Spatiotemporally localized solitons in resonantly absorbing Bragg reflectors

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We predict the existence of spatiotemporal solitons (“light bullets”) in two-dimensional self-induced transparency media embedded in a Bragg grating. The “bullets” are found in an approximate analytical form, their stability being confirmed by direct simulations. These findings suggest new possibilities for signal transmission control and self-trapping of light.

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Light propagation in periodic dielectric structures exhibits a variety of unique regimes that are technologically promising: nonlinear filtering, switching and distributed feedback amplification [1]. Of particular interest are gap solitons (GSs), i.e., moving or standing self-localized field structures centered in a band gap of the grating. These self-localized field structures arise due to the interplay between the medium nonlinearity and resonant Bragg reflections. Their spectrum is tuned away from the Bragg resonance by the nonlinearity at sufficiently high field intensities. Theoretical studies of gap solitons in Bragg gratings with Kerr nonlinearity have been followed up by their experimental observation in a nonlinear optical fiber with the grating written on it [2]. Gap solitons have also been theoretically studied in gratings with second harmonic generation [3].

A principally different mechanism giving rise to gap solitons has been found in studies of models consisting of a periodic array of thin layers of resonant two-level systems (TLS’s) separated by half-wavelength nonabsorbing dielectric layers, i.e., a resonantly absorbing Bragg reflector (RABR) [4] (Fig. 1). Such a RABR has been shown, for any Bragg reflectivity, to produce a vast family of stable solitons, both standing and moving ones. As opposed to the $2\pi$ solitons arising in self-induced transparency (SIT), i.e., near-resonant field-TLS interaction in a uniform medium [5–7], gap solitons in a RABR can have an arbitrary pulse area [8]. The existence of GS solutions can only be consistently demonstrated in a RABR with thin active TLS layers. By contrast, a recent attempt to obtain such solutions in a periodic structure uniformly filled with active TLS’s is physically doubtful, and fails for many parameter values.

The potential applications of GSs are based on the system’s ability to filter out (by Bragg reflection) all pulses except for those satisfying the GS dispersion condition, as well as the control of pulse shape and velocity [9]. It would be clearly desirable to supplement these advantages by immunity to transverse diffraction of the pulse, i.e., to achieve simultaneous transverse as well as longitudinal self-localization in the structure. This calls for the consideration of “light bullets” (LBs), multi-dimensional solitons that are localized in both space and time. In the last decade they have been theoretically investigated in various nonlinear optical media [10, 11], and the first experimental observation of a quasi two-dimensional (2D) bullet was recently reported [12]. In a recent work [12], we have predicted that uniformly 2D and 3D self-induced transparency (SIT) media can support stable light bullets.

In the present work, we aim to extend this investigation to RABR’s of the kind shown in Fig. 1 so as to combine LB and GS properties in resonantly absorbing media. We find that a RABR with any Bragg reflectivity and any absorption cross-section can support the propagation of attenuated stable LBs, which are closely related to the bullets we have found in uniform SIT media [12]. It should be noted that 2D LBs supported by a combination of the Bragg reflector with a different (second-harmonic-generation) nonlinearity were theoretically investigated in Ref. [13].

We start by considering a 2D SIT medium with a spatially-varying refractive index $n(z,x)$, which is described by the lossless Maxwell-Bloch equations [14].

\[
-i\varepsilon_{xx} + n^2 \varepsilon_z + \varepsilon_\tau + i(1 - n^2) \varepsilon - P = 0, \tag{1a}
\]

\[
\varepsilon_\tau - \varepsilon W = 0, \tag{1b}
\]

\[
W_\tau + \frac{1}{2}(\varepsilon^*P + P^*\varepsilon) = 0. \tag{1c}
\]

Here $\varepsilon$ and $P$ are the slowly varying amplitudes of the electric field and polarization, $W$ is the inversion, $z$ and $x$ are the longitudinal and transverse coordinates (measured in units of the resonant-absorption length), and $\tau$ is time (measured in units of the input pulse duration). The Fresnel number, which governs the transverse diffraction in the 2D and 3D propagation, has been incorporated in $x$ and the detuning of the carrier frequency $\omega_0$ from the central atomic-resonance frequency was absorbed in $\varepsilon$ and $P$. The Fresnel number $F$, detuning $\Delta \Omega$, and wave vector $k_0 \equiv \omega_0/c$ can be brought back into Eqs. (1) by the transformation $\varepsilon(\tau, z, x) \rightarrow (2/k_0) \varepsilon(\tau, z, x) \exp(-i\Delta \Omega \cdot \tau)$, $P(\tau, z, x) \rightarrow$
of the two-level atoms. The parameter $\eta$ is a ratio of resonant-absorption length in the two-level medium to the Bragg reflection length, and can be expressed as $\eta = a_1 \omega_c \tau_0 / \alpha$.

In the 1D case, a family of exact soliton solutions to Eqs. (3a) and (3b) was found in Ref. [6]: $\Sigma(\pm) = \left(\alpha; -\sqrt{\alpha^2 + 2} \right) \text{sech}(\Theta(t, z) \cdot \text{exp}(i\Phi), \quad P = 2\text{sech}(\Theta(t, z) \cdot \text{tanh}(\Theta(t, z)) \cdot \text{exp}(i\Phi), \quad W = 2\sec^2(\Theta(t, z) - 1), \quad \text{where } \Theta(t, z) = \alpha \tau + \sqrt{\alpha^2 + 2} z + \Theta_0, \quad \Phi \equiv \eta \Theta(t, z)^2 + \phi, \quad M \equiv -\alpha \sqrt{\alpha^2 + 2}, \quad \text{and } \Delta \equiv \eta (\alpha^2 + 1).

The shape of the fields $\Sigma^+$ and $\Sigma^-$ in this solution is similar to the SG soliton in the uniform 1D SIT medium. Inspired by this analogy and the fact that there exist LIs in the uniform 2D SIT medium which reduces to the SG soliton in 1D [20], we search for a LB solution to the 2D equations (3), which reduces to the exact soliton in 1D. To this end, we try the following approximation,

$$\Sigma^+ = 2 \alpha \sqrt{\text{sech}(\Theta_1 \cdot \text{sech}(\Theta_2 \cdot e^{i\eta M \tau + i\eta N z + i\tau / 4}, \quad (\text{4a})$$

$$\Sigma^- = -2 \alpha \sqrt{\alpha^2 + 2} \sqrt{\text{sech}(\Theta_1 \cdot \text{sech}(\Theta_2 \cdot e^{i\eta M \tau + i\eta N z + i\tau / 4}, \quad (\text{4b})$$

$$P = \sqrt{\text{sech}(\Theta_1 \cdot \text{sech}(\Theta_2)^2 + 1 / 4 \alpha^2 C^4 (\text{tanh}(\Theta_1 - \text{tanh}(\Theta_2)^2 - 2 (\text{sech}(\Theta_1 + \text{sech}(\Theta_2)^2)^2)^{1/2} e^{i\eta M \tau + i\eta N z + i\tau / 4}, \quad (\text{4c})$$

with $\Theta_1(\tau, z) \equiv \alpha \tau + \sqrt{\alpha^2 + 2} z + \Theta_0 + C x, \quad \Theta_2(\tau, z) \equiv \alpha \tau + \sqrt{\alpha^2 + 2} z + \Theta_0 - C x$, the phase $\nu$ and coefficient $C$ being real constants.

The ansatz (3) satisfies Eqs. (3a) and (3b) exactly, while Eqs. (3c) are satisfied to order $|C|^2$, which requires that $|C|^2 \ll 1$. The ansatz is relevant for arbitrary $\eta$, admitting both weak ($\eta \ll 1$) and strong ($\eta > 1$) reflectivities of the Bragg grating, provided that the detuning remains small with respect to the gap frequency, or $\eta \ll \omega_c / (\alpha + 1)$. Comparison with numerical simulations of Eqs. (3), using (3) as an initial configuration, tests this analytical approximation and shows that it is indeed fairly close to a numerically exact solution; in particular, the shape of the bullet remains within 98% of its originally presumed shape after having propagated a large distance, as is shown in Fig. (5).
FIG. 2. The forward-propagating electric field of the 2D “light bullet” in the Bragg reflector, $|E_\parallel|$, vs. time $\tau$ and transverse coordinate $x$, after having propagated the distance $z = 1000$. The parameters are $\alpha = 1$, $C = 0.1$ and $\Theta_0 = -1000$. The field is scaled by the constant $\hbar/4\pi\mu_0$.

We stress that 2D or 3D LB solutions of the variable-separated form $\Sigma^+ \sim \Sigma^- \sim f(\tau, z) \cdot g(x) \cdot \exp(iA\tau + iBx)$, with constant $A$ and $B$. We briefly discuss experimental conditions under which LBs can be observed in RABRs. The incident pulse should have uniform transverse intensity within its diameter $d$. For the transverse diffraction to be strong enough, one needs $\alpha_{\text{eff}} d^2/\lambda_0 < 0$, where $\alpha_{\text{eff}}$ and $\lambda_0$ are the inverse resonant-absorption length and carrier wavelength, respectively [23]. For $\alpha_{\text{eff}} \sim 10^3 \text{ m}^{-1}$ and $\lambda_0 \sim 10^{-4} \text{ m}$, one thus requires a diameter $d < 10^{-4} \text{ m}$, which implies that the transverse medium size $L_x \sim \lambda$ is a few $\mu\text{m}$.

In order to realize a RABR, thin layers of rare-earth ions [23] embedded in a semiconductor structure with a spatially-periodic RI [24] may be used. The two-level structures are the inverse resonant-absorption length and carrier lifetime, respectively [23]. In a RABR with transverse size $10 \mu\text{m}$, LBs depicted in Fig. 2 are localized on the time and transverse-length scales $\sim 10^{-13} \text{ s}$ and $1 \mu\text{m}$. Cryogenic conditions strongly extend the dephasing time $T_2$ and thus the LB lifetime, well into the $\mu\text{sec}$ range [23]. The construction of suitable structures constitutes a feasible experimental challenge.

To conclude, we have studied light bullets in SIT media embedded in a resonantly-absorbing Bragg reflector. Light bullets in a multi-dimensional Bragg reflector have the potential of serving as a novel type of optical filters, which stably transmit selected signal frequencies through their spectral gap and block others. They can also be used to both spatially and temporarily localize light in certain frequency bands.

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