Charge and spin addition energies of one dimensional quantum dot.

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We derive the effective action for a one dimensional electron island formed between a double barrier in a single channel quantum wire including the electron spin. Current and energy addition terms corresponding to charge and spin are identified. The influence of the range and the strength of the electron interaction and other system parameters on the charge and spin addition energies, and on the excitation spectra of the modes confined within the island is studied. We find by comparison with experiment that spin excitations in addition to non-zero range of the interaction and inhomogeneity effects are important for understanding the electron transport through one dimensional quantum islands in cleaved-edge-overgrowth systems.

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I. INTRODUCTION

Recently, considerable progress in the science and technology of semiconductor nanostructures has been made. The experimental realization of one-dimensional (1D) quantum wires has opened new routes to investigating the influences of interactions and impurities on electron transport at low temperatures where quantum effects dominate. In spite of numerous theoretical results obtained for the Luttinger liquid model, which have been used for interpreting the data, complete physical understanding of the experiments has not been achieved yet. It is therefore useful to study microscopically the combined effects of interactions and impurities on 1D quantum transport within more realistic models and to compare the results with experimental data.

The Luttinger liquid model allows to take into account interactions between Fermions in 1D exactly. The linear conductance of a clean, infinitely long (spin degenerate) Luttinger liquid has been predicted to be $G = 2e^2/\kappa$ (1), which have been used for interpreting the data, complete physical understanding of the experiments has not been achieved yet. It is therefore useful to study microscopically the combined effects of interactions and impurities on 1D quantum transport within more realistic models and to compare the results with experimental data.

We derive the effective action for a one dimensional electron island formed between a double barrier in a single channel quantum wire including the electron spin. Current and energy addition terms corresponding to charge and spin are identified. The influence of the range and the strength of the electron interaction and other system parameters on the charge and spin addition energies, and on the excitation spectra of the modes confined within the island is studied. We find by comparison with experiment that spin excitations in addition to non-zero range of the interaction and inhomogeneity effects are important for understanding the electron transport through one dimensional quantum islands in cleaved-edge-overgrowth systems.

When the electron density in the GaAs/AlGaAs-quantum wires fabricated by using the cleaved-edge-overgrowth technique is decreased by applying a voltage to an external gate, eventually even the lowest electronic sub-band can be depopulated and the region of Coulomb blockade reached. Here, the mean electron density is sufficiently low, such that only very few maxima of the random potential of the impurities are higher than the Fermi level. A 1D quantum island can be formed between two potential maxima in such a wire. Electron transport is then dominated by charging effects. At temperatures lower than the charging energy, the linear conductance shows discrete peaks corresponding to transferring exactly one electron through the quantum island.

In this region, it has been detected that the temperature dependence of the intrinsic width $\Gamma$ of several conductance peaks is modified by the correlations between the electrons and, instead of being independent of $T$, shows non-analytical power-law behaviour ($\lambda > 0$)

$$\Gamma(T) \propto T^{-\lambda}.$$ (2)

Such a behaviour has been predicted in the sequential tunneling regime for resonant transport through a quantum dot coupled to a spinless Luttinger liquid and for a double barrier in a Luttinger liquid.

In non-linear transport, collective excited states of the electrons in the island can contribute to the quantum wire which eventually determine the conductance. However, at lower temperatures, temperature-dependent deviations from the universal conductance steps have been found

$$\delta G \propto T^\kappa$$ (1)

($\kappa < 0$). This has been attributed to weak scattering at the random impurity potential in a Luttinger liquid.
non-linear conductance showed an interaction-induced non-analytical temperature dependence consistent with the above Luttinger liquid behaviour \[2\]. Thus, correlations beyond the phenomenological charging model seem to be present in these semiconductor quantum wires. This is also indicated by recent results of Raman scattering experiments, which strongly suggest that semiconductor quantum wires are very probably dominated by non-Fermi liquid behaviour \[23\]. In view of the existing broad theoretical understanding of 1D non-Fermi liquids \[24\], it is highly desirable to provide quantitative results for charging and correlation phenomena in these systems by using reasonably realistic and controllable models, including non-zero range interaction between the electrons as well as the potential of the impurities.

Efforts in this direction have been made by studying transport through a single barrier in a Luttinger liquid including long range interactions \[27–30\]. Charging in the presence of two impurities has been studied for spinless electrons with interaction range much smaller than the distance between the two impurities \[31\]. The crossover from a Luttinger liquid with long-range interaction to a Wigner crystal in the presence of two symmetrically arranged potential barriers has been investigated \[32\].

In the present paper, we establish the fundamentals of a transport theory for 1D quantum dots embedded in a 1D electron system of correlated electrons with spin which turns out to be considerably different from the spinless case. Especially, we concentrate here on the energies for charge and spin additions to the electron island formed between two strong impurities in a single-channel quantum wire of diameter \(d\) with an electron interaction of non-zero range. The quantum wire is described by using the Luttinger liquid model with spin \[20\]. The bias electric field is assumed to have an arbitrary spatial shape. The interaction is assumed as a 3D Coulomb potential, which we project to 1D by using the Gaussian wave function associated with the lowest state in a parabolic, cylindrically symmetric confinement potential. Screening is introduced by an infinite metallic plane at a distance \(D\) parallel to the wire. For comparison, we comment also on results obtained for a projected screened 3D Coulomb interaction.

With the imaginary-time path-integral method, we determine the effective action in which charging and transport contributions are identified. The strength and the range of the interaction, \(V_0\) and \(D\), respectively, and the length of the 1D quantum island – the distance between two delta-function-like potential barriers \(a\) – determine characteristic energy scales related to the charge and spin excitations that appear in the model. We find that the characteristic energy related to the addition of charge to the island – the charging energy – always increases with increasing interaction range until it saturates when the latter exceeds considerably the length of the island. On the other hand, the energy related to the addition of a spin – the spin addition energy – is approximately the same as that of non-interacting particles induced by the Pauli principle. This is due to the smallness of the (short-ranged) exchange interaction.

We observe that the spatial shape of the electric field enters the effective action. The corresponding term can be interpreted as simulating the influence of the voltage at a gate electrode in experimental realizations.

Using our results, we have analyzed recent experimental data \[13\]. We attempted to deduce the parameters of the quantum wire used in the experiment quantitatively. We found that it is not possible to identify a region of parameters which is consistent with all of the experimental findings. We conclude that depending on the quantity considered, the frequency spectra of the collective excitations or the temperature dependence of the peaks in the differential conductance, experiments probe the interaction in different regions of the 1D sample, namely within the island itself or within the whole quantum wire extending along the edge of the sample. This indicates that, together with spin and finite interaction range, inhomogeneity effects have to be taken into consideration for understanding the quantum transport in these 1D semiconductor systems.

The paper is organized as follows. Section \(\text{II}\) briefly describes the model. In Section \(\text{III}\), the effective action is provided and discussed. In Section \(\text{IV}\) we comment on the conditions current transport. Quantitative results for the charging energy are given in Section \(\text{V}\). Finally, we analyze experimental data in Section \(\text{VI}\).

II. THE MODEL

A. The Hamiltonian

We use the bosonization technique \[2,28\] for interacting electrons with spin in 1D. The quantum dot is described by a double barrier consisting of two delta-function potentials \(V_i \delta(x - x_i)\) at \(x_i (i = 1, 2)\) where \(x_1 < x_2\). An external electric field \(E(x, t) = -\partial_x U(x, t)\) is assumed to induce transport. The Hamiltonian is

\[
H = H_0 + H_B + H_U. \tag{3}
\]

The first term describes the interacting electrons within the Luttinger liquid model \[31\].

\[
H_0 = \frac{\hbar v F}{2} \int dx \left[ \Pi^2_\rho(x) + (\partial_x \vartheta_\rho(x))^2 \right] + \frac{1}{\pi} \int dx \int dx' \partial_x \vartheta_\rho(x) V(x - x') \partial_{x'} \vartheta_\rho(x') + \frac{\hbar v_F g_\sigma}{2} \int dx \left[ \Pi^2_\sigma(x) + \frac{1}{g_\sigma^2} (\partial_x \vartheta_\sigma(x))^2 \right], \tag{4}
\]

where \(v_\sigma\) is defined below and
\[ g_\sigma = \left( \frac{1 + \eta_{\text{ex}}}{1 - \eta_{\text{ex}}} \right)^{1/2}, \tag{5} \]

with the exchange interaction matrix element \( \eta_{\text{ex}} \equiv V(2k_F)/2\pi \hbar v_F. \) In Eq. (4), the electrons are represented by conjugate bosonic fields \( \Pi_\rho, \vartheta_\rho, \) and \( \Pi_\sigma, \vartheta_\sigma \) associated with the collective charge and spin density excitations, respectively. The system length is \( L \) and the Fermi velocity \( v_F. \) The interaction, \( V(x - x'), \) is a projection of a modified 3D Coulomb interaction onto the \( x \)-direction (see below). It has the Fourier transform \( \hat{V}(q) \) which is the dominant quantity in the dispersion relation of the charge excitations [33].

\[ \omega_\rho(q) = v_F |q| \left[ 1 - \eta_{\text{ex}}^2 + (1 + \eta_{\text{ex}}) \frac{2\hat{V}(q)}{\pi \hbar v_F} \right]^{1/2} \equiv v_\rho(q)|q|, \tag{6} \]

The dispersion relation of the spin excitations,

\[ \omega_\sigma(q) = v_F |q| \left[ 1 - \eta_{\text{ex}}^2 \right]^{1/2} \equiv v_\sigma|q|, \tag{7} \]

contains as the dominant part the exchange interaction which is generally very small compared with \( \hat{V}(q) \) at small \( q \) [23,33]. This implies that the group velocity of the spin excitations, \( v_\sigma, \) is very close to the Fermi velocity, \( g_\sigma \approx 1. \) In the following, \( \eta_{\text{ex}} \) will therefore be omitted whenever it is only a small correction.

The contribution of the two localized impurities is

\[ H_B = \rho_0 \sum_{i=1,2} V_i \cos \left[ 2k_F x_i + \sqrt{2\pi} \vartheta_\rho(x_i) \right] \times \cos \left[ \sqrt{2\pi} \vartheta_\sigma(x_i) \right] \tag{8} \]

where \( \rho_0 = 2k_F/\pi \) is the mean electron density. Equation (9) is the potential energy of the impurities corresponding to the number density of charges

\[ \rho(x) = \rho_\uparrow(x) + \rho_\downarrow(x) = \rho_0 + \sqrt{\frac{2}{\pi}} \vartheta_\rho(x) + \rho_0 \cos \left[ 2k_F x + \sqrt{2\pi} \vartheta_\rho(x) \right] \cos \left[ \sqrt{2\pi} \vartheta_\sigma(x) \right]. \tag{9} \]

The second term in Eq. (9) accounts for the slowly varying part of the charge fluctuations. The third term represents the charge density wave involved in the \( 2k_F \) backscattering interference between left and right moving electrons. It also couples the charge with the long wave length part of the density of spins,

\[ \rho_\uparrow(x) - \rho_\downarrow(x) \approx \sqrt{\frac{2}{\pi}} \vartheta_\uparrow \vartheta_\sigma(x), \tag{10} \]

which is considered here with respect to a zero mean value. When calculating the energy of the impurities, one obtains also a contribution due to the second term in (9), besides the above \( H_B. \) As this represents a forward scattering process, it can be eliminated by a unitary transformation and will not be considered any further.

On the other hand, if a potential is slowly varying on the scale \( k_F^{-1} \) its dominant contribution is mainly due to the long wave length part of the density (9). We assume this to be the case for the bias electric field. The corresponding term in the Hamiltonian is, with the elementary charge \( e(>0) \)

\[ H_U = -e \sqrt{\frac{2}{\pi}} \int \text{d}x U(x,t) \partial_x \vartheta_\rho(x). \tag{11} \]

The presence of the two localized impurity naturally separates the charge and spin degrees of freedom at “bulk” positions \( x \neq x_1, x_2 \) from those at the barriers. It is useful to introduce symmetric and antisymmetric variables for particle (\( \nu = \rho \)) and spin densities (\( \nu = \sigma \))

\[ N_{\nu}^\pm = \sqrt{\frac{2}{\pi}} \vartheta_\nu(x_2) \pm \vartheta_\nu(x_1). \tag{12} \]

The quantity \( N_{\rho}^- \) is associated with the fluctuations of the particle number within the island as compared to the mean particle number \( n_0 = \rho_0(x_2 - x_1). \) The corresponding excess charge is \( Q = -eN_{\rho}^- \); \( N_{\sigma}^- \) represents the \( z \)-component of the fluctuation of the number of spins within the island corresponding to a change of spin \( N_{\sigma}^-/2 \). The numbers of imbalanced particles and spins between left and right leads are represented by \( N_{\nu}^+. \) The DC current-voltage characteristic can then be evaluated by considering the stationary limit of the charge transfer through the dot in the presence of an external voltage

\[ I = \frac{e}{2} \lim_{t \to \infty} \langle \hat{N}_{\rho}^+(t) \rangle. \tag{13} \]

The brackets \( \langle \cdots \rangle \) include a thermal average over the collective excitations at \( x \neq x_1, x_2, \) and a statistical average performed with the reduced density matrix for the degrees of freedom at \( x = x_1, x_2. \)

B. The Interaction Energy

We consider in the following two models which can be used for discussing experimental results quantitatively.

1. Model 1.

Because of gates and surrounding charges in the experiments done on quantum wire, screening is always present for the interaction between the electrons which normally interact via the 3D Coulomb interaction. Although the geometry of the gates and the wire are certainly more difficult in the experiment [33], we assume
in the following that we can describe the screening as being solely due to an infinite metallic plane at a distance $D$ in the $y$-direction parallel to the $(x,z)$-plane. The quantum wire is assumed in the $x$-direction. The effective electron-electron interaction energy can then be calculated by using the method of image charges

$$V(r - r') = \frac{V_0}{|r - r'|} - \frac{V_0}{\sqrt{(x - x')^2 + (z - z')^2 + (y + y' - 2D)^2}}, \quad (14)$$

with $V_0 = e^2/4\pi\varepsilon_0\varepsilon$ and the dielectric constant $\varepsilon_0\varepsilon$.

We assume for simplicity a parabolic confinement of the electrons perpendicular to the wire. The effective interaction in the lowest subband can then be obtained from (14) by projecting with a normalized Gaussian confinement wave function corresponding to the diameter $d$ of the wire. The Fourier transform of the resulting effective interaction potential is for $D \gg d$

$$\hat{V}(q) = V_0 \left[ e^{i q^2 r^2 / 4} E_1 \left( \frac{d^2 q^2}{4} \right) - 2K_0(2Dq) \right], \quad (15)$$

where $E_1(z)$ is the exponential integral and $K_0(z)$ the modified Bessel function [34]. This expression shows explicitly how the gate screens the Coulomb interaction. In the limit $qd \to 0$, one obtains the finite value $\hat{V}(q \to 0) = 2V_0 \left[ \gamma / 2 + \log(2D/d) \right]$ ($\gamma = 0.57722$ Euler constant). This implies a finite interaction constant

$$g_\rho \equiv \frac{V_0}{\rho(q \to 0)} = \left[ 1 + \eta \gamma + 2\eta \log \left( \frac{2D}{d} \right) \right]^{-1/2}. \quad (16)$$

where $\eta = 2V_0 / \pi \hbar v_F$. The 1D equivalent of the unscreened Coulomb interaction is obtained for $D \to \infty$.

2. Model 2.

We also consider the screened Coulomb interaction

$$\hat{V}(r - r') = V_0 e^{-\alpha |r - r'|}, \quad (17)$$

with a phenomenological screening length $\alpha^{-1}$. The Fourier transform of its Gaussian projection to 1D is [30]

$$V(q) = V_0 e^{(d^2/4)(q^2 + \alpha^2)} E_1 \left( \frac{d^2}{4} q^2 + \alpha^2 \right), \quad (18)$$

and the corresponding interaction constant

$$g_\rho = \left[ 1 - \eta \gamma + 2\eta \log \left( \frac{2}{\alpha d} \right) \right]^{-1/2}. \quad (19)$$

III. THE EFFECTIVE ACTION

In order to evaluate the current-voltage characteristic one has to perform a thermal average over the “bulk modes” at $x \neq x_1, x_2$. This can be done with the imaginary-time path integral method [33]. The result of the integration is an effective action $S_{\text{eff}}$ which depends only on the four variables defined in (12). In the continuous limit ($L \to \infty$), with the inverse temperature $\beta = 1/k_B T$ one obtains

$$S_{\text{eff}}[N_\rho^+, N_\rho^-] = \int_0^{\hbar\beta} d\tau H_B[N_\rho^+, N_\rho^-]$$

$$+ \sum_{\tau = \pm} \sum_{\nu = \rho, \sigma} \left[ \int_0^{\hbar\beta} d\tau d\tau' N_\nu^+(\tau) K_\nu^+(\tau - \tau') N_\nu^-(\tau') \right. \left. \right]$$

$$\quad - \delta_{\rho,\sigma} \int_0^{\hbar\beta} d\tau N_\rho^+(\tau) \mathcal{L}^+(\tau). \quad (20)$$

The Fourier transforms, at Matsubara frequencies $\omega_n = 2\pi n / \hbar \beta$, of the dissipative kernels $K_\nu^+(\tau)$ and of the effective “forces” $\mathcal{L}^+(\tau)$ are determined by the dispersion relations of the collective modes [1] and [3], respectively,

$$[K_\rho^+(\omega_n)]^{-1} = \frac{8\sqrt{\rho q}}{\hbar \pi^2} \int_0^{\infty} dq \frac{1 + \cos[q(x_1 - x_2)]}{\omega_n \omega_n + \omega_n^2(q)}, \quad (21)$$

$$\mathcal{L}^+(\omega_n) = \frac{4e v_F}{\hbar \pi^2} K_\rho^+(\omega_n) \int_{-\infty}^{\infty} dx \mathcal{E}(x, \omega_n)$$

$$\times \int_0^{\infty} dq \frac{\cos[q(x - x_2)] + \cos[q(x - x_1)]}{\omega_n^2 + \omega_n^2(q)}. \quad (22)$$

Both, $K_\rho^+$ and $\mathcal{L}^+$ contain the collective bulk modes which introduce the interaction effects to be described below. First of all, we note that $K_\rho^+(\omega_n \to 0) = 0$ [21,35]. On the other hand, $K^-_\rho(\omega_n \to 0) \neq 0$. The latter describe the costs in energy for changing the numbers of charges and/or spins on the island between the potential barriers. The corresponding Euclidean action is

$$S_0[N_\nu^-, N_\nu^-] = \sum_{\nu = \rho, \sigma} \frac{E_\nu}{2} \int_0^{\hbar\beta} d\tau (N_\nu^-)^2, \quad (23)$$

with the characteristic energies

$$E_\nu = 2K^-_\rho(\omega_n \to 0) \quad (\nu = \rho, \sigma). \quad (24)$$

For $\nu = \rho$, this corresponds to the charging energy that is supplied/gained, in order to transfer/remove one charge to/from the island as compared with the mean value, $N_\rho^- = \pm 1$. Correspondingly, for $\nu = \sigma$, the spin addition energy $E_\sigma$ is needed/gained in order to change the spin by exactly $\pm 1/2$. The Coulomb interaction in the dispersion relation of the charge excitations, increases considerably
the charging energy $E_\rho$, in comparison with the spin addition energy $E_\sigma$, which is only influenced by the (small) exchange interaction. We always expect $E_\rho > E_\sigma$.

The frequency dependent parts of the kernels describe the dynamical effects of the external leads and of the correlated excited states in the dot. Their influence is described by spectral densities $J_v^\pm(\omega)$ related via analytic continuation to the imaginary-time kernels 20,21:

$$J_v^\pm(\omega) = \frac{2}{\pi \hbar} \text{Im} K_v^\pm(\omega_n \to +i\omega).$$

Due to the non-zero range of the interaction, analytic expressions for these densities are not available. However, one can always extract their limits for $\omega \to 0$,

$$J_v^\pm(\omega \to 0) = \frac{A_v^\pm(g_0)}{4g_0} \omega,$$

where $A_v^\pm = g_0^2(E_\rho/E_0)^2$ ($E_0 = \hbar \pi v_0/2a$), and for the three other combinations of indices $A_v^\pm = 1$. This limit describes the dissipative influence of the low-frequency charge and spin excitations in the external leads, $x < x_1$ and $x > x_2$. It holds also for finite frequencies. However, these must be smaller than the frequency scale corresponding to the range of the interaction, and smaller than the characteristic excitation energy of the correlated electrons in the dot.

Let us now discuss the driving forces. In general, $L^\pm(\tau)$ depend in a quite complicated way on the dispersion of the collective modes and on the shape of the electric field. We focus on the DC limit where it is sufficient to evaluate the Fourier components for $\omega_n \to 0$. In this case, the quantity $L^+(\tau)$, which acts on the total transmitted charge, depends only on the integral of the time independent electric field over the entire system, the source-drain voltage $U \equiv \int_{-\infty}^{\infty} dx \mathcal{E}(x)$,

$$L^+(\tau) = \frac{eU}{2}.$$  

Since $L^+$ is the part of the effective force that generates the current transport, this result generalizes the one obtained previously for only one impurity 22. It can be easily derived from Eq. (22) by using the relation

$$\frac{e^2}{\hbar \pi x} \int_0^\infty dq \frac{\omega_n(1 \pm \cos qx)}{\omega_n^2 + \omega_n^2(q)} = \sigma_0(0, \omega_n) \pm \sigma_0(x, \omega_n).$$

Here, $\sigma_0(x, \omega_n)$ is the frequency dependent non-local conductivity of the Luttinger liquid per spin channel 23, with the DC limit $\sigma_0(0, 0) = g_0 e^2/\hbar$.

On the other hand, $L^-(\tau)$ acts on the excess charge on the island, it does not generate a current. It depends on the spatial shape of the electric field and can formally be written in terms of the total charge $Q_\mathcal{E}$ accumulated between the points $x_1$ and $x_2$ in the absence of the barriers as a consequence of the presence of the DC electric field

$$L^- = \frac{e Q_\mathcal{E}}{e},$$

where the charge is given by

$$Q_\mathcal{E} = 2 \int_{-\infty}^{\infty} dx' \mathcal{E}(x') \times \lim_{\omega \to 0} \left[ \frac{\sigma_0(x_1 - x', -i\omega) - \sigma_0(x_2 - x', -i\omega)}{i\omega} \right].$$

Equivalently, this can also be understood in terms of addition energies. By introducing explicitly in Eqs. (22, 23) the dependence on the interval considered when evaluating the addition energies, one finds

$$L^-(\tau) = \frac{e}{2} \frac{Q_\mathcal{E}(x_1 - x_2)}{E_\rho(x_1 - x_2)} \times \int_{-\infty}^{\infty} dx \mathcal{E}(x) \left[ \frac{1}{E_\rho(x - x_1)} - \frac{1}{E_\rho(x - x_2)} \right].$$

It is reasonable to assume $x_{2,1} = \pm a/2$. If the effective electric field has inversion symmetry, $L^-$ vanishes. Without inversion symmetry, the electric field generates an effective charge on the island which will influence the total current via coupling between $N_\rho^+$ and $N_\rho^-$ due to the impurity Hamiltonian $H_B$. Physically, this externally induced charge may be thought of as being generated by a voltage $V_C$ applied to an external gate which electrostatically influences the charge on the island. Thus, the above Eq. (23) can be interpreted as a term representing the effect of the gate voltage in the phenomenological theory of Coulomb blockade.

**IV. CONDITIONS FOR CURRENT TRANSPORT**

So far, we have discussed the influence of the spin and charge bulk modes on the four variables $N_\rho^\pm$ that describe the quantum dot. In order to calculate explicitly the electrical current one has to solve the equations of motion for the $N_\rho^\pm$. This is beyond the scope of the present work and will be discussed elsewhere. Here, we briefly describe the results that are needed in the following.

For barriers much higher then the charging energy, $V_i \gg E_\rho$, the dynamics is dominated by tunneling events connecting the minima of $H_B$ in the 4D ($N_\rho^+, N_\rho^-, N_\sigma^+, N_\sigma^-$)-space 22. For equal barriers, $V_1 = V_2 = V$, the impurity Hamiltonian is

$$H_B = \rho_0 V \left[ \cos \frac{\pi N_\rho^+}{2} \cos \frac{\pi N_\rho^-}{2} \cos \frac{\pi (n_0 + N_\rho^-)}{2} \cos \frac{\pi N_\sigma^-}{2} + \sin \frac{\pi N_\rho^+}{2} \sin \frac{\pi N_\rho^-}{2} \sin \frac{\pi (n_0 + N_\rho^-)}{2} \sin \frac{\pi N_\sigma^-}{2} \right].$$

(32)
The transitions between the minima of this function of four variables correspond to different physical processes of transferring electrons from one side to the other of the quantum dot. At very low temperature, the dominant processes transfer the electron coherently through the dot. In particular, when the number of particles in the island is an odd integer there is spin degeneracy, \( N^\sigma = \pm 1 \). The island acts as a localized magnetic impurity, similar as in the Kondo effect \( [7] \). This leads to a Kondo resonance in the transport through the island \( [17,38] \).

On the other hand, if the temperature is higher than the tunneling rate through the single barrier, the dominant processes are sequential tunneling events \( [19,21] \). The transfer of charge occurs via uncorrelated single-electron hops into, and out of the island, associated with corresponding changes in the total spin. This is precisely the regime that recently has become accessible by using cleaved-edge-overgrowth quantum wires \( [15] \). In this region, one has to consider minima which correspond to pairs of even or odd \( N^\rho \) and \( N^\sigma \). The dominant transport processes are those which connect minima via transitions \( N^\rho \rightarrow N^\rho \pm 1 \) associated with changes of the spin \( N^\sigma \rightarrow N^\sigma \pm 1 \). For each of these processes also the external charge and the spin change by \( N^\rho \rightarrow N^\rho \pm 1 \) and \( N^\sigma \rightarrow N^\sigma \pm 1 \).

The degeneracy of these minima is lifted by the charge and spin addition energies \( E^\rho \) and \( E^\sigma \) which force the system to select favourable charge and spin states in the island. These selections become essential at low temperatures, \( k_B T < E_v \), when current can flow only under resonant conditions. The latter can be achieved in experiment by tuning external parameters, like the source-drain voltage or the gate voltage, in order to create degenerate charge states in the island.

### A. Linear Transport

In the linear regime, \( U \to 0 \) for \( T = 0 \), starting with the island occupied by \( n \) electrons, we expect that another electron can enter and leave only if the difference between the ground state energies of \( n + 1 \) and \( n \) electrons is aligned with the chemical potential of the external semi-infinite Luttinger systems. The ground state of an even number of electrons in the 1D island has the total spin 0. On the other hand, the ground state of an odd number of electrons can be assumed to have the spin \( N^\sigma = \pm 1 \) \( [24,39] \). This implies the resonance condition

\[
U(n + 1, \pm s_{n+1}) - U(n, \pm s_n) = 0 \tag{33}
\]

with \( U(n, \pm s_n) \) the ground state energies with \( n \) particles and total spins \( s_n = 0 \) (\( n \) even) or \( s_n = 1/2 \) (\( n \) odd), respectively.

With the above charge, spin and external gate terms, Eqs. \( (24) \) and \( (29) \), these conditions become

\[
E^\rho \left( n - n_0 - n_G + \frac{1}{2} \right) + (-1)^n E^\sigma \frac{2 \delta}{\rho^2} = 0. \tag{34}
\]

The variable \( n_G = e V_G \delta / E^\rho \) represents the number of induced particles due to the coupling to a gate at which the voltage \( V_G \) is applied, with a proportionality factor \( \delta \) which can be determined experimentally. The zero of energy has been assumed to be given by the external chemical potential in Eq. \( (33) \).

From the above expression one can see that the distance of the peaks of the linear conductance when changing the gate voltage are given by \( \Delta V_G = (E^\rho + (-1)^n E^\sigma) / e \delta \approx E^\rho / e \delta \) since \( E^\sigma \ll E^\rho \). Having independent information on \( \delta \) one can extract the value of the charging energy \( E^\rho \) (in principle also for the spin addition energy \( E^\sigma \)) from the experimental data.

In order to evaluate the current as a function of temperature and/or source-drain voltage, one needs to consider the behavior of the spectral densities given in Eq. \( (23) \). In the sequential tunneling regime, one can show that only their sum enters the transport \( [23] \):

\[
J(\omega) = \sum_{r=\pm} \sum_{\nu=\rho,\sigma} J^r_\nu(\omega). \tag{35}
\]

The frequency behavior of this determines the current-voltage characteristics both in the linear and in the nonlinear regimes (see below). In particular, it determines the exponent of the power-law dependencies of the current as a function of temperature and/or the bias voltage.

For temperatures lower than the excitation energy in the quantum dot, the low-frequency behavior of the spectral density is mainly determined by the charge and spin excitations in the leads. Thus, the power-law dependence of the conductance peaks is dominated by the interaction in the regions of the quantum wire outside of the electron island. From Eqs. \( (22,23) \) one obtains

\[
J(\omega) \approx J_{\text{leads}}(\omega) = \frac{\omega}{2} \left( \frac{1}{g^\rho} + \frac{1 + A^\rho}{2g^\rho} \right). \tag{36}
\]

Eq. \( (36) \) generalizes the results obtained previously for spinless electrons and zero-range interaction \( [13,21] \), where \( J(\omega) \approx \omega / g^\rho \). We conclude that the presence of the spin in the leads introduces an effective interaction strength

\[
\frac{1}{g^\text{eff}} = \frac{1}{4} \left( \frac{1 + A^\rho}{g^\rho} + \frac{2}{g^\sigma} \right), \tag{37}
\]

which determines the exponents of the power laws.

For example, it has been shown for spinless electrons \( [13,21] \) that the intrinsic width \( \Gamma(T) \) of the linear conductance Coulomb peak depends on the temperature. For low temperatures this has been found to be given by \( \Gamma(T) \propto T^{1/2} \). Correspondingly, with spin, one finds instead

\[
\Gamma(T) \propto T^{1/2g^\sigma}. \tag{38}
\]
B. Non-Linear Transport

One can investigate the non-linear current-voltage characteristic by increasing the source drain voltage $U$. In this case, the current-voltage characteristic shows the Coulomb staircase associated with transitions between successive ground states of the electrons in the quantum dot. In addition, fine structure appears which reflects the excitation spectra of the correlated electrons. They can be due to collective charge and spin modes or to particular polarizations of the spins, $N_{\sigma^-} \neq 0, \pm 1$. The former, for a voltage drop $U$ at the potential barriers (fixing the chemical potential in one of the leads to be equal to that in the dot), has a maximum periodicity

$$U = \mu_0(n+1) - \mu_0(n) = E_\rho + (-1)^{n+1} E_\sigma$$ \hspace{1cm} (39)

where $\mu_0(n)$ is the electrochemical potential of $n$ electrons on the quantum island.

In the present model, the possible particle-hole excitations are collective spin and charge modes confined within the island. In a completely isolated island, the corresponding energy spectrum would be discrete, $\omega_\rho(q_m)$ and $\omega_\sigma(q_m)$, due to the discretization of the wave number $q_m = \pi m/a$ associated with the confinement. Due to the coupling to the electrons in the leads via the interaction, these levels are broadened. In the following argument, we assume that this broadening is negligible.

The screened Coulomb interaction causes a non-linear dispersion relation for the charge modes in the infinite Luttinger system. This leads to non-equidistant charge excitation energies in the quantum dot,

$$\Delta \epsilon_\rho(q_m) = \hbar [\omega_\rho(q_{m+1}) - \omega_\rho(q_m)] .$$ \hspace{1cm} (40)

Their explicit values depend on the ratio between the distance $a$ and the range of the interaction, $D$ or $a^{-1}$.

For values of $a$ much larger than this range, the first excited charge modes are equidistant. They are given by the charge-sound velocity for $q \to 0$, $v_\rho = v_F / g_\rho$,

$$\Delta \epsilon_\rho = \frac{\hbar \pi v_\rho}{a} = \frac{\hbar \pi v_F}{ag_\rho} = \frac{2 E_0}{g_\rho} .$$ \hspace{1cm} (41)

In the opposite limit, the non linear dispersion is already affecting strongly the first excitation. This results in a value smaller than Eq. (41), for model 1,

$$\Delta \epsilon_\rho = \frac{\hbar \pi v_F}{a} \sqrt{1 - \gamma \eta + 2 \eta \log \left( \frac{2 a}{\pi d} \right)} \equiv \frac{2 E_0}{f_\rho} .$$ \hspace{1cm} (42)

For the spin excitations, the dispersion in the infinite system is linear and the discrete spectrum is equidistant

$$\Delta \epsilon_\sigma = \frac{\hbar \pi v_\sigma}{a} \equiv 2 E_0 \frac{v_\sigma}{v_F}$$ \hspace{1cm} (43)

with the spin mode velocity $v_\sigma (\approx v_F)$.

V. THE CHARGING ENERGY

The above discussion emphasizes the importance of charge and spin addition energies, and of the charge and spin excitation energies for the linear and non-linear transport properties. It is therefore crucial to evaluate these quantities microscopically and determine their dependencies on the model parameters, especially in view of comparisons with the experimental data of [13].

We analyze now in more detail the charging and spin energies of the Luttinger island as a function of the parameters of above Model 1. In addition to the interaction strength $V_0$, which contains as an essential quantity the dielectric constant $\varepsilon$, we have the Fermi energy $E_F$ and three geometrical parameters: the distance $D$ between the 1D system and the infinite metallic plane, the diameter $d$ of the wire and the length $a$ of the island.

The spin addition energy can be easily evaluated from the simple dispersion relation Eq. (46)

$$E_\sigma = \frac{\pi \hbar v_F}{2 a g_\sigma} .$$ \hspace{1cm} (44)

It is the same as the addition energy of non-interacting particles which is related to the Pauli principle and due to the quasi-discretization of the spin energies in the dot.

The charging energy $E_\rho$ is evaluated numerically from Eq. (24) as a function of $a/d$. It is shown in Fig. 3 for different ratios $D/d$ and different interaction parameters $V_0/d$ with $d = 10$nm and $d = 20$nm. Different $V_0/d$ correspond to different $\varepsilon$. Changing $E_F$ between 2meV and 4meV, values that cover the experimentally relevant region, changes the curves in Fig. 3 only less than 10%. For small $a/d$, the charging energy diverges. We consider only the region $a/d > 1$. Asymptotically,

$$\frac{E_\rho}{E_0} \approx \left[1 + \eta \gamma + 2 \eta \log \left( \frac{2 D}{d} \right) \right] \equiv \frac{1}{g_\rho^2} (D \ll a) ,$$ \hspace{1cm} (45)

$$\frac{E_\rho}{E_0} \approx \left[1 - \eta \gamma + 2 \eta \log \left( \frac{2 a}{\pi d} \right) \right] \equiv \frac{1}{f_\rho^2} (D \gg a) .$$ \hspace{1cm} (46)

$E_\rho$ is only weakly influenced by $D/d$: changing $D/d$ by a two orders of magnitude changes $E_\rho$ only by about 30%.

The charging energy always increases with increasing interaction range $D$, because the cost of energy for putting additional electrons into the island increases. Interaction ranges much larger than $a$ do not change the charging energy because only the short-range part of the repulsion between the electrons contributes. Therefore, $E_\rho$ approaches the asymptotic value of Eq. (46) (Fig. 3, curves for largest $D/d$). For strong Coulomb interaction ($\eta = 2 V_0 / \pi \hbar v_F \gg 1$) and $a \gg d$, this is

$$E_\rho = \frac{e^2}{2 \pi \epsilon_0 a} \log \left( \frac{2 a}{d} \right) \equiv \frac{e^2}{C} .$$ \hspace{1cm} (47)
Here, the $C$ is the classical self-capacitance of a cylinder of the length $a$. The stronger the interaction (increasing $V_0$) the larger is $E_{\rho}$, very similar to the behaviour of a classical capacitor with a dielectric, $C \propto \varepsilon$.

Very similar results, with $d/D$ replaced by $\alpha d$, are obtained by using Model 2. Basically, this tells that screening effects may well be described by a global parameter, irrespective of the underlying microscopic model.

As mentioned earlier, the non-linear transport shows Coulomb steps with widths given approximately by $E_{\rho}$, and fine structure due to the excited states. For a first estimate of the strength of the interaction it is very useful to estimate how many excited states are present within the energy window given by $E_{\rho}$. Figure 2 shows the ratios between the energy $E_{\rho}$ and the first collective excited (spin or charge) state in the electron island $\Delta \varepsilon_{\nu}$ ($\nu = \sigma, \rho$) for Model 1 as a function of the interaction constant $g_{\rho}$. The change in $g_{\rho}$ is obtained changing the distance $D$ from the gate, $g_{\rho} \rightarrow 0$ corresponds to $D \rightarrow \infty$. The ratio $E_{\rho}/\Delta \varepsilon_{\sigma}$ is always smaller than $E_{\rho}/\Delta \varepsilon_{\sigma}$.

We expect to observe within the window $E_{\rho}$ much more levels due to spin excitations than due to charge excitations; $E_{\rho}/\Delta \varepsilon_{\nu}$ always increases when $g_{\rho}$ decreases (increasing $D$). It saturates for $D \gg a$,

\[
\frac{E_{\rho}}{\Delta \varepsilon_{\rho}} = \frac{1}{2f_{\rho}}, \quad \frac{E_{\rho}}{\Delta \varepsilon_{\sigma}} = \frac{v_{\rho}}{v_{\sigma}} \frac{1}{2f_{\rho}}. \tag{48}
\]

In the opposite limit, $D \ll a$, the behavior is described by the asymptotic expressions

\[
\frac{E_{\rho}}{\Delta \varepsilon_{\rho}} = \frac{1}{2g_{\rho}}, \quad \frac{E_{\rho}}{\Delta \varepsilon_{\sigma}} = \frac{v_{\rho}}{v_{\sigma}} \frac{1}{2g_{\rho}}. \tag{49}
\]

VI. COMPARISON WITH EXPERIMENT

In this section, we compare our results with the experimental data obtained in [14]. In that work, results of the temperature dependence of the intrinsic width of the conductance peaks in the region of Coulomb blockade on quantum wires fabricated with the cleaved-edge-overgrowth technique have been reported. The data have been found to be consistent with the power law

\[
\Gamma(T) \propto T^{1/g^* - 1}, \tag{50}
\]

with $g^* \approx 0.82$ and $g^* \approx 0.74$ for a peak closer to the onset of the conductance and the next lower peak, respectively. In addition, information about the excited energy levels of correlated electrons have been obtained by measuring the non-linear current voltage characteristics. A minimum of five excited levels have been observed for a given electron number. Presumably, the number of excited levels is even larger since the majority of the excited island states is generally not clearly observed in non-linear transport due to matrix element effects [14].

The data have been analyzed previously by assuming that within the quantum wire, a quantum dot has been accidentally formed between two maxima of the random potential of impurities. The electron spin has been neglected. The parameters estimated from the experimental setup are: length of wire $L \approx 5 \mu m$; length of the electron island $a \approx 100 - 200$ nm; (non-spherical) diameter of the wire $d \approx 10 - 25$ nm; distance to the gate $D \approx 0.5 \mu m$. The charging energy, as estimated from the distance between the conductance peaks, has been given as $E_C \approx 2.2$ meV. Since the observed conductance peaks are equidistant within 10% we deduce that $E_{\sigma} \ll E_{\rho}$. A rough estimate of the Fermi energy is $E_F \approx 3$ meV. With these parameters, the value of the interaction constant has been estimated as $g_{\rho} \approx 0.4$ [15], clearly inconsistent with the above $g^*$ determined from the temperature dependence of the conductance peaks.

Using our above microscopic approach which includes the influence of the electron spin and finite range of the interaction, we confirm that there is a discrepancy, though the estimate for $g_{\rho}$ from the temperature dependence of the conductance peaks seems to be slightly improved.

First, we estimate the length of the island $a$ from the charging energy in Fig. 4. As $E_{\rho}$ is relatively insensitive with respect to changes of $D/d$ and $E_F$ in the experimentally relevant region of parameters, we assume $D/d = 50$ with $E_F = 3$ meV and $d = 20$nm. Furthermore, $V_0/d = 6.02$ meV for $\epsilon = 12$. With the above 22 meV we find $a/d \approx 16$. This is consistent with the value given in [14] within the uncertainties.

By taking into account the spin and using an interaction of a finite range, $g^*$ has to be identified with the effective interaction $g_{\text{eff}}$ in Eq. (37). Since for the spin excitations $g_{\sigma} \approx 1$, we find for $g^* = 0.82$ and 0.74 a interaction constant $g_{\text{eff}}^* \approx 0.6 \pm 20\%$ by solving Eq. (37) for $E_{\rho}(g_{\rho})/E_0$ and comparing with $E_{\rho}(g_{\rho})/2E_0 = E_{\rho}/\Delta \varepsilon_{\sigma}$ in Fig. 2. This would imply that $D \approx d$ (Eq. (41)). On the other hand, by using Eq. (14) with the above parameters, especially $D \approx 50d$, we find a interaction constant $g_{\rho} \approx 0.25 \pm 20\%$ depending only weakly on $D/d$. This is clearly not a quantitatively consistent result. The discrepancy can be reduced by changing $\epsilon$. Also, taking into account the exchange correction $\eta_{\text{ex}} (g_{\rho} \neq 1)$ gives some improvement. However, within reasonable values of all of the parameters, one is in any case forced to conclude $g_{\rho}^* \approx 2g_{\rho}$.

Furthermore, with $g_{\rho}^* = 0.6$, one reads from Fig. 2 $E_{\rho}/\Delta \varepsilon_{\rho} \approx E_{\rho}/\Delta \varepsilon_{\sigma} \approx 1$. Thus, about 1 to 2 excited states corresponding to a given electron number should be observed in non-linear transport spectroscopy. This is also not consistent with the experiment. In order to observe a larger number of excited states, the interaction constant must be considerably smaller, $g_{\rho} < 0.3$, such that the number of spin excited states within the given window of $E_{\rho}$ is increased (cf. Fig. 3). Taking into
account the exchange interaction \((v_x < v_F)\) would additionally decrease \(\Delta E_p\) (and \(E_p\)) increase \(E_p\), and thus increase the number of excited states.

Despite the above inconsistency, the energetically lowest of the excited states seen in the non-linear transport are predicted by our approach to correspond to spin excitations. Without the latter there is not at all any possibility to understand the results from the non-linear transport even when using parameters that give consistent results for the interaction constants. Given this scenario, only the energetically highest experimentally observed state would correspond to a charge excitation. The latter could be identified, for instance, with the state denoted in [15] as “strongly coupled excited state”.

One might conclude that the part of above quantitative discrepancy is due to the Luttinger model being questionable for quantum wires in the extreme low-density limit. However, one can also conclude that the data obtained from non-linear transport spectroscopy have to be interpreted by using a different interaction constant than that obtained from the temperature dependence of the transport. This is also supported by the theoretical derivation: the temperature behaviour of the conductance peaks is dominated by the excitations in the whole quantum wire, while the quasi-discrete excitation spectra are showing the interaction in the region of the quantum island. We are thus forced to conclude that inhomogeneity effects and the influence of the contacts are an important issue for the understanding of the correlations in the electron transport in these cleaved-edge-overgrowth quantum wires, in addition to spin effects.

VII. CONCLUSION

In conclusion, we have derived the effective action for a quantum dot formed by a double barrier potential with a realistic long range interaction between the electrons, and, most importantly, including the electron spin. We have identified an effective driving force acting on the charge of the electrons, which turned out to be independent of the shape of the driving electric field. Mass terms originating from the dissipative degrees of freedom in the Luttinger liquid have been found for both the charge and the spin. They are identified with the charging energy in the case of the addition of charge and with a spin addition energy for the spin. Also, an effective force has been found which, in the DC limit, is only non-zero when the driving electric field and/or the barriers are asymmetric. This influences the transport via the coupling of modes at the potential barriers. It can be attributed to charging of the electron island via an external gate.

We have made an attempt to interpret quantitatively a recent experiment which includes linear as well as non-linear transport data in the region of Coulomb blockade. We find that even taking into account the electron spin it is impossible to consistently fit all of the experimental data with the same interaction parameter. It seems that different interaction strengths have to be used for explaining the temperature dependence of the transport on the one hand, and the excitation spectrum of the quantum dot on the other.

A possible conjecture to solve this puzzle is to assume that the temperature behaviour of the conductance peaks is dominated by the interaction among the electrons in the quantum wire along the edge of the whole sample, in contrast to the excitation spectra which feel the interaction strength near the quantum dot. This is qualitatively consistent with the physical origin of the temperature dependence of the conductance, namely the bulk modes outside the electron island. This conjecture would also explain why \(g_p^{*} \approx 0.6\), as estimated from the temperature behaviour, is larger than that obtained from the observed number of the excited states in the island, \(g_p < 0.3\). The interaction is weaker in the leads due to screening induced by the contact regions with the attached 2D electron gases while it can be assumed to be stronger within the island due to the absence of nearby 2D electron gases.

The above interpretation also predicts that the energetically lowest collective excitations in the quasi-1D quantum island observed in the experiment are very likely spin excitations. The energies of the lowest observed collective excited states are so small that it is hard to believe that they can be related to charge modes since \(v_p > v_x\), given the above estimates for the range and strength of the interaction is of the correct order. This is consistent with a previous suggestion based on a semi-phenomenological model in which exact diagonalization and rate equations have been combined in order to describe the non-linear transport through a 1D quantum island containing only few electrons [24].

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FIG. 1. Charging energy $E_p$ in meV for Model 1 as a function of $a/d$ (Fermi energy $E_F = 3$ meV, effective electron mass $m = 0.067m_0$). Top: $d = 10$ nm; interaction strengths $V_0/d = 24.08$ meV (dashed-dotted), 12.04 meV (full lines), 6.02 meV (dotted), corresponding to $\epsilon = 6$, 12, 24 (equivalent to $\eta = 2V_0/\pi \hbar v_F = 2.04, 1.02, 0.51$, respectively), and $D/d = 1000, 100, 10$ (top to bottom). Bottom: $d = 20$ nm; interaction strengths $V_0/d = 12.04$ meV (dashed-dotted), 6.02 meV (full lines), 3.01 meV (dotted) and $D/d = 500, 50, 5$ (top to bottom).
FIG. 2. Ratios $E_\rho/\Delta \epsilon_\nu$ ($\nu = \sigma, \rho$; upper/lower curves for fixed $a/d$) as a function of the interaction constant $g_\rho$ for $a/d = 20, 60, 500$ with $\epsilon = 12$, $d = 20$ nm, $m = 0.067 m_0$, $g_\sigma = 1$. 
