Plasma Physics of the Very Local Interstellar Medium

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Abstract. Models of the large-scale heliosphere show that the very local interstellar medium (VLISM), bounded by the heliopause and either a bow wave or bow shock, experiences significant heating by the deposition of neutral hydrogen (H) originating from the hot inner heliosheath. As hot neutrals stream into the interstellar medium, they experience charge exchange with the background cooler interstellar protons, creating a population of energetic (∼1 keV) pickup ions. Similarly, fast neutrals created in the supersonic solar wind stream into the local interstellar medium and create a pickup ion population in the VLISM. The importance of these pickup ions is thought to manifest itself in the creation of the IBEX ribbon. Like the outer regions of the supersonic solar wind and the inner heliosheath, the VLISM is a pickup ion mediated plasma. Here we derive a model of a pickup ion mediated plasma using an approach analogous to a Chapman-Enskog expansion. We derive the anomalous heat flux and obtain a three-fluid model comprising electrons, thermal protons, and pickup ions. We investigate waves in a pickup ion mediated VLISM plasma, comparing the basic properties to those of the better known two-fluid model.

1. Introduction

The interstellar plasma upwind of the heliopause is mediated by energetic PUIs. It was noted already by Zank et al., 1996 [1] that energetic neutral H created via charge exchange in the inner heliosheath (IHS) and fast solar wind could “splash” back into the VLISM where they would experience a secondary charge exchange. The secondary charge exchange of hot and/or fast neutral H with cold (∼6300 K [2]) VLISM protons leads to the creation of a hot or suprathermal PUI population in the VLISM. The heating of the VLISM has been discussed in detail by Zank et al., 2013 [3] who pointed out that the heating of the VLISM plasma would result in an increase of the sound speed with a concomitant weakening or even elimination of the bow shock (yielding instead a bow wave). Zirnstein et al., 2014 [4] have extended the Zank et al., 2010 [5] model by including the multiple PUI populations that contribute to the heating of the VLISM [3]. Here, we restrict our attention to neutrals created in both the IHS and supersonic solar wind, i.e., with typical speeds of ∼100 km/s or ∼400 km/s, that experience secondary charge exchange in the VLISM. The PUIs form a tenuous ($n_p \approx 5 \times 10^{-5}$ cm$^{-3}$ [4]) suprathermal component in the VLISM.

Coulomb collisions can equilibrate the background VLISM thermal plasma and the PUI protons. Assume the background VLISM is a Maxwellian plasma comprised of thermal protons
and electrons. PUIs then satisfy the ordering

\[ v_{ts} \ll v_p < v_{te}, \]

where \( v_{ts/e} \) denotes the background proton/electron thermal speed respectively and \( v_p \) the PUI speed. For PUIs scattering collisionally off a Maxwellian distribution of background protons, the collision frequency becomes

\[ \nu_{ps} = \frac{n_s e^4 \ln \Lambda}{2\pi e^2 m_p v^3 s}, \]

illustrating the well-known \( v^{-3} \) dependence with PUI speed. By contrast, PUI collisional scattering off a Maxwellian electron background yields a larger collision frequency which is given by

\[ \nu_{pe} = \frac{n_e e^4 \ln \Lambda m_e^{1/2}}{2(2\pi)^{3/2} e^2 (kT_e)^{3/2} m_p}. \]

If the collisional time scale exceeds the characteristic flow time of the plasma region of interest, \( \tau_f \approx L/U \), where \( L \) is the size of the region and \( U \) the characteristic velocity, then the PUI distribution will not equilibrate with the background plasma. Expressions (1) and (2) are necessary to determine whether one needs to introduce a model that distinguishes the PUIs from the background plasma protons. In the case of the supersonic solar wind, Isenberg, 1986 [6] used related arguments to justify the introduction of a form of multi-fluid model to describe a coupled solar wind - PUI plasma. Isenberg [6] assumed instantaneous isotropization of the PUI distribution at the time of creation. While a reasonable approximation when considering the effects of PUIs on a large-scale flow such as the supersonic solar wind, to understand the impact of PUIs on the basic plasma physical system requires that the PUI-scattering time scale not be neglected. As we discuss below, PUI scattering yields the dominant heat flux and dissipative (collisionless viscosity) processes in a collisionless coupled thermal plasma - PUI system.

For the VLISM plasma, we take the background proton and electron number density to be approximately \( n_{s/e} \sim 0.08 \text{ cm}^{-1} \) and the PUI speed to be \( \sim 100 \text{ km/s} \) (i.e., H created in the IHS). This yields a lower limit on the PUI-proton collision time of \( \tau_{ps} = (\nu_{ps})^{-1} \sim 1.3 \times 10^9 \text{ s} \). For a length scale of 75 AU, and typical flow speed of 15 km/s, the characteristic flow time is \( \tau_f \sim 7.5 \times 10^8 \text{ s} \). For PUI-electron collisions, we assume an electron temperature, \( T_e \sim 10,000 \text{ K} \). Consequently, \( \tau_{pe} \sim 3.2 \times 10^8 \text{ s} \), which is comparable to the characteristic flow time \( \tau_f \) on a scale of 75 AU. Thus, neither proton nor electron collisions can equilibrate a PUI-thermal plasma in the VLISM on scales smaller than at least 75 AU.

2. Governing model

In deriving a multi-fluid model that includes PUIs, we shall assume that the distribution functions for the background protons and electrons are each Maxwellian, which ensures the absence of heat flux or stress tensor terms for the background plasma. The exact continuity, momentum, and energy equations governing the thermal electrons and protons are therefore given by

\[ \frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{u}_e) = 0; \]

\[ m_e n_e \left( \frac{\partial \mathbf{u}_e}{\partial t} + \mathbf{u}_e \cdot \nabla \mathbf{u}_e \right) = -\nabla P_e - e n_e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}); \]

\[ \frac{\partial P_e}{\partial t} + \mathbf{u}_e \cdot \nabla P_e + \gamma_e P_e \nabla \cdot \mathbf{u}_e = 0, \]
for the electrons, and

\[
\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{u}_s) = 0; \tag{6}
\]

\[
m_p n_s \left( \frac{\partial \mathbf{u}_s}{\partial t} + \mathbf{u}_s \cdot \nabla \mathbf{u}_s \right) = -\nabla P_s + e n_s (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}); \tag{7}
\]

\[
\frac{\partial P_s}{\partial t} + \mathbf{u}_s \cdot \nabla P_s + \gamma_s P_s \nabla \cdot \mathbf{u}_s = 0, \tag{8}
\]

for the protons. Here \(n_{e/s}, \mathbf{u}_{e/s}, \) and \(P_{e/s}\) are the macroscopic fluid variables for the electron/proton number density, velocity, and pressure respectively, \(\gamma_{e/s}\) the electron/proton adiabatic index, \(\mathbf{E}\) the electric field, \(\mathbf{B}\) the magnetic field, and \(e\) the charge of an electron.

Pickup ions initially form an unstable distribution that excites Alfvénic fluctuations. The self-generated fluctuations and \textit{in situ} turbulence serve to scatter PUIs in pitch-angle. The Alfvén waves and magnetic field fluctuations both propagate and convect with the bulk velocity of the system \(\mathbf{U} = \mathbf{U}(\mathbf{u}_e, \mathbf{u}_p, n_e, n_s, n_p, m_e, m_p),\) where \(n_p\) and \(\mathbf{u}_p\) refer to PUI variables. The PUIs are governed by the Fokker-Planck transport equation with a collisional term \(\delta f/\delta t|_c,\)

\[
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{e}{m_p} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_v f = \frac{\delta f}{\delta t|_c}, \tag{9}
\]

for average electric and magnetic fields \(\mathbf{E}\) and \(\mathbf{B}\). We assume that the velocity \(\mathbf{v}\) of PUIs is always non-relativistic. On transforming the transport equation (9) into a frame that ensures there is no change in PUI momentum and energy due to scattering, assuming that the cross-helicity is zero, and introducing the random velocity \(\mathbf{c} = \mathbf{v} - \mathbf{U},\) we obtain

\[
\frac{\partial f}{\partial t} + (U_i + c_i) \frac{\partial f}{\partial x_i} + \left[ \frac{e}{m_p} (\mathbf{E} + \mathbf{U} \times \mathbf{B})_i + \frac{e}{m_p} (\mathbf{c} \times \mathbf{B})_i - \frac{\partial U_i}{\partial t} - (U_j + c_j) \frac{\partial U_i}{\partial x_j} \right] \frac{\partial f}{\partial c_i} = \left. \frac{\delta f}{\delta t|_c} \right|.
\]

(10)

The velocity \(\mathbf{U}\) is still unspecified so we choose \(\mathbf{U}\) such that \(\mathbf{E}' \equiv \mathbf{E} + \mathbf{U} \times \mathbf{B} = 0.\) This assumption corresponds to choosing

\[
\mathbf{U}_\perp = \mathbf{U} - \mathbf{U}_\parallel = \frac{\mathbf{E} \times \mathbf{B}}{B^2} \equiv \mathbf{U}, \tag{11}
\]

since we choose \(\mathbf{U}_\parallel = 0\) (\(\mathbf{U}_\parallel\) is parallel to \(\mathbf{B}\) and therefore arbitrary). This corresponds to expressing (9) in the guiding center frame. The use of the velocity \(\mathbf{U}\) then yields

\[
\frac{\partial f}{\partial t} + (U_i + c_i) \frac{\partial f}{\partial x_i} + \left[ \frac{e}{m_p} (\mathbf{c} \times \mathbf{B})_i - \frac{\partial U_i}{\partial t} - (U_j + c_j) \frac{\partial U_i}{\partial x_j} \right] \frac{\partial f}{\partial c_i} = \left. \frac{\delta f}{\delta t|_c} \right|.
\]

(12)

By taking moments of (12), we can derive the evolution equations for the macroscopic PUI variables, such as the number density \(n_p = \int f d^3c,\) velocity \(n_p u_{p_i} = \int c_i f d^3c,\) and so on. Although unspecified for now, we shall assume that moments of the collisional term \(\delta f/\delta t|_c\) are zero. This can be checked against the particular scattering model that we use below. The zeroth moment of (12) yields the continuity equation for PUIs,

\[
\frac{\partial n_p}{\partial t} + \frac{\partial}{\partial x_i} (n_p (U_i + u_{p_i})) = 0, \tag{13}
\]

where \(\mathbf{u}_p\) is the PUI bulk velocity in the guiding center frame. For the first moment, we multiply (12) by \(c_j\) and integrate over velocity space. This yields, after a little algebra,

\[
\frac{\partial}{\partial t} \left( n_p (U_j + u_{p_j}) \right) + \nabla \cdot [n_p \mathbf{U} (U_j + u_{p_j}) + n_p \mathbf{u}_p U_j] + \frac{\partial}{\partial x_i} \int c_i c_j f d^3c = \frac{e}{m_p} n_p \varepsilon_{jkl} u_{p_k} B_l, \tag{14}
\]
where \( \varepsilon_{ijk} \) is the Levi-Civita tensor.

To close equation (14), we need to evaluate the PUI distribution function \( f \), which requires that we solve (12). In solving (12), we assume 1) that the PUI distribution is gyrotropic, and 2) that scattering of PUIs is sufficiently rapid to ensure that the PUI distribution is nearly isotropic. We can therefore average (12) over gyrophase, obtaining the “focused transport equation” for non-relativistic particles [8]. Details of the derivation can be found in Ch. 5 of [7]. To solve the gyrophase-averaged transport equation requires that we specify the scattering or collisional operator. We make the simplest possible choice, which is the isotropic pitch-angle diffusion operator,

\[
\frac{\partial}{\partial \mu} \left( \nu_s (1 - \mu^2) \frac{\partial f}{\partial \mu} \right),
\]

where \( \mu = \cos \theta \) is the cosine of the particle pitch-angle \( \theta \), and \( \nu_s = \tau_s^{-1} \) is the scattering frequency. The form of the scattering operator (15) allows us to solve the focused transport equation using a Legendre polynomial expansion of the distribution function \( \partial f/\partial \mu \). We can therefore average (12) over gyrophase, obtaining the “focused transport equation” for\( \phi \)(and of course gyrophase equation using a Legendre polynomial expansion of the distribution function \( D/Dt \) the magnetic field, and \( c \) is the particle random speed, \( c \) are functions of position, time, and particle random speed \( c \) i.e., independent of pitch-angle \( \mu \) (and of course gyrophase \( \phi \)). Of particular importance is the retention of the large-scale velocity, acceleration, and shear terms. These terms are often neglected in the derivation of the transport equation describing \( f_0 \) (for relativistic particles, the transport equation is the familiar cosmic ray transport equation). Thus, the second term in (18) is typically neglected. However, the velocity derivative term in the moment equation (18) is well known in relativistic transport theory for cosmic rays and is known as the relativistic heat inertia term [e.g. 11,12,13]. In the context of deriving a multi-fluid model, retaining these terms is essential to derive the correct multi-fluid formulation for PUIs. By introducing

\[
\int c_i c_j f d^3 c = \int (c_i - u_{p_i})(c_j - u_{p_j}) f d^3 c + n_p u_{p_i} u_{p_j} \equiv \int c'_i c'_j f d^3 c + n_p u_{p_i} u_{p_j}
\]

we show that

\[
\frac{\partial}{\partial x_i} \int c'_j f_0 d^3 c = \frac{1}{m_p} \frac{\partial}{\partial x_i} (\delta_{ij} P_p), \quad \text{and} \quad \int c'_j \mu f_1 d^3 c = 0,
\]

where

\[
P_p \equiv m_p \frac{4\pi}{3} \int c^2 f_0 d^2 dc.
\]

Consequently, the PUI stress tensor is identically zero at the first-order and the pressure tensor is isotropic, \( \delta_{ij} P_p \). Including the second-order terms in the Legendre polynomial expansion of the gyrophase-averaged transport equation does in fact yield a non-zero collisionless stress tensor. The PUI momentum equation to first-order can therefore be expressed as

\[
\frac{\partial}{\partial t} \left( n_p \left( U_j + u_{p_j} \right) \right) + \frac{\partial}{\partial x_i} \left[ n_p \left( U + u_p \right) \left( U_j + u_{p_j} \right) + \frac{1}{m_p} \delta_{ij} P_p \right] = \frac{e}{m_p} n_p \varepsilon_{jkl} u_{pk} B_l.
\]
To derive the transport equation for $P_p$, we multiply (12) by $\frac{1}{2}c^2$ and integrate over $d^3c$. We then use (16) - (18) to evaluate the various integrals. Introducing $c' = c - u_p$ as before, we find
\[
\int \frac{1}{2}c^2 f_0 d^3c = \frac{3}{2} \frac{1}{m_p} P_p + \frac{1}{2} n_p u_p^2 ,
\]
for example. Similarly, we find that the heat flux $q_i(x, t)$ can be expressed as
\[
q_i(x, t) \equiv \int \frac{1}{2} c^2 c' f d^3c' = \frac{1}{2} \int c^2 c f d^3c - \frac{5}{2} \frac{1}{m_p} u_p P_p - \frac{1}{2} n_p u_p^2 u_p .
\]
It then follows that
\[
\int \frac{1}{2} c^2 c' f_0 d^3c = \pi \int c^3 \mu b_i f_0 c^2 dc' = 0 ,
\]
and
\[
\int \frac{1}{2} c^2 c' f_1 d^3c = - \frac{2\pi}{3} \int c^2 \kappa_{ij} \frac{\partial f_0}{\partial x_j} c^2 dc' = - \frac{1}{2} \kappa_{ij} \frac{\partial P_p}{\partial x_j} = q_i(x, t) .
\]
In (23), we introduced the spatial diffusion coefficient
\[
\kappa_{ij} = b_i c^2 \tau_s \frac{\partial}{\partial x_j} ,
\]
valid with PUI speed-averaged form $\kappa_{ij} \equiv K_{ij}$ The collisionless heat flux for PUIs is therefore described in terms of the PUI pressure gradient and consequently the averaged spatial diffusion introduces a PUI diffusion time and length scale into the multi-fluid system.

We obtain, after some algebra, the transport equation for the PUI pressure
\[
\frac{\partial P_p}{\partial t} + (u_p + U) \cdot \nabla P_p + \frac{5}{3} P_p \nabla \cdot (u_p + U) = \frac{1}{3} \nabla \cdot (K \cdot \nabla P_p) ,
\]
illustrating that the PUI heat flux yields a spatial diffusion term in the PUI equation of state. The PUI system of equations is properly closed and correct to the first-order. The PUI total energy equation may be expressed as
\[
\frac{\partial}{\partial t} \left( \frac{3}{2} P_p + \frac{1}{2} n_p (u_p + U)^2 \right) + \frac{\partial}{\partial x_i} \left[ \frac{1}{2} n_p (u_p + U)^2 (u_{pi} + U_i) + \frac{5}{2} P_p (u_{pi} + U_i) - \frac{1}{2} \kappa_{ij} \frac{\partial P_p}{\partial x_j} \right] = \frac{e}{m_p} \varepsilon_{ijk} n_p u_p B_k (u_{pi} + U_i) .
\]
The full system of PUI equations is given by (13), (21), and (25) or (26). It is not particularly illuminating to work in the guiding center frame, and we simplify (13), (21), and (25), (26), by letting
\[
U_p = u_p + U .
\]
The RHS of equations (21) and (26) is proportional to $u_p \times B$, which becomes
\[
(U_p - U) \times B = E + U_p \times B ,
\]
since $E$ was perpendicular to $B$ by construction. Hence the PUI fluid equations can be written in the more familiar form
\[
\frac{\partial n_p}{\partial t} + \nabla \cdot (n_p U_p) = 0 ;
\]
\[
\frac{\partial}{\partial t} (n_p U_p) + \nabla \cdot [n_p U_p U_p + IP_p] = \frac{e}{m_p} n_p (E + U_p \times B) ;
\]
\[
\frac{\partial}{\partial t} \left( \frac{3}{2} P_p + \frac{1}{2} n_p U_p^2 \right) + \nabla \cdot \left[ \frac{1}{2} n_p U_p^2 U_p + \frac{5}{2} P_p U_p - \frac{1}{2} K \cdot \nabla P_p \right] = \frac{e}{m_p} n_p U_p \cdot (E + U_p \times B) ,
\]
which is the form we use below. Similarly, we have
\[ \frac{\partial P_p}{\partial t} + U_p \cdot \nabla P_p + \frac{5}{3} P_p \nabla \cdot U_p = \frac{1}{3} (\nabla \cdot K \cdot \nabla P_p) . \] (30)

The full thermal electron-thermal proton-PUI multi-fluid system is therefore given by equations (3) - (8) and (27) - (29) or (30), together with Maxwell’s equations
\[ \frac{\partial B}{\partial t} = -\nabla \times E; \] (31)
\[ \nabla \times B = \mu_0 J; \] (32)
\[ \nabla \cdot B = 0; \] (33)
\[ J = e \left( n_s u_s + n_p U_p - n_e u_e \right), \] (34)
where \( J \) is the current and \( \mu_0 \) the permeability of free space. The diffusion tensor is assumed to be of a simple diagonal form (i.e., we do not include the off-diagonal terms associated with drift and curvature - see the discussion in [7]) and we specify
\[ K = \begin{pmatrix}
\kappa_\perp & 0 & 0 \\
0 & \kappa_\perp & 0 \\
0 & 0 & \kappa_\parallel
\end{pmatrix} ; \] (35)
\[ k \cdot K \cdot k = k^2 (\kappa_\perp \sin^2 \theta + \kappa_\parallel \cos^2 \theta); \] (36)
\[ \kappa_\perp = \eta \frac{1}{3} \Omega_p C_0^2, \quad \kappa_\parallel = \frac{1}{3} \Omega_p C_0^2, \] (37)
and \( \eta = 0.01 \). In estimating the diffusion coefficients (37) from (24), we choose a typical PUI speed for the VLISM and assume that the scattering time can be approximated by the PUI gyrofrequency.

3. Linear wave modes in the VLISM
Here we consider wave modes admitted in the PUI mediated VLISM as described by the multi-fluid equations (3) - (8) and (27) - (34). We linearize the PUI multi-fluid equations using
\[ n_e/s/p = n_{e0}/s_{0}/p_{0} + \delta n_e/s/p; \quad u_e/s = \delta u_e/s; \quad U_p = \delta U_p; \quad P_e/s/p = P_{e0}/s_{0}/p_{0} + \delta P_e/s/p; \]
\[ B = B_0 + \delta B = B_0 \hat{z} + \delta B; \quad E = \delta E, \]
and seek plane wave solutions \( \propto \exp[i(\omega t - k \cdot x)] \), where \( \omega \) is the frequency, and \( k = (k_x, 0, k_z) \) the wave number. The neglect of electron inertia yields an 11th-order polynomial dispersion relation,
\[ \sum_{n=0}^{11} A_n \left( \frac{\omega}{\Omega_p} \right)^n = 0, \] (38)
where the coefficients are cumbersome and given elsewhere. We introduce the proton gyrofrequency \( \Omega_p = eB_0/m_p \), the electron number density \( n_0 \), the density ratio \( \alpha \equiv n_{e0}/n_0 \) and thus \( n_{p0} = n_0(1 - \alpha) \). The square of the Alfvén speed is defined with the electron number density \( n_0 \) as
\[ V_A^2 = \frac{B_0^2}{\mu_0 n_0 m_p} . \]
and the thermal proton, electron, and PUI sound speeds are defined as

\[
C_s^2 = \frac{\gamma_s P_s \rho_s}{\rho_s n_{s0} m_p}, \\
C_e^2 = \frac{m_e \gamma_e P_{e0}}{m_p \rho_{e0}} = \frac{\gamma_e P_{e0}}{n_{e0} m_p}, \\
C_p^2 = \frac{\gamma_p P_{p0}}{\rho_{p0}} = \frac{\gamma_p P_{p0}}{n_{p0} m_p},
\]

and \(\gamma_{e/s/p}\) refers to the appropriate adiabatic index.

By way of example, the dispersion relation for parallel propagating waves factors into three polynomials. The first corresponds to dispersive Alfvén waves, i.e., ion-cyclotron or whistler modes, satisfying

\[
\omega^4 - (2k^2 V_A^2 + \frac{k^4 V_A^4}{\Omega_p^2})\omega^2 + k^4 V_A^4 = \left(\omega^2 + \frac{k^2 V_A^2}{\Omega_p} \omega - k^2 V_A^2\right) \left(\omega^2 - \frac{k^2 V_A^2}{\Omega_p} \omega - k^2 V_A^2\right) = 0, \tag{39}\]

with solutions

\[
\omega = \pm \frac{k^2 V_A^2}{2\Omega_p} + k V_A \sqrt{1 + \left(\frac{k V_A}{2\Omega_p}\right)^2} \quad \text{(yields } \omega > 0); \tag{40}
\]

\[
\omega = \pm \frac{k^2 V_A^2}{2\Omega_p} - k V_A \sqrt{1 + \left(\frac{k V_A}{2\Omega_p}\right)^2} \quad \text{(yields } \omega < 0). \tag{41}
\]

The ion-cyclotron mode satisfies \(\omega \rightarrow \pm \Omega_p\) in the limit that \(k \rightarrow \infty\), whereas the whistler mode satisfies \(\omega \rightarrow \pm \infty\) in the same limit (since electron inertia is neglected).

The second factorization corresponds to a hybrid proton-PUI mode

\[
\omega = \pm \Omega_p. \tag{42}
\]

The remaining polynomial dispersion relation couples the proton and PUI sound modes, and is given by

\[
\omega^5 - \frac{i}{3} k \cdot \mathbf{K} \cdot \mathbf{k} \omega^3 - (C_e^2 + C_s^2 + C_p^2)k^2 \omega^3 + \frac{i}{3} k \cdot \mathbf{K} \cdot \mathbf{k}(C_e^2 + C_s^2)k^2 \omega^2
\]

\[
+ \left((1 - \alpha)C_e^2 C_s^2 + C_p^2 (C_s^2 + \alpha C_e^2)\right)k^4 \omega - \frac{i}{3} k \cdot \mathbf{K} \cdot \mathbf{k}(1 - \alpha)k^4 C_e^2 C_s^2 = 0. \tag{43}
\]

For the VLISM, we have approximately \(\alpha \simeq 1\). The dispersion relation is therefore usefully rewritten as

\[
\left(\omega^4 - \frac{i}{3} k \cdot \mathbf{K} \cdot \mathbf{k} \omega^3 - (C_e^2 + C_s^2 + C_p^2)k^2 \omega^2 + \frac{i}{3} k \cdot \mathbf{K} \cdot \mathbf{k}(C_e^2 + C_s^2)k^2 \omega + C_p^2 (C_s^2 + \alpha C_e^2)k^4\right) \omega
\]

\[
= (1 - \alpha)k^4 C_e^2 C_s^2 \left(\frac{i}{3} k \cdot \mathbf{K} \cdot \mathbf{k} - \omega\right). \tag{44}
\]

The diffusion term \(k \cdot \mathbf{K} \cdot \mathbf{k}\) can be expressed as \(\kappa_i k^2\) for parallel wave propagation.

For a characteristic speed \(V_0\), we may exploit the length scale introduced by the diffusion coefficient \(\kappa_i\) to consider long wavelength and short wavelength solutions of (44). For the long wavelength limit, we obtain five solutions, one of which is a purely damped mode. Another
Therefore, beyond some critical angle $\theta$, the order of the modes will always be $\omega_A < \omega_f < \omega_{sp} < \omega_{fp}$, i.e., red, green, blue, cyan, black. However, as $\theta$ increases, the frequency of the fast proton (blue) mode increases too with $\omega_f \rightarrow k \sqrt{V_S^2 + V_A^2}$ and the frequency of the slow pickup ion (cyan) mode decreases with $\omega_{sp} \rightarrow 0$. Therefore, beyond some critical angle $\theta_c$, the fast proton mode is faster than the slow pickup ion mode (i.e., the blue mode will find itself above the cyan mode). This happens at the critical angle

$$\cos^2 \theta_c = \frac{V_A^2}{C_p^2} \left( 1 + \frac{V_S^2}{V_A^2} - \frac{V_S^2}{C_p^2} \right).$$  

(48)
Figure 1. In the left and right panels, the horizontal dotted lines correspond to the asymptotes $\omega = \Omega_p \cos \theta$ and $\omega = \Omega_e \cos \theta$. **Top panel:** $\theta = 0^\circ$, 2-fluid wave modes (left); 3-fluid with diffusion wave modes showing the real frequency (middle panel), and the damping rate (right panel). **Second panel:** $\theta = 30^\circ$, 2-fluid (left); 3-fluid real frequency (middle), and damping rate (right). Note that the fast PUI mode (black) is evanescent at wave numbers greater than about $k \geq \Omega_p / V_A$. **Third panel:** $\theta = 70^\circ$, 2-fluid (left); 3-fluid real frequency (middle), and damping rate (right). **Fourth panel:** $\theta = 88^\circ$, 2-fluid (left); 3-fluid real frequency (middle), and damping rate (right). **Bottom panel:** $\theta = 90^\circ$, 2-fluid (left); 3-fluid real frequency (middle), and damping rate (right).
Furthermore, because $\omega_A < \omega_{sp}$ for all angles $\theta$ beyond the critical angle $\theta_c$, the order of the modes will always be $\omega_s < \omega_A < \omega_{sp} < \omega_f < \omega_f$, i.e., red, green, cyan, blue, black. Of course, this is only approximate since we are discussing the strict limits $\alpha \rightarrow 1$ and $\mathbf{k} \cdot \mathbf{K} \cdot \mathbf{k} \rightarrow 0$.

For numerical solutions of the dispersion relation in the VLISM, we assume a PUI density $n_p = 5 \times 10^{-8} \text{cm}^{-3}$ and an electron density $n_0 = 0.08 \text{cm}^{-3}$, so that $n_p/n_0 = 6.25 \times 10^{-4}$. Thus, the parameter $\alpha = 1 - n_p/n_0 = 0.999375$. We assume a VLISM magnetic field $B_0 = 3 \mu G = 0.3 \text{nT}$, which yields the Alfvén speed as $V_A = B_0/\sqrt{\mu_0 m_0 m_p} = 23.14 \text{km/s}$. The proton temperature is taken to be $T_s = 15,000 \text{K}$, the electron temperature $T_e = T_s$, and the polytropic indices $\gamma_s = \gamma_e = 5/3$. The proton and electron sound speeds are $C^2_s = \gamma_s k_B T_s/m_p$ and $C^2_e = \gamma_e k_B T_e/m_p$, which yields $C_s = C_e = 14.37 \text{km/s}$. Since we are assuming that fast neutrals from the supersonic solar wind and hot neutrals from the IHS are the origin of the PUIs in the VLISM, we use $U_0 \sim 200 \text{ km/s}$ to compute the pickup ion sound speed $C_p = U_0 \sqrt{\gamma_p/7} = 97.59 \text{ km/s}$. For the diffusion tensor, we use a characteristic PUI speed of $C_p \sim 200 \text{ km/s}$ to evaluate the diffusion coefficient. The perpendicular diffusion is suppressed relative to the parallel diffusion coefficient, and is given by $\kappa_\perp = \eta \kappa_\parallel$ with parameter $\eta = 0.01$.

Three columns of panels are shown in Fig. 1. The leftmost column shows numerical solutions of the standard two-fluid plasma model [9, 10] i.e., thermal electrons and protons only. The middle column shows plots of the real frequency as a function of wave number for the PUI-mediated 3-fluid plasma. The rightmost column shows the corresponding imaginary part of the frequency as a function of wave number of the PUI-mediated VLISM. The rows correspond to specific wave propagation directions, from parallel (top row) to perpendicular (bottom), as listed in the figure caption.

The 2-fluid dispersion relation is included for reference, so that the differences in the PUI-mediated model are apparent. The 2-fluid model for wave propagation angles $\theta < 90^\circ$ shows that three waves exist in the system: the fast (blue) and slow (red) magnetosonic modes and the Alfvén mode (green). These are clearly distinguishable for $0 < \theta < 90^\circ$. For large $k$, the parallel Alfvén mode tends to the asymptote $\omega = \Omega_p \cos \theta$, and the fast mode to $\omega = \Omega_e \cos \theta$. For oblique angles, the Alfvén mode (green) eventually resonates at the electron frequency $\Omega_e \cos \theta$. The 2-fluid perpendicular case is degenerate with only the fast mode remaining.

The PUI-mediated model possesses a richer set of wave modes than the 2-fluid case. Shown in the middle panels, the PUI-mediated dispersion relation now exhibits two further modes, a fast PUI (black) and slow PUI (cyan) mode. The PUI sound speed is larger than the thermal plasma sound speeds and so these dispersion curves lie above their thermal counterparts. For parallel propagation, the dispersive Alfvén waves (39) are unchanged from the 2-fluid case. The hybrid PUI mode ($\omega = \pm \Omega_p$) is now present (black curve). Three acoustic modes, the slow PUI mode (cyan, equation (45)), the fast thermal mode (blue, equation (46), and the slow thermal mode (red) exist at long wavelengths (small $k$), and are more clearly distinguished with increasing propagation angle. As illustrated in the top panel of the right column, and supported analytically by expressions (45) and (46), the PUI sound mode is damped and the fast thermal acoustic mode is not. The parallel propagating PUI acoustic mode exists only for $k \leq \Omega_p/V_A$, after which it is completely damped away and no longer propagates. By contrast, at short wave lengths, the slow thermal acoustic mode (equation (47) is coupled only weakly to the PUIs, the propagation speed being $\sim \sqrt{C^2_s + C^2_{ei}}$, but it is damped by PUIs as illustrated by the red curve in the top panel of the right column (see also equation (47)). For parallel propagation ($\theta = 0^\circ$), we note that various modes intersect.

With increasing propagation angles $\theta$, the five wave modes remain conceptually similar in terms of the classification but now, as illustrated in the panels of the leftmost column, all oblique modes are damped by PUIs, including the dispersive Alfvén mode. Also, despite not being damped when propagating parallel to $B_0$, the fast thermal acoustic mode is much more strongly damped than the slow thermal acoustic mode, despite the latter experiencing damping.
at $\theta = 0^\circ$. The fast and slow PUI modes are clearly distinguishable as $\theta$ increases. The fast PUI mode (black) deviates from the $\omega = \Omega_p \cos \theta$ asymptote with increasing $k$, but becomes evanescent shortly after peaking in frequency. The imaginary frequency of course becomes very large for this mode. The slow PUI acoustic mode frequency increases with increasing $k$, tending to the electron gyrofrequency asymptote $\omega = \Omega_e \cos \theta$. The fast and slow PUI acoustic modes can also intersect at certain propagation angles. As noted earlier, above a critical angle $\theta_c$ the fast thermal acoustic mode is faster than the slow PUI mode. This is illustrated in the fourth panel down of Fig. 1. Finally, for perpendicular propagation (last panel down), only the fast PUI and fast thermal acoustic modes remain, both experiencing damping. Above a certain value of $k$, the short wavelength fast PUI mode becomes evanescent.

4. Conclusions

A multi-fluid description of a PUI-mediated plasma has been developed. We have assumed fast scattering of PUIs by turbulence maintains a nearly isotropic PUI distribution. In deriving PUI closures, we followed a Chapman-Enskog-like procedure applied to a collisionless plasma. As a result, we find that PUIs introduce a non-negligible heat flux, which is expressed as spatial diffusion term in the PUI pressure with a diffusion tensor whose elements are proportional to the scattering time and reflect the magnetic field geometry of the system. Although not presented here, we have derived the collisionless viscosity transport coefficients for the PUI fluid.

By way of example, we have applied the new three-fluid model to investigate basic plasma physical processes in the VLISM. With Voyager 1 making in situ observations and IBEX making remote observations of the VLISM, this investigation of basic plasma properties of the VLISM is clearly important. Charge exchange of fast and hot neutral H created in the supersonic solar wind or IHS with VLISM protons leads to the creation of an energetic PUI population that is not equilibrated with the background thermal VLISM plasma within about 75 - 100 AU of the heliopause. Thus, in studying the VLISM, we need utilize a PUI-mediated plasma description.

We have investigated linear wave modes in the VLISM using the new multi-fluid model derived here. Instead of the usual three modes found in a 2-fluid plasma, a richer set of waves is found, which includes fast and slow PUI acoustic modes. Our numerical and analytical results show that PUIs damp all obliquely propagating wave modes, including some wave modes that are damped completely at short wavelengths. Our results suggest that PUIs are likely effective at damping waves in the VLISM. We do not take into account the possibility of Landau damping [14] of VLISM wave modes via the background thermal plasma, but this is likely to further dissipate interstellar medium fluctuations. However, we have not yet addressed the question of whether this efficient PUI damping manifests itself as increased dissipative heating of the VLISM.

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