Absorption and scattering of massless scalar waves by nonsingular static spherical black holes in conformal gravity

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Wave is an excellent probe to study black holes and extremely curved spacetime. In this paper, we look into the absorption and scattering of massless scalar waves by nonsingular black holes conformally related to the Schwarzschild black hole. This kind of black holes can be found as a solution to conformal gravity which retains the conformal symmetry of the spacetime. The partial and total absorption cross section, as well as the differential scattering cross section, are presented for black holes with different choices of conformal parameters. Although having identical null geodesics to Schwarzschild black hole, these black holes show unique signature of the wave absorption and scattering. The absorption generally increases with the conformal factor, while the high energy limit of the absorption cross section, namely the shadow of the black hole remains the same as the Schwarzschild black hole. The shift of the peaks in the oscillatory pattern of scattering seems more significant for larger conformal parameters, while the width of the glory peak remains rather close to the Schwarzschild case.

I. INTRODUCTION

As a great theoretical achievement, Einstein’s General Relativity (GR) works perfectly well in explaining various observations. A class of its solutions, black holes, also seems to prove present in binary systems and galactic centers by the growing evidence in recent years\cite{1–3}. Nonetheless, the long known challenges of GR, such as the dark contents of the universe and the quantization of the gravity, have motivated the search of alternative theories of gravity. The detection of gravitational waves (GWs)\cite{1, 2} and the direct observation of the center of our Galaxy by Event Horizon Telescope\cite{3} may thus herald a new era of testing those alternative gravity theories that
have different predictions from GR in strong field regime. One of the schemes is the conformal gravity which can ameliorate or eliminate the singularities in black hole solutions of GR\cite{4, 5}. This description of gravity is invariant under a conformal transformation of the metric

$$g_{\mu\nu} \rightarrow \hat{g}_{\mu\nu} = S(x)g_{\mu\nu},$$

where $S(x)$ is a nonsingular function depending on the spacetime coordinates. Conformal gravity can be constructed with, for example, the Weyl tensor \cite{4–9} or the dilaton\cite{10–12}. With this conformal symmetry, the singularities of spacetime can be avoided in the sense that they are artificial and rely on the choice of conformal gauges\cite{5, 12}, which is similar to situation of the apparent or coordinate singularities in GR.

However, the conformal symmetry is not really possessed by the realistic spacetime\cite{12}. For conformal gravity to be a physical description of gravitation, some sort of symmetry breaking mechanism is thus required, such that the spacetime can present itself in the current phase. In fact, if we consider specifically the conformal gravity constructed with the dilaton $\varphi$\cite{10–12},

$$S = \int d^4x \sqrt{-\hat{g}} \left( \varphi^2 \hat{R} + 6\hat{g}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right),$$

which is invariant under the transformation Eq.(1) with $\varphi \rightarrow S^{-1}\varphi$, the Einstein-Hilbert Lagrangian can be recovered as one of the possible vacua in the spontaneous breaking of the conformal symmetry in analogy with the Higgs mechanism\cite{12}. And in the strong field regime such as the regions close to the black holes, the spacetime may be still in its conformally invariant phase\cite{7, 11, 13}. It was shown that a class of nonsingular black holes from conformal gravity is compatible with the x-ray observation data\cite{14, 15}. Various aspects of this class of black holes including the formation\cite{16}, evaporation\cite{17} and perturbation\cite{18, 19} have also been discussed.

In this work, we adopt another approach to study this class of nonsingular black holes by considering their absorption and scattering of planar waves. The study of black hole scattering dates back to 1960s\cite{20–24}. Using waves as probes may provide deeper insights of the physics of black holes and high curvature spacetime, and increasing amount of attention has been attracted to these subjects related to various gravity theories and black holes in recent years. (See, e.g., Refs.\cite{25–40} and the references therein.) The scattered waves may even extract energy from charged or rotating black holes via the superradiant mechanism \cite{41–46}. Besides these theoretical interests, the experimental advances, including the historical detection of GWs\cite{1, 2} and the first observation of the supermassive black hole in M87 by Event Horizon Telescope\cite{3}, have also incentivized the growing investigation of the interaction between black holes and various waves. The numerical
techniques for related calculation have also been significantly improved\cite{47–49}. The pattern of the scattered waves may likely bear the information of the near-horizon structure of the black hole, and may one day be observed in the near future.

Following this path, in this paper, we utilize the partial wave formalism to examine the absorption and scattering of scalar waves by the nonsingular spherical static black holes in conformal gravity. The paper is organized as follows. In Sec. II, we briefly review the nonsingular black hole solution of the conformal gravity. Section III and IV are dedicated to the absorption and scattering cross sections of scalar waves, respectively. We conclude in Sec. V. Throughout the paper, we use the units that $c = 8\pi G = 1$, and assume that the impinging waves have no backreaction on the background.

II. NONSINGULAR STATIC SPHERICAL BLACK HOLES IN CONFORMAL GRAVITY

In conformal gravity, the spacetime singularity resides in the black hole can be considered as mathematical artificial and can be removed through conformal transformation. For static spherical case in GR, the Schwarzschild black hole is described by

$$d\tau_s^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2d\Omega^2, \quad (3)$$

where

$$f(r) = 1 - \frac{r_s}{r}, \quad (4)$$

and $r_s$ is the horizon radius. In this case, the intrinsic singularity at $r = 0$ can be removed by considering a conformally related metric

$$d\tau^2 = \hat{g}_{\mu\nu}dx^\mu dx^\nu = S(r)d\tau_s^2 \quad (5)$$

with a suitable conformal factor $S(r)$. One particular choice of $S(r)$ is\cite{12}

$$S(r) = \left(1 + \frac{L^2}{r^2}\right)^{2N}, \quad (6)$$

with a positive integer $N$ and a new length scale $L$. Obviously, the Schwarzschild metric is recovered when $L \ll r$. It is shown that $L$ may be restricted to $L \leq 0.6r_s$ for related cases by observation\cite{14, 15}. In this work, following the orders of magnitude in Refs.\cite{18}, we consider $N$ ranges up to 50 and $L$ ranges up to 0.6$r_s$. 
The spacetime described by Eq. (5) is nonsingular everywhere [12]. Particularly, at $r = 0$, the Ricci scalar $\hat{R}$ can be approximated as

$$\hat{R} \simeq \frac{24N^2r_s}{L^{4N}}r^{4N-3},$$

(7)

and the Kretchmann scalar $\hat{K} = \hat{R}_{\alpha\beta\gamma\delta}\hat{R}^{\alpha\beta\gamma\delta}$ as

$$\hat{K} \simeq \frac{12}{L^{8N}}(1 + 12N^2 - 16N^3 + 16N^4)\frac{r_s^2}{r^{8N-6}}.$$  

(8)

Since it is presumed that $N$ is a positive integer, these curvature invariants are thus regular everywhere. It is also shown that [12] the spacetime described by Eq. (5) is geodesically complete in that massless particles cannot reach the center for a finite value of the affine parameter and massive particles cannot reach the center in a finite proper time.

To consider the classical paths in static spherical spacetime conformally related to Schwarzschild metric, one can read from the metric Eqs. (3) and (5) that

$$\dot{\tau}^2 = S(r) \left[ f(r)\dot{t}^2 - \dot{r}^2 - r^2 \dot{\theta}^2 - r^2 \sin^2 \theta \dot{\phi}^2 \right] = \kappa,$$

(9)

where the overdot indicates the derivative with respect to certain affine parameter, and $\kappa = 0, 1$ for massless and massive particles, respectively. In the current case with $\kappa = 0$, the effect of the conformal factor $S(r)$ is eliminated. For a given impact parameter $b$, the null orbit equation can be written as

$$\left( \frac{du}{d\theta} \right)^2 = \frac{1}{b^2} - u^2 + r_s u^3,$$  

(10)

with $u \equiv 1/r$, namely the same orbit equation for massless particles as in the Schwarzschild case. Hence the deflection function $b(\theta)$ is also the same, and it can be approximated by [50]

$$\frac{b(\theta)}{r_s} \sim \frac{3}{2} \sqrt{3} + 1.74e^{-n}, \quad \text{for} \quad \theta \gtrsim \pi.$$  

(11)

The critical impact parameter, which leads to the unstable orbiting of massless particles trapped on the photon sphere, is thus also $b_s = \frac{3}{2} \sqrt{3} r_s$. The shadow area $A_{\text{shadow}}$ of the current black holes is therefore also $A_{\text{shadow}} = \frac{3}{4} \pi r_s^2$ as the Schwarzschild black hole.

However, as we will see in the following sections, despite the identical geometric-optics with the Schwarzschild case, the nonsingular static spherical black holes in conformal gravity will demonstrate distinct absorption and scattering cross sections.
III. ABSORPTION CROSS SECTION

To consider the interaction between massless scalar wave and the black hole described by Eq.(5), we firstly consider the absorption of the wave by the black hole utilizing the partial wave method.

A minimally coupled massless scalar field \( \Phi \) in curved spacetime is described by the covariant Klein-Gordon equation

\[
\nabla^\nu \nabla_\nu \Phi = \frac{1}{\sqrt{-g}} \partial_\nu \left( \sqrt{-g} \partial^\nu \Phi \right) = 0.
\]

Separating the variables of \( \Phi(t,r,\theta,\phi) \) as

\[
\Phi = \frac{\psi_{\omega l}(r)}{r \sqrt{S(r)}} Y_{lm}(\theta,\phi) e^{-i\omega t},
\]

where \( Y_{lm} \) is the spherical harmonic function of degree \( l \) and order \( m \), we have the equation for the radial function \( \psi_{\omega l}(r) \)

\[
\frac{d^2}{dx^2} \psi_{\omega l}(x) + \left[ \omega^2 - V_{\text{eff}} \right] \psi_{\omega l}(x) = 0,
\]

with the tortoise (Regge-Wheeler) coordinate \( x \)

\[
\frac{d}{dx} = f(r) \frac{d}{dr},
\]

and the effective potential related to the metric (5) for the angular momentum index \( l \)

\[
V_{\text{eff}} \equiv f(r) \left\{ \frac{l(l+1)}{r^2} + \frac{1}{r \sqrt{S}} \frac{d}{dr} \left[ f(r) \frac{d}{dr} \left( r \sqrt{S} \right) \right] \right\}.
\]

Some representative cases of the effective potential are plotted in Fig.1. For \( l = 0 \) part, which plays the major role in the low frequency absorption, one can see that the potential barrier may be lower than the Schwarzschild case for small conformal parameters \( L \) and \( N \), but as \( L \) or \( N \) gets larger, the height of the potential barrier increases, and will surpass the Schwarzschild barrier. Moreover, on the side of the potential barrier close to the horizon, there exists a potential well, whose depth also increases with the conformal parameter \( L \) and \( N \).

Since the effective potential is localized and vanishes both at the horizon \( (x \to -\infty) \) and in the regime far away from the horizon \( (x \to +\infty) \), we have the asymptotic solution to Eq.(14)

\[
\psi_{\omega l}(x) \simeq \begin{cases} 
\mathcal{T}_{\omega l} e^{-i\omega x} & (x \to -\infty), \\
\mathcal{R}_{\omega l} e^{i\omega x} + \mathcal{R}_{\omega l} e^{-i\omega x} & (x \to +\infty),
\end{cases}
\]

where the amplitude of the impinging wave has been normalized, and \( \mathcal{T}_{\omega l} \) and \( \mathcal{R}_{\omega l} \) are complex constants satisfying \( |\mathcal{T}_{\omega l}|^2 + |\mathcal{R}_{\omega l}|^2 = 1 \). In the intermediate region \(-\infty < x < +\infty\), one thus can match the two parts of Eq.(17) via Eq.(14) numerically.
FIG. 1: The dependence of the effective potential on the angular quantum number $l$ and the conformal parameters $L$ and $N$.
The partial and total absorption cross section are then given by

$$\sigma_{\text{abs}}^{(l)} = \frac{\pi}{\omega^2} (2l + 1) \left( 1 - |R_{\omega l}|^2 \right),$$  \hspace{1cm} (18)$$

and

$$\sigma_{\text{abs}} = \sum_{l=0}^{\infty} \sigma_{\text{abs}}^{(l)},$$  \hspace{1cm} (19)$$

respectively. In Figs. 2 and 3 we illustrate the partial \((l = 0)\) and total absorption cross sections of the nonsingular static spherical black holes in conformal gravity for various parameters \(L\) and \(N\). One can see that in the low frequency regime where \(l = 0\) part dominates, the absorption generally increases with the conformal parameters \(L\) and \(N\). This suggests that the net effect of the potential barrier and well shown in Fig.1 is generally stronger attraction for larger \(L\) or \(N\). Since the effective potential reflects the spacetime geometric, the dependence of the absorption on the conformal factor may be related to the conformally rescaled horizon, namely the ultimate absorbing surface. In fact, although the horizon still lies at \(r = r_s\) for any \(L\) and \(N\), the area of the horizon is no longer \(A_s = 4\pi r_s^2\). Instead, it should be rescaled by a conformal factor as the spatial slice of the metric is conformally transformed, i.e., \(A = S(r_s)A_s\). It can be checked that in the \(\omega \to 0\) limit, the absorption cross section tends to the rescaled area \(A\) of the horizon\cite{51–53}. Nonetheless, the crosses of the absorption lines in Figs. 2 and 3 embody the complicated wrestling of the higher potential barrier and deeper potential well seen in Fig.1.

In the high frequency regime \((\omega r_s \gg 1)\) where the wave length is much smaller than the size of the horizon, the absorption cross section should tend to the shadow area of the black hole\cite{54}. As mentioned in Sec.II, in the current cases the shadow area is identical for any choice of conformal parameters \(L\) and \(N\). It can be seen from Fig.3 that the absorption cross sections in different cases indeed tend to the same shadow area (the straight line) in high frequency limit, which is in concordance with the Schwarzschild shadow \(A_{\text{shadow}}\). Moreover, the oscillation of the absorption cross section with respect to the frequency \(\omega\) can be approximated by \(~ \text{sinc}(T_0\omega)\)\cite{54}, where \text{sinc}(x) is the sine cardinal \(\text{sin} x / x\), and \(T_0\) is the period for a massless particle to orbit on the photon sphere. Due to the identical classical null orbits, one can see from Fig.3 that the different cases demonstrate almost the same oscillation in the high frequency regime.
Partial absorption cross section for $l=0$ and $N=10$

Partial absorption cross section for $l=0$ and $L/r_s = 0.25$

**FIG. 2:** The partial absorption cross sections with $l = 0$ and various $N$ or $L$ for nonsingular static spherical black holes in conformal gravity.

### IV. SCATTERING CROSS SECTION

As for the scattering of the scalar wave by the black holes in conformal gravity, the scattering amplitude can be expressed as

$$g(\theta) = \frac{1}{2i\omega} \sum_{l=0}^{\infty} (2l + 1) \left[ e^{2i\delta_l(\omega)} - 1 \right] P_l(\cos \theta),$$  \hspace{1cm} (20)
FIG. 3: The total absorption cross sections with various $N$ and $L$ for nonsingular static spherical black holes in conformal gravity. The straight lines indicate the high frequency limit ($\omega r_s \to \infty$).

where

$$e^{2i\delta_l(\omega)} \equiv (-1)^{l+1} R_{\omega l}$$  \hspace{1cm} (21)
with $R_{\omega l}$ given in Eq.(17) indicates the phase shifts of the scattered waves and $P_l$ is the Legendre polynomial. The differential scattering cross section is given by

$$
\frac{d\sigma_{sc}}{d\Omega} = |g(\theta)|^2.
$$

(22)

Near the antipodal direction $\theta \approx \pi$, the scattering interference usually results in a bright spot. This is known as glory scattering, which can be approximated analytically by

$$
\frac{d\sigma_{g}}{d\Omega} \Bigg|_{\theta \approx \pi} \approx 2\pi \omega b_g^2 \left| \frac{db}{d\theta} \right|_{\theta = \pi} J_0^2 (\omega b_g \sin \theta).
$$

(23)

Here $b$ is the impact parameter and $b_g$ is the one that corresponds to the scattering angle $\theta = \pi$. $J_0$ is the Bessel function of the first kind with order 0.

In Fig.4 we plot the differential scattering cross sections of the nonsingular static spherical black holes in conformal gravity using Eq.(22), as well as the backward glory computed from Eq.(23). The width of the glory peak is prescribed by $b_g$ in the argument of the Bessel function in Eq.(23). For some black holes (see, e.g., Refs. [25, 28, 32, 37, 40]), the null orbits, and hence $b_g$, are altered by model parameters. Consequently, the widths of the glory peaks in these black holes are significantly different for various model parameters. However, as mentioned before, in current black hole of conformal gravity, the geometric optics are not different from the Schwarzschild spacetime, and the deflection equation $b(\theta)$ can still be described by Eq.(11). It follows that the widths of the glory peaks for various conformal parameters do not differ drastically like the other aforementioned black holes (see Fig.4).

The oscillatory behavior in the scattering cross section is known to come from wave orbiting. The peaks in the orbiting interference pattern are wider at smaller scattering angle. Since conformal transformation may rescale the arc length that the waves have traveled for a given scattering angle, the dependence of the width of peaks on the observing angle seems more significant for larger $L$ or $N$, namely the larger conformal factor $S(r)$. This effect also helps tell apart the actual scattering peaks and the glory approximation Eq.(23) of the Schwarzschild black hole in the observing angles slightly away from the antipodal direction. Moreover, the heights of the peaks in the scattering patterns are lower due to greater absorption (see Fig.3) for larger $L$ or $N$.

In Fig.5, we plot the differential scattering cross sections for $\omega r_s = 6.0$ and 12.0. Comparing the two graphs in Fig.5 and the upper panel of Fig.4, we can see that the difference of the oscillatory behavior in the scattering cross sections among the cases of different $L$ (Schwarzschild case corresponds to $L/r_s = 0$) becomes smaller as the frequency gets higher. This is because in the high

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1 In actual calculation, we have employed the method developed in Ref.[47] to improve the convergence.
FIG. 4: The differential scattering cross sections with $\omega r_s = 4.0$ and various $N$ and $L$ for nonsingular static spherical black holes in conformal gravity. The backward glory scattering is also plotted.

frequency regime, as the wave length becomes more negligible, the wave behaves more classically and can be better described by the same null orbit for different conformal factors.
In the present paper, we have looked into the absorption and scattering of impinging massless scalar waves by nonsingular static spherical black holes in conformal gravity. We have shown the

FIG. 5: The differential scattering cross sections with $\omega r_s = 6.0$ and 12.0 for nonsingular static spherical black holes in conformal gravity. For comparison, we have only plotted the scattering cross sections in the observing angle range $110^\circ \sim 180^\circ$ for the $\omega r_s = 12.0$ case.

V. CONCLUSIONS AND DISCUSSIONS

In the present paper, we have looked into the absorption and scattering of impinging massless scalar waves by nonsingular static spherical black holes in conformal gravity. We have shown the
absorption and scattering cross sections for typical choices of conformal parameters $L$ and $N$.

The classical orbits for massless particles are the same as those in the Schwarzschild spacetime. Therefore, when the wave length of the impinging wave is negligible compared to the horizon scale, it behaves a lot like in the Schwarzschild spacetime. This gives identical shadow areas of the nonsingular static spherical black holes.

Away from the high frequency limit, where the wave properties are somewhat significant, larger conformal parameters $L$ or $N$ means larger absorbing area at a given radius, which is the rescaled area $A$ of the horizon at the ultimate absorbing surface. This geometric effect on interaction between the impinging wave and the spacetime also incarnates itself in the effective potential, providing not only a higher barrier but also a deeper well for larger conformal parameter $L$ or $N$. The overall consequence is that the absorption cross section of the nonsingular static spherical black hole in conformal gravity generally increases with the conformal parameter $L$ and $N$. This difference in absorption for various conformal parameters is in concordance with the different heights of peaks in scattering cross section.

Conformal factor also leads to shift of the oscillatory peaks in the scattering cross section. In the literature concerning black hole scattering, the oscillatory pattern shifts are mainly due to the null geodesics altered by, e.g. the charge of the Reissner-Nordström black hole[28] or the Bardeen black hole[37], the coupling in Hořava-Lifshitz gravity[32], the tidal charge in the brane-world models[40], or even the angular momentum of the black hole if the wave impinges along the axial direction[25]. Hence in these cases the widths of the glory peaks are obviously different from the Schwarzschild black hole. However, this is not the case for the nonsingular static spherical black holes in conformal gravity. The conformal factor does not change the null orbits, but it rescales the distances the waves have traveled before they escape and interfere with each other. And hence the glory peaks are not significantly different in width like other aforementioned black holes. The shifts of peaks are perceivable only at the observing angle away from the antipodal direction.

With the rapidly growing observation and more accurate measurements of the interaction between black holes and all kind of waves, it is possible that the scattering and absorption patterns of black holes may be observed one day in the near future. Our study shows that if the spacetime is described by a conformal theory, the nonsingular black hole may demonstrate different signatures from the GR prediction, it may as well distinguish itself from other alternative gravitation theories. The absorption may increase and the scattering oscillation may shift, while the shadow and the glory peak remain close to the Schwarzschild case.

The investigation in this work may be extended to more complicated cases. In conformal gravity,
nonsingular rotating black holes have also been given in the literature [12, 14, 15], which may be a more realistic subject to consider absorption and scattering since black holes in reality are more likely to have spins. The interaction between waves and the Kerr black hole has been well described in GR[25, 35], and the conformally related case may also have unique characteristics. Moreover, the interactions between conformally nonsingular black holes and other kinds of waves, such as electromagnetic and gravitational waves, are worth further consideration as well. We will continue to study these issues in the future.

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