Effects of In-Medium NN Cross Sections on Particle Production

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Abstract. The in-medium modification of nucleon-nucleon (NN) cross sections is investigated by means of particle production in heavy ion collisions (HIC) at intermediate energies. In particular, the density dependence of the inelastic cross sections considerably affects the pion and kaon yields and their rapidity distributions. However, the \( \frac{\pi^-}{\pi^+} \)- and \( \frac{K^+}{K^0} \)-ratios depend only moderately on the in-medium behavior of the inelastic cross sections. It turns out that particle ratios seem to be robust observables in determining the nuclear equation of state (EoS) and, particularly, its isovector sector.

1. Introduction

The knowledge of the properties of highly compressed and heated hadronic matter is an important issue for the understanding of astrophysics such as the physical mechanism of supernovae explosions and the physics of neutron stars [1, 2]. HIC provide the unique opportunity to explore highly excited hadronic matter, i.e. the high density behavior of the nuclear EoS, under controlled conditions (high baryon energy densities and temperatures) in the laboratory [3].

Important observables have been the nucleon collective dynamics [3, 4] and the dynamics of produced particles such as pions and kaons [5]. However, the reaction dynamics is a rather complex process which involves the nuclear mean field (EoS) and binary 2-body collisions. In the presence of the nuclear medium the treatment of binary collisions represents a non-trivial topic. The NN cross sections for elastic and inelastic processes, which are the crucial physical parameters here, are experimentally accessible only for the free space and not for 2-body scattering at finite baryon density. Recent microscopic studies, based on the \( T \)-matrix approach, have shown a strong decrease of the elastic NN cross section [6] in the presence of a hadronic medium. These in-medium effects of the elastic NN cross section considerably influence the hadronic reaction dynamics [7]. Obviously the question arises whether similar in-medium effects of the inelastic NN cross sections may affect the reaction dynamics and, in particular, the production of particles (pions and kaons).

Since microscopic results are not available, we discuss here in a simple phenomenological way possible density modifications of the inelastic NN cross sections and their influences on particle multiplicities, rapidity distributions and ratios. We find a strong dependence of the yields and rapidity distributions on the in-medium modifications of the inelastic cross sections, but on the other hand, this effect is only moderate for particle ratios such as \( \frac{\pi^-}{\pi^+} \), and almost vanishes for \( \frac{K^+}{K^0} \). Therefore such ratios turn out to be robust observables in determining the nuclear EoS and, particularly, the isovector channel of the nuclear mean field [8, 9, 10].

2. The transport equation

In this chapter we briefly discuss the transport equation by concentrating on the treatment of the cross sections, which are the important parameters of the collision integral.

The theoretical description of HIC is based on the kinetic theory of statistical mechanics, i.e. the Boltzmann Equation [11]. The relativistic semi-classical analogon of this equation is the
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The free cross section ($k_F = 0$) is compared to the experimental total $np$ cross section [6].

Relativistic Boltzmann-Uehling-Uhlenbeck (RBUU) equation [12]

$$
\left[ k^\mu \partial^\nu_k + (k^\mu F^\mu\nu + M^* \partial^\nu_k M^*) \partial^\nu_k \right] f(x, k^*) = \frac{1}{2(2\pi)^9} \int \frac{d^3k_2}{E_{k_2}} \frac{d^3k_3}{E_{k_3}} \frac{d^3k_4}{E_{k_4}} W(kk_2|k_3k_4) \left[ f_3 f_4 \tilde{f}_2 - f f_2 \tilde{f}_3 \tilde{f}_4 \right]
$$

where $f(x, k^*)$ is the single particle distribution function. In the collision term the short-hand notation $f_i \equiv f(x, k_i^*)$ for the particle and $\tilde{f}_i \equiv (1 - f(x, k_i^*))$ and the hole-distribution is used. The collision integral exhibits explicitly the final state Pauli-blocking while the in-medium scattering amplitude includes the Pauli-blocking of intermediate states. The dynamics of the lhs of eq. (1), the drift term, is determined by the mean field. Here the attractive scalar field $\Sigma_S$ enters via the effective mass $M^* = M - \Sigma_s$ and the repulsive vector field $\Sigma_\mu$ via kinetic momenta $k_\mu^* = k_\mu - \Sigma_\mu$ and via the field tensor $F^{\mu\nu} = \partial^\mu \Sigma^\nu - \partial^\nu \Sigma^\mu$. The in-medium cross sections enter into the collision integral via the transition amplitude

$$
W = (2\pi)^4 \delta^4 (k + k_2 - k_3 - k_4)(M^*)^4|T|^2
$$

with $T$ the in-medium scattering matrix element.

In the kinetic equation (1) one should use both physical quantities, the mean field (EoS) and the collision integral (cross sections) according to the same underlying effective two-body interaction in the medium, i.e. the in-medium T-matrix; $\Sigma \sim \Re T\rho$, $\sigma \sim \Im T$, respectively $W \sim |T|^2$. However, in most practical applications phenomenological mean fields and cross sections have been used. In these models adjusting the known bulk properties of nuclear matter around the saturation point one tries to constrain the models for supra-normal densities with the help of heavy ion reactions [13, 14]. Medium modifications of the NN cross section are usually not taken into account which works, in comparison to experimental data, astonishingly well [13, 14, 15, 16]. However, in particular kinematics regimes a sensitivity of dynamical observables such as collective flow and stopping [7, 17] or transverse energy transfer [18] to the elastic NN cross section has been observed.

Fig. 1 shows the energy dependence of the in-medium neutron-proton ($np$) cross section at Fermi momenta $k_F = 0.0, 1.1, 1.34, 1.7 fm^{-1}$, corresponding to $\rho \sim 0, 0.5, 1, 2 \rho_0$ ($\rho_0 = 0.16 fm^{-3}$ is the nuclear matter saturation density) as found in relativistic Dirac-Brueckner (DB) calculations [9]. The presence of the medium leads to a substantial suppression of the cross section which is most pronounced at low laboratory energy $E_{lab}$ and high densities where the Pauli-blocking of intermediate states is most efficient. At larger $E_{lab}$ asymptotic values of 15-20 mb are reached.
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However, not only the total cross section but also the angular distributions are affected by the presence of the medium. The initially strongly forward-backward peaked \( np \) cross sections become much more isotropic at finite densities \[6\] which is mainly due to the Pauli suppression of soft modes (\( \pi \)-exchange) and correspondingly of higher partial waves in the T-matrix \[6\].

Obviously one expects similar in-medium effects for the inelastic NN cross sections mainly due to Pauli-blocking of intermediate scattering states and in-medium modified matrix elements. Such microscopic studies for inelastic processes are very rare or still in development \[19, 20\]. However, to explore the sensitivity we use here a rather simple parametrization which assumes a reduction of the inelastic NN cross section with increasing baryon density, in line with that of Fig. 1 for the elastic one which has previously been used in Ref. \[19\]. This can be achieved by assuming a factorization of the effective matrix element of the form

\[
|M_{\text{eff}}|^2 = \kappa(\rho, \beta)|M_{\text{vac}}|^2
\]

with \( M_{\text{vac}} \) the vacuum matrix element (taken from experimental free scattering data) and \( \kappa(\rho, \beta) \) a density dependent function depending on a quenching parameter \( \beta \). A very simple parametrization of the function \( \kappa(\rho) \) is shown in Fig. 2 (bottom panel) for different values of the parameter \( \beta \). The effect on the inelastic NN cross section is shown on the top of Fig. 2 for the choice \( \beta = -1 \). A significant reduction of the effective inelastic NN cross section (by a factor of 2) is observed with respect to that of the free case at a given baryon density \( \rho = 1.5\rho_0 \).

3. Results

We have applied the parametrization \[8\] in the collision integral of the transport equation \[11\] and analyzed the transport calculations in terms of particle production. At energies below 1.6 AGeV pions and kaons are produced, where the second ones do not significantly affect the reaction dynamics due to their small production cross sections. In particular, the kaons are treated perturbatively and can be produced from \( BB \rightarrow BYK \) and \( \pi B \rightarrow YK \). Here \( B \) stands for a nucleon or \( \Delta \) resonance and \( Y \) for a hyperon. Pions are created via the decay of the \( \Delta(1232) \) resonance. For more details we refer to \[8\]. For the nuclear mean field the \( N\bar{L}2 \) parametrization of the non-linear Walecka model \[21\] is adopted here with a compression modulus of 200 MeV and an effective Dirac mass of \( m^* = 0.82 M \) (\( M \) is the bare nucleon mass). The momentum dependence enters via the relativistic treatment through the vector components of the baryon self energy. In order to keep the discussion transparent, we do not apply here any isovector components of the baryon self energy, as it has been previously done in Ref. \[8\]. This implies no effective mass splitting.
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Figure 3. Time evolution (in units of fm/c) of the multiplicity of Δ-resonances (dashed lines) and pions (solid lines) for different β-values as indicated. The gray band represents the range of experimental data for central Au+Au reactions at 1 AGeV incident energy [22].

between protons and neutrons, and on the other hand, between the different isospin states of the Δ resonance (Δ−,0, Δ+,++) and of the hyperons (Λ, Σ±,0). Furthermore, pions are propagated in a coulomb field and kaons do not experience any potential.

We start the discussion with the time evolution of the multiplicities of the Δ resonances and of the pions, as can be seen in Fig. 3. The time evolution of the multiplicity of produced Δ-resonances is shown with their maximum around 15 fm/c which corresponds to the time of maximum compression. Due to the finite lifetime these resonances decay into pions (and nucleons) according $\Delta \rightarrow \pi N$ (some of these pions are re-absorbed in the inverse process, i.e. $\pi N \rightarrow \Delta$). This mechanism continues until all resonances have decayed leading to a final constant pion yield for times $t \geq 50$ fm/c (the so-called freeze-out time). After the freeze-out the pions can be measured experimentally. The experimental pion multiplicity is schematically shown in Fig. 2 by the gray band for central Au+Au collisions [22]. We observe an essential reduction of the pion multiplicity using the effective inelastic NN cross sections, in line with the previous Fig. 2. As an important result the calculations with the effective inelastic NN cross section describe the experimental data reasonably well for a quenching parameter of about $\beta = -1$. However, for a more realistic comparison one should use more microscopic calculations of the inelastic cross section, as it has been done in Ref. [20]. Such a progress is under study.

The strong in-medium dependence of the inelastic cross sections is shown also in the rapidity distributions of pions and kaons in Fig. 4. The rapidity distribution of kaons is even more stronger affected by the density behavior of the inelastic cross section with respect to that of pions. This is due to the fact that the leading channels for kaon production are $N\Delta \rightarrow BYK$ and pionic ones, which are both reduced when the in-medium dependent $\sigma_{inel}$ are used.

The question arises whether particle ratios are influenced by in-medium effects of the inelastic cross section. An answer on this question is of major importance, particularly for kaons and less for pions, since particle ratios have been widely used in determining the nuclear EoS at supranormal densities. relative ratios of kaons between different colliding systems have been used in determining the isoscalar sector of the nuclear EoS, see Refs. [5]. More recently, the $(\pi^-/\pi^+)$- and $(K^+/K^0)$-ratios have been used in exploring the high density behavior of the symmetry energy, i.e. the isovector part of the nuclear mean field [10, 9, 8].

Table 1 shows the $(\pi^-/\pi^+)$- and $(K^+/K^0)$-ratio for central ($b = 1$ fm) Au+Au collisions at 1.48 AGeV incident energy using both, the free and in-medium inelastic cross sections. The pionic ratio moderately depends on the density dependence of the inelastic cross section, but the $(K^+/K^0)$-ratio is not particularly affected by the in-medium effects. A possible reason for the different behavior between the pionic and kaonic ratios may be the fast pre-equilibrium emission of
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**Figure 4.** Rapidity distributions of $\pi^-$ (top-left), $\pi^+$ (top-right), $K^0$ (bottom-left) and $K^+$ (bottom-right) for central ($b = 1$ fm) Au+Au collisions at 1.48 AGeV incident energy. Results of transport calculations using the free (Solid lines) and in-medium (dashed lines) inelastic cross sections with $\beta = -1$ are shown.

| particle ratio | free $\sigma_{\text{inel}}$ | in-medium $\sigma_{\text{inel}}(\beta = -1)$ |
|----------------|-------------------------------|---------------------------------------------|
| $(\pi^-/\pi^+)$ | 1.63 (± 0.11)               | 1.92 (± 0.14)                             |
| $(K^+/K^0)$     | 0.77 (± 0.05)                | 0.74 (± 0.05)                             |

Table 1. In-medium dependence of the $(\pi^-/\pi^+)$- and $(K^+/K^0)$-ratios for central ($b = 1$ fm) Au+Au collisions at 1.48 AGeV incident energy.

the kaons. In particular, kaons are created very early during the formation of the high density phase and are emitted from the compression region without undergoing any interaction with the hadronic environment. Therefore one obviously expects a direct relation between the high density effects of the inelastic cross sections and the $(K^+/K^0)$-ratio. Pions, on the other hand, are strongly interacting with the medium via secondary re-absorption processes and thus are emitted from different stages of a collision which may influence the final $(\pi^-/\pi^+)$-ratio [23, 10]. It turns out that the strangeness ratio presents a robust observable to investigate the isovector character of the nuclear matter EoS.

4. Conclusions

We have investigated the role of the density dependence of the inelastic cross section on particle production in intermediate energy heavy ion collisions within a covariant transport equation of a Boltzmann type. Since microscopic studies on the in-medium behavior of inelastic cross sections are still rare, we have used here a simple phenomenological in-medium dependence of the inelastic cross sections. We have applied the transport equation to Au+Au collisions at intermediate relativistic energies below the kaon threshold energy.

Our studies have shown a strong sensitivity of the particle multiplicities and rapidity distributions of pions and kaons. In particular, a reduction by a factor of 2 for pions has been seen when the in-medium effects in the inelastic cross section are accounted for. Consequently, the kaon $(K^{0,+})$ yields decrease more than 50%. This is due to the assumption of a reduction of the inelastic cross section at high densities. As an interesting finding, the multiplicities of $K^0$ and $K^+$ are influenced in such a way that their ratio is independent on the density dependence of the inelastic cross sections. This may be due to the long mean free path of the $K^{0,+}$, whereas the pionic ratio, due to their strong secondary interaction processes with the hadronic environment, have shown a moderate dependence on the density behavior of the inelastic cross sections.

Certainly more systematic studies are necessary to investigate better the mechanism which
leads to a moderate relation between the in-medium dependence of the inelastic cross sections and the pion ratio. A comparison with more experimental data would be helpful in determining more precisely the density dependence of the inelastic cross sections, which will be an object of future studies. On this level of investigations we conclude that the $(K^+/K^0)$-ratio represents a robust observable in determining the nuclear matter EoS at supra-normal densities.

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