Teleportation via classical entanglement

Seyed Mohammad Hashemi Rafsanjani1*, Mohammad Mirhosseini1*, Omar S. Magaña-Loaiza1, and Robert W. Boyd1,2
1 Institute of Optics, University of Rochester, Rochester, New York 14627
2 Department of Physics, University of Ottawa, Ottawa, ON K1N6N5 Canada
(Dated: March 13, 2015)

We present a classical counterpart to quantum teleportation that uses classical entanglement instead of quantum entanglement. In our implementation we take advantage of classical entanglement among three parties: orbital angular momentum (OAM), polarization, and the radial degrees of freedom of a beam of light. We demonstrate the teleportation of arbitrary OAM states, in the subspace spanned by any two OAM states, to the polarization of the same beam. Our letter presents the first classical demonstration of a commonly-perceived–quantum phenomenon that requires entanglement among more than two parties.

Entanglement in quantum systems leads to many of the surprising consequences of the quantum mechanical description of nature. For many decades the aim of physicists has been to realize and confirm such phenomena in experimental settings. Such efforts came to fruition in a series of seminal observations that validated quantum mechanics and contradicted some classical counterparts such as local hidden variable theories [1–4].

The original proposal by Bennett et al. [15] takes advantage of three parties. One we call Alice, the other Bob, and a third party that we name Charlie. Initially Alice’s state is separable from the other two parties and Charlie and Bob share a joint maximally entangled state. One then performs a projective measurement on a joint Bell state of Alice and Charlie. The projection leads to the loss of some of the initial states; when we post-select on those measurements that have led to a specific Bell state, Bob’s state will be the same as the initial Alice’s state.

Teleportation allows us to transfer the state of one party to another party via a projection on a Bell state. In the following we introduce a scheme in which classical entanglement among three parties has been used to mediate transfer of coherence between two DoFs, and it is formally equivalent to teleportation. We report on the realization of this phenomenon, i.e. equivalent of teleportation, in an entirely classical system. In our implementation we teleport an arbitrary orbital angular momentum (OAM) state of a laser beam onto the polarization of the same laser beam. This, to the best of our knowledge, represents the first realization of this classical version of teleportation, and the extension of the concept of classical entanglement to more than two degrees of freedom.

The question then arises as to which of the phenomena enabled by multipartite entanglement could also be mimicked using the classical equivalent of multipartite entanglement, i.e. classical multipartite entanglement. Perhaps the most well-known consequence of quantum entanglement among more than two parties is the phenomenon of teleportation, that was first proposed by Bennett et al. [15] and was realized with quantum entanglement by Bouwmeester et al. [16]. Since then quantum teleportation has been realized in different experimental systems [17].

To realize classical teleportation we replace the three parties with three degrees of freedom of a single optical beam. In our implementation, the three degrees of freedom are the radial degree of freedom, polarization, and orbital angular momentum, that play the roles of Charlie, Bob, and Alice respectively. In Fig. 1 we have presented three examples of classical entanglement between different degrees of freedom of a spatial profile of a beam of light. These can be considered classical Bell states between different DoFs that we deal with in this paper. In principle any three DoFs can be used to realize entanglement as long as one can perform arbitrary joint

*These authors, SMHR and MM, have contributed equally to this work.
measurements on these observables/quantities. This requirement stems from the fact that in designating parties in an entangled state one has to be able to perform local operations on each of the parties. We emphasize that the designation of parties is embedded in the idea of entanglement, and a mere change of party designation can change the degree of entanglement for the same state \[18\].

In our realization we first produce a beam whose polarization and radial DoFs are entangled and both are separable from the OAM:

\[
\left[\gamma |l\rangle + \bar{\gamma} |-l\rangle\right] \otimes \left[|r_1, H\rangle + |r_2, V\rangle\right].
\]

(1)

We deliberately choose a different notation from the usual Dirac bra-ket notation to emphasize the fact that the DoFs are merely vector spaces. \(|H\rangle, |V\rangle\) denote the horizontal and vertical polarizations. The polarization of an optical field arises from the vectorial nature of electromagnetic field and techniques for its manipulation are easy to implement. \(|l\rangle\) denotes an OAM mode, which is defined via the helical phase structure \(e^{i\ell \phi}\). Although the OAM degree of freedom is commonly utilized in quantum optics \[19\ 20\], it naturally arises as a paraxial solutions to the Maxwell equations in cylindrical coordinates, and hence can be completely understood using classical physics. Finally \(|r_1\rangle, |r_2\rangle\) denote two radial modes, defined as two concentric, mutually exclusive, annular regions with a uniform intensity pattern. Note that \(|r_1\rangle, |r_2\rangle\) are orthogonal since there is no overlap between their corresponding spatial extents. Radial modes have also been the subject of a few recent investigations for their potential applicability in quantum communication. The transverse profile of a beam represented by Eq. \(1\) is depicted in Fig.2 (top). The dependence of the phase on the azimuthal angle is identical for both radial components since their OAM contents are the same.

Now that we have prepared the state in Eq. \(1\), the next step in the teleportation protocol is to implement a projective measurement onto one of the Bell states of

\[
(Charlie(radial))-Alice(OAM):
\]

\[
\Phi^\pm = (r_1, l) \pm (r_2, -l),
\]

(2)

\[
\Psi^\pm = (r_1, -l) \pm (r_2, l).
\]

Depending on our choice, the state of Bob (polarization) will be either \(\gamma |H\rangle \pm \bar{\gamma} |V\rangle\) for projection onto \(\Phi^\pm\), and \(\gamma |V\rangle \pm \bar{\gamma} |H\rangle\) for projection onto \(\Psi^\pm\) respectively. We choose to project onto \(\Phi^+\). In our projection we use a pinhole in the far field (setup below); thus after the projection the light emerging from the pinhole represents a single spatial mode that carries no orbital angular momentum. The transverse profile of such a beam is depicted in Fig.2 (bottom, right). The polarization of this beam is completely separable from the radial and OAM DoFs and the emerging beam’s polarization state reads:

\[
\gamma |H\rangle + \bar{\gamma} |V\rangle.
\]

(3)

We note that the final Bob (polarization) state carries the same information as the initial Alice (OAM) state in Eq. \(1\). This result is independent of the choice of the initial state. Although our derivation has assumed that Alice’s initial state to be a pure state, the derivation can be easily generalized to accommodate mixed states \[19\].
A schematic representation of the setup is given in Fig. 3. Our source of light is a cw He-Ne laser that emits at the wavelength of 633 nm. In order to produce the state prepared in Eq. 1 we first use a hologram to produce a coherent beam of two rings. The phase profiles of both rings are identical and match the OAM state that is to be teleported. In principle, one can choose to use any two orthogonal OAM state. In our realization we used the two OAM states $\{\ket{10}, \ket{-10}\}$. This choice minimizes the cross talk between the two states that often results from imperfect experimental realization of OAM projections.

The laser beam is then passed through a polarizer and then a half wave plate (HWP) whose aperture only covers the inner portion of the beam. The HWP is set to 45° in order to rotate the polarization of the inner disk to the orthogonal polarization. We name this combination a Bell-state synthesizer. The last HWP induces a phase difference between the two rings that can be cancelled by the spatial light modulator used for shaping the laser beam [Fig 3, SLM1].

The beam emerging from the last HWP can be set to possess an arbitrary state of OAM, along with a polarization structure that is maximally entangled to the radial DoF. As a result, the field can be formally described by Eq. 1. At this stage, we need to project onto a joint maximally entangled state of OAM and radial degrees of freedom in order to realize teleportation. The SLM allows for performing a projection onto the OAM state of $l = -10$ in the inner disk and a simultaneous projection over $l = 10$ for the outer annular ring. We use a phase-only liquid crystal SLM to shape the wavefront of the horizontal polarization component of the beam. To achieve a polarization-insensitive projection, we use the SLM in a double-pass geometry, with a HWP in between the two reflections for rotating the polarization of the beam by 90°. We use a lens with a focal length of 30 cm after the SLM to focus the beam onto a pinhole with a diameter of 5 microns. The beam emerging from the pinhole is approximately a single spatial mode with a polarization state that is related to the initial OAM state of Alice. A combination of a polarizer, quarter wave plate, and detector are used to measure the Stokes parameters and subsequently characterize the polarization state.

We test our protocol by first teleporting pure states of OAM. This has been done by converting the polarization Stokes parameters into a two-dimensional Jones vector and then finding the degree of similarity between the initial (OAM) state owned by Alice and the final (polarization) state detected by Bob. In Fig. 4 we report the fidelities between different initial OAM states and the polarization state that was measured at the end. The initial OAM states are chosen to be along the three primary axes of the Bloch sphere for a two-dimensional sub-space of $\{\ket{10}, \ket{-10}\}$. Namely, we have teleported the states $\{\ket{10}, \ket{-10}, \ket{10}+\ket{-10}, \ket{10}-\ket{-10}, \ket{10}+i\ket{-10}, \ket{10}-i\ket{-10}\}$. The ideal teleported states are then supposed to be the following polarization state respectively:

![Image of the setup](image-url)
FIG. 4. Fidelity of different teleported polarization states with the initial OAM states. All OAM states are in the subspace spanned by \( l = \pm 10 \). The three last states are mixed states. For brevity of notation, we have shown the unnormalized states.

\[
\begin{align*}
|10\rangle &= |\beta\rangle \\
|\pm 10\rangle &= |\alpha\rangle \\
|10\rangle + |\pm 10\rangle &= |\beta\rangle \\
|10\rangle - |\pm 10\rangle &= |\alpha\rangle \\
|10\rangle + i|\pm 10\rangle &= |\beta\rangle \\
|10\rangle - i|\pm 10\rangle &= |\alpha\rangle \\
3|\beta\rangle|\beta\rangle + 3|\alpha\rangle|\alpha\rangle &= 3|\beta\rangle|\beta\rangle + 3|\alpha\rangle|\alpha\rangle \\
3|\beta\rangle|\alpha\rangle + 3|\alpha\rangle|\beta\rangle &= 3|\beta\rangle|\alpha\rangle + 3|\alpha\rangle|\beta\rangle \\
3|\beta\rangle|\beta\rangle + 3i|\alpha\rangle|\alpha\rangle &= 3|\beta\rangle|\beta\rangle + 3i|\alpha\rangle|\alpha\rangle \\
3|\alpha\rangle|\alpha\rangle + 3i|\beta\rangle|\beta\rangle &= 3|\alpha\rangle|\alpha\rangle + 3i|\beta\rangle|\beta\rangle \\
3|\beta\rangle|\alpha\rangle + 3|\alpha\rangle|\beta\rangle &= 3|\beta\rangle|\alpha\rangle + 3|\alpha\rangle|\beta\rangle \\
3|\alpha\rangle|\alpha\rangle + 3i|\beta\rangle|\beta\rangle &= 3|\alpha\rangle|\alpha\rangle + 3i|\beta\rangle|\beta\rangle \\
\end{align*}
\]

Note that although the quantum density matrix is by definition a semi-positive definite matrix, the results of state tomography for a pure state often turns out to have negative eigenvalues [21]. This is primarily due to imperfect projective measurements and the noise in the experiment. We have used the maximum likelihood recovery algorithm to find a positive state that is the most probable given the data from the measurement. The average fidelity of teleported states with their corresponding initial states is approximately 99%, demonstrating a very good agreement with the theoretical predictions.

From a practical point of view pure states are an idealization; irrespective of how carefully a state is prepared, noise will inevitably render a pure state mixed. It is then significant if an implementation can also accommodate mixed states. Additionally, pure states are only a restricted set of physical states in the Hilbert space. The vast majority of states are mixed states [21]. Since we always project onto the same Bell OAM-radial state, our implementation allows us to also teleport the mixed states. For demonstration we have also teleported three typical mixed states. The states are chosen to be 0.75|\beta\rangle + 0.25|\alpha\rangle, 0.75|\alpha\rangle + 0.25|\beta\rangle, and 0.75|\alpha\rangle + 0.25|\beta\rangle. These OAM states are ideally teleported to the polarization states 0.75|H\rangle + 0.25|V\rangle, 0.75|D\rangle + 0.25|A\rangle, and 0.75|R\rangle + 0.25|L\rangle, respectively. In Fig. 4 we have reported the fidelities between the polarization states from the experiment with the ones from theory. Note that the fidelity between two mixed states is defined as \( F = \text{tr} \sqrt{\rho^{1/2} \sigma^{1/2}} \). The average fidelities for the three representative mixed states are found to be 99.33%, which confirms the accurate operation of our experimental realization.

Our realization, although classical, includes all aspects of the quantum teleportation except for non-locality. We believe this is further evidence that non-locality is a purely quantum–mechanical feature that cannot be explained through any classical explanation [22]. Nevertheless, the mathematical structure behind both classical and quantum teleportation are equivalent and further analysis and differentiation of both can help in developing a more comprehensive understanding of quantum teleportation. Considering that the formalism of classical teleportation applies to any three degrees of freedom, we anticipate this machinery can be utilized to transfer the state of other DoFs to another, or to realize other classical counterparts to entanglement-enabled phenomena, e.g. entanglement swapping. Furthermore, our setup provides the capability to map an arbitrary OAM state to a polarization state in a one-to-one fashion. This is the complimentary functionality of a q-plate [20]. Given the key role of q-plates [23] in some recent experiments in quantum information science, we predict our technique has potential for a wide range of applications in quantum information.

While entanglement is an essential part of the quantum paradigm, the mathematical machinery behind it can be utilized to describe phenomena that are not exclusively quantum mechanical. Thus many of the important phenomena that are enabled by entanglement may have counterparts in classical systems. To investigate which of such entanglement-enabled features can be replicated with classical entanglement, and which features, e.g. non-locality, can only be produced with quantum entanglement opens a new front in studying the boundary between quantum and classical physics.

From this point of view, it is of fundamental and practical importance to extend classical entanglement studies beyond two party scenarios. Here we have realized an experiment that can be understood as a classical version of a teleportation in a physical system that can be explained in the context of physical optics. We start with a beam of light with an arbitrary orbital angular momentum state that is entangled between radial and polarization degrees of freedom. We teleported the initial OAM state into the polarization degree of freedom of the beam. Our experiment is the first attempt at realizing phenomena that

\[
\begin{align*}
|H\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \\
|L\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle) \\
|V\rangle &= |\uparrow\rangle \\
|D\rangle &= |\downarrow\rangle \\
|A\rangle &= |\uparrow\rangle \\
|R\rangle &= |\downarrow\rangle \\
|L\rangle &= |\uparrow\rangle \\
|A\rangle &= |\downarrow\rangle \\
|R\rangle &= |\uparrow\rangle \\
|L\rangle &= |\downarrow\rangle \\
\end{align*}
\]
require handling of classical entanglement among more than two parties, and we believe that it has opened a door for many more to follow.

RWB acknowledges funding from the Canada Excellence Research Chairs program. OSML acknowledges support from CONACYT and the Mexican Secretaria de Educacion Publica (SEP).

* hashemi@pas.rochester.edu
mirhosse@optics.rochester.edu

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