Shell Model description of the $\beta\beta$ decay of $^{136}$Xe

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Abstract

We study in this letter the double beta decay of $^{136}$Xe with emission of two neutrinos which has been recently measured by the EXO-200 collaboration. We use the same shell model framework, valence space, and effective interaction that we have already employed in our calculation of the nuclear matrix element (NME) of its neutrinoless double beta decay. Using the quenching factor of the Gamow-Teller operator which is needed to reproduce the very recent high resolution $^{136}$Cs data, we obtain a nuclear matrix element $M^{2\nu}=0.025$ MeV$^{-1}$ compared with the experimental value $M^{2\nu}=0.019(2)$ MeV$^{-1}$.

Keywords: Shell Model, Double beta decay matrix elements

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The double beta decay is a rare process (second order in the weak interaction) which takes place when the single beta decay of the parent even-even nucleus to the neighbor odd-odd nucleus is forbidden by energy conservation or highly suppressed by the angular momentum selection rules. In addition it is one of the major sources of the mass scale of the neutrinos. Until the EXO-200 measure has been confirmed by KamLAND-Zen the 2$\nu$ neutrinoless decay which, if detected, will settle the nature (Majorana or Dirac) and the mass scale of the neutrinos. The EXO-200 measure only a few weeks ago.

Using the quenching factor of the Gamow-Teller operator which is needed to reproduce the very recent high resolution $^{136}$Cs data, we obtain a nuclear matrix element $M^{2\nu}=0.025$ MeV$^{-1}$ compared with the experimental value $M^{2\nu}=0.019(2)$ MeV$^{-1}$.

Table 1: Experimental 2$\nu$ $\beta\beta$ decay matrix elements

| Decay        | $M^{2\nu}$ (MeV$^{-1}$) | $T^{2\nu}_{1/2}$ (y) |
|--------------|-------------------------|----------------------|
| $^{48}$Ca $\rightarrow$ $^{48}$Ti | 0.047±0.003 | 4.4 $\times$ 10$^{19}$ |
| $^{76}$Ge $\rightarrow$ $^{76}$Se | 0.140±0.005 | 1.5 $\times$ 10$^{21}$ |
| $^{82}$Se $\rightarrow$ $^{82}$Kr | 0.098±0.004 | 9.2 $\times$ 10$^{19}$ |
| $^{96}$Zr $\rightarrow$ $^{96}$Mo | 0.096±0.004 | 2.3 $\times$ 10$^{19}$ |
| $^{100}$Mo $\rightarrow$ $^{100}$Ru | 0.246±0.007 | 7.1 $\times$ 10$^{18}$ |
| $^{116}$Cd $\rightarrow$ $^{116}$Sn | 0.136±0.005 | 2.8 $\times$ 10$^{19}$ |
| $^{128}$Te $\rightarrow$ $^{128}$Xe | 0.049±0.006 | 1.9 $\times$ 10$^{24}$ |
| $^{130}$Te $\rightarrow$ $^{130}$Xe | 0.034±0.003 | 6.8 $\times$ 10$^{20}$ |
| $^{136}$Xe $\rightarrow$ $^{136}$Ba | 0.019±0.002 | 2.1 $\times$ 10$^{23}$ |
| $^{150}$Nd $\rightarrow$ $^{150}$Sm | 0.063±0.003 | 8.2 $\times$ 10$^{18}$ |

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deviation of 100 keV. More details can be found in ref. [8].

It is well known that the effective Gamow-Teller operator \( \vec{\sigma}_{\pm} t^\pm \) for complete harmonic oscillator valence spaces can be approximated by \( q \cdot \vec{\sigma} \cdot t^\pm \). \( q \) is called the quenching factor and behaves as a sort of effective GT charge (see ref. [9] for a recent update of this topic). The value of \( q \) has been fitted throughout the nuclear chart and the resulting values are 0.82, 0.77, and 0.74 for the p, sd and pf shells. Asymptotically it tends to 0.7. With the quoted value of \( q \) for the pf-shell the half-life of the 2\( \nu \) double beta decay of \(^{48}\)Ca [10] could be predicted in perfect agreement with the later measured value [11]. The problem arises when we try to describe heavier emitters in which the minimal complete valence spaces (in the harmonic oscillator sense) are still out of reach computationally. A possible solution is to carry out a fit to all the experimentally available GT decays. We did this exercise in our valence space and the resulting value was \( q = 0.57 \). A more accurate way of estimating the quenching factor is by comparing the theoretical predictions for the Gamow Teller strength functions relevant for the process with the experimental results obtained in charge exchange reactions. These data were not available for the \(^{136}\)Xe case until the advent of the results of the \(^{136}\)Xe (\(^3\)He, \(t\))\(^{136}\)Cs reaction in the appropriate kinematics, which have been published very recently [12]. These results impact in our calculations in two ways; first because they give us the excitation energy of the first 1\(^+_2\) state in \(^{136}\)Cs, 0.59 MeV, unknown till now, which appears in the energy denominator of equation 2, and secondly because it makes it possible to extract directly the quenching factor adequate for this process.

In what follows, we use for the A=136 isobars the wave functions that result of the large scale shell model calculations in the same valence space and with the same effective interaction which we had used in our calculation of the 0\( \nu \) matrix elements of \(^{124}\)Sn, \(^{128}\)Te, \(^{130}\)Te and \(^{136}\)Xe in ref. [13]. First, we compare in Fig. 1 the theoretical running sum of the B(GT\(^-\)) strength of \(^{136}\)Xe with the experimental data from [12]. We have normalized the total theoretical strength in the experimental energy window to the measured one. This implies a quenching \( q = 0.45 \). Notice the very good agreement between the theoretical and experimental strength functions. If we had shifted the theoretical position of the first excited state of \(^{136}\)Xe to its experimental value, the quenching factor would have been slightly larger.

Then we compute the 2\( \nu \) matrix element with the quenching factor extracted above. The result is given in Figure 2 in the form of a running sum. The final matrix element \( M_{2\nu} = 0.025 \text{ MeV}^{-1} \) agrees nicely with the experimental value. However, one should bear in mind that the absolute normalization of the Gamow-Teller strength extracted from the charge exchange reactions may be affected by systematic errors, which could lead to modifications of the extracted quenching factor. Minor variants of the gen50:82 interaction which locally improve the quadrupole properties of \(^{136}\)Ba lead to \( q = 0.48 \) and \( M_{2\nu} = 0.021 \text{ MeV}^{-1} \). Comparing Figure 1 and Figure 2 it is evident that the nuclear matrix element \( M_{2\nu} \) does not saturate in the experimentally studied energy window. In fact, according to the calculation, about 40% of the total 2\( \nu \) matrix element comes from states above it.

For completeness, we present in Table 2 a compilation of the 2\( \nu \) matrix elements for which there are large scale shell model calculations. For the results of QRPA-like calculations see refs. [14, 15, 16, 17]. In the calcula-

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**Figure 1:** (color online) The running sum of the Gamow-Teller strength of \(^{136}\)Xe (energies in MeV). The theoretical strength is normalized to the experimental one.

**Figure 2:** The running sum of the 2\( \nu \) matrix element of the double beta decay of \(^{136}\)Xe (energies in MeV).
the values and are very much interaction independent, as in for the 2 interactions we adopt this value for them all. The results region. As there are no fits available with the other information from a fit to the Gamow-Teller decays in the value of the quenching factor $q=0.60$ was obtained in this section with a preliminary version of jun45 in ref. [25]. The (gcn28:50) and [24] (jun45). There is a published calculation of the 2 double beta decays (in MeV$^{-1}$). See text for the definitions of the valence spaces and interactions.

| $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$ | 0.047±0.003 | 0.74 | 0.047 | kb3 |
| $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$ | 0.047±0.003 | 0.74 | 0.048 | kb3g |
| $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$ | 0.047±0.003 | 0.74 | 0.065 | gcn28:50 |
| $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ | 0.140±0.005 | 0.60 | 0.116 | gcn28:50 |
| $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ | 0.140±0.005 | 0.60 | 0.120 | jun45 |
| $^{82}\text{Se} \rightarrow ^{82}\text{Kr}$ | 0.098±0.004 | 0.60 | 0.126 | gcn28:50 |
| $^{82}\text{Se} \rightarrow ^{82}\text{Kr}$ | 0.098±0.004 | 0.60 | 0.124 | jun45 |
| $^{128}\text{Te} \rightarrow ^{128}\text{Xe}$ | 0.049±0.006 | 0.57 | 0.059 | gcn50:82 |
| $^{130}\text{Te} \rightarrow ^{130}\text{Xe}$ | 0.034±0.003 | 0.57 | 0.043 | gcn50:82 |
| $^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$ | 0.019±0.002 | 0.45 | 0.025 | gcn50:82 |

![Figure 3: The running sum of the 2ν matrix element of the double beta decay of $^{48}\text{Ca}$ (energies in MeV).](image)

In addition, we have plotted in Figure 3 the running sum of the 2ν double beta decay of $^{48}\text{Ca}$, using the kb3 interaction, and in Figure 4 the same for the decays of $^{76}\text{Ge}$ and $^{82}\text{Se}$ using the gcn28:50 interaction. Notice that the patterns in the three cases are quite different and are also at variance with the results for the $^{136}\text{Xe}$ decay shown in Figure 2, reflecting the different structures of the intermediate nuclei. The $^{48}\text{Ca}$ decay is peculiar in one aspect, to be the only case in which there are important canceling contributions to the 2ν NME. In fact, if the contributions from all the states would have had the same sign, the matrix element would have more than doubled, thus needing a quenching factor $q=0.53$ instead of $q=0.74$, in line with our findings in the other valence spaces. In the remaining cases the sign coherence is nearly complete. However, it is difficult to make a more precise surmise about the extra quenching factor needed when incomplete valence spaces (in the harmonic oscillator sense) are used, based solely in this example. Concerning the energy at which the 2ν NME saturates, the differences among the four cases studied are relatively minor, and one can safely conclude that states beyond 10 MeV excitation energy above the first 1$^+$ state in the intermediate odd-odd nucleus do not contribute to $M^{2\nu}$.

![Figure 4: (color online) The running sum of the 2ν matrix element of the double beta decay of $^{76}\text{Ge}$ (red) and $^{82}\text{Se}$ (blue)(energies in MeV).](image)

Finally, for the decays of $^{128}\text{Te}$ and $^{130}\text{Te}$ we use the same valence space and interaction than for the decay of $^{136}\text{Xe}$. The quenching factor, as we have already mentioned, is obtained by a fit to the single Gamow-Teller
decays experimentally known in this region (and within our computational limits). Had we used the newer value from the analysis of the $^{136}$Xe charge exchange data, the values of the $2\nu$ matrix elements would have been:

$$M^{2\nu}(^{128}\text{Te})=0.037\text{ MeV}^{-1}$$
$$M^{2\nu}(^{130}\text{Te})=0.027\text{ MeV}^{-1}$$

In conclusion, we have shown that large scale shell model calculations which describe in detail the spectroscopic properties of large regions of the nuclear chart can also make accurate predictions (or postdictions) of the nuclear matrix elements of weak processes including the rarest ones; the $2\nu$ and $0\nu$ double beta decays. In the $2\nu$ case we have found that we can explain the experimental data provided the Gamow-Teller operator is renormalized (quenched) so as to reproduce the Gamow Teller single beta decays or the charge exchange results in the relevant regions. We have highlighted the case of $^{136}$Xe, whose $2\nu$ double beta decay half-life has been recently measured by the EXO-200 collaboration and confirmed by KamLAND-Zen.

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