Proposal to probe the Dzyaloshinskii-Moriya interaction by the propagation characteristics of spin waves in ferromagnetic thin films

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The Dzyaloshinskii-Moriya interaction (DMI) has attracted considerable recent attention owing to the intriguing physics behind and the fundamental role it played in stabilizing magnetic solitons, such as magnetic skyrmions and chiral domain walls. A number of experimental efforts have been devoted to probe the DMI, among which the most popular method is the Brillouin light scattering spectroscopy (BLS) to measure the frequency difference of spin waves with opposite wave vectors $\pm k$ perpendicular to the in-plane magnetization $m$. Such a technique, however, is not applicable for the cases of $k \parallel m$, since the spin-wave reciprocity is recovered then. For a narrow magnetic strip, it is also difficult to measure the DMI strength using BLS because of the spatial resolution limit of lights. To fill these gaps, we propose to probe the DMI via imaging the propagation characteristics of spin waves in ferromagnetic films. We show that the DMI can cause the non-collinearity of the group velocities of spin waves with $\pm k \parallel m$. In heterogeneous magnetic thin films with different DMIs, negative refractions of spin waves emerge at the interface under proper conditions. These findings enable us to quantify the DMI strength by measuring the angle between the two spin-wave beams with $\pm k \parallel m$ in homogeneous film and by measuring the incident and negative refraction angles in heterogeneous films. For a narrow magnetic strip, we propose a nonlocal scheme to determine the DMI strength via nonlinear three-magnon processes. We implement theoretical calculations and micromagnetic simulations to verify our ideas. The results presented here are helpful for future measurement of the DMI and for designing novel spin-wave spintronics devices.

I. INTRODUCTION

The Dzyaloshinskii-Moriya interaction (DMI) is the antisymmetry component of exchange couplings, which was initially proposed to explain the weak ferromagnetism of antiferromagnets [1, 2]. This interaction originates from spin-orbit coupling in magnetic materials with broken inversion symmetry, either in bulk or at the interface. Recently, the DMI has drawn extensive research interest due to two main reasons: (i) its fundamental role in stabilizing topological magnetic solitons, such as skyrmions [3–7] and chiral domain walls [8–12], which are suitable candidates for future spintronic applications; (ii) the intriguing physics associated with the nonreciprocal propagation of spin waves (magnons) [13–16], the elementary excitations in ordered magnets. The determination of the DMI is thus an important issue.

Several experimental schemes have been proposed to measure the DMI strength. For example, it can be quantified by imaging the profile of chiral domain walls [8–10] or by analyzing their dynamical behaviors [11, 17–20] when the driving electric currents and/or magnetic fields are applied. When the DMI is not strong enough to stabilize the inhomogeneous magnetic texture (such as the domain wall), the spin-wave excitation carries the unique information of the DMI. Recent experiments have demonstrated that the DMI constant can be determined by measuring the frequency difference $|\Delta \omega = \omega(k) - \omega(-k)|$ of spin waves with opposite wave vectors $\pm k$ perpendicular to the magnetization $(m)$ using the Brillouin light scattering spectroscopy (BLS) [20–25], the spin-polarized electron energy loss spectroscopy [26], and the propagating wave spectroscopy [27]. For the case of $k \parallel m$, the frequency difference of spin waves with $\pm k$ vanishes and these schemes are unfeasible. We note that spin-wave excitations in these experiments are in the long wavelength regime, where the anisotropic dipolar interaction cannot be ignored. The nonreciprocal nature of dipolar interactions may blur the quantification of the DMI. Moreover, for a magnetic strip with the width well below 100 nm, it is difficult to utilize the BLS to measure the DMI owing to the diffraction limit of lights. It is therefore necessary to develop new methods to measure the DMI for the situations mentioned above.

In this work, we propose to probe the DMI strength in ferromagnetic films via the propagation characteristics of spin waves. To this end, we systematically investigate the effects of the DMI on the propagation, scattering, and interaction of spin waves in different magnetic structures. We first consider a homogeneous ferromagnetic film, and find that the DMI induces a non-collinearity between the wave vector and group velocity $(v_g = d\omega/dk)$ of spin waves when $k$ is not perpendicular to $m$. Spin-wave canting induced by the DMI has been reported in ferromagnetic nanowires [28] with $v_g \parallel m$. Here, we predict another non-collinearity between two spin-wave beams with opposite wave vectors $\pm k \parallel m$. The angle between the two spin-wave beams is derived analytically. Since this non-collinearity comes from the DMI but not the dipolar interaction, we can exclusively determine the DMI strength by measuring the angle between the two beams. Inspired by recent advances of spatially modulated DMI in heterogeneous ferromagnetic films [29–31], we then investigate spin-wave scattering at the interface separating two co-planar ferromagnets with different DMIs. We focus on the exchange spin-wave region, where the nonlo-
cal dipolar effect can be approximated by local demagnetizing fields. We obtain the generalized Snell’s law, and show the emergence of both the negative refraction and the total reflection under proper spin-wave incident angles. These peculiar phenomena and the generalized Snell’s formula can be used to quantify the DMI strength by simply measuring the incident and refracted angles of spin-wave beams, which can be readily realized by direct imagings [32].

Recently, we developed a three-magnon interaction method to detect spin waves localized in the magnetic domain wall nanochannels [33]. The approach can be parallelly applied for probing the DMI in narrow magnetic strips. In general, the three-magnon process is triggered by the weak nonlocal magnetic dipole-dipole interaction in uniform ferromagnets [34]. It can also occur in magnetic textures such as skyrmions [35] and domain walls [33] without the dipolar interaction. Here, we consider another three-magnon effect induced by the DMI in uniform ferromagnets. The idea is analytically formulated with micromagnetic simulations performed to verify the theoretical predictions.

The structure of this paper is organized as follows. In Sec. II, we introduce the model and derive the dispersion relation of chiral spin waves in a ferromagnetic thin film. The group-velocity non-collinearity of two propagating spin-wave beams with anti-parallel wave vectors is presented in Sec. III. In Sec. IV, we investigate the spin-wave scattering across the interface of two ferromagnets with different DMIs. The generalized Snell’s law is derived. We show the emergence of the total reflection and the negative refraction phenomena under proper spin-wave incident angles. In Sec. V, three-magnon processes arising in a narrow magnetic strip with the DMI are studied. We demonstrate that these nonlinear effects can be utilized to accurately quantify both the DMI parameter and the magnetization inhomogeneity of the strip edge. Conclusions are drawn in Sec. VI. In the appendix, we derive the three-magnon interaction Hamiltonian induced by the DMI.

II. MODEL

We consider the spin-wave propagation in a magnetic thin film with the interfacial DMI of the following form [36],

\[ H_{DM} = \frac{2D}{\mu_0 M_s} [\nabla m_z - (\nabla \cdot m) \hat{z}] = \frac{2D}{\mu_0 M_s} \left( \frac{\partial m_z}{\partial x} \hat{x} - \frac{\partial m_z}{\partial y} \hat{y} - \frac{\partial m_x}{\partial x} \hat{x} - \frac{\partial m_y}{\partial y} \hat{y} \right) \]  

(1)

where \( D \) is the DMI constant, \( M_s \) is the saturation magnetization, and \( m = (m_x, m_y, m_z) \) is the unit magnetization vector. The magnetization dynamics is described by the Landau-Lifshitz-Gilbert (LLG) equation,

\[ \frac{\partial m}{\partial t} = -\gamma \mu_0 m \times H_{eff} + \alpha m \times \frac{\partial m}{\partial t} \]  

(2)

with the gyromagnetic ratio \( \gamma = 1.76 \times 10^{11} \text{ rad s}^{-1} \text{T}^{-1} \), the vacuum permeability \( \mu_0 \), and the Gilbert damping constant \( \alpha \). The effective field \( H_{eff} \) comprises of the exchange field, the DM field, the demagnetization field, and the external field. Although the DMI facilitates the inhomogeneous magnetic texture, when the external field is sufficiently strong, it is possible to stabilize a single-domain structure as the ground state. Given that the DMI has no effect on spin waves when \( m \) is perpendicular to the film plane [15, 37], an external field \( H_{ext} = H_0 \hat{y} \) is applied to make \( m \) in the film plane \( (m = +\hat{y}) \), as shown in Fig. 1. In the large \( k \) limit, one may neglect the nonlocal magnetostatic contribution, the spin wave dispersion relation can be obtained by solving the linearized LLG equation [37, 38],

\[ \omega(k) = \sqrt{(A^*k^2 + \omega_{\parallel}) (A^*k^2 + \omega_{\perp} + \omega_m) - D^*k_x} , \]  

(3)

with \( A^* = 2\gamma A/M_s \) with the exchange constant \( A \), \( \omega_{\parallel} = \gamma \mu_0 H_0 \), \( \omega_m = \gamma \mu_0 M_s \), \( D^* = 2\gamma D/M_s \), and \( k = (k_x, k_y) \) the wave vector of spin wave. Here we focus on the exchange spin waves with high frequency and simplify Eq. (3) to

\[ \omega(k) = A^*k^2 - D^*k_x + \omega_{\parallel} + \frac{\omega_m}{2} , \]  

(4)

and obtain the group velocity

\[ v_g = \frac{\partial \omega}{\partial k} = (2A^*k_x - D^*)\dot{x} + 2A^*k_y\dot{y} . \]  

(5)

III. SPIN WAVE PROPAGATION IN CHIRAL MAGNETIC FILM

The influence of the DMI on spin-wave propagation in a chiral ferromagnetic film can be analyzed by the dispersion relation and the isofrequency curve, as shown in Fig. 2. According to Eq. (4), we plot the spin wave dispersion relation \( \omega(k) \) in Fig. 2(a) for two representative cases \( k \perp m \) and \( k \parallel m \). In the case of \( k \perp m \) (\( k_y = 0 \)), a frequency difference of spin waves with \( \pm k \) (blue curve) is identified as \( \Delta \omega = 2D^*k_x \), which is widely used to measure the DMI strength in the BLS experiments. For
\( \mathbf{k} \parallel \mathbf{m} \) (\( k_x = 0 \)), we observe a curve (red curve) symmetric with respect to \( k_y = 0 \), which overlaps with the dispersion relation for \( D = 0 \) (dashed black curve) indicating the reciprocity of spin waves. Figure 2(b) shows the isofrequency curve of spin waves based on Eq. (4). In the presence of the DMI, the isofrequency circle shifts away from the origin in the \( \mathbf{k} \) space. For \( \mathbf{k} \perp \mathbf{m} \), the magnitudes of wave vectors with opposite directions are different, which indicates asymmetry of spin wave wavelength with respect to the propagation direction [38]. It is known that the wave vector determines the wave front while the group velocity represents the propagation direction. When the propagation direction (\( \mathbf{v}_{gx}^+ \) and \( \mathbf{v}_{gy}^- \)) of spin wave is along the magnetization, the wave vectors (\( \mathbf{k}^+ \) and \( \mathbf{k}^- \)) of spin waves become oblique with respect to the propagation direction, which is responsible for the spin-wave canting numerically observed in Ref. [28]. In the case of \( \mathbf{k} \parallel \mathbf{m} \) (\( k_x = 0 \)), the group velocity in Eq. (5) can be written as \( \mathbf{v}_g = -D^x \hat{x} + 2A^x k_y \hat{y} \). In the presence of the DMI (\( D \neq 0 \)), the group velocities (\( \mathbf{v}_g^+ \) and \( \mathbf{v}_g^- \)) of spin waves with opposite wave vectors (\( \mathbf{k}_g^+ \) and \( \mathbf{k}_g^- \)) are not collinear, as shown in Fig. 2(b). The angle between the two group velocities can be obtained:

\[
\theta = \arccos \left( \frac{\mathbf{v}_g^+ \cdot \mathbf{v}_g^-}{|\mathbf{v}_g^+||\mathbf{v}_g^-|} \right) = \arccos \left( \frac{(D^x)^2 - 4A^x(\omega - \omega_H - \omega_m)/2}{(D^x)^2 + 4A^x(\omega - \omega_H - \omega_m)/2} \right). \tag{6}
\]

Based on Eq. (6), the DMI strength can be evaluated by measuring the angle of the two spin-wave beams.

We confirm the above results using micromagnetic simulations code OOMMF including the interface DMI [39, 40]. We consider a ferromagnetic thin film with length 2000 nm, width 2000 nm, and thickness 1 nm, which lies in the \( x = y \) plane. Magnetic parameters of Permalloy were used in simulations: \( M_s = 8 \times 10^5 \text{ A/m}, A = 13 \text{ pJ/m}, \) and \( \alpha = 0.01 \). In the simulations, the Gilbert damping constant close to the film edges is set to linearly increase to 1.0 to avoid the spin-wave reflection by the boundaries [41]. We apply an external field \( \mu_0 H_0 = 1 \text{ T} \) along \( +\hat{y} \) that is sufficiently strong to saturate the magnetization in the film plane. Then, we stimulate the propagations of two spin-wave beams with opposite wave vectors parallel and antiparallel to the magnetization (\( \pm \mathbf{k} \parallel \mathbf{m} \)). To this end, we apply a sinusoidal monochromatic microwave source \( \mathbf{H}_{ext} = H_0 \sin(\omega t) \hat{z} \) in a narrow rectangular region (300 \times 10 \text{ nm}^2) [black bar shown in Fig. 3(a)], where the field amplitude \( H_0 \) has a Gaussian profile in the transverse direction \( (\hat{x}) \) [42]. Figure 3(a) shows two spin waves with \( \omega/2\pi = 80 \text{ GHz} \) and \( \pm \mathbf{k} \parallel \mathbf{m} \) in the presence of DMI (\( D = 3.0 \text{ mJ/m}^2 \)). It is clear to see that the group velocities \( \mathbf{v}_g^+ \) and \( \mathbf{v}_g^- \) of the two spin-wave beams are non-collinear. Both beams propagate towards the left, which is fully in line with formula \( \mathbf{v}_g = -D^x \hat{x} + 2A^x k_y \hat{y} \). The angle \( \theta = 120.5^\circ \) obtained from simulation is also consistent with the theoretical prediction Eq. (6) with a deviation less than 0.5%.

The angles between the two spin-wave beams measured from simulations (dots) as well as the ones obtained from Eq. (6) (curves) are plotted as a function of the excitation frequencies for different DMI constants, as shown in Fig. 3(b). Good agreements can be found, except at low frequencies (\( \omega/2\pi \lesssim 50 \text{ GHz} \)). For \( D = 0 \), the angle between the two spin-wave beams is always equal to 180°. This indicates that the non-collinearity of the two spin-wave beams observed above is exclusively induced by the DMI rather than the dipolar interaction. Thus, the DMI strength can be determined by measuring the angle between the two spin-wave beams, which can be realized by direct imagings in experiments [32].
FIG. 4. Schematic plot of the generalized Snell’s law for spin-wave scattering at the DMI interface. The magnetization of ultrathin film is saturated along +\( \hat{y} \) direction. The green (lower) and magenta (upper) circles correspond to the isofrequency curves in momentum space for spin-wave propagation in no-DMI and DMI regions, respectively.

IV. SPIN WAVE SCATTERING ACROSS THE INTERFACE OF TWO FERROMAGNETIC FILMS WITH DIFFERENT DMIS

We proceed to investigate the scattering of exchange spin waves across the interface of two ferromagnets with different DMIs. One effect of the spatially modulated DMI is the equilibrium spin canting at the DMI interface [29], which is caused by an additional effective field along \( \hat{z} \) axis originating from the DMI step [30]. Since spin cantings only occur at a narrow range around the DMI interface, its effect on the spin-wave propagation is rather weak. Hence, we view the magnetization of the heterogeneous film as uniform along the +\( \hat{y} \) direction. Based on Eq. (4), the isofrequency curves of spin waves propagation in no-DMI and DMI regions are plotted in \( \mathbf{k} \) space, as shown in Fig. 4. In the absence of the DMI (\( D = 0 \)), the spin-wave isofrequency curve at a given frequency \( \omega \) is a circle centered at the origin with the radius \( k^0_\omega = \sqrt{(\omega - \omega_H - \omega_m/2)/A^2} \). With the DMI (\( D \neq 0 \)), the isofrequency circle is shifted by \( \Delta = D^*/2A^* \) along +\( k_x \) axis and its radius increases to \( k^D_\omega = \sqrt{(k^0_\omega)^2 + \Delta^2} \). According to the conservation of momentum parallel with the interface, we obtain the generalized Snell’s law

\[
k^D_\omega \sin \theta_i = k^D_\omega \sin \theta_r + \Delta,
\]

where \( \theta_i \) and \( \theta_r \) are the incident angle and refraction angle of spin-wave beams with respect to the interface normal, as shown in Fig. 4. Similar results have been presented for spin waves propagation in different magnetic systems [43, 44]. For instance, the generalized Snell’s law describes the spin wave refracted at the domain wall in a chiral magnetic film in Ref. [43].

Micromagnetic simulations are performed to validate the generalized Snell’s law (7). The DMI strength of the upper (\( y > 0 \)) and lower (\( y < 0 \)) films are set to be \( D = 3.0 \, \text{mJ/m}^2 \) and \( D = 0 \), respectively. An external field \( \mu_0 H_0 = 1 \, \text{T} \) along +\( \hat{y} \) is applied to stabilize the uniform state. A spin-wave beam with 80 GHz is excited in a narrow rectangular area (300 x 10 nm\(^2\)) of the lower film (\( D = 0 \)). Figure 5(a) shows the simulation result for the incoming wave with an incident angle \( \theta_i = 60^\circ \). The incident spin wave is reflected at the interface and refracted through the interface with angle \( \theta_r = 60^\circ \). In the no-DMI region (lower), the directions of \( \mathbf{v}_g \) and \( \mathbf{k} \) for both the incident and reflection waves are always parallel to each other. But in the upper region, due to the anisotropy induced by the DMI, \( \mathbf{v}_g \) and \( \mathbf{k} \) are not collinear. To obtain more quantitative insights, we perform a spatial fast-Fourier-transformation (FFT) spectrum analysis of the dynamic magnetization (\( \delta m_z \)) over the region inside the dashed black square with the side length 700 nm in (a). The green and magenta circles are the isofrequency curves in no-DMI and DMI regions, from Eq. (4). The arrows represent the wave vectors of the incident, reflected, and refracted waves.

FIG. 5. (a) Micromagnetic simulation for spin-wave refraction and reflection at the DMI interface with the incident angle \( \theta_i = 60^\circ \). The black bar corresponds the source area of spin-wave excitations and the exciting field frequency is \( \omega/2\pi = 80 \) GHz. The thin and thick arrows denote the wave vector \( \mathbf{k} \) and group velocity \( \mathbf{v}_g \) of spin waves propagating in no-DMI region (green arrow) and DMI region (magenta arrow), respectively. (b) FFT power distribution of spin waves in the \( k_x - k_y \) plane. The FFT analysis is implemented over the region inside the dashed black square with the side length 700 nm in (a). The green and magenta circles are the isofrequency curves in no-DMI and DMI regions, from Eq. (4). The arrows represent the wave vectors of the incident, reflected, and refracted waves.
In the case of vertical incidence, the refracted angle is normal refraction, as shown in Fig. 6(b) with \( \theta_0 \) and refracted angles have the same sign corresponding to \( \theta_0 \) becoming positive and have the same sign with completely reflected. For incident angles at 34\(^\circ\) and 49.6\(^\circ\), there is no real solution for the refracted angle \( \theta_i \) and total reflection takes place. This situation is plotted in Fig. 6(a), showing that the incident spin wave with \( \theta_0 = -60^\circ \) cannot transmit into the DMI region (upper gray region) and is completely reflected. For -35.7\(^\circ\) < \( \theta_0 \) < 0\(^\circ\), the incident and refracted angles have the same sign corresponding to normal refraction, as shown in Fig. 6(b) with \( \theta_0 = -18^\circ \). In the case of vertical incidence, the refracted angle is -29.5\(^\circ\) rather than 0\(^\circ\), which is different from its optical analog, as illustrated in Fig. 6(c). For 0\(^\circ\) < \( \theta_0 \) < 34.4\(^\circ\), the refracted angle is negative. As an example, we set \( \theta_0 = 18^\circ \) in Fig. 6(d), and find that the refracted angle is \( \theta_i = -11.7^\circ \). Both the incident and refracted spin-wave beams are on the same side of the interface normal, which indicates the occurrence of negative refraction. For the incident angles at 34.4\(^\circ\) < \( \theta_0 \) < 90\(^\circ\), the refracted angles become positive and have the same sign with \( \theta_0 \), recovering the normal refraction again, as shown in Fig. 5(a) with \( \theta_0 = 60^\circ \).

As we have shown, total reflection and negative refraction can happen at the DMI interface for a certain range of the incident angles. Utilizing total reflection at the DMI interface, a spin-wave fiber or guide can be designed, analogous to the cases studied in Refs. [43] and [44]. It is worth noting that total refraction is not a unique feature at the DMI interface. In non-chiral ferromagnetic heterostructures with different materials parameters such as the exchange constant, saturation magnetization, and thickness, it can happen as well [45–48]. In contrast, the negative refraction is exclusively due to the existence of a DMI step for the exchange spin wave and would disappear without the DMI. By measuring the incident and negative-refracted angles, one can determine the DMI constant based on the generalized Snell’s law [Eq. (7)].

V. THREE-MAGNON INTERACTIONS ARISING IN A MAGNETIC STRIP

In the above discussions, we focused on how to measure the DMI in ferromagnetic thin films of large scales. Due to the big laser spot size subjected to the diffraction limit of lights in BLS, it is difficult to detect spin waves propagating in a rather narrow magnetic strip or nanowire. Here, we propose a novel method to measure the DMI parameter in magnetic strip by analyzing the spectrum of spin waves involving in the nonlinear three-magnon processes. To this end, we construct a heterogeneous ferromagnetic thin film with length 1800 nm, width 1000 nm, and thickness 1 nm, in which a DMI strip with the width \( w = 50 \) nm locates in the center while the rest parts have no DMI, as shown in Fig. 7(a). To ascertain the spin-wave spectrum in the DMI strip, we apply a sinc-function field \( \mathbf{h}(t) = h_0 \sin[\omega_H (t - t_0)]/[\omega_H (t - t_0)]\hat{z} \) for 10 ns with \( \mu_0 h_0 = 0.01 \) T, \( \omega/2\pi = 100 \) GHz, and \( t_0 = 1 \) ns, over the black bar with volume 10 \( \times \) 400 \( \times \) 1 \( \text{nm}^3 \) shown in Fig. 7(a). In Fig. 7(c), the dispersion relation is obtained by performing the FFT of the spatiotemporal oscillation of the \( z \)-component magnetization (\( \delta m_z \)) over the lattices along the DMI strip center (\( y = 900 \) nm) in Fig. 7(a). One can immediately see that the theoretical result (3) [solid black curve shown in Fig. 7(c)] does
confluence splitting

\[
\omega(k) = \sqrt{(A^*k^2 + \omega_H/\cos \delta)(A^*k^2 + \omega_H/\cos \delta + \omega_m)} - D'k_y \cos \delta. \tag{8}
\]

The simulated dispersion relation is well fitted by the above formula (8) with \(\delta \approx 20^\circ\) [see the solid yellow curve in Fig. 7(c)]. This approximation is also suitable for describing the dispersion relation of spin waves localized at the edge of ferromagnets in Ref. [15]. Interestingly, we find that the presence of the DMI reduces the gap of spin-wave band from \(\sqrt{\omega_H(\omega_H + \omega_m)/2\pi} \approx 39.7\) GHz without the DMI to 31.4 GHz with \(D = 3.0\) mJ/m\(^2\), as shown in Fig. 7(c). In other words, spin waves with frequencies in the range (31.4, 39.7) GHz will be localized in the DMI strip. This motivates us to consider the three-magnon processes in the strip channel, while the present authors have considered a similar issue but in magnetic domain wall channels in Ref. [33]. See the Appendix for the analytical derivation of the three-magnon interaction Hamiltonian generated by the DMI beyond the linear spin-wave approximation.

Our strategy is as follows: we first input a propagation spin wave \((\omega_i, k_i)\) in the lower part of the heterogeneous films, to interact with the localized spin wave \((\omega_b, k_b)\) bounded in the strip. In general, two kinds of three-magnon processes, i.e., confluence and splitting, can occur, as illustrated in Fig. 8. In the three-magnon processes, the energy and momentum parallel with the strip is conserved. Thus we have

\[
\omega_k = \omega_i + \omega_b, \quad (k - k_i - k_b) \cdot \hat{x} = 0, \tag{9}
\]

for the three-magnon confluence and

\[
\omega_1 + \omega_2 = \omega_i, \quad (k_1 + k_2 - k_i) \cdot \hat{x} = 0, \tag{10}
\]

for the three-magnon splitting. A stimulated splitting implies \(k_1 = k_b\) in (10). These results suggest that, with the help of the information of the incident and transmitted spin waves, we can uniquely infer the spectrum of spin waves localized in the DMI strip. It is noted that the three-magnon splitting process has a threshold frequency higher than the spin-wave band gap [33].

Next, we examine the three-magnon processes arising in the DMI strip by micromagnetic simulations. We apply two sinusoidal monochromatic microwave fields simultaneously to excite the propagating spin waves \((\omega_i, k_i)\) in the lower part of the magnetic film and the localized spin waves \((\omega_b, k_b)\) in the DMI strip, respectively [see Fig. 9(a)]. Here, we consider \(\omega_i/2\pi = 80\) GHz and \(\omega_b/2\pi = 35\) GHz and focus on the normal incident case, i.e., \(k_i \parallel \hat{y}\). Because of the conservation of both the energy and the momentum along the DMI strip (\(\hat{x}\)), the transmitted spin waves carry the information \((\omega_{i,b}, k_{i,b})\) from the incident

FIG. 8. Schematic picture of nonlinear three-magnon processes in the DMI strip. In dashed red square, it shows the three-magnon confluence of \(k_i\) and \(k_b\) into \(k\). In dashed blue square, we plot the stimulated three-magnon splitting of \(k_i\) into two modes \(k_1 = k_b\) and \(k_2\), assisted by a localized magnon \(k_b\) (gray arrow).

FIG. 9. (a) Micromagnetic simulations of three-magnon processes. The incident and localized spin waves are excited at the lower part of the magnetic film (horizontal black bar) and the right side of the DMI strip (vertical black bar), respectively. (b) Temporal FFT spectrum at a single lattice [black dot in (a)]. (c)~(e) Spatial FFT spectrum analyses for three peaks at (c) 80 GHz, (d) 45 GHz and (e) 115 GHz, observed in (b) where the incident spin-wave frequency is \(\omega_i/2\pi = 80\) GHz and the localized spin-wave frequency is \(\omega_b/2\pi = 35\) GHz. The FFT analysis is implemented over the region inside the dashed black square with the side length 700 nm in (a).
and localized spin waves. This result is confirmed by the temporal FFT spectrum at a single cell [the black dot in Fig. 9(a)], which shows three peaks at 45 GHz, 80 GHz, and 115 GHz, as plotted in Fig. 9(b). The main peak of 80 GHz is from the incident spin wave excited at the lower part of the magnetic film. Two relatively weaker peaks at 45 GHz and 115 GHz are due to the three-magnon splitting and confluence processes, which satisfy the energy conservation $\omega_k = \omega_1 \mp \omega_b$, respectively. The wave vectors of spin waves for three frequency peaks can be obtained by the spatial FFT spectrum analysis over the region inside the dashed black square in Fig. 9(a). FFT results are shown in Figs. 9(c)-9(e). The magnon wave vector at 80 GHz is $k_1 = 0.215 \hat{y}$ in the unit of nm$^{-1}$, which agrees with the dispersion relation Eq. (3). While the magnon wave vectors at 45 GHz and 115 GHz are $k = -0.045 \hat{x} + 0.063 \hat{y}$ and $0.045 \hat{x} + 0.296 \hat{y}$ in the units of nm$^{-1}$, respectively. According to the conservation of momentum parallel with the DMI strip, the $x$-components of the wave vectors $k$ of the transmitted spin waves for three-magnon confluence and stimulated splitting are $k_x$ and $-k_{b1}$, respectively. Therefore, we can determine the wave vector of the localized spin wave in the DMI strip, $k_b = 0.045 \hat{x}$ in unit of nm$^{-1}$, which is consistent with the direct FFT analysis in the strip.

Now, considering the inverse problem by assuming both the DMI constant $D$ and the canting angle $\delta$ are two unknown parameters in Eq. (8), only one group of $(\omega_b, k_b)$ is insufficient to determine them. We need another set of $(\omega_b, k_b)$ to completely quantify the DMI. To this end, we apply the same sinusoidal microwave field on the left side of the DMI strip, as shown in Fig. 10(a).

A different microwave field on the same side also serves the same purpose (not shown). Although they have the same frequency, the localized spin waves excited at two sides of the DMI strip carry different wave vectors due to their non-reciprocal nature, as shown in Fig. 9(a) and Fig. 10(a). Temporal FFT spectrum analysis at a single cell [the black dot in Fig. 10(a)] shows two peaks at 80 and 115 GHz in Fig. 10(b), which are from the incident spin wave and three-magnon confluence event discussed above. As compared with the FFT spectrum in Fig. 9(b), the frequency peak at 45 GHz disappears, but there is a very weak new peak at 70 GHz. The reason for the disappearance of 45 GHz peak is that the frequency of the incident spin wave is not high enough for generating the stimulated three-magnon splitting process. The appearance of 70 GHz peak is due to the frequency-doubling effect of the localized spin wave in the DMI strip. Based on the energy conservation in the three-magnon process, the frequency of the localized spin wave is the difference between two frequency peaks, i.e., $\omega_1/2\pi = 115 - 80 = 35$ GHz. The conservation of momentum parallel with the DMI strip indicates that the wave vector of the localized spin wave is $k_b = 0.171 \hat{x}$ in unit of nm$^{-1}$, as shown in Fig. 10(c). Substituting the two sets $(\omega_b, k_b)$ into the dispersion relation (8), and solving the following coupled equations

$$\begin{align*}
\omega_b &= \sqrt{(A^* k_{b1}^2 + \frac{\omega_H}{\cos \delta})(A^* k_{b2}^2 + \frac{\omega_H}{\cos \delta} + \omega_m)} - D^* k_{b1} \cos \delta, \\
\omega_b &= \sqrt{(A^* k_{b2}^2 + \frac{\omega_H}{\cos \delta})(A^* k_{b2}^2 + \frac{\omega_H}{\cos \delta} + \omega_m)} - D^* k_{b2} \cos \delta,
\end{align*}$$

we obtain the DMI constant $D = 3.1$ mJ/m$^2$ and $\delta = 22.7^\circ$, which is consistent with the input parameter $D = 3.0$ mJ/m$^2$ and the fitting $\delta = 20^\circ$ obtained earlier. These results suggest that the DMI of a narrow magnetic strip can be accurately probed by only detecting the spectra of both the incident and transmitted spin waves involving in the nonlinear three-magnon processes.

VI. CONCLUSION

To conclude, we systematically investigate the propagation characteristics of spin wave in various ferromagnetic mediums and structures. In homogeneous ferromagnetic thin films, we predict a non-collinearity of two spin-wave beams with $\zeta k \parallel m$, which solely comes from the DMI rather that the dipolar interaction. By measuring the angle between the two beams, one can determine the DMI parameter. We also consider a magnetic interface in the heterogeneous ultrathin films with different DMIs, and obtained the spin-wave Snell’s law confirmed by micromagnetic simulations. Total reflection and negative refraction are observed at the DMI interface for certain incident angles. The total reflection induced by the DMI can be used to design spin-wave fiber with
unidirectional transmission functionalities. Negative refraction found here is exclusively induced by the DMI. These DMI-induced effects would provide an alternative approach to BLS for probing the DMI strength. Moreover, we propose a nonlocal scheme to measure the DMI parameter in a narrow ferromagnetic strip or nanowire by three-magnon processes, which is not accessible for BLS due to the diffraction limit. Our results would be helpful to extend the present method for probing the DMI in experiments and for designing novel magnonic devices.

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APPENDIX

In this appendix, we derive the three-magnon interaction Hamiltonian arising from the DMI in uniform ferromagnets. We start from the interfacial DMI Hamiltonian

\[ \mathcal{H}_{\text{DM}} = D \int dr [m_z (\nabla \cdot \mathbf{m}) - (\mathbf{m} \cdot \nabla) m_z] \]

\[ = \frac{D}{M_s^2} \int dr \left( M_x \frac{\partial M_x}{\partial x} + M_y \frac{\partial M_y}{\partial y} - M_x \frac{\partial M_z}{\partial x} - M_y \frac{\partial M_z}{\partial y} \right), \]  

(12)

where \( \mathbf{M} = (M_x, M_y, M_z) \) is the magnetization and \( D \) is the DMI constant. The static magnetization lies in-plane and deviates from \( \hat{x} \) direction with an arbitrary angle \( \varphi \), as shown in Fig. 11. We represent \( \mathbf{M} \) in the form \( \mathbf{M}_0 + \mathbf{s}(\mathbf{r}, t) \), where \( \mathbf{M}_0 \) is the background static magnetization, and \( \mathbf{s} \) corresponds to the small oscillations of the magnetization against \( \mathbf{M}_0 \). It is convenient to introduce a new coordinate system in which \( \mathbf{e}_1 \) is always along the equilibrium direction of \( \mathbf{M} \):

\[
\begin{pmatrix}
\mathbf{e}_1 \\
\mathbf{e}_2 \\
\mathbf{e}_3
\end{pmatrix} = \begin{pmatrix}
\cos \varphi & \sin \varphi & 0 \\
-\sin \varphi & \cos \varphi & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
\hat{x} \\
\hat{y} \\
\hat{z}
\end{pmatrix}.
\]  

(13)

Expressing the magnetization in coordinates \( (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3) \) yields

\[
M_x = M_1 \cos \varphi - M_2 \sin \varphi,
\]

\[
M_y = M_1 \sin \varphi + M_2 \cos \varphi,
\]

\[
M_z = M_3,
\]

where \( M_1 = M_0 + s_1, M_2 = s_2, \) and \( M_3 = s_3 \).

Substituting (14) into (12), we obtain the expression of \( \mathcal{H}_{\text{DM}} \),

\[
\mathcal{H}_{\text{DM}} = \frac{D}{M_s^2} \int dr \left[ M_3 \cos \varphi \frac{\partial M_1}{\partial x} + M_3 \sin \varphi \frac{\partial M_1}{\partial y} - M_3 \sin \varphi \frac{\partial M_2}{\partial x} + M_3 \cos \varphi \frac{\partial M_2}{\partial y} \\
- (M_1 \cos \varphi - M_2 \sin \varphi) \frac{\partial M_3}{\partial x} - (M_1 \sin \varphi + M_2 \cos \varphi) \frac{\partial M_3}{\partial y} \right].
\]  

(15)

We shall express the components of \( \mathbf{s}(\mathbf{r}, t) \) in the rotated coordinates in terms of the Holstein-Primakoff operators \( a(\mathbf{r}) \) and \( a^+(\mathbf{r}) \):

\[
s_1 = -2\mu_B a^+ a,
\]

\[
s_+ = s_2 + is_3 = 2\mu_B \sqrt{2S} \left( 1 - \frac{a^+ a}{2S} \right)^{\frac{1}{2}} a \approx 2\mu_B \sqrt{2S} \left( a - \frac{a^+ a a}{4S} \right),
\]

\[
s_- = s_2 - is_3 = 2\mu_B \sqrt{2S} a^+ \left( 1 - \frac{a^+ a}{2S} \right)^{\frac{1}{2}} \approx 2\mu_B \sqrt{2S} \left( a^+ - \frac{a^+ a^+ a}{4S} \right),
\]  

(16)

where \( S = M_s/(2\mu_B) \) is the spin of an atom with the Bohr magneton \( \mu_B \). The operators \( a(\mathbf{r}) \) and \( a^+(\mathbf{r}) \) satisfy the Bose commutation relations, \([a(\mathbf{r}), a^+(\mathbf{r}')] = \delta(\mathbf{r} - \mathbf{r}')\), and are the annihilation and creation operators of spin waves,
terms indeed contain the odd orders of $a$ written as:
\[ M \]
Substituting (17) into (15), we note that the terms without respectively. We then have
\[ H_{\text{odd}} = \frac{D}{M_s^2} \int \left[ M_3 \cos \varphi \frac{\partial M_1}{\partial x} + M_3 \sin \varphi \frac{\partial M_1}{\partial y} - M_1 \cos \varphi \frac{\partial M_3}{\partial x} - M_1 \sin \varphi \frac{\partial M_3}{\partial y} \right] \]
We therefore have
\[ H_{\text{DM}}^{(3)} = \frac{iDS}{4\sqrt{2}} \int \left\{ \cos \varphi \left[ 4(a-a^+) \frac{\partial}{\partial x} (a^+a) - 4(a^+a) \frac{\partial}{\partial x} (a-a^+) - \frac{\partial}{\partial x} (a^+aa - a^+a^+) \right] \
+ \sin \varphi \left[ 4(a-a^+) \frac{\partial}{\partial y} (a^+a) - 4(a^+a) \frac{\partial}{\partial y} (a-a^+) - \frac{\partial}{\partial y} (a^+aa - a^+a^+) \right] \right\} \]

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