ABSTRACT
We consider a communication scheme in which two low earth orbit (LEO) satellites jointly transmit (at the same time and frequency) to a multi antenna land terminal (LT). This scheme can increase the achievable terminal throughput by up to a factor of 2, depending on the channel matrices. The implementation aspects of this scheme are well-known but the usefulness of this scheme for LEO satellites has yet to be studied. Because the satellite channel is dominated by its line-of-sight (LOS) component, the terminal’s ability to separate the two streams depends critically on the system’s instantaneous-configuration; i.e., the relative location of the satellites and the terminal antennas. Since the relative locations of LEO satellites vary rapidly, the throughput characterization is not straightforward. To characterize the network performance we consider two satellites having one antenna, transmitting to a single LT having multiple antennas. We introduce a novel stochastic framework that assumes the terminal orientation as random. Using the proposed framework, we show that if the terminal antennas are close, the network throughput is nearly independent of the terminal orientation. When the terminal antennas are sufficiently separated, the result is completely different. For this case, we define an outage event and calculate the outage probability. The definition of outage in our novel stochastic framework allows us to prove that dual satellite transmission can indeed increase the downlink throughput with high probability.

INDEX TERMS
Multiple input multiple output (MIMO), satellite communication, LEO satellites, cooperative transmission.

I. INTRODUCTION
LEO satellite communication (SatCom) is expected to play an important role in wireless communications by providing global-coverage, high-throughput and low-cost internet access [1], [2]. This includes remote rural areas, civil aviation, as well as commercial and cruise shipping lines. Thousands of LEO satellites are expected to be deployed in the next decade by different commercial and public institutions [3], [4]. The idea is to create a flexible, low-latency SatCom network that supplies high coverage, independent of the terrestrial infrastructure. To justify such a large investment, the network must provide a high information rate and high reliability. This requirement is even more pressing for airplanes and ships, where thousands of subscribers will only be served by a few satellites.

Multiple input multiple output (MIMO) communication is a mature technology with a well-established theory and practical algorithms (e.g., [5], [6]) which are incorporated into 5G [7] and other protocols [8]. To compete with terrestrial systems, SatCom will need to resort to MIMO technology and take advantage of the significant research achievements in this field [9]. It is therefore important to evaluate the potential MIMO performance gain and determine whether this technology is feasible in LEO SatCom. The key obstacle to incorporating MIMO into LEO-SatCom is its line-of-sight (LOS) channel characteristics.

MIMO shows a significant gain in terrestrial wireless communications, primarily in cases of rich-scattering environments and systems with sufficient antenna spacing at the transmitter and receiver; all of which guarantee well-conditioned channels. By contrast, satellite channels in high frequency bands are characterized by strong LOS components with negligible scattering. Since scattering is widely believed to be a prerequisite for well-conditioned MIMO channels, the feasibility of spatial multiplexing to the SatCom
channel has been questioned in the past [10] and is subject to contentious discussions in the scientific community.

In MIMO satellite systems, the transmit or receive antennas must be spatially separated to achieve spatial-multiplexing gain. It has been shown, both theoretically [11] and experimentally [12], that spatial multiplexing in LOS-dominant channels is possible if multiple satellites in different orbital positions cooperate or if the LT antennas are sufficiently far apart (at least several kilometers). This latter separation is not possible in most LEO applications, which renders spatial multiplexing from a single satellite to a single terminal infeasible (even if both have multiple antennas). In this paper, we focus on the first alternative; that is, two satellites that cooperate to spatially multiplex data to a single, multi-antenna, LT.

Moreover, in a LOS channel, the exact antenna location has a significant effect on MIMO performance. Hence, for GEO satellite MIMO systems, much work has been devoted to optimizing antenna placement [11] or to improving user grouping based on their locations [13]. However, such configurations with LEO satellites is impractical due to the high speed at which these satellites travel. It is thus crucial to analyze the potential multiplexing gain for LEO satellites.

In this paper, we propose a new approach to analyze the performance of LEO communication systems. We consider two satellites with limited means of cooperation, that transmit to a single land terminal (LT). In more explicit terms, each satellite transmits an independent data stream with its single antenna, where the only coordination is through coarse synchronization and by adjusting transmission rates. The LT, which is equipped with multiple antennas, is responsible for the separation and detection of the two data streams. Note that LEO satellites have a large angular velocity with respect to Earth and the distance between each satellites and the terminal varies with time. Thus, unlike the single satellite case, this small distance variation may cause significant changes in the channel phase, which affect the terminal’s ability to separate and detect the two data streams. Thus, good separation of the two data streams is not guaranteed in all satellite-terminal configurations.

To characterize the network performance, we present a novel stochastic framework for satellite communication. We incorporate randomness into the channel by considering the terminal orientation (azimuth rotation) as random. Using this stochastic framework, we obtain a closed-form expression for the channel distribution in the downlink of cooperative MIMO communication with two LEO satellites and a single LT that is equipped with a uniform circular array (UCA) of antennas.

Based on this characterization, we evaluate performance in the two extreme cases where the LT antennas are very close and very far from each other (i.e., when the UCA radius is small or large). For close antennas, we show that the throughput changes very slowly, and is nearly independent of the terminal orientation. Thus, the throughput can be well-predicted by a deterministic closed form expression.

This expression depends solely on the network parameters and on a normalized measure of the satellite separation.

For far antennas, the throughput can change rapidly as the satellites move. In this case, we characterize the performance of the distribution of the instantaneous rate for joint transmission. This distribution serves to characterize the outage probability or to evaluate the throughput of an adaptive network with timely feedback from the LT. The results show that in almost all scenarios, the outage probability is low. That is, dual satellite transmission can increase the downlink rate with high probability.

The rest of the paper is structured as follows. Section II presents the system model for MIMO in satellite communication. Section III details our statistical characterization of the deterministic LOS channel. Section IV furnishes the performance analysis for the cases of small and large antenna separation. Section V introduces supporting numerical results and Section VI notes our concluding remarks.

II. SYSTEM MODEL

We consider a downlink in a satellite communication network that consists of two LEO satellites, each having a single antenna, and a single fixed LT with \( M > 1 \) antennas. The two satellites transmit independent data streams simultaneously to the terminal. We assume, for simplicity, that the terminal decodes each stream using zero-forcing (ZF) and further assume single-user decoding (SUD); i.e., the terminal decodes the signal of each satellite independently, while treating the signal of the other as noise.

A. COORDINATE SYSTEM

We use two coordinate systems, Cartesian and Spherical, both centered at the terminal’s location. The Cartesian coordinate system is depicted in Figure 1, where \( \hat{k} \) is a unit vector normal to Earth (z-axis), \( \hat{i} \) is a unit vector pointing east (x-axis), and similarly, \( \hat{j} \) points north (y-axis). Given a point with Cartesian

\[ \text{FIGURE 1. The Cartesian coordinate system used here.} \]
The main factors characterizing the LOS channel are the distances between each LT antenna and each satellite. The main factors characterizing the LOS channel are the distance between antenna and the satellite. We further define

\[ \Delta_{m,\ell} = \rho_{m,\ell} - r_{\ell} \]

where the latter approximation neglects terms of order \( O(d^2/r_{\ell}^2) \) because \( r_{\ell} \gg d \).

The baseband signal, transmitted by satellite \( \ell \in \{1, 2\} \), is given by

\[ s_{\ell}(t) = \sum_{n} x_{\ell}[n]p(t - nT - \tau_{\ell})e^{-j2\pi f_{\ell}t} \]

where \( x_{\ell}[n] \) is the \( n \)-th symbol transmitted by satellite \( \ell \) (with \( E[|x_{\ell}[n]|^2] = \sigma^2_{x,\ell} \)), \( T \) is the symbol duration, \( \tau_{\ell} \) is satellite \( \ell \) timing offset, \( p(\cdot) \) is the pulse shape and \( f_{\ell} \) is the satellite hardware frequency-offset. The pulse shape, which is normalized, is chosen such that it does not induce inter symbol interference (ISI). Thus, the pulse auto-correlation satisfies \( R_p(nT) = \int p(t)p^*(t - nT)dt = \delta[n] \), where \( \delta[n] \) is the Kronecker delta function.

Using the LOS channel, and considering the large difference between the satellite distance and the LT antenna separation \( (r_{\ell} \gg d) \), the received baseband-signal at antenna \( m \) is modeled by

\[ y_m(t) = \sum_{\ell=1}^{2} Y_{\ell} s_{\ell} \left( t - \frac{r_{\ell}}{c} \right) e^{-j2\pi \left( \frac{r_{\ell} + \Delta_{m,\ell}}{c} + \frac{r_{\ell}}{c} \right)} + n_m(t) \]

where \( \Delta_{m,\ell} \) is a constant expressing the effect of transmitter hardware, antenna gains, and atmospheric and rain attenuation; \( c \) is the speed of light, \( f_{c} \) is the carrier frequency and \( \frac{r_{\ell}}{c} \frac{dr_{\ell}}{dt} \) is the Doppler frequency shift of satellite \( \ell \). The additive noise, \( n_m(t) \), is a complex white Gaussian with two sided spectral density of \( N_0 \).

Note that the model of (7) includes several standard approximations. First the gap between the distances from the satellite to the LT center and to its \( m \) antenna, \( \Delta_{m,\ell} \) (cf. (6)), is neglected in the attenuation term \( \left( \frac{r_{\ell}}{c} \right) \) and in the signal delay \((s(t - \tau_{\ell}))\). This small gap affects only the argument of the exponent, where it is multiplied by \( f_{c} \). Furthermore, the dependence of \( r_{\ell} \) in time is considered only in the Doppler shift. The Doppler shift is also assumed to be the same for all LT antennas due to their proximity.

The LT employs two synchronization circuits. Each circuit estimates (and tracks) the overall time offset \( \tau_{\ell} = \tilde{\tau}_{\ell} + \frac{r_{\ell}}{c} \) and frequency shift \( f_{c} = \tilde{f}_{c} + \frac{r_{\ell}}{c} \frac{dr_{\ell}}{dt} \) of one satellite. This can be done using various known schemes for synchronization in LEO satellite networks (which have been comprehensively investigated in the last few decades). For example, [14], [15], present synchronization for CDMA communication, where the inherent interference mitigation between different CDMA spreading codes allows a simple separation of the different satellite signals. Current works focus primarily on 5G networks and present synchronization algorithms that are robust to multi-satellite transmission and large Doppler shifts, while employing the standard 5G primary synchronization signals (e.g., [16]). Note that once the ZF equalizer is initialized, each circuit has a clean signal from its corresponding satellite. Thus, in the following, we assume that \( \tau_{\ell} \) and \( f_{c} \) are perfectly known at the LT, for \( \ell = \{1, 2\} \).
The receiver separately compensates for the delay, $\tau_\ell$, and frequency shift, $f_\ell$, of each satellite. This operation creates two different signal branches for every $m$, i.e.,

$$y_{m,\ell}(t) = y_m(t)e^{j2\pi(f_\ell t + \tau_\ell t)}$$

for $\ell \in \{1, 2\}$. Each signal branch employs a match-filter, matched to $p(t)$, and synchronized to its respective satellite. Thus, the sampling times at the branch that corresponds to satellite $\ell$ are $nT + \tau_\ell, n \in \mathbb{Z}$. The resulting signal is

$$y_{m,\ell}[n] = y_{m,\ell}(t) * p^n(t - nT + \tau_\ell)dt$$

$$= \int_{-\infty}^{\infty} y_{m,\ell}(t)p^n(t - nT + \tau_\ell)dt$$

$$= \frac{\gamma_\ell}{r_\ell}x_\ell[n]e^{-j2\pi f_\ell n} + \frac{\gamma^*_\ell}{r^*_\ell}x^*_\ell[n] + n_{m,\ell}[n]$$

where $\Bar{\ell} = 3 - \ell$ is the index of the other satellite and

$$x_\ell[n] = \sum_{n'} x_{\ell'}[n']e^{j2\pi f_{\ell'}(t_{\ell'} - t_{\ell})} \int_{-\infty}^{\infty} p(t - n'T - \tau_{\ell'})p^n(t - n'T)dt.$$ 

As the pulse shape $p(t)$ is normalized and ISI free, the resulting noise samples, $n_{m,\ell}[n]$, are identical independently distributed (i.i.d.) complex Gaussian random variables with zero mean and variance of $N_0$.

Next, we denote

$$n_\ell[n] = [n_{0,\ell}[n], \ldots, n_{M-1,\ell}[n]]^T$$

and

$$y_\ell[n] = [y_0,\ell[n], \ldots, y_{M-1,\ell}[n]]^T.$$ 

For $\ell = 1$ the latter can be written as

$$y_1[n] = \bar{H}[x_1[n], x_2[n]]^T + n_1[n]$$

where

$$\bar{H} = \begin{bmatrix}
\gamma_1 e^{-j2\pi f_1} e^{j\Delta \theta_1} \\
\gamma_2 e^{-j2\pi f_2} e^{j\Delta \theta_2} \\
\vdots \\
\gamma_n e^{-j2\pi f_n} e^{j\Delta \theta_n}
\end{bmatrix}$$

and $\lambda = c/f_\ell$ is the wavelength of the carrier frequency. Correspondingly, for $\ell = 2$, one obtains

$$y_2[n] = \bar{H}[x_1[n], x_2[n]]^T + n_2[n].$$

As both signals can be expressed in terms of $\bar{H}$, the ZF equalizer designed for $\bar{H}$ is able to separate the signals completely. We note that $\bar{H}$ is not affected by the difference in the satellite timings and Doppler shifts. Hence, the design of the ZF equalizer, as well as its performance, are not affected by the different synchronization of each satellite.

### C. Channel Characteristics and Signal to Noise Ratio

It is useful to write the channel matrix as

$$\bar{H} = H \cdot D$$

where

$$D = \begin{bmatrix}
\gamma_1 & 0 \\
0 & \gamma_2
\end{bmatrix}$$

is the gain-matrix, in which each diagonal entry corresponds to the gain of every satellite, and

$$H = \begin{bmatrix}
e^{2\pi f_1} \cos(\theta_1 - \gamma_1) \cos(\phi_1) & e^{2\pi f_1} \cos(\theta_2 - \gamma_2) \cos(\phi_2) \\
\vdots & \vdots \\
e^{2\pi f_2} \cos(\theta_1 - \gamma_{M-1}) \cos(\phi_1) & e^{2\pi f_2} \cos(\theta_2 - \gamma_{M-1}) \cos(\phi_2)
\end{bmatrix}$$

is the phase matrix, where we also substituted (5).

The terminal employs a ZF equalizer to separate the symbols transmitted from the two satellites. From (15), the ZF equalizer vector designated to decode the signal from satellite $\ell$, could be built as a function of $H$, only; i.e.,

$$p_{zf,\ell} = H(H^H H)^{-1} e_\ell, \quad \ell = \{1, 2\}$$

where $e_\ell \in \mathbb{R}^{2 \times 1}$ is a vector of all zeros except the $\ell$ entry that is equal to 1, and we assume that $(H^H H)^{-1}$ exists. Employing the ZF equalizer one obtains

$$\hat{x}_\ell[n] = \frac{\gamma_\ell}{r_\ell}x_\ell[n] + \tilde{n}_\ell[n]$$

where

$$\tilde{n}_\ell[n] = p_{zf,\ell}^H n_\ell[n].$$

Using (17), the terminal can decode the signal from satellite $\ell$ without any interference\(^3\) from the other satellite. The signal-to-noise ratio (SNR) for decoding the data sent by satellite $\ell \in \{1, 2\}$ is given by

$$\text{SNR}_\ell = \frac{\gamma^2_\ell \sigma^2_n}{r_\ell^2 p_{zf,\ell}^H p_{zf,\ell} N_0}.$$ 

It is convenient to compare the network throughput to a reference scenario in which only satellite $\ell$ serves the terminal, while the latter employs a maximal ratio combining equalizer. To compare the case of two transmitting satellites to that of a single satellite, we consider two different power constraints. In one case, every satellite has an independent power constraint and the single-satellite SNR is given by

$$\text{SNR}^s_\ell = \frac{\gamma^2_\ell \sigma^2_n M}{r^2_\ell N_0}.$$ 

While the independent power constraint may be suitable in some cases, it is not completely “fair” since the network transmits twice the power when transmitting from two satellites. Hence, the performance gain comes both from the use of MIMO topology and from the increased transmission power.

\(^3\)Assuming no channel estimation errors and well-conditioned channels.
Thus, to have a fair comparison, we also consider the case where the total transmission power is equal. That is, in the single satellite case, we allow the satellite to double the power. Namely, if the symbol energy for each satellite in the dual case is $\sigma^2$, then the symbol energy in the single satellite case is $2\sigma^2$ (3dB higher). Here, the single-satellite SNR is given by $\text{SNR}_s^{3\text{dB}} = \frac{2Y^2}{\sigma^2N_0}$.

As expected, without any interference, the single satellite SNR satisfies

\[
\text{SNR}_s \leq \text{SNR}_s^{3\text{dB}} = \frac{1}{2} \text{SNR}_s^{3\text{dB}} \quad (22)
\]

namely, the ZF equalizer induces an SNR loss with respect to the single satellite case. For convenience, in most of this paper we consider the independent power constraint (i.e., comparing to $\text{SNR}_s$). The analysis under the second constraint is the same up to a constant of 2. In the numerical section, we show that in most cases, the gain from the use of MIMO outperform single satellite even with the joint power constraint.

III. STATISTICAL CHARACTERIZATION OF THE DETERMINISTIC LOS CHANNEL

A. STATISTICAL CHARACTERIZATION OF THE RATES

The joint transmission scheme considered here, in which each satellite transmits an independent data stream, aims at increasing terminal throughput by utilizing the multiplexing gain. However, it induces an SNR loss (22). In the case of a single satellite, the throughput is given by

\[
R_S = \max_{\ell=1,2} \left\{ B \log_2 \left( 1 + \text{SNR}_s^{\ell} \right) \right\} \quad (23)
\]

where $B$ is the transmission bandwidth whereas in the two-satellite case it is given by

\[
R_D = \sum_{\ell=1}^{2} B \log_2 \left( 1 + \text{SNR}_s^{\ell} \right). \quad (24)
\]

It can be shown that

\[
0 \leq R_D \leq 2R_S. \quad (25)
\]

from which it follows that if the SNR loss is negligible, the throughput can indeed be (nearly) doubled. On the other hand, if the SNR loss is significant, the throughput can be even lower than the one obtained with a single satellite.

While (24) completely characterizes the network throughput, it does not provide insight into the problem. Because the throughput depends on the distances between every antenna and each satellite. Even small variations in the location or orientation can lead to a significant different throughput. It is therefore important to evaluate these variations in both cases where satellites can track variations and not. We consider two different models:

1) OUTAGE MODEL

In the outage model, the satellites do not track the instantaneous changes in the channel state. In this case, the network decides on a code rate, $R_O$ (which can vary from time to time, but not as fast as the channel does). Every codeword is encoded at rate $R_O$, and each satellite transmits a different part of the resulting codeword. We assume that the channel remains constant during the entire codeword and that full CSI is available at the LT (CSIR). If the instantaneous channel cannot support decoding at a rate of $R_O$, the decoding will fail, and we say that the terminal is in outage. In this case, it is important to predict what the success rate will be, i.e., what percentage of the codewords will be successfully decoded at the LT.

2) FEEDBACK MODEL

Here, we assume low-rate feedback from the LT to the satellites, which indicates the achievable rate at any given time. In many cases, such feedback is not a burden on the network and, therefore, practical. The required feedback rate depends on the distances between LT antennas. For example, Figure 10 indicates that the update rate should be of the order of tenths of a second for a UCA radius of $d = 30$ cm. As the link rate is typically several Mbps, a tiny fraction of that link can provide feedback with negligible delay.

In systems with such a timely feedback, satellites can precisely know the achievable data rate, and consequently, adapt the code-rate to ensure decoding success. In this model, the decoding will (almost) always be successful, but the instantaneous rate of the network will change significantly over time. Hence it is important to predict the characteristics of these variations.

In both models, to gain more insight, it is instructive to consider the network variability as random. To that end, we present a novel stochastic framework, where we assume that the locations of the terminal and the satellites are fixed and known, but the orientation of the terminal is random. More specifically, we assume that the terminal orientation angle, $\gamma_0$, is uniformly distributed over $[0, 2\pi]$. Our analysis shows that this type of randomness is sufficient to characterize the network throughput.

Thus, in the following we consider the SNR $\tilde{R}_s$ of Equation (20) as random. We further denote

\[
\tilde{R}_s = B \log(1 + \text{SNR}_s^{\ell}) \quad (26)
\]

that will be referred to as the “individual instantaneous rate”. Moreover, we refer to

\[
R_D = \bar{R}_1 + \bar{R}_2 \quad (27)
\]

as the “instantaneous rate”. The operational meaning of these $R_D$ is different in the two communication models. In the outage model, $R_D$ determines the probability for a given outage-rate threshold $R_O$. Explicitly, the outage probability is defined as

\[
P_O = \text{Pr}(R_D \leq R_O). \quad (28)
\]

In the feedback model, $R_D$ is an achievable rate. Thus, the distribution of $R_D$ indicates the network-throughput distribution (e.g., the average network throughput is $\bar{R} = \mathbb{E}[R_D]$).
B. ANALYSIS OF THE “INSTANTANEOUS RATE”

We now analyze the network performance under both the outage and feedback models, by characterizing the distribution of the “instantaneous rate” (cf. (27)). Comparing (20) and (21) it is convenient to write:

\[
\text{SNR}_\ell = \frac{\gamma_1^2 \sigma_s^2}{r_\ell^2 N_0 e^{d (H \mathbf{H})_\ell^{-1}}} = \frac{\gamma_1^2 \sigma_s^2 M (1 - |S_M|^2)}{r_\ell^2 N_0} = \text{SNR}_\ell^* (1 - |S_M|^2) \tag{29}
\]

where

\[
S_M = \frac{1}{M} \sum_{k=0}^{M-1} \text{exp} \left[ j2\pi d \left( \cos(\theta_1 - \gamma_k) \cos(\phi_1) - \cos(\theta_2 - \gamma_k) \cos(\phi_2) \right) \right] \tag{30}
\]

and it can be shown that

\[
0 \leq |S_M|^2 \leq 1. \tag{31}
\]

Note that unlike the two-satellite case, the SNR of the LT served by a single satellite is deterministic; i.e., does not depend on the terminal orientation. Thus it is sufficient to characterize the SNR loss

\[
|S_M|^2 = 1 - \frac{\text{SNR}_\ell^*}{\text{SNR}_\ell^*_R} \tag{32}
\]

since it is only random variable that effects the “instantaneous rate”. We also note that the SNR loss is independent of \( \ell \in \{1, 2\} \) (cf. (30)); i.e., independent of the serving satellite. Thus, \( R_D \) (24), can be written as

\[
R_D = 2 \left( B \log_2 \left( 1 + \text{SNR}_\ell^* (1 - |S_M|^2) \right) \right). \tag{33}
\]

We now characterize the distribution of the instantaneous rate through the evaluation of the outage probability. As shown in Appendix A, the outage probability \( P_O \), for an outage rate threshold \( R_O \), is given by

\[
P_O = \text{Pr}(R_D \leq R_O) = \text{Pr} \left( |S_M|^2 \geq \mu \right) \tag{34}
\]

where

\[
\mu = 1 + \frac{1}{2 \text{SNR}_1^*} + \frac{1}{2 \text{SNR}_2^*} - \sqrt{\left( \frac{1}{2 \text{SNR}_1^*} + \frac{1}{2 \text{SNR}_2^*} \right)^2 - \frac{(1 - 2 R_0/\beta)}{\text{SNR}_1^* \cdot \text{SNR}_2^*}} \tag{35}
\]

is the outage SNR loss. Note that \( \mu = 0 \) is equivalent to \( R_O = B \log_2 \left( 1 + \text{SNR}_1^* \right) + B \log_2 \left( 1 + \text{SNR}_2^* \right) \) and hence leads to an outage probability of \( 1 \). On the other hand, \( \mu = 1 \) is equivalent to \( R_O = 0 \), which means no outage events. Moreover, there exists a value \( 0 < \mu_1 < 1 \) for which \( R_O = R_S \) (cf. (23)), the maximum rate of the single satellite.

Another interesting case is when \( \text{SNR}_1 = \text{SNR}_2 \), where for every \( \mu_1 < 0.5 \) the throughput is improved in the two-satellite case in comparison to single satellite case (albeit only slightly in the low SNR regime).

As indicated by (34), the SNR-loss, \( |S_M|^2 \), is a key to determine the outage probability. Therefore, to evaluate whether simultaneous transmission improves system performance, we analyze the distribution of \( |S_M|^2 \).

C. STATISTICAL CHARACTERIZATION OF \( |S_M|^2 \)

We now characterize the statistical properties of \( |S_M|^2 \). Recalling that the receiver employs a UCA, (30) can be written as

\[
S_M = \frac{1}{M} \sum_{k=0}^{M-1} \text{exp} \left[ j2\pi d \left( \cos(\theta_1 - \gamma_k) \cos(\phi_1) - \cos(\theta_2 - \gamma_k) \cos(\phi_2) \right) \right].
\]

and, using trigonometric identities, we get

\[
S_M = \frac{1}{M} \sum_{k=0}^{M-1} \text{exp} \left[ j2\pi d \left( \cos(\theta_1) \cos(\gamma_0) + \frac{2k\pi}{M} \right) \right] \left( \sin(\theta_1) \sin(\gamma_0) + \frac{2k\pi}{M} \right) \cos(\phi_1) - \left( \cos(\theta_2) \cos(\gamma_0) + \frac{2k\pi}{M} \right) \cos(\phi_2) \right]
\]

\[
S_M = \frac{1}{M} \sum_{k=0}^{M-1} \text{exp} \left[ j2\pi d \left( \cos(\gamma_0) + \frac{2k\pi}{M} \right) \right] \left( \sin(\gamma_0) + \frac{2k\pi}{M} \right) \cos(\phi_2) \right]
\]

\[
S_M = \frac{1}{M} \sum_{k=0}^{M-1} \text{exp} \left[ j2\pi d \left( \cos(\gamma_0) + \frac{2k\pi}{M} \right) \cos(\phi_1) + \sin(\gamma_0) + \frac{2k\pi}{M} \right) \cos(\phi_2) \right]
\]

\[
S_M = \frac{1}{M} \sum_{k=0}^{M-1} \text{exp} \left[ j2\pi d \left( \cos(\gamma_0) + \frac{2k\pi}{M} \right) \cos(\phi_1) - \cos(\phi_2) \right] \right] + \sin(\gamma_0) + \frac{2k\pi}{M} \right) \cos(\phi_2) \right]
\]

\[
\text{Let:}
\begin{align*}
  c_1 &= \cos(\phi_1) \sin(\theta_1) - \cos(\phi_2) \sin(\theta_2) \\
  c_2 &= \cos(\phi_1) \cos(\theta_1) - \cos(\phi_2) \cos(\theta_2)
\end{align*}
\]

then

\[
S_M = \frac{1}{M} \sum_{k=0}^{M-1} \text{exp} \left[ j2\pi d \left( \cos(\gamma_0) + \frac{2k\pi}{M} \right) \right] c_2 + \sin(\gamma_0) + \frac{2k\pi}{M} \right) \right] \right]
\]

To simplify the exponent argument, denote

\[
c = [c_1, c_2]^T = [u \cos(\psi), u \sin(\psi)]^T
\]
The normalized projections of the satellite distance on plain $\Pi$.

where

$$\psi = \arctan \left( \frac{c_2}{c_1} \right)$$

(40)

and

$$u = \sqrt{c_1^2 + c_2^2}$$

$$= \left( \cos^2(\phi_1) + \cos^2(\phi_2) \right) - 2 \cos(\phi_1) \cos(\phi_2) \cos(\theta_1 - \theta_2) \right) ^{1/2}.$$ (41)

Using the latter notation one obtains

$$S_M = \frac{1}{M} \sum_{k=0}^{M-1} \exp \left( j2\pi \frac{ud}{\lambda} \cos(\gamma_k) + \frac{2k\pi}{M} + \psi \right).$$ (42)

The expression for $S_M$ in (42) is important as it provides a compact representation of the effect of different network features. This includes non-varying parameters such as the carrier wavelength, $\lambda$, UCA radius at the LT, $d$, and its number of antennas, $M$. The satellite locations, which are represented solely by $u$ and $\psi$, vary with time, but slowly enough to be considered constant during the analysis period.

From (41), $u$ can be interpreted as the length of an edge of a triangle, as shown in Fig 3, where the other two edges are equal to $\cos(\phi_1)$ and $\cos(\phi_2)$ and the angle between them is $\theta_1 - \theta_2$. To understand the relationship between $u$ and the system geometry, consider Figure 4, which depicts two unit-length vectors, each pointing from the LT location (point A) toward one of the satellites. Plain $\Pi$ is tangent to Earth at point A and $k$ is the normal to plain $\Pi$ at A. Note that the triangle in Figure 3 is the one created by projecting the normalized satellite position vectors (cf. Figure 4) into plain $\Pi$. Thus, the edge that connects these two projections has a length $u$. Note that 0 $\leq u \leq 2$ is the only parameter required to characterize the effect of the satellites location. Furthermore, $u = 0$ if and only if the two satellites are seen in exactly the same direction from the LT. Thus, $u$ is henceforth dubbed normalized satellite separation.

The randomness in $S_M$ follows only from the orientation angle $\gamma_0$. We therefore study the effect of $\gamma_0$ on the distribution of $S_M$ and through it, on the network throughput. Recall that $S_M$ affects the throughput only through its squared magnitude:

$$|S_M|^2 = S_M S_M^*$$

$$= \frac{1}{M^2} \sum_{k=0}^{M-1} \sum_{\ell=0}^{M-1} e^{-j2\pi \frac{ud}{\lambda} \cos(\gamma_0 + \frac{\ell\pi}{M})}.$$ (43)

Because the exact distribution of $|S_M|^2$ is difficult to evaluate, in the following we characterize it with simple closed form expressions for the two extreme cases: small and large values of $ud/\lambda$.

**IV. PERFORMANCE ANALYSIS**

**A. PERFORMANCE ANALYSIS FOR SMALL $ud/\lambda$.**

We now show that if $ud/\lambda$ is small, the variance of $S_M$ is very small. Thus, in this regime, $S_M$ can be well approximated by a deterministic function of $ud/\lambda$, and we can accurately predict the SNR-loss, $|S_M|^2$. To do so, we upper bound the variance of $S_M$ and show that it is significantly smaller than one (while 0 $\leq |S_M|^2 \leq 1$).

Recalling that $\gamma_0$, the rotation-angle of the first LT’s antenna, is uniformly distributed over $[0, 2\pi)$, adding a constant to $\gamma_0$ does not affect its distribution up to a modulo $2\pi$. Thus, $\gamma_0 + \frac{2\pi}{M} + \psi \mod 2\pi$ is also uniformly distributed over $[0, 2\pi)$. Hence, we can calculate the mean and the variance of $S_M$. Let $\alpha \sim U[0, 2\pi)$, then

$$E[e^{jc\sin(\alpha)}] = \frac{1}{2\pi} \int_0^{2\pi} e^{jc\sin(\alpha)} d\alpha = J_0(c)$$

(44)

where $c \in \mathbb{R}$ and $J_0(c)$ is the Bessel function of the first kind and 0th order. Thus, the mean of $S_M$ is:

$$E[S_M] = \frac{1}{M} \sum_{k=0}^{M-1} E[e^{j2\pi \frac{ud}{\lambda} \cos(\gamma_0 + \frac{2\pi}{M} + \psi)}] = J_0(2\pi ud/\lambda).$$ (45)
and, taking the expectation over (43), the variance of $S_M$ is:

$$
\text{Var}[S_M] = E\left[|S_M|^2\right] - E^2[S_M]
= \frac{1}{M^2} \sum_{k=0}^{M-1} \sum_{\ell=0}^{M-1} J_0 \left(2\pi \frac{ud}{\lambda} \sin \left(\frac{(k-\ell)\pi}{M}\right)\right)
- J_0^2 \left(2\pi \frac{ud}{\lambda}\right). \tag{46}
$$

Since the exact evaluation of the variance, (46), does not provide much insight, we turn to the calculation of an upper bound on the variance of $S_M$, which is presented in the following theorem:

**Theorem 1:** The variance of $S_M$ is upper bounded by

$$
\text{Var}[S_M] \leq 2 \left(\frac{\pi ud}{\lambda M!}\right)^2. \tag{47}
$$

Theorem 1 shows that for small $\frac{ud}{\lambda}$ the variance is negligible, and is further reduced as $M$ increases. Thus, in this regime, $S_M$ can be well approximated by its mean: $S_M \approx J_0(2\pi \frac{ud}{\lambda})$. To further demonstrate the relevance of this approximation, Figure 5 depicts the variance of $S_M$ for different numbers of receive antennas as a function of $\frac{ud}{\lambda}$. It shows that the variance decreases as the number of receive antennas increases, implying that $S_M$ approaches $E[|S_M|]$ as $M$ gets larger. Moreover, even for small values of $M$, the variance is very low. For example, when $M = 3$ the variance is below $10^{-2}$ for any $\frac{ud}{\lambda} < 0.2$. Figure 5 also depicts the upper bound on the variance (47). The figure shows that the bound is indeed useful, and that becomes tight for large $M$ or for small $\frac{ud}{\lambda}$.

Using Theorem 1, while approximating $S_M$ by its expectation (cf., (45)) and considering small values of $\frac{ud}{\lambda}$, it follows that

$$
\text{SNR}_\ell \approx \text{SNR}_\ell^2 \cdot \left(1 - |J_0(2\pi \frac{ud}{\lambda})|^2\right). \tag{48}
$$

and

$$
R_D \approx \sum_{\ell=1}^{2} B\log_2 \left(1 + \text{SNR}^2 \left[1 - |J_0(2\pi \frac{ud}{\lambda})|^2\right]\right). \tag{49}
$$

Recall that the Bessel function has its maximum at $J_0(0) = 1$ and it first zero at $x_0 = 2.405$. Thus, the system will have zero throughput for $\frac{ud}{\lambda} = 0$, whereas a maximum throughput is expected for $\frac{ud}{\lambda} = \frac{x_0}{\lambda} = 0.383$.

**Proof:** [Proof of Theorem 1] To derive the upper bound on the variance of $S_M$, we use the Taylor series of the Bessel function of the first kind. Thus, (46) can be written as

$$
\text{Var}[S_M] = \sum_{p=0}^{\infty} \frac{(-1)^p (2\pi \frac{ud}{\lambda})^{2p}}{2^{2p}(p!)^2} \cdot \frac{1}{M^2} \cdot \sum_{k=0}^{M-1} \sum_{\ell=0}^{M-1} \sin \left(\frac{(k-\ell)\pi}{M}\right)^{2p} \sum_{p=0}^{\infty} \frac{(-1)^p (2\pi \frac{ud}{\lambda})^{2p}}{2^{2p}(p!)^2}.
$$

Changing the variable in the last summation using $p = p_1 + p_2$, (50) is simplified as follows

$$
\text{Var}[S_M] = \sum_{2p=0}^{\infty} \frac{(-1)^p (2\pi \frac{ud}{\lambda})^{2p}}{2^{2p}(p!)^2} \cdot \frac{1}{M^2} \cdot \sum_{k=0}^{M-1} \sum_{\ell=0}^{M-1} \sin \left(\frac{(k-\ell)\pi}{M}\right)^{2p} \sum_{p=0}^{\infty} \frac{(-1)^p (2\pi \frac{ud}{\lambda})^{2p}}{2^{2p}(p!)^2} \cdot \frac{1}{2^{2p}} \cdot \frac{1}{2^{2p}} \cdot \frac{1}{2^{2p}}.
$$

(51)

where

$$
K_{M,p} = \frac{1}{M^2} \sum_{k=0}^{M-1} \sum_{\ell=0}^{M-1} \sin \left(\frac{(k-\ell)\pi}{M}\right)^{2p}. \tag{52}
$$

Before continuing, the following lemma is required.

**Lemma 1:** For every positive integer $M \geq 2$,

$$
K_{M,p} = \frac{(2p)!}{2^{2p}} \sum_{n=-\left\lfloor p/M \right\rfloor}^{\left\lfloor p/M \right\rfloor} \frac{(-1)^n M}{(p + n'M)! (p - n'M)!}. \tag{53}
$$

Proof: see Appendix B.

The left hand term in the parentheses of (51) can be simplified using Vandermonde’s identity:

$$
\sum_{k=0}^{r} \frac{m!}{k!(m-k)!} \frac{n!}{(n-r)!} \frac{r!}{(m+n-r)!} = \frac{(m+n)!}{r!(m+n-r)!}.
$$
Setting \( m = n = r = p \) and \( k = p_1 \), one obtains
\[
\sum_{p_1=0}^{p} \frac{(p!)^2}{(p!)^2((p - p_1)!)^2} = \frac{(2p)!}{(p!)^2}
\]
and by substituting (53) and (54) into (51), it follows that
\[
\text{Var}[S_M] = \sum_{p=0}^{\infty} (-1)^p (2\pi \frac{ud}{\lambda})^2p \frac{(2p)!}{(p!)^2} \left( \sum_{n'=\lceil \frac{p}{2} \rceil}^{\lfloor \frac{p}{2} \rfloor} \frac{(-1)^{n'M}}{(p + n'M)!} - \frac{1}{(p!)^2} \right)
\]
and with rewriting we get
\[
\text{Var}[S_M] = \sum_{p=0}^{\infty} (-1)^p (2\pi \frac{ud}{\lambda})^2p \frac{(2p)!}{(p!)^2} \left( \sum_{n'=\lceil \frac{p}{2} \rceil}^{\lfloor \frac{p}{2} \rfloor} \frac{(-1)^{n'M}}{(p + n'M)!} - \frac{1}{(p!)^2} \right)
\]
We next note that for \( n' = 0 \) the term in the sum within the parentheses of (55) equals \( 1/(p!)^2 \) and is canceled out with the right hand term. Noting, also, that summation terms indexed with \( n' = -a \) are identical to those with \( n' = a \), \( \forall a \in \{0, \ldots, \lfloor p/M \rfloor\} \), (55) can be written as
\[
\text{Var}[S_M] = \sum_{p=0}^{\infty} (-1)^p (2\pi \frac{ud}{\lambda})^2p \frac{(2p)!}{(p!)^2} \left( \sum_{n'=\lceil \frac{p}{2} \rceil}^{\lfloor \frac{p}{2} \rfloor} \frac{(-1)^{n'M}}{(p + n'M)!} - \frac{1}{(p!)^2} \right)
\]
where the latter inequality follows from \( \lfloor p/M \rfloor = 0 \) for \( p < M \).

It is now possible to bound the variance. Consider
\[
\text{Var}[S_M] = \sum_{n=0}^{\infty} \left( \mathbb{E}_{M,M} + \mathbb{E}_{M,M+2n+1} + \mathbb{E}_{M,M+2n+2} \right)
\]
where
\[
\mathbb{E}_{M,p} = (-1)^p (2\pi \frac{ud}{\lambda})^2p \frac{(2p)!}{(p!)^2} \left( \sum_{n'=\lceil \frac{p}{2} \rceil}^{\lfloor \frac{p}{2} \rfloor} \frac{(-1)^{n'M}}{(p + n'M)!} - \frac{1}{(p!)^2} \right)
\]
In Appendix C we show that \( \mathbb{E}_{M,M+2n+1} + \mathbb{E}_{M,M+2n+2} \leq 0 \) for any \( M > 1 \) and \( n \geq 0 \). Thus
\[
\text{Var}[S_M] \leq \mathbb{E}_{M,M} = \frac{(-1)^M (2\pi \frac{ud}{\lambda})^{2M} (2M)!}{2^{2M-1} (M!)^2} \sum_{n'=1}^{\lfloor M/M \rfloor} \frac{(-1)^{n'M}}{(M + n'M)!} (M - n'M)!
\]
where \( \sum_{n'=1}^{\lfloor M/M \rfloor} \frac{(-1)^{n'M}}{(M + n'M)!} (M - n'M)! = 2 \left( \frac{2\pi ud}{\lambda} \frac{2M}{M} \right)^2 \frac{1}{(M!)^2} (M + M)! (M - M)!
\]
which establishes the desirable result.

**B. PERFORMANCE ANALYSIS FOR ud/λ \( \gg 1 \)**

We now characterize the performance in the case where \( ud/\lambda \gg 1 \). Reviewing (42), we note that in this regime, tiny variations in the satellites’ locations induce corresponding small variations in \( u \) and \( \psi \). The latter, however, lead to significant fluctuations in \( S_M \). Thus, in this regime we expect the achievable joint transmission rate, \( R_D \), to vary rapidly with time (this is demonstrated in the numerical results section). Therefore, instead of trying to predict the exact value of the achievable rate, we turn to characterizing its statistical distribution.

Let
\[
\alpha_k \triangleq 2\pi \frac{ud}{\lambda} \cos \left( \gamma_0 + \frac{2\pi}{M} \gamma + \psi \right) \mod 2\pi
\]
then, (42) can be written as:
\[
S_M = \frac{1}{M} \sum_{k=0}^{M-1} e^{-j\alpha_k}.
\]
Using the theory of quantization error distribution (e.g., [17, p. 353]), if \( ud/\lambda \gg 1 \), the distribution of \( \alpha_k \) is approximately uniform. Furthermore, if \( \cos \left( \gamma_0 + \frac{2\pi}{M} \gamma + \psi \right) \neq \pm \cos \left( \gamma_0 + \frac{2\pi}{M} \gamma + \psi \right) \), then \( \alpha_k \) may be assumed to be statistically independent of \( \alpha_m \). This inequality is satisfied for any integer \( k \neq m \) as long as \( M \) is odd. We therefore consider two cases: odd \( M \) and even \( M \).

In the case of odd \( M \), \( S_M \) can be approximated by
\[
S_M \approx \frac{1}{M} \sum_{k=0}^{M-1} e^{-j\alpha_k}
\]
where \( \{\alpha_m : m = 0, \ldots, M - 1\} \) is a set of i.i.d. random variables with a uniform distribution over \([0, 2\pi]\). Using (34), the outage probability is given by
\[
P_O(\mu) = \Pr \left( |S_M|^2 > \mu \right)
\]
where \( \mu \) is the threshold SNR loss. Hence
\[
P_O(\mu) \approx \Pr \left( \left| \frac{1}{M} \sum_{k=0}^{M-1} e^{-j\alpha_k} \right|^2 > \mu \right)
\]
If $M$ is even, there is an inherent dependence between the antennas, because $\gamma_{m+M/2} = \gamma_m + \pi$ for every $m = 0, \ldots, M/2 - 1$. Thus

$$a_{m+M/2} = 2\pi \frac{ud}{\lambda} \cos \left( \gamma_0 + \frac{2k\pi}{M} + \psi + \pi \right) \mod 2\pi = a_m + \pi \mod 2\pi \quad (63)$$

and by combining the latter with (60), it follows that

$$S_M = \frac{2}{M} \sum_{k=0}^{M/2-1} \cos(\alpha_k). \quad (64)$$

Using the uniform-distribution approximation, once again, it follows that for even $M$ the outage probability, (34), can be approximated as

$$P_O(\mu) \approx \Pr \left( \left| \frac{2}{M} \sum_{k=0}^{M/2-1} \cos(\tilde{a}_k) \right|^2 > \mu \right). \quad (65)$$

The outage expressions in (62) and (65) can be further simplified if $M$ is sufficiently large, by applying the central limit theorem. In Appendix D we show that $E[S_M] = 0$ and $E[|S_M|^2] = \frac{1}{M}$, for all $M > 0$. However, if $M$ is odd, the central limit theorem yields a complex Gaussian distribution, whereas even $M$ results in a real Gaussian distribution. Hence, using the central limit theorem, for large enough $M$, the outage probability for $\mu > 0$ can be approximated by

$$P_O(\mu) \approx \begin{cases} e^{-M\mu} & \text{if } M \text{ odd} \\ 2Q(\sqrt{M\mu}) & \text{if } M \text{ even} \end{cases}. \quad (66)$$

The approximation of (66) is very simple and intuitive. It shows that in the large $du/\lambda$ regime, the main parameter that determines the success probability is the number of antennas. As the number of antennas becomes large, the outage probability goes to zero quite fast. Recall that (66) resulted from two subsequent approximations: the uniform distribution approximation and the central limit theorem. The uniform distribution approximation, which led to (62) and (65), becomes accurate as $du/\lambda$ increases, whereas the Gaussian approximation becomes accurate as $M$ increases. Nevertheless, in the next section we show, numerically, that these approximations are accurate and useful even with only $M = 3$ antennas. It is therefore possible to predict the system performance with a simple formula such as (66), which is very important for system management and optimization.

V. NUMERICAL RESULTS

In this section we present numerical results that demonstrate the usefulness and accuracy of the derived approximations. All simulations used parameters, which represent typical values in a SatCom system, as follows: the transmit and receive antenna gains are set to 45dBi and 10dBi, respectively. The transmit power (of each satellite) is 40dBm, the bandwidth is 10MHz and the noise power spectral density is $N_0 = -170dBm/Hz$ (at the terminal). The satellites orbit is 1000 km above Earth (implying an orbit radius of 7371Km) and the carrier frequency is 30GHz.

![FIGURE 6. Minimum and maximum of the SNR for one of the two transmit satellites over all the possible terminal orientation angles as a function of the UCA radius at the terminal, d. The figure also shows the single satellite performance, SNR, and the suggested approximation, (48).](image)

A. SMALL $ud/\lambda$

Section IV-A indicates that for small $ud/\lambda$, the SNR is well approximated by (48). This is demonstrated in Figure 6, which considers two satellites at $\theta_1 = 0^\circ$, $\phi_1 = 90^\circ$ and $\theta_2 = 0^\circ$, $\phi_2 = 60^\circ$, respectively. Thus, $u = 0.5$ (see (41)) and the first point of maximal performance is predicted at $d = 0.38\lambda/\mu = 0.76$ cm. The figure depicts the SNR for the signal received from satellite 1 when both satellites transmit independent data streams (joint transmission), as a function of the UCA radius, $d$. This scenario inherently suffers from SNR-loss because of zeroing the interference inflicted by the other satellite. In this figure, triangles depict the maximum and minimum SNR for random UCA orientations (namely, $\min_{\gamma \in [0,\pi]}(SNR)$ and $\max_{\gamma \in [0,\pi]}(SNR)$), whereas the solid line depicts the approximation of (48). For reference, the figure also depicts the SNR for satellite 1 when satellite 2 does not transmit (single satellite transmission). In this case the SNR remains constant as $d$ varies, because there is no interference.

The figure depicts the SNR for two different numbers of receive antennas that are organized in a UCA, $M = 3$ and $M = 8$. As expected for low $ud/\lambda$, the difference between the maximum and minimum SNR is negligible, and the approximation of (48) is very accurate. The figure demonstrates that in this case, for $M = 3$ antennas the approximation is very good up to $d = 0.6$ cm whereas for $M = 8$ antennas the approximation is good even beyond $d = 1$ cm.

The LT’s ability to cancel the interference depends on $ud/\lambda$. As predicted by (48), the SNR approaches zero when $d$ approaches 0, whereas the maximum SNR is achieved at $d = 0.76$ cm and is identical to the single satellite SNR. In this maximal case, joint transmission can achieve twice the rate of single satellite transmission.

To further illustrate the performance, we next consider the movement of the two LEO satellites in their orbit, while the
LT is fixed. We consider two satellites on the same orbit, that pass exactly above the terminal. The separation angle between the satellites is set to $1^\circ$ (with respect to the Earth’s center). The LT has a UCA of $M = 6$ receive antennas with radius $d = 2 \text{ cm}$, where the first antenna is oriented in the direction of the satellites’ orbit.

Due to the satellites’ movement, their normalized separation changes with time, ranging from $u = 0.128$ at time 0 (when both satellites are at an equal distance from the LT) to $u = 0.072$ at the figure edges. The change in $u$ brings about a change in the achieved rate, as predicted by (49). The figure depicts the approximation of (49) (in square markers) as well as the actually achievable rate evaluated by simulation (in blue solid line). Since the antenna separation and the satellite separation are quite small, the approximation is again very accurate.

For reference, we again show the instantaneous rate obtained by each satellite alone while the other is idle (dashed lines). As discussed above, we also wish to make a stricter comparison, where the total transmission power is equal; i.e., a single satellite transmission where we allow the satellite to transmit twice the power (3dB higher). The achievable rate for this case is marked in dotted lines, and is obviously better than the single satellite transmission with lower power. Nevertheless, the data rate with joint transmission is significantly higher than the single satellite case, even with the double transmit power.

Figure 7 also depicts the performance of the optimal linear minimum mean square error (MMSE) equalizer. As expected, the MMSE equalizer outperforms the ZF equalizer, but, the gap is very small. This result is compatible with the well known behavior of the ZF equalizer, whose performance approaches that of the MMSE at high SNR. This outcome indicates that spatial multiplexing-gain can be studied using the ZF equalizer.
\( \phi_2 = 85^\circ \) (thus, \( u = 0.1125 \)). The figure presents the probability of a 3 dB loss (\( \mu = 0.5 \)), for terminals with \( M = 2 \) and \( M = 5 \) antennas. It also depicts the two approximations, as in Figure 8. The figure shows that as the UCA radius increases, the uniform approximation becomes increasingly more accurate. In particular, for \( d > 100 \) cm (where \( ud/\lambda = 11.25 \)) the accuracy is very good. The Gaussian approximations accuracy increases with \( M \); hence, it is acceptable when \( M = 5 \) but quite poor if \( M = 2 \). Note that the probability for an SNR loss of more than 3dB converges to 50\% for \( M = 2 \) but is less than 10\% for \( M = 5 \).

Figure 10 considers the movement of two satellites on the same circular orbit with \( 10^\circ \) separation (one after the other). Again, the circular orbit passes exactly above the terminal. The terminal utilizes a UCA with \( M = 5 \) and a radius of 30 cm (\( ud/\lambda = 33.08 \) at \( t = 0 \)). The figure depicts the instantaneous rate using joint transmission from the two satellites, (33), the rates obtained by each satellite on its own while the other is idle, (23), and the rate when a single satellite uses twice the power (+3dB).

As explained above, the large \( ud/\lambda \) regime is characterized by rapid drops in the instantaneous rate, and rapid recoveries. Recall that the dual satellite instantaneous rate has two interpretations. It can represent the achievable rate in an adaptive system that uses timely feedback from the terminal on the link quality. Alternatively, it can show the capability of the system with a fixed transmission rate to support reliable detection (that is, a drop in the instantaneous rate in Figure 10 below the predetermined transmission rate indicates outage events).

The results show that most of the time, the instantaneous rate of joint transmission, \( R_D \), is significantly larger than with a single satellite. On average the maximum rate, obtained by a single satellite (with double transmit power) is 69.6Mbps whereas the average for joint transmission is 105.3Mbps.\(^4\) These demonstrate the contribution of joint transmission to the network throughput. As in other MIMO networks, this contribution will be even larger in networks with higher signal to noise ratios.

Finally, we note again that the ZF performance approaches that of the MMSE. This further demonstrates that the ZF analysis is useful in studying the significant spatial-multiplexing gain that lies in multi-user MIMO.

### VI. CONCLUSION

This paper explored the feasibility of MIMO in LEO satellite networks. We introduced a novel stochastic framework for satellite communication analysis, where we considered the terminal orientation as random. Using this stochastic framework, we obtained a closed-form expression for the channel distribution in the downlink of cooperative MIMO communication with two LEO satellites and a single LT equipped with a UCA of antennas.

Based on this characterization, we evaluated the performance in the two extreme cases: when the LT antennas are either very far or very close to each other (i.e., when the UCA radius is small or large). In the latter case, we showed that the throughput varies very slowly, and is practically independent of the terminal orientation. Thus, the throughput can be well-predicted by a deterministic closed-form expression, which solely depends on the network parameters and on a normalized measure of the satellite separation.

In the case of far antennas, we derived simple, yet accurate, approximations for the distribution of the instantaneous rate. These approximations were evaluated numerically and shown to predict the outage probability very well. Furthermore, they showed that in almost all scenarios, the outage probability is low, and dual satellite transmission can increase downlink rate significantly.

In the other case, where the distance between the LT receive antennas is small, we derived an upper bound on

\(^4\)In practice, an adaptive system will gain even slightly more, by using single satellite transmission when joint transmission leads to lower rates.
the variance of the SNR loss. Due to the low value of this upper bound, we concluded that the terminal rate is nearly independent of its orientation, and can easily be predicted by the normalized satellite separation (or more precisely by the quantity $\bar{u}d/\lambda$). We showed that the throughput reaches its maximum when the four-axis separation satisfies $\bar{u}d/\lambda = 0.38$. At this point, the throughput is twice the throughput of single satellite transmission; that is, the transmissions from the two satellites do not interfere with each other. For lower values $\bar{u}d/\lambda$, the throughput decreases, and joint transmission is typically not advantageous.

**APPENDIX A PROOF OF EQUATION (27)**

In this Appendix we prove (34) and derive the equation for the outage SNR loss, (35). Starting from (26) and substituting (28) and (29):

$$P_O = \Pr(R_O > \tilde{R}_1 + \tilde{R}_2) = \Pr(R_O > B \log(1 + \text{SNR}_1) + B \log(1 + \text{SNR}_2)) = \Pr(1 + \text{SNR}_1 + \text{SNR}_2 + \text{SNR}_3 \cdot \text{SNR}_2 < 2R_o/B) = \Pr(1 + (\text{SNR}_3^1 + \text{SNR}_2^3)(1 - |S_M|^2) + \text{SNR}_3^3 \cdot \text{SNR}_2^3 (1 - |S_M|^2)^2 < 2R_o/B)$$

(67)

which is equivalent to:

$$P_O = \Pr(|S_M|^2 < \vartheta_2)$$

(68)

where

$$\vartheta_1 = -\frac{1}{2\text{SNR}_1^3} - \frac{1}{2\text{SNR}_2^3} - \left(\frac{1}{2\text{SNR}_1^3} + \frac{1}{2\text{SNR}_2^3}\right)^2 - \frac{1}{(2\text{SNR}_1^3 \cdot \text{SNR}_2^3)} - (1 - 2R_o/B)$$

$$\vartheta_2 = -\frac{1}{2\text{SNR}_1^3} - \frac{1}{2\text{SNR}_2^3} + \left(\frac{1}{2\text{SNR}_1^3} + \frac{1}{2\text{SNR}_2^3}\right)^2 - \frac{1}{(2\text{SNR}_1^3 \cdot \text{SNR}_2^3)} - (1 - 2R_o/B)$$

(69)

As $\vartheta_1 < 0$ and $|S_M|^2 < 1$, therefore the interesting domain is

$$P_O = \Pr(|S_M|^2 > \mu)$$

(70)

where $\mu = 1 - \vartheta_2$.

**APPENDIX B PROOF OF LEMMA 1**

Starting from Equation (52) and using Euler’s formula:

$$K_{M,p} = \frac{1}{M^2} \sum_{k=1}^{M-1} \sum_{\ell=0}^{M-1} \left(\sin\left(\frac{(k - \ell)\pi}{M}\right)\right)^2$$

$$= \frac{(-1)^p}{M^2 2^{2p}} \sum_{k=0}^{M-1} \sum_{\ell=0}^{M-1} \left(\frac{e^{i\pi(\ell-k)/M} - e^{-i\pi(\ell-k)/M}}{M}\right)^2$$

(71)

Using the Binomial theorem, we expand the parentheses to the power of $2p$ into a sum:

$$K_{M,p} = \frac{(-1)^p}{M^2 2^{2p}} \sum_{k=0}^{M-1} \sum_{\ell=0}^{M-1} \sum_{n=0}^{2p} \left(\frac{2p}{n}\right) e^{i\pi(\ell-k)/M} e^{-i\pi(\ell-k)/M}$$

$$= \frac{(-1)^p}{M^2 2^{2p}} \sum_{n=0}^{2p} \left(\frac{2p}{n}\right) e^{-\pi(\ell-k)/M} e^{\pi(\ell-k)/M}$$

$$= \frac{(-1)^p}{M^2 2^{2p}} \sum_{n=0}^{2p} \left(\frac{2p}{n}\right) e^{-\pi(\ell-k)/M} e^{\pi(\ell-k)/M}$$

(72)

Note that the sum $\sum_{n=0}^{2p} e^{-\pi(\ell-k)/M}$ equals $M$ if $n$ equals $0$ and equals 0 for any other integer $n$. Thus, for values of $n$ that satisfy $(n-p) \mod M = 0$ we set: $n-p = n'M$, and we can write:

$$K_{M,p} = \frac{(-1)^p}{2^{2p}} \sum_{n=0}^{[p/M]} \left(\frac{2p}{n'}\right) e^{-\pi(\ell-k)/M} e^{\pi(\ell-k)/M}$$

$$= \frac{1}{2^{2p}} \sum_{n'=0}^{[p/M]} \left(\frac{2p}{n'}\right) e^{-\pi(\ell-k)/M} e^{\pi(\ell-k)/M}$$

(72)

**APPENDIX C PROOF OF THE VARIANCE BOUND**

In this appendix we prove that $\Delta_{M,n} \leq \Im_{M,M+2n+1} + \Im_{M,M+2n+2} \leq 0$, where $\Im_{M,p}$ is defined in (57). We start with

$$\Delta_{M,n} = \frac{(2\pi)^{\frac{M}{2}}(2M+2n+1)}{2^{(M+2n+1)}} \left(2(M+2n+1)\right)! \left[(M+2n+1)\right]^{M\frac{M}{2}} \sum_{k=1}^{[M\frac{M}{2}]} ((-1)^{M-k+2n+1})$$

$$= \frac{(2\pi)^{\frac{M}{2}}(2M+2n+1)}{2^{(M+2n+1)}} \left(2(M+2n+1)\right)! \left[(M+2n+1)\right]^{M\frac{M}{2}} \sum_{k=1}^{[M\frac{M}{2}]} ((-1)^{M-k+2n+1})$$

$$= \frac{(2\pi)^{\frac{M}{2}}(2M+2n+1)}{2^{(M+2n+1)}} \left(2(M+2n+1)\right)! \left[(M+2n+1)\right]^{M\frac{M}{2}} \sum_{k=1}^{[M\frac{M}{2}]} ((-1)^{M-k+2n+1})$$

(72)

and denote two positive constants, $\Psi = \frac{(2n)^{\frac{M}{2}}}{2^{(M+2n+1)}}$ and $\Phi = \frac{(2\pi)^{\frac{M}{2}}(2M+2n+1)}{2^{(M+2n+1)}}$, to get

$$\Delta_{M,n} = \Phi \sum_{k=1}^{[M\frac{M}{2}]} ((-1)^{M-k+2n+1})$$

$$+ \Psi \sum_{k=1}^{[M\frac{M}{2}]} ((-1)^{M-k+2n+1})$$

(72)
where \( \tilde{k} = \left\lfloor \frac{M+2n+1}{M} \right\rfloor \). The sum in the last line is either empty or consists one element (if \( \lfloor (M+2n+2)/M \rfloor = \lfloor (M+2n+1)/M \rfloor \)). Thus, we can upper bound \( \Delta_{M,n} \) by replacing the sum with the absolute value of the term with index \( k = \lfloor (M+2n+2)/M \rfloor \). Moreover, if \( \lfloor (M+2n+2)/M \rfloor > \lfloor (M+2n+1)/M \rfloor \) then \( (M+2n+2)/M \) is an integer and \( (M+2n+1)/M \) is not an integer. Therefore

\[
\Delta_{M,n} \leq \Phi \sum_{k=1}^{\tilde{k}} (M+2n+1+k)! (M+2n+2-kM)!
+ \Phi \psi \frac{(2M+4n+4)(2M+4n+3)}{2(M+2n+2)}
\]

(73)

Combining the two sums in (73), we get

\[
\Delta_{M,n} \leq \frac{\psi}{\Phi} \sum_{k=1}^{\tilde{k}} (-1)^{M(k+1)+1} (M+2n+1+k)! (M+2n+2-kM)!
\]

and collect pairs of terms in the sum of (74) using: \( \chi_{k,M,n} \triangleq \zeta_{2k-1,M,n} + \zeta_{2k,M,n} \). We get

\[
\Delta_{M,n} \leq \frac{\psi}{\Phi} \frac{\sum_{k=1}^{\lfloor (M+2n+1)/M \rfloor} \chi_{k,M,n}}{\lfloor (M+2n+1)/M \rfloor} \leq \frac{\psi}{\Phi} \frac{\sum_{k=1}^{\tilde{k}} \chi_{k,M,n} + \tilde{\chi}_{M,n}}{(M+2n+2)(2M+4n+2)}
\]

(75)

where

\[
\tilde{\chi}_{M,n} \triangleq \begin{cases} 
\chi_{\lfloor (M+2n+1)/M \rfloor,M,n} & \text{odd} \\
0 & \text{even}
\end{cases}
\]

is the last term in the sum, when the number of terms in the sum of (74) is odd.

**Lemma 2:** For every integer \( n \geq 0, M > 1 \):

a. For every \( 1 \leq k \leq \lfloor (M+2n+1)/M \rfloor - 1 \):

\[
\chi_{k,M,n} \leq 0
\]

b. For \( k = 1 \):

\[
\chi_{1,M,n} \leq -\psi 3M^2(2M+4n+3)
\]

(2M+2n+2)(2M+4n+2)(3M+2n+2)

(76)

c. \( \tilde{\chi}_{M,n} \) satisfies:

\[
\tilde{\chi}_{M,n} \leq \psi (2M+4n+3)
\]

(2M+4n+2)! (2M+4n+4)!

(77)

Proof: at the end of the Appendix.

Using part a. of Lemma 2 and (75), we can upper bound \( \Delta_{M,n} \), removing \( \chi_{k,M,n} \), for all \( 2 \leq k \leq \lfloor (M+2n+1)/M \rfloor - 1 \):

\[
\Delta_{M,n} \leq \frac{1}{\Phi} \leq \frac{1 + (M+2n+2)(2M+4n+3)}{(M+2n+2)(2M+4n+2)!} + \chi_{1,M,n} + \tilde{\chi}_{M,n}.
\]

(78)

Using also parts b. and c. of Lemma 2 we further bound \( \Delta_{M,n} \) by:

\[
\Delta_{M,n} \leq \frac{2(M+2n+2)(2M+4n+3)}{(M+2n+2)(2M+4n+2)!} - \frac{3M^2(2M+4n+3)}{(2M+4n+3)(3M+2n+3)}
\]

(79)

To continue with the upper bound, we decrease the numerator of the positive fraction and increase the denominator of the negative fraction:

\[
\Delta_{M,n} \leq \frac{1 + (M+2n+2)(2M+4n+3)}{(M+2n+2)(2M+4n+2)!} - \frac{3M^2(2M+4n+3)}{(2M+4n+3)(3M+2n+3)}
\]

(79)

where the last inequality used the fact that \( M > 1 \). Simplifying (79) further, we have

\[
\frac{\Delta_{M,n}}{2\psi \Phi (2M+4n+3)} \leq \frac{1}{(2M+4n+2)!} - \frac{1}{(2M+2n+2)}.
\]

Now, noticing that the factorial term contains all of the terms in the denominator of the second term, we conclude that for any \( n \geq 0 \) and \( M > 1 \):

\[
1 > \frac{(2M+2n+2)(2n+2)(3M+2n+3)(M+n+1)}{(2M+4n+2)!}.
\]

Recalling also that \( \psi \) and \( \Phi \) are positive we conclude that \( \Delta_{M,n} \leq 0 \). Which completes the proof.

**Proof:** [Proof of Lemma 2] The proof of Lemma 2 is divided to two parts: the first is for \( 0 \leq k \leq \lfloor (M+2n+1)/M \rfloor - 1 \):
1)/|M| − 1 and the second for k = [(M + 2n + 1)/|M|]. We start with the first:

\[ \chi_{k,n} = \chi_{k,M,n} + \chi_{k+1,M,n} \]

\[ = \frac{-1}{(M + 2n + 1 + kM!)(M + 2n + 1 - kM)!} \left( 1 - \frac{1}{(M + 2n + 2 + kM)(M + 2n + 2 - kM)} \right) \]

and with common denominator:

\[ \chi_{i,n} = \prod_{i=1}^M 2n + 1 - kM + i - \prod_{i=1}^M 2n + 1 + kM + i \]

\[ = \frac{-1}{(M + 2n + 2 + kM)(M + 2n + 1 - kM)!} \left( 1 - \frac{1}{(M + 2n + 2 + kM)(M + 2n + 1 - kM)} \right) \]

and because k ≤ [(2n + 1)/|M|] we can know that 1 ≤ (2n + 2 - kM) ≤ 2n + 2. Thus, we upper bound again

\[ \chi_{k,n} \leq \frac{-\Psi(2M + 4n + 3)}{(M + 2n + 2 + kM)(2M + 2n + 2 - kM)} \]

For k = 1

\[ \chi_{1,n} \leq \frac{-\Psi(2M + 4n + 3)3M^2}{(2M + 2n + 2)(2n + 2)^2(3M + 2n + 2)} \]

In the second part, where \((M + 2n + 1)/|M|\), if k is even then \(\tilde{\alpha}_M = 0\). Otherwise, if \((M + 2n + 1)/|M|\) is odd, we start with the negative element:

\[ \tilde{\chi}_M,n \]

\[ = \frac{-1}{(M + 2n + 1 + kM!)(M + 2n + 1 - kM)!} \left( 1 - \frac{1}{(M + 2n + 2 + kM)(M + 2n + 2 - kM)} \right) \]

\[ \leq \frac{-\Psi(2M + 4n + 3)}{(M + 2n + 2 + kM)(M + 2n + 2 - kM)} \]

where k = [(M + 2n + 1)/|M|]. Using the fact that \(M + 2n \leq kM \leq M + 2n + 1\) we get

\[ \tilde{\chi}_M,n \leq \frac{-\Psi(2M + 4n + 3)}{(2M + 4n + 2)!} \]

APPENDIX D P0 APPROXIMATION USING THE CENTRAL LIMIT THEOREM

Recall that \({\bar{a}_m : m = 0, \ldots, M - 1}\) is a set of i.i.d. random variables with a uniform distribution over \([0, 2\pi]\).

In the case where \(M\) is odd, the random variable \(\tilde{S}_M \triangleq \frac{1}{M} \sum_{k=0}^{M-1} e^{-\jmath \theta_k}\) converges to a complex Gaussian random variable with expectation

\[ E[\tilde{S}_M] = E \left[ \frac{1}{M} \sum_{k=0}^{M-1} e^{-\jmath \theta_k} \right] = 0 \]

and its variance is given by

\[ \text{Var}[\tilde{S}_M] = \frac{1}{M^2} \sum_{k=0}^{M-1} E[\cos^2(\tilde{\alpha}_k)] \]

\[ = \frac{1}{M} \] (80)

Thus, \(\tilde{S}_M \sim CN(0, \frac{1}{M})\) and \(|\tilde{S}_M|^2\) has an exponential distribution: \(|\tilde{S}_M|^2 \sim \exp(\lambda = M)\). The cumulative distribution function (CDF) of \(|\tilde{S}_M|^2\) is

\[ F_{|\tilde{S}_M|^2}(x) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & \text{o.w.} \end{cases} \] (81)

which leads to the upper line of (66).

If \(M\) is even, the random variable \(\tilde{S}_M \triangleq \frac{2}{M} \sum_{k=0}^{M/2-1} \cos(\alpha_k)\) converges to a real Gaussian random variable where the expectation of \(S_M\) is

\[ E[S_M] = E \left[ \frac{2}{M} \sum_{k=0}^{M/2-1} \cos(\tilde{\alpha}_k) \right] = 0 \] (82)

and variance

\[ \text{Var}[S_M] = E \left[ \frac{2}{M} \sum_{k=0}^{M/2-1} \cos(\tilde{\alpha}_k) \right]^2 \]

\[ = \frac{4}{M^2} \sum_{k=0}^{M/2-1} E[\cos^2(\tilde{\alpha}_k)] \]

\[ = \frac{1}{M} \] (83)

Thus, \(\tilde{S}_M \sim N(0, \frac{1}{M})\). In this case, it is possible to derive the CDF of \(|\tilde{S}_M|^2\) directly from the CDF of \(\tilde{S}_M^2\):

\[ F_{|S_M|^2}(x) = \Pr(|S_M|^2 \leq x) \]

\[ = \Pr(-\sqrt{x} \leq S_M \leq \sqrt{x}) \]

\[ = 1 - 2Q(\sqrt{x/M}). \] (84)
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