Tetra-Quark Interpretation of $X(3872)$ and $Z_c(3900)$ Revisited

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(Dated: July 11, 2018)

In relation to the newly observed bottom-strange $X(5568)^+$ mesons, we revisit our tetra-quark interpretation of $X(3872)$ and $Z_c(3900)$. It is discussed that our assignment of $X(5568)^+$ to charged components of iso-triplet bottom partners of $D_{s0}^+(2317)$ is compatible with the revised version of our tetra-quark interpretation of $X(3872)$ and $Z_c(3900)$.

After the discovery of $D_{s0}^+(2317)$ [1, 2], observations of many heavy mesons have been reported [3], and various interpretations of them have been proposed [4]. In addition, recently, charged (iso-triplet) bottom-strange mesons, $X(5568)^+$, have been observed [5], though not confirmed [6] yet, and then, they have been interpreted [7] as bottom partners of $D_{s0}^+(2317)$. In relation to the newly observed $X(5568)^+$, we revisit our tetra-quark interpretation of $X(3872)$ and its partners $Z_{s0}^{±0}(3900)$ [8] (or $X(3900)$ [3]) with an opposite charge conjugation ($C$) property.

Tetra-quark states are classified into the following four groups,

$$\{qq\bar{q}\bar{q}\} = \{[qq][\bar{q}\bar{q}] \oplus (qq)(\bar{q}\bar{q}) \oplus \{[qq][\bar{q}\bar{q}] \oplus (qq)[\bar{q}\bar{q}]\},$$

(1)
in the framework of $q = u, d, s$ [9] (and $c$ [10]), where parentheses and square brackets in the above equation imply symmetry and anti-symmetry, respectively, of flavor wavefunctions (wfs.) under exchange of flavors between them. Each term in the right-hand-side of Eq. (1) is again classified into two groups with $3_c \times 3_c$ and $6_c \times 6_c$ of the color $SU_c(3)$. Although a mixing between two states with $3_c \times 3_c$ and $6_c \times 6_c$ (which consist of quarks with common flavors and have the same quantum numbers) was considered at the scale of light meson mass $\langle \pi \rangle$, such a mixing is neglected at the the scale of heavy meson mass under consideration. Next, we take states with $3_c$, as the lower lying ones [10] and those with $6_c$, as the higher ones, because a force between two quarks is attractive when the two quark state is of $3_c$, $\{qq\}_{3_c}$, while repulsive when of $6_c$, $\{qq\}_{6_c}$. [11]. Regarding spin ($J$) of $[qq]$ and $(qq)$, its values are $J = 0$ and 1 for $[qq]_{3_c}$ and $(qq)_{3_c}$, and 1 and 0 for $[qq]_{6_c}$ and $(qq)_{6_c}$, respectively, in the flavor symmetry limit, for the reson that their wfs. should be totally anti-symmetric in the limit. However, one might worry about large breakings of the flavor $SU_f(3)$ and $SU_f(4)$ symmetries, as in meson masses. Nevertheless, such breakings are not necessarily serious in wfs., as seen below. A matrix element of flavor charge is given by a related form factor of vector current at zero momentum-transfer squared, and the form factor is normalized to be unity in the flavor symmetry limit. This implies that its deviation from unity provides a measure of flavor symmetry breaking under consideration. Their phenomenological and measured values have been summarized in [12] as follows. The form factor of strangeness-changing vector current taken between (|π⟩ and |K⟩ has been given by $f_{\pi K}^{\text{mK}}(0) = 0.961 \pm 0.008$ [13]. This implies that the flavor $SU_f(3)$ symmetry works well even in the world including charm mesons, and the $SU_f(4)$ symmetry breaking is not very serious in wfs. In this way, we take the above values of spin of [qq] and (qq) in the flavor symmetry limit [14]. As the result, spin and parity ($P$) of [qq][qq] and [(qq) + (qq)][qq] mesons are taken to be $J^P = 0^+$ and $1^+$, respectively, when they are of $3_c \times 3_c$, while $J^P = (0^+, 1^+, 1^+)$ and $1^+$, respectively, when of $6_c \times 6_c$. Here, it should be noted that axial-vector [qq][qq] states with $3_c \times 3_c$ disappear in the flavor symmetry limit, while our axial-vector [(qq) + (qq)][qq] states survive even in the limit. However, (qq)[qq] states are not considered in this short note, because existence of strange scalar mesons with the isospin $|I| = 3/2$ which can be given by $(nm)(sn)$ with $n = (u, d)$ [9] is not established yet [17].

In our earier works [18, 19], the flavorless axial-vector meson, $X(3872)$, was assigned to $X(+) \sim \{[\bar{c}n]_{3_c}^1[\bar{c}n]_{3_c}^2 + (cn)_{3_c}^1[\bar{c}n]_{3_c}^2\}_{|I|=0}$, $(n = u, d)$ with $3_c \times 3_c$ as the lowest iso-singlet hidden-charm axial-vector tetra-quark meson, where $1_c$ and $3_c$ mean the spin-singlet and spin-triplet, respectively. However, its measured mass seems to be too high (higher by about 1600 MeV than that of $D_{s0}^+(2317)$ which has been assigned to the scalar $F^+_1 \sim \{[\bar{c}n]_{3_c}^1[\bar{c}n]_{3_c}^1\}_{|I|=1}$, $(n = u, d)$ with the same $3_c \times 3_c$ [10]). In addition, existence of iso-triplet hidden-charm scalar mesons $\delta^c \sim \{[\bar{c}n]_{3_c}^1[\bar{c}n]_{3_c}^1\}_{|I|=1}$, $(n = u, d)$ with $3_c \times 3_c$, was predicted in our tetra-quark model [20], and their mass was very crudely estimated to be $m_{\delta^c} \simeq 3.3$ GeV by using a simple quark counting with the mass difference $\Delta_{cs} = m_c - m_s \simeq m_{n_c} - m_{p_c} \simeq 1.0$ GeV at the scale of charm meson mass and taking the mass $\simeq 2.3$ GeV of $D_{s0}^+(2317)$ as the input data, independently of the observation of an indication of $\eta^0$ peak around 3.2 GeV at the Belle [21]. (The $\eta^0$ peak is now named as $\delta^{0}(3200)$ for later convenience, and assigned to $\delta^{0}$.) Our quark counting seems to work, though still crude, because the predicted $m_{\delta^c} \simeq 3.3$ GeV reproduces considerably well the measured $m_{\delta^{0}(3200)} \sim 3.2$ GeV, as seen above. However, the mass of $\delta^{0}$ is much lower than that of $X(3872)$, in spite that their
Table I. Open- and hidden-charm scalar tetra-quark mesons and their flavor wavefunctions, where \( C \), \( S \) and \( |I| \) denote charm, strangeness and isospin quantum numbers, respectively. Their masses are estimated by using a quark counting, as discussed in the text. Tetra-quark states with \( * \) are of \( 6_c \times 6_c \), and \( J/\psi \) is written as \( \psi \). Notations of flavor wfs. whose overall normalization factors are dropped are explained in the text.

| \( C \) | \( S \) | \( |I| = 1 \) | \( |I| = 1/2 \) | \( |I| = 0 \) | Mass | Candidate or possible decay |
|---|---|---|---|---|---|---|
| 1 | 0 | \( \hat{F}_I \sim \{ [ cn ]_{6_c}^+ [ \bar{c} n ]_{5_c}^+ \}_{(3,3)}^1 \}_{|I| = 1} \) | \( \hat{F}_I \sim \{ [ cn ]_{6_c}^+ [ \bar{c} n ]_{6_c}^+ \}_{(3,3)}^1 \}_{|I| = 1} \) | \( \hat{F}_I \sim \{ [ cn ]_{6_c}^+ [ \bar{c} n ]_{5_c}^+ \}_{(3,3)}^1 \}_{|I| = 0} \) | \( \sim 2.3 \text{ GeV} \) (\( \dagger \)) | \( D_{s0}^+(2317) \) |
| | | \( \hat{D} \sim \{ [ cn ]_{6_c}^+ [ \bar{u} d ]_{5_c}^+ \}_{(3,3)}^1 \}_{|I| = 1} \) | \( \hat{D} \sim \{ [ cn ]_{6_c}^+ [ \bar{u} d ]_{5_c}^+ \}_{(3,3)}^1 \}_{|I| = 0} \) | \( \hat{D} \sim \{ [ cn ]_{6_c}^+ [ \bar{u} d ]_{5_c}^+ \}_{(3,3)}^1 \}_{|I| = 0} \) | \( \sim 2.7 \text{ GeV} \) | \( \hat{D} \) |
| -1 | 0 | \( \hat{E}^0 \sim \{ [ cn ]_{6_c}^+ [ \bar{u} d ]_{5_c}^+ \}_{(3,3)}^1 \}_{|I| = 1} \) | \( \hat{E}^0 \sim \{ [ cn ]_{6_c}^+ [ \bar{u} d ]_{5_c}^+ \}_{(3,3)}^1 \}_{|I| = 0} \) | \( \hat{E}^0 \sim \{ [ cn ]_{6_c}^+ [ \bar{u} d ]_{5_c}^+ \}_{(3,3)}^1 \}_{|I| = 0} \) | \( \sim 2.9 \text{ GeV} \) | weak decay |

(\( \dagger \)) Input data. (\( * \)) A tiny \( \eta \pi^0 \) peak observed in \( \gamma \gamma \) collisions at the Belle [21].

constituents have common flavors. This suggests that these two meson states have different structure with respect to the color degree of freedom, i.e., \( \delta^c \) with \( 3_c \times 3_c \) and \( X(3872) \) with \( 6_c \times 6_c \), and therefore, the mass difference (\( \Delta_{\text{color}} \)) between two states with \( 6_c \times 6_c \) and \( 3_c \times 3_c \), (whose constituent quarks have common flavors) is taken as \( \Delta_{\text{color}} \approx m_{X(3872)} - m_{6_c} \approx 0.6 \text{ GeV} \) in this note. (Mass differences arising from different structures of spin and flavor wfs. are suspected to be not very large at the scale of heavy meson mass.) Here, it should be noted that the estimated mass \( m_{6_c} \approx 3.3 \text{ GeV} \) of the lowest hidden-charm scalar meson in our model (and the measured \( m_{6_c} \sim 3.2 \text{ GeV} \) of its candidate) are much lower than the mass \( m_{X(3872)} \approx 3.7 \text{ GeV} \) of the lowest hidden-charm scalar meson in the diquark model [22] and in the unitarized chiral model [23]. Thus, the hidden-charm scalar meson might be a clue to select a realistic model of multi-quark mesons, and therefore, confirmation of \( \delta^c(3200) \) is awaited.

As discussed above, it is natural to consider that \( D_{s0}^+(2317) \) and \( \delta^c(3200) \) are of \( 3_c \times 3_c \), while \( X(3872) \) is of \( 6_c \times 6_c \). In addition, the mass of \( X(5568) \) has been reported as \( m_{X(5568)} = 5567.8 \pm 2.9^{+0.9}_{-1.9} \text{ MeV} \) [3]. If they truly exist and are charged components of iso-triplet bottom partners, \( X(5568)^{n,0} \sim \{ [ bn ]_{6_c}^+ [ \bar{c} n ]_{5_c}^+ \}_{(3,3)}^1 \}_{|I| = 0} \) (\( n = u, d \), of \( D_{s0}^+(2317) \) with \( 3_c \times 3_c \), it will be expected that the mass difference \( m_{X(5568)} - m_{D_{s0}^+(2317)} \) is not very far from \( (m_{6_c} - m_{n_c})/2 \) under the same quark counting as the above. Actually, their measured values \( m_{X(5568)} - m_{D_{s0}^+(2317)} \approx 3250 \text{ MeV} \) and \( m_{n_c} - m_{6_c}/2 \approx 3210 \text{ MeV} \), respectively, as expected. On the other hand, the mass of \( X(3872) \) with \( 6_c \times 6_c \) is much higher than that of \( \delta^c \) with \( 3_c \times 3_c \), as discussed above, and therefore, it is expected, from the same quark counting, that the mass difference \( m_{X(3872)} - m_{X(5568)} \) is much smaller than \( m_{6_c} \). In fact, the measured value \( m_{X(3872)} = 3.7 GeV \) of the mass difference \( m_{X(5568)} - m_{X(3872)} \) is much lower than \( m_{6_c} \). In this case, masses of \( \{ [ qq ]_{6_c}^3 [ \bar{q} q ]_{6_c}^3 [ \bar{q} q ]_{6_c}^3 \}_{(3,3,5)}^1 \) are expected to be high enough for their hadronic decay modes (which are allowed under the OZI-rule [24]) to be open. In contrast, masses of \( \{ [ qq ]_{3_c}^1 [ \bar{q} q ]_{3_c}^1 [ \bar{q} q ]_{3_c}^1 \}_{(3,3,5)}^1 \) will be much lower, so that most of them will decay through the electromagnetic interactions, except for some exceptional cases.
Open- and hidden-charm scalar mesons, $|cq\rangle_{1}^{+} [\bar{c}q\rangle_{1}^{+}$ and $|cq\rangle_{1}^{-} [\bar{c}q\rangle_{1}^{-}$, $(q = u, d, s)$, respectively, in our tetra-quark model have been studied in $^{10}$ and $^{26}$, where they were assumed to be of $3_{c} \times 6_{c}$. Therefore, we put an asterisk (*) on each symbol of tetra-quark mesons with $6_{c} \times 6_{c}$ to distinguish it from the corresponding one with $\bar{3}_{c} \times 3_{c}$, for example, $\bar{F}_{1}^{+} \sim \{\{cn\}_{3_{c}} \{\bar{s}n\}_{3_{c}}\}_{1_{c}}^{+} \{\bar{c}n\}_{6_{c}} \{\bar{c}n\}_{6_{c}}\}_{1_{c}}^{+}$ with $3_{c} \times 3_{c}$ and $\bar{F}_{1}^{+} \sim \{\{cn\}_{6_{c}} \{\bar{s}n\}_{6_{c}}\}_{1_{c}}^{+} \{\bar{c}n\}_{6_{c}} \{\bar{c}n\}_{6_{c}}\}_{1_{c}}^{+}$ with $6_{c} \times 6_{c}$, along with $^{3}$. We list this type of tetra-quark mesons in Table I, in which $D_{0}^{*}(2317)$ has been assigned to the iso-triplet $\bar{F}_{1}^{+}$, because it was observed in the $D_{s}^{+} \tau^{0}$ channel while no signal in the $D_{s}^{*+} \gamma$ channel. This fact means that its $D_{s}^{*+} \pi^{0}$ decay is much stronger than the radiative $D_{s}^{*+} \gamma$ as expected from the hierarchy of hadron interactions $^{25, 27}$. |isospin conserving hadronic int. | $\gg$ | electromagnetic interactions | $\gg$ | isospin non-conserving hadronic int. | . In contrast, if it were an iso-singlet state as in $^{3}$, it should decay dominantly through the electromagnetic interactions, because of the above hierarchy. In this case, it should be remembered that productions of the isosinglet $\bar{F}_{0}^{+}$ in $e^{+}e^{-}$ annihilations are expected to be suppressed in comparison with the iso-triplet $\bar{F}_{1}^{+}$ $^{27, 28}$. For these reasons, experiments should have detected $D_{0}^{*}(2317)$ in the $D_{s}^{*+} \gamma$ channel of $B$ decays. Nevertheless, it was discovered in the $D_{s}^{+} \pi^{0}$ channel in inclusive $e^{+}e^{-}$ annihilations $^{1}$ and $B$ decays $^{2}$, while no signal in the $D_{s}^{*+} \gamma$ channel. This implies that the assignment of $D_{0}^{*}(2317)$ to an iso-triplet state is quite natural. Possible decays of scalar tetra-quark mesons are tentatively listed in Table I, while only a part of them will be discussed later.

As to axial-vector mesons, we study only hidden-charm flavorless $[cq][\bar{c}q] \oplus [cq][\bar{c}q]$, $(q = u, d, s)$ mesons in this note, because $X(3872)$ and $Z_{c}(3900)$ have been observed. (The other members will be studied elsewhere.) Here, ideally mixed hidden-charm $[cq][\bar{c}q]$ and $(cq)[\bar{c}q]$ states belong to $60$- and $60$-plets, respectively, of $SU(4)$, and two flavorless states in $60$- and $60$-plets which consist of quarks with common flavors and have the same quantum numbers mix with each other to form $C$-parity eigenstates.

$$X_{I}^{(*)0}(\pm) = \frac{1}{\sqrt{2}} \{ X_{I}^{(*)0}(60) \pm X_{I}^{(*)0}(60) \} = \frac{1}{4} \left\{ \left( [uc]_{3c}(6c) \{\bar{u}c\}_{3c}(6c) \right) - \left( [dc]_{3c}(6c) \{\bar{d}c\}_{3c}(6c) \right) \right\}$$

$$X^{(*)}(\pm) = \frac{1}{2} \{ X^{(*)}(60) \pm X^{(*)}(60) \} = \frac{1}{4} \left\{ \left( [uc]_{3c}(6c) \{\bar{u}c\}_{3c}(6c) \right) + \left( [dc]_{3c}(6c) \{\bar{d}c\}_{3c}(6c) \right) \right\}$$

$$X^{s(*)}(\pm) = \frac{1}{2} \{ X^{s(*)}(60) \pm X^{s(*)}(60) \} = \frac{1}{2\sqrt{2}} \left\{ \left( [se]_{3c}(6c) \{\bar{s}c\}_{3c}(6c) \right) \pm \left( [se]_{3c}(6c) \{\bar{s}c\}_{3c}(6c) \right) \right\}$$

where an asterisk * has been put on each symbol of axial-vector states with $6_{c} \times 6_{c}$, as in the scalar mesons, and the arguments $\pm$ denote the $C$-parity eigenvalues. Although we assigned $X(3872)$ to $X(\pm)$ and studied its decay property in our earlier works $^{18, 19}$, we now revise the assignment, i.e., $X(3872) = X^{(*)}(\pm)$, as discussed before. In this case, the old assignment of $Z_{c}(3900)$ to $X_{I}(-)$ $^{20}$ also should be revised, i.e., $Z_{c}(3900) = X_{I}^{*-}$. When $m_{X^{(*)}} = m_{X}(3872) \simeq 3.9$ GeV is taken as the input data, the masses of $X_{I}^{(*)}$, $X^{(*)}$ and $X^{s(*)}$ are very crudely estimated as $m_{X_{I}^{(*)}} \simeq m_{X^{(*)}} \simeq m_{X^{(*)}} \simeq 3.9$ GeV and $m_{X^{s(*)}} \simeq 4.1$ GeV by using the same quark counting, where the mass difference $\Delta_{sn} = m_{s} - m_{n} \simeq m_{D^{*+}} - m_{D^{*+}} \simeq 0.1$ GeV at the scale of charm meson mass has been taken.

A gross approach to the properties of tetra-quark mesons will be seen by replacing each of them into a sum of products of $\{qq\}$ pairs, and then, replacing a colorless spin-singlet $\{qq\}_{c}^{S}$ by a pseudoscalar meson with the corresponding flavor and iso-spin quantum number and a spin-triplet $\{qq\}_{c}^{T}$ by a vector meson as in $^{3, 25}$, where contributions of products of colored $\{qq\}_{c}$ pairs will be dropped in contrast to $^{3}$, and $J/\psi$ will be written as $\psi$. We list a part of results on decompositions of tetra-quark states under consideration, i.e., hidden-charm axial-vector $X^{(*)}$, $X_{I}^{(*)}(\pm)$ and $X^{s(*)}$, and a hidden-charm scalar $\sigma^{cc*}$ with $6_{c} \times 6_{c}$ below. (As to decompositions of tetra-quark states with $3_{c} \times 3_{c}$, a part of them have been listed in our earlier works $^{18, 19, 23, 24, 30}$.)

1. Hidden-charm axial-vector tetra-quark mesons $X^{(*)}$, $X_{I}^{(*)}(\pm)$ and $X^{s(*)}$ with $6_{c} \times 6_{c}$:

$$X^{(*)} = \frac{1}{2\sqrt{6}} \left\{ 2(\bar{\psi}\omega - \omega\psi) - \left( [D^{0}\bar{D}^{*0} + D^{*0}\bar{D}^{0}) - (\bar{D}^{0}D^{*0} + D^{*0}\bar{D}^{0}) \right] \right\}$$

$$- \left[ (D^{+}\bar{D}^{*-} + D^{*-}\bar{D}^{+}) - (\bar{D}^{-}D^{*+} + D^{*-}\bar{D}^{*+}) \right] + \cdots$$

$$X^{*}(-) = \frac{1}{2\sqrt{3}} \left\{ - (\eta_{c}\omega - \omega\eta) - (\psi\eta_{0} - \eta\psi) \right\}$$

$$+ \left( [D^{0}\bar{D}^{*0} - \bar{D}^{*0}D^{0}) - (D^{*+}\bar{D}^{*-} - \bar{D}^{*+}D^{*-}) \right] + \cdots.$$


\( X^\pm_1(+) = \frac{1}{2\sqrt{6}} \left\{ 2(\psi^0 - \rho^0 \psi) - [(D^0 \bar{D}^{*0} + \bar{D}^* D^0) - (D^0 D^{*0} + \bar{D}^{*0} \bar{D}^0)] \\
+ [(D^+ \bar{D}^{*-} + D^{*-} D^-) - (D^- D^{*+} + \bar{D}^{*-} \bar{D}^+)] \right\} + \cdots, \quad (7) \\
X^0_1(-) = \frac{1}{2\sqrt{3}} \left\{ (\psi n_0 - \rho^0 n_c) - (\psi \pi^0 - \pi^0 \psi) \\
- [D^0 D^{*0} - \bar{D}^{*0} \bar{D}^0] - [D^+ \bar{D}^{*-} - D^{*-} D^-] \right\} + \cdots, \quad (8) \\
X^{*+} = \frac{1}{2\sqrt{3}} \left\{ \sqrt{2}(\psi \phi - \phi \psi) - [D_s^+ D_s^{*-} - D_s^{*-} D_s^-] - [D_s^+ D_s^{*-} - D_s^{*-} D_s^-] \right\} + \cdots, \quad (9) \\
X^{*-} = \frac{1}{2\sqrt{3}} \left\{ \sqrt{2}(\psi \phi - \phi \psi) - [\psi n_0 - n_c \psi] - [n_c \psi - \phi n_c] \right\} + \cdots, \quad (10) \\
\)

where \( n_0 \) and \( n_c \) have been given by \( n_0 = \eta_c \cos(\chi + \theta_p) + \eta' \sin(\chi + \theta_p) \) and \( n_c = -\eta_n \sin(\chi + \theta_p) + \eta' \cos(\chi + \theta_p) \) under the ordinary \( \eta \eta' \) mixing with the mixing angle \( \theta_p \) \( \frac{\pi}{2} \), and \( \chi \) satisfies \( \cos(\chi) = \sqrt{1/3} \) and \( \sin(\chi) = \sqrt{2/3} \).

2. \( \hat{\sigma}^{sce} \) as a typical example of \([qq][\bar{q}q]\) type of scalar tetra-quark mesons with \( 6_c \times 6_c \):

\( \hat{\sigma}^{sce} = \frac{1}{2\sqrt{6}} \left\{ \sqrt{3}(\eta n_0 + n_c \eta) - \sqrt{3}(D_s^+ D_s^{*-} + D_s^{*-} D_s^-) + (\psi \phi + \phi \psi) - (D_s^+ D_s^{*-} + D_s^{*-} D_s^-) \right\} + \cdots. \quad (11) \)

(Decompositions of the other members of \([qq][\bar{q}q]\) and \([qq][\bar{q}q]\) \( \oplus [qq][\bar{q}q] \) will be presented elsewhere.)

Although \( X(3872) \) is now assigned to \( X^+(+) \) with \( 6_c \times 6_c \), as discussed before, it is seen from Eq. (5) that its possible decay modes are not drastically changed and the confirmed decay modes of \( X(3872) \), i.e., the isospin conserving \( X(3872) \rightarrow DD^* \oplus DD^* \rightarrow DD_{c\bar{c}} \), \( X(3872) \rightarrow \psi \omega \rightarrow \psi \pi^+ \pi^- \pi^0 \), the radiative \( X(3872) \rightarrow \psi \omega \rightarrow \psi \pi^+ \pi^- \pi^0 \) (under the vector meson dominance, VMD \( [3] \)) and the isospin non-conserving \( X(3872) \rightarrow \psi \omega \rightarrow \psi \rho^0 \rightarrow \psi \pi^+ \pi^- \) (through the \( \omega \rho^0 \) mixing), are reproduced as in the old assignment \( [19] \), because their flavor wfs. are not changed, in contrast to their color and spin wfs. Regarding their rates, we do not study them at the present stage, because their experimental informations seem to be not sufficiently definite yet \( [3] \). (We need more definite informations for numerical analyses.) Here, it should be noted that a role of the \( \omega \rho^0 \) mixing in the isospin non-conserving \( X(3872) \rightarrow \psi \rho^0 \rightarrow \psi \pi^+ \pi^- \) decay \( [19] \) should not be neglected, because it plays essential roles in the observed \( \omega \rightarrow \pi^+ \pi^- \pi^0 \) decay \( [3] \) and in isospin non-conserving nuclear forces \( [22] \). Under the revised assignment \( X^+(+) = X(3872) \), Eq. (7) implies that its isospin triplet partner \( X^+(+) \) with the same \( C \)-parity has an isospin conserving \( X^+(+) \rightarrow \psi \rho \rightarrow \psi \pi \pi \) decay. Therefore, it is expected that its rate is very large and hence, its width is very broad, as in the old assignment \( [33] \), and hence, very high statistics will be needed to observe the iso-triplet partners of \( X(3872) \) with the same \( C \) property. Although the assignment of \( Z_{c\bar{c}}^{0/}\bar{c}(3900) \) as iso-triplet partners of \( X(3872) \) with opposite \( C \) property is also revised, i.e., \( Z_{c\bar{c}}^{0/}\bar{c}(3900) = X^{0/\pm}(+) \), their OZI-rule-allowed decay modes are not drastically changed again. On the other hand, \( X^+(+) \) and \( X^+(+) \) (which were previously assigned to \( X(3872) \) and \( Z_{c\bar{c}}^{0/}\bar{c}(3900) \)) are now expected to have approximately degenerate masses much lower (by \( \Delta_{c\bar{c}} \approx 0.6 \) GeV) than the above \( X^+(+) \) and \( X^+(+) \) (i.e., very crudely \( m_{X^+(+) \sim m_{X^+(+) \sim 3.3} \) GeV. As the result, \( X^+(+) \) cannot have any OZI-rule-allowed decay modes but will decay dominantly through the electromagnetic interactions, while \( X^{0/\pm}(+) \) might be able to decay exceptionally into \( \psi \pi^\pm \) final states, if their mass is truly higher than the \( \psi \pi \) threshold. (Although \( X^+(+) \) and \( X^+(+) \) in the above are not \( C \)-parity eigenstates, they are partners of the \( C \)-parity eigenstates \( X^0(+) \) and \( X^0(+) \), respectively, in each iso-triplet and satisfy \( C\{X^0(+) \} = \pm \{X^0(+) \} \) under the charge-conjugation.) As seen in Eq. (5), \( X^0(+) \) couples to \( \psi \pi^0 \) while it does not directly couple to \( (DD^*)^0 \) and \( (DD^*)^0 \) in our model, in contrast to a molecular model \( [5] \). Therefore, the \( DD^* \) peak, \( Z_{c\bar{c}}(3885) \), which has been observed at the BESIII \( [35] \) might not be identified with \( Z_{c\bar{c}}(3900) \), i.e., it might be some kind of kinematical effect like a coupled-channel cusp \( [30] \).

Now we return to scalar mesons listed in Table I. As was seen in \( [29] \), the isospin conserving \( D^0 \rightarrow D^+_c \pi^0 \) is the dominant decay of \( D^+_c \) which is assigned to \( D^0_{c\bar{c}}(2317) \) \( [10] \). Although this is allowed under the OZI rule, its rate can be small, because tetra-quark states have a variety of color and spin configurations, and as the result, the \( D^+_c \) \( D^-_c \) coupling is suppressed at the scale of heavy meson mass, as discussed in \( [23, 37] \). On the other hand, a dominant decay of its iso-singlet partner \( D^+_c \) will be \( D^+_c \rightarrow D^+_c \omega \rightarrow D^+_c \gamma \), when the VMD is accepted. Therefore, its detection in the \( D^+_c \gamma \) channel in \( B \) decays is awaited, because its production in inclusive \( e^+e^- \) annihilation will be suppressed as discussed before. In addition, it is expected that non-strange partners \( D \sim \{\psi n|\bar{d}d\} \) (\( n = u, d \)) of \( D^0_{c\bar{c}}(2317) \) are narrow because of the same reason as the narrow width of \( D^+_c = D^0_{c\bar{c}}(2317) \). Their mass \( \sim 2.2 \) GeV estimated by using the same quark counting as the above is lower by about 100 MeV than the mass \( m_{D^0_{c\bar{c}}(2317)} = 2318 \pm 29 \) MeV \( [2] \) of the conventional \( D^0_\psi \sim 3P_0 \{\bar{c}u\}, (n = u, d) \) mesons, and it is expected that \( D \) decays dominantly into the same \( D \pi \)
final states as the observed $D_0^*$. This implies that the tetra-quark $\hat{D}$ and the conventional $D_0^*$ co-exist in the observed broad $D\pi$ enhancement around 2.3 GeV and $D_0^*$ occupies its major part, because its production rate is much higher and its width is much broader ($\Gamma_{D_0^*} = 267 \pm 40$ MeV [3]) than $\hat{D}$. (For more details, see [10].) Therefore, $\hat{D}$ will be observed as a tiny peak on the lower tail of the broad $D\pi$ enhancement arising from $D_0^*$. The above argument might be compared with a recent discussion [35] on two-pole structure of $D_{s0}^*(2400)$. Returning to the charm-strange scalar sector, we expect existence of the conventional $^3P_0\{c\bar{s}\}$ scalar meson, $D_{s0}^{++}$ [10]. Its mass is estimated as $m_{D_{s0}^{++}} \approx 2.4$ GeV by using the same quark counting with $\Delta m \approx 0.1$ GeV as the above. This result, though still very crude, is not very far from our earlier estimate by the QCD sum rule [39], and it is high enough to decay into the $DK$ final state.

As to hidden-charm scalar mesons, $\hat{\sigma}^{\prime \circ}$ is interesting, because an indication of its candidate has been observed [21], as mentioned before. It will be narrow, for the same reason as the expected narrow widths of $D_{s0}^{++}(2317)$ and $\hat{D}$. In addition, it is suspected that the $\eta \pi^0$ peak will be tiny, because the $\eta \pi^0$ decay of $\hat{\sigma}^{\prime \circ}$ is suppressed because of the OZI rule. This result is compatible with the observation of a tiny $\eta \pi^0$ peak around 3.2 GeV at the Belle, mentioned before. On the other hand, open- and hidden-charm $[cq][\bar{q}q]$ and $[cq][\bar{c}q]$ with $6_c \times 6_c$ might not be narrow, because their masses will be high enough for various strong decay channels to be open.

In summary, we have studied a part of open- and hidden-charm scalar tetra-quark mesons in addition to hidden-charm axial-vector tetra-quark ones, assigning $D_{s0}^{++}(2317)$ to $[\{cn\}_{1_c}^{|[\bar{n}]_s|}]_{I}_{I=1}^{|c} + (cn)_{1_c}^{|[\bar{n}]_s|} |_{I=0}$ and $X(3872)$ to $[\{cn\}_{1_c}^{|(\bar{n})_s|}]_{I=0} + (cn)_{1_c}^{|(\bar{n})_s|} |_{I=0}$. In this way, the measured large mass difference between $X(3872)$ and $D_{s0}^{++}(2317)$ has been naturally understood, in relation to the newly observed $X(5568)^\pm$. In addition, it has been discussed that our assignment of $D_{s0}^{++}(2317)$ to an iso-triplet state is quite natural. This assignment seems to be implicitly supported by the observation of charged $X(5568)^\pm$ as its iso-triplet bottom partners, that is, existence of $X(5568)$ as the bottom partner of $D_{s0}^{++}(2317)$ is quite natural in our model. Therefore, confirmation of existence of $X(5568)$ is awaited. In addition, this assignment expects existence of its neutral and doubly charged partners, $\hat{F}_0^0 = D_{s0}^{0}(2317)$ and $\hat{F}_s^{++} = D_{s0}^{++}(2317)$, while a recent experiment did not observe any indication of them [40]. However, no signal of $D_{s0}^{++}(2317)$ in the $D_s^+\gamma$ channel and no indication of $D_{s0}^{++}(2317)$ and $D_s^{++}(2317)$ are a serious (model-independent) dilemma in the $D_{s0}(2317)$ physics [27]. Therefore, observation of $D_{s0}^{0}(2317)$ and $D_s^{++}(2317)$ and/or a $D_s^{+}\gamma$ peak around $m_{D_{s0}^{++}(2317)}$ is strongly desired. Regarding hidden-charm scalar mesons, an indication of a tiny $\eta \pi^0$ peak around 3.2 GeV (called as $\hat{\sigma}^{\prime \circ}(3200)$ in this note) also has been understood easily in the present model, in contrast to the other existing models [22, 23]. Therefore, it is awaited that existence of $\hat{\sigma}^{\prime \circ}(3200)$ is confirmed.

Assignment of $X(3872)$ has been revised, i.e., $X(3872) = X^* (+)$ with $6_c \times 6_c$ in this note. However, its decay property has not drastically been changed and the experimentally confirmed decay modes have been reproduced. Assignment of $Z_{c0}^{++}(3900)$ as the iso-triplet partners of $X(3872)$ with an opposite $C$ property also has been revised, i.e., $Z_{c0}(3900) = X^*_c(-)$, and it has been seen that $Z_{c0}(3900) = X^*_c(-)$ can decay into the $\psi \pi$, while it does not directly couple to $DD^*$ and $DD^*$ in our model. This might imply that the observed $DD^*$ peak, $Z_{c0}(3885)$, should not be identified with $Z_{c0}(3900)$ but is some kind of kinematical effect.

The hidden-charm and -strangeness scalar $\hat{\sigma}^{\prime \circ \circ}$ and axial-vector $X^{*+}(+) with 6_c \times 6_c$, are interesting in relation to recently observed $\psi \phi$ resonances [41], because each of them couples to a $\psi \phi$ state, as seen in Eqs. [9] and [11]. However, they will be studied elsewhere in future. Detailed analyses in scalar, axial-vector and tensor mesons, $[\{q\bar{q}][c\bar{q}]_{3_c}^{(\bar{q})} |_{I=0}^{3_c}]_{1_c}$, with $6_c \times 6_c$, and open-charm axial-vector mesons, $[\{c\bar{q}][c\bar{q}]_{3_c}^{(\bar{q})} |_{I=0}^{3_c}]_{1_c}$, with $3_c \times 3_c$, (and with $6_c \times 6_c$) are left intact as our future subjects.

Acknowledgments

The author would like to appreciate Professor T. Hyodo for discussions and comments, and Professor H. Kunitomo for careful reading of the manuscript. He also would like to thank the Yukawa Institute for Theoretical Physics, Kyoto University, where this work was motivated during the workshop, YITP-T-15-08, on “Exotic hadrons from high energy collision”.

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