Origin of nonclassicality: observed state versus measurement

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According to Born’s rule quantum probabilities are given by the overlap between the system state and measurement states. We investigate whether typical nonclassical effects such as subPoissonian statistics and quadrature squeezing should be ascribed just to the system state or to the measurement states, which are typically highly non classical by themselves: number states and infinitely squeezed states. To this end we investigate whether non classicality still holds after replacing in Born’s rule number and quadrature states by classical-like states.

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Introduction.– Nonclassical effects are at the heart of the quantum theory. They are relevant both from fundamental as well as practical reasons, and nonclassicality is actually a resource for future quantum technologies.

Nonclassicality is revealed by peculiar effects in observed statistics. According to Born’s rule quantum statistics are determined in a symmetrical way both by the system state $|\psi\rangle$ and measurement states $|m\rangle$, this is $P(m|\psi) = |\langle m|\psi \rangle|^2$ [1]. Typically $|m\rangle$ are the eigenvectors of the measured observable.

This raises the question of whether nonclassicality is a property of the apparatus states $|m\rangle$ or a property of the state $|\psi\rangle$ being measured. This is specially pertinent since typically $|m\rangle$ are highly nonclassical states by themselves, say number states and infinitely squeezed states in photon-number and quadrature measurements [2,3]. Despite this natural and simple remark, the non classicality has been always ascribed to the system state.

In this work we reconsider this situation. To this end we consider that the statistic arises not by projection on nonclassical states, but on classical-like states. We may say that is the action of some kind of a Born’s demon that does not alter any other element of the quantum theory, just performs this little trick. We examine whether we can conclude that after this change the statistics is classical-like or nonclassical. We show that paradigmatic nonclassical effects vanish if the measurement states become classical-like. So subPoissonian statistics or quadrature squeezing no longer allows us to conclude that it is the state by itself that is nonclassical just the measurement basis.

As paradigmatic nonclassical tests we focus on subPoissonian number statistics, anti-correlation of photons, and quadrature squeezing. We shall adopt the customary universal definition stating that the state is nonclassical when the Glauber-Sudarshan $P$ is not a bona fide probability distribution [4]. This is not the only criterion, nor the most general [5,7], but it will serve well for our purposes.

Classical-like measurements.– Considering for simplicity a single-mode scenario of the measurement of an observable $M$ when the system state is $|\psi\rangle$ resulting in an statistics $P(m|\psi) = |\langle m|\psi \rangle|^2$. We will consider just two cases: light intensity assumed proportional to photon number $M = \hat{n} = a\dagger a$, $|m\rangle = |n\rangle$ with $\langle \hat{n}|n\rangle = n \langle n|$, and quadrature measurement $M = \hat{X} = (a\dagger + a)/\sqrt{2}$, $|m\rangle = |x\rangle$ with $\hat{X}|x\rangle = x|x\rangle$.

The classical-like scenario of these measurements is obtained by replacing the projection on $|m\rangle$ by the projection on the classical-like coherent states $|\alpha\rangle$ with $a|\alpha\rangle = \alpha|\alpha\rangle$. More specifically we replace number $\hat{n}$ by light intensity $\hat{n} \rightarrow I = |\alpha|^2$ and quadrature $\hat{X}$ by the real part of $\alpha = x + iy$. The statistics of such classical-like measurements are thus given by corresponding marginals of the Husimi $Q$ distribution

$$P(n|\psi) \rightarrow P_\alpha(n|\psi) = \frac{1}{2} \int_{2\pi} d\phi Q(\alpha = \sqrt{I}e^{i\phi}),$$

$$P(x|\psi) \rightarrow P_\alpha(x|\psi) = \int_{-\infty}^{\infty} dy Q(\alpha = x + iy),$$

and

$$Q(\alpha) = \frac{1}{\pi} |\langle \alpha|\psi \rangle|^2.$$

Alternatively we can just focus on moments, so that the classical-like measurement corresponds to the evaluation of anti-normally-ordered moments. That is for example that

$$\int d^2\alpha f(\alpha, \alpha\dagger) Q(\alpha) = \langle f_A(a, a\dagger) \rangle,$$

where $f_A(a, a\dagger)$ means that in a power series expansion of $f(\alpha, \alpha\dagger)$ the variables $\alpha, \alpha\dagger$ are replaced by the operators $a, a\dagger$ respectively, with all operator $a\dagger$ to the right of the operators $a$.

SubPoissonian statistics.– As a classic non classical test, it is known that subPoissonian statistics, $\Delta^2\hat{n} < \langle \hat{n} \rangle$, is incompatible with a bona fide $P(\alpha)$. The question is

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whether subPoissonian behavior still holds after the replacement \([\alpha] = \beta\). To this end we focus on the first two moments of \(I\):

\[
\langle I \rangle = \langle a a^\dagger \rangle = \langle \hat{n} \rangle + 1, \tag{5}
\]

\[
\langle I^2 \rangle = \langle a^2 a^\dagger^2 \rangle = \langle \hat{n}^2 \rangle + 2, \tag{6}
\]

so that

\[
\Delta^2 I = \langle I^2 \rangle - \langle I \rangle^2 = \Delta^2 \hat{n} + \langle I \rangle \geq \langle I \rangle. \tag{7}
\]

So if we replace quantum by classical-like measurement all states are Poissonian (number states) or superPoissonian (all the rest). So we may safely say that subPoissonian statistics holds only if the measurement itself is non classical.

**Anticorrelation.**—Next we move from subPoissonian statistics to anticorrelation of photo-counts in the typical scenario displayed in Fig. 1, as the flagship of quantum optics. There a single photon impinges on a lossless beam splitter and two joint intensity measurement are performed at the outputs of the beam splitter. Since the photon is indivisible, the detectors can never both trigger simultaneously so that \(\langle \hat{n}_1 \hat{n}_2 \rangle = 0\). This is maybe the most clear and simple evidence of the quantum nature of light [3].

In accordance with our objectives let us replace the two-mode quantum number statistics \(P(n_1, n_2|\psi)\) by the classical intensity distribution as marginal of the \(Q(\alpha_1, \alpha_2)\) function, leading to

\[
P(I_1, I_2|\psi) = (RI_1 + TI_2) e^{-I_1-I_2}, \tag{8}
\]

where \(R, T\) are the transmission and reflection coefficients with \(T + R = 1\), leading to

\[
\langle I_1 I_2 \rangle = 2, \tag{9}
\]

so the alleged quantum effect would not be never observed.

Along the same lines we can examine the Hong-Ou-Mandel effect illustrated in Fig. 2 [4], where two photons impinge simultaneously on the input ports of a lossless 50 % beam splitter. The quantum theory predicts the result \(\langle \hat{n}_1 \hat{n}_2 \rangle = 0\) again as an evidence of the quantum nature of light.

However, this result is not preserved if we replace the detectors by classical-like measurements. In such a case the joint statistics would be a a simple two-mode generalization of (1)

\[
P(I_1, I_2|\psi) = \frac{1}{4} (I_1^2 + I_2^2) e^{-I_1-I_2}, \tag{10}
\]

leading to

\[
\langle I_1 I_2 \rangle = 3. \tag{11}
\]

**Quadrature squeezing.**—As a further classic non classical test, it is known that quadrature squeezing, \(\Delta^2 \hat{X} < 1/4\) for \(\hat{X} = (a^\dagger + a)/2\), is incompatible with a bona fide \(P(\alpha)\). We carry out the analysis by replacing the quadrature distribution obtained by projection on highly nonclassical states by the marginal of the \(Q(\alpha = x + iy)\) function for the random variable \(x\) [2]. Computing the first two moments using the anti normal ordering when necessary we have

\[
\langle x \rangle = \langle \hat{X} \rangle, \quad \langle x^2 \rangle = \langle \hat{X}^2 \rangle + \frac{1}{4}, \tag{12}
\]

so that

\[
\Delta^2 x = \Delta^2 \hat{X} + \frac{1}{4} \geq \frac{1}{4}. \tag{13}
\]

Therefore, there would be no quadrature squeezing for any field state, and so no evidence of nonclassical behavior regarding this physical variable.

**Conclusions.**—We have shown that paradigmatic nonclassical effects vanish if the measurement states become classical-like. We can legitimately conclude that classic tests of nonclassical effects do not discriminate between the nonclassical properties of the system and measuring states. So subPoissonian statistics or quadrature squeezing no longer allows us to conclude that it is the state by itself that is nonclassical.
Seemingly the only consistent solution to this circular argument is that actually all the states are nonclassical, as revealed by recent approaches [5, 7].

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[1] In passing, this recalls the Goethe formulation of vision as the meeting of the inner and outer lights at the eye, as recalled by A. G. Zajonc, Goethe’s theory of color and scientific intuition, Am. J. Phys. 44, 327 (1976).
[2] N. G. Walker, Quantum Theory of Multiport Optical Homodyning, J. Mod. Opt. 34, 15 (1987); O. Jedrkiewicz, R. Loudon, and J. Jeffers, Retrodiction for coherent communication with homodyne or heterodyne detection, Eur. Phys. J. D 39, 129 (2006).
[3] A. Rivas and A. Luis, Nonclassicality of states and measurements by breaking classical bounds on statistics, Phys. Rev. A 79, 042105 (2009); A. Luis and A. Rivas, Independent nonclassical tests for states and measurements in the same experiment, Phys. Scr. T143, 014015 (2011).
[4] L. Mandel and E. Wolf, Optical Coherence and Quantum Optics (Cambridge University Press, 1995); M. O. Scully and M. S. Zubairy, Quantum Optics (Cambridge University Press, 1997).
[5] A. Luis, Nonclassical states from the joint statistics of simultaneous measurements, http://arxiv.org/abs/1506.07680; A. Luis, Nonclassical light revealed by the joint statistics of simultaneous measurements, Opt. Lett. 41, 1789 (2016); A. Luis, All states are nonclassical: entanglement of joint statistics, arXiv:1606.01478 [quant-ph].
[6] J. Vaccaro, Number-phase Wigner function on Fock space, Phys. Rev. A 52, 3474 (1995); L. M. Johansen, Nonclassical properties of coherent states, Phys. Lett. A 329, 184 (2004); L. M. Johansen, Nonclassicality of thermal radiation, J. Opt. B: Quantum Semiclassical Opt. 6, L21 (2004); L. M. Johansen and A. Luis, Nonclassicality in weak measurements, Phys. Rev. A 70, 052115 (2004); A. Luis, Nonclassical polarization states, Phys. Rev. A 73, 063806 (2006).
[7] A. Ferraro, L. Aolita, D. Cavalcanti, F. M. Cucchietti, and A. Acín, Almost all quantum states have nonclassical correlations, Phys. Rev. A 81, 052318 (2010); A. C. de la Torre, D. Goyeneche, and L. Leitao, Entanglement for all quantum states, Eur. J. Phys. 31, 325 (2010).
[8] P. Grangier, G. Roger and A. Aspect, Experimental Evidence for a Photon Anticorrelation Effect on a Beam Splitter: A New Light on Single-Photon Interferences Europhys. Lett. 1, 173– (1986)
[9] C. K. Hong, Z. Y. Ou and L. Mandel, Measurement of subpicosecond time intervals between two photons by interference, Phys. Rev. Lett. 59, 2044 (1987).