Difference in Differences and Ratio in Ratios for Limited Dependent Variables

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Difference in differences (DD) is widely used to find policy/treatment effects with observational data, but applying DD to limited dependent variables (LDV’s) Y has been problematic. This paper addresses how to apply DD and related approaches (such as “ratio in ratios” or “ratio in odds ratios”) to binary, count, fractional, multinomial or zero-censored Y under the unifying framework of ‘generalized linear models with link functions’. We evaluate DD and the related approaches with simulation and empirical studies, and recommend ‘Poisson Quasi-MLE’ for non-negative (such as count or zero-censored) Y and (multinomial) logit MLE for binary, fractional or multinomial Y.

Running Head: DD and RR for LDV.

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1 Introduction

Difference in Differences (DD) is one of the most popular research designs in social sciences. Not just in social sciences, DD has been gaining popularity also in natural sciences, as can be seen in, e.g., Jena et al. (2015), Cataife and Pagano (2017), and McGrath et al. (2019). There are various references for DD: Angrist and Krueger (1999), Shadish et al. (2002), Lee (2005, 2016a), Athey and Imbens (2006), Angrist and Pischke (2009), Lechner (2011), Lee and Kim (2014), Morgan and Winship (2014), Kim and Lee (2017), Lee and Sawada (2020), Kahn-Lang and Lang (2020), etc.

DD is basically for linear models with additive components, which makes applying DD to limited dependent variables (LDV’s) with nonlinear models problematic. This paper provides answers to this problem, using the unifying idea of ‘generalized linear models with link functions’.

Consider an outcome/response $Y_{it}$ for subject $i$ at time $t = 2, 3$, a time-constant treatment qualification dummy $Q_i$, and a binary treatment $D_{it}$; we set $t = 2, 3$ to avoid the confusion with dummy variable values $0, 1$. The hallmark of DD is that $D_{it}$ is the interaction of $Q_i$ and $1[t = 3]$: $D_{it} = Q_i 1[t = 3]$, where $1[A] \equiv 1$ if $A$ holds and $0$ otherwise. That is, only the $Q_i = 1$ group is treated at $t = 3$, and untreated otherwise.

DD can be implemented with panel data or repeated cross-sections (RCS). We use RCS in this paper, because RCS are easier to collect than panel data and also because our empirical study uses RCS. In typical RCS, there is a huge reservoir of subjects, and random sampling for a sample size $N$ is done each period. Hence, we can safely assume that each subject is sampled only once in RCS, and that the sampling dummy $S_i$ is independent of the other random variables;

$$D_i = Q_i S_i \quad \text{where} \quad S_i \equiv 1[i \text{ is sampled at } t = 3].$$

Let $Y_{it}^{d}$ be the potential version of $Y_{it}$ for $D_{it} = d = 0, 1$, and $Y_{i}^{d} = (1 - S_i)Y_{i2}^{d} + S_i Y_{i3}^{d}$ be the RCS potential response. Let $W_{it}$ denote covariates, and $W_i \equiv (1 - S_i)W_{i2} + S_i W_{i3}$ be the RCS covariates. Clearly, RCS variables are derived from the underlying panel model variables. Henceforth, we often omit the subscript $i$ indexing subjects.
As a preliminary, ignoring the covariates $W$ for a while, define for RCS:

$$
\mu_{QS} \equiv E(Y|Q, S) = \lambda^{-1}(\beta_2 + \beta_\tau S + \beta_q Q + \beta_d D), \quad \beta_\tau \equiv \beta_3 - \beta_2, \quad (1.1)
$$

where $\lambda(\cdot)$ is a ‘link function’ as in the generalized linear model (Nelder and Wedderburn 1972), $(\beta_2, \beta_3)$ are the period-(2,3) intercepts, $\beta_\tau$ is the time effect of $t = 3$ relative to $t = 2$, $\beta_q$ is the group effect of $Q = 1$, and $\beta_d$ is the desired treatment effect.

Since $(Q, S)$ generates four cells for the four parameters $(\beta_2, \beta_\tau, \beta_q, \beta_d)$ in (1.1), there seems no loss of generality in (1.1). However, (1.1) does include a restriction: $QS$ should not appear separately from the treatment $D$. If the group effect of $Q$ changes across time, then $QS$ becomes relevant other than through $D$. This restriction—no change in the group effect over time—is the well-known DD ‘parallel trend assumption’.

For continuous $Y$, $\lambda(\cdot)$ in (1.1) is the identity, so that $\mu_{QS} = \beta_2 + \beta_\tau S + \beta_q Q + \beta_d D$. For this, DD is

$$
\mu_{11} - \mu_{10} - (\mu_{01} - \mu_{00}) = (\beta_\tau + \beta_d) - \beta_\tau = \beta_d:
$$

DD removes $\beta_2 + \beta_\tau S + \beta_q Q$ to leave $\beta_d D$ that changes across both times and groups.

In practice, to account for the covariates $W$, a linear model such as

$$
E(Y|Q, S, W) = \beta_2 + \beta_\tau S + \beta_q Q + \beta_d D + \beta_w W \quad (1.2)
$$

is estimated to find the slope of $D$ as the treatment effect.

For LDV’s, the story changes much. E.g., consider $Y = 1[0 \leq Y^*]$ where $Y^*$ is the latent continuous outcome. With the $N(0,1)$ distribution function $\Phi(\cdot)$, the probit is

$$
E(Y|Q, S) = P(Y = 1|Q, S) = \Phi(\beta_2 + \beta_\tau S + \beta_q Q + \beta_d D). \quad (1.3)
$$

One way to stick to DD is estimating (1.3) to interpret $\beta_d$ as the effect on $Y^*$, not on $Y$. E.g., if $\beta_d = 2$, then $D$ shifts $Y^*$ by twice the standard deviation (SD) of $Y^*$. However, many practitioners desire the effect as a change in $P(Y = 1|Q, S)$, not in $Y^*$.

The ‘marginal effect’ that is a change of $P(Y = 1|Q, S)$ in (1.3) due to $D$ is

$$
\Phi(\beta_2 + \beta_\tau S + \beta_q Q + \beta_d) - \Phi(\beta_2 + \beta_\tau S + \beta_q Q). \quad (1.4)
$$
Ai and Norton (2003) noted that this is not the correct effect, but their criticism applies to the case of an interaction treatment, where both \( Q \) and \( S \) are genuine treatments and the interest is in the effect of taking both treatments (e.g., drugs) together. Differently from this, \( Q \) and \( S \) are not treatments per se in the usual DD, and \( D = QS \) just happened to be the way the treatment was implemented. Indeed, Puhani (2012, eq. (10)) showed that (1.4) with \( S = Q = 1 \) is a legitimate treatment effect of interest.

The complication involving (1.3) and (1.4) arises because DD is applied to a nonlinear model, despite that DD is appropriate for linear models. To drive home our point, consider the ‘log link’ \( \lambda(\cdot) = \ln(\cdot) \iff \lambda^{-1}(\cdot) = \exp(\cdot) \), with which (1.1) becomes

\[
\mu_{QS} = E(Y|Q, S) = \exp(\beta_2 + \beta_r S + \beta_q Q + \beta_d D). \tag{1.5}
\]

This is appropriate for non-negative responses. For (1.5), ‘ratio in ratios (RR)’ removes the time and group effects, and ‘RR minus one’ gives the proportional effect of \( D \):

\[
\frac{\mu_{11}}{\mu_{10}} / \frac{\mu_{01}}{\mu_{00}} - 1 = \exp(\beta_d) - 1. \tag{1.6}
\]

In practice, as the linear model (1.2) is used instead of the DD \( \mu_{11} - \mu_{10} - (\mu_{01} - \mu_{00}) \) to find \( \beta_d \), \( \exp(\beta_2 + \beta_r S + \beta_q Q + \beta_d D + \beta'_w W) \) is used instead of the RR in (1.6).

This paper makes the following contributions, some of which might have been known, although we cannot point out the exact references as DD has been applied widely. First, we adopt the unifying framework of generalized linear models with link functions. Second, we advocate RR for non-negative (such as count or zero-censored) responses based on the log link, and “ratio in odds ratios (ROR)” for binary, fractional or multinomial responses based on the ‘logit link’. Third, although ROR is difficult to interpret, we show that it becomes a proportional effect for “rare events”. Fourth, if more than two periods are available, we propose a simple test for the DD parallel trends and analogous assumptions for RR and ROR: test for zero slope of \( tQ \), as zero slope validates the DD parallel trends and analogous assumptions for RR and ROR.

Practitioners often ignore the LDV nature of \( Y \), and simply use a linear model for DD. One justification for this was provided by Lee (2018) for any response \( Y \) and a binary exogenous \( D \): under \((Y^0, Y^1) \perp D|W \) with ‘\( \perp \)’ for independence, it holds that

\[
Y = E(Y^0|W) + E(Y^1 - Y^0|W)D + error. \tag{1.7}
\]
This representation for exogenous $D$ was generalized for endogenous $D$ in Lee (2021). Then, justifying a linear model for LDV’s can be done by linearly approximating $E(Y^0|W)$ and $E(Y^1 - Y^0|W)$ in (1.7). However, this paper’s approach is using LDV’s as such without such approximations.

In the remainder of this paper, Sections 2 and 3 examine RR and ROR, respectively, where the covariates are controlled in addition to $(Q, S)$. Section 4 presents an empirical analysis for various health outcomes. Section 5 concludes this paper. The appendix contains proofs, as well as a simulation study to show that the usual linear-model DD is misleading for LDV’s whereas RR and ROR approaches work well.

## 2 Ratio in Ratios (RR) for Non-Negative Response

This section studies RR for non-negative responses including count and zero-censored responses. First, the identification aspect is examined. Second, although RR can be estimated nonparametrically replacing the conditional means in RR with sample analogs, this is not how RR would be estimated in practice; instead, a practical semiparametric estimator for RR is advocated. Third, several remarks are made.

### 2.1 Proportional Effect Identification with RR

To simplify notation when covariates $W$ are allowed for, define

$$
\mu_{QS}(w) \equiv E(Y|w, Q, S) \tag{2.1}
$$

where $E(Y|w, Q, S)$ is a shorthand for $E(Y|W = w, Q, S)$. With this, define RR conditional on $W = w$ analogously to (1.6) plus one:

$$
RR(w) \equiv \frac{\mu_{11}(w)}{\mu_{10}(w)} / \frac{\mu_{01}(w)}{\mu_{00}(w)}.
$$

The identification condition for $RR(w)$ is

$$
\left( \frac{E(Y_3^0|w, Q = 1)}{E(Y_2^0|w, Q = 1)} \right) / \left( \frac{E(Y_3^0|w, Q = 0)}{E(Y_2^0|w, Q = 0)} \right) = 1; \tag{ID_{RR}}
$$

keep in mind that $S$ is independent of the other random variables, and $ID_{RR}$ involves only untreated responses. In $ID_{RR}$, $E(Y_3^0|w, Q = 1)$ is a counterfactual, because only
\( E(Y^3_3|w, Q = 1) \) is realized for \( Q = 1 \) at \( t = 3 \). \( \text{ID}_{RR} \) is analogous to the usual DD identification condition (i.e., parallel trends) appropriate for linear models:

\[
E(Y^3_3|w, Q = 1) - E(Y^2_2|w, Q = 1) - \{E(Y^0_3|w, Q = 0) - E(Y^0_2|w, Q = 0)\} = 0. \quad (\text{ID}_{DD})
\]

The main point is that \( RR(w) - 1 \) is the ‘proportional effect on the treated at the post-treatment period \( t = 3 \)’, in view of the first and last expressions of the following:

\[
RR(w) - 1 = \frac{E(Y|w, Q = 1, S = 1)}{E(Y|w, Q = 1, S = 0)} - \frac{E(Y|w, Q = 0, S = 1)}{E(Y|w, Q = 0, S = 0)} - 1
\]

\[
= \left( \frac{E(Y^3_3|w, Q = 1)}{E(Y^2_2|w, Q = 1)} \right) \left( \frac{E(Y^0_3|w, Q = 0)}{E(Y^0_2|w, Q = 0)} \right) - 1 \quad \text{(as } S \text{ II ‘the other variables’)}
\]

\[
= \frac{E(Y^3_3|w, Q = 1)}{E(Y^0_3|w, Q = 1)} - 1 = \frac{E(Y^3_3 - Y^0_3|w, Q = 1)}{E(Y^0_3|w, Q = 1)} \quad \text{(under ID}_{RR}). \quad (2.2)
\]

If the dimension of \( W \) is low (or if \( W \) is discrete), \( RR(w) \) can be estimated nonparametrically by substituting nonparametric estimators into the four components of \( RR(w) \). In practice, however, typically the dimension of \( W \) is high, and thus we explore a simpler semiparametric exponential regression next—semiparametric because only \( E(Y|W, Q, S) \) is specified, not the full distribution of \( Y|(W, Q, S) \).

### 2.2 Poisson Quasi-MLE (QMLE)

In view of (1.5), suppose that a panel data exponential model holds for \( Y^d_t \):

\[
E(Y^d_t|W_t, Q) = \exp(\beta_t + \beta_w Q + \beta_d d + W'_t \beta_w) \quad (2.3)
\]

\[
\iff Y^d_t = \exp(\beta_t + \beta_w Q + \beta_d d + W'_t \beta_w + U_t), \quad \exp(U_t) \equiv Y^d_t/E(Y^d_t|W_t, Q);
\]

\( \beta_t \) is a time-varying intercept, and \( E\{\exp(U_t)|W_t, Q\} = 1 \) holds. \( \text{ID}_{RR} \) holds for (2.3):

\[
\left( \frac{E(Y^0_3|w, Q = 1)}{E(Y^0_2|w, Q = 1)} \right) / \left( \frac{E(Y^0_2|w, Q = 0)}{E(Y^0_2|w, Q = 0)} \right) = \frac{\exp(\beta_3 + \beta_q + w'\beta_w)}{\exp(\beta_2 + \beta_q + w'\beta_w)} / \frac{\exp(\beta_3 + w'\beta_w)}{\exp(\beta_2 + w'\beta_w)} = 1.
\]

Turn the panel data model (2.3) into the RCS model for \( Y^d \equiv (1 - S)Y^d_2 + SY^d_3 \):

\[
Y^d = \exp(\beta_2 + \beta_r S + \beta_q Q + \beta_d d + W'\beta_w + U), \quad (2.4)
\]

\( \beta_r \equiv \beta_3 - \beta_2, \quad W \equiv (1 - S)W_2 + SW_3, \quad U \equiv (1 - S)U_2 + SU_3; \)
(W, Q, S) is exogenous to U in the sense \( E\{\exp(U)|W, Q, S\} = 1 \). Take \( E(\cdot|W, Q, S) \) on the observed \( Y = (1 - D)Y^0 + DY^1 \): due to \( D = QS \),

\[
E(Y|W, Q, S) = (1 - QS) \cdot E(Y^0|W, Q, S) + QS \cdot E(Y^1|W, Q, S)
\]

\[
= \exp(\beta_2 + \beta_S S + \beta_Q Q + \beta_d D + W'\beta_w) \tag{2.5}
\]

\[
\implies RR(w) - 1 = \exp(\beta_d) - 1; \tag{2.6}
\]

the second equality can be verified by substituting \( D = QS = 0, 1 \) into both sides of the equality. We use RCS and the model (2.5) to estimate \( \beta_d \) and other parameters.

The constant treatment effect \( \beta_d \) can be easily allowed to be a function of \( W \), as in \( \beta_d(W_t) = \beta_{d0} + \beta_{dW}' W_t \) for parameters \( (\beta_{d0}, \beta_{dW}) \). Then we have \( \beta_d(W) = \beta_{d0} + \beta_{dW}' W \) in RCS, and (2.5) becomes

\[
E(Y|W, Q, S) = \exp\{\beta_2 + \beta_S S + \beta_Q Q + \beta_d(W)D + W'\beta_w\}
\]

\[
\implies RR(w) - 1 = \exp(\beta_d(w)) - 1 \quad \text{(proportional effect at } W = w).}
\]

For estimation, the simplest approach is the ‘Poisson Quasi-Maximum Likelihood Estimator (Poisson QMLE)’. The Poisson QMLE is the same as the Poisson MLE, except that the variance is estimated with a “sandwich-form” asymptotic variance estimator. The maximand for the Poisson QMLE is the same as that for Poisson MLE:

\[
\sum_i \{Y_i(X_i'\beta) - \exp(X_i'\beta)\}, \quad X_i \equiv (1, S_i, Q_i, D_i, W_i'), \quad b = (b_2, b_S, b_Q, b_d, b_w').
\]

The first order-condition at \( b = \beta \) is \( \sum_i \{Y_i - \exp(X_i'\beta)\}X_i = 0 \) where \( \beta \equiv (\beta_2, \beta_S, \beta_Q, \beta_d, \beta_w)' \), which holds due to \( E(Y|X) = \exp(X'\beta) \). The maximum is unique as the second order derivative \( -\sum_i X_iX_i' \exp(X_i'\beta) \) is n.d.: just under \( E(Y|X) = \exp(X'\beta) \), the Poisson QMLE is consistent for \( \beta \). The Poisson QMLE for exponential models was advocated in Lee (2005) and Santos Silva and Tenreyro (2006). For heterogeneous effects, we may use \( \beta_d(W) = \beta_{d0} + \beta_{dW}' W \) in the Poisson QMLE.

2.3 Remarks

Here we make a few remarks on the applicability of the above RR identification and Poisson QMLE to count and zero-censored responses. Bear in mind that the
semiparametric exponential regression model (2.5) requires no upper bound on $Y$.

**First**, instead of the difference effect $E(Y_3^1 - Y_3^0|w, Q = 1)$, examining the proportional effect in (2.2) can be beneficial (Yadlowsky et al. 2021). E.g., suppose $E(Y_3^0|w, Q = 1) = G(w)$ for a function $G(\cdot)$ and the proportional effect is a constant $\beta_d$. Then the difference effect $E(Y_3^1 - Y_3^0|w, Q = 1) = \beta_d G(w)$ introduces effect heterogeneity unnecessarily, compared with the simple $\beta_d$. Proportional effects for exponential models have been advocated in many studies: Lee and Kobayashi (2001), Dukes and Vansteelandt (2018) and Ciani and Fisher (2019), among others.

**Second**, suppose $Y = \exp(Y^*)$, $Y^* = \beta_2 + \beta_q S + \beta_q Q + \beta_d D + W' \beta_w + U$ and $E\{\exp(U)|W, Q, S\} = 1$. Then we can interpret $\beta_d$ as the DD effect on $Y^*$, whereas $\exp(\beta_d) - 1$ is the proportional effect on $Y$. However, for count responses such as $Y|(W, Q, S)$ generated by the Poisson distribution with $P(Y = y|X) = \{\exp(X'\beta)\}^y \exp\{-\exp(X'\beta)\}/y!$, there is no $Y^*$. In this Poisson case, the proportional effect interpretation on the observed $Y$ with RR is the only way to meaningfully interpret the slope $\beta_d$ of $D$ in the exponential model. This statement applies also to count responses based on other distributions such as Negative Binomial.

**Third**, if $\beta_q tQ$ with $\beta_q \neq 0$ appears as a regressor, then $\text{ID}_{RR}$ fails due to $\beta_q tQ$:

$$
\frac{(\exp(\beta_3 + \beta_q + 3\beta_q tQ + w' \beta_w))}{(\exp(\beta_2 + \beta_q + 2\beta_q tQ + w' \beta_w))} / \frac{(\exp(\beta_3 + w' \beta_w))}{(\exp(\beta_2 + w' \beta_w))} = \exp(\beta_q tQ) \neq 1. 
$$

Hence, using $tQ$ as an extra regressor is an easy way to test or allow for non-parallel trends or analogous conditions for RR. However, $tQ$ cannot be used if only two periods are available, because using $tQ$ is equivalent to using $QS$ which is $D$. With more than two periods available, there are two ways to entertain $\beta_q tQ$ as follows.

The first way is using $tQ$ as an extra regressor. For panel data, $tQ$ can be used as such, but for RCS, $Q_i^t \equiv Q_i \sum_t S_{it} t$ should be used instead, where $S_{it} = 1$ if $i$ is sampled in period $t$ and 0 otherwise. Intuitively speaking, the untreated group difference is allowed to change linearly with $tQ$ over time, and then any deviation from the change is taken as the treatment effect. With more periods, the allowed linear untreated trend difference can be expanded to quadratic ($t^2Q$), cubic ($t^3Q$), and so on.
The other way is using triple ratios, or “ratio in ratios in ratios (RRR)” generalizing RR, analogously to triple differences (Lee 2016b) to allow for non-parallel trends in DD. With \( t = 1, 2, 3 \) available, let \( m_{Q_t}(w) \equiv E(Y|w, Q, \text{sampled at } t) \) and

\[
\left( \frac{E(Y_{t}^0|w, Q = 1)}{E(Y_{t-1}^0|w, Q = 1)} \right) / \left( \frac{E(Y_{t}^0|w, Q = 0)}{E(Y_{t-1}^0|w, Q = 0)} \right) = \gamma \quad \text{for } t = 2, 3,
\]

(ID_{RRR})

which allows ID_{RR} to be violated when \( \gamma \neq 1 \) as follows. Observe

\[
RRR(w) = \left[ \frac{m_{13}(w)}{m_{12}(w)} \right] / \left[ \frac{m_{03}(w)}{m_{02}(w)} \right] \quad \text{for } E(X) \neq 0.
\]

The last two terms in \([\cdot]\) are both equal to \( \gamma \) to cancel each other. Hence, under ID_{RRR}, RRR identifies the same effect as RR identifies, even when ID_{RR} fails.

**Fourth**, consider a RCS zero-censored model:

\[
Y = \max(0, Y^*) = Y^*1[0 < Y^*], \quad Y^* \equiv \beta_2 + \beta_rS + \beta_qQ + \beta_dD + W^t\beta_w + U
\]

\[
\implies E(Y|X) = E(Y^*1[0 < Y^*]|X) \tag{2.8}
\]

Since \( E(Y|X) = E(Y^*1[0 < Y^*]|X) \) is non-negative without any upper bound, the exponential regression model (2.5) can be adopted, although it may not be as appealing as for count responses because the transformation \( \max(0, \cdot) \) is not smooth.

Santos Silva and Tenreyro (2011) showed that the exponential regression holds for (2.8) if \( Y_i = \sum_{j=1}^{M_i} Z_{ij} \), where \( M_i \) is a non-negative integer random variable such as Poisson count, and \((Z_{i1}, Z_{i2}, \ldots)\) are independent and identically distributed (iid) positive random variables with \( Z_{ij} \sim M_i[X_i; Y = 0] \) occurs if \( M = 0 \). Due to \( Z_{ij} \sim M_i[X_i, E(Y|X) = E(M|X)E(Z_j|X)
\]

\[
= \exp\{\alpha_2 + \beta_2 + (\alpha_r + \beta_r)S + (\alpha_q + \beta_q)Q + \beta_dD + W'(\alpha_w + \beta_w)\} \quad \text{if } E(M|X) = \exp(\alpha_2 + \alpha_rS + \alpha_qQ + W'\alpha_w),
\]

\[
E(Z_j|X) = \exp(\beta_2 + \beta_rS + \beta_qQ + \beta_dD + W'\beta_w).
\]
It is not clear what $Y^*$ is here, but the interpretation of $\exp(\beta_d) - 1$ as a proportional effect on $Y$ still holds regardless of what $Y^*$ might be.

A DD example for $Y_i = \sum_{j=1}^{M_i} Z_{ij}$ is that $Y_i$ is the expenditure on tobacco by person $i$ in a year, $Z_{ij}$ is the tobacco expenditure of person $i$ on day $j$, $M_i$ is the number of the tobacco-purchasing days for person $i$ in the year, $Q_i = 1$ if person $i$ is legally eligible to smoke, $W_i$ is individual traits of person $i$, and there is a smoking-discouraging policy $D_i = Q_i S_i$ implemented at $t = 3$ effectively increasing tobacco product prices. In this case, $M_i$ is how frequently tobacco is purchased which is unlikely to be affected by the policy, and $Z_{ij}$ is the day-$j$ purchase amount affected by the policy.

3 Ratio in Odds-Ratios (ROR)

This section studies ROR: we examine the identification aspect first, followed by logit-based estimation for binary and fractional responses. ROR is also applicable to multinomial response, but it is presented (along with a simulation study) in the appendix due to the complexity involving multiple equations and additional notation.

3.1 Proportional Odds Effect Identification with ROR

For binary $Y$, define the ‘odds conditional on $(W = w, Q = q, S = s)$’ for RCS as

$$R_{qs}(Y; w) \equiv \frac{P(Y = 1|w, Q = q, S = s)}{P(Y = 0|w, Q = q, S = s)}$$

which leads to

$$(3.1) R_{11}(Y; w) = R_{11}(Y_3; w), \quad R_{01}(Y; w) = R_{01}(Y_3; w),$$

$$R_{10}(Y; w) = R_{10}(Y_2^0; w), \quad R_{00}(Y; w) = R_{00}(Y_2^0; w).$$

Also define ‘Ratio in Odds-Ratios (ROR) conditional on $W = w$’:

$$\text{ROR}(Y; w) \equiv \left( \frac{R_{11}(Y; w)}{R_{10}(Y; w)} \right) / \left( \frac{R_{01}(Y; w)}{R_{00}(Y; w)} \right).$$

The identification condition to be invoked for ROR is

$$\text{ID}_{\text{ROR}} := \frac{R_{11}(Y_3^0; w)}{R_{10}(Y_2^0; w)} / \left( \frac{R_{01}(Y_3^0; w)}{R_{00}(Y_2^0; w)} \right) = 1.$$ (ID_{\text{ROR}})
where \( R_{11}(Y_3^0; w) \) is a counterfactual, because only \( R_{11}(Y_3^1; w) \) is realized. Doing analogously to (2.2), \( ROR(Y; w) - 1 \) is the ‘proportional odds effect on the treated at the post-treatment period \( t = 3' – 'on the treated' because \( R_{11} \) is for \( Q = 1 \) and \( S = 1 \):

\[
ROR(Y; w) - 1 = \left( \frac{R_{11}(Y; w)}{R_{10}(Y; w)} \right) / \left( \frac{R_{01}(Y; w)}{R_{00}(Y; w)} \right) - 1
\]

\[
= \left( \frac{R_{11}(Y_3^1; w)}{R_{10}(Y_3^0; w)} \right) / \left( \frac{R_{01}(Y_3^0; w)}{R_{00}(Y_3^0; w)} \right) - 1
\]

\[
= \frac{R_{11}(Y_3^1; w)}{R_{11}(Y_3^0; w)} \cdot \left( \frac{R_{11}(Y_3^0; w)}{R_{10}(Y_3^0; w)} \right) / \left( \frac{R_{01}(Y_3^0; w)}{R_{00}(Y_3^0; w)} \right) - 1
\]

\[
= \frac{R_{11}(Y_3^1; w)}{R_{11}(Y_3^0; w)} - 1 = \frac{R_{11}(Y_3^1; w) - R_{11}(Y_3^0; w)}{R_{11}(Y_3^0; w)} \quad \text{(under ID}_{ROR}. \quad (3.2)
\]

One disadvantage of ROR compared with RR is the difficulty in interpreting the ‘proportional odds effect’. For this, suppose \( Y = 1 \) is a rare event in the sense

\[
P(Y_3^0 = 0|w, Q = 1) \approx 1 \quad \text{for all } w;
\]

(3.3)
e.g., \( Y = 1 \) is a rare cancer occurrence such that \( P(Y_3^0 = 0|w, Q = 1) \approx 1 \) for all \( w \) and \( d = 0, 1 \). Under (3.3), \( ROR(Y; w) = R_{11}(Y_3^1; w)/R_{11}(Y_3^0; w) \) in (3.2) becomes

\[
\frac{P(Y_3^1 = 1|w, Q = 1)/P(Y_3^0 = 0|w, Q = 1)}{P(Y_3^0 = 1|w, Q = 1)/P(Y_3^0 = 0|w, Q = 1)} \approx \frac{P(Y_3^1 = 1|w, Q = 1)}{P(Y_3^0 = 1|w, Q = 1)}.
\]

Hence, the proportional odds effect in (3.2) becomes the proportional effect in (2.2):

\[
ROR(Y; w) - 1 \approx \frac{E(Y_3^1 - Y_3^0|w, Q = 1)}{E(Y_3^0|w, Q = 1)} \quad \text{under the rare event condition (3.3)}.
\]

\( ROR(Y; w) \) can be estimated nonparametrically by substituting sample analogs into the components of \( ROR(Y; w) \). However, as was the case for DD and RR(\( w \)), this is not what practitioners would do. Instead, we apply logistic regression next.

### 3.2 Logit for Binary Response

Consider the popular logistic binary choice panel data model for \( Y_t^d \):

\[
Y_t^d = 1[0 < \beta_t + \beta_q Q + \beta_d d + W_t' \beta_w + U_t], \quad U_t \sim \text{Logistic II } (Q, W_t).
\]

(3.4)
This yields the RCS model for \( Y^d = 1[0 < \beta_2 + \beta_r S + \beta_q Q + \beta_d d + W'\beta_w + U] \), which then yields the logistic RCS model for \( Y = (1 - D)Y^0 + DY \) as in (2.4) to (2.5):

\[
E(Y|W, Q, S) = (1 - QS) \cdot E(Y^0|W, Q, S) + QS \cdot E(Y^1|W, Q, S)
= \frac{\exp(\beta_2 + \beta_r S + \beta_q Q + \beta_d D + W'\beta_w)}{1 + \exp(\beta_2 + \beta_r S + \beta_q Q + \beta_d D + W'\beta_w)},
\]

the last equality can be verified by substituting \( D = QS, 0, 1 \).

The logistic panel data model for \( Y_{it}^0 \) gives

\[
R_{11}(Y_{3i}^0; w) = \exp(\beta_2 + \beta_r + \beta_q + w'\beta_w), \quad R_{01}(Y_{3i}^0; w) = \exp(\beta_2 + \beta_r + w'\beta_w),
R_{10}(Y_{2i}^0; w) = \exp(\beta_2 + \beta_q + w'\beta_w), \quad R_{00}(Y_{2i}^0; w) = \exp(\beta_2 + w'\beta_w).
\]

Hence, ID_{ROR} holds for the logistic panel data model: due to (3.6),

\[
ROR(Y^0; w) = \left( \frac{R_{11}(Y_{3i}^0; w)}{R_{10}(Y_{2i}^0; w)} \right) \cdot \left( \frac{R_{01}(Y_{3i}^0; w)}{R_{00}(Y_{2i}^0; w)} \right) = 1.
\]

Also, \( R_{11}(Y_{3i}^1; w) = \exp(\beta_2 + \beta_r + \beta_q + \beta_d + w'\beta_w) \) and \( R_{11}(Y_{3i}^0; w) \) in (3.6) give

\[
ROR(Y; w) - 1 = \{R_{11}(Y_{3i}^1; w)/R_{11}(Y_{3i}^0; w)\} - 1 = \exp(\beta_d) - 1.
\]

Estimate \( \beta_d \) with the logistic MLE with (3.5) to use \( \exp(\beta_d) - 1 \) as the proportional odds effect on \( Y \), which is also the proportional effect when \( Y = 1 \) is a rare event.

Suppose \( \beta_q_1 tQ \) with \( \beta_q \neq 0 \) appears as an extra regressor in (3.4). Then the parallel trends do not hold for the latent response \( Y^* \). The appearance of \( \beta_q tQ \) also ruins ID_{ROR} for binary \( Y \) because ID_{ROR} becomes (2.7), just as \( \beta_q tQ \) ruins ID_{RR} in (2.7). As in (2.7), using \( tQ \) is an easy way to test or allow for non-parallel trends in \( Y^* \). The comments made for (2.7) hold more or less the same for (3.4) and (3.5).

Suppose now that the slope of \( d \) in (3.4) is \( \beta_d(W_t) \), e.g., \( \beta_d(W_t) = \beta_{d0} + \beta_d'W_t \):

\[
Y_t^d = 1[0 < \beta_t + \beta_q Q + \beta_d(W_t) d + W_t'\beta_w + U_t].
\]

Then (3.5) and \( ROR(Y; w) - 1 \) become, respectively,

\[
E(Y|W, Q, S) = \frac{\exp\{\beta_2 + \beta_r S + \beta_q Q + \beta_d(W_t) D + W_t'\beta_w\}}{1 + \exp\{\beta_2 + \beta_r S + \beta_q Q + \beta_d(W_t) D + W_t'\beta_w\}},
ROR(Y; w) - 1 = \exp\{\beta_d(w)\} - 1.
\]
3.3 Logit for Fractional Response

When $Y$ takes on a value in $[0, 1]$, $Y$ is a fractional response; e.g., the proportion of asset invested in stocks. There are two types of fractional response: (i) $P(Y = 0 \text{ or } Y = 1) = 0$ and (ii) $P(Y = 0 \text{ or } Y = 1) > 0$. Since the logistic regression model (3.5) always gives a value in $(0, 1)$, the logistic regression can be adopted for type-(i) fractional response, regardless of whether (3.5) is derived from some latent $Y^*$ or not.

As for type (ii), analogously to $Y = \max(0, Y^*)$, we can use $Y = \max\{0, \min(Y^*, 1)\}$. Since the transformation $\max\{0, \min(\cdot, 1)\}$ is not smooth, one may object to adopting (3.5) for type (ii). However, as Santos Silva and Tenreyro (2011) justified adopting the exponential regression for $\max(0, \cdot)$, Papke and Wooldridge (1996) justified adopting the logistic regression for $\max\{0, \min(\cdot, 1)\}$.

Papke and Wooldridge maximize the logistic QMLE log-likelihood function for $b$:

$$
\sum_i \left\{ Y_i \ln \frac{\exp(X_i'b)}{1 + \exp(X_i'b)} + (1 - Y_i) \ln \frac{1}{1 + \exp(X_i'b)} \right\};
$$

$X_i$ and $b$ were defined for the Poisson QMLE. The first-order condition is

$$
\sum_i \left\{ Y_i - \frac{\exp(X_i'b)}{1 + \exp(X_i'b)} \right\} X_i = 0 \quad \text{(satisfied under (3.5)).}
$$

That is, the logistic QMLE applies to fractional response too, but as in Poisson QMLE, a “sandwich form” asymptotic variance estimator should be used. The maximum is unique, because the second-order matrix $-\sum_i X_i'X_i[\exp(X_i'b)/(1 + \exp(X_i'b))]^2$ is n.d.

4 Empirical Analysis

In this section, we estimate the effects of the Affordable Care Act Dependent Coverage Provision (‘DCP’) on various health outcomes. Under the DCP that went into effect in September 2010, dependents can remain on the parent’s private health plan until age 26. The treatment group is dependents aged 23-25, and the control group is dependents aged 27-29; 26 was excluded due to the treatment status ambiguity.

Our data came from the Behavioral Risk Factor Surveillance System (BRFSS) for years 2007-2013, which is health-related telephone surveys in the U.S. Almost the same
data were used in Barbaresco, Courtemanche and Qi (2015) (`BCQ', henceforth), with small differences occurring due to updates, imputed values, data cleaning, etc. As in BCQ, sampling weights are used in estimation and cluster-robust standard errors are reported in the tables below.

BCQ considered 18 outcomes, of which we use 12. Each outcome variable has a different sample size, as we replaced “Don’t Know” and “Refused” with missing values. With the sample size in \{\cdot\}, the 12 outcome variables are: (1) ‘any health insurance’ \{127618\}, (2) ‘any primary (care) doctor’ \{127533\}, (3) needed medical care in past year not taken due to cost (‘cost blocked care’) \{108433\}, (4) current smoker \{126557\}, (5) ‘risky drinker (in past 30 days)’ \{122035\}, (6) ‘obese (BMI≥30)’ \{121294\}, (7) ‘pregnant (while) unmarried’ \{40006\}, (8) ‘(alcoholic) drinks (in past) 30 days’ \{121845\}, (9) BMI \{121290\}, (10) days of last 30 not in good mental health (‘days poor mental’) \{125681\}, (11) days of last 30 not in good physical health (‘days poor physical’) \{125766\}, and (12) days of last 30 with health-related limitations (‘days health limits’) \{71079\}. The first seven outcomes are binary, and the remaining five are non-negative (counts or continuous).

Table 1 presents summary statistics on covariates: age, gender, race, marital status, education, state unemployment rate, ‘any DCP’ for whether the state has any DCP mandate although the dependent may not be covered, household income, the number of children, ‘cell phones only’ (vs. cell phone plus landline), student, and unemployed. Because the treatment group is younger than control group by 2 ∼ 6 years, the treatment group has fewer married, fewer college degree, lower household income, fewer children, more students, and more unemployed. Also, the treatment group has the lower state unemployment rate, higher any DCP, and higher cell phone only.

Let ‘Lin-DD’ stand for the usual linear model DD using (1.2). Table 2 shows the estimates for $\beta_{q\tau}$ (non-parallel trends) along with $\beta_d$ (treatment effect), although the effect of interest is the proportional effect $\exp(\beta_d) - 1$. Poisson QMLE estimates are $\tilde{\beta}_{q\tau}$ and $\tilde{\beta}_d$, whereas Lin-DD estimates ignoring the LDV nature are $\hat{\beta}_{q\tau}$ and $\hat{\beta}_d$. 
Table 1. Summary Statistics of Covariates: Mean & Standard Deviation (SD)

| Covariates                        | Treated Mean (SD) | Treated Mean (SD) | Control Mean (SD) | Control Mean (SD) |
|-----------------------------------|------------------|------------------|------------------|------------------|
| **Age** (age 23 omitted)          |                  |                  |                  |                  |
| Age 24                            | 0.35 (0.48)      | -                | 0.07 (0.26)      | 0.05 (0.22)      |
| Age 25                            | 0.32 (0.47)      | -                | 0.10 (0.30)      | 0.08 (0.27)      |
| Age 27                            | - 0.31 (0.46)    | $10K–$15K        | 0.12 (0.32)      | 0.10 (0.30)      |
| Age 28                            | - 0.35 (0.48)    | $15K–$20K        | 0.14 (0.35)      | 0.13 (0.34)      |
| Age 29                            | - 0.35 (0.48)    | $20K–$25K        | 0.16 (0.37)      | 0.16 (0.37)      |
| **Female**                        | 0.51 (0.50)      | 0.51 (0.50)      | 0.14 (0.35)      | 0.19 (0.39)      |
| **Race** (non-Hispanic whites omitted) |                  |                  |                  |                  |
| Black                             | 0.11 (0.31)      | 0.11 (0.32)      |                  |                  |
| Hispanic                          | 0.23 (0.42)      | 0.22 (0.41)      | 0.23 (0.42)      | 0.23 (0.42)      |
| Others                            | 0.09 (0.28)      | 0.08 (0.27)      | 0.16 (0.37)      | 0.23 (0.42)      |
| Married                           | 0.30 (0.46)      | 0.56 (0.50)      | 0.06 (0.23)      | 0.11 (0.31)      |
| **Number of children**            |                  |                  |                  |                  |
| Education (less than HS degree omitted) | 0.28 (0.45)      | 0.26 (0.44)      | 0.01 (0.09)      | 0.02 (0.12)      |
| High school (HS)                  | 0.31 (0.46)      | 0.37 (0.48)      | 0.70 (0.46)      | 0.67 (0.47)      |
| Non-4-yr coll.                    | 0.27 (0.44)      | 0.30 (0.49)      | 0.37 (0.48)      | 0.30 (0.46)      |
| Coll. graduate                    | 0.31 (0.46)      | 0.30 (0.49)      | 0.72 (2.72)      | 0.72 (2.73)      |
| State unemp. rate                 | 0.22 (2.72)      | 0.21 (2.69)      | 0.11 (0.31)      | 0.12 (0.32)      |
| Any DCP                           | 0.26 (0.44)      | 0.04 (0.20)      | 0.13 (0.34)      | 0.12 (0.32)      |

Three main findings emerge from Table 2, which are also seen in the simulation part of the appendix: (i) RR and ROR estimates differ much from Lin-DD estimates; (ii) the difference is overall greater for non-negative responses than for binary responses, as all signs are the same for binary responses but some signs differ for non-negative responses; and (iii) $\beta_q = 0$ is rejected in Lin-DD more often than in RR and ROR. Comparing RR, ROR and Lin-DD in their qualitative conclusions by testing for $\beta_d = 0$, they lead to the same qualitative conclusions, except for ‘drinks 30 days’.
| Outcome variable                  | RR and ROR | Lin-DD (Linear model DD) |
|----------------------------------|------------|--------------------------|
|                                  | $\tilde{\beta}_q\tau$ | $\tilde{\beta}_d$ | $\hat{\beta}_q\tau$ | $\hat{\beta}_d$ |
| **Binary response**              |            |                          |                      |                     |
| Any health insurance             | -0.011 (-0.32) | 0.415 (3.13) | -0.002 (-0.37) | 0.069 (2.84) |
| Any primary doctor               | 0.007 (0.28)  | 0.150 (1.32) | 0.001 (0.23)  | 0.030 (1.22)  |
| Cost blocked care                | 0.014 (1.56)  | -0.167 (-1.64)| 0.003 (2.12)  | -0.029 (-1.79) |
| Current smoker                   | -0.051 (-2.57)| 0.195 (2.98) | -0.009 (-2.74) | 0.034 (3.23)  |
| Risky drinker                    | 0.032 (1.78)  | -0.108 (-2.62)| 0.006 (2.17)  | -0.022 (-3.29) |
| Obese                            | 0.033 (2.02)  | -0.083 (-1.10)| 0.007 (1.95)  | -0.018 (-1.03) |
| Pregnant unmarried               | 0.008 (0.13)  | -0.074 (-0.42)| 0.000 (0.10)  | -0.003 (-0.36) |
| **Non-negative response**        |            |                          |                      |                     |
| Drinks 30 days                   | -0.037 (-1.22)| 0.091 (0.76) | -1.420 (-4.69) | 4.521 (3.19)  |
| BMI                              | -0.002 (-1.45)| 0.000 (-0.04) | -0.034 (-0.92) | -0.105 (-0.69) |
| Days poor mental                 | -0.006 (-0.23)| 0.002 (0.01) | -0.016 (-0.15) | 0.195 (0.41)  |
| Days poor physical               | 0.008 (0.18)  | -0.047 (-0.26)| -0.087 (-0.94) | 0.429 (1.14)  |
| Days health limits               | -0.001 (-0.02)| 0.017 (0.11) | 0.041 (0.35)  | 0.177 (0.35)  |

Turning to interpreting effect magnitude, proportional odds effects are a little hard to interpret; e.g., DCP increases the odds ratio of ‘any health insurance’ by 42%. This should not be taken as a drastic effect, because odds ratios can easily take on large values (and change much), which is, in fact, one of the reasons why some researchers prefer ratios to differences. Compared with the overall large magnitudes in proportional odds effects for binary responses, the proportional effect magnitudes for non-negative responses are in a much smaller scale and easy to interpret, ranging just over $-0.047$ to $0.091$; e.g., DCP increases ‘drinks 30 days’ by 9.1%. As an example for proportional odds effects becoming proportional effects for rare events, unmarried pregnancies are fairly rare ($4 \sim 5\%$) in our data, and consequently, we can interpret the ROR estimate $-0.074$ for ‘pregnant unmarried’ as a 7% decrease due to DCP.

BCQ checked out the parallel trend assumption with graphs plotting the pre-
treatment trends across the treatment and control groups. BCQ also estimated their models using different time periods or using more aggregated data. Whereas these are informal/indirect ways of testing for parallel trends, our approach of using $tQ$ as an extra regressor provides a formal test for parallel trends, as well as a simple way to allow for non-parallel trends. The $\tilde{\beta}_{q\tau}$ estimates in Table 2 reveal that parallel trend assumption in $Y^*$ and the analogous $ID_{RR}/ID_{ROR}$ assumption do not hold at least for ‘current smoker’ and ‘obese’, and Lin-DD rejects $\beta_{q\tau} = 0$ for even more outcomes.

| Outcome variable               | RR and ROR Estimate (tv) | Lin-DD Estimate (tv) |
|-------------------------------|--------------------------|----------------------|
| **Binary response**           |                          |                      |
| Any health insurance          | 0.375 (4.55)             | 0.061 (4.34)         |
| Any primary doctor            | 0.176 (5.34)             | 0.034 (4.87)         |
| Cost blocked care             | -0.109 (-1.30)           | -0.018 (-1.34)       |
| Current smoker                | 0.009 (0.25)             | 0.001 (0.25)         |
| Risky drinker                 | 0.011 (0.38)             | 0.002 (0.31)         |
| Obese                         | 0.037 (1.17)             | 0.009 (1.22)         |
| Pregnant unmarried            | -0.045 (-0.44)           | -0.002 (-0.49)       |
| **Non-negative response**     |                          |                      |
| Drinks 30 days                | -0.043 (-0.58)           | -0.708 (-0.49)       |
| BMI                           | -0.008 (-2.52)           | -0.228 (-3.15)       |
| Days poor mental              | -0.019 (-0.40)           | 0.137 (0.78)         |
| Days poor physical            | -0.019 (-0.43)           | 0.109 (2.15)         |
| Days health limits            | 0.014 (0.25)             | 0.327 (2.67)         |

To appreciate better how much difference allowing $\beta_{q\tau} \neq 0$ makes, Table 3 repeats Table 2 under the restriction $\beta_{q\tau} = 0$ (i.e., without using $tQ$ as a regressor). The differences between Tables 2 and 3 are huge both in terms of effect magnitude and t-value. In RR and ROR, only ‘any health insurance’ maintained its statistical significance, whereas ‘current smoker’ and ‘risky drinker’ become misleadingly insignificant by imposing $\beta_{q\tau} = 0$ falsely. Also, ‘any primary doctor’ and BMI become significant by
imposing $\beta_{qT} = 0$ unnecessarily. In Lin-DD as well, only ‘any health insurance’ maintains its statistical significance in Tables 2 and 3, whereas the statistical significance of seven other outcomes is switched.

The main finding in BCQ is that DCP increases ‘any health insurance’, ‘any primary doctor’ and ‘risky drinker’, but decreases BMI. This finding is similar to that of the RR and ROR column in Table 3, except for ‘risky drinker’ that is insignificant in Table 3. This similarity is due to $\beta_{qT} = 0$ assumed in both BCQ and Table 3.

Since Table 3 imposes the unnecessary restriction $\beta_{qT} = 0$, it is interesting to compare the finding in BCQ to that in Table 2. The RR and ROR column of Table 2 reveals significantly increasing effects on ‘any health insurance’ and ‘current smoker’, and a significantly decreasing effect on ‘risky drinker’. Hence, only the increasing effect on ‘any health insurance’ is shared by BCQ and the RR and ROR column of Table 2; the sign of ‘risky drinker’ changes across BCQ and the RR and ROR column of Table 2. Overall, the differences due to allowing $\beta_{qT} \neq 0$ are large.

**5 Conclusions**

Difference in Differences (DD) is one of the most popular approaches in finding the effect of a treatment $D$ on an outcome/response $Y$. However, DD is suitable for linear models, and consequently, applying DD to limited dependent variables (LDV’s), or more generally to nonlinear models, has been problematic. Many researchers with LDV’s simply ignore the LDV nature to use a linear model. The goal of this paper is to explore what can be done in this case, and this paper obtained the following findings, adopting the framework of generalized linear models with link functions.

First, when the LDV is a non-negative outcome such as count or zero-censored response, ‘ratio in ratios (RR)’ is more appropriate than DD, because exponential regression models appear naturally in this context, and RR removes the time and group effects to identify the treatment effect. The semiparametric ‘Poisson Quasi-MLE’ can be applied with $\beta_d D$ in the model, and $\exp(\beta_d) - 1$ is the unit-free proportional effect $E(Y_3^1 - Y_0^0|Q = 1)/E(Y_0^0|Q = 1)$, where $(Y_0^0, Y_3^1)$ are the potential outcomes in the
Second, when the LDV is binary, fractional or multinomial, ‘ratio in odds ratios (ROR)’ is more appropriate than DD, because “normalized” exponential regression models appear naturally in this context, and ROR removes the time and group effects to identify the treatment effect. The binary/multinomial logit MLE can be applied with $\beta_d D$, and $\exp(\beta_d) - 1$ is the unit-free proportional odds effect, which is not easy to interpret though, compared with the proportional effect. Nevertheless, for rare events (i.e., $P(Y = 0) \approx 1$), the proportional odds effect becomes the proportional effect.

ROR is not applicable to ordinal responses, which however can be reduced to binary responses in multiple ways, and then the overlapping information in those multiple ways can be combined with minimum distance estimation (see, e.g., Lee 2015).

Third, a simple interaction regressor $tQ$ where $t$ denotes time allows testing for the critical DD, RR and ROR identification conditions (for DD, the condition is called ‘parallel trends’). Namely, with $\beta_{q\tau}$ being the slope of $tQ$, if $\beta_{q\tau} = 0$, then the identification conditions hold. Viewed differently, instead of testing for the conditions, using $tQ$ as an extra regressor relaxes the identification conditions for DD, RR and ROR.

Our empirical study, as well as the simulation study in the appendix, revealed the importance of using RR or ROR instead of DD for LDV’s. The empirical study using as many as 12 outcome variables showed that RR and ROR give much different findings from DD. Also, using $\beta_{q\tau} tQ$ made big differences in empirical findings, compared with imposing the parallel-trend-type restriction $\beta_{q\tau} = 0$ unnecessarily.

APPENDIX

Simulation Study

Our simulation study addresses four LDV models: (i) positive continuous response, (ii) count response, (iii) zero-censored response with many zeros, and (iv) binary response. Poisson QMLE is applied to (i), (ii) and (iii), and logistic MLE to (iv); their estimates are compared with the usual linear model DD (‘Lin-DD’). Fractional response is not tried because it is not yet clear how to generate fractional responses subject to
the exponential regression model, and multinomial response is addressed separately in the next section because it is inconceivable to apply Lin-DD to multinomial response.

In the following, we explain (i) and Table A1 in detail, from which it will be clear how (ii), (iii) and (iv) are dealt with and how to interpret the other tables. In all cases, the effect of interest is \( \exp(\beta_d) - 1 \), which is the proportional (odds) effect, but we take \( \beta_d \) as the effect of interest because knowing \( \beta_d \) is equivalent to knowing \( \exp(\beta_d) - 1 \).

For (i) positive continuous response, we generate \( Y_{it} \) for \( t = 0, 1, 2, 3 \):

\[
Y_{it} = \exp(\beta_t + \beta_q Q_i + \beta_{q\tau} t Q_i + \beta_d D_{it} + U_{it}) \quad \text{where} \quad (A.1)
\]

\[
P(Q_i = 0) = P(Q_i = 1) = 0.5, \quad D_{it} = Q_i 1[t = 3], \quad U_{i0}, U_{i1}, U_{i2}, U_{i3} \text{ iid } N(0, 1),
\]

\[
\beta_0 = -2, \quad \beta_1 = -2, \quad \beta_2 = -1, \quad \beta_3 = -1, \quad \beta_q = 0.5, \quad \beta_{q\tau} = 0.5, \quad \beta_d = 0, 0.5;
\]

recalling \((2.7)\), \( \beta_{q\tau} = 0 \) makes the parallel trends hold in \( Y^* \) and \( \text{ID}_{RR} \) hold in \( Y \), but \( \beta_{q\tau} = 0.5 \) violates both. The simulation design is somewhat sensitive to the parameter values, as the exponential function can “blow up” sometimes to make Poisson QMLE fail to converge. The parameter values in \((A.1)\) are chosen to avoid this pitfall.

From the \( Y_{it} \)'s in \((A.1)\), the RCS response \( Y_i \) and its regressor \( X_i \) are obtained:

\[
S_i \text{ is the sampled period for } i, \quad S_{it} \equiv 1[S_i = t], \quad P(S_{it} = 1) = 0.25 \text{ for all } t,
\]

\[
Y_i = \sum_{t=0}^{3} S_{it} Y_{it}, \quad Q_i^t \equiv Q_i \sum_{t=1}^{3} S_{it}, \quad X_i \equiv (1, S_{i1}, S_{i2}, S_{i3}, Q_i, Q_i^t, D_i)' \quad (A.2)
\]

for parameters \( \{\beta_0 + \ln(1.64), \beta_1 - \beta_0, \beta_2 - \beta_0, \beta_3 - \beta_0, \beta_q, \beta_{q\tau}, \beta_d\} \);

1.64 comes from \( E\{\exp(U_{it})\} = 1.64 \) with \( U_{it} \sim N(0, 1) \), which appears due to

\[
E(Y_{it}|Q_i, S_i) = E\{\exp(\beta_i + \beta_q Q_i + \beta_{q\tau} t Q_i + \beta_d D_{it})\} \cdot E\{\exp(U_{it})\}.
\]

For the other LDV models, the RCS data are generated analogously.

Table A1 presents the simulation results with 5,000 repetitions for \( N = 250 \) and 10,000, where each entry consists of the absolute bias (|Bias|), SD, and Root Mean Squared Error (RMSE) for the \( \beta_{q\tau} \) and \( \beta_d \) estimates.

With \( N = 250 \), Lin-DD estimates \( \hat{\beta}_{q\tau} \) and \( \hat{\beta}_d \) do sometimes better than the Poisson QMLE \( \bar{\beta}_{q\tau} \) and \( \bar{\beta}_d \), but this is due to the low SD's; the |Bias| of Lin-DD \( \hat{\beta}_d \) is huge in several cases. With \( N = 10,000 \), the |Bias|’s for the Lin-DD estimates remain almost
the same as those with $N = 250$ whereas the gaps in SD between Lin-DD and Poisson QMLE are reduced, and consequently, Poisson QMLE does better than Lin-DD. Using $tQ$ solves the problem of IDRR violation for Poisson QMLE, but not for Lin-DD; $\hat{\beta}_{qτ}$ in Lin-DD is biased much even when $\beta_{qτ} = 0$. In short, Table A1 demonstrates that Lin-DD is highly biased when the true model is exponential for positive $Y$.

Table A1. Positive $Y$: |Bias|, SD and (RMSE)

| N = 250 | $\hat{\beta}_{qτ}, \hat{\beta}_d$: 0, 0 | $\beta_{qτ}, \beta_d$: 0.5, 0 | $\beta_{qτ}, \beta_d$: 0, 0.5 | $\beta_{qτ}, \beta_d$: 0.5, 0.5 |
|---------|---------------------------------|----------------|----------------|----------------|
| $\tilde{\beta}_{qτ}$ | 0.00 0.23 (0.23) | 0.00 0.24 (0.24) | 0.00 0.23 (0.23) | 0.00 0.24 (0.24) |
| $\tilde{\beta}_d$ | 0.01 0.60 (0.60) | 0.01 0.60 (0.60) | 0.01 0.60 (0.60) | 0.01 0.60 (0.60) |
| DD $\beta_{qτ}$ | 0.12 0.15 (0.19) | 0.48 0.33 (0.59) | 0.12 0.15 (0.19) | 0.48 0.33 (0.59) |
| DD $\beta_d$ | 0.08 0.46 (0.47) | 1.05 1.38 (1.73) | 0.08 0.56 (0.57) | 3.48 1.96 (4.00) |

| N = 10000 | $\tilde{\beta}_{qτ}$ | 0.00 0.04 (0.04) | 0.00 0.04 (0.04) | 0.00 0.04 (0.04) | 0.00 0.04 (0.04) |
| $\hat{\beta}_{qτ}$ | 0.00 0.04 (0.04) | 0.00 0.04 (0.04) | 0.00 0.04 (0.04) | 0.00 0.04 (0.04) |
| $\tilde{\beta}_d$ | 0.00 0.10 (0.10) | 0.00 0.10 (0.10) | 0.00 0.10 (0.10) | 0.00 0.10 (0.10) |
| DD $\beta_{qτ}$ | 0.12 0.02 (0.13) | 0.48 0.05 (0.49) | 0.12 0.02 (0.13) | 0.48 0.05 (0.49) |
| DD $\beta_d$ | 0.08 0.07 (0.11) | 1.03 0.22 (1.05) | 0.07 0.09 (0.11) | 3.44 0.30 (3.45) |

$\beta_{qτ} = 0$ for parallel trends in $Y^*$ & IDRR in $Y$; $\beta_d (\exp(\beta_d) - 1)$ is the desired effect; $\tilde{\beta}_{qτ}, \tilde{\beta}_d$: Poisson QMLE; $\hat{\beta}_{qτ}, \hat{\beta}_d$: linear-model DD.

For (ii) count response, similarly to (A.1), $Y_{it}$ is generated from the Poisson distribution with parameter $\exp(\beta_t + \beta_q Q_i + \beta_{qτ} tQ_i + \beta_d D_{it})$ for $t = 0, 1, 2, 3$. Then $Y_i$ and $X_i$ are generated as in (A.2), and Poisson QMLE is implemented, which is actually the Poisson MLE. The same parameters as in (A.2) are estimated except for the intercept because $\ln(1.64)$ is no more present. Table A2 presents the simulation results, and what was mentioned for Table A1 applies to Table A2 almost word to word.

For (iii) zero-censored response, we use (2.9) where $M_i \sim \text{Poisson}(1)$ with $P(M_i = 0) = 0.37$ and $Y_{it} = \sum_{j=0}^{M_i} Z_{ijt}$ with $Z_{ijt} = \exp\{\beta_t + \beta_q Q_i + \beta_{qτ} tQ_i + \beta_d D_{it} + N(0, 1)\}$. The same parameters as in (A.2) are estimated except for the intercept because $\exp(1)$ from $E(M)$ is added to $\beta_0$ in view of (2.9). Despite the big difference in the data generating processes, Table A3 differ little from Tables A1 and A2, and all comments
made for Tables A1 and A2 apply to Table A3 as well. The similarities in the findings from Tables A1-A3 seem to stem from the common exponential regression specification.

Table A2. Poisson Count Y: |Bias|, SD and (RMSE)

|       | $N = 250$                                                                 | $N = 10000$                                                                 |
|-------|---------------------------------------------------------------------------|------------------------------------------------------------------------------|
|       | $\beta_{q\tau}, \beta_d: 0, 0$                                         | $\beta_{q\tau}, \beta_d: 0, 0$                                             |
| $\tilde{\beta}_{q\tau}$ | $0.01 \ 0.40 \ (0.40)$                                                  | $0.00 \ 0.06 \ (0.06)$                                                     |
| $\tilde{\beta}_d$          | $0.00 \ 0.85 \ (0.85)$                                                  | $0.00 \ 0.12 \ (0.12)$                                                     |
| DD $\tilde{\beta}_{q\tau}$ | $0.08 \ 0.10 \ (0.13)$                                                  | $0.08 \ 0.02 \ (0.08)$                                                     |
| DD $\tilde{\beta}_d$        | $0.06 \ 0.31 \ (0.31)$                                                  | $0.05 \ 0.05 \ (0.07)$                                                     |

$\beta_{q\tau} = 0$ for parallel trends in $Y^* \& ID_{RR}$ in $Y$; $\beta_d (\exp(\beta_d) - 1)$ is the desired effect; $\tilde{\beta}_{q\tau}, \tilde{\beta}_d$: Poisson QMLE; $\hat{\beta}_{q\tau}, \hat{\beta}_d$: linear model DD.

Table A3. Zero-Censored Y: |Bias|, SD and (RMSE)

|       | $N = 250$                                                                 | $N = 10000$                                                                 |
|-------|---------------------------------------------------------------------------|------------------------------------------------------------------------------|
|       | $\beta_{q\tau}, \beta_d: 0, 0$                                         | $\beta_{q\tau}, \beta_d: 0, 0$                                             |
| $\tilde{\beta}_{q\tau}$ | $0.01 \ 0.31 \ (0.31)$                                                  | $0.00 \ 0.05 \ (0.05)$                                                     |
| $\tilde{\beta}_d$          | $0.01 \ 0.80 \ (0.80)$                                                  | $0.00 \ 0.12 \ (0.12)$                                                     |
| DD $\tilde{\beta}_{q\tau}$ | $0.12 \ 0.19 \ (0.22)$                                                  | $0.12 \ 0.03 \ (0.13)$                                                     |
| DD $\tilde{\beta}_d$        | $0.07 \ 0.59 \ (0.60)$                                                  | $0.08 \ 0.09 \ (0.12)$                                                     |

$\beta_{q\tau} = 0$ for parallel trends in $Y^* \& ID_{RR}$ in $Y$; $\beta_d (\exp(\beta_d) - 1)$ is the desired effect; $\tilde{\beta}_{q\tau}, \tilde{\beta}_d$: Poisson QMLE; $\hat{\beta}_{q\tau}, \hat{\beta}_d$: linear model DD.
For (iv) binary response, $Y_{it}$ is generated with Logistic error $U_{it}$:

$$Y_{it} = 1[0 < \beta_t + \beta_q Q_i + \beta_{qr} tQ_i + \beta_d D_{it} + U_{it}], \quad U_{i0}, U_{i1}, U_{i2}, U_{i3} \text{ are iid Logistic.}$$

‘$\beta_{qr} = 0$’ makes the parallel trends hold in term of $Y^*$, and makes $\text{ID}_{ROR}$ hold in terms of $Y$; $\beta_{qr} = 0.5$ violates both of these.

### Table A4. Binary $Y$: $|\text{Bias}|$, SD and (RMSE)

| $N$ = 250 | $\hat{\beta}_{qr}$, $\hat{\beta}_d$: 0, 0 | $\hat{\beta}_{qr}$, $\hat{\beta}_d$: 0.5, 0 | $\hat{\beta}_{qr}$, $\hat{\beta}_d$: 0, 0.5 | $\hat{\beta}_{qr}$, $\hat{\beta}_d$: 0.5, 0.5 |
|-----------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\hat{\beta}_{qr}$ | 0.01 0.50 (0.50) | 0.01 0.51 (0.51) | 0.01 0.50 (0.50) | 0.01 0.51 (0.51) |
| $\hat{\beta}_d$ | 0.02 1.16 (1.16) | 0.04 1.18 (1.19) | 0.04 1.15 (1.15) | 0.13 1.54 (1.55) |
| DD $\hat{\beta}_{qr}$ | 0.02 0.07 (0.08) | 0.35 0.08 (0.36) | 0.02 0.07 (0.08) | 0.35 0.08 (0.36) |
| DD $\hat{\beta}_d$ | 0.02 0.21 (0.21) | 0.02 0.21 (0.21) | 0.39 0.21 (0.45) | 0.43 0.20 (0.48) |

| $N$ = 10000 | $\hat{\beta}_{qr}$, $\hat{\beta}_d$: 0, 0 | $\hat{\beta}_{qr}$, $\hat{\beta}_d$: 0.5, 0 | $\hat{\beta}_{qr}$, $\hat{\beta}_d$: 0, 0.5 | $\hat{\beta}_{qr}$, $\hat{\beta}_d$: 0.5, 0.5 |
|-----------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\hat{\beta}_{qr}$ | 0.00 0.07 (0.07) | 0.00 0.07 (0.07) | 0.00 0.07 (0.07) | 0.00 0.07 (0.07) |
| $\hat{\beta}_d$ | 0.00 0.17 (0.17) | 0.00 0.17 (0.17) | 0.00 0.17 (0.17) | 0.00 0.17 (0.17) |
| DD $\hat{\beta}_{qr}$ | 0.02 0.01 (0.03) | 0.35 0.01 (0.35) | 0.02 0.01 (0.03) | 0.35 0.01 (0.35) |
| DD $\hat{\beta}_d$ | 0.02 0.03 (0.04) | 0.02 0.03 (0.04) | 0.39 0.03 (0.39) | 0.43 0.03 (0.43) |

$\beta_{qr} = 0$ for parallel trends in $Y^*$ & $\text{ID}_{ROR}$ in $Y$; $\beta_d (\exp(\beta_d) - 1)$ is the desired effect; $\hat{\beta}_{qr}, \hat{\beta}_d$: logit estimates; $\hat{\beta}_{qr}, \hat{\beta}_d$: linear-model DD.

Table A4 addresses binary $Y$. Since the logistic regression is used in Table A4 instead of the exponential regression in Tables A1-A3, the results in Table A4 differ much from those in Tables A1-A3. First, the overall magnitude of $|\text{Bias}|$ is much smaller than in Tables A1-A3. Second, surprisingly, when $\beta_{qr} = \beta_d = 0$, the Lin-DD estimates with almost zero bias do several times better than the logistic MLE estimates. Third, biases in Lin-DD are persistent even when $N$ increases to 10000, which implies that Lin-DD will be eventually dominated by logistic MLE for a large enough $N$. Nevertheless, less harm is seen in using Lin-DD for binary response, compared with the other LDV’s.

### Multinomial Logit for DD with Multinomial Response

*Identification*
For multinomial response $Y$ taking on a value among $0, 1, ..., C$ classes, define the ‘class-$c$ odds’ (with the base class 0) conditional on $(W = w, Q = q, S = s)$ as
\[
R^c_{qs}(Y; w) \equiv \frac{P(Y = c|w, Q = q, S = s)}{P(Y = 0|w, Q = q, S = s)} \quad \text{which implies}
\]
\[
R^c_{11}(Y; w) = R^c_{11}(Y^1_3; w), \quad R^c_{01}(Y; w) = R^c_{01}(Y^0_3; w),
\]
\[
R^c_{10}(Y; w) = R^c_{10}(Y^0_2; w), \quad R^c_{00}(Y; w) = R^c_{00}(Y^0_2; w),
\]
\[
\text{analogously to (3.1). Also define ‘class-$c$ ROR conditional on } W = w':
\]
\[
ROR^c(Y; w) \equiv \left( \frac{R^c_{11}(Y; w)}{R^c_{10}(Y; w)} \right) / \left( \frac{R^c_{01}(Y; w)}{R^c_{00}(Y; w)} \right).
\]

The identification condition for $ROR^c$ with multinomial response is
\[
ROR^c(Y^0; w) = \left( \frac{R^c_{11}(Y^0_3; w)}{R^c_{10}(Y^0_2; w)} \right) / \left( \frac{R^c_{01}(Y^0_3; w)}{R^c_{00}(Y^0_2; w)} \right) = 1. \quad \text{(ID$_{ROR^c}$)}
\]

As in (3.2), $ROR^c(Y; w) - 1$ is equal to the ‘class-$c$ proportional odds effect on the treated at the post-treatment period $t = 3$':
\[
ROR^c(Y; w) - 1 = \frac{R^c_{11}(Y^1_3; w) - R^c_{11}(Y^0_3; w)}{R^c_{11}(Y^0_3; w)} \quad \text{under ID$_{ROR^c}$}.
\]

Also, as in (3.3), if $Y = c \neq 0$ is a rare event in the sense of (3.3), then
\[
ROR^c(Y; w) - 1 \approx \frac{P(Y^1_3 = c|w, Q = 1) - P(Y^0_3 = c|w, Q = 1)}{P(Y^0_3 = c|w, Q = 1)} \quad \text{(A.3)}
\]
\[
\text{which is the class-$c$ proportional effect on the treated at the post-treatment period.}
\]

Estimation

In panel multinomial choice with classes $c = 0, 1, ..., C$, there are a few possibilities for regressors, depending on whether they vary across subjects, classes or times. Here, we consider three types of regressors: $A_i$ varying only across subjects (e.g., race), $H_{it}$ varying only across subjects and times (e.g., income), and $W_{ict}$ varying across subjects, classes and times (e.g., expense from choosing class $c$). Let the ‘latent utility from class $c$’ of subject $i$ at period $t = 2, 3$ be
\[
L^d_{ict} \equiv \beta_{tc} + \beta_{qc}Q_i + \beta_{de}d + \beta_{ac}A_i + \beta_{hc}H_{it} + \beta_{wc}W_{ict} + U_{ict}, \quad c = 0, 1, ..., C \quad \text{(A.4)}
\]
where the error terms ($U_{i02}, ..., U_{iC2}$, $U_{i03}, ..., U_{iC3}$) are iid with the type-I extreme value distribution, and independent of all regressors at all times (‘strict exogeneity’).

The potential choice $Y_{it}^d$ with $D = d$ is

$$Y_{it}^d = \sum_{j=0}^{C} (j \times 1[L_{ijt}^d > L_{ikt}^d \text{ for all } k \neq j]);$$

$Y_{it}^d$ takes on 0, 1, ..., $C$, depending on which class gives the maximum utility. Using (A.4), the choice probabilities for the untreated responses $Y_{it}^0 = 0, 1, ..., C$ are:

$$P(Y_{it}^0 = c|Q_t, A_t, H_{it}, W_{i0t}, ..., W_{iCt}) = \frac{\exp(\beta_{tc} + \beta_{qc}Q_t + \beta_{ac}^tA_t + \beta_{hc}^tH_{it} + \beta_{wc}^tW_{ict})}{\sum_{j=0}^{C} \exp(\beta_{tj} + \beta_{qj}Q_t + \beta_{aj}^tA_t + \beta_{hj}^tH_{it} + \beta_{wj}^tW_{ijt})} = \frac{\exp(\Delta \beta_{tc} + \Delta \beta_{qc}Q_t + \Delta \beta_{ac}^tA_t + \Delta \beta_{hc}^tH_{it} - \beta_{w0}^tW_{i0t} + \beta_{wc}^tW_{ict})}{1 + \sum_{j=1}^{C} \exp(\Delta \beta_{tj} + \Delta \beta_{qj}Q_t + \Delta \beta_{aj}^tA_t + \Delta \beta_{hj}^tH_{it} - \beta_{w0}^tW_{i0t} + \beta_{wj}^tW_{ijt})},$$

the second equality holds, dividing through by $\exp(\beta_{t0} + \beta_{q0}Q_t + \beta_{a0}^tA_t + \beta_{h0}^tH_{it} + \beta_{w0}^tW_{i0t})$ for the base class $c = 0$. The numerator of the last ratio becomes one for the base class. ID$_{RORc}$ holds for $P(Y_{it}^0 = c|\cdot)$, analogously to the proof for ID$_{ROR}$.

Analogously derive the model for $P(Y_{it}^1 = c|\cdot)$, which then gives (i omitted)

$$R_{11}^c(Y_{it}^d; w) = \exp(\Delta \beta_{tc} + \Delta \beta_{qc}Q_t + \Delta \beta_{ac}^tA_t + \Delta \beta_{hc}^tH_{it} - \beta_{w0}^tW_{i0t} + \beta_{wc}^tW_{ict}),$$

where $\Delta \beta_{dc} \equiv \beta_{dc} - \beta_{d0}$.

Since $P(Y_{it}^1 = c|\cdot)$ differs from $P(Y_{it}^0 = c|\cdot)$ only in the extra term $\Delta \beta_{dc}$, we get the class-$c$ proportional effect under the rare event condition (3.3):

$$\{R_{11}^c(Y_{it}^1; w)/R_{11}^c(Y_{it}^0; w)\} - 1 = \exp(\Delta \beta_{dc}) - 1.$$

In RCS, omitting the subscript $i$, we observe $Y \equiv (1 - S)Y_2 + SY_3$ where $Y_i (= 0, ..., C)$ is the realized choice at $t$, along with $Q$, $S$ and $A$, $H \equiv (1 - S)H_2 + SH_3$, $W_0 \equiv (1 - S)W_{02} + SW_{03}$, ..., $W_C \equiv (1 - S)W_{C2} + SW_{C3}$.

The RCS choice probabilities are, with $\Delta \beta_{rj} \equiv \Delta \beta_{3j} - \Delta \beta_{2j}$ for $j = 1, ..., C$,

$$P(Y = c|Q, S, A, H, W_0, ..., W_C) = \frac{\exp(\Delta \beta_{2c} + \Delta \beta_{rc}S + \Delta \beta_{qc}Q + \Delta \beta_{ac}^tA + \Delta \beta_{hc}^tH - \beta_{w0}^tW_0 + \beta_{wc}^tW_c)}{1 + \sum_{j=1}^{C} \exp(\Delta \beta_{2j} + \Delta \beta_{rj}S + \Delta \beta_{qj}Q + \Delta \beta_{aj}^tA + \Delta \beta_{hj}^tH - \beta_{w0}^tW_0 + \beta_{wj}^tW_j)}.$$
Noting $\Delta \beta_{20} = \Delta \beta_{30} = 0$, the numerator becomes one for the base class $c = 0$.

Because $D$ alters the choice probability for class $c$ by $\beta_{dc}$, the “net increase” in the propensity to choose class $c$ relative to the base class 0 is $\Delta \beta dc \equiv \beta dc - \beta d0$, not $\beta dc$. Estimate $\Delta \beta d1, ..., \Delta \beta dC$ with cross-section multinomial logit using the last display. Then, $\exp(\Delta \beta dc) - 1$ is the class-$c$ proportional odds effect relative to the class 0, and the class-$c$ proportional effect as well when $Y = c$ is a rare event in the sense of (3.3).

Simple Simulation Study for Multinomial Response

Our simulation study using the above $P(Y = c|Q, S, A, H, W0, ..., WC)$ with $C = 2$ has the following design (the error terms generated as in (A.4) and $H_d$ excluded):

$A \sim \text{Uniform}(-1, 1), \quad Wc2, Wc3$ for $c = 0, 1, 2$ are iid $N(0, 1), \quad P(Q = 1) = 0.5,$
$P(S = 1) = 0.5, \quad \beta_{20} = \beta_{30} = \beta_{q0} = \beta_{d0} = \beta_{a0} = \beta_{w0} = 0$ (for class 0),
$\beta_{21} = \beta_{22} = -4, \quad \beta_{31} = \beta_{32} = -5, \quad \beta_{q1} = \beta_{q2} = -0.5$ (for classes 1,2),
$\beta_{d1} = \beta_{d2} = 0.5, \quad \beta_{a1} = \beta_{a2} = 0.5, \quad \beta_{w1} = \beta_{w2} = 0.5$ (for classes 1,2).

That is, the class-0 parameters are all zero, and the parameters of classes 1 and 2 are the same. Due to $\beta_{20} = \beta_{30} = 0$ but $\beta_{21} = \beta_{22} = -4$ and $\beta_{31} = \beta_{32} = -5$ (much smaller intercepts for classes 1 and 2 relative to class 1), the events $Y = 1, 2$ are rare.

| Table A5. Multinomial $Y$: 3 Classes, $N = 10,000, 5,000$ Repetitions |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| **Class $c = 1$** | **Class $c = 2$** | **Class $c = 1$** | **Class $c = 2$** | **Class $c = 1$** | **Class $c = 2$** |
| **True, |Bias| SD, RMSE | AvgSE | **True, |Bias| SD, RMSE | AvgSE | **True, |Bias| SD, RMSE | AvgSE | **True, |Bias| SD, RMSE | AvgSE |
| $\Delta \beta_{2c}$ | -4.0, 0.019 | 0.15, 0.023 | 0.15 | -4.0, 0.030 | 0.15, 0.024 | 0.15 |
| $\Delta \beta_{q2c}$ | -1.0, 0.017 | 0.28, 0.078 | 0.28 | -1.0, 0.011 | 0.28, 0.077 | 0.28 |
| $\Delta \beta_{q3c}$ | -0.5, 0.004 | 0.23, 0.055 | 0.23 | -0.5, 0.006 | 0.23, 0.054 | 0.23 |
| $\Delta \beta_{dc}$ | 0.5, 0.004 | 0.41, 0.166 | 0.41 | 0.5, 0.007 | 0.41, 0.168 | 0.41 |
| $\Delta \beta_{ac}$ | 0.5, 0.003 | 0.16, 0.026 | 0.17 | 0.5, 0.005 | 0.17, 0.027 | 0.17 |
| $\beta_{w0}$ | 0.0, 0.001 | 0.07, 0.004 | 0.07 | 0.0, 0.001 | 0.09, 0.008 | 0.10 |
| $\beta_{wc}$ | 0.5, 0.000 | 0.09, 0.008 | 0.10 | 0.5, 0.001 | 0.09, 0.008 | 0.10 |

AvgSE is the average of the standard error estimates

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Table A5 presents the simulation results, where each entry consists of true values (True), |Bias|, SD, RMSE, and the average of the standard error estimates (AvgSE). Overall, biases are very small, and AvgSE’s are almost the same as the SD’s. With \( N = 10,000 \), the multinomial logit with RCS works well even for rare events \( Y = 1, 2 \).

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