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The macroeconomics of age-varying epidemics

Marta Giagheddu a,∗, Andrea Papetti b

a University of Lund, School of Economics and Management – Department of Economics, P.O. Box 7080, SE-220 07 Lund, Sweden
b Bank of Italy – Directorate General for Economics, Statistics and Research, Italy

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A B S T R A C T

We incorporate age-specific socio-economic interactions in a SIR macroeconomic model to study the role of demographic factors for the COVID-19 epidemic evolution, its macroeconomic effects and possible containment measures. We capture the endogenous response of rational individuals who choose to reduce inter- and intra-generational social interactions, consumption- and labor-related personal exposure to the virus, while not internalizing the impact of their actions on others. We find that social distancing measures targeted to the elderly (who face higher mortality risk and are not part of the labor force) are best suited to save lives and mitigate output losses. The optimal economic shutdown generates small gains in terms of lives saved and large output losses, for any given type of social distancing. These results are confirmed by calibrating the model to match real epidemic and economic data in the context of a scenario exercise.

1. Introduction

Disease epidemics force policy-makers to take fast decisions considering multiple dimensions of individual vulnerability to virus transmission. In the case of the novel COVID-19 epidemic, age is a particularly important dimension as older people tend to face a higher case fatality rate (Goldstein and Lee, 2020; Dudel et al., 2020). Countries with more inter-generational contacts may experience more transmissions to the elderly (Dowd et al., 2020), and thus a more disruptive epidemic.1 This paper introduces a model that can be classified as “epi-econ” (Boppart et al., 2022) or “epi-macro” (Dück et al., 2022), i.e. an extension of a standard epidemiological modeling structure to account for economic decisions in a macroeconomic environment, with a novel focus on age-heterogeneity. It provides an input to understand and potentially contain via public policy measures the evolution and the macroeconomic consequences of an epidemic where mortality rates differ by age.

Allowing for variation in contact rates by considering how subpopulations differ in their activity levels is of primary importance to understand the evolution and containment of epidemics with asymmetric impact on the population (Ellison, 2020). We contribute to this goal by developing a “SIR2-age macro” model whose distinctive feature is to consider age-heterogeneity in social and

1 Kuhn and Bayer (2020) find a positive correlation across countries between the case fatality rate and the share of 30 to 49 year-old people living with their parents.

2 SIR is acronym for Susceptible–Infected–Removed where we consider that an individual is “removed” from a bout of infection by either recovering or dying.
economic interactions. Merging the epidemiological SIR structure à la (Kermack and McKendrick, 1927) with a macroeconomic framework, Eichenbaum et al. (2021) (ERT, henceforth) developed a “SIR macro” model. This model revealed that susceptible rational agents severely reduce their consumption and hours worked to lower their own probability of getting infected. Meanwhile, agents do not internalize their impact on the overall spread of the infection thus motivating policy measures.

Our contribution is to build on the core of the ERT’s model with a set of extensions that focus on age-heterogeneity including endogenous choices on intra- and intergenerational contacts. Our framework combines the age-varying dimension of a purely epidemiological model (cf. Towers and Feng (2012)) with an economic framework that considers consumption, work and social activity as distinct choices. We evaluate how these extensions affect macroeconomic outcomes, and how these, in turn, depend on the design of age-specific containment policies. As in ERT, when the infection risk depends on the intensity of economic interactions, agents endogenously limit their economic activity, thereby reducing the infection spread. However, as a big share of deaths derives from the interaction between the more socially active young and the more vulnerable old, also the endogenous response of social activity can affect the infection spread. We show to what extent it is possible to preserve economic activity while saving lives via reducing the social contact component of the interaction instead of the economic one.

We consider two age-groups: aged 70 or more (elderly) and the rest of the population (young). As standard in SIR models, a susceptible agent that gets infected can either recover (in which case cannot get infected again) or die. In our model with two age-groups, the elderly face a higher mortality risk, they do not work and consume their fixed pension. All individuals are hand-to-mouth. We explicitly model economic and social interactions between agents that occur when consuming, working or in a residual category of activity. Each type of economic interaction has a different infection probability which, among other factors, depends on the endogenous daily number of contacts (between and within age-groups) which are calibrated on data by Mossong et al. (2017) for the pre-infection steady state.

We assess the epidemic evolution under different combinations of containment measures, with comparisons to the case of laissez-faire (i.e. no containment). The pandemic can be contained either by means of a consumption tax controlled by the government (as in ERT), referred to as “economic shutdown”, or alternatively, by “social distancing”, reducing the utility derived from social interactions. While the latter does not directly impair economic activity, the former makes consumption and hence production more expensive. In our model, social distancing captures mandated policies affecting the overall number of social contacts (e.g. use of personal protective equipment, prohibition of indoor gatherings, curfews, etc.).

Our model simulations suggest that targeted social distancing allows for a considerable reduction in the epidemic death toll. In our baseline model (when assuming that an effective vaccine is discounted by agents to arrive within one year since the outbreak of the epidemic), the no containment scenario is characterized by the highest death toll (0.29% of the population) and an average output loss in the first year of 1.9%. Using a generalized social distancing (i.e lowering the utility of social interactions equally for all types of contacts) lowers the corresponding output loss (0.8%) and death toll (0.08% of the population). Deviating from generalized social distancing, via milder restrictions on young–young interactions, results in higher deaths (0.14%) and higher output loss (1.4%). Finally, implementing a strict measure on the old–young interactions, keeping milder restrictions on young–young interactions, results in a 1.1% output loss (worse than generalized social distancing, but better than mild social distancing for young) and 0.05% deaths (better than generalized and mild on young distancing).

Without social distancing, an optimal economic shutdown is characterized by 0.17% deaths and 19.2% yearly output loss. The optimal level of economic shutdown depends on the efficacy of other restrictions in lowering the death toll. Given different social distancing measures, the optimal economic shutdown generates 11.3%, 14.3% and 5.9% average yearly output losses in the cases of generalized, milder on young–young interactions and stricter on young-old social distancing, respectively. In the face of these large losses in terms of yearly output, the implied optimal economic shutdown generates small gains in terms of lives saved for any type of social distancing, while its strictness gets diminished as soon as distancing penalizes especially young-old interactions. Our results suggest that targeting interventions to curb the interaction between the most vulnerable agents and the most socially active ones keeps low the death toll and reduces the degree of economic shutdown needed to maximize aggregate welfare.

We evaluate the model’s ability to replicate actual data in a simplified version of the framework using Italy, one of the countries firstly affected by the epidemic at the global level. We calibrate our model, including the containment policies, to target the number of excessive deaths (as obtained in Galeotti et al. (2020)) and to capture the 5.3% quarterly output loss in the first quarter of 2020. The simulations suggest that about three weeks were necessary to flatten the curve of total deaths due to COVID-19, and that absent any government intervention more than 0.4% of the population would have died in two months. To complement our theoretical results, given this epidemic evolution in line with the data, we evaluate the epidemic and its economic impact in a series of hypothetical post-lockdown scenarios corresponding to different types of social distancing measures for any given economic shutdown.

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3 See empirical evidence by Farboodi et al. (2021) for the United States, Andersen et al. (2020) for Denmark and Sweden.
4 In Section 3.3.1 we also show the case which allows for reinfection.
5 Data on the South Korean fatality rates (supposedly the most reliable, given the world’s highest per capita test rates for COVID-19) and estimates of the infection fatality rates for Italy (Garibaldi et al., 2020) (both used in our analysis) show that those aged 70 or more face a mortality probability upon infection of 22 to 28 times higher than the rest of the population. However, uncertainty remains on the mortality risk (Goldstein and Lee, 2020).
6 The design of the actual implementation of these targeted measures goes beyond the purpose of the present paper. It could entail innovation in how goods and services are delivered as well as dedicated personnel to the care of the psychological burden of isolation for vulnerable groups.
7 ISTAT (2020).
We recognize the uncertainty surrounding many of the model parameters. Hence, our quantitative results are better interpreted in terms of a relative order of magnitude within the internal modeling consistency. Our model points to the preference of differentiating containment measures by age, acting more on social distancing of the elderly from the young than on homogeneous economic measures. Finally, while the recommendations stemming from the model points to the benefits of reducing the number of contacts between the elderly and the young, there are clearly human costs associated with long-term isolation that we are not considering. In addition, when evaluating the imposition of restrictions, one needs to ponder other elements, outside of our framework, which can affect the conclusions and policy implementation, e.g. the likelihood of virus mutations deriving from free transmission.

This work contributes to the broad literature on the macroeconomic modeling of epidemics targeting the asymmetric impact of a viral infection on the population while allowing for economic choices to be endogenously affected by the epidemic dynamics. In particular, our work is mostly related to the literature developed since the onset of the COVID-19 epidemic that has extended the purely epidemiological models with a SIR structure to account for the choices of rational economic agents in a macroeconomic environment (Eichenbaum et al., 2021, 2022; Jones et al., 2021; Farboodi et al., 2021; Kaplan et al., 2020; Bodenstein et al., 2020; Krueger et al., 2022; Garibaldi et al., 2020; Kapicka and Rupert, 2022; Brotherhood et al., 2021; Glover et al., 2020; Boppart et al., 2022). Among the contributions that in this line of work focus on age-heterogeneity, our analysis complements models such as the one in Glover et al. (2020) as we allow for the likelihood of infection to increase with consumption focusing on the role of age-specific containment policies for aggregate health-output trade-offs, while not considering redistributive aspects. Compared to Brotherhood et al. (2021), we explicitly consider the empirical contact matrix by age that prevails in “normal” (i.e. pre-epidemic) times when assessing how individuals endogenously choose the level of social interactions, as a separate choice from economic interactions, once facing the risk of infection.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 details the main calibration of the model and the laissez-faire equilibrium in comparison to other workhorse SIR and SIR-macro models as well as to variants of the baseline models that take into account reinfection probability, endogenous mortality rate and unknown health status. Section 4 studies social distancing and the optimal economic shutdown, first separately and then as a policy mix, comparing with the cases in which vaccination is prioritized to either the elderly or the young. Section 5 applies the model to the Italian case to study different scenarios. Section 6 concludes.

2. The age-varying SIR macro model

To a purely epidemiological age-varying SIR model – which we will refer to as “SIR-age model” – we add macroeconomic interactions affecting the number of infected people thus leading to a “SIR-age macro model”. We build on ERT, extending their framework in two main ways: (i) first, while ERT focus on the working-age population only, we consider age as a key dimension of heterogeneity dividing the entire population into “young” and “old”; (ii) second, economic agents can not only choose consumption and hours worked as in ERT, but also the overall amount of (intra- and inter-generational) social activity which they value in their utility function per se (independently of consumption and hours worked).

2.1. The age-varying SIR block of the model

Individuals are either young (y) or old (o). The spread of the disease is characterized by an age-structured SIR model where, depending on age $a \in \{y,o\}$, in each time-period $t$ the population is divided into susceptible ($S_{a,t}$), infected ($I_{a,t}$), recovered ($R_{a,t}$) and deceased ($D_{a,t}$) so that the following set of equations holds:

\begin{align*}
S_{a,t+1} &= S_{a,t} - T_{a,t} \\
I_{a,t+1} &= I_{a,t} + T_{a,t} - (\pi_{a,y} + \pi_{a,d})I_{a,t} \\
R_{a,t+1} &= R_{a,t} + \pi_{a,y}I_{a,t} \\
D_{a,t+1} &= D_{a,t} + \pi_{a,d}I_{a,t} \\
N_T &= \sum_a N_{a,t}
\end{align*}

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8 A replication archive of “macro-epi” models is available online at https://www.epi-mmb.com/ (Dück et al., 2022). Contributions in the economic modeling of the epidemics have also focused on purely epidemiological models (Atkeson, 2020b,a; Fernández-Villaverde and Jones, 2022; Stock, 2020; Berger et al., 2020; Acemoglu et al., 2021; Alvarez et al., 2021; Rampini, 2020; Chikina and Pegden, 2020; Favero et al., 2020) and purely economic models (Guerrieri et al., 2022; Garibaldi et al., 2020; Kapicka and Rupert, 2022; Brotherhood et al., 2021; Glover et al., 2020; Boppart et al., 2022). Among the contributions that in this line of work focus on age-heterogeneity, our analysis complements models such as the one in Glover et al. (2020) as we allow for the likelihood of infection to increase with consumption focusing on the role of age-specific containment policies for aggregate health-output trade-offs, while not considering redistributive aspects. Compared to Brotherhood et al. (2021), we explicitly consider the empirical contact matrix by age that prevails in “normal” (i.e. pre-epidemic) times when assessing how individuals endogenously choose the level of social interactions, as a separate choice from economic interactions, once facing the risk of infection.

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10 Shutting down the possibility of economic interactions the model described in Section 2.1 becomes a standard age-varying SIR model as employed in the epidemiological literature (see e.g. Towers and Feng, 2012).
where \( N_t \) denotes the total number of alive individuals in the economy while \( \pi_{r,t} \) and \( \pi_{o,t} \) are parameters representing the probabilities of recovering and dying, respectively. The variable \( T_{a,t} \) represents the flow of infected people which depends on the macroeconomic interactions as described in Section 2.2.

The initial population is normalized to one, \( N_0 = 1 \). We assume that there is an initial shock \( \varepsilon \) to the total number of infected across the age groups according to: \( I_{o,0} = \varepsilon f_o \) and \( I_{y,0} = \varepsilon - I_{o,0} \), where \( f_o \) and \( f_y = 1 - f_o \) denote the initial fraction of old and young respectively.

### 2.2. Macroeconomic interactions

#### Households

There is a continuum of measure one of individuals differing in health status \( j \in \{s, i, r\} \) and age \( a \in \{y, o\} \), choosing consumption \( (c_{a,j}^t) \), hours worked \( (n'_{a,j}^t) \) and number of contacts among same-age \( (\kappa_{a,a,j}^t) \) and different-age \( (\kappa_{a,\mu,j}^t) \) peers in order to maximize the objective function:

\[
E_0 \sum_{t=0}^{\infty} \beta^t u(c_{a,j}^t, n'_{a,j}^t, \kappa_{a,a,j}^t, \kappa_{a,\mu,j}^t)
\]

subject to the budget constraints for the young and the old individuals:

\[
(1 + \mu_{c,j})c_{j,s}^t = w_c \phi_s n'_{j,s}^t + I_t,
\]

\[
(1 + \mu_{c,j})c_{j,o}^t = \bar{P} + \Gamma_t
\]

where \( 0 < \beta < 1 \) is the discount factor, \( \phi_s \) is a parameter capturing labor productivity depending on the health status, \( w_c \) is the real wage rate, \( \mu_{c,i} \) is a Pigouvian tax rate on consumption, \( \Gamma_t \) represents lump-sum transfers from the government and where we assume that the elderly do not work (i.e. \( n'_{a,j}^t = 0 \forall j, t \)) and receive a constant pension transfer proportional to the steady state consumption of the young: \( \bar{P} = \alpha c^t, 0 < \alpha < 1 \).

In each period, the number of newly infected people in each age-category is given by:

\[
T_{y,t} = \eta S_{y,s} \left[ \sum_{t}^i x_{yy,t} v_{y,y}^t \kappa_{y,y}^t \kappa_{y,o}^t \kappa_{y,o}^t + \sum_{t}^i x_{yo,t} v_{y,o}^t \kappa_{y,o}^t \kappa_{y,o}^t + \sum_{t}^i x_{yo,t} v_{y,o}^t \kappa_{y,o}^t \kappa_{y,o}^t \right] (2.7)
\]

\[
T_{o,t} = \eta S_{o,s} \left[ \sum_{t}^i x_{yo,t} v_{o,y}^t \kappa_{o,y}^t \kappa_{o,y}^t + \sum_{t}^i x_{oo,t} v_{o,o}^t \kappa_{o,o}^t \kappa_{o,o}^t + \sum_{t}^i x_{oo,t} v_{o,o}^t \kappa_{o,o}^t \kappa_{o,o}^t \right] (2.8)
\]

where each parameter \( \pi_{j,a}^t \) captures the weight to each type of infectious interaction while \( \eta \) is a parameter controlling the overall disease transmission rate. The variables \( x_{a,j}^t \) denote the average intra- and inter-generational contacts across all types of individuals as identified by the following contact matrix:

\[
X_{a,j} = \left[ \begin{array}{c} x_{yy,t} \ x_{yo,t} \ x_{oo,t} \end{array} \right] = \left[ \begin{array}{c} \sum_{j} x_{y,y} v_{y,y}^t \ \sum_{j} x_{y,o} v_{y,o}^t \ \sum_{j} x_{o,o} v_{o,o}^t \end{array} \right] (2.9)
\]

for health status \( j \in \{s, i, r\} \), where the share variables are, for \( J \in \{S, I, R\} \) and \( a \in \{y, o\} \):

\[
v''_{a,j} = \frac{J_{a,j}}{N_{a,t}}
\]

This matrix must satisfy reciprocity, i.e. the total number of contacts that the young share with the old must be equal to the total number of contacts that the old have with the young:

\[
N_{y,t} \sum_{j} x_{y,o} v''_{y,j} = N_{o,t} \sum_{j} x_{o,y} v''_{o,j}
\]

Eqs. (2.7) and (2.8) can be interpreted as follows. Consider, for example, that a young individual has on average \( x_{y,y} \) contacts per time-period with other young individuals, a fraction \( v_{y,y}^t \) of whom are infected. Only a fraction \( \eta \) of these contacts with infected turns out as actual new infection cases. And since there is a total of \( S_{y,s} \) susceptible individuals and interactions that are not due to consumption and work have a weight of \( \pi_{y,y}^t \) in the set of all possible interactions faced by the young, the total number of new infected young individuals due to reasons other than consumption and work is given by \( \eta S_{y,s} \pi_{y,y}^t x_{y,y} v''_{y,y} \). A similar reasoning

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11 Since our focus is on the short-run that pertains the outbreak of an epidemic we abstract from long-run issues such as transfers among individuals of different age classes as well as public debt sustainability. Notice that our focus on the epidemic entails that individuals do not die for reasons other than those associated with the epidemic itself.

12 Notice that in equilibrium group-specific aggregates for consumption, hours worked and number of social contacts coincide with the respective individual levels. Throughout the paper we keep a small-capital notation for both cases.

13 Setting \( \pi_{a,s}^t = \pi_{a,i}^t = \pi_{a,r}^t = \pi_{a,s}^t = 0 \) and \( \pi_{a,i}^t = \pi_{a,r}^t = 1 \) the model is an epidemiological age-varying SIR model without economic interactions. The ERT model is nested assuming that \( x_{y,y} = x_{y,o} = x_{o,o} = 0 \) and \( x_{y,y} = N_y \) for all \( t \) where \( N_y = N_c \) as well as \( T_{o,y} = T_{o,y} = S_{o}, I_{y} = I_{o} \) in (2.7) – since in ERT only the young population is considered.

14 For example, the first row of \( X \) denotes the average number of contacts per period that a young makes with a young \( (x_{y,y}) \) and with an old \( (x_{y,o}) \).

15 Notice that \( v''_{y,y} = 1 - v''_{y,o} - v''_{o,y} \).
applies to the other addends of the equations where the rate of infection during consumption and work activities is assumed to be proportional to the product of consumption and work levels chosen by infected and susceptible individuals.

According to their type – young or old who are either susceptible, infected or recovered – individuals satisfy the following dynamic programming.

**Susceptibles.** The lifetime utilities of young and old susceptible individuals are given by:

\[
U_{sj} = u' (c^s_j, n^s_j, x^s_j, y^s_j) + \beta \left[ (1 - r_{ss}) U^s_{j+1} + r_{ss} U^r_{j+1} \right] (1 - \delta_s) + \delta_i U^i_{j+1} + \delta_r U^r_{j+1}
\]  

(2.11)

\[
U_{o,j} = u' (c^o_j, n^o_j, x^o_j, y^o_j) + \beta \left[ (1 - r_{os}) U^o_{j+1} + r_{os} U^r_{j+1} \right] (1 - \delta_s) + \delta_i U^i_{j+1} + \delta_r U^r_{j+1}
\]  

(2.12)

where \(\delta_i\) is the per period probability of discovering a vaccine against the virus,\(^{16}\) and

\[
\tau_{ss} = \frac{T_s}{S_{ss}}
\]  

(2.13)

\[
\tau_{o,t} = \frac{T_o}{S_{ot}}
\]  

(2.14)

represent the age-dependent probabilities of becoming infected. Importantly, when choosing consumption, hours worked and contact level – thus impacting on the infection probability – a susceptible household takes as given the economy-wide aggregates (such as \(v^s_j, c^s_j, \nu^s_j, n^s_j, y^s_j\)). In this way, households do not internalize the impact of their choices on the economy-wide infections so that a Pigouvian tax might optimally reduce such an externality.

**Infected.** The lifetime utilities of young and old infected individuals are given by:

\[
U^i_{sj} = u' (c^i_j, n^i_j, x^i_j, y^i_j) + \beta \left[ (1 - r_{is} - r_{ir}) U^i_{j+1} + r_{is} U^r_{j+1} \right] + \beta U^r_{j+1}
\]  

(2.15)

\[
U^i_{o,j} = u' (c^i_j, 0, x^i_j, y^i_j) + \beta \left[ (1 - r_{os} - r_{or}) U^o_{j+1} + r_{os} U^r_{j+1} \right] + \beta U^r_{j+1}
\]  

(2.16)

where it is implicitly assumed that the utility from death is zero.

**Recovered.** The lifetime utilities of young and old recovered individuals are given by:

\[
U^r_{sj} = u' (c^r_j, n^r_j, x^r_j, y^r_j) + \beta U^r_{j+1}
\]  

(2.17)

\[
U^r_{o,j} = u' (c^r_j, 0, x^r_j, y^r_j) + \beta U^r_{j+1}
\]  

(2.18)

**Firms.** There is a continuum of competitive representative firms of unit measure that produce consumption goods \((Y)\) using hours worked \((H)\) according to the technology \(Y = AH_i\) to maximize profit:

\[
\max \left\{ AH_i - \omega_i H_i \right\}
\]

which leads to the optimal condition: \(\omega_i = A\).

**Government.** The government sets the consumption tax \(\mu_{ij}\) and distributes lump-sum transfers \(T_i\) according to the following budget constraint:

\[
\mu_{ij} C_i = T_i N_i
\]  

(2.19)

where \(C_i = c^i_j S_{ij} + c^s_j I_{ij} + c^r_j R_{ij} + c^o_j S_{ot} + c^i_j I_{ot} + c^o_j R_{ot}\)

(2.20)

**Clearing.** In equilibrium both households and firms solve their respective maximization problems, the government satisfies its budget constraint while both the goods and the labor market clear:

\[
C_i = AH_i + \bar{P} N_{o,t}
\]  

(2.21)

\[
n^s_{ij} S_{ij} + \bar{n}^i_{o,t} I_{ij} + n^o_{ot} R_{ij} = H_i
\]  

(2.22)

**Welfare.** When computing optimality of policy interventions we assume the following aggregate welfare in the first period of the epidemic:

\[
U_0 = S_{ij0} U^i_{s,j0} + I_{ij0} U^i_{s,j0} + S_{ij0} U^r_{s,j0} + I_{ij0} U^r_{s,j0}
\]  

(2.23)

where the terms \(U^i_{s,j0}\) and \(U^r_{s,j0}\) represent the lifetime utilities of susceptible and infected agents in each age groups.

**Functional form.** For age \(a \in \{y, o\}\) and health status \(j \in \{s, i, r\}\), we assume the utility function:

\[
u'(c^a_j, n^a_j, x^a_j, y^a_j) = \log c^a_j - \frac{\theta}{2} (n^a_j)^2 + \nu(x^a_j, y^a_j)
\]  

(2.24)

where \(\nu(x^a_j, y^a_j) = \psi^a_{da} \log[x^a_j] + \psi^a_{da} \log[y^a_j] - \frac{\nu^a_{da}}{x^a_{da}} - \frac{\nu^a_{da}}{z_{da}} - \log[\psi^a_{da} z_{da}] - \psi^a_{da} \log[\psi^a_{da} z_{da}] + \psi^a_{da} + \psi^a_{da}\)

\(^{16}\) As in ERT we assume that upon discovery the vaccine is administered to all the susceptibles in the country from the period of the discovery. Once a person is vaccinated this person becomes immune to the disease.
which, aside from the standard formulation for consumption and work, captures the utility return from social activity (independently of consumption and work) via the additive component \( r(x_{s_{i}}x_{w_{o}}) \). The utility is designed in such a way that it is costly for an individual to reduce the number of contacts relative to the respective pre-epidemic level. In this way, during an epidemic there can be a trade-off in the choice of an individual who, on the one hand, is willing to reduce the number of contacts to diminish the probability of being infected while, on the other hand, can do so only at a utility cost. We further assume that the utility weight on the endogenous choice for the number of contacts \( (\psi) \) can differ from the corresponding weight on the exogenous component \( (\hat{\psi}) \), where the latter is meant to capture the normal level prevailing in the pre-epidemic steady state. We will treat the case of imposed \( \psi < \hat{\psi} \) as a way to reproduce "social distancing" measures enforced by the government which make engaging in social activity more costly in terms of utility value. We consider \( 0 < \psi < \hat{\psi} \leq 1 \). The normal pre-epidemic numbers of contacts are denoted by the entries \( z \) of the contact matrix:

\[
Z = \begin{bmatrix} z_{yy} & z_{yo} \\ z_{yo} & z_{o0} \end{bmatrix}
\]

which can be deduced from the data and satisfies reciprocity: \( f_{e}z_{o} = (1 - f_{e})z_{yo} \). Notice that for \( \psi = \hat{\psi} \) the additive part of the utility function concerning social activity has an interior maximum of 0 when social activity is set to its steady state level, i.e. \( x_{s_{i}} = \psi x_{s_{i}} \). As reported later in the calibration Section 3.1, we assume \( \psi = \hat{\psi} = 1 \) so that the equilibrium number of all types of contacts in the pre-epidemic steady state is equal to the normal pre-epidemic number as captured by the matrix (2.25). Finally, we assume that individuals once infected have a preference that returns less utility for a given level of endogenous social activity, by setting \( 0 < \psi < \psi < \hat{\psi}^\prime \), a simple way to capture altruism.

3. Laissez-faire equilibrium

We first describe the laissez-faire equilibrium, i.e. the epidemic evolution in the absence of any restriction. We also describe the calibration choices of model parameters and in particular the contact matrix determining the level of social interactions across and within age-groups in the initial steady state.

3.1. Calibration

The initial share of young (those younger than 70) is set to \( f_{e} = 0.825 \). The contact matrix \( Z \) in Eq. (2.25) is based on Italian values from Mossong et al. (2017) contact survey data. A young person has on average about 19.1 contacts per day with other young and 1.3 contacts per day with the older group. An old person has on average 6.3 contacts per day with the young and 1.4 contacts per day with other elderly. We multiply these values by 7 since the frequency in the model is weekly. In the initial pre-infection steady state the population is composed only by susceptible individuals, yielding:

\[
\begin{align*}
\alpha^\prime &= 0.8 \\
\alpha^2 &= 0.8^2 \\
\alpha^3 &= 0.8^3 \\
\end{align*}
\]

We assume \( \alpha = 0.8 \), i.e. the steady state consumption of an old individual is 80% of the steady state consumption of a young. We assume that the utility weight on the steady-state level of social activity is identical across age-classes and health-statuses and equal to 1, i.e. \( \lambda_{a} = \hat{\psi}_{y}^{o} = 1 \) for all \( a \in \{y,o\} \). Following Brotherhood et al. (2021) who set the marginal utility of abstaining from social activity once infected in order to match an increase in time spent at home by 50% (based on Rizzo et al., 2013 who report the actual adherence to public recommendations based on the swine flu H1N1), we assume \( \psi_{a}^{o} = \psi_{o}^{o} = 0.5 \) while \( \psi_{a}^{y} = \psi_{y}^{o} = \psi_{y}^{o} = 1 \), for \( a \in \{y,o\} \).

We assume that the "removal" rate (the rate at which an infected individual either recovers or dies) is equal across age classes (what assumed for influenza by e.g. Towers and Feng, 2012), i.e. \( \xi_{y} + \xi_{o} = \xi_{y} + \xi_{o} \equiv \gamma \) and, similarly to ERT and Atkeson (2020b), we assume that it takes 18 days to either recover or die for both ages, i.e. \( \gamma = 7/18 \).

\[17\] The multivariate additive part of the function concerning social activity draws on the univariate utility function assumed by Farboodi et al. (2021), for x denoting overall social activity: \( r(x) = (x - x^*)^2 + 1 \) which has an interior maximum of zero achieved at \( x = x^* = 1 \).

\[18\] Compatible with the 2019 shares for Italy (source: United Nations World Population Prospects 2019).

\[19\] Dataset available via the R-package socialmixr. The main reference is Mossong et al. (2008). A contact is defined as "either skin-to-skin contact such as a kiss or handshake (a physical contact), or a two-way conversation with three or more words in the physical presence of another person but no skin-to-skin contact (a nonphysical contact)".

\[20\] See e.g. Towers and Feng (2012) and in the COVID-19 related epidemiological literature (Ferguson et al., 2020) and Fig. 7 in Appendix D.

\[21\] The contact matrix needs to respect a symmetric property such that \( f_{i}z_{o} = f_{j}z_{o} \), which implies: \( f_{i} = (f_{j}/(1 - f_{j}))z_{o} = 0.825/(1 - 0.825) = 0.825 \). Notice that young individuals have more contacts and there is a tendency for within age group interactions. These patterns are confirmed in Appendix E where we show the contact matrix for the whole sample in the Mossong et al. (2017) survey data (including also Germany, Luxembourg, Netherlands, Poland, United Kingdom, Finland, Belgium).

\[22\] This number reflects the life-cycle profile of consumption (see e.g. Fernandez-Villaverde and Krueger, 2007) with average consumption during retirement generally smaller than what one has during working-age periods.
Following ERT, we consider the case fatality rates by age reported by the South Korean Ministry of Health and Welfare\(^{23}\) interacting with them with the demographic Italian shares in year 2019. Hence we set the ratio between the probability of dying and the removal rate for the young and the old to be, respectively:

\[
\frac{\pi_{s,d}}{\tau} = 0.0045 \quad \frac{\pi_{o,d}}{\tau} = 0.1273
\]

We calibrate the parameter \(\eta\) in Eqs. (2.7)–(2.8) relying on the SIR-age model, with no economic interactions. In this case \(\eta\) represents the transmission rate obtained from the expression for the basic reproduction number \(R_0\)\(^{24}\) for a SIR-age model \(\text{à la} \) (Towers and Feng, 2012):

\[
\eta = \frac{\gamma R_0}{\max\{|\text{eig}(M)|\}}
\]

(3.1)

where \(\max\{|\text{eig}(M)|\}\) denotes the largest eigenvalue of the \(M\) matrix:

\[
M = \begin{bmatrix}
\frac{f_t}{\tau} & \frac{f_t}{\tau} & \frac{f_t}{\tau} \\
\frac{f_o}{\tau} & \frac{f_o}{\tau} & \frac{f_o}{\tau} \\
\frac{f_y}{\tau} & \frac{f_y}{\tau} & \frac{f_y}{\tau}
\end{bmatrix}
\]

We set the \(R_0\) to 1.59 such that 60% of the initial population either recovers or die,\(^{25}\) implying \(\eta = 0.0045\).

To calibrate all the \(\pi_{i,s}\) parameters we modify the approach in ERT to account for the age-varying nature of our model. We assume that consumption and work activities account for two-thirds of all the infection transmissions for each age class, i.e. \(\pi_{i,s}\), satisfy:

\[
\frac{\pi_{i,1} z_{yp}(c_o)^2}{\Pi_y} = \frac{\pi_{i,2} z_{yo}(c_o)^2}{\Pi_o} = \frac{\pi_{i,3} z_{yy}(n_o)^2}{\Pi_y} = 1/9,
\]

\[
\frac{\pi_{i,4} z_{yo}(c_o)^2}{\Pi_o} = \frac{\pi_{i,5} z_{yo}(c_o)^2}{\Pi_o} = 1/6
\]

where

\[
\Pi_y = \pi_{i,1} z_{yp}(c_o)^2 + \pi_{i,2} z_{yo}(c_o)^2 + \pi_{i,3} z_{yy}(n_o)^2 + \pi_{i,4} (z_{yp} + z_{yo})
\]

\[
\Pi_o = \pi_{i,1} z_{yp}(c_o)^2 + \pi_{i,2} z_{yo}(c_o)^2 + \pi_{i,3} (z_{yo} + z_{yo})
\]

A second set of conditions returns limit-values for the number of people who either recover or die at the end of the epidemic:

\[
\lim_{t \to \infty} \{R_{y,1} + D_{y,1}\} = 0.545 \quad \lim_{t \to \infty} \{R_{o,1} + D_{o,1}\} = 0.555
\]

which correspond to targeting \(\lim_{t \to \infty} \{R_{y,j} + D_{y,j} + R_{o,j} + D_{o,j}\} = 0.6\).

The remaining parameters are set to the values assumed in ERT. In particular, \(A = 39.835\), \(\theta = 0.001275\) so that in the pre-epidemic steady state the average working week is 28 h and the average weekly earnings is 58000/52 US dollars. The discount factor is set to \(\beta = 0.96152\) (as in ERT this number implies the value of a life to be 9.3 million 2019 US dollars in the pre-epidemic steady state). The relative productivity of infected people is assumed to be \(\psi = 0.8\) which captures the notion that 80% of infected people continue to work even if infected, as asymptomatic, while the remaining do not. The fraction of people initially infected \(\epsilon\) is set to 0.1%. In the laissez-faire equilibrium there is no policy intervention i.e. \(\mu_{i,s} = 0\) for all \(i\).

3.2. Comparison: SIR-age macro model vs SIR-age model

Considering a laissez-faire environment, we start by comparing the model dynamics in a SIR-age macro model with endogenous contacts, which represents our baseline, to both a SIR-age macro model with exogenous contacts and with a corresponding epidemiological SIR-age model without macroeconomic interactions. We further compare our results with those reported by the ERT’s SIR macro model which tackles only the working-age population and the macroeconomic choices pertaining to consumption and hours worked (not the endogenous choice of the number of contacts).

Fig. 1 shows that, in line with ERT, a model with macroeconomic interactions predicts fewer deaths and a sharper recession compared to a purely epidemiological model. We add that the pattern of this result hinges upon the choice of not only consumption and hours worked, as in ERT, but also the amount of social contacts which agents value per se. Furthermore, we add that considering also the elderly in the economy helps to explain a greater fatal incidence of the disease as well as a slightly less severe recession since the elderly are granted an (externally funded) pension income regardless of the epidemic dynamics — while the young rely uniquely on (self-produced) labor income to finance their consumption.

\(^{23}\) We consider data on April 5, 2020. See https://www.cdc.go.kr/board/board.es?mid=a3040200000008&bidx=0030&act=view&list_no=366739&tag=&nPage=1

\(^{24}\) \(R_0\) is defined as the average number of secondary infections produced by one infected individual during his/her entire period of infection in an entirely susceptible population.

\(^{25}\) As assumed by ERT and outlined by Angela Merkel in her March 11, 2020 speech https://www.nytimes.com/2020/03/11/world/europe/coronavirus-merkel-germany.html.
At the end of the transition, 0.87% of the initial population dies in the SIR-age macro model with exogenous contact versus 0.95% in the SIR-age model (where the figure for the elderly is 0.64% and 0.70%, respectively). However, in the SIR-age macro model with endogenous contacts the share of the population dying at the end of the transition is even lower at 0.43% of the initial population (0.27% for the elderly). Allowing agents to reduce their social contacts endogenously contributes to fewer infections and smaller number of deaths. As a reference, these numbers are both considerably larger than the ERT baseline where 0.27% of the initial population dies in their SIR macro model as they exclude those older than 70 while we focus explicitly on this age-class.

The recession in the macro-models where consumption and work choices affect transmission is much worse than in the purely epidemiological model (“SIR-age”): up to four times worse in the SIR-macro model with no old-age group and without endogenous contacts (ERT model). This gap is reduced when also the elderly are considered and agents are allowed to choose endogenously the number of social contacts. The average aggregate consumption in the first year of the epidemic falls by 1.92% and by 3.21% in the SIR-age macro with endogenous and exogenous contacts, respectively, compared to a fall by only 0.90% in the SIR-age model. From the peak-to-trough weekly aggregate consumption decreases by as much as 3.14% and by 7.17% in the SIR-age macro with endogenous and exogenous contacts, respectively, compared to a fall by only 1.84% in the SIR-age model. In the long-run aggregate consumption is permanently lowered by the death toll: 0.76% lower in the SIR-age macro model with exogenous contacts and 0.42% in the SIR-age macro model with endogenous contacts compared to 0.84% in the SIR-age model.

Despite the permanent effect of the death toll, the recession is relatively mild, with aggregate consumption after one year standing at −1.54% of its pre-infection level in the model with exogenous contacts and −2.42% of its pre-infection level in the model with endogenous contacts, which compare with −0.88% in the SIR-age model and with −2.81% in the ERT model.

Behind the lower peak in infections, deaths and consumption losses in the model with endogenous contacts (as compared to its exogenous contacts counterpart) there is the response of rational susceptible agents who reduce all types of contacts as the number of infected increases (see Fig. 10 in Appendix E) with a choice that results from an interesting trade-off for the determination of the spread of the virus since the young and the old face a different death risk from interactions but their choice on the number of contacts is interdependent.

26 Steady state per capita consumption is given by $C/N = C^0_τ[1 - (1 - \alpha)N_o/N]$. Young individuals consume the same in both the initial and final steady state ($C^0_τ$) and the elderly are given the same fraction of consumption ($\alpha C^0_τ$). Since the infection reduces the proportion of elderly in the economy ($N_o/N$), per capita consumption in the final steady state will be (slightly) higher than in the initial steady state.
3.3. Variants of the baseline model

In this section, we consider three variants of the baseline model that include: (i) a positive reinfection probability so that the model can feature recurrent waves of infection; (ii) a mortality rate that is endogenously determined as a function of the total number of infected agents in each period; (iii) the impossibility for agents to discern between their susceptible and infected statuses. While these variants probably represent steps that move the model closer to reality, they do not alter the substance of the outcomes. As such, we do not consider these variants in the subsequent sections.

3.3.1. Positive reinfection probability

While most of our analysis focuses on the evolution and the handling of the initial epidemic outbreak, it is informative to illustrate our SIR-age macro model dynamics in the presence of probability of reinfection after recovery.\textsuperscript{27}

From Fig. 11 in Appendix E, we observe additional waves of contagion when recovered agents can become susceptible again in the model with exogenous contacts. Furthermore, as more agents become susceptible the initial wave of infection is about 0.8 percentage points larger than in the model without reinfection probability (in Fig. 1). In the model, the additional waves are smaller and smaller in magnitude as the immunity duration is large enough to reduce the number of susceptibles in each wave. This corresponds to the observed pattern for the second wave onward in most countries (occurring after restrictions have been occasionally relaxed).\textsuperscript{28}

When agents can choose to adjust their social activity level (model with endogenous contacts), the infection waves are smoother. As a result less agents get infected in the initial wave and consumption is hence reduced to a lesser extent.

When it comes to containment policies analyzed in the next sections, the presence of additional contagion waves will imply larger optimal shutdown (as more people are susceptible). Social distancing will alter the additional waves in a similar way it does for the first wave. As our paper’s analysis focuses on the first wave and in addition no qualitative change to the direction of containment policies is induced by this alternative assumption, we focus our analysis below on the model without reinfection.

3.3.2. Endogenous mortality rate

As the limit to healthcare capacity is an important driver of mortality and one of the main reasons behind containment measures, we introduce in this section state dependent death probability. Similarly to the “medical preparedness” version of the model developed by ERT, we endogenize the case fatality rate so that it depends positively on a quadratic function of the fraction of infected people in the population. We assume that the rise of total infections affects equally the mortality probability of both young and old individuals, thus assuming that in the face of a saturation of the health care system hospitals do not distinguish by age the patients who can receive treatment.\textsuperscript{29} We calibrate the parameter governing the transmission of total infections to the mortality probabilities ($\kappa$, see Appendix C.2) so that young individuals face a peak case fatality rate which is about double compared to the baseline (see Fig. 12).\textsuperscript{30}

Fig. 12 in Appendix E illustrates that the SIR-age macro model with endogenous case fatality rates (light blue line with diamond marker) involves a larger recession and a higher final number of deaths than the baseline SIR-age macro model (black lines with circle marker). The reason is that individuals, internalizing a higher probability of dying due to hospitalization limits, cut more back on consumption and hours worked as well as on their purely social activities compared to the baseline. This results in a significantly smaller rate of infection for both the young and the old. However, compared to the baseline, aggregate consumption in deviation from steady state evaluated at the trough is about 16% lower, while the final death toll is about 21% higher. Young individuals bear the brunt of these additional deaths since, in relative terms, they face a much higher increase in the probability of dying compared to the elderly.

3.3.3. Unknown health status

We consider a variant of the model where individuals know imperfectly their health status. In particular, following Farboodi et al. (2021), we consider the extreme case where individuals do not distinguish between the susceptible and infected statuses when making their economic decisions (while they know when they have recovered from the disease). This amounts to solve a dynamic problem where the choices on consumption, hours worked and social activity of infected individuals are set equal to those of susceptible individuals at each point in time.\textsuperscript{31}

When the economic choices of the infected individuals are identical to those of the susceptible individuals, the system generates clearly more infectious cases since infected individuals tend to consume, work and socially interact more than in the baseline. This results in a final death toll which is about 9% higher, compared to the baseline, while the consumption fall is smaller at least in the short-run (see Fig. 13 in Appendix E).

\textsuperscript{27} The possibility of reinfection implies that in each period some of the recovered agents become susceptible again. See Appendix C.1 for details on how the main framework has been modified to allow for reinfection probability.

\textsuperscript{28} However, the comparability of model waves under laissez-faire equilibrium and the data is not feasible as the model in this part does not incorporate, among other features, restrictions and seasonal effects.

\textsuperscript{29} See Appendix C.2 for more details about the implementation of this model variant.

\textsuperscript{30} By design, the case fatality rates, given by $\pi_{a,t}$ for age $a \in \{y,o\}$, follow the same dynamics in terms of percentage point deviations from the initial value. Since the initial value is much smaller for the young, the relative increase of the case fatality rate (depicted in Fig. 12) is much bigger for the young compared to the old. The case fatality rates deviate by about 0.45 percentage points from the onset of the epidemic to the peak.

\textsuperscript{31} Coherently, we assume that all infected individuals do work fully ($\phi = 1$) and that the inability to identify the infectious status leads to no preference shift in favor of self-isolation ($\psi_w = \psi_m = \psi_c = \psi_s = 1$). See Appendix C.3 for further model details.
4. Analysis of containment policies

Based on the baseline SIR-age-macro model we consider different possible ways of containing the epidemic’s spread: (1) a consumption tax \( \mu_c \) inducing a reduction in the level of consumption and labor, referred to as “economic shutdown”; (2) the practice of “social distancing” via reducing the utility weight of social interactions. We also consider targeting the latter policy differently towards the two age groups. A combination of the policies in (1) and (2) is also considered.

4.1. Social distancing

We model mandated social distancing through an exogenous reduction in the utility parameter \( \psi \), reducing the utility derived from social contacts. This captures authorities’ adoption of enforced targeted measures to lower the number of social contacts. In general, measures have included e.g. recommendations or rules that modify social gatherings which may lower the utility derived from interactions such as reduced indoor capacity and masking rules. This measure can be age specific, targeting within and between cohorts interactions. Examples of the latter include lowering the number of visits to the elderly in the care facilities, alternatively limiting access to activities that elderly are more prone to participate in. Additional social distancing measures may consist of reducing cohabitation of elderly and young agents (e.g. providing alternative accommodation for younger or older agents to reduce shared spaces). In addition, providing contact-less services to elderly can allow for a substantial reduction in contacts during shopping or other activities. Finally, dedicated personnel can reduce the number of social contacts in case of necessary external care, reducing the aggregate contacts between age groups. While none of these measures requires police intervention, potential associated costs could be balanced by benefits given by continuity in service provision or consumption as well as reduced disruption of economic activity allowed by increased protection of the more vulnerable.

\[ \psi \text{ reduced by } 20\% \text{ with respect to the baseline ("no confinement").} \]

\[ \text{The scenario } \text{"mild on young, GEN on others" is the same as GEN, but the social distancing among young individuals is } 50\% \text{ less severe than in GEN. The scenario } \text{"mild on young, extreme on young-old, GEN on old-old" is the same as the previous scenario but assumes that almost no utility is derived from contacts between young and old individuals.} \]

In Fig. 2 we show the effect of different social distancing measures throughout the transition dynamics and we compare these scenarios with the results from the baseline SIR-age macro model with endogenous contacts (black lines with circle markers). We consider here the case with no vaccine in the horizon.

A generalized policy applied to all individuals (blue lines with diamond marker), modeled as a generalized 20% cut in the utility parameter related to social activity, flattens the epidemic curve (roughly halved death toll) with respect to a no confinement policy.
In addition, the drop in aggregate consumption with respect to the steady state after one year is about halved: 1.1% in the generalized policy as compared to 2.4% in the no confinement scenario, leading to a less pronounced while still prolonged recession. This reflects the smaller final death toll (about 0.23% of the initial population versus 0.43% in the baseline) which is in turn disproportionately borne by the old individuals.

The green line with a star marker in Fig. 2 shows the case in which the confinement measures restricting the contacts among young are milder: i.e. the reduction in the social activity utility parameter is 50% lower if it involves contacts among young individuals (while the generalized distancing applies to all other contacts). This implies that the utility parameter for social activity among young individuals is 90% of the baseline and it generates a severer epidemic and a sharper recession as compared to a generalized policy whereas the elderly bear disproportionately more the brunt of the death toll.

It is possible to impose a milder social distancing on contacts among young while containing the death toll and the recession via imposing an almost total isolation of the elderly from the young (a policy that in the model is implemented by imposing such an heavy burden on young-old interactions that the utility weight on these interactions is close to zero) as shown in the violet lines with down arrow marker in Fig. 2. This implies that the death toll is significantly reduced (0.11% of the initial population by the end of the epidemic) with a relatively higher share of young in the death toll.

4.2. Shutdown of economic activity

A shutdown of economic activity is implemented (in the absence of social distancing) via increasing the consumption tax parameter $\mu_c$. The optimal economic shutdown $\mu_c$ is the one that maximizes the expected utility of all agents in the economy, i.e. our welfare function. The optimal level of shutdown is considered for the cases in which since the onset of the epidemic the vaccine is expected to arrive within (i) one year; (ii) two years; (iii) eight years, corresponding to $\delta_v$ equal to $1/52$, $1/(52 \times 2)$ and $1/(52 \times 8)$, respectively.

Two main forces regulate the optimal shutdown level: (1) minimizing the number of deaths calls for a higher intensity of shutdown; (2) maximizing aggregate consumption calls for a lower intensity of the shutdown.

In Fig. 3 we see that when a vaccine is possible, it is optimal to immediately introduce severe containment measures. The closer in time the vaccine is expected, the more optimal it is to delay infections, as a larger number of susceptible will then benefit from the vaccination as can be observed in Fig. 3. The sooner a vaccine is likely to arrive, the less agents get infected and the later the infection peak will be (at week 37 and week 41 when a vaccine is available in two years or one year, respectively), with infection hitting a smaller fraction of the initial population at peak.
The sooner the vaccine is expected to be the larger the initial drop in consumption. Intuitively, knowing that a vaccine will be available soon, it is preferable to postpone consumption until when it is safe, hence the optimal shutdown curve is higher initially (green line with right arrow marker). It follows that the further away the vaccine is expected to be, the smaller is the immediate cost from optimally constraining the economic activity.

The further away in time the vaccine is expected to be, the more the optimal shutdown level follows the dynamics of contagion. Hence, for the case in which the vaccine is expected to arrive only very late (after 8 years in the case considered in Fig. 3), as the average contagion level rises the mortality-diminishing motive prevails over the consumption-rising motive in the welfare maximization, thus making it optimal to increase the shutdown level.

The balancing of the different motives underlying the optimal choice of the shutdown level tends to deliver an economy that eventually performs quite similarly, irrespective of the vaccine arrival rate. Hence, the final death toll is quite similar across the three cases depicted in Fig. 3 while the average output loss in the first two years since the onset of the epidemic amounts to about 14%, 16% and 15% in the case of optimal shutdown with vaccine in one year, two and eight years, respectively.

4.3. Optimal shutdown with social distancing

The containment policy combination (restricting both the economic activity and the utility from social activity) can be proxied by a joint implementation of a uniform economic shutdown and social distancing policies.

As both social distancing and economic shutdown contribute to flattening the infection curve with varying degrees of consumption losses, the different social distancing measures of Section 4.1 imply different optimal levels of economic shutdown, see Fig. 4 where we have assumed that a vaccine is expected within one year from the onset of the epidemic.

The policy mix minimizing consumption losses and deaths is not the one flattening the overall infection curve, but the one flattening the elderly infections curve the most (violet line with down arrow marker in the first panel of Fig. 4). The social distancing measure minimizing the optimal economic shutdown and the consumption reduction is the isolation of the old from the young population which also delivers the most favorable economic outcome standing at the one year horizon (4.6% losses with respect to the pre-epidemic equilibrium). As the old isolate from the more socially active young, the contagion possibilities are significantly
reduced therefore requiring weaker additional measures. Consumption drops immediately by only 6.8% (as compared to e.g. the 15.5% with generalized social distancing) and the overall death toll is limited to 0.11% of the total population.

The second lowest need of shutdown is obtained when a generalized (“GEN”) reduction by 20% of the utility parameter for social activity is implemented. However, with milder policies on the young–young contacts, as young agents have the largest number of social interactions (see Fig. 7), the loosening of social distancing for these agents implies a higher level of optimal enforced shutdown as compared to the generalized case. When young agents are allowed to interact in a socio-economic context the rise in infection calls for stricter economic shutdown measures generating the highest average output loss with respect to the pre-epidemic among the social distancing cases.

4.3.1. Alternative vaccination distribution

Due to limited availability, vaccination distribution has often prioritized the most vulnerable groups including the elderly. A vaccine can help to control an epidemic via both reducing the number of deaths and slowing the infection rate. The former objective is better achieved prioritizing the vaccination of the elderly, while the latter is better attained via prioritizing the vaccination of the young as shown in Bubar et al. (2021) and Buckner et al. (2021). Several countries have initially considered a combination of these principles in their vaccination strategies. In this section, we consider the case in which a vaccine is prioritized to either the young (delayed to the elderly) or the elderly (delayed to the young). To do so, for a given probability of receiving a vaccine corresponding to \(\delta\) we set \(\delta = 1/52\) (consistent with Fig. 4), we consider the arrival rate of the vaccine for the young \(\delta_{v,y}\) and for the elderly \(\delta_{v,o}\) such that the average vaccination rate using initial population shares is \(f_{v,y} \delta_{v,y} + f_{v,o} \delta_{v,o} = 1/52\). When the elderly are prioritized we set \(\delta_{v,o} = 2/52\) and we solve for the corresponding \(\delta_{v,y}\). When the young are prioritized we set \(\delta_{v,o} = 0.5/52\) and we solve for the corresponding \(\delta_{v,y}\) such that the average vaccination rate is 1/52 using the initial population shares.

As compared to the case where the expected time of vaccination is one year for everyone (\(\delta = 1/52\)), when the vaccine is available later for the young and earlier for the old (see Fig. 14 in Appendix E) we record a larger number of infected (driven by the young) and a larger number of deaths (driven by the elderly), except when young-old interactions are extremely limited. This follows from the fact that the optimal shutdown level is smaller over time as the elderly get vaccinated and consequently a smaller share of the relatively more risky population is susceptible. As a result of the smaller shutdown over time, consumption over the first year is harmed less. In the absence of an extreme distancing for young-old interactions, this translates into slightly larger deaths for both the young and the elderly. However, if distancing is extreme on young-old interactions then prioritizing vaccination to the elderly yields a death toll (violet line with down arrow marker in 14 in Appendix E) which is essentially equal to the case of no vaccine prioritization.

If the young population is instead prioritized in the vaccination (see Fig. 15 in Appendix E) the opposite logic holds. Now, compared to the case of vaccination priority to the elderly, more elderly are susceptible implying a larger optimal economic shutdown once the infection curve increases (even if smaller on impact) in all cases with no extreme distancing on young-old interactions. As a result, consumption decreases relatively more which in turn reduces infections and deaths in the society.

Overall, in the presence of optimal economic shutdown and in the absence of extreme distancing on young-old interactions, prioritizing the vaccination of the elderly does not reduce the death toll but reduces output losses, while prioritizing the vaccination of the young induces a larger shutdown that depresses consumption more but reduces infection and mortality more. When combining extreme distancing measures on young-old interactions with the optimal shutdown it does not make much of a difference, along both the economic and epidemic aggregate measures, to vaccinate with or without priority to any of the two age-groups.

4.4. Containment policies comparison

To summarize what we learned from the analysis of the containment policies in the previous sections, we compare different containment policy-mixes in terms of economic impact and death costs at the aggregate level. The comparison is done for the cases in which a vaccine is expected to be available in one year for all containment policies.

Fig. 5 shows the deaths after one year (two years horizon numbers in brackets) in percentage of the initial population (y-axis) and the average output loss (x-axis) for different containment policies. The no containment scenario is characterized by the highest death toll of 0.29% (0.42%) of the population and an economic loss of 1.92% (1.57%) of output as compared to the steady state.

In comparison to a no containment case, a generalized social distancing would attain a lower average output loss, 0.77% over one year (0.87% over two years) and a lower death toll of 0.08% (0.18%) of the initial population. A deviation from this generalized social distancing featuring a mild reduction in young people’s utility from social activity implies higher deaths, to 0.14% (0.27%) of the initial population and a higher average output loss of 1.37% (1.35%) as compared to generalized social distancing (however, it still represents an improvement in both dimensions as compared to no containment). A deviation from generalized social distancing via a stricter social measures for the young-old interactions is instead characterized by 1.10% (1.11%) average output loss – slightly worse than generalized social distancing, but better than mild social distancing for young – and 0.05% (0.10%) deaths relative to the initial population – better than both generalized and mild on young distancing.

An optimal shutdown measure is characterized by 0.17% (0.36%) deaths and 19.2% (14.1%) average consumption losses. In the presence of social distancing the optimal shutdown will generate 11.27%, 14.28% and 5.95% (8.38%, 10.66%, 4.48%), average output losses in the case of generalized social, milder on young and stricter on young-old distancing respectively. The corresponding death tolls in the three cases are 0.05%, 0.09% and 0.04% (0.14%, 0.23%, 0.09%). For any level of social distancing, the implied optimal economic shutdown generates small gains in terms of lives saved and large increases in terms of average output losses,
especially over one-year time. From the comparison across time horizons, we also can observe that the average output losses are consistently lower in the two year horizon.

Based on this comparison and the dynamic analysis from previous sections, one can conclude that targeted social distancing policies are better than economic shutdown at containing the death toll as well as the consumption losses. The reason for this lies in one key characteristic of our model: the presence of economic and age-specific social interaction as determinants of the transmission. Economic shutdowns contain the epidemic by making consumption more expensive, while social distancing acts directly on the number of contacts, thus being more targeted. So, an economic shutdown might force people to consume less, but as long as consumption happens with no social distancing it takes a higher aggregate economic cost to attain a given level of death toll as compared to social distancing. In Fig. 10 in Appendix E we observe that contacts among agents tend to be reduced more in response to a generalized social distancing than in response to an economic shutdown (as calibrated in the corresponding cases underlying Fig. 5) so that, generally, an economic shutdown needs to generate a much stronger recession than social distancing to attain a given desired level of contacts reduction.

5. Applying the model

We employ a version of the model which is meant to test its in-sample fit for the initial phases of the epidemic evolution as well as to investigate epidemic management scenarios in a country-specific setting, simulating the implementation of similar policies to the ones highlighted in the sections above. To do so, we target the epidemic and macroeconomic dynamics of the Italian case re-calibrating the model over the first 6 months of 2020.

We consider the first week of 2020 as the epidemic origin (similarly to Favero et al. (2020)) and we calibrate the model to match the number of effective deaths in week 10. According to our model it took two to three weeks to flatten the death curve after the imposed lockdown began (at the beginning of week 11). Absent the lockdown measures in Italy, March and April would have

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Fig. 5. SIR-age macro: containment policies comparison.

Note. Left panel: 1 year horizon. Right panel: 2 years horizon. The scenario “generalized (GEN)" assumes that the social activity utility parameter $\psi$ is reduced by 20% with respect to the baseline ("no confinement"). The scenario “mild on young, GEN on others” is the same as GEN, but the social distancing among young individuals is 50% less severe than in GEN. The scenario “mild on young, extreme on young-old, GEN on old-old” is the same as the previous scenario but assumes that almost no utility is derived from contacts between young and old individuals. In all cases we assume that a vaccine is expected to be available in one year from the outbreak of the epidemic.

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32 We employ a version of the SIR-age macro model with exogenous contacts (see Section 2) to facilitate the calibration of the model to the epidemic developments in Italy.

33 Details on the re-calibration are provided in Appendix D.

34 Data on COVID-19 infections and deaths are observed since week 9 (starting February 23). Week 11 (starting March 8) marks the beginning of the general shutdown of “inesential” economic activities and of enforced generalized social distancing: Lockdown. In week 19 (starting May 3) the government started by decree a post-lockdown period with milder containment measures while keeping a close monitoring of the total stock of infected (adjusting the measures of economic shutdown accordingly). May 3 is also when we stopped retrieving data on observed total deaths.
seen a higher death toll according to our model, killing more than 0.4% of the initial population in a matter of 2 months.\textsuperscript{35} To be consistent with the death-toll observed in the data we assume that during the lockdown phase all contacts among individuals of any age group get reduced to 46% of what would prevail in normal circumstances. To make our scenarios consistent also with the official estimate of the Italian economic recession (real GDP) in the first quarter of 2020 (–5.3% in 2020Q1 with respect to the previous quarter, ISTAT (2020)) we set $\mu_c = 0.625$ in all periods of the lockdown phase.\textsuperscript{36}

We then investigate different possibilities for social distancing and economic shutdown for the progressive release of economic activity. We assume that the government post-lockdown sets the intensity of the economic shutdown ($\mu_c$) in proportion to the observed stock of contemporaneous infected people:

$$\mu_{c,t} = \beta_c (I_{y,t} + I_{o,t})$$

(5.1)

aiming at capturing the ready-to-intervene attitude of the government.\textsuperscript{37} Assuming $\beta_c = 9$ in all periods $t$ in the post-lockdown phase, we obtain that the annual average growth rate of output in 2020 in the range of $-11.4\%$ to $-8.6\%$ for the three main social distancing scenarios analyzed in Fig. 6. This result compares, for example, with the $-9.1\%$ estimate provided by the International Monetary Fund (2020). For the second quarter of 2020 compared with the first quarter the model generates an economic recession in the range of $-18.8\%$ to $-12.2\%$. For comparison, the equivalent figure projected at the time by European Commission (2020) is $-13.6\%$.

We show the following possible scenarios for the post-lockdown phase:

(0) The reference case is a laissez-faire equilibrium (black continuous lines in Fig. 6) where individuals maintain the same number of weekly contacts prevailing in the pre-epidemic times and the consumption tax is always set to zero. Our SIR-age macro model predicts that the economy would have faced a rapid recession with a trough at week 17 and an annual average output

\textsuperscript{35} The official and effective data point to a total death-toll of 0.047% and 0.070% of the initial population by the end of week 18 (April 26 to May 2), respectively.

\textsuperscript{36} In particular, our model produces a recession in the first quarter of 2020 (compared to the previous quarter) of 5.3%, 5.2%, 4.9% corresponding to the blue dot-dashed, gray dashed and green dotted lines in Appendix Fig. 8, respectively.

\textsuperscript{37} Cf. “Phase 2, Press Conference of the Prime Minister, Giuseppe Conte”, April 26, 2020, available at http://www.governo.it/node/14518.
growth rate in 2020 of −2.1% (vs −1.2% in the SIR-age model with no macroeconomic interactions). The cost of this milder recession is a high death-toll in the long-run of more than 0.9% of the initial population, i.e. more than 0.6 million lives.

The three post-lockdown social distancing policies are:

1. **Uniformly careful** (blue dot-dashed lines in Fig. 6), as careful as in the lockdown phase. Compared to the laissez-faire equilibrium, the final death-toll is about 3 times smaller while the annual average output growth rate in 2020 is about 4 times more negative. About two-thirds of the final death toll occur among the elderly.

2. **Uniformly loose** (gray dashed lines in Fig. 6). The post lockdown social distancing is 50% looser than during the lockdown phase. Compared to scenario (1), this loosening doubles the final death toll, with more than two-thirds of deaths accounted by the elderly. The annual average output growth rate in 2020 is −11.4%, worse than under scenario (1) (−8.6%) as the number of total infected increases slightly.

3. **Careful with young-old, loose with young-young and old-old contacts** (green dotted lines in Fig. 6). We assume that the contacts among young and old individuals are two-thirds less than in the lockdown phase; the contacts among individuals of the same age are correspondingly two-thirds more. Under this scenario, the final death-toll is roughly the same as the one prevailing in scenario (1); there are more contacts, but the composition is changed and the elderly now account for a smaller share of the final death toll (44%). There is a new wave of infections similar to the one under scenario (2), but in this case the new infectious cases are not so lethal as they mostly occur among the young individuals. This results in an annual output growth rate at −10.6%. While part of this bigger output loss is due to the fact that more infected individuals decrease aggregate productivity and induce individuals (who discount a higher probability of being infected) to cut back on consumption and hours worked, the main reason is that we assume that the government sets the consumption tax proportionally to the number of contemporaneous infected. This is confirmed by the violet continuous line in Fig. 6 showing what would happen if the government was to set the consumption tax to zero in all post-lockdown periods. In this case the output loss in the second quarter (quarter on quarter) would be about 2.4 times smaller (compare with the green dotted line). 38

6. Concluding remarks

Combining an epidemiological framework (Towers and Feng, 2012) into a macroeconomic one (Eichenbaum et al., 2021), adding the possibility for economic agents to value and thus choose social activity per se (independently of consumption and hours-worked), our model suggests that differentiating by age the policies to contain an epidemic such as COVID-19 would be desirable. Compared to uniform social distancing and economic shutdown measures, age-targeted measures reduce the death toll while limiting the output losses. Hence, our work contributes to the view that to optimally contain an epidemic one needs to act on individuals’ observable characteristics such as age in designing the containment policies.

We arrived to these conclusions via a model that is calibrated for two age-groups, those aged 70 or more and the rest of the population, and that derives the main economic trade-off of social distancing from costs in terms of individual utility. Among the next steps, we envisage to consider more age-groups thus extending the scope of the analyzed policies and to incorporate additional economic trade-off channels of social distancing that might be particularly relevant when analyzing phases of economic reopening.

Finally, aspects such as the human costs of prolonged social distancing between young and old individuals or time-varying developments of the COVID-19 transmissibility are not considered in our framework and should be remembered when interpreting our results. Overall, given the uncertainty surrounding many of the model parameters, we recognize that our quantitative results are most valuable in terms of a relative order of magnitude.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. First order conditions

The Lagrangian for a susceptible young individual who chooses \( \{c^t_{y,j}, n^t_{y,j}, x^t_{yy,j}, x^t_{yo,j}\}_{t=0}^\infty \):

\[
\begin{align*}
\log c^t_{y,j} - \frac{\theta}{2} (n^t_{y,j})^2 + \psi^t_{yy} \log(x^t_{yy,j}) + \psi^t_{yo} \log(x^t_{yo,j}) - \frac{x^t_{yy,j}}{2} \frac{X^t_{yy,j}}{z_{yy,j}} + X^t_{yo,j} \frac{\bar{\psi}^t_{yy} + \bar{\psi}^t_{yo} - \bar{\psi}^t_{y,j} \log(\bar{\psi}^t_{yy,j} - {\bar{\psi}^t_{yo,j}})}{\log(\bar{\psi}^t_{yy,j} - {\bar{\psi}^t_{yo,j}})} - \lambda^t_{y,j} \left( \frac{1}{(1 + \mu_{c,j})} c^t_{y,j} - A \phi n^t_{y,j} - \Gamma_j \right) \\
- \lambda^t_{y,j} \left\{ n^t_{y,j} - \eta \left[ x^t_{yy,j} x^t_{yy,j} + x^t_{yy,j} x^t_{yy,j} + x^t_{yy,j} n^t_{y,j} \right] + \bar{\psi}^t_{yy,j} x^t_{yy,j} x^t_{yy,j} n^t_{y,j} + \bar{\psi}^t_{y,j} \left( x^t_{yy,j} x^t_{yy,j} + x^t_{yo,j} x^t_{yo,j} \right) \right\} \\
- \lambda^t_{y,j} \left\{ N_{y,j} x^t_{yo,j} - N_{o,j} x^t_{yo,j} \right\}
\end{align*}
\]

38 Throughout the simulations we keep the number of contacts among the elderly ("old-old contacts") at the same level of looseness prevailing for contacts among the young. However, the old-old contact channel is not very important in the model at a macroeconomic level as the elderly are fewer in the macroeconomy (initially 16.7% of the population) and have comparatively few contacts among themselves in normal times (see Fig. 7).
Similarly, for a susceptible old individual who chooses \( \{c_{o,z}^s, x_{o,y,z}^s, x_{o,y,z}^i, \lambda_{o,z}^s\} \)_{0:t}:
\[
\log c_{o,z}^s + \psi_{o,z}^s \log(x_{o,y,z}^s) + \psi_{o,y}^s \log(x_{o,y,z}^i) - \frac{x_{o,y,z}^s}{\bar{z}_{o,y}} - \frac{x_{o,y,z}^i}{\bar{z}_{o,y}} - \lambda_{o,z}^s \left(1 + \mu_{c,z}\right)c_{o,z}^s - \bar{P} - \bar{\Gamma}_z
\]
\[
- \lambda_{o,z}^s \frac{\eta}{\bar{z}_{o,y}} x_{o,y,z}^s w_{o,y,z}^s c_{o,z}^s + \lambda_{o,z}^s \left(n_{o,y,z}^s x_{o,y,z}^s v_{o,y,z}^i + \pi_{o,3}^s x_{o,y,z}^s v_{o,y,z}^i c_{o,z}^s + \pi_{o,3}^s \left(x_{o,y,z}^s v_{o,y,z}^i + x_{o,y,z}^i v_{o,y,z}^i\right)\right)\}
- \lambda_{o,z}^s \left( \gamma_{o,z}^s, x_{o,y,z}^s - N_{o,z} x_{o,y,z}^s \right)
\]

where:

\[
\begin{align*}
x_{y,y,z} &= x_{y,y,z}^s v_{y,y,z}^s + x_{y,y,z}^i v_{y,y,z}^i + x_{y,y,z}^s v_{y,y,z}^i \\
x_{o,y,z} &= x_{o,y,z}^s v_{o,y,z}^s + x_{o,y,z}^i v_{o,y,z}^i + x_{o,y,z}^s v_{o,y,z}^i \\
x_{o,o,z} &= x_{o,o,z}^s v_{o,o,z}^s + x_{o,o,z}^i v_{o,o,z}^i + x_{o,o,z}^s v_{o,o,z}^i
\end{align*}
\]

The first order conditions are:

**Susceptibles**

\[
c_{y,z}^s : \quad \frac{1}{c_{y,z}^s} - (1 + \mu_{y,z})c_{y,z}^s + \eta \left(\pi_{y,1} x_{y,y,z}^s v_{y,y,z}^s + \pi_{y,2} x_{o,y,z}^s v_{o,y,z}^i\right) - \lambda_{y,z}^s = 0 \tag{A.1}
\]

\[
n_{y,z}^s : \quad -\theta n_{y,z}^s + A \lambda_{y,z}^s + \eta \pi_{y,3} x_{y,y,z}^s v_{y,y,z}^i n_{y,z}^s - \lambda_{y,z}^s = 0 \tag{A.2}
\]

\[
\tau_{y,z} : \quad \beta \left(U_{y,z+1}^s - U_{y,z+1}^i\right)(1 - \delta_{y,z}) = \lambda_{y,z}^s \tag{A.3}
\]

\[
\lambda_{o,z}^s : \quad (1 + \mu_{c,z}) c_{y,z}^s = \lambda_{o,z}^s + \Gamma_z \tag{A.4}
\]

\[
x_{y,y,z}^s : \quad \psi_{y,z}^s \frac{x_{y,y,z}^s}{z_{y,y}} - \frac{1}{z_{y,y}} + \eta v_{y,y,z}^s n_{y,z}^s \left(\pi_{y,1} c_{y,z}^s + \pi_{y,3} n_{y,z}^s + \pi_{y,s}^s\right) - \lambda_{y,z}^s = 0 \tag{A.5}
\]

\[
x_{o,y,z}^s : \quad \psi_{y,z}^s \frac{x_{o,y,z}^s}{z_{o,y}} - \frac{1}{z_{o,y}} + \eta v_{o,y,z}^s n_{o,z}^s \left(\pi_{o,1} c_{y,z}^s + \pi_{o,3} n_{o,z}^s + \pi_{o,s}^s\right) - \lambda_{o,z}^s = 0 \tag{A.6}
\]

\[
\lambda_{o,z}^s : \quad c_{o,z}^s = \frac{\bar{P} + \Gamma_z}{1 + \mu_{c,z}} \tag{A.7}
\]

\[
\tau_{o,z} : \quad \beta \left(U_{o,z+1}^s - U_{o,z+1}^i\right)(1 - \delta_{y,z}) = \lambda_{o,z}^s \tag{A.8}
\]

\[
x_{o,o,z}^s : \quad \psi_{o,z}^s \frac{x_{o,o,z}^s}{z_{o,o}} - \frac{1}{z_{o,o}} + \eta v_{o,o,z}^s n_{o,z}^s \left(\pi_{o,1} c_{o,z}^s + \pi_{o,3} n_{o,z}^s + \pi_{o,s}^s\right) - \lambda_{o,z}^s = 0 \tag{A.9}
\]

\[
x_{o,o,z}^s : \quad \psi_{y,z}^s \frac{x_{o,o,z}^s}{z_{o,o}} - \frac{1}{z_{o,o}} + \eta v_{o,o,z}^s n_{o,z}^s \left(\pi_{o,1} c_{o,z}^s + \pi_{o,3} n_{o,z}^s + \pi_{o,s}^s\right) - \lambda_{o,z}^s = 0 \tag{A.10}
\]

\[
\lambda_{o,z}^s : \quad N_{o,z} (x_{o,y,z}^s v_{o,y,z}^i + x_{o,y,z}^i v_{o,y,z}^i + x_{o,o,z}^s v_{o,o,z}^i + x_{o,o,z}^i v_{o,o,z}^i) = N_{o,z} (x_{o,y,z}^s v_{o,y,z}^i + x_{o,y,z}^i v_{o,y,z}^i + x_{o,o,z}^s v_{o,o,z}^i + x_{o,o,z}^i v_{o,o,z}^i) \tag{A.11}
\]

**Infected**

\[
c_{y,z}^i : \quad \frac{1}{c_{y,z}^i} - (1 + \mu_{y,z})c_{y,z}^i = 0 \tag{A.12}
\]

\[
n_{y,z}^i : \quad -\theta n_{y,z}^i + \phi A n_{y,z}^i = 0 \tag{A.13}
\]

\[
\lambda_{y,z}^i : \quad (1 + \mu_{z}) c_{y,z}^i = A \phi n_{y,z}^i + \Gamma_z \tag{A.14}
\]

\[
x_{y,y,z}^i : \quad \psi_{y,z}^i \frac{x_{y,y,z}^i}{z_{y,y}} \tag{A.15}
\]

\[
x_{y,z}^i : \quad \psi_{y,z}^i \frac{x_{y,z}^i}{z_{y,z}} \tag{A.16}
\]

\[
\lambda_{o,z}^i : \quad c_{o,z}^i = \frac{\bar{P} + \Gamma_z}{1 + \mu_{c,z}} \tag{A.17}
\]

\[
x_{o,o,z}^i : \quad \psi_{o,z}^i \frac{x_{o,o,z}^i}{z_{o,o}} \tag{A.18}
\]

\[
x_{o,y,z}^i : \quad \psi_{o,y,z}^i \frac{x_{o,y,z}^i}{z_{o,y}} \tag{A.19}
\]
Recovered.

\[ c_{r,j}^r : \frac{1}{c_{r,j}} - (1 + \mu_{e,j}) x_{r,j}^r = 0 \]  
(A.20)

\[ n_{r,j}^r : -\theta n_{r,j}^r + \Lambda x_{r,j}^r = 0 \]  
(A.21)

\[ \lambda_{r,j}^r : (1 + \mu_{e,j}) c_{r,j}^r = \Lambda n_{r,j}^r + \Gamma_i \]  
(A.22)

\[ x_{y,y,j}^r : x_{y,y,j}^r = \psi_{y,y}^r z_{yy} \]  
(A.23)

\[ x_{y,o,j}^r : x_{y,o,j}^r = \psi_{y,o}^r z_{yo} \]  
(A.24)

\[ \lambda_{o,j}^r : c_{o,j}^r = \frac{\bar{P} + \Gamma_i}{1 + \mu_{e,j}} \]  
(A.25)

\[ x_{o,o,j}^r : x_{o,o,j}^r = \psi_{o,o}^r z_{oo} \]  
(A.26)

\[ x_{o,y,j}^r : x_{o,y,j}^r = \psi_{o,y}^r z_{oy} \]  
(A.27)

Appendix B. Computing the equilibrium

Following closely ERT, for a given sequence of containment rates \( \{\mu_{e,i}\}_{i=0}^F \) for some final horizon \( F \), guess sequences \( \{n_{r,i}^r, n_{y,j}^r, x_{y,j}^r, x_{o,o,j}^r, x_{o,y,j}^r\}_{i=0}^F \) compute the sequence of the remaining unknown variables in each of the following equilibrium equations:

\[ \lambda_{y,j}^r = \frac{\theta n_{y,j}^r}{A} \]  
(B.1)

\[ c_{y,j}^r = \frac{(1 + \mu_{e,j}) \lambda_{y,j}^r}{A} \]  
(B.2)

\[ \Gamma_i = (1 + \mu_{e,j}) c_{y,j}^r - \Lambda n_{y,j}^r \]  
(B.3)

\[ c_{o,j}^r = \frac{\bar{P} + \Gamma_i}{1 + \mu_{e,j}} \]  
(B.4)

\[ x_{y,y,j}^r = \psi_{y,y}^r z_{yy} \]  
(B.5)

\[ x_{y,o,j}^r = \psi_{y,o}^r z_{yo} \]  
(B.6)

\[ x_{o,o,j}^r = \psi_{o,o}^r z_{oo} \]  
(B.7)

\[ x_{o,y,j}^r = \psi_{o,y}^r z_{oy} \]  
(B.8)

\[ u_{y,j}^r = \log c_{y,j}^r - \frac{\theta}{2} (n_{y,j}^r)^2 + \psi_{y,y}^r \log [x_{y,y,j}^r] + \psi_{y,o}^r \log [x_{y,o,j}^r] - \frac{x_{y,y,j}^r - x_{y,o,j}^r}{z_{yy}} + \psi_{y,y}^r + \psi_{y,o}^r - \psi_{y,y}^r \log [\psi_{y,y}^r z_{yy}] - \psi_{y,o}^r \log [\psi_{y,o}^r z_{yo}] \]  
(B.9)

\[ u_{o,j}^r = \log c_{o,j}^r + \psi_{o,o}^r \log [x_{o,o,j}^r] + \psi_{o,y}^r \log [x_{o,y,j}^r] - \frac{x_{o,o,j}^r - x_{o,y,j}^r}{z_{oo}} + \psi_{o,o}^r + \psi_{o,y}^r - \psi_{o,o}^r \log [\psi_{o,o}^r z_{oo}] - \psi_{o,y}^r \log [\psi_{o,y}^r z_{oy}] \]  
(B.10)

Iterate backwards from the post-epidemic steady-state values of \( U_{y,j}^r, U_{o,j}^r, U_{y,F}^r = u_{y,j}^r/(1 - \beta), U_{o,F}^r = u_{o,j}^r/(1 - \beta) \):

\[ U_{y,j}^r = u_{y,j}^r + \beta U_{y,j+1}^r \]  
(B.11)

\[ U_{o,j}^r = u_{o,j}^r + \beta U_{o,j+1}^r \]  
(B.12)

Calculate the sequence for remaining unknowns in the following equations:

\[ \lambda_{y,j}^i = \frac{\theta n_{y,j}^i}{\phi A} \]  
(B.13)

\[ c_{y,j}^i = \frac{(1 + \mu_{e,j}) \lambda_{y,j}^i}{A} \]  
(B.14)

\[ c_{o,j}^i = \frac{\bar{P} + \Gamma_i}{1 + \mu_{e,j}} \]  
(B.15)

\[ x_{y,y,j}^i = \psi_{y,y}^i z_{yy} \]  
(B.16)

\[ x_{y,o,j}^i = \psi_{y,o}^i z_{yo} \]  
(B.17)

\[ x_{o,o,j}^i = \psi_{o,o}^i z_{oo} \]  
(B.18)

\[ x_{o,y,j}^i = \psi_{o,y}^i z_{oy} \]  
(B.19)

\[ u_{y,j}^i = \log c_{y,j}^i - \frac{\theta}{2} (n_{y,j}^i)^2 + \psi_{y,y}^i \log [x_{y,y,j}^i] + \psi_{y,o}^i \log [x_{y,o,j}^i] - \frac{x_{y,y,j}^i - x_{y,o,j}^i}{z_{yy}} + \psi_{y,y}^i + \psi_{y,o}^i - \psi_{y,y}^i \log [\psi_{y,y}^i z_{yy}] - \psi_{y,o}^i \log [\psi_{y,o}^i z_{yo}] \]  
(B.20)
Given initial values $I_{o,0} = \epsilon f_y, I_{y,0} = \epsilon - I_{o,0}, S_{y,0} = (1-\epsilon)f_y, S_{o,0} = 1 - \epsilon - S_{y,0}, N_{y,0} = f_y, N_{o,0} = f_o = 1 - f_y, R_{y,0} = R_{o,0} = 0, N_0 = N_{y,0} + N_{o,0} = 1$ iterate forward the following equations for $t = 0, 1, \ldots, F - 1$:

\begin{align}
\nu_{y,t}^{i} &= \frac{1}{\nu_{o,t}^{i}} - \frac{1}{\nu_{o,t}^{i}} \\
\pi_{y}^{i} &= \left( \sum_{j} x_{y,j,t}^{i} \nu_{y,j,t}^{i} \right) - x_{0y,t}^{i} \nu_{o,t}^{i} \\
\pi_{o}^{i} &= \left( \sum_{j} x_{o,j,t}^{i} \nu_{o,j,t}^{i} \right) - x_{0o,t}^{i} \nu_{o,t}^{i} \\
\nu_{o,t}^{i} &= \frac{1}{\nu_{o,t}^{i}} - \frac{1}{\nu_{o,t}^{i}} \\
\lambda_{y,t}^{i} &= \frac{\eta \nu_{y,t}^{i} \nu_{o,t}^{i}}{\eta \nu_{y,t}^{i} \nu_{o,t}^{i}} \\
\lambda_{o,t}^{i} &= \frac{\eta \nu_{y,t}^{i} \nu_{o,t}^{i}}{\eta \nu_{y,t}^{i} \nu_{o,t}^{i}} \\
\eta_{y} &= \frac{\pi_{y}^{i}}{\pi_{o}^{i}} \left( x_{y,t,j}^{i} + x_{o,t,j}^{i} \right) \\
\eta_{o} &= \frac{\pi_{o}^{i}}{\pi_{y}^{i}} \left( x_{y,t,j}^{i} + x_{o,t,j}^{i} \right) \\
T_{y,t} &= \eta S_{y,t} \left[ \pi_{y}^{i} x_{y,t,j}^{i} + \pi_{o}^{i} x_{o,t,j}^{i} \right] \\
T_{o,t} &= \eta S_{o,t} \left[ \pi_{y}^{i} x_{y,t,j}^{i} + \pi_{o}^{i} x_{o,t,j}^{i} \right] \\
u_{o,t}^{i} &= \log c_{o,t}^{i} + \psi_{o,t}^{i} \log[\pi_{o}^{i} x_{o,t,j}^{i}] - \frac{\pi_{o}^{i} x_{o,t,j}^{i}}{\pi_{o}^{i}} \nu_{o,t}^{i} \\
u_{y,t}^{i} &= \log c_{y,t}^{i} + \psi_{y,t}^{i} \log[\pi_{y}^{i} x_{y,t,j}^{i}] - \frac{\pi_{y}^{i} x_{y,t,j}^{i}}{\pi_{y}^{i}} \nu_{y,t}^{i} \\
S_{y,t+1} &= S_{y,t} - T_{y,t} \\
S_{o,t+1} &= S_{o,t} - T_{o,t} \\
I_{y,t+1} &= I_{y,t} + T_{y,t} - (\pi_{y}^{i} + \pi_{o}^{i}) I_{y,t} \\
I_{o,t+1} &= I_{o,t} + T_{o,t} - (\pi_{y}^{i} + \pi_{o}^{i}) I_{o,t} \\
R_{y,t+1} &= R_{y,t} + \pi_{y}^{i} I_{y,t} \\
R_{o,t+1} &= R_{o,t} + \pi_{o}^{i} I_{o,t} \\
\end{align}
Calculate the sequence of remaining unknown by the following equation:

\[ \lambda_{y,t}^* = \frac{1}{1 + \mu_{\lambda}} \left( \frac{\pi_{y,t}^s \lambda_{y,t}^s + \pi_{y,t}^o \lambda_{y,t}^o}{1 + \mu_{\lambda}} \right) \]  

(B.59)

Finally, given aggregate consumption:

\[ C_t = c_{y,t} S_{y,t} + c_{y,t} I_{y,t} + c_{y,t} R_{y,t} + c_{o,t} S_{o,t} + c_{o,t} I_{o,t} + c_{o,t} R_{o,t} \]  

(B.60)

use a gradient-based method to adjust the guesses \( \{n_i^s, n_i^o, y_{y,t}, y_{o,t}, x_{y,t}, x_{o,t}, x_{y,o,t}, x_{y,o,t} \} \) so that the following equations hold with arbitrary precision:

\[ (1 + \mu_{\lambda})c_{y,t} - A\phi n_i^s - \Gamma_i = 0 \]  

(B.61)

\[ -\theta n_i^s + A\lambda_i^s + \phi n_i^s y_{y,t} v_{y,t}^s \lambda_{y,t}^s = 0 \]  

(B.62)

\[ \mu_c C_t - \Gamma_t N_t = 0 \]  

(B.63)

\[ \lambda_{y,t} - \beta \left( U_{y,t+1} - U_{y,t+1}^o \right) \left( 1 - \delta_c \right) = 0 \]  

(B.64)

\[ \lambda_{o,t} - \beta \left( U_{o,t+1} - U_{o,t+1}^o \right) \left( 1 - \delta_o \right) = 0 \]  

(B.65)

\[ \frac{\psi_{y,o}}{x_{y,o,t}} - \frac{1}{x_{y,o,t}} + \eta v_{y,t} v_{o,t} \left( \pi_{y,t}^s c_{y,t}^o + \pi_{y,t}^o \right) \lambda_{y,t}^* - \lambda_{y,t}^* N_{y,t} y_{o,t} = 0 \]  

(B.66)

Appendix C. Variants of the baseline model
C.1. Positive reinfection probability

The model with reinfection probability discussed in Section 3.3.1 was obtained modifying the following equilibrium conditions. In the SIR block the recovered becomes susceptible with a probability $\sigma_{r,y} = \sigma_{r,o}$. This implies that Eqs. (B.42) and (B.43) in Appendix B (i.e. (2.1) in Section 2) become:

$$S_{y,t+1} = S_{y,t} - T_{y,t} + \sigma_{r,y} R_{y,t} \tag{C.1}$$

$$S_{o,t+1} = S_{o,t} - T_{o,t} + \sigma_{r,o} R_{o,t} \tag{C.2}$$

while Eqs. (B.46) and (B.47) in Appendix B ((2.3) in Section 2) become:

$$R_{y,t+1} = R_{y,t} + \pi_{y,d} I_{y,t} - \sigma_{r,y} R_{y,t} \tag{C.3}$$

$$R_{o,t+1} = R_{o,t} + \pi_{o,d} I_{o,t} - \sigma_{r,o} R_{o,t} \tag{C.4}$$

In addition, the present value of utility of recovered (Eqs. (B.11) and (B.12)) in Appendix B (i.e. (2.17) and (2.18) in Section 2) are consistently changed as follows:

$$U_{r,y,t} = u_{r,y,t} + \beta ((1 - \sigma_{r,y}) U_{r,y,t+1} + \sigma_{r,y} U_{s,y,t+1}) \tag{C.5}$$

$$U_{r,o,t} = u_{r,o,t} + \beta ((1 - \sigma_{r,o}) U_{r,o,t+1} + \sigma_{r,o} U_{s,o,t+1}) \tag{C.6}$$

C.2. Endogenous mortality rate

The endogenous mortality rate discussed in Section 3.3.2 has been implemented via substituting in the equilibrium conditions the parameter $\pi_{y,d}$ and $\pi_{o,d}$ with the following endogenous variables:

$$\pi_{y,d,t} = \pi_{y,d} + \kappa (I_{y,t} + I_{o,t})^2 \tag{C.7}$$

$$\pi_{o,d,t} = \pi_{o,d} + \kappa (I_{y,t} + I_{o,t})^2 \tag{C.8}$$

where $\pi_{y,d}$ and $\pi_{o,d}$ are now the corresponding exogenous components. This specification follows closely the “medical preparedness” version of the model developed by ERT.

C.3. Unknown health status

The variant discussed in Section 3.3.3 is a model where individuals do not distinguish between susceptible and infected statuses in their economic decisions. The dynamic equilibrium of the model is obtained by solving for the following five endogenous variables $\{n_{y,t}^i, n_{o,t}^i, x_{y,y,t}^i, x_{y,o,t}^i, x_{o,o,t}^i, x_{o,y,t}^i\}$ setting:

$$n_{y,t}^i = n_{y,t}^i$$

$$x_{y,y,t}^i = x_{y,y,t}^i$$

$$x_{y,o,t}^i = x_{y,o,t}^i$$

$$x_{o,o,t}^i = x_{o,o,t}^i$$

$$x_{o,y,t}^i = x_{o,y,t}^i$$

which substitute the respective equations in the system composed by (B.1) to (B.66) where (B.61) is now omitted. We set $\phi^i = 1$, which is akin to assume that all infected individuals do work fully, and $\psi_{y,y}^i = \psi_{y,o}^i = \psi_{o,o}^i = \psi_{o,y}^i = 1$ thus assuming that the inability to identify the infectious status leads to no preference shift in favor of self-isolation.

Appendix D. Calibration details

See Fig. 7.

Fig. 7. Daily contact matrix: 2 age-groups, Italy.
Note. Elaboration on survey data from Mossong et al. (2017). The underlying matrix where each entry is multiplied by the relative demographic size of each age group is made symmetric employing the same demographic shares employed in the model, namely $f_y = 0.825, f_o = 1 - f_y$. 

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D.1. Calibration to the Italian case

The official data on COVID-19 deaths, published by the Protezione Civile of the Italian government, are available from February 24, 2020. As noted by Galeotti et al. (2020), the official numbers might underreport the actual death-toll of the virus in the Italian population. We build a counterfactual of the 2020 total deaths in the absence of COVID-19 based on the trend in the preceding five years and obtain an estimate of the COVID-19 effective number of deaths on a weekly basis. Throughout the calibration we aim at targeting these epidemiological data and we consider as goodness of fit whether our model’s simulation series lay in-between those two data curves in the lockdown phase.

The first step of the calibration we did is to set the value of $R_0$, the basic reproduction number, in the purely epidemiological ("SIR-age") model to a value consistent with the Italian case. While there is a notorious uncertainty surrounding this statistic, in an early study based on Lombardy (the Northern Italian region most hit by COVID-19) Cereda et al. (2020) report a value of 3.1 (95% CI, 2.9 to 3.2), which is the value we assume. Applying Eq. (3.1), this value of the reproduction number implies $\eta = 0.01129$.

We depart from ERT’s strategy of employing COVID-19 mortality rates on reported cases in South Korea, turning instead to the estimates of the infection fatality rate (IFR) by age for Italy provided by Rinaldi and Paradisi (2020). For the two age classes of our interest, their central estimates – once interacted with the Italian demographic shares – give the following probabilities of dying upon infection:

$$\frac{\pi_{y,d}}{\pi_{y,r} + \pi_{y,d}} = 0.00278 \quad \text{and} \quad \frac{\pi_{o,d}}{\pi_{o,r} + \pi_{o,d}} = 0.06274$$

Given their reported estimates, we consider that the elderly are those aged 71 or more (71+) while the rest of the population figures as young. The initial demographic shares, using data from the United Nations World Population Prospects 2019, are $f_y = 0.833$, $f_o = 1 - f_y = 0.167$.

Furthermore, we assume that it takes on average 14 days to either recover or die from the infection (cf. Eichenbaum et al. (2022)). Hence, given that our model is calibrated at the weekly frequency, we set $\pi_{y,r} + \pi_{y,d} = \pi_{o,r} + \pi_{o,d} = \gamma = 7/14$.

Given the same main parameter values of Section 3.1, we run the SIR-age model finding the following limiting values:

$$\lim_{t \to \infty} \{ R_{y,r} + D_{y,r} \} = 0.8189 \quad \text{and} \quad \lim_{t \to \infty} \{ R_{o,r} + D_{o,r} \} = 0.1214$$

that we used as new values in the SIR-age macro model to calibrate the $\pi_{\cdot,\cdot}$. To do so we had to choose the size of the initial shock to the total amount of infected. We found that a value of $\epsilon = 0.0000225$ gave a number of total deaths in the SIR-age macro model consistent with the value of total effective deaths in the first and second weeks observed in the data (see Fig. 8). This amounts to assume that in the first week of 2020 about 1360 people were infected.

Fig. 8. The lockdown phase calibration: data vs model.

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The methodology exploits the data released by the Italian Office for National Statistics (ISTAT), available at https://www.istat.it/it/archivio/240401 - the specific dataset used is "Dataset analitico con i decessi giornalieri", which comprises the daily total deaths (by any cause) in 2020 until April 4 in the 1689 Italian municipalities most affected by COVID-19. Contrary to Galeotti et al. (2020), we also impute a value to the effective number of deaths due to COVID-19 beyond April 4 (until May 3). We do so via applying the last observed ‘bias factor’ (the ratio of the stock of COVID-19 effective deaths over the official ones) which is 1.48. We find that this statistic was very high, at 25, in the first week of official data release (February 23 to 29) and then decreased progressively: 8.6, 3.9, 2.45, 1.86 and 1.48 (in the last week from March 29 to April 4).
In the SIR-age macro model we always set the vaccination probability $\delta_v$ to $1/52$ which implies that it takes on average 52 weeks (i.e. 1 year) for the vaccine to become available.\footnote{We do so because otherwise it would be hard to justify such a strict observed lockdown in Phase 1 (if not for other reasons such as the overrunning of the hospitalization capacity which is a factor that we do not consider in the current analysis). As also found by ERT, and as we have documented in the previous section, this assumption leaves essentially unaltered the resulting \textit{laissez-faire} equilibrium outcomes. It matters, however, for the implied optimal economic shutdown.}

Fig. 8 shows both the effective and the official data series for the total deaths caused by COVID-19 in Italy in the weeks corresponding to the period between February 24 and May 3, 2020, as percent of the initial population (according to United Nations World Population Prospects 2019 the Italian population stood at 60.55 million people in 2019).

**Appendix E. Additional figures**

See Figs. 9–15.

**Fig. 9.** Daily contact Matrix: \textbf{2 age-groups, full sample}.  
\textit{Note.} Elaboration on survey data from Mossong et al. (2017). Full sample comprises: Italy, Germany, Luxembourg, Netherlands, Poland, United Kingdom, Finland, Belgium. The underlying matrix (where each entry is multiplied by the relative demographic size of each age group) is made symmetric employing the same demographic shares employed in the main text, namely $f_y = 0.825$, $f_o = 1 - f_y$.

**Fig. 10.** Endogenous social contacts across selected policies.  
\textit{Note.} The three cases shown in the figure refer to: (a) no containment scenario, i.e. the baseline SIR-age macro with endogenous contacts (black line with circle marker); (b) generalized social distancing reducing the utility from social activity by 20% and vaccine in one year (orange line with diamond marker); (c) optimal shutdown without social distancing and vaccine in one year (green line with right arrow marker).
Fig. 11. **Laissez-faire** SIR-age macro model with reinfection probability.

*Note.* Assuming a probability of recovered to become susceptible equal to 1/35 corresponding to about 8 months of expected immunity.

Fig. 12. **Laissez-faire** SIR-age macro model with endogenous mortality.

*Note.* Model details in Appendix C.2. The relative case fatality rate is given by $\frac{\pi_{ad}}{\pi_{a'd}}$ for age $a \in \{y,o\}$.
Fig. 13. Laissez-faire SIR-age macro model with unknown health status. Note. Model details in Appendix C.3.

Fig. 14. SIR-age macro model: optimal shutdown $\mu_c$, with social distancing, vaccine priority to elderly – differences with respect to no-priority case. Note. The reference series for the no-priority case are those in Fig. 4. The optimal economic shutdown parameter $\mu_c$ is computed in the case in which a vaccine is offered on average in one year-time and earlier to old. The scenario “generalized (GEN)” assumes that the social activity utility parameter $\varphi$ is reduced by 20% with respect to the baseline (“no confinement”). The scenario “mild on young, GEN on others” is the same as GEN, but the social distancing among young individuals is 50% less severe than in GEN. The scenario “mild on young, extreme on young-old, GEN on old-old” is the same as the previous scenario but assumes that almost no utility is derived from contacts between young and old individuals. A vaccine is expected within one year from the onset of the epidemic.
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Fig. 15. SIR-age macro model: optimal shutdown $\mu_c$ with social distancing, vaccine priority to young – differences with respect to no-priority case.

Note. The reference series for the no-priority case are those in Fig. 4. The optimal economic shutdown parameter $\mu_c$ is computed in the case in which a vaccine is offered on average in one year-time and earlier to young. The scenario “generalized (GEN)” assumes that the social activity utility parameter $\psi$ is reduced by 20% with respect to the baseline (“no confinement”). The scenario “mild on young, GEN on others” is the same as GEN, but the social distancing among young individuals is 50% less severe than in GEN. The scenario “mild on young, extreme on young-old, GEN on old-old” is the same as the previous scenario but assumes that almost no utility is derived from contacts between young and old individuals. A vaccine is expected within one year from the onset of the epidemic.
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