CHARMLESS TWO-BODY $B$ MESON DECAYS IN FACTORIZATION ASSISTED TOPOLOGICAL AMPLITUDE APPROACH

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We analyze charmless two-body non-leptonic $B$ decays under the framework of factorization assisted topological amplitude approach. Unlike the conventional flavor diagram approach, we consider flavor $SU(3)$ breaking effect assisted by factorization hypothesis for topological diagram amplitudes of different decay modes, by factorizing out the corresponding decay constants and form factors. The non-perturbative parameters of topology diagram magnitudes $\chi$ and strong phase $\phi$ are universal that can be extracted by $\chi^2$ fit from current abundant experimental data of charmless $B$ decays. The number of free parameters and the $\chi^2$ per degree of freedom are both reduced comparing with previous analysis. With these best fitted parameters, we predict branching fractions and $CP$ asymmetry parameters of nearly 100 $B_{u,d}$ and $B_s$ decay modes. The long-standing $\pi\pi$ and $\pi K-CP$ puzzles are solved simultaneously.

1 Introduction

Charmless two-body non-leptonic $B$ decays are of importance for testing the standard model (SM). They can be used to study $CP$ violation via the interference of tree and penguin contributions. They are also sensitive to signals of new physics that would change the small loop effects from penguin diagrams. With regards to them, the BaBar, Belle and LHCb experiments have measured numerous data of branching fractions and $CP$ asymmetries of $B \to PP, PV$ decays, where $P(V)$ denotes a light pseudoscalar (vector) meson. On the theoretical side, it requires complicated study of non-perturbative strong QCD dynamics in the charmless $B$ decays, which not only involve tree topologies but also more complicated penguin loop diagrams.

Based on the leading order power expansion of $\Lambda_{QCD}/m_b$, the QCD factorization (QCDF)$^1$, the perturbative QCD (PQCD)$^2$, and the soft-collinear effective theory (SCET)$^3$ have been developed to study the charmless $B$ decays. However, some puzzles encountered at the leading power of $\Lambda_{QCD}/m_b$ in these factorization approaches, for example, (I) the predicted branching fractions for color-suppressed tree-dominated decays $B^0 \to \pi^0\pi^0, \rho^0\pi^0$ are too small comparing with experimental data, that is the so-called $\pi\pi$ puzzle, (II) some direct $CP$ asymmetries of $B \to PP, PV$ decays are inconsistent with experiment in signs, such as $K\pi$ puzzle. Although some soft and sub-leading power of $\Lambda_{QCD}/m_b$ effects were taken into account in the QCDF$^4$ and the PQCD$^5$, the $B \to \pi\pi$ puzzle was still left in the conventional factorization theorem. Unlike these perturbative approaches, some model-independent approaches were introduced to analyze the charmless $B$ decays, such as global $SU(3)/U(3)$ flavor symmetry analysis$^6$ and flavor topological diagram approach based on flavor $SU(3)$ symmetry$^7$. Nowadays, $SU(3)$ breaking effects have to be considered to compare the theoretical results with the precise experimental data. It is also observed in the flavor topological diagram analysis that they have to fit three different sets of parameters for the three types of $B$ decays respectively$^7$ due to large difference.
between pseudo-scalar and vector final states of $B \to PP$, $B \to PV$ and $B \to VP$ decays. There are too many parameters to be fitted thus its prediction power is limited.

In view of the above complexity and incompleteness in power correction of factorization approaches and the limitation of the conventional flavor topological diagram approach, a new method called factorization-assisted-topological-amplitude (FAT) approach was proposed in studying the two-body hadronic decays of $D$ mesons. Aiming to include all non-factorizable QCD contributions compared to factorization approaches, it adopts the formalism of flavor topological diagram approach. However, different from the conventional flavor topological diagram approach, it had included $SU(3)$ breaking effect in each flavor topological diagram assisted by factorization hypothesis, further reducing the number of free parameters by fitting all the decay channels and the precision of the FAT approach then not limited to the order of flavor $SU(3)$ breaking effect. In the following, we will analyze the charmless $B \to PP, PV$ decays in the FAT approach.

2 The Amplitudes of $B \to PP, PV$ decays in FAT Approach

The charmless two body $B$ decays are induced by the quark level diagrams classified by leading order (tree diagram) and 1-loop level (penguin diagram) weak interactions. For different $B$ decay final states, the tree level weak decay diagram can contribute via different orientations: the so-called color-favored tree emission diagram $T$, color-suppressed tree emission diagram $C$, $W$-exchange tree diagrams $E$ and $W$-annihilation tree diagrams $A$, respectively. Similarly, the 1-loop penguin diagram can also be classified as 5-types: color-favored QCD penguin emission diagram $P$, color-suppressed QCD penguin emission diagram $P_C$, penguin-annihilation diagram $P_A$, the time-like penguin diagram $P_E$ and electro-weak penguin emission diagram $P_{EW}$. The three categories of $B \to PP, PV$ and $VP$ decays parameterized as three sets of parameters in the conventional topological diagram approach, will be parameterized as only one set of universal parameters in the FAT approach.

The $T$ topology is proved factorization to all orders of $\alpha_s$ expansion in QCD factorization approaches and SCET. Their numerical results also agree to each other in different approaches. Thus, to reduce one free parameter, we will just use their theoretical results from QCD calculation, not fitting from the experiments:

$$T^{P_1P_2} = iG_F\sqrt{2}V_{ub}V_{uq}a_1(\mu)f_{P_2}(m_B^2 - m_{P_1}^2)F_0^{B_{P_1}}(m_{P_2}^2),$$
$$T^{PV} = \sqrt{2}G_FV_{ub}V_{uq}a_1(\mu)f_{V}m_VF_1^{B-P}(m_V^2)(\varepsilon_V^* \cdot p_B),$$
$$T^{VP} = \sqrt{2}G_FV_{ub}V_{uq}a_1(\mu)f_{P}m_VA_0^{B-V}(m_V^2)(\varepsilon_V^* \cdot p_B),$$

where the superscript of $T^{P_1P_2}$ denote the final mesons with two pseudoscalar mesons, and $T^{PV(VP)}$ for recoiling mesons are pseudoscalar meson (vector meson) with one pseudo-scalar and one vector meson final states. $a_1(\mu)$ is the effective Wilson coefficient of four quark operators with QCD corrections. $f_{P_2}(f_P)$ and $f_V$ are the decay constants of the emitted pseudoscalar meson and vector meson, respectively. $F_0^{B_{P_1}}(F_1^{B-P})$ and $A_0^{B-V}$ are the form factors of $B \to P$ and $B \to V$ transitions, respectively. $\varepsilon_V^*$ is the polarization vector of vector meson and $p_B$ is the 4-momentum of $B$ meson. For the color suppressed $C$ topology, we parameterize its magnitude and associate phase as $\chi_C$ and $e^{i\phi_C}$ in $B \to PP, VP$ decays and $\chi_C^C e^{i\phi_C^C}$ in $B \to PV$, respectively to distinguish cases in which the emitted meson is pseudo-scalar or vector meson:

$$C^{P_1P_2} = iG_F\sqrt{2}V_{ub}V_{uq}\chi_C e^{i\phi_C}f_{P_2}(m_B^2 - m_{P_1}^2)F_0^{P_{P_1}}(m_{P_2}^2),$$
$$C^{PV} = \sqrt{2}G_FV_{ub}V_{uq}\chi_C e^{i\phi_C}f_{V}m_VF_1^{P-B}(m_V^2)(\varepsilon_V^* \cdot p_B),$$
$$C^{VP} = \sqrt{2}G_FV_{ub}V_{uq}\chi_C e^{i\phi_C}f_{P}m_VA_0^{P-V}(m_V^2)(\varepsilon_V^* \cdot p_B),$$

(2)
where the decay constants and form factors $f_P$, $f_V$, $f_{BP}$, $f_{1B-P}$ and $A_{B-V}$ characterizing the $SU(3)$ breaking effects are factorized out. The $W$-exchange $E$ topology is non-factorizable in QCD factorization approach that is expected smaller than emission diagrams as power suppressed. We use $\chi^E$, $e^{i\phi^E}$ to represent the magnitude and its strong phase for all decay modes:

$$E_{BPBP} = \frac{G_F}{\sqrt{2}} V_{ub} V_{eq}^{\dagger} \chi^E e^{i\phi^E} f_B m_B^2 \left( \frac{f_{BP}}{f^2_\pi} \right),$$

$$E_{PVVP} = \sqrt{2} G_F V_{ub} V_{eq}^{\dagger} \chi^E e^{i\phi^E} (\mu) f_B m_V \left( \frac{f_{PV}}{f^2_\pi} \right) (\mu_B, \mu_B).$$

We will ignore $A$ topology, as its contribution is negligible as discussed in 7.

Similarly, we parameterize the corresponding penguin diagrams with 8 parameters: chiral enhanced penguin amplitude $\chi^P$ and its phase $\phi^P$ excluding the factorizable leading power contribution of the $P$ topology, flavor singlet penguin amplitude $\chi^P_C$, $\chi^P_V$ and their phases $\phi^P_C$, $\phi^P_V$ for the pseudo-scalar and vector meson emission, respectively, the penguin annihilation amplitude $\chi^{PA}$ and its phase $\phi^{PA}$ for the vector meson emission only. The contribution from $P_E$ diagram is argued smaller than $P_A$ diagram, which can be ignored reliably in decay modes not dominated by it. Similar to $T$ and leading power contribution from the $P$ topology, we calculate $P_{EW}$ topology, the largest contribution from EW-penguin contribution, in QCD factorization approaches.

3 Numerical results and discussion

With the experimental data of 37 branching fractions and 11 $CP$ asymmetry parameters\textsuperscript{10}, we do a global fit to extract the 14 parameters. The best-fitted values and the corresponding $1\sigma$ uncertainty are:

$$\chi^C = 0.48 \pm 0.06, \quad \phi^C = -1.58 \pm 0.08, \quad \chi^C' = 0.42 \pm 0.16, \quad \phi^C' = 1.59 \pm 0.17,$$

$$\chi^E = 0.057 \pm 0.005, \quad \phi^E = 2.71 \pm 0.13, \quad \chi^P = 0.10 \pm 0.02, \quad \phi^P = -0.61 \pm 0.02,$$

$$\chi^{PA} = 0.059 \pm 0.008, \quad \phi^{PA} = 1.51 \pm 0.09,$$

\begin{equation}
\end{equation}

with $\chi^2/d.o.f = 45.2/34 = 1.3$. This $\chi^2$ per degree of freedom is smaller than the conventional flavor diagram approach\textsuperscript{7}, even though with much more parameters than us. The mapping of well-known QCDF-amplitudes introduced in 11 and topological diagrams amplitudes in FAT approaches were compared in table 1. It is apparent that there are large differences between results fitted from experimental data in the FAT approaches and the calculated results in the QCDF, especially for the strong phases. Later we will show that the small strong phases, $\phi^C$ and $\phi^{CA}$ from QCDF are the main reason for the $\pi \pi$ and $\pi K$ puzzles.

Using the fitted parameters in eq.(4), we give the numerical results of branching fractions and the direct $CP$ and mixing-induced $CP$ asymmetries of charmless $B_s \to PP, PV$ decays shown in the tables of ref.\textsuperscript{12} Nearly 100 channels are provided to be tested in the future experiments. Similar to the conventional topological diagram approach\textsuperscript{7}, the long-standing puzzle of large
$B^0 \to \pi^0\pi^0$ branching ratio can be resolved well attributed to the appropriate magnitude and phase of $C$ in FAT approach compared with the small magnitude of $C = \alpha_2 = 0.20^{+0.17}_{-0.11}$ by perturbative calculation in QCDF. However, $\|T^{\pi\pi}\| : |C^{\pi\pi}| = 1 : 0.47$ in FAT approach is not as large as the one in ref.\textsuperscript{7}, where the ratio even reached 0.97 in Scheme C. The branching fractions of pure penguin decays $B^- \to K^-K^0$, $B^0 \to K^0\bar{K}^0$ given in the FAT approach are in much better agreement with experimental data than the previous conventional flavor diagram approach\textsuperscript{7}, as we have considered the flavor $SU(3)$ breaking effect. With a large strong phase for sub-leading contribution $C$ in FAT approach, the $K\pi$ puzzle can also resolved. This again implies large power corrections or large non-perturbative QCD corrections in the $C$ diagram of $B \to \pi K$ decays.

The flavor $SU(3)$ breaking effect considered here in every topology amplitude between $B \to \pi\pi$ and $B \to \pi K$ is around 10% and larger than 20% in corresponding $B \to PV$ models. The difference between $\pi$ and $\rho$ meson emission is indeed much larger than the so called flavor $SU(3)$ breaking effect between $\pi$ and $K$ meson due to the meson decay constant $f_\rho > f_K$ and more larger characterized by the $K$ and $K^*$ decay constant.

4 Conclusions

We studied charmless two-body hadronic $B$ decays in factorization assisted topological amplitude approach. By using the factorization results for $T$ and $P_{EW}$ diagrams, there were 6 parameters $\chi^C(\phi^C), \chi^{C'}(\phi^{C'})$ and $\chi^E(\phi^E)$ for tree diagrams $C, E$ and 8 parameters $\chi^P(\phi^P), \chi^{Pc}(\phi^{Pc}), \chi^{P'c}(\phi^{P'c})$ and $\chi^{P4}(\phi^{P4})$ for QCD-penguin diagrams to be fitted from 48 measured data of branching ratios and $CP$ asymmetry parameters of the $B \to PP$, $PV$ decays together. The $\chi^2$ per degree of freedom is smaller than the conventional flavor diagram approach, even with much more free parameters in their approach. With the fitted parameters, we predicted branching fractions of nearly 100 charmless $B_{(s)} \to PP$, $PV$ decay modes and their $CP$ asymmetry parameters. The long-standing puzzles of $\pi\pi$ branching ratios and $\pi K$ $CP$ asymmetry have been resolved consistently with not too large color suppressed tree diagram contribution $\chi^C$. The flavor $SU(3)$ breaking effect between $\pi$ and $K$ were approximately 10%, even more than 20% in $\rho$ and $K^*$ meson case.

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