Temperature in the Throat

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ABSTRACT: We study the temperature of extended objects in string theory. Rotating D-branes in warped Calabi-Yau throats have induced metrics with thermal horizons and Hawking temperatures a la Unruh effect. We solve the equations of motion for slow rotating probe branes and derive their induced metrics in the UV/IR solutions of warped conifold throats. Our analysis shows that horizons and temperatures of expected features form on the world volume of the rotating probe brane in terms of conserved charges in the UV solutions of the conifold throat. In certain limits, we find world volume horizons and temperatures of the form similar to those of rotating probes in the AdS throat.

KEYWORDS: String theory, world volume black holes; Hawking temperatures.

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1. Introduction

Gauge/gravity duality [1–3] has been at the frontier of research in string theory over the past decade. According to gauge/gravity duality, the nearby geometry of a stack of $N$ D3-branes on the smooth ten-dimensional Minkowski space, the $\text{AdS}_5 \otimes S^5$ geometry, is dual to $\mathcal{N} = 4$ super Yang-Mills theory (SYM). This duality implies that $\mathcal{N} = 4$ SYM is described at high temperatures by black holes in $\text{AdS}_5$ [4, 5]. The duality also led to the construction of more realistic $\mathcal{N} = 1$ supersymmetric warped throat string geometries dual to gauge theories with confinement and chiral symmetry breaking [6–8] (see also [9, 10]).

The prototypical example of such warped throat geometries is the Klebanov-Strassler (KS) throat, [6], produced by placing $N$ regular D3-branes and $M$ fractional D3-branes (wrapped D5-branes) not at a smooth point but at a generic Calabi-Yau singularity, the conifold point. In this set up, the presence of regular and fractional background branes, respectively, reduce the supersymmetry to $\mathcal{N} = 1$ and break conformal invariance, and the singularity is removed by conifold deformation. Finally,
these branes dissolve into R–R and NS–NS fluxes producing a fully regular supergravity solution with a large hierarchy, consisting of a warped throat region that smoothly closes off at the tip, in the IR, where warping approaches a constant. This implies confinement and chiral symmetry breaking in the dual gauge theory. Although the exact KS solution is noncompact, it is well approximated by a compact solution, [11], where the UV end of the throat is attached to the compact internal Calabi-Yau space, which sets the UV/IR hierarchy of the throat. Far from the tip of the throat, the KS solution is well-approximated by the Klebanov-Tseylin (KT) solution, [7], with logarithmic warping for which the dual gauge theory is nonconformal but chiral. In the limit where the fractional branes are removed, the KT solution reduces to the Klebanov-Witten (KW) throat solution, [8], dual to \( \mathcal{N} = 1 \) superconformal gauge field theory. The metrics of these geometries asymptote AdS space in the UV and are equivalent in the mid throat region. In the IR region, however, the KT and KW solutions are singular at the tip while the KS caps off smoothly. The fact that the pure KS solution has confinement and chiral symmetry breaking at zero temperature raises a problem: At finite temperature one expects chiral symmetry restoration in the gauge theory and by gauge/gravity duality there has to be a corresponding black hole solution on the gravity side. These led to nonextremal generalizations of the supergravity solution, [12–14], showing that at sufficiently high temperature the system develops a horizon with a corresponding Hawking temperature (see also [9]).

Despite the fact that thermal horizons and Hawking temperatures in string theory arise predominantly from black hole and brane solutions on the background spacetime, studies over the past few years show the presence of horizons and temperatures resulting from the induced world volume metrics on probe D-branes moving in the background spacetime [15–22]. In [15] the appearance of horizons on accelerated D-branes was analysed in general terms. The presence of horizons on the world-sheet and on time-dependent D-branes in flat spacetime was analyzed in [16–19] and [20], respectively. Horizons on D-branes from accelerated observers in pure AdS space-time were studied in [21]. Moreover, in [22] it was shown that induced metrics on world volume probe D-branes in AdS\(_5\) have horizons with a characteristic Hawking temperature even if there is no black hole in the bulk.

The appearance of horizons and temperatures on accelerating probe branes is in fact the usual Unruh effect, [23], not for point particles but for extended objects. In this setup the original supergravity/string solution remains unchanged and the full dynamics is given by the world volume dynamics of the probe brane, which is straightforward to solve. Such an analysis has thus far been limited to the example of AdS\(_5\) \(\otimes\) S\(_5\) background (e.g. see [22]). In this work, we study the horizon and temperature of rotating probes in the warped throat backgrounds known as the warped conifold throats, [6–8], emerging from \( \mathcal{N} = 1 \) string solutions, [11]. Since these conifold throats have less symmetry and are well approximated to a compact string solution, rotating probes in such backgrounds are expected to have thermal
horizons with characteristic Hawking temperatures of distinct type, albeit unknown.

For simplicity we consider the simplest probe, a D1-brane, which is rotating slowly around spheres inside conifold throats. We solve the brane equations of motion and derive the horizons and temperatures from the induced world volume metrics on probe branes in the UV/IR solutions of the throat. We find that horizons and temperatures of expected features form on the world volume of the probe brane in terms of conserved charges at large radii, far above the IR, in UV solutions, including the KT and KW throats. We find, in general, that the temperature of the rotating probe is always finite, positive definite, and increasing/decreasing with expanding/shrinking horizons. Due to the Logarithmic warping, we find that the temperature of the rotating probe in the KT solution is more or less constant and given in terms of the flux and deformation parameter, which set the profile of the throat. In the absence of logarithmic warping, we find that the induced metric on the probe in the KW solution gives horizons and temperatures similar to those in the $AdS_5 \otimes S^5$ throat.

Our paper is organized as follows. In Section 2 we introduce our basic set up, including the general supergravity solution and the general probe D-brane action in such a solution. In Section 3 we consider the KS throat and solve the brane equations of motion in the very deep IR region of the KS throat and analyze the horizon and temperature from the induced metric on the probe. In Sections 4 & 5 we repeat this analysis and derive the horizons and temperatures of rotating probes in the KT and KW throats. In Section 6 we summarize and conclude with future outlook.

2. General basic set up: IIB theory and Dp-brane action

For our warped supergravity background, we consider the Calabi-Yau flux compactification of type IIB theory containing a warped throat region that smoothly closes off in the IR and is attached to the compact Calabi-Yau space at the UV end [11]. Type IIB string theory contains NS-NS fields, \{g_{MN}, \phi, B_{MN}\}, and R–R forms, \{C_0, C_2, C_4\}. The Type IIB action in the Einstein form takes the form

$$S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{|g|} \left( R_{10} - \frac{\partial_M \tau \cdot \partial^M \bar{\tau}}{(2\text{Im}\tau)^2} - \frac{|G_3|^2}{12 \text{Im}\tau} - \frac{|\tilde{F}_5|^2}{4 \cdot 5!} \right) + \frac{1}{8i\kappa_{10}^2} \int \frac{1}{\text{Im}(\tau)} C_4 \wedge G_3 \wedge G_3^* + S_{\text{loc}}. \quad (2.1)$$

Here $S_{\text{loc}}$ stands for localized contributions from D-brane and orientifold planes; $G_3 = F_3 - \tau H_3$ is the combination of R–R and NS–NS field strengths, given respectively as $F_3 = dC_2$ and $H_3 = dB_2$; $\tau = C_0 + i e^{-i\Phi}$ is the axion-dilaton; $\kappa_{10}^2 = \frac{1}{2} (2\pi)^7 \alpha' g_s^2$ is the ten-dimensional gravitational coupling; $\tilde{F}_5 = *_{10} F_5 = dC_4 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$ is the self-dual five form with $*_{10}$ being the ten-dimensional Hodge-star operator;
\( R_{10} \) is the ten-dimensional Ricci-scalar. The ten-dimensional warped metric and the self-dual five-form take the form \[ 11 \]

\[
\begin{align*}
  ds_{10}^2 &= \gamma_{MN} dX^M dX^N = h^{-1/2}(y) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(y) g_{mn} dy^m dy^n, \quad (2.2) \\
  \tilde{F}_5 &= (1 + \ast_{10}) \left[ d\alpha(y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \right]. \quad (2.3)
\end{align*}
\]

Here \( \alpha(y) \) is a scalar quantity and \( h(y) \) is the warp factor. The Laplace equation and Bianchi identity imply that the localized sources to saturate BPS like identity and, \( \alpha = h^{-1} \) and \( \ast_6 G_3 = iG_3 \). Such a string solution is called imaginary self-dual.

The action of a \( Dp \) brane is the sum of the DBI and CS actions, which in the above background takes the form

\[
S_{Dp} = S_{\text{DBI}} + S_{\text{CS}}, \quad (2.4)
\]

\[
S_{Dp} = -g_s T_p \int d^{p+1} \xi e^{-\Phi} \sqrt{-\det(\gamma_{ab} + F_{ab})} + g_s T_p \int C_{p+1}. \quad (2.5)
\]

Here we note that \( F_{ab} = B_{ab} + 2\pi\alpha' F_{ab} \) is the gauge invariant field strength, \( B_{ab} \) is the pullback of \( B_{MN} \) onto the D-brane world volume, \( \xi^a \) are the world volume coordinates of the probe brane, \( T_p = [(2\pi)^p g_s(\alpha')^{(p+1)/2}]^{-1} \) is the tension of the \( Dp \) brane, \( g_s \) is the string coupling and \( \gamma_{ab} = \partial_a X^M \partial_b X^N \gamma_{MN} \).

In what follows we first consider explicit examples of (2.2) describing infinite throat configurations which are, nonetheless, in good approximation to finite throats attached to the compact internal Calabi-Yau space at the UV. This sets the UV/IR hierarchy of the throat, and determines the size of the internal space. We then evaluate (2.3) in such explicit backgrounds for slow rotating branes, assuming that there are no gauge fields on the world volume of the probe brane, \( F_{ab} = 0 \). This gives us the equations of motion for slow rotating branes, and by solving them we compute their induced metrics and thermal horizons, giving their temperature.

### 3. Temperature in the Klebanov-Strassler throat

#### 3.1 The Klebanov-Strassler solution

As our first example, we take the Klebanov-Strassler (KS) throat geometry \[ 6 \] (see also \[ 9, 10 \]), also known as the warped deformed conifold. The deformed conifold is a nonsingular and noncompact Calabi-Yau threefold defined by a hypersurface in \( \mathbb{C}^4 \)

\[
\sum_{a=1}^{4} z_a^2 = \epsilon^2, \quad z_a \in \mathbb{C}^4, \quad (3.1)
\]

and by a radial coordinate
\[
\sum_{a=1}^{4} |z_a|^2 = \epsilon^2 \cosh \eta = r^3. \tag{3.2}
\]

Here \( \eta \) is the ‘radial’ coordinate on the conifold and \( \epsilon \) is the deformation parameter of the conifold, which can be made real by phase rotation. In the limit \( \epsilon \to 0 \), Eq. (3.1) gives the singular conifold and describes a cone over a five-dimensional Einstein, base manifold \( T^{1,1} \) of topology \( S^2 \times S^3 \) with both \( S^2 \) and \( S^3 \) spheres shrinking to zero size at the tip of the cone, \( r = 0 \). The topology of the base \( T^{1,1} \) is parametrized in a standard way by a set of five Euler angles \( \{ \theta_i, \phi_i, \psi \} \), where \( 0 \leq \theta_i \leq \pi, 0 \leq \phi_i \leq 2\pi, 0 \leq \psi \leq 4\pi \) \((i = 1, 2)\). The \( S^2 \times S^3 \) topology can be then identified as:

\[
S^2 : \quad \psi = 0, \quad \theta_1 = \theta_2, \quad \phi_1 = -\phi_2; \quad \text{and} \quad S^3 : \quad \theta_2 = \phi_2 = 0. \tag{3.3}
\]

In the nonsingular limit, at the tip, \( \eta \simeq 0 \), the \( S^2 \) shrinks to zero size while the \( S^3 \) remains of finite size with radius \( \epsilon^{2/3} \) amounting to the deformation by \( \epsilon \), which removes the singularity of the tip. The deformed conifold contains two independent three-cycles: the \( S^3 \) at the tip, known as the \( A \)-cycle, and the Poincare dual three-cycle, known as the \( B \)-cycle.

The Kähler potential on the deformed conifold derived from (3.2) reads [24]

\[
k(\eta) = \frac{\epsilon^{4/3}}{2^{1/3}} \int_0^\eta d\eta' [\sinh(2\eta') - 2\eta']^{1/3} \tag{3.4}
\]

The metric on the deformed conifold, which is derived from this Kähler potential, takes the form [24, 25]

\[
ds_6^2 = \frac{1}{2} \epsilon^{4/3} K(\eta) \left[ \frac{1}{3K(\eta)^3} \left\{ d\eta^2 + (g^5)^2 \right\} + \cosh^2 \frac{\eta}{2} \left\{ (g^3)^2 + (g^4)^2 \right\} + \sinh^2 \frac{\eta}{2} \left\{ (g^1)^2 + (g^2)^2 \right\} \right], \tag{3.5}
\]

where

\[
g^{1,3} = \frac{e^1 \mp e^3}{\sqrt{2}}, \quad g^{2,4} = \frac{e^2 \mp e^4}{\sqrt{2}}, \quad g^5 = e^5 \quad \tag{3.6}
\]

with

\[
e^1 = -\sin \theta_1 d\phi_1, \quad e^2 = d\theta_1, \quad e^3 = \cos \psi \sin \theta_2 d\phi_2 - \sin \psi d\theta_2, \quad e^4 = \sin \psi \sin \theta_2 d\phi_2 + \cos \psi d\theta_2, \quad e^5 = d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2. \quad \tag{3.7}
\]
and

\[ K(\eta) = \frac{(\sinh(2\eta) - 2\eta)^{1/3}}{2^{1/3} \sinh \eta}. \quad (3.8) \]

In terms of this, the proper radial coordinate which measures the actual distance up the throat in the six-dimensional, internal metric is given by

\[ r(\eta) = \frac{\epsilon^{2/3}}{\sqrt{6}} \int_0^\eta \frac{dx}{K(x)}. \quad (3.9) \]

The warping in this background is produced by placing N D3-branes, sourcing the self-dual R–R five-form field strength \( \tilde{F}_5 \), and M D5-branes wrapping a vanishing two-cycle given by the collapsing \( S^2 \) sphere, sourcing R–R three-form field strength \( F_3 \), on the conifold geometry. There is also a nontrivial NS–NS three-form field, \( H_3 \), and the R–R zero-form, \( C_0 \), and the dilaton field, \( \phi \), vanish on this background. The background three-form fluxes of the KS solution are quantized

\[ \frac{1}{(2\pi)^2 \alpha'} \int_A F_3 = M, \quad \frac{1}{(2\pi)^2 \alpha'} \int_B H_3 = N, \quad (3.10) \]

where \( M \gg 1 \) and \( N \gg 1 \) denote integers. Due to the presence of these fluxes, there is a backreaction on the geometry, producing the ten-dimensional warped line element [6] (see also [9, 10])

\[ ds_{10}^2 = h^{-1/2}(\eta) ds_4^2 + h^{1/2}(\eta) ds_6^2. \quad (3.11) \]

Here \( h(\eta) \) is the warp factor, \( ds_4^2 \) is the usual, four-dimensional Minkowski spacetime metric, and the internal metric \( ds_6^2 \), Eq. (3.5), is a strongly warped and deformed throat, which interpolates between a regular \( \mathbb{R}^3 \times S^3 \) tip, to an \( \mathbb{R} \times T^{1,1} \) cone in the UV. At small \( r \) one has:

\[ r \sim \frac{\epsilon^{2/3}}{3^{1/6} \cdot 2^{5/6}}, \quad K \simeq \left( \frac{2}{3} \right)^{1/3}, \quad (3.12) \]

where the internal metric (3.5) along the proper distance (3.9) smoothly rounds off with a finite \( S^3 \) of radius \( \epsilon^{2/3}/(12)^{1/6} \). Inspection of the metric (3.5) in terms of the proper radial coordinate (3.9) shows that at large \( r \), or for \( \eta \simeq 10 - 15 \), the throat explicitly takes the form of a cone \( \mathbb{R} \times T^{1,1} \). Hence \( \epsilon^{2/3} \) gives the radius of the nonsingular \( S^3 \) at the bottom of the throat, and the scale at which the throat asymptotes the \( T^{1,1} \) cone: It sets the IR scale of the geometry. Like in the deformed conifold, the IR geometry is smooth and the A-cycle is finite in size, with radius \( r_A = \sqrt{g_s M \alpha'} \), so the supergravity approximation remains valid near the tip provided that \( g_s M \gg 1 \). The other background fields are [6] (see also [9, 10])
\begin{align}
B_2 &= \frac{g_s M \alpha'}{2} \left[ f(\eta) g^1 \wedge g^2 + k(\eta) g^3 \wedge g^4 \right], \\
H_3 &= \frac{g_s M \alpha'}{2} \left[ d\eta \wedge (f' g^1 \wedge g^2 + k' g^3 \wedge g^4) + \frac{1}{2} (k - f) g^5 \wedge (g^1 \wedge g^3 + g^2 \wedge g^4) \right], \\
F_3 &= dC_2 = \frac{M \alpha'}{2} \left[ g^5 \wedge g^3 \wedge g^4 (1 - F) + g^5 \wedge g^1 \wedge g^2 F + F' d\eta \wedge (g^1 \wedge g^3 + g^2 \wedge g^4) \right], \\
\tilde{F}_5 &= \mathcal{F}_5 + \ast \mathcal{F}_5 = B_2 \wedge F_3 + dC_4, \\
dC_4 &= g_s^{-1} d(h^{-1}) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3.
\end{align}

The explicit form of the functions appearing above is [6] (see also [9, 10]):

\begin{align}
F(\eta) &= \frac{\sinh \eta - \eta}{2 \sinh \eta}, \\
f(\eta) &= \frac{\eta \cosh \eta - 1}{2 \sinh \eta} (\cosh \eta - 1), \\
k(\eta) &= \frac{\eta \cosh \eta - 1}{2 \sinh \eta} (\cosh \eta + 1), \\
h(\eta) &= 2^{2/3} (g_s M \alpha')^2 \epsilon^{-8/3} \int_{\eta}^{\infty} dx \frac{x \cosh x - 1}{\sinh^2 x} (\sinh x \cosh x - x)^{1/3}.
\end{align}

### 3.2 Induced metric and Hawking temperature in the KS throat

In order to analyze the temperature of the rotating probe in the deep IR region of the throat, i.e., at small radii near the bottom of the throat, we note that in this limit the warp factor is constant \( h_0 = a_0 (g_s M \alpha')^2 2^{2/3} \epsilon^{-8/3} \) with \( a_0 \approx 0.71805 \) and the ten-dimensional background metric (3.5) takes the form [6] (see also [9, 10])

\begin{align}
\epsilon^{4/3} \left( \frac{2}{(2)^{1/3} a_0^{1/2} (g_s M \alpha')} \right) (dx^2 - dt^2) + a_0^{1/2} 6^{-1/3} (g_s M \alpha') \left\{ \frac{1}{2} \eta^2 + \frac{1}{2} (g^5)^2 + (g^3)^2 + (g^4)^2 + \frac{\eta^2}{4} [(g^1)^2 + (g^2)^2] \right\}.
\end{align}

We also note that near the bottom of the throat the \( S^2 \) sphere shrinks to zero size while the \( S^3 \) sphere remains finite, as discussed. To obtain the explicit form of the metric near the bottom of the throat, we may therefore consider an \( S^3 \) round (3.3), and for simplicity we may also fix \( \theta_1 = \theta = \pi/2 \) (which together imply \( g^{2,4} = 0 \)). The full background metric then reads
\[ ds_{10}^2 \rightarrow \frac{\epsilon^{4/3}}{(2)^{1/3}a_0^{1/2}(g_Ma')} (dx^2 - dt^2) \]
\[ + (2^{-1}a_0^{1/2}6^{-1/3})(g_Ma') \left\{ d\eta^2 + d\psi^2 + B(\eta)d\phi^2 \right\}, \]

(3.23)

where \( B(\eta) = 1 + \eta^2/4 \). To evaluate the action of the probe D1-brane in the background (3.23), note that near the bottom of the throat (3.18) is \( F \simeq 1/2 \), and so from (1.13) one infers that in the background (3.23) (where \( g^{2,4} = 0 \)) \( C_2 \) is locally constant. Also note that the brane extends in \( t \)- and \( \eta \)-directions, while localized at \( x = 0 \). Evaluating (2.5) in the background (3.23), then gives the action of the brane, spinning slowly in the \( \psi \)- and \( \phi \)-direction, in the deep IR region of the KS throat as

\[ S_{D1} \equiv -g_s T_{D1} \int d^2 \xi \sqrt{-\det \gamma_{ab}} \]
\[ = -g_s T_{D1} \frac{\epsilon^{4/3}}{2(12)^{1/3}} \int d^2 \xi \sqrt{1 + B(\eta)(\phi')^2 - \frac{a_0(g_Ma')^2 B(\eta)}{(24\epsilon^4)^{1/3}} \phi^2 + (\psi')^2 - \frac{a_0(g_Ma')^2}{(24\epsilon^4)^{1/3}} \psi^2} \]
\[ = -g_s T_{D1} \frac{\epsilon^{4/3}}{2(12)^{1/3}} \int d^2 \xi \left[ 1 + \frac{1}{2} B(\eta)(\phi')^2 - \frac{a_0(g_Ma')^2 B(\eta)}{2(24\epsilon^4)^{1/3}} \phi^2 + \frac{1}{2}(\psi')^2 - \frac{a_0(g_Ma')^2}{2(24\epsilon^4)^{1/3}} \psi^2 \right]. \]

(3.24)

Here we note that \( \phi(\eta, t) \) and \( \psi(\eta, t) \) specify the world-volume of the brane with their higher-order derivatives dropped by considering the limit of small velocities, giving the final line in (3.24) (after Taylor expansion). The leading-order brane equations of motion then take the form

\[ \frac{\partial}{\partial \eta} [B(\eta)\phi'(\eta, t)] = \frac{a_0(g_Ma')^2}{(24\epsilon^4)^{1/3}} \frac{\partial}{\partial t} \left[ B(\eta)\dot{\phi}(\eta, t) \right], \]

(3.25)
\[ \frac{\partial}{\partial \eta} [\psi'(\eta, t)] = \frac{a_0(g_Ma')^2}{(24\epsilon^4)^{1/3}} \frac{\partial}{\partial t} \left[ \dot{\psi}(\eta, t) \right]. \]

(3.26)

Now, consider solutions of the form

\[ \psi(\eta, t) = \omega_1 t + \xi_1(\eta) = \omega_1 t + \eta + \psi_0, \]

(3.27)
\[ \phi(\eta, t) = \omega_2 t + \xi_2(\eta) = \omega_2 t - 2 \tan^{-1} \left( \frac{\eta}{2} \right). \]

(3.28)

Put these into the background and obtain the induced metric on the D1-brane as
\[ ds_{\text{ind}}^2 = -\frac{a_{\eta 0}^{1/2}(g_* M \alpha') \Omega(\eta)}{2 \cdot 6^{1/3}} dt^2 + \frac{a_{\eta 0}^{1/2}(g_* M \alpha')}{2 \cdot 6^{1/3} B(\eta)} (1 + 2B(\eta)) \, d\eta^2 \\
+ \frac{a_{\eta 0}^{1/2}(g_* M \alpha') \Delta \omega}{2 \cdot 6^{1/3}} dt \, d\eta, \quad \text{with} \]
\[ \Omega(\eta) = \frac{(24 \epsilon^4)^{1/3}}{a_0(g_* M \alpha')^2} - (\omega_1^2 + B(\eta) \omega_2^2), \quad \Delta \omega = \omega_2 - \omega_1. \]

To eliminate the cross term in this metric, introduce a new coordinate

\[ \tau = t - \frac{a_{\eta 0}^{1/2}(g_* M \alpha') \Delta \omega}{2 \cdot 6^{1/3}} \int \frac{d\eta}{\Omega(\eta)}. \] (3.31)

The metric (3.29) then becomes

\[ ds_{\text{ind}}^2 = -\frac{a_{\eta 0}^{1/2}(g_* M \alpha') \Omega(\eta)}{2 \cdot 6^{1/3}} d\tau^2 + \frac{a_{\eta 0}^{1/2}(g_* M \alpha')}{2 \cdot 6^{1/3} B(\eta) \Omega(\eta)} \left\{ (1 + 2B(\eta)) \left[ \frac{a_{\eta 0}^{1/2}(g_* M \alpha') \Omega(\eta)}{2 \cdot 6^{1/3}} \right] + B(\eta) \Delta \omega^2 \right\} d\eta^2. \] (3.32)

One can go on and find the location of the horizon in the induced metric by the usual methods, setting \( g^{\eta m} = 0 \). However, as we will show in the following, some unexpected features appear in this case. First note that this equation has no solutions for \( \eta \) if both angular momenta are zero. Therefore with no rotations there will be no world volume horizon as expected. Next consider the case with \( \omega_2 \neq 0 \) and \( \omega_1 = 0 \). One can see that for small values of \( \omega_2 \), the horizon appears at very large values of \( \eta \). As \( \omega_2 \) increases, the horizon moves towards smaller values of \( \eta \) and in the limit of large \( \omega_2 \) it will hit \( \eta \approx 0 \).

This tells us that the world volume black hole nucleates at large values of \( \eta \) with a horizon that grows by increasing the angular momentum. The horizon grows in two directions, one away from the bottom of throat and one towards it. The former part is outside the regime of validity of the metric we have considered in this section and the latter is described by the induced metric above. The feature just described predicts a specific value for angular momentum for which the horizon reaches the bottom of the throat. When this happens, the world volume of the brane that is rotating in the IR region of the throat is completely inside the world volume black hole.

For \( \omega_2 = 0 \) and \( \omega_1 \neq 0 \) we will never have a world volume black hole. This is due to the fact that in the IR region of the throat, the \( \psi \) angle will have a constant (\( \eta \) independent) warping factor. However, once the linear velocity in the \( \psi \) direction reaches that of speed of light the metric degenerates as expected.
All of this can be seen by studying the following equation

\[ g^{\eta\eta}(\eta_0) = 0 \rightarrow \eta_0 = \left( \frac{2}{\omega_2} \right) \sqrt{\frac{(24\epsilon^4)^{1/3}}{a_0(g_sM\alpha')^2} - (\omega_1^2 + \omega_2^2)} \]  

(3.33)

This relation already shows the upper bound on the angular velocities

\[ \omega_1^2 + \omega_2^2 < \frac{(24\epsilon^4)^{1/3}}{a_0(g_sM\alpha')^2}. \]  

(3.34)

One should have in mind that, as explained above, the bound on \( \omega_2 \) is due to the maximum radius of horizon in the IR region before hitting the bottom of throat whereas the bound on \( \omega_1 \) is a result of relativistic bound on linear velocity.

The upshot of this section is that all nontrivial features of the world volume metric that arise because of accelerating the brane, especially for slow rotations, occur at large values of \( \eta \) which can be explained by studying the Klebanov-Tseytlin throat. This will be the subject of next section\(^1\).

4. Temperature in the Klebanov-Tseytlin throat

4.1 The Klebanov-Tseytlin solution

Far from the tip of the cone, where \( \eta \) is large, the deformation of the conifold can be neglected and Eq. (3.1) reduces the constraint equation of the singular conifold

\[ \sum_{a=1}^{4} z_a^2 = 0. \]  

(4.1)

In this limit, one may introduce another radial coordinate \( r \) through

\[ r^2 = \frac{3}{2^{2/3}} \epsilon^{4/3} \exp(2\eta/3). \]  

(4.2)

The Kähler potential is then given by [24]

\[ k = \frac{3}{2} \left( \sum_{a=1}^{4} |z_a|^2 \right)^{2/3} = \frac{3}{2} r^2 = r^2. \]  

(4.3)

\(^1\)The nucleation of black hole at \( \eta \sim \infty \) and appearance of a horizon that approaches the bottom of throat for larger rotations can also be seen by a naive calculation of the horizon temperature. If one wrongly interprets \( \eta_0 \) as appearing in (3.33) as the radius of horizon, a standard calculation of the horizon temperature gives

\[ T_H = \frac{\langle g^{\eta\eta} \rangle}{4\pi} \bigg|_{\eta=\eta_0} = \frac{6^{1/3}\omega_2}{2\pi a_0^{1/2}(g_sM\alpha')\Delta \omega^2} \sqrt{\frac{(24\epsilon^4)^{1/3}}{a_0(g_sM\alpha')^2} - (\omega_1^2 + \omega_2^2)}. \]

This equation predicts that for larger rotations we will have smaller temperatures which is clearly wrong.
The metric on the singular conifold, which is derived from this Kähler potential, takes the form \[24\]

\[ds_6^2 = d\hat{r}^2 + \hat{r}^2 ds_{T^{1,1}}, \quad \text{with} \quad ds_{T^{1,1}}^2 = \frac{1}{9}(g^5)^2 + \frac{1}{6} \sum_{i=1}^4 (g^i)^2,\]  

(4.4)

where \(g^i\)'s are given by Eq. (3.6) - (3.7). Far from the bottom of the throat (tip of the cone), where the deformation parameter can be neglected, by which Eq. (3.1) reduces to Eq. (4.1), the KS throat solution joins the Klebanov-Tseytlin (KT) throat solution [7] (see also [9, 10]). In this limit, the ten-dimensional warped line element is that of the KT throat and takes the form [7] (see also [9, 10])

\[ds_{10}^2 = h^{-1/2}ds_4^2 + h^{1/2}(d\hat{r}^2 + \hat{r}^2 ds_{T^{1,1}}^2),\]  

(4.5)

where \(ds_{T^{1,1}}^2\) is given by (4.4). The warp factor, \(h\), and other background fields take the form [7] (see also [9, 10])

\[B_2 = \frac{3g_s M\alpha'}{4} \left[ \ln \frac{\hat{r}}{\hat{r}_{\text{UV}}} \right] (g^1 \wedge g^2 + g^3 \wedge g^4),\]  

(4.6)

\[H_3 = dB_2 = \frac{3g_s M\alpha'}{4\hat{r}} d\eta \wedge (g^1 \wedge g^2 + g^3 \wedge g^4),\]  

(4.7)

\[C_2 \rightarrow \frac{M \alpha' \psi}{2} (g^1 \wedge g^2 + g^3 \wedge g^4)\]  

(4.8)

\[F_3 = dC_2 = \frac{M\alpha'}{4} g^5 \wedge (g^1 \wedge g^2 + g^3 \wedge g^4),\]  

(4.9)

\[\hat{F}_5 = \mathcal{F}_5 + \ast \mathcal{F}_5,\]  

(4.10)

\[\mathcal{F}_5 = B_2 \wedge F_3 = 27\pi(\alpha')^2 N_e \text{Vol}(T^{1,1}),\]  

(4.11)

\[N_e = N + \frac{3(g_s M)^2}{2\pi} \ln \frac{\hat{r}}{\hat{r}_{\text{UV}}},\]  

(4.12)

\[\ast \mathcal{F}_5 = dC_4 = g_s^{-1} d(h^{-1}(\hat{r})) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3\]  

(4.13)

\[h(\hat{r}) = \frac{27\pi \alpha'^2}{4\hat{r}^4} \left[ g_s N + \frac{3(g_s M)^2}{2\pi} \left( \ln \frac{\hat{r}}{\hat{r}_{\text{UV}}} + \frac{1}{4} \right) \right].\]  

(4.14)

4.2 Induced metric and Hawking temperature in the KT throat

In order to analyze the horizon and temperature of the rotating probe in the UV solution, i.e., at large radii far above the IR deformations of KS, away from the bottom of the throat, but well inside the throat, we work within the range \(\epsilon^{2/3} \ll \hat{r} \ll \hat{r}_{\text{UV}}\). \(^*\)

Here we note that according to (1.2) for \(\eta_{\text{UV}} \simeq 10 - 15^2\) one has \(\hat{r}_{\text{UV}} \simeq 10^2 \epsilon^{2/3}\), which sets the maximum radial distance for the throat, the UV scale of the geometry, where

\(^*\)See also Section 3, the discussion below (3.12).
the throat is attached to the compact Calabi-Yau space. We may also write the warp factor (4.14) as [7] (see also [9, 10])

\[ h(\hat{r}) = \frac{L^4}{\hat{r}^4} \ln(\hat{r}/\epsilon^{2/3}), \quad L^4 = \frac{81(g_s M\alpha')^2}{8}, \]  

\[ (4.16) \]

giving the ten-dimensional background metric (4.5) of the form [7] (see also [9, 10])

\[ ds^2 = \frac{\hat{r}^2}{L^2 \sqrt{\ln(\hat{r}/\epsilon^{2/3})}} (dx^2 - dt^2) + \frac{L^2 \sqrt{\ln(\hat{r}/\epsilon^{2/3})}}{\hat{r}^2} d\hat{r}^2 + \frac{L^2}{\hat{r}^2} \sqrt{\ln(\hat{r}/\epsilon^{2/3})} ds_{7,1}^2. \]

\[ (4.17) \]

Note that due to logarithmic dependence this metric becomes singular at \( \hat{r} = \frac{\epsilon^2}{3} \).

Also, note that because \( T_{1,1} \) expands slowly toward large \( \hat{r} \), the curvatures decrease there so that corrections to the supergravity solution become negligible. Thus even when \( g_s M \) is very small, the supergravity solution considered here is reliable for sufficiently large radii where \( g_s N_e \gg 1 \) (see Eq. (4.12)) [7] (see also [9, 10]).

Considering the same \( S^3 \) cycle as in the previous section, we obtain this full background metric of the form

\[ ds_{10}^2 = \frac{\hat{r}^2}{L^2 \sqrt{\ln(\hat{r}/\epsilon^{2/3})}} (dx^2 - dt^2) + \frac{L^2 \sqrt{\ln(\hat{r}/\epsilon^{2/3})}}{\hat{r}^2} (d\hat{r}^2 + \frac{\hat{r}^2}{6} d\phi^2 + \frac{\hat{r}^2}{9} d\psi^2), \]

\[ (4.18) \]

To evaluate the action of the probe D1-brane in the background (1.18), note that, far from the bottom of the throat, from (1.18) one infers that in the background (4.18) (where \( g_2^2 = 0 \)) \( C_2 \) is locally vanishing. Also note that the brane extends in \( t \)- and \( \hat{r} \)-directions, while localized at \( x = 0 \). Evaluating (2.3) in the background (4.18), then gives the action of the brane, spinning in the \( \psi \)- and \( \phi \)-direction, in the KT throat in the form:

\[ S_{D1} = -g_s T_{D1} \int d^2 \xi \sqrt{-\det \gamma_{ab}} \]

\[ = -g_s T_{D1} \int d^2 \xi \left[ 1 + \frac{\hat{r}^2 (\phi')^2}{6} - \frac{L^4}{6 \hat{r}^2} \ln(\hat{r}/\epsilon^{2/3}) \phi'^2 + \frac{\hat{r}^2 (\psi')^2}{9} - \frac{L^4}{9 \hat{r}^2} \ln(\hat{r}/\epsilon^{2/3}) \psi'^2 \right] \]

\[ = -g_s T_{D1} \int d^2 \xi \left[ 1 + \frac{\hat{r}^2 (\phi')^2}{12} - \frac{L^4}{12 \hat{r}^2} \ln(\hat{r}/\epsilon^{2/3}) \phi'^2 + \frac{\hat{r}^2 (\psi')^2}{18} - \frac{L^4}{18 \hat{r}^2} \ln(\hat{r}/\epsilon^{2/3}) \psi'^2 \right]. \]

\[ (4.19) \]

Here we note that \( \phi(\hat{r}, t) \) and \( \psi(\hat{r}, t) \) specify the world-volume of the brane with their higher-order derivatives dropped by considering the limit of small velocities, giving the final line in (4.19) (after Tylor expansion). The leading-order brane equations of motion then take the form
\[
\frac{\partial}{\partial \hat{r}} \left[ \frac{\hat{r}^2 \psi' (\hat{r}, t)}{9} \right] = \frac{\partial}{\partial t} \left[ \frac{L^4}{9 \hat{r}^2} \ln \left( \hat{r} / \epsilon^{2/3} \right) \psi (\hat{r}, t) \right],
\]
\[
\frac{\partial}{\partial \hat{r}} \left[ \frac{\eta^2 \phi' (\hat{r}, t)}{6} \right] = \frac{\partial}{\partial t} \left[ \frac{L^4}{6 \hat{r}^2} \ln \left( \hat{r} / \epsilon^{2/3} \right) \phi (\hat{r}, t) \right].
\]

As before, consider solutions of the form
\[
\psi (\hat{r}, t) = \omega_1 t + g (\hat{r}) = \omega_1 t - \frac{\omega_1}{\hat{r}} + \psi_0,
\]
\[
\phi (\hat{r}, t) = \omega_2 t + f (\hat{r}) = \omega_2 t - \frac{\omega_2}{\hat{r}} + \phi_0.
\]

Putting these into the background, gives the induced metric on the brane as
\[
ds_{\text{ind}}^2 = - \left[ \hat{r}^2 - L^4 \ln (\hat{r} / \epsilon^{2/3}) \omega^2 \right] dt^2 + \sqrt{L^4 \ln (\hat{r} / \epsilon^{2/3})} \left( \frac{1}{\hat{r}^2} + \frac{\omega^2}{\hat{r}^4} \right) d\hat{r}^2
\]
\[+ \frac{2\omega^2}{\hat{r}^2} \sqrt{L^4 \ln (\hat{r} / \epsilon^{2/3})} dt d\hat{r}, \quad \omega^2 = \frac{\omega_1^2}{9} + \frac{\omega_2^2}{6}.
\]

To eliminate the cross term in this metric, we may consider a coordinate transformation of the form
\[
\tau = t - \omega^2 \int \frac{d\hat{r}}{\hat{r}^2 (\hat{r}^2 - L^4 \ln (\hat{r} / \epsilon^{2/3}) \omega^2)}.
\]

The induced metric (4.24) then takes the form
\[
ds_{\text{ind}}^2 = - \left[ \hat{r}^2 - L^4 \ln (\hat{r} / \epsilon^{2/3}) \omega^2 \right] d\tau^2 + \sqrt{L^4 \ln (\hat{r} / \epsilon^{2/3})} \left[ \frac{\omega^2 + \hat{r}^2 - L^4 \ln (\hat{r} / \epsilon^{2/3}) \omega^2}{\hat{r}^2 (\hat{r}^2 - L^4 \ln (\hat{r} / \epsilon^{2/3}) \omega^2)} \right] d\hat{r}^2.
\]

To obtain the horizon, we set from this metric \( g_{\hat{r}\hat{r}} = 0 \), which gives:
\[
\hat{r}_H^2 - L^4 \omega^2 \ln (\hat{r}_H / \epsilon^{2/3}) = 0.
\]

This equation can have at most two (real positive) zeros. The value and number of these zeros depends on the value of the conserved charge. Clearly, at \( \omega = 0 \) no zero of (4.27) appears and there will be no horizon. Now let \( \omega_c \) be the minimum, critical value of conserved charge for which at least one zero of (4.27) appears. Inspection of (4.27) shows that for \( \omega_c \sim \epsilon^{2/3} / L^2 \), the equation (4.27) will have one solution in the IR region, close to the tip of the throat and near the singularity of KT, \( \hat{r}_c \sim \epsilon^{2/3} \).
We can also see this more directly by obtaining an explicit solution of Eq. (4.27). By expanding around $\omega = 0$ and consider the leading terms, we obtain the solution \(^3\):

$$\hat{r}_H = L^2 \omega \sqrt{|\ln(\sqrt{2} e^{2/3} / L^2 \omega)|}, \tag{4.28}$$

Here again it is clear that for $\omega_c \simeq e^{2/3} / L^2$ we get $\hat{r}_c \simeq e^{2/3}$. Now, increase $\omega$. For any value of conserved charge $\omega > \omega_c$, we will have two solutions for (4.27), one smaller than $\hat{r}_c$ and one larger, denoted by $\hat{r}_<$ and $\hat{r}_>$ respectively. Increasing the value of $\omega$ continuously, further decreases (increases) the value of $\hat{r}_<$ ($\hat{r}_>$). One may first be tempted to interpret this result as the appearance of a double horizon. However, one should keep in mind that by the validity range of the KT solution, (4.13), the zero at $\hat{r}_<$ should be discarded. The conclusion of this analysis is that by rotating the brane inside the throat, a world volume black hole nucleates around the KT singularity and by increasing the angular momentum the horizon grows in size. For rotations $\omega \simeq (10^2 - 10^3)^{1/2} e^{2/3} / L^2$, the horizon approaches the UV region, far from the tip and the KT singularity, and will be of the size of the UV scale of the geometry, $\hat{r}_H \to 10^2 e^{2/3}$.

To obtain the Hawking temperature, we Wick-rotate $\tau$ into a Euclidean time, and after a straightforward calculation we get:

$$T_H = \left. \frac{(g^{\hat{r}\hat{r}})'}{4\pi} \right|_{\hat{r} = \hat{r}_H} = \frac{\hat{r}_H (2\hat{r}_H^2 - L^4 \omega^2)}{4\pi (\omega L)^2 \sqrt{\ln (\hat{r}_H / e^{2/3})}}, \tag{4.29}$$

where $\hat{r}_H$ is the horizon discussed above. Inspection of (4.29) shows that the temperature of the black hole solution on the probe, $T_H$, is always real, positive definite and finite. This is because within intermediate scales, (4.13), $\hat{r}_H$ neither hits the KT singularity in the IR nor extends beyond the UV cutoff, with the temperature scaling roughly as $T_H \gtrsim L^2 e^{2/3}$ for $\hat{r}_H \to 10^2 e^{2/3}$ and $T_H \lesssim L^2 e^{2/3}$ for $\hat{r}_H \to e^{2/3}$. This means that away from the mid throat region the temperature of the probe, $T_H$, is uniformly continuous and more or less constant, not approaching hierarchically smaller, or larger values. One can also see that $T_H$ increases/decreases continuously with expanding/shrinking $\hat{r}_H$, as expected.

The acceptable values of $T_H$ follow from the validity range for $\hat{r}_H$. In addition, since the KT supergravity solution is reliable for sufficiently large radii even if $g_s M$ is very small, the temperature scales smaller, or larger, depending on whether

\(^3\)Here we note that Eq. (4.27) can be rearranged into a Lambert’s transcendental equation of the form $\ln \tilde{X}_H = (e^{4/3} / L^4 \omega) X^2_H$ whose solution is given in terms of Lambert W-function as $\tilde{X}_H = \exp[-W(-2 e^{4/3} / L^4 \omega^2)]/2$. The expansion of the Lambert W-function $W(X)$ about $X = \infty$, or $\omega = 0$, is $W(X) = -\ln[1/|X|] - \ln[-\ln[1/|X|]] - \ln[-\ln[-\ln[1/|X|]]]/\ln[1/|X|]^2 - \cdots$. Considering the first two leading terms in this expansion and replacing $X$ by the argument of the exponent of $\tilde{X}_H$ one obtains (4.28), after rearranging back.
\( g_s M \gg 1 \) (making \( L^2 \gg 1 \)), or \( g_s M \ll 1 \) (making \( L^2 \ll 1 \)) is considered. Finally, note that for \( g_s M \ll 1 \) our Hawking temperature (4.29) takes the general form \( T_H \simeq \frac{\hat{r}^3}{4\pi \omega^2 L^2} \sqrt{\ln \left( \frac{\hat{r}_H}{2^{2/3}} \right)} \). The denominator of this modulo \( 4\pi \omega^2 \) equals the denominator of the Hawking temperature discussed in [12–14] (cf. Eq. (89) of [9]), but its numerator is very different. Here it grows with the cube of \( \hat{r}_H \) whereas there (cf. Eq. (89) of [9]) the numerator of \( T_H \) grows linearly with \( \hat{r}_H \). Remember that the black hole here lives on a rotating probe brane whereas in [12–14] there are no probes and the black hole lives in the supergravity background itself.

5. Temperature in the Klebanov-Witten throat

5.1 The Klebanov-Witten solution

In the absence of \( M \) wrapped D5-branes the KT throat solution joins the Klebanov-Witten throat solution [8]. The KW throat is the simplest conifold throat background. The ten-dimensional metric on the KW throat takes the form [8]

\[
\begin{align*}
\sum_{10}^2 &= h^{1/2} g_{\mu\nu} dx^\mu dx^\nu + h^{1/2} (d\hat{r}^2 + \hat{r}^2 ds_{T_{1,1}}^2),
\end{align*}
\]

where the warp factor reads

\[
\begin{align*}
\hat{r} &= \frac{L^4}{\hat{r}^4}, \quad \text{and} \quad L^4 \equiv \frac{27\pi}{4} g_s N(\alpha')^2.
\end{align*}
\]

The other background fields are similar to those of the KT throat with the crucial difference that logarithmic warping is set to vanish.

5.2 Induced metric and Hawking temperature in the KW throat

Considering the same \( S^3 \) cycle as before, we obtain this full background metric of the form

\[
\begin{align*}
\sum_{10}^2 &= \frac{\hat{r}^2}{L^2} \left( dx^2 - dt^2 \right) + \frac{L^2}{\hat{r}^2} \left( d\hat{r}^2 + \frac{\hat{r}^2}{6} d\phi^2 + \frac{\hat{r}^2}{9} d\psi^2 \right),
\end{align*}
\]

As in the previous sections, we may evaluate the action of the brane noting that the brane extends in \( t \) - and \( \hat{r} \) -directions, while localized at \( x = 0 \). Evaluating (2.3) in the background (5.3) (where there is no \( C_2 \) since \( M = 0 \)), gives the action of the brane, spinning in the \( \psi \) - and \( \phi \)-direction, in the KW throat

\[
\begin{align*}
S_{D1} &= -g_s T_{D1} \int d^2 \xi \sqrt{-\det \gamma_{ab}}
\\&= -g_s T_{D1} \int d^2 \xi \sqrt{1 + \frac{\hat{r}^2 (\psi')^2}{6} - \frac{L^4}{6 \hat{r}^2} \hat{\phi}^2 + \frac{\hat{r}^2 (\psi')^2}{9} - \frac{L^4}{9 \hat{r}^2} \hat{\psi}^2}
\\&= -g_s T_{D1} \int d^2 \xi \left[ 1 + \frac{\hat{r}^2 (\psi')^2}{12} - \frac{L^4}{12 \hat{r}^2} \hat{\phi}^2 + \frac{\hat{r}^2 (\psi')^2}{18} - \frac{L^4}{18 \hat{r}^2} \hat{\psi}^2 \right].
\end{align*}
\]

\[-15-\]
As in the KT throat, here we note that $\phi(\hat{r}, t) \quad \text{and} \quad \psi(\hat{r}, t)$ specify the world-volume of the brane with their higher-order derivatives dropped by considering the limit of small velocities, giving the final line in (5.4) (after Taylor expansion). The leading-order brane equations of motion then take the form

$$\frac{\partial}{\partial \hat{r}} \left[ \hat{r} \psi'(\hat{r}, t) \right] = \frac{\partial}{\partial t} \left[ \frac{L^4}{6\hat{r}^2} \psi(\hat{r}, t) \right], \quad (5.5)$$

$$\frac{\partial}{\partial \hat{r}} \left[ \eta^2 \phi'(\hat{r}, t) \right] = \frac{\partial}{\partial t} \left[ \frac{L^4}{6\hat{r}^2} \phi(\hat{r}, t) \right]. \quad (5.6)$$

As before, consider solutions of the form

$$\psi(\hat{r}, t) = \omega_1 t + g(\hat{r}) = \omega_1 t - \frac{\omega_1}{\hat{r}} + \psi_0, \quad (5.7)$$

$$\phi(\hat{r}, t) = \omega_2 t + f(\hat{r}) = \omega_2 t - \frac{\omega_2}{\hat{r}} + \phi_0. \quad (5.8)$$

Putting these into the background, gives the induced metric on the brane as

$$ds^{2}_{ind} = -\left[ \hat{r}^2 - L^4 \overline{\omega}^2 \right] dt^2 + L^2 \left( \frac{1}{\hat{r}^2} + \overline{\omega}^2 \right) d\hat{r}^2 + \frac{2\overline{\omega}^2}{\hat{r}^2} L^2 dt d\hat{r}, \quad \overline{\omega}^2 = \frac{\omega_1^2}{9} + \frac{\omega_2^2}{6}. \quad (5.9)$$

To eliminate the cross term in this metric, we may consider a coordinate transformation of the form

$$\tau = t - \overline{\omega}^2 \int \frac{d\hat{r}}{\hat{r}^2(\hat{r}^2 - L^4 \overline{\omega}^2)}. \quad (5.9)$$

The induced metric (5.9) then takes the form

$$ds^{2}_{ind} = -\left[ \hat{\rho}^2 - L^4 \overline{\omega}^2 \right] d\tau^2 + L^2 \left( \overline{\omega}^2 + \hat{\rho}^2 - L^4 \overline{\omega}^2 \right) d\hat{\rho}^2. \quad (5.10)$$

Using the induced metric (5.10), we can derive the horizon and temperature of the rotating probe in the KW solution as:

$$\hat{\rho}_H^2 - L^4 \overline{\omega}^2 = 0, \quad (5.11)$$

$$T_H = \frac{\hat{\rho}_H}{2\pi} = \frac{L^2}{2\pi} \sqrt{\frac{\omega_1^2}{9} + \frac{\omega_2^2}{6}}. \quad (5.12)$$

Clearly, Eq. (5.11) has one (real positive) zero, forming a single horizon, $\hat{\rho}_H = L^2 \overline{\omega}$. This $\hat{\rho}_H$ has a form similar to $\hat{r}_H$ in KT (e.g. see (4.28)), but shrinking/expanding
linearly with $\overline{\omega}$, since there is no logarithmic warping, and therefore changing more rapidly, by which $T_H$ in KW increases/decreases faster than $T_H$ in KT. But, since the validity range is the same, as in KT, $\hat{r}_H$ and $T_H$ are constrained by UV/IR scales of the throat, and are therefore finite, not approaching arbitrary large, or small, values.

It is also clear from Eq. (5.11)-(5.12) that for rotating probes in KW, $\hat{r}_H$ and $T_H$ have a form very similar to those for rotating probes in $AdS_5 \otimes S^5$, in particular, increasing/decreasing linearly with $\overline{\omega}$, but supressed by prefactors $1/9$ and $1/6$, relative to $\hat{r}_H$ and $T_H$ of rotating probes in $AdS_5 \otimes S^5$. Also note that $AdS_5 \otimes S^5$ extends from $r = 0$ to $r = \infty$, by which $\hat{r}_H$ and $T_H$ can increase linearly with $\overline{\omega}$ without bound whereas in KW $\hat{r}_H$ and $T_H$ are subject to the validity range of the UV solution and therefore constrained by the UV/IR scales of the throat.

6. Discussion

In this paper we studied the horizons and temperatures of black hole solutions on the world volume of rotating probe branes in warped Calabi-Yau throats. We considered the KS, KT and KW solutions as explicit examples of Calabi-Yau throats. The motivation of our work was to study Unruh effect for extended objects in these throat backgrounds. The focus of our paper was on a UV/IR consistent analysis of horizons and temperatures on the world volume of the probe brane in these throat backgrounds. We analyzed and derived the horizons and temperatures of slow rotating branes from their induced metrics in the KS, KT and KW solutions, where in the KS case we focused on the very deep IR region.

We found that black hole solutions with horizons and temperatures of expected features form on the world volume of the probe brane in the UV solutions, consisting of the KT and KW solutions. In both cases we found single horizons with finite temperatures. For horizons approaching the UV/IR ends of the throat, we found that the temperature of the rotating probe in the KT solution is more or less constant and determined by the flux and deformation parameter, which set the profile of the throat. In the KW solution we found horizons and temperatures similar to those of rotating probes in $AdS_5 \otimes S^5$.

We did not discuss in this work the dual gauge field theory of our results. For instance, we could consider gauge theory on the probe in the KW theory whose conformal field theory is known in detail, compute the related temperature on the probe and find agreement with the supergravity result for the temperature on the probe discussed here. The gauge theory approach can shed light on the temperature for extended objects which we may consider in a separate work.

Our analysis in this paper can be extended in several ways. One immediate extension would be to study world volume horizons and temperatures of higher dimensional rotating branes which are not point like in the extra dimensions and make comparison with the results obtained in this paper. The other possible extension
would be to study world volume horizons and temperatures of rotating branes in throat subject to moduli stabilization, which induce corrections to the ISD supergravity solution and hence to the action of the probe brane. Finally, we note that the complete understanding of the temperature of extended objects in the throat would require the consideration of the full square root structure of the kinetic terms along the entire throat with their equations of motion solved, perhaps, numerically. We leave these for future study.

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