Determination of optimum gear ratios of a two-stage helical gearbox with second stage double gear sets

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Abstract. This paper introduces a study on the optimum calculation of the gear ratios of a two-stage helical gearbox with second stage double gear sets. In this study, for finding the optimum gear ratios of the gearbox, an optimization problem was conducted. In the optimization problem, the objective function was the acreage of the cross section of the gearbox. Besides, the effect of the input parameters including the total gearbox ratio, the wheel face width coefficient, the allowable contact stress and the output torque were considered. For investigation of the influence of these factors on the optimum gear ratios, an “experiment” was designed and a computer program was built for conducting the “experiment”. Also, models for determination of the optimum gear ratios of the gearbox were proposed. By using these models, the gear ratios can be calculated accurately and simply.

1. Introduction
In optimum gearbox design, the determination of the optimum gear ratios is very important. The reason of that is the dimension, the mass, and the cost of a gearbox depend strongly on the gear ratios. Hence, until now there have been many researches on the determination of the gear ratios.

For a two-stage helical gearbox, V.N. Kudreavtev et al. [1] presented a graph method (Figure 1) for finding the gear ratio of the first stage $u_1$. It is imagined that the determination of $u_1$ by using the graph in Figure 1 is very complicated. In addition, for determining $u_1$, we must select the coefficient $\lambda c^3$ (with $\lambda c^3 = 0.6$ to 4) so it is quite complicated and the gear ratio can not reach to optimum values. G. Milou et al. [2] proposed “a practical method” in which the gear ratios were found from practical data. Based on the data from gearbox factories, it was noted that the two-stage gearbox weight is minimum if the ratio of two center distances $a_{w2} / a_{w1}$ ranged from 1.4 to 1.6 [2]. Therefore, the optimal gear ratios were proposed in the tabulated form. In practice, the most common method for determining the optimum gear ratios is the model method in which the optimum gear ratio model is usually found based on the results of the optimization problem. In the optimization problem, the objective can be the minimum volume of gears [3], the minimum cross section dimension of the gearbox [4], the minimum gearbox mass [5] or the minimum mass of gears [6].

For two-stage helical gearbox with second stage double gear sets, Vu Ngoc Pi used the model method for determination of the optimum gear in order to get the minimum cross section [7] and the minimum gearbox length [8]. However, in these studies, the influence of the input parameters on the...
optimum gears was still not evaluated.

![Figure 1](image1)

**Figure 1.** The gear ratio of the stage 1 versus the total gearbox ratio [1].

This paper introduces a study for the determination of the optimum gear ratios of a two-stage helical gearbox with second stage double gear sets. In this study, an optimization problem in which the objective is the minimum gearbox cross-sectional acreage was carried out. In addition, the effect of the input factors on the optimum gear ratios was investigated.

2. Optimization problem

![Figure 2](image2)

**Figure 2.** Calculation schema.

With a two stage helical gearbox with second stage double gear sets, the acreage of cross section is calculated by (Figure 2):

\[ A = L \cdot h \]  

(1)

where, \( L \) and \( h \) are determined by the following equations:

\[ L = \frac{d_{w1}}{2} + a_{w1} + a_{w2} + \frac{d_{w2}}{2} \]  

(2)

\[ h = \max(d_{w21}, d_{w22}) \]  

(3)

In equations (2) and (3), \( a_{w1} \) and \( a_{w2} \) are the center distance of the first and the second stage; \( d_{w1} \), \( d_{w21} \) and \( d_{w22} \) are pitch diameters (mm) of the first and the second stage, respectively. The diameters \( d_{w1} \), \( d_{w21} \) and \( d_{w22} \) can be calculated as [9]:
\[ d_{w1} = 2 \cdot a_{w1} \cdot (u_t + 1) \] (4)
\[ d_{w1} = 2 \cdot a_{w1} \cdot u_t \cdot (u_t + 1) \] (5)
\[ d_{w22} = 2 \cdot a_{w2} \cdot u_2 \cdot (u_2 + 1) \] (6)

Also, for this gearbox we have:
\[ u_2 = \frac{u_g}{u_1} \] (7)

In which, \( u_g \) is the total gearbox ratio; \( u_1, u_2 \) are the gear ratios of the first and the second stages.

From (2), (3), (4), (5) and (6) the following equations are given:
\[ L = f(a_{w1}, a_{w2}, u_1, u_2, u_g) \] (8)
\[ h = f(a_{w1}, a_{w2}, u_1, u_2, u_g) \] (9)

Thus, the optimization problem is defined as
\[ \text{minimize } A = L \cdot h \] (10)

With the following constraints
\[ 5 \leq u_g \leq 30 \] (11)

From equations (8), (9) and (10), it is known that in for solving the optimization problem it is necessary to determine the center distances of the first stage \( a_{w1} \) and the second stage \( a_{w2} \).

2.1. Calculating the center distance of the first stage
The center distance of the first stage \( a_{w1} \) can be found by the following equation [9]:
\[ a_{w1} = K_u \cdot (u_t + 1) \cdot \left( \frac{T_{11} \cdot k_{H\beta}}{[\sigma_H]^2 \cdot u_t \cdot \psi_{bal}} \right)^{1/3} \] (12)

Where,
- \( K_{H\beta} \) is the contact load ratio for pitting resistance; for the first stage of the gearbox \( k_{H\beta} = 1.02 \pm 1.28 \) [9]. Hence, we can choose \( k_{H\beta} = 1.1 \);
- \([\sigma_H]\) - the allowable contact stress (MPa); In practice, \([\sigma_H] = 350 \ldots 410 \) (MPa);
- \( k_u \) is the material coefficient; Normally, the gear material is steel, \( k_u = 43 \) [9];
- \( \psi_{bal} \) is the coefficient of wheel face width; for the for the first stage of the gearbox \( \psi_{bal} = 0.3 \ldots 0.35 \);

From the moment equilibrium condition of the gearbox we have:
\[ T_{out} = T_{11} \cdot \eta_{hg} \cdot \eta_{be} \cdot u_g \] (13)

In which, \( \eta_{hg} \) is the helical gear transmission efficiency (\( \eta_{hg} = 0.96 \ldots 0.98 \) [9]); \( \eta_{be} \) is the transmission efficiency of a pair of rolling bearing (\( \eta_{be} = 0.99 \ldots 0.995 \) [9]). Choosing \( \eta_{hg} = 0.97 \) and \( \eta_{be} = 0.992 \) and substituting them into (13) gets:
\[ T_{11} = 1.0887 \cdot T_{out} / u_g \] (14)

Substituting (14) and \( k_{H\beta} = 1.1 \) into (12) we have:
\[ a_{w1} = 45.6635 \cdot (u_t + 1) \cdot \left( \frac{T_{out}}{[\sigma_H]^2 \cdot u_t \cdot u_g \cdot \psi_{bal}} \right)^{1/3} \] (15)

2.2. Calculating the center distance of the second stage
The center distance of the second stage \( a_{w2} \) can be determined by [9]:

\[ a_{w1} = K_u \cdot (u_t + 1) \cdot \left( \frac{T_{11} \cdot k_{H\beta}}{[\sigma_H]^2 \cdot u_t \cdot \psi_{bal}} \right)^{1/3} \] (12)
\[ a_{w2} = K_u \cdot (u_2 + 1) \left( \frac{T_{12} \cdot k_{H} \cdot H}{\sigma_H \cdot u_2 \cdot \psi_{ba2}} \right)^{1/3} \]  

(16)

In addition, for the second stage we have

\[ T_{out} = 2 \cdot T_{12} \cdot \eta_{bg} \cdot \eta_{ba}^2 \cdot u_2 \]

Choosing \( \eta_{bg} = 0.97 \) and \( \eta_{ba} = 0.992 \) as in section 2.1 and substituting them into (17) gives

\[ T_{12} = 0.5238 \cdot T_{out} / u_2 \]

(18)

Substituting (18), \( k_u = 43 \) and \( k_{H} = 1.1 \) (as in section 2.1) into (16) gets:

\[ a_{w2} = 35.7812 \cdot (u_2 + 1) \left( \frac{T_{out}}{\sigma_H \cdot u_2 \cdot \psi_{ba2}} \right)^{1/3} \]

(19)

2.3. Experimental work

To explore the influence of the input parameters on the optimum gear ratios, an “experiment” was designed and conducted. For the design of the experiment, a 2-level full factorial design was carefully chosen. Table 1 presents 5 input factors which were selected for the investigation. Consequently, there are \( 2^5 = 32 \) number of tests for conducting. From equation (10) and (11), a computer program was built for performing the experiment. The various levels of input factors and the results of the output of the program (the optimum gear ratio of the first stage \( u_1 \)) are shown in Table 2.

Table 1. Input parameters.

| Factor                          | Code | Unit | Low | High |
|--------------------------------|------|------|-----|------|
| Total gearbox ratio            | \( u_g \) | -    | 5   | 30   |
| Coefficient of wheel face width of stage 1 | \( x_{ba1} \) | -    | 0.3 | 0.35 |
| Coefficient of wheel face width of stage 2 | \( x_{ba2} \) | -    | 0.35 | 0.4 |
| Allowable contact stress       | AS   | MPa  | 350 | 410  |
| Output torque                  | \( T_{out} \) | Nmm | \( 10^5 \) | \( 10^6 \) |

3. Optimization results and discussions

For evaluating the effect of input factors on the response and the relative strength of the effect, a graph of the main effect of each input factor is plotted in Figure 3. From the Figure, it is found that the optimum gear ratio \( u_1 \) depends strongly on the total gearbox ratio \( u_g \). It increases significantly with the increase of the total gearbox ratio. Besides, it also affected by the coefficient of wheel face width of the first and the second stages (\( \psi_{ba1} \) and \( \psi_{ba2} \)). In addition, the optimum gear ratio is not affected by the allowable contact stress of the helical gear set AS, and the output torque \( T_{out} \).

Table 2. Experimental plans and output response.

| StdOrder | RunOrder | CenterPt | Block | \( u_g \) | Xba1 | Xba2 | AS (MPa) | Tout (Nm) | \( u_1 \) |
|----------|----------|----------|-------|--------|------|------|---------|----------|--------|
| 25       | 1        | 1        | 1     | 5      | 0.3  | 0.35 | 410     | 1000     | 2.17   |
| 6        | 2        | 1        | 1     | 30     | 0.3  | 0.4  | 350     | 100      | 6.86   |
| 21       | 3        | 1        | 1     | 5      | 0.3  | 0.4  | 350     | 1000     | 2.08   |
| 3        | 4        | 1        | 1     | 5      | 0.35 | 0.35 | 350     | 100      | 2.29   |
The Pareto chart of the standardized effects from the largest to the smallest effect is shown in Figure 4. It was found from this graph, the bars that represent factors including the total gearbox ratio (factor A), the coefficients of wheel face width of the first and the second stages (factors B and C) and the interactions between them (AB and AC) cross the reference line. Hence, these input factors are statistically significant at the 0.05 level with the response model (the gear ratio of the first stage $u_1$).

Also, because the bars which represent the allowable contact stress of the helical gear set (factor D), and the output torque (factor E) do not cross the reference line, these factors (D and E) do not affect the response model.
Figure 5 shows the Normal Plot of the standardized effects. This graph is used to describe which effects increase or decrease the response. From Figure 5, it was found that the total gearbox ratio (factor A) is the most significant factor for the optimum gear ratio. In addition, the total gearbox ratio and the coefficient of wheel face width of the first stage (factor B) have a positive standardized effect. If they change from the low level to the high level of the factors, the optimum gear ratio of the first stage increases. Also, the coefficient of wheel face width of the second stage (factor C) has a negative standardized effect. The optimum gear ratio decreases when it increases.

Figure 6 presents the estimated effects and coefficients for the optimum gear ratio $u_1$. From the figure, the total gearbox ratio $u_g$, the coefficient of wheel face width of the first and the second stage ($\psi_{ba}$ and $\psi_{ba2}$) and their interactions have P-values lower than 0.05. Therefore, these factors are significant to the optimum gear ratio. In addition, the relation between the optimum gear ratio and these significant effect factors can be described as follows:

$$u_1 = 0.5238 + 0.201277 \cdot u_g + 3.1952 \cdot \psi_{ba} + 0.6585 \cdot \psi_{ba2} + 0.205704 \cdot u_g \cdot \psi_{ba} - 0.179448u_g \cdot \psi_{ba2} - 5.148 \cdot \psi_{ba} \cdot \psi_{ba2}$$  \hspace{1cm} (20)

The above estimated model fit the data very well because the adj-R2 and pred-R2 are in the high values (Figure 6).
Equation (20) is used to determine the optimum gear ratio of the first stage $u_1$. After having $u_1$, the gear ratio of the second stage can be found by $u_2 = u_g / u_1$.

4. Conclusions
The minimum cross-sectional acreage of a two-stage helical gearbox with second stage double gear sets can be obtained by determination of optimum gear ratios of the gearbox.

Models for determining the optimum gear ratios of a two-stage helical gearbox with second stage double gear sets were introduced for getting the minimum cross-sectional acreage of the gearbox.

The optimum gear ratios of the gearbox can be calculated accurately and simple as the estimated models are explicit.

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References
[1] Kudryavtsev V N, Derzhavets Yu A and Glukharev E G 1971 Design and calculation of gear reducers. Reference guide (Saint-Petersburg: Mechanical Engineering) p 328
[2] G Milou, G Dobre, F Visa and H Vitila 1996 Optimal Design of Two Step Gear Units, regarding the Main Parameters VDI Berichte No 1230 p 227
[3] A N Petrovski, B A Sapiro and N K Saphonova 1987 About optimal problem for multi-step gearboxes (in Russian) Vestnik Mashinostroenie No 10 p 13
[4] Vu Ngoc Pi 2001 A method for optimal calculation of total transmission ratio of two step helical gearboxes Proc. of the National conf. on Engineering Mechanics (Ha Noi) pp 12-15
[5] Romhild I Linke H Gezielte Auslegung 1992 Von Zahnradgetrieben mit minimaler Masse auf der Basis neuer Berechnungsverfahren Konstruktion 44 pp 229- 236
[6] V N Pi and V Q Dac 2004 A new and effective method for optimal calculation of total transmission ratio of two step helical gearboxes, School of Computational Sciences and Engineering: Theory and Applications, March 3 (Ho Chi Minh city: Vietnam) pp 103-106
[7] Vu Ngoc Pi 2008 A Study on Optimal Determination of Partial Transmission Ratios of Helical Gearboxes with Second-Step Double Gear-Sets Int. J. of Mathematical, Physical and Engineering Sciences Volume 2 Number 2 pp 99-102
[8] Vu Ngoc Pi, Optimal calculation of partial ratios of helical gearboxes with second-step double gear-sets, Proc. of the 3rd IASME / WSEAS International Conference on CONTINUUM MECHANICS (CM’08) pp 63-66
[9] T Chat and L V Uyen 1998 Design and calculation of Mechanical Transmissions (in Vietnamese), (Hanoi: Educational Republishing House) p 229