Randomly inhomogeneous Luttinger liquid: Fluctuations of the tunnel conductance.

A. Gramada and M. E. Raikh

Department of Physics, University of Utah, Salt Lake City, Utah 84112

Abstract

The Luttinger liquid in which the concentration of electrons varies randomly with coordinate is considered. We study the fluctuations of the tunnel conductance, caused by the randomness in the concentration. If the concentration changes slowly on the scale of the Fermi wavelength, its prime role reduces to the scattering of the plasmon waves, propagating along the system. As a result of such a scattering, plasmons get localized. We show that the localization length, $l_\omega$, of a plasmon with frequency $\omega$ is inverse proportional to the square of the interaction strength and changes with frequency as $l_\omega \propto \omega^{-2}$. If the relative variation of the concentration is small, the randomness–induced correction to the tunnel conductance, $\delta G(V)$, where $V$ is the applied bias, can be expressed through the spectral characteristics of the localized plasmons. The magnitude of the correction, $\langle (\delta G^2)^{1/2} \rangle / G$, increases with $V$ as $\sqrt{V}$. The typical period of the fluctuations in $\delta G(V)$ is of the order of $V$. At a fixed $V$, the correlator of $\delta G$ at different points of the liquid falls off with distance as a power law and oscillates with the period which is one half of the wavelength of a plasmon with frequency $\omega = eV/h$.

PACS Numbers: 73.20.Dx, 73.40.Gk

Typeset using REVTeX
I. INTRODUCTION

It is well-known that the density of states in a pure one-dimensional interacting electron gas (Luttinger liquid) vanishes as a power law in the vicinity of the Fermi level: \( \nu(\omega) \propto \omega^\kappa \), where the exponent is determined by the interaction strength. Such a behavior should reveal itself in the dependence of the differential tunnel conductance on the applied bias: \( G(V) = dI/dV \propto \nu(eV) \). It is obvious that the presence of a disorder would perturb the local value of the density of states and, thus, cause some random correction, \( \delta G(V) \), to the conductance. Then the relevant questions are:

i) What is the typical magnitude of \( \delta G \) for a given realization of the disorder?

ii) How the values of \( \delta G \) at different voltages are correlated?

iii) How the correlation between the values of \( \delta G \) at the same voltage, but at different points of the liquid, falls off with increasing separation between the points?

These questions are addressed in the present paper. We will consider the case of a smooth disorder. Namely, we will assume that the correlation radius is much larger than the Fermi wavelength. This assumption simplifies the problem drastically, since it permits one to neglect the backward scattering of electrons and, hence, to view the disorder as a random variation of the electron concentration with coordinate. The key to understanding the role of the disorder is provided by the bosonization procedure, which allows to describe the low-energy excitations of the system in terms of bosonic excitations (plasmons), propagating along the liquid. Since the velocity of a plasmon depends on the concentration of electrons, the spatial variation of the concentration would give rise to the backscattering of plasmons. In other words, in the presence of a disorder, a plasmon with frequency \( \omega \) acquires a finite mean free path \( l_\omega \). It is important to note that a smooth disorder, for which the backscattering of electrons at the Fermi level is suppressed, might, ultimately, be quite efficient in backscattering of plasmons with wavelengths of the order of the correlation radius.

For non-interacting electrons in one dimension it is established that even a weak disorder
leads to the localization of all eigenstates. Then the mean free path acquires the meaning of
the localization radius. The same conclusion applies, certainly, to plasmons. The difference
is, however, that an electron becomes more and more localized as its energy decreases,
whereas for plasmons the situation is the opposite: the lower is the frequency, the weaker
is the localization. Since the dispersion law of a plasmon is linear: $\omega = sk$, where $s$ is the
sound velocity, the density of the plasmon states is frequency–independent. The reason for
suppression of the localization for the low–frequency plasmons is that the matrix element of
the backscattering vanishes in the limit of long wavelengths. In particular, we show that for
small $\omega$ the frequency dependence of the mean free path is $l_\omega \propto \omega^{-2}$ and, correspondingly,
the product $kl_\omega$, which determines the localization strength, increases as $1/\omega$ when $\omega$
go to zero. We also show that $l_\omega$ turns to infinity when the interactions are switched off, thus
revealing that the backscattering of plasmons is possible only due to interactions.

If the relative variation of the concentration is small, the disorder–induced correction
to $\nu(\omega)$ can be expressed in terms of the spectral characteristics of the localized plasmons
(such as local density of states). On the other hand, these characteristics were the subject
of detailed studies in application to the localized electrons. By utilizing the approach
developed in Refs. we calculate the variance $(\delta G^2)^{1/2}$, and the two–point correlator
$\langle \delta G(x)\delta G(0) \rangle$. We find that the ratio $(\delta G^2)^{1/2}/G$ increases with voltage as $\sqrt{V}$ and that the
characteristic “period” of change of $\delta G(V)$ with voltage is of the order of $V$. We also find
that the two–point correlator falls off with $x$ as a power law and oscillates with the period
$\delta x = \pi sh/eV$, which is one half of the wavelength of a plasmon with frequency $\omega = eV/\hbar$.

The paper is organized as follows. In the next section the formula for the mean free
path of a plasmon is derived. In Section we calculate the correlator of fluctuations of the
tunnel conductance. Section concludes the paper.
II. MEAN FREE PATH OF A PLASMON

The Hamiltonian of a Luttinger liquid with concentration of electrons, \( n(x) \), being a function of coordinate has the form

\[
\hat{H} = \int_0^\infty dx \left[ \frac{\hat{p}^2(x)}{2mn(x)} + \frac{1}{2} \left( V_0 + \frac{\pi^2 \hbar^2}{m} n(x) \right) \left( \frac{d(n\hat{u})}{dx} \right)^2 \right], \tag{1}
\]

where \( \hat{u} \) and \( \hat{p}(x) \) are, correspondingly, the displacement and conjugate momentum ([\( \hat{u}(x), \hat{p}(x') \]) = i\hbar \delta(x - x') \); \( V_0 \) is the effective interaction strength: \( V_0 = \int dx V(x) \). The Hamiltonian can be reduced to a system of harmonic oscillators

\[
\hat{H} = \sum_\mu \left[ \frac{\hat{p}_\mu^2}{2m} + \frac{m\Omega_\mu^2}{2} \hat{Q}_\mu^2 \right], \tag{2}
\]

by means of the following transformation

\[
\hat{u}(x) = \sum_\mu \frac{1}{\sqrt{n(x)}} \Phi_\mu(x) \hat{Q}_\mu, \quad \hat{p}(x) = \sum_\mu \sqrt{n(x)} \Phi_\mu(x) \hat{P}_\mu. \tag{3}
\]

Here \( \Phi_\mu \) are the eigenfunctions of the operator \( \hat{D} \) which is defined as

\[
\hat{D} \Phi_\mu = -\sqrt{n(x)} \frac{d}{dx} \left[ \left( V_0 + \frac{\pi \hbar^2}{m} n(x) \right) \frac{d}{dx} \left( \sqrt{n(x)} \Phi_\mu \right) \right] = \Omega_\mu^2 \Phi_\mu. \tag{4}
\]

The eigenvalues of \( \hat{D} \) determine the frequencies, \( \Omega_\mu \), of the oscillators. If the concentration is constant \( (n(x) = n_0) \), the solutions of (4) are the plane waves

\[
\Phi_\mu^0 = \frac{e^{ikx}}{\sqrt{L}}, \tag{5}
\]

with a linear spectrum \( \Omega_\mu = \Omega_\mu^0 = \sqrt{\frac{2m}{\pi \hbar v_F}} (L \) is the normalization length). Substituting (3) into (4), we get the standard expression for the sound velocity

\[
s = v_F \sqrt{1 + \frac{V_0}{\pi \hbar v_F}}, \tag{6}
\]

where \( v_F = \pi \hbar n_0 / m \) is the Fermi velocity.

Assuming that the relative variation of the concentration is small, \( |n(x) - n_0| \ll n_0 \), the expression for the mean free time, \( \tau_\omega \), for a plasmon with frequency \( \omega \) is given by the golden rule.
\[ \frac{1}{\tau_\omega} = \frac{\pi}{\omega} \sum_\nu \left| \langle \Phi^{(0)}_\mu | \hat{D} | \Phi^{(0)}_\nu \rangle \right|^2 \delta(\Omega^{(0)}_\nu^2 - \omega^2), \]  

(7)

where \( \Phi^{(0)}_\mu \) is the plane wave with \( k_\mu = \omega/s \) and \( \langle \cdots \rangle \) stands for the averaging over the fluctuations of \( n(x) \). The factor \( 1/\omega \) in (7) appears since we define the mean free time as \( 1/\tau_\omega = \text{Im} \Omega_\mu = \text{Im} \Omega^2_\mu/2\omega \). Keeping only the first order terms in the difference \( n(x) - n_0 \), we get the following expression for the matrix element

\[ \langle \Phi^{(0)}_\mu | \hat{D} | \Phi^{(0)}_\nu \rangle = \frac{V_0}{mL} \frac{(\omega/s)^2}{2n_0^2} \int_{-\infty}^{+\infty} dx \ (n - n_0) e^{2i\omega x/s}. \]  

(8)

The energy conservation, insured by the \( \delta \)-function in (7), requires that \( k_\nu = -\omega/s \) (backscattering). Then we have \( \Phi^{(0)*}_\mu = \Phi^{(0)}_\nu \) and, as it can be easily seen, the second term in the matrix element vanishes identically. As a result, the matrix element takes the form

\[ \langle \Phi^{(0)}_\mu | \hat{D} | \Phi^{(0)}_\nu \rangle = \frac{V_0}{mL} \frac{(\omega/s)^2}{2n_0^2} \int_{-\infty}^{+\infty} dx \ (n - n_0) e^{2i\omega x/s}. \]  

(9)

We see that the matrix element for backscattering is proportional to the interaction strength, which reflects the fact that this process is possible only due to interactions. The mean free path, \( l_\omega \), defined as \( l_\omega = s\tau_\omega \), can be found after substituting (9) into (7)

\[ \frac{1}{l_\omega} = \left( \frac{s^2 - v_F^2}{s^2} \right)^2 \frac{(\bar{n}^2)^2}{2n_0^2} \left( \frac{\omega}{s} \right)^2 \int_{-\infty}^{+\infty} dx K \left( \frac{|x|}{R_c} \right) e^{2i\omega x/s}, \]  

(10)

where we have introduced the correlator of the fluctuations of \( n(x) \)

\[ \langle n(x) - n_0)(n(x') - n_0) \rangle = (\bar{n}^2)K \left( \frac{|x - x'|}{R_c} \right). \]  

(11)

Here \( R_c \) is the correlation radius and \( (\bar{n}^2) \) is the mean square fluctuation of \( n(x) \), so that \( K(0) = 1 \).

It is instructive to rewrite (10) for the product \( kl_\omega = \omega l_\omega/s \), which measures the effective scattering strength. Assuming the gaussian form of the correlator, \( K(z) = e^{-z^2} \), we obtain

\[ (kl_\omega)^{-1} = \sqrt{\pi} \left( \frac{s^2 - v_F^2}{s^2} \right)^2 \frac{(\bar{n}^2)^2}{2n_0^2} \left( \frac{\omega R_c}{s} \right) e^{-\omega^2 R_c^2/s^2}. \]  

(12)
Note that the last factor in (12) is a function of the argument \( \omega R_c/s \) and this function does not exceed unity. Thus, the product \( kl_\omega \) is large for any \( \omega \). Obviously, the backscattering is ineffective when \( \omega R_c/s > 1 \), i.e. when the correlation radius \( R_c \) exceeds the wave length of the plasmon. In the opposite limit, \( \omega R_c/s < 1 \), the mean free path behaves as \( l_\omega \sim \omega^{-2} \). As it was mentioned in the Introduction, the real meaning of \( l_\omega \) is the localization length of a plasmon with frequency \( \omega \). The fact that \( kl_\omega > 1 \) allows one to apply to localized plasmons, the description developed earlier for localized electrons just in this limit. This is done in the next section.

III. CORRELATOR OF THE FLUCTUATIONS OF THE TUNNEL CONDUCTANCE

Once the Hamiltonian is diagonalized, the derivation of the formula for the density of states becomes standard. The operator \( \Psi^+(x_0) \), creating an electron at point \( x_0 \), can be presented in the form

\[
\Psi^+(x_0) = \exp \left\{ -\frac{i}{\hbar} \sum_\mu \left( \alpha_\mu(x_0) \hat{P}_\mu + \beta_\mu(x_0) \hat{Q}_\mu \right) \right\},
\]

where the coefficients \( \alpha_\mu \) and \( \beta_\mu \) are defined as

\[
\alpha_\mu(x_0) = \int_{x_0}^{+\infty} dx \frac{\Phi_\mu}{\sqrt{n(x)}}, \quad \beta_\mu(x_0) = \pi \sqrt{n(x_0)} \Phi_\mu(x_0).
\]

The calculation of the Green function \( \langle \Psi(t)\Psi^+(0) \rangle \) does not differ from that for a pure Luttinger liquid and leads to the following expression for the density of states

\[
\nu(\omega) = \frac{1}{\pi} \text{Re} \int_0^\infty dt e^{i\omega t} \langle \Psi(t)\Psi^+(0) \rangle = \frac{1}{\pi} \text{Re} \int_0^\infty dt e^{i\omega t} e^{-W(t)},
\]

where \( W \) is the sum over eigenmodes

\[
W(t) = \sum_\mu \left( \frac{m|\alpha_\mu|^2\Omega_\mu}{2\hbar} + \frac{\hbar|\beta_\mu|^2}{2m\Omega_\mu} \right) \left( 1 - e^{-i\Omega_\mu t} \right).
\]

If \( n(x) \) is constant so that the eigenfunctions are given by (5), Eq. (15) reproduces the known result for the density of states: \( \nu_0(\omega) \propto \omega^\kappa \), with
\[ \kappa = \frac{1}{2} \left( \frac{s}{v_F} + \frac{v_F}{s} \right) - 1 = \frac{1 + \frac{v_0}{2\pi \hbar v_F}}{\sqrt{1 + \frac{v_0}{\pi \hbar v_F}}} - 1 \]  

(17)

Indeed, for plane waves we have \(|\alpha_\mu|^2 = 1/k_\mu^2 L n_0, |\beta_\mu|^2 = \pi^2 n_0 / L\) and we get the following expression for the function \( W \)

\[ W_0(t) = \frac{1}{2} \left( \frac{s}{v_F} + \frac{v_F}{s} \right) \int_0^\infty \frac{dk}{k} (1 - e^{-ikt}) e^{-r_0 k} = \frac{1}{2} \left( \frac{s}{v_F} + \frac{v_F}{s} \right) \ln \left( \frac{r_0 + ist}{r_0} \right), \]

(18)

where \( r_0 \) is the cutoff parameter. Substituting this expression into (15) leads to (17).

To find the disorder-induced correction to \( \nu_0(\omega) \), we treat the difference \( W(t) - W_0(t) \) as a perturbation and expand the exponent in (15) to the first order. This gives

\[ \delta \nu = -\frac{1}{\pi} \Re \int_0^\infty dt e^{i\omega t} \left( \frac{r_0}{r_0 + ist} \right)^{\kappa+1} \left[ W(t) - W_0(t) \right]. \]

(19)

To calculate the integral over \( t \), it is convenient to present the denominator in (19) in the form

\[ \frac{1}{(r_0 + ist)^{\kappa+1}} = \frac{1}{\Gamma(\kappa + 1)} \int_0^\infty dzz^{\kappa} e^{-z(r_0 + ist)}. \]

(20)

After substituting (20) into (19), both integrations, over \( z \) and \( t \) can be easily carried out, and one obtains

\[ \delta \nu(x_0, \omega) = \frac{1}{\Gamma(\kappa + 1)} \left( \frac{r_0}{s} \right)^{\kappa+1} \left\{ \sum_\mu \left[ \frac{m|\alpha_\mu|^2 \Omega_\mu}{2\hbar} + \frac{\hbar |\beta_\mu|^2}{2m\Omega_\mu} \right] (\omega - \Omega_\mu)^\kappa \right. \\
- (\kappa + 1) \int_0^\infty \frac{dq}{q} (\omega - sq)^\kappa \right\}. \]

(21)

Note that the constant \( \alpha_\nu \) can be expressed through the derivative \( \frac{d\Phi_\mu}{dx} \bigg|_{x=x_0} \) by integrating the equation (4) from \( x_0 \) to \( \infty \). Then, one gets

\[ \alpha_\mu = -\frac{s^2}{\Omega_\mu^2} \frac{1}{\sqrt{n_0}} \left( \frac{d\Phi_\mu}{dx} \right) \bigg|_{x=x_0}. \]

(22)

To study the correlation properties of \( \delta \nu \), we introduce the following local densities

\[ \rho_1(\varepsilon, x_0) = \sum_\mu |\Phi_\mu(x_0)|^2 \delta(\varepsilon - \Omega_\mu), \]

(23)

\[ \rho_2(\varepsilon, x_0) = s^2 \sum_\mu \frac{1}{\Omega_\mu^2} \left| \frac{d\Phi_\mu(x_0)}{dx} \right|^2 \delta(\varepsilon - \Omega_\mu). \]

(24)
The average values of these densities are equal

$$\overline{\rho_1} = \overline{\rho_2} = \frac{1}{\pi s}.$$  \hspace{1cm}  (25)

Then the correction $\delta \nu$ can be rewritten in terms of the fluctuations $\delta \rho_1 = \rho_1 - 1/\pi s$ and $\delta \rho_2 = \rho_2 - 1/\pi s$

$$\delta \nu(x_0, \omega) = \frac{1}{\Gamma(\kappa + 1)} \left( \frac{r_0}{s} \right)^{\kappa + 1} \frac{\pi s}{2} \int_0^\omega \frac{d\omega_1}{\omega_1} (\omega - \omega_1)^\kappa \left[ \frac{v_F}{s} \delta \rho_1(\omega_1, x_0) + \frac{s}{v_F} \delta \rho_2(\omega_1, x_0) \right].$$  \hspace{1cm}  (26)

Correspondingly, the correlator of $\delta \nu$ at different points is expressed through the correlators of the fluctuations $\delta \rho_1, \delta \rho_2$

$$\overline{\delta \nu(x_1, \omega) \delta \nu(x_2, \omega)} = \left[ \frac{1}{\Gamma(\kappa + 1)} \left( \frac{r_0}{s} \right)^{\kappa + 1} \frac{\pi s}{2} \right]^2 \times \int_0^\omega \frac{d\omega_1}{\omega_1} \int_0^\omega \frac{d\omega_2}{\omega_2} (\omega - \omega_1)^\kappa (\omega - \omega_2)^\kappa \left( \frac{v_F^2}{s^2} F_1 + \frac{s^2}{v_F^2} F_2 + 2 F_{12} \right),$$  \hspace{1cm}  (27)

where the correlators $F_1, F_2$ and $F_{12}$ are defined as

$$F_1 = \overline{\delta \rho_1(x_1, \omega_1) \delta \rho_1(x_2, \omega_2)},$$  \hspace{1cm}  (28)

$$F_2 = \overline{\delta \rho_2(x_1, \omega_1) \delta \rho_2(x_2, \omega_2)},$$  \hspace{1cm}  (29)

$$F_{12} = \overline{\delta \rho_1(x_1, \omega_1) \delta \rho_2(x_2, \omega_2)}.$$  \hspace{1cm}  (30)

The correlator $F_1$ is, in fact, the correlator of the fluctuations of the local density. In application to localized electrons, it was studied in Refs. 8,9 using, correspondingly, the Berezinskiǐ technique and the technique developed by Berezinskiǐ and Gor’kov. In the limit $\omega_1 \tau_{\omega_1} \gg 1, \omega_2 \tau_{\omega_2} \gg 1$, the correlator is nonzero only if the difference $\omega_1 - \omega_2$ is small enough: $|\omega_1 - \omega_2| \sim 1/\tau_{\omega_1}$. For distances $z = |x_1 - x_2| \ll \ell_{\omega_1}, \ell_{\omega_2}$, the expression for $F_1$, obtained in Ref. 8, reads

$$F_1 = \frac{1}{3\pi^2 s^2} \left[ \frac{\pi}{2\tau_{\omega_1}} \delta(\varepsilon) \left( 3 - 2 \sin^2 \left( \frac{\omega_1 z}{s} \right) \right) + 2 \sin^2 \left( \frac{\omega_1 z}{s} \right) \left( C(\varepsilon) - 1 \right) \right],$$  \hspace{1cm}  (31)

where $\varepsilon = \omega_1 - \omega_2$ is the difference between the two frequencies and the function $C(\omega)$ is given by
\[ C(\varepsilon) = (2\varepsilon_\tau_\omega)^2 \int_0^\infty dy \frac{\cos(2\varepsilon_\tau_\omega y)}{y + 1} = (2\varepsilon_\tau_\omega)^2 \int_0^\infty dq e^{-q} \frac{q}{q^2 + 4\varepsilon^2_\tau_\omega}. \]  

(32)

As it was noted in Refs. 9, 8, 13, at \( z = 0 \) and \( \varepsilon \neq 0 \), we have \( F_1 = 0 \), i.e. the correlation is absent.

It is easy to establish that the correlator \( F_2 \) is equal to \( F_1 \). Concerning the correlator \( F_{12} \), we did not find the expression for this correlator in the literature. So, we have calculated it using the Berezinskiĭ-Gor’kov technique and obtained the following expression

\[ F_{12} = \frac{1}{3\pi^2 s^2} \left[ \frac{\pi}{2\tau_\omega} \delta(\varepsilon) \left( 1 + 2\sin^2 \left( \frac{\omega_1 z}{s} \right) \right) + 2\cos^2 \left( \frac{\omega_1 z}{s} \right) (C(\varepsilon) - 1) \right]. \]  

(33)

It is seen that in contrast to \( F_1 \), the correlator \( F_{12} \) is non-zero at \( z = 0 \) and \( \varepsilon \neq 0 \). For finite \( \varepsilon \), both correlators are proportional to \((C(\varepsilon) - 1)\) and decay with increasing \( \varepsilon \) as \((\varepsilon_\tau_\omega)^{-2}\).

To calculate the double integral in (27), we make use of the fact that the correlators \( F_1, F_2 \) and \( F_{12} \) are sharp functions of \( \varepsilon \), i.e. the major contribution to the integral comes from the domain \(|\omega_1 - \omega_2| \sim 1/\tau_\omega \ll \omega_1, \omega_2\). This allows to put \( \omega_2 = \omega_1 \) in all other factors and to extend the integration over \( \varepsilon = \omega_1 - \omega_2 \) to \((-\infty, \infty)\). Noting that the integral of \((1 - C(\varepsilon))\) is equal to \( \pi/\tau_\omega \), we obtain

\[ \int_{-\infty}^\infty d\varepsilon \left( \frac{v_F^2}{s^2} F_1 + \frac{s^2}{v_F^2} F_2 + 2F_{12} \right) = \frac{1}{2\pi s^2 \tau_\omega} \left( \frac{s}{v_F} - \frac{v_F}{s} \right)^2 \cos \left( \frac{2\omega_1 z}{s} \right). \]  

(34)

Note that the coefficients in front of \( F_1, F_2 \) and \( F_{12} \) in (27) have combined into the factor \((s/v_F - v_F/s)^2\) which is proportional to \(V_0^2\) at small \( V_0\). Using (34), the expression for the correlator \( \overline{\delta\nu(x_1, \omega)\delta\nu(x_2, \omega)} \) takes the form

\[ \frac{\pi}{8} \left( \frac{s}{v_F} - \frac{v_F}{s} \right)^2 \left[ \frac{1}{\Gamma(k + 1)} \left( \frac{v_0}{s} \right)^{k+1} \right]^2 \int_0^\omega d\omega_1 \left( \omega - \omega_1 \right)^{2k} \cos \left( \frac{2\omega_1 z}{s} \right). \]  

(35)

Apparently, the factor \( 1/\omega_1^2 \) in (35) diverges at small \( \omega_1 \). However, this divergence is compensated by the frequency dependence of the mean free time, which at small \( \omega_1 \) behaves as \( 1/\omega_1^2 \). Substituting \( \tau_\omega \) in (35) and introducing the new variable \( w = \omega_1/\omega \), we get the final result

\[ \int_{-\infty}^\infty d\varepsilon \left( \frac{v_F^2}{s^2} F_1 + \frac{s^2}{v_F^2} F_2 + 2F_{12} \right) = \frac{1}{2\pi s^2 \tau_\omega} \left( \frac{s}{v_F} - \frac{v_F}{s} \right)^2 \cos \left( \frac{2\omega_1 z}{s} \right). \]  

(34)
where the function $R$ is defined as

$$R(\tilde{z}) = (2\kappa + 1) \int_0^1 dw (1 - w)^{2\kappa} e^{-w^2(\frac{2\kappa}{s})^2} \cos(w \tilde{z}).$$

(37)

Here, $\tilde{z}$ is the dimensionless distance

$$\tilde{z} = \frac{2\omega}{s}(x_2 - x_1).$$

(38)

The function $R$ is defined in such a way that, in the only interesting limit $\omega R_c/s \ll 1$, it turns to 1 at $\tilde{z} = 0$. This function determines the coordinate dependence of the correlator and, consequently, the coordinate dependence of the correlator of the fluctuations of the tunnel conductance

$$\frac{\delta G(x)\delta G(0)}{\langle \delta G(0) \rangle^2} = R\left( \frac{2eV}{\bar{s}h} x \right).$$

(39)

The function $R$ is plotted in Fig.1. With increasing distance, it falls off and oscillates. The asymptotic behavior of $R(\tilde{z})$ at $\tilde{z} \gg 1$ is as follows

$$R(\tilde{z}) \approx \frac{\Gamma(2\kappa + 2)}{\tilde{z}^{2\kappa + 1}} \sin(\tilde{z} - \pi \kappa).$$

(40)

The spatial period of the oscillations of $R$ is $\delta x = \pi s\bar{h}/eV$ and it decreases with increasing voltage. It is to be emphasized that such a behavior of the correlator of $\delta G$ (slow decay and oscillations) is entirely due to interactions. In the absence of the interactions, the correlator would simply reproduce the correlator of the fluctuations of $n(x)$, i.e. it would decay monotonously at distance $\sim R_c$ which is much smaller than $\delta x$.

At $\tilde{z} = 0$, the formula (27) defines the variance of $\delta \nu$. It is convenient to rewrite it for the relative magnitude of the fluctuations of $\delta G$

$$\frac{\langle (\delta G)^2 \rangle^{1/2}}{G} = \frac{\pi^{3/4}}{4(2\kappa + 1)^{1/2}} v_F \left( 1 - \frac{v_F^2}{s^2} \right)^2 \frac{(\delta n_s)^{1/2}}{n_0} \left( \frac{\omega R_c}{s} \right)^{1/2}$$

(41)
Naturally, the variance \( (\delta G^2)^{1/2} \) is proportional to the magnitude of the fluctuations of the density. Less trivial is that \( \langle 1 - v_F^2/s^2 \rangle^2 \) which is proportional to \( V_0^2 \) at small \( V_0 \), and a small factor \( (\omega R_c/s)^{1/2} \) (note that \((\Pi)\) is written in the limit \( \omega \ll s/R_c \)). This factor determines the voltage dependence: \( (\langle \delta G^2 \rangle^{1/2}/G \propto \sqrt{V} \). Qualitatively, this dependence can be interpreted as follows. Formula \((26)\) shows that all plasmon modes with frequencies smaller than \( \omega \) contribute to \( \delta \nu(x_0, \omega) \). Obviously, the major contribution comes from plasmons for which the center of localization lies within the localization radius from the point \( x_0 \). Let us consider some frequency strip \((\omega_1 - \Delta, \omega_1 + \Delta)\) centered at \( \omega_1 < \omega \) and with the width \( \Delta \ll \omega_1 \). Then, the localization radius for all modes within this strip is approximately \( l_{\omega_1} \). For a spatial interval of length \( l_{\omega} \), the typical frequency spacing between modes is \( 1/\rho l_{\omega_1} \sim 1/\tau_{\omega_1} \). Correspondingly, the average number of modes within the strip is \( \Delta \tau_{\omega_1} \) and, hence, the relative fluctuation of this number is \( \sim (\Delta \tau_{\omega_1})^{-1/2} \). The final estimate for the fluctuation of \( \delta \nu \) energies if we set \( \Delta \sim \omega_1 \sim \omega \). Then, one gets \( \delta \nu/\nu = \delta G/G \sim (\omega \tau_\omega)^{-1/2} \approx (k l_{\omega})^{-1/2} \). Substituting \((12)\) for \( k l_{\omega} \), we reproduce the voltage dependence in \((11)\). However, the estimate and the result of the calculation still differ by a factor \( (1 - v_F^2/s^2) \). This factor originates from the specific details of the structure of the eigenfunctions (see the comment after Eq. \((34)\)), and we cannot interpret it qualitatively. Note in conclusion of the section, that since all frequencies from 0 to \( \omega \) give, roughly, comparable contributions to \( \delta \nu(x_0, \omega) \), it will change significantly only when the frequency doubles. This means that at fixed \( x_0 \) the characteristic period of fluctuations in \( \delta G \) as a function of voltage is of the order of \( V \).

IV. CONCLUSIONS

The main result of the present paper is that the randomness in the concentration of electrons in the Luttinger liquid causes a random component in the tunnel conductance which changes semiperiodically along the liquid, with the period depending on the applied bias. This behavior results from the fact that at a fixed bias, \( V \), the frequencies of plas-
mons, responsible for the correction to the density of states, are strictly limited by $eV/h$. Correspondingly, the oscillating behavior of the correlator of the fluctuations reflects the distribution of density in a plasmon with maximal frequency. If the conductance is studied as a function of bias, the disorder would cause fluctuations with characteristic period of the order of $V$. The fluctuations disappear as the wavelength of the plasmon with frequency $eV/h$ becomes smaller than the spatial scale of the change of the concentration, $R_c$. There is also a limitation from low voltages, imposed by the finite length of the liquid, $L$. Namely, our calculation applies when the mean free path of a plasmon, $l_\omega$, is smaller than $L$. At low enough frequencies $l_\omega$ exceeds $L$ and the disorder does not play any role. For such frequencies the oscillations of the tunnel conductance with voltage have their origin in the size quantization of plasmons, the period of oscillations being $\pi hs/eL$. These oscillations were studied in Ref. 10.

Note finally, that the approach developed in the present paper can be extended to the case of the multichannel Luttinger liquid. In the latter case, the scattering of plasmons between the channels, caused by a disorder, should be taken into account.

ACKNOWLEDGMENTS

One of the authors (M.R.) is grateful to I. L. Aleiner and A. I. Larkin for a very useful discussion.
REFERENCES

* also at University of Bucharest, Romania

1 A. Luther and I. Peschel Phys. Rev. B 9, 2911 (1974).

2 Y. Oreg and A. M. Finkel’stein, Phys. Rev. Lett. 76, 4230 (1996).

3 D. C. Mattis and E. H. Lieb, J. Math. Phys. 6, 304 (1965).

4 F. D. M. Haldane, J. Phys. C 14, 2585 (1981).

5 N. F. Mott and W. D. Twose, Adv. Phys. 10, 107 (1961).

6 V. L. Berezinskiï, Zh. Eksp. Teor. Fiz. 65, 1251 (1973) [Sov. Phys. JETP 38, 620 (1974)].

7 V. L. Berezinskiï and L. P. Gor’kov, Zh. Eksp. Teor. Fiz. 77, 2489 (1979) [Sov. Phys. JETP 50, 1209 (1979)].

8 E. P. Nakhmedov, V. N. Prigodin, and Yu. A. Firsov, Zh. Eksp. Teor. Fiz. 92, 2133 (1987) [Sov. Phys. JETP 65, 1202 (1987)].

9 L. P. Gor’kov, O. N. Dorokhov, and F. V. Prigara, Zh. Eksp. Teor. Fiz. 84, 1440 (1983) [Sov. Phys. JETP 57, 838 (1983)].

10 Yu. V. Nazarov, A. A. Odintsov, and D. V. Averin, Bull. American Phys. Soc. 41, 376 (1996).

11 A. Gramada, M. E. Raikh, preprint cond-mat/9604090

12 G. Mahan Many-Particle Physics, 2nd ed. (Plenum, New York, 1990).

13 B. L. Al’tshuler and V. N. Prigodin, Zh. Eksp. Teor. Fiz. 95, 348 (1989) [Sov. Phys. JETP 68, 198 (1989)].

14 Actually there is no qualitative understanding by now, why $F_1$ turns to zero at coinciding point.\[\square\]
K. A. Matveev and L. I. Glazman, Phys. Rev. Lett. 70, 990, (1993).
FIGURES

FIG. 1. The correlation function of tunnel conductance is plotted as a function of the dimensionless distance $\tilde{z}$ for $s/\nu_F = 1.2$
