Abstract—The roll-out of stochastic renewable energy sources (RES) undermines the efficiency of power system and market operations. This paper proposes an approach to derive electricity prices that internalize RES stochasticity. We leverage a chance-constrained AC Optimal Power Flow (CC AC-OPF) model, which is robust against RES uncertainty and is also aware of the resulting variability (variance) of the system state variables. Using conic duality theory, we derive and analyze energy and balancing reserve prices that internalize the risk of system limit violations and the variance of system state variables. We compare the risk- and variance-aware prices on the IEEE 118-node testbed.

I. INTRODUCTION

Power systems and electricity markets struggle to accommodate the massive roll-out of renewable energy sources (RES), which are stochastic in nature and impose additional risks on the system operations and market-clearing decisions. The current industry practice to mitigate these risks is based on procuring additional reserves, which are selected based on exogenous and often ad-hoc policies (e.g., 95-percentile rule in ERCOT, [1], or (5+7) rule in CAISO, [2]).

Alternatively, such risk assessments can be carried out endogenously, i.e. while optimizing operational and market-clearing decisions, using high-fidelity prediction and historical data parameterizing the RES stochasticity. Bienstock et al. [3] proposed a risk-aware approach to solving an Optimal Power Flow (OPF) problem that uses chance constraints (CC) to internalize the RES stochasticity and risk tolerance of the system operator to violating system constraints. Since [3], the CC-OPF has been shown to scale efficiently for large networks [4], accommodate various assumptions on the RES stochasticity (e.g. parametric distributions and distributional robustness) [4]–[6], as well as to accurately account for AC power flow physics. [7], [8]. However, this framework has primarily been applied to risk-aware operational planning in a vertically integrated environment, neglecting market considerations. From a market design perspective, RES stochasticity has been primarily dealt with using scenario-based stochastic programming, e.g. [9]–[11], which is more computationally demanding than chance constraints, [3].

With the exception of our recent work in [12], [13], chance constraints have so far been overlooked in electricity pricing applications. The chance-constrained market design proposed in [13] leads to a stable robust equilibrium that, unlike scenario-based approaches in [9]–[11], guarantees desirable market properties, i.e. welfare maximization, revenue adequacy and cost recovery, under various assumptions on the RES stochasticity. Therefore, the resulting energy and reserve prices make it possible to better approximate real-time operating conditions for look-ahead dispatch applications, thus improving consistency between look-ahead and real-time stages. However, [13] neglects network constraints, an important modeling feature for real-life market applications.

This paper uses a chance-constrained AC OPF (CC AC-OPF) from [7] to derive network-aware electricity prices that internalize the RES stochasticity with the intention to produce more accurate signals to market participants. This convex formulation allows the use of duality theory to derive risk-aware marginal-cost-based prices, which are similar to traditional determinstic locational marginal prices (LMPs) based on linear duality. [14]. Furthermore, the CC AC-OPF can explicitly consider reactive power and voltage support services and analyze their role in the deliverability of active power, thus supporting the design of a more “complete” electricity market, [15], [16]. Completing the market by allowing all assets and services (active and reactive power, reserve capacity, transmission and voltage support) to be transacted, [16], makes it possible to co-align technical needs and requirements imposed by the physical aspects of power system operations and price signals received by market participants. We also extend the CC AC-OPF to follow a variance-aware dispatch paradigm, introduced in [7], to compute variance-aware prices and analyze the relationship between the system cost, risk and variance.

II. MODEL FORMULATION

This paper builds on the AC-CCOPF model presented in [7] with model assumptions and modifications explained below.

A. Preliminaries

Consider a transmission network with set of nodes $\mathcal{N}$, set of lines $\mathcal{L}$, set of generators $\mathcal{G}$ and set of renewable generators $\mathcal{U}$ (e.g. wind or commercial solar farms). For simplicity of notation, we assume that each node hosts one conventional and one renewable generator, such that $\mathcal{G} = \mathcal{U} = \mathcal{N}$. We denote the set of PQ and PV nodes as $\mathcal{N}^{PQ}, \mathcal{N}^{PV} \subset \mathcal{N}$ and index reference ($\theta V$) node as $i = ref$. Nodes without generation or with more than one generator can be handled by setting the generation limit to zero or by changing notations, respectively; both modification will not affect the proposed method. Let vectors $p_G$ indexed as $p_{G,i}$, $p_D$ indexed as $p_{D,i}$, and $p_V$ indexed as $p_{V,i}$, denote the total active power output of conventional generators, the total active power demand and the active power injections from renewable generation at every node. The corresponding reactive power injections are denoted
and we require $\sum_{i \in G} \alpha_i = 1$ to balance the system. Vector $\alpha$ collects all $\alpha_i, i \in G$. The response of reactive power generation $q_{G,i}(\omega)$, voltage magnitudes $v_i(\omega)$ and voltage angles $\theta_i(\omega)$ is determined by the type of node $i$. At PV nodes $v_i(\omega) = v_i, \forall i \in N^{PV}$, is controlled and $q_{G,i}(\omega), \theta_i(\omega), \forall i \in N^{PV}$, are implicitly determined by power flow equations $F(p, q, v, \theta)$. Similarly, at PQ nodes $q_{G,i}(\omega) = q_{G,i}, \forall i \in N^{PQ}$, is controlled and $v_i(\omega), \theta_i(\omega), \forall i \in N^{PQ}$, are implicitly determined by power flow equations $F(p, q, v, \theta)$. Finally, at the $\theta'$ node $\forall i \in N^{\theta'}$, $\forall \theta' \in N^{\theta'}$, and reactive power response at the $\theta'$ node is also determined implicitly by power flow equations $F(p, q, v, \theta)$. The resulting active and reactive power flows are implicitly given by $f_{ij}^p(\omega) = f_{ij}^p(v(\omega), \theta(\omega))$ and $f_{ij}^q(\omega) = f_{ij}^q(v(\omega), \theta(\omega))$.  

**D. Production Cost**

The production cost of each generator is approximated by a quadratic function, (19):

$$c_i(p_{G,i}) = c_{2,i}(p_{G,i})^2 + c_{1,i}p_{G,i} + c_{0,i}$$

and, for the compactness of derivations, we denote $c_{2,i} = 1/2b_i$, $c_{1,i} = a_i/b_i$, and $c_{0,i} = a_i^2/2b_i$. Given uncertainty $\omega$ and the response in (3), the expected operating cost is:

$$\mathbb{E}[c_i(g_i^p(\omega))] = c_{i}(p_{G,i}) + \frac{a_i^2}{2b_i}S^2.$$  

**E. Linearization of AC Power Flow Equations**

As discussed in Section II-C, some system state variables are determined implicitly by the non-linear, non-convex AC power flow equations in (2), which do not permit a direct solution. Therefore, we linearize $F(p, q, v, \theta) = 0$ around a given (forecast) operating point using Taylor’s theorem as in [7]. Let $(\bar{p}, \bar{q}, \bar{f}_p, \bar{f}_q, \bar{v}, \bar{\theta})$ be the linearization result, then the nodal power injections and line flows are:

$$p_i = \bar{p}_i + J_{i}^{p,v}(\bar{v}, \bar{\theta})v + J_{i}^{p,\theta}(\bar{v}, \bar{\theta})\theta \quad \forall i \in U$$

$$q_i = \bar{q}_i + J_{i}^{q,v}(\bar{v}, \bar{\theta})v + J_{i}^{q,\theta}(\bar{v}, \bar{\theta})\theta \quad \forall i \in U$$

$$f_{ij}^{\bar{v},\bar{\theta}} = J_{ij}^{f,v}(\bar{v}, \bar{\theta})v + J_{ij}^{f,\theta}(\bar{v}, \bar{\theta})\theta$$

where $J_{i}^{p,v}, J_{i}^{p,\theta}, J_{i}^{q,v}, J_{i}^{q,\theta}, J_{ij}^{f,v}, J_{ij}^{f,\theta}$ are row-vectors of sensitivity factors describing the change of active and reactive nodal injections as functions of $v$ and $\theta$ derived from the AC power flow linearization. Similarly, the response of voltages, flows and reactive power outputs to $\omega$ can be modeled as (see Appendix A):

$$q_{G,i}(\omega) = q_{G,i} + [R_i^{p}(I - a\omega^T) + X_i^q] \text{diag}(\gamma)\omega$$

$$v_i(\omega) = v_i + [R_i^{p}(I - a\omega^T) + X_i^q] \text{diag}(\gamma)\omega$$

$$f_{ij}^q(\omega) = f_{ij}^q + [R_{ij}^{p}(I - a\omega) + X_{ij}^q] \text{diag}(\gamma)\omega$$

$$f_{ij}^p(\omega) = f_{ij}^p + [R_{ij}^{p}(I - a\omega) + X_{ij}^q] \text{diag}(\gamma)\omega$$

where row-vectors $R_i^{p}, R_i^{q}, R_{ij}^{p}, R_{ij}^{q}$ map adjustments of the respective variables to active power changes, row-vectors $X_i^q, X_{ij}^q$ map adjustments of the respective variables to reactive power changes and $I$ is the identity matrix. Note that sensitivity vectors $R_i^{p}, R_i^{q}, R_{ij}^{p}, R_{ij}^{q}$ can be zero, if $i$ is a PV or PQ node, and depend on a chosen linearization point.
F. Chance Constrained Optimal Power Flow

For a given operating point \((p_G, q_G, v, \theta, \gamma, \alpha)\) the system will respond to any realization of \(\omega\) according to (5), (6)–(13). To ensure that this system response does not violate the physical system limits with a high probability, we formulate the following chance constraints:

\[
\mathbb{P}(p_G^{\text{min}} \leq p_G(\omega) \leq p_G^{\text{max}}) \geq 1 - 2\epsilon_p \quad i \in \mathcal{G} \quad (14)
\]
\[
\mathbb{P}(q_G^{\text{min}} \leq q_G(\omega) \leq q_G^{\text{max}}) \geq 1 - 2\epsilon_q \quad i \in \mathcal{G} \quad (15)
\]
\[
\mathbb{P}(v_i^{\text{min}} \leq v_i(\omega) \leq v_i^{\text{max}}) \geq 1 - 2\epsilon_v \quad i \in \mathcal{N} \quad (16)
\]
\[
\mathbb{P}((f^p_{ij}(\omega))^2 + (f^q_{ij}(\omega))^2 \leq (\sigma_{ij}^{\text{max}})^2) \geq 1 - \epsilon_f \quad i, j \in \mathcal{L}, \quad (17)
\]

where \(\epsilon_p, \epsilon_q, \epsilon_v, \epsilon_f < 1/2\) can be chosen to tune the risk level associated with the individual chance constraints (14)–(17). Using (10)–(13), we can obtain computationally tractable reformulations of chance constraints (14)–(17), [3], [7], [18], and formulate the deterministic equivalent of the CC-AC-OPF:

\[
\text{EQV-CC} : \quad \min_{p_G, q_G} \mathbb{E}_\mathcal{G} \left[ \sum_{i \in \mathcal{G}} c_i(p_G, i) + \sum_{i \in \mathcal{G}} \frac{\alpha_i^2}{2b_i} S^2 \right] \quad (18a)
\]

s.t.

\[
(\chi_i) : \quad \sum_{i \in \mathcal{G}} \alpha_i = 1 \quad (18d)
\]
\[
(\delta^p_i) : \quad p_G - p_G^\epsilon - z_i \epsilon_p S \leq p_G^{\text{max}} \quad i \in \mathcal{G} \quad (18e)
\]
\[
(\delta^q_i) : \quad q_G - q_G^\epsilon - z_i \epsilon_q S \leq q_G^{\text{max}} \quad i \in \mathcal{G} \quad (18f)
\]
\[
(\delta^q_i) : \quad q_G - q_G^\epsilon - z_i \epsilon_q S \leq q_G^{\text{max}} \quad i \in \mathcal{G} \quad (18g)
\]
\[
(\chi_i) : \quad \left\| (R_i^p - \rho_i^p \epsilon^T + X^p_i \text{diag}(\gamma)) \Sigma^{1/2} \right\|_2 \leq t_i^p \quad i \in \mathcal{G} \quad (18h)
\]
\[
(\nu^p_i) : \quad R_i^p \alpha = \rho_i^p \quad i \in \mathcal{G} \quad (18i)
\]
\[
(\nu^q_i) : \quad v_i + z_i \epsilon_v t_i^q \leq v_i^{\text{max}} \quad i \in \mathcal{N} \quad (18k)
\]
\[
(\nu^q_i) : \quad v_i + z_i \epsilon_v t_i^q \leq v_i^{\text{max}} \quad i \in \mathcal{N} \quad (18l)
\]
\[
(\nu^q_i) : \quad \left\| (R_i^q - \rho_i^q \epsilon^T + X^q_i \text{diag}(\gamma)) \Sigma^{1/2} \right\|_2 \leq t_i^q \quad i \in \mathcal{G} \quad (18m)
\]
\[
(\nu^q_i) : \quad R_i^q \alpha = \rho_i^q \quad i \in \mathcal{N} \quad (18n)
\]
\[
(\nu^q_i) : \quad (a_{ij}^p)^2 + (a_{ij}^q)^2 \leq (\sigma_{ij}^{\text{max}})^2 \quad i, j \in \mathcal{L} \quad (18o)
\]
\[
(\xi_{ij}^{p,+}) : \quad -a_{ij}^p + z_i \epsilon_v t_{ij}^p \leq f_{ij}^p \quad i \in \mathcal{L} \quad (18p)
\]
\[
(\xi_{ij}^{q,+}) : \quad -a_{ij}^q + z_i \epsilon_v t_{ij}^q \leq f_{ij}^q \quad i \in \mathcal{L} \quad (18q)
\]
\[
(\xi_{ij}^{p,0}) : \quad z_{ij} f_{ij}^p \leq a_{ij}^p \quad i \in \mathcal{L} \quad (18r)
\]
\[
(\xi_{ij}^{q,0}) : \quad z_{ij} f_{ij}^q \leq a_{ij}^q \quad i \in \mathcal{L} \quad (18s)
\]

where Greek letters in parentheses in (18e)–(18w) denote dual multipliers of constraints. Objective (18a) minimizes the expected cost as in (5). Eqs. (18b)–(18c) are the active and reactive power balances and flows based on the linearized AC power flow equations. Eq. (18d) is the balancing reserve adequacy constraint and (18e)–(18w) are the deterministic reformulation of chance constraints (14)–(17). Constraints (18e)–(18f) limit the active power production \(p_G\), and the amount of reserve \(\alpha_i z_i \epsilon_p S\) provided by each generator, [13], [20]. Risk parameters are given by \(z_i = \Phi^{-1}(1 - \epsilon)\), where \(\Phi^{-1}(1 - \epsilon)\) is the \((1 - \epsilon)\)-quantile of the standard normal distribution, if \(\omega\) follows a normal distribution. Although less restrictive assumptions on the distribution of \(\omega\) can be invoked in (18), e.g. of means of non-Gaussian parametric distributions [5] or distributionally robust formulations [4], [13], this paper assumes normally distributed forecast errors for the sake of presentation clarity. The standard deviation of reactive power outputs, voltage levels and flows resulting from the uncertainty and the system response is given by the SOC constraints (18b), (18m) and (18v). Given the convexity of the SOC constraints, auxiliary variables \(t_i^p, t_i^q\), \(t_{ij}^p, t_{ij}^q\) relate these standard deviations to the reactive output limits (18e)–(18w), voltage bounds (18x)–(18t) and flow limits (18p)–(18n). Due to its quadratic dependency on the uncertain variable, the chance constraint in (17) requires a more complex reformulation than (14)–(16). To accommodate this reformulation, we follow [7] and introduce auxiliary variables \(a_{ij}^p, a_{ij}^q\) and risk parameters \(\epsilon_{ij}/2, \epsilon_{ij}/5\) (i.e. \(\epsilon_i/\epsilon_j\) divided by 2.5 and 5), respectively. This yields an inner approximation of (17) that ensures feasibility of the AC OPF with desired confidence \(1 - \epsilon_f\) and the conservatism of the approximation can be tuned by adapting the divisor (2.5 and 5). [7]. Note that the two-sided chance constraints in (14)–(17) are expressed as one-sided chance constraints in (18a)–(18w) since simultaneous violations of both the upper and lower capacity or voltage limits are physically impossible. Auxiliary variables \(\rho_i^p, \rho_i^q, \rho_{ij}\) and constraints (18j), (18n) and (18w) have been introduced to simplify subsequent derivations. As a result, (18) includes convex quadratic objective and second-order conic constraints. Although it can be reformulated into a convex conic program to gain computational tractability, [21], the form in (18) allows for a clear presentation below.

III. RISK-AWARE PRICING

The EQV-CC endogenously trades off the expected operating point \((p_G, q_G, v, \theta, \gamma, \alpha)\) and the risk of system limit violations defined by the choice of parameters \(z_{ij}, z_{ij}, z_{ij}/2.5, z_{ij}/5\). Since the EQV-CC is a convex program, we can use its dual form to compute the marginal prices for active and reactive power, and balancing reserve that internalize this trade-off.
A. Prices with Chance Constraints on Generation

First, we consider the multipliers of the EQV-CC given as:

\[
\text{GEN-CC :} \quad \min_{p_G,q_G} \sum_{i \in \mathcal{N}} c_i p_G(i) + \sum_{i \in \mathcal{N}} \delta_i^2 \quad (19a)
\]

s.t.

\[
\begin{align*}
(\delta_i^+,\delta_i^-) : & \quad \quad q_{G,i}^\min \leq q_{G,i} \leq q_{G,i}^\max \\
(\mu_i^+,\mu_i^-) : & \quad v_i^\min \leq v_i \leq v_i^\max \\
(\eta_{ij}) : & \quad (f_{ij}^p)^2 + (f_{ij}^q)^2 \leq (s_{ij}^\max)^2,
\end{align*}
\]

where, relative to the EQV-CC in (18), chance constraints are only enforced on active power generation limits and reactive power, voltage and power flow constraints are enforced deterministically by (19b)–(19d). In other words, the GEN-CC replicates a traditional deterministic OPF that allocates the committed reserve given by (19b)–(19d). In other words, the GEN-CC replicates a traditional deterministic OPF that allocates the committed reserve given by (19b)–(19d).

Using the GEN-CC, we compute the following prices:

**Proposition 1.** Consider the GEN-CC in (19). Let \( \lambda_i^p, \lambda_i^q \) be dual multipliers of the nodal active and reactive power balance at node \( i \) in (18b). Then \( \lambda_i^p \) and \( \lambda_i^q \) are given as:

\[
\lambda_i^p = \frac{p_G(i) + a_i}{b_i} + \delta_i^p - \delta_i^p^- \quad \quad \lambda_i^q = \delta_i^q - \delta_i^q^-.
\]

**Proof.** The first order optimality conditions of (19) for \( p_G,i, q_G,i, \alpha_i, f_{ij}^p, f_{ij}^q \) are:

\[
\begin{align*}
(p_G,i) : & \quad \lambda_i^p + (\delta_i^p - \delta_i^p^-) = \frac{p_G(i) + a_i}{b_i} & i \in \mathcal{G} \\
(q_G,i) : & \quad \lambda_i^q + (\delta_i^q - \delta_i^q^-) = 0 & i \in \mathcal{G} \\
(\alpha_i) : & \quad \lambda_i q_e S(\delta_i^p + \delta_i^q) + \frac{a_i}{b_i} S^2 & i \in \mathcal{G} \\
(f_{ij}^p) : & \quad 2 f_{ij}^p \eta_{ij} + \beta_{ij} = 0 & i,j \in \mathcal{L} \\
(f_{ij}^q) : & \quad 2 f_{ij}^q \eta_{ij} + \beta_{ij} = 0 & i,j \in \mathcal{L}.
\end{align*}
\]

Eqs. (20)–(21) follow directly from (22a)–(22b). \( \square \)

Dual multiplier \( \lambda_i^p \) of the active power balance, itemized in (20), is interpreted as the real power LMP at node \( i \) and a function of production cost coefficients \( a_i, b_i \) and scarcity rents \( \delta_i^p, \delta_i^p^- \) related to active generation limits. Dual multiplier \( \lambda_i^q \) of the reactive power balance, itemized in (21), is interpreted as the reactive power LMP given by scarcity rent \( \delta_i^q, \delta_i^q^- \) related to reactive generation limits. Although there is no explicit production cost for reactive power in (18a), providing reactive power can have a non-zero value, if at least one reactive power limit is binding. Further, Proposition 4 shows that both \( \lambda_i^p \) and \( \lambda_i^q \) in (20)–(21) do not explicitly depend on uncertainty and risk parameters.

In contrast, the price for balancing reserve explicitly depends on the uncertainty and set risk levels:

**Proposition 2.** Consider the GEN-CC in (19). Let \( \chi \) be the dual multiplier of the balancing adequacy constraint in (18d). Then \( \chi \) is given as:

\[
\chi = \frac{1}{\sum_{i \in \mathcal{G}} b_i \left( S^2 + z_{e,i} S \sum_{i \in \mathcal{G}} b_i (\delta_i^p + \delta_i^p^-) \right)}.
\]

**Proof.** Using (18d) to eliminate \( \alpha_i \) in (22c) yields (23). \( \square \)

Dual multiplier \( \chi \) of (18d) is interpreted as the price for balancing reserve, because it enforces sufficiency of the system-wide reserve. As per (23), \( \chi \) is an explicit function of the uncertainty \( S^2 = \chi^T \Sigma \) and risk parameter \( z_{e,i} \). Notably, the balancing reserve price is always non-zero, if there is uncertainty in the system (i.e. \( S > 0 \)), even if all constraints (18a)–(18d) are inactive, i.e. \( \delta_i^{p^-} = \delta_i^{q^-} = 0, \forall i \in \mathcal{G} \). In this case, \( \chi \) is independent of the risk parameters and is determined by the total uncertainty \( S^2 \) weighted by the total marginal generator cost \( \sum_{i \in \mathcal{G}} b_i \alpha_i q_e S = z_{e,i} S \) among individual generators, see (7).
\[(\alpha_i): \quad \chi = z_{\rho}S(\sigma_{p}^{2} + \delta_{p}^{2}) + \sum_{j \in G} \nu_{ij}^{p}[R_{ij}^{p} \nu_{ij}^{p} + \sum_{j \in G} \nu_{ij}^{p}[R_{ij}^{p} \nu_{ij}^{p}]
+ \sum_{j \in \mathbb{K}} \nu_{ijk}^{p}[R_{ijk}^{p} \nu_{ijk}^{p} \nu_{ijk}^{p} \nu_{ijk}^{p} = \frac{\alpha_i}{b_i} \sqrt{2}\]
\[i \in \mathbb{G} \quad (28a)\]
\[(t_{ij}^{p}): \quad z_{\rho}(\delta_{p}^{2} + \delta_{q}^{2}) - \zeta_{ij}^{q} = 0 \quad i \in \mathbb{G} \quad (28b)\]
\[\rho_{ij}^{q} = \frac{(R_{ij}^{p} - \rho_{ij}^{p} e^{T} + X_{ij}^{q} \text{diag}(\gamma))\Sigma_{e}}{\|R_{ij}^{p} - \rho_{ij}^{p} e^{T} + X_{ij}^{q} \text{diag}(\gamma)\|^{2}} \quad i \in \mathbb{G} \quad (28c)\]

The result (i) follows directly from the proof of Proposition 1. The result (ii) follows from (28a) by eliminating \(\alpha_i\) using (18d).

\[\rho_{ij}^{q} = \frac{(R_{ij}^{p} - \rho_{ij}^{p} e^{T} + X_{ij}^{q} \text{diag}(\gamma))\Sigma_{e}}{\|R_{ij}^{p} - \rho_{ij}^{p} e^{T} + X_{ij}^{q} \text{diag}(\gamma)\|^{2}} \quad i \in \mathbb{G} \quad (28d)\]
\[\rho_{ij}^{q} = \frac{(R_{ij}^{p} - \rho_{ij}^{p} e^{T} + X_{ij}^{q} \text{diag}(\gamma))\Sigma_{e}}{\|R_{ij}^{p} - \rho_{ij}^{p} e^{T} + X_{ij}^{q} \text{diag}(\gamma)\|^{2}} \quad i \in \mathbb{G} \quad (28e)\]

Specifically, metric \(V(\cdot)\) penalizes the variance of state variables and, thus, can be used to trade-off the overall system variance and the expected operating cost in the system as discussed in (17). We define metric \(V(\cdot)\) as:

\[V(t_{ij}^{p}, t_{ij}^{p}, t_{ij}^{f}) = \sum_{ij \in \mathbb{L}} \left(\Psi_{ij}^{p}(t_{ij}^{p})^{2} + \Psi_{ij}^{q}(t_{ij}^{q})^{2}\right),\]

where \(\Psi_{ij}^{p}, \Psi_{ij}^{q}, \Psi_{ij}^{f}, \Psi_{ij}^{e}\) are variance penalty factors in the units of \([\text{MW}]^{2}\), \([\text{MVAr}]^{2}\), \([\text{MW}]^{2}\), and \([\text{MVAr}]^{2}\), respectively. Note that active power standard deviation \(t_{ij}^{p}\) is already controlled by the generation cost and the constraints on \(\alpha_i\).

**Proposition 4.** Consider the VA-CC in (29). Let \(\lambda_{p}^{p}, \lambda_{q}^{q}\) be dual multipliers of the nodal active and reactive power balance at node \(i\) as in (18b). Further, let \(\chi\) be the dual multiplier of the balancing adequacy constraint in (18d). Then (i) \(\lambda_{p}^{p}\) and \(\lambda_{q}^{q}\) are given by (20)–(21) and (ii) \(\chi\) is given as:

\[\chi = \frac{1}{\sum_{i \in G} b_i(\delta_{p}^{2} + \delta_{q}^{2}) \sum_{i \in G} b_i(\delta_{p}^{2} + \delta_{q}^{2}) + \sum_{i \in G} b_i(y_{ij}^{p} + y_{ij}^{q} + y_{ij}^{f} + y_{ij}^{e})},\]

where:

\[y_{ij}^{p} = \sum_{j \in \mathbb{G}} \left[R_{ij}^{p}\right] \zeta_{ij}^{p} \frac{(R_{ij}^{p} + X_{ij}^{q} \text{diag}(\gamma))\Sigma_{e} - R_{ij}^{p} \alpha S^{2}}{\sigma_{q_{ij}}(\alpha, \gamma)}\]

\[y_{ij}^{q} = \sum_{j \in \mathbb{G}} \left[R_{ij}^{p}\right] \zeta_{ij}^{q} \frac{(R_{ij}^{p} + X_{ij}^{q} \text{diag}(\gamma))\Sigma_{e} - R_{ij}^{p} \alpha S^{2}}{\sigma_{q_{ij}}(\alpha, \gamma)}\]

\[y_{ij}^{2} = \sum_{j \in \mathbb{G}} \left[R_{ij}^{p}\right] \zeta_{ij}^{q} \frac{(R_{ij}^{p} + X_{ij}^{q} \text{diag}(\gamma))\Sigma_{e} - R_{ij}^{p} \alpha S^{2}}{\sigma_{q_{ij}}(\alpha, \gamma)}\]

\[\zeta_{ij}^{p} = z_{\rho}(\delta_{p}^{2} + \delta_{q}^{2}) - 2\sigma_{q_{ij}}(\alpha, \gamma)\Psi_{ij}^{p}\]

\[\zeta_{ij}^{q} = z_{\rho}(\mu_{p}^{2} + \mu_{q}^{2}) - 2\sigma_{q_{ij}}(\alpha, \gamma)\Psi_{ij}^{q}\]

\[\zeta_{ij}^{2} = z_{\rho}(\mu_{p}^{2} + \mu_{q}^{2}) - 2\sigma_{q_{ij}}(\alpha, \gamma)\Psi_{ij}^{2}\]

\[\omega = f^{p}, f^{q}\]
Proof. The first-order optimality conditions of (29) for \( p_{G,i} \), \( q_{G,i} \), \( \alpha_i \), \( f_{ij}^p \), \( f_{ij}^q \) and auxiliary variables are:

\[
(22a), (22b), (28c), (28d) \quad \text{and} \quad (28b)-(28d)
\]

\[
(\alpha_i) : z_{\epsilon_i} - \delta_i^p + \delta_i^q - \epsilon_i \rightarrow \lambda
\]

\[
+ \sum_{j \in \mathcal{G}} v_{ij}^p [R_{ij}^p] + \sum_{j \in \mathcal{L}} v_{ij}^q [R_{ij}^q] = (1 - \Psi) \alpha_i S^2
\]

\[i \in \mathcal{G}, \phi = f^p, f^q \quad (38a)
\]

\[
(t_i^q) : z_{\epsilon_i} - \delta_i^p + \delta_i^q - \epsilon_i = 2 t_i^q \Psi_i^q \quad i \in \mathcal{G} \quad (38b)
\]

\[
(t_i^q) : z_{\epsilon_i}(\mu_i^p + \mu_i^q) - \epsilon_i = 2 t_i^q \Psi_i^q \quad i \in \mathcal{N} \quad (38c)
\]

\[
(t_{ij}) : z_{ij}^{\epsilon_i} + z_{ij}^{\epsilon_j} \quad i \in \mathcal{L}, \phi = f^p, f^q \quad (38d)
\]

The result (i) follows directly from the proof of Proposition 1. The result (ii) follows from re-arranging (38a) using (18d) to eliminate \( \alpha_i \). Note that terms \( v_{ij}^p, v_{ij}^q, v_{ij}^p, v_{ij}^q \) are given by (28c), (28d) and (28b) and terms (35)–(37) follow from (38b)–(38d). Similarly to the proof of Proposition 3, \( t_i^{\epsilon_i} = \sigma_v(\alpha, \gamma) \), if \( \xi_i > 0 \) as per (18m), \( t_i^{\epsilon_i} = \sigma_{f,sp,\phi}(\alpha, \gamma) \), if \( \xi_i > 0 \) as per (18m). Also, increasing conservatism of the model increases system-wide balancing reserve price \( \lambda \) for both values of \( \epsilon \).

Relative to the results of Proposition 3 terms \( y_{ij}^p, y_{ij}^q, y_{ij}^{\epsilon_i} \) now include an inherent trade-off between the risk of limit violation and the absolute standard deviations weighted by penalty factors \( \Psi_i^p, \Psi_i^q, \Psi_i^{\epsilon_i}, \Psi_i^{f,\phi} \), see (35)–(37). Since dual multipliers \( \xi_i^{\epsilon_i}, \xi_i^{\epsilon_j}, \xi_i^p, \xi_i^q, \xi_i^{f,\phi} \) must be non-negative by definition, the scarcity rents of reactive power \( \delta_i^p, \delta_i^q \), voltage magnitude \( \mu_i^p, \mu_i^q \), active power flows \( \xi_i^p, \xi_i^q \), and reactive power flows \( \xi_i^{f,\phi} \) and risk parameters \( z_{\epsilon_i}, z_{\epsilon_j}, z_{\epsilon_j} \) set an upper bound to the standard deviations \( \sigma_{\epsilon_i}, \sigma_{\epsilon_j}, \sigma_{f,\phi,\epsilon_i}, \sigma_{f,\phi,\epsilon_j} \) weighted by the penalty factors.

V. CASE STUDY

We conduct numerical experiments using the modified 118-node IEEE test system from [7], which includes 11 wind farms with the total forecast power output of 1196 MW (≈ 28.2% of the total active power demand). As in [4, 7], the wind power forecast error is zero-mean with the standard deviation of \( \sigma_{p,w,i} = 0.125p_{w,i} \), \( \forall i \in \mathcal{U} \). In addition to the GEN-CC, EQV-CC and VA-CC, we solve a deterministic AC OPF (reference) case using the forecast renewable generation and \( \alpha_i = 0, \forall i \in \mathcal{G} \). All calculations have been performed for risk levels \( \epsilon = 0.1 \) and \( \epsilon = 0.01 \) assuming that \( \epsilon_p = \epsilon_q = \epsilon = \epsilon_f = 0.01 \). Additionally, the VA-CC has been computed for various values of \( \Psi = \{0.1, 1, 10, 100, 1000\} \) assuming that \( \Psi_i^p = \Psi_i^q = \Psi, \forall i \in \mathcal{N} \) and \( \Psi_i^{f,\phi} = \Psi_i^{f,\phi}, \forall i \in \mathcal{L} \). All models are implemented in Julia using JuMP [23] and the code and input data are reported in [24]. The linearization point (see Section II-E) has been obtained as described in [7] using the IPOPT solver, [25], and the chance-constrained models have been solved using the MOSEK solver, [26].

A. Cost and Price Analysis

Table I compares the results of the deterministic, GEN-CC, EQV-CC and VA-CC cases for different values of \( \epsilon \) and \( \Psi \). As expected, the objective value and expected generation cost increase as we introduce additional chance constraints and increase the value of \( \Psi \), thus internalizing the cost of re-dispatch to ensure larger security margins and lower variance of state variables. Similarly to the results in [17], which uses DC power flow assumptions, increasing variance penalty factor \( \Psi \) does not significantly raise the expected generation cost. This observation suggests that this reduction in state variable variances is achieved by adjustments to those variables which are not limited by binding constraints in the optimal solution. In other words, the variance of variables related to non-binding constraints can be controlled without significantly affecting the optimal values of other variables. Note that the variance of variables related to binding chance constraints is a priori controlled by the violation tolerance of these constraints.

Also, increasing conservatism of the model increases system-wide balancing reserve price \( \lambda \) for both values of \( \epsilon \). For example, in the GEN-CC, the value of \( \lambda \) is only driven by chance constraints on power output limits of generators, as per Proposition 2, while the EQV-CC and VA-CC introduce additional components (e.g. reactive power, voltage and flow variances) to price \( \lambda \) as per Propositions 3 and 4. Location-specific prices \( \lambda_i^p \) and \( \lambda_i^q \) for all network nodes are displayed in Fig. 1(a), where Figs. 1(b)–(c) map the relative difference between \( \lambda_i^p \) for the VA-CC case with \( \Psi = 100 \) and \( \epsilon = 0.01 \) and the deterministic case. At the majority of nodes, prices \( \lambda_i^p \) (indicated by the box-plots in Fig. 1) remain within 32–38 $/MWh. Note that unlike \( \lambda \), which significantly increases for more conservative models, prices for \( \lambda_i^p \) and \( \lambda_i^q \) do not vary as much as conservatism increases. This corresponds to our findings in Propositions 1, 4 which show that active and reactive power prices do not explicitly depend on the uncertainty and risk parameters. However, at some nodes, prices \( \lambda_i^p \) and \( \lambda_i^q \) in the GEN-CC and VA-CC cases exhibit larger deviations, e.g. see \( \lambda_i^p \) at nodes 20 and 23, which are also in proximity of wind farms, as shown in Fig. 1(a). A resulting high flow variance on the line between nodes 19 and 23 causes price differentiation at nodes 19, 20, 21 and 23, 24, 25.

B. Analysis of Variance of State Variables

Table I shows how the aggregated variance of state variables \( \sum_i \sigma_{x_i}^2, \sum_i \sigma_{\epsilon_i}^2, \sum_i \sigma_{\epsilon_j}^2, \sum_i \sigma_{\epsilon_j}^2 \) change relative to the EQV-CC case as penalty \( \Psi \) increases. Even if \( \Psi \) is set to a small value, the variance of state variables reduce significantly, without a large increase in the objective function, expected generation cost, and prices \( \lambda_i^p \) and \( \lambda_i^q \). Furthermore, as the value of \( \epsilon \) increases, the relative reduction in variances of all state variables slightly reduces. The effect of variance penalty \( \Psi \) on prices is two-fold. First, it does not affect prices \( \lambda_i^p \) and \( \lambda_i^q \) relative to the EQV-CC case. Second, system-wide balancing price \( \lambda \) which internalizes the variance penalties as per Proposition 1 increases with penalty \( \Psi \).
| Risk Level | Model | Det | GEN-CC | EQV-CC | VA-CC (Ψ = ψ^p = ψ^q = ψ^v = ψ^f, ∀i, ∀ij) |
|-----------|-------|-----|--------|--------|------------------------------------------------|
| Ψ = 0.1  | Objective [$] | 91103.22 | 91107.33 | 92237.67 | 92237.74 | 92238.30 | 92243.86 | 92296.91 | 92764.30 |
|           | Exp. Gen. Cost [$] | 91103.22 | 91107.33 | 92237.67 | 92237.74 | 92238.30 | 92243.86 | 92296.91 | 92764.30 |
|           | Δ rel. to EQV-CC | 100.000% | 100.000% | 100.000% | 100.000% | 100.000% | 100.000% | 100.000% | 100.000% |
|           | χ [$] | 8.72 | 28.10 | 28.11 | 28.23 | 29.40 | 40.35 | 125.54 |
|           | ∆∑_i σ^2_\psi_{G,i} [%] | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% |
|           | ∆∑_i σ^2_\psi_{V,i} [%] | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% |
|           | ∆∑_ij σ^2_\psi_{f_{pq}} [%] | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% |
|           | ∆∑_ij σ^2_\psi_{f_{iq}} [%] | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% |
| Ψ = 0.01 | Objective [$] | 91103.22 | 91107.71 | 93744.95 | 93745.01 | 93745.57 | 93751.17 | 93805.19 | 94281.35 |
|           | Exp. Gen. Cost [$] | 91103.22 | 91107.71 | 93744.95 | 93744.95 | 93744.95 | 93744.95 | 93747.04 | 93772.27 |
|           | Δ rel. to EQV-CC | 100.000% | 100.000% | 100.000% | 100.000% | 100.000% | 100.000% | 100.000% | 100.000% |
|           | χ [$] | 9.74 | 25.93 | 26.03 | 26.95 | 37.47 | 126.42 |
|           | ∆∑_i σ^2_\psi_{G,i} [%] | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% |
|           | ∆∑_i σ^2_\psi_{V,i} [%] | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% |
|           | ∆∑_ij σ^2_\psi_{f_{pq}} [%] | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% |
|           | ∆∑_ij σ^2_\psi_{f_{iq}} [%] | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% |

Fig. 1: (a) Active and reactive power prices $\lambda^p_i$ and $\lambda^q_i$ for the deterministic, GEN-CC and EQV-CC cases and VA-CC with $\Psi = 100$ for risk level $\epsilon = 0.01$. The orange line within the blue box represents the median value, the left and right edges of the box represent the first and third quartiles and the outliers are plotted as circles. (b) Difference of active power prices $\lambda^p_i$ in the VA-CC ($\Psi = 100$) relative to the deterministic case (in %). (c) Magnification of the area indicated by the dotted rectangle in (b).

VI. CONCLUSION

This paper described an approach to internalize RES stochasticity and risk parameters in electricity prices. Using SOC duality, these risk- and variance-aware prices are derived from a chance-constrained AC-OPF and are itemized in terms of active and reactive power, voltage support and power flow components. We proved that active and reactive power prices do not explicitly depend on uncertainty and risk parameters, while expressions for balancing reserve prices explicitly include these parameters. Further, introducing variance penalties on the system state variables has been shown to internalize the trade-off between variance, risk and system cost at a modest increase in the expected operating cost. The results have been demonstrated and analyzed on the modified IEEE 118-node testbed. Future work includes extensions of the proposed market-clearing model to account for risk-averse strategies of market participants, enable risk trading instruments using our preliminary work in [27], and to account for multi-period trading horizons.
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