Anomalous inverse bremsstrahlung heating of laser-driven plasmas

Mrityunjay Kundu
Institute for Plasma Research, Bhat, Gandhinagar - 382 428, Gujarat, India
E-mail: mkundu@ipr.res.in

Abstract. Absorption of laser light in plasma via electron-ion collision (inverse bremsstrahlung) is known to decrease with the laser intensity as $k^{-3/2}$ or with the electron temperature as $T_e^{-3/2}$ where Coulomb logarithm $\ln \Lambda = 0.5 \ln(1 + k_{\text{max}}^2/\nu_{\text{max}}^2)$ in the expression of electron-ion collision frequency $\nu_{\text{ei}}$ is assumed to be independent of ponderomotive velocity $v_0 = E_0/\omega$ which is unjustified. Here $k_{\text{min}}^{-1} = v_{th}/\max(\omega, \omega_p)$, and $k_{\text{max}}^{-1} = Z/v_{th}^2$ are maximum and minimum cut-off distances of the colliding electron from the ion, $v_{th} = \sqrt{T_e}$ is its thermal velocity, $\omega, \omega_p$ are laser and plasma frequency. Earlier with a total velocity $v = (v_0^2 + v_{th}^2)^{1/2}$ dependent $\ln \Lambda(v)$ it was reported that $\nu_{ei}$ and corresponding fractional laser absorption ($\alpha$) initially increases with increasing intensity, reaches a maximum value, and then fall according to the conventional $I_0^{-3/2}$ scaling. This anomalous increase in $\nu_{ei}$ and $\alpha$ may be objected due to an artifact introduced in $\ln \Lambda(v)$ through $k_{\text{min}}^{-1} \propto v$. Here we show similar anomalous increase of $\nu_{ei}$ and $\alpha$ versus $I_0$ (in the low temperature and under-dense density regime) with quantum and classical kinetic models of $\nu_{ei}$ without using $\ln \Lambda$, but a proper choice of the total velocity dependent inverse cut-off length $k_{\text{max}} \propto v^2$ (in classical case) or $k_{\text{max}} \propto v$ (in quantum case). For a given $I_0 < 5 \times 10^{14}$ Wcm$^{-2}$, $\nu_{ei}$ versus $T_e$ also exhibits so far unnoticed identical anomalous increase as $\nu_{ei}$ versus $I_0$, even if the conventional $k_{\text{max}} \propto v_{th}^2$, or $k_{\text{max}} \propto v_{th}$ is chosen. However, for higher $T_e > 15$ eV, anomalous growth of $\nu_{ei}$ and $\alpha$ disappear. The total velocity dependent $k_{\text{max}}$ in kinetic models, as proposed here, may explain anomalous increase of $\alpha$ with $I_0$ measured in some earlier laser-plasma experiments. This work may be important to understand collisional absorption in the under-dense pre-plasma region due to low intensity pre-pulses and amplified spontaneous emission (ASE) pedestal in the context of laser induced inertial confinement fusion.

1. Introduction

Collisional absorption [1, 2, 3, 4] through electron-ion collision (inverse bremsstrahlung) occurs almost all the time in the sub-relativistic laser field. However, there is no universal agreement for the electron-ion collision frequency ($\nu_{ei}$) in the laser field [5, 6, 7]. In the conventional scenario, Coulomb logarithm $\ln \Lambda = 0.5 \ln(1 + k_{\text{min}}^2/\nu_{\text{max}}^2)$ in the expression of $\nu_{ei}$ is assumed to be independent of the ponderomotive velocity $v_0 = E_0/\omega$ and the laser field strength $E_0$ which is clearly unjustified. Here $k_{\text{min}}^{-1} = v_{th}/\max(\omega, \omega_p)$, and $k_{\text{max}}^{-1} = Z/v_{th}^2$ are maximum and minimum cut-off distances of the colliding electron from the ion; $\omega, \omega_p$ are laser and plasma frequency, and $v_{th} = \sqrt{T_e}$ is the thermal velocity of an electron. Unless explicitly noted, we use atomic units (a.u.) i.e., $|e| = m_e = \hbar = 4\pi\epsilon_0 = 1$, where $e$ and $m_e$ are charge and mass of an electron, $\hbar$ is the reduced Planck constant, and $\epsilon_0$ is the permittivity of the free space. As a consequence, for a given $\omega_p, T_e$, and $\omega$, one finds that $\nu_{ei}$ initially remains almost constant [6, 8, 9] with increasing $v_0/v_{th}$ up to some value of $v_0/v_{th}$, then decreases as $v_0^{-3/2}$ when $v_0/v_{th} > 1$ (see Fig.1 in Ref. [6]). In terms of the laser intensity $I_0 \propto v_0^2$, $\nu_{ei}$ remains almost constant up to an intensity $I_c$, then it decreases as $I_0^{-3/2}$ when $I_0 \gg I_c$. The corresponding fractional absorption $\alpha$ (defined as the ratio

[Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.]
of the absorbed laser energy to the incident laser energy) of the laser pulse vary with \( I_0 \) in the same way as \( v_{ei} \) versus \( I_0 \) since \( \alpha \propto v_{ei} \). In some experiments [11][10], however, with normally incident s-polarized laser light, \( \alpha \) was found to increase with \( I_0 \) up to a maximum value corresponding to an intensity \( I_c \), and a decrease when \( I_0 > I_c \). This anomalous increase of \( \alpha \) versus \( I_0 \) was explained [11] with a classical scattering model (CSM) [7] using total velocity \( v = (v_0^2 + v_{th}^2)^{1/2} \) dependent \( \ln \Lambda(v) \). But, CSMs which use \( \ln \Lambda \) have deficiencies. They do not match with more accurate kinetic models [6][8][9][12][13] of \( v_{ei} \) which has no \( \ln \Lambda \) dependence. Kinetic models use only one cut-off \( k_{\text{max}} \). Therefore, the artifact in \( \ln \Lambda \) through \( k_{\text{min}} \) is questionable for CSMs. A large unphysical value of \( v_{ei} \) may result with CSM compared to the kinetic models in the low temperature regime of our interest. To overcome those limitations of CSM, we use kinetic models and show that anomalous variation of \( v_{ei} \) and \( \alpha \) with \( I_0 \) is possible to explain in the regime of low temperature (\(< 15 \text{ eV}\)) and under-dense densities for a given laser wavelength, provided \( k_{\text{max}} \) is chosen in some appropriate form. By choosing the conventional classical cut-off \( k_{\text{max}} = v_{th}/\sqrt{Z} \) (see Refs.[6][8]) or the quantum cut-off \( k_{\text{max}} = 2v_{th} \), we find anomalous increase of \( v_{ei} \) versus \( T_e \) for a given \( I_0 \). Then we propose a modified \( k_{\text{max}} = v_f/\sqrt{Z} \) (for the classical case) or \( k_{\text{max}} = 2v_f \) (for the quantum case) which depends on \( E_0 \) and \( \omega \), and show anomalous increase of \( v_{ei} \) vs \( I_0 \) for a given \( T_e < 15 \text{ eV} \) as in our earlier work [11] using CSM. The proposed \( k_{\text{max}} \) is useful to explain the low intensity growing part of the experimental absorption curve in Ref.[10]. We assume uniformly charged hydrogen like plasma with \( Z = 1 \), density \( n_e = Zn_i = n_p = 10^{28}/m^3 \), and \( \omega/\omega_p = 5 \) giving the laser wavelength \( \lambda \approx 66 \text{ nm} \) as considered in Ref.[8]. Corresponding critical density is \( n_e \approx 2.52 \times 10^{23} \text{ cm}^{-3} \).

2. Kinetic models of \( v_{ei} \)

Assuming the Maxwellian velocity distribution for electrons and stationary ions, the quantum mechanical expression [8] of energy absorption rate \( \dot{\varepsilon} \) is given by

\[
\dot{\varepsilon} = \frac{8\sqrt{2}\pi^2 n_e \omega}{v_{ei}^2} \omega^2 \sum_{n=1}^{\infty} n^2 \int_{0}^{\infty} dk \frac{\exp[-\frac{1}{2}(n\omega/kv_{ei})^2 - \frac{1}{2}(k/k_B)^2]}{k^3 |\varepsilon_{\text{RPA}}(k,n\omega)|^2} \int_{0}^{1} J_n^2(k_0 z) dz, \tag{1}
\]

which relates \( v_{ei} = 4\pi(\omega^2/\omega_p^2)\varepsilon/|\varepsilon|^2 \). Here \( \varepsilon_{\text{RPA}}(k,n\omega) \) is the quantum mechanical dielectric function (which is a complex quantity) in the random phase approximation (RPA) due to Lindhard [15]. \( v_{ei} \) is the electron velocity, \( k_B = 2v_e/\hbar \) (note \( \hbar = 1 \) in a.u.) is the inverse of the DeBroglie wavelength \( \lambda_e \), and \( k \) is the order of the Bessel function \( J_n(\mu) \), \( r_0 = E_0/\omega^2 \) is the excursion of a free electron in the laser field, and \( k \) signifies the distance of the electron from the colliding ion. The integrals and the summation (in Eq.(1)) are computed numerically with upper limits of \( n = n_{\text{max}} \), and \( k = k_{\text{max}} \).

Conventionally \( k_{\text{max}} = v_{th}/\sqrt{Z} \) [6][8] is chosen. \( \varepsilon_{\text{RPA}}(k,n\omega) \) can be obtained [14] from

\[
\varepsilon_{\text{RPA}}(k,n\omega) = 1 + (\omega_p/\omega_e)^2 [f(x - \frac{n\omega}{2}) - f(x + \frac{n\omega}{2})]/2q, \tag{2}
\]

where \( f(x) = \pi^{-1/2} \int_{-\infty}^{\infty} dt \exp(-t^2)/(t - x) \) is the plasma dispersion function, \( x = \omega/\sqrt{2}kv_{ei} \), and \( q/2 = k/\sqrt{2}k_B \). \( f(x) \) is related to the dawson’s integral \( D(x) = \exp(-x^2) \int_{0}^{x} dt \exp(t^2) = \frac{-2D(x) + i\sqrt{\pi} \exp(-x^2)}{2} \).

Calculation of \( \varepsilon_{\text{RPA}} = \varepsilon_1 + i\varepsilon_2 \), leads to write real and imaginary parts as \( \varepsilon_1(k,\omega) = 1 + \frac{1}{\sqrt{2}}(\omega_p/\omega_e)^2(\omega/\omega_e)^2G_2(k,\omega) \), \( \varepsilon_2(k,\omega) = \sqrt{\frac{\pi}{2}}(\omega_p/\omega_e)^2(\omega/\omega_e)^3G_1(k,\omega) \), where

\[
G_1(k,\omega) = \exp\left[-(\omega/k_B)^2(\omega/\omega_e)^2\right] J_0^2(kr_0z)/J_0^2(k_r z),
\]

and

\[
G_2(k,\omega) = \exp\left[-\frac{(\omega/\omega_e)^2(\omega/\omega_e)^2}{2}\right] J_0^2(kr_0z)/J_0^2(k_r z).
\]

Adding \( f(x) \) for \( h \) to \( \omega \) (for \( h \to 0 \)) classical expressions of \( G_1, G_2, \varepsilon_1, \varepsilon_2 \) are obtained as \( G_1^{\text{CI}}, G_2^{\text{CI}}, \varepsilon_1^{\text{CI}}, \varepsilon_2^{\text{CI}} \) respectively. Corresponding classical \( \varepsilon_{\text{RPA}}^{\text{CI}} = \varepsilon_1^{\text{CI}} + i\varepsilon_2^{\text{CI}} \) gives

\[
\varepsilon_{\text{CI}}^{\text{CL}} = 8\sqrt{2}\pi^2 n_e n_i v_{ei}^2 \omega^2 \sum_{n=1}^{\infty} n^2 \int_{0}^{\infty} dk \frac{\exp[-(n\omega/\sqrt{2}v_{ei})^2/2]}{k^3 |\varepsilon_{\text{RPA}}(k,n\omega)|^2} \int_{0}^{1} J_n^2(k_0 z) dz. \tag{3}
\]

Assuming \( |\varepsilon_{\text{RPA}}^{\text{CI}}|^2 \approx 1 \), the Dawson-Oberman model of \( v_{ei} \) can be deduced from Eq.(3) as

\[
\varepsilon_{\text{Dawson}}^{\text{CL}} = 8\sqrt{2}\pi^2 n_e n_i v_{ei}^2 \omega^2 \sum_{n=1}^{\infty} n^2 \int_{0}^{\infty} k^{-3} \exp[-(n\omega/\sqrt{2}v_{ei})^2/2] dk \int_{0}^{1} J_n^2(k_0 z) dz. \tag{4}
\]

We use Eqs.(1), (3), (4) to obtain \( v_{ei} \).
3. Results for \( v_{ei} \)

For numerical calculations of \( v_{ei} \) we choose \( n_{\text{max}} = 20 \), and \( k_{\text{max}} \) is discretized up to \( n_{\text{max}} = 100 \) distinct values such that \( k_{\text{max}} = n_{\text{max}} \Delta k \), and \( k = m \Delta k \). Above parameters are kept unchanged unless mentioned explicitly. Special attention is given to the low temperature regime where anomalous variation is found to be pronounced.

Figure 1 shows normalized frequency \( v_{ei}/\alpha_p \) versus normalized velocity \( v_0/v_{th} \) using expressions (1), (3), (4) represented by “Bornath-Q”, “Bornath-C”, “Dawson-C” respectively with (a) \( T_e = 10 \) eV and (b) \( T_e = 15 \) eV. Here, conventional \( k_{\text{max}} = v_{th}^2/|Z| \) has been used. For \( v_0/v_{th} > 1 \), classical approximations (“Bornath-C” and “Dawson-C”) match very well with the quantum result (“Bornath-Q”), but they under-estimate the frequency for \( v_0/v_{th} < 1 \) at low temperature where quantum results are important. In all cases, however, the universally accepted feature is that \( v_{ei}/\alpha_p \) remains almost constant up to a value of \( v_0/v_{th} \approx 1 \), and then it drops rapidly when \( v_0/v_{th} \geq 1 \) as in Ref.[8] (see Fig. 1 of Ref.[8]). The comparison between Figs 1(a), 1(b) shows that the conventional relation \( k_{\text{max}} = v_{th}^2/|Z| \) is increased from 10 to 15 eV. This anomalous increase of frequency with temperature is a new finding in this work within the existing models. To show it more clearly, Fig 2 depicts \( v_{ei}/\alpha_p \) versus \( T_e \) for a fixed intensity \( I_{\text{inc}} / c^2 \approx 10^{14} \text{Wcm}^{-2} \). It shows that \( v_{ei}/\alpha_p \) increases monotonically with increasing \( T_e \) up to a maximum about \( T_e \approx 35 \) eV (indicated by a shaded region), and then \( v_{ei}/\alpha_p \) decreases as \( T_e \) is increased beyond \( T_e \approx 35 \) eV. Just after the maximum the rate of decrease of \( v_{ei}/\alpha_p \) is much slower than the conventional \( v_{ei}/\alpha_p \propto T_e^{-3/2} \) fall, shown by a thin dashed line. Similar anomalous variation can also be shown for the quantum case with DeBroglie wavelength \( \lambda_B \) and the corresponding cut-off \( k_{\text{max}} = 1/\lambda_B = 2v_{th} \).

So far, the effect of laser field strength has been disregarded in \( k_{\text{max}} \) (and in earlier works) which is not justified. \( k_{\text{max}} \) should depend on parameters of the laser field through the ponderomotive velocity \( v_0 \) in some form and the above conventional relation \( k_{\text{max}} = v_{th}^2/|Z| \) should be modified depending on the total velocity \( v = (v_0^2 + v_{th}^2)^{1/2} \) of the electron. Replacing \( v_{th} \) by \( v \) in \( k_{\text{max}} \), and taking \( k_{\text{max}} = v^2/|Z| \) we show below that the conventional \( I_0^{3/2} \) decrease of \( v_{ei} \) is also not obeyed in the low temperature and low intensity regime. The traditional quantum cut-off \( k_{\text{max}} = 2v_{th} \) also does not depend on laser parameters and needs modification when electrons are driven by the laser field. For the quantum case, we use total velocity dependent \( k_{\text{max}} = k_B = 2v \) to study absorption of laser light in under-dense plasma in Sec.4.

4. Anomalous absorption of light wave in under-dense plasma

Anomalous increase of \( v_{ei} \) with temperature or intensity occurs in the low temperature and low intensity regime which can be used to find absorption of laser light in an under-dense plasma slab. The fractional absorption \( |I|/I_{\text{inc}} \) of light at a normal incidence can be written as \( \alpha = 1 - I/I_{\text{inc}} = 1 - \exp(-2\kappa L) \), where \( I_{\text{inc}}, I \) are the incident and the transmitted intensity of light, \( \kappa = (n/n_e)v_{ei}/v_g = c\sqrt{T}/n_i \) is the group velocity of light, and \( L \) is the thickness of the plasma slab. For \( \kappa \ll 1, \alpha \approx 2(n/n_e)v_{ei}/v_g \) holds which shows \( \alpha \) should vary similar to \( v_{ei} \) w.r.t. \( T_e \) and \( I_0 \) for a fixed plasma density and laser frequency. For illustration we assume \( L = 200\lambda \) and keep other parameters as above. Figure 2 shows that anomalous frequency increase is possible if \( T_e \) is less than the value at which respective frequency maximum occurs.
Figure 3 shows variation of $\nu_{ei}/\omega_p$, and corresponding $\alpha$ against $I_0$ using (i) $T_e = 15$ eV with the classical cut-off $k_{\text{max}} = v^2/Z$ (in a,b), and (ii) $T_e = 5$ eV with the quantum cut-off $k_{\text{max}} = 2v$ (in c,d). In both cases we have now used total velocity dependent cut-offs. It is seen that $\nu_{ei}$ and $\alpha$ grow hand in hand up to a maximum value with increasing intensity up to $\approx 10^{16}$ Wcm$^{-2}$ (with the classical cut-off) and $\approx 5 \times 10^{15}$ Wcm$^{-2}$ (with the quantum cut-off), and after that they fall together. This anomalous growth of $\alpha$ was found in some experiments [10] and an explanation was provided [11] for this fact with a classical scattering model [7] using total velocity dependent $\ln\Lambda(v)$. Here we find similar anomalous increase of $\alpha$ versus $I_0$ with more accurate kinetic models. But, discrepancy persists among these models which may be as large as 20% (for $\nu_{ei}$) in the low intensity regime. When $I_0$ exceeds $I_c$ corresponding to the maximum of $\nu_{ei}$ (or $\alpha$), discrepancy among these models disappears.

5. Summary
Electron-ion collision frequency ($\nu_{ei}$) and corresponding fractional absorption ($\alpha$) of laser pulses in an under-dense plasma has been re-examined with classical and quantum kinetic models of $\nu_{ei}$. Conventionally, it is believed that $\nu_{ei}$ and $\alpha$ should decrease as $T_e^{-3/2}$ and $I_0^{-3/2}$ w.r.t. $T_e$ and $I_0$ respectively. Contrarily, it is found to be true only in the high temperature ($> 15$ eV) and high intensity $> 5 \times 10^{15}$ Wcm$^{-2}$ regime. The conventional $k_{\text{max}}$ only depends on the thermal velocity [6,8], which is not justified. It must depend on the ponderomotive velocity $v_0 = E_0/\omega$. For a given $T_e < 15$ eV and a total velocity $v = (v_0^2 + v^2)^{1/2}$ dependent $k_{\text{max}}$, as the intensity is increased from a lower value $< 5 \times 10^{15}$ Wcm$^{-2}$, the frequency $\nu_{ei}$ is also found to increase monotonically up to a maximum value similar to $\nu_{ei}$ versus $T_e$ at a constant $I_0$. The fractional absorption $\alpha$ which is almost proportional to $\nu_{ei}$, follows the same anomalous variation w.r.t. $T_e$ and $I_0$ as $\nu_{ei}$. Thus, in both ways, we show growth of $\nu_{ei}$ and $\alpha$ with $T_e$ and with $I_0$ up to a maximum using classical and quantum cut-off $k_{\text{max}}$. Anomalous increase of $\nu_{ei}$ with $T_e$ and with $I_0$ were not pointed out earlier with kinetic models, because low temperature regime was not investigated in detail.

6. References
[1] Shalom Eliezer, *The Interaction of High-Power Lasers with Plasmas*, (IOP Publishing, Bristol, 2002).
[2] L. Schlessinger and J. Wright, Phys. Rev. A 20, 1934 (1979).
[3] P. Hilse, M. Schlanges, Th. Bornath, and D. Kremp, Phys. Rev. E 71, 056408 (2005).
[4] M. Moll, M. Schlanges, Th. Bornath, and V. P. Krainov, New Journal of Physics 14, 065010 (2012).
[5] V. P. Silin, Sov. Phys. JETP 20, 1510 (1965).
[6] C. D. Decker, W. B. Mori, J. M. Dawson, and T. Katsouleas, Phys. Plasmas 1, 4043 (1994).
[7] P. Mulser, F. Cornolli, E. Bésuelle, and R. Schneider, Phys. Rev. E 63, 016406 (2000).
[8] Th. Bornath, M. Schlanges, P. Hilse, and D. Kremp, Phys. Rev. E 64, 026414 (2001).
[9] A. Brantov, W. Rozmus, R. Sydora, C. E. Capjack, V. Yu. Bychenkov et al., Phys. Plasmas 10, 3385 (2003).
[10] D. Riley, L. A. Gizzi, A. J. Mackinnon, S. M. Viana, and O. Willi, Phys. Rev. E 48, 4855 (1993).
[11] M. Kundu, Phys. Plasmas 21, 013302 (2014).
[12] John Dawson and Carl Oberman, Phys. Fluids 5, 517 (1962).
[13] S.H. Kim and H.E. Wilhelm, Phys. Fluids 25, 668 (1982).
[14] A.V. Latyshev, A.A. Yushkanov, arXiv:1003.2531 [math-ph].
[15] J. Lindhard, Dan. Mat.-Fys. Medd. 28, No. 8 (1954).
[16] M. Kundu, Phys. Rev. E 91, 043102 (2015).