Global Sensitivity Analysis of Ultimate Limit States of Stainless Steel Structural Members

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Abstract. The article presents global Sobol sensitivity analysis of a rolled member in tension made from austenitic chromium-nickel stainless steel of type 1.4307 / AISI 304 L. The statistical characteristics of yield strength and of the geometry of the rolled steel IPE cross-section are presented on the basis of published experimental research. The sensitivity analysis showed the dominant effect of the yield strength on the static resistance. The second dominant variable is the flange thickness. Higher-order sensitivity indices oriented at detecting the presence of interaction effects between input variables are very small. The characteristics of other types of sensitivity analyses oriented at quantiles or the probability of failure are discussed, especially in terms of a higher proportion of higher-order sensitivity indices. The results of Sobol sensitivity analysis of stainless steel are compared with similar results of carbon steels.

1. Introduction
One of the most widely used methods for examining the relationships between inputs and outputs of stochastic computational models is Sobol sensitivity analysis [1, 2]. Regarding civil engineering, the popularity of Sobol sensitivity analysis has a growing trend that correlates with other disciplines [3]. However, there exist a number of tasks where the analysis of variance is not sufficient to comprehensively identify the influence of input variables on the system response [4]. A typical example is the analysis of structural reliability, which is assessed using the failure probability [5, 6] or design quantiles [7, 8]. A second example is time-dependent probability analysis [9-12], which presents methods adapted to optimizing system reliability. A third example is MCDM [13-15]. Although the analysis of variance (ANOVA) is not sufficient to completely identify the effect of input variables on reliability, comparative studies [16] show that variance is partially (and possibly sufficiently) empathic to a number of model-based inferences. These conclusions are also confirmed by studies comparing global Sobol sensitivity analysis with global quantile-oriented sensitivity analysis [17, 18], where both methods yield the same or very similar sensitivity order of total indices, while the interaction effects may vary. Although new methods of sensitivity analysis are being successfully developed [8, 19], Sobol sensitivity analysis is unlikely to be relegated in the near future to minority methods for studying the influence of input variables on the model output.

The results of the sensitivity analysis of the model output depend on both the quality of the computational model and on the correct identification of the input random variables. Experimental research on the mechanical properties of materials is an important part of the simulation process of abstract models of load-bearing structural systems during the examination of structural reliability of
frame resistance [20], steel girders [21], carbon steels columns [22], stainless steels columns [23], fatigue limit states [24], fatigue life [25], reinforcement corrosion [26], bridge corrosion [27], column buckling [28], buckling interactions [29], lateral-torsional buckling [30, 31], thin-walled members [32], anchor bolts [33], seismic fragility [34] and mechanical strains [35].

The use of new materials presents new challenges for research. New types of stainless steel exhibit useful properties, however, the design of such structures brings uncertainties in the values of material characteristics such as yield strength or Young’s modulus. The aim of this paper is the sensitivity analysis of the influence of input random variables on the model output. One of the objectives of sensitivity analysis is to identify the differences between the influence of input random characteristics on the reliability of structural members.

2. Material and geometrical properties

Steel 1.4307 / AISI 304 L is an austenitic chromium-nickel stainless steel with a low carbon content. This steel has good corrosion resistance to uniform corrosion and to many slightly corrosive organic and inorganic chemicals.

The minimum specified yield stress values are defined in the corresponding material standard EN3 (2008) as the characteristic values corresponding to the 5% confidence limit. Eurocode 3 (2008) determines the characteristic value of $\sigma_{0.2}$ for hot rolled stainless steel grade 1.4307 as 200 MPa, where $\sigma_{0.2}$ is the material 0.2% proof stress. The goal of the analysis is the resistance of the IPE80 cross-sectional member under tension.

Is it possible to identify the statistical characteristics of yield strength even though we do not have a large number of samples from experiments? According to article [36], the statistical characteristics of yield strength for different stainless steel plates and sheet materials are: mean value is 1.22 $f_{0.2}$, variation coefficient is 0.061. However, these are not the rolled profile shapes studied in this article. In the experimental research on carbon steel [37,38], it was observed that hot rolled IPN and IPE profiles have higher yield strength and lower variation coefficient in the rolling direction than sheets in the general rolling direction. This is due to production technology and better processing during rolling. The identification of Ramberg-Osgood non-linear material model parameters for hot-rolled stainless steel grade 1.4307 (AISI 304L) was performed in [39]. Reference data (stress-strain relation) were procured experimentally from normalized specimens. The parametric finite element model was developed using ANSYS Parametric Design Language, and the subsequent optimization process was performed using the software platform OptiSLang. The yield strength value of 257 MPa identified in [39] is considered as the mean value of the yield strength in this article. The standard deviation of the yield strength is considered as 0.061-257 MPa = 15.7 MPa in accordance with [36]. It can be noted that the variation coefficient of 0.061 of yield strength of stainless steel is close to the value of 0.057 identified in [37] for a large number of samples taken from a third of the flange of hot rolled carbon steel IPE profiles. The geometric characteristics of IPE80 stainless steel cross-section are considered according to the geometry measurements of rolled steel profiles published in [36].

| No. | Characteristics | Symbol | Mean value | Standard deviation | Type of pdf |
|-----|----------------|--------|------------|--------------------|-------------|
| 1   | Yield strength  | $f_y$  | 257 MPa    | 15.7 MPa           | Lognormal   |
| 2   | Height          | $H$    | 80 mm      | 0.56 mm            | Gauss       |
| 3   | Width           | $B$    | 46 mm      | 0.322 mm           | Gauss       |
| 4   | Web thickness   | $t_w$  | 3.8 mm     | 0.103 mm           | Gauss       |
| 5   | Flange thickness| $t_f$  | 5.2 mm     | 0.161 mm           | Gauss       |
It can be noted that the standard deviation of the IPE80 cross-section form carbon steel according to [38] would be: standard deviation of height is 0.352 mm, standard deviation of width is 0.454 mm, standard deviation of web thickness is 0.148 mm and the standard deviation of flange thickness is 0.238 mm.

3. **Sobol sensitivity analysis**

The comprehensive notion of sensitivity analysis facilitating the analysis of the effects of arbitrary subgroups of input parameters (doubles, triples, etc.) on an observed output was proposed by Ilja M. Sobol [1, 2]. In this paper, the effects of input parameters (random inputs $X$) on the load-carrying capacity (random output $Y$) were evaluated using Equations (1) and (2):

$$S_i = \frac{\nu(\mu(Y|X_i))}{\nu(Y)} \quad (1)$$

$$S_{ij} = \frac{\nu(\mu(Y|X_i, X_j))}{\nu(Y)} - S_i - S_j \quad (2)$$

Other sensitivity indices are calculated similarly [1, 2]. The sum of all indices is equal to one. The five input random variables listed in Table 1 lead to thirty-one sensitivity indices.

4. **Numerical results**

The static resistance is computed as the product of the yield strength and the cross-sectional area according to Equation (3). The cross-sectional area is idealized using a shape composed of rectangles. This idealization is sufficiently accurate and has been applied, for example, in study [40].

$$Y = f_y(2B t_f + (H - 2t_f)t_w) \quad (3)$$

The Latin Hypercube Sampling method (LHS) was applied for the computation of sensitivity indices. The model output $Y$ is the load-carrying capacity computed in each run of the LHS method. The estimation of the sensitivity index is based on double-nested-loop simulations. The procedure can be explained on sensitivity index $S_i$ according to Equation (1). The outer loop generates random realizations $X_i$ and estimates the variance from conditional realizations of the arithmetic mean of the model output $Y$. Five thousand simulation runs are used in the outer loop. The inner loop assumes $X_i$ and the realizations of the other random variables are generated using fifty thousand runs of the LHS method. The inner loop estimates the random realizations of the arithmetic means of the model output $Y$. Estimate $V(Y)$ is computed using fifty thousand steps of the LHS method. A description of the algorithm for the estimation of sensitivity indices is, for example, in [40]. Other sensitivity indices are estimated similarly. The results of the sensitivity analysis show the dominant influence of the yield strength on the resistance of the bar in tension. The dominant sensitivity indices are as follows $S_i = 0.892, S_2 = 0.002, S_3 = 0.005, S_4 = 0.02, S_5 = 0.08$. The sum of the first-order sensitivity indices is 0.999. The other sensitivity indices are very small and are approximately equal to zero.

5. **Discussion**

The height and width of the cross-section have very little influence on the static resistance: $S_i = 0.002, S_1 = 0.005$. Both input random variables have a very small coefficient of variation compared to other input variables. It can be expected that the input random variables can also have a similar effect on the design quantile. The design quantile is also influenced by skewness and kurtosis as shown in [7, 8], but variance is a key static variable with priority influence.

Sobol sensitivity indices are understandable because they have a small proportion of higher-order sensitivity indices. This is a big difference from indices oriented to design quantiles [7, 8, 17] or the
probability of failure [5, 6] where the proportion of interaction effects is relatively high. The high proportion of higher-order sensitivity indices requires the estimation of total indices as shown, for example, in case study [7]. The results of sensitivity analysis, in which all the influence is concentrated in the sensitivity index of the last row, where all input random variables are fixed, are very incomprehensible.

Compared to carbon steel, the yield strength of stainless steel is attained by gradual loading with a non-linear stress-strain diagram, which does not affect the results of the sensitivity analysis, because Young’s modulus is not part of the computational formula. If we were to study compressed members with consideration to the effect of buckling, then the material nonlinearity could have some effect on the resulting resistance. It is questionable whether geometric nonlinearity can have the same or possibly higher effect on the static resistance. Knowledge of non-linear behaviour can have a significant impact on the direction of further research and verification of criteria for the design of steel structures that are not as precise as for carbon steels. Sensitivity analysis is an important part of this process.

6. Conclusions
The influence of yield strength of austenitic chromium-nickel stainless steel 1.4307 / AISI 304 L is dominant in the case of the studied bar in tension. The second dominant variable is the flange thickness. The statistical characteristics of rolled stainless steel bars are similar to those of carbon steel bars, but are not the same. The variation coefficient of yield strength of the stainless steel considered in this article is slightly higher than that of carbon structural steel. This may be reflected in the estimation of Sobol sensitivity indices. It can be noted that other types of sensitivity indices oriented to the design quantiles or the probability of failure may be more or less different, especially when identifying interaction effects. What affects the design quantile may not have the same effect on resistance and vice versa.

The influence of residual stress and fatigue response of stainless steel bars will be studied in further research. Stainless steel structures are relatively recent and fatigue degradation has not yet been appropriately observed and studied in-depth. It is questionable whether corrosion resistance as a good property is not complemented by other less desirable properties that are different from carbon steels and which are not desirable for load-bearing structures in the long term. The reliability analysis of load-bearing stainless steel structural elements is more demanding, both in terms of numerical models and in terms of experiments. Numerous other random imperfections can influence the limit states of stainless steel load-bearing elements and frames. To identify these imperfections and their statistical characteristics, a large number of experiments are needed, which is very economically difficult to ensure for building structures. It is therefore necessary to look for ways to effectively combine the knowledge from new experiments with the experience and knowledge gained from theoretical research using numerical simulations on computers.

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