Dasher, A. Suki; Hermida, A.; Wong, Tian An

The three gap theorem and periodic functions. (English) [Zbl 1493.11022]

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Summary: The Three Gap Theorem, also known as the Steinhaus Conjecture, is a classical result on the combinatorics of the fractional part function, and has since been generalized in many ways. In this paper, we pose a new problem related to these results: for which other periodic functions does an analogue of the Three Gap Theorem hold? We prove analogous results for certain classes of piecewise-linear periodic functions and demonstrate the existence of functions for which no bound exists on the number of gap lengths.

MSC:

11B05 Density, gaps, topology

Keywords:

piecewise-linear periodic functions; three gap theorem

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