Reply to comment “On the test of the modified BCS at finite temperature”

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The applicability, predictive power, and internal consistency of a modified BCS (MBCS) model suggested by Dang and Arima have been analyzed in details in $^1$. It has been concluded that “The T-range of the MBCS applicability can be determined as far below the critical temperature $T_c$, i.e., $T \ll T_c$. Unfortunately, the source of our conclusions has been misrepresented in $^2$ and referred to MBCS predictions at $T >> T_c$.

Since above $T_c$ particles and holes contribute to an MBCS gap with opposite signs, the model results are rather sensitive to details of a single particle spectra (s.p.s.) (e.g., discussion in Sec. IV. A. 1. of $^3$). As so, it is indeed possible to find conditions when the MBCS simulates reasonable thermal behavior of a pairing gap. This can be achieved, e.g., by introducing some particular $T$-dependence of the s.p.s. (entry 1 in $^2$) or by adding an extra level to a picket fence model (PFM) (entry 2 in $^2$). But such results are very unstable and accordingly, the model has no predictive power.

Dang and Arima explain poor MBCS results for the PFM ($N = \Omega = 10$) discussed in $^1$ by referring to strong asymmetry in the line shape of the quasiparticle-number fluctuations $\delta N_j$ above $T \sim 1.75$ MeV (symmetry of $\delta N_j$ is announced as a criterion of the MBCS applicability.) The space limitation is blamed for that in $^2$. Remember, particle-hole symmetry is an essential feature of the PFM with $N = \Omega$. Thus, strong asymmetry is reported from the MBCS calculation in an ideally symmetric system.

It has been found that a less symmetrical example $N = 10$, $\Omega = 11$ satisfies better the MBCS criterion $^2$. Indeed, the model mimics behavior of a macroscopic thermodynamical and statistical mechanical definition of entropy, respectively. The calculations have been performed for neutron system of $^{120}$Sn with a realistic s.p.s.

Another example of the MBCS thermodynamical inconsistency is shown below. We calculate the system entropy $S$ as

\[ S_1 = \int_0^T \frac{1}{t} \cdot \partial \mathcal{E} / \partial t \, dt \]

and

\[ S_2 = - \sum_j (2j + 1) \cdot [n_j \ln n_j + (1 - n_j) \ln (1 - n_j)] \]

where $n_j$ are thermal quasiparticle occupation numbers. In Fig. 2 we compare $S_1$ and $S_2$ quantities which refer to thermodynamical and statistical mechanical definition of entropy, respectively. The calculations have been performed for neutron system of $^{120}$Sn with a realistic s.p.s.

It is not possible to distinguish by eye $S_1$ and $S_2$ in the FT-BCS calculation (solid curve in Fig. 2) represents both...
FIG. 2: Entropy of the neutron system in $^{120}$Sn calculated within the FT-BCS (solid curve) and MBCS (dashed and dot-dashed curves). Notice the logarithmic $y$ scale of the main figure and linear $y$ scale of the insert. See text for details.

We stress that low $T$ part is presented in Fig. 2. Dramatic disagreement between $S_1$(MBCS) and $S_2$(MBCS) representing the system entropy remains at higher $T$ as well but we do not find it necessary to extend the plot: the model obviously does not describe correctly a heated system even at $T \sim 200$ keV.

We show in the insert of Fig. 2 another MBCS prediction: entropy $S_1$ decreases as temperature increases. This result is very stable against variation of the pairing strength $G$ within a wide range and contradicts the second law of thermodynamics.

Finally, we repeat, the conclusion in [1] that “The $T$-range of the MBCS applicability can be determined as far below the critical temperature $T_c$” is based on the analysis of the model predictions from $T << T_c$ and not on $T >> T_c$ results as presented in [2].

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[1] V.Yu. Ponomarev and A.I. Vdovin Phys. Rev. C 72, 034309 (2005).
[2] N. Dinh Dang and A. Arima, arXiv: nucl-th/0510004
[3] N. Dinh Dang and A. Arima, Phys. Rev. C 68, 014318 (2003).
[4] Dang and Arima put in doubts the validity of the PFM calculations in [1] claiming that “The limitation of the configuration space with $\Omega = 10$ causes a decrease of the heat capacity $C$ at $T_M > 1.2$ MeV . . . Therefore, the region of $T > 1.2$ MeV, generally speaking, is thermodynamically unphysical.” [2]. It is well-known that such a behavior of the heat capacity is a characteristic feature of finite systems of bound fermions and “does not concern the validity of statistical mechanics” [O. Civitarese, G.G. Dussel, and A.P. Zuker, Phys. Rev. C 40, 2900 (1989)].