Quantum space-time fluctuations and primary state diffusion

by

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Abstract

Nondifferentiable fluctuations in space-time on a Planck scale introduce stochastic terms into the equations for quantum states, resulting in a proposed new foundation for an existing alternative quantum theory, primary state diffusion (PSD). Planck-scale stochastic space-time structure results in quantum fluctuations, whilst larger-scale curvature is responsible for gravitational forces. The gravitational field and the quantum fluctuation field are the same, differing only in scale. The quantum mechanics of small systems, classical mechanics of large systems and the physics of quantum experiments are all derived dynamically, without any prior division into classical and quantum domains, and without any measurement hypothesis. Unlike the earlier derivation of PSD, the new derivation, based on a stochastic space-time differential geometry, has essentially no free parameters. However many features of this structure remain to be determined. The theory is falsifiable in the laboratory, and critical matter interferometry experiments to distinguish it from ordinary quantum mechanics may be feasible within the next decade.
1. Introduction

This paper is intended to be read in conjunction with the original paper on primary state diffusion (Percival, 1994b: PSD1), whose contents are briefly summarised in the next four paragraphs.

PSD1 describes an alternative quantum theory (PSD) developed from quantum state diffusion theory (Gisin & Percival, 1992,1993a,b; Percival 1994a), in which both quantum and classical behaviour are derived from the same dynamics. Decoherence, localization (reduction) and the interaction of ‘quantum’ system with ‘classical’ apparatus are derived from the dynamics, and no independent measurement hypothesis or interpretation is required. In PSD, unlike the earlier quantum state diffusion theory, the diffusion is independent of any division between system and environment or observed system and observer. Primary state diffusion, like many similar theories referred to in PSD1 and in sections 4 and 5 of this paper, provides a common dynamical basis for classical and quantum systems and for their interaction, for example in the laboratory measurement of the state of a quantum system.

The distinguishing feature of PSD1 is that the diffusion of quantum states, which dominates on very small times scales, is primary, and the usual Schrödinger evolution of the state vector is derived from it. In the nonrelativistic theory, the stochastic differential equation for the state vector incorporates a time-dependent fluctuation and results in energy localization. This energy localization produces decoherence, but does not produce the phase space (including position) localization needed for the dynamics of classical systems. But special relativity requires that the quantum fluctuations should depend on space as well as time. This space dependence of the fluctuations gives the position localization.

The theory of PSD1 contains an undetermined time constant \( \tau_0 \), whose value was conjectured to be approximately the Planck time:

\[
T_{P\ell} = (\hbar Gc^{-5})^{1/2} \approx 5 \times 10^{-44} \text{ s}. \tag{1.1}
\]
Order of magnitude estimates then show localization of wave packets of caesium atoms over a distance of 15 cm, probably within the range of feasible matter interferometry experiments within the next decade.

The theory of PSD1 was derived without any reference to the structure of space-time.

In this paper it is shown that PSD leads naturally to stochastic differential relations for time, and thus, by a necessary extension, for space-time. This leads directly to a derivation of the previously conjectured value of $\tau_0$.

In the new formulation of PSD of this paper, the foundations are changed and the theory is derived from stochastic fluctuations of space-time on the scale of the Planck length and time and below. On these scales the time-time component of the transformation from a Minkowski universe to our universe resembles the transformation from the time variable to the space variable of a Brownian motion. This transformation is not differentiable and requires the Itô differential calculus to replace ordinary differential calculus.

By invariance the same may be said of the other components of the transformation, and of the intrinsic geometry of space-time, which is then a stochastic differential geometry. But these remain to be determined.

The representation of quantum fluctuations by a change in the differential structure of space-time differs from proposals using a quantized gravitational field (e.g. Grishchuk, Haus & Bergman, 1992) and from proposals to retain a smooth differential structure and rely on wormholes (Ellis, Mohanty & Nanopoulos, 1989).

In the original PSD equation (2.12), the fluctuations $d\xi$ are coupled to the quantum system through the hamiltonian; that is, relativistically, they are coupled to matter through the mass. But the gravitational field is also coupled to matter through the mass. This suggests that quantum fluctuations and gravity might be different manifestations of the same field. In general relativity, gravity comes from the large scale structure of space-time, so we should expect the quantum fluctuations to come from the small scale structure of space-time.

This results in a semiclassical theory in which matter is treated quantally, and space-time is represented classically. The probabilistic quantum properties of matter follow from the stochastic space-time fluctuations, which drive the quantum state through the resultant diffusion term which is added to the Schrödinger equation. With this hypothesis, we obtain a space-time primary state diffusion theory in which the results of the original PSD theory are rederived, with the one difference that $\tau_0$ is shown to be close to the Planck time $T_P\ell$. The only important constants to appear in the theory are the velocity of light, the gravitational constant $G$ and the Planck constant $\hbar$ for quantum mechanics.
However, this space-time PSD theory is far from complete, since it is worked out in detail only for the internal quantum dynamics of those systems for which all but the time-time component of space-time transformations can be neglected, and the effects of the spatial dependence of these components can also be neglected (Pauli 1967 p150, Weinberg 1972 p78). For a more general theory, nonrelativistic quantum state diffusion theory could be used as guide. For nonlinear systems, the fluctuations in the classical phase space trajectories of means of dynamical variables can be obtained. These fluctuations do not satisfy Hamilton’s equations, so we should not expect the space-time fluctuations of PSD to satisfy Einstein’s equations. However, since the large-scale structure of space-time is derived from the Planck-scale structure, the approximate validity of Einstein’s equations in the large will put a constraint on the small-scale theory, just as the Schrödinger equation in the large put a constraint on the form of the quantum state diffusion in PSD1.

The development of modern alternative quantum theories has been hampered because they have little or no contact with experiment. Either their predictions agree with those of ordinary quantum theory, or they contain parameters whose values can be shifted to ensure that agreement, for any experiments in the foreseeable future. This is true of the original PSD theory of PSD1. By contrast space-time primary state diffusion theory has essentially no free parameters and could be falsified by matter interferometry experiments within the next decade (PSD1, Section 9).

Section 2 contains a short account of Langevin-Itô theory (see Gardiner, 1985, 1991), quantum state diffusion and PSD, but the reader who needs to study the details, particularly the property of localization and the resultant derivation of classical mechanics from quantum mechanics with state diffusion, should consult the references in that section.

Section 3 is the key section, since it introduces fluctuating space-time and a related equivalence principle, which replaces the equivalence principle of general relativity (GR) on scales less than or equal to the Planck scale. In this section it is shown that the resultant equations for a quantum state are the PSD equations with $\tau_0 \approx T_P \ell$.

Sections 4 and 5 discuss alternative quantum theories and experimental tests. Section 4 reviews gravitational theories of quantum decoherence, including that of Károlyházy(1966), which most resembles space-time PSD. Section 5 is a guide to other current sharp formulations of quantum mechanics, as defined by Bell (1987, p171) to mean a formulation that provides a uniform description of the micro and macro worlds. The emphasis is on critical experiments needed to distinguish such sharp theories from ordinary quantum mechanics, including the critical matter interferometry experiments for space-time PSD.
2. Classical diffusion and quantum state diffusion

Itô has developed a very elegant theory and differential calculus for stochastic processes that satisfy Langevin equations, described for physicists in the books of Gardiner. For example when a particle with displacement $s$ diffuses in one dimension with no drift, as in Brownian motion, the mean of the distance traversed is zero and the mean square is proportional to the time. So for small displacements $ds$,

$$Mds(t) = 0, \quad M[ds(t)]^2 = a^2 dt,$$  \hspace{1cm} (2.1)

where $a$ is a constant and $M$ represents a mean over the ensemble of all possible processes. For more complicated stochastic processes, it is convenient to introduce a standard normalized stochastic differential $dw$, with zero mean and mean square equal to the time, so that

$$Mdw = 0, \quad M(dw)^2 = dt.$$  \hspace{1cm} (2.2)

For the example of one-dimensional Brownian motion with drift velocity $v$ and diffusion constant $a^2$, the Langevin-Itô differential equation is written as

$$ds = v dt + adw,$$  \hspace{1cm} (2.3)

where the coefficient of $dt$ gives the drift and the coefficient of $dw$ the diffusive part of the motion.

For applications to quantum mechanics it is convenient to consider two-dimensional isotropic diffusion as one-dimensional complex diffusion in the complex $z$-plane. The standard normalized complex stochastic differential is

$$d\xi = d\xi_R + id\xi_I,$$  \hspace{1cm} (2.4)

where the real and imaginary parts satisfy

$$Md\xi_I = Md\xi_R = 0, \quad Md\xi_R^2 = Md\xi_I^2 = dt/2.$$  \hspace{1cm} (2.5)

More convenient is the equivalent complex form of the conditions:

$$Md\xi = 0, \quad M(d\xi)^2 = 0, \quad M|d\xi|^2 = dt,$$  \hspace{1cm} (2.6)

where the statistics of the motion is unaffected if $d\xi$ is multiplied by a phase factor of unit norm.

The Langevin-Itô equation of a particle with drift and diffusion at position $z(t)$ in the complex plane is

$$dz = v dt + ad\xi.$$  \hspace{1cm} (2.7)
Both $v$ and $a$ can be complex, but the physics is unaffected by the (gauge) transformation for which $a$ is multiplied by a phase factor.

Also for more general differential stochastic processes, the drift is the term containing $dt$ and the diffusion the term containing $d\xi$. For sufficiently small times, the diffusion, which is proportional to $(dt)^{1/2}$, dominates the drift, which is proportional to $dt$, and when using the Itô calculus great care must be taken to ensure that quadratic terms with a factor $|d\xi|^2$ are included. This problem is overcome in the alternative but equivalent Stratonovich calculus (Gardiner 1985), at the price of a slightly more difficult interpretation of the results, but the Itô form is used here.

Quantum state diffusion represents the stochastic evolution of individual systems of an ensemble in states $|\psi\rangle$, when the ensemble as a whole is usually represented by a density operator $\rho$. In quantum state diffusion theory, unlike the usual density operator theory, care must be taken to distinguish the quantum expectation $\langle ... \rangle$ for an individual system from the mean $M$ over the ensemble.

The simplest master equation for $\rho$, with no Hamiltonian evolution, is

$$\dot{\rho} = L \rho L^\dagger - \frac{1}{2}L^\dagger L \rho - \frac{1}{2}\rho L^\dagger L,$$

(2.8)

where $L$ is a Lindblad operator which may or may not be self-adjoint.

The corresponding state diffusion equation is

$$|d\psi\rangle = \left( (L^\dagger) L - \frac{1}{2} L^\dagger L - \frac{1}{2} \langle L^\dagger L \rangle \right) |\psi\rangle dt + (L - \langle L \rangle) |\psi\rangle d\xi,$$

(2.9)

where $\langle L \rangle = \langle \psi | L | \psi \rangle$. Gisin and Percival (1992) derive the state diffusion equation (2.9) from the master equation (2.8). The derivation requires $|\psi\rangle$ to be normalized, and also requires the state diffusion equations, like the master equation, to be invariant under a unitary gauge transformation $L' = u L$ in operator space. With these constraints, the set of equivalent state diffusion equations corresponding to a given master equation is unique.

For sufficiently short times the QSD evolution is dominated by the diffusion term, which represents one-dimensional complex diffusion on the sphere of normalized quantum states.

For a nonrelativistic system with hamiltonian $H$, the PSD master equation and primary state diffusion equation have the forms (2.8, 2.9) with

$$L = \tau_0^{1/2} \frac{H}{\hbar} + i \tau_0^{-1/2} I,$$

(2.10)

where $I$ is the identity operator, but it is more convenient to write them explicitly in terms of the Hamiltonian as the transformed equations (PSD1)

$$\hbar \dot{\rho} = -i[H, \rho] + \tau_0 (H \rho H - \frac{1}{2} H^2 \rho - \frac{1}{2} \rho H^2)$$

(2.11)
\[ \hbar |\psi\rangle = (-iH_\Delta dt - \frac{4}{5}\tau_0 H_\Delta^2 dt + \tau_0^{\frac{1}{2}} H_\Delta d\xi)|\psi\rangle, \quad (2.12) \]

where
\[ H_\Delta = H - \langle \psi |H|\psi \rangle. \quad (2.13) \]

Note that in PSD1 the transformed equation corresponding to (2.12) has a factor \( \frac{1}{2} \) missing.

The additional fluctuation term destroys the Fourier relation between energy representation and time representation. In addition to the usual exponential of the Fourier transform there is a fluctuating term. In a relativistic theory this must also be true of momentum and position representation. So for high frequencies and large wave numbers, the usual simple relations between these representations break down.
3. Fluctuating space-time

According to the equivalence principle of general relativity (GR), around every space-time point of our universe there is a sufficiently small region with a smooth transformation relating it to a flat Minkowski universe.

It is commonly agreed that in our universe, space-time has quantum fluctuations on the Planck scale, and that by quantum indeterminacy these fluctuations become relatively larger on smaller scales. It follows that they cannot be differentiable. Accordingly there are no locally inertial Lorentz frames in our universe, and no small region around any point with a smooth transformation relating it to a Minkowski universe. As we go down in scale into the Planck domain, space-time become less flat, not more so. The GR equivalence principle applies only on scales significantly larger than the Planck scale.

Space-time PSD is based on a stochastic differential structure and a fluctuating space-time metric. These are quantum fluctuations, but they are expressed in terms of non-differentiable relations between classical space-times. All other quantum fluctuations are consequences of these space-time fluctuations. Space-time PSD is therefore made of two parts: the properties of a Minkowski universe and its Lorentz frames, and the properties of our universe that are obtained by local nondifferentiable space-time transformations between the Minkowski universe and ours.

Just as in GR, our universe can be locally related to a Minkowski universe by a space-time transformation, even down to the Planck scale and below. But whereas on a macro scale, the space-time of our universe and the transformations appear to be smooth and differentiable, so on the Planck scale the space-time and the transformations are fluctuating and nondifferentiable. In the present semiclassical theory they resemble Brownian motion and satisfy Langevin-Itô stochastic differential relations.

In a Lorentz frame in the Minkowski universe with time $\tilde{t}$, elementary quantum systems at low velocity are represented by states that satisfy the Schrödinger equation,

$$\hbar \hat{d} |\psi\rangle = -iH \hat{d} |\psi\rangle$$

(3.1)

and the more subtle systems satisfy the flat space-time laws of quantum field theory. There is no classical dynamics because there is no quantum state diffusion and no localization. In PSD, as in other sharp theories, there is only one world, which includes both quantum and classical dynamics, but for PSD the existence of the latter depends on the localization in phase space that comes from the quantum state diffusion.

To help us in the journey from the Minkowski universe to our universe it is convenient to introduce an intermediate stage. This is the uniformly fluctuating universe or UFU. The local transformation from the Minkowski universe to the UFU
is stochastic, nondifferentiable and nonuniform in space-time, but it has uniform statistical properties in space-time. This represents quantum fluctuations. The local transformation from the UFU to our universe is deterministic, smooth and nonuniform in space-time. This represents large-scale gravity.

The fluctuation equivalence principle (FEP) that replaces the usual GR equivalence principle says that around every space-time point of our universe there is a sufficiently small region with a smooth transformation relating it to the UFU. This sufficiently small region is normally very large compared to the Planck scale, but the transformation applies on all scales, and is the equivalent, in the new theory, of the smooth transformations of GR.

We look at the new quantum physics coming from the nondifferentiable transformations between the flat Minkowski universe and the UFU. Since the theory is not complete, we do not have the statistics of all components of this transformation, nor of its space dependence, which would then describe the statistical metric structure of the UFU in full. We consider only a local time-time component of the transformation from Minkowski space-time with time $t$ to UFU space-time with time $\bar{t}$.

Suppose that on far sub-Planck scales, much smaller than the Planck scale, the transformation is dominated by the fluctuation term, that on far super-Planck scales it is dominated by the identity transformation, and that on the Planck scale the two effects are about equal. The time-time component of the fluctuating transformation is then

$$d\bar{t} = dt + \tau_1^+ d\xi,$$

where $\tau_1$ is a universal time constant. The simplest assumption is that this is the Planck time $T_P\ell$, but a small numerical factor $C$ of order unity, like $2\pi$, cannot be excluded, so

$$\tau_1 = C T_P\ell.$$  \hspace{1cm} (3.3)

In classical dynamics a real time-time transformation has no local physical effect other than a change in the time coordinate that results in a time dilation or contraction with respect to other frames, as in the usual time dilation of relativity. But for complex time transformations of quantum systems this is no longer so. They produce nonphysical changes in the norm of the state vector, which must be removed. This is done through a set of conditions on the state vector of space-time PSD which must be satisfied in all frames. These conditions are included in the principles of space-time PSD, or STP, which must define the time evolution of the state uniquely. For the UFU the principles are

STP1. The state vector $|\psi\rangle$ satisfies a nonlinear Langevin-Itô diffusion equation, where the right side is an operator on $|\psi\rangle$. 

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STP2. On far sub-Planck scales the operator is given by the transformation from the flat Minkowski universe, except for an additional scalar times the unit operator, which is needed to preserve normalization.

STP3. On far super-Planck scales the skew-adjoint part of the operator is given by the transformation from the Minkowski universe. This is the Schrödinger operator.

STP4. The norm of $|\psi\rangle$ is preserved.

The principles for our universe are obtained from the STP principles for the UFU and the new equivalence principle.

From STP1-3, using the Schrödinger equation (3.1) in Minkowski space, and the transformation (3.2), the UFU state vector satisfies an equation of the form

$$\hbar |d\psi\rangle = (-iH dt + R(\psi) dt + \tau_1^{1/2} H d\xi - s(\psi) d\xi)|\psi\rangle,$$

where $R(\psi)$ is a self-adjoint operator.

It follows from STP4 that

$$0 = d\langle\psi|\psi\rangle = 2\text{Re}\langle\psi| - iH + R(\psi)|\psi\rangle dt + 2\text{Re}(\langle\psi|\tau_1^{1/2} H - sI|\psi\rangle d\xi)$$

$$+ \langle\psi| (\tau_1^{1/2} H - sI)^\dagger (\tau_1^{1/2} H - sI)|\psi\rangle dt$$

and since $d\xi$ can vary, its coefficient must be zero, so $s = \tau_1^{1/2} \langle\psi|H|\psi\rangle$ and

$$R(\psi) = -\frac{i}{2} \tau_1 H^2_\Delta,$$

with $H_\Delta$ defined by (2.13), so the state vector in the uniformly fluctuating universe satisfies the primary state diffusion equation (2.12) with

$$\tau_0 = \tau_1 = CT_{Pl},$$

as required.

The Lindblad-Gisin condition of PSD1, requiring the derived density operator equation to be of Lindblad (1976) form, is not needed here as one of the principles of space-time PSD. It is satisfied automatically.

For a restricted set of problems, space-time PSD unites space-time structure and the foundations of quantum theory. It does so by abandoning one of the basic principles of each. The principle of equivalence of general relativity relating our universe to a Minkowski universe is replaced by a different principle relating our universe to a uniformly fluctuating universe. The Fourier relations between energy-momentum and space-time representations in quantum mechanics are also lost. So for these small scales the theory of space-time and quanta must be built again on the new foundations.
4. Theories of quantum decoherence from gravity

In Feynman’s lectures on gravitation (Feynman et al 1961-2, p12), he said

‘The extreme weakness of quantum gravitational effects now poses some philosophical problems; maybe nature is trying to tell us something here, maybe we should not try to quantize gravity. . . . It is still possible that quantum theory does not absolutely guarantee that gravity has to be quantized. . . . In this spirit I would like to suggest that it is possible that quantum mechanics fails at large distances and for large objects. Now, mind you, I do not say that I think that quantum mechanics does fail at large distances, I only say that it is not inconsistent with what we know. If this failure is connected with gravity, we might speculatively expect this to happen for masses such that \( GM^2/\hbar c = 1 \), or \( M \) near \( 10^{-5} \) g’

This is the Planck mass. He continues later:

‘If there was some mechanism by which the phase evolution had a little bit of smearing in it, so it was not absolutely precise, then our amplitudes would become probabilities for very complex objects. But surely, if the phases did have this built in smearing, there might be some consequences to be associated with this smearing. If one such consequence were to be the existence of gravitation itself, then there would be no quantum theory of gravitation, which would be a terrifying idea for the rest of these lectures.’

This theme was taken up by Károlyházy (1966; Károlyházy Frenkel and Lukács, 1986), who connected stochastic reduction of the wave function with gravity through an imprecision in the space-time structure, just as it is here, but the imprecision comes from the presence of large masses, so the resultant decoherence is much weaker. There the matter rested for many years.

Recently Diósi and Lukács (1993a, 1993b) showed that the Fourier expansion of the small scale fluctuations in Károlyházy’s theory leads to unacceptably large fluctuations in energy density. The same objection applies to space-time PSD. However, the objection appears to be based on the Fourier relation between space-time and energy-momentum, which is no longer valid in PSD or related theories. If it were, then the white noise of the fluctuations \( d\xi \) would produce infinite energies even in the nonrelativistic quantum state diffusion theory (Gisin and Percival, 1992), and it does not. Nevertheless the objection shows how important it is to find a replacement for these Fourier relations.

Penrose (1986) based his gravitational theory of quantum decoherence on the entropy relations of Hawking (1982), and the need for gravitational entropy. A wave function that is sufficiently spread out in space, and gets coupled to a larger system which produces significant Weyl curvature, represents a significant rise in the gravitational entropy, and then reduction takes place.
Diósi (1987, 1989, 1992) emphasized that both gravity and quantum mechanics must be changed because neither is valid on all scales, and suggested a classical fluctuating gravitational field, whose fluctuations are given by Heisenberg’s indeterminacy principle. Ghirardi, Grassi & Rimini (1989) showed that Diósi’s theory requires a fundamental length, which they provided in a modified theory.

Ellis, Mohanty & Nanopoulos (1989) base their theory on a non-unitary modification of the Hamiltonian evolution equation caused by wormholes interacting with a microscopic system.

Significant effects which might distinguish any of these theories from ordinary quantum mechanics appear only for systems with mass approaching the Planck mass, and seem unlikely to be detectable in the foreseeable future. In space-time PSD the effects appear for much smaller systems.
5. Sharp quantum theories and experiment

In a letter of 27 May 1926 to Schrödinger on his recently proposed wave equation, H A Lorentz pointed out that a wave packet, which when moving with the group velocity should represent a particle, ‘can never stay together and remain confined to a small volume in the long run. The slightest dispersion in the medium will pull it apart in the direction of propagation, and even without that dispersion it will always spread more and more in the transverse direction. Because of this unavoidable blurring a wave packet does not seem to be very suitable for representing things that we want to ascribe a rather permanent existence.’

This shows the difficulty of using Schrödinger wave packets to represent classical free particles. The delocalization or dispersion spreads a wave packet in position and nonlinearities in the dynamics then spread it in momentum too. The dispersion is even stronger when there are particle interactions. And it is exactly this difficulty that quantum state diffusion, PSD and similar theories overcome, since quantum state diffusion works in the opposite direction, as shown by extensive numerical evidence, and theorems (Percival, 1994a). Quantum state diffusion localizes individual quantum systems into regions of phase space whose size is determined by Planck’s constant.

Bell (1987, p171) uses ‘sharp formulation of quantum mechanics’ to mean a theory that provides a uniform description of the micro and macro worlds. For many decades most physicists believed that there was no sharp formulation of quantum mechanics, despite the existence of the sharp pilot wave formulation of de Broglie and Bohm. Largely due to the influence of Bell, this is no longer the case. The problem now is that there are too many different sharp quantum theories.

Apart from the gravitational theories of the previous section, there are those like the pilot wave theory and many-worlds theories that are in principle indistinguishable experimentally from ordinary quantum theory. There are the theories of consistent or decoherent histories (Griffiths 1984), for which the differences are either absent (Omnès, 1994) or believed to be so small that they could never be detected by experiment (Dowker & Halliwell 1992; Gell-Mann & Hartle 1993). A relation between these theories and quantum state diffusion for open systems is shown by Diósi et al (1995).

But for us the important sharp theories are those like PSD that localize the de Broglie wave packets, and so overcome Lorentz’s problem explicitly. These theories might be distinguished from ordinary quantum theory experimentally. Some experimental tests have been discussed by Ellis et al, (1984) and Pearle (1984). However most of the theories have free parameters. For example the theories of Ghirardi, Rimini and Weber (1986) and Ghirardi, Pearle and Rimini (1990) are two-parameter theories. Recently Pearle and Squires (1995) have suggested that the gravitational
curvature scalar causes the wave function collapse in these theories. Those theories which depend on energy localization (Bedford and Wang 1975, 1977; Gisin 1989; Milburn 1991), including PSD1, are one-parameter theories, in which the parameter determines the rate of energy localization.

In these theories, the set of parameters can be chosen with a wide range of values that are consistent with all experiments to date. The range can be reduced by further experiment, but in every case there are possible values which are so extreme that there is no hope of measuring them. There is no accessible critical experiment that is guaranteed to distinguish between any of these sharp theories and ordinary quantum theory, which is frustrating for those who might want to test them experimentally.

The original primary state diffusion theory of PSD1 had one free time parameter $\tau_0$, which was conjectured to be close to the Planck time. That conjecture is here confirmed by changing the foundations of the theory, but leaving the rest of PSD theory intact, so that a value of $\tau_0$ close to the Planck time is derived. With these new foundations, PSD becomes a rigid theory with no free parameters, except for a factor $C$ of order unity, and with reasonable prospects for critical experiments using matter interferometry. It was shown in PSD1 that matter interferometry experiments have already been done that were only about a factor of 2000 from distinguishing between space-time PSD and ordinary quantum mechanics (Kasevich & Chu 1991), that is, showing one or the other to be wrong. Matter interferometry is a rapidly developing field, so space-time PSD could be subjected to such a critical experimental test within the next decade.

This would be a laboratory experiment in quantum gravity.

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