On the selection of an actuating device for a gas-dynamic drive of an object, performing a flat turning maneuver

A B Kondratyev
Moscow Aviation Institute (National Research University), 4, Volokolamskoe shosse, Moscow, 125993, Russia
E-mail: kondr48@mail.ru

Abstract. This paper contains an analysis of the control efficiency influence on the actuating medium supply amount of the gas-dynamic drive of an object, performing a flat turning maneuver, with two different types of actuating devices; it also defines the required momentum impulses. The required momenta are determined depending on the initial position of the object and the time of its transition to the final position. The influence of the actuating device time constant on the control efficiency has been evaluated, and the value of the relative operation speed, at which the engine time constant does not affect the transition time from the initial state to the final one, has been defined.

1. Introduction
While solving the problem of synthesizing the control system (or the orientation system) of an aircraft, which is equipped with a gas-dynamic drive, the selection of a feasible control algorithm allows to generate an unambiguous correlation of the implementation of these algorithms with the power indicators of the drive, and, therefore, with the type and parameters of its actuating device. For modern systems, containing a gas-dynamic drive, the volume and mass of the actuating medium source are comparable to the volume and mass of the engine and auxiliary equipment; therefore, the actuating medium supply amount in its source, and, thus, the momentum can serve as a quantitative measure of the influence of control algorithms for the systems of the same type. This paper reviews two types of actuating devices: with controllable (CAD) and uncontrollable (UAD) flow of the actuating medium. As an example of the first type of actuating devices, a vortex amplifier is reviewed. For the second type of devices, the paper reviews a tristable supersonic actuating jet engine, based on the Coanda effect, which is the effect of a gas jet “sticking” to a stationary wall due to the ejecting properties of the gas jet itself [1, 2]. These actuating devices were selected due to the fact that they allow simultaneously changing both the direction of the thrust vector and the value of the generated thrust.

2. Control efficiency determination
In general, the control problem during the performance of a flat turning maneuver has a multivalued solution, since the selected control laws that meet the optimum control criteria in terms of operation speed or flow rate do not give an answer, concerning either the type of the
actuating device or the choice of the energy carrying medium. Unambiguity can be achieved for specific classes of objects and engine types through the introduction of additional limitations and application conditions. In this case, such limitations include the following: restrictions on the phase coordinates, restrictions on the dimensions and weight of the entire control system, indication of the energy carrier type, preferable for operation.

Objects, equipped with a gas-dynamic drive and performing a flat turning maneuver, always have limitations, imposed on the initial phase coordinates, due to the object rotation angle being a multiple of either $\pi$ or $2\pi$ rad and due to the presence of a limiting (maximum) initial velocity, predicted by means of statistical processing of the real object behavior. The limitations, imposed on the dimensions and weight of the entire system, lead to the fact that the stored energy and, therefore, the available impulse momentum are also limited. The required control algorithms (synthesized control laws) are found using the quality criteria, taking into account the constraints on the available momentum impulse and the limitations, related to the initial phase coordinates [3, 4].

The selection of the required control efficiency, optimum in terms of operation speed or actuating medium flow rate, for an object, equipped with such a drive and performing a turning maneuver, is reviewed in [5, 6]. This paper provides the results of the control efficiency analysis and defines the required momentum impulses.

Figure 1. Control efficiency influence on the required amount of the actuating medium for CAD.

The value of the required control efficiency $\mu = M_y / J$, where $J$ is the object moment of inertia, $M_y$ is the control moment, depending on the initial state and transition time, is defined, using equations [6, 7]:

$$\theta_0 = \frac{1}{4} \mu T^2 + \frac{1}{2} \dot{\theta}_0 \chi T - \frac{\dot{\theta}_0^2}{4\mu},$$

where

$$\chi = \text{sign}(\dot{\theta}_0, \ddot{\theta}) = \begin{cases} +1, & \text{if sign } \dot{\theta}_0 = \text{sign } \dot{\theta}(t), \\ -1, & \text{if sign } \dot{\theta}_0 = - \text{sign } \dot{\theta}(t), \end{cases}$$

$$T = \frac{\theta_{0\text{max}}}{\mu} \text{sign} \left( \frac{\dot{\theta}_{0\text{max}}}{\dot{\theta}_{0\text{max}}} \right) \sqrt{\frac{\dot{\theta}_{0\text{max}}}{\mu}} + \frac{\dot{\theta}_{0\text{max}}^2}{2\mu^2},$$

$\theta_0, \dot{\theta}_0$—initial phase parameters; $T$—transition time; $\theta_{0\text{max}}, \dot{\theta}_{0\text{max}}$—limit initial phase parameters.
If transition durations in the actuating devices to be compared are considered equal and the same initial states are used, then it is possible to calculate the corresponding control efficiency values. Besides, the amount of calculations can be reduced significantly if the initial state has the following form:

$$\bar{\theta}_0 = (\theta_0, \dot{\theta}_0\chi = 0).$$

For CAD, the influence of the control efficiency $\mu_1 = M_{y1}/J$ on the actuating medium supply amount is estimated, using the following equation:

$$X = \frac{G}{G_0} = \pm \frac{\bar{\theta}_0 + 2 \sqrt{\frac{\ddot{\theta}_0}{\mu} + \frac{|\ddot{\theta}_0|\mu_1}{\mu}} \left(1 - \sqrt{1 - \frac{1}{\mu_1}}\right)}{\mu_1},$$

(2)

where $\bar{\theta}_0 = \theta_0/\mu T^2$, $\ddot{\theta}_0 = \dot{\theta}_0/\mu T$ are dimensionless initial phase parameters of the object; $X$ is the relative supply amount of the actuating medium.

Figure 1 shows a set of curves, illustrating the equation (2) for the different values of initial phase parameters of the object: curves 1, 2, 3, and 4 for $\dot{\theta}_0/\dot{\theta}_{0\text{max}} = -1; 0.5; 0; +1$, respectively.

3. Analysis of the control efficiency influence

An analysis of the control efficiency influence on the actuating medium amount allows to make the following conclusions:

- An increase in the control momentum with fixed transition time ($T = \text{const}$) allows, in the limit, to reduce the required amount of the actuating medium twice as compared with the necessary amount of the actuating medium, determined for the system that meets the optimum operation speed criterion.

- It is feasible to increase the control momentum 2.5–3 times ($\mu_1 = (2.5 \div 3)\mu$). Any further increase does not result in a significant change of the required actuating medium supply amount. In this case, the only control type that becomes possible and implementable is the one, optimum in terms of the actuating medium flow rate.

The consumption of the actuating medium in the gas-dynamic drive (which is equal to its flow rate multiplied by the operating time) is directly tied to the required momentum, since there is a one-to-one relationship between the flow rate and the drive thrust. Therefore, it can be assumed that the momentum and the operating time of the CAD and UAD type drive motors are interlinked by the following ratio:

$$I = M_y T;$$

$$I_1 = M_y T_1 + \gamma M_0 (T - T_1),$$

(3)

where $\gamma = M_0/M_{y1}$ is the momentum regulation range in CAD; $M_0$ is the minimum momentum in CAD without any control command; $I$ and $I_1$ are momentum impulses in UAD and CAD.

If the initial and final states of the object, as well as the transition time, are the same, then it becomes possible to correlate the control efficiency and moment impulses in UAD and CAD under different algorithms, using the following equations [6, 8]:

$$\frac{1}{4} \mu T^2 + \frac{1}{2} \dot{\theta}_0\chi T - \frac{\dot{\theta}_0^2}{4\mu} = \frac{\mu T_1 + \dot{\theta}_0\chi T}{2} \cdot (T - T_1) + \frac{1}{4} \mu_1 T_1 + \frac{\dot{\theta}_0\chi T_1}{2} - \frac{\dot{\theta}_0^2}{4\mu_1};$$

(4)

$$\frac{I}{I_1} = \frac{1 - \ddot{\theta}_0^2}{2(2 - k)} \left(1 + \gamma \frac{1 - k}{k}\right) \left[1 + \frac{\ddot{\theta}_0^2 k(2 - k)}{(1 - \ddot{\theta}_0^2)^2}\right],$$

(5)
where the relative time of full operation of CAD drive motor in relation to the UAD operation time is as follows:

\[ k = \frac{T_1}{T} = 1 - \sqrt{1 - \frac{\mu - \frac{\mu_0}{\mu_1} \dot{\theta}_0 (\mu_1 - \mu)}{\mu_1}}. \] (6)

Figure 2 shows the ratio of momentum impulses, calculated using equation (5) depending on the relative operating time of the drive motor at different regulation range values \( \gamma = 20; 10; 5; 3.3; 2.5 \). Solid curves correspond to the relative initial speed \( \dot{\theta}_0 = 0 \) and dashed curves correspond to the relative initial speed of \( \dot{\theta}_0 = 0.2 \) and \( \dot{\theta}_0 = 0.4 \) and the regulation range of \( \gamma = 10 \).

\[ I/I_1 = f(\gamma) \] Figure 2. Momentum impulses ratio.

\[ I_1/I = f(\gamma) \] Figure 3. Momentum impulses dependence on the regulation range.

Figure 3 demonstrates the inverse dependence \( I/I_1 = f(\gamma) \) on the regulation range, which was obtained by rearranging the extreme points shown in figure 2. The mentioned characteristic demonstrates that substantial benefits are observed at significant regulation ranges \( \gamma \leq 0.3 \) and at short operating times of the actuating devices; besides, such benefits are achieved by increasing the control efficiency. However, the mentioned studies, dedicated to evaluating the time constant of the actuating devices, indicate that an increase in control efficiency shall be accompanied either by a simultaneous increase of the relative operation speed \( \alpha \), or by a further enhancement of the control efficiency. The relative operation speed value changes in accordance with the equation \( \alpha = T/T_m \), where \( T_m \) is the time constant of the drive motor, \( T \) is the motor operation time.

This conclusion is obtained through the analysis of the dependence, which can be achieved by the joint review of the equations, correlating the initial state \((\dot{\theta}_0, \theta_0)\) and the active operation time of the actuating devices. Assuming \((\dot{\theta}_0 = 0, \theta_0 = 0)\), we obtain the following equation:

\[ \theta_0 = \mu_0 T^2 / 4 - \mu_0 (1 - e^{-0.5\tau T}). \] (7)

The combined review of equations (1) and (7) enables to represent the control efficiency in the following form, provided that \( \dot{\theta}_0 = 0 \)

\[ \mu_0 = (\alpha - 1 + e^{-\alpha}) = \mu \tau. \] (8)

Equation (8) allows to estimate the influence of the actuating device time constant on the control efficiency and find such a value of the relative operation speed, at which the time constant of the motor does not affect the transition time from \((\dot{\theta}_0, \theta_0)\) state into the end one.
The analysis of equation (5) indicates the following:

- The transition from the control mode, optimum in terms of operation speed, to the one, optimum in terms of flow rate, provides savings in the momentum impulse and, consequently, in the supply amount of the actuating medium.
- With a supply source that allows for ideal deep regulation $\gamma = \infty$ ($G_{\text{min}} = 0$), which is, for example, a compressed gas source, the benefits reach their maximum level and enable to reduce the required amount of the actuating medium twice (at $\bar{\theta}_0 = 0$).
- For solid propellant systems, the transition to the control mode, optimum in terms of flow rate, provides real benefits only at significant regulation ranges $\gamma = 3.3$.
- The characteristics of the impulse ratio variation depending on the relative operation time $K$ are defined by the regulation range value $\gamma$. For $\gamma \neq 0$, the impulse ratio is not defined by a monotonic function and has a minimum, which depends on $\gamma$ and $\theta_0$.
- If the values of the relative time of full motor operation remain the same, the presence of the initial speed reduces the impulse ratio value in comparison with the case when $\theta_0 = 0$.

It is worth mentioning that the transition to the control mode, optimum in terms of flow rate, is implemented by increasing the thrust of the drive motor and is necessarily accompanied by an increase in the motor operation speed.

![Figure 4](image-url)

**Figure 4.** Dependence of the regulation range on the relative operation time for CAD and UAD.

Taking into consideration the constraints on the impulse momentum, $\int_0^m M_0 dt \leq I$ and, therefore, the constraints on the impulse ratio $I_1/I$, it is possible to correlate the regulation range with the relative operation time for CAD and UAD

$$\frac{I_1}{I} = \frac{1 - \bar{\theta}_0^2}{2(2 - k)} \left( 1 + \gamma \frac{1 - k}{k} \right) \left[ 1 + \sqrt{\frac{\bar{\theta}_0^2 k(2 - k)}{(1 - \bar{\theta}_0^2)^2}} \right] \leq 1.$$  

If $\bar{\theta}_0 = 0$, the equation (7) turns into the following inequation:

$$\gamma \leq K.$$  

(10)
The expressions (9) and (10) allow to evaluate the benefits of each type of CAD and UAD at the predetermined relative motor operation time of engine and known regulation range \( \gamma \) for CAD, as well as formulate the requirements to the CAD type motors.

In this case, it should be noted that the presence of the initial speed \( \dot{\theta}_0 \neq 0 \) reduces the required regulation range, which substantially decreases the relative operation time value, at which CAD is preferable to CAD, as compared to the case when \( \dot{\theta}_0 = 0 \).

Figure 4 demonstrates the relationship (9) for the initial speed values of \( \bar{\dot{\theta}}_0 = 0.2; 0.4 \). It also contains a dashed line, demonstrating the dependence \( \gamma = k \). The allowable \( \gamma \) values are marked by hatching.

4. Conclusion
Therefore, based on the conducted studies, an analytic dependence was obtained, linking the required momentum impulses for a gas-dynamic drive, equipped with an actuating device of CAD or UAD type, which allows to select the motor type or to evaluate the power benefits of a particular motor type for the known regulation range and a predetermined transition time value.

References
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