A SIMPLE APPROACH FOR THE SOLUTION OF FUZZY MULTI OBJECTIVE TRAVELLING SALESMAN PROBLEM

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Abstract We propose a simple method for solving multi objective travelling salesman problem whose decision parameters are expressed as triangular fuzzy number. We apply arithmetic operation and ranking for the parametric form of triangular fuzzy number. The proposed gives preference solution to the decision maker without change to classical travelling salesman problem. A numerical example is provided to show the efficiency of the propose method.

Key words: Travelling salesman problem, fuzzy number, fuzzy arithmetic, fuzzy ranking.

1. Introduction
A sales rep regularly should visit various urban communities beginning from his base camp. The separation, time, cost between each pair of urban communities are thought to be known. The issue of finding the most limited separation, least time and least cost if the sales rep begins from his base camp and goes through every city under his locale precisely once and come back to the Headquarters. In this paper, we have utilized Hungarian to determine triangular fuzzy number without changing over them to traditional triangular fuzzy number issue so that shipping cost, time, distance is minimum Bellman and Zadeh\cite{2} first proposed by the concept of a fuzzy decision making.

2. Preliminaries

\textbf{Definition 2.1.} A fuzzy set \( \tilde{A} \) defined on \( X \) is a collection of ordered pair

\[ \tilde{A} = \{(x, \mu_{\tilde{A}}(x)): x \in X\} \] where \( \mu_{\tilde{A}}(x) \) is a membership value of \( X \)

\textbf{Definition 2.2.} A fuzzy set \( \tilde{a} \) defined on the set of real numbers \( \mathbb{R} \) is said to be a fuzzy number if its membership function \( \tilde{a}: \mathbb{R} \to [0,1] \) has the following characteristics

(i) \( \tilde{a} \) is convex
(ii) \( \tilde{a} \) is normal
(iii) \( \tilde{a} \) is Piecewise continuous.

\textbf{Definition 2.3.} A fuzzy number \( \tilde{a} \) on \( \mathbb{R} \) is said to be a triangular fuzzy number (TFN) or linear fuzzy number if its membership function \( \tilde{a}: \mathbb{R} \to [0,1] \) has the following characteristics:

...
We denote this triangular fuzzy number by \( \bar{a} = (a_1, a_2, a_3) \). We use \( F(R) \) to denote the set of all triangular fuzzy numbers. Also if \( m = a_2 \), represents the modal value or midpoint, \( \alpha = (a_2 - a_1) \) represents the left spread and \( \beta = (a_3 - a_2) \) represents the right spread of the triangular fuzzy number \( a = (a_1, a_2, a_3) \), then the triangular fuzzy number \( a \) can be represented by the triplet \( \bar{a} = (\alpha, m, \beta) \). i.e. \( \bar{a} = (a_1, a_2, a_3) = (\alpha, m, \beta) \). We use \( F(R) \) to denote the set of all triangular fuzzy numbers.

### 2.1 Ranking of Triangular Fuzzy Numbers

Many different approaches for the ranking of fuzzy numbers have been proposed in the literature. Abbasbandy and Hajjari [1] proposed a new ranking method based on the left and the right spreads at some \( \alpha \)-levels of fuzzy numbers.

For an arbitrary triangular fuzzy number \( \bar{a} = (a_1, a_2, a_3) = (a_0, a_+, a^*) \) with parametric form \( \bar{a} = (a(r), a(r)) \), we define the magnitude of the triangular fuzzy number \( \bar{a} \) by

\[
\text{Mag}(\bar{a}) = \frac{1}{2} \int_{0}^{1} \left( (a) + a + a_{0} \right) f(r) \, dr
\]

\[
= \frac{1}{2} \int_{0}^{1} \left( a^* + 4a_{0} - a_{+} \right) f(r) \, dr.
\]

where the function \( f(r) \) is a non-negative and increasing function on \([0,1]\) with \( f(0)=0 \), \( f(1)=1 \) and \( \int_{0}^{1} f(r) \, dr = \frac{1}{2} \). The function \( f(r) \) can be considered as a weighting function. In real life applications, \( f(r) \) can be chosen by the decision maker according to the situation. In this paper, for convenience we use \( f(r) = r \).

For any two triangular fuzzy numbers \( \bar{a} = (a_0, a_+, a^*) \) and \( \bar{b} = (b_0, b_+, b^*) \) in \( F(R) \), we define the ranking of \( \bar{a} \) and \( \bar{b} \) by comparing the \( \text{Mag}(\bar{a}) \) and \( \text{Mag}(\bar{b}) \) on \( R \) as follows:

(i) \( \bar{a} \geq \bar{b} \) if and only if \( \text{Mag}(\bar{a}) \geq \text{Mag}(\bar{b}) \)

(ii) \( \bar{a} \leq \bar{b} \) if and only if \( \text{Mag}(\bar{a}) \leq \text{Mag}(\bar{b}) \)

(iii) \( \bar{a} \approx \bar{b} \) if and only if \( \text{Mag}(\bar{a}) = \text{Mag}(\bar{b}) \)

### 2.2 Arithmetic operation on triangular Fuzzy Numbers

Ming Ma et al. [7] have proposed a new fuzzy arithmetic based upon both location index and fuzziness index functions. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice \( L \). That is for \( a, b \in L \) we define \( a \lor b = \max \{a, b\} \) and \( a \land b = \min \{a, b\} \).
For arbitrary triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$ and $* = \{+, -, \times, \div\}$, the arithmetic operations on the triangular fuzzy numbers are defined by $\tilde{a} * \tilde{b} = (a_0 * b_0, a_* \lor b_*, a^* \lor b^*)$.

In particular for any two triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$, we define

(i) Addition: $\tilde{a} + \tilde{b} = (a_0, a_*, a^*) + (b_0, b_*, b^*) = (a_0 + b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$

(ii) Subtraction: $\tilde{a} - \tilde{b} = (a_0, a_*, a^*) - (b_0, b_*, b^*) = (a_0 - b_0, \min\{a_*, b_*\}, \min\{a^*, b^*\})$

(iii) Multiplication: $\tilde{a} \times \tilde{b} = (a_0, a_*, a^*) \times (b_0, b_*, b^*) = (a_0 \times b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$

(iv) Division: $\tilde{a} / \tilde{b} = (a_0, a_*, a^*) / (b_0, b_*, b^*) = (a_0 / b_0, \min\{a_*, b_*\}, \min\{a^*, b^*\})$

3. Main result

Fuzzy Multi objective linear programming problem with triangular fuzzy number

A fuzzy multi objective linear programming problem involving triangular fuzzy numbers can be defined as follows:

$$\max \tilde{Z} = (\tilde{Z}_1, \tilde{Z}_2, ..., \tilde{Z}_n)^T$$

where $\tilde{Z}_1 = \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_j$, $\tilde{Z}_2 = \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_j$, ..., $\tilde{Z}_n = \sum_{j=1}^{n} \tilde{c}_{nj} \tilde{x}_j$

subject to $\sum_{j=1}^{n} \tilde{a}_{ij} \tilde{x}_j \leq \tilde{b}_i$ for all $i = 1, 2, ..., m$

and $\sum_{j=1}^{n} \tilde{a}_{ij} \tilde{x}_j \geq \tilde{b}_i$ for all $i = m+1, ..., n$ (2)

Practically, it is difficult to reach fuzzy optima for all objective function subject to the given constraints in problem (2) and (3).
3.1. Algorithm to solve travelling salesman problem

Step 1: Express the given multi objective travelling salesman problem in parametric form.
Step 2: Convert the multi objective travelling salesman problem into a single objective travelling salesman problem by giving suitable weight to the objectives.
Step 3: Solve the reduced single objective travelling salesman problem without converting to Crisp form.
Step 4: Optimal solution of the single objective travelling salesman problem is also optimal solution to multi objective travelling salesman problem

4. Numerical example

A travelling salesman has to visit 4 cities in the following multi objective fuzzy travelling salesman problem. He wishes to start from a particular city, visit each city once and then return to his starting point the following matrix display the fuzzy travelling cost matrix our aim is to minimized the cost, time, distance.

Table 1: Cost matrix of the multi objective travelling salesman problem involving triangular fuzzy numbers

|     | A       | B       | C       | D       |
|-----|---------|---------|---------|---------|
| A   | ∞       | (18,20,22) | (14,15,16) | (7,9,11) |
|     | (4,5,6) | (6,5,4)  | (3,5,7)  | (1,3,5)  |
|     | (2,4,6) | (3,5,7)  | (2,1,3)  | (2,3,5)  |
| B   | (18,20,22) | ∞       | (29,30,31) | (8,10,12) |
|     | (3,5,7)  | (2,5,8)  | (2,3,4)  | (2,3,4)  |
|     | (3,4,5)  | (2,3,4)  | (1,3,5)  | (1,3,5)  |
| C   | (13,15,17) | (28,30,32) | ∞       | (18,20,22) |
|     | (3,5,7)  | (2,3,4)  | (18,20,22) | (8,10,12) |
|     | (4,5,6)  | (2,3,4)  | (1,2,3)  | (2,3,4)  |
| D   | (9,11,13) | (7,10,13) | (19,20,21) | ∞       |
|     | (2,3,4)  | (1,3,5)  | (9,10,11) |          |
|     | (1,2,3)  | (2,3,4)  | (1,2,3)  |          |

Table 2: Cost matrix in which triangular fuzzy numbers are expressed in their parametric form

|     | A       | B       | C       | D       |
|-----|---------|---------|---------|---------|
| A   | ∞       | (20,2-2r,2-2r) | (15,1-1r,1-1r) | (9,2-2r,2-2r) |
|     |         | (5,1-1r,1-1r) | (5,1-1r,1-1r) | (3,2-2r,2-2r) |
|     |         | (4,2-2r,2-2r) | (5,2-2r,2-2r) | (2,1-1r,1-1r) |
| B   | (20,2-2r,2-2r) | ∞       | (30,1-1r,1-1r) | (10,2-2r,2-2r) |
|     | (5,2-2r,2-2r) |         | (5,3-3r,3-3r) | (3,2-2r,2-2r) |
|     | (4,1-1r,1-1r) |         | (3,1-1r,1-1r) |          |
| C   | (15,2-2r,2-2r) | (30,2-2r,2-2r) | ∞       | (20,2-2r,2-2r) |
|     | (5,2-2r,2-2r) | (5,1-1r,1-1r) |         | (20,2-2r,2-2r) |
|     | (5,1-1r)    | (5,1-1r)   |         | (20,2-2r,2-2r) |
| D   | (11,2-2r,2-2r) | (10,3-3r,3-3r) | (20,1-1r,1-1r) | ∞       |
|     | (3,1-1r,1-1r) | (3,2-2r,2-2r) | (10,1-1r,1-1r) |          |
|     | (2,1-1r,1-1r) | (3,1-1r,1-1r) | (2,1-1r,1-1r) |          |
Table 3: Cost matrix in which triangular fuzzy numbers are given preferred weights

|     | A                | B                         | C                         | D                |
|-----|------------------|---------------------------|---------------------------|------------------|
| A   | ∞                | (10,1-r,1-r)              | (15,0,3-0.3r,0.3-0.3r)    | (4.5,1-r,1-r)    |
|     |                  | (1.5,0.3-0.3r,1-0.3r)     | (1.5,0.3-0.3r,0.3-0.3r)   |                  |
|     |                  | (0.8,0.4-0.4r,0.4-0.4r)   | (1.0,0.4-0.4r,0.4-0.4r)   |                  |
| B   | (10,1-r,1-r)    | ∞                         | (5,1-0.5r,0.5-0.5r)       |                  |
|     | (1.5,0.6-0.6r,0.6-0.6r) |              | (1.5,0.9-0.9r,0.9-0.9r)   |                  |
|     | (0.8,0.2-0.2r,0.2-0.2r) |              | (0.6,0.2-0.2r,0.2-0.2r)   |                  |
| C   | (7.5,1-r,1-r)   | (15,1-r,1-r)              | ∞                         |                  |
|     | (1.5,0.6-0.6r,0.6-0.6r) |              | (3,0.6-0.6r,0.6-0.6r)     |                  |
|     | (1.0,0.2-0.2r,0.2-0.2r) |              | (0.4,0.2-0.2r,0.2-0.2r)   |                  |
| D   | (5,1-r,1-r)     | (5,1-1.5r,1.5-1.5r)       | (10,0.5-0.5r,0.5-0.5r)    | ∞                |
|     | (0.9,0.3-0.3r,0.3-0.3r) |              | (3,0.3-0.3r,0.3-0.3r)     |                  |
|     | (0.4,0.2-0.2r,0.2-0.2r) |              | (0.4,0.2-0.2r,0.2-0.2r)   |                  |

Table 4: Multi objective travelling salesman problem is converted to a single objective travelling salesman problem

|     | A                | B                         | C                         | D                |
|-----|------------------|---------------------------|---------------------------|------------------|
| A   | ∞                | (12.3,1-r,1-r)            | (10,0.5-0.5r,0.5-0.5r)    | (5.8,1-r,1-r)    |
| B   | (12.3,1-r,1-r)  | ∞                         | (17,1-0.9r,0.9-0.9r)      | (6.5,1-r,1-r)    |
| C   | (10,1-r,1-r)    | (17,1-1-r,1-r)            | ∞                         | (13,4,1-r,1-r)   |
| D   | (6.8,1-r,1-r)   | (6.5,1-1.5r,1.5-1.5r)     | (13,4,0.5-0.5r,0.5-0.5r)  | ∞                |

Table 5: Row reduced Cost matrix

|     | A                | B                         | C                         | D                |
|-----|------------------|---------------------------|---------------------------|------------------|
| A   | ∞                | (6.5)                     | (4.2,-0.5+0.5r,-0.5+0.5r) | 0                |
| B   | (5.8)            | ∞                         | (10.6,-0.1+0.1r,-0.1+0.1r)| 0                |
| C   | 0                | (7.1)                     | ∞                         | (3.4)            |
| D   | (0.3,-0.5+0.5r,-0.5+0.5r) | 0                      | (6.9,-1+r,-1+r)          | ∞                |
**Table 6:** Column reduced Cost matrix

|     | A        | B         | C               | D               |
|-----|----------|-----------|-----------------|-----------------|
| A   | ∞        | (6.5)     | 0               | 0               |
| B   | (5.8)    | ∞         | (6.4, 0.4-0.4r, 0.4-0.4r) | 0               |
| C   | 0        | (7.1)     | ∞               | (3.4)           |
| D   | (0.3,-0.5+0.5r,-0.5+0.5r) | 0         | (2.7,-0.5+0.5r,-0.5+0.5r) | ∞               |

The Optimum solution is **A → D → B → C → A**

∴  Fuzzy optimal distance = (3.2-2r,2-2r)+(5.3-3r,3-3r)+(5.2-2r,2-2r)+(3.2-2r,2-2r) = (16.3-3r,3-3r)

Fuzzy optimal cost = (9.2-2r,2-2r)+(30,1-1-r)+(15.2-2r,2-2r)+(10.3-3r,3-3r) = (64.3-3r,3-3r)

Fuzzy optimal time = (2.1-1-r)+(3.1-1-r)+(5.1-1-r)+(3.1-1-r) = (13.1-1-r)

| The value of r | The optimal distance | Optimal cost | Optimal time |
|---------------|----------------------|--------------|--------------|
| r=0           | (13,16,19)           | (61,64,67)   | (12,13,14)   |
| r=0.5         | (14,16,17)           | (62,64,65)   | (12,13,13)   |
| r=1           | (16,16,16)           | (64,64,64)   | (13,13,13)   |

5. Conclusion

In the Travelling Salesman problem all the parameters are triangular fuzzy numbers. In this paper, we have reduced the multi objective travelling salesman problem into a single objective travelling salesman problem by giving suitable weights for the decision parameters. We have solved the reduced single objective travelling salesman problem involving triangular fuzzy numbers without converting to classical travelling salesman problem so that the distance, cost and time is minimum. The proposed method provide an optimal solution which is sharper than the solution obtained by other methods. It is to be seen that the Decision maker have the versatility of picking r∈[0,1] depending on the condition and his craving by applying the proposed technique.

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