Research Article
Some Hermite-Hadamard Type Inequalities for Harmonically s-Convex Functions

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We establish some estimates of the right-hand side of Hermite-Hadamard type inequalities for functions whose derivatives absolute values are harmonically s-convex. Several Hermite-Hadamard type inequalities for products of two harmonically s-convex functions are also considered.

1. Introduction

Let \( f : I \subseteq \mathbb{R} \to \mathbb{R} \) be a convex function and \( a, b \in I \) with \( a < b \); then

\[
\frac{f\left(\frac{a+b}{2}\right)}{2} \leq \frac{1}{b-a} \int_a^b f(t) \, dt \leq \frac{f(a) + f(b)}{2}. \tag{1}
\]

Inequality (1) is known as the Hermite-Hadamard inequality.

In [1], Hudzik and Maligranda considered the class of functions which are \( s \)-convex in the second sense. This class of functions is defined as follows.

A function \( f : [0, \infty) \to [0, \infty) \) is said to be \( s \)-convex in the second sense if the inequality

\[
f(ax + (1-a)y) \leq ax^s f(x) + (1-a)^s f(y) \tag{2}
\]

holds for all \( x, y \in [0, \infty), a \in [0, 1] \) and for some fixed \( s \in (0, 1] \).

It can be easily seen that, for \( s = 1 \), \( s \)-convexity reduces to ordinary convexity of functions defined on \( [0, \infty) \).

In [2], Dragomir and Fitzpatrick established a variant of Hermite-Hadamard inequality which holds for the \( s \)-convex functions in the second sense.

**Theorem 1** (see [2]). Suppose that \( f : [0, \infty) \to [0, \infty) \) is an \( s \)-convex function in the second sense, where \( s \in (0, 1] \) and let \( a, b \in [0, \infty), a < b \). If \( f \in L[a, b] \), then the following inequalities hold:

\[
2^{s-1} f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a} \int_a^b f(t) \, dt \leq \frac{f(a) + f(b)}{2}. \tag{3}
\]

Some generalizations, improvements, and extensions of inequalities (1) and (3) can be found in the recent papers [2–18].

In [16], İscan investigated the Hermite-Hadamard type inequalities for harmonically convex functions.

**Definition 2** (see [16]). Let \( I \subseteq \mathbb{R} \setminus \{0\} \) be a real interval. A function \( f : I \to \mathbb{R} \) is said to be harmonically convex, if

\[
f\left(\frac{xy}{tx + (1-t)y}\right) \leq tf(y) + (1-t)f(x), \tag{4}
\]

for all \( x, y \in I \) and \( t \in [0, 1] \). If the inequality in (4) is reversed, then \( f \) is said to be harmonically concave.

**Theorem 3** (see [16]). Let \( f : I \subseteq \mathbb{R} \setminus \{0\} \to \mathbb{R} \) be a harmonically convex function and \( a, b \in I \) with \( a < b \). If \( f \in L(a, b) \), then one has

\[
f\left(\frac{2ab}{a+b}\right) \leq \frac{ab}{b-a} \int_a^b \frac{f(x)}{x^2} \, dx \leq \frac{f(a) + f(b)}{2}. \tag{5}
\]
Theorem 4 (see [16]). Let \( f : I \subseteq \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R} \) be a differentiable function on \( I^0 \) (\( I^0 \) is the interior of \( I \)), \( a, b \in I \) with \( a < b \), and \( f' \in L[a, b] \); then

\[
\frac{f(a) + f(b)}{2} - \frac{ab}{b-a} \int_a^b \frac{f(x)}{x^2} \, dx = \frac{ab(b-a)}{2} \int_0^1 \frac{1 - 2t}{(tb + (1-t)a)^2} f'(tb + (1-t)a) \, dt.
\]  

(6)

In [19], İşcan investigated the Hermite-Hadamard type inequalities for harmonically \( s \)-convex functions.

Definition 5 (see [19]). Let \( I \subseteq \mathbb{R} \setminus \{0\} \) be a real interval. A function \( f : I \subseteq (0, \infty) \rightarrow \mathbb{R} \) is said to be harmonically \( s \)-convex, if

\[
f\left( \frac{xy}{tx + (1-t)y} \right) \leq t^s f(y) + (1-t)^s f(x),
\]  

(7)

for all \( x, y \in I \), \( t \in [0, 1] \) and for some fixed \( s \in (0, 1] \). If the inequality in (7) is reversed, then \( f \) is said to be harmonically \( s \)-concave.

Theorem 6 (see [19]). Let \( f : I \subseteq (0, \infty) \rightarrow \mathbb{R} \) be a harmonically \( s \)-convex function and \( a, b \in I \) with \( a < b \). If \( f \in L(a, b) \), then one has

\[
2^{s-1} f \left( \frac{2ab}{a+b} \right) \leq \frac{ab}{b-a} \int_a^b \frac{f(x)}{x^2} \, dx = \frac{f(a) + f(b)}{s+1}.
\]  

(8)

In [20], Pachpatte established two new Hermite-Hadamard type inequalities for products of convex functions asserted by Theorem 7.

Theorem 7 (see [20]). Let \( f \) and \( g \) be real-valued, nonnegative, and convex functions on \([a, b]\). Then

\[
\frac{1}{b-a} \int_a^b f(x) g(x) \, dx \leq \frac{1}{3} M(a, b) + \frac{1}{6} N(a, b),
\]

\[
2f \left( \frac{a+b}{2} \right) g \left( \frac{a+b}{2} \right) \leq \frac{1}{b-a} \int_a^b f(x) g(x) \, dx + \frac{1}{6} M(a, b) + \frac{1}{3} N(a, b),
\]  

(9)

where \( M(a, b) = f(a)g(a) + f(b)g(b) \) and \( N(a, b) = f(a)g(b) + f(b)g(a) \).

For more results concerning the Hermite-Hadamard inequality, we refer the reader to [21–25] and the references cited therein.

In this paper, we establish some estimates of the right-hand side of Hermite-Hadamard type inequalities for functions whose derivatives absolute values are harmonically \( s \)-convex. Moreover, we provide several Hermite-Hadamard type inequalities for products of two harmonically \( s \)-convex functions.

2. Inequalities for Harmonically \( s \)-Convex Functions

We recall the following special functions.

The gamma function is as follows:

\[
\Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} \, dt, \quad x > 0;
\]  

(10)

the beta function is as follows:

\[
\beta(x, y) = \int_0^1 (1-t)^{y-1} t^{x-1} \, dt, \quad x > 0, \ y > 0,
\]

\[
\beta(x, y) = \frac{\Gamma(x) \Gamma(y)}{\Gamma(x+y)};
\]  

(11)

the hypergeometric function is as follows:

\[
\mbox{}_2F_1(x, y; c; z) = \frac{1}{\beta(y, c-y)} \int_0^1 t^{y-1} (1-t)^{c-y-1} (1-zt)^{-x} \, dt,
\]  

(12)

\[|z| < 1, \ c > y > 0.\]

Our main results are given in the following theorems.

Theorem 8. Let \( f : I \subseteq (0, \infty) \rightarrow \mathbb{R} \) be a differentiable function on \( I^0 \) such that \( f' \in L[a, b] \), where \( a, b \in I^0 \) with \( a < b \). If \( |f'|^q \) is harmonically \( s \)-convex on \([a, b]\) for some fixed \( s \in (0, 1] \), \( q \geq 1 \), then

\[
\left| \frac{f(a) + f(b)}{2} - \frac{ab}{b-a} \int_a^b \frac{f(x)}{x^2} \, dx \right| \leq \frac{ab(b-a)}{2} C_1^{-1/q}(a, b) \left[ \frac{C_2(s; a, b)}{C_3(s; a, b)} \right]^{\frac{q-1}{q}},
\]  

(13)

where

\[
C_1(a, b) = b^{-2} \left( \mbox{}_2F_1 \left( 2, 2; 3; 1 - \frac{a}{b} \right) - \mbox{}_2F_1 \left( 2, 1; 2; 1 - \frac{a}{b} \right) \right) + \frac{1}{2} \mbox{}_2F_1 \left( 2, 1; 3; \frac{1}{2} \left( 1 - \frac{a}{b} \right) \right),
\]
Proof. Let $A_s = ta + (1 - t)b$. Using Theorem 4, the power mean inequality, and the harmonically s-convexity of $|f'|^q$, we have

\[
\left| \frac{f(a) + f(b)}{2} - \frac{ab}{b - a} \int_a^b \frac{f(x)}{x^2} dx \right| \\
\leq \frac{ab(b - a)}{2} \int_0^1 \frac{|1 - 2t|}{A^2_t} |f'(\frac{ab}{A_t})| dt \\
\leq \frac{ab(b - a)}{2} \left( \int_0^1 \frac{|1 - 2t|}{A^2_t} dt \right)^{1-1/q} \\
\times \left( \int_0^1 \frac{|1 - 2t|}{A^2_t} \left| f'(\frac{ab}{A_t}) \right|^q dt \right)^{1/q} \\
\leq \frac{ab(b - a)}{2} K_1^{-1/q} \\
\times \left( \int_0^1 \frac{|1 - 2t|}{A^2_t} \left[ |1 - t|^q + t^q |f'(b)|^q \right] dt \right)^{1/q} \\
\leq \frac{ab(b - a)}{2} K_2^{1/q} (K_2 |f'(b)|^q + K_3 |f'(a)|^q)^{1/q},
\]

where

\[
K_1 = \int_0^1 \frac{|1 - 2t|}{A^2_t} dt, \\
K_2 = \int_0^1 \frac{|1 - 2t|}{A^2_t} t^q dt, \\
K_3 = \int_0^1 \frac{|1 - 2t|}{A^2_t} (1 - t)^q dt.
\]

Calculating $K_1$, $K_2$, and $K_3$, we find

\[
K_1 = \int_0^1 \frac{|1 - 2t|}{A^2_t} dt \\
= \int_0^{1/2} \frac{1 - 2t}{A^2_t} dt + \int_0^{1/2} \frac{2t - 1}{A^2_t} dt \\
= \int_0^1 \frac{2t - 1}{A^2_t} dt + 2 \int_0^1 \frac{1 - 2t}{A^2_t} dt \\
= 2 \int_0^1 tA_s^2 dt - \int_0^1 A_s^2 dt \\
+ \int_0^1 (1 - u) b^{-2} \left( 1 - u \frac{1}{2} \left( 1 - \frac{a}{b} \right) \right)^2 du \\
= b^{-2} \left( 2F_1 \left( 2; 2; 3; 1 - \frac{a}{b} \right) \right. \\
- \left. \frac{1}{2} b^{-2} \left( 2F_1 \left( 2; 1; 2; 1 - \frac{a}{b} \right) \right) \right) \\
= C_1 (a, b).
\]

Similarly, we get

\[
K_2 = \int_0^1 \frac{|1 - 2t|}{A^2_t} t^q dt \\
= \int_0^1 \frac{2t - 1}{A^2_t} t^q dt + 2 \int_0^{1/2} \frac{1 - 2t}{A^2_t} t^q dt \\
= 2 \int_0^1 t^q A_s^2 dt - \int_0^1 t^q A_s^2 dt \\
+ \frac{1}{2} \int_0^1 (1 - u) b^{-2} \left( 1 - u \frac{1}{2} \left( 1 - \frac{a}{b} \right) \right)^2 du \\
= \frac{2b^{-2}}{s + 1} 2F_1 \left( 2, s + 2; s + 3; 1 - \frac{a}{b} \right) \\
- \frac{b^{-2}}{s + 1} 2F_1 \left( 2, s + 1; s + 2; 1 - \frac{a}{b} \right) \\
+ \frac{b^{-2}}{2} \int_0^1 (1 - u) b^{-2} \left( 1 - u \frac{1}{2} \left( 1 - \frac{a}{b} \right) \right)^2 du \\
= C_2 (a, b),
\]

and

\[
K_3 = \int_0^1 \frac{|1 - 2t|}{A^2_t} (1 - t)^q dt \\
= \int_0^1 \frac{2t - 1}{A^2_t} (1 - t)^q dt + 2 \int_0^{1/2} \frac{1 - 2t}{A^2_t} (1 - t)^q dt.
\]
\[ \leq 2 \int_0^1 (1-t)^2 A_i^{-2} dt \\
- \int_0^1 (1-t)^4 A_i^{-2} dt + 2 \int_0^{1/2} 1 - 2t A_i^2 dt \\
= b^{-2} \left( \frac{2}{s+1} \frac{s}{s+2} F_1 \left( \frac{2,2,s+3,1-a}{b} \right) \right) \\
- \frac{1}{s+1} 2F_1 \left( 2,1,s+2,1-\frac{a}{b} \right) \\
+ \frac{1}{2} 2F_1 \left( 2,1,3,\frac{1}{2} (1-\frac{a}{b}) \right) \\
= C_3 (s; a,b). \] (19)

This completes the proof of Theorem 8.

Theorem 9. Let \( f : I \subseteq (0, \infty) \rightarrow \mathbb{R} \) be a differentiable function on \( I \) such that \( f' \in L[a,b] \), where \( a,b \in I^0 \) with \( a < b \). If \( f'' \) is harmonically \( s \)-convex on \( [a,b] \) for some fixed \( s \in (0,1] \), \( q > 1 \), then

\[ \frac{f(a) + f(b)}{2} - \frac{ab}{b-a} \int_a^b \frac{f(x)}{x^2} dx \leq \frac{a(b-a)}{2b} \left( \frac{1}{p+1} \right)^{1/p} \]

\[ \times \left( \left( 2F_1 \left( \frac{2q,s+1,s+2,1-a}{b} \right) \right)[f''(b)]^q \right) \]

\[ + \frac{1}{s+1} 2F_1 \left( \frac{2q,1,s+2,1-a}{b} \right)[f''(a)]^q \right) \]

\[ \times (s+1)^{-1/q}, \] (20)

where \((1/p) + (1/q) = 1.\)

Proof. Let \( A_t = ta + (1-t)b \). Utilizing Theorem 4, the Hölder inequality, and the harmonically \( s \)-convexity of \( f'' \), we have

\[ \frac{f(a) + f(b)}{2} - \frac{ab}{b-a} \int_a^b \frac{f(x)}{x^2} dx \]

\[ \leq \frac{ab(b-a)}{2} \int_0^1 \frac{1}{A_i^2} \left[ f'(\theta) \right] \left| f''(b) \right|^q d\theta \]

\[ \leq \frac{ab(b-a)}{2} \left( \left( \int_0^1 \frac{1}{A_i^2} \right)^{1/p} \left( \left( \int_0^1 \frac{1}{A_i^2} \right)^{1/q} \right)^{1/q} \right) \]

\[ \leq \frac{ab(b-a)}{2} K_4^{1/p}. \]

This completes the proof of Theorem 9.

Theorem 10. Let \( f, g : [a,b] \rightarrow [0,\infty), a,b \in (0,\infty), a < b \), be functions such that \( f, g, fg \in L[a,b] \). If \( f \) is harmonically \( s_1 \)-convex and \( g \) is harmonically \( s_2 \)-convex on \( [a,b] \) for some fixed \( s_1, s_2 \in (0,1] \), then

\[ \frac{ab}{b-a} \int_a^b \frac{f(x)g(x)}{x^2} dx \]

\[ \leq \frac{1}{1 + s_1 + s_2} M(a,b) \Gamma \left( \frac{1 + s_1}{s_1 + s_2 + 2} \right) N(a,b), \] (23)

where \( M(a,b) = f(a)g(a) + f(b)g(b) + N(a,b) = f(a)g(b) + f(b)g(a). \)

Proof. Since \( f \) is harmonically \( s_1 \)-convex and \( g \) is harmonically \( s_2 \)-convex on \( [a,b] \), then for \( t \in [0,1] \) we get

\[ f \left( \frac{ab}{ta + (1-t)b} \right) \leq t^{s_1} f(b) + (1-t)^{s_1} f(a), \]

\[ g \left( \frac{ab}{ta + (1-t)b} \right) \leq t^{s_2} g(b) + (1-t)^{s_2} g(a). \] (24)
From (24), we get
\[
\begin{align*}
\frac{ab}{t a + (1 - t) b} \frac{ab}{t a + (1 - t) b} \\
\leq t^{s_1 + s_2} f(b) g(b) + (1 - t)^{s_1 + s_2} f(a) g(a) \\
+ t^s (1 - t)^{s_2} f(b) g(a) + (1 - t)^{s_1} t^s f(a) g(b).
\end{align*}
\]
\[ (25) \]

Integrating both sides of the above inequality with respect to \( t \) over \([0, 1]\), we obtain
\[
\begin{align*}
\int_0^1 \frac{ab}{t a + (1 - t) b} \frac{ab}{t a + (1 - t) b} \, dt \\
= \frac{ab}{b - a} \int_a^b \frac{f(x) g(x)}{x^2} \, dx \\
\leq \frac{1}{s_1 + s_2 + 1} \left[ f(a) g(a) + f(b) g(b) \right] \\
+ f(a) g(b) \int_0^1 (1 - t)^{s_1 + t^{s_2}} \, dt \\
+ f(b) g(a) \int_0^1 t^s (1 - t)^{s_2} \, dt \\
= \frac{1}{s_1 + s_2 + 1} M(a, b) + \beta(1 + s_1, s_2 + 1) N(a, b).
\end{align*}
\]
\[ (26) \]

The proof of Theorem 10 is completed.

Remark 11. Taking \( s_1 = s_2 = 1 \) in Theorem 10, we obtain
\[
\frac{ab}{b - a} \int_a^b \frac{f(x) g(x)}{x^2} \, dx \leq \frac{1}{3} M(a, b) + \frac{1}{6} N(a, b).
\]
\[ (27) \]

Remark 12. Choosing \( s_1 = s_2 = 1 \) and \( g \equiv 1 \) in Theorem 10 gives
\[
\frac{ab}{b - a} \int_a^b \frac{f(x)}{x^2} \, dx \leq \frac{f(a) + f(b)}{2},
\]
\[ (28) \]

which is the right-hand side inequality of (5).

Theorem 13. Let \( f, g : [a, b] \rightarrow [0, \infty), a, b \in (0, \infty), a < b, \) be functions such that \( f, g, fg \in L[a, b] \). If \( f \) is harmonically \( s_1 \)-convex and \( g \) is harmonically \( s_2 \)-convex on \([a, b]\) for some fixed \( s_1, s_2 \in [0, 1]\), then
\[
\begin{align*}
2^{s_1 + s_2 - 1} f\left(\frac{2ab}{a + b}\right) g\left(\frac{2ab}{a + b}\right) \\
\leq \frac{ab}{b - a} \int_a^b \frac{f(x) g(x)}{x^2} \, dx + M(a, b) \frac{(1 + s_1)}{(s_1 + s_2 + 2)} \\
+ \frac{1}{s_2 + s_1 + 1} N(a, b),
\end{align*}
\]
\[ (29) \]

where \( M(a, b) = f(a) g(a) + f(b) g(b) \) and \( N(a, b) = f(a) g(b) + f(b) g(a) \).

Proof. Using the harmonically \( s \)-convexity of \( f \) and \( g \), we have for all \( x, y \in [a, b] \)
\[
\begin{align*}
f\left(\frac{2xy}{x + y}\right) &\leq \frac{f(y) + f(x)}{2^s}, \\
g\left(\frac{2xy}{x + y}\right) &\leq \frac{g(y) + g(x)}{2^s}.
\end{align*}
\]
\[ (30) \]

Choosing \( x = ab/(tb + (1 - t)a) \) and \( y = ab/(tb + (1 - t)a) \), we have
\[
\begin{align*}
f\left(\frac{2ab}{a + b}\right) g\left(\frac{2ab}{a + b}\right) \\
&\leq f\left(\frac{ab}{tb + (1 - t)a}\right) + f\left(\frac{ab}{ta + (1 - t)b}\right) \\
&\times g\left(\frac{ab}{tb + (1 - t)a}\right) + g\left(\frac{ab}{ta + (1 - t)b}\right) \\
&= \frac{1}{2^{s_1 + s_2}} \left[ f\left(\frac{ab}{tb + (1 - t)a}\right) g\left(\frac{ab}{tb + (1 - t)a}\right) \\
&+ f\left(\frac{ab}{ta + (1 - t)b}\right) g\left(\frac{ab}{ta + (1 - t)b}\right) \\
&+ g\left(\frac{ab}{tb + (1 - t)a}\right) f\left(\frac{ab}{ta + (1 - t)b}\right) \\
&+ g\left(\frac{ab}{ta + (1 - t)b}\right) f\left(\frac{ab}{tb + (1 - t)a}\right) \right] \\
&\leq \frac{1}{2^{s_1 + s_2}} \left[ f\left(\frac{ab}{tb + (1 - t)a}\right) g\left(\frac{ab}{tb + (1 - t)a}\right) \\
&+ f\left(\frac{ab}{ta + (1 - t)b}\right) g\left(\frac{ab}{ta + (1 - t)b}\right) \right] \\
&\times \left[ t^s f(a) + (1 - t)^s f(b) \right] \\
&\times \left[ (1 - t)^s g(a) + t^s g(b) \right] \\
&+ \left[ (1 - t)^s f(a) + t^s f(b) \right] \\
&\times \left[ t^s g(a) + (1 - t)^s g(b) \right] \\
&= \frac{1}{2^{s_1 + s_2}} \left[ f\left(\frac{ab}{tb + (1 - t)a}\right) g\left(\frac{ab}{tb + (1 - t)a}\right) \\
&+ f\left(\frac{ab}{ta + (1 - t)b}\right) g\left(\frac{ab}{ta + (1 - t)b}\right) \right] \\
&\times \left[ t^s (1 - t)^s + t^s (1 - t)^s \right] M(a, b) \\
&+ \left[ (1 - t)^s + t^s \right] N(a, b).
\end{align*}
\]
\[ (31) \]
Integrating the resulting inequality with respect to $t$ over $[0,1]$, we get

$$
 f \left( \frac{2ab}{a+b} \right) \leq \frac{1}{2^{s_1+s_2}} \left[ \int_0^1 f \left( \frac{ab}{tb + (1-t)a} \right) g \left( \frac{ab}{tb + (1-t)a} \right) dt 
 + \int_0^1 f \left( \frac{ab}{ta + (1-t)b} \right) g \left( \frac{ab}{ta + (1-t)b} \right) dt \right] 
 + \frac{1}{2^{s_1+s_2}} \left\{ M(a,b) \int_0^1 \left[ t^{s_1}(1-t)^{s_2} + (1-t)^{s_1}t^{s_2} \right] dt 
 + N(a,b) \int_0^1 \left[ (1-t)^{s_2+s_1} + t^{s_2+s_1} \right] dt \right\}.
$$

That is,

$$
 f \left( \frac{2ab}{a+b} \right) \leq \frac{ab}{b-a} \int_a^b \frac{f(x)g(x)}{x^2} dx
 + \frac{1}{2^{s_1+s_2}} \left\{ M(a,b) \int_0^1 \left[ t^{s_1}(1-t)^{s_2} + (1-t)^{s_1}t^{s_2} \right] dt 
 + N(a,b) \int_0^1 \left[ (1-t)^{s_2+s_1} + t^{s_2+s_1} \right] dt \right\}.
$$

(32)

From

$$
 \int_0^1 \left[ t^{s_1}(1-t)^{s_2} + (1-t)^{s_1}t^{s_2} \right] dt = \frac{2}{s_2+s_1+1},

\int_0^1 \left[ (1-t)^{s_2+s_1} + t^{s_2+s_1} \right] dt = \frac{2}{s_2+s_1+1},
$$

(34)

we get

$$
 2^{s_1+s_2} f \left( \frac{2ab}{a+b} \right) \leq \frac{ab}{b-a} \int_a^b \frac{f(x)g(x)}{x^2} dx + M(a,b) \beta \left( s_1 + 1, s_2 + 1 \right) 
 + N(a,b) \frac{1}{s_2+s_1+1}.
$$

(35)

This completes the proof of Theorem 13.

Remark 14. Putting $s_1 = s_2 = 1$ in Theorem 13 gives

$$
 2f \left( \frac{2ab}{a+b} \right) \leq \frac{ab}{b-a} \int_a^b \frac{f(x)g(x)}{x^2} dx + \frac{1}{6} M(a,b) + \frac{1}{3} N(a,b).
$$

(36)

Remark 15. If we take $s_1 = s_2 = 1$ and $g \equiv 1$ in Theorem 13, then we obtain

$$
 2f \left( \frac{2ab}{a+b} \right) \leq \frac{ab}{b-a} \int_a^b \frac{f(x)g(x)}{x^2} dx + \frac{f(a) + f(b)}{2}.
$$

(37)

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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