This paper studies disclosure games with general disclosure rules. A sender observes a piece of evidence about an unknown state and tries to influence the posterior belief of a receiver by disclosing evidence with possible omission. We characterize the unique equilibrium value function of the sender given any disclosure rule. Applying this characterization, we study left-censored disclosure, where evidence is a sequence of signals, and the sender can truncate evidence from the left. In equilibrium, seemingly sub-optimal messages are disclosed, and the sender’s disclosure contains the longest truncation that yields the maximal difference between the number of favorable and unfavorable signals. These findings are results of coordination among senders with different evidence endowment.

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Seldom, very seldom does complete truth belong to any human disclosure; seldom can it happen that something is not a little disguised, or a little mistaken; but where, as in this case, though the conduct is mistaken, the feelings are not, it may not be very material.

—Jane Austen, *Emma*

1. Introduction. Eliciting information from a biased party is challenging. Verifiability of information alleviates the problem by eliminating lies from disclosure, but rarely can we exclude omission or holdback of information. For example, a car dealer may emphasize the fuel efficiency of a car but neglect the fact that it lacks certain safety features; a researcher may draw conclusions using only the subsample of young adults; a job seeker may include five years of employment history in resume but leave out earlier experiences. In each case,
the disclosure bears true information about the subject of interest, but skepticism remains
due to possible omission of information.

Another observation from the above examples is that different ways of omitting information
are feasible in different contexts. The car dealer can omit any inferior aspects of the
car, while the researcher can only reasonably exclude observations based on the values of
certain observables. The job seeker can include work experiences in a most recent period but
usually cannot write a resume with gaps between jobs. How does the disclosure rule affect
information receiver’s skepticism and outcomes of communication?

In this paper, we study verifiable disclosure games, allowing for general disclosure rules.
A sender is endowed with a piece of hard evidence about an unknown state of the world
and tries to influence the posterior belief of a receiver by strategically disclosing evidence.
A disclosure rule describes all feasible ways of omitting information, i.e. for every possible
evidence endowment, a set of messages that the sender can disclose. The disclosure rule
is common knowledge, and the sender cannot commit to fully disclosing her evidence en-
dowment. Hence, the receiver is skeptical about the disclosed information and believes that
unfavorable information may be concealed.

Given any disclosure rule, we characterize the unique equilibrium value function of the
sender. Applying this general result, we study the following leading example of left-censored
disclosure.

1.1. Leading example. New hedge funds are often privately offered in incubation periods
before being publicly offered. The main goal of incubation is to accumulate trading records–
when a hedge fund turns full fledged, the manager can disclose (part of) the trading record
from the incubation period to potential investors as hypothetical returns.

In this leading example, we consider the interaction between a hedge fund manager (the
sender) who has a trading record from an incubation trial and a representative investor (the
receiver). For simplicity, the hedge fund is either a success or a failure, and the trading record
is a dated sequence of binary signals–high or low–each indicates the return on a trading day
in the incubation period. If the hedge fund is a success, it has frequent high returns, whereas
in the case of a failure, it has high returns with lower frequency. The fund manager’s goal is
to persuade the investor that the new hedge fund is a success (i.e. to increase the investor’s
posterior belief that the hedge fund is a success) by selectively disclosing the trading record.

The disclosure rule in place permits left censoring. Since the investor does not know
the starting date of the incubation period (we assume that the ending date is commonly
known as the current date or when the public offering is filed), the manager can truncate
the trading record from the left and disclose a set of most recent signals. This may be
profitable if the trading record contains relatively more low returns at the beginning of the incubation period. From the investor’s perspective, the incubation period starts no later than the earliest date reported but is unknown by how much earlier, and the investor cannot distinguish a left-censored report from a trading record that is shorter to begin with. This leads to the investor’s skepticism about the disclosed record.

1.2. Preview of results and related literature. We model disclosure rules as preorders on a fixed evidence space. This representation of disclosure rules satisfies the standing assumptions in the literature of verifiable disclosure (e.g. Bull and Watson (2007), Ben-Porath and Lipman (2012), and Hart et al. (2017)), but is more general in that the evidence space in our model need not be finite. Given any disclosure rule, we show that all equilibria are payoff equivalent and characterize the unique value function of the sender. This characterization is similar to that in Rappoport (2017), but our result uses a straightforward construction and applies to games with countably infinite evidence space. The construction can be used to construct an equilibrium from the equilibrium value function.

Using the general characterization result, we study a model of left-censored disclosure. The equilibrium has the following features. First, there is no unravelling. That is, full disclosure of the trading record is not an equilibrium. Grossman (1981), Milgrom (1981), and Milgrom and Roberts (1986) show that when disclosure is credible, and the sender’s bias is known to the receiver, all information is revealed. In the leading example, however, the investor does not become fully informed about the trading record. The reason is in line with the explanation by Dye (1985), Shin (1994, 2003), and Dziuda (2011): the receiver is uncertain about how well informed the sender is, and the sender can pretend to possess less evidence rather than less favorable evidence.

Second, initial low returns are disclosed in equilibrium. The manager’s equilibrium value depends on the maximal difference between the number of high returns and the number of low returns among all truncations of the trading record from the left. However, the manager may reveal a longer truncation than the one that obtains this maximal difference. In terms of face value, this longer truncation lower the manager’s payoff, but in equilibrium, the investor is not influenced by the redundant unfavorable evidence at the beginning of the report. This result is explained as coordination between senders with different evidence endowment. If all senders with trading records that have a same optimal truncation report the same, the persuasiveness of this message becomes so undermined that senders with some evidence endowment will find it optimal to report differently. Instead, senders report different messages in equilibrium, including those seemingly sub-optimal ones, such that all on-path messages yield a higher payoff than deviating.
This explanation complements previous studies which explain similar observations. Dziuda (2011) models a disclosure game with strategic and honest senders. The honest type always reveals the evidence without omission, so her message is discounted less by the receiver. The strategic type therefore has an incentive to mimic the honest type by reporting some seemingly unfavorable evidence. Guttman et al. (2014) study the dynamic disclosure of firm performance, and show that due to dynamic considerations, later reports are interpreted as more favorable although the timing of obtaining signals is independent of the firm’s value. Our model does not have an honest type or dynamic considerations. It is also worth noting here that in our model, the informativeness of each signal is the same, so the result does not arise because the receiver values earlier signals less.

Third, when two truncations both yield the maximal difference between the number of high and low returns, the equilibrium message always contains the longer truncation (it may be even longer, as is explained above). This is not due to any intrinsic value of the length of evidence. In fact, the length of trading record per se is uninformative about the quality of the hedge fund in our model, and there is no tie-breaking rule that favors longer messages. The result is again due to coordination between senders with different evidence endowment. In equilibrium, senders with longer evidence endowment also report longer, so that senders with shorter evidence endowment who cannot make longer reports pool on reporting shorter truncations.

The rest of the paper is organized as follows. Section 2 presents our main model, and Section 3 characterizes the unique value function of the sender. We present and solve a model of the leading example in Section 4. The last section concludes. All proofs are relegated in the appendix.

2. The Model. There are two stages. Two players, a sender (she) and a receiver (he), move sequentially. At the outset of the game, a state of the world \( \omega \in \{G, B\} \) is realized with probability \( \pi_0 \in (0, 1) \) on \( \omega = G \). Neither player observes the realized state \( \omega \), and the prior \( \pi_0 \) is common knowledge. In the first stage, the sender observes a piece of hard evidence, denoted by \( e \), about the realized state. She then reports a message \( m \) to the receiver. In the second stage, the receiver observes the message and takes an action \( a \).

2.1. Evidence. The evidence space \( E \) is exogenously given and contains finite or countably many elements. Let \( F_G \) and \( F_B \) be two distributions supported on \( E \) (i.e. \( F_G(e) > 0 \) and \( F_B(e) > 0 \) for all \( e \in E \)). In the first stage, the sender observes a piece of evidence \( e \in E \), which is referred to as her evidence endowment. It is a random draw from either \( F_G \) or \( F_B \), depending on the realized state. If \( \omega = G \), \( e \) is drawn from distribution \( F_G \); if \( \omega = B \), it is drawn from distribution \( F_B \).
2.2. Messages and Disclosure Rules. After observing her evidence endowment \( e \), the sender reports a message \( m \in M(e) \) to the receiver, where \( M(e) \) denotes the set of feasible messages given her evidence endowment \( e \). Naturally, the collection of the sender’s feasible messages \( \{M(e)\}_{e \in E} \) describes the disclosure rule of the game. We consider a broad class of disclosure rules satisfying the following standing assumptions:

(A1) \( M(e) \subset E \) for all \( e \in E \);

(A2) (Reflexivity) \( e \in M(e) \) for all \( e \in E \);

(A3) (Transitivity) If \( e' \in M(e) \) and \( e'' \in M(e') \), \( e'' \in M(e) \);

(A4) For all sequences \( \{e_n\}_{n=1}^{\infty} \) in \( E \) such that \( M(e_1) \supset M(e_2) \supset \ldots \), there exists \( N \geq 1 \) such that \( M(e_n) = M(e_N) \) for all \( n \geq N \).

One way to interpret these assumptions is to view evidence and messages as physical documents (Bull and Watson, 2004). The sender is endowed with a set of documents and is able to omit some documents and submit the remainder to the receiver. (A2) says that the sender is always allowed to submit all documents without omission. (A3) says that if there are two senders handling documents sequentially, both abiding by the disclosure rule, then any omission that can be done by two senders can be done by just one sender under the disclosure rule. (A4) says that even if there are infinitely many senders who can sequentially omit part of the documents, the process of omission must stop in finite steps. One implication of (A4) is that in any subset of the evidence space, there exists at least one minimal evidence to which further omission of information is not possible.

Under (A1) through (A3), a disclosure rule can be equivalently defined as a preorder \( \sim \) on \( E \) such that \( e' \sim e \) if and only if \( e' \in M(e) \).

An equivalence of (A4) in terms of this preorder is as follows.

(A4') For all sequences \( \{e_n\}_{n=1}^{\infty} \) in \( E \) such that \( e_1 \sim e_2 \sim \ldots \), there exists \( N \geq 1 \) such that \( e_N \sim e_n \) for all \( n \geq N \).

In the remainder of the paper, we shall abstract away the specific structure of the message space, and refer to a disclosure rule simply by a preorder on the evidence space satisfying

\[\text{(A1)} \] through (A3) are standard in the literature (see, for example, Ben-Porath and Lipman (2012) and Hart et al. (2017)), and they are without loss of generality if normality is assumed (Bull and Watson, 2007). A disclosure rule \( \{M(e)\}_{e \in E} \) satisfies normality if for every \( e \in E \), there exists \( m_e \in M(e) \) such that \( m_e \in M(e') \Rightarrow M(e) \subset M(e') \) for all \( e' \in E \). That is, if a sender with evidence \( e \) can distinguish herself from a sender with evidence \( e' \) (using some message in \( M(e) \) but not in \( M(e') \)), then she can do so using a “maximal message” \( m_e \). Hence, it is without loss of generality to consider only maximal messages, i.e. \( \{m_e\}_{e \in E} \), which can be mapped onto from the evidence space \( E \).

\[\text{(A4)} \] is a technical assumption. It is automatically satisfied when the evidence space is finite. To the best of my knowledge, no previous paper has studied disclosure games with countably infinite evidence spaces where (A4) is violated.

A preorder \( \sim \) is a binary relation satisfying reflexivity (\( e \sim e \) for all \( e \)) and transitivity (\( e'' \sim e' \sim e \Rightarrow e'' \sim e \)).
Given a disclosure rule \( \preceq \), the sender who observes evidence \( e \) reports a message \( m \preceq e \) to the receiver.

2.3. The receiver’s action. In the second stage, the receiver observes the sender’s message \( m \) and takes an action \( a \in \mathbb{R} \).

2.4. Payoffs. The receiver has quadratic loss utility function and maximizes his expected payoff, i.e. \( u_R(a, \omega) = -(a - x_\omega)^2 \), where \( x_G = 1 \) and \( x_B = 0 \). The receiver’s optimal action therefore is to choose action equal to his posterior belief on \( \omega = G \).

The sender’s payoff equals the receiver’s action, i.e. \( u_S(a, \omega) = a \). That is, the sender has an incentive to persuade the receiver that the state is good. Specifically, the sender’s evidence endowment \( e \) and the realized state \( \omega \) are payoff irrelevant to the sender.

2.5. Strategies. A (pure) strategy of the sender is \( s : E \rightarrow E \) such that \( s(e) \preceq e \) for all \( e \in E \). A mixed strategy of the sender is \( \sigma : E \rightarrow \Delta(E) \) such that \( \text{supp}(\sigma(\cdot|e)) \subset LC(e) \) for all \( e \in E \), where \( LC(e) = \{ m : m \preceq e \} \) is the lower contour set of \( e \) with respect to the disclosure rule.

A (pure) strategy of the receiver is \( a : E \rightarrow \mathbb{R} \). A system of beliefs for the receiver is \( \mu : E \rightarrow [0, 1] \), where \( \mu(m) \) is the receiver’s posterior belief on \( \omega = G \) after receiving message \( m \). Since the receiver’s problem admits a unique solution given any posterior belief, the receiver never uses mixed strategies in equilibrium.

2.6. Equilibrium. A disclosure game may have multiple Nash equilibria, including some trivial ones. For example, if the sender can always omit all evidence and report an empty message, there exists a Nash equilibrium where the sender always reports this empty message and the receiver chooses the prior regardless of the message. Following [Hart et al. 2017], we focus on truth-leaning equilibria.

A truth-leaning equilibrium (or simply, an equilibrium) is a collection of the sender’s strategy, the receiver’s strategy, and the receiver’s system of beliefs \( (\sigma^*, a^*, \mu) \), such that:

1. (Sender optimality) Given \( a^* \),

\[
\text{supp}(\sigma^*(\cdot|e)) \subset \arg\max_{m \preceq e} a^*(m)
\]

for all \( e \in E \);

2. (Receiver optimality) Given \( \mu \),

\[
a^* = \mu;
\]
3. (Bayesian consistency) Given $\sigma^\star$, the receiver’s posterior belief at any on-path message $m \in \bigcup_{e \in E} \text{supp}(\sigma^\star(\cdot|e))$ is given by the Bayes’ rule

$$
\mu(m) = \frac{\sum_{e \in \text{UC}(m)} \sigma^\star(m|e) F_G(e) \pi_0}{\sum_{e \in \text{UC}(m)} \sigma^\star(m|e)[F_G(e)\pi_0 + F_B(e)(1 - \pi_0)]},
$$

where $\text{UC}(m) = \{e : m \preceq e\}$ is the upper contour set of $m$;

4. (Truth-leaning) Given $a^\star$,

$$
e \in \arg\max_{m \preceq e} a^\star(m) \Rightarrow \sigma^\star(e|e) = 1;
$$

5. (Off-path beliefs) The receiver’s belief at any off-path message $m$ is

$$
\mu(m) = \frac{F_G(m)\pi_0}{F_G(m)\pi_0 + F_B(m)(1 - \pi_0)}.
$$

In other words, a truth-leaning equilibrium is a Perfect Bayesian Equilibrium that satisfies the following additional conditions:

1. (Truth-leaning) The sender strictly prefers disclosing her evidence endowment $e$ to disclosing other messages if reporting $e$ is optimal;

2. (Off-path beliefs) If an off-path message $m$ is reported, the receiver believes that the sender is disclosing her evidence endowment without omission, i.e. $e = m$.

Several remarks are in order regarding the equilibrium concept. First, truth-leaning equilibria can be viewed as limit points of Nash equilibria in perturbed games where (i) there is an infinitesimal probability that the sender’s evidence endowment is revealed to the receiver regardless of what she chooses to disclose, and (ii) there is an infinitesimal reward for revealing the evidence endowment. Second, a truth-leaning equilibrium is receiver optimal and has the same outcome as in a mechanism where the receiver can commit to a reward policy. That is, there is no value of commitment to a reward policy for the receiver. Lastly, condition 2 above is a tie-breaking rule that applies only when reporting the evidence endowment is optimal—it does not require the sender to report the optimal message that omits the least information if reporting the evidence endowment is not optimal.

2.7. Notations. We conclude this section by introducing two auxiliary functions and some notations related to disclosure rules. Let $\nu$ be a set function on $E$ such that for all
\(A \subset E\) and \(A \neq \emptyset\),

\[
(6) \quad \nu(A) = \frac{F_G(A)\pi_0}{F_G(A)\pi_0 + F_B(A)(1 - \pi_0)}.
\]

If \(A = \{e\}\) is a singleton, we write it as a function, and \(\nu(e)\) is the sender’s private belief on \(\omega = G\) after observing evidence \(e\), derived from the Bayes’ rule. By definition, it is also the receiver’s posterior belief at an off-path message in a truth-leaning equilibrium. Given any equilibrium, let \(V\) be a function such that \(V(e) = \max_{m \preceq e} a^*(m)\) for all \(e \in E\). This is referred to as the sender’s value function.

Given a disclosure rule \(\preceq\), we denote the asymmetric part by \(\prec\) (i.e. \(e \prec e' \iff e \preceq e', e' \not\preceq e\)), and the equivalence relation by \(\sim\) (i.e. \(e \sim e' \iff e \preceq e', e' \preceq e\)). Let \(A \subset E\) and \(L \subset A\) be nonempty. We say \(L\) is a lower contour set in \(A\) if for all \(e \in L\) and \(e' \in A\), \(e' \not\preceq e \Rightarrow e' \in L\).

3. Main Results. In Theorem \(\Pi\) we show that the sender’s equilibrium value function is unique. That is, all equilibria of a disclosure game are payoff equivalent. Moreover, we characterize the sender’s equilibrium value function by three conditions.

**Theorem 1.** A function \(V : E \rightarrow [0, 1]\) is the sender’s value function in an equilibrium if and only if it satisfies the following conditions:

1. \(V : (E, \preceq) \rightarrow ([0, 1], \leq)\) is non-decreasing;
2. \(\nu(V^{-1}(x)) = x\) for all \(x \in \text{Range}(V)\);
3. \(\nu(L) \geq x\) for all \(x \in \text{Range}(V)\) and all lower contour sets \(L\) in \(V^{-1}(x)\).

Moreover, the function satisfying the above conditions is unique.

Rappoport (2017) shows a similar result with finite evidence space under a similar setup. A contribution of this paper is to provide an alternative and more straightforward proof using induction which applies to disclosure games with infinite evidence spaces. The induction process, described below and detailed in the proof of Theorem \(\Pi\) can be used to construct equilibrium strategies from the sender’s equilibrium value function.

We outline the proof for the “if” part here. That is, given a function \(V\) satisfying conditions 1 though 3 of Theorem \(\Pi\) we want to find an equilibrium such that \(V\) is the sender’s value function. First, notice that in any equilibrium, if \(V(e) \leq \nu(e)\) at some \(e \in E\), then the sender reports \(e\) when her evidence endowment is \(e\), and \(a^*(e) = \mu(e) = V(e)\); if \(V(e) > \nu(e)\), then \(e\) as a message is off-path, and \(a^*(e) = \mu(e) = \nu(e)\). Since \(\nu\) is determined solely by \(F_G\) and \(F_B\), given any \(V\), \(a^*\) and \(\mu\) can be pinned down in any candidate equilibrium. The remaining question is to find a reporting strategy \(\sigma^*\) that is optimal and truth-leaning, and is consistent with \(\mu\) and the Bayes’ rule at on-path messages. To do so, we consider the
following induction process. Suppose that we have a candidate strategy of the sender defined on a lower contour set $L$ in $E$, satisfying the following two properties: (i) sender optimality and truth-leaning are satisfies on $L$, and (ii) the receiver’s posterior belief at each on-path message $m \in L$ calculated by the Bayes’ rule is at least $V(m)$. If $L = E$, the candidate strategy is indeed an equilibrium strategy, and $V$ is the sender’s value function in this equilibrium. If $L \subset E$, condition 3 of Theorem 1 ensures that there are “slacks” in property (ii), and we can extend the candidate reporting strategy beyond $L$. Indeed, in the proof of Theorem 1 we construct another candidate strategy defined on a lower contour set $\tilde{L} \supset L$ in $E$ that satisfies properties (i) and (ii), so we may continue with the induction process. If the evidence space is finite, the induction process solves an equilibrium strategy. If the evidence space is not finite, using transfinite induction, we show existence of an equilibrium such that $V$ is the sender’s value function.

Since the sender’s value function is the same in all equilibria, we shall use the term without referring to a specific equilibrium. Naturally, given the sender’s value function, we can partition the evidence space by the sender’s equilibrium value. The following corollary formalizes this idea using the notion of ordered partition and states an equivalent result of Theorem 1.

**Corollary 2.** An equilibrium exists if and only if there exists an ordered partition of $E$, denoted by $(A, \leq_P)$, satisfying:

1. $\nu : (A, \leq_P) \to ([0,1], \leq)$ is strictly increasing;
2. For all $A_1, A_2 \in A$ and $e_1 \in A_1, e_2 \in A_2$, $e_1 \preceq e_2 \Rightarrow A_1 \leq_P A_2$;
3. For all $A \in A$ and all lower contour sets $L$ in $A$, $\nu(L) \geq \nu(A)$.

Moreover, the sender’s value function $V$ satisfies $V(e) = \nu(A(e))$ for all $e \in E$, where $A(e) \in A$ is such that $A(e) \ni e$. The ordered partition satisfying the above conditions is unique.

We refer to the ordered partition $(A, \leq_P)$ in Corollary 2 as the *equilibrium partition*. Corollary 2 provides useful conditions to verify whether an ordered partition is the equilibrium partition, but it provides little insight on existence of equilibrium or how to solve the equilibrium partition. On the other hand, [Hart et al. (2017)] show that truth-leaning equilibrium exists in disclosure games with finite evidence spaces using the standard existence argument of Nash equilibrium, but their result does not help constructing an equilibrium either. Although we are unable to show existence when the evidence space is countably infinite, we further characterize the equilibrium partition in the following theorem and propose

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4 An ordered partition (Stanley, 1999) is a partition with a total order defined on the sets of the partition. Recall that a total order $\leq$ (on a set $X$) is a partial order that is also connex ($\forall x, x' \in X$, $x \leq x'$ or $x' \leq x$).
an induction process which solves the equilibrium partition for a broad class of disclosure
games, including those with finite evidence spaces and the model of left-censored disclosure
presented in Section 4.

**Theorem 3.** An ordered partition \((A, \leq_P)\) of \(E\) is the equilibrium partition if and only
if every \(A \in A\) is the largest lower contour set in \(E \setminus \bigcup_{A' \in P A} A'\) that minimizes \(\nu\). That is, \(A\) solves

\[
\min_L \nu(L) \quad \text{s.t.} \quad L \text{ is a lower contour set in } E \setminus \bigcup_{A' \in P A} A',
\]

and for all \(L\) that solves (7), \(L \subset A\).

Following Theorem 3, we propose an induction process to construct the equilibrium
partition. First, solve the minimization problem

\[
\min_L \nu(L) \quad \text{s.t.} \quad L \text{ is a lower contour set in } E.
\]

As is shown in the proof of Theorem 3, if the above problem admits a minimizer, then it
has a largest minimizer. Denote by \(A_0\) this largest minimizer. Now suppose that for some
\(n \in \mathbb{N}\), \(A_0, A_1, \ldots, A_n\) are such that every \(A_k\) is the largest lower contour set in \(E \setminus \bigcup_{i<k} A_i\)
that minimizes \(\nu\). We solve the minimization problem

\[
\min_L \nu(L) \quad \text{s.t.} \quad L \text{ is a lower contour set in } E \setminus \bigcup_{i \leq n} A_i.
\]

If the above minimization problem admits a minimizer, then it has a largest minimizer. Let
\(A_{n+1}\) be this largest lower contour set, and we can proceed by induction.

By Theorem 3, if the equilibrium partition is finite, then the above induction process ends
after finite steps, and the resulted partition \(A = \{A_0, A_1, \ldots, A_n\}\), along with the total order
\(\leq_P\) such that \(A_i \leq_P A_j \iff i \leq j\), is the equilibrium partition. Specifically, the above induction
process solves the equilibrium partition when the evidence space is finite. If the equilibrium
partition is not finite, but sets in \(A\) can be indexed by \(\mathbb{N}\) such that \(A_i \leq_P A_j \iff i \leq j\),
then every \(A_n\) is solved by the above induction process. Conversely, if every \(A_n\) in the above
induction process is well defined, and \(\bigcup_{n \in \mathbb{N}} A_n = E\), then an equilibrium exists, and the
equilibrium partition is given by \(A = \{A_n\}_{n \in \mathbb{N}}\) and \(\leq_P\) such that \(A_i \leq_P A_j \iff i \leq j\).

**4. Disclosure of Left-Censored Data.** This section models the leading example of
left-censored disclosure in Section 3. The hedge fund manager is the sender, and the investor
is the receiver. The two possible states, \( G \) and \( B \), corresponds to the hedge fund being a success and a failure, respectively. We first specify the evidence space and the disclosure rule, and then discuss the equilibrium features.

4.1. Evidence. The evidence space \( E = \bigcup_{t=0}^{\infty} \{g, b\}^t \). A piece of evidence \( e \in E \) is a sequence of binary signals. A typical element of length \( L > 0 \) is denoted by \( e = (s_1, s_2, \ldots, s_L) \), where each \( s_t \in \{g, b\} \). This can be interpreted as a trading record of the incubation trial, and each signal \( s_t \) represents the return on the \( t \)-th trading day—\( s_t = g \) stands for high return, and \( s_t = b \) stands for low return.

There is a “null” evidence (i.e. the evidence of length zero), denoted by \( \emptyset \). For every \( e \in E \), let \( L(e) \) denote the length of \( e \), \( G(e) \) the number of \( g \)'s in \( e \), \( B(e) \) the number of \( b \)'s in \( e \), and \( D(e) = G(h) - B(h) \) the difference between the number of \( g \)' and \( b \)'s. Moreover, for every \( e \) of length \( L > 0 \) and \( 0 < k \leq L \), let \( e|_k = (s_{L-k+1}, s_{L-k+2}, \ldots, s_L) \) denote a truncation of \( e \) from the left. As a convention, \( e|_0 = \emptyset \) for all \( e \in E \).

Let the two distributions \( F_G \) and \( F_B \) be as follows:

\[
F_\omega(e) = (1 - p - q)p^K q^{L(e)-K}
\]

for all \( e \in E \) and \( \omega \in \{G, B\} \), where \( K = G(e) \) if \( \omega = G \) and \( K = B(e) \) if \( \omega = B \), and \( p, q \) are such that \( 1 > p > q > 0 \) and \( 1 > p + q > 0 \).

Given the specification in (8), the length of evidence follows a geometric distribution and is not informative about the realized state. Given the length of evidence, each signal is an independent draw from a Bernoulli distribution—with probability \( \frac{p}{p+q} \), it indicates the realized state; with the complement probability \( \frac{q}{p+q} \), it indicates the alternative state. The informativeness of each signal is constant over time and symmetric across states.

4.2. Left censoring disclosure rule. Given her evidence endowment, the sender can disclose a set of most recent signals and conceal earlier signals, i.e. she can report a truncation of her evidence endowment. This defines the left censoring disclosure rule. Formally, it is a partial order \( \preceq \), such that \( e_1 \preceq e_2 \Leftrightarrow e_1 = e_2|_{L(e_1)} \).

4.3. Equilibrium characterization. The sender’s value function is given by the following proposition.

**PROPOSITION 4** (Equilibrium value function). Let \( N : E \rightarrow \mathbb{N} \) be defined by

\[
N(e) = \max_{k \leq L(e)} D(e|_k)
\]
An equilibrium exists, and the sender’s value function is

\[ V(e) = \frac{1}{1 + \frac{1-\pi_0}{\pi_0} \left( \frac{2}{p} \right)^{N(e)+1}}. \]

where \( \alpha = \frac{1-\sqrt{1-4pq}}{2} \).

The proposition states that in any equilibrium, given an evidence endowment \( e \), the sender’s equilibrium value is determined by \( N(e) \), which is the maximal difference between the number of \( g \)'s and the number of \( b \)'s among all truncations. Senders with different evidence endowment but a same value of \( N \) receive the same payoff in any equilibrium.

We now characterize the sender’s equilibrium strategy. A seemingly optimal strategy for the sender with evidence endowment \( e \) is to report \( e|_k \) such that \( D(e|_k) = N(e) \). Disclosing any other message \( m \) appears to be sub-optimal, since from the receiver’s perspectives, \( m \) is worse than \( e|_k \) if interpreted at face value (i.e. \( \nu(m) < \nu(e|_k) \)). However, this is not necessarily the case. In fact, the sender sometimes report longer messages which include redundant and seemingly unfavorable information at the beginning of the messages. The receiver, on the other hand, is not influenced by the unfavorable information in on-path messages and acts the same as if only \( e|_k \) is reported.

Notice that for all \( e \in E \), \( \nu(e) \geq V(e) \) if \( D(e) \geq N(e) - n^* \), where

\[ n^* = \log_{p/q} \left( \frac{p-\alpha}{q-\alpha} \right) - 1 > 0 \]

is a constant determined solely by the model parameters \( p \) and \( q \). Hence, in any equilibrium, an evidence endowment \( e \) is disclosed without omission so long as \( D(e) \geq N(e) - n^* \). This demonstrates our first point. That is, seemingly unfavorable information may be reported on-path. Specifically, if

\[ (p + q)(1 - p - q) \leq pq, \]

then \( n^* \geq 1 \), and in any equilibrium, some seemingly unfavorable information is disclosed. For instance, evidence \( e = (b) \) is disclosed without omission in any equilibrium under the parametric assumption \( (11) \), even though reporting the null evidence is feasible and seemingly better in that \( \nu(\emptyset) > \nu((b)) \). This result may appear similar to the result in Dziuda (2011), but the strategic reasoning underlying these results are different. In Dziuda (2011), the sender discloses unfavorable signals in order to be perceived as being honest. In the current paper, it is an act of coordination between senders with different evidence endowment: if
all senders whose evidence endowment has a same optimal truncation pool on reporting the same message, the persuasiveness of the message is so undermined that some senders will find it profitable to deviate. Instead, senders pool on several messages, including those seemingly sub-optimal ones, such that all of them have the same persuasiveness and yield a strictly higher payoff than deviating to other feasible messages.

Secondly, when there are multiple truncations that yield the maximal difference between the number of $g$’s and the number of $b$’s, the sender’s message always contains the longest truncation. That is, in any equilibrium, $L(m) \geq \max(\text{argmax}_k D(e|_k))$ for all $e \in E$ and $m \in \text{supp}(\sigma^*(\cdot|e))$. For instance, the sender with evidence endowment $(b, b, b, g, b)$ will always include the two most recent signals $(g, b)$ in her message, although both truncations $(g, b)$ and $\emptyset$ contain the same number of $g$’s and $b$’s. This result follows from the fact that $\nu(E_0^\nu) = \nu(E_0)$, where $E_0^\nu$ is the set containing the null evidence and all evidence such that every truncation has strictly more $b$’s than $g$’s, and $E_0$ is the set of evidence such that $N(e) = 0$. Notice that $E_0^\nu$ is a lower contour set in $E_0$. Therefore, no evidence $e \in E_0 \setminus E_0^\nu$ can report a message $m \in E_0^\nu$ with positive probability in any equilibrium: after accommodating all reports from $e \in E_0^\nu$, the sender’s posterior beliefs at on-path messages in $E_0^\nu$ is already $\nu(E_0)$; any further pooling from evidence outside the set $E_0^\nu$ will make the posterior beliefs at these messages too low. Specifically, since $(b, b, b, g, b) \in E_0 \setminus E_0^\nu$, and $\emptyset, (b) \in E_0^\nu$, the sender with evidence endowment $(b, b, b, g, b)$ will disclose at least the two most recent signals $(g, b)$ in her message.

Lastly, since all evidence $e$ with a same value of $N(e)$ yield the same sender’s equilibrium value, the receiver’s action is not affected by the unfavorable information revealed by the sender in equilibrium. Unless the message contains so many bad signals that it is not an on-path message, the receiver will act based solely on the favorable information in the message, i.e. his posterior depends on $N(m)$ instead of $D(m)$.

The proposition below summarizes the results.

**Proposition 5 (Equilibrium strategies).** In any equilibrium $(\sigma^*, a^*, \mu)$, for all $e \in E$:

1. $a^*(e) = \mu(e) = \min\{V(e), \nu(e)\}$, where $V$ is given by (9);
2. $\sigma^*(e|e) = 1$ if and only if $D(e) \geq N(e) - n^*$, where $n^* > 0$ is given by (10);
3. If $\sigma^*(m|e) > 0$, then $L(m) \geq \max(\text{argmax}_k N(e|_k))$.

4.4. **Comparative statics.** The following proposition shows how the sender’s equilibrium value change with respect to parameters $p$ and $q$. Notice that increasing, say $p$, while holding $q$ constant introduces two effects: on the one hand, it increases the informativeness of each signal; on the other hand, the expected length of the sender’s evidence endowment increases. We introduce the following notations to disentangle the two effects. Let $\gamma = \frac{p}{q} > 1$ be the
informativeness of each signal, and $\kappa = \frac{1}{1 - p - q}$ the expected length of the sender’s evidence endowment. Given $\gamma > 1$ and $\kappa > 1$, $p$ and $q$ are determined.

**Proposition 6 (Comparative statics).** The following are true:

1. $V(e)$ is decreasing in $\kappa$ for all $e \in E$.
2. If $N(e) = 0$, $V(e)$ is decreasing in $\gamma$. If $N(e) > 0$, there exists $\hat{\gamma} > 1$ such that $V(e)$ is increasing in $\gamma$ when $\gamma > \hat{\gamma}$, and the threshold $\hat{\gamma} \rightarrow 1$ as $N(e) \rightarrow \infty$.
3. Fixing $e \in E$ and $q \in (0, \frac{1}{2})$, there exists $\hat{p} \in [q, 1 - q)$ such that $V(e)$ is increasing in $p$ if $p < \hat{p}$ and decreasing in $p$ if $p > \hat{p}$, and the threshold $\hat{p}$ is non-decreasing in $N(e)$.

The proposition shows different effects of increasing the informativeness of signals and increasing the length of evidence on the sender’s equilibrium value. As is demonstrated in Figure 1(a) for $N(e) = 2$, increasing the expected length of the sender’s evidene endowment always decreases the sender’s equilibrium value, whereas the sender may benefit from increased informativeness of signals. When $\kappa$ is small, the sender’s equilibrium value is monotone increasing in $\gamma$; when $\kappa$ is large, the sender’s equilibrium value first decreases and then increases as $\gamma$ increases. The proposition shows that there exists a threshold $\hat{\gamma} > 1$, independent of $\kappa$, such that the sender’s value increases in $\gamma$ for all $\gamma > \hat{\gamma}$ and $\kappa > 1$. An equivalent interpretation of the result is that the effect of increasing the informativeness of signals depends on the sender’s evidence endowment. Fixing $\gamma > 1$, there is a threshold $\hat{n} \geq 1$ such that the sender benefits from increasing the informativeness of signals if her evidence endowment is such that $N(e) > \hat{n}$. That is, increasing the informativeness of signals
benefits the sender if her evidence endowment is relatively favorable.

Combining the two competing effects, the effect of increasing $p$ on the sender’s equilibrium value depends on the sender’s evidence endowment and is generally not monotonic. This is demonstrated in Figure 1(b). If the sender’s evidence endowment is relatively unfavorable (specifically, if $N(e) = 0$), increasing $p$ decreases the sender’s equilibrium value. If the sender’s evidence endowment is relatively favorable, the sender’s equilibrium value first increases and then decreases as $p$ increases. When $p$ approaches $1 - q$, the expected length of the sender’s evidence endowment tends to infinity. Regardless of what the sender discloses, the receiver believes that there are infinitely many signals being omitted, and his skepticism becomes so overwhelming that the sender receives almost zero payoff in equilibrium.

5. Conclusion. There are many situations where communication relies on hard evidence. However, even when information is verifiable, a party may withhold key information from others if their interests are not perfectly aligned. In this paper, we model disclosure rules as preorders on a countable evidence space and study a unified framework of disclosure games.

We show that in any disclosure game, all equilibria are payoff equivalent, and the unique equilibrium value function of the sender is characterized by three conditions. Related to this result, we present two induction processes. The first one can be used to construct an equilibrium from the sender’s equilibrium value function, and the second process solves the sender’s equilibrium value function for a broad class of disclosure games.

Applying these general results, we study a model of left-censored disclosure. In this model, evidence is a sequence of signals, and the sender is allowed to disclose a truncation of her evidence endowment to the receiver. In equilibrium, there is no unravelling. Moreover, seemingly sub-optimal messages are reported by the sender, and the sender’s equilibrium strategy favors longer messages over shorter ones, although the length of evidence has no intrinsic value in our model. Lastly, we show that increasing the expected length of evidence always decreases the sender’s equilibrium value, while the effect of increasing the informativeness of signals depends on the sender’s evidence endowment.

Appendix A. Omitted Proofs

A.1. Preliminaries. We will repeatedly use Zorn’s lemma in the proofs of our results. The statement of the lemma is as below. Recall that given a partially ordered set $(X, \leq)$, a chain is a totally ordered subset of $X$, an upper bound of a subset $A \subset X$ is an element $x \in X$ such that $x \geq y$ for all $y \in A$, and a maximal element of $X$ is an element $x \in X$ such
that $y \geq x \Rightarrow y = x$.

**Lemma A.1 (Zorn’s lemma).** Let $X$ be a partially ordered set. If every chain in $X$ has an upper bound, then $X$ has a maximal element.

By $(A4')$ and Zorn’s lemma, for all nonempty sets $A \subset E$, the set of minimal elements $\min(A, \preceq)$ is nonempty.

**A.2. Proof of Theorem 1.** We divide the proof into a sequence of lemmas. Lemma A.2 and Lemma A.3 show that a function $V : E \rightarrow [0, 1]$ is the sender’s value function in an equilibrium if and only if it satisfies conditions 1 through 3 of Theorem 1. Lemma A.4 shows that the value function is unique.

**Lemma A.2 (Necessity).** Let $(\sigma^*, a^*, \mu)$ be an equilibrium, and $V$ the sender’s value function. Then $V$ satisfies conditions 1 through 3 of Theorem 1.

**Proof.** Condition 1 is by definition. If $e_1 \preceq e_2$, $LC(e_1) \subset LC(e_2)$. Hence, $V(e_1) = \max_{m \in LC(e_1)} a^*(m) \leq \max_{m \in LC(e_2)} a^*(m) = V(e_2)$. $V$ is non-decreasing.

For condition 2, let $M_x = \bigcup_{e \in V^{-1}(x)} supp(\sigma^*(\cdot|e))$ be the set of on-path messages which yield sender value $x$. For all $m \in M_x$,

$$x = \mu(m) = \frac{\sum_{e \in V^{-1}(x)} \sigma^*(m|e)F_G(e)\pi_0}{\sum_{e \in V^{-1}(x)} \sigma^*(m|e)[F_G(e)\pi_0 + F_B(e)(1 - \pi_0)]}. \quad (A.1)$$

Rearranging and summing $(A.1)$ over $m \in M_x$ yields $x = \nu(V^{-1}(x))$.

For condition 3, let $M_L = \bigcup_{e \in L} supp(\sigma^*(\cdot|e))$ be the set of on-path messages reported by senders with evidence endowment in $L$. By assumption, $M_L \subset L$. Hence, for all $m \in M_L$ and $e \in V^{-1}(x) \setminus L$, if $\sigma^*(m|e) > 0$, reporting $e$ is not optimal at evidence endowment $e$. Therefore, $e$ is off-path as a message, and

$$x = V(e) > \nu(e) = \frac{F_G(e)\pi_0}{F_G(e)\pi_0 + F_B(e)(1 - \pi_0)}. \quad (A.2)$$

Rearranging and summing $(A.1)$ and using $(A.2)$,

$$x = \frac{\left(\sum_{e \in L} + \sum_{e \in V^{-1}(x) \setminus L}\right) (\sigma^*(m|e)F_G(e)\pi_0)}{\left(\sum_{e \in L} + \sum_{e \in V^{-1}(x) \setminus L}\right) (\sigma^*(m|e)[F_G(e)\pi_0 + F_B(e)(1 - \pi_0)])} \leq \frac{\sum_{e \in L} \sigma^*(m|e)F_G(e)\pi_0}{\sum_{e \in L} \sigma^*(m|e)[F_G(e)\pi_0 + F_B(e)(1 - \pi_0)].} \quad (A.3)$$

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Rearranging and summing (A.3) over $m \in M_L$ yields the desired result, i.e. $x \leq \nu(L)$.

**Lemma A.3 (Sufficiency).** Let $V : E \to [0, 1]$ be a function satisfying conditions 1 through 3 of Theorem 1. Then there exists an equilibrium in which the sender’s value function is $V$.

**Proof.** Notice that in any equilibrium $(\sigma^*, a^*, \mu)$ and for all $e \in E$, one of the following two cases happens:

1. The sender with evidence endowment $e$ reports $e$, and $a^*(e) = \mu(e) = V(e) \leq \nu(e)$;
2. $e$ is off-path as a message, and $a^*(e) = \mu(e) = \nu(e) < \nu(e)$.

Therefore, given $V$, any candidate equilibrium $(\sigma, a, \mu)$ in which the sender’s value function is $V$ must satisfy

(A.4) \[ a(e) = \mu(e) = \min\{V(e), \nu(e)\} \]

for all $e \in E$. Moreover, the set of on-path messages is $O := \{e : V(e) \leq \nu(e)\}$. Since $\nu$ depends solely on $F_G$ and $F_B$, and $V$ is given, the receiver’s strategy and beliefs are determined by (A.4). The remainder of the proof constructs a sender’s strategy that is sender optimal and truth-leaning, and makes $\mu$ consistent on-path. We do so by induction.

Suppose that for some lower contour set $L$ in $E$, we have a candidate strategy. That is, suppose that we have a collection of distributions $\{\sigma(\cdot|e)\}_{e \in L}$ satisfying the following:

(A.5) \[ \text{supp}(\sigma(\cdot|e)) \subset LC(e) \cap V^{-1}(V(e)) \cap O; \]

(A.6) \[ \sigma(e|e) = 1 \text{ for all } e \in O; \]

(A.7) \[ \mu(m) \leq \frac{\sum_{e \in L} \sigma(m|e)F_G(e)\pi_0}{\sum_{e \in L} \sigma(m|e)[F_G(e)\pi_0 + F_B(e)(1 - \pi_0)]} \text{ for all } m \in O \cap L. \]

If $L = E$, $(\sigma, a, \mu)$ is an equilibrium in which the sender’s value function is $V$. Sender optimality and truth-leaning are guaranteed by (A.5) and (A.6), respectively. Suppose that Bayesian consistency is not satisfied. Then at some $m^* \in O$, (A.7) holds with strict inequality. Let $x = V(m^*)$. Rearranging and summing (A.7) over $m \in V^{-1}(x) \cap O$, and noticing that at $m^*$, (A.7) holds with strict inequality, we have

\[ x < \frac{F_G(V^{-1}(x))\pi_0}{F_G(V^{-1}(x))\pi_0 + F_B(V^{-1}(x))(1 - \pi_0)} = \nu(V^{-1}(x)) = x, \]

A proof for this observation can be found in Hart et al. (2017).
Some evidence endowment in \( e \) violating (A.7). Following the above process, we show below that we can extend the candidate sender’s strategy to \( \tilde{L} := L \cup \{ e^* \} \), where \( [e^*] = \{ e : e \sim e^* \} \) is the equivalence class of \( e^* \). That is, we define \( \sigma(\cdot|e) \) for all \( e \in [e^*] \) such that the collection \( \{ \sigma(\cdot|e) \}_{e \in \tilde{L}} \) satisfies (A.5) through (A.7).

Let \( m_1, m_2, \ldots \) be the elements in \( P := \tilde{L} \cap V^{-1}(V(e^*)) \cap O \) such that \( m_i < m_j \Rightarrow i < j \). We extend the candidate strategy as follows. First, let \( \sigma(m|e) = 0 \) for all \( e \in [e^*] \) and all \( m \not\in P \), and let \( \sigma(e|e) = 1 \) and \( \sigma(e'|e) = 0 \) for all \( e \in [e^*] \cap P \) and all \( e' \neq e \). By doing so, (A.5) and (A.6) are satisfied. If \( [e^*] \subset P \), (A.7) is also satisfied, since for all \( e \in [e^*] \),

\[
\mu(e) \leq \nu(e) = \frac{F_G(e)\pi_0}{F_G(e)\pi_0 + F_B(e)(1 - \pi_0)}.
\]

Hence, we already have an extension. If \( [e^*] \not\subset P \), let \( e_1, e_2, \ldots \) be the elements of \( [e^*] \setminus P \), and define \( \sigma(m_i|e_j) \) recursively as follows:

\[
\sigma(m_j|e_i) = \max \left\{ f_i(m_j), 1 - \sum_{k=1}^{j-1} \sigma(m_k|e_i) \right\},
\]

with the initial condition

\[
\sigma(m_1|e_1) = \max \left\{ \frac{S(m_1)}{-F_G(e_1)\pi_0(1 - \mu(m_1)) + F_B(e_1)(1 - \pi_0)\mu(m_1)}, 1 \right\},
\]

where

\[
f_i(m_j) = \frac{S(m_j) - \sum_{k=1}^{i-1} \sigma(m_j|e_k)[-F_G(e_k)\pi_0(1 - \mu(m_j)) + F_B(e_k)(1 - \pi_0)\mu(m_j)]}{-F_G(e_i)\pi_0(1 - \mu(m_j)) + F_B(e_i)(1 - \pi_0)\mu(m_j)},
\]

and

\[
S(m_j) = \sum_{e \in \tilde{L} \cup \{ m_j \}} \sigma(m_j|e)[F_G(e)\pi_0(1 - \mu(m_j)) - F_B(e)(1 - \pi_0)\mu(m_j)].
\]

Intuitively, \( S(m_j) \geq 0 \) is the slackness in (A.7) at \( m_j \). If \( S(m_j) > 0 \), then the sender with some evidence endowment in \( [e^*] \setminus P \) can assign positive probability on reporting \( m_j \) without violating (A.7). Following the above process, \( \{ \sigma(\cdot|e) \}_{e \in \tilde{L}} \) constructed satisfies (A.5) through (A.7). Moreover, \( \tilde{L} \) by definition is a lower contour set in \( E \). We are left to check that \( \sigma(\cdot|e_i) \) is well-defined, i.e. \( \sum_j \sigma(m_j|e_i) = 1 \) for all \( i \). Suppose not, then at some \( e_i; \sum_j \sigma(m_j|e_i) < 1 \).

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Moreover, it must be the case that there remains no slackness at any \( m_j \), i.e.

\[
(A.8) \quad V(e^*) = \mu(m_j) = \frac{\sum_{e \in \tilde{L}} \sigma(m_j|e)F_G(e)\pi_0}{\sum_{e \in \tilde{L}} \sigma(m_j|e)[F_G(e)\pi_0 + F_B(e)(1 - \pi_0)]}
\]

for all \( m_j \). Notice that \( \sum_j \sigma(m_j|e_i) < 1 \) and \( \nu(e_i) < V(e_i) = V(e^*) \). Therefore, rearranging \( (A.8) \) and summing over \( j \), we have

\[
V(e^*) > \nu(\tilde{L} \cap V^{-1}(V(e^*))),
\]

which is a contradiction to condition 3 of Theorem 1. Hence, \{\( \sigma(\cdot|e) \}_{e \in \tilde{L}} \} constructed through the above process extends \{\( \sigma(\cdot|e) \}_{e \in L} \} and is a candidate strategy defined on a lower contour set of \( E \).

We can now proceed with transfinite induction. Let \( \mathcal{S} \) be the space of all candidate strategies defined on some lower contour set of \( E \). Let \( \Sigma_1 = \{\sigma_1(\cdot|e)\}_{e \in L_1} \) and \( \Sigma_2 = \{\sigma_2(\cdot|e)\}_{e \in L_2} \) be two typical elements in \( \mathcal{S} \). We can define a partial order, denoted by \( \sqsubseteq \), on \( \mathcal{S} \), such that \( \Sigma_1 \sqsubseteq \Sigma_2 \) if and only if \( \Sigma_2 \) extends \( \Sigma_1 \), i.e. \( L_1 \subseteq L_2 \), and \( \sigma_1(\cdot|e) = \sigma_2(\cdot|e) \) for all \( e \in L_1 \). For every chain \( \mathcal{C} = \{\Sigma_\alpha\}_{\alpha \in \mathcal{I}} \) in \( \mathcal{S} \), it has an upper bound \( \{\hat{\sigma}(\cdot|e)\}_{e \in \tilde{L}} \) defined on \( \tilde{L} := \bigcup_{\alpha \in \mathcal{I}} L_\alpha \), where \( \hat{\sigma}(\cdot|e) = \sigma_\alpha(\cdot|e) \) for some \( \alpha \in \mathcal{I} \) such that \( L_\alpha \ni e \). By Zorn’s lemma, \( \mathcal{S} \) has a maximal element. Let \( \hat{\Sigma} = \{\hat{\sigma}(\cdot|e)\}_{e \in \tilde{L}} \) be one such maximal element, where \( \tilde{L} \) is a lower contour set in \( E \). If \( \tilde{L} \neq E \), we can extend \( \hat{\Sigma} \). This contradicts the assumption that \( \hat{\Sigma} \) is a maximal element. Hence, \( \tilde{L} = E \), and by the earlier argument, \( (\hat{\sigma}, \mathbf{a}, \mu) \) is an equilibrium in which the sender’s value function is \( V \).

It remains to show that \( \mathcal{S} \) is nonempty, i.e. there exists a candidate strategy. Notice that the extension process discussed above applies even when \( L = \emptyset \). When \( L = \emptyset \), let \( e^* \) be a minimal element of \( E \), and \( P = [e^*] \cap O \) is nonempty by condition 2 of Theorem 1. Following the extension process, we can construct a candidate strategy defined on \([e^*]\). Hence, \( \mathcal{S} \) is nonempty.

**Lemma A.4 (Uniqueness).** The function that satisfies conditions 1 through 3 of Theorem 1 is unique.

**Proof.** Suppose, contrary to our claim, that \( V \neq W \) are two functions that satisfy conditions 1 through 3 of Theorem 1. Without loss of generality, there exist \( e_1, e_2 \) such that

\[\text{Since } \mathcal{C} \text{ is totally ordered, for all } \alpha, \alpha' \in \mathcal{I}, \Sigma_\alpha \sqsubseteq \Sigma_{\alpha'} \text{ or } \Sigma_{\alpha'} \sqsubseteq \Sigma_\alpha. \text{ Therefore, if } e \in \text{ both } L_\alpha \text{ and } L_{\alpha'}, \sigma_\alpha(\cdot|e) = \sigma_{\alpha'}(\cdot|e). \text{ We can define } \hat{\sigma}(\cdot|e) \text{ using any } \alpha \in \mathcal{I} \text{ such that } L_\alpha \ni e.\]


\( V(e_1) = V(e_2) \) and \( W(e_1) < W(e_2) \)

Let

\[
L_w = \bigcup_{x \leq W(e_1)} W^{-1}(x), \quad U_w = \bigcup_{x \geq W(e_2)} W^{-1}(x),
\]

\[
L_v = \bigcup_{y \leq V(e_1)} V^{-1}(y), \quad U_v = \bigcup_{y \geq V(e_2)} V^{-1}(y).
\]

By condition 1 of Theorem 1, \( L_w \) is a lower contour set in \( E \). Therefore, for all \( y \geq V(e_2) \) and \( y \in Range(V) \), \( V^{-1}(y) \cap L_w \) is a lower contour set in \( V^{-1}(y) \). By condition 3 of Theorem 1, \( \nu(V^{-1}(y) \cap L_w) \geq y \geq V(e_2) \). Hence, \( \nu(U_v \cap L_w) \geq V(e_2) \). But \( U_v \) is an upper contour set in \( E \). Therefore, for all \( x \leq W(e_1) \) and \( x \in Range(W) \), \( U_v \cap W^{-1}(x) \) is an upper contour set in \( W^{-1}(x) \), so \( \nu(U_v \cap W^{-1}(x)) \leq x \leq W(e_1) \). Hence, \( \nu(U_v \cap L_w) \leq W(e_1) \). That is,

(A.9) \[ V(e_2) \leq \nu(U_v \cap L_w) \leq W(e_1). \]

Similarly, we can show that

(A.10) \[ W(e_2) \leq \nu(U_w \cap L_v) \leq V(e_1). \]

The inequalities in (A.9) and (A.10) contradict the assumptions that \( W(e_1) < W(e_2) \) and \( V(e_1) = V(e_2) \). Hence, the function that satisfies conditions 1 through 3 of Theorem 1 is unique.

A.3. Proof of Corollary 2

**Proof.** Given an equilibrium and the sender’s value function \( V \), we can define an ordered partition \( (\mathcal{A}, \leq_p) \) of \( E \) as follows: \( \mathcal{A} := \{ V^{-1}(x) : x \in Range(V) \} \), and \( A_1 \leq_p A_2 \iff \nu(A_1) \leq \nu(A_2) \). By construction, the ordered partition \( (\mathcal{A}, \leq_p) \) satisfies conditions 1 and 3 of Corollary 2. By Theorem 1, for all \( A_1, A_2 \in \mathcal{A} \) and \( e_1 \in A_1, e_2 \in A_2 \) such that \( e_1 \not\leq_p e_2 \), \( \nu(A_1) = V(e_1) \leq V(e_2) = \nu(A_2) \). That is, \( A_1 \leq_p A_2 \), and condition 2 of Corollary 2 is satisfied.

Given any ordered partition \( (\mathcal{A}, \leq_p) \) of \( E \) that satisfies conditions 1 through 3 of Corollary 2 we can define \( V : E \to [0, 1] \) as follows: \( V(e) = \nu(A) \) for all \( A \in \mathcal{A} \) and \( e \in A \). By construction, \( V \) satisfies conditions 1 through 3 of Theorem 1 so it is the sender’s value function in an equilibrium.

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7Otherwise, either for all \( e_1 \neq e_2, V(e_1) \neq V(e_2) \) and \( W(e_1) \neq W(e_2) \), or \( V(e_1) = V(e_2) \iff W(e_1) = W(e_2) \). In the former case, \( V(e) = W(e) = \nu(e) \) for all \( e \in E \), which is a contradiction. In the latter case, for all \( x \in Range(V) \), \( \exists y \in Range(W) \) such that \( V^{-1}(x) = W^{-1}(y) \) (and vice versa). But by condition 2 of Theorem 1 \( x = \nu(V^{-1}(x)) = \nu(V^{-1}(y)) = y \). Hence, \( V = W \), which is a contradiction.
Let \((A, \leq_p)\) and \((A^*, \leq_p^*)\) both satisfy conditions 1 through 3 of Corollary 2. Suppose that \(A \neq A^*\). Then, without loss of generality, there exist \(e, e' \in E, A, A' \in A, A \neq A'\), and \(A^* \in A^*\) such that \(e \in A, e' \in A', e, e' \in A^*\). Hence, \(V(e) = V(e') = V(A^*)\). But \(V(e) = \nu(A) \neq \nu(A') = V(e')\). Contradiction reached. Hence, \(A = A^*\). By condition 1 of Corollary 2 \(A_1 \leq_p A_2 \iff \nu(A_1) \leq \nu(A_2) \iff A_1 \leq_p^* A_2\) for all \(A_1, A_2 \in A = A^*\). Hence, \(\leq_p\) is also identical to \(\leq_p^*\). That is, \((A, \leq_p) = (A^*, \leq_p^*)\). Uniqueness is shown.

A.4. Proof of Theorem 3.

Proof. For the “if” part, let \((A, \leq_p)\) be an ordered partition of \(E\) such that every \(A \in A\) is the largest minimizer of (7). We show that \((A, \leq_p)\) satisfies conditions 1 through 3 of Corollary 2 and therefore is the equilibrium partition. Suppose that condition 1 is violated. Then there exist \(A_1 <_p A_2\) such that \(\nu(A_1) \geq \nu([A_1, A_2])\), where \([A_1, A_2] = \cup_{A_1 \leq_p A' \leq_p A_2} A'\). This contradicts the assumption that \(A_1\) is the largest minimizer of (7). Hence, condition 1 is satisfied. Let \(A_1, A_2 \in A, e_1 \in A_1, e_2 \in A_2\) such that \(e_1 \not\subset e_2\). Since \(A_2\) is a lower contour set in \(E \setminus \cup_{A' <_p A_2} A'\), \(e_1 \in A_2\) or \(e_1 \in \cup_{A' <_p A_2} A'\). Hence, \(A_1 \leq_p A_2\); condition 2 is satisfied. Let \(A \in A\), and \(L\) a lower contour set in \(A\). Since \(L\) is a lower contour set in \(E \setminus \cup_{A' <_p A} A'\), by minimization, \(\nu(L) \geq \nu(A)\); condition 3 is satisfied.

For the “only if” part, suppose that \((A, \leq_p)\) is the equilibrium partition, we are to show that every \(A \in A\) is the largest minimizer of (7). To obtain a contradiction, consider two separate cases. First, suppose that for some \(A_0 \in A\), \(A_0\) does not solve (7). Then there exists a lower contour set \(L\) in \(E \setminus \bigcup_{A' <_p A_0} A'\) such that \(\nu(L) < \nu(A_0)\). Notice that \(\{A \cap L\}_{A \geq_p A_0}\) is a partition of \(L\), and by assumption, \(\nu(A \cap L) \geq \nu(A) \geq \nu(A_0)\) for all \(A \geq_p A_0\) such that \(A \cap L \neq \emptyset\). Hence, \(\nu(L) \geq \nu(A_0)\), a contradiction. Second, suppose that there exists \(A_0 \in A\) that is not the largest minimizer of (7). Then there exists \(A_0' \not\subset A_0\) that solves (7). Since \(A_0' \setminus A_0\) is an upper contour set in \(A_0'\), \(\nu(A_0' \setminus A_0) \leq \nu(A_0') = \nu(A_0)\). But \(\{A \cap A_0'\}_{A \geq_p A_0}\) is a partition of \(A_0' \setminus A_0\), and by assumption, \(\nu(A \cap A_0') \geq \nu(A) > \nu(A_0)\) for all \(A >_p A_0\) such that \(A \cap A_0' \neq \emptyset\). Hence, \(\nu(A_0' \setminus A_0) > \nu(A_0)\), a contradiction.

Remark. If (7) admits a minimizer, it has a (unique) largest minimizer. Denote by \(L\) the set of minimizers to (7), and suppose that it is not empty. Let \(\underline{\nu}\) be the minimum of (7). For every chain \(\{L_\alpha\}_{\alpha \in \mathcal{J}}\) in \((L, \subset)\), there is an upper bound \(\bar{L} := \cup_{\alpha \in \mathcal{J}} L_\alpha\) in \((L, \subset)\). By Zorn’s lemma, there exists a maximal element of \((L, \subset)\). Let \(L^*\) be one maximal element. Suppose that \(L^*\) is not the greatest element in \((L, \subset)\), then there exists \(L' \in L\) such that \(L' \not\subset L^*\). If \(L'\) and \(L^*\) are disjoint, it is easy to see that \(\nu(L^* \cup L') = \underline{\nu}\), so \(L^* \cup L' \in L\). If \(L' \cup L^* \neq \emptyset\), by minimization, \(\nu(L' \cup L^*) \geq \underline{\nu}\), so \(\nu(L' \setminus L^*) \leq \underline{\nu}\) and \(\nu(L^* \cup L') \leq \underline{\nu}\). But \(L^* \cup L'\) is a lower contour set in \(E \setminus \bigcup_{i \leq n} A_i\), so \(\nu(L^* \cup L') = \underline{\nu}\), and \(L^* \cup L' \in L\). This is
a contradiction to maximality of $L^*$, as $L^* \cup L' \supseteq L^*$. Hence, $L^*$ is the largest minimizer of (7).

A.5. Proof of Proposition 4.

**Proof.** Since $N$ is non-decreasing and $p > q$, $V$ is non-decreasing. Let $E_n = \{e \in E : N(e) = n\}$ for every $n \in \mathbb{N}$. For all $x \in \text{Range}(V)$, $V^{-1}(x) = E_n$ for some $n \in \mathbb{N}$. We are to verify conditions 2 and 3 of Theorem 1 for every $E_n$.

Notice that $E_0 = \emptyset \cup \{(b, h) : h \in E_0 \cup E_1\}$, and $E_n = \{(g, h) : h \in E_{n-1}\} \cup \{(b, h) : h \in E_{n+1}\}$ for all $n \geq 1$. Hence,

\[
F_\omega(E_n) = (1 - p - q) + p^1(\omega = B) q^1(\omega = G) \cdot (F_\omega(E_0) + F_\omega(E_1));
\]

\[
F_\omega(E_n) = p^1(\omega = G) q^1(\omega = B) \cdot F_\omega(E_{n-1}) + p^1(\omega = B) q^1(\omega = G) \cdot F_\omega(E_{n+1}) \text{ for all } n \geq 1;
\]

\[
\sum_{n=0}^{\infty} F_\omega(E_n) = 1.
\]

It follows that

\[
F_G(E_n) = \frac{q - \alpha}{q} \left(\frac{\alpha}{q}\right)^n; \quad F_B(E_n) = \frac{p - \alpha}{p} \left(\frac{\alpha}{p}\right)^n.
\]

Hence, for all $n \in \mathbb{N}$ and $e \in E_n$,

\[
\nu(E_n) = \frac{1}{1 + \frac{1 - \pi_0}{\pi_0} \frac{p - \alpha}{q - \alpha} \left(\frac{q}{p}\right)^{n+1}} = V(e);
\]

condition 2 is satisfied.

We now verify condition 3. Notice that for all $n \in \mathbb{N}$ and $e \in E_n$, $e$ can be uniquely decomposed into two parts, as follows:

\[
e = (e_0, e_{\min}^n),
\]

where $e_0 \in E_0$, and $e_{\min}^n \in \text{min}(E_n, \preceq)$. Therefore, every lower counter set $L$ in $E_0$ can be partitioned into at most countably many subsets, each having the form

(A.11) \[
\{(e_0, e_{\min}^n) : e_0 \in L_0\},
\]

where $e_{\min}^n \in \text{min}(E_n, \preceq)$, and $L_0$ is a lower contour set in $E_0$. Hence, it suffices to show
that, for all sets \( L' \) of the form (A.11), \( \nu(L') \geq \nu(E_n) \). Notice that
\[
\nu(L') = \frac{1}{1 + \frac{1 - \pi_0}{\pi_0} \frac{p_0}{F_n(L_0)} \left( \frac{q}{p} \right)^n},
\]
and
\[
\nu(E_n) = \frac{1}{1 + \frac{1 - \pi_0}{\pi_0} \frac{p_0 - \alpha}{q_0 - \alpha} \left( \frac{q}{p} \right)^{n+1}}.
\]

It is equivalent to show that \( \nu(L_0) \geq \nu(E_0) \) for all lower contour sets \( L_0 \) in \( E_0 \).

Suppose that this is not true. Then for some small \( \varepsilon > 0 \), there exists a lower contour set \( L_0 \) in \( E_0 \) such that \( \nu(L_0) \leq \nu(E_0) - \varepsilon \). Without loss of generality, choose
\[
\varepsilon < \nu(E_0) - \frac{1}{1 + \frac{1 - \pi_0}{\pi_0} \frac{p_0 - \alpha}{q_0 - \alpha}}.
\]

Denote by \( \mathcal{L}_\varepsilon \) the nonempty set of all lower contour sets \( L_0 \) in \( E_0 \) such that \( \nu(L_0) \leq \nu(E_0) - \varepsilon \). Notice that every chain \( \{L_\alpha\}_{\alpha \in \mathcal{S}} \) in \( (\mathcal{L}_\varepsilon, \subset) \) has an upper bound \( \bigcup_{\alpha \in \mathcal{S}} L_\alpha \). Therefore, there exists a maximal element \( L_\varepsilon^* \) in \( \mathcal{L}_\varepsilon \) by Zorn’s lemma. To obtain a contradiction, let us discuss the following two cases.

If there exists \( e^* \in \min(E_0 \setminus L_\varepsilon^*, \prec) \) such that \( D(e^*) < 0 \), then \( \tilde{L}_\varepsilon := L_\varepsilon^* \cup \{(e_0, e^*) : e_0 \in E_0\} \) is a lower contour set in \( E_0 \), and \( \nu(\tilde{L}_\varepsilon) \leq \nu(E_0) - \varepsilon \), since
\[
\nu(\{(e_0, e^*) : e_0 \in E_0\}) = \frac{1}{1 + \frac{1 - \pi_0}{\pi_0} \frac{p_0 - \alpha}{q_0 - \alpha} D(e^*) + 1} \leq \frac{1}{1 + \frac{1 - \pi_0}{\pi_0} \frac{p_0 - \alpha}{q_0 - \alpha}} < \nu(E_0) - \varepsilon.
\]

Hence, \( \tilde{L}_\varepsilon \in \mathcal{L}_\varepsilon \). But \( L_\varepsilon^* \subset \tilde{L}_\varepsilon \), a contradiction to maximality of \( L_\varepsilon^* \).

If \( D(e^*) = 0 \) for all \( e^* \in \min(E_0 \setminus L_\varepsilon^*, \prec) \), then
\[
E_0 \setminus L_\varepsilon^* = \bigcup_{e^* \in \min(E_0 \setminus L_\varepsilon^*, \prec)} \{(e_0, e^*) : e_0 \in E_0\}.
\]

Since \( \nu(\{(e_0, e^*) : e_0 \in E_0\}) = \nu(E_0) \) for all \( e^* \in \min(E_0 \setminus L_\varepsilon^*, \prec) \), \( \nu(E_0 \setminus L_\varepsilon^*) = \nu(E_0) \). Hence, \( \nu(L_\varepsilon^*) = \nu(E_0) \), a contradiction.

Therefore, \( \nu(L_0) \geq \nu(E_0) \) for all lower contour set \( L_0 \) in \( E_0 \), and condition 3 is verified. \( \square \)

**A.6. Proof of Proposition 5**

**Proof.** 1 is shown in the proof of Theorem 11 as (A.4), and 2 is by calculation found in the main text. 3 is clearly true if at \( e \), it is optimal to disclose without omission. Hence, the proof
is concluded once we show that 3 holds for evidence endowment \( \hat{e} \) such that \( \nu(\hat{e}) < V(\hat{e}) \). Let \( \hat{e} \in E_n \), and let \( 0 \leq k_1 < k_2 < \cdots < k_l = k^* \) be the elements in \( \text{argmax}_k D(\|k) \). For each \( 1 \leq i \leq l \), let \( K_i \) be \( \{(e_{i0}, \hat{e}|e_{i0} \in E_{\hat{e}}) \} \), where \( E_{\hat{e}} = \{\emptyset\} \cup \{e : D(e|e_{i0}) < 0, \forall 0 < k \leq L(e)\} \).

It is equivalent to show that \( \text{supp}(\sigma^*(\cdot|e_{i0})) \subset K_i \), so that \( \hat{e}|e_{i0} \not\succ \) for all \( m \in \text{supp}(\sigma^*(\cdot|e_{i0})) \).

We show this by induction.

First, notice that for all \( e \in K_1 \), sender optimality requires that \( \text{supp}(\sigma^*(\cdot|e)) \subset K_1 \).

Now, assume that for some \( i < l \), we have \( \text{supp}(\sigma^*(\cdot|e)) \subset K_j \) for all \( e \in K_j \) and \( 1 \leq j \leq i \).

Suppose that there exist \( e \in K_{i+1}, j \leq i \), and \( m \in K_j \), such that \( \sigma^*(m|e) > 0 \), and argue by contradiction. Rearranging (A.8) and summing over all on-path messages in \( K_j \), we have:

\[
V(\hat{e}) = \frac{\left( \sum_{e \in K_j} \sum_{m \in K_j} + \sum_{e \notin K_j} \sum_{m \in K_j} \right) (\sigma^*(m|e)F_G(e)\pi_0)}{\left( \sum_{e \in K_j} \sum_{m \in K_j} + \sum_{e \notin K_j} \sum_{m \in K_j} \right) (\sigma^*(m|e)[F_G(e)\pi_0 + F_B(e)(1 - \pi_0)])}.
\]

By assumption,

\[
\frac{\sum_{e \in K_j} \sum_{m \in K_j} \sigma^*(m|e)F_G(e)\pi_0}{\sum_{e \in K_j} \sum_{m \in K_j} \sigma^*(m|e)[F_G(e)\pi_0 + F_B(e)(1 - \pi_0)]} = \nu(K_j) = V(\hat{e}),
\]

and

\[
\frac{\sum_{e \notin K_j} \sum_{m \in K_j} \sigma^*(m|e)F_G(e)\pi_0}{\sum_{e \notin K_j} \sum_{m \in K_j} \sigma^*(m|e)[F_G(e)\pi_0 + F_B(e)(1 - \pi_0)]} < V(\hat{e}).
\]

Therefore, \( V(\hat{e}) < V(\hat{e}) \), which is a contradiction. Hence, it must be the case that for all \( e \in K_{i+1}, \text{supp}(\sigma^*(\cdot|e)) \subset K_{i+1} \). By induction, this holds for \( K_l \). Since \( \hat{e} \in K_l, \text{supp}(\sigma^*(\cdot|\hat{e})) \subset K_l \).

**A.7. Proof of Proposition 6.**

**Proof.** For 1, notice that

\[
\frac{p - \alpha}{q - \alpha} = \frac{2p - 1 + \sqrt{1 - 4pq}}{2q - 1 + \sqrt{1 - 4pq}}
\]

\[
= \frac{\left( \sqrt{1 - 4pq} + 2p - 1 \right) \left( \sqrt{1 - 4pq} - 2q + 1 \right)}{(1 - 4pq) - (2q - 1)^2}
\]

\[
= \frac{2(p + q) - 8pq + 2(p - q)\sqrt{1 - 4pq}}{4q(1 - p - q)}
\]

\[
= \frac{1}{2}\left( \gamma + 1 \right) \kappa - 2 \frac{\gamma}{\gamma + 1} \left( \kappa - 1 \right) + \frac{1}{2} \left( \gamma - 1 \right) \sqrt{\Delta}
\]

\[
= \frac{(\gamma - 1)^2}{2(\gamma + 1)} \kappa + 2 \frac{\gamma}{\gamma + 1} + \frac{1}{2} \left( \gamma - 1 \right) \sqrt{\Delta},
\]

\begin{equation}
(A.12)
\end{equation}
where
\[ \Delta = \kappa^2 - \frac{4\gamma}{(\gamma + 1)^2} (\kappa - 1)^2 \]
is increasing in \( \kappa \). Hence, (A.12) is increasing in \( \kappa \), and \( V(e) \) given by (9) is decreasing in \( \kappa \).

For 2, let us first consider the case of \( N(e) = N > 0 \). Notice that \( \sqrt{\Delta} \) is bounded from above by \( \kappa \) and from below by \( \frac{\gamma - 1}{\gamma + 1} \). Applying the lower bound,
\[
\frac{p - \alpha}{q - \alpha} > \frac{(\gamma - 1)^2}{\gamma + 1} \kappa + \frac{2\gamma}{\gamma + 1}.
\]

Applying both bounds and omitting the negative term of \( \frac{1}{\kappa} \),
\[
\frac{\partial}{\partial \gamma} \left( \frac{p - \alpha}{q - \alpha} \right) = \frac{1}{2} \kappa - \frac{2}{(\gamma + 1)^2} (\kappa - 1) + \frac{1}{2} \sqrt{\Delta} + \frac{1}{2} \frac{\gamma - 1}{(\gamma + 1)^3} (\kappa - 1)
\]
\[
< \frac{\gamma^2 + 2\gamma - 1}{(\gamma + 1)^2} \kappa + \frac{3}{(\gamma + 1)^2}.
\]

Hence,
\[
\frac{\partial}{\partial \gamma} \left[ \frac{p - \alpha}{q - \alpha} \left( \frac{q}{p} \right)^{N+1} \right] = \frac{1}{\gamma^{N+1}} \left[ \frac{\partial}{\partial \gamma} \left( \frac{p - \alpha}{q - \alpha} \right) - (N + 1) \frac{p - \alpha}{q - \alpha} \right]
\]
\[
\leq \frac{1}{\gamma^{N+1}} \left[ \frac{\gamma^2 + 2\gamma - 1}{(\gamma + 1)^2} \kappa + \frac{3}{(\gamma + 1)^2} - (N + 1) \left( \frac{\gamma - 1}{\gamma + 1} \kappa + \frac{2}{\gamma + 1} \right) \right]
\]
\[
= \frac{[-N\gamma^3 + (N + 3)\gamma^2 + N\gamma - (N + 1)]\kappa - \gamma[2(N + 1)(\gamma + 1) - 3]}{\gamma^{N+2}(\gamma + 1)^2}.
\]

Notice that \( -N\gamma^3 + (N + 3)\gamma^2 + N\gamma - (N + 1) \) has three zeros: \( \gamma_1 < 0 < \gamma_2 < 1 < \gamma_3 \). When \( \gamma \geq \gamma_3 \), the last line of (A.14) is negative for all \( \kappa > 1 \). By continuity, there exists \( \hat{\gamma} \in [1, \gamma_3] \) such that when \( \gamma \geq \hat{\gamma} \), the first line of (A.14) is negative, so \( V(e) \) given by (9) is increasing in \( \gamma \). Notice that \( \gamma_3 \) is strictly decreasing in \( N \) and converges to 1 as \( N \to \infty \). Therefore, \( \hat{\gamma} \to 1 \) as \( N \to \infty \). Lastly, we observe that \( \hat{\gamma} > 1 \). By (A.12), \( \frac{p - \alpha}{q - \alpha} \to 1 \) as \( \gamma \downarrow 1 \); by (A.13),
\[
\frac{\partial}{\partial \gamma} \left( \frac{p - \alpha}{q - \alpha} \right) \to \frac{1}{2} + \frac{1}{2} \sqrt{2\kappa - 1} \text{ as } \gamma \downarrow 1; \text{ therefore, by the first equality in (A.14),}
\]
\[
\frac{\partial}{\partial \gamma} \left[ \frac{p - \alpha}{q - \alpha} \left( \frac{q}{p} \right)^{N+1} \right] \to \frac{1}{2} + \frac{1}{2} \sqrt{2\kappa - 1} - \frac{1}{2} - N
\]
as \( \gamma \downarrow 1 \). Hence, for \( \kappa > 2N^2 + 2N + 1 \), the right hand side of (A.13) is positive. By
continuity, there is a small neighborhood to the right of 1 where $V(e)$ is decreasing in $\gamma$, so $\hat{\gamma} > 1$.

For the case of $N(e) = 0$, we compute the following:

\begin{align}
(A.16) \quad \frac{\partial}{\partial p} \left[ p - \alpha \left( \frac{q}{p} \right)^{N+1} \right] &= \left( \frac{q}{p} \right)^{N+1} \frac{1}{q - \alpha} \left[ 1 + \frac{p - q}{q - \alpha} \left( 1 - 2\alpha \right) - \frac{(N + 1)(p - \alpha)}{p} \right], \\
(A.17) \quad \frac{\partial}{\partial q} \left[ p - \alpha \left( \frac{q}{p} \right)^{N+1} \right] &= \left( \frac{q}{p} \right)^{N+1} \frac{1}{q - \alpha} \left[ (N + 1)\frac{p - \alpha}{q} - \frac{p - q}{q - \alpha} - \frac{p - q}{q - \alpha} \frac{p}{q - \alpha} \left( 1 - 2\alpha \right) \right].
\end{align}

When $N = 0$, the bracket on the right hand side of (A.16) is positive, and the bracket on the right hand side of (A.17) is negative. Hence, if $N(e) = 0$, $V(e)$ is decreasing in $p$ and increasing in $q$, so it is decreasing in $\gamma$.

We continue to show 3. Notice that the bracket on the right hand side of (A.16) is increasing in $p$ and decreasing in $N$. Hence, there exists $\hat{p} \in [q, 1 - q]$ such that (A.16) is negative when $p < \hat{p}$ and positive when $p > \hat{p}$. Moreover, $\hat{p}$ is non-decreasing in $N$. That is, $V(e)$ is increasing in $p$ when $p < \hat{p}$ and decreasing in $p$ when $p > \hat{p}$. Finally, notice that as $p \uparrow 1 - q$, $\kappa \to \infty$. Hence, by (A.12), $\frac{p - \alpha}{q - \alpha} \to \infty$, and $V(e) \to 0$. Therefore, $V(e)$ is decreasing in $p$ on a small neighborhood to the left of $1 - q$, so $\hat{p} < 1 - q$.

\begin{flushright}
\Box
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References

Ben-Porath, E. and Lipman, B. L. (2012). Implementation with partial provability. *Journal of Economic Theory*, 147(5):1689–1724.

Bull, J. and Watson, J. (2004). Evidence disclosure and verifiability. *Journal of Economic Theory*, 118(1):1–31.

Bull, J. and Watson, J. (2007). Hard evidence and mechanism design. *Games and Economic Behavior*, 58(1):75–93.

Dye, R. A. (1985). Disclosure of nonproprietary information. *Journal of Accounting Research*, 23(1):123–145.

Dziuda, W. (2011). Strategic argumentation. *Journal of Economic Theory*, 146(4):1362–1397.

Grossman, S. J. (1981). The informational role of warranties and private disclosure about product quality. *The Journal of Law and Economics*, 24(3):461–483.

Guttman, I., Kremer, I., and Skrzypacz, A. (2014). Not only what but also when: A theory of dynamic voluntary disclosure. *American Economic Review*, 104(8):2400–2420.

Hart, S., Kremer, I., and Perry, M. (2017). Evidence games: Truth and commitment. *American Economic Review*, 107(3):690–713.

Milgrom, P. and Roberts, J. (1986). Relying on the information of interested parties. *The RAND Journal of Economics*, 17(1):18–32.
Milgrom, P. R. (1981). Good news and bad news: Representation theorems and applications. *The Bell Journal of Economics*, 12(2):380–391.

Rappoport, D. (2017). Evidence and skepticism in verifiable disclosure games. *Working Paper*.

Shin, H. S. (1994). The burden of proof in a game of persuasion. *Journal of Economic Theory*, 64(1):253–264.

Shin, H. S. (2003). Disclosures and asset returns. *Econometrica*, 71(1):105–133.

Stanley, R. P. (1999). *Enumerative Combinatorics, Volume 2*. Cambridge University Press.