM}$:

$$\left( \frac{\langle \phi \rangle}{\langle \bar{\phi} \rangle} \right)^{-1} = -\frac{\langle K^{-1}_{MM} F_M \rangle}{M_P m^2 (q_{\bar{\phi}} + 1)}.$$

Eq. (48) is derived from the minimization condition of the potential. Assuming a normalized Kähler, we find (from other minimization condition of the potential)

$$\frac{\langle K^{-1}_{MM} F_M \rangle}{M_P} \simeq m^2,$$

and therefore

$$\langle \bar{\phi} \rangle \simeq \langle \phi \rangle.$$
We then estimate that $\langle \bar{\phi} \rangle / \langle \phi \rangle$ lays between $\sqrt{\varepsilon}$ and 1, where the lower bound comes from gravity contributions (see above eq. (8)). The upper and lower value of $\langle \bar{\phi} \rangle / \langle \phi \rangle$ correspond to the scenario (a) and (b) respectively.

For the model of section 5, we just have to make the replacement $\phi \bar{\phi} \rightarrow (\phi \bar{\phi} + \lambda H \bar{H})$ in eq. (12). Unlike the case where $m$ was taken to be a frozen parameter, eqs. (33), we find that now $B_\mu$ can be different from zero:

$$B_\mu = \frac{\langle K^{-1}_{MM} F_M \rangle}{M_P}.$$  

(51)

Due to the unknown strong effects on Kähler metric, we cannot determined $B_\mu$ exactly; we can only estimate from eq. (49) that its value is close to $m^2$. For $\lambda = 1$ we must have a massless scalar (the PGB) associated with the breaking of the global $SU(3)_L$ symmetry. This implies that $\text{Det} M_H^2 = 0$ and therefore $|B_\mu| = \bar{m}_H \bar{m}_H$. We then see that the value of $B_\mu$ can be inferred (at tree-level) from the PGB condition. The PGB is now the linear combination

$$\frac{1}{\sqrt{\bar{m}_H^2 + \bar{m}_H^2}} \left( \bar{m}_H H + \bar{m}_H \bar{H}^\dagger \right).$$  

(52)
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