ON THE EVOLUTION OF THE IONIZING EMMISSIVITY OF GALAXIES AND QUASARS REQUIRED BY THE HYDROGEN REIONIZATION

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ABSTRACT

The average rate of emission of ionizing radiation per unit volume (or emissivity) in the universe can be calculated as the ratio of the intensity of the ionizing background to the mean free path of ionizing photons. The intensity of the background is measured from the mean transmitted flux of the Ly\(\alpha\) forest, and the mean free path is measured from the abundance of Lyman limit systems, which has been observed so far up to \(z = 4\). This yields an emissivity that is not larger than seven ionizing photons per Hubble time for each atom in the universe at \(z = 4\), which may reasonably arise from QSOs and star-forming galaxies. In order for the reionization to end by \(z = 6\), and assuming that the clumping factor of ionized gas during the reionization epoch is close to unity, this ionizing comoving emissivity cannot decline from \(z = 4\) up to \(z \sim 9\) by more than a factor 1.5. If the clumping factor were much larger than unity, then the emissivity would need to rapidly increase with redshift. Unless the ionizing emissivity increases substantially from \(z = 4\) to \(z \sim 10-20\), the Thomson optical depth to the cosmic microwave background must be in the range \(0.045 < \tau < 0.09\).

Subject headings: cosmology: theory — diffuse radiation — galaxies: formation — intergalactic medium

1. INTRODUCTION

The highest redshift quasar known at the present time, at \(z = 6.28\), is the first one to show a complete Gunn-Peterson trough (Becker et al. 2001; Djorgovski et al. 2001). This suggests that the reionization of hydrogen was probably completed near the epoch \(z = 6\). This conclusion is not yet completely certain because an ionized medium can produce a complete Gunn-Peterson trough if the neutral fraction is everywhere high enough (owing to a low intensity of the ionizing background). However, the abrupt change with redshift near \(z = 6\) of the fraction of transmitted flux in the Ly\(\alpha\) region of the spectrum (Becker et al. 2001) implies a rapid increase of the mean free path of ionizing photons with time (Fan et al. 2002), at least in the region of the line of sight to the \(z = 6.28\) QSO, just as expected when the last low-density regions of the intergalactic medium are ionized (e.g., Gnedin 2000).

Ending reionization by \(z = 6\) requires that at least one ionizing photon has been emitted to the intergalactic medium for each baryon in the universe, in addition to the amount needed to compensate for any recombinations that have taken place. This implies that the emissivity of ionizing photons (defined as the mean number of photons emitted per unit time and volume) should be greater than the mean number density of baryons divided by the age of the universe at \(z \approx 6\), unless the emissivity had declined rapidly with time at \(z > 6\).

It is possible to infer the emissivity at lower redshift by using the intensity of the ionizing background measured from the mean transmitted flux in the Ly\(\alpha\) forest and the mean free path of ionizing photons determined from the observed abundance of Lyman limit systems. The first of these two quantities, the intensity of the ionizing background, can be measured in the context of the present theoretical understanding of the Ly\(\alpha\) forest as arising from a photoionized intergalactic medium that follows the gravitational evolution of primordial fluctuations (see the review by Rauch 1998 and references therein). The observed mean transmitted flux can then be used to infer the Ly\(\alpha\) scattering optical depth at the mean hydrogen density and in Hubble expansion, \(\tau_u\):

\[
\tau_u = 4.65 \times 10^4 \frac{\Omega_b h (1 - Y) H_0}{H(z)} x_{\rm H_1} = 2.12 \left( \frac{\Omega_b h^2}{0.02} \right)^{2/3} \left( \frac{0.3}{\Omega_m} \right)^{1/2} T_4^{-0.7} \Gamma_{-12}^{-9/2} \left( \frac{1 + z}{5} \right)^{9/2}.
\]

Here, \(x_{\rm H_1}\) is the hydrogen neutral fraction at mean density, \(T_4\) is the gas temperature in units of \(10^4\) K, and \(\Gamma_{-12}\) is the photoionization rate due to the cosmic background in units of \(10^{-12}\) s\(^{-1}\). When the value of \(\Omega_b h^2\) was still highly uncertain, the value of \(\tau_u\) inferred from observations of the mean transmitted flux, \(\tau_u \approx 1.0 \pm 0.2\) at \(z = 3\) (see Rauch et al. 1997 and Table 2 in McDonald et al. 2000), implied a lower limit to the value of \(\Omega_b h^2\) given by the lower limit on \(\Gamma_{-12}\) that was inferred from the observed QSOs. Now, \(\Omega_b h^2\) has been more accurately measured from the primordial deuterium abundance (O’Meara et al. 2001) and the cosmic microwave background (CMB) spectrum of fluctuations (de Bernardis et al. 2002; Pryke et al. 2002; Spergel et al. 2003), and we can use the value of \(\tau_u(z)\) determined from the Ly\(\alpha\) forest to constrain the intensity of the ionizing background and its evolution with redshift.

The mean free path of ionizing photons is related to the abundance of Lyman limit systems, which have column densities \(N_{\rm H_1} > 1.6 \times 10^{17}\) cm\(^{-2}\). The reason is that the column density distribution of absorption systems is approximately of the form \(f(N_{\rm H_1}) dN_{\rm H_1} \propto N_{\rm H_1}^{-1.5} dN_{\rm H_1}\) (e.g., Petitjean et al. 1993), so the average rate of absorption is dominated by high column densities, up to the column density at which the absorption systems become optically thick. The abundance of Lyman limit systems has been reasonably well measured up to redshift \(z = 4\) (§2). We then compare this emissivity to the
observations of known sources at this redshift (§ 3). Then in § 4 we find the consequences for the evolution of the emissivity at higher redshift to satisfy the requirement of reionizing the IGM by \( z = 6 \). Finally, in § 5 the consequences for the Thomson optical depth of the CMB photons are discussed. We will use the flat cosmological model with \( \Omega_m = 0.3, h = 0.65 \), and \( \Omega_b h^2 = 0.022 \).

2. THE IONIZING EMISSIVITY AT \( z = 4 \)

In this section we evaluate the emissivity of ionizing radiation at redshift \( z = 4 \). In terms of the photoionization rate \( \Gamma \) and the mean free path of the ionizing photons \( \lambda_i \), the photon emissivity is given by \( \epsilon_i = \Gamma/(\lambda_i \sigma) \), where \( \sigma \) is a frequency-averaged cross section. The result we find in this paper is that the emissivity at \( z = 4 \) has a rather small value, implying that it must not have declined much with redshift, up to a substantially earlier epoch at \( z > 6 \), in order to satisfy the requirement that reionization was completed by \( z \approx 6 \). With that in mind, we evaluate the maximum allowed value of \( \Gamma \) and of \( \epsilon_i \) at \( z = 4 \), which is the highest redshift at which the mean free path \( \lambda_i \) has been observationally determined.

2.1. The Photoionization Rate

The most recent measurements of the mean transmitted flux through the Ly\( \alpha \) forest, \( F \), were reported by Bernardi et al. (2003) from a large sample of QSOs from the Sloan Digital Sky Survey (SDSS). From their Figure 4, the effective optical depth [defined as \( F = \exp(-\tau_{\text{eff}}) \)] at \( z = 4 \) is \( \tau_{\text{eff}} = 1.02 \pm 0.02 \), where we have estimated the error from the error bars and scatter of their measured points at various redshift bins. Hence, \( F < 0.37 \) at \( z = 4 \). Note that previous values of \( F \) from a small sample of high-resolution spectra (Schaye et al. 2000; McDonald et al. 2000) were higher, but this was probably because their method underestimated the QSO continuum flux, which becomes increasingly difficult to set by fitting regions between absorption lines as the redshift increases.

We now use the results of McDonald & Miralda-Escudé (2001, hereafter MM01) to infer the photoionization rate \( \Gamma_{-12} \). According to their Figure 2, the value \( F < 0.37 \) implies a photoionization rate of \( \Gamma_{-12} < 0.40 \) at \( z = 4.5 \). This value needs to be corrected from the model used in MM01 to the model we use here and to \( z = 4 \), where we choose all the parameters to maximize \( \Gamma_{-12} \) and therefore infer an upper limit. First of all, the relationship between \( F \) and \( \tau \) in equation (1) depends on the amplitude of the mass power spectrum at the Jeans scale of the Ly\( \alpha \) forest: a lower amplitude of fluctuations implies that the gas is less concentrated in collapsed regions that correspond to saturated absorption lines, and a greater fraction of the gas is left in low-density regions, where it contributes more efficiently to decreasing the mean transmitted flux. The measured amplitude of the power spectrum of the Ly\( \alpha \) forest (see Croft et al. 2002 and references therein) is 0.86 times lower than the amplitude in the model that was used by MM01 (as shown in McDonald et al. 2000), which had the parameters \( \sigma_8 = 0.79 \) and \( \beta = 0.95 \) for the power spectrum. In other words, the model used by MM01 would match the Ly\( \alpha \) forest power spectrum amplitude found by Croft et al. if its normalization had been changed to \( \sigma_8 = 0.86 \times 0.79 = 0.68 \), leaving all other parameters unchanged. Since the amplitude of mass fluctuations grows proportionally to the scale factor \( a \) (the effects of the vacuum energy and the radiation are negligible at \( z = 4 \)), the model with normalization \( \sigma_8 = 0.68 \) has the same relation of \( F \) and \( \tau \) at \( z = 4 \) as the model with normalization \( \sigma_8 = 0.79 \) at \( z = (1 + 4)/0.86 - 1 = 4.8 \). It was also found in MM01 that the value of \( \tau \) inferred from a fixed \( F \) depends on the amplitude approximately as \( \sigma_8^2 \) (this can in fact be checked by comparing the values of \( \Gamma_{-12} \) at \( z = 4.5 \) and \( z = 5.2 \) for fixed \( F \) in Fig. 2 of MM01). Because the limit \( \Gamma_{-12} < 0.4 \) was derived at \( z = 4.5 \), and \( \Gamma_{-12} \propto \sigma_8^{-2} \), the power spectrum amplitude correction is a factor \( (5.8/5.5)^2 \). We note that here we have taken the central measured value of Croft et al. of the power spectrum normalization, instead of a lower value within their error bar (which would further raise the upper limit to \( \Gamma_{-12} \) by about 20%), because other measurements of the power spectrum amplitude favor a higher value instead (see Seljak 2002; Van Waerbeke et al. 2001; Croft et al. 2002; Jarvis et al. 2003; Spergel et al. 2003).

Second, the inferred \( \Gamma_{-12} \) depends on the gas temperature. MM01 assumed \( T_4 = 2 \), but measurements of the gas temperature from the Ly\( \alpha \) forest spectra have given values in the range \( 1.2 < T_4 < 2 \) (Theuns et al. 2002 and references therein). Although the optical depth of a uniform medium is proportional to \( T^{-0.7} \) (eq. [1]) because of the temperature dependence of the recombination coefficient, the relation between \( F \) and \( \tau \) is also affected by the temperature because thermal broadening spreads some of the absorption from saturated spectral regions to other regions with less absorption. We use the result found in McDonald et al. (2001, their § 7.4) that thermal broadening causes a change in \( \tau \) of 27% of that due to the recombination coefficient, in the opposite direction, therefore giving an effective dependence \( \Gamma_{-12} \propto T^{-0.5} \). If we assume the lowest value of the temperature among the reported measurements, \( T_4 = 1.2 \), then the temperature correction for the upper limit of \( \Gamma_{-12} \) is \( (2/1.2)^{0.5} \).

Finally, we correct for all other parameters used in MM01 (\( \Omega_b h^2 = 0.02, h = 0.65, \Omega_m = 0.4, z = 4.5 \)) to the ones used here (\( \Omega_b h^2 = 0.022, h = 0.65, \Omega_m = 0.3, z = 4 \)). The result of all these corrections (see eq. [1]) for the upper limit to \( \Gamma_{-12} \) at \( z = 4 \) is

\[
\Gamma_{-12}(z = 4) < 0.40 \left( \frac{5.8}{5.5} \right)^2 \left( \frac{2}{1.2} \right)^{0.5} \left( \frac{0.022}{0.02} \right)^2 \approx 0.53.
\]

2.2. The Mean Free Path

Storrie-Lombardi et al. (1994) found that the rate of Lyman limit systems (defined as absorption systems with column density \( N_{H_1} > 1.6 \times 10^{17} \) cm\(^{-2} \)) at \( z = 4 \) is \( dN_{\text{LL}}/dz = 3.3 \pm 0.6 \). In other words, the mean spacing between Lyman limit systems is \( \lambda_{\text{LL}} = cH^{-1}(z)/(1 + z) \) (\( 3.3 \pm 0.6 \) = (0.06 ± 0.01)cH\(^{-1}(z) \)) at \( z = 4 \). To find the relationship of \( \lambda_{\text{LL}} \) to the mean free path of ionizing photons, we assume that the column density distribution of the absorbers is \( f(N_{H_1})dN_{H_1} \propto N_{H_1}^{-5/3}dN_{H_1} \) (Petitjean et al. 1993). The absorption probability per unit length, \( \lambda_{\text{LL}}^{-1} \), for a photon at the hydrogen ionization edge frequency
\( \nu = \nu_{H_i} \) is
\[
\frac{1}{\lambda_0} = \int_0^\infty \frac{d \tau \tau^{-1.5}(1 - e^{-\tau})}{\int_0^\infty d \tau \tau^{-1.5} \lambda_{LL}} = \frac{\sqrt{\pi}}{\lambda_{LL}},
\]
where \( \tau = N_{H_i}/(1.6 \times 10^{17} \text{ cm}^2) \). At a higher frequency \( \nu \), and approximating the photoionization cross section as proportional to \( \nu^{-3} \), the mean free path is increased to \( \lambda_{0} \approx (\nu/\nu_{H_1})^{1.5} \).

We note here that our mean free path in equation (3) is 1.5 times longer than that adopted in Madau, Haardt, & Rees (1999) and in Schirber & Bullock (2003). For some reason, their opacity corresponds to a Lyman limit system abundance of five per unit redshift, larger than reported by Storrie-Lombardi et al. (1994).

### 2.3. The Emissivity

The photoionization rate is related to the emissivity \( \epsilon_\nu \), the cross section \( \sigma_\nu \), and the mean free path as a function of frequency by
\[
\Gamma = \int_{\nu_{H_1}}^\infty d\nu \epsilon_\nu \lambda_\nu \sigma_\nu.
\]
Here we use the approximation that the mean free path is much less than the horizon distance. (The exact relation between the emissivity and background intensity involves an integration over time with redshift effects, a full knowledge of the spectrum, and reemission; see Miralda-Escudé & Ostriker 1990; Haardt & Madau 1996; Fardal, Giroux, & Shull 1998. We treat reemission approximately in § 2.4.)

Assuming that \( \epsilon_\nu \propto \nu^{-1} \), \( \lambda_\nu \propto \nu^{-1} \), and \( \sigma_\nu \propto \nu^{-3} \), we have
\[
\Gamma = \epsilon_0 n_{H_1} \sigma_0 (1 + \eta)^{1.5},
\]
where \( \sigma_0 = 6.3 \times 10^{-18} \text{ cm}^2 \) and \( \epsilon_0 \) is the emissivity per unit frequency at the ionization edge. The emissivity integrated over frequency is
\[
\epsilon_i = \int d\nu \epsilon_\nu = \epsilon_0 / \eta.
\]

The emissivity can be easily expressed as the number of ionizing photons emitted per Hubble time and for each atom in the universe:
\[
\frac{\epsilon_i(z = 4)}{H_0 n_{at}} = \frac{\sqrt{\pi} (\eta + 1.5) \Gamma}{(0.06 \pm 0.01) \eta \sigma_0 n_{at} (1 + z)^3},
\]
where \( n_{at} \) is the comoving number density of atoms (adding hydrogen and helium), \( n_{at} = \Omega_{b} \rho_{crit} / m_{H}(1 - 3 Y/4) = 2.03 \times 10^{-7} \text{ cm}^{-3} \), and \( H_0 \) is the Hubble constant at \( z = 4 \).

Using \( \Gamma_{12} < 0.53 \), taking the lower limit for the mean free path (to obtain an upper limit for the emissivity), and assuming \( \eta = 1.5 \), the final result is
\[
\frac{\epsilon_i(z = 4)}{H_0 n_{at}} < 7.6.
\]
In other words, there are fewer than eight ionizing photons being emitted per Hubble time and for each atom at \( z = 4 \).

### 2.4. Effect of Reemission from the Ly\( \alpha \) Absorbers

Some of the photons that contribute to the photoionization rate in the intergalactic medium arise from reemission by the Ly\( \alpha \) absorption systems (see Haardt & Madau 1996). For every hydrogen atom that is photoionized by absorbing a photon, and assuming ionization equilibrium, a recombination will take place, and about 40% of the recombinations are directly to the ground state, producing another ionizing photon. However, the majority of the ionizing photons that are not very close to the hydrogen ionization edge are absorbed deep within Lyman limit systems, and the photons produced by recombinations in these regions are close to the ionization edge [with a typical frequency separation from the Lyman limit of \( \Delta \nu/\nu \approx kT/(13.6 \text{ eV}) \approx 0.1 \) and not likely to escape from the Lyman limit system. Because of this, reemission by hydrogen recombination increases the background intensity by only \( \sim 10\% \) (as is found, for example, by using the approximation that the reemitted photons escape only when produced in an absorber of column density \( N_{H_1} < 2/\sigma_0 \), which is the column density for which the typical optical depth for the photon to escape is unity).

Some photons are also emitted by helium recombinations. He\( \iota \) makes little difference to the total budget of ionizing photons, because for each absorption by He\( \iota \), one photon that ionizes hydrogen is produced by the subsequent recombination. Because we did not include absorption by He\( \iota \) in our previous estimate of the mean free path, we do not include the reemission either.

Finally, photons with energy above 4 ryd are mostly used to photoionize He\( \iota \), and each He\( \iota \) recombination can produce between one and three photons capable of ionizing hydrogen. About 25% of the recombinations that are not directly to the ground state go to \( n = 2 \), producing a Balmer continuum photon, which can ionize hydrogen. In addition, all the recombinations produce either a Lyman series photon or two continuum photons when the last transition is from \( 2s \) to \( 1s \); in 8% of the recombinations, two continuum photons that can both ionize hydrogen are produced (see Osterbrock 1989, Tables 2.8, 4.11, and 4.12). Hence, on average 1.33 ionizing photons are produced for each He\( \iota \) that is photoionized. For a source spectrum \( \epsilon_\nu \propto \nu^{-1} \), with \( \eta = 1.5 \), the fraction of photons used to ionize He\( \iota \) is 1/8, so He\( \iota \) reemission increases the emissivity by a factor 1 + 0.33/8 = 1.04. This factor can be reduced during the He\( \iota \) reionization, when the rate of recombinations is less than the rate of photoionizations.

Based on this discussion of the effect of reemission from absorbers, we reduce our upper limit in equation (6) for the total emissivity \( \epsilon_i \) by a factor 1.14 to obtain the upper limit for the direct emissivity from sources \( \epsilon_{at} \) at \( z = 4 \) (rounded up to one digit given the various uncertainties discussed):
\[
\frac{\epsilon_{at}(z = 4)}{H_0 n_{at}} < 7.
\]

### 3. COMPARISON WITH THE EMISSIVITY FROM LYMAN BREAK GALAXIES AND QSOs

#### 3.1. QSOs

Fan et al. (2001) measured the following QSO luminosity function at high redshift (\( z \gtrsim 3.5 \)) from SDSS:
\[
\psi_\nu(L) dL = 7.8 \times 10^{-8} \text{ Mpc}^{-3} \left( \frac{L}{L_{26}} \right)^{-\gamma} \left( 10^{-0.47(z-3)} \right) \frac{dL}{L_{26}},
\]
where \( L_{26} \) is the luminosity of a QSO with absolute AB magnitude \( M_{1450} = -26 \) at \( \lambda = 1450 \) Å, and \( \gamma = 2.58 \). Using a power-law spectral index \( \eta_0 = 1.5 \) for the QSO spectrum at \( \lambda < 1450 \) Å (Teller et al. 2002 find \( \eta_0 = -1.57 \) at \( \lambda < 1200 \) Å for radio-quiet QSOs, and the spectral slope is shallower at longer wavelengths; e.g., Schmidt, Schneider, & Gunn 1995), the luminosity \( L_{26} \) expressed as the number
of ionizing photons (with \( \lambda < 912 \) Å) emitted is \( L_{-26} = 5.48 \times 10^{50} \) photons s\(^{-1}\). The luminosity function in equation (8) was observed down to \( L \approx 0.5L_{-26} \) by Fan et al. (2001); the total emissivity from QSOs clearly depends on the luminosity at which the slope of \( \psi_Q(L) \) becomes shallower than \( L^{-2} \). If we assume the form \( \psi_Q(L) \propto \left(\frac{L}{L_{Q*}}\right)^{-4} + \left(\frac{L}{L_{Q*}}\right)^{-1} \), which produces a similar slope of \( \psi(L) \) at low luminosities \( (L < L_*) \) as observed at lower redshift (e.g., Boyle, Shanks, & Peterson 1988; Boyle et al. 2000), then the comoving emissivity is

\[
\epsilon_{\text{QSO}} = 1.45 \times 10^{-24} \left(\frac{L_{-26}}{L_{*}}\right)^{-2} 10^{-0.47(z-3)} \times \int_0^\infty \frac{x}{x^{3/2}+1} \, dx \text{ photons s}^{-1} \text{ cm}^{-3}. \tag{9}
\]

Expressing this as the number of photons emitted per Hubble time and per atom at \( z = 4 \), we find

\[
\frac{\epsilon_{\text{QSO}}}{H(z=4)n_{\text{at}}} = 0.61 \left(\frac{L_{-26}}{L_{*}}\right)^{-2}. \tag{10}
\]

A lower limit to the emissivity from QSOs is obtained if \( L_* \approx 0.5L_{-26} \), \( \epsilon_{\text{QSO}}(Hn_{\text{at}}) \geq 1 \). In order not to exceed the upper limit obtained in the last section, \( \epsilon/H(z=4)n_{\text{at}} < 7 \), we must require \( L_{Q*} > 0.015L_{-26} \). We note, however, that low-luminosity QSOs may be intrinsically self-absorbed at the Lyman limit, reducing their contribution to the ionizing emissivity (see Alam & Miralda-Escudé 2002).

The number of faint active galactic nuclei has been constrained by Steidel et al. (2002), who find that their abundance at \( z = 3 \) and at an apparent AB magnitude \( R \approx 23.9 \) (or luminosities \( L \approx 0.018L_{-26} \)) is about 3% of the Lyman break galaxy abundance at a magnitude \( R = 24.6 \) at \( z = 3 \). Using the Lyman break galaxy luminosity function given in the next subsection, we find that this implies that the turnover of the QSO luminosity function should occur at \( L_{Q*} = 0.046L_{-26} \), assuming again the form \( \psi_Q \propto \left(\frac{L}{L_{Q*}}\right)^{-4} + \left(\frac{L}{L_{Q*}}\right)^{-1} \). If the value of \( L_{Q*} \) were the same at \( z = 4 \) as at \( z = 3 \) (but with the QSO abundance having declined as \( 10^{-0.47z} \) at all luminosities, as in eq. [8]), QSOs would contribute about half of the total emissivity required at \( z = 4 \) from equation (7). There are also limits from the Hubble Deep Field (Conti et al. 1999), but these probe QSOs at much lower luminosities and do not provide a stronger constraint on \( L_{Q*} \). In summary, the available observations are consistent with a contribution to the ionizing emissivity from QSOs that does not exceed the upper limit in equation (7).

### 3.2. Lyman Break Galaxies

The galaxy ultraviolet luminosity function at high redshift has been measured by Steidel et al. (1999). They fit the luminosity function to a Schechter function,

\[
\phi(L) dL = \phi_\alpha \left(\frac{L}{L_\alpha}\right)^{-\alpha} e^{-L/L_\alpha} dL/L_\alpha. \tag{11}
\]

After converting to the flat cosmological model with \( \Omega_m = 0.3 \), \( h = 0.65 \), the parameters they find are \( \phi_\alpha = 8.6 \times 10^{-5} \) Mpc\(^{-3}\), \( \alpha = 1.6 \), and \( L_\alpha = 1.4 \times 10^{47} \) ergs s\(^{-1}\) Hz\(^{-1}\) at \( \lambda = 1700 \) Å. To convert this to the ionizing luminosity, we use the models of Bruzual & Charlot (1993), for which the luminosity per unit frequency decreases by a factor 10 from \( \lambda = 1700 \) to 912 Å, for an age of 10\(^6\) yr (these models assume a Salpeter mass function up to a maximum mass of 125 M\(_{\odot}\) and solar metallicity). Using also the approximation of \( L_\nu \propto \nu^{-2} \) at \( \lambda < 912 \) Å, we find the ionizing luminosity is \( L_\nu = 1.06 \times 10^{54} \) photons s\(^{-1}\).

The form of the galaxy luminosity function (eq. [11]) is highly uncertain at \( z \leq 4 \), because the observations of Steidel et al. (1999) are not deep enough to reach lower luminosities, and galaxies down to \( 10^{-0.8}L_* \) are detected only in the Hubble Deep Field (e.g., Madau et al. 1996). The Hubble Deep Field counts at \( z = 4 \) are significantly below the prediction from the model in equation (11), but this may be due to galaxy clustering (Steidel et al. 1999; see their Fig. 8). If we include only galaxies with luminosities larger than \( (L_*, 10^{-0.8}L_*, 10^{-0.8}L_*) \), the total emissivity inferred from equation (11) is

\[
\epsilon_{\text{gal}}(H(z=4)n_{\text{at}}) = (3.4, 8.5, 13.8)f_{\text{esc}} \tag{12}
\]

Steidel, Pettini, & Adelberger (2001) found that Lyman break galaxies may have escape fractions of ionizing photons as large as 50%, from a possible detection of Lyman continuum flux after co-adding spectra of several galaxies. If this result were true, the luminous galaxies with \( L > 10^{-0.8}L_* \) would already produce the maximum emissivity in equation (7). Alternatively, \( f_{\text{esc}} \) may be much lower (see Giallongo et al. 2002; note also that the value \( f_{\text{esc}} \approx 0.5 \) measured by Steidel et al. 2001 was for the subset of the bluest quartile of the Lyman break galaxies, which may not be representative), and lower luminosity objects may be responsible for the bulk of the emissivity. More accurate determinations of the escape fraction, the abundance of Lyman limit systems, and the inferred photoionization rate from the Ly\(\alpha\) forest transmitted flux will be needed to resolve this question.

### 4. Constraints on the Emissivity Evolution to End Reionization at \( z = 6 \)

The highest redshift QSO known at the present time (Becker et al. 2001) shows a complete Gunn-Peterson trough at \( z = 6 \), but the spectra of all the sources at lower redshift have detectable transmitted flux blueward of Ly\(\alpha\), indicating that most of the volume in the intergalactic medium had to be reionized by \( z = 6 \). We now address the constraint implied by this fact on the evolution of the ionizing emissivity, in view of its value at \( z = 4 \) (eq. [7]).

#### 4.1. The Reionization Equation

Reionization proceeds by the creation of ionized regions around individual sources (Arons & Wingert 1972). The mean free path of ionizing photons through the neutral medium is typically very short compared to the distance between sources, so reionization proceeds by the growth of the ionized regions and can be modeled in terms of the fraction of the volume that is ionized at time \( t \), \( y(t) \), assuming that the remaining fraction \( 1 - y \) is completely neutral. Moreover, because the emitted photons are used to ionize atoms almost immediately after their emission, it follows that the change in the fraction of atoms that are ionized in the universe is equal to the ionizing photon emission rate per atom minus the average recombination rate of atoms. If
\[ \epsilon_i(t) \equiv \epsilon_i(t)/n_{\text{at}} \] is the comoving ionizing photon emissivity per atomic nucleus, \( R(t) \) is the average recombination rate per atom in ionized regions (which includes the effect of an effective clumping factor of the ionized gas), and \( F_i \) is the fraction of baryons that actually need to be ionized in the ionized fraction \( y \) of the universe (with the remaining \( 1 - F_i \) fraction being the baryons that are in dense systems, such as stars or dense gas in galaxies, that do not need to be ionized by external sources), the equation for the evolution of \( y \) is (Madau et al. 1999; Miralda-Escudé, Haehnelt, & Rees 2000)

\[
\frac{d(yF_i)}{dt} = \epsilon_i - R_y \ .
\]

This equation (apart from the factor \( F_i \)) is analogous to the one obtained for the problem of an expanding cosmological \( \text{H} \) region around an ionizing source, which was discussed by Shapiro & Giroux (1987) and Donahue & Shull (1987), where \( y \) behaves as the volume of the \( \text{H} \) region produced by a source of luminosity proportional to \( \epsilon_i(t) \). It is also identical to the equation of radiative transfer, where \( yF_i \) plays the role of the intensity, and \( RF_i^{-1} \) and \( \epsilon_y \) play the role of the absorption and emission coefficients, respectively. The solution is found through the substitutions

\[
y(t) = F_i^{-1} \int_{t_i}^{t} dt' \epsilon_i(t') \exp \left[ \frac{R_y t_i}{c} \right] .
\]

where \( \tau(t) = \int_{0}^{t} dt' R_i^{-1} R(t') \), and \( t_i \) is the initial time when the first sources turn on. Assuming that any clumping factor is constant and that \( \Omega_m \) is \( \simeq 1 \) [e.g., for the cosmological constant model with a present matter density \( \Omega_m = 0.3 \), we have \( \Omega_m(z = 4) = 0.982 \)], the recombination rate varies as \( R(t) \propto r^{-2} \). Choosing \( t_4 \equiv t(z = 4) = \frac{4}{3} H_4^{-1} \) as a fiducial time and \( R_4 = R(t_4) \), we have

\[
y(t) = F_i^{-1} \int_{t_i/t_4}^{t/i} \frac{d(t' \tau)}{t_4} \frac{2 \epsilon_i(t')}{3H_4} \exp \left[ \frac{R_y t_4}{c} \tau \left( \frac{t_4}{t} - \frac{t_4}{t'} \right) \right] .
\]

4.2. The Recombination Rate

Before presenting solutions of the reionization equation, we discuss the value that we adopt for the recombination rate \( R \). The average recombination rate is affected by the clumping factor of the ionized gas, which we define as \( C_i = (n_{\text{at}}^2)_{\text{IGM}}/(n_{\text{IC}}^2)_{\text{IGM}} \). The subscript IGM is used here to denote that the average is to be taken only over those regions of space where matter is predominantly photoionized by photons that have escaped to the intergalactic medium. This restriction is necessary because if we did not impose it, the clumping factor would be dominated by regions of dense gas that are locally ionized by sources that were not included in the emissivity in equation (6), precisely because the radiation from these sources is locally absorbed. In fact, in the absence of any restriction on the regions over which one must average to obtain the clumping factor of ionized matter, the clumping factor would obviously be very large and dominated by the densest stars. Once stars are eliminated from consideration, there are still \( \text{H} \) regions that form in dense clouds in the interstellar medium of galaxies around massive young stars, in which all the emitted photons are immediately absorbed locally. This emission is irrelevant for the reionization of the intergalactic medium, and it is best not to include it in equation (6) and at the same time not to include the dense gas ionized locally in the clumping factor. This definition of the clumping factor inevitably involves an arbitrary choice in defining when a photon is considered to be escaped, and therefore when a region is predominantly ionized by escaped photons rather than local ones. In practice, a photon can be defined to have escaped once it reaches an intergalactic region with gas density below some critical value. The calculation of the clumping factor can be obtained only from full radiative transfer simulations of reionization, but its value can differ among studies by large factors depending on the definition that is adopted (see Gnedin 2000 and references therein). If the restriction that escaped photons dominate the radiation intensity is not imposed, then very large values of the clumping factor can be obtained, which could only be used consistently by including the locally absorbed photons in the emissivity.

Here we use the simple model of Miralda-Escudé et al. (2000) for the clumping factor, which uses the approximation that the gas is ionized only up to an overdensity \( \Delta_i = \rho_i/\rho \) and all the gas at higher densities is neutral or otherwise is ionized locally and therefore does not contribute to the clumping factor. The overdensity \( \Delta_i \) is related to the mean free path \( \lambda_i \) that a photon can traverse before being absorbed. During reionization, the mean free path \( \lambda_i \) should be of order the size of the \( \text{H} \) regions or of order the typical separation between neighboring sources. For \( \lambda_i = 1000 \text{ km s}^{-1} \), or \( \sim 10 \text{ Mpc} \) at \( z = 6 \) in comoving units, the critical overdensity is \( \Delta_i \approx 5 \), and the clumping factor is very close to unity (see Figs. 2d and 6 in Miralda-Escudé et al. 2000).

It has been pointed out by Haiman, Abel, & Madau (2001) that before reionization is complete, a large number of low-mass halos should be present and should contain dense gas accreted before it was heated by reionization. The effect of these low-mass halos would be to decrease the mean free path at a fixed overdensity \( \Delta_i \) and to widen the distribution of the overdensity. To estimate the plausible change in the clumping factor due to these low-mass halos, we can assume that the structure of the atomic intergalactic medium before reionization is self-similar to that of the ionized intergalactic medium but with the scales reduced in proportion to the Jeans scale \( \lambda_\text{J} \), so that the clumping factor should depend only on the ratio \( \lambda_i/\lambda_\text{J} \) and the amplitude of the primordial density fluctuations at the Jeans scale. If the temperature increases by a factor 100 during reionization (from \( \sim 100 \) to \( \sim 10^4 \text{ K} \)), the Jeans length increases by a factor 10. The amplitude of primordial density fluctuations in cold dark matter (CDM) with \( \Omega_m = 0.3 \) and \( h = 0.65 \) decreases by a factor \( \sim 2 \) between the scales of 0.05 and 0.5 Mpc (roughly corresponding to the Jeans scales at these two temperatures). Therefore, the clumping factor of a medium in which all the low-mass halos that formed from scales intermediate between the Jeans scales of the atomic and ionized medium have survived, at \( z = 7 \) and \( \lambda_{i,7} = 10^5 \text{ km s}^{-1} \), should be the same as the clumping factor of a medium with no such surviving halos, evaluated at \( z = 3 \) and \( \lambda_i = 10^4 \text{ km s}^{-1} \). From Figures 2b and 6 of Miralda-Escudé et al. (2000), we find this clumping factor to be 7. We note that this is an upper limit to the true clumping factor, because the gas in the low-mass halos that might increase
the clumping factor above the value of \( \sim 1 \) found previously is likely to evaporate on a timescale short compared to the Hubble time, once these halos have been reached by an ionization front (Shapiro & Raga 2001). The gas may also be expelled by a small amount of internal star formation even before the halo is reached by external ionizing radiation.

Finally, a discussion is needed to decide if the case A or case B recombination coefficient should be used. Ionizing photons of frequency \( \nu \) will typically be absorbed in regions of column density \( N_{\text{HI}} \sim 1.6 \times 10^{17} \text{ cm}^{-2} (\nu/140\text{MHz})^2 \), where \( \nu_{\text{HI}} \) is the Lyman limit frequency. The sources typically have power-law spectra, and most of the emitted photons have frequencies substantially above \( \nu_{\text{HI}} \), especially if the spectra are hardened by internal absorption. Most of the photons emitted by direct recombinations to the ground state, which have frequencies very close to the Lyman limit, will be reabsorbed in the same system (in addition, among the small fraction of photons from direct recombinations to the ground state that escape the system in which they have been produced, some will be redshifted below the Lyman limit and will not be absorbed again). Hence, these photons do not contribute to increasing the cosmic ionizing background intensity for the most part. An alternative way to understand this argument is that even though in principle we are in a case B situation because the recombination photons are reabsorbed, in practice the inclusion of the photons from recombinations increases the clumping factor by the ratio of the case A to case B recombination coefficients because these photons increase the ionization in dense regions, therefore yielding a recombination rate equivalent to using case A. Therefore, we use the case A recombination coefficient.

4.3. Results for the Reionization History

We now compute \( y(\tau) \) assuming models where the emissivity varies as \( \epsilon_\nu \propto (1+z)^{-3} \propto \tau^{-2/3} \) for \( z > 4 \) and has the value \( \epsilon_\nu/H_\nu = 7 \) at \( z = 4 \), i.e., the maximum allowed value found previously (eq. [7]). We choose the recombination coefficient \( \alpha = 3.15 \times 10^{-13} \text{ cm}^3 \text{s}^{-1} \), valid for case A and \( T = 1.5 \times 10^4 \text{ K} \), and electron density \( n_e = 6.95 \times 10^{-5} \text{ cm}^{-3} \) at \( z = 6 \), valid for \( \Omega_m h^2 = 0.022 \) and if helium is only once ionized. This yields a recombination rate per Hubble time \( R(z=6)/H(z=6) = 1.02 \). In some cases, we increase this recombination rate by a clumping factor \( C_i \). We fix the fraction of baryons in the ionized regions that actually need to be ionized to \( F_i = 0.9 \) (independent of redshift), in reasonable agreement with the fraction of baryons that seem to be present in damped Ly\( \alpha \) systems at \( z \approx 4 \) (Lanzetta et al. 1991; Storrie-Lombardi, McMahon, & Irwin 1996) and the fraction of baryons above the critical overdensity \( \Delta_m \approx 5 \) at \( z = 6 \) in the model discussed above for the clumping factor (Miralda-Escude et al. 2000, Fig. 2d).

We solve equation (15) starting at \( y = 0 \) at some initial redshift \( z_i \) when the emissivity is assumed to turn on suddenly. The parameters of the model that we vary are \( z_i \), \( \beta \), and \( C_i \), and we require all models to reach \( y = 1 \) (i.e., the end of reionization) at \( z = 6 \).

The results of five representative models are shown in Figure 1. When \( C_i = 1 \), assuming the emissivity to remain constant satisfies the requirement of completing reionization by \( z = 6 \) when the emissivity is turned on at \( z = 8.7 \). With only a slight decline of the emissivity, the initial redshift needs to be pushed to much higher values. For \( \beta = 0.63 \), which corresponds to a decline of the comoving emissivity by a factor of only \( \sim 1.5 \) between \( z = 4 \) and \( z = 9 \), the sources need to start at very high redshift to reach \( y = 1 \) at \( z = 6 \). Naturally, changes in the emissivity at redshifts higher than \( \sim 15 \) are irrelevant for the requirement that reionization ends at \( z = 6 \), because most of the atoms ionized at this high redshift will recombine again. We therefore conclude that, for the case of no clumping, the comoving emissivity at \( 6 < z \leq 9 \) cannot be much lower than its maximum value allowed at \( z = 4 \). In other words, the sources responsible for the emission of ionizing radiation cannot decrease their overall comoving emissivity by more than a factor \( \sim 1.5 \) between \( z = 4 \) and \( z = 9 \). Allowing now for a clumping factor, an increase to \( C_i = 2.1 \) already requires the model with constant emissivity \( (\beta = 0) \) to maintain the emissivity up to a very high initial redshift. A model where the same amount of radiative energy is emitted at every Hubble time [i.e., \( H(z) \propto (1 + z)^{1.3} \)], starting at very high \( z_i \), results in complete reionization at \( z = 6 \) when \( C_i = 5.8 \). Therefore, for a clumping factor as large as \( \sim 7 \), as suggested previously for a model where the ionized gas in all low-mass halos is retained, the emissivity would need to substantially increase with redshift.

4.4. Comparison to the Observed Evolution at \( z < 4 \)

We have shown that by combining the measurement of the ionizing emissivity \( \epsilon_\nu \) at \( z = 4 \) with the requirement that the reionization must end by \( z = 6 \), we can infer that the emissivity does not decrease with redshift by more than a factor \( \sim 1.5 \) from \( z = 4 \) to \( z = 9 \). We can now ask how this evolutionary constraint compares to the evolution at \( z < 4 \) that is implied by observations of the Ly\( \alpha \) forest decrement...
and the mean free path between Lyman limit systems. From MM01, the photoionization rate (or proper intensity of the background) declines roughly as \((1 + z)^{1.5}\) from \(z = 2.5\) to \(z = 4\). The abundance of Lyman limit systems per unit redshift is proportional to \((1 + z)^{1.5}\) (Storrie-Lombardi et al. 1994), implying that the proper mean free path varies as \((1 + z)^{-0.5}\) essentially constant within the likely errors of measurement for both the proper intensity and mean free path evolution. Therefore, the results of § 4.3 show that if the clumping factor is low, this constant emissivity needs to be maintained roughly at the same value up to \(z \sim 9\), while if the clumping factor is much larger than 1, the emissivity needs to modify its evolution and increase strongly with redshift at \(z > 4\).

4.5. The Thomson Optical Depth of the Cosmic Microwave Background

The optical depth of CMB photons to electron scattering, \(\tau_{es}\), is given by

\[
\tau_{es} = \int_{0}^{z_i} \frac{c}{dz} \frac{dF_i}{dz} n_{e0} (1 + z)^3 \sigma_{es} \gamma(z),
\]

where \(\sigma_{es}\) is the Thomson cross section for electron scattering. The values of \(\tau_{es}\) for the five models in Figure 1 are indicated in the figure. We have assumed \(F_i = 0.9\) at all redshifts; the 10% of the matter not included is supposed to be either gas that remains atomic or molecular in self-shielded regions or stars.

These results show that the minimum allowed value of \(\tau_{es}\), which occurs for models where the emissivity turns on at a redshift not much larger than 6, is \(\approx 0.045\). The maximum values of the optical depth depend on the emissivity at high redshift. If the emissivity does not increase with redshift, then the ionized fraction of the volume at high redshift needs to be small due to the fast rate of recombinations \([\text{i.e., for constant emissivity, } y \propto (1 + z)^{-3}\), and the integral in eq. (16) converges rapidly at high \(z\). For a model with a constant emissivity up to \(z_i = 20\) and clumping factor equal to 1, the optical depth can be increased only up to \(\tau_{es} = 0.09\) (in this model, reionization ends at \(z = 7.5\), but the reionization end could always be delayed by increasing the clumping factor as the fraction \(y\) reaches a value close to unity).

An emissivity increasing with redshift can obviously produce larger optical depths, and as mentioned before, the value of the emissivity at \(z \approx 15\) does not affect the epoch at which reionization ends, so in principle the optical depth could be arbitrarily high if enough ionizing sources were present at very high redshift. However, in the flat CDM model with \(\Omega_m = 0.3\), which is strongly supported by all observations of large-scale structure, the stars form only at \(z \approx 20\) from \(\sigma\) peaks collapsing into the first low-mass halos (e.g., Couchman & Rees 1986; Bromm, Coppi, & Larson 1999; Abel, Bryan & Norman 2000; Ricotti, Gnedin, & Shull 2002; Venkatesan, Tumlinson, & Shull 2003), and so the emissivity cannot continue to increase up to a very high redshift. At the same time, observations indicate a cosmic star formation rate in galaxies that starts to decline with redshift at \(z > 2\) (Madau, Pozzetti, & Dickinson 1998), although the ionizing emissivity might increase with redshift even when the star formation rate decreases because of an increasing escape fraction for ionizing photons.

As this paper was being refereed, the results of the Wilkinson Microwave Anisotropy Probe (WMAP) mission were announced. Kogut et al. (2003) give a result \(\tau_e = 0.16 \pm 0.04\) from a model-independent analysis, although the result and the error can change when fitted to different CDM models (Spergel et al. 2003). The value of \(\approx 0.08\) implied by a constant emissivity with clumping factor of 1 up to very high redshift would represent a 2 \(\sigma\) deviation from the value of Kogut et al. (2003). A value as high as \(\tau_e = 0.16\) would clearly imply a substantial increase of the emissivity up to \(z \sim 20\), when only a small fraction of the mass has collapsed into halos that can form stars in the CDM model.

5. Conclusions

The observed mean transmitted flux of the Ly\(\alpha\) forest, when combined with independently measured values of the mean baryon density, the amplitude of the power spectrum, and the gas temperature, yields a measurement of the intensity of the ionizing background. The abundance of Lyman limit systems determines the mean free path of ionizing photons. These two quantities together determine the ionizing emissivity. We have argued in this paper that the emissivity measured in this way at \(z = 4\) is at most seven ionizing photons per atom per Hubble time. We have then shown that the emissivity over the redshift range \(6 < z \lesssim 9\) cannot be much lower than the value at \(z = 4\) in order that the universe be reionized by \(z = 6\), if the clumping factor of ionized gas during the reionization epoch is close to unity. If the clumping factor is larger than unity, then the ionizing emissivity must increase with redshift between \(z = 4\) and \(z \sim 9\).

The electron scattering optical depth must be at least \(0.045\), given the presence of ionized intergalactic gas at \(z < 6\) and a reasonable minimum redshift range over which sources ionize the universe at \(z > 6\). If the emissivity of ionizing photons does not increase with redshift at \(z > 4\), then the optical depth must be less than \(0.09\). Measurements of the optical depth from the CMB spectrum of temperature and polarization fluctuations therefore provide a powerful diagnostic to decide if the peak of the comoving ionizing emissivity occurred at \(z \approx 4\) (with a broad plateau extending up to \(z \sim 9\) so that the end of reionization can be reached by \(z = 6\)), or at a much earlier epoch. The recent WMAP result has an error that is still too large to reach a definite conclusion, but if a value \(\tau_e \approx 0.16\) is confirmed, it implies that the comoving emissivity continues to rise up to \(z \approx 20\).

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