Quantum Solution to the Extended Newcomb’s Paradox

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We regard the Newcomb’s Paradox as a reduction of the Prisoner’s Dilemma and search for the considerable quantum solution. The all known classical solutions to the Newcomb’s problem always imply that human has freewill and is due to the unfair set-up(including strategies) of the Newcomb’s Problem. For this reason, we here substitute the asymmetric payoff matrix to the general form of the payoff matrix (\(M\)) and consider both of them use the same quantum strategy. As a result we obtained the fair Nash equilibrium, which is better than the case using classical strategies. This means that whether the supernatural being has the precognition or not depends only on the choice of strategy.

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\section{I. INTRODUCTION}

A quantum game theory is a natural extension to the quantum world of the ubiquitous classical game theory. Von Neumann pointed out \cite{1}, a game is simply the totality of the rule which describe it. The elements of the rule such as the rational choices(strategies) are one of the main component of a general game. Moveover a game assumed that each individual player is trying to maximize his own advantage, without concern for the well-being of the other players. After the theory of the general game has been broken down into details \cite{2}, we can meet several games to play more efficiently for his profits. For example, von Neumann’s the mathematical approaches help us rational decision for the economic behavior, which strategies are fit and what’s the causal relationship. The Darwin’s theory for the ‘survival of fittest’ is maybe the another well-known game, the evolutionary theory, describe the biological evolution in nature, it is described by the Evolutionarily Stable Strategy(ESS) \cite{2}.

Here, we present two games known as the Prisoner’s dilemma(PD) \cite{4} and the Newcomb’s Paradox(NP) \cite{5}. Those game have some irrational consequence, i.e., the players cannot concurrently increase their payoff using a certain strategy in the PD. Also in the NP case, the supernatural being(SB)’s prophecy fail to facing human’s freewill surprisingly. Game theorists maybe call this ambiguous situation as a dilemma or paradox for a given competition. We will solve the NP under the well-known PD solution, which means the PD’s symmetric consideration helpful to grasp the NP.

The classical games, as long as its history, form a various kinds of game contained certain rules and it also have studied deeply. Its solutions of the dilemmatic view point are generally solved by the Nash equilibrium governed by a dominant strategy. While the usual quantum games \cite{4,6} have comparatively short history, but their solution shows the similar Nash equilibrium pattern—the changed Nash equilibrium can be compared to the classical one. Meyer has already proved \cite{6} that an optimal quantum strategy in a two-person zero-sum game surpass against to an optimal classical (mixed) strategy. If the participant, Alice, is using a probabilistic (mixed) quantum strategies against to Bob’s the deterministic classical strategies, then Alice always win to Bob on the game. Furthermore a two-person static game has a Nash equilibrium for the quantum versus quantum strategy. For consider the NP, SB’s deterministic prophecy gives an unfair situation for the human’s probabilistic strategies. So we need to reconstruct the original payoff to the new reasonable payoff structure.

\section{II. CLASSICAL STRATEGY AND ITS SOLUTION}

\textbf{A. NP and PD in the Classical Game}

In the PD each of the players, Alice and Bob, must independently decide whether they choose to defect (strategy \(D\)) or cooperate (strategy \(C\)). Depending on their decision, each player receives a certain payoff \(\pi_6\). The strategy \(D\) is the dominant and will be in equilibrium, i.e., \(\{\sigma_A, \sigma_B\} = \{D, D\}\) and \(\{\pi_A(\sigma_A), \pi_B(\sigma_B)\} = \{1, 1\}\) in this game. A dominant strategy for Alice is defined by a strategy \(\sigma_A\) such that the payoff \(\pi_A\) has the property \(\pi_A(\sigma_i, \sigma_B) \geq \pi_A(\sigma_A', \sigma_B')\) for all \(\sigma_A', \sigma_B' \in S_A, \sigma_B' \in S_B\) provided such a strategy exists, where \(i, j \in N\). The strategy set \(S_k \simeq \{\sigma_i\}\) contains the all strategies \(i \in N\), where \(N\) is the number of the all possible strategies in the game and \(k\) denote the players \(A\) or \(B\). Moreover Nash equilibrium, corresponding to the payoff \(\{1, 1\}\), is a combination of strategies \(\{\sigma_A, \sigma_B\}\) such that neither party can increase his or her payoff by unilaterally departing from the given

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equilibrium point \( i.e., \Pi_A(\Sigma_A, \Sigma_B) \geq \Pi_A(\Sigma'_A, \Sigma'_B) \) and \( \Pi_B(\Sigma_A, \Sigma_B) \geq \Pi_B(\Sigma'_A, \Sigma'_B) \). Another main concept of a game theory is the Pareto optimality, a pair of payoffs \{\Sigma_A, \Sigma_B\} = \{3, 3\}, if it is not jointly dominated by another point, and if neither party can increase his or her payoff without decreasing the payoff to the other party. Someone perhaps ask that the strategy \( M_1 \) will fill up the Box1 with \$1 000 000, and Box2 which contains either nothing or \$1 000 000. The human may pick either Box1 or both. However, at some time before the choice is made by human, \( SB \) has predicted what the human’s decision will be made \( F \). Thereafter SB will fill up the Box1 with \$1 000 000 if \( SB \) predicts human to take it, or with nothing if \( SB \) predicts human to take both boxes. The constructionally important notion of the NP is no other than its one-person game as the restricted two-person game PD’s \( S \). We can see the original asymmetric payoff matrix for human in Table II.

### Extended form of NP and PD

Von Neumann referred to the fact that game theory only concern with the relation of the elements of the payoff matrix. So that in this paper, we can consider the extended version of the NP and PD. The general form of the corresponding payoff matrix is denoted by \( \mathcal{M} = M_1 + M_2 \) where \( M_1 = \begin{pmatrix} \alpha & 0 \\ 0 & \gamma \end{pmatrix} \) and \( M_2 = \begin{pmatrix} 0 & \delta \\ \beta & 0 \end{pmatrix} \). \( M_2 \) will be considered as two special case, an asymmetric payoff matrix and the quasi-skew symmetric matrix. Moreover we fix the matrix elements, in the payoff matrix, along to \( \beta > \alpha > \gamma > \delta \) and \( \delta \leq 0 \).

### IV. QUANTUM STRATEGY

#### A. Quantum Strategies and Quantum Circuit

A general quantum game \( \Gamma \) is consisted of the player \( k \), strategy \( S_k \in S_k \subset \hat{U}_k \equiv \hat{U}(\theta, \phi) \) and payoff \( \Pi_k \).
like as the general classical game \[4\]. But the strategic space \(S_k\) is comparatively large space chosen from \(\hat{U}\), \(S_k \subset \hat{U}_k \subset \hat{U}\), \(\hat{U} \equiv \{\hat{U}(\theta, \phi) | \theta \in [0, \pi] \) and \(\phi \in [0, \pi/2]\}\} in Hilbert space \(\mathbb{H}\), compare to the classical deterministic space \(S_k\). The quantum games are following the next trivial stages. First, the well-known initial states are prepared by a umpire or the public renouncement, because the fairness of the game is ensured by its open source of the payoff. In PD, we set the initial state as two qubit state \(|00\rangle\). Next step, both player conflict to win the game with the given strategy set \(\hat{S}_k \subset \hat{U}_k\), which is the quantum mechanical operation. Main point of a game maybe make out what is the best strategy in the confiction. Finally, the last step is some detection of the game result by quantum measurement.

We denote the initial state of the game by \(|\psi_0\rangle = \hat{J}|00\rangle\), where \(\hat{J}\) is a unitary operation executed by an umpire and the adjustment of the parameter \(\tau\) gives a separable or entangled states. The unitary operation \(\hat{J}\) is described by \(\hat{J} = e^{i\tau \hat{U} \otimes \hat{U}/2}\), where \(\tau \in [0, \pi/2]\) and \(\hat{U} \in \hat{U}(\theta, \phi)\) stand for the selected operation by an umpire. In fact, \(\tau\) is said to be a measure for the game’s entanglement i.e., \(\tau = \pi/2\) means the maximally entangled state. If we have prepared the initial game setup such as the separable or entangled state, then we can play PD in each player’s strategic space \(\hat{S}_k\). Each player may execute own’s qubit by \(\hat{\Sigma}_k \in \hat{S}_k\) for increment of his payoff, of course the player let to moves secretly. It provide to be sufficient to restrict the strategic space to the two-parameter set of 2 \(\times 2\) matrices,

\[
\hat{U}(\theta, \phi) = \begin{pmatrix}
    e^{i\phi} \cos \theta/2 & \sin \theta/2 \\
    -\sin \theta/2 & e^{-i\phi} \cos \theta/2
\end{pmatrix}
\]

where \(0 \leq \theta \leq \pi\) and \(0 \leq \phi \leq \pi/2\). From the result of the Eisert et al. \[4\] \[10\], where various kinds of strategy are introduced, we can find the general quantum leaps in the game theory. Specially the strategy \(\hat{Q} \otimes \hat{Q} \equiv \hat{U}(0, \pi/2) \otimes \hat{U}(0, \pi/2)\) give a new Nash equilibrium, against to classical strategy \(\hat{D} \otimes \hat{D} \equiv \hat{U}(\pi, 0) \otimes \hat{U}(\pi, 0)\). Here, we will briefly depict the evolution of qubit state. The prepared initial state of the two qubit, to play on the game, is denoted by \(|\psi_0\rangle = \hat{J}|00\rangle\). Next the participants, Alice and Bob, play own’s qubit using the unitary operation \(\hat{U}_k\). The state of the game is prepared, after the previous strategic process, like that

\[
|\psi_f\rangle = \hat{J}^\dagger (\hat{U}_A \otimes \hat{U}_B)|\psi_0\rangle
\]

where \(|\psi_f\rangle\) means the final state of the game, see figure in

Before we measure the final state, the Alice’s payoff can be expected as \(\Pi_A = \alpha P_{\hat{C}C} + \beta P_{\hat{D}C} + \delta P_{\hat{C}D} + \gamma P_{\hat{D}D}\) where \(P_{\hat{A}\hat{B}} = \langle (|\hat{A}\hat{B}| \psi_f\rangle |^2\rangle\) is the measurable probability with corresponding payoff as the coefficient respectively. If each qubit is measured by appropriate device, then each player obtains the corresponding payoff which is a resultant value of the quantum PD. For example, the case of \(\tau = \pi/2\) and \(\hat{Q} \otimes \hat{Q}\) build up to the new Nash equilibrium \(\{\alpha, \alpha\}\) from the classical equilibrium \(\{\gamma, \gamma\}\). That is, the classical Pareto optimal correspond to the new stable point of the PD.

Now we will compute the new Nash equilibrium in the PD, which is occurred by the some additional quantum strategy such as Walsh-Hadamard transformation \(H\),

\[
H = \frac{1}{\sqrt{2}} \begin{pmatrix}
    1 & 1 \\
    1 & -1
\end{pmatrix}
\]

or Pauli operations

\[
\sigma_x = \begin{pmatrix}
    0 & 1 \\
    1 & 0
\end{pmatrix}, \quad \sigma_y = \begin{pmatrix}
    0 & -i \\
    i & 0
\end{pmatrix}, \quad \sigma_z = \begin{pmatrix}
    1 & 0 \\
    0 & -1
\end{pmatrix}
\]

First of all the initial state \(|\psi_0\rangle\) let to be the maximally entangled state, changing the variational value \(\tau\) from \(0\) to \(\pi/2\) and the choice of \(\hat{U}\). For instance, if the entangled measure \(\tau\) fix to \(\pi/2\) and the umpire’s choice \(\hat{U}\) select to \(\hat{D} \otimes \hat{D}\), then the value \(\hat{J}\) is locked up \[4, 10\] like that \(\hat{J} = e^{i\pi \hat{D} \otimes \hat{D}/4} = \frac{1}{\sqrt{2}} (\hat{I} + i\hat{D} \otimes \hat{D})\), where \(\hat{I}\) means the identity operation. Therefore, we obtain the maximally entangled state, denoted by \(|\psi_f\rangle = \hat{J}|00\rangle = \frac{1}{\sqrt{2}} (|00\rangle + i|11\rangle)\), and sequentially get to the separable final state \(|\tilde{\psi}_f\rangle\), by the disentangling operation \(\hat{J}^\dagger\) after the local operations \(\hat{U}_A = \hat{D}\) and \(\hat{U}_B = \hat{D}\) are done of each player respectively i.e., \(|\tilde{\psi}_f\rangle = \frac{1}{\sqrt{2}} \hat{J}^\dagger (11) + i(00)\rangle = |11\rangle\).

Although we perform the measurement by a certain observable \(\hat{\Omega}_{A(\hat{B})}\) with the combinations of

\[
\{\alpha|00\rangle\langle00|, \delta(\beta)|01\rangle\langle01|, \beta(\delta)|10\rangle\langle10|, \gamma|11\rangle\langle11|\},
\]

the Nash equilibrium,

\[
\begin{align*}
\Pi_A &= \text{tr} (\hat{\Omega}_A|\tilde{\psi}_f\rangle\langle\tilde{\psi}_f|) \\
\Pi_B &= \text{tr} (\hat{\Omega}_B|\tilde{\psi}_f\rangle\langle\tilde{\psi}_f|) = \{\gamma, \gamma\}
\end{align*}
\]

with the probability 1, is obtained on this traditional strategy. This is only the classical consequence. If we choose the some less quantum strategy and full quantum strategies such as the Hadamard transformation \(H\) and Pauli matrices \(\{\sigma_x, \sigma_y, \sigma_z\}\), then the new Nash equilibrium is created, that is, we will choose the extended strategic space \(\hat{S}\). Given the entangled state

\[
|\tilde{\psi}_0\rangle = \frac{1}{\sqrt{2}} (|00\rangle - i|11\rangle)
\]

with \(\tilde{\hat{J}} = e^{i\pi (\sigma_y \otimes \sigma_y)/4}\) transform to the final state via the local operation \(\tilde{\hat{U}}_k = \hat{H} \in \hat{U}_k(\theta, \phi),\)

\[
|\tilde{\psi}_f\rangle = \frac{1}{2} (|00\rangle - |01\rangle - |10\rangle + |11\rangle)
\]
The final state (10) gives $\tilde{\Pi}_A = \text{tr}(\hat{\Omega}_A |\tilde{\psi}_f\rangle\langle\tilde{\psi}_f|) = \frac{1}{4}(\alpha + \delta + \beta + \gamma)$, that is,

$$\{\tilde{\Pi}_A, \tilde{\Pi}_B\} = \{\frac{1}{4}(\alpha + \delta + \beta + \gamma), \frac{1}{4}(\alpha + \beta + \delta + \gamma)\}. (11)$$

Equation (11) is always superior to the classical equilibrium $\{\frac{1}{4}(\alpha + \gamma), \frac{1}{4}(\alpha + \gamma)\}$. But the value $\delta$ equal to $-\beta$, the skew-symmetric case, only gives classical payoff. If the each player choose another strategy $\hat{U}_k = \sigma_z$, then the final state will be

$$|\tilde{\psi}_f\rangle = \frac{1}{\sqrt{2}}\hat{f}(\sigma_x A \otimes \sigma_x B)(|00\rangle + i|11\rangle) = |00\rangle \quad (12)$$

and the expected payoff is $\{\tilde{\Pi}_A, \tilde{\Pi}_B\} = \{\alpha, \alpha\}$. This value is the new Nash equilibrium within the quantum strategy, i.e., $\{	ilde{\Pi}_A(\sigma_z, \sigma_z), \tilde{\Pi}_B(\sigma_z, \sigma_z)\} \succeq \{\Pi_A(\sigma_z, \hat{U}_B), \Pi_B(\hat{U}_A, \sigma_z)\}$ for all $\hat{U}_k$. We can conclude that the allowed $SB$’s quantum strategies always predict the human’s choice (diagonal elements), the solution of the game. Although the classical solution to the PD using repeated game gives a equilibrium point in $\{\alpha, \alpha\}$ as like as Eisert et al., the quantum solution to the PD is one-shot solvable. After the expected payoff is reported with the PD, we can confirm the solution of the NP unambiguously.

B. Quantum NP Solution

The quantum solutions to the PD, two-person complete information game, directly correspond to the NP, if we consider the game NP only its reduced one-person game. The Nash equilibrium change $\{\gamma, \gamma\}$ to $\{\frac{1}{4}(\alpha + \delta + \beta + \gamma), \frac{1}{4}(\alpha + \delta + \beta + \gamma)\}$ on the Hadamard strategy. Furthermore the strategy $\sigma_z$ creates the new optimal equilibrium $\{\alpha, \alpha\}$, corresponding to the classical Pareto optimal. Therefore we can adapt the solution of the NP from PD, as the payoff $\Pi_k = \alpha$. Another words the human’s unique choice $Box 1$ is restricted by quantum mechanical $SB$’s prophecy, maybe $SB$’s prediction is always corrected as the diagonal elements in the given payoff matrix. This result also can be calculated by the convex combination of the human’s pure choice $\tilde{\Pi}^*$, let’s the human choose the $Box 1$ with a certain probability $p \in [0, 1]$ and $Box 2$ with a probability $1 - p$, then the intermediate state of the NP (using the Hadamard transform $H$) is given by

$$|\tilde{\psi}_f\rangle\langle\tilde{\psi}_f| = \frac{1}{2} \left( \begin{array}{cc} 1 & 2p - 1 \\ 2p - 1 & 1 \end{array} \right). \quad (13)$$

So that the expected payoff of the NP is $\tilde{\Pi}_A = \frac{1}{2}[\alpha + (2p - 1)\delta + (2p - 1)\beta + \gamma]$, coincide with the previous value in the PD. If one player choose the quantum strategy, such as $\sigma_z$ occur a previous situation, the Nash equilibrium $\Pi_A = \alpha$, this is the predicted value by $SB$’s $\Pi_B = \alpha$.

IV. CONCLUSION

In this paper, we used the payoff matrix with arbitrary variables and also described the performance of quantum strategies, which enable to represent classical strategies. Especially we regarded the one-person NP as a two-person PD problem and then, under quantum strategies, obtained more reasonable and uplifted solutions over classical strategies. In the same strategies, we found out the Nash equilibrium always lies along the fair values, regardless of the payoff matrix. That is, the winner is never determined by the payoff, but by the level of strategies. Therefore the Newcomb’s paradox is not a paradox under the fair situation.

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[1] J. von Neumann, and O. Morgenstern, (Princeton University Press, Princeton, NJ, 1947).
[2] R. B. Myerson, (MIT Press, Cambridge, MA, 1991).
[3] A. Iqbal, and A. H. Toor, Phys. lett. A 280, 249 (2001).
[4] J. Eisert, M. Wilkens, and M. Lewenstein, Phys. Rev. Lett 83, 3077 (1999).
[5] E. W. Piotroksi, and J. Sladkowski, International Journal of Quantum Information 1 (2003) 395.
[6] D. A. Meyer, Phys. Rev. Lett 82, 1052 (1999).
[7] Assume that the $SB$’s prophecy and human’s decision are expected almost simultaneously.
[8] C. Schmidt-Petri, In Proceedings Philosophy of Science Assoc. 19th Biennial Meeting - PSA2004. PSA 2004 Contributed Papers.
[9] J. O. Grabbe, e-print [quant-ph/0506219]
[10] S. C. Benjamin and P. M. Hayden, Phys. Rev. Lett 87, 069801 (2001).