Beaming Effects in Gamma-Ray Bursts

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Abstract

Based on a refined generic dynamical model, we investigate afterglows from jetted gamma-ray burst (GRB) remnants numerically. In the relativistic phase, the light curve break could marginally be seen. However, an obvious break does exist at the transition from the relativistic phase to the non-relativistic phase, which typically occurs at time 10 to 30 days. It is very interesting that the break is affected by many parameters, especially by the electron energy fraction (\(\xi_e\)), and the magnetic energy fraction (\(\xi_B^2\)). Implication of orphan afterglow surveys on GRB beaming is investigated. The possible existence of a kind of cylindrical jets is also discussed.

1 Introduction

Researches on afterglows from long gamma-ray bursts (GRBs) have shown that they are of cosmological origin. The standard fireball model, which incorporates internal shocks to explain the main bursts and external shocks to account for afterglows, becomes the most popular model (for recent reviews, see van Paradijs et al. 2000). Some GRBs localized by BeppoSAX satellite have implied isotropic energy release of more than \(10^{54}\) ergs, leading many theorists to deduce that GRB radiation must be highly collimated (Castro-Tirado et al. 1999; Huang 2000; Halpern et al. 2000; Dai & Gou 2001; Dai & Cheng 2001; Gou et al. 2001; Ramirez-Ruiz & Lloyd-Ronning 2002; Zhang & Mészáros 2002).

To differentiate a jet from an isotropic fireball, we must resort to the afterglow light curves. When the bulk Lorentz factor of a jet drops to \(\gamma < 1/\theta\), with \(\theta\) the half opening angle, the edge of the jet becomes visible, the light curve will steepen by \(t^{-3/4}\). This is called the edge effect (Mészáros & Rees 1999; Panaitescu & Mészáros 1999; Kulkarni et al. 1999). In addition, the lateral expansion of a relativistic jet will make the break even more precipitous. So it is generally believed that afterglows from jetted GRBs are characterized by an obvious break in the light curve at the relativistic stage.

In this talk, we use our refined dynamical model to study the jet effect on the afterglow light curves. The possible existence of cylindrical jets is also discussed.
The importance of the non-relativistic phase of fireball expansion has been stressed by Huang et al. (1998a, b). In the literature, it is generally believed that the following equation can depict the evolution of GRB remnants,

\[
\frac{d\gamma}{dm} = -\frac{\gamma^2 - 1}{M},
\]

where \(m\) is the rest mass of the swept-up medium, \(M\) is the total mass in the co-moving frame, including internal energy. However, Huang et al. (1999) pointed out that during the non-relativistic phase of an adiabatic expansion, Eq. (1) cannot reproduce the familiar Sedov solution. This is clearly shown in Fig. 1.

Huang et al. (1999) have proposed a refined equation,

\[
\frac{d\gamma}{dm} = -\frac{\gamma^2 - 1}{M_{ej} + \epsilon m + 2(1-\epsilon)\gamma m},
\]

where \(M_{ej}\) is the initial baryon mass ejected from the GRB central engine, and \(\epsilon\) is the radiative efficiency. For an adiabatic fireball, \(\epsilon = 0\); and for a highly radiative one, \(\epsilon = 1\). Huang et al. (1999) have shown that Eq. (2) is correct for both radiative and adiabatic fireballs, and in both ultra-relativistic and non-relativistic phases (c.f. Fig. 1).

Using the refined dynamical model, the evolution of the beamed ejecta can be described by (Huang et al. 2000a, b, c):

\[
\frac{dR}{dt} = \beta c\gamma(\gamma + \sqrt{\gamma^2 - 1}),
\]
\[
\frac{dm}{dR} = 2\pi R^2(1 - \cos \theta) n m_p, \quad (4)
\]
\[
\frac{d\theta}{dt} = c_p (\gamma + \sqrt{\gamma^2 - 1}), \quad (5)
\]
\[
\frac{d\gamma}{dm} = -\frac{\gamma^2 - 1}{M_{ej} + \epsilon m + 2(1 - \epsilon)\gamma m}, \quad (6)
\]
\[
c_s^2 = \hat{\gamma}(\hat{\gamma} - 1)(\gamma - 1) \frac{1}{1 + \hat{\gamma}(\gamma - 1)} c^2, \quad (7)
\]
where \(m_p\) is the proton mass, \(c_p\) is the co-moving sound speed, \(\hat{\gamma} \approx (4\gamma + 1)/(3\gamma)\) is the adiabatic index, and \(c\) is the speed of light. Below we will consider only adiabatic jets, for which \(\epsilon \equiv 0\).

### 3 Beaming Effects

A strong blastwave will be generated due to the interaction of the jet and the ISM. Synchrotron radiation from the shock accelerated ISM electrons gives birth to afterglows. As usual we assume that the magnetic energy density in the co-moving frame is a fraction \(\xi_B^2\) of the total thermal energy density \(B^2/8\pi = \xi_B^2 c^4\), and that electrons carry a fraction \(\xi_e\) of the proton energy. This means that the minimum Lorentz factor of the random motion of electrons in the co-moving frame is \(\gamma_{e,\text{min}} = \xi_e (\gamma - 1) m_p (p - 2)/(m_e (p - 1)) + 1\), where \(p\) is the index characterizing the power law energy distribution of electrons, and \(m_e\) is the electron mass.

For convenience, let us define the following initial values or parameters as a set of “standard” parameters: initial energy per solid angle \(E_0/\Omega_0 = 10^{54}\) ergs/4\(\pi\), \(\gamma_0 = 300\), \(n = 1\) cm\(^{-3}\), \(\xi_B^2 = 0.01\), \(p = 2.5\), \(\xi_e = 0.1\), \(\theta_0 = 0.2\), \(\theta_{\text{obs}} = 0\), \(D_L = 10^6\) kpc, where \(\theta_{\text{obs}}\) is the angle between the line of sight and the jet axis, and \(D_L\) is the luminosity distance.

We have followed the evolution of jetted GRB remnants numerically and calculated their afterglows (Huang et al. 2000a, c). Fig. 2 shows the evolution of \(\gamma\) for some exemplary jets. We see that the ejecta will cease to be highly relativistic at time \(t \sim 10^5 - 10^6\) s. This gives strong support to our previous argument that we should be careful in discussing the fireball evolution under the simple assumption of ultra-relativistic limit (Huang et al. 1998a, b, 1999, 2000a, b, c).

Fig. 3 illustrates the effect of \(\xi_e\) on the optical (R-band) light curves. In no case could we observe the theoretically predicted light curve steepening (with the break point determined by \(\gamma \sim 1/\theta\)) during the relativistic stage itself. The reason is: at time of \(\gamma \sim 1/\theta\), the jet is already in its mildly relativistic phase and it will become non-relativistic soon after that, so the break due to the edge effect and the lateral expansion effect does not have time to emerge during the relativistic phase (Huang et al. 2000a, c). Further more, since \(\gamma\) is no longer much larger than 1, conventional theoretical analyses (under the assumption of \(\gamma \gg 1\)) are not proper. However, when \(\xi_e\) is small, an obvious break does appear in the light curve, but it is clearly due to the relativistic-Newtonian transition. The simulations by many other authors do not reveal such breaks, because their models
Figure 3: The effect of $\xi_e$ on the R-band light curve. The solid line corresponds to a jet with “standard” parameters. Other lines are drawn with only $\xi_e$ altered (Huang et al. 2000c).

Figure 4: The effect of $\xi_B$ on the R-band light curve (Huang et al. 2000c).

are not appropriate for Newtonian expansion. When $\xi_e$ is large, the break disappears. This is not difficult to understand. According to the analysis in the ultra-relativistic limit, the time that the light curve peaks scales as $t_m \propto \xi_e^{4/3}(\xi_B^2)^{1/3}$. Fig. 3 shows this trend qualitatively. In the case of $\xi_e = 1.0$, $t_m$ is as large as $\sim 10^5$ s, then we can not see the initial power law decay (with timing index $\alpha \sim 1.1$) in the relativistic phase, it is hidden by the peak. So the break disappears. We should also note that in all cases, light curves during the non-relativistic phase are characterized by quick decays, with $\alpha \geq 2.1$. This is quite different from isotropic fireballs, whose light curves steepen only slightly after entering the Newtonian phase (i.e., $\alpha \sim 1.3$).

Fig. 4 illustrates the effect of $\xi_B^2$ on the optical light curves. Interestingly but not surprisingly, we see that $\xi_B^2$ has an effect similar to $\xi_e$: for small $\xi_B^2$ values, there are obvious breaks at the relativistic-Newtonian transition points; but for large $\xi_B^2$ values, the break disappears, we could only observe a single steep line with $\alpha \geq 2.1$. This is also due to the dependence of $t_m$ on $\xi_B^2$ (Huang et al. 2000c).

We have also investigated the effects of other parameters such as $\theta_0$, $n$, $p$ on the optical light curves (Huang et al. 2000c). When $\theta_0 \geq 0.4$, the light curve becomes very similar to that of an isotropic fireball and no steep break exists. In the case of a dense medium ($n \geq 10^3$ cm$^{-3}$), the expansion becomes non-relativistic quickly, so that the break also disappears and we could only observe a quick decay with $\alpha \geq 2.1$. But generally speaking, $p$ does not affect the break notably.

Optical afterglows from GRB 990123, 990510 are characterized by an obvious break in the light curve, and afterglows from GRB 970228, 980326, 980519, 991208 faded rapidly. We suggest that these phenomena are due to beaming effects. We have fitted these afterglows based on our refined jet model and find that the observations can be reproduced easily with a universal initial half opening angle $\theta_0 \sim 0.1$ (Fig. 5). The obvious light curve break in GRB 990123 is due to the relativistic-Newtonian transition of the beamed ejecta, and the rapidly fading afterglows come from synchrotron emissions during the mildly relativistic and non-relativistic phases. We thus strongly suggest that the rapid
fading of afterglows currently observed in some GRBs is evidence for beaming in these cases.

4 Orphan Afterglows

The concept of orphan afterglows was first clearly proposed by Rhoads (1997), who pointed out that due to relativistic beaming effects, γ-ray radiation from the vast majority of jetted GRBs cannot be observed, but the corresponding late time afterglow emission is less beamed and can safely reach us. These afterglows are called orphan afterglows, which means they are not associated with any detectable GRBs. The ratio of the orphan afterglow rate to the GRB rate might allow measurement of the GRB collimation angle (Totani & Panaitescu 2002). Great expectations have been put on this method (Nakar et al. 2002; Vanden Berk et al. 2002). In fact, the absence of large numbers of orphan afterglows in many surveys has been regarded as evidence that the collimation cannot be extreme (Rhoads 1997; Greiner et al. 1999).

However, there is a difficulty associated with the method (Huang et al. 2002a): there
should be many “failed gamma-ray bursts (FGRBs)”, i.e., baryon-contaminated fireballs with initial Lorentz factor $\gamma_0 < 100$. BeppoSAX team has reported the discovery of several anomalous events named as fast X-ray transients, X-ray rich GRBs, or even X-ray-GRBs. They resemble usual GRBs except that they are extremely X-ray rich (Frontera et al. 2000). Observational data on this kind of events are being accumulated rapidly. We propose that these events are probably just FGRBs (Huang et al. 2002a). FGRBs cannot be observed in gamma-rays, but their long-lasting afterglows are detectable, thus they will also manifest themselves as orphan afterglows. In short, we cannot omit an important fact: if GRBs are really due to isotropic fireballs, then there should be much more failed GRBs. So, the simple discovery of orphan afterglows does not necessarily indicate that GRBs are beamed (Huang et al. 2002a).

Theoretically, when orphan afterglows are really discovered observationally, it is still risky to conclude that GRBs are beamed. We should study these orphans carefully to determine whether they come from FGRBs or Jetted GRBs. Unfortunately, this is not an easy task. The major problem is that for orphan afterglow observations, the derivation of a $\log S_\nu$ $-$ $\log t$ light curve is not direct: we do not know the trigger time so that the exact value of $t$ for each observed data point cannot be determined (Huang et al. 2002a).

In Figure 6, we compare the theoretical $\log S_\nu$ $-$ $\log t$ light curves of optical afterglows from FGRBs and jetted but off-axis GRBs directly. To investigate the influence of the uncertainty in trigger time, we also shift the light curve of FGRBs by $t \pm 3$ d, $t \pm 10$ d and $t \pm 30$ d intentionally. From the dashed curves, we can see that the shape of the FGRB afterglow light curve is seriously affected by the uncertainty of the trigger time.
But fortunately, these dashed curves still differ from the theoretical light curve of the jetted GRB orphan markedly, i.e., they are much flatter at very late stages. This means it is still possible for us to discriminate them. In Figure 7, similar results to Figure 6 are given, but this time the light curve of the jetted GRB orphan is shifted. Again we see that the two kinds of orphans can be discriminated by their late time behaviour.

Figures 6 and 7 explain what we should do when an orphan afterglow is discovered. First, we have to assume a trigger time for it arbitrarily, so that the logarithmic light curve can be plotted. We then need to change the trigger time to many other values to see how the light curve is affected. In all our plots, we should pay special attention to the late time behaviour, which will be less affected by the uncertainty in the trigger time. If the slope tends to be $\sim -1.0 - -1.3$, then the orphan afterglow may come from an FGRB event. But if the slope tends to be steeper than $\sim -2.0$, then it is very likely from a jetted but off-axis GRB (Huang et al. 2002a).

However, we must bear in mind that it is in fact not an easy task. First, to take the process we need to follow the orphan as long as possible, and the simple discovery of an orphan is obviously insufficient. Note that currently optical afterglows from most well-localized GRBs can be observed for only less than 100 days. It is quite unlikely that we can follow an orphan for a period longer than that. Second, since the orphan is usually very faint, errors in the measured magnitudes will seriously prevent us from deriving the slope. Due to all these difficulties, a satisfactory light curve is usually hard to get for most orphans (Huang et al. 2002a). We see that measurement of the GRB beaming angle using orphan searches is not as simple as we originally expected. In fact, it is impractical to some extent (Huang et al. 2002a).

Anyway, there are still some other possible solutions that may help to determine the onset of an orphan afterglow (Huang et al. 2002a). Firstly, of course we should improve our detection limit so that the orphan afterglow could be followed as long as possible. The longer we observe, the more likely that we can get the true late-time light curve slope. Secondly, we know that FGRBs usually manifest themselves as fast X-ray transients or X-ray-GRBs. If an orphan can be identified to associate with such a transient, then it is most likely an FGRB one. In this case, the trigger time can be well determined. Thirdly, maybe in some rare cases we are so lucky that the rising phase of the orphan could be observed. For a jetted GRB orphan the maximum optical flux is usually reached within one or two days and for an FGRB orphan it is even within hours. Then the uncertainty in trigger time is greatly reduced. Additionally, a jetted GRB orphan differs markedly from an FGRB one during the rising phase. The former can be brightened by more than one magnitude in several hours (see Figures 6 — 7), while the brightening of the latter can hardly be observed. So, if an orphan afterglow with a short period of brightening is observed, then it is most likely a jetted GRB orphan. Of course, we should first be certain that it is not a supernova.

Fourthly, valuable clues may come from radio observations. In radio bands, the light curve should be highly variable at early stages due to interstellar medium scintillation, and it will become much smoother at late times. So the variability in radio light curves provides useful information on the trigger time. And fifthly, in the future maybe gravitational wave radiation or neutrino radiation associated with GRBs could be detected due to
progresses in technology, then the trigger time of an orphan could be determined directly and accurately. Sixthly, the microlensing effect may be of some help. Since the size of the radiation zone of a jetted GRB orphan is much smaller than that of an FGRB one, they should behave differently when microlensed (Huang et al. 2002a).

Finally, although a successful detection of some orphan afterglows does not directly mean that GRBs be collimated, the negative detection of any orphans can always place both a stringent lower limit on the beaming angle for GRBs and a reasonable upper limit for the rate of FGRBs (Huang et al. 2002a).

Figure 8: R band afterglows from beamed GRB ejecta without lateral expansion ($v_\perp \equiv 0$). The dotted line corresponds to a conical jet with $p = 2.5$, other lines are for cylindrical jets which differ only in the parameter $p$. The dash-dotted, solid and the dashed line correspond to $p = 2.2, 2.5$ and $3.0$ respectively. The breaks at $t \sim 10^9$ s in the light curves for cylindrical jets are due to cooling of electrons (Cheng et al. 2001).

Figure 9: R band afterglows from beamed GRB ejecta with lateral expansion ($v_\perp \equiv c_s$). The dotted line corresponds to a conical jet with $p = 2.5$, other lines are for cylindrical jets which differ only in the parameter $p$. The dash-dotted, solid and the dashed line correspond to $p = 2.2, 2.5$ and $3.0$ respectively. Note that the cylindrical jets here are already non-relativistic when $t > 10^6$ s (Cheng et al. 2001).

5 Cylindrical Jets

Nearly all previous discussions on beaming effects in gamma-ray bursts have assumed a conical geometry. However, more and more observations on relativistic jets in radio galaxies, active galactic nuclei, and “microquasars” in the Galaxy have shown that many of these outflows are not conical, but cylindrical, i.e., they maintain constant cross sections at large scales. Thus it is necessary to discuss the possibility that gamma-ray bursts may be due to highly collimated cylindrical jets, not conical ones. In fact, this idea has already been suggested as GRB trigger mechanism by Dar (1998).
Dynamical evolution of cylindrical jets and their afterglows have been discussed in great detail by Cheng et al. (2001). Both analytical and numerical results are presented. It is shown that when the lateral expansion is not taken into account, a cylindrical jet typically remains to be highly relativistic for $\sim 10^8 - 10^9$ s. During this relativistic phase, the optical afterglow decays as $S_\nu \propto t^{-p/2}$ at first. Then the light curve steepens to be $S_\nu \propto t^{-(p+1)/2}$ due to cooling of electrons. After entering the non-relativistic phase (i.e., $t \geq 10^{11}$ s), the afterglow is $S_\nu \propto t^{-(5p-4)/6}$. But if the cylindrical jet expands laterally at co-moving sound speed, then the decay becomes $S_\nu \propto t^{-p}$ and $S_\nu \propto t^{-(15p-21)/10} - t^{-(15p-20)/10}$ in the ultra-relativistic and non-relativistic phase respectively. Note that in both cases, the light curve turns flatter after the relativistic-Newtonian transition point, which differs markedly from the behaviour of a conical jet.

It is suggested that some gamma-ray bursts with afterglows decaying as $t^{-1.1} - t^{-1.3}$ may be due to cylindrical jets, not necessarily isotropic fireballs (Huang et al. 2002b).

6 Concluding Remarks

We investigate beaming effects in GRBs numerically. It is found that the light curve break of optical afterglows is not obvious within the relativistic phase. But an obvious break can truly be found at the relativistic-Newtonian transition point. Further investigations show that the break is parameter dependent, but afterglows from a jet in the non-relativistic phase are uniformly characterized by a quick decay (such as $t^{-2}$). It is also shown that measure of GRB beaming by using orphan afterglow surveys is not as easy as original expectancy.

As for the geometry of GRB beaming, we suggest that the jet could also be cylindrical. Afterglows from cylindrical jets with or without lateral expansion have been discussed. It is shown that those GRBs with a flat afterglow light curve could be well accounted for by a cylindrical jet without lateral expansion.

The most difficult enigma in GRBs is the progenitor. Beaming effects can provide important clues. Other helpful hints may come from X-ray observations (Antonelli et al. 2000).

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