Improbability of DUrca process constraints EOS

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Abstract According to recent observational and theoretical progresses, the DUrca process (direct Urca process) may be excluded from the category of neutron star cooling mechanisms. This result combined with the latest nuclear symmetry energy experiments, will provide us an independent way of testing the EOS (equation of state) for supranormal density. For example, soft EOSs such as FPS will probably be excluded.

Key words: equation of state—neutrinos—stars: neutron

1 INTRODUCTION

Ever since 1965, MUrca process (Modified Urca process) has become the standard cooling mechanism of neutron stars (Bahcall, Wolf 1965; Yakovlev, Kaminker, Gnedin et al. 2001; Page, Geppert, Weber 2006). This has not changed until the 1980s. Boguta first noted this thing (Boguta 1981), and argued that DUrca process (direct Urca process) is possible in a relativistic nuclear theory. Lattimer et al. made a thorough investigation about the nucleon and hyperon DUrca process (Lattimer, Pethick, Prakash et al. 1991; Madappa Prakash, Manju Prakash, Lattimer et al., 1992; Pethick 1992). According to their researches, the critical density of the DUrca process is determined by the nuclear symmetry energy. If the central density of a neutron star is above that density, then it will cool via the DUrca process, which is much faster than the MUrca process. While for a given neutron star, its central density is determined by the EOS (equation of state). This means that any information about DUrca process will provide us an independent way of knowing something about the neutron star core EOS. Of course, it is only of prospective use in the 1990s. But there are dramatic changes recently.

Tsuruta’s group has made systematical comparisons between observations and theories of neutron star cooling. In their point of view, nucleon DUrca process as well as kaon ones may already be excluded (Tsuruta, Teter, Takatsuka et al. 2002; Tsuruta 2004, Tsuruta 2006). Thus if one EOS permits DUrca process, then it will probably be excluded. The improbability of DUrca process provides us an independent way of testing the EOS. There are also tremendous progresses about nuclear symmetry energy (Li, Chen 2005; Chen, Ko, Li 2005; Li, Chen, Ko 2006). This two joined together give us deep insight into the neutron star core EOS. For example, soft EOS such as FPS may be excluded. Before looking into the EOS, we will review some details about the DUrca process.

2 THE DURCA PROCESS

Several minutes after a neutron star’s borning, it enters the neutrino cooling epoch. DUrca process is the simplest neutrino emission process (Gamow, Schoenberg 1941, Pethick 1992). It is simply decay of neutrons and successive electron captures.

\[
\begin{align*}
n & \rightarrow p + e^- + \bar{\nu} \\
p + e^- & \rightarrow n + \nu.
\end{align*}
\]  

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Table 1: DUrca Process Critical Density for Different Values of q.

| q      | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 |
|--------|-----|-----|-----|-----|-----|
| n_c/n_n | 0.62 | 0.75 | 0.80 | 0.85 | 0.85 |
| n_c (fm^{-3}) | 0.58 | 0.44 | 0.37 | 0.32 | 0.30 |

Whereas, if another nucleon is present as a bystander particle, this becomes the traditional MUrca process (Chiu, Salpeter 1964; Bahcall, Wolf 1965, Yakovlev, Kaminker, Gnedin et al. 2001). Since MUrca involves two additional fermions, it is 5–6 orders slower than that of DUrca. MUrca process can proceed without difficulty, conserving both energy and momentum, of course with the sacrifice of a slower rate. For DUrca process to occur, there is a minimal proton concentration \( x \) in order to meet the momentum conservation condition. We will follow Lattimer’s treatment here.

The momentum of the emitted neutrinos and antineutrinos is of order \( k T/c \), where \( T \) is the neutron star’s internal temperature taken to be \( 10^{10} \) K. While the typical Fermi momentum is of order \( \approx 100 \text{ MeV} \). Thus, the momentum conservation condition is \( p_{Fp} + p_{Fe} > p_{Fe} \). Noting that \( n_i \propto p_i^2 \), for a \( np \)e matter, \( n_p = n_e \) as a consequence of charge neutrality, or \( p_{Fe} = p_{Fe} \). So, the momentum conservation becomes \( 2p_{Fp} > p_{Fe} \), or \( n_p > 1/8n_n \). We define the proton concentration as \( x = n_p/(n_p+n_n) \). Therefore we obtain the threshold for DUrca process to proceed \( x \geq 1/9 \). If the proton concentration exceed that threshold, a neutron star will cool via the rapid DUrca process.

For a neutron star, the actual proton concentration is determined by the microscopic interaction, such as the isospin dependent part of the three body interaction. Nuclear symmetry energy is well among the list.

Selecting a schematic model. The energy per baryon can be expanded quadratically around the symmetry value \( x = 1/2 \),

\[
\epsilon(n, x) = \epsilon(n, 1/2) + S_v(n)(1 - 2x)^2 + \cdots .
\]

Where \( n \) is the number density of baryons, \( S_v(n) \) is the bulk symmetry energy. The above expansion is a good approximation for all \( x \), at any density (Lattimer, Pethick, Prakash et al. 1991, and reference therein).

We are considering a system in \( \beta \) equilibrium, the chemical potentials of the fermions have the relation (Shapiro, Teukolsky 1983)

\[
\mu_i = \mu_n - \mu_p = -\frac{\partial \epsilon}{\partial x}.
\]

Where \( \mu_i \) stands for the chemical potential of the \( i \)th Fermi system. Substitute the above expansion of the energy, we get the equation which determines the equilibrium proton concentration,

\[
\hbar c(3\pi^2nx)^{1/3} = 4S_v(n)(1 - 2x) .
\]

We may adopt a power law nuclear symmetry energy,

\[
S_v = S_0 \left( \frac{n}{n_s} \right)^q.
\]

Where \( S_0 \) is bulk symmetry energy at nuclear saturation density \( n_s = 0.16 \text{ fm}^{-3} \).

A power law symmetry energy has recently been approved by nuclear diffusion experiment at subnormal density (Li, Chen 2005; Chen, Ko, Li 2005). So we are on the edge to see if there is DUrca process in the interior of neutron stars. Corresponding to the minimum proton concentration, the critical density is

\[
\frac{n_c}{n_p} = \left[ 1.71(30 \text{ MeV})/S_0 \right]^{1/(q-1/3)} .
\]

Where \( n_c \) is the critical number density corresponding to setting \( x = x_c = \frac{1}{7} \). For a conservative consideration (Li, Chen, Ko 2006),

\[
32 \text{ MeV}(n/n_s)^{0.7} < S_v < 32 \text{ MeV}(n/n_s)^{1.1} .
\]

Selected values of \( n_c \) are given in table 1. Combined with the EOS, we can say whether there is DUrca process in the interior of neutron stars. But whether it exists, can only be inferred from neutron star cooling observations. That is the subject of the next section.
Fig. 1 Thermal Evolution Curves from Tsuruta (2006). In Fig. 1a (left panel) the dotted and solid curves refer to the standard cooling of $M = 1.4 M_\odot$ neutron stars with and without heating, respectively, while the dot-dashed and dashed curves are for hyperon cooling of 1.6 and 1.8$M_\odot$ stars, respectively. In Fig. 1b (the right panel) the solid, dot-dashed and dashed curves refer to pion cooling of 1.4, 1.6 and 1.8$M_\odot$ stars, respectively. In the same figure the dotted curve refers to thermal evolution of a 1.4$M_\odot$ pion star with heating. The vertical bars refer to temperature detection data with error bars, while the downward arrows refer to the upper limits. The more accurate detection data are shown with numbers, for (1) the Vela pulsar, (2) PSR 0656+14, (3) Geminga, and (4) PSR 1055-52. The rest of the data shown are more rough estimates. Some of more interesting among these are shown with letters, as (A) Cas A point source, (B) the Crab pulsar, (C) PSR J0205+6449 in 3C58, (F) RX J0822-4300, (G) 1E1207.4-5209, (I) PSR 1046-58, (N) RX J1856-3754, and (R) PSR 1929+10. (©)By permission of the author.

3 IMPROBABILITY OF DURCA

In a series of papers, Tsuruta et al. has made clear their conclusions (e.g. Tsuruta, Teter, Takatsuka et al. 2002; Tsuruta 2004; Tsuruta 2006). Three points can be summarized,

1. Soft EOS, such as BPS should be excluded from neutron star mass measurements.

2. Nucleon and kaon D Urca process should be excluded especially for the Vela data.

3. Pion cooling is consistent with both observation and theory.

A graphical summary is given Figure[1]

Lattimer & Prakash (2006) have also made a survey of the neutron star mass-radius relation. Their result is for Tsuruta’s. The surviving EOSs all supports large masses. Recently, Özel (2006) has made a compound analysis of the neutron star EXO 0748-676. From its stringent mass radius relation, only the stiffest EOSs are consistent with the measurement. Conservatively, we consider both the medium and stiff EOSs from now on. Using point 2 of Tsuruta’s conclusion, neutron star cooling could provide us an independent way of testing the EOS.

Following the above discussion of DUrca process, we can calculate the critical mass for a specific EOS, above which DUrca process will turn on in the interior of neutron stars. If the critical mass is smaller than $1.4 M_\odot$, then the EOS may probably be excluded. Table[2] gives the critical neutron star mass, calculated for different EOSs.
### Table 2  DUrca Critical Mass for Different EOS.

| q  | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 |
|----|-----|-----|-----|-----|-----|
| FPS| 1.18| 0.84| 0.64| 0.50| 0.45|
| SLy| 1.50| 1.09| 0.84| 0.65| 0.58|
| RMF210| 1.27| 1.16| 1.07| 0.98| 0.93|
| RMF240| 1.40| 1.26| 1.16| 1.05| 0.99|
| RMF300| 1.55| 1.40| 1.28| 1.16| 1.09|
| TNI2u| 1.01| 0.76| 0.61| 0.50| 0.46|
| TNI6u| 1.24| 0.94| 0.76| 0.62| 0.56|
| TNI3u| 1.40| 1.07| 0.86| 0.69| 0.62|

**Fig. 2** EOS for RMF210, RMF240, RMF300, from Bottom to Top Respectively. The behavior of TNI2u, TNI6u, TNI3u is similar.

The critical mass is given in units of $1 M_\odot$. FPS (Friedman Pandharipande Skyrme) and SLy (Skyrme Lyon effective interaction) are EOSs taken from Haensel & Potekhin (2004). RMF means relativistic mean field theory. The quantities 210, 240, 300 are the compression modulus. They are taken from Ma (2002), Glendenning (1997). The EOSs TNI2u, TNI6u, TNI3u (Three Nucleon Interaction, u stands for universal inclusion) are taken from Takatsuka et al. (2006), which Tsuruta’s group have used to get their conclusions. For a comparison of the EOSs, see Figure 2.

When the critical mass is smaller than $1.4 M_\odot$, a normal neutron star will cool via the rapid DUrca process. Of course, it is in contradiction to Tsuruta’s conclusions. So, soft EOS, such as FPS, RMF210, TNI2u, may be excluded. Even medium EOS TNI6u is in danger.

On the other hand, when we choose several stiff EOS, such as SLy, RMF300, these EOS meet the improbability of DUrca process. This time there is a tendency for smaller q values (Li, Chen 2005). A smaller q, 0.6, 0.7, etc, satisfies better the astronomical requirement. This may be tested by further nuclear symmetry energy measurement. But it may take many years.

Excluding the soft EOSs is consistent with Tsuruta (2006), and Lattimer & Prakash (2006). Our result can also be compared with Özel’s (2006). In the case of neutron star cooling, not only the stiffness of the EOS determines, but also the composition. The key point is, we present an independent way of testing the EOS, from the improbability of DUrca process. Following this treatment, we can separate EOSs more likely from those less likely. Before we come to the end, there are several points to note.

### 4 DISCUSSIONS

There are four points to note about.
1. The presence of muon. When we incorporate muons into our consideration, we get a larger minimal proton concentration, but smaller critical density (Lattimer, Pethick, Prakash et al. 1991). So the exclusion of soft EOSs is strengthened.

2. About hyperons. Hyperon DUrca process only add to the more effective nucleon DUrca process (Madappa Prakash, Manju Prakash, Lattimer et al. 1992). Since nucleon DUrca process is reconsidered, hyperon ones may be subtle (Takatsuka, Nishizaki, Yamamoto et al. 2006).

3. The consistent problem. The symmetry energy here is an extrapolation of subnormal density experiment. When we choose a fixed form of symmetry energy, we also fix the EOS to some degree. Maybe combined with compression modulus data, one can make a seemingly more consistent calculation. But it will not bother us that, this is an independent way of testing the EOS.

4. The presence of quark matter. In this case, things will be more complicated. First, the threshold of quark DUrca process is not a simple ingredient fraction. Quark-quark interaction must be taken into account (Pethick 1992). Second, the cooling scenario in the presence of quark matter have some considerable difference with that of a neutron star (Page 2006). Moreover, deconfinement heating must be included (Yuan, Zhang 1999; Kang, Zheng 2007). Since Tsuruta's exclusion of DUrca process is done in the frame of nucleon processes, plus pion and kaon condensation. If we want to extrapolate our conclusions here, e.g. to that of hybrid stars, it will be a systematic project. This may be the scope of further studies.

Excluding soft EOSs is a general tendency of neutron star researches. The preference of smaller q values needs further studies. As Lattimer said sixteen years ago, *The continuing attempts to observe thermal radiation from neutron stars will have important implications for these properties of nuclear matter*. That is what we try to do here.

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