Formalization and study of the game-theoretic model of the optimal distribution of workers in enterprises

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Abstract. The paper presents a cooperative game about the appointment for the study of the optimal distribution of workers in the enterprise. Many workers and many enterprises are given. In this model, workers can team up, work in different enterprises and perform different jobs. Workers, teaming up and working in different enterprises, receive wages. The sum of these wages at all enterprises forms the income of the team. It is required to distribute income optimally between employees. A numerical example of finding the Shapley vector is compiled, which, depending on the productivity of workers performing work, shows what income each employee should receive.

1. Introduction
Each employee has his own productivity of performing work at the enterprise, as well as as part of various teams $G$. employee productivity $i$ work $l$ denote $\alpha_{i,l} \in \mathbb{Z}^+$, (\(\alpha_{i,l}\) the amount of money that can earn $i$ worker performing $l$ work.) (\(i = 1,2,...,q; l = 1,2,...,k\)). The performance of employees performing work is recorded in the productivity matrix for employees: $A_{\text{med}} = (\alpha_{i,l})$. The values of the productivity of workers in the various teams are calculated by the formula [1-5]:

$$\sum_{i,j} \alpha_{i,j} x_{i,j} = \beta_{i,j}^S = V_{h_j} (G),$$

where $G = (1,...,g), x_{i,j} = 1$, if the employee $i$ will do the job $l$ at the enterprise $h_j$, otherwise $x_{i,j} = 0$.

Reducing the number of people in the team, we find the values of the maximum productivity of the team for all enterprises and form a matrix of maximum productivity, the elements of which will indicate the maximum productivity of the team at each of the enterprises. The number of teams that can be formed is determined by the formula:
\[ P = \sum_{r_1 + r_2 + \cdots + r_m = n} C_n^{r_1} \cdot C_n^{r_2} \cdot C_n^{r_3} \cdot \cdots \cdot C_n^{r_m}. \]

Where \( r_k \in [1, n - l + 1], k = (1, \ldots, l). \)

Based on these considerations, we construct the characteristic function \( V(G) \), where \( G \subseteq S. \) The value of the characteristic function will be considered the sum of the maximum productivity of the teams at each of the enterprises [6-7].

Such models are considered in the theory of cooperative games. As a solution to this model, it is proposed to find the Shapley vector, which shows what part of the income each of the coalition members should receive. The Shapley vector is determined by the formula:

\[ \Phi(v) = \sum_{T \subseteq C \subseteq M} \frac{(m-t)! (t-1)!}{(m)!} (v(T) - v(T \setminus \{i\})). \]

2. Cooperative appointment game

2.1. Informal model description

The paper describes a model of the optimal distribution of workers. Many employees asked \( S = \{s_1, \ldots, s_m\} \) and many enterprises \( H = \{h_1, \ldots, h_n\}. \) In this model, workers can team up, work in different enterprises and perform different jobs. Several workers may be assigned to one enterprise depending on the profile of the enterprise, the number of work performed at the enterprise, and the need for employees. Workers, teaming up in different enterprises, receive wages. The sum of these wages at all enterprises form the income of the team. It is required to divide this income between employees in an optimal way [8-10].

2.2. Formal model building

Consider a game model in which workers will be players. Each player (employee) has his own productivity of performing work in the enterprise, as well as in various teams \( G. \) Performance of the employee \( i \) work \( l \) denote \( \alpha_{i,l} \in Z^+ \) (\( \alpha_{i,l} \) - the amount of money that can earn \( i \) worker performing \( l \) work), Where \( Z^+ \) - many positive integers \((i = 1, \ldots, q; l = 1, \ldots, k). \) The performance of employees performing work is recorded in the productivity matrix for employees [11]:

\[
A = \begin{pmatrix}
\alpha_{11} & \alpha_{12} & \cdots & \alpha_{1l} \\
\alpha_{21} & \alpha_{22} & \cdots & \alpha_{2l} \\
\vdots & \vdots & \ddots & \vdots \\
\alpha_{q1} & \alpha_{q2} & \cdots & \alpha_{ql}
\end{pmatrix},
\]

where the rows of matrix \( A \) correspond to the numbers of workers, and the columns to the numbers of jobs. At the intersection of the row and column is the element that corresponds to the performance of execution \( i \) employee \( l \) work.

The performance values of the brigades depends on the productivity of each employee and we calculate by the formula [12]:

\[
\sum_{i,j} \alpha_{i,j} x_{i,j} = \beta_{i,j} = V_{h_j}(G),
\]

where \( G = (1, \ldots, g), x_{i,j} = 1, \) if the employee \( i \) will do the job \( l \) at the enterprise, otherwise. Because each employee performs work in an enterprise with different productivity, it is necessary to find such a
distribution of workers in the enterprise in order to obtain maximum productivity. Reducing the number of people in the team, we find the values of the maximum productivity of the teams for all enterprises and form a matrix of maximum productivity:

\[
\begin{pmatrix}
\beta_{1h} & \beta_{1h} & \cdots & \beta_{1h} \\
\beta_{2h} & \beta_{2h} & \cdots & \beta_{2h} \\
\vdots & \vdots & \ddots & \vdots \\
g & \beta_{gh} & \beta_{gh} & \cdots & \beta_{gh}
\end{pmatrix}
\]

where \(( g = 2^m - 1 \) all kinds of coalitions \( \subset S \)). Elements of the maximum productivity matrix indicate the maximum productivity of the teams in each of the enterprises. The number of teams that can be formed at each enterprise is determined by the formula [13-15]:

\[
P = \sum_{r_1 + r_2 + \cdots + r_l = n} C_n^{r_1} \cdot C_n^{r_2} \cdots C_n^{r_l} \cdot C_n^{n-(r_1 + r_2 + \cdots + r_l)},
\]

where \( r_k \in [1, n-l+1], k = (1, \ldots, l) \).

Based on these considerations, we construct the characteristic function \( V(G) = \sum_{h \in G} \max V_{h_i}(G) \), where \( G \subset S \). The value of the characteristic function will be considered the sum of the maximum productivity of the teams at each of the enterprises. In this model, it is necessary to find the Shapley vector from the values of the characteristic function, which will show how much each player should receive. The Shapley vector is determined by the formula [16-18]:

\[
\Phi_i(v) = \sum_{\{T \in \mathcal{M} \}} \frac{(m-t)(t-1)!}{(m)!} (v(T) - v(T \setminus \{i\})).
\]

3. Numerical example
Let there be 5 workers and 7 enterprises. Enterprises need the following employees: drivers, mechanics and loaders. To form teams, workers must be able to do these three jobs. The employment department at the enterprise evaluates each employee on a 6-point scale (the assessment is made in rubles). Drivers depending on their categories, mechanics by rank, movers by experience and physical data. Suppose the first worker can do these three jobs with productivity \( S_1 = (5, 3, 2) \). From the productivity of the first employee, we can see that he is a good driver, an average mechanic and a poor loader. The second worker has productivity \( S_2 = (3, 5, 1) \). Third \( S_3 = (4, 5, 2) \). Fourth \( S_4 = (6, 4, 2) \). Fifth \( S_5 = (3, 2, 1) \). Based on these data, we form a performance matrix, which will take the form:

\[
\begin{array}{ccc}
1 & 2 & 3 \\
5 & 3 & 2 \\
3 & 5 & 1 \\
4 & 5 & 2 \\
6 & 4 & 2 \\
3 & 2 & 1 \\
\end{array}
\]

Suppose that the first enterprise needs workers who can do all 3 jobs, the 2nd - the first and second, the 3rd - the first and third, the 4th - the second and third, the 5th - only the first, the 6th - only the second, the 7th - only the third. We will go over all possible options for the formation of brigades and see what the values of maximum productivity are equal to. Brigades can be formed as follows:
1) workers a, b, c will perform the first work, employee d the second, e third; the productivity of such a team will be equal to \( V(1) = 17 \);

2) workers a, b perform the first work, c, d the second, e the third; the performance of this team is \( V = 18 \);

3) employee a first, b, c, d second, e third; performance \( V = 20 \).

Going through all the options, we see that the first enterprise receives a maximum productivity \( \max V(5) = 22 \) when appointing the first and fourth workers to the first enterprise, the second and third workers to the second enterprise and the fifth to the third enterprise. We write it as follows

\[
V(1) \mid_{l(a,d),2(b,c),3(e)} = 22.
\]

Next, the maximum productivity of all possible cases of the formation of teams for other enterprises is calculated:

1) \( V(2) \mid_{l(a,d,e),2(b,c)} = 24 \),
\[
V(3) \mid_{l(a,c,d),3(b,e)} = 17,
\]
\[
V(4) \mid_{l(b,c,d),3(a,e)} = 17,
\]
\[
V(5) \mid_{l(a,b,c,d,e)} = 21,
\]
\[
V(6) \mid_{l(a,b,c,d,e)} = 19,
\]
\[
V(7) \mid_{3(a,b,c,d,e)} = 8.
\]

Also, reducing the number of employees, the maximum productivity at each of the enterprises is calculated, we get:

2) without a worker e:
\[
V(1) \mid_{l(d),2(b,c),3(a)} = 18,
\]
\[
V(2) \mid_{l(a,d),2(b,c)} = 21,
\]
\[
V(3) \mid_{l(a,d),3(b,c)} = 14,
\]
\[
V(4) \mid_{l(b,c),3(a,d)} = 14,
\]
\[
V(5) \mid_{l(a,b,c,d)} = 18,
\]
\[
V(6) \mid_{l(a,b,c,d)} = 17,
\]
\[
V(7) \mid_{3(a,b,c,d)} = 8,
\]

3) without a worker d:
\[
V(1) \mid_{l(d),2(b,c),3(a,e)} = 16,
\]
\[
V(2) \mid_{l(a,e),2(b,c)} = 18,
\]
\[
V(3) \mid_{l(a,d),3(c,e)} = 11,
\]
\[
V(4) \mid_{l(b,c),3(a,d)} = 13,
\]
\[
V(5) \mid_{l(a,b,c,d,e)} = 15,
\]
\[
V(6) \mid_{l(a,b,c,d,e)} = 15,
\]
\[
V(7) \mid_{3(a,b,c,d,e)} = 6,
\]

4) without a worker c:
\[
V(1) \mid_{l(d),2(b,c),3(e)} = 17,
\]
\[
V(2) \mid_{l(a,e),2(b,c)} = 18,
\]
\[
V(3) \mid_{l(a,d),3(c,e)} = 13,
\]
\[
V(4) \mid_{l(b,c),3(a,d)} = 12,
\]
\[
V(5) \mid_{l(a,b,c,d,e)} = 17,
\]
\[
V(6) \mid_{l(a,b,c,d,e)} = 14,
\]
\[
V(7) \mid_{3(a,b,c,d,e)} = 6,
\]

5) without working b:
\[
V(1) \mid_{l(d),2(c),3(a,e)} = 17,
\]
\[
V(2) \mid_{l(a,d),2(c)} = 18,
\]
\[
V(3) \mid_{l(c,d),3(a,e)} = 13,
\]
\[
V(4) \mid_{l(c,d),3(a,e)} = 12,
\]
\[
V(5) \mid_{l(a,c,d,e)} = 18,
\]
\[
V(6) \mid_{l(a,c,d,e)} = 14,
\]

6) without working a:
\[
V(1) \mid_{l(d),2(b,c),3(e)} = 17,
\]
\[
V(2) \mid_{l(d,e),2(b,c)} = 19,
\]
\[
V(3) \mid_{l(c,d),3(b,e)} = 12,
\]
\[
V(4) \mid_{l(b,d),3(c,e)} = 13,
\]
\[
V(5) \mid_{l(b,c,d,e)} = 16,
\]
\[
V(6) \mid_{l(b,c,d,e)} = 16,
\]
\[ V(7) \mid _{S(a,c,d,e)} = 7, \quad V(7) \mid _{S(b,c,d,e)} = 6. \]

Next, we consider for coalitions of workers where \( S = 3 \):

7) without working \( d \) and \( e \):

8) without working \( c \) and \( e \):

9) without working \( c \) and \( d \):

10) without working \( b \) and \( e \):

11) without working \( b \) and \( d \):

12) without working \( b \) and \( c \):

13) without working \( a \) and \( e \):

14) without working \( a \) and \( c \):

15) without working \( a \) and \( b \):

16) without working \( a \) and \( d \):

| Coalition | Value |
|-----------|-------|
| 1) \( V(1) \mid _{l(a),2(b),3(c)} \) = 12 | 8) \( V(1) \mid _{l(a),d),2(b),3(c)} \) = 13 |
| 2) \( V(2) \mid _{l(a),2(b),c} \) = 15 | 9) \( V(1) \mid _{l(a),2(b),3(e)} \) = 11 |
| 3) \( V(3) \mid _{l(a,c),3(b)} \) = 10 | 10) \( V(1) \mid _{l(d),2(c),3(a)} \) = 13 |
| 4) \( V(4) \mid _{l(2,b),3(a)} \) = 12 | 11) \( V(1) \mid _{l(a),2(b),3(e)} \) = 10 |
| 5) \( V(5) \mid _{l(a,b,c)} \) = 12 | 12) \( V(1) \mid _{l(a,d),2(b)} \) = 16 |
| 6) \( V(6) \mid _{l(2,a),b,c} \) = 13 | 13) \( V(1) \mid _{l(d),2(b),3(c)} \) = 12 |
| 7) \( V(7) \mid _{3(b,a,c)} \) = 5 | 14) \( V(1) \mid _{l(d),2(b),3(c)} \) = 12 |
|             | 15) \( V(1) \mid _{l(d),2(c),3(e)} \) = 12 |
|             | 16) \( V(1) \mid _{l(d),2(c),3(e)} \) = 12 |
\[ V(1) |_{i(c),2(b),3(e)} = 10, \]
\[ V(2) |_{i(c),2(b),e} = 13, \]
\[ V(3) |_{i(b),e,3(c)} = 8, \]
\[ V(4) |_{2(b),e,3(c)} = 11, \]
\[ V(5) |_{i(b),e} = 10, \]
\[ V(6) |_{2(b),e} = 11, \]
\[ V(7) |_{3(b),e} = 5, \]

When forming teams of two people, the productivity of teams in the first enterprise will be zero, since the number of jobs in the enterprise exceeds the number of people in the team.

17) Without workers \( c, d \) and \( e \): 18) without workers \( b, d \) and \( e \): 19) without workers \( a, d \) and \( e \):

\[ V(1) = 0, \quad V(2) |_{i(a),2(b)} = 10, \quad V(3) |_{i(a),3(b)} = 6, \]
\[ V(4) |_{2(b),3(a)} = 7, \quad V(5) |_{i(a),b} = 8, \quad V(6) |_{2(a),b} = 8, \]
\[ V(7) |_{3(a),b} = 3, \]

20) without workers \( b, c \) and \( e \): 21) without workers \( a, c \) and \( e \): 22) without workers \( a, b \) and \( e \):

\[ V(1) = 0, \quad V(2) |_{i(d),2(a)} = 9, \quad V(3) |_{i(d),3(a)} = 8, \]
\[ V(4) |_{2(d),3(a)} = 6, \quad V(5) |_{i(d),a} = 11, \quad V(6) |_{2(d),a} = 11, \]
\[ V(7) |_{3(d),a} = 4, \]

23) without workers \( b, c \) and \( e \): 24) without workers \( a, c \) and \( e \): 25) without workers \( a, b \) and \( e \):

\[ V(1) = 0, \quad V(2) |_{i(c),2(d)} = 7, \quad V(2) |_{i(d),2(b)} = 8, \]
\[ V(2) |_{i(d),2(c)} = 8, \]
\( V(3) \mid_{1(a),3(d)} = 6, \quad V(3) \mid_{1(b),3(d)} = 4, \quad V(3) \mid_{1(c),3(d)} = 6, \)
\( V(4) \mid_{2(d),3(a)} = 4, \quad V(4) \mid_{2(b),3(d)} = 6, \quad V(4) \mid_{2(c),3(d)} = 6, \)
\( V(5) \mid_{1(a),3(d)} = 8, \quad V(5) \mid_{1(b),3(d)} = 6, \quad V(5) \mid_{1(c),3(d)} = 7, \)
\( V(6) \mid_{2(a),3(d)} = 5, \quad V(6) \mid_{2(b),3(d)} = 7, \quad V(6) \mid_{2(c),3(d)} = 7, \)
\( V(7) \mid_{3(a),d} = 3, \quad V(7) \mid_{3(b),d} = 2, \quad V(7) \mid_{3(c),d} = 3. \)

26) without workers \(a, b\) and \(c\):
\( V(1) = 0, \)
\( V(2) \mid_{1(d),2(e)} = 8, \)
\( V(3) \mid_{1(d),3(e)} = 7, \)
\( V(4) \mid_{2(d),3(e)} = 5, \)
\( V(5) \mid_{1(d),3(e)} = 9, \)
\( V(6) \mid_{2(d),e} = 6, \)
\( V(7) \mid_{3(d),e} = 3. \)

If workers work one at a time in enterprises, then they will not be able to satisfy the first, second, third and fourth enterprises. The productivity values of these enterprises for workers will be zero.

27) Without workers \(b, c, d\) and \(e\):

28) without workers \(a, c, d\) and \(e\):

29) without workers \(a, b, d\) and \(e\):

30) without workers \(a, b, c\) and \(e\):

6) without workers \(a, b, c\) and \(d\):
\( V(1) = 0, \quad V(2) = 0, \quad V(3) = 0, \quad V(4) = 0. \)
\[ V(5) \mid_{1(d)} = 6, \quad V(5) \mid_{1(e)} = 3, \]
\[ V(6) \mid_{2(d)} = 4, \quad V(6) \mid_{2(e)} = 2, \]
\[ V(7) \mid_{3(d)} = 2, \quad V(7) \mid_{3(e)} = 1. \]

We write the results obtained in the performance matrix. Summing up the values of the maximum productivity of brigades at each enterprise, we consider the unit of productivity 1 ruble, we obtain the values of the characteristic function \([19-21]\). Values of the characteristic function for all coalitions

\[ V(S) = \sum_{i} V_{h_i}(G): \]

\[ V(a; b; c; d; e) = 131; \quad V(a; b; c; d) = 113; \quad V(a; b; c; e) = 97; \quad V(a; b; d; e) = 101; \quad V(a; c; d; e) = 104; \]
\[ V(b; c; d; e) = 103; \quad V(a; b; c) = 79; \quad V(a; b; d) = 83; \quad V(a; c; d) = 86; \quad V(b; c; d) = 84; \quad V(a; b; e) = 67; \]
\[ V(a; c; e) = 70; \quad V(b; c; e) = 68; \quad V(a; d; e) = 71; \quad V(b; d; e) = 73; \quad V(c; d; e) = 76; \quad V(a; b) = 42; \quad V(a; c) = 45; \quad V(b; c) = 41; \quad V(a; d) = 45; \quad V(b; d) = 46; \quad V(c; d) = 49; \quad V(a; e) = 33; \quad V(b; e) = 33; \quad V(c; e) = 36; \]
\[ V(d; e) = 38; \quad V(a) = 10; \quad V(b) = 9; \quad V(c) = 11; \quad V(d) = 12; \quad V(e) = 6. \]

Shapley vector matters \( \Phi = \{26,75; 25,68; 26,75; 31,25; 18,65\} \), Where \( i = (1;\ldots;5) \) and shows the amount that each of the employees (players) will receive.

4. Conclusion
Thus, a model of the optimal distribution of workers in the enterprise is built. Based on a given performance matrix of work performance for workers, a matrix of maximum productivity of work performance by workers and teams at each enterprise is built [22-23]. The characteristic function is constructed and the Shapley vector is found. Based on the constructed model, we can construct an algorithm for specifying the characteristic function and finding the Shapley vector, programmed in a programming language.

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