Abstract. Determining the redshift of astronomical sources is fundamental for cosmological studies, since it allows deriving distances to the observer. However, the distance-redshift relationship commonly used is obtained from the isotropic, homogeneous Friedmann-Lemaître-Robertson-Walker (FLRW) metric. By fitting this distance-redshift relations to Supernovae Ia data, used by many authors as standard candles for distance measurements, it was shown that the expansion of the universe is accelerating, since the addition of equations of state $p < -\frac{1}{3} \rho$ in the cosmological model was required. Whether this dark energy is a cosmological constant or a quintessence is still to be determined, and is the objective of some current and future dark energy studies.

However, the Universe is not homogeneous. The gravitational lens effect changes the luminosity of distant sources, and it may also change significantly the angular diameter distance, thus modifying the luminosity distance, changing the distance-redshift relation with respect to that of an homogeneous universe. This effect can be considered an additional source of dispersion of the data, affecting the determination of the dark energy equation of state.

We are developing a test for studying departures from the FLRW distance-redshift relation, and its impact on the determination of the dark energy equation of state.

1. Introduction.
The Friedmann-Lemaître-Robertson-Walker with $\Lambda$ (AFLRW) Cosmology is the current cosmological paradigm. Within it, the goal of determining the large scale geometrical structure of the Universe requires estimating the Hubble parameter $H_0$, the mass density parameter $\Omega_m$ and the cosmological constant $\Omega_\Lambda$.

However, AFLRW Cosmology is homogeneous an isotropic on all scales, so a fourth parameter is needed for taking into account the inhomogeneity of the real Universe. This inhomogeneity affects the model via gravitational lensing.

2. Angular Diameter And Distance Measurement [1]
In a FLRW Cosmology, expressions can be derived for the observed angular diameter $\Theta$ and the observed brightness of an object with a known absolute diameter and absolute brightness, as a function of the redshift $z$. The function $\Theta(z)$ has a remarkable feature, the presence of a minimum when $z$ is approximately equal to $1/2$. Moreover, the maximum of the function $f(z) = rH/c\Theta$ (being $r$ the radius of the object) corresponds to a minimum of the angle $\Theta$. Such
behavior of $\Theta$ is caused by the curvature of space due to the matter filling the Universe, so in a homogeneous Universe, observations of $\Theta(z)$ can provide an independent determination of the average density of matter $\rho_0$.

The angular diameter $\Theta$ subtended by an object AB is determined by the two rays AO and BO emitted by the outer points of the object.

**Figure 1.** A mass situated between the rays bends the latter in such a way that $\Theta$ is increased. This type of bending is the physical reason for the features of the variation of $\Theta$ with $z$. The density of matter within the cone of light rays must be the same as the mean density of matter in the Universe; only in this case will the angle $\Theta$ be the same as that obtained in the uniform case.

**Inhomogeneous Universe**

For approximating the effect of the non-uniformity of density in the determination of the function $\Theta(z)$ let’s assume that we neglect the mass of the intergalactic medium by comparison with the mass of matter in galaxies. In that case, the angular diameter of objects with no galaxies within the cone subtended by them at the observer is $\Theta_1(z) \neq \Theta(z)$ and $\Theta_1(z)$ does not have a minimum.

**Figure 2.** Also, for extended objects, sized AB and that have a galaxy C within their cone, the angular diameter is given by $\Theta_2(z)$ and a ring-shaped blurred image appears, in the case of perfect alignment, or a more complex pattern if C does not lie on the axis of the cone.

In this case, it is not possible to derive any fixed expression for $\Theta_2(z)$, since $\Theta_2$ depends on the mass of C and the relative distances between AB, O and C.

$\Theta(z)$ represents, in a nonhomeneous Universe, a weighted mean of $\Theta_1(z)$ and all possible values of $\Theta_2(z)$. The blurring of extended sources in the non-homogeneous case makes it difficult to determine $\Theta(z)$.

### 3. Filling parameter

Then, homogeneous matter inside an observing beam of light, gravitationally focuses the beam in a very different way than a set of clumps of equal total mass. The simplest correction for this gravity-light effect leads to the previously mentioned fourth parameter: $\nu$ [2],[3],[4]

$$\frac{\rho_I}{\rho_0} = \frac{\nu(\nu + 1)}{6}; \ 0 \leq \nu \leq 2$$

($\nu = 0$ is for standard 100%-filled-beam FLRW case and $\nu = 2$ is for the empty-beam case).

The matter density of the Universe, $\rho_0$ is seen as a result of two contributions: homogeneous matter, $\rho_H$ (i.e. intergalactic medium) and inhomogeneous matter (i.e. galactic contribution): $\rho_0 = \rho_H + \rho_I$

In the distance-redshift articles series by Kantowski et al. (2000, 2001, 2003) the procedure required to obtain the distance-redshift relationship for partially filled beam FLRW observations is described. Moreover, the simplified results for the special cases $\nu = 0, 1, 2$ are calculated. They
also fit these new analytic Hubble curves to supernovae (SNe) data trying to determine both the mass parameter $\Omega_m$ and the beam-filling parameter $\nu$, and the data available comes out to be inadequate for limiting $\nu$.

In our work, we are tackling further studies for estimating realistic values of the filling parameter. In a realistic inhomogeneous Universe $\nu$ is a function of the direction and distance the observer is looking at. The further and more homogeneous region we look, the smaller is $\nu$.

Let’s assume the simplest case: a circular 2D non-uniform, non-isotropic field of point particles placed in an equispaced grid. We model non-uniformity by removing some of the particles, but maintaining uniformity away from the observer, as shown in the picture. We assume all particles have the same mass $m = 1$ and $R_5 = 1$. In that case, an estimation of densities and filling parameters is straightforward. The total and homogeneous densities are: $\rho_0 = 55.39$ and $\rho_H = 61.43$.

The results obtained agree with the expectation. For the annulus, $\nu$ values range from $\nu(R_1) = 1.24$ to $\nu(R_5 - R_4) = 0.15$, approaching zero as space far from the center gains uniformity. For the $\alpha_1$ case, $\nu$ values range from $\nu(R_1) = 1.50$ to $\nu(R_5 - R_0) = 0.70$, therefore non-uniformity is more evident. In the $\alpha_2$ case, all the values are smaller than 1, since this region is highly uniform.

In our estimations of $\nu$ for more realistic values, our first step is testing average inhomogeneity using the Millenium Simulation database [5] and the second step is using real catalogues, like the Supernova Cosmology Project [6].

One tricky point of the $\nu$ estimation is the direction-depending mass distribution calculation. Also, developing a numerical method for measuring distance-redshift relationship is required.

But on sufficiently large scales, the Universe is homogeneous and isotropic, so a estimation of the inhomogeneity length-scale is also important. Sarkar et al. [7] find this length-scale between 60 and 70 $h^{-1}Mpc$, in good agreement with earlier works.

4. Dark Energy equation of State

One of the big unknows of Cosmology is dark energy. Not only its nature is still to be established, but its equation of state is not fixed yet.

The dark energy equation of state relates density with pressure, through the $\omega$ parameter: $p = \omega \rho c^2$. A useful method of limiting the $\omega$ parameter is fitting observational data of good standard candles to distance-redshift relations. The main problem of this method are uncertainties, given the fact that constraining the dark energy equation of state requires an accurate knowledge of $H_0$, $\Omega_m$, the curvature and the distances. So, we are going to estimate uncertainties due to real inhomogeneity, compare them with already known uncertainties, and quantify whether dark energy equation of state could be measured with future data.

5. References

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