Optimal dynamic control for a maglev vehicle moving on multi-span guideway girders

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ABSTRACT

An optimal control algorithm using a virtual tuned mass damper called virtual TMD to control the levitation force of a magnetic system is developed for resonance suppression of a maglev vehicle moving on multi-span guideway girders. Since the optimal dynamic parameters of a TMD in vibration control are well developed, the optimal tuning gains required to control the magnetic oscillations of the maglev bogie can be directly used and fed back to the maglev control system. To address the dynamic interaction analysis from the maglev vehicle to the guideway girders and vice versa, the entire coupling system is decomposed into two subsystems, one is the moving vehicle subsystem and another the stationary guideway subsystem. Then, an incremental–iterative procedure associated with the Newmark method is presented to solve the two sets of subsystem equations. Finally, the control effectiveness and parametric studies of the optimal virtual TMD scheme on resonance reduction of the moving maglev vehicle are demonstrated.

KEYWORDS: guideway, maglev dynamics, vibration control, virtual TMD

1. INTRODUCTION

Since the 1970s, Japan and Germany have been developing magnetic levitation (maglev) transport for their potential necessity of growing cities in the future [1]. In contrast to conventional guided ground transport of rolling wheel/track contact mode that needs ballasted sleepers or non-ballasted slab tracks with rail fastenings to stabilize or fix the rail gauge, the levitation and guidance modes of maglev transport have frictionless benefits of energy savings and low noise levels over the rail-contact travel besides speed. As a maglev vehicle travels over a flexible guideway girder, magnetic dynamics between the two coupling subsystems is of great interest in conducting vehicle–guideway interaction system of maglev transport [1–9]. Over the past two decades, numerous engineering researchers have devoted themselves to interaction dynamics of maglev vehicle/guideway systems. Cai and Chen [1] have reviewed various aspects of dynamic characteristics, magnetic suspensions, vehicle stability and suspension controls of maglev vehicle/guideway system. Considering magnetic dynamics, Zheng et al. [2, 3] developed two types of vehicle/guideway coupling models with controllable magnetic suspension systems to observe the dynamic phenomena of divergence, flutter and collision for a maglev vehicle traveling on a flexible guideway. Zhao and Zhai [4] simulated a TR06 carriage as a 10-degree-of-freedom (10-DOF) multi-body vehicle to study the stochastic vibration of a maglev vehicle traveling on elevated guideway girders. Focusing on magnetic controls, such as LQR, PID and PD algorithms, Yau [5–7] proposed a series of simplified vehicle models to investigate the interaction responses of the maglev vehicle/guideway system considering various vibration scenarios that include earthquakes, wind actions or support settlements. Concerning the vibration reduction of a flexible guideway girder, Zhou et al. [8] introduced a tuned mass damper (TMD) scheme onto the girder as a feed-forward compensator to suppress the coupled resonance of stationary maglev vehicle–bridge. Wang et al. [9] presented a framework for dynamic coupling analysis of a high-speed maglev train moving on a series of curved viaducts. Their studies indicated that in addition to track radius, cant deficiency would be another key issue to affect the structural safety of the viaduct. However, to the authors’ knowledge, relatively little research attention so far seems to have been devoted to the resonant phenomenon of a maglev vehicle running on multi-span guideway girders.

As a vehicle moves on a series of equal-span (L) bridges at constant speed v, it may be excited by the bridges during its travel. When any of the frequencies (f v) of the vehicle coincides with the exciting frequency (v/L) caused by the vibrating bridges, bridge-induced resonance will occur on the vehicle [5, 22]. To reduce the guideway-induced resonance on a moving maglev vehicle, an idea of virtual TMD scheme [10] is adopted to regulate the magnetic forces in the maglev system. By decomposing the entire maglev vehicle/guideway system into two separated...
subsystems, an incremental–iterative procedure \[11\] associated with the Newmark method is presented to carry out nonlinear dynamic analysis of the maglev vehicle/guideway system. From the present studies, the control effectiveness of the proposed virtual TMD scheme in reducing resonant vibration of a maglev vehicle moving on multi-span guideways will be verified.

2. THEORETICAL FORMULATION

As shown in the schematic model in Fig. 1, a test maglev vehicle modeled by a 2-DOF system is traveling over a flexible guideway girder. For illustration, the following symbols are used: \(c\) = damping coefficient; \(E_l\) = flexural rigidity; \(G_r\) = magnetic force to lift up the vehicle; \(h\) = air gap; \(c_r\) = secondary damping; \(k_r\) = secondary spring stiffness; \(L\) = span length; \(u,(x, t)\) = beam deflection, \((M_v, m_b)\) = masses of the maglev vehicle and magnetic wheel, and \((u_v, u_b)\) = displacements of the vehicle and magnetic wheel. In addition, the notations of \((m, c, k)\) with the subscript “t” represent the tuning parameters of mass, damping and stiffness of a virtual TMD that is virtually mounted on the magnetic bogie for levitation control, as shown in Fig. 1. Considering the dominant vibration of planar maglev vehicle/guideway coupling system, only the vertical motion of the dynamic system is of concern. For analytical formulation, the following assumptions are considered \[5–7, 11\]:

1. The maglev vehicle is idealized as a 2-DOF system with a magnetic bogie supported by a magnetic force (see Fig. 1).
2. The guideway girder is modeled as a linear elastic Bernoulli–Euler beam with uniform section.
3. Allowable levitation gap between the magnetic wheel and guideway rails is of free contact.
4. The time lag (delay) in the maglev control system is neglected.

2.1 Motion-dependent nature of magnetic forces

In this study, the electromagnetic suspension mode is selected to suspend the vehicle by a magnetic force from electromagnet \[12\], as shown in Fig. 2. It shows a schematic model of an electromagnet suspended by a magnetic force \(G_z(i, h)\). Here, \((i, h)\) denote electric current and air gap, respectively. For small fluctuations of air gap and induced current around the desired levitation gap and control current of \((i_0, h_0)\), that is \(|\Delta i/i_0| < 1\) and \(|\Delta h/h_0| < 1\), the magnetic force \(G_z(i, h)\) and control voltage \(V\) can be approximated as \[13–16\]

\[
G_z(i, h) = \kappa_0 \left( \frac{i(t)}{h(t)} \right)^2 \approx G_0 + 2G_0 \left[ \frac{\Delta i(t)}{i_0} - \frac{\Delta h(t)}{h_0} \right],
\]

\[
V = L_0 \frac{d(i/h)}{dt} + R_0 i = V_0 + \Delta V,
\]

\[
\Delta V \approx 2G_{z0} \frac{h_0}{i_0} \frac{d}{dt} \left( \frac{\Delta i}{i_0} - \frac{\Delta h}{h_0} \right) + V_0 \Delta i/i_0,
\]

where \(G_0 = \kappa_0 (i_0/h_0)^2\) is the static magnetic force to lift up the magnet, \(L_0 = 2G_0(i_0/h_0)^2\) is the magnet inductance, \(\kappa_0\) is the electromechanical coupling factor \[15\], \(R_0\) is the electric resistance, \(h = h_0 + \Delta h\) is the air gap, \(i = i_0 + \Delta i\) is the control current, \(\Delta i\) is the deviation of induced current and \(\Delta h\) is the levitation increment of air gap.

2.2 Equations of the maglev vehicle/guideway system

Considering the linear magnetic force and control voltage shown in Section 2.1, the equations of motion and control voltage for the 2-DOF maglev vehicle moving on a beam (see Fig. 1) are given by \[5–7\]

\[
\ddot{m}_u + c \dot{u} + E_1 u'''' = -G_z(i, h) \times \delta (x - vt),
\]

\[
\begin{bmatrix}
M_v & m_b & u_v \\
-1 & c_v & -c_v & 0 \\
0 & -k_v & -k_v & 0 \\
0 & -k_r & -k_r + S_b & 0 \\
\end{bmatrix}
\begin{bmatrix}
\dot{u}_v \\
\dot{u}_b \\
\dot{u}_v \\
\dot{u}_b \\
\end{bmatrix}
\begin{bmatrix}
\dot{u}_v \\
\dot{u}_b \\
\dot{u}_v \\
\dot{u}_b \\
\end{bmatrix}
\begin{bmatrix}
0 \\
F_v \\
0 \\
\end{bmatrix},
\]

\[
\Delta V = 2G_{z0} \frac{h_0}{i_0} \left[ \frac{d(i)}{i_0 dt} - \frac{d(\Delta h)}{h_0 dt} \right] + V_0 \frac{\Delta i}{i_0},
\]

where

\[
S_b = \frac{2G_{z0}}{h_0},
\]

\[
F_v = \frac{-2G_{z0}}{h_0} (u_v(v, t) - r(v, t))
\]
and \((\bullet)’ = \partial(\bullet)/\partial x, (\bullet) = \partial(\bullet)/\partial t\) and \(\delta(\bullet)\) is the Dirac’s delta function. The levitation gap \(h(t)\) at the maglev vehicle moving to the position \(x = vt\) of the guideway girder is

\[
h(t) = h_0 + \Delta h,
\]

\[
\Delta h = u_c(t) - u_c(vt, t) + r(vt)
\]

and \(r(x)\) is the vertical irregularity of guide rail. As indicated in Eq. (4), the tuning force of \(2G_{z0}(\Delta i/\Delta t)\) represents the induced control gains to regulate the magnetic force \(G_z(i, h)\). It is possible to design a compensator to provide a tuning gain for the maglev system that can be predicted by the feedback force from a virtual TMD. In the following, the virtual TMD-based control algorithm to regulate the magnetic force will be implemented.

### 2.3 Virtual TMD and tuning forces

TMD is an efficient tool to control mechanical vibrations [17]. Thus, the control force of the TMD can be regarded as a tuning input or dynamic absorber to control the oscillation of the dynamic system. In this study, a virtual tuning algorithm that can control the magnetic force is proposed. Let us suppose a virtual TMD \((m_v)\) is mounted on the maglev bogie, as shown in the bogie mass \(m_b\) of Fig. 1. Then, the vehicle equations shown in Eq. (4), associated with the virtual TMD are rewritten as

\[
\begin{bmatrix}
M_v & m_b \\
m_b & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_c \\
\dot{u}_b
\end{bmatrix}
+ \begin{bmatrix}
c_v & -c_v \\
-c_v & c_v
\end{bmatrix}
\begin{bmatrix}
\dot{u}_c \\
\dot{u}_b
\end{bmatrix}
+ \begin{bmatrix}
k_v & -k_v \\
-k_v & k_v + S_v
\end{bmatrix}
\begin{bmatrix}
\dot{u}_c \\
\dot{u}_b
\end{bmatrix}
= \begin{bmatrix}
0 \\
F_v
\end{bmatrix}
- \begin{bmatrix}
0 \\
m_v \ddot{u}_c
\end{bmatrix}
\]

\[
(\Delta i) = -\frac{m_v \ddot{u}_c}{2G_{z0}}.
\]

Here, the subscript “des.” represents the desired parameters using Eq. (9) to determine the corresponding control current and voltage by the following procedure.

### 3. DETERMINATION OF OPTIMAL CONTROL VOLTAGE

Using the desired current shown in Eq. (9), the instantaneous desired voltage equation of the magnet at current time \(t + \Delta t\) of Eq. (5) is rewritten as

\[
\Delta V_{\text{des}} = V_0 \left(\frac{\Delta i}{l_0}\right)_{\text{des}} + 2G_{z0} \frac{\dot{h}_0}{l_0} \left(\frac{\Delta i + \Delta \dot{i}}{l_0 \times \Delta t}\right)_{\text{des}} - \frac{(\Delta h)_{\text{des}}}{h_0}
\]

Combing the vehicle equations of Eq. (4) with the optimal control voltage equation of Eq. (10) yields the following matrix equation of motion for the maglev vehicle associated with the virtual TMD scheme:

\[
\begin{bmatrix}
M_v & m_b \\
m_b & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_c \\
\dot{u}_b
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_v & -\varepsilon_v \\
-\varepsilon_v & \varepsilon_v
\end{bmatrix}
\begin{bmatrix}
\dot{u}_c \\
\dot{u}_b
\end{bmatrix}
+ \begin{bmatrix}
k_v & -k_v \\
-k_v & k_v + S_v
\end{bmatrix}
\begin{bmatrix}
\dot{u}_c \\
\dot{u}_b
\end{bmatrix}
= \begin{bmatrix}
0 \\
F_v
\end{bmatrix}
- \begin{bmatrix}
0 \\
2G_{z0} \ddot{u}_c(vt, t)
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}
+ \begin{bmatrix}
0 \\
\Delta V_{\text{des}}
\end{bmatrix}.
\]

Once the desired control voltage shown in Eq. (10) is obtained (from the tuning force of the virtual TMD), the dynamic response of the maglev vehicle moving on a guideway girder can be computed and controlled using Eq. (11), in which the desired control voltage \((\Delta V_{\text{des}})\) was obtained from Eq. (10) to tune the magnetic force acting on the maglev bogie of the moving vehicle. A computational flowchart with virtual TMD control algorithm has been shown in Fig. 3, the control block diagram of which indicates that the unbalance forces (gains) between the internal system and external excitation are adopted as feedback gains (control forces) to adjust the tuning gains of the maglev system.

### 4. SOLUTION FOR THE BEAM–VEHICLE EQUATIONS

The dynamic deflection \(u_c(x, t)\) of a simple beam can be expressed as \(u_c(x, t) = \sum_{n=1} q_n(t) \sin(n\pi x/L)\). Here, \(q_n(t)\) means the generalized coordinate associated with the nth vibration mode. By the Galerkin method [18], the generalized equation of motion for the nth mode system becomes

\[
\ddot{q}_n + 2\xi \omega_n \dot{q}_n + \omega_n^2 q_n = \frac{-2mL}{g} G_z(i, h) \sin \frac{n\pi vt}{L},
\]

where \(\xi\) is the modal damping ratio and \(\omega_n = (n\pi/L)^2 \sqrt{EI/m}\) is the nth natural frequency. Considering the motion-dependent nature of magnetic forces, as the coupled equations shown in Eqs (11) and (12), an iterative method is adopted to solve the dynamic equations of the maglev vehicle/guideway system. The incremental–iterative analysis involves three phases: predictor, corrector and equilibrium checking [11]. Figure 3 shows the present computational flowchart to carry out nonlinear interaction dynamic analysis of the maglev vehicle/guideway system, in which the control gains produced by the virtual TMD scheme have been included. Details concerning the incremental–iterative procedure for nonlinear dynamic analysis of maglev vehicle/guideway interaction are available in [11]. In this regard, Newmark’s \(\beta\) method [19, 20] is first employed to discretize the generalized equations of motion in Eqs (11) and (12) into two equivalent incremental stiffness equations. Then, the incremental–iterative procedure is carried out to compute the response of the maglev vehicle–guideway system for each time step.
5. GUIDEWAY IRREGULARITY

To account for the random nature and characteristics of guide rail irregularity in practice [20, 21], the following power spectrum density function [20] is given to simulate the vertical profile of track geometry variations:

\[
S(\omega) = \frac{A_v \Omega_e^3}{(\Omega^2 + \Omega_e^2)(\Omega^2 + \Omega_c^2)}, \tag{13a}
\]

\[
r(x) = \sqrt{2(\Delta \Omega)} \sum_{n=1}^{N} \left[ \sqrt{S(\Omega_n)} \cos(2\Omega_n \pi x + \psi_n) \right], \tag{13b}
\]

where \(\Omega\) represents the spatial frequency, \(\Omega_e (=0.8246\text{ rad/m})\) and \(\Omega_c (=0.0206\text{ rad/m})\) are the relevant constants depending on irregularity type, \(A_v (=1.08 \times 10^{-6}\text{ m}^2\text{ rad/m})\) relies on the irregularity type and rail quality, \(\psi_n\) is a random variable uniformly distributed between 0 and \(2\pi\), and \(\Delta \Omega\) is the spatial frequency increment. Figure 4 shows the vertical profile of track irregularity for the simulation of rail geometry variations in this study.

6. ILLUSTRATIVE EXAMPLES

For theoretical demonstration, let the 2-DOF vehicle model cross over six-span simple beams at constant speed \(v\), as indicated in Fig. 1. Tables 1 and 2 show the properties of the guideway girder and maglev vehicle studied herein. In the following examples, the Newmark integration method of constant average acceleration, i.e. with \(\beta = 0.25\) and \(\gamma = 0.5\), will be employed to compute the dynamic response of the maglev vehicle/guideway system for a time step of 1 ms. Then, the control effectiveness of the proposed virtual TMD scheme on tuning the magnetic force of a moving maglev vehicle on multi-span guideway girders will be conducted.

6.1 Suppression of resonance

As a vehicle moves on a series of simple beams with identical span length \(L\) at constant speed \(v\), the running vehicle
may experience repetitive excitations transmitted from the beam oscillations with an exciting frequency \( v/L \). Once the exciting frequency coincides with one of the vehicle frequencies \( (f_r) \), resonance may take place on the running vehicle, i.e. \( v/L = f_r \) [11, 22]. Thus, the corresponding primary resonant speed is denoted as \( v_{r1} = f_rL \). In general, the primary resonant speed is much higher than the operating speed of ground guided transport. Thus, the second sub-resonant response of a maglev vehicle traveling over a series of guideway girders will be adopted in this example, i.e. \( v_{r2} = f_rL/2 \) [22]. Let us neglect the irregularity of guide rail, that is \( r(x) = 0 \). With the sub-resonant speed of \( (v_{r2}) \) listed in Table 2, the time-history responses of vertical acceleration for the magnetic bogie \( (M_b) \) and vehicle \( (M_v) \) have been plotted in Fig. 5, respectively. As indicated, the response of the uncontrolled bogie tends to amplify as time increases, which is indicative of the occurrence of sub-resonance. Moreover, one can observe two cyclic oscillations from the response as the magnetic bogie passes through a span of the multi-unit guideway girders.

To suppress the resonant responses, let us adopt the virtual TMD scheme proposed in Sections 2 and 3, and consider the virtual mass ratio \( \mu (=m_b/m_v = 0.1) \). The Den Hartog optimal tuning parameters [23] of optimal damping ratio \( (\xi_{opt} = 18.5\%) \) and target frequency ratio \( (\beta_{opt} = f_r/f_v = 0.91) \) are applied to the virtual TMD scheme, as shown in Table 3. Then, the corresponding time-history acceleration responses are also depicted in Fig. 5. As indicated, the control performance of the virtual TMD is developed to reduce vibration of the maglev bogie. It means that the motion of the beam excited is situated in anti-phase vibration with respect to the oscillation of the moving bogie so that the resonant response of the vehicle is mitigated. On the other hand, the response reduction of the lumped mass \( M_v \) is insignificant due to soft secondary spring \( k_v \), as listed in Table 2. Considering tuning gains of the moving maglev vehicle, the control performance of the present virtual TMD scheme to reduce the vehicle’s response will be further investigated in the following examples.

### 6.2 Control effectiveness of the virtual TMD scheme

In the following examples, the same maglev vehicle model adopted in the previous example is considered. Figure 6 illustrates the maximum acceleration \( (a_{v,max}) \) of the maglev vehicle with respect to the speed \( (v) \) from 100 to 400 km/h. The plot of the maximum vehicle’s acceleration \( (a_{v,max}) \) versus speed \( (v) \) is denoted as the \( a_{v,max}–v \) plot. As can be seen, one noticeable resonant peak appears at the sub-resonant speed of 280 km/h \((=f_rL/2)\) on the \( a_{v,max}–v \) plot. In addition, there exist other minor sub-resonant peaks on the \( a_{v,max}–v \) plot as well. To suppress these sub-resonant peaks, the proposed virtual TMD scheme is applied. The numerical results have been depicted in the \( a_{v,max}–v \) plots of Fig. 6 as well. The numerical results demonstrate the present virtual TMD scheme is effective to suppress the resonant peaks in the \( a_{v,max}–v \) plots.

In addition to response analysis conducted previously, Fig. 7 illustrates the maximum \((h_{max})\) and minimum \((h_{min})\) ratios of air gaps to the static levitation gap \((h_0)\) with respect to speed \( (v) \) for the magnetic bogie controlled/uncontrolled by the virtual TMD scheme. As indicated, if the magnetic bogie was controlled...
by the virtual TMD, its maximum and minimum air gaps (see blue lines) will be located within the feasible operating range of \(0.5 < h_{\text{min}} / h_0 < 1, 1 < h_{\text{max}} / h_0 < 1.5\) so that the suspended bogie can work to run on guided rails, but the uncontrolled maglev bogie fails at the sub-resonant speed of 280 km/h. For this, the response of the uncontrolled bogie is out of control and the nonlinear magnetic force with respect to the electric current \((i)\) and air gap \((h)\) shown in Eq. (1) should be taken into account in re-evaluating the magnetic force. Thus, the present virtual TMD scheme can develop an effective tuning gain to stabilize and control the resonant vibration of the maglev bogie.

To examine the control performance of the present virtual TMD scheme on tuning the magnetic force, different virtual mass ratios of \((0.1, 0.2, 0.3)\) are considered in the following studies. The corresponding optimal damping ratio \((\xi_{\text{opt}})\) and target frequency ratio \((\beta_{\text{opt}})\) are listed in Table 3. The peak responses of magnetic force, acceleration and air gap of the maglev vehicle are plotted in Figs 8–10, respectively. As can be seen in Fig. 8, more virtual mass ratio can offer more control efforts to the maglev system so that more suppression on the vibration and levitation gap of the maglev bogie can be achieved, as shown in Figs 9 and 10.

7. CLOSING REMARKS

In this study, an alternative control strategy of virtual TMD scheme is proposed to suppress the resonance of a maglev vehicle running on multi-span guideway girders. To perform the nonlinear dynamic response analysis of the maglev vehicle/guideway coupling system, an incremental–iterative procedure associated with the Newmark method is presented by decomposing the vehicle/guideway coupling system into two subsystems, that is the moving subsystem of the maglev vehicle and the supporting subsystem of the guideway girders. From the present studies, several conclusions are reached as follows:

1. The present virtual TMD scheme provides an alternative control approach to reduce the multiple resonant peaks of a moving maglev vehicle.
2. The control effectiveness of the virtual TMD scheme is verified to reduce the resonance of a maglev vehicle moving on multi-span guideway girders.
3. Once the dynamic response of the maglev bogie in resonance is out of control, the corresponding nonlinear magnetic force in terms of the electric current and air gap should be taken into account in evaluating the magnetic force to lift up the magnetic bogie.
4. Although larger virtual mass ratio of the virtual TMD scheme can achieve better control effectiveness on vibration suppression and levitation band of the moving maglev bogie, more tuning gains to control the maglev system would be required.
To conduct the magnetic vibration control analytically, the maglev vehicle model is idealized as a 2-DOF system for preliminary study of resonance suppression using the proposed virtual TMD scheme in this paper. A more realistic and sophisticated model of a multibody maglev vehicle considering electromagnetic suspension magnets can be carried out in future studies.

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REFERENCES
1. Cai Y, Chen SS. Dynamic characteristics of magnetically-levitated vehicle systems. Applied Mechanics Reviews 1997;50:647–670.
2. Zheng XJ, Wu JJ, Zhou YH. Numerical analyses on dynamic control of five-degree-of-freedom maglev vehicle moving on flexible guideways. Journal of Sound and Vibration 2000;235:43–61.
3. Zheng XJ, Wu JJ, Zhou YH. Effect of spring non-linearity on dynamic stability of a controlled maglev vehicle and its guideway system. Journal of Sound and Vibration 2005;279:201–215.
4. Zhao CF, Zhai WM. Maglev vehicle guideway vertical random response and ride quality. Vehicle System Dynamics 2002;38(3):185–210.
5. Yau JD. Response of a maglev vehicle moving on a series of guideways with differential settlement. Journal of Sound and Vibration 2009;324:816–831.
6. Yau JD. Interaction response of maglev masses moving on a suspended beam shaken by horizontal ground motion. Journal of Sound and Vibration 2010;329(2):171–188.
7. Yau JD. Response of a maglev vehicle moving on a two-span flexible guideway. Journal of Mechanics 2010;26(1):95–103.
8. Zhou D, Li J, Hansen CH. Suppression of the stationary maglev vehicle–bridge coupled resonance using a tuned mass damper. Journal of Vibration and Control 2012;19(2):191–203.
9. Wang ZL, Xu YL, Li GQ, Chen SW, Zhang XL. Dynamic analysis of a coupled system of high-speed maglev train and curved viaduct. International Journal of Structural Stability and Dynamics 2018;18(11):1850143.
10. Wu ST, Shao YJ. Adaptive vibration control using a virtual-vibration-absorber controller. Journal of Sound and Vibration 2007;305:891–903.
11. Yang YB, Yau JD. An iterative interacting method for dynamic analysis of the maglev train–guideway/foundation-soil system. Engineering Structures 2011;33:1013–1024.
12. Zhang Z, She L, Zhang L, Shang C, Chang W. Structural optimal design of a permanent-electromagnetic suspension magnet for middle-low-speed maglev trains. IET Electric System Transport 2011;1(2):61–68.
13. Meisenholder SG, Wang TC. Dynamic Analysis of an Electromagnetic Suspension System for a Suspended Vehicle System. FRA-RT-73-1. Redondo Beach, CA: TRW System Group, 1972.
14. Nagurka ML, Wang SK. A superconducting maglev vehicle/guideway system with preview control. ASME Journal of Dynamic System, Measurement and Control 1997;119:638–649.
15. Sinha PK. Electromagnetic Suspension, Dynamics and Control. London, UK: Peter Peregrinus Ltd, 1987.
16. Trumper DL, Olson SM, Subrahmanyan PK. Linearizing control of magnetic suspension systems. IEEE Transaction of Control System Technique 1997;5(4):427–438.
17. Warburton GB. Optimum absorber parameters for minimizing vibration response. Earthquake Engineering and Structural Dynamics 1981;9:251–262.
18. Fryba L. Vibration of Solids and Structures under Moving Loads, 3rd edn. London, UK: Thomas Telford, 1999.
19. Newmark NM. A method of computation for structural dynamics. ASCE Journal of Engineering Mechanics Division 1959;85:67–94.
20. Yang YB, Yau JD, Wu YS. Vehicle–Bridge Interaction Dynamics. Singapore: World Scientific, 2004.
21. Shi J, Wei Q, Zhao Y. Analysis of dynamic response of the high-speed EMS maglev vehicle–guideway coupling system with random irregularity. Vehicle System Dynamics 2007;45:1077–1095.
22. Yang YB, Yau JD. Vertical and pitching resonance of train cars moving over a series of simple beams. Journal of Sound and Vibration 2015;337:135–149.
23. Den Hartog JP. Mechanical Vibrations, 4th edn. New York: McGraw-Hill, 1956.