Spectrum of orientifold QCD in the strong coupling and hopping expansion approximation

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A R T I C L E   I N F O

Article history:
Received 29 December 2009
Received in revised form 4 May 2010
Accepted 5 May 2010
Available online 12 May 2010
Editor: L. Alvarez-Gaumé

Keywords:
Lattice gauge theory
Orientifold planar equivalence
Meson spectrum
Strong coupling expansion

A B S T R A C T

We use the strong coupling and hopping parameter expansions to calculate the pion and rho meson masses for lattice Yang–Mills gauge theories with fermions in irreducible two-index representations, namely the adjoint, symmetric and antisymmetric. The results are found to be consistent with orientifold planar equivalence, and leading order $1/N_c$ corrections are calculated in the lattice phase. An estimate of the critical bare mass, for which the pion is massless, is obtained as a function of the bare coupling. A comparison to data from the two-flavour SU(2) theory with adjoint fermions gives evidence for a bulk phase transition at $\beta_c \sim 2$, separating a pure lattice phase from a phase smoothly connected to the continuum.

1. Introduction

Recently, there has been an ongoing effort to use lattice techniques for studying properties of gauge theories beyond QCD, with a large number of colours ($N_c \gg 3$) and/or fermions in higher dimensional representations. The earliest motivation for this has been the proposed ‘orientifold planar equivalence’ [1,2] which asserts that certain sectors of pairs of gauge theories coincide at infinite $N_c$. A precise statement can be made for the case of ‘adjoint QCD’, ‘symmetric QCD’ and ‘antisymmetric QCD’ (by which we mean theories whose action is the same as QCD but with the fermions in the respective representation), which we collectively refer to as ‘orientifold theories’. In this case, planar equivalence predicts that the bosonic observables in adjoint QCD exactly coincide with C-even observables in symmetric and antisymmetric QCD at infinite $N_c$ (for a general review, see [3]; for a lattice formulation, see [4]). It has been shown that the only necessary and sufficient condition for this matching is that charge conjugation symmetry is not spontaneously broken in any of the theories [5–8].

It is interesting that by taking fermions in the adjoint, the one-flavour theory is identically $N = 1$ Super Yang–Mills, and so we can copy analytical predictions obtained using supersymmetry to the other two theories [9]. Furthermore, for $N_c = 3$, the antisymmetric theory becomes one-flavour fundamental QCD, suggesting a pathway for making real predictions in a close relative to real QCD — provided $N_c = 3$ is ‘close’ to infinity. However, today this question can only be addressed by measuring the size of $1/N_c$ corrections non-perturbatively using lattice methods. In the case of pure Yang–Mills or quenched QCD with fundamental fermions, many studies have found that the corrections are indeed small (for a review, see [10]). There has been much less work on two-index fermions, though studies are now beginning to appear. In particular, in [11], a quenched lattice simulation of the quark condensate in orientifold theories was carried out, and a comparison with the analytic expression from [12] supports the equivalence. Note however that for two-index fermions, the quenched theory and the dynamical theory are different at infinite $N_c$, so a definitive result has yet to appear.

Orientifold theories have also gained attention as candidates for Beyond the Standard Model physics, as their dynamics is potentially very different to QCD. In particular, there are proposals for ‘Technicolor’ models of Dynamical Electro-Weak Symmetry Breaking where the Higgs is replaced by a composite bound state of strongly coupled higher dimensional fermions. A recent concrete example making use of such a Higgs sector is Minimal Walking Technicolor [13], which in our language is just SU(2) adjoint QCD with two flavours. Numerical lattice studies have already been carried out to determine the non-perturbative dynamics of this theory [14–22], and there is mounting evidence of a conformal infra-red fixed point, or at least near-conformal behaviour — a requirement for the walking scenario. There have also been numerical investigations of conformal behaviour in the case of SU(3) symmetric fermions [23–28].
Since, however, unlike QCD, there is no experimental data to
guide the interpretation of numerical results, it is important to
learn as much as possible by analytical means. A weak-coupling
analysis has already been performed [29] and, among other con-
clusions, gives perturbative estimates for the ratio of $\lambda$ parameters,
$\Lambda_{\text{lat}}/\Lambda_{\text{QCD}}$, and for the additive renormalisation of the quark
mass and of fermion bilinears in this regime. The present Letter is
written in the same spirit, and looks at the opposite side of the lattice
phase diagram by studying the meson spectrum in the strong cou-
pling regime.

It is clear that a lattice strong coupling expansion has no rel-
evance in the continuum limit, however our goal is to establish analytic
results which, first, will give a starting point for choosing
simulation parameters and, second, will provide information
on the phase structure of the lattice theory. In particular, we
derive formulæ against which numerical data can be compared to
ensure simulations are not in an ‘unphysical’ phase (in the sense
of not having a continuum limit).

In addition, the meson masses provide explicit observables for
which to check orientifold planar equivalence at infinite $N_c$. For-
maHy, a general proof of planar equivalence that holds to all orders
in the strong coupling and hopping expansion has already been
presented [4], and the results of this Letter should be considered
a special case. The benefit of our direct calculation is that it provides
explicit expressions for the meson masses at finite $N_c$.

2. Strong coupling and hopping expansion approximations

Here we set up the notation and outline the strategy. Discretis-
ing using Wilson fermions in lattice units ($a = 1$), the action and
Dirac operator is

$$S_g = S_g + \sum_{x,y} \bar{\psi}(x)D(x,y)\psi(y), \quad (2.1)$$

$$S_g = -\frac{1}{2g^2} \sum_{x,y,\mu,\nu} \text{Tr}[U(x,\mu)U(x+\mu,\nu)U^\dagger(x+\nu,\mu)U^\dagger(x,\nu)]$$
$$+ \text{h.c.},$$

$$D(x,y) = \delta_{xy} - K(x,y),$$

$$K(x,y) = 2\kappa \left[ P_\mu^- V(x,\mu)\delta_{x,y+\mu} + P_\mu^+ V(x-\mu,\mu)\delta_{x,y-\mu} \right], \quad (2.2)$$

where $P_\mu^\pm = (1 \pm \gamma_\mu)/2$ are the standard projectors, we have set
the Wilson parameter $r$ to 1, and the expansion parameter $\kappa$ is
related to the bare quark mass by the usual relation $2\kappa = 1/(4 + m_0).
We do not specify the number of flavours beyond stating $N_f \geq 2$,
as the final result will coincide with the quenched result to the
order we work to in the hopping expansion. The gauge part of the
action is the standard sum over elementary plaquettes, with
the links $U$ always transforming in the fundamental representa-
tion of the gauge group. In the Dirac operator the links $V$ are
in an arbitrary representation, which we will take to be either the
fundamental or a two-index irreducible representation.

We want to compute the two-point meson correlator in the
triplet channel,

$$G_{\alpha\beta\gamma\delta}(x,y) = \langle (\bar{\psi}_\alpha(x)\gamma_\mu \psi_\beta(x))\bar{\psi}_\gamma(y)\psi_\delta(y) \rangle$$
$$= -\frac{1}{2} \int \mathcal{D}[U] \text{det} D \Gamma_{\alpha\beta\gamma\delta}^{-1}(x,y)\Gamma_{\mu\nu\gamma\delta}(x,y)e^{iS}, \quad (2.3)$$

where $\psi$ and $\psi'$ are different fermion flavours and colour indices
are contracted to make colour singlet mesons.

The procedure for computing the correlator in the strong
coupling and hopping expansion is, in principle, well established. One
expands the quark propagators and fermion determinant as a series
in $\kappa$, and the gauge action in powers $1/g^2$. This joint ex-
ansion consists of all possible paths linking the two spacetime
points $x$ and $y$, decorated with arbitrary insertions of plaquettes.
The new feature appearing in our calculation is that, in general,
links $V$ from the fermions will appear together with links $U$ from
the gauge plaquettes, and this will affect the Haar integrals. The
two-link integral $U(N_c)$ for two irreducible representations is

$$\int \mathcal{D}[U|R^a[U]] R^b[U^\dagger] \delta_{ij} = \frac{1}{d_R} \delta^{ab}\delta_{3ij}, \quad (2.4)$$

where $R^a[U]$ denotes the link $U$ in representation $a$, and $d_R$ is
the dimension of the representation. The difference between $U(N_c)$
and $SU(N_c)$ will be discussed in Section 3.3.

The technical details for expanding (2.3) are given in [30],
where the method was used to compute the pion and rho masses
in standard QCD. The basic idea is to define a recursion relation
between paths of length $L$ and $L - 1$:

$$G_{\alpha\beta\gamma\delta}(n,0) = \sum_{n',\sigma,x} M_{\alpha\sigma\beta}(n,n') G_{\sigma\gamma\delta\beta}(n',0), \quad (2.5)$$

where $M$ contains the factors describing the propagation between
sites $n$ and $n'$. The full correlator is then simply

$$G_{\alpha\beta\gamma\delta}(n,0) = \sum_{L=0}^{\infty} G_{L\alpha\beta\gamma\delta}(n,0), \quad (2.6)$$

and it can then be shown (e.g. by looking at (2.5) in Fourier space)
that it is given by an infinite geometric series which depends only
on $M$:

$$G_{\alpha\beta\gamma\delta}(n,0) = \sum_{L=0}^{\infty} G_{L\alpha\beta\gamma\delta}(n,0) = (\delta_{\alpha\beta}\delta_{\gamma\delta} - M_{\alpha\beta}\delta_{\gamma\delta}(n,0))^{-1}. \quad (2.7)$$

We can look at the desired channels by inserting the appropriate
$\Gamma$ matrices,

$$G_{AB}(n,0) = \sum_{\alpha\beta\gamma\delta} (\Gamma_A)_{\alpha\beta}(\Gamma_B)_{\gamma\delta} G_{\alpha\beta\gamma\delta}(n,0), \quad (2.8)$$

and extract the masses by looking in momentum space for poles
in $G_{AB}$ (or, equivalently, zeros in its inverse) for a particle at rest
with momentum $p_\mu = (im,0)$.

3. Computing the masses

3.1. Leading order

In the framework of Section 2, the problem reduces to writing
an expression for $M(n,n')$. To lowest order, the only contribution
is the $k = 2$ diagram in Table 1. The corresponding expression is

$$M_{\alpha\beta\gamma\delta}(n,n') = (2\kappa)^2 \sum_{\mu} \left[ P_\mu^- \bar{\psi}_\alpha(x) P_{\mu}^+ \gamma_\rho \delta_{n+\mu,n'} \right]$$

$$+ \left[ P_{\mu}^- \bar{\psi}_\alpha(x) P_\mu^+ \gamma_\rho \delta_{n-\mu,n'}, \quad (3.1)$$

We have omitted the links $V$, as the colour contribution does not
depend on the shape of the path and can be factorised and easily
treated separately, giving a constant factor $d_R$. Substituting (3.1)
into (2.7) and taking the Fourier transform, we obtain the expres-
sion.
representation and the gauge links are in the fundamental. There are analogous diagrams with \( \hat{\mu} \leftrightarrow -\hat{\mu} \) and \( \nu \leftrightarrow -\nu \). In the fundamental representation \( q = 2 \), while for two-index representations \( q = 4 \).

| Order | Diagram | Order | Diagram |
|-------|---------|-------|---------|
| 0/\( g_0 \)| 0 | 2/\( g_0 \)| \( q \) |
| \( q \) | \( 6 \) | \( 4 \) | \( q \) |

\[
G_{\alpha\beta\gamma\delta}(n, 0) = -d_R \int_{-\pi}^{\pi} \frac{d^4p}{(2\pi)^4} e^{ip\nu} \tilde{G}_{\alpha\beta\gamma\delta}(p),
\]

\[
\tilde{C}_{\alpha\beta\gamma\delta}^{-1}(p) = \delta_{\alpha\delta} \delta_{\beta\gamma} - (2\kappa)^2 \sum_\mu (P^+ _\mu)_{\alpha\delta} (P^+ _\mu)_{\gamma\beta} e^{ip\mu} + (P^- _\mu)_{\alpha\delta} (P^- _\mu)_{\gamma\beta} e^{ip\mu}.
\]

Contracting (3.2) with \( \Gamma \) matrices then gives the momentum space correlator. Detailed expressions can be found in [30]; we simply quote the final result after substituting \( p_\mu = (im, \bf{0}) \) and solving \( \det \tilde{G}^{-1} = 0 \):

\[
\cosh m_\pi = 1 + \frac{(1 - 16\kappa^2)(1 - 4\kappa^2)}{8\kappa^2(1 - 6\kappa^2)},
\]

\[
\cosh m_\rho = 1 + \frac{(1 - 12\kappa^2)(1 - 8\kappa^2)}{8\kappa^2(1 - 6\kappa^2)}.
\]

where, in analogy to QCD, we call the pseudoscalar particle \( \pi \) and the vector \( \rho \). These results are well known, and, to this order, there is no dependence on representation.

3.2. Next to leading order

We now consider the remaining diagrams in Table 1, up to \( O(\kappa^6) \). The higher order terms contain ‘fermion squares’ which must be filled by plaquettes from the expansion of the gauge action. In the fundamental representation, one simply expands to first order, bringing down one plaquette to satisfy (2.4). The calculation for this representation has already been done in [30]; we will discuss how to modify it in the case of different fermion representations.

The first thing to notice is that first order diagrams, with one plaquette within the fermion squares, are all zero because of the orthogonality condition (2.4), since the fermions are in a two-index representation and the gauge links are in the fundamental. There are no other diagrams at order \( 1/\kappa^2 \) so the entire order vanishes and we must consider the next one. At second order, we can place two gauge plaquettes on the lattice. Overlapping them inside the fermion square (in the correct orientation, which is representation dependent) yields a non-zero contribution.

For concreteness, consider the colour factor for the symmetric representation. The gauge integrals can be done by writing the reducible product of two overlapping fundamental links in terms of symmetric links \( S \) and antisymmetric links \( A \),

\[
U_{ac}U_{bd} = S_{(ab), (cd)} + A_{(ab), (cd)}.
\]

where \( (ab) \) indicates the symmetric combination, with \( b \geq a \), and \( [ab] \) indicates the antisymmetric combination, with \( b > a \). Defining a new basis such that \( S_{(ab), (cd)} = S_{ij} E_{a(i} E_{d)j} \) (and similarly for the antisymmetric) with \( i, j \) running from 1 to \( d_R \), the gauge integrals take the form

\[
\int \mathcal{D}[U](S_{ij} + A_{ij})S^T_{ij}.
\]

By orthogonality, the second term is zero, and the first is given by (2.4). The argument is identical for antisymmetric fermion links, with the first term being zero and the second given by (2.4). Thus, we are back in a case exactly analogous to the fundamental. The same diagrams contribute, and, in particular, the spinorial contribution is the same. However, the colour integrals are modified owing to the higher order expansion of the gauge action. There is a factor 2! from the Taylor expansion, and a combinatorial factor 1 \( \times \) 1 exactly one way to place the two gauge plaquettes. This gives an overall colour factor of \( 1/(2d_R^3) \) for the symmetric and antisymmetric representations.

The case of the adjoint is similar, except for the fact that the two inserted gauge plaquettes must run in opposite orientations. There are now two ways to do this, so there is an extra factor of 2 which cancels the 2! from the Taylor expansion. The result for the adjoint is thus \( 1/g_0^3 \).

Having obtained the colour factor, we can construct \( M \) from the diagrams in Table 1 (intermediate expressions can be found in [30]), and extract the poles by solving \( \det \tilde{G}^{-1} = 0 \). This gives the meson masses,

\[
cosh m_\pi = 1 + \frac{1 - 20\kappa^2 + 64\kappa^4 - 48\kappa^2 + 96\kappa^4}{8\kappa^2(1 - 6\kappa^2)},
\]

\[
cosh m_\rho = 1 + \frac{1 - 20\kappa^2 + 96\kappa^4 - 12\kappa^2 + 384\kappa^4}{8\kappa^2(1 - 6\kappa^2 + 6\kappa^2(1 - 11\kappa^2 + 48\kappa^4))}.
\]

where the constant \( \epsilon_K \) depends on the representation in the following way:

\[
\text{fundamental} \quad \epsilon_K = \frac{1}{N_c g_0^2},
\]

\[
\text{adjoint} \quad \epsilon_K = \frac{1}{d_R g_0^2}, \quad d_R = N_c^2 - 1.
\]

\[
\text{symmetric} \quad \epsilon_K = \frac{1}{2d_R g_0^2}, \quad d_R = N_c(N_c + 1)/2.
\]

\[
\text{antisymmetric} \quad \epsilon_K = \frac{1}{2d_R g_0^2}, \quad d_R = N_c(N_c - 1)/2.
\]

To this order, the meson correlators in the other channels do not contain poles leading to real-valued masses.

3.3. \( U(N_c) \) vs \( SU(N_c) \)

The masses as calculated in this section only make use of the \( U(N_c) \) integral (2.4), so the results strictly apply only to \( U(N_c) \) gauge theories, not \( SU(N_c) \). If one is interested in the large-\( N_c \) limit, this is not a problem as the singlet part of \( U(N_c) = SU(N_c) \times U(1) \) decouples, and both groups give the same result. However, for small gauge groups, one must also take into account contributions of the form

\[
\int \mathcal{D}[U]U_{i_1j_1}U_{i_2j_2} \cdots U_{i_{N_c}j_{N_c}} = \frac{1}{N_c!} \epsilon_{i_1j_2 \cdots i_Nj_N} \epsilon_{j_1j_2 \cdots j_Nc}.
\]

With all links expressed in terms of the fundamental representation, the diagrams enumerated in Table 1 allow for up to four
superimposed gauge links — two from the two-index fermions and another two from the insertions of (up to) two plaquettes. Eq. (3.12) is non-zero for $N_c$ superimposed links, so this integral will contribute for $N_c = 2, 3, 4$. For $N_c \geq 5$, to this order in the strong coupling expansion, $SU(N_c)$ coincides with $U(N_c)$. In addition, representations for some small gauge groups are equivalent to each other, and need to be considered as special cases. We look at each individually:

- **SU($N_c$) adjoint** — For the adjoint representation, the full contribution is captured by including the two orientations of the gauge plaquettes, as we have done in Section 3.2, and Eq. (3.12) plays no role. The result (3.9) is therefore unchanged for all $N_c$.
- **SU(2) antisymmetric** — This representation is just the singlet, so the theory is simply the free fermion theory. A strong coupling expansion is meaningless in this case, and we discard it completely.
- **SU(2) symmetric** — For SU(2), the symmetric and adjoint representations are unitarily equivalent. As the result for the adjoint has already been argued to be correct, the result for the symmetric must be the same. Alternatively, we can work directly in the symmetric representation, adding the two diagrams in Fig. 1(a). The two diagrams turn out to be equal, and this provides the factor of 2 needed to give $\epsilon_R = 1/(d_2 g_2^3)$.
- **SU(3) antisymmetric** — This representation is unitarily equivalent to SU(3) fundamental. The correct result must therefore be (3.8), namely $\epsilon_R = 1/(N_c g_2^3)$. The difference comes about because the orthogonality condition (2.4) does not vanish to order $1/g_2^3$ as it does for the other two-index representations.
- **SU(3) symmetric** — From (3.12), the only extra three-link diagram which could contribute is shown in Fig. 1(b). However, in this representation, the symmetrisation of the indices leads to the vanishing of the diagram. Thus (3.10) is correct without modification for SU(3).
- **SU(4) antisymmetric** — This case is similar to SU(2) symmetric: the same two diagrams in Fig. 1(a) contribute and they are both equal, giving an extra factor of 2 compared to $U(4)$. The result is $\epsilon_R = 1/d_2 g_2^4$.
- **SU(4) symmetric** — As for SU(3) symmetric, the symmetrisation of the indices makes the diagram on the right in Fig. 1(a) vanish, so (3.10) is valid as it stands.

### 4. Discussion

The Eqs. (3.7) give analytical predictions for the pion and rho masses in the strong coupling and hopping parameter expansions. We can use them to make a few general observations which may help in future lattice studies of orientifold theories. The critical value of $\kappa$ where the pion vanishes can be calculated as a function of the bare coupling and is found to be

$$\kappa_c \approx \frac{1}{4} \left(1 - \frac{3}{32} \epsilon_R\right).$$

Thus, moving away from the infinite coupling limit has the effect of reducing $\kappa_c$ below 1/4 (although the first order correction is very small). Note also that at $\kappa_c$ the pion mass is zero but the rho mass remains finite. Indeed, the rho mass is always above the pion mass (Fig. 2), and the strong coupling phase is qualitatively similar to fundamental QCD. Furthermore, at any given $\kappa$, there is a definite ordering in the masses (both for the pion and the rho); the different factors $\epsilon_R$ are such that the symmetric is always heaviest, followed by the adjoint and then the antisymmetric (Fig. 3). While this has no physical significance, it is relevant for numerical simulations, as it tells us that the values of $\kappa$ needed to approach the chiral limit will be similarly ordered, with symmetric highest and antisymmetric lowest.

It is interesting to compare (3.7) with numerical lattice data. Fig. 4 plots the pion mass for the SU(2) adjoint theory, using data from [16] for two-flavour dynamical simulations supplemented by our own quenched simulations (also on a $16 \times 8^3$ lattice). The strong coupling line is plotted for $\beta = 0.5$, but in practice does not move significantly in the range $\beta = 0.5-3.0$. For $\beta = 0.5$, deep in the strong coupling phase, the lattice data falls on top of the strong coupling prediction (note that this is not a fit as (3.7) has

![Fig. 1.](image1) Extra diagrams appearing in SU($N_c$), not present in U($N_c$). The solid lines are coming from the fermions, and are symmetrised or antisymmetrised. The dotted lines are insertions of plaquettes in the fundamental representation. All the diagrams which include squares in Table 1 have analogues to the above.
The strong coupling curve is plotted for good agreement with the strong coupling prediction. In contrast, at large masses, the weak dependence on $m_0$ to be explained by going to higher order in the hopping expansion. For small masses, which is likely a small underestimated systematic error in the data. With this understood, as the factors $\epsilon$ in (4.2) is borne out in this phase. The dynamical simulations are slightly puzzling: for $\beta = 1.5$, one would expect the quenched and dynamical results to coincide for large masses, as the effects of the fermion determinant become negligible. This is not observed, suggesting either that the mass is simply not large enough, or that there could be a small underestimated systematic error in the data. With this uncertainty in mind, we can say that the data for $0.5 < \beta < 1.75$ is good agreement with the strong coupling prediction. In contrast, for $\beta > 2$, there are significant departures from strong coupling, both in the magnitude of the masses and even in the qualitative behaviour as one approaches light quark masses. This is consistent with the finding in [16] that there is a bulk phase transition at $\beta_c \sim 2$, with a strong coupling lattice phase possessing no continuum limit for $\beta < \beta_c$, and a phase smoothly connected to the continuum for $\beta > \beta_c$.

The results are also consistent with orientifold planar equivalence, as the factors $\epsilon_R$ for the two-index representations all tend to the same value as $N_c \to \infty$. We have computed analytic expressions for masses of the $\pi$ and the $\rho$ mesons in the strong coupling and hopping expansion (large mass) approximations, for fermions in the three irreducible two-index representations. In the limit $N_c \to \infty$, the three converge to the same value, as predicted by the formal proof of orientifold planar equivalence on the lattice presented in [4]. In addition, we have extracted the leading $1/N_c$ corrections, which in the strong coupling phase are expressed only in terms of the dimensionality of the representation, $d_R$ (and the bare quark mass).

The results are already useful in understanding the lattice phase structure emerging from Monte Carlo simulations. By comparing with recent numerical determinations of meson masses in two-flavour SU(2) adjoint QCD, we find evidence of two phases, supporting the conclusions of [16]. The theory has a bulk phase transition with a strong coupling lattice phase, having no continuum limit, on one side ($\beta < 2$), and a phase smoothly connected to the continuum on the other ($\beta > 2$). As more simulations are performed for as yet unstudied theories, it is our hope that the results of this Letter will help in recognising the phase structure and locating the correct region of parameter space to use for extracting continuum physics.

Acknowledgements

I am greatly indebted to A. Patella and B. Lucini for their insights and careful reading of the manuscript, and to A. Armoni for numerous discussions and thoughtful analysis. I would also like to thank M. Lüscher and the CERN Theory Department, where the bulk of this work was carried out. The quenched simulations were performed using the HIfRep code described in [14], and I am grateful to its authors.

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