Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

Elsevier hereby grants permission to make all its COVID-19-related research that is available on the COVID-19 resource centre - including this research content - immediately available in PubMed Central and other publicly funded repositories, such as the WHO COVID database with rights for unrestricted research re-use and analyses in any form or by any means with acknowledgement of the original source. These permissions are granted for free by Elsevier for as long as the COVID-19 resource centre remains active.
Fractional order epidemiological model of SARS-CoV-2 dynamism involving Alzheimer’s disease

Emmanuel Addai a, Lingling Zhang a,b, Ama Kyerewaa Preko b, Joshua Kiddy K. Asamoah c

a Department of Mathematics, Taiyuan University of Technology, Shanxi Taiyuan 030024, PR China
b Department of Mathematics, Zhejiang Normal University, Zhejiang Jinhua, 321004, PR China
c Department of Mathematics, Kwame Nkrumah University of Science and Technology, Kumas, Ghana

1. Introduction

No doubt, the merciless outbreaks of infectious disease pandemics have impacted humanity’s history. Entire nations and civilizations have gone off the face of the earth throughout history. Beginning with biblical plagues of Egypt in the twentieth century “cocoliztli” epidemics killed some 13 million people, completely destroying the Mesoamerican native population, as well as the Black Death pandemic that erupted in Europe in 1348 BC, killed over 25 million people in just five years [1]. The pandemic influenza viruses of 1918–1919 spread across all part of the world, killing an estimated 40 million people worldwide. AIDS, measles, malaria, and tuberculosis have all been re-emerging epidemics in recent decades, killing millions of infected individuals each year. According to a report on the global AIDS epidemic released by the World Health Organization (WHO) and the United Nations AIDS Programme (UNAIDS), an estimated 37.7 million individuals, including 1.7 million children, were living with HIV at the end of 2020 [2]. SARS-CoV-2 pandemic has stabilized world progress and put the globe into silent mode, in the twenty-first century, where rapid technological and theoretical advancements have substantially strengthened our armament in prevention and control of epidemics. The SARS-CoV-2 virus causes Coronavirus (COVID-19), a recent infectious disease. The emergence of new to severe respiratory difficulties is one of the most well-known symptoms of this condition, with the patient recovering without therapy in some circumstances. This virus causes symptoms such as fever, cough, loss of taste. The infected individuals sometimes show less common symptoms such as sore throat, headache, rash, among others. COVID-19’s age-related transmissibility has become a public health concern in this context. Adults over the age of 60 have had the most serious health problems, with deadly effects for those over the age of 80. This is attributable to the high prevalence of underlying health issues in older people [3–5]. Coronavirus, like all viruses, evolves throughout time. The majority of the changes have no effect on the virus’s properties. Some of these changes, however, may have an effect on the virus’s features, such as how easily it spreads, the severity and scope of the sickness it causes, or the efficacy of vaccines, therapeutic drugs, and diagnostic tests. The authors in [6] considered COVID-19 model where a brief details of infection were discussed. They showed that the main source of infection was the seafood industry. In [7], they also analyzed the impact of non-pharmaceutical interventions on the dynamics COVID-19 involving optimal control strategy. The authors in [8] modeled a new COVID-19 model incorporated quarantine and isolation. They discussed relevant mathematical results.

Quarantine is one of the most effective strategies to limit the transmission route of this disease, because the symptoms of the sickness may not emerge on the infected individual at all times, allowing the virus to spread more swiftly. According to mental health experts, the global SARS-CoV-2 virus pandemic quarantine implemented on millions of people is neither simple nor overstated since it is a one-of-a-kind and unprecedented action that restricts day-to-day movement of people. Many people suffer from psychological and neurological problems as a result of this predicament, especially those who do not deal with it...
positively. Recently, there has been a worldwide concern of the rising tide of the memory difficulties among all population. According to previous studies, depression in the old age has been connected to cognitive impairment and the risk of Alzheimer’s disease [9–12]. Alzheimer’s disease is a brain disorder that causes memory and cognitive function to deteriorate over time. Alzheimer’s disease is a neuro degenerative disorder having higher fatality in the elderly due to delayed treatment caused by a lower detection rate. The lockdown has affected people’s memories not only the elderly population but all ages. Although there are presently no medical treatments for this disease, numerous medications promise to delay or drastically diminish the symptoms and harmful consequences it has on the patient.

Many medical research [13,14] have mentioned the impact of the SARS-CoV-2 pandemic on Alzheimer’s disease and increased morbidity and death in infected patients with recent Neuroinflammation. SARS-CoV-2 has accelerated the progression of brain inflammatory neurodegeneration, and elderly people are more prone to poor consequences following SARS-CoV-2 infection due to the immune response and excessive inflammation. It has been observed that societies or communities found themselves in an isolated routine during the SARS-CoV-2 pandemic are already experiencing memory difficulties. Furthermore, some studies have found that communities that were isolated during the SARS-CoV-2 pandemic are at an increased risk for Alzheimer’s Disease, and that this severity may create an exceptionally high-risk profile for certain demographics like Asian Americans, African Americans, and Hispanic Americans [15].

When dealing with an epidemic, it is important to anticipate future, and understand how to control the epidemic’s progress. Hence, numerous academicians from several fields have contributed to the analysis, creation of models, and study of significant COVID-19 prevention strategies. In the past, mathematical models were employed to simulate SARS-CoV-2 epidemics (see for example [16–19]). Even while results from the use of equations of the integer-order have been somewhat successful, results from the use of equations of the fractional-order are still preferable in terms of how closely they relate to reality when they depict real phenomena. Fractional calculus has grown in popularity and importance as a result of its proved applications in system biology and several sectors of science [20]. In fractional calculus, all non-negative order derivatives and integrals are allowed. Fractional derivatives have the advantage of not being a local attribute [21]. Not surprisingly, fractional-order models are commonly used in epidemiology to understand the complexity of infectious diseases. In the realm of mathematical biology, the Caputo–Fabrizio (CF) fractional-order operator has been used over Atangana–Baleanu, beta derivatives, and a few mathematical biology, the Caputo–Fabrizio (CF) fractional-order operator is given over Atangana–Baleanu, beta derivatives, and a few mathematical biology, the Caputo–Fabrizio (CF) fractional-order operator.

The aforementioned literature on the importance of CF fractional derivative on dynamical systems, we formulate a novel model for SARS-CoV-2 pandemic incorporating Alzheimer’s disease. The memory and heredity features are aim of dealing with fractional-order systems in our model, which gives us a more realistic method to biological systems, that help in deal with complicated behavioral patterns of biological systems. The memory function allows fractional order models to incorporate more knowledge from the past, allowing for more accurate prediction and translation. The major goal of this research is to determine the scope of the SARS-CoV-2 epidemic, to forecast what might happen in the future and how to prevent the disease from spreading, and to determine the influence of COVID-19 quarantine on Alzheimer’s disease. Motivated greatly by these and considering the aforementioned papers, there is no single mathematical formulation for SARS-CoV-2 pandemic incorporating Alzheimer’s disease, and this research reveal some critical qualitative information about the disease’s course.

The structure for this paper is as follows: In Section 2, the definitions and some significant properties of the CF fractional operator are given. In Section 3, we described the model in classical and CF fractional order perspective. In Section 4, we determined the classical analysis of our proposed model, thus, positivity, boundedness and invariant region. In Section 5, we discussed the existence and uniqueness results using Banach and Krasnoselskii’s type fixed point theorem. In Section 6, we examined the proposed model stability under HU stability type. In Section 7, by employing Lagrange interpolation, we analyze the numerical solutions of the proposed model. In Section 8, we used the numerical results in Section 9 and run the numerical simulation. Finally, Section 9 sum-up our results and suggestions are given for future studies.

2. Preliminaries

In this section, we recall some critical concepts, lemmas, and definitions to study our proposed model.

Definition 2.1 ([32,40]). Let \( Y \in H^1(a, b) \), \( b > a \), and \( \mu \in (0, 1) \). Then the CF derivative can be defined as

\[
\begin{align*}
\mathcal{D}_\sigma^{\mu} Y(x) &= \frac{G(\mu)}{1 - \mu} \int_a^x (x - s)^{-\mu} G(\sigma, \mu, s) Y(s) \, ds,
\end{align*}
\]

Here, \( G(\mu) \) is a normalization function, where \( G(0) = G(1) = 1 \). The fractional integral of CF is defined by:

\[
\begin{align*}
\mathcal{I}_\sigma^{\mu} Y(x) &= \int_0^x \frac{(1 - \mu)}{(1 - \mu) G(\mu)} Y(s) \, ds + \frac{2 \mu}{(2 - \mu) G(\mu)} \int_0^x Y(s) \, ds \sigma \geq 0.
\end{align*}
\]

Lemma 2.2 ([41,42,48]). Suppose that \( y(x) \in L_2(0, T) \), then the solution of fractional differential equation

\[
\begin{align*}
\mathcal{D}_\sigma^{\mu} Y(x) &= y(x), x \in [0, T],
\end{align*}
\]

is given by

\[
\begin{align*}
Y(x) &= Y_0 + \frac{2(1 - \mu)}{2(1 - \mu) G(\mu)} y(x) + \frac{2 \mu}{2 - \mu} \int_0^x y(s) \, ds \sigma \geq 0.
\end{align*}
\]
disease infected population $E$, at least one solution $t > 0$, where we subdivided into mutually exclusive compartments, which are

3. Model formulation

We consider the entire human population $N$ at time $t$ thus, $N(t)$, where we subdivided into mutually exclusive compartments, which are susceptible humans with moderate risk of Covid-19 and Alzheimer’s disease infection $S_1(t)$, thus young population (<60 years old), susceptible humans with high risk of Covid-19 and Alzheimer’s disease infection $S_2(t)$, thus aged population (≥60 years old), exposed population $E(t)$, asymptotically infected population $I_1(t)$, symptomatically infected population $I_2(t)$, individuals in isolation centers $Q(t)$, recovered population $R(t)$ and individuals who have had a Alzheimer’s disease $A_2(t)$. Thus, $N(t)$, is given by $N(t) = S_1(t) + S_2(t) + E(t) + I_1(t) + I_2(t) + Q(t) + R(t) + A_2(t)$. Table 1 gives detail interpretation of parameters in the model, while we assumed the state variables and parameters to be all positive. Hence, our proposed model is described by the following system of differential equations;

\[
\begin{align*}
\frac{dS_1(t)}{dt} &= aN - (\mu_1 + \gamma_1 + \gamma_2)S_1 - \lambda S_1 - \phi S_1, \\
\frac{dS_2(t)}{dt} &= (1 - a)N + \phi S_1 - \eta \lambda S_2 - (\mu_1 + \gamma_1 + \gamma_2)S_2, \\
\frac{dE(t)}{dt} &= \lambda (S_1 + \eta S_2) - (\mu_1 + \delta_1 + \delta_2)E, \\
\frac{dI_1(t)}{dt} &= \psi E - \delta_1E - (\mu_1 + \xi_1 + \xi_2)I_1, \\
\frac{dI_2(t)}{dt} &= (1 - \psi)E - (\mu_1 + \xi_1 + \xi_2)I_2, \\
\frac{dQ(t)}{dt} &= \gamma_1 (S_1 + S_2) + \theta_1 I_1 + \theta_2 I_2 - (\mu_1 + \mu_2)Q, \\
\frac{dR(t)}{dt} &= \sigma_1 I_1 + \sigma_2 I_2 - (\nu + \mu_1)R, \\
\frac{dA_2(t)}{dt} &= \gamma_2 (S_1 + S_2) + \nu R + \xi_1 I_1 + \xi_2 I_2 + mQ - (\mu_1 + \mu_3)A_2, \\
\end{align*}
\]

For $t > 0$ with the initial conditions $S_1(0) = S_{100}$, $S_2(0) = S_{200}$, $E(0) = E_{00}$, $I_1(0) = I_{100}$, $I_2(0) = I_{200}$, $Q(0) = Q_{00}$, $R(0) = R_{00}$, $A_2(0) = A_{200}$, subject to $\min(S_1, S_2, E, I_1, I_2, Q, R, A_2) \geq 0$, and we assume that dimension of both model sides are the same.

4. Basic dynamics of the model

We examine the dynamics of the positivity, boundedness, and invariance of the proposed model’s solutions in this section. In an epidemic model, it is important to evaluate population survival and the expansion that is naturally constrained by scarce resources. As a result, we demonstrate the following theorems.

Theorem 1. Let $S_1(0) \geq 0$, $S_2(0) \geq 0$, $E(0) \geq 0$, $I_1(0) \geq 0$, $I_2(0) \geq 0$, $Q(0) \geq 0$, $R(0) \geq 0$, $A_2(0) \geq 0$, such that the solution set $\Psi = \{(S_1(t), S_2(t), E(t), I_1(t), I_2(t), Q(t), R(t), A_2(t)) : t \in \mathbb{R}_+\}$, of the proposed model (1) are positive for all $t > 0$. Furthermore

\[
\limsup_{t \to \infty} N(t) \leq \frac{A}{\mu}.
\]
Proof. Let $t_1 = \sup\{S_1(0) \geq 0, S_2(0) \geq 0, E(0) \geq 0, I_A(0) \geq 0, I_S(0) \geq 0, Q(0) \geq 0, R(0) \geq 0, A_P(0) \geq 0\} \in [0, t]$. Let us now consider the model's initial dynamical Eq. (2) where $t > 0$. Keep in mind that the starting values are higher than zero, then,

$$\frac{dS_i}{dt} = aA - (\mu_i + \gamma_1 + \gamma_2)S_i - \lambda S_i - \phi S_i, \quad i = 1, 2.$$

For simplicity, we let $M_1^* = [\mu_1 + \gamma_1 + \gamma_2 + \lambda + \phi]$. Thus, this implies

$$\frac{dP_1}{dt} + M_1^* S_1(t) = aA.$$

This lead to

$$\frac{d}{dt}(S_1(t)\exp\int_0^t M_1^*(s)ds)) = (aA)\exp\int_0^t M_1^*(s)ds).$$

So, we get

$$S_1(t) = S_1(0)\exp\left(-\int_0^t M_1^*(s)ds\right) + \exp\left(-\int_0^t M_1^*(s)ds\right)\int_0^t (aA)\exp\left(\int_0^s M_1^*(s)ds\right)ds.$$

Therefore, we proved that $S_1(0) > 0$ for all $t > 0$. Similarly, we can prove that $S_2(0) > 0, E(0) \geq 0, I_A(0) \geq 0, I_S(0) \geq 0, Q(0) \geq 0, R(0) \geq 0, A_P(0) \geq 0$ for all $t > 0$.

Now, we get the following expression for boundedness $\dot{N} = A - \mu N, E(0) \geq 0, I_A(0) \geq 0, I_S(0) \geq 0, Q(0) \geq 0, R(0) \geq 0, A_P(0) \geq 0$ for all $t > 0$.

Considering invariant, we let $\Phi = \Omega \subset R_x \times R_x$ where

$$\Psi = \{(S_1(t), S_2(t), E(t), I_A(t), I_S(t), Q(t), R(t), A_P(t)) \in R_x^8 : N(t) \leq A\mu\}.$$

Now, for positively invariant of $R^8_x$,

$$\frac{dN}{dt} = A - \mu N,$$

solving this equality we get as follows

$$N(t) \leq N(0)e^{-\mu t} + \frac{A}{\mu}(1 - e^{-\mu t}).$$

Therefore,

$$\limsup_{t \to \infty} N(t) \leq \frac{A}{\mu}.$$
Further, let say $B = C([-\infty, \infty])$ is the Banach space, supposing that the following assumptions hold

$(H_1)$ There exist a nonnegative constant $Q_\epsilon$, $W_\epsilon$, and $k_\epsilon \in [0, 1]$ such that

$$\Phi(t, N(t)) \leq Y_\epsilon |N|k_\epsilon + Z_\epsilon.$$ 

$(H_2)$ There exist a nonnegative constant $C_\epsilon > 0$ for all $N, \bar{N} \in B$, then

$$|\Phi(t, N(t)) - \Phi(t, \bar{N}(t))| \leq C_\epsilon |N - \bar{N}|.$$ 

Also, let us define operator $T_\epsilon : B \to B$, such that

$$T_\epsilon N(t) = M_1^* N(t) + M_2^* N(t).$$

basically, we can see that

$$\begin{aligned}
M_1^* N(t) &= N_0(t) + \frac{2(1 - \mu_\epsilon)}{2(1 - \mu_\epsilon)G(\mu_\epsilon)} \Phi(t, N(t)), \\
M_2^* N(t) &= \frac{2\mu_\epsilon}{(2 - \mu_\epsilon)G(\mu_\epsilon)} \int_0^t \Phi(s, N(s)) ds.
\end{aligned}$$

From this knowledge (8) can be written as

$$T_\epsilon N(t) = N_0(t) + \frac{2(1 - \mu_\epsilon)}{2(1 - \mu_\epsilon)G(\mu_\epsilon)} \Phi(t, N(t)) + \frac{2\mu_\epsilon}{(2 - \mu_\epsilon)G(\mu_\epsilon)} \int_0^t \Phi(s, N(s)) ds.$$ 

**Theorem 3.** Suppose that $(H_1)$ and $(H_2)$ hold, such that

$$\frac{2(1 - \mu_\epsilon)}{2(1 - \mu_\epsilon)G(\mu_\epsilon)} C_\epsilon < 1,$$ 

then the SARS-CoV-2 transmission model incorporate Alzheimer’s disease has at least one solution.

**Proof.** For simplicity, we divide the proof into two steps.

Step 1. We proof that operator $M_1^*$ is contraction. Then, let $\bar{N} \in B$, where $B = \{N \in W_\epsilon : \|N\| \leq \phi, \phi > 0\}$ is a close convex set, thus

$$\begin{aligned}
|M_1^* N(t) - M_1^* \bar{N}(t)| &= \frac{2(1 - \mu_\epsilon)}{2(1 - \mu_\epsilon)G(\mu_\epsilon)} \max_{\mu_\epsilon \in [0, 1]} |\Phi(t, N(t)) - \Phi(\sigma, \bar{N}(\sigma))|, \\
&\leq \frac{2(1 - \mu_\epsilon)}{2(1 - \mu_\epsilon)G(\mu_\epsilon)} C_\epsilon \|N - \bar{N}\|.
\end{aligned}$$

Thus,

$$\|M_1^* N(t) - M_1^* \bar{N}(t)\| \leq \frac{2(1 - \mu_\epsilon)}{2(1 - \mu_\epsilon)G(\mu_\epsilon)} C_\epsilon \|N - \bar{N}\|.$$ 

Hence $M_1^*$ is contraction since

$$\frac{2(1 - \mu_\epsilon)}{2(1 - \mu_\epsilon)G(\mu_\epsilon)} C_\epsilon < 1.$$ 

Step 2. We proof that $M_2^*$ is compact and also continuous, for all $N \in B$, then $M_2^*$ will be continuous as $N$ is continuous, thus

$$\|M_2^* N(t)\| = \max_{t \in [0, 1]} \frac{2\mu_\epsilon}{(2 - \mu_\epsilon)G(\mu_\epsilon)} \int_0^t |\Phi(s, N(s))| ds.$$ 

$$\leq \frac{2\mu_\epsilon}{(2 - \mu_\epsilon)G(\mu_\epsilon)} \int_0^t |\Phi(s, N(s))| ds.$$ 

$$\leq \frac{2\mu_\epsilon}{(2 - \mu_\epsilon)G(\mu_\epsilon)} |Y_\epsilon| K^{k_\epsilon} + Z_\epsilon.$$ 

Hence $M_2^*$ is boundedness. For equicontinuous, let $t_1, t_2 \in [0, \eta]$, such that

$$\begin{aligned}
|(M_2^* N(t_1)) - (M_2^* N(t_2))| &= \frac{2\mu_\epsilon}{(2 - \mu_\epsilon)G(\mu_\epsilon)} \max_{t \in [0, 1]} \int_0^t |\Phi(s, N(s))| ds \\
&\leq \frac{2\mu_\epsilon}{(2 - \mu_\epsilon)G(\mu_\epsilon)} |Y_\epsilon| K^{k_\epsilon} + Z_\epsilon |t_1 - t_2|.
\end{aligned}$$

As $t_1 \to t_2$, then $|(M_2^* N(t_1)) - (M_2^* N(t_2))| \to 0$ which make operator $M_2^*$ an equicontinuous and compact by Arzela–Ascoli theorem. Therefore by Lemma 2.3 the existence for the SARS-CoV-2 transmission model incorporate Alzheimer’s disease has at least one solution.

**Theorem 4.** Suppose that a nonnegative integer $A_\epsilon > 0$ such that

$$A_\epsilon = \left[ \frac{2(1 - \mu_\epsilon)}{2(1 - \mu_\epsilon)G(\mu_\epsilon)} L_\epsilon + \frac{2\mu_\epsilon}{(2 - \mu_\epsilon)G(\mu_\epsilon)} \eta \epsilon \right] < 1,$$

then operator $T_\epsilon$ has a unique fixed point.

**Proof.** Let $N, \bar{N} \in W_\epsilon$, then we say

$$\|T_\epsilon N - T_\epsilon \bar{N}\| \leq \|M_1^* \bar{N} - M_1^* N\| + \|M_2^* \bar{N} - M_2^* N\|.$$ 

$$\leq \frac{2(1 - \mu_\epsilon)}{2(1 - \mu_\epsilon)G(\mu_\epsilon)} \max_{t \in [0, 1]} |\Phi(t, N(t)) - \Phi(t, \bar{N}(t))| + \frac{2\mu_\epsilon}{(2 - \mu_\epsilon)G(\mu_\epsilon)}$$

$$\times \max_{t \in [0, 1]} \int_0^t |\Phi(s, N(s))| ds - \int_0^t |\Phi(s, \bar{N}(s))| ds.$$ 

$$\leq \frac{2(1 - \mu_\epsilon)}{2(1 - \mu_\epsilon)G(\mu_\epsilon)} C_\epsilon + \frac{2\mu_\epsilon}{(2 - \mu_\epsilon)G(\mu_\epsilon)} \|N - \bar{N}\|.$$ 

$$\leq A_\epsilon \|N - \bar{N}\|.$$ 

Hence, by Banach contraction principle, $T_\epsilon$ has a unique fixed point. Consequently, the SARS-CoV-2 transmission model incorporate Alzheimer’s disease has unique solution.

6. Hyers–Ulam (HU) stability results for the SARS-CoV-2 transmission model incorporate Alzheimer’s disease

Stability is one of crucial component of differential equation. There has been many stability-type concepts that study dynamical systems, the HU stability-type concept has recently used for many epidemiological models, due to the approximation properties in the solutions which reduce the burden of getting exact solutions, for example (see [52]) and references therein. To assess and analyze the SARS-CoV-2 transmission model incorporating Alzheimer’s disease, we apply the concept of HU stability-type to get approximate solution for our proposed model.

**Definition 6.1.** The proposed model is HU stable if for $\delta > 0$ and letting $N \in W_\epsilon$ be any solution of below inequality

$$C_F D_\epsilon^\delta N(t) - \Phi(t, N(t)) \leq \delta, \forall t \in [0, \eta],$$

and with a unique solution $N$ of problem (16) with a positive constant $C_F > 0$, such that,

$$\|N - \bar{N}\| \leq C_F \delta, \forall t \in [0, \eta].$$

**Definition 6.2.** Given a function $\psi \in C(R, R)$, such that $\psi(0) = 0$ for any solution $N$ of (16) and $N$ be unique solution of (16), then

$$\|N - \bar{N}\| \leq \psi(\delta),$$

then the system (16) is generalized HU stable.

**Remark 6.3.** Suppose $\chi(t) \in C([0, \eta], R)$, we say $N \in W_\epsilon$ satisfies inequality (18) suppose that,

(i) $|\chi(t)| \leq \delta, \forall t \in [0, \eta],$

(ii) $C_F D_\epsilon^\delta N(t) = \Phi(t, N(t)) + \chi(t), \forall t \in [0, \eta].$

Now, we consider the resulting perturbation equation of system (16) as follows;

$$C_F D_\epsilon^\delta N(t) = \Phi(t, N(t)) + \chi(t).$$

$$\{N(0) = \bar{N}_0\}$$

The below Lemma is necessary for our proves.
Fig. 1. Numerical trajectory of the CF-fractional order derivative, $\mu^*$ of the model (2).

### Table 2

Parameter values.

| Parameter | Value     | Parameter | Value     |
|-----------|-----------|-----------|-----------|
| $\lambda$ | 0.99      | $\alpha$  | 0.98      |
| $\rho_1$  | 0.000001  | $\rho_2$  | 0.0019    |
| $\gamma_1$| 0.0039    | $\eta$    | 0.82      |
| $\delta_1$| 0.008     | $\tau$    | 0.010     |
| $\delta_2$| 0.0002    | $\sigma_1$| 0.01      |
| $\psi_1$  | 0.0056    | $\psi_2$  | 0.0076    |
| $\psi$    | 0.0002    | $\psi$    | 0.08      |
| $\rho$    | 0.2       | $\phi$    | 0.67      |
| $\sigma_1$| 0.019     | $\sigma_2$| 0.0251    |

### Lemma 6.4.

From Eq. (30), we say the following result hold. Thus,

$$|N(t) - T_p^\phi(t, N(t))| \leq \left[ \frac{2(1 - \mu_1)}{2(1 - \mu_1)G(\mu_1)} + \frac{2\mu_1}{(2 - \mu_1)\Gamma(\mu_1)} \eta \right] \delta.$$

### Proof.

Consider Lemma 6.4, relatively, solution for Eq. (19) is given as;

$$N(t) = N_0 + C^F \int_t^0 \Phi(t, N(s)) + C^F \int_t^0 \chi(s).$$

Now, with the help of (10), we deduce that

$$|N(\sigma) - T_p^\phi(t, N(\sigma))| \leq \left[ \frac{2(1 - \mu_1)}{2(1 - \mu_1)G(\mu_1)} |\chi| \sigma + \frac{2\mu_1}{(2 - \mu_1)\Gamma(\mu_1)} \eta \right] \delta.$$

### Theorem 5.

Suppose that the system (16) is HU stable, if there exist

$$\eta \Gamma(\mu_1) \eta \Gamma(\mu_1) < 1.$$
Fig. 2. Numerical trajectory of the CF-fractional order derivative, \( \mu \), of the model (2).

**Proof.** With the help from Lemma 6.4, let \( N \in W_0^\ast \) be any solution and \( \bar{N} \in W_0^\ast \) be unique solution for considered problem (17), then

\[
\|N(t) - \bar{N}(t)\| \leq \|N(t) - T_p N(t)\| + \|T_p N(t) - T_p \bar{N}(t)\|
\]

\[
\leq \frac{2(1 - \mu_c)}{2(1 - \mu_c)G(\mu_c)} \frac{2\mu_c}{(2 - \mu_c)G(\mu_c)} \eta \delta
\]

\[
+ \frac{2(1 - \mu_c)}{2(1 - \mu_c)G(\mu_c)} \frac{2\mu_c}{(2 - \mu_c)G(\mu_c)} \eta \|N - \bar{N}\| \tag{21}
\]

Thus,

\[
\|N(t) - \bar{N}(t)\| \leq \frac{2(1 - \mu_c)}{2(1 - \mu_c)G(\mu_c)} \frac{2\mu_c}{(2 - \mu_c)G(\mu_c)} \eta \delta
\]

\[
+ \frac{2(1 - \mu_c)}{2(1 - \mu_c)G(\mu_c)} \frac{2\mu_c}{(2 - \mu_c)G(\mu_c)} \eta \|N - \bar{N}\|
\]

\[
\leq \frac{2(1 - \mu_c)}{2(1 - \mu_c)G(\mu_c)} \frac{2\mu_c}{(2 - \mu_c)G(\mu_c)} \eta \delta
\]

Hence, we conclude that, the SARS-CoV-2 transmission model incorporate Alzheimer’s disease has at least one solution. Consequently, it is generalized HU stable. \( \square \)

7. Numerical scheme

In this section, we present the numerical results for SARS-CoV-2 transmission model incorporate Alzheimer’s disease base on the Lagrange interpolation. The Cauchy problem of the CF fractional derivative can be given as;

\[
\frac{d^\mu N(t)}{dt^\mu} = \Phi(t, N(t)). \tag{22}
\]

On the other hand, we can be expressed (22) as

\[
N(t) = N_0 + \int_0^t \frac{(1 - \mu_c)}{G(\mu_c)} \Phi(s, N(s)) + \frac{\mu_c}{G(\mu_c)} \times \int_0^s \Phi(s, N(s))ds. \tag{23}
\]

Taking (23) at the point \( t_{z+1} = (z + 1)h \) and \( t_z = z_h, z = 0, 1, 2, 3, \ldots, \) with \( h \) being the time step, we have
Substituting Eq. (29) into Eq. (28), then the generalize numerical scheme of CF is as follows;

\[\forall x_{n+1} = \forall x_n + \left( \frac{1}{4} \mu_n \frac{G(\mu_n)}{G(\mu)} \right) \Phi(t_n, \forall x_{n+1}) + \frac{\mu_n}{G(\mu)} \times \int_{t_n}^{t_{n+1}} \Phi(s, \forall x_n) ds.\]

Taking (24) and (25) results in

\[\forall x_{n+1} - \forall x_n = \left( \frac{1}{4} \mu_n \frac{G(\mu_n)}{G(\mu)} \right) \left\{ \Phi(t_n, \forall x_{n+1}) - \Phi(t_{n+1}, \forall x_{n+1}) \right\} + \frac{\mu_n}{G(\mu)} \times \int_{t_n}^{t_{n+1}} \Phi(s, \forall x_n) ds.\]

Eq. (26) in two-step Lagrange polynomial gives

\[\forall x_{n+1} = \forall x_n + \left( \frac{1}{4} \mu_n \frac{G(\mu_n)}{G(\mu)} \right) \left\{ \Phi(t_n, \forall x_{n+1}) - \Phi(t_{n+1}, \forall x_{n+1}) \right\} + \frac{\mu_n}{G(\mu)} \times \int_{t_n}^{t_{n+1}} \Phi(s, \forall x_n) ds.\]

The above Eq. (27) leads to

\[\forall x_{n+1} = \forall x_n + \left( \frac{1}{4} \mu_n \frac{G(\mu_n)}{G(\mu)} \right) \left\{ \Phi(t_n, \forall x_{n+1}) - \Phi(t_{n+1}, \forall x_{n+1}) \right\} + \frac{\mu_n}{G(\mu)} \times \int_{t_n}^{t_{n+1}} \frac{\Phi(t_n, \forall x_n)}{h} (s - t_n) ds.\]

Solving the integrals in Eq. (28) yields

\[\int_{t_n}^{t_{n+1}} (s - t_n) ds = \frac{3}{2} h^2,\]

\[\int_{t_n}^{t_{n+1}} (s - t_n) ds = \frac{1}{2} h^3.\]
\[ A_{D_{n+1}} = A_{D_{n}} + \left[ \frac{(1-\mu_{*})}{G(\mu_{*})} + \frac{3h\mu_{*}}{2G(\mu_{*})} \right] \Phi(t_{n}, A_{D_{n}}) - \left[ \frac{(1-\mu_{*})}{G(\mu_{*})} + \frac{h\mu_{*}}{2G(\mu_{*})} \right] \Phi(t_{n-1}, A_{D_{n-1}}). \]  

(38)

8. Numerical simulation and discussion

In this section, we provide the numerical solutions to the CF fractional order model for SARS-CoV-2 involving Alzheimer’s disease, using the Adams–Bashforth method, taken account of the following initial conditions; \( S(0) = 18; S_{2}(0) = 15; E(0) = 10; I_{1}(0) = 8; I_{2}(0) = 10; Q(0) = 15; R(0) = 7; A(t)(0) = 9 \), with parameter values in Table 2. Parameters have a crucial influence in disease propagation in numerical solutions, and which fractional order \( \mu \) indicates the best memory effect. As a result, using the parameter values in Table 2, the trajectory of each compartment through time has been simulated for various values of \( \mu \). Furthermore, graphics for significantly influence parameters have also been obtained. For different values of \( \mu \), with parameter values Figs. 1–4 gives graphical presentation of our proposed SARS-CoV-2 pandemic and Alzheimer’s disease. Figs. 1 and 2 subplots, demonstrates the fall and up in different compartment population for the different \( \mu \) values, and it is worth noting that, the forecast is dependent on the value of the fractional order \( \mu \), implying that the epidemic’s evolution and control are linked to memory. For increasing values of \( \mu \), in Fig. 1(a,b), and (a,c), we observed that susceptible (young age) individuals, susceptible (old age) individuals, symptomatic infected, and exposed individuals all have a declining attitude with no convergency at the end of the simulation time. This behavior in a real-world or biological viewpoint is inevitable as a result of people having no or less knowledge about SARS-CoV-2 associated with Alzheimer’s disease and also the emergence and rapid transmission of the SARS-CoV-2 with different variants coming up, especially Omicron variant. In Fig. 2(b) and (d), we observed increasing behavior of the trajectories as the fractional order \( \mu \) reduce from 1. This implies that a high rate of isolation has significant impact on Alzheimer’s disease, and hence when there is too much quarantine, the rate of infected person end up with Alzheimer’s disease. For different values of \( \gamma_{1} \) and \( \gamma_{2} \), the time-dependent fluctuation of the Alzheimer’s disease class has been explored in Fig. 3. We notice that an increase in \( \gamma_{2} \) increases the number of Alzheimer’s disease individuals, and an increase in \( \gamma_{1} \) has minimal effect on the Alzheimer’s disease compartment. We see in Fig. 4 an increase in \( m \) increases the number of Alzheimer’s disease individuals. Similarly, we see in Fig. 5 an increase in \( \xi_{1} \) and \( \xi_{2} \) increases the number of Alzheimer’s disease individuals.

9. Conclusion

To assess and analyze the SARS-CoV-2 transmission model incorporating Alzheimer’s disease, a mathematical model has been developed and analyzed under CF fractional order derivative. Initial formulation of the model uses a traditional integer order differential system, which is later expanded to include a CF fractional order differential system. We determined the classical analysis of our proposed model, thus, (positivity, boundedness and invariant region). By using the fixed point theorem of Banach and Krasnoselskii type, the proposed system has been proved to have at least one unique solution. Due to complicated nature of SARS-CoV-2 and its effect associated with Alzheimer’s disease, we discussed the proposed model stability under HU stability type to get approximate solution. Numerical analysis and simulations are carried out to check the actual behavior of our proposed model using the Adams–Bashforth technique and the CF fractional order derivative. Base on our numerical findings, if the number of symptomatic and asymptomatic infected individuals in quarantine can be reduced by using techniques such as short-days-quarantine, enforcing nationwide vaccination, rapid antigen testing, improving isolation centers, then it will be possible to control the SARS-CoV-2 and its effect associated with Alzheimer’s disease. In this regard, this paper gives insight to future research possibilities. In the future, because this work is not supported by real data, alternative types of fractional operators can be considered with real data and also optimal control section and sensitivity analysis are encourage.

CRediT authorship contribution statement

Emmanuel Addai: Conceptualization, Methodology, Formal analysis, Software, Visualization, Writing – original draft, Writing – review & editing. Lingling Zhang: Supervision, Methodology, Formal analysis, Visualization, Writing – review & editing. Ama Kyerewaa Preko: Formal analysis, Visualization, Writing – review & editing. Joshua Kiddy K. Asamoah: Supervision, Methodology, Formal analysis, Software, Visualization, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Funding

This paper is supported by Key R&D program of Shanxi Province, PR China (International Cooperation, 201903D421042) and Research Project Supported by Shanxi Scholarship Council of China (2021–030).
References

[1] C.I. Siettos, L. Russo, Mathematical modeling of infectious disease dynamics, Virus Res. (2013) 295–306, http://dx.doi.org/10.1016/j.virus.2014.

[2] World Health Organization AIDS Report, Fact sheets about HIV AIDS, 2021, https://www.who.int/news-room/fact-sheets/detail/hiv-aids.

[3] A. Sigdel, A. Bista, N. Bhattachar, et al., Depression, anxiety and depression-anxiety comorbidity amid covid-19 pandemic: an online survey conducted during lockdown in Nepal, 2020, http://dx.doi.org/10.21046/2020.04.27.20081695, MedRxiv [Preprint].

[4] M.Z. Ahmed, O. Ahmed, Z. Aibao, et al., Epidemic of COVID-19 in China and associated psychological problems, Asian J. Psychiatr. 51 (2020) 102092, http://dx.doi.org/10.1016/j.ajp.2020.102092.

[5] N. Media, S. Pardini, I. Slongo, et al., COVID-19 and depressive symptoms in students before and during lockdown, 2020, http://dx.doi.org/10.1016/j.ajp.2020.102092.

[6] M.A. Khan, A. Atangana, Modeling the dynamics of novel coronavirus (2019-ncoV) with fractional derivative, Alexandr. Eng. J. 59 (4) (2020) 2379–2389, http://dx.doi.org/10.1016/j.ajchao.2020.02.033.

[7] S. Ullah, M.A. Khan, Modeling the impact of non-pharmaceutical interventions on the dynamics of novel coronavirus with optimal control analysis with a case study, Chaos Solitons Fract. 139 (2020) 110075, http://dx.doi.org/10.1016/j.chaos.2020.110075.

[8] M.A. Khan, A. Atangana, E. Alzahrani, et al., The dynamics of COVID-19 with quarantined and isolation, Adv. Differential Equations 2020 (2020) 425, http://dx.doi.org/10.1186/s13662-020-02882-9.

[9] J.K. Djernes, Prevalence and predictors of depression in populations of elderly: a review, Acta Psychiatr. Scand. 113 (2006) 37287, http://dx.doi.org/10.1111/j.1600-0447.2006.00770.x.

[10] D. Li, D. Zhang, J. Shao, et al., A meta-analysis of the prevalence of depressive symptoms in chinese older adults, Arch. Gerontol. Geriatr. 58 (2014) 1–5, http://dx.doi.org/10.1371/journal.pone.0137643.

[11] M.F. Gerlins, R.A. Schoevers, A.T.F. Beekman, et al., Depression and risk of cognitive decline and Alzheimer’s disease, Br. J. Psychiatr. 176 (2000) 568–575, http://dx.doi.org/10.1192/bjp.176.6.568.

[12] N. Sach-Ericson, T. Joiner, E.A. Plant, et al., The influence of depression on cognitive decline in community-dwelling elderly persons, Am. J. Geriatr. Psychiatr. 13 (2005) 4028, http://dx.doi.org/10.1097/00019442-20050500-00009.

[13] S.X. Naughtona, U. Ravala, G.M. Pasinetti, Potential novel role of COVID-19 in Alzheimer’s disease and related dementia, Am J. Geriatric Psych. 28 (7) (2020) 712–721.

[14] S.K. Asamoah, C.S. Bonna, R. Seida, Z. Jin, Mathematical analysis of the effects of controls on transmission dynamics of SARS-CoV-2, Alexandr. Eng. J. 59 (6) (2020) 5069–5078, http://dx.doi.org/10.1016/j.ajae.2020.09.053.

[15] F. Bozkurt, F. Ozkose, Stability analysis of macrophage-tumor interaction with piecewise constant arguments, in: AIP Conference Proceedings, Vol. 1648, AIP Publishing LLC, 2015, 850035.

[16] S. Dewanjee, J. Vallamkondu, et al., A mathematical model of coronavirus disease (COVID-19) containing asymptomatic and symptomatic classes, Results Phys. 21 (2021) 103776, http://dx.doi.org/10.1016/j.rinp.2021.103776.

[17] R. Zarin, A. Yusuf, Analysis of fractional COVID-19 epidemic model under Caputo operator, Math. Methods Appl. Sci. (2021) http://dx.doi.org/10.1002/mma.7924.

[18] A. Atangana, D. Baleanu, New fractional derivatives with nonlocal and non-singular kernel: theory and application to heat transfer model, Therm. Sci. 22 (2) (2016) 763–769, http://dx.doi.org/10.2298/TSCI160111018A.

[19] A. Atangana, J.F. Gomez-Aguilar, A new derivative with normal distribution kernel: Theory, methods and applications, Physica A: Stat. Mech. Appl. 476 (2017) 1–14, http://dx.doi.org/10.1016/j.physa.2017.02.016.

[20] Y. Khan, M.A. Khan, N. Faraz, et al., A fractional bank competition model in Caputo–Fabrizio derivative through newton polynomial approach, Alexand. Eng. J. 60 (1) (2021) 711–718.

[21] J.K.K. Asamoah, E. Okyere, et al., Non-fractional and fractional mathematical analysis and simulations for Q fever, Chaos Solitons Fractals 156 (2022) 111821, http://dx.doi.org/10.1016/j.chaos.2022.111821, 2022.

[22] M. Higazy, M.A. Alyami, New Caputo-Fabrizio fractional order SEIASqEqHR model for COVID-19 epidemic transmission with genetic algorithm based control strategy, Alex. Eng. J. 59 (2020) 4719–4736, http://dx.doi.org/10.1016/j.ajae.2020.08.034.

[23] J.D. Djida, A. Atangana, More generalized groundwater model with space-time Caputo fractional differentiation, Numer. Methods Partial Differ. Equ. 33 (5) (2017) 1616–1627.

[24] M.A. Khan, C. Alfiniyah, E. Alzahrani, et al., Analysis of dengue model with fractal-fractional Caputo-Fabrizio operator, Adv. Difference Equ. 2020 (2020) 1–23.

[25] I.A. Baba, Existence and uniqueness of a fractional order tuberculosis model, Eur. Phys. J. Plus 134 (2019) 489, http://dx.doi.org/10.1140/epjp/i2019-13009-1.

[26] S. Qureshi, A. Yusuf, A.A. Shaikh, M. Inc, D. Baleanu, Mathematical modeling for adsorption process of dye removal nonlinear equation using power law and exponentially decaying kernels, Chaos 30 (2020) http://dx.doi.org/10.1063/1.5121845.

[27] M. Sher, K. Shah, et al., Computational and theoretical modeling of the transmission dynamics of novel COVID-19 under Mittag-Leffler Power Law, Alexandria Eng. J. 59 (2020) 3133–3147, http://dx.doi.org/10.1016/j.aej.2020.07.014.

[28] K.M. Owolabi, A. Atangana, Mathematical modeling and analysis of fractional epidemic models using derivative with exponential kernel, in: Fractional Calculus and Applied Analysis, vol. 22, Special issue on fractional differential equations and applications, Bulgarian Academy of Sciences-Ser. Math., Plovdiv, 2019, pp. 2067–2076.

[29] A. Atangana, J.F. Gomez-Aguilar, A new derivative with normal distribution exponent dynamics of novel COVID-19 under Mittag-Leffler Power Law, Alexandria Eng. J. 59 (2020) 4719–4736, http://dx.doi.org/10.1016/j.ajae.2020.08.034.

[30] J.D. Djida, A. Atangana, More generalized groundwater model with space-time Caputo fractional differentiation, Numer. Methods Partial Differ. Equ. 33 (5) (2017) 1616–1627.

[31] M.A. Khan, C. Alfiniyah, E. Alzahrani, et al., Analysis of dengue model with fractal-fractional Caputo-Fabrizio operator, Adv. Difference Equ. 2020 (2020) 1–23.

[32] I.A. Baba, Existence and uniqueness of a fractional order tuberculosis model, Eur. Phys. J. Plus 134 (2019) 489, http://dx.doi.org/10.1140/epjp/i2019-13009-1.

[33] S. Qureshi, A. Yusuf, A.A. Shaikh, M. Inc, D. Baleanu, Mathematical modeling for adsorption process of dye removal nonlinear equation using power law and exponentially decaying kernels, Chaos 30 (2020) http://dx.doi.org/10.1063/1.5121845.

[34] M. Sher, K. Shah, et al., Computational and theoretical modeling of the transmission dynamics of novel COVID-19 under Mittag-Leffler Power Law, Alexandria Eng. J. 59 (2020) 3133–3147, http://dx.doi.org/10.1016/j.ajae.2020.07.014.

[35] K.M. Owolabi, A. Atangana, Mathematical modeling and analysis of fractional epidemic models using derivative with exponential kernel, in: Fractional Calculus and Applied Analysis, vol. 22, Special issue on fractional differential equations and applications, Bulgarian Academy of Sciences-Ser. Math., Plovdiv, 2019, pp. 2067–2076.
[34] I. Ahmed, I. Baba, et al., Analysis of Caputo fractional-order model for COVID-19 with lockdown, Adv. Differential Equations 2020 (1) (2020) 114, 110007.

[35] A.A. Khan, Z. Hammouch, D. Baleanu, Modeling the dynamics of hepatitis E via the Caputo-fabrizio derivative, Math. Model. Nat. Phenom. 14 (2019) 311, http://dx.doi.org/10.1051/mmnp/2018074.

[36] D. Baleanu, S.M. Aydog, et al., On modelling of epidemic childhood diseases with the Caputo-Fabrizio derivative by using the Laplace Adomian decomposition method, Alex. Eng. J. 59 (2020) 3029–3039, http://dx.doi.org/10.1016/j.aej.2020.05.007.

[37] R. Zarin, A. Khan, Fractional-order dynamics of Chagas-HIV epidemic model with different fractional operators, AIMS Math. 7 (10) 18897–18924, http://dx.doi.org/10.3934/math.2022104.

[38] J.P. Olumuyiwa, Y. Abdullahi, K. Oshinubi, et al., Fractional order of pneumococcal pneumonia infection model with Caputo Fabrizio operator, Results Phys. 29 (2021) 104581, http://dx.doi.org/10.1016/j.rinp.2021.104581.

[39] M.A. Dokuyucu, A fractional order alcoholism model via Caputo–Fabrizio derivative, AIMS Math. 5 (2) 781–797, http://dx.doi.org/10.3934/math.2020053.

[40] V. SinghPanwar, P.S.S. Uduman, J.F. Gómez-Aguilar, Mathematical modeling of coronavirus disease COVID-19 dynamics using CF and ABC non-singular fractional derivatives, Chaos Solitons Fractals 145 (2021) 110451.

[41] P. Pandey, J.F. Gómez-Aguilar, M.K.A. Kaabar, et al., Mathematical modeling of COVID-19 pandemic in India using Caputo–Fabrizio fractional derivative, Comput. Biol. Med. 145 (2022) 105518, http://dx.doi.org/10.1016/j.compbiomed.2022.105518.

[42] P. Verma, J.F. Gómez-Aguilar, -NCoV) system with variable Caputo–Fabrizio fractional order, Chaos. Solitons & Fractals 142 (2020) (2019) 110451.

[43] S. Ullah, M.A. Khan, et al., A fractional model for the dynamics of tuberculosis infection using Caputo-Fabrizio derivative, Discrete Cont. Dyn. Syst. 13 (2020) 975–993, http://dx.doi.org/10.3934/dcdss.2020057.

[44] H. Abboubakar, P. Kumar, N.A. Rangaig, S. Kumar, A malaria model with Caputo–Fabrizio and Atangana-Baleanu derivatives, Int. J. Mod. Simul. Sci. Comput. 12 (2021) 2150013, http://dx.doi.org/10.1142/S1793962221500136.

[45] H. Abboubakar, P. Kumar, N.A. Rangaig, S. Kumar, A malaria model with Caputo-Fabrizio and Atangana-Baleanu derivatives, Int. J. Mod. Simul. Sci. Comput. 12 (2021) 2150013, http://dx.doi.org/10.1142/S1793962221500136.

[46] E. Bonyah, M. Juga, et al., Fractional dynamics of coronavirus with comorbidity via Caputo–Fabrizio derivative, Commun. Math. Biol. Neurosci. 2022 (2022) Article-ID.

[47] J.O. Peter, Transmission dynamics of fractional order brucellosis model using Caputo–Fabrizio operator, Int. J. Differ. Eqs. (2020) 1–11, http://dx.doi.org/10.1155/2020/2791380, 2791380.

[48] L.L. Zhang, E. Addai, et al., Fractional-order Ebola-Malaria coinfection model with a focus on detection and treatment rate, Comput. Math. Methods Med. (2022) 6502598, http://dx.doi.org/10.1155/2022/6502598.

[49] M.Z. Ullah, A.K. Atahran, D. Baleanu, An efficient numerical technique for a new fractional tuberculosis model with nonsingular derivative operator, J. Taibah Univ. Sci. 13 (1) (2019) 1147–1157.

[50] G.T. Tilahun, W.A. Woldegerimab, N. Mohammed, A fractional order model for the transmission dynamics of hepatitis B virus with two-age structure in the presence of vaccination, Arab J. Basic App. Sci. 28 (1) (2021) 87–106, http://dx.doi.org/10.1080/25765299.2021.1896423.

[51] W. Lin, Global existence theory and chaos control of fractional differential equations, J. Math. Anal. Appl. 332 (2007) 709–726, http://dx.doi.org/10.1016/j.jmaa.2006.10.040.

[52] E. Addai, L.L. Zhang, J. Ackora-Prah, et al., Fractal-fractional order dynamics and numerical simulations of a Zika epidemic model with insecticide-treated nets, Physica A (127809) (2022) http://dx.doi.org/10.1016/j.physa.2022.127809.