Research Article

Study on Fluid-Structure Interaction of Flexible Membrane Structures in Wind-Induced Vibration

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A strongly coupled monolithic method was previously proposed for the computation of wind-induced fluid-structure interaction of flexible membranous structures by the authors. How to obtain the accurate solution is a key issue for the strongly coupled monolithic method. Projection methods are among the commonly used methods for the coupled solution. In the work here, to impose initial pressure boundary conditions implicitly defined in the original momentum equations in classical projection methods when dealing with large-displacement of membranous structures, a modified factor is introduced in corrector step of classical projection methods and a new modified projection method is obtained. The solution procedures of the modified projection method aimed at strongly coupled monolithic equations are given, and the related equations are derived. The proposed method is applied to the computation of a two-dimensional fluid-structure interaction benchmark case and wind-induced fluid-structure interaction of a three-dimensional flexible membranous structure. The performance and efficiency of the modified projection method are evaluated. The results show that the modified projection methods are valid in the computation of wind-induced fluid-structure interaction of flexible membranous structures, with higher accuracy and efficiency compared with traditional methods. The modified value has little effects on the computation results whereas iteration times has significant effects. Computation accuracy can be improved greatly by increasing iteration times with less increase in computation time and little effects on stability with the modified projection method.

1. Introduction

The fluid-structure interaction of wind and membrane structure is one of the cutting-edge topics that have been widely concerned by researchers but have not been studied in depth. With the rapid improvement of computer hardware and software technology, numerical simulation methods have developed into an important tool for analyzing the fluid-solid coupling of wind and membrane structures. At present, numerical methods for studying fluid-structure interaction in wind vibration of membrane structures include weak coupling partitioning method, strong coupling partitioning method, and strong coupling integral method [1]. The weakly coupled partitioned method is to solve the fluid control equation and the structural control equation separately in each time step, and then the aerodynamic load acting on the fluid domain model is transferred to the structural domain model through the interface between the fluid and the structure, so as to predict the displacement of the structure. The displacement of the structure is then transferred to the fluid domain as a new load, and the process is repeated until the result converges to the specified value. The strongly coupled partitioned method is to add an iteration cycle based on solvers in the fluid domain and the structural domain, solving the nonlinear equations in each time step, and calculating the value of the variables in the whole field. These methods, respectively, integrate the governing equations explicitly and implicitly in time. In strongly coupled monolithic method, the governing equations for the flow and
structure domains are discretized together, and the nonlinear system of equations is solved as a whole [2, 3].

Most scholars at home and abroad have studied the fluid-structure interaction in wind-induced vibration of membrane structures by strongly coupled partitioned method or weakly coupled partitioned method [3–8]. The research studies on the computation of fluid-structure interaction problem by strongly coupled monolithic methods are still very limited [9,10], but the strongly coupled monolithic method shows its advantages in terms of stability and accuracy. Typically, the implicit coupling is needed for the large-scale structure deformation to ensure the numerical stability of the FSI solver [11]. In order to account structural dynamics in an FSI system, previous studies successfully integrated finite-element-based structural solver with existing flow solvers. For example, Bhardwaj and Mittal [12] proposed an FSI solver by coupling a sharp-interface IB method and an open-source finite-element solver (Tahoe), using an implicit partitioned approach. Employing this solver, they validated the FSI benchmark, proposed by Turek and Hron; cylinder attains self-sustained oscillation in a laminar channel flow. Similarly, Tian et al. [13] proposed a versatile FSI solver which could handle large-scale FID of a flexible structure. They carried out several validations with established benchmarks and demonstrated the three-dimensional capability of the solver. Bailoor et al. [14] coupled a compressible flow solver with an open-source finite-element solver (Tahoe) to simulate blast loading on thin plates. Very recently, Furquan and Mittal [15] numerically studied two side-by-side flexible splitter plates attached to square cylinders using a deforming-spatial-domain/stabilized space-time flow solver coupled with a finite-element open-source structural dynamics solver. The authors [16] derived the strong coupled monolithic equations for fluid-structure interaction calculation of wind and membrane structures and analyzed the fluid-structure interaction of wind vibration of a membrane structure with typical shapes. The results showed that the strongly coupled monolithic method has advantages in both accuracy and stability in computation of fluid-structure interaction. However, since the strongly coupled monolithic method needs nonlinear equations to represent the fluid-structure interaction system, it is usually necessary to employ the Newton–Raphson method to solve the equations after linearization. For solving the equations, a large amount of computation time is spent on the repeated integration of the Jacobian matrix and the Newton modification of the solution to the corresponding linear system. This often leads to a large amount of computation time, and how to solve the strongly coupled monolithic equations is among one of the key issues to be solved.

In the work here, considering the characteristics of undergoing large deformations of flexible membrane structures, the traditional projection method is modified to solve the previously proposed strongly coupled monolithic equations by the authors. Based on the classical projection method, the iterative correction of the pressure correction factor is introduced into the correction step to satisfy the pressure constraint conditions implicitly defined in the original momentum method. The solution procedures of the modified projection method aimed at strongly coupled monolithic equations are given, and the related equations are derived. The proposed modified projection method is used to calculate the classical two-dimensional fluid-structure benchmark problem. The wind-induced fluid-structure interaction effects of a three-dimensional flexible membrane structure are obtained employing the proposed method.

2. Governing Equations and Boundary Conditions

2.1. Fluid Governing Equations. The incompressible viscous fluid is governed by the Navier–Stokes equations, consisting of momentum conservation and continuity equation. The governing equations for the structure are described by a total Lagrange (TL) formulation and a large deformation theory. A linear elastic model is introduced to deal with the data transfer at the interface, which is governed by semidiscretized finite element equations. These equations can be found in Sun and Gu [16]. The fluid governing equations are Navier–Stokes equation (abbreviated as N-S equation) for incompressible viscosity, namely, the continuum equation and the momentum equation:

\[ \nabla \cdot v_f = 0, \]

\[ \rho_f \left( \frac{\partial v_f}{\partial t} + v_f \cdot \nabla v_f \right) = \nabla \cdot \sigma_f + f_f^B, \]  \hspace{1cm} (1)

where \( v_f \) is the fluid flow velocity, \( \nabla \) is the spatial gradient, \( \rho_f \) is the fluid density, \( \sigma_f \) is the fluid's full stress tensor (pressure and viscous force), \( f_f^B \) is the fluid volume force.

2.2. Structural Governing Equations. The membrane structure can be regarded as an hyperelastic model and is described by the total Lagrangian equation:

\[ \nabla \cdot \sigma_L + f_s = 0. \]  \hspace{1cm} (2)

The above equations are based on the expression of the initial undeformed configuration, where \( \sigma_f \) is the stress tensor of Piola–Lagrange, which is the stress with respect to the deformation configuration of the undeformed surface.

2.3. Fluid and Structure Coupling. A linear elastic model is introduced to deal with the deformation of the fluid domain, solve the data transfer problem at the interface of the fluid domain and the structural domain, and realize the coupling of fluid and structure. The equation of the linear elastic model is

\[ \nabla \cdot \sigma_{LE} = 0, \]

\[ E_{LE} = \frac{1}{2} \left( h_{LE} + (h_{LE}^T)^T \right), \]

\[ \sigma_{p(ij)}^{LE} = \sigma_{c(ij)}^{LE} + \sigma_{ijkl}^{LE} E_{kl}^{(LE)}, \]

where \( \sigma_{LE} \) is the linear elastic model Piola–Lagrange stress tensor; \( E_{LE} \) is the linear elastic model Green–Lagrange strain tensor; and \( \sigma_{p}^{LE} \) is the linear elastic model Piola–Kirchhoff stress tensor. The boundary conditions of the above equation are as follows:
where $u_{\text{LE}}$ represents the displacement of the linear elastic model on the interface and $\Gamma_{\text{FSI}}$ represents the fluid-solid interface; that is, the elastic displacement on the interface is zero. The coupling condition of fluid and structure at the interface is

$$
u_{\text{F}} = \nu_{\text{S}} , \text{on} \Gamma_{\text{FSI}},$$

$$v_{\text{FSI}} = v_{\text{FS}} , \text{on} \Gamma_{\text{FSI}},$$

$$\sigma_{c} \cdot n_{c} + \sigma_{f} \cdot n_{f} = 0 \text{on} \Gamma_{\text{FSI}}.$$  

Finally, along the interface, linear elastic model displacements are implicitly imposed to be equal to structural displacements in a strong sense:

$$u_{\text{FS}} = u_{\text{S}} \text{ (on)}.$$  

(7)

The monolithic equations of the fluid-structure are solved using projection methods, which are commonly used methods for the coupled solution. But for large-displacement problems for flexible membranous structures, to impose initial pressure boundary conditions implicitly defined in the original momentum equations in classical projection method it is weak to obtain an accurate solution. Thus, the key issue here is how to modify the classical projection method considering large-displacement of membranous structures to obtain an accurate solution of the strongly coupled monolithic equations.

$$u_{\text{LE}} = 0 \text{on} \Gamma_{\text{FSI}},$$  

where $u_{\text{FSI}}$ represents the displacement of the linear elastic model on the interface and $\Gamma_{\text{FSI}}$ represents the fluid-solid interface; that is, the elastic displacement on the interface is zero. The coupling condition of fluid and structure at the interface is

$$\nu_{\text{F}} = \nu_{\text{S}} , \text{on} \Gamma_{\text{FSI}},$$

$$v_{\text{FSI}} = v_{\text{FS}} , \text{on} \Gamma_{\text{FSI}},$$

$$\sigma_{c} \cdot n_{c} + \sigma_{f} \cdot n_{f} = 0 \text{on} \Gamma_{\text{FSI}}.$$  

where $\nu_{s}$ is the structural velocity, $\nu_{t}$ is the unknown structural displacement, $\sigma_{c}$ is the Cauchy stress, $\sigma_{f} = F_{a}^{T} \sigma_{f} F_{a}^{T}$ $F = \text{det}(F)$ is the Jacobian determinant, and $n_{c} = -n_{f}$, $n_{f}$ is the unit vector in the direction of the external normal of the fluid boundary.

2.4. The Strongly Coupled Global Equation for a Fluid-Solid Coupled System. The strongly coupled monolithic equations $f_{\text{FSI}}$ of the FSI in wind-induced fluid-structure interaction of flexible membranous structures read

$$f_{\text{F}} = \int_{\Omega_{f}} \left( \omega \nabla \nu_{f} + -\zeta \cdot \rho_{f} \left( \frac{\partial v_{f}}{\partial t} + v_{f} \cdot \nabla v_{f} \right) - \nabla \zeta \cdot \sigma_{f} + \zeta \cdot \nu_{f} \right) dQ = 0,$$

$$f_{\text{S}} = \int_{\Omega_{s}} \left( \nabla \sigma_{t} - \eta \cdot f \right) dQ = 0,$$

$$f_{\text{LE}} = \int_{\Omega_{\text{le}}} \left( \nabla \sigma_{le} \right) dQ = 0,$$

$$f_{\text{FSI}} = \int_{\Gamma_{\text{FSI}}} \left( \eta \cdot n_{f} \right) d\Gamma_{f} = 0,$$

$$f_{\text{S}} = \int_{\Gamma_{\text{FSI}}} \left( \eta \cdot n_{f} \right) d\Gamma_{f} + \int_{\Omega_{\text{le}}} \left( \sigma_{le} \cdot \nu_{f} \right) d\Gamma_{f} = 0,$$

where $\nu_{s}$ is the structural velocity, $\nu_{t}$ is the unknown structural displacement, $\sigma_{c}$ is the Cauchy stress, $\sigma_{f} = F_{a}^{T} \sigma_{f} F_{a}^{T}$ $F = \text{det}(F)$ is the Jacobian determinant, and $n_{c} = -n_{f}$, $n_{f}$ is the unit vector in the direction of the external normal of the fluid boundary.

3. Solution to Strongly Coupled Monolithic Equations

The projection method in fluid dynamics is a common method for solving the nonlinear coupling equations. The projection method is generally divided into two steps of a prediction step and a correction step. For details of the corrector and prediction steps, please refer to reference [17]. Generally, the additional velocity calculated in the prediction step does not satisfy the nondispersion condition. For this reason, iterative correction is performed in the correction step so that the constraint conditions are satisfied. Although the traditional projection method can solve the pressure field and velocity field separately, it cannot satisfy the initial boundary conditions of pressure that are implicitly defined in the original momentum method, especially for the large deformation problem [18]. Therefore, this paper introduces the pressure correction factor into the projection method to solve the above problems.

First, the initial velocity pressure is set before the iteratively calculating $(\nu^{0}, p^{0}, u^{0})$, and $\nu^{0} = \nu^{0}, f_{a, \text{proj}} = 0$, and the additional velocity field $\nu^{a,n+1}$ at the moment of $(n+1)$ is solving by the following equation:

$$F_{\nu} \left( \nu^{a,n+1}, \phi_{a} \right)_{\Omega} + f_{\text{LE}} \left( \nu^{a,n+1}, \phi_{a} \right)_{\Omega}^{T} + u_{c} \left( \nu^{a,n+1}, \phi_{a} \right)_{\Omega}^{T}$$

$$- \lambda_{c} \left( \left( \nabla \cdot \nu^{a,n+1} \right)_{\Omega}, \nabla \cdot \phi_{a} \right)_{\Omega}$$

$$= F_{\nu} \left( \nu^{a,n+1}, \phi_{a} \right)_{\Omega} + \left( \eta f_{a, \text{proj}} + \delta p_{a, \text{proj}}, \nabla \cdot \phi_{a} \right)_{\Omega} \forall \phi_{a} \in S_{a}(\Omega),$$

where $\nu^{a,n+1}$ is the projection term in the correction step, $\delta p_{a, \text{proj}}$ is the pressure correction factor, which is used to apply
the initial boundary conditions of the pressure to satisfy the
original momentum equation, \( \nabla p \) is the additional speed in
the prediction step, and \( \lambda_1 \) is the correction coefficient.

\[
\begin{align*}
F_{LE}(\tilde{v}_a, \phi_a) &= u^f(\tilde{v}_{a,k}^{\text{r+1}}, \phi_a) - (V \cdot \phi_a)\tilde{v}_{a,k}^{\text{r+1}}, \phi_a) - \mathbf{f}_F, \\
u^f(\tilde{v}_a, \psi_a) &= \Delta t u^f(\tilde{v}_{a,k}^{\text{r+1}}, \phi_a) - \mathbf{f}_S^{\text{FSI}}.
\end{align*}
\]

(9)

Update the pressure correction factor \( \delta \rho_{a,p}^{n+1} \) at \((n+1)\) time:

\[
\delta \rho_{a,p}^{k,n+1} = \delta \rho_{a,p}^{k-1,n+1} + \lambda_2 \left( \nabla \cdot \tilde{v}_{a,k}^{\text{r+1}} \right).
\]

(10)

When the additional velocity field \( [\tilde{v}_{a,k}^{\text{r+1}} - \tilde{v}_{a,k}^{\text{r+1}}] \) between two adjacent iteration steps is less than the specified
tolerance, the calculation is stopped and the following steps are
entered.

According to the calculated additional velocity field \( \tilde{v}_{a,k}^{\text{r+1}} \) at the moment of \((n+1)\), the following formula is used
to calculate the projection term \( \rho_{a,p}^{n+1} \) in the projection step:

\[
\begin{align*}
\partial t(\tilde{V}_a, \phi_a) + \partial t(\tilde{V}_a, \phi_a) + F_{LE}(\tilde{V}_a, \phi_a) &= \left( V \cdot \phi_a \right) \tilde{V}_a, \phi_a + u^f(\tilde{V}_a, \phi_a) \\
\left( \nabla \delta \rho_{a,p}^{n+1} \right) \nabla \phi_a &= \frac{\rho_0}{\Delta t} \left( \nabla \cdot \tilde{v}_{a,k}^{\text{r+1}}, \phi_a, \right), \forall \psi_a \in S_a(\Omega).
\end{align*}
\]

(11)

Finally, project the velocity predicted above into the
nonscattered vector field space to obtain the velocity,
pressure, and displacement of the nonlinear equations:

\[
\begin{align*}
v_f &= \tilde{v}_{a,k}^{\text{r+1}} - \frac{\Delta t}{\rho_0} \delta \rho_{a,p}^{n+1} \Omega_f, \\
p &= \rho_0 n + \delta \rho_{h,p}^{n+1} \Omega_f, \\
u_s &= u_{a,s} + \Delta t v_f^{n+1}.
\end{align*}
\]

(12)

4. Examples

4.1. Two-Dimensional Fluid-Structure Interaction. Here, a
benchmark case of two-dimensional fluid-structure inter-
action [18] is studied by the proposed method, whose details
are not described here. The settings and parameters are as
follows: the density of fluid water in the experiment
\( \rho_f = 1000 \text{ kg/m}^3 \), the solid is similar to rubber material,
and the geometric dimensions are shown in Figure 1. The test is
divided into three tests: FSI1, FSI2, and FSI3. The FSI1 test is a
steady state test, and the FSI2 and FSI3 tests are two-di-

4.2. Analysis of Wind-Induced Vibration Fluid-Structure
Interaction of Three-Dimensional Flexible Membrane Structure.
Taking the typical saddle-shaped membrane structure as an
example, the above strong coupling overall method program
is used to analyze and calculate the wind-induced vibration
response. The calculation diagram of the membrane struc-
ture is shown in Figure 4, and its basic parameters are as
follows: span \( L = 20 \text{ m} \), height \( H = 5 \text{ m} \), rise-span ratio \( \gamma / L = 1/8 \), pretension \( T = 2.0 \text{ kN/m} \), film thickness \( 1 \text{ mm} \), mass
per unit area \( g = 1.25 \text{ kg/m}^2 \), tensile stiffness is
\( Et = 8.0 \times 105 \text{ N/m} \), shear stiffness \( Gt = 1.2 \times 104 \text{ N/m} \), and
Poisson’s ratio is \( 0.3 \). Because the performance of modified
projection method is mainly studied in this paper, the in-
fluence of turbulence is not considered. Taking a typical
three-dimensional saddle-shaped membrane structure as an
example, the above strong coupling overall method and
preprocessor are used to analyze and calculate the wind-
induced vibration response. The specific parameter settings
in the calculation are the same as those in [16] and will not be
repeated here.

Before the fluid-structure interaction computation, a
dynamic analysis of the saddle membrane structure analyzed
in the paper is carried out. We have got some grid pictures
showing the first four vibration modes and frequencies
results of the membrane structures, as shown in Figure 5.
Figure 1: Geometrical dimensions of classical fluid-structure coupling problem.

Figure 2: Comparison of displacement history of elastic beam end node A in different directions.

Figure 3: Comparison of lift and drag forces acting on cylinders and beams.
Table 1: Comparison of error norm of elastic beam end node A.

| Number of iterations | Adapting methods       | $|d_x - d_x^0|/10^4$ | $|d_y - d_y^0|/10^4$ |
|----------------------|------------------------|----------------------|----------------------|
| 10                   | Present method          | $1.34e-4$            | $2.46e-4$            |
|                      | Newton–Raphson method  | $2.65e-2$            | $2.88e-2$            |
| 16                   | Present method          | $5.37e-6$            | $4.51e-6$            |
|                      | Newton–Raphson method  | $3.37e-4$            | $6.44e-4$            |
| 22                   | Present method          | $3.23e-7$            | $2.55e-7$            |
|                      | Newton–Raphson method  | $1.22e-4$            | $4.34e-4$            |
| 28                   | Present method          | $4.17e-8$            | $3.92e-8$            |
|                      | Newton–Raphson method  | $3.23e-5$            | $2.79e-5$            |
| 34                   | Present method          | $2.61e-8$            | $1.94e-8$            |
|                      | Newton–Raphson method  | $1.22e-5$            | $1.04e-5$            |

Note: $d_x^0$ and $d_y^0$ are analytical displacements.

Figure 4: Geometry of saddle membrane structure.

Figure 5: First four vibration modes of the saddle membrane. (a) First vibration mode ($f = 2.87$ Hz). (b) Second vibration mode ($f = 3.96$ Hz). (c) Third vibration mode ($f = 4.05$ Hz). (d) Fourth vibration mode ($f = 4.31$ Hz).
In order to illustrate the computational efficiency and performance of the modified projection method in this paper, firstly, the influence of the modified value in the modified projection method on the wind pressure coefficient of the membrane structure is analyzed, and the partition of the saddle membrane structure is shown in Figure 6.

Table 2 shows the error norm when comparing the zonal wind pressure coefficient (0° wind angle) of the Newton–Raphson method with different correction values selected in this paper. The zonal wind pressure coefficient of the Newton–Raphson method comes from literature [16], and the number of iterations is 10.

After analyzing Table 2, the following conclusions can be drawn:

1. The zonal wind pressure coefficient obtained by the modified projection method in this paper is very close to that obtained by the Newton–Raphson method, which proves the correctness of the calculation method for the three-dimensional film structure wind-induced fluid-structure coupling.

2. The value of the corrected value $\lambda_1$ of the modified projection method in this paper does not have a great influence on the wind pressure coefficient. It can be seen that the error norm of the wind pressure coefficient does not change significantly with the increase of the corrected value $\lambda_1$. However, it is found that the increase of the correction value will lead to a significant increase in computer time and increase the risk of computational instability. Therefore, it is recommended that the correction value should be within a certain suitable range.

Table 3 shows the influence of the change of iteration times on the wind pressure coefficient, and the error norm change of the zonal wind pressure coefficient compared with the Newton–Raphson method at 0° wind direction angle is also calculated. Modifier value $\lambda_1 \approx 10^3$.

It can be seen from Table 3 that the calculation accuracy of the modified projection method is relatively high, and the number of iterations has an important influence on the wind pressure coefficient: the larger the number of iterations, the smaller the error norm of the wind pressure coefficient, and the more accurate the result. But it is important to pay attention to the balance between the accuracy of the calculation results and the calculation time. In the calculation, it was found that the average number of iterations increased by about 20%, the calculation accuracy increased by about 19%, and the calculation time increased by about 7%. It shows that the modified projection method is used to calculate the wind-induced fluid-solid coupling wind pressure coefficient of the membrane structure. The speed of calculation accuracy is higher than the number of iterations and time-consuming calculation. Therefore, if a more accurate result is needed, it can be achieved by increasing the number of iterations if the computational hardware conditions permit.

At the same time, in order to further illustrate the calculation accuracy and efficiency of the modified projection method in this paper, this paper also compares the wind-induced vibration response of the first 40s of the point in the membrane structure calculated by different solution methods under different grid numbers. With regard to the relative residual error $R$ and $T$ (hour) when using the same number of iterations, where the correction value of the modified projection method $\lambda_1 \approx 10^3$, the comparison results are shown in Table 4 and Figure 7.
Table 2: The effect of different correction values on the average wind pressure coefficient (mean).

| Partition | Newton–Raphson method | The method in this paper takes the error norm of different correction values |
|-----------|-----------------------|--------------------------------------------------------------------------|
|           |                       | 10e3 | 10e4 | 10e5 |
| A1        | −0.52                 | 2.14e−4 | 1.22e−4 | 3.23e−4 |
| A2        | −0.34                 | 1.49e−4 | 2.03e−5 | 2.98e−4 |
| A3        | −0.22                 | 3.23e−4 | 1.29e−4 | 1.22e−5 |
| A4        | −0.54                 | 2.33e−5 | 1.78e−4 | 3.22e−4 |
| B1        | −1.56                 | 3.21e−4 | 1.44e−5 | 2.03e−4 |
| B2        | −1.35                 | 0.92e−4 | 1.68e−4 | 2.43e−4 |
| B3        | −0.86                 | 2.31e−4 | 1.09e−5 | 1.65e−4 |
| C1        | −0.76                 | 2.33e−4 | 1.02e−5 | 2.11e−5 |
| C2        | −0.15                 | 0.94e−4 | 3.76e−4 | 2.76e−5 |
| C3        | −0.13                 | 3.66e−4 | 1.34e−5 | 3.22e−4 |
| D1        | 0.18                  | 1.75e−5 | 0.54e−5 | 3.18e−4 |
| E1        | −1.28                 | 0.43e−5 | 1.29e−4 | 0.87e−5 |

Table 3: The effect of different iteration times on the average wind pressure coefficient (mean).

| Partition | Newton–Raphson method | The error norms of different iterations in this method |
|-----------|-----------------------|-----------------------------------------------------|
|           |                       | 10   | 18   | 26   | 34   |
| A1        | −0.52                 | 2.14e−4 | 1.22e−5 | 0.46e−7 | 0.98e−8 |
| A2        | −0.34                 | 1.49e−4 | 1.25e−6 | 3.23e−7 | 4.12e−8 |
| A3        | −0.22                 | 3.23e−4 | 2.75e−5 | 0.87e−7 | 3.23e−8 |
| A4        | −0.54                 | 2.33e−5 | 2.37e−6 | 1.75e−7 | 2.57e−8 |
| B1        | −1.56                 | 3.21e−4 | 4.45e−5 | 3.54e−7 | 0.85e−8 |
| B2        | −1.35                 | 0.92e−4 | 3.76e−5 | 4.34e−7 | 2.53e−8 |
| B3        | −0.86                 | 2.31e−4 | 3.93e−5 | 1.86e−8 | 5.94e−9 |
| C1        | −0.76                 | 2.33e−4 | 5.23e−6 | 3.75e−7 | 5.74e−8 |
| C2        | −0.15                 | 0.94e−4 | 3.76e−6 | 2.76e−7 | 0.45e−8 |
| C3        | −0.13                 | 3.66e−4 | 1.34e−5 | 3.22e−6 | 1.54e−7 |
| D1        | 0.18                  | 1.75e−5 | 3.24e−6 | 4.65e−7 | 2.95e−8 |
| E1        | −1.28                 | 0.43e−5 | 4.73e−6 | 1.76e−7 | 4.62e−8 |

Table 4: Relative residuals and time-consuming under different grid numbers.

| Total number of grids (ten thousand) | Newton–Raphson method | Present method |
|--------------------------------------|-----------------------|----------------|
|                                      |                       |                |
|                                      | R                     | T              | R             | T              |
| 31                                   | 4.419 × 10^−5         | 149            | 1.122 × 10^−5 | 84             |
| 33                                   | 4.204 × 10^−5         | 178            | 8.329 × 10^−6 | 93             |
| 36                                   | 3.986 × 10^−5         | 209            | 5.788 × 10^−6 | 103            |
| 39                                   | 3.795 × 10^−5         | 247            | 3.866 × 10^−6 | 114            |

The following can be seen from Table 4 and Figure 7:

1. Under the same grid accuracy and iteration times, regardless of whether the traditional Newton–Raphson method or the method in this paper is used, as the grid refinement increases, the relative residuals of the calculation are gradually reduced, and the computation time is also increased. However, the relative residual error and machine time consumption of the modified projection method in this paper are lower than those of the traditional Newton–Raphson method, which indicates a higher accuracy and efficiency of the proposed method.

2. For Newton–Raphson method, when the grid accuracy increases by about 10% on average, the accuracy increases by about 5%, while the computation time increases by about 20% on average, and the stability is basically not affected, indicating that the fineness of grid division has little effect on the accuracy and stability.

3. When using the modified projection method in this paper, when the grid accuracy is increased by about 10% on average, the calculation accuracy is increased by about 30% on average, while the time spent on the machine is only increased by about 10% on average, and the stability is basically not affected. It shows
that, under the condition that the accuracy of the grid is improved, the calculation accuracy and efficiency of the modified projection method in this paper are significantly improved. It also shows that the calculation result of the method in this paper is highly dependent on the grid.

5. Conclusion

In this paper, according to the characteristics of flexible membrane structure undergoing large deformation, the traditional projection method is modified to solve the above strong coupling integral equation, which is applied to the two-dimensional fluid-solid coupling problem and the calculation of the wind-induced fluid-structure interaction of the three-dimensional flexible membrane structure. The main conclusions are as follows:

1. The modified projection method can be used to calculate the wind-induced fluid-structure coupling of flexible membrane structures, and its calculation accuracy and efficiency are higher than the traditional Newton–Raphson method.

2. The modified value of the modified projection method affects the results a little, while the calculation of iteration times is an important factor affecting the results.

3. The calculation result of the modified projection method is highly dependent on the grid, and the accuracy can be greatly improved by increasing the number of iterations in the computation.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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