Fitting DVCS amplitude in moment-space approach to GPDs

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Abstract: We describe small-$x_{\text{Bj}}$ deeply virtual Compton scattering measurements at HERA in terms of generalized parton distributions at leading order of perturbation series.

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1 Introduction

Deeply virtual Compton scattering (DVCS), $\gamma^*(q_1) p(P_1) \rightarrow \gamma(q_2) p(P_2)$, is viewed as the cleanest process to access generalized parton distributions (GPDs), which encode a partonic description of the nucleon, cf. Refs. \cite{3,4}. In the kinematics of H1 and ZEUS collider experiments at HERA, the DVCS cross section is to a large extent dominated by the flavor singlet part of the helicity conserved Compton form factor (CFF) $\mathcal{S}_H$:

\[ \frac{d\sigma}{dt}(W,t,Q^2) \approx \frac{4\pi\alpha^2}{Q^4}\xi^2 \left| \mathcal{S}_H(\xi, t = \Delta^2, Q^2) \right|^2 \bigg|_{\xi = Q^2/(2W^2+Q^2)}. \]  

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Here, $W$ is the c.o.m. energy, $\Delta = P_2 - P_1$ is the momentum transfer, $-Q^2 = q_1^2$ is the incident photon virtuality, and $\xi \approx x_{Bj}/2$ is a Bjorken-like scaling variable.

The CFF $^s\mathcal{H}$ factorizes further into a convolution of the partonic, i.e., hard scattering, amplitude $C = (^sC, ^gC)$ and GPDs $H = (^sH, ^gH)$, ($\Sigma=$singlet quark, $G=$gluon),

\[ ^s\mathcal{H}(\xi, t, Q^2) = \int_{-1}^{1} dx \ C(x, \xi, Q^2/\mu^2, \alpha_s(\mu)) \ H(x, \eta = \xi, t, \mu^2), \quad (2) \]

where the skewness parameter $\eta = -\Delta \cdot q/(P_1 + P_2) \cdot q$ is set equal to $\xi$. The factorization scale $\mu$ separates short- and long-distance dynamics and is often taken as $\mu = Q$. The scale dependence is governed by evolution equations. They also tell us that the evolution effect depends on the $(x, \eta)$ GPD shape itself. Note that gluons do not directly enter the DVCS amplitude at leading order (LO), but rather drive the evolution of singlet quarks.

Since the momentum fraction $x$ is integrated out in the amplitude (2), GPDs cannot be directly revealed. In the quest for a realistic GPD model, GPD values along the two trajectories $\eta = 0$ and $\eta = x$ play a prominent role. Namely, GPDs on these trajectories are given at LO by DIS structure function and imaginary part of DVCS amplitude, respectively. In this sense they are experimentally measurable. Thus, a modelling strategy that places emphasis on these trajectories has a good chance to be efficient in a global DVCS fit and to capture the physical GPD content, giving us a link to a partonic interpretation.

It is the objective of this work to find GPDs that satisfy the well-known theoretical constraints [3, 4] and provide a good fit to all available H1 and ZEUS DVCS data. Our particular concern is the LO description, where so far this goal has not been reached. Since the real part of the CFF $^g\mathcal{H}$ can be calculated from a dispersion relation, our outcome might be relevant for the real part in fixed target kinematics, too. Beyond LO these DVCS data can be described within specific GPD models, see, e.g., Refs. [6, 7].

### 2 Conformal moment-space approach

It is convenient to work with conformal GPD moments. For integral conformal spin $j + 2$ they are defined by convolution with Gegenbauer polynomials $C_j^n(x)$, e.g., for quarks:

\[ H_j^q(\eta, t, \mu^2) \equiv \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} dx \ \eta^j C_j^{3/2}(x/\eta) \ H^q(x, \eta, t, \mu^2), \quad (3) \]

\[ ^1 \]These trajectories appear in a GPD sum rule family, which can serve as powerful modelling tool [5].
where \( q \) is the flavor index. The normalization ensures that in the forward limit \( \Delta \rightarrow 0 \) the conformal moments simply reduce to familiar Mellin moments of parton distribution functions (PDFs). Some of the advantages of working with conformal moments are

- Conformal moments evolve autonomously at LO and, in a special factorization scheme, even at next-to-leading order (NLO).
- Powerful analytic methods of complex \( j \) plane are available (similar to complex angular momentum). This also allows to build a stable and fast routine for fitting.
- New possibilities for GPD modelling, which makes direct contact to the \( t \)-channel SO(3) partial wave expansion, Regge phenomenology, and lattice measurements.

In moment space the convolution formula (2) yields formally a divergent series over the integral conformal spin. Analogously to a SO(3) partial wave expansion, it can be resummed by means of a Mellin-Barnes integral representation \[8, 9, 7\]

\[
S\mathcal{H}(\xi, t, Q^2) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \xi^{-j-1} \left[ i + \tan \left( \frac{\pi j}{2} \right) \right] C_j(Q^2/\mu^2, \alpha_s(\mu)) H_j(\xi, t, \mu^2). \tag{4}
\]

GPD moments \( H_j \), cf. Eq. (3), can be decomposed in \( t \)-channel SO(3) partial waves, i.e., Wigner matrices \( d_{0,\nu}^J(\cos \theta) \), which are labelled by angular momentum \( J \) and hadron helicity differences \( (\nu = 0, \pm 1) \) \[10, 3, 7\]. The cosine of the scattering angle \( \theta \) might be approximated by \(-1/\eta\). We include an effective leading Regge pole at \( \alpha(t) = \alpha_0 + \alpha' t \) in the partial wave amplitudes and parameterize their residual \( t \)-dependence by a dipole with cut-off mass \( M_J \) (or, alternatively, with an exponential \( t \)-dependence). Our model (for assignments see Ref. \[3\]) is given at a input scale \( Q_0 \) and reads for integral \( j \):

\[
H_j(\eta, t, Q_0^2) = \sum_{j=J_{\text{min}}}^{J+1} h_j^I \frac{1}{J - \alpha(t)} \frac{1}{(1 - t M_J^2)^2} \eta^{j+1-J} d_{0,\nu}^J(-1/\eta), \tag{5}
\]

where \( h_j^I \) are strengths of partial waves. The PDF Mellin moments \( h_j^{I=j+1}/(j + 1 - \alpha_0) \) are fixed from DIS, the remaining parameters are constrained by DVCS fits.

In practice the parameter space in the ansatz (5) must be further reduced. We employ three GPD models: i. taking only the leading \( J = j + 1 \) SO(3) partial wave (l-PW), ii. including the next-leading \( J = j - 1 \) one (nl-PW), and iii. performing a model dependent resummation of SO(3) partial waves (Σ-PW). To set up the third
model, we assume that the GPDs in the vicinity of $x = \eta$ behave as $\frac{1}{\eta}(\frac{x+\eta}{1+\eta})^{1-\alpha(t)}$. Relevant for us is the small $\eta$ behavior of the resulting conformal moments for complex-valued $j$ [8], which is given by a series of ‘conformal’ daughter poles $j = \alpha(t) - 1 - n$ with $n = 2, 4, 6, \cdots$. All of them contribute to the leading Regge behavior of the CFF $H$. The strength of the non-leading SO(3) partial waves in the nl- and Σ-PW model for sea quarks and gluons is controlled by skewness parameters. These parameters allow us to adjust the normalization of CFFs. Note that Mellin-Barnes and “dual” [10] GPD parameterizations are related. In the latter $\rho = j+1-J$ is taken as a parameter and a ‘forward-like’ momentum fraction $z$-integral replaces the $j$-integral [11]. Our l- and nl-PW models are equivalent to a minimalist ($\rho = 0$) and a minimal ($\rho = \{0, 2\}$) “dual” model, respectively [11].

3 Results

The l-PW model is the simplest one with a rigid normalization, i.e., $H$ at $Q_0$ and $t = 0$ is given by the unpolarized sea quark PDF enhanced by the Clebsch-Gordan coefficient

$$\frac{2^{1+\alpha_0}\Gamma(\alpha_0 + 3/2)}{\sqrt{\pi}\Gamma(\alpha_0 + 2)} \sim 1.5.$$  \hspace{1cm} (6)
Figure 2: (a) the skewness ratio $r$, cf. Eq. (7), for fixed $x = 10^{-3}$ (thin) and alternative ratio $R$, cf. Eq. (5), for fixed $W = 82$ GeV (thick), (b) and (c) effective exponential $t$-slope (9) versus $Q^2$ and $W$, respectively, together with experimental data from Ref. [13]; (d) transverse width (10) of sea quarks at the input scale $Q^2 = 4$ GeV$^2$ (thin) and evolved at $Q^2 = 10$ GeV$^2$ (thick). The model parameter for l (dotted), nl (dashed), and $\Sigma$ (solid) SO(3)-PW models, see text, are obtained from LO fits such as in Fig. 1.

Within such a simplified model the $t$-slope is poorly described at LO, since it is used to adjust the normalization, whereas at NLO or beyond good fits are obtained [7]. To have a LO description, one has to include non-leading SO(3) partial waves. Both of our flexible GPD models (nl-PW and $\Sigma$-PW) allow for good DVCS fits at LO, illustrated for the $\Sigma$-PW model in Fig. 1 and beyond. The $t$-dependence is well described with $\alpha' = 0.15$/GeV$^2$ and $M_j \sim 0.6$ GeV, taken at the initial scale $Q_0 = 2$ GeV. Note that to some extent these parameters are correlated. The last panel shows fit to DIS structure function $F_2$, which fixes the normalization $h_{j+1}^0$ of leading SO(3) partial wave and intercept $\alpha_0$. We add that gluonic GPD is not well constrained, e.g., its $t$-slope can be taken from $J/\psi$ data [12].

We define a skewness ratio of singlet quark GPD on the $\eta = x$ and $\eta = 0$
trajectories:

\[ r(x, Q^2) \equiv \frac{\Sigma H(x, \eta = x, t = 0, Q^2)}{\Sigma H(x, \eta = 0, t = 0, Q^2)}. \]  

Resulting from our LO fit in Fig. 1, \( r \) versus \( Q^2 \) is plotted in Fig. 2(a) for the nl- and \( \Sigma \)-PW model as thin dashed and solid curves, respectively. One notes that for both models \( r \sim 1 \) over the wide \( Q^2 \) lever arm, i.e., a skewness effect is almost absent at LO and the models are not distinguishable. In these models the gluonic \( r \) ratio is considerably smaller than one and can even be negative. The ratio \( R \) (thick curves) of imaginary parts of DVCS amplitude and DIS structure function might be considered as an observable [13]. This alternative measure of the skewness effect in the LO approximation reads

\[ R \approx \frac{H(x, x, t = 0, Q^2)}{H(2x, 0, t = 0, Q^2)}. \]  

Note that \( R \approx 2^{\alpha(0, Q^2)} r \) with an intercept \( \alpha(0, Q^2 \sim 4 \text{ GeV}^2) \sim 1.2 \). Numerous GPD models and the claim \( r \sim 1.5 \) [17], which is implemented in the l-PW model (dotted curves) and arises from the Clebsch-Gordan coefficient [6], are disfavored by our LO fits. As mentioned above, we are even concerned about their uses in fixed target kinematics, if described at LO [18]. We emphasize that l-PW models with \( r \sim 1.5 \) can describe the DVCS data beyond LO [7], where the corresponding gluonic ratio is about one.

The \( t \)-dependence of the DVCS data can be directly fitted within an exponential ansatz, where a \( Q \) dependent \( t \)-slope and a vanishing \( \alpha' \) has been found [13]. We display in Fig. 2 (b) and (c) the H1 measurements [13] and the effective exponential \( t \)-slope

\[ b_{\text{eff}} = \frac{1}{-0.7 \text{ GeV}^2} \ln \frac{d^2 \sigma_{\text{DVCS}}(W, t = -0.8 \text{ GeV}^2, Q^2)}{d^2 \sigma_{\text{DVCS}}(W, t = -0.1 \text{ GeV}^2, Q^2)}. \]  

evaluated from a \( \Sigma \)-PW model (solid) with dipole ansatz and \( \alpha' = 0.15/\text{GeV}^2 \) as well as a nl-PW model (dashed) with exponential ansatz and \( \alpha' = 0 \). Models describe the present DVCS data set with \( \chi^2/\text{d.o.f.} \approx 101./98 \) and \( \chi^2/\text{d.o.f.} \approx 98./98 \), respectively. We emphasize that the differences between the nl- and \( \Sigma \)-PW model, visible in Fig. 2 mainly originate from the implementation of \( t \)-dependence. The decrease of the \( t \)-slope with growing photon virtuality \( Q \), shown in panel (b), entirely arises from evolution. The flatness of the \( t \)-slope with respect to the \( W \)-dependence, see panel (c), does not exclude a small \( \alpha' \) value. Note that due to the evolution the \( \alpha' \) value at the scale \( Q^2 = 10 \text{ GeV}^2 \) is smaller than the input value \( \alpha' = 0.15/\text{GeV}^2 \). We add that from our fits a large value of \( \alpha' \sim 0.8/\text{GeV}^2 \) for the dominant sea quark contribution is disfavored, cf. Ref. [18].
Present small-$x_{\text{Bj}}$ DVCS data are well described by different GPD models and thus a partonic interpretation w.r.t. the skewness dependence, i.e., the distribution of SO(3) partial waves, and the $t$-dependence can not be unique. For the transverse width \[ b^2 \] of sea quarks this is illustrated in Fig. 2 (d). Here the differences for the transverse width of the dipole (solid) and exponential (dashed) $t$-dependent model are caused by their extrapolation descriptions of experimental data to $t = 0$. We also show the evolution of the transverse width from $Q^2 = 4 \text{GeV}^2$ (thin) to $Q^2 = 10 \text{GeV}^2$ (thick). Thereby, the slope $\alpha'$ decreases, compare thick and thin solid curves.

We remark that our models also reproduce the preliminary H1 result on the dominant twist-two $\cos \phi$ harmonic for the beam charge asymmetry \[ 20 \]. Since this asymmetry is proportional to the real part of a CFF combination, its description gives us confidence in the Regge behavior of the DVCS amplitude. Unfortunately, we could not obtain a useful bound for the CFF $E$ or the related anomalous gravitomagnetic moment, which would shed light on the partonic decomposition of the proton spin.

In conclusion, we successfully describe the small $x_{\text{Bj}}$ DVCS data within flexible GPD models from the perspective of $t$-channel physics or Regge phenomenology. Thereby, a LO description requires the inclusion of non-leading SO(3) partial waves, while beyond this order they are not essential. The interplay of both $t$ and $\xi$ dependence with $Q^2$ evolution is in our GPD models compatible with data. In a partonic LO interpretation the dominant sea quark GPD possesses almost no skewness effect over a wide $Q^2$ lever arm. This feature can be realized within various models. We also found that gluons, driving the evolution at small $x$, are relatively suppressed (even a negative gluonic GPD on the trajectory $\eta = x$). The functional form of the $t$-dependence in the GPD models can not be fully pinned down from present DVCS data. In particular, a small, however, non zero, $\alpha'$ can not be excluded and the residual $t$-dependence might be given by a dipole ansatz.

For a global fit of DVCS data, a good LO description of small-$x_{\text{Bj}}$ DVCS data is valuable at present. Whether one prefers a quark interpretation of DVCS data in a specific factorization scheme, in which absence of skewness effect holds at any order, or likes to resolve the gluonic content within radiative corrections in the standard scheme, is primarily a convention, yielding a GPD reparameterization.
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