Role of shape on the forces on an intruder moving through a dense granular medium

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ABSTRACT
We use numerical simulation to investigate the effect an intruder’s shape has on the drag and lift forces that it experiences while moving through a granular medium composed of polydisperse disks of mean diameter $d$. The intruder velocity, $v$, was varied from 0.1 $\sqrt{dg}$ to 20 $\sqrt{dg}$. For frictionless particles ($\mu = 0.0$), there is a gradual increase in drag force with increasing $v$, whereas, for frictional systems ($\mu = 0.1, 0.5$), a constant drag regime appears at low velocities. The drag force depends weakly on the object’s shape provided that its cross-section is constant. The drag force depends linearly on the immersion depth, while the lift force varies weakly with depth for certain shapes. The lift experienced by the object is a strong function of its shape at a given velocity. The shape has an effect on the distribution of contacts around the surface of the intruder, which may result in a strong lift for certain shapes. We also present and discuss the force profiles around the intruder surface.

KEYWORDS
Granular medium; intruder shapes; drag and lift forces; discrete element method

1. Introduction

The granular matter is a collection of discrete macroscopic particles that exhibit properties of solids or fluids depending on the volume fraction (Jaeger, Nagel, and Behringer 1996). It is present everywhere around us in multiple forms, such as sand in the deserts or as grains in food industries. Dry granular matter exhibits properties that resemble those of Newtonian fluids, such as capillary action (Fan, Parteli, and Pöschel 2017), Magnus effect (Kumar, Dhiman, and Reddy 2019), Kevin-Helmholtz (Goldfarb, Glasser, and Shinbrot 2002), and Rayleigh-Taylor (Vinningland et al. 2007) instabilities. In addition, understanding the forces on objects moving in fluids has been an active area of research for the last few decades, as it has applications in the development of vehicles in the automobile (Humincic and Huminic 2017) and aerospace industries (Bushnell 2003), etc. Though drag and lift forces are well-understood in fluids, they have been less investigated in granular media.

Drag is the retarding force exerted on a body by the surrounding medium. Studies performed for various configurations (Albert et al. 1999, 2001; Ding, Gravish, and Goldman 2011) in granular media to understand the drag include an intruder object moving horizontally or vertically at low (Albert et al. 1999) or high velocities (Potiguar and Ding 2013). In the slow velocity regime, the drag force is either independent of, or weakly dependent on, the intruder’s velocity (Albert et al. 1999). At higher velocity, the drag force increases monotonically with the intruder velocity. Albert et al. (2001) studied objects of different shapes and found that the difference in drag for any pair was <20% in the low-velocity regime. This work, however, did not explore the behavior at higher velocities. In a recent study of intruders of various shapes in a slow granular silo flow, it was observed that the dimensionless number characterizing the drag force varied significantly with shape (Katsuragi, Anki Reddy, and Endo 2018).

In addition to drag, objects may also experience a lift force while moving in a granular medium. However, there have been very few studies devoted to understanding this phenomenon (Ding, Gravish, and Goldman 2011; Potiguar 2011; Guillard, Forther, and Pouliquen 2014; Debnath, Rao, and Nott 2017). Ding, Gravish, and Goldman (2011) stated that a symmetric object, such as a cylinder, when dragged within a granular medium, experiences a weak lift force that varies linearly with the depth. Potiguar (2011) reported that in a dilute granular flow, there was no net lift force on a circular obstacle, while the net lift force on an asymmetric obstacle depends non-monotonically on the intruder velocity and depth. Guillard, Forterre, and Pouliquen (2014) studied the lift force on a cylinder moving horizontally in a granular medium under gravity. They observed that the lift force saturates at depths greater than the cylinder diameter and noted that the gravitational pressure gradient breaks the moving intruder’s up/down symmetry, thereby modifying the flow around its surface. Debnath, Rao, and Nott (2017) observed that the lift force for a disk-shaped intruder rises with the immersion depth and reaches a constant value at...
larger depths. It was argued that the lift is the result of the asymmetry in the dilation and shear rate in the regions above and below the intruder.

The studies mentioned above are relevant for understanding animal locomotion in granular media. Subsurface motion is essential for sand-dwelling animals to shelter from high temperatures in deserts during the daytime or to escape from predators. Animals, such as sand lizards propel themselves using undulatory movements (Maladen et al. 2009; Ding et al. 2012). Hence, it is crucial to address the effect of shape on drag, and the induced lift, as the shape of the object determines the strength of the jammed region built in front of the intruder (Albert et al. 2001).

The objective of the present work is to understand the shape and friction dependence of the drag and lift forces on an intruder immersed in a granular medium as a function of its velocity and depth. The paper is organized as follows. We provide details about the simulation method in section 2, results and discussion in section 3, followed by our conclusions in section 4.

2. Simulation methodology

In this work, we employed the Discrete Element Method (DEM) (Cundall and Strack 1979) to study the forces on variously shaped objects being dragged through a granular medium in the presence of gravity. The simulations were performed in two dimensions with periodic boundary conditions in the x-direction. A gravitational field of magnitude \( g \) acts along the negative y-direction, and a wall, composed of particles of diameter \( d \), confines the simulation system along \( y = 0 \). The density (mass per unit area) is taken as \( \rho \). In the simulations, the physical quantities are scaled by the acceleration due to gravity \( g \), the mean particle diameter \( d \), and the density \( \rho \). The intruder position is fixed, and then some 63,000 particles are randomly poured into the system from above. The diameter of the particles ranges uniformly from 0.9 to 1.1\( d \) to avoid crystallization. The particles are allowed to settle under the influence of gravitational and dissipative forces (discussed later) until the system’s energy reaches a minimum. The simulation system spans 300\( d \) along the x-direction while a free surface is present at the height of about 195\( d \) measured from the bottom. The depth of the center of mass of the intruder from the free surface is denoted by \( h \) (see Figure 2).

The intruder is displaced at a constant velocity \( v \) along the positive x-direction for a total distance of 1200\( d \). We considered seven different shapes in our study: a square (S1), a rectangle (S2), a disk (S3), an ellipse with a major axis aligned with the direction of motion of the intruder (S4), an ellipse with minor axis aligned with the x-direction (S5), an equilateral triangle with the edge pointing opposite to the moving direction (S6) and an equilateral triangle with the edge pointing along the moving direction (S7): See Figure 1. The maximum cross-section of all shapes has the same length of 10\( d \). Figure 2 shows an initial configuration for S6 at a depth \( h = 105d \). Each of these shapes is created from a set of particles, for example, the square shape (S1) is made by arranging twenty layers of particles, with each layer consisting of 20 particles, each particle with a diameter of 0.5\( d \), and a total of 400 particles are used. The rectangle shape (S2) is created by arranging twenty layers of particles, with each layer having thirty particles having a total of 600 particles. The disk shape (S3) is created by placing seven consecutive layers of particles in a circular form having 339 particles in total. Similarly, the elliptical shape (S4 and S5) is created by placing the particles on the circumference of the ellipse, and then consecutive layers of particles are placed to make the desired shape. The S4 shape (ellipse-x-major) is created using 595 particles, while for the S5 shape (ellipse-y-major), 224 particles are used. Lastly, the equilateral triangles (S6 and S7) are created out of 231 particles. The particles that are used for creating the shapes have a diameter of 0.5\( d \) each. The intruder object as a whole is considered a single entity, and rigid body dynamics govern its momentum. The total force (torque) on the rigid intruder is computed at each timestep as the sum of the forces (torques) on its constituent particles. We maintained the same cross-section facing the flow direction and also

Figure 1. The various shapes are considered in our study. In the simulations, they move from left to right.

Figure 2. Initial configuration for one of the shapes (backward triangle, S6) at a depth \( h = 105d \). Periodic boundary conditions are applied in the x-direction. The system is confined by a wall at \( y = 0 \) composed of particles of size 1\( d \) (blue particles), while the top surface is unconstrained. The origin is located at the left bottom corner.
maintained the same mass for each intruder even though we used different numbers of particles to construct the intruder shapes.

In the DEM technique, the positions and velocities of each particle are updated at regular intervals of time by integrating the equations of motion using the velocity Verlet algorithm. We consider contact and gravitational forces. The normal \( f^n_{ij} \) and tangential \( f^t_{ij} \) components of the contact forces between particles \( i \) and \( j \) with masses \( m_i \) and \( m_j \) were calculated with the following expressions (Brilliantov et al. 1996; Silbert et al. 2001).

\[
\begin{align*}
    f^n_{ij} &= \frac{d_i d_j}{2(d_i + d_j)} \sqrt{\xi_{ij}} (k_n \xi_{ij} \hat{n}_{ij} - m_{\text{eff}} \gamma_n r^n_{ij}) \\
    f^t_{ij} &= -\frac{d_i d_j}{2(d_i + d_j)} \sqrt{\xi_{ij}} (k_t \Delta s_{ij} + m_{\text{eff}} \gamma_t r^t_{ij})
\end{align*}
\]

where \( d_i \) and \( d_j \) are the diameters of particles \( i \) and \( j \), the spring constant in the normal direction \( k_n = 2 \times 10^6 \text{N/m} \), and the spring constant in the tangential direction \( k_t = 2.54 \times 10^6 \text{N/m} \). The normal and tangential damping coefficients, \( \gamma_n \) and \( \gamma_t \), were taken as \( \gamma_n = \gamma_t = 3200 \text{N s/m} \). The upper limit of \( f^t_{ij} \) is restricted to \( m_{\text{eff}} \gamma_t \) for considering the slipping between contacts. The coefficient of friction \( \mu \) was varied between 0 to 0.5. \( \hat{n}_{ij} \) is the unit vector along the line joining the centers of particles \( i \) and \( j \) and \( \xi_{ij} \) is the overlap between the two particles in the normal direction. \( \Delta s_{ij} \) stands for the accumulated tangential displacement vector. Relative velocities in the normal and tangential directions are represented as \( r^n_{ij} \) and \( r^t_{ij} \) respectively. Finally, \( m_{\text{eff}} = \frac{m_i m_j}{m_i + m_j} \) is the effective mass. The model adopted is consistent with a velocity-dependent coefficient of restitution (Schwager and Pöschel 1998).

The forces on the intruder are recorded at time intervals of \( 5 \times 10^{-4} \sqrt{d/g} \). The simulation length corresponds to the time it takes for an intruder to travel \( 1200d \) for a specific velocity. All the simulations were performed with LAMMPS (Plimpton 1995) (https://lammps.sandia.gov/index.html) and OVITO (Stukowski 2010) was used for post-simulation visualization.

### 3. Results and discussion

In this section, we present the results for all seven intruder shapes (see Figure 1). The intruder moves at a constant velocity \( v \) along the positive \( x \)-direction at depth \( h \). The study was carried out for various velocities \( v \), depths \( h \), and friction coefficients \( \mu \). Subsection 3.1 presents the drag on the intruder for various velocities \( v \) and coefficient of friction \( \mu \) at a constant depth \( h = 105d \). Subsection 3.2 addresses the particle contacts on the intruder for various \( v \) and \( \mu \) at a constant depth \( h = 105d \) and kinetic drag regimes. Subsection 3.3 provides the details on the lift force experienced by the intruder for various \( v \) and \( \mu \) at a constant depth \( h = 105d \). Subsection 3.4 explores the depth dependence of forces on the intruder for velocities \( v/\sqrt{d/g} \) = 1, 5, and \( \mu = 0.1 \). Lastly, Subsection 3.5 examines the distribution of forces around the intruder for various velocities \( v \) at a fixed \( h \) and \( \mu \). Reported quantities, such as drag \( F_D \), lift \( F_L \), and red the number of contacts \( N_c \), has been calculated by averaging over several configurations after the intruder achieves a mean steady-state behavior, i.e., the instantaneous drag fluctuates around a well-defined mean.

#### 3.1. Drag on the moving intruder

This subsection presents and discusses the drag on the moving intruder at several velocities \( v \) and coefficients of friction \( \mu \). The drag \( F_D \) against velocity \( v \) (log-log plot) for \( \mu = 0, 0.1, \) and 0.5 is shown in Figures 3(a–c), respectively, for all the shapes. Additionally, \( \psi = F_D/v^2 \) is shown in the insets of each figure.

A minimum force, known as the yield drag (Hilton and Tordesillas 2013; Takehara and Okumura 2014; Kumar et al. 2017), is required to initiate the motion of an object in granular media. Therefore, we can write the drag as \( F_D = F_Y + F_K \) where \( F_Y \) is the yield drag necessary to initiate the motion, while \( F_K \) is the kinetic drag. Assuming that the drag for the lowest velocity at which we performed our simulations gives us the yield drag, \( F_Y \) is highest for \( S2 \) followed by \( S1 \) for all \( \mu \). The other shapes have nearly identical yield drag (within seven percent of each other) for all values of \( \mu \) considered.
It is known from a few published studies (Hilton and Tordesillas 2013; Takehara and Okumura 2014; Wang, Yang, and Du 2015; Kumar et al. 2017; Takada and Hayakawa 2017) that the drag regimes and drag laws in granular media are strongly correlated with the intruder’s velocity. In the present work, we observe that, without friction, the drag increases gradually with \( v \) (see Figure 3(a)). However, in the presence of friction, specifically, \( \mu = 0.1 \) and 0.5, we observe a constant drag regime at low velocities (see Figures 3(a,b)). Hilton and Tordesillas (2013) and Kumar et al. (2017) studied these drag regimes in the context of a dimensionless Froude number \( Fr \), which is the ratio of two timescales associated with the falling of grains in the wake and the forward motion of the intruder (assuming the intruder dimensions are much larger than the grains). They suggested that the constant drag regime exists for \( Fr < 1 \). Applying their individual definitions of \( Fr \) to our shape S3, Hilton and Tordesillas (2013) predicts that the constant drag regime should persist until \( \sqrt{v/dg} \approx 1.12 \), while Kumar et al. (2017) predicts the constant drag regime to exist until \( \sqrt{v/dg} \approx 1.23 \). Both definitions predict a constant drag regime in the velocity range very close to what we have observed in our study. While it is not clear how this definition of \( Fr \) can be extended to non-spherical intruders, we observe that the constant drag regime occurs for almost the same velocity range for all the shapes.

Beyond the constant drag regime that is observed at low intruder velocities for frictional system [see Supplementary Material (SM) Figure S1 for \( FD \) against low intruder velocity \( v \) in a log-log plot in SM], the drag on the intruder increases with velocity (see Figure S2 for \( FD \) against high intruder velocity \( v \) in a semi-log plot). The slope of the curve on the log-log plot seems to be varying with \( v \) suggesting that, for some constant \( a \), \( FD \propto \sqrt{v} \) is not valid for a granular medium. The kinetic drag contribution to the drag \( FK \) dominates yield drag \( FY \) at high velocities. This can be observed in the insets of Figure 3 where \( \psi = FY/v^2 \) approaches a constant value. We shall elaborate further on \( FK \) and the drag regimes in the next subsection. By comparing directly the values of \( FD \) at higher velocities (\( v > 10\sqrt{dg} \)), we note that the drag forces are in the order: \( FD_{S1} \approx FD_{S2} \approx FD_{S6} > FD_{S3} > FD_{S5} > FD_{S7} > FD_{S4} \), where \( FD_{S,i} \) is the drag \( FD \) for shape \( S_i \). Even though the differences are not large, this trend is observed consistently for all velocities higher than \( 10\sqrt{dg} \), and for all \( \mu \) considered in our study. At velocities lower than \( 1.23 \) (\( \sqrt{v/dg} < 1.23 \)), the trend is: \( FD_{S2} > FD_{S1} \geq FD_{S6} \geq FD_{S5} \approx FD_{S3} \approx FD_{S7} \approx FD_{S4} \).

Albert et al. (2001), in their study of the effect of the shape of a slowly moving object in a granular medium on jamming of grains around the object, highlighted that (a) streamlining the intruder significantly reduces the resistance offered by the granular medium to its motion, and (b) the increase in the drag on intruders that are longer in the flow direction is due to the creation of a more jammed state in front of the intruder. Ding, Gravish, and Goldman (2011) demonstrated that the local surface stress on an intruder is approximately equal to that on a plate oriented at the same angle as the local surface and moving at the same velocity and depth. Therefore, one could calculate the forces on an intruder by summing up the individual contributions of these stresses.

At low velocities (see Figure 3), it is evident that streamlining an object significantly reduces the drag. An example of this would be the shapes S2 (a rectangle) and S4 (an ellipse) of \( 20d \times 10d \) along \( x \) and \( y \), respectively. Since an ellipse is more streamlined than a rectangle of similar dimensions, the latter has a higher drag than the former. The streamlining of a body reduces the drag since the force chains applying a force of \( f_{sc} \) at a point \( P \) on the intruder contributes only \( f_{sc} \sin \gamma \) to the drag, where \( \gamma \) is the tangential orientation of the intruder at point \( P \) with \( x \)-axis (\( \gamma = 0 \) corresponds to the direction of motion). If the force chains developed were equally strong, streamlining of a body would significantly reduce drag.

Moreover, similar shapes, such as S3, S4, and S5 have identical drag at low velocities suggesting that viscosity is not a major contributor to the drag force in granular media (Albert et al. 2001). Although S4 is the longest shape and creates the most jammed force chains in front of itself compared to S3 and S5, the force chains tend to occur more laterally, thus, contributing less to the drag. However, it must be emphasized that if the bodies are equally streamlined, the drag increases with the length of the intruder at low velocities, as can be seen from \( FD \) for shapes S1 and S2 (Figure 3). This is because longer shapes delay the collapse of force chains in the intruder’s wake, allowing those with higher stress to be formed in front of the intruder. Also, since the forward-facing surfaces of S1, S2, and S6 are blunt (\( \sin \gamma = 1 \)), the force chains contribute more to drag than the other shapes. Moreover, triangle S6 allows the force chains to collapse almost immediately and, thus, has a lower drag than S1 or S2. S7 is also streamlined and, therefore, has less drag than S1 or S2 at lower velocities. This will be shown in later subsections.

As previously stated, at higher velocities, the drag forces are in the order \( FD_{S1} \approx FD_{S2} \approx FD_{S6} > FD_{S5} > FD_{S7} > FD_{S4} \). Firstly, S1, S2, and S6 are subject to an approximately equal drag, unlike at low velocities. Interestingly, Ding, Gravish, and Goldman (2011) showed that stress acting on a flat plate being dragged in granular media is identical to the small element on the intruder with a similar orientation to the flow \( \gamma \), irrespective of the shape of the intruder. Since the front faces (side facing the flow) of these three intruders are oriented at the same angle to the flow, they experience the same magnitude of stress on the front face. Additionally, the other faces contribute very little to the drag as they do not experience contact with the grains. This leads to the three shapes, S1, S2, and S6, having an identical drag. It is noteworthy that, at low velocities, the drag is not equal for the three shapes because they hinder the jamming and collapse of grains around them.

Other shapes with curved front surfaces experience a lower drag than S1, S2, and S6. The curved surfaces contribute sin \( \gamma < 1 \) times the normal force on the intruder’s surface to the drag. Therefore, the more blunt an object is, the more drag it experiences. Hence, S5 has a lower drag than...
S1, S2, and S6 but higher than S3. As the shape becomes less blunt, the drag is further reduced, with S4 having a lower drag than S3. Interestingly, S7 also has a higher $F_D$ than S4 because two of the sides of S7 are tilted at an angle of $\pi/3$ to the flow. This is very close to the angle at which a plate moved through granular media experiences the maximum stress (Ding, Gravish, and Goldman 2011). S3 and S7 experience similar drag due to the summation of drag components being almost similar. We discuss the force profiles on the intruders in section 3.5.

### 3.2. Number of contacts, kinetic drag, and drag regimes

To understand the forces acting on an intruder, it is imperative to reflect on the role of the grains in contact with the intruder: it is these grains that are ‘directly’ responsible for the forces. The average number of grain contacts, $N$, changes with the intruder’s shape as well as its velocity. Therefore, in this subsection, we present our results and discussion on $N$ and the relation between $N$, $F_K$, and the drag regimes.

The number of contacts $N$ is calculated by averaging over several configurations after the intruder has reached a steady-state behavior, i.e., when the drag force fluctuates around a well-defined mean. Of course, $N$ is a strong function of the shape of the intruder, but it does not imply that two shapes with the same number of contacts at a given $v$ experience equal forces. For example, a blunt and a streamlined object may have a similar number of contacts.

The variation of $N$ with $v$ is shown in Figures 4(a–c) on a semi-log plot for $\mu = 0$, 0.1, and 0.5, respectively. Two distinct regimes can be identified: an exponential one in which $N \propto e^{\nu v}$, with $\epsilon$ constant, and a second in which $N = N_\infty$ is constant. The latter seems to be in the same velocity range in which the $v^2$ dependence of drag force is expected. Although we do not exactly understand the physical picture behind this saturation, it is consistently present for all the shapes and the $\mu$ considered in our study. However, this saturation value can be approximated as $N_\infty \approx \Phi l_{cs}/d$, where $\Phi$ is the packing fraction of the bed and $l_{cs}$ is the cross-section length perpendicular to the direction of motion. Let $\phi = N_\infty/N_0$ where $N_0$ is the number of contacts in the yield limit, which is roughly equal to $\Phi S/d$ where $S$ is the perimeter of the obstacle. Therefore, $\phi \approx l_{cs}/S$. We have compared this rough theoretical estimate of $\phi$ with that obtained from the numerical simulations in Table 1. Additionally, we observe that $\epsilon \approx -0.1$ as can be seen in Figures 4(a–c) and the value of $\epsilon$ for each individual shape and $\mu$ is presented in Table 2.

Based on plots of $F_D$ and $N$ vs. $v$ for $\mu \geq 0$ and by comparing their respective behaviors for all the intruder shapes considered in the present study, we propose a three-regime model for the average drag force $F_D$ acting on the intruder. In the first, which is observed in the range $1 < v/\sqrt{\Phi g} < 4$, the drag is constant; the second occurs in the range $8 < v/\sqrt{\Phi g} < 12$ and the third for $10 < v/\sqrt{\Phi g} < 12$. In the remainder of this subsection, we explore the dependence of the kinetic drag on the velocity for each of these regimes.

Each contact exerts a force on the intruder whose magnitude depends on its location with respect to the intruder’s line of motion. It is difficult to correlate the average number of contacts $N$ with the force on the intruder. A previous study (Hilton and Tordesillas 2013) proposed $F_K \propto Nv^2$ with $\zeta = 1$, but they considered only the first two drag regimes. Here we assume that $\zeta$ can vary depending on the regime:

\[ F_K = 0 \]  \text{Regime I:}  \\
\[ F_K = zv^3 N/N_0 = z \exp(\epsilon v) v^3 \]  \text{Regime II:}  \\
\[ F_K = \beta v^3 N/N_0 = \beta v^3 l_{cs}/S \]  \text{Regime III:}

where $z$ and $\beta$ are constants. Parameters corresponding to the best fit of these equations to our simulation data, along with their coefficient of determination, are presented in Table 2.

### Table 1. Comparison of $\phi$ obtained from numerical simulations and the theoretical estimate, $l_{cs}/S$.

| Shape | $\phi_{\text{simulations}}$ | $\phi = l_{cs}/S$ |
|-------|-----------------------------|-------------------|
| S1    | 0.20                        | 0.25              |
| S2    | 0.24                        | 0.20              |
| S3    | 0.22                        | 0.32              |
| S4    | 0.17                        | 0.21              |
| S5    | 0.27                        | 0.37              |
| S6    | 0.30                        | 0.33              |
| S7    | 0.34                        | 0.33              |

Figure 4. The variation of the mean number of contacts $N$ on an intruder with the velocity at (a) $\mu = 0$, (b) $\mu = 0.1$, and (c) $\mu = 0.5$ for all the shapes in our study.
Table 2. The values of fits and constants.

| Shape | $\mu$ | $\zeta_1$ | $\zeta_2$ | $R^2_1$ | $R^2_2$ |
|-------|-------|-----------|-----------|---------|---------|
| S1    | 0.0   | 0.133     | 1.064     | 0.981   | 1.970   | 0.983   |
|       | 0.1   | 0.120     | 1.319     | 0.993   | 2.168   | 0.973   |
|       | 0.5   | 0.099     | 1.296     | 0.999   | 2.411   | 0.976   |
| S2    | 0.0   | 0.115     | 1.106     | 0.984   | 2.244   | 0.974   |
|       | 0.1   | 0.119     | 1.403     | 0.997   | 2.591   | 0.991   |
|       | 0.5   | 0.109     | 1.440     | 0.961   | 2.420   | 0.986   |
| S3    | 0.0   | 0.126     | 1.130     | 0.986   | 1.752   | 0.922   |
|       | 0.1   | 0.123     | 1.290     | 0.997   | 1.938   | 0.942   |
|       | 0.5   | 0.099     | 1.186     | 0.991   | 2.094   | 0.955   |
| S4    | 0.0   | 0.132     | 1.126     | 0.993   | 2.143   | 0.988   |
|       | 0.1   | 0.133     | 1.301     | 0.997   | 2.300   | 0.978   |
|       | 0.5   | 0.116     | 1.363     | 0.991   | 2.188   | 0.975   |
| S5    | 0.0   | 0.122     | 1.063     | 0.991   | 1.422   | 0.905   |
|       | 0.1   | 0.113     | 1.25      | 0.994   | 1.746   | 0.945   |
|       | 0.5   | 0.090     | 1.191     | 0.979   | 2.022   | 0.945   |
| S6    | 0.0   | 0.122     | 1.106     | 0.981   | 1.296   | 0.966   |
|       | 0.1   | 0.113     | 1.327     | 0.998   | 1.601   | 0.951   |
|       | 0.5   | 0.094     | 1.296     | 0.993   | 2.08    | 0.976   |
| S7    | 0.0   | 0.092     | 1.001     | 0.994   | 1.579   | 0.951   |
|       | 0.1   | 0.099     | 1.292     | 0.997   | 1.807   | 0.911   |
|       | 0.5   | 0.073     | 1.225     | 0.983   | 2.147   | 0.932   |
| Mean  | 0.0   | –         | 1.001     | –       | 1.579   | –       |
|       | 0.1   | –         | 1.292     | –       | 1.807   | –       |
|       | 0.5   | –         | 1.225     | –       | 2.147   | –       |

Figure 5. The variation of lift on an intruder with the velocity at (a) $\mu = 0$, (b) $\mu = 0.1$, and (c) $\mu = 0.5$ for all the shapes in our study.
It is generally observed that above a certain velocity, particle contact only occurs on the leading side of the intruder with flow detachment from both its top and bottom surfaces. This flow detachment plays an important role in the decrease of $F_L$ beyond a certain $v$ for shapes $S_1$ (square), $S_2$ (rectangle), and $S_4$ (ellipse with the major axis aligned with the $x$-direction), compared to the other intruders. The lift force on the equilateral triangle $S_7$ (with an edge pointing to the moving direction), disk $S_3$, and ellipse major $S_5$ shapes varies little with $v$ for $\mu = 0.0$. This could be due to the maximum particle interaction being concentrated on the frontal part. Interestingly, a negative $F_L$ is observed for an equilateral triangle with its edge pointing opposite to its direction of motion ($S_6$) in the low-velocity regime. In this case, particles traversing the edges of the blunt surface exert more force on the upper inclined surface than on the lower one. This is because the particles fall on the upper inclined surface of $S_6$, while the particles have to move against gravity to reach the lower inclined surface. It is also evident that the lift force saturates for most of the shapes in the high-velocity regime.

For systems with friction coefficients $\mu = 0.1$ and 0.5, the lift forces on $S_1$, $S_2$, and $S_4$ are higher than the other shapes in the intermediate velocity regime. The reason is the flow detachment and the larger contact surface around the top and bottom region of the intruder. The lift force on shapes $S_3$, $S_5$, and $S_6$ shows little variation for $\mu = 0.1$ for a range of $v$, while for $\mu = 0.5$ the lift force is higher for $S_3$ and $S_5$, in the low-velocity regime and then gradually saturates in the high-velocity regime. Unlike the other shapes, the net lift force on $S_6$ is small for all values of $\mu$ and is negative in the low-velocity regime.

The two equilateral triangles $S_6$ and $S_7$ have different orientations (edges pointing opposite and along the moving direction, respectively). This significantly impacts the $F_L$ experienced by the two shapes, as seen in Figure 5.
Albert et al. (2001) observed a nonlinear depth dependence of the drag force on a discrete object moving at a low velocity immersed in a granular bed at a depth of 40–150 mm (equivalent to 44–166 granular bed particles depth). Guillard, Forsterre, and Pouliquen (2013) observed a depth-independent drag force on a cylindrical object immersed deep (120 particle lengths or more) in a granular medium and rotated about the vertical axis. A few studies (Potiguar and Ding 2013; Guillard, Forsterre, and Pouliquen 2015; Panaitescu, Clotet, and Kudrolli 2017) also reported a linear depth dependence. In the context of these results, we compare \( F_L \) on the different intruder shapes as a function of their immersion depth in Figures 7(a,b). The results are shown for two intruder velocities 15 and 5 at \( \mu = 0.1 \). In the simulations, the intruder is placed at six different depths \( h/d = 15, 45, 70, 105, 140, 170 \). \( F_D \) increases linearly with an increase in \( h/d \) for all the intruders considered here, confirming that the drag is proportional to the hydrostatic pressure. In a granular medium, the number of particles above the intruder increases with an increase in its depth, thus increasing hydrostatic pressure. The \( F_D \) for the various heights \( h/d \) at 15 and 5 is ordered: \( F_{D, S3} > F_{D, S1} > F_{D, S4} \approx F_{D, S7} \approx F_{D, S5} \approx F_{D, S6} \). The same trend is also observed for 5. The S1 and S2 experience the maximum drag for all \( h/d \) for the two \( v \) shown in the plots while the other shapes have identical \( F_D \) within five percent of each other at a particular depth. Moreover, the difference between the highest (S2) and lowest (S7) drag forces calculated for one random depth is not more than 25%.

It has been reported in the literature that the lift force either saturates (Guillard, Forsterre, and Pouliquen 2014; Debnath, Rao, and Nott 2017) or increases (Ding, Gravish, and Goldman 2011) with the immersion depth of intruder in a granular medium. To determine the shape and depth dependence, we plot the lift force \( F_L \) as a function of \( h/d \) for two intruder velocities, \( v/\sqrt{d_0} = 1 \) and 5, at \( \mu = 0.1 \) in Figures 7(c,d). Minimal change in lift force for intruders S3, S4, and S5 is observed at \( v/\sqrt{d_0} = 1 \). The lift force on intruders S1 and S2 increases sharply below a certain depth for \( v/\sqrt{d_0} = 1 \). For the same velocity, \( F_L \) on S7 increases up to a certain depth and then saturates with a further increase in the depth. For \( v/\sqrt{d_0} = 5 \) there is a fluctuation in lift force for all the intruders except for S6 which shows a gradual decrease in \( F_L \) with an increase in \( h/d \). S6 and S7 with the same geometry but different orientations in the \( x \)-direction have the lowest and highest lift forces, respectively for almost all the depths considered. The lift force acting on S6 moving at \( v/\sqrt{d_0} = 1 \) shows little variation with depth, while at \( v/\sqrt{d_0} = 5 \) it decreases with the intruder depth.

While the results of Figure 7 suggest that the relation between drag and depth can be easily understood, the same is not true for the lift force, even though both forces result from the repulsive interactions between granular particles and the intruder. The lift force experienced by the intruder also depends upon the number of particle contacts at its upper and lower surface. So the forces depend on the specific region of contact around the intruder surface. Thus, if the force acting perpendicularly on the lower surface is largely due to the particle contacts, then the intruder experiences a positive \( F_L \).
and otherwise, it has a negative $F_l$. In the next section, we examine the force distribution on the intruder surface and how it influences the drag and lift forces.

### 3.5. Force profile along the surface of the intruder

To develop a better understanding of how the lift and drag forces acting around the periphery of a moving intruder, we have examined the force distribution as a function of the angle of contact, $\theta$ relative to the center of the intruder. The geometry and coordinate system for the shapes $S_1, S_4, S_6$, and $S_7$, are shown in Figure 8, and the force distributions for all objects are shown in Figure 9. Angular positions ranging from 0 to $\pi$ correspond to the upper half of the intruder, while the range $-\pi < \theta < 0$ corresponds to the bottom half of the intruder. $\theta = 0$ is in front of the intruder and $\theta = \pm \pi$ is in the back.

Let us examine the distribution of the normalized drag force shown in Figure 9(a). We first note that the normalized distributions vary only weakly with the intruder velocity. The drag force experienced by the intruder is mainly due to particle contacts at the front. Thus, the shapes $S_1, S_2$...
and S6, which present a blunt face in the direction of movement, experience a large contribution to the drag for $|\theta| < \pi/4$ (S1 and S2) and $|\theta| < \pi/3$ (S6). A sharp increase in drag is observed at the leading edges $\theta = \pm \pi/4$ (S1 and S2) and $\theta = \pm \pi/3$ (S6). This happens because when the blunt-faced intruder moves within the medium, it pushes the particles along with it, and the colliding particles then gradually slide on the intruder surface along the leading corner edges. Due to this, there is maximum perpendicular contact at the edges, and the drag force is higher in that region. There are more particle contacts at the backside of the intruder at low velocities compared with the case of high velocities (no visible contacts as shown in Figure 6). Though the particles exert minimal contact forces on the trailing edges as they just slide down due to gravity after contacting the leading surface of the intruder. Thus the contribution to the drag force is almost zero for $\pi/4 < |\theta|$ (S1 and S2) and $\pi/3 < |\theta|$ (S6). The shapes S3, S4, S5 and S7 all have a maximum drag force for $\theta = 0$. The curved intruders S3 and S5 have a maximum drag force for $\theta = 0$. The shapes S4 and S7, which have a pointed edge in the direction of motion, experience a higher drag at $\theta = 0$. The triangle S7 experiences a maximum drag on its front edges, with a strong spike around $\theta = 0$. There is also a slight asymmetry with a stronger drag force on the lower face. As expected, the drag force on the trailing face is very small.

We now consider the distribution of the (un-normalized) lift force shown in Figure 9(b). The shapes S1, S2, and S6, exhibit similar lift force profiles for $|\theta| > \pi/4$ as they present a blunt face in the leading direction. The rectangle S2 experiences high lift force for $-\pi/4 < \theta < -\pi/3$. The curved intruders S3, S4, and S5 exhibit a higher positive lift force suggesting a larger number of particle contacts on their lower surface. This is due to their more streamlined form compared to other shapes. The equilateral triangles S6 and S7 exhibit different lift force profiles. The former, with its blunt front face, experiences essentially zero lift on its trailing sides ($\theta > \pi/3$), while S7 experiences strong positive lift forces on its lower leading face $-2\pi/3 < \theta < 0$ and a weaker negative lift force on its upper leading face, $0 < \theta < 2\pi/3$. We also note that the lift force profile varies weakly with velocity for shapes S5, S6, and S7, while the variation is more significant for the other shapes. At high velocities, the magnitude of the lift force decreases due to flow detachment (Potiguar and Ding 2013). These observations are consistent with the variation of the total lift force with velocity shown in Figure 5.

4. Conclusions

We have presented extensive numerical simulation results of an intruder dragged horizontally through a granular medium to understand the drag and lift forces it experiences as a function of its velocity ($v$), immersion depth ($h/d$), and shape. The drag force gradually increases with $v$ in frictionless systems ($\mu = 0$), while we observe a constant drag regime at low velocities when friction is present. For a fixed cross-section, the drag force depends weakly on the intruder shape. In contrast, the lift force has a strong shape dependence. It may increase in a certain velocity range, but we observe a decrease in the lift force at higher velocities. The intruder shape has a major effect on the distribution of contacts around its surface, which explains the strong lift experienced by certain shapes. The force profiles around the intruder surface, resulting from granular contacts, exhibit a strong angular dependence.

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Disclosure statement

There are no conflicts to declare.

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