The Lightest Scalar Nonet as Higgs Bosons of Strong Interactions

Nils A. Törnqvist∗

Department of Physical Sciences
University of Helsinki
POB 64, FIN–00014, Finland

Abstract

I discuss how an extra light scalar meson multiplet could be understood as an effective Higgs nonet of a hidden local $U(3)$ symmetry. There is growing evidence that low energy data requires in addition to a conventional $3P_0 \bar{q}q$ nonet near 1.4 GeV, another light scalar nonet-like structure below 1 GeV, $(\sigma(600), a_0(980), f_0(980), \kappa)$, which could be interpreted as such a Higgs nonet.

Pacs numbers: 11.15.Ex, 11.30.Hv, 12.39.Fe, 14.80.Cp

The mesons with vacuum quantum numbers are known to be crucial for a full understanding of the symmetry breaking mechanisms in QCD, and presumably also for confinement. The lightest scalar mesons have been controversial since their first observation over thirty years ago. Due to the complications of the nonperturbative strong interactions there is still no general agreement as to where are the $q\bar{q}$ states, whether there is necessarily a glueball among the light scalars, and whether some of the too numerous scalars are multiquark, $K\bar{K}$ or other meson-meson bound states. The main problem is that there are too many light scalars below 1.5 GeV.

A likely solution [3] is that in addition to a $q\bar{q}$ nonet and a glueball above 1.2 GeV, there is another nonet of more complicated nature below 1 GeV $(\sigma(600), a_0(980), f_0(980), \kappa(\sim 800))$

∗e-mail: nils.tornqvist@helsinki.fi
i.e., 18 scalar states in all. The latter nonet should have large 4-quark and meson-meson components. There is a heated current debate as to whether the $\sigma$ and especially the $\kappa$ really are true resonances or just due to very strong attractions in the $\pi\pi$ and $K\pi$ channels. Here we do not want to enter into this debate, we shall only assume that one can approximately model these effects by effective fields. We discussed this with Close in more detail in a recent review [3].

The $\sigma$ is sometimes called a Higgs boson of strong interactions since in a simple NJL model and in a linear sigma model the $\sigma$ acts like a Higgs giving the constituent $u$ and $d$ quarks most of their mass, and one has the celebrated Nambu relation $m_{\sigma} = 2m_{u}^{\text{const}}$. But, in such models one generally breaks only a global symmetry spontaneously. For a true analogy with the Higgs mechanism one should have a local symmetry which is broken spontaneously or dynamically. Can one construct [4] such a model?

I shall argue that two coupled linear sigma models may provide a first step for an understanding of this and of such a proliferation of 18 light scalar states. After gauging a hidden $U(3)$ symmetry one can then look at the lightest scalars as Higgs-like bosons for the nonperturbative low energy strong interactions.

Let me first remind the reader of the simple $U(N_f) \times U(N_f)$ linear sigma model [5] which includes one scalar and one pseudoscalar multiplet. As well known this agrees with chiral perturbation theory at the lowest order in $p^2$ [6], but includes explicit scalars. The scalar nonet is put into the hermitian part of a $3 \times 3$ matrix $\Phi$ and the pseudoscalar nonet into the anti-hermitian part of $\Phi$. One has $\Phi = S + iP = \sum_{a=0}^{8}(\sigma_a + ip_a)\lambda_a/\sqrt{2}$, where $\lambda_a$ are the Gell-Mann matrices, and $\lambda_0 = (2/N_f)^{1/2}1$. Then the potential

$$V(\Phi) = -\frac{1}{2}\mu^2\text{Tr}[\Phi\Phi^\dagger] + \lambda\text{Tr}[\Phi\Phi^\dagger\Phi\Phi^\dagger] + \lambda'(\text{Tr}[\Phi\Phi^\dagger])^2 + \mathcal{L}_{SB},$$

(1)

where $\lambda'$ is a small parameter compared to $\lambda$ (which breaks the scalar singlet mass from that of the octet) and where $\mathcal{L}_{SB}$ contains a flavor symmetry breaking term $\propto \text{Tr}(\Phi M_q + M_q\Phi^\dagger)$ (where $M_q$ is the diagonal matrix composed of $m_u, m_d, m_s$, and an $U_A(1)$ breaking term $\propto (\det\Phi + \det\Phi^\dagger)$, is not a too bad representation of the lightest pseudoscalars and
scalars, already at the tree level. If five of the six parameters are fixed by the experimental $m_\pi^2, m_K^2, (m_\eta^2 + m_\eta'^2), f_\pi$ and $f_K$, one finds with a small sixth parameter ($\lambda'$) the scalar nonet to be near 1 GeV (a very broad $\sigma$ near 650 MeV, an $a_0$ at 1040 MeV, an $f_0$ near 1200 MeV, and a very broad $\kappa$ near 1120 MeV [7]). This is quite reasonable considering that unitarizing a similar model can, and in fact and does [8], shift these states in the second sheet by hundreds of MeV. The essential features we recall here is that neglecting the $U_A(1)$ term one has, after a shift to the minimum $\Phi \to \Phi + v1$ (where $v^2 = m^2/(4\lambda) + O(m_q)$, a nearly massless pseudoscalar nonet of squared mass of $O(m_q)$ and a massive scalar nonet of squared mass $= 2m^2 + O(m_q)$.

Now, for two scalar nonets in a chiral model we need two such $3 \times 3$ matrices $\Phi$ and $\hat{\Phi}$. (Let the scalar $q\bar{q}$ states above 1 GeV be in $\Phi$, and let those below 1 GeV be in $\hat{\Phi}$). Then model both $\Phi$ and $\hat{\Phi}$ by a gauged linear sigma model, but with different sets of parameters ($\mu^2, \lambda$) and ($\hat{\mu}^2, \hat{\lambda}$). For $\Phi$ without any symmetry breaking nor a $\lambda'$ term we have simply

$$L(\Phi) = \frac{1}{2} \text{Tr}[D_\mu \Phi D_\mu \Phi^\dagger] + \frac{1}{2} \mu^2 \text{Tr}[\Phi \Phi^\dagger] - \lambda \text{Tr}[\Phi \Phi^\dagger \Phi \Phi^\dagger],$$

(2)

and similarly for $\hat{\Phi}$:

$$\hat{L}(\hat{\Phi}) = \frac{1}{2} \text{Tr}[D_\mu \hat{\Phi} D_\mu \hat{\Phi}^\dagger] + \frac{1}{2} \hat{\mu}^2 \text{Tr}[\hat{\Phi} \hat{\Phi}^\dagger] - \hat{\lambda} \text{Tr}[\hat{\Phi} \hat{\Phi}^\dagger \hat{\Phi} \hat{\Phi}^\dagger].$$

(3)

Neglect to begin with the gauging. We have doubled the spectrum and initially we have two scalar, and two pseudoscalar multiplets, altogether 36 states for three flavors.

These lagrangians are invariant under a global symmetry: $\Phi \to L\Phi R$ and $\hat{\Phi} \to L\hat{\Phi} R$, where $L$ and $R$ are independent $U(3) = SU(3) \times U(1)$ transformations. If there were no coupling between $\Phi$ and $\hat{\Phi}$ the symmetry would be even larger as the $U(3)$ transformations on $\Phi$ could be independent of those on $\hat{\Phi}$. We refer to that symmetry as the relative symmetry. But, it is natural to introduce a small coupling [9]) between the two sets of multiplets, which breaks this relative symmetry [4].

The full effective Lagrangian for both $\Phi$ and $\hat{\Phi}$ thus becomes,

$$L_{tot}(\Phi, \hat{\Phi}) = L(\Phi) + \hat{L}(\hat{\Phi}) + \frac{\epsilon^2}{4} \text{Tr}[\Phi \hat{\Phi}^\dagger + h.c.].$$

(4)
If $\Phi_a$ is interpreted as $q\bar{q}$ and $\hat{\Phi}_a$ as $q\bar{q}q\bar{q}$ states then the $\epsilon^2$ term would allow for $q\bar{q} \rightarrow q\bar{q}q\bar{q}$ transitions [10]. This Lagrangian is still invariant under the above $U(3) \times U(3)$ symmetry, but not under the relative symmetry when $\Phi$ is transformed differently from $\hat{\Phi}$.

Now as a crucial assumption (differently from [9]), let both $\Phi$ and $\hat{\Phi}$ have vacuum expectation values (VEV), such that $v = <\sigma_0> / \sqrt{N_f} \neq 0$ and $\hat{v} = <\hat{\sigma}_0> / \sqrt{N_f} \neq 0$ even if $\epsilon = 0$. Then one has $v^2(\epsilon) = (\mu^2 + \epsilon^2 \hat{v}/v)/(4\lambda)$, and $\hat{v}^2(\epsilon) = (\hat{\mu}^2 + \epsilon^2 v/\hat{v})/(4\hat{\lambda})$. If $\epsilon$ would vanish all pseudoscalars would be massless, but with $\epsilon \neq 0$ the $2 \times 2$ submatrix between two pseudoscalars with same flavor becomes:

$$m^2(0^+) = \begin{pmatrix} 4\lambda v^2(\epsilon) - \mu^2 & -\epsilon^2 \\ -\epsilon^2 & 4\hat{\lambda}\hat{v}^2(\epsilon) - \hat{\mu}^2 \end{pmatrix} = +\epsilon^2 \begin{pmatrix} \hat{v}/v & -1 \\ -1 & v/\hat{v} \end{pmatrix}, \quad (5)$$

which is diagonalized by a rotation $\theta = \arctan(v/\hat{v})$, such that the eigenvalues are 0 and $\epsilon^2 v\hat{v}/(v^2 + \hat{v}^2)$:

$$\begin{pmatrix} c & -s \\ s & c \end{pmatrix} m^2(0^+) \begin{pmatrix} c & s \\ -s & c \end{pmatrix} = \epsilon^2 \begin{pmatrix} v^2 + \hat{v}^2/v\hat{v} \\ 1 \\ 0 \end{pmatrix}. \quad (6)$$

Here $s = \sin \theta \propto v$ and $c = \cos \theta \propto \hat{v}$. Thus the two originally massless pseudoscalar nonets mix through the $\epsilon^2$ term, with a mixing angle $\theta$, such that one nonet remains massless, while the other nonet obtains a mass $\epsilon^2(v^2 + \hat{v}^2)/v\hat{v}$. This is, of course, just what is expected, since we still have one exact overall $U(3) \times U(3)$ symmetry, while the relative symmetry is broken through the $\epsilon$ term.

The approximation is valid only if neither $v$ nor $\hat{v}$ vanishes. Thus one has one massive $|\pi>$ and one massless $|\hat{\pi}>$ would-be pseudoscalar multiplet. Denoting the the original pseudoscalars $|p>$ and $|\hat{p}>$, we have $|\pi> = c|p> - s|\hat{p}>$, and $|\hat{\pi}> = s|p> + c|\hat{p}>$. The mixing angle is determined entirely by the two vacuum expectation values, and is large if $v$ and $\hat{v}$ are of similar magnitudes, independently of how small $\epsilon^2$ is, as long as it remains finite. On the other hand the scalar masses and mixings are only very little affected if $\epsilon^2/(\mu^2 - \hat{\mu}^2)$ is small. They are still close to $\sqrt{2}\mu$ and $\sqrt{2}\hat{\mu}$ as in the uncoupled case.

In order that this should have anything to do with reality, one must of course get rid
of the massless Goldstones. By gauging the overall axial symmetry (\( \Phi \to H \Phi H \) and \( D_\mu = \partial_\mu - ig/2(\lambda_a A_a + A_a \lambda_a) \)) and reparameterizing the fields

\[
\Phi = v1 + (\sigma_a + ip_a) \frac{\lambda_a}{\sqrt{2}} \to H'[v1 + (\sigma_a + ic\pi_a) \frac{\lambda_a}{\sqrt{2}}]H',
\]

(7)

\[
\hat{\Phi} = \hat{v}1 + (\hat{\sigma}_a + i\hat{p}_a) \frac{\lambda_a}{\sqrt{2}} \to H'[^{\hat{v}}1 + (\hat{\sigma}_a - is\pi) \frac{\lambda_a}{\sqrt{2}}]H'.
\]

(8)

Here \( H' \) is a fixed gauge for the axial symmetry

\[
H' = \exp\left[ \frac{i\pi_a \lambda_a}{2\sqrt{2} (v^2 + \hat{v}^2)} \right].
\]

(9)

The validity of these reparametrizations can be seen most easily by expanding \( H' = 1 + i\frac{c}{2\hat{v}} \frac{\pi_a \lambda_a}{\sqrt{2}} \ldots = 1 + i\frac{c}{2\hat{v}} \frac{\pi_a \lambda_a}{\sqrt{2}} \ldots \). Thus by choosing a special gauge for the hidden symmetry \( H \) the \( \hat{\pi}_a \) fields vanish from the spectrum. The axial symmetry \( H \) remains as a hidden symmetry while the \( \hat{\pi} \) fields are gauged away. But, these degrees of freedom enter instead as longitudinal axial vector mesons and give these mesons (an extra) mass \( (m_A^2 = 2g^2(v^2 + \hat{v}^2)) \). This is like the conventional Higgs mechanism and it has similarities to the original Yang-Mills theory and the work of Bando et al. [11] on hidden local symmetries, in that mesons are gauge bosons, but is different both in the scalar particle spectrum and in the realization of the hidden symmetry. The axial vector-pseudoscalar-scalar couplings (\( APS \)) can be read off from the lagrangian. For the \( \sigma \) multiplet one finds \( gcA_{\mu,\alpha}[\pi_b \partial_\mu \sigma_c + \sigma_b \partial_\pi_c]Tr(\lambda_a \lambda_b \lambda_c)_{+}/4 \), while for the \( \hat{\sigma} \) multiplet \( c \) is replaced by \( s \). Other trilinear couplings also follow, in particular for scalar to 2 pseudoscalar couplings (\( SPP \)) one has: \( g_{\sigma_a \pi_b \pi_c} = vcsTr(\lambda_a \lambda_b \lambda_c)_{+}/\sqrt{2} \) and, \( g_{\sigma_a \pi_b \pi_c}/g_{\sigma_a \pi_b \pi_c} = v/\hat{v} = \tan \theta \).

Now, having gauged away the massless Goldstones one can interpret the massive pseudoscalars as the physical pseudoscalars. The would-be axial current related to the overall hidden symmetry is like the \( \hat{\pi} \) gauged away, while the explicitly broken relative symmetry defines a current which is only "partially conserved" when \( \epsilon \) differs from 0. Denoting the axial vector current obtained from \( \mathcal{L}(\Phi) \) by \( j_{A\mu,a} = \sqrt{N_f}v\partial_\mu p_a + \ldots \) and the one from \( \hat{\mathcal{L}}(\hat{\Phi}) \) by \( \hat{j}_{A\mu} = \sqrt{N_f}\hat{v}\partial_\mu \hat{p}_a + \ldots \), then both currents would before gauging be conserved if \( \epsilon = 0 \)
because of the masslessness of both 0− nonets. Adding the ϵ term the sum \( j_{A\mu} + \hat{j}_{A\mu} \) would still be exactly conserved, because of the \( H \) symmetry and since it would be \( \propto \partial_{\mu} \hat{\pi} \), but this current is like the \( \hat{\pi} \) gauged away. On the other hand \( j_{A\mu,a} \) or \( \hat{j}_{A\mu,a} \) alone is only “partially conserved”, \( \partial_{\mu} j_{A\mu,a} = -\partial_{\mu} \hat{j}_{A\mu,a} = \sqrt{N_f} v \hat{v} / \sqrt{(v^2 + \hat{v}^2)m^2_{\pi} \pi_a} \), because the \( \epsilon^2 \) term explicitly breaks the relative symmetry when the \( \pi \) nonet obtains mass. Identifying this with PCAC one has

\[
\begin{align*}
  f_\pi &= \frac{\sqrt{N_f} v \hat{v}}{\sqrt{(v^2 + \hat{v}^2)}} , \\
  m^2_{\pi} &= \epsilon^2 (v^2 + \hat{v}^2)/v\hat{v} .
\end{align*}
\]  

Comparing this with the conventional relation \( m^2_\pi = 2B\hat{m}_q \), where \( \hat{m}_q \) is the average chiral quark mass one sees that \( \epsilon^2 \) should be proportional to \( \hat{m}_q \). In fact a natural way to break flavor symmetry is obtained by replacing

\[
\frac{\epsilon^2}{4} \text{Tr}[\Phi \hat{\Phi}^\dagger + h.c.] \rightarrow \frac{B'}{2} \text{Tr}[\Phi M_q \hat{\Phi}^\dagger + h.c.] ,
\]  

Then the \( v1 \) and \( \hat{v}1 \) will be replaced by a diagonal matrix with elements \( v_{ii} \) which includes corrections due to unequal quark masses and satisfy \( v^2_{ii} = (\mu^2 + 2B'm_q \hat{v}_i/v_{ii})/(4\lambda)\) \( \hat{v}_i = u\bar{u}, d\bar{d}, s\bar{s} \) and a similar equation for \( \hat{v}_i \). For small \( m_{qi} \) one then recovers the usual relations that squared pseudoscalar masses are \( \propto (m_{qi} + m_{qj}) \), whereas the two scalar nonets as well as the vectors get split by the equal spacing rule.

If the \( \hat{\sigma} \) nonet is predominantly of the 4-quark form of Jaffe \(^{12}\) it should, with the appropriate symmetry breaking term, before unitarization obey the inverted mass spectrum with the \( a_0(980) \) and \( f_0(980) \) as the heaviest followed by the \( \kappa \) and the \( \sigma(600) \).

Also, if \( \hat{v}_{i} << v_{i} \) one recovers for the lighter multiplet (\( \hat{\sigma}_a \)) the predictions of the simple \( U(3) \times U(3) \) model discussed above in connection with Eq. (1). (The term \( \text{det} \Phi + h.c. \) in \( \mathcal{L}_{SB} \) of Eq. (1) would here be replaced by \( \propto \text{det} \Phi \hat{\Phi} + h.c. \) From the fact that \( \hat{\mu} < \mu \) and that the \( SPP \) couplings of the lower multiplet should be larger than the heavier one expects, in fact, that \( \hat{v}_a < v_a \) or \( \tan \theta_i > 1 \), but it is crucial that both \( v_{ii} \neq 0 \), and \( \hat{v}_{ii} \neq 0 \).

The main prediction of this scheme is that one have doubled the light scalar meson
spectrum, as seems to be experimentally the case. Of course in order to make any detailed comparison with experiment one must include loops and unitarize the model, which is not a simple matter as the couplings are very large.

The dichotomic role of the pions in conventional models, as being at the same time both the Goldstone bosons and the $q\bar{q}$ pseudoscalars, is here resolved in a particularly simple way: One has originally two Goldstone-like pions, out of which only one remains in the spectrum, and which is a particular linear combination of the two original pseudoscalar fields.

Both of the two scalar multiplets remain as physical states and one of these (formed by the $\sigma(600)$ and the $a_0(980)$ in the case of two flavors), or the $\sigma$, $a_0(980)$, $f_0(980)$ and the $\kappa$ in the case of three flavors can then be looked upon as effectively a Higgs multiplet of strong nonperturbative interactions when a hidden local symmetry is spontaneously broken.

One may ask is there any other source for the symmetry breaking term (12), except for the chiral quark masses put in by hand? The Syracuse group [9] argues for instanton effects. Another way of reasoning is that with quarks and quark loops there would be anomalous couplings $AVV$ for each flavor [13]. Anomaly related loops (like $P \rightarrow VV \rightarrow P$) could then be another source of the symmetry breaking.

I. ACKNOWLEDGEMENTS

Support from EU-TMR program, contract CT98-0169 is gratefully acknowledged. I thank J. Schechter, Syracuse for emphasizing that it should be the axial vectors which get mass through this mechanism.
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