Phenomenological effects of CP conserving Higgs bosons self couplings in SM×S(3)

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Abstract. A detailed analysis of a minimal $S_3$-invariant extension of the Standard Model including an extended $S_3$-Higgs sector is performed. In this model, we study the trilinear Higgs couplings and its dependence on the details of the model, even when the lightest Higgs boson mass is taken to be a fixed parameter. We study quantitatively the trilinear Higgs couplings, and compare these couplings to the corresponding Standard Model trilinear Higgs coupling in some regions of the parameter space. A precise measurement of the trilinear Higgs self coupling will also make it possible to test this extended $S(3)$-Standard Model which has a different trilinear Higgs couplings as compared to the Standard Model. Finally, partial numerical results of the phenomenological Higgs effects are presented.

1. Introduction

In the Standard Model (SM) the electroweak gauge bosons and the fundamental matter particles acquire masses through the interaction with a scalar field. The Higgs boson is a essential part of the SM upon the spontaneous breaking of the electroweak symmetry. The minimal version of the SM contains one complex Higgs doublet, resulting in one physical neutral CP-even Higgs boson $h_{SM}$ after electroweak symmetry breaking (EWSB). Although their existence is a basic piece of the theory and the Higgs potential is very simple and sufficient to describe a realistic model of mass generation, this may not be the final form of the theory [1]. The minimal supersymmetric extension of the SM (MSSM) contains a scalar Higgs sector corresponding to that of a Two-Higgs-Doublet Model (2HDM). Models with Higgs doublets (and singlets) possess the important phenomenological property that
\[ \rho = \frac{M_W}{M_Z \cos \theta_W} = 1 \] up to finite radiative corrections. In these days when it is to
be seen if the new boson at a mass of 125 GeV, observed in experiment CMS at the LHC,
corresponding to the scaler of the Standard Model (SM), is of ultimate importance to analyze
the mechanism of spontaneous symmetry breaking with models that have more number of
scalar bosons [2].

Each family of fermions enters independently in the SM, so that to understand the
replication of generations and to reduce the number of free parameters, usually more
symmetry is introduced in the theory. In this direction interesting work has been done with
the addition of discrete symmetries to the SM (see for instance [3] and references therein, for
a review on the subject). It is noticeable that many interesting features of masses and mixing
of the SM can be understood using a minimal discrete group, namely the permutational group
\( S_3 \) [4].

Prior to the introduction of the Higgs boson, the SM is chiral and invariant with respect
to any permutation of the left and right quark and lepton fields. After the introduction of
the Higgs boson in the theory, this field may be treated as an \( S_3 \) singlet \( H_3 \), but then, only
one fermion in each family can acquire mass. To give mass to all fermions and, at the same
time, preserve the \( S_3 \) flavour symmetry of the theory, an extended flavoured Higgs sector is
required with three Higgs \( SU(2) \) doublets, one in a singlet and the other two in a doublet
irreducible representation of \( S_3 \) [5].

Up to now, the particle observed at the LHC is a particle in the physical spectrum of the
Higgs boson of the SM. It is not known if there is one or many Higgs bosons. An indication
of the presence of one Higgs boson or an extended Higgs sector, as the one proposed in the
\( S_3 \)-invariant extension of the Standard Model, could be found at the Large Hadron Collider
[6].

Models with more than one Higgs doublet, with or without supersymmetry, have been
studied extensively. For a review of supersymmetric and two Higgs-doublet models, see for
instance [7]. Different aspects of several three and more Higgs doublets models have also been
studied, with and without discrete symmetries (see for instance [8]). In particular, in refs. [9]
it was shown that in two-Higgs doublet models, at tree level, the potential minimum that
preserves electric charge and CP symmetries, when it exists is stable and is the global one.
Many of these models are not concerned with the unsolved problem of family replication,
though. There are also analysis of different aspects of the Higgs potential of various discrete
flavour groups, see for instance [10].

We study the trilinear Higgs couplings and its dependence on the details of the model,
with an extended \( S_3 \)-Higgs sector, even when the lightest Higgs boson mass is taken to be a
fixed parameter. We study quantitatively the trilinear Higgs couplings, and compare these
couplings to the corresponding Standard Model trilinear Higgs coupling in some regions of
the parameter space. This paper is organized as follows: in section 2 we present some remarks
about the $S_3$ flavour symmetry and its Lagrangian; the Higgs potential for this model: the Higgs sector of the $S_3$ extended model, and the form of the Higgs mass matrix; in section 3 we focus on the trilinear self-couplings of neutral Higgs bosons; whereas the details of the numerical results are presented in sections 4; and finally in section 5 we present our conclusions.

2. The $S(3)$ extended Higgs doublet model

The Lagrangian $L_\Phi$ of the Higgs sector is given by

$$ L_\Phi = [D_\mu H]^2 + [D_\mu H_1]^2 + [D_\mu H_2]^2 - V(H_1, H_2, H_S), \tag{1} $$

where $D_\mu$ is the usual covariant derivative. The scalar potential $V(H_1, H_2, H_S)$ is the most general Higgs potential invariant under $SU(3)_C \times SU(2) \times U(1)_Y \times S_3$. The analysis of the stability properties of the potential $V$ is of great relevance to study the phenomenological implications of this model. In here we will concentrate on the analysis of the potential $V$ following the lines of previous work done in the vacuum stability of multi-Higgs models [9]. In our case, the discrete flavour symmetry $S_3$ simplifies the analysis.

There are many different ways of writing the Higgs potential for this model, but for the purpose of this work the best basis is

$$ H_1 = \begin{pmatrix} \phi_1 + i\phi_4 \\ \phi_7 + i\phi_{10} \end{pmatrix}, \ H_2 = \begin{pmatrix} \phi_2 + i\phi_5 \\ \phi_8 + i\phi_{11} \end{pmatrix}, \ H_S = \begin{pmatrix} \phi_3 + i\phi_6 \\ \phi_9 + i\phi_{12} \end{pmatrix}, \tag{2} $$

the numbering of the real scalar $\phi$ fields is chosen for convenience in writing the mass matrices for the scalar particles and the subscript $S$ is the flavor index for the Higgs field singlet. Now a simple matter to write down the potential is

$$ V = \mu_1^2 (x_1 + x_2) + \mu_0^2 x_3 + ax_1^2 + bx_2^2 + c(x_1 + x_2)^2 + d(x_1 - x_2)^2 + 2e x_6 x_5 [x_6 x_5 + f(x_3^2 + x_6^2 + x_8^2 + x_9^2)] + g \left[ x_1^2 + x_2^2 + 4x_4^2 + 2h (x_3^2 + x_6^2 - x_8^2 - x_9^2) \right]. \tag{3} $$

where the $\mu_{0,1}^2$ parameters have dimensions of mass squared, the real couplings $a, \cdot \cdot \cdot, h$ are dimensionless and the invariants $x_i$, the potential $V$ depends on the fields $\phi_i$ through $x_i$, considering our assignment as

$$ x_1 = H_1^1 H_1, \quad x_4 = \Re \left( H_1^1 H_2 \right), \quad x_7 = \Im \left( H_1^1 H_2 \right), $$
$$ x_2 = H_1^2 H_2, \quad x_5 = \Re \left( H_1^1 H_S \right), \quad x_8 = \Im \left( H_1^1 H_S \right), $$
$$ x_3 = H_1^3 H_S, \quad x_6 = \Re \left( H_1^2 H_S \right), \quad x_9 = \Im \left( H_1^2 H_S \right). \tag{4} $$

If the Higgs potential $S_3$ invariant Eq.(3) is bounded from below, being a quartic polynomial function it will certainly have a global minimum somewhere. We can two types of minima: the
“trivial” one, for which the Higgs acquires zero VEV’s, and the usual one, where electroweak symmetry breaking occurs, away from the origin, for

$$\phi_7 = v_1, \quad \phi_8 = v_2, \quad \phi_9 = v_3, \quad \phi_i = 0, \quad i \neq 7, 8, 9,$$

defined as the normal minimum, with VEV’s which do not have any complex relative phase.

Where, $v_1$, $v_2$ and $v_3$ are the VEV’s of the unbroken Higgs fields.

The minimization constraints may then be determined by demanding the vanishing of $\partial V/\partial \phi_i$, then we get the mass parameter by

$$\mu_1^2 = -(b + f + 2h) v_3^2 - 2(e + g) (v_1^2 + v_2^2) + \frac{3e(v_1^2 - 2v_1v_2 - v_2^2)v_3}{v_1 - v_2},$$

and

$$\mu_0^2 = -\left[2av_3^2 + (b + f + 2h)(v_1^2 + v_2^2) - e \left(\frac{3v_1^2 - v_2^2}{v_3}\right)v_2\right].$$

As well as, we have $v_1 = \sqrt{3}v_2$.

The Higgs-bosons masses in this model are obtained by diagonalizing the $12 \times 12$ matrix, with $M_H^2 = M_{II}$ and $M_{IJ}$, $I, J = C, S, P$, are all $3 \times 3$ sub-matrices. From

$$(M_H^2)_{ij} = \frac{1}{2} \left. \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} \right|_{\text{min}},$$

with $i, j = 1, 12$. We have

$$M_H^2 = \text{diag}(M_C^2, M_C^2, M_S^2, M_P^2).$$

The matrix charged Higgs is two times degenerate,

$$M_C^2 = \begin{pmatrix}
    c_{11} & c_{12} & c_{13} \\
    c_{21} & c_{22} & c_{23} \\
    c_{31} & c_{32} & c_{33}
\end{pmatrix},$$

where

$$c_{11} = -4gv_2^2 - v_3(4ev_2 + (f + 2h)v_3)$$
$$c_{12} = 2\sqrt{3}v_2(2gv_2 + ev_3)$$
$$c_{13} = \sqrt{3}v_2(2ev_2 + (f + 2h)v_3)$$
$$c_{22} = -12gv_2^2 - v_3(8ev_2 + (f + 2h)v_3)$$
$$c_{23} = v_2(2ev_2 + (f + 2h)v_3)$$
$$c_{33} = -4v_2^3 + (f + 2h)v_3) v_3.$$

The mass matrix for the CP-even Higgs scalars,

$$M_S^2 = \begin{pmatrix}
    s_{11} & s_{12} & s_{13} \\
    s_{21} & s_{22} & s_{23} \\
    s_{31} & s_{32} & s_{33}
\end{pmatrix},$$
where

\[ s_{11} = 12(c + g)v_2^2 \]
\[ s_{12} = 2\sqrt{3}v_2(2(c + g)v_2 + 3ev_3) \]
\[ s_{13} = 2\sqrt{3}v_2(3ev_2 + (b + f + 2h)v_3) \]
\[ s_{22} = 4v_2((c + g)v_2 - 3ev_3) \]
\[ s_{23} = 2v_2(3ev_2 + (b + f + 2h)v_3) \]
\[ s_{33} = 4av_2^2 - 8ev_3^2/v_3. \]

The mass matrix for the CP-odd Higgs pseudoscalar,

\[
 M_P^2 = \begin{pmatrix}
 p_{11} & p_{12} & p_{13} \\
 p_{22} & p_{23} \\
 p_{33}
 \end{pmatrix},
\]

where

\[ p_{11} = -4((d + g)v_2^2 + ev_3v_2 + hv_3^2) \]
\[ p_{12} = 2\sqrt{3}v_2(2(d + g)v_2 + ev_3) \]
\[ p_{13} = 2\sqrt{3}v_2(ev_2 + 2hv_3) \]
\[ p_{22} = -12(d + g)v_2^2 - 8ev_2v_3 - 4hv_3^2 \]
\[ p_{23} = 2v_2(ev_2 + 2hv_3) \]
\[ p_{33} = -8v_2^2(ev_2 + 2hv_3)/v_3. \]

After diagonalizing the mass matrices the masses of the physical scalars/pseudoscalars are obtained. In our analysis we are not taking into account the parameter space with negative eigenvalues solutions for the squared masses of the physical Higgs fields. Of the original twelve scalar degrees of freedom, three Goldstone bosons (\(G^\pm\) and \(G\)) are absorbed by \(W^\pm\) and \(Z\). The remaining nine physical Higgs particles are three \(CP\)-even scalar (\(h\) and \(H_1\), \(H_2\), with \(m_h \leq m_{H_1} \leq m_{H_2}\)), two \(CP\)-odd scalar (\(A_1\), and \(A_2\), with \(m_{A_1} \leq m_{A_2}\)), and two charged Higgs pair (\(H_{1,2}^\pm\), mass degenerate). Their physical masses and the respective Higgs-scalar mixing are related to the weak parameters of the Higgs potential as follows. We start considering the mass matrix for the \(CP\)-even Higgs scalars Eq.(11).

Defining the physical mass eigenstates \(m_{h}^2, m_{H_1}^2, \) and \(m_{H_2}^2\), the masses are found from the diagonalization process

\[
 M^2_{\text{diag}} = R^T M^2_{\text{even}} R = \text{diag}(m_h^2, m_{H_1}^2, m_{H_2}^2). \tag{13}
\]

The masses for the \(CP\)-even Higgs scalars are:

\[ m_h^2 = -18ev_2v_3 \]
\[ m_{H_1,2}^2 = (M_a^2 + M_c^2) \pm \sqrt{(M_a^2 - M_c^2)^2 + (M_b^2)^2}, \tag{14} \]
where
\[ M_a^2 = v_2 (8(c + g)v_2 + 3ev_3) \]
\[ M_b^2 = 4v_2 (3ev_2 + (b + f + 2h)v_3) \]
\[ M_c^2 = -\frac{4ev_3^2}{v_3^2} + 2av_3^2, \tag{15} \]
and mixing angles are defined as:
\[ \tan 2\alpha_3 = \frac{M_b^2}{M_a^2 - M_c^2} \tag{16} \]
As well as,
\[ R = O \cdot P \cdot Q, \tag{17} \]
\[
O = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad P = \begin{pmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\theta & s_\theta \\ 0 & -s_\theta & c_\theta \end{pmatrix}, \tag{18} \]
where
\[ \tan \theta = \frac{s_\theta}{c_\theta} = \frac{2s_{13}}{\sqrt{3}(m_{H_1}^2 - s_{33})} = \frac{M_b^2}{2M_a^2 - m_{H_2}^2} \tag{19} \]
which \( s_{13} \) and \( s_{33} \) from Eq.(11).

To generate the correct \( W^\pm \) and \( Z^0 \) masses, with the assignments \( v_1^2 + v_2^2 + v_3^2 = v^2 \) has to hold, where \( v = 246 \) GeV, and \( v_1 = \sqrt{3}v_2 \). Also, the \( v_i \) are taken real, i.e., we assume that spontaneous \( CP \) violation does not occur. The squared-mass parameters \( \mu_0^2 \) and \( \mu_1^2 \) can be eliminated by minimizing the scalar potential. The resulting squared masses for the CP-odd Higgs pseudoscalar are
\[ m_{A_1}^2 = -16(d + g)v_2^2 - 10ev_2v_3 - 4hv_3^2, \quad m_{A_2}^2 = -\frac{2(ev_2 + 2hv_3)(4v_2^2 + v_3^2)}{v_3}. \]
can be parametrized with
\[ \tan \omega_3 = \frac{2v_2}{v_3}, \tag{20} \]
where \( s\omega_3 = 2v_2/v, \ c\omega_3 = v_3/v. \)

The charged Higgs boson masses are
\[ m_{H_1^\pm}^2 = -(10ev_2 + (f + 2h)v_3) v_3, \quad m_{H_2^\pm}^2 = -\frac{v^2}{v_3} (2ev_2 + (f + 2h)v_3). \]

Higgs boson masses are not determined \( a \) \( priori \) within the theory and their decay patterns depend strongly on the masses, in order to determine the possible decay modes and branching ratios, it is necessary to investigate the change of mass spectrum with respect
to the quadrilinears $a, \ldots, h$ couplings. For simplicity, we assume in our numerical analysis for the following dimensionless parameters

$$a = 1, \quad b = 1, \quad c = 1, \quad d = -1, \quad e = -5/3, \quad f = 3.25, \quad g = 1, \quad h = 1,$$

and $v_2 = 246(\sin \omega_3)/2$ GeV, $v_3 = 246 \cos \omega_3$ GeV, $-\pi \leq \omega_3 \leq \pi$. The Figure 1 shows the masses of three CP-even Higgs scalars with respect to $\omega_3$.

![The CP–even Higgs boson masses](image)

**Figure 1.** The masses of three CP–even Higgs scalars with respect to $\omega_3$. The solid line is for $H^0_1$, the dashed line is for $H^0_2$ and the dotted line is for $h^0$.

The quadrilinears $a, \ldots, h$ couplings are related to the masses of the Higgs bosons by

$$a = \frac{9M^2_1 v_3^2 - 2m^2_{h_0} v_2^2}{18v_3^4}$$

$$b = \frac{2m^2_{h_0} v_2 + 9M^2_0 v_3}{36v_2 v_3^2} + \frac{m^2_{H^\pm}}{v^2}$$

$$c = \frac{m^2_{H^\pm}}{4v^2} - \frac{m^2_{h_0} - 9(m^2_{H^+_1} - m^2_{H^\pm} + 2M^2_0)}{144v_3^2}$$

(21)
where
\[ M_a^2 = \frac{1}{4} \left( m_{H_1^0}^2 + m_{H_2^0}^2 - (m_{H_1^0}^2 - m_{H_2^0}^2) \cos 2\alpha_3 \right) \]
\[ M_b^2 = (M_a^2 - M_c^2) \tan 2\alpha_3 \]
\[ M_c^2 = \frac{1}{4} \left( m_{H_1^0}^2 + m_{H_2^0}^2 + (m_{H_1^0}^2 - m_{H_2^0}^2) \cos 2\alpha_3 \right) . \]

As well as,
\[ d = \frac{1}{16} \left( \frac{m_{A_2}^2 - m_{A_1}^2 + m_{H_1^+}^2 - m_{H_2^+}^2}{v_2^2} - \frac{m_{A_2}^2 - m_{H_2^+}^2}{4v^2} \right) \]
\[ e = -\frac{m_{h_0}^2}{18v_2v_3} \]
\[ f = \frac{1}{18} \left( \frac{9(m_{A_2}^2 - 2m_{H_2^+}^2)}{v^2} + \frac{m_{h_0}^2}{v_3^2} \right) \]
\[ g = \frac{4m_{h_0}^2 - 9(m_{H_1^+}^2 - m_{H_2^+}^2)}{144v_2^2} - \frac{m_{H_2^+}^2}{4v^2} \]
\[ h = \frac{1}{36} \left( \frac{m_{h_0}^2}{v_3^2} - \frac{9m_{A_2}^2}{v^2} \right) . \]

In Figure 2 shows the behavior of the links quartic function of the reason for expectation values \( v_2 \) and \( v_3 \) given by \( \omega_3 \), Eq.(20). You can specify a value of \( \omega_3 \) and from this, determine a mass spectrum of Higgs bosons. For example, \( \omega_3 = 1 \), quadrilinears couplings take values:
\[ a = 3.58, \quad b = 5.60, \quad c = 3.40, \]
\[ d = 1.36, \quad e = -0.06, \quad f = -3.57, \quad g = -2.39, \quad h = -0.23, \]
fixing values mass spectrum:
\[ m_{h_0} = 125 \text{ GeV}, \quad m_{H_1} = 550 \text{ GeV}, \quad m_{H_2} = 336 \text{ GeV}. \]
\[ m_{H_1^+} = 687 \text{ GeV}, \quad m_{H_2^+} = 488 \text{ GeV}. \]
\[ m_{A_1^0} = 430 \text{ GeV}, \quad m_{A_2^0} = 225 \text{ GeV}. \]

3. Trilinear Self-Couplings of Neutral Higgs Bosons

The measurement of the Higgs self-coupling is crucial to determine the Higgs potential. The self-couplings are uniquely determined in the SM by the mass of the Higgs boson which is
Figure 2. The quadrilinears $a, \ldots, h$ couplings with respect to $\omega_3$, in the range $-\pi/2 \leq \omega_3 \leq \pi/2$. The $\omega_3$ values close to zero are on the order of tenths, $\tan \omega_3 \sim 0.09$. When $\omega_3$ approaches to $\pi/2$ diverges $\tan \omega_3$. Higher values are acceptable for $\tan \omega_3 = 5$. If $\tan \omega_3 = 1$, and then the mass spectrum of bosons Higgs is: $m_{h_0} = 125 \text{ GeV}$, $m_{H_1} = 550 \text{ GeV}$, $m_{H_2} = 315 \text{ GeV}$, $m_{A_1^0} = 492 \text{ GeV}$, $m_{A_2^0} = 232 \text{ GeV}$, $m_{H_1^0} = 756 \text{ GeV}$, $m_{H_2^0} = 491 \text{ GeV}$. But if $\tan \omega_3 = 5$ as $\omega_3 = 1.3734$, then the mass spectrum of bosons Higgs is: $m_{h_0} = 125 \text{ GeV}$, $m_{H_1} = 550 \text{ GeV}$, $m_{H_2} = 305 \text{ GeV}$, $m_{A_1^0} = 356 \text{ GeV}$, $m_{A_2^0} = 57 \text{ GeV}$, $m_{H_1^0} = 623 \text{ GeV}$, $m_{H_2^0} = 437 \text{ GeV}$ related to the quadrilinear coupling $\lambda$ by $M_H = \sqrt{2\lambda v}$. The trilinear and quadrilinear vertices of the Higgs field $H$ are given by the coefficients:

$$
\lambda_{HHH} = \lambda v = \frac{M_H^2}{2v} \quad \lambda_{HHHH} = \frac{\lambda}{4} = \frac{M_H^2}{8v^2}.
$$

(24)

The following definitions are often used:

$$
\lambda_{ijk} = \frac{-i \partial^3 V}{\partial H_i \partial H_j \partial H_k},
$$

(25)

which are most easily obtained from the corresponding derivatives of $V$ in Eq.(3) with respect to the fields $\{\phi_i\}$ with $i = 1, \cdots, 12$. We can then write the trilinear couplings in terms of the derivatives of the potential (3) with respect to $\phi_i$ and the elements of the rotation matrix $R$ Eq.(17) as

$$
\lambda_{ijk} = N \sum_{lmn} R_l R_{jm} R_{kn} \frac{\partial^3 V}{\partial \phi_l \partial \phi_j \partial \phi_k},
$$

(26)

where the indices $l, m, n$ refer to the weak field basis, and $l \leq m \leq n = 1, 2, 3$, $N$ is a factor of $n!$ for $n$ identical fields. We now proceed to obtain these couplings in an explicit form.
The trilinear self-couplings $a_{iyn}$ among the neutral Higgs bosons even can be written as

$$
\begin{align*}
a_{1,1,1} &= 6\sqrt{6}(c + g)v_2, \\
a_{1,1,2} &= \sqrt{2}(2(c + g)v_2 + 3ev_3), \\
a_{1,1,3} &= \sqrt{2}(3ev_2 + (b + f + 2h)v_3), \\
a_{1,2,1} &= 2\sqrt{6}(c + g)v_2, \\
a_{1,2,2} &= \sqrt{2}(b + f + 2h)v_2, \\
a_{1,2,3} &= 3\sqrt{6}v_2, \\
a_{2,2,1} &= \sqrt{2}(2(c + g)v_2 - ev_3), \\
a_{2,2,2} &= \sqrt{2}(b + f + 2h)v_3 - 3ev_2), \\
a_{2,2,3} &= 6\sqrt{2}av_3.
\end{align*}
$$

Then, we here have self-couplings to three Higgs bosons even substituting for the elements of the rotation matrix, Eq.(17), one obtains

$$
\begin{align*}
\lambda_{1,1,1} &= 6v (\lambda_1 \omega_3 + \lambda_2 \omega_3), \\
\lambda_{3,3,3} &= 6v (\lambda_5 \omega_3 + \lambda_6 \omega_3), \\
\lambda_{1,1,3} &= 2v (\lambda_9 \omega_3 + \lambda_{10} \omega_3), \\
\lambda_{1,2,3} &= v (\lambda_{13} \omega_3 + \lambda_{14} \omega_3), \\
\lambda_{2,2,3} &= 2v (\lambda_{17} \omega_3 + \lambda_{18} \omega_3), \\
\lambda_{1,2,3} &= 2v (\lambda_{19} \omega_3 + \lambda_{20} \omega_3).
\end{align*}
$$

Where $\lambda_1, \ldots, \lambda_{20}$ depend to parameters quadrilinear couplings of the Higgs potential Eq.(3) and the mixing angle $\theta$ Eq.(19). For example, one can use the results of appendix to compute the first $\lambda_{h_0h_0h_0} = \lambda_{1,1,1}$, these results are given en the appendix,

$$
\lambda_{h_0h_0h_0} = \frac{\sqrt{3}}{4} v \left[ 2(18as_\theta^3 + (b + f + 2h)(3c_\theta^2 + 1)s_\theta - 9cc_\theta^3 + 3cc_\theta) \omega_3 + (3(b + f + 2h)(c_\theta - 1)s_\theta^2 + 2(c + g)(9c_\theta^3 - 3c_\theta^2 + c_\theta - 3) - 3e(c_\theta(c_\theta + 1) - 1)s_\theta) \omega_3 \right].
$$

4. Numerical results

Here we present numerical results considering that is one a CP-even boson Higgs scalar behaves like the Higgs boson of the standard model, where the mass of the lightest Higgs scalar is significantly smaller than the masses of the other Higgs bosons of the model, We assume it is light and its mass is around 125 GeV. In our analysis we propose to $H_1^0$ as our candidate, it is a parameter free. The other bosons Higgs have masses consequently the order $O(500)$ GeV and without loss of generality and for the sake of simplicity, consider that they have no equal masses. The Figure 3 shows the trilinear couplings by CP-even Higgs Bosons in function the mixing angle $\theta$, Eq.(19), where tan $\omega_3 = 1$. The coupling of higher intensity corresponds to $\lambda_{h_0h_0h_0}$ and $\lambda_{H_2H_2H_2}$, whereas that $\lambda_{H_1H_1H_1}$ to be very small,

$$
\lambda_{h_0h_0h_0} \sim \lambda_{H_2H_2H_2} > \lambda_{H_1H_1H_1},
$$

followed by $\lambda_{h_0H_0H_2}$, $\lambda_{h_0H_0H_1}$ ($\lambda_{h_0H_2H_2}$) and then the other. We proposed that the lightest is $H_1^0$ and its trilinear coupling is the lower intensity. Although there are couplings equal in
Figure 3. The trilinear couplings by CP-even Higgs bosons, $\lambda_{ijk}/(125 \text{ GeV})^2$ with $i, j, k = h_0, H_1, H_2$, in function the mixing angle $-\pi \leq \theta \leq \pi$, Eq.(19), where $\tan \omega_3 = 1$.

shape, they are different in intensity. Comparing these results with the SM trilinear coupling only. We see that our values are within acceptance ranges in the literature, but they are
richer for a Higgs potential analysis, enabling better understand the spontaneous symmetry breaking weak.

\[ \lambda_{ijk}/(125 \text{ GeV})^2 \text{ with } i, j, k = h_0, H_{1,2}, \text{ in function the mixing angle } -\pi \leq \theta \leq \pi, \text{ and } -\pi \leq \omega_3 \leq \pi. \]

**Figure 4.** The trilinear couplings by CP-even Higgs bosons, \( \lambda_{ijk}/(125 \text{ GeV})^2 \) with \( i, j, k = h_0, H_{1,2} \), in function the mixing angle \( -\pi \leq \theta \leq \pi \), and \( -\pi \leq \omega_3 \leq \pi \).

Furthermore, when comparing the self couplings of the Higgs boson as a function of the two mixing angles \( \theta \) and \( \omega_3 \), there is a symmetry, Figure 4. This allows us characterize different Higgs bosons. The mass of the Higgs boson is fixed through \( \omega_3 \) and couplings are determined also by mixing Higgs bosons depending on \( \theta \). Our results are normalized \( M_h = 125 \text{ GeV} \) in Figure 3, but with either parameterized three scalar bosons dependence is observed. See figure such. We can analyze the behavior of the car couplings, depending on the mass of the boson \( M_{H_2} \) through \( \omega_3 \), see Figure 5.

\[ \lambda_{ijk}/(m_{H_2} \text{ GeV})^2 \text{ with } i, j, k = h_0, H_{1,2}, \text{ in function the mixing angle } -\pi/2 \leq \theta \leq \pi/2, \text{ and } 0.1 \leq \omega_3 \leq 1.5. \]

**Figure 5.** The trilinear couplings by CP-even Higgs bosons, \( \lambda_{ijk}/(m_{H_2} \text{ GeV})^2 \) with \( i, j, k = h_0, H_{1,2} \), in function the mixing angle \( -\pi/2 \leq \theta \leq \pi/2 \), and \( 0.1 \leq \omega_3 \leq 1.5 \).
5. Conclusions
We studied only the scalar sector assuming the pseudoscalars to be too heavy to be relevant. In this work we have analyzed the complete scalar sector of an $S_3$ flavour model. We deal with three $CP$-even, two $CP$-odd and two sets of charged scalar particles. We have improved our potential minimization technique which enabled us to explore a larger region of the allowed parameter space. We have studied in detail the trilinear couplings of the lightest Higgs boson of this model. Within the allowed domain of the parameters space of the model, the trilinear Higgs couplings have a strong dependence on $\tan \omega_3 = 2v_2/v_3$ and tan $\theta$. The extended Higgs spectrum in $S_3$ models gives rise numbers of trilinear couplings. The $hhh$ coupling can be measured in $hh$ continuum production linear colliders as at $e^+e^-$. 

Appendix

\[
\lambda_{hahbho} = 6v(\lambda_1 s\omega_3 + \lambda_2 c\omega_3), \\
\lambda_1 = \frac{3\sqrt{3}}{4}(b + f + 2h)(c_\theta - 1)s_\theta^2 \\
\lambda_2 = \frac{2\sqrt{3}}{4}(18a s^3_\theta + [(b + f + 2h)s_\theta - 3c_\theta] (3c_\theta^2 + 1)) + 2(c + g)(9c_\theta^3 - 3c_\theta^2 + c_\theta - 3) - 3e(3c_\theta(c_\theta + 1) - 1)s_\theta \\
\lambda_{H_1H_1H_1} = 6v(\lambda_3 s\omega_3 + \lambda_4 c\omega_3), \\
\lambda_3 = \frac{1}{4}[(b + f + 2h)(c_\theta + 3)s_\theta^2 + 6(c + g)(c_\theta^3 + c_\theta^2 + c_\theta + 9) - 3e((c_\theta - 3)c_\theta - 3)s_\theta] \\
\lambda_4 = \frac{1}{2}(6as_\theta^3 + (b + f + 2h)(c_\theta^2 + 3)s_\theta - 3ec_\theta(c_\theta^2 - 3)) \\
\lambda_{H_2H_2H_2} = 6v(\lambda_5 s\omega_3 + \lambda_6 c\omega_3), \\
\lambda_5 = 2((b + f + 2h)c_\theta^2 + 6(c + g)s_\theta^2 + 3ec_\theta s_\theta) \\
\lambda_6 = -4(6ac_\theta^3 + s_\theta^2((b + f + 2h)c_\theta + 3es_\theta)) \\
\lambda_{hahbH_1} = 2v(\lambda_7 s\omega_3 + \lambda_8 c\omega_3), \\
\lambda_7 = \frac{1}{4}(3(b + f + 2h)(3c_\theta + 1)s_\theta^2 + 2(c + g)(c_\theta(3c_\theta(9c_\theta + 1) - 5) + 27) - 3e(9c_\theta^2 - 3c_\theta + 5)s_\theta) \\
\lambda_8 = \frac{1}{4}(54as_\theta^3 + (b + f + 2h)(9c_\theta^2 - 5)s_\theta - 3ec_\theta(9c_\theta^2 + 5)) \\
\lambda_{hahbH_2} = 2v(\lambda_9 s\omega_3 + \lambda_{10} c\omega_3), \\
\lambda_9 = \frac{1}{2}(2s_\theta(3(b - 1 - c_\theta) + (9c - f + 9g - 2h)c_\theta - 2c + f - 2g + 2h)c_\theta + g) + 3(b + f + 2h)s_\theta^2 + 2cs_\theta - 9e(2c_\theta + 1)s_\theta^2 + 3ec_\theta(3c_\theta(c_\theta + 1) - 1)) \\
\lambda_{10} = 6(-9a + b + f + 2h)c_\theta s_\theta^2 - (b + f + 2h)(3c_\theta^2 + c_\theta) - 3e(9c_\theta^2 - 1)s_\theta
\[
\begin{align*}
\lambda_{h_0 H_1 H_1} &= 2v (\lambda_{12} \omega_3 + \lambda_{12} \omega_3), \\
\lambda_{11} &= \frac{\sqrt{3}}{4} ((b + f + 2h)(3c_\theta + 5) + 2(c + g)(9c_\theta^3 + 5c_\theta^2 + c_\theta - 27) \\
&+ 3e((5 - 3c_\theta)c_\theta + 1)s_\theta) \\
\lambda_{12} &= \frac{\sqrt{3}}{2} (18a_\theta^3 + (b + f + 2h)(3c_\theta^2 + 1)s_\theta - 9ec_\theta^3 + 3ec_\theta) \\
\lambda_{h_0 H_2 H_2} &= 2v (\lambda_{13} \omega_3 + \lambda_{14} \omega_3), \\
\lambda_{13} &= \sqrt{3} (-2((b(1 + c_\theta) - 9c_\theta - 2c + c_\theta f - 9c_\theta g + 2(c_\theta + 1)h + f - 2g)c_\theta + g)s_\theta \\
&+ (b + f + 2h)s_\theta^3 - 2c_\theta - 3e(1 - 2c_\theta)s_\theta^2 + 3ec_\theta) \\
\lambda_{14} &= 2\sqrt{3} ((2(-9a + b + f + 2h)s_\theta^2 - (b + f + 2h)(c_\theta^2 - 1) - 9ec_\theta^3 - 3e_\theta) \\
\lambda_{H_1 H_1 H_2} &= 2v (\lambda_{15} \omega_3 + \lambda_{14} \omega_3), \\
\lambda_{15} &= \sqrt{3} (-2((b + f - 9g + 2h)c_\theta - 9c_\theta + c_\theta + g)s_\theta^2 + (b + f + 2h)(c_\theta - 1)c_\theta^2 \\
&+ 3ec_\theta) \\
\lambda_{16} &= 2\sqrt{3}s_\theta (18ac_\theta^3 - (b + f + 2h)(2c_\theta^2 - s_\theta^2) - 9ec_\theta^3 s_\theta) \\
\lambda_{H_1 H_2 H_2} &= 2v (\lambda_{17} \omega_3 + \lambda_{18} \omega_3), \\
\lambda_{17} &= \frac{1}{2} (-2((c_\theta + 3) + (-9c + f - 9g + 2h)c_\theta - 6c + 3f - 6g + 6h)c_\theta - 3g)s_\theta \\
&+ (b + f + 2h)s_\theta^3 + 6c_\theta + 3e(3 - 2c_\theta)s_\theta^2 + 3ec_\theta) \\
\lambda_{18} &= 2(-9a + b + f + 2h)c_\theta s_\theta^2 - (b + f + 2h)(c_\theta^2 + 3)c_\theta - 9ec_\theta^3 - 3e_\theta) \\
\lambda_{H_2 H_2 H_2} &= 2v (\lambda_{19} \omega_3 + \lambda_{20} \omega_3), \\
\lambda_{19} &= 2((-b + f - 9g + 2h)c_\theta + c(9c_\theta + 3) + 3g)s_\theta^2 \\
&+ (b + f + 2h)(c_\theta + 3)c_\theta^2 + 3ec_\theta) \\
\lambda_{20} &= 2(18ac_\theta^2 - (b + f + 2h)(2c_\theta^2 - s_\theta^2) - 9ec_\theta^3 s_\theta) \\
\end{align*}
\]

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