Multifragmentation reactions and properties of hot stellar matter at sub-nuclear densities.

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Abstract

We point out similarity of thermodynamic conditions reached in intermediate-energy nuclear collisions and in supernova explosions. We show that a statistical approach, which has been previously applied for nuclear multifragmentation reactions, can be very useful for description of the electro-neutral stellar matter. Then properties of hot unstable nuclei extracted from analysis of multifragmentation data can be used for construction of a realistic equation of state of supernova matter.

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A type II supernova explosion is one of the most spectacular events in astrophysics, with huge energy release of about $10^{53}$ erg or several tens of MeV per nucleon [1]. When the core of a massive star collapses, it reaches densities several times larger than the normal nuclear density $\rho_0 = 0.15$ fm$^{-3}$. The repulsive nucleon-nucleon interaction gives rise to a bounce-off and creation of a shock wave propagating through the in-falling stellar material. This shock wave is responsible for the ejection of a star envelope that is observed as a supernova explosion.

During the collapse and subsequent explosion the temperatures $T \approx (0.5 \div 10)$ MeV and baryon densities $\rho_B \approx (10^{-5} \div 2)\rho_0$ can be reached. As shown by many theoretical studies, a liquid-gas phase transition is expected in nuclear matter under these conditions. It is remarkable that similar conditions can be obtained in energetic nuclear collisions studied in terrestrial laboratories. This fact gives a ground to use well established models of nuclear reactions, after certain modifications, for describing matter states in the course of supernova explosions. On the other hand, the supernova physics stimulates investigations of specific reaction mechanisms and exotic nuclei.

As demonstrated by several authors (see e.g. [2, 3]), present hydrodynamical simulations of the core collapse do not produce successful explosions, even when neutrino heating and convection effects are included. On the other hand, it is known that nuclear composition is extremely important for understanding the physics of supernova explosions. In particular, the weak reaction rates and energy spectra of emitted neutrinos are very
sensitive to the presence of heavy nuclei (see e.g. [4, 5, 6, 7]). This is also true for the 
equation of state (EOS) used in hydrodynamical simulations since the shock strength is 
mainly diminished due to the dissociation of heavy nuclei.

The EOS of supernova matter is under investigation for more than 20 years. A most 
popular EOS which is presently used in supernova simulations was derived in refs. [8, 9]. 
However, it is based on properties of cold or nearly cold nuclei ($T < \sim 1$ MeV) extracted 
from nuclear reactions at low (solid state target) density (i.e., $\rho_B \sim 10^{-14}\rho_0$). On the 
other hand, properties of nuclei may change significantly at high temperatures, baryon 
and lepton densities associated with supernova explosions. For more realistic description 
of supernova physics one should certainly use experience accumulated in recent years by 
studying intermediate-energy nuclear reactions. In particular, multifragmentation reactions 
provide valuable information about hot nuclei in dense environment. We believe 
that properties of equilibrated transient systems produced in these reactions are similar 
to those expected in supernova explosions. Another shortcoming of calculations presented 
in refs. [8, 9] is that they reduce an ensemble of hot nuclei to a single ”average nucleus”. 
This is a crude approximation which may strongly distort the true statistical ensemble. 
Therefore, an urgent task now is to derive a more realistic EOS of hot and dense stellar 
matter which can be applied for a broad range of thermodynamic conditions. The first 
step in this direction was made in our previous paper [10]. A similar model was also used 
ref. [11] where, however, only cold nuclei in long-lived states were considered.

In the supernova environment, as compared to nuclear reactions, new important 
ingredients should be taken into consideration. The matter at stellar scales is electrically 
neutral, therefore, electrons must be included to balance a positive nuclear charge. Ener-
getic photons are also present in hot matter and they can change the nuclear composition 
via photo-nuclear reactions. Finally, the flavor content of matter can be affected by a 
strong neutrino flux from the newly-born protoneutron star. The crucial question for the-
oretical description is what degree of equilibration is reached in different reactions. Our 
estimates show that at temperatures and densities of interest the characteristic reaction 
times for nuclear interactions vary within the range from $10$ to $10^6$ fm/c, that is very short 
compared to the characteristic time of the explosion (about $100$ ms [2]). The assumption 
of nuclear equilibration is fully justified for these conditions, and therefore, the nuclear 
composition can be determined from a statistical model. The rate of photo-nuclear reac-
tions depends strongly on the density, and at very low densities, less than $10^{-5} - 10^{-6}\rho_0$, 
these reactions are more efficient than nuclear interactions. The weak interactions are 
much slower. It is most likely that at high densities, $\rho_B > \sim 10^{-3}\rho_0$, neutrinos are trapped in 
a nascent neutron star, but at lower densities they stream freely from the star. The weak 
processes are entirely responsible for the neutrino and electron content of the matter. For 
example, the electron capture may not be in equilibrium with nuclear reactions at small 
densities [12]. Therefore, an adequate treatment is needed to discriminate various condi-
tions with respect to the weak reactions [10]. We conclude that the statistical approach 
can be applied for nuclear reactions, but possible deviations from equilibration for weak 
and electromagnetic interactions should be explicitly taken into account.

Statistical models have been proved to be very successful in nuclear physics. They are
used in situations when an equilibrated source can be defined in the reaction. The most famous example of such a source is the 'compound nucleus’ introduced by Niels Bohr in 1936. The standard compound nucleus picture is valid only at low excitation energies when evaporation of light particles and fission are the dominating decay channels. However, this concept cannot be applied at high excitation energies, \( E^* > 2-3 \text{ MeV/nucleon} \), when the nucleus breaks up fast into many fragments. Several versions of the statistical approach have been proposed for the description of such multifragmentation reactions (see e.g. [13, 14, 15]). As was demonstrated in many experiments (see e.g. [16, 17, 18, 19, 20, 21]), an equilibrated source can be formed in this case too, and statistical models are generally very successful in describing its properties. Furthermore, systematic studies of such highly excited systems have brought important information about the nuclear liquid-gas phase transition [22, 23].

As a basis for our study we take the Statistical Multifragmentation Model (SMM), see a review [15]. Presently, the SMM is the only model of multifragmentation which can be applied in the thermodynamical limit for infinite systems [24]. This makes possible to use it for astrophysical conditions. The model assumes statistical equilibrium at a low-density freeze-out stage. It considers all break-up channels composed of nucleons and excited fragments taking into account the conservation of baryon number, electric charge and energy. Light nuclei with mass number \( A \leq 4 \) are treated as elementary particles with only translational degrees of freedom ("nuclear gas"). Nuclei with \( A > 4 \) are treated as heated liquid drops. In this way one may study the nuclear liquid-gas coexistence in the freeze-out volume. The Coulomb interaction of fragments is described within the Wigner-Seitz approximation. Different channels \( f \) are generated by Monte Carlo sampling according to their statistical weights, \( \propto \exp S_f \), where \( S_f \) is the entropy of channel \( f \). After the break-up the Coulomb acceleration and the secondary de-excitation of primary hot fragments is taken into account.

An important advantage of the SMM is that it includes all break-up channels ranging from the compound nucleus to vaporization\(^1\), and one can study the competition between them. As was shown already in first publications [15, 25], multifragmentation dominates over compound nucleus at high excitation energies, and must be taken into account in order to explain multiple fragment production. Most clearly this is demonstrated in Fig. 1 where we show entropies of two extreme disintegration modes of \(^{238}\text{U}\), namely the compound nucleus (CN) and the vaporization (V) channels, as functions of excitation energy. One can clearly see that the CN channel dominates at low excitation energies but the V channel wins at excitation energies above 12 MeV/nucleon. However, in fact the CN channel dies out at much lower excitation energies, 2-3 MeV/nucleon, when the channels containing a heavy residue (HR) and/or several intermediate-mass fragments (IMFs: \( 4 < A < 50 \)) have a higher entropy. The entropy for the whole ensemble of fragmentation channels is also shown in Fig. 1 (solid line). One can see, for instance, that at 5 MeV/nucleon this ensemble has a 0.2 higher entropy per nucleon than the CN channel. This means that the relative probability of the CN channel is

\(^1\)By vaporization we mean all channels which contain only light particles (\( A \leq 4 \)).
\[ \exp \Delta S = \exp (-0.2 \cdot 238) \approx 2 \cdot 10^{-21}. \] This is a general trend, and it cannot be reversed by another description of the compound nucleus \[26].\) We emphasize that the difference

- \[\begin{align*}
\text{SMM (grand-canon.)} \\
2^{38}U
\end{align*}\]

Figure 1: Entropy per nucleon for different disintegration channels of \(2^{38}U\) as a function of excitation energy per nucleon. Calculations are performed within the grand-canonical version of the SMM (details see in refs. \[15,25\]), at the freeze-out density \(\rho = \rho_0/3\). Dashed and dotted lines correspond to compound nucleus and vaporization \((A \leq 4)\) channels, respectively. The total entropy, obtained by summing over all break-up channels, is shown by solid line.

between the solid and dotted lines in Fig. 1 is caused entirely by the presence of hot heavy and intermediate-mass fragments. From the figure one can conclude that their contribution remains significant even at very high excitation energies, up to 30 MeV/nucleon, and nuclear entropies up to 4 units per baryon. This observation is very relevant for the physics of supernova explosions since the survival of relatively heavy nuclei may help reviving the shock wave. Indeed, if nuclei can exist under such extreme conditions, the energy losses for dissociation of initial nuclei will be considerably reduced.

For astrophysical applications it is important that the SMM, besides fragment partitions, can describe well the neutron content of fragments \[16\]. Generally, in the case of neutron-rich sources, the SMM predicts neutron-rich hot primary fragments. As calculations show \[27,28\], they keep a part of their neutron excess even after de-excitation. New experiments give evidence for production of such unusual neutron-rich nuclei, which,
however, should be quite common for supernova matter. For example, Fig. 2 shows the neutron-to-proton ratio \((N/Z)\) for fragments produced in fragmentation of \(^{238}\text{U}\) with energy 1 GeV/nucleon on Pb and Ti targets \([29]\). The experiment was performed with Fragment Separator (FRS) at GSI. Fission and spallation fragments were excluded from the analysis in order to guarantee selection of multifragmentation-like events. One can see that the observed neutron content of fragments with \(Z < 60\) is larger than expected from the standard EPAX parametrization, which is a result of spallation-like processes considered previously as the main mechanism for production of these fragments. Moreover, at \(Z < 30\) it becomes larger than the neutron content of stable nuclei. These results are fully consistent with previous findings of the ALADIN collaboration \([16]\).

Figure 2: Mean neutron-to-proton ratio versus charge of fragments produced in multifragmentation-like break-up of \(^{238}\text{U}\) with energy 1 GeV/nucleon on Pb and Ti targets. Points are experimental data obtained on Fragment Separator at GSI \([29]\). The SMM calculations are shown by dashed (primary hot fragments) and solid (fragments after secondary de-excitation) lines. Dash-dotted line corresponds to stable nuclei, dotted line is the EPAX phenomenological parametrization for nuclei produced by spallation.

In Fig. 2 we demonstrate that the SMM can quantitatively describe this trend by predicting at the same time primary hot fragments with very large \(N/Z\). In recent years the interest to isospin degrees of freedom has considerably increased and strong experimental programs to study isospin-asymmetric nuclei exist now at GSI (Germany), TAMU (USA), NSCL/MSU (USA), INFN (Italy), GANIL (France) and other laboratories. These studies will give information about the symmetry energy in hot nuclei, e.g. via the isoscaling phenomenon \([30]\). Presently, there are indications of essential decrease of the symmetry
energy in hot nuclei [28, 31]. A small discrepancy shown in Fig. 2 between the theory and 
experiment for fragments with \( Z < 20 \) can be attributed to this effect: lower symmetry 
energy in the beginning of the secondary evaporation cascade can increase the neutron 
richness of final cold fragments [32]. This possibility should be kept open when adjusting 
the model parameters for best description of hot neutron-rich nuclei produced in nuclear 
reactions. Then these results can be used for constructing a reliable equation of state for 
nuclear matter at supernova conditions.

According to present knowledge, supernovae of type II are triggered by the collapse 
of massive stars after formation of a big enough iron core. After the bounce of infalling 
matter off a dense core a shock wave is generated, it propagates outwards leaving behind 
a highly compressed and heated matter. This matter consists of various nuclear species 
as well as free neutrons, protons, electrons, photons and possibly trapped neutrinos [11]. 
Calculation of the equation of state of supernova matter presents a challenge to modern 
nuclear physics. Due to the presence of leptons the EOS is essentially modified as compared 
with the case of pure nuclear matter. At densities below \( 0.5 \rho_0 \) uniform nuclear 
matter breaks into fragments and nucleons immersed in the uniform electron-positron 
plasma. The EOS should cover a broad range of baryon densities and temperatures ex-
pected during the core collapse and subsequent explosion.

In order to describe nuclear composition of supernova matter we have modified the 
SMM by including electrons and neutrinos in the statistical ensemble [10]. The model 
was formulated for different assumptions concerning the weak reaction rates that makes 
it flexible to be used for different stages of a supernova explosion. The most important 
difference of our approach as compared with the previous ones [8, 9] is that we consider the 
whole ensemble of hot primary nuclei, but not only an average nucleus characterizing the 
liquid phase. In Fig. 3 we present the SMM predictions for charge-to-mass ratios (Z/A) 
and mass distributions of hot nuclei for typical supernova conditions. We have performed 
calculations for different temperatures and densities at fixed lepton (neutrinos+electrons) 
to baryon (\( Y_L \)), or electron to baryon (\( Y_e \)) fractions. One can see that at low temperatures 
(\( T = 1 \) MeV) the mass distributions have usually three peaks: at A=1 (free nucleons), A=4 
(\( \alpha \)-particles) and a large A~100 corresponding to heavy nuclei. This is typical picture for 
a gas-liquid coexistence region. The distribution of heavy nuclei in this case can be well 
approximated by a Gaussian distribution. However, at higher temperatures (\( T = 3 \) MeV) 
the mass distributions can be very broad (see dashed line in the bottom left panel), and 
they become closer to a power law (for A>4). As seen from the top left panel, the Z/A ratios vary from about 0.45 at low density to about 0.25 at higher densities.

We stress that a great variety of neutron-rich, and even exotic (large mass—small 
charge) nuclei can be formed during the explosion. Our analysis [10] shows that decreasing 
the symmetry energy has a strong influence on the nuclear composition, and favors 
formation of neutron-rich nuclei. Therefore, properties of hot neutron-rich nuclei in super-
nova environments should be re-considered in accordance with most recent laboratory 
experiments, in particular, regarding the symmetry energy and level densities.

Besides of a more realistic EOS there are other important aspects of supernova dynam-
ics which are sensitive to the ensemble of hot nuclei. In particular, the neutrino opacity [7]
Figure 3: Mean charge-to-mass ratios (left top panel), and mass distributions of hot primary fragments (other panels) calculated with the SMM generalized for supernova conditions. Left panels are calculations for temperature $T = 3$ MeV and fixed lepton (electrons+neutrinos) fraction $Y_L = 0.2$ per nucleon. Right panels are calculations for temperature $T = 1$ MeV and fixed electron fraction $Y_e$ assuming that neutrinos escape. Lines correspond to baryon densities (in units of the normal nuclear density $\rho_0 = 0.15 \text{ fm}^{-3}$) shown in the figure.

and the electron capture rate [12] are very sensitive to the nuclear structure effects. Our approach can be easily generalized to incorporate the nuclear shell effects which should play a role at temperatures below 1 MeV, and disappear at high excitation energies. It is very promising to investigate consequences of new shells at very large neutron excess and possible production of heavy, or even superheavy, nuclei under supernova conditions.

Also, the nucleosynthesis in supernova environments may proceed differently if the
ensemble of hot nuclei will provide new 'seeds' for the r-process. Previous r-process calculations were based, as a rule, on a limited number of stable 'seed' nuclei \[33\]. On the other hand, as one can see in Fig. 3, a broad variety of primary hot nuclei should be produced at the nuclear equilibrium stage during the explosion. After ejection from the hot and dense environment, they will undergo de-excitation. As known from nuclear experiments, at initial temperature of 1–3 MeV this is achieved by evaporating only a few nucleons. We expect that in the course of evaporation certain isotopes will be enhanced by the shell effects. At later stages these new cold seed nuclei will be further processed in the standard s- and r-processes. We believe that this two-stage mechanism of nucleosynthesis may explain some puzzles in the observed nuclear abundances.

In conclusion, in this paper we have pointed out a similarity of nuclear matter states reached in heavy-ion reactions and in supernova explosions. The information on properties of hot fragments and their production mechanisms can be used for construction of a reliable equation of state of stellar matter. It is important that the improved EOS can also be applied for other astrophysical processes, where the appropriate densities and temperatures of the matter occur, for example, in the neutrino-driven protoneutron star wind during the first seconds after explosions \[34\]. We hope that in the future our improved EOS will be implemented into modern hydrodynamical simulations of supernova dynamics \[2\]. In this way, it will be possible to perform more realistic calculations of supernova explosions and synthesis of heavy elements.

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