MODELING THE HEATING AND COOLING OF WZ SAGITTAE FOLLOWING THE 2001 JULY OUTBURST

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ABSTRACT

Far Ultraviolet Spectroscopic Explorer and Hubble Space Telescope Space Telescope Imaging Spectograph spectra of the dwarf nova WZ Sge, obtained during and following the early superoutburst of 2001 July over a time span of 20 months, monitor changes in the components of the system during its different phases. The synthetic spectral fits to the data indicate a cooling in response to the outburst of about 12,000 K, from ~28,000 down to ~16,000 K. The cooling timescale $\tau$ (of the white dwarf temperature excess) is on the order of ~100 days in the early phase of the cooling period, and increases to $\tau \approx 850$ days toward the end of the second year following the outburst. In the present work, we numerically model the accretional heating and subsequent cooling of the accreting white dwarf in WZ Sge. The best compressional heating model fit is obtained for a 1.2 $M_\odot$ white dwarf accreting at a rate of $9 \times 10^{-9} M_\odot$ yr$^{-1}$ for 52 days. However, if one assumes a lower mass accretion rate or a lower white dwarf mass, then compressional heating alone cannot account for the observed temperature decline, and other sources of heating have to be included to increase the temperature of the model to the observed value. We quantitatively check the effect of boundary layer irradiation as such an additional source.

Subject headings: novae, cataclysmic variables — stars: individual (WZ Sagittae) — white dwarfs

1. INTRODUCTION

WZ Sge is a well-studied dwarf nova (DN, a subclass of Cataclysmic Variables CVs) with extreme properties. It has the largest outburst amplitude, shortest orbital period, longest outburst recurrence time, lowest mass Roche-lobe filling secondary, and lowest accretion rate of any class of DNe (Howell et al. 1999; Howell et al. 2002). In addition it is the brightest DN and arguably the closest CV, with a distance of 43 pc (J. Thorstensen 2001, private communication). WZ Sge was observed to go into outburst in 1913, 1946, and 1978, and it was consequently assumed to have an outburst period of about 33 years. The 2001 July 23 outburst, first reported by T. Ohshima (see Ishioka et al. (2001)), was therefore 10 years earlier than expected.

Following the nomenclature of Patterson et al. (2002), the optical light curve (Fig. 1) of the system during the 2001 July outburst exhibits a “plateau” phase (this phase is really a slow decline phase), which lasted for about 24 days. During this period the system brightness underwent a steady decline with a rate of about 0.1 mag day$^{-1}$, falling from $M_v \approx 8.2$ to 10.7 in about 24 days, a sign that the accretion was taking place at a slowly decreasing rate. It was then followed by a sharp drop (itself lasting a few days) of the visual magnitude from $m_v \approx 11$ down to 13, where it stayed for about 3 days: the “dip.” During the dip, accretion had either stopped completely or dropped considerably. Between days 29 and 52 of the outburst, the system then underwent 12 successive “echo” outbursts: the “rebrightening” phase. On the 53rd day, the system started to cool without any other noticeable outburst event: the “cooling” phase (in Fig. 1 the light curve is shown for $t < 75$ days only, since for $t > 53$ days $M_v$ is a monotonously decreasing function of time, as the star cools).

The 2001 outburst light curve of WZ Sge is remarkably similar to its 1978 outburst light curve, with practically the same initial decline rate of 0.1 mag day$^{-1}$ (Bailey 1979). However, in 1978 the plateau and rebrightening phases lasted about 30 days each against 25 days each in the 2001 outburst (i.e., the outburst in 1978 lasted about 10 days longer than in 2001). This may be related to the fact that in 2001 WZ Sge erupted after only 23 years of quiescence, while in 1978 it erupted after 33 years of quiescence. The 2001 outburst was not as strong as in 1978. From the International Ultraviolet Explorer archive we found that the continuum level of the spectrum has a flux density $F_\lambda$ (estimated around $\lambda = 1700$ Å) of about $F_\lambda \approx 2 \times 10^{-11}$ ergs s$^{-1}$ cm$^{-2}$ Å$^{-1}$ during the early
phase of the 1978 outburst, or about 3 times larger than in the 2001 outburst at the same epoch and wavelength $F_{\lambda} \approx 7 \times 10^{-12}$ erg s$^{-1}$ cm$^{-2}$ Å$^{-1}$. During the cooling phase, the flux density $F_{\lambda}$ was also larger in 1978 than in 2001 at the same epoch (Slevinsky et al. 1999). The 1946 eruption, however, was very different, with no apparent rebrightening phase, while the plateau phase had small amplitude variations during the whole duration of the outburst.

The main purpose of the present work is to try to account for the observed sequence of temperatures using a numerical model for the accretional heating and subsequent cooling of the accreting white dwarf (WD). In the next section we address the issue of the accuracy of the temperature determination during the outburst and cooling periods, together with an overview of the estimates of the WD temperature and accretion rate. We present the code we use to carry out the simulations of the heating and cooling of the WD in § 3. The results are presented in § 4. In the last section we discuss the possible origin of the observed high temperatures and slow cooling of the WD.

### 2. The Accretion Rate and White Dwarf Temperature Determination

In the present work we consider the observations of WZ Sge carried out in the far-ultraviolet (FUV) for which the temperature of the accreting WD and/or the mass accretion rate of the system were assessed. These observations monitor changes in the FUV component of the system during its different phases, over a time span of 20 months and reveal a cooling of the WD in response to the outburst of about 12,000 K. Namely, we consider four Far Ultraviolet Spectroscopic Explorer (FUSE) and 11 Hubble Space Telescope Space Telescope Imaging Spectograph (HST STIS) observations as follows: one FUSE observation obtained during the plateau phase, one HST observation obtained during the dip, two HST and one FUSE observations obtained during the rebrightening phase, and two FUSE with eight HST observations obtained during the cooling phase. A total of 15 observations together with their references are recapitulated in Table 1.

In order to estimate the temperature one generates a grid of theoretical spectra for different values of the surface temperature, gravity and composition, using the synthetic spectra generator codes TLUSTY and SYNSPEC (Hubeny 1988; Hubeny et al. 1994; Hubeny & Lanz 1995). One then masks regions of the observed spectrum that are not characteristic of a WD atmosphere (such are emission lines for example), and a $\chi^2$ fitting technique is used to find the best-fit model. The mass accretion rate is estimated in a similar manner using different options in the synthetic spectra generator codes TLUSTY (the older version is known as TLDISK) and SYNSPEC.

#### Mass Accretion Rates

The following mass accretion rates were estimated in the references mentioned in Table 1 and should be considered only as an order of estimate. From Table 1, we see that the mass accretion rate estimate during the plateau

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**TABLE 1**

| Observation Number | Date       | Day | Instrument | $T_{\text{wd}}$ (1000 K) | $M$ ($M_\odot$ yr$^{-1}$) | Phase          | Reference |
|-------------------|------------|-----|------------|--------------------------|---------------------------|---------------|-----------|
| 1.                 | 2001 Jul 30| 7   | FUSE       | ...                      | (1–3)E–09                 | Plateau       | 1         |
| 2.                 | 2001 Aug 19| 27  | STIS       | 25.0                     | ...                       | Dip           | 2         |
| 3.                 | 2001 Aug 22| 30  | STIS       | ...                      | 5E–10                     | Rebrightening*| 2         |
| 4.                 | 2001 Sep 07| 46  | FUSE       | 42.0                     | ...                       | Rebrightening*| 1         |
| 5.                 | 2001 Sep 07| 46  | FUSE       | ...                      | 1E–09                     | Rebrightening*| 1         |
| 6.                 | 2001 Sep 11| 50  | STIS       | Table 2                 | 3E–10                     | Rebrightening*| 3         |
| 7.                 | 2001 Sep 29| 68  | FUSE       | 25.0                     | ...                       | Cooling       | 1         |
| 8.                 | 2001 Oct 10| 79  | STIS       | Table 2                 | ...                       | Cooling       | 3         |
| 9.                 | 2001 Nov 07| 107 | FUSE       | 23.0                     | ...                       | Cooling       | 1         |
| 10.                | 2001 Nov 10| 110 | STIS       | Table 2                 | ...                       | Cooling       | 3         |
| 11.                | 2001 Dec 11| 141 | STIS       | Table 2                 | ...                       | Cooling       | 3         |
| 12.                | 2002 Apr 15| 266 | STIS       | Table 2                 | ...                       | Cooling       | 3         |
| 13.                | 2002 Jun 05| 317 | STIS       | Table 2                 | ...                       | Cooling       | 3         |
| 14.                | 2002 Aug 27| 400 | FUSE       | Table 2                 | ...                       | Cooling       | 3         |
| 15.                | 2002 Nov 01| 466 | STIS       | Table 2                 | ...                       | Cooling       | 3         |

* The observations on day 30 and day 46 coincide with the first and 10th echo outburst of the rebrightening phase, respectively. The observation made on day 50 was made during a relatively “low” state between the 11th and the 12th outburst in the rebrightening phase. The observation on day 46 was modeled twice: once with a WD atmosphere only and once with an accretion disk only. The observation on day 50 was modeled once with a WD atmosphere only and once as a combination of an accretion disk and a WD atmosphere.

**REFERENCES**—(1) Long et al. 2003; (2) Knigge et al. 2002; (3) see Table 2.
phase is rather modest, on the order of \((1 - 3) \times 10^{-9} \, M_\odot \, \text{yr}^{-1}\) (Long et al. 2003) on day 7 into outburst (Table 1). During the dip, the WD might have a temperature of around 25,000 K (Knigge et al. 2002) on day 27. This is more than 10,000 K above its quiescence temperature, which is \(\approx 14,500\, \text{K}\). During the rebrightening phase, the mass accretion rate seems to be somewhat smaller than during the plateau and only peaks at \(1 \times 10^{-9} \, M_\odot \, \text{yr}^{-1}\) (Long et al. 2003) on day 46. We put these values into the table, although we feel that they should be considered only as rough estimates rather than actual values since they reflect our inability to assess \(M\) and \(T_{\text{wd}}\) during this epoch. This is because during the different phases of the outburst, additional components (such as the accretion disk and other obscuring material ejected during outburst) contaminate and veil (Long et al. 2004) the spectrum of the WD. However, during the cooling phase, one expects to “see” mainly the WD with little or no contribution from the disk and/or other (possibly masking) additional components. During this epoch, the WD is exposed and its temperature decreases.

Because of that, we decide to use for our modeling the values of the WD temperature obtained during the cooling phase only. However, discrepancies of up to 5000 K exist between the temperature estimates of Sion et al. (2003a) and Long et al. (2004) for the observation carried out on day 50, when the WD was revealed, but apparently also partially masked or/and possibly with an accretion disk component. The rest of the observations in the cooling phase are consistent within 2000 K.

**Temperatures.—** We elaborate here a little more on the temperature, using the three different approaches (denoted \(T_a\), \(T_b\), and \(T_c\) in Table 2).

These were all observations using *HST STIS*. In \(T_a\) we mask the N v region of the spectrum when needed (together with less affected regions varying from spectrum to spectrum) and use the latest version of the synthetic spectra generator codes TLUSTY and SYNSPEC (Hubeny 1988; Hubeny et al. 1994; Hubeny & Lanz 1995). In \(T_b\) the spectral fitting technique uses the same masking regions for all the spectra, together with the new version of the code; and the values for \(T_b\) are taken from Long et al. (2004), which uses a third and different masking technique together with an earlier version of the code. In Table 2, we also list the flux integrated over the wavelength over the entire spectral range of STIS, to the power 1/4, for all the epochs, which is proportional to the effective temperature (although each integrated flux can be relatively over- or underestimated).

In Figure 2 we draw the temperatures listed in Table 2 as follows: \(T_a\) is represented by stars, \(T_b\) is represented by squares, \(T_c\) is represented by plus signs, and the temperature estimated through the flux is represented by circles, where it has been arbitrarily scaled in order to fit the last data point for \(T_b\). The two triangles represent the *FUSE* data points. The 2001 September datum has temperature estimates ranging from \(\approx 27,000\) up to \(\approx 32,000\, \text{K}\). For that particular epoch the spectrum was taken between the last two echo outbursts; obviously the WD was not the only component and in addition might have been partially masked. For that reason we decided not to take into consideration the data points that were not known accurately, or that were taken during periods when the WD might still have been veiled. In our numerical modeling we will try to model the cooling temperature as listed in Table 2, with some reservations about the 2001 September entry.

The cooling of the WD is initially pretty rapid and then slows down significantly. If we assume an exponential cooling law \(T = T_{\text{inf}} + \Delta T \times e^{-t/\tau}\) (with \(T_{\text{inf}} = 14,500\, \text{K}\)), equivalent to \(\Delta T(t) = \Delta T \times e^{-t/\tau}\), where \(\Delta T(t)\) is the excess temperature of the WD at time \(t\), then we find that \(\tau\) is not constant, i.e., the cooling is not exponential. In Tables 1 and 2, \(\tau\) is counted from the beginning of the cooling period, day 52 into outburst; however, it is usually agreed to count the cooling time from the beginning of the cooling phase.

**FIG. 2.—** Modeling of the heating and cooling of WZ Sge. The temperature (in Kelvins) of the WD is drawn as a function of time (in days) since the start of the outburst (2001 July 23). The solid line represents the compressional heating model, for 1.2 \(M_\odot\) WD with an initial temperature of 14,500 K, accreting at a rate of \(9 \times 10^{-9} \, M_\odot \, \text{yr}^{-1}\) for 52 days. The stars, squares, and plus signs denote the temperatures \(T_a\), \(T_b\), and \(T_c\), respectively, listed in Table 2. The triangles indicate the *FUSE* data points, and the circles represent the temperature estimates using the flux values (see text).

**TABLE 2**

STIS TEMPERATURE ESTIMATES OF WZ SGE

| Observation Number | Date       | Day | \(T_a^a\) (1000 K) | \(T_b^a\) (1000 K) | \(T_c^a\) (1000 K) | (Flux)\(^{1/4}\) |
|--------------------|------------|-----|-------------------|-------------------|-------------------|---------------|
| 5..................| 2001 Sep 11| 50  | 31.9              | 27.0              | 28.2              | 3.8 \times 10^{-3} |
| 7..................| 2001 Oct 10| 79  | 25.2              | 23.6              | 23.4              | 3.3 \times 10^{-3} |
| 9..................| 2001 Nov 10| 110 | 23.7              | 22.4              | 22.1              | 3.1 \times 10^{-3} |
| 10.................| 2001 Dec 11| 141 | 22.6              | 21.6              | 20.7              | 2.9 \times 10^{-3} |
| 11.................| 2002 Apr 15| 266 | 19.5              | 18.8              | 18.1              | 2.5 \times 10^{-3} |
| 12.................| 2002 Jun 05| 317 | 18.8              | 18.0              | 17.4              | 2.5 \times 10^{-3} |
| 13.................| 2002 Aug 27| 400 | 17.8              | 17.4              | 16.7              | 2.4 \times 10^{-3} |
| 14.................| 2002 Nov 01| 466 | 17.5              | 17.0              | 16.3              | 2.3 \times 10^{-3} |
| 15.................| 2003 Mar 23| 608 | 17.2              | 16.6              | 15.9              | 2.2 \times 10^{-3} |

\(^a\) \(T_a\) and \(T_b\) were estimated in this work (see text) while \(T_c\) is from Long et al. 2004.
Therefore, we set here $t \rightarrow t - 52$ days, and we denote the cooling time at time $t = n$ days by $\tau^n$. We find $\tau^{13} \approx (70 \pm 10)$ days, $\tau^{20} \approx (190 \pm 30)$ days, $\tau^{30} \approx (325 \pm 40)$ days, and $\tau^{50} \approx (850 \pm 280)$ days.

It is interesting to note that the observed cooling following the 2001 July outburst is faster than the cooling observed after the 1978 outburst. Slevinsky et al. (1999) found a WD temperature of 20,500 K, 220 days after the outburst against $\approx 19,000$ K at the same epoch for the 2001 outburst (e.g., observing epoch of 2002 April, day 266). At $\approx 500$ days after the 1978 outburst the temperature was still around 17,500 K, while the present observations seem to indicate a lower temperature ($\approx 17,000$ K) 466 days after the 2001 outburst.

3. NUMERICAL MODELING

In order to model the accretional heating and subsequent cooling of the WD in WZ Sge, we use a one-dimensional quasi-static evolutionary code without hydrodynamics (quasi-static assumption). It is an updated version of the quasi-static stellar evolution code of Sion (1995). The code includes time-variable accretion, OPAL opacities, and boundary layer irradiation, which indirectly accounts for the stellar rotational velocity (see eq. [1] below). Stellar rotation, however, is not included anywhere else in the code. In this code, the WD is computed all the way down to the core, in the region well below the nuclear burning region. Initial models are constructed by the fitting-point method and the resulting initial model down to a depth where $\rho = 10^6$ g cm$^{-3}$ is stored as input for the evolution code. For a given WD mass and an initial effective surface temperature ($T_{\text{eff}}$), we chose $R_{\text{wd}}$ to be the theoretical equilibrium radius. The models built in this manner are initially in equilibrium, and if no matter is accreted onto the WD surface, the WD parameters do not undergo any change on the timescale studied here. All other details of the code can be found in Sion (1995) and references therein.

Numerical simulations are carried out by switching on accretion for the duration of a DN outburst interval and then shutting it off to follow the cooling of the WD. In this way the effects of compressional heating and boundary layer irradiation can be assessed quantitatively. The matter is assumed to accrete "softly" with the same entropy as the WD outer layers. From theoretical considerations, one expects the accreting matter, as it transits through the boundary layer region, to increase its temperature (since the boundary layer temperature is high: $T_{\text{BL}} \gg T_*$). In addition in the boundary layer region, the radial velocity is large, and the accreting matter can also heat up by shock. For these reasons the soft accretion assumption is actually not justified, since this advected energy is nonnegligible. However, in our treatment of the boundary layer irradiation, we take into account part of this energy by assuming that a fraction of the energy liberated in the boundary layer ($L_{\text{BL}}$) is absorbed by the outer layer of the star: it is included as a source in the energy equation in the outer layer of the star.

The treatment of the boundary layer irradiation is done as follows. The energy liberated in the boundary layer is given by

$$L_{\text{BL}} = \frac{1}{2} L_{\text{acc}} \left[ 1 - \frac{\Omega_s}{\Omega_K(R_s)} \right]^2$$

(Kluźniak 1987), where

$$L_{\text{acc}} = \frac{GM_r \dot{M}}{R_s}$$

is the total accretion energy, $G$ is the gravitational constant, $M_r$ is the mass of the star, $R_s$ is the radius of the star, $\dot{M}$ is the mass accretion rate, $\Omega_s$ is the angular rotation rate of the star, and $\Omega_K(R_s)$ is the Keplerian angular velocity at 1 stellar radius. Equation (1) can be used as long as the disk is geometrically thin and optically thick and extends to the stellar surface. In the present case, one expects the stellar rotation rate to be the rotation rate of the WD: $\Omega_s = \Omega_{\text{wd}}$.

We assumed that only a fraction of the boundary layer luminosity is irradiating the star, namely,

$$L_{\text{irr}} = \alpha \frac{L_{\text{BL}}}{2}.$$  

A value $\alpha = 1$ means that half of the BL luminosity is lost in space, while the other half is absorbed by the star. Assuming a value $\alpha = 0.5$ means that only 25% of the BL luminosity is absorbed by the star. Here we choose $\alpha = 0.5$, which is the value used in the work of Shaviv & Starrfield (1987) and about half the estimated value of Regev & Shara (1989), who used $\alpha \approx 0.2 L_{\text{acc}}$. We then assume different rotation rates

$$\eta = \frac{\Omega_s}{\Omega_K(R_s)},$$

For a nonrotating star $\eta = 0$ and the BL luminosity is exactly half the accretion energy $L_{\text{BL}} = L_{\text{acc}}/2$, while for a star rotating near break-up, $\eta = 1$ and $L_{\text{BL}} = 0$. In this work we took values ranging from $\eta = 1$ (for no boundary layer irradiation, in order to check only the effect of the compressional heating) down to $\eta = 0.05$ (when the star is slowly rotating and boundary layer irradiation is the main source of heating). Clearly a smaller value of $\alpha$ will require a smaller value of $\eta$ in order to keep the same amount of BL irradiation in a specific model, and vice versa. However, the compressional heating results obtained in this work are not at all affected by the value of $\alpha$ used in the simulations.

In the simulations it is assumed that the accretion and heating of the WD is uniform rather than being restricted to the equatorial region. In addition, the transfer of angular momentum (by shear mixing) into the WD is neglected.

4. RESULTS

As previously remarked in § 2, during the outburst, as accretion takes place at a high rate, the star’s photospheric emission is overwhelmed by the emission of the hot components (mainly the inner disk), which makes it difficult to assess the exact temperature of the star and its rotation rate $\Omega_{\text{wd}}$. However, on day 53, the system is found in a low state and starts to cool. During that time, the accretion rate has probably dropped to its quiescence level.

Numerical simulations (Godon & Sion 2002) have shown that the temperature increase due to BL irradiation is sustained only during accretion, and when the accretion is turned off, the star rapidly radiates away the BL energy absorbed in its outermost layer. However, the temperature increase due to compressional heating takes place deeper in the outer layers of the star, and it takes many days (months) for the star to cool. Therefore, we assume that in the cooling phase the observed elevated temperature of the star is due to the compressional heating it has endured during the outburst phase alone.

Since BL irradiation and compressional heating take place at different depths in the outer layers of the star, and on
We run models with different WD mass, namely, 15,000 up to 28,000 K (or more, see Table 2) around day 53. The WD for a model with accretion rate are larger than expected, we decided to check that occurs during the outburst phase, therefore setting $\eta = \Omega_{wd}/\Omega_{K} = 1$ in the simulations. The exact mass accretion rate of the system is not known, and it changes with time during the outburst. From the optical light curve in Figure 1, it seems very likely that initially the mass accretion rate is very high at the onset of the outburst and decreases steadily during the plateau phase. At present, our code only simulates a constant accretion rate, but at any time the accretion can be turned off to model the cooling. Consequently, we model the outburst phase (plateau + rebrightening) by accreting at a constant rate for 52 days, after which accretion is turned off and the WD cools. From Table 1, we expect the mass accretion rate during the run for a few days only.

The maximum temperature reached by the model during the run.

$b$ The maximum temperature reached by the model during the run.

$c$ Models for which the effect of boundary layer irradiation is taken into account ($\Omega_{wd} < 1$) are run for a few days only.

| Model Number | $M_{wd}$ ($M_\odot$) | $\log R_{wd}$ (cm) | $T_{wd}$ (1000 K) | $\dot{M}$ ($M_\odot$ yr$^{-1}$) | $\dot{T}_{wd}$ (1000 K) | $\Omega_{wd}$ ($\Omega_\odot$) | Length$^c$ (days) |
|--------------|---------------------|-------------------|------------------|-----------------|---------------------|----------------|---------------|
| 1.............| 0.7                 | 8.95              | 15.0             | 1E–08           | 21.0                | 1.0           | 52            |
| 2.............| 1.0                 | 8.78              | 15.0             | 1E–08           | 25.0                | 1.0           | 52            |
| 3.............| 1.2                 | 8.60              | 14.5             | 9E–09           | 34.0                | 1.0           | 52            |
| 4.............| 1.2                 | 8.60              | 14.5             | 5E–09           | 27.5                | 1.0           | 52            |
| 5.............| 1.0                 | 8.78              | 18.0             | 2E–11           | 20.0                | 0.2           | ...           |

**Fig. 3.**—Same as Fig. 2, but here the solid line represents a model with a mass accretion rate of $5 \times 10^{-9} M_\odot$ yr$^{-1}$ and an initial temperature of 15,000 K (model 4 in Table 3).
itself larger by a factor of \( \approx 5 \), than the average value determined from the spectral fits to the observations which was \( \approx 1.7 \times 10^{-9} \, M_\odot \, \text{yr}^{-1} \) (25 days of accretion at a rate \( \dot{M} = 3 \times 10^{-9} \, M_\odot \, \text{yr}^{-1} \), followed by 25 days of intermittent accretion at an average rate of \( \approx 5 \times 10^{-10} \, M_\odot \, \text{yr}^{-1} \).

When we assumed a smaller WD mass of only \( 1 \, M_\odot \), we found that even with a high mass accretion rate of \( 10^{-8} \, M_\odot \, \text{yr}^{-1} \) the model could not account for the observed temperature. For that model, we simulated a second source of heating (in addition to compressional heating): boundary layer irradiation at a quiescence mass accretion rate, corresponding to the mass accretion rate during the cooling period. The mass accretion rate we obtained (\( 2 \times 10^{-11} \, M_\odot \, \text{yr}^{-1} \); Table 3) to fit the observations is in agreement with a recent analysis of an X-ray observation of WZ Sge. Hasenkopf & Eracleous (2002) presented results of a uniform analysis of all the ASCA X-ray observations of nonmagnetic CVs. For WZ Sge they estimated an X-ray luminosity of about \( L_X \approx 2.7 \times 10^{30} \, \text{ergs} \, \text{s}^{-1} \) in the range 0.5–10 keV, assuming a distance of 69 pc. Rescaling this value to the better estimate of 43 pc (and also to be consistent with the distance assumed in this work) leads to \( L_X \approx 7 \times 10^{30} \, \text{ergs} \, \text{s}^{-1} \) for the X-ray luminosity. Hasenkopf & Eracleous (2002) found that optically thin boundary layer models (Popham 1999) provide the best description of the data. Since the disk is radiating in both +z and –z directions, the X-ray luminosity is at least half the boundary layer luminosity, namely, \( L_X = \frac{1}{2} L_{BL} \), and if the star does partially mask the inner disk and boundary layer (since \( i = 78^\circ \)), then one has \( L_X < \frac{1}{2} L_{BL} \). For this reason we assume \( L_{BL} \approx 3 L_X \approx 2 \times 10^{31} \, \text{ergs} \, \text{s}^{-1} \); using the Kluzniak (1987) relation for \( L_{BL} \), an angular velocity of 1200 km s\(^{-1}\), and a 1.0 \( M_\odot \), we find the quiescence mass accretion rate from the X-ray observation to be \( \dot{M} = 6 \times 10^{-12} \, M_\odot \, \text{yr}^{-1} \), about 3 times smaller than our estimates.

Another possibility that we are unable to assess quantitatively is the slow release of rotational kinetic energy from the outer layer of the star (Sparks et al. 1993). If the outer layer of the star (equatorial belt) has been spun up during the outburst, then rotational kinetic energy can be released during the cooling phase as this fast rotating layer spins down. This effect could account for an additional source of heating of the WD occurring during the cooling phase.

In this work we have shown that compressional heating alone can account for the cooling curve of WZ Sge following the 2001 July superoutburst only if the WD in WZ Sge is massive (\( M_{\text{wd}} = 1.2 \, M_\odot \)) and the outburst accretion rate is large (\( \approx 10^{-8} \, M_\odot \, \text{yr}^{-1} \)). If this is not the case, then compressional heating alone is not enough to account for the observed decrease in the WD temperature, and we suggest boundary layer irradiation from a quiescent accretion disk and the slow release of rotational kinetic energy from a fast rotating accretion belt as possible additional sources of heating of the WD to account for the observed temperatures.

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