The Effect of a Non-Isotropic Flux of Very High
Energy Cosmic Rays on the values of Mean Shower Maxima

Pantea Davoudifar; Keihanak Rowshan Tabari
Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), P.O.Box:55134-441, IRAN
E-mail: dfpantea@riaam.ac.ir

Abstract. In our previous works we described a statistical method to interpret the results of extensive air shower simulations. For an isotropically distributed flux of cosmic rays, we used this method to deduce diagrams of mean values of shower maxima versus energy decades. To have a more realistic result, we considered the effect of a non-isotropic flux of cosmic rays at different energy ranges. This effect was considered as a weight factor deduced from a set of observed data. We discussed about the effect of this weight factor on our final resulted diagrams of mean shower maxima and for different interaction models compared the resulted distributions of very high energy cosmic ray’s mass composition.

1. Introduction
Entering the geomagnetic field, charged particles and ultra high energy $\gamma$-rays ($E_\gamma \geq 10^{19}$eV) are affected by the Earth magnetic field. For instance, the deflection of low energy charged particles in the Earth magnetic field, cause the azimuthal asymmetry or interaction of ultrahigh energy $\gamma$-rays with geomagnetic field cause pre-showering effect. In an array of cosmic ray’s detectors with a horizontal geometry, the event rate is also distorted due to azimuthal event rate modulation in the geomagnetic field.

Obviously, considering a set of simulated Extensive Air Showers (EASs), there is an statistical distribution of depth of shower maxima, $X_{\text{max}}$, over zenith angle intervals [1]. Using CORSIKA (version 7.37) [2] simulation, we studied this distribution for different hadronic interaction models and re-produced the diagrams of mean depth of shower maxima versus energy decades. The resulted diagrams then were used to study the cosmic ray mass composition, in which an isotropic flux of cosmic rays were supposed [1].

Anyhow the zenith angle dependence of the arrival rate of showers of size$>N$, $I(> N,X,\theta)$, for most of the experiments were considered to have a form of: $I(> N,X,\theta) = I(> N,X,0)\cos^n\theta[m^{-2}s^{-1}deg^{-1}]$ in which $X$ is the vertical depth of experiment [3]. Though, such a phenomena has not a direct effect on the calculated [1] mean depth of shower maxima, $\langle X_{\text{max}} \rangle$; but it may cause a tolerance on the average values of $X_{\text{max}}$ when the statistical fluctuations over zenith angle intervals are considered. In this paper, the order of magnitude of this possible tolerances are investigated.
2. The Method
For Yakutsk array, different fits to experimental data are available for the energy ranges of $E < 10^{18} eV$, $E > 10^{18} eV$, $E > 10^{19} eV$ and $E > 4 \times 10^{19} eV$ [4, 5]. For this array, it was also shown that the azimuthal effect on event rate at $E > 10^{17} eV$ is approximately the same in the whole energy range [5].

At the top of atmosphere and in the energy range of $\sim 10^{18} - 5 \times 10^{19} eV$, one may consider the event rate per degrees as:

$$n(N, 0, \theta) = n(0, 0) \cos^n(\theta) [s^{-1} deg^{-1}]$$

in which $n(N, 0, \theta) \equiv I(>N, 0, \theta) \times (1 \ m^2)$ is the event rate per degree at the top of atmosphere with a vertical depth equal to 0. As a result, using data from Yakutsk array (for energies higher than $10^{19} eV$) [6] the expected event rates for different zenith angle intervals are given in table 1.

Table 1. Expected Event Rates in Different Zenith Angle Intervals. The values are multiplied by a factor of $\sim 26$.

| Interval | 0 - 2° | 8 - 10° | 18 - 20° | 24 - 26° | 30 - 32° | 34 - 36° | 44 - 46° | 58 - 60° |
|----------|--------|---------|---------|---------|---------|---------|---------|---------|
|          | 100    | 1645    | 3120    | 3988    | 4593    | 4950    | 5195    | 4912    |

Simulating the total number of $N$ showers for each zenith angle interval and dividing the atmospheric depth to intervals of $10 \ gr/cm^2$, the occurrence frequencies of depth of shower maxima were counted. We used the second set of formulas were presented previously [1]:

$$\langle X_{\max} \rangle = \sum_{i=1}^{n} \omega_i X_{\max,i}$$

in which $n$ is the number of intervals (i.e. $\sum_{i=1,...,n} frequency_i = N$) and for energies in the range of $\sim 10^{19} - 5 \times 10^{19}$, $N$ were chosen from table 1. Here $\omega_i$ is a weight factor equivalent to $\omega_i \equiv \frac{frequency_i}{X_{\max}}$. No functional fits are considered as the number of simulated showers are now different. As before, zenith angle dependency may be removed using a function of the form of:

$$X_{\max}(\theta) = a \cos \theta + b$$

and $\langle X_{\max} \rangle$ has the value of $\sim a + b$.

Omitting the zenith angles of lower than $\sim 10^0$, it is possible to use less simulations. But as the most uncertainties occur in this range, an important part of data may be lost. So we did this study for 5 fixed energies ($1 \times 10^{18} eV$, $5 \times 10^{18} eV$, $1 \times 10^{19} eV$, $5 \times 10^{19} eV$ and $1 \times 10^{20} eV$) for $p$ and Fe primaries. Interaction models were QGSJETII (QGSJET-II-04) [7] and FLUKA [8]. Thinning parameters was set to $10^{-4}$. No SLANT option was considered and using of Gaisser-Hillas curves [9] were avoided.

3. Discussion and Results
The effect of this non-isotropic event rate is a decrease in the values of statistical errors by increasing zenith angle. In fact, a fit of formula [3] to the distribution of mean values of $X_{\max}$ versus zenith angle now is possible by establishing a standard deviation, $\lambda_\theta$, for each point (figure 1) which is now related to variable values of $N(\theta)$. For the studied energies these values were ranged from 10 down to 2 for a primary proton of $5 \times 10^{19} eV$, and 9 down to 2 for a primary proton of $1 \times 10^{18} eV$. In comparison, the standard deviation had a constant value for an isotropic distribution of the event rates.
Figure 1. Distribution of depth of shower maxima in different zenith angle intervals for a 100 $EeV$ proton (QUGSJET-II-04 in Combination with FLUKA were used).

Anyhow, in the case of a Non-Isotropic distribution of event rates; when the distribution of depth of shower maxima experiences higher fluctuations for lower zenith angle intervals, a fit of the form of formula 3 eliminates this effect and the resulted mean value of depth of shower maxima differs not much than 2% of its previous value [1]. For instance, for a 100 $EeV$ proton using a non-isotropic distribution caused an upper value of $\langle X_{max} \rangle$ by 5 gr/cm$^2$ which is less than 1% of its previous calculated value for an isotropic distribution.

The percentage of these differences with respect to previous calculated values [1] were presented in table 2.

This research shows that in the method we used [1] for calculating diagrams of $\langle X_{max} \rangle$ versus energy decades, the use of a non-isotropic event rate of primaries may affect the results on mass composition of primaries only slightly. In fact in the energy range of $10^{19}eV$ and above, these values (i.e. the values of $\langle X_{max} \rangle$) may be shifted to larger values of $\langle X_{max} \rangle$ by less than 8 gr/cm$^2$.
Table 2. The calculated percentage of differences, $P\%$ with respect to previously calculated values for different primaries.

| primary type | $1 \times 10^{18}\text{eV}$ | $5 \times 10^{18}\text{eV}$ | $1 \times 10^{19}\text{eV}$ | $5 \times 10^{19}\text{eV}$ | $1 \times 10^{20}\text{eV}$ |
|--------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| $p$          | 2%                            | 2%                            | 1%                            | 1%                            | 1%                            |
| $Fe$         | 2%                            | 2%                            | 2%                            | 1%                            | 1%                            |

for a proton primary and 7 $\text{gr/cm}^2$ for an Iron nuclei (see table I of [1]), and as these values are much less than the magnitude of experimental uncertainties [10, 11, 12, 13, 14, 15, 16], this has not a considerable effect on mass composition studies at the highest energies. Same result were attained for SYBILL 1.6 and SYBILL 2.1 [1, 17]

References
[1] P. Davoudifar, and K. Rowshan Tabari, 2013, J. Phys.: Conf. Series 410 012087
[2] D. Heck et al., (1998), Report FZKA 6019, Forschungszentrum Karlsruhe, http://www-ik.fzk.de/corsika/physics description/corsika phys.html
[3] P.K.F. Grieder, 2010, Extensive Air Showers, High Energy Phenomena and Astrophysical Aspects (Springer Heidelberg Dordrecht London New York) Volumw I, p 255
[4] N.N. Efimov, A.A. Mikhailov, M.I. Pravdin, 1983, Proc. of the 18th Int. Cosmic Ray Conf. (Bangalore), vol 2, p.149
[5] A.A. Ivanov, et al., 1999, Proc. of the 26th Int. Cosmic Ray Conf. (Salt Lake City), vol 1, p.403
[6] N.N. Efimov, et al., 1988, Catalogue of Highest Energy Cosmic Rays, Giant Extensive Air Showers, World Data Center C2 for Cosmic Rays Institute of Physical and Chemical Research, Wako, Saitama, Japan; Online catalogue, http://eas.ysn.ru/catalog/yakutsk-array-data
[7] N.N. Kalmykov and S.S. Ostapchenko, 1993, Physics of Atomic Nuclei, 56, 346; N.N. Kalmykov, S.S. Ostapchenko, and A.I. Pavlov, 1994, Bulletin of Russian Academy of Science (Physics), 58, 1966; 1997, Nuclear Physics B (Proc. Suppl.), 52B, 17; S.S. Ostapchenko, 2011, Physical Review D, 83, 044018
[8] A. Ferrari, P.R. Sala, A. Fasso, and J. Ranft, 2005, CERN-2005-010, INFN TC-05/11, SLAC-R-773; G. Battiston, S. Muraro, P.R. Sala, F. Cerutti, A. Ferrari, S. Roesler, A. Fasso, and J. Ranft, 2007, Proc. of the Hadron Shower Simulation Workshop 2006, Fermilab 6-8 September 2006; M. Albrow, R. Raja eds., AIP Conf. Proc. 896, 31-49
[9] T.K. Gaisser, A.M. Hillas, 1977, Proc. of the 15th Int. Cosmic Ray Conf., Plovdiv, Bulgaria, 8, 353
[10] V.M. Grigoryev, et al., 1990, Proc. of the 21th Int. Cosmic Ray Conf., 9, 206
[11] I.V. Doronina, et al., 1990, Proc. of the 21th Int. Cosmic Ray Conf., 3, 150
[12] M.N. Dyakonov, et al., 1991, Proc. of the 22nd Int. Cosmic Ray Conf., 4, 351
[13] M.N. Dyakonov et al., 1993, Proc. of 23rd Int. Cosmic Ray Conf., Calagary, Canada, vol 4, 303
[14] S. Knurenkov et al., 200), Proc. of the 27th Int. Cosmic Ray Conf., Hamburg, Germany, HE 1.3, 177
[15] A.A. Ivanov, et al.,(200), Proc. of the 29th Int. Cosmic Ray Conf., Pune, India, 6, 241
[16] S.P. Knurenko and A. Sabourov,(201), Astrophysics and Space Science Transactions, 7, 251
[17] J. Engel, T.K. Gaisser, P. Lipari, and T. Stanev, (1992), Physical Review D, 46, 5013; R.S. Fletcher, T.K. Gaiser, P. Lipari, and T. Stanev, (1994), Physical Review D, 50, 5710; R. Engel, (1999), Proc. of 26th Int. Cosmic Ray Conf., 1, 415