Horizon news function and quasi-local energy-momentum flux near black hole

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Abstract
From the 'quasi-local' definition of horizons, e.g. isolated horizon and dynamical horizon, the consequence quasi-local energy-momentum near horizons can be observed by using the idea of frame alignment. In particular, we find the horizon news function from the asymptotic expansion near horizons and use this to describe the gravitational flux and change of mass of a black hole.

1 Introduction

In this paper we apply a similar method of asymptotic expansion near null infinity (Newman-Unti [6]) to the geometry near a black hole to study gravitational radiation. The definition of 'quasi-local' horizons (isolated horizon and dynamical horizon) are given by Ashtekar and Krishnan [1, 2]. The covariant expression of quasi-local energy expression is given by Nester-Witten two form [4] which refers to an implicit reference frame when observing the quasi-local quantities. What is a good reference frame near a strong gravitating field such as a black hole? To tackle this problem, we use the concept and idea of frame alignment to find the asymptotically constant spinors which is done by Bramson [3] for null infinity and try to move this work on the horizon in Section 2. We then be able to use the spin frame $Z_{\Lambda A} = (\lambda_A, \mu_A)$ to give a definition of the energy-momentum for a rotating dynamical horizon in Section 3. Particularly, we find the horizon news function dominates the gravitational radiation of a black hole in Section 4.

2 Constant spinors near quasi-local horizons

We find the compatible conditions for asymptotically constant spinor $\lambda_A$ mainly include the Duggan-Mason’s holomorphic conditions and a time-related condition.
Different types of quasi-local horizons require different conditions \(^7\). For example, the compatible conditions of a rotating dynamical horizon are

\[
\dot{\lambda}_0^0 = 0, \Rightarrow \dot{\lambda}_0^0 - \epsilon_0 \lambda_0^0 = 0 \quad (1)
\]

\[
\dot{\lambda}_1^0 + \sigma_0 \lambda_1^0 = 0, \quad \dot{\lambda}_1^0 - \mu_0 \lambda_0^0 = 0, \quad (2)
\]

\[
\dot{\lambda}_1^0 = -\ddot{\lambda}_0^0 \quad (3)
\]

where include a time-related condition (1) and Dougan-Mason’s holomorphic conditions (2) and an extra condition (3). Next, we will focus on the issues of a rotating dynamical horizon.

3 Quasi-local energy-momentum of a rotating dynamical horizon

By using the compatible constant spinor conditions for a rotating dynamical horizon, i.e., the Dougan-Mason’s holomorphic conditions (2) in Section 2, we can get the quasi-local momentum integral near a rotating dynamical horizon based on Nester-Witten two form is

\[
I(r') = \frac{1}{4\pi} \oint_S (\rho \lambda_0 \tilde{\lambda}_0^0 + \rho \lambda_1 \tilde{\lambda}_1^0) dS
\]

\[
= \frac{1}{4\pi} \oint_S [-\mu_0 \lambda_0 \lambda_0^0 + O(r')] dS_\Delta \quad (4)
\]

where we use \( \rho = O(r'), \mu = \mu_0 + O(r'), dS_\Delta = \lim_{r' \to 0} dS_{r'} \) and \( dS_\Delta(v) \).

By using the result of the asymptotic expansion (NR7) \(^7\), we get the quasi-local momentum integral on a rotating dynamical horizon

\[
I(r) = -\frac{1}{4\pi} \oint \frac{1}{2\pi} \left[ \dot{\lambda}_0^0 - \dot{\mu}_0 + \dot{r} \Delta (\mu_0^2 + \sigma'_0 \sigma'_0) + \ddot{\delta}_0 \pi_0 + \pi_0 \ddot{\pi}_0 - \sigma_0 \sigma'_0 \right] \lambda_0^0 \lambda_0^0 dS_\Delta. \quad (5)
\]

4 Energy-momentum flux near a rotating dynamical horizon

Now, in order to make our calculation easier and use the approximate Kerr in Bondi coordinate and the slow rotating Kerr-Vaidya as our basis. We make a coordinate choice \( r_\Delta(v) = -\frac{1}{\mu_0} \) and make the assumption that \( \sigma'_0 = \kappa_0 = 0 \) to simplify our calculation of flux formula. Here we will also need the time related condition (1) of constant spinor of dynamical horizon in Section 2 and re-scale it. Therefore \( \dot{\lambda}_0^0 = 0 \). We note that this rescaling of spin frame is to chose a ‘permissible’ time so that the energy flux can only depend on the news function. It’s very tedious but straightforward to calculate the flux expression. It largely depends on
the non-radial NP equations and the second order NP coefficients. We substitute these equations back into the energy-momentum flux formula to simplify our flux expression (See [7] for the detail).

Apply time derivative on (4), we get

\[ \dot{I}(r_\Delta) = \frac{1}{4\pi} \int \mu_0 \lambda_0^0 \bar{\lambda}_0^0 dS_\Delta. \]  

(6)

Here we can see the fact that \( \dot{\mu}_0 \) is related with the mass loss of gain, hence it is the news function. Integrate the above equation with respect to \( v \) and use \( \dot{\mu}_0 = \dot{\tau}_\Delta (2) \frac{2 \pi}{R} \), we then have

\[ dI(r_\Delta) = \frac{1}{8\pi} \int (2) R \lambda_0^0 \bar{\lambda}_0^0 dS_\Delta dr_\Delta. \]  

(7)

Ashtekar’s total flux formula[2] is

\[ F_{\text{matter}} + F_{\text{grav}} = \frac{1}{16\pi} \int (2) RN d^3V. \]  

(8)

where \( F_{\text{matter}} + F_{\text{grav}} \) is equal to flux \( dI \) and \( d^3V = dr_\Delta dS \) on horizon. Therefore, if \( N = 2\lambda_0^0 \bar{\lambda}_0^0 \), then our flux formula from equation (4) is completely the same with Ashtekar-Krishnan’s formula (8).

Apply time derivative on (5) and integrate the equation with respect to \( v \), we have

\[ dI(r_\Delta) = -\frac{1}{4\pi} \int \left\{ \frac{\mu_0}{2\epsilon_0 \dot{\tau}_\Delta} [\sigma_0 \bar{\sigma}_0 + 4 \dot{\tau}_\Delta \pi_0 \bar{\pi}_0 + \Phi_{00}^0] \right. \\
+ \left. \frac{2}{\epsilon_0 \dot{\tau}_\Delta} [\sigma_0 \pi_0^2 + \bar{\sigma}_0 \bar{\pi}_0^2] \right\} \lambda_0^0 \bar{\lambda}_0^0 dS_\Delta dr_\Delta. \]  

(9)

where \( dv = \frac{dr_\Delta}{\dot{\tau}_\Delta} \). The total flux of Ashtekar-Krishnan [2] in terms of NP in our gauge [7] is

\[ F_{\text{total}} = \frac{1}{4\pi} \int [||\sigma||^2 + ||\pi||^2 + \Phi_{00}] Nd^3V. \]  

(10)

In order to compare with Ashtekar’s expression, therefore, if we choose \( N = 2\lambda_0^0 \bar{\lambda}_0^0 \), and \( -\frac{\mu_0}{2\epsilon_0 \dot{\tau}_\Delta} = 2 \), then (9) becomes

\[ dI(r_\Delta) = \frac{1}{4\pi} \int \left\{ [\sigma_0 \bar{\sigma}_0 + 4 \dot{\tau}_\Delta \pi_0 \bar{\pi}_0 + \Phi_{00}^0] + \frac{4}{\mu_0} [\sigma_0 \pi_0^2 + \bar{\sigma}_0 \bar{\pi}_0^2] \right\} N dS_\Delta d\bar{k}_\Delta. \]

However, here we have an extra term which is the coupling of the shear \( \sigma_0 \) and \( \pi_0 \).
5 Conclusion

The Nester-Witten two-form with the compatible conditions of constant spinors on 'quasi-local' horizons and together with the results from the asymptotic expansion near 'quasi-local' horizons give us the quasi-local energy-momentum and flux expressions on 'quasi-local' horizons. Dougan-Mason’s holomorphic conditions tell us how to gauge fix the quasi-local expression on each cross section of 'quasi-local' horizons. The time related condition tells us how the quasi-local expressions change with time along horizon. The news function which dominates the gravitational radiation near a rotating dynamical horizon can be understood as the time derivative of the expansion of the incoming null tetrad $\dot{\mu}_0$ in equation (6). The gravitational flux for a rotating dynamical horizon is obviously positive (mass gain) from our formula if the dominate energy condition holds. The shear square term in equation (9) which is related with the gravitational radiation near dynamical horizon in our formula matches the results of energy flux cross event horizon by using the perturbation method [Hawking and Hartle]. The $\pi$ square term is related with angular momentum contribution [See Ashtekar and Krishnan [2]]. There’s a shear and $\pi$ coupling term in my flux formula for a rotating dynamical horizon which is an extra term in Ashtekar-Krishnan’s formula.

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