Finding heavy Higgs bosons heavier than $2m_W$ in dileptonic $W$-boson decays

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We reconsider observables for discovering a heavy Higgs boson (with $m_h > 2m_W$) via its dileptonic decays $h \rightarrow WW \rightarrow ℓνℓν$. We show that observables generalizing the transverse mass that take into account the fact that both of the intermediate $W$ bosons are likely to be on-shell give a significant improvement over the variables used in existing searches. We also comment on the application of these observables to other decays which proceed via narrow-width intermediates.

The LHC collaborations have now excluded a heavy Higgs boson, with mass between 127 and 600 GeV, at the 95% confidence level, assuming that its production and decay proceed according to the predictions of the Standard Model. Nevertheless, there is a clear motivation to continue the search in this mass region. Indeed, theories beyond the Standard Model, with additional degrees of freedom, typically predict a lower cross section times branching ratio for Higgs production and decay in any given channel, for three reasons. Firstly, the branching ratio will necessarily be reduced whenever there are extra states into which a given Higgs boson may decay. Secondly, in theories with an extended scalar sector, mixing reduces the cross section to produce a given mass eigenstate. Thirdly, since the dominant production mechanism for Higgs bosons at the LHC is via loop-level gluon fusion, the production cross section will be changed by the presence of additional coloured states in the theory. If one of these states plays the rôle of cancelling the quadratic divergence in the Higgs mass, coming from a loop of top quarks, then it necessarily interferes destructively with the dominant contribution to gluon fusion in the Standard Model, which also comes from a top quark loop. Furthermore, it is observed (though sadly no theorem to the effect has been proven) that the cross-section is reduced in all models in which the Higgs boson is composite.

One way to search for a heavy Higgs boson is to look for evidence of it decaying to the fully leptonic final state $ℓνℓν$ (where $ℓ$ is an electron or muon) which results from an intermediate pair of (possibly virtual) $W$-bosons. A key step in many such analyses comprises a selection, or cut, based on a variable that should have a strong correlation with the mass of the Higgs boson in ‘signal’ events. Typically this variable will be a transverse mass of some kind, of which many have been described in the literature. The transverse mass variable $m_T$ of Ref. [13] (see also [8, 14]) is used in current LHC $h \rightarrow WW \rightarrow ℓνℓν$ searches.

It has previously been observed that a kinematic variable $m_T^{\text{bound}}$ (which, for reasons that will become clear, we refer to henceforth as $m_T^{\text{lower}}$) which demands intermediate mass-shell constraints for the tau leptons might prove advantageous in $h \rightarrow ττ$ decays. As was indicated in that paper, the same variable (after the trivial replacement $m_τ \rightarrow m_W$) might also prove useful in the context of $h \rightarrow WW$ when a heavy Higgs boson has mass greater than $2m_W$, since the intermediate $W$-bosons should be produced close to their mass shells. Substantiation of this suggestion was left open to further study and one of our intentions here is to perform that follow-up. We also intend to go somewhat further, in providing a complete discussion of the variables that characterize the kinematics of $h \rightarrow WW \rightarrow ℓνℓν$ decays, in a sense that will be made explicit below. We will find that the pre-existing variables $m_T$, $m_T^\text{upper}$, and $m_T^\text{lower}$ provide a complete description in cases where neither or either, respectively, of the intermediate $W$-bosons is produced on-shell. A complete description of the cases when both of the $W$-bosons are produced on-shell requires not only the extant variables $m_T^{\text{lower}}$ and $m_T^{\text{upper}}$, but also a new variable, which we denote $m_T^{\text{bound}}$. Our simulations suggest, moreover, that $m_T^{\text{bound}}$ gives a significant improvement in discovery potential for $h \rightarrow WW \rightarrow ℓνℓν$.

Having finished our preamble, we now describe more explicitly how variables of this kind may provide a complete characterization of the kinematics of events. Since the decay of the parent Higgs boson involves invisible neutrinos, one cannot find a variable that measures its mass directly; rather the best one can do is to find variables that bound the mass of the parent in some way. Myriad variables of this type may exist and what one would like to do, presumably, is to find the variable or variables that give the optimal bounds on the mass in an

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¹ The variable $m_T$ was referred to as $m_{T^{\text{true}}}$ in [13] to distinguish it from other, approximate transverse mass variables in use at

² In the mass range $m_W < m_h < 2m_W$, one might reasonably assume that one or other of the $W$ bosons is produced on shell [17]. However, simulations reported in [18] found no significant improvement in discovery potential when $m_T^{\text{upper}}$ was substituted for $m_T$ in existing ATLAS collaboration searches.
event.

This search for the optimal variables can be done in a more-or-less definitive way if one is willing to restrict one’s attention to kinematics alone. That is to say, suppose one assumes that a signal event corresponds to some decay topology (here the decay of a parent Higgs into two intermediate $W$ resonances, followed by decays of each intermediate into a combination of visible and invisible daughter particles). One may then write down the various kinematic constraints (corresponding to energy-momentum conservation and the mass-shell conditions) and ask which values of the *a priori* unknown masses are allowed, in the sense that the kinematic constraints then admit solutions in which the unmeasured momenta and energies in the event are real and real-positive, respectively. The resulting allowed mass region is an event observable and encodes all of the information which can be gleaned about the masses in an event using kinematics alone. The boundary of the allowed region can be described in terms of one or more relations between the masses, which are themselves event observables and which give the optimal bounds on the mass in an event. Given multiple events, the allowed mass region is obtained as the intersection of the allowed regions for each event and is itself an observable.3

This abstract recipe leads, in many cases, to simple, pre-existing observables. In the canonical example of a decay of a parent to a single invisible daughter (together with possibly multiple visible particles), the observable that results is the familiar transverse mass, $m_T$, invoked in the discovery of the $W$-boson in its decay to a charged lepton and a neutrino; for single parent decays into multiple invisibles, the resulting variable is $m_H$; for identical decays of a pair of parents into invisible particles, the variable is $m_{T2}$.

Thus, these variables encode all of the information that is available from kinematic considerations alone (subject to an assumed decay topology) and there is no point in trying to devise further variables to glean further information from kinematics.

In the next Section, we firstly try to prove the equivalence between the kinematic constraints with one intermediate on-shell and the variable $m_T^\star$ and secondly the equivalence between the constraints with both intermediates on-shell and the variables $m_{T1}^{\text{lower}}$ and $m_{T2}$. Our first attempt at a proof succeeds: $m_T^\star$ is equivalent to all of the information contained in the kinematic constraints when one intermediate is on-shell. But for the topology with both intermediates on shell, we fail. Indeed, we show that $m_{T1}^{\text{lower}}$ and $m_{T2}$ do not capture all of the information in the kinematic constraints. Rather there is a third, distinct variable, which we call $m_{T1}^{\text{upper}}$, and which gives an upper bound on the mass of the Higgs (or whatever parent particle is being considered).

In what follows we show that while $m_{T1}^{\text{lower}}$ only marginally enhances the discovery potential for a Higgs boson using current ATLAS collaboration search strategies, $m_{T1}^{\text{upper}}$ gives a significant improvement. In an Appendix, we discuss issues related to the existence of $m_{T1}^{\text{lower}}$ and $m_{T1}^{\text{upper}}$.

### KINEMATIC EQUIVALENCE

Since we are concerned with the three transverse masses $m_T, m_T^\star$ and $m_{T1}^{\text{lower}}$, it is helpful to recall and compare their definitions. All three make the assumption that the observed $l\nu l\nu$ final state resulted from the decay of a single progenitor particle or resonance (i.e. the Higgs in the case of the signal) whose mass is, *a priori*, unknown. All three assert that the sum of the transverse momenta of the neutrinos should equal the observed missing transverse momentum. All three accept that: (i) the way the transverse momentum is shared between the two neutrinos is unknown, and (ii) that the longitudinal momentum component of each neutrino is unknown. Finally, all three transverse masses are defined to be the greatest possible lower bound for the unknown mass of that progenitor particle, subject to consistency and any remaining constraints.5 It is only in these remaining constraints that the transverse masses differ: $m_T^\star$ applies none, $m_T$ permits only neutrino momenta that place at least one $W$-boson on mass shell, while $m_{T1}^{\text{lower}}$ allows only neutrino momenta that place both $W$-bosons on mass shell. In short, history would have been kinder to us had $m_T^\star$ and $m_{T1}^{\text{lower}}$ been named $m_{T1}^W, m_{T1}^{WW}$, and $m_{T1}^{WW}$ respectively. The variables and constraints are summarized in Table II.

We now attempt to prove the equivalence between the

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3 Whether this observable is the optimal observable for discovering the parent is unclear. To settle this, one would first have to define what one meant by an optimal discovery observable. For discussion, see e.g. 24 27.

4 The equivalence between variables and kinematics is also important for the purposes of determining which particle masses can be measured using kinematics alone: as an example, for a (possibly parent-produced) parent particle that decays off-shell to multiple invisible particles, it is a theorem that one can – at least in principle – measure the parent mass and the sum of the masses of the invisible daughter particles using kinematic information alone 24 27.

5 Equivalently, they may be defined to be the smallest possible progenitor mass observable by a search over all unknown neutrino momentum components that satisfy the remaining constraints. Indeed, this brute force search technique is used in the authors’ implementations of $m_T^\star$ and $m_{T1}^{\text{lower}}$ calculators. These C++ implementations, free of external dependencies, are available on request. A simple analytic answer is available for $m_T$ – see Eq. (1) of 10.
variables \(m_T^\ast\) and \(m_{T_{\text{lower}}}^\ast\) and the corresponding kinematic constraints.

### One intermediate on-shell

The theorem to be proved is a formalization of the colloquial: \(m_T^\ast\) is equivalent to all the information contained in the kinematic constraints. To wit, in a notation in which \(p^\mu = (E, p_T, q)\):

**Theorem 1** The kinematic constraints \([1]\) to \([3]\) and either \([4]\) or \([5]\) admit a solution with momenta in \(\mathbb{R}\) and energies in \(\mathbb{R}^+\) iff:

\[
m_h \geq m_T^\ast.
\]

Here, \(m_T^\ast\) is defined as the minimum value of

\[
m_h \geq m_T^\ast
\]

obtained by varying the unobserved momenta in \(\mathbb{R}\) and energies in \(\mathbb{R}^+\). This definition makes the necessity of \(m_h \geq m_T^\ast\) obvious; it is the sufficiency of the condition that requires deliberation.

So, does \(m_h \geq m_T^\ast\) imply that the kinematic constraints have a solution? One way to show this would be to consider every \(m_h \geq m_T^\ast\) and explicitly construct a solution therefor. An easier way is to show that one can find solutions corresponding to arbitrarily large \(m_h\) and then to invoke continuity and the intermediate value theorem to show that one can find solutions for any \(m_h\) between \(m_T^\ast\) and \(\infty\).

Let us then exhibit a solution for arbitrarily large \(m_h\) that satisfies constraints \([1]\) to \([4]\) (if the other \(W\) is on-shell, the required solution can be obtained by interchanging primed and unprimed quantities). It is given by

\[
p^\mu_\nu = \lim_{q'_\nu \to \infty} (\sqrt{q'^2_\nu + p'^2_T}, p_T, q'_\nu),
\]

\[
p^\nu_\nu = |q_\nu|, (0, q_\nu)
\]

where \(q_\nu\) is given by either of

\[
q_\nu = \frac{m_{W}^2}{2p_T^2} \left(q_l \pm \sqrt{q_l^2 + p_T^2}\right).
\]

This yields a value of \(m_h\) given by

\[
m_h^2 = \lim_{q'_\nu \to \infty} (m_{W}^2 + 2(p_l + p_{\nu}) \cdot p'_l + 2(p'_l + p_l + p_{\nu}) \cdot p'_l)
\]

\[
\rightarrow 2((E'_l + E_l + |q_\nu|) \sqrt{q'^2_\nu + p'^2_T}
\]

\[
- (p'_T + p_{T} \cdot p_{T} - (q'_l + q_l + q_\nu)q'_\nu))
\]

\[
\rightarrow 2((E'_l - q'_l) + (E_l - q_l) + (|q_\nu| - q_\nu))q'_\nu,
\]

where in the last line we assumed \(q'_\nu > 0\) without loss of generality (this amounts to a choice of which proton beam constitutes the +z direction). The terms in parentheses in the coefficient are each positive semi-definite and so no cancellation of the coefficient of \(q'_\nu\) can occur. Thus \(m_h\) grows without bound unless all terms vanish. To do so, both leptons must have vanishing transverse momenta, in which case they would fall outside the detector acceptance and this would not be identified as a di-lepton plus missing momentum event.

### Both intermediates on-shell

It is now easy to show that an analogous theorem cannot be proven for \(m_{T_{\text{lower}}}^\ast\), which applies to the topology in which both constraints \([4]\) and \([5]\) are satisfied. The reason is that one cannot find solutions to the kinematic constraints corresponding to arbitrarily large \(m_h\). Indeed, one can easily convince oneself that to get arbitrarily large \(m_h\), at least one of the neutrino momenta must become arbitrarily large. But each neutrino is now produced in the decay of a \(W\)-boson, which simultaneously results in a charged lepton of finite (and measured) momentum. Now, there is only one way in which a two-body decay can produce one daughter of finite momentum and another with infinite momentum in the lab frame: the rest frame of the \(W\) must be infinitely boosted exactly.
in the direction of motion of the neutrino (as measured in the W rest frame). This doesn’t work, because the lepton momenta are then not only finite but arbitrarily small, in contrast with their measured values in a generic event.

Thus, there is more information in the kinematic constraints than is captured by \( m_{T}^{\text{lower}} \) alone. In particular, since \( m_h \) cannot reach arbitrarily large values in events, there is a distinct variable, obtained by maximizing the sum of lepton and neutrino momenta in events, subject to the above constraints. This variable will be bounded below by \( m_h \) and we call it \( m_{T}^{\text{upper}} \). By the intermediate value theorem, this variable, together with \( m_{T}^{\text{lower}} \) does contain all of the information in kinematics.

### MONTE CARLO SIMULATIONS

Simulations, similar to those performed in Ref [13], are used to test the extent to which using \( m_{T}^{\text{lower}} \) and \( m_{T}^{\text{upper}} \) might be expected to enhance the statistical significance of a \( h \to WW \) signal above the dominant WW continuum background and the \( t\bar{t} \) background. We use the HERWIG 6.505 [32, 33] Monte Carlo generator, with LHC beam conditions (\( \sqrt{s} = 7 \) TeV). Our version of the generator includes the fix to the \( h \to WW \) cross section for all values of \( m_h \) and \( \tau \rightarrow \ell \nu \) decay mode.

The discovery potential using separately \( m_{T} \), \( m_{T}^{\text{lower}} \), or \( m_{T}^{\text{upper}} \), is shown in Figure 1. The first result is that when \( m_h \approx 2 m_W \) the doubly-on-shell lower bound variable, \( m_{T}^{\text{lower}} \) has significantly better sensitivity than the simple transverse mass \( m_{T} \) (or indeed the singly-on-shell variable \( m_{T}^{*} \), which is almost indistinguishable from \( m_{T}^{\text{upper}} \)). Secondly, we observe that the distribution of the doubly-constrained upper bound \( m_{T}^{\text{upper}} \) shows a markedly greater discovery potential again, relative to either \( m_{T} \) or \( m_{T}^{\text{lower}} \). Since any event satisfying the constraints of Table IV will generate both a lower and an upper bound, and since those bounds are not closely correlated (Figure 2) each adds information relative to the other.

### OTHER FINAL STATES

We note that it is also possible to construct equivalent bounds for any other decays of the form \( A \to B + C \) where \( B \) and \( C \) are produced near their mass shell and each decays to an invisible particle and one or more visible particles. A case relevant to LHC Higgs physics is the decay mode \( h \to \tau \tau \), followed by semi-invisible decay of each of the \( \tau \) leptons. Previous simulations [15] indicated that adding the tau mass constraints led to improved discrimination of the signal from the dominant \( Z \to \tau \tau \)

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7 The variable \( x_i \) is the momentum fraction of the \( i \)th tau carried by its daughter lepton and \( m_{T}^{\tau \tau} \) is the di-tau invariant mass. They are calculated using the approximation that each \( \tau \) was collinear with its daughter lepton.
CONCLUSION

When a heavy higgs boson with mass $> 2m_W$ decays via $h \rightarrow WW \rightarrow l\nu l\nu$, we have shown that one can use knowledge about the narrow width of the intermediate $W$ bosons to construct both upper and lower bounds on $m_h$. We demonstrate that these two bounds together make maximal use of the kinematic information about $m_h$ in each event. Since the two bounds are reasonably uncorrelated, the ATLAS and CMS experimental collaborations should be able to obtain improved kinematic sensitivity in an analysis which simultaneously makes use of both bounds.

We note that the same suite of variables can find use for discovery or mass determination in other decays of the form $A \rightarrow B + C$ followed by semi-invisible decay of each narrow-width intermediate particle.

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Appendix: Remarks on the existence of $m_T^{\text{lower}}$ and $m_T^{\text{upper}}$

Note that there exist ‘input momenta’ for which it is not possible to satisfy all the constraints required by $m_T^{\text{lower}}$ (and $m_T^{\text{upper}}$, to which the following arguments equally apply). For events containing such momenta, a value of $m_T^{\text{lower}}$ cannot be computed. In effect, the variable does not exist as those events are incompatible with the assumptions at face value. It is known [15] that $m_T^{\text{lower}}$ exists if and only if $m_{T2} < m_W$. The condition $m_{T2} < m_W$ can also be violated in $m_h > 2m_W$ events with sufficiently large detector mis-measurements, even if both $W$-bosons are produced on-shell. The largest source of mis-measurement is the missing transverse momentum, whose distribution can have a large, non-Gaussian tail. Though not discussed in [15], this potential problem might be thought more worrisome in the case of $h \rightarrow \tau\tau$ decays, since the typical size of an LHC $p_T$ mis-measurement greatly exceeds $m_\tau$. It is thus hard to see, naively, how the condition $m_{T2} < m_\tau$ may be maintained. Nonetheless, one is assisted by a

background, using the lower bound $m_T^{\text{lower}}$ [15]. However further simulations indicate that there seems to be little additional discrimination available from the upper bound in this case, perhaps because of the large hierarchy in masses between $m_h$ and $m_\tau$.

Note also that the variables $m_T$ and $m_T^* = m_T$ exist for all input momenta, and are therefore usable even when $m_T^{\text{lower}}$ is not. See discussion in [15].
property of the \( m_{T^2} \) variable under which large mis-
measurements of \( \not{p}_T \) can lead, in a non-negligible number
of events, to small changes (relative to \( m_T \)) in the value of
\( m_{T^2} \), provided that the intermediate particles in the
production process are sufficiently light. Indeed, in the
limit the intermediate pair of particles are massless, their
decay products are collinear and so the true \( \not{p}_T \) vector
lies in the smaller of the two sectors bounded by the
two visible transverse momenta. It follows that for such
configurations \( m_{T^2} \) must vanish, as there exists a split-
ing of the \( \not{p}_T \) (in fact the ‘true’ splitting) for which the
transverse mass of each pair of decays is zero. That \( m_{T^2} \)
is ‘small’ for well-measured events in this limit is not sur-
prising – after all, \( m_{T^2} \) is supposed to be bounded above
by the mass of either member of the pair, and we are
taking the limit in which those masses go to zero. What
is surprising, if anything, is that \( m_{T^2} \) can be forced ex-
actly to zero in a large number of events, even for finite
yet small intermediate particle masses. In this sense we
can say that \( m_{T^2} \) goes to zero faster than \( m_T \). It fol-

\[ m_{T^2} \to 0 \text{ faster than } m_T \]

This goes some way towards explaining why it was
found in [15] that \( m_{T^2}^{\text{lower}} \) could remain well-defined for
a large number of \( h \to \tau\tau \) events, even in the presence
of momentum mis-measurements. For \( h \to WW \) events
this issue is much less of a concern; we find that \( m_{T^2}^{\text{lower}} \)
exists for all but a few per cent of the \( h \to WW \) signal
events we simulate. We attribute the greater frequency
with which \( m_{T^2}^{\text{lower}} \) exists to the greater ease with which
the momentum mis-measurement errors (which have a
natural scale far below \( m_W \)) may be incorporated into
the intermediate particle mass constraints (whose natu-
ral scale is \( m_W \)).

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