Objective properties from subjective quantum states: Environment as a witness

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We study the emergence of objective properties in open quantum systems. In our analysis, the environment is promoted from a passive role of reservoir selectively destroying quantum coherence, to an active role of amplifier selectively proliferating information about the system. We show that only preferred pointer states of the system can leave a redundant and therefore easily detectable imprint on the environment. Observers who—as it is almost always the case—discover the state of the system indirectly (by probing a fraction of its environment) will find out only about the corresponding pointer observable. Many observers can act in this fashion independently and without perturbing the system: they will agree about the state of the system. In this operational sense, preferred pointer states exist objectively.

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The key feature distinguishing the classical realm from the quantum substrate is its objective existence. Classical states can be found out through measurements by an initially ignorant observer without getting disrupted in the process. By contrast, an attempt to discover the state of a quantum system through a direct measurement generally leads to a collapse 1,2,3; after a measurement, the state will be what the observer finds out it is, but not—in general—what it was before. Thus, it is difficult to claim that quantum states exist objectively in the same sense as their classical counterparts 4,5,6.

It is by now widely appreciated that decoherence, caused by persistent monitoring of a system by the environment, can single out a preferred set of states. In simplest models, such pointer states 7,8,9,10,11 are (often degenerate) eigenstates of the pointer observable which commutes with the system-environment interaction 12. This concept can be generalized using the predictability sieve: only pointer states evolve predictably in spite of the openness of the system 13,14,11. They exist in the sense that in absence of any perturbations—save for the monitoring by the environment—they or their dynamically evolved descendants will continue to faithfully describe the system. Thus, when an observer knows what are the pointer states, he can learn which of them represents the system without perturbing it. However, when an observer ignorant of what pointer states are attempts to find out the state of the system directly, he still faces, even in the presence of decoherence, the danger of collapsing its wave packet.

Here, we build on the idea that a direct measurement of the system is not how observers gather data about the Universe: rather, a vast majority (if not all) of our information is obtained indirectly by probing a small fraction of the environment 15,11,11. One may think that this twist in the story can be accounted for by adding a few links to the von Neumann chain 2, but this is not the case: we shall show that the monitoring environment acquires information about the system selectively. More importantly, this selective spreading of information through the environment—in essence “quantum Darwinism” 11—accounts for the objective existence of some preferred quantum states: by probing the system indirectly, hence without perturbing it, many independent observers can obtain reliable information, but only about the pointer states.

This letter is organized as follows: first, we introduce our operational definition of objectivity. We then state necessary and sufficient conditions for the objective existence of an observable in the context of einselection (environment-induced superselection). Next, these requirements are translated into an information theoretic framework, and proven to imply a unique observable: the usual pointer observable. This is our key result. Finally, we show that, because of quantum Darwinism, information about pointer states is robust and, hence, objective.

An operational definition of objectivity for a property of a quantum system should not rely on pre-existence of an underlying reality as it is presumed in the classical setting. Rather, we demand that an objective property of the system of interest should be (i) simultaneously accessible to many observers, (ii) who should be able to find out what it is without prior knowledge and (iii) who should arrive at a consensus about it without prior agreement.

As we already mentioned, the collapse of the wave packet following a direct measurement generally precludes this. However, when the system of interest S interacts with an environment E composed of many subsystems, E = ⊗k=1 Ek, the situation changes dramatically. When a property leaves a complete and redundant imprint on the environment, all three criteria are satisfied: many copies are available, hence simultaneous accessibility (i)
is straightforward. Moreover fractions of the environment can be measured without perturbing either $S$ or the rest of $E$. Therefore, ignorant observers can vary their measurements independently, corroborate their own results and arrive at a common description of properties of the system. Hence, owing to redundancy, prior knowledge is not necessary to (iii) reach consensus. The existence of an objective property requires the presence of its complete and redundant imprint in the environment as necessary and sufficient conditions. Our approach will focus on the study of the correlations between parts of the environment and the system of interest.

**Information theoretical framework.** A natural way to characterize such correlations is to use the mutual information $I(\sigma : e)$ between an observable $\sigma$ of $S$ and a measurement $e$ on $E$. In short, $I(\sigma : e)$ measures one’s ability to predict the outcome of measurement $\sigma$ on $S$ after having “looked at the environment” through $e$. For a given density matrix $\rho^{SE}$ of $S \otimes E$, the measurement results are random variables characterized by a joint probability distribution

$$p(\sigma_i, \epsilon_j) = \text{Tr}\{(\sigma_i \otimes \epsilon_j)\rho^{SE}\},$$

where $\sigma_i$ and $\epsilon_j$ are the spectral projectors of observables $\sigma$ and $e$. By definition, the mutual information is the difference between the initially missing information about $\sigma$ and the remaining uncertainty about $\sigma$ when $e$ is known. Using Shannon entropy as a measure of missing information, $H(\sigma) = -\sum_i p(\sigma_i) \log p(\sigma_i)$ and $H(\sigma, e) = -\sum_{i,j} p(\sigma_i, \epsilon_j) \log p(\sigma_i, \epsilon_j)$, the mutual information is

$$I(\sigma : e) = H(\sigma) + H(e) - H(\sigma, e). \tag{2}$$

The information about observable $\sigma$ of $S$ that can be optimally extracted from $m$ environmental subsystems is

$$\hat{I}_m(\sigma) = \max_{\{e \in \mathcal{M}_m\}} I(\sigma : e) \tag{3}$$

where $\mathcal{M}_m$ is the set of all measurements on those $m$ subsystems. In general, $\hat{I}_m(\sigma)$ will depend on which particular $m$ subsystems are considered. For simplicity, we will assume that any typical $m$ environmental subsystems yield roughly the same information. This may appear to be a strong assumption, but, as we discuss below, relaxing it does not affect our main conclusions. By setting $m$ to the total number $N$ of subsystems of $E$, we get the information content of the entire environment. Then,

$$\hat{I}_N(\sigma) \approx H(\sigma) \tag{4}$$

expresses the completeness prerequisite for objectivity: all (or nearly all) missing information about $\sigma$ must be in principle obtainable from all of $E$.

However, as a consequence of basis ambiguity, information about many observables $\sigma$ can be deduced by an appropriate measurement on the entire environment. Therefore, completeness, Eq. (4), while a prerequisite for objectivity, is not a very selective criterion (see Fig. 11 for illustration). To claim objectivity, it is not sufficient to have a complete imprint of the candidate property of $S$ in the environment. There must be many copies of this imprint that can be accessed independently by many observers: information must be redundant.

**Redundancy and its consequences.** To obtain a measure of redundancy, one must count the number of copies of the information about $\sigma$ present in $E$. Redundancy is thus quantified by the number of disjoint subsets of $E$ containing almost all—all but a fraction $\delta$—of the information about $\sigma$ available from the entire $E$:

$$R_\delta(\sigma) = N/m_\delta(\sigma). \tag{5}$$

Above $m_\delta(\sigma)$ is the smallest number $m$ of typical environmental subsystems that contain almost all the information about $\sigma$, i.e. $\hat{I}_m(\sigma) \geq (1 - \delta)\hat{I}_N(\sigma)$.

The key question now is: What is the structure of the set $O$ of observables that are completely, $I_N(\sigma) \approx H(\sigma)$, and redundantly, $R_\delta(\sigma) \geq 1$ with $\delta \ll 1$, imprinted on the environment? The answer is provided by the following theorem.

**Theorem:** The set $O$ is characterized by a unique observable $\pi$, called by definition the maximally refined observable: the information $\hat{I}_m(\sigma)$ about any observable $\sigma$ in $O$ obtainable from a fraction of $E$ is equivalent to the information about $\sigma$ that can be obtained through its correlations with the maximally refined observable $\pi$:

$$\hat{I}_m(\sigma) = I(\sigma : \pi) \tag{6}$$

for $m_\delta(\pi) \leq m \ll N$.

**Outline of the proof for perfect records, $\delta = 0$.** Let $\sigma^{(1)}$ and $\sigma^{(2)}$ be two observables in $O$ for $\delta = 0$. Since $\sigma^{(1)}$ and $\sigma^{(2)}$ can be inferred from two disjoint fragments of $E$, they must commute. Similarly, let $\epsilon^{(1)}$ (resp. $\epsilon^{(2)}$) be a measurement acting on a fragment of $E$ that reveals all the information about $\sigma^{(1)}$ (resp. $\sigma^{(2)}$) while causing minimum disturbance to $\rho^{SE}$. Then, $\epsilon^{(1)}$ and $\epsilon^{(2)}$ commute, and can thus be measured simultaneously. This combined measurement gives complete information about $\sigma^{(1)}$ and $\sigma^{(2)}$. Hence, for any pair of observables in $O$, it is possible to find a more refined observable which is also in $O$. The maximally refined observable $\pi$ is then obtained by pairing successively all the observables in $O$. By construction $\pi$ satisfies Eq. (5) for any $\sigma$ in $O$.

Note that the Theorem does not guarantee the existence of a non trivial observable $\pi$: when the system does not properly correlate with $E$, the set $O$ will only contain the identity operator.

In fact, this Theorem can be extended to nearly perfect records for assumptions satisfied by usual models of decoherence. The proof is based on the recognition that only the already familiar pointer observable can
imprinting is sharply peaked around the pointer observable. Redundancy is a very selective criterion. The number of copies off independently by measuring distinct fragments of the environment. For all values of the action, redundancy is high only for the observables chosen in $[0, \pi]$ when the interaction action is far-from-optimal measurement strategy. Information about any other observable $\sigma(\mu)$ is restricted by our theorem to be equal to the information brought about it by the pointer observable $\sigma_2^\mu$, Eq. (5).

have a redundant and robust imprint on $E$. The Theorem can be understood as a consequence of the ability of the pointer states to persist while immersed in the environment. This resilience allows the information about the pointer observables to proliferate, very much in the spirit of the “survival of the fittest”.

Two important consequences of this theorem follow. (i) An observer who probes only a fraction of the environment is able to find out the state of the system as if he measured $\pi$ on $S$. (ii) Information about any other observable $\sigma$ of $S$ will be inevitably limited by the available correlations existing between $\sigma$ and $\pi$. In essence, our theorem proves the uniqueness of redundant information, and therefore the selectivity of its proliferation.

Quantum Darwinism—the idea that the perceived classical reality is a consequence of the selective proliferation of information about the system [11]—is consistent with previous approaches to einselection, such as the predictability sieve, but goes beyond them. The existence of redundant information about the system, induced by specific interactions with the environment, completely defines what kind and how information can be retrieved from $E$: Eq. (4) shows that the most efficient strategy for inferring $\sigma$ consists in estimating $\pi$ first, and deducing from it information about $\sigma$. It also explains the emergence of a consensus about the properties of a system. Observers that gain information about $\pi$—the only kind of information available in fragments of $E$—will agree about their conclusions: their measurement results are directly correlated with $\pi$, and are therefore correlated with each other. Hence, observers probing fractions of the environment can act as if the system had a state of its own—an objective state (one of the eigenstates of $\pi$). By contrast, such consensus cannot arise for superpositions of pointer states, e.g. Schrödinger cats, since information about them can only be extracted by probing the whole environment, and thus cannot be obtained independently by several observers. Objectivity comes at the price of singling out a preferred observable of $S$ whose eigenstates are redundantly recorded in $E$. Cloning of quanta is not possible [16], but amplification of a preferred observable happens almost as inevitably as decoherence, and leads to objective classical reality. The impossibility of cloning and the capacity for amplification imply selection—Darwinian “survival of the fittest”.

Emergence of objectivity exemplified. In Figure 1, we show for a specific model the redundancy, $R_\delta=0.1$, as a function of the inferred observable $\sigma(\mu)$ (whose eigenstates are tilted by an angle $\mu$ from the pointer ones) and of the interaction action, $a_k = g_k t = a$, redundant imprinting is sharply peaked around the pointer observable. Redundancy is a very selective criterion. The number of copies of relevant information is high only for the observables $\sigma(\mu)$ falling inside the theoretical bound (see text) indicated by the dashed line. b) Information about $\sigma(\mu)$ extracted by an observer restricted to local random measurements on $m$ environmental subsystems (e.g. $\epsilon = \xi_1 \otimes \ldots \otimes \xi_m$ where each $\xi_k$ is chosen at random). The interaction action $a_k = g_k t$ is randomly chosen in $[0, \pi/4]$ for each $k$. Because of redundancy, pointer states—and only pointer states—can be found out through this far-from-optimal measurement strategy. Information about any other observable $\sigma(\mu)$ is restricted by our theorem to be equal to the information brought about it by the pointer observable $\sigma_2^\mu$, Eq. (5).
for our simulation, the above equation indicates that only observables with $|\mu| < 0.23$ leave a redundant imprint on the environment: the objective properties of the system are unique. This bound is in excellent agreement with our numerical results. Surprisingly, and as confirmed by our simulation, the interaction action $\alpha_k$ only plays a role in setting the value of the redundancy at its maximum, but does not affect the selectivity of our criterion. Which observable becomes objective is largely decided by the structure of the interaction Hamiltonian, (i.e. the set of pointer states), but not by its details such as strength and duration of the interaction. This ensures the stability of the pointer observable deduced from redundancy.

Robustness of information. Objective information must be extractable through “realizable”—hence, not necessarily optimal—measurements for many observers to arrive at an operational consensus about the state of a system. For instance, human eyes can only measure photons separately, yet we can still learn about the position of objects. This is considered for our model in Fig. 1c. Here, even local (i.e. spin by spin) random measurements eventually acquire the entire information available in $E$ about the pointer states. Though surprising, this result naturally follows from quantum Darwinism and the fact that high redundancy protects information against a wide range of errors. Almost any observable of $S$ is completely imprint on the environment (see Fig. 1b). However, as our theorem establishes and Fig 1b illustrates, only the observables which are “close” to the maximally refined pointer observable $\pi = \sigma_S^z$, can be imprinted redundantly in the environment. Therefore only information about pointer states can tolerate errors, i.e. can be extracted by non-optimal measurements. In short, not only is the information about the pointer observable easy to extract from fragments of the environment, it is impossible to ignore!

Clearly, for the emergence of objective properties, it is much more important to know that $R_\delta(\sigma) \gg 1$ than to know its precise value: the whole idea of redundancy is that it allows one to be sloppy in decoding the message and still “get it right”. This is our proposed explanation for the robustness of the classical. Thus, the essence of our conclusions is unaffected by the assumption (see Eq. (3)) of even spreading of information in the environment: our result depends only on the existence of multiple records of the same information in disjoint fragments of the environment, not on what these fragments are.

Similarly, $R_\delta(\sigma)$ depends on the tensor product of $E$ into subsystems, which is a priori arbitrary. However, several considerations suggest locality as guide line for the definition of elementary subsystems. For instance, our access to the information content of the environment is restricted by the fundamental Hamiltonians of nature which are local. Moreover, various observers occupy, and therefore monitor, different spatial regions. Hence, an operational notion of redundancy should reflect spreading of information in space. Again, details of the partition are not important as we are only interested in the typical behavior of redundancy. These issues will be discussed in more details in a forthcoming paper.

Quantum Darwinism capitalizes on some ideas that arose in the context of decoherence and einselection, but goes beyond them in an essential fashion. Existence of records in the environment has been noted before, and the fact that it is easiest to find out about the pointer observable has been also appreciated. Here, however, we have described an even more dramatic turn of events—environment as a broadcast medium—which may seem fanciful until we realize that it describes rather accurately what happens in the real world. For instance, human observers acquire all of their visual data by intercepting a small fraction of their photon environment. An operational notion of objectivity emerges from redundant information as it enables many independent observers to find out the state of the system without disturbing it. Furthermore, objective observables are robust—insensitive to changes in the strategy through which the environment is interrogated, as well as to variations of the strength and duration of the interaction between $S$ and $E$, etc. One might regard quantum Darwinism as a fully quantum implementation of Bohr’s idea about the role of amplification in the transition from quantum to classical.

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[1] N. Bohr, Nature 121, 580 (1928).
[2] J. von Neumann, Mathematische Grundlagen der Quantenmechanik (Springer, Berlin, 1932).
[3] P. A. M. Dirac, The Principles of Quantum Mechanics, (Clarendon Press, Oxford, 1947), 4th ed.
[4] N. Bohr, in Atti del Congresso Internazionale dei fisici, Como-Pavia-Roma, volume secondo, 565, (1927).
[5] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47, 777 (1935); N. Bohr, Phys. Rev. 48, 696-702 (1935).
[6] C. A. Fuchs and A. Peres, Phys. Today 53, 70 (2000).
[7] W. H. Zurek, Prog. Theor. Phys. 89, 281 (1993).
[8] D. Giulini et al., Decoherence and the Appearance of a Classical World in Quantum Theory (Springer, Berlin, 1996).
[9] W. H. Zurek, Phil. Trans. R. Soc. Lond. A 356, 1793 (1998); W. H. Zurek, Ann. der Phys. 9, 855 (1998).
[10] J.-P. Paz and W. H. Zurek, in Coherent Atomic Matter Waves, Les Houches Lectures, (Springer, Berlin, 2001).
[11] W. H. Zurek, Rev. Mod. Phys. 75, 715 (2003).
[12] W. H. Zurek, Phys. Rev. D 24, 1516 (1981); Phys. Rev. D 26, 1862 (1982).
[13] T. M. Cover and J. A. Thomas, Elements of Information Theory (John Wiley, New York, 1991).
[14] H. Everett III, Rev. Mod. Phys. 29, 454 (1957); A. Kent, Int. J. Mod. Phys. 5, 1745-1762 (1990).

[15] H. Ollivier, D. Poulin and W. H. Zurek, arXiv: quant-ph/0408125.

[16] W. K. Wootters and W. H. Zurek, Nature 299, 802 (1982); D. Dieks, Phys. Lett. 92A, 271 (1982).

[17] M. Gell-Mann and J. B. Hartle, in Proceedings of the 4th Drexel conference on quantum non-integrability (International Press, Cambridge MA, 1997); J. J. Halliwell, Phys. Rev. D 60, 105031 (1999).

[18] D. A. R. Dalvit, J. Dziarmaga, and W. H. Zurek Phys. Rev. Lett. 86, 373-376 (2001)

[19] N. Bohr, in Atomic Physics and Human Knowledge, p.83, (John Wiley & Sons, New York, NY 1958).