Multiplicity distribution and normalized moments in p–p collisions at LHC in forward rapidity using Weibull model

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The measured charged particle multiplicity distribution in the forward rapidity region of the p–p collisions at √s = 0.9 TeV, 7 TeV and 8 TeV at the LHC have been described using the Weibull distribution function. The higher order (up to 5th order) normalized moments and the factorial moments are also calculated using the extracted parameters. The multiplicity distributions in forward region are observed to be well described by the Weibull regularity and the higher order moment calculations confirm the violation of KNO scaling that has been observed at mid-rapidity and lower energies. The Weibull parameters and moments for p–p collisions at √s = 13 TeV are also estimated for the forward region.

I. INTRODUCTION

Recently, the ALICE (A Large ion collider Experiment) experiment at LHC (Large Hadron Collider) has measured the charged particle multiplicity distribution in wider pseudo-rapidity ranges for different event classes, INEL (inelastic events), INEL > 0 (same as INEL with the presence of at least one charged particle for |η| < 1.0 and NSD (non-singly diffractive events) [1]. This particular measurement allowed extending the previous measurements of other LHC experiments to a wider η coverage enabling to reach higher multiplicity and kinematic phase space together with previous measurement by CMS [2], ALICE [3], and LHCb [4]. The forward region is also relevant in view of being sensitive to interesting low Bjorken-x dynamics and multi-particle interactions [5].

The charged particle multiplicity distribution in full phase space for p–p collisions are influenced by the global conservation laws (namely energy-momentum, charge etc). However, the distributions in restricted phase space are less influenced by the global constraints and are sensitive to the dynamics of particle production and local fluctuations [6, 7]. The predictions of several QCD-inspired Monte Carlo event generators such as PYTHIA [8], PHOJET [9] etc. were used to compare the data. It was found that these models underestimated the data [2, 3]. Therefore, it becomes relevant to study the distributions in different event classes and varied phase space intervals to allow constraining the existing Monte-Carlo model parameters and understand the underlying dynamics of multi-particle production. The multiplicity distribution at LHC energies are observed to deviate from Poissonian shape and thus signals toward the existence of correlations [10, 11]. These multi-particle correlations can be studied through the normalized moments and factorial moments of the distribution. The study of moments is also sensitive to the KNO (Koba, Nielsen and Olesen) scaling i.e the energy independence of moments of various orders would imply the observance of KNO scaling [12]. In this work, Weibull distribution [13, 14] has been used to describe the multiplicity distribution of charged particles in pp collisions at √s = 0.9 TeV, 7 TeV and 8 TeV as measured by ALICE experiment in wider η intervals (forward region) for different event classes. The study was extended to calculate the higher moments to confirm the violation of KNO scaling as reported in previous measurements for central rapidity regions [2, 3, 15].

II. MOMENTS OF WEIBULL DISTRIBUTION

The probability density distribution of Weibull for a continuous random variable x is given by-

\[ f(x; \lambda, k) = \frac{k}{\lambda} \left( \frac{x}{\lambda} \right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} \] (1)

where k is the shape parameter and λ is the scale parameter of the distribution.

The nth raw moment is given by:

\[ m_n = \lambda^n \Gamma \left( 1 + \frac{n}{k} \right) \] (2)

The mean of the distribution, i.e. \( m_1 \) is denoted...
by \( \langle x \rangle \) and is given by,

\[
\langle x \rangle = \lambda \Gamma \left( 1 + \frac{1}{k} \right) \tag{3}
\]

The \( n^{th} \) factorial moment of a variable \( x \) is defined as:

\[
\begin{align*}
 f_n &= \left\langle \frac{x^n}{(x-\eta)!} \right\rangle \\
 &= \langle x(x-1)(x-2)\ldots(x-n+1) \rangle \tag{4}
\end{align*}
\]

The \( n^{th} \) normalized moment \( C_n \) and normalized factorial moments \( F_n \) are defined as:

\[
C_n = m_n/m_1^n; \quad F_n = f_n/m_1^n \tag{5}
\]

The first few normalized moments and factorial moments can be found in [13].

### III. ANALYSIS AND RESULTS

The present work is based on the analysis of the ALICE experiment which has measured the charged particle multiplicity distribution over a wide \( \eta \) interval\((-3.4 < \eta < 5.0)\) in \( p-p \) collisions at \( \sqrt{s} = 0.9, 7, \) and \( 8 \) TeV at the LHC[1]. The measurement was done for three classes of events. The first class consisted of all inelastic collisions\((INEL)\) while the second class known as \( INEL > 0 \) selects the same events as the first one with an additional requirement of at least one track in the region \( |\eta| < 1.0 \). The third class is the non-single diffractive (\( NSD \)) class where the number of single-diffractive events were greatly reduced by requiring the detection of charged particles in both the odrometers. The extension in \( \eta \) coverage also enabled the measurement to reach higher multiplicity and a different phase space when compared to previous results published by ALICE[2]. The present analysis only considers two different event classes namely the \( NSD \) and \( INEL > 0 \). The multiplicity distributions in two different event classes are fitted with Weibull distribution for five different \( \eta \) intervals i.e. \( |\eta| < 2.0, |\eta| < 2.4, |\eta| < 3.0, |\eta| < 3.4 \) and \(-3.4 < \eta < 5.0 \). Figure 2 shows the multiplicity distributions fitted with Weibull function for \( 0.9 \) TeV, \( 7 \) TeV, and \( 8 \) TeV, respectively. The first few bins are excluded from the fits as there is an increase in the number of events due to diffractive events. The Weibull distribution seems to describe the data quite well. Table II gives the details of the fitting parameters and the quality of fit. This is in agreement with the previous results where the fit described the data well for central rapidity[13]. The variation of \( \lambda \) and \( k \) as a function of beam energy is shown for the studied \( \eta \) intervals in Figure 2. One can observe that the values of \( \lambda \) increase with beam energy as well as with the width of \( \eta \) interval. The \( k \) values slightly decrease with an increase of collision energy while it shows an increasing trend with an increase in width of \( \eta \) interval. The trend of both \( \lambda \) and \( k \) with beam energy over all \( \eta \) intervals is similar to previous observations[13–15].

The normalized raw moments and the factorial moments (up to fifth order) have also been calculated using the extracted parameters using the equation 5. Fig 3 shows the variation of normalized moments \( (C_n) \) for \( NSD \) and Fig 4 shows the same for \( INEL > 0 \) event class. Fig 5 and 6 shows the variation of factorial moments \( (F_n) \) with beam energy for different \( \eta \) intervals for \( NSD \) and \( INEL > 0 \) event classes, respectively. It can be seen for all pseudorapidity intervals studied, \( C_2(\text{and } F_2) \) remains constant with energy while \( C_3(\text{and } F_3) \) shows a small increase with increasing energy. However, \( C_4 \) (and \( F_4 \)) and \( C_5 \) (and \( F_5 \)) show an increase with increasing energy, which becomes stronger for larger \( \eta \) intervals. This is observed in both the event classes and signals towards the violation of KNO scaling in wider \( \eta \) intervals. The observation is in agreement with the previous experimental results. The values of \( k \) for \( p-p \) collisions at \( 13 \) TeV are extrapolated from the parametrization of the variation of the parameters as shown in Fig 2. The estimated values of normalized moments and factorial moments for different \( \eta \) ranges are listed in Table II. The predictions are important for future experimental measurements of multiplicity distribution and its moments in the forward region for \( p-p \) collisions at \( \sqrt{s} = 13 \) TeV.

### IV. CONCLUSION

The charged-particle multiplicity in forward rapidity as measured by ALICE experiment for \( \sqrt{s} = 0.9 \) TeV, \( 7 \) TeV, and \( 8 \) TeV is well described by the two parameter Weibull distribution. The extracted fitting parameters were used to calculate the normalized moments and the factorial moments (up to 5th order) of the distribution for various \( \eta \) intervals. The model predictions of the higher moments indicate towards a strong violation of KNO scaling in the forward region in the two considered event classes. This observation is in agreement with the previous measurement at central rapidity.
Table I. The values of $k$, $\lambda$ and $\chi^2/\text{ndf}$ obtained from the fits of multiplicity distributions using Weibull function in $p$–$p$ collisions for different $\eta$ intervals at different energies as measured by ALICE experiment at LHC [1].

| $\sqrt{s}(\text{TeV})$ | Event class | $\eta$ Range | $k$     | $\lambda$ | $\chi^2/\text{ndf}$ |
|------------------------|-------------|--------------|---------|-----------|---------------------|
| 0.9                    | NSD         | $|\eta| < 2$ | 1.265 ± 0.007 | 17.54 ± 0.091 | 1.56               |
|                        |             | $|\eta| < 2.4$ | 1.2676 ± 0.01 | 21.16 ± 0.368 | 1.01               |
|                        |             | $|\eta| < 3$  | 1.2695 ± 0.013 | 26.028 ± 0.42 | 0.703              |
|                        |             | $|\eta| < 3.4$ | 1.31 ± 0.012 | 29.59 ± 0.56 | 0.82               |
| INEL>0                 |             | $|\eta| < 2$ | 1.269 ± 0.01 | 21.23 ± 0.481 | 1.77               |
|                        |             | $|\eta| < 2.4$ | 1.248 ± 0.013 | 25.65 ± 0.53 | 0.43               |
|                        |             | $|\eta| < 3.4$ | 1.286 ± 0.012 | 29.09 ± 0.63 | 0.62               |
|                        |             | $-3.4<\eta<5.0$ | 1.35 ± 0.012 | 33.68 ± 0.61 | 0.91               |
| 7                      | NSD         | $|\eta| < 2$ | 1.068 ± 0.007 | 25.72 ± 0.224 | 1.37               |
|                        |             | $|\eta| < 2.4$ | 1.07 ± 0.01 | 30.58 ± 0.37 | 0.7                |
|                        |             | $|\eta| < 3$  | 1.107 ± 0.013 | 38.58 ± 0.57 | 0.45               |
|                        |             | $|\eta| < 3.4$ | 1.12 ± 0.013 | 43.52 ± 0.84 | 0.62               |
|                        |             | $-3.4<\eta<5.0$ | 1.151 ± 0.014 | 51.05 ± 0.9 | 0.419              |
| INEL>0                 |             | $|\eta| < 2$ | 1.075 ± 0.007 | 25.65 ± 0.21 | 1.45               |
|                        |             | $|\eta| < 2.4$ | 1.104 ± 0.009 | 31.24 ± 0.33 | 0.89               |
|                        |             | $|\eta| < 3$  | 1.15 ± 0.01 | 40.34 ± 0.67 | 0.64               |
|                        |             | $|\eta| < 3.4$ | 1.17 ± 0.011 | 45.27 ± 0.9 | 0.57               |
|                        |             | $-3.4<\eta<5.0$ | 1.18 ± 0.013 | 52.88 ± 0.97 | 0.52               |
| 8                      | NSD         | $|\eta| < 2$ | 1.03 ± 0.007 | 27.93 ± 0.23 | 1.03               |
|                        |             | $|\eta| < 2.4$ | 1.06 ± 0.01 | 33.61 ± 0.39 | 0.72               |
|                        |             | $|\eta| < 3$  | 1.08 ± 0.01 | 42.62 ± 0.44 | 0.67               |
|                        |             | $|\eta| < 3.4$ | 1.09 ± 0.01 | 44.7 ± 0.57 | 0.35               |
|                        |             | $-3.4<\eta<5.0$ | 1.12 ± 0.013 | 52.93 ± 0.68 | 0.46               |
| INEL>0                 |             | $|\eta| < 2$ | 1.06 ± 0.007 | 26.13 ± 0.22 | 1.1                |
|                        |             | $|\eta| < 2.4$ | 1.09 ± 0.011 | 32.82 ± 0.7 | 0.62               |
|                        |             | $|\eta| < 3$  | 1.116 ± 0.013 | 41.33 ± 0.78 | 0.54               |
|                        |             | $|\eta| < 3.4$ | 1.13 ± 0.02 | 46.52 ± 1.04 | 0.48               |
|                        |             | $-3.4<\eta<5.0$ | 1.15 ± 0.02 | 55.15 ± 1.1 | 0.42               |

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FIG. 1. (Color online) The charged particle multiplicity distribution measured by ALICE experiment in different $\eta$ intervals for $p$–$p$ collisions at $\sqrt{s} = 0.9$ TeV, 7 TeV, and 8 TeV. The solid lines show the Weibull fit to the measured data.

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FIG. 2. The variation of Weibull parameters $k$ and $\lambda$ with center of mass energy in $p-p$ collisions. The variation is parameterized by a power law of the form $A + B \cdot \log (\sqrt{s})^C$, shown by the dashed curve.

FIG. 3. The variation of the normalized moments for NSD event class with collision energy as obtained from Weibull calculations for different $\eta$ intervals.
FIG. 4. The variation of the normalized moments for $INEL > 0$ event class with collision energy as obtained from Weibull calculations for different $\eta$ intervals.

FIG. 5. The variation of the normalized factorial moments for $NSD$ event class with collision energy as obtained from Weibull calculations for different $\eta$ intervals.
FIG. 6. The variation of the normalized factorial moments for $INEL > 0$ event class with collision energy as obtained from Weibull calculations for different $\eta$ intervals.

| Event class | $\eta$ Range | $k$ | $\lambda$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ |
|-------------|---------------|-----|-----------|-------|-------|-------|-------|-------|-------|-------|-------|
| NSD         | $|\eta| < 2.0$  | 30.447 | 0.987 | 2.026 | 6.201 | 25.411 | 130.597 | 1.994 | 6.005 | 24.220 | 122.526 |
|             | $|\eta| < 2.4$  | 36.643 | 1.021 | 1.960 | 5.70  | 21.958 | 105.193 | 1.932 | 5.540 | 21.032 | 99.297  |
|             | $|\eta| < 3.0$  | 46.731 | 1.046 | 1.914 | 5.378 | 19.855 | 90.6133 | 1.893 | 5.254 | 19.16  | 86.376  |
|             | $|\eta| < 3.4$  | 48.463 | 1.051 | 1.906 | 5.318 | 19.471 | 88.0322 | 1.885 | 5.199 | 18.80  | 84.0159 |
|             | $-3.4 < \eta < 5.0$ | 57.814 | 1.08  | 1.859 | 5.318 | 19.471 | 88.0322 | 1.841 | 4.896 | 16.93  | 71.827  |
| INEL>0      | $|\eta| < 2.0$  | 28.233 | 1.030 | 1.943 | 5.580 | 21.163 | 99.6   | 1.907 | 5.374 | 19.99  | 92.259  |
|             | $|\eta| < 2.4$  | 35.652 | 1.053 | 1.903 | 5.295 | 19.321 | 87.028 | 1.874 | 5.133 | 18.428 | 81.646  |
|             | $|\eta| < 3.0$  | 45.223 | 1.081 | 1.857 | 4.984 | 17.39  | 74.479 | 1.834 | 4.858 | 16.72  | 70.605  |
|             | $|\eta| < 3.4$  | 50.862 | 1.103 | 1.824 | 4.764 | 16.08  | 66.307 | 1.804 | 4.653 | 15.509 | 63.095  |
|             | $-3.4 < \eta < 5.0$ | 60.50  | 1.108 | 1.815 | 4.708 | 15.75  | 64.295 | 1.798 | 4.615 | 15.27  | 61.63   |

TABLE II. The values of $\lambda$, $k$, normalized moments and factorial moments of various order obtained from the fits of multiplicity distributions using Weibull function in p–p collisions for different $\eta$ intervals at 13 TeV.