High flux symmetry of the Spherical Hohlraum with Octahedral 6LEHs at a Golden Hohlraum-to-capsule Radius ratio

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In the present Letter, we investigate a spherical hohlraums with octahedral six laser entrance holes (LEHs) for inertial fusion, which has advantages over the conventional hohlraums of cylindrical geometry since it contains only one cone at each LEH and the problems caused by the beam overlap and crossed-beam energy transfer can be eliminated and the backscattering can be reduced. In particular, our study indicates that at a specific hohlraum-to-capsule radius ratio, i.e., the golden ratio, the flux asymmetry on capsule can be significantly reduced. From our study, this golden octahedral hohlraum has robust high symmetry, low plasma filling and low backscattering. Though the golden octahedral hohlraum needs 30% more laser energy than traditional cylinder for producing the ignition radiation pulse of 300 eV, it is worth for a robust high symmetry and low backscattering. The proposed octahedral hohlraum is also flexible and can be applicable to diverse inertial fusion drive approaches. As an application, we design an ignition octahedral hohlraum for the hybrid drive.

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Introduction—The hohlraum is crucial for the inertial fusions of both indirect drive [1,2] and the hybrid indirect-direct drive proposed recently (HID) [3]. In the indirect drive approach, the hohlraum is first heated by laser beams to a few million Kelvin and then the energy flux of the transferred X-ray radiation compress the deuterium-tritium capsule at a convergence ratio of 25 to 45, making the nuclear fuel finally burn in a self-sustained way. In the corresponding hohlraum design, the hohlraum shape, size and the number of Laser Entrance Hole (LEH) are optimized to balance tradeoffs among the needs for capsule symmetry, the acceptable hohlraum plasma filling, the requirements for energy and power, and the laser plasma interactions. Among many requirements, the energy coupling and flux symmetry are of most concern. A higher energy coupling will economize the input energy and increase the fusion energy gain. More importantly, a very uniform flux from the hohlraum on the shell of capsule is mandatory because a small drive asymmetry of 1% [2] can lead to the failure of ignition. Actually, the small flux asymmetry will be magnified during the compression process due to the varied kinds of instabilities and results in a serious hot-cold fuel mixture that can dramatically lessen the temperature or density of the hot spot for ignition.

Various hohlraums with different shapes have been proposed and investigated, such as cylinder hohlraum [1,2], rugby hohlraum [5–10] and elliptical hohlraum [11]. These hohlraums are elongated with a length-to-diameter ratio greater than unity and have cylindrically symmetry with two LEHs on the ends. Among all above hohlraums, the cylindrical hohlraums are used most often in inertial fusion studies and are chosen as the ignition hohlraum on NIF [3,12,13], though it breaks the spherical symmetry and leads to cross coupling between the modes.

Intuitively, spherical hohlraum has the feature of the most symmetry compared to other geometric shapes. In the late 1990s, experiments on hohlraum with 6 LEHs obviously exhibited its advantage in high uniformity of the radiation flux on the capsules’ surface [14], while the theoretical investigations of this kind of hohlraum design are in lack. Soon after that, the first experiment on hohlraum with 4 LEHs of tetrahedral symmetry was conducted at OMEGA [15], while the theoretical study showed that it needs two sets of laser beams in order to minimize the flux asymmetry by varying the relative power [16].

In this Letter, we investigate the spherical hohlraum with octahedral 6 LEHs for the first time from the theoretical side, addressing the most important issue of the flux symmetry. We find a golden hohlraum-to-capsule radius ratio of 5.14, at which the flux asymmetry can be reduced to about 0.1%. We call the hohlraum as golden octahedral hohlraum. From our study, there is a robust high symmetry inside such a golden octahedral hohlraum during the capsule implosion. In addition, the golden octahedral hohlraum contains only one cone at each LEH and the backscattering can be small without any beam phasing [3]. The golden octahedral hohlraum also has low plasma filling, which further benefits for a low backscattering. However, a larger volume of the hohlraum needs a little more laser energy to drive a golden octahedral hohlraum than to drive a traditional cylinder for generating same radiation. Nevertheless, it is worth to exchange some laser energy for a robust high symmetry. The octahedral hohlraum design can be implemented on the OMEGA laser and will be conducted on SG laser facilities in 2014. As an application, we design a golden octahedral hohlraum for the hybrid drive using the expended plasma-filling model and view factor model.

Spherical Hohlraum with Octahedral 6LEHs—For convenience, we consider that octahedral hohlraum has two poles and an equator though it is round. In the octahedral hohlraum, there are six LEHs, one at per pole and four along the equator coordinately. In the hohlraum system, we define $\theta$ as polar angle and $\phi$ as azimuthal angle. We use $R_H$ to denote the hohlraum radius, $R_C$ the capsule radius, $R_L$ the LEH radius and $R_Q$ the quad radius at LEH. Here, we assume the quad shape at LEH to be a circle. Each quad through a LEH is characterized by $\theta_L$ and $\phi_L$, where $\theta_L$ is the opening angle that the quad makes with the LEH normal direction and
\(\phi_L\) is the azimuthal angle about the normal of the LEH. The relative fluxes of the laser spot, the hohlraum wall and LEH are denoted as \(F_{\text{spot}}\), \(F_{\text{wall}}\) and \(F_{\text{LEH}}\), respectively. Usually, we take \(F_{\text{spot}} : F_{\text{wall}} : F_{\text{LEH}} = 2 : 1 : 0\), unless declaring. Fig. 1 shows the scenography of the octahedral hohlraum with six LEHs and laser spot of 48 quads and its pattern in the \(\theta/\phi\) plane, by taking \(R_H/R_C=5.1\), \(R_C=1\) mm, \(R_L=1\) mm, \(R_Q=0.3\) mm and \(\theta_L=55^\circ\). From our calculation, the flux asymmetry is about 0.1% on a capsule of 1.1 mm radius.

Golden ratio—We firstly use a simple model to prove that a golden hohlraum-to-capsule radius ratio exists for an octahedral hohlraum, at which the flux asymmetry can reach its minimum. In this simple model, the LEHs are treated as negative sources, and the wall and laser spots are treated as a homogeneous background by neglecting their flux difference. Only considering the negative effect of LEH on capsule, we present in Fig. 2 a schematic of a capsule inside an octahedral hohlraum. The capsule is concentric with the octahedral hohlraum with their center at point O. On capsule, there are two kinds of points which see LEH most different, such as points A and B in the figure. The normal of point A is in the same direction as that of the LEH centered on point M, while the normal of point B has equal angles with that of three LEHs, centered on points L, M and N, respectively. Hence, we can study the flux asymmetry on capsule by comparing the irradiation on points A and B.

The flux irradiated on a capsule point is mainly decided by the solid angle of the source opened to that point and the angle of the connecting line with respect to the normal of the capsule point. By denoting the LEH area as \(S\), the solid angle of LEH M seen by point A is \(d\Omega_A = S/(R_H - R_C)^2\), and the solid angle of LEH N seen by point B is \(d\Omega_B = S\cos\alpha/l^2\). Here, \(l\) is the length of line BN, and \(\alpha\) is the angle of BN with respect to the normal of N. We use \(\beta\) to denote the angle between the normal of N and the normal of B, then the angle of BN with respect to the normal of B is \(\alpha + \beta\). Note that both LEH M and LEH L open the equal solid angle to point B as LEH N. Then the quality of the illumination on the capsule can be quantified approximately by:

\[
f = 0.5 \times |3\cos^3\alpha \times \cos(\alpha + \beta) \times \frac{d\Omega_B}{d\Omega_A} - 1|\]  \( (1) \)

The variation of \(|f|\) as \(R_H/R_C\) is presented in Fig. 3. As shown, \(|f|\) reaches its minimum at \(R_H/R_C = 5.14\). It predicts the emergence of the minimum flux asymmetry at the golden hohlraum-to-capsule radius ratio.

Calculations with view factor model—To certify the above theoretical prediction, we further exploit the view factor model to calculate the radiation flux on the shell of the capsule numerically. We define ratio \(|\Delta F/F|\), in which \(\Delta F = 0.5 \times (F_{\text{max}} - F_{\text{min}})\) and \(F\) is the average value of flux \(F\) upon the capsule. The black solid line shown in Fig. 3 is variation of \(|\Delta F/F|\) as \(R_H/R_C\) on the capsule shown in Fig. 2, which is inside an octahedral hohlraum with \(R_L=1\) mm, \(R_Q=0.5\) mm and \(\theta_L=55^\circ\). As indicated, an asymmetry minimum do exist for \(|\Delta F/F|\) at \(R_H/R_C = 5.1\) quite close to the simple model. Using \(F(P)\) to denote the total flux \(F\) at
ical harmonic decomposition. We further define $Y_{lm}(\theta, \phi)$, where $Y_{lm}$ is spherical harmonic functions. We define $C_{lm}$ as $R_H/R_C$ for the same model in Fig. 3. As shown, $C_{44}$ and $C_{40}$ dominate the capsule flux asymmetry, except around the golden ratio where the asymmetry is dominated by $C_{46}$, $C_{84}$ and $C_{88}$ with values much smaller than 0.1%. Notice that $C_{2m}$ is on noise level and can be thoroughly neglected inside an octahedral hohlraum, quite different from the case inside a cylindrical geometry. The minimums of $C_{4m}$ at $R_H/R_C = 5$, $C_{6m}$ at $R_H/R_C = 4$ and $C_{8m}$ at $R_H/R_C = 6$ are due to the asymmetry smoothing factor on capsule inside a concentric spherical hohlraums. According to Ref. [1], the smoothing factor depends on mode number $l$ but not on the directional mode number $m$ because the choice of the direction of the polar axis is arbitrary for spherical symmetry. Here, it is worth to mention the 2LEH cylindrical and 4LEH spherical hohlraums, in which the symmetries are dominated respectively by $Y_{2m}$ and $Y_{5m}$, while the smoothing factors of $Y_{2m}$ and $Y_{5m}$ are much less reduced, especially at $R_H/R_C \leq 5$.

In order to distinguish the asymmetry contributions from LEH and laser spot, we calculate spherical hohlraums with only octahedral 6LEHs and only 48 quads, respectively. As shown in Fig. 3, the asymmetry contributed by the LEHs is significantly larger than that by the spots, and the asymmetry is mainly decided by the LEHs. Obviously, the asymmetry minimum is thoroughly due to the six LEHs of the octahedral hohlraum. That is why $R_H/R_C$ of the minimum asymmetry from the simple model agrees so well with that from view factor model. According to our calculations, $R_H/R_C$ of the minimum asymmetry has small deviation from 5.14 under different LEH-to-hohlraum radius ratio and different arrangement of laser beams. We call $R_H/R_C = 5.14$ as the golden ratio of the octahedral hohlraum. Notice that $|\Delta F|/|F|$ is around 0.2% at $R_H/R_C \geq 4.7$, which means that the golden octahedral hohlraum has robust symmetry not only at the early stage of capsule implosion when $R_H/R_C$ becomes a little small due to wall plasma expansion, spot motion and expansion of the outer layers of the capsule, but also during the stages of capsule inward acceleration and ignition when $R_H/R_C$ becomes very large. Here, it is worth to mention the pioneer work on 6LEH spherical hohlraum fielded at the ISKRA-5 facility with 12 laser beams [14], in which the ratio is taken as $R_H/R_C = 7$. Obviously, it costed double energy as compared to that designed at the golden ratio, while the corresponding symmetry was not the best.

In addition to the advantage in high robust symmetry, the golden octahedral hohlraums also have superiority on low backscattering and low plasma filling. As we mentioned above, the spherical octahedral hohlraums contain only one cone at each LEH, so the issues of beam overlapping and crossed-beam transfer do not exist. Thus, the backscattering can be remarkable decreased without any beam phasing, which therefore leads to a higher laser absorption efficiency for a spherical hohlraum than for a cylinder. In addition, the volume of a golden octahedral hohlraums is 125 times of that capsule volume, more than 2 times of that of the traditional cylindrical hohlraum which is about 50 to 60 times of the capsule volume. Thus, the plasma filling inside such an octahedral hohlraum is obviously lower than inside a cylinder, which further benefit a low backscattering. Of cause, it needs more laser energy to drive a larger hohlraum for producing same radiation. Nevertheless, it is worth to spend some more laser energy to get a robust high symmetry. As an example, we compare the laser energy required for generating an ignition radiation pulse of 300 eV inside a golden octahedral hohlraum with that inside a traditional cylinder. We consider the ignition target recently designed for NIF [13] and use the expended plasma-filling model [11, 17, 19] to calculate the required laser energy and the plasma filling inside the hohlraums. According to Ref. [13], the cylindrical uranium hohlraums with dimensions of 5.75mm in diameter and 9.4mm in length are used for a DT capsule of $R_C=1.13$ mm. To have same LEH area, we take the $R_L = 1.732$ mm for cylinder and $R_L = 1$ mm for the golden octahedral hohlraum. Here, we do not consider the backscattering and assume that the conversion efficiency from laser to x-ray is 87% for both hohlraums. From our calculation, it needs 1.5 MJ absorbed laser energy by using the golden octahedral hohlraum with $n_e = 0.067$, and 1.1 MJ by using the cylinder with $n_e = 0.094$. Here, $n_e$ is electron density in unit of the critical density, and the plasma filling criterion is $n_e = 0.1$ [17]. Obviously, it costs more than 30% laser energy by using a golden octahedral hohlraum, but it is available on both NIF and LMJ.

**Laser arrangement and constraints**— We define the hohlraum pole axis as $z$ axis. Axis $x$ is defined by the centers of two opposite LEHs on equator, and $y$ is defined by the other two. We name the LEHs centered on $z$ axis as LEH1 and LEH6, on $y$ axis as LEH2 and LEH4, and on $x$ axis as LEH3 and LEH5. Each quad through a LEH is characterized by $\theta_L$ and $\phi_L$. There is only one cone in our design, so all quads coming from the six LEHs have the same $\theta_L$. The chooses of $\theta_L$ and $\phi_L$ are not only related to the ratios of $R_H/R_C$, $R_L/R_C$, $R_Q/R_C$ but also interactional. There are three constraints which govern the quad arrangement. First, the lasers can not hit to the opposite half sphere in order to have a short transfer distance inside hohlraum for suppressing the increase of LPI, which limits the opening angle $\theta_L > 45^\circ$. Second, the laser can not enter the hohlraum at a very shal-

![Fig. 4: (color online) Variations of $C_{lm}$ as $R_H/R_C$.](image-url)
low angle in order to avoid absorbing by blowoff from the wall and making uncleanance of the hole. The latter requires $\theta_L < \arcsin((R_H - R_Q)/h)$, where $h = \sqrt{R_H^2 - R_L^2}$. For model in Fig.1, it requires $\theta_L < 65^\circ$. Third, a laser beams cannot cross and overlap with other beams.

We use $N_Q$ to denote the quad number per LEH. The quads come in each LEH coordinately around LEH axis at the azimuthal angles of $\phi_L k = 360^\circ/N_Q \ (k = 1, ..., N_Q)$. Here, $\phi_L$ is azimuthal angle deviated from x axis in the $xy$ plane for LEH1 and LEH6, from x axis in the $xz$ plane for LEH2 and LEH4, and from y axis in the $yz$ plane for LEH3 and LEH5. From the geometrical symmetry, we have $0^\circ < \phi_L < 360^\circ/2N_Q$. In order to avoid overlapping between laser spots and transferring out of neighbor LEHs, we usually take $\phi_L$ around $360^\circ/4N_Q$. In Fig. 1, we take $\phi_L = 11.25^\circ$.

Application—The proposed octahedral hohlraum is flexible and can be applicable to diverse inertial fusion drive approaches such as indirect and hybrid indirect-direct drives. As an application, we design a golden octahedral hohlraum for the hybrid drive by using the extended plasma-filling model and view factor model. In the HID model[4], the fuel capsule is first compressed by indirect-drive x rays, and then by both x rays and direct drive lasers. According to the HID model, a four-step radiation pulse with the fourth step of 260 to 270 eV and 1.7 ns is required for a capsule with radius $R_C = 850 \mu m$. Shown in Fig. 5 is the initial design of laser energy and $R_H$ by using the extended plasma-filling model. Here, $R_L$ is taken as 1 mm. The two semi-empirical criterions used here are: $n_e \leq 0.1$ and $n_{1B} \equiv (l/\sqrt{2})/(\lambda_{1B}) \leq 1$. The latter is related to the inverse bremsstrahlung absorption length $\lambda_{1B}$. Here, $l$ is the transfer distance of laser beam inside a spherical hohlraum. As shown, it needs absorbed laser energy of 0.5 to 0.6 MJ to produce a 260 to 270 eV radiation pulse inside a golden octahedral hohlraum with $n_e$ and $n_{1B}$ well meeting the criterions. The dependence of the capsule asymmetry on $R_H/R_C$, $R_L$, $R_Q$, $\theta_L$ and the relative flux of laser spot to hohlraum wall is important for choosing the optimum design of hohlraum. Using the view factor model, we study the variations of $|\Delta F/F|$ as these quantities for the HID model. The results indicate that $|\Delta F/F|$ is smaller than 0.2% at the golden ratio in the ranges of $R_H/R_C$, $R_L$, $R_Q$ and $\theta_L$ concerned in our model.

In summary, we have investigated the spherical hohlraum with octahedral 6LEHs at a golden hohlraum to capsule radius ratio, which has very high and robust symmetry on capsule, with significantly lower plasma filling and lower backscattering as compared to the cylindrical counterpart. Inside a golden octahedral hohlraum, it is $Y_{8m}$ which dominates the asymmetry. It needs about 30% more laser energy to drive such a golden octahedral hohlraum than to drive a cylinder for producing same radiation, but it is worth to exchange such available laser energy for a robust high symmetry. The above novel spherical hohlraum design has important implications for laser inertial fusion and is expected to be conducted on SG laser facilities in near future.

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