Locating the pseudogap closing point in cuprate superconductors: absence of entrant or reentrant behavior

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Current descriptions of the pseudogap in underdoped cuprates envision a doping-dependent transition line \(T^*(p)\) which descends monotonically towards zero just beyond optimal doping. There is much debate as to the location of the terminal point \(p^*\) where \(T^*(p)\) vanishes, whether or not there is a phase transition at \(T^*\) and exactly how \(T^*(p)\) behaves below \(T_c\) within the superconducting dome. One perspective sees \(T^*(p)\) cutting the dome and continuing to descend monotonically to zero at \(p_{\text{crit}} \approx 0.19\) holes/Cu — referred to here as ‘entrant behavior’. Another perspective derived from photoemission studies is that \(T^*(p)\) intersects the dome near \(p_{\text{crit}} \approx 0.23\) holes/Cu then turns back below \(T_c\), falling to zero again around \(p_{\text{crit}} \approx 0.19\) — referred to here as ‘reentrant behavior’. By examining thermodynamic data for \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}\) we show that neither entrant nor reentrant behavior is experimentally supported. Rather, \(p_{\text{crit}} \approx 0.19\) sharply delimits the pseudogap regime and for \(p < 0.19\) the pseudogap is always present, independent of temperature. Similar results are found for \(\text{Y}_n\text{Ba}_{2n+1}\text{Cu}_{3n+2}\) \(\Omega_{-\delta}\). For both materials \(T^*(p)\) is not a temperature but a crossover scale, \(\approx E^*(p)/2k_B\), reflecting instead the underlying pseudogap energy \(E^*(p)\) which vanishes as \(p \rightarrow 0.19\).

1. Introduction

Hole-doped cuprate superconductors, at and below optimal doping, are characterised by the opening of a partial gap in the electronic density of states, the so-called pseudogap \(^3\) \(^4\), which profoundly affects all spectroscopic properties and, below the transition temperature \(T_c\), results in an abrupt crossover from ‘strong’ to ‘weak’ superconductivity \(^5\) \(^6\). Recently, it has become evident that the underlying behavior involves a change of the Fermi surface (FS) from a large FS with area \(1 + p\) to either Fermi arcs \(^11\) or small hole pockets on the zone diagonal near the antiferromagnetic zone boundary having area \(p \approx 0.1\). Evidence for this change can be found in angle-resolved photoelectron spectroscopy (ARPES) \(^12\) \(^13\) quasiparticle interference in scanning tunneling spectroscopy \(^8\) \(^9\) and the probable crossover observed in the normal state at very high magnetic field from Hall number \(n_H = 1 + p\) to \(n_H = p \approx 0.1\).

It has long been known that the apparent characteristic pseudogap temperature, \(T^*\), below which pseudogap effects are often reported, falls with increasing hole concentration, \(p\), and vanishes at a critical doping, \(p^* = 0.19\) holes/Cu \(^14\) \(^15\). Despite an intensive search no specific heat anomaly has been reported at \(T^* \approx 0.19\), thus implying that \(T^*(p)\) is not a thermodynamic phase transition line. Indeed this line was originally reported as an energy scale, \(E^*(p)\), which descends to zero at \(p^*\), not a temperature scale \(^17\) \(^18\). Later reports however suggest that there is indeed some kind of mean-field transition occurring in the vicinity of \(T^*\) with various order-parameter-like properties observed to vanish there. The measurements include polarised neutron scattering \(^19\), polar Kerr effect \(^20\), ARPES \(^21\), time-resolved reflectivity \(^22\), resonant ultrasound spectroscopy \(^23\) and susceptibility nematicity \(^24\). The last two techniques were presented as clear evidence of a thermodynamic transition \(^23\) \(^24\), although this interpretation has been questioned in the former case \(^16\) and any thermodynamic effects are very weak. Of special relevance to the present work, several prominent studies on \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}\) (Bi-2212) identified the termination point of the pseudogap as located at a much higher doping of \(p^* \approx 0.23\), at or near a proposed Lifshitz transition from a hole-like to electron-like Fermi surface \(^22\). These include ARPES \(^26\), Raman \(^27\) and transport \(^28\) studies.

1.1 Preliminary remarks

In light of these contradictory ideas it is important to emphasize several points:

(i) the fact that a correlation may set in within the pseudogap ‘domain’ does not mean that this correlation is the underlying cause of the pseudogap. The pseudogap may simply create conditions conducive to this correlation. Thus, for example, a pseudogap-induced change of the Fermi surface can allow nesting \(q\)-vectors that induce charge ordering \(^21\).

(ii) in all the above-noted cases the electronic entropy at \(T^*\) is already strongly reduced by the pseudogap, reflecting a large fraction of lost states near the Fermi level occurring from well above \(T^*\). These lost states (or entropy reduction) are never recovered to the highest temperatures investigated (\(\approx 300\) K). The entropy reduction is very large so these are not weak precursor fluctuations, and notably no anomaly is seen in the specific heat at \(T^*\).
This is also mirrored in the spin susceptibility observed by NMR Knight shifts. Like the electronic entropy it has a marked temperature dependence and is strongly reduced by the pseudogap even well above $T^*$ [18, 30, 31] and extending to 400 K and more. Likewise, below $p \approx 0.19$ the static susceptibility is suppressed and remains so to at least 400 K [32]. These last authors plotted $T \times \Delta \chi(p, T) = T \times [\chi(p, T) - \chi(p_{ref}, T)]$ versus $T$ (where $p_{ref} \geq 0.19$) to obtain a series of flat $T$-independent lines displaced progressively downwards as soon as $p$ falls below 0.19 holes/Cu. This downward displacement is a direct measure of the gap magnitude $E^*$. In no case is the ‘lost susceptibility’ recovered to a temperature of at least 350 K. These results collectively imply the preexisting presence of the pseudogap far above $T^*$ and that the above-noted mean-field-like correlations set in within the pseudogap state. Whether the lost states referred to above arise from gapping or scattering-incoherence (or both) remains to be resolved [4, 33, 34]. Note also that this suppression up to 400 K (where the width of the Fermi function, $4k_B T \approx 130$ meV) tends to rule out a states-conserving gap such as spin, charge or pair density waves [5].

(iii) the claim by various groups [20, 28] that for Bi2212 $p^* \approx 0.23$ in the strongly overdoped region close to the proposed van Hove/Lifshitz transition contradicts the long-standing and overwhelming thermodynamic evidence that $p^*$ lies in the lightly overdoped region near 0.19 holes/Cu. As doping is reduced below this value we see [3] an abrupt collapse in (a) the condensation energy; (b) the jump $\Delta \gamma_c$ in specific heat coefficient at $T_c$; (c) the superfluid density [4, 33]; (d) the Knight shift at $T_c$ [30, 31]; (e) the critical fields, $H_{c1}$, $H_c$ and $H_{c2}$ [30, 51]; and (f) the self-field critical current, $J_{c}(sf)$ [30, 37] - all signifying an abrupt crossover from strong superconductivity to weak superconductivity (note - this is not to be confused with “weak-coupling” superconductivity which we discuss later in connection with calculations of free energies below $T_c$). This crossover is not merely of theoretical interest - these dramatic changes all combine to impact on the fine tuning of conductors for practical and commercial applications [33]. Any claims to the contrary need to critically address this well-established result, and so far have failed to do so.

1.2 Scope

The purpose of the present work is both to reinforce these points (as they seem to be forgotten) and to examine the thermodynamic behaviour in the neighbourhood of $p^*$ within the superconducting dome. If $T^*$ delineates the opening of the pseudogap, as claimed, and if the pseudogap is responsible for a large loss of spectral weight as is obvious from thermodynamic, NMR, infrared [32] and superfluid density measurements, then there should be radical changes in the superconducting state when the temperature falls below $T^* < T_c$. On cooling below $T_c$ one would expect the condensation free energy, critical fields and superfluid density to initially grow as if there were no pseudogap, consistent with the strong superconductivity seen in the overdoped region. Then, on crossing the $T^*$ line the mooted opening of the pseudogap would deplete this spectral weight such that these thermodynamic parameters would grow much more slowly on further cooling and perhaps even reduce in magnitude. Indeed, if there is a mean-field phase transition at $T^*$ then the $T$-dependent slope of these properties will change discontinuously. We refer to this general behaviour as ‘entrant’ where the slope of $T^*(p)$ below $T_c$ remains negative as depicted in the inset to Fig. 1(b) and in the following we present a search for such behaviour.

Additionally, we also test the converse of this, namely, if the slope of $T^*(p)$ below $T_c$ were positive as depicted by the gray line in Fig. 1(a). We refer to this as ‘reentrant’ behaviour. The motivation for this scenario is found in the ARPES study by Vishik et al. [26]. These authors claim to observe the pseudogap out to $p \approx 0.23$ just above $T_c$. $T^*(p)$ is then suggested to ‘back-bend’, adopting a positive slope and falling to zero at $p^* = 0.19$. In this case, between $0.19 < p < 0.23$, cooling below $T_c$ means that the initial onset of superconductivity just below $T_c$ is already within the pseudogap state and all measures (critical fields, condensation energy and superfluid density) should then indicate weak superconductivity. However, on crossing the reentrant $T^*(p)$ line the superconductor will exit the pseudogap state into a strong superconductivity regime in which these measures are no longer suppressed and they will grow much more rapidly with further decreasing temperature. Such reentrant behaviour has both theoretical support [40] and is experimentally observed in Ba(Fe$_{1-x}$Co$_x$)$_2$As$_2$ [41].

Complementing this view, Raman scattering in $B_{1g}$ symmetry has been interpreted to suggest that the pseudogap just above $T_c$ does indeed persist to $p \approx 0.23$, the putative location of the van Hove singularity, before closing [27]. This would give added support to a reentrant phase diagram [26]. Either way, entrant or reentrant behaviour should give a complex, non-BCS-like $T$-dependence of critical fields, condensation energy and superfluid density providing either a downturn or a boost, respectively, to these properties on traversing the $T^*$ line. If there is a thermodynamic transition at $T^*$ then these changes will be abrupt. Moreover, as the pseudogap is effective in causing a large reduction in electronic entropy these effects should be substantial.

2. Results and analysis

We focus on the generic system Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (Bi-2212) used in the ARPES [26], Raman [27] and high-field
transport so that we are discussing the same materials. However, we find essentially identical results also for Y$_{0.8}$Ca$_{0.2}$Ba$_2$Cu$_3$O$_{7-\delta}$ (Y-123) which, like Bi-2212, can also be overdoped. The measurements use a high-resolution differential technique that allows the electronic specific heat to be separated from the much larger lattice term. The electronic specific heat coefficient, $\gamma(T)$, and electronic entropy, $S(T)$, obtained in this way have been reported for Bi-2212 in three different compositions: Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$, Bi$_2$Sr$_2$Ca$_2$Y$_{0.3}$Cu$_3$O$_{7-\delta}$, and Bi$_{1.7}$Pb$_{0.3}$Sr$_2$CaCu$_2$O$_{8+\delta}$ where the second material allows lower doping while the third allows higher doping than the parent material. It was found that, for the same doping state, these three materials yielded essentially identical thermodynamic parameters so we conclude that the thermodynamic properties are independent of cation cross-substitution (and associated minor disorder) and are essentially dependent only on doping.

### 2.1 Entrant or reentrant behavior.

Firstly, we address the question of entrant versus reentrant behavior as illustrated in the inset panels in Fig. 1 and test this using the detailed $T$-dependence of the condensation free energy. In order to determine the condensation free energy one wishes to integrate $\Delta S = S_n - S_s$ from well above $T_c$ (where superconducting fluctuations have disappeared) down to $T = 0$. Here $S_n$ is the normal-state entropy and $S_s$ is the superconducting-state entropy identical to our measured electronic entropy. To determine $S_n$ we take the ARPES-derived electronic dispersion and calculate the density of states $N(E)$ assuming a rigid dispersion to account for the shift in Fermi level with doping. From the electronic density of states we calculate $S_n$ as described earlier using the standard formula for weakly interacting Fermions:

$$S_n = -2k_B \int_0^\infty [f \ln(f) + (1 - f) \ln(1-f)] N(E) \, dE \quad (1)$$

where $f$ is the Fermi function.

The pseudogap is treated as before and $S_n(T)$ is fitted to the normal-state experimental data, the only fitting parameters being the magnitude of the pseudogap, $E^*(p)$, and the distance away from the van Hove singularity, $E_{\text{vs}} - E_F$. The fits are shown by the solid curves in Fig. (a) and (b) and are seen to be an excellent description of the experimental entropy above $T_c$. Fig. 1 shows the condensation free energy, $\Delta F(T)$, calculated by integrating $\Delta S = S_n - S_s^{\text{exp}}$ for seven different doping states. The magenta curves in panels (a) and (b) are at critical doping $p = p^* = 0.188$ while (a) shows three other curves for higher doping ($p = 0.194, 0.20, 0.21$) which straddle the putative reentrant $T^*(p)$ line of Vishik et al. and (b) shows three curves for lower doping ($p = 0.182, 0.176, 0.169$) which straddle the putative entrant $T^*(p)$ line reported, for example, by Naqib et al. The data curves are plotted mostly as linear segments (rather than smooth spline curves) but the quality of the data is illustrated in panel (a) for the most heavily doped sample (olive green curve) where just every fourth data point is plotted using the green crosses.

The black dashed curves in both panels are our calculated mean-field near-weak-coupling $d$-wave BCS free energy calculations (dashed curves) rule out both the reentrant and entrant scenarios. The small deviations near $T_c$ are due to superconducting fluctuations.

![Figure 1](image.png)

**FIG. 1:** The two scenarios tested in this work (grey lines in insets): (a) ‘reentrant’ where the putative pseudogap line $T^*(p)$ meets the $T_c(p)$ phase curve then back-bends to fall to zero at critical doping $p^* = 0.19$; and (b) ‘entrant’ where the putative pseudogap line $T^*(p)$ continues to fall monotonically to zero at $p^* = 0.19$, continuing the trend above $T_c$. Colored curves in the main panels show the measured condensation free energy, $\Delta F(T)$, obtained by integrating the electronic entropy, $\Delta S(T)$. The four curves in each panel are at doping levels indicated by the colour-coded vertical lines on the phase diagrams shown in the insets. The smoothness of the data and its excellent agreement with near-weak-coupling $d$-wave BCS free energy calculations (dashed curves) rule out both the reentrant and entrant scenarios. The small deviations near $T_c$ are due to superconducting fluctuations.
haviour and the observed condensation free energy across the entire T-range except close to $T_c$ where superconducting fluctuations are the cause of the small discrepancies. We note in passing that $\Delta F(T)$ is surprisingly insensitive to fluctuations and that these are much more evident in $\Delta U(T)$ and $\Delta S(T)$ \cite{11}. In the underdoped samples the discrepancy due to fluctuations is larger, mainly due to the fact that $T^{\text{mf}}_c$ is so much greater than $T_c$ here, but also because the Padamsee model does not include an antinodal pseudogap which affects the $T$-dependence near $T_c$. However it is important to note that these small departures seen here in the underdoped region below $p_{\text{crit}}$ are the opposite to what would be expected in the entrant scenario. On cooling towards and below $T_c$ we see that $\Delta F(T)$ actually rises more slowly at first (due to fluctuations) then develops its full weight, with the $T$-dependence over most of the range below $T_c$ following the simple near-weak-coupling mean-field behavior.

Lastly, the color-coded circles on each curve show where the putative $T^*$ is expected from the two insert figures. Notably, there are no knees or kinks observed in $\Delta F(T)$ at these points.

The excellent match between mean-field behaviour and the observed condensation energy is significant in light of the mooted reentrant (a) or entrant (b) behaviour. In the former case $\Delta F(T)$ should rise more slowly at first as though heading for a small ground-state value (reduced due to the pseudogap) then abruptly upturn at $T^*$ (see circles) as the superconductor moves out of the reentrant pseudogap state. Instead all of these overdoped samples follow the canonical behaviour for a single order parameter, all exhibiting strong superconductivity with a similar ground-state condensation energy. Indeed it has been shown that across this region the BCS ratio $\Delta U(0)/\gamma \hbar k_B T_c^{\text{mf}}$ adopts a constant value of $\approx 0.17$, as expected for weak-coupling $d$-wave superconductivity \cite{17}. The small decrease in $\Delta F(0)$ at the highest doping is therefore simply due to the fall in $T_c$ on the overdoped side. In contrast, panel (b) shows a rapid fall in $\Delta F(0)$ as doping falls below $p^*$. Note that the lowest doping is still slightly above optimal doping, so this fall is indeed very rapid. This is due to the abrupt opening of the pseudogap at $p^*$ removing antinodal states that would otherwise be available for superconductivity. It is abundantly clear that these states are removed at all temperatures below $T_c$ and not just at a putative $T^*$ (circles) below $T_c$. Despite this rapid fall in $\Delta U(0)$, each of the curves rises monotonically, free of any semblance of a knee and consistent with a single-order-parameter mean-field behaviour as shown by the dashed curves. This conclusion is independent of the details of the normal-state entropy fits noted above, provided the entropy is a smooth function of $T$ (which it is). The measured entropy itself is smooth and free of knees or anomalies.

In short, neither entrant nor reentrant behaviour is supported. The data is entirely consistent with the pseudogap being present at all temperatures when $p < p^*$, and absent at all temperatures when $p > p^*$, i.e. the pseudogap is independent of $T$ \cite{3, 12, 48} and dependent only on $p$. The same is true above $T_c$ where, for $p < 0.19$, the lost entropy associated with the pseudogap is never recovered at least up to 300 K and the ‘lost susceptibility’ (as determined by NMR Knight shift and relaxation rate data) is never recovered up to 450 K.

2.2 The value of $p^*$.

Now we address more closely the value of $p^*$ where the pseudogap closes. For Bi2212 it was established long

FIG. 2: Thermodynamic data for Bi-2212: (a) $\gamma^{\text{Cu}}$ Knight shift \cite{53} expressed in entropy units using $aw_X$; the doping, $p$, is annotated. Dashed curves: normal-state pseudogap fits with $E^*/k_B = 160$, 78 and 8 K. (b) Electronic entropy coefficient $S/T$ for 11 doping states (listed) from \cite{12}, based on data from \cite{3}. Curves for critical doping where the pseudogap closes, optimal doping and most underdoped are shown by red, blue and green, respectively, and correspond to the color coding in (a). All curves are calculated \cite{12} using the full ARPES-derived dispersion involving Fermi arcs as applicable to both normal and superconducting states, while those in Fig. 1 use the non-weak-coupling Padamsee approximation for $\Delta F(T)$ (see text). (c) The change, $\Delta \gamma(13) = \gamma(13) - \gamma(0)$, in specific heat coefficient, $\gamma$, between 0 and 13 T for the doping states listed at right. Color coding is the same as above.
ago by examination of many physical properties that
$p^* = 0.19$ holes/planar Cu [14]. The data is summarised
in Loram et al. [3], in Fig. 1 of Tallon et al. [49] and
Fig. 1 of Storey et al. [48]. Despite this robust determi-
nation there remains a widespread view, promoted most
recently by Legros et al. [28], that this $T^*(p)$ line ex-
tends deep into the overdoped region, joining the $T_c(p)$
phase curve projecting to zero just where $T_c \to 0$. In
some cases these $T^*$ values are not temperatures at all
but spectroscopically-determined energy scales divided
by $k_B$. More of this later. In other cases these $T^*$ values
are obtained by identifying `kinks’ or downturns in the $T$-
dependence of various physical properties. The problem
of course is that above $T_c$ one must take superconducting
fluctuations into account. Experimentally these are seen
most easily from a field-dependent downturn in the in-
plane electrical resistivity and a diamagnetic contribution
to the static susceptibility. It is not widely recognised
that in overdoped Bi2212 superconducting fluctuations
can also reduce the electronic density of states (DOS) at
the Fermi level [53] and hence decrease the spin sus-
ceptibility and affect all other properties which depend on
the DOS. Although we are not aware of an explicit theo-
retical treatment, in our work a decrease in the measured
electronic entropy as $T_c$ is approached from above must
mean that there is a decrease in the electronic DOS. This
must be true because there is no other source of entropy
once the phonon part has been correctly subtracted.

The point is illustrated in some detail in Fig. 2 where
the $^{63}$Cu Knight shift data of Ishida et al. [52] is repro-
duced. This data was used by Legros et al. to augment
their ‘phase diagram’ (see their Supplementary Informa-
tion) which was largely adapted from Vishik et al. [20].
Also shown is the $^{17}$O Knight shift data from Ishida’s
Fig. 3 so that spin shifts are presented for overdoped
($p = 0.195$), optimal doped ($p \approx 0.16$) and underdoped
($p = 0.129$), as annotated. The scale on the RHS shows
the Knight shift referenced to the $T$-independent orbital
shift. Note that a spin shift of $^{17}$K = 0.2 (˚) corresponds
to $^{63}$K = 0.177 (˚). The small vertical arrows show the
$T^*$ values where Ishida et al. identified the opening of
the pseudogap (in our opinion erroneously). The dashed
curves are our pseudogap fits to their normal-state data
with $E^*/k_B = 8$, 78 and 160 K, respectively. (In the
first of these $E^*$ has already vanished but we retain the
small value of $E^*/k_B = 8$ K to illustrate below that large
curvature, as seen in this instance, can only occur at low
temperature when $E^*$ is very small and not at 90 K as
inferred by Ishida).

The LHS scale shows the spin susceptibility in en-
tropy units, $a_W \chi_s$, obtained from multiplying by the
Wilson ratio for weakly interacting Fermions, $a_W =
(\pi k_B/\mu_B)^2/(3\mu_0)$. As already noted, panel (b) of Fig. 2
shows the experimental [2] and modelled [43] electronic
entropy coefficient calculated for Bi2212 from the ARPES
derived dispersion [25]. To better expose the detail this
plot shows only every 20th experimental data point. No-
tably, the absolute magnitudes in panels (a) and (b) are
very similar, underscoring the previous observation [17]
that $S/T$ and $\chi_s$ are much the same in Fermi units, con-
sistent with the elementary excitations being weakly in-
teracting Fermions. Note that this is also true for many
strongly correlated heavy Fermion systems [54].

As noted, the two small arrows in Fig. 2(a) indicate $T^*$
values inferred by Ishida et al. [52] for the optimal and
overdoped samples. The two points are reproduced in the
plot of $T^*(p)$ reported by Legros et al. [28] with the over-
doped sample showing $T^* \approx 110$ K, well above $T_c \approx 79$ K.
However, the normal-state fits (dashed curves) combined
with the corresponding entropy data (red data and curves
in panel (b)) show that the downturn just above $T_c$ is
associated with superconducting fluctuations which reduce
both the spin susceptibility and the measured entropy
via their effect on the electronic DOS. It is unrelated to
the pseudogap. This is even more evident in the specific
heat coefficient, $\gamma(T)$ where the fluctuation contribution
is symmetric about $T_c$ [46, 47] - see also Fig. 2(c). A
lingering criticism of the electronic specific heat data is
whether the differential technique really has succeeded
in accurately backing-off the much larger phonon con-
tribution. To meet this concern we show in panel (c) the
change, $\Delta\gamma(H)$, in $\gamma$ between zero external field and
$\mu_0 H = 13$ T. This automatically eliminates any residual
phonon contribution and we see in the field-dependent
anomaly the effect of a magnetic field in suppressing fluc-
tuations above and below $T_c$. The fluctuation range for
the sample at critical doping (red curve) is shown by the
light red shading and this fluctuation range is reproduced
by the shading in panels (a) and (b). (Note that despite
the same doping state the $T_c$ values are somewhat differ-
ent for the single crystal in (a) with $T_c = 79$ K and the
polycrystal in (b) with $T_c = 83.9$ K. Such differences be-
 tween crystals and polycrystals are not uncommon. They
probably arise from the presence of slightly more in-plane
defects in state-of-the-art single crystals.) In summary,
the similarities between the downturns in $^{63}$K/$T$ and
the measured entropy just above $T_c$ together with the
evidence from tunnelling studies in overdoped Bi2212
[51, 52] lead us to conclude that these downturns arise
from superconducting fluctuations which reduce the elec-
tronic DOS, and that the pseudogap has already closed
at this doping level, i.e. $E^*/k_B = 0$ (not $T^* \approx 110$ K).

To pursue this in even more detail we consider the evolu-
tion of $S/T$ with doping shown in panel (b). For the
most underdoped sample (green data and curves) the
pseudogap is large, with $E^*/k_B = 322$ K, and $S/T$
is broadly curved over a comparable temperature range.
With increasing doping $E^*$ falls and the curvature in-
creases, but as it does so the region of high curvature
progresses towards $T = 0$. It simply is not possible with
any realistic pseudogap model for there to be a pseudo-
gap region of high curvature at 90 K as seen in the red
data points in panel (a). The downward curvature would necessarily begin around 180 K. Such a narrow region of downturn can only be found at low temperature when \( E^* \) is small, as seen in the black \( p = 0.182 \) curve in panel (b) where \( E^*/k_B = 51 \) K, or in the red dashed curve in panel (a) where \( E^*/k_B = 8 \) K. Typically the region of maximum curvature lies near a temperature of about \( E^*/k_B \) and extends over a region of \( \pm \frac{1}{2} E^*/k_B \). That the pseudogap opens abruptly at \( p^* = 0.19 \) can clearly be seen in the data for \( \Delta \chi_H(H=13) \) in panel (c) where, below this doping, the size of the electronic anomaly falls suddenly. As noted, the identification of a \( T^* \) simply from any downward curvature above \( T_c \) is naive and is easily confused with superconducting fluctuations.

As a final comment here, the Raman group of Sachtou et al. \[53\] very recently concluded that the pseudogap \( T^*(p) \) line collapses vertically in the superconducting state but not until the strongly overdoped range \( 0.222 \leq p \leq 0.226 \) just at the proposed location of the van Hove point. However, it is clear from thermodynamic and superfluid density \[33\] measurements that the strongly entropy-depleting pseudogap is completely absent beyond \( p^* = 0.19 \) and these authors must be observing some spectral feature in the Raman response other than the pseudogap, with this feature disappearing at the Lifshitz transition. The discussion in the present paper strongly suggests that the Raman response for \( p > 0.19 \) is influenced either by this proposed Lifshitz transition or by a gap-like feature arising from superconducting fluctuations that is also seen in intrinsic tunnelling data \[51,52\].

2.3 The pseudogap ‘phase diagram’.

As a final topic we wish to discuss the individual data points in the pseudogap \( T - p \) ‘phase diagram’ of Vishik \[20\] and Legros \[28\] as widely used by others. The \( T^* \) data points from SIS tunneling in both figures are not temperatures but energy gaps divided by \( k_B \). Further, these gaps in the overdoped region are superconducting gap magnitudes and therefore unrelated to the pseudogap. Resistivity-derived \( T^* \) values in Legros et al. are evaluated in the usual way from downturns in resistivity data taken from Oda et al. \[56\]. However, closer scrutiny shows that the downturn for optimal doping should be lower (if treated consistently over all dopings) and the overdoped downturn is, again, superconducting fluctuations as described above.

The c-axis resistivity data points are due to Watanabe et al. \[57\] who find a semiconducting-like upturn on cooling below a certain \( T^*(\rho_c) \) value. However, in an earlier publication these authors concluded in similar studies “we find that the onset of the semiconducting \( \rho_c(T) \) does not coincide with the opening of the spin gap seen in the \( \rho_s(T) \)” \[51\]. Note that within a tunnelling model for c-axis transport \[51,58\], such an upturn is in fact anticipated with the onset of superconducting fluctuations due to the reduction in the electronic DOS. Moreover the highest doping value of \( T^* \) was explicitly identified by these authors \[53\] as lying below the temperature of superconducting fluctuations, just as we have asserted.

Finally, the STS-derived \( T^* \) data points in both Vishik and Legros are due to Gomes et al. \[59\] and these simply map out the onset of the depression in DOS caused by superconducting fluctuations, not by the closure of the pseudogap. These data points map nicely onto the superconducting ‘pairing temperature’ inferred by Kondo et al. \[34\] and onto the doping-dependent mean-field transition temperature \( T_{\text{p}}^{\text{MF}}(p) \) determined from an entropy conservation treatment of the fluctuation specific heat \[40\]. In short, we believe these various reported \( T^*(p) \) ‘phase diagrams’ which merge with the \( T_c(p) \) phase curve in the heavily overdoped region are incorrect and highly misleading.

There are relatively few systematic studies which distinguish between the pseudogap and superconducting fluctuations, using for example an applied magnetic field, impurity substitution or by using a suitable fitting procedure for the overall \( T \)-dependence. Kokanović et al. \[60\] implement the latter, taking advantage of the broad temperature scale for the pseudogap compared with the relatively narrow domain of superconducting fluctuations. More precisely, plotting \( \chi_c(T) - \chi_{ab}(T) \) versus \( T \) (Fig. 3(a) of ref. \[61\]) eliminates an isotropic Curie term, \( C/T \), and allows the diamagnetic fluctuation contribution, which is much pronounced in \( \chi_c(T) \), to be seen more clearly. Alloul et al. \[61\] and Naqib et al. \[13\] apply a field to identify and suppress fluctuations. Both studies show that the \( T^*(p) \) line cuts through the fluctuation pairing temperature above \( T_c \), trending towards zero as \( p \to 0.19 \). However, the latter study, on epitaxial thin films of \( Y_{O.8}Ca_{O.2}Ba_2Cu_3O_{7-δ} \), was able to track \( T^* \) below \( T_c \) by combining progressive Zn substitution with their field studies. \( T^*(p) \) was thereby found to cut the \( T_c(p) \) phase curve and continue undetected towards zero as \( p \to 0.19 \). Even so we consider this as reflecting an underlying \( p \)-dependent energy scale which vanishes, rather than a closing temperature for the pseudogap.

3. Conclusions

In summary, in a search for either entrant or reentrant behaviour we have examined the condensation free energy, \( \Delta F(T) \), of \( Bi_2Sr_2CaCu_2O_{8+δ} \) at seven closely spaced doping states either side of critical doping, \( p^* = 0.19 \), where the pseudogap closes. In every case we observe \( \Delta F(T) \) to follow closely the weak-coupling \( T \)-dependence for a single \( d \)-wave order parameter. There is no obvious enhancement in \( \Delta F(T) \) on crossing a putative backbending \( T^*(p) \) line in the reentrant scenario,
nor is there any obvious suppression in $\Delta F(T)$ on crossing a putative monotonically decreasing $T^*(p)$ line in the entrant scenario. One simply observes a strong reduction in the overall amplitude of the entire $\Delta F(T)$ curve once the pseudogap opens as $p$ falls below 0.19, with no change in its mean-field-like, single-order-parameter shape. Note that our main conclusions do not depend on the specific mean-field model used here for $\Delta F(T)$. For example, an alternative interpretation involving pair-breaking below $T_c$ and Gaussian fluctuations above $T_c$ would also give smooth behavior in $\Delta F(T)$ i.e. no anomalies in the measured entropy, allowing us to rule out both entrant and reentrant behavior. In this alternative interpretation the postulated normal-state pseudogap between $p = 0.19$ and 0.23 [20, 52] is here ascribed to Gaussian superconducting fluctuations at higher $T$ [52] that cross over to critical fluctuations as $T_c$ is approached from below. We conclude that $T^* = 0.19$ is the temperature-independent location where the pseudogap abruptly opens or closes, consistent with the observed fact that the pseudogap-induced lost entropy for $p < 0.19$ is never recovered to well above room temperature. We reiterate that the pseudogap line often drawn on the phase diagram is actually the pseudogap energy scale (expressed as $E^*/k_B$) which falls with increasing doping and vanishes at $p^* = 0.19$. The pseudogap is still present and fully developed above this line, and there seems to be some kind of thermodynamically-weak mean-field (nematic?) transition near $E^*/k_B$ [24] that occurs within the preexisting pseudogap.

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