UNIFICATION OF COUPLINGS IN THE MSSM

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Abstract

I briefly summarize the main features related to the unification of gauge and Yukawa couplings in the MSSM, emphasizing the predictions derived in this framework, which are of special interest in the light of present and near–future experiments.

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The Minimal Supersymmetric extension of the Standard Model (MSSM) provides a solution to the hierarchy problem of the Standard Model (SM), related to the existence of quadratic divergences associated with the vacuum expectation value of the Higgs field which determines the fermion and gauge boson masses. Moreover, the MSSM can be derived as an effective theory in the framework of supersymmetric Grand Unified Theories (GUTs) [1], involving not only the strong and electroweak interactions but gravity as well. A strong indication for the realization of this physical picture in nature is the excellent agreement between the value of the weak mixing angle $\sin^2 \theta_W$ predicted by supersymmetric gauge coupling unification and the measured value [2]-[7]. In addition to the unification of gauge couplings, the unification of bottom and tau Yukawa couplings appears naturally in most minimal supersymmetric GUTs and it determines the value of the top Yukawa coupling at low energies, providing an explanation for the heaviness of the top quark mass [4, 8]-[11]. In the small to moderate $\tan \beta$ regime ($\tan \beta = v_2/v_1$, the ratio of the two Higgs vacuum expectation values), the condition of bottom-tau Yukawa unification implies a strong attraction of the top quark mass to its infrared fixed point, yielding a strong correlation between the top quark mass and $\tan \beta$.

The infrared fixed point solution has interesting implications for the Higgs, stop and chargino sectors of the theory [12]-[16]. In the large $\tan \beta$ regime, the phenomenology is more complex, mainly because of important supersymmetric threshold corrections to the bottom mass [17, 18]. Of special interest in the large $\tan \beta$ region are theories with a richer symmetry structure, where all three Yukawa couplings of the third generation unify, yielding a prediction for both $M_t$ and $\tan \beta$ [19].

Minimal gauge coupling unification at a scale $M_{GUT}$ of the order of $10^{16}$ GeV provides a prediction of one low energy gauge coupling as a function of the other two. One can then predict $\sin^2 \theta_W(M_Z)$ or, equivalently, consider as inputs the experimental values of $\alpha_{em}(M_Z)$ and $\sin^2 \theta_W(M_Z)$, and determine the value of the strong gauge coupling $\alpha_s(M_Z)$ to check if it is within the experimentally accepted range, say 0.11–0.13. The experimental prediction for $\sin^2 \theta_W(M_Z)$ in the modified $\overline{MS}$ scheme, in the limit of sufficiently heavy supersymmetric particles, can be given as a function of the electroweak parameters $G_F$, $M_Z$, $\alpha_{em}$ and the top quark mass; it also depends to a lesser extent on the Higgs mass [3, 6]:

$$\sin^2 \theta_W(M_Z) = 0.23166 + 5.4 \times 10^{-6}(m_h - 100) - 2.4 \times 10^{-8}(m_h - 100)^2$$
$$-3.03 \times 10^{-5}(M_t - 165) - 8.4 \times 10^{-8}(M_t - 165)^2 \pm 0.0003. \quad (1)$$

Performing the running of the gauge couplings from the GUT scale down to $M_Z$ by considering the beta functions up to two-loops, one has also to include properly the one-loop supersymmetric threshold corrections associated with the decoupling of the supersymmetric particles at intermediate scales between $M_{GUT}$ and $M_Z$. For each gauge coupling, the one-loop threshold correction to $1/\alpha_i(M_Z)$ reads

$$\frac{1}{\alpha_i^{thr.}} = \sum_{\eta, M_\eta > M_Z} \frac{b_i^\eta}{2\pi} \ln \left( \frac{M_\eta}{M_Z} \right), \quad (2)$$

where the summation is over all sparticles and heavy Higgs bosons with masses $M_\eta$ larger than $M_Z$, and $b_i^\eta$ is the contribution of each sparticle and heavy Higgs to the
one–loop beta function coefficient of the gauge coupling $\alpha_i$. A detailed analysis shows that the above effect of supersymmetric thresholds can be described in terms of one single scale $T_{\text{SUSY}}$ [3] which, considering different characteristic mass scales for squarks ($m_{\tilde{q}}$), gluinos ($m_{\tilde{g}}$), sleptons ($m_{\tilde{l}}$), electroweak gauginos ($m_{\tilde{W}}$), Higgsinos ($m_{\tilde{H}}$) and the heavy Higgs doublet ($m_H$), can be given as

$$T_{\text{SUSY}} = m_H \left( \frac{m_{\tilde{W}}}{m_{\tilde{g}}} \right)^{\frac{28}{19}} \left[ \left( \frac{m_H}{m_{\tilde{H}}} \right)^{\frac{3}{19}} \left( \frac{m_{\tilde{W}}}{m_{\tilde{H}}} \right)^{\frac{4}{19}} \left( \frac{m_{\tilde{l}}}{m_{\tilde{g}}} \right)^{\frac{3}{19}} \right] \tag{3}$$

The above relation holds whenever all the particles involved have masses above $M_Z$. If a mass is below $M_Z$ it should be replaced by $M_Z$ in the computation of the threshold corrections to $\alpha_s(M_Z)$. The value of the strong gauge coupling at low energies is then determined as a function of $\sin^2 \theta_W$, $\alpha_{\text{em}}$ and $T_{\text{SUSY}}$. It follows that

$$\frac{1}{\alpha_s(M_Z)} = \frac{1}{\alpha_{\text{sUSY}}^s(M_Z)} + \frac{19}{28\pi} \ln \left( \frac{T_{\text{SUSY}}}{M_Z} \right), \tag{4}$$

where $\alpha_{\text{sUSY}}^s(M_Z)$ would be the value of the strong gauge coupling at $M_Z$ if the theory were exactly supersymmetric down to the scale $M_Z$.

Observe that $T_{\text{SUSY}}$, Eq. (3), has only a slight dependence on the squark, slepton and heavy Higgs masses and a very strong dependence on the overall Higgsino mass as well as on the masses of the gauginos associated with the electroweak and strong interactions. In models with universal gaugino masses at the grand unification scale it follows that, $T_{\text{SUSY}} \simeq m_H \left( \alpha_2(M_Z)/\alpha_s(M_Z) \right)^{3/2} \simeq |\mu|/6$, where $\mu$ characterizes the Higgsino mass in the case of negligible mixing in the neutralino/chargino sector. Hence, if all supersymmetric masses are $\lesssim 1$ TeV, the effective supersymmetric scale $T_{\text{SUSY}}$ is $\lesssim$ the weak scale.

The unification condition implies the following numerical correlation [3, 7, 11],

$$\sin^2 \theta_W(M_Z) \simeq 0.2326 - 0.25 (\alpha_s(M_Z) - 0.123) \pm 0.0020 \tag{5}$$

where the central value corresponds to an effective supersymmetric threshold scale $T_{\text{SUSY}} = M_Z$ and the error is the estimated uncertainty in the prediction arising from a variation in $T_{\text{SUSY}}$ from 15 GeV to 1 TeV. Therefore, Eqs. (1) and (5) imply that the predictions from minimal gauge coupling unification agree with the experimental data provided that

$$\alpha_s(M_Z) = 0.1268 + 1.21 \times 10^{-4} (M_t - 165) + 3.36 \times 10^{-7} (M_t - 165)^2 - 2.16 \times 10^{-5} (m_h - 100) + 9.6 \times 10^{-8} (m_h - 100)^2 \pm 0.009. \tag{6}$$

For values of the top quark mass within the present experimental range, the above $M_t$–$\alpha_s$ correlation translates into acceptable predictions for $\alpha_s(M_Z)$, which have an important dependence on the range of the supersymmetric spectrum. Table 1 illustrates these results, including the two–loop effects of top and bottom Yukawa couplings in the running of $\alpha_s$.

The predicted value of $\alpha_s(M_Z)$ from unification may be further modified if some sparticle masses are $\mathcal{O}(M_Z)$. Indeed, not only the leading–log contributions but the full
Table 1: Gauge coupling unification predictions for $\alpha_s(M_Z)$, for given values of $\sin^2 \theta_W$ (correlated with $M_t$), $m_h = M_Z$ and $T_{SUSY} = 1$ TeV ($M_Z$)

| $M_t$[GeV] | $\sin^2 \theta_W(M_Z)$ | $\alpha_s(M_Z)$ |
|------------|------------------------|-----------------|
| 150        | 0.2321                 | 0.116 (0.125)   |
| 170        | 0.2315                 | 0.118 (0.127)   |
| 195        | 0.2306                 | 0.122 (0.131)   |

one–loop threshold contributions from SUSY loops should be included when extracting the couplings from the data [5]. The main additional effects come from light sfermions and are given by [5]-[7]

$$\frac{\delta \sin^2 \theta_W}{\sin^2 \theta_W} \simeq \frac{\cos^2 \theta_W}{\sin^2 \theta_W - \cos^2 \theta_W} \left( \frac{\delta \alpha_{em}}{\alpha_{em}} + \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} \right), \quad (7)$$

where $\Pi_{ij}$ are the vacuum polarization contributions to the gauge bosons. (The standard model contributions and the leading–log contributions of sparticles heavier than the $Z$ boson are not included in Eq. (7) since they were already taken into account previously.) Observe that, apart from a correction to the Z mass and a possible small correction to $\alpha_{em}$, the above expression is proportional to the parameter $\Delta \rho(0) = \Pi_{WW}(0)/M_W^2 - \Pi_{ZZ}(0)/M_Z^2$. In the MSSM it follows that $(\Delta \rho(0))^{SUSY} \geq 0$ and, after considering the additional terms in Eq. (7), one still obtains $\delta \sin^2 \theta_W \leq 0$ in most of the parameter space. This translates into an increase, with respect to the results from table 1, in the values of $\alpha_s(M_Z)$ predicted from supersymmetric grand unification, which may be important when the supersymmetric spectrum is sufficiently light. Larger values of $\alpha_s(M_Z)$ may however be in conflict with the data. (One should also keep in mind that large corrections to $\Delta \rho(0)$ are disfavoured by present experimental data, particularly for large values of the top quark mass $M_t \geq 175$ GeV.) High energy threshold corrections may be helpful in moderately lowering the $\alpha_s(M_Z)$ prediction. In fact, in the above I have considered the minimal gauge coupling unification scenario, that is, neglecting all possible GUT/M$_{Planck}$ scale threshold corrections in the running of the couplings. Perturbations to the unification relations are in general strongly model dependent, and they must be appropriately computed once a given high energy model is specified [20].

For the present experimental range of values for the top quark mass [21], $M_t = 180 \pm 12$ GeV, the condition of bottom-tau Yukawa coupling unification implies either low values of $\tan \beta$, $1 \leq \tan \beta \leq 3$, or very large values of $\tan \beta = O(m_t/m_b)$ [8]-[11]. Most interesting is the fact that to achieve b-τ unification, $h_b(M_{GUT}) = h_\tau(M_{GUT})$, etc.
large values of the top Yukawa coupling, $h_t$, at $M_{GUT}$ are necessary in order to compensate for the effects of the strong interaction renormalization in the running of the bottom Yukawa coupling. These large values of $h_t^2(M_{GUT})/4\pi \simeq 0.1$–1 are exactly those that ensure the attraction towards the infrared (IR) fixed point solution of the top quark mass $[1, 4, 10]$. In fact, the strength of the strong gauge coupling as well as the experimentally allowed range of values of the bottom mass play a decisive role in this behaviour $[11]$. For values of $\alpha_s(M_Z) \geq 0.115$, which are those preferred by the condition of minimal gauge coupling unification, the strong gauge coupling is sufficiently strong to demand a large value of the top Yukawa coupling to contravene its renormalization effect on the bottom mass $[4, 9, 10]$. A larger $M_b$, for instance, will be associated to a larger bottom Yukawa coupling allowing unification for the same values of $\alpha_s$ with a weaker top Yukawa coupling, relaxing hence the infrared fixed point attraction. In summary, in the low $\tan \beta$ case one obtains that for the presently allowed values of the electroweak parameters and the bottom mass, $b$-$\tau$ unification implies that the top quark mass must be within ten per cent of its infrared fixed point values. A mild relaxation of exact unification ($0.9 \leq h_b/h_t|_{M_{GUT}} \leq 1$) still preserves this feature, especially for $M_b \leq 4.95$ GeV $[22, 23]$. In the large $\tan \beta$ region, instead, one has $h_b = \mathcal{O}(h_t)$ and, hence, within the context of $b$-$\tau$ unification, the infrared fixed point attraction is much weaker than for low values of $\tan \beta$.

As mentioned above, the fixed point solution, $h_t = h_t^{IR}$, is obtained for large values of the top Yukawa coupling at the grand unification scale, which still remain in the perturbative regime. For $M_{GUT} \simeq 10^{16}$ GeV one obtains at low energies $(h_t^{IR})^2/4\pi \simeq (8/9)\alpha_s(M_Z)$, and the running top quark mass tends to its infrared fixed point value $m_t^{IR} = h_t^{IR} v \sin \beta$, with $v \simeq 174$ GeV. Hence, relating the running top quark mass $m_t$ with the pole top quark mass $M_t$ by considering the appropriate QCD corrections, for $\alpha_s(M_Z)$ in the range $0.11$–$0.13$ and small or moderate $\tan \beta$ values, one has $[13]$

\[
\begin{align*}
    m_t^{IR}(M_t) & \simeq 196 \text{GeV} \left[ 1 + 2 (\alpha_s(M_Z) - 0.12) \right] \sin \beta ; \\
    M_t^{IR} & = m_t^{IR}(M_t) \left[ 1 + \frac{4\alpha_s(M_Z)}{3\pi} + \mathcal{O}(\alpha_s^2) \right]. \quad (8)
\end{align*}
\]

The infrared fixed point structure also plays a decisive role in the evolution of the fundamental mass parameters of the theory, which has important implications on the Higgs and supersymmetric particle spectra $[12, 13, 14]$. The $M_t$-$\tan \beta$ relation, Eq. (8), associates to each value of $M_t$ the lowest possible value of $\tan \beta$ consistent with the validity of perturbation theory up to scales of order $M_{GUT}$. In addition, the infrared fixed point solution, $h_t \rightarrow h_t^{IR}$, induces an infrared fixed point value of the trilinear coupling $A_t$ $[14, 24]$ $[1, 4, 10]$. One has $A_t \simeq A_0 \left[ 1 - h_t^2/(h_t^{IR})^2 \right] - M_{1/2}[4 - 2h_t^2/(h_t^{IR})^2]$, with $A_0 = A_t(M_{GUT})$ and $M_{1/2}$ the common gaugino mass at $M_{GUT}$.

\footnote{1 Smaller values of $\alpha_s(M_Z)$, as could be obtained in the presence of large high energy threshold corrections in the running of the couplings, would change this picture.}

\footnote{2In Eq. (8), low energy SUSY threshold corrections, which may have a mild incidence in the definition of the infrared fixed point solution of the top quark mass, has been neglected $[23, 24]$.}

\footnote{3Nontrivial fixed points can also be present at high energies, leading to predictions for some supersymmetry breaking parameters at the GUT scale $[26]$.}
Hence, $A_{\text{IR}}^t \simeq -2M_{1/2}$ and this results in a very weak dependence of the whole spectrum on the parameter $A_0$. In addition, the combination of soft supersymmetry breaking mass parameters $M_{\text{QU}}^2 = m_Q^2 + m_U^2 + m_{H_2}^2$ has an IR fixed point behaviour as well: $M_{\text{IR}}^{M_{\text{QU}}^2} \simeq \sqrt{6.5}M_{1/2}$. Moreover, the condition of a proper electroweak symmetry breaking determines the supersymmetric mass parameter $\mu$ at the IR in terms of $\tan \beta$, $M_{1/2}$ and the common scalar mass $m_0$ at the unification scale. Therefore, in the case of universal mass parameters, $M_{1/2}$ and $m_0$ at $M_{\text{GUT}}$, for any given value of the top quark mass at the IR fixed point, the whole spectrum is basically determined in terms of only these two high–energy parameters. The effect of non-universal $m_0$ on the spectrum is, however, very interesting to study as well [13, 27].

The most interesting consequence of the IR fixed point $M_t$–$\tan \beta$ relation is associated with the lightest CP-even Higgs mass predictions in the MSSM [12, 14]. Indeed, for $\tan \beta$ larger than 1, the lowest tree level value of the lightest Higgs mass, $m_h$, is obtained at the lowest value of $\tan \beta$. Hence, since in any theory consistent with perturbative unification the IR fixed point solution assigns the lowest possible $\tan \beta$ value to each $M_t$, by the same token, the IR fixed point solution is associated with the lowest value of the tree level mass of the lightest Higgs boson consistent with the theory. Even after the inclusion of radiative corrections, the upper bound on the lightest Higgs mass is considerably reduced at the fixed point solution: considering $M_t = 160, 170, 180, 190$ GeV, for a characteristic supersymmetric mass scale of 1 TeV, the upper limit yields $m_h^{\text{IR}} \leq 80, 90, 110, 127$ GeV [28], which is considerably smaller than the upper bound one would obtain in the general MSSM framework, $m_h \leq 120, 125, 132, 140$ GeV [28], respectively. This is of particular interest for experimental searches. In fact, for $M_t \lesssim 175$ GeV, if the infrared fixed point top quark mass solution is realized in nature, the lightest CP-even Higgs mass must be within the reach of LEP2 for $\sqrt{s} = 192$ GeV [29].

The infrared fixed point solution is also experimentally appealing in relation to the sparticle spectrum. Considering the behaviour of the mass parameters of the theory it follows that light charginos, $m_{\tilde{\chi}^+} \lesssim 80$ GeV, and light right–handed stops, $m_{\tilde{t}} \lesssim 120$ GeV, may be present in the theory. This is of particular interest for direct experimental searches, and it can induce important positive corrections in $R_b = \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})$, which can ameliorate the present discrepancy between the experimentally measured value and the SM prediction for this quantity [14, 31, 32]. To obtain sizeable corrections to $R_b$, the lightest chargino must be mainly Higgsino like. Within the IR fixed point solution this necessarily demands to relax the universality condition for the scalar soft supersymmetry breaking masses at the unification scale [13].

In the context of $b$–$\tau$ Yukawa coupling unification it is clear that, possible large radiative corrections to the bottom mass are crucial in determining the top quark mass and $\tan \beta$ predictions. In general, one assumes that the top and bottom quarks couple each to only one of the Higgs doublets and hence $m_t(M_t) = h_t(M_t)v_2$ and $m_b(M_t) = h_b(M_t)v_1$, with $v_i$ the vacuum expectation value of the Higgs $H_i$. However, a coupling of the bottom (top) quark to the neutral component of the Higgs $H_{2(1)}$ may

\[ \text{Very large corrections to } R_b \text{ imply, however, that the value of } \alpha_s(M_Z) \text{ derived from experiments is shifted towards lower values, enhancing the necessity for large high-energy threshold corrections to the running of the gauge couplings in the framework of unification.} \]
be generated at the one-loop level, and, for large values of $\tan \beta \gtrsim 40$, since $v_2 \gg v_1$, large corrections to the bottom mass may be present \[17, 18, 32,\]

$$m_b = h_b v_1 + \Delta h_b v_2 \equiv \tilde{m}_b(1 + K \tan \beta).$$  \tag{9}$$

$\Delta m_b = K \tan \beta$ receives contributions from stop–chargino and sbottom–gluino loops, the latter being the dominant ones. The magnitude of $\Delta m_b$ is strongly dependent on the supersymmetric spectrum and its sign is generally governed by the overall sign of $\mu \times m_{\tilde{g}}$.

$$\Delta m_b = m_{\tilde{g}} \tan \beta \left[ \frac{2\alpha_s}{3\pi} I_1(m_{\tilde{q}_1}^2, m_{\tilde{q}_2}^2, m_{\tilde{g}}^2) + \frac{A_t}{m_{\tilde{g}} (4\pi)^2} I_2(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2, \mu^2) \right],$$  \tag{10}$$

where $m_{\tilde{q}_i}$ are the squark mass eigenstates and the integral factor is $I_i = K_i/a_{\text{max}}$, with $a_{\text{max}}$ the maximum of the squared masses and $K_i = 0.5 - 0.9$ depending on the mass splitting. Using the relation $m_{\tilde{g}} \simeq 2.6 - 2.8 M_{1/2}$ and the fact that from the renormalization group equations $A_t$ is in general of opposite sign and of $\mathcal{O}(M_{1/2})$, it follows that there is a partial cancellation between the two terms in Eq. (10). Although important, such partial cancellation is by far not sufficient to render the bottom mass corrections small. Hence, in the large $\tan \beta$ region, the bottom mass corrections need to be appropriately computed and their final effect on the predictions from $b$–$\tau$ Yukawa coupling unification will depend on the particular supersymmetric spectrum under consideration.

Large values of $\tan \beta$ are also interesting from the point of view of precision measurements \[30, 31, 33\]. Indeed, the fit to the measured value of $R_b$ can also be improved for large values of $\tan \beta \simeq m_t/m_b$, particularly if the CP-odd Higgs mass is below 70 GeV. This is due to the large one-loop positive corrections associated with the neutral CP-odd Higgs scalar sector of the theory, which become important when the supersymmetric bottom quark Yukawa coupling is enhanced. Experimentally, low values of the CP-odd Higgs mass, $m_A \lesssim m_Z$, are clearly very interesting from the point of view of Higgs searches, since they imply that both the lightest CP-even and the CP-odd Higgs masses will be within the reach of LEP2, $m_h \simeq m_A$. The charged Higgs mass is approximately determined through the CP-odd Higgs mass value, $m_{H^\pm}^2 \simeq m_A^2 + M_W^2$, as well, and hence, strong constraints can be obtained on the Higgs spectrum by considering the charged Higgs contributions to the branching ratio BR($b \to s\gamma$). Even taking into account in a very conservative way the QCD uncertainties associated with the BR($b \to s\gamma$) (assuming 40% QCD uncertainties), for $m_{H^\pm} \lesssim 130$ GeV the $b \to s\gamma$ decay rate becomes larger than the presently allowed experimental values \[34\], unless supersymmetric particle contributions suppress the charged Higgs enhancement of the decay rate. The most important supersymmetric contribution to this rare bottom decay mode comes from the chargino-stop one-loop diagram \[35\]. The chargino contribution to the $b \to s\gamma$ decay amplitude depends on the soft supersymmetry breaking mass parameter $A_t$ and on the supersymmetric mass parameter $\mu$, and for very large values of $\tan \beta$, it is given by \[36, 37\],

$$A_{\tilde{\chi}^0} \simeq \frac{m_{\tilde{\chi}}^2 A_t \mu}{m_{\tilde{\chi}}^2 m_{\tilde{\chi}}^2} \tan \beta G \left( \frac{m_{\tilde{\chi}}^2}{\mu^2} \right),$$  \tag{11}$$
where $G(x)$ is a function that takes values of order 1 when the characteristic stop mass $m_\tilde{t}$ is of order $\mu$ and grows for lower values of $\mu$. One can show that, for positive (negative) values of $A_t \times \mu$ the chargino contributions are of the same (opposite) sign as the charged Higgs ones [18]. Hence, to partially cancel the light charged Higgs contributions rendering the $b \to s\gamma$ decay rate acceptable, negative values for $A_t \times \mu$ are required. As follows from Eq. (10) and the discussion below, this requirement has direct implications on the corrections to the bottom mass and, after a detailed analysis, one concludes that it puts strong constraints on models with Yukawa coupling unification [18, 37]. Performing a $\chi^2$ fit to precision data, it follows that a light Higgs mass regime with unification of the three Yukawa couplings of the third generation at $M_{GUT}$ is possible but, the soft supersymmetry breaking parameters at high energies need to be highly non-universal. For moderate values of the soft supersymmetry breaking parameters and $\alpha_s(M_Z) \simeq 0.125, 0.120, 0.115$, the top quark mass is in the range $M_t > \sim 180, 170, 160$ GeV, respectively [18, 38].

In this talk I have considered the MSSM with three generations of quark and lepton superfields. The framework of unification of couplings can also be explored in models with four generations. Different fourth-generation low energy supersymmetric scenarios with a heavy [39] or a light ($M_t \simeq M_W$) [40] top quark have been studied. In these cases one obtains interesting predictions for some of the third- and fourth-generation particle masses, which can be tested at the Tevatron or at the next run of the LEP collider.

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