Greenberger-Horne-Zeilinger paradoxes for $N$ quNits

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In this paper we show the series of Greenberger-Horne-Zeilinger paradoxes for $N$ maximally entangled $N$-dimensional quantum systems.

I. INTRODUCTION

The Greenberger-Horne-Zeilinger (GHZ) correlations, discovered in 1989, started a new chapter in the research related to entanglement. To a great extent this discovery was responsible for the sudden renewal in the interest in this field, both in theory and experiment. All these developments finally led to the first actual observation of three qubit GHZ correlations in 1999 [1], and as a by product, since the experimental techniques involved were of the same kind, to the famous teleportation experiment [2].

In [3, 4] it has been shown that GHZ paradox can be extended to the correlations observed in quantum systems consisting of $N + 1$ maximally entangled $N$-dimensional quantum objects (so called quNits), where $N$ is an arbitrarily high integer number. The existence of the GHZ paradox for three maximally entangled three dimensional quantum systems has been shown in [5]. It is worth mentioning that two different approaches have been presented in [3, 4] to derive GHZ paradoxes. In [3] the series of paradoxes has been derived for the correlation function that results from the correlations observed in so called unbiased symmetric multiport beamsplitters (for the description of such devices see below) whereas [4] presents derivation based on the relations between operators.

In this paper we would like to show that GHZ-type paradoxes exist also in the case of correlations expected in gedanken experiments involving $N$ maximally entangled quNits. This is an important question as the existence of such paradoxes allows us to deeper understand the structure of quantum entanglement, which is the fundamental resource in quantum information as well as the nature of non-classicality of quantum correlations (by non-classicality we understand lack of local realistic description of such correlations; the alternative measure of non-classicality can be a measure based on the notion of non-separability).

To this end, we shall study a GHZ-Bell type experiment in which one has a source emitting $N$ quNits in a specific entangled state of the property, that the quNits propagate towards one of $N$ spatially separated non conventional measuring devices operated by independent observers. Each of the devices consists of an unbiased symmetric multiport beam splitter [6] (with $N$ input and $N$ exit ports), $N$ phase shifters operated by the observers (one in front of each input), and $N$ detectors (one behind each exit port).

II. UNBIASED MULTIPORT BEAMSPLITTERS

An unbiased symmetric $2N$-port beam splitter is defined as an $N$-input and $N$-output interferometric device which has the property that a beam of light entering via single port is evenly split between all output ports. I.e., the unitary matrix defining such a device has the property that the modulus of all its elements equals $\frac{1}{\sqrt{N}}$.

An extended introduction to the physics and theory of such devices is given in [6]. Multiport beam splitters were introduced into the literature on the EPR paradox in [11, 12] in order to extend two qubit Bell-phenomena to observables described as operators in Hilbert spaces of the dimension higher than two. In contradistinction to the higher than 1/2 spin generalizations of the Bell-phenomena [13–19], this type of experimental devices generalize the idea of beam-entanglement [20–23]. Unbiased symmetric multiport beam splitters are performing unitary transformations between "mutually unbiased" bases in the Hilbert space [24–26]. They were tested in several recent experiments [27, 28], and also various aspects of such devices were analyzed theoretically [29, 30].

We shall use here only multiport beam splitters which have the property that the elements of the unitary transformation which describes their action are given by

$$U_{m,m'}^N = \frac{1}{\sqrt{N}} \gamma_N^{(m-1)(m'-1)},$$

where $\gamma_N = \exp(\frac{2\pi}{N})$ and the indices $m, m'$ denote the input and exit ports. Such devices were called in [3] the Bell multiports.
III. QUANTUM MECHANICAL PREDICTIONS

Although in this paper we consider only the situation in which there is an equal number of observers and input ports in each multiport beamsplitter it is instructive to derive necessary formulas for the more general case in which there are \( M \) observers each operating 2\( N \) port beamsplitter. The initial \( M \) quNIt state has the following form:

\[
|\psi(M)\rangle = \frac{1}{\sqrt{N}} \sum_{m=1}^{N} \prod_{l=1}^{M} |m\rangle_l,
\]

(2)

where \(|m\rangle_l\) describes the \( l\)-th quNIt being in the \( m\)-th beam, which leads to the \( m\)-th input of the \( l\)-th multiport. Please note, that only one quNIt enters each multiport. However, each of the quNIts itself is in a mixed state (with equal weights), which gives it equal probability to enter the local multiport via any of the input ports.

The state (2) seems to be the most straightforward generalization of the GHZ states to the new type of observables. In the original GHZ states the number of their components (i.e., two) is equal to the dimension of the Hilbert space describing the relevant (dichotomic) degrees of freedom of each of the quNIts. This property is shared with the EPR-type states proposed in [3] for a two-multiport Bell-type experiment. In this case the number of components equals the number of input ports of each of the multiport beam splitters. We shall not discuss here the possible methods to generate such states. However, we briefly mention that the recently tested entanglement swapping [1] [5] [6] technique could be used for this purpose.

As it was mentioned earlier, in front of every input of each multiport beam splitter one has a tunable phase shifter. The initial state is transformed by the phase shifters into

\[
|\psi(M)\rangle' = \frac{1}{\sqrt{N}} \sum_{m=1}^{N} \prod_{l=1}^{M} \exp(i\phi_l^m) |m\rangle_l,
\]

(3)

where \( \phi_l^m \) stands for the setting of the phase shifter in front of the \( m\)-th port of the \( l\)-th multiport.

The quantum prediction for probability to register the first photon in the output \( k_1 \) of an \( 2N \)-port device, the second one in the output \( k_2 \) of the second such device \( \ldots \), and the \( M\)-th one in the output \( k_M \) of the \( M\)-th device is given by:

\[
P_{QM}(k_1, \ldots, k_M|\phi_1^1, \ldots, \phi_M^M) = (\frac{1}{N})^{M+1} |\sum_{m=1}^{N} \exp(i \sum_{l=1}^{M} \phi_l^m) \prod_{k=1}^{M} \gamma_N^{(m-1)(k-1)}|^2 \]

\[= (\frac{1}{N})^{M+1} \left[N + 2 \sum_{m>m'} \cos \left( \sum_{l=1}^{M} \Delta \Phi_{l,k_l}^{m,m'} \right) \right],
\]

(4)

where \( \Delta \Phi_{l,k_l}^{m,m'} = \phi_l^m - \phi_l^{m'} + \frac{2\pi}{N}(k_l - 1)(m - m') \). The shorthand symbol \( \vec{\phi}_k \) stands for the full set of phase settings in front of the \( k\)-th multiport, i.e. \( \phi_1^k, \phi_2^k, \ldots, \phi_N^k \).

To efficiently describe the local detection events let us employ a specific value assignment method (called Bell number assignment; for a detailed explanation see again [3]), which ascribes to the detection event behind the \( m\)-th output of a multiport the value \( \gamma_N^{m-1} \), where \( \gamma_N = \exp(i\frac{2\pi}{N}) \). With such a value assignment to the detection events, the Bell-type correlation function, which is the average of the product of the expected results, is defined as

\[
E(\phi_1^1, \ldots, \phi_M^M) = \sum_{k_1, \ldots, k_M=1}^{N} \prod_{l=1}^{M} \gamma_N^{(k_l-1)(m_l-1)} P(k_1, \ldots, k_M|\phi_1^1, \ldots, \phi_M^M)
\]

(5)

and as we shall see for the quantum case it acquires particularly simple and universal form (which is the main purpose for using this non-conventional value assignment).

The easiest way to compute the correlation function for the quantum prediction employs the mid formula of (4):

\[
E_{QM}(\phi_1^1, \ldots, \phi_M^M) = \]

\[= (\frac{1}{N})^{M+1} \sum_{k_1, \ldots, k_M=1}^{N} \prod_{l=1}^{M} \gamma_N^{(k_l-1)(m_l-1)} \exp \left( i \sum_{n=1}^{M} (\phi_n^m - \phi_n^{m'}) \right) \]

\[\times \prod_{l=1}^{M} \gamma_N^{(k_l-1)(m_l-1)} \]

(6)
Now, one notices that \( \sum_{k=1}^{N} \gamma_N^{(k-1)(m-m'+1)} \) differs from zero (and equals to \( N \)) only if \( m - m' + 1 = 0 \) modulo \( N \). Therefore we can finally write:

\[
E_{QM}(\vec{\phi}_1, \ldots, \vec{\phi}_M) = \sum_{m=1}^{N} \exp(i \sum_{l=1}^{M} \phi_l^{m, m+1}),
\]

where \( \phi_l^{m, m+1} = \phi_l^m - \phi_l^{m+1} \) and the above sum is modulo \( N \), i.e., \( \phi_l^{N+1} = \phi_l^1 \).

One can notice here a striking simplicity and symmetry of this quantum correlation function \( \vec{\phi} \). It is valid for all possible values of \( M \) (number of quNits) and for all possible values of \( N \geq 2 \) (number of ports). For \( N = 2 \), it reduces itself to the usual two qubit, and for \( N = 2, M \geq 2 \) the standard GHZ type multi-qubit correlation function for beam-entanglement experiments, namely \( \cos(\sum_{l=1}^{M} \phi_l^{1, 2}) \). The Bell-EPR phenomena discussed in \( \vec{\phi} \) are described by \( \vec{\phi} \) for \( M = 2, N \geq 3 \).

Even for \( N = 2, M = 1 \) the function \( \vec{\phi} \) describes the following process. Assume that a traditional four-port 50-50 beam splitter, is fed a single photon input in a state in which is an equal superposition of being in each of the two input ports. The value of \( \vec{\phi} \) is the average of expected photo counts behind the exit ports (provided the click at one of the detectors is described as +1 and at the other one as -1), and of course it depends on the relative phase shifts in front of the beam splitter. In other words, this situation describes a Mach-Zehnder interferometer with a single photon input at a chosen input port. For \( N = 3, M = 1 \) the same interpretation applies to the case of a generalised three input, three output Mach-Zehnder interferometer described in \( \vec{\phi} \), provided one ascribes to firings of the three detectors respectively \( \gamma_3 = \alpha \equiv \exp(i \frac{2\pi}{3}) \), \( \alpha^2 \), and \( \alpha^3 \).

The described set of gedanken experiments is rich in EPR-GHZ correlations (for \( N \geq 2 \)). To reveal the above, let us first analyze the conditions (i.e. settings) for such correlations. As the correlation function (\( \vec{\phi} \)) is an average of complex numbers of unit modulus, one has \( |E_{QM}(\vec{\phi}_1, \ldots, \vec{\phi}_M)| \leq 1 \). The equality signals a perfect EPR-GHZ correlation. It is easy to notice that this may happen only if

\[
\sum_{l=1}^{M} \phi_l^{1, 2} = \sum_{l=1}^{M} \phi_l^{2, 3} = \cdots = \sum_{l=1}^{M} \phi_l^{M, 1} = \gamma_N^k,
\]

where \( k \) is an arbitrary natural number. Under this condition \( E(\vec{\phi}_1, \ldots, \vec{\phi}_M) = \gamma_N^k \). This means that only those sets of \( M \) spatially separated detectors may fire, which are ascribed such Bell numbers having the property that their product is \( \gamma_N^k \). Knowing, which detectors fired in the set of \( M - 1 \) observation stations, one can predict with certainty which detector would fire at the sole observation station not in the set.

### IV. GHZ PARADOXES FOR \( N \) MAXIMALY ENTANGLED QUANTS.

Now we show the derivation of GHZ paradoxes for \( N \) maximally entangled quNits. However, first it is instructive to consider the simplest case, which is three qutrits. The correlation function for such an experiment reads

\[
E_{QM}(\vec{\phi}_1, \vec{\phi}_2, \vec{\phi}_3) = \frac{1}{3} \sum_{k=1}^{3} \exp\left(i \sum_{l=1}^{3} (\phi_l^{k} - \phi_l^{k+1}) \right).
\]

The experimenters perform three distinctive experiments. In the first run of the experiment we allow the observers to choose the following settings of the measuring apparatus, \( \vec{\phi}_1 = \vec{\phi}_2 = (0, \frac{2\pi}{3}, \frac{2\pi}{3}) = \vec{\phi}, \vec{\phi}_3 = (0, 0, 0) = \vec{\phi}' \) in the second run they choose \( \vec{\phi}_2 = \vec{\phi}_3 = \vec{\phi}, \vec{\phi}_1 = \vec{\phi}' \) whereas in the third run they fix the local settings of their triters on \( \vec{\phi}_1 = \vec{\phi}_3 = \vec{\phi}, \vec{\phi}_2 = \vec{\phi}' \).

Now, let us calculate the numerical values of the correlation function for each experimental situation. We easily find that for all three experiments this value, due to the special form of the correlation function, is the same and reads

\[
E_{QM}(\vec{\phi}, \vec{\phi}, \vec{\phi}) = E_{QM}(\vec{\phi}', \vec{\phi}, \vec{\phi}) = E_{QM}(\vec{\phi}, \vec{\phi}', \vec{\phi}) = \exp(-\frac{2\pi}{3}) = \alpha^2,
\]

i.e., we observe perfect correlations.
The (deterministic) local hidden variable correlation function for this type of experiment must have the following structure:

\[ E_{HV}(\vec{\phi}_1, \vec{\phi}_2, \vec{\phi}_3) = \int \prod_{k=1}^{3} I_k(\vec{\phi}_k, \lambda) \rho(\lambda) d\lambda. \] (10)

The hidden variable function \( I_k(\vec{\phi}_k, \lambda) \), which determines the firing of the detectors behind the \( k \)-th multiport, depends only upon the local set of phases, and takes one of the three possible values \( \alpha, \alpha^2, \alpha^3 = 1 \) (these values indicate which of the detectors is to fire), and \( \rho(\lambda) \) is the distribution of hidden variables. The local realistic description of the experiment is only possible if the local hidden variable correlation function defined above equals that of quantum ones for the given set of phase shifts.

Because in each experiment described above we observe perfect correlations, i.e., the quantum mechanical correlation function takes value \( \alpha^2 \), the local realistic description is possible if and only if one has

\[ E_{HV}(\vec{\phi}, \vec{\phi}, \vec{\phi}) = E_{HV}(\vec{\phi}', \vec{\phi}, \vec{\phi}) = E_{HV}(\vec{\phi}, \vec{\phi}', \vec{\phi}) = \alpha^2. \]

Taking into account the structure of local hidden variable correlation function one obtains the set of three equations

\[ I_1(\vec{\phi}, \lambda) I_2(\vec{\phi}, \lambda) I_3(\vec{\phi}', \lambda) = \alpha^2 \]
\[ I_1(\vec{\phi}', \lambda) I_2(\vec{\phi}, \lambda) I_3(\vec{\phi}, \lambda) = \alpha^2 \]
\[ I_1(\vec{\phi}, \lambda) I_2(\vec{\phi}', \lambda) I_3(\vec{\phi}, \lambda) = \alpha^2 \] (11)

After multiplication of the above equations one arrives at:

\[ \prod_{k=1}^{3} I_k(\vec{\phi}', \lambda) \prod_{k=1}^{3} I_k(\vec{\phi}, \lambda)^2 = (\alpha^2)^3 = 1, \] (12)

which can be also written in the following form

\[ \prod_{k=1}^{3} I_k(\vec{\phi}', \lambda) = \prod_{k=1}^{3} I_k(\vec{\phi}, \lambda). \] (13)

We have used the property of the hidden variable functions: \( I_k(\vec{\phi}, \lambda)^2 = I_k(\vec{\phi}, \lambda)^* \) for every \( \lambda \). However, because \( E_{QM}(\vec{\phi}', \vec{\phi}', \vec{\phi}') = 1 \) we must also have

\[ \prod_{k=1}^{3} I_k(\vec{\phi}', \lambda) = 1, \] (14)

which, because of (13), gives

\[ \prod_{k=1}^{3} I_k(\vec{\phi}, \lambda) = 1 \] (15)

for every \( \lambda \). This in turn implies that

\[ E_{QM}(\vec{\phi}, \vec{\phi}, \vec{\phi}) = 1, \] (16)

which means that local hidden variables predict perfect correlations for the experiment when all observers set their local settings at \( \vec{\phi} \). However, the quantum prediction is that

\[ E_{QM}(\vec{\phi}, \vec{\phi}, \vec{\phi}) = -\frac{1}{3}. \] (17)

Therefore, we have the contradiction: \( 1 = -\frac{1}{3} \).

This contradiction is of the different type than the one derived in [3] although it has been obtained in the similar way, i.e., the perfect correlations have been used to derive it (equations (11) and (12)). Here local hidden variables imply a certain perfect correlation, which is not predicted by quantum mechanics, whereas in [3] as well as in [4] the contradiction is that both theories predict perfect correlations but of the different type.

Now, we employ the above procedure for the case when we have an arbitrary odd number of multiports and quNits \( N = 2m + 1 \). As we have seen in in the above derivation the crucial point is to find the proper phases.
for the multiports. Let us choose for the first gedanken experiment the following ones \( \vec{\phi}_1 = \vec{\phi}_2 = \vec{\phi}_3 = ... = \vec{\phi}_{2m} = (0, \vec{\phi}, \vec{\phi}, ..., \vec{\phi}) \).

As before, in the next run of the experiment we choose the same phases but we change the role of observers such that in the second run the first one chooses \( \vec{\phi} \) while the rest of them choose \( \vec{\phi}_i \), in the third run the third one chooses \( \vec{\phi} \) while the rest of them choose \( \vec{\phi}_i \). etc. Again, the value of the correlation function for each experiment is the same

\[
E_{QM}(\vec{\phi}, ..., \vec{\phi}) = E_{QM}(\vec{\phi}, ..., \vec{\phi}) = \ldots = E_{QM}(\vec{\phi}, ..., \vec{\phi})
\]

\[
= \frac{1}{2m+1} \left[ 2m \exp \left( -i \frac{2m+1}{2m+1} \pi \right) + \exp \left( i \frac{2m+1}{2m+1} \pi \right) \right] = \gamma_N^{m}
\]

(18)

Using (18) and the structure of the hidden variables correlation function, i.e., \( E_{HV}(\vec{\phi}_1, ..., \vec{\phi}_N) = \int_{\Lambda} \prod_{k=1}^{N} I_k(\vec{\phi}_k, \lambda) \rho(\lambda) d\lambda \), we arrive at the set of \( N \) equations \( I_i(\vec{\phi}, \lambda) \prod_{k \neq l}^{N-1} I_k(\vec{\phi}, \lambda) = \gamma_N^{-m} \) \( (l = 1, 2, \ldots, N) \). After multiplying them by each other we obtain

\[
\prod_{k=1}^{2m+1} I_k(\vec{\phi}, \lambda) = \prod_{k=1}^{2m+1} I_k(\vec{\phi}, \lambda)^{-2m} = \prod_{k=1}^{2m+1} I_k(\vec{\phi}, \lambda)
\]

(19)

for every \( \lambda \). Because \( E_{QM}(\vec{\phi}, ..., \vec{\phi}) = 1 \) one must have \( \prod_{k=1}^{2m+1} I_k(\vec{\phi}, \lambda) = 1 \), which according to (19) gives \( \prod_{k=1}^{2m+1} I_k(\vec{\phi}, \lambda) = 1 \) for every \( \lambda \). Thus, local hidden variables imply the following perfect correlation \( E_{QM}(\vec{\phi}, ..., \vec{\phi}) = 1 \), whereas quantum mechanics gives

\[
E_{QM}(\vec{\phi}, ..., \vec{\phi}) = \frac{1}{2m+1} \left[ 2m \exp \left( -i \frac{2m+1}{2m+1} \pi \right) + \exp \left( i \frac{2m+1}{2m+1} \pi \right) \right] = \frac{-2m+1}{2m+1}.
\]

(20)

Therefore, for each \( m \) one obtains the untruly identity \( 1 = \frac{-2m+1}{2m+1} \), which in the limit of \( m \to \infty \) becomes \( m = -1 \).

For the even number of quNits and the multiports \( N = 2m \) \( (m > 2) \) we choose the following phases: \( \vec{\phi}_1 = \vec{\phi}_2 = \vec{\phi}_3 = ... = \vec{\phi}_{N-1} = (0, \vec{\phi}, \vec{\phi}, ..., \vec{\phi}, \vec{\phi}) = \vec{\phi}, \vec{\phi}_N = (0, 0, 0, \ldots, 0) = \vec{\phi} \). One easily finds that

\[
E_{QM}(\vec{\phi}, ..., \vec{\phi}) = E_{QM}(\vec{\phi}, ..., \vec{\phi}) = \ldots = E_{QM}(\vec{\phi}, ..., \vec{\phi})
\]

\[
= \frac{1}{2m} \left[ (2m-1) \exp \left( -i \frac{2m-1}{2m-1} \pi \right) + \exp \left( i \frac{2m-1}{2m-1} \pi \right) \right] = -1
\]

(21)

and

\[
E_{QM}(\vec{\phi}, ..., \vec{\phi})
\]

\[
= \frac{1}{2m} \left[ (2m-1) \exp \left( -i \frac{2m-1}{2m-1} \pi \right) + \exp \left( i \frac{2m(2m+1)}{2m+1} \pi \right) \right]
\]

\[
= \frac{1}{2m} \left[ (2m-1) \exp \left( -i \frac{2m-1}{2m-1} \pi \right) + 1 \right].
\]

(22)

Applying the same reasoning as above one has \( 1 = \frac{1}{2m} \left[ (2m-1) \exp \left( -i \frac{2m-1}{2m-1} \pi \right) + 1 \right] \). Again, for each \( m > 2 \) one has a contradiction, which in the limit of \( m \to \infty \) becomes \( 1 = -1 \).

V. CONCLUSIONS

We have derived the series of paradoxes in which the number of observers and the dimension of the Hilbert space describing each subsystem is the same. The derivation of the paradoxes relies on the perfect correlations observed in the system but the resulting contradiction between quantum mechanics and local realism manifests itself in the fact that local realism predicts a certain perfect correlation whereas quantum mechanics does not.

An interesting feature of the paradoxes is that they naturally split into two parts: the even and the odd number of observers (input ports). In each part the final contradiction has different numerical values. All paradoxes do not vanish with the growing dimension \( N \) of the Hilbert space of each quNit.

Furthermore, for the case of two observers one cannot derive the GHZ paradoxes with the method presented here (for this case we have so called Hardy paradox but for non-maximally entangled state [5]).
In conclusion we state that the multiport beam splitters, and the idea of value assignment based Bell numbers, lead to a strikingly straightforward generalisation of the GHZ paradox for $N$ entangled quNits. These properties may possibly find an application in future quantum information and communication schemes (especially as GHZ states are now observable in the lab [1]).

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