Peculiar Velocities of Galaxy Clusters

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ABSTRACT
We investigate the peculiar velocities predicted for galaxy clusters by theories in the cold dark matter family. A widely used hypothesis identifies rich clusters with high peaks of a suitably smoothed version of the linear density fluctuation field. Their peculiar velocities are then obtained by extrapolating the similarly smoothed linear peculiar velocities at the positions of these peaks. We test these ideas using large high resolution N–body simulations carried out within the Virgo supercomputing consortium. We find that at early times the barycentre of the material which ends up in a rich cluster is generally very close to a high peak of the initial density field. Furthermore the mean peculiar velocity of this material agrees well with the linear value at the peak. The late-time growth of peculiar velocities is, however, systematically underestimated by linear theory. At the time clusters are identified we find their rms peculiar velocity to be about 40% larger than predicted. Nonlinear effects are particularly important in superclusters. These systematics must be borne in mind when using cluster peculiar velocities to estimate the parameter combination \( \sigma_\delta \Omega^{0.6} \).

Key words: cosmology: theory – large-scale structure of Universe, cosmology: theory – dark matter, galaxies: clusters

1 INTRODUCTION
The motions of galaxy clusters are thought to result from gravitational forces acting over the very large scales on which superclusters are assembled. The rms deviations from uniformity on such scales appear to be small, and so may be adequately described by the linear theory of fluctuation growth. For a linear density field of given power spectrum the rms peculiar velocity is proportional to \( \sigma_\delta \Omega^{0.6} \) where \( \Omega \) is the cosmic density parameter and \( \sigma_\delta \), the rms mass fluctuation in a sphere of radius \( 8 h^{-1} \) Mpc, is a conventional measure of the amplitude of fluctuations (e.g. Peebles 1993). (As usual the Hubble constant is expressed as \( H_0 = 100h \) km/sec/Mpc.) Distance indicators such as the Tully-Fisher or \( D_L-\sigma \) relations allow the peculiar velocities of clusters to be measured, thus providing a direct estimate of this parameter combination (see, for example, Strauss & Willick 1996).

Essentially the same parameter combination can also be estimated from the abundance of galaxy clusters (e.g. White et al. 1993) and a comparison of the two estimates could in principle provide a check on the shape of the assumed power spectrum and on the assumption that the initial density field had gaussian statistics. In practice this is difficult because of the uncertainties in relating observed cluster samples to the objects for which quantities are calculated in linear theory or measured from N-body simulations. The standard linear model was introduced by Bardeen et al. (1986; hereafter BBKS). It assumes that clusters can be identified with “sufficiently” high peaks of the linear density field after convolution with a “suitable” smoothing kernel. The peculiar velocity of a cluster is identified with the linear peculiar velocity of the corresponding peak extrapolated to the present day. In the present paper we study the limitations both of this model and of direct N-body simulations by comparing their predictions for clusters on a case by case basis.

In the next section we summarize both the linear predictions for the growth of peculiar velocities and the BBKS formulae for the values expected at peaks of the smoothed density field. Section 3 then presents our set of N-body simulations and outlines our procedures for identifying peaks in the initial conditions and clusters at \( z = 0 \). Section 4 begins by studying how well peaks correspond to the initial barycentres of clusters; we then show that the smoothed
linear velocity at a peak agrees well with the mean linear velocity of its cluster; finally we show that the growth of cluster peculiar velocities is systematically stronger at late times than linear theory predicts. A final section presents a brief discussion of these results.

2 LINEAR PREDICTIONS FOR THE PECULIAR VELOCITIES OF PEAKS

2.1 The Growth of Peculiar Velocities

According to the linear theory of gravitational instability in a dust universe (Peebles 1993), the peculiar velocity of every mass element grows with cosmic scale factor $a$ as

$$v \propto a D ,$$

where $a(t)$ is obtained from the Friedman equation

$$\left( \frac{\dot{a}}{a} \right)^2 = \Omega_0 a^{-3} + (1 - \Omega_0 - \Lambda_0) a^{-2} + \Lambda_0 ,$$

and $D(t)$ is the growth factor for linear density perturbations, $\delta(x, t) = D(t) \delta_0 (x)$, $\Omega_0$ and $\Lambda_0$ are the density parameter and the cosmological constant at $z = 0$, respectively, and we define $a = 1$ at this time. A number of accurate approximate forms are known for the relations between $D$ and $a$ and can be used to cast the scaling of eq. (1) into a more convenient form. We write

$$\frac{D}{D_0} = \frac{\Omega_0 a^{-3} + (1 - \Omega_0 - \Lambda_0) a^{-2} + \Lambda_0}{\Omega_0 + \Lambda_0} ,$$

and substitute for $d\Omega_0/dt$ from the Friedman equation [2]. Lahav et al. (1991) give an approximation for $dD/da$ in the combination

$$f(a) \equiv \frac{dD}{da} D \approx \left( \frac{\Omega_0 a^{-3}}{\Omega_0 a^{-3} + (1 - \Omega_0 - \Lambda_0) a^{-2} + \Lambda_0} \right)^{0.6} .$$

For $a = 1$ this gives the standard factor $f \approx \Omega_0^{0.6}$ which appears when predicting the peculiar velocities produced by a given overdensity field. Carrol et al. (1992) used this result to derive an approximation for $D(a)$ itself,

$$D \approx a g(a) ,$$

where

$$g(a) = \frac{5}{2} \frac{\Omega(a)}{\Omega_0^{4/7}(a) - \Lambda(a)} + \left( 1 + \frac{\Omega(a)}{\Omega_0^{4/7}(a)} \right) \left( 1 + \frac{\Lambda(a)}{\Omega_0^{4/7}(a)} \right) ,$$

with

$$\Omega(a) = \frac{\Omega_0}{a + \Omega_0 (1 - a) + \Lambda(a^3 - a)} ,$$

$$\Lambda(a) = \frac{\Lambda_0}{a + \Omega_0 (1 - a) + \Lambda(a^3 - a)} .$$

Combining these equations, we obtain an explicit approximation for the growth of peculiar velocities,

$$v \propto f(a) g(a) a^2 \sqrt{\Omega_0 a^{-3} + (1 - \Omega_0 - \Lambda_0) a^{-2} + \Lambda_0} .$$

For the simple Einstein-de Sitter case where $\Omega_0 = 1$ and $\Lambda = 0$, these formulae reduce to the exact results $D = a \propto t^{2/3}$ and $v \propto \sqrt{a}$.

Recently, Eisenstein (1997) has shown that the exact solutions for $D$ and $f(a)$ can be given explicitly in terms of elliptic integrals. He also shows that the above approximations always have fractional errors better than 2% if $\Omega > 0.1$. We therefore work with the simpler approximate formulae in the present paper.

2.2 The Velocities of Peaks

The idea that the statistical properties of nonlinear objects like galaxy clusters can be inferred from the initial linear density field was developed in considerable detail in the monumental paper of Bardeen et al. (1986; BBKS). If the initial fluctuations are assumed to be a gaussian random field, they are specified completely by their power spectrum, $P(k)$. Similarly, any smoothed version of this initial field is specified completely by its own power spectrum, $P(k)W^2(kR)$, where $W(kR)$ is the Fourier transform of the spherical smoothing kernel and $R$ is a measure of its characteristic radius. In particular, BBKS showed how the abundance and rms peculiar velocity of peaks of given height can be expressed in terms of integrals over $P(k)W^2(kR)$. The difficulty in connecting this model with real clusters lies in the ambiguity in deciding what smoothing kernel, characteristic scale, and peak height are appropriate. Typically the smoothing kernel is taken to be a gaussian or a top-hat, $R$ is chosen so that the kernel contains a mass similar to the minimum mass of the cluster sample, and the height is assumed sufficient for a spherical perturbation to collapse by $z = 0$.

The smoothed initial peculiar velocity field is isotropic and gaussian with a three-dimensional dispersion given by

$$\sigma_\ell (R) = H \Omega^{0.6} \sigma_{-.}(R) ,$$

where, in the notation of BBKS, $\sigma_j$ is defined for any integer $j$ by

$$\sigma_j^2 (R) = \frac{1}{2\pi^2} \int P(k) W^2(kR) k^{2j+2} dk .$$

The rms peculiar velocity at peaks of the smoothed density field differs systematically from $\sigma_\ell$; BBKS show that it is given by

$$\sigma_p (R) = \sigma_\ell (R) \sqrt{1 - \sigma_0^2/\sigma_p^2} .$$

Note that this expression does not depend on the height of the peaks. As shown in BBKS, the velocities of peaks are statistically independent of their height.

Throughout this paper we will approximate the power spectra of CDM models by the parametric expression of Bond & Efstathiou (1984),

$$P(k, \Gamma) = \frac{Ak}{\{1 + [ak/\Gamma + (bk/\Gamma)^{3/2} + (ck/\Gamma)^{3/2}]^{\nu}\}^{2/\nu} - \Gamma} ,$$

where $a = (6.4/\Gamma) h^{-1}$ Mpc, $b = (3.0/\Gamma) h^{-1}$ Mpc, $c = (1.7/\Gamma) h^{-1}$ Mpc, $\nu = 1.13$, and the shape parameter $\Gamma$ is given for the models discussed below by

$$\Gamma = \left\{ \begin{array}{ll}
\Omega_0 h / 0.861 + 3.8 (m_{10}^2 \tau_2) \{3/2\}^{1/2} & \text{for } \tau_{CDM}, \\
\Omega_0 h & \text{otherwise.}
\end{array} \right.$$
power spectrum are actually better fit by slightly smaller values of $\Gamma$ than we assume (Sugiyama 1995).

The normalisation constant in equation (13) can be related to the conventional normalisation $\sigma_8$ by noting that

$$\sigma_8 \equiv \sigma_0(8h^{-1}{\text{Mpc}})$$

and using equation (10) with a top-hat window function, $W_{\text{TH}}(x) = 3(x\sin x - \cos x)/x^3$. This corresponds to the linear fluctuation amplitude extrapolated to $z = 0$ and can be matched to observation by fitting either to the cosmic microwave background fluctuations measured by COBE or to the observed abundance of rich galaxy clusters. The models of this paper are normalised using the second method (c.f. Eke et al. 1996), as reflected by the $\sigma_8$ values given in Table 1 together with the other parameters defining the models.

In the following linear density fields are smoothed either with a top-hat or with a gaussian. In the latter case the window function is $W_G(x) = \exp(-x^2/2)$. It is unclear for either filter how $R$ should be chosen in order to optimize the correspondence between peaks and clusters. We follow previous practice in assuming that cluster samples contain all objects with mass exceeding some threshold $M_{\text{min}}$, and then choosing $R$ so that the filter contains $M_{\text{min}}$. Hence $M_{\text{min}} = 4\pi p R^3/3$ in the top-hat case and $M_{\text{min}} = (2\pi)^{3/2} p R^3$ in the gaussian case. The simulations analysed here have $\Omega_0 = 0.3$ or 1.0, and we will isolate cluster samples limited at $M_{\text{min}} = 3.5 \times 10^{14} h^{-1}{\text{M}_\odot}$, the value appropriate for Abell clusters of richness one and greater (e.g. White et al. 1993). A detailed discussion of filtering schemes can be found in Monaco (1998) and references therein.

Table 2 gives characteristic filter radii $R$ and values of $\sigma_v$ and $\sigma_p$ from equations (11) and (13) for both smoothings and for all the cosmological models we consider in this paper; the velocity dispersions are extrapolated to the linear values predicted at $z = 0$. The difference between $\sigma_v$ and $\sigma_p$ has often been ignored in the literature when predicting the peculiar velocities of galaxy clusters (e.g. Croft & Efstathiou 1994; Bahcall & Oh 1996; Borgani et al. 1997); for our models the two differ by about 15%. Notice also that with our choice of filter radii, gaussian smoothing predicts $rms$ peculiar velocities about 10% smaller than top-hat smoothing.

### Table 1. The Virgo models

| Model   | $\Omega$ | $\Lambda$ | $h$ | $\sigma_8$ | $\Gamma$ | $z_{\text{start}}$ |
|---------|----------|-----------|-----|------------|---------|-------------------|
| OCDM    | 0.3      | 0.0       | 0.7 | 0.5        | 0.51    | 0.85              |
| ACDM    | 0.3      | 0.7       | 0.7 | 0.90       | 0.21    | 0.90              |
| SCDM    | 1.0      | 0.0       | 0.5 | 0.51       | 0.50    | 0.90              |
| $r$CDM  | 1.0      | 0.0       | 0.5 | 0.51       | 0.21    | 0.90              |

are completed using a task farm approach. This T3D version currently includes an SPH treatment of gas dynamics, but this was not used for the simulations of this paper.

A second version of HYDRA, based on CRAY’s shared memory and message passing architecture, has been written by MacFarland et al. (1997). This can run on CRAY T3E’s but does not currently include refinement placing.

The simulations used here were run on the Cray T3D and T3E supercomputers at the computer center of the Max Planck Society in Garching and at the Edinburgh Parallel Computing Centre.

### 3.2 The Simulation Set

A set of four matched N-body simulations of CDM universes was completed in early 1997. Each follows the evolution of structure within a cubic region $240 h^{-1}{\text{Mpc}}$ on a side using $256^3$ equal mass particles and a gravitational softening of $30 h^{-1}{\text{kpc}}$. The choices of cosmological parameters correspond to standard CDM (SCDM), to an Einstein-de Sitter model with an additional relativistic component ($r$CDM), to an open CDM model (OCDM), and to a flat low density model with a cosmological constant (ACDM). A list of the parameters defining these models is given in Table 1.

In all models the initial fluctuation amplitude, and so the value of $\sigma_8$, was set by requiring that the models should reproduce the observed abundance of rich clusters. Further details of this choice and of other aspects of the simulations can be found in Jenkins et al. (1998). Note that each Fourier component of the initial fluctuation field had the same phase in each of these four simulations. As a result there is an almost perfect correspondence between the clusters in the four models.

Because of their finite volume, these simulations contain no power at wavelengths longer than $240 h^{-1}{\text{Mpc}}$. Furthermore, Fourier space is sampled quite coarsely on the largest scales for which they do contain power, and so realisation to realisation fluctuations on these scales can be significant. The size of the effects can be judged from Table 2 where we list the values of $\sigma_v$ and $\sigma_p$ obtained for each model when the theoretical power spectrum is replaced in equations (11) and (13) by the initial power spectrum of the model itself. These are systematically smaller than the values found before. The difference is primarily a reflection of the loss of large-scale power.

### 3.3 The Selection of Peaks

We identify peaks in the initial conditions of the simulations by binning up the initial particle distribution on a $128^3$ mesh using a cloud-in-cell (CIC) assignment and then smoothing with a gaussian or a top-hat with characteristic scale $R$ corresponding to $M_{\text{min}} = 3.5 \times 10^{14} h^{-1}{\text{M}_\odot}$. A peak

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Table 2. For each of the models, the following quantities are given: the radius $R$ (second and fifth column) of the filter used in eq. (12); the three-dimensional velocity dispersions $\sigma_v$ and $\sigma_p$ (third, fourth, sixth, and seventh column) obtained using eqs (11) and (13) with the given filter radii; the three-dimensional velocity dispersions $\sigma_v$ and $\sigma_p$ (eighth, ninth, eleventh, and twelfth column) obtained using eqs (11) and (13) with the given filter radii and the power spectra of the simulations themselves; the $\text{rms}$ linear overdensity $\Delta$ (tenth and thirteenth column) smoothed with the given filter radii and extrapolated to $z = 0$; the number of clusters $N_{\text{Cl}}$ (fourteenth column) found in the simulations at $z = 0$; the three-dimensional velocity dispersions of peaks (fifteenth and sixteenth column) in the initial conditions of the simulations using the given filters; the three-dimensional linear velocity dispersions of clusters extrapolated to $z = 0$; and the three-dimensional measured velocity dispersion of clusters at $z = 0$. The radii are given in Mpc/$h$, the velocity dispersions in km/sec. Top Hat and Gaussian filters are abbreviated as TH and G, respectively.

| Model | $R$ | $\sigma_v$ | $\sigma_p$ | $R$ | $\sigma_v$ | $\sigma_p$ | $\Delta$ | $\sigma_v$ | $\sigma_p$ | $\Delta$ | $N_{\text{Cl}}$ | $\sigma_{\text{Peak}}$ | $\sigma_{\text{Peak}}$ | $\sigma_{\text{lin}}$ | $\sigma_{z=0}$ |
|-------|----|-----------|-----------|----|-----------|-----------|--------|-----------|-----------|--------|---------------|----------------|--------------------|----------------|-------------|
| OCDM  | 10.3 | 390 | 349 | 6.6 | 366 | 315 | 351 | 300 | 0.94 | 321 | 258 | 0.96 | 62 | 253 | 266 | 280 | 407 |
| $\Lambda$CDM | 10.3 | 413 | 370 | 6.6 | 387 | 334 | 371 | 318 | 0.98 | 340 | 272 | 1.03 | 69 | 296 | 323 | 300 | 439 |
| SCDM  | 6.9 | 381 | 334 | 4.4 | 349 | 290 | 375 | 325 | 0.58 | 342 | 278 | 0.60 | 92 | 308 | 318 | 307 | 425 |
| $\tau$CDM | 6.9 | 509 | 464 | 4.4 | 485 | 430 | 464 | 412 | 0.57 | 437 | 371 | 0.58 | 70 | 392 | 399 | 398 | 535 |

Figure 1. The mass of the clusters in our simulations against the height of the corresponding peaks in the initial conditions, once these are smoothed with a top-hat with the characteristic radius listed in Table 2. All clusters with mass greater than $3.5 \times 10^{14} h^{-1} M_\odot$ and all peaks with height greater than $\nu = 1.5$ are shown. There are 351, 239, 84, and 83 unmatched peaks in the SCDM, $\tau$CDM, $\Lambda$CDM, and OCDM model, respectively.

is then taken to be any grid point at which the smoothed density is greater than that of its 26 nearest neighbours. The dimensionless height of a peak, $\nu$, is defined by dividing its overdensity by the $\text{rms}$ overdensity, $\Delta$, which we list in Table 2. Again, within the matched set there is a close correspondence between the peaks found in the four models. In addition, the peaks found with gaussian smoothing correspond closely to those found with top-hat smoothing.

Particle peculiar velocities are binned up and smoothed in an identical way and the peculiar velocity of a peak is taken to be the value at the corresponding grid point. In Table 2 we list the $\text{rms}$ peculiar velocity of the peaks found in each model. Again this is scaled up to the value expected at $z = 0$ according to linear theory. It differs slightly from the value predicted by inserting the power spectrum of the simulation directly into equation (13) because there are realisation to realisation fluctuations depending on the phases of the Fourier components. As it should, the $\text{rms}$ peculiar velocity averaged over all grid points agrees very well with
the value found by putting the simulation power spectrum into equation (11).

3.4 The Selection of Clusters

We define clusters in our simulations in the same way as White et al. (1993). High-density regions at \( z = 0 \) are located using a friends-of-friends group finder with a small linking length \((b=0.05)\), and their barycentres are considered as candidate cluster centres. Any candidate centre for which the mass within 1.5 \( h^{-1} \) Mpc exceeds \( M_{\text{min}} \) is identified as a candidate cluster. The final cluster list is obtained by deleting the lower mass candidate in all pairs separated by less than 1.5 \( h^{-1} \) Mpc. In the following we will normally consider only clusters more massive than \( M_{\text{min}} = 3.5 \times 10^{14} h^{-1} M_\odot \).

The number of clusters found in each simulation is listed in Table 2. As already noted, the individual clusters in the different simulations of the matched set correspond closely. Despite the normalisation to cluster abundance it appears as though the SCDM model has significantly more clusters than the others. This is a reflection of its steeper power spectrum together with the value of \( M_{\text{min}} \) we have chosen. For \( M_{\text{min}} = 5.5 \times 10^{14} h^{-1} M_\odot \) all the models have about 20 clusters.

We define the peculiar velocity of each cluster at \( z = 0 \) to be the mean peculiar velocity of all the particles within the 1.5 \( h^{-1} \) Mpc sphere. The peculiar velocity of the cluster at earlier times is taken to be the mean peculiar velocity of these particles. Consistent with this, we define the position of the cluster at each time to be the barycentre of this set of particles. At \( z = 0 \) this is very close to, but not identical with the cluster centre as defined above. We give the \( \sigma_8 \) values of the initial (linear) and final (\( z = 0 \)) peculiar velocities of the clusters in each of our models in Table 2. The initial values have been scaled up to the linear values predicted at \( z = 0 \). It is clear that these substantially underestimate the actual values, a result we discuss in more detail below. We note that the present-day properties of clusters in these simulations are considered in much more detail in Thomas et al. (1998).

4 COMPARISON OF THE PEAK MODEL WITH SIMULATIONS

4.1 The Cluster-Peak Connection

The extent to which dark haloes can be associated with peaks of the smoothed initial density field is somewhat controversial. Frenk et al. (1988) concluded that, for appropriate choices of filter scale and peak height, the correspondence is good, whereas Katz et al. (1993) claimed that “there are many groups of high mass that are not associated with any peak”. The result of correlating the peaks in the initial conditions of our simulations with the initial positions of clusters is illustrated in Fig. 1. We consider a peak and a cluster to be associated if their separation is less than 4 \( h^{-1} \) Mpc (comoving). We find that the barycentres of 70% and 80% of the clusters with masses exceeding \( 3.5 \times 10^{14} h^{-1} M_\odot \) lie within of a peak with \( \nu > 1.5 \) for the low and high \( \Omega \) models, respectively.

Fig. 1 shows that there is, as expected, a correlation between the height of a peak and the mass of the corresponding cluster. In addition, combining the peak heights with the \( \Delta \) values from Table 2, we see that the extrapolated linear overdensities of the peaks at redshift zero are similar but somewhat larger than the threshold value of 1.69 used in the standard Press-Schechter approach to analysing structure formation.

4.2 Linear Peculiar Velocities of Peaks and Clusters

Given the excellent correspondance between peaks of the smoothed linear density field and the initial positions of clusters, it is natural to compare the smoothed peculiar velocity at a peak with the mean initial peculiar velocity of its associated cluster. We show such a comparison in Fig. 2, again based on top-hat smoothing of both position and peculiar velocity fields using the characteristic radii listed in Table 2. All velocities are scaled up to the expected value at \( z = 0 \) according to linear theory. The correlation is clearly excellent in all cases, and is similar if gaussian rather than top-hat smoothing is used. The \( \sigma_8 \) difference in peculiar velocity between a cluster and its associated peak is 16%, 16%, 23%, and 17% of the corresponding \( \sigma_8 \) value listed in Table 2 for the OCDM, \( \Lambda \)CDM, SCDM and \( \tau \)CDM simulations respectively. The somewhat larger percentage for the SCDM model is probably a consequence of the greater influence of small-scale power in this case.

4.3 The Growth of Cluster Peculiar Velocities

If cluster peculiar velocities grew according to linear theory the scaled initial velocities discussed in the last section and plotted in Fig. 2 would correspond to the actual velocities of the clusters at \( z = 0 \). In Fig. 3 we show scatter diagrams in which these two velocities are plotted against each other. It is evident that in fact the agreement is quite poor and that there is a systematic trend for the true cluster velocity to be larger than the extrapolated linear value. This is reflected in the substantial difference between the \( \sigma_8 \) values of these two quantities listed in Table 2. It is presumably a consequence of nonlinear gravitational forces accelerating the clusters.

Some confirmation of this is provided by Fig. 4 where we plot the peculiar velocity in units of its initial value for five clusters from each of our cosmologies. At early times the peculiar velocities all grow as expected from linear theory (indicated in the figures by a dotted line) but at later times the behaviour is more erratic and most clusters finish with larger velocities than predicted.

Further evidence that late–time nonlinear effects are responsible for this discrepancy comes from Fig. 3. In this plot all clusters that have a neighbour within 10 \( h^{-1} \)Mpc are indicated with a diamond while more isolated clusters are indicated by a cross. It is evident that deviations from linear theory are substantially larger for the “supercluster” objects than for the rest. These objects also have systematically larger peculiar velocities at \( z = 0 \). Their \( \sigma_8 \) peculiar velocity is around 20 to 30% larger than that of the sample as a whole.

For the \( \tau \)CDM model, we have run a second realization
Figure 2. The initial peculiar velocities of clusters in each of our four cosmogonies are compared to the linear peculiar velocities of their associated peaks. The linear peculiar velocity field was smoothed with a top–hat in the same way as the density field in order to obtain the peak peculiar velocities.

of the power spectrum. We have extracted a cluster sample in the same fashion as described above. The \( \sigma_{z=0} \) peculiar velocity of the clusters at \( z = 0 \) is \( \sigma_{z=0} = 511 \) km/sec. The extrapolated \( \sigma_{z=0} \) linear peculiar velocity is \( \sigma_{z=0} = 394 \) km/sec. These numbers are very close to the values obtained for the first realization. Although two simulations are not a good statistical sample, we conclude that there is no realization dependence of the mis–match between the extrapolated linear and the actual peculiar velocities of galaxy clusters.

It might be thought that this anomalous acceleration of clusters at late times was a consequence of the relatively small radius, \( 1.5 h^{-1} \) Mpc, which we use to define our clusters. Material could, perhaps be ejected asymmetrically from this region during the merging events by which clusters form. We have searched for such effects by redefining clusters to be all the material contained within a radius of 3 or 5 \( h^{-1} \) Mpc and then repeating the analysis for the same set of objects as before. In most cases this turned out to make very little difference to either the initial or the final velocities measured, and it did nothing to reduce the discrepancy between them. The relevant nonlinear effects are acting on significantly larger scales. We repeated this procedure going as far out as 25 \( h^{-1} \) Mpc from the cluster center. At a radius of \( 10 h^{-1} \) Mpc, the difference between the \( \sigma_{rms} \) peculiar velocity and the extrapolated \( \sigma_{rms} \) linear peculiar velocity is only 10%. By a radius of 20 \( h^{-1} \) Mpc, the numbers have finally converged.

The discrepancy between the \( \sigma_{rms} \) peculiar velocity of clusters and their extrapolated \( \sigma_{rms} \) linear peculiar velocity is independent of any smoothing of the density field. With our choice of smoothing filter, the linear peculiar velocities of our clusters match those of their associated peaks as well as the \( \sigma_{rms} \) value predicted by linear theory when the simulated realization of the power spectrum and the proper expression for the peculiar velocities (eq. 13) is used. Previous work (e.g. Borgani et al. 1997) has tried to match N–body data with linear theory by tuning the filter scale. Our results undermine the physical basis for such procedure.

5 CONCLUSIONS

We have investigated the peculiar velocities predicted for galaxy clusters by theories in the Cold Dark Matter family. A widely used hypothesis identifies rich clusters with high peaks of a smoothed version of the linear density fluctuation field. Their peculiar velocities are then obtained by extrapolating the similarly smoothed linear peculiar velocities at the positions of these peaks. We have tested this using a set of four large high–resolution N–body simulations. We identify galaxy clusters at \( z = 0 \) and then trace the particles they consist of back to earlier times. In the initial density field, the barycenters of 70% and 80% of the clusters with masses exceeding \( 3.5 \times 10^{14} h^{-1} M_\odot \) lie within \( 4 h^{-1} \) Mpc (comoving) of a peak with \( \nu > 1.5 \) for the low and high \( \Omega \) models, respectively. Furthermore, the mean linear peculiar velocity of the material which forms a cluster at \( z = 0 \) agrees well with the value at that peak.

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Figure 3. The initial peculiar velocities of clusters in each of our four cosmogonies, scaled up to $z = 0$ using linear theory, are compared to their actual peculiar velocities at $z = 0$. Diamonds denote clusters which have a neighbour within $10\ h^{-1}\text{Mpc}$ while crosses denote more isolated clusters.

However, the late-time growth of peculiar velocities is systematically underestimated by linear theory. At the time clusters are identified, i.e. at $z = 0$, we find that the \textit{rms} peculiar velocity is about 40\% larger than predicted. Nonlinear effects are particularly important in superclusters; the \textit{rms} values for clusters which are members of superclusters are about 20\% to 30\% larger than those for isolated clusters.

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Figure 4. The evolution with expansion factor $a$ of the ratio $|\vec{v}(a)|/|\vec{v}_0|$ for five clusters from each of our four cosmogonies (solid lines) is compared with the evolution predicted by linear theory (dotted line). In some of the cases, merging leads to abrupt changes in this ratio – the most impressive case can be seen for one of the SCDM clusters.

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