Letter

Protecting and enhancing spin squeezing from decoherence using weak measurements

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Abstract

We propose an efficient method to protect spin squeezing under the action of amplitude-damping, depolarizing and phase-damping channels based on measurement reversal from weak measurement, and consider an ensemble of \(N\) independent spin-1/2 particles with exchange symmetry. We show that spin squeezing can be enhanced greatly under three different decoherence channels and spin-squeezing sudden death can be avoided undergoing amplitude-damping and phase-damping channels. Moreover, we find that the spin-squeezing parameters can be completely recovered to its initial value under amplitude-damping channel by changing weak measurement strength irrespective of sufficiently strong decoherence. For the phase-damping channel, the spin-squeezing parameter can be recovered to a certain stationary value in certain conditions. Our results provide an active way to suppress decoherence and enhance the spin squeezing under decoherence, which is rather significant for quantum metrology in open systems.

Keywords: spin squeezing, weak measurement, sudden death, decoherence

(Some figures may appear in colour only in the online journal)

1. Introduction

Spin squeezing has attracted a lot of attention in both the theoretical and experimental fields for decades [1–8]. An important application of spin squeezing is to detect quantum entanglement [9–11]. Due to the fact that spin squeezing is relatively easy to generate and measure [2, 12–14], spin-squeezing parameters are multipartite entanglement witnesses in a general sense. Lots of efforts have been devoted to find relations between spin squeezing and entanglement [4–7, 15–17]. Another application of spin squeezing is to improve the precision of measurements such as leading-noise reduction [18] and improving atomic sensor precision [19]. Thus, spin-squeezed states are useful resources for quantum metrology and other quantum information processing tasks. However, the interactions between the system and its environment usually cause decoherence. In practice, decoherence is inevitable and harmful to spin squeezing and entanglement [20–26].

Analogous to the definition of entanglement sudden death (ESD) [27] and distillability sudden death (DSD) [28], spin squeezing can also suddenly vanish with different lifetimes for some decoherence channels, showing in general different vanishing times in multipartite correlations of quantum many-body systems. Wang et al [25] have found that, under local
decoherence, spin squeezing also appears as sudden death similar to the discovery of pairwise entanglement sudden death. A method to protecting and enhancing spin squeezing via continuous dynamical decoupling has been proposed by Chaudhry and Gong [29].

Weak measurements are generalizations of von Neumann measurements and are associated with positive operator valued measures (POVM). For weak measurements [30, 31], the information extracted from the quantum system is deliberately limited, thereby keeping the measured state alive (i.e. without completely collapsing towards an eigenstate). Thus, it would be possible to reverse the initial state with some operations. Weak measurement is very useful and can help understanding many counterintuitive quantum phenomena, for example, Hardy’s paradoxes [32]. Recently, the weak measurement has been applied as a practically implementable method for protecting entanglement, quantum fidelity of quantum states undergoing decoherence [33–37] and improving payoffs in the quantum games in the presence of decoherence [38]. Lee et al. [39] investigated how weak measurement affects the spin squeezing generated by the two-axis countertwisting Hamiltonian. Their study are combining weak measurement with Hamiltonian evolution instead of decoherence. The study on protecting spin squeezing under the action of decoherence and avoiding spin-squeezing sudden death with the technique of weak measurements is not involved so far.

Motivated by recent studies of decoherence effects on spin squeezing and the application of weak measurement, we propose an efficient method to avoid spin-squeezing sudden death via measurement reversal from weak measurement, and consider an ensemble of $N$ independent spin-1/2 particles with exchange symmetry.

The rest of this paper is organized as follows. In section 2, we introduce the initial state from the one-axis twisting Hamiltonian and then, in section 3, the definitions of spin squeezing and concurrence. In section 4, we show how the spin squeezing could be enhanced under the action of three different decoherence channels by weak measurements. Both analytical and numerical results are given. We offer our conclusions in section 5.

2. Initial state

We consider an ensemble of $N$ spin-1/2 particles with ground state $|0\rangle$ and excited state $|1\rangle$. This system has exchange symmetry, and its dynamical properties can be described by the collective operators

$$J_\alpha = \frac{1}{2} \sum_{k=1}^{N} \sigma_{\alpha k}$$

for $\alpha = x, y, z$. Here, $\sigma_{\alpha k}$ are the Pauli matrices for the $k$th qubit.

We choose the initial state as a standard one-axis twisted state [1]

$$|\psi(0)\rangle = e^{-i\theta J_z^T}|\downarrow \ldots \downarrow\rangle$$

This state is prepared by the one-axis twisted Hamiltonian $H = \chi J_z^T$, with the coupling constant $\chi$, and $\theta = 2\chi t$ the twist angle.

3. Spin squeezing and concurrence

There are several spin-squeezing parameters, but we list only three typical and related ones as follows [1–5]:

$$\xi_1^2 = \frac{4(\Delta J_{||})^2}{N}$$

$$\xi_2^2 = \frac{N^2}{4\langle \hat{J}^2 \rangle}$$

$$\xi_3^2 = \frac{\lambda_{min}}{\langle \hat{J}^2 \rangle - \frac{N}{2}}$$

Here, the minimization in the first equation is over all directions denoted by $\hat{n}_i$, which are perpendicular to the mean spin direction $\langle \hat{J}\rangle / \langle \hat{J}^2 \rangle$; $\lambda_{min}$ is the minimum eigenvalue of the matrix $\Gamma = (N-1)\gamma + C$

where

$$\gamma_{ij} = C_{ij} - \langle J_i \rangle \langle J_j \rangle$$

for $k, l \in \{x, y, z\} = \{1, 2, 3\}$

is the covariance matrix and $C = [C_{ij}]$ with

$$C_{ij} = \frac{1}{2} \langle [J_i, J_j] \rangle$$

is the global correlation matrix. The parameters $\xi_1^2$, $\xi_2^2$ and $\xi_3^2$ were defined by Kitagawa and Ueda [1], Wineland et al. [2], and Tóth et al. [4], respectively. If $\xi_3^2 < 1$ ($\xi_3^2 > 1$) spin squeezing occurs, and we can safely say that the multipartite state is entangled.

For states with a well-defined parity (even or odd), we now express the squeezing parameters in terms of local expectations and correlations [7, 25]

$$\xi_1^2 = 1 + 2(N-1)(\langle \sigma_1 \sigma_2 \rangle - \frac{1}{2})$$

$$\xi_2^2 = \frac{\xi_1^2}{\langle \sigma_1^2 \rangle}$$

$$\xi_3^2 = \frac{\min(\xi_1^2, \xi_2^2)}{(1 - N^{-1})(\langle \sigma_1^2 \rangle + N^{-1})}$$

where

$$\zeta^2 = 1 + (N-1)(\langle \sigma_1 \sigma_2 \rangle - \langle \sigma_1 \rangle \langle \sigma_2 \rangle)$$

For convenience, hereafter we use

$$\zeta_k^2 = \max(0, 1 - \zeta_k^2), k \in \{1, 2, 3\}$$
to characterize spin squeezing. With the above definition, spin squeezing occurs when \( \zeta^2 > 0 \).

The close relations between the spin squeezing and pairwise entanglement are meaningful and help to understand quantum correlations. To study relationships between spin squeezing and pairwise entanglement under decoherence, we should consider pairwise entanglement in symmetric multi-qubit systems. We take the concurrence [40] as such a measure. The concurrence is defined as

\[
C = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)
\]

(14)

where \( \lambda_i \) are the square roots of eigenvalues of \( \tilde{\rho}\tilde{\rho}^\dagger \). Here \( \rho \) is the reduced density matrix of the system, and

\[
\tilde{\rho} = (\sigma_i \otimes \sigma_j)\rho (\sigma_i \otimes \sigma_j)
\]

(15)

where \( \tilde{\rho} \) is the conjugate of \( \rho \).

Due to the exchange symmetry, pairs of particles can be extracted from a symmetric state of two-level systems. Since the mean spin of the initial state (2) is along the \( z \)-direction, the two-spin reduced density matrix can be written as a block-diagonal form [7]

\[
\rho_{zz} = \begin{bmatrix} u & w \n w & v \end{bmatrix} \oplus \begin{bmatrix} y & w \n w & y \end{bmatrix}
\]

(16)

in the basis \( |00\rangle, |11\rangle, |01\rangle, |10\rangle \), where

\[
u_\pm = \frac{1}{4}(1 \mp 2(\sigma_{zz} + \langle \sigma_1 \sigma_2 \rangle))
\]

\[
w = \frac{1}{4}(1 - \langle \sigma_1 \sigma_2 \rangle)
\]

\[
u = \langle \sigma_+ \sigma_2^+ \rangle
\]

\[
y = \langle \sigma_+ \sigma_2^- \rangle
\]

(17)

the concurrence is given by

\[
C = \max\{0, 2(|u| - w), 2(y - \sqrt{u_v})\}.
\]

(18)

From the above expressions of the spin-squeezing parameters and concurrence, we notice that if we know the expectation \( \langle \sigma_z \rangle \), and the correlations \( \langle \sigma_+ \sigma_- \rangle \), \( \langle \sigma_+ \sigma_2^+ \rangle \), and \( \langle \sigma_+ \sigma_2^- \rangle \), all the squeezing parameters and concurrence can be determined.

4. Protecting spin squeezing from decoherence by weak measurements

We propose a scheme to protect spin squeezing under the action of decoherence channels by using weak measurement. The scheme is weak measurement \( M + \) decoherence channel + weak measurement \( N \).

The effect of quantum channels on the state of a system is a completely positive and trace-preserving map that is described in terms of Kraus operators.

\[
\rho_m = |\psi(0)\rangle\langle\psi(0)| \rightarrow \epsilon_{\text{channel}}(\rho_m) = \sum_l E_l|\psi(0)\rangle\langle\psi(0)|E_l^\dagger.
\]

(19)

The operator \( E_l \) satisfies the CPTP relation \( \sum_l E_l^\dagger E_l = I \).

In order to protect and improve the spin squeezing, we should perform weak measurement \( M \) and measurement reversal \( N \), before and after the decoherence channel, respectively. The two weak measurements can be written, respectively, as a non-unitary quantum operation [41]

\[
M = \begin{bmatrix} 1 & 0 \\
0 & m \end{bmatrix} \quad N = \begin{bmatrix} n & 0 \\
0 & 1 \end{bmatrix}
\]

(20)

where the measurement strengths \( m, n \in [0, \infty) \). \( M \) is the projective measurement when \( m = 0 \). And when \( n \in (0,1) \), it is a measurement partially collapsing the quantum system to the ground state. When \( m \in (1, \infty) \), \( M \) partially collapses the quantum system to the excited state. Usually a weak measurement can be parametrized as \( M = \text{diag}(1, \sqrt{1-p}, 1) \), and the measurement reversal operator is written as \( N = \text{diag}(\sqrt{1-p}, 1, 1) \). In our article, for convenience and generality, we use \( M = \text{diag}(1, m) \) and \( N = \text{diag}(n, 1) \) with \( m, n \in [0, \infty) \).

After these weak measurements being implemented, the state becomes

\[
\Theta(\rho_m) = \frac{N_{\text{channel}}(M\rho_m M^\dagger)N^\dagger}{\text{Tr}(N_{\text{channel}}(M\rho_m M^\dagger)N^\dagger)}
\]

(21)

where \( \epsilon_{\text{channel}} \) is defined by equation (19). Since the local decoherence and weak measurement are independent and identical, the exchange symmetry is not affected by the decoherence and weak measurement. The spin squeezing can then calculated by the dynamics of the local expectations and correlations. It is easy to check that an expectation value of the operator \( A \) can be calculated as

\[
\langle A \rangle = \text{Tr}[\Theta(\rho_m)A] = \text{Tr}[\Theta^+(A)\rho_m].
\]

(22)

Thus, we can calculate the expectation value via the above equation, which is very similar to the standard Heisenberg picture.

4.1. Amplitude-damping channel

A single qubit Kraus operators for amplitude-damping channel(ADC) is

\[
E_0 = \sqrt{p}|0\rangle\langle 0| + |1\rangle\langle 1|, \quad E_1 = \sqrt{1-p}|1\rangle\langle 0|
\]

(23)

where \( p = 1 - s, s = \exp(-\gamma t/2) \) and \( \gamma \) is the damping rate.

Based on the above approach and the Kraus operators for the ADC given by equation (23), when \( sn^2 + p = m^2 \), we find the evolutions of the following expectations under decoherence using weak measurement (see appendix for details):

\[
\langle \sigma_z \rangle = [sn^2(\sigma_{z0}) - p]F_1
\]

\[
\langle \sigma_+ \sigma_- \rangle = sn^2(\sigma_+ \sigma_-)F_1^2
\]

\[
\langle \sigma_+ \sigma_2^+ \rangle = [sn^2(\sigma_+ \sigma_2^+) - 2sn^2 p(\sigma_{z0}) + p^2]F_1^2
\]

\[
\langle \sigma_+ \sigma_2^- \rangle = [sn^2(\sigma_+ \sigma_2^-) - 2sn^2 p(\sigma_{z0}) + p^2]F_1^2
\]

(24)
where $\langle \sigma_1 \rangle_0 = -\cos N^{-1}(\theta/2)$, $\langle \sigma_1 \sigma_2 \rangle_0 = 2^{-1}(1 + \cos N^{-2}(\theta))$, $F_1 = s n^2 + p = m^2$. Substituting the relevant expectation values and the correlation function into equations (9)–(11) leads to the explicit expression of the spin-squeezing parameters

$$\xi_1^2 = 1 - \frac{s n^2 n C_0(0)/F_1^2}{1}, \quad (25)$$

$$\xi_2^2 = \frac{\xi_1^2}{[s n^2(\langle \sigma_1 \rangle_0 - p)/F_1]^2}, \quad (26)$$

$$\xi_3^2 = \frac{\xi_1^2}{(1 - N^{-1})Q_1 + N^{-1}}, \quad (27)$$

where $C_0(0) = (N - 1)C_0$, $C_0 = \frac{1}{4}[(1 - \cos N^{-2}\theta)^2 + 16 \sin^2 (\theta/2) \cos^{2N-4}(\theta/2)]^{1/2} - 1 + \cos N^{-2}\theta$.

The expression of concurrence can be simplified to [25]

$$C_r = 2(N - 1) \max \{0, |u|/F_1^2 - w \} \quad (28)$$

where $u = -\frac{1}{4}s n^2 n Q_{12}, -s n^2 n u_0, w = \frac{1}{4}(1 - \langle \sigma_1 \sigma_2 \rangle_0)$, with $Q_{12} = \frac{1}{2}(1 - \cos N^{-2}\theta)$, $u_0 = -\frac{1}{8}(1 - \cos N^{-2}\theta) - \frac{i}{2} \sin(\theta) \cos N^{-2}(\theta)$.

In figure 1, we plot the spin-squeezing parameters and concurrence against the decoherence strength $p$ under amplitude damping channel for different weak measurement strengths $m = 2, 4, 30$. It clearly shows that as the decoherence strength $p$ increases, the spin squeezing decreases without weak measurement. For the smaller value of $\theta$, there is no ESD and spin-squeezing sudden death (SSSD). They vanish only in the asymptotic limit (see figure 1(a)). However, we are able to enhance spin-squeezing parameters and the concurrence greatly by using weak measurement. Especially, they don’t disappear in the asymptotic limit (i.e. $p = 1$). Moreover, with the increase of $m$, spin-squeezing parameters and the concurrence becomes a fixed value respectively. The spin-squeezing parameters and the concurrence can be completely recovered to its initial value respectively regardless of the decoherence when weak measurement strength is large (see figure 1(d)). It seems that decoherence has no effect on the spin-squeezing parameters and the concurrence. This result can be explained as follows. According to $s n^2 + p = m^2$, we have $n^2 \gg 1$ when the weak measurement strength $m^2 \gg 1$. And, we obtain $s n^2 = m^2$. From equation (24), we can obtain the expectations as follows

$$\langle \sigma_1 \sigma_2 \rangle = \langle \sigma_1 \sigma_2 \rangle_0 \quad \frac{Q_1}{Q_0} = \langle \sigma_1 \sigma_2 \rangle = 1 \quad (29)$$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Spin-squeezing parameters $\varsigma_2^2$ (dash-dotted line), $\varsigma_3^2$ (dashed line) and the concurrence $C_r$ (solid line) versus the decoherence strength $p$ for the amplitude-damping channel with $\theta = 0.1\pi$, $N = 12$. (a) Without weak measurement; (b) weak measurement strength $m = 2$; (c) $m = 4$; (d) $m = 30$.}
\end{figure}
Thus, the spin-squeezing parameters and concurrence can be calculated as

$$
\begin{align*}
\zeta_1^2 & = 1 - C_r(0) \\
\zeta_2^2 & = \frac{\zeta_1^2}{\langle \sigma_z \rangle_0^2} \\
\zeta_3^2 & = \zeta_4^2 \\
C_r & = \zeta_3^2 = C_r(0).
\end{align*}
$$

(30)

So, the spin-squeezing parameters and the concurrence can be completely recovered to its initial values respectively when weak measurement strength is very large. The overlap of the solid line and the dashed line in figure 1(d) due to the fact that the spin squeezing $\zeta_3^2$ and the concurrence $C_r(0)$ are equivalent for the initial state equation (2).

We plot the spin-squeezing parameters and concurrence against the decoherence strength $\rho$ under amplitude damping channels for different weak measurement strength $m$ with $\theta = 1.8\pi$, $N = 12$ in figure 2. For larger values of $\theta$, as the decoherence strength $\rho$ increases, the spin squeezing decreases until it suddenly vanishes, so the phenomenon of ESD and SSSD occurs when there is no weak measurement (see figure 2(a)). However, the spin-squeezing parameters and concurrence can be improved greatly by using weak measurement. Moreover, with the increase of $m$, the phenomenon of ESD and SSSD can be avoided. When the measurement strength $m$ is very large, the spin-squeezing parameters and the concurrence can be completely recovered to its initial value respectively no matter what the decoherence parameter is (see figure 2(d)).

4.2. Depolarizing channel

A single qubit Kraus operators for depolarizing channel (DPC) is

$$
\begin{align*}
E_0 & = \sqrt{1 - p'}I, & E_i & = \frac{p'}{3} \sigma_i \\
E_2 & = \sqrt{\frac{p'}{3}} \sigma_1, & E_3 & = \sqrt{\frac{p'}{3}} \sigma_2
\end{align*}
$$

(31)

where $p' = 3p/4$ and $I$ is the identity operator.

From equation (22) and the Kraus operators for the DPC given by equation (31), when $m = 1$, we find the evolutions of the following expectations under decoherence using weak measurement (see appendix for details):
where
\[
\langle n_z \rangle = \frac{1}{2} [(n^2 + s)\langle n_z \rangle_0 + (n^2 - 1)]/F_2
\]
\[
\langle n_1 n_2 \rangle = s^2 n^2 \big(\langle n_1 n_2 \rangle_0 - \langle n_1 \rangle_0 \langle n_2 \rangle_0 \big)/F_2^2
\]
\[
\langle n_1, n_2 \rangle = \frac{1}{4} [(n^2 + s)^2 \langle n_1, n_2 \rangle_0 + 2(n^2 - 1)(n^2 + s)\langle n_1 \rangle_0 + (n^2 - 1)^2]F_2^2
\]
\[
Q_2 = \langle \sigma_1, \sigma_2 \rangle = (s^2 n^2 (1 - (\langle n_1, n_2 \rangle_0 + \frac{1}{4}(n^2 + s)^2 (\langle n_1 \rangle_0 + (n^2 - 1))) + 2(n^2 - 1)(n^2 + s)\langle n_1 \rangle_0 + (n^2 - 1)^2)/F_2^2
\]
where \( F_2 = \frac{1}{2} [(n^2 + s)\langle n_z \rangle_0 + (n^2 + 1)] \). Substituting the relevant expectation values and the correlation function into equations (9)–(11) leads to the explicit expression of the spin-squeezing parameters
\[
\xi_1^2 = 1 - s^2 n^2 C_{1}(0)/F_2^2;
\]
\[
\xi_2^2 = \frac{\xi_1^2}{(1/s^2 (n^2 + s)\langle n_1 \rangle_0 + (n^2 - 1))/F_2^2}^2
\]

The expression of concurrence can be simplified to [25]
\[
C_r = 2(N - 1) \max \{0, |u|/F_2^2 - w\}
\]
where, \( u = -\frac{1}{2} s^2 n^2 Q_{12} - s^2 n^2 u_0 \).

In figure 3, we plot the spin-squeezing parameters and concurrence against the decoherence strength \( p \) under depolarizing channel with \( \theta = 1.8\pi, N = 12 \). We can see that similar to amplitude damping channel, the spin squeezing decreases as the decoherence strength \( p \) increases without weak measurement. And, the phenomenon of ESD and SSSD occurs (see figure 3(a)). However, we are able to improve the spin-squeezing parameters \( \xi_3^2 \) and the concurrence greatly by using weak measurement. The larger is the weak measurement strength \( n \), the later is the vanishing time. And when weak measurement strength is very large, the spin-squeezing parameter \( \xi_3^2 \) and the concurrence vanish approximately in the asymptotic limit (see figure 3(d)). We note that with the increase of weak measurement strength \( n \), the spin-squeezing parameter \( \xi_2^2 \) becomes
more and more weak until it is zero. This means that in our model, the parameter $\xi^2 < 1$ implies the existence of pairwise entanglement, while $\xi^2 < 1/2$ does not.

4.3. Phase-damping channel

A single qubit Kraus operators for phase-damping channel (PDC) is

$$E_0 = \sqrt{\bar{p}} I, \ E_1 = \sqrt{\bar{p}} |0\rangle \langle 0|, \ E_2 = \sqrt{\bar{p}} |1\rangle \langle 1|$$ (37)

From equation (22) and the Kraus operators for the PDC given by equation (37), when $n^2 - 1 = m^2 + 1$, we find the evolutions of the following expectations under decoherence using weak measurement (see appendix for details):

$$\langle \sigma_{z_0} \rangle = [(m^2 + 1)\langle \sigma_{z_0} \rangle_0 + 1]/F_3$$
$$\langle \sigma_{x_0\sigma_{x_0}} \rangle = s^2 m^2 n^2 \langle \sigma_{x_0\sigma_{x_0}} \rangle_0 /F_3^2$$
$$\langle \sigma_{x_1\sigma_{0}} \rangle = [(m^2 + 1)^2 \langle \sigma_{x_1\sigma_{0}} \rangle_0 + 2(m^2 + 1)\langle \sigma_{x_0} \rangle_0 + 1]/F_3^3$$

$$Q_0 = \langle \sigma_{\chi_0} \sigma_{\chi_0} \rangle = s^2 m^2 n^2 (1 - \langle \sigma_{x_0\sigma_{x_0}} \rangle_0) + (m^2 + 1)^2 \langle \sigma_{x_1\sigma_{0}} \rangle_0 + 2(m^2 + 1)\langle \sigma_{x_0} \rangle_0 + 1]/F_3^3$$ (38)

where $F_3 = m^2 + 1 + \langle \sigma_{x_0} \rangle_0$. Substituting the relevant expectation values and the correlation function into equations (9)–(11) leads to the explicit expression of the spin-squeezing parameters

$$\xi^2_1 = 1 - s^2 m^2 n^2 C_r(0)/F_3^2;$$ (39)

$$\xi^2_2 = \xi^2_1 / [(m^2 + 1)\langle \sigma_{x_0} \rangle_0 + 1]/F_3^2$$ (40)

$$\xi^2_3 = (1 - N^{-1})Q_0 + N^{-1}.$$ (41)

The expression of concurrence can be simplified to [25]

$$C_r = 2(N - 1) \max \{0, \ |u|/F_3^2 - w \}$$ (42)

where, $u = -\frac{1}{2} s^2 m^2 n^2 Q_0, - s^2 m^2 n^2 w$.

In figure 4, we plot the spin-squeezing parameters and concurrence against the decoherence strength $p$ under phase-damping channel with $\theta = 1.8\pi, N = 12$. We can see that similar to amplitude-damping and depolarizing channels, the spin squeezing decreases as the decoherence strength $p$ increases without weak measurement. And the phenomenon of ESD and SSSD occurs (see figure 4(a)). However, we are able to enhance the spin-squeezing parameters $\xi^2_2$ and the concurrence greatly, and to avoid the phenomenon of ESD and SSSD by using weak measurement. Moreover, when weak...
measurement strength $m$ is small, the spin-squeezing parameter $\zeta_2$ and the concurrence becomes a fixed value respectively regardless of the decoherence although the spin-squeezing parameter $\tilde{\zeta}_2$ becomes zero (see figure 4(d)). This result can be explained as follows. When the weak measurement strength $m^2 \ll 1$, according to $n^2 - 1 = m^2 + 1$, we have $n^2 = 2$. So, we obtain $F_3 = 1 + \langle \sigma_z \rangle_0$ and $2m^2 n^2 \ll F_3^2$. From equation (38), we can obtain the expectations as follows

$$\langle \sigma_z \sigma_z \rangle = \langle \sigma_z \rangle_0 + 2 \langle \sigma_z \rangle_0 + 1/F_3^2$$

$$Q_3 = \langle \sigma_z \sigma_z \rangle = \langle \sigma_z \rangle_0 + 2 \langle \sigma_z \rangle_0 + 1/F_3^2.$$  

(43)

Thus, the spin-squeezing parameters and concurrence can be calculated as

$$\zeta_2 = 1,$$

$$\tilde{\zeta}_1 = \tilde{\zeta}_2 = 1,$$

$$\tilde{\zeta}_2 = \frac{1}{(1-N^{-1})Q_3 + N^{-1}}.$$

$$C_r = \frac{1}{2}N^{-1}(1 + 2 \langle \sigma_z \rangle_0 + 1/F_3^2 - 1).$$  

(44)

So, the spin-squeezing parameter $\zeta_2$ and the concurrence can be recovered to certain stationary values respectively and the spin-squeezing parameter $\tilde{\zeta}_2 = 0$ when weak measurement strength $m$ is very small.

We also note that with the decrease of weak measurement strength $m$, the spin-squeezing parameter $\tilde{\zeta}_2$ becomes more and more weak until it is zero. This means that in our model, the parameter $\tilde{\zeta}_2 < 1$ implies the existence of pairwise entanglement, while $\zeta_2 < 1$ does not. This result is the same as that discussed in the case of depolarizing channel.

5. Conclusion

In this paper, we have proposed an efficient method to protect spin squeezing under the action of amplitude-damping, depolarizing and phase-damping channels based on measurement reversal from weak measurement, and have considered an ensemble of $N$ independent spin-1/2 particles with exchange symmetry. We have shown that spin squeezing can be enhanced greatly under three different decoherence channels and spin-squeezing sudden death can be avoided undergoing amplitude-damping and phase-damping channels. Moreover, we have found that the spin-squeezing parameters can be completely recovered to its initial value under amplitude-damping channel by changing weak measurement strength irrespective of sufficiently strong decoherence. For the phase-damping channel, the spin-squeezing parameter can be recovered to certain stationary value in certain conditions. Our work is of great significance for quantum metrology in open systems and other quantum information processing tasks where the research objects are subject to noise environments.

Some experimental implementation of spin squeezing are described in [13, 14]. Moreover, Weak measurements have been experimentally realized in various physical systems, e.g. solid systems [32], superconducting phase qubits [42] and linear optic devices [33]. We hope that our proposal can be realized by using such or other systems. Although we have illustrated our protocol for spin squeezing with one-axis twisted states, it would be interesting to see the effect of weak measurement on spin squeezing with two-axis twisted states under decoherence. And, we believe that weak measurement scheme can be used to improve spin squeezing in some more general channels.

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Author contributions

X P Liao contributed to the initial idea. X P Liao and M S Rong performed the computations. X P Liao and M F Fang discussed the results. X P Liao wrote the paper. All authors reviewed the manuscript.

Additional information

Competing financial interests: The authors declare no competing financial interests.

Appendix

Derivations of the evolution of the correlations and expectations under decoherence with weak measurements are given below.

From equation (22), by using the cyclic nature of the trace

$$\text{Tr}(AB...CDE) = \text{Tr}(EAB...CD)$$

$$= \text{Tr}(DEAB...C)$$

$$= ...$$  

(A.1)

we obtain the expectation value of the operator $A$

$$\langle A \rangle = \text{Tr}[A\Theta(\rho_{\text{in}})]$$

$$= \frac{\text{Tr}[\text{N\text{channel}}(M_{\text{in}}^{M'}N^*)]}{\text{Tr}[\text{N\text{channel}}(M_{\text{in}}^{M'}N^*)]}$$

$$= \text{Tr}[\text{N\text{channel}}(M_{\text{in}}^{M'}N^*)]$$

$$= \frac{E}{F}.$$  

(A.2)

where $E = \text{Tr}[(\sum_k M^1 E_k^{*} N^1 \text{ANE}_k M)\rho_{\text{in}}]$, $F = \text{Tr}[(\sum_k M^1 E_k^{*} N^1 \text{ANE}_k M)\rho_{\text{in}}]$.

For an arbitrary matrix
\[
A = \begin{bmatrix}
  a & b \\
  c & d
\end{bmatrix},
\]
from the Kraus operators (23) for the ADC, we obtain
\[
F = \text{Tr}\left[\begin{bmatrix}
  sn^2 + p & 0 \\
  0 & m^2
\end{bmatrix}\rho_m\right]
\] (A.3)
\[
E = \text{Tr}\left[\begin{bmatrix}
  asn^2 + dp & bmn^2 \sqrt{s} \\
  cmn^2 \sqrt{s} & dmn^2
\end{bmatrix}\rho_m\right]
\] (A.4)
when \(sn^2 + p = m^2\), it is straightforward to find
\[
F_1 = sn^2 + p
\] (A.5)
and
\[
\Theta^+(\sigma_x) = mn\sqrt{s}\sigma_x/(sn^2 + p) \quad \text{for} \quad \mu = x, y
\] (A.6)

As we considered independent and identical decoherence channels and weak measurements acting separately on each spin, the evolution correlations and expectations in equation (24), are obtained directly from the above and the following equations
\[
\sigma_{x}\pm = \frac{1}{2}(\sigma_{x} \pm i\sigma_{y})
\] (A.7)
\[
\bar{\sigma}_1 \cdot \bar{\sigma}_2 = \sigma_1\sigma_{2x} + \sigma_1\sigma_{2y} + \sigma_1\sigma_{2z}
\] (A.8)
\[
E = \text{Tr}\left[\begin{bmatrix}
  \frac{1}{2}(p - pn^2 + 2n^2) & 0 \\
  0 & \frac{m^2}{2}(pn^2 - p + 2)
\end{bmatrix}\rho_m\right]
\] (A.9)
when \(m = 1\), it is straightforward to find
\[
F_2 = \frac{1}{2}(n^2 + 1) + \frac{1}{2}(n^2/s - s)(\sigma_x)_0
\] (A.10)
and
\[
\Theta^+(\sigma_x) = ns\sigma_x/(1/2(n^2 + 1) + 1/2(n^2/s - s)(\sigma_x)_0)\] (A.11)
\[
\Theta^+(\sigma_z) = [(m^2 + 1)\sigma_z + 1]/[(m^2 + 1) + (\sigma_z)_0]
\] (A.12)

From the Kraus operators (37) for the PDC, we obtain
\[
F = \text{Tr}\left[\begin{bmatrix}
  n^2 & 0 \\
  0 & m^2
\end{bmatrix}\rho_m\right]
\] (A.13)
\[
E = \text{Tr}\left[\begin{bmatrix}
  an^2 & bmn^2 \\
  cmn^2 & dmn^2
\end{bmatrix}\rho_m\right]
\] (A.14)
when \(n^2 - 1 = m^2 + 1\), it is straightforward to find
\[
F_1 = (m^2 + 1) + (\sigma_z)_0
\] (A.15)
and
\[
\Theta^+(\sigma_\mu) = mns\sigma_\mu/[\(m^2 + 1\) + (\sigma_\mu)_0] \quad \text{for} \quad \mu = x, y
\] (A.16)

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