Effective potential in non-supersymmetric $SU(N) \times SU(N)$ gauge theory
and interactions of type 0 D3-branes

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Abstract

We study some aspects of short-distance interaction between parallel D3-branes in
type 0 string theory as described by the corresponding world-volume gauge theory. We
compute the one-loop effective potential in the non-supersymmetric $SU(N) \times SU(N)$
gauge theory (which is a $Z_2$ projection of the $U(2N) \mathcal{N} = 4$ SYM theory) representing
dyonic branes composed of $N$ electric and $N$ magnetic D3-branes. The branes of
the same type repel at short distances, but an electric and a magnetic brane attract,
and the forces between self-dual branes cancel. The self-dual configuration (with the
positions of the electric and the magnetic branes, i.e. the diagonal entries of the adjoint
scalar fields, being the same) is stable against separation of one electric or one magnetic
brane, but is unstable against certain modes of separation of several same-type branes.
This instability should be suppressed in the large $N$ limit, i.e. should be irrelevant for
the large $N$ CFT interpretation of the gauge theory suggested in [hep-th/9901101].

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1 Introduction

In recent papers \cite{1, 2, 3, 4} it was suggested that a study of D3-branes in non-supersymmetric type 0 string theory may be useful in attempts to extend the string/gravity – large \(N\) gauge theory duality \cite{5, 6} to non-supersymmetric Yang-Mills theories.

Type 0B theory \cite{7} has unconstrained Ramond-Ramond 5-form field and thus contains both electric and magnetic D3-branes \cite{8, 2} which may be combined to form dyonic branes \cite{2, 4}. The field theory of light string modes corresponding to parallel \(N\) electric or \(N\) magnetic D3-branes is a truncation of \(\mathcal{N} = 4\) SYM theory in which all fermions are excluded, i.e. is \(U(N)\) gauge theory coupled to 6 adjoint scalars \cite{2}. It is asymptotically free, and was argued in \cite{3} to have an IR fixed point at infinite coupling.

In \cite{2} the string-theory cylinder diagram expression for the potential between same-type D3-branes was found and its large-distance limit (dominated by lightest states of the closed-string channel) was compared with the corresponding interaction potential in the effective low-energy gravitational theory. It was found that at large distances the branes attract because of the contribution of the bulk tachyon field (but would repel if the closed-string tachyon is removed from the spectrum).

In the supersymmetric type II theory the leading non-vanishing term in the interaction potential between (e.g., moving) branes has the same behavior at short and at large distances \cite{9, 10}. This property is based on a certain non-renormalization theorem (cf. \cite{11}), e.g., the coefficient of the \(\frac{1}{\sqrt{x}}\) term in the string expression (which, in general, is expected to be a function of \(\frac{x}{\sqrt{\alpha'}}\)), i.e. to receive contributions from massive open string modes) turns out to be a constant \cite{9}. Since this non-renormalization is a consequence of supersymmetry, there is no reason to expect a similar ‘small distance – large distance’ relation in the non-supersymmetric type 0 theory case.

The expression for the cylinder amplitude \cite{2} seems to imply (depending on a regularization of short-distance divergence) that branes of the same type repel at short distances. The force between electric and magnetic branes is the same in value but opposite in sign \cite{4}, i.e. they attract at short distances.

As in the case of type II theory \cite{9}, the short distance (\(\Delta x < \sqrt{\alpha'}\)) interaction between D-branes should be dominated by the light open string modes, i.e. should be described by the one-loop effective potential in the corresponding gauge theory\cite{1}. Indeed, the presence of a repelling force between two electric (or two magnetic) branes can be seen directly from the corresponding field-theory calculation of the scalar effective potential in \cite{12} (assuming that one-loop induced masses are fine-tuned to zero).

A self-dual type 0 D3-brane is found by putting together an electric and a magnetic D3-brane \cite{2}. Since an open string connecting these two branes is a fermion \cite{8, 2}, the resulting low-energy world-volume field theory contains not only the massless bosons for each of the branes but also the massless fermions \cite{4}. The perturbative type 0 string theory calculation of the interaction potential between two such self-dual branes gives (just as in the case of type IIB D3-branes \cite{13}) the vanishing result at all distances \cite{4}. Moreover, the potential

\[ \frac{\Delta x}{\sqrt{\alpha'}} < \frac{1}{\sqrt{\alpha'}}. \]

\footnote{It is important here that the open string channel does not contain tachyon \cite{1, 2, 8}. Truncation to massless open string modes is possible when the masses of the stretched strings are much less than those of excited open string modes, \(\frac{\Delta x}{\sqrt{\alpha'}} < \frac{1}{\sqrt{\alpha'}}\).}
between an electric (or magnetic) brane and a self-dual brane also vanishes. These results may be interpreted as being due to cancellations between the bosonic (electric-electric and magnetic-magnetic) repulsion and the fermionic (electric-magnetic) attraction.

The field theory on \( N \) electric and \( N \) magnetic parallel type 0 D3-branes contains, in addition to the \( U(N) \times U(N) \) gauge field and 6 adjoint scalars, also 4 Weyl fermions in the \((N,\overline{N})\) representation of \( U(N) \times U(N) \) and 4 Weyl fermions in the \((\overline{N},N)\) representation. This non-supersymmetric gauge theory may be interpreted as a special \( Z_2 \) projection of the \( U(2N) \) \( N=4 \) SYM theory: one is to keep the fields invariant under change of sign of fermions combined with the global \( U(2N) \) gauge transformation \( X \rightarrow I X T^{-1} \), \( T = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \) (\( I \) is the \( N \times N \) identity matrix). The \( T \)-transformation changes signs of the off-diagonal \( N \times N \) blocks of \( U(2N) \) matrices, implying that the resulting \( SO(6) \) invariant theory should contain only diagonal (\( U(N) \times U(N) \)) bosons and off-diagonal (bifundamental) fermions.

As was argued in [4], in the large \( N \) limit this \( SU(N) \times SU(N) \) non-supersymmetric gauge theory (with \( U(1) \times U(1) \) part assumed to be decoupled) is expected to be a conformal field theory. It was checked that the one-loop gauge coupling beta-function indeed vanishes, while the 2-loop beta-function and the planar parts of the one-loop renormalizations of the scalar potential and the Yukawa coupling matrix vanish to the leading order in large \( N \).

Our aim below is to further explore some perturbative properties of this gauge theory. We shall compute the one-loop effective potential for the diagonal scalar fields representing short-distance interaction between separated branes. The resulting expression will be a generalization of the potential in the purely-electric theory case: it will have a simple structure of the sum of the bosonic (electric-electric and magnetic-magnetic) and the fermionic (electric-magnetic) contributions. In agreement with the string-theory result that the self-dual branes do not interact, we will find that the potential indeed vanishes in the case when the scalar field backgrounds for the two \( SU(N) \) groups (i.e. the positions of the electric and the magnetic constituents) are taken to be equal.

Having found the expression for the one-loop effective potential (Sect.2), we shall then analyze stability of the self-dual configuration of branes corresponding to the same diagonal entries of the two sets of 6 scalar fields (Sect.3). While the self-dual configuration is stable against separation of one electric or one magnetic brane, we shall find that it is unstable against separation of several same-type (electric or magnetic) branes. The instability disappears at finite temperature \( T \) (Sect.4): the stack of coincident like-charge branes becomes a metastable state, and dissociation of a multiply charged brane into elementary constituents ceases to be energetically favorable at some \( T = T_c \) (this is similar to the finite-temperature restoration of spontaneously broken symmetry in the Higgs model). As we shall discuss in Sect.5, this instability is likely to be suppressed in the large \( N \) limit, i.e. should be irrelevant for the large \( N \) CFT interpretation of the \( SU(N) \times SU(N) \) gauge theory suggested in [4].

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2 Similar (but somewhat more complicated, having less global symmetry) large \( N \) conformal ‘orbifold’ gauge theories were considered in [14, 15, 16].

3 For example, when expressed in terms of the ’t Hooft coupling \( \lambda = g_{YM}^2 N \) the 2-loop RG equation becomes \( \frac{d}{d \ln \mu} \lambda = \frac{b_2}{N^2} \lambda^3 \), i.e. running is suppressed in the large \( N \), \( \lambda = \) fixed limit.
2 One-loop effective potential

To write down the action of the $SU(N) \times SU(N)$ gauge theory and to compute the corresponding quantum effective potential it is simplest to view it as a reduction to 4 dimensions of a non-supersymmetric ten-dimensional gauge theory (this is indeed the way how it originates from string theory). The latter $D = 10$ gauge theory is the $(-1)^F \cdot \mathcal{I}$ projection of the $\mathcal{N} = 1$, $D = 10$ supersymmetric $U(2N)$ Yang-Mills theory.

The field-theory content (i.e. the low-energy degrees of freedom on parallel $N$ electric and $N$ magnetic D3-branes) is then described by the $U(N) \times U(N)$ $D = 10$ gauge potentials and ten-dimensional Majorana-Weyl spinors in $(N, \bar{N})$ and $(\bar{N}, N)$ representations of $U(N) \times U(N)$. Gauge potentials are embedded in $U(2N)$ diagonally

$$A_M = \left( \begin{array}{cc} A_{(e)M} & 0 \\ 0 & A_{(m)M} \end{array} \right),$$

where the $N \times N$ Hermitian matrices $A_{(e)M}$ and $A_{(m)M}$ describe massless modes of open strings connecting electric with electric and magnetic with magnetic branes, respectively ($M = 0, \ldots, 9$). All fields depend only on 4 ‘parallel’ coordinates $x^\mu$ ($\mu = 0, 1, 2, 3$). The internal components of the gauge potentials

$$A_i = \left( \begin{array}{cc} \Phi_{(e)i} & 0 \\ 0 & \Phi_{(m)i} \end{array} \right)$$

are the adjoint scalars ($i = 1, \ldots, 6$).

The fermions fill off-diagonal blocks of the $U(2N)$ matrices

$$\Psi = \left( \begin{array}{cc} 0 & \psi \\ \psi^\dagger & 0 \end{array} \right),$$

where $\Psi_{(em)} = \psi$ and $\Psi_{(me)} = \psi^\dagger$ correspond to the massless modes of fermionic strings stretched between electric and magnetic branes. They satisfy the $D = 10$ chirality constraint: $\Gamma_{11} \psi = \psi$, $\Gamma_{11} \psi^\dagger = \psi^\dagger$. The 4-d action written in the ten-dimensional notation is

$$S = \frac{1}{2g_{YM}^2} \int d^4x \left( \text{tr} F_{MN}^2 + 2i\psi^\dagger \Gamma^0 \Gamma^M D_M \psi \right),$$

where $\Gamma^M$ are the 10-d Dirac matrices. The covariant derivative $D_M = \partial_M + i[A_M, \cdot]$ acts on the fermions $\psi$ as follows:

$$D_\mu = \partial_\mu + i(A_{(e)\mu} \otimes 1 - 1 \otimes A_{(m)\mu}^T), \quad D_i = \partial_i + i(\Phi_{(e)i} \otimes 1 - 1 \otimes \Phi_{(m)i}^T).$$

\footnote{For simplicity, we shall first assume that the gauge group is $U(N) \times U(N)$ as it directly follows from the ‘D-branes in flat space picture’. The truncation to the $SU(N) \times SU(N)$ case will be easy to do in the final one-loop expressions since the bosons of the two groups do not mix and the contributions of both $U(1)$’s decouple. The difference between $SU(N)$ and $U(N)$ cases is irrelevant in the large $N$ limit.}

\footnote{In more detail, the 10-d real MW fermions of $U(2N)$ gauge theory are represented by $\Psi_I$, $\Gamma_1 \Psi_I = \Psi_I$, $I = 1, \ldots, 4N^2$. To describe the projection one multiplies them by the Hermitian $2N \times 2N$ matrix generators $T^I$ of $U(2N)$ and then sets the diagonal entries to zero. The off-diagonal field $\psi$ is thus a complex Weyl spinor. The matrix form of the fermionic action is obtained by using that $\text{tr}(T^I T^J) = \frac{1}{2} \delta^{IJ}$.}

\footnote{Here the trace is in the fundamental representation and the canonical gauge theory coupling is related to the string coupling by $g_{YM}^2 = 4\pi g_s$.}
Note that $A_\mu$ and $\Phi_i$ are Hermitian, i.e. $A^* = A^T$, $\Phi^* = \Phi^T$.

As in the $\mathcal{N} = 4$ SYM theory, the classical scalar potential

$$\text{tr} \left( [\Phi_{(e)i}, \Phi_{(e)j}]^2 + [\Phi_{(m)i}, \Phi_{(m)j}]^2 \right)$$  \hspace{1cm} (2.6)

has a minimum at $[\Phi_{(e)i}, \Phi_{(e)j}] = 0$, $[\Phi_{(m)i}, \Phi_{(m)j}] = 0$. The classical moduli space of the world-volume theory is thus described by constant transverse coordinates of $N$ electric and $N$ magnetic D3-branes

$$\Phi_{(e)i} = \text{diag}(y_{(e)i}^1, ..., y_{(e)i}^N), \quad \Phi_{(m)i} = \text{diag}(y_{(m)i}^1, ..., y_{(m)i}^N),$$  \hspace{1cm} (2.7)

where $y_i$ (having mass dimension 1) are related to the string coordinates $x_i$ by $y_i = \alpha'^{-1}$. In the case of the $SU(N) \times SU(N)$ theory $\Phi_{(e)i}$ and $\Phi_{(m)i}$ are traceless, i.e.,

$$\sum_{a=1}^{N} y_{(e)i}^a = 0, \quad \sum_{a=1}^{N} y_{(m)i}^a = 0.$$  \hspace{1cm} (2.8)

Our aim below is to compute the one-loop effective action $\Gamma = \int d^4x V_{\text{eff}}(\Phi)$ in this constant scalar field background, i.e. the corresponding effective potential \cite{19}. This effective potential vanishes in $\mathcal{N} = 4$ SYM theory where the classical moduli space is not deformed at the quantum level, but is non-trivial in the non-supersymmetric ‘projected’ theory (2.4).

The bosonic part of the effective potential (which describes interactions of like-charge branes) was computed in \cite{12}:

$$V_{\text{eff}}^{(bos)} = \frac{1}{2} \sum_{a,b=1}^{N} \left[ V(|y_{(e)i}^a - y_{(e)i}^b|) + V(|y_{(m)i}^a - y_{(m)i}^b|) \right].$$  \hspace{1cm} (2.9)

Here $|y| = (y_i y_i)^{1/2}$ and the two-body interaction potential $V$ is given by the loop integral

$$V(r) = 8 \int \frac{d^4p}{(2\pi)^4} \ln \left( p^2 + r^2 \right) = - \frac{8}{(4\pi)^2} \int_0^{\infty} \frac{dt}{t^3} e^{-tr^2}. \hspace{1cm} (2.10)$$

This integral diverges in the UV region ($t \rightarrow 0$) and requires renormalization. We shall assume that appropriate counterterms cancel quartic and quadratic power-like divergencies and promote the logarithm of the cutoff to the logarithm of a characteristic energy scale $\Lambda$ (which should be of order of $\frac{1}{\sqrt{\alpha'}}$ in the context of comparison with string theory). In general, $V = c_0 \Lambda^4 + c_1 \Lambda^2 r^2 + \frac{1}{4\pi} r^4 \ln \frac{r^2}{\Lambda^2}$. One may fine-tune the renormalization-dependent constant $c_1$ (i.e., the coefficient of the one-loop adjoint-trace terms $\text{Tr}[\Phi_{(e)i}^2 + \Phi_{(m)i}^2]$) to zero which would correspond to keeping the scalar fields massless at the one-loop level. While in the purely electric theory this fine tuning may seem unnatural (in the absence of supersymmetry adjoint scalars may get masses at loop level \cite{2}), it is formally possible (see also footnote below eq. (2.13) and Section 3 for a discussion of this point). Fortunately, this fine-tuning
will not be needed in the ‘self-dual’ $SU(N) \times SU(N)$ theory we are actually interested in where the quadratic mass divergences will cancel out (quartic divergences will also cancel out in the large $N$ limit \cite{4}). Anticipating that, we shall assume the following expression for the renormalized 2-body potential $V$ (2.10)

$$V(r) = \frac{1}{4\pi^2} r^4 \ln \frac{r^2}{\Lambda^2}.$$  \hspace{1cm} (2.11)

This potential is repulsive at short distances and attractive at large ones, see fig. 1 (note that the potential becomes attractive at short distances if one adds the mass term $c_1 \Lambda^2 r^2$). This bosonic part of the potential can be compared with the cylinder string amplitude describing the interaction between the two parallel electric branes \cite{2}.

$$V_{\text{str}}^{(\text{bos})} = -\frac{1}{(4\pi)^2} \int_0^\infty \frac{dt}{2(2\pi\alpha')^2} e^{-\frac{r^2}{2\pi\alpha'}} \left( \left[ \frac{f_3(q)}{f_1(q)} \right]^8 - \left[ \frac{f_4(q)}{f_1(q)} \right]^8 \right), \quad q = e^{-\pi t}, \hspace{1cm} (2.12)$$

where we used the notation of \cite{21} (and suppressed the space-time volume factor). The short-distance ($r \ll \sqrt{\alpha'}$) limit of this expression is obtained by expanding the integrand for $t \to \infty$ (i.e. $q \to 0$) \cite{4} $f_1(q) = q^{1/12}(1-q^2+\ldots)$, $f_3(q) = q^{-1/24}(1+q+\ldots)$, $f_4(q) = q^{-1/24}(1-q+\ldots)$. Keeping only the leading term, i.e. the contribution of the massless open-string modes, we get (we set $2\pi\alpha' = 1$)

$$V_{\text{str}}^{(\text{bos})} = -\frac{1}{(4\pi)^2} \int_0^\infty \frac{dt}{2\ell^3} e^{-r^2t} (16 + \ldots), \hspace{1cm} (2.13)$$

which is thus the same as the potential (2.10) calculated in field theory. At distances of order $\sqrt{\alpha'}$ the integral over $t$ (2.12) is divergent at zero ($q \to 1$) because of the closed-string tachyon, i.e. at string scales the effects of the tachyon condensation \cite{2} should become important. \cite{5}

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8 Equivalent result is obtained taking the limit $x \to 0$, $\alpha' \to 0$, $y = \frac{r}{\alpha'} = $ fixed, implying again that $t \to \infty$.

9 The issue of UV regularization of the $t$-integral is closely related to that of the tachyon condensation.
Figure 2: Interaction potential (2.16) between an electric and a magnetic brane.

The contribution of the Weyl fermion $\psi$ in (2.4) to the effective action is

$$\Gamma^{(\text{ferm})} = -\frac{1}{2} \text{Tr} \ln \left( \left( -D^2 - \frac{i}{2} F_{MN} \Gamma^{MN} \right) \frac{1 + \Gamma_{11}}{2} \right).$$

(2.14)

For the constant commuting ($F_{MN} = [D_M, D_N] = 0$) real scalar background (2.7) one has

$$\Gamma^{(\text{ferm})} = -8 \text{Tr} \ln(-D^2).$$

(2.15)

The eigenvalues of $D_\mu = \partial_\mu$ and $D_i$ (2.5) are $i p_\mu$ and $i(y^{a}_{(e)i} - y^{b}_{(m)i})$. The fermionic contribution to the effective potential $V_{\text{eff}}$ is thus

$$V^{(\text{ferm})}_{\text{eff}} = -8 \sum_{a,b=1}^{N} \int \frac{d^4p}{(2\pi)^4} \ln \left[ p^2 + |y^{a}_{(e)} - y^{b}_{(m)}|^2 \right] = -\sum_{a,b=1}^{N} V(|y^{a}_{(e)} - y^{b}_{(m)}|),$$

(2.16)

where the two-body interaction potential $V$ is the same as in the bosonic case (2.10). The loop of the massless fermionic string states leads to the interaction between the electric and the magnetic branes equal in magnitude, but opposite in sign, to the bosonic loop interaction between same-type branes (fig. 2). This is in agreement with the full string-theory result [2, 4] for the electric–magnetic brane interaction which is given by (2.12) taken with the opposite sign.

We are assuming that the bosonic and fermionic determinants are regularized in the same way, i.e. the scale $\Lambda$ is the same in both cases. The string-theory (or $\mathcal{N} = 4$ SYM projection) origin of the theory under consideration implies that this is indeed the right regularization prescription. As a result, the effective potential will vanish for the self-dual configurations of branes (see eq. (2.17) below), in agreement with the vanishing of the corresponding cylinder amplitude in string theory.
Collecting together the contributions of the bosonic and the fermionic degrees of freedom we finally get:

\[
V_{\text{eff}} = \frac{1}{2} \sum_{a,b=1}^{N} \left[ V(|y^a_e - y^b_e|) + V(|y^a_{(m)} - y^b_{(m)}|) - 2V(|y^a_e - y^b_{(m)}|) \right]
\]

\[
= \frac{1}{8\pi^2} \sum_{a,b=1}^{N} \left[ |y^a_e - y^b_e|^4 \ln \frac{|y^a_e - y^b_e|^2}{\Lambda^2} + |y^a_{(m)} - y^b_{(m)}|^4 \ln \frac{|y^a_{(m)} - y^b_{(m)}|^2}{\Lambda^2} - 2 |y^a_e - y^b_{(m)}|^4 \ln \frac{|y^a_e - y^b_{(m)}|^2}{\Lambda^2} \right]. \tag{2.17}
\]

In a generic regularization prescription for (2.10) the \(N^2\) part of the coefficient of the \(\Lambda^4\) term here cancels out [4] while the \(\Lambda^2\) term is

\[
(V_{\text{eff}})_{\Lambda^2} = \frac{1}{2} c_1 \Lambda^2 \sum_{a,b=1}^{N} \left[ |y^a_e - y^b_e|^2 + |y^a_{(m)} - y^b_{(m)}|^2 - 2 |y^a_e - y^b_{(m)}|^2 \right]. \tag{2.18}
\]

Using that \(|y^a_e - y^b_e|^2 = y^a_e y^b_e - 2y^a_e y^b_e + y^a_{(m)} y^b_{(m)}\) and (2.8) it is easy to see that this combination vanishes in the \(SU(N) \times SU(N)\) case, \((V_{\text{eff}})_{\Lambda^2} = 0\). The coefficient of the logarithmic divergence in (2.17) can be transformed with the help of (2.8) into the following form

\[
(V_{\text{eff}})_{\ln \Lambda^2} = -\frac{1}{8\pi^2} \ln \Lambda^2 \left( 2 \left[ \sum_{a=1}^{N} (y^a_e y^a_{(m)} - y^a_{(m)} y^a_e) \right]^2 + \frac{1}{4} \left[ \sum_{a=1}^{N} (\sum_{j=1}^{N} (y^a_i y^a_j - y^a_j y^a_i) \right]^2 \right). \tag{2.19}
\]

Note that the remaining dependence on \(\Lambda\) has non-planar ‘double-trace’ form – the ‘planar’ or ‘\(N\)tr’ part has cancelled out (\(\frac{1}{8\pi^2} N \sum_{a=1}^{N} (2 |y^a_{(e)}|^4 + 2 |y^a_{(m)}|^4 - 2 |y^a_{(e)}|^4 - 2 |y^a_{(m)}|^4) = 0\)) in agreement with the general conclusion in [4, 1]. The remaining finite part of the effective potential also originates from non-planar one-loop graphs and thus is subleading at large \(N\) (see also Sect.5).

We shall discuss some properties of (2.17) in Section 3 and its finite-temperature generalization in Section 4.

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11 The double-trace term originating from non-planar diagrams (with two scalar legs on one boundary of a loop, and two – on another) is subleading in the large \(N\) limit – the traces of the external or background fields are finite for \(N \to \infty\) (equivalently, one needs to separate a factor of \(N\) in the divergent part of the one-loop effective action to combine it with the gauge coupling into \(N g^2_{YM}\) which is to be fixed in the large \(N\) limit). We are grateful to I. Klebanov for a discussion of this issue and also for suggesting to put the \(\ln \Lambda^2\) term in (2.17) in the form (2.11), which is a special case of the ‘double-trace’ expression

\[
(V_{\text{eff}})_{\ln \Lambda^2} = -\frac{1}{8\pi^2} \ln \Lambda^2 \left( 2 \left[ \text{tr}(\Phi_{(e)i} \Phi_{(e)i}) - \text{tr}(\Phi_{(m)i} \Phi_{(m)i}) \right]^2 + 4 \left[ \text{tr}(\Phi_{(e)i} \Phi_{(e)j}) - \text{tr}(\Phi_{(m)i} \Phi_{(m)j}) \right]^2 \right). \tag{2.11}
\]

Although the counterterm of this form when added to the bare action seems to give the leading order \(N^2\) contributions, it actually produces only subleading contribution for \(N \to \infty\) due to the large \(N\) factorization. In calculating diagrams or in the Schwinger-Dyson equations one of the traces in the double-trace expression can be factorized and replaced by its vacuum average which is zero in the self-dual vacuum. Similar remarks apply to the \(U(N) \times U(N)\) theory (which is equivalent to \(SU(N) \times SU(N)\) one in the large \(N\) limit). In particular, the quadratic divergence term here, while non-vanishing, has (like the logarithmic term) the double-trace form, \((V_{\text{eff}})_{\Lambda^2} = -c_1 \Lambda^2 [\text{tr}(\Phi_{(e)i}) - \text{tr}(\Phi_{(m)i})]^2\), and thus is subleading at large \(N\).
3 Some properties of effective potential

There are some obvious properties of the expression (2.17) for the effective potential. First, the whole potential (and, in particular, (2.19)) vanishes for a self-dual configuration of D-branes, i.e. when the positions of the electric and the magnetic branes are taken to be the same,

\[ y_{(e)i} = y_{(m)i} . \]  

This is in agreement with the vanishing of the corresponding string-theory cylinder amplitude [2, 4]. It also follows from (2.17) that the self-dual branes do not interact either with electric or with magnetic branes.

In the case of the bosonic theory on purely electric branes, the effective potential (2.9) (defined according to (2.11)) exhibits the typical Coleman-Weinberg behavior [19] – the \( U(N) \) world-volume symmetry appears to be spontaneously broken to \( \left[U(1)\right]^N \) by radiative corrections. This means that the stack of electric D3-branes is unstable against separation; in the equilibrium configuration all \( N \) branes are separated by distances of order \( \Lambda \).

The potential of the self-dual brane theory (2.17) expanded near the self-dual point (3.1) has stable, valley-type, and unstable directions. Separating a single electric (or magnetic) brane away from the rest of \( N - 1 \) electric and \( N \) magnetic branes gives the same attractive force (for \( \Delta y \ll \Lambda \)) as between a single electric and a single magnetic brane, i.e. this is a stable direction. An example of a valley in field space is a separation of some number of self-dual branes \( (y_{(e)}^{s_i} = y_{(m)}^{s_i}, s = 1, ..., M) \) from the remaining \( (N - M) \) self-dual ones.

If we allow the electric and magnetic branes to be separated, the effective potential can become negative due to the repulsion of the same-type branes at short distances. This repulsion makes self-dual configuration of branes unstable. An example of an unstable direction is obtained by separating two electric branes along some axis by distances \( \pm \rho \) from the remaining stack of coinciding \( N - 2 \) electric and \( N \) magnetic branes. The energy density of such configuration is then the same as of the system of two electric branes at positions \( y = \rho \) and \( y = -\rho \) and two coinciding magnetic branes at the origin \( y = 0 \) (note that the \( N \)-dependent contributions in (2.17) cancel out)

\[ E(\rho) = V_{\text{eff}} = \frac{3 \rho^4}{\pi^2} \ln \frac{2^{8/3} \rho^2}{\Lambda^2} . \]  

The self-dual point \( \rho = 0 \) is a local maximum, rather than a minimum, of the energy \( E(\rho) \) (see fig. 1). The branes thus tend to separate by distances of order \( \Lambda \). The behavior of the effective potential for \( \rho \sim \Lambda \) is non-perturbative in field theory, so an equilibrium configuration of branes cannot be reliably determined in the one-loop approximation.

The above calculation of the effective potential is not limited to the case of equal numbers of electric and magnetic branes. The world-volume theory on a dyonic brane formed by \( Q \)

\[ \begin{aligned} \footnotesize \text{12} & \text{ This type of instability depends, of course, on particular form of the two-body interaction potential. For example, it is absent in the case of the} \left[U(1)\right]^N \text{ as the forces in the neutral system of two} \ \end{aligned} \]

\[ \begin{aligned} \footnotesize \text{13} & \text{ Let us note that the electric-magnetic duality symmetry can be interpreted from the world-volume point of view as a} \ Z_2 \ \text{ symmetry interchanging} \ A_{(e)} \text{ with} \ A_{(m)}, \Phi_{(e)} \text{ with} \Phi_{(m)}, \text{ and} \psi \text{ with} \psi^\dagger \text{ in (2.4). Instability may be interpreted as a spontaneous breaking of this} \ Z_2 \ \text{ symmetry.} \end{aligned} \]
electric and $P$ magnetic branes has $U(Q) \times U(P)$ gauge bosons and adjoint scalars and fermions in the $(Q, \bar{P})$ and $(\bar{Q}, P)$ representations. The effective potential in the generic case is given by the same eq. (2.17) with the appropriate ranges of summation ($a = 1, \ldots, Q$ and $b = 1, \ldots, P$).  

### 4 Effective potential at non-zero temperature

Generalization of the effective potential to the case of non-zero temperature is straightforward: the integration over $p_0$ component of momentum in (2.10) should be replaced by summation over Matsubara frequencies, even for bosons and odd for fermions. The proper-time representation for the bosonic loop integral (2.10) at finite temperature changes to (see, e.g., [24])

$$
V_B(r, T) = \frac{8}{(4\pi)^2} \int_0^\infty \frac{dt}{t^3} e^{-r^2t} \left( \frac{i}{4\pi tT^2} \right). 
$$

The thermal effective potential in the purely electric theory is thus

$$
V_{\text{eff}}^{(el)} = \frac{1}{2} \sum_{a,b=1}^N V_B(||y^a - y^b||, T). 
$$

The temperature qualitatively changes the behavior of the effective potential. At short distances it becomes attractive, since combining the high-temperature expansion of the thermal correction to the effective potential [24] (the expansion parameter is $\frac{1}{T}$) with its

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14 The dyonic theory ‘interpolates’ between the purely electric and the self-dual theory. This theory is not asymptotically free: its one-loop gauge theory beta functions for the two $U(N)$ couplings are proportional to $\frac{3}{2} (Q - P)$ and $\frac{3}{2} (P - Q)$, i.e. have opposite signs.
zero-temperature value (2.11) we get:
\[
V_B(r, T) = -\frac{8\pi^2}{45} T^4 + \frac{2}{3} T^2 r^2 - \frac{4}{3\pi} T r^3 - \frac{1}{2\pi^2} r^4 \ln \frac{\Lambda}{T} + \ldots .
\] (4.3)

The temperature is assumed to be small compared to \( \Lambda \) (i.e. \( r < T < \Lambda \)) and the coefficient in front of \( r^4 \) is written down with logarithmic accuracy. The region \( 0 < r < r_0 \) where forces between branes are attractive is very small – the potential has a maximum (see fig. 3) at
\[
r_0 = \frac{\pi T}{\sqrt{\frac{2}{3} \ln \frac{\Lambda}{T}}}. \quad (4.4)
\]

Although at large \( r \) the contribution of massive open string states can no longer be neglected and large-distance interaction of D-branes is not under control in the world-volume field theory, one expects, as in [12], that the potential has a global minimum at some \( r \sim \Lambda \). The location of the potential peak, \( r_0 \), increases with the temperature, along with the height of the potential barrier. One also expects that the free energy in the global minimum should grow [23]. At some temperature \( T = T_c \sim \Lambda \), the free energies at zero and at the symmetry-breaking minimum become equal to each other and for \( T > T_c \) the global minimum of the free energy is at \( r = 0 \) (see fig. 3). This is the well-known picture of symmetry restoration at finite temperature [25].

In the high-temperature phase the \( U(N) \) symmetry is restored, i.e. the equilibrium configuration corresponds to all \( N \) branes accumulating at one point. Similar conclusion is reached in the case of the type IIB theory D3-branes [20]. The strong-coupling counterpart of this fact is non-existence of stable separated-brane supergravity solutions in the non-extremal case – when the energy of the system is larger than the total charge, branes attract and form a single black brane.

In the \( U(N) \times U(N) \) theory of \( N \) electric and \( N \) magnetic branes the finite temperature effective potential has the same structure as at \( T = 0 \), eq. (2.17),
\[
V_{\text{eff}} = \frac{1}{2} \sum_{a,b=1}^N \left[ V_B(|y^a_e - y^b_e|, T) + V_B(|y^a_m - y^b_m|, T) - 2V_F(|y^a_e - y^b_m|, T) \right],
\] (4.5)
but now the fermionic and bosonic two-body potentials \( V_B \) and \( V_F \) are not the same. The proper-time representation for the fermionic loop contribution is
\[
V_F(r, T) = -\frac{8}{(4\pi)^2} \int_0^\infty dt \frac{1}{t^3} e^{-r^2t} \theta_4 \left( \frac{i}{4\pi t T^2} \right),
\] (4.6)
so that at short distances [24]
\[
V_F(r, T) = \frac{7\pi^2}{45} T^4 - \frac{1}{3} T^2 r^2 - \frac{1}{2\pi^2} r^4 \ln \frac{\Lambda}{T} + \ldots .
\] (4.7)

As we have seen in the previous section, at zero temperature the self-dual vacuum (3.1) of the world-volume gauge theory is unstable because of the repulsive forces between same-type constituent branes. For the same reasons as in the purely bosonic theory, we expect that this instability disappears at some critical temperature of order \( \Lambda \) (the self-dual vacuum is metastable at any non-zero temperature). As in the type II theory case, at finite temperature separated self-dual branes start attracting and should form a single-center cluster.
5 Discussion

In this paper we considered some aspects of short-distance interactions between D3-branes in type 0 string theory described by the corresponding world-volume field theory. We concentrated on the self-dual branes in flat space at weak coupling, i.e. on the perturbative $SU(N) \times SU(N)$ gauge theory of [4]. We computed the one-loop effective potential in this field theory and found that the self-dual D3-brane configuration (3.1) is perturbatively unstable – constituent electric and magnetic branes tend to separate to distances $\alpha' \Lambda$ which should naturally be of order $\sqrt{\alpha'}$.

In the large $N$ limit this $SU(N) \times SU(N)$ theory is expected to have a dual representation in terms of the classical type 0 string theory on $AdS_5 \times S^5$ background with $N$ units of electric and $N$ units of magnetic 5-form flux [4]. This implies that the large $N$ limit of this gauge theory should represent a 4-d CFT [4].

The perturbative instability of the self-dual configuration discussed above should not modify this conclusion – it is to be absent in the part of the effective potential which is dominant in the large $N$ limit.

The $SU(N) \times SU(N)$ theory has a classical moduli space with coordinates being positions of the branes $(y^a(e), y^a(m))$. Together with $N$ and $g_{YM}^2$ they play the role of parameters of the theory. The CFT should be defined by a fixed point in the whole parameter space: $N \to \infty$, $N g_{YM}^2 = \lambda$, $y^a(e) = y^a(m)$. The formal one-loop quantum-mechanical instability of the self-dual point should be irrelevant in this context: the large $N$ CFT may be defined by a proper set of conformal composite operators (and their correlation functions) which, roughly, does not include operators vanishing for $\Phi(e) = \Phi(m)$. Like for the ‘non-planar’ logarithmically divergent terms $(\text{tr}\Phi^2(e) - \text{tr}\Phi^2(m))^2$ in the one-loop effective action (2.19), the contributions of such operators in correlation functions should vanish in the conformal limit. That means that fluctuations with $\text{tr}\Phi^2(e) \neq \text{tr}\Phi^2(m)$ should be effectively forbidden, i.e. the above instability should be suppressed.

As was recently pointed out [27, 28], the non-supersymmetric $SU(N) \times SU(N)$ theory interpreted as a $Z_2$ projection of the $U(2N) N = 4$ SYM theory in [4] is also a special case of $Z_2$ orbifolds of the $N = 4$ SYM theory considered in [13, 16] with $Z_2$ here being in the center of the $R$-symmetry group $SU(4)$. This implies [28] that all planar graphs in this theory are the same as in the $U(2N) N = 4$ SYM theory restricted to invariant external states. In particular, all planar graph contributions to the scalar effective potential should vanish, as they do in the SYM theory when restricted to the classical or on-shell (constant commuting) values of scalars. As a consequence, the instability discussed above should be absent in the large $N$ theory to all orders in perturbation theory.

\[\lambda\]

Indeed, it may seem natural to ignore the effect of separation of a few electric or magnetic branes from a cluster of large number $(2N)$ of branes. Simultaneous separation of a large number $M \sim N$ of branes should be statistically suppressed.

We are grateful to I. Klebanov for this suggestion.

Here one is interested in the classical value of the effective potential (or vacuum energy) and not in what kind of scalar operators are induced by loops. For example, planar and non-planar 1-loop graphs induce $N\text{tr}(\ldots)$ and $\text{tr}(\ldots)\text{tr}(\ldots)$ operators which produce the same-order $N^2$ contributions when formally inserted back in the loop expansion.
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References

[1] A.M. Polyakov, “The Wall of the Cave”, hep-th/9809057; “String theory and quark confinement,” Nucl. Phys. B (Proc. Suppl.) 68 (1998) 1, hep-th/9711002.
[2] I.R. Klebanov and A.A. Tseytlin, “D-Branes and Dual Gauge Theories in Type 0 String”, hep-th/9811035.
[3] I.R. Klebanov and A.A. Tseytlin, “Asymptotic Freedom and Infrared Behavior in the Type 0 String Approach to Gauge Theory”, hep-th/9812089.
[4] I.R. Klebanov and A.A. Tseytlin, “A non-supersymmetric large N CFT from type 0 string theory”, hep-th/9901101.
[5] J. Maldacena, “The Large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2 (1998) 231, hep-th/9711200, S.S. Gubser, I.R. Klebanov, and A.M. Polyakov, “Gauge theory correlators from noncritical string theory,” Phys. Lett. B428 (1998) 105, hep-th/9802109; E. Witten, “Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. 2 (1998) 253, hep-th/9802150.
[6] I.R. Klebanov, “From Threebranes to Large N Gauge Theories”, hep-th/9901018.
[7] L. Dixon and J. Harvey, “String theories in ten dimensions without space-time supersymmetry”, Nucl. Phys. B274 (1986) 93; N. Seiberg and E. Witten, “Spin structures in string theory”, Nucl. Phys. B276 (1986) 272.
[8] O. Bergman and M. Gaberdiel, “A Non-supersymmetric Open String Theory and S-Duality”, Nucl. Phys. B499 (1997) 183, hep-th/9701137.
[9] M.R. Douglas, D. Kabat, P. Pouliot and S.H. Shenker, “D-branes and Short Distances in String Theory”, Nucl. Phys. B485 (1997) 85, hep-th/9608024.
[10] M.R. Douglas and W. Taylor, “Branes in the bulk of Anti-de Sitter space,” hep-th/9807225.
[11] C. Bachas and E. Kiritsis, “F^4 terms in N=4 string vacua”, Nucl.Phys.Proc.Suppl. B55 (1997) 194, hep-th/9611203.
[12] K. Zarembo, “Coleman-Weinberg mechanism and interaction of D3-branes in type 0 string theory”, hep-th/9901106.
[13] J. Polchinski, “Dirichlet-Branes and Ramond-Ramond Charges”, Phys. Rev. Lett. 75 (1995) 4724, hep-th/9510017.
[14] M. Douglas and G. Moore, “D-branes, quivers, and ALE instantons”, hep-th/9603167.
[15] S. Kachru and E. Silverstein, “4d conformal field theories and strings on orbifolds”, Phys. Rev. Lett. 80 (1998) 4855, hep-th/9802183.
[16] A. Lawrence, N. Nekrasov and C. Vafa, “On conformal field theories in four dimensions”, Nucl. Phys. B533 (1998) 199, hep-th/9803013; M. Bershadsky, Z. Kakushadze and C. Vafa, “String Expansion as Large N Expansion of Gauge Theories”, hep-th/9803076; M. Bershadsky and A. Johansen, “Large N limit of orbifold field theories”, Nucl. Phys. B536 (1998) 141, hep-th/9803243.
[17] E. Witten, “Bound States Of Strings And p-Branes”, Nucl. Phys. B460 (1996) 335, hep-th/9510135.
[18] F. Gliozzi, J. Scherk and D. Olive, “Supersymmetry, Supergravity Theories and the Dual Spinor Model”, Nucl. Phys. B122 (1977) 253; L. Brink, J.H. Schwarz and J. Scherk, “Supersymmetric Yang-Mills Theories”, Nucl. Phys. B121 (1977) 77.
[19] S. Coleman and E. Weinberg, “Radiative corrections as the origin of spontaneous symmetry breaking,” Phys. Rev. D7 (1973) 1888.
[20] E.S. Fradkin and A.A. Tseytlin, “Quantum properties of higher dimensional and dimensionally reduced supersymmetric theories,” Nucl. Phys. B227 (1983) 252.
[21] J. Polchinski, “TASI Lectures on D-Branes”, hep-th/9611050.
[22] J. Minahan, “Glueball Mass Spectra and Other Issues for Supergravity Duals of QCD Models”, hep-th/9811156.
[23] J. Minahan, “Asymptotic freedom and confinement from type 0 string theory”, hep-th/9902074.
[24] J.I. Kapusta, “Finite-temperature field theory” (Cambridge University Press, 1989).
[25] D.A. Kirzhnits, “Weinberg model in the hot universe”, Sov. Phys. JETP. Lett. 15 (1972) 529 [Pis’ma ZhETF 15 (1972) 745]; D.A. Kirzhnits and A.D. Linde, “Macroscopic consequences of the Weinberg model”, Phys. Lett. B42 (1972) 471.
[26] A.A. Tseytlin and S. Yankielowicz, “Free energy of $N = 4$ super Yang-Mills in Higgs phase and non-extremal D3-brane interactions”, hep-th/9809032.
[27] E. Silverstein, private communication.
[28] N. Nekrasov and S. Shatashvili, “On Non-Supersymmetric CFT in Four Dimensions”, hep-th/9902110.