Nonlinear optical properties in a nanoring: quantum size and magnetic field effect

Shijun Liang1, Wenfang Xie1, H A Sarkisyan2,3, A V Meliksetyan4 and Huaya Shen5

1 Department of Physics, College of Physics and Electronic Engineering, Guangzhou University, Guangzhou 510006, People’s Republic of China
2 Russian-Armenian (Slavonic) State University, Yerevan, Armenia
3 Yerevan State University, Yerevan, Armenia
4 North Carolina Central University, NC, USA
5 China Institute for Radiation Protection, Taiyuan 030006, People’s Republic of China

E-mail: shijun_liang@163.com

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Abstract
We have studied the nonlinear optical absorption and the nonlinear optical rectification of an exciton in a nanoring in the presence of magnetic flux. The calculation results show that one can control the properties of nonlinear optical absorption and nonlinear optical rectification of a nanoring by tuning the outer and inner radius. Moreover, we find that the nonlinear optical properties of a nanoring can be modulated by the magnetic flux through the nanoring.

1. Introduction
As we know, a charged particle walking through a region where only magnetic flux exists acquires a phase. We call this the Aharonov–Bohm effect (ABE). Since Aharonov and Bohm [1] proposed an experiment to assess the manifestations of the electromagnetic potential in quantum mechanics in 1959, some work [2–4] has been undertaken by researchers, and they concluded that all observable phenomena are modulated by the magnetic flux through the region, and that these phenomena with respect to \( \Phi/\Phi_0 \) demonstrate a period with magnetic flux quantum \( \Phi_0 = \hbar c/e \). With the advance of lithography and semiconductor growth techniques, nanoring-like devices [5–8] have been realized experimentally. Due to the unique character of a nanoring, a self-assembled quantum ring has become the best ideal candidate to observe Aharonov–Bohm (AB) oscillation [9–11].

In 1995, Chaplik [12] first put forward the prediction that the neutral exciton can exhibit AB oscillator behavior in a one-dimensional quantum ring. And subsequently similar results were found in the literature [13] where a short-range interaction potential has been employed. Over the last decade, some researchers paid their attention to the AB effect of a neutral exciton in a two-dimensional quantum ring. For instance, Hu et al [9] theoretically studied the magnetic field effects on an exciton in an InAs nanoring; they found that the AB effect of the exciton only exists in a finite (but small) width nanoring in the presence of Coulomb correlation. Ribeiro et al [14] presented an experimental magneto-photoluminescence study of type-II InP/GaAs self-assembled quantum dots, which reveals AB type oscillations for neutral excitons when the angular momentum of the hole ground state ranges from \( l_h = 0 \) to 1, 2, 3. Dias da Silva et al [15] reported impurity effects on the AB optical signatures of neutral quantum ring magnetoexcitons. Their results, on the one hand, show that the energy and oscillator strength of neutral excitons are strongly modulated by the magnetic field. On the other hand, a scattering impurity enhances the photoluminescence (PL) intensity on otherwise ‘dark’ magnetic field windows and nonzero PL emission appears for a wide magnetic field range even at zero temperature. Recently Ding et al [10] have made a magneto-PL study of a single self-assembled semiconductor quantum ring which is realized by \( \text{in situ} \) AsBr\(_3\) etching [16] and experimentally observed AB type oscillations in the photoluminescence energy. As mentioned above, studies on the AB effect are mainly focused on the energy and oscillator strength of an exciton in a nanoring. Research into the AB
effect on nonlinear optical properties has been rarely reported so far. Understanding the nonlinear optical properties of an exciton in a nanoring is of significance for designing optical devices. Hence, it is of interest to study the nonlinear optical properties of an exciton in a nanoring. In the present work, we will focus on studying nonlinear optical absorption and nonlinear optical rectification of an exciton in a nanoring.

The paper is organized as follows: in section 2 we describe the model and theoretical framework, section 3 is dedicated to the results and discussion, and, finally, our conclusions are given in section 4.

2. Model and calculations

2.1. Exciton in a nanoring

Here we are concerned with a nanoring with Winternitz–Smorodinsky confinement potential. In the presence of a magnetic field applied along the z-direction, the Hamiltonian of a single particle reads

\[ \hat{H} = \frac{1}{2\mu} \left( \mathbf{p} + e\mathbf{A} \right)^2 + V(r). \]  

In equation (1), \( \mu \) is the effective mass, \( e \) is the electron charge, \( c \) is the speed of light in vacuum, \( \mathbf{A} \) is the magnetic vector potential. \( V(r) \) is given by [17]:

\[ V(r) = \frac{\alpha}{r^2} + \beta r^2 - 2\sqrt{\alpha\beta}, \quad \text{if } \alpha(\varphi) = \text{constant}. \]  

It should be noted that \( \alpha \) and \( \beta \) are the confinement potential parameters which are connected with the outer radius \( R_2 \) and inner radius \( R_1 \) by the following relations [17]:

\[ \beta R_1^2 + \alpha R_1^2 - 2\sqrt{\alpha\beta} = U, \]
\[ \beta R_2^2 + \alpha R_2^2 - 2\sqrt{\alpha\beta} = U, \]  

where for an electron \( U = 0.19952 \text{ meV} \) and for a hole \( U = 0.29928 \text{ meV} \). When the magnetic vector potential \( \mathbf{A} \) is chosen as \( \mathbf{A} = \frac{1}{2} Br\hat{z} + \Phi \frac{\hat{z}}{r} \) and polar coordinates are applied, the Hamiltonian can be rewritten as

\[ \hat{H} = -\frac{\hbar^2}{2\mu} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \left( \frac{\partial}{\partial \varphi} + i \frac{\Phi}{\Phi_0} \right)^2 \right] + \frac{eB}{2\mu e} \left( \frac{\partial}{\partial \varphi} + i \frac{\Phi}{\Phi_0} \right) - \frac{e^2 B^2 r^2}{8\mu^2 c^2} + V(r), \]

where \( \Phi_0 = \frac{e}{c} \) is the magnetic flux quantum. According to the Schrödinger equation

\[ H\Psi = E\Psi. \]  

We can obtain the wave function and energy spectrum of a single particle as follows:

\[ \Psi(r, \varphi) = 1 \sqrt{\underbrace{\Gamma[n_t + M + 1]}_{2M+1}} \frac{\Gamma[M + 1]}{2M+1}! \frac{2^{\frac{M}{2}}}{\pi} \]
\[ \times \left( \frac{r}{\lambda} \right)^M e^{-\frac{r^2}{4\lambda^2}} \left( -n_t, M + 1; \frac{r^2}{2\lambda^2} \right) e^{i\omega p}, \]

where

\[ E_{n,m} = \left( n_t + \frac{M}{2} + \frac{1}{2} \right) \hbar \omega - \frac{m - \Phi}{\hbar^2} \]  

In equation (7), \( F(x, y; z) \) is the confluent hypergeometric function. \( n_t = 0, 1, 2, \ldots \) is the radial quantum number, \( m = 0, \pm 1, \pm 2, \ldots \) is the magnetic quantum number.

Here

\[ \omega = \sqrt{\frac{\omega^2_c + 8\beta}{\mu}}, \]
\[ \lambda = \frac{h}{\mu \omega}, \]

denote the effective cyclotron frequency and effective magnetic length, respectively.

For an exciton in a nanoring, the Hamiltonian can be written as

\[ H = \frac{1}{2\mu_e} \left( \mathbf{p} + e\mathbf{A} \right)^2 + \frac{1}{2\mu_h} \left( \mathbf{p} - e\mathbf{A} \right)^2 + V(r) - \frac{e^2}{\epsilon |r_e - r_h|}. \]

Here we assume that the electron and hole are confined by the same potential. And the last term on the right-hand side of equation (12) is assumed to be a perturbation. According to the perturbation method, we can obtain the correction from the Coulomb interaction to energy [17].

2.2. Oscillator strength

The oscillator strength [18] is a very important physical quantity in the study of the optical properties that are related to the electronic dipole-allowed transitions. Generally, the oscillator strength \( P_\| \) is defined as

\[ P_\| = \frac{4\mu}{\hbar^2} E_{\|} |M_\||^2, \]

where \( E_{\|} = E_\| - E_\| \) denotes the energy difference between the initial state and the final state, and \( M_\| = \langle \Psi_\| | r | \Psi_\| \rangle \) is the electric dipole moment of the transition from the \( \Psi_\| \) state to the \( \Psi_\| \) state.

2.3. Nonlinear optical rectification and nonlinear optical absorption in a nanoring

Based on the density matrix approach and the perturbation expansion method, the second-order nonlinear optical rectification coefficient is given by [19, 20]
The difference in size between the outer radius and the inner radius increases. Here we define the width of a nanoring with respect to the inner radius for a fixed outer radius, while the transition energy is reduced for a fixed inner radius as the outer radius increases. In addition, this phenomenon. The electron and hole are confined to the nanoring and Coulomb interaction is dominant. Figure 3 shows the initial and the final state when the exciton moves in a narrower radius when \( r_1 \) is greater than 15 nm. It should be noted that the transition energy suddenly becomes very large when \( r_1 \) is very narrow (around 1 nm). The physical origins of these behaviors can be interpreted in the following way. For a wider ring, an exciton moving in the ring can be considered to be two-dimensional, while for a narrower ring, one can take the exciton as one-dimensional. A two-dimensional nanoring evolves into a one-dimensional one when the width of the nanoring continuously decreases. The transformation from a two-dimensional nanoring to a one-dimensional one is accompanied by variations in the Coulomb interaction between the electron and the hole, which greatly influence the energy level of the system. In addition, in figure 1, we note that the dashed line coincides with the full line, which shows that the magnetic field has little influence on the transition energy between the electron and the hole, which greatly influence the energy level of the system. In addition, in figure 1, we note that the dashed line coincides with the full line, which shows that the magnetic field has little influence on the transition energy of an exciton. Coulomb interaction between the electron and the hole and may lead to such a result. Figure 2 shows the oscillator strength as a function of the outer radius \( R_2 \), and inner radius \( R_1 \), respectively, for two different magnetic fields \( B = 5 \, \text{T}, 20 \, \text{T} \). We find from this figure that the behaviors of the oscillator strength with respect to the inner or outer radius are similar to those shown in figure 1. The reasons are that there is a greater probability of transition between the initial and the final state when the exciton moves in a narrower ring and Coulomb interaction is dominant. Figure 3 shows the oscillator strength as a function of the magnetic flux \( \Phi/\Phi_0 \) with \( R_1 = 5 \, \text{nm} \) and \( R_2 = 25 \, \text{nm} \). It is easily seen from this figure that the oscillator strength decreases with increasing \( \Phi/\Phi_0 \). But Aharonov–Bohm oscillations of the exciton are not observed in the nanoring. We can provide a reason for this phenomenon. The electron and hole are confined to the same potential in the nanoring and move over the same path, leading to zero electric dipole moment [23]. The magnetic flux through the nanoring has an effect on the wave function of the electron and hole. Hence, the probability of transition between different states in a nanoring is dependent on the magnetic flux.

### 3. Results and discussion

Our calculations are performed for an Al_{0.4}Ga_{0.6}As nanoring. The parameters [17, 22] chosen in this work are the following: \( \mu_e = 0.067\mu_0, \mu_h = 0.082\mu_0 \) where \( \mu_0 \) is the free electron mass, \( \rho = 3 \times 10^{23} \, \text{m}^{-3} \), \( n_t = 3.2, \epsilon = 12.9. \)

In figure 1, we plot the transition energy as a function of the inner radius \( R_1 \), and outer radius \( R_2 \), respectively, for two different magnetic fields \( B = 5 \, \text{T}, 20 \, \text{T} \). From this figure, we can find that the transition energy has a continuous increase with respect to the inner radius for a fixed outer radius, while the transition energy is reduced for a fixed inner radius as the outer radius increases. Here we define the width of a nanoring as the difference in size between the outer radius and the inner radius, \( \Delta r = R_2 - R_1 \). It should be noted that the transition energy is not almost affected by variations of the inner or outer radius when \( \Delta r \) is greater than 15 nm. Also it can be easily seen that the transition energy suddenly becomes very large when \( \Delta r \) is very narrow (around 1 nm). The physical origins of these behaviors can be interpreted in the following way. For a wider ring, an exciton moving in the ring can be considered to be two-dimensional, while for a narrower ring, one can take the exciton as one-dimensional. A two-dimensional nanoring evolves into a one-dimensional one when the width of the nanoring continuously decreases. The transformation from a two-dimensional nanoring to a one-dimensional one is accompanied by variations in the Coulomb interaction between the electron and the hole, which greatly influence the energy level of the system. In addition, in figure 1, we note that the dashed line coincides with the full line, which shows that the magnetic field has little influence on the transition energy between the electron and the hole, which greatly influence the energy level of the system. In addition, in figure 1, we note that the dashed line coincides with the full line, which shows that the magnetic field has little influence on the transition energy of an exciton.

In the present work, we also performed calculation of the intensity-dependent resonant peaks of the nonlinear optical absorption and nonlinear optical rectification coefficient which are given by [21]

\[
\chi_0^2 = \frac{4\sigma_0 M_0^2 \delta_0(E_{\text{In}}^2 (1 + \frac{T_1}{T_2} ) + [h \omega + (\frac{h}{T_2})^2 ](\frac{T_1}{T_2} - 1))}{e_0[(E_{\text{In}} - h \omega)^2 + (\frac{h}{T_2})^2 ][(E_{\text{In}} + h \omega)^2 + (\frac{h}{T_2})^2 ]}
\]

(14)

In the present work, we also performed calculation of the intensity-dependent resonant peaks of the nonlinear optical absorption and nonlinear optical rectification coefficient which are given by [21]

\[
\chi_0^2 = \frac{\xi_0(\omega = \omega_{\text{In}}, I = 0)}{\xi_0(\omega = \omega_{\text{In}}, I = 0)} = \frac{2\sigma_0 M_0^2 \delta_0 T_1 T_2}{\epsilon_0 h^2},
\]

(15)

and

\[
\alpha_{\text{max}} = \alpha(\omega = \omega_{\text{In}}, I = 0) = \frac{\sigma_0 M_0^2 T_2}{h^2 \epsilon_0 c n_t},
\]

(16)

where \( \epsilon_0 \) is the vacuum permittivity, \( n_t \) is the refractive index of the semiconductor. \( \sigma_0 \) denotes the electron density in the nanoring. In addition, \( T_1 \) and \( T_2 \) are the longitudinal and transverse relaxation times, respectively. In the calculation, we set the relaxation times at \( T_1 = 1 \, \text{ps}, T_2 = 0.2 \, \text{ps} \).
Figure 3. Oscillator strength as a function of magnetic flux $\Phi/\Phi_0$ with $R_1 = 5$ nm and $R_2 = 25$ nm.

Figure 4. Nonlinear optical absorption is presented as a function of inner radius $R_1$, and outer radius $R_2$, respectively, for magnetic field $B = 5$ T.

In figure 4, the nonlinear optical absorption is presented as a function of inner radius $R_1$, and outer radius $R_2$, respectively, for magnetic field $B = 5$ T. We note that the nonlinear optical absorption is not sensitive to variations of the inner or outer radius for a wider ring, while the nonlinear optical absorption instantly becomes extremely large when $\Delta r$ is reduced to be quite narrow and can be taken as a one-dimensional nanoring. The strong Coulomb interaction in a one-dimensional nanoring contributes to this phenomenon. Figure 5 shows the nonlinear optical absorption with respect to the magnetic flux $\Phi/\Phi_0$ with three different widths of ring. It is clearly seen that the effect of magnetic flux on the nonlinear absorption can be tuned by the width of a ring. So we can tune the outer and inner radius to meet the special needs of an experiment, which is practically significant in designing devices.

Figure 5. Nonlinear optical absorption with respect to magnetic flux $\Phi/\Phi_0$ with three different widths of ring.

In figure 5, the nonlinear optical absorption is presented as a function of magnetic flux $\Phi/\Phi_0$ with three different widths of ring. We note from this figure that the magnitude of the resonant peak of nonlinear optical absorption has a blue shift, which is modulated by the width of a ring. In addition, we also observe that the magnitudes of the resonant peak are far less than the results from the literature [21] where a parabolic confinement potential is employed to confine the electron and hole. And this indicates that the Winternitz–Smorodinsky potential is not realistic in crystal. Figure 7 shows the nonlinear optical rectification as a function of the inner radius $R_1$, and outer radius $R_2$, respectively, for magnetic field $B = 5$ T. When the outer radius is set at 31 nm, the nonlinear optical rectification is negative and decreases with the inner radius ranging from 0 to 5 nm. And then the nonlinear optical rectification monotonically increases with increasing inner radius. In addition, we also see that the nonlinear optical rectification monotonically decreases with the outer radius for a fixed $R_1$. Finally, we show the nonlinear optical rectification as a function of the magnetic flux $\Phi/\Phi_0$ for $R_1 = 5$ nm and $R_2 = 50$ nm in figure 8. From this figure, we can see that the nonlinear rectification increases with respect to $\Phi/\Phi_0$. And no AB oscillation is observed in this figure.

Figure 6. Nonlinear optical rectification is plotted as a function of photon energy with magnetic field for three different widths of ring.

Figure 7. Nonlinear optical rectification as a function of the inner radius $R_1$, and outer radius $R_2$, respectively, for magnetic field $B = 5$ T. When the outer radius is set at 31 nm, the nonlinear optical rectification is negative and decreases with the inner radius ranging from 0 to 5 nm. And then the nonlinear optical rectification monotonically increases with increasing inner radius. In addition, we also see that the nonlinear optical rectification monotonically decreases with the outer radius for a fixed $R_1$. Finally, we show the nonlinear optical rectification as a function of the magnetic flux $\Phi/\Phi_0$ for $R_1 = 5$ nm and $R_2 = 50$ nm in figure 8. From this figure, we can see that the nonlinear rectification increases with respect to $\Phi/\Phi_0$. And no AB oscillation is observed in this figure.
Figure 7. Nonlinear optical rectification as a function of inner radius $R_1$, and outer radius $R_2$, respectively, for magnetic field $B = 5$ T.

Figure 8. Nonlinear optical rectification as a function of magnetic flux $\Phi/\phi_0$ for $R_1 = 5$ nm and $R_2 = 50$ nm.

4. Summary

We have investigated the effects of the magnetic flux and the size of the quantum ring on the transition energy, oscillator strength, nonlinear optical absorption and nonlinear optical rectification of an exciton in a nanoring with Winternitz–Smorodinsky confinement potential. The results show that the transition energy, oscillator strength, nonlinear optical absorption and nonlinear optical rectification are strongly affected by the width of the nanoring, and that one can control these properties of a nanoring by modulating the outer or inner radius to meet special needs in designing electro-optical devices. Furthermore, it is found that the magnetic flux through the nanoring can tune the transition energy, oscillator strength, nonlinear optical absorption and nonlinear optical rectification. In addition, we should note the role of Coulomb interaction in such a nanoring when the two-dimensional nanoring evolves into a one-dimensional one.

In conclusion, the properties of an exciton in a nanoring are strongly dependent on the width of the ring and the magnetic flux. Finally, we hope that our research can stimulate further studies on nonlinear optical properties of nanorings.

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