A Case Study on Three-Dimensional Single-Beacon Localization for Autonomous Underwater Vehicles

Huapeng Yu\textsuperscript{a} and Chao Han\textsuperscript{b}
National Innovation Institute of Defense Technology, Academy of Military Sciences, Beijing 100010, China
\textsuperscript{a}hpyu_qtxy@163.com; \textsuperscript{b}fatal_han@163.com

Abstract. Circular trajectory is most frequently used when an autonomous underwater vehicle calibrates positions of underwater acoustic beacons or corrects its own navigation errors using beacons. However, the space-time effect on control and navigation data processing and the impact of the angle of attack on three-dimensional localization performance of AUVs have rarely been mentioned in previous studies. To improve the three dimensional localization performance of AUVs with a single-beacon aiding, methods for space-time trade-offs from a control perspective and determination of the optimal angle of attack are investigated in this study. Through theoretical deduction aided with numerical simulation, concerned quantitative values with their corresponding preconditions are given, and some general conclusions which can greatly improve the navigation capability of AUVs have been made.

1. Introduction
For long-duration underwater navigation or reliable control and guidance of the booming autonomous underwater vehicles (AUVs), there is a great demand for acoustic beacon aiding [1, 2]. Meanwhile, homing and navigating using a single-beacon is always the most cost-effective and robust method in complex underwater environments [1, 3-7]. For the geometry of a single-beacon, circular trajectory is easy to realize and be controlled for AUVs. There are lots of published works to show the various applications of circular trajectories [5-10]. However, it should be pointed out that few published studies have discussed in-depth the three-dimensional single-beacon localization issue for AUVs using circular trajectories. Most researchers always project the three-dimensional localization problem to the planar coordinates for simplification. Despite the aid of depth sensors, ignoring the impact of angle of attack on localization performance yet to be possible. Our research is motivated by the fact that, the space-time effect on control and navigation data processing in the planar systems and the impact of the angle of attack on three-dimensional localization performance of AUVs under the circular trajectory case need to be studies sufficiently.

The remaining of this paper is arranged as follows. In Section II, we give some detailed preliminaries and problem statements for AUV navigation with single-beacon aiding. Theoretical analysis about the circular trajectory case study on three-dimensional localization for AUVs is comprehensively presented in Section III. Numerical simulation and theoretical deduction are performed and discussed in Section IV. Finally, Section V summarizes the analysis of the preceding sections.

2. Preliminaries and problem setup
Figure 1 illustrates the system configuration. Using a single beacon, the AUV maneuvers relatively to accomplish certain tasks, such as homing, position correcting, etc. In Figure 1, we denote the radius of rotation of the AUV in circular trajectory as $D$, the speed of the AUV as $V$, the depth of the AUV as $d$, the angle-of-attack of the AUV as $\theta$, and the sideslip angle of the AUV as $\psi$.

Take positioning of an unknown beacon using an AUV as an example. Let the position of the single beacon be $P_b = [x_b, y_b, z_b]^T$ and the real-time position of the AUV be $P_{AUV} = [x_{AUV}, y_{AUV}, z_{AUV}]^T$, we can obtain the following expression with the acoustic range measurement $r$ [3, 4]:

$$2A_bP_b = \begin{bmatrix} (r_1)^2 - (r_2)^2 + [P_{AUV,2}]^2 - [P_{AUV,1}]^2 \\ (r_2)^2 - (r_1)^2 + [P_{AUV,3}]^2 - [P_{AUV,2}]^2 \\ (r_1)^2 - (r_2)^2 + [P_{AUV,4}]^2 - [P_{AUV,3}]^2 \end{bmatrix}$$

$$A_p = \begin{bmatrix} (P_{AUV,2} - P_{AUV,1})^T \\ (P_{AUV,3} - P_{AUV,2})^T \\ (P_{AUV,4} - P_{AUV,3})^T \end{bmatrix}$$

(1)

To obtain a unique solution of equation (1), the rank of the left coefficient matrix $A_p$ of $P_b$ matrix must be 3, that is $|A_p| = (P_{AUV,2} - P_{AUV,1}) \cdot (P_{AUV,3} - P_{AUV,2}) \times (P_{AUV,4} - P_{AUV,3}) \neq 0$. Therefore, it is illustrated that, only when the changes of angle of attack and sideslip angle of the AUV all occur and these changes are not synchronized, we can obtain the unique solution of the beacon position, which is the basis of the analysis in this paper.

By linearizing the range measurement about a certain point $P_{AUV,0}$, we can describe the system model for correcting navigation error of the AUV with a single beacon [4, 8-10]:

$$r = \left| P_{AUV,0} - P_b \right| + C\left( P_{AUV,0} \cdot P_b \right) \delta P_{AUV} + w_r, \quad w_r \cdot N(0, \sigma_r)$$

(2)

where, $\delta P_{AUV}$ represents the three-dimensional incremental travel displacement, $w_r$ denotes range measurement noise, and

$$C\left( P_{AUV,0} \cdot P_b \right) = 2\left[ (x_{AUV,0} - x_b)/r \quad (y_{AUV,0} - y_b)/r \quad (z_{AUV,0} - z_b)/r \right]$$

(3)

Thus, the localization performance can be evaluated in the Cramér Rao lower bound (CRLB) form through assembling multiple measurements estimation performance in the Cramér Rao lower bound (CRLB) form:

$$E = \left[ C\left( P_{AUV,0} \cdot P_b \right) \right]^T \sigma_r^{-1} C\left( P_{AUV,0} \cdot P_b \right)$$

(4)

This tool would be used and discussed in-depth later.
3. Three-dimensional localization solution

3.1. Sampling period from a control perspective

First, to obtain the optimal numerical solution of the sampling period, we only consider the navigation and control of an AUV in the planar rectangular coordinate system.

Assuming the sampling period is $T$, the Jacobian can be then evaluated at the fixed position of the single beacon. Without losing generality, the initial projection is considered to be located on the $x$ axis (Figure 1) of the rectangular coordinate system centered on the beacon. Thus, we have:

$$
C\left(P_{AUV,b}, P_b\right) = -\frac{2}{D} \begin{bmatrix}
D & 0 \\
D \cos(VT/D) & -D \sin(VT/D) \\
D \cos(2VT/D) & -D \sin(2VT/D)
\end{bmatrix}
$$

The CRLB representing the geometry of the single beacon can be written as follows:

$$
\sigma^2 = \left[ C\left(P_{AUV,b}, P_b\right) \right]^T \left[ C\left(P_{AUV,b}, P_b\right) \right]^{-1}
$$

$$
= \frac{\sigma^2}{4 \text{DET}} \begin{bmatrix}
\sin(VT/D)^2 + \sin(2VT/D)^2 & \sin(2VT/D)/2 + \sin(4VT/D)/2 \\
\sin(2VT/D)/2 + \sin(4VT/D)/2 & 1 + \cos(VT/D)^2 + \cos(2VT/D)^2
\end{bmatrix}
$$

where, the mathematical symbol “DET” denotes solving the determinant for a matrix; $\sigma$, represents the range measurement noise variances between the AUV and the single beacon. Apparently, $\sigma$, on both axes have been assumed to equal in the planar rectangular coordinate system for simplification.

A scalar metric commonly known in the field of Global Positioning System (GPS) applications, the dilution of position ($DOP$), is used here to further predict the navigation performance. From equation (6), a navigation performance index in $DOP$ form can be obtained. And, the resulting performance index equation can be expressed as:

$$
DOP = \frac{3\sigma}{4 \left[ \sin^2 \frac{VT}{D} + \sin^2 \frac{2VT}{D} \right] \left[ 3 - \left( \sin^2 \frac{VT}{D} + \sin^2 \frac{2VT}{D} \right) \right] - \left[ \frac{1}{2} \sin^2 \frac{2VT}{D} + \frac{1}{2} \sin^2 \frac{4VT}{D} \right]^2}
$$

3.2. Determination of the optimal angle of attack

In order to analyze the impact of the angle of attack on three-dimensional localization performance, the depth of the AUV ($d$) needs to be considered. Besides, to intuitively investigate the system performance, the Jacobian $C\left(P_{AUV,b}, P_b\right)$ is expressed in the single-beacon centered polar coordinate system as follows:

$$
C^{3D}\left(P_{AUV,b}, P_b\right) = 2 \begin{bmatrix}
e_1^e & e_2^e & e_3^e & e_4^e \\
e_1^\theta & e_2^\theta & e_3^\theta & e_4^\theta \\
\sin \theta \cos \psi_1 & \sin \theta \sin \psi_1 & \cos \theta \\
\sin \theta \cos \psi_2 & \sin \theta \sin \psi_2 & \cos \theta \\
\sin \theta \cos \psi_3 & \sin \theta \sin \psi_3 & \cos \theta \\
\sin \theta \cos \psi_4 & \sin \theta \sin \psi_4 & \cos \theta
\end{bmatrix}
$$

where, the subscripts of $e$, $\theta$ and $\psi$ denote continuous multiple measurements.

Similarly, the range measurement noise level projected on the three polar coordinate axes are assumed to be the same, and their variances are denoted as $\sigma_r$. Then, we can derive the CRLB estimation.
Using the general matrix operating formula
\[
\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix} - \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix} \text{DET} \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix}
\]
where, the superscript of a matrix represents its adjoint matrix, and
\[
M_1 = \sum_{i=1}^{4} \begin{bmatrix} \sin^2 \theta \cos^2 \psi_i & \sin^2 \theta \cos \psi_i \sin \psi_i \\ \sin^2 \theta \cos \psi_i \sin \psi_i & \sin^2 \theta \sin^2 \psi_i \end{bmatrix}
\]
\[
M_2 = \sum_{i=1}^{4} \begin{bmatrix} -\sin 2\theta_i \cos \psi_i \\ -\sin 2\theta_i \sin \psi_i \end{bmatrix}
\]
\[
M_3 = \sum_{i=1}^{4} \begin{bmatrix} \sin 2\theta_i \cos \psi_i \\ \sin 2\theta_i \sin \psi_i \end{bmatrix}
\]
\[
M_4 = \sum_{i=1}^{4} \cos^2 \theta_i
\]

Using the general matrix operating formula \( \text{DET} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \text{DET}(D)\text{DET}(A - BD^{-1}C) \), a conclusion can be made that the determinant of the matrix \( \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix} \) is the largest when the following conditions hold:
\[
\sum_{i=1}^{4} \sin^2 \theta_i \sin 2\psi_i = 0, \quad \sum_{i=1}^{4} \sin^2 \theta_i \cos 2\psi_i = 0
\]
\[
\arg \min \left\{ \sum_{i=1}^{4} \begin{bmatrix} \sin 2\theta_i \cos \psi_i \\ \sin 2\theta_i \sin \psi_i \end{bmatrix}, \sum_{i=1}^{4} \begin{bmatrix} \sin 2\theta_i \cos \psi_i \sin \psi_i \\ \sin 2\theta_i \sin \psi_i \sin \psi_i \end{bmatrix}, \sum_{i=1}^{4} \cos^2 \theta_i \right\}
\]

Thus,
\[
\text{DET} \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix} = \sum_{i=1}^{4} \cos^2 \theta_i \left( 2 - \frac{1}{2} \sum_{i=1}^{4} \cos^2 \theta_i \right)^2
\]

Under the predefined condition (14), the CRLB estimation is the lowest, and thus the localization performance is the best-case estimation.

4. Results and discussions

4.1. Sampling period

To analytically solve the derived navigation performance index equation (7) in DOP form is rather complicated. In this subsection, we will determine the sampling period by synthesizing theoretical deduction and simulation.

As mentioned before, the control of the AUV and data processing of navigation are key issues for autonomous underwater travel. From a control perspective, the incremental rudder angle for a circular trajectory can be expressed as \( \omega = \frac{VT}{D} \). Therefore, through numerical simulation, we can derive the relative DOP value with incremental rudder angle variation. From the simulation results, Figure 2 is plotted for intuitive investigation on characteristics of rudder control scheme, and Figure 3 is plotted for comparative analysis.
Figure 2. DOP value versus incremental rudder angle and locally enlarged drawing.

Figure 3. A commonly used navigation and correction Configuration.

In Figure 2, it can be seen that, the DOP values reaches their local extreme points when the incremental rudder angle accumulates up to integer multiple of π/3 or 2π/3. Apparently, the AUV localization performance in a circular trajectory of the single-beacon geometry are strongly relative with control maneuver scheme, which are determined by the speed $V$, the radius of rotation $D$ and the sampling period $T$ of the AUV. Compared to Figure 3, it is obviously that the DOP values decrease rapidly with the distance the AUV travelled during the sampling period and increase linearly with the radius of rotation. Subsequently, the AUV can determine its data processing period for optimal navigation and control using real-time online measurements about $D$ and $V$, which has provided the AUV with a degree of decision and control intelligence.

4.2. The optimal angle of attack

By getting the maximum value of the determinant $\det \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix}$, the lowest CRLB estimation of $\Lambda^{10}$ can be obtained, which means that the localization error of the AUV would be restrained to the greatest extent.

Substituting $\sum \frac{\partial^2 x}{\partial \theta^2} \theta = \xi$ into the derive determinant, we have:

$$\det \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} = \xi \left( 2 - \frac{1}{2} \xi \right)^2$$ (16)
Differentiating equation (16), it can be easily concluded that the derived determinant reaches its maximum value when $\xi = 4/3$. Then, when a constant angle of attack is chosen, the optimal $\theta$ value is determined:

$$\cos^2 \theta = 1/3 \Rightarrow \theta = 54.74^\circ$$  \hspace{1cm} (17)

5. Conclusions

This study makes a thorough analysis on the three-dimensional single-beacon localization problems for AUVs in circular trajectory. Theoretical and numerical methods for space-time trade-offs from a control perspective and determination of the optimal angle of attack of the AUV are proposed and verified. Our future work will focus on how to apply these methods to in-motion usage of the AUV with a moving single beacon and to evaluate the sea current influences on three-dimensional localization.

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