Irreducible Hamiltonian BRST approach to topologically coupled abelian forms

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Abstract

An irreducible Hamiltonian BRST approach to topologically coupled \( p \)- and \((p+1)\)-forms is developed. The irreducible setting is enforced by means of constructing an irreducible Hamiltonian first-class model that is equivalent from the BRST point of view to the original redundant theory. The irreducible path integral can be brought to a manifestly Lorentz covariant form.

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1 Introduction

The typical feature of \( p \)-form gauge theories, namely, the reducibility allows their link with string theory and supergravity models [1]–[6]. Recently, \( p \)-form gauge theories have attracted attention in relation with their characteristic cohomology [7] and also with their applications in higher dimensional bosonisation [8]. From the point of view of the BRST quantization, theories involving \( p \)-forms implies the introduction of ghost fields with ghost number greater that one (ghosts of ghosts, etc.), and, in the meantime, of a pyramid

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of non-minimal variables [9]–[17]. Interacting $p$-forms were analyzed within the reducible Hamiltonian BRST framework in [18], being obtained the ghost and auxiliary field structures necessary at the antifield BRST quantization.

The main result of this paper consists in proving that it is possible to quantize $p$-form gauge theories with topological coupling along an irreducible Hamiltonian BRST procedure. Our method basically relies on replacing the original redundant first-class model with an irreducible one, and on further quantizing the resulting irreducible first-class system accordingly the standard Hamiltonian BRST lines. The derivation of the irreducible first-class theory relies on requiring that all the antighost number one co-cycles from the Koszul-Tate homology identically vanish under a convenient redefinition of the antighost number one antighosts while the number of physical degrees of freedom is kept unchanged with respect to the initial model. As a consequence of our analysis, the two theories are found physically equivalent, which further allows (from the BRST point of view) the substitution of the Hamiltonian BRST quantization of the reducible model with that of the irreducible system. Initially we approach topologically coupled abelian $p$- and $(p + 1)$-forms described by a quadratic action [19] and then discuss the more general case of interacting abelian forms with topological coupling, inferring an irreducible Lagrangian formulation implied by our Hamiltonian approach that can be conveniently applied to the interacting case. Although the idea of transforming a set of reducible first-class constraints into an irreducible one is addressed in [13], [20], it has not been either developed or applied until now to the irreducible quantization of this type of models.

The paper is organized in four sections. In Section 2 we focus on the construction of an irreducible Hamiltonian first-class theory starting with topologically coupled abelian $p$- and $(p + 1)$-form gauge fields described by a quadratic action within the homological context of the Koszul-Tate differential, and provide the associated irreducible Hamiltonian BRST symmetry. We then find by means of standard BRST Hamiltonian arguments that it is permissible to replace the redundant Hamiltonian BRST symmetry with the irreducible one, and infer the irreducible path integral with the help of a suitable gauge-fixing fermion. Section 3 is devoted to the extension of our irreducible procedure to the interacting case. There, we work with a model of irreducible Hamiltonian first-class system and find that the resulting Lagrangian gauge theory displays some manifestly Lorentz covariant irreducible gauge transformations. The Lagrangian setting is adequate for
an irreducible approach to higher-order interacting gauge theories with topological coupling. In Section 4 we expose the final conclusions.

2 Irreducible Hamiltonian BRST analysis

In this section we construct the path integral for topologically coupled abelian $p$- and $(p + 1)$-form gauge fields in the context of an irreducible Hamiltonian BRST procedure. In view of this, we perform the canonical analysis of the starting quadratic Lagrangian action and observe that this model is subject to some abelian first-class constraints that are $p$-stage reducible. The first-step of our irreducible approach consists in the construction of an irreducible first-class set of constraints corresponding to the initial redundant ones based on homological aspects. This purpose is attained by means of making the original antighost number one co-cycles from the reducible Koszul-Tate complex to vanish identically under a proper redefinition of the antighost number one antighosts, and, in the meantime, by maintaining the initial number of physical degrees of freedom unchanged with respect to the irreducible background. The implementation of these conditions yields an abelian irreducible first-class constraint set, an associated first-class Hamiltonian and, moreover, provides an irreducible Koszul-Tate complex corresponding to the original reducible one. The construction is realized in a gradual manner starting with the cases $p = 1$ and $p = 2$, and is further generalized to arbitrary values of $p$. Next, we show that the irreducible BRST symmetry exists as it satisfies the general grounds of homological perturbation theory. In the sequel we investigate the correlation between the reducible and irreducible Hamiltonian BRST symmetries and prove that the physical observables underlying the reducible and irreducible theories coincide, which enables the substitution of the Hamiltonian BRST quantization of the original model with the BRST quantization of the irreducible system. Finally, we realize the Hamiltonian BRST quantization of the irreducible model by using an appropriate gauge-fixing fermion and non-minimal sector, inferring the irreducible path integral, which is local and manifestly Lorentz covariant.
2.1 Canonical analysis of the reducible model

We start with the quadratic Lagrangian action

\[ \tilde{S}_0^L \left[ A^{\mu_1...\mu_p}, H^{\mu_1...\mu_{p+1}} \right] = \int d^{2p+2}x \left( -\frac{1}{2\cdot(p+1)!} F_{\mu_1...\mu_{p+1}}^2 - \frac{1}{2\cdot(p+2)!} F_{\mu_1...\mu_{p+2}}^2 \right), \]  

where \((F_{\mu_1...\mu_{p+1}}, F_{\mu_1...\mu_{p+2}})\) stand for the field strengths respectively corresponding to the antisymmetric tensor fields \((A^{\mu_1...\mu_p}, H^{\mu_1...\mu_{p+1}})\), and \(\varepsilon_{\mu_1...\mu_{2p+2}}\) denote the completely antisymmetric symbol in \((2p+2)\) dimensions. The notation \(F_{\mu_1...\mu_{p+1}}^2\) signifies \(F_{\mu_1...\mu_{p+1}} F_{\mu_1...\mu_{p+1}}\), and the same for the other square. It is worthnote that the topological coupling present in the third term from the right-hand side of (1) is a generalization of the Chern-Simons coupling introduced by Jackiw, Deser, et al [21]–[23].

From the canonical approach of (1), one infers the first-class constraints

\[ \tilde{\gamma}_{i_1...i_{p-1}}^{(1)} \equiv \pi_{0i_1...i_{p-1}} \approx 0, \]  

\[ \tilde{\gamma}_{i_1...i_p}^{(1)} \equiv \Pi_{0i_1...i_p} \approx 0, \]  

\[ \tilde{G}_{i_1...i_{p-1}}^{(2)} \equiv -p \partial^j \pi_{ij_1...j_{p-1}} + \]  

\[ (-)^{p+1} \frac{M}{(p-1)! \cdot (p+2)!} \varepsilon_{0i_1...i_{p-1}j_1...j_{p+2}} F^{j_1...j_{p+2}} \approx 0, \]  

\[ \tilde{G}_{i_1...i_p}^{(2)} \equiv -(p+1) \partial^j \Pi_{ij_1...j_p} \approx 0, \]

and the canonical Hamiltonian

\[ \tilde{H} = \int d^{2p+1}x \left( -\frac{p!}{2} \pi_{i_1...i_p} \pi^{i_1...i_p} - \frac{(p+1)!}{2} \Pi_{i_1...i_{p+1}} \Pi^{i_1...i_{p+1}} + \right) \]

\[ \frac{M}{p!} \varepsilon_{0i_1...i_{p+1}j_1...j_p} \Pi^{i_1...i_{p+1}} A^{j_1...j_p} + \frac{1}{2\cdot(p+1)!} F_{i_1...i_{p+1}} F^{i_1...i_{p+1}} + \]

\[ \frac{1}{2\cdot(p+2)!} F_{i_1...i_{p+2}} F^{i_1...i_{p+2}} + \frac{M^2}{2\cdot p!} A_{i_1...i_p} A^{i_1...i_p} + \]

\[ A^{0i_1...i_{p-1}} \tilde{G}_{i_1...i_{p-1}}^{(2)} + H^{0i_1...i_p} \tilde{G}_{i_1...i_p}^{(2)} \).
The secondary constraints (4) and (5) are \((p - 1)\), respectively, \(p\)-stage reducible, with the reducibility relations given by

\[
Z_{i_1 \ldots i_{p-1} j_1 \ldots j_{p-2}}^{(2)} \tilde{G}^{(2)}_{i_1 \ldots i_{p-1} j_1 \ldots j_{p-2}} = 0, \quad \tilde{Z}_{i_1 \ldots i_p}^{i_1 \ldots i_p} \tilde{G}^{(2)}_{i_1 \ldots i_p} = 0, \tag{7}
\]

\[
\bar{Z}_{i_1 \ldots i_{p-1} j_1 \ldots j_{p-2}}^{i_1 \ldots i_{p-1} j_1 \ldots j_{p-2}} \tilde{Z}^{j_1 \ldots j_{p-1}}_{i_1 \ldots i_{p-1}} = 0, \quad k = 1, \ldots, p - 2, \tag{8}
\]

\[
\bar{Z}_{i_1 \ldots i_{p-1} j_1 \ldots j_{p-2}}^{i_1 \ldots i_{p-2} j_1 \ldots j_{p-1}} \tilde{Z}^{j_{p-1}}_{i_1 \ldots i_{p-1}} = 0, \quad k = 1, \ldots, p - 1, \tag{9}
\]

and the \(k\)th order reducibility functions expressed by

\[
Z_{i_1 \ldots i_{p-k} j_1 \ldots j_{p-k-1}}^{i_1 \ldots i_{p-k} j_1 \ldots j_{p-k-1}} = \frac{1}{(p-k-1)!} \partial_{i_1} \delta_{i_2} \ldots \delta_{i_{p-k}} \bar{Z}^{j_1 \ldots j_{p-k-1}}_{j_1 \ldots j_{p-k}} \delta_{j_{p-k}}, \quad k = 1, \ldots, p - 1, \tag{10}
\]

\[
\tilde{Z}_{i_1 \ldots i_{p-k+1} j_1 \ldots j_{p-k}}^{i_1 \ldots i_{p-k}} = \frac{1}{(p-k)!} \partial_{i_1} \delta_{i_2} \ldots \delta_{i_{p-k+1}} \bar{Z}^{j_1 \ldots j_{p-k}}_{j_1 \ldots j_{p-k}} \delta_{j_{p-k}}, \quad k = 1, \ldots, p. \tag{11}
\]

The notations \(\pi_{0 i_1 \ldots i_p}\) and \(\pi_{i_1 \ldots i_p}\) signify the canonical momenta conjugated with the corresponding \(A\)'s, while \(\Pi_{0 i_1 \ldots i_p}\) and \(\Pi_{i_1 \ldots i_p+1}\) stand for the canonical momenta associated with the \(H\)'s. The notation \([i_1 \ldots i_{p-k}]\) signifies antisymmetry with respect to the indices between brackets. In the sequel we work with the conventions \(f^{i_1 \ldots i_m} = f\) if \(m = 0\), and \(f^{i_1 \ldots i_m} = 0\) if \(m < 0\).

2.2 Construction of irreducible constraints

Initially, we obtain an irreducible model corresponding to topologically coupled abelian \(p\)- and \((p + 1)\)-form gauge fields by means of homological arguments and by requesting the preservation of the number of physical degrees of freedom with respect to the redundant model. In this context, we derive an irreducible first-class set associated with the reducible constraints (4–5).

In order to clarify the main aspects linked to our irreducible treatment, we gradually investigate the cases \(p = 1\) and \(p = 2\), and then generalize the construction to an arbitrary \(p\).

2.2.1 The case \(p = 1\)

The constraints (4 5) take in this situation the concrete form

\[
\tilde{G}^{(2)} \equiv -\partial^j \pi_j + \frac{M}{6} \varepsilon_{0jkl} F^{jkl} \approx 0, \tag{12}
\]
\[ \bar{G}_i^{(2)} \equiv -2 \partial^j \Pi_{ji} \approx 0, \quad (13) \]

and are first-stage reducible, the reducibility relations being expressed by

\[ Z^i \bar{G}_i^{(2)} \equiv \partial^i \bar{G}_i^{(2)} = 0. \quad (14) \]

The reducible Hamiltonian BRST symmetry \( s_R = \delta_R + D_R + \cdots \) involves two crucial graded differentials. One of them \( (\delta_R) \) is called the Koszul-Tate differential and realizes an homological resolution of smooth functions defined on the first-class constraint surface. Its graduation is governed by the antighost number \( (\text{antigh}) \), and we have that \( \text{antigh} (\delta_R) = -1 \). The main property of \( \delta_R \) is the acyclicity at non-vanishing antighost numbers. The other one \( (D_R) \) is known as a model of exterior derivative along the gauge orbits and accounts for the gauge invariances implied by the presence of the first-class constraints. The degree of \( D_R \) is named pure ghost number \( (\text{pure gh}) \), and is defined like \( \text{pure gh} (D_R) = 1 \). In the case \( p = 1 \) the reducible Koszul-Tate complex includes the antighost number one fermionic antighosts \( P_2 \) and \( P_{2i} \), being defined through the relations

\[ \delta_R z^A = 0, \quad (15) \]
\[ \delta_R P_2 = -\bar{G}^{(2)}, \quad (16) \]
\[ \delta_R P_{2i} = -\bar{G}_i^{(2)}, \quad (17) \]

where \( z^A \) is any of the fields \( A^\mu, H^{\mu\nu} \) or their momenta. With the help of the definitions (17) and the reducibility relations (14), it follows that there appear a non trivial co-cycle in the homology of \( \delta_R \), of the type

\[ \bar{\mu} = \partial^i P_{2i}. \quad (18) \]

In order to restore the \( \delta_R \)-exactness of this co-cycle and thus the acyclicity of the Koszul-Tate differential it is necessary to enhance the antighost spectrum with the antighost number two bosonic antighost \( \bar{\lambda} \) and to set

\[ \delta_R \bar{\lambda} = -\partial^i P_{2i}. \quad (19) \]

The idea of transforming this reducible model into an irreducible one is based on redefining the antighost number one antighosts \( P_{2i} \) involved with the co-cycle (18) like

\[ P_{2i} \rightarrow P'_{2i} = D^j_i P_{2j}, \quad (20) \]
such that the new co-cycle of the type (18), namely,
\[ \bar{\mu}' = \partial^i P'_{2i}, \]  
vanishes identically. As a consequence, the new co-cycle \( \bar{\mu}' \) is trivial without adding the antighost number two antighost \( \bar{\lambda} \), hence the resulting model is irreducible. In view of this we choose the matrix \( D^j_i \) to satisfy the properties
\[ \partial^i D^j_i = 0, \]  
\[ D^j_i \bar{G}^{(2)}_j = \bar{G}^{(2)}_i. \]  
Taking into account (17), (20) and (23), we have that
\[ \delta P'_{2i} = -\bar{G}^{(2)}_i, \]  
while the properties (22) yield that \( \bar{\mu}' \) is indeed vanishing
\[ \bar{\mu}' \equiv 0. \]  
In (24) we used the notation \( \delta \) instead of \( \delta_R \) in order to emphasize the irreducibility of the new approach. Thus, if the equations (22, 23) possess solutions, then the co-cycle \( \bar{\mu}' \) vanishes identically and the theory becomes irreducible, the presence of the antighost \( \bar{\lambda} \) being useless. The solution to the equations (22, 23) exists and is given by
\[ D^j_i = \delta^j_i - \frac{\partial^j \partial_i}{\Delta}, \]  
where \( \Delta = \partial^k \partial_k \). Replacing (26) in (24) we arrive at
\[ \delta P_{2i} - \frac{\partial_i}{\Delta} \delta \left( \partial^j P_{2j} \right) = -\bar{G}^{(2)}_i. \]  
The relations (27) describe the action of the Koszul-Tate differential underlying an irreducible model. At this point we explore the request on the equality between the numbers of physical degrees of freedom associated with the reducible and irreducible theories. The original reducible theory has three physical degrees of freedom, while the irreducible theory possesses two physical degrees of freedom as the set (13) will be replaced by a corresponding set of three independent first-class constraints. This is why we need to
supplement the original field/momentum spectrum of the irreducible theory
with an extra canonical bosonic pair, to be denoted by \((H, \Pi)\). With these
supplementary variables at hand, the number of physical degrees of freedom
associated with the irreducible model is now equal to three. We demand that
\(\Pi\) is the non vanishing solution to the equation
\[
\delta \left( \partial^i P_{2i} \right) = \Delta \Pi.
\] (28)
The last condition together with the invertibility of \(\Delta\) guarantee the irre-
ducibility of the new theory because the last equation possesses non-vanishing
solutions if and only if \(\delta \left( \partial^i P_{2i} \right) \neq 0\), hence if and only if (18) is not a co-cycle.
Inserting (28) in (27) we infer that
\[
\delta P_{2i} = -\bar{G}^{(2)}_i + \partial_i \Pi \equiv -\bar{\gamma}^{(2)}_i,
\] (29)
which signify the definitions of \(\delta\) on the antighost number one antighosts
associated with an irreducible model possessing the irreducible constraints
\[
\bar{\gamma}^{(2)}_i \equiv \bar{G}^{(2)}_i - \partial_i \Pi \approx 0,
\] (30)
instead of the reducible constraints (13) of the original theory. In conclu-
sion, we constructed an irreducible first-class constraint set corresponding to
topologically coupled abelian one- and two-form gauge fields, of the type
\[
\bar{\gamma}^{(1)} \equiv \pi_0 \approx 0, \quad \bar{\gamma}^{(1)}_i \equiv \Pi_{0i} \approx 0,
\] (31)
\[
\bar{\gamma}^{(2)} \equiv -\partial^i \pi_j + \frac{M}{24} \varepsilon_{ijklm} F^{jkl} \approx 0, \quad \bar{\gamma}^{(2)}_i \equiv -2\partial^i \Pi_{ji} - \partial_i \Pi \approx 0.
\] (32)

2.2.2 The case \(p = 2\)
The constraints (13) are given in this case by
\[
\bar{G}^{(2)}_i \equiv -2\partial^j \pi_{ji} - \frac{M}{24} \varepsilon_{ijklm} F^{jklm} \approx 0,
\] (33)
\[
\bar{G}^{(2)}_{ij} \equiv -3\partial^j \Pi_{kij} \approx 0,
\] (34)
and are second-stage reducible, the first-stage reducibility relations being
given by
\[
\partial^i \bar{G}^{(2)}_i = 0, \quad 2\partial^i \bar{G}^{(2)}_{ij} = 0.
\] (35)
By introducing the fermionic antighosts $\mathcal{P}_{2i}$ and $P_{2ij}$ of antighost number one, the Koszul-Tate operator acts like

$$\delta_R \mathcal{P}_{2i} = -\tilde{G}_{i}^{(2)},$$

$$\delta_R P_{2ij} = -\tilde{G}_{ij}^{(2)},$$

while its action on the original fields/momenta is vanishing. The reducibility relations (35) yield the antighost number one non trivial co-cycles

$$\tilde{\nu} \equiv \partial^i \mathcal{P}_{2i},$$

$$\bar{\nu}_j \equiv 2 \partial^i P_{2ij}.$$  

In order to restore the acyclicity of $\delta_R$ we add the bosonic antighost number two antighosts $\tilde{\lambda}$ and $\bar{\lambda}_i$, and put

$$\delta_R \tilde{\lambda} = -\partial^i \mathcal{P}_{2i},$$

$$\delta_R \bar{\lambda}_j = -2 \partial^i P_{2ij}.$$  

Because of the second-stage reducibility relation, there appear a supplementary non trivial co-cycle at antighost number two

$$\tilde{\nu} \equiv \partial^i \tilde{\lambda}_i,$$

which is ‘killed’ by means of introducing the fermionic antighost number three antighost $\bar{\lambda}$ through

$$\delta_R \bar{\lambda} = -\partial^i \bar{\lambda}_i.$$  

The passing to the irreducible model goes along the line employed at the case $p = 1$, namely, we enforce that the objects $\tilde{\nu}$ and $\bar{\nu}_j$ are not closed in terms of the irreducible Koszul-Tate differential $\delta$, therefore not co-cycles. This request can be satisfied by adding the bosonic canonical pairs $(A, \pi)$, $(H^i, \Pi_i)$ whose momenta are the non vanishing solutions to the equations

$$\delta \left( \partial^i \mathcal{P}_{2i} \right) = \Delta \pi,$$

$$\delta \left( 2 \partial^i P_{2ij} \right) = \Delta \Pi_j.$$  

Applying $\partial^j$ on (44) it follows that $\Delta \left( \partial^j \Pi_j \right) = 0$, which further leads to

$$\partial^j \Pi_j = 0,$$
on account of the invertibility of $\triangle$. The prior relation is nothing but a new constraint of the irreducible theory

$$\bar{\gamma}^{(2)} \equiv -\partial^j \Pi_j \approx 0,$$

(47)

which is necessary in order to maintain the number of physical degrees of freedom for the irreducible model equal with that of the redundant theory. Indeed, the number of independent constraints (33–34) is equal to ten, hence the reducible model displays ten physical degrees of freedom. The irreducible model will possess thirty independent constraint functions corresponding to the reducible set (33–34) plus the supplementary pairs $(A, \pi), (H^i, \Pi_i)$, which gives eleven physical degrees of freedom. It is precisely the presence of the new first-class constraint (44) that restores the number of physical degrees of freedom associated with the irreducible theory to ten. We notice that the constraint function $\bar{\gamma}^{(2)}$ is irreducible with respect to (33–34), such that it does not induce further antighost number one co-cycles. By introducing its antighost $P_2$ (which is fermionic of antighost number one), the corresponding action of the irreducible Koszul-Tate operator reads as

$$\delta P_2 = -\bar{\gamma}^{(2)}.$$

(48)

Next, we perform the redefinition of the antighosts $P_{2i}$ and $P_{2ij}$ in such a way that the new co-cycles of the type (38–39) identically vanish. In this light, we remark that the constraint functions in (33–34) are separately reducible, such that the redefinition of the antighosts $P_{2i}$ and $P_{2ij}$ can be done in a way that does not mix these fields, namely,

$$P_{2i} \rightarrow P_{2i}' = D^j_i P_{2j},$$

(49)

$$P_{2ij} \rightarrow P_{2ij}' = D^{kl}_{ij} P_{2kl}.$$  

(50)

We demand that the matrices $D^j_i$ and $D^{kl}_{ij}$ are subject to the conditions

$$\partial^i D^j_i = 0, \ 2\partial^i D^{kl}_{ij} = 0,$$

(51)

$$D^j_i \tilde{G}^{(2)}_j = \tilde{G}^{(2)}_i, \ D^{kl}_{ij} \tilde{G}^{(2)}_{kl} = \tilde{G}^{(2)}_{ij}. $$

(52)

On the one hand, with the help of the conditions (52) and using (49–50) we find that

$$\delta P_{2i}' = -\tilde{G}^{(2)}_i.$$

(53)
\[ \delta P'_{2i} = -\bar{G}^{(2)}_{ij}, \quad (54) \]

while, on the other hand, the properties (51) yield that the new co-cycles of the type (38–39) vanish identically, i.e.,

\[ \partial^i P'_{2i} \equiv 0, \quad (55) \]

\[ 2\partial^i P'_{2ij} \equiv 0. \quad (56) \]

The solution to the equations (51–52) exists and is expressed by (26) for \( D^j_i \) and by

\[ D^{kl}_{ij} = \frac{1}{2} \left( \delta^k_i \delta^l_j - \frac{1}{\Delta} \delta^l_m \delta^k_j \partial^m \partial^l \right). \quad (57) \]

Substituting the solutions (26) and (57) in the relations (53–54) and recalling that \( (\pi, \Pi_i) \) are the non vanishing solutions to the equations (44–45), we obtain

\[ \delta P_{2i} = -\bar{G}^{(2)}_i + \partial_i \pi \equiv -\tilde{\gamma}^{(2)}_i, \quad (58) \]

\[ \delta P_{2ij} = -\bar{G}^{(2)}_{ij} + \frac{1}{2} \partial_i \Pi_j \equiv -\tilde{\gamma}^{(2)}_{ij}, \quad (59) \]

which emphasize the irreducible constraints deriving from the reducible set (33–34) under the form

\[ \tilde{\gamma}^{(2)}_i \equiv \tilde{G}^{(2)}_i - \partial_i \pi \approx 0, \quad (60) \]

\[ \tilde{\gamma}^{(2)}_{ij} \equiv \tilde{G}^{(2)}_{ij} - \frac{1}{2} \partial_i \Pi_j \approx 0. \quad (61) \]

In conclusion, the irreducible model attached to two- and three-forms with topological coupling is pictured by the irreducible first-class constraint set

\[ \tilde{\gamma}^{(1)}_i \equiv \pi_{0i} \approx 0, \quad \tilde{\gamma}^{(1)}_{ij} \equiv \Pi_{0ij} \approx 0, \quad (62) \]

\[ \tilde{\gamma}^{(2)}_i \equiv -2 \partial^j \pi_{ji} - \frac{M}{24} \varepsilon_{0ijklm} F^{jklm} - \partial_i \pi \approx 0, \quad (63) \]

\[ \tilde{\gamma}^{(2)}_{ij} \equiv -3 \partial^k \Pi_{kij} - \frac{1}{2} \partial_i \Pi_j \approx 0, \quad (64) \]

\[ \tilde{\gamma}^{(2)} \equiv -\partial^j \Pi_j \approx 0. \quad (65) \]
2.2.3 Generalization to arbitrary \( p \)

Now, we are in the position to generalize the irreducible construction to arbitrary values of \( p \). The first step resides in deriving a reducible theory involving more fields. To this end, we introduce the antisymmetric bosonic canonical pairs

\[
\left( A^{j_1 \ldots j_{p-2k-2}}, \pi_{j_1 \ldots j_{p-2k-2}} \right)_{k=0, \ldots, c}, \quad \left( H^{i_1 \ldots i_{p-2k-1}}, \Pi_{i_1 \ldots i_{p-2k-1}} \right)_{k=0, \ldots, a},
\]

and, acting accordingly some homological arguments similar to those used previously, we infer the following irreducible first-class set corresponding to (4–5)

\[
\tilde{\gamma}^{(2)}_{i_1 \ldots i_{p-2k-1}} \approx 0, \quad k = 0, \ldots, a,
\]

\[
\tilde{\gamma}^{(2)}_{i_1 \ldots i_{p-2k-1}} \equiv - (p - 2k + 1) \partial^i \Pi_{i_1 \ldots i_{p-2k}} - \partial_{[i_1 \Pi_{i_2 \ldots i_{p-2k}]} \approx 0, \quad k = 0, \ldots, b,
\]

with

\[
\tilde{\gamma}^{(2)}_{i_1 \ldots i_{p-2k-1}} \equiv \begin{cases} \tilde{G}^{(2)}_{i_1 \ldots i_{p-1}} - \partial_{[i_1 \pi_{i_2 \ldots i_{p-1}}]}, & k = 0, \\ - (p - 2k) \partial^i \pi_{i_1 \ldots i_{p-2k-1}} - \partial_{[i_1 \pi_{i_2 \ldots i_{p-2k-1}}]}, & k = 1, \ldots, a, \end{cases}
\]

where we employed the notations

\[
a = \begin{cases} \frac{p}{2} - 1, & \text{if } p \text{ even,} \\ \frac{p-1}{2}, & \text{if } p \text{ odd,} \end{cases}, \quad b = \begin{cases} \frac{p}{2}, & \text{if } p \text{ even,} \\ \frac{p-1}{2}, & \text{if } p \text{ odd,} \end{cases}, \quad c = \begin{cases} \frac{p}{2} - 1, & \text{if } p \text{ even,} \\ \frac{p-3}{2}, & \text{if } p \text{ odd.} \end{cases}
\]

In order to infer a manifestly covariant path integral for the irreducible theory it is still necessary to add some supplementary canonical pairs subject to some additional constraints such that on the one hand the entire set of resulting constraints is first-class and irreducible, and, on the other hand, the number of physical degrees of freedom of the irreducible theory remains unchanged as compared to that of the redundant model. First, we introduce the antisymmetric bosonic canonical pairs

\[
\left( A^{0i_1 \ldots i_{p-2k-3}}, \pi_{0i_1 \ldots i_{p-2k-3}} \right)_{k=0, \ldots, d}, \quad \left( H^{0i_1 \ldots i_{p-2k-2}}, \Pi_{0i_1 \ldots i_{p-2k-2}} \right)_{k=0, \ldots, c},
\]

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subject to the constraints

$$\pi_{0_{i_1 \cdots i_{p-2k-3}}} \approx 0, \ \Pi_{0_{i_1 \cdots i_{p-2k-2}}} \approx 0,$$

where \( d \) is defined by

$$d = \begin{cases} \frac{p}{2} - 2, & \text{if } p \text{ even,} \\ \frac{p-3}{2}, & \text{if } p \text{ odd.} \end{cases}$$

We redenote the constraints (2–3) together with (71) by

$$\tilde{\gamma}_{i_1 \cdots i_{p-2k-1}}^{(1)} \equiv \pi_{0_{i_1 \cdots i_{p-2k-1}}} \approx 0, \ k = 0, \cdots, a, \ (72)$$

$$\tilde{\gamma}_{i_1 \cdots i_{p-2k}}^{(1)} \equiv \Pi_{0_{i_1 \cdots i_{p-2k}}} \approx 0, \ k = 0, \cdots, b. \ (73)$$

Thus, the irreducible model is described until now by the irreducible abelian first-class constraints (67–68) and (72–73). We take the first-class Hamiltonian with respect to these constraints under the form

$$\tilde{H}' = \int d^{2p+1}x \left( -\frac{p!}{2} \pi_{i_1 \cdots i_p} \pi_{i_1 \cdots i_p} + \frac{(p+1)!}{2} \Pi_{i_1 \cdots i_{p+1}} \Pi_{i_1 \cdots i_{p+1}} + \frac{M}{p!} \pi_{0_{i_1 \cdots i_{p+1}j_1 \cdots j_p}} \Pi_{i_1 \cdots i_{p+1}} A_{j_1 \cdots j_p} + \frac{1}{2 \cdot (p+1)!} F_{i_1 \cdots i_{p+1}} F_{i_1 \cdots i_{p+1}} + \frac{1}{2 \cdot (p+2)!} F_{i_1 \cdots i_{p+2}} F_{i_1 \cdots i_{p+2}} + \frac{M^2}{2 \cdot p!} A_{i_1 \cdots i_p} A_{i_1 \cdots i_p} + \sum_{k=0}^{a} A_{0_{i_1 \cdots i_{p-2k-1}}} \tilde{\gamma}_{i_1 \cdots i_{p-2k-1}}^{(2)} + \sum_{k=0}^{b} H_{0_{i_1 \cdots i_{p-2k}}} \tilde{\gamma}_{i_1 \cdots i_{p-2k}}^{(2)} \right). \ (74)$$

Second, to every pair (74) we associate two more antisymmetric bosonic pairs, respectively denoted by

$$\left( B^{(1)i_1 \cdots i_{p-2k-2}}, \pi_{i_1 \cdots i_{p-2k-2}}^{(1)} \right), \ \left( B^{(2)i_1 \cdots i_{p-2k-2}}, \pi_{i_1 \cdots i_{p-2k-2}}^{(2)} \right), \ k = 0, \cdots, c, \ (75)$$

$$\left( V^{(1)i_1 \cdots i_{p-2k-1}}, \Pi_{i_1 \cdots i_{p-2k-1}}^{(1)} \right), \ \left( V^{(2)i_1 \cdots i_{p-2k-1}}, \Pi_{i_1 \cdots i_{p-2k-1}}^{(2)} \right), \ k = 0, \cdots, a, \ (76)$$

which we demand to be constrained by

$$\tilde{\gamma}_{i_1 \cdots i_{p-2k-2}}^{(1)} \equiv \pi_{i_1 \cdots i_{p-2k-2}}^{(1)} \approx 0, \ k = 0, \cdots, c, \ (77)$$

$$\tilde{\gamma}_{i_1 \cdots i_{p-2k-1}}^{(1)} \equiv \Pi_{i_1 \cdots i_{p-2k-1}}^{(1)} \approx 0, \ k = 0, \cdots, a. \ (78)$$
\[
\tilde{\gamma}^{(2)}_{i_1 \ldots i_{p-2k-2}} \equiv -(p - 2k - 1) \pi^{(2)}_{i_1 \ldots i_{p-2k-2}} \approx 0, \quad k = 0, \ldots, c, \quad (79)
\]
\[
\tilde{\gamma}^{(2)}_{i_1 \ldots i_{p-2k-1}} \equiv -(p - 2k) \Pi^{(2)}_{i_1 \ldots i_{p-2k-1}} \approx 0, \quad k = 0, \ldots, a. \quad (80)
\]

In the meantime, it is well-known that one can always add to a set of first-class constraints any combination of first-class constraints whose coefficients determine an invertible matrix without afflicting the theory. We notice that from the concrete form of the constraint functions in (67–68) one can express the momenta \((\pi_{i_1 \ldots i_{p-2k-2}})_{k=0,\ldots,c}\), respectively, \((\Pi_{i_1 \ldots i_{p-2k-1}})_{k=0,\ldots,a}\) under the form

\[
\pi_{i_1 \ldots i_{p-2k-2}} = -\frac{1}{\Delta} \left( \partial^i \tilde{\gamma}^{(2)}_{i_1 \ldots i_{p-2k-2}} + \frac{1}{p - 2k - 2} \partial_{[i_1} \tilde{\gamma}^{(2)}_{i_2 \ldots i_{p-2k-2}]} \right), \quad (81)
\]
\[
\Pi_{i_1 \ldots i_{p-2k-1}} = -\frac{1}{\Delta} \left( \partial^i \tilde{\gamma}^{(2)}_{i_1 \ldots i_{p-2k-1}} + \frac{1}{p - 2k - 1} \partial_{[i_1} \tilde{\gamma}^{(2)}_{i_2 \ldots i_{p-2k-1}]} \right). \quad (82)
\]

Thus, in view of the above observation, we can redefine the constraints (77–78) through

\[
\tilde{\gamma}^{(1)}_{i_1 \ldots i_{p-2k-2}} \equiv \pi^{(1)}_{i_1 \ldots i_{p-2k-2}} - \pi^{(1)}_{i_1 \ldots i_{p-2k-2}} \approx 0, \quad k = 0, \ldots, c, \quad (83)
\]
\[
\tilde{\gamma}^{(1)}_{i_1 \ldots i_{p-2k-1}} \equiv \Pi^{(1)}_{i_1 \ldots i_{p-2k-1}} - \Pi^{(1)}_{i_1 \ldots i_{p-2k-1}} \approx 0, \quad k = 0, \ldots, a. \quad (84)
\]

The introduction of the pairs (75–76) is motivated by the fact that our irreducible Hamiltonian formalism is intended to lead to some corresponding Lagrangian gauge transformations that are manifestly Lorentz covariant. Thus, we need to replace the gauge parameters associated with the first-stage reducibility functions in the reducible context by some other parameters that render the Lorentz covariance of the Lagrangian gauge variations of the fields from the irreducible framework. These parameters are offered precisely by the presence of the supplementary first-class constraints (79–80) and (83–84). As a consequence of the above redefinitions, the theory having the constraints (77–78), (72–73), (79–80) and (83–84) is still irreducible, first-class, abelian, and has the same number of physical degrees of freedom like the original model. The first-class Hamiltonian with respect to these irreducible constraints can be taken under the form

\[
\tilde{H}'' = \int d^{2p+1}x \left( \sum_{k=0}^{c} A^{(2)}_{i_1 \ldots i_{p-2k-2}} \pi^{(2)}_{i_1 \ldots i_{p-2k-2}} + \sum_{k=0}^{a} H^{(2)}_{i_1 \ldots i_{p-2k-1}} \Pi^{(2)}_{i_1 \ldots i_{p-2k-1}} + \right.
\]

\[
\left. \cdots \right). \quad (85)
\]

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\[
\sum_{k=0}^{c} B^{(2)i_1 \ldots i_{p-2k-2}} \left( \partial^2 \gamma^{(2)}_{i_1 \ldots i_{p-2k-2}} + \frac{1}{p-2k-2} \partial_{i_1} \gamma^{(2)}_{i_2 \ldots i_{p-2k-2}} \right) + \\
\sum_{k=0}^{a} V^{(2)i_1 \ldots i_{p-2k-1}} \left( \partial^2 \gamma^{(2)}_{i_1 \ldots i_{p-2k-1}} + \frac{1}{p-2k-1} \partial_{i_1} \gamma^{(2)}_{i_2 \ldots i_{p-2k-1}} \right) + \\
\tilde{H}' \equiv \int d^{2p+1}x \tilde{h}''.
\]

(85)

In this manner, we constructed an irreducible model (described by the first-class constraints (67–68), (72–73), (79–80), (83–84) and by the first-class Hamiltonian (85)) associated with topologically coupled \( p \)- and \((p+1)\)-forms.

### 2.3 Irreducible Hamiltonian BRST symmetry

In this subsection we focus on the construction of the Hamiltonian BRST symmetry for the irreducible model derived in the above. The irreducible BRST differential \( s_I \) has a simple structure due to the abelian character of the irreducible first-class constraint set, containing only the irreducible Koszul-Tate operator \( \delta \) and the exterior derivative along the gauge orbits \( D \). The irreducible Koszul-Tate complex contains the fermionic antighost number one minimal antighosts

\[
\left( P_{1i_1 \ldots i_{p-2k-1-1}}, P_{2i_1 \ldots i_{p-2k-2}} \right)_{k=0, \ldots, a}, \left( P_{1i_1 \ldots i_{p-2k-2}}, P_{2i_1 \ldots i_{p-2k-2}} \right)_{k=0, \ldots, c},
\]

(86)

\[
\left( P_{1i_1 \ldots i_{p-2k}}, P_{2i_1 \ldots i_{p-2k-2}} \right)_{k=0, \ldots, b}, \left( P_{1i_1 \ldots i_{p-2k-1-1}}, P_{2i_1 \ldots i_{p-2k-1}} \right)_{k=0, \ldots, a},
\]

(87)

respectively associated with the first-class constraints (72), (67), (83), (73), (68), (74) and (84). The definitions of \( \delta \) acting on the variables in the minimal Koszul-Tate complex take the usual form

\[
\delta z^A = 0,
\]

(88)

\[
\delta P_{\Delta i_1 \ldots i_{p-2k-1-1}} = -\tilde{\gamma}^{(\Delta)}_{i_1 \ldots i_{p-2k-1}}, \quad \Delta = 1, 2, \ k = 0, \ldots, a,
\]

(89)

\[
\delta P_{\Delta i_1 \ldots i_{p-2k-2}} = -\tilde{\gamma}^{(\Delta)}_{i_1 \ldots i_{p-2k-2}}, \quad \Delta = 1, 2, \ k = 0, \ldots, c,
\]

(90)

\[
\delta P_{\Delta i_1 \ldots i_{p-2k}} = -\tilde{\gamma}^{(\Delta)}_{i_1 \ldots i_{p-2k}}, \quad \Delta = 1, 2, \ k = 0, \ldots, b,
\]

(91)

\[
\delta P_{\Delta i_1 \ldots i_{p-2k-1-1}} = -\tilde{\gamma}^{(\Delta)}_{i_1 \ldots i_{p-2k-1}}, \quad \Delta = 1, 2, \ k = 0, \ldots, a,
\]

(92)
where $z^A$ can be any of the original field/momenta or newly added canonical variable from the pairs (66), (70) or (75–76). These definitions ensure the acyclicity at non-vanishing antighost numbers, as well as the nilpotency of $\delta$, as required by the BRST formalism. The longitudinal complex involves the minimal ghost spectrum

$$\left( \eta_{i_1\cdots i_{p-2k-1}}, \eta_{i_1\cdots i_{p-2k-2}} \right)_{k=0,\ldots,a}, \left( \eta_{i_1\cdots i_{p-2k-2}}, \eta_{i_1\cdots i_{p-2k-2}} \right)_{k=0,\ldots,c},$$

\hspace{1cm} (93)

$$\left( C^{i_1\cdots i_{p-2k}}, C^{i_1\cdots i_{p-2k}} \right)_{k=0,\ldots,a}, \left( C^{i_1\cdots i_{p-2k-1}}, C^{i_1\cdots i_{p-2k-1}} \right)_{k=0,\ldots,c},$$

\hspace{1cm} (94)

where all the fields are fermionic, with pure ghost number one, and respectively correspond to the first-class constraints (72), (67), (83), (79), (73), (68), (84), (80). The definitions of $D$ acting on the variables from the longitudinal complex read as

$$DF = \sum_{\Delta=1}^{a} \left( \sum_{k=0}^{a} \left[ F, \tilde{\gamma}^{(\Delta)}_{i_1\cdots i_{p-2k-1}} \right] \eta_{i_1\cdots i_{p-2k-1}} + \right. \sum_{k=0}^{c} \left[ F, \tilde{\gamma}^{(\Delta)}_{i_1\cdots i_{p-2k-2}} \right] \eta_{i_1\cdots i_{p-2k-2}} + \sum_{k=0}^{b} \left[ F, \tilde{\gamma}^{(\Delta)}_{i_1\cdots i_{p-2k}} \right] C^{i_1\cdots i_{p-2k}} + \sum_{k=0}^{a} \left[ F, \tilde{\gamma}^{(\Delta)}_{i_1\cdots i_{p-2k-1}} \right] C^{i_1\cdots i_{p-2k-1}} \right),$$

\hspace{1cm} (95)

$$DG^\Gamma = 0,$$

\hspace{1cm} (96)

where $F$ is any function of $z^A$, and $G^\Gamma$ generically denotes the minimal ghost spectrum (93–94). The exterior derivative along the gauge orbits is found strongly nilpotent. By enhancing the action of $\delta$ to the ghosts through

$$\delta G^\Gamma = 0,$$

\hspace{1cm} (97)

and the action of $D$ to the antighosts (86–87) (which we globally denote by $\mathcal{P}_\Gamma$) like

$$D\mathcal{P}_\Gamma = 0,$$

\hspace{1cm} (98)

the homological perturbation theory [25–28] guarantees the existence of the irreducible Hamiltonian BRST symmetry, $s_I = \delta + D$, that is nilpotent, $s_I^2 = 0$. The BRST differential is graded accordingly the ghost number ($gh$), defined like the difference between the pure ghost number and the antighost
number. This completes the construction of an irreducible BRST symmetry for the irreducible model deriving from topologically coupled $p$- and $(p+1)$-forms. The next step is to establish the relationship between the irreducible BRST symmetry built here and the standard reducible Hamiltonian BRST symmetry of the original model.

### 2.4 Classical relationship between the reducible and irreducible models

In order to clarify the link between the reducible and irreducible BRST symmetries we show that the two models are physically equivalent. A simple count indicates that the numbers of physical (independent) degrees of freedom of the reducible, respectively, irreducible models coincide. Thus, we have to investigate only the equality between the sets of physical observables corresponding to the irreducible and reducible systems. (We recall that a classical observable is a gauge invariant function.) First, we show that any observable corresponding to the irreducible model is also an observable for the reducible one. To this end, we start with an observable $F$ of the irreducible theory, that should verify the equations

\[
\left[ F, \tilde{\gamma}^{(1)}_{i_1 \ldots i_{p-2k-1}} \right] \approx 0, \quad k = 0, \ldots, a,
\]

\[
\left[ F, \tilde{\gamma}^{(2)}_{i_1 \ldots i_{p-2k+1}} \right] \approx 0, \quad k = 0, \ldots, a,
\]

\[
\left[ F, \tilde{\gamma}^{(1)}_{i_1 \ldots i_{p-2k-2}} \right] \approx 0, \quad k = 0, \ldots, b,
\]

\[
\left[ F, \tilde{\gamma}^{(1)}_{i_1 \ldots i_{p-2k-1}} \right] \approx 0, \quad k = 0, \ldots, a,
\]

\[
\left[ F, \tilde{\gamma}^{(2)}_{i_1 \ldots i_{p-2k-2}} \right] \approx 0, \quad k = 0, \ldots, b.
\]

The equations \((99)\) induce that $F$ does not involve, at least weakly, the fields $\left( A^{01\ldots i_{p-2k-1}} \right)_{k=0,\ldots,a}$ and $\left( H^{01\ldots i_{p-2k}} \right)_{k=0,\ldots,b}$. On the other hand, the equations \((102)\) coupled with the relations \((81-82)\) and \((100-101)\) lead to

\[
\left[ F, \tilde{\pi}^{(1)}_{i_1 \ldots i_{p-2k-2}} \right] \approx 0, \quad k = 0, \ldots, c,
\]

\[
\quad \left[ F, \tilde{\pi}^{(1)}_{i_1 \ldots i_{p-2k-1}} \right] \approx 0, \quad k = 0, \ldots, a.
\]

\[
\left[ F, \tilde{\pi}^{(2)}_{i_1 \ldots i_{p-2k-2}} \right] \approx 0, \quad k = 0, \ldots, c,
\]

\[
\quad \left[ F, \tilde{\pi}^{(2)}_{i_1 \ldots i_{p-2k-1}} \right] \approx 0, \quad k = 0, \ldots, a.
\]

Thus, the equations \((103)\) and \((104)\) indicate that $F$ does not depend, also at least weakly, on the newly added fields $\left( B^{(1)i_1\ldots i_{p-2k-2}}, B^{(2)i_1\ldots i_{p-2k-2}} \right)_{k=0,\ldots,c}$. 

...
and \( (V^{(1)i_1\cdots i_{p-2k-1}}, V^{(2)i_1\cdots i_{p-2k-1}}) \) \( k=0,\ldots,a \). Let us investigate now the conditions (100) and (101). For definiteness, we approach here the case \( p \) even, the other one being solved in a similar manner. We begin with the last relation (100) (assuming that \( p \) is even)

\[
-2\partial_y^i [F(x), \pi_{ii_1}(y)] - \partial_{i_1}^y [F(x), \pi (y)] \approx 0. \tag{105}
\]

Applying \( \partial_y^i \) on (105), we infer

\[
-\partial_y^i \partial_{y}^i [F(x), \pi (y)] \approx 0,
\]

which further yields

\[
[F(x), \pi (y)] \approx 0. \tag{106}
\]

Substituting (106) in (105), we get

\[
\partial_y^i [F(x), \pi_{ii_1}(y)] \approx 0. \tag{107}
\]

Applying \( \partial_y^i \) on the next relation (100), namely,

\[
-2\partial_y^i [F(x), \pi_{ii_1 i_{i_2}}(y)] - \partial_{i_1}^y [F(x), \pi_{i_{i_2}i_1}(y)] - \partial_{i_2}^y [F(x), \pi_{i_{i_3}i_1}(y)] - \partial_{i_3}^y [F(x), \pi_{i_{i_3}i_2}(y)] \approx 0, \tag{108}
\]

and using (107), we derive

\[
[F(x), \pi_{i_{i_2}i_3}(y)] \approx 0,
\]

hence

\[
[F(x), \pi_{i_{i_2}i_3}(y)] \approx 0. \tag{109}
\]

Replacing the above result in (108) and reprising the same program on the next relations (100), we are led to

\[
[F(x), \pi_{i_1\cdots i_{p-2k-2}}(y)] \approx 0, \; k = 0, \ldots, c, \tag{110}
\]

which, inserted into the first equation (100), yield

\[
[F(x), \bar{G}^{(2)}_{i_1\cdots i_{p-1}}(y)] \approx 0. \tag{111}
\]

If we act along the same line, but starting from the last equation (101), we will accordingly arrive at

\[
[F(x), \Pi_{i_1\cdots i_{p-2k-1}}(y)] \approx 0, \; k = 0, \ldots, a, \tag{112}
\]

which, substituted in (101) for \( k = 0 \) imply

\[
[F(x), \bar{G}^{(2)}_{i_1\cdots i_p}(y)] \approx 0. \tag{113}
\]
The equations (110) and (112) indicate that $F$ does not depend, at least weakly, on the fields $(A_{11...i_p-2k-2})_{k=0,...,c}$ and $(H_{11...i_p-2k-1})_{k=0,...,a}$. We can summarize the prior results by stating that if $F$ denotes an observable of the irreducible theory, then it does not depend on any of the newly introduced fields (66), (70) and (75–76). Moreover, it satisfies the relations

\[ [F(x), \tilde{G}_{i_1...i_{p-1}}(y)] \approx 0, \quad [F(x), \bar{G}_{i_1...i_p}(y)] \approx 0, \]  
(114)

(see (99) for $k = 0$), and also (111), (113), which are precisely the equations verified by an observable of the reducible model. All these show that if $F$ is an observable of the irreducible theory, then it is also an observable of the redundant system. The converse is valid, too, because any observable of the redundant model checks the equations (111), (113–114), and does not depend on the newly added canonical pairs, such that (99–103) are automatically satisfied. Thus, as both the irreducible and reducible models display the same physical observables, the zeroth order cohomological groups of the reducible and irreducible BRST symmetries, $s_R$ and $s_I$, are equal

\[ H^0(s_R) = H^0(s_I). \]  
(115)

In view of this, the reducible and irreducible models are equivalent from the BRST formalism point of view, i.e., from the point of view of the basic requirements of the BRST symmetry, $s^2 = 0$ and $H^0(s) = \{\text{physical observables}\}$. As a consequence, we can substitute the reducible Hamiltonian BRST symmetry for the original system by that of the irreducible theory. This further implies that at the BRST quantization level we can also replace the Hamiltonian BRST quantization of topologically coupled abelian $p$- and $(p+1)$-forms with that of the irreducible first-class theory.

### 2.5 Hamiltonian BRST quantization of the irreducible theory

In the sequel we rely on the last conclusion and investigate the Hamiltonian BRST quantization of the irreducible model. The minimal antighost and ghost spectra are offered by (86–87) and (93–94). It is convenient to work with the non-minimal sector

\[ \left( P^{i_1...i_{p-2k-1}}_{\bar{\eta}}, \bar{\eta}_{i_1...i_{p-2k-1}} \right), \left( P_{\bar{\eta}^1}^{i_1...i_{p-2k-1}}, \bar{\eta}^1_{i_1...i_{p-2k-1}} \right), \quad k = 0, \ldots, a, \]  
(116)
\[
\left( P_b^{i_1 \ldots i_{p-2k-1}} , b_{i_1 \ldots i_{p-2k-1}} \right) , \left( P_{b_1}^{i_1 \ldots i_{p-2k-1}} , b_{1 \ldots i_{p-2k-1}} \right) , k = 0, \ldots, a, \tag{117} \\
\left( P_C^{i_1 \ldots i_{p-2k}} , C_{i_1 \ldots i_{p-2k}} \right) , \left( P_{C_1}^{i_1 \ldots i_{p-2k}} , C_{1 \ldots i_{p-2k}} \right) , k = 0, \ldots, b, \tag{118} \\
\left( P_{b_1}^{i_1 \ldots i_{p-2k}} , \tilde{b}_{1 \ldots i_{p-2k}} \right) , \left( P_{b_1}^{i_1 \ldots i_{p-2k}} , \tilde{b}_{1 \ldots i_{p-2k}} \right) , k = 0, \ldots, b. \tag{119} 
\]

The variables (17), (19) are bosonic and have ghost number zero. The fields from (10), (18) are fermionic, the \( P \)'s possessing ghost number one, while the \( \tilde{\eta} \)'s and \( \tilde{C} \)'s have ghost number minus one. The non-minimal BRST canonical generator of the irreducible Hamiltonian BRST symmetry reads as

\[
\tilde{\Omega} = \int d^{2p+1} x \left( \sum_{\Delta = 1}^{2} \left( \sum_{k=1}^{p} \eta_{\Delta}^{i_1 \ldots i_{p-k}} z_{\Delta}^{i_1 \ldots i_{p-k}} + \sum_{k=1}^{p+1} C_{\Delta}^{i_1 \ldots i_{p-k+1}} z_{\Delta}^{i_1 \ldots i_{p-k+1}} \right) + \sum_{k=0}^{a} \left( P_{\tilde{\eta}}^{i_1 \ldots i_{p-2k-1}} b_{i_1 \ldots i_{p-2k-1}} + P_{\tilde{\eta}}^{i_1 \ldots i_{p-2k-1}} b_{1 \ldots i_{p-2k-1}} \right) + \sum_{k=0}^{b} \left( P_{\tilde{C}}^{i_1 \ldots i_{p-2k}} \tilde{b}_{i_1 \ldots i_{p-2k}} + P_{\tilde{C}}^{i_1 \ldots i_{p-2k}} \tilde{b}_{1 \ldots i_{p-2k}} \right) \right), \tag{120}
\]

while the BRST-invariant extension of \( \tilde{H}'' \) has the form

\[
\tilde{H}_B'' = \tilde{H}'' + \int d^{2p+1} x \left( \sum_{k=0}^{c} \frac{1}{p-2k+1} \eta_{i_1 \ldots i_{p-2k-2}} P_{2i_1 \ldots i_{p-2k-2}} + \frac{1}{p-1} \eta_{i_1 \ldots i_{p-1}} \partial_{[i_1} P_{2i_2 \ldots i_{p}]} - \sum_{k=1}^{a} \frac{1}{p-2k+1} \eta_{1 \ldots i_{p-2k-2}} P_{2i_1 \ldots i_{p-2k-2}} + \frac{1}{p-1} \eta_{i_1 \ldots i_{p-1}} \partial_{[i_1} P_{2i_2 \ldots i_{p}]} \right) - \sum_{k=0}^{c} \eta_{i_1 \ldots i_{p-2k-2}} \left( \frac{p-2k}{p-2k+1} \eta_{i_1 \ldots i_{p-2k-2}} P_{2i_1 \ldots i_{p-2k-2}} + \frac{1}{p-2k+1} \partial_{[i_1} P_{2i_2 \ldots i_{p}]} \right) + \sum_{k=0}^{c} \eta_{i_1 \ldots i_{p-2k-2}} \left( \frac{p-2k}{p-2k+1} \eta_{i_1 \ldots i_{p-2k-2}} P_{2i_1 \ldots i_{p-2k-2}} + \frac{1}{p-2k+1} \partial_{[i_1} P_{2i_2 \ldots i_{p}]} \right) + \sum_{k=0}^{b} \frac{1}{p-2k} C_{1 \ldots i_{p-2k}} P_{2i_1 \ldots i_{p-2k}} - \sum_{k=0}^{a} \frac{1}{p-2k} C_{1 \ldots i_{p-2k-1}} P_{2i_1 \ldots i_{p-2k-1}} + \sum_{k=1}^{b} \frac{1}{p-2k} \eta_{i_1 \ldots i_{p-2k}} P_{2i_1 \ldots i_{p-2k}} + \frac{1}{p-2k} \partial_{[i_1} P_{2i_2 \ldots i_{p-2k}]} \right) - \sum_{k=0}^{a} \frac{1}{p-2k} \eta_{i_1 \ldots i_{p-2k-1}} P_{2i_1 \ldots i_{p-2k-1}} + \frac{1}{p-2k} \partial_{[i_1} P_{2i_2 \ldots i_{p-2k-1}]}. \tag{121}
\]
In order to fix the gauge we choose the gauge-fixing fermion

\[ \tilde{K} = \int d^{2p+1}x \left( \mathcal{P}_{1i_1 \ldots i_{p-1}} \left( \partial_i A^{i_1 \ldots i_{p-1}} + \frac{1}{p-1} \partial^{[i_1} B^{(1) i_2 \ldots i_{p-1}]} \right) + \right. \\
\left. \sum_{k=1}^{a} \mathcal{P}_{1i_1 \ldots i_{p-2k-1}} \left( \partial_i B^{(1) i_1 \ldots i_{p-2k-1}} + \frac{1}{p-2k-1} \partial^{[i_1} B^{(1) i_2 \ldots i_{p-2k-1}]} \right) + \right. \\
\left. (-)^{p+1} \sum_{k=0}^{c} \mathcal{P}_{1i_1 \ldots i_{p-2k-2}} \left( (p-2k-1) \partial_i A^{i_1 \ldots i_{p-2k-2}0} + \partial^{[i_1} A^{i_2 \ldots i_{p-2k-2}]} \right) + \right. \\
\left. \sum_{k=0}^{a} \mathcal{P}_{b_1^i \ldots i_{p-2k-1}} \left( \partial_i H^{i_1 \ldots i_{p-2k-1}} - \tilde{\eta}_{i_1 \ldots i_{p-2k-1}} + \tilde{\eta}^{1}_{1 \ldots i_{p-2k-1}} \right) + \right. \\
\left. \sum_{k=0}^{b} \mathcal{P}_{1i_1 \ldots i_{p-2k}} \left( \partial_i V^{(1) i_1 \ldots i_{p-2k}} + \frac{1}{p-2k} \partial^{[i_1} V^{(1) i_2 \ldots i_{p-2k}]} \right) + \right. \\
\left. (-)^{p} \sum_{k=0}^{b} \mathcal{P}_{b_1^i \ldots i_{p-2k-1}} \left( (p-2k) \partial_i H^{i_1 \ldots i_{p-2k-1}0} + \partial^{[i_1} H^{i_2 \ldots i_{p-2k-1}]} \right) + \right. \\
\left. \sum_{k=0}^{b} \mathcal{P}_{b_1^i \ldots i_{p-2k}} \left( \partial_i V^{(1) i_1 \ldots i_{p-2k}} + \partial^{[i_1} V^{(1) i_2 \ldots i_{p-2k}]} \right) \right). \tag{122} \]

The corresponding path integral, resulting after some computation, will be

\[ Z_{\tilde{K}} = \int \mathcal{D}H^{\mu_1 \ldots \mu_p + 1} \mathcal{D}A^{\mu_1 \ldots \mu_p} \times \]
\[ \left( \prod_{k=0}^{a} \mathcal{D}V^{(1) \mu_1 \ldots \mu_{p-2k-1}} \right) \left( \prod_{k=0}^{c} \mathcal{D}B^{(1) \mu_1 \ldots \mu_{p-2k-2}} \right) \times \]
\[ \left( \prod_{k=0}^{b} \mathcal{D}b_1^{\mu_1 \ldots \mu_{p-2k}} \mathcal{D}C^{\mu_1 \ldots \mu_{p-2k}} \mathcal{D}C^{(1) \mu_1 \ldots \mu_{p-2k}} \right) \times \]
\[ \left( \prod_{k=0}^{a} \mathcal{D}b_2^{\mu_1 \ldots \mu_{p-2k-1}} \mathcal{D}b_1^{\mu_1 \ldots \mu_{p-2k-1}} \mathcal{D}b_1^{\mu_1 \ldots \mu_{p-2k-1}} \right) \exp iS_{\tilde{K}}. \tag{123} \]
where

\[
S_K = \tilde{S}_0^L + \int d^{2p+2}x \left(-\sum_{k=0}^a \tilde{\eta}_{\mu_1\cdots\mu_{p-2k-1}} \Box \eta_2^{\mu_1\cdots\mu_{p-2k-1}} - \sum_{k=0}^b \tilde{C}_{\mu_1\cdots\mu_{p-2k}} \Box C_{2}^{\mu_1\cdots\mu_{p-2k}} + b_{\mu_1\cdots\mu_{p-1}} \left( \partial_{\mu} A^{\mu_1\cdots\mu_{p-1}} + \frac{1}{p-1} \partial_{\mu_1} B^{(1)\mu_2\cdots\mu_{p-1}} \right) + \sum_{k=1}^a b_{\mu_1\cdots\mu_{p-2k-1}} \left( \partial_{\mu} B^{(1)\mu_1\cdots\mu_{p-2k-1}} + \frac{1}{p-2k-1} \partial_{\mu_1} B^{(1)\mu_2\cdots\mu_{p-2k-1}} \right) + \tilde{b}_{\mu_1\cdots\mu_{p}} \left( \partial_{\mu} \tilde{H}^{\mu_1\cdots\mu_{p}} + \frac{1}{p} \partial_{\mu_1} \tilde{V}^{(1)\mu_2\cdots\mu_{p}} \right) + \sum_{k=1}^b \tilde{b}_{\mu_1\cdots\mu_{p-2k}} \left( \partial_{\mu} \tilde{V}^{(1)\mu_1\cdots\mu_{p-2k}} + \frac{1}{p-2k} \partial_{\mu_1} \tilde{V}^{(1)\mu_2\cdots\mu_{p-2k}} \right) \right) ,
\]

and \( \Box = \partial^\mu \partial_\mu \). The action \( \tilde{S}_0^L \) is nothing but the original action, expressed by \([123, 124]\). We mention that in obtaining \([123, 124]\) we performed the identifications

\[
B^{(1)\mu_1\cdots\mu_{p-2k-2}} \equiv \left( A^{0i_1\cdots i_{p-2k-3}}, B^{(1)i_1\cdots i_{p-2k-2}} \right), \quad k = 0, \ldots, c , \quad (125)
\]

\[
b_{\mu_1\cdots\mu_{p-2k-1}} \equiv \left( \tilde{\eta}_{1i_1\cdots i_{p-2k-2}}, b_{1i_1\cdots i_{p-2k-1}} \right), \quad k = 0, \ldots, a , \quad (126)
\]

\[
\eta_2^{\mu_1\cdots\mu_{p-2k-1}} \equiv \left( \tilde{\eta}_2^{i_1\cdots i_{p-2k-2}}, \eta_2^{i_1\cdots i_{p-2k-1}} \right), \quad k = 0, \ldots, a , \quad (127)
\]

\[
\tilde{\eta}_{\mu_1\cdots\mu_{p-2k-1}} \equiv \left( -\mathcal{P}_{i_1\cdots i_{p-2k-2}}, \tilde{\eta}_{i_1\cdots i_{p-2k-1}} \right), \quad k = 0, \ldots, a , \quad (128)
\]

\[
V^{(1)\mu_1\cdots\mu_{p-2k-1}} \equiv \left( H^{0i_1\cdots i_{p-2k-2}}, V^{(1)i_1\cdots i_{p-2k-1}} \right), \quad k = 0, \ldots, a , \quad (129)
\]

\[
\tilde{b}_{\mu_1\cdots\mu_{p-2k}} \equiv \left( \tilde{\Pi}_{i_1\cdots i_{p-2k-1}}, \tilde{b}_{i_1\cdots i_{p-2k}} \right), \quad k = 0, \ldots, b , \quad (130)
\]

\[
C_2^{\mu_1\cdots\mu_{p-2k}} \equiv \left( C_2^{i_1\cdots i_{p-2k-1}}, C_2^{i_1\cdots i_{p-2k}} \right), \quad k = 0, \ldots, b , \quad (131)
\]

\[
\tilde{C}_{\mu_1\cdots\mu_{p-2k}} \equiv \left( -\tilde{P}_{i_1\cdots i_{p-2k-1}}, \tilde{C}_{i_1\cdots i_{p-2k}} \right), \quad k = 0, \ldots, b . \quad (132)
\]

It is easy to check that the gauge-fixed action \([124]\) has no residual gauge invariances. Hence, following our irreducible treatment, we inferred a path integral for topologically coupled \(p\) and \((p+1)\)-form gauge fields that involves no ghosts for ghosts, and, in addition, is Lorentz covariant.
3 Irreducible treatment for interacting theories with topological coupling

In the sequel we extend our irreducible treatment to interacting gauge theories with topological coupling. A possibility would be to investigate the canonical analysis of the interacting theory and then develop an irreducible method along the lines exposed in the previous section. A major difficulty in implementing this program is that the interaction terms may involve higher order derivatives of the fields, which would make the canonical approach too complicated. An alternative that surpasses this inconvenient is to analyze whether our irreducible Hamiltonian procedure induces a corresponding irreducible Lagrangian version, and, if the answer is affirmative, to solve the interacting case within the irreducible Lagrangian context. We will see that this idea can be consistently enforced, our irreducible Hamiltonian scheme for topologically coupled $p$- and $(p+1)$-form gauge fields allowing indeed an irreducible Lagrangian formalism that maintains the space-time locality and Lorentz covariance of the resulting gauge-fixed action. The manifest covariance will be restored precisely due to the introduction in the theory of the supplementary canonical pairs (75–76). While in the Hamiltonian background the distinction between primary and secondary constraints is not significant, this aspect becomes important at the Lagrangian level in order to obtain the gauge transformations of the Lagrangian action. This is why in what follows we work with a model of irreducible Hamiltonian theory in the case of topological coupling in the framework of which we assume that (72), (73), (83) are primary constraints whose consistencies respectively imply the secondary ones (67), (68), (79). The derivation of the gauge transformations of our irreducible model involves three steps. First, we write down the associated extended action

$$
\tilde{S}^{\mu E} = \int d^{2p+2}x \left( \sum_{k=0}^{b} \hat{A}^{j_1\ldots j_{p-2k}} \pi_{j_1\ldots j_{p-2k}} + \sum_{k=0}^{a+1} \hat{H}^{i_1\ldots i_{p-2k+1}} \Pi_{i_1\ldots i_{p-2k+1}} + \sum_{k=0}^{d} \hat{A}^{0i_1\ldots i_{p-2k-3}} \pi_{0i_1\ldots i_{p-2k-3}} + \sum_{k=0}^{c} \hat{H}^{0i_1\ldots i_{p-2k-2}} \Pi_{0i_1\ldots i_{p-2k-2}} + \sum_{\Delta=1}^{2} \sum_{k=0}^{c} \hat{B}^{(\Delta)i_1\ldots i_{p-2k-2}} \pi_{(\Delta)i_1\ldots i_{p-2k-2}} + \sum_{\Delta=1}^{2} \sum_{k=0}^{a} \hat{V}^{(\Delta)i_1\ldots i_{p-2k-1}} \Pi_{(\Delta)i_1\ldots i_{p-2k-1}} - \tilde{h}'' \right)
$$
\[
\sum_{\Delta=1}^2 \sum_{k=0}^a \tilde{\gamma}_{i_1 \ldots i_{p-2k-1}}(\Delta) \tilde{u}(\Delta)_{i_1 \ldots i_{p-2k-1}} - \sum_{\Delta=1}^2 \sum_{k=0}^b \xi_{i_1 \ldots i_{p-2k}}(\Delta) \tilde{u}(\Delta)_{i_1 \ldots i_{p-2k}} - \sum_{\Delta=1}^2 \sum_{k=0}^c \xi_{i_1 \ldots i_{p-2k-2}}(\Delta) \tilde{u}(\Delta)_{i_1 \ldots i_{p-2k-2}} - \sum_{\Delta=1}^2 \sum_{k=0}^a \xi_{i_1 \ldots i_{p-2k-1}}(\Delta) \tilde{u}(\Delta)_{i_1 \ldots i_{p-2k-1}}
\]

, (133)

and determine its gauge invariances. In the last relation \( \tilde{h}'' \) is given by (85), while the \( \tilde{u}(\Delta) \)'s and \( \bar{u}(\Delta) \)'s represent the Lagrange multipliers of the corresponding constraints. Second, on the one hand with the help of the extended action (133) we infer the so-called total action by setting zero all the multipliers carrying the index (2) (and associated by virtue of our choice with the secondary constraints of the irreducible model), and, on the other hand, we determine the gauge invariances of the total action by taking all the gauge variations of the multipliers associated with the secondary constraints to vanish. Third, we deduce the Lagrangian action for the irreducible model together with its gauge invariances by eliminating all the momenta and the remaining Lagrange multipliers on their equations of motion resulting from the total formalism. In addition, we notice that the fields carrying the superscript (2) and also \((A_{j_1 \ldots j_{p-2k-2}})_{k=0, \ldots, c}, (H_{i_1 \ldots i_{p-2k-1}})_{k=0, \ldots, a} \) are auxiliary variables, hence we can remove them from the irreducible model. As a result of this three-step algorithm, we get that the Lagrangian action implied by the irreducible Hamiltonian theory is nothing but the original action

\[
\tilde{S}_0^{\mu L} \left[ A^{\mu_1 \ldots \mu_p}, H^{\mu_1 \ldots \mu_{p+1}}, B^{(1)\mu_1 \ldots \mu_{p-2k-2}}, V^{(1)\mu_1 \ldots \mu_{p-2k-1}} \right] =
\tilde{S}_0^{\mu L} \left[ A^{\mu_1 \ldots \mu_p}, H^{\mu_1 \ldots \mu_{p+1}} \right],
\]

while the corresponding gauge transformations, which can be checked to be irreducible, are expressed by

\[
\delta_\epsilon A^{\mu_1 \ldots \mu_p} = \partial [\mu_1 \ddot{\epsilon}^{\mu_2 \ldots \mu_p}],
\]

\[
\delta_\epsilon B^{(1)\mu_1 \ldots \mu_{p-2k-2}} = \partial [\mu_1 \ddot{\epsilon}^{\mu_2 \ldots \mu_{p-2k-2}}] +
(p-2k-1) \partial_\mu \ddot{\epsilon}^{\mu_1 \ldots \mu_{p-2k-2}}, k = 0, \ldots, c,
\]

\[
\delta_\epsilon H^{\mu_1 \ldots \mu_{p+1}} = \partial [\mu_1 \ddot{\epsilon}^{\mu_2 \ldots \mu_{p+1}}],
\]

\[
\delta_\epsilon V^{(1)\mu_1 \ldots \mu_{p-2k-1}} = \partial [\mu_1 \ddot{\epsilon}^{\mu_2 \ldots \mu_{p-2k-1}}] +
(p-2k) \partial_\mu \ddot{\epsilon}^{\mu_1 \ldots \mu_{p-2k-1}}, k = 0, \ldots, a,
\]

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where the identifications (123) and (129) have also been employed. The
gauge parameters involved with (135–138) are defined by
\[ \tilde{e}^{\mu_1 \cdots \mu_{p-2k-1}} \equiv \left( \tilde{e}^{1 \cdots i_{p-2k-2}}, \tilde{e}^{1 \cdots i_{p-2k-1}} \right), \quad k = 0, \ldots, a, \tag{139} \]
\[ \tilde{e}^{\mu_1 \cdots \mu_{p-2k}} \equiv \left( \tilde{e}^{1 \cdots i_{p-2k-1}}, \tilde{e}^{1 \cdots i_{p-2k}} \right), \quad k = 0, \ldots, b, \tag{140} \]
where the parameters \((\tilde{e}^{1 \cdots i_{p-2k-2}}, \tilde{e}^{1 \cdots i_{p-2k-1}})\) correspond to the constraints (29),
respectively, (27), while \((\tilde{e}^{1 \cdots i_{p-2k-1}}, \tilde{e}^{1 \cdots i_{p-2k}})\) are associated with the
constraints (80), respectively, (68). We remark that the gauge variations
(135) and (137) involved with the original fields \(A^{\mu_1 \cdots \mu_p}\) and \(H^{\mu_1 \cdots \mu_{p+1}}\) are
nothing but the gauge invariances of the original action (1). However,
although these transformations alone are reducible, the entire set of
gauge transformations (135–138) connected to the larger field spectrum is irre-
ducible. In this manner we constructed an irreducible Lagrangian model
originating in our irreducible Hamiltonian approach addressed in the previ-
sion section. It can be shown that we can recover the relations (123–124)
in the framework of the antifield-BRST quantization of this irreducib le
Lagrangian model by using an appropriate non-minimal sector and gauge-fixing
fermion. The non-minimal solution to the master equation for the irreducib le
Lagrangian system reads as
\[
\tilde{S}'' = \tilde{S}_0'' + \int d^{2p+2}x \left( A^{* \mu_1 \cdots \mu_p} \partial^{[\mu_1} \eta^{\mu_2 \cdots \mu_p]} + H^{* \mu_1 \cdots \mu_{p+1}} \partial^{[\mu_1} C^{\mu_2 \cdots \mu_{p+1}]} + \right. \\
\left. \sum_{k=0}^{c} B^{(1)}_{\mu_1 \cdots \mu_{p-2k-2}} \left( \partial^{[\mu_1} \eta^{\mu_2 \cdots \mu_{p-2k-2}]} + (p - 2k - 1) \partial^{[\mu_1} \eta^{\mu_2 \cdots \mu_{p-2k-2}]} \right) + \right. \\
\sum_{k=0}^{a} \gamma^{(1)}_{\mu_1 \cdots \mu_{p-2k-1}} \left( \partial^{[\mu_1} C^{\mu_2 \cdots \mu_{p-2k-1}]} + (p - 2k) \partial^{[\mu_1} C^{\mu_2 \cdots \mu_{p-2k-1}]} \right) - \\
\sum_{k=0}^{a} \tilde{\eta}^{(a)}_{\mu_1 \cdots \mu_{p-2k-1}} \tilde{b}^{[\mu_1} \eta^{\mu_2 \cdots \mu_{p-2k-1}]} + \sum_{k=0}^{b} \tilde{C}^{(b)}_{\mu_1 \cdots \mu_{p-2k}} \tilde{b}^{[\mu_1} \eta^{\mu_2 \cdots \mu_{p-2k}]} \right), \tag{141} \]
where \((\eta^{\mu_1 \cdots \mu_{p-2k-1}})_{k=0,\ldots,a}\) and \((C^{\mu_1 \cdots \mu_{p-2k}})_{k=0,\ldots,b}\) signify the Lagrangian pure
ghost number one ghosts, the star variables denote the antifields of the cor-
responding fields, and the other variables belong to the non-minimal sector.
If we choose a gauge-fixing fermion of the type
\[
\psi = -\int d^{2p+2}x \left( \sum_{k=0}^{a} \tilde{\chi}^{[\mu_1} \eta^{\mu_2 \cdots \mu_{p-2k-1}]} + \sum_{k=0}^{b} \tilde{\chi}^{[\mu_1} \eta^{\mu_2 \cdots \mu_{p-2k}]} \right), \tag{142} \]
where the functions $\tilde{\chi}^{\mu_1\cdots\mu_p-1}$ and $\bar{\chi}^{\mu_1\cdots\mu_p-2k}$ are expressed by
\begin{align}
\tilde{\chi}^{\mu_1\cdots\mu_p-1} &= \partial_\mu A^{\mu_1\cdots\mu_p-1} + \frac{1}{p-1} \partial^{\mu_1} B^{(1)\mu_2\cdots\mu_p-1}, \quad (143) \\
\tilde{\chi}^{\mu_1\cdots\mu_p-2k-1} &= \partial_\mu B^{(1)\mu_1\cdots\mu_p-2k-1} + \frac{1}{p-2k-1} \partial^{\mu_1} B^{(1)\mu_2\cdots\mu_p-2k-1}, \quad k = 1, \ldots, a, \quad (144) \\
\bar{\chi}^{\mu_1\cdots\mu_p} &= \partial_\mu H^{\mu_1\cdots\mu_p} + \frac{1}{p} \partial^{\mu_1} V^{(1)\mu_2\cdots\mu_p}, \quad (145) \\
\bar{\chi}^{\mu_1\cdots\mu_p-2k} &= \partial_\mu V^{(1)\mu_1\cdots\mu_p-2k} + \frac{1}{p-2k} \partial^{\mu_1} V^{(1)\mu_2\cdots\mu_p-2k}, \quad k = 1, \ldots, b, \quad (146)
\end{align}
and eliminate all the antifields from (141) with the help of (142) we are led precisely to (123–124) modulo the identifications
\begin{align}
\eta^{\mu_1\cdots\mu_p-2k-1} &\equiv \eta^{\mu_1\cdots\mu_p-2k-1}, \quad (147) \\
C^{\mu_1\cdots\mu_p-2k} &\equiv C^{\mu_1\cdots\mu_p-2k}. \quad (148)
\end{align}
In consequence, we emphasized how our irreducible Hamiltonian procedure gives rise to an irreducible covariant Lagrangian approach for topologically coupled $p$- and $(p+1)$-form gauge fields that outputs the path integral derived in the Hamiltonian context. Taking into consideration this result, the interaction case can be solved in a direct manner. Indeed, if one adds to the Lagrangian action (1) some interaction terms which are invariant under the original reducible gauge transformations (135) and (137), then the starting point toward an irreducible Lagrangian approach to the interacting system is represented by the interacting Lagrangian action subject to the irreducible gauge transformations (135–138) of the broader field spectrum. The main point is that even if the interaction terms involve higher-order derivatives of the fields, this does not afflict in any way our procedure as the interacting Lagrangian action satisfies the same Noether identities like in the absence of the interaction. Therefore, the non-minimal solution to the master equation results from (141) in which we replace $\hat{S}_0^f$ with the action of the interacting Lagrangian model under study. Consequently, we can still employ the gauge-fixing fermion (142), which will produce a gauge-fixed action of the type (124) excepting the starting Lagrangian action that must contain the gauge-invariant interaction terms. Moreover, our formalism can yet be extended to interacting theories like the ones discussed above which contain
more sorts of abelian $p$-form gauge fields. These theories are important in order to derive all consistent interactions between $p$-form gauge fields [24]. In this light, our irreducible Hamiltonian procedure gives rise to an irreducible Lagrangian approach which proves to be efficient at the irreducible investigation of interacting theories with topological coupling.

4 Conclusion

In this paper we develop a consistent irreducible Hamiltonian BRST treatment of $p$-form gauge theories with topological coupling. We start with a quadratic action describing topologically coupled abelian $p$- and $(p + 1)$-form gauge fields and construct an irreducible Hamiltonian first-class model that is equivalent at the BRST quantization level with the starting redundant theory. The irreducibility is enforced in the background of the Koszul-Tate complex via making all the initial antighost number one co-cycles of the Koszul-Tate differential to vanish identically under a proper ‘rotation’ of the antighost number one antighosts such that the total number of physical degrees of freedom does not vary. The irreducible Hamiltonian analysis of the initial quadratic action presents the desirable feature that it induces a corresponding irreducible Lagrangian version, which, in turn, is the most natural framework for investigating higher-order interacting Lagrangian gauge theories with topological coupling. Finally, we remark that our analysis covers the free case in the limit $M \to 0$.

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