On the nitrogen abundance of FLIERs: the outer knots of the planetary nebula NGC 7009

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\textbf{ABSTRACT}

We have constructed a 3D photoionisation model of a planetary nebula (PN) similar in structure to NGC 7009 with its outer pair of knots (also known as FLIERs –fast, low-ionization emission regions). The work is motivated by the fact that the strong \([\text{N}\,\text{ii}]\,\lambda6583\) line emission from FLIERs in many planetary nebulae has been attributed to a significant local overabundance of nitrogen. We explore the possibility that the apparent enhanced nitrogen abundance previously reported in the FLIERs may be due to ionization effects. The model is constrained by the results obtained by Gonçalves, Corradi, Mampaso & Perinotto (2003) from the analysis of both HST \([\text{O}\,\text{iii}]\) and \([\text{N}\,\text{ii}]\) images, and long-slit spectra of NGC 7009. Our model is indeed able to reproduce the main spectroscopic and imaging characteristics of NGC 7009’s bright inner rim and its outer pairs of knots, assuming \textit{homogeneous elemental abundances} throughout the nebula, for nitrogen as well as all the other elements included in the model.

We also study the effects of a narrow slit on our non-spherically symmetric density distribution, via the convolution of the model results with the profile of the long-slit used to obtain the spectroscopic observations that constrained our model. This effect significantly enhances the \([\text{N}\,\text{ii}]\)/\(\text{H}\,\beta\) emission, more in the FLIERs than in the inner rim.

Because of the fact that the \((\text{N}\,\text{+}/\text{N})/(\text{O}\,\text{+}/\text{O})\) ratio predicted by our models are 0.60 for the rim and 0.72 for the knots, so clearly in disagreement with the \(\text{N}\,\text{+}/\text{N} = \text{O}\,\text{+}/\text{O}\) assumption of the ionization correction factors method (icf), the icfs will be underestimated by the empirical scheme, in both components, rim and knots, but more so in the knots. This effect is partly responsible for the apparent inhomogeneous N abundance empirically derived. The differences in the above ratio in these two components of the nebula may be due to a number of effects including charge exchange –as pointed out previously by other authors– and the difference in the ionization potentials of the relevant species –which makes this ratio extremely sensitive to the shape of the local radiation field. Because of the latter, a realistic density distribution is essential to the modelling of a non-spherical object, if useful information is to be extracted from spatially resolved observations, as in the case of NGC 7009.

\textbf{Key words:} Atomic data - ISM: abundances - planetary nebulae: individual (NGC 7009).

\section*{1 INTRODUCTION}

It is well known that planetary nebulae (PNe) possess a number of small-scale structures that, at variance with their large-scale structures, are prominent in low-ionization emission lines, such as \([\text{N}\,\text{ii}]\) and \([\text{O}\,\text{ii}]\). In terms of morphology and kinematics, the low-ionization structures (LISs, for a review see Gonçalves 2004) may appear as knots, filaments, jets, and isolated features that, in some cases, move with the same velocity of the ambient nebula, or supersonically...
Table 1. Main physical/chemical parameters from Paper 1.

| Parameter          | Eastern | Western | Eastern | Western | Whole Nebula |
|--------------------|---------|---------|---------|---------|--------------|
| $N_e$[S ii] (cm$^{-3}$) | 5,500   | 5,900   | 2,000   | 1,300   | 4,000        |
| $N_e$[Cl iii] (cm$^{-3}$) | 5,200   | 5,900   | -       | 1,900   | 1,300        |
| $T_e$(O iii) (K)    | 10,000  | 10,200  | 9,600   | 10,400  | 10,100       |
| $T_e$[N ii] (K)     | 10,400  | 12,800  | 11,000  | 11,700  | 10,300       |
| $T_e$[S ii] (K)     | -       | -       | 7,100   | 9,400   | -            |
| He/H               | 1.08(-1) | 1.16(-1) | 1.02(-1) | 9.55(-2) | 1.11(-1)     |
| O/H                | 4.5(-4)  | 4.82(-4) | 5.8(-4)  | 4.5(-4)  | 4.71(-4)     |
| N/H                | 7.0(-5)  | 1.8(-4)  | 3.8(-4)  | 2.5(-4)  | 1.7(-4)      |
| Ne/H               | 1.1(-4)  | 1.1(-4)  | 1.1(-4)  | 1.3(-4)  | 1.1(-4)      |
| S/H                | 6.1(-6)  | 4.9(-6)  | 1.39(-5) | 9.3(-6)  | 8.3(-6)      |

Empirically derived parameters, for the different structures of the NGC 7009, for both, the Eastern and Western sides of the nebula, with “R”, “K” and “NEB” standing for rim, outer knots and the whole nebula (see Tables 1 and 3 of Paper 1).

For over ten years, the strong [N ii] line emission from LISs has been attributed to a significant local over-abundance of nitrogen (Balick et al. 1993, 1994, 1998; and references therein). Balick et al. (1993) interpreted the N-enrichment in FLIERs as evidence of their origins in recent high-velocity ejections of the PN central star. The above work and successively Balick et al. (1994), which included the derivation of the ionical and elemental abundances of LISs, pointed out “an apparent enhancement of nitrogen relative to hydrogen by factors of 2-5” in FLIERs. However, since then, further work casted doubts over the latter statement (Hajian et al. 1997; Alexander & Balick 1997; Gonçalves et al. 2003; Perinotto et al. 2004).

Abundances in PNe can be derived from the analysis of collisionally excited line (CEL) or optical recombination line (ORL) spectra using empirical methods or tailored photoionisation models (e.g. Stasińska 2004) or a combination of the two. For some elements (e.g. He) and ions (e.g. N$^+$ if only the optical spectrum is available) the use of ORL is mandatory, whilst for others the use of CELs may be unavoidable. But, when both CEL and ORL determinations are available, the former have been commonly preferred because they are generally stronger and easier to detect than ORLs. Moreover, there is a well known discrepancy between CEL- and ORL-abundances, as well as electron temperature determinations (Liu et al. 1995). Empirical abundance analysis rely on ionization correction factors (icfs) to account for the unseen ions (e.g. Kingsburgh & Barlow, 1994). Results obtained with the icf method can be somewhat uncertain in some cases, particularly when they are applied to spatially resolved long-slit spectra (Alexander & Balick 1997), as has been the case for the work of Balick et al. (1994); Hajian et al. (1997) and Gonçalves et al. (2003) (hereafter Paper 1).

In fact, the abundances derived in Paper 1 and shown here in Table 1, from optical long-slit spectra of NGC 7009, using the icf scheme of Kingsburgh & Barlow (1994), showed only a marginal evidence for overabundance of N/H in the outer knots of the nebula (the ansae), reinforcing the doubts over previous results (Balick et al. 1994) where the N/H enhancement of a factor of 2-5 in the ansae were reported.

One of the major shortcomings of empirically-determined chemical abundances lies in the fact that a number of assumptions on the ionization structure of the gas need to be made in order to obtain the icfs. A preferred alternative would be the construction of a tailored photoionisation model for a given object, aiming to fit the emission line spectrum and, in the case of spatially resolved objects, projected maps in a number of emission lines.

NGC 7009, the “Saturn Nebula”, is a PN comprising a bright elliptical rim. Its small-scale structures include a pair of jets and two pairs of low-ionization knots. On a larger scale, it is known that NGC 7009 possess a tenuous halo with a diameter of more than 4 arcmin (Moreno et al. 1998), whose inner regions display a system of concentric rings (Carradi et al. 2004) like those observed in NGC 6543 and other few PNe (Balick et al. 2001). High-excitation lines dominate the inner regions along the minor axis, while emission from low-ionization species is enhanced at the extremities of the major axis. The ionization structure is further enriched by the fact that the low-excitation regions present strong variations in excitation level and clumpiness. NGC 7009 was classified as an oxygen-rich PN (Hyung & Aller 1995a), with an O/C ratio exceeding 1, and anomalous N, O, and C abundances (Baker 1983; Balick et al. 1994; Hyung & Aller 1995a). Its central star is an H-rich O-type star, with effective temperature of 82,000 K (Méndez et al. 1992; Kingsburgh & Barlow 1992). The kinematics of NGC 7009 was studied first by Reay & Atherton (1985) and Balick et al. (1987), who showed that the ansae are expanding near the plane of the sky at highly supersonic velocities. The derived inclination of the inner (caps) and outer (ansae) knots, with respect to the line of sight, are $i \cong 51^\circ$ and $i \cong 84^\circ$, respectively (Reay & Atherton 1985). More recently, Fernández et al. (2004) have measured the proper motion and kinematics of the ansae in NGC 7009, assuming that they are equal and opposite from the central
star, obtaining \( V_{\text{exp}} = 114 \pm 32 \ \text{km s}^{-1} \), for the distance of \( \sim 0.86 \pm 0.34 \ \text{kpc} \).

In this paper we will present a simple 3D photoionisation model, aiming at reproducing the observed geometry and spectroscopic “peculiarities” of a PN like NGC 7009, exploring the possibility that the enhanced \([\text{N} \ ii]\) emission observed in the outer knots may be due to ionization effects. We will use the 3D photoionisation code MOCASSIN (Ercolano et al. 2003a) together with the long-slit data presented in Paper 1 and the HST images available for this nebula. Our model and the observational data are described in Section 2. The results for our main model are presented and discussed in Section 3, while in Section 4 we discuss an alternative model, aimed at highlighting the relevance of the geometry and density distribution in such a kind of modelling. The work is briefly summarised in Section 5, where our final conclusions are also stated.

2 METHOD

2.1 Observational data

The observations used to constrain the photoionisation model were described in detail in Paper 1. These included HST \([\text{O} \ ii]\) and \([\text{N} \ ii]\) images as well as Isaac Newton Telescope long-slit, intermediate dispersion spectra, along the PN major axis (P.A. = 79°). For further details see Paper 1.

We refer to Figure 1 of Paper 1, the HST \([\text{O} \ ii]\) and \([\text{N} \ ii]\) images of NGC 7009, in which the nebular features are outlined. Here we will employ the same nomenclature introduced in Paper 1. See also the left panel of Figure 2, in Section 3.5.

2.2 The 3D photoionisation code: MOCASSIN

The nebula was modeled using the 3D photoionisation code, MOCASSIN, of Ercolano et al. (2003a). The code employs a Monte Carlo technique to the solution of the radiative transfer of the stellar and diffuse field, allowing a completely geometry-independent treatment of the problem without the need of imposing symmetries or approximations for the transfer of the diffuse component.

The reliability of the code was demonstrated via a set of benchmarks described by Péquignot et al. (2001) and Ercolano et al. (2003a). A number of axy-symmetric planetary nebulae have already been modeled using MOCASSIN, examples include NGC 3918 (Ercolano et al. 2003a), NGC 1501 (Ercolano et al. 2004) and the H-deficient knots of Abell 30 (Ercolano et al. 2003c).

2.3 Input parameters

A thorough investigation of the vast parameter space was carried out in this work. This involved experimenting with various gas density distributions, central star parameters and nebular elemental abundances. The model input parameters that best fitted all observational constraints are summarised in Table 2 and discussed in more detail in the following subsections.

A common problem when studying galactic PNe is the large uncertainties associated with the distance estimates, that propagate to the determination of the nebula geometry and central star parameters. We adopted a distance of 0.86 kpc for our models of NGC 7009, as computed by Fernández et al. (2004) from the weighted average of 14 values determined with statistical methods; the value was quoted with an error of \( \pm 0.34 \) kpc.

2.3.1 Stellar parameters

After having experimented with various stellar atmosphere models to describe the ionising continuum, we reverted to using a blackbody of \( T_{\text{eff}} = 80,000 \ \text{K} \) and \( \log L_\star = 3.50 \), as this resulted in the best fit of the nebular emission line spectrum. Méndez et al. (1993) determined the \( T_{\text{eff}} \) and \( \log g \) for the central star of NGC 7009 using non-LTE model atmosphere analysis of the stellar H and He absorption line profiles, finding \( T_{\text{eff}} = 82,000 \ \text{K} \) and \( \log L_\star = 3.97 \), in solar units. Méndez et al. (1992) assumed a distance of 2.1 kpc in their analysis which therefore resulted in their value for the stellar luminosity being higher than that inferred from our modelling.

2.3.2 Elemental Abundances

The nebular elemental abundances used for the photoionisation model are listed in Table 2, where they are given by number, relative to H. Although the MOCASSIN code can handle chemical inhomogeneities, these were not included in our models as they proved to be not necessary to reproduce the CEL spectra of the R and K regions of NGC 7009.

The values shown result from an iterative process, where the initial guesses at the elemental abundances of He, N, O, Ne and S, taken from Paper 1 (see Table 1), and those of C and Ar, from Pottasch (2000), were successively modified to fit the spectroscopic observations.

2.3.3 Density distribution

The simplest possible density distribution model was constructed in order to demonstrate that the spectroscopic peculiarities often found in LISs can be the product of simple (and well-known) photoionisation effects. The case of NGC 7009 is taken as an example, but the emphasis is not in the construction of a detailed model for this object in particular. With this in mind we described the nebula by an ellipsoidal rim with a H number density, \( N_H \), peaking to 9000 \( \text{cm}^{-3} \) in the short axis direction exponentially decreasing to a minimum value of 4000 \( \text{cm}^{-3} \) in the long axis direction. The short and long axes of the inner and outer

| Table 2. Input parameters for the model. |
|-----------------------------------------|
| \( L_\star \) \((L_\odot)\) \( 3136 \) | \( \text{N/H} \) \( 2.0(-4) \) |
| \( T_{\text{eff}} \) \( (\text{K}) \) \( 80,000 \) | \( \text{O/H} \) \( 4.5(-4) \) |
| \( R_\text{in} \) \( (\text{cm}) \) \( 0.0 \) | \( \text{Ne/H} \) \( 1.06(-4) \) |
| \( R_\text{out} \) \( (\text{cm}) \) \( 3.88(17) \) | \( \text{S/H} \) \( 0.9(-5) \) |
| \( \text{He/H} \) \( 0.112 \) | \( \text{Ar/H} \) \( 1.2(-6) \) |
| \( \text{C/H} \) \( 3.2(-4) \) | \( \text{Fe/H} \) \( 5.0(-7) \) |

Abundances are given by number, relative to H.
ellipsoids measure $3.84 \times 10^{16}$ cm and $9.99 \times 10^{16}$ cm, and $7.06 \times 10^{16}$ cm and $1.84 \times 10^{17}$ cm, respectively, at the distance assumed for NGC 7009. The rim is surrounded by a spherical shell of less opaque, homogeneous density gas, with $N_H = 1600 \text{ cm}^{-3}$. The diameter of the sphere is equal to the long axis of the outer ellipsoid defining the rim. Cylindrical jets, $1.75 \times 10^{16}$ cm in diameter, connect the rim to a pair of disk-shaped knots aligned at a distance of $3.49 \times 10^{17}$ cm from the central star, along the long axis of the ellipsoid. The cylindrical jets widen into cone-shapes at the knot ends in order to simulate the effect of material accumulating at the knots, as suggested by the $HST$ images (particularly for knot K4, as seen in the right panels of Figure 2). The diameter of the base of the cones equals that of the disk-shaped knots. The centres of the $3.49 \times 10^{16}$ cm diameter circular disks representing the knots are aligned with the centres of the cylindrical jets (hence they are seen almost edge on). The width of the disks is assumed to be $3.88 \times 10^{16}$ cm in our model, although only a fraction of these is ionised, as is clear from the right panels of Figure 2. The H number density in the jets and knots is taken to be homogeneous and equal to $1250 \text{ cm}^{-3}$ and $1500 \text{ cm}^{-3}$, respectively, consistently with the values derived in Paper 1.

Results from the ellipsoidal rim and the spherical outer shell are combined into a single $R$-component to enable us to carry out a direct comparison with the slit spectra from Paper 1, as we show in Table 1.

The jets ($J$-component) are included in our simulation as the radiation field has to be transferred through them before reaching the outer ansae ($K$-component). However, given that the emission detected from this region is very faint and that such structures may not be in equilibrium, we take our results for the $J$-component as very uncertain and omit them from any further discussion.

Finally, neither the inner caps nor the tenuous halo were included in our model. The former because they do not lie on the same axis as the $K$-component (Reay & Atherton 1985), and are therefore not expected to have a major influence on the ionisation structure of the outer knots. And the latter because it is very faint, and therefore it is not expected to contribute significantly to the integrated emission line spectrum.

Figure 1 shows model profiles along the major axis, in agreement with our assumed density, as $N_e/N_H$ ratio is correlated to the level of ionisation of the gas (see Section 3.4).

3 RESULTS

The best fit to the observed spectra was obtained assuming the parameters given in Table 2, as discussed in Section 2.1. Table 3 shows the predicted and observed intensities of some important collisionally excited lines, in which values are given relative to that of $H\beta$ $= 100$ for each nebular component (R and K) and integrated over the nebula (NEB).

The model $H\beta$ fluxes are given in the first row of the table. The dereddened line intensities quoted in the $Obs$ column of Table 3 were obtained from the spectroscopic data presented in Table 1 of Paper 1, by using a logarithmic extinction constant $c_{H\alpha}=0.16$ (Gonçalves et al. 2003) and the reddening law of Cardelli et al. (1989). For each line intensity, in each component, the upper row (of the $Obs$ column) shows the values for the North-East (R1 and K1) regions of NGC 7009, while those for the South-Western R2 and K4 components are given in the lower row (see Figure 1 and Table 1 of Paper 1).

The line intensities predicted by our model were convolved with a long-slit profile (assumed to be rectangular) aligned along the long axis of the PN. This is a necessary correction for an extended object with a complex geometry, such as that of NGC 7009, if any meaningful conclusions regarding the ionisation and temperature structures are to be gained from the comparison of the model with the observations. The dimensions of $1.5''$ vs. $4'$ at a distance of 0.86 kpc were assumed in order to be consistent with Paper 1. For each nebular component listed in Table 3 we give the slit and no-slit line intensities in adjacent columns.

The absolute value for the observed $H\beta$ flux of NGC 7009 of $3.197 \times 10^{-10} \text{ erg cm}^{-2} \text{ s}^{-1}$, quoted in Table 3, was obtained from the VLA radio recombination line flux (Garay et al. 1983). This flux should be compared to the nebula-integrated value (NEB) predicted by our model for the no-slit case. A very good agreement (better than 2%) is found.

3.1 Narrow slit effects

From the comparison of the model slit and no-slit columns it appears that some emission lines are more affected than others. This is very easy to understand if one considers the electron temperature distribution and the physical extension of the various regions where each of the relevant ionic species are most abundant.

He $\lambda \lambda 4686$, for example, is enhanced by $\sim 31\%$ in the slit results for the R component and $\sim 29\%$ overall. The opposite behaviour is shown by the He I lines for which the slit results show a $\sim 5\%$ depletion for He $\lambda \lambda 5876$ and He $\lambda \lambda 6678$ in both the R component and NEB component. Similar behaviours are observed for other species as well, particularly we note the enhancement of intensities in the slit column for [N II], [S II] and [S III] lines. We also note that different lines from the same ionic species appear to be enhanced/depleted by different amounts, like [S II] $\lambda 4069$, $\lambda 4076$, enhanced by $\sim 10-12\%$ and $\lambda 6717, \lambda 6731$ enhanced by $\sim 15-18\%$. This is due to the different sensitivities of the various transitions to changes in the electron temperatures.

3.2 Comparison of the emission lines spectrum

We show in the last three columns of Table 3 a comparison of our model with the observations integrated over the whole slit. Whilst a satisfactory agreement is obtained for many emission lines, some discrepancies do remain, including the case of the [N II] and the [S II] lines. These discrepancies can be readily understood by noticing the different emission lines intensities measured for the North-East and the South-West sides of the nebula (Table 1 of Paper 1). In our models we have assumed the nebula to be symmetric about the x-, y- and z-axis and cannot therefore reproduce such asymmetries, which will be reflected into the integrated nebular spectrum for the species that are affected the most. This is not a great concern for us, given that a detailed model...
Table 3. Model and observed dereddened spectra of R, K and NEB. Line intensities are normalized to H\(\beta\)=100.

| Line Identification (Å) | Model no-slit | R Model slit | Obs. | Model no-slit | K Model slit | Obs. | Model no-slit | NEB Model slit | Obs. |
|------------------------|---------------|--------------|------|---------------|--------------|------|---------------|----------------|------|
| H\(\beta\)(10\(^{-13}\) erg cm\(^{-2}\) s\(^{-1}\)) | 3119 | 397.1 | - | 7.93 | 2.94 | - | 3136 | 405.7 | 3197* |
| [N\(\text{ii}\)] 3726.0 + 3728.8 | 5.38 | 5.88 | 15.5 | 7.34 | 213. | 241. | 204. | 6.06 | 8.21 | 24.2 |
| [Ne\(\text{iii}\)] 3868.7 | 34.5 | 34.8 | 51.7 | 44.6 | 40.5 | 40.1 | 35.3 | 34.6 | 34.9 | 48.9 |
| [S\(\text{ii}\)] 4068.6 | 0.31 | 0.35 | 1.50 | 2.36 | 4.95 | 5.46 | 47.3 | 5.04 | 4.00 | 1.98 |
| [S\(\text{ii}\)] 4076.4 | 0.10 | 0.11 | 0.87 | 0.11 | 1.63 | 1.80 | 1.83 | 1.00 | 0.13 | 1.11 |
| [O\(\text{iii}\)] 4363.2 | 8.54 | 8.55 | 7.76 | 8.65 | 10.2 | 9.88 | 6.94 | 8.55 | 8.59 | 8.15 |
| He\(\text{i}\) 4685.7 | 16.8 | 22.0 | 26.0 | 23.4 | 0.00 | 0.00 | 1.00 | 16.7 | 21.6 | 15.8 |
| **[Ar\(\text{iv}\)] 4711** | 3.92 | 3.87 | 5.48 | 4.96 | 1.50 | 1.40 | 2.19 | 3.91 | 3.84 | 4.10 |
| [Ar\(\text{iv}\)] 4740.2 | 5.19 | 5.24 | 5.56 | 4.54 | 1.27 | 1.19 | 1.14 | 5.17 | 5.18 | 3.92 |
| H\(\beta\) 4861.3 | 100. | 100. | 100. | 100. | 100. | 100. | 100. | 100. | 100. | 100. |
| [N\(\text{ii}\)] 5006.8 | 1197 | 1172 | 1162 | 1125 | 1315 | 1272 | 1238 | 1198 | 1177 | 1206 |
| [Cl\(\text{ii}\)] 5517.7 | 0.35 | 0.36 | 0.43 | 0.43 | 1.09 | 1.08 | 0.00 | 0.35 | 0.37 | 0.54 |
| [Cl\(\text{ii}\)] 5537.9 | 0.57 | 0.59 | 0.53 | 0.55 | 1.04 | 1.03 | 0.00 | 0.57 | 0.59 | 0.64 |
| [N\(\text{i}\)] 5754.6 | 0.10 | 0.11 | 0.14 | 0.18 | 3.04 | 3.48 | 6.49 | 3.91 | 3.91 | 0.46 |
| He\(\text{i}\) 5875.7 | 14.1 | 13.4 | 13.9 | 14.1 | 15.8 | 15.9 | 18.8 | 14.1 | 13.5 | 14.5 |
| [S\(\text{ii}\)] 6312.1 | 1.47 | 1.55 | 1.27 | 1.14 | 4.04 | 3.99 | 3.89 | 1.48 | 1.59 | 1.68 |
| [N\(\text{ii}\)] 6583.4 | 5.87 | 6.64 | 7.09 | 5.60 | 167. | 193. | 355. | 6.39 | 8.44 | 27. |
| He\(\text{i}\) 6678.1 | 3.99 | 3.82 | 3.87 | 3.95 | 4.48 | 4.49 | 7.49 | 3.99 | 3.83 | 3.97 |
| [S\(\text{ii}\)] 6716.5 | 0.46 | 0.53 | 0.48 | 0.38 | 21.9 | 24.2 | 36.8 | 0.53 | 0.79 | 2.33 |
| [S\(\text{ii}\)] 6730.8 | 0.84 | 0.99 | 0.84 | 0.69 | 29.3 | 32.4 | 51.0 | 0.94 | 1.32 | 3.85 |

The Model spectra are given with and without considering the narrow slit effect. Upper rows give the observed intensities of the North-East (R1 and K1) part of the nebula, while those for the South-West (R2 and K4) zone are given in the lower rows. See Figure 1 of Paper i for the sizes of R1, K1, R2 and K4. “0.00” as the model predicted He\(\text{ii}\)\(\lambda\)4686 emission and as the observed [Cl\(\text{iii}\)]\(\lambda\)5517,5737 from K, former means that model does not produce any such emission and latter means that such lines were not detected in the spectra of the knots. *The absolute value of the observed H\(\beta\) flux for the whole nebula was obtained from Garay et al. (1989). **From the spectra in Paper i what we actually measured was [Ne\(\text{iii}\)]\(\lambda\)3967.5+H\(\text{e}\)\(\lambda\)4686 and [Ar\(\text{iv}\)]\(\lambda\)4711+He\(\text{i}\)\(\lambda\)4713.

Specific to NGC 7009 is not being sought, our goal being the construction of a model able to explain the apparent enhancement of some low-ionisation species in LISs of PNe such as NGC 7009 in terms of photoionisation effects only, without the need of assuming enhancements in the total elemental abundances of those regions. For this reason, in the modelling we aimed at reproducing the spectra of the individual regions (R and K) with values falling between or being close to one of the observational data for North-East or the South-West regions.

A good agreement is shown in the rim and shell (namely R, as justified in Section 2.3.2) with nearly all predicted lines falling at intermediate values between the two sides of the nebula (upper and lower rows of the Obs column), or being
Gonçalves, Ercolano, et al.

Table 4. Mean temperatures (K) weighted by ionic species.

| Element | I  | II | III | IV | V  | VI | VII |
|---------|----|----|-----|----|----|----|-----|
| H       | 10.002 | 10.378 | 10.625 | 10.562 | 10.106 | 10.380 | 10.106 |
| He      | 9.832 | 9.955 | 12.238 | 9.603 | 10.562 | 10.565 | 9.914 |
| C       | 9.800 | 9.874 | 10.089 | 10.733 | 12.887 | 12.813 | 10.378 |
| N       | 9.798 | 9.875 | 10.098 | 10.706 | 12.757 | 13.158 | 13.072 |
| O       | 9.795 | 9.863 | 10.073 | 12.452 | 12.962 | 13.183 | 13.301 |
| Ne      | 9.805 | 9.920 | 12.209 | 12.590 | 13.026 | 13.269 | 13.312 |
| S       | 9.793 | 9.845 | 10.021 | 10.423 | 11.341 | 13.021 | 13.072 |

For each element, from the upper to the lower row, we show values for R, K and NEB, respectively.

3.3 Mean temperatures

Mean temperatures weighted by the ionic abundances are given in Table 4. Expressions for the volume integrated fractional ionic abundances (discussed in Section 3.4) as well as those for the mean temperatures weighted by ionic abundances were defined in Ercolano et al. (2003a), their eqs. (2) and (3). The values listed in Table 4 show that the temperature structure of the nebula is well reproduced by our model. Figures of \( T_\text{e} \[\text{O} \text{iii}\] \) obtained for regions R, K and NEB are within the empirical values determined in Paper I (see Table 1) for the two sides of the nebula, namely 10,100 K, 10,000 K, and 10,100 K for rim, knots and NEB, respectively. Similarly, the values of \( T_\text{e} \[\text{N} \text{ii}\] \) of the three components agree with the empirical values to better than 5%, while for \( T_\text{e} \[\text{S} \text{ii}\] \) the agreement is to within 33% and 11% for the two knots, due to our model underestimating of the \( \text{[S} \text{ii]} \) auroral lines (see Section 3.2).

We confirm that the electron temperature distribution is fairly homogeneous across the various regions of the nebula, see for instance Rubin et al. (2002) and Sabbadin et al. (2002). The mean \( T_\text{e} \) for the whole nebula, from the neutral (I) to the highly ionized ions (VII), for all species, are respectively 10,200±200 K, 10,050±150 K, 10,050±850 K, 11,400±1050 K, 12,600±700 K, 13,100±200 K and 12,650±1,300 K.

Our model results for \( N_\text{e} \) and \( T_\text{e} \) compare well with the many estimates available from the literature; in particular \( T_\text{e} \[\text{O} \text{iii}\] \) and \( T_\text{e} \[\text{N} \text{ii}\] \) for the rim, can be compared to the values of Hyung & Aller (1995a,b); Mathis et al. (1998); Kwitter & Henry (1998); Luo et al. (2001); Sabbadin et al. (2004); Hyung & Aller (1995a,b); Mathis et al. (1998); Kwitter & Henry (1998); Gonsalves et al. (2002). As for the outer knots, \( T_\text{e} \[\text{N} \text{ii}\] \) and \( T_\text{e} \[\text{O} \text{iii}\] \) were found as being 8,100 K and 11,500 K by Balick et al. (2001), 9,300 K and 9,100 K by Kwitter & Henry (1998) and 13,350 K and 10,000 K by Gonsalves et al. (2002).
Nitrogen abundance of the NGC 7009’s outer knots

Table 5. Averaged fractional ionic abundances.

| Element | Ion | i  | ii | iii | iv | v  | vi | vii |
|---------|-----|----|----|-----|----|----|----|----|
| H       |     | 5.40(-4) | 0.999 | 1.04(-2) | 0.989 | 6.46(-4) | 0.999 |
| He      |     | 7.00(-4) | 0.814 | 0.185 | 7.39(-3) | 0.992 | 7.74(-4) | 0.817 | 0.181 |
| C       |     | 3.10(-6) | 1.08(-2) | 0.555 | 0.428 | 5.45(-3) |
| N       |     | 1.37(-6) | 5.84(-3) | 0.546 | 0.443 | 4.12(-3) | 1.00(-5) |
| O       |     | 4.83(-6) | 9.72(-3) | 0.862 | 0.125 | 3.13(-3) | 1.11(-6) |
| Ne      |     | 3.56(-6) | 1.07(-2) | 0.917 | 7.11(-2) | 7.20(-4) |
| S       |     | 6.84(-7) | 8.13(-3) | 0.343 | 0.546 | 9.89(-2) | 2.64(-3) | 9.97(-6) |
| Ar      |     | 1.52(-7) | 1.25(-3) | 0.363 | 0.613 | 1.90(-2) | 2.86(-3) | 2.51(-5) |

For each element the first row is for the R, the second is for K, and the last one is for NEB.

3.4 Fractional ionic abundances

Results for the fractional ionic abundances are shown in Table 5. Hydrogen and helium are fully at least singly-ionized in R, K and NEB, and significant fractions of the heavy elements are in higher ionization stages in the rim as well as in the nebula as a whole (NEB). We also note the lower ionization of the knots. An important issue that should be noted here is the N/N$^+$ ratio being higher than the O/O$^+$ ratio by a factor of 1.39 in the knots, 1.66 rim and 1.61 in the total nebula. This result is at variance with the N/N$^+$ ratio of Paper i, whose discrepancy reaches 70%.

In Figure 1 we show the ionic abundance profiles of the model, as well as the electron density and temperature profiles along the axis which includes the knots. The top-left panel in this Figure, reflects the N_e (continuous line) and T_e (dashed line) variations through the long axis of the nebula. It shows that N_e peaks at the innermost region of the R-component, having a mean value through the component of 4,100 cm$^{-3}$. The J-component as well as the K-component have mean N_e of 1385 cm$^{-3}$ and 1640 cm$^{-3}$, respectively, while somewhat smoothed profiles are present at the edges. The electron density profile is a result of the ionization structure combined with the density distribution assumed in Section 2.3.3. As for the T_e profiles, although varying somewhat within the components, have mean values that are in close agreement with the values measured in Paper 1, see numbers quoted in the previous subsection. This will be discussed further in Section 4.

Figure 1 also highlights the strong dependence of the ionization level on the geometry and density distribution of the gas. It is therefore clear that an apparent overabundance of N$^+$ in the knots can be produced by providing the correct gas opacity to screen this region from the direct stellar photons. We should add at this point that an even larger N$^+$ abundance can be obtained by further enhancing the gas density at the rim-jet interface, without significantly changing the [S ii] density ratio in the R-component.
Figure 1. Model profiles along the major axis. From left to right and top to bottom, the electron density and temperature, as well the ionic fraction for He, C, N, O and S are shown. The position of the \( R \)-component (9.9 \( \times 10^{16} \) up to 1.84 \( \times 10^{17} \) cm) and that of the \( K \)-component (3.49 \( \times 10^{17} \) up to 3.87 \( \times 10^{17} \) cm) are marked in each panel. After the inner cavity the inner wall of the ellipsoidal shell is encountered, this has \( N_{H} = 4,000 \) cm\(^{-3} \) and \( N_{e} \sim 5,000 \) cm\(^{-3} \). The value of \( N_{e} \) decreases through the first shaded area that represents the ellipsoidal shell, as the level of ionisation of the gas decreases. After the first shaded area we find the jet (\( N_{H} = 1,250 \) cm\(^{-3} \), \( N_{e} \sim 1,500 \) cm\(^{-3} \)). At the end of the jet we find the knot, where and the \( N_{H} = 1,500 \) cm\(^{-3} \), while \( N_{e} \) reaches about 1,700 cm\(^{-3} \).

3.5 Visualization of the model results

NGC 7009 has been observed with the HST WFPC2 with filters centred in the [O \textsc{iii}]\( \lambda 5007 \) and [N \textsc{ii}]\( \lambda 6583 \) emission lines (see Section 2.1). These archive images, already published in Paper i, are compared to the model predicted emission maps in Figure 2. The maps were produced at an inclination of 84° with respect to the line of sight, as indicated by the kinematics of the PN polar axis (Reay & Atherton 1985), for [O \textsc{iii}] (right top panel) and [N \textsc{ii}] (lower right panel) emission lines. First of all we call the readers attention to the fact that the polar bubble, which appear in the HST [O \textsc{iii}] image, at fainter intensity levels, as an extension of the shell, and also the inner pair of knots, \( K_{2} \) and \( K_{3} \) were not considered in our modelling. Thus, excluding the polar bubble and the inner knots, the images and our maps are at least qualitatively in good agreement (note that a photometric comparison is beyond the scope of this paper).

From the maps we note the higher excitation of the equatorial axis of the PN as compared to the polar one, because of the higher density of the rim, and following the pattern known from previous studies, as stated in the Introduction. One also clearly sees that the [N \textsc{ii}] map is more extended than the [O \textsc{iii}] one, as expected from a nebula excited by a central star. Finally, one notes that the knots are fainter (as compared to the inner regions) in [O \textsc{iii}] and brighter in [N \textsc{ii}], due to the enhanced recombination in the knots. All the above suggest that our model can satisfactorily reproduce the main features in the images of NGC 7009, that could in principle be extended to other PNe with a pair of outer low-ionization knots (see, for more examples, Gonçalves et al. 2001).

4 ALTERNATIVE MODEL

As noted in previous sections, the ionisation structure of the model is strongly dependent on the input 3D density distribution. Because of that, here we also present the results obtained with a slightly different geometry, which we call the alternative model. The only difference in terms of input density distribution between our initial model and the alternative one is that in the latter the jets start 3.3% further away from the central star than in the previous model. Since the H number density in the jet is four times lower than the value in the rim at the position of the polar axis, our change results into an increase in the line optical depth along the path connecting the central star to the outer pair of knots. The intensity of the ionizing stellar field reaching the knots is therefore decreased in our alternative model, hence resulting in a lower ionization level. Other model parameters were also adjusted in order to optimise our fit of the spectrum and these are summarised in Table 6. More importantly, with this alternative model we aimed at reproducing the more extreme [N \textsc{ii}] emission of the Eastern knot of the nebula (Knot1 in Paper i and in Figure 2).

As in the case of our original model discussed in Section 3, we found a good agreement of the alternative model with the dereddened fluxes of Paper i, for the rim, the knots as well as for the whole nebula (NEB). Again, most of the
Figure 2. Left panel: HST [O iii] and [N ii] images of NGC 7009 on a logarithmic intensity scale; slit position for the spectra discussed in Section 3 is indicated by short lines (P.A. = 79°), while labels mark the position of the several structures, where “K”, “J” and “R” stand for “knot”, “jet”, and “rim”, respectively. Right panel: project emission maps from the model; the top map is [O iii] and the bottom one is [N ii]; maps (in arbitrary units) are not on the same colour intensity scale.

Table 6. Input parameters for the alternative model.

| Parameter | Value |
|-----------|-------|
| $L_\ast$ ($L_\odot$) | 3316 |
| $T_{eff}$ (K) | 82,000 |
| $R_{in}$ (cm) | 0.0 |
| $R_{out}$ (cm) | 3.88 x 10^{17} |
| He/H | 0.12 |
| C/H | 3.5 x 10^{-4} |
| $N/H$ | 2.8 x 10^{-4} |
| $O/H$ | 4.5 x 10^{-4} |
| $Ne/H$ | 1.0 x 10^{-4} |
| $S/H$ | 0.9 x 10^{-5} |
| $Ar/H$ | 1.5 x 10^{-6} |
| $Fe/H$ | 1.25 x 10^{-6} |

Abundances are given by number, relative to H.

line intensities are within 10-30% of one of the sides of NGC 7009, or between the two values of the Eastern and Western sides, as quoted in Table 7, with the exceptions of the [S ii]λλ4068,4076 and [S ii]λλ6717,6731 doublets, as pointed out in Section 3.2.

The mean $T_e$ of the alternative model also compare very nicely with the observed values, namely, $T_e$[O iii] and $T_e$[N ii] of 10,200 K and 9,950 K, 10,600 K and 10,600 K, and 10,200 K and 10,100 K, for rim, knots and NEB, respectively. And the $N_e$ are automatically matched with the empirical values, since they are constraining the model as input parameters.

The corresponding ionisation structure is shown in Table 8, from which we obtain the N/N$^+$ and O/O$^+$ ratios of the different zones in the nebula, as being: 1.64 (R-component), 1.33 (K-component) and 1.56 (NEB), that, as above, are in contradiction with the ratios adopted in the empirical $icf$ scheme. We chose to include Table 8 in the paper, although qualitatively similar to Table 5, to allow for model $icfs$ to be derived at a later time should one wish to do so.

Finally, the projected emission maps obtained from this model are not shown here as they are qualitatively identical to those of our original model (right panels of Figure 2).

5 CONCLUSIONS

This work focused on the study of the apparent N overabundance in the outer knots of NGC 7009 with respect to the main nebular rim and shell. Alexander & Balick (1997) and Gruenwald & Viegas (1998) showed that long-slit data may give spurious overabundances of N and other elements in the outer regions of model PNe. These authors have identified the different charge-exchange reaction rates of N and O as the main responsible for the effect in the low-ionization regions of the nebulae, therefore affecting the (N$^+$/N)/(O$^+$/O) ratio, that as discussed in Section 3.4, is commonly used to obtain the $icf$ for nitrogen. However, as shown by Mampaso (2004) charge-exchange reactions can, in this case, account at most by 20% of the nitrogen overabundance of the knots of NGC 7009.

In this paper we have presented a model that was able to reproduce the main spectroscopic characteristics of the various spatial regions of NGC 7009 without the need of assuming an inhomogeneous set of abundances. We investigated the importance of taking into account the effects of a narrow slit, and our results in Table 3 and 7 show that the convolution of the model results with the profile of the narrow slit used for the observations presented in Paper 1 results in the [N ii] emission being enhanced respect to H$\beta$ in all regions of the nebula, with the effect being slightly more pronounced in the knots.

The (N$^+$/N)/(O$^+$/O) predicted by our models are 0.60, 0.72 and 0.62 (or alternatively, 0.61, 0.75 and 0.64) for the
| Line Identification (Å) | Model no-slit | R Model slit | Obs. | Model no-slit | K Model slit | Obs. | Model no-slit | NEB Model slit | Obs. |
|------------------------|--------------|-------------|------|--------------|-------------|------|--------------|---------------|------|
| Hβ(10^{-13} erg cm^{-2} s^{-1}) | 3120 | 398.5 | - | 7.95 | 2.95 | - | 3137 | 407.1 | 3197* |
| [Oii] 3726.0 + 3728.8 | 5.37 | 5.95 | 15.5 | 257. | 299. | - | 204. | 6.18 | 8.81 | 24.2 |
| [Neii] 3868.7 | 109. | 110. | 106. | 123. | 122. | - | 82.3 | 109. | 110. | 108. |
| **[Neii] 3967.5** | 33.8 | 34.0 | 51.7 | 38.1 | 37.9 | - | 35.3 | 33.8 | 34.1 | 48.9 |
| [Si] 4068.6 | 0.32 | 0.36 | 1.50 | 5.89 | 6.72 | - | 7.43 | 0.33 | 0.42 | 1.98 |
| [Si] 4076.4 | 0.10 | 0.12 | 0.87 | 1.16 | 2.36 | - | 2.36 | 0.11 | 0.14 | 1.11 |
| [Oiii] 4363.2 | 9.11 | 9.13 | 7.76 | 9.86 | 9.39 | - | 6.94 | 9.12 | 9.16 | 8.15 |
| Hei 4685.7 | 18.4 | 24.0 | 26.0 | 23.4 | 4.96 | - | 1.00 | 18.3 | 23.5 | 15.8 |
| **[Ariv] 4711** | 4.28 | 4.23 | 5.48 | 1.49 | 1.38 | - | 2.19 | 4.27 | 4.19 | 4.10 |
| [Ariv] 4740.2 | 5.67 | 5.73 | 5.56 | 1.26 | 1.17 | - | 1.14 | 5.65 | 5.65 | 3.92 |
| Hβ 4861.3 | 100. | 100. | 100. | 100. | 100. | - | 100. | 100. | 100. | 100. |
| [Oii] 5006.8 | 1234 | 1208 | 1162 | 1225 | 1205 | - | 1238 | 1234 | 1212 | 1206 |
| [Clii] 5517.7 | 0.35 | 0.36 | 0.43 | 1.07 | 1.05 | - | 0.00 | 0.35 | 0.37 | 0.54 |
| [Clii] 5537.9 | 0.57 | 0.59 | 0.53 | 1.03 | 1.01 | - | 0.00 | 0.57 | 0.60 | 0.64 |
| [Nii] 5754.6 | 0.15 | 0.16 | 0.14 | 0.18 | 5.33 | - | 6.49 | 0.16 | 0.22 | 0.46 |
| Hei 5875.7 | 15.0 | 14.3 | 13.9 | 14.1 | 17.0 | - | 18.8 | 15.0 | 14.4 | 14.5 |
| [Siii] 6312.1 | 1.50 | 1.57 | 1.27 | 1.14 | 3.97 | - | 3.89 | 1.50 | 1.61 | 1.68 |
| [Nii] 6583.4 | 8.12 | 9.32 | 7.09 | 5.60 | 296. | - | 355. | 9.03 | 12.5 | 27. |
| Hei 6678.1 | 4.25 | 4.06 | 3.87 | 3.95 | 4.81 | - | 7.49 | 4.26 | 4.08 | 3.97 |
| [Sii] 6716.5 | 0.45 | 0.54 | 0.48 | 26.0 | 28.9 | - | 36.8 | 0.54 | 0.85 | 2.33 |
| [Sii] 6730.8 | 0.84 | 1.00 | 0.84 | 0.69 | 34.9 | - | 51.0 | 0.96 | 1.41 | 3.85 |

As in Table 2, with the top/bottom rows given intensities of the Eastern/Western size of the nebula.

rim, knots and the whole nebula, respectively, all at variance with the icf assumption of unity for this ratio. The icfs will therefore be underestimated by the empirical scheme, both in the case of the R and K components, but more so in the former (by a factor of 1.21). Therefore this effect may partly be responsible for the apparent inhomogeneous N abundance derived from observations. The differences in the (N+/N)/(O+/O) in these two components may be due to a number of effects, including charge exchange, as stated above, and the difference in the ionization potentials of the relevant species, which makes the (N+/N)/(O+/O) ratio extremely sensitive to the shape of the local radiation field. For this reason, a realistic density distribution is essential to the modelling of a non-spherical object, if useful information is to be extracted from spatially resolved observations. The density distribution of the gas modifies the shape of the local radiation field via the gas opacities and (as demonstrated in Section 3.4 and Section 4) matching the emission line spectrum of a given nebular region relies, among other things, on the careful construction of a model capable of providing the correct optical depths in various directions.

Our main conclusions discussed here may also be extended to other PNe exhibiting FLIERs, such as NGC 6543.
Table 8. Averaged fractional ionic abundances for the alternative model.

| Ion | Element | I   | II  | III | IV  | V   | VI  | VII |
|-----|---------|-----|-----|-----|-----|-----|-----|-----|
| H   | 5.53(-4)| 0.999 | 1.38(-2)| 0.986 | 6.91(-4)| 0.999 |
| He  | 6.95(-4)| 0.809 | 0.190 | 8.99(-3)| 0.991 |
| C   | 3.19(-6)| 1.06(-2)| 0.542 | 0.440 | 6.48(-3)| 1.62(-5)|
| N   | 1.39(-6)| 5.70(-3)| 0.533 | 0.455 | 5.08(-3)| 1.62(-5)|
| O   | 4.90(-6)| 9.40(-3)| 0.856 | 0.130 | 3.94(-3)| 1.95(-6)|
| Ne  | 3.42(-6)| 1.02(-2)| 0.910 | 7.85(-2)| 1.03(-3)|
| S   | 7.12(-7)| 8.03(-3)| 0.333 | 0.548 | 0.105 | 3.38(-3)| 1.58(-5)|
| Ar  | 5.51(-5)| 1.22(-3)| 0.351 | 0.621 | 2.17(-2)| 3.94(-3)| 4.39(-5)|

Top to bottom rows give values for R, K and NEB, respectively.

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