Stabilization of double tearing mode growth by resonant magnetic perturbations

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Abstract

It is well known that for non-monotonic profiles of the safety factor $q$ with two $q = mn$ resonant surfaces inside the plasma ($mn$ being the poloidal/toroidal mode numbers), the low-$m$ double tearing modes (DTMs) are usually unstable, especially for plasmas with a high bootstrap current fraction as required for the steady operation of advanced scenarios. The effect of applied resonant magnetic perturbations (RMPs) on the $m/n = 2/1$ DTM growth is investigated numerically in this paper using two-fluid equations. The DTM growth is found to be stabilized by moderate static $m/n = 2/1, 4/2$ or $6/3$ RMPs below their penetration threshold if the distance between the two resonant surfaces and the local plasma rotation velocity at the outer resonant surface are sufficiently large. The outer magnetic island is stabilized due to the change of the local plasma current density gradient around the outer resonant surface caused by RMPs, while the inner island growth is stabilized by the bootstrap current perturbation in the negative magnetic shear region. The mode stabilization is more effective for a higher electron temperature, indicating a possible method to improve the DTM stability in a fusion reactor.

Keywords: resonant magnetic perturbation, error field, double tearing mode, magnetic island, plasma current density gradient, two-fluid physics, advanced scenario

1. Introduction

The internal transport barrier (ITB) has mostly been observed in tokamak experiments for non-monotonic profiles of the safety factor $q$ with negative magnetic shear in the central region, characterized by the significantly improved plasma confinement in the core, associated with a high bootstrap current fraction, as observed in [1–9]. Advanced tokamak scenarios with non-monotonic $q$-profiles are proposed for steady-state operations of ITER, making use of an ITB and the associated large bootstrap current.

Both theories and experiments indicate, however, that such $q$-profiles are subject to magnetohydrodynamic (MHD) instabilities, driven by the plasma pressure or current density gradient [3–17]. Even for a plasma pressure well below the ideal MHD instability limit, double tearing modes (DTMs), especially the $m/n = 2/1$ DTM ($mn$ being the poloidal/toroidal mode number), can be unstable with two resonant surfaces of the same $m/n$ value inside the plasma [4–6, 10–15]. Due to the coupling of the perturbations associated with the inner and outer resonant surfaces, the DTM is usually more unstable than the neoclassical tearing mode (NTM) in monotonic $q$-profiles [12–14]. Although differential plasma rotation weakens the coupling of these perturbations, the outer magnetic island can grow even without the coupling [12–14]. The perturbed bootstrap current has a stabilizing effect on the inner island, located in the negative magnetic shear region, but a destabilizing effect on the outer one in the positive magnetic shear region, leading to so-called neoclassical DTMs [13, 14]. Once the outer island grows to a sufficiently large width, the inner island will be locked to the outer one and the two islands grow together [14], similar to the case of forced magnetic reconnection in a monotonic $q$-profile by a large error field [18–25].
simultaneous growth of both islands will flatten the plasma pressure between the two resonant surfaces and possibly cause disruption [4–6, 14, 15]. Even in the case of an ideal mode driven by the plasma pressure gradient in the linear phase, magnetic islands will be formed when the mode growth is slowed down in the nonlinear phase such that non-ideal effects become important [26]. Therefore, suppression of the outer island growth is important for the stability of non-monotonic $q$-profiles and for extrapolating the ITB regime to a fusion reactor.

Similar to the stabilization of NTMs by electron cyclotron current drive (ECCD) [27–34], the DTM can also be stabilized by a sufficiently large driven current at the outer resonant surface. Recent studies predicted that due to plasma density perturbations, the deposition width of the electron cyclotron wave would be several times larger in ITER [35], leading to a weaker stabilizing effect due to a smaller driven current density at the resonant surface. In addition, the slow-down time of fast electrons generated by the wave will be much longer in a reactor plasma than that in existing experiments. This will cause a larger broadening of the driven current density profile due to the spatial transport of fast electrons before they slow down, and a further increase in the wave power required for mode stabilization. Exploring efficient ways to stabilize the DTM growth would be desirable, especially for plasmas with a high bootstrap current fraction as needed for steady operation, since the required driven current or wave power for mode stabilization by ECCD is proportional to the local bootstrap current density.

It is well known that sufficiently large RMPs or error fields can cause mode locking or field penetration [18–25]. Moderate $m/n = 2/1$ RMPs, however, were found to stabilize 2/1 tearing modes with monotonic $q$-profiles in experiments [19, 36]. This stabilization was observed from the single-fluid simulation for the electron temperature of a few 100 eV due to the non-uniform mode rotation caused by RMPs [19, 20, 36].

Recently, the effect of the RMPs on the NTM growth was studied numerically using two-fluid equations [37–39]. When the local bi-normal electron fluid velocity at the resonant surface is sufficiently large, especially for plasma rotation in the ion drift direction, the growth of $m/n = 2/1$ NTMs is found to be suppressed in a similar way by moderate static $m/n = 2/1, 4/2$ or 6/3 RMPs for an electron temperature of a few keV. The stabilization is dominated by the change in the local plasma current density gradient at the $q = 2$ surface caused by RMPs, resulting from the nonlinear coupling of the helical shielding current to magnetic and plasma velocity perturbations [38].

Considering the importance of DTM stabilization for the stability of advanced scenarios, the effect of applied RMPs on the DTM growth is studied in this paper, based on the two-fluid equations and large aspect ratio approximation [40]. It is found that in the presence of bootstrap current, the DTM growth can also be stabilized by static RMPs of moderate amplitude below their penetration threshold for an electron temperature of a few keV or higher, if the distance between the two resonant surfaces and the plasma rotation velocity at the outer resonant surface are sufficiently large. The mode stabilization is more effective for a higher electron temperature, indicating a possible application in a fusion reactor.

In the next section, our theoretical model and input parameters are described. The nonlinear growth of the 2/1 DTM without applying RMPs is studied in section 3. The effects of RMPs on the DTM growth are presented in section 4. The discussion and summary are in the last section.

2. Theoretical model

Most of our numerical calculations are based on four-field equations [40], including the important effects for a small magnetic island growth such as the electron inertia, bootstrap current density, electron diamagnetic drift and the associated ion polarization current [41–47]. In addition, some results are obtained by solving the electron heat transport equation together with the four-field equations, to further take the electron temperature perturbations into account. These equations are described in appendix A of reference [38].

The radial equilibrium $q$-profiles used for our calculations are shown in figure 1. The green curve shows the case where the inner and outer $q = 2$ surfaces are located at $r_{e1} = 0.437 \, a$ and $r_{e2} = 0.598 \, a$, respectively, with a normalized distance $\Delta \rho = (r_{e2} - r_{e1})/a \approx 0.16$, where $a$ is the plasma minor radius. By increasing the amplitude of the plasma current density while keeping its radial profile unchanged, the $q$-profiles are downwards shifted, as shown by the red curve with $\Delta \rho \approx 0.31$ ($r_{e1} = 0.365 \, a$, $r_{e2} = 0.670 \, a$) and the black curve with $\Delta \rho \approx 0.35$ ($r_{e1} = 0.341 \, a$, $r_{e2} = 0.695 \, a$).

The following input parameters are used if not otherwise mentioned: the equilibrium electron temperature $T_e = 2$ keV, electron density $n_e = 3 \times 10^{19} \{0.8[1 - (r/a)^2]^2 + 0.2\} \, m^{-3}$, $a = 0.5 \, m$, plasma major radius $R = 1.7 \, m$, and toroidal magnetic field $B_t = 2 \, T$. The perpendicular plasma momentum and particle transport coefficients are assumed to be $\mu = 0.2 \, m^2 \, s^{-1}$ and $D_\perp = \mu/5$, and a larger plasma viscosity for the $m/n = 0/0$ component, 20 m$^2$ s$^{-1}$, is used, considering the neoclassical damping of poloidal plasma rotation [19, 38]. The electron diamagnetic drift frequencies at the two resonant surfaces, $f_{e1}$ and $f_{e2}$, are around the level of 1 kHz for all the $q$-profiles shown in figure 1, calculated from the above parameters and equation (B6) of reference [38]. With increasing $\Delta \rho$ from 0.16 to 0.35, $f_{e1} = 0.991, 0.972, 0.959$
and 0.949 kHz ($f_{c2} = 1.074, 1.093, 1.102$ and 1.107 kHz) for $m = 2$. By varying the amplitude of the momentum source in the plasma vorticity equation, equation (A3) of reference [38], different poloidal equilibrium plasma rotation (electric drift) frequency is included.

3. Nonlinear 2/1 DTM growth without applied RMPs

Before studying the effect of RMPs, it is helpful to first look into the nonlinear DTM growth without externally applied RMPs for the equilibrium $q$-profiles shown in figure 1.

When the equilibrium plasma rotation velocity and bootstrap current are taken to be zero in the calculations, the time evolution of the normalized outer (solid curve) and inner (dashed) 2/1 magnetic island width are plotted in figure 2(a) for the equilibrium $q$-profile with $\Delta \rho = 0.35$ (black), 0.31 (red), 0.25 (blue) and 0.16 (green). The time is normalized to the resistive time $\tau_0$. The inner and outer island widths are calculated by

$$W_{m/n} = 4[\psi_{m/n}/(B_p q/q)]^{1/2}$$

with $m/n = 2/1$ at the inner and outer $q = 2$ surfaces, respectively, where $\psi_{m/n}$ is the helical flux perturbation of the $m/n$ component, the prime is for the radial gradient, and $B_p$ is the poloidal field. In this case, the mode frequency is determined by the local electron diamagnetic drift frequency. The difference between the diamagnetic drift frequencies at the inner and outer resonant surfaces is small, ranging from 83 to 158 Hz with increasing $\Delta \rho$ from 0.16 to 0.35. It is seen that for the equilibrium $q$-profile with a smaller value of $\Delta \rho$, the islands grow faster, and the difference between the outer and inner island growth is smaller due to a stronger coupling of the two islands. The outer island grows faster than the inner one, as expected [12, 13].

The results shown in figure 2(b) include the equilibrium plasma rotation in the calculations while keeping the other input parameters unchanged from those in figure 2(a). In this case, the mode frequency is determined by the local electron fluid frequency due to both the electron diamagnetic and electric drifts. The differences between the frequencies at the inner and outer resonant surfaces are 0.63, 0.95, 1.17 and 1.34 kHz with increasing $\Delta \rho$ from 0.16 to 0.35. The frequency difference weakens the coupling of the two islands and leads to a slower growth of the inner island in the early phase when the outer island is small, while the outer island growth is nearly unaffected. Once the outer island becomes sufficiently large in the later phase, the inner island is driven to grow together with it.

The results shown in figure 2(c) further include the bootstrap current in the calculations while keeping the other input parameters the same as those in figure 2(b). With increasing values of $\Delta \rho$ from 0.16 to 0.35 a, the bootstrap current density fractions at the inner (outer) resonant surfaces are $f_{b1} = 0.28, 0.27, 0.26$ and 0.25 ($f_{b2} = 0.37, 0.39, 0.41$ and 0.41), respectively. The outer island grows faster compared to that obtained with $f_b = 0$ shown in figure 2(b). Once its width reaches 0.03–0.04 a, the inner and outer islands are locked and grow together, despite the stabilizing role of the bootstrap current perturbation in the inner island growth [13, 14].

Although the DTM grows faster in the early phase for the equilibrium $q$-profile with a shorter distance between two $q = 2$ surfaces as shown in figure 2, it saturates at a smaller amplitude after the fast nonlinear growth phase. Corresponding to figure 2(b), radial profiles of the 0/0 component of the electron density, $n_e(n_0)/n_0$, after nonlinear mode saturation are shown in figure 3(a). For a smaller value of $\Delta \rho$, $\Delta \rho = 0.16$ (solid black curve), only the local electron density profile between the two $q = 2$ surfaces is flattened by the DTM. The density profile is hollow due to the radial outward (inward) expansion and motion of the inner (outer) island. For a larger value of $\Delta \rho$, $\Delta \rho = 0.31$ (red), however, the electron density profile is flattened over a much larger region by the DTM. The dashed curve shows the original equilibrium electron density profile. It should be mentioned that the perpendicular particle diffusivity, $D_{\perp} = 0.04$ m$^2$ s$^{-1}$, is fixed in our calculations, which could be different from experimental values, especially when considering that the local plasma turbulence level and particle transport could be significantly changed as a result of the larger local electron density gradient temporally generated by the fast DTM growth, as shown in figure 3.
Local radial profiles of the averaged safety factor, $q_0/\rho_0$, calculated using only the $mnl=0/0$ component poloidal field, after nonlinear mode saturation, are shown in figure 3(b). The local $q_0/\rho_0$ profile between two $q = 2$ surfaces is flattened by the DTM for $\Delta \rho = 0.16$ (solid black curve), similar to experimental observations [5]. For a large value of $\Delta \rho$, $\Delta \rho = 0.31$ (solid red curve), however, the $q_0/\rho_0$ profile is flattened up to the region near the magnetic axis. The dashed black (red) curve shows the original equilibrium $q$-profile for $\Delta \rho = 0.16$ ($\Delta \rho = 0.31$). The horizontal dotted line marks the $q = 2$ value.

The free magnetic energy driving the DTM is mainly from the region between the two resonant surfaces [10, 48], being proportional to their distance. A larger distance corresponds to more available free magnetic energy, allowing the DTM to grow to a large amplitude and being much more harmful to the confinement. More detailed results about nonlinear DTM growth will be reported in a separate paper.

4. Effect of RMPs on DTM growth

An $m/n$ component static RMP is taken into account in our calculations via the boundary condition

$$\psi_{m/n}|_{r=a} = \psi_{m/n} a B_t \cos(m\theta + n\phi), \quad (2)$$

where $\psi_{m/n}$ is the normalized (to $aB_t$) $\psi_{m/n}$ amplitude at $r = a$, and $\theta$ and $\phi$ are the poloidal and toroidal angles.

The shielding of applied RMPs depends on the local bi-normal equilibrium electron fluid velocity at the outer resonant surface $r_{x2}$. The normalized frequency at $r_{x2}$ is defined as

$$\omega_n = \left(1 - \omega_{n0}/\omega_{e0}\right), \quad (3)$$

where $\omega_{n0}$ ($\omega_{e0}$) is the local equilibrium electron diamagnetic (electric) drift frequency. The electron fluid velocity is in the ion drift direction for $\omega_n < 0$. Different $\omega_n$ values are used in the calculations by varying the momentum source in the plasma vorticity equation, equation (A3) of reference [38], while keeping the equilibrium electron diamagnetic drift frequency unchanged.

In the following section 4.1, the effects of $m/n = 4/2$ RMPs on the 2/1 DTM growth will be shown. The stabilization of the 2/1 DTM growth by $m/n = 2/1$ and 6/3 RMPs will be described in section 4.2, including also the 3/1 DTM stabilization by 6/2 RMPs. The results obtained for a higher equilibrium electron temperature, $T_e = 5$ keV, are shown in section 4.3. Further, results including the electron temperature perturbations in the calculations will be presented in section 4.4.

4.1. Effect of $m/n = 4/2$ RMPs on the 2/1 DTM growth

An example of the stabilization of the 2/1 DTM growth by $m/n = 4/2$ RMPs is shown in figure 4 for $\Delta \rho = 0.35$ and $\omega_n = -1.57$, keeping the other input parameters unchanged from those in figure 2(c). The time evolution of $W_{2/1}$ (solid curves) and $W_{4/2}$ (dashed curves), calculated from equation (1) at the outer $q = 2$ surface with $m/n = 2/1$ and 4/2, respectively, is shown in figure 4(a). It should be noted that when $W_{2/1} < W_{4/2}$, $W_{4/2}$ is the 4/2 island width, while $W_{2/1}$ only represents the normalized local amplitude of ($\psi_{4/2}/\rho_{4/2}$). With the 4/2 RMP amplitude $\psi_{4/2} = 5 \times 10^{-5}$ (red curves), $W_{2/1}$ grows more slowly than that without applying the RMP shown by the blue curve, but the result is about the same as that without the RMP in the later nonlinear saturation phase. Increasing the RMP amplitude to $\psi_{4/2} = 10^{-4}$ (black), the outer 4/2 island saturates at a width of $\approx 0.0075 \Delta$, while $W_{2/1}$ decays in time. It is seen that a moderate 4/2 RMP can suppress the 2/1 DTM growth. If a too large RMP is applied, however, the RMP has penetrated in, leading to the growth of the outer 4/2 island first.

Corresponding to the case with $\psi_{4/2} = 10^{-4}$ in figure 4(a), the time evolution of $W_{2/1}$ (solid) and $W_{4/2}$ (dashed) at both the outer (black) and inner (red) $q = 2$ surfaces is shown in figure 4(b). It is seen that $W_{2/1}$ and $W_{4/2}$ at the inner $q = 2$ surface decay in time.

Corresponding to figure 4(b) at the time $t = 6.4 \times 10^{-4} \tau_R$, local radial profiles of the real (solid curve) and imaginary (dashed curve) parts of the normalized helical flux of the
Figure 5. Corresponding to figure 4(b) with $\psi_{a,4/2} = 10^{-4}$ at the time $t = 6.4 \times 10^{-4} \tau_R$, local radial profiles of (a) the real (solid curve) and imaginary (dashed curve) parts of the normalized helical flux of the $m/n = 4/2$ (black) and 2/1 (red) components; (b) the normalized plasma current density of the $m/n = 0/0$ (solid black curve) and 4/2 (red curve) component. The solid (dashed) red curve is the real (imaginary) part. The dashed black curve is the original equilibrium plasma current density. The vertical dotted line marks the outer $q = 2$ surface.

$m/n = 4/2$ (black) and 2/1 (red) components are shown in figure 5(a). Due to the plasma shielding current, the amplitude of the 4/2 component helical flux is reduced by more than two orders of magnitude at the outer resonant surface marked by the vertical dotted line, and it further decreases to about zero at the inner resonant surface. The amplitude of the 2/1 component helical flux is much smaller, since the 2/1 mode decays in time. The local radial profile of plasma current density of the $m/n = 0/0$ component, $j_{b0/0}$, around the outer $q = 2$ surface is shown in figure 5(b) by the solid black curve. The dashed black curve is the original equilibrium plasma current density without applying the RMP, being negative in our calculations. The red curves show the real (solid) and imaginary (dashed) parts of the shielding current density of the $m/n = 4/2$ component. Due to the nonlinear coupling of the helical shielding current to magnetic and plasma velocity perturbations [38], the local $j_{b0/0}$ gradient is significantly changed by the 4/2 RMP around the outer $q = 2$ surface marked by the vertical dotted line, being reversed on the inner side but increased on the outer side, similar to that with a localized ECCD at the outer $q = 2$ surface in the plasma current direction. This leads to a stabilizing effect against the outer island growth. In addition, the large radial gradient of the shielding current density of the $m/n = 4/2$ component induced by RMPs is also stabilizing against the outer 2/1 island growth for the electron fluid velocity in the ion drift direction [38]. Both effects lead to the stabilization of the outer island growth. As the RMP is shielded by the outer $q = 2$ surface, the 0/0 component plasma current density changes little at the inner $q = 2$ surface. However, without the destabilization from a large outer island, the inner island growth is stabilized by the perturbed bootstrap current in the reversed magnetic shear region [13, 14].

Extensive nonlinear calculations based on four-field equations have been made for different equilibrium plasma rotation velocities and the 4/2 RMP amplitudes, while keeping the other input parameters unchanged from those in figure 4. The results are shown in the $(\omega_n - \psi_{a,4/2})$ plane in figure 6. The black circles (red squares) are the cases where the 2/1 DTM growth is not stabilized by the 4/2 RMP. The DTM is stabilized in a region for $-\omega_n > 0$ (the electron fluid velocity in the ion drift direction). When the value of $|\omega_n|$ is sufficiently large, the stabilization is also found for $-\omega_n < 0$. Due to the dependence of the modified current density profile on the $\omega_n$ value [38, 39], the stabilization regions are asymmetric on the two sides of $\omega_n = 0$. When the local electron fluid velocity is not sufficiently large in the electron drift direction, the change in the local current density gradient is destabilizing [38]. The upper boundary of the stabilization region is determined by the 4/2 RMP penetration threshold, being larger for a larger value of $|\omega_n|$ [49–50], while the lower boundary is determined by the minimum 4/2 RMPs required for the mode stabilization. Outside the stabilization window, moderate RMPs can still slow down the DTM growth, e.g. as shown by the red curves in figure 4(a). If there is no equilibrium plasma rotation, then rotating RMP is required to stabilize the DTM, such that the relative rotation between the local electron fluid and RMP is in the stabilization region.

Differing from the two-fluid simulation results, no 2/1 DTM stabilization by 4/2 RMPs has been found from single-fluid simulations for $|\omega_n| \lesssim 5$.

When the bootstrap current density is taken to be zero in the calculations, the time evolution of $W_{2/1}$ at the inner (black curve) and outer (red curve) $q = 2$ surface is shown in figure 7 for $\Delta \rho = 0.35$, $\psi_{a,4/2} = 1.75 \times 10^{-4}$ and $\omega_n = -2.08$. The other input parameters are the same as those in figure 4. As there is no stabilizing effect from the bootstrap current perturbation in the negative magnetic shear region in this case, the inner 2/1 island slowly grows. When it grows to a width of about 0.05 a at $t = 3.8 \times 10^{-3} \tau_R$, the outer island is destabilized by the inner one and grows together, similar to the penetration of a large error field (but from the inner side rather than from the outside). Before the destabilization, the outer 4/2 island width (green) saturates at $\sim 0.01$ a, and the $W_{2/1}$ at the outer $q = 2$ surface is suppressed at a low level, since the local plasma current density gradient around the outer $q = 2$ surface is changed by 4/2 RMPs, similar to the case shown in figure 5. As a comparison, the blue curves show the growth of the outer (solid) and inner (dashed) 2/1 island width without applying RMPs. In this case, the outer 2/1 island grows much earlier, and the inner 2/1 island grows, following the outer one. It is seen that a finite...
The blue curves show the time evolution of the outer (solid) and inner (dashed) 2/1 island width for $f_0 = 0$, $\Delta \rho = 0.35$ and $\omega_a = -2.08$ without applying RMPs. With $\psi_{a,2/1} = 1.75 \times 10^{-4}$, $W_{2/1}$ at the inner and outer $q = 2$ surface are shown by the black and red curves, and the outer 4/2 island width is shown by the green curve.

Figure 8. Similar to figure 6, but for $f_0 = 0$. The blue circles (red squares) are the cases where the outer island growth is (not) stabilized by the 4/2 RMP, before the inner 2/1 island grows up.

Bootstrap current density at the inner resonant surface is required for the DTM stabilization, since the applied RMPs generate helical shielding current only at the outer resonant surface.

Next, we keep the other input parameters the same as those in figure 6, while taking the bootstrap current density to be zero in the calculations, and the results are shown in figure 8. The blue circles (red squares) are the cases where the outer island growth is (not) stabilized by the 4/2 RMP before the inner 2/1 island grows up. The outer island is eventually destabilized by the inner one for the cases shown by blue circles, similar to what is demonstrated in figure 7. Compared to figure 6, the stabilization region for the outer island growth in figure 8 is smaller for $-\omega_n > 0$ but larger for $-\omega_n < 0$, indicating that the bootstrap current density perturbation plays a role in causing the asymmetry in the results shown in figure 6 for different directions of the electron fluid velocity.

The results shown in figure 9 are similar to figure 6, but using the equilibrium $q$-profile with a shorter distance between the two resonant surfaces, $\Delta \rho = 0.31$. The black circles (red squares) are again the cases where the 2/1 DTM growth is (not) stabilized by the 4/2 RMP. The stabilization region is smaller than that for the equilibrium $q$-profile with $\Delta \rho = 0.35$ shown in figure 6. No stabilization is found for plasma rotation in the electron drift direction ($-\omega_n < 0$) with $|\omega_a| \leq 6$. For a smaller value of $\Delta \rho$, the DTM is more unstable due to a stronger coupling of the inner and outer islands, as seen from figure 2, so that the stabilizing effect by 4/2 RMPs is relatively weaker. For the equilibrium $q$-profile with a sufficiently small value of $\Delta \rho$ ($= 0.25$), RMPs are found to slow down the 2/1 DTM growth but not stabilize it, even for plasma rotations in the ion drift direction with $|\omega_a| \leq 6$.

4.2. Effect of the 2/1 and 6/3 RMP on the 2/1 DTM growth

Applying an $m/n = 2/1$ RMP with $\psi_{a,2/1} = 2 \times 10^{-4}$ for $\Delta \rho = 0.31$, $\omega_n = -1.66$, $f_{b1} = 0.26$ and $f_{b2} = 0.41$, while keeping the other input parameters the same as those in figure 4, the time evolution of the outer (solid black curve) and inner (dashed black curve) 2/1 island width is shown in figure 10. The outer 2/1 island growth is suppressed by the RMP, saturating at a small width of $\sim 0.01$ a, while the inner 2/1 island is even smaller. The 2/1 island growth without applying the RMP is shown with the red curves for comparison.

Corresponding to the black curves in figure 10, figure 11 shows the local radial profile of $j_{b0}$ in steady state (solid black curve). The dashed curve is the original equilibrium plasma current density. The red curve is for the $j_{b0}$ of another case with $\psi_{a,4/2} = 2 \times 10^{-4}$ and $\omega_n = -1.66$, for which the 2/1 DTM growth is stabilized by the 4/2 RMP. The local $j_{b0}$ gradient is changed in a similar way by the 2/1 or 4/2 RMPs around the outer $q = 2$ surface, marked by the vertical dotted line.
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\( / \) 2 RMP has a larger window, and the minimum 6/2 RMP

amplitude required for the 3/1 DTM stabilization is smaller,

since higher-\( m \) modes are usually less unstable than the 2/1

DTM. The 6/2 RMPs are also found to stabilize the 3/1 DTM

for the equilibrium \( q \)-profile with a shorter distance of the two

resonant surfaces, \( \Delta \rho = 0.25 \). For a sufficiently small value of

\( \Delta \rho (0.16) \), however, no 3/1 DTM stabilization by 6/2 RMPs

has been found for |\( \omega_n | \leq 5 |$

4.3. Effect of the equilibrium electron temperature

The results shown in sections 4.1 and 4.2 are obtained for an equilibrium electron temperature of \( T_e = 2 \) keV. Decreasing the equilibrium electron temperature to \( T_e = 300 \) eV while keeping the other input parameters unchanged, no DTM stabilization by RMPs was found in the calculations for |\( \omega_n | \) values up to 20.

Increasing the equilibrium electron temperature to \( T_e = 5 \) keV while keeping the other input parameters unchanged, a much stronger stabilizing effect of the RMPs is found, as shown by the stability diagram in the \( (\omega_n - \psi_{a,4/1}) \) plane for \( \Delta \rho = 0.31 \) in figure 13. The black (red) symbols are for the equilibrium electron temperature \( T_e = 5 \) keV (\( T_e = 2 \) keV). The circles (squares) are the cases where the 2/1 DTM growth is (not) stabilized by the 4/2 RMP. Compared to that obtained for \( T_e = 2 \) keV, the stabilization region is much larger for \( T_e = 5 \) keV, existing also for the electron fluid velocity in the electron drift direction with \( -\omega_n = -3 |$

Using the equilibrium \( q \)-profile with a shorter distance between the two resonant surfaces, \( \Delta \rho = 0.25 \), the 2/1 DTM growth is also found to be stabilized by moderate 4/2 RMPs for the equilibrium electron temperature \( T_e = 5 \) keV (but not for \( T_e = 2 \) keV as mentioned in section 4.1). For the equilibrium \( q \)-profile with \( \Delta \rho (0.16) \), however, 4/2 RMPs are found to slow down the 2/1 DTM growth but not to stabilize it for |\( \omega_n | \leq 6 |$

We apply the 6/3 RMP for the equilibrium electron temperature \( T_e = 5 \) keV, with the other input parameters being the same as those in figure 12, and the results are shown in figure 14. The black circles (red squares) are the cases where the 2/1 DTM growth is (not) stabilized by the 6/3 RMP. Compared to the case obtained with \( T_e = 2 \) keV shown by red symbols in figure 12, a much larger stabilization region by the 6/3 RMP is also found for a higher electron temperature.

Local radial profiles of \( j_{0,0} \) in steady state are shown in figure 15. The solid black curve is for the case with \( T_e = 2 \) keV, \( \psi_{a,4/2} = 2 \times 10^{-4} \), \( \Delta \rho = 0.31 \), \( \omega_n = -1.66 \), \( f_{b1} = 0.26 \) and \( f_{b3} = 0.41 \). The red, green and blue curves are for \( T_e = 5 \) keV and \( \omega_n = -1.55 \), obtained with \( \psi_{a,4/2} = 10^{-4} \), \( 5 \times 10^{-5} \) and \( \psi_{a,6/3} = 10^{-4} \), respectively, keeping the other input parameters the same as those for the solid black curve. The dashed curve is the original equilibrium plasma current

By applying 6/3 RMPs, the 2/1 DTM growth can also be stabilized. The stability diagram in the \( (\omega_n - \psi_{a}) \) plane is shown in figure 12. The red solid circles (empty squares) are the cases in which the 2/1 DTM growth is (not) stabilized by 6/3 RMPs. The black solid circles (empty squares) are the cases where the 2/1 DTM growth is (not) stabilized by 4/2 RMPs, shown here for comparison. To save computation time, the calculations for the 6/3 RMPs have been performed only for one \( \omega_n \) value, which is the same as that of the neighboring black symbols. To have a clear view, however, the red symbols are plotted at a slightly different \( \omega_n \) value (the magenta and green symbols in figure 12 are plotted in the same way). The stabilization window by applying 6/3 RMPs is similar to that for 4/2 RMPs, but the required 6/3 RMP is somewhat larger. The magenta circles (squares) in figure 12 are the cases where the 2/1 DTM growth is (not) stabilized by 2/1 RMPs. The stabilizing window is somewhat smaller compared to that for applying a 4/2 RMP.

Calculations have also been made to study the effect of the 6/2 RMP on the 3/1 DTM growth, using an upwards-shifted equilibrium \( q \)-profile such that the radial locations of the two \( q = 3 \) surfaces are the same as those of the \( q = 2 \) surfaces with \( \Delta \rho = 0.31 \) as shown in figure 1, while keeping the other input parameters unchanged. Without applying RMPs, the 3/1 DTM is unstable. The green circles (squares) in figure 12 are the cases where the 3/1 DTM growth is (not) stabilized by the 6/2 RMP. Compared to the 2/1 DTM stabilization by the 4/2 RMP, the stabilization of the 3/1 DTM by the 6/2 RMP has a larger window, and the minimum 6/2 RMP amplitude required for the 3/1 DTM stabilization is smaller, since higher-\( m \) modes are usually less unstable than the 2/1 DTM. The 6/2 RMPs are also found to stabilize the 3/1 DTM for the equilibrium \( q \)-profile with a shorter distance of the two resonant surfaces, \( \Delta \rho = 0.25 \). For a sufficiently small value of \( \Delta \rho (0.16) \), however, no 3/1 DTM stabilization by 6/2 RMPs has been found for |\( \omega_n | \leq 5 |$

Figure 11. Radial profiles of the (normalized) \( j_{0,0} \) in steady state for \( \psi_{a,2/1} = 2 \times 10^{-4} \) (solid black curve) and \( \psi_{a,4/1} = 2 \times 10^{-4} \) (red curve), with \( \Delta \rho = 0.31 \), \( \omega_n = -1.66 \), \( f_{b1} = 0.26 \) and \( f_{b3} = 0.41 \). The dashed curve is the original equilibrium plasma current density.

Figure 12. Stability diagram in the \( (\omega_n - \psi_q) \) plane for \( \Delta \rho = 0.31 \), \( f_{b1} = 0.26 \) and \( f_{b3} = 0.41 \). The solid circles (empty squares) are the cases where the 2/1 DTM growth is (not) stabilized by the 4/2 (black), 6/3 (red) and 2/1 (magenta) RMP. The green circles (squares) are the cases where the 3/1 DTM growth is (not) stabilized by the 6/2 RMP using an upwards-shifted equilibrium \( q \)-profile such that the radial locations of two \( q = 3 \) surfaces are the same as those of the \( q = 2 \) surfaces with \( \Delta \rho = 0.31 \) shown in figure 1, while keeping the other input parameters unchanged. The red, magenta and green symbols have the same value of \( \omega_n \) as the neighboring black ones, but are plotted at a slightly different value for clearer presentation.

Compared to the case obtained with \( \omega_n = -3 |$

Using the equilibrium \( q \)-profile with a shorter distance between the two resonant surfaces, \( \Delta \rho = 0.25 \), the 2/1 DTM growth is also found to be stabilized by moderate 4/2 RMPs for the equilibrium electron temperature \( T_e = 5 \) keV (but not for \( T_e = 2 \) keV as mentioned in section 4.1). For the equilibrium \( q \)-profile with \( \Delta \rho (0.16) \), however, 4/2 RMPs are found to slow down the 2/1 DTM growth but not to stabilize it for |\( \omega_n | \leq 6 |$

We apply the 6/3 RMP for the equilibrium electron temperature \( T_e = 5 \) keV, with the other input parameters being the same as those in figure 12, and the results are shown in figure 14. The black circles (red squares) are the cases where the 2/1 DTM growth is (not) stabilized by the 6/3 RMP. Compared to the case obtained with \( T_e = 2 \) keV shown by red symbols in figure 12, a much larger stabilization region by the 6/3 RMP is also found for a higher electron temperature.

Local radial profiles of \( j_{0,0} \) in steady state are shown in figure 15. The solid black curve is for the case with \( T_e = 2 \) keV, \( \psi_{a,4/2} = 2 \times 10^{-4} \), \( \Delta \rho = 0.31 \), \( \omega_n = -1.66 \), \( f_{b1} = 0.26 \) and \( f_{b3} = 0.41 \). The red, green and blue curves are for \( T_e = 5 \) keV and \( \omega_n = -1.55 \), obtained with \( \psi_{a,4/2} = 10^{-4} \), \( 5 \times 10^{-5} \) and \( \psi_{a,6/3} = 10^{-4} \), respectively, keeping the other input parameters the same as those for the solid black curve. The dashed curve is the original equilibrium plasma current
density. The DTM growth is stabilized by RMPs for these four cases, similar to that shown in figure 4(b). A higher electron temperature corresponds to a smaller plasma resistivity, so that the change in the plasma current density profile by the RMPs is larger, leading to a stronger stabilizing effect of the RMPs as shown in figures 13 and 14. Applying the 4/2 or the 6/3 RMP causes a similar change in the local plasma current density profile.

Calculations have also been made to study the effect of the 6/2 RMP on the 3/1 DTM growth for the equilibrium electron temperature of $T_e = 5$ keV, using upwards-shifted equilibrium $q$-profiles as mentioned before. Compared to the 2/1 DTM stabilization by the 4/2 RMP, the stabilization of the 3/1 DTM by the 6/2 RMP is again found to have a larger window, existing also for the equilibrium $q$-profile with a short distance of the two resonant surfaces, $\Delta \rho = 0.16$, indicating that the higher-$m$ DTM growth can be more easily stabilized by RMPs for a higher electron temperature.

4.4. Effect of electron heat transport

The electron temperature perturbation is neglected for the results presented in sections 4.1–4.3. This perturbation is included in this sub-section by solving the electron heat transport equation together with the four-field equations, described in appendix A of reference [38]. The results are shown in figure 16 for the equilibrium electron temperature $T_e = 2$ keV, $\Delta \rho = 0.31$, $f_{b1} = 0.28$, $f_{b2} = 0.25$, the ratio between the parallel and perpendicular heat conductivity $\chi_\parallel/\chi_\perp = 10^5$, $\chi_\perp = \mu$, and a parabolic profile of the equilibrium electron temperature. The other input parameters are the same as those in figure 9. The black circles (red squares) are the cases where the 2/1 DTM growth is (not) stabilized by the 4/2 RMP. The results are similar to that shown in figure 9, with asymmetry on the two sides of $\omega_n = 0$.

5. Discussion and summary

With a relatively large distance between two resonant surfaces, the DTM can greatly degrade the confinement, as shown in figure 3. In this case, our results reveal that the 2/1 DTM growth can be stabilized by static 4/2, 2/1 or 6/3 RMPs of moderate amplitude ($10^{-5}$ to $10^{-4}B_t$) below their penetration threshold if the local electron fluid velocity at the outer $q = 2$ surface is sufficiently large. The stabilization window is larger for the plasma rotation in the ion drift direction.

More importantly, when increasing the equilibrium electron temperature from $T_e = 2$ keV to $T_e = 5$ keV, the stabilization of the DTM growth by RMPs is found to have a larger window
and extend to the equilibrium $q$-profile with a shorter distance between two resonant surfaces, as shown in figures 13 and 14. A high electron temperature (a small plasma resistivity) poses a major problem in the nonlinear MHD instability calculations but is required for simulating fusion plasmas, since the dominating physics mechanism can be different for different plasma resistivities. The stabilization window of the 3/1 DTM growth by RMPs is always larger than that of the 2/1 DTM, indicating that the DTM growth can be more easily stabilized by RMPs for the non-monotonic $q$-profile with the minimum $q$ value above 2.

For the equilibrium $q$-profile with a short distance between two resonant surfaces, $\Delta \rho \leq 0.16$, no 2/1 DTM stabilization by RMPs has been found for the equilibrium electron temperature $T_e = 5$ keV. However, in this case only the local plasma pressure is flattened by the DTM, as shown in figure 3, so DTM stabilization might not be necessary. The resistive wall effect has been neglected in this paper, since this effect is not expected to be important for the growth of a small magnetic island. For the case with multiple low-$m$ resonant surfaces inside the plasma, a more sophisticated RMP spectrum, including the appropriate RMP amplitude for multiple helicities, will be required to stabilize the DTMs of multiple helicities.

It was planned in the ITER design to use 18 external saddle coils to reduce the sum of the error field of the 2/1, 3/1 and 1/1 components to $5 \times 10^{-5} B_i$ [52]. The results shown in figure 15 imply that even without applying RMPs, a small error field in this range can still significantly change the local current density gradient at the outer resonant surface with an electron temperature of $\sim 10$ keV. Taking the neutral beam injection and the intrinsic torque into account, the ITER plasma is predicted to rotate in the ion drift direction with a frequency being much higher than the electron diamagnetic frequency [53]. If this is the case, static 2/1, 4/2 or 6/3 RMPs or an error field of the order of $10^{-3} B_i$ will have a stabilizing effect on the DTM growth, revealing a positive role of moderate error field in a fusion reactor. On the other hand, the prediction of ITER plasma rotation might be subject to a certain level of uncertainty. In the case that the local plasma rotation frequency is around or lower than the electron diamagnetic drift frequency, the mode-locking threshold will be quite low [54, 55], and an error field in the order of $10^{-2} B_i$ will be destabilizing for the plasma rotation in the electron drift direction due to an unfavorable change in the local current density gradient in this case [38]. As the stabilization window depends on the relative rotation between the RMP and the local electron fluid at the outer resonant surface, rotating RMPs in the electron drift direction can widen the stabilization window. However, considering the shielding from the blanket and vacuum vessel in a fusion reactor, significant engineering effort is required in order to generate rotating RMPs of the order of $10^{-2} B_i$, similar to the generation of a fast-changing horizontal field by in-vessel coils for stabilizing the vertical instability in ITER [56].

It is well known that the externally applied RMPs can suppress or mitigate edge-localized modes [57–65], tearing modes [19, 20, 36–39] and sawteeth [66]. Rotating RMPs have been utilized to avoid mode locking and disruption [67–69]. Together with the results presented in this paper, one would expect that a weakly three-dimensional equilibrium, utilizing moderate RMPs of optimized spectrum and frequency, can be more stable against MHD instabilities than the usual axisymmetric tokamak equilibrium.

In summary, it is found in this paper that the $m/n = 2/1$ DTM growth is stabilized by static $m/n = 2/1, 4/2$ or 6/3 RMPs of moderate amplitude for a sufficiently large distance between the two resonant surfaces and the electron fluid velocity at the outer $q = 2$ surface. The stabilization window is larger for a higher electron temperature and for the local electron fluid velocity in the ion drift direction. The stabilization of the 3/1 DTM growth by RMPs has a larger window than that of the 2/1 DTM. The outer magnetic island growth is stabilized due to the change in the local plasma current density gradient around the outer $q = 2$ surface by RMPs, while the inner island growth is stabilized by the bootstrap current perturbation.

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### References

[1] Levinton F.M. et al 1995 Phys. Rev. Lett. 75 4417
[2] Strait E.J. et al 1995 Phys. Rev. Lett. 75 4421
[3] Chu M.S. et al 1996 Phys. Rev. Lett. 77 2710
[4] Chang Z. et al 1996 Phys. Rev. Lett. 77 5553
[5] Günter S. et al 2000 Nucl. Fusion 40 1541
[6] Wolf R.C. 2003 Plasma Phys. Control. Fusion 45 R1
[7] Litaudon X. 2006 Plasma Phys. Control. Fusion 48 A1
[8] Joffrin E. et al 2003 Nucl. Fusion 43 1167
[9] Iida K. and Fujita T. 2018 Plasma Phys. Control. Fusion 60 033001
[10] Goodall D.H.J. and Wesson J.A. 1984 Plasma Phys. Control. Fusion 26 789
[11] Pritchett P.L., Lee Y.C. and Drake J.F. 1980 Phys. Fluids 23 1368
[12] Yu Q. 1996 Phys. Plasmas 3 2898
[13] Yu Q. 1997 Phys. Plasmas 4 1047
[14] Yu Q. and Günter S. 1999 Nucl. Fusion 39 487
[15] Ishii Y., Azumi M. and Kishimoto Y. 2002 Phys. Rev. Lett. 89 205002
[16] Guo W., Ma J. and Yu Q. 2019 Plasma Phys. Control. Fusion 61 075011
[17] Ma J., Guo W., Yu Z. and Yu Q. 2017 Nucl. Fusion 57 126004
[18] Nave M.F.F. and Wesson J.A. 1990 Nucl. Fusion 30 2575
[19] Hender T.C. et al 1992 Nucl. Fusion 32 2091
[20] Fitzpatrick R. 1993 Nucl. Fusion 33 1049
[21] Butterly R.J., Benedetti M.D., Hender T.C. and Tubbing B.J.D. 2000 Nucl. Fusion 40 807
[22] La Haye R.J., Fitzpatrick R., Hender T.C., Morris A.W., Scoville J.T. and Todd T.N. 1992 Phys. Fluids B 4 2098
[23] Wolfe S.W. et al 2005 Phys. Plasmas 12 056110
[24] Koslowski H.R., Liang Y., Krämer-Flecken A., Löwenbrück K., Hellermann M.V., Westerhof E., Wolf R.C. and Zimmermann O. (the TEXTOR team) 2006 Nucl. Fusion 46 L1
[25] Zohm H., Kallenbach A., Bruhns H., Fussmann G. and Klüber O. 1990 Europhys. Lett. 11 745
[26] Yu Q., Günter S., Lackner K. and Maraschek M. 2012 Nucl. Fusion 52 063020
[27] Zohm H. et al 1999 Nucl. Fusion 39 577
[28] Gantenbein G. et al 2000 Phys. Rev. Lett. 85 1242
[29] Günter S. et al 1999 Plasma Phys. Controlled Fusion 41 b231
[30] La Haye R.J. et al 2002 Phys. Plasmas 9 2051
[31] Isayama A. et al 2000 Plasma Phys. Control. Fusion 42 L37
[32] Maraschek M. et al 2007 Phys. Rev. Lett. 98 025005
[33] Yu Q., Günter S., Giruzzi L.K., Lackner M. and Zabiego M. 2000 Phys. Plasmas 7 312
[34] Yu Q., Zhang X.D. and Günter S. 2004 Phys. Plasmas 11 1960
[35] Poli E. et al 2015 Nucl. Fusion 55 013023
[36] Hu Q., Yu Q., Rao B., Ding Y.H., Hu X. and Zhuang G. 2012 Nucl. Fusion 52 063020
[37] Yu Q., Günter S. and Lackner K. 2011 Nucl. Fusion 51 073030
[38] Yu Q., Günter S. and Finken K.H. 2009 Phys. Plasmas 16 042301
[39] Wesson J.A. 1978 Nucl. Fusion 18 87
[40] Yu Q. and Günter S. 2009 Nucl. Fusion 49 062001
[41] Yu Q. and Günter S. 2011 Nucl. Fusion 51 073030
[42] Yu Q., Günter S. and Finken K.H. 2009 Phys. Plasmas 16 042301
[43] Hender T.C. et al 2007 Nucl. Fusion 47 S128
[44] Wilson H.R., Connor J.W., Hastie R.J. and Hegna C.C. 1996 Phys. Plasmas 3 248
[45] Sauter O. et al 1997 Phys. Plasmas 4 1654
[46] Waelbroeck F.L., Connor J.W. and Wilson H.R. 2001 Phys. Rev. Lett. 87 215003
[47] Fitzpatrick R., Waelbroeck F.L. and Militello F. 2006 Phys. Plasmas 13 122507
[48] Hu Q., Nazikian R., Grierson B.A., Logan N.C., Paz-Soldan C. and Yu Q. 2020 Nucl. Fusion 60 076001
[49] Li J. et al 2020 Nucl. Fusion 60 126002
[50] Volpe F. et al 2015 Phys. Rev. Lett. 115 175002
[51] Zohm H., Kallenbach A., Bruhns H., Fussmann G. and Klüber O. 1990 Europhys. Lett. 11 745
[52] Yu Q. and Günter S. 2009 Nucl. Fusion 49 062001
[53] Gantenbein G. et al 2000 Phys. Rev. Lett. 85 1242
[54] Günter S. et al 1999 Plasma Phys. Controlled Fusion 41 b231
[55] La Haye R.J. et al 2002 Phys. Plasmas 9 2051
[56] Isayama A. et al 2000 Plasma Phys. Control. Fusion 42 L37
[57] Maraschek M. et al 2007 Phys. Rev. Lett. 98 025005
[58] Yu Q., Günter S., Giruzzi L.K., Lackner M. and Zabiego M. 2000 Phys. Plasmas 7 312
[59] Yu Q., Zhang X.D. and Günter S. 2004 Phys. Plasmas 11 1960
[60] Poli E. et al 2015 Nucl. Fusion 55 013023
[61] Hu Q., Yu Q., Rao B., Ding Y.H., Hu X. and Zhuang G. 2012 Nucl. Fusion 52 063020
[62] Yu Q., Günter S. and Lackner K. 2018 Nucl. Fusion 58 054003
[63] Yu Q., Günter S. and Lackner K. 2021 Nucl. Fusion 61 036040
[64] Yu Q. 2020 Nucl. Fusion 60 084001
[65] Hazeltine R.D., Kotschenreuther M. and Morrison P.J. 1985 Phys. Fluids 28 2466
[66] Smolyakov A.I. 1993 Plasma Phys. Control. Fusion 35 657
[67] Wilson H.R., Connor J.W., Hastie R.J. and Hegna C.C. 1996 Phys. Plasmas 3 248
[68] Sauter O. et al 1997 Phys. Plasmas 4 1654
[69] Waelbroeck F.L., Connor J.W. and Wilson H.R. 2001 Phys. Rev. Lett. 87 215003
[70] Fitzpatrick R., Waelbroeck F.L. and Militello F. 2006 Phys. Plasmas 13 122507
[71] Yu Q. 2010 Nucl. Fusion 50 025014
[72] Wesson J.A. 1978 Nucl. Fusion 18 87
[73] Yu Q. and Günter S. 2009 Nucl. Fusion 49 062001
[74] Yu Q. and Günter S. 2011 Nucl. Fusion 51 073030
[75] Yu Q., Günter S. and Finken K.H. 2009 Phys. Plasmas 16 042301
[76] Hender T.C. et al 2007 Nucl. Fusion 47 S128
[77] Wilson H.R., Connor J.W., Hastie R.J. and Hegna C.C. 1996 Phys. Plasmas 3 248
[78] Isayama A. et al 2000 Plasma Phys. Control. Fusion 42 L37
[79] Maraschek M. et al 2007 Phys. Rev. Lett. 98 025005
[80] Yu Q., Günter S., Giruzzi L.K., Lackner M. and Zabiego M. 2000 Phys. Plasmas 7 312
[81] Yu Q., Zhang X.D. and Günter S. 2004 Phys. Plasmas 11 1960
[82] Poli E. et al 2015 Nucl. Fusion 55 013023
[83] Hu Q., Yu Q., Rao B., Ding Y.H., Hu X. and Zhuang G. 2012 Nucl. Fusion 52 063020
[84] Yu Q., Günter S. and Lackner K. 2018 Nucl. Fusion 58 054003
[85] Yu Q., Günter S. and Lackner K. 2021 Nucl. Fusion 61 036040
[86] Yu Q. 2020 Nucl. Fusion 60 084001
[87] Hazeltine R.D., Kotschenreuther M. and Morrison P.J. 1985 Phys. Fluids 28 2466
[88] Smolyakov A.I. 1993 Plasma Phys. Control. Fusion 35 657
[89] Wilson H.R., Connor J.W., Hastie R.J. and Hegna C.C. 1996 Phys. Plasmas 3 248
[90] Sauter O. et al 1997 Phys. Plasmas 4 1654
[91] Waelbroeck F.L., Connor J.W. and Wilson H.R. 2001 Phys. Rev. Lett. 87 215003
[92] Fitzpatrick R., Waelbroeck F.L. and Militello F. 2006 Phys. Plasmas 13 122507
[93] Yu Q. 2010 Nucl. Fusion 50 025014