Forced convection flow over a moving porous and non-uniform cylinder of variable thickness

Nadeem Jan, Dil Nawaz Khan Marwat and Sajid Rehman

Abstract
Forced convection flow of viscous fluid over a moving, porous, and heated cylinder of variable thickness is studied in this paper. The non-uniform cylinder is placed vertically in a quiescent fluid. All the field quantities (the normal and axial velocities and temperature), defined at the wall or surface of cylinder, are variable and may have non-linear form. An appropriate set of transformations is developed on the analogies of systematic classical symmetries for a system of partial differential equations (PDE's), whereas, they are equipped with the representative of each velocity component, temperature function, and similarity variable. However, the PDE's along with the boundary conditions (BC's) are reduced into a system of boundary value problem (BVP) of ordinary differential equations (ODE's) by using these new and unseen similarity variables. The system of non-linear coupled ODE's, combined with the BC's is solved, whereas, the numerical results are obtained for different values of the existing governing parameters. All the field quantities, skin friction, rate of heat transfer at the surface of the cylinder are evaluated with the help of bvp4c package of MATLAB and the results are shown in different graphs. The present simulation generalizes all types of forced convection flow problems, over a porous and heated cylinder of variable (uniform) radius, when it is stretched (shrunk) with linear and non-linear (uniform) velocity. However, the simulated problem gives rise to a new set of problems and that they are valid for non-linear (linear and uniform) injection (suction) velocity.

Keywords
Forced convection, heated, porous, stretched (shrunk) cylinder of variable radius

Date received: 30 September 2021; accepted: 13 May 2022

Handling Editor: Chenhui Liang

Introduction
Mixed convection flows happen in many industrial applications and natural process, for example, wind flow on the surface of solar receivers, cooling of electronic devices by exhaust fans, cooling of nuclear energy system in an emergency shutdown, installation of heat exchangers in an area of small velocity of air and flows in the deep cavities and chambers. Mixed convection in flow is usually occur in the other field of sciences such as geology, astrophysics, biology, oceanography, chemical processes, and crystal-growth techniques in ocean and atmosphere.

Convective flows over cylinder have been studied extensively by many researchers such as Nield et al.,1 Ioan Pop et al.,2 and Vafai.3 Chen and Mucoglu4 studied the force convective flow over a vertical and uniformly heated cylinder, however, they examined the behavior of fluid motion and diffusion of heat under the influences of buoyancy forces. Moreover, Mucoglu...
and Chen\(^5\) extended and modified the concept of Chen and Mucoglu\(^4\) and solved the problem for uniform surface heat flux. Furthermore, solutions of such flow models have been found within the boundary layers by many researchers such as Blottner.\(^6\) Bui and Cebeci\(^7\) examined the convection in flow of mixed type on a heated, circular, and vertical cylinder. Researchers also used the finite difference method and found the solution of some special problem, composed of mixed convection flow over a vertical cylinders, such as Lee et al.,\(^8\) Merkin and Mahmood et al.,\(^9,10\) Wang and Kleinstreuer,\(^11\) Kumari and Nath,\(^12\) Ali and Al-Yousef,\(^13\) Rahman and Mulolami.\(^14\) Note that the mixed convection flow problems have been solved for other geometries and such flow along a horizontal circular cylinder placed in porous medium with constant wall temperature was studied by Nazar et al.\(^15\) The mixed convection flow of nano-fluid within momentum and thermal boundary layers was examined by Nazar et al.,\(^16\) Patial et al.,\(^17,18\) Prakash et al.,\(^19\) Swati,\(^20\) and Wang.\(^21\) Most recently, the forced convection flows along a vertical and heated cylinder has been studied by Pandey and Kumar,\(^22\) Khan and Malik,\(^23\) Hayat et al.,\(^24\) Sinha et al.,\(^25\) and Khan et al.\(^26\)

In this paper, a generalize model problem of forced convection flow is simulated over a variably porous and heated cylinder of non-uniform radius. Furthermore, the vertical cylinder is stretched/shrunk with the variable and non-linear velocity in a quiescent fluid. Note that this approach modifies the previous simulations of Jan et al.\(^27,28\) All the thermodynamic and kinetic properties, defined at the surface of porous, heated, and moving cylinder of variable radius are non-uniform (i.e. they have linear and non-liner form). The flow field variables are evaluated within a thin boundary layers, therefore, the equations of motion are simplified through the boundary layer approximations and similarity transformation. Also appropriate boundary conditions (BC’s) are imposed at the surface of cylinder. Moreover, keeping in view the physics of the problem and field variables, the incompressible continuity, momentum, and energy equations are simplified using similarity transformation and they are converted into a system ODE’s. The non-linear coupled ODE’s are then solved by bvp4c method of MATLAB. The current modeled problem is the generalization of previous problems of natural (forced) convection in flow of viscous fluid over a stretching (shrinking), porous and heated cylinder of variable (uniform) thickness.

**Formulation of problem**

Consider the steady, laminar two-dimensional, and axisymmetric flow of a viscous and incompressible fluid over a porous, moving, and heated cylinder of non-uniform radius. Moreover, all the field variables, defined at the wall of the duct, including the surface of the cylinder are non-uniform and non-linear. The schematic diagram of the simulated model is given in Figure 1.

Note that the surface temperature is greater than ambient temperature, whereas, all the thermal properties are uniform. The continuity, momentum and energy equation along with BC’s are taken in cylindrical coordinate system in order to simulate the problem in appropriate manner and the system of relevant governing PDE’s takes the following form:\(^4\)

\[
\begin{align*}
  v_r + u_z + vr^{-1} &= 0, \\
  u u_z + v u_r &= \frac{\nu}{\rho} (ru_{rr} + u_r) + g \beta (T - T_w), \\
  u T_z + v T_r &= \frac{k}{\rho} (r T_{rr} + T_r).
\end{align*}
\]

where, the subscripts denote the partial differentiation of the dependent variables w.r.t. independent variables. Note that equations (2) and (3) represents the boundary layer form of momentum and energy equations, respectively, whereas, the convective term in the momentum equation comes from Boussinesq approximation and the energy equation is free from the viscous dissipation term. The BC’s are

\[
\begin{align*}
  u(r, z) &= V_1(z), \quad v(r, z) = V_2(z) \quad T(r, z) = T_w(z), \quad \text{at} \quad r = R \\
  u(r, z) &= 0, \quad T(r, z) = T_w, \quad \text{as} \quad r \to \infty,
\end{align*}
\]

where, \(\nu\) and \(\kappa\) are kinematic viscosity and thermal conductivity, also \(T_w\) and \(T_w\) are called temperature at the surface of cylinder and ambient region respectively, acceleration due to gravity is denoted by \(g\) and the coefficient of thermal expansion is denoted by \(\beta\). Moreover, the variable temperature of the cylinder is represented by \(T_w(z)\). The stretching (shrinking) velocity is demonstrated by \(V_1(z) = u_0(c_1 + a_0^2 z)^{\alpha_0}\), where \(a_0 = \frac{m_1 + m_2}{m_1 + m_2 + 2}\) and injection (suction) velocity is represented by \(V_2(z) = \frac{k_s}{R^2}\). Note that \(U_0 > 0(0 < 0)\) and \(k_s > 0(0 < 0)\) are the coefficients of velocity components taken in \(z\) and \(r\) directions, whereas, \(T\) denotes the temperature distribution respectively. In equations (1)–(3), \(u\) and \(v\) are the representatives of stretching (shrinking) and injection (suction) velocities respectively. In equations (1)–(4), we employed proper functions and similarity variable to transform them into a system of BVP of ODE’s. Therefore, we introduced the following variables for velocity components, temperature, and similarity function.

\[
\begin{align*}
  \nu(r, z) &= U_1 f(\eta) + V_2(z), \quad u(r, z) = U_2 g(\eta) \\
  \theta(\eta) &= \frac{T(r, z) - T_w}{T_w - T_w}, \quad \eta = \frac{k_3 s(r) - R(z)}{k_s R(z)}.
\end{align*}
\]
However, the known variables in equation (5) are expressed in the following form:

\[
U_1 = s_1 w_1, \quad U_2 = s_2 w_2, \quad s_1 = j_1 s_1^0, \quad s_2 = j_2 s_2^0,
\]

\[
w_1 = k_3 R^{-(1+m)}, \quad w_2 = k_2 R^{m}, s = s = \left( \frac{r}{a_0} \right)^m,
\]

\[
R(z) = R = (\alpha (c_1 + a_0^{-1}z)^{\frac{1}{2}}), \quad T_w(z) = T_{\infty} + T_0 R(z)^N,
\]

The non-uniform radius of the cylinder is denoted by \(R\) and defines surface geometry. The new independent variable \(\eta\) is so chosen that both the independent variables \(r\) and \(z\) are changed significantly. The radial, axial velocities and temperature functions are denoted by \(f, g,\) and \(\theta\) respectively. Note that the constants \(j_1, j_2, k_2, k_3, k_4,\) and \(k_5\) are the controlling parameters, whereas, \(a_0\) and \(k_1\) represent characteristic length and curvature parameter, respectively. Moreover, \(m, m_2, n_1,\) \(n_2, N,\) and \(\alpha = 2 + m_2 + m n_2\) are different parameters appeared as exponents of variables in equation (6). The variables in equations (5) and (6) are substituted into equations (1)–(3) and we get the set of following ODE's:

\[
\alpha_1(1 + k_1 \eta)^{-1-n_1} + \alpha_2(1 + k_1 \eta)^{-1}f + \alpha_3(1 + k_1 \eta)^{-1+1/m-n_1} + n_2 g + f' + \alpha_4(1 + k_1 \eta)^{1/m-n_1} + n_2 g' = 0.
\]

\[
\left( \frac{\theta_2}{1 + k_1 \eta} \right)^2 - \left( \frac{\theta_0}{1 + k_1 \eta} \right)^2 = \left( \frac{\theta_2}{1 + k_1 \eta} \right)^2 m_2 = 0.
\]

\[
Gr(1 + k_1 \eta)^{-2+2/m-n_2} \theta(\eta) + \left( k_1 + 2 \theta_4 \right) \left( \frac{\theta_0}{1 + k_1 \eta} \right)^{-1/m} \theta' + \left( \frac{\theta_1 \theta_3}{1 + k_1 \eta} \right)^{-1/m-n_1} g' + \left( \frac{\theta_2 \theta_3}{1 + k_1 \eta} \right)^{-2/m-n_2} \theta'' = 0.
\]

\[
(1 + k_1 \eta)^{\theta''} + \left( k_1 - \frac{Pr \theta_0 \theta_3}{1 + k_1 \eta} \right) \theta' = - \left( \frac{Pr \theta_0 \theta_3}{1 + k_1 \eta} \right)^{1/m-n_1} \theta' + \left( \frac{Pr \theta_0 \theta_3}{1 + k_1 \eta} \right)^{-1/m} f \theta' - \left( \frac{Pr \theta_0 \theta_3}{1 + k_1 \eta} \right)^{1/m-n_2} g \theta + \left( \frac{Pr \theta_0 \theta_3}{1 + k_1 \eta} \right)^{-2/m-n_2} g \theta' = 0.
\]

where, \(\theta_0, \theta_1\) and \(\theta_2\) are dimensionless numbers. On the other hand, the prime denotes the derivatives of unknown functions w.r.t. \(\eta\). The non-dimensionalized quantities in equations (7)–(9) are:

\[
\alpha_1 = \frac{\theta_0 \theta_3}{\theta_1}, \quad \alpha_2 = \theta_3 + \theta_5, \quad \alpha_3 = \frac{\theta_2 \theta_2 m_2}{\theta_1}, \quad \alpha_4 = \frac{\theta_2}{\theta_1}, \quad \theta_0 = \frac{a_0 k_4}{k_3^{1/m}}, \quad \theta_1 = \frac{a_0 k_4}{k_3^{1/m}}, \quad \theta_2 = \frac{d_0 + a_0 j_2 k_2^{1/m} + n_2}{k_3^{2/m} n_2}, \quad \theta_3 = \frac{k_1}{m}, \quad \theta_4 = k_1 n_2, \quad \theta_5 = k_1 n_1, \quad \theta_7 = \frac{a_0}{k_3^{2/m}}, \quad Gr = \frac{\theta_2^2 g' T_0 \beta \theta_0^3}{\theta_2^2}.
\]

where, \(Pr = \frac{\nu}{\kappa}\) and \(Gr\) are called Prandtl and modified Grashof number. The transformations in equations (5) and (6) are also used in equation (4) and we obtained the following BC's:

\[
f(c_2) = 0, \quad g(c_2) = \theta_6, \quad g(\infty) = 0, \quad \theta(c_2) = 1, \quad \theta(\infty) = 0, \quad \theta_6 = \frac{U_0}{\alpha_5}, \quad \alpha_5 = \frac{(j_2 k_2)(1 + \alpha)^{m_2 + m_2}}{m_2}.
\]

Here we have new quantities \(g, f, \) and \(\theta,\) whereas, they have replaced by field variable \(u, v,\) and \(T\) respectively. These kinematic and thermodynamic properties of field are determined by solving the system of ODE's in equations (7)–(9) subject to the BC's in equation (11).

**Evaluation of shear stress and rate of heat transfer at the wall of cylinder**

Skin friction and rate of heat transfer at wall are defined as:

\[
C_g = \frac{2 \tau}{\mu \rho U_w^2}, \quad Nu = \frac{r \xi_s}{\kappa (T_w - T_\infty)},
\]

\[\text{where} \quad \tau = \mu \left( \frac{\partial u}{\partial r} \right)_{r = R}, \quad \xi_s = - \kappa \left( \frac{\partial T}{\partial r} \right)_{r = R}.
\]

Skin friction coefficient is represented by \(C_g\) (non-dimensional) and \(Nu\) (non-dimensional) denotes Nusselt number. These physical entities are transformed into \(g'(c_2)\) and \(-\theta'(c_2),\) in view of the transformations given in equation (5) eventually we get the final forms of them as:

\[
g'(c_2) = \frac{\theta_3 \tau a_1^{1+m-n_1} (c_1 + a_0^{-1} z)^{-1+3/m}}{k_3 j_2 \rho \kappa a_1^{3/m}} - a_0^m \theta_4 \theta_0, \quad \theta'(c_2) = \frac{\theta_3 \xi_s (c_1 + a_0^{-1} z)^{-1+5/m} a_0^m + m a}{T_0 \kappa a_1^{5/m}}.
\]
Graphs and discussion

In this section, we discussed the numerical solution of equations (7)–(11), whereas, this solution of the problem is obtained by bvp4c package of MATLAB. Multiple graphs are drawn in this paper from the numerical solution for different values of the parameters, whereas, the key results have been discussed in Figures 2 to 9. Effects of curvature parameter ($k_1$) are seen on the profiles of velocity $g(h)$ and temperature $u(h)$ in Figures 2 and 3 respectively for fixed value of other dimensionless numbers. The momentum and thermal boundary layers are increased with the increase of curvature $k_1$. Moreover, the temperature profiles are also graphed against $h$ for the same set of parameters values. Similarly, effects of different $Gr$ are seen on the velocity and temperature profiles in Figures 4 and 5 respectively. It has been observed that the thickness of thermal boundary layers is decreased with the increasing of $Gr$. Note that the large values of $Gr$ present a weak bond between the fluid molecules. Therefore, it also decreases the strength of internal friction due to a strong enough gravity (accelerate the velocity of fluid). In laminar flow, the buoyancy parameter is highly effective within the boundary layer, formed during the moving and porous cylinder of non-uniform radius.
Remember that the positive values of Gr support the fluid motion. It is known fact that the buoyancy forces are producing the motion of fluid near the surface of cylinder, therefore, the increase and decrease in the velocity is observed at the vicinity of wall and the boundary layer behavior in the profiles is obvious due to the increase and decrease of Gr, however, the variation in velocity depends upon the value of thermal numbers. Figure 6 shows that the rate of heat transfer is increased with the increasing of Prandtl (Pr) number. Figure 7 demonstrates the graph of skin friction against $k_1$ for different values of stretching parameter $\theta_2$. Since higher Pr means that the fluid will be more viscous and vice versa. Effects of parameter $Gr$ is observed on the rate of heat transfer in Figure 8, however, heat transfer coefficient $-\theta'(0)$ is calculated against $k_1$ and $Pr$ respectively for different values of $Gr$. It is observed that the profiles are increased with the increasing of $Gr$, remember that on increase in $Gr$ implies an increase in the buoyancy force. The graphs are increased linearly with the increase of $Gr$, however, it shows that the buoyancy force is increasing. In Figure 9, effects of parameter $n_2$ are seen on velocity curves. Note that $n_2$ is appeared in the exponent of different functions and determines the types and nature of non-linearity of stretching (shrinking) velocities and wall temperature at the surface. With the increase of $n_2$ the boundary layer thicknesses are decreased. Similar effects of $n_2$ are observed on temperature functions for the same set of parameters but the graph is not presented here.

Comparison with literature

The current modeled problem in equations (7)–(9) is exactly matched with the published results of Poply et al.,29 Rangi and Ahmad,30 Vajravelu et al.,31 Mukhopadhyay and Ishak,20 and Mukhopadhyay and Gorla32 for given values of parameters, that is, when we fixed the parameters such that: $m = 2$, $n_1 = -1/2$, $n_2 = 0$, $j_1 = a_0^2 a_0^{-1}$, $j_2 = a_0 a_0^{-m_2}$, $k_1 = 2 \sqrt{\frac{v}{\alpha}}, k_2 = \frac{1}{k_1}$, $k_3 = \sqrt{\frac{v}{\alpha}}, k_4 = 0$, $k_5 = 1$, $c_1 = 0$, $\alpha = m_2$ and large
values of $m_2$. Moreover, if $k_1 = 0$ and $Gr = 0$ the problem reduces to the boundary layer flow along a stretching flat plate which was considered by Ali.$^{33}$ Furthermore, if $k_1 = 0$ and $Gr = 0$ the analytical solution of the thermal field was given by Grubka and Bobba.$^{34}$ In Table 1, the numerical values of skin friction and rate of heat transfer are noted for different parameters.

## Conclusion

We have concluded the main results and new observations with the following remarks:

1. The present model generalizes the problems of forced (convection) flow, over a vertical porous, moving and heated cylinder of non-uniform radius and thickness. Note that radius of the cylinder, its surface and the three field variables, defined at the wall are variable and may have linear, non-linear, uniform, algebraic and non-algebraic forms. Therefore, the diverse nature of all such problems is main objective of present simulation.

2. There are two different types of parameters in the final version of ODE’s, one set of parameters determines the non-linear nature of the field variables and wall thickness, whereas, the second set of parameters explores the set of dimensionless quantities.

3. We have seen indirect proportionality between velocity and temperature profiles with curvature parameter, however, for thermal parameters, there is direct and inverse proportionality between velocity and temperature profiles.

4. Skin friction and rate of heat transfer are graphed against different parameters and their linear, non-linear increasing and decreasing behaviors are observed.

5. Moreover, this research work can be extended to a corrugated cylinder of variable radius and the model may solve for exponentially variable thickness of wall of cylinder, whereas, the simulated problem can also be solved for non-Newtonian and Nano fluids flow easily. Furthermore, these investigations can be carried out for mixed convection flow and double diffusive convection in flow as well.

## Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

## Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

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**Appendix**

**Nomenclature**

| Symbol | Description |
|--------|-------------|
| $\theta_0$ | injection (suction) parameter |
| $\theta_1$ | supports the injection (suction) |
| $\theta_2$ | supports the stretching (shrinking) |
| $\theta_3$, $\theta_4$, $\theta_5$, $\theta_6$, $\theta_7$ | dimensionless parameters |
| $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$, $\alpha_5$ | dimensionless parameters |
| $c_5$ | power index parameter |
| $m$, $m_2$, $m_3$ | power indices |
| $n_1$, $n_2$ | characteristic length |
| $a_0$, $k_1$, $c_1$, $j_1$, $j_2$, $k_2$, $k_3$, $k_4$, $k_5$ | curvature parameter |
| $Pr$ | controlling parameters |
| $\text{Prandtl number}$ | controlling parameters |

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\( R(z) \) nonuniform (variable) radius of the cylinder

\( T \) temperature distribution

\( T_w(z) \) nonuniform temperature of the cylinder

\( T_\infty \) uniform temperature at ambient region

\( u, v \) velocity components

\( U_1, U_2 \) velocities at the surface of cylinder

\( U_0 \) coefficient of stretching/shrinking velocity

\( V_1(z) \) stretching(shrinking) velocity of cylinder

\( V_2(z) \) injection(suction) velocity

\( q_s \) rate of heat transfer

\( r, z \) cylindrical coordinates

\( \eta \) similarity variable

\( \kappa \) thermal conductivity

\( \mu \) viscosity of the fluid

\( \nu \) kinematic viscosity of the fluid

\( \rho \) density of the fluid

\( \tau_s \) shear stress

\( \Delta T \) temperature difference, \( T_w - T_\infty \)

**Function notation**

\( f \) representative of radial velocity

\( g \) representative of axial velocity

\( \theta \) temperature distribution

**Subscripts**

\( r, z \) partial derivative w.r.t. \( r \) and \( z \)

\( \infty \) ambient region