Self-Similar Solution of Hot Accretion Flows with Ordered Magnetic Field and Outflow

De-Fu Bu\textsuperscript{1,2,3},* Feng Yuan\textsuperscript{1,2,*} and Fu-Guo Xie\textsuperscript{1,2,3,*}

\textsuperscript{1}Shanghai Astronomical Observatory, Shanghai 200030, China; \textsuperscript{2}Joint Institute for Galaxy and Cosmology (JOINGC) of SHAO and USTC; \textsuperscript{3}Graduate School of the Chinese Academy of Sciences, Beijing 100039, China; 

ABSTRACT
Observations and numerical magnetohydrodynamic (MHD) simulations indicate the existence of outflows and ordered large-scale magnetic fields in the inner region of hot accretion flows. In this paper we present the self-similar solutions for advection-dominated accretion flows (ADAFs) with outflows and ordered magnetic fields. Stimulated by numerical simulations, we assume that the magnetic field has a strong toroidal component and a vertical component in addition to a stochastic component. We obtain the self-similar solutions to the equations describing the magnetized ADAFs, taking into account the dynamical effects of the outflow. We compare the results with the canonical ADAFs and find that the dynamical properties of ADAFs such as radial velocity, angular velocity and temperature can be significantly changed in the presence of ordered magnetic fields and outflows. The stronger the magnetic field is, the lower the temperature of the accretion flow will be, and the faster the flow rotates. The relevance to observations is briefly discussed.

Key words: accretion, accretion discs – magnetohydrodynamics: MHD – ISM: jets and outflow – black hole physics

1 INTRODUCTION
Advection-dominated accretion flows (hereafter ADAFs) have been studied extensively (e.g., Narayan & Yi 1994, 1995; Abramowicz et al. 1995; see Narayan, Mahadevan & Quataert 1998 and Kato, Fukue & Mineshige 1998 for reviews). It is now rather well established that this accretion mode exists in the quiescent and hard states of black hole X-ray binaries and low-luminosity active galactic nuclei (see Narayan 2005, Yuan 2007, Ho 2008, and Narayan & McClintock 2008 for recent reviews). ADAFs only exist below a critical mass accretion rate. Below this accretion rate, the rate of radiative cooling is so weak that the viscous heating is balanced by the advection. With the increase of accretion rate, radiation becomes more and more important until it becomes equal to the viscous heating at this critical rate. In this case, the energy advection is equal to zero (Narayan, Mahadevan & Quataert 1998). Above this critical accretion rate, up to another limit close to the Eddington accretion rate—luminous hot accretion flows (LHAFs)—was found which is a natural extension of ADAFs (Yuan 2001, 2003). In this solution, the flow is able to remain hot because the radiative cooling rate, although it is strong, is still lower than the sum of compression work and viscous heating. Note that the cool thin solution—standard thin disk—always exists in the whole range of accretion rates of ADAFs and LHAFs. In the present work we only focus on the hot accretion flows—ADAFs, but our discussion should hold for LHAFs. In the early version of ADAFs, the accretion rate is assumed to be a constant, i.e., there is no outflow throughout the region. The magnetic field is included, but only its stochastic component, while the large-scale ordered field is not considered.

Great developments have been achieved since the discovery of ADAFs. One is the realization of the existence of outflows in ADAFs, i.e., only some fraction of the accretion material available at the outer boundary is actually accreted into the central black hole (Narayan & Yi 1994, 1995; Blandford & Begelman 1999; Stone, Begelman, Pringle 1999; Igumenshchev & Abramowicz 1999; Stone & Pringle 2001). The physical reason for the origin of the outflow is believed to be that the Bernoulli parameter of the flow is positive (Narayan & Yi 1994; Blandford & Begelman 1999). Another possible mechanism of the outflow origin is associated with large-scale ordered magnetic fields, e.g., through the magnetocentrifugal force (Blandford & Payne 1982; Henriksen & Valls-Gabaud 1994; Fiege & Henriksen 1996).
in the center of our Galaxy supply strong evidence for the existence of outflows. From Chandra observations combined with Bondi accretion theory, we can predict the accretion rate at the Bondi radius. Polarization observations at radio wavebands, however, indicate that the accretion rate at the innermost region must be significantly smaller than the Bondi value (Yuan, Quataert & Narayan 2003). Therefore, a large amount of material must be lost into outflows.

Another interesting result of numerical magnetohydrodynamic (MHD) simulations of the hot accretion flow is that a large-scale ordered magnetic field exists in the inner regions of ADAFs. Independent of the initial configuration of the magnetic field (toroidal or poloidal) in the main body of the accretion flow the field is primarily toroidal, with weak radial and vertical components. This large-scale structure is imposed on the stochastic component of the magnetic field on small scales (Machida, Hayashi & Matsumoto 2000; Hirose et al. 2004).

Both outflows and large-scale magnetic fields can affect the dynamics of ADAFs significantly. For example, both of them can effectively transfer angular momentum. These are alternative mechanisms in addition to the turbulence mechanism associated with the magnetorotational instability (MRI) proposed by Balbus & Hawley (1991; 1998), Stone & Norman (1994; see also Mouschovias & Paleologou 1980) investigate the angular momentum transfer by magnetic braking effects associated with a large-scale magnetic field. If the specific internal energy of the outflow is different from that of the inflow where the outflow originates, the outflow acts as an extra cooling or heating term in the accretion flow, as discussed phenomenologically by Blandford & Begelman (1999). Xie & Yuan (2008) parameterize the outflow properties and systematically investigate the effects of the outflow on the dynamics of the inflow, in absence of the large-scale magnetic field.

It is thus necessary to investigate the dynamics of ADAFs with coexistent outflows and large-scale magnetic fields. Several works have been done recently under the self-similar approximation (Akizuki & Fukue 2006; Abbassi, Ghanbari & Najjar 2008; Zhang & Dai 2008) or global solution (Oda et al. 2007). All these works consider the dynamical effects of the outflow by adopting the form \( M \propto r^{4+1/2} \) (e.g., Eq. [21]; Blandford & Begelman 1999) to describe the accretion rate while all other effects such as the probable angular momentum transfer by outflows are neglected. In Akizuki & Fukue (2006), Oda et al. (2007), and Abbassi, Ghanbari & Najjar (2008), only the toroidal component of the large-scale magnetic field is considered; thus, the large-scale magnetic field in their model only supplies an additional force in the radial direction, while it is unable to transfer angular momentum. In Zhang & Dai (2008), although all the three components of the large-scale magnetic field are included explicitly, their solutions unfortunately violate the magnetic divergence-free condition when \( s \neq 0 \).

In this paper, we investigate the self-similar solutions of ADAFs with coexistent outflows and large-scale magnetic fields. We assume the large-scale magnetic field has both \( z \) and \( \phi \) components, hence, the large-scale magnetic field does not only affect the radial force balance of the accretion flow, but also helps to remove angular momentum. Following Xie & Yuan (2008), we take into account the dynamical effects of outflows by considering the differences of specific angular momentum and specific internal energy between inflow and outflow. The paper is organized as follows. In §2 we present the basic MHD equations, which include ordered magnetic fields and outflows. We address in §3 further assumptions, and achieve a set of self-similar equations. Numerical results are presented in §4. We summarize the paper in §5.

2 BASIC EQUATIONS

In cylindrical coordinates \((r, \phi, z)\), we investigate the steady-state, axisymmetric \((\partial/\partial t = \partial/\partial \phi = 0)\) magnetized advection-dominated accretion flows with outflows. The general MHD equations in Gaussian units read (e.g., Lovelace, Romanova, & Newman 1994),

\[
\frac{dp}{dt} + \rho \nabla \cdot \mathbf{v} = 0, \tag{1}
\]

\[
\frac{d\mathbf{v}}{dt} = -\frac{\nabla p}{\rho} - \nabla \psi + \frac{1}{c^2} \left( \frac{\mathbf{J} \times \mathbf{B}}{\rho} + \mathbf{F}_{vis} \right), \tag{2}
\]

\[
\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J}, \tag{3}
\]

\[
\nabla \cdot \mathbf{B} = 0. \tag{4}
\]

The induction equation of the magnetic field reads,

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left( \mathbf{v} \times \mathbf{B} - \frac{4\pi}{c} \eta \mathbf{J} \right). \tag{5}
\]

The final equation is the energy equation,

\[
\rho \left( \frac{de}{dt} - \frac{p}{\rho^2} \frac{dp}{dt} \right) = q_{vis} - q_{rad} \equiv f \ q_{vis}. \tag{6}
\]

Here \( \psi \) is the gravitational potential; \( \mathbf{F}_{vis} \) is the viscous force per unit volume; \( \mathbf{B} \) is the magnetic field; \( \eta \) is the magnetic diffusivity; \( \mathbf{J} \) is the current density; \( \varepsilon \) is the specific internal energy of the accretion flow; \( q_{vis} \) and \( q_{rad} \) are the viscous heating and radiative cooling rates; \( \rho \), \( \mathbf{v} \) and \( p \) have their usual meanings. For simplicity, we use the Newtonian potential and neglect the self-gravity of the accreting gas. The advection factor, \( f \) (\( 0 \leq f \leq 1 \)), describes the fraction of the viscous dissipation energy which is stored in the accretion flow and advected into the central black hole rather than radiated away.

From the numerical MHD simulations (e.g., Machida, Hayashi & Matsumoto 2000; Hirose et al. 2004), it is reasonable to decompose the magnetic field into one large-scale component and one fluctuating or turbulent component. The general picture that emerged from the simulations is that the main body of the accretion flow is governed by a toroidal magnetic field, especially in its inner region, while regions near the poles are primarily governed by a poloidal magnetic field (see Fig. 6 in Hirose et al. 2004), which is mainly in the vertical direction. Thus, we only consider the \( \phi \) and \( z \) components \((B_\phi, B_z)\) of the large-scale magnetic field and neglect the radial component \((B_r)\). We describe the effects of the fluctuating component of the magnetic field in transferring the angular momentum and in dissipating the energy through the usual \( \alpha \) description. Specifically, the viscous heating rate \( q_{vis} \) in Eq. (6) is associated with the turbulent component instead of the large-scale ordered component of the magnetic field. This is because numerical simulations show that the dissipation mainly comes from the thermalization of the magnetic energy via magnetic reconnection.
Self-similar solution of magnetized ADAFs with outflow

(Hirose et al. 2006), which is obviously associated with the turbulent component.

Following Lovelace, Romanova & Newman (1994), we assume $B_z$ is an even function of $z$. Furthermore, we neglect the vertical gradient of $B_z$ in the disc. We take $B_\phi$ to be an odd function of $z$. We therefore have,

\[
\frac{\partial B_z}{\partial z} = 0, \tag{7}
\]

\[
B_\phi = B_\phi|_{z=H}, \quad B_\phi|_{z=-H} = -B_\phi|_{z=-H}, \tag{8}
\]

\[
B_0 = B_\phi|_{z=H}, \tag{9}
\]

where $H$ is the half-height of the accretion disc defined as $H \equiv c_s r / v_K (1 + \beta_1)^{1/2}$, where $c_s$ is the isothermal sound speed, $v_K$ is the Keplerian velocity and $\beta_1$ is defined below in Eq. (22). It will reduce to its traditional form of $H = c_s r / v_K$ in absence of the toroidal component ($B_\phi$) of the ordered magnetic field (Eq. (13)). Under the above assumptions, the divergence-free condition of the magnetic field (Eq. (1)) is automatically satisfied.

The specific angular momentum and specific internal energy of the outflow are in general different from those of the inflow where the outflow originates. A two-dimensional calculation is ideal but complicated and out of the scope of our current work. Instead, we follow the method illustrated in Xie & Yuan (2008) and use several parameters ($\xi_1, \xi_2, \xi_3$; see below for definitions) to characterize the properties of the outflow. In this way, we integrate the above MHD equations in the vertical direction and get the following 1.5-dimensional equations to describe the inflow dynamics (Xie & Yuan 2008).

The equations of the conservation of mass and momentum are,

\[
dM(r) = \eta_1 4\pi r \rho v_{z,w}, \tag{10}
\]

\[
\frac{v_r}{r} \frac{dv_r}{dr} + \frac{1}{2\pi \Sigma} \frac{dM(r)}{dr} (v_{r,w} - v_r) = \frac{v_{\phi}^2}{r} - \frac{GM}{r^2} \frac{1}{\Sigma} \frac{d(\Sigma \beta_1)}{dr} - \frac{1}{4\pi \Sigma} \times
\]

\[\left[ \frac{d}{dr} (HB_\phi^2) + \frac{1}{3} \frac{d}{dr} (HB_\phi^1) + \frac{2}{3} \frac{d}{dr} H \right]. \tag{11}\]

\[
\frac{1}{r} \Sigma v_r \frac{d(rv_r)}{dr} = \frac{1}{2\pi} \frac{dM(r)}{dr} (v_{\phi,w} - v_\phi) = \frac{1}{r^2} \Sigma \frac{d}{dr} \left( \frac{\Sigma \beta_1 c_s^2 v^3}{v_K} \right) \frac{B_0 B_\phi}{2\pi}. \tag{12}\]

The static equilibrium in the vertical direction is achieved between the $z$-component of the gravitational force and the (gas and magnetic) pressure gradient force,

\[
\frac{GM}{r^3} = (1 + \beta_1)c_s^2. \tag{13}\]

The tension term associated with $B_z$ is $B_r dB_z/dr$. It is neglected since we have assumed $B_r = 0$. The energy equation is

\[
v_r \frac{dc_r^2}{dr} \frac{v_{\phi}^2}{\rho} \frac{d\rho}{dr} + \frac{1}{2\pi \Sigma} \frac{dM(r)}{dr} (v_{z,w} - v_z) = \int \frac{\alpha \beta_1 c_s^3}{v_K} \left( \frac{d\Omega}{dr} \right)^2. \tag{14}\]

Here $v_{r,w}$, $v_{\phi,w}$ and $v_{z,w}$ denote the $r$, $\phi$ and $z$ components of the velocity of the outflow when it is just launched from the accretion flow (Xie & Yuan 2008), and $\epsilon_w$ and $\epsilon$ correspond to the specific internal energy of the outflow and that of the inflow, respectively. Following Xie & Yuan (2008), we assume $v_{r,w} = \xi_1 v_r$, $v_{\phi,w} = \xi_2 v_\phi$ and $\epsilon_w = \xi_3 \epsilon$. Parameter $\gamma$ is the ratio of the specific heats, which goes from the relativistic value 4/3 to the typical monatomic gas value 5/3. In our calculations we set $\gamma = 4/3$. We find our results are not sensitive to the value of $\gamma$.

The second terms on the left-hand side of Eqs. (11) & (12) are the momenta (radial and angular) taken away from or deposited into the inflow by the outflow (Xie & Yuan 2008), and the third term on the left-hand side of Eq. (11) is the internal energy taken away from or deposited into the inflow by the outflow. If all these $\xi$s are equal to unity, our model will reduce to those neglecting the discrepancies of specific momenta and internal energy between the inflow and the outflow (e.g., Akizuki & Fukue 2006; Abbasi, Ghanbari & Najjar 2008; Zhang & Dai 2008).

Now we discuss the induction equation (Eq. (5)). This equation describes the growth or escape rate of the magnetic field by dynamo and diffusion. We define the advection rates of the two components of the magnetic flux as,

\[
\Phi_r = \int v_r B_\phi dz, \tag{15}\]

\[
\Phi_\phi = \int rv_r B_\phi dz. \tag{16}\]

For a steady-state accretion system considered here, if the dynamo and the diffusion effects are neglected, the above two quantities will be constant. However, in a realistic accretion disc, the above two quantities can vary with radius due to the presence of the dynamo and the diffusion effects.

3 SELF-SIMILAR SOLUTIONS

We seek self-similar solutions in the following forms (e.g., Narayan & Yi 1995),

\[
v_r = -c_1 \alpha v_K, \tag{17}\]

\[
v_\phi = c_2 v_K, \tag{18}\]

\[c^2_s = c_3 v^2_K. \tag{19}\]

We assume the surface density ($\Sigma \equiv 2\rho H$) to have the form, $\Sigma = \Sigma_0 r^s$, where $\Sigma_0$ and $s$ are two constants, so the accretion rate takes the form,

\[
\dot{M} = 2\pi \alpha c_1 \Sigma_0 \sqrt{GM} r^{s+1/2}. \tag{21}\]

Note that $s = -1/2$ will lead to solutions without outflows.

We assume the magnetic pressure on the surface of the accretion flow to be proportional to the gas pressure in the following way,

\[
\frac{\left( B_\phi |_{z=H} \right)^2}{\rho c^2_s} = \beta_1, \tag{22}\]

\[
\frac{\left( B_r |_{z=H} \right)^2}{\rho c^2_s} = \beta_2. \tag{23}\]
\( \beta_1 \) and \( \beta_2 \) are two parameters. The induction equations will be satisfied if the dynamo and the diffusion of the magnetic field satisfy the self-similar conditions required by the advection rate (Eqs. (15) & (16)). Typical values of \( \beta_1 \) and \( \beta_2 \) lie in the range 0.01 – 1 in hot accretion flows (e.g., De Villiers, Hawley & Krolik 2003; Beckwith et al. 2008), but here we also consider the magnetically-dominated case \((\beta_1, \beta_2 > 1)\). On one hand, the MHD numerical simulation by Machida, Nakamura & Matsumoto (2006) shows that when thermal instability happens in an ADAF, the thermal pressure rapidly decreases while the magnetic pressure increases due to the conservation of magnetic flux. This will result in large \( \beta_1 \) and \( \beta_2 \) and forms a magnetically-dominated accretion flow. In addition, an initially weak large-scale field at the outer region can be amplified due to the compression of the flow as it is accreted inward. This can also result in large \( \beta_1 \) and \( \beta_2 \). Of course, in this case, the MRI will be suppressed, but we still keep \( \alpha \) in equations for simplicity and this treatment will not affect our results significantly for our purpose.

The momentum conservation and the vertical equilibrium equations (Eqs. (11 – 13)) now reduce to,

\[
\frac{1}{2} c_1^2 \alpha^2 - (s + \frac{1}{2})(\xi_1 - 1)c_2^2 \alpha^2 = c_2^2 - 1 - (s - 1)c_2 - \beta_2 c_1(s - 1) - \frac{1}{3} \beta_1 c_3(s - 1) - \frac{2}{3} \beta_1 c_3, \tag{24}
\]

\[
\alpha c_1 c_2 - 2\alpha(s + \frac{1}{2})(\xi_2 - 1)c_1 c_2 = 3\alpha c_2 c_3(1 + s) + 4\sqrt{c_3} \frac{\beta_1 \beta_2}{1 + \beta_1}, \tag{25}
\]

\[
H/r = \sqrt{(1 + \beta_1)c_1}. \tag{26}
\]

The energy equation becomes,

\[
c_1 \left[ \frac{1}{\gamma - 1} + (s + 1) \frac{(s + \frac{4}{3})(\xi_1 - 1)}{\gamma - 1} \right] = \frac{9}{4} f c_2^2. \tag{27}
\]

Obviously for given values of \( \alpha, f, s, \beta_1 \) and \( \beta_2 \), Eqs. (24), (25) and (27) form a closed set of equations of \( c_1, c_2 \) and \( c_3 \), which will determine the dynamics of the accretion flow.

4 RESULTS

The first and the second terms on the right-hand side of Eq. (25) are angular momentum transfer due to turbulent-viscosity (MRI) and the large-scale magnetic field, respectively. We define a new parameter \( c_4 \) to account for the ratio of angular momentum transport due to these two mechanisms,

\[
c_4 = \frac{3\alpha(1 + s)\sqrt{1 + \beta_1 c_2} \sqrt{c_3}}{4\sqrt{\beta_1 \beta_2}}. \tag{28}
\]

The large-scale magnetic field will be the dominant mechanism in angular momentum transport when \( c_4 \) is below unity.

We first show in Fig. 1 the effects of the large-scale magnetic field on the accretion flow. We set \( \alpha = 0.1, f = 1.0 \) and \( s = -0.5 \) (no outflows). The solid, dotted and long-dashed lines correspond to \( \beta_2 = 0.01, 1.0 \) and 10, respectively. We can see from the figure that as \( B_\phi (\beta_1) \) increases, the radial \((c_1)\) and angular \((c_2)\) velocities increase, while the temperature \((c_3)\) and the turbulent-viscosity contribution to angular momentum transport \((c_4)\) decrease. The decrease of \( c_4 \) is obvious and easy to understand, since the large-scale magnetic field should be more important in transporting angular momentum as its strength increases. We find that the magnetic pressure gradient force (the fourth and fifth terms on the right-hand side of Eq. (22)) is a centrifugal force, while the magnetic stress force (the sixth term on the right-hand side of Eq. (22)) is a centripetal force. For the wide range of \( \beta_1 \) and \( \beta_2 \) considered here, the gradient of the total pressure (gas plus magnetic pressure) serves as a centrifugal force, and it exceeds the magnetic centripetal force. All these terms are proportional to the sound speed (or, equivalently, the temperature \( c_3 \)). The solutions to Eqs. (24)–(27), as shown in Fig. 1, indicate that an increase in \( B_\phi (\beta_1) \) results in a decrease in temperature \((c_3)\). This in turn leads to a decrease of the effective centrifugal force, and subsequently an increase of the infalling velocity \( v_r \), as shown in the upper-left panel of Fig. 1. Combined with the energy equation (Eq. (27)), the angular velocity will increase correspondingly.

The change of the \( z \) component of the ordered magnetic field has similar effects. It is evident from Figure 1 that an increase in the \( z \) component of the magnetic field \( B_x (\beta_2) \) will lead to a decrease in temperature \((c_3)\). Due to the decrease of temperature, the effective centrifugal force decreases, so the infalling velocity increases. From the energy equation (Eq. (27)), the angular velocity will increase correspondingly. We thus come to a general conclusion that, the stronger the magnetic field is, the faster the accretion flow rotates and falls into the central object, and the lower the temperature will be (see Figs. 2 & 3 for situations with outflows).

The decrease of temperature of ADAFs after taking into account the large-scale magnetic field may have an important observational implication. The X-ray spectra of the hard state of black hole X-ray binaries and some AGNs (such as type 1 Seyfert galaxies) are well described by a power-law form with a high-energy cutoff. This kind of spectrum is widely accepted as originating from the thermal Comptonization process in a hot plasma which is likely to be the region of a hot accretion flow where most of the radiation originates (see review by Zdziarski & Gierliński 2004). The value of the cutoff energy is determined by the temperature of the accretion flow, while the slope of the power-law distribution is determined by the product of temperature and optical depth. From fitting the observed X-ray spectrum, we can well constrain these two parameters, namely the optical depth and temperature. On the other hand, we can also calculate their values from the hot accretion flow model. Yuan & Zdziarski (2004) find that while the theoretical prediction is roughly consistent with observational results, for some sources the temperature obtained from observation, \( \sim 40 – 60 \text{ keV} \), is significantly lower than that the lowest value predicted by the hot accretion flow models, \( \gtrsim 80 \text{ keV} \) (see Fig. 1 in Yuan & Zdziarski 2004). Our current work indicates that this discrepancy may be solved by including the large-scale magnetic field (Fig. 1) and the outflow (Figs. 2 & 3). Unfortunately, it is difficult to give a quantitative estimation of how strong the magnetic field is required to be.
Figure 1. The dynamics of the accretion flow in absence of outflows. Parameters are set as $s = -0.5, \alpha = 0.1$ and $f = 1$. The solid, dotted and long-dashed lines correspond to $\beta_2 = 0.01, 1$ and $10$, respectively. The bottom-right panel shows the ratio of the angular momentum transport due to turbulence (MRI), to that due to the large-scale magnetic field (see Eq. (28) for definition).

to solve the discrepancy. This is because our current work is based on the self-similar assumption which does not hold in the innermost transonic region of the accretion flow where most of the radiation originates.

Before examining in detail the effects of the outflow on the dynamics of the inflow, we first check the effects of the discrepancy of the radial velocity between the outflow and the inflow (indicated by $\xi_1$). Consistent with Xie & Yuan (2008), we find that for relatively weak outflows (e.g., $s < 0$) and reasonable values of $\xi_1$, the effects are of minor importance. Thus for the discussions below, we set $\xi_1 = 1$.

We present in Fig. 2 the effects of angular momentum discrepancy between the inflow and the outflow. We set $\alpha = 0.1, f = 1, \xi_1 = \xi_3 = 1$ and $\beta_2 = 0.01$. Parameter $s = 0.5$ means $\dot{M} (r) \propto r$, which corresponds to a relatively strong outflow (Blandford & Begelman 1999). The dotted, solid and long-dashed lines correspond to $\xi_2 = 0.8, 1$ and $1.2$, respectively. Parameter $\xi_2 > 1$ means that the outflow helps to remove angular momentum from the inflow.

The accretion flow has distinctive properties between $\beta_1 < 1$ and $\beta_1 > 1$. When $\beta_1$ is below unity, the effects of the large-scale magnetic field can be neglected. As $\xi_2$ increases from 0.8 to 1.2, we can see from Fig. 2 that the radial and angular velocities increase while the temperature decreases. We can understand these results in the following way. From Eq. (24), we see that $c_1$ and $c_2$ will increase or decrease together (note the magnetic terms are negligible). If both of them would decrease, from Eq. (24) we know that $c_3$ must increase; while this is in conflict with what Eq. (25) implies if $\xi_2$ increases. Thus both $c_1$ and $c_2$, i.e., the radial and angular velocities, must increase when $\xi_2$ increases, and the temperature decreases. Physically, the increase of the radial velocity is explained by the decrease in the gas pressure gradient force with the decreasing temperature. We note that these self-similar results are consistent with the global solu-
Figure 2. The dynamics of the accretion flow when the outflow has different angular velocity from that of the inflow. Parameters are $\beta_2 = 0.01, \alpha = 0.1, f = 1, s = 0.5$ and $\xi_1 = \xi_3 = 1.0$. The accretion rate $\dot{M}$ is then proportional to $r$. The dotted, solid and long-dashed lines correspond to $\xi_2 = 0.8, 1$ and $1.2$, respectively. The bottom-right panel shows the ratio ($c_4$; Eq. (28)) of the angular momentum transport due to MRI, to that due to the ordered magnetic field.

From the upper-right panel of Fig. 2, we see that the accretion flow will be super-Keplerian when the toroidal magnetic field is very strong (i.e., $\beta_1 > 1$). This is because the strong magnetic stress force serves as a centripetal force.

Figure 3 shows the case when the specific internal energy of the outflow is different from that of the inflow. We take parameters $\alpha = 0.1, f = 1, \beta_2 = 0.01, s = 0.5$ and $\xi_1 = 1$, as before. Unlike the case shown in Fig. 2, here we set $\xi_2 = 1$. The dotted, solid and long-dashed lines correspond to $\xi_3 = 0.8, 1$ and $1.2$, respectively. The outflow then plays an extra cooling role for the inflow if its specific internal energy is higher than that of the inflow ($\xi_3 > 1$), or a heating role otherwise. We can see from the figure that for the case of a weak magnetic field ($\beta_1 < 1$), when $\xi_3$ increases from 0.8 to 1.2, the temperature ($c_3$) and radial velocity of the inflow decrease while the angular velocity increases. The reason is as follows. From Eq. (24) we know $c_1$ and $c_3$ must increase or decrease together. If both would increase with increasing $\xi_3$, from Eq. (27) we know that $c_2$ must decrease, which is in conflict with Eq. (27). The results of a strong magnetic field ($\beta_1 > 1$) can be understood in a similar way.
flow is super-Keplerian. This is because, for the cases of strong outflows \((s = 0.5; M \propto r)\) considered in Figs. 2 & 3, the “centrifugal force” (gradient of the gas plus magnetic pressure) is much weaker than in the case without outflows (Fig. 1); while the “centripetal force” (magnetic stress force) does not vary too much.

Finally, we note that in the limits of \(f, \beta_1, \beta_2 \to 0, s \to -0.5\) and \(\xi_1, \xi_2, \xi_3 \to 1\), i.e., the accretion flow is radiatively efficient, and is devoid of outflows and magnetic fields, our equations will lead to the solution with \(v_r \to 0, c_s \to 0\) and \(v_\phi \to v_K\), which corresponds to a standard thin disc, as expected.

### 5 SUMMARY

Numerical MHD simulations show that both large-scale ordered magnetic fields and outflows exist in the inner regions of an ADAF. In this paper, we have investigated their influences on the dynamics of the accretion flow in a self-similar approach. We assume that in addition to the stochastic component, the magnetic field has a strong large-scale ordered component. The magnetic pressure and stress forces associated with the large-scale field thus will affect the dynamics of the accretion flow. We adopt the conventional \(\alpha\) description to mimic the angular momentum and heating effects associated with the tangled magnetic field. We find that when the large-scale magnetic field gets stronger, the radial and angular velocities of the accretion flow increase, while the temperature decreases. The large-scale field will be more important than the turbulent-viscosity in transferring angular...
momentum even when the field is moderately strong (Fig. 1).

We have obtained the self-similar solutions with both large-scale magnetic fields and outflows. The outflow does not only change the radial profile of the mass accretion rate, but is also able to take away angular momentum and energy from the inflow. We parameterize the physical properties of the outflow, namely its radial velocity, specific angular momentum and specific internal energy. We find that when the specific angular momentum of the outflow increases, the temperature of the inflow decreases, while the radial and angular velocities increase (decrease) when the magnetic field is weak (strong) (Fig. 2). When the specific internal energy of the outflow increases, the temperature and radial velocity of the inflow decrease, while the angular velocity increases (decreases) when the large-scale magnetic field is weak (strong) (Fig. 3). When both outflows and large-scale magnetic fields are present, we find that the inflow could be super-Keplerian if the field is very strong.

The decrease in temperature of the inflow in presence of large-scale magnetic fields and/or outflows could explain a puzzle that the predicted temperature of hot accretion flows is higher than that obtained from fitting the observational data.

6 ACKNOWLEDGMENTS

We are grateful for a discussion with Ramesh Narayan. We thank the anonymous referee for his/her helpful suggestions and careful reading, which improve the manuscript. This work was supported in part by the Natural Science Foundation of China (grants 10773024, 10833002, 10821302, and 10825314), One-Hundred-Talents Program of Chinese Academy of Sciences, and the National Basic Research Program of China (2009CB824800).

REFERENCES

Abbassi, S., Ghanbari, J., & Najjar, S. 2008, MNRAS, 388, 663
Abramowicz, M. A., Chen, X., Kato, S., Lasota J.-P., & Regev, O. 1995, ApJ, 438, L37
Akizuki, C., & Fukue, J. 2006, PASJ, 58, 469
Blandford, R. D., & Begelman, M. C. 1999, MNRAS, 303, L1
Blandford, R. D., & Payne, D. G. 1982, MNRAS, 199, 883
Balbus, S., & Hawley, J. F. 1991, ApJ, 376, 214
Balbus, S., & Hawley, J. F. 1998, RvMP, 70, 1
Beckwith, K., Hawley, J. F., & Krolik, J. H. 2008, ApJ, 678, 1180
De Villiers, J.-P., Hawley, J. F., & Krolik, J. H. 2003, ApJ, 599, 1238
Fiege, J. D., & Henriksen, R. N. 1996, MNRAS, 281, 1005
Hirose, S., Krolik, J. H., De Villiers, J. P., & Hawley, J. F. 2004, ApJ, 606, 1083
Hirose, S., Krolik, J., & Stone, J. 2003, ApJ, 640, 901
Henriksen, R. N., & Valls-Gabaud, D. 1994, MNRAS, 266, 681
Ho, L. 2008, ARA&A, in press (arXiv:0803.2268)
Igumenshchev, I. V. & Abramowicz, M. A. 1999, MNRAS, 303, 309
Kato, S., FuKue, J., & Mineshige, S., 1998, Black Hole Accretion Disks, Kyoto Univ. Press, Kyoto
Lovelace, R. V. E., Romanova, M. M., & Newman, W. I. 1994, ApJ, 437, 136L
Machida, M., Hayashi, M. R., & Matsumoto, R. 2000, ApJ, 532, L67
Machida, M., Nakamura, K. E., & Matsumoto, R. 2006, PASJ, 58, 193
Mouschovias, T. Ch., & Paleologou, E. V. 1980, ApJ, 237, 877
Narayan, R. 2005, ApSS, 300, 177
Narayan, R., Mahadevan, R., & Quataert, E. 1998, in “Theory of Black Hole Accretion Disks”, eds. M.A. Abramowicz, G. Bjornsson, J. E. Pringle, Cambridge Univ. Press, Cambridge, p. 148
Narayan, R., & McClintock, J. E. 2008, in “Jean-Pierre Lasota, X-ray binaries, accretion disks and compact stars” New Astronomy Reviews, eds. M.A. Abramowicz and O. Straub (Elsevier, 2008). arXiv:0803.0322
Narayan, R., & Yi, I. 1994, ApJ, 428, L13
Narayan, R., & Yi, I. 1995, ApJ, 452, 710
Oda, H., Machida, M., Nakamura, K. E., & Matsumoto, R. 2007, PASJ, 59, 457
Stone, J. M., & Norman, M. L. 1994, ApJ, 433, 746
Stone, J. M., & Pringle, J. E. 2001, MNRAS, 322, 461
Stone, J. M., Pringle, J. E., & Begelman, M. C. 1999, MNRAS, 310, 1002
Xie, F. G., & Yuan, F. 2008, ApJ, 681, 499
Yuan, F. 2001, MNRAS, 324, 119
Yuan, F. 2003, ApJ, 594, L99
Yuan, F. 2007, in “The Central Engine of Active Galactic Nuclei”, ASP Conference Series, Vol. 373, eds. Luis C. Ho and Jian-Min Wang, p.95
Yuan, F., Quataert, E., & Narayan, R. 2003, ApJ, 598, 301
Yuan, F., & Zdziarski, A. A. 2004, MNRAS, 354, 953
Zdziarski, A. A., & Gierliński, M. 2004, PThPS, 155, 99
Zhang, D., & Dai, Z. G. 2008, MNRAS, 388, 1409