A model of amorphous and nano-crystalline ribbon processing by planar-flow casting

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Abstract. This paper provides a mathematical model describing the processing of amorphous and nano-crystalline ribbon. The developed model is based on averaging of the equations of non-equilibrium thermodynamics over a forming zone. The model describes an effect of the various technological parameters on the geometry and temperature of the zone. These parameters are: the overpressure, the speed of rotation of a cooling disc, the value of a gap, the thickness of a nozzle, the initial overheating, the atmosphere pressure, and the evenness of the disc surface. Model allows to take into account various random factors such as vibrations, etc. Considered time-dependent quantities are: the temperature of the formation zone, the length of the zone, the thickness of the ribbon, the curvature radius of the rear part of the zone, the nozzle outflow speed. Model takes into account the following physical properties as determining the process: viscosity, density, surface tension, heat capacity, temperature of solidification, heat transfer. Relief of the free and contact surfaces can also be regarded as parameters of the model. Correlation between the quantities mentioned above is considered without introduction of any phenomenological constants.

1. Introduction.
Formulation of initial and boundary conditions for Navier-Stokes equations is the main problem in models of planar-flow casting. The initial conditions are determined by the following technological parameters:

\[ \Delta P \quad \text{overpressure in the nozzle} \]
\[ d \quad \text{thickness of the nozzle} \]
\[ V_d \quad \text{cooling disc speed of rotation} \]
\[ H \quad \text{value of the gap} \]
\[ T_0 \quad \text{the melt temperature in the nozzle} \]

The main task is to establish the relations between geometrical parameters of the ribbon formation zone (including thickness of ribbon) and the technological parameters. These relations should contain physical properties determining a type of a melt and material of a disc.

An approach that we use for solution of this problem consists of the averaging of the Navier-Stokes and heat transfer equations over the formation zone, that allows us to reduce the problem to the solution of the balance equations (mass, momentum and temperature). In fact, we formulate boundary conditions using the structure of hydrodynamic equations. It should be noted
that we do not solve the initial equations. This simplification results in the loss of information regarding the distributions of speed and temperature in the formation zone. Nevertheless, the obtained model is sufficient for the technological computations.

2. Ribbon formation zone.  
Formation zone is shown in figure 1. The zone has the following structure.  
(i) Line 1–2′–2 is the edge of the nozzle (see figure 1). Here \( V_e \) is the nozzle outflow speed, and \( d \) is the thickness of the nozzle. Pressure on the line is defined by Bernoulli equation:

\[
P_d = \Delta P - \frac{\rho V_e^2}{2},
\]

where \( \rho \) is the density of the melt.  
We assume that the liquid is ideal near the nozzle. Since the thickness of the nozzle is far less than its width, we consider a two-dimensional problem in the axis section of the zone (with respect to rotation of the disc) (See Figure 1).  
(ii) Let us denote \( x \)– and \( y \)–coordinates of the velocity by \( V_x \) and \( V_y \), resp. Then in the domain 2–3–4, \( V_y \) starts from some mean value \( \bar{V}_x \) and decreases to zero. Point 4 is a stagnation one, \([1]\). In this point, \( V_y \) changes its sign from \( - \) to \( + \).  
(iii) Section 5–5′ is the end of the formation zone. The domain 5–5′–6–6′ is a two-phase ribbon with thickness \( \delta \).  
(iv) Line 3–4–5 is the planar front side of the formation zone. Therefore, on this line we have \( P = 0 \).  
(v) Line 1–9–8 is the convex rear part of the formation zone. Let \( r \) be its curvature radius and \( P_H \) the pressure there. \( P_H \) and \( r \) satisfy the following relation:

\[
P_H = \frac{\sigma}{r},
\]

where \( \sigma \) is the surface tension of the melt.
(vi) On the line 9–2’ we have \( V_x = 0 \).

(vii) Consider a line where the melt touches the disc surface first (8–7). In the domain near this line the continuity condition does not hold:

\[
div \vec{V} \neq 0
\]

In [2] it is shown that the condition \( \text{div}\vec{V} = 0 \) implies that the line 9–8 is a quarter of an ellipse, in which the ratio of principal axes is approximately 1 : 20. Such a geometry of the rear part of the formation zone is impossible because of (2). Hence, in the domain 8–7 there can be periodic breaks of the flow of the melt [3], where air cavities with the size of 2–5 microns appear.

(viii) In the domain 7–5′–5″ the mean speed of the melt, \( \vec{V}_x \), increases to the disc speed, \( V_d \). We assume that Prandtl theory of boundary layer is applicable here, and, in particular, we have the Blasius relation:

\[
L = \frac{\delta^2 L}{\nu},
\]

where

- \( \delta_L \) - initial thickness of the liquid layer of the ribbon
- \( \delta_S \) - thickness of the solid layer of the ribbon (Figure 1)
- \( \nu \) - kinematic viscosity
- \( L \) - length of the formation zone.

(ix) The line 7–5′–6 is a solidification front. We assume that the shape of the front is planar one. The position of point 7 is determined by the solution of the one-dimensional heat transfer equation on the layer of thickness \( H \) under the following assumptions:

\[
V_y \frac{\partial T}{\partial y} \neq 0.
\]

The boundary conditions are:

\[
T = T_0, \quad y = H, \quad \frac{\partial T}{\partial y} = \alpha T, \quad y = 0,
\]

where \( \alpha = 10^6 \), [4]. It is established that under the nozzle the temperature of the melt does not decrease to the temperature of solidification \( T_S \). The distance 8–7 equals \( d \). The distance \( z \) was defined in [4] as a solution of one-dimensional heat transfer equation with an assumption that

\[
V_y \frac{\partial T}{\partial y} = 0.
\]

This equation describes the cooling of the ribbon of thickness \( \delta \), when the heat sink on the free surface is a radiation and that on the contact surface is a transmission (\( \alpha = 10^6 \)). The length of the two-phase part of the ribbon equals:

\[
z = \frac{c_m V_d \delta}{\alpha} \ln \left( \frac{T}{T_s} \right),
\]

where

- \( c_m \) - heat capacity of the melt per unit volume
- \( T \) - mean temperature of the zone.
The researching of amorphous melts density in every aggregative state (liquid, crystalline, amorphous) shows that $T_S$ is close to the liquidus temperature.

(x) It follows from the configuration of the solidification front that

$$\frac{\delta - \sqrt{\frac{L \nu}{V_x}}}{L} = \frac{\delta}{L + z}$$  \hspace{1cm} (5)

Relations (4) and (5) connect the length of the zone, $L_\alpha$ and the thickness of the ribbon, $\delta$.

(xi) Since in what follows we use the averaged balance equations, we introduce mean speeds $\bar{V}_x$ and $\bar{V}_y$. Since the flow near the nozzle is ideal, the streamlines are close to hyperbolas:

$$V_y \approx \frac{y}{H} V_c$$

or

$$\bar{V}_y = \frac{1}{2} V_c.$$  \hspace{1cm} (6)

The mass balance gives

$$\bar{V}_x = \frac{d}{H} V_c.$$  \hspace{1cm} (7)

Since the pressure depends linearly on the height in the interval from the nozzle to the disc surface, the reaction (pressure $P_0$, Figure 1), the pressure in the rear part of the zone ($P_H$) and $P_d$ satisfy the following relation:

$$P_H = \frac{1}{2} (P_0 + P_d)$$  \hspace{1cm} (8)

3. Balance equations in the formation zone.

We average the following equations over the formation zone (Figure 1):

$$\begin{aligned}
\frac{\partial V_x}{\partial t} + V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} &= - \frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \left( \frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial y^2} \right) \\
\frac{\partial V_y}{\partial t} + V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} &= - \frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \left( \frac{\partial^2 V_y}{\partial x^2} + \frac{\partial^2 V_y}{\partial y^2} \right) \\
\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} &= 0 \\
\frac{\partial T}{\partial t} + V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} &= \frac{\lambda}{c_m} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right).
\end{aligned}$$  \hspace{1cm} (9)

The details of the averaging are shown in [4]. The following system is the result of this process:

$$\begin{aligned}
\frac{dS}{dt} &= V_c d - V_d \delta \\
\frac{d(S \bar{V}_y)}{dt} &= \bar{V}_y \frac{d^2 V_y}{\rho} + \frac{P_d}{\rho} - \frac{P_0}{\rho} \\
\frac{d(S \bar{V}_x)}{dt} &= \frac{H}{2} \left( \frac{P_d}{\rho} + \frac{P_0}{\rho} \right) - 2 V_x \bar{V}_d \sqrt{L \frac{\nu}{V_d}} \\
\frac{d(ST)}{dt} &= T_0 V_c d - \frac{\alpha}{c_m} LT - TV_d \delta.
\end{aligned}$$  \hspace{1cm} (10)
where

\[ S = H d + \frac{1}{2} L H \]

Let us introduce the following dimensionless values:

\[ u = \frac{V_z}{V_d}; \quad x = \frac{\delta}{d}; \quad y = \frac{T}{T_0}; \quad \mathcal{L} = \frac{L}{d}; \quad h = \frac{H}{d}; \]

\[ \tau = \frac{t V_d}{d}; \quad p = \frac{\Delta P}{\rho V_d^2}; \quad \pi_0 = \frac{P_0}{\rho V_d^2}; \quad \pi_H = \frac{P_H}{\rho V_d^2}; \]

\[ c = \frac{T_0}{T_s}; \quad s = \frac{s}{d^2}; \quad \beta = \frac{\alpha d}{c_m \nu}; \quad \gamma = \frac{\nu}{V_d d}. \]

Using the assumptions from Section 2 we obtain the following main system for our process:

\[
\begin{aligned}
&\frac{d(\dot{S}u)}{d\tau} = u - x \\
&\frac{1}{2} \frac{d(\dot{S}u)}{d\tau} + \frac{2}{h} \frac{d((1 + \frac{1}{2} \mathcal{L})u)}{d\tau} = 2p - 4uh^{-2}\sqrt{L\gamma} \\
&\frac{d(\dot{S}y)}{d\tau} = u - \beta \mathcal{L} y - xy,
\end{aligned}
\]

(11)

where the following relations hold:

\[ x^2 \ln(cy) - \sqrt{L\gamma} x \ln(xy) - \mathcal{L}\sqrt{L\gamma} = 0 \]

\[ \dot{S} = h \left( 1 + \frac{1}{2} \mathcal{L} \right) \]

The pressure in the rear part and the reaction are equal:

\[ \pi_0 = p + \frac{1}{4} xu - \frac{1}{4} \dot{S} \frac{du}{dt} \]

\[ \pi_H = p + \frac{1}{4} xu - \frac{1}{4} u^2 - \frac{1}{4} \dot{S} \frac{du}{dt}. \]

(12)

We integrate (11) in two cases. First one is a free flow condition, in which:

\[ u = \sqrt{2p} = \text{const.} \]

In this case as shown in Figure 2 the height, \( h = h_\tau \), varies. If the gap is sufficiently large then the height attains its maximal value, and vertical increasing of the formation zone stops. From (11) we can see that \( h_\tau = 2 \) as \( \tau \to \infty \). Thus, for \( H \geq 2d \) we have a free flow from the nozzle with \( u = \sqrt{2p} \). The equations (11) are numerically solved for the unknowns \( h, \mathcal{L}, y \) and parameters \( p, \beta, \gamma, c \).

Typical solutions are shown in Figure 4. These solutions are damped self-excited oscillations. The main property is that damping time increases from \( 10^{-3} \) to \( 10^{-2} \) to \( 10^{-1} \) seconds as \( \alpha \) varies. For any set of parameters \( (p, \beta, \gamma, c) \) there exists a maximal value \( \alpha \), under which the stable focus (which is a solution of (11)) breaks.

The second case where we integrate our equations is used in the production of the amorphous metallic ribbon. This is the case of the excited formation of the zone. The main condition of the case is

\[ H \leq d. \]
\[ h \leq 1 \]

\[ V_e \leq \frac{\sqrt{2p}}{h} \]

\[ \text{Figure 2. Dynamics scheme of ribbon formation (free flow)} \]

\[ \text{Figure 3. Dynamics scheme of ribbon formation (excited formation)} \]

\[ \text{Figure 4. Zone length (L), outflow speed (u) and temperature (y) on time for free flow condition.} \]

If \( h \leq 1 \) then in the beginning of the casting we have a free flow from the nozzle \( (u = \sqrt{2p}) \); \( h_t \) increases from zero to the given value \( h \). At the time \( t_2 \) (Figure 3) the process goes to the conditions of the excited formation. \( V_e \) begins to decrease, and \( h \) becomes a parameter of the equations (11). The computations show that the interval \( 0 \leq t \leq t_2 \) (see Figure 3) is far less than the time of achieving stationary condition. Hence, the equations (11) can be solved for the unknowns \( L, u, y \) with initial time \( t_2 \). Parameters of the problem are \( p, \beta, \gamma, c, h \).

Typical solutions are shown in Figure 5. The solutions are rapidly damped self-excited oscillations. The time of achieving stationary condition is of order \( 10^{-4} \div 10^{-3} \) sec. Comparison
Figure 5. Zone length ($L$), outflow speed ($u$) and temperature ($y$) on time for excited formation condition.

Figure 6. Connection between technological parameters and physical properties during certain thickness ribbon obtaining

of the solutions (Figure 4 and Figure 5) shows that for the excited formation ($h \leq 1$) we have far greater stability of the formation zone on the initial stage. However, inductive solving of (11) including initial stage $t < t_2$ (see Figure 2) of the free flow shows the possibility of interrupting the formation process. This happens when $\alpha$ is close to the critical value or the overheating is small, $c \sim 1$. 
4. **Analysis of stationary solutions of the balance equations.**

In stationary condition the equations (11) give a system of algebraic equations:

\[
\begin{align*}
  u - x &= 0 \\
  p - 2uh^{-2}\sqrt{L}\gamma &= 0 \\
  u - \beta L y - xy &= 0 \\
  x^2 \ln(cy) - \sqrt{L}\gamma x \ln(xy) - L\sqrt{\gamma} &= 0
\end{align*}
\]  

(13)

If we define

\[ f = \frac{\delta L}{\delta} \]  

(Figure 1)  

(14)

then we can substitute

\[ \gamma L = x^2 f^2 \]

into the equations (13). Then (13) takes the following form:

\[
\begin{align*}
  u &= x \\
  fx^2 &= h^2 P \\
  \frac{1-y}{y} &= \beta x f^2 \\
  f &= \frac{\ln(yc)}{1-y} + \ln(yc)
\end{align*}
\]  

(15)

The technological problem is to obtain amorphous ribbon of the given thickness \( \delta_0(x_0) \).

Substitution \( x = x_0 \) into (15) gives

\[
\begin{align*}
  u &= x_0 \\
  A &= \frac{\ln(yc)}{1-y} + \ln(yc) \\
  \frac{1-y}{y} &= BA^2
\end{align*}
\]  

(16)

Excluding unknown \( y \) from (16) gives the implicit function:

\[ A = F(B,c), \]  

(17)

where

\[ A = \frac{H^2}{\delta_0^2} \frac{\Delta P}{2\rho V_d^2}, \quad B = \frac{\alpha \delta_0}{\kappa_{mT}}, \quad c = \frac{T_0}{T_s} \]

Dependence of \( A \) on \( B \) for different \( c \) is shown in Figure 6. Function (17), in fact, gives the solution of the main problem of our model. Namely, we evaluate technological parameters \( (\Delta P,V_d,H,T_0) \) for obtaining amorphous ribbon of the given thickness \( \delta_0 \). Our melt is determined by the values of physical properties \( (\nu,\rho,\sigma,\epsilon_p,\lambda_p) \) on the cooling disc; these values secure certain heat sink \( (\alpha,\lambda_d) \).

Geometric parameters of the zone are determined as follows. The thickness of the nozzle, \( d \), is determined by technology. The curvature radius of the rear part, \( r \), is given by formulas (2), (12). The value of the gap, \( H \), is defined by (17). The length of liquid zone equals

\[ L = \frac{\delta_0 V_d}{\nu} f^2. \]  

(18)
Formula (4) gives the length of the two-phase ribbon.

Function \( f \) depending on the mean temperature of the formation zone \( y \) and the overheating \( c \) plays an important role in our model. It defines geometric proportions of the zone:

\[
\frac{\delta L}{\delta} = f, \quad \frac{z}{L} = \frac{f}{1-f}, \quad \frac{z + L}{L} = \frac{1}{1-f}
\]  

(19)

If \( f \to 1 \), then we have \( T \to T_0 \) and \( z \to \infty \). The formation zone is almost completely of a liquid phase. Momentum transfer is determined by hydrodynamics of the melt. If \( f \to 0 \), then we have \( T \to T_S \) and \( z \to 0, L \to 0 \). The formation zone is solid (solid shell of the amorphous melt of thickness \( \delta \)). In the first case, we shall observe the sprinkling of the melt. Drops will crystallize in the air turning into powder. In the second case, we can watch the flow of crystallized alloy chips.

The functional dependence (17) shown in Figure 6 is universal under the assumptions made. Therefore, regardless of the melt type or material and size of the cooling disc we can assert that there exists a formation zone of optimal size and proportions. In other words, there exists an optimal value or variation interval for the function \( f \). Since the conditions \( f \to 1 \) and \( f \to 0 \) lead to an unstable process (the formation is lacking), it is reasonable to assume that \( f \) is close to 0.5. Thus, the choice of technological parameters is based on the formula:

\[
\frac{H^2 \delta P}{\delta_0 2\rho V_d^2} = 0.5 = f
\]

(20)

This, for instance, implies (see Figure 6) that for copper discs we should use far greater overheating than for steel ones. The heat sink of copper discs is far more intensive. Hence,

\[ B_{Cu} > B_{St} \]

5. Conclusion.

This model allows one to analyze planar flow casting in details. It is possible to work out some algorithms to compute technological parameters in various problems of amorphous melts production. This model is a basis for virtual experiments with animation close to the actual state of affairs. It is important to note that none of the observed experimental facts contradicts our model. Moreover, resting on our reasonings they can be qualitatively explained and numerically calculated.

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