Fracture Analyses of Cracked Delta Eye Plates in Ship Towing

Xiangbing Huang, Xingling Huang and Jizheng Sun

Department of Naval Architecture, Naval University of Engineering, Wuhan, Hubei Province 430033, P.R. China
Email:alinghuang@163.com

Abstract. Based on fracture mechanics, a safety analysis approach is proposed for cracked delta eye plates in ship towing. The static analysis model is presented when the delta eye plate is in service, and the fracture criterion is introduced on basis of stress intensity factor, which is estimated with domain integral method. Subsequently, three-dimensional finite element analyses are carried out to obtain the effective stress intensity factors, and a case is studied to demonstrate the reasonability of the approach. The results show that the classical strength theory is not applicable to evaluate the cracked plate while fracture mechanics can solve the problem very well, and the load level, which a delta eye plate can carry on, decreases evidently when it is damaged.

1. Introduction
Accidents usually occur in ship towing as a result of structure failure, the risk has been studied by some researchers [1-2]. Delta eye plate is one of the main towing assembly components, and it could be damaged in operation sometimes. However, it has been evaluated as an intact structure before, and in this way may, improper conclusions could be drawn. Therefore, fracture analyses will be conducted in the present study for cracked delta eye plates and the results will be compared with the classical strength theory.

2. Methodology
2.1. Problem Description
Delta eye plate is widely used in ship towing system, and one terminate is connected to towing hawser assembly, then the others are in connection to the wire rope bridles of tug. The arrangement of a delta eye plate is shown in Figure. 1(a). Stress concentration usually happens at the edges of holes, especially for the hole A where the cracks initiate. The configuration of the cracked delta eye plate is presented in Figure. 1(b).

The plate is subjected to tensile stresses \(\sigma_1, \sigma_2\) and \(\sigma_3\) when in service, and the stresses should satisfy the following conditions

\[
\begin{align*}
\sigma_1 - (\sigma_2 + \sigma_3) \cos \theta &= 0 \\
\sigma_2 &= \sigma_3
\end{align*}
\]

where \(\sigma_1\) is tensile stress from towed ship, and \(\sigma_2\) and \(\sigma_3\) are tensile stresses from wire rope bridles of tug. It is clear form Figure. 1(b) and Eq. (1) that the cracked plate is under mode I and II loadings.

The delta eye plate’s geometric parameters are as follows: \(H=500\text{mm}, h=260\text{mm}, W=260\text{mm}, t=90\text{mm}, R=50\text{mm}, a/R=0.1, 0.3,…, 1.1\) and \(\theta=30^\circ\). The plate’s material is assumed to be homogeneous and isotropic with Poisson’s ratio \(\nu=0.3\) and the Young’s modulus \(E=210\text{GPa}\).
2.2. Fracture Criterion
For a crack subjected to combined mode I and II loading, the fracture criterion can be given as

\[ K_{e} \leq K_{c} \]  

\[ K_{\text{eff}} = \sqrt{K_{I}^2 + K_{II}^2} \]  

where \( K_{IC} \) is fracture toughness, \( K_{I} \), \( K_{II} \) are the mode I and II stress intensity factors (SIF), and \( K_{\text{eff}} \) is effective stress intensity factor.

A domain integral expression proposed by Li et al (1985) [3] and Shih et al (1986) [4] is used to compute \( K_{I} \) and \( K_{II} \) along the crack front in three-dimensional models.

\[ K_{I}^2 + K_{II}^2 = \frac{E \cdot J}{(1 - \nu^2)} \]  

where \( J \) is the domain integral.

Geometry correction function (GCF) is considered as the non-dimensional stress intensity factor, and can be defined as

\[ \beta_{\text{eff}} = \frac{K_{\text{eff}}}{\sigma_{I} \sqrt{\pi \alpha}} \]
where $\beta_{\text{eff}}$ is effective geometry correction function and $a$ is the length of each crack.

![Global FE model](image1.png) ![FE mesh near the crack tip](image2.png)

**Figure. 2** Three-dimensional FE model of the delta eye plate

### 3. Finite Element Implementation

The delta eye plate models were analyzed in finite element (FE) code ABAQUS (ABAQUS 6.13) which used the domain integral method to compute $K_I$ and $K_{II}$ respectively [5-6].

Due to symmetry, only half of the plate was modeled in ABAQUS with 8-node linear brick with reduced integration element (C3D8R). There were 20 elements used along the plate thickness direction, and the singularity elements were placed at the position of the crack tips. The convergences of $K_I$ and $K_{II}$ were studied to check the number of elements, and there were about 117,840 elements in each FE model, varying slightly depending on the size of cracks. The typical three-dimensional FE model is shown in Figure. 2.

### 4. Results and Discussion

#### 4.1. Stress Analysis of Intact Delta Eye Plate

An intact delta eye plate was studied in finite element method to validate the locations in which cracks usually originate. When $\sigma_1 = 100$MPa, the stress state of the plate is shown in Figure. 3(a). It can be observed that the maximum stress is approximately in the horizontal plane of the hole A, and cracks easily initiate in this region. The result also proves that the assumption of crack location is reasonable.

![Intact plate](image3.png) ![Cracked plate](image4.png)

**Figure 3.** Deformation patterns of intact and cracked delta eye plates
4.2. Stress Intensity Factors of Cracked Delta Eye Plate

GCF is non-dimensional stress intensity factor, and employed widely in fracture analysis. The distribution of GCF along plate thickness direction is not constant in three-dimensional models, but it approaches maximum at the mid-plane. The three-dimensional cracked delta eye plates have been analyzed in finite element method, and the typical deformation pattern is presented in Figure 3(b), then the GCFs at the mid-plane are shown in Figure 4.

It can be found that the evolution of $\beta_{\text{eff}}$ is not monotonous. It decreases quickly at first, and then increase slowly with the increment of the ratio $a/R$. The stress intensity factors can be conveniently derived from Figure 4 when the stress $\sigma_i$ is determined.

4.3. Case Study

In a tug, the delta eye plate’s technical parameters are as follows: (a) Safe load level 120MPa; (b) the yield strength of the material $\sigma_Y$ = 330MPa; (c) The fracture toughness of the material $K_{IC} = 50$ MPa·(m)$^{1/2}$, i.e. 1581.14 MPa·(mm)$^{1/2}$. The delta eye plate’s safe load level is 120MPa. However, the real load is usually several times than safe load level. Thus the plate will be studied at several different loading conditions.

For intact delta eye plate, the maximum stress ($\sigma_{\text{max}}$) is calculated with FEM, and compared with the yield strength of the material to determine whether safe or not, then the results are summarized in Table 1.

For cracked delta eye plate, the effective stress intensity factors have been calculated with Eq. (5) and Figure 4 at various loading levels, as shown in Figure 5. According to fracture criterion, i.e. Eq. (2), the safe conditions of the plate are presented in Table 1.

![Figure 4. GCFs at the mid-plane of delta eye plates](image)

![Figure 5. SIFs at various load levels](image)
Table 1. Safe conditions of intact and cracked delta eye plates

| Load levels (σ₁) | Intact delta eye plate Safe: σₘₐₓ<0.90σᵧ | Cracked delta eye plate Safe: Kₑₑᵣ<𝐾ᵢC |
|------------------|----------------------------------------|----------------------------------------|
| 100 MPa          | σₘₐₓ=148.39MPa Safe                     | Safe                                   |
| 120 Mpa          | σₘₐₓ=178.07MPa Safe                     | Safe when (a/R)<1.0                    |
| 180 Mpa          | σₘₐₓ=267.11MPa Safe                     | Safe when (a/R)<0.50                   |
| 200MPa           | σₘₐₓ=296.80MPa Safe                     | Safe when (a/R)<0.30                   |

It is clear from Table 1 that although the load is in safe level (120MPa), the cracked plate failure will happen when the cracks exceed a certain length. The safe load, which the cracked delta eye plate can carry on, is much less than the intact plate’s. Therefore, classical strength theory is not applicable for cracked plates, and the safety analysis of cracked structures should be on the basis of fracture mechanics.

5. Conclusions
Fracture analyses were carried out for cracked delta eye plates, and compared with the analysis results of classical strength theory. The classical strength theories are applicable for intact plates, but overestimate the cracked plates’ safety. It seems that the classical strength theory should be considerate when dealing with cracked problems, and fracture mechanics can solve them very well.

6. References
[1] Huang XB, Huang XL. Safety Analysis and Decision Making Methodology in Ship Towing System Design//Applied Mechanics and Materials. Trans Tech Publications, 2012, 201: 1013-1016.
[2] Huang XL, Huang XB. A Risk-Based Conceptual Design Method for Submarine Rescue Vehicle//Applied Mechanics and Materials. Trans Tech Publications, 2012, 201: 477-482.
[3] Li FZ, Shih CF, Needleman A. A comparison of methods for calculating energy release rates. Engineering Fracture Mechanics, 1985, 21(2): 405-421.
[4] Shih CF, Moran B, Nakamura T. Energy release rate along a three-dimensional crack front in a thermally stressed body. International Journal of Fracture, 1986, 30(2): 79-102.
[5] Huang X, Liu Y, Huang X, Dai Y. Crack arrest behavior of central-cracked stiffened plates under uniform tensions. International Journal of Mechanical Sciences, 2017, 133: 704–719.
[6] Huang X, Liu Y, Dai Y. Characteristics and effects of T-stresses in central-cracked unstiffened and stiffened plates under mode I loading. Engineering Fracture Mechanics, 2017, In Press, http://dx.doi.org/10.1016/j.engfracmech.2017.09.017.