An unambiguous test of positivity at lepton colliders

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The diphoton channel at lepton colliders, \( e^+e^- \to \gamma\gamma \), has a remarkable feature that the leading new physics contribution comes only from dimension-eight operators. This contribution is subject to a set of positivity bounds, derived from fundamental principles of Quantum Field Theory, such as unitarity, locality and analyticity. These positivity bounds are thus applicable to the most direct observable — the diphoton cross sections. This unique feature provides a clear, robust, and unambiguous test of these principles. We estimate the capability of various future lepton colliders in probing the dimension-eight operators and testing the positivity bounds in this channel. We show that positivity bounds can lift certain degeneracies among the effective operators and significantly change the perspectives of a global analysis. We also perform a combined analysis of the \( \gamma\gamma/Z\gamma/ZZ \) processes in the high energy limit and point out the important interplay among them.

Introduction.— Positivity bounds on the coefficient of the Standard Model (SM) Effective Field Theory (SMEFT) operators arise from the assumption that their UV completion obeys fundamental principles of quantum field theory (QFT), such as unitarity, locality and analyticity. Testing these bounds at colliders is difficult, as they generally only apply to effects of dimension-eight (dim-8) or higher operators \([11,13]\). Their contribution to a given process is typically subleading compared with those from dimension-six (dim-6) operators, making their measurements experimentally challenging even with the use of differential observables \([12,14]\). For dim-6 operators, such bounds do not exist without explicit assumptions on the UV model \([15,19]\).

In this letter, we identify a specific process, \( e^+e^- \to \gamma\gamma \) (or \( \mu^+\mu^- \to \gamma\gamma \)), in which the dim-8 operators provide the leading new physics contribution, and a test of the positivity bounds can be unambiguously carried out. The measurements of this simple process at lepton colliders thus have profound implications. A confirmed violation of the positivity bounds would be more revolutionary than any particle discovery, as it indicates a break down of the positivity bounds would be more revolutionary than any particle discovery, as it indicates a break down of the foundations of QFT.

The di-photon channel.— We start by arguing that the leading new physics contributions to \( e^+e^- \to \gamma\gamma \) appear at dim-8. In SMEFT, the dominant contribution of new physics typically comes from the interference term between SM and dim-6 operators. However, such contribution to \( e^+e^- \to \gamma\gamma \) are strongly suppressed. Neglecting the electron mass, the tree level amplitude in the SM can only take the \( A(f^+f^-\gamma^+\gamma^-) \) helicity configuration. Here the superscripts denote the helicity. On the contrary, a tree-level single insertion of dim-6 operators to this process could only be generated by a dipole operator, which leads to a different fermion helicity configuration, and does not generate an interference term \([20]\).

At the one-loop level, several dim-6 contributions may arise, but they are all strongly constrained by other measurements. With the additional suppression of a loop factor, these contributions can be safely discarded. For instance, the operator \( O_{3W} = \pm g_{abc} W^a_\mu W^b_\nu W^c_{\rho\mu} \) contributes to \( A(f^+f^-\gamma^+\gamma^-) \) at one loop, but it can be very-well probed by the measurements of the \( e^+e^- \to WW \) process. A rough estimation with the projections from Ref.\([21]\) suggests that its impact on the diphoton cross section is at most around \( \delta \sigma_{\gamma\gamma}/\sigma_{\gamma\gamma} \sim 10^{-7} \), much smaller than the expected precision at a realistic lepton collider. Other contributions are either stringently constrained by \( Z \)-pole measurements or suppressed by the small electron mass and Yukawa coupling. While the 4-fermion operators involving two electron and two top-quark fields are poorly constrained at the tree level, their contribution to \( A(f^+f^-\gamma^+\gamma^-) \) is forbidden by the angular momentum selection rules, since they could not produce the \( J = 2 \) state of the two photons \([22]\). Contributions with two insertions of dim-6 operators are formally indistinguishable from dim-8 operators. At the tree level, the only such contribution which is not equivalent to a contact dim-8 operator insertion comes from two insertions of electron dipole couplings \([53]\). They are strongly constrained by the \( g_\mu - 2 \) and the electric dipole moment measurements \([23,24]\) and can be safely ignored.

Dim-8 operators, on the other hand, could generate amplitudes with the same helicities as the SM tree-level contribution, therefore providing the leading new physics contribution to the \( e^+e^- \to \gamma\gamma \) process. Remarkably, this contribution is completely fixed by its helicity amplitude: it must be a contact interaction proportional to \( E^4/\Lambda^4 \), with \( \Lambda \) the scale of the potential new physics. As we will show later, this feature is crucial for the direct
connection between the positivity bounds and the diphoton cross sections. On the other hand, dim-8 operators involving Higgs fields do not contribute to the same helicity amplitude even with the Higgs vev switched on, while those with a different helicity configuration will not interfere with the SM.

Naively, one may expect that the dim-6 contributions from new physics will be first observed in some other processes, which degrades the motivation to look for dim-8 deviations in the diphoton channel. This is not true for two reasons. First, testing the positivity of dim-8 operators provides more fundamental information about the nature of new physics, namely whether it is consistent with the QFT framework, which one cannot tell from a SMEFT analysis truncated at dim-6. For this reason, an observation of dim-6 deviation in a different process would only strengthen the motivation to test dim-8 deviations in $e^+e^− → \gamma\gamma$. Second, dim-6 effects from different UV particles might be suppressed due to dynamics, certain symmetries, or accidental cancellation. On the contrary, constraining dim-8 effects would lead to unambiguous exclusion limits on each individual UV particle as long as the QFT framework is valid, thanks to the positivity bounds.

Amplitudes and operators.— Denoting with $e$ the electric coupling, $v$ the Higgs vev, the amplitude of the diphoton process can be written as

$$A(f^+ f^- \gamma^+ \gamma^-)_{\text{SM}+ds} = 2e^2 \left( \frac{242}{13} \frac{23}{24} \right) + \frac{a}{v^4} [13][23][24]^2,$$

where the effective parameter $a$ depends on the dim-8 coefficients. Replacing $f$ by $\bar{e}_L$ and $e_R$, and denoting the corresponding dim-8 parameters $a = a_{L,R}$, we have

$$A(\bar{e}_L e_L \gamma^+ \gamma^-)_{ds} = \frac{a_L}{v^4} [13][23][24]^2,$$

$$A(e_R \bar{e}_R \gamma^+ \gamma^-)_{ds} = \frac{a_R}{v^4} [13][23][24]^2. \quad (2)$$

One could work in the amplitude basis and directly connect Equation 2 with the massless amplitudes of the $W$ and $B$ fields in the unbroken electroweak phase. Alternatively, one could consider the following five operators (using the conventions in Ref [7]):

$$O^{(8)}_{\ell L} = -\frac{1}{4} (\bar{e}_L \gamma^\rho (p \, D^\rho) \ell_L + \text{h.c.}) B_{\mu \nu} B^\rho_{\mu},$$

$$O^{(8)}_{e B} = -\frac{1}{4} (\bar{e}_R \gamma^\rho (p \, D^\rho) e_R + \text{h.c.}) B_{\mu \nu} B^\rho_{\mu},$$

$$O^{(8)}_{\ell W} = -\frac{1}{4} (\bar{e}_L \gamma^\rho (p \, D^\rho) \ell_L + \text{h.c.}) W^a_{\mu \nu} W^{a \rho}_{\mu},$$

$$O^{(8)}_{e W} = -\frac{1}{4} (\bar{e}_R \gamma^\rho (p \, D^\rho) e_R + \text{h.c.}) W^a_{\mu \nu} W^{a \rho}_{\mu},$$

$$O^{(8)}_{\ell BW} = -\frac{1}{4} (\bar{e}_L \sigma^a \gamma^\rho (p \, D^\rho) \ell_L + \text{h.c.}) B_{\mu \nu} W^{a \rho}_{\mu}. \quad (3)$$

which generate $a_L$ and $a_R$, given by

$$a_L = \frac{v^4}{\Lambda^4} \left( \cos^2 \theta_W c_{1B}^{(8)} - \cos \theta_W \sin \theta_W c_{\ell BW}^{(8)} + \sin^2 \theta_W c_{\ell IW}^{(8)} \right),$$

$$a_R = \frac{v^4}{\Lambda^4} \left( \cos^2 \theta_W c_{eB}^{(8)} + \sin^2 \theta_W c_{eW}^{(8)} \right), \quad (4)$$

where $\theta_W$ is the weak mixing angle and $\mathcal{L}_\text{dim-8} = \sum_i c_i^{(8)} O_i^{(8)}$. They are the only relevant operators in the full dim-8 basis (with slightly different conventions), not only for diphoton but also for $A(\bar{e}_L e_L V_1^+ V_2^-)_{ds}$ and $A(e_R \bar{e}_R V_1^+ V_2^-)_{ds}$ amplitudes, where $V_{1,2} = Z, \gamma$.

Positivity bounds on cross sections.— Rotating the diphoton amplitude to the elastic process $e^+ e^- \rightarrow e^+ e^-$, the dispersion relation implies

$$\frac{d^2}{ds^2} A(e^+ e^- \rightarrow \gamma \gamma)|_{s \rightarrow 0} \geq 0. \quad (5)$$

Since the dim-8 amplitudes are proportional to $s^2$, this requires that

$$a_L \geq 0, \quad a_R \geq 0. \quad (6)$$

These bounds can be directly related to the cross section measurements. Since the helicities of the two photons can not be measured, we work with the folded distribution $d\sigma = d\sigma(\cos \theta) + d\sigma(-\cos \theta)$,

$$d\sigma(e^+ e^- \rightarrow \gamma \gamma) = \frac{(1 - P_c^-)(1 + P_c^+)}{4} \frac{e^4}{4\pi s} \left[ \frac{1 + c_\theta^2}{1 - c_\theta^2} + a_L \frac{s^2(1 + c_\theta^2)}{4e^2v^4} \right] + \frac{(1 + P_c^-)(1 - P_c^+)}{4} \frac{e^4}{4\pi s} \left[ \frac{1 + c_\theta^2}{1 - c_\theta^2} + a_R \frac{s^2(1 + c_\theta^2)}{4e^2v^4} \right], \quad (7)$$

where $s$ is the square of the center-of-mass energy, $P_c^-$ ($P_c^+$) is the polarization of the electron (positron) beam, and $c_\theta \equiv \cos \theta$. It is now clear that the positivity bounds $a_L \geq 0, a_R \geq 0$ have a simple consequence, namely

$$\sigma(e^+ e^- \rightarrow \gamma \gamma) \geq \sigma_{\text{SM}}(e^+ e^- \rightarrow \gamma \gamma), \quad (8)$$

for any beam polarizations and any $\cos \theta$. We see that the $e^+ e^- \rightarrow \gamma \gamma$ channel is a special one, in that the positivity of Wilson coefficients can be directly translated into positivity in realistic observables, without being contaminated by any other non-positive operators. The diphoton process thus provides a simple, clear, and unambiguous test of the fundamental principles of QFT.

It is also interesting to note that the measurements at LEP2 display an overall signal strength of $e^+ e^- \rightarrow \gamma \gamma$ to be about 1.5 standard deviations below the SM expectation. While statistically insignificant, it exhibits a small tension with the positivity bound.
Collider reach.— To estimate the reach at future lepton colliders, we perform a simple binned analysis in the range $\cos \theta \in [0, 0.95]$, with a bin width of 0.05, and consider only statistical uncertainties. We expect the largest background to be the Bhabha scattering ($e^+e^- \rightarrow e^+e^-$), with a cross section of almost 2 orders of magnitude larger than $e^+e^- \rightarrow \gamma\gamma$. Assuming a sufficiently small rate ($\ll 1\%$) for an electron to be misidentified as a photon, this background is negligible since both electrons need to be misidentified as a photon for an event to be selected. The cut on the minimal production polar angle ($\cos \theta < 0.95$) is also very effective in removing the beamstrahlung and ISR effects. As a validation of our analysis, we apply it to the LEP 2 run scenarios and find a very good agreement with the result in Ref. [30] (with a $\lesssim 10\%$ difference in the reach on $\Lambda$).

To illustrate the interplay between the measurements and the positivity bounds, we show the $\Delta \chi^2 = 1$ contours in Figure 1 for collider scenarios CEPC/FCC-ee 240 GeV and ILC 250 GeV, with their details summarized in Table I. According to Equation 7 if the beams are unpolarized ($P_{e^-} = P_{e^+} = 0$), only the combination $a_L + a_R$ is probed, leaving a flat direction $a_L = -a_R$. It can be lifted by having multiple runs with different beam polarization, as for example at the ILC. Clearly, for the purpose of testing positivity, beam polarization would be desirable, because it allows for testing the sign of $a_L$ and $a_R$ (or left/right-handed polarized cross sections) individually. Without beam polarization, one could still probe the positivity of $a_L + a_R$ (or the total cross section).

On a different ground, assuming that the SMEFT has a UV completion consistent with the QFT principles, positivity bounds can be used to resolve the degeneracy between $a_L$ and $a_R$ by constraining both simultaneously, even without beam polarization, as clearly illustrated in Figure 1. We emphasize that this is a general feature that are also applicable to many other processes, such as the 4-fermion [12] or the Higgs ones [31]. Positivity thus provides important information for future global SMEFT analyses, complementary to the experimental inputs.

High energy lepton colliders can probe these operators even further. The reach on $\Lambda$ scales with the energy $E$ and luminosity $L$ as

$$\Lambda_2 \sim \left( \frac{E}{L} \right)^{\frac{1}{2}} \left( \frac{L_2}{L_1} \right)^{\frac{1}{4}},$$

assuming all other variables, such as beam polarizations, are the same for the two scenarios 1 and 2. The energy enhancement $E^4/\Lambda^4$ on the dim-8 contribution is slightly offset by the decrease of SM cross section with respect to energy, resulting in an energy dependence with power 3/4 for the reach on $\Lambda$.

In Figure 2, we show the 95% CL reach on $\Lambda_8 \equiv v/a^\frac{4}{5}$ for various collider scenarios, where $a = a_L, a_R$ is defined in Equation 1 and 2. $\Lambda_8$ corresponds directly to the scale of new physics which modifies the $e^+e^- \rightarrow \gamma\gamma$ amplitudes. When positivity is violated, $\Lambda_8$ indicates the scale of the new physics which generates this violation, analogous to the $\Delta$ parameter of Ref. [12]. The band in Equation 3 covers integrated luminosities of 1 to 5 ab$^{-1}$ and various beam polarization scenarios. The circles represent the best reach for each collider scenario listed in Table I. The LEP2 [30] reach is also shown, assuming a SM central value for the measurements. For linear colliders, the triangle (square) shows the reach from $a_L$ ($a_R$) in a simultaneous fit of the two parameters.
and $a_R$ can be simultaneously constrained, and the corresponding $L_L$ and $\Lambda_R$ are both shown in Figure 2 in addition to the best reach. We also find that a Tera-Z program or a 10 ab$^{-1}$ WW threshold run could have a reach on $\Lambda$ that is only slightly worse than the one of the 240 GeV run, assuming only statistical uncertainties.

| $f \mathcal{L} dt$ [ab$^{-1}$] |      |      |      |
|-------------------------------|------|------|------|
| unpolarized                   | 91 GeV | 161 GeV | 240 GeV | 365 GeV |
| CEPC                          | 8    | 2.6  | 5.6  |
| FCC-ee                        | 150  | 10   | 5    | 1.5   |
| ILC                          | 250 GeV | 350 GeV | 500 GeV |
| (-0.8, +0.3)                  | 0.9      | 0.135      | 1.6      |
| (+0.8, -0.3)                  | 0.9      | 0.045      | 1.6      |
| (±0.8, ±0.3)                  | 0.1      | 0.01       | 0.4      |
| CLIC                         | 380 GeV | 1.5 TeV | 3 TeV  |
| (-0.8, 0)                     | 0.5      | 2        | 4       |
| (+0.8, 0)                     | 0.5      | 0.5      | 1       |
| muon collider                | 10 TeV | 30 TeV |
| unpolarized                  | 10                    | 90        |

TABLE I: A summary of the run scenarios of future lepton colliders considered in our analysis with the corresponding integrated luminosity. For ILC and CLIC, the numbers in the brackets are the values of beam polarizations $P(e^-, e^+)$. For simplicity, we assume the Z-pole or WW-threshold runs are at one single energy. Numbers are taken from Refs. [21, 32]. Further details can be found in Refs. [30, 31].

Similar analyses can be carried out for muon colliders, which probe a different set of operators associated with the muon. One should also keep in mind that the constraints on muon dipole operators are significantly weaker than those of the electron ones. We find that, with the current bounds on the muon electric dipole moment [24], two insertions of the electric dipole operator could generate a deviation in the $\mu^+\mu^- \rightarrow \gamma\gamma$ cross section comparable to the expected precision reach. However, future improvements on the muon electric dipole moment measurement could make this contribution irrelevant for the $\mu^+\mu^- \rightarrow \gamma\gamma$ measurements at muon colliders [39, 40]. On the contrary, the current muon $g_\mu - 2$ measurement [41, 42] sufficiently constrains the magnetic dipole operator, so that its contribution in $\mu^+\mu^- \rightarrow \gamma\gamma$ can be safely ignored, independent of whether the apparent discrepancy with the SM is confirmed.

Interplay with $Z\gamma$ and $ZZ$ measurements.— The operators in Equation 3 also contribute to the $Z\gamma$ and $ZZ$ processes, and a combined analysis could provide additional information. These processes are, however, more complicated due to the massive $Z$ boson, which enables contributions to multiple helicity states from both SM and dim-8 operators (including those responsible for neutral triple-gauge-boson couplings [33, 40]). Dim-6 operators could also contribute at the tree level via modifications of the $Ze^+e^-$ couplings. However, at very high energies ($\sqrt{s} \gg m_Z$), the $Z$ boson is effectively massless, and $++$ final helicity states dominates the $Z\gamma$ and $ZZ$ cross sections [7]. In this limit, the $ZZ$ process also exhibits a similar positivity bound, namely that

$$\sigma(e^+e^- \rightarrow ZZ) \geq \sigma_{SM}(e^+e^- \rightarrow ZZ). \quad (10)$$

For the $Z\gamma$ process, we focus on the CP-even contribution and consider the elastic amplitude $A(eV \rightarrow eV)$, where $V$ is an mixed state of $\gamma$ and $Z$, with arbitrary mixing coefficients [50]. The dispersion relation requires

$$(a_L^{Z\gamma})^2 \leq a_L^{ZZ}a_L^{\gamma\gamma}, \quad (a_R^{Z\gamma})^2 \leq a_R^{ZZ}a_R^{\gamma\gamma}, \quad (11)$$

where $a_{L,R}^{Z\gamma}$ ($a_{L,R}^{ZZ}$) is defined as in Equation 2 with $\gamma^+\gamma^-$ replaced by $Z^+\gamma^-$ ($Z^+Z^-$), together with $a_{L,R} \rightarrow a_{L,R}^{\gamma\gamma}$ to distinguish them. This implies a simple relation among the $Z\gamma$, $\gamma\gamma$ and $ZZ$ cross sections for any fixed collider scenario,

$$(\Delta\sigma_{Z\gamma})^2 \leq 4\Delta\sigma_{\gamma\gamma}\Delta\sigma_{ZZ}, \quad (12)$$

where $\Delta\sigma \equiv \sigma - \sigma_{SM}$. The bounds on the $\gamma\gamma$, $ZZ$ and $Z\gamma$ cross sections are illustrated in Figure 3. Since $\Delta\sigma_{Z\gamma}$, $\Delta\sigma_{ZZ}$ are generated by only two operators $O_{eB}^{(8)}$ and $O_{eW}^{(8)}$, the purely right-handed polarized cross sections only occupy a plane in the parameter space, as shown in Figure 3. This implies additional bounds on them, which can for instance be written as $\Delta\sigma_{Z\gamma}^2 \leq \Delta\sigma_{ZZ}^2 \leq \frac{1}{\tan^2\theta_W} \Delta\sigma_{ZZ}^2$.

FIG. 3: A graphical illustration of the positivity bounds in Equation 8, 10 and 12 with $\Delta\sigma \equiv \sigma - \sigma_{SM}$. Scales are arbitrary. $\sqrt{s} \gg m_Z$ is always assumed. The purely right-handed polarized cross sections occupy only the red plane. The allowed and forbidden regions are separated by the green surface.

We perform a combined $\gamma\gamma/Z\gamma/ZZ$ global analysis with all five operators in Equation 3 restricting ourselves...
to colliders with $\sqrt{s} > 1$ TeV. The individual and global 95% CL reaches on the scale $\Lambda$ of the five operators are shown in Figure 4. Positivity bounds play an important role in the global fit. For the muon collider with unpolarized beams, only three combinations of the five operator coefficients are constrained. The degeneracy can be lifted only by imposing the positivity bounds. On the other hand, multiple runs with beam polarizations at CLIC already leave no flat directions in the fit, but imposing positivity bounds nevertheless rules out a large region of the parameter space. The latter is not reflected in Figure 4, which only shows the magnitudes of the scales.

![Figure 4](image)

**Figure 4:** The 95% CL reach on the scale $\Lambda$ of the five operators in Equation 3 from a combined $\gamma\gamma/Z\gamma/ZZ$ analysis at very high energy lepton colliders. The run scenarios can be found in Table 1. A global fit without positivity bounds is not possible for the muon collider which has no beam polarization.

**Violation of positivity.** — The observation of $\sigma(e^+e^-\rightarrow \gamma\gamma) < \sigma_{SM}(e^+e^-\rightarrow e^+e^-)$ does not necessarily establish the violation of positivity bounds. It is important to consider the possibility that the EFT description itself is invalid, for instance, due to the contribution to $e^+e^-\rightarrow \gamma\gamma$ from light new particles. In this channel, however, it is difficult for such particles to generate a sizable destructive interference term while evading the current and future search constraints. A $t$-channel fermion exchange, for example, only generates a constructive interference. One possibility can be an $s$-channel exchange of a light composite spin-2 particle, which could be very-well probed by the resonance search $e^+e^-\rightarrow X\gamma/XZ$. Measuring the diphoton process at multiple center of mass energies also help probe or exclude these light particle contributions. After all other possibilities are excluded, the result would then indicate the break down of the fundamental principles of QFT.

**Summary and outlook.** — In this letter, we have shown that positivity bounds require that the diphoton cross section at lepton colliders must be larger than the SM prediction, which offers a rare opportunity to clearly and unambiguously test of the fundamental principles of QFT, such as unitarity, causality, and the Lorentz invariance of the UV physics. While high energy colliders provide the best reaches, such probes are unambiguous and robust even for a collider at relatively low center-of-mass energies (240-250 GeV), a feature that is unique for the diphoton process. Alternatively, imposing these positivity bounds could lift the degeneracies among operators, which is also applicable to other processes, demonstrating that positivity provides important information for future global analyses with dim-8 operators.

Hadron colliders, such as the LHC or a future 100-TeV collider, has a large center-of-mass energy and could potentially provide competitive reaches on the same type of dim-8 operators associated with the quarks [7] [14]. However, due to the rapid reduction of statistics at high energy from the parton distribution function, care must be taken to ensure the validity of the EFT expansion, which the positivity bounds rely on [58]. On the other hand, a potential future high energy photon collider [48] could measure the reverse process $\gamma\gamma \rightarrow ff$ for different fermion final states, and probe a wide range of operators and their associated positivity bounds. We leave the detailed analyses of these colliders to future studies.

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[1] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, and R. Rattazzi, JHEP 10, 014 (2006), hep-th/0602178.
[2] B. Bellazzini, JHEP 02, 034 (2017), 1605.06111.
[3] C. de Rham, S. Melville, A. J. Tolley, and S.-Y. Zhou, Phys. Rev. D 96, 081702 (2017), 1702.06134.
[4] C. de Rham, S. Melville, A. J. Tolley, and S.-Y. Zhou, JHEP 03, 011 (2018), 1706.02712.
[5] V. Chandrasekaran, G. N. Remmen, and A. Shahbazi-Moghaddam, JHEP 11, 015 (2018), 1804.03153.
[6] C. de Rham, S. Melville, A. J. Tolley, and S.-Y. Zhou, JHEP 03, 182 (2019), 1804.10624.
[7] B. Bellazzini and F. Riva, Phys. Rev. D 98, 095021 (2018), 1806.09640.
[8] Q. Bi, C. Zhang, and S.-Y. Zhou, JHEP 06, 137 (2019), 1902.08977.
[9] G. N. Remmen and N. L. Rodd, JHEP 12, 032 (2019), 1908.09845.
[10] G. N. Remmen and N. L. Rodd (2020), 2004.02885.
[11] C. Zhang and S.-Y. Zhou (2020), 2005.03047.
[12] B. Fuks, Y. Liu, C. Zhang, and S.-Y. Zhou (2020), 2009.02212.
[13] K. Yamashita, C. Zhang, and S.-Y. Zhou (2020), 2009.04490.
[14] S. Alioli, R. Boghezal, E. Mereghetti, and F. Petriello, Phys. Lett. B 809, 135703 (2020), 2003.11615.
[15] I. Low, R. Rattazzi, and A. Vichi, JHEP 04, 126 (2010), 0907.5413.
[16] A. Falkowski, S. Rychkov, and A. Urbano, JHEP 04, 073 (2012), 1202.1532.
[17] B. Bellazzini, L. Martucci, and R. Torre, JHEP 09, 100 (2014), 1405.2960.
[18] J. Gu and L.-T. Wang (2020), 2008.07551.
[19] J. P. Delahaye, M. Diemoz, K. Long, B. Mansoulié, G. Bennett et al. (Muon g-2), Phys. Rev. D 95, 065014 (2017), 1607.05236.
[20] A. Azatov, R. Contino, C. S. Machado, and F. Riva, JHEP 12, 117 (2019), 1907.04311.
[21] V. Andreev et al. (ACME), Nature 562, 3/2018.
[22] M. Jiang, J. Shu, M.-L. Xiao, and Y.-H. Zheng (2020), 2005.00008.
[23] T. Han, D. Liu, I. Low, and X. Wang (2020), 2008.12204.
[24] C. Hays, A. Martin, V. Sanz, and J. Setford, JHEP 02, 119 (2013), 1302.3415.
[25] J. Delahaye, M. Diemoz, K. Long, B. Mansoulié, N. Pastrone, L. Rivkin, D. Schulte, A. Skrinsky, and A. Wulzer (2019), 1901.06150.
[26] A. Crivellin, M. Hoferichter, and P. Schmidt-Wellenburg, Phys. Rev. D 98, 113002 (2018), 1807.11484.
[27] M. Abe et al., PTEP 2019, 053C02 (2019), 1901.03047.
[28] G. Bennett et al. (Muon g-2), Phys. Rev. D 73, 072003 (2006), hep-ex/0602035.
[29] P. Zyla et al. (Particle Data Group), PTEP 2020, 083C01 (2020).
[30] G. Gounaris, J. Layssac, and F. Renard, Phys. Rev. D 61, 073013 (2000), hep-ph/9910395.
[31] C. Degrande, JHEP 02, 101 (2014), 1308.6323.
[32] J. Ellis, S.-F. Ge, H.-J. He, and R.-Q. Xiao, Chin. Phys. C 44, 063106 (2020), 1902.06631.
[33] J. Ellis, H.-J. He, and R.-Q. Xiao (2020), 2008.04298.
[34] K. Fuji et al. (LCC Physics Working Group) (2016), 1607.03829.
[35] E. Adli et al. (ALEGRO) (2019), 1901.10370.
[36] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, JHEP 10, 085 (2010), 1008.4884.
[37] H. Elvang and Y.-t. Huang (2013), 1308.1697.
[38] L. J. Dixon, in Theoretical Advanced Study Institute in Elementary Particle Physics: Particle Physics: The Higgs Boson and Beyond (2014), pp. 31–67, 1310.5353.
[39] C. Cheung, in Proceedings, Theoretical Advanced Study Institute in Elementary Particle Physics : Anticipating the Next Discoveries in Particle Physics (TASI 2016): Boulder, CO, USA, June 6-July 1, 2016 (2018), pp. 571–623, 1708.03872.
[40] One could also choose a basis, such as the Warsaw basis [19], in which the only dim-6-squared contribution comes from the dipole operators. We assume such a basis is always chosen, and the $f^+f^-\gamma^+\gamma^-$ contact interaction is a dim-8 contribution.
[41] The insertion of a Higgs vev makes the amplitude effective at a lower mass dimension, in which case a contact term for $\mathcal{A}(f^+f^-\gamma^+\gamma^-)$ could not be written down. This can also be explicitly checked with the basis in Refs. [28, 29].
[42] See e.g. Refs. [50,52] for recent reviews of the helicity amplitude formalism and the bracket notation of spinors. Alternatively, one could impose the bounds on the Wilson coefficients in Equation 3 by replacing $\gamma$ or $Z$ with an arbitrary superposition of $B$ and $W$ fields. This leads to $c_{LB}^{(8)} \geq 0$, $c_{LW}^{(8)} \geq 0$, $c_{cB}^{(8)} \geq 0$ and $c_{cW}^{(8)} \geq 0$ from $eB$ and $eW$ scattering, and $4c_{LB}^{(8)}c_{LW}^{(8)} \geq (c_{LB}^{(8)})^2$ from all other mixings [7]. They are equivalent to the bounds on the $a$ parameters.
[43] “When you have excluded the impossible, whatever remains, however improbable, must be the truth.” – Sherlock Holmes.