Two-Terminal Spin Filter Using Quantum Dot with Spin-Orbit Interaction in Magnetic Field

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Abstract. We propose a two-terminal spin filter utilizing a quantum dot with spin-orbit interaction and magnetic field. First we examine a quantum dot with two energy-levels, as a minimal model, and obtain an analytical expression for the spin-dependent conductance. When the spacing between the two levels is smaller than the level broadening due to the tunnel coupling to the leads, a largely spin-polarized current is generated around the current peaks of Coulomb oscillation. Next, we perform a numerical simulation using a realistic model for a quantum dot and tunnel barriers and evaluate the efficiency of our spin filter.

1. Introduction
A spin injection without ferromagnetic materials is an important issue for spin-based electronics, “spintronics.” The spin-orbit (SO) interaction can be a key ingredient for the purpose. For conduction electrons in direct-gap semiconductors, an external potential \( U(r) \) results in the Rashba SO interaction, \( H_{SO} = (\lambda/\hbar)\sigma \cdot (p \times \nabla U) \), where \( p \) is the momentum operator and \( \sigma \) is the Pauli matrices indicating the electron spin \( s = \sigma/2 \). The coupling constant \( \lambda \) is enlarged by the band effect in narrow-gap semiconductors, such as InAs [1].

In our previous paper [2], we investigated the generation of spin-polarized current through a semiconductor quantum dot (QD) with strong SO interaction in the absence of magnetic field. When the QD is connected to two leads (with single mode in each lead), the spin-polarized current is not created. If the QD is connected to more than two leads, it works as a spin filter: When an unpolarized current is injected to the QD from a lead, spin-polarized currents are ejected to the rest of the leads. The spin polarization of the current is enlarged around the current peaks of the Coulomb oscillation when the energy-level spacing in the QD is smaller than the level broadening due to the tunnel coupling to the leads.

In the present paper, we investigate the spin filter of the QD in the presence of magnetic field. In a conventional geometry of two terminal QD [3, 4], the spin filter works in a magnetic field. First, we examine a QD with two energy-levels as a minimal model and obtain an analytical expression for the spin-dependent conductance. It elucidates the condition for the large spin polarization. Next, we perform a numerical calculation using a realistic model for a QD and tunnel barriers on the tight-binding model [5] to evaluate the efficiency of our spin filter.

2. Model and calculation
In the presence of magnetic field, the single-electron Hamiltonian of the QD is written as 
\[
H_{\text{dot}} = \left( \frac{\mathbf{p} - e\mathbf{A}}{2m^*} \right)^2 + U(\mathbf{r}) + H_{SO},
\]
where \( U(\mathbf{r}) \) is the confining potential of the QD.
The Zeeman effect is neglected in a weak magnetic field. We also neglect terms of $A^2$ and $A \times \nabla U$:

$$H_{\text{dot}} = \frac{p^2}{2m^*} + U(r) + \frac{\hbar |e|}{2m^*} B \cdot l + \frac{\lambda}{\hbar} \sigma \cdot (p \times \nabla U). \quad (1)$$

Here, we have used the symmetric gauge $A = (B \times r)/2$ and $\hbar l = r \times p$.

We examine a QD with two levels, connected to two leads, as shown in Fig. 1(a). The energy levels in the QD are $\varepsilon_1$ and $\varepsilon_2$ in the absence of magnetic field and SO interaction. The wavefunctions of the states, $(r|1)$ and $(r|2)$, are real. Since $l$ and $p \times \nabla U$ in eq. (1) are pure imaginary operators, they have off-diagonal elements only; $\hbar |e|/(2m^*) (2|B \cdot l|) \equiv ib/2$ and $(\lambda/\hbar) \sigma \cdot (2p \times \nabla U) \equiv i \sigma_1 \cdot h_{\text{SO}}/2$. If the quantization axis of spin is taken in the direction of $h_{\text{SO}}$, we obtain

$$H_{\text{dot}} = \sum_{\sigma = \pm} (d_{1,\sigma}^\dagger, d_{2,\sigma}^\dagger) \left( \varepsilon_d - \frac{\Delta}{2} \tau_z + \frac{b + \sigma \Delta_{\text{SO}}}{2} \tau_y \right) \left( d_{1,\sigma}^\dagger, d_{2,\sigma}^\dagger \right), \quad (2)$$

where $d_{1,\sigma}^\dagger$ and $d_{j,\sigma}^\dagger$ are the creation and annihilation operators of an electron with orbital $j$ and spin $\sigma$, respectively. $\varepsilon_d = (\varepsilon_1 + \varepsilon_2)/2$, $\Delta = \varepsilon_2 - \varepsilon_1$, and $\Delta_{\text{SO}} = |h_{\text{SO}}|$. The Pauli matrices, $\tau_y$ and $\tau_z$, are introduced for the pseudo-spin representing levels 1 and 2.

The state $|j\rangle$ ($j = 1, 2$) in the QD is connected to lead $\alpha$ (= S, D) by tunnel coupling, $V_{\alpha,j}$, which is real. We assume a single channel of conduction electrons in the leads. The tunnel Hamiltonian is $H_T = \sum_j \sum_{\alpha,k} (V_{\alpha,j} d_{j,\sigma}^\dagger c_{\alpha k,\sigma} + \text{h.c.})$, where $c_{\alpha k,\sigma}$ is the annihilation operator of an electron with state $k$ and spin $\sigma$ in lead $\alpha$. We introduce a unit vector, $\mathbf{e}_\alpha = (V_{\alpha,1}, V_{\alpha,2})^T/V_\alpha$, with $V_\alpha = \sqrt{(V_{\alpha,1})^2 + (V_{\alpha,2})^2}$ for the tunnel coupling to lead $\alpha$. Its strength is characterized by the level broadening, $\Gamma_\alpha = \pi \nu_\alpha (V_\alpha)^2$, where $\nu_\alpha$ is the density of states in lead $\alpha$. The total Hamiltonian is $H = \sum_\alpha \sum_{k,\sigma} \varepsilon_k e_k^\dagger c_{\alpha k,\sigma}^\dagger c_{\alpha k,\sigma} + H_{\text{int}} + H_T$, where $H_{\text{int}}$ describes the Coulomb interaction between electrons in the QD.

We calculate the spin-dependent current to lead D when an unpolarized current is injected into the QD from lead S. In the vicinity of the Coulomb peaks, where the resonant tunneling takes place, the electron-electron interaction in the QD, $H_{\text{int}}$, can be neglected in a good approximation. Then we obtain an analytical expression for the conductance $G_\pm$ for spin $\sigma = \pm$ in the direction of $h_{\text{SO}}$.

### 3. Calculated results

We focus on two extreme cases, (I) $\Delta \gg \Gamma_S, \Gamma_D$ and (II) $\Delta \ll \Gamma_S, \Gamma_D$, when $b, \Delta_{\text{SO}} < \Gamma_S, \Gamma_D$.

In case (I), the level spacing $\Delta$ in the QD is much larger than the level broadening due to the tunnel coupling. In this case, the conductance $G_\pm$ consists of two Lorentzian peaks as a function of $\varepsilon_d = (\varepsilon_1 + \varepsilon_2)/2$, reflecting the resonant tunneling through one of the energy levels. Around

![Figure 1](image-url)
Figure 2. Calculated results using two-level model for a quantum dot. Panels (a) and (c) indicate the conductance $G_{\pm}$ as a function of the energy level $\varepsilon_d = (\varepsilon_1 + \varepsilon_2)/2$, by solid (broken) line. The level broadenings by the tunnel coupling to leads S and D are $\Gamma_S = \Gamma_D \equiv \Gamma ((\varepsilon_S,1/\varepsilon_S,2 = 1/2, \varepsilon_D,1/\varepsilon_D,2 = -3)$. The level spacing is (a) $\Delta = 5\Gamma$ and (c) $0.2\Gamma$. $b = 0.5\Gamma$ and $\Delta_{SO} = 0.2\Gamma$. Panels (b) and (d) show $G_+ - G_-$ when $b/\Gamma = 0.1$ (solid), 0.2 (broken), and 0.5 (dotted line). The other parameters are the same as in panels (a) and (c), respectively.

Figure 3. Calculated results using a realistic model for a quantum dot and tunnel barriers shown in Fig. 1(b). $\lambda = 1.171$ nm$^2$, $W = 50$ nm, $B = 100$ mT, $\lambda_F = W/3$, $L_{QPC} = \lambda_F$, and $L_{QD} = 4\lambda_F$. Panel (a) indicates the conductance $G_{\pm}$ as a function of the gate voltage $V_g$ by solid (broken) line, whereas panel (b) shows $G_+ - G_-$. A peak through level 1, which appears at $\varepsilon_1 \approx \varepsilon_F$ (Fermi energy in the leads), the spin-polarized current ($\propto G_+ - G_-$) is given by

$$G_+ - G_- \approx -\frac{e^2}{4\hbar} \frac{\Gamma_{S,1} \Gamma_{D,1} b \Delta_{SO}}{\Delta} \frac{2(\varepsilon_F - \varepsilon_1)}{((\varepsilon_F - \varepsilon_1)^2 + \Gamma_1^2)^2}, \tag{3}$$

where $\Gamma_{a,1} = \Gamma_a (\varepsilon_{a,1})^2$ ($a = S, D$) and $\Gamma_1 = \Gamma_{S,1} + \Gamma_{D,1}$. $G_+ - G_-$ shows a peak-dip structure as a function of $\varepsilon_d$, around $\varepsilon_1 = \varepsilon_F$. The maximum of $|G_+ - G_-|$ is $(3\sqrt{3}/8)(4\Gamma_{S,1} \Gamma_{D,1}^2 b \Delta_{SO}/\Gamma_1 \Delta)$ in units of $e^2/h$, which is much smaller than unity when $\Delta \gg \Gamma_{S,1}, \Gamma_{D,1} > b, \Delta_{SO}$. Around the peak through level 2, $G_+ - G_- - G_+ - G_-$ shows a dip-peak structure.

In a typical situation, spin-dependent conductance $G_{\pm}$ and their difference $G_+ - G_-$ are shown in Figs. 2(a) and (b), respectively, as functions of $\varepsilon_d$. We set $\Gamma_S = \Gamma_D = \Gamma$. The conductance shows the Coulomb oscillation when $\Delta = 5\Gamma$. Note that we neglect the Coulomb interaction in the QD, $H_{QD}$, and hence underestimate the distance between the peaks. Around the current peak through level 1, the spin-polarized current ($\propto G_+ - G_-$) shows a peak-dip behavior in accordance with eq. (3). The height and depth of $G_+ - G_- - G_+ - G_-$ increase with an increase in magnetic field $b$. Around the current peak through level 2, the enhancement of $|G_+ - G_-|$ is also seen.

In case (II), where the level spacing $\Delta$ is much smaller than the level broadening, two energy-levels contribute to a single conductance peak. The height of the peak reflects the interference between the wavefunctions through two levels. In the vicinity of the peak, where $|\varepsilon_F - \varepsilon_d|^2 \ll \Gamma_S \Gamma_D$, we obtain

$$G_+ - G_- \approx \frac{e^2}{4\hbar} \frac{\Gamma_{S}^2 \Gamma_{D}^2 \Gamma_{S}^3 \Gamma_{D}^3 + (\varepsilon_F - \varepsilon_d)^2 (\Gamma_{S}^2 + \Gamma_{D}^2) \varepsilon}{\Gamma_{S}^2 \Gamma_{D}^2 \varepsilon^2 + (\varepsilon_F - \varepsilon_d)^2 ((\Gamma_{S} + \Gamma_{D})^2 - 2\Gamma_{S} \Gamma_{D} \varepsilon)^2}, \tag{4}$$
with $\epsilon \equiv (e_{11} e_{22} - e_{12} e_{21})^2$. The maximum of $G_+ - G_- = 4b \Delta_{SO}/(\Gamma S \Gamma_D \epsilon)$ in units of $e^2/h$, which can be much larger than in the case of (I).

Figures 2(c) and (d) present $G_+$ and $G_+-G_-$ when $\Delta = 0.2 \Gamma$. The conductance $G_\pm$ shows a single peak, which is induced by the resonant tunneling through two energy-levels, as mentioned above. The peak height strongly depends on the spin direction, which results in a large spin-polarized current ($\propto G_+ - G_-)$. In Fig. 2(d), $G_+ - G_-$ shows a single peak, which increases in height with increasing $b$. When $b = 0.5 \Gamma$, $G_+ - G_- \approx 0.3 e^2/h$ at the peak, which is much larger than that in Fig. 2(b).

4. Numerical simulation
To confirm the generation of a largely spin-polarized current discussed in the previous section, we perform a numerical simulation for a spin-filtering device fabricated on semiconductor heterostructures, shown in Fig. 1(b). The QD is formed by quantum point contacts (QPCs) on a wire of width $W$. The edge of the quantum wire is represented by a hard-wall potential.

In the QD, the electrostatic potential is tuned by the gate voltage $V_g$. The device is described using the tight-binding model, which discretizes the real space in two dimensions. The magnetic field is included in the Peierls phase factors of the transfer integrals. The SO interaction is included in the hopping terms [5]. Using the Green’s function and Landauer-Büttiker formula, we calculate the spin-dependent conductance $G_\pm$ (see Ref. 6 for detail). We assume that $\lambda = 1.171 \text{nm}^2$ for InAs [1] and $W = 50 \text{nm}$. The magnetic field is $B = 100 \text{mT}$. The Fermi wavelength is $\lambda_F = W/3$. $L_{QPC} = \lambda_F$ and $L_{DD} = 4\lambda_F$ in Fig. 1(b).

Figure 3(a) shows the conductance $G_\pm$ as a function of $V_g$. In the gate-voltage range of the figure, we find four conductance peaks, which are 10th to 13th peaks (12th peak is very thin; the peak-peak distance is underestimated by the neglect of $H_{int}$). The level spacing between 10th and 11th is relatively small, which results in an enhancement of $G_+ - G_- \approx \Delta V_g/\epsilon_F = 0.54$ in Fig. 3(b). On the other hand, the 13th level is far from the other levels, which corresponds to case (I) in the previous section. Around $\Delta V_g/\epsilon_F = 0.48$, $G_+ - G_- \approx \Delta V_g/\epsilon_F = 0.54$ in Fig. 3(b). On the other hand, the 13th level is far from the other levels, which corresponds to case (I) in the previous section.

5. Conclusions
We have examined a spin filter using a QD with SO interaction and magnetic field, in a conventional geometry of two terminals. First we have investigated a QD with two levels and obtained an analytical expression for the spin-dependent conductance. A largely spin-polarized current is observed when the level spacing is smaller than the level broadening. Next, the numerical simulation has been performed using a realistic model, which shows an enhanced spin-polarized current in accordance with our analytical result.

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References
[1] Winkler R 2003 Spin-Orbit Coupling Effects in Two-Dimensional Electron and Hole Systems (Springer)
[2] Eto M and Yokoyama T 2010 J. Phys. Soc. Jpn. 79 123711
[3] Takahashi S, Deacon R S, Yoshida K, Oiwa A, Shibata K, Hirakawa K, Tokura Y and Tarucha S 2010 Phys. Rev. Lett. 104 246801
[4] Nadj-Perge S, Frolov S M, Bakkers E P A M and Kouwenhoven L P 2010 Nature 468 1084
[5] Yokoyama T and Eto M 2009 Phys. Rev. B 80 125311
[6] Yokoyama T and Eto M (in preparation)