I. INTRODUCTION

The quantum measurement problem is at the heart of the foundations of quantum theory, which has not been fully tackled [1–3]. The difficult nature of the measurement problem is mainly due to the linearity of quantum theory that admits the superposition of quantum states, which conflicts with our everyday reality [4, 25]. The measurement of a quantum system in the superposition state will result in different outcomes with statistics determined by the Born rule [5]. This inherently irreversible probabilistic cannot be explained by the unitary evolution of a quantum system according to the Schrödinger’s equation [6]. Various interpretations have been proposed to reconcile the issue. The well-known ones include the Copenhagen interpretation [7, 8], the many-world interpretation [9–11], the De Broglie-Bohm theory [12–14], and the decoherence theory [15–17]. Apart from these interpretations, many specific dynamical models have also been proposed and investigated in detail for elucidating the quantum measurement [3].

e.g., the von Neumann model [18], quantum statistical models [19–21], and system-pointer-bath models [22–24].

There has been a recurring interest in the measurement topic due to the rapid development of quantum information science and technology in the past several decades [26–35]. Significant progress has been made in the manipulation of qubits in various physical systems, e.g., superconducting circuits [36–38], trapped ions [39–41], atoms in optical lattice [42–44], and quantum optics/electronics systems [45–47]. Advances in quantum technology imply that simulating quantum dynamics with quantum computers has become feasible nowadays [48–50]. Since classical reality in quantum measurement is believed to emerge from the building blocks of quantum, the microscopic system plus macroscopic measurement apparatus must be described quantum mechanically according to mainstream measurement theory [1–3]. It is thus natural to ask whether and how quantum computers can be used to simulate and investigate the quantum measurement process. While general-purpose and universal quantum computers are still far away, current noisy intermediate-scale quantum (NISQ) processors and more advanced ones in the near future have already provided this opportunity [51].

In order to simulate with quantum computers, qubit models of the measurement process are necessary. Here, we propose two dynamical models with a scalable number of qubits. The system to be measured is a single qubit, and the measurement device is modeled as a collection of qubits. The first model is motivated by the single-photon detection in quantum optics, in which the system qubit is de-excited after the measurement.

The second model is related to the non-destructive spin measurement, and the system qubit can be used for subsequent measurements. In both models, if the system qubit is in the definite state |0⟩ or |1⟩, the corresponding final state of the measurement qubits will be in a collective state either |0⟩₀ m or |1⟩₀ m [64]. Given an arbitrary superposition state of the system qubit: α|0⟩₀ ± β|1⟩₀, the first model leads to the final state of the compound system as

\[ |\Psi(t_f)\rangle_{\text{first model}} = |0\rangle₀ \otimes (\alpha |0\rangle_m + \beta |1\rangle_m) \]

while the second model results in a cat-like final state

\[ |\Psi(t_f)\rangle_{\text{second model}} = \alpha |0\rangle₀ |0\rangle_m + \beta |1\rangle₀ |1\rangle_m \]

In both models, the interaction will lead to a proliferation of the quantum information from a single qubit to a collection...
of qubits—a prototype of the measurement process. Since the models are in the paradigm of unitary evolution, the purpose of this work is not to solve the measurement problem but to explore the prospect of using quantum computers to study the emergence of classical reality in the measurement process.

II. DESCRIPTION OF QUBIT MEASUREMENT MODELS

The first model originates from the phenomenology of the single-photon detection in quantum optics. In the photodetector, a single photon leads to the excitation of many electrons which creates a macroscopic observable current. We model the continuous translational degrees of freedom of electrons by discrete two-level systems, i.e., qubits. The model is illustrated in Fig. 1a. It is a tree network of qubits, and they are interacting with each other via a three-qubit interaction, i.e., each qubit is coupled to two qubits in the next layer. The first qubit, or the system qubit, is viewed as the single photon (or the atom that generates the single photon). The last layer of the network mimics the electrons that generate the observed current of the photodetector.

The Hamiltonian of the entire network reads

$$\hat{H} = \sum_i \omega_i \hat{\sigma}_i^Z + h g_i \hat{\sigma}_i^A \hat{\sigma}_i^A \hat{\sigma}_i^B \hat{\sigma}_i^B + h g_2 \hat{\sigma}_Z \hat{\sigma}_C \hat{\sigma}_C \hat{\sigma}_C \hat{\sigma}_C + \cdots + \text{h.c.} \tag{3}$$

Here $\omega_i$ is the transition frequency of each qubit, $\hat{\sigma}_X \equiv (\hat{\sigma}_X \pm \hat{\sigma}_Y)/2$ are ladder operators defined by Pauli operators $\hat{\sigma}_X$ and $\hat{\sigma}_Y$; $g_{1,2,3,\ldots}$ represent the coupling strength, and h.c. means Hermitian conjugate. Each term in the Hamiltonian describes the process of de-exciting one qubit and exciting two qubits in the next layer, while the conjugate part describes the process of de-exiting two qubits and exciting one qubit in the previous layer. In this model, the total number of qubits equals to $N_{\text{qubits}} = 2N_{\text{layers}} - 1$ with $N_{\text{layers}}$ being the number of layers. As $N_{\text{qubits}}$ increases, the dimension of Hilbert space of qubits increase exponentially, and thus it is impossible to simulate such model efficiently with classical computers.

The dynamical evolution depends on the coupling strength $g_i$. We control them such that if qubit $A$ is in the excited state $|1\rangle$ initially, the qubits in the last layer will end up in a collective excitation state $|1\rangle$. The other qubits in the network will go through some intermediate state and be in the ground state at the end of the simulation. The simplest realisation, which can also be implemented by the quantum circuit shown later, is to make $g_i$ being pulse functions with the same amplitude of $g$ and the same duration of $\tau = \pi/(2g)$. In this case, only two layers are turned on for each period of $\tau$ when the qubits involved undergo a complete state transfer. Take the example of three layers with 7 qubits, we have

$$g_1(t) = g [\Theta(t) - \Theta(t - \tau)] ,$$

$$g_2(t) = g_3(t) = g [\Theta(t - \tau) - \Theta(t - 2\tau)] \tag{4}$$

with $\Theta(t)$ the Heaviside function. If the initial state of the network is

$$|\Psi(t_0)\rangle = (\alpha|0\rangle + \beta|1\rangle)\otimes|0\rangle_m . \tag{5}$$

When the simulation ends at $t_f = 2\tau$, we will obtain the final state shown in Eq. (1).

The second model, as shown in Fig. 1b, describes the spin measurement, which is motivated by Stern-Gerlach experiment. We use a collection of $2N$ spins as the measurement device to probe the state of the system spin directly, in which they interact with the system spin $\hat{\sigma}_Z$ via Ising-type Hamiltonian:

$$\hat{H}(t) = -\sum_{mn} J_{mn}(t) \hat{\sigma}_Z^m \hat{\sigma}_Z^n - \sum_{mn\neq s} J_{mn} \hat{\sigma}_Z^m \hat{\sigma}_Z^n - h(t) \sum_m \hat{\sigma}_Z^m , \tag{6}$$

where $h(t)$ represents the transverse field $B$ applied to the measurement spins.

We apply a strong field at $t < t_0$: $h(t) = h_0 \Theta(t_0 - t)$ to prepare the state of the measurement spins in $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ with $|0\rangle$ and $|1\rangle$ representing spin-up and spin-down, respectively. The system spin is prepared in a superposition state: $\alpha|0\rangle + \beta|1\rangle$. The resulting state of the entire system is thus given by

$$|\Psi(t_0)\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes |\rangle^{\otimes 2N} . \tag{7}$$
At $t = t_0$, the transverse field is turned off and we turn on the interaction between the system spin and the measurement spins: $J_{s q}(t) = J_{s q} > 0$, and the Hamiltonian becomes zero-field Ising Hamiltonian

$$\hat{H}(t \geq t_0) = \hat{H}_{\text{Ising}} = - \sum_{ij} J_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z. \quad (8)$$

The ground state of the above Hamiltonian is double degenerate, i.e., $|0, 0, ..., 0\rangle$ and $|1, 1, ..., 1\rangle$. Therefore, if we can drive the entire system into its ground state, we will obtain the final state shown in Eq. (2). However, the dynamical evolution of spins system under the Ising Hamiltonian does not lead to the ground state. This is because the initial state in Eq. (7) can be recast as

$$|\Psi(t_0)\rangle = \frac{1}{2^n} \left( |E_0\rangle + \sum_i |E_i^0\rangle + \sum_{m,n} \sum_{i,j} |E_{m,n}\rangle + \cdots + |E_N\rangle \right), \quad (9)$$

where $|E_0\rangle \equiv \alpha|0, 0, ..., 0\rangle + \beta|1, 1, ..., 1\rangle$ is the ground state of $\hat{H}_{\text{Ising}}$, and $|E_i^0\rangle$ is the excited state. The dynamical evolution under $\hat{H}_{\text{Ising}}$ only introduces relative phase for different eigenstates. In order to obtain the desired final ground state $|E_0\rangle$, we introduce a control field acting on the measurement spins. Here we adopt Dicke-type Hamiltonian in the mean field approximation [52, 53]:

$$\hat{H}_{\text{Dicke}} = \hbar \sum_{k=1}^{2N} \phi(t) \hat{\sigma}_k^X. \quad (10)$$

We want to design the time-dependent control field $\phi(t)$ such that the coefficients of the excited states $|E_i^0\rangle$ decay away and only the lower energy states, e.g., $|E_0\rangle$ will remain by the end of evolution. In the following, we show that the target evolution can be well approximated with a novel variational quantum circuit.

### III. QUANTUM CIRCUITS OF TWO MODELS

The key to realize the above two qubit measurement models with quantum computers is constructing the corresponding quantum circuits, which are shown in Fig. 2. We use 7 qubits as an example and the circuits can be extended to the case with more qubits. In the circuits, $q_s$ represent the system qubit and the other qubits are the measurement qubits.

Fig. 2a shows the quantum circuit of 3 layer qubits tree network model with 7 qubits. The core module of the circuit is the realization of Hamiltonian term proportional to $g \hat{\sigma}_-^s \hat{\sigma}_+^s \hat{\sigma}_z^s$. With the pulsed interaction described in the previous section, an interaction duration of $\tau = \pi/(2g)$ results in de-exciting one qubit and exciting two qubits in the next layer. Its equivalent gate operation, shown in Fig. 2a, consists of two CNOT gates and one Toffoli gate.

Take the process involving $\hat{\sigma}_+^s \hat{\sigma}_-^s \hat{\sigma}_z^s$ for instance, two CNOT gates with $q_1$ as the qubit are executed sequentially and then Toffoli gate with $q_1, q_2$ as the control qubits is executed. If $q_s$ is in the ground state $|0\rangle$, $q_1, q_2$ will not be excited and stay in the ground state. If instead $q_s$ is in the excited state $|1\rangle$, the gate operation guarantees that $q_1, q_2$ be excited to $|1\rangle$ and meanwhile $q_s$ be de-excited to the ground state $|0\rangle$. For the general case that $q_s$ is prepared in the superposition state $\alpha|0\rangle + \beta|1\rangle$ by using the gate operation $R_\gamma(\theta)$, the state of $q_1, q_2$ after operations be $\alpha|0\rangle_1|0\rangle_2 + \beta|1\rangle_1|1\rangle_2$. Using a similar analysis, the final state of $q_s, q_1, q_2, q_5, q_6$ in Fig. 2a will be $\alpha|0\rangle_5|0\rangle_4|0\rangle_3|0\rangle_2 + \beta|1\rangle_5|1\rangle_4|1\rangle_3|1\rangle_2$.

Extending the above analysis to the situation with more layers, the final state of the measurement qubits (the last layer in the network) will indeed be $\alpha|0\rangle + \beta|1\rangle$, i.e., the one shown in Eq. (1); all the other qubits will be in the ground state $|0\rangle$. The quantum circuit for the tree network model thus demonstrates how the quantum information of the system qubit can be transferred or, in some sense, augmented to the measurement qubits. Note that the model only emulates the ideal single photon detection. In the realistic situation, however, there also exists dark counts and imperfect detections. The former describes clicks of the detector even when there were no sig-
prepared in di

ential photon coming in; the latter refers to the situation when the detector does not response to the signal photon. These two phenomena can be accounted for in the quantum circuit if we introduce a probabilistic flip gate operation $X$ after the three-qubit interaction, which is also shown in Fig. 2a.

For the spin measurement model, unlike the previous model, there is no specific interaction duration that could lead to one meaningful state transition. Here we propose to approximate the dynamical evolution by implementing the idea of variational quantum circuit [54]. The circuit evolution is given by

$$\hat{U} |\Psi(t_0)\rangle = \hat{U}(\beta_1) \hat{U}(\gamma_1) \cdots \hat{U}(\beta_p) \hat{U}(\gamma_p) |\Psi(t_0)\rangle .$$

(11)

Here $\hat{U}(\gamma) = e^{i \gamma \sum_i \sigma_i^y \sigma_i^z}$ and $\hat{U}(\beta) = e^{i \beta X_{\text{tot}} \sigma_i^x}$, which are related to the evolution under $\hat{H}_{\text{Ising}}$ and $\hat{H}_{\text{Dicke}}$, respectively. It is interesting to note that this variational quantum circuit is very similar to the circuit of quantum approximate optimization algorithm (QAOA) [55] except that there is no single qubit rotation $e^{i \beta \sigma_i^x}$ on the system qubit $q_s$. We limit ourselves to the case that $\hat{H}_{\text{Ising}}$ only takes the nearest-neighbor interaction. In Fig. 2b, we show the corresponding circuit for 7 qubits. The Hardmard gates on the measurement qubits $q_1 \cdots q_6$ and the single qubit rotation gate on the system qubit $q_s$ prepares the initial state shown in Eq. (7). Quantum operations within the blue dotted line realizes the evolution $\hat{U}(\beta) \hat{U}(\gamma)$, and it is repeated $p$ times in the circuit.

To obtain the desired target final state shown in Eq. (2), we need to specify the $2p$ parameters $\{\vec{\gamma}, \vec{\beta}\}$ for given circuit depth $p$. Since our target is to obtain the ground state of the Ising model, we adopt the idea of variational quantum algorithm in which optimal parameters $\{\vec{\gamma}, \vec{\beta}\}$ are determined by using classical optimizer that minimizes the expectation value of Ising Hamiltonian [54–57]. Specifically, we prepare the parameterized quantum circuit with a finite $p$ as shown in Fig. 2b, and then obtain the optimized parameters $\{\vec{\gamma}^*, \vec{\beta}^*\}$ using a classical optimizer, e.g., COBYLA to minimize the expectation value $\langle \Psi(\vec{\gamma}, \vec{\beta})| \hat{H}_{\text{Ising}} |\Psi(\vec{\gamma}, \vec{\beta})\rangle$. Finally, the optimal parameters $\{\vec{\gamma}^*, \vec{\beta}^*\}$ are fixed in the model circuit for the final sampling with different input state $q_s$.

IV. SIMULATION RESULTS

Here we present and discuss the corresponding simulation results. We simulate the corresponding quantum circuits of two models by using IBM qiskit package [58]. Fig. 3a shows the result for the 7 qubits tree network without introducing the qubit flip error $X$. It matches the theoretical prediction of the final state shown in Eq. (1) for different initial states of the system qubit $q_s$. In the Appendix, we also present the results with different flip error probability to simulate dark counts and imperfect detection. Interestingly, those imperfections do not break the mirror symmetry between the final probability distributions for two different initial states $\alpha|0\rangle + \beta|1\rangle$ and $\beta|0\rangle + \alpha|1\rangle$, respectively.

For the spin measurement model, we use the variational quantum circuit with a finite circuit depth $p$. In general, the larger $p$ is, the better the simulation results are. However, more time will be needed to optimize $2p$ parameters $\{\vec{\gamma}, \vec{\beta}\}$. In practice the performance of quantum computers will also limit the depth that can be efficiently implemented. For 7 qubits, we choose $p = 3$ that leads to a good performance. More qubits results with $p = 12$ are shown in Appendix. To determine the optimal $2p$ parameters $\{\vec{\gamma}^*, \vec{\beta}^*\}$, we execute the parameterized quantum circuit with $2p$ initial guess for these parameters and obtain a final state $|\Psi(\vec{\gamma}, \vec{\beta})\rangle$; we calculate the expectation value $\langle \Psi(\vec{\gamma}, \vec{\beta})| \hat{H}_{\text{Ising}} |\Psi(\vec{\gamma}, \vec{\beta})\rangle$. The classical optimizer, e.g., COBYLA, is called to update parameters to minimize the above expectation value until the condition of convergence is satisfied. Even though the optimal values would depend on the

FIG. 3. Simulation results for a: the tree network model, and b: the spin measurement model (second row) for 7 qubits. The system qubit is prepared in different initial states shown on the top. The bar diagram indicates the probability of different final states (indicated by the labels on the horizontal axis) for all the qubits. The optimal parameters of spin measurement circuit are determined by variational optimization with $q_s = |0\rangle$. Once optimal parameters are obtained, we fixed them in quantum circuit and run simulation for other states of $q_s$. 

\[ q_s = |0\rangle \quad q_s = \cos \frac{\pi}{8} |0\rangle + \sin \frac{\pi}{8} |1\rangle \quad q_s = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \quad q_s = \cos \frac{3\pi}{8} |0\rangle + \sin \frac{3\pi}{8} |1\rangle \quad q_s = |1\rangle \]
initial state of the system qubit, it is interesting that the dependence is not strong. As shown in Fig. 3b, we have optimised for the initial state of $q_s$ being $|0\rangle$ and obtained a final state very close to the desired one $|0, 0, 0, 0, 0, 0\rangle$. The results for other input initial states still match well with the expected outcome. Therefore, after the optimal parameters $(\vec{\gamma}, \vec{\beta})$ are found for one specific initial state, we can perform measurement of $q_s$ with arbitrary unknown quantum state $\alpha|0\rangle + \beta|1\rangle$, which is ideal for the physical realisation.

V. DISCUSSION AND CONCLUSION

The quantum measurement problem is one of the most fundamental issues in quantum theory. The rapid development of quantum information science in recent years has already provided many insights into solving the measurement problem [28–32]. The purpose of this work is to motivate the simulations of the measurement process with scalable and programmable quantum computers. Since the randomness in measurement outcomes cannot be explained within the current framework of quantum theory, it should be noted that quantum simulations based upon linear and unitary evolution alone cannot solve the problem. The quantum computers, however, can help explore the possible quantum-to-classical boundary in the quantum measurement process. In particular, the scalability of quantum computers implies that we can study complex dynamical characteristics of the measurement process when more and more qubits are included. Such a platform could also verify various measurement theories, e.g., quantum Darwinism which emphasizes the importance of redundancy in environment for the emergence of classical objective reality [34]. Besides, the impossibility to perform a large scale simulation with many qubits offer a new way to demonstrate quantum advantages in the NISQ era.

In conclusion, we have proposed two qubit measurement models. The qubit tree network model, which is inspired by the single photon detection in quantum optics, describes how the state information of a single system qubit can be propagated and amplified to the last layer of network that consists of many qubits. The state of the system qubit is destroyed after the measurement process in this model. In contrast, the spin measurement model describes how the information of the system qubit is mapped to the measurement qubits without being destroyed. So far, we have been focusing on the circuit models for gate-based quantum computers. It is, however, also possible to consider analog quantum computers or simulators [59–63]. Taking Rydberg atom-based programmable quantum simulator [60, 61] as example, we may arrange atoms in the tree network configuration, and take advantage of the blockade effect to realize the three-qubit interaction. Similarly, due to the natural description of ion interaction with Ising Hamiltonian, it may be more suitable to consider ion trap-based quantum simulator [62, 63] for the spin measurement model. Therefore, we can perform simulations of these two measurement models in both gate-based and analog quantum computers to explore the limit of quantum realm.

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FIG. 4. Qubit tree network model with flip error. Results of different flip probability $P$ of qubits tree network circuit with a: $P=0.01$, b: $P=0.02$, c: $P=0.05$, d: $P=0.1$.

FIG. 5. Variational simulation of spin measurement model with different qubits number and fixed circuit layer of $p=12$. Results of different qubits number a: $N=15$, b: $N=17$, c: $N=19$.

Appendix A: More Simulation Results