Time Variation of the Fine Structure Constant Driven by Quintessence

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There are indications from the study of quasar absorption spectra that the fine structure constant $\alpha$ may have been measurably smaller for redshifts $z > 2$. Analyses of other data (\textsuperscript{149}Sm fission rate for the Oklo natural reactor, variation of \textsuperscript{187}Re $\beta$-decay rate in meteorite studies, atomic clock measurements) which probe variations of $\alpha$ in the more recent past imply much smaller deviations from its present value. In this work we tie the variation of $\alpha$ to the evolution of the quintessence field proposed by Albrecht and Skordis, and show that agreement with all these data, as well as consistency with WMAP observations, can be achieved for a range of parameters. Some definite predictions follow for upcoming space missions searching for violations of the equivalence principle.

INTRODUCTION

Independent observations of a number of absorption systems in the spectra of distant quasars (QSOs) seem to indicate that $\alpha$, the fine structure constant of quantum electrodynamics, is slowly increasing over cosmological time scales \cite{1,2}. Specifically, the experiments indicate that averaged over redshifts $0.2 < z < 3.7$, there is a $5.7\sigma$ deviation of the fine structure constant from its present value, namely $\Delta \alpha/\alpha = -0.57 \pm 0.10 \times 10^{-5}$ \textsuperscript{3}. On the other hand, terrestrial and solar system measurements provide several constraints on recent rate of variation of $\alpha$ : (1) Analyses of the resonant fission reaction rate in the naturally occurring reactor at Oklo in Gabon provide a bound $-0.9 \times 10^{-7} < \Delta \alpha/\alpha < 1.2 \times 10^{-7}$ over the era $z < 0.14$ at the 95\% CL \textsuperscript{4}. (2) Based on plausible assumptions, new estimates of the age of iron meteorites ($z \approx 0.45$) combined with a measurement of the Os/Re ratio resulting from the radioactive decay \textsuperscript{187}Re $\to$ \textsuperscript{187}Os have allowed a narrowing in the uncertainty of the average decay rate over the age of the meteorite \textsuperscript{5}. This has been translated \textsuperscript{6} into a strong bound $\Delta \alpha/\alpha < 3 \times 10^{-7}$, following the original suggestion of Peebles and Dicke \textsuperscript{7}. (3) Recently, three years of observations of hyperfine spectra using atomic fountain clocks have allowed a 1\% bound $\dot{\alpha}/\alpha < 1.6 \times 10^{-15}$ yr\textsuperscript{-1} for the present time-rate of variation of $\alpha$ \textsuperscript{8}. (4) Additional bounds in this category have been derived \textsuperscript{9}, \textsuperscript{10}, but these are weaker than the ones listed above. Finally, there are constraints resulting from cosmological considerations: (1) Limits on the temperature fluctuations of the cosmic microwave background (CMB) could lead to a measurement with experimental sensitivity of $|\Delta \alpha/\alpha| < 10^{-2} - 10^{-3}$, at $z \sim 1000$ \textsuperscript{11}. Analysis of data from the Wilkinson Microwave Anisotropy Probe (WMAP) provides a bound $-0.06 < \Delta \alpha/\alpha < 0.02$ at 95\% CL \textsuperscript{12}. (2) Big bang nucleosynthesis (BBN) considerations place bounds on $|\Delta \alpha/\alpha|$ on the same order of magnitude as those from CMB, though at much larger redshift, $z \sim 10^{9} - 10^{10}$ \textsuperscript{13}. Over the last years, a second set of observations (most recently from WMAP \textsuperscript{13}) has accumulated which indicate that the universe is spatially flat to within 1\%. In addition, luminosity distance measurements of Type Ia supernovae strongly imply the presence of some unknown form of energy density, related to otherwise empty space, which appears to dominate the recent gravitational dynamics of the universe and yields a stage of cosmic acceleration \textsuperscript{14}. We still have no solid clues as to the nature of such dark energy (or perhaps more accurately dark pressure), but in recent years it has been associated with a dynamical scalar field $\phi$ evolving in a potential $V(\phi)$ \textsuperscript{15}, generally called “quintessence” \textsuperscript{16}.

The universality of gravitational interactions implies that one may expect the Lagrangian below the Planck scale to contain non-renormalizable couplings of $\phi$ to standard model fields \textsuperscript{17}. In particular, the free Lagrangian for the electromagnetic field tensor $F_{\mu\nu}$ will be modified to

$$\tilde{\mathcal{L}}_{em} = -\frac{1}{4} Z_{\phi}(\phi/M_{Pl}) F_{\mu\nu} F^{\mu\nu} ,$$

with $M_{Pl} = (8\pi G)^{-1/2}$, the reduced Planck mass. On expansion about the present value $\phi_{0}$ of $\phi$, this becomes

$$\tilde{\mathcal{L}}_{em} = -\frac{1}{4} \left(1 + \frac{\Delta \phi}{M_{Pl}} + \ldots \right) F_{\mu\nu} F^{\mu\nu} ,$$

with $\Delta \phi = \phi - \phi_{0}$ and $\kappa = \partial_{\phi} Z_{\phi}|_{\phi_{0}}$. The field renormalization $A_{\mu} \to A_{\mu}/Z_{\phi}^{1/2}$ to obtain a canonical kinetic energy, generates an effective charge $\epsilon/Z_{\phi}^{1/2}$. Expansion to linear order about the present value $\epsilon_{0}$, leads to

$$\frac{\Delta \alpha}{\alpha} = -\kappa \frac{\Delta \phi}{M_{Pl}} .$$

Compatibility between the Oklo/meteorite/atomic clock and QSO measurements can greatly constrain the dynamics of $\phi$: its evolution should slow considerably between the quasar era and the present epoch \textsuperscript{18}. 

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If the field $\phi$ driving the variation in $\alpha$ is a quintessence field, then its evolution is further constrained by observation. In particular, it must provide about 70% of the total energy density at present. Its equation of state, $w_\phi \equiv p_\phi/\rho_\phi$ ($p_\phi \equiv$ pressure, $\rho_\phi \equiv$ energy density), is most strongly constrained by WMAP observations: $w_\phi < -0.78$ at the 95%CL [13]. Additionally, radiation dominance at the time of BBN must be maintained – no more than 20% of the energy density at that time can reside in quintessence [10].

In this work we search for a model of quintessence in which the evolution of the scalar field $\phi$ in its potential $V(\phi)$ drives the variation in $\alpha$ [20]. The particular case where $\phi$ is the dilaton was examined in [21]. With such a dynamics, however, the universal coupling of the dilaton greatly constrains the variation in $\alpha$, making it difficult to comply with the QSO data. An alternative proposal [22], where the dynamics of the scalar field resides in a non-trivial Kähler potential, can provide a variation of $\alpha$ compatible with observation. However, in this work we wish to pursue a path based on sums of exponential potentials, which can be more explicitly linked to string theories (more on this below). It is also important to stress that our approach differs fundamentally from models where the variations of the scalar field are primarily driven by its couplings to non-relativistic matter [23, 24] (generically referred to as Bekenstein-type models [25]).

Before proceeding, we take note of a discussion [26] which relates this type of variation in $\alpha$ to a large shift in the cosmological constant, and hence questions its viability. In the spirit of [27] we adopt here a more wait-and-see position, since present field-theoretic considerations all require fine-tuning to solve the cosmological constant problem, and may need to be totally supplanted (perhaps by self-tuning mechanisms, such as described in [28]).

**ALBRECHT-SKORDIS COSMOLOGY**

**Quintessence Phenomenology**

An interesting model for quintessence has been presented by Albrecht and Skordis [24]. The scalar field evolves in a potential (hereafter we adopt natural units $8\pi G = 1$)

$$V(\phi) = V_\rho(\phi) \exp[-\lambda \phi] ,$$

with an economic polynomial factor,

$$V_\rho(\phi) = (\phi - B)^\beta + A ,$$

in which the constants $A, B, \beta$ and $\lambda$ are phenomenologically determined. Because of the polynomial factor, this potential differs in a critical manner from the much-studied pure exponential: although the tracking properties are similar, it allows sufficient radiation dominance during BBN while evolving to quintessence dominance in the present epoch, all largely independent of initial conditions. For the particular case $\beta = 2$, the field is trapped in a minimum yielding a permanently accelerated universe if $\lambda^2 A < 1$, whereas if $\lambda^2 A > 1$ (and for a small region $0 < 1 - \lambda^2 A \ll 1$), the accelerated era is transient [30]. In what follows we set $\beta = 2$.

Exponential potentials are ubiquitous in 4-D field theory descendants of string/M theory [31]. Additionally, sums of exponentials have been proposed [32] in earlier attempts at stabilizing the dilaton and allowing supersymmetry breaking through gaugino condensates. In order to avoid the difficulties associated with identifying quintessence as the dilaton [21], in what follows we associate $\phi$ with moduli related to compactification, since, unlike dilatons, the latter need not be universally coupled to matter and gauge fields. The origin of the polynomial factor form of the potential in Eq. (4) can be linked to a non-trivial Kähler term in an effective 4-D supergravity theory [33]. However, in such a case an explicit connection to string theory has not been successfully established. On the other hand, string/M theory does provide motivation for superpotentials which are sums of exponentials [34]. A much-discussed recent example occurs in 11-dimensional supergravity, with a geometry consisting of a warped product of our 4 dimensional space-time and an internal compact 7-dimensional hyperbolic manifold whose volume is proportional to the dual of the field strength. Upon dimensional reduction, the effective potential in 4 dimensions consists of two exponentials, the first proportional to the 7-dimensional Ricci scalar, the second to the volume of the compact space. The exponents are proportional to the breathing modulus of the warp factor.

With this in mind, we can think of the Albrecht-Skordis potential as the limiting case of three exponentials

$$V(\phi) = C e^{-(\lambda+\epsilon)(\phi-B)} - 2C (1 - \epsilon^2 A/2) e^{-\lambda(\phi-B)} + C e^{-(\lambda-\epsilon)(\phi-B)} ,$$

where $\epsilon^2 C = e^{-\lambda B}$, and $\epsilon^2 \ll 1$. In order that the reduction to a 4-D theory generate three exponentials with a single modulus, it is necessary to split the compactification of the internal space so that the resulting internal manifolds have different warping factors with a single collective coordinate. Additional dilatonic degrees of freedom can arise if one starts from 10-D string theory (such as Type IIA or Type IIB), and these need to be constrained or stabilized. We are currently studying these possibilities and the results will be presented elsewhere [30].
Cosmological Evolution of $\alpha$

In order to follow the evolution of the fine structure constant, we need to trace the temporal behavior of $\phi$ since the quasar epoch, while at the same time requiring that the field provide a successful model for quintessence. This evolution follows from the 4-D effective action

$$ S = \int d^4x \sqrt{-g}\left\{ \frac{R}{2} + {\mathcal L}_\phi + {\mathcal L}_{\text{vis}} + [Z_F(\phi) - 1] {\mathcal L}_{\text{em}} \right\}. $$

As usual, $R$ is the Ricci scalar derived from the metric tensor $g_{\mu\nu}$,

$$ {\mathcal L}_\phi = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi), $$

where $V(\phi)$ is the Lagrangian of visible matter (baryons, photons, and also baryonic and neutrino dark matter), and\n
$$ {\mathcal L}_{\text{em}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}. $$

Note that we have omitted terms coupling $\phi$ to the other matter fields because we are searching for a non-Bekenstein solution, i.e., the potential is primarily driving the evolution of $\phi$.

The equation of motion for $\phi$ then reads

$$ \doteqdot \phi + 3H \dot{\phi} = -\frac{\partial V}{\partial \phi}, $$

where

$$ H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{3} \left[ \frac{1}{2} \phi^2 + V(\phi) + \rho_m + \rho_r \right], $$

with $\rho_m$ and $\rho_r$ the matter and radiation energy densities, respectively. It is more convenient to consider the evolution in

$$ x = \ln a = -\ln(1+z), $$

with the present value of the scale parameter $a_0 = 1$. Denoting by a prime derivatives with respect to $x$, the equation of motion for $\phi$ becomes

$$ \ddot{\phi} + \frac{\dot{\phi}^2}{1 - \phi^2/6} + 3 \phi' + \frac{\phi' \phi''}{2} + 3 \frac{\partial V}{\partial \phi} = 0, $$

where $\rho = \rho_m + \rho_r$. Quantities of importance are the dark energy density

$$ \rho_\phi = \frac{1}{2} H^2 \phi^2 + V, $$

generally expressed in units of the critical density ($\Omega \equiv \rho/\rho_c$)

$$ \Omega_\phi = \frac{\rho_\phi}{3H^2}, $$

and

$$ H^2 = \left[ \frac{V + \rho}{3} \right] \left[ 1 - \frac{\phi^2}{6} \right]^{-1}. $$

The equation of state is

$$ w_\phi = \left[ \frac{H^2 \phi^2 - V}{2} - \frac{H^2 \phi^2}{2} + V \right]^{-1}. $$

For $\rho_m$ and $\rho_r$ we adopt the expressions

$$ \rho_m = C e^{-3x}, \quad \rho_r = 10^{-4} C e^{-4x} f(x), $$

where $C = \Omega_{m,0} \rho_{c,0}$ and $f(x)$ parameterizes the $x$-dependent number of radiation degrees of freedom. In order to interpolate the various thresholds appearing prior to recombination (among others, QCD and electroweak), we adopt a convenient phenomenological form,

$$ f(x) = \exp(-x/15). $$

In natural units $\rho_{c,0} = 2.3 \times 10^{-120} h_0^2$. Finally, we set our constants $\Omega_{m,0} = 0.3$, $h_0 = 0.7$ in accordance with WMAP observations

As described in [29], for a wide range of parameters this potential allows a plausible cosmological behavior independent of initial conditions. For definiteness, in this work we take the initial kinetic energy in the field $\phi$ to equal its initial potential energy. The remaining degrees of freedom $\lambda$ and $A$ will be used in order to study the variation of the fine structure constant over the history of the universe. We have found that the recent variation of $\phi$ is most directly controlled by the curvature of the potential at its minimum. This is given by the mass of the scalar field,

$$ m_\phi^2 = 2 \left( 1 - K \right) e^{-\lambda B - K}, $$

where

$$ K = 1 - \sqrt{1 - 4AX^2}. $$

We will state our results in terms of

$$ \tilde{m}_\phi^2 \equiv \frac{m_\phi^2}{H_0^2} = \frac{3 \Omega_{\phi,0} \lambda^2 \left( 1 - K \right)}{K}. $$

For fixed values of $\lambda$ and $\alpha$, the value of $B$ is fixed by requiring that the current dark energy density constitute a fraction $\Omega_{\phi,0}$ of the critical density.

Now, Eq. (13) is integrated for a range of values of $\alpha$ and $\tilde{m}_\phi^2$, from $\alpha = 10^{-30}$ to the present epoch. Of these, a small subset has been found to be of interest with respect to the data on the variation of $\alpha$ presented in the Introduction. Within this subset, two types of solution can be identified: in the first, the variation of $\alpha$ shows an oscillatory behavior between the present and QSO epochs, which reflects (through Eq. (19)) the oscillatory behavior
\[ \Delta \alpha / \alpha \text{ x } 10^{-4} \]

![Graph](image)

**FIG. 1:** The solid line indicates the variation of \( \Delta \alpha / \alpha \) for \( \tilde{m}_{\phi}^2 = 52.5 \), \( \lambda = 8.5 \), \( B = 32.0 \), and \( \kappa = -3.3 \times 10^{-4} \). The cosmological evolution of \( \alpha \) is superimposed over a binned-data sample from 72 QSO absorption systems: The points indicated by * correspond to two HI 21 cm and molecular absorption systems. Those points assume no change on the proton g-factor, and therefore should be interpreted with caution. The 7 squares are binned results for 49 QSO absorption systems. The single point indicated by • represents the average over 21 QSO SiIV absorption doublets using the alkali doublet method. The embedded box details the behavior of \( \Delta \alpha / \alpha \) for small values of \( z \). The solid horizontal lines indicate the bound derived from the nuclear reactor at Oklo, whereas the dotted line in the middle indicates the predicted average value of \( \Delta \alpha / \alpha \) within \( 0 < z < 0.14 \).

of \( \phi \) in the potential well; in the second, the motion of \( \phi \) as it comes to its present value is overdamped, so that the change in \( \alpha \) is a monotonically decreasing function of \( z \).

In Fig. 1, we show an example of the oscillatory behavior, for the set of parameters \( \tilde{m}_{\phi}^2 = 52.5 \), \( \lambda = 8.5 \), \( B = 32.0 \), and \( \kappa = -3.3 \times 10^{-4} \). Although the fit to the QSO data is not particularly good (\( \chi^2 / \text{d.o.f.} = 16/7 \)), a solution of this type (with some \( V_\phi \) of higher order in Eq. 21) can lead to an interesting prediction for future observations: \( |\Delta \alpha / \alpha| \) diminishes with increasing redshift beyond \( z = 3 \). The fit is consistent with the Oklo bound (see the inset), but it fails to comply with the new meteorite bound.

Further comparison with data must include compliance with measured bounds on violation of the equivalence principle, in the form of limits on composition-dependent inertial forces [21, 22, 10]. These can translate into an upper bound on \( \kappa \). Along these lines, Olive and Pospelov [23] parametrize the \( \phi \)-dependent modification of the electromagnetic Lagrangian via an equation analogous to Eq. 21:

\[
\tilde{L}_{\text{em}} = -\frac{1}{4} (1 + \zeta F \frac{\Delta \phi}{M_*} + \ldots) F_{\mu \nu} F^{\mu \nu},
\]

where \( M_* \) is the analogue of \( M_{\text{Pl}} \) in the \( \phi \) sector, and the field \( \phi \) in Eq. 20 is defined to have a canonical kinetic energy. Comparing Eqs. 21 and 23, we find \( \kappa = 4 \zeta \sqrt{\tilde{m}_{\phi}^2 / \omega} \), where \( \omega \equiv M_*^2 / 2M_{\text{Pl}}^2 \). The limit derived in 20 is \( \kappa < 7 \times 10^{-4} \).

We can see that our value, \( \kappa = 3.3 \times 10^{-4} \), is consistent with the one given in Eq. 21, but close: taking this model seriously would suggest that composition-dependent inertial forces could be observed in an improved round of experiments.
The third constraint mentioned in the Introduction is the variation of atomic clock measurements. In terms of our variables, the fractional change in the present epoch is

$$\frac{\dot{\alpha}}{\alpha} = \kappa H_0 \phi_0,$$

(25)

where for this set of parameters $\phi_0 = -7.8 \times 10^{-3}$. This implies $\dot{\alpha}/\alpha = 1.8 \times 10^{-16}$ yr$^{-1}$, well within the bound stated in the Introduction.

Finally, as can be seen in Fig. 2, the change in $\phi$ between BBN and the present is $\sim 10$. On the assumption that the dominant variation in $Z_F(\dot{\phi})$ is linear over this domain, the fractional change in $\alpha$ is well within the sensitivity of present analysis. A similar statement holds for the CMB.

We now turn to our second example, shown in Fig. 3 in which $m_\phi^2 = 10.5$, $\lambda = 6$, $B = 45.4$, and $\kappa = 2.1 \times 10^{-4}$. This solution passes muster on several counts: (1) The fit to the QSO data is acceptable—$\chi^2/d.o.f = 9.8/7$, corresponding to 20% CL. (2) As can be seen in the embedded box in Fig. 3, the low-$z$ model results are in good agreement with both Oklo and meteorite constraints. It is also consistent with equivalence principle bounds, the value of $\kappa$ being more than a factor of three below the limits discussed above. It should be stressed that less than an order of magnitude improvement in the experimental sensitivity for composition-dependent inertial forces may provide a direct test for this type of model. As in the previous case, this solution is also consistent with limits imposed by atomic clocks: we find $\phi_0 = -2.4 \times 10^{-3}$, giving $\dot{\alpha}/\alpha = 3.5 \times 10^{-17}$ yr$^{-1}$. (3) Finally, the model also gives variations in $\alpha$ during the BBN and recombination epochs which are well within present sensitivities: for $z = 1100$ (CMB), we find (retaining the linear expansion of $Z_F(\dot{\phi})$) that $\Delta \alpha/\alpha = -4.4 \times 10^{-4}$; for $z = 10^{10}$ (BBN), we obtain $\Delta \alpha/\alpha = -0.0025$.

**DISCUSSION AND CONCLUSIONS**

(1) In this work we have attempted to associate the possible temporal variation of $\alpha$, the electromagnetic fine structure constant, as indicated in absorption spectra of QSOs, with the temporal evolution of the quintessence field which is responsible for the present dark energy content of the visible universe. The outstanding obstacle in maintaining this association is the observation that $\Delta \alpha$, the deviation of $\alpha$ from its present value, is much smaller in the recent past (say $z < 0.45$) compared to the variation indicated by the higher-$z$ QSO data. The reconciliation can be effected if the quintessence field has undergone a rapid slowing in the recent past. The quintessence model that we study (the Albrecht-Skordis model) has precisely such a property. We illustrate our results with an example (Fig. 1) which can simultaneously fit the QSO data and comply with the upper bounds on $\Delta \alpha$ from the Oklo and meteorite analyses and atomic clock measurements. This class of solutions presents several inevitable predictions: (a) deviations from universal free fall should be observed when experimental sensitivity is improved by a factor of 10 (b) QSO measurements at ever-larger redshifts should continue to show a monotonic decrease in $\Delta \alpha$; in this type of solution the quintessence field is just receding from its first turning point in its damped oscillation about the fixed point. Another type of behavior (example in Fig. 1) is seen in solutions which do not satisfy the meteorite bound: there the quintessence field has completed a few oscillations, allowing a return of $\alpha$ to its present value at large redshift.

(2) The evolution of the quintessence field, and therefore of $\alpha$, is determined by the quintessence potential. As noted in the text, this is in contrast to Bekenstein-type models where changes in $\alpha$ are driven by the matter density. The latter allows local spatial variation which could provide different dynamics for $\Delta \alpha$ in our local environment (Oklo and meteorites) and over cosmological scales (QSO data). In the quintessence model, only the zero mode is relevant, and this option of spatial variation is much suppressed.

(3) Comparison with Ref. 24, in which the evolution of $\phi$ is driven by both matter and by a harmonic scalar potential, shows that we require a much larger value of
for agreement with data. The origin of this difference is of interest, since it highlights the constraints imposed by requiring that $\phi$ fulfill its role as a quintessence field. Small values of the mass ($\lesssim H$) will flatten the potential well to such an extent that the field escapes entrapment and leads to an exit from the de Sitter phase. Moreover, in our model, $\phi$ is not initially placed at the equilibrium point, and thus for a shallow well will have considerable velocity during the present era, leading to a strong disagreement with the low $z$ data.

(4) Because the coupling $\kappa$ of the scalar field to the electromagnetic Langrangian plays a dual role in determining both the variation of $\alpha$ and the violation of the equivalence principle, planned experiments on universal free fall can directly test the viability of our model. These include the mission MICROSCOPE from the Centre National d’Etudes Spatiales (CNES) expected to fly in 2005 [13] and the National Aeronautics and Space Agency (NASA) and European Space Agency (ESA) mission STEP (Satellite Test of the Equivalence Principle) [14].

(5) Our prediction for the present rate of variation of $\alpha$ is an order of magnitude below present atomic clock sensitivities. Improvements in the accuracy of such measurements to the range of $10^{-16}$ yr$^{-1}$, of the order of our result, are anticipated for the near future [8].

(6) Amusingly, we can project the evolution of $\alpha$ forward in time. For our model in Fig. $\phi$ is at a turning point in the potential well of $V$ and consequently $\phi$ (and $\alpha$) would reverse motion in the future. For the model in Fig. $\phi$ is already decreasing at present and will continue to do so in the near future.

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[1] J. K. Webb, V. V. Flambaum, C. W. Churchill, M. J. Drinkwater and J. D. Barrow, Phys. Rev. Lett. 82, 884 (1999) [arXiv:astro-ph/9803165]; J. K. Webb et al., Phys. Rev. Lett. 87, 091301 (2001) [arXiv:astro-ph/0012539]; M. T. Murphy, J. K. Webb, V. V. Flambaum and S. J. Curran, Astrophys. Space Sci. 283, 577 (2003) [arXiv:astro-ph/0210532].

[2] Possible ambiguities in the interpretation of the “many-multiplet” results in Ref. $D$ have been pointed out by J. D. Bekenstein, [arXiv:astro-ph/0301566]. Very recently, a method has been proposed for constraining $\Delta T$ based on microwave transitions in the OH molecule. The single species nature of such observations offers the possibility of bypassing the above-mentioned systematic uncertainties. J. Darling. [arXiv:astro-ph/0305550].

[3] Throughout the paper we omit any subscript on the fine structure constant, with the understanding that $\alpha$ denotes the present value.

[4] T. Damour and F. Dyson, Nucl. Phys. B 480, 37 (1996) [arXiv:hep-ph/9606486]; Y. Fujii et al., Nucl. Phys. B 573, 377 (2000) [arXiv:hep-ph/9809549]. Limits on the time variation of $\alpha$ from studies of the isotopic ratios of Sm in the Oklo reactor were first derived by A. I. Shlyakhter, Nature 25, 340 (1976).

[5] M. I. Smoliar, J. K. Walker, and J. W. Morgan, Science 271, 1099 (1996).

[6] K. A. Olive, M. Pospelov, Y. Z. Qian, A. Coccia, M. Casse and E. Vangioni-Flam, Phys. Rev. D 66, 045022 (2002) [arXiv:hep-ph/0205269].

[7] P. J. Peebles and R. H. Dicke, Phys. Rev. 128, 1962 (1962).

[8] H. Marion et al., Phys. Rev. Lett. 90, 150801 (2003) [arXiv:physics/0212112]. See also S. Bize et al., Phys. Rev. Lett. 90, 150802 (2003).

[9] See e.g., P. Sisterna and H. Vucetich, Phys. Rev. D 41 (1990) 1034.

[10] S. Hannestad, Phys. Rev. D 60, 023515 (1999) [arXiv:astro-ph/9810102]; M. Kaplinghat, R. J. Scherrer and M. S. Turner, Phys. Rev. D 60, 023516 (1999) [arXiv:astro-ph/9810153]; R. A. Battye, R. Crittenden and J. Weller, Phys. Rev. D 63, 043505 (2001) [arXiv:astro-ph/0008265]; P. P. Avelino, C. J. Martins, G. Rocha and P. Viana, Phys. Rev. D 62, 123508 (2000) [arXiv:astro-ph/0008416].

[11] C. J. Martins, A. Melchiorri, G. Rocha, R. Trotta, P. P. Avelino and P. Viana, [arXiv:astro-ph/0302209].

[12] E. W. Kolb, M. J. Perry and T. P. Walker, Phys. Rev. D 33, 869 (1986); L. Bergstrom, S. Iguri and H. Rubinstein, Phys. Rev. D 60, 045005 (1999) [arXiv:astro-ph/9902157]; P. P. Avelino et al., Phys. Rev. D 64, 103505 (2001) [arXiv:astro-ph/0102144].

[13] D. N. Spergel et al., [arXiv:astro-ph/0302209].

[14] A. G. Riess et al. [Supernova Search Team Collaboration], Astron. J. 116, 1009 (1998) [arXiv:astro-ph/9805201]; S. Perlmutter et al. [Supernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999) [arXiv:astro-ph/9812133]; N. A. Bahcall, J. P. Ostriker, S. Perlmutter and P. J. Steinhardt, Science 284, 1481 (1999) [arXiv:astro-ph/9906463].

[15] B. Ratra and P. J. Peebles, Phys. Rev. D 37, 3406 (1988).

[16] I. Zlatev, L. M. Wang and P. J. Steinhardt, Phys. Rev. Lett. 82, 896 (1999) [arXiv:astro-ph/9807002].

[17] S. M. Carroll, Phys. Rev. Lett. 81, 3067 (1998) [arXiv:astro-ph/9806099].

[18] T. Chiba and K. Kohri, Prog. Theor. Phys. 107, 631 (2002) [arXiv:hep-ph/0110186].

[19] K. A. Olive, G. Steigman and T. P. Walker, Phys. Rept. 333, 389 (2000) [arXiv:astro-ph/0005320].

[20] Some models in which the scalar field driving the time variation of $\alpha$ is not tied to quintessence have also been discussed. See, e.g., A. Kostelecky, R. Lehnert and M. Perry, [arXiv:astro-ph/0212003]; Q. G. Huang and M. Li, JHEP 0305, 026 (2003) [arXiv:hep-ph/0302208].

[21] T. Damour, F. Piazza and G. Veneziano, Phys. Rev. Lett. 89, 081601 (2002) [arXiv:gr-qc/0204094]; T. Damour, F. Piazza and G. Veneziano, Phys. Rev. D 66, 046007
(2002) [arXiv:hep-th/0205111].
[22] C. Wetterich, Phys. Lett. B 561, 10 (2003) [arXiv:hep-ph/0301261].
[23] K. A. Olive and M. Pospelov, Phys. Rev. D 65, 085044 (2002) [arXiv:hep-ph/0110377].
[24] C. L. Gardner, [arXiv:astro-ph/0305080].
[25] J. D. Bekenstein, Phys. Rev. D 25 (1982) 1527.
[26] T. Banks, M. Dine and M. R. Douglas, Phys. Rev. Lett. 88, 131301 (2002) [hep-ph/0112059].
[27] P. Langacker, G. Segre and M. J. Strassler, Phys. Lett. B 528, 121 (2002) [hep-ph/0112233]; T. Dent and M. Fairbairn, Nucl. Phys. B 653, 256 (2003) [arXiv:hep-ph/0112279].
[28] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, Phys. Lett. B 480, 193 (2000) [hep-th/0001197]; S. Kachru, M. B. Schulz and E. Silverstein, Phys. Rev. D 65, 045021 (2000) [hep-th/0001206].
[29] A. Albrecht and C. Skordis, Phys. Rev. Lett. 84, 2076 (2000) [arXiv:astro-ph/0004321].
[30] J. Barrow, R. Bean and J. Magueijo, Mon. Not. Roy. Astron. Soc. 316, L41 (2000) [arXiv:astro-ph/0004321].
[31] For a quintessence scenario see e.g., M. Gasperini, F. Piazza and G. Veneziano, Phys. Rev. D 65, 023508 (2002) [arXiv:gr-qc/0108016].
[32] B. de Carlos, J. A. Casas and C. Munoz, Nucl. Phys. B 399, 623 (1993) [arXiv:hep-th/9204012].
[33] E. J. Copeland, N. J. Nunes and F. Rosati, Phys. Rev. D 62, 123503 (2000) [arXiv:hep-ph/0005222]; P. Brax, J. Martin and A. Riazuelo, Phys. Rev. D 64, 083505 (2001) [arXiv:hep-ph/0104210].
[34] See e.g., M. Cvetic et al., Nucl. Phys. B 558, 96 (1999) [arXiv:hep-th/9903214]; M. Cvetic, J. T. Liu, H. Lu and C. N. Pope, Nucl. Phys. B 560, 230 (1999) [arXiv:hep-th/9905096]; H. Lu and C. N. Pope, Phys. Lett. B 467, 67 (1999) [arXiv:hep-th/9906168]; M. Cvetic, H. Lu and C. N. Pope, Phys. Rev. Lett. 83, 5226 (1999) [arXiv:hep-th/9906221]; H. Lu, C. N. Pope and T. A. Tran, Phys. Lett. B 475, 261 (2000) [arXiv:hep-th/0001209]; M. Cvetic, H. Lu and C. N. Pope, Nucl. Phys. B 574, 761 (2000) [arXiv:hep-th/9910252]; M. Cvetic, H. Lu and C. N. Pope, Nucl. Phys. B 584, 149 (2000) [arXiv:hep-th/0002099]; C. Nunez, I. Y. Park, M. Schvellinger and T. A. Tran, JHEP 0104 (2001) 025 [arXiv:hep-th/0103080]; J. D. Edelestein and C. Nunez, JHEP 0104, 028 (2001) [arXiv:hep-th/0103167]; C. P. Burgess, C. Nunez, F. Quevedo, G. Tasinato and I. Zavala, [arXiv:hep-th/0305211).
[35] See e.g., M. S. Bremer, M. J. Duff, H. Lu, C. N. Pope and K. S. Stelle, Nucl. Phys. B 543, 321 (1999) [arXiv:hep-th/9807051]; P. K. Townsend and M. N. Wohlfarth, [arXiv:hep-th/0303097].
[36] B. de Carlos, J. A. Casas and C. Munoz, Nucl. Phys. B 558, 96 (1999) [arXiv:hep-th/9807051]; P. K. Townsend and M. N. Wohlfarth, arXiv:hep-th/0303097.
[37] L. Anchordoqui, H. Goldberg and C. Nunez, work in progress.
[38] M. T. Murphy, J. K. Webb, V. V. Flambaum, M. J. Drinkwater, F. Combes and T. Wiklind, Mon. Not. Roy. Astron. Soc. 327, 1244 (2001) [arXiv:astro-ph/0101159].
[39] M. T. Murphy et al., Mon. Not. Roy. Astron. Soc. 327, 1208 (2001) [arXiv:astro-ph/0012419].
[40] M. T. Murphy, J. K. Webb, V. V. Flambaum, J. X. Prochaska and A. M. Wolfe, Mon. Not. Roy. Astron. Soc. 327, 1237 (2001) [arXiv:astro-ph/0112421].
[41] G. R. Dvali and M. Zaldarriaga, Phys. Rev. Lett. 88, 091303 (2002) [arXiv:hep-ph/0108217].
[42] It is noteworthy that a variation in α alters the redshift of the CMB last-scattering, whereas evolution in φ changes the conformal distance to a fixed redshift. Therefore, these two effects can compensate one another in generating the CMB anisotropy spectrum. G. Huey, S. Alexander and L. Pogosian, Phys. Rev. D 65, 083001 (2002) [arXiv:astro-ph/0110562].
[43] J. D. Barrow and D. F. Mota, Class. Quant. Grav. 20, 2045 (2003) [arXiv:gr-qc/0212052]; D. F. Mota and J. D. Barrow, arXiv:astro-ph/0306047.
[44] P. Touboul et al., C. R. Acad. Sci. Paris 2 (série IV), 1271 (2001).