Dynamic Asymmetric Communication

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Abstract

We show how any dynamic instantaneous compression algorithm can be converted to an asymmetric communication protocol, with which a server with high bandwidth can help clients with low bandwidth send it messages. Unlike previous authors, we do not assume the server knows the messages’ distribution, and our protocols are the first to use only one round of communication for each message.

Key words: Data compression; asymmetric communication.

1 Introduction

Internet users usually download more than they upload, and many technologies have asymmetric bandwidth — greater from servers to clients than from clients to servers. Adler and Maggs considered whether a server can use its greater bandwidth to help clients send it messages. They proved it can, assuming it knows the messages’ distribution. We argue this assumption is often both unwarranted and, fortunately, unnecessary.

Suppose a number of clients want to send messages to a server. At any point, the server knows all the messages it has received so far; each client only knows its own messages and does not overhear communication between other clients and the server. Thus, the server may be able to construct a good code but the clients individually cannot. Adler and Maggs assumed the server, after receiving a sample of messages, can accurately estimate the distribution of all the messages. This assumption let them simplify the problem: Can the server help a single client send it a message drawn from a distribution known to the server? Given a representative sample of messages and a protocol for this simpler problem, the server can just repeat the protocol for each remaining message. In fact, it can even do this in parallel.
Adler and Maggs gave protocols for the simpler problem in which the server uses its knowledge to reduce the expected number of bits the client sends to roughly the entropy of the distribution. Their work has been improved and extended by several authors [13,9,11,5], whose results are summarized in Table 1 and used in the Infranet anti-censorship system [7,8]. However, while implementing Infranet, Wang [12] found the distribution of the messages (webpage requests) changed over time — the sample was unreliable.

| References | Bits sent by Server | Bits sent by Client | Rounds |
|------------|---------------------|---------------------|--------|
| [3,11]     | $3\lceil\log n\rceil$ | $1.09H + 1$        | $1.09H + 1$ |
| [3]        | $O(\log n)$         | $O(H + 1)$         | $O(1)$ |
| [13]       | $(H + 2)\lceil\log n\rceil$ | $H + 2$ | $H + 2$ |
| [13]       | $O(2^k H \log n)$   | $H + 2$            | $(H + 1)/k + 2$ |
| [9]        | $kH\lceil\log n\rceil + 1$ | $H \log_{k-1} k + 1$ | $H/\log k + 1$ |
| [5]        | $(k + 2)\lceil\log n\rceil$ | $H \log_{k+2} k + 2$ | $H/\log(k+2) - 1 + 1$ |

Table 1

Suppose a server tries to help a client send it one of $n$ messages, chosen according to a distribution with entropy $H$ that is known to the server but not the client. Adler and Maggs [3], Watkinson, Adler and Fich [13], Ghazizadeh, Ghodsi and Saberi [9] and Bose, Krizanc, Langerman and Morin [5] gave protocols for this problem with expected-case upper bounds as shown; the last three protocols take a parameter $k \geq 1$. This table is based on one given by Bose et al. but we use a different notation.

2 Dynamic compression and asymmetric communication

In this section we show how algorithms for dynamic instantaneous data compression — e.g., dynamic Huffman coding [10] and move-to-front compression [4] — can be converted to dynamic asymmetric communication protocols. By dynamic protocols, we mean ones not needing the server to know the messages’ distribution.

For dynamic instantaneous compression (also called prefix-free compression), an encoder makes a single pass over a string $S$ and writes a codeword after reading each character; a decoder can later make a single pass over the encoding and writes a character of $S$ after reading each codeword. Implicitly or

1 Table 1 does not include a recent paper by Adler [11], in which he considered a harder version of the original problem with many clients: Can the server take advantage of correlations between messages? He showed it can, but used the even stronger assumption that the server knows the probability distribution over entire sequences of messages.
explicitly, the encoder and decoder each maintain a binary tree, called a code-tree, in which left edges are labelled with 0s, right edges are labelled with 1s and leaves are labelled with the characters in the alphabet; the binary string on the path from the root to a leaf is the codeword for that leaf’s label. When the encoder reads a character $a$, it writes the codeword for $a$ and updates its code-tree; when the decoder reads the codeword for $a$, it writes $a$ and updates its code-tree. Notice that, as long as the encoder and decoder initialize and update their code-trees according to the same rule, then the encoder’s code-tree when writing a codeword and the decoder’s code-tree when reading that codeword are the same — so the decoder always writes the same characters the encoder has read.

**Theorem 1** Suppose a number of clients want to send a sequence $S$ of messages, drawn from a set of size $n$, to a server. Furthermore, suppose there is a dynamic instantaneous compression algorithm that, when applied to $S$, produces a $B$-bit encoding. Then for any $k \geq 1$, there is a dynamic asymmetric communication protocol with which the server sends at most $O(n^{1/k} \log n)$ bits to each client and the clients send, in total, at most $kB + 2|S|$ bits.

**PROOF.** The server builds a code-tree $T$ in the same way the encoder would. For each message $s$ in $S$, the server truncates $T$ at depth $\lceil (\log n)/k \rceil$ to obtain a tree $T'$ (some or all of whose leaves may not be labelled), then sends $T'$ to the client that has $s$. Since $T'$ has at most $2^{\lceil (\log n)/k \rceil} \leq 2n^{1/k}$ leaves, the server can send it to the client in $O(n^{1/k} \log n)$ bits.

The client examines $T'$ and, if it finds $s$ labelling a leaf, responds with 1 followed by the codeword for $s$ according to $T'$ (and $T$); if not, it responds with 0 followed by the $\lceil \log n \rceil$-bit representation of $s$’s index in the set of possible messages. Notice that, in both cases, the server receives enough information to recover $s$; once it has done this, the server updates $T$ in the same way the encoder would. Thus, $T$ is always the same as if maintained by the encoder.

Let $b$ be the length of the codeword for $s$ in $T$. If $b \leq \lceil (\log n)/k \rceil$, then the client sends $b + 1$ bits; otherwise, the client sends $\lceil \log n \rceil + 1 < kb + 2$ bits. Since the compression algorithm encodes $S$ in $B$ bits, all the clients together send, in total, at most $kB + 2|S|$ bits.  

3 Incompressibility and asymmetric communication

An advantage of our conversion in Theorem 1 from dynamic instantaneous compression to dynamic asymmetric communication, is that the resulting protocols use only one round of communication for each message; i.e., the server
sends a truncated code-tree and the client responds with either a codeword or an index. Reducing the number of rounds needed is useful because, as Adler, Demaine, Harvey and Pătrașcu \cite{adler2015towards} wrote:

Any time savings obtained from reducing the number of bits sent by the client could easily be lost by the extra latency cost induced by multiple rounds in the protocol, particularly in long-distance networks, such as satellites, where communication has very high latency.

However, in the same paper, they proved a lower bound for protocols that use few rounds. They considered the simpler asymmetric communication problem — in which the server knows the distribution (with entropy $H$) over the $n$ possible messages and there is only one client. Lower bounds for this simpler problem also hold for dynamic asymmetric communication. They proved protocols that use $o\left(\frac{\log\log n}{\log\log\log n}\right)$ rounds with high probability and with which the client is expected to send at most $O(H + 1)$ bits cannot have a $2^{(\log n)^{1-\epsilon}}$ upper bound on the number of bits the server sends, for any $\epsilon > 0$.

For the special case of single-round protocols, a stronger, earlier lower bound was proved by Adler and Maggs \cite{adler1994message}: Protocols with which the client is expected to send at most $kH$ bits cannot have a $\frac{9}{30}n^{1/(20k)} \log n$ upper bound on the number of bits the server sends. We now prove a nearly tight lower bound for this special case, assuming transmissions are self-delimiting; we show afterward that assumption is not necessary.

**Theorem 2** There does not exist a single-round asymmetric communication protocol with which, given a probability distribution $P$ with entropy $H$ over a set of $n$ possible messages, the server sends $O(n^{1/k-\epsilon})$ bits in the worst case and the client sends at most $kH + o(\log n)$ bits in the expected case, for any $k \geq 1$ and $\epsilon > 0$.

**PROOF.** For the sake of a contradiction, assume there does exist such a protocol. Let $S = s_1, \ldots, s_m$ be a sequence of $m = n^{1/k-\epsilon/2}$ messages chosen uniformly at random and let $P$ be the normalized distribution of messages in $S$. By assumption, the server sends sends $O(n^{1/k-\epsilon}) \subset o(m)$ bits and, by the definition of entropy,

$$H \leq \log m = \left(\frac{1}{k} - \frac{\epsilon}{2}\right) \log n.$$  

Let $a'$ denote the client’s response when it has message $a$. Since $P$ is the normalized distribution of the messages in $S$, the length of $s'_1 \cdots s'_m$ is $m$

\footnote{This does not contradict the second row of Table \ref{table:protocols}. Adler and Maggs’ second protocol uses $O(1)$ rounds in the expected case but not with high probability.}
times the expected length of the client’s transmission — at most
\[ m \left( kH + o(\log n) \right) = \left( 1 - \frac{\epsilon k}{2} \right) m \log n + o(m \log n) \]

bits.

Since transmissions are self-delimiting, we can store \( S \) as the server’s transmission followed by \( s'_1 \cdots s'_m \); thus, we can store \( S \) in \( (1 - \frac{\epsilon k}{2}) m \log n + o(m \log n) \) bits. However, by a simple counting argument, storing \( S \) takes \( m \log n \) bits in the average case. \( \square \)

Even if the channel between the server and the client indicates the end of transmissions — so the client’s response need not be self-delimiting — Theorem 2 still applies. To see why, assume there is a single-round protocol for such channels, with which the server sends \( O(n^{1/k-\epsilon}) \) bits and the client is expected to send at most \( kH + o(\log n) \) bits. We can make all transmissions self-delimiting by prefacing each transmission by its length encoded in Elias’ gamma code [6]; in this code, the codeword for the positive integer \( i \) has length \( 2\lfloor \log i \rfloor + 1 \). Notice the resulting protocol still takes a single round; since \( O(\log n^{1/k} + \log n^{1/k-\epsilon}) = O(n^{1/k-\epsilon}) \), the server’s self-delimiting transmission is still \( O(n^{1/k-\epsilon}) \) bits; by Jensen’s Inequality and because \( H \leq \log n \), we have \( kH + 2 \log(kH) + 1 + o(\log n) = kH + o(\log n) \) and the expected length of the client’s self-delimiting transmission is still at most \( kH + o(\log n) \) bits. However, Theorem 2 forbids the existence of such a protocol.

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