Investigation of the problems of parametric identification of nanosatellite motion model

I V Belokonov, I A Lomaka

Inter-University Department of Space Research, Samara University, 34 Moskovskoye shosse, Samara 443086, Russia

igorlomaka63@gmail.com

Abstract. In this paper, we discuss problems that exist during nanosatellite identification procedure. These problems are: choice of data collection interval that allows to identify required parameters with desired error, study of dependence model between identification parameters. We studied solution of these problems on the example of variable-mass SamSat-M nanosatellite which is equipped with propulsion unit. We obtained analytical model of parameters dependences and provided results of identification of fuel level.

1. Introduction

The restrictions of nanosatellite include low energy and limited space for payload. These circumstances force the use of transformable structures on board the nanosatellite (deployable solar panels, rods with equipment, etc.). At the same time, the inertial and aerodynamic characteristics of the nanosatellite also change.

Practice of using cheap electronic components that do not have radiation resistance is widespread in nanosatellite development. As a result, nanosatellite can operate in an abnormal mode. Determination of the characteristics of the controlled design parameters that reflect the design features of the nanosatellite, in some cases, allows to determine the causes and consequences of abnormal processes occurring on board.

Thus, the implementation of modern space missions using nanosatellite necessitates the solution of new inverse problems of parametric identification on board the complex, the results of which can be used in the loop of the motion control system to improve the quality of the control process and in the operational analysis of flight information to identify signs of abnormal functioning.

These problems can be solved using special algorithms for controlling the design parameters of the nanosatellite based on the identification of the vector of parameters of the onboard mathematical model of angular motion: mass-inertial characteristics, parameters of rotational motion, as well as the coefficients of external moments under the influence of which this motion is realized.

In this paper we have developed and generalized ideas obtained in [1].
2. Problem formulation

2.1. Motion model

The nonlinear attitude dynamics of a rigid satellite is described by Euler equations (1). In this paper, it is assumed that aerodynamic drag and gravity gradient forces are the most strong forces producing torques for a nanosatellite. The dynamics equations are defined as:

\[
\begin{align*}
\dot{\omega}_x &= \mu (\omega_x \omega_y - v \omega_z) + M_{xx} \\
\dot{\omega}_y &= -\lambda (\omega_x \omega_y - v \omega_z) + \frac{\lambda}{I_z + \mu} M_{yy} \\
\dot{\omega}_z &= -\left(1 - \lambda + \frac{\lambda}{I_z + \mu}\right) (\omega_x \omega_y - v \omega_z) + \frac{\lambda}{I_z + \mu} M_{zz}
\end{align*}
\]

(1)

where \( \lambda = I_x / I_z \), \( \mu = (I_y - I_z) / I_z \) - dimensionless inertia coefficients [2], \( v = 3\mu / r^3 \) - gravitation torque coefficient, \( M_{ij} = (K_s \times V_e) V_e \rho \) - aerodynamic torque, \( K_s = \frac{1}{2I_s} C_s S d \) - aerodynamic torque coefficients related to \( I_s \), \( \rho \) - atmospheric density, \( V_e = [V_x, V_y, V_z] \) - orbital velocity vector in body coordinate system, \( a_j \) - elements of rotation matrix \( A \), \( d \) - vector between center of mass and center of pressure. The nanosatellite quaternion kinematics can be described by the following differential equations:

\[
\dot{q} = \overline{\Omega} q
\]

(2)

where \( q \) - orientation quaternion, \( \overline{\Omega} \) - matrix of angular velocities described in [2].

2.2. Parameter definition

Usually it is convenient to identify parameters that are directly included in nanosatellite model of angular motion. In general these estimated parameters are:

- initial angular velocity vector \( W \);
- initial attitude parameters (quaternion or Euler angles) \( U \);
- inertial parameters of nanosatellite (inertial moments or inertial coefficients) \( P \);
- coefficients of external moments (aerodynamics, magnetic, etc.) \( K \).

All of this parameters can be combined into vector of rotation motion model parameters \( b \):

\[
b = [W \ U \ P \ K].
\]

There are also some specific nanosatellite design parameters (solar panel deployment, amount of fuel in propulsion unit, etc) that are useful to monitor during mission, these parameters are included in additional vector \( \gamma \). Parameters \( \gamma \) are not directly included into motion model (1), (2), but they also affect on parameters \( b \).

Identification process is based on the idea of minimizing square differences between observed measurements and measurement model which has variable parameters. Parameter’s values which provide minimum of differences are the most close to real. This process can be described by equation (3).

\[
J(b, T) = \sum_{\alpha=X,Y,Z} \sum_{i=1}^{N} (C_\alpha(t_i) - \hat{C}_\alpha(b)(t_i))^2 \rightarrow \min,
\]

(3)
where $C_\alpha(t_i)$ - observed measurements, $\hat{C}_\alpha(b,t_i)$ - measurement model, $\alpha$ - measurement axis, $T$ - time interval of data collection, $N=T/\Delta t$ - number of measurements, $\Delta t$ - measurement time step.

2.3. Measurement models

Effectiveness of identification directly depends on sensor characteristics. Sensors used on nanosatellites can be divided into two groups according to the physics of measurement: inertial sensors and vector sensors. The inertial sensors include gyroscope (GYR) and accelerometer. The vector ones include three-axis magnetometer (TAM), horizon sensor, solar sensor, star tracker, etc.

The measurement system, which includes a combination of GYR and TAM, has received the greatest application at nanosatellite design [3], because these sensors have low cost, acceptable accuracy and small dimensions.

In this paper, GYR measurements $\omega(t_i)$ are simulated by solving the dynamic Euler equations (1). Stochastic measurement errors (measurement noise) are simulated by adding to $\omega(t_i)$ random variables vector $\omega_n(\sigma_\omega)$, which components are independent and distributed according to the normal laws with zero mathematical expectations and given standard deviation $\sigma_\omega$. Thus, GYR measurements model is written in the form:

$$C_\omega(b,t) = \omega(t_i) = \omega(b,t) + \omega_n(\sigma_\omega).$$

(4)

TAM sensor measurements are simulated using the following equation:

$$C_\beta(b,t) = \beta(t_i) = A(b,t)B_{orb} + \omega_n(\sigma_\beta).$$

(5)

where $A(b,t)$ is a rotation matrix from orbital coordinate system to body coordinate system, $B_{orb}$ - Earth magnetic field induction vector in orbital coordinate system calculated by IGRF model, $\omega_n(\sigma_\beta)$ - measurement noise.

Thus the objective function (3) is transformed to:

$$J_\omega(b,T) = \sum_{a=x,y,z} \sum_{i=1}^{N} (\omega_{a}(t_i) - \hat{\omega}_{a}(b,t_i))^2 \rightarrow \min,$$

(6)

in case of using GYR sensor, $\omega_{a}(t_i)$ - obtained angular velocity measurements, $\hat{\omega}_{a}(b,t_i)$ - measurements calculated by model (4). And for magnetic measurements it is described by:

$$J_\beta(b,T) = \sum_{a=x,y,z} \sum_{i=1}^{N} (\beta_{a}(t_i) - \hat{\beta}_{a}(b,t_i))^2 \rightarrow \min,$$

(7)

where $\beta_{a}(t_i)$ - obtained measurements, $\hat{\beta}_{a}(b,t_i)$ - measurements calculated by (5).

2.4. Identification problems formulation

Thus we can formulate following identification problems:

- Choice of $b$ that allows to calculate $\gamma$;
- Definition of model $b = f(\gamma)$;
- Choice of data collection interval $T$.

Solution of these problems is illustrated on the example of SamSat-M nanosatellite which is under developing in Samara University.

3. Problems illustration

The mission goal of SamSat-M is testing of orbital maneuver technology and propulsion unit (PU) [4]. The design parameters of SamSat-M change during orbital flight according to PU operation. Dimensionless coefficient of inertia $\mu$ changes in range $[-0.81 -0.78]$ and $\lambda$ - $[0.979 0.982]$. Fuel mass of PU is 0.4 kg while nanosat mass is 3.2 kg.
In the mission for the flight-testing of PU, the main monitoring parameter is fuel level \( h_f \) \((\gamma = [h_f])\), which changes in range \([0 \ 0.07]\) m. The SamSat-M nanosatellite does not have high-accuracy onboard sensors and orientation determination algorithms. It also has a significant margin of static stability. Therefore, dynamic parameters \( W \), orientation parameters \( U \), inertial parameters \( P \) and aerodynamic moment coefficients \( K \) are included in the vector of estimated parameters. So in this case the vector of estimated parameters has the form: \( \mathbf{b} = [W \ U \ P \ K] \) or in extended form:

\[
\begin{bmatrix}
    b_1 \\
    b_2 \\
    b_3 \\
    b_4 \\
    b_5 \\
    b_6 \\
    b_7 \\
    b_8 \\
    b_9 \\
    b_{10} \\
    b_{11} \\
    b_{12} \\
    b_{13} \\
    b_{14} \\
    b_{15} \\
    b_{16}
\end{bmatrix}
\]

3.1. Choosing parameter \( b_i \) for estimating \( \gamma \)

Components of vector \( \mathbf{b} \) have different sensitivity to vector \( \gamma \), so sensitivity study leads to analysis of sensitivity matrix \( H \) (or matrix of sensitivity coefficients) which is written in the following form:

\[
H = \begin{bmatrix}
    \frac{\partial b_1}{\partial \gamma_1} & \cdots & \frac{\partial b_i}{\partial \gamma_j} & \cdots & \frac{\partial b_k}{\partial \gamma_1} \\
    \vdots & \ddots & \vdots & \ddots & \vdots \\
    \frac{\partial b_i}{\partial \gamma_j} & \cdots & \frac{\partial b_k}{\partial \gamma_j} \\
    \vdots & \ddots & \vdots & \ddots & \vdots \\
    \frac{\partial b_i}{\partial \gamma_k} & \cdots & \frac{\partial b_k}{\partial \gamma_k}
\end{bmatrix}_{k \times m}
\]

Of the sensitivity coefficients, the largest modulus is selected for the corresponding \( \gamma_j \), and the corresponding parameter of the motion model is used to calculate the design parameter. In this example matrix \( H \) has a form:

\[
H = \begin{bmatrix}
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

Zeros mean that initial angular velocities and orientation angles do not depend on fuel level \( h_f \). To find values of matrix \( H \) we should define model \( \mathbf{b} = f(\gamma) \).

3.2. Definition of model \( \mathbf{b} = f(\gamma) \)

We obtain models \( \mathbf{b} = f(\gamma) \) and calculate matrix \( H \). The most sensitive value to \( h_f \) is dimensionless coefficient of inertia \( \mu \) the model of which is written as follows:

\[
\mu(h_f) = \frac{\left( I_{xx} - I_{zz} + M_s \left( X_{cm} - \frac{M_c X_{cm}}{Q_7} \right) + Q_6 \right) - Q_1 + 0.5 m_f r_f^2 + m_f \left( \frac{M^2 X^2}{(M_s + m_f)^2} - Q_4 \right) - Q_2 - Q_3}{I_{xx} + Q_1 + Q_2 + Q_3}
\]

where \( Q_1 = \frac{m_f (h_f^2 + 3 r_f^2)}{12} \), \( Q_2 = m_f \left( \frac{h_f}{2} - Q_5 + L_4 \right)^2 + Q_4 \), \( Q_3 = M_s (Q_6 + Y_{cm}^2) \), \( Q_4 = \frac{M^2 Z^2}{(M_s + m_f)^2} \), \( Q_5 = \frac{1}{Q_7} m_f (0.5 h_f + L_4) + M_s Y_{cm} \), \( Q_6 = (Z_{cm} - \frac{M Z_{cm}}{Q_7})^2 \), \( Q_7 = m_f + M_s \),

\( m_f = \pi h_f r_f^2 \rho_f \) - fuel mass, \( h_f \) - fuel level, \( \rho_f \) - fuel density, \( r_f \) - fuel tank radius, \( [I_{xx} \ I_{ys} \ I_{zz}] \) - satellite inertia moments without fuel, \( L_4 \) - distance between satellite front edge and satellite fuel tank front edge. Comparison of derivatives values are show in figure 1.
Thus we can say that parameter $\mu$ is the most sensitive one and it will be used to calculate fuel level.

3.3. Choice of data collection interval $T$

To calculate time interval of data collection it is necessary to study how the objective function (3) is sensitive to parameter $\mu$. Sensitivity can be calculated as partial derivative $\frac{\partial J(b,T)}{\partial \mu}$. As nanosatellite is non-linear dynamic object this derivative is calculated numerically. We assume vector $\Delta b$ with all constant parameters except variable $\mu$ and vector $b_{true}$, which consists of true parameter values. We also assume small addition $\Delta \mu = 0.0001$. Thus we obtain derivative using the following equation:

$$\frac{\partial J(b,T)}{\partial \mu} = \frac{J(\Delta b,T) - J(b_{true},T)}{\Delta \mu},$$

(8)

it should be mentioned that $J(b_{true},T) = 0$ because there are no differences between obtained measurements and model measurements. Examples of derivatives $\frac{\partial J_{\omega}(b,T)}{\partial \mu}$ and $\frac{\partial J_{B}(b,T)}{\partial \mu}$ are shown in figure 2.

Figure 1. Derivatives values.

Figure 2. Derivatives $\frac{\partial J_{\omega}(b,T)}{\partial \mu}$ and $\frac{\partial J_{B}(b,T)}{\partial \mu}$. 
Figure 2 shows that the sensitivity of objective function to inertial coefficient $\mu$ is increasing with the increase of the interval of data collection. It should be mentioned that derivative calculation should be made several times due to a priori uncertainty of some $b$ parameters, for example angular velocities and attitude. In this work we calculate derivatives for 400 times and average the result.

To calculate maximum estimation error of fuel level we use formula:

$$\Delta h_{f,\text{max}} = 3 \left( \frac{\partial \mu}{\partial h_f} \right)^{-1} \sigma \left( \frac{\partial J(b,T)}{\partial \mu} \right)^{-1}. \quad (9)$$

If it is necessary to estimate components of vector $b$ the following equation should be used:

$$\Delta b_i = 3 \sigma^2 \left( \frac{\partial J(b,T)}{\partial b_i} \right)^{-1}.$$

4. Results
According to formula (9) we achieve the following results in figures 3 and 4:

![Figure 3. $\Delta h_f$ estimation error for different GYR sensor.](image1)

![Figure 4. $\Delta h_f$ estimation error for different TAM sensors.](image2)

As it can be seen estimation error decreases due to increasing of information content of measurements. Obtained results can be used to choose time interval of data collection according to acceptable estimation error.
5. **Discussion and Conclusion**

The study of identification problem was made. It shows how they effect on identification procedure. The paper provides an efficient approach of how to obtain model of parameter dependence and choice of interval of data collection. The process of obtaining numerical derivative of objective function is shown.

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