Exact and Asymptotic solution of a steady two dimensional boundary layer of a Micropolar fluid flow past a moving wedge

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Abstract. In this paper, we propose an analytic solution of a boundary value problem which models a steady, laminar, two dimensional, boundary layer flow of an incompressible and viscous micropolar fluid over a moving wedge. The governing non-linear partial differential equations are converted into highly non-linear ordinary differential equations using similarity transformations. An analytical exact solution obtained for particular values of parameters are then extended to obtain an exact solution for more general values of the parameters involved. We also propose asymptotic solution of the Micropolar boundary layer flow. The results thus obtained are compared with those of direct numerical solutions, which show a good agreement. The results are discussed in terms of velocity profiles and wall shear stresses for various physical parameters.

Keywords: Laminar Boundary layer, Similarity transformations, Exact solution, Asymptotic solution

1. Introduction

In the past few decades, the interest in non-Newtonian fluids such as paints, polymers, physiological fluids, liquid crystals, gel propellants and colloidal suspensions, has increased rapidly and tremendously, mainly due to its association with applied sciences and its indispensable role not only in theory but also in its many industrial applications such as aerodynamics, the extrusion of polymer and plastic sheets, the cooling of metallic plate, continuous stretching of plastic films etc. As the physical behavior of these fluids falls outside the coverage of classical field theories described in Yuan[25], classical continuum approach remains inadequate to explain the complex mechanical behavior of such fluid due to the presence of particles called microelements which exhibits micromotion and therefore led to the development of microcontinuum theories. Significant among them is the micropolar fluid theory of Eringen[6]. Micropolar fluid is a subclass of simple microfluids as explained by Eringen in [7], in which the effects of deformation of the fluid microelements are negligible but microrotational effects are prominent. The micropolar fluid theory constitutes an important branch of non-Newtonian fluid dynamics and is also considered to be one of the well established theories for fluids with microstructure. Micropolar theory includes the effects arising due to micromotion of the particles. The theory provides a mathematical model for accurately simulating the flow characteristics of the aforesaid non-Newtonian fluids.
The complexities in solving field equations are well known. The equations for motion in the boundary layer therefore had to be simplified. The theory thus developed is known as Prandtl’s boundary layer theory which is primarily based on Prandtl’s concept, that the effect of viscosity is predominant only in a very thin layer close to the wall. The Navier-Stokes equations are simplified to a great extent to a mathematically amenable form, called boundary layer equations.

The boundary layer theory helps in improving the design of airfoils, wings, wind tunnel contours, supersonic nozzles, control surfaces etc. This process of simplification was later applied to micropolar fluid flows. The concept of boundary layer in micropolar fluids was studied by Peddieson and McNitt [2] prior to Willson [3]. Further studies of the boundary layers under different conditions include [8-12] in which authors have dealt with the physical models numerically. Rees and Bassom [12] studied Blasius boundary layer of micropolar fluid flow past a flat plate. Kim [26] has analyzed boundary layer of micropolar fluid flow past a fixed wedge with constant surface temperature whereas Kim and Kim [11] has studied the model with constant surface heat flux. Ishak et al. [10] improvised and extended work of Kim and Kim [11] by studying the effect of variable magnetic field with constant surface heat flux for a fixed wedge. Following Kim [26] and Kim and Kim [11], the author also employed similarity transformations to convert partial differential equations into nonlinear ordinary differential equations and also Keller-Box method to solve the system numerically. In the present paper, attempt has been made to give an exact analytical and also asymptotic solution to the boundary layer equation in the micropolar fluid flow past a moving wedge [28].

2. Formulation:

Under usual boundary layer approximation [13] the governing equations for the steady, two-dimensional, laminar, incompressible, viscous, micropolar fluid flow past a wedge moving with a constant velocity \( U_w(x) \) in the non-dimensional form with the absence of body forces and body couples, are

Conservation of mass:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

Conservation of momentum:

\[
\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = U \frac{\partial U}{\partial x} + (\mu + \chi) \frac{\partial^2 u}{\partial y^2} + \chi \frac{\partial w}{\partial y}
\]  

Conservation of angular momentum:

\[
\rho j \left( u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} \right) = -\frac{\partial}{\partial y} \left( v \frac{\partial w}{\partial y} \right) - k \left( 2w + \frac{\partial u}{\partial y} \right)
\]  

Conservation of micro-inertia:

\[
u \frac{\partial s}{\partial x} + v \frac{\partial s}{\partial y} = 0
\]  

where \( u \) is the velocity component in the \( x \) direction, taken along the surface of the wedge and velocity component \( v \) in the \( y \) direction perpendicular to it. \( w \) is the non-dimensional component of gyration vector normal to the \( xy \)-plane and assigning the
variable relation between \( w \) and the surface shear stress\([1]\), boundary conditions on velocity and microrotation are

\[
at \quad y = 0 \quad u = U_w(x), \quad v = 0, \quad w = -\frac{1}{2} \frac{\partial u}{\partial y} \quad (5)
\]

\[
as \quad \frac{y}{\delta} \to \infty \quad u \to U, \quad v \to 0, \quad w \to 0 \quad (6)
\]

The free stream velocity \( U \) is given by the power law \( U = U_\infty x^m \) with \( U_\infty \) and \( m \) as constants. The stretching surface velocity \( U_w(x) \) satisfies the power-law relation \( U_w(x) = U_0 w x^m \). \( \nu \) denotes the kinematic viscosity of the fluid, \( \rho \) is the fluid density, \( \mu \) is the viscosity, \( \gamma \) is given by \( \gamma = (\mu + \chi/2) s \), where \( s \) is the microinertia density considered as \( s(x,y) \) \([4]\) and not as a constant as regarded by many recent authors (Rees and Bassom\([12]\)). Introducing the stream function \( \psi(x,y) \) with \( u = \frac{\partial \psi}{\partial y} \) and \( v = -\frac{\partial \psi}{\partial x} \) and adopting the coordinate transformations from the variables \((x,y)\) to the new dimensionless similarity variables\((9)\), the boundary layer equations transform to the following non-linear ordinary differential equations

\[
(1 + k) f''' + 2m f'' + \frac{2m}{m+1} (1 - f'^2) + k h' = 0 \quad (7)
\]

\[
(1 + \frac{k}{2}) (ih')' + i \left( fh' - \frac{3m-1}{m+1} hf' \right) - k (2h + f'') = 0 \quad (8)
\]

\[
(1 - m) if' - \frac{m+1}{2} fi' = 0 \quad (9)
\]

under the boundary conditions

\[
 f(0) = 0, \quad f'(0) = -\lambda, \quad i(0) = 0, \quad h(0) = -\frac{1}{2} f''(0) \quad (10)
\]

\[
f'(\infty) \to 1, \quad h(\infty) \to 0 \quad (11)
\]

where \( \eta \) is a new similarity variable, \( f(\eta) \) is the non-dimensional stream-function and \( \lambda(= -\frac{U_w}{U_\infty}) \) is the ratio of free stream velocity and boundary velocity. \( \lambda < 0 \) corresponds to wedge moving in the direction of stream velocity whereas and \( \lambda > 0 \) corresponds to that of the opposite direction, \( k = \frac{\mu}{\nu} \) where \( \mu = \rho \gamma \), is the dimensionless viscosity ratio\([22]\). The solution of micro inertia density equation\((9)\) satisfying boundary conditions\((10)\) is

\[
i = Af^{\frac{2(1-m)}{m+1}} \quad (12)
\]

where \( A = C^{\frac{2}{m+1}} \) is a non dimensional constant of integration.

Using the boundary condition \( i(0) = 0 \) leads to \( i(\eta) = 0 \) which is a trivial solution, in which case \((8)\) reduces to

\[
h = -\frac{1}{2} f'' \quad (13)
\]

substituting \((12)\) and \((13)\) in \((7)\) we get

\[
\left(1 + \frac{k}{2}\right) f''' + ff'' + \frac{2m}{m+1} (1 - f'^2) = 0 \quad (14)
\]

In this paper, we obtain the solution analytically, asymptotically and numerically of this equation with the boundary conditions

\[
f(0) = 0, \quad f'(0) = -\lambda \quad f'(\infty) \to 1 \quad (15)
\]
where primes denote differentiation with respect to $\eta$. and the stream wise pressure gradient is favorable pressure gradient when $m > 0$ and adverse pressure gradient when $m < 0$ whereas Blasius flow over a flat plate when $m = 0$[12]. The flow corresponding to stagnation point when $m = 1$[8].

3. Analytical solution:

Inspired by the work of Kudenatti([18],[19]) on solving a boundary value problem for the Falkner- skan equation[5] analytically, which is strikingly close to single hump solution([21],[24]) which is a similarity solution of the Burger’s equation, we seek exact solution[16] of (14) with (15). Exact Solution of (14) for $m = -\frac{1}{3}$[23] is obtained by integrating (14) twice and applying the boundary conditions (15) which results in a Riccati type equation and leads to the solution of (14) as

$$f(\eta) = \eta + \delta - \frac{\delta e^{-(1+\frac{k}{2})^{-1}(\frac{\eta^2}{2}+\delta \eta)}}{1 - \frac{\delta(1+\frac{k}{2})^{-\frac{1}{2}}}{2} e^{\frac{\eta^2}{2(1+\frac{k}{2})}} \sqrt{\frac{\pi}{2}} \left(\text{erf}\left(\frac{(1+\frac{k}{2})^{-\frac{1}{2}}(\eta+\delta)}{\sqrt{2}}\right) - \text{erf}\left(\frac{(1+\frac{k}{2})^{-\frac{1}{2}} \delta}{\sqrt{2}}\right)\right)}$$

(16)

provided $\delta^2 = -\frac{2(1+\lambda)}{(1+\frac{k}{2})}$.

To obtain an exact analytical solution of the system (14) with (15) for different values of $m$ and $k$, we rewrite the solution (16) as

$$f(\eta) = \eta + \delta - \frac{\delta G(\eta)}{G(\eta)}$$

(17)

where

$$G(\eta) = e^{\frac{\delta^2}{2}(1+\frac{k}{2})} - \frac{\delta}{2} \left(1 + \frac{k}{2}\right)^{-\frac{1}{2}} \sqrt{\frac{\pi}{2}} \left(\text{erf}\left(\frac{(1+\frac{k}{2})^{-\frac{1}{2}}(\eta+\delta)}{\sqrt{2}}\right) - \text{erf}\left(\frac{(1+\frac{k}{2})^{-\frac{1}{2}} \delta}{\sqrt{2}}\right)\right)$$

(18)

Substituting (17) into (14) and (15), we get

$$\left(1 + \frac{k}{2}\right) (G^2 G'' - 6GG'G'' + 6G'^3) + (\eta + \delta)G^2G'' - 2(\eta + \delta)GG'^2$$

$$- \frac{4m}{m+1} G^2 G' + \delta \left(\frac{2}{m+1}\right) G'^2 - \delta GG'' = 0$$

(19)

with the boundary conditions

$$G(0) = 1, \quad G'(0) = \delta \frac{2}{1}, \quad G'(\infty) = 0.$$

(20)

The solution of (19) for $m = -\frac{1}{3}$ subject to (20) is given by (16).

The error and exponential functions in equation (18) are entire functions with infinite radius of convergence about $\eta = 0$ and therefore can be expanded using Taylor series.
Further the solution (17) which is in series representation\cite{20} for \(m = -\frac{1}{3}\) plays an important role in further analysis for general values of \(m\). Thus we let
\[
G(\eta) = \sum_{n=0}^{\infty} a_n \eta^n
\]
for general \(m\) and \(k\). Substituting (21) into (19) and equating the coefficients of \(\eta^n\) to zero we get the coefficients \(a_n\) and in general
\[
a_{n+3} = \frac{-1}{(1 + \frac{k}{2})(n+3)(n+2)(n+1)} (-1 + \frac{k}{2}) \delta \sum_{i=0}^{n} (i+1)((i+2)a_{n-i}a_{i+2}
\]
\[
+ ( -\frac{2}{m+1})(n-i+1)a_{n-i+1}\delta_{i+1})
\]
\[
+ \sum_{j=0}^{n-1} \sum_{i=0}^{n-j} (1 + \frac{k}{2})(j+3)(j+2)(j+1)a_{n-j-i}a_{i}a_{j+3}
\]
\[
+ \sum_{i=0}^{n-1-j} (j+1)((j+2)a_{i}a_{j+2} - 2(i+1)a_{i+1}a_{j+1})a_{n-1-j-i}
\]
\[
+ \sum_{j=0}^{n} \sum_{i=0}^{n-j} (j+1)(-6(1 + \frac{k}{2})(j+2)(i+1)a_{n-j-i}a_{i+1}a_{j+2}
\]
\[
+ 6(1 + \frac{k}{2})(i+1)(n-j-i+1)a_{n-j-i}a_{i+1}a_{j+1}
\]
\[
+ (1 + \frac{k}{2})\delta(j+2)a_{n-j-i}a_{i+1}a_{j+2}
\]
\[
- 2(1 + \frac{k}{2})\delta(i+1)a_{n-j-i}a_{i+1}a_{j+1} - \frac{4m}{m+1}a_{n-j-i}a_{i+1}a_{j+1})
\]
(22)
where \(n = 1, 2, 3, \ldots\) and the coefficients \(a_n\) have been expressed in terms of \(a_2, \delta, k, m\).

The value of coefficient of skin friction \(a_2\) that satisfies the derivative boundary condition at far away from the wall has to be determined. This is same as determining the value of either \(a_2\) of series (21) or \(f''(0)\) of the system (14) and (15) as they are intrinsically related to each other by the following
\[
a_2 = \frac{f''(0) + (1 + \frac{k}{2})\delta}{2(1 + \frac{k}{2})}\]
(23)

The coefficients \(a_n\) consists of two arbitrary constants, namely \(f''(0)\) and \(\delta\). For \(m = -\frac{1}{3}\), we match the series (21) with the exact solution (16) which gives \(\delta = \sqrt{-\frac{2(1+N)}{(1+\frac{k}{2})}}\). This constant \(\delta\) plays an important role in this analysis. The solution of equation (14) exists only when the expression under the square root in \(\delta\) is positive.

The other constant \(f''(0)\) or \(a_2\) needs to be determined. Thus, (14 - 15) have infinite solutions in the form of (21). The constant \(f''(0)\) is determined in the following manner. We integrate (14) from \(\eta = 0\) to \(\eta = \infty\) and use (15) to get
\[
\int_{0}^{\infty} \left( f' - f'^2 + \frac{2m}{m+1}(1 - f'^2) \right) d\eta = f''(0).
\]
(24)
Since skin friction $f''(0)$ appears on both sides of (23) and (24) it has to be determined iteratively using an appropriate initial approximations for it, taken from the known exact solution (16), (20) and (24) for all values of $k$, $m$ and $\lambda$. $f''(0)$ converges when the derivative condition at far distance in (15) is satisfied (Kudenatti et al 2013). It is known that the series behaves well for small values of $\eta$ enabling its integration. So Pade’s approximants are used for the summation of the series. With an initial approximation of $f''(0)$ and a fewer iterations, $f''(0)$ can be obtained up to desired accuracy without any difficulty by numerically integrating the integral relation. Thus we obtain an exact solution of the equation for all the values of $m$ and $k$. To prove the robustness of the method the values of skin friction $f''(0)$ obtained analytically are compared with that of direct numerical solution of the equation (14) with boundary condition (15) obtained using Keller Box method (Cebeci[14]), based on finite difference. It is observed that results agree well with the Numerical solution for all the values of parameters.

4. Asymptotic solution:

We analyze the far-field behavior of (14) asymptotically for which we study large $\eta$ behavior i.e. $|f'(\eta)| \ll 1$ as $\eta \to \infty$ because the derivative boundary condition $f'(\eta)$ becomes linear as $\eta$ increases away from zero. This helps us to define a new function

$$f(\eta) \sim \eta + E(\eta)$$

(25)

where $E(\eta)$ and their derivatives are assumed to be small. Substituting (25) with $f'(\eta) = 1 + E'(\eta) = 1 + F(\eta), f''(\eta) = E''(\eta) = F'(\eta)$ and $f'''(\eta) = E'''(\eta) = F''(\eta)$ into (14) with the boundary conditions (15) and linearizing the resulting ordinary differential equation, we get

$$(1 + \frac{k}{2})F'' + \eta F' - 2\beta F = 0$$

(26)

and boundary conditions take the form

$$F(0) = -(1 + \lambda), \quad F(\infty) = 0$$

(27)

whose solution eventually results in, Kummer’s equation [27] with solution involving confluent hypergeometric series[26]. Thus the solution to (26) is given by

$$F(\eta) = (1 + \lambda) \left( -M \left( \frac{2m}{m + 1} \frac{1}{2}, -\frac{1}{2(1 + \frac{k}{2})} \eta^2 \right) + \sqrt{2} \frac{\Gamma(1 + \frac{2m}{m+1})}{\Gamma(\frac{1}{2} + \frac{2m}{m+1})} \eta M \left( \frac{1}{2} - \frac{2m}{m + 1} \frac{3}{2}, -\frac{1}{2(1 + \frac{k}{2})} \eta^2 \right) \right)$$

(28)

The solution in terms of $f(\eta)$ is given by

$$f'(\eta) = 1 + E'(\eta) = 1 + F(\eta)$$

(29)

5. RESULTS AND DISCUSSIONS

The graphs displayed in figure 1 are obtained using the exact method and the velocity profile suggest that velocity decreases as the micopolar parameter $k$ increases satisfying
the boundary condition at infinity, for fixed parameters \( m \) and \( \lambda \). Further the thickness of the boundary layer decreases with the increase of the pressure gradient at the wall (\( \eta = 0 \)) and hence increases the skin friction \( f''(0) \). Figure 1 also shows the angular velocity profile \( h(\eta) \) which decreases as \( \eta \) increases satisfying boundary condition at the far away distance from the surface (\( h(\infty) = 0 \)).

Table 1 gives the values of skin friction coefficient \( f''(0) \) obtained analytically with that of numerical solution using Keller-box method varying values of \( k \) and \( \lambda \). We observe that solutions show an excellent agreement, for all the values. Also, as the absolute value of stretching parameter increases numerical value of skin friction also increases.

Table 1. Comparison of the skin friction \( f''(0) \) obtained by analytical method and numerical method.

| \( \lambda \) | \( k = 0.0 \) Exact | \( k = 0.0 \) Numerical | \( k = 1.0 \) Exact | \( k = 1.0 \) Numerical | \( k = 2.0 \) Exact | \( k = 2.0 \) Numerical | \( k = 3.0 \) Exact | \( k = 3.0 \) Numerical |
|--------------|---------------------|-------------------------|---------------------|-------------------------|---------------------|-------------------------|---------------------|-------------------------|
| -1.1         | -0.127960           | -0.127056               | -0.104505           | -0.103897               | -0.080498           | -0.080059               | -0.080498           | -0.080059               |
| -1.2         | -0.261577           | -0.259434               | -0.213726           | -0.212156               | -0.184866           | -0.183902               | -0.1656112          | -0.164591               |
| -1.3         | -0.399999           | -0.396961               | -0.326596           | -0.324633               | -0.282865           | -0.281407               | -0.253272          | -0.251861               |
| -1.4         | -0.543585           | -0.539479               | -0.443876           | -0.441201               | -0.384595           | -0.382462               | -0.344147          | -0.342311               |
| -1.5         | -0.692410           | -0.686841               | -0.565430           | -0.561739               | -0.489511           | -0.486963               | -0.437862          | -0.435848               |

Asymptotically obtained velocity curves for various negative values of pressure gradient are shown in figure 2, but the micropolar parameter \( k \) and stretching parameter \( \lambda \) are kept fixed. Curves plotted for negative pressure gradient show undershoot (\( f'(\eta) < 1 \) for some \( \eta \)), as they steeply decrease crossing their boundary region and then increase to satisfy the end condition. Values in table 2 show that asymptotically obtained skin friction agrees well with that of numerical solution. We also observe that absolute skin friction increases with the absolute stretching parameter.
Figure 2. Asymptotically obtained velocity profiles \( f'(\eta) \) with \( \eta \) for different negative values of \( m \).

Table 2. Comparison of the skin friction \( f''(0) \) obtained by asymptotic method and numerical method.

| \(-\lambda\) | \( k = 0.0 \) | \( k = 0.5 \) | \( k = 1.0 \) |
|-------------|-------------|-------------|-------------|
|             | Asymptotic  | Numerical   | Asymptotic  | Numerical   |
| 1.1         | -0.159577   | -0.160648   | -0.143885   | -0.130294   | -0.131481   |
| 1.2         | -0.319154   | -0.327295   | -0.285460   | -0.293153   | -0.260588   | -0.267889   |
| 1.3         | -0.478731   | -0.499763   | -0.428190   | -0.447647   | -0.390882   | -0.409079   |
| 1.4         | -0.638308   | -0.677886   | -0.570920   | -0.6072172  | -0.521176   | -0.554916   |
| 1.5         | -0.797885   | -0.861511   | -0.713650   | -0.771727   | -0.651470   | -0.705275   |

Figure 3. Numerical velocity profiles \( f' (\eta) \) with \( \eta \) for different values of material parameter \( m \).
The DNS velocity profiles against $\eta$ shown in figure 3 for fixed $\lambda$ and $k$ but for varying pressure gradient, exhibits similar nature as in figure 1 which are obtained analytically using Error and Exponential functions. This further emphasizes the exactness of proposed analytic method. Figure 4 describes the variation of the velocity profiles $f'(\eta)$ with $\eta$ for different $k$ and $m = 1$. One should note that the results for $\lambda > -1$ could be obtained by the Keller Box method wherein exact method fails to give a solution. The solutions approach the mainstream flow from below. In this case the wedge moves slower than the mainstream flow. It is clearly seen that boundary layer thickness is found to be thinner for smaller values of $k$, as also observed in figure 1. The microrotation profile indicates that the absolute value of dimensionless angular velocity or microrotation decreases continuously with $\eta$ and becomes zero far away from the wall, satisfying the boundary condition ($h(\infty) = 0$).

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