Higgs Mass Corrections in the SUSY B-L Model with Inverse Seesaw

A. Elsayed1,2, S. Khalil1,3, and S. Moretti4,5

1Center for Theoretical Physics at the British University in Egypt, Sherouk City, Cairo 11837, Egypt.
2Department of Mathematics, Faculty of Science, Cairo University, Giza, 12613, Egypt.
3Department of Mathematics, Faculty of Science, Ain Shams University, Cairo, 11566, Egypt.
4School of Physics and Astronomy, University of Southampton, Highfield, Southampton SO17 1BJ, UK.
5Particle Physics Department, Rutherford Appleton Laboratory, Chilton, Didcot, Oxon OX11 0QX, UK.

In the context of the Supersymmetric (SUSY) B − L (Baryon minus Lepton number) model with an inverse seesaw mechanism, we calculate the one-loop radiative corrections due to right-handed (s)neutrinos to the mass of the lightest Higgs boson when the latter is Standard Model (SM)-like. We show that such effects can be as large as \( \mathcal{O}(100) \) GeV, thereby giving an absolute upper limit on such a mass around 200 GeV. The importance of this result from a phenomenological point of view is twofold. On the one hand, this enhancement greatly reconciles theory and experiment, by alleviating the so-called ‘little hierarchy problem’ of the minimal SUSY realisation, whereby the current experimental limit on the SM-like Higgs mass is very near its absolute upper limit predicted theoretically, of 130 GeV. On the other hand, a SM-like Higgs boson with mass below 200 GeV is still well within the reach of the Large Hadron Collider (LHC), so that the SUSY realisation discussed here is just as testable as the minimal version.

PACS numbers:

The Higgs boson is the last missing particle in the SM. Higgs boson discovery at the LHC is, therefore, crucial for its validity as a low energy approximation of a new physics scenario valid to high energy scales. A possibility for the latter emerges in SUSY theories, wherein the Higgs mechanism is retained for mass generation and multiple Higgs bosons appear in order to cancel anomalies. In addition, the stabilization of the Higgs mass against loop corrections (gauge hierarchy problem) is possibly the strongest motivation for a SUSY theory of nature. Hence, Higgs boson discovery at the LHC is also crucial for SUSY as a whole. A consequence of a SUSY Higgs sector is the existence of a stringent upper bound on the mass of the lightest SUSY Higgs boson, \( h \), when the latter is SM-like. In the Minimal Supersymmetric Standard Model (MSSM), this value is \( m_h \lesssim 130 \) GeV. Therefore, non-observing at the LHC a SM-like Higgs boson lighter than 130 GeV would rule out the MSSM.

In detail, in the MSSM, the mass of the lightest Higgs state can be approximated, at the one-loop level, as

\[
m_h^2 \leq M_{Z}^2 + \frac{3g^2}{16\pi^2 M_W^2} \frac{m_{\tilde{t}}^4}{\sin^2 \beta} \log \left( \frac{m_{\tilde{b}}^2}{m_{\tilde{t}}^2} \right),
\]

where \( g \) is the \( SU(2) \) gauge coupling. \( m_{\tilde{t}, \tilde{b}} \) are the two stop physical masses. The ratio of the Electroweak (EW) Vacuum Expectation Values (VEVs) is given by \( \tan \beta = v_2/v_1 \). Note that the factor 3 in the above top-stop correction is due to color. If one assumes that the stop masses are of order TeV, then the one-loop effect leads to a correction of order \( \mathcal{O}(100) \) GeV, which implies that

\[
m_h^{\text{MSSM}} < \sqrt{(90 \text{ GeV})^2 + (100 \text{ GeV})^2} \approx 135 \text{ GeV}.
\]

It is worth mentioning that the two-loop corrections reduce this upper bound by a few GeVs, to the aforementioned 130 GeV or so value [2].

Experimental evidence now exists for physics beyond the SM, in the form of neutrino oscillations, which imply neutrino masses [3]. In turn, the latter imply new physics beyond not only the SM, but also the MSSM. Right-handed neutrino superfields are usually introduced in order to implement the seesaw mechanism, which provides an elegant solution for the smallness of the left-handed neutrino masses. Right-handed neutrinos, which are heavy, can naturally be implemented in the SUSY \( B-L \) extension of the SM (hereafter, the ‘SUSY \( B-L \) model’ for short), which is based on the gauge group \( SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \). In this scenario, the scale of \( B-L \) symmetry breaking is related to the soft SUSY breaking scale [1]. Thus, the right-handed neutrino masses are naturally of order TeV and the Dirac neutrino masses must be less than \( 10^{-4} \) GeV (i.e., they are of order the electron mass) [6]. Nevertheless, due to the smallness of Dirac neutrino Yukawa couplings, the right-handed neutrino sector has very suppressed interactions with the SM particles. Therefore, the predictions of such a SUSY \( B-L \) model (i.e., with standard seesaw mechanism) remain close to the MSSM ones. In particular, the discussed MSSM prediction for the lightest Higgs boson mass upper bound remains intact. Same conclusion is obtained in the context of the minimal Supersymmetric seesaw model, where the right-handed neutrino masses are of order \( 10^{13} \) GeV [6].

The SUSY \( B-L \) model with inverse seesaw, where Dirac neutrino Yukawa couplings are of order 1, has recently been considered [7]. The superpotential of the leptonic sector associated to this model is given by

\[
W = Y_L L H_1 E^c + Y_L L H_2 N^c + Y_S N^c \chi_1 S_2 + \mu H_1 H_2 + \mu' \chi_1 \chi_2.
\]
where $\chi_{1,2}$ are SM singlet superfields with $B - L$ charges +1 and -1, respectively. Therefore, $U(1)_{B-L}$ is spontaneously broken by the VEV of the scalar components of these superfields. $N_i$ are three SM singlet chiral superfields with $B - L$ charge = -1, introduced to cancel the $U(1)_{B-L}$ anomaly. The fermion components of $N_i$ account for right-handed neutrinos. Finally, chiral SM singlet superfields $S_{1,2}$ with $B - L$ charge = +2, -2 are considered to implement the inverse seesaw mechanism. Note that a $\mathbb{Z}_2$ symmetry is assumed in order to prevent the interactions between the field $S_1$ and any other field.

After $B - L$ and EW symmetry breaking, the neutrino Yukawa interaction terms lead to the following expression:

$$\mathcal{L}''_{\nu} = m_D \bar{\nu}_L N^c + M_N \bar{N}^c S_2,$$

(4)

where $m_D = Y_{\nu} \nu \sin \beta$ and $M_N = Y_S \nu' \sin \theta$. Light neutrino masses are related to a small mass term $\mu_S S_2^c S_2$, with $\mu_S \lesssim \mathcal{O}(1)$ KeV, which can emerge at the $B - L$ scale from a non-renormalizable term in the superpotential, $\lambda S_2^c S_2$, with $M_1$ an intermediate scale of order $\mathcal{O}(10^7)$ GeV. Note that the non-renormalizable scale $M_1$ can be related to a more fundamental scale and couplings of the $S_2$ and $\chi$ fields with integrated out fields and a suppression factor. In this case one can write for instance $1/M_1^2 \sim \lambda^4/M_2^4$, therefore, if $\lambda \sim \mathcal{O}(0.01)$, then $M_1$ is of order $10^{12}$ GeV. Thus, the Lagrangian of neutrino masses, in the flavor basis, is given by:

$$\mathcal{L}''_{\nu} = m_D \bar{\nu}_L N^c + M_N \bar{N}^c S_2 + \mu_S S_2^c S_2.$$

(5)

In the basis $\{\nu_L, N^c, S_2\}$, the $3 \times 3$ neutrino mass matrix of one generation takes the form:

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M_N \\ 0 & M_N & \mu_S \end{pmatrix}. \quad (6)$$

Therefore, the following light and heavy neutrino masses are given by

$$M_\nu^2 = \begin{pmatrix} m_D^2 + \frac{M_N^2 \cos \beta}{2} + \frac{M_2^2 \cos \theta}{2} & m_D(A_\nu + \mu \cot \beta) & m_D M_N \\ m_D(A_\nu + \mu \cot \beta) & m_D^2 + M_N^2 - \frac{M_2^2 \cos \theta}{2} & m_D M_N \\ m_D M_N & m_D M_N & m_N^2 + M_2^2 \cos \theta \end{pmatrix}. \quad (13)$$

If one chooses the A-terms such that elements 12 and 23 vanish, then the sneutrino masses can be written as

$$m_1^2 = d,$n_2^2 = \frac{1}{2} \left((a + f) + \sqrt{(a - f)^2 + 4c^2}\right),$$

$$m_3^2 = \frac{1}{2} \left((a + f) - \sqrt{(a - f)^2 + 4c^2}\right), \quad (14)$$

$$m_{\nu} = \frac{m_D^2 \mu_S}{M_N^2 + m_D^2}, \quad (7)$$

$$m_{\nu_{H',h'}} = \pm \sqrt{M_N^2 + m_D^2 + \frac{1}{2} M_2^2 \mu_S}. \quad (8)$$

In the limit of neglecting $\mu_S$, the neutrino masses are approximated as

$$m_{\nu}^2 \simeq 0, \quad m_{\nu_{H',h'}}^2 \simeq m_D^2 + M_N^2. \quad (9)$$

Further, in this type of model, the heavy neutrinos may have large couplings to SM particles, leading to very interesting phenomenological implications [3].

The sneutrino mass matrix is obtained from the sneutrino scalar potential, which is given by

$$V_{\text{scalar}} = V_F + V_D + V_{\text{soft}}, \quad (10)$$

where $V_F$ is defined as usual as $|\partial W/\partial \phi|^2$ and

$$V_D = \frac{M_2^2}{\cos 2\beta} \nu^2 \bar{\nu}_L + M_2^2 \cos \theta \nu^2 \bar{\nu}_L + \frac{1}{\cos 2\beta} \nu^2 \bar{\nu}_L + \frac{1}{\cos 2\beta} \nu^2 \bar{\nu}_L + \frac{1}{\cos 2\beta} \nu^2 \bar{\nu}_L + \frac{1}{\cos 2\beta} \nu^2 \bar{\nu}_L,$$

(11)

where $M_{2'}$ is the mass of the $B - L$ neutral gauge boson $Z'$, given by $M_{2'}^2 = 4g''(\sqrt{2})^2$. From the LEP II experimental limits, one finds $M_{2'}/g'' > 6$ TeV [4]. Finally, $V_{\text{soft}}$ is defined as

$$V_{\text{soft}} = m_D^2 \sum \phi^2 + \frac{1}{2} M_{1/2} \sum \bar{\chi}_i \chi_i + \left[ A_0 \left( \nu_L \bar{N}^c \tilde{L}_H + \nu_L \bar{E}^c \tilde{L}_H + \nu_L \bar{N}^c \tilde{S}_2 \right) \right] + \frac{1}{2} \left( \mu H_1 H_2 + \mu' \chi_1 \chi_2 \right) \right] + \text{h.c.}. \quad (12)$$

Here, the sum in the first term runs over $\phi = H_1, H_2, \chi_1, \chi_2, \tilde{L}, \tilde{E}^c, \tilde{N}, \tilde{S}_1, \tilde{S}_2$ and the sum in the second term runs over the gauginos: $\lambda_i = B, W^a, \tilde{g}^a, Z'$. In general, one finds that the sneutrino mass matrix, for one generation, can be written as a $3 \times 3$ matrix, with entries multiplied by the identity $2 \times 2$ matrix [10], i.e., with one generation, one obtains two left-handed sneutrinos $\tilde{\nu}_{L,1,2}$ and four right-handed sneutrinos $\tilde{\nu}_{H,3,4,5,6}$.

$$m^2 = \left( \frac{m_D^2 + \frac{M_N^2 \cos \beta}{2} + \frac{M_2^2 \cos \theta}{2}}{m_D M_N} \right). \quad (13)$$

where

$$c = m_D M_N,$n_2^2 = \frac{1}{2} \left((a + f) + \sqrt{(a - f)^2 + 4c^2}\right),$$

$$f = m_3^2 + M_N^2 + M_2^2 \cos \theta. \quad (15)$$

If one assumes that $m_L = m_N = m_S = \tilde{m}$ and neglects
the $D$-term, then the sneutrino masses can be written as
\[ m_{\tilde{\nu}_{1,2}}^2 = \tilde{m}^2, \quad m_{\tilde{\nu}_{3,4,5,6}}^2 = m_D^2 + M_N^2 + \tilde{m}^2. \quad (16) \]

It is important to note that, unlike the squark sector, where only the third generation (stops) has a large Yukawa coupling with the Higgs boson, hence giving the relevant contribution to the Higgs mass correction, all three generations of the (s)neutrino sector may lead to important effects since the neutrino Yukawa couplings are generally not hierarchical. Also, due to the large mixing between the right-handed neutrinos $N_i$ and $S_{2j}$, all the right-handed sneutrinos $\tilde{\nu}_H$ are coupled to the Higgs boson $H_2$, hence they can give significant contribution to the Higgs mass correction. In this respect, it is useful to note that the stop effect is due to the running of 6 degrees of freedom (3 colors times 2 stop eigenvalues) in the Higgs mass loop corrections, while in case of right-handed sneutrinos we have, in general, 12 degrees of freedom (3 generations times 4 eigenvalues).

As example of a generic $3 \times 3$ neutrino Yukawa coupling, $Y_\nu$, we consider $Y_\nu = m_D/v_2$, with the Dirac neutrino mass matrix $m_D$.

\[ m_D = U_{\text{MNS}} \sqrt{m_{\text{phys}}} \mathcal{R} \mu_2^{-1} M_N, \quad (17) \]

where $R$ is an arbitrary orthogonal matrix and $U_{\text{MNS}}$ is the light neutrino mixing matrix. If we assume that $R = I_{3 \times 3}$ and $\sqrt{m_{\text{phys}}} / \mu_2 \sim O(0.1)$, then we find $Y_\nu \simeq U_{\text{MNS}}$. Note that here we assume a hierarchical $\mu_2$ in order to account for a possible hierarchy between light neutrino masses. For simplicity, we also assume universal Majorana neutrino masses, $M_N = \text{diag}(M, M, M)$. In this case, one can easily verify that the 12 right-handed sneutrinos have a very similar mass coupled to with the Higgs boson $H_2$ with order one Yukawa.

The one-loop radiative correction to the effective potential is given by the relation
\[ \Delta V = \frac{1}{64\pi^2} \text{STr} \left[ M^4 \left( \log \frac{M^2}{Q^2} - \frac{3}{2} \right) \right], \quad (18) \]
where $M^2$ is the field dependent generalized mass matrix and $Q$ is the renormalization scale, to be fixed from minimization conditions. The supertrace is defined as follow:
\[ \text{STr}(M^2) = \sum_i (-1)^{2J_i}(2J_i + 1) f(m_i^2). \]
Here $m_i^2$ is a field dependent squared mass eigenvalue of a particle with spin $J_i$. Therefore $\Delta V$, due to one generation of neutrinos and sneutrinos, is given by
\[ \Delta V_{\nu,\tilde{\nu}} = \frac{1}{64\pi^2} \sum_{i=1}^6 m_i^4 \left( \log \frac{m_i^2}{Q^2} - \frac{3}{2} \right) - \sum_{i=1}^3 m_i^4 \left( \log \frac{m_{\tilde{\nu}_i}^2}{Q^2} - \frac{3}{2} \right). \quad (19) \]

The first sum runs over the sneutrino mass eigenvalues, while the second sum runs over the neutrino masses (with vanishing $m_\nu$). In case of three generations, these sums should be from 1 to 18 and from 1 to 9, respectively. In case of our above example, where $Y_\nu \sim U_{\text{MNS}}$, one finds that the total $\Delta V$ is given by three times the value of $\Delta V$ for one generation. This factor then compensates the color factor of (s)top contributions.

The one-loop minimization conditions are given by $\partial V / \partial m^2 = 0$, with $i = 1, 2$ and $V = V_0 + \Delta V_{\text{MSSM}}$. In order to retain the minimization conditions as $\partial V / \partial m_i^2 = 0$, either we choose a renormalization scale $\tilde{Q}$ such that $\left. \frac{\partial \Delta V_{\nu,\tilde{\nu}}}{\partial m^2} \right|_{Q = \tilde{Q}} = 0$, then we must evaluate the Higgs mass correction at this scale, or we define the mass parameters $m_i^2$ in the potential $V(H_1, H_2)$ as
\[ m_i^2 = m_i^2|_{\text{tree}} - \frac{1}{2H_i} \frac{\partial \Delta V_{\nu,\tilde{\nu}}}{\partial H_i} \big|_{H_i = \nu_i}. \quad (20) \]

In this case, the genuine $B - L$ correction to the CP-even Higgs mass matrix, due to the (s)neutrinos, at any renormalization scale $Q$, is given by
\[ \Delta M_{ij}^2 = \frac{1}{2} \left( \frac{\partial^2 (\Delta V)_{\nu,\tilde{\nu}}}{\partial H_i \partial H_j} - \frac{\delta_{ij}}{2H_i} \frac{\partial \Delta V_{\nu,\tilde{\nu}}}{\partial H_i} \right) \big|_{H_i = \nu_i}. \quad (21) \]
As known, the complete effective potential is scale independent. However, the effective potential at one-loop level contains implicit dependence on the scale. The $Q$-dependence is approximately cancelled by neglecting the $D$-term and imposing the minimization conditions as explained above.

From the (s)neutrino masses, given in Eqs. (16) and (18), one can easily show that
\[ \delta M_{11} = \delta M_{12} = \delta M_{21} = 0, \quad (21) \]
\[ \delta M_{22} = \frac{1}{16\pi^2} \left[ \left( \frac{\partial m_{\nu_1}^2}{\partial v_2} \right)^2 \log \frac{m_{\nu_1}^2}{Q^2} - \left( \frac{\partial m_{\nu_2}^2}{\partial v_2} \right)^2 \log \frac{m_{\nu_2}^2}{Q^2} \right] \]
\[ = \frac{m_D^4}{4\pi^2 v_2^2} \log \frac{m_{\nu_1}^2}{m_{\nu_2}^2}. \quad (22) \]
Therefore the complete one-loop matrix of squared CP-even Higgs masses will be given by $M_{\text{tree}} + \Delta M$, with
\[ \Delta M = \begin{pmatrix} 0 & \delta_1^2 + \delta_2^2 \\ 0 & \delta_2^2 + \delta_3^2 \end{pmatrix}, \]
where $\delta_i^2$ refers to the (s)top contribution presented in Eq. (10) and $\delta_\nu^2$ is the (s)neutrino correction given in Eq. (22). In this case, the lightest Higgs boson mass is given by
\[ m_h^2 = 1/2 \left( M_A^2 + M_Z^2 + \delta_1^2 + \delta_\nu^2 \right) \left[ 1 - \sqrt{1 - \frac{4M_A^2 M_Z^2 \cos^2 2\beta + (\delta_1^2 + \delta_\nu^2)(M_A^2 \sin^2 \beta + M_Z^2 \cos^2 \beta)}{(M_A^2 + M_Z^2 + \delta_1^2 + \delta_\nu^2)^2}} \right]. \quad (23) \]
If $\tilde{m} \simeq \mathcal{O}(1)$ TeV, $Y_\nu \simeq \mathcal{O}(1)$ and $M_N \simeq \mathcal{O}(500)$ GeV, one finds that $\delta_2^2 \simeq \mathcal{O}(100)^2$, thus the Higgs mass will be of order $\sqrt{(90)^2 + \mathcal{O}(100)^2 + \mathcal{O}(100)^2} \simeq 170$ GeV.

In Fig. 1 we show the Higgs mass, $m_h$, as a function of the trilinear couplings $A_N$ and $A_S$, which contribute to the off-diagonal elements of the sneutrino mass matrix (39). For simplicity, we assume $A_N = A_S = A_0$. It turns out that, for a large value of $A_0$, $m_h$ can be enhanced by 20 GeV. In Fig. 2 we display the dependence of $m_h$ on $A_0$ for $Y_\nu = 0.8, 1, 1.1, 1.2$, $\tilde{m} = 1$ TeV and $M_N = 500$ GeV.

In conclusion, we have calculated the one-loop radiative corrections to the lightest SM-like Higgs boson mass, due to the right-handed (s)neutrinos in a SUSY $B - L$ extension of the SM with inverse seesaw mechanism. We have shown that the upper bound on the Higgs mass can be enhanced to be around 200 GeV. This enhancement alleviates a possible conflict between the experimental limits from Higgs searches at the LHC and the absolute upper limit predicted in MSSM theoretically, of 130 GeV. It is remarkable that our result remains valid for any model beyond the MSSM with TeV scale right-handed neutrinos (including Left-Right, Pati-Salam and other models derived from $SO(10)$).

The work of A.E. and S.K. is partially supported by the Science and Technology Development Fund (STDF) project 1855 and the ICTP project 30. The work of S.M. is partially supported by the NExT Institute. S.K. acknowledges an International Travel Grant from the Royal Society (London, UK). A.E. thanks W. Abdallah for fruitful discussions.

For $M_A \gg M_Z$ and $\cos 2\beta \simeq 1$, one finds that

$$m_h^2 \simeq M_Z^2 + \delta_t^2 + \delta_\nu^2.$$  (24)

FIG. 1: Lightest Higgs boson mass versus the right-handed sneutrino mass for $M_N = 500$ GeV, $Y_\nu = 0.8, 1, 1.1, 1.2$ (for curves from bottom to up respectively).

FIG. 2: Lightest Higgs boson mass as a function of the trilinear coupling $A_N$ for $M_N = 400$ GeV, $\tilde{m} = 1$ TeV, and $Y_\nu = 0.8, 1, 1.1, 1.2$ (for curves from bottom to up respectively).

[1] J. R. Ellis, G. Ridolfi and F. Zwiner, Phys. Lett. B 257, 83 (1991); J. R. Ellis, G. Ridolfi and F. Zwiner, Phys. Lett. B 262, 477 (1991); H. E. Haber, R. Hempfling, Phys. Rev. Lett. 66, 1815-1818 (1991); Y. Okada, M. Yamaguchi, T. Yanagida, Phys. Lett. 326, 54-58 (1991).
[2] S. Heinemeyer, W. Hollik and G. Weiglein, Phys. Lett. B 455, 179 (1999); M. S. Carena, H. E. Haber, S. Heinemeyer, W. Hollik, C. E. M. Wagner and G. Weiglein, Nucl. Phys. B 580, 29 (2000); J. R. Espinosa and R. J. Zhang, Nucl. Phys. B 586, 3 (2000).
[3] R. Wendell et al. [Kamiokande Collaboration], Phys. Rev. D 81, 092004 (2010).
[4] S. Khalil and A. Masiero, Phys. Lett. B 665, 374 (2008).
[5] S. Khalil, J. Phys. G 35, 055001 (2008); L. Basso, A. Belyaev, S. Moretti and C. H. Shepherd-Themistocles, Phys. Rev. D 80, 055030 (2009); P. Fileviez Perez, T. Han and T. Li, Phys. Rev. D 80, 073015 (2009).
[6] S. Heinemeyer, M. J. Herrero, S. Penaranda and A. M. Rodriguez-Sanchez, JHEP 1105, 063 (2011).
[7] S. Khalil, Phys. Rev. D 82, 077702 (2010).
[8] W. Abdallah, A. Awad, S. Khalil, H. Okada, [arXiv:1105.1047]
[9] M. Carena, A. Daleo, B. A. Dobrescu and T. M. P. Tait, Phys. Rev. D 70, 093009 (2004); T. Appelquist, B. A. Dobrescu and A. R. Hopper, Phys. Rev. D 68, 035012 (2003).
[10] S. Khalil, H. Okada and T. Toma, [arXiv:1102.4249].