Impacts of Nuclear Symmetry Energy on Sound Speeds and Hadron-Quark Phase Transitions via Spinodal Decompositions in Dense Neutron-Rich Matter

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Using a meta model for nuclear Equation of State (EOS) with its parameters constrained by astrophysical observations and terrestrial nuclear experiments, we examine effects of nuclear EOS especially its symmetry energy $E_{\text{sym}}(\rho)$ term on the speed of sound squared $C_s^2(\rho)$ and the hadron-quark transition density $\rho_t$ where $C_s^2(\rho_t)$ vanishes via the spinodal decomposition mechanism in both dense neutron-rich nuclear matter relevant for relativistic heavy-ion collisions and the $n+p+e+\mu$ matter in neutron stars at $\beta$-equilibrium. Unlike in nucleonic matter with fixed values of the isospin asymmetry $\delta$, in neutron stars with a density dependent isospin profile $\delta(\rho)$ determined consistently by the $\beta$ equilibrium and charge neutrality conditions, the $C_s^2(\rho)$ almost always show a peak and then vanishes at $\rho_t$ strongly depending on the high-density behavior of $E_{\text{sym}}(\rho)$ if the skewness parameter $J_0$ characterizing the stiffness of high-density symmetric nuclear matter (SNM) EOS is not too far above its currently known most probable value of about $-190 \text{ MeV}$ inferred from recent Bayesian analyses of neutron star observables. Moreover, in the case of having a super-soft $E_{\text{sym}}(\rho)$ that is decreasing with increasing density above about twice the saturation density of nuclear matter, the $\rho_t$ is significantly lower than the density where the $E_{\text{sym}}(\rho)$ vanishes (indicating the onset of pure neutron matter formation) in neutron star cores.

I. INTRODUCTION

The energy per nucleon $E(\rho, \delta)$ in cold neutron-rich nuclear matter of density $\rho$ and isospin asymmetry $\delta = (\rho_n - \rho_p)/\rho$ where $\rho_n$ and $\rho_p$ are the densities of neutrons and protons, respectively, can be written as

$$E(\rho, \delta) = E_0(\rho) + E_{\text{sym}}(\rho) \cdot \delta^2 + \mathcal{O}(\delta^4) \quad (1)$$

where $E_0(\rho)$ is the energy per nucleon in symmetric nuclear matter (SNM) while the symmetry energy $E_{\text{sym}}(\rho)$ encodes the energy cost to make the matter more neutron-rich $E_{\text{sym}}(\rho)$. Both the $E_0(\rho)$ and $E_{\text{sym}}(\rho)$ are important for the stiffness and stability of neutron-rich nuclear matter. For example, within the minimal model of neutron stars consisting of neutrons (n), protons (p), and electrons (e) the incompressibility of $n+p+e$ matter at $\beta$ equilibrium is approximately

$$K_\mu(\rho) = \rho \frac{d^2 E_\text{sym}}{d \rho^2} + 2 \rho \frac{d E_\text{sym}}{d \rho} + \frac{\delta^2}{2} \left[ 2 \rho \frac{d^2 E_\text{sym}}{d \rho^2} + 2 \rho \frac{d E_\text{sym}}{d \rho} - 2 E_{\text{sym}}(\rho) (\rho \frac{d E_\text{sym}}{d \rho})^2 \right].$$

When $K_\mu(\rho)$ is negative the corresponding speed of sound $C_s(\rho)$ becomes imaginary, indicating the onset of the so-called spinodal decomposition. When this happens, the density fluctuations in the dynamically unstable region grow exponentially. At zero temperature, the $K_\mu$ can become negative at both sub-saturation and supra-saturation densities depending on the characteristics of both $E_0(\rho)$ and $E_{\text{sym}}(\rho)$. At low densities, the onset of spinodal decomposition in either the $n+p+e$ matter or purely nucleonic matter is often used to identify the crust-core transition in neutron stars or a liquid-gas phase transition related to the nuclear multifragmentation phenomenon in intermediate energy heavy-ion reactions. In hot dense hadronic matter formed in relativistic heavy-ion reactions, a high-density spinodal decomposition has been associated with a first-order hadron-quark phase transition, see, e.g., Refs. Many potential signatures of such a transition have been proposed. For example, the resulting time delay of expansion and/or the rapid growth of fluctuations after the spinodal decomposition may lead to the disappearance of elliptic flow or a minimum in the excitation function of collective flow or an enhanced production of composite particles in relativistic heavy-ion collisions. In cold neutron star matter, when the high-density SNM EOS $E_0(\rho)$ is not very soft and the symmetry energy $E_{\text{sym}}(\rho)$ is very soft, the spinodal decomposition was also found to occur in $n+p+e$ matter at supra-saturation densities indicating possibly a hadron-quark phase transition in the cores of neutron stars. Between the crust-core and hadron-quark transition densities in neutron stars, the $K_\mu$ ($C_s^2$) peaks at certain density depending on the EOS of neutron-rich matter. There have been many extensive studies on the possible first-order phase transition in neutron stars, see, e.g., Refs. for two examples.

While the effects of EOS on the crust-core (liquid-gas) transition density and pressure have been extensively studied, see, e.g., reviews in Refs. little is known about the hadron-quark transition density. In fact, determining the latter and the associated phase structures of dense matter has been a long standing goal of both nuclear physics and astrophysics. It is well known that QCD is still facing fundamental difficulties at finite baryon densities to predict accurately the hadron-quark transition density. Often in models considering a first-order hadron-
quark phase transition in heavy-ion reactions and/or neutron stars, the transition density \( \rho_t \) is normally used as a free model parameter, see, e.g., Refs. [22, 31], that may be inferred from observational data. Sometimes parameterized \( C_s(\rho) \) functions are used as inputs in constructing the EOSs to be tested against experimental data. Alternatively, the \( \rho_t \) may be obtained by examining the high-density behavior of \( C_s^2 \) and its possible vanishing point in the hadronic phase.

In this work, we study the \( C_s^2(\rho) \) and the \( \rho_t \) via the spinodal decomposition mechanism in both dense neutron-rich nucleonic matter relevant for relativistic heavy-ion collisions and the \( n + p + e + \mu \) matter in neutron stars at \( \beta \)-equilibrium using a meta model for nuclear EOS with its parameters constrained by astrophysical observations and terrestrial nuclear experiments. We focus on effects of nuclear EOS especially its symmetry energy \( E_{\text{sym}}(\rho) \) term on the \( C_s^2(\rho) \) and \( \rho_t \). Because of the long time needed for \( \beta \)-equilibrium, it is the partial derivative of pressure \( \partial P(\rho, \delta)/\partial \rho \) at constant \( \delta \) determines the \( C_s^2 \) in relativistic heavy-ion reactions while the \( \partial P(\rho, \delta)/\partial \rho \) determines that in neutron stars with their density dependent isospin profile \( \delta(\rho) \) determined consistently by the \( \beta \)-equilibrium and charge neutrality conditions. We found that the \( C_s^2(\rho) \) in neutron stars almost always show a peak and then vanishes at a \( \rho_t \) strongly depending on the high-density behavior of \( E_{\text{sym}}(\rho) \) if the skewness parameter \( J_0 \) characterizing the stiffness of high-density SNM EOS is not too far above its currently known most probable value of about \(-190 \text{ MeV} \).

The rest of this paper is organized as follows. In the next section, we summarize the major ingredients and available constraints of a meta EOS model for nucleonic matter and neutron star matter. We also recall definitions of a few relevant physical quantities. We present and discuss our results in section IV. We then conclude in section IV.

II. SUMMARY OF A META MODEL EOS FOR NEUTRON-RICH MATTER

In a meta model for neutron-rich matter [32], the SNM EOS \( E_0(\rho) \) and symmetry energy \( E_{\text{sym}}(\rho) \) in Eq. (1) are parameterized respectively according to

\[
E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2} \frac{\rho - \rho_0}{3\rho_0}^2 + \frac{J_0}{6} \frac{\rho - \rho_0}{3\rho_0}^3,
\]

where \( E_0(\rho_0) = -15.9 \text{ MeV} \) and

\[
E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + L \left( \frac{\rho - \rho_0}{3\rho_0} \right)^2 + \frac{K_{\text{sym}}}{2} \frac{\rho - \rho_0}{3\rho_0}^2 + \frac{J_{\text{sym}}}{6} \frac{\rho - \rho_0}{3\rho_0}^3.
\]

Such models have been widely used in Bayesian inferences of nuclear EOS from neutron star observables or forward modelings of the structures of neutron stars and nuclei as well as their mergers and collisions. The parameters involved have the asymptotic boundary conditions that they become Taylor expansion coefficients when the above parameterizations are used near the saturation density \( \rho_0 \) of SNM and \( \delta = 0 \). Thus, the fiducial values of the incompressibility \( K_0 = 9\rho_0^2 \partial^2 E_0(\rho)/\partial \rho^2 \big|_{\rho=\rho_0} \) and skewness \( J_0 = 27\rho_0^3 \partial^3 E_0(\rho)/\partial \rho^3 \big|_{\rho=\rho_0} \) of nuclear EOS \( E_0(\rho) \) at \( \rho_0 \), the magnitude \( E_{\text{sym}}(\rho_0) \), slope \( L = 3\rho_0 \partial E_{\text{sym}}(\rho)/\partial \rho \big|_{\rho=\rho_0} \), curvature \( K_{\text{sym}} = 9\rho_0^2 \partial^2 E_{\text{sym}}(\rho)/\partial \rho^2 \big|_{\rho=\rho_0} \) and skewness \( J_{\text{sym}} = 27\rho_0^3 \partial^3 E_{\text{sym}}(\rho)/\partial \rho^3 \big|_{\rho=\rho_0} \) of nuclear symmetry energy \( E_{\text{sym}}(\rho) \) at \( \rho_0 \) respectively, from astrophysical observations and terrestrial nuclear experiments provide a guide and some constraints on the parameters of the meta model EOS.

In this work, we fix the low-density EOS parameters at their currently known most probably values, e.g., \( K_0 = 240 \text{ MeV} \) and \( E_{\text{sym}}(\rho_0) = 31.7 \text{ MeV} \) but vary the high-density parameters within their current uncertain ranges if known. For example, the \( L \) parameter is relatively well determined to be around \( L = 57.7 \pm 19 \text{ MeV} \) based on surveys of over 80 analyses of various data of nuclear experiments and astrophysical observations available up to 2021 [34, 50]. While several analyses have found strong supports for \( K_{\text{sym}} \) to be around \(-100 \pm 100 \text{ MeV} \) [34, 50], a larger range is still often used in the literature. As to the \( J_{\text{sym}} \) parameter, only some theoretical predictions put it in the range between -200 to +800 MeV [40, 43]. To our best knowledge, none of the analyses of neither astrophysical nor nuclear physics data has provided a reliable constraint on \( J_{\text{sym}} \). Consequently, the high-density behavior of \( E_{\text{sym}}(\rho) \) above about \( 2\rho_0 \) is largely open [36, 44].

To see how diverse the metal model \( E_{\text{sym}}(\rho) \) can be, a few examples are shown in Fig. 1 by varying the \( K_{\text{sym}} \) and \( J_{\text{sym}} \) parameters in their uncertainty ranges discussed above. It is seen that the generated \( E_{\text{sym}}(\rho) \) ranges from being super-soft (first increases up to about \( 2\rho_0 \) then start decreasing with increasing density and can even become negative at high densities) to super-hard (first decreases up to about \( -2 \rho_0 \) then start increasing with increasing density and can even become negative at high densities).
stiff (increases with increasing density much faster than linear). Interestingly, one can even simulate a “cusp” in $E_{\text{sym}}(\rho)$ due to a topology change as predicted by a pseudo-conformal symmetry theory [45]. To our best knowledge, currently neither experimental findings nor fundamental physics principles can rule out any of the high-density behaviors of $E_{\text{sym}}(\rho)$ illustrated in Fig. [1]

It has been recognized that the poorly known isospin-dependent tensor force and the corresponding short-range nucleon-nucleon correlation as well as the spin-isospin dependence of three-body force in neutron-rich matter are among the most important origins of the wide open behavior of high-density symmetry energy [10]. Impacts of the diverse high-density behaviors of $E_{\text{sym}}(\rho)$ on properties of neutron stars and heavy-ion collisions have also been studied in several works, see, e.g., Refs. [36, 45–54].

On the other hand, there are several available constraints on the $J_{0}$ parameter which characterizes the stiffness of SNM EOS at densities above about (2–3)$\rho_{0}$. For example, recent Bayesian analyses of neutron star radii from LIGO and NICER observations have inferred the most probable value of $J_{0}$ to be about $J_{0} = -190^{+40}_{-40}$ at 68% confidence level [56, 57]. While its most probable value extracted from a Bayesian analysis of nuclear collective flow in relativistic heavy-ion collisions is $J_{0} = -180^{+100}_{-100}$ MeV at 68% confidence level [58]. Moreover, it was found that to support a neutron star of mass 2.17 $M_{\odot}$ a minimum value of about $J_{0} = -180$ MeV is needed [54]. While the causality requirement can provide an upper limit on $J_{0}$, the range of about $-90 < J_{0} < 200$ MeV obtained in this way depends strongly on the correlation between the high-density SNM EOS and symmetry energy [59]. We will thus explore effects of $J_{0}$ in a large range around its most probable value of about -190 MeV.

The energy density in neutron stars consisting of neutrons, protons, electrons, and muons at $\beta$-equilibrium is given by

$$\epsilon(\rho, \delta) = \rho[E(\rho, \delta) + M_{N}] + \epsilon_{l}(\rho, \delta),$$

where $M_{N}$ represents the average nucleon mass and $\epsilon_{l}(\rho, \delta)$ denotes the lepton energy density. The pressure is then

$$P(\rho, \delta) = \rho^{2} \frac{d(\rho(\rho, \delta)/\rho)}{d\rho}.$$

The pressure in neutron stars at $\beta$-equilibrium becomes barotropic once the density-profile of the isospin asymmetry $\delta(\rho)$ is obtained self-consistently from the $\beta$-equilibrium condition

$$\mu_{n} - \mu_{p} = \mu_{e} = \mu_{\mu} \approx 4\delta E_{\text{sym}}(\rho)$$

and the charge neutrality condition $\rho_{p} = \rho_{e} + \rho_{\mu}$. The chemical potential $\mu_{i}$ for a particle $i$ is obtained from $\mu_{i} = \partial \epsilon(\rho, \delta)/\partial \rho_{i}$. We notice that in nucleonic matter at fixed isospin asymmetries relevant for heavy-ion reactions, the nucleonic part of the above equations gives the corresponding pressure $P(\rho, \delta)$.

For easy of our following discussions, it is useful to recall that the pressure in the n+p+e model of neutron stars is given by

$$P(\rho, \delta) = \rho^{2} \left[ \frac{dE_{\text{SNM}}(\rho)}{d\rho} + \frac{dE_{\text{sym}}(\rho)}{d\rho} \right] \delta^{2} + \frac{1}{2} \rho \left( 1 - \delta \right) \rho E_{\text{sym}}(\rho)$$

and the isospin profile $\delta(\rho)$ (or the corresponding proton fraction $x_{p}(\rho) = (1-\delta)/2$) at $\beta$-equilibrium is determined completely by the $E_{\text{sym}}(\rho)$ via

$$x_{p}(\rho) = 0.048 \left[ \frac{E_{\text{sym}}(\rho)}{E_{\text{sym}}(\rho_{0})} \right]^{3} \left( \frac{\rho}{\rho_{0}} \right) (1 - 2x_{p}(\rho))^{3/2}.$$
longer than the dynamical timescale. The difference between the adiabatic and equilibrium sound speeds determines the g-mode frequency. In fact, the high-density behavior of $E_{\text{sym}}(\rho)$ was found to have a significant effect on the g-mode frequency, see, e.g., Ref. [62] and the review in Section 7.1 of Ref. [29].

We notice that there are many interesting studies on the sound speed in different phases of dense matter or neutron stars assumed to have multiple components under various conditions, see, e.g., Refs. [39, 45, 64–77] and references therein. As outlined above, our main goal here is rather conservative. We focus on studying the density dependence of the speeds of sound and their possible vanishing points.

III. RESULTS AND DISCUSSIONS

In the following, we present our results on the speeds of sound in nucleonic matter and neutron star matter separately. We examine effects of the EOS parameters on the density dependence of the speeds of sound and their possible vanishing points.

A. Speed of sound in neutron-rich nucleonic matter with fixed isospin asymmetries

Shown in Fig. 2 is the speed of sound $C_{NM}^2$ in nucleonic matter with fixed isospin asymmetries of $\delta = 0.1, 0.3$ and 0.6, respectively, as a function of density using the symmetry energy $E_{\text{sym}}(\rho)$ functions with $J_{\text{sym}}$ varying between 0 and 800 MeV as shown in the left panel of Fig. 1. All other EOS parameters are fixed at their currently known most probable values as we discussed earlier. As one expects, as the matter becomes more neutron-rich, effects of $J_{\text{sym}}$ become larger. Interestingly, with relatively small $\delta$ and/or $J_{\text{sym}}$ values, the $C_{NM}^2$ shows a peak.

From inspecting the expression of Eq. (10) for $C_{NM}^2$ in the limiting case of SNM, one can see that while the denominator keeps increasing with density, the incompressibility $K$ in the numerator peaks at certain density since the $K_0$ is positive but $J_0$ is negative. Thus, the $C_{NM}^2$ in SNM normally has a peak. With the parameters $L$ and $K_{\text{sym}}$ fixed, the isospin asymmetric energy, pressure, and its derivative all increase with positive and increasing $J_{\text{sym}}$. Their net effect is adding a positive and continuously increasing contribution to $C_{NM}^2$, making its peak disappears gradually when the $E_{\text{sym}}(\rho)$ becomes very stiff for large $J_{\text{sym}}$ values. Of course, if one uses negative $J_{\text{sym}}$ values leading to super-soft $E_{\text{sym}}(\rho)$ functions, the $C_{NM}^2$ not only has a peak but also vanishes at a lower density $\rho_t$ as we shall discuss next.

As shown in Fig. 1 with small or negative $J_{\text{sym}}$ values, the $E_{\text{sym}}(\rho)$ is super-soft and vanishes at some point or becomes negative at higher densities. It is thus interesting to compare the hadron-quark transition density $\rho_t$ where $C_{NM}^2(\rho_t)=0$ and the critical density where $E_{\text{sym}}(\rho)=0$. It is known that a vanishing $E_{\text{sym}}(\rho)$ in nucleonic matter indicates the onset of the so-called isospin separation instability, namely it is energetically more favorable to split SNM into pure neutron matter and proton matter when the $E_{\text{sym}}(\rho)$ becomes negative [47, 50]. Its possible ramifications in heavy-ion reactions and neutron stars have been studied in several works, see, e.g., Refs. [34, 45, 55].

Shown in Fig. 3 is a comparison of the two transition densities where either $C_{NM}^2(\rho_t)=0$ or $E_{\text{sym}}(\rho)=0$ as a function of $K_{\text{sym}}$ with several small $J_{\text{sym}}$ values all...
Kish quickly, the critical density for $E_{\text{sym}}$ is small leading to very soft $E_{\text{sym}}$ about 2$\rho_d$ determining the high-density behavior of SNM. A stiff $E_{\text{sym}}$ will lead to super-soft symmetry with EOS parameters leading to super-soft $E_{\text{sym}}$. Unless the matter is very neutron-rich (with $\delta_{\text{sym}}$ close to its most probable value of -190 MeV) the $E_{\text{sym}}$ function created with $K_{\text{sym}} = -230$ MeV and $J_0 = 800$ MeV. The results are shown in Fig. 4. It is seen that the peak in the speed of sound gradually disappears if the $J_0$ is far from its most probable value of -190 MeV such that the SNM EOS becomes very stiff at high densities. This effect is stronger for more neutron-rich matter as a very large $J_{\text{sym}}$ of 800 MeV is used. These features are easily understood from the terms involved in the expression for $C_{NM}^2$ in Eq. (10).

In short, in nucleonic matter with fixed isospin asymmetries, if the $J_0$ is close to its most probable value of -190 MeV and the $E_{\text{sym}}(\rho)$ is not too stiff, the $C_{NM}^2(\rho)$ becomes zero at high densities. However, if the $E_{\text{sym}}(\rho)$ is super-stiff especially with high isospin asymmetries the $C_{NM}^2(\rho)$ will keep increasing with density until reaching the causality limit. As we shall illustrate and discuss next, the physical consequence of the last case will be changed completely when the $\beta$-equilibrium condition is introduced in building the EOS for neutron stars.

**B. Speed of sound in charge neutral $npe\mu$ matter in neutron stars at $\beta$-equilibrium**

Besides the contributions from leptons, the most important difference between the EOSs of nucleonic matter at fixed $\delta$ and neutron star matter is the consequence of charge neutrality and $\beta$-equilibrium in the latter. It is well known that in neutron stars at $\beta$-equilibrium, the isospin profile $\delta(\rho)$ is uniquely determined by the Eq. (11). As mentioned earlier, the resulting $\delta(\rho)$ approaches one (zero) when the $E_{\text{sym}}(\rho)$ becomes zero (very high or stiff). This result is also easy to understand qualitatively from the energy conservation point of view. As shown in Eq. (11), the $E(\rho, \delta) \propto E_{\text{sym}}(\rho) \cdot \delta^2$. To conserve the total energy, if the $E_{\text{sym}}(\rho)$ is high (low) at certain density $\rho$ the $\delta(\rho)$ there will be low (high). In both cases, the same isospin asymmetry $\delta$ of the whole system is just being distributed to different density regions according to the well-known isospin fractionation mech-
anism in asymmetric nuclear matter \[11, 78, 81]. Considering two connected regions with local densities \( \rho_1 \) and \( \rho_2 \), respectively, the chemical equilibrium condition requires \( E_{\text{sym}}(\rho_1)\delta(\rho_1) = E_{\text{sym}}(\rho_2)\delta(\rho_2) \). Thus, for a given \( E_{\text{sym}}(\rho) \) function, the local isospin asymmetries will adjust themselves according to the relative symmetry energy in those regions.

Consequently, regardless of the high-density behavior of \( E_{\text{sym}}(\rho) \) (super-soft or super-stiff), the equilibrium speed of sound \( C_s^2(\rho) \) in neutron stars at \( \beta \)-equilibrium always show a peak with its position depends on the values of \( J_{\text{sym}} \) and other EOS parameters as we shall discuss in more detail. Therefore, compared to the results shown in the previous subsection, it is clearly seen that the high-density behaviors of sound speeds in neutron stars at \( \beta \)-equilibrium and nucleonic matter with fixed \( \delta \) values are very different.

Shown in Fig. 5 are the density profile of isospin asymmetry \( \delta(\rho) \) (upper panel) and the corresponding squared speed of sound in neutron stars at \( \beta \)-equilibrium with \( J_{\text{sym}} \) varying between 0 and 800 MeV but other parameters fixed at their currently known most probable values indicated.

FIG. 5: The density profile of isospin asymmetry \( \delta(\rho) \) (upper panel) and the corresponding squared speed of sound \( C_s^2(\rho) \) in unit of \( c^2 \) in neutron stars at \( \beta \)-equilibrium with \( J_{\text{sym}} \) varying between 0 and 800 MeV but other parameters fixed at their currently known most probable values indicated.

Again, it is interesting to compare the critical density where the symmetry energy \( E_{\text{sym}}(\rho) \) =0 with that where the sound speed \( C_s^2 = 0 \) as functions of \( K_{\text{sym}} \) with \( J_{\text{sym}} = 0, -100 \) and -200 MeV, respectively, and all other parameters fixed at their currently known most probable values. It is seen that in the whole EOS parameter space considered, the hadron-quark phase transition density \( \rho_t \) is always lower than the critical density where \( E_{\text{sym}}(\rho) \) =0. Consequences and/or observational evidences of this finding deserve further studies.

C. Hadron-quark transition density via spinodal decomposition in neutron stars at \( \beta \)-equilibrium

We now focus on examining the hadron-quark transition density \( \rho_t \) where \( C_s^2(\rho_t) = 0 \). First, we fix the \( J_0 \) at its most probable value of -190 MeV and study the dependence of \( \rho_t \) on the slope \( L \), curvature \( K_{\text{sym}} \), and skewness \( J_{\text{sym}} \) parameters of \( E_{\text{sym}}(\rho) \) within their current uncertainty ranges in Fig. 6 Overall, a softer \( E_{\text{sym}}(\rho) \) leads to a smaller \( \rho_t \). Approaching the lower
limits of $K_{\text{sym}}$ and $J_{\text{sym}}$ where the $E_{\text{sym}}(\rho)$ is super-soft, the $\rho_t$ can be as low as about 2$\rho_0$. Such a low hadron-quark phase transition density is actually consistent with its lower boundary inferred from several recent Bayesian analyses of neutron star observables \[82\] \[-86\]. However, at the other limit where both the $K_{\text{sym}}$ and $J_{\text{sym}}$ take large positive values, the $E_{\text{sym}}(\rho)$ is super-stiff and the resulting $L$ parameter from 40 to 80 MeV causes an approximately 30% change in $\rho_t$. The wide range of $\rho_t$ due to the uncertainty of the $E_{\text{sym}}(\rho)$ especially at high densities signifies again the need to better constrain the density dependence of nuclear symmetry energy.

Next, we examine in Fig. 8 effects of $J_0$ on $\rho_t$ while the $E_{\text{sym}}(\rho)$ parameters are also varied around their most probable values. For exploration purposes only, in one of the calculations we purposely used a very low $J_0$ of -290 MeV leading to an EOS that is not stiff enough to support a neutron star of mass 2.01 $M_\odot$. We also extended the lower limit of $K_{\text{sym}}$ to -400 MeV that is far below its 1σ lower boundary of $K_{\text{sym}} \approx -100 \pm 100$ MeV from several surveys of its constraints in the literature \[87\]. It is seen that even with such a low $K_{\text{sym}}$ (which controls the behavior of $E_{\text{sym}}(\rho)$ around (1 – 2)$\rho_0$) value, the hadron-quark transition may still happen at densities below about (1.5 – 3)$\rho_0$. However, as the $K_{\text{sym}}$ increases in the case of using a very stiff SNM EOS (e.g., with $J_0 = -90$ MeV shown in the left panel far above its most likely value of -190 MeV used in the middle panel) and a large $J_{\text{sym}}$, the hadron-quark transition may not occur as indicated by the ending of the $\rho_t$ curves with $J_{\text{sym}} = 200$ MeV in the left panel.

At the other extreme of using a very soft SNM EOS with $J_0 = -290$ MeV shown in the right panel), the transition density is again dominated by the SNM EOS and becomes less dependent on the $E_{\text{sym}}(\rho)$ as indicated by the tendency of saturation at the high $K_{\text{sym}}$ limit in the right panel in Fig. 8. These features indicate that there are strong interplays and correlations among the SNM EOS and $E_{\text{sym}}(\rho)$ parameters as one expects.

In short, unlike in nucleonic matter with fixed $\delta$ values, the speed of sound in neutron stars at $\beta$-equilibrium almost always show a peak as long as the $J_0$ characterizing the stiffness of high-density SNM EOS is not too far above its currently known most probable value of about -190 MeV. Moreover, the hadron-quark transition density $\rho_t$ from the spinodal decomposition mechanism strongly depends on the high-density behavior of nuclear symmetry energy $E_{\text{sym}}(\rho)$.

IV. SUMMARY

In summary, within a meta EOS model we studied the speed of sound and the possible hadron-quark transition in both nucleonic matter with fixed isospin asymmetries and the $n + p + e + \mu$ matter in neutron stars at $\beta$-equilibrium. While in nucleonic matter with fixed isospin asymmetries the speed of sound may continuously increase with density and does not undergo a spinodal decomposition especially for neutron-rich matter if the $E_{\text{sym}}(\rho)$ is very stiff, the sound speed in neutron star matter at $\beta$-equilibrium almost always show a peak at certain density depending strongly on the high-density behavior of $E_{\text{sym}}(\rho)$ if the skewness $J_0$ of SNM EOS is not too far above its currently known most probable value of about -190 MeV. Moreover, the corresponding hadron-quark transition density from the spinodal decomposition mechanism depends sensitively on the high-density behavior of nuclear symmetry energy $E_{\text{sym}}(\rho)$. Furthermore, in the case of using a super-soft $E_{\text{sym}}(\rho)$, the hadron-quark transition occurs at a density lower than the critical density above which the neutron star cores become pure neutron matter.

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FIG. 8: Effects of the skewness $J_0$ of SNM EOS on the hadron-quark transition density in neutron stars at $\beta$-equilibrium through the spinodal decomposition mechanism.

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