Edge magnetoplasmons in graphene: Effects of gate screening and dissipation

Alexey A. Sokolik\textsuperscript{1,2} and Yurii E. Lozovik\textsuperscript{1,2,3,4}\textsuperscript{*}

\textsuperscript{1}Institute for Spectroscopy, Russian Academy of Sciences, 142190 Troit\k{e}k, Moscow, Russia
\textsuperscript{2}National Research University Higher School of Economics, 109028 Moscow, Russia
\textsuperscript{3}Dukhov Research Institute of Automatics (VNIIA), 127055 Moscow, Russia
\textsuperscript{4}Moscow Institute of Physics and Technology, 141700 Dolgoprudny, Moscow region, Russia

Magnetoplasmons on graphene edge in quantizing magnetic field are investigated at different Landau level filling factors. To find the mode frequency, the optical conductivity tensor of disordered graphene in magnetic field is calculated in the self-consistent Born approximation, and the nonlocal electromagnetic problem is solved using the Wiener-Hopf method. Magnetoplasmon dispersion relations, velocities and attenuation lengths are studied numerically and analytically with taking into account the screening by metallic gate and the energy dissipation in graphene. The magnetoplasmon velocity decreases in the presence of nearby gate and oscillates as a function of the filling factor because of the dissipation induced frequency suppression occurring when the Fermi level is located near the centers of Landau levels, in agreement with the recent experiments.

I. INTRODUCTION

Two-dimensional plasmons on graphene offer ample opportunities of applications due to wide tunability of their properties achieved by changing the doping level, confining charge carriers, by nanostructuring graphene or combining it with metal electrodes \textsuperscript{1,3}. In magnetic field the plasmon resonance splits into two magnetoplasmon modes, as found using the terahertz spectroscopy of graphene disks \textsuperscript{4,5}. The higher-frequency mode can be treated in the quasiclassical limit as two-dimensional plasma oscillations acquiring a frequency enhancement due to confining action of magnetic field \textsuperscript{6,7}. The lower-frequency mode is localized near graphene edge and propagates only in one direction determined by a magnetic field direction, so it can be guided along the edges and used to design plasmon circuits. Edge magnetoplasmons and possibilities of their manipulation were extensively studied in semiconductor-based quantum Hall systems (see, e.g., \textsuperscript{8–10} and references therein).

In several recent experiments \textsuperscript{11–15} the time-domain measurements of edge magnetoplasmon propagation on graphene were carried out, which allowed to directly determine their velocities. Similarly to that in semiconductor quantum wells \textsuperscript{8,11}, the velocity shows pronounced oscillations as a function of the Landau level filling factor, decreasing at non-integer fillings where the system is conducting and the dissipation is present. The presence of nearby metallic gate was also showed to reduce the plasmon velocity. Although the general theory of magnetoplasmons \textsuperscript{16,17} allows to estimate their velocities, the effects of screening and dissipation are insufficiently studied from the theoretical point of view. The existing approaches for graphene magnetoplasmons \textsuperscript{11,14,18} rely either on analytical formulas applicable for a clean system, or use the Drude approximation for graphene conductivity, that cannot describe the oscillating filling-factor dependencies originating from discreteness of Landau levels.

In this paper we provide the theoretical treatment of edge magnetoplasmons in graphene with taking into account gate screening, dissipation and the filling-factor dependence of graphene optical conductivity in quantizing magnetic field. In Sec. \textsuperscript{II} we consider the electromagnetic part of the problem solved using the Wiener-Hopf method and estimate magnetoplasmon frequencies both in the absence and in the presence of dissipation. In contrast to conventional calculations of a complex frequency accounting for the damping, we consider a real frequency and a complex wave vector. In Sec. \textsuperscript{III} we calculate the conductivity tensor of disordered graphene in quantizing magnetic field using the self-consistent Born approximation and length gauge which allow us to remove the $1/\omega$ divergence of conductivity and provide the qualitatively correct description of both its low-frequency and filling-factor dependencies, both being crucial to the theory of edge magnetoplasmons. In Sec. \textsuperscript{IV} we show the results of numerical calculations of the magnetoplasmon dispersions and analyze how the velocity depends on the Landau level filling factor and on the distance between graphene and metallic gate. In agreement with the experiments \textsuperscript{11–15}, we find the oscillating behavior of the velocity, which is suppressed by dissipation at non-integer Landau level fillings, and general reduction of the velocity by the gate screening. Our conclusions are presented in Sec. \textsuperscript{V}.

II. ELECTROMAGNETIC PROBLEM

Consider the magnetoplasmon wave propagating along the graphene edge, which is directed parallel to the $y$ axis, with the frequency $\omega$ and wave vector $q$ (Fig. \textsuperscript{I}). To take into account the damping, we assume the complex wave vector $\tilde{q} = q + i\alpha$ instead of a complex frequency. It is done in order to avoid complications arising in many-body calculations of the retarded conduc-
where

\[
Z(\tilde{q}, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{\tilde{q} \, dk}{k^2 + \tilde{q}^2} \ln \left\{ 1 + \frac{4\pi i\sigma_{xx}(\omega)}{\varepsilon \omega} \frac{\sqrt{k^2 + \tilde{q}^2}}{1 + \coth(d \sqrt{k^2 + \tilde{q}^2})} \right\}.
\]

Here \( \tilde{q} \) is assumed to have a positive real part, otherwise we need to replace it by \(-\tilde{q}\).

Introducing the dimensionless quantity \( \eta = \frac{4\pi \tilde{q} \sigma_{xx}(\omega)}{i\omega\varepsilon} \) [18], we can find \( Z \) as a function of \( \eta \) and \( \tilde{q}d \). We are interested in the long-wavelength and low-frequency limit, when \( \tilde{q}, \omega \to 0 \) and \( \eta \approx \sigma_{xx}(0) \). In the case of low dissipation we can assume \( |\eta| \ll 1, \tilde{q} \approx q \), and calculate the analytical asymptotics [17] of \( Z \) in this limit:

\[
Z \approx -\frac{\eta}{2\pi} \log \left( \frac{-4\epsilon d}{\eta} \right), \quad qd \gg e^{-\gamma_E}, \quad (6)
\]

\[
Z \approx -\frac{\eta}{2\pi} \log \left( \frac{-4\epsilon^{1+\gamma_E} qd}{\eta} \right), \quad |\eta| \approx qd \ll e^{-\gamma_E}, \quad (7)
\]

\[
Z \approx \sqrt{-\eta qd}, \quad qd \ll \frac{|\eta|}{4\epsilon^{1+\gamma_E}}. \quad (8)
\]

where \( \gamma_E \approx 0.577 \) is the Euler gamma constant. Eq. (8) corresponds to the case where the gate is either absent or too far to influence the magnetoplasmons. Eq. (7) corresponds to the opposite limit of local capacitance approximation [19], when (2) reduces at small \( d \) to the local relationship for the plane capacitor: \( \varphi(x) = 4\pi \rho(x) / \varepsilon \). Substituting (3)–(5) to (4), we obtain the dispersion relations for the edge magnetoplasmons [17]:

\[
\omega = -\frac{2q\sigma_{yy}(0)}{\varepsilon} \ln \frac{2q}{w}, \quad qd \gg e^{-\gamma_E}, \quad (9)
\]

\[
\omega = -\frac{2q\sigma_{yy}(0)}{\varepsilon} \ln \frac{2e^{1+\gamma_E} qd}{w}, \quad \varepsilon qd \ll \frac{2q}{w} e^{-\gamma_E}, \quad (10)
\]

\[
\omega = -\frac{2q\sigma_{yy}(0)}{\varepsilon} \sqrt{\frac{2d}{w}}, \quad qd \ll \frac{qw}{2e^{1+\gamma_E}}. \quad (11)
\]

where \( w = -\frac{2\pi}{\varepsilon} |d| \ln \sigma_{xx}(\omega) / \omega d| \rvert_{\omega=0} \). Since \( \sigma_{xx}(0) < 0 \) at \( B \propto e_z \), we have the edge mode propagating in the positive \( y \) direction (Fig. 1).

In the presence of dissipation the expressions (9)–(11) are inaccurate because in the long-wavelength and low-frequency limit probed in the experiments [11] the damping rate dominates the frequency therefore the real part of \( \sigma_{xx} \) dominates the imaginary part connected with \( w \) (in other words, the dissipation dominates electron inertial motion). Using (6) and (8) in (4) at \( \omega \to 0 \), we obtain the approximations for dispersion law and attenuation rate \( \alpha \). At large graphene-to-gate distance \( qd \gtrsim 1 \) we obtain

\[
\omega = \frac{\pi q\sigma_{xx}(0)}{\epsilon \varepsilon \text{Im} Y}, \quad \alpha = -\frac{\text{Re} Y}{\text{Im} Y}, \quad (12)
\]
where $Y$ is the complex solution of the equation $1 - iXY \ln Y = 0$ with $Re Y < 0$, $Im Y > 0$, and $X = -2e\sigma_{xy}(0)/\pi\sigma_{xx}(0)$. At very small distances, when $q d \lesssim 0.01$, we obtain

$$\omega = \frac{8\pi q^2 \sigma_{xy}(0) d}{\varepsilon \sigma_{xx}(0)}, \quad \alpha = q.$$  \hspace{1cm} (13)

Note that, in contrast to the long-wavelength limit of the solution in Ref. [19] with complex $\omega$ and real $q$, where $\omega$ is purely imaginary, here we have the oscillations highly damped in space with $\tilde{q} = q(1 + i)$. At larger distances or wave vectors, when $q d \gtrsim 0.01$, the dispersion becomes linear and attenuation rate decreases.

III. OPTICAL CONDUCTIVITY IN MAGNETIC FIELD

To calculate the optical conductivity tensor $\sigma_{\alpha\beta}$ in disordered graphene in quantizing magnetic field we use the version of the self-consistent Born approximation [20] which allows us to take into account both formation of Landau levels and their disorder-induced broadening. In this approximation the single-electron Green functions are dressed by interaction with random disorder potential, which results in broadening of each Landau level, and then the current vertex is modified by a disorder ladder. Direct application of the Kubo formula to calculate the current response to the oscillating vector potential $A = (e/\omega)E \propto e^{-i\omega t}$ provides the dynamical conductivity

$$\tilde{\sigma}_{\alpha\beta}(q, \omega) = i\tilde{g}_{\alpha\beta}^{R}(q, \omega)$$  \hspace{1cm} (14)

in terms of the Fourier transform $G_{j_\alpha j_\beta}(q, \omega) = -iS^{-1} \int \frac{d\mathbf{r} d\mathbf{r}'}{2\pi^2} \int_0^\infty dt e^{-i(q(r-r')+i\omega t)} G_{\alpha\beta}(\mathbf{r}, \mathbf{r}', 0))$ of the retarded Green function of the current $j_\alpha(\mathbf{r}, t)$, $j_\beta(\mathbf{r}', 0))$) of the current vertex $\psi(r, t)$ and the components of the Heisenberg field operator of massless Dirac electrons in the valley $K$ of graphene, $\tilde{g}_{\alpha\beta} \approx 10^6$ m/s is the Fermi velocity, $g = 4$ is the degeneracy over the valleys and spin projections, $S$ is the system area.

However, as noted in Ref. [22], due to the prefactor $1/\omega$ in (14) this form of conductivity does not obey the Kramers-Kronig relations and diverges in the limit $\omega \to 0$ which we are interested in. To obtain a correct low-frequency behavior we need either to calculate $\sigma_{xy}$ in the $p \cdot A$ gauge with paying attention to its causality properties of to use the $E \cdot r$ gauge. The latter can be applied in the dipole long-wavelength limit $qd \ll 1$ probed in the experiments [11,13], where $l_H = \sqrt{hc/eB}$ is the magnetic length defining the spatial scale of Landau level wave functions. Using the commutation relations $[H, r_n] = -i\hbar v_F \sigma_n$ and the spectral representation

$$G_{j_\alpha j_\beta}^{R}(0, \omega) = \int_{-\infty}^{+\infty} d\omega' \frac{S_{j_\alpha j_\beta}(\omega')}{\omega - \omega' + i\delta},$$  \hspace{1cm} (15)

we find that both methods give the same result for the optical conductivity

$$\sigma_{\alpha\beta}(\omega) = \frac{i\tilde{g}_{\alpha\beta}^{R}}{\hbar} \int_{-\infty}^{+\infty} d\omega' \frac{S_{j_\alpha j_\beta}(\omega')}{\omega' - \omega' + i\delta}$$  \hspace{1cm} (16)

at $q \to 0$ in terms of the spectral function $S_{j_\alpha j_\beta}$; here $\delta \to +0$.

The self-consistent Born approximation provides the following expression for the Green function of currents (see also [21,22]):

$$G_{j_\alpha j_\beta}^{R}(0, \omega) = -\frac{e^2 \tilde{g}_{\alpha\beta}^{2}}{2\pi^2 l_H^2} \sum_{n_1 n_2} \int_{-\infty}^{+\infty} F_{n_1 n_2}^{R} dz \frac{1 - \gamma_{n_1 n_2} G_{n_1 n_2}^{R} \gamma_{n_1 n_2} G_{n_1 n_2}^{R}}{1 - \gamma_{n_1 n_2} G_{n_1 n_2}^{R}} \left\{ \nu_{F}(z) G_{n_1} \text{Im} G_{n_2} + \frac{\nu_{F}(z + \omega) \text{Im} G_{n_1} G_{n_2}^{*} + \nu_{F}(z + \omega) \text{Im} G_{n_1} G_{n_2}^{*}}{1 - \gamma_{n_1 n_2} G_{n_1 n_2}^{R}} \right\},$$  \hspace{1cm} (17)

where $G_{n_1} = G_{n_1}^{R}(z + \omega)$ and $G_{n_2} = G_{n_2}^{R}(z)$ are the retarded Green functions of electrons on Landau levels with the numbers $n_1, n_2 = 0, \pm 1, \pm 2, \ldots$, $\nu_{F}(z) = \text{sgn}(\hbar^2 - m^2)\nu_{F}(z)$ is the Fermi-Dirac distribution, $\gamma_{n_1 n_2} = (2\pi l_H^2/2)^{1/2} \sum_k \langle \langle \psi_{n_1 k'}^{*}(U|\psi_{n_1 k}|\psi_{n_2 k'}^{*}(U|\psi_{n_2 k}) \rangle \rangle_U$ is the matrix element of the disorder potential $U$ averaged over its realizations, which appears in the disorder ladder, $\psi_{nk}$ is the electron state on the nth Landau level with the kth guiding center index. The factor $F_{n_1 n_2}^{R} = 2\nu_{F}(z + \omega)\nu_{F}(z + \omega)$ corresponding to the Lorentzian spectral density $\rho_{n}(\omega) = (\hbar^2 - m^2)/(\hbar^2 - E_n)^2 + \Gamma^2$ instead of half-elliptic densities of the Kramers-Kronig relations and behaves correctly in the $\omega \to 0$ limit:

$$\sigma_{\alpha\beta}(\omega) = \frac{i\tilde{g}_{\alpha\beta}^{R}}{2\pi l_H^2} \sum_{n_1 n_2} \int dz_1 dz_2 \rho_{n_1}(z_1) \rho_{n_2}(z_2) \frac{\nu_{F}(z_2) - \nu_{F}(z_1)}{(z_1 - z_2)(\omega - z_2 - z_1 + i\delta)},$$  \hspace{1cm} (18)

The integrals in (18) can be calculated analytically in the limit $T \to 0$, $\nu_{F}(z) \to \Theta(\mu - h \omega)$, corresponding to the experiments [11,13] carried out at cryogenic temperatures. The chemical potential $\mu$ can be connected with the Landau level filling factor: $\nu = \sum_{n} \frac{1}{2} \text{sgn}(n) + \int dz \nu_{F}(z) \rho_{n}(z)$; $\nu$ is zero for undoped graphene, where
the 0th Landau level is half-filled, and increases by 4 for each fully filled Landau level because of the fourfold degeneracy of electron states, so it equals $4n + 2$ when the $n$th level is completely filled and $4n$ when the $n$th level is half-filled. We do not take into account Zeeman splitting and formation of fractional Hall states because they become important either in very high magnetic fields of in very clean samples.

In the limit of high doping or low magnetic field, when the chemical potential $\mu$ is located between $E_n$ and $E_{n+1}$, a single intraband transition $n \rightarrow n + 1$ provides a major contribution to the low-frequency behavior of Landau levels [Fig. 2(b,d)]. The conductivity behavior at low frequencies is close to the Drude model predictions, especially at high filling factors. At non-integer filling of Landau levels [Fig. 2(a,c)], the conductivity deviates from the Drude model. Note the marked increase of $\sigma_{xx}$ at low frequencies $|\omega| \lesssim \Gamma$ indicating the dissipation due to intralevel transitions. The second important difference is the positive derivative $d \Im \sigma_{xx}(\omega)/d\omega|_{\omega=0} > 0$ at the half-integer filling in Fig. 2(a), which is indicative of effectively free electrons moving within the Landau level at $|\omega| \lesssim \Gamma$, being in contrast to the negative $d \Im \sigma_{xx}(\omega)/d\omega|_{\omega=0} < 0$ at the integer filling in Fig. 2(c) typical to bound electrons. In the latter case this derivative is related to the quantity $w$ in [17], which can be interpreted [17] as a distance where the energy of two-dimensional plasma oscillations is comparable with the cyclotron energy. At half-integer fillings, when the dissipation is significant, this quantity has no such meaning, and the formulas (19)–(20) are inapplicable as well.

Although our conductivity demonstrates the qualitatively correct low-frequency properties and properly takes into account the Landau level quantization, further improvement is needed to achieve quantitative agreement with the experiment. For example, the peaks in the static $\rho_{xx}(0)$ [see inset in Fig. 2(b)] and the simultaneously occurring rising parts in the dependence of $\sigma_{xy}(0)$ on $\nu$ between the quantized plateaus [inset in Fig. 2(c)] are characteristic to conducting states at non-integer Landau level fillings are broader than in the typical quantum Hall effect measurements [11, 22]. From the other side, the broadening of these peaks at nonzero frequencies studied in semiconductor quantum wells [26, 27] should also been taken into account. The conductivity model which explicitly includes consideration of localized and extended states would provide more accurate results in the low-frequency region.

IV. CALCULATION RESULTS

Using the formulas (19), (20), (15), we can calculate numerically the dispersion relations $\omega(q)$ and attenuation rates $\alpha(q)$ at different filling factors $\nu$ and graphene-to-gate distances $d$ to study the effects of gate screening and dissipation. We take the parameters $B = 12$ T, $\Gamma = 5$ meV, $\varepsilon = 4$, which are close to the experimental conditions [11, 13] where Landau quantization is well developed.

In Fig. 3 we show typical examples of $\omega(q)$ and $\alpha(q)$ calculated in the absence of the gate screening, at $d = \infty$, with the full Landau-level-based conductivities (15) and within the Drude model (19)–(20) at the same carrier density. At integer Landau level fillings $[\nu = 6$, Fig. 3(b,d)] $\omega$ and $\alpha$ are, respectively, slightly higher and significantly lower than in the Drude model. This indicates that the dissipation, which suppresses $\omega$ and increases $\alpha$, is lower than in the Drude model due to the inter-Landau level gap. At half-integer Landau level fill-
ings \([\nu = 4, \text{Fig. 3(a,c)}]\) the situation is opposite: the dissipation caused by the intralevel transitions slightly suppresses \(\omega\) and significantly increases \(\alpha\) in comparison with the Drude model. Note also the pronounced dissipation-induced decrease of \(\omega\) and increase of \(\alpha\) at \(\nu = 4\) in comparison with \(\nu = 6\). The numerical calculations in both models are close to the analytical approximation \((12)\) where the corresponding conductivities at \(\omega = 0\) are substituted. For comparison we plotted the magnetoplasmon frequency calculated with the Drude conductivity of a clean \((\gamma = 0)\) system, which is higher than in the disordered system and agrees with the analytical approximation \((11)\) very well.

The similar calculation results in the presence of the screening gate at \(d = 200\) nm are shown in Fig. 4. We see the overall suppression of \(\omega\) in comparison with the ungated case, which becomes even stronger in the presence of dissipation. At integer \([\nu = 6, \text{Fig. 3(b,d)}]\) and half-integer \([\nu = 4, \text{Fig. 4(a,c)}]\) Landau level fillings we again see the effect of, respectively, decreased and enhanced dissipation on \(\omega\) and \(\alpha\). The analytical approximation \((13)\) predicting highly damped mode with quadratic dispersion at small \(d\) is applicable only at very low distances or wave vectors, \(qd \lesssim 0.01\), while at higher \(q\) the dispersion laws become linear. Calculations with the Drude conductivity of a clean system agree with \((13)\) and provide considerably higher \(\omega\).

In Fig. 5 we show the edge magnetoplasmon phase velocity \(\nu = \omega/q\) and quality factor \(Q = q/2\alpha\) calculated at \(q = 0.2\) \(\mu m^{-1}\) (where dispersions are almost linear) as functions of the filling factor \(\nu\) both in the absence and in the presence of the metallic gate. As expected from the aforementioned dissipation-induced frequency suppression, we observe the dips in both \(\nu\) and \(Q\) when the Fermi level is located in the centers of Landau levels \((\nu = 4, 8, 12, 16)\). In Fig. 5(a) these dips are slightly displaced to the left, perhaps, due to the general rising trend of \(v(\nu)\), and also demonstrate some extra oscillations, which can be an artefact of the model used to calculate the conductivity. On the contrary, when the Fermi level is located in the middle of any inter-Landau level gap \((\nu = 2, 6, 10, 14)\), \(v\) and \(Q\) have peaks due to reduced dissipation. The oscillations of \(v(\nu)\) and \(Q(\nu)\) occur around the smooth results of the quasiclassical Drude model, which is insensitive to how the individual Landau levels are filled. Comparison with the calculations for a clean system shows that the gate screening not only reduces the velocity, but also enhances the dissipation-induced suppression of \(v\) and \(Q\). As in the previous picture, the analytical formula \((12)\) well describes the dispersion and damping at \(d = \infty\), while the formula \((13)\) for small \(d\) is applicable only in the regions of low velocity and high dissipation.

V. CONCLUSIONS

We considered the magnetoplasmon modes propagating along graphene edge in quantizing magnetic fields in the presence of the grounded metallic gate. The relationship between real frequencies and complex wave vectors (with the imaginary part responsible for the damp-
FIG. 5: Edge magnetoplasmon velocities $v$ (a,b) and quality factors $Q$ (c,d) as functions of the Landau level filling factor $\nu$ at $q = 0.2 \mu m^{-1}$ in the absence [$d = \infty$, (a,c)] and in the presence [$d = 200 \text{ nm}$, (b,d)] of the gate screening at $B = 12 \text{T}$, $\Gamma = 5 \text{ meV}$, $\varepsilon = 4$. The curves show numerical calculation results with Landau-level based (LL) and Drude conductivities, and with Drude conductivity in a clean system, and the points show analytical approximations. The notations for curves and points on the panels (a,c) and (b,d) are the same as in, respectively, Figs. 3 and 4.

The optical conductivity tensor $\sigma_{\alpha\beta}(\omega)$ was calculated at low temperatures using the self-consistent Born approximation for graphene in magnetic field in the limit of short-range impurities and with the assumption of Lorentzian broadening of Landau levels. To obtain a physically correct behavior of the conductivity at low frequencies, the length (or $E\cdot r$) gauge was used. The quasiclassical Drude approximation to the conductivity, valid in the limit of large number of filled Landau level, was also considered for comparison. The magnetoplasmon dispersions, attenuation rates, velocities and quality factors were calculated both in the absence and in the presence of the gate at different Landau level filling factors.

The analysis of the calculation results allows us to make the following conclusions:

(a) At integer filling of Landau levels ($\nu = 2, 6, 10, \ldots$), where the Fermi level is located in the interlevel gap, the dissipation is low, and the frequency, velocity and life time of the edge magnetoplasmon are increased in comparison to the predictions of the Drude model. On the contrary, at half-integer filling of Landau levels ($\nu = 4, 8, 12, \ldots$), where the Fermi level lies within a broadened Landau level, the dissipation is enhanced due to intralevel transitions, and the frequency, velocity and life time of the edge magnetoplasmon are suppressed. So our approach predicts the oscillations of the edge magnetoplasmon velocity when the filling factor is changed, which are superimposed on the smooth trend of growing velocity as the carrier density increases. The commonly used Drude conductivity model describes only the latter because does not take into account Landau level quantization.

(b) The electric field screening caused by a nearby metallic gate decreases the frequency and velocity of the mode, and makes its dissipation-induced suppression at non-integer filling of Landau levels much more pronounced.

(c) The analytical formulas [11–12] for the mode frequency, which are frequently used to analyze the experimental data [11–13], are applicable only for very clean systems. The quantity $w$ entering these formulas lose its meaning at half-integer Landau level fillings, when the dissipation is significant. Eq. (12) can be used instead in the case when the gate screening is negligible. The important feature of the experimental conditions is that plasmons are probed at relatively low frequencies not exceeding the terahertz range, so any realistic rate of dissipation due to inter-Landau level transitions dominates the frequency and substantially modify mode properties in comparison with the predictions for a clean system.

Our conclusions are in qualitative agreement with the experimental data [11–13], where the suppression of the magnetoplasmon velocities at non-integer Landau level fillings was observed. Our approach assume the abrupt edge of graphene and does not take into account the details of spatial structure of Landau level wave functions near the edge, which include formation of incompressible stripes [19], edge channels [9, 14, 28], and electron drift due to electric field normal to the edge [12, 13]. Analysis of these details requires an essential complication of our approach and is beyond the scope of this paper.

To study the effects of Landau level filling on the edge magnetoplasmon we used the model for the optical conductivity providing qualitatively correct description of the low-frequency behavior and the filling-factor dependence. A refined model, which takes into account the localized states and their scaling properties, can provide an additional insight into the theory of edge magnetoplasmons in quantum Hall regime and bring the results of our approach into better quantitative agreement with the experiments.

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