Improving Students’ Mathematical Argumentation Skill Through Infusion Learning Strategy

Lia Budi Tristanti¹, Toto Nusantara²

¹Mathematics Education, STKIP PGRI Jombang, Indonesia
²Mathematics Education, Universitas Negeri Malang, Indonesia

*btlia@rocketmail.com, toto.nusantara.fmipa@um.ac.id

Abstract. This study aimed to develop infusion learning strategy to improve students’ mathematical argumentation skill. One problem of mathematical learning in college was that students had low competence to solve problems, particularly in argumentation. To cope with that issue, therefore, infusion learning strategy was developed. It aimed to assist students in improving their argumentation skills. The stages of infusion learning strategy were based on theory of argument by Walton. This study applied Research & Development (R&D) method with procedures of development by Borg & Gall which had been modified into 10 stages, but it was restricted to 6th stage. The instrument of this study used validation sheet, test of mathematical argumentation, and interview guidelines. The result referred to a valid product of infusion learning strategy which implementation could improve students’ mathematical argumentation. Another result of this study found that students’ argumentation skill progressed/improved.

1. Introduction
Argumentation and verification are two interrelated components in learning and understanding mathematics. One part of understanding mathematics is developing mathematical arguments and proofs before making evaluation [1]. Mathematical argumentation and verification aim to reveal the truth of conclusion making [2]. The general characteristic function of mathematical argumentation and verification referred to rational justification that aims to convince universal audience [3].

A valid argument is deductive which premises were based on definition, theorem, or verified actual facts [4] [5] [6][7]. In college level, however, students are found still having non-deductive arguments such as structural intuitive and inductive ones [8]. Such fact shows that they have no competence to make a scheme of valid arguments. Similarly, [9] [10] [11] [12] [13] argue that students’ mathematical arguments are incomplete yet, as not all students use deductive argument and they are confused distinguishing between valid and invalid evidence. Students with non-deductive arguments have low capability to develop evidence in formal way. Furthermore, they overlook evidence validation process. They feel confused to start and have no idea how to verify evidence in formal way [14].

The use of non-deductive argument makes students difficult to manipulate mathematical expressions which are equivalent to other forms. This condition leads to failure in making formal argument. As the result, they may fail to develop their cognitive skill. Following Piaget, the development of cognitive skill for students in 20-40 age range is on formal operational stage. Piaget describes those people thought as “hypothetico deductive” which means that they could develop...
hypotheses and design an experiment to verify them [15]. Therefore, students’ mathematical argumentation should be improved.

Responding to that issue, it needs to develop a strategy of Infusion Learning that aims to improve students’ mathematical argumentation skill. The stages of this strategy are based on Walton’s theory. As presented in Figure 1, [16] argues that reasoning activities happen in making arguments, and it may occur in either dialogue or non-dialogue.

Walton’s theory is applied in the stages of infusion learning strategy which aims to assist students to develop their mathematical argumentation skill. The learning stages of this strategy are as follow.

a. Reasoning
A mathematical problem that deals with argumentation is provided for students. They are asked to think actively to construct ideas and apply them to solve the problem.

b. Argument Not In Dialog
Students are asked to show and confirm their ideas using arguments addressed to themselves. They should convince themselves and thus, it leads to an approach and debate inside.

c. Argument In Small Dialogue
Students get engaged in a small group discussion consisting of three students. The group formation is based on heterogeneity of ideas in solving argumentation problem. Then, students are asked to have a critical discussion and each member of the group should show the right ideas with arguments addressed to another member. This small dialogue aims to make students speak clearly and convince other members.

d. Arguments in Class Dialogue
A student is asked to present his arguments in class and the other students give their response to it. This class dialogue aims to assist students to speak clearly and convince a number of students collectively (at least 10 students).

2. Methodology
This study aimed to develop Infusion Learning Strategy to improve students’ mathematical argumentation skill. It used Research & Development (R&D) method with development procedures by Borg & Gall modified into 10 stages including potency and problem, data collection, product design, design validation, design revision, product trial, product revision, trial run, product revision, and massive production [17]. In this case, however, it was restricted up to 6th stage. This simplification and restriction was due to limited time and personnel [18]. The subject of this study was the students of STKIP PGRI Jombang, particularly those in class 2019B.

The instrument of this study involved validation sheet, test of mathematical argumentation, and interview guidelines. Validation sheet was used for data collection of validity assessment on infusion learning strategy and teaching media by experts and practitioners. Based on the validation result, the mean scores of indicators and aspect for each experts and practitioners were all calculated. Furthermore, the total mean of validation score (Va) would be confirmed by category classification interval of infusion learning strategy based on [19], as presented on Table 1.
Table 1. Range and Criteria of Validation Result ($V_a$)

| Range of Mean Score on Validation Result ($V_a$) | Criteria      |
|------------------------------------------------|---------------|
| 1.0 $\leq V_a < 1.8$                           | Not valid     |
| 1.8 $\leq V_a < 2.6$                           | Less valid    |
| 2.6 $\leq V_a < 3.2$                           | Valid         |
| 3.2 $\leq V_a \leq 4.0$                        | Very valid    |

The criterion to decide that infusion learning strategy had adequate level of validity was score $V_a$ for all minimum aspects classified into valid category. Otherwise, it needed revision as validators’ suggestion or by reviewing the aspects with low scores. Subsequently, re-validation and re-analysis should be conducted. The stages run over and over again until $V_a$ score reached, at least, the minimum range of valid category.

The data of students’ mathematical argumentation skill was used to describe their skill on mathematical argumentation using either deductive or non-deductive arguments. This data was all collected through several instruments including mathematical argumentation test and interview guidelines. The elements of students’ mathematical argumentation skill were as follow.

a. The completeness of mathematical argumentation, that is: revealing facts/claim, revealing warrant and making conclusion

b. The quality of mathematical argument would be confirmed by having students use deductive argument correctly.

The data of students’ mathematical argumentation skill was analyzed quantitatively by giving scores on each of the elements. The scoring guidelines were as follow.

- Score 2, if the students revealed the elements correctly
- Score 1, if the students revealed the elements wrongly
- Score 0, if the students did not reveal any element

Furthermore, the conclusion of completeness level and quality of students’ mathematical argumentation skill (i.e., TPM) was made based on several criteria presented in Table 2 [20], as follow.

Table 2. Range and Criteria of TPM

| Range of mean score of validation result (TPM) | Criteria         |
|----------------------------------------------|------------------|
| $90 \leq TPM \leq 100$                      | Very good        |
| $70 \leq TPM < 90$                           | Good             |
| $40 \leq TPM < 70$                           | Less good        |
| $0\% \leq TPM < 40$                          | Not good         |

3. Result and Discussion

The study realized a potency of problem that: the first problem is some students of STKIP PGRI Jombang were found still using non-deductive arguments. This condition corresponded to some previous studies by [8] [21] [22] that many students in college level were found using non-deductive arguments such as intuitive and structural intuitive arguments. The second problem is Some students were found capable to construct mathematical arguments in complete way. This issue was in accordance to what [9] called as malfone. The third problem, in the process of teaching and learning, the lecturer gave a task that should be done immediately in group and asked the students to discuss to complete it. Hence, they were not ready yet to understand the task. as the result, they had no idea to complete the task.

The fourth problem is the students were not exclusively trained to express statements in the form of mathematical arguments. They were not trained to prepare any valid mathematical arguments to convince others in a dialogue. Their mathematical argumentation skill was built during their learning process with teacher in class. It was based on [23] [24] [25] [26] that teachers’ treatments might develop students’ mathematical argumentation skill, since they were able to encourage their students to describe, note, and justify their arguments during class discussion. Students’ arguments were dependent on their learning culture in class, features of tasks, and types of reasoning.
their teacher stressed on. The lecturers’ and students’ activities were also influenced by the applied teaching and learning strategy.

In the stage data collection, any information on literature reviews from several textbook references and supporting journals was all collected. Then, a product would be developed in the form of infusion learning strategy. The stages of this strategy were based on Walton’s theory [16], while the analysis of mathematical argumentation completeness was based on Toulmin’s theory [27].

After collecting information from field studies and literature reviews, the data was then used as references to plan the development of infusion learning strategy. It was also used as references to analyze the needs of infusion learning strategy to be developed. Moreover, designing infusion learning strategy was about to come. It was based on Walton’s theory [16]. It involved several stages such as reasoning, argument not in dialog, argument in small dialog and arguments in class dialogs.

The validity of infusion learning strategy considered two aspects including the content and the construct. This strategy was assessed by three validators (V-1, V-2 and V-3). Those three validators gave scores in a validation sheet in addition to some notes for revision. The mean scores of each validity assessment aspect of infusion learning strategy were presented in Table 3.

| No. | Assessment Aspects   | Validator 1 | Validator 2 | Validator 3 | Mean  |
|-----|----------------------|-------------|-------------|-------------|-------|
| 1   | Rationality          | 3.17        | 3.17        | 3.29        | 3.21  |
| 2   | Theoretical Base     | 3.00        | 3.29        | 3.17        | 3.15  |
| 3   | Model Component      | 3.29        | 3.17        | 3.29        | 3.25  |
| 4   | Instruction          | 3.00        | 3.17        | 3.29        | 3.15  |
|     | **Total mean**       | **3.12**    | **3.20**    | **3.26**    | **3.19** |

Table 3 showed the mean score of the total aspects was \( \mu = 3.19 \). Referring to the classification of validity level in Table 1, this strategy was classified into valid level.

The revised product which became a draft of qualified infusion learning strategy must be tested to see the improvement of students’ mathematical argumentation skill. This product trial was conducted in a small group. It was conducted in STKIP PGRI Jombang, particularly in class of space geometry. Infusion learning strategy was implemented three times in class 2019B. 30 students participated in the class. The result of product trial was as follow.

| Mathematical argumentation skill | Meeting 1 | Meeting 2 | Meeting 3 |
|---------------------------------|-----------|-----------|-----------|
| The completeness of Students’   | Revealing fact/claim | 12 | 17 | 25 |
| mathematical argumentation     | Revealing warrant | 11 | 15 | 23 |
|                                | Making conclusion | 11 | 14 | 21 |
| Using deductive argument correctly | 9 | 14 | 19 |
| Mean of TPM                     | 65 | 71 | 77 |

According to Table 4, it showed that implementing infusion learning strategy may improve students’ mathematical argumentation skill in each of the meetings. In meeting 3, 25 students (83%) were found capable to reveal facts/claim, 23 students (77%) were capable to reveal warrant, 21 students (70%) were capable to make conclusion, and the mean score of TPM= 77. Referring to the predetermined criteria of completeness level of students’ mathematical argumentation skill, the result was classified into good category. However, 19 students (63%) used deductive argument correctly. Referring to the predetermined criteria of students’ mathematical argumentation quality level, it was classified into less good category.

The result of this study found that implementing infusion learning strategy could improve students’ mathematical argumentation skill, and it also showed that their argumentation skill was improved. The correlation between argumentation and verification in mathematics was as a rational justification [3],
since the reasoning process during argumentation played an important role in verification construction [23] [24] [28].

Implementing infusion learning strategy was found capable to improve students’ argumentation and verification skills. Arguments and evidence in mathematics were developed when someone wanted to convince himself and others about the truth of a statement [29]. Therefore, this study could be used as reference to improve students’ mathematical argumentation skill through infusion learning strategy. However, further researches would still be necessary to see the effectiveness and impact of this strategy in improving students’ argumentation and verification skill in bigger sample.

Before implementing infusion learning strategy, a lecture should make sure his students’ initial competence in basic course and argumentation. They used their initial competence to produce mathematical arguments. Otherwise, students might not know how to start constructing evidence if they had no initial competence in argumentation. [30] argued that the common fault of making argument was that students had no idea how to start writing evidence.

4. Conclusion
This study showed that the development of infusion learning strategy had met the criteria of valid category, and its implementation could improve students’ mathematical argumentation skill. The stages of infusion learning strategy involved reasoning, argumentation not in dialogue, argumentation in small dialogue, and argumentation in class dialogue. To drill, develop, and improve students’ mathematical argumentation skill, the researchers suggested to implement this strategy in mathematics learning. This study brought some implications to researches and practices such as (1) the result of this study could be used as inspiration for education observer particularly to issues dealing with mathematical argumentation for further researches, and (2) infusion learning strategy could be used as an alternative way to explore students’ mathematical argumentation skill in learning math which might eventually bring positive impacts for their skills in solving argumentation problems.

Acknowledgement
This article was a work (postgraduate research program) supported by DRPM KEMENRISTEKDIKTI 2020. The researchers would like to show gratitude to DRPM for funding this research so that this article could be published in internationally reputable journal or proceeding. The researchers also wanted to show gratitude to the Head of State University of Malang and STKIP PGRI Jombang who had allowed this postgraduate research to be conducted.

References
[1] . National Council of Teachers of Mathematics (NCTM). "Principles and Standards of School Mathematics". Reston, VA: Author, 2000.
[2] . Aberdein, A. "The Uses of Argument in Mathematics". Argumentation, 19(3), 287-301, 2005.
[3] . Pedemonte, B. "How can the relationship between argumentation and proof be analysed?". Educational studies in mathematics, 66(1), 23-41, 2007.
[4] . Rodd, M. M. "On Mathematical Warrants: Proof Does Not Always Warrant, and a Warrant May Be Other Than a Proof". Mathematical Thinking and Learning, 2(3), 221-244, 2000.
[5] . Harel, G. "The development of mathematical induction as a proof scheme: A model for DNR-based instruction." Learning and teaching number theory: Research in cognition and instruction, 2, 185-212, 2001.
[6] . Tall, D. "Introducing Three Worlds of Mathematics". For the Learning of Mathematics, 23 (3). 29–33, 2004.
[7] . Lodder, A. R. "Law, Logic, Rhetoric: a Procedural Model of Legal Argumentation". Logic, Epistemology, and the Unity of Science. pp. 569-588. Springer Netherlands, 2009.
[8]. Inglis, M., Mejia-Ramos, J. P., & Simpson, A. "Modelling Mathematical Argumentation: The Importance of Qualification". Educational Studies in Mathematics, 66(1), 3-21, 2007.
[9]. Fuat, F., Nusantara, T., Hidayanto, E and Irawat, S. "The Exploration Of Argument Scheme Expression In Students' Proof Construction". International Journal Of Scientific & Technology Research, 9 (01), 2369-2372, 2020.
[10]. Selden, A., & Selden, J. "Validations of proofs considered as texts: Can undergraduates tell whether an argument proves a theorem?". Journal for research in mathematics education, 34(1), 4-36, 2003.
[11]. Weber, K. "Mathematics majors' perceptions of conviction, validity, and proof". Mathematical thinking and learning, 12(4), 306-336, 2010.
[12]. Inglis, M., & Alcock, L. "Expert and novice approaches to reading mathematical proofs". Journal for Research in Mathematics Education, 43(4), 358-390, 2012.
[13]. Hodds, M., Alcock, L., & Inglis, M. "Self-explanation training improves proof comprehension". Journal for Research in Mathematics Education, 45(1), 62-101, 2014.
[14]. Alcock, L., & Weber, K. "Referential and syntactic approaches to proving: Case studies from a transition-to-proof course". Research in collegiate mathematics education VII, 93-114, 2010.
[15]. Slavin, R. E. "Educational psychology: Theory and practice". Boston: Allyn & Bacon, 2006.
[16]. Walton, D. "Fundamentals of Critical Argumentation". Cambridge University Press, 2005.
[17]. Borg, W. R., & Gall, M. D. "Instructor's Manual for Educational Research: To Accompany Educational Research: an Introduction". Longman, 1983.
[18]. Sugiyono. "Educational Research Methods Quantitative, Qualitative, and R&D Approaches". Bandung: Alfabeta, 2011.
[19]. Arikunto, S. "Fundamentals of Educational Evaluation. Revised Edition". Jakarta: Bumi Aksara. 2009.
[20]. Arifin, Z. "Learning Evaluation: Principles, Techniques, and Procedures". Bandung: PT Remaja Rosdakarya Offset, 2011.
[21]. Tristanti, L.B., Sutawidjaja, A., As'ari, A.R., & Muksar, M. "The Construction of Deductive Warrant Derived from Inductive Warrant in Preservice-Teacher MathematicalArgumentations". Educational Research and Reviews, 11(17),1696-1708, 2016.
[22]. Tristanti, L. B., Sutawidjaja, A., As'ari, A. R., & Muksar, M. "Types of Warrant in Mathematical Argumentations of Prospective-Teacher". International Journal of Science and Engineering Investigations, 6(68), 96-101, 2017.
[23]. Boero, P., Douek, N., Morselli, F., & Pedemonte, B. "Argumentation and proof: A contribution to theoretical perspectives and their classroom implementation". Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education (Vol. 1, pp. 179-204). Belo Horizonte, Brazil: PME, 2010.
[24]. Boero, P. "Argumentation and mathematical proof: A complex, productive, unavoidable relationship in mathematics and mathematics education". International newsletter on the teaching and learning of mathematical proof, 7(8), 1999.
[25]. Whitenack, J & Yackel, E. "Making Mathematical Arguments in the Primary Grades: The Importance of Explaining and Justifying Ideas". Teaching Children Mathematics. 8 (9), 524-527, 2002.
[26]. Conner, A.M. "Student Teachers’ Conceptions of Proof and Facilitation of Argumentation in Secondary Mathematics Classrooms". The Pennsylvania State University: Disertasion. 2007.
[27]. Toulmin, S. "The Uses of Argument". UK: Cambridge University Press. 2003.
[28]. Durand-Guerrier, V., Boero, P., Douek, N., Epp, S. S., & Tanguay, D. "Argumentation and proof in the mathematics classroom". Proof and proving in mathematics education (pp. 349-
367). Springer, Dordrecht. 2011.

[29] Hanna, G. "Proofs that prove and proofs that explain". *Proceedings of the 13th Conference of International Group for the Psychology of Mathematics Education*, Vol. 2, pp. 45-51, 1989

[30] Stavrou, S. G. "Common Errors and Misconceptions in Mathematical Proving by Education Undergraduates". *Issues in the Undergraduate Mathematics Preparation of School Teachers, 1*, 2014.