Spin-Statistics Connexion, Neutrinos, and Big Bang Nucleosynthesis

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Abstract

We show how the $^4$He-abundance in the early Universe can be used to demonstrate that macroscopic samples of neutrinos in thermal equilibrium are indeed distributed according to Fermi-Dirac statistics.
Cosmology is an excellent testing ground for theories and models of Particle Physics [1]. Fundamental laws of Nature as well ambitious theoretical constructs have been scrutinized and confronted against cosmological data. Conversely, Particle Physics provides important clues to cosmological problems (e.g. provides candidates to dark matter, provides explanations to primordial baryogenesis, etc.) A paradigm of this fruitful symbiosis is the momentous prediction from Big Bang Nucleosynthesis (BBN) on the number of light neutrino species [2].

A fundamental result in relativistic quantum field theory is the celebrated spin-statistics theorem. Particles with half-integer spin obey Fermi-Dirac statistics and particles with zero or integer spin obey Bose-Einstein statistics. This theorem is deeply rooted in very basic principles such as relativistic invariance, locality and microcausality and its experimental verification (and/or the confirmation of its consequences) is very important. Of course, the experimental evidence of the spin-statistics connexion for ordinary matter in macroscopic samples is overwhelming. Electrical (e.g. superconductivity) and thermal (e.g. specific heats) properties of metals at low temperatures can only be explained if electrons in matter obey Fermi-Dirac statistics. Superfluidity too, is a reflection of the spin properties of matter. The planckian blackbody spectrum of radiation is just another phenomenological manifestation of the spin-statistics theorem and traces back to the Bose-Einstein character of statistical ensembles of photons.

Neutrinos, however, are not usually found in macroscopic samples and held in thermal equilibrium at a given temperature in a laboratory. Thus, their statistical behaviour is difficult to be experimentally established. Hence, a direct verification of the spin-statistics connexion in the thermodynamic sense is still lacking in the case of neutrinos. Sure enough, they are spin 1/2 particles, as a large amount of data coming both from nuclear reactors and particle accelerators do convincingly demonstrate. It would be nice then, as is the case for other elementary constituents, to reveal the statistical mechanics of neutrinos.

Two systems in Nature do contain a large macroscopic ensemble of neutrinos in thermal equilibrium. A hot supernova core is one of them. The other is the early Universe. In the cosmic system, one can find a period in the early history of the Universe where the spin-statistics connexion of neutrinos has observational consequences. In the present paper we shall see that it is possible to decide whether or not a macroscopic collection of neutrinos
in thermal equilibrium at temperature $T$ do indeed follow Fermi-Dirac statistics. BBN and in particular the primordial helium abundance $Y_P$ provides the basic tool for the analysis of this question as we try to explain in the rest of this paper.

A fundamental postulate of Quantum Mechanics is that systems of identical particles are described by either symmetric or antisymmetric wave functions. Systems of particles in thermal equilibrium whose states are described by symmetric (antisymmetric) wavefunctions are distributed according to Bose (Fermi) distribution functions. Now, it is a phenomenological fact that electrons (and other particles with half-integer spin) are described by antisymmetric wave functions (and obey Fermi-Dirac statistics when assembled in large numbers and held in thermal equilibrium) and photons (and other particles with integer spin) are described by symmetric wavefunctions (and follow Bose-Einstein statistics). That this should be so and not the other way around is the content of the Pauli spin-statistics theorem, which follows very generally from relativistic quantum field theory as already mentioned above. The actual purpose of this investigation is to realize that this is observationally the case for neutrinos. So, our intention is not, to doubt about the fermionic nature of neutrinos but rather, to explicitly display in a real physical system a characteristic phenomenon associated to their nature. Namely, a large collection of neutrinos in thermal equilibrium at temperature $T$ is Fermi-Dirac distributed.

The helium-4 and other light element (i.e. deuterium, helium-3 and lithium-7) primordial abundances have been periodically evaluated and confronted with data embracing observations of ever increasing quality. The most recent analysis can be found in references 3 to 9. The theoretical calculation of the primordial helium abundance is by now standard and the basic arguments can be read in the textbooks (see e.g. ref. 10). The key quantity to be computed is the fraction $X_n$ of neutrons to all nucleons as the Universe cools from equilibrium temperatures well above neutrino freeze-out ($T >> 1 \text{ MeV}$) down to the temperature where nucleosynthesis takes place ($T \sim 0.1 \text{ MeV}$). $X_n$ is found from the evolution equation

$$\frac{dX_n}{dT} = -\lambda(n \rightarrow p)X_n + \lambda(p \rightarrow n)(1 - X_n)$$

where $\lambda(p \leftrightarrow n)$ are the weak rates (per nucleon) that interconvert protons and neutrons. These rates contain the microphysics (weak interaction
probabilities) and the macrophysics (thermalization). It is this latter component that we shall manipulate.

The general strategy to test the statistical character of neutrinos in the cosmic sample will be to use Bose-Einstein statistics as an alternative templet in order to check how dependent the helium abundance actually is on the Fermi-Dirac distribution functions (it could well be that the choice of statistics were irrelevant). We do not consider here other kinds of statistics (e.g. parastatistics) which could also lead to sound field theories. An other place where the choice of statistics matters -apart from the rates $\lambda(n \leftrightarrow p)$ above- is the law $T = T(t)$ that enters eq. (1) since the expansion of the Universe depends on the fermionic/bosonic effective degrees of freedom. Here we choose all three neutrino species (in the rates only the electron neutrino enters) to obey Bose statistics.

In order to contrast the Fermi behaviour against an alternative behaviour one should solve eq. (1) using neutrinos with the right (F-D) statistics and find also the neutron fraction in the case of neutrinos with the wrong (B-E) statistics, i.e. we replace everywhere $(e^{E/\kappa T_{\nu}} + 1)^{-1}$ by $(e^{E/\kappa T_{\nu}} - 1)^{-1}$ and replace Pauli blocking factors $1 - f_{FD}$ for neutrinos wherever they appear in the conventional calculation (see ref. 10) by stimulated emission factors $1 + f_{BE}$. One may check at this point the self-consistency of using B-E statistics for neutrinos by proving that, at high temperature, detailed balance is satisfied, i.e. $\lambda(n \rightarrow p) = \lambda(p \rightarrow n)$, and hence equilibrium can be maintained. Indeed, for $(m_n - m_p)/\kappa T << 1$, with $T = T_\gamma = T_{\nu}$,

$$\lambda(n \rightarrow p) = \lambda(p \rightarrow n) = \text{const} \times \int_{-\infty}^{\infty} dq \, q^4 f(q/\kappa T)$$ (2)

where

$$f(x) \equiv \pm(1 + e^{-x})^{-1}(1 - e^{x})^{-1}$$

the upper (lower) sign for negative (positive) $x$.

We have implemented the above modifications in Kawano’s version [11] of the BBN code by Wagoner [12] to obtain the relative effect on the primordial abundances of $^4He$, $D$, $^3He$ and $^7Li$ induced by the change in statistics. We define the quantities,

$$\frac{\Delta a_i}{a_i} \equiv \frac{a_i(BE) - a_i(FD)}{a_i(FD)} \quad i = 1, 2, 3, 4$$ (3)
where \( a_1 \equiv Y_P \), \( a_2 \equiv [D/H]_P \), \( a_3 \equiv [{}^3{}He/H]_P \) and \( a_4 \equiv [{}^7{}Li/H]_P \). Our results are shown in figure 1.

Being the number of light neutrino species no longer a free parameter (but rather, fixed by LEP data to be 3) the only free parameter in our BBN calculation in \( \eta_{10} \), related to the baryon density \( \Omega_B \) of the Universe through

\[
\Omega_B = 0.0036 h^{-2} (T/2.726)^3 \eta_{10} 
\]

where \( T \) is the microwave background temperature today and \( h \) defines the Hubble parameter \( H = 100 h \text{ km sec}^{-1} / \text{Mpc} \). Therefore, figure 1 shows the quantities \( \Delta a_i/a_i \) as a function of \( \eta_{10} \). The effect on the helium abundance \( Y_P \) -the one we are primarily interested in since it is the one which is more severely constrained by observation- is negative and almost 4%. This number is the net result of two competing sources: a change in the weak interaction rates and a change in the expansion rate. The variation due to the Hubble expansion is necessarily positive because trading fermionic degrees of freedom for bosonic degrees of freedom implies an acceleration of the expansion rate and, as a consequence a larger amount of \( {}^4{}He \) produced. In fact, it amounts to \(+2.3\%\) relative increase as we have explicitly checked. The dominating effect lies therefore in the \( \lambda(n \leftrightarrow p) \) rates which turns out to be negative.

We are now prepared to confront our results with observation. The basic observational quantity is \( Y_P \) and we shall use the primordial abundances of other light elements \( (D, {}^3{}He \text{ and } {}^7{}Li) \) to help us fixing a value or range of values for \( \eta_{10} \). The recent detection of the Lyman \( \alpha \) line of deuterium in a Quasar Absorption System (QAS) at high redshift \((z=3.32)\) can be interpreted as determining the primordial deuterium abundance:

\[
[D/H]_P = (1.9 - 2.5) \times 10^{-4} 
\]

It is an order of magnitude larger than the measurement by the Hubble Space Telescope of the D abundance in the local interstellar medium (LISM):

\[
[D/H]_{\text{LISM}} = (1.6^{+0.07}_{-0.18}) \times 10^{-5} 
\]

Although strictly speaking there is no contradiction between eqs. (5) and (6), since eq. (6) has to be interpreted as a lower bound on the primordial D abundance

\[
[D/H]_P \geq 1.5 \times 10^{-5}, 
\]
nonetheless they cannot be made to agree when extrapolating eq. (6) deep into the past by using galactic chemical evolution models.

If one takes eq. (5) seriously then a very good overall agreement between theory and data can be achieved with a value of $\eta_{10} = 1.6 \pm 0.1$ derived from the matching of BBN and the QAS deuterium abundance measurement \cite{10}. The dependence of $[D/H]_P$ on $\eta_{10}$ is much stronger than it is the case for $Y_P$ and, hence, small variations on deuterium abundance associated to the change in statistics as the ones shown in fig. 1, do not lead to appreciable modifications in the determination of $\eta_{10}$. Therefore, we can use the same $\eta_{10}$ range in both, Fermi-Dirac and Bose-Einstein cases.

The above determination of $\eta_{10}$ in turn implies a band of values for the helium abundance $Y_P$. We display our result in fig. 2. In the figure we also collect data associated to recent analysis of observations of the helium abundance. The errors shown are $1\sigma$ errors and we refer to the original work [refs. 3-7] for a full discussion of their nature and for an appraisal of the techniques of analysis involved. We also include a weighted average (full circle). The gray shaded column on the right is the standard BBN prediction and the darker column on the left is our result for neutrinos with the wrong statistics (the width of the bands reflects both the spread in $\eta_{10}$ and the experimental uncertainty in the neutron half life: $\tau_n = 887 \pm 2.0 \ sec$ \cite{17}). To $1\sigma$, the spin-statistics connexion for neutrinos is confirmed (i.e. we can exclude the alternative templet).

However, we should investigate the dependence of our statement on the chosen value of $\eta_{10}$. As already mentioned above, other sets of data suggest lower deuterium abundances, which in turn mean higher values for $\eta_{10}$. A reasonable window in the intermediate segment of $\eta_{10}$ values should be $2.5 \leq \eta_{10} \leq 3$. In the high end of $\eta_{10}$ we may take $\eta_{10}=6$ which is comfortably larger than the upper limit $\eta_{10} = 5.27$ derived from matching primordial $^7Li$ abundance and BBN \cite{8}. Figure 3 shows the low, intermediate and high domains of $\eta_{10}$. The vertical dotted lines are the $1\sigma$ limits on the observed (average) helium abundance (the fat error bar in fig. 2). The gray and dark-gray boxes correspond to standard BBN results (“right” statistics) and to the results of our analysis (neutrinos with ”wrong” statistics), respectively. A comment on the interpretation of this figure may serve as a concluding summary on the content of the paper.

First, note that the low part of fig. 3 is just a restatement of fig. 2. It best exemplifies the purpose of this work, i.e. to show that BBN has the
potential to decide empirically whether neutrinos verify the spin-statistics theorem. The middle part of the figure tells us that B-E statistics (the alternative templet) cannot be ruled out which, of course, means that the F-D statistics of neutrinos cannot be demonstrated. It shows also that, to $1\sigma$, even standard BBN can be in trouble. Better data are therefore required. Finally, the top part of fig. 3 informs us that standard BBN does not work (even at the $2\sigma$ level) and hence the statistics issue is irrelevant in this case.

A last comment is in order. No matter what the final fate of the controversial deuterium data [13,14] shall be, the avalanche of new and better data in the coming years will certainly permit a positive discrimination of neutrino statistics. As an illustration of this point, we note that a modest 20% reduction of the error bars in the helium data suffices to make the distance between boxes in fig. 3 larger then the 1-$\sigma$ error interval.

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Figure Captions

**Fig. 1.** The various quantities $\Delta a_i/a_i$ as defined in the text, as a function of $\eta_{10}$.

**Fig. 2.** Confronting F-D statistics (gray band) and B-E statistics (dark-gray band) with He-4 data from: ref. 3 (circle), ref. 4 (diamonds) ref. 5 (triangles), ref. 6 (squares), ref. 7 (inverted triangle). The full circle is the average value. Here, $\eta_{10} = 1.6 \pm 0.1$.

**Fig. 3.** Our analysis displayed for three domains of $\eta_{10}$. The two vertical dotted lines bracket the observationally allowed He-4 abundances as given by the average abundance shown in fig. 2.
\[ \eta_{10} = 1.6 \pm 0.1 \]

Fractional helium-4 abundance by mass

Fig. 2
Fig. 3

\[ \eta_{10} = 6.0 \]

\[ 2.5 \leq \eta_{10} \leq 3.0 \]

\[ \eta_{10} = 1.6 \pm 0.1 \]

fractional helium–4 abundance by mass