Conference Proceedings / Title

Variable displacement alpha-type stirling engine

V M Homutescu1, D T Bălănescu1, C E Panaite1 and M V Atanasiu1

1 Automotive and Mechanical Engineering Department, “Gheorghe Asachi” Technical University of Iasi, Iasi, Romania

E-mail: mariohomutescu@gmail.com

Abstract. The basic design and construction of an alpha-type Stirling engine with on load variable displacement is presented. The variable displacement is obtained through a planar quadrilateral linkage with one on load movable ground link. The physico-mathematical model used for analyzing the variable displacement alpha-type Stirling engine behavior is an isothermal model that takes into account the real movement of the pistons. Performances and power adjustment capabilities of such alpha-type Stirling engine are calculated and analyzed. An exemplification through the use of the numerical simulation was performed in this regard.

1. Introduction

The Stirling engines were named after their inventor, who patented them exactly 200 years ago (in 1816). Stirling engines are external combustion machines with pistons with reciprocating movement. These engines use heat regeneration. The theoretical Stirling engine runs after a thermodynamic cycle comprising two isothermal processes connected by two isochoric processes.

In the last sixty years, due to the continuous development of science and technology, the interest for such engines has risen again, especially in the specific utilization fields that came up for them [1], [2], [3]. During these sixty years, new Stirling engines with different configurations were developed. Nowadays Stirling engines are used to power electric generators of various size, compressors or pumps, for motorizing submarines, yachts or even torpedoes. A very actual use of the Stirling engine is as a key component of small and medium heat and power cogeneration units [1].

The Stirling engine configuration proposed and analyzed in the paper is a modification of the standard alpha-type configuration [2], [3], and has a motive drive that allows the variation of the displacement under load [4], [5]. In consequence, it allows the continuous variation of the work yielded by the engine.

2. Variable Displacement Alpha-Type Stirling Engine

The variable displacement alpha-type Stirling engine (VDATSE) schematic diagram is presented in figure 1. A VDATSE functional unit has the following main components: a power piston cylinder 8, a power piston 7, a displacer cylinder 17, a displacer 1 and the heat exchangers (a heater 3, a regenerator 4 and a cooler 5). The heater is placed inside the high temperature heat source (e.g. inside a combustion chamber). The low temperature heat exchanger is cooled with water or air.

The working agent is placed inside five functional spaces. The expansion space 2 is places between the displacer and its cylinder head. The compression space 6 is placed between the power piston and its cylinder head. The last three functional spaces are to be found inside the three heat exchangers.
Figure 1. Schematic diagram of the alpha-type Stirling engine with variable displacement: 1 - displacer; 2 - expansion chamber; 3 - cooler; 4 - regenerator; 5 - heater; 6 - compression chamber; 7 - power piston; 8 - power piston cylinder; 9 - power piston rod; 10 - triangular plate; 11 - adjustment bar; 12 - leaning bar; 13 - nut; 14 - screw; 15 - crankshaft; 16 - displacer rod; 17 - displacer cylinder.

Figure 2. The modified motive drive and its main dimensions.
The motive drive has a crankshaft 15. Between the power piston rod 9 and the crankshafts 15 the side MN of the triangular plate 10 has been supplementary introduced. The end T of the triangular plate is socketed with the adjustment bar 11. The adjustment bar is also socketed with the leaning bar 12. The leaning bar 12 can turn for several degrees around the fixed socket V. The socket U between the bars 11 and 12 is placed inside the nut 13. The socket U can rotate along a circle arc due to the adjustment screw 14. The screw 14 is spun from outside through the nuts 13. The final effect is that the end T of the triangular plates 10 shifts and thus the length of the “equivalent rod” MP_p is modified, resulting in the displacement variation of the analyzed Stirling engine.

The displacer piston is equipped with a regular slider-crank mechanism.

In figure 2 are presented the most important dimensions of the motive drive that allows the on-load displacement modifications.

3. Hypotheses and physical model

The model chosen for analyzing the functioning of the VDATSE belongs to the family of the isothermal models. Isothermal models are complex enough to correctly evaluate the possible maximum performances that could be obtained by the Stirling machines (both engines and refrigerators), and are also simple enough in order to neglect the nonessential influences over the functioning. All isothermal models share the following hypotheses:

- the working agent is the ideal gas;
- there are no gas leaks towards exterior during functioning - the mass of working agent is constant;
- at thermodynamic level all cycle functional processes are time independent;
- there is no heat exchange between the metallic parts of the machine;
- the processes taking place inside the cooler and heater are isothermal;
- the temperature of the gas inside the expansion chamber is equal with the temperature inside the heater, is constant and equal to the one of the displacer and that of the displacer cylinder;
- the temperature of the gas inside the compression chamber is equal with the temperature inside the cooler, is constant and equal to the one of the power piston and to the one of the power piston cylinder;
- the heat regeneration is an ideal process (with 100% regeneration efficiency); the temperature of the agent trapped inside regenerator is constant, being calculated as logarithmic (or other) mean;
- the instantaneous pressure is identical in all spaces occupied by the agent, its value varies along the cycle;
- the movements of the pistons are the real movements of the motive drive.

![Figure 3. The temperature-related hypotheses of the isothermal physico-mathematical model of the Stirling engine.](image)

Hypotheses implying temperatures inside the Stirling engine (presented in figure 3) show that only isothermal processes take place inside the functional chambers; thus, denomination of the isothermal model to the described physico-mathematical model is confirmed.
4. Mathematical model

The pressure variation law for the isothermal model results from the equation of state applied for each of the five functional spaces (from which the masses inside each chamber are expressed) and from the agent total mass ($m_t$) conservation equation:

$$p(\alpha, \psi) = m_t R \left( \frac{V_e(\alpha) + V_h}{T_h} + \frac{V_r(\alpha, \psi)}{T_reg} + \frac{V_c(\alpha, \psi) + V_k}{T_k} \right)^{-1}. \quad (1)$$

Notations in relation (1) are: $R$ = gas constant; $m_t$ = agent total mass; $p$ = pressure; $V$ = volume; $T$ = temperature. The following subscripts were used: $c$, $e$ = compression and expansion chambers; $reg$ = regenerator, $h$ = heater; $k$ = cooler; $t$ = total; $d$ = displacer piston; $p$ = power piston.

Variation laws of the expansion and compression chamber volume, namely $V_e(\alpha)$ and $V_c(\alpha, \psi)$, are evaluated from kinematic considerations [5], in relation with figure 2.

The work yielded is the sum of the works exchanged in the expansion and the compression spaces:

$$L = L_e + L_c = \int_0^{2\pi} p(\alpha) \, dV_e + \int_0^{2\pi} p(\alpha) \, dV_c. \quad (2)$$

The indicated power of the engine is

$$P = \frac{n}{60} L, \quad (3)$$

where $n$ is the rotational speed, in rpm.

Calculated with the isothermal model, the thermal efficiency of any Stirling engine is always equal with the maximum possible thermal efficiency, namely the one of the Carnot cycle working between the same extreme temperatures (due to the isothermal processes and to the ideal regeneration of the heat). So, at the VDATSE, the thermal efficiency calculated with the isothermal model will not depend on the adjustment angle $\psi$.

5. Results of the numerical simulation of the VDATSE and discussion

For a VDATSE (figure 1 and figure 2) described by the following values: crank radius $r = 0.0385$ m; eccentricity $e = 1.6 \, r$; displacer rod length $l_{1d} = 3 \, r$; power piston rod length $l_{1p} = 2.5 \, r$; $l_2 = l_3 = l_4 = 2 \, r$; $l_4 = 3 \, r$; $d_1 = 2.5 \, r$; $\gamma = 50^\circ$; $\delta_1 = 190^\circ$; pistons diameter $D = 0.073$ m; $V_h = V_k = 0.05 \, V_{max}$; $V_{reg} = 1.2 \, V_{max}$, where $V_{max}$ is the maximum volume of the expansion chamber; $m_t = 0.0025$ kg of hydrogen; $T_h = 773$ K; $T_k = 310$ K and $n = 1500$ rpm, for an adjustment angle $\psi$ between $250^\circ$ ... $295^\circ$, the performances presented in the following pictures were obtained. $T_{reg}$ was estimated as arithmetic mean of $T_h$ and $T_k$. The subscript max stands for the maximum value.

$$V_e(\alpha, \psi) \quad [10^{-3} \text{ m}^3]\quad (a)$$

$$V_e(\alpha) \quad [10^{-3} \text{ m}^3]\quad (b)$$

Figure 4. Volume variation: (a) - for compression space; (b) - for expansion space.
Figure 4 presents the variation of the compression chamber (a) and expansion chamber (b) volumes during a complete cycle (any cycle of a Stirling machine needs one full rotation to complete, so the crankshaft position angle $\alpha$ varies between 0° and 360° during the entire cycle). The volume of the compression chamber is affected by the modified motive drive, and is finally causing the intended power variation. The volume variation inside the expansion chamber is independent of the adjustment angle $\psi$, because the displacer comes with its own regular slider-crank mechanism (figure 1).

The curves inside figure 5 show the positions of the dead centers of the power piston for the entire variation interval of the adjustment angles. Both dead centers are dependent by the adjustment angle. The power piston cylinder head must be placed above the maximum position of the TDC against the adjustment angle. So, the dead space of the power piston cylinder depends on the angle $\psi$. The diagrams show that the changes of the total volume occupied by the working gas are obtained mainly due to the movements of the power piston bottom dead center (BDC).

**Figure 5.** Position of the power piston dead centers.  **Figure 6.** Variation of pressure inside VDATSE.

Figure 6 presents the cyclic variation of the pressure inside VDATSE, for several adjustment angles. The maximum pressure inside VDATSE increases with $\psi$ because the dead space inside power piston cylinder is progressive smaller for larger values of the adjustment angles. The minimum pressure decreases with $\psi$, because the compression volume and the total volume occupied by the working agent inside the engine increases with $\psi$.

**Figure 7.** Indicator diagrams: (a) - for compression space; (b) - for expansion space.

By comparing the diagrams from figure 7 (a) and (b) it can be seen that areas of the indicator diagram for both compression and expansion chambers increases when $\psi$ increases. This increasing is more accentuate for the area of the indicator diagram of the expansion chamber. This conclusion is strengthened by the numerical data presented in table 1.
Table 1. Work exchanged inside VDSE chambers.

| Adjustment angle $\psi$ | Unit | Work in expansion chamber $L_e$ [J] | Work in compression chamber $L_c$ [J] | Total work $L$ [J] | $100 \frac{L}{L_{\text{max}}}$ [%] |
|-------------------------|------|-----------------------------------|--------------------------------------|-----------------|----------------------|
|                         | 250º | 856.7                             | -343.6                               | 513.1           | 58.7                 |
|                         | 270º | 1058.7                            | -424.6                               | 634.1           | 72.5                 |
|                         | 295º | 1459.0                            | -585.1                               | 873.9           | 100                  |

The indicator diagrams for the entire volume of working agent from figure 8 show the large dead volume, peculiar to the nowadays Stirling engines. This large dead space is caused by the inner volume of the regenerator, which is usually larger than the volume swept by displacer piston. The diagrams clearly show that the variation of the total volume occupied by the working agent inside the VDATSE increases with the adjustment angle $\psi$.

![Figure 8. Indicator diagrams for the entire engine.](image)

Variations of the works $L_e, L_d$ and $L$ exchanged and of the power $P$ produced by VDATSE, as function of the adjustment angle (calculated with an isothermal model for the peculiar engine configuration, used in exemplification) are presented in figure 9 and in figure 10. The power yielded was calculated for a possible rotational speed of 1500 rpm.

The VDATSE studied in this numerical example can adjust the load (for adjustment angle $\psi$ between 250º and 295º) within the ratio $P_{\text{max}} / P_{\text{min}} = 1.7$.

6. Conclusions

The on load variation of the power produced by an alpha-type Stirling engine was obtained by varying the displacement of the power piston. The displacement variation was achieved by gearing the power piston with a planar quadrilateral linkage with one on load movable ground link.
Although the isothermal model is a theoretical one, it allows to obtain a coherent image of the VDATSE functioning.

The application of the model on the VDATSE shows that work produced in the expansion chamber increases quicker than work consumed in the compression chamber (the work consumed is taken in absolute value) when the adjustment angle increases. Besides, increasing of total gas volume with adjustment angle leads to an increase of work produced. The maximum pressure of the working agent also increases when adjustment angle increases.

Calculations show that VDATSE can adjust the load within the ratio of 1.7. By optimizing the dimensions of VDATSE motive drive, an increase of the maximum/minimum power ratio is expected.

A possible use of VDATSE could be in small combined heat and power (CHP) plants, where the rotational speed of both the electric generator and the engine must remain constant. Such small CHP plant can function with a constant amount of fuel. The adjustment capacities of VDATSE would change the ratio between the power and heat produced in this case.

7. References
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Acknowledgments
This work was partly supported by the project POSCCE-A2-O2.2.1-2009-4-ENERED, ID nr. 911, co-financed by the European Social Fund within the Sectoral Operational Program “Increase of Economic Competitiveness”.