Systematics of radial excitations in heavy-light hadrons

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Abstract
Some simple expectations for the quark mass dependence of radial excitation energies of heavy-light hadrons based on consideration of non-relativistic quantum mechanics are discussed. Experimental and theoretical results are reviewed in light of these expectations. Some new lattice QCD results for masses of $\Lambda_b$ and $\Sigma_b$ baryons are presented.
1 Introduction

The calculation of excited state masses in lattice QCD is quite challenging. In recent years, using large operator bases and advanced analysis methods some results for radial excitation energies of hadrons containing a single heavy (charm or bottom) quark have been obtained. Assessing these calculations is also not easy since they are often done without the continuum limit or physical quark mass extrapolation having been made. As well, experimental information about radial excitations of heavy-light hadrons is very fragmentary. In any case, simply making a number-by-number comparison of different results may not provide the most insight into the physics of these systems. Some heuristics that enable one to see global qualitative trends may be more informative than individual number comparisons.

In this note we discuss some simple “rules of thumb” for the behaviour of radial excitation energies in different heavy-light systems. These are motivated by consideration of the nonrelativistic quark model and are obtained making severe simplifying assumptions. However, the rules of thumb need not be exact. Rather, they serve to focus our attention on the questions we should be asking as we compare different calculations with each other and with experimental data.

In Sect. 2 expectation for the quark mass dependence of radial excitation energies of heavy-light hadrons is discussed using nonrelativistic quantum mechanics as a guide. Experimental information on heavy-light excitation energies is reviewed in Sect. 3. A sample of quark model calculations are discussed in Sect. 4. These serve to assess the validity of the rules of thumb presented in Sect. 2 and to provide a comparison to the lattice QCD results reviewed in Sect. 5. A lattice QCD calculation for \( \Lambda_b \) and \( \Sigma_b \) baryons employing a free-form smearing method is outlined in the Appendix.

2 Scaling in nonrelativistic quantum mechanics

In this section we review the behaviour of excitation energy as a function of constituent mass in the framework of nonrelativistic quantum mechanics. This is then used to develop some “rules of thumb” for how heavy-light hadron masses may be expected to depend on quark masses.

The scaling argument follows the quark model discussions of Ref. [1, 2, 3]. Consider a simple power law potential \( V(r) = cr^\nu \), the Schrödinger equation for S-waves can be written in the form

\[
-\frac{1}{2\mu} \frac{d^2 u}{dr^2} + cr^\nu u = Eu
\]

where \( \mu \) is the reduced mass. Multiply by \( 2\mu \) and rescale \( r \) using

\[
\rho = \frac{r}{(2\mu c)^{\nu}}.
\]

The goal is to find \( p \) such that all the explicit \( \mu \) dependence can be moved to
the right hand side of (1). Using (2) it is easy to find the necessary condition

\[ 1 + 2p + p\nu = 0 \]  

which gives \( p = -\frac{1}{\nu+2} \). Under these manipulations the energy \( E \) goes to \( 2\mu(2\mu c)^{2p}E \). Since this combination of factors must be independent of \( \mu \) it implies that

\[ E \sim \frac{1}{\mu^{1+2p}} = \frac{1}{\mu^{\nu+2}}. \]  

(4)

In nonrelativistic quantum mechanics energy levels for a given system can be shifted by a constant amount (by adding a constant to the potential) but energy differences should obey (4) in any case.

For a power law potential with \(-2 < \nu < 0\) the excitation energy will increase when \( \mu \) increases while for a confining potential \( \nu > 0 \) the excitation energy decreases with increasing \( \mu \). The common quark model potentials have a Coulomb part \((\nu = -1)\) for which energy would increase as \( \mu \) and a linear confining part \((\nu = 1)\) which gives energies proportional to \( \mu^{-1/3} \).

It will be assumed that in a heavy-light hadron that the hadron size is sufficiently large that the confining part of the potential determines the mass dependence of the excitation energy. Furthermore, to include baryons in the discussion it is assumed that the two light quarks within a singly-heavy baryon act as an effective diquark with constituent mass greater than the constituent mass of a single light quark. Then considering the reduced mass

\[ \mu = \frac{m_q}{M_Q} \]  

where \( M_Q \) and \( m_q \) are the heavy and light masses respectively, we can get the following rules of thumb:

1. Keeping the light mass(es) fixed and increasing the heavy mass will decrease the energy of the radial excitation. For example, the radial excitation energy of a B meson will be smaller than that of a D meson.

2. Keeping heavy mass fixed and increasing the light quark mass, that is, going from \( u,d \) quarks to strange will decrease the excitation energy.

3. The radial excitation energy of a singly-heavy baryon will be less than that of a heavy-light meson containing the same heavy quark flavour. For example, the excitation energy of \( \Lambda_c \) will be less than that of D.

If empirical data proves to be consistent with these rules of thumb they would provide a useful guide with which to assess in a broad way the results of calculations of the spectrum. A large discrepancy with these rules would be a signal that a simple understanding of the physics is not correct. How the rules are violated may provide some clues of where to search for the correct explanation.
Table 1: Experimental values for radial excitation energies.

| Energy          | Excitation energy[MeV] |
|-----------------|------------------------|
| $\Delta E(D^0)$ | 674.6(8.2)             |
| $\Delta E(D^{*0})$ | 601.7(3.0)             |
| $\Delta E(D^{*+})$ | 611.0(4.7)             |
| $\Delta E(D^0)$ | 606.4(5.6)             |
| $\Delta E(D^{*+})$ | 619.9(4.9)             |
| $\Delta E(D^0)$ | 597(4)                 |
| $\Delta E(D_s)$ | 480(4)                 |
| $\Delta E(\Lambda_c(2765))$ | 652.9(1.5)             |
| $\Delta E(\Lambda_c(2940))$ | 502(4)                 |

3 Experimental results

At present there are no observed candidates for radial excitations of heavy-light hadrons with a b quark so rule no. 1 can not be tested empirically.

In the charm meson sector, the BaBar Collaboration has observed candidates for the radial excitations of the $D^0$, $D^{*0}$, and $D^{*+}$ mesons\[4\]. Using these results the radial excitation energies are calculated including isospin averages where data are available and are given in Table I. As well as individual pseudoscalar and vector meson excitation energies the value using the spin averaged mass $M = (M(J = 0) + 3M(J = 1))/4$ is given for the $D$ meson. In the charm-strange sector an excited state with $J^\pi = 1^-\bar{\pi}$ has been observed\[5, 6\] which we interpret as the radial excitation of $D_s^*$. As yet there is no identified candidate for the radial excitation of $D_s$. Comparing the values of $\Delta E(D^{*0})$ and $\Delta E(D_s^*)$ one sees that the rule of thumb no. 2 is at best very weakly satisfied.

In the charm baryon sector no excited states have been positively identified as radial excitations. The PDG\[6\] particle listings show five excited states of $\Lambda_c$. Three of these have $J^\pi \neq 1/2^+$. The excitation energies of the other two states, for which spin and parity are undetermined, are given in Table 1. Chen et al.\[7\] advocate the identification of $\Lambda_c(2765)$ with the radial excitation. The PDG also list five $\Xi_c$ states whose spins and parities are unknown\[6\]. Chen et al. suggest that $\Xi_c(2980)$ is the first radial excitation\[7\]. The excitation energy for this state is noted in Table 1. The identification of radial excitations from \[7\] would be consistent with the expectation that baryon excitation energies are smaller than those of mesons (rule of thumb no. 3) and with quark model calculations as will be seen in the next section. It would also suggest that rule of thumb 2 is weakly violated by charmed baryons.

4 Quark models

In this section some results from quark models are presented. The purpose here is not to review the myriad of such calculations that have been done but to
Table 2: Radial excitation energies in MeV of charm mesons from some quark model calculations. The overline indicates a weighted average of pseudoscalar and vector meson values.

|     | Ref. [8] | Ref. [9] | Ref. [10] | Ref. [11] |
|-----|---------|---------|----------|----------|
| $\Delta E(D)$  | 700     | 666     | 721      | 710      |
| $\Delta E(D^*)$ | 600     | 595     | 685      | 622      |
| $\Delta E(D_s)$ | 625     | 613     | 694      | 644      |
| $\Delta E(D^*_s)$ | 690     | 689     | 731      | 720      |
| $\Delta E(D_s)$ | 600     | 595     | 694      | 620      |
| $\Delta E(D^*_s)$ | 622     | 613     | 703      | 645      |

Table 3: Radial excitation energies in MeV of bottom mesons from some quark model calculations. The overline indicates a weighted average of pseudoscalar and vector meson values.

|     | Ref. [8] | Ref. [9] | Ref. [10] | Ref. [11] |
|-----|---------|---------|----------|----------|
| $\Delta E(B)$  | 590     | 545     | 607      | 610      |
| $\Delta E(B^*)$ | 560     | 523     | 596      | 580      |
| $\Delta E(B)$ | 567     | 529     | 599      | 588      |
| $\Delta E(B_s)$ | 590     | 573     | 612      | 604      |
| $\Delta E(B^*_s)$ | 560     | 549     | 598      | 573      |
| $\Delta E(B_s)$ | 567     | 555     | 602      | 596      |

show a sample of results that will serve as a point of comparison and contrast to the lattice QCD results to be discussed in the next section.

Table 2 shows calculated excitation energies for charmed mesons. The results from different calculations (done over a period of about 25 years) are fairly consistent with Ref. [10] perhaps showing a significant variation in the vector meson channel. The results are in reasonable agreement with experimental values. There is no appreciable difference in these models between the $D$ and the $D_s$ systems. Rule no. 2 is certainly not satisfied so likely the assumptions made were overly simple. All these calculations incorporate some relativistic effects. As well, the assumption that the confining potential dominates in determining the light quark mass dependence may not be adequate here.

Table 3 gives bottom meson results. The different models are reasonably consistent and, as in the charm sector, rule no. 2 for the light quark mass dependence is not evident. Comparing the results of Table 2 and Table 3 one sees very clearly the heavy quark mass dependence expected from rule no. 1.

The radial excitation energies for singly-heavy baryons calculated in some potential models are listed in Table 4. As expected, the excitation energy for bottom baryons is less than for charm baryons. Also, baryonic excitation energies are smaller than mesonic ones. The calculated values of $\Delta E(\Lambda_c)$ support the identification of $\Lambda_c(2765)$ as a radial excitation as mentioned in Sect. 4.
Table 4: Radial excitation energies in MeV of singly-heavy baryons from some quark model calculations.

| Ref.  | Ref.  | Ref.  | Ref.  | Ref.  | Ref.  | Ref.  |
|-------|-------|-------|-------|-------|-------|-------|
| $\Delta E(\Lambda_c)$ | 510 | 497 | 377 | 523 | 483 | 572 |
| $\Delta E(\Sigma_c)$ | 450 | 488 | 345 | 503 | 458 | 569 |
| $\Delta E(\Xi_c)$ | | | | | 483 |
| $\Delta E(\Lambda_b)$ | 460 | 372 | 495 | 466 | 535 |
| $\Delta E(\Sigma_b)$ | 405 | 338 | 461 | 405 | 520 |
| $\Delta E(\Xi_b)$ | | | | | 463 |

Table 5: Radial excitation energies in MeV of heavy-light mesons from recent lattice QCD calculations.

| Ref.  | Ref.  | Ref.  | Ref.  | Ref.  | Ref.  |
|-------|-------|-------|-------|-------|-------|
| $\Delta E(D)$ | 697(26) | | | | |
| $\Delta E(D^*)$ | 642(29) | | | | |
| $\Delta E(D)$ | 655(26) | | | | |
| $\Delta E(D_s)$ | 754(27)(10) | 766(39)(50) | | | |
| $\Delta E(D^*_s)$ | 693(21)(10) | 719(43)(80) | | | |
| $\Delta E(D_s)$ | 708(20)(10) | 731(41)(73) | | | |
| $\Delta E(B)$ | | | | 617(42) | 791(93) |
| $\Delta E(B^*)$ | | | | 636(39) | | |
| $\Delta E(B)$ | | | | 632(38) | | |
| $\Delta E(B_s)$ | | | | 594(14) | 566(57) |
| $\Delta E(B^*_s)$ | | | | 595(15) | | |
| $\Delta E(B_s)$ | | | | 595(15) | | |

5 Lattice QCD simulations

The calculation of excited state energies within the framework of lattice QCD is very challenging. However, in the past decade large scale simulations using large operator bases, variational methods and advanced analysis techniques have started to yield results. There is also difficulty in comparing different lattice simulations with each other, with calculations done in other approaches and with experiment. Lattice simulations are carried out with a variety of different discretized actions. Often they are done on single gauge field ensembles i.e., without continuum extrapolation and at light ($u,d$) quark masses that are considerably larger that physical. Notwithstanding these limitations, we believe it is worthwhile to assess the state of lattice QCD simulations for heavy-light hadron radial excitations in view of the rules of thumb proposed in Sect. 2 and of the results reviewed in Sect. 3 and 4.

Table 5 summarizes results of recent lattice simulations for heavy-light me-
Table 6: Radial excitation energies in MeV of heavy-light baryons from recent lattice QCD calculations.

|                  | Ref. [23] | Ref. [24] | This work |
|------------------|-----------|-----------|-----------|
| $\Delta E(\Lambda_c)$ | 903(76)(80) | 781(18)   |           |
| $\Delta E(\Sigma_c)$ | 676(95)(98) | 758(22)   |           |
| $\Delta E(\Xi_c)$   | 792(39)(46) |           |           |
| $\Delta E(\Lambda_b)$ |           |           | 740(92)   |
| $\Delta E(\Sigma_b)$ |           |           | 643(60)   |

The calculations in Ref. [18] and Ref. [19] were done using different gauge field ensembles and different quark masses so these results do not inform us about the $D$ versus $D_s$ comparison suggested by rule no. 2. Ref. [19] and Ref. [20] used the same gauge field ensemble but the calculational methods were completely different. However, the results are completely compatible. In Ref. [20] a potential was first determined and this was then used in the Schrödinger equation to calculate energies. Ref. [19] used the standard technique of extracting masses from the Euclidean time dependence of meson two-point functions. Comparison of these charm meson results with Table 1 shows that lattice QCD values to be somewhat larger than experimental values. Comparing to Table 2, one sees that lattice QCD results are not inconsistent with potential model calculations.

The calculations in Ref. [19] and Ref. [21] were done using the same gauge field ensemble although the lattices actions used for the heavy quark were different, Fermilab clover and NRQCD for the charm and bottom quarks respectively. The expected decrease of the excitation energy in going from charm to bottom is clearly exhibited. We note also that the lattice QCD results of Ref. [21] are fairly compatible with potential model calculations.

The results of Ref. [22] are quite interesting. The value for $\Delta E(B)$ is determined rather poorly but it does appear to be somewhat of an outlier. It doesn’t quite fit into the pattern established by experiment, potential models or other lattice QCD simulations. The difference between $\Delta E(B)$ and $\Delta E(B_s)$ is large (although only about 2σ significant) and the value of $\Delta E(B)$ seems contrary to rule of thumb no. 1 which is satisfied in all other calculations.

Results of recent lattice QCD calculations of excitation energies of charm baryons are given in Table 6. The calculations of Ref. [23] were done at a single lattice spacing, about 0.075fm, and extrapolated to physical quark masses. The calculations of Ref. [24] were done with $u, d$ quarks corresponding to a pion mass of about 379MeV on an anisotropic lattice with temporal and spatial lattice spacings of 0.034fm and 0.12fm. Taken at face value they are at variance with the expectation that baryon excitations are smaller than meson excitations and with potential model calculations. They suggest quite strongly that the $\Lambda_c(2765)$ is not the radial excitation of $\Lambda_c$. Also shown are the results from an exploratory study of single bottom baryons using the same lattice setup as in Ref. [21] (see the Appendix). As in the charm sector, the lattice simulation yields a bottom baryon excitation energy larger than expected from
quark models and larger than calculated for heavy-light mesons. Should these patterns persist with improvements in lattice simulations and confirmation by new experimental information that would present a significant challenge to our understanding of heavy-light hadrons.

6 Summary

Some simple “rules of thumb” for the quark mass dependence of radial excitation energies, motivated by nonrelativistic quantum mechanics, were proposed. Experimental results, quark models and lattice QCD calculations were reviewed in light of these expectations. In quark models and lattice QCD bottom mesons have smaller excitation energies than charm mesons as expected. This is not yet confirmed experimentally. As well, in quark models baryons have smaller excitation energies than mesons containing the same heavy flavour. However, the expectation that increasing the light quark mass, that is, going from $u,d$ to strange should decrease excitation energy is not evident.

For the most part, quark models and lattice QCD simulations give a pattern of quark mass dependence in heavy-light mesons which is compatible with available experimental results. The rules of thumb allow one to spot results which are possible outliers. For example, the results of Ref. [22] for excitation energy of $B$ and $B_s$ do not fit the pattern established by other calculations.

The results of recent lattice QCD calculations of heavy-light baryon excitation energies, Table 6, seem to be at variance with the expectation that they should be smaller than excitation energies of heavy-light mesons. A confirmation of this pattern would challenge the quark model as a guide to heavy-light baryon spectroscopy.

We hope that this note will provide motivation for more study of excited heavy-light hadrons to fill the gaps in experimental information and in lattice QCD simulations.

Appendix

Free-form smearing [25] allows for the construction of correlation functions selectively enhancing the contribution of particular states. A variation of this method was shown to be quite effective in the calculation of the spectrum of bottomonium and B mesons[21]. Here we explore the efficacy of free-form smearing in the simulation of single bottom baryons.

The essential idea of free-form smearing is to start at a single lattice source point $y$ and smear the quark field over the whole spatial lattice volume using the reweighting formula

$$\hat{\psi}_y(x) = \frac{\hat{\psi}_y(x)}{\langle |\hat{\psi}_y(x)| \rangle} f(x - y)$$

where $f(x - y)$ is an arbitrary profile function. The field $\hat{\psi}_y(x)$ is obtained by starting at the source point $y$ and extending the field $\psi(y)$ by multiplying by
gauge field links following minimal paths to all spatial site $s$ in the source time slice

$$\tilde{\psi}_y(x) = \sum_{\text{minimal paths}} U(x \rightarrow y) \psi(y).$$  \hfill (7)

The smeared fields $\tilde{\psi}_y(x)$ are used as the source quark fields in the construction of the hadron two-point functions.

In our study of mesons [21] the profile functions were chosen to have the shape of Coulomb wavefunctions, that is, $f(x - y)$ is $G = e^{-\frac{r}{a_0}}$ and $E = e^{-\frac{r}{a_0}}(r - b)$ for S-wave ground and excited states respectively where $r$ is the shortest distance between $y$ and $x$ in a periodic box. The range $a_0$ and node position $b$ can be adjusted to improve the isolation of ground and excited states.

In this work we explore the application of free-form smearing to the calculation of spin-$1/2$ $\Lambda_b$ and $\Sigma_b$ baryon masses. The bottom quark is described using lattice NRQCD as in Ref. [21] and is not smeared in the correlation function construction. The light quarks are simulated with the clover action using code from the DD-HMC package [26] and can be unsmeared or smeared at the source using either a ground state($G$) or an excited state($E$) profile function.

This yields six correlation functions with different source smearing for each baryon interpolating operator. The baryon operators used in this work are

$$\Lambda_b = \frac{1}{\sqrt{6}} \epsilon^{abc} [q_a^T C\gamma_5 q'_b] Q_c + [q'_a^T C\gamma_5 Q_b] q'_c - [q'_a^T C\gamma_5 Q_b] q'_c$$  \hfill (8)

$$\Sigma_b = \epsilon^{abc} [q_a^T C\gamma_5 q_b] Q_c$$  \hfill (9)

where $Q$ is the heavy $b$-quark field and $q, q'$ are light ($u, d$) fields. The so-called heavy or nonrelativistic lambda (see (3) in [27]) was also considered but was not used in the final analysis. The relativistic forms used here allow for both positive and negative parity baryon states to be simulated.

It is natural to consider the heavy quark as acting approximately as a static color source and to smear the light quarks about it. Ideally one would like to explore baryon operators which incorporate correlations between the light quarks to mimic, for example, a quark-diquark structure as commonly used in quark model calculations of baryon spectra. We do not attempt to do this here. Smearing was applied independently to each light quark. This may be a limitation of the present approach.

The lattice setup was the same as used in [21]. An $N_f = 2 + 1$ flavour dynamical gauge field ensemble from the PACS-CS Collaboration [28] was used. The lattice was $32^3 \times 64$ with a lattice spacing of 0.0907(13)fm determined by the PACS-CS Collaboration. The pion mass is 156(7) MeV for the light quarks used in the simulation. Other parameters are described in [21].

Correlation functions for positive and negative parity baryons were calculated for 198 gauge field configurations averaging over 16 source time positions for each configuration. It was found that correlation functions without smearing did not provide useful data. The effective simulation energies did not reach a plateau before the signal disappeared into noise. Correlators with light quark source smearing profiles GG, EE, and GE were analyzed.
Figure 1: Effective simulation energies in lattice units.
Figure 2: Simulation energies in lattice units for the two lowest states from different fits.

Figure 2 shows the effective simulation energies. A variety of constrained multi-exponential fits were done. For the positive parity correlators points up to $t = 15$ were included. For negative parity correlators only times up to 11 were used. The results were quite robust with regard to initial time (2 or 3), number of exponential terms (3 or 4) and choice of priors. Representative fit values for ground and first excited state simulation energies are shown in Fig. 2.

The mass difference $\Sigma_b(1/2^+) - \Lambda_b(1/2^+)$ was found to be 213(42) MeV consistent with the experimental value $^{[6]}$ 194(3) MeV. The mass difference $\Lambda_b(1/2^-) - \Lambda_b(1/2^+)$ is poorly determined 344(105) MeV compared to the experimental value $^{[6]}$ 293(1) MeV. For $\Sigma_b(1/2^-) - \Sigma_b(1/2^+)$ we have a prediction of 252(60) MeV. The radial excitation energies of the positive parity states are given in Table 6.

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References

[1] C. Quigg and J. L. Rosner. Phys. Lett. 71B, 153 (1977).

[2] G. Feldman, T. Fulton and A. Devoto, Nucl. Phys. B514, 441 (1978).
[3] C. Quigg and J. L. Rosner. Phys. Rept. 56, 167 (1979).
[4] P. del Amo Sanchez et al. (BaBar Collaboration), Phys. Rev. D 82, 111101 (2010).
[5] R. Aaij et al. (LHCb Collaboration), JHEP 1210, 151 (2012).
[6] K. A. Olive et al. (Particle Data Group), Chin. Phys. C 38, 090001 (2014).
[7] B. Chen, K.-W. Wei and A. Zhang, Eur. Phys. J. A 51, 82 (2015).
[8] S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985).
[9] T. A. Lähde, C. J. Nyfält and D. O. Riska, Nucl. Phys. A674, 114 (2000).
[10] M. Di Pierro and E. Eichten, Phys. Rev. D 64, 114004 (2001).
[11] D. Ebert, R. N. Faustov and V. O. Galkin Eur. Phys. J. C 66, 197 (2010).
[12] S. Capstick and N. Isgur, Phys. Rev. D 34, 2809 (1986).
[13] S. Migura, D. Merten, B. Metsch and H.-R. Petry, Eur. Phys. J. A 28, 41 (2006).
[14] H. Garcilazo, T. Vijande and A. Valcarce, J. Phys. G 34, 961 (2007).
[15] W. Roberts and M. Pervin, Int. J. Mod. Phys. A 23, 2817 (2008).
[16] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 84, 014025 (2011).
[17] T. Yoshida, E. Hiyama, A. Hosaka, M. Oka and K. Sadato, arXiv:1510.01067 [hep-ph].
[18] D. Mohler, S. Prelovsek and R. M. Woloshyn, Phys. Rev. D 87, 034501 (2013).
[19] D. Mohler and R. M. Woloshyn, Phys. Rev. D 84, 054505 (2011).
[20] T. Kawanai and S. Sasaki, Phys. Rev. D 92, 094503 (2015).
[21] M. Wurtz, R. Lewis and R. M. Woloshyn, Phys. Rev. D 92, 054504 (2015).
[22] F. Bernardoni, B. Blossier, J. Bulava, M. Della Morte, P. Fritzsch, N. Gar- ron, A. Gérardin and J. Heitger et al., Phys. Rev. D 92, 054509 (2015).
[23] P. Pérez-Rubio, S. Collins and G. S. Bali, Phys. Rev. D 92, 034504 (2015).
[24] M. Padmanath, R. G. Edwards, N. Mathur and M. J. Peardon, PoS LAT- TICE2014 084 (2015).
[25] G. M. von Hippel, B. Jäger, T. D. Rae and H. Wittig, JHEP 1309, 14 (2013).
[26] M. Lüscher, Comput. Phys. Commun. 165, 199 (2005).
[27] N. Mathur, R. Lewis and R. M. Woloshyn Phys. Rev. D 66, 014502 (2002).
[28] S. Aoki, et al. [PACS-CS Collaboration], Phys. Rev. D 79, 034503 (2009).