Heat transfer behavior caused by temperature difference in reciprocating sliding contact

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Abstract
In order to study the heat transfer characteristics between two rough surfaces of two contacting blocks with different bulk temperatures and sliding reciprocating motion, a two-dimensional heat transfer model was used to analyze the dimensionless average heat flux, considering thermal contact conductance. The results of a series simulations were presented, covering a wide range of operating parameters including dimensionless amplitude \( \tilde{S}_0 \), dimensionless frequency \( \tilde{\omega} \), and measurements of interface conductance \( \tilde{J} \). The results show that the dimensionless average heat flux increases with the increase of dimensionless frequency and amplitude, and the dimensionless average heat flux rises sharply in the low range of \( \tilde{\xi} \) and approaches to a steady state approximation when \( \tilde{\xi} > 71 \), \( 500 \) and \( \tilde{S}_0 > 8 \).

Keywords
Frequency, amplitude, thermal contact conductance, reciprocating sliding contact

Introduction
Surface heating at a sliding contact interface has long been of interest. Most studies¹–⁸ involving this aspect have focused on unidirectional sliding. In contrast, the heat source is time-varying, and the sliding motion is periodic in sliding contact involving reciprocating motion, which is anticipated to behave differently not only
in terms of its different tribological characteristics but also in its thermal response. Such contact conditions are often found in many applications, such as in reciprocating pistons in cylinders, in sealing components in reciprocating pumps, or in sliding bears.

In the range of a typical oscillatory amplitude of a fretting contact, 10–100 µm, there is a response of the nominally static joints to vibration. However, when the amplitude of motion is greater than the size of the body, some regions experience periodical contacting and separating. Depending on the amplitude of the motion, the heat transfer behavior and temperature distribution in the heated zone are quite different. Greenwood and Alliston-Greiner analyzed the surface temperature in fretting contact by separating the sinusoidal heat source into constant and periodic heat inputs using a Fourier transform. Based on the Green’s function method, Tian and Kennedy presented that the surface temperature rise in oscillatory sliding can be assumed as the sum of the nominal temperature rise and the cyclic local temperature rise in an oscillatory contact. Kalin and Vizintin pointed out that the flash temperature of a low-amplitude fretting motion is much different from that of a high-amplitude fretting motion. In their particular case, the flash temperature averages from 45°C to 768°C, corresponding to the motion amplitude from the lowest to the highest.

Besides amplitude, the frequency of the sliding motion also significantly affects the surface temperature and the heat flux at the interfaces. Wen and Khonsari investigated the periodic oscillations of the surface temperature rise with different types of heat sources (circular, rectangular, and parabolic) over a wide range of frequencies and amplitudes. They suggested that a lower dimensionless frequency or a higher dimensionless amplitude results in a greater oscillation of the overall surface temperature. Simulation and experimental results show that an increase in the oscillation frequency yields a higher average contact temperature rise.

Significant efforts have been devoted to study the heat transfer and temperature field generated by friction in fretting contact. However, until today, there has been no specific study on the relationship of the heat exchange caused by differences in body temperatures versus frequency and amplitude. In this case, the regions of different temperatures are juxtaposed to enhance heat transfer relative to the static case. To some extent, motion is a form of convective heat transfer. At the same time, well-known imperfect contact to static solids should be considered in reciprocating sliding contact. The thermal contact conductance of non-static contacts has been identified. The authors investigated the thermal contact conductance, considering speed, mean contact pressure, and surface roughness.

This article studied the heat transfer between two objects due to temperature differences. A finite element model was implemented in the general finite element code ABAQUS to analyze the heat transfer of the reciprocating contact under dry sliding conditions. The changes in the heat flux were correlated to the changes in the mechanical parameters of the reciprocating sliding contact, such as amplitude, frequency, surface roughness, and pressure. The effective thermal contact conductance was believed to provide more efficient and accurate calculations.
Consider a system consisting of a stationary (lower) body $\Omega_2$ in contact with a sliding (upper) body $\Omega_1$ undergoing a reciprocating motion. Referring to Figure 1, assume that the upper body $\Omega_1$ oscillates sinusoidally along the $x$ direction according to

$$V(t) = V_0 \sin (\omega t)$$

where $V_0$ is the velocity amplitude, $\omega$ is the angular frequency, and $t$ is time.

The corresponding displacement is given by

$$S = S_0 [1 - \cos (\omega t)]$$

where $S_0$ is the amplitude of the reciprocating motion. The length of the upper body $\Omega_1$ is $l_1$ which is much shorter than that the length of the lower body $\Omega_2$, that is, the contact length depends on $l_1$. Consider heat exchange at the interfaces due to the existence of a temperature difference $T_0 = T_1 - T_2$ between the two bodies, where $T_1$ and $T_2$ are bulk initial temperatures of body $\Omega_1$ and body $\Omega_2$, respectively. Therefore, the total heat transferred depends on the sliding displacement determined by equation (2).

The contact interface on the microscale appears as a thermal resistance on the macroscale. In its inverse form, thermal contact conductance can be considered as characterizing the transfer of heat across the contacting interfaces.\textsuperscript{19}
Based on the principle of conservation of energy, the mean heat flux of the interface can also be obtained as the product of the temperature change \( \Delta T \) and the thermal contact conductance \( h_c \), which is defined by equation (3)\(^{20,21} \) as follows

\[
q_y = h_c \Delta T
\]  

where the thermal contact conductance \( h_c \) was derived from our earlier paper\(^{18} \) and defined as

\[
h_c = C\sqrt{V}
\]  

where \( C \) is a combined factor consisting of thermal and physical parameters, and a brief description is shown in Appendix 1.

**Heat conduction analysis**

Incomplete contact occurs at the interface of the two bodies. The transient behavior with constant thermophysical properties is performed by the following governing equations.

For the upper body \( \Omega_1 \) \((0 < y \leq g_2)\)

\[
\frac{\partial^2 T(y,t)}{\partial y^2} = \frac{1}{k_1} \frac{\partial T(y,t)}{\partial t} \text{ in } 0 < y < g_2, \ t > 0
\]  

\[
T(y,0) = T_1 \text{ at } 0 < y < g_2, \ t = 0
\]  

\[
T(g_2,t) = T_1 \text{ at } y = g_2, \ t \gg 0
\]  

For the lower body \( \Omega_2 \) \((-g_1 \leq y < 0)\)

\[
\frac{\partial^2 T(y,t)}{\partial y^2} = \frac{1}{k_2} \frac{\partial T(y,t)}{\partial t} \text{ in } -g_1 < y < 0, \ t > 0
\]  

\[
T(y,0) = T_2 \text{ at } -g_1 < y < 0, \ t = 0
\]  

\[
T(-g_1,t) = T_2 \text{ at } y = -g_1, \ t \gg 0
\]

where \( y = g_2 \) corresponds to the top surface of the upper body \( \Omega_1 \) and \( y = -g_1 \) denotes the bottom surface of the lower body \( \Omega_2 \).

**Dimensional considerations**

The equations were solved numerically; thus, it is crucial to make use of dimensionless variables to extend the generality of the results. The following dimensionless parameters are used to derive the solution
\[ \begin{align*}
\tilde{x} &= \frac{x}{l_1}; \tilde{y} = \frac{y}{l_1}; \tilde{S}_0 = \frac{S_0}{l_1} \\
\tilde{t} &= \frac{kt}{l_1^2}; \tilde{\omega} = \frac{\omega l_1^2}{k}
\end{align*} \]

(11)

which ensures that the evolution of the sliding contact is the same for all the parameters in dimensionless space-time. The product of \( \tilde{\omega} \) and \( \tilde{S}_0 \) can be interpreted as a Peclet number in the case of a reciprocation (i.e. \( \text{Pe} = \tilde{\omega} \tilde{S}_0 \)).

The sliding velocity is normalized as

\[ \tilde{V} = \frac{l_1}{k} V = \tilde{V}_0 \sin (\tilde{\omega} \tilde{t}) \]

(12)

where the dimensionless parameter \( \text{Pe} = \tilde{V}_0 = V_0 l_1 / k \) is known as the Peclet number.

For the problem with the local temperature change \( \Delta T \), a dimensionless temperature can be denoted as

\[ \tilde{T} = \frac{T}{\Delta T} \]

(13)

In these dimensionless coordinates, the heat conduction equation in the dimensionless form is

\[ \nabla^2 \tilde{T} = \frac{\partial \tilde{T}}{\partial \tilde{t}} \]

(14)

According to the Fourier law, the heat flux per unit area along axial direction is

\[ q_y = -K \frac{\partial T}{\partial y} = -\frac{K \Delta T}{l_1} \frac{\partial \tilde{T}}{\partial \tilde{y}}, \text{where} \quad \tilde{q}_y = -\frac{\partial \tilde{T}}{\partial \tilde{y}} \]

(15)

Substituting equations (3), (4), and (11)–(13) into equation (15) gives

\[ \tilde{q}_y = -(\tilde{V}_0 \sin (\tilde{\omega} \tilde{t}))^{1/2} \xi \]

(16)

In this dimensionless problem, three dimensionless parameters affect the heat flow, and they are

\[ \tilde{S}_0 = \frac{\tilde{V}_0}{\tilde{\omega}}; \quad \tilde{\omega} = \frac{\omega l_1^2}{k}; \quad \xi = \frac{C l_1^{1/2} k^{1/2}}{K} \]

(17)

where the amplitude of the dimensionless oscillation \( \tilde{S}_0 \) depends on the relative motion of the bodies and the geometry of the upper body \( \Omega_1 \), which plays an important role in the heat transfer between the two contacting surfaces. If \( \tilde{S}_0 \) keeps constant, a higher frequency of the reciprocating motion accordingly requires a larger velocity amplitude. The dimensionless frequency \( \tilde{\omega} \) denotes a time scale, and
the dimensionless combined parameter $\xi$ indicates a measure of the interface conductance due to surface roughness and contact pressure.

**Numerical solution**

The finite element results of the heat conduction problem described in the “Statement of the problem” section were obtained using the two-dimensional model shown in Figure 2, which consisted of two blocks with the identical contact surfaces. The lower body $\Omega_2$ is stationary, supporting the upper body $\Omega_1$ to move under a uniform pressure, and its speed varies with a sinusoidal function. The initial temperatures of the upper body $\Omega_1$ and the lower body $\Omega_2$ are 200°C and 0°C, respectively. During the reciprocating sliding process, the temperatures of the top surface of the upper body $\Omega_1$ and the bottom surface of the lower body $\Omega_2$ remain unchanged. Insulation boundary conditions (no heat loss) are assumed at all other boundaries except for the contact elements. Then, heat is conducted from the high temperature body to the low temperature body through the contact patches.

The ABAQUS finite element software package was used to predict the thermal behavior of the reciprocating sliding contact model. The mesh was constructed using two-dimensional continuum elements in the ABAQUS/Standard library consisting of a coupled temperature–displacement four-node solid elements CPE4T with full integration and hourglass control. The user-defined subroutine GAPCON was called to define the effective thermal contact conductance and to establish the heat transfer between the sliding contact surfaces. This effective thermal contact

![Figure 2. (a) Finite element model. (b) Partial amplification drawing.](image)
conductance includes all the geometric details of the micro-topography, the contact pressure, and the sliding speed.

It should be noted that the number of elements used on the contact interface significantly affects the heat exchange at the contacting interface. The surface temperature of the upper contact center of the upper body $\Omega_1$ was used to verify the simulation results. Figure 3 shows the dimensionless temperature changes of the six different node numbers (i.e. 729, 972, 1428, 2187, 2916, and 3645) on the top surface of the lower body $\Omega_2$ with the cycles. It can be seen that at the beginning of the contact, the dimensionless temperature drops rapidly and then rises slightly. The graph contains 410 cycles, each corresponding to a dimensionless time of 104.89. After 400 cycles, the fluctuations of all the dimensionless temperatures become extremely small, indicating that the heat transfer conditions become stable. Based on the inset in Figure 3, a comparison of the dimensionless temperature between a model with 2916 contact nodes and a model with 3645 contact nodes indicates a relative error of less than 0.1%. A good mesh should be fine enough to provide an adequate solution accuracy without excessive computational time. Therefore, 2916 nodes were used for the contact surface of the lower body $\Omega_2$. A similar procedure was used to determine the number of nodes on the contact surface of the upper body $\Omega_1$, and 32 nodes were used in this study.

**Results and discussions**

A series of results were presented to study the effects of the dimension frequency $\dot{\omega}$, the dimensionless amplitude $\tilde{S}_0$, and a measure of the interface conductance $\xi$. Table 1 shows the input data for the simulations, where the first two columns show

![Figure 3. Dimensionless nodal temperature variations at the center of the contact of the upper body $\Omega_1$ by using six different node number models for $\tilde{S}_0 = 0.32$, $\dot{\omega} = 10^{5.69}$, and $\xi = 7150$.](image-url)
the range of values for each parameter, and the third column presents the fixed values for the corresponding parameters when the other two parameters are considered.

The normalized average heat flux at different dimensionless frequencies versus $\xi$ at a fixed dimensionless amplitude $\tilde{S}_0 = 0.32$ is illustrated in Figure 4. All curves are approximately parallel to each other, indicating that the average dimensionless heat flux sharply rises in the low range of $\xi$ and researches steady state over a wide range of $\xi$. The same phenomenon appears in Figure 6 when $\xi > 71,500$. In addition, for the same $\xi$, the average dimensionless heat flux increases as the dimensionless frequency $\tilde{\omega}$ increases. Depending on the dimensionless $\xi$ and frequency, these changes in heat flux can be combined by equation (18), which is well suited to the points in Figure 4.

Table 1. Dimensionless input data.

| Parameter | Low    | High   | Fixed |
|-----------|--------|--------|-------|
| $\tilde{\omega}$ | $10^{3.39}$ | $10^{6.39}$ | $10^{4.69}$ |
| $\tilde{S}_0$  | 0.16   | 15.92  | 0.32  |
| $\tilde{\xi}$  | 71.5   | 715,000| 7150  |

Figure 4. Comparisons of the average dimensionless heat flux with $\xi$ for various values of dimensionless frequencies, where $\tilde{\omega}_1 = 10^{3.39}$, $\tilde{\omega}_2 = 10^{4.09}$, $\tilde{\omega}_3 = 10^{4.69}$, $\tilde{\omega}_4 = 10^{5.39}$, and $\tilde{\omega}_5 = 10^{5.69}$, respectively.
A comparison of the simulated and calculated average dimensionless heat fluxes is shown in Figure 5. In the figure, error lines with relative errors of $\pm 10\%$ and $\pm 20\%$ are also presented. Detailed analysis indicates that 84\% of the data is within a relative error of $\pm 20\%$, and 64\% of the data are within a relative error of $\pm 10\%$. It can also be seen that an excellent fitting result can be obtained under the conditions of $\xi > 7150$ and $\tilde{\omega} > 10^{4.69}$.

When $\xi = 7150$, the effects of dimensionless amplitude and amplitude frequency on the average dimensionless heat flux are shown in Figure 6. The average dimensionless heat flux increases as the dimensionless frequency increases. The dimensionless average heat flux appears to increase faster with frequencies in the calculations of lower amplitudes (for $\tilde{\omega} < 10^{5.39}$) than in the calculations of higher amplitudes. It should be noted that when the dimensionless frequency remains the same value, the normalized heat flux increases as the dimensionless amplitude increases. Physically, the amplitude increases and the heat exchanges over a larger surface, resulting in a greater average dimensionless heat flux.

From the comparative study of Figure 7, it was found that all of these average dimensionless heat fluxes showed an increasing trend with an increase in dimensionless amplitude $\tilde{S}_0$, and the growth rate gradually decreased. When the dimensionless amplitude exceeds 2, the curves of $\xi_3 = 7150$, $\xi_4 = 71,500$, and $\xi_5 = 715,000$ are close to each other. Meanwhile, when the parameter $\xi$ is relatively large ($\xi > 715$), the relationship between the dimensionless average heat flux and the amplitude is nonlinear. If $\xi$ is smaller, the nonlinearity decreases, causing the heat flux to be proportional to the amplitude. It is noted that a larger

$$q = \frac{\xi}{7207.65 + \xi} \cdot \left( \frac{17.06\omega}{169030.94 + \omega} + 1.03 \right)$$

(18)
\( j = 0.7150 \) yields a slower heat flux increase in a large amplitude, demonstrating that the thermal conductance is dominant. Besides, the output of the average dimensionless heat flux approaches a constant value when \( \dot{j} = 0.7150 \) and \( S_0 = 0.8 \).

**Figure 6.** Comparisons of the average dimensionless heat flux with the dimensionless frequency for various dimensionless amplitudes of reciprocations, where \( \tilde{S}_{0-1} = 0.16, \tilde{S}_{0-2} = 0.32, \tilde{S}_{0-3} = 0.64, \tilde{S}_{0-4} = 1.59, \) and \( \tilde{S}_{0-5} = 6.37 \), respectively.

\( \ddot{\xi}(\ddot{\xi} > 7150) \) yields a slower heat flux increase in a large amplitude, demonstrating that the thermal conductance is dominant. Besides, the output of the average dimensionless heat flux approaches a constant value when \( \ddot{\xi} > 715, 500 \) and \( S_0 > 8 \).
Conclusion
The heat transfer affected by the oscillatory sliding motion under a bulk temperature difference was investigated. The average heat flux at the interface was characterized by three dimensionless parameters that represent the amplitude of motion, frequency, and the interface conductance due to surface roughness. The computational results of the parametric study for various dimensionless operating parameters are presented. Simulation results show the following:

1. Increasing the dimensionless frequency $\tilde{\omega}$ or dimensionless amplitude $\tilde{S}_0$, thereby increasing the Peclet number Pe, can result in a higher dimensionless average heat flux. And relative rapid growth appears under the conditions of $\tilde{\omega} < 10^{5.39}$ and $\tilde{S}_0 < 8$.

2. For a fixed dimensionless amplitude $\tilde{S}_0 = 0.32$, the average dimensionless heat flux can be expressed by a simple mathematical equation of two variables, dimensionless frequency $\tilde{\omega}$ and $\tilde{\xi}$. The equation obtained in this study is suitable for predicting heat flux, especially under the conditions of $\tilde{\xi} > 7150$ and $\tilde{\omega} > 10^{4.69}$.

3. When the dimensionless frequency $\tilde{\omega}$ remains unchanged, the dimensionless average heat flux sharply rises in the low range of $\tilde{\xi}$ and approaches to the steady state approximation when $\tilde{\xi} > 71,500$ and $\tilde{S}_0 > 8$, indicating that a constant interface conductance can be assumed in the finite element solution.

We suggest

1. Conducting further experimental studies to compare with the simulation results.
2. Conducting the heat transfer analysis of frictional heating without temperature difference.

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Appendix 1

The thermal contact conductance \( h_c \) has already been defined as a function of the sliding speed, the mean contact pressure, and the surface parameters, as\(^{18}\)

\[
h_c = \frac{4.26C_1C_2\sqrt{R_1R_2}\sqrt{V}}{(C_1 + C_2)(R_1 + R_2)^{3/4}(\sigma_i^2 + \sigma_j^2)^{3/8}} \frac{p_{nom}}{E^*}
\]

(19)

where \( p_{nom} \) is the mean nominal contact pressure; \( C_i = K_i/\sqrt{k_i} \), \( k_i \) is the thermal diffusivity, \( K_i \) is the thermal conductivity; \( R_i \) is the summit radius of the asperity; \( \sigma_i \) is the summit height standard deviation, \( i = 1, 2 \); and \( E^* \) is the composite elastic modulus.

If the two surfaces are identical, equation (19) reduces to

\[
h_c = C\sqrt{V}
\]

(20)

where \( C = \frac{0.98KR_1^{1/4}p_{nom}}{\sqrt{k\sigma_i^{3/4}} E^*} \).