Magnetic spin–orbit coupling and mass transfer rates in the polars

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Accepted 1999 January 13. Received 1999 January 13; in original form 1998 June 24

ABSTRACT
The magnetic torques resulting from an asynchronous white dwarf in an AM Herculis binary are considered. The magnetic field induced in the secondary leads to an orbital torque which affects the evolution rate of its Roche lobe. An over-synchronous primary transfers angular momentum to the orbit which significantly reduces the rate of shrinkage of the lobe, for a range of asynchronous periods. Although the effect on the total evolution time of the binary is small, the temporarily reduced accretion rate has important consequences for the condition to approach synchronism.

Key words: accretion, accretion discs – magnetic fields – binaries: close – stars: magnetic fields – novae, cataclysmic variables – white dwarfs.

1 INTRODUCTION
The AM Herculis binaries, also known as the polars, are an important class of magnetic cataclysmic variables. The absence of accretion discs in the AM Her systems is unique amongst close binary stars. Related to this is the orbital synchronization of the spin of their strongly magnetic white dwarf primary stars. At least four systems are known to have significant degrees of asynchronism, possibly owing to the disruptive effect of recent nova explosions. However, these systems are observed to be spinning towards synchronism (e.g. Geckeler & Staubert 1997; Schwope et al. 1997).

The major luminosity source in AM Her binaries is the hot, localized accretion column which forms above the white dwarf surface. This results from magnetic field channelling of the supersonic stream originating from the L1 region of the secondary star. Incoming material passes through a standing shock at the top of the column and undergoes compressional heating. Hard X-rays are emitted from the hot post-shock material, most being reprocessed in the stellar surface to a softer component. All radiations emitted from the accretion column have their observed intensities modulated owing to the rotation of the white dwarf. This effect can be used to measure the rotation period to high accuracy.

As in all close binaries, the mechanism for maintaining mass transfer via Roche lobe overflow from the secondary is believed to be orbital angular momentum loss arising from gravitational waves and magnetic wind braking. The latter process generally becomes most effective at orbital periods >3 h. The continuous removal of orbital angular momentum results in a shrinkage of the Roche lobe of the secondary, enabling the L1 region to remain in contact with the shrinking stellar surface resulting from the mass loss. It was pointed out by King (1997) that in the discless AM Her systems magnetic orbital coupling of an asynchronous primary will affect the orbital evolution and hence the mass transfer rate. The magnitude of this effect was estimated by King & Cannizzo (1998). It is shown here that, for plausible system parameters, asynchronism of the magnetic white dwarf can result in significant perturbations to the mass transfer rate. The detailed analysis shows the dependence of the effect on the degree of asynchronism, with an over-synchronous primary causing a reduction in the mass transfer rate. The present paper considers such modifications to the gravitational radiation driven mass transfer rate.

In Section 2 the magnetic field solution resulting from a asynchronous primary is presented. Section 3 applies this solution to calculate the magnetic orbital torque and the torque on the secondary star. In Section 4 the angular momentum evolution equations are formulated, and the modification to the mass transfer rate is calculated for a range of degrees of asynchronism. The results are discussed in Section 5.

2 THE INDUCED MAGNETIC FIELD
The magnetic field resulting from the induction of electric currents in the secondary star by an asynchronous primary was found by Campbell (1983). This field was used to find the torque on the primary star, which spins it towards synchronism. The present paper uses the field solution to find the orbital torque and the torque on the secondary. The relation between the three magnetic torques enables the orbital evolution and consequent modification to the mass transfer rate to be found. Only the necessary details of the induced field solution will be presented here (for more details see Campbell 1983, 1997b).

The primary is taken to have a centred dipole magnetic field with moment \( \mathbf{m}_p \). It is noted that the inclusion of higher multipoles would
only add small components to the torque, since their associated fields fall more rapidly with distance. Fig. 1 shows the orbital frame used, together with the magnetic orientation angles \((\alpha, \beta)\) and the spherical coordinates \((r, \theta, \phi)\) centred on the secondary. The secondary and primary masses are denoted \(M_s\) and \(M_p\), while \(\Omega\) and \(D\) are the orbital angular velocity and separation. The synodic (i.e. relative to the orbit) angular velocity of the primary is \(\omega = \omega_k\), and hence \(\beta = \omega t\). The unit magnetic moment is therefore

\[
\mathbf{m} = \sin \alpha \cos \omega t \mathbf{i} + \sin \alpha \sin \omega t \mathbf{j} + \cos \alpha \mathbf{k},
\]

where the Cartesian unit vectors are those relative to the primary origin, shown in Fig. 1.

The time-varying magnetic field of the primary induces a field in the diffusive secondary obeying the induction equation

\[
\nabla \wedge (\eta \nabla \wedge \mathbf{B}) = - \frac{\partial \mathbf{B}}{\partial t},
\]

where \(\eta\) is the diffusivity. The red dwarf secondary will be essentially fully convective and hence a turbulent origin for \(\eta\) is appropriate. It is noted that the associated magnetic force is small in the main body of the secondary so, for a tidally synchronized star, the velocity term can be ignored in the induction equation. The medium between the stars is taken to be essentially a vacuum. Allowing for a conducting magnetosphere will not change the qualitative nature of the torques, although it may enhance their magnitudes (see Campbell 1997b).

The magnetic field may be expressed in the generalized poloidal form

\[
\mathbf{B} = \nabla \wedge [\nabla \wedge (\Phi \mathbf{r})],
\]

where \(\Phi\) is a poloidal scalar. This scalar can be expanded in a basis of radial functions and spherical harmonics as

\[
\Phi = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm} G_l(r) Y_l^m(\theta, \phi) e^{i\omega t},
\]

where \(a_{lm}\) are constants. It is convenient to express the functions \(G_l(r)\) in the form

\[
G_l(r) = C_l(r) \exp[i\delta_l(r)],
\]

and hence

\[
\Phi = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm} C_l(r) Y_l^m(\theta, \phi) \exp[i(\omega t + \delta_l(r))].
\]

To make the problem mathematically tractable, the diffusion equation is solved in a sphere of radius \(R_s\) equal to the mean radius of the distorted secondary. It is a reasonable approximation to expand \(\Phi\) in terms up to \(l = 2\). The poloidal scalar of the field of the primary is then

\[
\Phi_p = -\frac{\mu_0 m_p \sin \alpha}{8 \pi D^3} r^2 P_1^1(2 \cos \phi \sin \omega t + \sin \phi \cos \omega t) + \frac{\mu_0 m_p \alpha}{8 \pi D^3} r^3 \left[ P_2^0(1 \cos 2 \phi \sin \omega t + \frac{1}{3} \sin 2 \phi \cos \omega t) \right],
\]

where \(\mu_0\) is the magnetic permeability and \(P_j^m(\theta)\) are associated Legendre functions.

The magnetic field exterior to the secondary resulting from its induced current source has an associated scalar

\[
\Phi_s = \frac{1}{r} P_1^1(\cos(\alpha_1 \sin \omega t + \alpha_2 \cos \omega t)) + \frac{1}{r} P_1^1(\sin(\alpha_3 \sin \omega t + \alpha_4 \cos \omega t)) + \frac{1}{r} P_2^0(\beta_1 \sin \omega t + \beta_2 \cos \omega t)
\]

\[+ \frac{1}{r} P_2^2(\cos(\gamma_1 \sin \omega t + \gamma_2 \cos \omega t)) + \frac{1}{r} P_2^2(\sin(\gamma_3 \sin \omega t + \gamma_4 \cos \omega t)),
\]

where \(\alpha, \beta, \gamma\) are associated Legendre functions.
where $\alpha$, $\beta_i$ and $\gamma_i$ are constants. The poloidal scalar in the secondary star then takes the form

$$
\Phi = C_1 P_1^1 \cos \phi (A_1 \sin (\omega t + \delta_1) + A_2 \cos (\omega t + \delta_2)) - \frac{1}{2} C_1 P_1^1 \sin \phi (A_2 \sin (\omega t + \delta_1) - A_1 \cos (\omega t + \delta_1))
$$

$$
+ C_2 P_2^2 (B_1 \sin (\omega t + \delta_2) - B_2 \cos (\omega t + \delta_2)) - \frac{1}{2} C_2 P_2^2 \cos 2 \phi (B_1 \sin (\omega t + \delta_2) + B_2 \cos (\omega t + \delta_2))
$$

$$
+ \frac{1}{3} C_2 P_2^2 \sin 2 \phi (B_2 \sin (\omega t + \delta_2) - B_1 \cos (\omega t + \delta_2)),
$$

where subscripts s denote surface values here, and the $\omega$-dependent coefficients $A_i$ and $B_i$ involve surface values of the functions $C_i(r), \delta_i(r)$ and their first derivatives.

### 3 MAGNETIC TORQUES

#### 3.1 The primary torque

The foregoing field solution was used by Campbell (1983) to find the torque on the primary star and to relate this to the dissipation of currents in the secondary. The torque is given by

$$
T_{mp} = m_p \land B_s(r_p),
$$

where $B_s(r_p)$ is the induced field of the secondary at the position of the primary. The field solution yields the torque, averaged over a synodic period $2\pi/\omega$, as

$$
T_{mp} = -\frac{5 \mu_0 m_p^2 R_1^3 \sin^2 \alpha}{4 \pi D^6} f(\omega/\eta) k,
$$

where

$$
f(\omega/\eta) = \frac{3}{4} (C_1^2 \delta_1) F_1 + \left(\frac{R_s}{D}\right)^2 (C_2^2 \delta_2) F_2,
$$

with $F_1$ and $F_2$ functions of the surface values $C_i, C_i^l$ and $\delta_i$. Fig. 2 shows $f(\omega/\eta)$, illustrating that it has a single maximum.

The dimensionless function $f(\omega/\eta)$ depends purely on the similarity variable

$$
\alpha_s = \left(\frac{2|\alpha|}{\eta}\right)^{1/2} R_s.
$$

It follows that $a_s \sim (\tau_d P_{syn})^{1/2}$, where $\tau_d$ is the magnetic diffusion time through the secondary and $P_{syn} = 2\pi/\omega$. Synodically averaged torques are appropriate for $t_{sn} \gg P_{syn}$, where $t_{sn}$ is the synchronization time-scale given by

$$
t_{sn} = \frac{I|\alpha|}{|T_{mp}|}
$$

with $I$ the moment of inertia of the primary.
3.2 The orbital torque

The present paper calculates the orbital magnetic torque and the torque on the secondary, and relates these to \( T_{mp} \). The consequences for mass transfer can then be investigated.

The spatial dependence of the secondary’s field \( B_s \) results in a net force on the primary star given by

\[
F = \left[ \nabla (m_p B_s) \right]_{r=r_p},
\]

where

\[
B_s = \nabla \left( \frac{\partial \Phi}{\partial r} \right).
\]

It follows from (1) that \( \dot{m}_p \cdot B_s = \sin \alpha \cos \omega t (i \cdot B_s) + \sin \alpha \sin \omega t (j \cdot B_s) + \cos \alpha (k \cdot B_s) \), and the synodic time average of the last term vanishes, since it contains only linear terms in \( \sin \omega t \) and \( \cos \omega t \). Substituting the first two terms in (15), using (8) and (16) and time-averaging, leads to the force components at \( r, \theta, \phi = (D, \pi/2, 0) \) as

\[
F_r = \frac{3m_p}{D^4} \left[ \alpha_1 + \frac{\alpha_2}{2} - \frac{2}{D} (\beta_1 - 6\gamma_1 - 4\gamma_2) \right] \sin \alpha,
\]

\[
F_\theta = \frac{3m_p}{2D^4} \left[ \alpha_2 - \alpha_1 - \frac{1}{D} (\beta_2 - 14\gamma_2 + 16\gamma_3) \right] \sin \alpha,
\]

with \( F_\phi = 0 \). An equal and opposite force is exerted on the centre of mass of the secondary.

The force \( F \) has a non-central component \( F_\phi \) which leads to an orbital torque. Adding the primary and secondary orbital torques gives a total magnetic torque about the binary centre of mass of

\[
T_{mo} = -Dj' \cdot F.
\]

Noting that at the centre of the primary \( \dot{r} = -j' \) and \( \dot{\phi} = i' \), it follows that

\[
T_{mo} = DF_0 k.
\]

The continuity of \( B \) at the surface of the secondary requires continuity of \( \Phi \) and \( \partial \Phi / \partial r \) at \( r = R_s \). Applying these conditions, using (7)–(9), leads to the relations

\[
\alpha_1 = R_s C_4 A_2, \quad \alpha_2 = -\frac{1}{2} \alpha_2, \quad \beta_2 = R_s^2 C_2 B_2, \quad \gamma_2 = -\frac{1}{2} \beta_2, \quad \gamma_3 = \frac{1}{3} \beta_2.
\]

Using these in (18) and (20) gives

\[
T_{mo} = \frac{m_p R_s \sin \alpha}{D^4} \left[ \frac{9}{4} C_4 A_2 - 20 \left( \frac{R_s}{D} \right) C_2 B_2 \right] k.
\]

The forms of \( A_2 \) and \( B_2 \) then lead to

\[
T_{mo} = \frac{m_p R_s^2 \sin^2 \alpha}{\pi D^6} \left[ \frac{27}{16} (C_4^2 \delta_1) F_1 + \frac{25}{8} \left( \frac{R_s}{D} \right)^2 (C_2^2 \delta_2) F_2 \right] k,
\]

where the \( \omega \)-dependent functions in the brackets are the same as those occurring in (12) for \( T_{mp} \), apart from their coefficients.

3.3 The secondary torque

The magnetic torque acting about the centre of mass of the secondary is given by

\[
T_{ms} = \frac{1}{\mu_0} \int_{V_2} r \wedge [(\nabla \wedge B) \wedge B] dV,
\]

where \( V_2 \) is the stellar volume. It follows from (2)–(4) that

\[
\nabla \wedge B = \frac{i \omega}{\eta} \hat{r} \wedge \nabla \Phi.
\]

Substituting this in (23), and expanding the vector products, yields

\[
T_{ms} = \frac{\omega}{\mu_0} \int_{V_2} \frac{ir}{\eta} B_\phi \hat{r} \wedge \nabla \Phi dV.
\]

Equations (3) and (4) give

\[
B_\phi = \frac{1}{r^2} \hat{L} \cdot \hat{\Phi} = -\frac{1}{r^2} \left[ \frac{1}{\sin \theta \partial \theta} \left( \sin \theta \frac{\partial \Phi}{\partial \phi} \right) + \frac{1}{\sin^2 \theta \partial^2 \Phi} \right].
\]

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while

\[
\hat{L} \wedge \nabla \Phi = \frac{1}{r} \frac{\partial \Phi}{\partial \theta} \sin \theta - \frac{1}{r} \frac{\partial \Phi}{\partial \phi} \cos \theta, \tag{27}
\]

with

\[
\hat{\theta} = \cos \theta \sin \phi \hat{i} - \cos \theta \cos \phi \hat{j} - \sin \theta \hat{k},
\]

\[
\hat{\phi} = \cos \phi \hat{i} + \sin \phi \hat{j}.
\]

Using (26)–(28) in (25), and noting that only the \(k\)-component integral will be non-vanishing, leads to

\[
T_{ms} = \frac{\omega}{m_0} \left[ \frac{i}{r} (L^2 \Phi) \frac{\partial \Phi}{\partial \phi} \right] dV. \tag{29}
\]

The torque integral in (29) is evaluated by noting that each harmonic of \(\Phi\) in (4) has the property \(L^2 \Phi_{lm} = l(l+1)\Phi_{lm}\), and that multiplication of \(\Phi\) by \(i\) gives a phase shift of \(\pi/2\) in the exponential. Then, using (9) for \(\Phi\), evaluating the angular integrals noting the orthogonality of \(Y^{l}_{m}(\theta, \phi)\), and time-averaging over \(2\pi/\omega\), yields

\[
T_{ms} = -\frac{4\pi}{m_0} \left[ \frac{1}{3} \left( A_1^2 + A_2^2 \right) \int_{0}^{\infty} \frac{C_i^2}{r} \, dr + \frac{24}{5} \left( B_1^2 + B_2^2 \right) \int_{0}^{\infty} \frac{C_i^2}{r} \, dr \right] k, \tag{30}
\]

where \(A_i, B_i\) are the \(\omega\)-dependent coefficients occurring in (9). The functions \(C_i(r)\) have the property

\[
\int_{0}^{\infty} \frac{C_i^2}{r} \, dr = \frac{1}{\omega} \langle C_i^2 \rangle_k, \tag{31}
\]

(see Campbell 1983) where the subscript \(s\) denotes the stellar surface. Using this and the forms of \(A_i\) and \(B_i\) in (30) gives the secondary magnetic torque

\[
T_{ms} = -\frac{3 \mu_0 m_p^3 R_s^3 \sin^2 \alpha}{4\pi D^4} \left[ \left( C_i^2 \right)_l A_1^2 + \frac{5}{2} \left( \frac{R_s^2}{D} \right)^2 \left( C_i^2 \right)_l A_2^2 \right] k, \tag{32}
\]

where the functions \(A_1\) and \(A_2\) are the same as those occurring in (12) and (22).

Adding the primary, secondary and orbital magnetic torques given by (11), (22) and (32) yields

\[
T_{ms} + T_{mp} + T_{mo} = 0. \tag{33}
\]

This relation gives the magnetic coupling, owing to an asynchronous primary, between the stellar spins and the orbital motion. It states that no angular momentum is lost from the binary as a result of the action of these torques. The spin–orbit coupling enables the primary star to exchange angular momentum and energy with the orbit as it spins towards synchronism.

A similar torque balance to (33) was found by King, Frank & Whitehurst (1990) for the synchronous dipole interaction between intrinsic fields of the primary and secondary. As in the dissipative asynchronous case considered here, the non-central force on the primary arises from the spatial dependence of the field of the secondary in which it lies.

### 4 Mass Transfer Rates

#### 4.1 The angular momentum equations

The angular momentum evolution equations for the secondary, primary and orbit are

\[
\dot{L}_s = T_{ms} + T_{tid}, \tag{34}
\]

\[
\dot{L}_p = T_{mp} + T_{c}, \tag{35}
\]

\[
\dot{L}_{orb} = T_{mo} + T_{gr} - T_{tid} - T_{c}. \tag{36}
\]

These equations are appropriate when the primary is asynchronous and are time averages over a synodic period \(2\pi/\omega\). The accretion torque is then

\[
T_s = A^2 \Omega M_p k, \tag{37}
\]

where \(A\) is the distance of the centre of the primary from the \(L_1\) point, being related to the orbital separation \(D\) by the fitted formula

\[
A = [0.50 - 0.23 \log (M_p/M_f)] D \tag{38}
\]

(Plavec & Kratochvıl 1964). The tidal torque \(T_{tid}\) only requires a small degree of asynchronism of the secondary, and couples its spin to the orbital motion. The gravitational radiation torque is given by

\[
T_{gr} = \frac{32 G}{5c^3} \left( \frac{M_s M_p}{M} \right)^2 \left( \frac{r}{M} \right)^{5} k \tag{39}
\]

(Landau & Lifshitz 1951), where \(c\) is the speed of light. This torque drains angular momentum from the orbit, causing a decrease in \(D\) and hence an increase in \(\Omega\).
In binaries containing accretion discs, the rate of transfer of orbital angular momentum owing to mass transfer from the secondary, via $L_1$, is balanced by a backflow of angular momentum from the disc at the tidal radius. No such orbital feedback occurs in a discless AM Her binary, so there is a net transfer of orbital angular momentum to the primary, and a consequently large accretion torque. This transfer is represented by the accretion torque terms appearing in (35) and (36), with a negative contribution in the latter orbital equation.

The magnetic wind braking torque, believed to be effective at orbital periods $>3\,\text{h}$, is not considered in the present paper. Its form is less certain than that arising from gravitational radiation, partly due to lack of knowledge of the dependence of the secondary’s surface magnetic field on its rotation rate (Campbell 1997a). Magnetic torques resulting from the interaction of the primary with a permanent (i.e. non-induced) magnetic field emanating from the secondary do not appear in (34)–(36). Such fields may be important in the maintenance of synchronism of the primary. However, the asynchronous situation is considered here and the torques resulting from permanent fields have vertical components with vanishing synodic averages (Campbell 1997b).

The magnetic torque $T_{ms}$ appearing in (34) will cause some spin evolution of the secondary away from synchronism. However, the tidal torque will become operable and cancel $T_{ms}$, keeping the star rotating close to the orbital angular velocity $\Omega$, with $L_s = \theta$. Hence (34) becomes

$$T_{ms} + T_{tid} \approx 0,$$

with the vector notation now dropped since all torques are in the $z$-direction. Eliminating $T_{ms}$ between (33) and (40) yields

$$T_{mo} \approx T_{gr} - T_{a} - T_{mp},$$

and is comparable to the orbital evolution time-scale of $L_{orb}/|L_{orb}|$. The simple mass–radius relation

$$R_s \approx R_L,$$

will be adopted here, with $n = 1.1$ (Kippenhahn & Weigert 1990).

Equations (43), (44) and (47) enable the orbital angular momentum to be expressed as

$$L_{orb} = M_s M_p \left(\frac{GD}{M}\right)^{1/2}.$$

A lobe-filling secondary has $R_s = R_L$, and (45) and (47) give

$$\frac{L_{orb}}{L_{orb}} = \left(\frac{4}{3} - \frac{M_s}{M_p}\right) \frac{M_s}{M_p}.$$
Use of (38), (51) and (52) in (37) yields the accretion torque as
\[
\dot{\Omega}_a = \frac{6}{n^{1/2}} \left( \frac{GM}{M_p} \right)^{1/2} \left[ \frac{0.5 - 0.23 \log \left( \frac{M_s}{M_p} \right)}{M_p} \right]^{1/3} M M_s.
\] (53)

Equations (48) and (49) give the orbital angular momentum derivative
\[
L_{orb} = \frac{10^{16}}{3} n^{1/2} \left( \frac{GM}{M_p} \right)^{1/2} \left( \frac{4 - 7 \left( \frac{M_s}{M_p} \right)^{1/3} M M_s}{M_p} \right).
\] (54)

Substituting (51) and (52) for \(Q\) and \(D\) in the gravitational radiation torque expression (39) leads to
\[
T_{gr} = -\frac{0.436}{n^{1/2}} \left( \frac{GM}{M_p} \right)^{1/2} \left( \frac{4 - 7 \left( \frac{M_s}{M_p} \right)^{1/3} M M_s}{M_p} \right)^{1/3} M.
\] (55)

Finally, using (47) and (52) to eliminate \(R_s\) and \(D\) in (11) for the primary magnetic torque, together with \(m_p = 2 \pi (B_0) R_p^2 / \mu_0\), yields
\[
T_{mp} = -\frac{\pi}{20 \mu_0 n} \left( \frac{M_p}{R_p} \right)^3 (B_0)^2 R_p^6 \left( \frac{M_s}{M_p} \right)^2 \frac{M}{M_s} f(\alpha) \sin^2 \alpha.
\] (56)

**Figure 3.** The magnetically reduced mass transfer rate, normalized by the standard gravitational radiation rate, for \(P = 3\ h, M_p = 0.7 M_\odot\) and \((B_0)_p = 45\ MG\).

**Figure 4.** The magnetically reduced mass transfer rate, normalized by the standard gravitational radiation rate, for \(P = 2\ h, M_p = 0.6 M_\odot\) and \((B_0)_p = 32\ MG\).
where the asynchronous dependence is contained in $a_s$. A relation between the mass of a lobe-filling secondary and the orbital period follows from (44), (47) and (50) as

$$\frac{M_s}{M_\odot} = \frac{1}{9n^{1/2}} \left(\frac{P}{\hbar}\right)\left(\frac{Q}{\Omega}\right)^{1/2},$$

(57)

Using this and (47) to eliminate $R_s$ in (13) gives

$$a_s = \frac{4.6 \times 10^{16}}{n^{3/2}Q_s^{1/2}} \left(\frac{P}{\hbar}\right)^{1/2} \left(\frac{|\omega|}{\Omega}\right)^{1/2},$$

(58)

where $|\omega/\Omega|$ is the degree of asynchronous of the primary.

Substituting (53)–(56) in (42), and simplifying, gives the mass loss rate of the secondary as

$$\dot{M}_s = \frac{4.68 \times 10^{-10} \mu^{2/3}(1 - \mu)^2 - 5.73n^{1/2}Qf(a_s)}{n^3 \mu^{3/2}(4 - 7\mu) - 3(1 - 0.45 \log[\mu(1 - \mu)])} M_\odot \text{yr}^{-1},$$

(59)

where $\mu = M_s/M$ and

$$Q = \left(\frac{(B_0)_{\text{p}}}{40 \text{ MG}}\right)^2 \left(\frac{R_p}{9.7 \times 10^6 \text{ m}}\right)^6 \left(\frac{M}{0.79 M_\odot}\right)^{-4}.$$

(60)

Examples of the variation of $\dot{M}_s$ with asynchronism are shown in Figs 3 and 4, normalized by the standard gravitational radiation driven accretion rate

$$\dot{M}_{gr} = -8.0 \times 10^{-11} \left(\frac{1 - \mu^2}{(4 - 7\mu)^{1/2}}\right) M_\odot \text{yr}^{-1}.$$  

(61)

For the parameters considered, which are typical values, the mass transfer rate is significantly reduced for a range of $\omega/\Omega$ around the value at which $f(\omega/\eta)$ has its maximum (cf. Fig. 2). For higher and lower values of $\omega/\Omega$ the ratio $|\dot{M}_s|/|\dot{M}_{gr}|$ exceeds unity. This is caused by the effect of the accretion torque term in (42) which extracts angular momentum from the orbit, owing to the absence of a disc, and hence enhances $|\dot{M}_s|$. It is noted that the stellar surface in the $L_1$ region adjusts on the local dynamical time-scale, $\tau_{\text{dyn}}$, determined by the sound speed. Hence the mass transfer rate can adjust essentially instantaneously to changes in the Roche lobe size on time-scales significantly longer than $\tau_{\text{dyn}}$ (e.g. Lubow & Shu 1975; Edwards & Pringle 1987).

The typical spin evolution time-scale for the primary to move along the asynchronous curves shown in Figs 3 and 4 is $t_{\text{syn}} \sim 10^3$ yr, where $t_{\text{syn}}$ is the synchronization time given by (14). Since the mass transfer time-scale is $\tau_{\text{M}} \sim 10^3$ yr, the effect of the lowered accretion rate on the binary evolution is small. However, its effect on the synchronization process can be significant. This is because a critical value of $|\dot{M}_s|$ exists, denoted $\dot{M}_c$, above which the accretion torque exceeds the synchronizing torque $|T_{mp}|$ for all values of $\omega/\Omega$. At $|\dot{M}_s| = \dot{M}_c$ the curves of $T_{mp}$ and $T_a$ touch at the maximum of $f(\omega/\eta)$. Because $|\dot{M}_c|$ is significantly reduced in this region by the effect considered here, it is less likely to exceed $\dot{M}_c$.

Figs 5 and 6 show the magnitudes of the torques $T_a$ and $T_{mp}$, both normalized with respect to typical values of $10^{27}$ N m. The critical mass transfer rate corresponds to the minimum of $T_a$, exceeding the maximum of $T_{mp}$. The value of $\dot{M}_c$ is raised owing to the spin–orbit coupling effect considered here. $|\dot{M}_s|$ can exceed $\dot{M}_c$ for $M_p > 0.7 M_\odot$ and lower primary star magnetic fields.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{torque_plot}
\caption{The magnetic and accretion torques on the primary, normalized by $10^{27}$ N m, for $P = 3$ h, $M_p = 0.6 M_\odot$ and $(B_0)_{\text{p}} = 30$ MG.}
\end{figure}
It is noted that the condition $|M_s| < M_c$ is necessary for spin-down towards synchronism, but does not ensure the attainment of synchronism. The right-hand intersection of the torque curves shown in Figs 5 and 6 is an unstable spin equilibrium. If the angular velocity of the primary lies beneath this point, its spin will evolve towards the left-hand intersection which represents a stable spin equilibrium and a slightly asynchronous state. As explained by Campbell (1997b), certain conditions are needed to attain synchronism, including a stable, non-dissipative locking mechanism and a suitable magnetic diffusivity in the secondary. It should be emphasized that such conditions are independent of the mass transfer rate lowering effect considered in the present paper.

For an under-synchronous primary the magnetic spin–orbit coupling acts to increase the inertial space angular momentum and energy of the star, and spin it towards synchronism. The transfer of orbital angular momentum will enhance the accretion rate and associated torque which, in this case, aids the synchronisation process.

It is noted that the inclusion of magnetospheric currents flowing between the stars is likely to enhance the strength of the magnetic torques considered here. However, they should not affect the qualitative nature of the torque dependences on the degree of asynchronism. In particular, the induced torque magnitudes should decrease at high and low degrees of asynchronism.

As the primary approaches synchronism, provided that the attainment conditions outlined above are met, the induced magnetic torque reaches zero and hence $M_s$ and the accretion torque become independent of $\omega$. Corotation must be maintained by a non-dissipative torque which cancels $T_a$ in a stable manner. The synodically time-averaged angular momentum equations (34)–(36) do not then apply. Torque balance ensures that $L_s = 0$ and $L_p = 0$, while $L_{orb}$ evolves under the action of $T_{gr}$ and possibly a magnetic wind braking torque for longer period systems.

5 CONCLUSIONS

The work presented here shows that, for asynchronous white dwarfs in discless binary systems, the mass transfer rate can have a significant dependence on the degree of asynchronism. For an over-synchronous primary, this facilitates the approach to synchronism by lowering $|M_s|$ for a range of $\omega/\Omega$ about the value at which the induced magnetic torque has its maximum. In the case of an under-synchronous primary $|M_s|$ is enhanced, but this also aids the approach of the star to corotation. The torque becomes independent of $\omega$ as corotation is attained. Hence different forms are appropriate for the accretion torque in the cases of synchronous and asynchronous primaries.

ACKNOWLEDGMENT

The author thanks the Astrophysics Institute, Potsdam, for financial support and hospitality during the period over which this work was done.

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Figure 6. The magnetic and accretion torques on the primary, normalized by $10^{27}$ N m, for $P = 1.5\ h, M_p = 0.7M_\odot$ and $(B_0)_p = 34$ MG.
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