1. Introduction

Impedance matching is an important aspect in the design of microwave and millimeter wave circuitry since impedance mismatches may severely deteriorate the overall performance of electronic systems. In high-power applications, the standing electromagnetic wave resulting from mismatch in a transmission line is highly undesirable as it leads to amplitudes of voltage and current which might be several times higher than those in a matched line. This can lead to disruption or even damage of the dielectric in the transmission line. A reflected electromagnetic wave can also result in frequency pulling of signal generators connected to the mismatched transmission line, thereby shifting the oscillation frequency from the desired.

In transceiver applications, antenna mismatch leads to signal power loss and lower signal-to-noise ratio, thereby deteriorating the overall transmit or receive performance. When designing low-noise amplifiers, it is often required to control the input network mismatch. Generally, it is not possible to design an amplifier which has the optimum input impedance for minimum noise figure equal in value to the optimum impedance for maximum gain. The input network is then should be mismatched in order to provide a low-noise operation.

Impedance transformers can also be effectively used to improve selectivity of resonant circuits and are very useful in filter design. Low values of source and load impedance decrease the loaded quality factor Q and increase the bandwidth of a given resonant circuit. This makes it very difficult to design even a basic LC high-Q resonant circuit for use between two very low values of source and load resistances. A common method to overcome this problem is to use impedance transforming circuits to present the resonant circuit with a source or load resistance that is much larger than what is actually present. Consequently, by utilizing impedance transformers, both the Q of the resonator and its selectivity can be increased.

Matching a complex impedance in a wide frequency range is most commonly achieved by using one of the following techniques:
- passive two-port networks consisting of reactive components;
- passive two-port networks consisting of resistive components;
Wideband matching can also be achieved by means of ferrite circulators in which the reflected wave is guided to an absorbing load, and ferrite isolators in which the transmission losses are different for the incident and reflected waves. For a wideband matching, it is preferable to place the matching network as close as possible to the load, as it is illustrated in Fig. 1.

This concept will be demonstrated in the later section 3.1 by considering an example of matching a complex load using shunt stubs.

In this chapter, different techniques for wideband matching are presented. Sections 2 thru 4 briefly present some of the well-known matching techniques while the use of coupled transmission lines for wideband matching is treated in depth in Section 5. The first part of the chapter includes a discussion of resistive and reactive lumped elements in Section 2, different types of stub matching in Section 3, and the use of series of transmission lines in Section 4. Since these techniques are all thoroughly treated in the literature, only the design-considerations relevant for applying the techniques for wideband matching are treated here while the reader is referred to the literature for specifics, such as the relevant formulas for calculating the values of the different components.

The use of coupled transmission lines for wideband impedance matching is not as widely used as the techniques described in sections 2 thru 4. Hence, in Section 5, a detailed presentation of this technique is given.
2. Matching Using Resistive and Reactive Lumped Elements

Resistive elements or attenuators can be effectively used to lower the level of the reflected signal from the load in a very wide frequency range. It should be noted, though, that the efficiency of such matching networks is low because they attenuate not only the reflected but also the incident wave. Another type of matching network uses lumped reactive components to match a complex load impedance to a desired complex impedance. For moderate bandwidths, the component values of two-element matching networks can be found relatively easy by first choosing a pair of initial values on the basis of the Smith Chart and then applying computational optimization. To increase the bandwidth, more than two reactive elements are required. The synthesis and optimization of multi-element wideband matching circuits can be accomplished by means of software tools, which are currently available in a wide variety. The implementation of this type of transformers in microwave and millimeter wave range is limited due to the low Q-factor of lumped components. Therefore, lumped element matching is usually employed only at low frequencies, or in applications where compact size is a major requirement, e.g., in monolithic microwave integrated circuits design (Kinayman & Aksun, 2005).

3. Stub matching

This section is dedicated to matching circuits that use open-circuited or short-circuited transmission line sections, connected in parallel with the load or transmission feed line. This is a well developed matching technique which is often used in microwave and millimeter wave circuits. In this section, some of the important operational principles and properties of shunt stub matching circuits are discussed. More detailed analysis of this type of matching technique is available in the literature (Pozar, 1998), (Kinayman & Aksun, 2005).

3.1 Single-Stub Matching

This is one of the most simple and convenient ways of matching a transmission line with a load which has real or complex impedance. This method was developed by Tatarinov V. V. in 1931 and is widely used for narrow-band matching in microwave and millimeter wave applications. It consists of a short circuited or open circuited stub and a piece of transmission line between the load and the stub. An example of the single-stub matching circuit is shown in Fig. 2. There are several choices of electric distance $\theta_d$ from the load to the matching stub. In the first case (Fig. 2 (a)), the distance between the load and matching stub is chosen as short as possible while this distance is chosen to be several times longer in the second case (Fig. 2 (b)). The responses of these two matching circuits are shown in Fig. 3. The 10 dB reflection loss bandwidth of the circuit in Fig. 2 (a) is 10.3 % while the same parameter for the circuit in Fig. 2 (b) is equal to 1.9 %. Thus, by using $\theta_d = 56.85^\circ$ instead of $\theta_d = 282.05^\circ$, the bandwidth is increased by more than a factor of 5.

There is also a difference in the wideband response of the matching circuits. The circuit with long distance between the load and the matching stub demonstrates more passbands in the same frequency range.
3.2 Double-Stub Matching

Single-stub matching can match any load impedance, but it requires a variable electric length of the transmission line between the load and the stub. This poses practical difficulties for adjustable tuners.

![Diagram of double-stub matching](image)

Fig. 2. Two single-stub matching solutions. (a) wideband, (b) narrowband. The load is matched at $f_0 = 1$ GHz.

![Graph of S11 magnitude](image)

Fig. 3. Magnitude of $S_{11}$ versus offset frequency for the matching circuits in Fig. 2. Here, $f_0 = 1$ GHz is the center frequency of operation.

Therefore, it would be more useful to have the length fixed and still be able to match a wide range of load impedances. This can be achieved with double-stub matching, as shown in Fig. 4, which allows for an arbitrary electric distance between the load and the stub.
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It should be noted that stub spacings near 0 or $\lambda_g/2$ (where $\lambda_g$ is the guided wavelength) lead to matching networks that are very frequency sensitive (Pozar, 1998), and consequently, very narrowband. In practice, stub spacings are usually chosen as odd number of $\lambda_g/8$, for example $\lambda_g/8$, $3\lambda_g/8$ or $5\lambda_g/8$.

3.3 Triple-Stub Matching

The double-stub matching circuit can not match all load impedances. For a specified distance between two stubs, the matching is possible only for limited values of loads, which depend on amplitude and phase of the standing wave. This limitation arises from the fact, that the stub itself can not change the real part of the impedance at the point of connection to the transmission line.

This limitation can be overcomed by using a triple-stub matching as the one shown in Fig. 5.

Fig. 4. Double-stub matching. The first stub can be placed at arbitrary distance from the load.

Fig. 5. Triple-stub matching. The first stub can be placed at arbitrary distance from the load.
It allows for an arbitrary distance between the load and the stub and also allows to match arbitrary load impedance. The operation of triple-stub matching circuit can be treated as a combination of two double-stub matching circuits and stub spacings are usually chosen as $\lambda_g/4$.

4. Series Transmission Line Matching

This section is dedicated to matching circuits that use series transmission lines, such as single section quarter-wave transformer, multisection transformers, and tapered transmission lines.

4.1 The Quarter-Wave Transformer

The quarter-wave transformer is one of the most simple and practical circuits for impedance matching, especially for matching of real load impedances. It is also possible to match a complex load using the quarter-wave transformer, but this requires an additional length of transmission line between the load and the quarter-wave transformer to transform the complex load impedance into a real impedance. A circuit employing a quarter-wave transformer is shown in Fig. 6.

Fig. 6. A single section quarter-wave matching transformer.

One of the main drawbacks of this transformer is the requirement to have available a transmission line with an impedance of $Z_1 = \sqrt{Z_01 Z_02}$. In some cases, e.g., matching with coaxial cable, the required quarter wave transmission line calls for a nonstandard value of the characteristic impedance.

4.2. Transformers with Fixed Values of Characteristic Impedance

Another useful type of series transformers are those which are based on transmission lines with the same characteristic impedances as the lines which should be matched. Such transformers are convenient for interconnection of standard lines as well as transmission lines with different geometry, where realization of transmission lines of arbitrary characteristic impedance involve difficulties.

The simplest realization of such transformer is shown in Fig. 7 (a) and described in detail by (Aizenberg et al., 1985).
It allows for an arbitrary distance between the load and the stub and also allows to match arbitrary load impedance. The operation of triple-stub matching circuit can be treated as a combination of two double-stub matching circuits and stub spacings are usually chosen as $\lambda_g/4$.

### 4. Series Transmission Line Matching

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Fig. 7. Transformers with fixed values of characteristic impedance consisting of (a) two sections and (b) four sections.

This transformer consists of two transmission line sections. The characteristic impedances of these lines are the same as impedances of lines to be matched. The length of one section is

$$l = \frac{\lambda_g}{2\pi} \theta = \frac{\lambda_g}{2\pi} \arctan \left( \frac{1}{\sqrt{n + \frac{1}{n} + 1}} \right),$$

where $n = Z_{02}/Z_{01}$ is the transformation ratio.

For small values of $n$, the value of $l$ approaches $\lambda_g/12$, implying that the total length of the transformer approaches $\lambda_g/6$. For increasing $n$, the value of $l$ approaches 0.

The operating frequency band of the described transformer is about 5% narrower in comparison to the quarter-wave transformer, and its length for practical values of $n$ is 1.5 - 2 times shorter.

The response of the transformer in Fig. 7 (a) is shown in Fig. 8 (curve (a)) and compared to the response of the conventional quarter-wave transformer (Fig. 8 curve (b)). For transformation ratio 2:1 the electrical length of the section in Fig. 7 (a) is equal to 28.1°. The achieved for this ratio bandwidth at 20 dB return loss level is 31%.
Fig. 8. Comparison of matching characteristic of quarter-wave transformer. The most common configuration of the transformer is shown in Fig. 9.

A more broadband stepped impedance transformer is shown in Fig. 7(b)). It consists of four sections with the length of the outermost sections being shorter than the length of sections in the middle. Fig. 8 (curve (c)) shows the magnitude of $S_{11}$ for a transformer with the following parameters: the transformation ratio is 2:1; $\theta_1/\theta_2 = l_1/l_2 = 0.35$.

Here

$$\theta_1 = \frac{2\pi l_1}{\lambda_6}, \quad \text{and} \quad \theta_2 = \frac{2\pi l_2}{\lambda_6}$$

are the electrical lengths of the sections in Fig. 7(b).

The total length of the transformer is $2l_1 + 2l_2 = 0.346\lambda_6$. The achieved bandwidth at 20 dB return loss level is 71%. The bandwidth and inband reflection level of this type of transformer depend on length of the sections (Aizenberg et al., 1985).

### 4.3 Tapered Transmission Lines

As described above, the bandwidth of the quarter-wave transformer is limited. In order to extend its operating frequency band, multisection transformers, with different characteristic impedance in each section, may be used. In contrast to the transformers described in the previous section, the lengths of the sections used in the multisection transformer can be chosen equal to each other. The desired reflection coefficient response as a function of frequency can be achieved by properly choosing the characteristic impedance of the transmission line sections. In the limit of an infinite number of sections, the multisection transformer becomes a continuously tapered line. There are many ways to choose the taper profile. By changing the type of taper, one can obtain different passband characteristics. Several taper profiles may be considered: linear, exponential, triangular, and so on.
For a given taper length, the Klopfenstein taper has been shown to be optimum in the sense that the reflection coefficient is minimum over the passband (Pozar, 1998). Alternatively, for a specified level of reflection coefficient, the Klopfenstein taper yields the shortest matching section. However, it should be noted that the response of this taper has equal level of ripples in its passband.

In many cases, the relation between the physical dimensions and the characteristic impedance of a guiding structure is complicated and the generation of an optimal tapering configuration is thus not a trivial task. This implies that a linear or exponential tapering of the physical dimensions of the transmission line is often chosen for practical implementations.

5. Coupled Line Transformers

In recent years, coupled transmission lines have been suggested as a matching element due to greater flexibility and compactness in comparison to quarter wavelength transmission lines (Jensen et al., 2007). It has been demonstrated that matching real and complex loads with coupled lines leads to more compact realizations and could therefore become important at millimeter-wave frequencies for on-chip matching solutions. Another area where coupled line structures are useful is matching of antenna array structures, as successfully demonstrated by (Jaworski & Krozer, 2004).

As it was shown above, the quarter-wave transformer is simple and easy to use, but it has no flexibility beyond the ability to provide a perfect match at the center frequency for a real-valued load, although a complex load of course can be matched by increasing the overall length of the transformer. The coupled line section provides a number of variables which can be utilized for matching purposes. These variables are the even and odd mode impedances and loads of the through and coupled ports. This loading can be done in form of a feedback connection which provides additional zeros for broadband matching.

These variables can be chosen to provide a perfect match or any desired value of the reflection coefficient at the operating frequency. The bandwidth of the coupled line transformer can be further increased in case of mismatch. In addition, it is also possible to match a complex load.

In the lower GHz range the loading of the through and coupled ports can be done with lumped elements which allows for easy matching of both real and imaginary impedance values. At higher frequencies it is not possible to use lumped elements, but the difference between the even and odd mode impedances is a parameter which makes it possible to turn a mixed real and imaginary control load at the through port into a purely imaginary one, which can be implemented with a transmission line stub.

5.1 Symmetric Coupled Line Section

Coupled line impedance transformers are very useful at millimetre wave frequencies where they successfully perform direct current blocking and can handle large impedance transformation avoiding transverse resonances which occur in a conventional low impedance quarter-wave transformer. The most common configuration of the transformer is shown in Fig. 9.
In this configuration, the diagonal terminals of the coupled line section are loaded with generator \((Z_g)\) and load \((Z_L)\) impedances. The opposite terminals are open circuited. In this standard configuration however, the electrical performance of the coupled lines transformer in terms of insertion loss and bandwidth can not compete with performance of the corresponding quarter-wave transformer (Mongia et al., 1999).

5.2 Asymmetric Coupled Line Section

Symmetric coupled lines represent a restricted configuration of the more general class of coupled lines. They allow for a simpler analysis, however, for wideband applications asymmetric coupled lines are preferable. For example, the bandwidth of a forward-wave directional coupler using asymmetric coupled transmission lines is greater than the one formed using symmetric ones (Jones & Bolljahn, 1956).

In this section the design of a wideband impedance transformer based on asymmetric coupled lines is described. The considered wideband impedance transformer is based on asymmetric, uniform coupled lines in nonhomogeneous medium. A microstrip line is one of the most commonly used classes of transmission lines in nonhomogeneous medium. Edge-coupled microstrip lines are shown in Fig. 10.

![Edge-coupled microstrip lines](image)

Fig. 9. Symmetric coupled transmission line transformer.

Fig. 10. A coupled microstrip line four-port.
For the purpose of analysis, this coupled line four-port is transformed to a two-port network with arbitrary load using impedance matrix representation. The investigations presented in this book are only for the most commonly used configuration, when diagonal terminals of the coupled lines are loaded with generator and load impedances. Thus, the entire circuit can be represented as a two-port network, which performs impedance transformation between a generator impedance $Z_g$ connected to a port 1 and a load impedance $Z_L$ connected to a port 3, as shown in Fig. 11.

As it can be seen in Fig. 11, the network consists of the coupled line four-port network described by an impedance matrix $[Z]$ and arbitrary load matrix at opposite terminals described by matrix $[Z'']$. In practice, ports 2 and 4 are in general either short-circuited or open-circuited with a corresponding representation of the two-port network $[Z'']$.

The magnitude of $S_{11}$ is equal to

$$|S_{11}|_{dB} = 20 \log \left( \frac{Z_{IN}(Z_{ij} + Z_{ij}'' + Z_L) - Z_g}{Z_{IN}(Z_{ij} + Z_{ij}'' + Z_L) + Z_g} \right),$$

(3)

where $Z_{IN}$ is the input impedance of the transformer, which is a function of the load impedance $Z_L$, impedance matrix elements of coupled lines $Z_{ij}$ and the arbitrary load $Z_{ij}''$ ($i$ and $j$ are the indexes of the matrix elements). Using the general impedance matrix representation for coupled lines (Tripathi, 1975) and boundary conditions at ports 2 and 4 the input impedance is expressed by

$$Z_{IN} = Z_{11} + Z_{12} \cdot a_1 + Z_{14} \cdot b_1 - \frac{(Z_{13} + Z_{12} \cdot a_2 + Z_{14} \cdot b_2)^2}{Z_{33} + Z_{32} \cdot a_2 + Z_{34} \cdot b_2 + Z_L},$$

(4)

where

$$a_1 = \frac{Z_{41}(Z_{24} + Z_{12}'' - Z_{21}(Z_{44} + Z_{22}''))}{(Z_{22} + Z_{11})(Z_{44} + Z_{22}'')(Z_{24} + Z_{12}')(Z_{42} + Z_{21}')},$$

(5a)
\[ a_2 = \frac{Z_{43}(Z_{24} + Z''_{12}) - Z_{23}(Z_{44} + Z''_{22})}{(Z_{22} + Z''_{11})(Z_{44} + Z''_{22}) - (Z_{24} + Z''_{12})(Z_{42} + Z''_{21})}, \]  
\quad (5b)

\[ b_1 = -\frac{Z_{41}}{(Z_{44} + Z''_{22})} - \frac{(Z_{42} + Z''_{21})}{(Z_{44} + Z''_{22})}a_1, \]  
\quad (5c)

\[ b_2 = -\frac{Z_{43}}{(Z_{44} + Z''_{22})} - \frac{(Z_{42} + Z''_{21})}{(Z_{44} + Z''_{22})}a_2. \]  
\quad (5d)

A total number of six quantities is required to describe asymmetric coupled lines (Mongia et al., 1999), being: \( Z_{c1} \) and \( Z_{c4} \), which are, respectively, the characteristic impedances of line 1 for \( c \) and \( \pi \) modes of propagation; \( \gamma_c \) and \( \gamma_\pi \), the propagation constants of \( c \) and \( \pi \) modes; \( R_c \) and \( R_\pi \), the ratios of the voltages on the two lines for \( c \) and \( \pi \) modes. Thus, the elements of the impedance matrix are given by

\[ Z_{11} = Z_{44} = \frac{Z_{c1}\coth(\gamma_c l)}{1 - R_c/R_\pi} + \frac{Z_{\pi1}\coth(\gamma_\pi l)}{1 - R_\pi/R_c}, \]  
\quad (6a)

\[ Z_{12} = Z_{21} = Z_{34} = Z_{32} = \frac{Z_{c1}R_c\coth(\gamma_c l)}{1 - R_c/R_\pi} + \frac{Z_{\pi1}R_\pi\coth(\gamma_\pi l)}{1 - R_\pi/R_c}, \]  
\quad (6b)

\[ Z_{13} = Z_{31} = Z_{24} = Z_{42} = \frac{Z_{c1}R_c\csch(\gamma_c l)}{1 - R_c/R_\pi} + \frac{Z_{\pi1}R_\pi\csch(\gamma_\pi l)}{1 - R_\pi/R_c}, \]  
\quad (6c)

\[ Z_{14} = Z_{41} = \frac{Z_{c1}\csch(\gamma_c l)}{1 - R_c/R_\pi} + \frac{Z_{\pi1}\csch(\gamma_\pi l)}{1 - R_\pi/R_c}, \]  
\quad (6d)

\[ Z_{22} = Z_{33} = \frac{Z_{c1}R_c^2\coth(\gamma_c l)}{1 - R_c/R_\pi} + \frac{Z_{\pi1}R_\pi^2\coth(\gamma_\pi l)}{1 - R_\pi/R_c}, \]  
\quad (6e)
\[ Z_{23} = Z_{32} = \frac{Z_{c1} R_c^2 \text{csch}(\gamma c l)}{1 - \frac{R_c}{R_\pi}} + \frac{Z_{\pi1} R_\pi^2 \text{csch}(\gamma \pi l)}{1 - \frac{R_\pi}{R_c}}, \quad (6f) \]

where \( l \) is the length of the coupled line section, as it is shown in Fig. 10. These relations are substituted into (5) and (4) to calculate the input impedance and finally the reflection coefficient of the transformer.

From relation (3) it can be seen that the matching properties of the transformer depend not only on coupled line parameters, but also on load of ports 2 and 4, which are described by elements \( Z''_{ij} \). This dependence introduces additional degree of freedom during design procedure and can be used to expand the bandwidth of the impedance transformer, as shown below.

**Loading With Transmission Line**

As an example, terminals 2 and 4 can be loaded with a microstrip transmission line. The impedance matrix of the transmission line with characteristic impedance \( Z_0 \), length \( l \), and propagation constant \( \gamma \) is given by

\[
\begin{bmatrix}
Z_0 \coth(\gamma l) & \frac{Z_0}{\sinh(\gamma l)} \\
\frac{Z_0}{\sinh(\gamma l)} & Z_0 \coth(\gamma l)
\end{bmatrix}
\]

(7)

The transformer configuration is shown in Fig. 12.

**Fig. 12.** Schematic illustration of the transformer based on coupled line section and a transmission line load.
In order to simplify further calculations, the transmission lines are considered to be lossless, and electrical lengths of the coupled line section \((\theta_c + \theta_n)/2\) and the microstrip transmission line are assumed equal, resulting in

\[
\gamma l = j\beta l = j\theta, \quad \gamma cl = j\theta c, \quad \gamma \pi l = j\theta \pi, \quad (8)
\]

\[
\theta = (\theta_c + \theta_n)/2, \quad (9)
\]

where \(\theta_c\) and \(\theta_n\) are the electrical lengths of the coupled line section for \(c\) and \(\pi\) mode respectively. \(\theta\) is a function of frequency and can be used for the analysis of the spectrum of the transformer reflection coefficient. The response (3) for the transformer of Fig. 12 is shown in Fig. 13.

![Graph](image)

**Fig. 13.** Response of transformer shown in Fig. 12. The transformation ratio is 1:2.

As it can be seen in Fig. 13, this transformer configuration exhibits an additional minimum in the magnitude of \(S_{11}\) in comparison to the traditional impedance transformer based on coupled line section with open-circuited terminals (Kajfez, 1981). These minima are non-uniformly distributed in the frequency domain. This is due to the differences in electrical lengths \(\theta_c\) and \(\theta_n\) for two coupled line modes in nonhomogeneous medium.

For the case of homogeneous medium the propagation constants for the two modes are equal, \(\gamma_c = \gamma \pi\), and hence the electrical lengths for the two propagating modes are also equal. It is therefore possible to obtain three equidistant reflection zeros in the spectrum of the reflection coefficient. Because transmission lines in a homogeneous medium are a special case of transmission lines in a nonhomogeneous medium the expressions given above are also valid for response calculations.

It can be depicted from the data in Fig. 14 that the transformer provides wideband operation with uniformly distributed reflection zeros in the frequency domain. In addition, the distance between the zero locations can be varied by adjusting the parameters of the structure.
For the case of homogeneous medium the propagation constants for the two modes are
\[ \gamma_c = \frac{\theta}{c}, \quad \gamma_\pi = \frac{\pi}{c}, \]
and hence the electrical lengths for the two propagating modes are also equal. The electrical length of the transformer is equal to a quarter wavelength at the center frequency. Comparing the results in Fig. 13 and Fig. 14 it can be deduced that the impedance transformer in nonhomogeneous medium has approximately the same bandwidth as the one in homogeneous medium. However, in many cases, like for example in surface mount technology, it is more useful to deal with microstrip structures.

**Loading With Stepped Impedance Transmission Line**

The differences in electrical lengths of the coupled lines in nonhomogeneous medium can be compensated by introducing a stepped impedance transmission line, as it is shown in Fig. 15.

![Fig. 15. Schematic illustration of the wideband impedance transformer.](image)

The transformer consists of asymmetric coupled lines described by the electrical parameters \( Z_{c1}, Z_{c2}, Z_{\pi1}, Z_{\pi2}, \) which are, respectively, the characteristic impedances of line 1 and 2 for the \( c \) and \( \pi \) modes of propagation; \( \theta_c \) and \( \theta_\pi \) the electrical lengths for the \( c \) and \( \pi \) modes; \( R_c \) and \( R_\pi \) the ratios of the voltages on the two lines for the \( c \) and \( \pi \) modes. The stepped impedance transmission line consists of two equal length transmission lines with characteristic
impedances $Z_{01}$ and $Z_{02}$, as shown in Fig. 15. The electrical length of each transmission line is set to be half the electrical length of the coupled line section to reduce the number of design parameters.

For the purpose of analysis, this structure is transformed into a two-port network with arbitrary load using an impedance matrix representation. Thus, the entire circuit can be represented as a two-port network, which performs impedance transformation between a generator impedance $Z_g$ connected to the port 1 and a load impedance $Z_l$ connected to the port 3, as shown in Fig. 11. The magnitude of $S_{11}$ at the port 1 is defined by (3). The input impedance $Z_{IN}$ in (3) is calculated using relations (4)-(6) together with the corresponding elements of the impedance matrix $[Z'']$ for the stepped impedance transmission line. A series connection of two transmission lines shown in Fig. 16 can be described as a connection of two two-port networks.

\[
(Z^{1*}) = Z_{01}, l_1, \gamma_1
\]

\[
(Z^{2*}) = Z_{02}, l_2, \gamma_2
\]

Fig. 16. Series connection of transmission lines.

The impedance matrices of the transmission lines with characteristic impedances $Z_{01}$, $Z_{02}$, lengths $l_1, l_2$ and propagation constants $\gamma_1, \gamma_2$ are given by

\[
[Z^{(1)}] = \begin{bmatrix}
Z_{11}^{(1)} & Z_{12}^{(1)} \\
Z_{21}^{(1)} & Z_{22}^{(1)}
\end{bmatrix} = \begin{bmatrix}
Z_{01} \coth(\gamma_1 l_1) & \frac{Z_{01}}{\sinh(\gamma_1 l_1)} \\
\frac{Z_{01}}{Z_{01}} & Z_{01} \coth(\gamma_1 l_1)
\end{bmatrix},
\]

\[
[Z^{(2)}] = \begin{bmatrix}
Z_{11}^{(2)} & Z_{12}^{(2)} \\
Z_{21}^{(2}(Z_{22}^{(2)}/Z_{11}^{(2)}/Z_{12}^{(1)})} & Z_{02} \coth(\gamma_2 l_2) & \frac{Z_{02}}{\sinh(\gamma_2 l_2)} \\
\frac{Z_{02}}{Z_{02}} & Z_{02} \coth(\gamma_2 l_2)
\end{bmatrix}.
\]

Impedance matrix for the overall circuit in Fig. 16 is derived using boundary conditions at the common terminal. At this terminal the voltages of two two-ports are equal, and currents are equal and oppositely directed.

Thus, impedance matrix elements are found to be:

\[
Z''_{11} = Z_{11}^{(1)} - \frac{(Z_{12}^{(1)})^2}{Z_{11}^{(2)}/Z_{12}^{(1)}}
\]

\[
= Z_{01} \coth(\gamma_1 l_1) - \frac{Z_{01}^2}{(Z_{01} \coth(\gamma_1 l_1) + Z_{02} \coth(\gamma_2 l_2)) \cdot \sinh^2(\gamma_1 l_1)},
\]

\[
(12a)
\]
The impedance matrices of the transmission lines with characteristic impedances and lengths are shown in Fig. 11. The magnitude of $S_{11}$ and $S_{22}$ for two-port networks is described as a connection of two transmission lines shown in Fig. 16 can be derived using boundary conditions at the common terminal. At this terminal the voltages of two two-ports are equal, and currents through these terminals are equal and oppositely directed. Impedance matrix for the overall circuit in Fig. 16 is derived using boundary conditions at the terminal.

Thus, impedance matrix elements are found to be:

$$Z_{11}^* = Z_{21}^* = \frac{Z_{12}^{(1)} Z_{12}^{(2)}}{Z_{11}^{(2)} + Z_{11}^{(1)}}$$

$$= \frac{Z_{01} Z_{02}}{(Z_{01} \coth(\gamma_1 l_1) + Z_{02} \coth(\gamma_2 l_2)) \cdot \sinh(\gamma_1 l_1) \cdot \sinh(\gamma_2 l_2)},$$

$$Z_{12}^* = Z_{21}^* = \frac{(Z_{12}^{(2)})^2}{Z_{11}^{(2)} + Z_{11}^{(1)}},$$

$$= Z_{02} \coth(\gamma_2 l_2) \cdot \frac{Z_{02}^2}{(Z_{01} \coth(\gamma_1 l_1) + Z_{02} \coth(\gamma_2 l_2)) \cdot \sinh^2(\gamma_2 l_2)}.$$

In case of transmission lines with equal electrical length $\theta/2$, (12) can be rewritten as

$$Z_{11}^* = Z_{01} \coth\left(\frac{\theta}{2}\right) - \frac{2Z_{01}^2}{(Z_{01} + Z_{02}) \cdot \sinh(\theta)},$$

$$Z_{12}^* = Z_{21}^* = \frac{2Z_{01} Z_{02}}{(Z_{01} + Z_{02}) \cdot \sinh(\theta)},$$

$$Z_{22}^* = Z_{02} \coth\left(\frac{\theta}{2}\right) - \frac{2Z_{02}^2}{(Z_{01} + Z_{02}) \cdot \sinh(\theta)}.$$

These equations are used for the calculation of elements of the matrix $[Z^*]$ in Fig. 11. Thus, the analysis of the structure now can be performed using (3).

It can be depicted from the data in Fig. 17 that the transformer provides wideband operation, and the electrical length of the transformer is equal to a quarter wavelength at the center frequency.

![Fig. 17. Response of the 50-100 Ω impedance transformer shown in Fig. 15.](image-url)
In addition, the distance between the minima locations $\Delta \theta$ can be varied by adjusting the parameters of the structure. This distance $\Delta \theta$ characterizes operating frequency bandwidth of the transformer. The characteristics of the transformer for three different values of $\Delta \theta$ are shown in Fig. 17. As it can be seen, the in-band level of the reflection coefficient depends on parameter $\Delta \theta$. The estimation of the maximum level of the return loss between minima for different transformation ratios can be found using the data shown in Fig. 18.

![Fig. 18. The minimum level of the return loss between minima in Fig. 17.](image)

As expected, the level of in-band return loss for the transformer increases with reducing of transformation ratio, and reaches the absolute maximum at $Z_L/Z_g = 1$.

### 5.3 Multisection Coupled Line Transformers

To further increase the bandwidth, it is possible to create an impedance transformer using more coupled line sections connected in series. The example of a microstrip two section impedance transformer is shown in Fig. 19.

![Fig. 19. Layout of the 12.5-50 Ω multi section impedance transformer.](image)
The total electrical length of the transformer is equal to half a wavelength at the center frequency. The response of the transformer is shown in Fig. 20.

![Graph showing the response of the 12.5-50 Ω impedance transformer](image)

**Fig. 20.** Response of the 12.5-50 Ω impedance transformer shown in Fig. 19.

The transformer exhibits six minima in the spectrum of reflection coefficient. The achieved fractional matching bandwidth is beyond a decade at -10 dB reflection coefficient level. The distance between the minima locations can be varied by adjusting the parameters of the structure.

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Modelling and computations in electromagnetics is a quite fast-growing research area. The recent interest in this field is caused by the increased demand for designing complex microwave components, modeling electromagnetic materials, and rapid increase in computational power for calculation of complex electromagnetic problems. The first part of this book is devoted to the advances in the analysis techniques such as method of moments, finite-difference time-domain method, boundary perturbation theory, Fourier analysis, mode-matching method, and analysis based on circuit theory. These techniques are considered with regard to several challenging technological applications such as those related to electrically large devices, scattering in layered structures, photonic crystals, and artificial materials. The second part of the book deals with waveguides, transmission lines and transitions. This includes microstrip lines (MSL), slot waveguides, substrate integrated waveguides (SIW), vertical transmission lines in multilayer media as well as MSL to SIW and MSL to slot line transitions.

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