In this paper we examine Casimir effect in the case of tachyonic field, which is connected with particles with negative four-momentum square \(m^2 < 0\). We consider here only the case of the one dimensional, scalar field. In order to describe tachyonic field, we use the absolute synchronization scheme preserving Lorentz invariance. The renormalized vacuum energy is calculated by means of the Abel-Plana formula. Finally, the Casimir energy and Casimir force as the functions of distance are obtained. In order to compare the resulting formula with the standard one, we calculate the Casimir energy and Casimir force for massive, scalar field \(m^2 > 0\).

\[ x'^0 = x^0 / W^0. \]  

I. INTRODUCTION

Tachyons are hypothetical particles which move faster than light. Recently, a number of papers was published, suggesting that tachyons can play a role in cosmology, as well as in neutrino physics [1, 2]. In order to describe the kinematics of this kind of particles, formalism based on the absolute synchronization scheme should be used. Now, we briefly describe main results related to the absolute synchronization scheme. Derivation of these results and exhaustive description can be found in [3, 4].

The main idea is based on the well known fact, that the definition of the time coordinate depends on the procedure used to synchronize clocks. Furthermore, the form of Lorentz transformations depends on the synchronization scheme. In absolute synchronization a new concept, the preferred frame arise. It is intimately connected with tachyons [5]. Possibly, this distinguished inertial frame can be identified with cosmic background radiation frame.

In the formalism presented here, the transformation of coordinates between inertial frames takes the following form:

\[ x'(u') = D(\Lambda, u)x(u), \]  

where \(\Lambda\) is an element of Lorentz group, \(u\) is the four-velocity of the preferred frame, and \(D(\Lambda, u)\) is a matrix depending on \(\Lambda\) and \(u\).

Transformation law for the four-velocity of the preferred frame, which follows from Eq. (1), is defined by

\[ u' = D(\Lambda, u)u. \]  

Matrix \(D(\Lambda, u)\) for any rotation \(R\) is given by

\[ D(R, u) = \begin{pmatrix} 1 & 0 \\ 0 & R \end{pmatrix}, \]  

while for the boosts \(w\) it takes the form

\[ D(w, u) = \begin{pmatrix} \frac{1}{\sqrt{1 - w^2}} & 0 \\ -w & 1 + \frac{w^2}{\sqrt{1 - w^2}} - u^0 w \otimes u_T \end{pmatrix}, \]  

where \(w\) is the four-velocity of the system \(x'\) with respect to the system \(x\). Hence, in the absolute synchronization scheme, we get the Lorentz transformation law for time coordinate in the form

\[ x'^0 = x^0 / W^0. \]  

Contrary to the standard procedure of synchronization, the transformation law (6) is well defined not only for subluminal velocities but also for superluminal. It is continuous, and does not produce ‘transcendental’ tachyons moving with \(|v| = \infty\). This property and preservation of absolute causality make absolute synchronization scheme useful for description of tachyons.

The line element \(dx^2 = g_{\mu\nu}(u)dx^\mu dx^\nu\) is invariant under Lorentz transformations if the covariant metric tensor is expressed as follows:

\[ g_{\alpha\beta}(u) = \begin{pmatrix} 1 & u^0 u^T \\ u^0 u & -I + u \otimes u_T (u^0)^2 \end{pmatrix}, \]  

where \(u = (u^0, u)\) is four-velocity of the preferred frame. Covariant metric tensor is given by

\[ g^{\alpha\beta}(u) = \begin{pmatrix} (u^0)^2 & u^0 u^T \\ u^0 u & -I \end{pmatrix}. \]  

In the case of tachyons, square of the four-momentum

\[ k_0 k^\alpha = -m^2, \]  

leads to the dependence of the energy \(k_0\) on momentum \((k_1, k_2, k_3) = k\) in the form

\[ k_0 = (u^0)^{-1} \left[ -u_k + \sqrt{(uk)^2 + k^2 - m^2} \right], \]  

whereas contravariant energy \(k^0\) is

\[ k^0 = u^0 \sqrt{(uk)^2 + k^2 - m^2}. \]
Sign of $k^0$ is positive and Lorentz invariant not only for massive particles but also for tachyons. Values of momentum $k$ fulfill the condition in the form

$$k^2 > \frac{m^2}{1 + (u k / |k|)^2}. \quad (12)$$

Values of $k$ lie outside of the oblate spheroid with half-axes $m, mu^0$ and with the symmetry axis parallel to $u$.

A more exhaustive discussion of the absolute synchronisation scheme in the language of frame bundles is given in [4]. Another applications in physics can be found in [6] (EPR correlations) and [3] (thermodynamics).

II. ONE DIMENSIONAL TACHYONIC FIELD

The quantum description of free tachyonic field was given in [3].

In this article we consider only scalar field in one space-dimension. In this case Eqs. (10) and (11) take the form

$$k_0 = (u^0)^{-1} \left[ -u^1 k_1 + \frac{1}{a^2} \sqrt{k_1^2 - (u^0 m)^2} \right], \quad (13)$$

$$k^0 = \sqrt{k_1^2 - (u^0 m)^2}. \quad (14)$$

Whereas the condition in Eq. (12) reduces to

$$k_1^2 > \frac{m^2}{1 + (u^1)^2},$$

which, with the help of the identity $(1 + u^1)^2(u^0)^2 = 1$, leads to the condition

$$k_1^2 > (mu^0)^2. \quad (15)$$

The Lagrangian density of the scalar field can be written in the following form:

$$\mathcal{L} = \frac{1}{2} (g^{ab} \partial_a \phi \partial_b \phi + m^2 \phi^2) =$$

$$= \frac{1}{2} \left[ (u^0)^2 (\partial_0 \phi)^2 + u^0 u^1 (\partial_0 \phi \partial_1 \phi + \partial_1 \phi \partial_0 \phi) +$$

$$- (\partial_1 \phi)^2 + m^2 \phi^2 \right]. \quad (16)$$

and canonical momentum is

$$\pi = \frac{\partial \mathcal{L}}{\partial (\partial_0 \phi)} = (u^0)^2 \partial_0 \phi + u^0 u^1 \partial_1 \phi. \quad (17)$$

Hamiltonian density is given by

$$\mathcal{H} = \frac{1}{2} \left[ (u^0)^2 (\partial_0 \phi)^2 + (\partial_1 \phi)^2 - m^2 \phi^2 \right]. \quad (18)$$

Scalar field in one dimension is represented by the operator:

$$\phi(x, u) = \frac{1}{\sqrt{2\pi}} \int dk |(e^{ikx} a^+(k) + e^{-ikx} a(k)),$$  

where $d\mu(k)$ is an invariant measure defined by

$$d\mu(k) = dk_0 dk_1 \Theta(k^0) \delta(k^2 + m^2) = dk_0 dk_1 \frac{\delta(k^0 - \omega_k)}{2\omega_k}, \quad (20)$$

while the coefficient $\omega_k$ is given by

$$\omega_k = \sqrt{k_1^2 - (u^0 m)^2}. \quad (21)$$

By means of (20), the Fourier decomposition of the field $\phi(x, u)$ takes the form

$$\phi(x, u) = \frac{1}{\sqrt{2\pi}} \int \frac{dk_1}{2\omega_k} (e^{ikx} a^+(k_1, u) + e^{-ikx} a(k_1, u)), \quad (22)$$

where the integration range $\Gamma$ is determined by Eq. (15).

III. CASIMIR EFFECT FOR TACHYONIC FIELD IN THE PREFERRED FRAME

Now, let us consider the one dimensional, scalar, tachyonic field in the limited region of length $a$ (Fig. 1) with Dirichlet boundary conditions:

$$\phi(x^0, 0) = \phi(x^0, a) = 0. \quad (23)$$

In the frame moving relative to preferred frame, energy of the particle Eq. (13) is not an even function of momentum $k_1$. Hence, two waves which form the standing wave (and moving in different directions) have different length and speed. This fact causes some difficulties in calculations. In order to avoid these difficulties, in this section only Casimir energy in preferred frame ($u^0 = 1$) is computed.

![FIG. 1: One dimensional field.](image)

Taking into account the boundary condition Eq. (23) we obtain the Fourier expression of the operator $\phi$ such that

$$\phi(x^0, x^1) = i \sum \frac{\sin k_n x^1}{\omega_k} \left[ e^{-ik_0 a} a_n^+ - e^{ik_0 a} a_n \right], \quad (24)$$

where $k_n = \pi n / a$. The summation in Eq. (24) is over every $n$ which fulfill condition (15). Operators $a_n$ and $a_n^+$ are annihilation and creation operators which satisfy well known commutation relations:

$$[a_n, a_m^+] = [a_n^+, a_m^+] = 0, \quad (25)$$

$$[a_n, a_m] = 2 \delta_{n, m} \omega_n. \quad (26)$$

Hamiltonian of tachyonic field (in preferred frame) can be expressed as

$$H = \frac{1}{2} \int_0^a dx \left[ (\partial_0 \phi)^2 + (\partial_1 \phi)^2 - m^2 \phi^2 \right]. \quad (27)$$
On inserting operator $\phi$ from Eq. (24) with its derivatives into Eq. (27) and integrating with respect to variable $x^1$, we find

$$H = \sum_{k^2 > m^2} \frac{1}{2} \omega_k^{-2} (k^2 - m^2 + \omega_k^2) (a_k^+ a_n + a_n a_k^+) = \sum_{k^2 > m^2} \frac{1}{2} (a_k^+ a_n + a_n a_k^+).$$

If we calculate vacuum energy as an expectation value of $H$, we get

$$E = \langle 0 | H | 0 \rangle = \frac{i}{2} \sum_{k^2 > m^2} \sqrt{k^2 - m^2},$$

where $| 0 \rangle$ is a vacuum state, defined by equation $a_n | 0 \rangle = 0$. It is well defined and stable state (See [3]).

If we compare Eq. (29) with Eq. (13) we can see that the vacuum energy (Eq. 30), also in the case of tachyonic fields, is infinite. Therefore we introduce the renormalized energy (Eq. 31), which is finite.

$$E_c(a) = \frac{\pi}{a} \sum_{n=0}^{\infty} \sqrt{(n + n_0)^2 - \lambda^2 a^2} - \frac{a}{\pi} \int_{n_0}^{\infty} dk \sqrt{k^2 - m^2} = \frac{a}{\pi} \int_{n_0}^{\infty} dk \sqrt{k^2 - m^2}.$$

The last stage follows as a result of obvious equation $\lim_{x \to \infty} [\lfloor x \rfloor / x] = 1$.

The vacuum energy (Eq. 30), also in the case of tachyonic fields, is infinite. Therefore we introduce the renormalized “Casimir energy” as a difference between vacuum energy in the two configurations shown in Fig. 2. Parameter $L$ is the total length of considered region. In this case the Casimir energy is

$$E_c(a) = \frac{\pi}{a} \left[ \sum_{n=0}^{\infty} \sqrt{(n + n_0)^2 - \lambda^2 a^2} - \frac{a}{\pi} \int_{n_0}^{\infty} dk \sqrt{k^2 - m^2} \right].$$

Furthermore, applying of the formula (35) we finally get

$$E_c(a) = -\frac{\pi}{a} \times \left[ \sqrt{2} \int_{0}^{\infty} dt \sqrt{\frac{(n_0^2 - \lambda^2 t^2)^2 + 4n_0^2 t^2 - n_0^2 + \lambda^2 + t^2}{e^{2\pi t} - 1}} + \frac{1}{2} \lambda^2 \ln \left( \frac{n_0 + \sqrt{n_0^2 - \lambda^2}}{\lambda} \right) + \frac{1}{2} (n_0 - 1) \sqrt{n_0^2 - \lambda^2} \right].$$

The numerical computation of the expression (36) are shown in Figs 3-6.
The Fig. 4 shows Casimir energy (Eq. 34) as a function of the parameter \( a \) (Recall that \( a \) is the distance between two points where \( \phi \) vanishes). The presented curve corresponds to \( m = 1 \) and \( u_0 = 1 \) (preferred frame).

Since the dependence on \( m \) enters to the expression (34) only through parameter \( \lambda \), therefore for other values of \( m \) the Casimir energy function will have the same character, only its horizontal and vertical scale will change.

The Fig. 5 shows Casimir force (as a function of the distance \( a \), \( \lambda \)-parameter). Numerical calculations show, that presented function behaves like the power function \( c \lambda^\alpha \) where \( \alpha = -0.5 \).

To test the solutions presented on Figs 3-6, we have computed expression in Eq. (34) by three other methods, which are not presented here. There were among others: the method of zeta function and the method of generalized Abel-Plana formula. All gave the same results.

We realize that the results presented above seem strange. Nowadays, tachyons are still hypothetical objects. Maybe it is another reason which causes that tachyons does not exist. However, this question is still open.

IV. CASIMIR EFFECT FOR MASSIVE FIELD IN THE PREFERRED FRAME

In this section we calculate Casimir energy in the case of the massive field \( (m^2 > 0) \) using the absolute synchronization scheme. Our purpose is the comparison of the results obtained in previous paragraph for tachyons and the well known ones for massive particles \[8, 9, 10\]. Analogously as on last section we consider only one dimensional, scalar field.

The formula on the square of four-momentum for massive particle

\[ k^\alpha k^\alpha = m^2, \]  

leads to the following dependence of the energy \( k_0 \) on momentum \( k \):

\[ k_0 = (u^0)^{-1} \left[ -uk + \sqrt{(uk)^2 + k^2 + m^2} \right]. \]  

Lagrangian density of field can be written as

\[ \mathcal{L} = \frac{1}{2}(g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - m^2 \phi^2) = \]

\[ = \frac{1}{2} \left[ (u^0)^2 (\partial_0 \phi)^2 + u^0 u^1 (\partial_0 \phi \partial_1 \phi + \partial_1 \phi \partial_0 \phi) + - (\partial_1 \phi)^2 - m^2 \phi^2 \right]. \]
Scalar, one dimensional and massive field, similarly to the tachyonic case (Eq. 23), can be expressed by

\[ \phi(x, u) = \frac{1}{\sqrt{2\pi}} \int d\mu'(k)(e^{ikx}a^+(k) + e^{-ikx}a(k)), \]  

(41)

where the invariant measure \( d\mu'(k) \) takes now the form

\[ d\mu'(k) = dk_0dk_1 \Theta(k^0) \delta(k^2 - m^2) = dk_0dk_1 \frac{\delta(k^0 - \omega'_k)}{2\omega'_k}, \]  

(42)

and the coefficient \( \omega'_k \) is given by

\[ \omega'_k = \sqrt{k^2 + (u^0m)^2}. \]  

(43)

(Index ‘prime’ is added to differentiate expressions from their tachyonic equivalents.)

The operator \( \phi \), with the measure \( d\mu'(k) \) (Eq. 42), can be written as

\[ \phi(x, u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk_1 \frac{\sin k_0x}{2\omega'_k} [e^{-ik_0x}a^+_n - e^{ik_0x}a_n], \]  

(44)

Contrary to the case of the tachyonic field, we have a full range of integration because the relation \( \omega'_k \) does not lead to any restrictions for momentum values.

In this section we compare the Casimir energy for massive field with results from Section III and again we consider only Casimir effect in preferred frame (\( u^0 = 1 \)).

Hamiltonian of the field in a finite region of length \( a \) with boundary condition (23) can be expressed by

\[ H = \frac{1}{2} \int_{0}^{a} dx [\partial_0^2 + (\partial^1)^2 + m^2 \phi^2]. \]  

(45)

Fourier decomposition of \( \phi \) is defined by

\[ \phi(x^0, x^1) = \frac{i}{\sqrt{a}} \sum_{n=-\infty}^{\infty} \sin k_nx^1 \frac{2\omega_k}{\omega'_k} [e^{-ik_0x^0}a^+_n - e^{ik_0x^0}a_n], \]  

(46)

where \( k_n = \pi n/a \). Operators \( a_n \) and \( a^+_n \) fulfill the following commutation relations:

\[ [a_n, a_m] = [a^+_n, a^+_m] = 0, \]  

(47)

\[ [a_n, a^+_m] = 2\delta_{n,m} \omega'_k. \]  

(48)

On inserting \( \phi \) from Eq. (46) and its derivatives into Eq. (45) we get (after integrating with the respect to \( x^1 \)) the Hamiltonian in the form:

\[ H = \frac{1}{10} \omega'_k \sum_{n=-\infty}^{\infty} \frac{1}{4} \delta^2(k^0_0 + m^2 + k^2_n)(a^+_n a_n + a_n a^+_n) = \]  

\[ = \frac{1}{4} \sum_{n=-\infty}^{\infty} \frac{1}{4} (a^+_n a_n + a_n a^+_n), \]  

(49)

Now, the vacuum energy is given by

\[ E = \langle 0 \rangle H(0) = \frac{1}{2} \sum_{n=-\infty}^{\infty} \sqrt{k^2_n + m^2}. \]  

(50)

After the change of the summation limit, we finally obtain

\[ E(a) = \sum_{n=1}^{\infty} \sqrt{\left( \frac{\pi n}{a} \right)^2 + m^2}. \]  

(51)

Renormalized Casimir energy (a difference between a vacuum energy in two configurations from Fig 1) takes the form

\[ E_c(a) = \pi \left( \sum_{n=0}^{\infty} \sqrt{n^2 + \lambda^2} - \int_{0}^{\infty} \sqrt{n^2 + \lambda^2} \right), \]  

(52)

where \( \lambda = am/\pi \).

Using the Abel-Plana formula \( 55 \) we finally receive

\[ E_c(a) = -\frac{\pi}{a} \int_{\lambda}^{\infty} \frac{\sqrt{n^2 - \lambda^2}}{\exp(2\pi n) - 1} dn. \]  

(53)

The expression in Eq. 53 is the well known formula, describing the Casimir effect for massive field (\( m^2 > 0 \)), in standard procedure of synchronization (See \( 9, 10 \)).

Below we present Figs 7-9, which are counterparts of the Figs 3-5. The Casimir energy as a function of \( m \), the Casimir energy as a function of \( a \) and Casimir force are shown respectively.

![FIG. 7: Casimir energy for massive field as a function of mass m, corresponding to a = \( \pi \).]

![FIG. 8: Casimir energy for massive field as a function of the distance a, corresponding to a = \( \pi \).]

V. CASIMIR EFFECT FOR TACHYONS IN NONTRIVIAL TOPOLOGY

In this section, we again consider the one dimensional, scalar, tachyonic field in the limited region of length \( a \) (Fig. 1).
FIG. 9: Casimir force for massive field as a function of the distance \(a\), corresponding to \(m = 1\).

But now we impose boundary conditions

\[ \phi(x^0,0) = \phi(x^0,a), \quad \partial_t (x^0,0) = \partial_t (x^0,a), \] (54)

which describe the identification of the boundary points \(x^1 = 0\) and \(x^1 = a\). In this section we consider Casimir energy in arbitrary frame which is moving relative to preferred frame with fourvelocity \(u\).

Taking into account the boundary condition Eq. (54) we obtain the Fourier expression of the operator \(\phi\) such that

\[ \phi(x^0,x^1) = \frac{1}{\sqrt{a}} \sum_{k_2^2 > (mu^0)^2} \frac{1}{2\omega_k} [e^{ikx}a_n^+ + e^{-ikx}a_n], \] (55)

where \(k_n = 2\pi n/a\). In this case Hamiltonian of tachyonic field can be expressed as

\[ H = \frac{1}{2} \int_0^a dx \left[ (u^0)^2 (\partial_0 \phi)^2 + (\partial_1 \phi)^2 - m^2 \phi^2 \right]. \] (56)

On inserting operator \(\phi\) from Eq. (55) with its derivatives into Eq. (56) and integrating with respect to variable \(x^1\), we find

\[ H = \sum_{k_2^2 > (mu^0)^2} \frac{1}{8} \omega_k^{-2} (u^0 k_0)^2 - m^2 + k_0^2 (a_n^2 + a_n^2). \] (57)

If we calculate vacuum energy as an expectation value of \(H\), we get

\[ E = \langle 0 | H | 0 \rangle = \frac{1}{2} (u^0)^{-2} \sum_{k_2^2 > (mu^0)^2} \left( -u^0 a k_n + \sqrt{k_n^2 - (u^0 m)^2} \right). \] (58)

After changing the limits of summation, Eq. (58) can be written as

\[ E(a,u) = (u^0)^{-2} \sum_{n = n_0}^{\infty} \left( \frac{2\pi n}{a} \right)^2 - (mu^0)^2, \] (59)

where \(n_0 = [mu^0 a/2\pi]\). (Function \([x]\) gives the smallest integer greater than or equal to \(x\).)

The renormalized Casimir energy (a difference between vacuum energy in the two configurations shown in Fig. 2.) is given by

\[ E_c(a,u) = (u^0)^{-2} \times \left( \sum_{n=n_0}^{\infty} \sqrt{ (2\pi n/a)^2 - (mu^0)^2 } - \frac{a}{2\pi} \int_0^{\infty} dk k^2 - (mu^0)^2 \right). \] (60)

Introducing parameter \(\lambda(u) = amu^0/2\pi\) and introducing the new variable of integration \(n = ak/2\pi\) we get

\[ E_c(a,u) = \frac{2\pi}{a(u^0)^2} \left( \sum_{n=n_0}^{\infty} n^2 - \lambda^2(u) - \int_0^{\infty} dn n^2 - \lambda^2(u) \right), \] (61)

where \(n_0 = [amu^0/2\pi] = [\lambda]\). The expression (61) is equal to expression (54) multiplied by factor \(2(u^0)^{-2}\). Hence, in the preferred frame \((u^0 = 1)\), energy (61) is two times larger than energy (54), showed in Figs 3,4. Since the dependence on \(u^0\) enters to the expression (61) only through parameter \(\lambda\) and factor \(\pi/a(u^0)^2\), therefore for other values of \(u^0\) (in moving frame relative to preferred frame) the Casimir energy function will have the same character, only its horizontal and vertical scale will change.

VI. COMPARISON OF RESULTS FOR TACHYONIC AND OTHER FIELDS-CONCLUSIONS

Let us compare the results for tachyonic field (Section III) with the results obtained for the massive ones (Section IV). In the case of the massive field \(m^2 > 0\) we get monotonic and differentiable dependence of energy and force (as a functions of distance). Force is always attractive and smoothly goes to zero when the distance approaches infinity.

In the tachyonic case the situation is completely different. Indeed, there is no monotonicity at obtained expressions. Energy as well as force, show quasi-oscillatory behaviour and change own sign many times. Furthermore, the force is not a differentiable function of distance. There are some ‘jumps’, where force goes to infinity.

FIG. 10: Casimir energy for tachyonic and massive fields in preferred frame.
Let us compare the behaviour of the Casimir energy as a function of the length $a$. The Fig. 10 shows Casimir energy for the tachyonic and massive fields. This figure is made in SI units for mass of particle $m = 1\text{ eV}$. First (upper) curve shows Casimir energy for massive field. In this case Casimir energy decay exponentially with distance $a$. Second (lower) curve shows Casimir energy for the tachyonic field (averaged over oscillations). As we mentioned previously, in this case energy behaves like a power function.

For full comparison, we present on the Fig. 11 Casimir energy for massless field. This figure is made in the same units as Fig. 10. In case of the one dimensional, massless field, formula for Casimir energy takes well known form: $E_c(a) = -\pi/12a$ (See [2]).

The behaviour of Casimir energy (averaged over oscillations) for tachyonic case is more similar to massless field than to massive field. In both cases (tachyonic and massless) we have slow decay of the Casimir Energy with distance $a$. Contrary, in case of massive field, Casimir energy decay fast (exponentially) with distance $a$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig11}
\caption{Casimir energy for massless field in preferred frame.}
\end{figure}

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