The Universal Deceleration and Angular Diameter Distances to Clusters of Galaxies

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ABSTRACT

We show how the virial theorem can be applied to the hot gas in clusters of galaxies to obtain a yardstick, which could then be used to determine cosmological parameters. This yardstick relies on the assumptions of hydrostatic equilibrium and that the gas fraction is approximately constant. The constancy is checked empirically from a local population of clusters. By using the observed parameters consisting of temperature, surface brightness and radial profile $\beta$, one can calculate the expected core radius. Comparing it to the observed angular size, one can in principle calibrate the cosmological deceleration parameter $q_0$. We test this method on a small sample of 6 clusters, and show its promise and accuracy. The preliminary implications would be to suggest $q_0 \approx 0.85 \pm 0.29$ with $1 - \sigma$ statistical error bars, with several systematic uncertainties remaining. Taken at face value, this would argue against a cosmological constant. The method is robust to errors in the measurement of the core radius as long as the product of the central density and the core radius squared $\rho_0 r_c^2$ are well determined. New lensing and X-ray data can dramatically improve on the statistics.

1. Introduction

According to the standard model of the hot big bang (Peebles 1993), the universe began with small primordial fluctuations which grew through gravitational instability. The model invokes the existence of gas and dark matter, which on scales larger than $10^6$ solar masses obey the same dynamical equations in linear theory. After the perturbations grow non-linear at a redshift $z \approx 10$ when the universe was a few percent of its current age, our understanding of structure formation becomes incomplete. Stars, quasars and galaxies form from the collapsing and cooling gas, while the dark matter is non-linearly stable and

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remains in halos around the galaxies. At some point clusters of galaxies formed, containing more mass in hot gas than in stars or galaxies. Because most are very hot \((T \approx 10^8 \text{K})\), that gas is unable to cool and contract efficiently, and would remain in hydrostatic equilibrium balanced by pressure and gravity in the same way as the dark matter. Detailed analyses ([White et al 1993]) show that we know of no mechanism to keep the gas out of the potential well, nor to collapse it significantly relative to the dark matter, and we would thus think that the gas would constitute the same fraction of the mass in clusters as they did globally, except for the small fraction which condensed to form stars. This is confirmed in numerical simulations ([Pen 1996], [Navarro et al 1994], [Lubin et al 1996]). We parametrize the Hubble constant as \(50h_{50}\text{km/sec/Mpc}\), where \(h_{50}\) is believed to lie between 1-1.6. The first attempt to use this hypothesis was by Henry and Tucker (1979), who derived a gas fraction of 18% and suggested its use for cosmological purposes. In this letter we use the better data collected since then to actually estimate the deceleration parameter \(q_0\).

By analyzing a local sample of clusters, we find the gas fraction \(f_g\) from an Einstein IPC sample of 5 clusters together with COMA to be \(f_g = 0.178 \pm 0.015 h_{50}^{-3/2}\), with an intrinsic scatter of only 20%. The estimate assumes that the gas fraction is constant at all radii. This gas fraction is consistent with earlier estimates ([White et al 1993], [White and Fabian 1995]). While the constant gas fraction assumption certainly introduces errors, we find empirically that this error must be less than the 20% observed in the data. Furthermore, by assumption this fraction does not depend on the radius at which it is measured. The observed radial profiles are determined from X-ray data, which puts most weight near one core radius. There we would expect clusters to be well relaxed, and the gas fraction to be representative of its global value.

In this article we use a detailed model of hydrostatic equilibrium to calculate the core radius of clusters from their observable properties, assuming a constant gas fraction. By comparing it with the angular size, we obtain the angular diameter distance to the cluster, which provides a direct measure of the deceleration parameter \(q_0\). Taken at face value, the small test sample suggests \(q_0 = 0.85 \pm 0.29\) if we assume the sample of three distant clusters at a redshift of \(z \sim 0.5\) to have the same gas fraction as the local sample. This preliminary result is still subject to large systematic uncertainties. The situation is expected to improve rapidly as current X-ray data at intermediate redshift \((z \gtrsim 0.4)\) is collected and reduced. The value determined with this method is consistent with recent determinations by Perlutter (1996a,b) using type 1a supernovae.

We use the notation \(\Omega_b, \Omega_m\) to describe the fraction of baryons and total non-relativistic matter relative to the critical density.
2. Estimation of gas fraction.

We will use the observed emission weighted temperatures $< T >_X$, central surface brightness $\Sigma_0$, angular core radii $r_c$, and radial power law index $\beta$ to determine the desired parameters. In order to apply the virial theorem and hydrostatic equilibrium, we need to further assume that gas traces mass, which has been found to be a good approximation to simulated clusters.

For x-ray clusters, a popular fit to the density profile is that of a $\beta$-model (Jones and Forman 1984) which assumes spherical symmetry of the mass distribution. The model fits the density profile to be $\rho = \rho_0 (1 + r^2/r_c^2)^{-3\beta/2}$. $\rho_0$ is the central density and $r_c$ is called the core radius. Projected onto two dimensions, we obtain the surface density $\Sigma = \Sigma_0 (1 + r^2/r_c^2)^{-3\beta/2 - 1/2}$. If we assume the sphere to be self-gravitating, we obtain the pressure

$$P = 4\pi G\rho_0^2 r_c^2 \int_{r/r_c}^{\infty} (1 + u^2)^{-3\beta/2} u^{-2} \int_0^u (1 + v^2)^{-3\beta/2} v^2 dv du.$$ (1)

At $r \gg r_c$, we note that $P \propto r^{-6\beta+2}$, and for typical $\beta \approx 2/3$, the pressure drops rapidly. Therefore the behavior of the cluster far outside the core radius does not affect the interior pressure significantly. Similarly, we can derive the temperature profile $T = \mu m_p P/\rho k$, and we can express the central density in terms of the luminosity weighted temperature assuming pure bremsstrahlung $< T >_X \equiv \int \rho^2 T^{3/2} / \int \rho^2 T^{1/2}$ to be

$$\rho_0 = \frac{k < T >_X}{4\pi G r_c^2 \mu m_p H(\beta)}$$ (2)

where $H(\beta)$ is shown in figure 1. We note that this formulation is only accurate for $\beta > 0.65$, and for $\beta < 4/7$ the emission weighted temperature $H(\beta)$ formally diverges. Using standard values for the parameters, we can now calculate the gas fraction

$$f_g = 9.883 H(\beta) \left( \frac{n_e}{10^{-3}\text{cm}^{-3}} \right) \left( \frac{r_c}{\text{Mpc}} \right)^{2} \left( \frac{\text{keV}}{T} \right) h_{50}^{-3/2},$$ (3)

where $n_e$ is the central electron density. We have assumed the mean molecular weight to be $\mu = 0.63$.

At cosmological distances, we can similarly invert (3) to yield the core radius. The dimensionless deceleration parameter is defined as $q_0 = -\ddot{a}/a^2$ in terms of the scale factor $a = 1/(1 + z)$. In order to facilitate translations of observed and published data, we assume that the observational quantities were fitted using $h_{50} = 1$, $q_0 = 0.5$ in a matter dominated universe, as the majority of authors seem to prefer. The observable quantities are the central surface brightness, angular core radius and temperature. Only the second quantity
has a cosmological \( q_0 \) dependence. In order to convert the central X-ray surface brightness into a density, we need to divide the surface brightness by a length, and take its square root. This introduces a dependence on angular diameter distance to the inverse one half power. The \( r_c \) term in (3) depends on the square of the angular diameter distance. Putting it together, we find a dependence on the angular diameter distance to the \( 3/2 \) power.

We have now derived the cosmological dependence

\[
fg(q_0) = \left(1 + \left(\frac{1}{4} - \frac{q_0}{2}\right)z\right)^{3/2} \times fg(q_0 = \frac{1}{2})
\]  

(4)

which is independent of the Hubble constant \( h \). Strictly speaking equation (4) is only correct in the limit \( z \to 0 \). If the deceleration were constant \( q = q_0 \), for example if \( \Omega_m = 1 \) or \( \Omega_m = 0 \), then equation (4) is accurate to better than 10% for \( z < 4 \).

3. Sample data set.

To implement this simple prescription, we used the Jones and Forman cluster sample (Jones and Forman 1984). In order to obtain an accurate data set, we restricted ourselves to clusters where \( L_X > 10^{44} \) erg/sec, \( \beta > 0.6 \) and \( r_c \) is determined to better than a factor of 2. This leaves us with a set of five clusters: A85, A2199, A2255, A2256 and A2319. The respective gas mass fractions derived from formula (3) are 0.22, 0.14, 0.15, 0.17, 0.22. In addition, we used the data from (Briel et al. 1992) for the COMA cluster in order to compare our method to the well studied numbers from White et al. (White et al. 1993). That data yielded a gas fraction of 0.17, which is consistent with the previous estimates by White et al. All numbers are expressed for \( h_{50} = 1 \), and scale as \( h_{50}^{3/2} \). If we use equal weighting, the mean gas fraction is \( f_g = 0.178 \) with a standard deviation of \( \sigma = 0.034 \). That would imply a 20% RMS cluster to cluster variation in the gas fraction. The mean \( f_g \) estimate from our sample of \( N = 6 \) clusters would then be accurate to \( \sigma/\sqrt{N-1} \approx 0.015 \). This fluctuation is consistent with the simplest hypothesis that the gas fraction is constant for all clusters in the sample, with the variation arising solely from the error in measurement of the cluster properties. Detailed weighting of the observational error bars are difficult to implement from the published data since each of the error terms are correlated. Nevertheless, we see that this crude method yields robust results with a small scatter, which allows us to apply it to measure cosmological parameters.

The highest redshift clusters satisfying the same selection criteria that our literature search revealed were A370 at a redshift \( z = 0.373 \) (Bautz et al. 1994, Struble and Rood 1987) and CL0016+16 at \( z = 0.5545 \) (Neumann and Böhringer 1996). For A370, the authors of
that paper were able to measure the temperature accurately, but due to the nature of the ASCA X-ray satellite, the spatial profile is still subject to uncertainties. We use their best estimates $r_c = 0.25\, \text{Mpc}$, $T = 8.8\, \text{keV}$, $\beta = 0.65$ and $n_e = 6.5 \times 10^{-3}\, \text{cm}^{-3}$. Using (3) and (4) implies $f_g = 0.189(1 + (q_0^2 - 1)z^{3/2})$. If we assume that the gas fraction is drawn from the same distribution as the local sample, we obtain $q_0 = 0.71^+0.68_{-0.64}$ with $1 - \sigma$ errors.

Cl0016+16 is a better candidate due to its higher redshift. Neumann and Böhringer used both the ROSAY HRI and PSPC to measure the cluster parameters. For the HRI the best fit values are $\beta = 0.8$, $r_c = 0.372\, \text{Mpc}$, $n_e = 6.5 \times 10^{-3}\, \text{cm}^{-3}$, from which we deduce a gas fraction $f_g = 0.20$ for $q = 1/2$. The PSPC parameters are slightly different, $\beta = 0.68$, $r_c = 0.283\, \text{Mpc}$, $n_e = 7.6 \times 10^{-3}\, \text{cm}^{-3}$, for which $f_g = 0.24$. We use the average of those two values $f_g = 0.22$, and apply the same procedure as above to obtain $q_0 = 0.98^{+0.31}_{-0.39}$, again at $1 - \sigma$. The HRI and PSPC gas fractions vary from their average by 10%, which is smaller than the local scatter of 19% which we used to determine the error bars. It is difficult to quantify how errors should be added or treated, and they should not be considered rigorous in any sense. While the published values for the $\beta$ model parameters include error bars, these are strongly correlated. The model depends only strongly on the asymptotic radial profile of the gas, which is well determined in the fits. The observational errors in the determination of $r_c$ and $n_e$ are correlated in such a way as to preserve the asymptotic radial behavior, $n_e \sim r_c^{3\beta}$. In principle it is possible to obtain intrinsic errors by fitting the models directly to the raw data. For now, the most robust estimator is still the local sample. Some systematic errors could of course enter, the largest of which being an inherent evolution in the gas fraction of clusters. Neglecting such possibilities allows us to formally combine the two observations and derive a combined $q_0 = 0.91 \pm 0.34$.

A more difficult example is RXJ1347.5-1145 (Schindler et al 1996). At a redshift $z = 0.451$ this cluster was found to have a strong cooling flow, and formally $\beta = 0.56$. Our formula (3) diverges, and therefore does not apply for this value of $\beta$. While a significant portion of the emission may come from the cooling flow (the authors estimate 43%), the total gas mass interior to 240 kpc is well constrained. The authors obtained a gas fraction at that radius for a flat universe of $f_g = 0.19$. Their inferred gas mass fraction increases with radius since the dark matter was assumed to be isothermal, while the gas was assumed scale as $\rho \propto r^{-1.68}$. Due to a cooling flow at the center of the cluster, the radial profile cannot be inferred accurately. The gas mass is only directly well determined at the 240 kpc radius, even at 1 Mpc the uncertainties are large, probably more than a factor of two. Our Ansatz assumed that the gas fraction is independent of radius, so we would infer here at the most accurate radius that $f_g(q_0 = 0.5) = 0.19h_50^{-3/2}$. Applying (4) we find $q_0 = 0.69^{56}_{-52}$ for this cluster.
Combining all three clusters now formally gives $q_0 = 0.85 \pm 0.29$. Conversely, if we allow the cluster gas fraction to vary freely in time and assume, then a $q_0 = -1/2$ would imply that clusters of galaxies at a redshift $z \sim 0.5$ had a gas fraction 50% higher than clusters do today. We note that the distant cluster sample only had a small scatter in their relative gas fraction, no bigger than that of the local sample.

4. Systematic Uncertainties.

Historically the measurement of $q_0$ has been plagued by evolutionary effects (Misner et al. 1973). Objects at cosmological distances represent the universe when it was younger, and could certainly have been systematically different in the past. It has been difficult to measure the evolution of objects separately from the evolution of space-time. The current estimate relies on the gas fraction remaining constant, which is justified by the similarity in the equation of state between collisionless dark matter and an ideal gas, as well as numerical simulations which confirm these guesses. But here we do not have to believe blindly. Near term high resolution temperature measurements with the AXAF X-ray satellite to be launched in 1998 will measure the gas fraction of the local population to high accuracy. Recent measurements of gravitational lensing have been able to reconstruct the mass distribution in distant clusters, in fact a redshift of 1/2 is ideal for such measurements. The strong arcs observed in A370 (Grossman and Narayan 1989) would certainly be compatible with an assumed constant gas fraction, but here we still have systematic uncertainties involved. Comprehensive modelling combining weak and strong lensing can ultimately test the assumption of constant gas fraction between clusters at similar distance, and constancy in the radial profile.

The cluster sample was chosen from the literature, which does not have an objective selection function. If some clusters had a higher gas fraction at some fixed temperature, these would also be most X-ray luminous and might be preferably selected. It is not clear that the distant cluster sample had similar selection criteria as the local sample. It is therefore by no means statistically fair to simply compare the averages of gas fractions in these two populations and draw definitive conclusions. In the absence of a quantitative selection function, however, we can only ignore the possibility of this effect. The only reassurance is that the scatter in cluster gas fractions is small, both in the local as well as in the distant sample.

We have assumed that the gas is spherically symmetric and in hydrostatic equilibrium. Since the cores of clusters are overdense by about $10^4$ relative to the cosmic density, the age of the cluster is a hundred fold longer than the dynamical time $t_d \approx 1/\sqrt{G\rho}$. Since
any perturbations are expected to damp on a dynamical time, one would expect the assumptions of hydrostatic equilibrium to hold to high accuracy. Numerical simulations (Pen 1996) indicate that the kinetic energy fraction in the core is indeed typically less than 10% of the thermal energy. We can make some estimates to the expected presence of substructure. Henry et al 1992 observed no evolution in the cluster luminosity function to a redshift of $z = 0.35$. We can thus assume that the clusters are at least half as old as the universe. Let us consider two extreme cases. If they grew by a constant rate of continuous accretion, we would find all clusters with the same amount of non-equilibrium energy of a few percent. The opposite extreme is that clusters formed by sudden mergers of equal mass objects. In that case, a few percent of clusters would be just merging, and be maximally out of equilibrium. The remaining ninety-some percent would be in very good hydrostatic equilibrium. The truth is probably somewhere inbetween. There may be a few clusters just undergoing mergers, and all clusters may have some small amount of substructure. But we see that the effect must be quite small, unless we are unlucky and happen to be selecting the few non-relaxed clusters. We would need to wait until we collect a larger sample size at cosmological redshift before we can know for sure. Occasionally subclumps could appear near the cores of clusters due to chance projections.

Similarly, one would expect the core to be more spherical than the cluster as a whole, since the core has had more time to undergo relaxation. The assumptions of hydrostatic equilibrium can be tested using the X-ray satellite AXAF by comparing line ratios to line widths.

Core radii are often difficult to measure accurately, and the formal error bars on the fits in the local sample is larger than the inferred gas fraction scatter. The reason for this behavior is an almost perfect correlation between the error in fitting the core radius and the inferred central density. The parameter which is usually well determined is the surface brightness at radii slightly larger than the core radius, where most photons are measured. Errors in the core radius and central density will tend to correlate in a way to preserve this surface density. In the case that $\beta = 2/3$, which is in the typical range for clusters, any such correlated change have no effect at all on the total gas fraction. Of course, the $\beta$ model parametrization provides a convenient notation to describe the density outside the core radius, but if we wish to have uncorrelated measurements, one should perhaps parametrize the surface density just outside the core radius separately.

These arguments might explain the small observed scatter in the gas fraction. There are undoubtably an even greater number of effects which would contribute to increase the measured scatter, but empirically we have found that all those effects either cancel, or contribute at most 20% scatter to the deduced gas fraction.
In simulations (Pen 1996) we find that the dark matter is more centrally concentrated than the gas, even though they trace each other quite well beyond a core radius. This is also found in lensing studies (Kneib 1996), and can be explained in terms of the second law of thermodynamics for gases. While the entropy of a fluid element must increase along its flow line (in the absence of cooling), the dark matter has no such constraint. On the contrary, the latter settles to high density cores due to the process of violent relaxation, which locally violates the second law of thermodynamics. During the formation of a cluster in a time dependent potential, the fast moving particles leave the center, while the slow ones preferentially remain there. This Maxwellian demon leads to more centrally concentrated dark matter as the cluster forms. The sign of this effect is to lower the velocity dispersion of the dark matter relative to the gas interior to the core radius, which means we might systematically underestimate the gas fraction using the procedure described here. In simulations, this effect would affect our estimate by $\approx 20\%$, but this offset is independent of redshift. Once high resolution resolved temperature profiles are measured with AXAF, we can test the structure of actual clusters.

At radii much smaller than the core radius it appears that gas does not trace total mass. While clusters are well fit by the $\beta$ model with well defined core radius $r_c \approx 200h^{-1}_{50}$ kpc (Jones and Forman 1984), lensing indicates that the dark matter continues to increase in density inward (Kneib et al 1996). This had led to some activity, including questioning of the hypothesis of hydrostatic equilibrium (Babul and Katz 1993, Loeb and Mao 1994). The presence of lenses were inconsistent with an isothermal gas in hydrostatic equilibrium at the measured temperature. As Babul and Katz point out, the discrepancy can be resolved completely if one allows for an 80% increase in the temperature from the core radius at $r_c = 230h^{-1}_{50}$kpc to the arc radius $\theta_a = 80h^{-1}_{50}$kpc. When emission weighted, the increase in the projected temperature profile is only 49%, where $\sim 5\%$ of the photons would originate from the hotter region, and may not be currently detectable. By itself, this would imply an entropy inversion, and such a cluster would be convectively unstable. However, a combination of temperature gradient and density gradient within the observational errors could reduce or eliminate the discrepancy. Additional masses behind the cluster could also contribute to the inferred gravitational mass. Moreover, weak lensing studies for A2218 (Squires et al 1996a) yield mass estimates consistent with the gas mass estimates at the arc radii.

The method proposed in this article depends on observable quantities near the core radius. This makes the estimate robust, and less dependent on cosmological uncertainties. At larger radii, the assumption of hydrostatic equilibrium and spherical symmetry breaks down, and the clusters are observed to have significant substructure. At smaller radii, the discrepancy between dark matter and gas increases, as we discussed above. Clusters may
provide a handle as a cosmic meter in the region near the core radius.

There is an additional possibility that the gas fraction in clusters changed due to non-gravitational processes, such as heating, cooling, or gas injected from galaxies. While cooling flows are an interesting phenomenon observed in several clusters, most clusters do not exhibit cooling flows. Current estimates indicate that up to 30-50% of X-ray clusters may have cooling flows, amongst which 10% of the X-ray emission may arise from such an effect (Fabian 1994). These effects are small compared to the error budget in our estimates. They would lead to a slight increase of the gas fraction at the center of the cluster core, violating the assumption stated earlier that gas traces total mass at all radii. Gas injection from galaxies would cause the gas fraction to increase in time, raising the inferred value of \(q_0\). Heating could lower the gas fraction in clusters today. The primary signature of such an effect would be that colder clusters should have a much lower gas fraction than hot clusters. In fact, the local sample shows no dependence of gas fraction on cluster temperature.

Squires et al (1996b) used weak lensing to measure the radial mass profile for A2163 at \(z = 0.201\), and found it consistent with the gas tracing total mass at all radii. They inferred the mean gas fraction to be \(f_g = 0.2h_{50}^{-3/2}\), consistent with the local sample. At this redshift, no constraints on \(q_0\) are obtained with this cluster. Similarly, Squires et al (1996a) found that gas traces total mass for A2218 at \(z = 0.17\), with \(f_g \approx 0.11 \pm 0.07h_{50}^{-3/2}\), still consistent with our hypothesis. While weak lensing probes the mass profile well, it allows a systematic error in normalizing the absolute mass. Thus the current data appears consistent with all our assumptions.

## 5. Conclusions

We have presented a new promising method for measuring angular diameter distance and determining the deceleration parameter \(q_0\). Using the Ansatz of constant gas fraction, we are able to obtain a small scatter of only 20% in the observed local cluster sample selected objectively from a published catalog (Jones and Forman 1984). This is sufficient to obtain interesting cosmological constraints using only three clusters at moderate redshift \(z \sim 0.5\).

Using data which is currently being collected by ASCA and ROSAT, the cluster sample will increase tenfold. Most importantly for the future, we need a systematic study of cluster gas fraction from samples with clear selection criteria, such as an X-ray flux limited sample. Weak lensing data is improving rapidly for clusters at redshifts near \(z=0.4\), which in the near future may provide us with an accurate measure of the deceleration parameter \(q_0\).
(Tyson and Fischer 1995, Squires et al 1996a,b). In the longer term future when AXAF is launched, more detailed modeling will become possible. The current analysis is still subject to large and systematic uncertainties, but we have demonstrated that even with the limited available published data, interesting cosmological constraints can be derived. This strategy will provide an independent measure of \( q_0 \) to compare with supernovae searches (Perlmutter et al 1996a) and the Sunyaev-Zeldovich effect (Silk and White 1978).

If gas traces mass, as simulations suggest, then \( \Omega_b / \Omega_m > f_g \). The high gas fraction \( f_g \approx 0.178 h_{50}^{-3/2} \) together with big bang nucleosynthesis \( \Omega_b h_{50}^2 = 0.05 \) (Walker et al 1991) would imply a dark matter fraction \( \Omega_m < 0.28/\sqrt{h_{50}} \). Such a low abundance of dark matter would suggest the presence of possibly a cosmological constant or negative spatial curvature. The curvature scenario predicts a deceleration parameter \( 0 < q_0 < 1/2 \), while the cosmological constant would imply \( q_0 \approx -1/2 \). A low value of \( \Omega_0 \) has also been proposed for several other reasons (Ostriker and Steinhardt 1995), but the choice between a cosmological constant or spatial curvature has so far been aesthetical. A measurement of \( q_0 \) would ultimately distinguish these scenarios. Using the small sample described above the hyperbolic spatial curvature model has a higher probability of fitting the observations, but we cannot rule out a cosmological constant with the available data.

Acknowledgements. I thank Avi Loeb, Tsafrir Kolatt, Alexey Vilkhein, Christine Jones and William Forman for useful discussions. This work was supported by the Harvard Society of Fellows and the Harvard-Smithonian Center for Astrophysics.

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Fig. 1.— The dimensionless weighting factor as a function of radial profile. The cross indicates the value of an isothermal sphere of zero core radius $\beta = 2/3$, $H(\beta = 1/2)$. 