Economic and mathematical model of the profit of sugar production from beets

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Abstract. The economic sector of sugar production from beets in Russia is currently operating in a crisis of overproduction. The volume of sugar produced over the past few years exceeds the demand of the country’s population and the need for related processing industries. This economic situation led to a decrease in the market price of sugar, the formation of a deficit of working capital of enterprises. It arose a risk for unprofitable work of sugar factories. One of the main business processes that determine the efficiency of the company's operation is to make a profit. Based on the analysis of the specifics of sugar production from beets, a mathematical model of the sugar factory's profit was developed. The developed model takes into account all the components of the business process of making a profit that determine the efficiency of the sugar factory. Based on the developed mathematical model, it is possible to analyze the break-even operation of a sugar factory and determine the break-even point. The obtained mathematical model allows solving problems of optimal enterprise management, optimal design and reconstruction of sugar plants, and conducting financial audits. With the help of the developed mathematical model, the production activity of three sugar factories of different production capacities and different degrees of technical equipment is studied. The economic parameters of break-even operation of the studied enterprises are determined. Comparative analysis has shown the decisive role of effective production technologies in the break-even value.

1. Introduction. Tasks Research.
The volume of sugar produced in Russia over the past few years from domestic raw materials exceeds the needs of the population and processing industries that use sugar as raw materials. Over the past two years, the amount of unrealized sugar exceeded 1 million tons. The Market price of sugar began to decline, and there was a risk of unprofitable operation of sugar factories. There was a shortage of working capital, resulting in the bankruptcy of enterprises and the termination of production activities. The current economic situation makes increased demands on the efficiency of enterprise management through the use of mathematical methods of planning and forecasting.
2. Justification and development of an economic and mathematical model of the profit of sugar production from beets.

The main goal of the company's production activity is to make a profit. Therefore, it is advisable to consider the solution of enterprise management problems on the basis of an economic profit model. In general, the amount of profit of the enterprise \( P \) is defined as the difference between the revenue from the sale of manufactured products \( R \) and the total cost of production \( C \):

\[
P = R - C.
\]

On the basis of equation (1), using mathematical methods [1-3], laws of technological processes and basic principles of production accounting [5-7], it is possible to build an economic model of sugar production from beets.

In theoretical Economics [7] there are two types of profit: accounting and economic. Accounting profit is used for the preparation of external reports of the company to the state control authorities. The method of calculating accounting profit is regulated by government regulations and international standards. Economic profit and its analysis are designed to identify unused production reserves to improve the economic efficiency of the enterprise through effective management. The structure of economic profit is not regulated and depends on the main factors that determine the production cycle. The main feature of sugar production from beets is seasonality. The production season includes three periods: growing beets, harvesting and processing beets, and overhauling equipment before the start of the next season.

From the point of view of economic theory, seasonal production can be considered short-term, with perfect competition [7] in which it is provided for the sale of sugar at a constant selling price of Russia. Under these conditions, the revenue from the sale of 1 ton of sugar for all Russian factories must be constant, equal to the price of sugar. The economic profit of an individual sugar factory can be increased by increasing the production of sugar and the amount of processed beet.

Revenue from the sale of \( R \) at a constant cost of sugar is determined by the equation:

\[
R = p_s \cdot A \cdot t \cdot \frac{Y}{100},
\]

where \( p_s \) - the selling price of sugar, \( \text{R/t} \); \( A \) is the production capacity of the plant, \( \text{t.beet/day} \); \( t \) is the duration of the season, \( \text{day} \); \( Y \) is the yield of sugar, \( \% \).

The total cost of production \( C \) consists of two components: fixed costs \( F \), which do not depend on the volume of production and variable costs \( V \):

\[
C = F + V.
\]

Fixed costs \( F \) depend on the tasks that the mathematical model of profit is used to solve. There are three main tasks:

- design and reconstruction of sugar factories;
- management the production of sugar;
- conducting an audit to determine inefficient operations in the management of sugar production.

The goal of solving the first task is to determine the optimal productivity of a sugar factory, determine the costs of construction or reconstruction, on which the amount of depreciation depends. The cost of construction or reconstruction determines the payback period, the amount of depreciation \( D \) and the amount of credit \( Cr \), which is necessary to perform construction and installation work.

When solving enterprise management tasks, fixed costs should also include depreciation charges, short-term loans, and the amount of the company's rent. The peculiarity of the sugar factory is that the processing of raw materials is carried out within 4-6 months of the current production season. Sales of finished products and receipt of revenue occur throughout the calendar year. The factory is forced to store the produced product, receiving unproductive storage costs. In addition, significant funds are withdrawn from circulation. In addition, significant funds are withdrawn from circulation. This reduces the liquidity of working capital. Fixed costs must include \( Ar \) accounts receivable. Seasonality,
continuity of the technological process, aggressive impact of products on equipment and communications is reflected in the structure of fixed costs. After the end of the current season and before the start of the next season, major repairs of the equipment are carried out with the replacement of worn-out components. To implement this technical measure, a budget working capital fund is created. These funds, as well as the Sf salary fund, must be included in the fixed costs.

With this in mind, fixed costs can be expressed by the equation:

\[ F = D + Cr + Re + Ar + B + Sf, \]  

where \( D \) is the depreciation (wear and tear) of fixed assets, \( R \); \( Cr \) – repayment of loans, \( R \); \( Re \) – amount of lease payment for land use, water resources, \( R \); \( Ar \) - accounts receivable, \( R \); \( B \) – budget investment enterprises on the season, including repair costs, the cost of purchasing filter fabrics, caustic soda, defoamers, sulphur and sulphuric acid, the descaling, the costs of the laboratory, the cost of limestone and fuel for his firing, \( R \); \( Sf \) – salary fund of the production season, \( R \).

Variable costs include the total cost of raw materials, energy, and auxiliary materials, minus the amount from the sale of production molasses and granulated pulp.

The calculation of variable costs is based to the performance of the production capacity of the plant \( A \), the duration of the production season \( t \) and yield of sugar \( Y \). An analytical expression is obtained for the calculation of variable costs of sugar beet production in the form [2]:

\[ V = \left( \frac{100 \cdot p_{sb}}{Y} + \frac{(Fc + Fc_{p}) \cdot p_{f}}{14.1 \cdot Y} + \frac{1000 \cdot p_{p} \cdot M}{m_{p} \cdot Y} - \frac{Pl \cdot p_{pl}}{Y} \right) A \cdot Y \cdot Y_{100}, \]  

where \( p_{sb} \) – the price of beet, \( R/t \); \( Fc \) - consumption of conditional fuel for production of heat and electric energy, \%; \( Fc_{p} \) – consumption of conditional fuel for drying of sugar beet pulp, \%; \( p_{f} \) - the price of fuel, \( R/m^{3}.gas \); \( p_{p} \) – the price of the packaging, \( R \); \( m_{p} \) is the mass of packaging, \( kg \); \( M \) – the yield of molasses (a waste product), \%; \( p_{m} \) - the price of molasses, \( R/t \); \( Pl \) – yield of the granulated pulp, \%; \( p_{pl} \) - the price of granulated pulp, \( R/t \).

Equation (5) reflects the total technological cost price of produced sugar [3].

Substituting the ratios (2), (4) and (5) in equation (1), we get the equation of the economic profit of the sugar factory for the production season without taxes:

\[ P = p_{s} \cdot A \cdot Y \cdot Y_{100} - F - V. \]  

Equation (6) is a mathematical model of the economy of seasonal sugar production. Using this model, you can solve the three problems listed above.

The maximum profit value can be obtained by differentiating equation (6) by the value of the first derivative profit. If the derivative is positive, then a function (6) has a maximum, if it is negative, a function has a minimum. It is difficult to use this method to find the profit extremum, since all the profit factors (6) are variables and the maximum profit must be for a function of many variables. A simpler solution is to determine the maximum profit using the break-even point [8].

When working in the sugar industry in conditions of overproduction, the law of perfect competition is violated [7], enterprises seek to increase sales by reducing the price of sugar. This raises the problem of determining the price of sugar, to which you can reduce. This price of sugar is called a break-even price.

To ensure minimal losses, you must constantly control the break-even point for the analyzed enterprise. The break-even point of an enterprise is determined by an integral indicator of the ratio of fixed and total costs [8]. This point can be calculated using equation (6). The equation for calculating the integral coefficient will be as follows:

\[ \gamma = \frac{F}{F + V}. \]
From equation (7), you can determine the unknown fixed costs $F$ of a sugar factory:

$$F = \frac{\gamma}{V - \gamma}. \quad (8)$$

The use of the integral coefficient $\gamma$ allows us to determine the value of fixed costs $F$, defined by equation (4) through variable costs, the values of which are determined by equation (5). After substituting equation (8) into equation (6), we get the final equation of the sugar factory's profit model for the production season in the form:

$$P = p_r \cdot \frac{V}{100} - \frac{\gamma}{1-\gamma} V - V. \quad (9)$$

The solution of equation (9) shows that at a certain value of the coefficient expressed as a percentage, the company's profit becomes zero, which is the break-even point. To determine the break-even point of a sugar factory, we made calculations using equation (9) for three plants of different production capacity at different beet prices.

For the three plants under study, we made calculations using equation (9) to determine the break-even point for two values of the beet price of 2500 R and 2500 R, when the sugar price changes from 26 000 R/t to 16 000 R/t.

3. Results

The results of the calculations are presented in tables 1-3.

Table 1. Results of break-even modeling for a plant with a capacity of $A = 10000$ t.beet/day

| The price of sugar, R | Fixed costs F, R. | Variable costs, R. | Break-even point | Revenue per season, R |
|-----------------------|------------------|--------------------|------------------|----------------------|
| The price of beet is 2250 R. |                |                    |                  |                      |
| 26000                 | 1897054954       | 2002929825         | 0.9471           | 3899984779           |
| 24000                 | 1597136242       | 2002929825         | 0.7974           | 3600066067           |
| 22000                 | 1297097354       | 2002929825         | 0.6476           | 3300027179           |
| 20000                 | 997058466        | 2002929825         | 0.4978           | 2999988291           |
| 18000                 | 697019578        | 2002929825         | 0.3480           | 2699949403           |
| 16000                 | 396980691        | 2002929825         | 0.1982           | 2399910516           |
| The price of beet is 2500 R. |                |                    |                  |                      |
| 26000                 | 1647116994       | 2252929825         | 0.7311           | 3900046819           |
| 24000                 | 1347026742       | 2252929825         | 0.5979           | 3599956567           |
| 22000                 | 1047161782       | 2252929825         | 0.4648           | 3300091607           |
| 20000                 | 747071529        | 2252929825         | 0.3316           | 3000001354           |
| 18000                 | 446981277        | 2252929825         | 0.1984           | 2699911102           |
| 16000                 | 147116317        | 2252929825         | 0.0653           | 2399910516           |

Table 2. Results of break-even modeling for the reconstructed plant with a capacity of $A = 3000$ t.beet/day

| The price of sugar, R | Fixed costs F, R. | Variable costs, R. | Break-even point | Revenue per season, R |
|-----------------------|------------------|--------------------|------------------|----------------------|
| The price of beet is 2250 R. |                |                    |                  |                      |
| 26000                 | 569092451        | 600878947          | 0.9471           | 1169991398           |
| 24000                 | 479140872        | 600878947          | 0.7974           | 1080001819           |
| 22000                 | 389129206        | 600878947          | 0.6476           | 990008153            |
| 20000                 | 299117540        | 600878947          | 0.4978           | 899996487            |
| 18000                 | 209105873        | 600878947          | 0.3480           | 809984820            |
| 16000                 | 119094207        | 600878947          | 0.1982           | 719973154            |
| The price of beet is 2500 R. |                |                    |                  |                      |
| 26000                 | 494135098        | 675878947          | 0.7311           | 1170014045           |
| 24000                 | 404108022        | 675878947          | 0.5979           | 1079986969           |
| 22000                 | 314148534        | 675878947          | 0.4648           | 990027481            |
| 20000                 | 224121458        | 675878947          | 0.3316           | 900000405            |
| 18000                 | 134094383        | 675878947          | 0.1984           | 809973330            |
| 16000                 | 44134895         | 675878947          | 0.0653           | 720013842            |
Table 3. Results of break-even modeling for an unreconstructed plant with actual capacity $A = 2700$ t.beet/day

| Price of sugar, R | Fixed costs F, R | Variable costs, R | Break-even point | Revenue per season, R |
|------------------|-----------------|-----------------|-----------------|---------------------|
| 26000            | 402827396       | 650141052       | 0.6196          | 1052968448         |
| 24000            | 321884835       | 650141052       | 0.4951          | 972025887          |
| 22000            | 240877260       | 650141052       | 0.3705          | 891018312          |
| 20000            | 159869684       | 650141052       | 0.2459          | 810010737          |
| 18000            | 78862109        | 650141052       | 0.1213          | 729003162          |
| 16000            | -               | -               | -               | -                   |

| 26000            | 335353663       | 717641052       | 0.4673          | 1052994716         |
| 24000            | 254331989       | 717641052       | 0.3544          | 971973041          |
| 22000            | 173382078       | 717641052       | 0.2416          | 891023130          |
| 20000            | 92360403        | 717641052       | 0.1287          | 810001456          |
| 18000            | 11338728        | 717641052       | 0.0158          | 728979781          |
| 16000            | -               | -               | -               | -                   |

4. Discussion of results

Initial data for calculations were formed based on the results of the work of sugar factories in the Central black earth region of Russia over the past 5 years. Technological parameters of products were determined according to the data of the Union of sugar producers of Russia. The price of fuel is determined by state regulations, in particular the price of gas is 5800 R/1000 m$^3$. The price of sugar beet is determined by contracts. For leading sugar producers, 50 % of beet is harvested by their own agricultural firms at a cost of 2000 R/t, 50 % is supplied by agricultural producers at a price of 2500 R/t. Therefore, when modeling break-even, the average price of sugar beet is taken as two values: 2250 R/t and 2500 R/t. The price of sugar is a market indicator that can take different values. The calculations used a multivariate approach to modeling break-even with a price change from 26,000 R/t to 16,000 R/t.

Using these initial data, with equation (9), it was obtained the parameters of break-even operation of three sugar plants are determined. The first two plants (tables 1, 2) with a production capacity of 10000 tons of beet per day and 3000 tons of beet per day were reconstructed with the introduction of modern technologies. The third plant of the project in 1985 was partially reconstructed with the replacement of a four-hull evaporation plant with a five-hull one. The plant does not have deep pressing presses and the plant has to work with high pumping of diffusion juice (120-130 %). This leads to a decrease in the production capacity of the heat scheme and the thermal power plant to 2700 tons of beet per day. The plant does not use the full heat of thermal waste, which leads to increased fuel consumption. These features of production schemes are reflected in the results of calculating the parameters of break-even operation of the studied plants.

Tables 1-3 present in value terms the planned values of fixed and variable costs, revenue (sales volumes) and the corresponding break-even indicator. The tables show that the value of variable costs depends only on the price of beet and does not depend on the price of sugar. This is because variable costs are determined only by the efficiency of the technologies used in the plant. For reconstructed plants with a production capacity of 10000 tons of beet per day and 3000 tons of beet per day, using modern efficient technologies, the cost of variable costs is the same and does not depend on the market price of sugar. The break-even point for these plants is the same. This leads to the conclusion that factories with a performance that differs three times, can sell sugar at the same price.

In contrast to these plants, the third plant has significantly lower break-even rates. To avoid large losses, the third plant is forced to sell sugar at a higher price than the first and second plants.
There is a law of perfect competition in the market, according to which all producers must sell sugar at the same price, and sugar from a third factory that has a higher price, will not be sold. In order to sell the resulting sugar, the third plant will be forced to lower the price below the break-even point, incurring losses that may lead it to bankruptcy. Data on fixed costs allows you to create a balance of fixed costs by defining the optimal plant management condition, which is written in the form of equation (4) with the corresponding calculated value of this value.

The calculated value of fixed costs allows us to formulate a condition for optimal management of a sugar plant: \[ D + Cr + Re + Ar + B + Sf = F. \]

5. Conclusions
An equation is obtained that determines the break-even operation of a sugar beet plant, which allows solving the problems of optimal enterprise management, optimal design and reconstruction of sugar plants, and conducting financial audits.

Using the example of the evaluation of three sugar factories, it is shown that in conditions of overproduction of sugar, plants that use inefficient technologies can go bankrupt.

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