Twist-3 Distribute Amplitude of the Pion in QCD Sum Rules

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We apply the background field method to calculate the moments of the pion two-particles twist-3 distribution amplitude (DA) $\phi_p(\xi)$ in QCD sum rules. In this paper, we do not use the equation of motion for the quarks inside the pion since they are not on shell and introduce a new parameter $m_0^p$ to be determined. We get the parameter $m_0^p \approx 1.30\text{GeV}$ in this approach. If assuming the expansion of $\phi_p(\xi)$ in the series in Gegenbauer polynomials $C_n^{1/2}(\xi)$, one can obtain its approximate expression which can be determined by its first few moments.

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I. INTRODUCTION

The perturbative QCD theory has been applied extensively to inclusive and exclusive processes during the past decades. The structure functions in inclusive processes, $F_i(x, Q^2)$, can be solved by matching the theoretical predictions with the experimental values at different $Q^2$ regions. Unlike the structure functions, the distributive amplitude in exclusive processes, $\phi_i(x, Q^2)$, can not easy to be measured directly from experiments. The distribution amplitudes in hadrons are the key ingredients and provide the universal non-perturbative inputs for many exclusive processes. Although distribution amplitudes satisfy the renormalization group equation or the QCD evolution equation at short distances \cite{1}, their solutions depend on the initial conditions $\phi_i(x, Q^2_0)$, which are determined by the non-perturbative theory.

In Ref. \cite{2}, QCD sum rules were used to study the leading-twist distribution amplitude of the pion at the first time. They worked out the first few moments of the twist-2 distribution amplitude of the pion and used them to restrict its shape. After that, much work have been done to study the leading twist distribution amplitude of pion and their conclusion is that the experimental data favors the asymptotic $\phi_{as}(\xi) = \frac{3}{2}(1 - \xi^2)$ which is the solution of the QCD evolution equation as
$Q^2 \to \infty$.

In the past twenty years, the studies on leading-twist distribution amplitude of the pion also had been done by many papers in Ref. [1, 2, 3, 4]. From the naive point of view, the contribution of higher twist distribution amplitudes is suppressed by a factor $1/Q^2$ in large momentum transfer processes. In many cases, we can consider solely the contribution from leading-twist distribution amplitude, but not involve the impact of higher-twist distribution amplitudes. However the leading twist contribution to explain the data for the exclusive processes at the present energy region ($Q^2 > 4\text{GeV}^2$) leave a lot of space to discuss the contributions from higher twist terms. For example, the contribution of twist-3 distribution amplitudes to the pion electromagnetic form factor, although it is suppressed by the factor $1/Q^2$, is comparable with and even larger than the contribution from leading-twist distribution amplitude of the pion at intermediate energy region of $Q^2$ being 2-40 GeV$^2$ in Ref [5]. In the semileptonic B meson decays processes, the $B \to \pi$ transition form factor were calculated in light cone QCD sum rules in the literature [6, 7]. The $B \to \pi$ transition form factor in Ref [6] contain twist-2, twist-3 and twist-4 contributions. From the numerical results of Ref. [6] we can see that the contribution of the twist-3 distribution amplitudes is about 30-50% and the contribution of the twist-4 distribution amplitudes is about 5% to $B \to \pi$ transition form factor. Although one can avoid to calculate the twist-3 contribution by choosing an adequate chiral current correlator [7] to make the contribution of the twist-3 distribution amplitudes in the $B \to \pi$ transition form factor vanish automatically, one can’t ignore their contributions once and for all. On the other hand, we can get the sum rules for the $B \to \pi$ transition form factor $f_{B\pi}^+(q^2)$ which is determined by the twist-3 distribution amplitudes solely if choosing an adequate chiral correlator [8]. It means that the contribution from the twist-3 distribution amplitudes to the $f_{B\pi}^+(q^2)$ has the same order of magnitude as that from the leading twist distribution amplitude. Therefore one has to study the twist-3 distribution amplitudes $\phi_p(\xi), \phi_\sigma(\xi), \phi_{3\pi}(\alpha_i)$ and higher twist distribution amplitudes of the pion and other meson.

The pion twist-3 distribution amplitudes have been studied in Ref. [9, 10] in the chiral limit, based on the techniques of nonlocal product expansion and conformal expansion and including corrections in the meson-mass. Ref. [10] studied the vector meson twist-3 distribution amplitudes in details. However they employ the equations of motion of on-shell quarks in the meson to get two relations among two-particles twist-3 distribution amplitudes of the pion, $\phi_p(\xi)$ and $\phi_\sigma(\xi)$, and three-particle twist-3 distribution amplitude $\phi_{3\pi}(\alpha_i)$ of the pion. Then, they took the moments of three-particles twist-3 distribution amplitude calculated in QCD sum rules and obtained the approximate forms for three twist-3 distribution amplitudes of the pion. The question is whether
one can use the equation of motion of the quark inside the meson since the quarks are not on-shell. The relations between \( \phi_p(\xi), \phi_\sigma(\xi) \) and \( \phi_3(\alpha_i) \) cannot be satisfied exactly in this point of view.

In this paper, we do not apply the quark equation of motion and calculate the moments of the twist-3 distribution amplitudes \( \phi_p(\xi) \) directly in QCD sum rules approach. We employ the QCD in the background field \([11]\) to calculate the relevant correlator. The \( \phi_p(\xi) \) is defined as follow:

\[
\langle 0 | \bar{d}(z) \gamma_5 \exp \left\{ ig \int_{-z}^{z} dx^\mu A_\mu \right\} u(-z) | \pi^+ + (q) \rangle = m^p_0 f_\pi \frac{1}{2} \int_{-1}^{1} d\xi \phi_p(\xi) e^{i\xi (z \cdot q)} + \cdots ,
\]

where the dots refer to those higher twist terms omitted here, and \( m^p_0 \) is a parameter which is to be determined in our calculation. This parameter has nothing to do with the masses of the quarks and/or meson. If we assume the expansion of \( \phi_p(\xi) \) in the series in Gegenbauer polynomials \( C^{1/2}_n(\xi) \) one can obtain the approximate expressions of the twist-3 distribution amplitudes of the pion which can be determined by their moments.

For another twist-3 distribution amplitude \( \phi_\sigma(\xi) \), some words are in order. We found that, results calculated directly by QCD sum rules method depend on the Borel parameter seriously. That is to say, there is no way to make the \( \pi \)-contribution dominate. So we do not consider this case in this paper.

Finally, it needs to be pointed out that, just as shown in Ref. \([12]\), axial currents in a correlator would be coupled to objects like instantons. And this may cause some complication in the calculation and make the results unreliable. We will not explore their influences in this paper, some more work will be done in later paper.

This paper is organized as follows: The calculation of two-point correlation function is presented in Sec. II. We calculate moments of \( \phi_p(u) \) in Sec. III. Sec. IV is devoted to build a possible model on the twist-3 distribution amplitude. Sec. V is reserved for discussion and summary.

**II. CALCULATION OF TWO-POINT CORRELATION FUNCTION**

In order to calculate two-point correlation function, we apply the background field approach which were expressed in Ref. \([11]\) explicitly. They called the QCD in background fields. In that framework, the non-vanishing vacuum condensates are described by the classical fields, while the corresponding quantum fields are quantized in the Furry representation and the physical states are built upon the physical QCD vacuum through the action of creation operators. One can derive the propagators of quarks and gluons as a perturbative series in gauge invariant form. The propagators
are the solution of equations,

\[(i\gamma^\mu(\partial_\mu - igA_\mu) - m)S_F(x, 0) = \delta^4(x),\]

\[(g_{\mu\nu}(D_\mu D^\nu)^{AB} + 2f^{ABC}G^{C}_{\mu\nu})S_{\nu\sigma}^{BD}(x, 0) = \delta^{AD}g_{\mu\sigma}\delta^4(x).\]

To the aim of calculation of first few moments in this paper, we need only consider following terms in the two propagators respectively,

\[S^{ab}_F(x, 0) = \frac{i\gamma^\mu x^\mu}{2\pi^2 x^2} \delta^{ab} + \frac{m}{4\pi^2 x^2} \delta^{ab} - \frac{\gamma^\alpha\gamma^\mu x^\mu\gamma^\beta}{16\pi^2 x^2} g(G^A_{\alpha\beta}(0)T^A)^{ab} + \frac{\ln(-x^2)}{48\pi^2} g\gamma^\mu \left[G_{\alpha\mu;\alpha\nu}(0)T^A\right]^{ab}\]

\[S^{AB}_\mu(x, 0) = \frac{g_{\mu\nu}}{4\pi^2 x^2} \delta^{AB},\]

where \(a, b = 1, 2, 3\), \((\bar{G}_{\mu\nu})^{AB} = -if^{ABC}G^{C}_{\mu\nu}(A, B, C = 1, 2, \ldots, 8), G^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu + gf^{ABC}A^B_\mu A^C_\nu\) is the classical field strength, \(A^A_\mu\) is the classical field of the gluon.

We define a two-point correlation function,

\[\Pi_{\Gamma_1\Gamma_2}(q) = i \int d^4x e^{iq\cdot x}\langle 0 \mid T \{j_{\Gamma_1}(x)j_{\Gamma_2}(0)\} \mid 0\rangle,\]

where \(\Gamma_1\) and \(\Gamma_1\) are Dirac matrices, and the vacuum state is the physical one.

For the light quark current, the OPE of two-point correlation function is as follows:

\[i \int d^4x e^{iq\cdot x}\langle 0 \mid T \{j_{\Gamma_1}(x)j_{\Gamma_2}(0)\} \mid 0\rangle = C^{(f)}_{\Gamma_1\Gamma_2}(q^2)I + \frac{m\bar{\psi}\psi}{\pi}C^{(m)}_{\Gamma_1\Gamma_2}(q^2)0 + \frac{G^{(G^2)}_{\Gamma_1\Gamma_2}(q^2)}{\pi}G^20 + \cdots ,\]

where we ignore the higher power of light quark mass \(m_q\) and only reserve the linear \(m_q\) term. \(C_{\Gamma_1\Gamma_2}\)'s in Eq.(6) are Wilson coefficients which can be worked out in perturbative QCD theory.

For calculating the moments of \(\phi_p(\xi)\), we take as usual the two-point correlation functions:

\[i \int d^4x e^{iq\cdot x}\langle 0 \mid T \{j_{5}^{(2n)}(x, z)\bar{j}^{(0)}_5(0)\} \mid 0\rangle \equiv (z \cdot q)^{2n}\int_{p_0}^{2n,0}(q^2),\]

where the currents are defined as

\[j_{5}^{(2n)}(x, z) = \bar{d}(x)\gamma_5(iz \cdot \vec{D})^{2n}u(x),\]

with \(\vec{D} = \vec{\partial}_\mu - igA^a_\mu T^a\). The relation between the light quark currents \(j_{5}^{(2n)}(x, z)\) and the twist-3 distribute amplitude of the pion will be given in the next section.

On the one hand, one can calculate the correlation functions in QCD framework perturbatively. Following the familiar procedure \[\text{by applying operator expansion in the background field approach, corresponding Feynman diagrams in Fig.1, with condensates up to dimension-6 and the}\]
perturbative contribution part to the lowest order, we have

\[
I_p^{2n,0}(-Q^2) = \frac{3}{8\pi^2} \frac{1}{2n+1} Q^2 \ln\left(\frac{-Q^2}{\mu^2}\right) - \frac{2n - 1}{2} \frac{(m_u + m_d)\langle \bar{\psi}\psi\rangle}{Q^2} + 2n + 3 \frac{\langle \mu^2 G^2 \rangle}{Q^2} - \frac{\sqrt{\alpha_s \bar{\psi}\psi}^2}{Q^4} \frac{16\pi}{81} (21 + 8n(n + 1)).
\] (8)

On the other hand, one can express the two-point correlation functions in hadronic representation by inserting the complete intermediate states with the same quantum numbers as those of corresponding currents which is \( j_5^{2n} \) in this case. By isolating the pole term of the lowest pseudoscalar pion, the hadronic representation can be obtained in the following,

\[
i \int d^4x e^{iq\cdot x} \langle 0 | T \left\{ j_5^{(2n)}(x, z) j_5^{(0)+}(0) \right\} | 0 \rangle = \sum_{\pi^H} \langle 0 | j_5^{(2n)}(z) | \pi^H \rangle \langle \pi^H | j_5^{(0)+}(0) | 0 \rangle
\] (9)

where \( \pi^H \)'s are higher exciting hadronic states.

### III. SUM RULES FOR THE MOMENTS OF TWIST-3 DISTRIBUTION AMPLITUDE

Twist-3 distribution amplitude \( \phi_p(\xi) \) of the pion is given as the leading terms by gauge invariant elements of nonlocal hadronic matrices in Eqs.(1). Furthermore, the elements of nonlocal hadronic matrices can be expanded near light cone \( z^2 = 0 \),

\[
\langle 0 | \bar{d}(z) \gamma_5 \exp \left\{ ig \int_{-z}^{z} dx^\mu A_\mu \right\} u(-z) | \pi^+(q) \rangle = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \langle 0 | \bar{d}(0) \gamma_5 (z \cdot \vec{n})^n u(0) | \pi^+(q) \rangle.
\] (10)

Substituting Eq.(14) into Eq.(1), we have the following relations,

\[
\langle 0 | j_5^{(2n)}(z) | \pi \rangle = f_\pi m_0^{\pi}(\xi^{2n}_p)(z \cdot q)^{2n}
\] (11)

where the moments of \( \phi_p(\xi) \) is defined as,

\[
(\xi^{2n}_p) = \frac{1}{2} \int_{-1}^{1} \xi^{2n}\phi_p(\xi) d\xi.
\] (12)

Therefore the spectrum can be taken as approximately,

\[
\text{Im} I_p^{2n,0}(q) = \pi \delta(q^2 - m_\pi^2) (m_0^{\pi})^2 f_\pi^2 (\xi^{2n}_p) \cdot (\xi^0_p) + \pi \frac{3}{8\pi^2} \frac{1}{2n + 1} q^2 \theta(q^2 - s_\pi),
\] (13)

where \( s_\pi \) is the threshold parameter and should be set to a value between \( m_\pi^2 \) and its first exciting state \( \pi' \).
To get the sum rules for moments, we employ dispersion relation for $I(Q^2)$,

$$
\frac{1}{\pi} \int ds \ \text{Im} I^{2n,0}_p(s) = I^{2n,0}_p(-Q^2),
$$

and assume the contribution of the lowest state $\pi$ dominant in the sum rule. Making a Borel transformation upon above dispersion relation, we can get an improved one

$$
\frac{1}{\pi M^2} \int ds \ e^{-\frac{s}{M^2}} \text{Im} I^{2n,0}_p(s) = \hat{L}_M I^{2n,0}_p(-Q^2),
$$

(14)

where $M$ is the parameter used in the Borel transformation with $M = Q^2/n$ fixed, $\hat{L}_M = \lim_{Q^2,n \to \infty} \frac{1}{(n-1)!} (Q^2)^n (-\frac{d}{dQ^2})^n$.

Inserting Eq.(13) and Eq.(8) into the left and right hand side of Eq.(14) respectively, we obtain the sum rule for moments of $\phi_p(\xi)$,

$$
\langle \xi_p^{2n} \rangle \cdot \langle \xi_p^0 \rangle = \frac{M^4}{(m_0^p)^2 f_\pi^2 m_\pi^2 / M^2} \left[ \frac{3}{8\pi^2} \frac{1}{2n+1} \left( 1 - \left( 1 + \frac{8\pi}{M^2} \right) e^{-\frac{8\pi}{M^2}} \right) - \frac{2n-1}{2} (m_u + m_d) \langle \bar{\psi} \psi \rangle \right.
+ \frac{2n + 3 (\alpha_s G^2)}{24} \frac{16\pi}{M^4} (21 + 8n(n + 1)) \left( \langle \sqrt{\alpha_s \bar{\psi} \psi} \rangle \right)^2 \left( \langle \sqrt{\alpha_s \bar{\psi} \psi} \rangle \right)^2.
$$

(15)

As we can see, the first term in Eq.(15) is the pure perturbative results with condensate contributions omitted completely.

**IV. A POSSIBLE MODEL OF THE PION TWIST-3 DISTRIBUTION AMPLITUDE**

Before proceeding further we need to fix input parameters. For the condensate parameters, we take as usual,

$$
(m_u + m_d) \langle \bar{\psi} \psi \rangle \simeq -1.7 \times 10^{-4} GeV^4,
$$

$$
\langle \frac{\alpha_s}{\pi} G^2 \rangle \simeq 0.014 GeV^4,
$$

$$
\langle \sqrt{\alpha_s \bar{\psi} \psi} \rangle^2 \simeq 1.83 \times 10^{-4} GeV^6.
$$

(16)

The parameter $s_\pi$ in Eq.(15) should be chosen to make moments and $m_0^p$ most stable against $M^2$ in a certain range. This can be done by considering the ratios of $\langle \xi_p^{2(n+1)} \rangle$ to $\langle \xi_p^{2n} \rangle$. The advantage is that the ratios depend only on $s_\pi$ and $M^2$, but not $m_0^p$ and $\langle \xi_p^0 \rangle$. This make us determine $s_\pi$ and $M^2$ on a more general ground at first. The results are shown in Fig.2. It is shown from these figures that $s_\pi$ affect the results a little and $M^2$ should be taken, at least, $1.2 GeV^2$. The key point to avoid the concept of on-shell equations of motion is that the introduced $m_0^p$ appearing in the
The above numerical results show that the dependence of the parameter is about 25% as $M^2$ is in the region we chose. It is obvious that the fourth moment of $\phi_p(\xi)$ is more unreliable. The above numerical results also show that the parameter $m_0^b$ is smaller than $m_0^2/(m_u + m_d) \simeq 1.78 - 3.92 GeV$ which was used in [4, 9] when the equation of motion of on-shell quarks are employed.

In principle, infinite number of moments are needed to determine twist-3 distribution amplitude of the pion $\phi_p(\xi)$ completely, that is beyond one’s capability by now in this method. Also we have neglected contributions from higher dimension condensates in the QCD calculation and higher exciting resonances in the hadronic representation. Thus the prediction for more moments is not reliable. We will take the above three moments into consideration in the following analysis and expand twist-3 distribution amplitudes $\phi_p(\xi)$ of the pion in Gegenbauer polynomials to first few terms,

$$\phi_p(\xi) = A_p + B_p \ C_2^{1/2}(\xi) + C_p \ C_4^{1/2}(\xi),$$

(18)

where $C_n^{1/2}(\xi)$ are Gegenbauer polynomials. $A_p, B_p, C_p$ are three coefficients to be determined through the moments given in previous section. We obtain the two group of equations,

$$\frac{1}{2} \int_{-1}^{1} d\xi \xi^{2n}(A_p + B_p \ C_2^{1/2}(\xi) + C_p \ C_4^{1/2}(\xi)) = \langle \xi_p^{2n} \rangle \quad n = 0, 1, 2.$$  

(19)

Solving the above equations, it is easy to find these coefficients in twist-3 distribution amplitude $\phi_p(\xi)$,

$$A_p = \langle \xi_p^0 \rangle \quad , \quad B_p = -\frac{9}{32}(\langle \xi_p^0 \rangle - 3\langle \xi_p^2 \rangle) \quad , \quad C_p = \frac{9}{16}(3\langle \xi_p^0 \rangle - 30\langle \xi_p^2 \rangle + 35\langle \xi_p^4 \rangle).$$  

(20)

Substituting the value Eq.(17) into Eq.(20), the $\phi_p(\xi)$ can be obtained. (when $s_\pi = 1.7 GeV^2$, we have, $A_p \simeq 1, B_p \simeq (0.05) - (0.17), C_p \simeq (-1.52) - (-0.371)$. We can compare Eq.(18) with the results of Ref. [9],

$$\phi_p(\xi) = 1 + 30R \ C_2^{1/2}(\xi) + \frac{3}{2} R(4\omega_{2,0} - \omega_{1,1} - 2\omega_{1,0}) \ C_4^{1/2}(\xi),$$  

(21)
where \( R = \frac{m_u + m_d}{m^2} \) and the parameters \( \omega 's \) can be found from the moments of \( \phi_{3\pi} \) in QCD sum rules. Comparison Eq.\((21) \) with our results is expressed in Fig.\( 3 \). The distribution amplitude depicted in the dashed line contains contributions from the second and the fourth moment. But it should be pointed out that the fourth moment calculated in this paper is less reliable than the second one. The dotted line depicts the one only contains the contribution from the second moment of \( \phi_p(\xi) \). Finally, Our result is closed to the asymptotic form, \( \phi_p^{AS}(\xi) = 1 \).

V. SUMMARY AND DISCUSSION

In applying QCD to exclusive processes, form factor and decay amplitude are determined by the convolution of a perturbative hard scattering amplitude with a non-perturbative distribution amplitude of given twist. The study of the leading twist and non-leading twist distribution amplitudes is a crucial task for predicting exclusive processes more precisely. The asymptotic behavior of exclusive processes is determined by the leading twist distribution amplitude only as \( Q^2 \) is very large. One has to consider the contributions from the non-leading twist at the present energy regions. Many people have claimed that the contributions from the twist-3 distribution amplitudes of the pion make a sizeable corrections to the exclusive processes. Therefore it is helpful to study the twist-3 distribution amplitudes of the pion in different approaches in order to understand their behaviors.

In this paper, we apply the background field approach to calculate the OPE of the two-point correlation functions related to the moments of twist-3 distribution amplitudes of the pion. With the aid of the improved dispersion relations, we obtain the sum rules of the moments of \( \phi_p(\xi) \). Inputting all sorts of parameters required in sum rules with the contribution of condensate to dimension-6, we worked out first three moments of \( \phi_p(\xi) \). It is shown that the main contribution to moments is from pertubative term, the contributions from condensate terms are relatively small, In the previous approach \([9, 10]\), it was based on the techniques of nonlocal operator product expansion and conformal expansion. They applied the equations of motion to impose relations between the higher twist distribution amplitudes and \( \phi_{3\pi} \). We emphasize that the equations of motion can not be satisfied since the quarks are not on mass-shell within hadrons.

To avoid imposing equations of motion of on-shell quark, it is not felicitous, we introduce a new parameter \( m_0^p \) in the definitions of moments of \( \phi_p(\xi) \). Because of the emergence of the new parameter, we can not determined all moments if we don’t know \( m_0^p \). The strategy we take is to require normalization condition \( \langle \xi_0^p \rangle = 1 \), and then we obtain \( m_0^p \) and other moments. In this
paper, we have made some approximations, such as the lowest pole dominates and the higher dimension condensates are negligible, also we haven’t calculated the radiative corrections. Based on these approximations, we obtain the parameter $m_0^P \simeq 1.30 GeV \pm 0.06 GeV$ which is smaller than $m_\pi^2/(m_u + m_d)$. However, it is closed to the phenomenological value \[13\].

Using the values of the parameter $m_0^P$ and the moments, the twist-3 distribution amplitude $\phi_p(\xi)$ expanded in Gegenbauer polynomial can be obtained,

$$\phi_p(\xi) = 1 + 0.137 \ C_2^{1/2}(\xi) - 0.721 \ C_4^{1/2}(\xi).$$

Its behavior is closed to the shapes of $\phi_p^{AS}(\xi)$ in Ref.\[4\].

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FIG. 1: Several types of Feynman diagrams calculated in this paper. The gluon condensations are included in the Fermion propagators implicitly and the quark condensations are depicted as the crosses in the diagrams.

FIG. 2: The ratios of $\langle \xi_{p}^{2(n+1)} \rangle$ to $\langle \xi_{p}^{2n} \rangle$ versus Borel parameter $M$. The upper curves are the ratios with $n=1$ and the lower curves are the ratios with $n=0$, while the solid, the dashed and the dotted curves correspond to three threshold values: (a) $s_\pi = 1.7GeV^2$; (b) $s_\pi = 1.6GeV^2$ (c) $s_\pi = 1.5GeV^2$, respectively.
FIG. 3: The twist-3 distribution amplitude $\phi_p(\xi)$. The solid-curve is the one of Ref. 9, the dashed curve corresponds to Eq. (18) and the dotted curve corresponds to the one only contained the second moment in this paper.