Protecting quantum systems from decoherence with unitary operations

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Abstract. Decoherence is a fundamental obstacle to the implementation of large-scale and low-noise quantum information-processing devices. We suggest an approach for suppressing errors by employing preprocessing and postprocessing unitary operations, which precede and follow the action of a decoherence channel. In contrast to quantum error correction and measurement-based methods, the suggested approach relies on specifically designed unitary operators for a particular state without the need in ancillary qubits or postselection procedures. We consider the case of decoherence channels acting on a single qubit belonging to a many-qubit state. Preprocessing and postprocessing operators can be either individual, which is acting on the qubit effected by the decoherence channel only, or collective, which is acting on the whole multiqubit state. We give a classification of possible strategies for the protection scheme, analyze them, and derive expressions for the optimal unitary operators providing the maximal value of the fidelity regarding initial and final states. Specifically, we demonstrate the equivalence of the schemes where one of the unitary operations is individual while the other is collective. We then consider the realization of our approach for the basic decoherence models, which include single-qubit depolarizing, dephasing, and amplitude damping channels. We also demonstrate that the decoherence robustness of multiqubit states for these decoherence models is determined by the entropy of the reduced state of the qubit undergoing the decoherence channel.

Keywords: quantum information processing; decoherence; quantum channels.
effect, specific measurements, and many other methods of control. These methods are useful under certain conditions. However, they encounter some difficulties for the experimental realization. A particular task is improving the level of entanglement in a distributed quantum state after its degradation due to losses in communication lines, which can be solved efficiently with the use of quantum catalysis. Unfortunately, the entanglement level increases at the cost of employing postselection.

Here we propose a technique for protecting quantum states from decoherence, which is free from using ancillary information carriers or postselection procedures (see Fig. 1). First, we use a "preprocessing" procedure for preparing a given known quantum state of the system in a specific form. Next, we use a "postprocessing" operation, which follows the action of a "decoherence channel." These operations can be implemented as unitary operators, and their particular form can be efficiently constructed based on prior knowledge of the state under the protection and decoherence channel, i.e., the method is "state-dependent." We study the case of local decoherence channels, which act on a particular qubit of the \( n \)-qubit input state. The preprocessing and postprocessing unitary operators are considered to be either "individual," that is acting on the same qubit, or "collective," that is acting on the whole \( n \)-qubit state. As a model for decoherence processes, we consider single-qubit depolarizing, dephasing, and amplitude damping channels. In our consideration, the main studied characteristic is the fidelity regarding input and output states. We present an analysis of possible strategies for the protection scheme and derive expressions for the optimal unitary operators providing the maximal value of the fidelity regarding initial and final states. Specifically, we show the equivalence of the schemes where one of the unitary operations is individual while the other is collective.

We also consider relations between the forms of the reduced state of the qubit undergoing an individual decoherence process and the losses of the fidelity of the whole \( n \)-qubit state. We demonstrate that in the case of performing preprocessing and postprocessing, this kind of robustness to decoherence is determined by the linear entropy of the reduced state. This feature appears to be crucial in the case where one can select a part of the whole quantum state, which is affected by the decoherence. It is important to note that the suggested scheme is able in principle to supplement existing error correction and error suppression techniques.

Our paper is organized as follows. In Sec. 2, we present a general framework for a method of protecting quantum states from decoherence with unitary operations. In Sec. 3, we apply the presented method for the case of acting basic single-qubit decoherence channels, which include depolarizing, dephasing, and amplitude damping channels. In Sec. 4, we discuss the obtained results and conclude.

## 2 Protecting States from Decoherence with Unitary Operations

Consider a \( n \)-qubit quantum system, which consists of \( n \) subsystems with indices \( 1, \ldots, n \). Let the whole system be initialized in a pure state written in the following form:

\[
|\Psi\rangle_{1, \ldots, n} = \sum_{x \in \{0,1\}^n} c_x |x\rangle_{1, \ldots, n}, \tag{1}
\]

where complex coefficients \( \{c_x\} \) obey a standard normalization condition \( \sum_{x \in \{0,1\}^n} |c_x|^2 = 1 \). Then let a particular \( \kappa \)'th qubit (\( \kappa \in \{1, \ldots, n\} \)) undergo a decoherence process described by
completely positive trace-preserving (CPTP) map $\mathcal{E}$, which we refer to as a decoherence channel. The initial state Eq. (1) then turns into a new (generally mixed) state as follows:

$$\rho_{1\ldots,n}^{(k)} = \text{Id}_{1\ldots,n}/\{k\} \otimes \mathcal{E}_k[\ket\Psi_{1\ldots,n}],$$  

(2)

where $\text{Id}_{1\ldots,n}/\{i\}$ denotes the identical channel acting on all qubits except $k$'th and $\mathcal{E}_k$ is a decoherence channel acting on $k$'th qubit. As the main target characteristic, we consider the fidelity regarding input and output states, which is given as the following expression:

$$F := \langle \Psi_{1\ldots,n} | \rho_{1\ldots,n}^{(k)} | \Psi_{1\ldots,n} \rangle.$$  

(3)

In the general case, the state $\rho_{1\ldots,n}^{(k)}$ has less than one fidelity with respect to $|\Psi_{1\ldots,n}\rangle$.

The main goal of our work is to develop a method for improving the fidelity of the state after decoherence by means of employing unitary operations acting on qubits. We study using two types of unitary operations:

- individual operations acting on $k$'th qubit only and
- collective operations acting on the whole $n$-qubit system.

Specifically, we consider employing two unitary operations just before and after an impact of the decoherence channel $\mathcal{E}$. We refer to these operations as preprocessing and postprocessing unitary operators. Since each of the processing operators can be either individual or collective, we obtain four possible strategies:

1. “both individual” scheme: both operations are individual;
2. “individual-then-collective” scheme: preprocessing operation is individual, whereas the postprocessing operation is collective;
3. “collective-then-individual” scheme: preprocessing operation is collective, whereas the postprocessing operation is individual; and
4. “both collective” scheme: scheme with two collective operations.

We denote $k$'th qubit as $Q$ and the subsystem of all the rest qubits $\{1,\ldots,n\}/\{q_i\}$ as $R$. Using the Schmidt decomposition, the initial state Eq. (1) can be written in the following form:

$$|\Psi\rangle_QR = \sum_{i=0,1} \sqrt{\lambda_i} \ket{\psi_i}_Q \otimes \ket{\zeta_i}_R,$$  

(4)

where a pair of states $\{\ket{\psi_0}, \ket{\psi_1}\}$ forms the orthonormal basis in the Hilbert space of $Q$, $\{\ket{\zeta_0}, \ket{\zeta_1}\}$ are the two orthogonal normalized vectors in the space of $R$, $\lambda_0$ and $\lambda_1$ are the non-negative real numbers such that $\lambda_0 + \lambda_1 = 1$ and $\lambda_0 \geq \lambda_1$, which always can be achieved by an appropriate choice of $\{\ket{\psi_i}\}$ and $\{\ket{\zeta_i}\}$. To characterize the information properties of the considered quantum system, we also employ linear entropy, von Neumann entropy, and min-entropy:

$$S_{\text{lin}}(\rho) := 1 - \text{tr}(\rho^2),$$  

(5)

$$S_{\text{vN}}(\rho) := -\text{tr}(\rho \log \rho),$$  

(6)

$$S_{\text{min}}(\rho) := -\log \Lambda_{\text{max}}(\rho),$$  

(7)

where $\log$ stands for the base-2 logarithm, $\rho$ is a density matrix of arbitrary dimension, and $\Lambda_{\text{max}}(\rho)$ is the operator giving the largest eigenvalue of $\rho$. We note that von Neumann entropy and min-entropy are special cases of the Rényi entropy:
$$S_\alpha(\rho) := \frac{1}{1 - \alpha} \log \text{tr}(\rho^\alpha),$$  \hspace{1cm} (8)

for $\alpha \to 1$ and $\alpha \to +\infty$, correspondingly.

The initial reduced state of qubit $Q$ is as follows:

$$\rho_Q := \text{tr}_R |\Psi\rangle_{QR} \langle \Psi| = \sum_{i=0,1} \lambda_i |\psi_i\rangle_Q \langle \psi_i|.$$  \hspace{1cm} (9)

The considered entropies for the state $\rho_Q$ then take the following forms:

$$S^Q_{\text{vN}} := S_{\text{vN}}(\rho_Q) = -\lambda_0 \log \lambda_0 - (1 - \lambda_0) \log(1 - \lambda_0),$$  \hspace{1cm} (10)

$$S^Q_{\text{lin}} := S_{\text{lin}}(\rho_Q) = 1 - \sum_{i=0,1} \lambda_i^2 = 2\lambda_0(1 - \lambda_0),$$  \hspace{1cm} (11)

$$S^Q_{\text{min}} := S_{\text{min}}(\rho_Q) = -\log \lambda_0.$$  \hspace{1cm} (12)

All these quantities can be expressed via single parameter $\lambda_0$ only, and they monotonously decrease with $\lambda_0$ as it ranges from $1/2$ to 1. Thus, there are one-to-one correspondences between linear entropy, von Neumann entropy, and min-entropy for the qubit case. We also note that for the pure state $|\Psi\rangle_{QR}$, the von Neumann entropy $S^Q_{\text{vN}}$ equals to the entropy of entanglement and the quantum discord$^{27,38-40}$ with respect to the $R-Q$ partitioning.

All four strategies for our protection scheme, which are depicted in Fig. 2, are considered and the corresponding values of maximal achievable fidelity are derived.

### 2.1 Both Individual Scheme

Here we consider the scheme depicted in Fig. 2(a) step by step. First, we apply, as a preprocessing procedure, the following individual unitary operation $U_{\text{ind}}$:

$$U_{\text{ind}} = \sum_{i=0,1} |\psi_i\rangle \langle \psi_i|.$$  \hspace{1cm} (13)

![Fig. 2](image-url)
where \( \{|\phi_i\rangle\} \) is a new orthonormal basis in the qubit space. After acting operation \( U_{\text{ind}} \), the state of the whole system is as follows:

\[
|\Phi\rangle_{QR} = \sum_{i=0,1} \sqrt{\lambda_i} |\phi_i\rangle_Q \otimes |\zeta_i\rangle_R. \tag{14}
\]

Then, after the action of the decoherence channel \( E \), state Eq. (14) transforms as follows:

\[
\rho^\text{dec}_{QR} = \sum_{i,j} E(|\phi_i\rangle_Q \langle \phi_j|) \otimes |\zeta_i\rangle_R \langle \zeta_j|. \tag{15}
\]

As a postprocessing procedure, we apply the second individual unitary operator \( V_{\text{ind}} \) of the following form:

\[
V_{\text{ind}} = \sum_{i=0,1} |\psi_i\rangle \langle \chi_i|, \tag{16}
\]

where \( \{|\chi_i\rangle\} \) is another orthonormal basis.

Using the fact that \( |\zeta_n\rangle \) and \( |\zeta_1\rangle \) are orthogonal and normalized, we obtain the following expression for the resulting fidelity:

\[
F_{\text{ind ind}} = \langle \Psi |_{QR} (V_{\text{ind}} \otimes \mathbb{I}) \rho^\text{dec}_{QR} (V_{\text{ind}} \otimes \mathbb{I}) |\Psi\rangle_{QR}
\]

\[
= \sum_{i,j} \lambda_i \lambda_j |\chi_i\rangle_Q E(|\phi_i\rangle_Q \langle \phi_j|) |\chi_j\rangle_Q. \tag{17}
\]

where \( \mathbb{I} \) stands for \( 2^{n-1} \)-dimensional identity operator acting in the space of the subsystem \( R \).

The maximal value of the fidelity can be obtained by optimization over both unitary operators and can be written in the following form:

\[
F^\text{opt}_{\text{ind ind}} = \max_{U_{\text{ind}}, V_{\text{ind}}} F_{\text{ind ind}} = \max_{\{|\phi_i\rangle\}, \{|\chi_i\rangle\}} \left[ \sum_{i,j} \lambda_i \lambda_j |\chi_i\rangle_Q E(|\phi_i\rangle_Q \langle \phi_j|) |\chi_j\rangle_Q \right]. \tag{19}
\]

We note that \( F^\text{opt}_{\text{ind ind}} \) depends on \( \{\lambda_i\} \) defining the initial reduced state of the qubit \( Q \).

### 2.2 Individual-Then-Collective Scheme

The second scheme, which is depicted in Fig. 2(b), begins with applying the individual unitary operator \( U_{\text{ind}} \) and decoherence channel \( E \) in the same way as in the both individual scheme. We then obtain the same state of the system given in Eq. (15). We can write its spectral decomposition given as the following expression:

\[
\rho^\text{dec}_{QR} = \sum_{i=0}^{2^{n-1}} p_i |\xi_i\rangle_{QR} \langle \xi_i|. \tag{20}
\]

where \( \{|\xi_i\rangle\} \) forms an orthonormal basis in whole \( 2^n \)-dimensional Hilbert space, and we assume that \( p_i \leq p_j \) for \( i > j \). Since we are able to apply an arbitrary unitary transformation \( V_{\text{col}} \) to the whole state of the system, we can achieve the maximal possible fidelity (for a fixed form of \( U_{\text{ind}} \)) equal to the largest eigenvalue of \( \rho^\text{dec}_{QR} \). It can be written using min-entropy of \( \rho^\text{dec}_{QR} \) in the following form:

\[
F^\text{prelim}_{\text{ind col}} = p_0 = \Lambda_{\max} (\rho^\text{dec}_{QR}) = 2^{-S_{\max}(\rho^\text{dec}_{QR})}. \tag{21}
\]

This value of the fidelity can be achieved with \( V_{\text{col}} \) transforming the pure state with the highest eigenvalue \( |\xi_0\rangle_{QR} \) into the initial state \( |\Psi\rangle_{QR} \). Thus, we arrive to the optimal form of \( V_{\text{col}} \) given as...
\[ V_{\text{col}}^{\text{opt}} = |\Psi\rangle \langle \xi_0 | + [. . .], \]  

(22)

where \([. . .]\) stands for any appropriate remaining part of the unitary operator.

The maximal value of the fidelity for the whole individual-then-collective scheme can be obtained by remaining optimization over \(U_{\text{ind}}\):

\[ F_{\text{ind col}}^{\text{opt}} := \max_{U_{\text{ind}}} F_{\text{prelim ind col}} = \max_{U_{\text{ind}}} \left[ 2^{S_{\text{min}}(\rho_{\text{dec}}^{QR})} \right] = \exp \left[ -\ln(2) \min_{\{|\phi_i\rangle\}} S_{\text{min}}(\tilde{\rho}_{\text{dec}}^{QR}) \right]. \]  

(23)

### 2.3 Collective-Then-Individual Scheme

The third scheme, as shown in Fig. 2(c), can be considered as the second scheme in the reverse order. To study this scheme, it is useful to write the action of the map \(E\) via Kraus operators as follows:

\[ E(\rho) = \sum_k A_k \rho A_k^\dagger, \]  

(24)

where \(\{A_k\}\) obeys the standard CPTP condition \(\sum_k A_k A_k^\dagger = 1\), where \(\mathbb{1}\) stands for the identity operator in the qubit space.

The resulting fidelity for some \(U_{\text{col}}\) and \(V_{\text{ind}}\) takes the following form:

\[ F_{\text{col ind}} = \sum_k \langle \Psi |_{QR} (V_{\text{ind}} \otimes \mathbb{1}) A_k U_{\text{col}} |\Psi\rangle_{QR} |\Psi\rangle_{QR} \left( V_{\text{ind}} \otimes \mathbb{1} \right) A_k \rangle_{QR} \]

\[ = \sum_k \langle \Psi |_{QR} U_{\text{col}} A_k (V_{\text{ind}} \otimes \mathbb{1}) \times |\Psi\rangle_{QR} \left( V_{\text{ind}} \otimes \mathbb{1} \right) A_k U_{\text{col}} |\Psi\rangle_{QR}. \]  

(25)

One can see that the resulting expression is the same as the fidelity in the individual-then-collective scheme up to the following change (see Fig. 3):

\[ V_{\text{col}} \leftrightarrow U_{\text{col}}^\dagger, \quad U_{\text{ind}} \leftrightarrow V_{\text{ind}}^\dagger, \quad E \leftrightarrow \tilde{E}, \]  

(26)

where \(\tilde{E}\) is a conjugate map to \(E\) of the following form:

\[ \tilde{E}[\rho] := \sum_k A_k^\dagger \rho A_k. \]  

(27)

The maximal fidelity in the collective-then-individual scheme is given by the similar expression as in Eq. (23):

\[ F_{\text{col ind}}^{\text{opt}} := \exp \left[ -\ln(2) \min_{\{|\phi_i\rangle\}} S_{\text{min}}(\tilde{\rho}_{\text{dec}}^{QR}) \right], \]  

(28)

where

\[ \tilde{\rho}_{\text{dec}}^{QR} := \sum_{i,j} \tilde{E}(\langle \phi_i |_Q \langle \phi_j |_Q \otimes | \zeta_j \rangle_R \langle \zeta_j |_R), \]  

(29)

\[ |\Psi\rangle \]
\[ |\Psi\rangle \]
\[ U_{col} \]
\[ \tilde{E} \]
\[ \tilde{E} \]
\[ V_{\text{ind}} \]
\[ V_{\text{ind}} \]
\[ U_{\text{col}}^\dagger \]

**Fig. 3** Equivalence between collective-then-individual and individual-then-collective schemes.
We then show that the maximal fidelities given in Eqs. (28) and (23) are the same. Let $|\xi^{\text{opt}}\rangle$ and $\{|\phi_i^{\text{opt}}\rangle\}$ are such that $F_{\text{ind col}}^{\text{opt}}$ achieves its maximal value of the following form:

$$F_{\text{ind col}}^{\text{opt}} = \sum_{k,i,j} \lambda_i \lambda_j \langle \xi^{\text{opt}} | R (A_k |\phi_i^{\text{opt}}\rangle \langle \phi_j^{\text{opt}} | A_k \otimes | \xi_j^{R} \rangle) | \xi^{\text{opt}} \rangle R.$$  

(30)

One can see that $|\xi^{\text{opt}}\rangle_R$ is an eigenvector corresponding to the largest eigenvalue of $\rho_{\text{dec}}^{\text{opt}}$ obtained after optimal individual operation defined by $\{|\phi_i^{\text{opt}}\rangle\}$.

We introduce matrix elements of Kraus operators in the basis of $\{|\phi_i^{\text{opt}}\rangle\}$:

$$a_{ij}^\xi := \langle \phi_i^{\text{opt}} | A_k |\phi_j^{\text{opt}}\rangle,$$

(31)

and also introduce the overlap between $|\xi^{\text{opt}}\rangle$ and the vector set $\{|\phi_i^{\text{opt}}\rangle \otimes | \xi_j \rangle \rangle_i$:

$$C_{ij} := (\langle \phi_i^{\text{opt}} | \otimes | \xi_j \rangle | \xi^{\text{opt}} \rangle).$$

(32)

Then expression Eq. (30) takes the following form:

$$F_{\text{ind col}}^{\text{opt}} = \sum_{k,i,j,m,n} \lambda_i \lambda_j C_{mi} a_{mi} a_{nj} C_{nj} = \sum_k \left( \sum_{i,m} \lambda_i C_{mi} a^k_{mi} \right) \left( \sum_{j,n} \lambda_j C_{nj} a^k_{nj} \right)$$

$$= \sum_k \left| \sum_{i,m} \lambda_i C_{mi} a^k_{mi} \right|^2.$$

(33)

Let us then return to the collective-then-individual scheme. As we demonstrated above, it is equivalent to individual-then-collective with the only difference that the map $\tilde{E}$ is changed by the dual map $\tilde{E}$. One can write the fidelity of the collective-then-individual scheme in the form similar to Eq. (30):

$$\tilde{F}_{\text{col ind}} = \sum_{k,i,j} \lambda_i \lambda_j \langle \tilde{\xi}^{\text{opt}} | R (A_k |\tilde{\phi}_i\rangle \langle \tilde{\phi}_j | A_k \otimes | \tilde{\xi}_j \rangle) | \tilde{\xi}^{\text{opt}} \rangle.$$

(34)

where $|\tilde{\xi}\rangle$ is some $2^m$-dimensional normalized vector and $\{|\tilde{\phi}_i\rangle\}$ is an orthonormal basis in qubit space. We can set

$$|\tilde{\phi}_i\rangle := |\phi_i^{\text{opt}}\rangle,$$

(35)

and choose $|\tilde{\xi}\rangle$ such that

$$\langle \phi_i^{\text{opt}} | \tilde{\phi}_j \rangle | \tilde{\xi} \rangle = C_{ji}.$$

(36)

Then we obtain the following expression for the fidelity:

$$\tilde{F}_{\text{col ind}} = \sum_{k,i,j,m,n} \lambda_i \lambda_j C_{mi} a^k_{mi} a^k_{nj} C_{nj} = \sum_k \left| \sum_{j,n} \lambda_j C_{nj} a^k_{nj} \right|^2 = F_{\text{ind col}}^{\text{opt}}.$$  

(37)

On the one hand, we have the following expression:

$$F_{\text{col ind}}^{\text{opt}} \geq \tilde{F}_{\text{col ind}} = F_{\text{ind col}}^{\text{opt}}.$$  

(38)

On the other hand, by employing a similar line reasoning we can obtain $F_{\text{col ind}}^{\text{opt}} \geq F_{\text{col ind}}^{\text{opt}}$. Finally, we arrive at the following relation: $F_{\text{col ind}}^{\text{opt}} = F_{\text{col ind}}^{\text{opt}}$, i.e., there is a full equivalence of the performance of both schemes. The observed identity reveals a time-symmetry aspect of the transformation. That is why, of all our subsequent considerations, we will take into account the individual-then-collective scheme only since all the results for collective-then-individual scheme can be obtained from it straightforwardly.
2.4 Both Collective Scheme

Finally, we consider a scheme, which is presented in Fig. 2(d). Although this scheme may seem far from practical application, it is important from a theoretical point of view since it provides an upper bound on the fidelity level achieved in the considered class of schemes.

We can use any operator \( U_{\text{col}} \) in order to "reprepare" any desired state before acting of the decoherence channel. In this way, the optimal strategy is to choose \( U_{\text{col}} \) such that the resulting pure state is affected by \( \mathcal{E} \) as less as possible. Since \( \mathcal{E} \) acts individually on \( Q \), the best fidelity can be achieved with \( U_{\text{col}} \) transforming \( |\Psi\rangle_{QR} \) in a separable state of the following form:

\[
U_{\text{col}} |\Psi\rangle_{QR} = |\gamma\rangle_Q \otimes |\xi\rangle_R.
\]

with some normalized pure states \( |\gamma\rangle_Q \) and \( |\xi\rangle_R \) in two-dimensional (2-D) and \( 2^n - 1 \)-dimensional Hilbert spaces correspondingly.

Since the second unitary \( V_{\text{col}} \) can turn any pure state into \( |\Psi\rangle \), the best final fidelity will be achieved with \( |\gamma\rangle \) minimizing a min-entropy of \( \mathcal{E}(|\gamma\rangle \langle \gamma|) \), denoted as

\[
|\gamma^{\text{opt}}\rangle := \arg \min_{|\gamma\rangle} S_{\text{min}}[\mathcal{E}(|\gamma\rangle \langle \gamma|)].
\]

We note that the minimum for min-entropy is the same as a minimum for linear and von Neumann entropies since \( \mathcal{E} \) is a qubit channel:

\[
|\gamma^{\text{opt}}\rangle = \arg \min_{|\gamma\rangle} S_{\text{min}}[\mathcal{E}(|\gamma\rangle \langle \gamma|)] = \arg \min_{|\gamma\rangle} S_{\text{vN}}[\mathcal{E}(|\gamma\rangle \langle \gamma|)].
\]

We then obtain the following expression for the maximal fidelity as follows:

\[
F_{\text{opt}}^{\text{col}} = \Lambda_{\text{max}}[\mathcal{E}(|\gamma^{\text{opt}}\rangle \langle \gamma^{\text{opt}}|)] = \exp\{\ln(2)S_{\text{min}}[\mathcal{E}(|\gamma^{\text{opt}}\rangle \langle \gamma^{\text{opt}}|)]\}.
\]

This value is achieved under the action of unitary operators of the following form:

\[
U_{\text{col}} = |\gamma^{\text{opt}}\rangle \otimes |\xi\rangle \langle \gamma| + [...],
\]

\[
V_{\text{col}} = |\Psi\rangle \langle \Theta| \otimes |\xi\rangle \langle \gamma| + [...],
\]

where \( |\Theta\rangle \) stands for an eigenstate of \( \mathcal{E}(|\gamma^{\text{opt}}\rangle \langle \gamma^{\text{opt}}|) \) corresponding to largest eigenvalue \( \Lambda_{\text{max}}[\mathcal{E}(|\gamma^{\text{opt}}\rangle \langle \gamma^{\text{opt}}|)] \) and [...] stands for the arbitrary appropriate remaining part of the unitary operators.

In conclusion of the section, we note the following relation between all the considered approaches:

\[
F_{\text{opt}}^{\text{ind}} \leq F_{\text{opt}}^{\text{col}} \leq F_{\text{opt}}^{\text{ind}} = F_{\text{opt}}^{\text{col}}.
\]

It holds true since the scheme with two individual operations is a particular case for individual-then-collective (or collective-then-individual) scheme, and the latter schemes are particular cases for the scheme with two collective operations.

3 Applications for the Basic Decoherence Channels

Here we apply the above-described schemes to main decoherence models, particularly depolarizing, dephasing, and amplitude damping quantum channels. We derive the values of maximal achievable fidelities depending on the strength of particular decoherence channel and the structure of the reduced state \( \rho_Q \) of the qubit affected by this channel. Without loss of generality, we consider the initial state \( |\Psi\rangle_{QR} \) of the quantum system in the following form:

\[
|\Psi\rangle_{QR} = \sum_{i=0,1} \sqrt{\lambda_i} |i\rangle_Q \otimes |\xi_i\rangle_R.
\]
where $|0\rangle$ and $|1\rangle$ form a standard computational basis. It can be done since one can always add an individual unitary operator to the preprocessing operator, which rotates the eigenstates of the reduced state $\rho_Q$ to any set of orthonormal states.

### 3.1 Depolarizing Channel

The action of depolarizing channel on an arbitrary qubit state $\rho$ is as follows:

$$\mathcal{E}(\rho) = (1 - p)\rho + \frac{p}{2} \mathbb{1} \text{tr}(\rho), \quad (47)$$

where $\mathbb{1}$ is the 2-D identity matrix and $p \in [0,1]$ is a parameter describing a depolarization strength. It can be interpreted as turning the state into the maximally mixed state $\mathbb{1}/2$ with probability $p$, and leaving it untouched with probability $1 - p$.

#### 3.1.1 Both individual scheme

For any qubit unitary operator $u$, we have the following property of the depolarizing channel:

$$\mathcal{E}(u \rho u^\dagger) = u \mathcal{E}(\rho) u^\dagger. \quad (48)$$

That is why one can set $U_{\text{ind}} := 1$ in both individual as well as individual-then-collective scheme since all the necessary operations can be released with the use of postprocessing unitary operator. The state after decoherence takes the following form:

$$\rho_{QR}^{\text{dec}} = (1 - p)|\Psi\rangle_{QR} \langle \Psi| + p^\frac{1}{2} \mathbb{1} \sum_i \lambda_i |\zeta_i\rangle_R \langle \zeta_i|. \quad (49)$$

One can easily check that the maximal fidelity in the both individual scheme is achieved with $V_{\text{ind}} = 1$, and it is as follows:

$$F_{\text{ind ind}}^{\text{opt}} = (1 - p) + p_{\frac{1}{2}} \sum_i \lambda_i^2 = 1 - p - (1 + S_{\text{lin}}^Q). \quad (50)$$

where $S_{\text{lin}}^Q$ is a linear entropy of the reduced state of $Q$ in the initial state [see Eq. (11)]. Thus, we obtain that the fidelity decreases linearly with the growth of decoherence strength and linear entropy, as shown in Fig. 4(a1).

#### 3.1.2 Individual-then-collective scheme

The state after the depolarizing channel in the individual-then-collective scheme with $U_{\text{ind}} := 1$ is given in Eq. (49). The optimal fidelity in the considered scheme equals to its largest eigenvalue and it is as follows:

$$F_{\text{ind col}}^{\text{opt}} = \frac{1}{2} - \frac{1}{4} p + \frac{1}{4} \sqrt{(p - 2)^2 - 2p(4 - 3p)S_{\text{lin}}^Q}. \quad (51)$$

The behavior of the fidelity $F_{\text{ind col}}^{\text{opt}}$ is presented in Fig. 4(a1), and the difference $F_{\text{ind col}}^{\text{opt}} - F_{\text{ind ind}}^{\text{opt}}$ is shown in Fig. 4(a2). We note that both fidelities $F_{\text{ind ind}}^{\text{opt}}$ and $F_{\text{ind col}}^{\text{opt}}$ decrease as $S_{\text{lin}}^Q$ increases. The difference between fidelities obtained in the individual-then collective scheme and the both individual scheme increases with $p$ and achieves its maximum for $p = 1$ and $S_{\text{lin}}^Q \approx 0.375$.

The subtle question is how to obtain the optimal form of the postprocessing operator $V_{\text{col}}$. This operator has to turn the eigenstate $|\tilde{\zeta}_0\rangle_{QR}$, which corresponds to the largest eigenvalue of $\rho_{QR}^{\text{dec}}$ to the initial state $|\Psi\rangle_{QR}$. For the state given in Eq. (49), we have the following representation of $|\tilde{\zeta}_0\rangle_{QR}$ in the basis $\{|0\rangle_Q \otimes |\zeta_0\rangle_R, |0\rangle_Q \otimes |\zeta_1\rangle_R, |1\rangle_Q \otimes |\zeta_0\rangle_R, |1\rangle_Q \otimes |\zeta_1\rangle_R\}$:
with

\[ \nu = \arctan \frac{2F_{\text{ind col}} - (2 - p)\lambda_0}{2(1 - p)\sqrt{\lambda_0^2/2}}. \]  

To transform \(|\xi\rangle_{QR}\) into \(|\Psi\rangle_{QR}\), which in the same basis has the following form:

\[ |\Psi\rangle_{QR} = \begin{bmatrix} \sqrt{\lambda_0} \\ 0 \\ 0 \\ \sqrt{\lambda_1} \end{bmatrix}, \]  

one can employ the operator as follows:

\[ V_{\text{col}} = W(2\gamma). \]

where

\[ W(\theta) := \begin{bmatrix} \cos(\theta/2) & 0 & 0 & \sin(\theta/2) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta/2) & 0 & 0 & \cos(\theta/2) \end{bmatrix}. \]
with
\[ \gamma := -\arccos \langle \xi_0 | \Psi \rangle = -\arccos \left( \sqrt{\lambda_0} \cos v + \sqrt{\lambda_1} \sin v \right). \] (57)

The operator \( W(\theta) \) can be constructed with a circuit presented in Fig. 5, where we use the following notations:

\[ X := |1\rangle \langle 0| + |0\rangle \langle 1|, \quad X := |\xi_1\rangle \langle \xi_0| + |\xi_0\rangle \langle \xi_1|. \] (58)

The rotation operation around \( y \) axis of the Bloch sphere,

\[ R_y(\theta) = \begin{bmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix}, \] (59)

in the central controlled-unitary gate is assumed to be performed if the subsystem \( R \) is in the state \( |\Psi_1\rangle \).

### 3.1.3 Both collective scheme

Since in the case of depolarizing channel the action of individual unitary operation cannot affect the resulting entropy, as it follows from the property Eq. (48), we can set the preprocessing unitary as follows:

\[ U_{col} := W(2\varsigma), \quad \varsigma := \arctan \sqrt{\lambda_1/\lambda_0}, \] (60)

which turns the state \( |\Psi\rangle_{QR} \) into the state \( |0\rangle_Q |\xi_0\rangle_R \). Then after acting the depolarizing channel, we obtain:

\[ \rho_{QR}^{\text{dec}} = \left( 1 - \frac{p}{2} \right) |0\rangle_Q \langle 0|_Q + \frac{p}{2} |1\rangle_Q \langle 1|_Q \otimes |\xi_0\rangle_R \langle \xi_0|, \] (61)

with the largest eigenvalue equal to \( 1 - p/2 \) and corresponding eigenvector \( |0\rangle_Q |\xi_0\rangle_R \). Thus, finally, we have the following expression:

\[ F_{\text{col col}}^{\text{opt}} = 1 - \frac{p}{2}. \] (62)

It is achieved if the operators are as follows:

\[ V_{col} := U_{col}^\dagger = W(-2\varsigma). \] (63)

where \( \varsigma \) is given in Eq. (60).

The unavoidable decrease of the fidelity can be explained by an unavoidable entropy production effect inherent to the depolarizing channel. The behavior of \( F_{\text{col col}}^{\text{opt}} \) is also presented in Fig. 4(a1).
3.2 Dephasing Channel

The action of the dephasing channel on a qubit state $\rho$ has the following form:

$$\mathcal{E}(\rho) = (1 - p)\rho + p(\rho_{00}|0\rangle\langle 0| + \rho_{11}|1\rangle\langle 1|),$$

(64)

where $\rho_{ii} = \langle i|\rho|i\rangle$ are the diagonal elements of $\rho$ and $p \in [0,1]$ is again a strength of decoherence. One can note that the dephasing channel destroys nondiagonal elements of the density matrix $\rho$ with probability $p$ and does not change the state with probability $1 - p$. In contrast to the depolarizing channel, we see that the dephasing channel does not affect the diagonal states. This feature can be efficiently employed in protection schemes.

3.2.1 Both individual scheme

It is easy to check that the optimal fidelity for the both individual scheme is achieved for $U_{\text{ind}} = V_{\text{ind}} = 1$ since these operations left the reduced state of $Q$ in the safe diagonal form. For the identity preprocessing operator, we obtain the state after dephasing as follows:

$$\rho_{\text{dec}Q} = (1 - p)|\Psi\rangle_{QR}\langle \Psi| + p\sum_{i}\lambda_{i}|i\rangle_{Q}\langle i|\xi_{R}\langle \xi|.$$  

(65)

After applying the identity, postprocessing the value of fidelity takes the following form:

$$F_{\text{opt ind ind}} = (1 - p) + p\sum_{i}\lambda_{i}^{2} = 1 - pS_{\text{lin}}^{Q},$$

(66)

One can see that in the case of the depolarizing channel [Eq. (50)], the fidelity in dephasing channel decreases linearly with a growth of $p$ and $S_{\text{lin}}^{Q}$ [see also Fig. 4(b1)].

3.2.2 Individual-then-collective scheme

In the individual-then-collective scheme, the optimal individual operator is again the identity operator $U_{\text{ind}} = 1$, which provides the state after decoherence given in Eq. (65). Its maximal eigenvalue determines the maximal fidelity as follows:

$$F_{\text{ind col}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - 2p(2 - p)S_{\text{lin}}^{Q}},$$

(67)

and the corresponding eigenstate has the same form as in Eq. (52) with the only difference that

$$\upsilon := \arctan\frac{F_{\text{opt ind col}} - \lambda_{0}}{(1 - p)\sqrt{S_{\text{lin}}^{Q}/2}}.$$  

(68)

Thus, the optimal collective postprocessing operator is given in Eq. (55) with $\gamma$ calculated using Eq. (57) with the updated value of $\upsilon$ from Eq. (68).

We demonstrate the behavior of the fidelity $F_{\text{ind col}}$ in Fig. 4(b1) together with the difference $F_{\text{ind col}} - F_{\text{ind ind}}$ in Fig. 4(b2). One can see that the maximal difference in the case of the dephasing is two times higher than the one for depolarizing channel and is achieved at the same point $p = 1$, $S_{\text{lin}}^{Q} \approx 0.375$.

3.2.3 Both collective scheme

One can see that the state $|0\rangle_{Q}\xi_{0}\rangle_{Q}$ is not affected by dephasing channels, and that is why we can employ the preprocessing and postprocessing collective operators in the forms given in Eqs. (60) and (63), correspondingly. They allow achieving the maximal fidelity $F_{\text{opt col col}} = 1$. 

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3.3 Amplitude Damping Channel

We complete our consideration of the basic decoherence models with an amplitude damping channel. We can write its action in terms of Kraus operators:

\[
A_1 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{bmatrix}.
\] (69)

Here \(p \in [0,1]\) is again a decoherence strength. The amplitude damping process corresponds to the relaxation of the “excited” state \(|1\rangle\) to the “ground” state \(|0\rangle\). We note that there is an apparent asymmetry between the behavior of \(|0\rangle\) and \(|1\rangle\): \(|0\rangle\) remains untouched by the channel, whereas \(|1\rangle\) undergoes a transformation to \(|0\rangle\). Here we would like to remember that according to our convention we have \(\lambda_0 \geq \lambda_1\) in the initial state [Eq. (46)]. It corresponds to the fact that we assume that \(Q\) has the ground level to be populated more than the excited one.

3.3.1 Both individual scheme

The optimal fidelity in the both individual scheme is achieved for the \(U_{\text{ind}}\) and \(V_{\text{ind}}\) being equal to identity operators: \(U_{\text{ind}} = V_{\text{ind}} = 1\). The state after the decoherence takes the following form:

\[
\rho_{QR}^{\text{dec}} = \begin{bmatrix}
\lambda_0 & 0 & 0 & \sqrt{\lambda_0 \lambda_1 \sqrt{p}} \\
0 & p \lambda_1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\sqrt{\lambda_0 \lambda_1 \sqrt{p}} & 0 & 0 & \sqrt{p} \lambda_1
\end{bmatrix},
\] (70)

where \(p = 1 - p\) and the matrix is written in the basis \{\(|0\rangle \otimes |ζ_0\rangle_R, |0\rangle \otimes |ζ_1\rangle_R, |1\rangle \otimes |ζ_0\rangle_R, |1\rangle \otimes |ζ_1\rangle_R\}\.

The resulting fidelity takes the following form:

\[
F_{\text{ind-ind}}^{\text{opt}} = \lambda_0^2 + \lambda_1^2 - \lambda_1^2 p + 2 \lambda_0 \sqrt{1 - p \lambda_1}.
\] (71)

By using the substitution,

\[
\lambda_0 = \frac{1}{2} + \frac{1}{2} \sqrt{1 - 2S_{\text{lin}}^Q},
\] (72)

we obtain the following expression:

\[
F_{\text{ind-ind}}^{\text{opt}} = \sqrt{1 - p S_{\text{lin}}^Q} + \left(1 - \frac{p}{2}\right) \left(1 - S_{\text{lin}}^Q\right) + \frac{p}{2} \sqrt{1 - 2S_{\text{lin}}^Q}.
\] (73)

We see that the fidelity decreases with the growth of \(S_{\text{lin}}^Q\) like in the case of depolarizing and dephasing channels, as shown in Fig. 4(b1).

3.3.2 Individual-then-collective scheme

In the individual-then-collective scheme, the best fidelity is also achieved for \(U_{\text{ind}} = 1\). The maximal fidelity is given by the largest eigenvalue of state [Eq. (70)]:

\[
F_{\text{ind-col}}^{\text{opt}} = 1 - p(1 - \lambda_0) = 1 - p \left(1 - \frac{1}{2} - \frac{1}{2} \sqrt{1 - 2S_{\text{lin}}^Q}\right).
\] (74)

The corresponding eigenstate has the same form as in Eq. (52) with

\[
\nu := \arctan \left[ \lambda_1 \sqrt{\frac{2(1 - p)}{S_{\text{lin}}^Q}} \right].
\] (75)
The optimal collective postprocessing operator is then given by the same expression [Eq. (55)] with $\gamma$ calculated by Eq. (57) with updated value of $\nu$ given in Eq. (75).

We show the behavior of $F_{\text{ind, col}}$ in Fig. 4(c1). The corresponding difference $F_{\text{ind, col}} - F_{\text{ind, ind}}$ is presented in Fig. 4(c2). The maximal advantage of individual-then-collective (collective-then-individual) schemes over both individual schemes in the case of the amplitude damping channel is two times higher than the one for dephasing channel and is achieved for $p = 1$ and maximally mixed reduced state of $Q$ with $S_{\text{lin}}^Q = 0.5$.

### 3.3.3 Both collective schemes

As we have already emphasized above, the state $|0\rangle$ is not affected by the amplitude damping channel, and that is why we can employ the preprocessing and postprocessing collective operators in the form given in Eqs. (60) and (63), correspondingly. As in the case of dephasing channel, they allow achieving the maximal unit fidelity $F_{\text{col, col}} = 1$.

### 4 Discussion and Conclusion

We summarize the main results of this work. We have suggested the method for improving fidelity based on the class of state-dependent operations protecting the system from decoherence. Specifically, we have considered the case where the decoherence affects a single $\kappa$’th qubit from an $n$-qubit pure state while the operators can be employed either on the same $\kappa$’th qubit or on the whole state. We have shown that two schemes of this class, namely individual-then-collective and collective-then-individual, provide the same maximal achievable levels of the fidelity.

We have considered these schemes for three basic decoherence models given by depolarizing, dephasing, and amplitude damping channels. The main results on the comparison between all the strategies of error suppression for various decoherence models are summarized in Table 1. For all the channels, we have seen that for given decoherence strength the maximal fidelities in all the schemes except both collective is expressed with the linear entropy of the qubit under decoherence $S_{\text{lin}}^Q$. In particular, we have obtained that the larger the linear entropy, the lower is the fidelity.

This feature provides an answer to the question: which qubit from the whole $n$-qubit system is the most vulnerable and which qubit is the most robust in the sense of preserving the whole $n$-qubit state after qubit decoherence. It turns out that the best choice for $\kappa$ is defined by the qubit whose reduced state has the lowest linear entropy. For the qubit case, the reduced state with the lowest linear entropy is automatically the state with the lowest min-entropy, von Neumann entropy, entropy of entanglement, and quantum discord. A promising direction for future research may be also to consider other various measures of decoherence.

The obtained results surely have a clear intuitive explanation since the more the entropy of the reduced state of the qubit is, the more it is entangled to remaining qubits of the system, and the more it is informationally important to the whole state. However, we would like to mention that one should be very careful with such kind of intuitive conclusions. For example, if we turn to speak about quantum correlations (say, entanglement), as it was shown in Refs. 24–26, in the

| Table 1 | The maximal achievable fidelities in all the considered approaches and decoherence models as functions of decoherence strength $\rho$ and initial linear entropy of the reduced state of the decohered qubit $S_{\text{lin}}^Q$. |
|---------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|
|         | Both individual | Individual-then-collective and collective-then-individual | Both collective |
| Depolarizing | $1 - p(1 + S_{\text{lin}}^Q)/2$ | $1/2 - p/4 + \sqrt{(p - 2)^2 - 2p(4 - 3p)S_{\text{lin}}^Q}/4$ | $1 - p/2$ |
| Dephasing  | $1 - pS_{\text{lin}}^Q$ | $1/2 + \sqrt{1 - 2p(2 - p)S_{\text{lin}}^Q}/2$ | $1$ |
| Amplitude damping | $\sqrt{1 - pS_{\text{lin}}^Q} + (1 - p/2)(1 - S_{\text{lin}}^Q) + p\sqrt{1 - 2S_{\text{lin}}^Q}/2$ | $1 - p(1 - \sqrt{1 - 2S_{\text{lin}}^Q})/2$ | $1$ |

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case of depolarizing channel and mixed two-qubit state, the state with the largest entropy may be more robust in the sense of preserving initial correlations.

Finally, we would like to mention that the considered schemes seem to be effective in the framework of quantum communication protocols, where several parties employ an entangled state distributed between them. If the communication channels have a different degree of decoherence, then the whole protocol can be adjusted in such a way that the most robust parts of the entangled state go through the noisiest communication paths.

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