Gravitational Wave During Slowly Evolving

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The ekpyrotic slow contraction or the slow expansion might be responsible for the adiabatical production of the nearly scale invariant curvature perturbation. However, the tensor perturbation generated is generally strongly blue, which implies that it is negligible on large scale. Thus it has been still thought that the detection of primordial tensor perturbation will rule out the relevant models. Here, we will show a counterexample. We will illustrate that in a model of the slow evolution, due to the rapid change of the gravitational coupling, both the curvature perturbation and the tensor perturbation can be nearly scale invariant. The resulting ratio of the tensor to scalar is in a regime which can be detected by the coming or planned experiments. We argue that the result is similar to that of inflation because the background evolution given here is actually conformally equivalent to the inflationary background.

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The observations indicated that the primordial curvature perturbation is nearly scale invariant. How generating it has been still a significant issue, especially for single field. The curvature perturbation on large scale consists of a constant mode and a mode dependent of time. When one of which is dominated and scale invariant, the spectrum of perturbation will be scale invariant.

The quadratic action of the curvature perturbation $\mathcal{R}$ is generally given by $[2],[3],[4],[5], \int d\eta d^3x \frac{\alpha^2 Q_R}{c_R} \left( \mathcal{R}^2 - c_R^2 \left( \partial\mathcal{R} \right)^2 \right), \quad (1)$

where $Q_R$ and $c_R^2$ are determined by the evolution of background field. $Q_R > 0$ and $c_R^2 > 0$ are required to avoid the ghost and gradient instabilities. The equation of $\mathcal{R}$ is $[6],[7],\quad u_k'' + \left( c_R^2 k^2 - \frac{z_R''}{z_R} \right) u_k = 0, \quad (2)$

after defining $u_k \equiv z_R R_k$, where the prime is the derivative for $\eta$, $z_R \sim a Q_R^{1/2} / c_R$. The scale invariance of $\mathcal{R}$ requires $\quad (3)$

$z_R \sim \frac{a Q_R^{1/2}}{c_R} \sim \frac{1}{\eta_* - \eta} \quad \text{for constant mode}$

$\quad (4)$

or $(\eta_* - \eta)^2 \quad \text{for increasing mode}$

has to be satisfied, where initially $\eta \ll -1$. In certain sense, both evolutions are dual $[8]$. Noting the results will be different if $c_R^2$ is changed $[9],[10],[11],[12],[13]$, however, we will not involve it here, see $[14],[15]$ for recent discussions. In principle, both $a$ and $Q_R$ can be changed, and together contribute the change of $z$. However, only one among them is changed while another is hardly changed might be the most interesting.

We have generally $Q_R = M_p^2 \epsilon$ for single field action $P(X, \varphi) [2]$. While the case is slightly complicated for the most general single field $[2],[3],[5]$. However, as has been showed, if $\epsilon \leq 0$, we actually can manage to obtain $Q_R \simeq M_p^2 |\epsilon| [14]$. When the scale factor is rapidly changed while $\epsilon$ is nearly constant, the constant mode is responsible for inflation $[17]$, while the increasing mode is for the matter contraction $[8],[18],[19]$.

However, the case can also be inverse. When $a$ is slowly evolving, the scale invariant spectrum of curvature perturbation can be adiabatically generated for $\epsilon \gg 1 [20]$ or $\epsilon \ll -1 [21]$. The evolution with $\epsilon \gg 1$ corresponds the slow contraction, which is that of ekpyrotic universe $[22]$. While $\epsilon \ll -1$ gives the slow expansion, which has been applied for island universe $[23]$. In certain sense, in Ref. $[21]$ it was for the first time observed that the slow expansion might adiabatically induce the scale invariant spectrum of curvature perturbation, see $[24]$ for that induced by the entropy perturbation.

It is generally thought that for constant $|\epsilon|$, the result depends of the physics around the exiting $[27]$. However, when $\epsilon$ is rapidly changed, the thing is altered, see $[20]$ for the case of the ekpyrotic slow contraction, and criticism for it $[27]$. In general, during the slow contraction or the slow expansion with rapidly changed $\epsilon$, the scale invariant adiabatical perturbation can be naturally induced by its increasing mode $[23]$, or its constant mode $[20],[30]$. Though both cases give the scale invariant spectrum, both pictures are distinct. In $[28]$, initially $|\epsilon| \gg 1$, and then is rapidly decreasing, the slow evolution ends when $|\epsilon| \sim 1$. While in $[20],[26],[31]$, initially $|\epsilon| \ll 1$, and then is rapidly increasing. In $[15]$, it is argued that only a finite range of the scale invariant mode can be generated in the regime of the validity of perturbation theory. Whether the conclusion is applicable to that induced by the increasing mode $[16],[25]$ is interesting for studying. However, even if considering the constant mode, the larger range of the scale invariant mode might be also obtained by a series of the phases of different slow evolutions.

Not as the curvature perturbation, the tensor perturbation is generally strongly blue during the slowly evolving, which implies that it is negligible on large scale. Thus it has been still thought that the detection of primordial tensor perturbation will rule out the relevant models of the slow evolution. However, is it certain that
the tensor perturbation is not scale invariant? We will revisit the reason of having this result.

The quadratic action of the tensor perturbation $h_{ij}$ is

$$S_2 \sim \int \frac{a^2 Q_T}{c_T^2} \left( h_{ij}'^2 - c_T^2 (\partial h_{ij})^2 \right), \quad (5)$$

where $Q_T > 0$ and $c_T^2 > 0$ are required to avoid the ghost and gradient instabilities. This action has same shape with that of the curvature perturbation. Thus the scale invariance of $h_{ij}$ requires

$$z_T \sim \frac{a Q_T^{1/2}}{c_T} \sim \frac{1}{\eta_* - \eta} \text{ for constant mode} \quad (6)$$

or $(\eta_* - \eta)^2$ for increasing mode \quad (7)

has to be satisfied. The results will be different if $c_T^2$ is changed, however, as for the curvature perturbation, we will not involve this case here. In principle, both $a$ and $Q_T$ can be changed. $Q_T = M_p^2$ for $P(X, \varphi)$. We generally have not $Q_T \sim Q_R$, since $Q_R \sim M_p^2 |\epsilon|$ and $|\epsilon|$ is rapidly changed. Thus it is impossible that during the slow evolution, i.e. $a$ is hardly changed, both Eqs.(3) and (6) are simultaneously satisfied. During the slow expansion, $z_T \sim a$ is hardly changed, which implies a strongly blue spectrum.

During the inflation, both $\mathcal{R}$ and $h_{ij}$ are scale invariant. The reason is simple, since only $a$ is rapidly changed, which just equally appears in $z_T$ and $c_T$. This is the crux of the matter. This implies that the scale invariance of both $\mathcal{R}$ and $h_{ij}$ can be obtained simultaneously, only when the variable, which is required to be rapidly changed, equally appears in $Q_R$ and $Q_T$. When $Q_R \sim M_p^2 |\epsilon|$ and $Q_T = M_p^2$ for $P(X, \varphi)$ are brought into mind, it is intuitional that for hardly changed $a$ and constant $|\epsilon| \gg 1$, if

$$M_{\text{eff}} \sim \frac{1}{\eta_* - \eta} \text{ for constant mode} \quad (8)$$

or $(\eta_* - \eta)^2$ for increasing mode \quad (9)

can be managed, both $\mathcal{R}$ and $h_{ij}$ are scale invariant might be obtained. We will discuss one possible implement in the following.

The change of $M_{\text{eff}}$ can be obtained by a nonminimal coupling of the scalar field to the gravity. The nonminimal coupling has been investigated for a long history, e.g.\cite{32} for a review. Recently, the nonminimal coupling inflation has been studied in \cite{31,33}, and the most general single field inflation in \cite{34,35}. However, here the nonminimal coupling is introduced for a different purpose. When the nonminimal coupling with the gravity is considered, we will study whether the result is actually as imagined. Here, we only consider the case of (8), since only it is consistent with the argument in Ref.\cite{32}.

We will only consider the slow expansion. However, perhaps the similar calculation is applicable to the slow contraction, in which a bounce is required \cite{37,38,39,40}. We begin with the action

$$\mathcal{L} \sim \frac{M_{\text{eff}}^2 R}{2} + \frac{1}{2} (\partial \varphi)^2 - \frac{1}{4} \lambda \varphi^4, \quad (10)$$

where $M_{\text{eff}}^2 = \xi \varphi^2$ and $\xi$ is a constant to be determined. The sign before $(\partial \varphi)^2$ is reverse, however, as will be showed, there are not the ghost instability for the perturbation. The calculation is essentially similar to that in \cite{21}. The background evolution of the slow expansion is given in \cite{21,41},

$$a \sim \frac{1}{(t_* - t)^{1/|\epsilon|}}, \quad H = \frac{1}{|\epsilon|(t_* - t)} \quad (11)$$

for negative $\epsilon$ and $|\epsilon| > 1, \epsilon \ll -1$. Noting for $|\epsilon| \ll 1, a$ corresponds to that of inflation. When the nonminimal coupling is considered, we require this background can be still valid. The equations of background and field are given in e.g.\cite{32}. When $3H^2 \varphi^2 \ll 6H \dot{\varphi} \varphi$ is neglected, the Friedmann equation is simplified as

$$H \sim -\frac{1}{3} \dot{\varphi}^2 + \frac{1}{3} \lambda \varphi^4 \sim 3 \frac{d}{dt} M_{\text{eff}}^2 \sim \frac{1}{|\epsilon|(t_* - t)}. \quad (12)$$

This implies

$$\dot{\varphi} \sim \left( 1 + O\left( \frac{1}{|\epsilon|} \right) \right) \frac{\sqrt{2/\lambda}}{1/(t_* - t)^{1/|\epsilon|+1}}. \quad (13)$$

and $\frac{1}{3} \dot{\varphi}^2 + \frac{1}{3} \lambda \varphi^4 = 0$ generally leads $\varphi \sim \sqrt{\frac{2}{\lambda}} \frac{1}{1/(t_* - t)^{1/|\epsilon|+1}}$. The deviation $\sim 1/(t_* - t)$ is required to accurately give Eq.(12). Thus $\varphi \sim \frac{1}{(t_* - t)}$ for $\epsilon \ll -1$. The result is consistent with $3H^2 \varphi^2 \ll 6H \dot{\varphi} \varphi$, since

$$H \varphi \sim \frac{1}{|\epsilon|(t_* - t)^2} \ll \dot{\varphi} \sim \frac{1}{(t_* - t)^2} \quad (14)$$

for $\epsilon \ll -1$.

When Eqs.\cite{11} and \cite{13} are considered, $\xi$ can be determined by the equation of $\dot{H}$, which is

$$2M_{\text{eff}}^2 \dot{H} - \left( \frac{d}{dt} M_{\text{eff}}^2 \right) H = \dot{\varphi}^2 - \frac{d^2}{dt^2} M_{\text{eff}}^2, \quad (15)$$

where $M_{\text{eff}}^2 = \xi \varphi^2$. The left side of this equation is 0 up to $1/|\epsilon|$ order, which requires both terms in the right side should set off up to $1/|\epsilon|$ order. Thus we have

$$\xi \sim \frac{1}{6} \left( 1 + \frac{1}{3|\epsilon|} \right), \quad (16)$$

which implies that for $|\epsilon| \gg 1$, $\xi = 1/6$ is just that of conformal coupling constant. Thus here the corresponding theory \cite{10} is actually nearly conformal invariant.

The equation of $\varphi$ approximately is $\ddot{\varphi} \sim \lambda \varphi^3$. Thus the equation of the perturbation $\delta \varphi$ is $\dot{\delta \varphi} - 3\lambda \varphi^2 \delta \varphi \sim 0$. The dominated solution of $\delta \varphi$ is

$$\delta \varphi \sim \frac{1}{(t_* - t)^2} \quad (17)$$
which is the result that \( \varphi \) has weight 1 [42]. This perturbation can be resummed into a constant timeshift of background, which thus is harmless.

There might be other fluids and also anisotropy. However, their energies \( \sim 1/a^n, n > 0 \) generally do not increase, since the universe is still expanding. Thus they will not destroy the background. Thus the solutions [11] and [12] can be stable.

When the slow expansion ends, the energy of field should be released to reheat the universe. Here, the reheating is similar to that after inflation. The evolution of hot big bang cosmology begins after the reheating. However, \( M_{\text{peff}}^2 \sim M_P^2 \) before the reheating should be set up. This might be done by considering

\[
M_{\text{peff}}^2 \sim M_P^2/(1 + \frac{M_P^2}{\varphi^2}),
\]

which implies \( M_{\text{peff}}^2 \sim \varphi^2 \) for \( \varphi \ll M_P \), and \( M_{\text{peff}}^2 \sim M_P^2 \) for \( \varphi \sim M_P \). Thus initially \( M_{\text{peff}} \) is small, which implies the gravity is stronger, since \( G_{\text{Newton}} \sim 1/M_{\text{peff}}^2 \). However, this is consistent with the argument in Ref.[39]. \( \varphi \) is increased all along during the slow expansion. Thus when the slow expansion is over, we have

\[
\varphi_f \sim \sqrt{\frac{2}{\lambda}} \frac{1}{(t_s - t_f)} \sim M_P.
\]

Thus \( |t_f| \sim O(1) \sim 1/\sqrt{\lambda M_P} \). As will be showed, \( \lambda \sim 1/10^{10} \), which implies \( |t_s| \sim 10^9 t_P \).

The quadratic actions of \( \mathcal{R} \) and \( h_{ij} \) for the most general single field, including the nonminimal coupling case, are given in Ref.[3]. We will follow Ref.[3] for the calculation and definition of both perturbations. Here, \( Q_T \) is given by \( Q_T = M_{\text{peff}}^2 \sim \frac{c_2^2}{3} \), and \( c_2^2 = 1 \). Thus

\[
z_T = 0.5 \frac{a Q_T^{1/2}}{c_T} \sim \frac{a}{2 \sqrt{3} \lambda (t_s - \eta)},
\]

since \( t_s \sim t^{1/|\epsilon|+1} \) for \( \epsilon \ll -1 \). Thus the spectrum is nearly scale invariant. The amplitude of tensor perturbation is given by

\[
\mathcal{P}_T = \frac{k^3}{2\pi^2} \left| \frac{u_{kT}}{z_T} \right|^2 \sim \frac{6}{\pi^2} \lambda,
\]

where \( u_{kT} \sim \frac{1}{\sqrt{2} \lambda |t_s - \eta|} \), all quantities are calculated around \( k^2 \sim z_T^2/T \), since this spectrum is induced by the constant mode.

The calculation of the curvature perturbation is slightly complicated, \( Q_R = G_R c_R^2 \) and \( c_R^2 = \frac{\kappa}{3 \epsilon} \). \( G_R \) is given by

\[
G_R = \frac{\frac{1}{6} \lambda^2 \varphi^2 - 3 M_{\text{peff}}^2 H^2 - 6 H \varphi M_{\text{peff}} M_{\text{peff}, \varphi}}{(M_{\text{peff}}^2 H + \varphi M_{\text{peff}} M_{\text{peff}, \varphi})^2} M_{\text{peff}}^4 + 3 M_{\text{peff}}^2
\]

\[
\sim \frac{1}{6|\epsilon|} \varphi^2,
\]

where \( M_{\text{peff}}^2 = \lambda \varphi^2 \) and \( \xi \) is given by [16]. It is found that only that with \( 1/|\epsilon| \) order is left, since all terms with \( 0/|\epsilon| \) is just set off. Thus \( G_R > 0 \), independent of the sign of \( \epsilon \). However, if \( M_{\text{peff}}^2 = M_P^2 \), \( M_{\text{peff}, \varphi} = 0 \), we have \( G_R = M_P^2 \). Thus if \( \epsilon < 0 \), \( G_R < 0 \). Here, the advantage of the rapidly changed \( M_{\text{peff}} \) is obvious, because it is its change that alters the sign of \( G_R \), and leads \( G_R > 0 \). In similar reason, \( F_R \) is given by

\[
F_R = \frac{1}{a} \left( M_{\text{peff}}^2 H + \varphi M_{\text{peff}} H M_{\text{peff}, \varphi} \right) - M_{\text{peff}}^2 \sim \frac{1}{6|\epsilon|} \varphi^2,
\]

which implies \( c_R^2 = \frac{F_R}{Q_R} = 1 \) and \( Q_R = \frac{1}{6|\epsilon|} \varphi^2 \sim \frac{1}{(t_s - t_f)} \sim Q_T \). Thus if we assume \( Q_R \sim M_{\text{peff}}^2 \), we will have

\[
M_{\text{peff}} \sim \frac{1}{(t_s - \eta)},
\]

which is just claimed in (8). \( z_R \) is given by

\[
z_R = \frac{\sqrt{2} a Q_R^{1/2}}{c_R} \sim \frac{\sqrt{2} a}{\sqrt{3} \lambda |(t_s - \eta)|}.
\]

Thus similarly \( \mathcal{R} \) is nearly scale invariant, and is also induced by the constant mode. The amplitude is

\[
\mathcal{P}_R = \frac{k^3}{2\pi^2} \left| \frac{u_{kR}}{z_R} \right|^2 \sim \frac{3}{8\pi^2} \lambda |\epsilon|,
\]

which is only dependent of \( |\epsilon| \). It is interesting to notice that if we replace \( |\epsilon| \) with \( 1/|\epsilon| \), Eq.(20) is just that of minimal coupling inflation with constant \( \epsilon \). In principle, the larger \( |\epsilon| \) gives the smaller \( r \). However, perhaps \( |\epsilon| \) can not be arbitrary large, or there will be the strong couple problem [13]. Here, with Refs.[34],[35], this can be discussed similarly in detail, which is left for the future. In general, if \( |\epsilon| \sim 10^2 \), Eq.(20) implies \( r \sim 0.16 \). The result is in a regime which can be detected by the coming or planned experiments and CMB observations. \( \mathcal{P}_R^{1/2} \sim 10^{-5} \) requires \( \lambda \sim 1/10^{10} \) for \( |\epsilon| \sim 10^2 \). Thus the only left and adjusted parameter of the model is fixed, and there is not additional finetuning.

The freezed horizons of \( \mathcal{R} \) and \( h_{ij} \), outside which the perturbation modes freeze, are defined by \( c_{R, T}^2 k^2 \sim \frac{z_{R, T}}{z_{R, T}} \). The results are plotted in Fig.1. Here, the perturbation mode naturally leaves the freezed horizons of \( \mathcal{R} \) and \( h_{ij} \), and the Hubble horizon. Thus this model can be responsible for the emergence of scale invariant primordial perturbations on large scale.

We reclarify the significance of the rapidly changed gravitational coupling. When the gravitational coupling
which naturally leads the scale invariance of tensor perturbation. The cost is that the background evolution is altered, but luckily is still slowly expanding. However, this cost is just interesting, since it simultaneously leads the scale invariance of $R$.

Here, the results might be incredible. However, the essential might be simple. We still enjoy in Jordan frame, which actually can be conformally transformed to Einstein frame by the redefinition of metric. Then it can be found the background evolution is just inflation. Thus the background given here is actually conformally equivalent with the inflationary background. This explains the equivalence of result to that of inflation, since the physical predictions for observables are independent of the frames, even if the description of backgrounds is completely different e.g. $[40]$. In $[17]$, it is argued that when the universe is dominated by the single component, the perturbation is conformal invariant fully nonperturbatively. The conformal invariant of perturbations implies that the different background evolution having equivalent observable to that of inflation might be designed by a conformal transformation of inflationary background. The model showed here is just such a nontrivial example, which will be studied in detail in the coming.

In conclusion, it is found that, in a phase of slowly expanding, both the curvature perturbation and the tensor perturbation can be scale invariant, due to the rapid change of the gravitational coupling, and the resulting ratio of the tensor to scalar is in a regime which can be detected by the coming or planned experiments. Here, we only bring one of all possible implements of the slow evolution, however, in principle, there could be other effective actions of the single scalar field $[3],[8],[18]$, giving the similar result, which might be interesting for further exploring. This work in certain sense highlights the fact again that identifying the evolution of primordial universe might be a more subtle task than expected.

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$$\ln a \sim \frac{1}{\lambda M_P^2(t_* - t)^2}, \quad H \sim \frac{1}{\lambda M_P^2(t_* - t)^3},$$

$$\mathcal{L} \sim \varphi^2 \left( \dddot{h}^2_{ij} - \frac{1}{a^2} (\partial h_{ij})^2 \right),$$

which is consistent with $[11]$ for $M_P \sim \frac{1}{(t_* - t)}$. Thus the expansion is exponentially slow. It is found that this case that $R$ is strongly blue. In pseudoconformal universe, the result is similar $[42]$, though in a phase of slow contraction.

In Refs. $[42],[44],[45]$, it is imagined that the scale invariance of $R$ is obtained by the conversion of the perturbation of a light field $\chi$. This light field has the conformal coupling $\varphi^2 (\partial \chi)^2$. The mechanism here is actually similar, however, for the graviton, since the coupling actually equals $\mathcal{L} \sim \varphi^2 \left( \dddot{h}^2_{ij} - \frac{1}{a^2} (\partial h_{ij})^2 \right)$.

\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{fig1}
\caption{The evolutions of $a$ for Eq.\,(11) (black line), and Eq.\,(27) (gray line) with respect to time. The freeze horizons of $R$ and $h_{ij}$ (brown line), and the Hubble horizon (green line) are also plotted for (11). We see when the gravitational coupling is hardly changed, the expansion is exponentially slow, while when the coupling is rapidly changed, the exponentially slow expansion is interrupted and becomes power law. This might be analogous with the extended inflation $[13]$.}
\end{figure}

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