Ratchet effect in dc SQUIDs

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We analyzed voltage rectification for dc SQUIDs biased with ac current with zero mean value. We demonstrate that the reflection symmetry in the 2-dimensional SQUID potential is broken by an applied flux and with appropriate asymmetries in the dc SQUID. Depending on the type of asymmetry, we obtain a rocking or a simultaneously rocking and flashing ratchet, the latter showing multiple sign reversals in the mean voltage with increasing amplitude of the ac current. Our experimental results are in agreement with numerical solutions of the Langevin equations for the asymmetric dc SQUID.

Directed molecular motion in the absence of a directed net driving force or a temperature gradient in biological systems has drawn much attention to the so-called Brownian motors \textsuperscript{1,2} where nonequilibrium fluctuations can induce a net drift of particles along periodic structures which lack reflection symmetry. Such structures are denoted as ratchets.

In general, a ratchet system is characterized by the equation of motion of an overdamped particle in a periodic potential \( w(x,t) \)

\[
\frac{dx}{dt} = -\frac{\partial}{\partial x}[w(x,t) - F_d(t)x] + F_u(t) \equiv -\frac{\partial}{\partial x}u(x,t) + F_u(t) \tag{1}
\]

with \( w(x,t) \neq w(-x,t) \) for all choices of origin. Here, \( \xi \) is a friction coefficient, \( x \) a cyclic coordinate, \( F_d(t) \) a driving force and \( F_u(t) \) represents a force with zero mean value due to thermal noise with Gaussian distribution. One can classify different types of ratchet systems depending on the actual form of \( w(x,t) \) and \( F_d(t) \) (see e.g. \textsuperscript{3}). The \textit{rocked thermal ratchet} \textsuperscript{4} is obtained if one chooses \( w(x,t) \) to be time independent and \( F_d(t) \) to be either stochastic or deterministic. For the \textit{flashing ratchet} a time dependent potential \( w(x,t) \) is chosen which, in the simplest case, is of the form \( w(x,t) = w_0(x)w_d(t) \), where \( w_d(t) \) again can be stochastic or deterministic. The common feature of all ratchet systems is their rectifying property: it is possible to extract directed motion from nonequilibrium fluctuations or periodic excitations with vanishing time average. Although considerable theoretical work exists in particular for stochastically driven ratchets, only few experiments have been realized e.g. by means of dielectric \textsuperscript{5} and optical potentials \textsuperscript{6}.

The periodicity of the Josephson coupling energy \( E_J \) with respect to the phase difference \( \delta \) of the macroscopic wave function across a Josephson junction is an ideal prerequisite to build a ratchet system. Within the resistively and capacitively shunted junction model the equation of motion of \( \delta \) is equivalent to the motion of a particle in the so-called tilted washboard potential \( U(\delta,t) = -\frac{\Phi_0}{2\pi I_0} \{ I_0 \cos \delta + I(t)\delta \} \) (see e.g. \textsuperscript{7}). Here, \( \Phi_0 \) is the flux quantum and \( I_0 \) is the maximum Josephson current across the junction. An external force causing the tilt is obtained by a bias current \( I \), and the mass and friction coefficient are determined by the junction capacitance \( C \) and the junction resistance \( R \), respectively. In the strongly overdamped limit \( \beta_C \equiv \frac{2\pi}{\Phi_0} I_0 R^2 C \ll 1 \) \( C \) can be neglected. The voltage \( V = \frac{d\Phi_0}{dt} \frac{I_0}{2\pi} \) across the junction represents the velocity of the particle via the second Josephson equation. For a short Josephson junction with a current phase relation which is an odd function, \( U(\delta) \) is always an even function and no ratchet potential is obtained. However, voltage rectification may be induced by asymmetric fluctuations as proposed in \textsuperscript{8}. To obtain a ratchet potential for Josephson junction systems, two ratchet configurations based on \textit{coupled} Josephson junctions have been proposed. Zapata et al. \textsuperscript{9} considered an asymmetric three junction dc SQUID, with vanishing loop inductance \( L \) thereby coupling the phase differences across the junctions rigidly via the applied flux \( \Phi_a \). Due to the doubled phase shift in the SQUID arm with the two junctions in series the reflection symmetry in the effective 1-dimensional potential is broken. Falo et al. \textsuperscript{10} proposed a 1-dimensional Josephson junction array

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with spatially alternating critical junction currents and alternating loop inductances. They showed that a Josephson fluxon experiences a ratchet potential as it moves along the array. This effect has been confirmed experimentally very recently [12].

In this letter we show that the well known voltage rectification in asymmetric dc SQUIDs [13] is a consequence of broken symmetry in the two dimensional SQUID potential due to these asymmetries and the application of magnetic flux. In contrast to the model proposed by Zapata et al. [10] this results in a quasi two dimensional ratchet potential. We demonstrate its experimental realization by using a two junction high transition temperature (Tc) dc SQUID with small but finite L. In our experiments we use an asymmetry αf in the critical currents \( I_0 = (1 - \alpha_f)I_0 \) and \( I_0^a = (1 + \alpha_f)I_0^a \) of the two junctions and/or an asymmetry αL which introduces a self-field effect, i.e. a flux component \( \Phi_{self} \equiv \alpha_L \Phi_0 I/L \) with \( L \equiv I_0^a + I_0 \). A finite value of \( \alpha_f \) can be realized for example with different inductances in the left and the right arm of the SQUID loop; our definition however is more general and accounts for all kinds of self field generation.

The extension of the tilted washboard potential model in two dimensions has been used to provide insight into the dynamics of both, symmetric dc SQUIDs [14] and dc SQUIDs with an \( I_0 \) asymmetry [15]. The SQUID potential \( U_s \)

\[
U_s(\delta_1, \delta_2, t) = -\frac{\Phi_0}{2\pi} \{ I_0^f \cos \delta_1 + I_0^f \cos \delta_2 - \frac{I}{4\pi \delta L} [\delta_2 - \delta_1 - 2\pi (\frac{\Phi_0}{\delta L} + \alpha_L \frac{I}{L})] \}^2 + \frac{I(t)}{2} (\delta_1 + \delta_2)
\]

contains three basic terms, (i) the two cosine terms representing \( E_J \), (ii) the quadratic magnetic energy term and (iii) the external force term proportional to \( I \). Here, \( \delta_1, \delta_2 \) are the phase differences across the two Josephson junctions and \( \delta_L \equiv I/L/\Phi_0 \) is the normalized inductance. An asymmetry \( \alpha_L \neq 0 \) introduces a difference in \( E_J \) for the two junctions thus deforming the cosine part of the potential landscape, while \( \alpha_L \neq 0 \) shifts the minimum of the parabolic magnetic energy term proportional to \( I \). Transforming into more appropriate phase coordinates along the direction of the bias current \( \delta \equiv (\delta_1 + \delta_2)/2 \) and the applied flux \( \varphi \equiv (\delta_2 - \delta_1)/2 \) and using \( i \equiv I/I_L \) and \( f_a \equiv \frac{\Phi_0}{\Phi_0} \) yields in analogy to eq. (1)

\[
w_s(\delta, \varphi, t) = w_s(\delta, \varphi, t) - i(t) \delta
\]

In this coordinates, the voltage \( V \) across the SQUID is determined by \( \frac{d\varphi}{dt} \). A ratchet effect, i.e. voltage rectification for an external excitation with zero mean value, will occur if the system is not invariant under parity transformations with respect to \( \delta \), that is for

\[
w_s(\delta_0 + \delta, \varphi_0 + \varphi) \neq w_s(\delta_0 - \delta, \varphi_0 + \varphi) \quad \text{and} \\
w_s(\delta_0 + \delta, \varphi_0 + \varphi) \neq w_s(\delta_0 - \delta, \varphi_0 - \varphi).
\]

for arbitrary origin \( (\delta_0, \varphi_0) \). Eq. (3) satisfies these conditions, i.e. \( w_s \) is a ratchet potential in \( \delta \), if \( f_a \neq \frac{\Phi_0}{\Phi_0} \wedge (\alpha_L \neq 0 \vee \alpha_L \neq 0); (n = 0, 1, 2, ...). For given \( \alpha_f, \alpha_L \), the applied flux \( f_a \) is an experimentally controllable parameter, which allows to change the parity of \( w_s \). The type of asymmetry of the dc SQUID driven by a fluctuating or periodic bias current \( i(t) \) determines the nature of our ratchet system. For \( \alpha_f \neq 0, \alpha_L = 0 \), \( w_s \) is time independent, thus we have a rocking ratchet system. For \( \alpha_f \neq 0, w_s \) becomes time dependent since the minimum of the magnetic energy term shifts proportional to \( i(t) \). In that case we have a simultaneously flashing and rocking ratchet.

Figure 1 shows contour plots of the 2-dimensional SQUID potential \( u_s \) for \( \alpha_L = 0 \) and different values of \( \alpha_f \) and bias current \( i \). For \( \alpha_I = 0 \) (upper row), the potential has reflection symmetry with respect to \( \delta \), and with finite \( i \) we have \( u_s(\delta, \varphi; i) = u_s(-\delta, \varphi; -i) \) and thus \( V(i) = -V(-i) \). For \( \alpha_I = 0.3 \) (lower row) no reflection symmetry is present and in fact, for \( i = +0.9 \) no local minima are found i.e. the solution for the phase is unbounded, for \( i = -0.9 \) local minima are present and no voltage appears.

The signature of a ratchet effect can be observed in the \( V(\Phi_a) \) characteristics of an asymmetric SQUID \( (\alpha_f \neq 0) \) in form of a flux shift \( \Delta \Phi \) (see Fig. 2) between the two curves obtained by biasing the SQUID with the currents \( \pm I \) with an appropriate choice of \( |I| \approx I_c \) (experimental details are given below). A flux shift \( \Delta \Phi \neq n \Phi_0 \) corresponds to \( V(I, \Phi_a) \neq -V(-I, \Phi_a) \) for all \( \Phi_a \neq \frac{n}{2} \Phi_0 \) \( (n = 0, 1, 2, ... \)). Thus the bias current (or time-) reversal symmetry is broken, which is true for every tilted ratchet potential: for certain values of the external force the magnitude of the velocity of the system depends on the sign of the force. Note however, that \( V(I, \Phi_a) = -V(-I, -\Phi_a) \) always holds. Applying an ac excitation at angular frequency \( \omega \) with zero mean to a SQUID with \( \Delta \Phi \neq n \Phi_0 \) results in a finite mean voltage, which is maximum for \( \Phi_a = \frac{2n-1}{2} \Phi_0 \) and zero for \( \Phi_a = \frac{n}{2} \Phi_0 \). Obviously the maximum mean voltage with ac bias is largest if \( \Delta \Phi = 0.5 \Phi_0 \).

Next we analyze the dependence of \( \Delta \Phi \) on \( \alpha_L, \alpha_f, \beta_L \) and \( I \). With self field effect and symmetric junctions \( (\alpha_L \neq 0, \alpha_f = 0) \), \( \Delta \Phi \) between a pair of \( V(\Phi_a; \pm I) \) curves is simply proportional to the bias current and \( \alpha_L: \Delta \Phi = 2 \Phi_0 \alpha_L I/I_c \).
For $\alpha_L = 0$, $\alpha_I \neq 0$, $\Delta \Phi$ depends on $\alpha_I$, $\beta_L$ and can be obtained as follows. The flux value $\Phi_1$ (cf. Fig. 2) for $I \sim I_c$ is equal to the flux value of maximum critical current $I_c = I_1 + I_2$. $I_c$ is reached if the circulating current in the loop equals $\alpha_I I_0$, corresponding to an applied flux $\frac{1}{2} \Phi_0 \alpha_I \beta_L$. From our definition it follows that $\Delta \Phi = \Phi_0 \alpha_I \beta_L$. For a SQUID with given inductance this relation can be used to determine the junction asymmetry $\alpha_I$ by measuring $\Delta \Phi$ and $I_c$. We note that $\Delta \Phi \neq 0$ independent on $I$ is frequently observed for our high-$T_c$ SQUIDs which is not surprising, since the spread of the critical currents of the two junctions can be substantial.

To obtain more insights into the dynamics of the system we performed numerical simulations to solve the coupled Langevin equations of the asymmetric dc SQUID [13] with $\beta_L < 1$ and with thermal fluctuations due to Nyquist noise in the junction resistors which adds white noise with zero mean and Gaussian distribution. We use the normalized noise parameter $\Gamma \equiv \frac{2 k_B T}{\pi \Phi_0 I}$ with thermal energy $k_B T$. In our simulations we chose the normal resistance of the two junctions $R_n = 2 R_n/(1 - \alpha_I)$, $R_n^2 = 2 R_n/(1 + \alpha_I)$ thus keeping the characteristic voltage $V_c \equiv I R_n = I_1 R_n^2 = I_2 R_n^2$ and the corresponding intrawell relaxation frequency $\omega_c = \frac{2 e}{\Phi_0} V_c$ of each junction constant ($R_n$ is the SQUID normal resistance). This choice corresponds to an $I_0$ asymmetry due to different junction areas. We calculated the normalized mean voltage $< v >$ for an ac excitation $i(t) = i_{ac} \sin(\omega t)$ with normalized amplitude $i_{ac} = I_{ac}/I_c$. The integration period $T$ was chosen to be $T > 2\pi/\omega$ (for $\omega < \omega_0$). We first discuss the rocking ratchet case ($\alpha_I \neq 0, \alpha_L = 0$). Here, we find $< v >$ ($i_{ac}$) characteristics as shown in Fig. 3, which are strikingly similar to those calculated for a 1-dimensional rocking thermal ratchet [3]. For $\Gamma = 0$ (Fig. 3(a)] we find for adiabatically slow excitation, $\tilde{\omega} = 0.001$ ($\tilde{\omega} = \omega/\omega_0$), a lower ($i_{l}$) and an upper ($i_{u}$) threshold for $i_{ac}$. For $i_{ac} < i_{l}$, no mean voltage appears. For $i_{l} < i_{ac} < i_{u}$ the mean voltage increases monotonically with increasing $i_{ac}$ and for $i_{ac} > i_{u}$ the voltage decreases monotonically. For $\tilde{\omega} = 0.01$, the overall shape of $< v >$ ($i_{ac}$) is similar to the $\tilde{\omega} \rightarrow 0$ case, however, voltage steps at $< v > = n\tilde{\omega}$ appear. These steps can be interpreted as Shapiro steps [3], where the phase dynamics synchronizes with the external excitation. For a given $i_{ac}$ and within one excitation period the phase "rolls" $m\pi$ in one current direction and $k\pi$ in the opposite current direction ($m, k$: integer). On average the phase evolves $(m-k)2\pi$ per excitation period and hence $< v > = (m-k)\tilde{\omega}$. Within $i_{l} < i_{ac} < i_{u}$, we have $k = 0$ and $m$ increases with increasing $i_{ac}$. For $i_{ac} > i_{u}$ both, $m$ and $k$ increase with increasing $i_{ac}$, while $(m-k)$ exhibits a switching behavior [see inset Fig. 3(a)] with an overall decrease with increasing $i_{ac}$. As $i_{ac} \rightarrow \infty$ we have $(m-k) \rightarrow 0$. With increasing frequency the value of $(m-k)$ decreases. For $\tilde{\omega} = 0.1$ a finite voltage appears in certain $i_{ac}$-intervals only, where plateaus of equal voltage appear, i.e. $(m-k) = 0.1$. A further increase of $\tilde{\omega}$ confines the locking condition to obtain $(m-k) > 0$ to smaller $i_{ac}$-intervals. For $\tilde{\omega} = 0.5$, without noise, no mean voltage is predicted. If one introduces small thermal fluctuations ($\Gamma = 0.03$) [see Fig. 3(b)], the condition for $(m-k) > 0$ is relaxed and a stochastic resonance like effect sets in (for a recent review see [13]). While the maximum voltage for each frequency is reduced the sharp plateaus observed for $\Gamma = 0$ smear out. Furthermore, even for $\tilde{\omega} = 0.5$ there exists an $i_{ac}$-range wherein $< v > \neq 0$.

We next consider the flashing ratchet case ($\alpha_L \neq 0$). Here, a novel feature appears: the mean voltage undergoes multiple sign reversals with increasing amplitude $i_{ac}$ as shown in Fig. 4. The envelope of the peak voltages exhibits an oscillatory damped behavior, and with finite $\Gamma = 0.03$ the step like behavior is again smeared out. An additional $I_0$-asymmetry [Fig. 3(b)] further complicates the $< v >$ ($i_{ac}$)-characteristics. For $\Gamma = 0$, the oscillation period of the envelope is clearly smaller as in Fig. 3(a), however, this oscillation is less obvious if one adds small thermal fluctuations ($\Gamma = 0.03$). In any case a finite $\Gamma$ induces further sign changes in $< v >$ ($i_{ac}$).

To test our model we performed measurements on a YBa$_2$Cu$_3$O$_7$ thin film dc SQUID with 24° bicrystal Josephson junctions on a SrTiO$_3$ substrate. At $T \approx 78K$, we found $I_c = 172$ $\mu$A ($\Gamma \approx 0.04$), $R_n = 1.34$ $\Omega$, $V_c = 232$ $\mu$V, $\omega_0/2\pi = 112$GHz and $\beta_L \approx 0.07$. From the estimated geometric inductance we obtain $L \approx 30$ $\mu$H. However, we note that due to the unknown contribution from the kinetic inductance, $L$ may be substantially larger. All measurements were carried out in a magnetically shielded cryostat. To apply the ac bias current we used two different techniques: in the adiabatic frequency regime up to 500kHz the ac current was fed via a twisted pair of wires to the SQUID. At higher frequency (12.31GHz) the microwave power was guided by a semi rigid coaxial cable into the cryostat. At the end of the coaxial cable an antenna structure was positioned about 1cm above the SQUID. In this geometry the microwave coupling is not well defined, which implies that both, in- and out- of phase currents are induced across the two junctions. The latter corresponds to a circulating current in the SQUID loop. Hence, a substantial self-field effect can be generated which is not present in the adiabatic experimental setup. For details of the experimental setup and SQUID layout see [13].

The measured mean voltage vs. $I_{ac}$ at adiabatically slow excitation ($\omega/2\pi = 50$kHz) is shown in Fig. 4(a) together with the numerical simulation result for $L = 60$ pH corresponding to $\beta_L = 5.1$. The measured flux shift $\Delta \Phi / \Phi_0 \approx 0.5$ (see Fig. 3) yields a junction asymmetry of $\alpha_I = \Delta \Phi / \Phi_0 \beta_L \approx 0.1$. The good agreement is a further confirmation of a ratchet effect in our system. In contrast to the adiabatic case a much more complex behaviour occurs at $\omega/2\pi = 12.31$GHz ($\tilde{\omega} \approx 0.1$) as shown in Fig. 4(b). The mean voltage oscillates with increasing ac amplitude and multiple sign reversals occur. At this frequency the ac bias is not well defined and the self-field effect can play a major role. In fact, the qualitative agreement with the simulation results shown in Fig. 4(b) for $\Gamma = 0.03$ suggests that the
assumption of a finite value of $\alpha_L$ is reasonable. We believe that the observed behaviour can be ascribed to a flashing rocked ratchet system.

In conclusion, we propose asymmetric dc SQUIDs as a model system for the study of the ratchet effect. We show that the voltage rectification effect in dc SQUIDs occurs due to broken reflection symmetry in the 2-dimensional SQUID potential and present a model which accounts for this observation by introducing a critical current asymmetry and a self field effect. The latter, although found to be responsible for a rich and complex dynamic behaviour, requires better control of high frequency coupling to the SQUID in further experiments. For the asymmetry due to different critical junction currents we obtain a simple rule to design SQUIDs with a large ratchet effect. It should be pointed out that the observed ratchet effect is no genuine property of high-$T_c$ SQUIDs. In fact, using conventional low-$T_c$ SQUIDs might have advantages due to the superior low spread junction technology. However the voltage scale on which the ratchet effect occurs is determined by the $I_cR_n$ product which is larger for high-$T_c$ SQUIDs [16]. Finally we emphasize that the observation of a ratchet effect in asymmetric dc SQUIDs opens up the perspective for a variety of experimental studies in a system with straightforward detection of directed motion, simply by measuring voltage. The system offers experimental control over important parameters such as amplitude and frequency of the external force, damping coefficient and thermal noise parameter. Furthermore, the impact of different types of fluctuations might be interesting to investigate.

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Fig. 1 Contour plot of the SQUID potential $u_s$ ($\beta_L = 1.5$, $f_a = 0.25$) with and without $I_0$ asymmetry (upper row: $\alpha_I = 0$, lower row: $\alpha_I = 0.3$) and for both directions of bias current $i$. Arrows indicate direction of motion along phase trajectories (dashed lines).

Fig. 2 Measured dc voltage $V$ vs. applied flux $\Phi_a$ for different values of bias current $I$ with a flux shift $\Delta \Phi = [\Phi_1 - \Phi_2] \approx 0.5\Phi_0$, with $\Phi_{1,2}$ defined as the flux values of corresponding extrema.

Fig. 3 Calculated mean voltage $<v>$ across dc SQUID with $I_0$ asymmetry ($\alpha_I = 0.3$) vs. ac amplitude $i_{ac}$ without thermal noise (a) and for $\Gamma = 0.03$ (b) for different frequencies and $f_a = 0.25$. The inset in (a) shows an expanded view for $\hat{\omega} = 0.01$ at $i_{ac} \approx 1$. The labels for $\hat{\omega}$ in (b) are also valid for (a).

Fig. 4 Calculated mean voltage $<v>$ across dc SQUID with self-field effect vs. ac amplitude $i_{ac}$ without thermal fluctuations (thick line) and for $\Gamma = 0.03$ (dots); $f_a = 0.25$.

Fig. 5 Mean voltage across dc SQUID vs. ac amplitude at $f_a = 0.25$: (a) comparison of experiment and calculation in the adiabatic limit; (b) experiment for $\hat{\omega} \approx 0.1$. 

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