Universal property of single topological singularity dynamics

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Abstract

Using the random matrix theory we demonstrate explicitly the insensitivity of the transverse force on a moving vortex to non-intrinsic and non-magnetic impurities in a superconductor.

vortex dynamics, transverse force, Berry phase

I. MOTIVATION

The Berry phase acquired by a moving vortex suggests that the total transverse force on a moving vortex is proportional to the superfluid density at zero temperature, irrespective whether or not impurities are presented [1,2]. This prediction has been confirmed by a direct measurement of the transverse force in dirty superconductors [3]. Though the relaxation time approximation which accounts for impurity effects in vortex dynamics has been showed to inapplicable [4], the drastic reduction the total transverse force by impurities were believed right [5].

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To examine this question further, an exact formulation has been developed [6,7]. The distribution of impurities is assumed to be homogeneous and the variation of impurity potential is only appreciable on a scale much smaller than size of the vortex core. If the vortex is allowed to move slowly against the ionic lattice background, to the leading order of the vortex velocity \( v \), the total force acting on the vortex has the form \( \mathbf{F} = B \mathbf{v} \times \hat{z} - \eta \mathbf{v} \).

Within the BCS theory, the transverse coefficient \( B \) of the total transverse force is explicitly expressed in terms of the quasiparticle wave functions:

\[
B = i\hbar \sum_k \int d^3x \left\{ f_k \nabla_v u_k(x) \times \nabla_v u_k^*(x) - (1 - f_k) \nabla_v v_k(x) \times \nabla_v v_k^*(x) \right\} \cdot \hat{z}.
\] (1)

Here \( f_k = 1/(1 + e^{\beta E_k}) \) is the Fermi distribution function. The wave functions \( \{\Psi_k(x)\} \) and the corresponding eigenvalues \( \{E_k\} \) are determined by the usual Bogoliubov-de Gennes equation, \( \mathbf{H} \Psi_k(x) = E_k \Psi_k(x) \), with \( \Psi_k(x) = \begin{pmatrix} u_k(x) \\ v_k(x) \end{pmatrix} \) and the system Hamiltonian

\[
\mathbf{H} = \begin{pmatrix} H & \Delta \\ \Delta^* & -H^* \end{pmatrix}.
\]

Here \( H = -(\hbar^2/2m)\nabla^2 - \mu_F + V(x) \), and \( V(x) \) the impurity potential. The order parameter \( \Delta \) is determined self-consistently. Now we are ready to explicitly demonstrate that the coefficient \( B \) for the total transverse force is insensitive to impurities as first observed through the Berry phase calculation.

**II. DEMONSTRATION**

In the presence of impurities, the replacement \( \nabla_v \rightarrow -\nabla \) cannot be directly used in Eq.(1) because of the implicit impurity dependence. Instead, we expand \( \Psi_k \) in terms of eigenfunction \( \{\bar{\Psi}_l\} \) of \( \bar{\mathbf{H}} \), a corresponding Hamiltonian to \( \mathbf{H} \) without the impurity potential \( V(x) \) and with impurity averaged \( \Delta \),

\[
\Psi_k = \sum_l a_{kl} e^{i\delta_{kl}} \bar{\Psi}_l.
\] (2)
Here \( \{a_{kl}\} \) and \( \{\delta_{kl}\} \) are the modulus and phases of the expansion coefficients. They are functions of the vortex position as well as the positions of impurities. We assume these coefficients to be described separately by two independent random matrices, making use of the randomness of impurities. \( \hat{H} \) still has a dependence on impurities through the impurity averaged order parameter \( \Delta \): \( \hat{H} = \begin{pmatrix} \bar{H} & \Delta \\ \Delta^* & -\bar{H}^* \end{pmatrix} \), with \( \bar{H} = -(\hbar^2/2m)\nabla^2 - \mu_F \), \( \bar{H}\bar{\Psi}_l = \bar{E}_l \bar{\Psi}_l \). Since \( \{\bar{\Psi}_\mu\} \) form a complete set, the expansion coefficients \( \{a_{kl}e^{i\delta_{kl}}\} \) form a unitary matrix, and \( \sum_l a_{kl}^2 = 1 \). For \( \bar{\Psi}_l \) we can use the replacement \( \nabla_v \rightarrow -\nabla \). Away from the vortex core the value of the order parameter is the same as that in the clean case, guaranteed by the Anderson theorem. Using Eq.(2), Eq.(1) becomes

\[
B = -i\hbar \sum_k \sum_{l,l'} \int d^3 x \left\{ f_k \nabla_v a_{kl} e^{i\delta_{kl}} \bar{u}_l(x) \times \nabla_v a_{kl'} e^{-i\delta_{kl'}} \bar{u}_{l'}^*(x) \right\} \cdot \hat{z} > .
\]  

Here \( < ... > \) stands for the impurity average over the expansion coefficients. Then,

\[
B = -i\hbar \sum_k \sum_l < a_{kl}^2 > \int d^3 x \left\{ f_k \nabla_v \bar{\psi}_l(x) \times \nabla_v \bar{\psi}_{l}^*(x) \right\} \cdot \hat{z} .
\]  

Terms containing derivative to vortex position in the expansion coefficients have been averaged to zero. Now the replacement of \( \nabla_v \rightarrow -\nabla \) can be used, and turning the area integral into the line integral we have, at zero temperature, the desired result \( B = 2\pi \hbar \rho_0 \), because

\[
\sum_{k,E_k>0} \sum_l < a_{kl}^2 > |\bar{v}_l(|x-x_v| \rightarrow \infty)|^2 = \sum_{k,E_k>0} < |v_k(|x-x_v| \rightarrow \infty)|^2 >= \rho_0 .
\]  

The above second equality is the Anderson theorem. Eq.(5) can also be reached from the envelop wave function argumentation [9]. The insensitivity of the transverse force to impurities can also be demonstrated from core state transitions [7].

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Finally, we point out that the impurity effect on the transverse force were supposed on the scale of vortex core level space \([\mathcal{E}]\), which is extremely small comparing to the Fermi energy. There is indeed no effect at all to the superfluid density.

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