Confinement in the Three-dimensional Anisotropic $t$-$J$

Model: A Mean-Field Study

Sanjoy K. Sarker

Department of Physics and Astronomy

The University of Alabama

Tuscaloosa, Alabama 35487

Abstract

We consider the anisotropic $t$-$J$ model with the c-axis parameters $t_c$ and $J_c$ different from their in-plane counterparts, $t$ and $J$. Within the slave-fermion mean-field approximation it is shown that the spiral state exhibits charge-confinement in the intermediate $\delta$ regime for a range of values of $t_c/t$. In the confined state the hopping amplitude $\langle c_{i\sigma}^\dagger c_{j\sigma} \rangle = 0$ along the c direction so that c-axis resistivity is infinite at $T = 0$. 

1
The normal-state resistivity of the cuprate superconductors is metallic in the \( ab \) plane but is characteristic of an insulator along the \( c \) direction, leading to the possibility of the remarkable phenomenon of confinement \([1, 2]\). By continuity, such a behavior is not expected to occur in a Fermi liquid. In addition, confinement is at the heart of the proposed pair-tunneling mechanism which is in essence a deconfining process \([3]\). Theoretical treatments so far have been focused on a collection of weakly coupled Hubbard chains \([2]\). In this paper we study confinement in a collection of Hubbard planes, more specifically in the anisotropic \( t-J \) model described by the Hamiltonian

\[
H = -\sum_{ij} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \frac{1}{2} \sum_{ij} J_{ij} [S_i \cdot S_j - n_i n_j]. \tag{1}
\]

Here the in-plane hopping parameter \( t_{ab} \equiv t \) and the exchange interaction \( J_{ab} \equiv J \) are in general different \( t_c \) and \( J_c \), the corresponding quantities along the \( c \) axis. Not all the parameters are free since the \( t-J \) model is thought to be derived from an underlying Hubbard model \( (J_{ij} = 4t_{ij}^2/U) \) so that \( J_c/J = (t_c/t)^2 \). There are therefore three independent parameters: \( t/J, \xi \equiv t_c/t \) and the hole density \( \delta \).

Confinement is presumably intimately connected with spin-charge separation. While model (1) is not exactly solvable, in two dimensions a number of approximate ground states have been proposed that exhibit spin-charge separation. Here we study the spiral states in the Schwinger-boson slave-fermion representation: \( c_{i\sigma} = h_i^\dagger b_{i\sigma} \), where \( h_i^\dagger \) creates a fermionic hole and \( b_{i\sigma} \) destroys a bosonic spin \([4]\). We will impose the constraint \( h_i^\dagger h_i + \sum_\sigma b_{i\sigma}^\dagger b_{i\sigma} = 1 \) on the average. A mean-field decomposition leads to following hamiltonians:

\[
H_h = 2 \sum_{ij} t_{ij} B_{ij} h_i^\dagger h_j \tag{2}
\]

\[
H_b = \sum_{i j \sigma} t_{ij} D_{ij} b_{i\sigma}^\dagger b_{j\sigma} - \sum_{i j \sigma} J_{ij} A_{ij} \sigma b_{i\sigma} b_{j-\sigma}. \tag{3}
\]

Here \( D_{ij} = \langle h_i^\dagger h_j \rangle \) is the average hopping amplitude. This is associated with ferromagnetic backflow \( B_{ij} = \langle b_{i\sigma}^\dagger b_{j\sigma} \rangle \). And \( A_{ij} = \frac{1}{2} < (b_{i\uparrow} b_{j\downarrow} - b_{i\downarrow} b_{j\uparrow}) > \) represents the antiferromagnetic correlations associated with the exchange term. We have shown that in two dimensions this competition gives rise to \([4, 5]\) an incommensurate spiral metallic state that evolves
continuously from the Neel state at $\delta = 0$ to a ferromagnetic state at large $t\delta/J$. The spiral state is favored over double spiral and canted states and is stabilized against phase separation and domain walls by Coulomb repulsion \[3, 4\].

In the present problem the in-plane amplitudes ($D_{ab}, B_{ab}$ and $A_{ab}$) are in general different from those along the $c$ directions ($D_c, B_c$ and $A_c$). We have solved the mean-field equations numerically for various values of $t_{ab}/J_{ab}, \xi = t_c/t_{ab}$ and $\delta$. One self-consistent solution is found to be the usual spiral metallic state which evolves continuously out of the Neel state. For this state all the mean-field amplitudes including $D_c$ and $B_c$ are nonzero and hence there is no confinement. In addition, the state with $D_c = 0$ (and hence $B_c = 0$) is also a self-consistent solution. This is clearly a direct consequence of spin-charge separation. Charge can propagate in the $ab$ plane in this case ($D_{ab} \neq 0$). Hence $D_c$ can be thought of as the order-parameter for deconfining transitions.

While spin-charge separation can lead to confinement, it does not guarantee that such a state is favored energetically. The ground-state energy is given by

$$E_G = 8tD_{ab}B_{ab} - 4JR_{ab} + 4t_cD_cB_c - 2J_cR_c,$$  \hspace{1cm} (4)

where $R_{ij} = A_{ij}^2 - B_{ij}^2/2 + (1 - \delta)^2/8$. In general $D$ and $B$ have opposite signs and $R > 0$. For physically interesting values of $t - J$, we find that the unconfined phase has a lower energy both at small and large $\delta$. But, as shown in Fig. 1, for intermediate $\delta$ there is a region in the $\delta-t_c/t$ plane where the confined state is favored. Interestingly, for fixed $t/J$ and $\delta$, the mean-field state is unconfined at small $t_c/t$, and with increasing $t_c/t$ there is a first-order transition to a confined state. This is because as $t_c$ decreases, $J_c \propto t_c^2$ decreases more rapidly. This favors ferromagnetic alignment and deconfinement. These results are summarized in Fig. 2.

For $\delta$ not too large the hole hopping amplitude $D$ in the isotropic 3-D case is given by

$$D_3(\delta) \approx -\delta + \frac{(6\pi^2)^{2/3}}{10}\delta^{5/3}.$$  

In two dimensions, $D_2(\delta) \approx -\delta + \pi\delta^2/2$. Hence, the hopping energy per bond can be lower in two dimensions. When $t_c$ and $\delta$ are not too small, the system can gain maximum exchange energy in the $c$ direction and maximum kinetic energy in the ab plane by having $B_c = D_c = 0$.  

3
To summarize, we have shown that spiral state in the anisotropic $t$-$J$ model exhibits charge confinement. For a more realistic treatment one needs to include fluctuations that destroy long-range magnetic order and reconstruct the Luttinger-Fermi surface, as shown previously for the 2-D model [7]. Such fluctuations are likely to bring the region of stability of the confined state to smaller values of $\delta$ and $t_c/t$. Nonetheless, there are some interesting consequences of confinement in our simple theory. (1) In the confined state the magnetic correlations along the $c$ direction is peaked at $Q_c = \pi$, while the correlations in the $ab$ plane remains incommensurate. (2) The $c$-axis resistivity is strictly infinite at $T = 0$, since $\langle c_i^{\dagger} c_j \rangle = -D_{ij} B_{ij} = 0$ along the $c$ direction. However, at finite $T$ or frequency there will be an incoherent contribution to the conductivity. Such a contribution will be activated if there is a spin gap.
References

[1] P. W. Anderson, preprint.

[2] D. G. Clarke, S. P. Strong and P. W. Anderson, Phys. Rev. Lett. 72, 3218 (1994).

[3] S. Chakravarty, A. Sudbo, P. W. Anderson and S. P. Strong, Science 261, 337 (1993).

[4] C. Jayaprakash, H.R. Krishnamurthy, and S.K. Sarker, Phys. Rev. B40, 2610 (1989).

[5] S. K. Sarker, Phys. Rev. B 46, 8617 (1992).

[6] F. M. Hu, S. K. Sarker and C. Jayaprakash, Phys. Rev. B 50, 17901 (1994).

[7] S. K. Sarker, Phys. Rev. B46, 8617 (1992).
Figure Captions

Fig. 1. Phase diagram in the $\delta-t_c/t$ plane for two $t/J = 3$ (squares) and $t/J = 5$ (diamonds). In each case, the confined state ($D_c = 0$) has a lower energy in the V-shaped region. The transition is first order.

Fig. 2. The hole-hopping amplitude $D_c/(-\delta)$ and the ordering wavevector along the c-direction $Q_z \equiv \frac{Q_c}{2\pi}$ (diamonds) vs $t_c/t$. Note that $Q_c$ is essentially zero (ferromagnetic) in the unconfined phase. Close to the transition it acquires a spiral character, and at the transition jumps to $\pi$ corresponding to antiferromagnetic correlations in the confined phase.