Quark Diffusion and Baryogenesis at the Electroweak Phase Transition

James M. Cline
School of Physics and Astronomy, University of Minnesota
Minneapolis, MN 55455, USA

Abstract
Baryogenesis at the electroweak phase transition may take place through CP-violating reflections of quarks from expanding bubbles of the broken symmetry phase. We formulate and approximately solve the transport equations for the reflected quark asymmetries. The results are comparable to a previous Monte Carlo calculation and allow for analytic estimates of the baryon asymmetry $\Delta B$ when the bubble walls have a small velocity. Our method predicts no velocity-dependence for $\Delta B$ in the small $v$ limit, unlike results obtained from a simplified treatment based on the diffusion equation.
There is currently much interest in the possibility that the baryon asymmetry of the universe was created during the electroweak phase transition, using its first-order nature as the means for departure from thermal equilibrium, as is required in any theory of baryogenesis [1-4]. There are two paradigms, depending on whether the bubbles of true vacuum which nucleate in the symmetric phase during the transition have relatively thin or thick walls. In the thin-wall regime which is investigated here, CP-violation in the Higgs sector causes an asymmetry between fermions and antifermions reflected from the bubble walls. To leading order in \( \alpha_{\text{weak}} \) it consists of equal and opposite excesses of right-handed and left-handed fermions, due to CPT conservation, so that the net baryon or lepton number in the disturbance is zero, but its chirality is nonzero. This asymmetry is subsequently converted to a baryon asymmetry by sphaleron interactions. The expanding wall overtakes the baryon asymmetry and incorporates it into the true vacuum phase where it is preserved because of the freezeout of sphalerons.

To compute the resulting baryon asymmetry in this scenario, it is necessary to know the distribution functions of the fermions and antifermions in front of the wall in some detail. So far several treatments have been given, including a Monte Carlo simulation of the diffusion of reflected quarks [3] (hereafter called CKN), and the diffusion equation [4]. Our aim is to complement these approaches by starting with the exact Boltzmann equation and solving for the distributions with the help of a few reasonable approximations. We will find the profile for the reflected quark asymmetry, estimate the effect of nonvanishing domain wall thickness on the baryon asymmetry, and show that the present method disagrees with the diffusion equation approach concerning the velocity dependence of the final result.

Our starting point to compute the transport of reflected quarks with distribution functions \( f_i \) is the Boltzmann equation,

\[
\frac{\partial}{\partial t} + \frac{\mathbf{p}}{E} \cdot \nabla \mathbf{x} \right] f_i(\mathbf{x}, \mathbf{p}) = C_i
\]

where \( C_i \) is the collision term.\(^1\) The latter is the standard difference between two terms,\(^1\)

---

\(^1\)We have ignored force terms that would give rise to the screening of hypercharge, because any deviations \( \delta f_i \) due to screening would be of the form \( y_i \phi \), where \( y_i \) is the hypercharge of particle \( i \) and \( \phi \) is the hypercharge potential to be screened. Such a deviation would have no effect on the biasing of sphaleron interactions because they conserve hypercharge.
which scatter the particle of interest respectively into and out of an element of phase space,

\[ C_i = \sum_{\text{channels}} \frac{1}{2E_i} \int d\Pi_1 d\Pi_2 d\Pi_3 \left\{ (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p) |M_{12\to 3p}|^2 f(p_2) f_i(x, p_1) 
- (p_1 \leftrightarrow p) \right\}; \quad d\Pi_j \equiv d^3 p_j/(16\pi^2 E_j), \]

ignoring Pauli blocking factors. \( f(p) \) is the nearly-equilibrium distribution function for the quark or gluon off of which the species \( i \) of interest is scattering. Of course there is one of approximately 20 channels where the former is also the quark of flavor \( i \), in which case \( f_i \) would appear quadratically, but ignoring this makes only a small error. We have also omitted decay processes because the thermal masses of the gauge bosons are below the threshold for producing two thermal quarks, and the rate for Higgs boson decays into fermions is smaller than scatterings mediated by gauge bosons, due to the large number of scattering channels.

Because the scattering processes are dominated by low momentum transfer, due to the near masslessness of exchanged gauge bosons in the thermal plasma, it is possible to simplify the collision term using the Fokker-Planck approximation [3]. The key observation is that in the absence of a thermal mass \( m_g \) for the gluon, \( C_i \) would diverge logarithmically in the infrared because the matrix element has the leading behavior \( |M|^2 \sim g^4 s^2/(t - m_g^2) \) in terms of Mandelstam variables. We will isolate the logarithm and discard the corrections that would vanish as \( m_g \to 0 \). Let \( p \) denote the three-momentum of the scattered quark, and define \( p_2 = p' \); \( p_3 = p' + q \); so \( p' \) is the momentum of the background quark or gluon and \( q \) is the momentum transfer. By substituting \( p_1 = p + q \) in the first term on the right hand side of (2) and \( p_1 = p - q \) in the second term, and doing the integral over \( d\Pi_3 \) using the momentum-conserving delta function, we get

\[ C_i = \frac{1}{16(2\pi)^5} \int d^3 q \left\{ G(p + q, q) - G(p, q) \right\}; \quad (3) \]

\[ G(p, q) = \frac{f_i(x, p)}{|p||p - q|} \int \frac{d^3 p' f(p')}{|p'||p' + q'|} \delta(|p| + |p'| - |p - q| - |p' + q|) |M_{p,p'\to p-q,p'+q}|^2 \]

as the contribution from a single scattering channel; we will sum over channels in the end.

The next step is to approximate the integrand of (3) as

\[ \left[ q \cdot \partial_p + \frac{1}{2}(q \cdot \partial_p)^2 \right] G(p, q) \quad (4) \]
by Taylor-expanding. This is a good approximation since we expect the scattering to be dominated by small momentum transfer. One finds that only these first two terms are logarithmically sensitive to the gluon mass; the higher ones are convergent. It is fairly straightforward to evaluate them, with the result that the collision term can finally be expressed as

$$C_i = D \partial_p \cdot \left( \partial_p f_i(x, p) + \frac{\beta p}{E} f_i(x, p) \right).$$

(5)

This form is quite general and appears in nonrelativistic scattering processes as well [6].

The diffusion coefficient emerging from this reduction is

$$D = \frac{20\zeta(3)}{\pi} \alpha_s^2 T^3 \int \frac{dq^2}{q^2},$$

(6)

where the factor of 20 comes from weighting the multiplicity of different scattering channels by their respective squared matrix elements and thermal statistical factors (8 × 1 × 1 for gluons, 18 × 4/9 × 3/4 for quarks and the same for antiquarks). To evaluate the logarithm we take the Debye screening mass $m_g^2 = 8\pi \alpha_s T^2$ for the lower limit, the thermally averaged, squared, center of mass energy $s \cong 20T^2$ for the upper limit, and $\alpha_s = 0.1$, with the result $D = 0.16T^3$. Since we have ignored weak interactions, $D$ is the same for both quark chiralities. Note that $D$ differs in dimension and meaning from the usual diffusion coefficient, since it multiplies derivatives with respect to momentum rather than position.

It is convenient to work in the rest frame of the wall, where the distribution will be stationary so that the time derivative can be neglected. In this frame we should Lorentz-transform the momentum variables in (1) and (5), but this complicates the solution of the transport equation, so we will take the limit of small wall velocities $v$, where the difference between the momenta in the rest frame of the universe and that of the bubble wall can be neglected. It will turn out that the asymmetry we seek between quarks and antiquarks is of order $v$ so that simply dropping the time derivative in eq. (1) amounts to ignoring higher order corrections in $v$.

The term linear in derivatives in eq. (5) is cancelled if we write $f_i = e^{-\beta E/2} \hat{f}_i$. Also, since we are interested in the asymmetries between particles and antiparticles, $\delta f \equiv f_i - \bar{f}_i$, we
must subtract from eq. (1) the same equation for \( f_\bar{\bar{\eta}} \). The solution will have the form

\[
\delta f(p, z) = e^{-\beta E/2} \sum_{n=0}^{\infty} \int_{k_0}^\infty dke^{-k|z|} \left( \theta(z) A^+_n g^+_n(p) + \theta(-z) A^-_n g^-_n(p) \right).
\]

The differential equation for \( g^+_n \) is

\[
\left( \frac{\partial^2}{\partial p^2} - \frac{\beta^2 p^2}{4E^2} + \frac{\beta}{2} \left( \frac{3}{E} - \frac{p^2}{E^3} \right) \pm \frac{k p_z}{D E} \right) g^+_n = 0,
\]

where one must remember that \( E = p \) for \( g^+_n \) but \( E = (p^2 + m^2)^{1/2} \) for \( g^-_n \) because the electroweak symmetry is broken only to the left of the wall. However, we will solve (8) only to leading order in the quark mass \( m \). Then, similarly to our neglect of the wall velocity in the Fokker-Planck equation, we can set \( E = p \) to discard higher order corrections in \( m \), since the solution \( \delta f \) is homogeneous in a source term that vanishes as \( m \to 0 \). Throughout this paper \( m \) is understood to be the quark mass in the broken symmetry phase at the critical temperature \( T_c \), which is smaller than the usual mass because the Higgs field VEV is smaller at \( T_c \) than at \( T = 0 \).

Using parabolic coordinates \( p_z = \frac{1}{2}(\xi - \eta) \), \( p = \frac{1}{2}(\xi + \eta) \), the massless limit of eq. (8) is separable and it can be solved exactly:

\[
g^+_n(\xi, \eta) = e^{-a_+ \eta/2 + ia_- \xi/2} F_1(-n; 1; a_+ \eta) F_1(c_n; 1; -ia_- \xi); \quad a_\pm = (k/D \pm \beta^2/4)^{1/2}; \quad c_n = 1/2 - (i/a_-) (\beta/2 - (n + 1/2)a_+),
\]

and \( g^-_n(\xi, \eta) = g^+_n(\eta, \xi) \). The requirement that \( n \) be an integer in eq. (9) is to insure good behavior as \( \eta \to \infty \). The functions \( g^\pm_n \) form a complete set for \( k \geq k_0 \equiv \beta^2 D/4 \approx 0.04T \). Note that the spatial extent of the diffusion layer will therefore be given by the distance scale \( k_0^{-1} \), from eq. (9).

Let us now discuss the boundary conditions for the distributions, which express the conservation of flux at the bubble wall. For example, since left-handed particles are reflected into right-handed particles and vice versa (because of angular momentum conservation), the distribution function for left-handed quarks satisfies

\[
f^+_{iL}(p_z) = \mathcal{R}_{R \to L} f^+_{iR}(-p_z) + \mathcal{T}_L f^-_{iL}(p_z) \quad (p_z > m),
\]

and
where the superscripts indicate that $f_i$ is evaluated at $z = 0+$ or $z = 0-$ with respect to the wall, $\mathcal{R}_{R\rightarrow L}$ is the probability for the reflection of right- to left-handed quarks, and $\mathcal{T}_L$ is the transmission probability for left-handed quarks. Subtracting the distribution function of the antiparticle to obtain $\delta f^+ \equiv f^+_L - f^+_\bar{L}$, remembering that $t_R$ has the corresponding asymmetry $-\delta f^+$, and doing the same for the distributions on the $z < 0$ side of the wall, one obtains

$$\delta f^+(\pm p_z) - \delta f^-(\pm p_z) = \mp \mathcal{R}_0 \left( \delta f^+(p_z) + \delta f^-(p_z) \right) + S(p_z) \quad (p_z > m), \quad (11)$$

where $\mathcal{R}_0$ is the mean value of the two reflection coefficients $\mathcal{R}_{R\rightarrow L}, \mathcal{R}_{L\rightarrow R}$, and the source term $S(p_z)$ depends on their difference $\Delta \mathcal{R}$, and on that of the unperturbed distribution functions:

$$S(p_z) = \left( f^-(p_z) - f^+(p_z) \right) \Delta \mathcal{R}(p_z) \cong 2 \beta v p_z e^{-\beta p_z} \Delta \mathcal{R}(p_z). \quad (12)$$

Here we used the equilibrium distribution functions $f(\pm p_z) \cong e^{-\beta(\pm v p_z)}$ on either side of the wall, expanding to first order in the velocity $v$. The difference between positive and negative $p_z$ is due to our working in the rest frame of the bubble wall.

As one would intuitively expect, the solution to the boundary conditions (14) requires that $\delta f^-(p_z) = -\delta f^+(p_z)$: there is a dipole layer of left-handed quark excess at the bubble wall, since the net number of chiral quarks is conserved and remains zero during the process of reflection. This means that $A_{kn}^- = -A_{kn}^+$, and the boundary condition becomes an integral equation for $A_{kn}^+$, which can be solved using the orthogonality of the eigenfunctions $g_{kn}^\pm(\xi, \eta)$:

$$A_{kn}^+ = N_{kn}^{-1} \int_0^\infty d\xi \int_0^\xi d\eta (\xi - \eta) e^{\beta p_z/2} S(p(\xi, \eta)) \left[ g_{kn}^+(\xi, \eta) - g_{kn}^+(\eta, \xi) \right]. \quad (13)$$

Here $N_{kn}$ is the normalization of the orthogonality integral of the $g_{kn}^+$’s, $N_{kn} = (D/8a_+ \sin(\pi c_{kn})$.

We thus have an approximate solution for the left-handed quark asymmetry in the symmetric phase, which enables us to compute the baryon asymmetry that sphalerons will be driven to produce. A convenient measure is the baryon-to-entropy ratio. In CKN it is found
that in a theory with \( n \) Higgs doublets (we will assume \( n = 2 \)),

\[
\frac{\rho_B}{s} \simeq \frac{3 \times 10^{-7} \kappa}{(2n + 1)v} \beta^2 \delta n,
\]

where \( \kappa \) is the dimensionless constant \( (\kappa \leq 1) \) characterizing the rate of sphaleron interactions in the symmetric phase, and \( \delta n \) is the number of reflected quarks per unit area in front of the wall,

\[
\delta n = \int d^3 p \int_{z_0}^{\infty} dz \delta f(p, z).
\]

Notice that in our approximation of small wall velocities, \( \delta n \sim S(p) \sim v \) so the baryon asymmetry is independent of \( v \) in this limit.

The lower limit \( z_0 \) of the spatial integration in eq. (15) requires some explanation since it does not appear in CKN: due to the finite thickness of the domain wall, the spatial region in which baryogenesis takes place is reduced. It was estimated in CKN that the baryon number violating interactions effectively turn off, in going from the symmetric to the broken phase, when the Higgs field reaches a value of \( g/4\pi \) of its full VEV. For a profile of the form \( \phi_c(1 + \tanh(z/\Delta)) \) this occurs at \( z = z_0 \approx 1.5\Delta \). If the Higgs potential is parametrized as \( \lambda \phi^2(\phi - \phi_c)^2 \) at the critical temperature, then \( \Delta = (2/\lambda)^{1/2} \phi_c^{-1} \), and the experimental lower limit on the mass of the Higgs boson gives \( \sqrt{\lambda} > 0.085 \). We assume that this bound is saturated, as well as the bound \( \phi_c > T_c \), which assures that sphaleron interactions in the broken phase are sufficiently suppressed to avoid washing out the baryon asymmetry on cosmological timescales. It is convenient to express \( z_0 \) in units of the characteristic width of the diffusion layer. The result is \( z_0 k_0 = 1.0 \).

In order to evaluate the density \( \delta n \) of reflected quarks in front of the wall, one must assume a form for the reflection asymmetry \( \Delta R(p_z) \). By analyzing existing results \( \text{[3][7]} \) it can be shown that \( \Delta R \) is rather well fitted by the form

\[
\Delta R(p_z) = N \theta(\epsilon) e^{\alpha \epsilon} e^{-\epsilon/w}; \quad \epsilon \equiv p_z - m.
\]

The parameters \( N \), \( w \) and \( \alpha \) depend only on the quark mass, and \( \alpha \) is negligible for a particle as heavy as the top quark \( \text{[8]} \). Then, although the explicit expression for \( \delta n \) is rather
complicated, it takes a simple expression in the limit that the temperature is much larger than any other mass scales in the problem,

$$\delta n = \frac{Nwv}{D\beta} \left\{ 31.7w^3, \quad T \gg w \gg m_q \right\} \cdot 14.3m_q^3, \quad T \gg m \gg w \right\} s(z_0), \quad (17)$$

after numerically evaluating the sum/integral appearing in eq. (17). The suppression factor $s(z_0)$ due to finite wall thickness is shown in figure 1. For the value of $z_0$ determined above we see that $s(z_0)$ is approximately 0.12, which is a source of suppression that has not previously been taken into account.

To compare to the Monte Carlo computation of $\delta n$ by CKN, we can use their determination of the reflection asymmetry in our result, eq. (17). For example, in the most favorable case when the width of the wall is assumed to be $m^{-1}$ and CP violation is maximal, they find a reflection asymmetry with $N = 0.68$ and $w = 0.44m$, from which we obtain $\rho_B/s = 1.9 \times 10^{-7} \kappa(m/T_c)^4$. Our result is quite close to that of CKN for small wall velocities ($v = 0.1$) and $m/T_c < 1$, but for larger $m$ it begins to exceed their result. Of course in this region our approximations are no longer valid, but for a top quark of mass 170 GeV the Yukawa coupling is $y \cong 1$, and $m/T_c = y\phi_c/T_c \cong 1$.

We have computed the shape of the profile $\delta n(z)$ in the same approximation as used to obtain the integrated value of $\delta n$ above. It is shown in figure 2, where one sees that at a distance of $k_0^{-1} = 25/T$ it has dropped to 5% of its maximum value, and at $2k_0^{-1}$ to 0.8%. Near the wall it falls off faster than an exponential, but asymptotically approaches exponential decay far from the wall. It is interesting to compare the width of the profile with the mean free path of quarks due to QCD scatterings, which we have computed to be $\lambda = 3T^{-1}$. Figure 1 indicates that the profile falls by a factor of $e$ in a similar distance, $4T^{-1}$, comparable to the diffusion length obtained using the position-space formulation of the diffusion problem [8].

The behavior of $\delta n(z)$ we find is quite different from what would have been predicted from the diffusion equation,

$$\left( \partial_t - D_z \partial_z^2 \right) n(z - vt) = 0, \quad (18)$$
where $D_z$ is the usual position-space diffusion coefficient of order $1/T$, not to be confused with our $D$. The solution has the form

$$n(z - vt) = n_0 \exp(-v(z - vt)/D_z)$$  \hspace{1cm} (19)

for $z > vt$, the region in front of the wall. Thus in the small $v$ limit the distribution approaches a constant in space, whereas ours has a finite extent (albeit vanishing height) even as $v \to 0$. This leads to at least one extra power of $v^{-1}$ for the baryon asymmetry in the diffusion equation approach, due to the integration over $z$ of the density in (19).

How do we understand the discrepancy between the two approaches? To bolster our confidence we can look for the same behavior in a simplified version of the same physical system, by working in one dimension and taking the infinite temperature limit, holding $D$ fixed. Then eq. (8) becomes

$$(\partial^2_p \mp \frac{kp}{D|p|}) g^\mp_k = 0,$$  \hspace{1cm} (20)

whose solutions are mere exponential and trigonometric functions. In this case the integral for $A_k^+$ analogous to (13) can be done explicitly and $\delta n(z)$ can be reduced to a single integral over $u \equiv (k/D)^{1/2}$,

$$\delta n(z) = 2N \pi d \int_0^\infty \frac{du}{u} e^{-zu^2} \left( \frac{w + u}{w^2 + u^2} - \frac{1}{w + u} \right),$$  \hspace{1cm} (21)

whose large distance behavior goes like $z^{-1/2}$. Again we see that the profile is not constant in position space even at zero velocity (although the length scale $T^2/D$ we had previously is now gone because of taking the $T \to \infty$ limit). However, we find that if we replaced the boundary condition (11) at the wall by one that did not depend on momentum, by letting the source term $S(p_z)$ be a constant (let $w \to 0$), then the solution for $A_k$ would be a delta function at $k = 0$, and eq. (21) would indeed give a constant in $z$-space! So we have discovered one reason the two methods disagree: the Boltzmann equation demands that the boundary condition for the distributions at the bubble wall are functions of momentum,

\footnote{In fact there should be a second factor of $v^{-1}$ because by Fick’s law, the flux is $J = -D_z dn/dz = vn_0$ at the bubble wall, so $n_0$ is the flux at the wall divided by $v$. This extra factor does not appear in refs. [4] or [10].}
whereas the diffusion equation has integrated over the momenta from the outset, and so cannot account for such details of the distribution. Of course the full Boltzmann equation should be the more accurate of the two methods. It would be interesting to compare the two predictions with the Monte Carlo method of CKN, but since the latter display data only for a single small value of \( v \), it is impossible to measure their \( v \)-dependence in the region of \( v = 0.1 \).

In summary, we have given an analytic expression for the baryon asymmetry due to quarks reflecting from domain walls during the first order electroweak phase transition, by solving the Boltzmann equation for the reflected particles. The solution is valid for slowly moving walls and temperatures which are large compared to the quark masses or the width in momentum space for the reflection asymmetry between the quarks and antiquarks. Although the top quark mass is actually not small compared to the critical temperature, the fact that we obtained quantitative agreement with the previous results of CKN for \( m/T \) as large as unity is encouraging since we don’t expect it to be greater than this. Furthermore \( m \) vanishes (up to thermal loop contributions) in the symmetric phase outside the bubble, the region most important to the estimate of baryon production, so it might be hoped that the small-mass approximation is better than expected parametrically. An interesting feature of our expression is its insensitivity to the velocity of the expanding bubble wall, relative to predictions based on the diffusion equation.

Our result makes it possible to estimate the baryon asymmetry \( \Delta B \) directly in terms of the quark reflection asymmetry \( \Delta R(p_z) \), an independent calculation which is being repeated because of apparently conflicting calculations in the literature. The quantitative results of electroweak baryogenesis through quark reflection hinge upon this since the momentum-space width of \( \Delta R(p_z) \), on which \( \Delta B \) depends linearly, falls quite sharply with increasing quark mass, whereas for small masses \( \Delta B \) is suppressed by a factor of \( m^3 \). Further application of the present work will be given in.

I would like to thank Larry McLerran and Sonia Paban for important contributions during the early stage of this work, which was supported in part by DOE grant DE-AC02-83ER-40105. I also thank Kimmo Kainulainen and Alejandro Ayala for helpful discussions.
References

[1] M.E. Shaposhnikov, JETP Lett. 44 (1986) 465; Nucl. Phys. B287 (1987) 757; Nucl. Phys. B299 (1988) 797;
L. McLerran, Phys. Rev. Lett. 62 (1989) 1075;
A.G. Cohen, D.B. Kaplan and A.E. Nelson, Phys. Lett. B245 (1990) 561; Phys. Lett. B263 (1991) 86; Nucl. Phys. B349 (1991) 727;
L. McLerran, M. Shaposhnikov, N. Turok and M. Voloshin, Phys. Lett. B256 (1991) 451.
N. Turok and J. Zadrozny, Phys. Rev. Lett. 65 (1990) 2331; Nucl. Phys. B358 (1991) 471.
M. Dine and S. Thomas, Santa Cruz preprint SCIPP 94/01 (1994).

[2] G.R. Farrar and M.E. Shaposhnikov, Phys. Rev. Lett. 70 (1993) 2833; erratum-ibid. 71 (1993) 210; CERN preprint CERN-TH.6734/93, hep-ph/9305275 (1993);
M.B. Gavela, P. Hernandez, J. Orloff and O. Pène, Mod. Phys. Lett. 9A 1795 (1994);
E. Sather and P. Huet, SLAC preprint SLAC-PUB-6479, hep-ph/9404302.

[3] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Nucl. Phys. B373 (1992) 453.

[4] M. Joyce, T. Prokopec and N. Turok, Princeton preprint PUPT-1437 (1994).

[5] S.Yu. Khlebnikov, Phys. Lett. B300 (1993) 376;
A.G. Cohen, D.B. Kaplan and A.E. Nelson, Phys. Lett. B294 (1992) 57;
M. Joyce, T. Prokopec and N. Turok, Princeton preprint PUPT-1436 (1994).

[6] L.D. Landau and E.M. Lifshitz, Statistical Physics (Oxford; Pergamon Press, 1978) 89.

[7] K. Funakubo A. Kakuto, S. Otsuki, K. Takenaga and F. Toyoda, Saga University preprint SAGA-HE-55 (1994), to appear in Phys. Rev. D50.

[8] J.M. Cline, K. Kainulainen and A. Vischer, in preparation.

[9] H. Heiselberg, Phys. Rev. Lett. 72 (1994) 3013.
[10] A.F. Heckler, Univ. of Washington preprint astro-ph/9407064 (1994).
Fig. 1: logarithm of the suppression factor due to finite wall thickness. $z_0$ is proportional to the bubble wall thickness (see text), and is in units of the diffusion length $k_0^{-1} \approx 25T^{-1}$. 
Fig. 2: The reflected quark asymmetry profile in arbitrary units, versus distance in front of the wall, in units of $k_0^{-1} \approx 25T^{-1}$. 