Single and double inclusive cross-sections for nucleus-nucleus collisions in the perturbative QCD

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Abstract. Single and double inclusive cross-sections in nucleus-nucleus collisions are derived in the perturbative QCD with interacting BFKL pomerons in the quasi-classical approximation.

1 Introduction

With the advent of colliders the study of particle production in nucleus-nucleus collisions has acquired a prominent role both in experimental and theoretical studies. Single and double inclusive cross-sections and correlations related to them draw naturally most attention. In the framework of the perturbative QCD they can be studied in the approach based on interacting BFKL pomerons developed for nucleus-nucleus collisions in [1, 2, 3]. Intuitive considerations plus experience with the Local Reggeon Field Theory (LRFT) allow to guess the structure of the single inclusive cross-section as emission either from the central pomeron or from the two adjacent triple pomeron vertexes in the convolution of two sets of fan diagrams propagating from the center to the projectile or target. However no rigorous demonstration of this structure has been given in the literature. Still worse is the situation with the double inclusive cross-section for which even in the LRFT one has a very complicated formula (see [4]).

In this paper we aim at filling this gap. We derive formulas for both single and double inclusive cross-sections in nucleus-nucleus collisions in the perturbative QCD approach with interacting BFKL pomerons. Part of the problem which involves particle emission from the pomerons is ideologically rather similar to the LRFT (although considerably more complicated technically). So to treat this problem we shall use the cut pomeron formalism developed within the LRFT [5], which we appropriately generalize for the BFKL pomerons. A new problem is emission from the triple pomeron vertex. For the single inclusive we find that it is indeed described by simple formulas used in our previous studies ([6]). However the double inclusive cross-sections with a single emission from the vertex lead to a complicated expression including different parts of the cut emission vertex found in [7].
Numerical calculations of the single inclusive cross-section in nucleus-nucleus collisions, with vertex emission taken into account, were earlier reported in [6] (as mentioned, without rigorous justification of the formulas). As to the double inclusive cross-sections and correlations, due to their complexity, we do not attempt here to use them for numerical studies. These studies present a separate difficult problem and are postponed for future publications.

The paper is organized as follows. In the next section we generalize the cut pomeron formalism for BFKL pomerons. The derivation closely follows that in the LRFT ([4, 5]) with inevitable complications due to the non-local structure of the BFKL pomeron. Next in Section 3 we solve the equation of motion and find working formulas for the single and double inclusive cross-section corresponding to emission from the pomeron. In Section 4 we discuss emission from the vertex and derive formulas for the single and double inclusive cross-sections which involve such emission. Finally we discuss our results in the last section.

2 Cut pomeron formalism and inclusive cross-sections for nucleus-nucleus scattering in the perturbative QCD

2.1 Fields and Lagrangian

Pomerons are described by two fields \( \psi \) and \( \psi^\dagger \), which depend on rapidity \( y \), relative gluon momentum \( q \) and point \( b \) in the transverse space. We shall restrict ourselves with the purely forward case relevant for collisions off large nuclei. Then \( b \) is conserved during the collision. Correspondingly we do not indicate it explicitly unless it may lead to confusion. After transition to real fields the generating functional for the Green functions is written as

\[
Z = \int D\psi D\psi^\dagger e^{-A},
\]

where action \( A \) is an integral over \( y, q \) and \( b \):

\[
-A = \int dyd^2bd^2qL(\psi, \psi^\dagger)
\]

and the Lagrangian density \( L \) is

\[
L(y, q, b) = \psi^\dagger Q\psi + \lambda[(K\psi^\dagger) \cdot \psi^2 + (K\psi) \cdot \psi^\dagger^2] + g\delta(y)\psi + f\delta(Y-y)\psi^\dagger.
\]

Here \( Q \) is essentially the BFKL operator

\[
Q = 2K\left(\frac{\partial}{\partial y} + H\right),
\]

with \( H \) the BFKL Hamiltonian, and

\[
K = \nabla_q^2q^4\nabla_q^2.
\]

Operator \( K \) is conformal invariant and commutes with \( H \). The triple-pomeron coupling \( \lambda \) is

\[
\lambda = \frac{4\alpha_s^2N_c}{\pi}.
\]
Finally $g(y, b)$ and $f(y, b, B)$ represent the coupling of the pomeron to the colliding nuclei. $g(y, b)$ describes this coupling to the target A (at zero rapidity)

$$g(y, b) = AT_A(b)\rho(q),$$

where $T_A(b)$ is the profile function of nucleus A with its center at the origin in the transverse plane and $\rho(q)$ is the colour density of the nucleon. $f(y, b, B)$ describes the coupling to the projectile at rapidity $Y$ and its center at point $B$ (the impact parameter of the collision)

$$f(y, b, B) = BT_B(B - b)\rho(q).$$

To study different cuts we introduce the cut pomeron formalism following the scheme developed in [5] for the local pomeron. We introduce 6 fields

$$\psi_{\pm}, \psi_c, \psi_{\mp}, \psi_{\mp}^\dagger, \psi_{c}^\dagger$$

which describe pomerons to the left (right) of the cut (subindex +(-)) and cut pomerons (subindex c). To fulfill the AGK rules the Lagrangian of these fields is to be taken as

$$L_c = L_c^0 + L_c^I + L_c^E.$$  

Here the unperturbed Lagrangian is

$$L_c^0 = \sum_{i=\pm,c} \epsilon_i \psi_i^\dagger Q \psi_i,$$

with

$$\epsilon_{\pm} = -\epsilon_c = 1.$$

The interaction involves several terms

$$L_c^I = \lambda(K\psi_{\mp}^\dagger \cdot \psi_{\mp}^2) + \lambda(K\psi_+ \cdot \psi_+^2) + \lambda(K\psi_- \cdot \psi_-^2) + \lambda(K\psi_c \cdot \psi_c^2)$$

$$- 2\lambda(K\psi_{\mp}^\dagger \psi_+ + \psi_-) - 2\lambda(K\psi_c \psi_+^\dagger + \psi_-^\dagger)$$

$$+ \sqrt{2}\lambda(K\psi_{\mp}^\dagger \psi_{\mp}^\dagger + \psi_c^\dagger \psi_c^\dagger) + \sqrt{2}\lambda(K\psi_c \psi_+^\dagger + \psi_-^\dagger + \psi_{\mp}^\dagger + \psi_{\mp}^\dagger).$$

Finally the external Lagrangian is

$$L_c^E = g\delta(y)(\psi_+ + \psi_- - \sqrt{2}\psi_c) + f\delta(Y - y)(\psi_{\mp}^\dagger + \psi_{\mp}^\dagger - \sqrt{2}\psi_{c}^\dagger).$$

### 2.2 Inclusive cross-sections

To study inclusive cross-sections we additionally introduce interaction of the cut pomeron with the emitted particles. As is well-known, the inclusive cross-section corresponding to emission of a gluon jet from the pomeron can be written as a double integral

$$I(y, \kappa) \equiv \frac{(2\pi)^3 d\sigma}{dy d^2\kappa} = 2 \int d^2b d^2q_1 d^2q_2 \delta^2(q_1 + q_2 - \kappa)\eta(q_1, q_2)p_A(y, q_1, b)p_B(y, q_2, b),$$

where $p_A(B)(y, q, b)$ is the pomeron coupled to the target (projectile) and

$$\eta(q_1, q_2) = \frac{16\pi^2 \alpha_s N_c}{\kappa^2} q_1^2 q_2^2 \nabla_1^2 \nabla_2^2.$$
This expression can be conveniently rewritten in the coordinate space in the local form, in terms of the 'non-amputated' pomeron

\[ P(y, r, b) = 2\pi r^2 \int \frac{d^2q}{2\pi} e^{iqr} p(y, q, b) \]  
(16)

as

\[ I(y, \kappa) = \frac{8\alpha_s N_c}{\kappa^2} \int d^2b d^2r P_A(y, r, b) \nabla^2 e^{i\kappa \cdot r} \nabla^2 P_B(y, r, b). \]  
(17)

To generate inclusive cross-sections we add a term to the action

\[ -\Delta A_c = \int dyd^2bd^2q_1d^2q_2 \xi(y, q_1, q_2, b) \psi_c^\dagger(y, q_1, b) \psi_c(y, q_2, b). \]  
(18)

Differentiation in \( \xi \) then gives insertions into the cut pomeron propagator at rapidity \( y \) and transverse point \( b \) with momenta \( q_1 \) and \( q_2 \). As in [4] we denote the total action as

\[ A_c(\xi) = A_c + \Delta A_c. \]  
(19)

It follows from unitarity that

\[ A_c(\xi = 0) = 0 \]  
(20)

Also one finds that the generating functional of the amplitudes at \( \xi \neq 0 \) is given by

\[ T_c(\xi) = 1 - S + \frac{1}{2} \left( e^{-A_c(\xi)} - 1 \right) \]  
(21)

where \( S \) is just the \( S \) matrix at \( \xi = 0 \) [4] (also see Appendix 1).

The single and double inclusive cross-sections are obtained as

\[ I_1(y, \kappa) = 2 \int d^2b d^2q_1d^2q_2 \delta^2(q_1 + q_2 - \kappa) \eta(q_1, q_2) \frac{\delta T_c(\xi)}{\delta \xi(y, q_1, q_2, b)} \bigg|_{\xi=0} \]  
(22)

and

\[ I_2(y, \kappa|\kappa') = 2 \int d^2b d^2b' d^2q_1d^2q_2 \delta^2(q_1 + q_2 - \kappa) d^2q_1' d^2q_2' \delta^2(q_1' + q_2' - \kappa') \eta(q_1, q_2) \eta(q_1', q_2') \frac{\delta^2 T_c(\xi)}{\delta \xi(y, q_1, q_2, b) \delta \xi(y', q_1', q_2', b')} \bigg|_{\xi=0}. \]  
(23)

Next we use the property proven in [4] that at any values of \( \xi \), due to the equations of motion,

\[ -\frac{\delta A_c(\xi)}{\delta \xi(y, q_1, q_2, b)} = \psi_c^\dagger(y, q_1, b, \xi) \psi_c(y, q_2, b, \xi). \]  
(24)

This together with (20) immediately gives a simple formula for the single inclusive cross-section:

\[ I_1(y, q) = \int d^2b d^2q_1d^2q_2 \delta^2(q_1 + q_2 - q) \eta(q_1, q_2) \psi_c^\dagger(y, q_1, b, \xi = 0) \psi_c(y, q_2, b, \xi = 0). \]  
(25)

The formula for the calculation of the double functional derivative at \( \xi = 0 \) becomes:

\[ \frac{\delta^2 T_c(\xi)}{\delta \xi(y, q_1, q_2, b) \delta \xi(y', q_1', q_2', b')} \bigg|_{\xi=0} = \frac{1}{2} \left[ \psi_c^\dagger(y, q_1, b, \xi) \psi_c(y, q_2, b, \xi) \psi_c^\dagger(y', q_1', b', \xi) \psi_c(y', q_2', b', \xi) \right. \]  

\[ \left. + \frac{\delta}{\delta \xi(y, q_1, q_2, b)} \left( \psi_c^\dagger(y', q_1', b', \xi) \psi_c(y', q_2', b', \xi) \right) \right] \bigg|_{\xi=0}. \]  
(26)

So the whole problem is reduced to finding \( \psi_c, \psi_c^\dagger \) and their first derivatives in \( \xi \) at \( \xi = 0 \).
2.3 Unitary transformation to new variables and equations of motion

Considerable simplification can be achieved if, following \[4\], we make a unitary transformation of our fields introducing new fields $\phi_\pm, \phi_0$ and conjugate fields $\pi_\pm, \pi_0$ by

$$\psi_\pm = \phi_\pm, \quad \psi_c = -\frac{1}{\sqrt{2}}(\phi_0 - \phi_+ - \phi_-),$$

$$\psi^\dagger_\pm = \pi_\pm + \pi_0, \quad \psi^\dagger_c = \sqrt{2}\pi_0.$$  

(27)

This transforms $L_c$ into a new Lagrangian

$$L'_c = \sum_{i=+,-,0} \pi_i Q\phi_i + \lambda \sum_{i=+,-,0} \left((K\pi_i) \cdot \phi_i^2 + (K\phi_i) \cdot \pi_i^2\right) + \lambda \left(\pi_+\pi_0 \cdot K(\phi_0 + \phi_+ - \phi_-) + \pi_-\pi_0 \cdot K(\phi_0 - \phi_+ + \phi_-) + \pi_+\pi_- \cdot K(\phi_+ + \phi_- - \phi_0)\right).$$  

(28)

The coupling to the nuclei takes the form

$$L'_E = g\delta(y)\phi_0 + f\delta(y - Y)(\pi_+ + \pi_-)$$  

(29)

and the part of action depending on $\xi$ becomes

$$-\Delta A_c = \int dyd^2q_1d^2q_2\xi(y, q_1, q_2)\pi_0(q_1)\left(\phi_+(q_2) + \phi_-(q_2) - \phi_0(q_2)\right).$$  

(30)

Variation of the action with respect to our fields gives equations of motion. It can be seen that they are totally identical for $\pm$ fields. So it is sufficient to write them down for $\phi = \phi_+ = \phi_-, \phi_0, \pi = \pi_+ = \pi_- \text{ and } \pi_0$. They are (at point $y, q$):

$$\pi Q + 2\lambda K(\pi^2) + 2\lambda\phi K\pi + \int d^2k\xi(y, q, k)\pi_0(k) = 0,$$  

(31)

$$\pi_0 Q + \lambda K(\pi_0^2 - \pi^2 + 2\pi_0\pi) + 2\lambda\phi_0 \cdot K\pi - \int d^2k\xi(y, q, k)\pi_0(k) + g\delta(y) = 0,$$  

(32)

$$Q\phi + \lambda K\phi^2 + \lambda\pi \cdot K(4\phi - \phi_0) + \lambda\pi_0 \cdot K\phi_0 + f\delta(Y - y) = 0,$$  

(33)

$$Q\phi_0 + \lambda K\phi_0^2 + 2\lambda(\pi_0 + \pi) \cdot K\phi_0 + \int d^2k\xi(y, q, k)\left(2\phi(k) - \phi_0(k)\right).$$  

(34)

3 Inclusive cross-sections due to emission from the pomeron

3.1 Solution of the equation of motion: fields at $\xi = 0$ and single inclusive cross-sections

Our first task is to find the solutions of our equations of motion at $\xi = 0$, which knowledge is sufficient for the single inclusive cross-section according to (25).

If $\xi = 0$ then the equations for $\pi$ and $\phi_0$ do not contain driving terms, so that these fields are identically zero:

$$\pi(y, q) = \phi_0(y, q) = 0.$$  

(35)
The two remaining equations decouple:

$$\pi_0 Q + \lambda K (\pi_0^2) + g\delta(y) = 0 \quad (36)$$

and

$$Q\phi + \lambda K\phi^2 + f\delta(Y - y) = 0. \quad (37)$$

These are the standard BK equations for the sum of fan diagrams $\chi$ which go to the point $(y, q)$ from the target

$$\pi_0(y, q) = \chi(y, q, g) \quad (38)$$

or from the projectile

$$\phi(y, q) = \chi(Y - y, q, f) \equiv \bar{\chi}. \quad (39)$$

Put into the expression (25) they give the commonly used fact orized expression for the single inclusive cross-section as a convolution of two sets of fan diagrams:

$$I_1(y, \kappa) = 2 \int d^2q_1d^2q_2\delta^2(1 + q_2 - \kappa)\eta(q_1, q_2)\chi(y, q, b)\bar{\chi}(y, q_2, b) \quad (40)$$

or, in the coordinate space, in terms of $Z(y, r, b)$ related to $\chi(y, q, b)$ similarly to (16):

$$Z(y, r, b) = 2\pi r^2 \int d^2q e^{iqr}\chi(y, q, b), \quad (41)$$

as

$$I(y, \kappa) = \frac{8\alpha_s N_c}{\kappa^2} \int d^2qd^2rZ(y, r, b)\nabla^2 e^{i\kappa r} \nabla^2 \bar{Z}(y, r, b). \quad (42)$$

### 3.2 Derivative of the fields in $\xi$ at $\xi = 0$ and double inclusive cross-sections

To set up equations for the derivatives of the fields in $\xi$ at $\xi = 0$ we have to differentiate Eqs. (31) - (34) in $\xi(y_1, q_1, q_2, b_1)$ and then put $\xi = 0$. The field derivatives and the equations as a whole will depend on 7 variables: $y, q, b$ and $y_1, q_1, q_2, b_1$, which we shall show explicitly only when it is necessary. Thus say $\frac{\delta\pi}{\delta\xi}$ will in fact mean $\frac{\delta\pi}{\delta\xi}(y_1, q_1, q_2, b_1)$. With these notations we get the equations:

$$\left( - \frac{\partial}{\partial y} + H + \lambda \bar{\chi} \right)2K\frac{\delta\pi}{\delta\xi} + \delta(y - y_1)\delta^2(b - b_1)\delta^2(q - q_1)\chi(q_2) = 0, \quad (43)$$

$$2K\left( - \frac{\partial}{\partial y} + H + \lambda \bar{\chi} \right)\frac{\delta\pi}{\delta\xi} + 2\lambda \frac{\delta\phi_0}{\delta\xi} K\chi + 2\lambda K\left( \frac{\delta\pi}{\delta\xi} \right) - \delta(y - y_1)\delta^2(b - b_1)\delta^2(q - q_1)\chi(q_2) = 0, \quad (44)$$

$$2K\left( \frac{\partial}{\partial y} + H + \lambda \bar{\chi} \right)\frac{\delta\phi_0}{\delta\xi} + 4\lambda \frac{\delta\pi}{\delta\xi} K\bar{\chi} + \lambda \chi \cdot K\frac{\delta\phi_0}{\delta\xi} = 0. \quad (45)$$

$$\left( \frac{\partial}{\partial y} + H + \lambda \bar{\chi} \right)2K\frac{\delta\phi_0}{\delta\xi} + + 2\delta(y - y_1)\delta^2(b - b_1)\delta^2(q - q_1)\bar{\chi}(q_2) = 0. \quad (46)$$

To simplify these multivariable equations we first note that the $b$ dependence of the derivatives is trivial: obviously they all are proportional to $\delta^2(b - b_1)$. So we separate this factor and consider the derivatives at a fixed point $b$ which need not be shown explicitly.
Next we return to our expression for the double inclusive cross-section. Obviously it consists of two terms. One is just the product of two single inclusive cross-sections

\[ I_2^{(1)}(y_1, \kappa_1|y_2, \kappa_2) = I_1(y_1, \kappa_1)I(y_2, \kappa_2). \]  

(47)

The other term comes from the second term in [26] and has the form

\[ I_2^{(2)}(y, \kappa|y', \kappa') = \int d^2b d^2b'd^2q_1 d^2q_2 \phi (q_1 + q_2 - \kappa) d^2q_1' d^2q_2' \delta^2(q_1' + q_2' - \kappa') \eta(q_1, q_2) \eta(q_1', q_2') \]

\[ \frac{\delta}{\delta \xi(y, q_1, q_2, b)} \psi(y', q_1', b', \xi) \psi(y', q_2', b' \xi) \bigg|_{\xi=0} \]

\[ = \int d^2q_1 d^2q_2 \delta^2(q_1 + q_2 - \kappa) d^2q_1' d^2q_2' \delta^2(q_1' + q_2' - \kappa') \eta(q_1, q_2) \eta(q_1', q_2') \]

\[ \left( \psi(y', q_1', b', \xi = 0) \frac{\delta \psi(y', q_2', b', \xi)}{\delta \xi(y, q_1, q_2, b)} \bigg|_{\xi=0} + \psi(y', q_1', b', \xi = 0) \frac{\delta \psi(y', q_2', b', \xi)}{\delta \xi(y, q_1, q_2, b)} \bigg|_{\xi=0} \right). \]  

(48)

Due to factor \( \delta^2(b-b') \) contained in the derivatives the double integrations in \( b \) and \( b' \) in fact turn into one over the common point \( b \). Taking this into account and suppressing this common argument \( b \), in terms of fields \( \phi \) and \( \pi \) we find

\[ I_2^{(2)}(y, \kappa|y', \kappa') = \int d^2b d^2q_1 d^2q_2 \phi (q_1 + q_2 - \kappa) d^2q_1' d^2q_2' \delta^2(q_1' + q_2' - \kappa') \eta(q_1, q_2) \eta(q_1', q_2') \]

\[ \left( \chi(y', q_1') \frac{\delta (2\phi(y', q_2', \xi) - \phi_0(y', q_2', \xi))}{\delta \xi(y, q_1, q_2)} \bigg|_{\xi=0} + 2\chi(y', q_1') \frac{\delta \pi_0(y', q_2', \xi)}{\delta \xi(y, q_1, q_2)} \bigg|_{\xi=0} \right), \]  

(49)

where it is assumed that factor \( \delta^2(b-b') \) has been dropped from the derivatives. From this formula we can conclude that we do not need our derivatives in \( \xi(y_1, q_2) \) at all values of its arguments but rather integrated over \( q_1 \) and \( q_2 \) with weight \( \delta(q_1 + q_2 - \kappa) \eta(q_1, q_2) \). Correspondingly we define (suppressing the argument \( b \))

\[ \Pi_0(y, q, y_1, \kappa) = \int d^2q_1 d^2q_2 \delta^2(q_1 + q_2 - \kappa) \eta(q_1, q_2) \frac{\delta \pi_0(y, q, \xi)}{\delta \xi(y_1, q_1, q_2)} \bigg|_{\xi=0}, \]  

(50)

\[ \Pi(y, q, y_1, \kappa) = \int d^2q_1 d^2q_2 \delta^2(q_1 + q_2 - \kappa) \eta(q_1, q_2) \frac{\delta \pi(y, q, \xi)}{\delta \xi(y_1, q_1, q_2)} \bigg|_{\xi=0}, \]  

(51)

\[ \Phi_0(y, q, y_1, \kappa) = \int d^2q_1 d^2q_2 \delta^2(q_1 + q_2 - \kappa) \eta(q_1, q_2) \frac{\delta \phi_0(y, q, \xi)}{\delta \xi(y_1, q_1, q_2)} \bigg|_{\xi=0}, \]  

(52)

\[ \Phi(y, q, y_1, \kappa) = \int d^2q_1 d^2q_2 \delta^2(q_1 + q_2 - \kappa) \eta(q_1, q_2) \frac{\delta \phi(y, q, \xi)}{\delta \xi(y_1, q_1, q_2)} \bigg|_{\xi=0}, \]  

(53)

In view of expression [49] we also introduce

\[ \Psi(y, q, y_1, \kappa) = 2\Phi(y, q, y_1, \kappa) - \Phi_0(y, q, y_1, \kappa). \]  

(54)

In terms of these quantities we find

\[ I_2^{(2)}(y, \kappa|y', \kappa') \]
\[ = \int d^2b d^2q_1 d^2q_2 \delta^2(q_1 + q_2 - \kappa') \eta(q_1, q_2) \left( \chi(y', q_1') \Psi(y', q_2', y, \kappa) + 2 \tilde{\chi}(y', q_1') \Pi_0(y', q_2', y, \kappa) \right). \] 

(55)

Integrating our equations (43) - (46) over \( q_1 \) and \( q_2 \) with weight \( \delta^2(q_1 + q_2 - \kappa) \eta(q_1, q_2) \) we get the following system

\[
\left( -\frac{\partial}{\partial y} + H + \lambda \chi \right) 2K \Pi + \delta (y - y_1) \eta(q, \kappa - q) \chi(y, \kappa - q) = 0, \tag{56}
\]

\[
2K \left( \frac{\partial}{\partial y} + H + \lambda \chi \right) \Pi_0 + 2\lambda \Phi_0 \cdot K \chi + 2\lambda K (\chi \Pi) - \delta (y - y_1) \eta(q, \kappa - q) \chi(y, \kappa - q) = 0, \tag{57}
\]

\[
2K \left( \frac{\partial}{\partial y} + H + \lambda \chi \right) \Psi + 8\lambda \Pi \cdot K \chi + 2\lambda K (\tilde{\chi} \Phi_0) - 2 \delta (y - y_1) \eta(q, \kappa - q) \tilde{\chi}(y, \kappa - q) = 0, \tag{58}
\]

\[
\left( \frac{\partial}{\partial y} + H + \lambda \chi \right) 2K \Phi_0 + 2\delta (y - y_1) \eta(q, \kappa - q) \tilde{\chi}(y, \kappa - q) = 0. \tag{59}
\]

One observes that the first and last equations determine \( \Pi \) and \( \Phi_0 \) in terms of the known \( \chi \) and \( \tilde{\chi} \), after which the second and third equation allow to find \( \Pi_0 \) and \( \Psi \).

Eqs. (56) - (59) are convenient for numerical calculations. However one can also formally express the solution in terms of Green function of the operators \( \partial / \partial y + H + \lambda \chi \) and \( \partial / \partial y + H + \tilde{\chi} \) to obtain formulas which can be compared with [4]

### 3.3 Formal solution

We define the Green functions:

\[
G = \left( \frac{\partial}{\partial y} + H + \lambda \chi \right)^{-1}, \quad G^T = \left( -\frac{\partial}{\partial y} + H + \lambda \chi \right)^{-1},
\]

\[
\tilde{G} = \left( \frac{\partial}{\partial y} + H + \lambda \tilde{\chi} \right)^{-1}, \quad \tilde{G}^T = \left( -\frac{\partial}{\partial y} + H + \lambda \tilde{\chi} \right)^{-1} \tag{60}
\]

Each Green function is an integral operator in \((y, q)\) space, so that e.g. for \( G \) the kernel is \( G(y, q|y', q') \). In our formulas rapidities and momenta enter asymmetrically, so in many cases we shall suppress the momenta but leave the rapidities, considering \( G_{yy'}(q|q') \) as an operator in the momentum space with the kernel \( G_{yy'}(q|q') \).

In these notation solution of Eqs. (56) and (59) is immediate

\[
\Pi(y) = -\frac{1}{2} K^{-1} G_{yy_1}^T (\eta \chi)_{y_1}, \tag{61}
\]

\[
\Phi_0(y) = -K^{-1} G_{yy_1} (\eta \tilde{\chi})_{y_1}. \tag{62}
\]

We put these solutions into Eq. (57) multiplied by \((2K)^{-1}\) to obtain

\[
\left( -\frac{\partial}{\partial y} + H + \lambda \chi \right) \Pi_0 - \lambda K^{-1} (K \chi(y)) K^{-1} G_{yy_1} (\eta \tilde{\chi})_{y_1}
\]

\[
- \frac{1}{2} \chi \cdot K^{-1} \tilde{G}_{yy_1}^T (\eta \chi)_{y_1} - \delta (y - y_1) \frac{1}{2} K^{-1} (\eta \chi y_1) = 0. \tag{63}
\]
Figure 1: Diagrams corresponding to the non-trivial part $I_2^{(2)}$ of the double emission from pomerons. Crosses mark $\chi$ and $G$, full circles mark $\tilde{\chi}$ and $\tilde{G}$.

Applying operator $G^T$ we get

$$\Pi_0 = \lambda \left[ G^T K^{-1}(K\chi)K^{-1}G \right]_{y'y_1} (\eta\tilde{\chi})_{y_1} + \frac{1}{2} \lambda \left[ G^T \chi K^{-1}\tilde{G}^T \right]_{y'y_1} (\eta\chi)_{y_1} + \frac{1}{2} G^T_{yy_1} K^{-1}(\eta\chi)_{y_1}. \quad (64)$$

Similar operations with Eq (58) first give the equation

$$\frac{\partial}{\partial y} + H + \lambda \tilde{\chi} \Psi - 2\lambda K^{-1}(K\tilde{\chi})K^{-1}\tilde{G}^T_{y,y_1}(\eta\chi)_{y_1} - \lambda \tilde{\chi} K^{-1}G_{y,y_1}(\eta\tilde{\chi})_{y_1} - \delta(y-y_1)K^{-1}\eta\tilde{\chi}_{y_1} = 0, \quad (65)$$

which after application of operator $\tilde{G}$ gives

$$\Psi = 2\lambda \left[ \tilde{G} K^{-1}(K\tilde{\chi}) \cdot K^{-1}\tilde{G}^T \right]_{y,y_1}(\eta\chi)_{y_1} + \lambda \left[ \tilde{G} \tilde{\chi} K^{-1}G \right]_{y,y_1}(\eta\tilde{\chi})_{y_1} + \tilde{G}^T_{yy_1} K^{-1}\eta\tilde{\chi}_{y_1}. \quad (66)$$

One observes that $\Psi$ is obtained from $2\Pi_0$ by the substitutions

$$G \rightarrow \tilde{G}^T, \quad \tilde{G} \rightarrow G^T, \quad \chi \leftrightarrow \tilde{\chi}. \quad (67)$$

Putting the obtained expressions for the field derivatives (61), (61), (64) and (66) into (55) and using the property (67) we obtain the part $I_2^{(2)}$ of the inclusive cross-section as

$$I_2^{(2)}(y,\kappa|y',\kappa') =$$

$$\int d^2b d^2q_1' d^2q_2' \delta^2(q_1' + q_2' - \kappa') \eta(q_1', q_2')\tilde{\chi}_{y',q_1'} \left[ 2\lambda \left[ G^T K^{-1}(K\chi)K^{-1}G \right]_{y',q_2'|y,q_2} \right] + \left[ G \rightarrow \tilde{G}^T, \quad \tilde{G} \rightarrow G^T, \quad \chi \leftrightarrow \tilde{\chi} \right]. \quad (68)$$

The obtained expression (68) is similar to the one in the framework of LRFT [4], except that in our non-local case all quantities are operators also in the momentum space and that in various places there appear operators $K$ and $K^{-1}$ acting in this space. Its diagrammatic illustration is presented in Fig. 1. In this figure external lines with crosses (circles) show sums of fan diagrams propagating towards the target $\chi$ (projectile $\tilde{\chi}$). Correspondingly the internal lines marked with crosses (circles) show the Green functions $G$ ($\tilde{G}$). Horizontal lines indicate the two observed particles. To the diagrams shown in Fig.1 one should add similar ones with the target and projectile interchanged.

From the practical point of view expression (68) is not very useful due to multiple integrations in $(y,q)$ space. Numerical solution of evolution equations (56)-(59) seems more promising.
4 Emission from the vertex

To include emission from the triple-pomeron vertex we have to add new parts to the Lagrangian which describe this emission. In the splitting vertex the pomeron before the split has always to be cut. The two emerging pomerons may be both cut and uncut. In accordance with the structure of the interaction in Eq. (12) the vertex emission part of the action is to be taken as

\[ -A_\gamma = \int dyd^2bd^2q_1d^2q_2d^2q_3 L_\gamma + h.c., \]  

(69)

where

\[ L_\gamma = \sqrt{2} \psi^\dagger_c(q_1) \left( \gamma_d(q_1|q_2, q_3) \psi_+(q_2) \psi_-(q_3) - \sqrt{2} \gamma(q_1|q_2, q_3) \psi_c(q_2)(\psi_+(q_3) + \psi_-(q_3)) \right) \]

\[ + \gamma_c(q_1|q_2, q_3) \psi_c(q_2) \psi_c(q_3) \right). \]  

(70)

The part explicitly shown corresponds to the emission from the splitting vertex. The part corresponding to the emission from the merging vertex is indicated as h.c.. The vertex functions \( \gamma_d, \gamma \) and \( \gamma_c \) describe emissions from the diffractive, single and double cuts of the triple pomeron vertex respectively. They are real functions of the three relative momenta of the joining pomerons and carry factor \( \delta^2(\kappa + q_1 - q_2 - q_3) \) where \( \kappa \) is the momentum of the emitted jet. They are different and their form has been found in \([1]\) and reproduced in Appendix 2. The fields in (70) are supposed to be taken at the same rapidity \( y \) and transverse point \( b \), which dependence is not shown explicitly. In the perturbation expansion the new interaction term \( A_\gamma \) has to be taken the number of times which corresponds to the number of the emissions from the vertex. So for the single inclusive cross-section we have to take it only once and for the double inclusive cross-section at most twice.

The single inclusive cross-section corresponding to the emission from the vertex is obtained from (70) just by substituting the fields by solutions of the equations of motion (31) - (34) at \( \xi = 0 \) and integrating over \( b \). For the fields \( \psi \) and \( \psi^\dagger \) these solutions are

\[ \psi_\pm = \tilde{\chi}, \quad \psi_c = \sqrt{2} \tilde{\chi}, \quad \psi^\dagger_\pm = \chi, \quad \psi^\dagger_c = \sqrt{2} \chi. \]  

(71)

Thus we find a contribution

\[ I_1^{(\gamma)}(y, \kappa) = 2 \int d^2b \prod_{i=1}^3 d^2q_i \left( \gamma_d(q_1|q_2, q_3) - 4\gamma(q_1|q_2, q_3) + 2\gamma_c(q_1|q_2, q_3) \right) \]

\[ \left( \chi(q_1)\tilde{\chi}(q_2)\tilde{\chi}(q_3) + \tilde{\chi}(q_1)\chi(q_2)\chi(q_3) \right). \]  

(72)

The total vertex

\[ \gamma^{tot} = \gamma_d - 4\gamma + 2\gamma_c \]  

(73)

has a simple form in the coordinate space. If we introduce the coordinate vertex \( \Gamma(r_1|r_2, r_3) \) acting on non-amputated pomerons according to the relation

\[ \int d^2q_1d^2q_2d^2q_3 \gamma(q_1|q_2, q_3)\tilde{\chi}(q_2)\tilde{\chi}(q_3) = \int d^2r_1d^2r_2d^2r_3 Z(r_1)\Gamma(r_1|r_2, r_3)\tilde{Z}(r_2)\tilde{Z}(r_3), \]  

(74)
where \( \chi(q) \) and \( Z(r) \) are related by (11), then one finds (7)

\[
\Gamma^{\text{tot}}(r_1 | r_2, r_2) = -\frac{2\alpha_s N_c}{\kappa^2} \nabla_1^2 e^{i\kappa r_1} \nabla_2^2 \delta^2(r_2 - r_1) \delta^2(r_3 - r_1).
\] (75)

It corresponds to the expression first obtained in (8) as a contribution additional to the emission from the pomeron.

Passing to the double inclusive cross-section we first find a contribution corresponding to the emission from two vertices, which is obtained by taking a product of two interactions (70) with different external momenta \( \kappa \) and \( \kappa' \), at different rapidities \( y \) and \( y' \) and transverse points \( b \) and \( b' \), substituting in them the fields by the solution of the equations of motion and integrating over both transverse points \( b \) and \( b' \). As a result we obviously find a product of two single inclusive cross-sections (72):

\[
I_2^{\gamma\gamma}(y, \kappa | y', \kappa') = I_1^{\gamma}(y, \kappa) I_1^{\gamma}(y', \kappa').
\] (76)

To find the mixed contribution in which one jet is emitted from the vertex and the other from the pomeron we have to consider the theory with the interaction term \( \Delta A_c \), Eq. (18), and once differentiate in \( \xi \):

\[
I_2^{\gamma}(y, \kappa | y', \kappa') = \int d^2 b d^2 b' \prod_{i=1}^3 d^2 q_i d^2 q'_i d^2 q'_2 \delta^2(q'_1 + q'_2 - \kappa') \eta(q'_1, q'_2)
\]

\[
\left[ \frac{\delta}{\delta \xi(y', q'_1, q'_2, b')} L_\gamma(y, b, \kappa, q_1, q_2, q_3, \xi) e^{-A_c(\xi)} \right]_{\xi=0} + \left( y \leftrightarrow y', \kappa \leftrightarrow \kappa' \right).
\] (77)

Differentiation in \( \xi \) will give two terms. One comes from the differentiation of the exponential. After integrations over \( b' \) and \( q'_1, q'_2 \) with weight \( \eta \) this differentiation will give the single inclusive cross-section \( I_1(y', \kappa') \) corresponding to emission from the pomeron. Factor \( L_\gamma \) at \( \xi = 0 \) will generate the single inclusive cross-section (72). As a result this part gives a contribution

\[
I_1^{\gamma}(y, \kappa) I_1(y', \kappa') + I_1^{\gamma}(y', \kappa') I_1(y, \kappa).
\] (78)

If we introduce the total single inclusive cross-section

\[
I_1^{\text{tot}}(y, \kappa) = I_1(y, \kappa) + I_1^{\gamma}(y, \kappa)
\] (79)

then collecting (47), (78) and (76) we find a factorized contribution to the double inclusive cross-section

\[
I_2^{\text{fact}}(y, \kappa | y', \kappa') = I_1^{\text{tot}}(y, \kappa) I_1^{\text{tot}}(y', \kappa').
\] (80)

The second part of \( I_2^{\gamma\gamma} \) will come from the differentiation in \( \xi \) of the fields inside \( L_\gamma \), the exponential factor giving unity. Differentiation in \( \xi \) together with integrations over \( q'_1 \) and \( q'_2 \) with weight \( \eta \) will substitute fields in accordance with Eqs. (50) - (54). If we define

\[
D \equiv \int d^2 q_1 d^2 q_2 \delta^2(q_1 + q_2 - \kappa) \eta(q_1, q_2) \frac{\delta}{\delta \xi(y_1, q_1, q_2)} |_{\xi=0}
\] (81)

then in terms of fields \( \psi \) and \( \psi^\dagger \) we find

\[
D \psi_c = \frac{1}{\sqrt{2}} \Psi, \quad D \psi_{\pm} = \Phi, \quad D \psi^\dagger_c = \sqrt{2} \Pi_0, \quad D \psi^\dagger_{\pm} = \Pi + \Pi_0.
\] (82)
We recall that the derivatives are proportional to $\delta^2(b-b')$, so that the double integration over $b$ and $b'$ turns into a single one. We then obtain the following expression for the non-factorized part of the double inclusive cross-section $I^{(\gamma)}_2$

$$I^{(\gamma,nf)}_2(y,\kappa|y',\kappa') = \int d^2b \prod_{i=1}^3 d^2q_i \left\{ 2\gamma^\text{tot}(q_1|q_1,q_3)\Pi_0(q_1)\tilde{\chi}(q_2)\tilde{\chi}(q_3) 
+ 2\chi(q_1)\tilde{\chi}(q_3) \left[ \gamma^\text{tot}(q_1|q_2,q_3)\Psi(q_2) + \left( \gamma_d(q_1|q_2,q_3) - 2\gamma(q_1|q_2,q_3) \right)\Phi_0(q_2) \right] 
+ \gamma^\text{tot}(q_1|q_1,q_3)\Psi(q_1)\chi(q_2)\chi(q_3) 
+ 4\tilde{\chi}(q_1)\chi(q_3) \left[ \gamma^\text{tot}(q_1|q_2,q_3)\Pi_0(q_2) + \left( \gamma_d(q_1|q_2,q_3) - 2\gamma(q_1|q_2,q_3) \right)\Pi(q_2) \right] \right\} 
+ \left( y \leftrightarrow y', \kappa \leftrightarrow \kappa' \right).$$

In this expression it is assumed that the derivative fields $\Phi_0$, $\Pi$, $\Psi$ and $\Pi_0$, apart from the argument explicitly shown, depend on their 'own' rapidity $y$ and transverse point $b$ and also on rapidity $y'$ and external momentum $\kappa'$ which enter in their definitions (50) - (54). Due to property (67) the last two terms in (83) are obtained from the first two ones by interchanging the target and projectile. Graphical illustration of $I^{(\gamma,nf)}_2$ is presented in Fig. 2 which shows diagrams corresponding to the six terms in (83). The notations are as in Fig. 1. To the diagrams shown in Fig. 2 one has to add the diagrams which are obtained by the interchange of the target and projectile corresponding to the last two terms in (83) and also diagrams from the the interchange $y \leftrightarrow y'$, $\kappa \leftrightarrow \kappa'$.

5 Conclusions

We have derived expressions for the single and double inclusive cross-sections in nucleus-nucleus collisions in the framework of the perturbative QCD with interacting BFKL pomeron, in the quasi-classical approximation (without loops). The cross-sections include terms with emissions both from the pomeron and from the triple pomeron vertex. The obtained single inclusive cross-sections are simple. As expected from the AGK rules
they reduce to emissions from the central pomeron in the convolution of two sets of fan diagrams connecting it with the projectile and target and from the two neighboring vertices by which this pomeron splits into fans. This expected form of the single inclusive cross-sections was used in the calculations in [6].

In contrast, the expressions for the double inclusive cross-sections are much more complicated. A part of it is just the product of two single inclusive cross-sections corresponding to independent emissions of two jets from different points in the nuclear overlap transverse space. The other part however includes all sorts of rescattering corrections and needs summation of diagrams of a structure different from fans. It also includes emissions from the triple pomeron vertex having different forms for different cuts passing through the vertex. We have set up evolution equations which allow to find the rescattering corrections to the double inclusive cross-sections. Different forms of emission from the vertex have been found in our paper [7]. So in principle the theoretical basis for the calculation of the double inclusive cross-section is completed. However practical computations along these lines seem to be quite complicated and apparently present a separate (and formidable) task.

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7 Appendix 1. The unitarity equation with cut pomeron

The derivation with BFKL pomeron follows that in [4] for local pomeron. The starting point is the equation which tells that the sum of all Green functions in the cut pomeron theory is equal to the sum of all Green function in the original theory, provided the couplings of the fields to themselves and external sources are chosen in accordance with the AGK rules. It is written as

$$\sum_{n,m} g^n G^{(n,m)} \frac{f^m}{m!} = \sum_{n_i,m_j} i g^n (\sqrt{2})^{n_c-1} G^{(n_i,m_j)} i f^m (\sqrt{2})^{m_c-1} \frac{n_i! n_j!}{m_i! m_j!}.$$ (84)

Here $G^{(n,m)}$ are the Green function in the original theory with $n$ incoming and $m$ outgoing pomeron and $G^{(n,m)}$ are the Green function in the cut pomeron theory, where $i, j = +, -, c$, $n_i$ is the number of incoming pomeron of sort $i$, $n = n_+ + n_+ + n_c$ and $m_j$ is the number of outgoing pomeron of sort $j$, $m = m_+ + m_+ + m_c$. Symbol $\Sigma'$ means that one should drop terms with $n_+ = n_+ = 0$ and $m_+ = m_+ = 0$ which are absent in the cut theory. The Green functions are assumed to be operators acting in the rapidity-transverse momentum space, convoluted with the external sources given by functions $g$ and $f$.

One then uses obvious properties

$$G^{(n_+ = n_+ = n_c = 0; m_+ = m_+ = m_c = 0)} = G^{(n,m)},$$ (85)

$$G^{(n_+ = n_+ = n_c = 0; m_+ = m_+ = m_c = 0)} = 0$$ (86)
to consider the sum \((84)\) without restrictions on the summations over \(n_i\) and \(m_j\) and multiplied by 2. One has
\[
\sum_{n_i,m_j} i g^n(\sqrt{2})^{n_c} G^{(n_i,m_j)} \frac{i f^m(\sqrt{2})^{m_c}}{m_+!m_-!m_c!} = 2 \sum_{n,m>0} \sum_{n_i,m_j} i g^n(\sqrt{2})^{n_c-1} G^{(n_i,m_j)} \frac{i f^m(\sqrt{2})^{m_c-1}}{m_+!m_-!m_c!} + 2 \sum_{n,m>0} \frac{g^n}{n!} G^{n,m} \frac{f^m}{m!} + 1. \tag{87}
\]
where in the first sum of the second equality \(n = n_+ + n_- + n_c\) and \(m = m_+ + m_- + m_c\).

However according to \((84)\) the first term is exactly equal to the second with the opposite sign. So we are left with
\[
\sum_{n_i,m_j} i g^n(\sqrt{2})^{n_c} G^{(n_i,m_j)} \frac{i f^m(\sqrt{2})^{m_c}}{m_+!m_-!m_c!} = 1. \tag{88}
\]

The sum in \((88)\) is just the total \(S\) matrix in the cut pomeron theory, so that the equality \((88)\) means that action \(A_c(\xi) = 0\).

In the theory with the action \(A_c(\xi)\) the amplitude \(T_c(\xi)\) is
\[
T_c(\xi) = \frac{1}{2} \sum_{n_i,m_j} g^n(\sqrt{2})^{n_c} G^{(n_i,m_j)}(\xi) \frac{f^m(\sqrt{2})^{m_c}}{m_+!m_-!m_c!}. \tag{89}
\]
Adding and subtracting terms with \(n_+ = n_c = 0\) and \(m_- = m_c = 0\) and using properties \((85)\) and \((86)\) we find
\[
T_c(\xi) = \frac{1}{2} \sum_{n_i,m_j} g^n(\sqrt{2})^{n_c} G^{(n_i,m_j)}(\xi) \frac{f^m(\sqrt{2})^{m_c}}{m_+!m_-!m_c!} - \sum_{m,n} \frac{g^n}{n!} G^{n,m} \frac{f^m}{m!} + \frac{1}{2}, \tag{90}
\]
where again in the first sum \(n = n_+ + n_- + n_c\) and \(m = m_+ + m_- + m_c\). The first sum is just one half of \(e^{-A_c(\xi)}\) while the second is \(e^{-A}\) in the original theory, so that Eq. \((90)\) leads to Eq. \((21)\).

### 8 Appendix 2. Cut vertex functions \(\gamma_d, \gamma\) and \(\gamma_c\)

The three cut vertex functions \(\gamma_d, \gamma\) and \(\gamma_c\) which enter \((70)\) can be most conveniently written in the coordinate space as functions \(\Gamma_d, \Gamma\) and \(\Gamma_c\) acting on non-amputated pomeron functions according to \((74)\). To simplify notations we introduce a vector
\[
h(r_1, r_2) = \frac{r_1}{r_1^2} - \frac{r_1 + r_2}{(r_1 + r_2)^2}. \tag{91}
\]
From the results obtained in \((71)\) we find
\[
\Gamma_d(r_1 | r_2, r_3) = \frac{2\alpha_s N_c}{(2\pi)^2} \nabla^4 \left\{ e^{i\kappa(r_2-r_3)} h(r_2, r_1) h(r_3, r_1) - e^{i\kappa(r_2-r_3+r_1)} h(r_2, r_1) h(r_3, -r_1) \right\}
\]
Due to symmetry properties of the pomerons, these expressions have to be symmetrized

\[ +4i\pi e^{ikr}(1 - e^{ikr_1}) \frac{K}{K^2} h(r_2, r_1) \delta^2(r_3 - r_1) + 4\pi^2 \frac{K^2}{K^2} (1 - e^{ikr_1}) \delta^2(r_2 - r_1) \delta^2(r_3 - r_1) \]

\[ - \frac{4\pi^2}{K^2} \nabla^{-2} e^{ikr_1} \nabla^2 \delta^2(r_2 - r_1) \delta^2(r_3 - r_1) \} , \]

\[ \Gamma(r_1|r_2,r_3) = \frac{\alpha_s N_c}{2(2\pi)^2} \nabla^4 \]

\[ \{ e^{ikr_2 - r_3} h(r_2, r_1) h(r_3, r_1) - 2e^{ikr_2 - r_3 + r_1} h(r_2, r_1) h(r_3, -r_1) \]

\[ -6i\pi i^{ikr_2} \frac{K}{K^2} h(r_2, r_1) \delta^2(r_3 - r_2) - 3i\pi e^{ikr_2}(1 - e^{ikr_1}) \frac{K}{K^2} h(r_2, r_1) \delta^2(r_3 - r_1 - r_2) \]

\[ -3e^{ikr_3} h(r_2, r_1) h(r_2 + r_3, r_1) + 2i\pi e^{ikr_2}(1 - e^{ikr_1}) \frac{K}{K^2} h(r_2, r_1) \delta^2(r_3 - r_1) \]

\[ + \frac{4\pi^2}{K^2} (1 + e^{ikr_1}) \delta^2(r_2 - r_1) \delta^2(r_3 - r_1) - \frac{16\pi^2}{K^2} \nabla^{-2} e^{ikr_1} \nabla^2 \delta^2(r_2 - r_1) \delta^2(r_3 - r_1) \} , \]

\[ \Gamma_c(r_1|r_2,r_3) = \frac{2\alpha_s N_c}{(2\pi)^2} \nabla^4 \]

\[ \{ - e^{ikr_2 - r_3 + r_1} h(r_2, r_1) h(r_3, -r_1) - 3e^{ikr_3} h(r_2, r_1) h(r_2 + r_3, r_1) \]

\[ -3i\pi e^{ikr_2}(1 - e^{ikr_1}) \frac{K}{K^2} h(r_2, r_1) \delta(r_3 - r_1 - r_2) + 2i\pi e^{ikr_2} \frac{K}{K^2} h(r_2, r_1) \delta(r_3 - r_2) \]

\[ + 2i\pi e^{ikr_2} \frac{K}{K^2} h(r_2, r_1) \delta(r_3 - r_1) - 16\pi^2 \nabla^{-2} e^{ikr_1} \delta^2(r_2 - r_1) \delta^2(r_3 - r_1) \]

\[ - \frac{8\pi^2}{K^2} \nabla^{-2} e^{ikr_1} \nabla^2 \delta^2(r_2 - r_1) \delta^2(r_3 - r_1) \} . \]

Due to symmetry properties of the pomerons, these expressions have to be symmetrized in \( r_2 \) and \( r_3 \). Also all exponentials have to be substituted by their real and imaginary parts:

\[ e^{ikr} \rightarrow \cos kr, \quad ie^{ikr} \rightarrow -\sin kr, \]

which makes the cut vertexes real.

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