Medium Effects in Cooling of Neutron Stars and $3P_2$ Neutron Gap

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Abstract. We study the dependence of the cooling of isolated neutron stars on the magnitude of the $3P_2$ neutron gap. It is demonstrated that our “nuclear medium cooling” scenario is in favor of a suppressed value of the $3P_2$ neutron gap.

Key words. Dense baryon matter, neutron stars, medium effects, nucleon gaps, pion softening, heat transport.

1. Introduction

Theoretical study of the neutron star (NS) cooling has been started long ago in pioneering works of Tsuruta & Cameron 1965 and Bahcall & Wolf 1965. It has been argued that the one-nucleon so called direct Urca (DU) processes, $n \rightarrow p\bar{e}\nu$, $p\bar{e} \rightarrow n\nu$, are forbidden up to sufficiently high density and the main role play the two-nucleon so called modified Urca (MU) processes, like $nn \rightarrow npe\bar{\nu}$ and $np \rightarrow ppe\bar{\nu}$. As the result of many works the so called “standard” scenario of NS cooling emerged, where the main process responsible for the cooling is the modified Urca process $MU_{nn} \rightarrow npe\bar{\nu}$ calculated using the free one pion exchange (FOPE) between nucleons, see Friman & Maxwell (1979). By an order of magnitude less contribution to the neutrino emissivity is given by the nucleon bremsstrahlung (NB) processes. The main process among NB processes responsible for the cooling is neutron-neutron bremsstrahlung $nn \rightarrow nn\nu\bar{\nu}$, less effective is the neutron-proton bremsstrahlung $nn \rightarrow npnu\bar{\nu}$, and still less effective is the proton-proton one $pp \rightarrow ppnu\bar{\nu}$. This scenario explains only the group of slow cooling data. To explain the group of rapid cooling data the “standard” scenario was supplemented by one of the so called “exotic” processes either with pion condensate, or with hyperons, or involving the DU reactions, see Tsuruta (1979), Shapiro & Teukolsky (1983) and references therein. All these processes may occur only for densities higher than a critical density, $(2 \div 6) n_0$, depending on the model, where $n_0$ is the nuclear saturation density.

Also the pair breaking and formation (PBF) processes permitted in nucleon superfluids have been suggested. Flowers et al. (1976) calculated the emissivity of the $1S_0$ neutron pair breaking and formation (nPBF) process and Voskresensky & Senatorov (1987) considered a general case. Neutron and proton pair breaking and formation processes (nPBF) were incorporated within a closed diagram technique including medium effects. Numerical estimates are valid both for $1S_0$ and $3P_2$ superfluids. Schaab et al. (1997) have shown that the inclusion of the PBF processes into the cooling code may allow to describe the “intermediate cooling” group of data (even if one artificially suppressed medium effects). Thus the “intermediate cooling” scenario arose. Then the PBF processes were incorporated in the cooling codes of other groups, elaborating the “standard plus exotics” scenario, see Tsuruta et al. (2002), Yakovlev et al. (2004a), Page et al. (2004). However medium effects were for simplicity disregarded. Some papers included the possibility of internal heating that results in a slowing down of
the cooling of old pulsars, see Tsuruta (2004) and Refs. therein. However, paying the price for a simplification of the consideration, calculations being performed within the “standard plus exotics” scenario, did not incorporate in-medium effects. Recently Page et al. (2004) called this approach the "minimal cooling" paradigm.

The necessity to include in-medium effects into the NS cooling problem is a rather obvious issue. It is based on the whole experience of condensed matter physics, of the physics of the atomic nucleus and it is called for by the heavy ion collision experiments, see Migdal et al. (1990), Rapp & Wambach (1994), Ivanov et al. (2001). The relevance of in-medium effects for the NS cooling problem has been shown by Voskresensky & Senatorov (1984), (1986), (1987), Senatorov & Voskresensky (1987), Migdal et al. (1990), Voskresensky (2001) who calculated emissivity of the MU, NB and PFB processes taking into account in-medium effects. We further call these processes medium modified Urca process (MMU), medium nucleon bremsstrahlung (MNB) and medium pair breaking and formation (MPBF) processes. The efficiency of the developed “nuclear medium cooling” scenario for the formation (MPBF) processes. The efficiency of the development of the new cooling code first by Schaab et al. (1997) and then by Blaschke et al. (2004). In the latter paper it was shown that it is possible to fit the whole set of cooling data available by today. Besides the incorporation of in-medium effects into the pion propagator and the vertices, it was also exploited that the $3P_2$ neutron gaps are dramatically suppressed. The latter assumption was motivated by the analysis of the data (see Figs 12, 15, 20 – 23 of Blaschke et al. (2004)) and by recent calculations of the $3P_2$ neutron gaps by Schwenk & Friman (2004). However more recent work of Khodel et al. (2004) suggested that the $3P_2$ neutron pairing gap should be dramatically enhanced, as the consequence of the softening of the pion propagator. Thus results of calculations of Schwenk & Friman (2004) and Khodel et al. (2004), which both had the same aim to include medium effects in the evaluation of the $3P_2$ neutron gaps, are in a deep discrepancy with each other.

Our aim here is to check consequences of a possibility of enhanced $3P_2$ neutron pairing gap within the "nuclear medium cooling" scenario following the work of Blaschke et al. (2004). The paper is organized as follows. In section 2 we start with a brief recapitulation of the Landau-Migdal Fermi liquid approach. Subsection 2.1 calculates $NN$ interaction amplitude. In subsection 2.2 we separate the in-medium pion mode yielding the main contribution to the $NN$ interaction for $n > n_0$. Subsection 2.3 demonstrates how important is the renormalization of the weak interaction in the medium. In subsection 2.4 we show internal inconsistencies of the FOPE model, which is the base of the "standard" scenario. Then in section 3 we recapitulate main processes as they are treated within the "nuclear medium cooling" scenario. In section 4 we discuss calculations of the pairing gaps and how gaps affect the emissivities of different processes. Then in section 5.1 we discuss the cooling model of Blaschke et al. (2004). In section 6 we present emissivities of the main processes, the heat capacity and thermal conductivity contributions in the scenario of Khodel et al. (2004), when the neutron processes are assumed to be frozen. Then in section 7 we show our numerical results. Concluding remarks are formulated in section 8.

2. Medium effects. Nuclear Fermi liquid

2.1. $NN$ interaction. Hard and soft modes

At temperatures of our interest ($T \ll \varepsilon_F$, $\varepsilon_F$) neutrons and even protons are only slightly excited above their Fermi seas and all the processes occur in a narrow vicinity of the Fermi energies $\varepsilon_F$. Quasiparticle approximation is fulfilled for nucleons. In such a situation Fermi liquid approach seems to be the most efficient one. Within this approach the long-scale phenomena are treated explicitly whereas short-scale ones are described by the local quantities expressed via phenomenological so called Landau-Migdal parameters. We deal with the baryon densities $n_0 > n > 0.5 n_0$. The value $n_0 \approx 0.5 m^3_{\pi}$, where $m_{\pi} = 140$ MeV is the pion mass, $\hbar = c = 1$. At such densities related to intermolecular distances $d \sim 1/n^{1/3} \sim (0.5 \div 1.3) 1/m_{\pi}$ processes important for the description of the $NN$ interaction correspond to typical energies and momenta $\omega, k \lesssim few m_{\pi}$. We call them the long-range processes and treat explicitly. These are nucleon particle-hole processes, $\Delta$ isobar-nucleon hole processes (since the typical energy of the isobar is of the order of the mass difference $m_{\Delta} - m_{NN} \approx 2.1 m_{\pi}$) and processes related to the excitation of the pion (the typical excitation energy is of the order of the pion mass). Thus we explicitly present loop diagrams which sharply depend on the energy and momentum for $\omega, k \lesssim few m_{\pi}$ of our interest.

Using above argumentation of Fermi liquid theory (see Landau (1956), Migdal (1967), Migdal et al. (1990)) the retarded $NN$ interaction amplitude is presented as follows (see also Voskresensky (2001) for further details)

\[ \begin{align*}
\bar{\Psi} \Psi &= \bar{\psi} \psi + \bar{\Delta} \Delta, \\
\bar{\Delta} \Delta &= \bar{\psi} \psi + \bar{\Delta} \Delta, \\
\end{align*} \]

where

\[ \begin{align*}
\bar{\psi} \psi &= \bar{\psi} \psi + \bar{\Delta} \Delta, \\
\bar{\Delta} \Delta &= \bar{\psi} \psi + \bar{\Delta} \Delta. \\
\end{align*} \]

The solid line presents the quasiparticle Green function of the nucleon, whereas double-line, of the $\Delta$ isobar.\footnote{Diagrams with open particle lines have sense only if quasiparticle approximation is valid. Otherwise one needs to use a closed diagram technique developed by Voskresensky & Senatorov (1987), Knoll & Voskresensky (1995), (1996).}
Thus most long-range diagrams of the particle-hole and ∆-nucleon hole types are summed up in (1). Other long-range term comes from the pion. The double-wavy line in (2) corresponds to the exchange of the free pion with inclusion of the contributions of the residual S wave πNN interaction and ππ scattering, i.e., the residual irreducible interaction to the nucleon particle-hole and delta-nucleon hole. The latter contributions we have taken into account in (1). Thus the full particle-hole, delta-nucleon hole and pion irreducible block (first block in (2)) is by its construction essentially more local than contributions given by explicitly presented graphs. If we would like to calculate the irreducible block in (2) we would need to introduce σ, ω, ρ exchanges and to somehow treat diagrams with multiple pion lines. All these terms are less energy-momentum dependent for ω, k ≲ few mπ then those we treat explicitly. Up to now we did not do any approximations, except the quasiparticle approximation for the nucleons. Now comes the main approximation (or better to say ansatz). Using the locality argument, one assumes that the Landau-Migdal parameters, fnn, fnp in scalar channel and gnn, gnp in spin channel, which parameterize the first block in (2), are approximately constants. Their values are extracted from analysis of experimental data. In a more extended approach these parameters should certainly be calculated as functions of the density, neutron and proton concentrations, energy and momentum.

The part of interaction involving ∆ isobar is analogously constructed

\[
\begin{align*}
\gamma \to [ \begin{array}{c}
\gamma \\
\alpha \\
\beta 
\end{array} ] \\
\xrightarrow{\text{∆}} \begin{array}{c}
\alpha \\
\beta 
\end{array} 
\end{align*}
\]  
\tag{3}

The main part of the N∆ interaction is due to the pion exchange. Although information on local part of the N∆ interaction is rather scarce, one can conclude (Migdal et al. (1990), Suzuki et al. (1999)) that the corresponding Landau-Migdal parameters are essentially smaller than those for NN interaction. Therefore for simplicity we neglect the first graph in r.h.s. of (3).

Straightforward resummation of (1) in neutral channel yields (Voskresensky & Senatorov (1987), Migdal et al. (1990))

\[
\Gamma_{\alpha\beta}^{\pi} = C_0 \left( f^{\alpha\beta}_{\pi} + Z^{\alpha\beta}_{\pi} \sigma_1 \cdot \sigma_2 \right) + f^{\pi}_n \mathcal{T}^{\alpha\beta}_{nn} (\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k}),
\]  
\tag{4}

\[
\begin{align*}
\mathcal{F}^{\alpha\beta}_{\pi} &= f_{n\alpha}(f_{n\beta}), \\
Z^{\alpha\beta}_{\pi} &= g_{nn}\gamma(g_{nn}), \\
\mathcal{T}^{\alpha\beta}_{nn} &= \gamma^2(g_{nn})D^R_{\pi}, \\
\mathcal{T}^{\alpha\beta}_{pp} &= \gamma^2(p\pi)D^R_{\pi}, \\
\gamma^{-1}(x) &= 1 - 2xC_0A^R_{\pi n}, \\
f_{nn} &= f_{pp} = f + f', \\
f_{np} &= f - f', \\
g_{nn} &= g_{pp} = g + g', \\
g_{np} &= g - g',
\end{align*}
\]  
\tag{5}

\[
\begin{align*}
f_{nn} &= f_{pp} = f + f', \\
f_{np} &= f - f', \\
g_{nn} &= g_{pp} = g + g', \\
g_{np} &= g - g',
\end{align*}
\]  
\tag{6}

\[
A_{n\beta} = \frac{\beta}{\alpha},
\]  
\tag{7}

\[
A_{nn}(\omega \approx q) \approx m_\pi^2(4\pi^2)^{-1} \left( \frac{1 + v_{Fn}}{1 - v_{Fn}} - 2v_{Fn} \right),
\]  
\tag{8}

where Saperstein & Tolokonnikov (1998), Fayans & Zawischa (1995), Borzov et al. (1984), including...
quasiparticle renormalization pre-factors, derived the values $f \approx 0$, $f' \approx 0.5 \div 0.6$, $g \approx 0.05 \pm 0.1$, $g' \approx 1.1 \pm 0.1$.

Typical energies and momenta entering $NN$ interaction of our interest are $\omega \gtrsim 0$ and $k \gtrsim p_{FN}$. Then one estimates $\gamma(g_{nn}, \omega \approx 0, k \approx p_{FN}, n = n_{0}) \approx 0.35 \div 0.45$. For $\omega = k \approx T$ typical for the weak processes with participation of $\nu \nu$ one has $\gamma^{-1}(g_{nn}, \omega \approx k \approx T, n = n_{0}) \approx 0.8 \div 0.9$.

### 2.2. Virtual Pion Mode

The pion is strongly affected by nucleon surrounding. It is obvious already from the fact that typical densities of our interest are $n \sim 1$ and the pion-nucleon coupling constant is $f_{N \pi} = 1$ in units $m_{\pi} = \hbar = c = 1$. Obviously perturbation theory is not applicable in this case. Instead we continue to use Fermi liquid approach suitable for the description of the strong interaction.

Straightforward resummation of diagrams (1), (2) yields the following re-summed Dyson equation for pions

\[ \pi = \pi + \pi_{\text{res}} + \Pi_{R_{\text{res}}}^{R} \]  

The $\pi N \Delta$ full-dot-vertex includes a phenomenological background correction due to the presence of the higher resonances, $\Pi_{R_{\text{res}}}^{R}$ is the residual retarded pion self-energy that includes the contribution of all the diagrams which are not presented explicitly in (9), as $S$ wave $\pi NN$ and $\pi \pi$ scatterings (included by double-wavy line in (2)). For zero temperature the main contribution to $\Pi_{R_{\text{res}}}^{R}$ is given by the Weinberg-Tomozawa term which has a simple analytic form. A part of $\Pi_{R_{\text{res}}}^{R}$ related to $\pi \pi$ fluctuations is important for the description of the vicinity of the pion condensation critical point. It is calculated explicitly. Other contributions are rather small and can be neglected, as it follows from the comparison of the theory predictions with different atomic nucleus data, see Migdal et al. (1990).

The full $\pi NN$ vertex takes into account $NN$ correlations

\[ \pi = \pi + \pi_{\text{res}} + \Pi_{R_{\text{res}}}^{R} \]  

Due to that the nucleon particle-hole part of $\Pi_{\pi 0}$ is $\propto \gamma(g_{nn})$ and the nucleon particle-hole part of $\Pi_{\pi \pm}$ is $\propto \gamma(g')$. The value of the $NN$ interaction in the pion channel is determined by the full pion propagator at small $\omega$ and $k \approx p_{FN}$, i.e. by the quantity

\[ (\omega^*)^2(k) = -(D_{\pi}^{R})^{-1}(\omega = 0, k, \mu_{\pi}) \].

Typical momenta of our interest are $k \approx p_{FN}$. Indeed the momenta entering the $NN$ interaction in MU and MMU processes are $k = p_{FN}$, the momenta governing the MNB are $k = k_{m}$ (Voskresensky & Senatorov (1986)) where the value $k_{m} \approx (0.9 \div 1)p_{FN}$ corresponds to the minimum of $(\omega^*)^2(k)$. The quantity $\omega^* \equiv \omega^*(k_{m})$ has the meaning of the effective pion gap, i.e., in another words, an effective pion mass. The effective pion gap $\omega^*$ demonstrates how much the virtual (particle-hole) mode with pion quantum numbers is softened at given density. The quantity $\omega^*^2(p_{FN}(n))$ replaces the value $(\pi^2 + g_{F_{\pi}}^2)$ in the case of the free pion propagator used for the calculation of the MU process by Friman & Maxwell (1979). It is different for $\pi^0$ and for $\pi^\pm$ since neutral and charged channels are characterized by different diagrams permitted by charge conservation, thus also depending on the value of the pion chemical potential, $\mu_{\pi^0} \neq \mu_{\pi^\pm} \neq 0$, $\mu_{\pi^0} = 0$. For $T \ll \epsilon_{F_{\pi}} \approx \epsilon_{p_{F_{\pi}}}$, one has $\mu_{\pi^0} = \mu_{e} = \epsilon_{F_{\pi}} - \epsilon_{p_{F_{\pi}}}$, as follows from equilibrium conditions for the reactions $n \rightarrow p\pi^0$ and $n \rightarrow p\pi^\pm$.

As follows from numerical estimates of different $\gamma$ factors entering (4) and (7), the main contribution to $NN$ interaction for $n > n_0$ is given by the resummed medium one pion exchange (MOPE) diagram

![Fig. 1. Nucleon - nucleon correlation factor $\Gamma$ and squared of the effective pion gap $\omega^*$ with pion condensation (branches 1a, 2, 3) and without (1a, 1b).](image)

Fig.1 shows the behavior of the pion gap for $n < n_c^{PU}$, where $n_c^{PU}$ is the critical density for the pion condensation. In this work we for simplicity do not distinguish between different possibilities of $\pi^0$, $\pi^\pm$ condensations and the so called alternative-layer-structure, see Voskresensky & Senatorov (1984), Migdal et al. (1990).
and Umeda et al, 1994. Thus we assume that $\pi^0$ and $\pi^-$ condensations occur at the very same critical density $n^c_{\text{PU}}$. Although the value $n^c_{\text{PU}}$ depends on different rather uncertain parameters we following Blaschke et al. (2004) further assume $n^c_{\text{PU}} \approx 3 n_0$, cf. discussion by Migdal et al. (1990). The curve 1b demonstrates the possibility of a saturation of pion softening and the absence of pion condensation for $n > n^c_{\text{PU}}$ (this possibility could be realized, e.g., if Landau-Migdal parameters increased with the density). This pion gap (from curves 1a+1b) is the value that determines the pion Green function for the pion excitations. Curves 2, 3 demonstrate the possibility of pion condensation for $n > n^c_{\text{PU}}$. The continuation of the branch 1a for $n > n^c_{\text{PU}}$, the branch 2, shows the reconstruction of the pion dispersion relation on the ground of the condensate state. This pion gap is the value that determines the pion Green function for the pion excitations on the ground of the pion condensate vacuum. In presence of the pion condensate (for $n > n_{\text{c}}$) the value $\omega^*$ from the curve 2 enters emissivities of all processes with pion excitations in initial, intermediate and final reaction states.

In agreement with a general trend known in condensed matter physics fluctuations dominate in the vicinity of the critical point of the phase transition and die out far below and above the critical point. The jump from the branch 1a to 3 at $n = n^c_{\text{PU}}$ is due to the first order phase transition to the $\pi$ condensation, see discussion of this point by Dyugaev (1975), (1982), Voskresensky & Mishustin (1981), (1982), Migdal et al. (1990). The $|\omega^*|$ value on the branch 3 is proportional to the amplitude of the pion condensate mean field (the line with the cross in standard notation of the diagram technique).

The observation that the pion condensation appears by the first order phase transition needs a comment. With the first order phase transitions in the systems with several charged species is associated the possibility of the mixed phase, see Glendenning 1992. The emissivity is increased within the mixed phase since efficient DU-like processes due to nucleon re-scattering on the new-phase droplets are possible. However Voskresensky et al. 2002, (2003), Maruyama et al. 2003, (2005a) demonstrated that, if exists, the mixed phase is probably realized only in a narrow density interval due to the charge screening effects. Thereby to simplify the consideration we further disregard the possibility of the mixed phase. We also disregard the change in the equation of state (EoS) due to the pion condensation assuming that the phase transition is rather weak.

Also in Fig. 1 the density dependence of the correlation factor $\Gamma$ is presented. Approximately for the correlation factor entering the emissivity of the MMU process one has $\Gamma(n) \approx 1/[1 + C(n/n_0)^{1/3}]$, $C \approx 1.4 \div 1.6$. Note that this value $\Gamma$ is an averaged quantity ($\Gamma^a = \Gamma^a_{\text{MMU}} G^a_{\text{PU}}$). Actually correlation factor $\Gamma^a$ entering the emissivity of the MMU process (see eq. (23) below) looks more involved and depends on the energy-momentum transfer, being different for vertices connected to the weak coupling (the correlation factor related to the weak coupling vertices $\Gamma_{\nu \rightarrow \phi}$ is rather close to unity) and for vertices related to the pure strong coupling ($\Gamma_\phi$ is slightly less than above introduced factor $\Gamma$), see estimates of energy-momentum dependence of correlation factors at the end of previous subsection. We see that vertices are rather strongly suppressed (and this suppression increases with the density) but the softening of the pion mode is enhanced ($\omega_{\pi}^2 > m_{\pi}^2$) for $n \approx 0.5 \div 0.8 n_0$. Such a behavior is motivated both theoretically and by analysis of nuclear experiments, see Migdal et al. (1990).

Please note that even with full microscopic calculations the functions $\Gamma(n)$ and $\omega^*(n)$ contain large uncertainties. These uncertainties come mainly from simplifications inherent in the Landau-Migdal approach to nuclear forces where the Landau-Migdal parameters are constants. However it seems us completely misleading to disregard medium effects only basing on the fact of existing rather large uncertainties in their knowledge. We believe that further more detailed comparison of the theory and experiment will allow to reduce these uncertainties.

### 2.3. Renormalization of the weak interaction.

The full weak coupling vertex that takes into account $NN$ correlations is determined by (10) where now the wavy line should be replaced by the lepton pair. Thus for the vertex of our interest, $N_1 \rightarrow N_2 l \bar{\nu}$, we obtain, see Voskresensky & Senatorov (1987), Migdal et al. (1990),

$$ V_\beta = \frac{G}{\sqrt{2}} [\bar{\gamma}(f') l_0 - g_A \bar{\gamma}(g') l \sigma], $$

for the $\beta$ decay and

$$ V_{nn} = -\frac{G}{2\sqrt{2}} [\bar{\gamma}(f_{nn}) l_0 - g_A \bar{\gamma}(g_{nn}) l \sigma], $$

$$ V_{pp} = \frac{G}{2\sqrt{2}} [\kappa_{pp} l_0 - g_A \bar{\gamma}_{pp} l \sigma], $$

for processes on the neutral currents $N_1 N_2 \rightarrow N_1 N_2 \nu \bar{\nu}$, $\kappa_{pp} = \gamma_{pp} = (1 - 4g_C A_{nn}) \gamma(g_{nn}),$}

$\gamma_{pp} = c_V - 2f_{np} \gamma(f_{nn}) C_0 A_{nn},$}

$\gamma_{pp} = (1 - 4g_C A_{nn}) \gamma(g_{nn}),$}

$$ V_{pp} = V_{pp}^N + V_{pp}^\gamma, G \approx 1.17 \cdot 10^{-5} \text{ GeV}^{-2}$$

is the Fermi weak coupling constant, $c_V = 1 - 4\sin^2 \theta_W$, $\sin^2 \theta_W \approx 0.23$, $g_A \approx 1.26$ is the axial-vector coupling constant, and $\kappa_{pp} = \gamma_{pp} = (1 - \gamma_{pp}) \gamma(g_{pp})$ is the lepton current. The pion contribution $\sim q^2$ is small for typical $|q| \approx T$ or $p_{pe}$, and for simplicity is omitted. In medium the value of $g_A$ (i.e. $g_A^a$) slightly decreases with the density.

The $\gamma$ factors renormalize the corresponding vacuum vertices. These factors are essentially different for different processes involved. The matrix elements of the neutrino/antineutrino scattering processes $N\nu \rightarrow N\nu$ and of MNB behave differently in dependence on the energy-momentum transfer and whether $N = n$ or $N = p$ in the weak coupling vertex. Vertices
are modified by the correlation factors (5) and (8). For \( N = n \) these are \( \gamma(g_{nn}, \omega, q) \) and \( \gamma(f_{nn}, \omega, q) \) leading to an enhancement of the cross sections for \( \omega > qf_{nn} \) and to a suppression for \( \omega < qf_{nn} \).

Renormalization of the proton vertex (vector part of \( V_{pp}^N + V_{pp}^+ \)) is governed by the processes, see Voskresensky & Senatorov (1987), Voskresensky et al. (1998), Migdal et al. (1990), Schaab et al. (1997), and it was shown that MnPBF and MpPBF processes may give contributions of the same order of magnitude. Finally, with electron-electron hole (second diagram (17) and neutron-neutron hole (first diagram (17))) correlations included we recover this statement and numerical estimate of Voskresensky & Senatorov (1987), Schaab et al. (1997).

Paper Kolomeitsev & Voskresensky (1999) gives another example demonstrating that, although the vacuum branching ratio of the kaon decays is \( \Gamma(K^- \rightarrow e^- + \nu_e)/\Gamma(K^- \rightarrow \mu^- + \nu_\mu) \approx 2.5 \times 10^{-5} \), in medium (due to lambda-proton hole, \( \Lambda \rho^{-1} \), decays of virtual \( K^- \)) it becomes to be of order of unit. Thus we again see that in dependence of what reaction channel is considered in-medium effects may or strongly enhance the reaction rates or substantially suppress them. Ignorance of these effects may lead to quite misleading results where one struggling for numerical factors \( \sim 1 \) may lose by orders of magnitude larger factors.

### 2.4. Inconsistencies of FOPE model

Since FOPE model became the base of the "standard" scenario for cooling simulations, which is still in use, we would like to demonstrate principal inconsistencies of the model for the description of interactions in dense \((n \gg n_0)\) baryon medium (see Voskresensky & Senatorov (1986), Voskresensky (2001)).

The only diagram in FOPE model which contributes to the MU and NB is

![Diagram](image)

Dots symbolize FOPE. This is first available Born approximation diagram, i.e. second order perturbative contribution in \( f_{\pi N} \) coupling. In order to be theoretically consistent one should use perturbation theory up to the very same second order in \( f_{\pi N} \) for all the quantities. E.g., pion spectrum is then determined by pion polarization operator expanded up to the very same order in \( f_{\pi N} \):

\[
\omega^2 \simeq m_\pi^2 + k^2 + \Pi^R_0(\omega, k, n),
\]

(19)

The value \( \Pi^0(\omega, k, n) \) is easily calculated containing no any uncertain parameters. For \( \omega \rightarrow 0, k \simeq p_F \) of our interest and for isospin symmetric matter

\[
\Pi^R_0(\omega, k, n) = -\alpha_0 - i\beta_0 \omega + \frac{2m_N p_F k^2 f_{\pi N}^2}{ \pi^2} > 0, \quad \beta_0 \simeq \frac{m_\pi^2 k^2 f_{\pi N}^2}{ \pi} > 0.
\]

Replacing this value to (19) we obtain a solution \( i\beta_0 \omega \simeq (\omega^*)^2(2) \) with \( \text{Im} \omega < 0 \) for \( k \simeq k_m \) already for \( n > 0.3n_0 \).
that would mean appearance of the pion condensation \((\omega^*)^2(k_m) < 0\). Indeed, the mean field begins to increase with the time passage \(\varphi \sim \exp(-\text{Im} \omega \cdot t) \sim \exp(\alpha t), \alpha > 0\), until repulsive \(\pi \pi\) interaction will not stop its growth. But it is experimentally proven that there is no pion condensation in atomic nuclei, i.e. even at \(n = n_0\).

The puzzle is solved as follows. FOPE model does not work for such densities. One should replace FOPE by the full \(NN\) interaction given by (4), (7). Essential part of this interaction is due to MOPE with vertices corrected by \(NN\) correlations, see (12). Also the particle-hole, \(NN^{-1}\), part of the pion polarization operator is corrected by \(NN\) correlations. Thus

\[
\gamma \approx \Pi^0(\omega, k, n) \gamma(g', \omega, k, n) \tag{21}
\]

being suppressed by the factor \(\gamma(g', \omega = 0, k \simeq p_F, n \simeq n_0) \approx 0.35 \div 0.45\). Final solution of the dispersion relation (19), now with full \(\Pi\) instead of \(\Pi^0\), yields \(\text{Im} \omega > 0\) for \(n = n_0\) whereas the solution with \(\text{Im} \omega < 0\), which shows the beginning of pion condensation, appears only for \(n > n_{\pi} > n_0\).

### 3. Medium effects in neutrino radiation processes

#### 3.1. Medium effects in two-nucleon processes

Medium effects essentially modify the contributions of all processes. Main contributing diagrams are

![Diagram of two-nucleon processes]

It was shown in Voskresensky & Senatorov (1984), (1986), (1987), Migdal et al. (1990), see also a more recent review Voskresensky (2001), that the main contribution to the MMU process actually comes from the pion channel of the reaction \(nn \to npe\pi\) (first diagram (22)), where \(e\nu\bar{\nu}\) are radiated from the intermediate pion exchanging nucleons. A less contribution comes from the \(NN^{-1}\) intermediate reaction states (second diagram), and only much less contribution for \(n \gtrsim n_0\) comes from the nucleon of the leg of the reaction (third diagram, which naturally generalizes the corresponding MU(FOPE) contribution (18)).

Moreover, due to the pion softening (medium modification of the pion propagator) the matrix elements of the MMU process are further enhanced with the increase of the density towards the pion condensation critical point, see Fig. 1. Roughly, the emissivity of MMU reaction acquires then a factor (mainly due to the pion decay channel of MMU)

\[
\frac{\varepsilon_{\nu}[\text{MMU}]}{\varepsilon_{\nu}[\text{MU}]} \sim 10^{3/2} \frac{(n/n_0)^{10/3} \Gamma^6(n)}{[\omega^*(n)/m_\pi]^8}, \tag{23}
\]

where the pre-factor \((n/n_0)^{10/3}\) arises from the phase space volume.

A different enhancement factor arises for the MNB processes, where radiation from intermediate reaction states (see first two diagrams (22)) is forbidden:

\[
\frac{\varepsilon_{\nu}[\text{MNB}]}{\varepsilon_{\nu}[\text{NB}]} \sim 10^{4/3} (n/n_0)^{4/3} \frac{\Gamma^6(n)}{[\omega^*(n)/m_\pi]^5}. \tag{24}
\]

The value \(\omega^*\) entering eqs. (23) and (24) is determined by the curve 1a for \(n < n_{\pi}\) and by the curve 2 for \(n > n_{\pi}\) if condensate is present. If condensate is assumed to be absent one should use the continuation 1b of the curve 1a.

#### 3.2. Medium effects in one-nucleon DU-like processes

##### 3.2.1. MNPBF processes

The one-nucleon processes with neutral currents given by the second diagram (16) for \(N = (n, p)\) are forbidden at \(T > T_{cN}\) by energy-momentum conservations but they can occur at \(T < T_{cN}\), where \(T_{cN}\) is the critical temperature for the nucleon-nucleon \((NN)\) pairing. Then necessary energy and momentum can be taken from breaking and formation of the Cooper pair. However they need special techniques to be calculated, see Flowers et al. (1976), Voskresensky & Senatorov (1987). Their calculation is easily done in terms of closed diagrams with normal and anomalous Green functions, see Voskresensky & Senatorov (1987). These diagrams contain only one nucleon loop. It clearly demonstrates that these processes are indeed one-nucleon-like processes rather than two-nucleon NB-like processes, as one sometimes interprets them. Due to the one-nucleon origin there appears a huge (for the gap \(\Delta \gtrsim 2\) MeV) pre-factor \(\sim 10^{29}\) in their emissivity, see eq. (37) below. Also the typical temperature pre-factor \(\sim T^2\) that one re-writes following rough estimate of Flowers et al. (1976) is actually misleading. Instead there is a \(\Delta^2(T/\Delta)^{1/2}\) pre-factor.

These processes \((\text{MnPBF}) n \to n\nu\bar{\nu}\) and \((\text{MpPB}) p \to p\nu\bar{\nu}\) play very important role in the cooling of superfluid NS, see Voskresensky & Senatorov (1987), Senatorov & Voskresensky (1987), Migdal et al. (1990), Schaab et al. (1997), Blaschke et al. (2004). Due to the full vertices in (16) their appear extra \(\Gamma_2\) factors in the MPBF emissivity. Medium effects are not as important for the \(n \to n\nu\bar{\nu}\) process changing the emissivity by a factor of the order of one but they increase the emissivity...
of the $p \rightarrow p\nu\bar{\nu}$ process by two orders of magnitude, see eq. (17).

### 3.2.2. Pion and kaon condensate processes

The $P$ wave pion condensate can be of three types: $\pi^+_c$, $\pi^+_s$, and $\pi^0$ with different values of the critical densities $n_{c\pi} = (n_{c\pi^+}, n_{c\pi^0}, n_{c\pi^0})$, see Migdal (1978). Thus above the threshold density for the pion condensation of the given type, the neutrino emissivity of the MMU process (22) is to be supplemented by the corresponding PU processes

\[
\begin{align*}
    n & \rightarrow p \nu \pi^- \quad (\pi^-) \\
    n & \rightarrow p \nu \pi^0 \quad (\pi^0) \\
    n & \rightarrow p \nu \pi^+ \quad (\pi^+) \ldots (25)
\end{align*}
\]

The wavy line with the cross is associated with the amplitude of the pion condensate mean field. It is to be stressed that contrary to FOPE model, the MOPE model of Voskresensky & Senatorov (1986) consistently takes into account the pion softening effects for $n < n_{c\pi}$ and both the pion condensation and pion softening effects on the ground of the condensate for $n > n_{c\pi}$. As we have mentioned, in our numerical calculations we for simplicity assume that $n_{c\pi} = n_{c\pi PU} \simeq 3 n_0$ is the same for neutral and charged condensates. For vertices in (25) we use the values presented above. Due to that emissivities of the PU processes are suppressed by the $\Gamma^2T^2_{\alpha\beta}$ factors. In case of rather weak condensate field for non-superfluid matter all processes with charged currents yield contributions to emissivity of the same order of magnitude, whereas processes on neutral currents are significantly suppressed, see Voskresensky & Senatorov (1984), (1986), Leinson (2004).

For $n > n_{c\pi}^{NU}$ the kaon condensate processes come into play. Most popular is the idea of the $S$ wave $K^-$ condensation (e.g. see Brown et al (1988), Tatsumi (1988)) which is allowed at $\mu_e > m_{K^-}$ ($m_{K^-}$ is the effective kaon mass) due to possibility of the reaction $e \rightarrow K^-\nu$. Analogous condition for the pions, $\mu_e > m_{\pi^-}$ ($m_{\pi^-}$ is the effective pion mass), is not fulfilled owing to a strong $S$ wave $\pi NN$ repulsion, cf. Migdal (1978), Migdal et al. (1990), (again intermediate effect!) otherwise $S$ wave $\pi^-$ condensation would occur at smaller densities then $K^-$ condensation. The neutrino emissivity of the $K^-$ condensate processes is given by equation analogous to charged pions however with a different $NN$ correlation factor and an additional suppression factor due to a small contribution of the Cabibbo angle.

The phase structure of dense NS matter might be very rich, including $\pi^0$, $\pi^\pm$ condensates and $K^0$, $K^-$ condensates in both $S$ and $P$ waves (Kolomeitsev & Voskresensky 2003); coupling of condensates Umeda et al, 1994; charged $\rho$-meson condensation (Voskresensky 1997, Kolomeitsev & Voskresensky 2004); fermion condensation yielding an efficient DU-like process in the vicinity of the pion condensation point (with the emissivity $\varepsilon_\nu \sim 10^{27} T_9^5$, $m_N^* \propto 1/T$), see Voskresensky et al. (2000); hyperonization, see Takatsuka & Tamagaki (2004); quark matter with different phases, like so called 2SC, CFL, CSL, plus their interaction with meson condensates, see K. Rajagopal & F. Wilczek 2000, Blaschke et al. (2001), Grigorian et al. (2004) and refs therein; and different mixed phases. In the present work, as in Blaschke et al. (2004), we suppress all these possibilities of extra efficient cooling channels except a PU process on the pion condensate. Other choices are effectively simulated by our PU choice.

### 3.2.3. Other resonance processes

There are many other reaction channels allowed in the medium. E.g., any Fermi liquid permits propagation of zero sound excitations of different symmetry related to the pion and the quanta of a more local interaction determined via Landau-Migdal parameters $f_{\alpha,\beta}$ and $g_{\alpha,\beta}$. These excitations being present at $T \neq 0$ may also participate in the neutrino reactions. The most essential contribution comes from the neutral current processes, see Voskresensky & Senatorov (1986), given by first two diagrams of the series

\[
\begin{align*}
    n \rightarrow \nu \pi \nu \pi \ldots (26)
\end{align*}
\]

Here the dotted line is zero sound quantum of appropriate symmetry. These are the resonance processes (second one of the DU-type) analogous to those processes going on the condensates with the only difference that the rates of reactions with zero sounds are proportional to the thermal occupations of the corresponding spectrum branches whereas the rates of the condensate processes are proportional to the modulus squared of condensate mean field. Contribution of the resonance reactions is as a role rather small due to a small phase space volume ($g \sim T$) associated with zero sounds. Please also bear in mind an analogy of the processes (26) with the corresponding phonon processes in the crust.
3.2.4. DU processes

The proper DU processes in matter, as \( n \rightarrow pe^-\bar{\nu}_e \) and \( pe^- \rightarrow n\nu_e \),

\[
\begin{array}{c}
e
\end{array}
\begin{array}{c}p
\hline
\end{array}
\begin{array}{c}
\bar{\nu}_e
\end{array}
\begin{array}{c}p
\hline
\end{array}
\begin{array}{c}
en
\end{array}
\begin{array}{c}
e
\hline
\end{array}
\begin{array}{c}
\nu
\end{array}
\begin{array}{c}n
\hline
\end{array}
\tag{27}
\]

should also be treated with the full vertices. They are forbidden up to the density \( n_e^{DU} \) when triangle inequality \( p_{Fp} < p_{F\bar{e}} + p_{Fp} \) begins to fulfill. For traditional EoS like that given by the variational theory by Akmal et al. (1998) DU processes are permitted only for \( n > 5n_0 \). Due to the full vertices in (27) their appear extra \( \Gamma_{w-s}^2 \) factors in the DU emissivity.

4. Gaps

In spite of many calculations which have been performed, the values of nucleon gaps in dense NS matter are poorly known. This is the consequence of the exponential dependence of the gaps on the density dependent potential of the in-medium \( NN \) interaction. This potential is not sufficiently well known. Gaps that we have adopted in the framework of the ”nuclear medium cooling” scenario, see Blaschke et al. (2004), are presented in Fig. 2. Thick dashed lines show proton gaps which were used in the work of Yakovlev et al. (2004a) performed in the framework of the “standard plus exotics” scenario. In their model proton gaps are artificially enhanced (that is not supported by any microscopic calculations) just to get a better fit of the data. We use their “1p” model. Neutron \( 3P_2 \) gaps presented in Fig. 2 (thick dash - dotted lines) are the same, as those of “3nt” model of Yakovlev et al. (2004a). We will call this choice, the model I. Thin lines show \( 1S_0 \) proton and \( 3P_2 \) neutron gaps from Takatsuka & Tamagaki (2004), for the model AV18 by Wiringa et al. (1995) (we call it the model II). We take the same \( 1S_0 \) neutron gap in both models I and II (thick solid line), as it was calculated by Ainsworth et al. (1989) and was previously used by Schaab et al. (1997) within the cooling code. Blaschke et al. (2004) have used both models I and II within the “nuclear medium cooling” scenario. As it was checked there, since the \( 1S_0 \) neutron pairing gap exists only within the crust, dying for baryon densities \( n \geq 0.6n_0 \), its effect on the cooling is rather minor. Opposite, the effect on the cooling arising from the proton \( 1S_0 \) pairing and from the neutron \( 3P_2 \) pairing, with gaps reaching up to rather high densities, is pronounced. The NS cooling essentially depends on the values of the gaps and on their density dependence. Findings of Schulze et al. (1996), Lombardo & Schulze (2000), who incorporated in-medium effects, motivated us to check the possibility of rather suppressed \( 1S_0 \) neutron and proton gaps. For that aim we introduced pre-factors for \( 1S_0 \) neutron and proton gaps which we varied in the range \( 0.2 \pm 1 \), see Figs. 18 and 19 of Blaschke et al. (2004).

4.1. Possibilities either of strong suppression or strong enhancement of \( 3P_2 \) gap

Recently Schwenk & Friman (2004) have argued for a strong suppression of the \( 3P_2 \) neutron gaps, down to values \( \lesssim 10 \text{ keV} \), as the consequence of the medium-induced spin-orbit interaction. They included important medium effects, as the modification of the effective interaction of particles at the Fermi surface owing to polarization contributions, with particular attention to spin-dependent forces. In addition to the standard spin-spin, tensor and spin-orbit forces, spin non-conserving effective interactions were induced by screening in the particle-hole channels. Furthermore a novel long-wavelength tensor force was generated. The polarization contributions were computed to second order in the low-momentum interaction \( V_{\text{low } k} \). These findings motivated Blaschke et al. (2004) to suppress values of \( 3P_2 \) gaps shown in Fig. 2 by an extra factor \( f(3P_2, n) = 0.1 \). Further possible suppression of the \( 3P_2 \) gap is almost not reflected on the behavior of the cooling curves.

Contrary to expectations of Schwenk & Friman (2004) a more recent work of Khodel et al. (2004) argued that the \( 3P_2 \) neutron pairing gap should be dramatically enhanced, as the consequence of the strong softening of the pion propagator. According to their estimation, the \( 3P_2 \) neutron pairing gap is as large as \( 1 \pm 10 \text{ MeV} \) in a broad region of densities, see Fig. 1 of their work.

Note that in order to apply these results to a broad density interval both models may need further improvements. The model of Schwenk & Friman (2004) was developed to describe not too high densities. It does not incorporate higher order nucleon-nucleon hole loops and the \( \Delta \) isobar contributions and thus it may only partially include the pion softening effect at densities \( \lesssim n_0 \). Contrary, the model of Khodel et al. (2004) uses a simplified analytic
expression for the effective pion gap \((\omega^*)^2(k_m)\), valid near the pion condensation critical point, if the latter occurred by a second order phase transition. The latter assumption means that \((\omega^*)^2(k_m)\) is assumed to be zero in the critical point of the phase transition. Outside the vicinity of the critical point the parameterization of the effective pion gap that was used can be considered only as a rough interpolation. Actually the phase transition is of first order and evaluations of quantum fluctuations done by Dyuaev (1982) show that the value of the jump of the effective pion gap in the critical point is not as small. Moreover repulsive correlation contributions to the NN amplitude have been disregarded. In the pairing channel under consideration, already outside a narrow vicinity of the pion condensation critical point, the repulsion originating from the pion softening. Notice that, if the pairing gap enhancement occurred only in a rather narrow vicinity of the pion condensation critical point, it would not affect the results of Blaschke et al. (2004). In the latter work two possibilities were considered (see Fig. 1): i) a saturation of the pion softening with increase of the baryon density resulting in the absence of the pion condensation and ii) a stronger pion softening stimulating the attraction originating from the pion softening. Notice that, if the pairing gap enhancement occurred only in a rather narrow vicinity of the pion condensation critical point, it would not affect the results of Blaschke et al. (2004). In the latter work two possibilities were considered (see Fig. 1): i) a saturation of the pion softening with increase of the baryon density resulting in the absence of the pion condensation and ii) a stronger pion softening stimulating the occurrence of the pion condensation for \(n > n_c \approx 3 n_0\). In both cases the effective pion gap was assumed never approaching zero and undergoing a not too small jump at the critical point from a finite positive value \((\omega^*)^2 \simeq 0.3 m_\pi^2\) to a finite negative value \((\omega^*)^2 \simeq -0.1 m_\pi^2\). The reason for such a strong jump is a strong coupling. If it were so, a strong softening assumed by Khodel et al. (2004) would not be realized. However, due to uncertainties in the knowledge of forces acting in strong interacting nuclear matter and a poor description of the vicinity of the phase transition point we can’t exclude that the alternative possibility of a tiny jump of the pion gap exists. Therefore we will check how these alternative hypotheses may work within our ”nuclear medium cooling” scenario. Thus avoiding further discussion of the theoretical background of the models, we investigate the possibility of a significantly enhanced \(3P_2\) neutron pairing gap and of a partially suppressed proton \(1S_0\) gap, as it has been suggested by Khodel et al. (2004). To proceed in the framework of our “nuclear medium cooling” scenario we introduce the enhancement factor of the original \(3P_2\) neutron pairing gap \(f(3P_2, n)\), and a suppression factor of the proton \(1S_0\) gap \(f(1S_0, p)\). We do not change the neutron \(1S_0\) gap since in any case its effect on the cooling is minor.

4.2. Suppression of the emissivity by superfluid gaps

Generally speaking, the suppression factors of superfluid processes are given by complicated integrals. As it was demonstrated by Sedrakian (2005) on the example of the DU process, these integrals are, actually, not reduced to the so called \(R\)-factors, see Yakovlev et al. (2004a). However, for temperatures essentially below the critical temperature the problem is simplified. With an exponential accuracy the suppression of the specific heat is governed by the factor \(\xi_{nn}\) for neutrons, and \(\xi_{pp}\) for protons:

\[
\xi_{ii} \approx \exp\left[-\Delta_{ii}(T)/T\right], \quad \text{for } T < T_{ci}; \quad i = n, p, \quad (28)
\]

and \(\xi_{ii} = 1\) for \(T > T_{ci}\), \(T_{ci}\) is the corresponding critical temperature. We do not need a higher accuracy to demonstrate our result. Therefore we will use these simplified factors.

For the emissivity of the DU process the suppression factor is given by \(\min\{\xi_{nn}, \xi_{pp}\}\), see Lattimer et al. (1991). The same suppression factor appears for other one nucleon processes, as PU and KU and the resonance processes on zero sounds, see second diagram (26). Suppression factors for two nucleon processes follow from this fact and from the diagrammatic representation of different processes within the closed diagram technique by Voskresensky & Senorov (1987), Knoll & Voskresensky (1995), (1996). These are: \(\xi_{nn} \cdot \min\{\xi_{nn}, \xi_{pp}\}\) for the neutron branch of the MU process (and for the medium modified Urca process, MMU); \(\xi_{pp} \cdot \min\{\xi_{nn}, \xi_{pp}\}\) for the corresponding proton branch of the process; \(\xi_{nn}^2\) for the neutron branch of the (medium modified) nucleon bremsstrahlung (MnB) and \(\xi_{pp}^2\) for the corresponding proton branch of the bremsstrahlung (MpB). Thus, for \(\Delta_{nn} \ll \Delta_{pp}\) both neutron and proton branches of the MMU process are frozen for \(T < T_{cn}\) due to the factors \(\xi_{nn}^2\) and \(\xi_{pp}^2\), respectively. Zero sound and other phonon processes shown by the first diagram (26) are not suppressed by the \(\xi_{ii}\) factors. However their contribution to the emissivity is very small due to the smallness of the available phase space volume.

5. Cooling model of Blaschke et al. (2004)

5.1. EoS and structure of NS interior, crust, surface

5.1.1. NS interior

We will exploit the EoS of Akmal et al. (1998) (specifically the Argonne \(V18 + \delta v + U\times^*\) model), which is based on the most recent models for the NN interaction with the inclusion of a parameterized three-body force and relativistic boost corrections. Actually we adopt a simple analytic parameterization of this model given by Heiselberg & Hjorth-Jensen 1999 (HHJ). The latter uses the compressional part with the compressibility \(K \approx 240\) MeV, and a symmetry energy fitted to the data around nuclear saturation density, and smoothly incorporates causality at high densities. The density dependence of the symmetry energy is very important since it determines the value of the threshold density for the DU process. The HHJ EoS fits the symmetry energy to the original Argonne \(V18 + \delta v + U\times^*\) model yielding \(n_{DU}^c \approx 5.19 n_0\) \((M_{DU}^c \approx 1.839 M_\odot)\). The original Argonne EoS allows for the neutron pion condensation (for \(n > 2n_0\)), that only slightly affects the energy density.
One may disregard this small change. This EoS does not allow for the charged pion condensation. The HHJ parameterization of the EoS does not include \( \pi \) condensation effects. We further assume that the pion condensation (neutral and charged) occurs for \( n > 3n_0 \) (see discussion in Blaschke et al. (2004)). We assume a minor effect of the pion condensation on EoS and disregard it. We also disregard changes of the isotopic composition due to the charged pion condensation. The latter effect would be small only if the charged pion condensate field were rather weak. Thus we assume that the HHJ parameterization of the EoS includes both mentioned effects or that they are negligible.

### 5.1.3. Envelope

The resulting cooling curves depend on the \( T_{\text{in}} - T_s \) relation between internal and surface temperatures in the envelope. Fig. 3 shows uncertainties existing in this relation. Calculation is presented for the canonical NS: \( M = 1.4M_\odot, R = 10 \text{ km} \) with the crust model HZ90 of Yakovlev et al. (2004b). Below we will show that a minimal discrepancy with the data is obtained with “our fit” model used by Blaschke et al. (2004). Using other choices like “Tsuruta law” \( T_{\text{sur}}^n = (10T_{\text{in}})^{2/3} \), where \( T_n \) and \( T_{\text{in}} \) are measured in K) only increases the discrepancy. To compare results with “our fit” model we use the upper boundary curve, \( \eta = 4 \cdot 10^{-16} \) and the lower boundary curve \( \eta = 4 \cdot 10^{-8} \). In Fig. 3 we also draw lines \( \eta = 1 \cdot 10^{-14} \) and \( \eta = 1 \cdot 10^{-11} \) as they are indicated in the corresponding Fig. 2 of Yakovlev et al. (2004b). In reality the selection of \( \eta = 4 \cdot 10^{-8} \) and \( \eta = 4 \cdot 10^{-16} \) as the boundaries of the uncertainty band seems to be a too strong restriction, see Yakovlev et al. (2004b). The limit of the most massive helium layer is achieved for \( \eta \sim 10^{-10} \).

On the other hand the helium layer begins to affect the thermal structure only for \( \eta > 10^{-13} \). Thus one could exploit \( 10^{-13} < \eta < 10^{-10} \), as a \( T_{\text{in}} - T_s \) band. We will use a broader band, as it is shown in Fig. 3. By this we simulate effect of maximum uncertainties in the knowledge of the \( T_{\text{in}} - T_s \) relation.

### 5.2. Main cooling regulators

We compute the NS thermal evolution adopting our fully general relativistic evolutionary code. This code was originally constructed for the description of hybrid stars by Blaschke et al. (2001). The main cooling regulators are the thermal conductivity, the heat capacity and the emissivity. In order to better compare our results with results of other groups we try to be as close as possible to their inputs for the quantities which we did not calculate ourselves. Then we add inevitable changes, improving EoS and including medium effects.

#### 5.2.1. Thermal conductivity

We take the electron-electron contribution to the thermal conductivity and the electron-proton contribution for normal protons from Gnedin & Yakovlev 1995. The total contribution related to electrons is then given by

\[
1/\kappa_e = 1/\kappa_{ee} + 1/\kappa_{ep}.
\]

For \( T > T_{cp} \) (normal ”n” matter), we have \( \kappa_{cp}^n = \kappa_{ep} \). For \( T < T_{cp} \) (superfluid ”s” matter) we use the expression

\[
\kappa_{cp}^s = \kappa_{ep}/\xi_{pp} > \kappa_{cp}^n,
\]

that gives a crossover from the non-superfluid case to the superfluid case. The vanishing of \( \kappa_{cp}^s \) for \( T \ll T_{cp} \) is a consequence of the scattering of superfluid protons on the electron impurities, see Blaschke et al. (2001). Following (29) we get \( \kappa_{cp}^n < \kappa_{cp}^s \).

For the neutron contribution,

\[
\kappa_n = 1/\kappa_{nn} + 1/\kappa_{np},
\]

we use the result of Baiko et al. 2001 that includes corrections due to the superfluidity. Although some medium

![Fig. 3. The relation between the inner crust temperature and the surface temperature for different models. Dash-dotted curves indicate boundaries of the uncertainty band. Notations of lines are determined in the legend. For more details see Blaschke et al. (2004) and Yakovlev et al. (2004b).](image)
effects are incorporated in this work, the nucleon-hole corrections of correlation terms and the modification of the tensor force are not included. This should modify the result. However, since we did not calculate $\kappa_n$ ourselves, we may only roughly estimate the modification: not too close to the critical point of the pion condensation the squared matrix element of the $N\bar{N}$ interaction $|M_{\text{med}}^2| \sim p_{F,n}^2 T_0^2/\omega^2$, see eq. (12) and values shown in Fig.1), is of the order of the corresponding quantity $|M_{\text{vac}}^2| \sim p_{F,n}^2/[m_\pi^2 + p_{F,n}^2]$ estimated with the free one pion exchange, whereas $|M_{\text{med}}^2|$ may significantly increase for $n \sim n_c^{\text{PU}}$. However Blaschke et al. (2004) checked that both increasing and decreasing of the thermal conductivity does not change the picture as the whole.

The proton term is calculated similar to the neutron one,

$$\kappa_p = 1/\kappa_{pp} + 1/\kappa_{np} .$$  (32)

The total thermal conductivity is the straight sum of the partial contributions

$$\kappa_{tot} = \kappa_e + \kappa_n + \kappa_p + ...$$  (33)

For the values of the gaps used in Blaschke et al. (2004) other contributions to this sum are smaller than those presented explicitly ($\kappa_e$, $\kappa_n$ and $\kappa_p$). Finally Blaschke et al. (2004) concluded that in their scenario transport is relevant only up to the first $10^3$ y.

5.2.2. Heat capacity

The heat capacity contains nucleon, electron, photon, phonon, and other contributions. The main in-medium modification of the nucleon heat capacity is due to the density dependence of the effective nucleon mass. We use the same expressions as Schaab et al. (1997). The main regulators are the nucleon and the electron contributions. For the nucleons ($i = n, p$), the specific heat is (Maxwell 1979)

$$c_i \sim 10^{20} (m_i^*/m_N) (n_i/n_0)^{1/3} \xi_i \ T_9 \ \text{erg cm}^{-3} \text{K}^{-1} ,$$  (34)

for the electrons it is

$$c_e \sim 6 \times 10^{19} (n_e/n_0)^{2/3} \ T_9 \ \text{erg cm}^{-3} \text{K}^{-1} .$$  (35)

Near the phase transition point the heat capacity acquires a pion fluctuation contribution. For the first order pion condensation phase transition this additional contribution contains no singularity, in difference with what would be for the second order phase transition, see Voskresensky & Mishustin (1982), Migdal et al. (1990). Finally, the nucleon contribution to the heat capacity may increase up to several times in the vicinity of the pion condensation point. The effect of this correction on global cooling properties is rather unimportant and simplifying we neglect it.

The symmetry of the $3P_2$ superfluid phase allows for the contribution of Goldstone bosons (phonons):

$$C_G \simeq 6 \cdot 10^{14} T_9^3 \ \text{erg cm}^{-3} \text{K}.$$  (36)

for $T < T_{n}(3P_2)$, $n > n_{cn}(3P_2)$. We include this contribution in our study although its effect on the cooling is rather minor. A similar contribution comes from other resonance processes permitted on zero sounds, see first diagram (26). In order not to introduce extra parameter dependence we will simulate the effect of all phonon and zero sound terms by the term (36).

5.2.3. Emissivity

We adopt the same set of partial emissivities as in the work of Schaab et al. (1997). The phonon contribution to the emissivity of the $3P_2$ superfluid phase, as well as the zero sound contribution, is negligible. The main emissivity regulators are the MMU, see above rough estimation (23), MnPBf and MpPBf, and MNB processes. For $n > n_{c}^{\text{PU}} \simeq 3 \ n_0$ the PU process becomes efficient and for $n > n_{c}^{\text{DU}} \simeq 5.19 \ n_0$ the DU process is dominating process.

Only qualitative behavior of the interaction shown in Fig. 1 is motivated by microscopic analysis whereas actual numerical values of the correlation parameter and the pion gap are rather uncertain. Thereby we vary the values $\Gamma(n)$ and $\omega^2(n)$ in accordance with our above discussion of Fig. 1. By that we check the relevance of alternative possibilities: a) no pion condensation and a saturation of the pion softening with increasing density, curves 1a+1b, and b) presence of pion condensation, curves 1a+2+3. We also add the contribution of the DU for $n > n_c^{\text{DU}}$.

All emissivities are corrected by correlation effects. The PU process contains an extra $\Gamma_3^2$ factor compared to the DU process (the emissivity of the latter is $\propto \Gamma_2^2 \omega_{\nu}^2 \omega_{\nu}^2$). Another suppression of PU emissivity comes from the fact that it is proportional to the squared pion condensate mean field $|\varphi|^2$. Near the critical point $|\varphi|^2 \sim 0.1$ increasing with the density up to $|\varphi|^2 \sim \omega_{\nu}^2/2$, where $f_{\nu} \simeq 93$ MeV is the pion decay constant. Finally, the PU emissivity is about 1-2 orders of magnitude suppressed compared to the DU one. Moreover, we adopt the same gap dependence for the PU process as that for the corresponding DU process. In superfluid matter all emissivity terms are suppressed by corresponding $\xi_i$ factors.

6. Main cooling regulators in scenario of Khodel et al. (2004)

In case when neutron processes are frozen the most efficient process is the MpPBf process, $p \rightarrow p\nu\bar{\nu}$, for $T < T_{cP}$. Taking into account medium effects in the weak coupling vertex, see eq. (17), we use the same expression for the emissivity of this process as has been used by Voskresensky (2001), Blaschke et al. (2004):

$$\varepsilon_{p[MpPBf]} \sim 10^{29} \frac{m_n^*}{m_N} \frac{p_{FP}}{p_{F}(n_0)} \left[ \frac{\Delta_{pp}}{\text{MeV}} \right]^{7/2} \times \left[ \frac{T}{\Delta_{pp}} \right]^{1/2} \frac{\xi_{pp}^2}{\text{erg cm}^3 \text{sec}} , T < T_{cP} .$$  (37)
We point out that this process contributes only below the critical temperature for the proton pairing. Inclusion of medium effects greatly enhances the vertex of this process compared to the vacuum vertex, see above discussion of this fact in subsection 2.3. Due to that a factor $\Gamma^2_{w-s} \sim 10^2$ arises, since the process may occur through $nn^{-1}$ and $ee^{-1}$ correlation states, with subsequent production of $\nu \bar{\nu}$ from the $nn^{-1}\nu \bar{\nu}$ and $ee^{-1}\nu \bar{\nu}$ channels rather than from a strongly suppressed channel $pp^{-1}\nu \bar{\nu}$, see Voskresensky & Senatorov (1987), Senatorov & Voskresensky (1987), Migdal et al. (1990), Voskresensky et al. (1998), Leinson (2000), Voskresensky (2001). Relativistic corrections incorporated in the description of the $pp^{-1}\nu \bar{\nu}$ vertex in Yakovlev et al. (2004a) also produce an enhancement but quite not as strong as that arising from medium effects in $nn^{-1}\nu \bar{\nu}$ and $ee^{-1}\nu \bar{\nu}$ channels. We point out that we see no arguments not to include these medium effects and we pay attention to only a moderate dependence of the result on the uncertainties in the knowledge of the strong interaction.

We also present here an explicit expression for the emissivity of the proton branch of the nucleon bremssstrahlung including medium effects, MpB, $pp \rightarrow pp\nu \bar{\nu}$. In case of suppressed neutron $3P_2$ gaps this process contributed much less than several others. However, in case when neutron reactions are frozen, the $pp \rightarrow pp\nu \bar{\nu}$ process becomes the dominating process for $T_{en} > T > T_{cp}$. The emissivity of the $pp \rightarrow pp\nu \bar{\nu}$ reaction takes the form (see Voskresensky & Senatorov (1986) for more details)

$$\epsilon(pB) \sim 10^{23} L_{pp}^2 I_{pp} \frac{Y_p^{5/3} \Gamma_{pp}^{1/3} T_s^{8}}{(n/v)^{5/3}} \text{erg cm}^3 \text{sec},$$

(38)

$T_s = T/10^9$ K, $m_p^*$ is the effective proton mass. Here we take $\Gamma_s \simeq \Gamma(n) \simeq 1/[1 + C(n/n_0)^{1/3}]$, $C \simeq 1.4 \div 1.6$. This factor takes into account $NN$ correlations in strong interaction vertices, $Y_p = n_p/n$ is the proton to nucleon ratio. As above, we for simplicity assumed that the value $k = k_m$, at which the squared of the effective pion gap $(\omega^*)^2(k)$ gets the minimum, is rather close to the value of the neutron Fermi momentum $p_F n$ (as it follows from the microscopic analysis of Migdal et al. (1990)). To simplify the consideration we take the same value of the effective pion gap for the given process as that for the MMU process (although in general case it is not so, and thus the result (38) proves to be essentially model dependent), cf. Blaschke et al. (2004),

$$I_{pp} \sim \frac{\pi}{64} \frac{(p_F n)^5}{p_F p} \omega^{*}(k_m) \frac{\omega^{*}(k_m)}{p_F n}.$$

(39)

We have checked that for $T < T_{cp}$ for the pairing gaps under consideration the MpB reaction contributes significantly less than the MpPBF process. It could be not the case only in a narrow vicinity of the pion condensation critical point, if pion condensation occurred with only a tiny jump of the effective pion gap in the critical point. However, even in this case there are many effects which could mask this abnormal enhancement.

For $n > n_c^{PU}$ the process $pp\nu \bar{\nu} \rightarrow pp\nu \bar{\nu}$ on the neutral pion condensate is still permitted. However its contribution is strongly suppressed, see Leinson (2004).

In case of frozen neutron degrees of freedom the specific heat is governed by protons and electrons, see eq. (34) for $i = p$ and (35). Here, we again suppress a contribution to the specific heat of a narrow vicinity of the pion condensation critical point due to the fact that in our scenario (see Fig. 1) the modulus of the squared effective pion gap $(\omega^*)^2$ is always larger than $\sim (0.1 \div 0.3) m^2$. With such an effective pion gap the pion contribution to the specific heat is not too strong and can be disregarded in order to simplify the consideration. For the second order phase transition (either for a first order phase transition but with a tiny jump of $|\omega^*|^2$ in the critical point), pion fluctuations would contribute stronger to the specific heat yielding a term $c_\pi \propto T_{PU}^3/\omega^*$ for $T - T_{PU}^3/T_c^U \ll T_c^U$, see Voskresensky & Mishustin (1982), Migdal et al. (1990).

In case when neutron processes are frozen the values $\kappa_n$ and $\kappa_p$ is suppressed and the thermal conductivity is reduced to the electron and proton contributions, see eqs. (29) and (32).

### 7. Numerical results

Now we will demonstrate results of our calculations of cooling curves. First we will present Fig. 21 of Blaschke et al. (2004), now Fig. 4. Cooling curves shown in this figure were calculated using "our fit" model of the crust, demonstrated by the solid curve in Fig. 3. Here and in the corresponding figures below the surface temperature is assumed to be red-shifted, as it is inferred by the observer from the radiation spectrum. Gaps are given by the model II of Fig. 2. However, the $3P_2$ gap is additionally suppressed by a factor $f(3P_2 n) = 0.1$, as motivated by calculations of Schwenk & Friman (2004). The calculation includes pion condensation for $n > n_c^{PU}$. Figure demonstrates a good fit of the data. If we used calculation disregarding possibility of pion condensation (see curves 1a+1b of Fig. 1) we would also get an appropriate fit of the data, cf. Fig. 20 of Blaschke et al. (2004). If we took the original $3P_2$ gap of the model II, we would not succeed to describe the data. The cooling then would be too fast, see Fig. 23 of Blaschke et al. (2004).

We will now check another possibility of ultraligh $3P_2$ neutron pairing gaps, as motivated by Khodel et al. (2004). In Figs. 5 and 6 we demonstrate the sensitivity of the results presented in Fig. 4 to the enhancement of the neutron $3P_2$ gap and to a suppression of the $1S_0$ proton gap, following the suggestion of Khodel et al. (2004). Again we use the calculation including pion condensation for $n > n_c^{PU}$. 

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is determined by curves 1a+2+3 of Fig. 1. The subsequent figures the value is given by "our fit" curve of Fig. 3. Here and in all subseuquent figures the value $T_1$ is the red-shifted temperature. NS masses are indicated in the legend. For more detail see Blaschke et al. (2004).

**Fig. 4.** Fig. 21 of Blaschke et al. (2004). Gaps are from Fig. 2 for model II. The original $3P_2$ neutron pairing gap is additionally suppressed by a factor $f(3P_2, n) = 0.1$. Pion gap is determined by curves 1a+2+3 of Fig. 1. The $T_s - T_{ia}$ relation is given by "our fit" curve of Fig. 3. Here and in all subsequent figures the value $T_1$ is the red-shifted temperature. NS masses are indicated in the legend. For more detail see Blaschke et al. (2004).

**Fig. 5.** Cooling curves according to the nuclear medium cooling scenario, see Fig. 4. Gaps are from Fig. 2 for model II but the $3P_2$ neutron pairing gap is additionally enhanced by a factor $f(3P_2, n) = 50$ and the $1S_0$ proton gap is suppressed by $f(1S_0, p) = 0.1$. Pion gap is determined by curves 1a+2+3 of Fig. 1. The $T_s - T_{ia}$ relation is given by "our fit" curve of Fig. 3.

We start with the "our crust" model and the model II for the gaps, using however the additional enhancement factor $f(3P_2, n) = 50$ for the neutron $3P_2$ gap. Introducing factors $f(1S_0, p) = 0.1$ and $f(1S_0, p) = 0.5$ we test the sensitivity of the results to the variation of the $1S_0$ proton gap. We do not change the value of the $1S_0$ neutron gap since its variation almost does not influence on the cooling curves for NS’s with masses $M > 1 M_\odot$, that we will consider.

Comparison of Figs. 4 – 6 shows that in all cases NS’s with masses $M \gtrsim 1.8 M_\odot$ cool similar in spite of the fact that $3P_2$ neutron and $1S_0$ proton gaps are varied in wide limits. This is because $3P_2$ neutron and $1S_0$ proton gaps disappear at the high densities, being achieved in the central regions of these very massive NS’s, see Fig. 2. Thus these objects cool down similar to non-superfluid objects. Extremely rapid cooling of stars with $M \gtrsim 1.84 M_\odot$ is due to the DU process, being very efficient in the normal matter. The latter process appears in the central region of NS’s with $M > 1.839 M_\odot$. Thereby we will notice that the cooling curves are very sensitive to the density dependence of the gaps. The difference in the cooling of NS’s with $M < 1.8 M_\odot$ in cases presented by Figs. 5 and 6 is the consequence of different values of proton gaps used in these two calculations. This difference is mainly due to the MpPBF processes. The larger the proton gap, the more rapid the cooling.

We checked that for stars with $M \lesssim 1.6 M_\odot$ for $T < T_{cn}$ for the $3P_2$ neutron pairing, a complete freezing of neutron degrees of freedom occurs already for $f(3P_2, n) \gtrsim 20$. Contributions to the emissivity and to the specific heat involving neutrons are fully suppressed then. For heavier stars ($M > 1.6 M_\odot$) a weak dependence on the value of the factor $f(3P_2, n)$ still remains even for $f(3P_2, n) > 100$ but the corresponding cooling curves lie too low to allow for an appropriate fit of the data. This difference between cooling of stars with $M < 1.6 M_\odot$ and $M > (1.6 \pm 1.7) M_\odot$ is due to the mentioned density dependence of the neutron $3P_2$ gap. The latter value smoothly decreases with increase of the density reaching zero for $n \gtrsim 4.5 n_0$ (the density 4.5 $n_0$ is achieved in the center of a NS of the mass $M = 1.7 M_\odot$). At densities slightly below 4.5 $n_0$ the gap is rather small. Therefore for stars with $M > (1.6 \pm 1.7) M_\odot$...
the scaling of the gap by a factor $f(3P_2, n)$ changes the size of the region where the gaps may affect the cooling. For stars with $M \lesssim 1.6 M_\odot$ gaps have finite values even at the center of the star. Thereby there exists a critical value of the factor $f(3P_2, n)$, such that for higher values of $f(3P_2, n)$ the cooling curves are already unaffected by its change.

Figs. 5 and 6 demonstrate that we did not succeed to reach appropriate overall agreement with the data getting too rapid cooling. The cooling of the old pulsars is not explained in all cases. Although the heating mechanism used by Tsuruta (2004) may partially help in this respect, the discrepancy between the curves and the data points seems to be too high, especially in Fig. 6. We see that in our regime of frozen neutron processes a better fit is achieved in Fig. 5, i.e., for a stronger suppressed proton gap (for $f(1S_0, p) = 0.1$). Actually we note that the discrepancy is even more severe, since to justify the idea of Khodel et al. (2004) we should exploit a softer pion propagator. Only a strong softening of the pion mode might be consistent with significant increase of the neutron $3P_2$ gap. On the other hand such an additional softening would immediately result in a still more rapid cooling. The work of Voskresensky et al. (2000) discussed the possibility of a novel very efficient process with the emissivity $\epsilon_p \propto T^5$, that would occur due to non-fermi liquid behavior of the Fermi sea in a narrow vicinity of the pion condensation critical point at the assumption of a strong pion softening. If we included this very efficient process, the disagreement with the data could be strongly enhanced. An enhancement of the specific heat due to pion fluctuations within the same proximity region of the pion condensation point can’t compensate the acceleration of the cooling owing to the enhancement of the emissivity. Khodel et al. (2004) used the value $n_c = 2 n_0$ for the critical density of the pion condensation. In case of the Urbana-Argonne equation of state that we exploit here (we use the HHJ fit of this equation of state that removes the causality problem, see Blaschke et al. (2004) for details) the density $n = 2 n_0$ is achieved in the central region of a NS with the mass $M \sim 0.8 M_\odot$. This means that all NS’s with $M \gtrsim 0.8 M_\odot$ would cool extremely fast and would not be seen in soft X rays.

Actually, we checked the whole interval of variation of $f(3P_2, n)$ and $f(1S_0, p)$ factors in the range $1 \div 100$ and $0.1 \div 0.5$ respectively. We verified that the variation of $f(3P_2, n)$ and $f(1S_0, p)$ factors in the whole mentioned range done within our parameterization of the effective pion gap does not allow to improve the picture (curves 1a+1b of Fig. 1). Using of the pion gap from branches 1a+1b does not allow to achieve a better fit. In all cases we obtain too fast cooling. To demonstrate this in Fig. 7 we show the cooling of a $1.4 M_\odot$ star for different values of the $f(3P_2, n)$ factor. The factor $f(1S_0, p)$ is taken to be 0.1. We see that for $f(3P_2, n) < 15 \div 0.2$ the curves rise with the increase of $f(3P_2, n)$ factor. For $f(3P_2, n) > 20$ the curves do not depend on $f(3P_2, n)$.

To check how the results are sensitive to uncertainties in our knowledge of the value (39) that determines the strength of the in-medium effect on the emissivity of the MpB process we multiplied (38) by a pre-factor $f$ (MpB) that we varied in a range $f$(MpB) $= 0.2 \div 5$. In agreement with the above discussion, for $f$(MpB) $< 1$, for temperatures $\log T_s[K] > 5.9$ the cooling curves are shifted upwards. Opposite, for $f$(MpB) $> 1$, for temperatures $\log T_s[K] > 5.9$ the cooling curves are shifted downwards. However independently of the value $f$(MpB) for $\log T_s[K] < 5.9$ curves are not changed. Thus it does not allow to diminish the discrepancy with the data.

Now we will check the efficiency of another choice of the gaps, as motivated by the model I, thick lines in Fig. 2. Compared to the model II the model I uses an artifically enhanced proton gap. Thereby, one can expect that the model I is less realistic than the model II. Also we pay attention to a different density dependence of the proton gap (it cuts off for densities $n \gtrsim 3n_0$ in the model I) compared to that given by the model II. However, as we have mentioned, uncertainties in existing calculations of the gaps are very high. Thus it is worthwhile to check different possibilities. Since the mentioned parameterization has been used by one of the groups working on the problem of cooling of NS’s, see Yakovlev et al. (2004a), we will consider consequences of this possibility as well.

Fig. 8 demonstrates our previous fit of the data within the model I, but for the original $3P_2$ neutron gap being suppressed by $f(3P_2, n) = 0.1$ (see Blaschke et al. (2004)). Again pion gap is determined by curves 1a+2+3 of Fig. 1.

![Fig. 7. Cooling curves of the neutron star with the mass 1.4 $M_\odot$ according to the nuclear medium cooling scenario, see Fig. 4. Gaps are from Fig. 2 for model II but the $3P_2$ neutron pairing gap is additionally enhanced by different factors $f(3P_2, n)$ (shown in Figure) and the $1S_0$ proton gap is suppressed by $f(1S_0, p) = 0.1$. Pion gap is determined by curves 1a+2+3 of Fig. 1. The $T_s = T_{in}$ relation is given by “our fit” curve of Fig. 3.](image-url)
If we took the original $3P_2$ gap of the model I, we would not succeed to describe the data. The cooling then would be too fast, see Fig. 22 of Blaschke et al. (2004). Therefore we will now check possibility of ultra-high $3P_2$ neutron pairing gaps, as motivated by Khodel et al. (2004).

As Fig. 5, Fig 9 uses factors $f(3P_2, n) = 50$ and $f(1S_0, p) = 0.1$ but now for the gap model I, and, as Fig. 6, Fig 10 uses $f(1S_0, p) = 0.5$ for the gap model I. Figs. 9 and 10 show that within the variation of the gaps of the model I the discrepancy with the data is still stronger compared to that for the above calculation based on the use of the model II. The difference between curves shown in Figs. 9 and 10 is less pronounced than for those curves demonstrated in Figs. 5 and 6 respectively. Indeed, as we have mentioned, the density dependence of the proton gap is different in models I and II. In the model II the proton gap reaches up to higher densities than in the model I (in the latter case the gap is cut already for $n \gtrsim 3n_0$). Thus in the case shown by Figs. 9 and 10 a non-superfluid core begins to contribute already for smaller values of the star mass. Therefore in both figures the corresponding cooling curves are almost the same for $M \gtrsim 1.6M_\odot$. Using of the gap from branches 1a+1b, i.e. disregarding the possibility of the pion condensation, does not allow to achieve a better fit.

The dependence of the results on the different choices of the $T_s - T_{in}$ relation is demonstrated by Figs. 11 and 12 for gaps based on a modification of the model II. For this demonstration we first took the upper boundary curve $\eta = 4 \cdot 10^{-16}$ and then the lower boundary curve $\eta = 4.0 \cdot 10^{-8}$ in Fig. 3. We show that these choices however do not allow to improve the fit. Comparing Figs. 11 ($\eta = 4 \cdot 10^{-16}$) and 5 (“our fit”) based on the very same modification of the model II we see that with the “our fit” crust model (Fig. 5) the deviation from the data points is less pronounced. We have checked that basing on the model I one arrives at the very same conclusion. An increase of the factor $f(1S_0, p)$ up to 0.5 still spoils the fit.

In Fig. 12 we use the lower boundary curve $\eta = 4.0 \cdot 10^{-8}$ of the Fig. 3. We further demonstrate that the selection of a different choice of the $T_s - T_{in}$ relation within the band shown in Fig. 3 does not allow to diminish discrepancy with the data. Contrary, this discrepancy just increases compared to that demonstrated by “our fit” model. Indeed, the cooling evolution for times $t \lesssim 10^5$ yr ($T_s \gtrsim 10^8$K) is governed by neutrino processes. Thus the
of Fig. 3. The evolution of NS’s for times $t > 10^8$ yr begins to be controlled by the photon processes. In the photon epoch ($t \gg 10^5$ yr) the smaller the $T_0$ value, the less efficient the radiation is. Thus for $t \gg 10^5$ yr the slowest cooling is obtained, if one uses the upper boundary curve $\eta = 4.0 \cdot 10^{-8}$ of Fig. 3. The “our crust” curve just simulates the transition from the one limiting curve to the other demonstrating the slowest cooling in the whole temperature interval shown in the figures.

We point out that in all cases the data are not explained within the assumption of an enhanced neutron $3P_2$ gap (for $f(3P_2, n) > 1$) and of a partially suppressed $1S_0$ proton gap (for $f(1S_0, p) = 0.1 \pm 0.5$).

8. Concluding remarks

Our aim was to consider the possibility of large $3P_2$ gaps within the same “nuclear medium cooling” scenario of Blaschke et al. (2004) that well described the cooling data in opposite assumption of suppressed $3P_2$ gaps. Therefore in the present work we did not incorporate possibilities of internal heating for old pulsars (see Tsuruta (2004)) and of existence of quark cores (see Blaschke et al. (2001), Grigorian et al. (2004) and refs therein).

The main problem with the given scenario is the following. At the frozen neutron contribution to the specific heat and to the emissivity the region of surface temperatures $T_0 > 10^6$K is determined by proton processes. The most efficient among them are the MpPBF process for $T < T_{cp}$ and MpB for $T > T_{cp}$. For proton gaps, which we deal with, the MpPBF process proves to be too efficient yielding too rapid cooling. Thus at least several slow cooling data points (at least data for old pulsars) are not explained. Note that some works ignore the mentioned above medium induced enhancement of the MpPBF emissivity that results in 10-100 times suppression of the rate. We omitted this possibility as not physical one. The origin of this enhancement is simple and associated with opening up of new reaction channels in the medium being forbidden in vacuum.

Concluding, we have shown that the “nuclear medium cooling” scenario of Blaschke et al. (2004) fails to appropriately fit the neutron star cooling data at the assumption of a strong enhancement of the $3P_2$ neutron gaps (we checked the range $f(3P_2, n) = 1 \div 100$) and for moderately suppressed $1S_0$ proton gaps (for $f(1S_0, p) = 0.1 \div 0.5$). On the other hand the very same scenario allowed us to appropriately fit the whole set of data at the assumption of a significantly suppressed $3P_2$ neutron gap (for $f(3P_2, n) \sim 0.1$). We observed an essential dependence of the results not only on the values of the gaps but also on their density dependence. We used the density dependence of the gaps according to the models I and II. The latter model is supported by microscopic calculations. We excluded an attempt to artificially fit the density dependence of the gaps trying to match cooling curves with the data. Although such an attempt could improve the fit, this way seems us rather not physical and we did not follow it. However we will greatly encourage further attempts of microscopic calculations of the gaps, which would take into account most important medium effects. With carefully treated gaps one could return to the simulation of the NS cooling.

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