Conditional phase gate and quantum state transfer via off-resonant quantum Zeno dynamics

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Abstract

We propose a scheme to realize the conditional phase gate (CPG) and quantum state transfer (QST) between two qubits (acted by nitrogen-vacancy (NV) centers) based on off-resonant quantum Zeno dynamics. We also consider the entanglement dynamics of two qubits in this system. Since no cavity photons or excited levels of the NV center is populated during the whole process, the scheme is immune to the decay of cavity and spontaneous emission of the NV center. The strictly numerical simulation shows that the fidelities of QST and CPG are high even in the presence of realistic imperfections.

Keywords: phase gate, quantum Zeno effect, N-V center
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1. Introduction

Reliable quantum state transfer (QST) and quantum phase gates between distant qubits have become a holy grail of quantum physics research, owing to its potential application in a scalable quantum information processing (QIP) [1]. So far, many proposals have been presented for QIP in different quantum systems [2-9]. Among these systems, the composite systems of nitrogen-vacancy (NV) centers embedded in nanocavities are considered to be a fresh tool for a room-temperature solid-state QIP [10]. The NV center in diamond has a long electron spin coherence time and can be manipulated by the optical or microwave pulses [11]. Since the ability to address individual NV center was proved, a lot of theoretical and experimental efforts have been

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devoted to QIP based on the composite microcavity-NV center systems \cite{12-15}. The significant advances in implementing various protocols for QIP can lead to long-distance quantum communication or the creation of a quantum computer in the future. Especially, Li et al \cite{16} have proposed a scheme for the realization of QST and entanglement with NV centers coupled to a high-Q whispering gallery modes (WGM) microresonator, which is processed via Raman transitions. Based on the weak coupling limit for pure state transport, Ajoy and Cappellaro \cite{17} have proposed a scheme for realizing perfect quantum transport in two separated NV centers.

The quantum Zeno effect \cite{18-20} occurs when a slowly evolving quantum system undergoes a rapid sequence measurement. In 2002, Facchi and Pascazio found the quantum Zeno dynamics \cite{21}, which took continuous couplings as a substitute for frequent measurements and hindered the evolution of the quantum system. Until now, the new finding has enlightened numerous schemes \cite{22, 24} to implement quantum gates, prepare quantum entangled states and transfer quantum information. For example, Shi et al \cite{25} have shown how to realize quantum information transfer for one atom to another in coupled-cavity system via quantum Zeno dynamics. Based on the quantum Zeno dynamics, these protocols above are robust against cavity decay. However, atomic spontaneous emission may affect these systems.

For the purpose of preventing the decoherence including the atomic spontaneous emission and cavity decay, off-resonant quantum Zeno dynamics should be used to remedy the defect in the resonant case \cite{26}. For instance, Zhang et al \cite{27} proposed a scheme to realize a $\sqrt{\text{swap}}$ gate via off-resonant quantum Zeno dynamics. Inspired by their works above, we will propose a scheme to implement QST and CPG between NV centers via the off-resonant quantum Zeno dynamics. Moreover, the entanglement dynamics of two qubit off-resonantly coupled to a cavity with weak driving classical fields will also be considered in the present paper. The present approach has the following merits: (i) The cavity field would not be excited in the whole process, and the interaction is a virtual-photon process. In other words, our model works well in the bad-cavity limit, which makes it more applicable to current laboratory techniques. (ii) The idea, which combines the advantages of the quantum Zeno dynamics and the large detuning between NV centers and cavity, will make the protocols also robust against NV center spontaneous decay.
2. model and effective dynamics

We begin by considering a system composed of two separated NV centers simultaneously interacting with a microcavity (e.g., microtoroidal resonator). The microcavity with high quality factor, serves as a quantum bus, as shown in Fig.1. The states of the NV center form a $\wedge$ configuration with the ground states $|m_s = +1\rangle = |g\rangle$, $|m_s = -1\rangle = |f\rangle$ and the excited state $|A_2\rangle = (|E_-\rangle + |E_+\rangle)/\sqrt{2} = |e\rangle$, where $|E_\pm\rangle$ are orbital states with angular momentum projection $\pm 1$ along the N-V axis. Another ground state $|m_s = 0\rangle = |i\rangle$ is an ancillary state. The transition $|f\rangle \rightarrow |e\rangle$ is driven by a microwave field with Rabi frequency $\Omega$ and detuning $\Delta_d$. While, the transition $|g\rangle \rightarrow |e\rangle$ couples to the cavity field with coupling strength $g_i$ (where $i = 1, 2$), and the corresponding detuning is $\Delta_c$. For convenience, we assume $g_1 = g_2 = g$ and $\Delta_d = \Delta_c = \Delta$ to be real. In the frame rotating with the cavity frequency $\omega_c$, the Hamiltonian of the combined system is given by ($\hbar = 1$),

$$H = H_c + H_l + H_{de},$$

$$H_c = \sum_{i=1,2} g_i (|e\rangle_i \langle g| + |g\rangle_i \langle e| a^+),$$

$$H_l = \sum_{i=1,2} \Omega_i (|e\rangle_i \langle f| + |f\rangle_i \langle e|),$$

$$H_{de} = \Delta (|e\rangle_1 \langle e| + |e\rangle_2 \langle e|),$$

where $a^+$ and $a$ are the creation and annihilation operators for the cavity mode, respectively. $H_c$ stands for the NV center-cavity interaction, and $H_l$ represents the interaction between the NV centers and the classical field.

If the initial state of the system is $|gf\rangle|0\rangle_c$, it will evolve in a close subspace spanned by $\{|gf\rangle|0\rangle_c, |ge\rangle|0\rangle_c, |gg\rangle|1\rangle_c, |eg\rangle|0\rangle_c, |fg\rangle|0\rangle_c\}$. Therefore the above Hamiltonian can be rewritten with eigenstates of the $H_c$ representation:

$$H = H_c + H_l + H_{de},$$

$$H_c = -\sqrt{2} g (|\psi_2\rangle \langle \psi_2| - |\psi_3\rangle \langle \psi_3|),$$

$$H_l = (1/\sqrt{2})(\Omega_2 |gf\rangle|0\rangle_c - \Omega_1 |fg\rangle|0\rangle_c) (|\psi_1\rangle),$$

$$+ (1/2)(\Omega_2 |gf\rangle|0\rangle_c + \Omega_1 |fg\rangle|0\rangle_c) (|\psi_2\rangle + |\psi_3\rangle) H.c.,$$

$$H_{de} = \Delta |\psi_1\rangle \langle \psi_1| + (\Delta/2) (|\psi_2\rangle + |\psi_3\rangle) (|\psi_2\rangle + |\psi_3\rangle).$$
where \( \{|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle \} \) are the eigenvectors of \( H_e \):

\[
|\psi_1\rangle = \frac{1}{\sqrt{2}}(|ge\rangle|0\rangle_c - |eg\rangle|0\rangle_c), \\
|\psi_2\rangle = \frac{1}{2}(|ge\rangle|0\rangle_c - \sqrt{2}|gg\rangle|1\rangle_c + |eg\rangle|0\rangle_c), \\
|\psi_3\rangle = \frac{1}{2}(|ge\rangle|0\rangle_c + \sqrt{2}|gg\rangle|1\rangle_c + |eg\rangle|0\rangle_c),
\]

and the corresponding eigenvalues are \( \lambda_1 = 0 \), \( \lambda_2 = -\sqrt{2}g \) and \( \lambda_3 = \sqrt{2}g \).

Next, we assume that \( U_c = e^{-iH_c t} \) is the unitary time evolution operator with respect to the Hamiltonian \( H_c \). Assuming \( \Omega_1 = \Omega_2 = \Omega \), after a calculation in the intermediate "picture", we obtain

\[
H_{eff}^I = U_c^\dagger H_I U_c = (\Omega/\sqrt{2})(|gf\rangle|0\rangle_c - |fg\rangle|0\rangle_c)\langle\psi_1| \\
+ (\Omega/2)(|gf\rangle|0\rangle_c + |fg\rangle|0\rangle_c)(\langle\psi_2|e^{i\sqrt{2}gt} + \langle\psi_3|e^{-i\sqrt{2}gt}) + H.c.,
\]

\[
H_{de}^I = U_c^\dagger H_{de} U_c = \Delta|\psi_1\rangle\langle\psi_1| + (\Delta/2)(|\psi_2\rangle\langle\psi_2| + |\psi_3\rangle\langle\psi_3|) \\
+ (\Delta/2)(|\psi_3\rangle e^{-i\sqrt{2}gt} + |\psi_3\rangle e^{i\sqrt{2}gt}).
\]

On the condition that \( \Omega << g \) and \( \Delta << 2\sqrt{2}g \), the terms in \( H_{eff}^I \) and \( H_{de}^I \) with high oscillating frequency \( \sqrt{2}g \), \( 2\sqrt{2}g \) can be safely discarded. Since \( |\psi_2\rangle \) and \( |\psi_3\rangle \) are decoupled from the initial state \( |gf\rangle|0\rangle_c \), the terms describing the interaction associated with these states can be omitted. So the effective Hamiltonian of the system can be given

\[
H_{eff} = (\Omega/\sqrt{2})(|gf\rangle|0\rangle_c - |fg\rangle|0\rangle_c)\langle\psi_1| \\
+ |\psi_1\rangle((|0\rangle\langle fg| - (0\langle gf|)) + \Delta|\psi_1\rangle\langle\psi_1|.
\]

If we consider \( \langle|g\rangle|f\rangle - |f\rangle|g\rangle \) as a stable level \( |G\rangle \), the above equation can be considered as an effective Hamiltonian of the two-level system interacting with a vacuum cavity field. As a result, the stable ground level \( |G\rangle \) is coupled to the excited level \( |\psi_1\rangle \) with a coupling constant \( \Omega \) and a detuning \( \Delta \). On the large detuning condition that \( |\Delta| \gg \Omega \), there is no transition between \( |G\rangle \) and \( |\psi_1\rangle \), the coupling only induces a Stark shift for each level. By adiabatically eliminating the excited state \( |\psi_1\rangle \), the final effective Hamiltonian governing the evolution of states \( |g\rangle|f\rangle|0\rangle_c \) and \( |f\rangle|g\rangle|0\rangle_c \) cannot be rewritten as

\[
H_{eff} = (\Omega^2/(2\Delta))(|gf\rangle\langle fg| + |fg\rangle\langle gf|)
\]
In order to validate the feasibility of the above physical model, we perform a direct numerical simulation of the Schrödinger equation with the full Hamiltonian in Eq. (1) and the effective Hamiltonian in Eq. (6). To satisfy the quantum Zeno dynamics $\Omega \ll g$ and large detuning $|\Delta| >> \Omega$, we set the parameters $\Omega = 0.05g$ and $|\Delta| = 0.5g$. We plot the time-dependent populations of the basic states $|gf\rangle|0\rangle_c$ (P1) and $|fg\rangle|0\rangle_c$ (P2) governed by the full Hamiltonian in Eq. (1) (green lines in Fig. 2) and the effective Hamiltonian in Eq. (6) (red lines in Fig. 2). It is shown that the population of the basic states governed by the effective Hamiltonian exhibits excellent agreement with that governed by the full Hamiltonian when the conditions are satisfied. Eventually, the simulation result of the full Hamiltonian is almost the same as that of the effective Hamiltonian when $\Omega = 0.01g$ and $|\Delta| = 0.2g$. It is appropriate that the above approximation for the Hamiltonian is reliable as long as $\Delta/\Omega$ is large enough. While the deviations decrease at the cost of the long evolution time. The larger the scaled ratio $\Delta/\Omega$ is, the longer the evolution time is. Considering the decoherence, we choose $\Omega = 0.05g$ and $|\Delta| = 0.5g$ to satisfy the requirement in the following.

3. Quantum State Transfer

We note that the above model can be used to realize QST. Two NV centers (1 and 2) are coupled to a microcavity. We assume the NV center 1 is in an arbitrary unknown state $\alpha |g\rangle_1 + \beta |f\rangle_1$, where $|\alpha|^2 + |\beta|^2 = 1$, while the NV center 2 is prepared in the state $|g\rangle_2$ and the cavity mode is initially in vacuum state $|0\rangle_c$. The initial state of the whole system can be described as $\Psi(0) = (\alpha |g\rangle_1 + \beta |f\rangle_1 |g\rangle_2 |0\rangle_c$. For the initial state $|g\rangle_1 |g\rangle_2 |0\rangle_c$, it is easily checked that the evolution is frozen, since $H |g\rangle_1 |g\rangle_2 |0\rangle_c = 0$. Governed by the effective Hamiltonian in Eq. (6), and for an interaction time $t$, the final state of the system becomes

$$\Psi(t) = \alpha |g\rangle_1 |g\rangle_2 |0\rangle_c + (\beta/2) [(1 - e^{-i(\Omega^2/\Delta)t}) |g\rangle_1 |f\rangle_2 |0\rangle_c$$

$$+ (1 + e^{-i(\Omega^2/\Delta)t}) |f\rangle_1 |g\rangle_2 |0\rangle_c].$$

By selecting the interaction time $t'$ to satisfy $(\Omega^2/\Delta)t' = \pi$, one will obtain
\[ |\Psi(t')\rangle = \alpha |g\rangle_1 |g\rangle_2 |0\rangle_c + \beta |g\rangle_1 |f\rangle_2 |0\rangle_c. \] (1)

The quantum state transfer from NV center 1 to NV center 2 has been realized.

To characterize this QST process, we utilize the fidelity which is given by

\[ F = \langle \Psi(t)|\rho|\Psi(t)\rangle, \] (2)

where \( |\Psi(t)\rangle \) is the final state described by the Schrödinger equation \( i\frac{d|\Psi(t)\rangle}{dt} = H |\Psi(t)\rangle \). Here \( H \) is the full Hamiltonian governed by Eq.(1). The large \( \Delta \) can ensure the whole system evolution governed by the effective Hamiltonian in Eq.(6). However, the large detuning prolongs the evolution time which will lead to the worse impacts of decoherence. Fig.3 plots the influences of the fluctuation of the detuning ratio \( \Delta/g \) and the Rabi frequency ratio \( \Omega/g \) on the fidelity of QST. In this scheme, we choose the median values \( \Delta/g = 0.5 \) and \( \Omega/g = 0.05 \), the corresponding fidelity of QST is larger than 0.998. The fluctuation of the detuning ratio \( \Delta/g \) and the Rabi frequency ratio \( \Omega/g \) almost does not affect the optimal fidelity of QST, which is always larger than 0.982.

In a realistic experiment, the spontaneous emission of the NV centers and cavity losses on the QST should be taken into account. In the following part, we present numerical simulation to show how the dissipation sources take effects. The master equation of the whole system can be expressed by

\[
\dot{\rho} = -i[H, \rho] + \frac{\kappa}{2} \sum_{j=1}^{2} \left(2a_j \rho a_j^+ - a_j^+ a_j \rho - \rho a_j^+ a_j\right) \\
+ \frac{\gamma}{2} \sum_{j=1}^{2} \sum_{i=f,g} \left(2\sigma_{ie}^j \rho \sigma_{ei}^j - \sigma_{ei}^j \sigma_{ie}^j \rho - \rho \sigma_{ei}^j \sigma_{ie}^j\right),
\]

where \( \kappa \) denotes the effective decay rate of the cavity. For simplicity, we assume \( \gamma_{ef} = \gamma_{eg} = \gamma \), and \( \gamma \) represents the branching ratio of the spontaneous decay from level \( |e\rangle \) to \( |f\rangle \) or \( |g\rangle \). By solving the master equation numerically, we obtain the relation of the fidelity versus the scaled ratio \( \gamma/g \) and \( \kappa/g \) in Fig.4 in the case of \( \Omega = 0.05g \) and \( |\Delta| = 0.5g \). Fig.4 shows that the fidelity of QST will decrease slowly with the increasing of cavity
decay and atomic spontaneous emission. The fidelity of QST is insensitive to cavity decay and atomic spontaneous emission since it is still about 91% even for $\gamma/g = 0.01$ and $\kappa/g = 0.2$. The physical principle behind this phenomenon is that once the quantum Zeno condition is satisfied, if the cavity is initially in the vacuum state, no photons are included in the intermediate state $|\psi_1\rangle$, i.e., the cavity field has not been excited. So the cavity decay terms in Eq.(10) have a little influence on the evolution of the encoded qubit states. What’s more, the further large detuning condition excludes the excited states of NV centres, so this process is also robust against NV center’s spontaneous emission.

4. **Conditional phase gate**

A new type of quantum conditional phase gate (CPG) has been proposed by Zheng depends neither on dynamical phase shift nor a solid angle along a suitable loop [28]. Induced by the adiabatic evolution along dark eigenstates, such type of non-geometric CPG is robust against moderate fluctuations of experimental parameters. Recently, Lacour et al [29] gave a scheme to realize arbitrary state controlled-unitary gate based on fractional stimulated Raman adiabatic passage. Schemes have also been proposed for reliable realization of CPG for two atoms [30] or NV centers [31] separately trapped in two distant cavities via adiabatic passage.

Now, we will show how to realize a non-geometric CPG via off-resonant quantum Zeno dynamics. The quantum information is encoded in $|f\rangle_1 |i\rangle_2 |0\rangle_c$ for the NV center 1, while in $|g\rangle_1 |i\rangle_2 |0\rangle_c$ for the NV center 2. The cavity field is initially in vacuum state. For the initial state $|f\rangle_1 |i\rangle_2 |0\rangle_c$, $|i\rangle_1 |g\rangle_2 |0\rangle_c$ and $|i\rangle_1 |i\rangle_2 |0\rangle_c$, it is easily checked that the evolution is frozen. If the initial state of the system is $|f\rangle_1 |g\rangle_2 |0\rangle_c$, considering the phase of the classical fields, it will evolve in according to Eq.(6),

$$
|f\rangle_1 |g\rangle_2 |0\rangle_c \rightarrow \frac{e^{i(\phi_1-\phi_2)/2}}{2} [(1 - e^{-i(\Omega^2/\Delta)t}) |g\rangle_1 |f\rangle_2 |0\rangle_c + (1 + e^{-i(\Omega^2/\Delta)t}) |f\rangle_1 |g\rangle_2 |0\rangle_c].
$$

In step 1, the phases of the laser pulses (1 and 2) are set to be equal and the interaction time satisfies $(\Omega^2/\Delta)t_1 = \pi$. In step 2, the phase of the laser 2 takes a circularly polarized $\pi$ rotation, while the phase of the laser 1 remains
unchanged. After an interaction time \( t_2 = t_1 \), the state of the system will evolve to

\[
|f\rangle_1 |g\rangle_2 |0\rangle_c \rightarrow |g\rangle_1 |f\rangle_2 |0\rangle_c \rightarrow e^{i\pi} |f\rangle_1 |g\rangle_2 |0\rangle_c. \tag{3}
\]

Thus, after a total period \( T = 2\pi\Delta/\Omega^2 \), \( |f\rangle_1 |g\rangle_2 |0\rangle_c \) returns to the initial state with an additional phase shift \( \pi \). Then we obtain

\[
\begin{align*}
|fg\rangle |0\rangle & \rightarrow -|fg\rangle |0\rangle, \\
|fi\rangle |0\rangle & \rightarrow |fi\rangle |0\rangle, \\
|ig\rangle |0\rangle & \rightarrow |ig\rangle |0\rangle, \\
|ii\rangle |0\rangle & \rightarrow |ii\rangle |0\rangle,
\end{align*}
\tag{4}
\]

which corresponds to two-qubit conditional phase gate. Taking advantage of the quantum Zeno dynamics, the cavity mode keeps in vacuum state during the whole process, so the decay of the cavity is largely suppressed.

The phase gate is exercised by modifying the phases of the laser pulses, so the fluctuation of the shift time of pulses influences the fidelity of the phase gate, as is shown in Fig.5. The result shows the optimal fidelity of conditional phase gate is almost unaffected even when the shift time \( \delta t/t \leq \pm 0.1 \). The CPG is free of the laser field strength and insensitive to the fluctuation of the shift time, which will reduce the difficulty in the experiment.

We numerically simulate the full Hamiltonian model in Eq.(1) to show how the dissipation sources take effects, as is shown in Fig.6. The fidelity of CPG is insensitive to cavity decay and atomic spontaneous emission since it is still about 94.5\% even for \( \gamma/g = 0.01 \) and \( \kappa/g = 0.2 \).

5. Entanglement Dynamics of this System

Sabrina et al \[32\] have proposed a strategy to fight against the deterioration of the entanglement using the quantum Zeno effect in the resonant case. They also have tested in the off-resonant regime, protecting the entanglement will become more efficient than that in the resonant limit \[26\]. In this paper, We investigate the entanglement dynamics of two qubits using the off-resonant quantum Zeno dynamics. If the initial state of the system is \( |\Psi(0)\rangle = (\alpha|gf\rangle + \beta|fg\rangle)|0\rangle_c \), based on the physical model above, the system will evolve with respect to the effective Hamiltonian in Eq.(6). For choosing an interaction time \( t \) and \( \lambda = \Omega^2/\Delta \), the final state of the system becomes
\[
|\Psi(t)\rangle = e^{-i(\lambda/2)t}\left[(\alpha \cos \frac{\lambda}{2} t + i\beta \sin \frac{\lambda}{2} t)|gf\rangle + (i\alpha \sin \frac{\lambda}{2} t + \beta \cos \frac{\lambda}{2} t)|fg\rangle\right]|0\rangle_c.
\]

In the standard basis, the reduced density matrix, obtained from the density operator $|\Psi(t)\rangle\langle\Psi(t)|$ after tracing over the cavity mode degrees of freedom, takes the form

\[
\rho(t) = \frac{1}{4}
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & a & b & 0 \\
0 & c & d & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\] (5)

where \(a = 2(\alpha^2 + \beta^2) + 2(\alpha^2 - \beta^2) \cos \lambda t, b = 4\alpha\beta + 2i(\alpha^2 - \beta^2) \sin \lambda t, c = 4\alpha\beta - 2i(\alpha^2 - \beta^2) \sin \lambda t,\) and \(d = 2(\alpha^2 + \beta^2) - 2(\alpha^2 - \beta^2) \cos \lambda t.\)

We use the concurrence \(C(t)\) [33], ranging from 0 for separable state to 1 for maximally entangled states, to quantify the amount of entanglement encoded into the two qubit system. The explicit analytic expression of \(C(t)\) can be obtained from the reduced density matrix of Eq.(16). The concurrence takes a form

\[
C(t) = \sqrt{(ac^* + a^*b)(cd^* + b^*d)/8}
\] (6)

Assuming here the weight factor ratio of initial state \(r = \alpha/\beta, r = 1\) is for the maximally entangled states. We can get different \(r\) via changing the Rabi frequencies of classical fields \(\Omega_1/\Omega_2\) in Eq.2. We now look at the entanglement dynamics versus \(r\) and the interaction time \(gt\), as is shown in Fig.7. It is observed that the amplitude of concurrence collapse and revive versus the interaction time. When \(r\) is close to 1, the amplitude of concurrence is larger than 0.9. Particularly, when \(r = 1\), the amplitude of concurrence keeps on 1 and is independent of the interaction time, which is important in QST. The maximum of \(C(t)\) can reach 1 in a specific time, which is immune to the changing of \(r\) and \(gt\). While when \(r \rightarrow 0\) and \(r >> 1\), the minimum of \(C(t)\) becomes slightly. The above results indicate in our model, the QST can reach a high fidelity in a special time, and that is independent of weight factor of initial states, while when \(r \rightarrow 1\), the QST keeps on high fidelity which depends on the initial states. Due to the quantum Zeno dynamics
and the large detuning between NV centers and cavity field, the interaction is a virtual process and the system only evolves in a close quantum Zeno subspace spanned by \(|gf⟩|0⟩_c, |fg⟩|0⟩_c\). In all the processes, the excited state of NV centers and cavity field have not been populated. Only \(|gf⟩|0⟩_c\) and \(|fg⟩|0⟩_c\) have the population, which has been shown in Fig.1. From the discussions above, it is worth stressing that the concurrence in this system crucially depends on the initial state.

To check the influence of the classical field and the detuning, we numerically simulate the concurrence versus \(λ = Ω^2/Δ\) and the interaction time \(gt\), for \(r = 1/3\), as is shown in Fig.8. When \(λ → 0\), the \(C(t)\) is always small \((<0.4)\). In this case, the detuning is too large, there would be no coupling between the NV centers and the cavity mode, the concurrence only depends on the initial states of the system \((r)\) and is immune to the interaction time. In the off-resonant regime, such as \(λ = 0.01g\), the high concurrence can keep on in a long time, i.e., the oscillating period \(T\) is long. In this case, the detuning meets the requirement of our model, so the time evolution is only in the subspace composed of the initial states of the system. In the far off-resonant regime, such as \(λ = 0.05g\), by contrast, the values of \(T\) become short. The shorter \(T\) is, the larger the decay rate of concurrence is. In this case, the system becomes disentangled more easily. From the discussions above, we can conclude using the off-resonant quantum Zeno dynamics to protect the entanglement will become more efficient than that in the resonant case.

6. Experimental Feasibility and Conclusion

Now, we discuss the schematic setup and the theoretical model of proposed scheme may be experimentally realized with quantum optical devices, such as microsphere cavity [34] and superconducting resonator [35]. In order to get identical N-V centers, we adjust the energy levels of different N-V centers by an external magnetic field. The NV centers are put near the equator of a microsphere cavity and interact with the cavity via the evanescent fields. The coupling constant between the NV centers and the cavity ranges from hundreds of MHz to several GHz in the experiment [36]. The Q factor of the microsphere cavity can have a value exceeding \(2 \times 10^6\), leading to a photon leakage rate \(κ = ω/Q \sim 2π \times 120\) MHz [37]. The spontaneous decay rate of the NV center is \(γ \sim 2π \times 15\) MHz [38]. The coupling between N-V centers and a microsphere cavity has been realized with the relevant cavity QED parameters \([g, γ, κ, Ω, ∆]/2π = [1, 0.015, 0.12, 0.05, 0.5]\) GHz, with which the
corresponding fidelity of the QST and CPG can reach 97%. The decoherence time being longer than 600\(\mu s\) at room temperature has been observed for individual NV centers \[39\]. The QST operation time is about 200\(\text{ns}\) with the parameters above, which is shorter than the decoherence time of NV center.

In summary, based on off-resonant quantum Zeno dynamics, we have provided a scheme for implementing quantum state transfer and conditional phase gate between two NV centers strongly coupled to a microcavity. By numerical calculation, we have demonstrated that the present scheme is immune to the excited levels and cavity photons.

7. acknowledge

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Figure 1: (Color online) The schematic setup for implementing QST and CPG with cavity-NV centers system. Configuration of the NV center level structure and relevant transitions. The ground state $|m_s = +1\rangle = |g\rangle$, $|m_s = -1\rangle = |f\rangle$ and the excited state $|A_2\rangle = (|E_-\rangle |1\rangle + |E_+\rangle |-1\rangle)/\sqrt{2} = |e\rangle$. The state $|m_s = 0\rangle = |i\rangle$ is an ancillary state.
Figure 2: (Color online) The population of the basic states $|gf\rangle|0\rangle_c$ (P1) and $|fg\rangle|0\rangle_c$ (P2) governed by the full Hamiltonian in Eq.(1) (green lines) and the effective Hamiltonian in Eq.(6) (red lines), where (a) $\Omega = 0.05g$, $|\Delta| = 0.5g$ (b) $\Omega = 0.01g$, $|\Delta| = 0.2g$.

Figure 3: (Color online) The influences of the fluctuation of the detuning ratio $\Delta/g$ and the Rabi frequency ratio $\Omega/g$ on the fidelity of QST, the median values are $\Delta/g = 0.5$ and $\Omega/g = 0.05$. 
Figure 4: (Color online) The fidelity of QST as a function of the scaled NV center spontaneous emission $\gamma/g$ and scaled cavity decay $\kappa/g$ in the case of $\Omega = 0.05g$ and $|\Delta| = 0.5g$.

Figure 5: (Color online) The influences of the fluctuation of the shift pulse time $\delta t/t$ on the fidelity of non-geometric CPG, in the case of $\Omega = 0.05g$ and $|\Delta| = 0.5g$.

Figure 6: (Color online) The fidelity of non-geometric CPG as a function of the scaled NV center spontaneous emission $\gamma/g$ and scaled cavity decay $\kappa/g$ in the case of $\Omega = 0.05g$ and $|\Delta| = 0.5g$. 
Figure 7: (Color online) The concurrence versus the interaction time $gt$ and the weight factor ratio of initial state $r$. $r = 1$ is for the maximally entangled states.

Figure 8: (Color online) The concurrence versus $\lambda/g$ and the interaction time $gt$, for $r = 1/3$. 