Nonspectator effects in $B \to K^{*}\gamma$ within the vector quark model.

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Abstract

The tree level FCNC due to the presence of an additional generation of vector quarks result in the leading order nonspectator contributions to rare $B \to K^{*}\gamma$ decay mode. These tree level contributions are sensitive only to $b \to s$ nonunitary parameter $U_{sb}^a$ and therefore, provide a direct constraint on this model parameter. We obtain the isospin asymmetry between $\bar{B}^0 \to \bar{K}^{*0}\gamma$ and $B^- \to K^{*-}\gamma$ to be $\Delta_{0^-} = -0.03 \times \Re(U_{sb}^a V_{tb} V_{ts}^*)$ and the direct CP asymmetry between $B^+ \to K^{*+}\gamma$ and $B^- \to K^{*-}\gamma$ to be $A_{CP}^{VQM} = 0.27 \frac{|U_{sb}^a|}{|V_{tb} V_{ts}^*|} \sin \theta \sin \phi_s$, where $\theta$ is the weak phase of $U_{sb}^a$ and $\phi_s$ is the strong phase of decay amplitude. We predict a direct CP asymmetry of around a few percent if the current experimental difference between $\Delta_{0^-}$ and $\Delta_{0^+}$ is to be explained by the presence of the additional vector quarks.

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I. INTRODUCTION

The precision measurement of the radiative decay mode $B \to K^*\gamma$ has provided an exciting opportunity to test the Standard Model (SM) and beyond. Besides the branching ratio, the isospin asymmetry in this process which is defined as:

$$\Delta_0^- = \frac{\Gamma(B^0 \to \bar{K}^*0\gamma) - \Gamma(B^- \to K^+\gamma)}{\Gamma(B^0 \to \bar{K}^*0\gamma) + \Gamma(B^- \to K^+\gamma)},$$

(1)
could prove to be an important observable for examining the SM as well as discriminating between various new physics scenarios. The data from Belle[1] and Babar[2] point to isospin asymmetries of at most a few percent and consistent with zero within the experimental error:

$$\Delta_0^- = +0.051 \pm 0.044\text{(stat.)} \pm 0.023\text{(sys.)} \pm 0.024(R^{+/-}) \text{ (Babar)},$$

(2)

$$\Delta_0^+ = +0.012 \pm 0.044\text{(stat.)} \pm 0.026\text{(sys.)} \text{ (Belle)},$$

(3)

where $\Delta_0^+$ is defined as in eq.(1) but using the charge conjugate modes. The last error in eq.(2) is due to the uncertainty in the ratio of the branching fractions of the neutral and charged B meson production in $\Upsilon(4S)$ decays. This asymmetry is due to the non-spectator contributions and has been estimated to be around a few percent in the SM within the QCD factorization approach in Refs. [3] and [4], Brodsky-Lepage formalism [5] and the perturbative QCD method in Ref. [6]. The more accurate measurement of the isospin asymmetry in the near future and a better understanding of the SM prediction for this observable should provide a sensitive testing venue for possible models of new physics. One such model is the extension of the SM with an extra generation of iso-singlet quarks[7]. Unlike the three generations of ordinary quarks in the SM, both the left- and the right-handed components of the quarks of this additional generation are invariant under $SU(2)_L$ gauge group. Therefore, the flavor changing weak interactions of these exotic quarks proceeds only through mixing with ordinary quarks and this results in the non-unitarity of the extended $4 \times 4$ quark mixing matrix and thus non-vanishing flavor changing neutral currents (FCNC) at the tree level. Isospin asymmetry in $B \to K^*\gamma$ transitions offers an excellent physical observable for constraining the parameters of this so-called vector quark model (VQM). As is shown in our result, nonspectator effects like the isospin and direct CP asymmetry in $B \to K^*\gamma$ offer the advantage of being sensitive to only one model parameter, namely the non-unitarity parameter $U_{sb}^{U}$ and therefore, can provide a good constraint on the size of the FCNC in the context of VQM irrespective of the masses of the additional quarks.
II. ISOINSPIN SYMMETRY BREAKING IN $B \to K^* \gamma$

The non-vanishing FCNC at the tree level leads to an additional contributing Feynmann diagram which is illustrated in Fig. II. The amplitude for $b\bar{q} \to s\bar{q}$ transition via $Z^0$ exchange in the VQM can be written as [7]:

$$A^{VQM} = \frac{ig}{2 \cos(\theta)} \left( -\frac{1}{2} U^{sb} \right) \bar{s}\gamma^\mu(1 - \gamma_5)b \times \frac{1}{M_Z^2}$$

$$\frac{ig}{2 \cos(\theta)} \left[ (I^q_W - Q_q\sin^2(\theta)\bar{q}\gamma^\mu(1 - \gamma_5)q - Q_q\sin^2(\theta)\bar{q}\gamma^\mu(1 + \gamma_5)q) \right] \ , \ (4)$$

where $U^{sb} = (V^\dagger V)_{sb}$ is a measure of the non-unitarity of the extended quark mixing matrix and $I^q_W$ is the third component of the weak isospin of quark flavor $q$. One can then write (4) in terms of the effective operators $O_3$ and $O_5$ which are defined as:

$$O_3 = \bar{s}_a\gamma^\mu(1 - \gamma_5)b_\alpha\bar{q}_\beta\gamma^\mu(1 - \gamma_5)q_\beta \ ,$$

$$O_5 = \bar{s}_a\gamma^\mu(1 - \gamma_5)b_\alpha\bar{q}_\beta\gamma^\mu(1 + \gamma_5)q_\beta \ . \ (5)$$

and therefore the contribution of the extra vector quarks results in additional terms in the Wilson coefficients $C_3$ and $C_5$ to the leading order in the strong coupling $\alpha_s$.

$$C^{VQM}_3 = \frac{U^{sb}}{V_{tb}V_{ts}^\ast}(I^q_W - Q_q\sin^2(\theta)) = \frac{U^{sb}}{V_{tb}V_{ts}^\ast} \begin{cases} 1/2 - 2/3\sin^2(\theta) = 0.35 \ldots q = up \\ -1/2 + 1/3\sin^2(\theta) = -0.42 \ldots q = down \end{cases} \ ,$$

$$C^{VQM}_5 = -\frac{U^{sb}}{V_{tb}V_{ts}^\ast} Q_q\sin^2(\theta) = \frac{U^{sb}}{V_{tb}V_{ts}^\ast} \begin{cases} -2/3\sin^2(\theta) = -0.15 \ldots q = up \\ 1/3\sin^2(\theta) = 0.08 \ldots q = down \end{cases} \ . \ (6)$$

With the upper bound $|U^{sb}| \lesssim 10^{-3}$ coming from the rare B decays [7], the additional contribution due to the tree level FCNC could be comparable to the SM value of these coefficients at $\mu = m_b$, i.e. $C_3 = 0.014$ and $C_5 = -0.041$.

Here an explanation is in order. Strictly speaking, one should include the extra terms given in eq. (6), which are proportional to the electric charge of the light quark, in the electroweak penguin operators $O_{7..10}$ [8]. However, since, as far as nonspectator effects to the leading order of $\alpha_s$ are concerned, one can ignore these operators within SM, we prefer to write the additional VQM-generated contributions in terms of the dominant QCD penguin operators. In any case, our results do not change had we followed the strict formulation of the problem.

Following the method of Ref. [3], one can write the nonspectator contributions to $B \to K^* \gamma$ amplitude as $A_q = b_q A_{lead}$, where $q$ is the flavor of the light anti-quark in the
B meson and $A_{lead}$ is the leading spectator amplitude. To leading order in the strong coupling constant $\alpha_s$, the main contribution to $B \to K^*\gamma$ is from the electromagnetic penguin operator $O_7$ and the factorizable amplitude $A_{lead}$ is proportional to the form factor $T_1^{B\to K^*}$ which parameterizes the hadronic matrix element of this operator to the leading order in $\Lambda_{QCD}/m_b$. $b_q$ is the parameter that depends on the flavor of the spectator and, in fact, this parameterization leads to a simple expression for the isospin asymmetry (as defined by eq. (1)) in terms of $b_q$:

$$\Delta_{0-} = \Re(b_d - b_u) \quad ,$$

Using the expression for $b_q$ which is derived within the QCD factorization method in Ref [3], we obtain the contribution of vector quarks to the isospin asymmetry as follows:

$$\Delta_{0-}^{VQM} = \Re \left( \frac{4\pi^2 f_B}{N_c m_b T_1^{B\to K^*} a_7^c} \frac{U_{tb}^{\ast} V_{ts}}{V_{tb} V_{ts}} \left[ -0.22 \frac{f_{K^*}^{\perp} F_{K^*}^{\perp}}{m_b} - 0.28 \frac{f_{K^*}^{\perp} m_{K^*}}{6 \lambda_B m_B} \right] \right) \quad .$$

The numerical input for the parameters of eq. (8) are tabulated in Table II which results in an isospin asymmetry due to the extra generation of quarks of the form:

$$\Delta_{0-}^{VQM} = -0.08 \left| \frac{U_{tb}^{\ast} V_{ts}}{V_{tb} V_{ts}} \right| \cos (\theta + \phi_s) \quad .$$

In the above formula, $\theta$ is the weak phase (CP odd) of the ratio $U_{tb}^{\ast} V_{ts}$ in the extended $4 \times 4$ quark mixing matrix. In a particular parametrization of the mixing matrix where $V_{tb}$ and $V_{ts}$ are taken to be real as in SM, $\theta$ is the phase of the nonunitarity parameter $U_{tb}^{\ast}$. On the other hand, $\phi_s$ is the strong phase (CP even) entering in eq. (8). For example, the imaginary part of the effective Wilson coefficient $a_7^c$ can be one possible source of this latter phase. We would like to point out that the extra contribution due to vector-like quarks to $\Delta_{0+}$, i.e.

$$\Delta_{0+}^{VQM} = -0.08 \left| \frac{U_{tb}^{\ast} V_{ts}}{V_{tb} V_{ts}} \right| \cos (\theta - \phi_s) \quad ,$$

is expected to be different from eq. (9) if $\phi_s$ is appreciable. It is interesting to see if the difference between $\Delta_{0-}$ and $\Delta_{0+}$, as reflected in eqs. (2) and (3), will persist in the future measurements of the isospin asymmetry. One could explain this within the vector quark model if $|U_{tb}^{\ast}|$ happens to be around its upper allowed limit, i.e. a few times $10^{-3}$, with $\theta \sim \phi_s \sim \pi/4$. In case that the strong phase $\phi_s$ is negligible (clearly this is the case if the phase of $a_7^c$ is the only CP even phase in this transition), the isospin asymmetry due to an extra generation of vector-like quarks is sensitive only to the magnitude and phase of
TABLE I: The numerical values of the parameters in eqn. (8).

| $T_1$ | $m_b$ | $\lambda_B$ | $f_{K^*}$ | $f_{K^*}^+\lambda_B$ | $m_B$ | $m_{K^*}$ | $F_\perp$ | $a_\gamma^x$ |
|-------|-------|-------------|-----------|----------------------|-------|-----------|-----------|-----------|
| 0.32  | 4.2 GeV | 0.35 GeV    | 0.226 GeV | 0.175 GeV            | 5.28 GeV  | 0.892 GeV | 1.21      | -0.41-0.03i |

FIG. 1: Tree level contribution to non-spectator processes in $B \rightarrow K^*\gamma$. Cross represents the alternative coupling of the emitted photon.

$U^{sb}_{\gamma}$, and therefore, with more precise experimental data becoming available in the future, this observable could serve to impose a stringent constrain on the important nonunitarity parameter of the vector quark model.

III. DIRECT CP VIOLATION WITHIN THE VQM

In case that the strong phase $\phi_s$ in eq. (9) is significant, one could look into another important observable in the $B \rightarrow K^{*}\gamma$ transition, i.e. direct CP violation, which in combination with isospin asymmetry help to constrain $U^{sb}_{\gamma}$. Direct CP asymmetry, which is defined as:

$$A_{CP} = \frac{\Gamma(B^+ \rightarrow K^{*+}\gamma) - \Gamma(B^- \rightarrow K^{*-}\gamma)}{\Gamma(B^+ \rightarrow K^{*+}\gamma) + \Gamma(B^- \rightarrow K^{*-}\gamma)}, \tag{11}$$

is nonzero if at least two different diagrams with non-identical weak and strong phases contribute to the decay process. In other words, for $B \rightarrow K^{*}\gamma$ transition, we should expect non-vanishing $A_{CP}$ from nonspectator processes if $b_q$ happens to include both strong as well
as weak phases. In this case, eq. (11) can be written as:

\[ A_{CP} = \Re(b_u - b_{\bar{u}}) \]  

(12)

The SM prediction for direct CP violation in \( B \to K^*\gamma \) is vanishingly small within the theoretical error. For example, using the perturbative QCD method, it is calculated to be \( A_{CP} = (0.62 \pm 0.13) \times 10^{-2} \) [J]. Therefore, any significant CP asymmetry is an indication of new physics. In our case, the contribution of extra vector-like quarks to eq. (12) within the QCD factorization method is obtained as follows:

\[ A_{VQM}^{CP} = \left( \frac{16\pi^2 f_B}{N_c m_b f_{1_{B \to K^*}^*}} \right) \left| \frac{U_{tb}^* V_{ts}}{\alpha_G V_{tb} V_{ts}} \right| \sin \theta \sin \phi_s \]  

(13)

Inserting the values given in Table I for the parameters in the above formula leads to a simple expression of the additional direct CP violation in terms of the nonunitarity parameter, its CP odd phase and the strong phase:

\[ A_{VQM}^{CP} = 0.27 \left| \frac{U_{tb}^* V_{ts}}{\alpha_G V_{tb} V_{ts}} \right| \sin \theta \sin \phi_s \]  

(14)

The available experimental data on this asymmetry, i.e. \( A_{CP} = 0.007 \pm 0.074 \pm 0.017 \) [2] has large errors and is consistent with zero. The combination of isospin (eqns. (9) and 10)) and direct CP (eq. (13)) asymmetries due to vector quarks leads to the following prediction: if the difference between \( \Delta_0^- \) and \( \Delta_0^+ \) is mainly due to extra vector quarks then one expects a direct CP asymmetry in \( B \to K^*\gamma \) of around 6 – 7%. It will be exciting to see if the more accurate experimental measurements in the future will result in a significant shift of the central value of \( A_{CP} \).

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