Abstract—In this paper, we devise adaptive decision schemes to detect targets competing against clutter and smart noise-like jammers (NLJs) which illuminate the radar system from the side lobes. Specifically, the considered class of NLJs generates a pulse of noise (noise cover pulse) that is triggered by and concurrent with the received uncompressed pulse in order to mask the skin echo and, hence, to hide the true target range. The detection problem is formulated as a binary hypothesis test and two different models for the NLJ are considered. Then, ad hoc modifications of the generalized likelihood ratio test are exploited where the unknown parameters are estimated by means of cyclic optimization procedures. The performance analysis is carried out using simulated data and proves the effectiveness of the proposed approach for both situations where the NLJ is either active or switched off.

Index Terms—Adaptive radar detection, alternating estimation, generalized likelihood ratio test, electronic countermeasures, electronic counter-countermeasures, noise cover pulse, noise-like jammers.

I. INTRODUCTION

Electronic countermeasures (ECMs) are active techniques aimed at protecting a platform from being detected and tracked by the radar [1]. This is accomplished through two approaches: masking and deception. Noncoherent jammers or noise-like jammers (NLJs) attempt to mask targets generating nondeceptive interference which blends into the thermal noise of the radar receiver. As a consequence, the radar sensitivity is degraded due to the increase of the constant false alarm rate threshold which adapts to the higher level of noise [1], [2]. In addition, this increase makes it more difficult to discover that jamming is taking place [3], [4]. On the other hand, the coherent jammers (CJs) transmit low-duty cycle signals intended to inject false information into the radar processor. Specifically, they are capable of receiving, modifying, amplifying, and retransmitting the radar’s own signal to create false targets maintaining radar’s range, Doppler, and angle far away from the true position of the platform under protection [1]–[3], [5].

Nowadays, radar designers have developed defense strategies referred to as electronic counter-countermeasures (EC-CMs) which are aimed at countering the effects of the enemy’s ECM and eventually succeeding in the intended mission. Such techniques can be categorized as antenna-related, transmitter-related, receiver-related, and signal-processing-related depending on the main radar subsystem where they take place [3]. The reader is referred to [3, and references therein] for a detailed description of the major ECCM techniques.

The first line of defense against jamming is represented by the radar antenna, whose beampattern can be suitably exploited and/or shaped to eliminate sidelobe false targets or to attenuate the power of NLJs entering from the antenna side lobes. In this context, famous antenna-related techniques capable of preventing jamming signals from entering through the radar side lobes are the so-called sidelobe blanking (SLB) and sidelobe canceling (SLC) [6]. In particular, suppression of NLJs can be accomplished via an SLC system. SLC uses an array of auxiliary antennas to adaptively estimate the direction of arrival and the power of the jammers and, subsequently, to modify the receiving pattern of the radar antenna placing nulls in the jammers’ directions. SLB and SLC can be jointly used to face with NLJs and CJs contemporaneously impinging on the side lobes of the victim radar [7]. In [6] it is also shown that a data dependent threshold, based on [8], outperforms a cascade of SLC and SLB stages. The detector proposed in [8] is a special case of the more general class of tunable (possibly space-time) detectors which have been shown to be an effective means to attack detection of mainlobe targets or rejection of CJs notwithstanding the presence of NLJs and clutter [8]–[20]. As a matter of fact, such solutions can be viewed as signal-processing-related ECCMs. A way to design tunable receivers relies on the so-called two-stage architecture; such schemes are formed by cascading two detectors (usually with opposite behaviors in terms of selectivity): the overall one declares the presence of a target in the cell under test only when data survive both detection thresholdings [9]–[12], [16]–[20]. Such detectors can also be used as classifiers: in this case, the first stage is less selective than the second one and it is used to discriminate between the null hypothesis and the alternative that a structured signal is present. In case of detection, the second stage is aimed at discrimination between mainlobe and sidelobe signals, as explicitly shown in [17] for the adaptive sidelobe...
blanker (ASB). Adaptive detection and discrimination between useful signals and CJs in the presence of thermal noise, clutter, and possible NLJ has also been addressed in [21]. Therein the CJ is assumed to belong to the orthogonal complement of the space spanned by the nominal steering vector (after whitening by the true covariance matrix of the composite disturbance). This approach, based on a modified adaptive beamformer orthogonal rejection test (ABORT), see also [20], [22], allows to investigate the discrimination capabilities of adaptive arrays when the CJ is not necessarily confined to the "sidelobe beam pattern," but might also be a mainlobe deception jammer. Another approach to deal with CJ is presented in [23], [24], where the CJ is modeled exploiting the subspace paradigm. A network of radars can be exploited to combat ECM signals. In this case, it is reasonable that, for a given CUT, only a subset of the radars receives ECM signals (CJs) as considered in [25].

Herein, we address adaptive detection in presence of noise cover pulse (NCP) jamming. The NCP is an ECM technique belonging to the class of noise-like jamming. Specifically, this kind of ECM generates a pulse of noise that is triggered by and concurrent with the received uncompressed pulse (see Fig. 1). To this end, several received radar pulses are used to estimate the pulse width (PW) and the pulse repetition interval (PRI) to predict the arrival time instant of the next pulse of the victim radar. The transmitted noise power is strong enough to mask the skin echo even after the radar performs the pulse compression, which is used to enhance the range resolution. It follows that, since the length of the transmitted pulse is much higher than the duration of a range bin, the NCP creates an extended-range return spread over many range bins that hides the true target range. Thus, it becomes of vital importance for a radar system to counteract the effects of an NCP attack. An ECCM technique against NCP is represented by the cover pulse channel (CPC) [26], which consists in using an auxiliary physical channel to track the NCP transmission rather than the skin return from the target. The main drawbacks of this technique are the degradation of the high-range resolution associated with the narrow pulses which result from the compression process and the exploitation of additional hardware resources. In order to overcome such limitations, in this paper we devise a signal-processing-related ECCM capable of detecting targets which compete against a NCP, while satisfying the original system requirements on range resolution. Besides, the proposed solution by its nature can reside in the signal processing unit of the system without the need of additional hardware. From a mathematical point of view, we formulate the detection problem as a binary hypothesis test where primary data (namely those containing target returns) are formed by a set of range bins which is representative of the uncompressed pulse length and such that target return is located in only one bin whereas all the primary range bins are contaminated by the NCP. As for the NCP, we consider two models. In the first case, the NCP is represented as a rank-one modification of the interference covariance matrix (ICM), while in the second case the presence of the NCP is accounted for by including a deterministic structured component in all the range bins. Moreover, we assume that a set of training samples are available to estimate the clutter and noise components of the ICM. These data are collected using a suitable number of guard cells surrounding those under test and related to the uncompressed pulse length. Then, we derive adaptive architectures exploiting ad hoc modifications of the generalized likelihood ratio test (GLRT) design criterion where the unknown parameters are estimated resorting to an alternating procedure. Specifically, we leverage the cyclic optimization paradigm described in [27]–[29]. Finally, we present numerical examples which highlight the effectiveness of the proposed solutions also in comparison with existing architectures which are somehow compatible with the considered problem.

The remainder of the paper is organized as follows: next section is devoted to the problem formulation and to the description of the two different models for the NCP. Section III contains the derivation of the detection architectures, whereas Section IV provides the performance assessment of the detectors (also in comparison to natural competitors). Concluding remarks and future research tracks are given in Section V.

A. Notation

In the sequel, vectors and matrices are denoted by boldface lower-case and upper-case letters, respectively. The symbols $\text{det}(\cdot)$, $\text{Tr} (\cdot)$, $\text{ctr} \{ \cdot \}$, $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^\dagger$ denote the determinant, trace, exponential of the trace, complex conjugate, transpose, and conjugate transpose, respectively. As to numerical sets, $\mathbb{R}$ is the set of real numbers, $\mathbb{R}^{N \times M}$ is the Euclidean space of $(N \times M)$-dimensional real matrices (or vectors if $M = 1$), $\mathbb{C}$ is the set of complex numbers, and $\mathbb{C}^{N \times M}$ is the Euclidean space of $(N \times M)$-dimensional complex matrices (or vectors if $M = 1$). $I_N$ stands for the $N \times N$ identity matrix, while $0$ is the null vector or matrix of proper dimensions. Let $f(x) \in \mathbb{R}$ be a scalar-valued function of vector argument, then $\partial f(x)/\partial x$ denotes the first derivative of $f(\cdot)$ with respect to $x$ arranged in a column vector. The Euclidean norm of a vector is denoted by $\| \cdot \|$. The $(k,l)$-entry (or $l$-entry) of a generic matrix $A$ (or vector $a$) is denoted by $A(k,l)$ (or $a(l)$). Finally, we write $x \sim \mathcal{CN}_N (m, M)$ if $x$ is an $N$-dimensional complex normal vector with mean $m$ and positive definite covariance matrix $M$. 

Fig. 1. Operating principle of NCP.
II. PROBLEM STATEMENT

Assume that the radar is equipped with a uniformly-spaced linear array (ULA) of \( N \) identical and isotropic sensors with inter-element distance equal to \( \lambda / 2 \), \( \lambda \) being the wavelength corresponding to the radar carrier frequency. For each sensor, the incoming signal is downconverted to baseband and, then, convolved with a conjugate time-reversed copy of the transmitted waveform (matched filter). The output of this filter is sampled to form the range bins of the area under surveillance. Thus, each range bin is represented by an \( N \times 1 \) waveform (matched filter). The output of this filter is sampled to form the range bins of the area under surveillance. Thus, each range bin is represented by an \( N \times 1 \) linear array (ULA) of \( N \) elements indexing the CUT and by \( \{ i \} \), \( z_i \in \mathbb{C}^{N \times 1} \), and \( \bar{z}_i \in \mathbb{C}^{N \times 1} \) with \( i \in \Omega \setminus \{ i \} \), and \( \bar{r}_k \in \mathbb{C}^{N \times 1} \) with \( k = 1, \ldots, K \), the vector containing the returns from the CUT, the vectors contaminated by the NCP jammer, but free of target components, and the secondary data, respectively. For further developments we assume that such vectors are statistically independent. Then, the problem of detecting the possible presence of a coherent return from a given cell is formulated in terms of the following hypothesis test

\[
\begin{aligned}
\mathcal{H}_0 : & \quad \{ z_i \sim \mathcal{C}_N (0, M + q q^\dagger), \quad i \in \Omega \setminus \{ i \}, \\
& \quad \bar{r}_k \sim \mathcal{C}_N (0, M), \quad k = 1, \ldots, K, \\
\mathcal{H}_1 : & \quad \{ z_i \sim \mathcal{C}_N (\alpha v \bar{\theta}_r, M + q q^\dagger), \quad i \in \Omega \setminus \{ i \}, \\
& \quad \bar{r}_k \sim \mathcal{C}_N (0, M), \quad k = 1, \ldots, K,
\end{aligned}
\]

where

- \( \alpha \in \mathbb{C} \) is an unknown deterministic factor accounting for target response and channel effects;
- \( v(\theta_r) = \frac{1}{\sqrt{N}} [ e^{j \pi \sin(\theta_r)} \cdots e^{j \pi (N-1) \sin(\theta_r)} ]^T \in \mathbb{C}^{N \times 1} \) is the known steering vector of the target with \( \theta_r \) the angle of arrival of the target\(^1\):
- for brevity, we omit the dependence of \( v \) on \( \theta_r \);
- \( M \in \mathbb{C}^{N \times N} \) is the unknown positive definite covariance matrix of thermal noise plus clutter;
- \( q \in \mathbb{C}^{N \times 1} \) is an unknown vector representing the contribution to the noise covariance matrix of the NCP jamming.

Some definitions that will be used in the next developments for problem (1) are now in order. Let \( Z_{\Omega,i} = [z_{i-H_1} \cdots z_{i-H_2}], Z_{\Omega,i} = [z_{\alpha,i} Z_{\Omega,i}] \) with \( z_{\alpha,i} = z_i - \alpha v \), and \( \bar{Z}_i = [\bar{z}_i Z_{\Omega,i}] \). Then, the probability density functions (PDFs) of \( Z_i \) under \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) are given by

\[
f_0(Z_i; q, M) = \frac{1}{[\pi^N \det (M + q q^\dagger)]^T} \chi \left\{ -(M + q q^\dagger)^{-1} Z_i; Z_i^\dagger \right\}
\]

and

\[
f_1(Z_i; \alpha, q, M) = \frac{1}{[\pi^N \det (M + q q^\dagger)]^T} \chi \left\{ -(M + q q^\dagger)^{-1} Z_{\alpha,i}; Z_{\alpha,i}^\dagger \right\},
\]

respectively, whereas the PDF of \( R = [r_1 \cdots r_K] \) under both hypotheses has the following expression

\[
f(R; M) = \frac{1}{[\pi^N \det (M)]^T} \chi \left\{ -M^{-1} R R^\dagger \right\}.
\]

Finally, let us define the likelihood function of the unknown parameters under \( \mathcal{H}_0, i = 0, 1, \) as

\[
\begin{aligned}
\mathcal{L}_0(q, M) & = f_0(Z_i; q, M), \\
\mathcal{L}_1(\alpha, q, M) & = f_1(Z_i; \alpha, q, M).
\end{aligned}
\]

Now, we formulate the detection problem from another perspective. Specifically, observe that the radar system, at each dwell, collects a realization of the NCP. Thus, it is reasonable to compare (1) with another detection problem formulated as

\[
\begin{cases}
\mathcal{H}_0 : \quad \{ z_i \sim \mathcal{C}_N (\beta q, M), \quad i \in \Omega \setminus \{ i \}, \\
& \quad \bar{r}_k \sim \mathcal{C}_N (0, M), \quad k = 1, \ldots, K,
\end{cases}
\]

\[
\begin{cases}
\mathcal{H}_1 : \quad \{ z_i \sim \mathcal{C}_N (\alpha v + \beta q, M), \quad i \in \Omega \setminus \{ i \}, \\
& \quad \bar{r}_k \sim \mathcal{C}_N (0, M), \quad k = 1, \ldots, K,
\end{cases}
\]

where

- \( \beta \in \mathbb{C} \) and \( \beta_i \in \mathbb{C} \) are unknown deterministic factors representative of the different jammer amplitudes;
- \( q \in \mathbb{C}^{N \times 1} \) is an unknown deterministic vector representing the contribution of the NCP jamming.

\(^1\)Note that the steering corresponds to a ULA with half-wavelength spacing.
Again, we have that
\( \alpha \in \mathbb{C} \) is an unknown deterministic factor accounting for the target response and channel effects;
\( \bm{v} \in \mathbb{C}^{N \times 1} \) is the known steering vector of the target;
\( \bm{M} \in \mathbb{C}^{N \times N} \) is the unknown positive definite covariance matrix of thermal noise plus clutter.

Furthermore, in this case, the PDF of \( \mathbf{Z}_i \), under \( \mathcal{H}_l \), \( l = 0, 1 \), exhibits the following expression
\[
f(z_i, \mathbf{Z}_{\Omega,i}; l) = \frac{1}{[\pi^N \det(M)]^H} \exp \left\{ -\text{Tr} \left[ M^{-1} \left( (z_i - l \alpha \bm{v} - \beta) (z_i - l \alpha \bm{v} - \beta) \right) + \sum_{i \in \Omega \setminus \{i\}} (z_i - \beta_i) (z_i - \beta_i) \right] \right\},
\]
where the likelihood functions under \( \mathcal{H}_l \), \( l = 0, 1 \), are given by
\[
\mathcal{L}_0(\beta, \beta_i, i \in \Omega \setminus \{i\}, M, q) = f(z_i, \mathbf{Z}_{\Omega,i}; 0, \beta, \beta_i, i \in \Omega \setminus \{i\}, M, q), \\
\mathcal{L}_1(\alpha, \beta, \beta_i, i \in \Omega \setminus \{i\}, M, q) = f(z_i, \mathbf{Z}_{\Omega,i}; \alpha, \beta, \beta_i, i \in \Omega \setminus \{i\}, M, q).
\]

### III. Detector Designs

In this section, we devise adaptive decision schemes for problems (1) and (7). To this end, observe that we cannot apply the Neyman-Pearson criterion since parameters \( \alpha, \beta, \beta_i, M \) and \( q \) are not known. For this reason, we have to resort to ad hoc solutions. In particular, we adopt the two-step GLRT-based design procedure: first we derive the GLRT for known \( M \); then we obtain an adaptive detector replacing the unknown matrix \( M \) with an estimate based on secondary data. Thus, the main problem to solve is to discriminate between the interference-only-hypothesis \( \mathcal{H}_0 \) and the signal-plus-interference-hypothesis \( \mathcal{H}_1 \) based on \( z_i \) and \( \mathbf{Z}_{\Omega,i} \) only (for known \( M \)).

#### A. An Adaptive Architecture for Problem (1)

The GLRT for known \( M \) is given by
\[
\max_{q} \mathcal{L}_1(\alpha, q, M) \quad \mathcal{L}_0(q, M) \geq \eta \quad \eta \rightarrow \eta_0,
\]
where \( \eta \) is the threshold\(^2\) to be set according to the desired value of the probability of false alarm (\( P_{fa} \)).

Maximization of the PDF under \( \mathcal{H}_0 \) can be conducted by using the following identities
\[
\det(M + q \mathbf{u}^\dagger) = \det(M) \det(I_N + M^{-1/2} q \mathbf{u}^\dagger M^{-1/2}) = \det(M) (1 + \mathbf{u}^\dagger \mathbf{u}),
\]
where \( \mathbf{u} = M^{-1/2} q \) and
\[
\text{Tr} \left[ (M + q \mathbf{u})^{-1} Z_i^\dagger Z_i^\dagger \right] = \text{Tr} \left[ M^{-1/2} (I_N + \mathbf{u}^\dagger \mathbf{u})^{-1} X_i^\dagger Z_i^\dagger \right] = \text{Tr} \left[ (I_N + \mathbf{u}^\dagger \mathbf{u})^{-1} X_i^\dagger X_i^\dagger \right] \quad \text{Tr} \left[ X_i^\dagger X_i^\dagger - \frac{\mathbf{u}^\dagger \mathbf{u}}{1 + \mathbf{u}^\dagger \mathbf{u}} X_i^\dagger X_i^\dagger \right]
\]
where the last equality in equation (12) is obtained using the matrix inversion lemma while \( X_i = M^{-1/2} [z_i, Z_{\Omega,i}] = [x_{\alpha,i}, X_{\Omega,i}] \). Maximization under the \( \mathcal{H}_1 \) hypothesis is conducted using identity (11), but replacing (12) with
\[
\text{Tr} \left[ (M + q \mathbf{u})^{-1} Z_{\alpha,i}^\dagger Z_{\alpha,i}^\dagger \right] = \text{Tr} \left[ X_{\alpha,i}^\dagger X_{\alpha,i}^\dagger - \frac{\mathbf{u}^\dagger \mathbf{u}}{1 + \mathbf{u}^\dagger \mathbf{u}} X_{\alpha,i}^\dagger X_{\alpha,i}^\dagger \right]
\]
where \( X_{\alpha,i} = M^{-1/2} Z_{\alpha,i} = [x_{\alpha,i}, X_{\Omega,i}] \) and, in particular, \( x_{\alpha,i} = M^{-1/2}(z_i - \alpha \mathbf{v}) \). It follows that the likelihood functions under \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) can be re-written as
\[
\mathcal{L}_0(q, M) = \frac{1}{[\pi^N \det(M)] (1 + \mathbf{u}^\dagger \mathbf{u})^H} \times \text{etr} \left\{ -X_i^\dagger X_i^\dagger + \frac{\mathbf{u}^\dagger \mathbf{u}}{1 + \mathbf{u}^\dagger \mathbf{u}} X_i^\dagger X_i^\dagger \right\}
\]
and
\[
\mathcal{L}_1(\alpha, q, M) = \frac{1}{[\pi^N \det(M)] (1 + \mathbf{u}^\dagger \mathbf{u})^H} \times \text{etr} \left\{ -X_{\alpha,i}^\dagger X_{\alpha,i}^\dagger + \frac{\mathbf{u}^\dagger \mathbf{u}}{1 + \mathbf{u}^\dagger \mathbf{u}} X_{\alpha,i}^\dagger X_{\alpha,i}^\dagger \right\}
\]
respectively.

Now, we focus on the maximization of the PDF under \( \mathcal{H}_0 \). To this end, observe that \( \mathbf{u} \), borrowing the approach developed in [30], can be represented as \( \mathbf{u} = \sqrt{p} \mathbf{u}_0 \) with \( p = \mathbf{u}^\dagger \mathbf{u} = ||\mathbf{u}_0||^2 > 0 \) and, hence, \( ||\mathbf{u}_0|| = 1 \). For future reference, we also define by \( \tilde{s} \) the \( N \)-sphere centered at the origin with unit radius; thus, condition \( \mathbf{u}_0^\dagger \mathbf{u}_0 = ||\mathbf{u}_0||^2 = 1 \) is equivalent to \( \mathbf{u}_0 \in \tilde{s} \).

It follows that
\[
\max_q \mathcal{L}_0(q, M) = \frac{1}{[\pi^N \det(M)]^H} \text{etr} \left\{ -X_i^\dagger X_i^\dagger \right\} \times \max_{\mathbf{u}_0 > 0} \left\{ \frac{p \mathbf{u}_0^\dagger \mathbf{u}_0 X_i^\dagger X_i^\dagger}{1 + p} \right\}
\]
Thus, for known \( \mathbf{u}_0 \), maximizing \( \mathcal{L}_0 \) with respect to \( p \) is tantamount to maximizing
\[
g(p) = \frac{1}{(1 + p)^H} \text{etr} \left\{ \frac{p \mathbf{u}_0^\dagger \mathbf{u}_0 X_i^\dagger X_i^\dagger}{1 + p} \right\}
\]
\(^2\)Hereafter, \( \eta \) denotes any modification of the original threshold.
with respect to \( p \geq 0 \). It can be shown that the maximum is attained at
\[
\hat{p} = \begin{cases} \frac{u_0^\dagger X_i X_i^\dagger u_0}{H} - 1, & \text{if } \frac{u_0^\dagger X_i X_i^\dagger u_0}{H} > 1, \\ 0, & \text{otherwise}, \end{cases}
\]
and is given by
\[
\max_{p \geq 0} g(p) = \begin{cases} \left[ \frac{H}{u_0^\dagger X_i X_i^\dagger u_0} \right]^H \exp \left\{ u_0^\dagger X_i X_i^\dagger u_0 - H \right\}, & \text{if } \frac{u_0^\dagger X_i X_i^\dagger u_0}{H} > 1, \\ 1, & \text{otherwise}. \end{cases}
\]

Now, we let
\[
h(x) = \begin{cases} \left[ \frac{H}{x} \right]^H e^{-x}, & x \in (H, +\infty), \\ 1, & x \in [0, H], \end{cases}
\]
and observe that it is a strictly increasing function of \( x \) over \([H, +\infty]\) (and constant over \([0, H]\)). It follows that to maximize \( \mathcal{L}_0 \) with respect to \( u \) it is sufficient to plug the maximizer of \( u_0^\dagger X_i X_i^\dagger u_0 \) with respect to \( u_0 \) into \( \max_{p \geq 0} g(p) \). Using the Rayleigh-Ritz theorem [31], we obtain
\[
\max_{u_0 \in \mathbb{S}} u_0^\dagger X_i X_i^\dagger u_0 = \lambda_1 \left( X_i X_i^\dagger \right),
\]
where \( \lambda_1(\cdot) \) denotes the maximum eigenvalue of the matrix argument and a maximizer for \( u_0 \) is a normalized eigenvector of the matrix \( X_i X_i^\dagger \) corresponding to \( \lambda_1(X_i X_i^\dagger) \). Thus, we can conclude that
\[
\max_{q \in \mathbb{S}} \mathcal{L}_0(q, M) = \frac{1}{|N\det(M)|^H \text{etr} \left\{ -X_i X_i^\dagger \right\}} \times \begin{cases} \left[ \frac{H}{\lambda_1(X_i X_i^\dagger)} \right]^H \exp \left\{ \lambda_1 \left( X_i X_i^\dagger \right) - H \right\}, & \text{if } \frac{\lambda_1(X_i X_i^\dagger)}{H} > 1, \\ 1, & \text{otherwise}. \end{cases}
\]

As for the optimization problem under \( \mathcal{H}_1 \), let us compute the logarithm of the likelihood function (14) neglecting the terms independent of \( \alpha \) and \( u \) to obtain
\[
g(\alpha, p, u_0) = -H \log (1 + u^\dagger u) - \text{Tr} \left[ \left( I_N - \frac{uu^\dagger}{1 + u^\dagger u} \right) S_{\alpha, \tilde{\alpha}} \right] - \text{Tr} \left[ \left( I_N - \frac{uu^\dagger}{1 + u^\dagger u} \right) S_{\alpha, \tilde{\alpha}} \right],
\]
\[
= -H \log (1 + u^\dagger u) + \frac{u^\dagger S_{\alpha, \tilde{\alpha}} u}{1 + u^\dagger u} + \frac{x_i^\dagger u}{1 + u^\dagger u} - x_i^\dagger x_i - \text{Tr} \left[ S_{\Omega, \tilde{\alpha}} \right]
\]
\[
= -H \log (1 + p) + \frac{p}{1 + p} u_0^\dagger S_{\alpha, \tilde{\alpha}} u_0
\]
\[
+ \frac{p}{1 + p} |x_i^\dagger u_0|^2 - x_i^\dagger x_i - \text{Tr} \left[ S_{\Omega, \tilde{\alpha}} \right],
\]
where \( S_{\alpha, \tilde{\alpha}} = X_{\alpha, \tilde{\alpha}} X_{\alpha, \tilde{\alpha}}^\dagger \) and \( x_{\alpha, \tilde{\alpha}} = x_{\alpha, \tilde{\alpha}} u_0^\dagger \).
It follows that maximizing \( \mathcal{L}_1(\alpha, q, M) \) with respect to \( \alpha \) and \( q \) is tantamount to
\[
\max_{\alpha, q, u_0} g(\alpha, q, u_0).
\]

However, this joint maximization with respect to \( \alpha \), \( p \), and \( u_0 \) is not an analytically tractable problem at least to the best of authors’ knowledge. For this reason, we resort to a suboptimum approach relying on alternating maximization [27]–[29]. Specifically, let us assume that \( u_0 = u_0^{(n)} \) and \( p = p^{(n)} \) are known, then it is not difficult to show that
\[
\alpha^{(n)} = \arg \max_{\alpha} g(\alpha, p^{(n)}, u_0^{(n)})
\]
\[
= \arg \min_{\alpha, \bar{\alpha}, \tilde{\alpha}} \left[ I_N - \frac{p^{(n)}}{1 + p^{(n)}} u_0^{(n)} (u_0^{(n)})^\dagger \right] x_{\alpha, \tilde{\alpha}}
\]
\[
= \frac{v_0^\dagger \left[ I_N - \frac{p^{(n)}}{1 + p^{(n)}} u_0^{(n)} (u_0^{(n)})^\dagger \right] x_\alpha}{v_0^\dagger v_0}
\]
where \( v_0 = M^{-1/2} v \). Now, let us exploit \( \alpha^{(n)} \) to estimate \( u_0 \) and \( p \), namely to solve the problem
\[
\max_{\alpha, p, u_0} g(\alpha^{(n)}, p, u_0).
\]
To this end, following the same line of reasoning as for \( \mathcal{H}_0 \), we obtain that
\[
\begin{bmatrix} p^{(n+1)} \\ u_0^{(n+1)} \end{bmatrix} = \arg \max_{\alpha, p, u_0} g(\alpha^{(n)}, p, u_0)
\]
\[
= \left[ \max \left\{ \lambda_1 \left( S_{\Omega, \tilde{\alpha}} + x_{\alpha^{(n)}, \tilde{\alpha}} x_{\alpha^{(n)}, \tilde{\alpha}}^\dagger / H - 1, 0 \right) \right\} b_{\tilde{\alpha}} \right],
\]
where we remember that \( \lambda_1(\cdot) \) is the maximum eigenvalue of the matrix argument, \( x_{\alpha^{(n)}, \tilde{\alpha}} \) is obtained replacing \( \alpha \) with \( \alpha^{(n)} \) in \( x_{\alpha, \tilde{\alpha}} \), and \( b_{\tilde{\alpha}} \) is a normalized eigenvector corresponding to \( \lambda_1 \left( S_{\Omega, \tilde{\alpha}} + x_{\alpha^{(n)}, \tilde{\alpha}} x_{\alpha^{(n)}, \tilde{\alpha}}^\dagger / H - 1, 0 \right) \).

\[3\] As a matter of fact, applying the classical maximum likelihood approach to \( g(\alpha, p, u_0) \) leads to difficult equations regardless the parameter optimization order as it stems from the expressions of the iterative parameter estimates given below.
Iterating the above estimation procedure, we come up with the following nondecreasing sequence
\[
\mathcal{L}_1(\alpha(0), p^{(0)}, M) \leq \mathcal{L}_1(\alpha(1), p^{(1)}, M) \leq \cdots \leq \mathcal{L}_1(\alpha(n), p^{(n)}, M), \tag{25}
\]
where we start using for \( q^{(0)} \) a normalized steering vector from a sideline direction and \( q^{(i)} = p(i) M^{(1)}/u_0(i) \), \( i = 1, \ldots, n \).
Since the likelihood under \( H_1 \) is bounded from above with respect to \( p, u_0 \), the nondecreasing sequence (25) converges as \( n \) diverges and, hence, a suitable stopping criterion can be defined. For example, a possible strategy might consist in continuing the procedure until
\[
\|q^{(n)} - q^{(n-1)}\| < \epsilon_q \quad \text{and/or} \quad |\alpha(n) - \alpha(n-1)| < \epsilon_\alpha. \tag{26}
\]

Another approach might be that the alternating procedure terminates when \( n > N_{\text{max}} \) with \( N_{\text{max}} \) the maximum allowable number of iterations. We will use the latter stopping criterion with \( N_{\text{max}} \) chosen in the next section.

To prove that the likelihood is bounded from the above, we re-write \( g \) as the sum of three (bounded above) functions, namely as
\[
g(\alpha, p, u_0) = -H \log(1 + p) - \text{Tr} \left[ S_{i\alpha} \right] + \frac{p}{1 + p} u_0^T S_{i\alpha} u_0
\]
\[
+ \frac{p}{1 + p} |x_{\alpha,i}^T u_0|^2 - x_{\alpha,i}^T x_{\alpha,i}
\]
\[
= g_1(p) + g_2(p, u_0) + g_3(\alpha, p, u_0) \tag{27}
\]
with
\[
g_1(p) = -H \log(1 + p) - \text{Tr} \left[ S_{i\alpha} \right], \tag{28}
\]
\[
g_2(p, u_0) = \frac{p}{1 + p} u_0^T S_{i\alpha} u_0, \tag{29}
\]
\[
g_3(\alpha, p, u_0) = \frac{p}{1 + p} |x_{\alpha,i}^T u_0|^2 - x_{\alpha,i}^T x_{\alpha,i}. \tag{30}
\]

Then, it is sufficient to observe that
• \( g_1(p) \leq 0, \forall p \geq 0; \)
• \( \forall p \geq 0, u_0 \in \mathcal{S} \) the second term \( g_2(p, u_0) \) can be trivially upperbounded as
\[
g_2(p, u_0) = \frac{p}{1 + p} u_0^T S_{i\alpha} u_0
\]
\[
\leq u_0^T S_{i\alpha} u_0 (\leq \lambda_1(S_{i\alpha})) \tag{31}
\]
and the right-most hand side attains a maximum since it is a continuous function of \( u_0 \) over a compact set (\( u_0 \) belongs to the \( N \)-sphere with unit radius);
• the third term can be re-written as
\[
g_3(\alpha, p, u_0) = -x_{\alpha,i}^T \left( I_N - \frac{p}{1 + p} u_0 u_0^\dagger \right) x_{\alpha,i}
\]

since the matrix \( I_N - \frac{p}{1 + p} u_0 u_0^\dagger \) is positive definite \( \forall p \geq 0 \) and \( u_0 \in \mathcal{S} \), it follows that \( g_3(\alpha, p, u_0) \leq 0, \forall \alpha \in \mathcal{C}, p \geq 0, u_0 \in \mathcal{S} \).
Finally, \( M \) can be estimated using secondary data \( R \) as
\[
\hat{M} = \frac{1}{K} RR^\dagger. \tag{32}
\]

**Algorithm 1:** Procedure to compute the decision statistic of R-NCP-D.

**Input:** \( z_i, Z_{\Omega,i}, R, u_0^{(0)}, p^{(0)}, v \)

**Output:** Decision statistic of R-NCP-D

1: Compute \( \hat{M} = \frac{1}{K} RR^\dagger \)

2: Compute \( \hat{X}_i = \hat{M}^{-1/2} \left[ z_i, Z_{\Omega,i} \right], \lambda_1(\hat{X}_i) \), and \( \hat{v}_0 = \hat{M}^{-1/2} v \)

3: Compute (19) with \( \hat{X}_i \) and \( \hat{M} \) in place of \( X_i \) and \( M \), respectively, and denote the result by \( \hat{L}_0 \)

4: Set \( n = 0 \)

5: Compute \( \alpha(n) \) using (22) with \( \hat{X}_i \) and \( \hat{v}_0 \) in place of \( X_i \) and \( v_0 \), respectively

6: If \( n < N_{\text{max}} \) go to step 7 else go to step 9

7: Compute \( p^{(n+1)} \) and \( u_0^{(n+1)} \) using (24) with \( \hat{X}_i \) and \( \hat{v}_0 \) in place of \( X_i \) and \( v_0 \), respectively

8: Set \( n = n + 1 \) and go to step 5

9: Return \( \mathcal{L}_1(\alpha(n), p^{(n)}, u_0^{(n)}, \hat{M})/\hat{L}_0 \)

This decision scheme is referred to in the following as random NCP detector (R-NCP-D). The steps to compute the decision statistic of R-NCP-D are summarized in Algorithm 1.

**B. An Adaptive Architecture for Problem (7)**

In this case, the GLRT for known \( M \) is given by
\[
\max_{\alpha, \beta, \beta, i \in \Omega \setminus \{ i \}, M, q} \mathcal{L}_1(\alpha, \beta, i, \Omega \setminus \{ i \}, q) \tag{33}
\]
\[
\mathcal{L}_0(\beta, \bar{i}, \Omega \setminus \{ i \}, q) \rightarrow \eta_i.
\]

Focusing on the maximization of the PDF under \( H_0 \), we observe that maximizing \( \mathcal{L}_0 \) is tantamount to maximizing^4
\[
w(\beta, \bar{i}, i \in \Omega \setminus \{ i \}, q) = \text{etr} \left\{ -M^{-1} \left( (z_i - \beta q)(z_i - \beta q)^\dagger \right) \right\} + \sum_{i \in \Omega \setminus \{ i \}} (z_i - \beta q)(z_i - \beta q)^\dagger \right\} \tag{34}
\]

Furthermore, it can be proved that
\[
\min_{\beta} (z_i - \beta q)^\dagger M^{-1}(z_i - \beta q)
\]
\[
= z_i^\dagger M^{-1} z_i - z_i^\dagger M^{-1} q q^\dagger M^{-1} z_i \tag{35}
\]
and
\[
\min_{\beta} \sum_{i \in \Omega \setminus \{ i \}} (z_i - \beta q)^\dagger M^{-1}(z_i - \beta q)
\]
\[
= \sum_{i \in \Omega \setminus \{ i \}} z_i^\dagger M^{-1} z_i - z_i^\dagger M^{-1} q q^\dagger M^{-1} z_i \tag{36}
\]

^4 Note that this optimization is a special case of one of the instances considered in [23].
Thus, the maximization of $w$ with respect to $\beta$ and $\beta_i$ leads to 
\[
\max_{\beta, \beta_i, i \in H \setminus \{\overline{i}\}} w(\beta, \beta_i, i \in \Omega \setminus \{\overline{i}\}, q) = \max_q \exp \left[ \frac{\sum_{i \in H \setminus \{\overline{i}\}} \left( z_i M^{-1} z_i - z_i M^{-1} z_i \right)}{q M^{-1} q} \right] 
\]
\[
+ \sum_{i \in H \setminus \{\overline{i}\}} \left( z_i M^{-1} z_i - z_i M^{-1} z_i \right) \right] 
\]
\[
= \max_q \exp \left[ \frac{\sum_{i \in H \setminus \{\overline{i}\}} \left( z_i M^{-1} z_i + \sum_{i \in H \setminus \{\overline{i}\}} z_i z_i^\dagger \right) M^{-1} q}{q M^{-1} q} \right] 
\]
\[
- \left( z_i^\dagger M^{-1} z_i + \sum_{i \in H \setminus \{\overline{i}\}} z_i M^{-1} z_i \right) \right] 
\]
\[
= \max_u \left\{ u^\dagger \left[ z_i z_i^\dagger + \sum_{i \in H \setminus \{\overline{i}\}} z_i z_i^\dagger \right] M^{-1} u \right\} 
\]
\[
- \left( z_i^\dagger x_i^\dagger + \sum_{i \in H \setminus \{\overline{i}\}} z_i^\dagger x_i^\dagger \right) \right\} 
\]
\[
= \exp \left\{ \lambda_1(\mathbf{S}_1) - \text{Tr} (\mathbf{S}_1) \right\} 
\]
where $\mathbf{S}_1 = x_i x_i^\dagger + \sum_{i \in H \setminus \{\overline{i}\}} x_i x_i^\dagger$ and we used the Rayleigh-Ritz theorem [31]. We also recall that $x_i = M^{-1/2} z_i$ and $u = M^{-1/2} q$.

Thus, we conclude that the compressed likelihood under $H_0$ is given by
\[
\max_{\beta, \beta_i, i \in H \setminus \{\overline{i}\}, q} L_0(\beta, \beta_i, i \in \Omega \setminus \{\overline{i}\}, M, q) 
= \frac{1}{[\pi^N \det(M)]^{\frac{N}{2}}} \exp \left\{ \lambda_1(\mathbf{S}_1) - \text{Tr} (\mathbf{S}_1) \right\}. 
\]

As far as the maximization of the likelihood function under $H_1$ is concerned, we first focus on $\alpha$ and observe that
\[
\min_\alpha \left( z_i - \alpha v - \beta q \right) M^{-1} (z_i - \alpha v - \beta q) 
= (z_i - \beta q) M^{-1} (z_i - \beta q) 
- \frac{(z_i - \beta q) M^{-1} w v \dagger M^{-1} (z_i - \beta q)}{v \dagger M^{-1} v}. 
\]

Thus, it turns out that
\[
\max_\alpha L_1(\alpha, \beta, \beta_i, i \in \Omega \setminus \{\overline{i}\}, M, q) = \frac{1}{[\pi^N \det(M)]^{\frac{N}{2}}} \exp \left\{ \frac{(z_i - \beta q) M^{-1} w v \dagger M^{-1} (z_i - \beta q)}{v \dagger M^{-1} v} 
- (z_i - \beta q) M^{-1} (z_i - \beta q) 
- \sum_{i \in H \setminus \{\overline{i}\}} (z_i - \beta q) M^{-1} (z_i - \beta q) \right\}. 
\]

To the best of authors’ knowledge, the maximization of (40) with respect to the remaining parameters cannot be conducted in closed form; in particular, maximizing with respect to $u$ first makes the resulting function of $\beta$ and $\beta_i$ difficult to optimize due to the fact that it contains matrix-valued functions of the parameters. For this reason, we exploit another alternating optimization procedure. To this end, we first re-write the partially-compressed likelihood as
\[
\max_\alpha L_1(\alpha, \beta, \beta_i, i \in \Omega \setminus \{\overline{i}\}, M, q) 
= \frac{1}{[\pi^N \det(M)]^{\frac{N}{2}}} \exp \left\{ -h(u, \beta, \beta_i, i \in \Omega \setminus \{\overline{i}\}) \right\} 
\]
with
\[
h(u, \beta, \beta_i, i \in \Omega \setminus \{\overline{i}\}) = (z_i - \beta u) \dagger P_{v_0} \dagger (z_i - \beta u) 
+ \sum_{i \in H \setminus \{\overline{i}\}} (x_i - \beta_i u) \dagger (x_i - \beta_i u), 
\]

where
\[
P_{v_0} = I_N - v_0 v_0 \dagger. 
\]

and, for the reader ease, we recall that $u = M^{-1/2} q$, $x_i = M^{-1/2} z_i$, $x_i = M^{-1/2} \overline{z}_i$, and $v_0 = M^{-1/2} v$.

Then, assuming that $\beta = \beta(\alpha)$ and $\beta_i = \beta_i(\alpha)$ are given, we can focus on the maximization with respect to $q$. To this end, setting zero to the first derivative of
\[
\max_\alpha L_1(\alpha, \beta, \beta_i, i \in \Omega \setminus \{\overline{i}\}, M, q) 
= \frac{1}{[\pi^N \det(M)]^{\frac{N}{2}}} \exp \left\{ -h(u, \beta, \beta_i, i \in \Omega \setminus \{\overline{i}\}) \right\}, 
\]

with respect to $u$ leads to
\[
- \beta^{(n)} v_0 \dagger x_i + |\beta|^{2} P_{v_0} \dagger u 
- \sum_{i \in H \setminus \{\overline{i}\}} \beta_i^{(n)} v_0 x_i + \sum_{i \in H \setminus \{\overline{i}\}} \beta_i^{(n)} v_0 x_i = 0; 
\]

then
\[
u^{(n)} = \arg \min_u \left\{ h(u, \beta^{(n)}(\alpha), \beta_i^{(n)}(\alpha), i \in \Omega \setminus \{\overline{i}\}) \right\} 
= \left( |\beta|^{2} P_{v_0} + \sum_{i \in H \setminus \{\overline{i}\}} \beta_i^{(n)} v_0 x_i \right)^{-1} \left( \beta^{(n)} P_{v_0} \dagger x_i + \sum_{i \in H \setminus \{\overline{i}\}} \beta_i^{(n)} v_0 x_i \right). 
\]

We make use of the following definition for the derivative of a real function $f(\alpha)$ with respect to the complex argument $\alpha = \alpha_r + j \alpha_i$, $\alpha_r, \alpha_i \in \mathbb{R}$ [32]
\[
\frac{\partial f(\alpha)}{\partial \alpha} = \frac{1}{2} \left( \frac{\partial f(\alpha)}{\partial \alpha_r} - j \frac{\partial f(\alpha)}{\partial \alpha_i} \right). 
\]
IV. PERFORMANCE ASSESSMENT

The aim of this section is to investigate the performance of the proposed algorithms in terms of probability of detection ($P_d$). To this end, we resort to standard Monte Carlo counting techniques by evaluating the thresholds to ensure a preassigned $P_f$ and the $P_d$ curves over $100/P_f$ and 1000 independent trials, respectively. Data are generated according to the model defined in problem (1), where

$$M = \sigma_n^2 I_N + p_c M_c.$$  \hfill (48)

In (48), $\sigma_n^2 I_N$ represents the thermal noise component with power $\sigma_n^2$ while $p_c M_c$ is the clutter component with $p_c$ the clutter power and $M_c$ the structure of the clutter covariance matrix; the clutter-to-noise ratio (CNR) is thus given by $\text{CNR} = p_c/\sigma_n^2$.

In the following, we set $\sigma_n^2 = 1$, $\text{CNR} = 20$ dB, and $M_c(i,j) = \rho^{(i-j)}$ with $\rho = 0.9$ (recall that $M_c(i,j)$ is the $(i,j)$th entry of $M_c$). We recall that the victim radar is equipped with a ULA. The target response is computed according to the signal-to-clutter-plus-noise ratio (SCNR), whose expression is

$$\text{SCNR} = |\alpha|^2 \langle v(0) | M^{-1} v(0) \rangle.$$  \hfill (49)

As for the NCP, we assume that, when it is present, it enters the antenna array response of the victim radar from the sidelobes with a power $p_j$ such that the jammer-to-noise ratio (JNR), defined as $\text{JNR} = p_j/\sigma_n^2$, is 30 dB. In order to select the amount of primary data, we consider system parameter values of practical interest, namely we assume that the radar system transmits a linear frequency modulated pulse of duration 3 ms and bandwidth 5 MHz. Now, since the sampling time at which the range bins are generated is $2 \cdot 10^{-7}$ s, then the uncompressed pulse covers 15 range bins. Using 5 additional guard cells, we set $H_1 = H_2 = 10$. For the reader ease, all the simulation parameters are summarized in Table I.

Finally, for comparison purposes, we also report the $P_d$ curves of the subspace detector (SD) proposed in [23], the adaptive matched filter (AMF) [33], which raises from a suitable modification of the derivation contained in Subsection III-B, which consists in forcing the constraint $q^T M_c^{-1} v = 0$ as shown in the appendix\(^6\), the adaptive coherence estimator (ACE) [34], [35] (also known as adaptive normalized matched filter), and Kelly’s GLRT [36]. It is also important to notice that in the presence of NCP attack, another (heuristic) competitor would consist in projecting data onto the orthogonal complement of the subspace spanned by the jammer steering vector and in exploiting transformed data to build up a decision statistic as the AMF. However, such architecture requires that an estimate of jammer

\(^6\)Recall from Table I that the jammer is located at 35° with respect to the array normal, leading to $\cos(\theta_j) = 0.07$ for $N = 8$ and $\cos(\theta_j) = 0.03$ for $N = 16$, where $\theta_j$ is the angle between target and jammer steering vectors in the whitened observation space.
AOA should be available. Now, due to the enormous amount of AOA estimation algorithms in the open literature, the choice of the “right” estimation procedure would be a critical issue affecting the detection performance. For this reason, we simplify the analysis assuming that jammer AOA is perfectly known and proceed by considering the following heuristic detector (HD)

\[
\frac{|v(\theta_j)U_j (U_j^H \hat{M} U_j)^{-1} U_j^H \tilde{z}_i|_2^2}{\frac{v(\theta_j)U_j (U_j^H \hat{M} U_j)^{-1} U_j^H v(\theta_j)}{H_0} \geq \frac{v(\theta_j)U_j (U_j^H \hat{M} U_j)^{-1} U_j^H v(\theta_j)}{H_1}}
\]  \tag{50}

where \(\theta_j\) is the AOA of the jammer and \(U_j \in \mathbb{C}^{N \times (N-1)}\) is a slice of unitary matrix such that \(P_{\perp j} = U_j U_j^H\) with \(P_{\perp j} = I_N - \frac{v(\theta_j)U_j (U_j^H \hat{M} U_j)^{-1} U_j^H v(\theta_j)}{v(\theta_j)U_j (U_j^H \hat{M} U_j)^{-1} U_j^H v(\theta_j)}\). The curves for the likelihood ratio test with known parameters, referred to as clairvoyant detector (CD), are also included since they represent an upper bound to the detection performance.

In the next subsection, we focus on the stopping criterion and provide suitable numerical examples aimed at establishing a reasonable iteration number for both cyclic optimization procedures.

A. Selection of the Maximum Number of Iterations

The detection performance, assessed in Subsection IV-B, are obtained using a preassigned number of iterations. Now, in order to select this value, in Figs. 3 and 4, we show the average norm of the difference between the estimates at the \(n\)th iteration and their respective values at the \((n-1)\)th iteration. The averages are evaluated over 1000 independent Monte Carlo trials assuming \(N = 8\), \(K = 12\), and SCNR = 20 dB. In both cases we model \(q\) as a narrowband plane wave impinging onto the antenna array from a direction whose azimuth, generated at random at each Monte Carlo trial, is uniformly distributed outside the antenna mainlobe. Inspection of the figures highlights that 10 iterations for each procedure ensure a variation of the estimated quantities less than or equal to \(10^{-5}\) and also represent a good compromise from the computational point of view. For this reason, in what follows we adopt this number for the computation of the \(P_d\) curves.

B. Detection Performance

In this subsection, we investigate the behavior of the proposed architectures in terms of \(P_{fa}\) sensitivity to the interference parameters and \(P_d\) versus the SCNR for two different scenarios, which are complementary. Specifically, the former assumes a jammer illuminating the victim radar from its sidelobes, whereas the latter consider a surveillance area free of intentional interferers. It is important to underline that the second case does not correspond to the design assumptions.

The sensitivity analysis results for the \(P_{fa}\) with respect to jammer AOA, JNR, and CNR are summarized in Tables II–IV. Specifically, they are obtained by setting the threshold for a nominal scenario with \(P_{fa} = 10^{-3}\) and estimating the actual \(P_{fa}\) under several instances of NCP power, NCP AOA, and clutter power. The values reported in the tables show that the actual \(P_{fa}\) of the proposed architectures does not experience a
significant variation with respect to the nominal value, namely $10^{-3}$, when the interference parameters take on value in the considered intervals. The observed trend suggests a method of practical appeal to set the detection thresholds. In fact, the latter can be computed resorting to a nominal scenario where the parameters are set using reference values.

Figs. 5–7 refer to the first scenario assuming $N = 8$ and different values for $K$. The common denominator of these figures is that R-NCP-D, D-NCP-D, and HD (which perfectly knows the jammer AOA) share the same performance, since the respective curves are overlapped, and outperform the remaining architectures except for the CD (as expected). The gain of the R-NCP-D and D-NCP-D over the SD is about 10 dB at $P_d = 0.9$. On the other hand, the AMF has the worst performance with a loss at $P_d = 0.9$ ranging from about 7 dB for $K = 12$ to about 5 dB for $K = 24$ with respect to the SD. The curves of ACE and Kelly’s GLRT are in between those of SD and AMF and intersect the latter. Comparison of the figures also points out that the $P_d$ curves move towards left as $K$ increases, which means that the $P_d$ is an increasing function of $K$ given the SCNR. Finally, the loss at $P_d = 0.9$ of the R-NCP-D/D-NCP-D with respect to the CD halves when $K$ goes from $K = 12$, which corresponds to a loss of about 8 dB, to $K = 24$, which results in a loss of about 4 dB.

The second scenario is accounted for in Figs. 8–10, which assume the same parameter setting as previous figures except for the presence of the jammer. Note that the figures do not contain the $P_d$ curve of HD since it cannot be used in this
scenario because the information about the jammer AOA is not available. In this case, the curves of R-NCP-D and D-NCP-D are no longer overlapped and, more important, the latter does not achieve $P_d = 1$ at least for the considered parameter values. The R-NCP-D continues to provide satisfactory performance outperforming the other counterparts. In fact, the comparison with previous figures highlights that the performance of R-NCP-D keeps more or less unaltered. On the other hand, for $d$ values than those of Fig. 6; the performance of SD seems insensitive to the considered parameter values (and the considered scenario), the SD is completely useless since the resulting $P_d$ values are close to zero, while the AMF and Kelly’s GLRT significantly improve their performance as $K$ increases ensuring about the same $P_d$ values of the R-NCP-D for $K = 3N$.

Finally, the comparison between Fig. 11 and Fig. 6 allows to appreciate the performance variations due to both $N$ and $K$ when their ratio is constant. In fact, Fig. 11 assumes $N = 16$ and $K = 32$, namely twice the analogous values of Fig. 6. It can be noted that all the considered architectures except for the SD provide higher $P_d$ values than those of Fig. 6; the performance of SD seems insensitive to the considered parameter change.

Summarizing, the above analysis singles out the R-NCP-D as an effective mean to face attacks of smart NLJs which transmit noise-like signals to cover the skin echoes from the platform under protection. As a matter of fact, the R-NCP-D outperforms all the other competitors either in the presence or absence of a noise cover pulse.

V. CONCLUSION

In this paper, two new detection architectures to account for possible NCP attacks have been proposed and assessed. Specifically, the first approach consists in modeling the NLJ contribution as a covariance component, while the second solution considers the realizations of the NLJ and handles them as deterministic signals. Ad hoc modifications of the GLRT have been devised for both scenarios where the unknown parameters are computed through alternating estimation procedures. The behavior of the two architectures has been first investigated resorting to simulated data adhering the design assumptions of the first approach and, then, they have been tested on data where the NCP is turned off. The analysis has singled out the R-NCP-D obtained with the first approach as the recommended solution for adaptive detection in the presence of clutter and NCP, since it can guarantee satisfactory performance in both the considered situations at the price of an additional computational load with respect to its competitors due to the iterations necessary to reach a stationary point through the alternating maximization.

Finally, future research tracks might encompass the application of the herein presented approach to the case of range-spread targets possibly embedded in non-Gaussian clutter.

APPENDIX A

ALTERNATIVE DERIVATION OF THE AMF

In this Appendix, we modify the derivation contained in Subsection III-B by imposing the constraint $u^\dagger v = 0$, namely the target steering vector and the NCP signature are orthogonal in the whitened space. For the reader convenience, let us recall that $u = M^{-1/2}q$, $S_1 = [x_i x_i^\dagger + \sum_{\ell \in \Omega(i)} x_{i,\ell} x_{i,\ell}^\dagger]$, $v_0 = M^{-1/2}v$, $x_i = M^{-1/2}z_i$, $x_i = \tilde{M}^{-1/2}z_i$.

Now, under $\mathcal{H}_0$, the maximization with respect to $\beta$ and $\beta_i$ of $w$ (given by (34)) leads to (35) and (36), respectively. Thus, assuming the orthogonality constraint, it is possible to reformulate the maximization of $w$ with respect to $u$ as

$$\max_{u: u^\dagger v_0 = 0} \exp \left\{ \frac{u^\dagger S_1 u}{u^\dagger u} - \text{Tr} \left[ S_1 \right] \right\}. \quad (51)$$

Let $U \in \mathbb{C}^{N \times N-1}$ be a slice of unitary matrix, namely $U^\dagger U = \mathbb{I}_{N-1}$, with columns forming a basis for the orthogonal complement of the subspace spanned by $v_0$. Given the orthogonality constraint, it follows that $u$ can be expressed in terms of a linear combination of the columns of $U$, i.e., $u = U\gamma$ with $\gamma \in \mathbb{C}^{(N-1) \times 1}$ the coordinate vector. Then, maximization (51) can be recast as

$$\max_{u: u^\dagger v_0 = 0} \exp \left\{ \frac{u^\dagger S_1 u}{u^\dagger u} - \text{Tr} \left[ S_1 \right] \right\} = \exp \left\{ \max_{\gamma} \frac{\gamma^\dagger U^\dagger S_1 U \gamma}{\gamma^\dagger \gamma} - \text{Tr} \left[ S_1 \right] \right\} = \exp \left\{ \lambda_1 \left( U^\dagger S_1 U \right) - \text{Tr} \left[ S_1 \right] \right\}, \quad (52)$$

where the last equality is due to the Rayleigh-Ritz theorem [31]. Thus, the solution to the constrained optimization problem, under $\mathcal{H}_0$, is

$$\max_{\beta, \beta_i, \ell \in \Omega(i), q} \mathcal{L}_0(\beta, \beta_i, i \in \Omega \setminus \{\hat{i}\}, M, q) = \frac{1}{\pi^N \det(M)^{\frac{1}{2}}} \exp \left\{ \lambda_1 \left( U^\dagger S_1 U \right) - \text{Tr} \left[ S_1 \right] \right\}. \quad (53)$$

On the other hand, under $\mathcal{H}_1$, we can start from the partially-compressed likelihood with respect to $\alpha$, given by (41). In fact, it is possible to show that, after optimization of the likelihood function with respect to $\alpha$, $\beta$, and $\beta_i$, we obtain

$$\max_{\alpha, \beta, \beta_i, \ell \in \Omega(i), q} \mathcal{L}_1(\alpha, \beta, \beta_i, i \in \Omega \setminus \{\hat{i}\}, M, q) = \frac{1}{\pi^N \det(M)^{\frac{1}{2}}} \exp \left\{ \frac{u^\dagger P^\dagger_{\mathcal{F}_0} x_i x_i^\dagger P^\dagger_{\mathcal{F}_0} u}{u^\dagger P^\dagger_{\mathcal{F}_0} v_0 u} \right\}.$$

Fig. 11. $P_d$ versus SCNR assuming $N = 16$, $K = 32$, and a jammer at $35^\circ$. 
\[
\begin{align*}
&+ \sum_{i \in \Omega_R(i)} x_i^u u^u x_i - \left( x_i^u P_v x_i + \sum_{i \in \Omega_R(i)} x_i x_i \right) \\
&= \frac{1}{\left(\pi^N \det(M)^{1/2}\right)} \exp \left\{ \frac{u^T S u}{u^T u} - \left( x_i^u P_v x_i + \sum_{i \in \Omega_R(i)} x_i x_i \right) \right\ }
\end{align*}
\] (54)

where the last equality comes from the orthogonality condition between \( u \) and \( v_0 \). Thus, it follows that
\[
\begin{align*}
\max_{u \neq u v_0 = 0} \max_{\alpha, \beta, \beta_i, \alpha \in \Omega \setminus \{ i \}, M, \eta}
& L_1(\alpha, \beta, \beta_i, i \in \Omega \setminus \{ i \}, M, \eta) \\
& = \frac{1}{\left(\pi^N \det(M)^{1/2}\right)} \times \exp \left\{ \lambda_1(U^T S U) - \left( x_i^u P_v x_i + \sum_{i \in \Omega_R(i)} x_i x_i \right) \right\ }
\end{align*}
\] (55)

where, again, the last equality comes from [31]. The final expression for the constrained GLRT-based architecture is obtained combining (53) and (55). Precisely, taking the log-likelihoods, the “constrained test (33)” is statistically equivalent to
\[
x_i^u P_v x_i \geq \eta_i, \quad \eta_i \in \eta_0, \quad (56)
\]
whose decision statistic, after replacing \( M \) with the sample covariance matrix based on secondary data, coincides with that of the AMF.

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