Chaotic scattering of a quantum particle
weakly coupled to a very complicated background

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Abstract

Effect of a complicated many-body environment is analyzed on the chaotic motion of a quantum particle in a mesoscopic ballistic structure. The dephasing and absorption phenomena are treated on the same footing in the framework of a model which is free of the ambiguities inherent to earlier models. The single-particle doorway resonance states excited via an external channel are damped not only because of the escape onto such channels but also due to ulterior population of long-lived background states, the resulting internal damping being uniquely characterized by the spreading width. On the other hand, the formation of the fine structure resonances strongly enhances the delay time fluctuations thus broadening the delay time distribution.

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Analog experiments with open electromagnetic microwave cavities [1, 2, 3, 4] on the one hand and extensive study of the electron transport through ballistic microstructures [3] on the other have drawn during last decade much attention to peculiarities of chaotic wave interference in open billiard-like set-ups. It is well recognized by now that the statistical approach [6, 7, 8] based on the random matrix theory (RMT) provides a reliable basis for description of the universal fluctuations characteristic of such an interference which, in particular, manifests itself in the single-particle resonance chaotic scattering and transport phenomena. At the same time, experiments with ballistic quantum dots reveal appreciable deviations from the predictions of RMT, which persist up to low temperatures indicating some loss of the quantum coherence. The known methods [9, 10] of accounting for this effect suffer deficiencies and ambiguities [11, 12] and give different results. An artificial prescription has been suggested in [12] to get rid of uncertainties and, simultaneously, to accord the models.

Below we propose a model of dephasing and absorption which from the very beginning do not suffer any ambiguity. We consider the environment (e.g. the walls of a ballistic quantum dot) to be a complicated many-body system with a very dense energy spectrum. The evolution of the extended system: the coupled particle and background, is described by means of an enlarged non-Hermitian matrix $\mathbf{H}$ of order $N = N(s) + N(e)$ with two blocks $H^{(r)} = H^{(r)} - \frac{i}{2} A^{(r)} A^{(r)\dagger}$, $r = s, e$ ($N(e) \gg N(s)$) along the main diagonal, which are mixed by the off-diagonal coupling $N(e) \times N(s)$-matrix $V$. The upper and lower diagonal blocks represent the non-Hermitian effective Hamiltonians of the uncoupled single-particle open system and of the environment respectively. The Hermitian matrices $H^{(r)}$ describe the closed counterparts (the corresponding mean level spacings satisfy the condition $D(r = s) \gg d(r = e)$) when the matrices $A^{(r)}$ are built of the amplitudes connecting the internal and channel states. The single-particle and background states have, when uncoupled, no common decay channels so that the coupling $V$ is purely Hermitian. Since the background states have no direct access to the observable outer channels and attain it only due to the mixing with the internal single-particle states, the latter play the role of doorway states.

The total $M \times M$ scattering matrix $S(E) = \mathbf{I} - i\mathbf{A}^\dagger (E - \mathbf{H})^{-1} \mathbf{A}$ is unitary so far as all the $M = M(s) + M(e)$ open channels, observable ($s$) as well as hidden ($e$), are taken into account. The transitions between $M(s)$ observable channels are described by the submatrix
\[ S(E) = I - i \mathcal{T}(E) \] where
\[ \mathcal{T}(E) = A^{(s)\dagger} \mathcal{G}^{(s)}_0(E) A^{(s)} \] (1)
and the matrix
\[ \mathcal{G}^{(s)}_0(E) = \frac{1}{E - \mathcal{H}^{(s)} - \Sigma^{(s)}(E)} \] (2)
is the upper left \( N^{(s)} \times N^{(s)} \) block of the resolvent \( \mathcal{G}(E) = (E - \mathcal{H})^{-1} \). The subscript \( D \) stands in (2) for "doorway". The self-energy matrix
\[ \Sigma^{(s)}(E) = V^{\dagger} \frac{1}{E - \mathcal{H}^{(e)}} V \equiv V^{\dagger} \mathcal{G}^{(e)}_0(E) V . \] (3)
includes all virtual transitions between the doorway states and the environment. Generally, this matrix is not Hermitian, hence the submatrix \( S \) is not unitary. The resonance spectrum \( \{ \mathcal{E}_\alpha = E_\alpha - \frac{1}{2} \Gamma_\alpha \} \), i.e. the eigenvalue spectrum of the matrix \( \mathcal{H} \), is found from the equation
\[ \text{Det}_{(s)} [ \mathcal{E} - \mathcal{H}^{(s)} - \Sigma^{(s)}(\mathcal{E}) ] = 0 . \] (4)

In what follows we chiefly analyze the temporal aspects of the scattering where the influence of the background shows up in the most full and transparent way. In particular, a straightforward calculation gives for the Smith delay-time submatrix \( Q = -i S^{\dagger} dS/dE \) in the \( M^{(s)} \)-dimensional subspace of the observable channels
\[ Q = b^{(s)\dagger} b^{(s)} = b^{(s)\dagger} b^{(s)} + b^{(e)\dagger} b^{(e)} = Q^{(s)} + Q^{(e)} \]
where the matrix amplitudes
\[ b^{(s)}(E) = \mathcal{G}(E) A^{(s)} ; \quad b^{(s)}(E) = \mathcal{G}^{(s)}_0(E) A^{(s)} ; \quad b^{(e)}(E) = \mathcal{G}^{(e)}_0(E) V b^{(s)}(E) \] (5)
have dimensions \( N \times M^{(s)} \), \( N^{(s)} \times M^{(s)} \) and \( N^{(e)} \times M^{(s)} \) respectively. The two contributions \( Q^{(s,e)} \) correspond to the modified due to the interaction with the background time delay within the dot and delay because of the virtual transitions into the background.

We further assume the coupling matrix elements \( V_{\mu n} \) to be random Gaussian variables, \( \langle V_{\mu m} \rangle = 0 ; \, \langle V_{\mu m} V_{\nu n}^* \rangle = \delta_{\mu \nu} \delta_{mn} \frac{1}{2} \Gamma_s \frac{1}{N^{(e)}} \) where \( \Gamma_s = 2 \pi \langle |V|^2 \rangle / d \) is the spreading width. The inequality \( \langle |V|^2 \rangle \gg d^2 \) is implied so that the interaction, though weak, is not weak enough for perturbation theory to be valid. Retaining the original notations also for the averaged quantities, we find for the Smith matrix in the main \( 1/N^{(e)} \) approximation
\[ Q = \Lambda b^{(s)\dagger} b^{(s)} = \Lambda Q^{(s)} \]
where the factor \( \Lambda \) and the doorway Green function \( \mathcal{G}^{(s)}_0 \) are expressed in terms of the loops
\[ g^{(e)}(E) = \frac{1}{N^{(e)}} \text{Tr} \mathcal{G}^{(e)}_0(E) \text{ and } l^{(e)}(E) = \frac{1}{N^{(e)}} \text{Tr} \left[ \mathcal{G}^{(e)}_0(\mathcal{G}^{(e)}_0(E) \right] \]
in the Hilbert space of the background states as
\[ \Lambda(E) = 1 + \frac{1}{2} \Gamma_s l^{(e)}(E) ; \quad \mathcal{G}^{(s)}_0(E) = \left[ E - \frac{1}{2} \Gamma_s g^{(e)}(E) - \mathcal{H}^{(s)} \right]^{-1} . \] (6)
The spectral equation (4) reduces in the same approximation to $N(s)$ similar equations originated each from an initial single-particle resonance.

As an instructive example, we consider first an embedded in the background isolated single-particle resonance with the bare complex energy $E_0 = \varepsilon_0 - \frac{i}{2}\gamma_0$, which decays onto an only channel. As to the environment whose spectrum is very dense and rich, its states are supposed to decay through a large number of weak statistically equivalent channels. The corresponding decay amplitudes $A^{(e)}_\mu$ are random Gaussian with zero means and variances $\langle A^{(e)}_\mu A^{(e)}_{\nu} \rangle = \delta_{\mu\nu} \delta_{\mu\nu} / M^{(e)}$. The widths $\gamma_e$ does not fluctuate since $M^{(e)} \gg 1$. The interaction $V$ redistributes the initial widths over exact resonances as $\Gamma_\alpha = f_\alpha \gamma_0 + (1 - f_\alpha) \gamma_e$ with the strength function $f_\alpha \equiv f(E_\alpha)$ obeying the condition $\sum_\alpha f_\alpha = 1$.

Peculiarities of the background energy spectrum are of no importance in what follows. Therefore we exploit the simplest uniform model \[13, 15\] with equidistant levels $\epsilon_\mu = \varepsilon_0 + \mu d - \frac{i}{2}\gamma_e$. Then the loops as well as the strength function can be calculated explicitly \[15\]. Depending on the magnitude of the coupling there exists two different scenarios. In the limit $\Gamma_s \gg \gamma_0 - \gamma_e$ (the natural assumption $\gamma_e \ll \gamma_0$ is accepted throughout the paper) of strong interaction all the individual strengths $f_\alpha$ are small, $f(E_\alpha) \leq 2d/\pi \Gamma_s$, and are distributed around the energy $\varepsilon_0$ according to the Lorentzian with the width $\Gamma_s$, $f_\alpha \propto L(\Gamma_s) (E_\alpha - \varepsilon_0)$. Thus the original doorway state fully dissolves in the sea of the background states. In the opposite limit of weak coupling, $\Gamma_s \ll \gamma_0 - \gamma_e$, which is the one of our interest, the strength $f_0 = 1 - \Gamma_s / (\gamma_0 - \gamma_e)$ remains large when the rest of them are small again $f(E_\alpha) \leq 2d \Gamma_s / \pi (\gamma_0 - \gamma_e)^2$ and distributed as $L_{(\gamma_0-\gamma_e)}(E_\alpha - \varepsilon_0)$. Therefore, only in the weak-coupling case the doorway state preserves individuality and can be observed through the outer channels.

First, we briefly consider the case of a stable background $\gamma_e = 0$. The matrix $S$ is then unitary since all resonances are excited and decay only onto the outer channels. In the single-channel scattering considered the delay time is related to the cross section $\sigma(E) = \gamma_0^2 |C_0^{(e)}(E)|^2$ as $Q(E) = \frac{\pi \Gamma_s}{\gamma_0} \sigma(E)$ and is equal to (we set $\varepsilon_0 = 0$ and have taken into account that $g^{(e)}(E) = \cot(\pi E/d)$; $l^{(e)}(E) = (\pi/d) \sin^{-2}(\pi E/d)$)

$$Q(E) = \gamma_0 \frac{\pi \Gamma_s / 2d + \sin^2(\pi E/d)}{[E \sin(\pi E/d) - \frac{i}{2} \Gamma_s \cos(\pi E/d)]^2 + \frac{i}{2} \gamma_0^2 \sin^2(\pi E/d)}.$$ (7)

It is seen that the coupling to the background causes in both the cross section as well as the delay time strong fine-structure fluctuations on the scale of the background level spacing $d$.
In particular, the delay time fluctuates between $Q(E = \epsilon_{\mu}) = (2\pi/d)(\gamma_{0}/\Gamma_{s}) \sim 1/\Gamma_{\mu}$ at the points of the fine structure levels and a much smaller value $Q(E \approx 0 \neq \epsilon_{\mu}) \approx (2\pi/d)(\Gamma_{s}/\gamma_{0})$ in between. Obviously, a particular fine-structure resonance cannot be resolved and only quantities averaged over an energy interval $d \ll \delta E \ll \Gamma_{s}$ are observed. The averaging yields for the time delay near the doorway energy

$$Q = \frac{4}{\gamma_{0} + \Gamma_{s}} + \frac{2\pi}{d}. \quad (8)$$

The first contribution comes from the doorway resonance when the second is just the background mean level density.

Let us now take into account that the excited background states are not stable and the background resonances strongly overlap, $\gamma_{e} \gg d$, even with no coupling to the doorway states. This smears out the fine-structure fluctuations thus reducing the loop functions to $g^{(e)}(E) \Rightarrow -i$ and $l^{(e)}(E) \Rightarrow 2/\gamma_{e} \uparrow \downarrow$. As opposed to the previous case, the unitarity of the scattering submatrix $S$ is broken. The structures narrower than $\gamma_{e}$ are excluded and all particles which delayed for a time larger than $1/\gamma_{e}$ are irreversibly absorbed and lost from the outgoing flow.

There exist two different temporal characteristics of the scattering process \[11\]. The first one is the decay rate $\gamma_{0} + \Gamma_{s}$ of the doorway state once excited through the incoming channel. This rate is readily seen from the scattering amplitude $T(E) = \gamma_{0} [E - \epsilon_{0} + i \frac{\pi}{2} (\gamma_{0} + \Gamma_{s})]^{-1}$. Since the doorway state is not an eigenstate of the total effective Hamiltonian $\mathcal{H}$ it fades out not only because the particle returns into the outer channel but also due to the internal transitions with formation of the exact fine structure resonances over which the doorway state spreads. During the time $t_{s} = 1/\Gamma_{s}$, the background absorbs the particle. After this, the particle can evade via one of the $e$ channels or be after a while emitted back into the dot and finally escape onto the outer channel. Since the particle reemitted by the chaotic background carries no phase memory, the time $t_{s}$ is just the characteristic time of dephasing. All the resonances, save the broad one with the width $\Gamma_{0} = \gamma_{0} - \Gamma_{s}$, have rather small widths and decay much slower. Just the resonances with the widths within the interval $\gamma_{e} < \Gamma_{a} \ll \Gamma_{s}$ contributes principally in the Wigner delay time which shows how long the excited system still returns particles into the initial channel. As a result, the delay time near the doorway energy equals

$$Q(E = \epsilon_{0}) = \frac{4\gamma_{0}}{(\gamma_{0} + \Gamma_{s})^{2}}. \quad (9)$$
where the enhance factor \( \Lambda = 1 + \Gamma_s / \gamma_e \). This factor is missing in the oversimplified consideration of [16] which implies that interaction with an environment yields only absorption.

Returning to the general consideration, we as usual model the unperturbed chaotic single-particle motion by the random matrix theory. The observable channels are considered below to be statistically equivalent and, to simplify the calculation, no time-reversal symmetry is suggested. We suppose below that \( d \ll \gamma_e \ll \min (D, \Gamma_s) \ll 1 \) (the radius of the semicircle). If, on the contrary, \( \gamma_e \gg \Gamma_s \) the transitions into the background become equivalent to irreversible decay similar by its properties to the decay into continuum. Only in such a limit of full absorption the factor \( \Lambda \to 1 \) and the approach of [16] is justified.

It is readily seen that to account for the interaction with the background the substitution \( \varepsilon \Rightarrow \varepsilon - i \Gamma_s \) should be done while calculating the two-point S-matrix correlation function \( C(\varepsilon) = S(E) \otimes S(\varepsilon) \). This immediately yields the connection \( C_V(t) = \exp(-\Gamma_st)C_0(t) \) between the Fourier transforms with and without interaction, the additional damping being as before caused by the spreading over the fine structure resonances. Contrary to the models with additional fictitious channels the exponential factor is an unambiguous consequence of our model.

The modification given in [16] of the method proposed in [17] allows us to calculate also the distribution \( \mathcal{P}(q) = (\pi M^{(s)})^{-1}\text{Im}\langle\text{tr}(q - Q - i0)^{-1}\rangle \) proper delay times (eigenvalues of the Smith matrix). The corresponding generating function is proportional to the ratio of the determinants of two \( 2N^{(s)} \times 2N^{(s)} \) matrices with the following structure (compare with [16])

\[
A(z) = -i (E - H^{(s)}) \sigma_3 + \frac{1}{2} (AA^\dagger + \Gamma_s) - \frac{\Lambda}{z} \left(1 + \sqrt{1 - \frac{\Gamma_s}{\Lambda} z \sigma_1}\right)
\approx -i (E - H^{(s)}) \sigma_3 + \frac{1}{2} AA^\dagger - (1 + \sigma_1) \left(\frac{\Lambda}{z} - \frac{\Gamma_s}{2}\right)
\]

where the variable \( z \) spans the complex \( q \)-plane. The square root sets [16] on the positive real axes the restriction \( q \leq \Lambda/\Gamma_s \approx 1/\gamma_e \). In the approximation of the second line valid if \( \Lambda \gg 1 \) we arrive to a simple relation

\[
\mathcal{P}_V(\tau) = \frac{1}{\Lambda}\mathcal{P}_0(\tau_V),
\]

where \( \tau_V \equiv (\tau/\Lambda) [1 - \pi (\Gamma_s/\Lambda D) \tau]^{-1} > 0 \) and \( \tau = \frac{D}{\pi q} \). This relation does not hold in the asymptotic region \( \tau_V \to \infty \) or \( \tau \to \Lambda D/\pi \Gamma_s \approx \frac{1}{\pi} D/\gamma_e \) where the more elaborate rigorous
expressions obtained in [16] must be used with the substitution \( \tau \Rightarrow \tau/\Lambda \) being made. For example in the single-channel scattering with the perfect coupling to the continuum the time delay distribution (see [19])

\[
\mathcal{P}_V(\tau) = e^{-1/\tau_V}/\tau_V^3
\]

reaches its maximum at the point \( \tau_V = 1/3 \) or \( \tau = (1/3)\Lambda \left(1 + \frac{2}{3}\Gamma_s/D\right)^{-1} \gg 1/3 \). The most probable delays shift towards larger values. The estimation just made is quantitatively valid only if \( \Gamma_s \ll D/2\pi \) and the maximum lies near the point \( \frac{1}{3}\Gamma_s/\gamma_e \) distant from the exact edge \( D/2\pi\gamma_e \) of the distribution. Under the latter restriction the approximate formula (11) describes well the bulk of the delay time distribution which becomes, roughly, \( \Lambda \) times wider. The condition noted is much less restrictive in the case of weak coupling to the continuum when the transmission coefficient \( T \ll 1 \) and the most probable delay time \( \tau \approx T\Gamma_s/4\gamma_e \ll D/2\pi\gamma_e \) as long as \( \Gamma_s \ll D/\pi\gamma_e \).

The approximation (11) works even better when the number of channels \( M^{(s)} \gg 1 \) and the delay times are restricted to a finite interval [17, 18]. The delay time scales in this case with \( M^{(s)}^{-1} \) and the natural variable looks as \( \tau = qM^{(s)}D/2\pi = \Gamma_W q/T \) with \( \Gamma_W \) being the Weisskopf width. The edges of the distribution (11) are displaced towards larger delays by the factor \( \Lambda \), \( \tau^{(+)\pm} = \Lambda\tau_{0}^{(+)\pm} \). One can readily convince oneself that the taken approximation remains valid in the whole interval \( \Delta\tau_V = \Lambda(\tau_{0}^{(+)\pm} - \tau_{0}^{(-)\pm}) \) under condition \( \Gamma_s \ll \Gamma_W \) which implies weak interaction with the background. The width of the delay time distribution broadens by the factor \( \Lambda \) due to the influence of the background.

At last, transport through a ballistic dot in the presence of a background is described by off-diagonal matrix elements \( T^{a,b}(E) \) of the matrix (11) with the doorway Green function

\[
\left[E + i\frac{\Gamma_s}{2} - \mathcal{H}^{(s)}\right]^{-1}
\]

This reproduces the results of the imaginary-potential model [10] with \( \gamma = \Gamma_s(2\pi/D) \) as the dimensionless dephasing rate. The unitarity is broken because of the leakage through \( e \)-channels. Contrariwise, if the background states are stable no loss of the flow takes place. This is in line with the voltage-probe model with zero total current \( I_{\phi} \) [12]. It must be noted that in this case probabilities rather than amplitudes should be averaged over the energy to get rid of the fine-structure fluctuations.

In summary, the influence of a very complicated environment on the chaotic single-particle scattering is analyzed. Unlike some earlier considerations, the coupling to the background is supposed to be purely Hermitian. Absorption takes place because of hidden decays of the background resonance states. The single-particle doorway states which are excited and observed through external channels are additionally damped with the rate \( \Gamma_s = 2\pi\langle|V|^2\rangle/d \) because of the spreading over the long-lived fine structure resonances. This rate uniquely de-
terminates the dephasing time during the particle transport through a ballistic microstructure. The formation of the fine-structure resonances strongly enhances delay time fluctuations thus, in particular, broadening the distribution of the proper delay times.

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