Weak Classical-Gravity Source in Standpoint Cosmology

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Abstract

Guided by a linearized approximation to Einstein theory, an interim prescription for “weak source of gravity” - - in “particle” energy-momentum distributed along standpoint light cone - - is formulated for (classical) standpoint cosmology.

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Introduction

An alternative cosmology featuring a notion of “standpoint” has been proposed\(^1\) and Hubble-scale predictions in “homogeneous-universe” approximation have been deduced.\(^2\) Going beyond the homogeneous approximation requires some equivalent to the Einstein relation between local energy-momentum tensor and metric curvature. In lieu of a systematic deduction from quantum underpinning, we here formulate an interim prescription - - through energy-momentum of “particles” - - for “weak local gravity source” in classical standpoint cosmology; for source distances small on Hubble scale, the prescription concurs with the linearized (weak gravity) approximation to Einstein theory. “Particle” source for the gravity experienced near standpoint is distributed along (backward and forward) standpoint light cone. Treatment of strong gravity will require attention to the non-Riemannian character of standpoint metric.

In the proposed prescription the gravitational constant $G$ appears in its traditional role of ad-hoc classical parameter. Reference (1) indicates on the other hand that, when and if classical standpoint cosmology is derived from quantum underpinning, $G$ on the scale of quantum-particle masses will relate inversely to the huge dimensionless dilation-generator expectation that gives the ratio between big-bang scale and the scale at which meaning began to develop for localized matter within (classical) spacetime. Hugeness of this scale ratio being essential in the quantum model to emergence of spacetime, smallness of $G$ appears essential to geometry. Classical correction to the “weak-gravity” prescription proposed here is then problematic. In our concluding section we emphasize that, because meaning for “particle energy-momentum” recognizes gravitation, “source of gravity” need not exhibit any explicit gravitational ingredient in addition to “particles”. Nevertheless, meaning for the “light cone” central to our prescription must be reconsidered when gravitational potential is large; in its dependence on light cone our prescription manifests a “flat-space” character with respect to gravitational-signal propagation.
II. Prescription

Because the wave function of standpoint quantum cosmology depends on the energy-momentum of particles whose trajectories intersect the standpoint’s light cone, it is natural to seek a representation of classical-gravity source distributed along this cone. In weak-gravity approximation to Einstein theory when sources are confined to a finite region of spacetime, a light-cone-source representation has been found for the quantity

\[ h_{\mu\nu}(x) \equiv g_{\mu\nu}(x) - \eta_{\mu\nu} \quad (II.1) \]

\((g_{\mu\nu}(x)\) being local metric tensor and \(\eta_{\mu\nu}\) Minkowski tensor), through a formula of Lienard-Wiechert type.\(^{(3)}\) We shall refer to the dimensionless \(h_{\mu\nu}(x)\) as “gravitational potential”. (Units with \(c = 1\) are implicit.) In adapting to standpoint cosmology, we consider metric in \(R\) spacetime at a standpoint labeled \(R\) and exploit the model feature that, in homogeneous-universe approximation, a certain “metric tensor” denoted \(g_{\mu\nu}(R)\) is equal to \(\eta_{\mu\nu}.\(^{(1)}\) Our prescription is correspondingly such that \(h_{\mu\nu}(R)\), defined by an analogue of (II.1), is generated by inhomogeneity of matter distribution.

Use of an integral formula for source rather than a differential equation is natural because the spacetime belonging to a standpoint is finite (compact) with most matter concentrated near the boundary.\(^{(1,2)}\) In Mach spirit one may think of this latter huge and maximally-distant source for metric near standpoint as “chiefly responsible” for the Minkowski component \(\eta_{\mu\nu}.\) More precisely, in building the gravitational potential at standpoint \(R\) (in \(R\) spacetime), defined by

\[ h_{\mu\nu}(R) \equiv g_{\mu\nu}(R) - \eta_{\mu\nu}, \quad (II.2) \]

from a formula of Lienard-Wiechert type in terms of deviation from homogeneous distribution of matter along standpoint light cone, nearby contributions (on Hubble scale) will dominate.

Before proceeding further let us recall from Reference (1) the quartic expression for metric at standpoint \(R\) in terms of \(g_{\mu\nu}(R)\), expressed through the coordinates \(x_\mu^R\) belonging to the standpoint:

\[ ds^4 = \left[ (\eta_{\mu\nu} - g_{\gamma\nu}(R)g_{\mu}^{\gamma}(R)) dx_\mu^R dx_\nu^R \right]^2 + 4\left[ g_{\mu\nu}(R) dx_\mu^R dx_\nu^R \right]^2. \quad (II.3) \]
Despite the non-Riemannian character of (II.3), we shall refer to $g_{\mu\nu}(R)$ as “metric tensor” because, for small $h_{\mu\nu}(R)$ when only terms up to first order in $h_{\mu\nu}(R)$ are kept, (II.3) reduces to the Riemannian form, $ds^2 = 2g_{\mu\nu}(R)dx^\mu_R dx^\nu_R$. The factor 2, irrelevant for local physics, affects “age of standpoint”, (2) Only standpoints for which $|h_{\mu\nu}(R)| << 1$ are covered by our interim prescription.

The coordinates $x^\mu_R$ of the compact spacetime belonging to the $R$ standpoint are limited by the constraint

$$0 \leq t_R \pm r_R \leq 2R,$$  \hspace{1cm} (II.4)

where

$$r_R \equiv |\vec{x}_R|,$$  \hspace{1cm} (II.5)

and $R$ is a positive real parameter that controls standpoint age. This limitation amounts to the interior of a double light cone. In its own spacetime the $R$ standpoint locates at $t_R = R, \vec{x}_R = 0$, i.e., at the double-cone’s center. The origin of this spacetime - - the point $t_R = 0, \vec{x}_R = 0$ - - is interpreted as “big bang” and is shared with all other standpoint spacetimes. References ((1) and (2) give the general rules for mapping of points within one standpoint spacetime onto other standpoint spacetimes.

Although the present paper considers only metric tensor at a standpoint for the spacetime of that standpoint, such specification (with metric invariance) when given for all standpoints provides the complete metric of the universe. Notice that general coordinate transformations - - hallmark of general relativity - - fail to be a feature of standpoint cosmology, being incompatible with the constraint (II.4).

For Einstein theory, the retarded Lienard-Wiechert gravitational potential at $x$ generated by a particle of energy-momentum $p^n_\mu = p^\mu$ is

$$h^R_{\mu\nu}(x) = G \frac{p^n_\mu p^n_\nu}{p^n \cdot (x^n - x)},$$  \hspace{1cm} (II.6)

where $x^n$ locates the intersection of “source-particle” trajectory with the backward light cone of the point $x$. The advanced potential is given by a similar formula. In the sense of Wheeler and Feynman(4) we require a superposition of advanced and retarded potentials. That is, the acceleration experienced by a test particle at $x$ combines its gravitational interaction with source particles
in past and in future. If a source-particle trajectory intersects both forward and backward light cones of the point $x$ with no intervening change of energy-momentum, its advanced and retarded potentials at $x$ are equal; but generally the retarded contributions from backward cone are independent of advanced contributions from forward cone. In Section IV below we comment further on the advanced potential.

Meaning for the term “particle” in this paper is classical, merely implying localization of (positive) energy within some region of 3-space. We require particle “size” to be small compared to distance from standpoint but otherwise put no upper limit on “particle diameter” and make no requirement as to “structure”. Discreteness of the “particle” concept is a convenient conceptual and notational device for representing “inhomogeneity” in matter distribution.

Our proposed analogue of (II.6) for standpoint spacetime (and standpoint Lorentz frame (1)) dovetails with the structure of standpoint cosmology when the point $x$ associates with $R$ standpoint and the 4-vectors $p^n$ and $x^n$ associate with particles whose trajectories intersect the $R$ standpoint (forward-backward) light cone. The Lorentz invariant denominator of (II.6) is exactly the quantity $\Phi_n(R)$ that has been called “action” in Reference (1). We propose as approximate gravitational potential at standpoint, generated by a (light-cone-intersecting) particle at distance from standpoint small on Hubble scale (small compared to $R$), the expression

$$G^p_{\mu\nu}(R)p^\nu_n(R)\Phi_n(R).$$

(II.7)

What about particle sources near the horizon --- at distances approaching $R/2$ on backward cone? Their number is so great according to Reference (1) as to generate a meaninglessly-huge gravitational potential $h_{00}(R)$ at standpoint if Formula (II.7) is used. But because “homogeneous distribution of sources”, a notion made precise below, corresponds to $h_{\mu\nu}(R) = 0$, and because mean energy density ($\sim \frac{1}{CR^2}$ (1)) is negligible at galactic and smaller scales, it is consistent to suppose potential at standpoint to be a sum over discrete contributions of the form (II.7) minus a corresponding continuous contribution from homogeneously-distributed matter. Such subtraction suppresses the effect on standpoint gravitational potential from matter at Hubble-scale distances, while leaving undamped “nearby” matter sources.
According to Peebles,\textsuperscript{(5)} homogeneity becomes a good approximation in the present universe at scales greater than $\sim 1\%$ of Hubble scale. Roughly speaking, then, our proposal calculates gravitational potential according to (II.7) from particle sources at distances less than $\sim 1\%$ of Hubble length and neglects the contribution from sources at larger distances. Precise meaning for “homogeneous” translates the foregoing rough statement into a quantitative prescription. Because angular isotropy is an aspect of “homogeneity”, subtraction is needed only for $h_{00}(R)$. For other components of the gravitational potential, large-distance damping of the discrete sum is automatic.
III. Homogeneous Distribution of Matter

We here recall the meaning of “homogeneous” as presented in References (1) and (2) for matter on the standpoint’s backward light cone. Analogous meaning applies to the forward cone.

Spatial location $\vec{r}_R$ on backward cone, with respect to standpoint, is related to a convenient dimensionless 3-vector $\vec{b}_R$, $0 \leq |\vec{b}_R| \leq 1$, by the formula

$$\vec{r}_R = \frac{1 + \sqrt{2} b_R}{(1 + b_R)^2} \tau_R \vec{b}_R,$$

(III.1)

where the quantity

$$\tau_R = \Omega^{1/2} R,$$

(III.2)

with

$$\Omega \equiv \left( \frac{1}{\sqrt{2}} + \frac{1}{2} \right)^{-2},$$

(III.3)

is age of standpoint. The dimensionless factor $\Omega$ gives mean energy density at standpoint (in the spacetime of that standpoint) in units of the standard model’s “critical density”. That is,\(^{(1)}\)

$$\rho_R = 3 \frac{1}{8\pi GR^2}$$

$$= \Omega \frac{3}{8\pi} \frac{H_0^2}{G},$$

(III.4)

with

$$H_0 = \tau_R^{-1}.$$

(III.5)

The parameter $\vec{b}_R$ is useful, partly because it controls (through a formula given in Reference (1)) the “Hubble-flow” velocity of homogeneously-distributed matter, but especially because expression through $\vec{b}_R$ of “homogeneous distribution” is simple, as is red shift observed at standpoint for light emitted by Hubble-flowing matter. (It has been shown\(^{(1)}\) that $\vec{b}_R$ is Hubble-flow velocity of source in a “flat” spacetime belonging to a standpoint of infinite age but the same spatial location as the $R$ standpoint.)

Defining a boost (or rapidity) parameter $0 \leq \beta_R \leq \infty$, equivalent to $b_R$, by

$$\beta_R \equiv \ln \left( \frac{1 + b_R}{1 - b_R} \right)^{1/2},$$

(III.6)
redshift $z_R$ is given by

$$z_R = e^{\beta_R} - 1,$$  \hspace{1cm} (III.7)

while “homogeneous distribution of matter” in boost space is proportional to

$$\sinh^2 \beta_R d\beta_R.$$  \hspace{1cm} (III.8)

Notice that, for distance of matter from standpoint small compared to $R$, \quad

$$\tilde{r}_R \approx \tau_R \tilde{b}_R,$$  \hspace{1cm} (III.9)

and

$$b_R \approx \beta_R \approx z_R,$$

while, at the other extreme, “horizon” corresponds to

$$r_R \to R/2,$$

$$b_R \to 1,$$

$$\beta_R \to \infty.$$  \hspace{1cm} (III.10)

An indefinite amount of matter in the universe is seen from (III.8) to concentrate near horizon. (A useful idea is that age of matter on standpoint backward light cone is $e^{-\beta_R} \tau_R$; “Mach sources” on the standpoint backward cone are thus extremely “young”.)

Using (III.4) for normalization, one easily computes the increment of potential $\Delta(b_{\text{max}})$ to be subtracted when calculating $h_{00}(R)$ through (II.7), corresponding to “nearby” homogeneously-distributed matter at distance less than $r_{\text{max}}$,

$$\Delta(b_{\text{max}}) \approx -\frac{3}{4}\Omega b_{\text{max}}^2$$

for $r_{\text{max}} << R$ ($b_{\text{max}} << 1$).  \hspace{1cm} (III.11)

As an example, for distances of the order of earth-sun separation such that $b_{\text{max}} \sim 10^{-14}$, the increment to be subtracted has order of magnitude $\Delta(b_{\text{max}}) \sim 10^{-28}$. Such a correction may be compared to the much larger, although still small, gravitational potential at earth generated by sun - of order $10^{-8}$. The homogeneous-matter subtraction at earth standpoint continues to be negligible for all galactic sources, although growing in relative importance with increasing scale. Only as $b_{\text{max}}$ approaches $10^{-2}$, does the subtraction begin substantially to damp the gravitational potential given by (II.7).
IV. Discussion

The interim prescription proposed here for weak local gravity facilitates inhomogeneity phenomenology in standpoint cosmology. With respect to galaxy and star formation, our prescription sustains previous work based on standard Einstein theory. Only for scales approaching Hubble scale is there novelty.

With regard to our prescription’s use of advanced as well as retarded potential, it may be recalled that a similar electromagnetic proposal by Wheeler and Feynman\(^{(4)}\) implied a boundary condition of all “radiation” from any particle being eventually absorbed by a “sink” of other particles. Such a notion dovetails in standpoint cosmology with indefinite accumulation near “future abyss” of matter serving as “gravitational sink”.\(^{(1,2)}\) As explained in References (1) and (2), “abyss” limits the standpoint’s forward light cone in a sense paralleling that by which “horizon” limits the backward.

Not addressed in this paper are inhomogeneities so large that associated gravitational potential \(h_{00}(R)\) approaches or exceeds magnitude 1 - - - inhomogeneities that in standard parlance are called “black holes”. Near such “strong” inhomogeneities meaning of “standpoint light cone” becomes obscure. We are hopeful of eventual illumination, through the non-Riemannian character of (II.3), of “black-hole” mysteries arising in standard theory.

Because, in the quantum underpinning of standpoint cosmology, meaning for spacetime attaches to particle location with respect to standpoint and meaning for localized energy-momentum attaches to “particle”, it is expected that summing (II.7), or some variant thereof, over all particles encompasses the full source of metric at standpoint. Meaning for “particle momentum-energy”, that is, includes local “gravitational potential.” (It is not generally true that \(p^n/E^n\) is particle velocity.) There is no “source of metric” apart from particles.

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