PAPER

Understanding the controllability of complex networks from the microcosmic to the macrocosmic

Peng Gang Sun\textsuperscript{1,2} and Xiaoke Ma\textsuperscript{1,2}

\textsuperscript{1} School of Computer Science and Technology, Xidian University, Xi’an, 710071, People’s Republic of China
\textsuperscript{2} Institute of Computational Bioinformatics, Xidian University, Xi’an, 710071, People’s Republic of China

E-mail: psun@mail.xidian.edu.cn and xkma@mail.xidian.edu.cn

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Abstract

From a microcosmic perspective, nodes as meta-structures (or ‘smallest units’) for the constitution of a graph are of great importance for understanding complex network-based systems. In this paper, we develop a new framework, which first of all defines the meta-structures of a graph in different levels and tries to depict a graph from a low to a high level of abstraction. Further, based on this framework we study the meta-structure-driven control model and try to understand the controllability of complex networks from a microcosmic to a macrocosmic perspective. Finally, we analyze the impact of the community strength of networks on meta-structure-driven control. The results for artificial networks and real-world networks indicate that for meta-structure-driven control, the number of driver nodes is dependent on the networks’ degree distribution, and the dependence weakens as the level of the meta-structures increases. In addition, the networks are easier to control as the community strength increases, while this monotonicity is not preserved as the level of the meta-structures increases. We also find that it is harder to control sparse and inhomogeneous networks as the level of the meta-structures increases.

1. Introduction

Many real systems can be modeled as complex networks/graphs, where elements in the systems and interactions between them correspond to nodes and edges, respectively [1–3]. Complex networks are of great importance for understanding complex network-based systems, e.g. the statistical mechanics of networks’ topology and dynamics provide us with a new insight into the understanding of real systems [1–3]. ‘Network science’ has permeated multiple disciplines, including economics, finance, sociology, biology and transport [4], and is attracting more attention from scholars in these fields. Nodes in graphs often correspond to the smallest function-independent units in systems. Two nodes are connected if the two corresponding units cooperate to perform a specific function. Through studying the structures of networks, we try to disclose the functions of systems [5–16].

Recently, the focus has shifted towards the controllability of complex networks [17–40]. According to traditional control theory, we define a dynamical system that is controllable if we select suitable inputs of external signals which can drive the system from any initial state to any desired final state in finite time [17–19]. Based on this definition, Liu et al [20] developed an important model, structural controllability for directed networks, and used a maximum matching algorithm to identify the minimum number of unmatched nodes that need to be driven by external signals for the whole system to achieve full control. Unmatched nodes are called driver nodes, and the number of these determines a system’s controllability [20]. Therefore, structural controllability is transformed into the identification of the minimum number of driver nodes [20]. The results showed that the minimum number of driver nodes is primarily determined by the networks’ degree distribution, and that driver nodes tend to deviate from high-degree nodes [20]. Based on this model, many works have been published on topics such as control centrality [21], exact controllability [22] and target control [23].

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The minimum dominating set (MDS) is another important model for the controllability of complex networks [24–29]. Nacher and Akutsu [27, 28] introduced the MDS to the study of controllability for undirected networks by assuming that each edge in a network is bi-directional, and they noted that a network is structurally controllable by selecting the nodes in the MDS as the driver nodes. Wuchty [29] applied the MDS-based model to study the controllability of protein interaction networks, and the results showed that proteins in the MDS tend to be essential, disease-related and virus-targeted genes. Remarkably, Nacher and Akutsu [27, 28] studied the relationship between the two models and observed that the former assumes that only driver nodes can be controlled directly through external signals, while the latter assumes that each driver node can control its associated edges independently, while each non-driver node is controllable if it is at least adjacent to a driver node. As mentioned by Nacher and Akutsu [27, 28], the difference in the two control models leads to different results, and the MDS-based model may lead to the identification of important nodes for the control of networks [27, 28]. Recently, Liu et al [30] reviewed the control principles of complex networks and also mentioned that the MDS-based model is capable of controlling an undirected network, and each node in the MDS can control all of its outgoing edges separately. From the discussion above, we can see that the MDS-based model provides us with a new insight into the control of network-based systems.

From a microcosmic perspective, nodes as meta-structures (or the ‘smallest units’) for the constitution of a graph are of great importance for understanding network-based systems [1–3]. Nodes, as the lowest level of abstraction, often correspond to the smallest function-independent units in systems. For a high level, two or more nodes as members of specific structures often form large and function-perfect units, which cooperate to perform a specific function, since the systems’ functions and structures are highly associated with each other [5–16]. In a high level, the control of large function-perfect units is a new way to control whole systems from a macrocosmic perspective. Therefore, we are motivated to develop a new framework, which first defines the meta-structures of a graph in different levels and tries to depict a graph from a low to a high level of abstraction. In a low level, meta-structures can be seen as nodes, and in a high level, meta-structures can be seen as links (also called 2-cliques) or triangles (also called 3-cliques). Further, based on this framework, we study the meta-structure-driven control model and try to understand the controllability of complex networks from a microcosmic to a macrocosmic perspective. The meta-structure-driven control model stresses that each non-driver node is controllable if it is at least adjacent to a meta-structure, and all the nodes belonging to the meta-structure are obliged to be driver nodes. Therefore, the MDS-based model extends to the identification of important structures in the control of networks, such as units. We also analyze the impact of the community strength of networks on meta-structure-driven control. Finally, we analyze the meta-structure-driven control of artificial networks and real-world networks.

The rest of the paper is organized as follows. In section 2, we define the meta-structures of complex networks in different levels. In section 3, we introduce the meta-structure-driven control model. In section 4, we analyze the meta-structure-driven control of artificial networks and real-world networks. The conclusion is provided in section 5.

2. Meta-structures of complex networks from the microcosmic to the macrocosmic

In this section, we define the meta-structures of complex networks in different levels, illustrate these structures as the smallest units for the constitution of complex networks and try to understand complex networks from a microcosmic to a macrocosmic perspective.

2.1. Nodes as meta-structures of complex networks

Most real systems can be modeled as complex networks, and complex networks can be seen as graphs, which provide us with a more vivid way to understand complex network-based systems [1–3]. In general, a graph consists of some nodes connected by edges, and nodes in the graphs often correspond to the smallest function-independent units in systems. Two nodes are connected if the two corresponding units cooperate to perform a specific function.

From a microcosmic perspective, nodes, as the smallest units, can be seen as the first-level meta-structures of graphs, which are of great importance for understanding real-world networks (see figure 1(a)). For instance, in social networks, a node often corresponds to a person, and two nodes are connected if they are friends, colleagues or relatives, etc. In biological networks, a node often corresponds to a protein/gene, and two proteins/genes are connected if they share similar molecular functions or take part in the same physiological processes. In World-Wide-Web networks, a node often corresponds to a web page, and two web pages are connected if they share similar topics [1–3].
2.2. Links as meta-structures of complex networks

From a macrocosmic view, we can see a graph consisting of links (also called 2-cliques), which can be seen as the second-level meta-structures of graphs, where a link (or a 2-clique) contains two nodes connected by an edge. In graph theory, links and edges are the same concept, while a 2-clique is called a link in this paper for a more vivid understanding. Links as the smallest units of graphs and also play an important role in understanding real-world networks (see figure 1(b)). For example, in sports and similar activities, a link (or 2-clique) often corresponds to a team containing two members who cooperate closely with each other, e.g. in social dancing, beach volleyball, badminton doubles and so on.

2.3. Triangles as meta-structures of complex networks

Communities exist widely in real-world networks [1–3, 5–16], and Radicchi et al [15] observed that many triangles exist within communities, and that these promote the full control of whole networks by control communities. Therefore, from a more macrocosmic perspective, a graph consists of triangles (or 3-cliques), which can be seen as the third-level meta-structures of graphs, where a triangle (or 3-clique) contains three nodes, and every two nodes are connected by an edge. Triangles, as the smallest units of graphs, are also very helpful for understanding real-world networks. For example, some social activities are based on families, where a family contains three members, a mother, a father and a child (see figure 1(c)). The three members in a team cooperate closely with each other to complete activities. In biological networks, a triangle motif often
corresponds to a functional unit that contains three closely associated genes. Of course, a specific structure that includes three nodes connected by two edges can also be viewed through the concept of meta-structures.

2.4. Illustration of different levels of meta-structures using real examples
Based on the discussions in the subsection above, we can see that different levels of meta-structures provide us with a new way to understand network-based systems. Here, we further illustrate different levels of meta-structures using real examples (figure 2). At different levels, the human body can be seen as systems, organs, tissues and cells (see figure 2(a)), and different levels of meta-structures provide us with different perspectives from which to understand the human body. Cells as meta-structures in the lowest level often correspond to the smallest function-independent units in the human body. For a high level, meta-structures often correspond to large and function-perfect units, and high-level meta-structures consist of low-level meta-structures, since the functions and structures in the human body are highly associated with each other. At a high level, the control of large function-perfect units is a new way to control the whole human body from a macrocosmic perspective. In addition, in a communication network, high-level meta-structures often correspond to the backbone of the network, and nodes that do not belong to the backbone communicate by passing messages through neighbors (driver nodes) that belong to the backbone. Therefore, the control of the backbone is of great importance for the control of the communication network (see figure 2(b)).
3. Controllability of complex networks from the microcosmic to the macrocosmic

Based on the different levels of complex-network meta-structures mentioned above, in this section we study the meta-structure-driven control model and try to understand the controllability of complex networks from a microcosmic to a macrocosmic perspective. First, we describe the MDS-based model, and then we introduce our framework, the meta-structure-driven control model. Finally, we solve the meta-structure-driven control model through binary integer programming.

3.1. Minimum dominating set-based model

In this paper, we use \( G(V, E) \) to denote an unweighted, undirected graph, where \( V \) and \( E \) correspond to the set of nodes and the set of edges, respectively. The MDS is defined as a minimum subset of nodes so that every node in the network is either adjacent to an element of the MDS or is an element of the MDS [24–29]. Formally, it can be expressed as follows.

**Definition 1** (minimum dominating set) \( V' \) is an MDS of \( G \), if (1) \( V' \) is a dominating set: \( \forall i \in V - V' \), \( \exists j \in V' \), \( (i, j) \in E \), and \( i \neq j \), where \( V' \subseteq V \), \( V' \neq \phi \), \( |V'| = n \) and \( i, j \in \{1, 2, \ldots, n\} \), and (2) \( V' \) is minimum: \( \forall V'' \subseteq D, |V''| \leq |V'| \), where \( D \) is the set containing all the dominating sets of \( G \).

3.2. Our framework

For the MDS-based model, called the node-driven control in this paper, a network is fully controlled if we dominate the driver nodes since non-driver nodes are implicitly controllable if they are at least adjacent to a driver node, where the nodes belonging to the MDS are called driver nodes [24–29].

Meta-structures of different levels often correspond to functional units of different levels in complex systems, and the systems are fully controlled if we control the functional units. Therefore, we are motivated to develop a new framework, meta-structure-driven control model, and to try to understand the controllability of complex networks from a microcosmic to a macrocosmic perspective. The framework stresses that one non-driver node is controllable if and only if (1) it is at least adjacent to one meta-structure, i.e. it is adjacent to at least one node, which is a member of the meta-structure, and (2) all the nodes contained in the meta-structure are obliged to be driver nodes. In figure 1, we illustrate the meta-structure-driven control model, and figures 1(a)–(c) correspond to the node-driven, link-driven and triangle-driven control of complex networks, respectively, where the meta-structures are highlighted in green. In figure 1(d), we illustrate the meta-structure-driven control of a graph consisting of nodes, links and triangles.

In the following, we determine the MDS of complex networks based on the meta-structure-driven control model and solve the problem using binary integer programming [24–29, 41, 42].

3.3. Node-driven control

Traditional control (also called node-driven control) provides a simple way to understand a network’s controllability, and non-driver nodes are controllable if they are at least adjacent to a driver node.

The minimum number of driver nodes based on node-driven control can be solved by binary integer programming [24–29].

\[
\min f = \sum_{i \in V} x_i
\]

subject to

\[
x_i + \sum_{j \in V} a_{ij} x_j \geq 1,
\]

\[
i, j = 1, 2, \ldots, n; \quad i \neq j,
\]

where \( A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \), \( X = [x_1, x_2, \ldots, x_n]^T \).

Binary integer programming (also called ‘0–1’ integer programming) is a mathematical optimization problem in which the variables in \( X \) are restricted to being either 0 or 1, e.g. \( x_i = 1 \) indicates \( i \in \text{MDS} \), and 0 otherwise. Equations (1) and (2) correspond to the objective function and the constraint, respectively.

3.4. Link-driven control

For link-driven control, one non-driver node is controllable if and only if (1) it is adjacent to one link, and (2) the two nodes contained in the link are obliged to be driver nodes.
For link-driven control, the MDS of a network can be solved by binary integer programming [24–29].

\[
\min f = \sum_{i \in V} x_i
\]

subject to

\[
x_i + \sum_{j,k \in V} a_{ij} a_{jk} x_j x_k \geq 1, \quad i, j, k = 1, 2, \ldots, n; \quad i \neq j \neq k.
\]

3.5. Triangle-driven control

For triangle-driven control, one non-driver node is controllable if and only if (1) it is adjacent to one triangle, and (2) the three nodes contained in the triangle are obliged to be driver nodes.

For triangle-driven control, the MDS of a network can be solved by binary integer programming [24–29].

\[
\min f = \sum_{i \in V} x_i
\]

subject to

\[
x_i + \sum_{j,k,l \in V} a_{ij} a_{jk} a_{kl} x_j x_k x_l \geq 1, \quad i, j, k, l = 1, 2, \ldots, n; \quad i \neq j \neq k \neq l.
\]

4. Results and discussions

In this section, we first analyze the meta-structure-driven control of Erdős–Rényi (E–R) networks [43] and scale-free (S-F) networks, and then discuss the impact of community strength on meta-structure-driven control. Finally, we test our control model on real-world networks.

4.1. Test on Erdős–Rényi and scale-free networks

Here, we generate E–R and S-F networks using the igraph R package based on the static model [44], where \( n = 0.5 \times 10^3 \), and each is averaged over ten realizations. In the following, we analyze the impact of some network parameters on the meta-structure-driven control.

4.1.1. Impact of average node degree on meta-structure-driven control

In this subsection, we study the meta-structure-driven control of E–R and S-F networks and analyze the impact of average node degree, \( \langle k \rangle \), on \( f_{driver} \), where \( f_{driver} \) is defined as the percentage of driver nodes for the control of the networks. Figures 3(a) and (b) show the results for \( f_{driver} \) as a function of \( \langle k \rangle \) for the E–R and S-F networks, respectively. In figure 3(a), we can see that \( f_{driver} \) decreases as \( \langle k \rangle \) increases, and the lower the level of the meta-structures is, the quicker the curve declines, e.g. the curve for node-driven control declines quickly compared with that for triangle-driven control. Similarly, the same findings can be observed for the S-F networks in figure 3(b). In addition, we also find that \( f_{driver} \) of the S-F networks decreases as the exponent, \( \gamma \), increases, and the higher the level of the meta-structures is, the more obvious the finding is, e.g. the finding is more observed more obviously for triangle-driven control than for node-driven control. Overall, the response of \( f_{driver} \) to the changes of \( \langle k \rangle \) weakens as the level of the meta-structures increases.

Figures 3(c)–(e) show \( \langle k_{driver} \rangle \) as a function of \( \langle k \rangle \), where \( \langle k_{driver} \rangle \) denotes the average degree of the driver nodes. We can see that \( \langle k_{driver} \rangle \) increases as \( \langle k \rangle \) increases. Compared with \( \langle k \rangle \), we find that high-degree nodes are more likely to be driver nodes for meta-structure-driven control. We also find that for sparse E–R networks, the \( \langle k_{driver} \rangle \) of link-driven control is greater than that of triangle-driven control, while for dense E–R networks, the result reverses, and this finding is observed more obviously for S-F networks. In addition, a phenomenon that can be observed between node-driven control and triangle-driven control is that there is no obvious difference for \( \langle k_{driver} \rangle \) compared with \( f_{driver} \). This is probably because triangle-driven control is more likely to avoid low-degree nodes, while low-degree nodes might be contained in node-driven control.

4.1.2. Impact of degree heterogeneity on meta-structure-driven control

In order to elaborate the impact of a network’s degree heterogeneity on \( f_{driver} \), we use a measure introduced by Liu et al [20], who showed that for structural controllability, \( f_{driver} \) is highly associated with the degree heterogeneity in the control of directed networks.

The degree heterogeneity, \( H \), is defined as the relative mean difference of a network’s degree distribution,

\[
H = \sum_{s} \sum_{t} \text{abs}(s - t) \cdot P(s) \cdot P(t) / \langle k \rangle,
\]

where \( P \) denotes a network’s degree distribution, \( s \) and \( t \) correspond to two variables ranging over all possible degrees of the nodes, and \( \text{abs}(\cdot) \) denotes the absolute value.
Here, we analyze the impact of $H$ on $f_{\text{driver}}$ for meta-structure-driven control. Figure 4(a) shows the results for $f_{\text{driver}}$ as a function of $H$ for the E–R networks, and we can see that the $f_{\text{driver}}$ of triangle-driven control shows a more sensitive response to the change of $H$, i.e. the higher the level of the meta-structures is, the more sensitive the response is. Similarly, the same finding can be observed for S-F networks in figure 4(b). In addition, we also find that the response of the S-F networks is more insensitive as the exponent, $\gamma$, increases.

Figures 4(c) and (d) show $\langle k_{\text{driver}} \rangle$ as a function of $H$, and we can see that the $\langle k_{\text{driver}} \rangle$ of node-driven control shows a more sensitive response to the change of $H$, i.e. the lower the level of the meta-structures is, the more sensitive the response is. Similarly, the same finding can be observed for the S-F networks in figure 4(d).

Based on the results above, we find that it is harder to control sparse and inhomogeneous networks as the level of the meta-structures increases, e.g. this finding is more obviously observed on the triangle-driven control than the node-driven control.

4.1.3. Scalability analysis for large networks

In order to test the scalability of the meta-structure-driven control model, we tested our model on large networks. The results in figure 3(f) show that as the level of the meta-structures increases, it is more time-consuming to find the corresponding MDS. Based on binary integer programming, node-driven control can be
Figure 4. The impact of $H$ on meta-structure-driven control of the E–R and S–F networks. (a) and (b) $f_{\text{driver}}$ as a function of $H$ for the E–R networks and the S–F networks respectively. (c) and (d) $<k_{\text{driver}}>$ as a function of $H$ for the E–R networks and the S–F networks respectively.

Figure 5. The results for meta-structure-driven control of the LFR benchmark. (a) $f_{\text{driver}}$ as a function of $\mu$ for fixed $(k)$. (b) $f_{\text{driver}}$ of the randomized versions of the networks based on the DDC, the NDC and the E–R. (c) $<k_{\text{driver}}>$ as a function of $\mu$ for fixed $(k)$. (d) $<k_{\text{driver}}>$ of the randomized versions of the networks based on the DDC, the NDC and the E–R.
seen as linear programming, while link-driven control and triangle-driven control belong to non-linear programming, and the complexity increases rapidly as the level of the meta-structures increases.

4.2. Test on networks with community structures

For traditional control, in our previous work we showed that networks with weaker communities need more driver nodes to establish control. Therefore, in this section we are motivated to elaborate on the impact of community strength on meta-structure-driven control.

4.2.1. Impact of community strength on meta-structure-driven control

Here, we try to elaborate on the impact of community strength on meta-structure-driven control. We use the Lancichinetti–Fortunato–Radicchi (LFR) benchmark [45] to generate networks with variable community strength [45].

Figure 5(a) shows the impact of community strength on $f_{driver}$. For node-driven control, we find in figure 5(a) that $f_{driver}$ increases as $mu$ increases, where $mu$ is the mixing parameter of the LFR benchmark, and community strength weakens as $mu$ increases [45]. However, this monotonicity is not preserved for link-driven control and triangle-driven control, i.e. as the level of the meta-structures increases, the networks with the strongest communities are not optimal for meta-structure-driven control. Figure 5(b) shows $<k_{driver}>$ as a function of $mu$ for fixed $<k>$, and we can see that overall $<k_{driver}>$ decreases as $mu$ increases for node-driven control, while for link-driven control and triangle-driven control, this finding does not hold.

4.2.2. Meta-structure-driven control of the networks in randomized versions

Further, we randomize the above networks based on three randomization procedures [46]: (1) node degree conservation (NDC); (2) degree distribution conservation (DDC); and (3) the E–R randomization procedure.

We can see in figure 5(b) that the results of the randomized version based on the DDC is closer to that of the
Figure 7. Robustness analysis for the LFR benchmark and the results for meta-structure-driven control of the karate network. (a) and (b) The control rate of the LFR benchmark. (c) and (d) $f_{\text{base}}$ and $\langle k_{\text{base}} \rangle$ of the karate network, respectively. Each point is averaged over 1000 realizations.

Figure 8. Illustration of meta-structure-driven control of the Zachary karate club network. (a), (b) and (c) Node-driven control, link-driven control and triangle-driven control, respectively. The network consists of two communities separated by dashed red curves, and the meta-structures for driving the full network are highlighted in red.
original networks than the NDC and the E–R ones, i.e. the number of driver nodes is dependent on the networks’ degree distribution. Further, the results indicate that the dependence weakens as the level of the meta-structures increases, e.g. the number of driver nodes is more dependent on the networks’ degree distribution for node-driven control than triangle-driven control. Similarly, for \( k_{\text{driver}} \) the same finding can be seen in figure 5(d).

In figure 6, we illustrate meta-structure-driven control of the LFR benchmark. Figures 6(a)–(c) correspond to node-driven control, link-driven control and triangle-driven control, respectively. The network consists of two communities, and the meta-structures for driving the full network are highlighted in red.

4.2.3. Robustness analysis
To test the robustness of meta-structure-driven control, we use the control rate, defined as the percentage of non-driver nodes controlled after the removal of edges. Figures 7(a) and (b) correspond to the control rate of the LFR benchmark with the increase of the fraction of removed edges, and the results show that as the level of the meta-structures increases, the control rate increases.

4.2.4. Meta-structure-driven control of real-world networks with community structure
Real-world networks provide a better benchmark for testing the performance of community detection methods. Here, we use the Zachary karate club network [47] to study the meta-structure-driven control model further. In the Zachary club network [47], Zachary considered 34 members of the network for two years. During his experiment, a disagreement occurred between vertex 1, the administrator of the club and vertex 33, the club’s instructor, which ultimately led to the division of the club, since the instructor left and started a new club by taking nearly half of the members of the original club with him. Figures 7(c) and (d) correspond to \( f_{\text{driver}} \) and \( k_{\text{driver}} \) of the real-world network and its randomized versions based on the NDC, DDC and E–R respectively. The findings are the same as those we observed for the artificial networks above.

In figure 8, we illustrate meta-structure-driven control of the Zachary karate club network. Figures 8(a)–(c) correspond to node-driven control, link-driven control and triangle-driven control, respectively. The network consists of two communities separated by dashed red curves, which are centered with node 1 and node 33, and the meta-structures for driving the full network are highlighted in red. Although node 10 belongs to the right community, it is misclassified into the left community by many community detection methods. In addition, from the viewpoint of network control, node 10 is driven by the meta-structures belonging to the left community.
We also study meta-structure-driven control of the dolphin [48] and football networks [6, 7], and the results are shown in figures 9 and 10, respectively. From the figures, we can see that high-level meta-structures tend to capture the cores of communities, and the control of whole networks through driving high-level meta-structures within communities provides a new insight into the controllability of complex networks.

5. Conclusions

In this paper, we develop a new framework, which first of all defines the meta-structures of a graph in different levels and tries to depict a graph from a low to a high level of abstraction. Further, based on the framework, we study the meta-structure-driven control model and try to understand the controllability of complex networks from a microcosmic to a macrocosmic perspective. Finally, we analyze the impact of the dynamics of networks on meta-structure-driven control. The results for artificial networks and real-world networks indicate that for meta-structure-driven control, the number of driver nodes is dependent on the networks’ degree distribution, and the dependence weakens as the level of abstraction of the meta-structures increases. In addition, the networks are easier to control as community strength increases, while this monotonicity is not preserved as the level of abstraction of the meta-structures increases. We also find that it is harder to control sparse and inhomogeneous networks as the level of abstraction of the meta-structures increases. In future work, we will study meta-structure-driven control of biological networks and try to understand the controllability of biological networks from a microcosmic to a macrocosmic perspective.

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References

[1] Strogatz S H 2001 Exploring complex networks Nature 410 268–76
[2] Albert R and Barabási A-L 2002 Statistical mechanics of complex networks Rev. Mod. Phys. 74 47–97
[3] Reichardt J and Bornholdt S 2006 Statistical mechanics of community detection Phys. Rev. E 74 016110
[4] Du W B, Zhou X L, Lordand O, Wang Z, Zhao C and Zhu Y B 2016 Analysis of the Chinese airline network as multi-layer networks Transportation Research Part E: Logistics and Transportation Review 89 108–16
[5] Girvan M and Newman M E J 2002 Community structure in social and biological networks Proc. Natl Acad. Sci. 99 7821–6
[6] Palla G, Derenyi I, Farkas I and Vicsek T 2005 Uncovering the overlapping community structure of complex networks in nature and society Nature 435 814–8
[7] Newman M E J 2006 Modularity and community structure in networks Proc. Natl Acad. Sci. 103 8577–82
[8] Newman M E J 2004 Detecting community structure in networks Eur. Phys. J. B 38 321–50
[9] Newman M E J and Girvan M 2003 Finding and evaluating community structure in networks Phys. Rev. E 69 026113
[10] Fortunato S 2010 Community detection in graphs Phys. Rep. 486 75–174
[11] Fortunato S, Latora V and Marchiori M 2004 Method to find community structures based on information centrality Phys. Rev. E 70 056104
[12] Danon L, Duch J and Diaz-Guilera A 2005 Comparing community structure identification J. Stat. Mech. 09 P09008
[13] Sun P G, Gao L and Yang Y 2013 Maximizing modularity intensity for community partition and evolution Inf. Sci. 236 82–93
[14] Rosvall M and Bergstrom CT 2007 An information-theoretic framework for resolving community structure in complex networks Proc. Natl Acad. Sci. 104 7332–37
[15] Radicchi F, Castellano C, Ceconi F, Loreto V and Parisi D 2004 Defining and identifying communities in networks Proc. Natl Acad. Sci. 101 2658–63
[16] Sun P G, Gao L and Han S 2011 Identification of overlapping and non-overlapping community structure by fuzzy clustering Inf. Sci. 181 1060–71
[17] Kalman RE 1963 Mathematical description of linear dynamical systems J. Soc. Indus. Appl. Math. Ser. A 1 152–92
[18] Luenberger D G 1979 Introduction to Dynamic Systems: Theory, Models, and Applications (New York: Wiley)
[19] Slotine J J and Li W 1991 Applied Nonlinear Control (New Jersey: NJ: Prentice-Hall)
[20] Liu Y Y, Slotine J J and Barabási A-L 2011 Controllability of complex networks Nature 473 167–73
[21] Liu Y Y, Slotine J J and Barabási A-L 2012 Control centrality and hierarchical structure in complex networks PLoS ONE 7 e44459
[22] Yuan Z, Zhao C, Di Z, Wang W-X and Lai Y-C 2012 Exact controllability of complex networks Proc. Natl Acad. Sci. 109 814–18
[23] Gao J, Liu Y Y, D’Souza R M and Barabási A-L 2014 Target control of complex networks Nat. Commun. 5 5415
[24] Nacher J C and Akutsu T 2013 Structural controllability of unidirectional bipartite networks Sci. Rep. 3 1647
[25] Nacher J C and Akutsu T 2014 Analysis of critical and redundant nodes in controlling directed and undirected complex networks using dominating sets Journal of Complex Networks 2 394–412
[26] Nacher J C and Akutsu T 2015 Structurally robust control of complex networks Phys. Rev. E 91 012826
[27] Nacher J C and Akutsu T 2012 Dominating scale-free networks with variable scaling exponent: heterogeneous networks are not difficult to control New J. Phys. 14 073005
[28] Nacher J C and Akutsu T 2016 Minimum dominating set-based methods for analyzing biological networks Methods 102 57–63
[29] Wuchty S 2014 Controllability in protein interaction networks Proc. Natl Acad. Sci. 11 7156–60
[30] Liu Y Y and Barabási A-L 2016 Control principles of complex networks Rev. Mod. Phys. 88 035006
[31] Sun P G 2015 Controllability and modularity of complex networks Inf. Sci. 325 20–32
[32] Cornelius S P, Kath W L and Motter A E 2013 Realistic control of network dynamics Nat. Commun. 4 1942
[33] Onnela JP 2014 Flow of control in networks Science 343 1325–6
[34] Ruths J and Ruths D 2014 Control profiles of complex networks Science 343 1373–6
[35] Masuda N 2015 Opinion control in complex networks New J. Phys. 17 033031
[36] Wang W-X, Ni X, Lai Y-C and Grebogi C 2012 Optimizing controllability of complex networks by small structural perturbations Phys. Rev. E 85 052811
[37] Yan G, Ren L, Lai Y-C, Lai C H and Li B 2012 Controlling complex networks—how much energy is needed? Phys. Rev. Lett. 108 218703
[38] Zhang X, Lv T, Yang X and Zhang B 2014 Structural controllability of complex networks based on preferential matching PLoS ONE 9 e112039
[39] Zhang X-F, Ou-Yang L, Zhu Y, Wu M-Y and Dai D-Q 2015 Determining minimum set of driver nodes in protein–protein interaction networks BMC Bioinform. 16 146
[40] Sun P G 2015 Co-controllability of drug–disease–gene network New J. Phys. 17 085009
[41] Land AH and Doig AG 1960 An automatic method of solving discrete programming problems Econometrica 28 497–520
[42] Du W B, Gao Y, Liu C, Zheng Z and Wang Z 2015 Adequate is better: particle swarm optimization with limited information Appl. Math. Comput. 268 832–8
[43] Erdös P and Rényi A 1960 On the evolution of random graphs Publ. Math. Inst. Hungarian Acad. Sci. 5 17–61
[44] Goh K-I, Kahng B and Kim D 2001 Universal behaviour of load distribution in scale-free networks Phys. Rev. Lett. 87 278701
[45] Lancichinetti A, Fortunato S and Radicchi F 2008 Benchmark graphs for testing community detection algorithms Phys. Rev. E 78 046110
[46] Brohee S, Faust K, Lima-Mendez G, Vanderstocken G and van Helden J 2008 Network analysis tools: from biological networks to clusters and pathways Nat. Protocols 3 1616–29
[47] Zachary W W 1977 An information flow model for conflict and fission in small groups J. Anthropol. Res. 33 452–73
[48] Lusseau D, Schneider K, Boisseau O J, Haase P, Slooten E and Dawson S M 2003 The bottlenose dolphin community of doubtful sound features a large proportion of long-lasting associations Behav. Ecol. Sociobiol. 54 396–405