Abstract
Recent work has shown that expander-based data center topologies are robust and can yield superior performance over Clos topologies. However, to achieve these benefits, previous proposals use routing and transport schemes that impede quick industry adoption. In this paper, we examine if expanders can be effective for the technology and environments practical in today's data centers, including use of traditional protocols, at both small and large scale, while complying with common practices such as over-subscription. We study bandwidth, latency and burst tolerance of topologies, highlighting pitfalls of previous topology comparisons. We consider several other metrics of interest: packet loss during failures, queue occupancy and topology degradation. Our experiments show that expanders can realize 3× more throughput than an equivalent fat tree, and 1.5× more throughput than an equivalent leaf spine topology, for a wide range of scenarios, with only traditional protocols. We observe that expanders achieve lower flow completion times, are more resilient to bursty load conditions like incast and outcast and degrade more gracefully with increasing load. Our results are based on extensive simulations and experiments on a hardware testbed with realistic topologies and real traffic patterns.

1 Introduction
Modern data centers (DCs) need to be designed for high performance and for high burst tolerance. To achieve the same, hyperscale DC architectures are based on Clos networks or Fat-trees [10] that provide full bandwidth (modulo oversubscription) e.g. Google’s Jupiter [41], Facebook’s DC fabric [5] and VL2 [23]. Small and medium DCs, where the 3-tier Clos topology is overkill, are commonly based on two-tiered leaf-spine designs. Recently, alternative topologies based on expander graphs[1] such as Jellyfish [43], Xpander [48], Slimfly [13] and LongHop [47], have been proposed for high performance.

To extract maximum benefits from expander networks, previous proposals have relied on non-traditional protocols. Multipath TCP (MPTCP) [51] and k-shortest path routing [53] was used in Jellyfish [43] and Xpander [48]. Kassing et. al. [34] demonstrated benefits of expanders using a hybrid of ECMP and Valiant load balancing when combined with flowlet switching [43, 33]. While all the above schemes are effective, they are not very widely used in today’s data centers and require additional mechanisms. Specifically, K-shortest path routing is feasible with today’s hardware but requires both control plane and data plane modifications; MPTCP requires operating system support and configuration which is often out of the control of the data center operator; and flowlet switching depends on flow size detection and dynamic switching of paths. We argue that dependence on these protocols impedes quick industry adoption.

In this paper, we examine if expander networks retain their performance advantages in environments practical for today’s data centers. Towards this end, we consider several factors. Traffic engineering should ideally not require host transport modifications. Routing should ideally be based on simple oblivious[2] schemes using commonly available mechanisms like shortest path routing, ECMP, segment routing, and MPLS. And finally, whereas (to the best of our knowledge) all past work on expander DCs has focused on comparing with larger three-tiered Clos networks, it is important to understand whether these topology designs can be applied to the much more numerous leaf-spine networks of small- and medium-scale DCs, with realistic oversubscription ratios.

To analyze performance for a wide range of cases, we introduce the C – S model as a synthetic workload (Section 5.3). The C – S model captures several frequently occurring application patterns in distributed systems besides a range of skewed and uniform patterns. We consider throughput and flow completion times as the main performance met-

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1Expanders are an extensively studied class of graphs that are in a sense maximally or near-maximally well-connected.

2Oblivious routing refers to path selection independent of the current traffic pattern.
rics. We test burst tolerance for different types of incast and outcast patterns (modeled as specific points in the $C - S$ model) and present experiments with a publicly available Facebook DC traffic trace [3]. We also present experiments on a physical testbed by emulating modestly sized topologies (up to 48 servers). We study several other metrics of interest-queue occupancy, traffic loss due to failures, topology degradation with increasing load and fraction of long distance links. We base our experiments on shortest path routing and TCP in the transport layer which constitute the most common setting in DCs. Additionally, we compare k-disjoint path routing, which can be implemented using segment routing [9] (Section 7.1). We use realistic baseline topologies, that are inspired by industry recommended sizing configurations [2].

Our results show that expanders can offer a very large performance improvement over Clos topologies, using only currently-supported protocols, for a wide range of traffic patterns in the $C - S$ model, particularly with realistic over-subscription ratios (e.g., $2 \times 4 \times$). Moreover, this performance advantage is present even for relatively small leaf-spine networks, such as an 8-leaf, 2-spine hardware testbed.

We found these results surprising, given that past work has required more advanced protocols, and has found generally larger benefits with larger scale [32]. There are several factors at play influencing these performance differences. Past work has discussed the expander’s advantage in lower mean path length (hence, delivering each packet requires less total bandwidth), and the disadvantage that its more complex structure means it will not attain its full potential without special routing protocols. However, we found two other effects to be critical. First, by moving from a two-tiered design to a flat design (while using the same physical equipment), expanders increase the bandwidth exiting each rack. In fact, we show analytically that the ToR over-subscription is $2 \times$ less in expanders than leaf-spines, irrespective of the number of leaf and spine switches, and is $4 \times$ less in a 3-tier fat tree (Section 4.2). This does not mean the expander always gets $2 \times$ higher throughput. But when two factors are combined – (1) over-subscription causing a bottleneck exiting the leaf-spine’s racks, and (2) a skewed traffic pattern causing this bottleneck at a minority of racks – the expander is effectively able to mask the over-subscription and behave closer to a non-blocking network. The oversubscribed setting also happens to be the more realistic scenario and has been overlooked in past work.

Second, use of the expander’s capacity is more efficient in oversubscribed networks. If the goal is to obtain a non-blocking (or full-bandwidth) network as in many recent studies [43, 34], expanders reach that level of performance only after hitting a regime of diminishing returns as more switches are added (Figure 1), since the DC is mostly host-bottlenecked by that point. Even with that inefficiency, expanders may edge out Clos networks in cost, but we find the advantage is larger in the oversubscribed regime.

A summary of our results are as follows:

- We show that expanders can yield as much as $3-4 \times$ higher throughput than fat trees and up to $2 \times$ that of leaf-spine networks for a wide range of traffic patterns, under realistic over-subscription ratios and with practical protocols.
- We demonstrate that the improvement in performance does not come at the cost of fairness; fairness in the expander was at least as good as in fat trees (Figure 3).
- We show that for a publicly available DC traffic trace [3], expanders result in significantly smaller flow completion times at the tail and degrade more gracefully than fat trees as the network is increasingly stressed.
- We show that expanders result in comparable amounts of transient packet loss to fat trees during link and switch failures.
- We demonstrate that expanders are more resilient to bursty load situations like incasts and outcasts resulting in smaller flow completion times and smaller queue sizes.
- We present modestly-sized experiments for topologies up to 48 servers emulated on a hardware testbed. Our hardware experiments compare throughput, flow completion times and the job completion time of a heavy shuffle operation and confirm our simulation results.

Our simulations are based on the htsim [38] and ns3 [8] simulators. We have made our code publicly available and provided scripts to reproduce experimental results for any topology (obfuscated for review). Overall, we believe our results show that expanders are highly promising for deployment using today’s hardware, even at moderate scale. Indeed, this scale may be especially practical in the near term, since concerns over wiring complexity are less relevant.

2 Background

Expanders are a class of graphs that have high expansion ratios [4] (defined as the ratio of the outgoing edges from
a set of nodes in a graph to the size of the set). Expander graphs prove[4] to have good connectivity and high bisection bandwidth, hence are an good option for datacenter networks. Singla et. al[43] first showed that random graphs (Jellyfish) are competitive to a Clos-based topology for datacenter networks. They demonstrated in [42] that random graphs are flexible to accommodate for heterogeneous switch hardware and come close to optimal performance for uniform patterns under a fluid flow routing model.

Asaf et. al[48] proposed the Xpander topology as a cabling-friendly alternative to Jellyfish, while matching its performance. Both Jellyfish and Xpander used K-shortest path routing and MPTCP [51] in the transport layer to demonstrate performance advantages. The Xpander topology has a larger number of long distance links than the random graph, albeit with more structure allowing cables to be bundled together neatly. Nevertheless, the large number of long distance links hinders manageability. In Section 3 we discuss a cabling solution to decrease the number of long distance links without compromising on expander quality.

Kassing et. al[34] demonstrated that expander networks can outperform fat trees for skewed traffic patterns using a combination of VLB, shortest path routing and flowlet switching[45,33]. More importantly, comparisons in Jellyfish[43,42] Xpander[48] and a recent evaluation[34] suffer from bottleneck at the end hosts. Furthermore, reliance on non-traditional protocols restricts the practical realization of these proposals. They also showed that under a fluid flow model where the host capacity is not modeled (and hence problems related to comparisons with full bandwidth do not arise), expanders can achieve around 3x more throughput than fat trees. Their work left an open question of designing better routing algorithms to realize this with oblivious routing schemes. In our work we show that in fact traditional routing algorithms suffice. However, the benefits are apparent only when the network is oversubscribed, rather than when the bottleneck lies primarily with hosts.

Jyothi et. al[32] showed that under the fluid flow model with ideal routing, the random graph outperforms the fat tree for near-worst case traffic matrices. The worst case traffic matrix is a permutation matrix since any traffic matrix in the hose model[17] can be written as a convex combination of all permutation matrices (using Birkhoff-von Neumann’s Theorem[13]). Although computing the exact worst case permutation matrix is computationally hard, longest matching traffic patterns are a good candidate. The comparisons in[32] are insightful since they don’t model the server-to-switch bottleneck and hence avoid the diminishing returns of the full-bandwidth scenario. However, the use of optimal routing and an idealized fluid-flow model does not give insights about the feasibility of practical routing schemes.

These works leave substantial room for improving the practicality of using expanders for data centers. It’s established that expanders can achieve superior performance; our goal is to examine if the gains still hold when used with traditional protocols. Answering that question is primarily a measurement study, guided towards practical settings. We next describe our methods for this study.

3 Experimental Setup

3.1 Topologies

Fat trees (k): We used standard fat trees[10] with k ports, oversubscribed at ToR by 1,2 and 4x. For oversubscribing, we assume that each ToR supports 4x more servers than a full bandwidth fat tree, if the oversubscription ratio is 4. We used k = 16 (320 switches, 4096 servers, 4x oversubscribed) for simulations in htsim[38] and k = 10 (125 switches, 1000 servers, 4x oversubscribed) for simulations in ns3.

Leaf-spine (x, y): We consider the following leaf spine topology with switch degree (x + y) for arbitrary parameters x and y,

- There are y spine switches, each of them connect to all leaf switches.
- There are (x + y) leaf switches, each connect to all y spine switches.
- Each leaf switch is connected to x servers.

As per recommended industry practices[2], we choose an oversubscription ratio = x/y = 3 with x = 24, y = 8 for experiments in the C-S model. The leaf spine topology built this way with parameters x = 48 and y = 16 is exactly same as an actual industry-recommended configuration[2]. We chose this recommended configuration in part because it uses leaf and spine switches with the same line speed, making comparisons more straightforward; we leave heterogeneous configurations to future work, but expect similar results.

Random Graph: To compare a random graph topology with a leaf-spine, we build a random graph with the exact same equipment by rewiring the baseline leaf spine topology, redistributing servers equally across all switches (including switches that previously served as spines) and applying a random graph to the remaining ports.

To compare a random graph with a full bandwidth fat tree, we build a random graph with the exact same equipment the fat tree, redistributing servers across all switches (including switches that constitute the upper layer core and aggregation layer) and applying a random graph to the remaining ports. To convert this to an oversubscribed topology, we add more servers to every rack of the random graph, distributing servers evenly across all racks. (Note that this follows the same over-subscription procedure as the fat tree except that it distributes the additional server ports across all switches rather than only the fat tree’s ToRs.)

Note that other expanders have very similar performance characteristics to the random graph[48] and hence our result
applies to all high-end expanders. In our text, we use the shorthand RRG for Random Regular Graph.

3.2 Routing and Transport

We used shortest path routing with ECMP and TCP in the transport layer for all sets of experiments. Additionally, for bandwidth experiments, we also evaluate k-shortest path routing [53] and k edge-disjoint path routing (henceforth, referred to as k-disjoint path routing) to contrast with shortest path routing. In Section 5, we discuss a simple practical variant of k-disjoint path routing that can be easily implemented with segment routing.

3.3 C-S Model

Traffic patterns in a datacenter have high variance and change rapidly with time [21]. Moreover, there is no underlying theme across datacenters [39, 12, 16]. Majority of the traffic was not found to be rack-local in Facebook’s datacenter [39], in contrast to findings in [12, 16, 21]. We showed that traffic pattern is highly skewed in Microsoft’s datacenters with only a very little fraction of racks carrying most of the traffic, while in Facebook’s datacenter [39], demand was wide-spread, uniform and stable across a longer time period.

Motivated by these arguments, we believe that there is a need to succinctly represent a wide range of workloads for topology evaluations. Previous works have used permutation matrices [43], near all patterns [43, 26], worst case longest matchings [32], skewed traffic patterns [21, 34] and snapshots from real datacenters. While all the above traffic matrices have different characteristics (worst case, skewed or uniform) and provide useful insights, we argue that these are very specific experimental points in a wide range of possibilities and hence do not portray the entire picture.

We propose a simple model to capture a wide range of scenarios. We pick a subset of client hosts, say set C and pack these clients into the fewest number of racks while randomly choosing the racks in the datacenter. Similarly, we pick a subset of server hosts, say set S and pack all servers into the fewest number of racks possible (avoiding racks that have been used for C). We wish to measure the network capacity between the sets C and S for all possible sizes of C and S. By varying the sizes of C and S, we claim that this model, which we call the C − S model, captures a wide range of patterns that commonly occur in applications.

- If C represented the map tasks and S the reduce tasks of a map-reduce job, then the C − S model can capture the shuffle operation between the map and reduce tasks.
- If C represented the parameter servers and S the workers for ML training, the C − S model can capture model updates from workers to the parameter servers and parameter reads from parameter servers back to the workers.
- Setting |S| = |C| = r, where r is the number of servers in a rack, captures rack-to-rack traffic pattern.
- With |C| << |S|, the C − S model captures a wide range of skewed traffic patterns.
- Uniform traffic patterns across the entire datacenter by setting |C| = |S| = n/2, where n is the total number of hosts.

4 Bandwidth capacity

We study bandwidth capacity in the C − S model. By covering a wide range of scenarios, the C − S model allows us to identify scenarios where expanders have an advantage over fat-trees and leaf spines, and where it doesn’t. We graph the C − S model using a heatmap, varying the size of sets C and S on the x and y-axis. Between every client in C to every server in S, we run a long running TCP connection with infinite data to send. Each cell in the heat map represents the average throughput per flow in the random graph normalized by the average throughput in the baseline topology. We plot two heatmaps in each case for (a) small runs comprising of smaller values of C and S, up to a few handful racks and (b) larger runs comprising of larger values of C and S, almost all the way till the entire datacenter. We used the simulator based on htsim from [43] for these experiments.

4.1 Comparisons with Fat tree

As noted in [34], a lot of bandwidth capacity remains unused in a fat tree when only a few racks are sending traffic. This is also evident from the heatmaps (Figures 2a−2f). For a large range of C and S, the expander gets drastically more throughput than the fat tree (3-4x in many cases, see Figure 2f).

4.1.1 Effect of oversubscription

Figures 2a, 2c and 2e illustrate the effect of oversubscription for small sizes of C and S in the C − S model. With an oversubscription ratio of 4, for smaller runs, the expander drastically outperforms the fat tree for all sizes of C and S (except for the red patch towards the bottom left). For runs (Figure 2f) with larger values of C and S, the expander network outperforms fat trees drastically in all cases. Also observe that when run with oversubscription 1 (Figure 2a), the difference between expander and fat trees are unapparent because the network is not the bottleneck, highlighting problems of comparisons with full bandwidth. The differences get increasingly evident as the oversubscription ratio is increased.
as effects of rack oversubscription come into the picture. Expanders significantly help out the hot racks by alleviating rack oversubscription.

An oversubscribed topology also brings out cases where expander graphs do not perform well (the red patches on bottom left of Figure 2e). As noted in [34], expander being a flat topology (all switches connect to servers) offers fewer shortest paths between a pair of nodes and hence when two racks send and receive data at full link speed, the expander network does not match fat tree’s performance with shortest path routing. This is the only region in the $C - S$ model where expander with shortest path routing does not match the performance of a fat tree.

### 4.1.2 Effect of routing scheme

Figures 2a, 2b and 2d illustrate the performance of random graphs with different routing strategies namely shortest paths, k-shortest paths and k-disjoint paths. As shown in Figure 2d, k-disjoint path routing eliminates the few cases where the random network is not able to match fat tree’s performance with shortest paths. One of the reasons is that k-disjoint routing distributes load evenly just at the edge of the network (network links at the ToR). Although naively k-disjoint paths require perfect source routing, we show that they can be approximated very accurately by combinations of two shortest paths, which can be easily implemented with MPLS or segment routing (Section 7.1). We emphasize that expander networks are already advantageous with ECMP and shortest paths in most cases. K-disjoint path routing might not be necessary for practical purposes.

The drastic improvement can be partly attributed to lesser rack oversubscription in an expander (more exit network ports per server in a rack, true for any flat network). Note that unlike a fat tree, the network links of a ToR switch in a flat network carry traffic that originated in the rack as well as traffic that originated elsewhere but routed through that ToR switch. Nevertheless, all network links can carry local outgoing traffic in the best case. This is especially true for
motivated by these arguments, we introduce the notion of uplink to downlink factor or UDF of a topology. Consider a topology \( T \) and a random graph topology \( R(T) \) built with the same equipment. For every top of the rack switch that contains servers, we look at the ratio of network ports and the number of server ports call this ratio NSR (for Network Server Ratio). We define UDF(\( T \)) as:

\[
UDF(T) = \frac{NSR(R(T))}{NSR(T)}
\]

Intuitively, NSR represents the outgoing network capacity per server in a rack. The UDF represents the best case scenario for the random graph when the outgoing links carry only traffic originated from the rack. It represents an upper bound on the performance of random graph as compared to the baseline topology.

We can easily compute the UDF for a fat tree with switch degree \( k \).

\[
UDF(T = FatTree(k)) = \frac{NSR(R(T))}{NSR(T)} = \left( \frac{k - (\frac{k^2}{4})}{s} \right) / \left( \frac{5k}{s} \right) \approx 4
\]

Note that number of server ports in one rack of a random graph built with the same equipment as a fat tree with switch degree \( k \) is \( \left( \frac{s^3}{3} \right) / \left( \frac{s}{6} \right) \) and the number of network ports in one rack is equal to \( (k - \text{server ports}) \). From Figures 2e and 2f random graphs achieve throughput that is very close to UDF: 4 in the case of fat trees.

4.1.3 Note on Fairness

We note that the differences in average throughput, highlighted in Section 4.1 with higher oversubscription ratios are not because the routing inefficiencies get masked by the presence of a large number of flows. If that were the case, some flows would utilize all existing capacity and the throughput distribution would be unfair. Figure 5 plots the CDF of throughputs of flow for \( |C| = 200, |S| = 200 \). It can be seen that Jain’s Fairness index of random graphs matches that of fat tree with k-disjoint path routing. With shortest path-routing, the throughput distribution of flows is strictly better for the random graph than the fat tree.

4.2 Comparisons with Leaf Spine

Leaf spine topologies are popular for small to medium tiered datacenters consisting of a few tens or hundreds of racks [2]. Leaf spine topologies are two-tiered tree based topologies where all lower layer ToR switches (leaves) are connected to all upper layer switches (spines). The servers are distributed across all the leaf switches. The topology provides multiple paths between any source and destination (one path through each spine) and is meant to provide good load balance.

In this section we compare leaf spine topologies to random graphs built with the same equipment. Figure 6 illustrates the throughput in the C − S model for small and large runs with \( x = 24 \) and \( y = 8 \). Each tile is normalized by the throughput for the leaf spine for that \( C \) and \( S \). As with fat trees, with
ECMP, random graph outperforms the leaf spine topology in all regions except the bottom left red patch of Figure 4a representing the bandwidth between two racks.

We can compute the UDF for leaf spine switches for arbitrary $x$ and $y$. We have,

$$NSR(T = \text{LeafSpine}(x, y)) = \frac{y}{x}$$

For the corresponding random graph $R(T)$ built with the same equipment,

$$NSR(R(T)) = \frac{(x+y) - \text{server ports per switch}}{\text{server ports per switch}}$$

$$= \frac{(x+y) - (x(x+y)/(x+2y))}{x(x+y)/(x+2y)}$$

$$= \frac{2y}{x}$$

Thus, $UDF(T = \text{LeafSpine}(x, y)) = \frac{NSR(R(T))}{NSR(T)} = 2$

It can be seen from Figures 4a and 4b that expanders indeed touch the upper bound of $UDF = 2$, against leaf spines, in the $C-S$ model. Observe that the $UDF$ of a leaf spine network is independent of the number leaf and spine switches. If a network has fewer spines and more leaves, the number of servers per rack are fewer but the aggregate uplink bandwidth at the ToRs is also lesser. These two factors cancel each other and hence, the UDF remains constant.

4.2.1 Effect of scale

To examine if the advantages of expanders hold at all scales, we experimented with increasing sizes of leaf spines. For traffic patterns, we chose multiple points in the $C-S$ model as a representative set for the entire space. Figure 5 plots the average throughput in the expander (normalized by the average throughput in the leaf spine for that size). It can be seen that the advantages of expanders over leaf spines in the $C-S$ model extend to all scales.

5 Flow completion time

Traffic in datacenters is often bursty and volatile [23, 41]. Incast (many-to-one) and outcast (one-to-many) patterns occur frequently in data processing pipelines. In the following subsection, we discuss multiple experiments in the $C-S$ model, with specific values of $C$ and $S$, to replicate different types of bursty incast and outcast situations. In each case, we use flow completion time as the performance metric. Every client in $C$ sends a 100 KB flow to every server in $S$. To get maximum
burstiness, we start all flows at the same time.

5.1 Incasts and Outcasts

Incasts and outcasts are challenging conditions for the network to deal with. Incasts (and outcasts) could be grouped into two categories [41]: (1) where the bottleneck is at the host and (2) bottleneck is in the network. Not much can be done by the network for the first kind, when the bottleneck is at the hosts. These type of incasts are mainly many-to-one server level patterns where the bottleneck is due to the traffic fanning in from the ToRs to a single host [41]. These constitute majority of the packet drops- 62.8% according to the study in [41]. We ran many-to-one (at the server level) incast patterns in ns3 and got roughly the same distribution of flow completion times for both fat trees and the random graph, which is expected since the bottleneck is at the hosts and not the network.

The second type of packet drops are caused mainly due to the ToR fan-in at the aggregation layer (35.9% of the packet drops [41]). These can occur for instance, if many servers are trying to send traffic to the same rack (but possibly different servers in the rack). To reproduce such incasts, we design an experiment involving 40 servers (2 racks in the fat tree) sending traffic to 20 other servers at the same time (\(|C| = 40, |S| = 20\) in the \(C - S\) model). We used the following settings for our experiments in ns3:

| Queue size | 500 packets (droptail) |
|------------|-----------------------|
| Flow size  | 100 KB                |
| Link speed | 1 Gbps                |
| Link delay | 1 µsec                |
| Packet size| 1 KB                  |
| RTO\(_{\text{min}}\) | 5 ms |
| Connection Time Out | 50 ms |

We used a larger queue size to ensure that the packet loss is not too high to render the network unusable. Figure 6a shows the distribution of flow completion times of the \(40 \times 20 = 800\) flows involved in the experiment. Clearly, the expander network results in significantly better flow completion times (by more than \(2 \times\) at the median). Figure 6b and 6c illustrate the queue lengths at the 5 most loaded link ports with time on the x-axis. We verified that all five of the most loaded queues in the fat-tree were at the downlink from the aggregation layer to the ToR. As can be seen in the figure, the traffic jam gets cleared more quickly in the expander graph (roughly twice as quickly except at the busiest link).

The third kind of packet drops are a result of oversubscription of ToR uplinks (around \(1.3\%\) as per [41]). Expanders, being flat networks, are better suited to handle such bursts as each rack supports fewer servers. To reproduce such outcast patterns, we ran an experiment with 20 servers (1 rack in the fat tree) sending traffic to 40 other servers (\(|C| = 20, |S| = 40\) in the \(C - S\) model). Figure 7a shows the distribution of flow...
5.2 Datacenter traffic trace

The C − S model served our purpose for characterizing throughput and latencies for a wide range of cases and identifying cases where a topology does well and where it doesn’t. In this section, we describe experiments with real traffic patterns based on packet traces from Facebook’s publicly available dataset [3].

5.2.1 Instrumentation

The Facebook dataset contains packet logs from multiple clusters, we focus on one of the clusters in the datacenter so that we have a complete traffic matrix for that cluster. We obtain aggregate traffic volumes across racks in that cluster (Figure 9), disregarding traffic going out of the cluster. Because of the high sampling ratio of 1:30000, we couldn’t derive any meaningful conclusions about the flow size distributions. To get the number of racks down to the number of racks in our fat tree topology (50 ToRs), we picked the busiest 50 racks out of 157 racks in the FB cluster to get a rack level traffic matrix (see Figure 9) for the fat tree. We found that the outgoing traffic volume from a rack has a near uniform distribution (Figure 10). We then split the traffic across a pair of racks equally across all pairs of servers in the two racks. This lead to a server level traffic matrix for the fat tree. To get a server level traffic matrix for the random graph from the server level matrix of the fat tree, we packed the busiest servers into a rack of the random graph, while randomly choosing racks. This process is illustrated in Figure 8. We run this server level traffic matrix in the ns3 simulator [8], by randomly starting flows across a span of 10 ms. We multiplied each entry in the traffic matrix by a normalizing factor, tuned so that the traffic matrix is neither too small nor too large. We used the following settings for our experiments:

| Parameter            | Value          |
|----------------------|----------------|
| Queue size           | 100 packets    |
| Link speed           | 1 Gbps         |
| Link delay           | 1 µsec         |
| Packet size          | 1 KB           |

5.2.2 Expanders degrade more gracefully

By increasing the load on the network, we assess how quickly the network degrades. After a certain load threshold, the network expectedly collapses owing to high packet loss. Things get significantly worse when TCP’s control packets start getting dropped. The network is no longer usable at this point. From a design point of view, aside from performance, one also needs to know the network’s limits for capacity building for worst case scenarios.

Figures 11a, 11b and 11c illustrate flow completion times for the traffic pattern described in Section 5.2.1. On the x-axis is the normalizing factor and hence the network load increases along the x-axis. The results suggest that not only do expanders have a better flow completion time than fat trees, but the network degrades more slowly and gracefully in expanders. In fact, expanders can support more than 2x the network load before degrading beyond being usable.

6 Traffic Loss on Failures

Fault recovery in modern data centers [49] is largely based on reconvergence of routing algorithms, resulting in traffic loss till routing tables are recomputed. To alleviate traffic loss, switches react locally to failures by avoiding failed next hops. This is possible only if the routing table has alternate next hops for a given destination. For instance, in a fat tree, a source to destination path comprises of an upward path from the source to a core switch and a downward path from the core switch to the destination. Alternate next hops are available only for links in the upward path. If a link on a downward path fails, then a packet traversing that path would get dropped. We quantify traffic loss due to a single link failure and a single switch failure till routing tables are recomputed. We assume local convergence, that is, switches are capable of avoiding a failed link if alternate next hops are available for that destination.
6.1 Model

We model traffic loss as follows. Assume one unit of traffic flow between a random source and destination pair. Further, assume that the path for the flow, is chosen according to an ideal hash function at each switch, that hashes based on the tuple identifying the flow. If traffic arrives at a switch that has no active next hops, the entire flow gets dropped. Let $F$ denote the event of a single link (switch) failure and $L$ denote the event of traffic getting dropped between a random source and destination pair. Then, traffic loss due to a single link (switch) failure can be given by $P[L|F] \times P[F]$. We have,

$$P[F] \approx \lambda \times \text{Number of links (switches)}$$

$\lambda$ is a constant that denotes the probability of a link (switch) failing in a given interval of time. $\lambda$ is also related to the Mean Time To Failure (MTTF) of a single link (switch).

We compute $P[L|F]$ empirically for shortest path routing, k-shortest and k-disjoint path routing. Traffic loss for k-shortest and k-disjoint path routing depends on the implementation, we assume complete source routing. Local convergence would imply that if there’s a path for which the first link has failed, then the source switch would avoid that route and choose from alternate paths. If there are no alternate paths, all packets in the flow would get dropped till the routing algorithm reconverges.

6.2 Single Link and Switch Failure

Traffic loss is determined by two factors - probability of hitting a failed link and the ability to locally reroute to alternate next hops. The probability of hitting a failed link is in fact, directly proportional to the average path length. The route lengths are largest in k-disjoint followed by k-shortest followed by shortest path routing which employs paths of optimal lengths. We see that the same is also reflected in

Figures 12 and 13 We emphasize that marginal differences in traffic loss for different routing schemes might not be of significant importance, since any traffic loss is short-lived lasting only till the routing algorithm reconverges. Both figures 12 and 13 convey the same underlying message: all schemes result in acceptable traffic loss in comparison to shortest path routing in a fat tree.

7 Hardware Experiments

In this section, we present experiments on a physical testbed consisting of 13 OpenFlow-enabled Pica8 switches, connecting to several VMs over a 1Gbps link. We emulate a leaf spine network with 3 spines, 7 ToR leaves and 28 servers, and a random graph topology with the same equipment. We study a shuffle operation, a common primitive among data processing jobs (e.g. to hash-partition keys in a map-reduce job). The shuffle operation involves an iperf [30] data transfer across every pair of nodes in the system. We used the following settings for the experiment:

| Transport          | TCP Window | Flow size | Link capacity | Routing | Load balancing |
|--------------------|------------|-----------|---------------|---------|----------------|
|                    | 416 KB     | 100 MB    | 1 Gbps        | Shortest path | ECMP           |

Figure 14a illustrates the distribution of flow completion times for the shuffle operation. The all-to-all heavy shuffle keeps the entire network busy and hence there is little room for spreading out the traffic. Hence, the expander doesn’t perform significantly better than the leaf spine. Note that the expander graph is still a little more efficient because it uses shorter paths.

Figures 14c and 14b illustrate coarse grained C−S model runs, comparing the simulator and the hardware results. As
7.1 Implementing k-disjoint Routing

Implementing k-disjoint and k-shortest paths routing is non-trivial with commodity switches. Many switches support only destination based look-ups, such as longest prefix matches on destination IP addresses. It can be easily seen that a network fabric with only destination based look-ups can not support k-disjoint path or k-shortest path routing trivially (see Figure 15). If matching on arbitrary bitmasks were allowed, then source routing could be implemented efficiently in openflow switches as shown in [31] and [25]. One could easily implement k-shortest path routing using source routing. However, most commodity openflow switches do not support matching on arbitrary bitmasks despite it being in the OpenFlow specification [46]. Alternatively, one could stack MPLS labels in the packet header to implement source routing. This has two problems a) it might blow up the packet header size since each MPLS label is 4 Bytes b) typically, there is a limit to the number of MPLS labels one can stack up in the packet header.

We take another approach that uses a simple observation. Call a path $P$ from source $s$ to destination $t$ expressible if there is an intermediate hop $u$ in $P$ such that $P_{s\rightarrow u}$ and $P_{u\rightarrow t}$ are shortest paths, where $P_{x\rightarrow y}$ denotes the portion of $P$ from $x$ to $y$. If the network uses standard ECMP routing, then the routing entry for $P$ can be expressed by attaching a single MPLS label for $u$ at source $s$. Note that pushing a MPLS label for $u$ and using ECMP between $s$ to $u$ and $u$ to $t$ is not the same as $P$ since there could be multiple shortest paths from $s$ to $u$ and $u$ to $t$. However, this only increases path diversity without increasing the path length and hence will only help. We found out that most of the k-disjoint paths in k-disjoint path routing could be expressed as a concatenation of two shortest paths. Figure 16 illustrates that the number of k-disjoint paths that are not expressible is less than 0.01%. Expressing routing paths by bouncing the packets off an intermediate hop also makes it possible to implement it using segment routing [9] which is increasingly getting adopted [1, 6].

8 Optimizing long distance links

Managing wiring complexity of an expander network poses a challenge for deployment, especially on large scale. While random graphs with more localized short distance links have been proposed as a solution to better manage the complexity [42], such networks would lose out on performance. In appendix A, we show that the expansion ratio of an expander (which can be thought of as a metric to judge the expander quality) decreases at least linearly with decreasing fraction of long distance links. We take another approach instead, one that is applicable for small to medium sized networks and does not make any compromise on the quality of the expander graph. We used the publicly available Metis graph partitioning library [7] to partition the network into $k$ multiple equal-sized clusters (in terms of the number of racks), minimizing the number of cross-cluster long distance links. Note that this problem is NP-hard in general, even for $k = 2$ clusters, so the Metis software uses heuristics that are known to work well in practice. Figure 17 shows the number of cross-cluster links after graph partitioning. Without partitioning, the expected number of cross-cluster links in a random graph, partitioned into $k$ clusters, would be $(k - 1)/k$. Thus, for a random network with 320 switches, smartly partitioning the network into $k = 5$ partitions brings down the number of long distance links from 80% to approximately 50% (see Figure 17). The partitioning trick doesn’t however work for hyperscale sized networks. As can be seen in Figure 17 as the network size increases, the fraction of cross cluster links after partitioning increases.
9 Related work

9.1 Tree-based designs

Apart from the standard 3-tier fat tree [10], a lot of research has been done to improve tree-based designs. LEGUP [15] proposed a design to build heterogeneous Clos networks for flexible expansion. F10 [36], although inherently similar to Fat trees, reacts quickly to failures and preserves performance even in the face of multiple failed nodes by co-designing routing algorithms, network and failure handling mechanism. BCube [24] recursively builds a tree-based server-centric architecture and is applicable mainly to container based datacenters that require high failure resilience.

9.2 Non-tree based static networks

Slimfly [13] optimizes for low network diameters and is also based on expander graphs. The proposal relies heavily on non-oblivious routing schemes that requires information about queues either locally or globally [13]. Further, it loses out on performance for having a smaller expansion ratio than Jellyfish or Xpander. Dragonfly [35] uses groups of high radix routers as virtual routers, effectively increasing the network radix to achieve low diameter. However, reliance on randomized routing based on Valiant’s algorithm to route packets across groups poses challenges for adoption. Designs based on structured sub-graphs and random links between the structured sub-graphs [40], inspired from the small-world distribution, were proposed to achieve high failure resilience and bandwidth. These designs have a lot in common with the random graph topology. The Flat-tree [52] architectures explores a convertible design between Clos topologies and random graphs to achieve the best of both worlds.

9.3 Dynamic Networks

Another research thrust has been towards dynamic networks, where link connections are configured dynamically based on the traffic load using free space optics. The argument for dynamic networks is the observation that traffic patterns in datacenters are highly skewed with only a few racks sending and receiving most of the traffic at any given point in time [21]. Proposals connecting racks with ceiling mirrors [27, 54], beam redirecting ceiling balls [21] that provide a large fan-out and using wireless links as an alternative to wired links [44, 20, 14, 50]. The advantage of such designs is that by varying the tilt of the reflecting surfaces, the incoming traffic beam can be redirected to a large number of racks. However, slow switching time of links (around 10-100 ms) poses a challenge in adoption of dynamic networks. Mordia [37] uses circuit scheduling to proactively assign circuit bandwidths and achieves a much lower switching time of around 11.5 µs.

9.4 Traffic Engineering

Several works in the past have aimed at improving load balance in Clos topologies to minimize flow completion times. Some of these ideas can be extended to expander networks. CONGA [11] uses flowlet switching [45], choosing paths based on congestion levels on the paths. PDQ [29] uses preemptive flow scheduling to simulate a number of scheduling schemes like shortest job first and earliest deadline first. Drill [22] makes fine grained decisions at the packet level based on load on the next hop. Presto [28] evenly distributes fixed sized flow-lets called flow-cells across all paths to the destination. Specific traffic engineering and load balancing schemes targeted towards expander networks would be an interesting avenue of research which we leave for future work.

10 Conclusion

We discovered that expander datacenters are extremely effective even with just traditional protocols, offering around 3-4× and 1.5-2× more bandwidth on average compared to fat trees (large scale) and leaf spine (small scale) networks respectively, for a wide range of cases. The advantages of expanders are even more pronounced with k-disjoint path routing, that we showed, can be implemented with segment routing. We examined extreme traffic situations like incasts and outcasts and showed that expanders are more resilient to such cases and degrade more gracefully as load increases. Furthermore, we analyzed several other metrics of interest: traffic loss on failures, time taken to complete a shuffle operation, queue occupancy and fraction of long distance links. Our results show that non-oblivious schemes or complex traffic engineering schemes are not required for building an expander datacenter. Our experimental results, based on
extensive simulation and hardware emulation, unanimously identify significant advantages of using expander-based datacenters over tree-based networks at both small scale and large scale with protocols that are realizable with today's hardware.

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A On Long distance links in Expander networks

The complexity of wiring expander based networks like Jellyfish and Xpander poses a hindrance for adoption. Often a datacenter will be divided into several clusters (for considerations of incremental expansion, simplifying monitoring and power supply). The metric we consider are number of long distance links or cross cluster links.

Intuitively, we expect the quality of expander networks to deteriorate with fewer long distance links. We formalize this intuition using the notion of edge expansion $h(G)$ of a graph $G$.

$$h(G) = \min_{|S| < \frac{\partial S}{2}} S$$

where, $\partial S$ is the number of edges going out of set $S$. Edge expansion can be interpreted as a metric to judge the quality
of the expander. Random graphs have high edge expansion ratio \cite{19} that asymptotically approaches \(d/2\). Theorem 1 suggests that the edge expansion decreases linearly with the fraction of long distance links.

**Theorem 1.** For a \(d\)-regular graph \(G\) with \(n\) nodes, partitioned into \(k\) equal-sized clusters,

\[
h(G) \leq \frac{dkf}{2(k-1)}
\]

where \(f\) = fraction of long distance links

**Proof.** We simply pick \(S\) to be all nodes in \(k/2\) clusters. There are \(\binom{k}{k/2}\) such sets. Any edge appears in \(\partial S\) for \(2\binom{k-2}{k/2-1}\) such sets. Thus, for at least one such set \(S\), we will have

\[
\partial S \leq \frac{ndf}{2} \times 2 \binom{k-2}{k/2-1} / \binom{k}{k/2} = \frac{ndkf}{4(k-1)}
\]

Thus,

\[
h(G) \leq \frac{\partial S}{|S|} \leq \frac{ndkf}{4(k-1)/n/2} = \frac{dkf}{2(k-1)}
\]

\(\square\)