Impact of CP phases on a light sbottom and gluino sector

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Abstract

We study a scenario in which light bottom squarks and light gluinos with masses in the range 2 − 5.5 GeV and 12 − 16 GeV, respectively, can coexist in the MSSM, without being in conflict with flavor-conserving low-energy observables. We find that in such a scenario, the anomalous magnetic moment of a muon could be as large as $10^{-9}$, if the theory conserves CP. However, if the theory violates CP, we conclude that not both, the gluino and bottom squark, can be light at the same time, after the neutron electric dipole moment constraint on Weinberg’s 3-gluon operator has been taken into account.
Inspired by the unforeseen excess of the bottom-quark production observed at the hadronic collider of Fermilab [1], Berger *et al.* [2] proposed a solution with light sbottom \( \tilde{b}_1 \) (\( m_{\tilde{b}_1} \approx 2.0 - 5.5 \) GeV) and light gluino \( \tilde{g} \) (\( m_{\tilde{g}} \approx 12 - 16 \) GeV). Interestingly, various problems can easily be avoided, such as by adopting the proper mixing angle of two sbottoms \( \tilde{b}_L \) and \( \tilde{b}_R \), the Z-peak constraint can be evaded; and also, the \( R_b \) contribution, arisen from sbottom-gluino loop, can be suppressed by considering the second sbottom \( \tilde{b}_2 \) being lighter than 180 GeV for the CP conserved case [3], while the mass of \( \tilde{b}_2 \) can be heavier in the CP violating one [4]. The amazing thing is that such light supersymmetric particles, so far, haven’t been excluded by experiments, even after including the data of precise measurements [5]. Moreover, for searching the signals of the light sbottom and gluino, many testable proposals are raised, e.g., the rate of \( \chi_b \) decaying to a pair of light bottom squarks [6], radiative \( B \) meson decays [7], the decays \( Z \rightarrow \tilde{b}_L^* \tilde{g} + \tilde{b}_1 \tilde{g} \) followed by \( \tilde{g} \rightarrow \tilde{b}_1^*/\tilde{b}_1 \) and \( e^+e^- \rightarrow q\bar{q}\tilde{g}\tilde{g} \) [8], as well as the running of strong coupling constant \( \alpha_s \) [9].

To further explore more impacts on other processes, it is necessary to investigate different systems instead of those in which the final states are directly associated with the light sbottom and gluino. It is known that supersymmetric models not only supply an elegant mechanism for the electroweak symmetry breaking and a solution to the hierarchy problem, but also guarantee the unification of gauge couplings at the scale of GUTs [10]. Therefore, besides the effects mentioned above, other contributions will also appear when considering different phenomena. Inevitably, new parameters will come out. To avoid introducing the irrelevant parameters, such as the mixing angles among different flavors of squark, we have to consider the processes in which the dependent parameters are still concentrated on the minimal set. The best candidates are the flavor-conserving processes.

One of the mysteries in the standard model (SM) is whether the Higgs mechanism plays an essential role for the symmetry breaking and the resultant of Higgs particle can be captured in future colliders. Based on the same philosophy of the symmetry breaking, the minimal supersymmetric standard model (MSSM) needs the second Higgs doublet field to balance the anomaly of quantum corrections, i.e., there are three neutral Higgs particles in MSSM, one of them is CP-odd \( (A^0) \) and the remains are CP-even \( (h \text{ and } H) \). It is obvious that besides the mixing angle of sbottoms and the masses of sbottom (gluino), the essential parameters in MSSM are \( A_{(b)} \), the trilinear SUSY soft breaking terms, \( \mu \), the mixing parameter of two Higgs superfields, \( m_{A^0(h,H)} \), the masses of corresponding Higgses, and \( \tan \beta \), defined by the
ratio of $v_u$ to $v_d$ in with $v_u(d)$ being the vacuum expectation of the Higgs field that couples to up (down) type quarks. And also, unlike the non-SUSY two-Higgs-doublet model, the mixing angle of two neutral Higgs fields, denoted by $\alpha$, is not an independent parameter and can be related to $\tan \beta$ \cite{11}.

The activity of searching for the Higgs particle and studying its properties is proceeding continuously \cite{12, 13}. In particular, the remarkable results with a large $\tan \beta$ have been investigated enormously because the exclusive characteristic can give the unification of bottom and tau Yukawa couplings and the realization of the top to the bottom mass ratio in GUTs \cite{14}. It has been found that if the SUSY soft breaking terms carry the explicit CP violating phases, due to the enhancement of large $A_{t(b)}$ and $\mu$, radiative effects can induce a sizable mixing between scalar and pseudoscalar such that the lightest Higgs boson could be $60 - 70$ GeV and thus escape the detection of detectors \cite{15}. Moreover, with the requirements satisfied with electroweak baryogenesis, a novel prediction on the muon electric dipole moment (EDM) of $10^{-24}$ e cm can be reached by the proposed experiment \cite{16}. In sum, it will be more exciting that if the light sbottom and gluino can be compatible with the Higgs physics with or without CP violation (CPV).

In order to further pursue the implications of the light sbottom and gluino, in this paper, we concentrate on two flavor-conserving processes: one is anomalous magnetic moment of muon, $\Delta a_{\mu}$, and the other is EDMs of muon and neutron. The former corresponds to CP conservation (CPC) while the latter is related to CPV. Since the results must be proportional to the mixing of $\tilde{b}_L$ and $\tilde{b}_R$, for generality, we describe the relationship between weak and physical eigenstates as

$$
\begin{pmatrix}
\tilde{b}_L \\
\tilde{b}_R
\end{pmatrix} = \begin{pmatrix}
1 & 0 \\
0 & e^{i\delta_b}
\end{pmatrix}
\begin{pmatrix}
\cos \theta_b & \sin \theta_b \\
-\sin \theta_b & \cos \theta_b
\end{pmatrix}
\begin{pmatrix}
\tilde{b}_1 \\
\tilde{b}_2
\end{pmatrix},
$$

(1)

where $\delta_b$ is the CP violating phase which could arise from the off-diagonal mass matrix element $m_b(A_b^* - \mu \tan \beta)$. To suppress the coupling $Z\tilde{b}_1\tilde{b}_1$, we set $\sin 2\theta_b = 0.76$ in our discussions. In the following analyses, we separate the problem into CPC and CPV cases.

1. CPC ($\delta_b = 0$):

Unlike non-SUSY two-Higgs doublet models, it is well known that besides the enhancement of large $\tan \beta$ or $1/\cos \beta$, there exists another enhanced factor $A_b$ in the couplings of $S = h$ and $H$ to bottom squarks \cite{11}. As described early, some novel consequences based
on both large factors have been displayed on the Higgs hunting. With these factors, we examine the implication on $\Delta a_\mu$ by considering the case of a light sbottom. In addition, for convenience, we adopt the decoupling limit with $m_{A^0} >> m_Z$ so that $\tan 2\alpha \approx \tan 2\beta$.

\[ i\Gamma_{\gamma\mu} = iA\gamma(q^2)[(q \cdot k)g_{\mu\nu} - q_{\mu}k_{\nu}], \]
\[ A\gamma(q^2) = N_c \alpha_{em}Q_b^2m_bA_b \sin \alpha \sin 2\theta_b \]
\[ \times \sum_{i=1,2} (-1)^{i+1} \int_0^1 \frac{dx x(1-x)}{M_{b_i}^2 - q^2x(1-x)} , \]

(2)

where $N_c = 3$ is the color number, $\alpha_{em}$ is the fine structure constant, $Q_b = -1/3$ is the charge of sbottom, and $m_b(M_{b_i})$ is the mass of bottom quark (squarks). Because we concentrate on the light sbottom case, we do not discuss the contributions of stops by setting their masses being heavy. Since $h$ couples to different sbottoms $\tilde{b}_L$ and $\tilde{b}_R$ but $\gamma$ couples to the same sbottoms, it is clearly inevitable to introduce the mixing angle $\theta_b$. In Eq. (2), we already neglect the smaller contribution related to $\mu \cos \alpha / \cos \beta$. By using the result of Eq. (2), via the calculation of the loop in Fig. (b), the anomalous magnetic moment of muon from two-loop Higgs-sbottom-sbottom diagrams is given by

\[ \Delta a_\mu = \frac{N_c \alpha_{em}Q_b^2 m_b^2 m_{A_b} \sin^2 \alpha}{16\pi^3} \frac{\sin 2\theta_b}{v^2 M_h^2 \cos^2 \beta} \]
\[ \times \sum_{i=1,2} (-1)^{i+1} F \left( \frac{M_{b_i}^2}{M_h^2} \right) , \]
\[ F(z) = \int_0^1 \frac{dx x(1-x)}{z - x(1-x)} \ln \frac{x(1-x)}{z} . \]

(3)
Immediately, we see that although this is a two-loop effect and there appears one suppressed factor $m_b/v \cdot (m_u/M_h)^2$, the $\Delta a_\mu$ could be enhanced in terms of $(A_b/v) \cdot \tan^2 \beta$. Due to $M_h \gg M_{\tilde{b}_1}$, one finds that $F(M_{\tilde{b}_1}^2/M_h^2) \approx 2(1 + \ln M_{\tilde{b}_1}/M_h)$. If we take $M_{\tilde{b}_2} = 180$ GeV and assume the lightest Higgs boson $100 < M_h < 140$ GeV, one gets that $F(M_{\tilde{b}_2}^2/M_h^2) \approx -0.25 \sim -0.16$. Hence, it is certain that $F(M_{\tilde{b}_1}^2/M_h^2) \gg F(M_{\tilde{b}_2}^2/M_h^2)$.

To understand the influence of free parameters $M_{\tilde{b}_1(h)}$, $\tan \beta$ and $A_b$, we present the $\Delta a_\mu$ as a function of $M_{\tilde{b}_1}$ for $A_b = 0.5$ and $1$ TeV with $\tan \beta = m_t/m_b$ [tan $\beta = 50$] in Fig. 2(a) and (b) [(c) and (d)], where the solid, dashed and dashed-dotted lines denote the results of $M_h = 100$, $120$ and $140$ GeV, respectively. Unlike non-SUSY models, which require the neutral Higgs to be a few GeV to fit $\Delta a_\mu$ [11], the lightest Higgs boson in SUSY models could be the values constrained by the current experiments. Interestingly, due to the light sbottom, a significant contribution to anomalous magnetic moment of muon without extremely tuning $A_b$ is shown up [17].

![Graphs showing $\Delta a_\mu$ as a function of $M_{\tilde{b}_1}$ for different values of $M_h$ and $A_b$.](image)

FIG. 2: $\Delta a_\mu$ (in units of $10^{-9}$) for (a) [(c)] $A_b = 0.5$ TeV and (b) [(d)] $A_b = 1$ TeV with $\tan \beta = m_t/m_b$ [tan $\beta = 50$], where the solid, dashed and dashed-dotted lines denote the results of $M_h = 100$, $120$ and $140$ GeV, respectively.

2. CPV($\delta_b \neq 0$):

CP problem has been investigated thoroughly in the kaon system since it was discovered in 1964 [18]. Now, CPV has been confirmed by Belle [19] and Babar [20] with high accuracy in the $B$ system. Although the mechanism of Kobayashi-Maskawa (KM) [21] phase in the SM is consistent with the CP measurements, the requirement of the Higgs mass of 60 GeV for the condition of the matter-antimatter asymmetry has been excluded by LEP. One of
candidates to deal with the baryogenesis is to use the SUSY theory. As known, any CP violating models will face the serious low energy constraints from EDMs of lepton and neutron, which are $T$ and $P$ violating observables and at elementary particle level usually are defined by $d_f \bar{f} \sigma_{\mu\nu} \gamma_5 f F^{\mu\nu}$, with $F^{\mu\nu}$ being the electromagnetic (EM) field tensor. We note that not only the EM field but also the chromoelectric dipole moment (CEDM) of gluon for colored fermions will contribute. Therefore, we have to examine the implication of the light sbottom on EDMs while $\delta_b$ is nonzero, i.e., $A_b$ and $\mu$ are complex.

Inspired by the previous mechanism for $\Delta a_\mu$, the similar effects with pseudoscalar $A^0$ instead of scalar $h$, shown in Fig. 1(a), will also contribute the EDMs of leptons and quarks. Since the effects have been analyzed by Refs. [16, 22], we directly summarize the formalisms for the fermion EDM and CEDM as

$$
\frac{d^f}{e} = \frac{N_c Q_b^2 \alpha_{em}}{64\pi^3} \frac{R_f Q_f m_\ell}{M_A^2} \xi_b \sum_{i=1,2} (-1)^{i+1} F \left( \frac{M_{b_i}^2}{M_A^2} \right),
$$

$$
\frac{d^C}{g_s} = \frac{1}{2Q_b^2 Q_f N_c \alpha_{em}} \frac{\alpha_s m_q}{m_\ell} \left( \frac{d^f}{e} \right),
$$

respectively, with $R_f = \tan \beta$ (cot $\beta$) for $T_3^f = -1/2$ (1/2) and $\xi_b = \sin 2\theta_b m_b \text{Im}(A_b e^{i\delta_b})/(v^2 \sin \beta \cos \beta)$. For simplicity, we set the effects of stops to be negligible. According to Eq. (4), we see that the lepton EDM of $d^f_\ell$ is proportion to the $m_\ell$. Due to the ratio $m_\mu/m_e \approx 205$, the strictest limit comes from the EDM of electron, with the current upper bound being $1.6 \times 10^{-27} \text{ e cm}$ [23]. Although the renormalization factor $(g_s(M_W)/g_s(\Lambda))^{32/23}$ for the EDM of neutron is around one order larger than $(g_s(m_W)/g_s(\Lambda))^{74/23}$ for the CEDM contribution, due to $\alpha_s$ and $1/(Q_b^2 Q_\ell)$ enhancements, the CEDM of quark is much larger than the EDM of quark. Hence, only the effects of the CEDM on neutron are considered. Since the unknown parameters for the EDM of electron and CEDM of neutron are the same, the values of parameters are taken to satisfy with the bound of the electron EDM. We present the results as a function of $M_{b_1}$ with $\tan \beta = 10$ (20) and $A_b = 500$ (200) GeV in Fig. 3 in which the solid, dashed and dashed-dotted lines correspond to $M_b = 100$, 120 and 140 GeV, respectively. We note that the origin of CP violation comes from $\text{Im}(A_b e^{i\delta_b})/|A_b|$ and it is set to be $O(10^{-1})$. From the figure, we see clearly that without fine tuning the CP phase to be tiny, the EDM of electron can be lower than the experimental bound and the CEDM of neutron is also not too far away from current limit. The effects become testable in experiments.
It seems that so far the scenario of the light sbottom with the mass of few GeV could give interesting results in $\Delta a_\mu$, $d_e^N$ and $d_N^C$. However, what we question is whether both light sbottom and gluino can coexist when the CP phases are involved. We find that the possibility encounters fierce resistance of the Weinberg’s 3-gluon operator, defined by

$$O = -\frac{1}{6}d^G f_{\alpha\beta\gamma}G_{\alpha\mu\rho}G_{\beta\nu}^\rho G_{\gamma\lambda\sigma}\varepsilon^{\mu\nu\lambda\sigma}$$

with $G_{\alpha\mu\rho}$ and $f_{\alpha\beta\gamma}$ being the gluon field-strength tensor and antisymmetric Gell-Mann coefficient, respectively. With the bottom, sbottom and gluino being the internal particles of the two-loop mechanism, the contribution of the Weinberg’s operator is given by

$$d^G = -\frac{3\alpha_s m_b}{2} \left( \frac{g_s}{4\pi} \right)^3 \sin 2\theta_b \frac{z_1 - z_2}{M_3^g} H(z_1, z_2, z_b) \sin \delta_b$$

where function $H$ is a two-loop integration, $z_i = M_{b_i}^2/M_3^g$ and $z_b = M_b^2/M_3^g$. Here, we have rotated away the phase of gluino. With naive dimensional analysis, the neutron EDM could be estimated by

$$d_N^G = (\epsilon_{\mu X}/4\pi)\eta_G d^G$$

with $\mu_X \sim 1.19$ GeV and $\eta_G \sim 3.4$ being the chiral symmetry breaking scale and renormalization factor of the Weinberg’s operator, respectively. Although the estimation still has a large theoretical uncertainty, our concern is on the problem of order of magnitude. Due to the result being proportional to $1/M_3^g$, we see that the lighter $M_3^g$ is, the larger $d^G$. It is worth mentioning that the similar tendency can be
also found in Ref. [26], in which the considered situation is via the gluino–stop–top-quark two-loop. For an illustration, we present the results in Table I with some given values for $M_{\tilde{b}_1}$ and $M_{\tilde{g}}$ and fixing $M_{\tilde{b}_2} = 180$ GeV and tan $\beta = 10$. From the table, we conclude that

| $(M_{\tilde{b}_1}, M_{\tilde{g}})$ | $(5, 16)$ | $(5, 1000)$ | $(100, 16)$ | $(100, 1000)$ |
|-----------------------------|-----------|-------------|-------------|----------------|
| $d_N^G / \sin \delta_b$    | $2.2 \cdot 10^4$ | $8.2$ | $1.7 \cdot 10^3$ | $8.0$ |

unless the CP violating phase $\delta_b$ is tuned to be of $O(10^{-4} - 10^{-3})$, the scenario of light sbottom and gluino with CPV will encounter the problem of naturalness.

In summary, we have extended the scenario of light sbottom to the flavor-conserved low energy physics and shown that by choosing proper values of tan $\beta$, $A_b$ and $M_h(A)$, the predictions of $\Delta a_\mu$ and EDMs of electron and neutron could satisfy with experiments. We have also found that except extreme fine tuning, the light sbottom and gluino cannot coexist after the neutron electric dipole moment constraint on the Weinberg’s 3-gluon operator has been taken into account. We note that it is possible that there exist some cancellations while we consider all possible contributions to the neutron EDM [27]. Nevertheless, it will depend on how to fine tuning the involved parameters and that is beyond our scope of the current paper.

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