Analytical and computational analytical mathematical models of the mosquito population in the middle zone of the Russian Federation

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Abstract. The objective is to obtain calculation and analytical mathematical models of the abundance of mosquitoes depending on the following factors: average monthly temperature over the year, average monthly precipitation over the year and average monthly atmospheric pressure over the year. The obtained models show the individual effect of each factor and their combined effect. The population of mosquitoes is most dependent on the average monthly precipitation. Its influence is 3.5 times greater than that of the average monthly temperature, and 6.4 times greater than that of the average atmospheric pressure. Also, models were obtained to enable calculation of the population size of mosquitoes without resorting to expensive field survey.

1. Relevance
Sanguinivorous Diptera sustain natural foci of vector-borne zoonoses. By themselves, the foci are a sophisticated parasitic system with several components, which is, in turn, a part of a much higher-order system. These systems are kept active and going due to interactions of carriers (mosquitoes), pathogens (viruses, bacteria, etc.) and disease and infestation circulators, i.e. vertebrates [1; 2].

Mathematical modelling for the population size dynamics of parasitic arthropods, particularly, mosquitoes, is especially relevant in the context of the increased incidence of human and animal dirofilariasis, malaria, dengue and other infections and infestations [3; 4]. Therefore, such a method enables one to take effective and timely preventive measures and predict the abundance of mosquitoes during a specific season in the geographical area under study. Mosquitoes are reservoirs and carriers of vector-borne disease pathogens. The population size dynamics and outbursts of vector-borne diseases can be effectively calculated using mathematical models [5; 6].

2. Materials and research methods
To build mathematical models, a type 2² multifactorial experiment was conducted in the field for 10 years (from 2009 through 2019) according to the standard methodology described in the works of V.V. Kalmykov [7].
3. Research results and their discussion
Calculation and mathematical models for population of small mammals were obtained, depending on
three factors: average monthly temperature, average precipitation and average atmospheric pressure
over a year in the Nonblack Soil Zone, as exemplified by the Kaluga Region.
In the Central region of the Russian Federation, including the Kaluga region, due to the lack of warm
days, there is a later gradual increase in the abundance of mosquitoes (by the last decade of May).
You should bear in mind that fluctuations in weather parameters (factors for mathematical modelling)
will only be relevant and adequate within the factor variation ranges.
In order to develop mathematical models, a full factorial experiment was conducted on the basis of
the obtained statistical data. Table 1 provides the values of the levels of factors.

Table 1. Factor variation ranges.

| Factors | –1 | 0     | +1     |
|---------|----|-------|--------|
| X1      | +4.57ºC | +6.55ºC | +7.57ºC |
| X2      | 31.6 mm | 49.5 mm | 64.14 mm |
| X3      | 741.0 mmhg | 745.5 mmhg | 750.0 mmhg |

X1 – average monthly temperature over the year (t ºC).
X2 – average monthly amount of precipitation over the year (S, mm).
X3 – average atmospheric pressure over the year (P, mm Hg).
Y response was the amount of mosquitoes of all species at the fixed observation points in all areas
of the Kaluga region.
In case of abnormal weather conditions, calculations of the mathematical model obtained may not
provide a correct forecast.
In order to rule out the influence of systematic errors caused by external conditions, it is advisable to
randomize the time when experiments with a specified experimental design are carried out. When
arranging the experiment, we kept in mind the need to assess the experiment variances. For this purpose,
experiments were duplicated, i.e. abundance of mosquitoes was measured in all areas of the region.
The matrix of the experimental design is presented in table 2.

Table 2. Experimental matrix with three factors.

| experiment number | X0 | X1 | X2 | X3 | X1X2 | X1X3 | X2X3 | X1X2X3 | Y1 | Y2 | Y3 | S² |
|-------------------|----|----|----|----|------|------|------|--------|----|----|----|----|
| 1                 | +  | +  | +  | +  | +    | +    | +    | +      | 1674 | 1638 | 1710 | 1296 |
| 2                 | +  | –  | +  | +  | –    | –    | +    | –      | 2112 | 2150 | 2174 | 977.3 |
| 3                 | +  | +  | –  | +  | –    | –    | +    | –      | 890  | 903  | 877  | 169  |
| 4                 | +  | –  | –  | +  | +    | –    | –    | +      | 1093 | 1116 | 1070 | 529  |
| 5                 | +  | +  | +  | –  | –    | –    | –    | –      | 1987 | 1949 | 2025 | 1444 |
| 6                 | +  | –  | +  | –  | –    | +    | –    | +      | 2345 | 2399 | 2291 | 2916 |
| 7                 | +  | +  | –  | –  | –    | –    | +    | +      | 987  | 969  | 1005 | 324  |
According to the purpose of the research, the horses were share on 3 groups: 1 - healthy horses (corneal transparency, specularity, and luster are preserved), 2 - horses with primary keratopathies (horses showed blepharospasm, conjunctival hyperemia and edema, abundant exudate, corneal ulcer or abscess, keratolysis), 3 - horses with secondary keratopathies (in horses there was a slight discharge of exudate from the conjunctival cavity, or corneal ulcer with uneven edges, changes in the transparency of the cornea that occurs during endothelial regeneration and stroma saturation with intraocular vascular fluid corneal infiltration).

\[ Y = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_{12} X_1 X_2 + b_{13} X_1 X_3 + b_{23} X_2 X_3 + b_{123} X_1 X_2 X_3 \]

After we remove statistically insignificant coefficients from the model, the final analytical mathematical model on a standardized scale takes on the following form:

\[ Y = 1529.167 - 144.667 X_1 + 508.667 X_2 - 78.583 X_3 - 62.667 X_1 X_2 - 23.917 X_1 X_3 - 49.583 X_2 X_3 \]

Figure 1 shows a graphical representation of the adhesion coefficients characterizing the effect of factors on the response.

The population of mosquitoes is most dependent on the average monthly precipitation. Its influence is 3.5 times greater than that of the average monthly temperature, and 6.4 times greater than that of the average atmospheric pressure. The “+” sign means that the abundance of mosquitoes is higher when there’s more precipitation. This provides for the high level during spring floods, due to melting of significant amounts of snow, and in the warm season it results in preservation of puddles, stagnant water, etc. The “−” sign before the average monthly temperature coefficient indicates a decrease in the abundance of mosquitoes when the temperature increases, which is explained by faster water evaporation during hot weather.

The paired effect of the interaction of average temperature and average monthly amount of precipitation also reduces the abundance of mosquitoes, and its significance is 2.3 times smaller than the influence of the temperature alone, and 8 times smaller than the influence of just the amount of precipitation.

After transformations, the calculated mathematical model for calculating the abundance of mosquitoes will take on the following form:
N = 42,673P + 609,111S + 2685,069t – 0,746PS – 3,543Pt – 2,829St

4. Conclusion
The model under consideration allows to calculate the population size of small mammals without resorting to expensive field survey, which may be of use in predicting the population size in the Kaluga region of the Russian Federation and other regions with similar climatic conditions.

By calculating the value using the model on a natural scale for zero-level factors, we can compare the obtained value with the experimental value of $b_0$. By the disagreement between these values one can judge the amount of the calculation model error.

By inserting the value of the factor corresponding to the zero level into the calculated mathematical model, namely:
mosquito individuals. In this regard, according to the experimental statistical data, 1529 mosquitoes were found under these conditions.

Error $\Delta = \frac{1531-1529}{1531} = 0,01$

Disagreement with the calculated model is 1%, which suggests adequacy of the calculated model obtained.

The advantage of the modelling applied makes it possible to take into account the aggregate effect of all factors, as well as their interaction effects, on the object under study.

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