Quark number susceptibilities and color screening at high temperatures

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Abstract. We discuss lattice calculations of quark number susceptibilities and the free energy of a static quark anti-quark pair in 2+1 flavor QCD at high temperatures using the HISQ action. We compare our lattice calculations with perturbative results.

1. Introduction

At high temperatures strongly interacting matter undergoes a deconfining transition to a new state, where the dominant degrees of freedom are quarks and gluons. At sufficiently high temperatures quarks and gluons are weakly interacting as a consequence of asymptotic freedom and the deconfined matter is called quark-gluon plasma in analogy with electromagnetic plasmas (for a recent review of this subject see Ref. [1]). The deconfinement transition is signaled by rapid increase in the energy density and the entropy density, indicating liberation of many degrees of freedom. Quark number susceptibilities are also a good probes of deconfinement as they show a rapid rise at the transition and at sufficiently high temperature are dominated by quark degrees of freedom [2, 3, 4, 5, 6, 7, 8, 9].

The deconfinement can also be characterized by onset of color electric screening that is analogous to the usual Debye screening\(^1\). Color screening can be studied in terms of the free energy of static quark anti-quark pair, which on the lattice is calculated in terms of the Polyakov loop correlators, or at asymptotically large distances in terms of the expectation value of the Polyakov loop [12].

In lattice QCD calculations at non-zero temperature discretization effects can be large. Therefore it is important to perform the calculations using improved actions at several lattice spacings. Because of relatively small computational costs staggered quark formulation is often used for these calculations. In this contribution we report on lattice calculations of the quark number susceptibilities and the free energy of static quark anti-quark pair using tree-level improved gauge action and highly improved staggered quark (HISQ) action on lattices with the temporal extent \(N_\tau = 4, 6, 8, 10\) and 12. The choice of the actions and of the simulation parameters is described in Ref. [13]. We extended the calculations for \(N_\tau = 6, 8\) and 12 to higher temperatures and also added calculations for \(N_\tau = 4\) and 10.

2. Quark number susceptibilities

Fluctuations and correlations of conserved charges are good probes of deconfinement because they are sensitive to the underlying degrees of freedom, i.e. they can tell whether the relevant

\(^1\) QCD and non-Abelian plasmas in general also screen static magnetic fields, see e.g. Refs. [10, 11] for discussions.
degrees of freedom of the system at a given temperature are hadronic or partonic. Here we consider quadratic fluctuations of light and strange quark numbers, also known as quark number susceptibilities

$$\chi_i(T) = \frac{1}{T^3V} \frac{\partial^2 \ln Z(T, \mu_i)}{\partial (\mu_i/T)^2} \bigg|_{\mu_i=0}.$$  

(1)

(2)

Here $Z(T, \mu_i)$ is the partition function and indices $i = l$ and $s$ stand for light and strange quark respectively. The numerical results for the strange and light quark number susceptibilities obtained with the HISQ action are shown in Fig. 1. The results for the strange quark number susceptibility for $T < 250$ MeV and $N_\tau = 6$, 8 and 12 are taken from Ref. [13]. We compare the strangeness fluctuations with the hadron resonance gas model including all resonances with masses smaller than 2.5 GeV. We also compare our results with lattice calculations performed with the stout action [9]. At low temperatures the lattice results agree with hadron resonance gas, while at high temperatures they approach a constant value which is 10% below the value for the ideal massless quark gas. Thus the temperature dependence of the strange quark number susceptibility shows the expected transition from hadronic degrees of freedom to quark degrees of freedom. For temperatures $T > 300$ MeV the difference between the strange and light quark number susceptibilities is quite small, i.e. less than 5%. For this reason the comparison with massless ideal gas is appropriate. In the right panel we show the light quark number susceptibility at high temperatures obtained with the HISQ action for $N_\tau = 6$, 8, 10 and 12. We also compare our results with the continuum-extrapolated stout data as well as with resummed perturbative calculations in next-to-leading log approximation (NLA) [14]. The $N_\tau = 8$ HISQ data are in good agreement with the continuum extrapolated stout data. On the other hand the $N_\tau = 10$ and 12 data are larger than the stout data. This may imply that for the HISQ action the continuum limit is approached from below and the continuum extrapolation for the stout action in Ref. [9] is not fully reliable. The $N_\tau = 12$ HISQ results agree well with the NLA perturbative results at the highest temperature. The $N_\tau = 6$ HISQ lattice data are significantly lower than other lattice results. This is due to cutoff effects which are large for $N_\tau = 6$. In fact, at high temperatures the cutoff effects in $N_\tau = 6$ data follow qualitatively the expectations based on the free field theory calculations [2, 4]. At quantitative level, however, the cutoff dependence is smaller than for the free quark gas as already pointed out in calculations with the p4 action [6]. It should be mentioned that the small deviations of quark number susceptibilities from the ideal gas value is indicative of the weakly interacting nature of the deconfined state. For strongly interacting plasma fluctuations of conserved charges are about half smaller than for the ideal gas [15].

3. Free energy of static quarks

In $SU(N)$ gauge theories there is a true deconfining phase transition related to $Z(N)$ symmetry. The order parameters of this phase transition are the expectation value of the Polyakov loop and the Polyakov loop correlator

$$L(T) = \langle \frac{1}{N_c} \text{Tr} W(\vec{x}) \rangle, \quad W(\vec{x}) = \prod_{\tau=0}^{N_\tau-1} U_0(\tau, \vec{x}),$$  

(3)

$$C_{PL}(r, T) = \frac{1}{N_c^2} \langle \text{Tr} W^\dagger(r) \text{Tr} W(0) \rangle.$$  

(4)

The Polyakov loop transforms non-trivially under $Z(N)$ transformation and the expectation value of the Polyakov loop is zero in the confined phase and non-zero above the transition
Figure 1. The strange (left) and light quark number (right) susceptibilities as function of the temperature calculated with the HISQ action. The solid black line in the left panel shows the results of the hadron resonance gas model. The band in the right panel shows the resummed perturbative results for the light quark number susceptibility in NLA [14]. We also show the continuum-extrapolated results for the stout action [8, 9].

The correlation function of the Polyakov loop is related to the free energy of a static quark anti-quark pair [12]. The qualitative change in the behavior of the Polyakov loop and its correlator above the phase transition temperature is related to color screening [12]. As it was pointed out already in Ref. [12] the Polyakov loop and the Polyakov loop correlator require renormalization to be interpreted as the free energy of an isolated static quark or the free energy of a static quark anti-quark, $(Q\bar{Q})$ pair. More precisely, they are related to the free energy difference of a system with static $Q\bar{Q}$ pair at some temperature and the system without $Q\bar{Q}$ pair at the same temperature. Since in the zero temperature limit the free energy of a static quark anti-quark pair should coincide with the static potential, the Polyakov loop renormalization is determined by the normalization constant of the static potential, namely

$$L_{\text{ren}}(T) = \exp(-c/(2T))L(T) = \exp(-F_{\infty}(T)/(2T)), \quad (5)$$

$$C_{PL}(r, T) = \exp(-F(r, T)/T + c/T), \quad F_{\infty}(T) = \lim_{r \to \infty} F(r, T), \quad (6)$$

where $c$ is an additive normalization that ensures that the static potential has a certain value at a given distance. In the above expressions we made explicit that the free energy of an isolated static quark is half the free energy of $Q\bar{Q}$ pair at infinite separation. In QCD the $Z(3)$ symmetry is broken by light dynamical quarks and the expectation value of the Polyakov loop is always non-zero. Physically this corresponds to the fact that the static quark can be screened by light dynamical quarks resulting in string breaking, the free energy of a static $Q\bar{Q}$ pair saturates at some value. In Fig. 2 we show the free energy of static quark anti-quark pair as function of the distance at different temperatures. To facilitate the comparison with zero-temperature potential we subtracted a temperature dependent constant $T \ln 9$ arising from the normalization of the Polyakov loop correlator in Eq. (4). The normalization constant $c$ has been previously calculated [13]. For the lowest temperature the free energy coincides with the zero-temperature potential calculated in Ref.[13] and is shown as the black line, up to distances where string breaking is expected to occur. At higher temperatures we see significant temperature dependence even at quite short distances. This feature of the $Q\bar{Q}$ free energy was also observed in $SU(3)$ gauge theory [16, 17]. To better understand the behavior of the free energy of a static $Q\bar{Q}$ pair it is worth to consider the so-called singlet free energy. The singlet free energy is useful for calculating
Figure 2. The free energy (left) and singlet free energy (right) of a static quark anti-quark pair as function of the separation \( r \) for different temperatures calculated with the HISQ action on \( 24^3 \times 6 \) lattice. The black line shows the zero-temperature potential [13]. The same color (symbol) coding is used for different temperature values in the two panels.

Quarkonium spectral functions at non-zero temperature in potential models [18, 19, 20], as direct lattice calculations of quarkonium spectral functions appear to be difficult [21, 22].

A static quark anti-quark pair could be either in a singlet or in an octet state, and thus (in a fixed gauge) one can define the so-called color singlet and octet free energy [23, 24]

\[
\exp(-F_1(r, T)/T + c/T) = \frac{1}{3} \langle \text{Tr}[W^\dagger(x)W(y)] \rangle,
\]

\[
\exp(-F_8(r, T)/T + c/T) = \frac{1}{8} \langle \text{Tr}W^\dagger(x)\text{Tr}W(y) \rangle - \frac{1}{24} \langle \text{Tr}[W^\dagger(x)W(y)] \rangle.
\]

Then the Polyakov loop correlator can be written as the thermal average over the singlet and the octet contributions [12, 23, 24]

\[
C_{PL}(r, T) = \frac{1}{9} \exp(-F_1(r, T)/T + c/T) + \frac{8}{9} \exp(-F_8(r, T)/T + c/T).
\]

At short distances, \( rT \ll 1 \), the singlet and octet free energies defined above coincide with the well-known perturbative singlet and octet potentials. The singlet and octet free energies can be also calculated at high temperature in the leading order HTL approximation and turn out to be gauge-independent [25]. However, it was not clear how to generalize the decomposition of the free energy into the singlet and octet contributions beyond the leading order. Recently using the effective theory approach, namely the potential non-relativistic QCD (pNRQCD) at finite temperature [26, 27], it was shown that this decomposition indeed holds at short distances [28]. The decomposition of the Polyakov loop correlator in Eq. (9) to large extent explains the strong temperature dependence of the free energy \( F(r, T) \). Even if the singlet and octet free energies are temperature-independent the Polyakov loop correlator may have a significant temperature dependence, especially when the singlet and octet contributions become comparable. We calculated the singlet free energy using Coulomb gauge. The numerical results are shown in Fig. 2. At short distances the singlet free energy agrees with the zero-temperature
potential as expected while at larger distances it approaches a constant value. The saturation of the free energy happens at smaller distances as temperature increases, as expected if color electric screening occurs. From the above discussions as well as from Fig. 2 it is also clear that the value of the singlet free energy at infinite separation is the same as for the free energy $F(r, T)$. The temperature dependence of the singlet free energy is much smaller than of the static $Q\bar{Q}$ free energy indicating that the octet contribution is significant at high temperatures. The qualitative features of the singlet free energy described above are not specific to the Coulomb gauge. In fact, the singlet free energy defined in terms of periodic Wilson loops [29] shows very similar behavior to the one calculated in the Coulomb gauge [30].

As discussed above the asymptotic value of the free energy (or singlet free energy) is related to the renormalized Polyakov loop. The numerical results for the renormalized Polyakov loop are shown in Fig. 3. Our calculations show that the renormalized Polyakov loop is remarkably insensitive to cutoff effects. We get a reliable estimate for the Polyakov loop expectation value already for $N_T = 4$. In the figure we compare our results with the calculations performed in $SU(3)$ gauge theory [31, 32]. The temperature dependence of the renormalized Polyakov loop in QCD and pure gauge theory is similar qualitatively but differences cannot be absorbed by scaling the temperature axis with the transition temperature. In particular in the transition region there are qualitative differences in the temperature dependence of the Polyakov loop. The increase in the expectation value of the Polyakov loop in the transition region is not as rapid in QCD as in pure gauge theory.

At high temperature the renormalized Polyakov loop becomes larger than one. This is expected from perturbation theory which predicts $F_\infty(T) = -4/3\alpha s m_D$. Here $m_D$ is the well-known leading order Debye mass $m_D = g T \sqrt{1 + N_f/3}$ with $N_f$ being the number of light quark flavors, $N_f = 3$ in our case. In Fig. 3 we compare our lattice results with NLO perturbative result of Refs. [33, 28]. As one can see, perturbation theory agrees with lattice results only for $T > 1$ GeV.

4. Conclusions
In this contribution we discussed the results on quark number susceptibilities and the free energy of static quark anti-quark pair obtained with the HISQ action on lattices with the temporal extent $N_T = 4, 6, 8, 10$ and 12. The lattice spacing dependence for quark number susceptibilities is small for $N_T \geq 8$, while for the free energy of static quark anti-quark pair it is small already for $N_T = 4$. This is presumably due to the fact that Polyakov loop is a gluonic quantity and cutoff effects are dominantly coming from the quark sector. The quark number susceptibilities show the expected change in degrees of freedom from hadronic to quark-like as the temperature increases. At high temperatures the lattice results on quark number susceptibilities agree well with resummed perturbative calculations. The increase in the renormalized Polyakov loop indicating the onset of color screening is slower than in quark number susceptibilities and agreement between the lattice results with perturbation theory is only found for $T > 1$ GeV.

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Figure 3. The renormalized Polyakov loop as function of the temperature calculated on different lattices with the HISQ action and compared with the results in pure glue theory. The dashed and solid lines show the next-to-leading perturbative results for 2+1 flavor QCD and SU(3) gauge theory, respectively.

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