Some directions beyond traditional quantum secret sharing

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Received 5 June 2007
Accepted for publication 16 April 2008
Published 21 May 2008
Online at stacks.iop.org/PhysScr/77/065007

Abstract
We investigate two directions beyond the traditional quantum secret sharing (QSS). Firstly, a restriction on QSS that comes from the no-cloning theorem is that any pair of authorized sets in an access structure should overlap. From the viewpoint of application, this places an unnatural constraint on secret sharing. We present a generalization, called assisted QSS (AQSS), where access structures without pairwise overlap of authorized sets are permissible, provided some shares are withheld by the share dealer. We show that no more than λ − 1 withheld shares are required, where λ is the minimum number of partially linked classes among the authorized sets for the QSS. Our result means that such applications of QSS need not be thwarted by the no-cloning theorem. Secondly, we point out a way of combining the features of QSS and quantum key distribution (QKD) for applications where classical information is shared by quantum means. We observe that in such case, it is often possible to reduce the security proof of QSS to that of QKD.

PACS number: 03.67.Dd

1. Introduction

Suppose the president of a bank, Alice, wants to give access to a vault to two vice-presidents, Bob and Charlie, whom she does not entirely trust. Instead of giving the combination to any one of them, she may desire to distribute the information in such a way that no vice-president alone has any knowledge of the combination, but both of them can jointly determine the combination. Cryptography provides the answer to this question in the form of secret sharing [1]. In this scheme, some sensitive data is distributed among a number of parties such that certain authorized sets of parties can access the data, but no other combination of players. A particularly symmetric variety of secret splitting (sharing) is called a threshold scheme: in a \((k, n)\) classical threshold scheme (CTS), the secret is split up into \(n\) pieces (shares), of which any \(k\) share forms a set authorized to reconstruct the secret, while any set of \(k - 1\) or fewer shares has no information about the secret. Blakely [2] and Shamir [3] showed that CTSs exist for all values of \(k\) and \(n\) with \(n \geq k\). By concatenating threshold schemes, one can construct arbitrary access structures, subject only to the condition of monotonicity (i.e. sets containing authorized sets should also be authorized) [4]. Hillery et al [5] and Karlsson et al [6] proposed the methods for implementing CTSs that use quantum information to transmit shares securely in the presence of eavesdroppers.

Subsequently, extending the above idea to the quantum case, Cleve et al [7], using the notion of quantum erasure correction [8–10], presented a \((k, n)\) quantum threshold scheme (QTS) as a method to split up an unknown secret quantum state \(|S\rangle\) into \(n\) pieces (shares) with the restriction that \(k > n/2\)—this inequality being needed to ensure that no two disjoint sets of players should be able to reconstruct the secret, in conformance with the quantum no-cloning theorem [11]. Quantum secret sharing (QSS) has been extended beyond QTS to general access structures [12, 13], but here none of the authorized sets shall be mutually disjoint: given a QSS access structure \(\Gamma = \{\alpha_1, \ldots, \alpha_t\}\) over \(N\) players, the no-cloning restriction entails that:

\[
\alpha_j \cap \alpha_k \neq \emptyset, \quad \forall j, k.
\] (1)

Potential applications of QSS include creating joint checking accounts containing quantum money [14], or sharing hard-to-create ancilla states [12], or performing...
secure distributed quantum computation [15]. A tri-qubit QSS scheme has recently been implemented [16]. The chances of practical implementation of QSS are improved by employing equivalent schemes that maximize the proportion of classical information processing [17, 18]. It has been shown that quantum teleportation [19] and entanglement swapping [20, 21] may be used to implement an \((n, n)\)-threshold scheme.

The requirement equation (1) places a restriction quite unnatural to applications, where we may more likely expect to find groups of people with mutual trust within the group, and hardly any outside it. First aspect of our present work is aimed at studying a way to overcome this limitation. In particular, in the sections 2 and 3, we show that allowing the dealer to withhold a small number of shares permits arbitrary access structures to be acceptable, subject only to monotonicity. We call this modified scheme ‘assisted QSS’ (AQSS), the shares withheld by the dealer being called ‘home shares’. While more general than conventional QSS, AQSS is clearly not as general as classical secret sharing, since it requires shares given to the (non-dealer) players, called ‘player shares’, to be combined with the home shares for reconstructing the secret.

In spite of this limitation, the modified scheme can be useful in some applications of secret sharing, in particular, those in which the secret dealer is by definition a trusted party and where reconstruction of the secret effectively occurs by reconvergence of shares at the dealer’s station. Further, it could be useful for schemes like circular QSS [22].

We note that the home share by themselves give no information. In the bank example above, access is allowed by the bank vault (which can be thought of effectively as the dealer, acting as the bank president’s proxy) if the secret reconstructed from the vice-presidents’ shares is the required password. The locker thus effectively serves as both the dealer and site of secret reconstruction. In AQSS, the player shares are combined with the home share(s) to reconstruct the secret. Clearly, this leads to no loss of generality in this type of QSS. Where the secret dealer is not necessarily trusted, such as in multiparty secure computation (MPSC), AQSS may be less useful, though here, again, only a more detailed study can tell whether MPSC cannot be turned into a suitable variant of AQSS.

It is assumed that all the \(n\) (quantum) shares are somehow divided among the \(N\) players. In an AQSS scheme, \(m < n\) shares are allowed to remain with the share dealer, as home shares. In order that AQSS departs minimally from conventional QSS, we further require that the number of home shares should be the minimum possible such that a violation of equation (1) can be accommodated. Thus, a conventional QSS access structure like \(\Gamma = \{ABC, ADE, BDF\}\), which as such conforms to the no-cloning theorem, will require no share assistance. A conventional QSS scheme is a special case of AQSS, in which the set of home shares is empty. We prove by direct construction that, by allowing for nonzero home shares, the restriction (1) does not apply to AQSS. Therefore, with share assistance, the only restriction on the access structure \(\Gamma\) in AQSS is monotonicity, as with classical secret sharing. Such constructions are described in detail in the sections 2 and 3.

Another cryptographic primitive where (multipartite) entanglement can be effectively used is that of quantum key distribution (QKD) [23, 24] and its generalization to \(n\) parties (\(n\)-QKD). Note that the \(n\)-QKD involves sharing a random key amongst \(n\) trustworthy parties unlike QSS, which splits quantum information among untrustworthy parties. Naturally, it would be an interesting extension to consider the situations where some kind of mutual trust may be present between sets of parties whereas parties being individually mistrustful, wherein it might be possible to combine the essential features of QKD and QSS. In the section 4, we discuss one such extension. We consider the problem of secure key distribution between two trustful groups where the individual group members may be mistrustful. The two groups retrieve the secure key string, only if all members cooperate with one another in each group. That is, how the two groups one of size \(k\) and the other of size \(n - k\) may share an identical secret key among themselves while an eavesdropper may cooperate with several (of course not all) dishonest members from any of the groups. If \(k = 1\), the result is equivalent to an \((n - 1, n - 1)\)-threshold secret sharing scheme.

The QKD–QSS connection is manifest in several earlier works [5, 22, 25]. Hillery et al [5] first introduced the idea of using a GHZ to implement a three-party secret sharing protocol. Bagherinezhad and Karimipour [25] extend a method of QKD with reusable entanglement [26] to QSS. Deng et al [22] extend the ping-pong QKD protocol [27] to a three-party circular QSS scheme. Sen et al [28] present a security for an \((n, n)\)-scheme involving on \(n\)-partite entangled states, based on the violation of Bell’s inequalities, even when the \(n\)-qubit correlations are weak. In contrast, we employ only bipartite states. From the theoretical perspective, this will provide the simplicity that we can build our protocol on top of QKD, which will help us reduce the security of our scheme to that of QKD. From a practical perspective, multiparty states employed for QSS earlier has exponentially low efficiency even in the noiseless scheme, since only rounds in which all participants measure the same observable, \(\sigma_i\) or \(\sigma_j\), are retained, with all other measurement possibilities discarded. In contrast, in our protocol, the key generation step will involve a measurement by all parties in the diagonal basis, so that no waste bits are produced through incompatible measurements by various parties.

2. Partial link classification

Given the access structure \(\Gamma = \{\alpha_1, \ldots, \alpha_r\}\), we divide all authorized sets \(\alpha_j\) into partially linked classes, each of which is characterized by the following two properties: (a) equation (1) is satisfied, if \(\alpha_j\) and \(\alpha_k\) belong to the same class; (b) for any two distinct classes, there is at least one pair \(j, k\), where \(\alpha_j\) belongs to one class and \(\alpha_k\) to the other, such that equation (1) fails.

We call a division of \(\Gamma\) into such classes as a partial link classification. The number of classes in a partial link classification gives its size. In general, neither the combinations nor size of partial link classifications are unique. We denote the size of the smallest partial link classification for a given \(\Gamma\) by \(\lambda\). If all authorized sets have mutual pairwise overlap, then \(\lambda = 1\) and the single partially linked class is, uniquely, \(\Gamma\) itself, and AQSS reduces to conventional QSS. If none of the \(\alpha_j\) sets have mutual pairwise overlap, then \(\lambda = r\) and
Given an access structure \( \Gamma = \{\alpha_1, \alpha_2, \ldots, \alpha_r\} \) with a minimum of \( \lambda \) partially linked classes among a set of players \( P = \{P_1, P_2, \ldots, P_N\} \), an AQSS scheme exists iff \( \Gamma \) is monotone. It requires no more than \( \lambda - 1 \) home shares.

**Theorem 1.** Given an access structure \( \Gamma = \{\alpha_1, \alpha_2, \ldots, \alpha_r\} \) with a minimum of \( \lambda \) partially linked classes among a set of players \( P = \{P_1, P_2, \ldots, P_N\} \), an AQSS scheme exists iff \( \Gamma \) is monotone. It requires no more than \( \lambda - 1 \) home shares.

**Proof.** It is known that if \( \lambda = 1 \), then there exists a conventional QSS to realize it [12]. Suppose \( \lambda > 1 \). To implement \( \Gamma \) (which represents a monotonic access structure), the dealer first employs a \((\lambda, 2\lambda - 1)\) majority function, assigning one share to each class. Recursively, each share is then subjected to a conventional QSS within each class. The remaining \( \lambda - 1 \) shares remain as home shares with the dealer. To reconstruct the secret, any authorized set can reconstruct the share assigned to its class, which combined with the home shares, is sufficient for the purpose. Clearly, since the necessity of the home share by itself fulfills the no-cloning theorem, authorized sets are not required to be mutually overlapping. Thus, monotonicity is the only constraint. \( \square \)

Some corollaries of the theorem are worth noting. First is that the number \((n = \lambda - 1)\) of home shares is strictly less than the number \((\geq N > \lambda)\) of player shares. A share \( q \) is ‘important’, if there is an unauthorized set \( T \) such that \( T \cup \{q\} \) is authorized. From the fact the theorem uses a threshold scheme (the \((\lambda, 2\lambda - 1)\) scheme) in the first layer, it follows that all the home shares are important.

As an illustration of theorem 1, we consider the access structure \( \Gamma = \{ABC, BD, EFG\} \), for which \( \lambda = 2 \). In the first layer, a \((2,3)\) scheme is employed to split \(|S|\) into three shares, with one share designated to the class \( C_1 = \{ABC, BD\} \) and the other to \( C_2 = \{EFG\} \). The last remains with the dealer. In the second layer, the first share is split-shared among members of \( C_1 \) according to a conventional QSS scheme. The second share is split-shared...
among players of $C_2$ according to a $((3, 3))$ scheme. Diagrammatically, this can be depicted as follows:

\[
\begin{align*}
((2, 3)) & : \{(r, 1, 1)\} \\
((3, 3)) & : \{(A, B, C), (B, D), (E, F, G)\} \\
((2, 2)) & : \{(F, G, H)\} \\
\end{align*}
\]

Note that given any $\Gamma$, even with disconnected elements, (so that the AS graph is not connected), there is an AQSS by simply adding a common player to all authorized sets, and designating him to be the dealer: e. g. $\Gamma = \{ABC, DE, FG\}$ giving $\Gamma' = \{ABCX, DE, FG\}$, where shares to $X$ would be designated as home shares. Thereby, the structure $\Gamma = \Gamma'_{|X}$, which denotes a restriction of $\Gamma'$ to members other than $X$, is effectively realized among the other players. However, this is not an efficient AQSS scheme because the number of resultant home shares are non-minimal, at least according to the recursive scheme outlined above. In all, it would require $3 + 2x$ shares, where $x$ is the number of instances in which $X$ appears in a maximal structure $\Gamma_{\text{max}}$ that includes $\Gamma'$. More generally, the requirement is a minimum of $r + (r - 1)x$ home shares, where $r$ is the number of authorized sets in $\Gamma$. A better method is for the dealer to employ a pure state scheme that implements $\Gamma_{\text{max}}$, retain all shares corresponding to $X$, while discarding all those corresponding to sets in $\Gamma_{\text{max}} - \Gamma'$. In all, this would require $3 + 2x$ shares, or, in general, $r$ shares. Better still, according to theorem 1, no more than $\lambda - 1 = 2$ home shares are needed. Clearly, in general, $\lambda - 1 < r$. These considerations suggest that $\lambda(\Gamma) - 1$ is the minimal number of home shares required to implement a QSS for $\Gamma$. We conjecture that this is indeed the case.

3. AQSS with quantum encryption

Finally, we note that practical AQSS can be made highly efficient in terms of using quantum resources by employing quantum encryption. Indeed, it is quite useful in QSS even outside the AQSS paradigm [17, 18]. Quantum encryption works as follows: suppose we have an $n$-qubit quantum state $|\psi\rangle$ and a random sequence $K$ of $2n$ classical bits. Each sequential pair of classical bits is associated with a qubit and determines which transformation $\hat{\sigma} \in I, \hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$ is applied to the respective qubit. If the pair is 00, $I$ is applied, if 01, $\hat{\sigma}_x$ is applied, and so on. To one not knowing $K$, the resulting $|\psi\rangle$ is a complete mixture and no information can be extracted out of it because the encryption leaves any pure state in a maximally mixed state, that is: $\frac{1}{2^d}\sum_{i=0}^{2^d-1} |i\rangle\langle i| \ket{i\rangle} = \sum_{i=0}^{d} \ket{i\rangle}\langle i| \sum_{j=0}^{d} \ket{j\rangle}\langle j| \sum_{k=0}^{d} \ket{k\rangle}\langle k| \sum_{l=0}^{d} \ket{l\rangle}\langle l|$. Any authorized set $\mathcal{K}$, the sequence of operations can be reversed and $|\psi\rangle$ recovered. Therefore, classical data can be used to encrypt quantum data.

In general, given $d$-dimensional objects, quantum encryption requires $d^2$ operators and a key of $2\log(d)$ bits per object to randomize perfectly [29]. In practice, such quantum operations may prove costly, and only near-perfect security may be sufficient. In this case, there exists a set of roughly $d \log(d)$ unitary operators whose average effect on every input pure state is almost perfectly randomizing, so that the size of the key can be reduced by about a factor of two [30].

The idea is quite simple: to share a quantum secret $|\psi\rangle$ according to access structure $\Gamma$, which does not necessarily satisfy the no-cloning condition, the dealer first encrypts the state to $|\overline{\psi}\rangle$ using classical bit string $K$. In the extreme case, the entire encrypted quantum state is treated as a single home share and transmitted to the reconstructor. String $K$ is then classically shared according to $\Gamma$. Any authorized set $\alpha$ can reconstruct $K$, and thus $|\overline{\psi}\rangle$, at the reconstructor’s location.

The above AQSS scheme with quantum encryption is much simpler than that based on partial linked classes proposed in section 2, however, it is interesting to note that the later scheme is stronger in the sense that the dealer (reconstructor) can give all his shares to some third party which might be untrustworthy and the secret still remains hidden even if all classical information leaks. Giving away the shares to third party is good from practical point of view as the dealer (reconstructor) might be limited by quantum memory requirements.

4. Combining QKD and QSS

In this section, we discuss our protocol for the two-group QKD—the problem of secure key distribution between two trustful groups where the individual group members may be mistrustful. The two groups retrieve the secure key string, only if all members cooperate with one another in each group. That is, how the two groups one of size $k$ and the other of size $n-k$ may share an identical secret key among themselves, while an eavesdropper may cooperate with several (of course not all) dishonest members from any of the groups. If $k = 1$, the result is equivalent to an $(n - 1, n - 1)$-threshold secret sharing scheme.

Note that the two-group QKD is trivially a classical secret sharing scheme if we involve a trusted third party, say, Lucy. Lucy will simply generate a random classical bit string. Since it is just a classical information, she makes two copies of it and splits one each amongst the two groups. Principles of quantum physics allow us, as in the case of 2-QKD [23, 24], to do away with the third party. We observe that the above problem essentially seems to be a combination of (a) 2-QKD between the two groups, each group being considered as a single party and (b) secret sharing in each group among their parties.

Our protocol works in two broad steps. In the first step, the $n$-partite problem is reduced to a two-party problem by means of a method for teleporting entanglement [31]. This creates a pure $n$-partite maximally entangled state among $n$-parties, starting from $n - 1$ EPR pairs shared along a spanning EPR tree, using only $O(n)$ bits of classical communication. This entanglement teleportation protocol exploits the combinatorial arrangement of EPR pairs to simplify the task of distributing multipartite entanglement. In the second step, as in the case of $n$-QKD, the Lo–Chau protocol [24] or the Modified Lo–Chau protocol [32] is invoked to prove the unconditional security of sharing nearly perfect EPR pairs between the two parties.
To this end, we will be using a state of the form:

\[
|\Psi| = \frac{1}{\sqrt{2}} (|00\cdots0\rangle + |11\cdots1\rangle), \tag{3}
\]
a maximally entangled \(n\)-partite state, represented in the computational basis.

Our protocol is motivated by a simple mathematical property possessed by multipartite states, unlike EPR pairs, which forces them to behave differently when measured in computational or diagonal basis. Let \(H\), \(\oplus\) and \(\otimes\) denote the Hadamard gate, the XOR operation and the tensor product respectively, then (with the presence of a proper normalizing factor in each expression),

\[
H^{\otimes s}|1\rangle^{\otimes s} = \sum_{x_1,x_2,\ldots,x_s} (-1)^{x_1+x_2+\cdots+x_s} |x_1x_2\cdots x_s\rangle,
\]

\[
H^{\otimes s}|0\rangle^{\otimes s} = \sum_{x_1,x_2,\ldots,x_s} |x_1x_2\cdots x_s\rangle,
\]

\[
\therefore H^{\otimes s}(|1\rangle^{\otimes s} + |0\rangle^{\otimes s}) = \sum_{x_1, x_2 \oplus, x_3 = 0} |x_1x_2\cdots x_s\rangle
\]

\[
= \left( \sum_{x_1, x_2 \oplus, x_3 = 0} |x_1x_2\cdots x_s\rangle \right) \times \left( \sum_{x_3, x_4, x_5, x_6 = 0} |x_3x_4x_5x_6\rangle \right)
\]

\[
+ \left( \sum_{x_1, x_2 \oplus, x_3 = 1} |x_1x_2\cdots x_3\rangle \right) \times \left( \sum_{x_4, x_5, x_6, x_7 = 0} |x_4x_5x_6x_7\rangle \right).
\]

We can observe by symmetry that the above factoring can be in fact done for any two groups of sizes \(s\) and \(n-s\), respectively. We are now ready to develop our protocol which involves the following steps:

(1) EPR protocol: Along the \(n-1\) edges of the minimum spanning EPR tree, EPR pairs are created. This involves pairwise quantum and classical communication between any two parties connected by an edge. Successful completion ensures that each of the two parties across a given edge share a nearly perfect singlet state \(\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)\). At the end of the run, let the minimum number of EPR pairs distilled along any edge of the minimum spanning EPR tree be \(2m\).

(2) The \(2m\) instances of the singlet state are then converted to the triplet state \(\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\), by the Pauli operator \(XZ\) being applied by the second party (called \(\mathcal{Y}\)) on his qubit.

(3) For each edge, the party \(\mathcal{Y}\) intimates the protocol leader (say ‘Lucy’) of the completion of step (2). Lucy is the one who starts and directs the following protocol for teleporting entanglement. Lucy can be from any of the two groups.

(4) Entanglement teleportation protocol: Using purely local operations and classical communication (LOCC), the \(n\) parties execute the teleportation protocol of [31], which consumes the \(n-1\) EPR pairs to produce one instance of the \(n\)-GHZ state equation (3) shared among them.

(5) A projective measurement in the diagonal basis is performed by all the parties on their respective qubits.

(6) Lucy decides randomly a set of \(m\) bits to be used as check bits, and announces their positions.

(7) All parties from a group assist to get one cbit corresponding to each check bit position by XORing their corresponding check bits. This gives an effective check bit corresponding to each check bit position. The two group then announce the value of their effective check bits. If too few of these values agree, they abort the protocol. We can note from the mathematics developed above that the effective check bits should agree after the diagonal basis measurement. Effective non-check bits are also calculated as above by XORing the non-check bits of the group members.

(8) Error correction is done as in for QKD between two trustful parties.

Proof of unconditional security (Sketch). The crucial element that simplifies the proof of the above protocol is that it can potentially be reduced to the proof of the security of sharing bipartite entanglement. This is because, beyond step (1), only LOCC are involved. We can thus exclude the involvement of a malicious eavesdropper Eve beyond step (1). Apart from correcting for quantum and classical noise, and the availability of an authenticated classical channel, the ability to detect Eve in this step suffices to secure the protocol against Eve.

The problem of secure bipartite entanglement generation has been extensively studied. For example, we may assume that step (1) is implemented using the Lo–Chau [24] or modified Lo–Chau protocol [32] (but by leaving out the final measurement step), which has been proven to be unconditionally secure.

Further, we need to mention the role of fault-tolerant quantum computation and of quantum error correcting codes during the execution of entanglement teleportation protocol of [31] for the following reason: suppose the probability of error on a bit is \(p\). Then, the probability of an error on the bit obtained by XORing all the \(s\) group members’ bits may be larger, given by: \(P = \sum_{r=1,3,5,\ldots} C(s,r)p^r(1-p)^{s-r}\), where \(C(s,r)\) is the number of all possible ways of selecting \(r\) elements from a set of \(s\) distinct elements. If \(P\) is too close to 0.5, then the effective channel capacity \(C_h\) for the protocol (given by \(C_h = 1 - H(P)\), where \(H(\cdot)\) is Shannon entropy) will almost vanish. Therefore, the quantum part of the protocol implementation should be very good to ensure that \(P\) is not too close to 0.5.

Of the XORed \(2m\) raw bits, \(m\) bits are first used for getting an estimate of \(P\), by obtaining the Hamming distance \(\delta\) between each group’s \(m\)-bit check string. If they are mutually too distant, the protocol run is aborted. If \(\delta\) is not too great (that is \(2\delta + 1 \geq d\)), it can be corrected with a classical code \(C(m,k,d)\), where \(m\) is block length, \(d\) is (minimum)
code distance and \( k/m \) is code rate. Each group decodes its XOR-ed \( m \)-bit string to the nearest codeword in \( C(m, k, d) \). This \( k \)-bit string is guaranteed with high probability to be identical between the two groups. An binary enumeration of the \( 2^k \) codewords of \( C(m, k, d) \) can be used as the actual key shared between the two groups.

Recalling the fact that the security of the QKD requires the two involved parties to be honest, one important point which should be carefully considered is whether the above reduction of the security proof to the bipartite case is impervious to the attacks when some subset of parties from one group collude with that from another group. It seems that the step (7) can ensure that the worst they can do is to sabotage the protocol. However, we feel that a more careful study is required before making any such claim, and we plan to do this in our future studies.

Acknowledgments

SKS thanks the LAMP Group, Raman Research Institute and NMR Research Center (SIF), Indian Institute of Science for supporting his visit, during which a part of this work was done.

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