A quantum reduction to Bianchi I models in loop quantum gravity

N. Bodendorfer

Faculty of Physics, University of Warsaw, Pasteur 5, 02-093, Warsaw, Poland
(Dated: October 22, 2014)

We propose a quantum symmetry reduction of loop quantum gravity to Bianchi I spacetimes. To this end, we choose the diagonal metric gauge for the spatial diffeomorphism constraint at the classical level, leading to a U(1) gauge theory, and quantise the resulting theory via loop quantum gravity methods. Constraints which lead classically to a suitable reduction are imposed at the quantum level. The dynamics of the resulting model turn out to be very simple and manifestly coincide with those of a polymer quantisation of a Bianchi I model for the simplest choice of full theory quantum states compatible with the Bianchi I reduction.

INTRODUCTION

Identifying symmetry reduced sectors within full theories is an important problem, since success in this endeavour usually allows one to perform computations which are otherwise intractable. Within loop quantum gravity, there has been a lot of recent interest in this subject, see e.g. [1–9]. Different strategies can be employed towards this goal, the most active one being the identification of suitable symmetry reduced states directly within the full, classically gauge fixed, quantum theory. While success along this route might be preferable, we will deal with another approach in this paper, which consists of proposing a symmetry reduction at the quantum level. This will call diffeomorphisms generated by such shift vectors “restricted”.

We would now like to go to the reduced phase space, coordinatised by \(q_{xx}, q_{yy}, q_{zz}\), and \(P_{xx}, P_{yy}, P_{zz}\), which is equivalent to using the Dirac bracket to solve our second class constraints. For this, we need to compute an expression for \(P_{xy}, P_{xz},\) and \(P_{yz}\) in terms of the reduced phase space variables by using the second class constraints. For, say, \(P_{xy}\), this can be achieved by solving the equation

\[
\frac{\delta}{\delta P_{xy}(x)} \left( P_{xy}[\omega_{xy}] + C_a [N^a] \right)_{q_{a\neq b}=0} = 0 \quad (2)
\]

for \(N^a\) as a function of \(\omega_{xy}\), where by \(P_{xy}[\omega_{xy}]\) we mean the smearing \(\int d^3 \sigma P_{xy}(\sigma) \omega_{xy}(\sigma)\). Given \(N^a\) as a function of \(\omega_{xy}\), we can evaluate \(P_{xy}[\omega_{xy}] := P_{xy}[\omega_{xy}] + C_a [N^a]\) at \(P_{x\neq y} = 0 = q_{c \neq d}\), and obtain the desired expression for \(P_{xy}\) on the reduced phase space. By construction, \(P_{xy}\) preserves the gauge fixing condition \(q_{a \neq b} = 0\). More explicitly, up to a boundary term, we have

\[
P_{xy}[\omega_{xy}] = \int \Sigma d^3 \sigma \left( P_{xx} \mathcal{L}_N q_{xx} + P_{yy} \mathcal{L}_N q_{yy} + P_{zz} \mathcal{L}_N q_{zz} \right)_{q_{a \neq b}=0}, \quad (3)
\]

where \(\mathcal{L}_N\) denotes the Lie derivative with respect to the vector field \(\vec{N}\).

For a general smearing \(P_{ab}[\omega_{ab}]\), we obtain from the equations

\[
2 \nabla (x N_y) = q_{yy} \partial_x N^y + q_{xx} \partial_y N^x = \omega_{xy} \quad (4)
\]

\[
2 \nabla (y N_z) = q_{zz} \partial_y N^z + q_{yy} \partial_y N^y = \omega_{yz} \quad (5)
\]

\[
2 \nabla (z N_x) = q_{xx} \partial_z N^x + q_{zz} \partial_z N^z = \omega_{xz} \quad (6)
\]

A general solution to these equations might be hard to find, however it is not needed for what follows. Instead, we will show that by choosing \(\omega_{a \neq b}\) appropriately, we

THE DIAGONAL METRIC GAUGE

We start with the ADM formulation of general relativity, that is with the phase space coordinatised by the spatial metric \(q_{ab}\) and its momentum \(P^{ab}\), living on the spatial slice \(\Sigma\), with the canonical Poisson brackets

\[
\{q_{ab}(\sigma), P^{cd}(\sigma')\} = \delta^c_{[a} \delta^d_{b]} \delta^{(3)}(\sigma, \sigma'), \quad (1)
\]

subject to the Hamiltonian constraint \(H\) and the vector constraint \(C_a := -2 \nabla_b P^b_a = 0\). We now introduce the gauge fixing \(q_{ab} = 0\) for \(a \neq b\), i.e. \(q_{ab} = \text{diag}(q_{xx}, q_{yy}, q_{zz})\), for the spatial diffeomorphism constraint, which is at least locally accessible [10]. We note that not all spatial diffeomorphisms are gauge fixed by this condition, but only those which do not preserve the off-diagonal components of \(q_{ab}\). In particular, \(C_a\) smeared with a lapse functions of the form \(\vec{N} = (N^x(x), N^y(y), N^z(z))\) is still a first class constraint. We will call diffeomorphisms generated by such shift vectors “restricted”.

We will call diffeomorphisms generated by such shift vectors “restricted”. To this end, we choose the diagonal metric gauge for the spatial diffeomorphism constraint at the classical level, also called an orthogonal system [11], which is a gauge fixing admitted by Bianchi I models. Our strategy will then be to quantise this model and impose a symmetry reduction at the quantum level. This strategy clearly separates the steps of gauge fixing and symmetry reduction, which is less transparent when performing both at the quantum level. We also discuss the related proposal of [6], which inspired us to the present paper in the first place. The approach taken in this paper is similar to the one in [11], where a reduction to spherical symmetry is achieved within a quantisation of general relativity in the radial gauge.
We can now choose along with the consistency condition and $\omega$ reduced phase space if of generators of spatial diffeomorphisms acting on the spatial diffeomorphisms, corresponds to a complete set can be repeated to generate an arbitrary above system, which still corresponds to full general relativity. 

and the spatial diffeomorphism constraint (for arbitrary shift vector) becomes, up to a boundary term, 

$$C_a[N^a] = \int_S d^3 \sigma E^a \mathcal{L}_N K_a + \ldots \quad (10)$$

... stands for terms of the form $\partial_a e_b, a \neq b$, and $\partial_a K_b, a \neq b$, which vanish for the special case of restricted diffeomorphisms, and originate from the fact that $\mathcal{L}_q q_{ab} \neq 2 e_b(\mathcal{L}_N e_b)$, even for $a = b$. Up to these terms, $K_a$ transforms as a one-form, and $E^a$ as a densitised vector. Since we would like to have an explicit interpretation of the action of $C_a$ as spatial diffeomorphisms, we will impose the additional constraints

$$\partial_a e_b = 0 = \partial_a K_b, \ a \neq b, \quad (11)$$

up to which (10) generates spatial diffeomorphisms. We note that these constraints are fully consistent with the Bianchi I symmetry. Moreover, they are first class with respect to the restricted diffeomorphisms. In fact, they e.g. impose $e_x = e_{\gamma}(x)$, such that the remaining $x$-dependence is removed by the restricted diffeomorphisms in $x$-direction. We will deal with imposing (11) later, and focus on the kinematical construction of the quantum theory now.

We will treat $K_a$ in analogy to the Ashtekar-Barbero connection in loop quantum gravity: we define the holonomies

$$h_\gamma(K) := \exp \left( i \int_\gamma K_a ds^a \right) \quad (12)$$

for arbitrary oriented paths $\gamma$. By smearing $E^a$ over two-surfaces, we obtain fluxes, and consequently a standard holonomy-flux algebra of a U(1) gauge theory. The only difference is that there is no Gauß law present which would enforce gauge invariance. It is also possible to introduce a Barbero-Immirzi-like parameter by rescaling $E^a, K_a$ accordingly. A similar system, Maxwell theory, has been quantised by the same methods in [13].

Quantisation can now be achieved via the Gelfand-Naimark-Segal construction by specifying the positive linear Ashtekar-Lewandowski functional $\omega_{\text{AL}}$, induced by the Ashtekar-Lewandowski measure $\mu_{\text{AL}}$ [14], on the holonomy-flux algebra. The Hilbert space is thus the space of generalised U(1) connections, without restricting to gauge invariance.

We can now define an operator measuring the area $A$ of a surface $S$ by substituting the flux operator in the expression $A(S) = \int_S E^a d^2 S_a$. The important difference to the usual SU(2) case [15] is that we do not need to define the area operator as $\int \sqrt{\text{flux}^2}$, since $E^a$ is gauge invariant as opposed to the SU(2) densitised triad $E_i^a$. Thus, in analogy to electric charge, the area operator of a closed surface (electric flux through $S$) we consider vanishes e.g. on a single Wilson loop, since the contributions coming from two intersections always cancel.

**QUANTUM THEORY**

We first need a set of variables suited for a loop quantum gravity type quantisation. We define $e_a := \sqrt{q_{aa}}$ with no summation implied. $E^a := \sqrt{\text{det} q} e^a$ corresponds to the densitised triad of the Ashtekar-Barbero variables. Next, we define $K_a := K_{\alpha x} e^x$, where $K_{\alpha b}$ is the extrinsic curvature, appearing in $P^{ab} = \frac{1}{\sqrt{\text{det} q}} \left( K^{ab} - q^{ab} K \right)$. The new Poisson brackets read

$$\{ K_a(\sigma), E^b(\sigma') \} = \delta^b_a \delta^{(3)}(\sigma, \sigma') \quad (9)$$
On this Hilbert space, the $h_s(K)$, seen as cylindrical functions, provide a basis and classically separate points on the configuration space. They are further subject only to the Hamiltonian constraint and the restricted set of spatial diffeomorphisms. In particular, these spatial diffeomorphisms are not sufficient to reduce our quantum states to diffeomorphism equivalence classes. Thus, after modding out the restricted spatial diffeomorphisms, the quantum states still know about the embedding information of the paths $\gamma$ into $\Sigma$, which is in stark contrast to the spatially diffeomorphism invariant Hilbert space of loop quantum gravity $[16]$. The reason is of course that we solved most of the spatial diffeomorphism constraint classically, which lead to a reduction from an SU(2) holonomy-flux algebra to a U(1) holonomy-flux algebra. A similar reduction has been observed in $[11]$ in the context of spherical symmetry, although in the choice of paths, as opposed to the gauge group.

We are now in a position to discuss a reduction of the proposed quantum theory to Bianchi I models. For this, we will implement two conditions. First $P_{a \neq b} P_{[\omega_a \neq \omega_b]} = 0$ together with the restricted spatial diffeomorphisms and the condition $[11]$, implies that we have to average with respect to all diffeomorphisms, leading to the spatially diffeomorphism invariant distributions of $[11]$. Thus, one part of the reduction is achieved by going over to spatial diffeomorphism invariance $[16]$. Next, we have to deal with $[11]$. Since e.g., $\partial_x E^x = \partial_y (e_y e_z)$ and $\partial_y K_x$ are second class with respect to each other, we cannot implement both of them strongly. Instead, we will implement only a suitable quantisation of

$$\partial_x E^x = \partial_y E^y = \partial_z E^z = 0,$$  \hspace{2cm} (13)$$

which form a first class subset (we “gauge unfixied” [17] $\partial_x K_0 = 0$ away). We note that this set of constraints is still weaker than imposing $\partial_x e_b = 0$, $a \neq b$, however it leads already to the desired result in the quantum theory, as we will see. In any case, these constraints have to be implemented in such a way that they are also first class with respect to the spatial diffeomorphisms. For $[13]$, this can be done in the following way:

We restrict the topology of $\Sigma$ to be a 3-torus $\mathbb{T}^3$. On such a torus, there are three distinct diffeomorphism equivalence classes of closed two-surfaces $S_z, S_y, S_x$, wrapping around a $\mathbb{T}^2 \subset \mathbb{T}^3$. We now demand that the action of $\hat{A}(S_{x,y,z})$ is diffeomorphism invariant, i.e. it does not depend on the chosen representative $S_{x,y,z}$ of the diffeomorphism equivalence class, and restrict our spin networks accordingly (see next paragraph). Given this restriction, the areas of the surfaces $S_x, S_y, S_z$ depend only on the chosen 2-torus $\mathbb{T}_x^2, \mathbb{T}_y^2, \mathbb{T}_z^2$. This corresponds to the area of, say, $\mathbb{T}_x^2$ being independent of the coordinate $x$, where in turn the area of $\mathbb{T}_y^2$ should be a function of $q_{yy}, q_{zz}$. A similar argument can be made for $\mathbb{R}^3$ topology by considering a fiducial cell.

The most elementary example $|n_x, n_y, n_z\rangle$ of a spin (or “charge” due to U(1)) network describing a Bianchi I universe with $\mathbb{T}^3$ topology and satisfying all the above constraints is to consider three Wilson loops with U(1)-labels $n_x, n_y, n_z$, which are wrapping around the $x, z, y$-directions of $\mathbb{T}^3$ respectively. We may furthermore require that these Wilson loops intersect in a single 6-valent vertex. More complicated states can now be constructed by adding Wilson loops, either wrapping around a torus, or contractible, thus changing the local properties of the spin network, e.g. adding new edges and changing the representation labels on existing edges. Our imposition of $[16]$ thus effectively enforces U(1) gauge invariance. With more hindsight, this would have been clear from the beginning: the Gauss law $\partial_a E^a = 0$ is the only diffeomorphism covariant expression that can be build by linearly combining $[13]$.

Up to the Hamiltonian constraint, which can be dealt with via deparametrisation, the three areas $A(S_x), A(S_y), A(S_z)$, as well as the total volume of $\Sigma$, are observables of our theory. A volume operator for a region $R$ can be defined by noting that $V(R) = \int_R \sqrt{E^x E^y E^z}$, and following the standard quantisation recipe of $[13]$. The operators $\hat{A}(S_x), \hat{A}(S_y), \hat{A}(S_z)$ measure the (absolute value of the) winding numbers of the spin network, weighted by the chosen representations, around the $x, y, z$ directions of $\mathbb{T}^3$. The volume operator is sensitive to more details of the state than the areas, e.g. it can detect the product of three contractible Wilson loops intersecting orthogonally in a 6-valent vertex, while this is invisible to $\hat{A}(S_x), \hat{A}(S_y), \hat{A}(S_z)$. It is therefore questionable whether the volume operator should be invoked as an observable, since the total volume of a Bianchi I universe can also be extracted as $\sqrt{A(S_x) A(S_y) A(S_z)}$. On $|n_x, n_y, n_z\rangle$, one can also define the holonomy observables $\hat{h}_s(K)$ for $\gamma$ coinciding with one of the three non-contractible Wilson loops defining the quantum state.

The Hamiltonian constraint now needs to be regulated in terms of holonomies and fluxes. As remarked before, we do not know the solution to $[4], [3]$, and $[4]$, prohibiting the construction of the full theory Hamiltonian constraint. However, on the space of spatially diffeomorphism invariant distributions, we have $P_{a \neq b} P_{[\omega_a \neq \omega_b]} = 0$, which suggests that one should just quantise the classical Hamiltonian with $P_{a \neq b} = 0$, such that it is consistent with all the constraints imposed. Under this condition, and in the diagonal metric gauge, the Hamiltonian constraint becomes $[16]$

$$H[N] = \int_{\mathbb{T}^3} dx \, dy \, dz \, N (e_x K_y K_z + e_y K_z K_x + e_z K_x K_y).$$  \hspace{2cm} (14)$$

In order to quantise it, we are going to adapt the methods developed in $[20],[21]$ to our case. Thiemann’s trick e.g. amounts to $e_x = 2|K_x, V\rangle$, where $V$ can be taken to be the total volume of the (compact, fiducial cell)
universe. Thus,

\[ H = 2\{K_x K_y K_z, V\}. \]  

(15)

\( K_z \) can be approximated in \( H[N] \) via holonomies as

\[ \int \gamma_x K_x \, dx \approx \frac{1}{2\hbar} (h_{\gamma_x}(K) - h_{\gamma_x}(-K)), \]

where \( \gamma_x \) is some path in the \( x \)-direction, and similar for \( y, z \).

We can now define the action of the Hamiltonian on \( |n_x, n_y, n_z\rangle \). We consider a graph-preserving regularisation, that is at a given point in \( \Sigma \), the holonomies \( h_{\gamma_x}(K), h_{\gamma_y}(K), h_{\gamma_z}(K) \) only act when already non-trivial holonomies are present [23].

The paths \( \gamma_{x,y,z} \) are thus chosen to coincide with the three holonomies defining \( |n_x, n_y, n_z\rangle \). Their action thus preserves the constraints [13].

The volume operator acts non-trivially only on the intersection point of the three holonomies. Quantisation is thus achieved by substituting the corresponding operators for the classical regularised expression of \( H[N] \) and the replacement \( \{\cdot, \cdot\} \to \frac{1}{\hbar}\{\cdot, \cdot\} \). We remark that the factor ordering implied by expression [13] leads to a symmetric operator. Furthermore, the Hamiltonian constraint commutes with all other constraints imposed.

We leave an extension of this prescription to more general quantum states for future work.

**RELATION TO A MINISUPERSPACE QUANTISATION**

For the simplest set of quantum states as described above, superpositions of \( |n_x, n_y, n_z\rangle \), the dynamics of the model coincides with that of a polymer quantisation [23] of the conjugate pairs \( \hat{\{h_{\gamma_a}(K), \hat{E}^b\}} = i\hbar \gamma_a(K) \delta^b_a \), \( a, b = x, y, z \), in a \( T^3 \) Bianchi I model, where \( \hat{h}_{\gamma_a}(K) = \exp\left(i \int_{\gamma_a} K_b \, dx^b \right) \) and \( \hat{E}^b = \frac{1}{\gamma} \int_S c_{de} \, dx^d \wedge dx^e \). The polymerisation scale is set by the free Barbero-Immirzi-like parameter that can be introduced (even independently for \( x, y, z \), leading to three free parameters) [23].

Results based on polymer quantisations, such as the resolution of the initial big bang singularity in loop quantum cosmology [24], can thus be transferred to a full theory setting.

**RELATION TO [6]**

A reduction to Bianchi I spacetimes within full loop quantum gravity was recently proposed in [6]. We will comment on the similarities of this proposal to ours. The main difference to our derivation is that the reduction in [6] is performed within a classically non-gauge fixed quantisation of loop quantum gravity in Ashtekar-Barbero variables, leading to \( SU(2) \) as the gauge group. However, [6] argues to gauge fix \( SU(2) \) to \( U(1) \) at the quantum level, so that the (effective) gauge group is \( U(1) \) as opposed to \( SU(2) \) in both cases. Next, [6] restricts to cubic spin networks, which are furthermore solutions to the diffeomorphism constraint. In our derivation, imposing \( P^{a,b} = 0 \) at the quantum level as a part of the reduction procedure, along with the modification by [11], leads to the same result, although the restriction to cubic graphs is not necessary, yet still convenient.

Also, there is no analogue of our condition of having \( A(S_x), A(S_y), A(S_z) \) independent of the choice of diffeomorphism-related surfaces. However, the authors of [6] explain that their model allows for inhomogeneities, which accounts for the missing of this condition. The equivalence of the dynamics is less clear and would require explicit calculation, which we leave for future research.

More concretely, one would like to understand the relation between the projections performed in [6] and the quantisation of the reduced phase space Hamiltonian subject to \( P^{a,b} = 0 \). However, at the kinematical level, we can conclude that the reduction proposed in [6] qualitatively agrees with our results.

An interesting point to consider is using the diagonal metric gauge for the full theory. This was discussed in [20], building on [6]. The problem here is of course that we would need to compute the Hamiltonian on the reduced phase space, which involves finding a general solution to [26]. In any case, our analysis shows that the spin networks would retain embedding information in the full theory, since the spatial diffeomorphism constraint is gauge fixed classically, and thus does not have to be solved at the quantum level.

**CONCLUSION**

We have presented a derivation of a Bianchi I sub sector of a quantisation of general relativity in the diagonal metric gauge, using quantisation methods of loop quantum gravity. Since the gauge group is \( U(1) \), the dynamics are significantly simpler than in the \( SU(2) \) case. In the case of the most simple quantum states, the evolution coincides with a polymer quantisation of the corresponding minisuperspace model. Furthermore, it will be interesting to compare our model in detail with results from loop quantum cosmology [24]. A comparison with another recently proposed reduction [6] has already been made at the kinematical level, finding qualitative agreement. The especially attractive feature of our model is its simplicity, being within the full theory while purely build on \( U(1) \). Issues like singularity resolution and the influence of the dynamics on coarse graining can thus be discussed explicitly.
Acknowledgements
This work was supported by a Feodor Lynen Research Fellowship of the Alexander von Humboldt-Foundation. Discussions with Emanuele Alesci, Jerzy Lewandowski, Jędrzej Świeżewski, and Antonia Zipfel are gratefully acknowledged.

norbert.bodendorfer@fuw.edu.pl

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