Weak versus deterministic macroscopic realism

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We construct a mapping of Bell and bipartite Leggett-Garg experiments for microscopic qubits onto a gedanken experiment for macroscopic qubits based on two macroscopically distinct coherent states. This provides an unusual situation where the dichotomic measurements (and associated hidden variables) involved in the Bell tests need only discriminate between two macroscopically distinct states of a system i.e. correspond to coarse-grained measurements that do not specify values to a level of precision of order $\sim \hbar$. Violations of macro-realism and macroscopic local realism are predicted. We show how one may obtain consistency with a weak form of macroscopic realism (wMR): that for a system prepared in a superposition of macroscopically distinct pointer eigenstates, the outcome of the coarse-grained pointer measurement $\lambda_M$ is predetermined. Macroscopic realism does not however hold in a deterministic fashion, where one assumes the predetermination of outcomes prior to the unitary rotations that define the choice of measurement setting in the Bell experiment. We illustrate an analogy with the Einstein-Podolsky-Rosen (EPR) argument, showing how wMR can be regarded as inconsistent with the completeness of quantum mechanics.

Understanding how to interpret the quantum superposition of two macroscopically distinguishable states has been a topic of interest for decades [1][2]. Schrodinger considered such a superposition, $|\psi\rangle = \frac{1}{\sqrt{2}}(|a\rangle + |d\rangle)$ where $|a\rangle$ and $|d\rangle$ are macroscopically distinct quantum states distinguished by a measurement $\hat{M}$ [1]. Schrodinger explained how the standard interpretation given to a quantum superposition introduces a paradox when applied to a macroscopic system. The system is interpreted as being in neither state $|a\rangle$ or $|d\rangle$ prior to measurement, suggesting it is somehow simultaneously in both states, which for a cat alive or dead is seen to be ridiculous.

Many studies have revealed how classical behaviour emerges from quantum mechanics through coarse-grained measurements [5][7]. In fact, recent work suggests the possibility of interpreting quantum mechanics using ontological models with an epistemic restriction of order $\hbar$ [8][12] — here, epistemic implies a fundamental lack of knowledge imposed on the observer. If applied to a system in a macroscopic superposition, this is appealing, since one may consider models involving a macroscopic hidden variable $\lambda_M$, which predetermines the outcome ($a$ or $d$) of the measurement $\hat{M}$, up to a value of order $\hbar$, sufficient to distinguish the two states. The system may then be interpreted as satisfying realism at a macroscopic level. This leads to the question of whether the assumption of $\lambda_M$ is valid and how it might be reconciled with the large number of reported violations of Leggett and Garg’s macrorealism (M-R) [13][17]— including a recent experiment conducted in a macroscopic superconducting regime [16].

In this paper, we address this question, by proposing tests of macroscopic realism based on superpositions of two macroscopically distinct coherent states (cat states), in which subtly different definitions of macroscopic realism can be distinguished. To establish the difference, we account for the unitary dynamics associated with the choice of a measurement setting (as in a polariser angle). We conclude that the $\lambda_M$ can be falsified, if attributed to the system prior to the determination of the measurement setting. By contrast, there is no inconsistency if the variables $\lambda_M$ apply to states prepared in a pointer superposition, after the unitary rotation determining the measurement basis.

In order to draw conclusions, we first define macroscopic realism in the most minimal sense. Weak macroscopic realism (wMR) is the assumption that the system prepared in a superposition of two macroscopically distinct quantum states $|a\rangle$ and $|d\rangle$ has a predetermined value for $\lambda_M$. The predetermination need not be specified to the level of $\hbar$, implying that the macroscopic hidden variable $\lambda_M$ can be introduced. The definition of wMR does not imply the system to be in one or other of the specific quantum states, $|a\rangle$ and $|d\rangle$, nor, indeed, in any quantum state, prior to measurement $\lambda_M$. We will elucidate further, by showing that the assumption of wMR leads to a paradox similar to that of Einstein, Podolsky and Rosen (EPR) [18]. If one assumes wMR, then the state with the predetermined value cannot be a quantum state. The cat paradox is then seen to be a demonstration of the inconsistency between wMR and the premise that quantum mechanics is a complete theory.

This motivates the principle question of whether wMR can be negated directly. Quantum mechanics predicts violation of Leggett-Garg inequalities, which falsifies macrorealism (M-R) [13]. M-R however postulates only macroscopic realism, but also macroscopic noninvasive measurability (NIM) — that one may determine the value of $\lambda_M$ without a subsequent macroscopic disturbance to the future dynamics of the variable $\lambda_M$. The latter assumption requires verification [17], but arguments justifying NIM often assume the system to be in one or other of two preparable, or quantum, states — which
is a stricter assumption than wMR [16]. However, recent work [19, 21] predicts violation of M-R for bipartite Leggett-Garg-Bell inequalities using spacelike-separated measurements, thus justifying NIM via the assumption of macroscopic locality (ML).

In this paper, we propose Leggett-Garg-Bell tests for which the combined assumption of wMR and ML is predicted to be violated. We find the violations may be viewed consistently with wMR, if defined carefully, with respect to the final pointer basis. Specifically, weak macroscopic realism (wMR) applies where the system is prepared in a superposition of the eigenstates of $\hat{M}$ at a given time $t$, and asserts the validity of the macroscopic hidden variable $\lambda_M$ in that case. Deterministic macroscopic realism (dMR) on the other hand applies where one assumes the predetermination $\lambda_M$ of the outcomes of the coarse-grained measurement $\hat{M}$ prior to the unitary rotation that determines the measurement setting. We see this is a stronger assumption than wMR that naturally incorporates ML, but does not assume the system to be in one or other of two quantum states. We show that quantum mechanics predicts the falsification of dMR.

We begin by considering the “cat state” [3, 22]

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left( |\alpha\rangle + i |\alpha\rangle \right)$$  \hspace{1cm} (1)

of a single-mode field or oscillator $A$. Here $|\pm \alpha\rangle$ are coherent states with $\alpha$ large and real. Quadrature phase amplitude measurements $\hat{X}_A = \frac{1}{\sqrt{2}} (\hat{a} + \hat{a}^\dagger)$ and $\hat{P}_A = \frac{1}{\sqrt{2}} (\hat{a} - \hat{a}^\dagger)$ are defined in a rotating frame, where $\hat{a}$, $\hat{a}^\dagger$ are the boson creation and destruction operators, in appropriate units where $h = 1$ [22]. The states $|\pm \alpha\rangle$ are distinguished by the measurement $\hat{M}$ (the “spin” of the system), taken as the sign $S_\alpha$ of the outcome of $\hat{X}_A$. $S_\alpha$ has value $+1$ if the outcome $X_A$ is positive and $-1$ otherwise. Weak macroscopic realism (wMR) specifies that the system (1) be in a state with a definite value for $\hat{M}$. A hidden variable $\lambda_\alpha^{(A)}$ is attributed to the system $A$ prior to the measurement $\hat{S}_\alpha^{(A)}$. The $\lambda_\alpha^{(A)}$ takes values $+1$ or $-1$ that predetermine the result for $\hat{M} = \hat{S}_\alpha^{(A)}$.

Weak macroscopic realism postulates the system to have a definite outcome for $\hat{S}_\alpha^{(A)}$, which implies it must be in a state sufficiently localised in $X_A$. If the state is to be a quantum state, we arrive at a constraint on the outcomes $P_A$ of measurement $\hat{P}_A$, for a given specified variance in $X_A$ associated with each spin. The distribution for $X_A$ gives two Gaussian hills each with variance $1/2$ (Figure 1). Supposing the system to be in a classical mixture of two states, one for each Gaussian, in accordance with wMR, then for each we specify $\Delta^2 X_A = 1/2$. If the two states are quantum states, then the uncertainty relation $\Delta X_A \Delta P_A \geq 1/2$ for each implies that the overall variance in $P_A$ satisfies $\Delta^2 P_A \geq 1/2$ [23]. Hence, the observation of $\Delta^2 P_A < 1/2$ leads to an EPR-type paradox, where, since the state consistent with wMR cannot also be consistent with the uncertainty principle, one argues either failure of wMR or else an incompleteness of quantum mechanics. For the cat state (1),

$$\Delta^2 P_A = \frac{1}{2} - 2\alpha^2 e^{-4\alpha^2}$$  \hspace{1cm} (2)

The paradox is obtained for all $\alpha$, albeit by a vanishingly small amount for larger $\alpha$ [24, 27].

We now examine how to test wMR directly. For our purposes, M-R postulates both wMR and NIM. We also consider the premise of macroscopic locality (ML). ML asserts that for spacelike separated events at two sites $A$ and $B$, the events at one site $B$ cannot change the value of the hidden variable $\lambda_\alpha^{(A)}$ at the site $A$, and vice versa.

We first show violations of M-R for the cat state (1) [28]. At a time $t_1 = 0$, we suppose system $A$ is prepared in $|\alpha\rangle$. The system then evolves according to the Hamiltonian $H_\alpha^{(A)} = \Omega \hat{A}^\dagger$ where $\Omega$ is a constant and $\hat{A} = \hat{a}^\dagger \hat{a}$. After a time $t_2 = \pi/4\Omega$, the system is in the state $\hat{U}^{(A)} |\alpha\rangle = e^{-i\pi/8} (\cos \pi/8 |\alpha\rangle + i \sin \pi/8 |\alpha\rangle)$ (3)

where $\hat{U}^{(A)} = U_A(t_2) = e^{-iH_\alpha^{(A)} t_2}/h$. After further evolution, at the time $t_3 = \pi/2\Omega$, the system is in the state $\hat{U}^{(A)} |\alpha\rangle = e^{-i\pi/4} \sqrt{2} (|\alpha\rangle + i |\alpha\rangle)$ (4)

where $\hat{U}^{(A)} = U_A(t_3)$. At each time $t_i$, we define the outcome $S^{(A)}_i$ for the spin $\hat{S}^{(A)}_i$ of system $A$. Assuming the system satisfies wMR (taking $\alpha \to \infty$), the value for the spin $S^{(A)}_i$ at the time $t_i$ is determined by a macroscopic hidden variable $\lambda^{(A)}_M$, which we denote by $\lambda^{(A)}_i$. Hence $\lambda^{(A)}_i$ has values $+1$ or $-1$. The two-time correlations are given as $\langle S^{(A)}_{i1} S^{(A)}_{i2} \rangle = \langle \lambda^{(A)}_i \lambda^{(A)}_j \rangle$ and the inequality $\langle \lambda^{(A)}_1 \lambda^{(A)}_2 \rangle - \langle \lambda^{(A)}_1 \lambda^{(A)}_3 \rangle + \langle \lambda^{(A)}_2 \lambda^{(A)}_3 \rangle \leq 1$ is always satisfied [13, 15]. The assumption of NIM implies the inequality could be measured, since the ideal measurement of $S^{(A)}_i$ at times $t_i$ allows determination of the value of $\lambda^{(A)}_i$ without subsequent disturbance to the system. Macrorealism

Figure 1. The probability distributions $P(X_A)$ and $P(P_A)$ for the cat state (1) with $\alpha = 2$. 

\[ \begin{align*}
\text{Figure 1. The probability distributions} & P(X_A) \text{ and } P(P_A) \text{ for the cat state (1) with } \alpha = 2.
\end{align*} \]
therefore implies the Leggett-Garg inequality [15]
\[
(S_1^{(A)} S_2^{(A)}) + (S_2^{(A)} S_3^{(A)}) - (S_1^{(A)} S_3^{(A)}) \leq 1
\] (5)

Since the outcome for \(S_1^{(A)}\) is known to be 1, the experimental prediction is \(S_1^{(A)} S_2^{(A)} = \cos(\pi/4)\) and \(S_1^{(A)} S_3^{(A)} = 0\). Establishing \((S_2 S_3)\) is not so clear, because it could be claimed that any real measurement at time \(t_2\) will affect the future dynamics. However, assuming the system is actually in one of the states \(|\alpha\rangle\) or \(|-\alpha\rangle\) at the time \(t_2\), the system will be in the state \(U_{\pi/4}^{(A)} |\alpha\rangle\) or \(U_{\pi/4}^{(A)} |-\alpha\rangle\) respectively, at the later time \(t_3\) [13]. This implies \((S_2^{(A)} S_3^{(A)}) = \cos(\pi/4)\), and the inequality (5) is violated. Of course, one sees from the paradox [2] that the system cannot actually be in either state \(|\alpha\rangle\) or \(|-\alpha\rangle\) at time \(t_2\), prior to measurement. Nonetheless, the macro-realist argues that as \(\alpha\) becomes large, the system is sufficiently close to \(|\alpha\rangle\) (or \(|-\alpha\rangle\)) that there should be negligible disturbance due to measurement.

The failure of macro-realism can be illustrated more convincingly if one is able to perform the measurement at the time \(t_2\) without any direct disturbance. Consider two spacelike separated systems \(A\) and \(B\) prepared at time \(t_1 = 0\) in the entangled state [4]
\[
|\psi_{\text{Bell}}\rangle_1 = N \left(|\alpha\rangle - \beta \right) - (\alpha\langle \beta|) \right)
\] (6)
where \(|\beta\rangle\) is a coherent state for system \(B\), \(N = \frac{1}{\sqrt{2}} \{1 - \exp(-2|\alpha|^2 - 2|\beta|^2)\}^{-1/2}\) and \(\alpha = \beta\). As for \(A\), we define 
\[
\hat{X}_A = \frac{i}{\sqrt{2}} (\hat{b} + \hat{b}^\dagger) \quad \text{and} \quad \hat{P}_A = \frac{i}{\sqrt{2}} (\hat{b} - \hat{b}^\dagger)
\]
for system \(B\). The systems evolve at each site independently according to the local Hamiltonians \(H_{\text{NL}} = \Omega \hat{b}^\dagger \hat{b}^\dagger\) and \(H_{\text{NL}} = \Omega \hat{b}^\dagger \hat{b}\) where \(\hat{n}_b = \hat{b}^\dagger \hat{b}\). After a time \(t_2\), the system is in the Bell state \(|\psi_{\text{Bell}}\rangle_2 = U_{\pi/4}^{(A)} U_{\pi/4}^{(B)} |\psi_{\text{Bell}}\rangle\), given by
\[
|\psi_{\text{Bell}}\rangle_2 = Ne^{-i\pi/4} \left(|\alpha\rangle - \beta \right) - (\alpha\langle \beta|) \right)
\] (7)
If evolved further, the system at time \(t_3\) is in the similar Bell state \(|\psi_{\text{Bell}}\rangle_3 = U_{\pi/8}^{(A)} U_{\pi/8}^{(B)} |\psi_{\text{Bell}}\rangle\). A measurement \(S_i^{(B)}\) can be performed at \(B\) at time \(t_i\), as for \(S_i^{(A)}\) (\(i = 1, 2, 3\)). For large \(\alpha\) and \(\beta\), the assumption wMR assigns to each system the hidden variables \(\lambda_i^{(A)}\) and \(\lambda_i^{(B)}\) that take the value +1 or -1 to predetermine the results for \(S_i^{(A)}\) and \(S_i^{(B)}\). The spin \(S_i^{(A)}\) can be measured, by inferring the value from a direct measurement of \(S_i^{(B)}\) at \(B\). This is because the antecorrelation between the spins at each \(t_i\) implies \(S_i^{(A)} = -S_i^{(B)}\). It is argued that this measurement is noninvasive to the system \(A\), based on the assumption of macroscopic locality (ML). A spacelike separation is assumed for all events occurring over the entire time interval from \(t_1\) to \(t_3\), implying no macroscopic changes occur to the outcomes at \(A\) at any time \(t_i\) due to measurement at \(B\). Assuming M-R, the inequality
\[
- \langle S_1^{(B)} S_2^{(A)} \rangle - \langle S_2^{(B)} S_3^{(A)} \rangle + \langle S_1^{(B)} S_3^{(A)} \rangle \leq 1
\] (8)
The predictions for \(\langle S_i^{(B)} S_j^{(A)} \rangle\) are identical to \(-\langle S_i^{(A)} S_j^{(A)} \rangle\), and hence a violation of (8) is predicted, as for (5). The predictions are based on the proposed measurements of \(X_A\) and \(X_B\) and are calculated by evaluating \(P(X_A, X_B)\) as shown in Figure 2. The violations are valid for arbitrarily large \(\alpha\), \(\beta\) and falsify the combined assumptions of wMR and ML. Hence, one cannot conclude violation of wMR directly.

However, one may falsify deterministic macroscopic realism (dMR) by considering the Leggett-Garg-Bell inequality
\[
- \langle S_1^{(B)} S_2^{(A)} \rangle + \langle S_2^{(B)} S_3^{(A)} \rangle + \langle S_1^{(B)} S_3^{(A)} \rangle - \langle S_1^{(B)} S_4^{(A)} \rangle \leq 2
\] (9)
Here, one considers four times, \(t_1 = 0, t_2 = \pi/4, t_3 = \pi/2\Omega\) and \(t_4 = 3\pi/4\). After a time \(t_4 = 3\pi/4\Omega\), the state \(|\alpha\rangle\) evolves to [28]
\[
U_{3\pi/4\Omega}^{(A)} |\alpha\rangle = e^{i\pi/8} \left(\cos 3\pi/8 |\alpha\rangle + i \sin 3\pi/8 |\alpha\rangle\right)
\] (10)
The system prepared in the Bell state [6] thus evolves after a time \(t_4\) to the Bell state \(|\psi_{\text{Bell}}\rangle_4 = U_{3\pi/4\Omega}^{(B)} U_{3\pi/4\Omega}^{(A)} |\psi_{\text{Bell}}\rangle_1\). This leads to predictions \(\langle S_3^{(B)} S_4^{(A)} \rangle = -\cos \pi/4\) and \(\langle S_1^{(B)} S_4^{(A)} \rangle = -\cos 3\pi/4\), and a violation of (9). Eq. (9) can be viewed as the original Leggett-Garg inequality \(\langle S_1^{(A)} S_2^{(A)} \rangle + \langle S_2^{(A)} S_3^{(A)} \rangle + \langle S_3^{(A)} S_4^{(A)} \rangle - \langle S_1^{(A)} S_4^{(A)} \rangle \leq 2\) derived in [13]. Similar to (8), to obtain (9) we justify the NIM premise using ML, and put \(S_4^{(B)} = -S_4^{(A)}\) for
times $t_1$ and $t_3$, based on the anti-correlation of the spins for the Bell states.

Alternatively, Eq. (9) is seen to be a macroscopic Bell inequality, where one measures the correlation $E(\theta_1, \theta_2) = \langle S_i^{(A)} S_j^{(B)} \rangle$. In the Bell case, the choice of two times of evolution for each system $A$ and $B$ ($t_2$ and $t_4$ for $A$, and $t_1$ and $t_3$ for $B$) is a choice between two measurement settings at each site. The Bell inequality (9) can be derived assuming deterministic macroscopic realism (dMR): each system $A$ and $B$ is simultaneously predetermined to be in one or other of two macroscopically distinct states, prior to the choice of measurement setting, so that two hidden variables $\lambda_1$ and $\lambda_2$ are ascribed to the system at the time $t_1$ [30, 31]. This assumption naturally incorporates macroscopic locality (ML), since it specifies that the result for the final measurement cannot be changed over the course of the unitary dynamics associated with the adjustment of measurement setting, at either site. The violation of (9) falsifies dMR.

We now ask how one can reconcile the violations of macrorealism (M-R) with the validity of weak macroscopic realism (wMR). For the Leggett-Garg-Bell test given by (8), it is clear that the systems are indeed prepared in the pointer superposition $|\psi_{Bell}\rangle$ at time $t_1$.

If wMR holds, then the value of the spins $S_i^{(A)}$ are to be given as $\lambda_i$ at the time $t_1$, in which case the violations must arise because the NIM / ML premise breaks down.

This is possible, because the unitary dynamics $U = e^{-iH_{mod}/\hbar}$ (which in the Bell test gives the choice of measurement setting) has a finite time duration. The dynamics transforms the Bell state $|\psi_{Bell}\rangle$ at time $t_1$ into a different Bell state $|\psi_{Bell}\rangle_2$ at time $t_2$, and then into a different state $|\psi_{Bell}\rangle_3$ at time $t_3$. The system given by the state $|\psi_{Bell}\rangle_1$ is not viewed to be simultaneously in all three pointer superpositions. One is therefore not required to assume deterministic macroscopic realism (dMR), which we have shown fails, by violation of (9).

In a model where wMR is valid, the simplest interpretation is that the violations for the macroscopically distinct outcomes emerge from the entangled initial state $|\psi_{Bell}\rangle_1$ over the timescales associated with the unitary rotations. This is consistent with results showing violation of Bell inequalities for trajectories at a microscopic level [32]. To explain, we examine Figure 2 for the Leggett-Garg-Bell test and ask what happens to the joint probabilities $P(X_A, X_B)$ depending on whether a measurement of $S_i^{(A)}$ takes place at time $t_1$ or time $t_2$. If the measurement of $S_i^{(A)}$ is made, then unitary evolution stops at time $t_1$ at site $B$ (to allow a record of the value $S_i^{(B)}$) and hence $-S_i^{(A)}$ and we see the results in the top sequence. If the measurement of $S_i^{(A)}$ takes place (lower sequence), then the unitary evolution continues at $B$ to the time $t_2$, and then ceases (so that the value $S_i^{(B)} = -S_i^{(A)}$ can be stored), while the system $A$ continues to evolve, after this time. Comparing the two sets of dynamics, we see that the choice to measure $S_i^{(A)}$ or $S_i^{(A)}$ makes a macroscopic difference, after further unitary dynamics, by significantly changing the joint probabilities $P(X_A, X_B)$ for macroscopically distinct outcomes at the later time $t_3$ (compare the far right plots).

In fact, unitary rotation at both sites is required to observe the macroscopic violations. We see this by comparing with the evolution of $\rho_{mix}$, the non-entangled 50/50 mixture of product states $|\pm \alpha\rangle \pm \beta\rangle$, which does not violate the inequalities. Any difference in $P(X_A, X_B)$ at the time $t_1 = 0$ is infinitesimal, being of order $\hbar e^{-|\alpha|^2}$. Similarly, we find no observable difference for the top sequence of Figure 2 involving a rotation at one site only [24]. However, the plots for the lower sequence become macroscopically different at the final time $t_3$ (Figure 3).

Finally, we address how the results are affected by the irreversible “collapse” stage of measurement, when the system $B$ is coupled to a detector. The unitary evolution, which precedes the collapse stage of the measurement, prepares the pointer superposition. In a model where wMR is valid, the macroscopic hidden variable $\lambda_i^{(A)}$ is fixed in value (+1 or −1). One then expects this gives a record of $\lambda_i^{(A)}$ at the time $t_1$ — which cannot be changed by the future collapse at $B$. Consistent with that argument, calculation shows it is possible to delay the final collapse stage of the measurement $S_i^{(B)}$ or $S_i^{(B)}$ at $B$ by any amount of time after $t_3$, and it makes no detectable difference to the probabilities $P(X_A, X_B)$, any corrections being of order $e^{-|\alpha|^2}$ [23].

It might be argued that one can perform delayed choice experiments which would counter the validity of wMR. The probabilities $P(X_A, X_B)$ depend on the local times $t_a$ and $t_b$ of the unitary interactions $U_A(t_a)$ and $U_B(t_b)$. Hence, one can delay the choice to measure either $S_i^{(A)}$ or $S_i^{(A)}$, until after $t_3$. Since this choice makes a macroscopic impact on the final probabilities at $t_3$, it might appear as though a retrocausal effect can induce a change to $\lambda_3$, thus making it macroscopically indeterminate. However, as with the delayed choice experiments for spin 1/2 qubits [33], a more complete analysis (based on the fact that there is unitary evolution at both sites after time $t_2$) negates this interpretation [24, 34].

In conclusion, we have shown how to falsify deterministic macroscopic (local) realism for superpositions of co-
herent states undergoing unitary evolutions. Consistency with a weak form of macroscopic realism can be retained however, if applied to the state produced after the choice of measurement setting.

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