The pulse and monochromatic light stimulation of semiconductor quantum wells

I. G. Lang
A. F. Ioffe Physical-Technical Institute, Russian Academy of Sciences, 194021 St. Petersburg, Russia

S. T. Pavlov
P.N. Lebedev Physical Institute, Russian Academy of Sciences, 119991 Moscow, Russia; pavlov@sci.lebedev.ru

The light reflectance and absorbance are calculated for a quantum well (QW) the width of which is comparable with the light wave length. The difference of the refraction coefficients of the quantum well and barriers is taken into account. The stimulating pulse form is arbitrary. An existence of two closely situated discrete excitation energy levels is supposed. Such energy level pare may correspond to two magnetopolaron states in a quantizing magnetic field perpendicular to the QW plane. The relationship of the radiative and non-radiative damping is arbitrary. The final results does not use the approximation of the weak Coulomb interaction of electrons and holes.

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I. INTRODUCTION.

At the light monochromatic and pulse irradiation of a quantum well there appear the characteristic peculiarities in the reflected and transmitted light waves carrying an information about the energy spectrum and lifetimes of the electron excitations [1-4]. The most interesting results are being obtained in the case of the discrete energy levels of the electronic system. Such situation is being realized in a quantizing magnetic field perpendicular to the QW plane or in the case of the excitonic states.

We are interested in the case of two closely located energy levels since at a pulse irradiation there appears an effect of the oscillations of the intensities of the reflected and transmitted light on the frequencies close to the difference of the energy levels [5]. Such situation appears, in particular, in the case of the magneto-phonon resonance [6], under condition

$$\omega_{LO} = j\omega_{e(h)H},$$

(1)

where $\omega_{LO}$ is the longitudinal optical (LO) phonon frequency, $e$ is the electron charge, $\omega_{e(h)H} = |e|H/|me_{e(h)}|$ is the cyclotron frequency, $me_{e(h)}$ is the electron (hole) effective mass, $H$ is the magnetic field. The number $j$ may be integer, what corresponds to a "classical" magnetopolaron, or fractional number (weak magnetopolaron) [7, 8].

For the high quality quantum wells the radiative damping $\gamma_r$ of the light absorption line may be comparable with the non-radiative damping $\gamma$ or exceed it. In such situation it is necessary to take into account all the orders on the electron - light interaction [5, 7, 9-26]. Our previous results (besides [20, 21, 23 - 26]) are true for comparatively narrow QWs, when it is satisfied the condition

$$kd << 1,$$

(2)

where $k$ is the module of the light wave vector, $d$ is the QW’s width. The calculations show that only the reflection and absorption peak positions depend on the QW’s width, but not their height and shape. In all the results, corresponding to the condition (2), the difference of the refraction indexes of the QW and barrier’s was not taken into account. We are going to show that for the narrow QWs the refraction index $\nu$ of the QW’s material is omitted from all the final results. There remains only the refraction index $\nu_1$ of the barrier’s material.

However, in the case of the wide enough QWs when

$$kd \geq 1,$$

(3)

the difference of $\nu$ and $\nu_1$ must be taken into account, what follows from the results of [20, 21, 23 - 26]. Let us remind that the QW’s width $d$ is limited from above by the demand of the size quantization of the electron movement along the axes $z$.

Let us estimate the role of the Coulomb interaction of electrons and holes. A separation of the variables $r_\perp$ and $z$ in calculations of the wave functions is possible, if the Coulomb interaction weakly influences the particle movement in $xy$ plane. It happens if

$$a_{exc}^2 >> a_H^2,$$

(4)
where \( a_{exc} = \hbar^2 \epsilon_0 / \mu e^2 \) is the Wannier-Mott exciton radius in the magnetic field absence, \( \epsilon_0 \) is the static permeability, \( \mu \) is the adjusted effective mass, \( a_H = c \hbar / |e| H \) is the magnetic length. Using the parameters of GaAs from [27], we obtain (in Å)

\[
a_{exc} = 146, \quad a_{H}^{\text{res}} = 57.2, \tag{5}
\]

where \( a_{H}^{\text{res}} \) corresponds to the resonant magnetic field \( H_{exc} = 20.2 \text{T} \), satisfying the condition (1) of the magnetopolaron resonance for an electron at \( j = 1 \). According to (5), we obtain \( (a_{H}^{\text{res}} / a_{exc})^2 \approx 0.154 \), i.e., the condition (4) is satisfied.

The influence of the Coulomb interaction on the movement of the electrons and holes in the \( xy \) plane had been considered in [28]. The Coulomb interaction influence on the particle movement along the \( z \) axes may be neglected, if

\[
a_{exc} > d. \tag{6}
\]

Let us check the compatibility of the conditions (6) and (3). For GaAs, for example, the energy gap is \( \hbar \omega_g \approx 1.52 \text{eV} \), the stimulating light frequency must be a little more of that value. The module \( k = \omega_0 \nu / c \) of the light wave vector equals \( k = 2.60 \times 10^5 \text{cm}^{-1} \). Using (5) and (6), we obtain

\[
kd < 0.38.
\]

In order to extend our theory on the case of the more wide QWs, when the Coulomb forces influence the movement of electrons and holes along the \( z \) axes, we do not concretize the exciton envelope wave function \( \phi(z) \) until obtaining the final results.

II. THE STIMULATING FIELD.

The permeability inside of the QW is \( \varepsilon = \nu^2 \), in barriers \( \varepsilon_1 = \nu_1^2 \) (Fig.1), where \( \nu, \nu_1 \) are the refraction coefficients. The electromagnetic wave propagates along the \( z \) axes perpendicular to the QW’s surface (the \( xy \) plane). Its electric field is as follows

\[
E_0(r, t) = E_0 e^{i} \int_{-\infty}^{\infty} d\omega e^{-i\omega(t-z\nu/c)} D_0(\omega) + c.c., \tag{7}
\]

where \( E_0 \) is the scalar amplitude, \( \epsilon_i \) is the polarization vector, \( c \) is the light speed in vacuum. The function \( D_0(\omega) \) determines the shape of the stimulating pulse and may be chosen as [5,17-19,21]

\[
D_0(\omega) = \frac{i}{2\pi} \left[ \frac{1}{\omega - \omega_i + i\gamma_1/2} - \frac{1}{\omega - \omega_i - i\gamma_2/2} \right].
\]

Under condition \( \gamma_1 = \gamma_2 = \gamma \) the light pulse is symmetrical [5,18,19,21], at \( \gamma_2 \to \infty \) it is asymmetrical and has a very steep front [16,17,19]. At \( \gamma \to 0 \) we obtain

\[
D_0(\omega) = \delta(\omega - \omega_i),
\]

what corresponds to the monochromatic irradiation.

III. THE ELECTRONIC EXCITATIONS IN A QW.

A QW is in a zero or quantizing magnetic field perpendicular to the QW’s surface at a zero temperature. The quasi-momentum matrix elements \( p_{cv} \), characterizing the inter-band transitions of an electron, are essential in the theory. As earlier [5,7,16-22] we use the following model. The vectors \( p_{cv} \) for two sorts of excitons (with indexes I and II) are as follow

\[
p_{cvI} = p_{cv}(e_x - ie_y)/\sqrt{2}, \quad p_{cvII} = p_{cv}(e_x + ie_y)/\sqrt{2}, \tag{8}
\]

where \( e_x \) and \( e_y \) are the unite vectors along the axis \( x \) and \( y \), respectively, \( p_{cv} \) is a real value. The model corresponds to the excitons with a participation of the heavy holes in the semiconductors with the zinc blend structure if the axes \( z \) is directed along the 4-th order axes [29,30]. Our results are applicable also to the excitons \( \Gamma_6 \times \Gamma_7 \) with a
participation of the holes from the valence band splitted by the spin-orbital interaction at an arbitrary direction of the \( z \) axes relatively the crystallographic axis \([31]\). If the circular polarization vectors of the stimulating light

\[
e_l = (e_x \pm ie_y)/\sqrt{2},
\]

are used, the conservation property of the polarization vector is performed:

\[
\sum_{I,II} p^*_c(e_l p_{cv}) = \sum_{I,II} p_{cv}(e_l p^*_c) = e_l p^2_{cv}.
\]

The form of the wave function \( F_\rho(r) \) of the electronic excitation at \( r_e = r_h = r \) in the effective mass approximation is essential in the theory. \( r_e(r_h) \) is the electron (hole) radius vector \([22]\). If an excitation is formed by the pair magnetopolaron - hole, the wave functions contain the phonon components \([8,32]\). In such a case the value \( F_\rho(r) \) is determined as follows: we multiply the function from the left at a phonon vacuum \( \langle 0 | \) and assume \( r_e = r_h = r \). Such a method is justified by the fact that at an exciton creation by light or at a light annihilation of an exciton with a participation of a magnetopolaron, it is essential only the term in the polaron wave function which does not contain the phonons.

Let us assume that the variables \( r_\perp \) and \( z \) may be separated, i.e.,

\[
F_\rho(r) = Q_\pi(r_\perp)\phi_\chi(z).
\] (9)

The separation of variables is possible, if the Coulomb interaction of electrons and holes influences weakly at their motion in the \( xy \) plain.

**IV. THE INDUCED CURRENT DENSITY.**

In order to calculate the electric fields on the left and on the right of the QW and the field inside of the QW, it is necessary, first of all, to calculate the density of the electric current induced inside of the QW. If we use (8), we obtain the average current density inside of the QW \([22]\)

\[
j_{1\alpha} = \frac{ie}{8\pi^2} \sum_\rho \gamma^0_{r\pi}(z) \int_{-\infty}^\infty d\omega e^{-i\omega t} \int_{-d/2}^{d/2} dz' \phi_\chi(z') E_\alpha(z', \omega) \left( \frac{1}{\omega - \omega_\rho + i\gamma_\rho/2} + \frac{1}{\omega + \omega_\rho + i\gamma_\rho/2} \right),
\] (10)

where \( \rho \) is the index of the electronic excitation in the QW (combining the indexes \( \pi \) and \( \chi \)), \( \hbar\omega_\rho \) is the excitation energy counted from the ground state energy, \( \gamma_\rho \) is the non-radiative damping.

The value \( \gamma^0_{r\pi} \) is a factor in the radiative damping expression. The radiative damping is determined as \([22,23]\)

\[
\tilde{\gamma}_{rp} = \gamma_{r\pi} |R_\chi(\omega_\rho \nu/c)|^2,
\] (11)

where

\[
\gamma_{r\pi} = \gamma^0_{r\pi}/\nu,
\] (12)

\[
R_\chi(k) = \int_{-d/2}^{d/2} dz \phi_\chi(z)e^{-ikz}.
\] (13)

The values \( \gamma_{r\pi} \) for an exciton consisting of a magnetopolaron and hole were calculated in \([7]\). For some another excitations they are given in \([22]\).

Equation (10) may be obtained from (46) from \([22]\), if one takes into account (12) and restricts the integration on \( z' \) by the limits \(-d/2\) and \( d/2 \), i.e., neglects by the tunnel penetration of the electronic excitations into the barrier (what, strictly speaking, corresponds to the infinitely deep QW).
V. THE EQUATION FOR THE ELECTRIC FIELD INSIDE OF THE QW.

Let us use the relationship

$$E(z, t) = -\frac{1}{c} \frac{\partial A(z, t)}{\partial t}, \quad \varphi = 0,$$

where $A(z, t)$ and $\varphi$ are the vector and scalar potentials, respectively. In the case of the model (8) the current (10) is transverse one. The $z$-component is absent, the induced charge density equals zero, hence it follows the condition $\varphi = 0$.

We start from the equation for the vector potential (see, for instance, [33, p. 439])

$$\frac{\partial^2 A}{\partial z^2} - \frac{\varepsilon}{c^2} \frac{\partial^2 A}{\partial t^2} = \frac{4\pi}{c} j(z, t) \tag{14}$$

Let us write the electric field as it follows

$$E(z, t) = \frac{e}{2\pi} \int_{-\infty}^{\infty} d\omega E(-i\omega) \mathcal{E}(z, \omega) + c.c. \tag{15}$$

Then with the help (10), (14) and (15) we obtain

$$\frac{\partial^2 \mathcal{E}(z, \omega)}{\partial z^2} + k^2 \mathcal{E}(z, \omega) = 2k_0 \phi(z) M(\omega) B(\omega), \tag{16}$$

VI. THE CASE OF TWO CLOSELY LOCATED ENERGY LEVELS. THE ELECTRIC FIELD INSIDE OF THE QW.

Let us limit the sum in (16) by two terms with numbers: $i = 1$ and $i = 2$. It is possible if the energy levels 1 and 2 are closely located and others levels are situated at $\Delta \omega$ far away, thus

$$\gamma_{1(2)} << |\Delta \omega|, \quad \gamma_{r1(2)} << |\Delta \omega|.$$  

Let us consider a case when the functions $\phi_i(z)$ for two excitations coincide, i.e.,

$$\phi_1(z) = \phi_2(z) = \phi(z). \tag{17}$$

For instance, such situation is realized for two closely located energy levels of an exciton in the magnetopolaron effect [12-14]. Under condition (17) equation (16) results in

$$\frac{\partial^2 \mathcal{E}(z, \omega)}{\partial z^2} + k^2 \mathcal{E}(z, \omega) = 2k_0 \phi(z) M(\omega) B(\omega), \tag{18}$$

where the designations

$$k = \frac{\omega \nu}{c}, \quad k_0 = \frac{\omega}{c}, \quad M(\omega) = \int_{-d/2}^{d/2} dz' \phi(z) \mathcal{E}(z, \omega),$$

$$B(\omega) = \sum_{i=1,2} \frac{\gamma_{i1}^0}{2} \left( \frac{1}{\omega - \omega_1 + i\gamma_i/2} + \frac{1}{\omega + \omega_1 + i\gamma_i/2} \right)$$

are introduced. According to [34, p. 85], the solution of the equation

$$\frac{\partial^2 y}{\partial z^2} + k^2 y = f(z)$$

is as follows

$$y = C_1 \cos kz + C_2 \sin kz + \frac{1}{k} \int_{z_0}^{z} dz' f(z') \sin k(z - z'). \tag{20}$$
Using (20), we obtain the Fourier transform of the electric field inside of the QW

$$\mathcal{E}(z, \omega) = Ae^{ikz} + Be^{-ikz} - \frac{i}{\nu} \mathcal{F}_k(z) M(\omega) B(\omega),$$  \hspace{1cm} (21)

where the designation

$$\mathcal{F}_k(z) = e^{ikz} \int_{-d/2}^{d/2} dz' \phi(z')e^{-ikz'} + e^{-ikz} \int_{d/2}^{d/2} dz' \phi(z')e^{ikz'}$$  \hspace{1cm} (22)

is introduced. Let us multiply (21) at $\Phi(z)$ and integrate from $-d/2$ to $d/2$. We obtain the equation for the value $M(\omega)$, the solution of which is as follows

$$M(\omega) = \frac{AR^*(k) + BR(k)}{1 + (i/\nu)B(\omega)J(k)},$$  \hspace{1cm} (23)

where

$$J(k) = \int_{-d/2}^{d/2} dz \Phi(z) \mathcal{F}_k(z),$$

$R(k)$ is determined in (13). Finally, having substituted (23) into (21), we obtain that inside of the QW

$$\mathcal{E}(z, \omega) = Ae^{ikz} + Be^{-ikz} - \frac{i}{\nu} \mathcal{F}_k(z) B(\omega) \frac{AR^*(k) + BR(k)}{1 + (i/\nu)B(\omega)J(k)}.$$  \hspace{1cm} (24)

where $A$ and $B$ are the constants which must be determined, using the boundary conditions on the edges of the QW.

**VII. THE ELECTRIC FIELD ON THE LEFT, ON THE RIGHT AND INSIDE OF THE QW.**

It is obviously that the Fourier components

$$\mathcal{E}_{left}(z, \omega) = \mathcal{E}_0(z, \omega) + \Delta \mathcal{E}_{left}(z, \omega), \quad \Delta \mathcal{E}_{left}(z, \omega) = Le^{-ik_1z}, \quad \mathcal{E}_{right}(z, \omega) = Re^{ik_1z}$$  \hspace{1cm} (25)

, correspond to the electric field outside of the QW. Here $k_1 = \omega n/c$, $L$ and $R$ are the constants,

$$\mathcal{E}_0(z, \omega) = 2\pi E_0 D_0(\omega)e^{ik_1z}.$$  \hspace{1cm} (26)

At $z = -d/2$ and $z = d/2$ the magnetic field and the tangential component of the electric field must be continuous. Since in the case of the model (8) the longitudinal components of the fields (along the $z$ axes) are absent, the boundary conditions may be written as four equations

$$\begin{align*}
\mathcal{E}_{left}(-d/2, \omega) &= \mathcal{E}(d/2, \omega), \\
\mathcal{E}_{right}(d/2, \omega) &= \mathcal{E}(d/2, \omega), \\
\frac{d\mathcal{E}_{left}(z, \omega)}{dz} \bigg|_{z=-d/2} &= \frac{d\mathcal{E}(z, \omega)}{dz} \bigg|_{z=-d/2}, \\
\frac{d\mathcal{E}_{right}(z, \omega)}{dz} \bigg|_{z=d/2} &= \frac{d\mathcal{E}(z, \omega)}{dz} \bigg|_{z=d/2}.
\end{align*}$$  \hspace{1cm} (27)

Having substituted (24) and (25) in (27), we solve the system of four equation relatively of the constants $A$, $B$, $L$, $R$. As a result we obtain the final expressions for the Fourier transforms of the electric fields

$$\mathcal{E}_{left}(z, \omega) = 2\pi E_0 D_0(\omega)e^{ik_1z} + 2\pi E_0 D_0 e^{-ik_1z} \left( 1 - e^{-i2kd}(\zeta^2 - 1) \right) - ig(\omega) \left[ e^{-ikd} \left( (\zeta - 1)^2 R^2(k) + (\zeta + 1)^2 R^2(k) \right) + 2(\zeta^2 - 1)|R(k)|^2 \right],$$  \hspace{1cm} (28)

$$\mathcal{E}_{right}(z, \omega) = 2\pi E_0 D_0(\omega)e^{ik_1z} \left( 1 - e^{-i(k+k_1)d}\zeta \right) \left[ \left( \frac{1 - i\omega}{|R(k)|^2} \right) \left( 1 - e^{-i2kd}(\zeta^2 - 1) \right) \right].$$  \hspace{1cm} (29)
Using the definition (31) of the function \( \gamma_d \) from the plane of the transparent dielectric plates, if the function \( \gamma_d \) may be used for an investigation of the transmission and reflection of light pulses following results for the Fourier transforms of the electric fields (compare with formulas from [35, p. 412])

We introduced the following designations

\[
\zeta = \nu / \nu_1, \quad g(\omega) = \sum_{i=1,2} \frac{(\gamma_{ri}^0 / 2\nu) L_i(\omega)}{1 + iJ(k) \sum_{i=1,2} (\gamma_{ri}^0 / 2\nu) L_i(\omega)},
\]

\[
L_i(\omega) = \frac{1}{\omega - \omega_i + i\gamma_i/2} + \frac{1}{\omega + \omega_i + i\gamma_i/2},
\]

\[
Z = e^{-2ikd}(\zeta + 1)^2 - (\zeta - 1)^2 + ig(\omega) \left\{ e^{-ikd}(\zeta^2 - 1)[R^2(k) + R^*2(k)] + 2(\zeta - 1)^2|R(k)|^2 \right\}.
\]

**VIII. THE LIMIT OF THE ABSENCE OF ELECTRONIC EXCITATIONS.**

Assuming

\[
\gamma_{r1}^0 = \gamma_{r2}^0 = 0,
\]

we move up to the situation, when light normally incident on a surface of a plane-parallel plate is reflected due to the difference of the refraction coefficients of the plate and medium. Let us stress that in such a case there is no any limitations on the plate depth \( d \) relatively of the light wave length. With the help of (28) - (30) we obtain the following results for the Fourier transforms of the electric fields (compare with formulas from [35, p. 412])

\[
\mathcal{E}_{left}^0(z,\omega) = 2\pi E_0 D_0(\omega) e^{ikz} + 2\pi E_0 D_0(\omega) e^{-ikz} Z_0^{-1} e^{ikd(1 - e^{-2ikd})(\zeta^2 - 1)},
\]

\[
\mathcal{E}_{right}^0(z,\omega) = 8\pi E_0 D_0(\omega) e^{ikz} Z_0^{-1} e^{-i(k + k_1)d},
\]

\[
\mathcal{E}^0(z,\omega) = 4\pi E_0 D_0(\omega) e^{-i(k + k_1)d/2} Z_0^{-1} \left[ e^{ikz} e^{-ikd}(\zeta + 1) + e^{-ikz}(\zeta - 1) \right],
\]

where

\[
Z_0 = e^{-2ikd}(\zeta + 1)^2 - (\zeta - 1)^2.
\]

Let us note that equations (34) - (36) may be used for an investigation of the transmission and reflection of light pulses from the plane of the transparent dielectric plates, if the function \( D_0(\omega) \) describes the stimulating pulse continuation and shape.

**IX. THE LIMIT OF EQUAL REFRACTION INDEXES OF THE BARRIERS AND QW.**

In the case of \( \nu = \nu_1 \) from (28) - (30) we obtain the results

\[
\mathcal{E}_{left}^{\nu_1}(z,\omega) = 2\pi E_0 D_0(\omega) e^{ikz} - 2\pi E_0 D_0(\omega) e^{-ikz} g(\omega) R^2(k),
\]

\[
\mathcal{E}_{right}^{\nu_1}(z,\omega) = 2\pi E_0 D_0(\omega) e^{ikz} - 2\pi E_0 D_0(\omega) e^{ikz} g(\omega) |R(k)|^2,
\]

\[
\mathcal{E}^{\nu_1}(z,\omega) = 2\pi E_0 D_0(\omega) \left[ e^{ikz} - iF_k(z) g(\omega) R^*(k) \right],
\]

Using the definition (31) of the function \( g(\omega) \), as well as the relationship

\[
J(k) = |R(k)|^2 + iq(k), \quad q(k = 0) = 0,
\]
we obtain from (37) and (38)

$$\mathcal{E}_{\text{left(right)}}(z, \omega) = \mathcal{E}_0(z, \omega) + \Delta \mathcal{E}_{\text{left(right)}}(z, \omega),$$

(40)

$$\Delta \mathcal{E}_{\nu=\nu_1}(z, \omega) = 2\pi E_0 \tilde{\mathcal{D}}_\nu(\omega) e^{-i(kz-\alpha)},$$

(41)

$$\Delta \mathcal{E}_{\nu=\nu_1}(z, \omega) = 2\pi E_0 \tilde{\mathcal{D}}_\nu(\omega) e^{ikz},$$

(42)

where

$$\tilde{\mathcal{D}}_\nu(\omega) = -iD_0(\omega) \frac{\tilde{\gamma}_{r1}/2 + (\tilde{\gamma}_{r2}/2) L_2(\omega)}{1 + i((\tilde{\gamma}_{r1}/2)L_1(\omega) + (\tilde{\gamma}_{r2}/2)L_2(\omega)) - \Delta_1 L_1(\omega) - \Delta_2 L_2(\omega)},$$

(43)

$$\tilde{\gamma}_{ri} = \frac{\gamma_0}{\nu} |R(k)|^2,$$

(44)

what coincide with the definition (11) in the approximation \( \omega = \omega_i \).

Let us note that in equation (32) for \( L_i(\omega) \) one has to neglect the second term (in order to avoid an exceeding precision), what justifies the approximation \( \omega \simeq \omega_i \).

In order to make clear the physical sense of the values \( \tilde{\gamma}_{r1} \) and \( \Delta_1 \), let us go to the case of the single energy level supposing \( \gamma_{r2}^0 = 0 \). Neglecting by the non-radiative term from \( L_1(\omega) \), we obtain

$$\tilde{\mathcal{D}}_\nu(\omega) = -iD_0(\omega) \frac{\tilde{\gamma}_{r1}}{\omega - (\omega_1 + \Delta_1 + i(\gamma_1 + \tilde{\gamma}_{r1})/2)},$$

(46)

whence it follows that \( \tilde{\gamma}_{r1} \) is the radiative lifetime, \( \Delta_1 \) is the correction to the excitation energy. However, if two energy levels are located closely enough, they influence each other. Equation (43) may be transformed into

$$\tilde{\mathcal{D}}_\nu(\omega) = -iD_0(\omega) \left( \frac{\tilde{\gamma}_{r1}/2}{\omega - \Omega_1 + iG_1/2} + \frac{\tilde{\gamma}_{r2}/2}{\omega - \Omega_2 + iG_2/2} \right),$$

(47)

where \( \tilde{\gamma}_{r1}, \Omega_i \) and \( G_i \) are the ”renormalized” values.

In the case of QWs and under condition \( kd << 1 \) we have

$$R(k) \simeq \int_{-d/2}^{d/2} dz \phi(z) = C, \quad \mathcal{F}_k(z) \simeq C, \quad J(k) \simeq C^2,$$

$$q(k) \simeq 0, \quad \Delta_1 \simeq \Delta_2 \simeq 0, \quad \tilde{\gamma}_{r1} \simeq \frac{\gamma_0}{\nu} C^2, \quad e^{i\alpha} = 1.$$  

(48)

Under condition \( \nu = \nu_1 \) we obtain from (43) for narrow QWs

$$\mathcal{D}_{\nu_1}(\omega) = -iD_0(\omega) \frac{C^2 \sum_{i=1,2} (\gamma_{ri}^0/2\nu) L_i}{1 + iC^2 \sum_{i=1,2} (\gamma_{ri}^0/2\nu) L_i},$$

(49)

where the index \( n \) corresponds to a narrow QW.
X. THE NARROW QW. THE DIFFERENT REFRACTION COEFFICIENTS IN THE BARRIERS AND QW.

Let us apply (48) and assume
\[ e^{-ikd} \simeq e^{-ik_1d} \simeq 1. \]
Then we obtain from (28) - (29)
\[
E_{n, left}(z, \omega) = 2\pi E_0 D_0(\omega/)e^{ik_1z} - 2\pi i E_0 D_0(\omega) e^{-ik_1z} \frac{g_n(\omega)C^2}{1 + ig_n(\omega)C^2(\zeta - 1)}, \tag{50}
\]
\[
E_{n, right}(z, \omega) = 2\pi E_0 D_0(\omega) \frac{1 - ig_n(\omega)C^2\zeta}{1 + ig_n(\omega)C^2(\zeta - 1)} = \text{const}, \tag{51}
\]
\[
E_n(z, \omega) = 2\pi E_0 D_0(\omega) e^{ik_1z} \frac{1 - ig_n(\omega)C^2}{1 + ig_n(\omega)C^2(\zeta - 1)}. \tag{52}
\]
where
\[
g_n(\omega) = \sum_{i=1,2} \frac{\gamma^0_{ri}/2\nu} {1 + iC^2 \sum_{i=1,2} \gamma^0_{ri}/2\nu} L_i \tag{53}
\]
Having substituted (53) into (50) -(52) and substituted the elementary transformations, we obtain the fields on the left and on the right of the QW (40), where
\[
\Delta E_{left n}(z, \omega) = 2\pi E_0 \tilde{D}_{nu_1}(\omega) e^{-ik_1z}, \tag{54}
\]
\[
\Delta E_{right n}(z, \omega) = 2\pi E_0 \tilde{D}_{nu_1}(\omega) e^{ik_1z}, \tag{55}
\]
and the field inside of the QW
\[
E_n(z, \omega) = E_0(z = 0, \omega) + 2\pi E_0 \tilde{D}_{nu_1}(\omega). \tag{56}
\]
where
\[
\tilde{D}_{nu_1}(\omega) = -i D_0(\omega) C^2 \sum_{i=1,2} \frac{\gamma^0_{ri}/2\nu_1} {1 + iC^2 \sum_{i=1,2} \gamma^0_{ri}/2\nu_1} L_i \tag{57}
\]
what distinguishes from the result (49) for narrow QWs at \( \nu = \nu_1 \) by the substitution of \( \nu \) by \( \nu_1 \). It means that at \( \nu \neq \nu_1 \) for narrow QWs \( kd << 1 \) the radiative damping contains the barrier refraction coefficient \( \nu_1 \), and the QW refraction coefficient \( \nu \) does not appear at all! The physical sense of the result is quite clear: Under condition \( kd << 1 \) we can go to the limit \( d \rightarrow 0 \), when the QW’s material is absent, but there exists the induced current corresponding to the exciton creation transitions.

Thus, it is proved that our results for the narrow QWs for the monochromatic and pulse irradiation [5,7,16-19] are true not only at \( \nu = \nu_1 \), but also at \( \nu \neq \nu_1 \), since our formulas contain only the barrier refraction coefficient.

XI. THE REFLECTANCE, TRANSMITTANCE AND ABSORBANCE AT A PULSE IRRADIATION.

The energy flux \( S(p) \), where \( p = t - z\nu_1/c \), corresponding to the electric field of the stimulating pulse, is as follows
\[
S(p) = e \frac{c\nu_1}{4\pi} E_0^2(z, t) = e S_0 P(p), \quad S_0 = \frac{c\nu_1}{2\pi} E_0^2. \tag{58}
\]
The transmitted flux on the right of the QW is
\[
S_{right}(p) = e \frac{c\nu_1}{4\pi} E_{right}^2(z, t) = e S_0 T(p). \tag{59}
\]
For the reflected flux (on the left of the QW) we obtain

\[ S_{\text{left}}(s) = -e \frac{c \nu_1}{4\pi} (\Delta E_{\text{left}}(z,t))^2 = -e_z S_0 R(s) \]  \hspace{1cm} (60)

where \( s = t + z \nu_1/c \). The dimensionless functions \( T(p) \) and \( R(s) \) determine the rates of the transmitted and reflected energy of the stimulating pulse.

The time-dependence of the absorption is determined by the dimensionless coefficient

\[ A(t) = P(t) - T(t) - R(t), \]  \hspace{1cm} (61)

if a time \( t \) of the pulse transition through the QW is small in comparison to the pulse continuation \( \Delta t \) (or if the pulse length along the axes \( z \) is much more in comparison to the QW’s width \( d \)). Let us check that the criterium is fulfilled for the wide enough QWs and pulses, duration of which is comparable with the value \( \bar{\hbar}/\Delta \omega \), where \( \Delta \omega \) is the distance between two polaron energy levels. Let us take, according to [5], \( \Delta \omega = 0.0065 \text{eV} \) (the corresponding pulse duration is \( \Delta t = \bar{\hbar}/\Delta \omega = 0.1 \text{ps} \)). The pulse time transition through the QW with the width \( d = 300 \text{Å} \), \( t \simeq d \nu_1 = 10^{-4} \nu_ \text{ps} \), i.e., the condition \( t \ll \Delta t \) is fulfilled, and the definition (60) is applicable.

Let us note that at a pulse irradiation the integral absorption (integrated on time from \( t = -\infty \) to \( t = \infty \)) equals 0, if the non-radiative damping \( \gamma = 0 \). For the monochromatic irradiation the coefficients \( T, R \) and \( A \) are constants, and \( A = 0 \) at \( \gamma = 0 \).

In [20,21,23-26] the light reflection and absorption are considered for comparatively wide QWs \((kd \geq 1)\). The results (the formulas and graphics) were obtained with the help of the "envelope" functions \( \phi_\chi(z) \) without taking into account the Coulomb interaction of electrons and holes. In the present work the calculations are performed without concretization the "envelope" functions. The difference of the refraction coefficients of a QW \( \nu \) and barriers \( \nu_1 \) is taken into account.

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1. H. Stolz. Time Resolved Light Scattering from Exitons. Springer Tracts In Modern Physics. Springer, Berlin (1994).
2. J. Shah. Ultrafast spectroscopy of semiconductors and semiconductor nanostructures. Berlin (1996).
3. H. Han, S.W. Koch. Quantum theory of the optical and electronic properties of semiconductors. World Scientific (1993).
4. E.Vorobiev, E.Ivchenko, D.A.Firsov, V.I.Shalygin. Optical Properties of Nanostructures. Nauka, St. Petersburg (2001).
5. D.A.Contreras-Solorio, S.T.Pavlov, L. I. Korovin, I.G.Lang. Phys. Rev. B., 62, 16815 (2000); cond-mat/0002229.
6. E. J. Johnson, D. M. Larsen, Phys. Rev. Letters 16, 655 (1966).
7. I.G.Lang, L.I.Korovin, D.A. Contreras-Solorio, S.T.Pavlov. Fiz.Tverdogo Tela, 44, 1681 (2002); cond-mat/0002229.
8. I.G.Lang, L.I.Korovin, S.T.Pavlov. Fiz.Tverdogo Tela, 47, 1704 (2005); cond-mat/0411692.
9. L.C. Andreani, F. Tassone, F. Bassani. Solid State Commun., 77, 641 (1991).
10. E.Ivchenko. Fiz.Tverdogo Tela, 33, 2388 (1991) [Physics of the Solid State (St. Petersburg), 33, 1344 (1991)].
11. E.Ivchenko, A.V.Kavokin. Fiz.Tverdogo Tela, 34, N 6, 1815-1822 (1992). Sov. Phys. Solid State 34, 968 (1992).
12. L.C. Andreani. Confined Electrons and Photons. Ed. by E. Burstein and C. Weisbuch, Plenum Press, N.Y., 1995.
13. F. Tassone, F. Bassani, L. C. Andreani. Phys. Rev. B45, N 11, 6025 (1992).
14. L.C. Andreani, G. Pansarini, A.V. Kavokin, M.R.Vladimirova. Phys. Rev. B 57, 4670 (1998).
15. I. G. Lang, V. I. Belitsky, M. Cardona. Phys. Stat. Sol. (A), 164, 307 (1997).
16. I. G. Lang, V. I. Belitsky. Physics Letters A 245, N 3-4, 329 (1998).
17. I. G. Lang, V. I. Belitsky. Solid State Commun. 107, 10, 577 (1998).
18. I.G. Lang, L.I. Korovin, D.A. Contreras-Solorio, S.T. Pavlov. Fiz.Tverdogo Tela, 42, 2230 (2000); cond-mat/0006364.
19. I.G. Lang, L.I. Korovin, D.A. Contreras-Solorio, S.T. Pavlov. Fiz.Tverdogo Tela, 43, 1117 (2001); cond-mat/0004178.
20. I.G. Lang, L.I. Korovin, D.A. Contreras-Solorio, S.T. Pavlov. Fiz.Tverdogo Tela, 43, N 11, 2091 (2001); cond-mat/0104262.
21. I.G. Lang, L.I. Korovin, D.A. Contreras-Solorio, S.T. Pavlov. Fiz.Tverdogo Tela, 44, 1681 (2002); cond-mat/0203390.
22. I.G. Lang, L.I. Korovin, S.T. Pavlov. Fiz.Tverdogo Tela, 46, 1706 (2004); cond-mat/0311180.
23. I.G. Lang, L.I. Korovin, S.T. Pavlov. Fiz.Tverdogo Tela, 48, 1693 (2006) [Physics of the Solid State (St. Petersburg), 2006, 48, 1795-1806]; cond-mat/0403519.
24. I.G. Lang, L.I. Korovin, S.T. Pavlov. Fiz.Tverdogo Tela, 48, 2208 (2006) [Physics of the Solid State (St. Petersburg), 2006, 48, 2337]; cond-mat/0605650.
25. I.G. Lang, L.I. Korovin, S.T. Pavlov. Fiz.Tverdogo Tela, 49, 1893 (2007); cond-mat/0704169.
26. I.G. Lang, L.I. Korovin, S.T. Pavlov. Fiz.Tverdogo Tela, 48, 2208 (2006) [Physics of the Solid State (St. Petersburg), 50, 328 (2008); cond-mat/0605650.
27. A. Garsia-Cristobal, A. Cantarero, C. Trallero-Giner, Phys. Rev. B 49, 13430 (1994).
28. I.V. Lerner, Yu. E. Lozovik. Zh. Eksp. i Teor. Fiz., 78, N 3, 1167 (1980) [Sov. Phys. JETP 51, 588 (1980)].
29. J.M. Luttinger, W. Kohn. Phys. Rev. 97, 869 (1955).
30. I.M. Tsidilkovsky. The Semiconductor Band Structure. Nauka, Moscow (1978).
31. I.G. Lang, L.I. Korovin, S.T. Pavlov. Z. Phys. B 133, 1169 (2008), V.; cond-mat/0703051.
32. I. G. Lang, V. I. Belitsky, A. Cantarero, L. I. Korovin, S. T. Pavlov, M. Cardona, Phys. Rev. B, 54, N 24, 17768 (1996).
33. E. Tamm. Foundations of the Electricity Theory. Moscow, 1966.
34. V.I. Smirnov. ... The higher mathematics, v. II, Moscow, p. 85.
35. L.D. Landau, E.M. Lifshitz. The condensed matter electrodynamics. Moscow, 1982, p. 412.
36. The partition of the RHS of (15) at the main and conjugated contributions we proceed so in order to obtain equation (26) for the exciting field.
37. In [26] Figs. 3a and 5 correspond to the case of narrow QWs. The dependence of results from $\zeta = \nu / \nu_1$ is determined only by there dependent from the coefficient $\nu_1$ at fixed $\nu$. 