The experimental observation of Beliaev damping in a Bose condensed gas

E. Hodby, O.M. Maragò, G. Hechenblaikner and C.J. Foot
Clarendon Laboratory, Department of Physics, University of Oxford,
Parks Road, Oxford, OX1 3PU,
United Kingdom.
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We report the first experimental observation of Beliaev damping of a collective excitation in a Bose-condensed gas. Beliaev damping is not predicted by the Gross-Pitaevskii equation and so this is one of the few experiments that tests BEC theory beyond the mean field approximation. Measurements of the amplitude of a high frequency scissors mode, show that the Beliaev process transfers energy to a lower lying mode and then back and forth between these modes. These characteristics are quite distinct from those of Landau damping, which leads to a monotonic decrease in amplitude. To enhance the Beliaev process we adjusted the geometry of the magnetic trapping potential to give a frequency ratio of 2 to 1 between two of the scissors modes of the condensate. The ratios of the trap oscillation frequencies $\omega_y/\omega_z$ and $\omega_z/\omega_x$ were changed independently, so that we could investigate the resonant coupling over a range of conditions.

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In the Beliaev damping process [1], one quantum of excitation converts into two quanta of a lower frequency mode, in a manner analogous to parametric down-conversion of light in nonlinear media. This process occurs readily in a homogeneous system such as superfluid helium where there are many very closely spaced low-lying energy levels. However, it has not previously been observed for a trapped Bose-condensed gas, which has discrete modes with well-resolved excitation energies. We have observed the Beliaev process for a scissors mode [3] of a BEC of rubidium atoms, when the mode resonantly couples to another mode at half its frequency. The centre of mass motion of an interacting cloud of atoms, such as the Bose condensate and any surrounding thermal cloud, has the same eigenfrequencies by Kohn’s theorem [6]. However, the frequencies of other collective excitations of the condensate, including the quadrupole modes, do not correspond to those of a harmonic oscillator because of the strong interatomic interactions. Six of the low lying collective excitations are described by the linearised set of equations:

$$\ddot{\mathbf{q}} = \mathbf{S} \mathbf{q}$$  \hspace{1cm} (1)

where the six components of the vector $\mathbf{q}$,

$$\mathbf{q} = (Q_{ii}, Q_{jj}, Q_{kk}, Q_{ij}, Q_{jk}, Q_{ik})$$  \hspace{1cm} (2)

are elements of the quadrupole tensor, $\mathbf{Q}$.

$$Q_{ij} = \langle x_i x_j \rangle$$  \hspace{1cm} (3)

The six eigenvalues of $\mathbf{S}$ give the frequencies of the six normal modes. Linear combinations of the diagonal elements of $\mathbf{Q}$ (e.g. $\langle x^2 \rangle$) describe the three normal modes with principal axes of fixed orientation. We will refer to these modes as $M_h, M_l, M_2$. In the case of the axially symmetric trap, where the angular momentum about the axis (m) is a good quantum number, these become the high and low lying $m=0$ modes and the $m=2$ mode respectively [5]. The three independent off-diagonal elements of $\mathbf{Q}$ (e.g. $\langle xy \rangle$) relate to the three scissors modes, which we label $M_{xy}, M_{xz}, M_{xz}$, with frequencies $\omega_{xy}, \omega_{yx}, \omega_{zz}$. The frequency spectrum has been calculated in [4] and is shown, in the hydrodynamic limit, in Fig. 1 as a function of trap geometry.

In general, the first and second harmonic oscillations involved in the down-conversion process are the scissors modes in the xy and xz planes respectively ($M_{xy}$ and $M_{xz}$). However, for the special case of an axially symmetric trap, $M_{xy}$ is degenerate with $M_2$ and the two cannot be distinguished. The frequencies of these two scissors modes in the hydrodynamic limit are given by...
\[ \omega_{xy} = \sqrt{\omega_x^2 + \omega_y^2} \quad \text{and} \quad \omega_{xz} = \sqrt{\omega_x^2 + \omega_z^2}. \]

Thus we observe a resonant coupling when the following condition is satisfied:

\[ 2 \times \sqrt{1 + \frac{\omega_y^2}{\omega_x^2}} = \sqrt{1 + \frac{\omega_z^2}{\omega_x^2}} \quad (4) \]

The scissors mode is a small angle, irrotational oscillation of the condensate, about a given axis, that occurs without a change of the cloud shape [3][4]. \( M_z \) is an out of phase oscillation in the \( x \) and \( y \) directions, with a very small motion in the \( z \) direction. This axial motion falls to zero amplitude in the cylindrically symmetric trap. References [3][4] show the geometries of the two oscillations [2].

We initially create Bose-Einstein condensates in an axially symmetric time-orbiting potential (TOP) trap \( (\omega_x = \omega_y = 127.6 \pm 1.0 \, \text{Hz}) \). The trap frequency ratio, \( \omega_z/\omega_x \), is 2.83 and hence the condensate has an oblate shape. After RF evaporative cooling, the condensate temperature is significantly below 0.5 \( T_c \) and the thermal cloud is no longer visible. A detailed description of our method for selectively exciting the \( xz \) scissors mode is given in [13]. In summary, we apply an additional bias field of amplitude \( B_z \) in the axial direction, oscillating in phase with the \( x \) component of the TOP bias field, \( B_x \). This enables the symmetry axis of the trapping potential to be tilted by an angle that depends on \( B_z/B_x \).

We first form the condensate in a standard TOP trap, then adiabatically tilt the trap to an angle \( \phi \) and then suddenly flip it to \( -\phi \). This excites a scissors mode oscillation of amplitude \( \theta = 2\phi \) in the \( xz \) plane, about the new equilibrium position.

However, the application of the \( z \) bias field has a second, more subtle effect [13]. The axial trapping frequency, \( \omega_z \), and hence the \( xz \) scissors mode frequency is reduced, as \( B_z \) is increased. The radial frequencies \( \omega_x \) and \( \omega_y \) are also reduced, but by a negligible amount. By changing \( B_z/B_x \) we are able to tune the \( xz \) scissors frequency into resonance with twice the \( xy \) scissors frequency. Since the excitation angle is coupled to \( B_z/B_x \), this is also changed, but always remains well within the small angle scissors mode limit defined in [6]. Figure 3 shows theoretically how the excitation amplitude, \( \theta \), and the frequency of the \( xz \) scissors mode, \( \omega_{xz} \), change as a function of \( B_z/B_x \) and hence \( \omega_z/\omega_x \).

Our first observation of Beliaev damping occurred in an axially symmetric trap, \( \omega_x = \omega_y \). In this case the \( xy \) scissors mode becomes degenerate with, and indistinguishable from the \( |m = 2\rangle \) quadrupole mode, with frequency \( \sqrt{2} \omega_x \) (Fig. 3). The resonance condition of Eq. (1) is then fulfilled when \( \omega_z/\omega_x = \sqrt{2} \).

We recorded the \( xz \) scissors mode oscillation using destructive absorption imaging. The angle of the cloud as a function of oscillation evolution time, was extracted from 2D Gaussian fits to absorption images of the expanded condensate. We fit an exponentially decaying sine wave to the angle versus time data and use the damping rate as a measure of the rate at which energy is flowing out of the mode. For values of \( \omega_z/\omega_x \) far from the resonance condition, we observe only a slow Landau damping rate of \( \sim 18 \, \text{Hz} \) (Fig. 4). This background damping rate increases with the mode frequency, \( \omega_{xz} \), as predicted in [14].

Close to resonance we observe two new features in the \( xz \) scissors mode. Firstly there is a sharp increase in the initial damping rate of the oscillation, as the Beliaev process becomes significant (Fig. 3b). Secondly, over longer observation periods \( (t > 30 \, \text{ms}) \), the amplitude envelope grows and then falls again, as energy continues to be transferred back and forward between the two modes, in a manner characteristic of non-linear coupling (Fig. 3c) [15]. To obtain a measure of the total damping rate, we fit the decaying sine wave up to, but not beyond, the first amplitude minimum. The maximum coupling rate occurs when \( \omega_z/\omega_x = 2.65 \pm 0.04 \), in very good agreement with the predicted value of \( \sqrt{2} \) (Fig. 4).

In the axially symmetric trap described so far, \( M_2 \) and \( M_{zy} \) are degenerate, but in a totally anisotropic trapping potential, \( \omega_x \neq \omega_y \), this degeneracy is broken (Fig. 4). To investigate coupling to the pure \( xy \) scissors mode, we repeated the experiment, looking for resonant behaviour in a range of triaxial traps. The trap geometry was chosen to be close to the resonance condition between \( M_{xy} \) and \( M_{xz} \) given in Eq. (4).

The condensate was formed in a symmetric TOP trap as before. However, whilst it was slowly tilted in the \( xz \) plane, it was also adiabatically deformed by ramping the amplitude of the \( y \) component of the TOP bias field. This made the trapping potential elliptical in the \( xy \) plane, with \( \omega_y/\omega_x \) between 0.6 and 1.05 [13]. For each value of \( \omega_y/\omega_x \), we determined the value of \( \omega_z/\omega_x \) for peak coupling. Figure 3 shows this data, with the resonant coupling condition superimposed. The agreement between experiment and theory over a range of trap geometries confirms that the correct damping process has been identified.

These observations do not rule out the possibility of down-conversion into the \( M_2 \) mode in an anisotropic potential, since our traps were specifically chosen for \( M_{xz} \) to be resonant with \( M_{xy} \) and not \( M_2 \). In fact, for traps with a very small anisotropy in the \( xy \) plane, so that both modes are close to resonance with the \( xz \) scissors mode, we notice a broadening of the peak in the initial damping rate. This is expected when a second down-conversion process, into the \( M_2 \) mode, is resonant in a slightly different trap geometry.

In non-linear optics, it is well-known that down-conversion produces output beams of squeezed light and analogous effects with matter waves may be explored using BEC. In previous work we observed up-conversion between the two \( m = 0 \) modes of a BEC in a cylindri-
cally symmetric potential \[17\] and interesting comparisons may be made between the two processes. Whilst the up-conversion process showed strong dispersion of the driving oscillation close to resonance, this was not observed in the measured values of \(\omega_{xz}\) in Fig. 2. Secondly, the return of energy to the original mode, shown strikingly in Fig. 3c, was not observed for the up-conversion process. Finally, whilst spontaneous up-conversion is predicted by the Gross-Pitaevskii equation, the reverse process of spontaneous down-conversion may not be described within this framework. This may be explained by the following simple argument \[15\]. The GPE contains a nonlinear term proportional to \(|\psi|^2\), so excitation of a pure mode with time dependence \(e^{i\omega t}\), leads to terms of angular frequency \(2\omega\), but not to terms in \(\omega/2\) and hence no down-conversion. However, if there is initially some amplitude at frequency \(\omega/2\), then nonlinear mixing with the oscillation at frequency \(\omega\) leads to a transfer of energy between the modes at \(\omega\) and \(\omega/2\).

In conclusion, we have observed Beliaev damping of the \(xz\) scissors mode. Measurements over a range of trap geometries confirm that this corresponds to resonant down-conversion into the \(xy\) scissors mode. In future experiments, we intend to probe the nature of the final squeezed state and investigate the role of the thermal cloud in the down-conversion process at finite temperature.

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\[12\] Note that in an axially symmetric trap down-conversion occurs into an equal superposition of the \(m = +2\) and \(m = -2\) modes, ensuring that the net angular momentum about the z axis is zero at all times. For a triaxial trap, angular momentum is not conserved and need not be considered in the coupling process.

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FIG. 2. The various effects of increasing the z bias field, $B_z$. $B_z/B_x$ is shown along the top axis and the trap frequency ratio, $\omega_z/\omega_x$, that results is shown along the lower x axis. The solid line is the theoretical value of the xz scissors frequency, $\omega_{xz}$, scaled against $\omega_x$. The solid circles show experimental values obtained by fitting sine waves to data such as in fig. 3a,b (left axis). The dotted line shows the theoretical excitation angle, $\theta$, whilst the open circles show the values obtained from measuring the tilt angle of the static trap (right axis).

FIG. 3. Experimental data showing the angle of a condensate in the xz scissors mode as a function of time. (a) and (b) are typical plots from cylindrically symmetric traps with different aspect ratios, fitted with exponentially decaying sine waves, from which the damping rates, $\Gamma$, were extracted. (a) $\omega_z/\omega_x = 2.36, \Gamma = 17 \pm 5$ Hz. (b) $\omega_z/\omega_x = 2.62, \Gamma = 57 \pm 8$ Hz. In (b) the trap geometry is very close to the resonance condition of Eq. [1] and the fast decay is primarily due to Beliaev damping. The conditions of (a) are far from resonance and so only the slow underlying Landau damping rate is observed. Figure (c) has the same initial data as (b) but continues to track the oscillation over a longer period. The transfer of energy back and forth between the two modes is clearly observed, as the amplitude of $M_{xz}$ falls and rises.

FIG. 4. Damping rate of the xz scissors mode as a function of the trap frequency ratio $\omega_z/\omega_x$, in an axially symmetric trap. The data is fitted with a Lorentzian peak plus a linear function. The Lorentzian gives the trap geometry for peak Beliaev damping. Far from resonance, the Landau damping rate is observed. The increase of the Landau damping rate with mode frequency is modeled by the linear part of the fit.
FIG. 5. Trap geometries for which $M_{xz}$ is resonant with twice the frequency of $M_{xy}$ (solid line) and $M_2$ (dashed line). The data points show the value of $\omega_z/\omega_x$ at which the peak damping rate was observed, for a given $\omega_y/\omega_x$. 