WG2 Conveners’ Report: $V_{td}$ and $V_{ts}$, $B$–$\bar{B}$ mixing, radiative penguin and rare (semi)leptonic decays

JM Flynn\textsuperscript{a}, M Paulini\textsuperscript{b} and S Willocq\textsuperscript{c}

\textsuperscript{a}School of Physics & Astronomy, University of Southampton

\textsuperscript{b}Department of Physics, Carnegie Mellon University

\textsuperscript{c}Department of Physics, University of Massachusetts–Amherst

We introduce the Working Group 2 proceedings contributions from the 2nd workshop on the CKM Unitarity Triangle and note their connection to the proceedings of the first workshop. The topic of WG 2 was the determination of the CKM matrix elements $V_{td}$ and $V_{ts}$ from $B$–$\bar{B}$ mixing, radiative penguin $B \to X_{s/d}\gamma$ decays and rare (semi)leptonic decays such as $B \to X_s\ell^+\ell^-$.

At the first CKM workshop, Working Group 2 focused on the present status of determining the Cabibbo-Kobayashi-Maskawa matrix elements $V_{td}$ and $V_{ts}$ from $B$ meson oscillations \cite{1}. On the experimental side, the status of the measurements of the $B^0_d$ oscillation frequency $\Delta M_d$ and the limits on the $B^0_s$ oscillation frequency $\Delta M_s$ were discussed. From the theoretical end, the focus was on the status and prospects of determining the non-perturbative parameters for $B$ meson mixing from lattice QCD calculations.

For the second CKM workshop at Durham, the experimental status of $B$ meson mixing and the near term prospects for measuring $B^0_s$ meson oscillations as well as the status of lattice QCD calculations were re-evaluated. However, the focus shifted towards methods of determining the CKM matrix elements $V_{td}$ and $V_{ts}$ from other sources in the next five years. In particular, the experimental prospects for measuring $|V_{td}|/|V_{ts}|$ from exclusive rare radiative decays $B \to K^+\gamma$ and $B \to \rho^+\gamma$ and their theoretical limitations were explored. Furthermore, rare (semi)leptonic decays $B \to X_s\ell^+\ell^-$ were discussed as summarized below.

## 1 $B$ Meson Mixing

In the Standard Model, $B^{0}$–$\bar{B}^{0}$ mixing occurs via second order weak processes. The mass difference $\Delta M_q$ between the two mass eigenstates $B_q$ and $B_{\bar{q}}$ of the neutral $B_q$ meson ($q = d, s$) can be determined within the Standard Model by computing the electroweak box diagram, where the dominant contribution is through top quark exchange:

$$\Delta M_q = \frac{G_F^2}{6\pi^2} m_{B_q} (\hat{B}_{B_q} F_{B_q}^2) \eta_B m_W^2 S_0(x_t) |V_{tq}|^2.$$  \hspace{1cm} (1)

Here, $\hat{B}_{B_q}$ is the bag parameter of the $B$ meson, $F_{B_q}$ is the weak $B$ decay constant, $\eta_B$ is a QCD correction and $S_0(x_t)$ is a slowly varying function of the top quark and $W$ boson mass.

The main uncertainty in determining $V_{td}$ from $\Delta M_d$ comes from the factor $\hat{B}_{B_d} F_{B_d}^2$ in Eq. (1). In the standard analysis of the Unitarity Triangle (UT), the $B^0_s$ mixing frequency $\Delta M_s$ is used in a ratio with $\Delta M_d$ defining the quantity $\xi$:

$$\frac{|V_{td}|}{|V_{ts}|} = \xi \frac{m_{B_s}}{m_{B_d}} \frac{\Delta M_d}{\Delta M_s}, \quad \xi = \frac{F_{B_s}}{F_{B_d}} \sqrt{\frac{\hat{B}_{B_d}}{\hat{B}_{B_s}}}.$$  \hspace{1cm} (1)

In the ratio the theoretical uncertainties are significantly reduced.

### 1.1 Experimental Status and Prospects for $B$ Mixing

The experimental status of $\Delta M_d$ is nicely summarized in the contribution to the proceedings by Ronga \cite{2}. The most recent world average value $\Delta M_d = (0.502 \pm 0.007) \text{ ps}^{-1}$ is dominated by results from the $e^+e^- B$ factories. It constitutes an impressive 1.4% measurement of the $B^0_d$ oscillation frequency. In the next 3–4 years, the BaBar and Belle experiments will collect datasets of about 500 fb$^{-1}$ each. The experimental precision on $\Delta M_d$ is expected to reduce to about 0.5% by then.

The future interest in $B$ mixing clearly lies in the discovery of $B^0_s$ oscillations. The current limit from LEP/SLC/CDF is $\Delta M_s > 14.4 \text{ ps}^{-1}$ at 95% C.L. with a combined sensitivity of 19.3 ps$^{-1}$. The place to observe $B^0_s$ mixing in the next few years is the Fermilab Tevatron. The contribution by Lucchesi \cite{3} summarizes the prospects of the CDF and DØ experiments to measure $\Delta M_s$ in Run II. Using data collected with the new hadronic track trigger, CDF has reconstructed the fully hadronic decay $B^0_s \to D^-_s \pi^+$ observing $44 \pm 11$ $B_s$ events with $D^-_s \to \phi\pi^-$, $\phi \to K^+K^-$. This sample is used first for initial measurements of $B^0_s$ production fraction and branching ratios. Work is in progress to
quantify the expectations for a measurement of $B^0_s$ mixing with the first Run II events now in hand. Both CDF and DØ were also able to collect initial samples of $B^0_s$ particles from semileptonic decays or from $B^0_s \to J/\psi \phi$ which are used for preliminary $B^0_s$ mass and lifetime measurements [6].

The prospects for measuring $\Delta M_s$ with the Atlas detector at the LHC were presented by Ghete [2]. A detailed treatment of systematic uncertainties was performed and the impact of changes to the luminosity target for the LHC start-up were discussed.

### 1.2 Lattice QCD Results for $\xi$

In the report from the first CKM workshop [11], the lattice QCD result for $\xi$ is quoted with an asymmetric upper error to account for the effects of chiral logarithms in extrapolating from simulated quark masses to their real-world values. For UT analyses this has the practical consequence of raising the central value of $\xi$. Ultimately the extrapolation can be controlled by simulations with lighter quarks, but in the interim, there is some profit in investigating ways to reduce the sensitivity to chiral logarithms. At the Durham meeting Bécirevic [5] highlighted one way to accomplish this, by considering the ratio $(F_{B_s}/F_{B_d})/(f_s/f_d)$, where the chiral logarithms largely cancel. However one has to assume that the cancellation, which is shown theoretically for light quarks, still holds in the region where one matches onto lattice results, evaluated for heavier quark masses. The current error on $\xi$ is 5–10% and will certainly be reduced with results from new simulations with lighter quarks. Experimental input from CLEOc on $F_{D_s}/F_D$ will also help.

## 2 Rare Radiative and Semileptonic $B$ Decays

At the first CKM workshop [11], discussion centered on rare radiative $B$ decays and is nicely summarized by Ali and Misiak [6] in the first workshop report. At the Durham meeting, the discussion was widened to include the rare semileptonic decays, addressing both CKM phenomenology and new physics reach. On the theory side, inclusive decays were reported on by Hurth [7], while Buchalla [8] and Lunghi [9] covered exclusive radiative decays. On the experimental side, Playfer [9] provided an overview, whereas Eigen [10] and Nakao [11] focused on results from BaBar and Belle, respectively.

The inclusive decays have a theoretically clean description and observables are dominated by partonic contributions. In the exclusive case, the difficult problem of evaluating quark operator matrix elements between hadronic states must be addressed. By contrast, experimentally the inclusive modes are more challenging to measure.

### 2.1 Inclusive Rare Radiative $B$ Decays

For $B \to X_s\gamma$ the NLL QCD calculation has a charm mass renormalization scheme dependence. The most recent calculations [12, 13] use the $\overline{MS}$ scheme for $m_c$. With this choice for $m_c$, the branching ratio (with a cut on photon energy) is $17$

$$BR(B \to X_s\gamma) = (3.70 \pm 0.30) \times 10^{-4}.$$ 

NNLL calculations are under study, to resolve the $m_c$ scheme-dependence issue.

Measurements of the branching ratio have been performed with several techniques, either fully inclusive (by requiring the presence of a high-momentum lepton in the event) or semi-inclusive (by fully reconstructing a fraction of all possible $X_s\gamma$ final states). Playfer [9] presented a world average value of

$$BR(B \to X_s\gamma) = (3.47 \pm 0.23 \pm 0.32 \pm 0.35) \times 10^{-4},$$

where the errors are due to statistics, experimental systematics and theory. This world average is in good agreement with the above theoretical prediction.

Most of the theoretical apparatus used for the $B \to X_s\gamma$ transition can be carried over to the $B \to X_d\gamma$ case, but one should take into account the possibility of long-distance contributions from intermediate $u$ quarks in the penguin loop. Several arguments suggest that these long-distance effects are small. Nonetheless, the theoretical status of $B \to X_d\gamma$ is not as clean as $B \to X_s\gamma$.

For CKM phenomenology, $b \to s$ transitions have no relevant impact unless unitarity is assumed, whereas $b \to d$ transitions could provide important constraints. In the ratio $R(dy/sy) = BR(B \to X_d\gamma)/BR(B \to X_s\gamma)$, a good part of the theoretical uncertainty cancels, making it valuable for CKM phenomenology, but also for new physics (since the suppression by $|V_{td}/V_{ts}|^2$ may not hold in extended models).

For more details (including a discussion of CP asymmetries) see the conference report by Hurth and Lunghi [7] and references therein.

### 2.2 Exclusive Rare Radiative $B$ Decays

Updates on exclusive rare radiative decays ($B \to V\gamma$ with $V = K^*, \rho, \omega$) and their new physics impact were presented at Durham by Buchalla [8] and Lunghi [9].

Much interest centers on the ratios

$$R^x(\rho/\gamma/K^*/\gamma) = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{(M_B^2 - M_{K^*}^2)^3}{(M_B^2 - M_{\rho}^2)^3} \rho^2 (1 + \Delta R^x)$$

Updates on exclusive rare radiative decays ($B \to V\gamma$ with $V = K^*, \rho, \omega$) and their new physics impact were presented at Durham by Buchalla [8] and Lunghi [9].

Much interest centers on the ratios

$$R^x(\rho/\gamma/K^*/\gamma) = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{(M_B^2 - M_{K^*}^2)^3}{(M_B^2 - M_{\rho}^2)^3} \rho^2 (1 + \Delta R^x)$$
and
\[ R^0(\rho\gamma/K^*\gamma) = \frac{1}{2} \left( \frac{V_{td}}{V_{ts}} \right)^2 \left( \frac{M_B^2 - M^2_{\rho}}{M_B^2 - M^2_{\rho^0}} \right)^2 (1 + \Delta R^0) \]

for charged and neutral decays respectively, where \( \zeta = \frac{\zeta_0}{\xi_0} \) is a ratio of form factors at \( q^2 = 0 \) and \( \Delta R_{\rho/0} \) account for explicit \( O(\alpha_s) \) corrections (together with annihilation contributions for charged \( B \) decays).

\( B \to K^*\gamma \) decays are well established experimentally but only limits exist for \( B \to \rho \gamma, \omega \gamma \) decays. This is partly due to the low expected branching ratio (~ 10^{-6}) and the difficulty in cleanly selecting \( \rho \) and \( \omega \) mesons. The best current limit on the combined charged and neutral modes is \( \text{BR}(B \to \rho \gamma) < 1.9 \times 10^{-6} \) at the 90% CL. This translates into an experimental 90% CL upper limit on the ratio of branching ratios,

\[ R(\rho\gamma/K^*\gamma) \equiv \frac{\text{BR}(B \to \rho \gamma)}{\text{BR}(B \to K^*\gamma)} < 0.047, \]

which is typically a factor of 2 above Standard Model estimates, but is already a significant constraint on beyond-the-SM scenarios.

The form factor ratio, \( \zeta \), has been estimated in a variety of sum-rule and quark model calculations. For the workshop, a value \( \zeta = 0.76 \pm 0.10 \) was adopted in [7] and \( 1/\zeta = 1.33 \pm 0.13 \) in [8]. This is a challenging quantity to evaluate in lattice QCD calculations, since reaching \( q^2 = 0 \) necessitates a large spatial momentum for the light meson, with the possibility of large discretization errors. One approach is to calculate the form factors for \( P \to V \gamma \) for a range of heavy pseudoscalar mesons, \( P \), masses below the \( B \) mass and then extrapolate. New preliminary lattice results [14] suggest a higher value for \( \zeta \) (less SU(3) breaking), leading to a stronger constraint on the unitarity triangle (see also the discussions in [7] and [8]). A further difficulty for lattice calculations will be to deal with the \( \rho \) and \( K^* \) in simulations where quark masses are small enough for them to decay.

Once measurements of both charged and neutral \( B \to \rho \gamma \) decays become available, isospin-violating ratios, with numerators proportional to \( 2\Gamma(B^0 \to \rho^0\gamma) - \Gamma(B^\pm \to \rho^\pm\gamma) \), could provide a useful CKM constraint [8].

The prospects for measuring \( \text{BR}(B \to \rho \gamma) \) at the \( B \) Factories are good. Both BaBar and Belle have sensitivity to observe a 5σ signal with integrated luminosities of 500 fb^{-1}, as should become available in 3 to 4 years. A determination of \( |V_{td}/V_{ts}| \) should then be possible with a precision of ~ 15 ~ 20%, including theoretical and experimental uncertainties. A measurement of this ratio from \( B \) mixing is expected to achieve higher precision but the determination from radiative penguin decays provides a complementary approach with different sensitivity to New Physics.

2.3 **Inclusive Rare Semileptonic Decay** \( B \to X_s \ell^+ \ell^- \)

Since the first workshop, there has been a lot of experimental progress in exploring the rare decays that proceed via a \( b \to s \ell^+ \ell^- \) transition. The exclusive decay \( B \to K \ell^+ \ell^- \) has been established by both BaBar and Belle, and the branching ratio for the inclusive decay \( B \to X_s \ell^+ \ell^- \) has been measured by Belle:

\[ \text{BR}(B \to X_s \ell^+ \ell^-) = (6.1 \pm 1.4^{+1.4}_{-1.3}) \times 10^{-6}, \]

for \( m(\ell^+ \ell^-) > 0.2 \text{ GeV}/c^2 \). This result is in good agreement with the SM-based predictions.

The decay \( B \to X_s \ell^+ \ell^- \) is dominated by perturbative corrections once the \( c \bar{c} \) resonances that show up as large peaks in the dilepton invariant mass spectrum are removed. In the perturbative ‘windows’ outside the resonance regions, a theoretical evaluation with a precision comparable to that in \( B \to X_s \gamma \) should be achievable, but it will be important to compare theory and experiment using the same energy cuts to avoid any extrapolation.

The partonic calculation has now been pushed to NNLL order [15] [16] [17], with the following result for the low-\( \delta \) (\( \delta = q^2/m_B^2 \)) window:

\[ \text{BR}(B \to X_s \ell^+ \ell^-)_{\delta<0.05,0.25} = (1.36 \pm 0.08) \times 10^{-6} \]

where the error is for the renormalization scale uncertainty. The calculation includes nonperturbative contributions scaling like \( 1/m_B^2 \) and \( 1/m_c^2 \). The NNLL contributions change the central value by more than 10% and significantly reduce some systematic errors (see [17] and references therein).

2.4 **Forward-Backward Charge Asymmetry in** \( B \to X_s \ell^+ \ell^- \) \( \text{and} \) \( B \to K \ell^+ \ell^- \)

The forward-backward asymmetry, \( A_{FB}(\delta) \), uses the angle between the \( \ell^+ \) and \( B \) in the lepton-pair rest frame. The position of the zero, defined by the value \( \delta_0 \) for which \( A_{FB}(\delta_0) = 0 \), is particularly interesting since it depends on the relative sign and magnitude of the Wilson coefficients \( C_7 \) and \( C_9 \) and is extremely sensitive to new physics effects.

In the inclusive case, NNLL calculations [15] [16] shifted the position of the zero and improved the precision of its location. They also improved the prediction of the \( \delta \) (or \( q^2 \)) shape of the asymmetry.

For the exclusive case, the asymmetry calculation involves values for ratios of form factors, hard-to-evaluate nonperturbative quantities. However, the position of the zero depends only on transverse form factors which are all related to a single function in the heavy (\( b \)) quark limit, and thus is quite well-determined [13].

In both inclusive and exclusive cases, discovery of a zero in \( A_{FB} \) would be extremely interesting.
Acknowledgments

We would like to thank all speakers and other participants in the Working Group as well as the organizers of this stimulating workshop.

References

1. M. Battaglia et al., eds., The CKM matrix and the unitarity triangle (CERN, 2003), hep-ph/0304132.
2. F.J. Ronga, in Ball et al. [19] hep-ex/0306061.
3. D. Lucchesi, in Ball et al. [19] hep-ex/0307025.
4. B. Epp, V.M. Ghete and A. Nairz, in Ball et al. [19] hep-ph/0307114.
5. D. Becirevic, in Ball et al. [19] hep-ph/0301072.
6. A. Ali and M. Misiak, Radiative Rare B Decays, in Battaglia et al. [11] chap. 6, p. 285, hep-ph/0304132.
7. T. Hurth and E. Lunghi, in Ball et al. [19] hep-ph/0307142.
8. S. Bosch and G. Buchalla, in Ball et al. [19].
9. S. Playfer, in Ball et al. [19] hep-ex/0308004.
10. G. Eigen, in Ball et al. [19].
11. M. Nakao, in Ball et al. [19] hep-ph/0307031.
12. P. Gambino and M. Misiak, Nucl. Phys. B611 (2001) 338, hep-ph/0104034.
13. A.J. Buras, A. Czarnecki, M. Misiak and J. Urban, Nucl. Phys. B631 (2002) 219, hep-ph/0203135.
14. D. Becirevic, in Ringberg Phenomenology Workshop on Heavy Flavours (2003) [http://www.th.mppmu.mpg.de/members/ahoang/ringberg2003/page07.html].
15. H.H. Asatrian, H.M. Asatrian, C. Greub and M. Walker, Phys. Rev. D66 (2002) 034009, hep-ph/0204341.
16. A. Ghinculov, T. Hurth, G. Isidori and Y.P. Yao, Nucl. Phys. B648 (2003) 254, hep-ph/0208088.
17. C. Bobeth, M. Misiak and J. Urban, Nucl. Phys. B574 (2000) 291, hep-ph/9910220.
18. M. Beneke, T. Feldmann and D. Seidel, Nucl. Phys. B612 (2001) 25, hep-ph/0106067.
19. P. Ball, J.M. Flynn, P. Kluit and A. Stocchi, eds., 2nd Workshop on the CKM Unitarity Triangle, IPPP Durham, April 2003 (Electronic Proceedings Archive eConf C0304052, 2003).