Decrumpling membranes by quantum effects

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Abstract. – The phase diagram of an incompressible fluid membrane subject to quantum and thermal fluctuations is calculated exactly in a large number of dimensions of configuration space. At zero temperature, a crumpling transition is found at a critical bending rigidity \(1/\alpha_c\). For membranes of fixed lateral size, a crumpling transition occurs at nonzero temperatures in an auxiliary mean field approximation. As the lateral size \(L\) of the membrane becomes large, the flat regime shrinks with \(1/\ln L\).

Introduction. – Amphiphilic molecules in aqueous solution form fluid bilayers with vanishing surface tension. This causes them to undergo strong shape fluctuations, governed by the Canham-Helfrich curvature energy \[\mathcal{H}_0 = \frac{1}{2\alpha_0} \int dS H^2,\] where \(dS\) is the surface element, \(H\) corresponds to the doubled mean curvature of the surface at each point, and \(1/\alpha_0\) is the bending rigidity.

Thermal undulations renormalize \(1/\alpha_0\) as follows \[\frac{1}{\alpha} = \frac{1}{\alpha_0} \left[ 1 - \frac{3}{4\pi} k_B T \alpha_0 \ln(\Lambda L) \right],\] where \(\Lambda\) is an ultraviolet wavevector cutoff set by the inverse width of the molecules in the membrane, and \(L\) is an infrared cutoff determined by its finite size. In practice, membranes occur in the form of spherical vesicles, and \(L^2\) is determined by their surface area. At finite temperatures, the model is only defined for finite planar surfaces.

For \(L\) larger than the de Gennes-Taupin persistence length \(\xi_p = \Lambda^{-1} \exp(4\pi/3k_B T \alpha_0)\) \[2\], the renormalized bending rigidity \(1/\alpha\) vanishes. Beyond the persistence length, the normal vectors of the surface are uncorrelated, and the membrane is crumpled. The renormalization group flow extracted from the perturbative result \[2\] as well as nonperturbative

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calculations in the large-$d$ limit \cite{11-14}, where $d$ is the dimension of the embedding space, exclude the possibility of a phase transition in the Canham-Helfrich model, even for tensionless membranes.

Recently, the model has been extended by a kinetic term to include quantum fluctuations \cite{15}. A one-loop renormalization group analysis showed that quantum effects stiffen the membrane. The ground state corresponds to a flat configuration, where the normal vectors of the surface are strongly correlated. The flat phase exists up to a critical temperature

$$T_c = \frac{4\pi}{3k_B} \frac{1}{\alpha}. \quad (3)$$

Above $T_c$, thermal fluctuations overcome quantum effects, and the membrane is always crumpled.

In this paper, we analyze the behavior of the quantum membrane exactly for very large dimension $d$ of the embedding space at all temperatures. Since the model is exactly solvable in this limit, we can calculate all of its relevant properties, in particular its order parameter and phase diagram.

**Definition of the model.** – The surface describing the membrane is parametrized by a vector field $\mathbf{X}(\vec{\sigma})$ in the $d$-dimensional embedding space, where $\vec{\sigma} = (\sigma_1, \sigma_2)$ is a two-dimensional parameter space. In this parametrization, the Hamiltonian (1) reads

$$H_0 = \frac{1}{2\alpha_0} \int d^2\sigma \sqrt{g}(\Delta \mathbf{X})^2, \quad (4)$$

where

$$g_{ab} = \partial_a \mathbf{X} \cdot \partial_b \mathbf{X} \quad (5)$$

is the metric induced by the embedding, and $g = \det[g_{ab}]$. The symbol $\partial_a$ ($a = 1, 2$) denotes the derivative with respect to the parameters $\sigma_1, \sigma_2$, and $\Delta = g^{-1/2}\partial_a g^{ab} g^{1/2}\partial_b$ is the Laplace-Beltrami operator. As in Ref. \cite{15}, we add to the Hamiltonian (4) a kinetic term to account for quantum fluctuations:

$$\mathcal{T} = \frac{1}{2\nu_0} \int d^2\sigma \sqrt{g} \dot{\mathbf{X}}^2, \quad (6)$$

where $\mathbf{X}$ is now time-dependent, $1/\nu_0$ is the bare mass density, and the dot indicates a time derivative.

The euclidean action describing the quantum membrane is thus

$$S_0 = \int d\tau d^2\sigma \sqrt{g} \left[ \frac{1}{2\nu_0} \dot{\mathbf{X}}^2 + \frac{1}{2\alpha_0} (\Delta \mathbf{X})^2 \right], \quad (7)$$

and the partition function $Z$ can be represented as a functional integral over all possible surface configurations $\mathbf{X}(\vec{\sigma}, \tau)$:

$$Z = \int \mathcal{D}\mathbf{X} \exp(-S_0[\mathbf{X}]/\hbar). \quad (8)$$

**Large-$d$ approximation.** – For large $d$ it is useful to consider $g_{ab}$ as an independent field \cite{14}, and impose relation (4) with help of a Lagrange multiplier $\lambda_{ab}$. We consider the case where the classical action will have an extremum around an almost flat configuration. In the $d$-dimensional generalization of the Monge parametrization of an almost flat surface, the metric tensor becomes
\[ g_{ab} = \delta_{ab} + \partial_a X \cdot \partial_b X. \]  

The partition function for the membrane can then be written as

\[ Z = \int \mathcal{D}g \mathcal{D}\lambda \mathcal{D}X \, e^{-S_0/\hbar}, \]  

with the euclidean action

\[ S_0 = \int d\tau d^2\sigma \sqrt{g} \left\{ \frac{1}{2\nu_0} \dot{X}^2 + r_0 + \frac{1}{2\alpha_0} \left[ (\Delta X)^2 + \lambda^{ab} (\delta_{ab} + \partial_a X \cdot \partial_b X - g_{ab}) \right] - \frac{c_0}{4} \lambda_{aa}^2 \right\}. \]  

We have included a bare surface tension \( r_0 \) to absorb infinities arising in the process of renormalization. The renormalized, physical surface tension \( r \) will be set equal to zero at the end of our calculations. We have further added a term proportional to \( \lambda_{aa}^2 \), with the proportionality constant \( c_0 \) being the in-plane compressibility of the membrane. This is also necessary to absorb infinities, and the renormalized compressibility \( c \) will be set equal to zero at the end to describe an incompressible planar fluid.

Note that the functional integration over the Lagrange multipliers \( \lambda_{ab} \) in (10) has to be performed along the imaginary axis for convergence.

The functional integral over all possible surface configurations \( X(\vec{\sigma}, \tau) \) in (10) is Gaussian, and can be immediately carried out, yielding an effective action

\[ S_{\text{eff}} = \tilde{S}_0 + S_1, \]  

with

\[ \tilde{S}_0 = \int d\tau d^2\sigma \sqrt{g} \left[ r_0 + \lambda^{ab} (\delta_{ab} - g_{ab}) - \frac{c_0}{4} \lambda_{aa}^2 \right], \]  

and

\[ S_1 = \frac{\hbar}{2} d \text{Tr} \ln \left[ -\partial_0^2 + \frac{\nu_0}{\alpha_0} (\Delta^2 - \partial_a \lambda^{ab} \partial_b) \right]. \]  

For large \( d \), the partition function (10) is dominated by the saddle point of the effective action (12) with respect to the metric \( g_{ab} \) and the Lagrange multiplier \( \lambda^{ab} \), and we are left with a mean-field theory in these fields. For very large membranes, translational invariance allows us to assume that this saddle point is symmetric and homogeneous \([12–14, 17]\), such that

\[ g_{ab} = \varrho_0 \delta_{ab}; \quad \lambda^{ab} = \lambda_0 g^{ab} = \frac{\lambda_0}{\varrho_0} \delta^{ab}, \]  

with constant \( \varrho_0 \) and \( \lambda_0 \). There, the functional trace in (14) becomes an integral \( \int d\tau d^2\sigma d\omega \dd^2q/(2\pi)^3 \) over the \( (2 + 1) \)-dimensional phase space, after replacing \( \partial_0^2 \rightarrow -\omega^2 \) and \( g^{ab} \partial_a \partial_b \rightarrow -q^2 \).

Zero Temperature Properties. – At zero temperature, the phase space integral in (14) yields

\[ S_1 = \frac{\hbar}{2} d \int d\tau d^2\sigma \varrho_0 \sqrt{\frac{\nu_0}{\alpha_0}} \left\{ \frac{\Lambda^4}{8\pi} + \frac{\lambda_0}{8\pi} \Lambda^2 + \frac{\lambda_0^2}{64\pi} \left[ 1 - 2 \ln \left( \frac{4\Lambda^2}{\lambda_0} \right) \right] \right\}, \]  

where the ultraviolet divergences of the integral have been regularized by a wavevector cutoff \( \Lambda \). The first term in (16) is a constant and renormalizes the surface tension to

\[ r = r_0 + \frac{\hbar d}{16\pi} \sqrt{\frac{\nu_0}{\alpha_0}} \Lambda^4. \]
The quadratically divergent term renormalizes the bending rigidity in the second term of (13).
The logarithmically divergent term proportional to \( \lambda^2 \) modifies the in-plane compressibility to

\[ c = c_0 + \frac{\hbar d}{32\pi} \sqrt{\frac{\nu_0}{\alpha_0}} \ln \left( 4e^{-1/2} \frac{\Lambda^2}{\mu^2} \right), \]

where \( \mu \) is a renormalization scale. We now set \( r \) and \( c \) equal to zero, to describe a tensionless incompressible membrane.

The renormalized effective action is then

\[ S_{\text{eff}} = \int d\tau d^2\sigma \lambda \left\{ \frac{1}{\alpha} \left( \frac{1}{\varrho} - 1 \right) + \frac{1}{\sqrt{\alpha c_c}} + \frac{a}{\sqrt{\alpha}} \lambda \left[ \ln \left( \frac{\lambda}{\bar{\lambda}} \right) - \frac{1}{2} \right] \right\}, \]

where we have defined the critical bending rigidity

\[ \frac{1}{\alpha_c} \equiv \frac{\hbar^2 d^2 v_0}{256\pi^2} \Lambda^4. \]

and the constants \( a \equiv h d\nu^{1/2}/(64\pi) \), \( \bar{\lambda} \equiv \mu^2 e^{-1/2} \). From the second-derivative matrix of \( S_{\text{eff}} \) with respect to \( \varrho \) and \( \lambda \) we find that the stability of the saddle point is guaranteed only for \( \lambda < \bar{\lambda} \). Note that the integration over \( \lambda_{ab} \) in (10) along the imaginary axis requires a maximum of (19) with respect to \( \lambda \) for stability.

The extremization of (19) with respect to \( \varrho \) leads to two different solutions for the saddle point, namely

\[ \lambda = 0 \quad \text{or} \quad \lambda \left[ \ln \left( \frac{\lambda}{\bar{\lambda}} \right) - \frac{1}{2} \right] = \frac{1}{a} \left( \frac{1}{\alpha^{1/2}} - \frac{1}{\alpha_c^{1/2}} \right). \]

These describe two different phases existing at zero temperature. For \( \alpha < \alpha_c \), \( \lambda = 0 \) is the only possible solution for the saddle point. This solution corresponds to the flat phase, as we shall verify below. For \( \alpha > \alpha_c \), on the other hand, there exists a solution of Eq. (22) for nonzero \( \lambda \). This solution corresponds to the crumpled phase. The behavior of the effective action (19) is shown in Fig. 1. As \( \alpha \) approaches the critical point from below, i.e. the membrane softens, \( \lambda \) becomes nonzero, and the surface crumples.

To determine the saddle point solution for \( \varrho \) we extremize (13) with respect to \( \lambda \). In the flat phase, where \( \lambda = 0 \), we obtain

\[ \varrho^{-1} = 1 - \left( \frac{\alpha}{\alpha_c} \right)^{1/2}, \]

showing that the total area of the membrane increases as \( \alpha \) approaches \( \alpha_c \) from below, with a crumpling transition at \( \alpha_c \). In the crumpled phase, \( \varrho \) is given by

\[ \varrho^{-1} = \left( \frac{\alpha}{\alpha_c} \right)^{1/2} - 1 - a\sqrt{\alpha} \lambda, \]

with nonzero \( \lambda \). As \( \alpha \) approaches \( \alpha_c \) from above, \( \lambda \) tends to zero, and \( \varrho \) goes again to infinity. The positivity of \( \varrho \) and the stability of the saddle point imply that there is an upper bound for the bending rigidity, given by

\[ \frac{1}{\alpha_c^{1/2}} = \frac{1}{\alpha_{\text{max}}^{1/2}} - a\bar{\lambda}, \]

below which an incompressible membrane becomes unstable. The behavior of \( \lambda \) in the two phases is shown in Fig. 2. The behavior of \( \varrho^{-1} \) is shown in Fig. 3.
Fig. 1 – Effective action at $T = 0$ in units of $a$. The physical solution of the saddle point equations (21), (22) lies at the maxima.

Fig. 2 – Physical branch of the solution of Eq. (22) for $\lambda$ as a function of the stiffness $\alpha^{-1}$. The dashed curve indicates the unstable extremum of the action.

**Finite-Temperature Properties.** – At finite temperature, the phase space integral in (14) involves a sum over the Matsubara frequencies [18]:

$$\omega_n = \frac{2\pi nk_B T}{\hbar}, \quad n = 0, \pm 1, \pm 2 \cdots$$

(26)

For small $\lambda_0$, a series expansion leads to

$$S_1 = \int dt d^2\sigma \sigma_0 \left\{ \frac{\hbar d}{16\pi} \left[ \frac{\nu_0}{\alpha_0} \Lambda^4 - \frac{\hbar d}{24} \sqrt{\frac{\alpha_0}{\nu_0}} \left( \frac{k_B T}{\hbar} \right)^2 \right] + \frac{\hbar d}{16\pi} \frac{\nu_0}{\alpha_0} \Lambda^2 
+ \lambda_0 \frac{d k_B T}{8\pi} \ln \left( \frac{L^2 k_B T}{\hbar} \sqrt{\frac{\alpha_0}{\nu_0}} \right) + a \frac{\lambda_0^2}{\alpha_0} \left[ 3 - 2\gamma + 2 \ln \left( \frac{\lambda_0 L^2 k_B T}{8\pi \Lambda^2 \hbar} \sqrt{\frac{\alpha_0}{\nu_0}} \right) \right] 
+ \frac{\hbar d}{2} \sqrt{\pi} \sum_{m=3}^{\infty} \frac{(-1)^{m+1} \lambda_0^m}{m^{2m/\pi^m}} \frac{\hbar}{k_B T} \left( \frac{\nu_0}{\alpha_0} \right)^{m-2} \frac{\Gamma(m-1)}{\Gamma(m/2)} \zeta(m-1) \right\}$$

(27)

As in the zero-temperature discussion, we absorb the logarithmic divergence by renormalizing the in-plane compressibility via Eq. (18), setting $c$ equal to zero for incompressible membranes.

The surface tension receives now a temperature dependent renormalization

$$r = r_0 + \frac{\hbar d}{16\pi} \sqrt{\frac{\nu_0}{\alpha_0}} \Lambda^4 - \frac{\hbar d}{24} \sqrt{\frac{\alpha_0}{\nu_0}} \left( \frac{k_B T}{\hbar} \right)^2,$$

(28)

and $r_0$ is chosen to make $r = 0$ for tensionless membranes at all temperatures.

Extremization of the renormalized combined effective action (13) and (27) with respect to $\rho$ leads again to two possible solutions for the saddle point, namely $\lambda = 0$ or $\lambda = \lambda_T$, with

$$\lambda_T \left[ \ln \left( \frac{\lambda_T}{\lambda} \right) - \frac{1}{2} \right] + \lambda_T \left[ 1 - \gamma + \ln \left( \frac{L^2 k_B T}{8\pi \hbar} \sqrt{\frac{\alpha}{\nu}} \right) \right] + 32\pi^{3/2} \sum_{m=3}^{\infty} \frac{(-1)^{m+1} \lambda_0^{m-1}}{m^{2m/\pi^m}} \frac{\hbar}{k_B T} \left( \frac{\nu}{\alpha} \right)^{m-2} \frac{\Gamma(m-1)}{\Gamma(m/2)} \zeta(m-1)$$




where

\begin{equation}
\frac{1}{\alpha_{T}^{1/2}} \approx \frac{1}{\alpha_{c}^{1/2}} \left[ \frac{1}{2} + \frac{1}{4} + \frac{dk_{B}T}{8\pi \alpha_{c} \ln \left( L_{2}^{2} k_{B} T \sqrt{\frac{\alpha_{c}}{\nu}} \right)} \right]
\end{equation}

is the critical bending rigidity at finite temperature. Alternatively, we find the critical temperature at a fixed bending rigidity \(1/\alpha\):

\begin{equation}
T_{c} \ln \left( L_{2}^{2} k_{B} T_{c} \sqrt{\frac{\alpha}{\nu}} \right) = \frac{8\pi}{dk_{B}} \left( \frac{1}{\alpha} - \frac{1}{\sqrt{\alpha_{c}}} \right),
\end{equation}

in qualitative agreement with the perturbative critical temperature in Eq. (3).

For \(\alpha < \alpha_{T}\), Eq. (29) has no solution for \(\lambda_{T}\). In this case, the membrane is in the flat phase, the only available solution for the saddle point being \(\lambda = 0\). For \(\alpha > \alpha_{T}\), \(\lambda_{T}\) is nonzero, and the membrane is crumpled.

Let us now examine the saddle point solutions for \(\varphi\). In the crumpled phase where \(\lambda = \lambda_{T}\) is nonzero, we may expand the effective action into a high-temperature series. Extremization with respect to \(\lambda_{T}\) leads to

\begin{equation}
\varphi_{T}^{-1} = \left[ \left( \frac{\alpha}{\alpha_{T}} \right)^{1/2} - 1 \right] \left[ 1 - \alpha^{1/2} \left( 1 - \sqrt{\frac{\alpha_{c}}{\alpha_{T}}} \right) \right] - a \sqrt{\alpha_{c} \lambda_{T}}
- \frac{h \alpha}{2} \sqrt{\pi} \sum_{m=3}^{\infty} \frac{(-1)^{m+1} \lambda_{T}^{m-1}}{2^{2m} \pi^{m}} \left( 1 - 2 \frac{h}{m} \right) \left( \frac{h}{k_{B} T} \right)^{m-2} \frac{\nu}{\alpha} \left( \frac{m-1}{\nu} \right)^{m-1} \frac{\Gamma(m-1)}{\Gamma(m-2)} \zeta(m-1).
\end{equation}

The positivity of \(\varphi\) and the stability of the saddle point again define an upper bound for the inverse bending rigidity, given by

\begin{equation}
\frac{1}{\sqrt{\alpha_{T_{\text{max}}}}} \approx \frac{1}{\sqrt{\alpha_{c}}} \left( \frac{1}{2} + \frac{1}{2} \frac{T}{T_{\text{stab}}} \right),
\end{equation}

with

\begin{equation}
k_{B} T_{\text{stab}} = \frac{4\pi}{d \alpha_{\text{max}} \ln(16\pi/\hbar d \sqrt{\alpha_{c} \lambda})}.
\end{equation}

For temperatures lower than \(T_{\text{stab}}\) the effective action becomes unstable if the rigidity is lower than \(1/\alpha_{T_{\text{max}}.}\) Above \(T_{\text{stab}}\), the membrane is stable at all rigidities.

In the flat phase, the situation is more delicate. For \(\lambda = 0\), \(\varphi\) can be calculated exactly, and we obtain

\begin{equation}
\varphi^{-1} = 1 - \frac{dk_{B}T}{8\pi} \ln \left[ \frac{\sinh \left( \frac{16\pi}{dk_{B}T \sqrt{\alpha_{c} \lambda}} \right)}{\frac{k_{B}T}{2} \sqrt{\frac{\alpha}{\nu}} L^{-2}} \right],
\end{equation}

with an infrared regulator \(L\) equal to the inverse lateral size of the membrane. For low temperatures, \(\varphi\) may be approximated by

\begin{equation}
\varphi^{-1} \approx \left[ 1 - \left( \frac{\alpha}{\alpha_{T}} \right)^{1/2} \right] \left[ 1 - \alpha^{1/2} \left( 1 - \sqrt{\frac{\alpha_{c}}{\alpha_{T}}} \right) \right].
\end{equation}
Fig. 3 – Behavior of $\varrho^{-1}$ for fixed $L^2$. In the flat phase, $\varrho$ is given by (35), and in the crumpled phase by (32). The transition happens at $\alpha_T$, where the flat phase becomes crumpled. The dashed lines show the behavior of $\varrho^{-1}$ at zero temperature.

Fig. 4 – Phase diagram of the quantum membrane. At $T = 0$, there is a crumpling transition as the rigidity $1/\alpha$ falls below $1/\alpha_c$. For membranes of fixed lateral size $L$, the crumpling transition still takes place at higher temperatures. The critical inverse rigidity tends asymptotically to zero as the temperature goes to infinity. Below the dotted line the membrane is still flat if its lateral size is smaller than the persistence length. The unstable region disappears for nonzero in-plane compressibility.

At high temperatures, however, the positivity of $\varrho$ is not guaranteed. For fixed, but high temperatures, and for fixed membrane lateral size $L$, there is a characteristic value of the inverse bending rigidity defined by

$$\alpha^* = \frac{8\pi}{dk_B T \ln (16\pi L^2 / \hbar d \sqrt{\nu \alpha_c})},$$

(37)

above which $\varrho$ changes sign, and (35) is no longer applicable. Interestingly, for all $L$ and at all temperatures $T$, the critical bending rigidity $1/\alpha_T$ is larger than $1/\alpha^*$, so that the crumpling transition still occurs. The behavior of $\varrho$ is depicted in Fig. 3 below.

Note that Eq. (37) reflects the existence of a persistence length. At fixed temperature, for $\alpha_T < \alpha < \alpha^*$, the membrane is flat at scales smaller than

$$L_p = \Lambda^{-1} \exp \left( \frac{4\pi}{dk_B T \alpha} \right),$$

(38)

and crumpled at larger scales. This agrees with the de Gennes-Taupin persistence length $\xi_p$.

As the projected area $L^2$ of the membrane approaches infinity, the root $\alpha_T$ of the branch $\varrho_{+}^{-1}$ (see Fig. 3) goes to zero, and the branch $\varrho_{+}^{-1}$ becomes unphysical.

The phase diagram of the quantum membrane is plotted in Fig. 4.

As the lateral size $L$ of the membrane goes to infinity, the inverse critical bending rigidity $\alpha_T$ goes to zero, and the crumpling transition is washed out. In the limit of infinite area, the ratio $\alpha^*/\alpha_T = 1$. The membrane is crumpled at large scales, and flat at scales smaller than the persistence length. Its behavior can thus be described by the classical Canham-Helfrich model alone.
Let us finally characterize the two phases in terms of the correlation functions between the normal vectors to the surface of the membrane. For the solution $\lambda = 0$, this correlation function coincides with the one we found perturbatively for the zero temperature case, namely

$$\langle \partial_a X(\vec{\sigma}, \tau) \cdot \partial_b X(\vec{\sigma}', \tau) \rangle \sim \frac{\delta_{ab}}{|\vec{\sigma} - \vec{\sigma}'|^3}. \quad (39)$$

This solution corresponds to the flat, low temperature phase, where the normal vectors are strongly correlated. Such behavior can incidentally also be obtained in the large-$d$ limit at high temperatures by adding curvature terms of higher orders to the Canham-Helfrich Hamiltonian to stabilize a negative bending rigidity $1/\alpha_0 \ [19–21]$.

For nonzero $\lambda = \lambda_T$ the correlation function behaves as

$$\langle \partial_a X(\vec{\sigma}, \tau) \cdot \partial_b X(\vec{\sigma}', \tau) \rangle \sim \delta_{ab} \exp(-\sqrt{\lambda_T}|\vec{\sigma} - \vec{\sigma}'|). \quad (40)$$

In this case, the normals to the membranes are uncorrelated beyond a length scale $\lambda_T^{-1/2}$. The exponential decay of the correlation function shows that this solution corresponds to the crumpled phase. The length scale $\lambda_T^{-1/2}$ may also be identified as the persistence length $\xi_p \ [14, 22]$.

**Summary.** We have analyzed the temperature behavior of a membrane subject to thermal and quantum fluctuations in the limit of large embedding space dimension. We found that at zero temperature there is a crumpling transition at some critical stiffness $1/\alpha_c$. For membranes of finite lateral size, quantum fluctuations are still relevant and a crumpling transition occurs also at nonzero temperature in an auxiliary mean field approximation. As the lateral size of the membrane goes to infinity, the transition disappears, and the membrane is always crumpled in spite of quantum fluctuations, this being a consequence of the infrared divergences in the flat phase.

**REFERENCES**

[1] Canham P. B., J. Theor. Biol., 26 (1970) 61.
[2] Helfrich W., Z. Naturforsch., 28c (1973) 693.
[3] Helfrich W., J. Phys. (Paris), 46 (1985) 1263.
[4] Peliti L. and Leibler S., Phys. Rev. Lett., 54 (1985) 1690.
[5] Kleinert H., Phys. Lett. A, 114 (1986) 263.
[6] Förster D., Phys. Lett. A, 114 (1986) 115.
[7] Helfrich W., J. Phys. (Paris), 48 (1987) 285.
[8] Förster D., Europhys. Lett., 4 (1987) 65.
[9] Kleinert H., J. Stat. Phys., 56 (1989) 227.
[10] de Gennes P. G. and Taupin C., J. Phys. Chem., 86 (1982) 2294.
[11] Kleinert H., Phys. Rev. Lett., 5819871915.
[12] Alonso F. and Espriu D., Nucl. Phys. B, 283 (1987) 393.
[13] Olesen P. and Yang S., Nucl. Phys. B, 283 (1987) 73.
[14] David F. and Guitter E., Nucl. Phys. B, 295 (1988) 332.
[15] Borelli M. E. S., Schakel A. M. J. and Kleinert H., Phys. Lett. A, 267 (2000) 201.
[16] Polyakov A. M., Nucl. Phys. B, 268 (1987) 406.
[17] Kleinert H., Phys. Lett. B, 189 (1987) 187; Phys. Rev. Lett., 58 (1987) 1915.
[18] Kleinert H., Path Integrals in Quantum Mechanics, Statistics and Polymer Physics (World Scientific, Singapore) 1995.
[19] Diamantini M. C. and Kleinert H., Smoothening Transition of Rough Surfactant Surfaces, Berlin preprint (1998), cond-mat/9806073.
[20] Diamantini M. C., Kleinert H. and Trugenberger C. A., Phys. Rev. Lett., 82 (1999) 267.
[21] Diamantini M. C., Kleinert H. and Trugenberger C. A., Floppy Membranes, Berlin preprint (1999), [cond-mat/9903021].
[22] Leibler S., Statistical Mechanics of Membranes and Surfaces, edited by Nelson D., Piran T. and Weinberg S. (World Scientific, Singapore) 1989.