The classification of imbalanced data has presented a significant challenge for most well-known classification algorithms that were often designed for data with relatively balanced class distributions. Nevertheless, skewed class distribution is a common feature in real world problems. It is especially prevalent in certain application domains with great need for machine learning and better predictive analysis such as disease diagnosis, fraud detection, bankruptcy prediction, and suspect identification. In this paper, we propose a novel tree-based algorithm based on the area under the precision-recall curve (AU-PRC) for variable selection in the classification context. Our algorithm, named as the “Precision-Recall Curve classification tree”, or simply the “PRC classification tree” modifies two crucial stages in tree building. The first stage is to maximize the area under the precision-recall curve in node variable selection. The second stage is to maximize the harmonic mean of recall and precision (F-measure) for threshold selection. We found the proposed PRC classification tree, and its subsequent extension, the PRC random forest, work well especially for class-imbalanced data sets. We have demonstrated that our methods outperform their classic counterparts, the usual CART and random forest for both synthetic and real data. Furthermore, the ROC classification tree proposed by our group previously has shown good performance in imbalanced data. The combination of them, the PRC-ROC tree, also shows great promise in identifying the minority class.

Keywords: Precision-Recall curve · Imbalanced data · PRC random forest · PRC-ROC random forest

1 Introduction

Knowledge Discovery and Data Mining (KDD) is an interdisciplinary area focusing on methodologies for extracting useful information from data. Classification is a fundamental tool in this domain. In the terminology of machine learning[1], classification is considered an instance of supervised learning, i.e., learning where a training set of correctly identified observations is available. The task of the constructed classifier is to predict class labels given any input information such as financial crisis prediction[2][3]. In this introductory section, we provide a brief overview of some well-developed classification algorithms to shed light on the complex issues of class imbalance.

Decision trees can be used to visually and explicitly represent decisions and the decision making process. A typical tree consists of three components. The inner tree nodes represent features with thresholds leading to the branching of edges eventually ended up in terminal nodes called leaves labeled with class decisions. In tree building, the decision tree model is built by recursively splitting the training data set based on a locally optimal criterion (typically information gain or gini impurity until certain stopping criteria are met. After building the decision tree, a tree pruning step is performed to prevent overfitting by reducing the size of the tree. In the presence of the class imbalance, more branches may need to be built in order to identify the minority class. CART is a well-known tree algorithm[4].

A random forest (RF) or random decision forest[5] is an ensemble learning method obtained by generating many decision trees and then combine their decisions. RF is a popular bagging algorithm which incorporates those trees[6].
types of randomness are built in the process. First, tree bagging is employed to select a bootstrap sample from original
the original data for each decision tree. The bootstrap resampling method was introduced by Bradley Efron\cite{7}. It is
implemented by sampling with replacement from an existing sample data independently with same size and conduct-
ing experiments among them. Second, a subset of features is randomly selected to generate the best split in each tree
building process. The reason in this is to decrease the correlation between trees and reduce overfitting problem.
A Support Vector Machine (SVM) introduced by Cortes is a discriminative classifier based on a separating
hyperplane\cite{8}. Given training data, for two-group (i.e., bi-class) classifications, the algorithm outputs an optimal
hyperplane where the margin is maximal. Akbani\cite{9} illustrates that SVMs can be ineffective in determining the class
boundary when the distribution is skewed.
A neural network is a series of algorithms that aims to discover the underlying relationships in a set of training data
through a process that mimics the way the human brain operates. It is a beautiful bio-inspired programming paradigm.
However, some experimental results demonstrate that its performance when dealing with imbalanced data is not satis-
factory. The principal reason is the minority class cannot be adequately weighted\cite{10}.

Besides these arguably the most fundamental classifiers, there are still many other classification algorithms which
we will not be able to introduce exhaustively here. Our goal is modest as we aim to develop a better decision tree
algorithm, and on top of which, a better random forest algorithm, and then compare them to these four original and
fundamental algorithms: CART, Random Forest (RF), SVM, and Neural Network, plus the weighted version of CART
and RF that in a way reflect the current state-of-the-art on combing class-imbalance. In the next section, we shall
introduce several common solutions to the class-imbalance problem.

2 Class imbalance problem

A number of solutions to the class imbalance problem have been presented in the literature. Basically they can be
divided into 3 broad categories, i.e., cost-sensitive learning, data level and algorithm level approaches.

2.1 Cost-sensitive learning

Cost-sensitive learning first introduced by Elkan\cite{11} has shown to be an effective technique for incorporating the
different misclassification costs into the classification process. The goal of this type of learning is to minimize the
total cost of misclassification. The key point is to assign different values to the false positives and negatives. If we
focus on identifying the minority which is the common case, the cost for labeling a positive example as negative can be
higher than the cost for labeling a negative example as positive. Weighted Random Forest is the extended version that
makes random forest more suitable for learning from extremely imbalanced data based on the idea of cost sensitive
learning\cite{12}\cite{13}. Kim\cite{14} proposed the weighted support vector machine which imposes weights to the loss term.
Likewise, some other learning algorithms have their cost-sensitive correspondings, such as cost-sensitive decision tree,
cost-sensitive neural network\cite{15}\cite{16}\cite{17}\cite{18}\cite{19}, etc.

2.2 Data level approaches

The data-level approaches can also be interpreted as resampling techniques. That is, we can either undersample the
majority class or oversample the minority class to attain the balanced distribution (the ratio of class distribution is 1:1).
Undersampling can be defined as removing some observations of the majority class. Some algorithms have been
applied to alleviate the skewness, e.g., random undersampling and cluster-based undersampling\cite{20}. However an
obvious drawback is that we are removing information that may be valuable which can then lead to an underfitting
problem.
Similarly, oversampling before splitting can achieve more minority observations. Well-known algorithms such as
SMOTE\cite{21} and ADASYN\cite{22} have been widely implemented in dealing with imbalanced classification problems.
However, it can cause overfitting and poor generalization.

2.3 Algorithm level approaches

It is always a good rule of thumb to modify the training procedure of algorithms to boost its performance with uneven
datasets. To develop an algorithmic solution, one needs in-depth knowledge of both the classifier learning algorithm
and the application domain. Based on the receiver operating characteristics (ROC), our group had invented the ROC
classification trees and the ROC random forest to identify the rare cases. Song suggested choosing the threshold with
the largest harmonic mean of sensitivity and specificity as the node splitting criterion\cite{23}\cite{24}. Yan\cite{25} extended the
bi-class classification ROC Tree and ROC RF\cite{23} to multiple classes. Our proposed algorithm PRC classification
trees, and later the PRC random forest, are developed from a different perspective as will be introduced in details in the following chapter.

### 2.4 Ensemble methods

It is well known that through combining classifiers one can improve the prediction accuracy. It is effective to embed the data level approaches in boosting procedure which is one of the most popular combination techniques. Sun\[26\] introduced the cost items into the learning framework of AdaBoost for improving the classification of imbalanced data\[27\]. Boosting has been proven to be a general and effective method for improving the accuracy of a given learning algorithm\[28\][29]. Nitesh\[30\] combined the SMOTE algorithm and the boosting procedure and WOTBoost\[31\] adjusted its oversampling strategy at each round of boosting to synthesize more targeted minority data samples. Similarly, RUSBoost alleviated class imbalance by introducing random undersampling techniques into the standard boosting procedure\[32\]. MEBoost mixed two different weak learners with boosting to improve the performance on imbalanced datasets\[33\]. So far, many learning methods have had their boosting counterparts, such as DataBoost\[34\], EUSBoost\[35\].

### 2.5 Performance evaluation

Performance metric plays a key role in evaluating the classifier. Additionally, it is a guideline for the modeling process of classifiers. Accuracy is a traditional measure to evaluate the performance. It may be misleading, however, for a data set with unbalanced class distribution. Suppose we give every observation the prevalent class label, the accuracy can be high but the performance for the rare class is very poor. The following is a brief summary on some commonly used performance metrics.

#### 2.5.1 Confusion matrix

In the bi-class scenario, positive and negative class samples can be categorized into four groups based on the classification process. This can be denoted as the confusion matrix shown in Table 1.

| Actually Positive | Predicted as Positive | Predicted as Negative |
|-------------------|-----------------------|-----------------------|
| True Positive (TP)|                       | False Negative (FN)  |
| Actually Negative | False Positive (FP)   | True Negative (TN)   |

Several measures can be derived from this matrix:

\[
\text{True positive rate (Recall/Sensitivity)} : TPR = \frac{TP}{TP + FN} \quad (1)
\]

\[
\text{False negative rate} : \text{FNR} = \frac{FN}{TP + FN} \quad (2)
\]

\[
\text{True negative rate (Specificity)} : \text{TNR} = \frac{TN}{TN + FP} \quad (3)
\]

\[
\text{False positive rate} : \text{FPR} = \frac{FP}{TN + FP} \quad (4)
\]

\[
\text{Positive predictive value (Precision)} : \text{PPV} = \frac{TP}{TP + FP} \quad (5)
\]

\[
\text{Negative predictive value} : \text{NPV} = \frac{TN}{TN + FN}. \quad (6)
\]

#### 2.5.2 F-measure

For situations where the performance of positive class is the priority, two measures of the indicators described earlier are crucial – the true positive rate and the positive predictive value. Specifically, the true positive rate is defined as recall referring to the percentage of positive class samples that are correctly classified. Correspondingly, the positive predictive value is defined as precision denoting the percentage of predictive positive class samples that are correctly
F-measure is devised to integrate these two measures. In principle, it is the harmonic mean of precision and recall:

\[ F_{\text{measure}} = \frac{2}{\text{recall}^{-1} + \text{precision}^{-1}}. \]  

(7)

In general, F-measure tends to be closer to the smaller one of the measures. Additionally, a high value indicates that both two measures are reasonably high and convincing.

3 Methodology

3.1 ROC Curve and the area under the ROC curve (AUC)

ROC curve is a performance measurement for classification problem at various thresholds settings. The curve is plotted based on the ROC space with True positive rate (TPR) against the false positive rate (FPR) where TPR is on y-axis and FPR is on the x-axis. In the ROC space, a ROC curve is created by connecting all the pairs of TPR and FPR at each threshold for a specific classifier. It is a common method to calculate the area under the ROC curve (AUC) to compare the measurement of separability among different classifiers. The higher AUC refers to a stronger ability of distinguishing between classes. There are several methods to perform a computational analysis of AUC estimators. The way we used in our analysis is to apply the trapezoidal estimators. The estimation is based on an approximation of the entire area by summing up all subareas and the formula is:

\[ \text{AUC}_{\text{trapezoid}} = \frac{1}{2} \cdot \sum_{i=1}^{n} (f_{i+1} - f_i) \cdot (t_{i+1} + t_i) \]

(8)

where \( f \) is the function of FPR and \( t \) is of TPR, the trajectory is partitioned in \( n-1 \) sections.

3.2 PRC and the area under the precision-recall curve (AUPRC)

For the classification problem, the performance is typically defined with the confusion matrix generated by the associated classifier. It is possible to compute the recall (sensitivity), precision and specificity. Similar to ROC curves described in the previous chapter, we connect all the pair of measurements at each threshold to make the PRC plot. Precision-Recall curve plots Precision which is the fraction of observations with a positive predicted value that are truly positive versus Recall, measuring the fraction of the examples with positive labels that get a positive predicted result.

The perfect classifier will have a PRC that passes through the upper right corner (corresponding to 100% precision and 100% recall). In general, the closer a point is to that position, the better the test is. Figure 1 and 2 show PRC and ROC curve for a perfect test respectively. To compare two classifiers based on PRC, the reasonable measure is the area under precision-recall curve (AUPRC). The higher it is, the better the performance of the classifier is. If the area under ROC (AUC) is less than 0.5, representing the baseline value of random guessing, we should convert the negative predictions into positive ones to get a better result. For AUPRC, the corresponding baseline value is the ratio of positive cases in the distribution expressed as

\[ \text{Baseline Value} = \frac{\text{positive cases}}{\text{positive cases} + \text{negative cases}}. \]

(9)

Algorithm 1 below shows the process of calculating the AUPRC and the computation in the case of it less than the baseline value. For the estimation of AUPRC, we also applied the trapezoidal approach:

\[ \text{AUPRC}_{\text{trapezoid}} = \frac{1}{2} \cdot \sum_{i=1}^{n} (r_{i+1} - r_i) \cdot (p_{i+1} + p_i) \]

(10)

where \( r \) is the function of recall and \( p \) is of precision, the trajectory is partitioned in \( n-1 \) sections.
Figure 1: PRC for a perfect test

Figure 2: ROC curve for a perfect test
The Precision-recall plot is more informative than the ROC plot when evaluating binary classifiers on imbalanced datasets. The Precision-recall curve is usually a tortuous curve, fluctuating up and down. Consequently, PRC tends to intersect each other much more often than the ROC curve. The main difference between them is that the number of true-negative results is not used for constructing the PRC. Ekelund [37] illustrates that the precision-recall curves are not impacted by the addition of patients without disease which are the true-negative cases. Evaluation of classifiers via receiver operating characteristics (ROC) curve have been commonly used. However, for imbalanced data, the ROC may tend to give an optimistic view since the classifier is more likely to assign the observation as the majority class. In many real-life problems, we care more about the true positive cases, e.g., fraud detection and disease diagnosis. The Precision-Recall plot is more informative than the ROC Plot when evaluating binary classifiers on imbalanced datasets [38]. Therefore, we propose the classification trees based on maximizing the area under the Precision-Recall curve.

3.3 Feature selection

Feature selection is deciding which variable to include in the model. The data features that are used to train tree-based models have a huge influence on the performance we can achieve. In the case of the PRC classification tree, it is done by Algorithm 2. It lets AUPRC decide which feature variable is useful to include in every split of tree node. Typically, the feature that yields the largest AUPRC value should be selected. In Algorithm 2, the function “AUPRC_calculation” is implemented by Algorithm 1. The below pseudo code illustrates the process of generating the suitable feature with the largest AUPRC value.

Algorithm 1 AUPRC_calculation

Input: Training data \((x_1, y_1), \ldots, (x_n, y_n)\), Feature set \(f_i \in F\), target vector \(y_i \in \{-1, +1\}\).
Output: Area under precision-recall curve (AUPRC) of feature \(f_i\).

1. sort data \((X[f_i], Y)\) by feature
2. set \(\text{uniq_values}_{f_i} \leftarrow \text{sort}(\text{unique}(f_i))\)
3. set \(\text{total_positives} \leftarrow \text{length}(\text{which}(Y == 1))\)
4. set \(\text{total_negatives} \leftarrow \text{length}(\text{which}(Y == -1))\)
5. set \(\text{PRC}_{\text{baseline}} \leftarrow \frac{\text{total_positives} + \text{total_negatives}}{\text{total_positives}}\)
6. set \(\text{recall_array} \leftarrow \text{rep}(0, \text{length}(\text{uniq_values}_{f_i}))\)
7. set \(\text{precision_array} \leftarrow \text{rep}(0, \text{length}(\text{uniq_values}_{f_i}))\)
8. set \(\text{AUPRC}_{f_i} \leftarrow 0\)
9. for \(j = 1\) to length(uniq_values_{f_i}) do
10. \(\text{indice} \leftarrow \text{which}(X[f_i] \leq \text{uniq_values}_{f_i}[j])\)
11. \(\text{recall_array}[j] \leftarrow \text{length}(\text{which}(X[f_i][\text{indice}, Y == 1]))\)
12. \(\text{precision_array}[j] \leftarrow \frac{\text{total_positives} - \text{length}(\text{which}(X[f_i][\text{indice}, Y == 1]))}{\text{length}(\text{indice})}\)
13. if \(\text{precision_array}[j] \text{ < PRC}_{\text{baseline}}\) then
14. \(\text{recall_array}[j] \leftarrow 1 - \text{recall_array}[j]\)
15. \(\text{precision_array}[j] \leftarrow \frac{\text{total_positives} - \text{length}(\text{which}(X[f_i][\text{indice}, Y == 1]))}{\text{length}(\text{indice})}\)
16. end if
17. if \(j = 1\) then
18. \(\text{AUPRC}_{f_i} \leftarrow \frac{\text{recall_array}[j] \cdot (1 + \text{precision_array}[j])}{2}\)
19. else
20. \(\text{AUPRC}_{f_i} \leftarrow \text{AUPRC}_{f_i} + \frac{(\text{recall_array}[j] \cdot \text{precision_array}[j] - \text{recall_array}[j - 1] \cdot \text{precision_array}[j - 1])}{2}\)
21. end if
22. end for
23. return \(\text{AUPRC}_{f_i}\)
24. end

The Precision-recall curve is usually a tortuous curve, fluctuating up and down. Consequently, PRC tends to intersect each other much more often than the ROC curve. The main difference between them is that the number of true-negative results is not used for constructing the PRC. Ekelund [37] illustrates that the precision-recall curves are not impacted by the addition of patients without disease which are the true-negative cases. Evaluation of classifiers via receiver operating characteristics (ROC) curve have been commonly used. However, for imbalanced data, the ROC may tend to give an optimistic view since the classifier is more likely to assign the observation as the majority class. In many real-life problems, we care more about the true positive cases, e.g., fraud detection and disease diagnosis. The Precision-Recall plot is more informative than the ROC Plot when evaluating binary classifiers on imbalanced datasets [38]. Therefore, we propose the classification trees based on maximizing the area under the Precision-Recall curve.
Algorithm 2 Feature_Selection_by_AUPRC

Input: Training data \((x_1, y_1), \ldots, (x_n, y_n)\), Feature set \(f_i \in F\), target vector \(y_i \in \{-1, +1\}\).

Output: Feature \(f_i\) with the largest AUPRC.

Require: AUPRC_calculation.

1: set \(Max_f \leftarrow NULL\)
2: set \(Max_{AUPRC} \leftarrow 0\)
3: for each feature \(f_i \in F\) do
4: \(Temp_{AUPRC} \leftarrow AUPRC_{calculation}(X[f_i], Y)\)
5: if \(Temp_{AUPRC} > Max_{AUPRC}\) then
6: \(Max_f \leftarrow f_i\)
7: \(Max_{AUPRC} \leftarrow Temp_{AUPRC}\)
8: end if
9: end for
10: return \(Max_f, Max_{AUPRC}\)
11: end

3.4 Threshold selection

It is important to calculate the optimum threshold for each split in the PRC Tree. This could be done by selecting the largest F1-score in the chosen feature variable. The F-score is a measure of accuracy which is the harmonic mean of precision and recall.

Note that the F-score does not take the true negatives into account. That’s the reason that it is more effective than other measures in the classification for imbalanced data. Algorithm 3 illustrates the procedure of threshold selection. The basic goal is to choose the cutoff point with the largest F1 after comparing all the scores for the threshold set of the feature achieved by Algorithm 2.

Algorithm 3 Threshold_Selection_by_F1-score

Input: PRC matrix \((\text{recall}_\text{array}, \text{precision}_\text{array})\) of selected feature \(f_i\), where \(f_i \in F\); Threshold set \(\Theta\).

Output: Selected splitting threshold \(\theta\).

Require: function HarmonicMean.

1: set \(Max_{F1} \leftarrow 0\)
2: set \(Max_{\theta} \leftarrow NULL\)
3: set \(uniq\_split_{f_i} \leftarrow \text{sort(unique}(f_i))\)
4: for \(j = 1\) to \(\text{length}(uniq\_split_{f_i})\) do
5: \(Temp_{F1} \leftarrow \text{HarmonicMean}(\text{recall}_\text{array}[j], \text{precision}_\text{array}[j])\)
6: if \(Temp_{F1} > Max_{F1}\) then
7: \(Max_{F1} \leftarrow Temp_{F1}\)
8: \(Max_{\theta} \leftarrow uniq\_split_{f_i}[j]\)
9: end if
10: end for
11: return \(Max_{\theta}\)
12: end

3.5 PRC Tree algorithm

The following Algorithm 4 is employed to generate the PRC Tree. The previous Algorithm 2 and 3 complete a node split process in each stage. Instead of measuring by traditional Gini impurity or information gain, the features and threshold are selected by AUPRC and F1-score respectively. Algorithm 4 “PRC_Tree” integrates the previous algorithms to generate each split node until the corresponding stopping criteria is met. Each node has scores for different classes, measuring the percentage of each class in it. It is called nodescore in the algorithm. The corresponding nodelabel is the majority class of this node. By doing so, it could be easy to achieve the majority target label in the Terminal node. Below is the pseudo code to build PRC classification tree recursively. The prediction of a PRC Tree, \(\mathcal{T}\), with \(K\) terminal nodes and depth \(L\), can be written as

\[
g(x_i; \hat{y}, K, L) = \sum_{i=1}^{K} \hat{y}_k \mathbb{1}_{\{x_i \in C_k(L)\}},
\]
where \( C_k(L) \) is one of the \( K \) partitions of the data. Each partition is a product of up to \( L \) indicator function of the features which are selected by AUPRC algorithm.

**Algorithm 4 PRC_Tree**

**Input:** Training data \((x_1, y_1), \ldots, (x_n, y_n)\), Feature set \( f_i \in F \), target vector \( y_i \in \{-1, +1\} \); stopping criterion (maximum tree depth, minimum leaf size); \( N_f \), the number of features used in each split.

**Output:** PRC tree.

**Require:** Feature_Selection_by_AUPRC and Threshold_Selection_by_F1-score

1. Do the nodescore and nodelabel for the root node.
2. if the stopping criterion is met then
3. return PRC tree
4. else
5. sample \( N_f \) features from the feature set \( F \)
6. set the selected features as \( F' \)
7. \((Max_f, Max_{AUPRC}) \leftarrow Feature\_Selection\_by\_AUPRC(X, Y, F')\)
8. \(Max_\theta \leftarrow Threshold\_Selection\_by\_F1\_score(Max_f, Max_{AUPRC})\)
9. \{\((X, Y)_{left}, (X, Y)_{right}\)\} \leftarrow Node\_Split(Max_f, Max_\theta)\)
10. apply the function \( PRC\_Tree(\text{maximumtreedepth} \leftarrow \text{maximumtreedepth} - 1) \) to the subsets \{\((X, Y)_{left}, (X, Y)_{right}\)\} recursively until resulting nodes are pure or the stopping criteria is met
11. end if
12. return PRC tree
13. end

### 3.6 PRC random forest algorithm

Random forests are an ensemble learning method for classification by constructing a multiple of classification trees. We can build our PRC random forest by treating the PRC tree as the base classifier. The PRC random forest has competitive predictive performance and provides a reliable feature importance estimate. Algorithm 5 “PRC_Random_Forest” states how to build the forests based on the PRC tree. The parameter \( N_t \) is used to decide the number of trees to form the “forest”. As discussed before, the number of features \( N_f \) for each node split is randomly chosen from the feature set \( F \). It can decrease the prediction error of the model by doing so. Below is the procedure of our “forests”.

**Algorithm 5 PRC_Random_Forest**

**Input:** Training data \((x_1, y_1), \ldots, (x_n, y_n)\), Feature set \( f_i \in F \), target vector \( y_i \in \{-1, +1\} \); Number of trees \( N_t \); Number of features for each node split \( N_f \).

**Output:** PRC random forest \( \mathcal{R} \).

**Require:** PRC_Tree

1. Set \( \mathcal{R} \leftarrow NULL \).
2. for \( j = 1 \) to \( N_t \) do
3. generate bootstrap sample \((X, Y)^j\)
4. for each node split, generate \( F' \) by randomly choosing \( N_f \) features from \( F \)
5. \( pre\_tree_j \leftarrow PRC\_Tree((X, Y)^j, F')\)
6. append \( pre\_tree_j \) to \( \mathcal{R} \)
7. end for
8. return PRC random forest \( \mathcal{R} \)
9. end

### 3.7 PRC-ROC Classification Trees

Each evaluation approach has itsown merits and demerits. As an aside, it is comprehensive to combine the PRC and ROC classification trees in meaningful and optimal ways. The following chapters will explore the method based on PRC and ROC curves in more detail.

#### 3.7.1 Feature selection

The one valuable feature of the RF is that we can use the out-of-bag (OOB) samples to make objective performance judgements based on the training data only. The OOB error is an error estimation technique often used to evaluate the
accuracy of a random forest. We will use the weighted average of both areas to do the feature selection. Based on the training data and OOB accuracy score, we can tune the parameter to find the optimal weighted parameter for a given data. The parameter can be expressed as

\[
a = \frac{(1 - OOB\_error_{PRC})}{(1 - OOB\_error_{PRC} + 1 - OOB\_error_{ROC})}.
\]

(12)

Hence, the weighted average of both areas becomes

\[
Area_{weighted} = a(AUPRC) + (1 - a)(AUC), \text{ where } 0 \leq a \leq 1.
\]

(13)

In the case of the PRC-ROC classification tree, it is done by Algorithm 7. Instead of AUPRC, it lets the weighted average of AUPRC and AUC decide which feature variable is useful to include in every split of tree node. The feature that yields the largest mean value should be selected in a similar fashion. Algorithm 6 shows how the function “AUC_calculation” is implemented. Then we achieves the algorithm by integrating it to the previous feature selection Algorithm 2.

---

Algorithm 6 AUC_calculation

**Input:** Training data \((x_1, y_1), \ldots, (x_n, y_n)\). Feature set \(f_i \in F\), target vector \(y_i \in \{-1, +1\}\).

**Output:** Area under curve (AUC) of feature \(f_i\).

1. sort data \((X[f_i], Y)\) by feature
2. set \(uniq\_values_{f_i} \leftarrow sort(unique(f_i))\)
3. set \(total\_positives \leftarrow length(which(Y == 1))\)
4. set \(total\_negatives \leftarrow length(which(Y == -1))\)
5. set \(tp\_array \leftarrow rep(0, length(uniq\_values_{f_i}))\)
6. set \(fp\_array \leftarrow rep(0, length(uniq\_values_{f_i}))\)
7. set \(AUC_{f_i} \leftarrow 0\)
8. for \(j = 1\) to \(length(uniq\_values_{f_i})\) do
9.   indice \leftarrow which\((X[f_i] \leq uniq\_values_{f_i}[j])\)
10.   \(tp\_array[j] \leftarrow length(which((X[f_i], Y)[indice,"Y"] == 1))\)
11.   \(fp\_array[j] \leftarrow length(which((X[f_i], Y)[indice,"Y"] == -1))\)
12.   if \(j == 1\) then
13.     \(AUC_{f_i} \leftarrow \frac{tp\_array[j] + fp\_array[j]}{2}\)
14.   else
15.     \(AUC_{f_i} \leftarrow AUC_{f_i} + \frac{(fp\_array[j] - fp\_array[j-1]) \cdot (tp\_array[j] + fp\_array[j-1])}{2}\)
16.   end if
17. end if
18. if \(AUC_{f_i} < 0.5\) then
19.   \(AUC_{f_i} \leftarrow 1 - AUC_{f_i}\)
20. end if
21. return \(AUC_{f_i}\)
22. end
Algorithm 7 Feature_Selection_by_AUPRC_AUC

**Input:** Training data \((x_1, y_1), \ldots, (x_n, y_n)\), Feature set \(f_i \in F\), target vector \(y_i \in \{-1, +1\}\).

**Output:** Feature \(f_i\) with the largest weighted average of AUPRC and AUC.

**Require:** AUPRC_calculation, AUC_calculation.

1. set \(Max_f \leftarrow NULL\)
2. set \(Max_{AUPRC\_AUC} \leftarrow 0\)
3. for each feature \(f_i \in F\) do
4. \(Temp_{AUPRC\_AUC} \leftarrow \text{Area}_{\text{weighted}}(\text{AUPRC\_calculation}(X[f_i], Y),\ \text{AUC\_calculation}(X[f_i], Y))\)
5. if \(Temp_{AUPRC\_AUC} > Max_{AUPRC\_AUC}\) then
6. \(Max_f \leftarrow f_i\)
7. \(Max_{AUPRC\_AUC} \leftarrow Temp_{AUPRC\_AUC}\)
8. end if
9. end for
10. return \(Max_f, Max_{AUPRC\_AUC}\)
11. end

3.7.2 Threshold selection

To calculate the optimum threshold for each split in the PRC-ROC Tree, we select the largest harmonic mean of three performance metrics, i.e., recall, precision and specificity in the chosen feature variable. The F-score in this case becomes the harmonic mean of precision, recall and specificity:

\[
F_3 = \frac{3}{\frac{1}{\text{recall}} + \frac{1}{\text{precision}} + \frac{1}{\text{specificity}}}.
\]  

Algorithm 8 illustrates the procedure of threshold selection. The main goal is to choose the cutoff point with the largest harmonic mean after comparing all the scores for the threshold set of the feature achieved by feature selection.

Algorithm 8 Threshold_Selection_by_F3-score

**Input:** PRC_AUC matrix (recall_array, precision_array, fp_array) of selected feature \(f_i\), \(f_i \in F\); Threshold set \(\Theta\).

**Output:** Selected splitting threshold \(\theta\).

**Require:** function HarmonicMean.

1. set \(Max_{F_3} \leftarrow 0\)
2. set \(Max_\theta \leftarrow NULL\)
3. set \(uniq\_split_{f_i} \leftarrow \text{sort(unique}(f_i))\)
4. for \(j = 1\) to length(uniq_split_{f_i}) do
5. \(Temp_{F_3} \leftarrow \text{HarmonicMean}(\text{recall_array}[j], \text{precision_array}[j], \text{fp_array}[j])\)
6. if \(Temp_{F_3} > Max_{F_3}\) then
7. \(Max_{F_3} \leftarrow Temp_{F_3}\)
8. \(Max_\theta \leftarrow \text{uniq_split}_{f_i}[j]\)
9. end if
10. end for
11. return \(Max_\theta\)
12. end

3.7.3 PRC-ROC Tree algorithm

The following Algorithm 9 is employed to generate the PRC-ROC Tree. The following algorithm “PRC-ROC_Tree” integrates the previous algorithms to generate each split node until the corresponding stopping criteria is met. Each node has scores for different classes, measuring the percentage of each class in it. It is called nodescore in the algorithm. The corresponding nodelabel is the majority class of this node. By doing so, it could be easy to achieve the majority target label in the Terminal node. Below is the pseudo code to build the classification tree recursively. The prediction of a PRC-ROC Tree, \(T\), with \(K\) terminal nodes and depth \(L\), can also be written as

\[
g(x_i; \hat{y}, K, L) = \sum_{i=1}^{K} \hat{y}_k \mathbb{1}_{\{x_i \in C_k(L)\}}.
\]
where $C_k(L)$ is one of the $K$ partitions of the data. Each partition is a product of up to $L$ indicator function of the features which are selected by AUPRC_AUC algorithm.

**Algorithm 9 PRC-ROC_Tree**

**Input:** Training data $(x_1, y_1), \ldots, (x_n, y_n)$. Feature set $f_i \in F$, target vector $y_i \in \{-1, +1\}$; stopping criterion (maximum tree depth, minimum leaf size); $N_f$, the number of features used in each split.

**Output:** PRC-ROC tree.

1. **Require:** Feature_Selection_by_AUPRC_AUC and Threshold_Selection_by_F3-score
2. **Do** the nodescore and nodelabel for the root node.
3. **if** the stopping criterion is met **then**
   4. **return** PRC-ROC tree
5. **else**
6. sample $N_f$ features from the feature set $F$
7. set the selected features as $F'$
8. $(Max_f, Max_{AUPRC_{AUC}}) \leftarrow$ Feature_Selection_by_AUPRC_AUC$(X, Y, F')$
9. $Max_\theta \leftarrow$ Threshold_Selection_by_F3-score$(Max_f, Max_{AUPRC_{AUC}})$
10. $(\{X, Y\}_{left}, \{X, Y\}_{right}) \leftarrow$ Node_Split$(Max_f, Max_\theta)$
11. **apply** the function $PRC-ROC_{Tree}(maximum_{treedepth} \leftarrow maximum_{treedepth} - 1)$ to the subsets $\{\{X, Y\}_{left}, \{X, Y\}_{right}\}$ recursively until resulting nodes are pure or the stopping criteria is met
12. **end if**
13. **return** PRC-ROC tree
14. **end**

### 3.7.4 PRC-ROC random forest algorithm

We can build our PRC-ROC random forest by treating the PRC-ROC tree as the base classifier in a similar way. The PRC-ROC random forest has competitive predictive performance and provides a reliable feature importance estimate. Algorithm 10 “PRC-ROC_Random_Forest” states how to build the forests based on the PRC-ROC tree. The parameter $N_t$ is used to decide the number of trees to form the “forest”. As discussed before, the number of features $N_f$ for each node split is randomly chosen from the feature set $F$. It can decrease the prediction error of the model by doing so.

**Algorithm 10 PRC-ROC_Random_Forest**

**Input:** Training data $(x_1, y_1), \ldots, (x_n, y_n)$. Feature set $f_i \in F$, target vector $y_i \in \{-1, +1\}$; Number of trees $N_t$; Number of features for each node split $N_f$.

**Output:** PRC-ROC random forest $\mathcal{R}$.

1. **Require:** PRC-ROC_Tree
2. **Set** $\mathcal{R} \leftarrow$ NULL
3. **for** $j = 1$ to $N_t$ **do**
4. generate bootstrap sample $(X, Y)^j$
5. for each node split, generate $F'$ by randomly choosing $N_f$ features from $F$
6. **append** $\text{prc_tree}_j$ to $\mathcal{R}$
7. **end for**
8. **return** PRC-ROC random forest $\mathcal{R}$
9. **end**

### 3.8 Complexity of algorithm

Let us consider a dataset with input size $n$. Time complexity is the measurement of the speed of the algorithm when performing for the input data. Random Forest is the ensemble model of decision trees. The situation is same to our proposed algorithms. For each tree, it will take $O(N_f + n\log(n))$, where $N_f$ is the number of features for each node split. For the random forest algorithm, the complexity would be $O(N_t * N_f + n\log(n))$ and $N_t$ is the number of the trees.
4 Experimental Studies

The experimental studies fall into two parts. Experiments in the first part are designed for simulation study. The goal is to evaluate the performance on classifying the imbalanced simulated data of binary classes. Experiments in the second part focus on several real-world data sets.

The following step after implementing our proposed algorithm is to find out how effective the model is based on the metrics and datasets. Five performance metrics are used to make evaluation between different algorithms in our analysis. Accuracy is the number of correct predictions made by the model over all kinds of predictions made. Specificity is a measure that tells us what proportion of the majority class, are predicted correctly. Precision shows us what proportion of the predicted minority class which is actually positive. Recall or sensitivity tells us the proportion of cases that actually belong to the minority class is identified by the algorithm as positive cases. F1-score, measuring both precision and recall, is the harmonic mean of them.

4.1 Simulation study

In the simulation experimental setting, the number of simulated data sets for classifying binary cases is set to 5,000 each. The feature variables for each class are generated from multivariate Gaussian distribution with specific mean and standard deviation. For our simulation, we divide the class-imbalance extent into 3 categories, namely “Mild”, “Moderate” and “Extreme”, according to the proportion of minority class. In general, the data set whose minority proportion is between 20% and 40% should be placed in the mild category. The criterion for moderate category is from 1% to 20% and the extreme case is when the proportion is less than 1%. Similarly, the dimension of features set to be 5 is considered as low feature category and 20 for the higher dimension. Furthermore, the noisy data may also make it difficult to learn the rare class. In the real world, data contains various types of errors, either random or systematic. Random errors are often referred to as noise. To complete the simulation setting, we also give the scenarios with noisy features for comprehensive comparison. Overall, we have two categories (i.e., easy-to-classify and hard-to-classify) in the simulated data and the following sections will give more detail about setting.

For the simulation study, we compared the performance of our proposed algorithms with that of CART, weighted CART and ROC Tree. The CART and weighted CART are implemented by functions in R package “rpart”[39]. Furthermore, we also compared the performance with random forest algorithms such as standard random forest and weighted version. Random forest (RF) is fit by the function in package “randomForest”[40] and weighted Random Forest (Weighted RF) is implemented in package “wsrf”[13].

4.1.1 Easy to classify, mild imbalanced data set with low feature dimension

In this scenario, the degree of class-imbalance is set to be mild, i.e., 30% of the simulated data points belong to the minority class. Five features are generated randomly from multivariate Gaussian distribution with specified standard deviation 1. The means of the features for the majority and minority class are set to 0 and 3 respectively. Figure 3 shows the distribution of the data with 2 features. Table 2 shows the performance metrics of all compared algorithms and it is divided in two. The first block is for single classification tree and the other is for random forest. The best results are denoted in bold respectively. Both PRC algorithms have better performance in F1 score and accuracy than ROC. The PRC is slightly better than CART. In addition, the PRC-ROC tree achieves the best performance in all aspects in the first block. PRC-ROC RF yields the highest recall, accuracy and F1-score among the random forest algorithms.

| Algorithms     | Recall   | Specificity | Precision | Accuracy | F1 Score |
|----------------|----------|-------------|-----------|----------|----------|
| CART           | 0.9782   | 0.9933      | 0.9846    | 0.9887   | 0.9814   |
| Weighted CART  | 0.9760   | 0.9722      | 0.9391    | 0.9733   | 0.9572   |
| ROC Tree       | 0.9214   | 0.9942      | 0.9860    | 0.9720   | 0.9526   |
| PRC Tree       | 0.9891   | 0.9904      | 0.9784    | 0.9900   | 0.9837   |
| PRC-ROC Tree   | 0.9934   | 0.9952      | 0.9891    | 0.9947   | 0.9912   |
| RF             | 0.9476   | 0.9981      | 0.9953    | 0.9827   | 0.9709   |
| Weighted RF    | 0.9934   | 0.9923      | 0.9827    | 0.9927   | 0.9880   |
| ROC RF         | 0.9476   | 0.9923      | 0.9819    | 0.9787   | 0.9644   |
| PRC RF         | 1.0000   | 0.9904      | 0.9786    | 0.9933   | 0.9892   |
| PRC-ROC RF     | 1.0000   | 0.9914      | 0.9807    | 0.9940   | 0.9903   |
4.1.2 Easy to classify, moderate imbalanced data set with higher feature dimension

In this scenario, the class-imbalance extent becomes moderate. The number of features changes from 5 to 15. Table 3 shows the performance metrics of all the algorithms and the best results are denoted in bold correspondingly. The ROC Tree is perfect in specificity and precision, but the performance of recall is not ideal. We can conclude that the PRC still outperforms both CART in this scenario. Furthermore, PRC-ROC is better than PRC in the five-dimensional evaluation. In the second block, the weighted RF work better than other algorithms under this situation.

Table 3: Summary of classification results under scenario 2 (Moderate imbalanced extent with higher feature dimension)

| Algorithms       | Recall  | Specificity | Precision | Accuracy | F1 Score |
|------------------|---------|-------------|-----------|----------|----------|
| CART             | 0.9556  | 0.9963      | 0.9627    | 0.9927   | 0.9591   |
| Weighted CART    | 0.9259  | 0.9905      | 0.9058    | 0.9847   | 0.9157   |
| ROC Tree         | 0.4667  | 1.0000      | 1.0000    | 0.9520   | 0.6364   |
| PRC Tree         | 0.9704  | 0.9985      | 0.9850    | 0.9960   | 0.9776   |
| PRC-ROC Tree     | 0.9778  | 0.9993      | 0.9925    | 0.9773   | 0.9851   |
| RF               | 0.9556  | 0.9832      | 0.8487    | 0.9807   | 0.8990   |
| Weighted RF      | 0.9259  | 1.0000      | 1.0000    | 0.9933   | 0.9615   |
| ROC RF           | 1.0000  | 0.9875      | 0.8882    | 0.9887   | 0.9408   |
| PRC RF           | 1.0000  | 0.9685      | 0.7584    | 0.9713   | 0.8626   |
| PRC-ROC RF       | 1.0000  | 0.9817      | 0.8437    | 0.9833   | 0.9152   |

4.1.3 Easy to classify, extreme imbalanced data set with low feature dimension

Under this scenario, only 1% of the simulated data points belong to the minority class. The parameter setting of features remains the same as scenario 1. Table 4 shows the performance metrics. The PRC achieves a higher number in recall than the weighted one. Both PRC methods apparently outclass than CART and ROC especially in recall.
PRC-ROC obtains larger values in specificity, precision and F1-score than other methods. Although the weighted RF achieves the best specificity and precision, PRC-ROC RF yields the highest recall, accuracy and F1-score among the random forest algorithms.

Table 4: Summary of classification results under scenario 3 (Extreme imbalanced extent with low feature dimension)

| Algorithms  | Recall | Specificity | Precision | Accuracy | F1 Score |
|------------|--------|-------------|-----------|----------|----------|
| CART       | 0.5833 | 0.9980      | 0.7000    | 0.9947   | 0.6363   |
| Weighted CART | 0.8333 | 0.9919      | 0.4545    | 0.9907   | 0.5882   |
| ROC Tree   | 0.3333 | 0.9980      | 0.5714    | 0.9927   | 0.4210   |
| PRC Tree   | **0.9167** | 0.9973      | 0.7333    | 0.9967   | 0.8148   |
| PRC-ROC Tree | 0.7500 | **0.9993** | **0.9000** |   **0.9973** | **0.8182** |
| RF         | 0.7500 | **0.9995** | **0.9000** |   **0.9973** | **0.8182** |
| Weighted RF | 0.5833 | **1.0000** | **1.0000** |   0.9967 | 0.7368   |
| ROC RF     | 0.8333 | 0.9973      | 0.7143    | 0.9960   | 0.7692   |
| PRC RF     | **1.0000** | 0.9966      | 0.7059    | 0.9967   | 0.8276   |
| PRC-ROC RF | **1.0000** | 0.9973      | 0.7500    |   **0.9973** | **0.8571** |

4.1.4 Hard to classify, mild imbalanced data set with higher feature dimension and noise

In this hard-to-classify scenario, the degree of class-imbalance is set to mild, i.e., 30% of the simulated data points belong to the minority class. Five features are generated randomly from multivariate Gaussian distribution with specifies standard deviation 1. The means of the features for the majority and minority class are set to 0 and -1 respectively. Figure 4 shows the distribution of the data with 2 features. We can see many overlaps compared to the previous setting. The setting for this scenario is with ten normal features and five noise features. We change their means to 1 to treat them as noisy features. Table 5 summarizes the performance metrics of the algorithms mentioned and the best performance is denoted in the same way. The results obviously indicate that in this specific case PRC-ROC surpasses other tree-algorithms in every metrics except the recall value. For the comparison of the random forest methods, PRC-ROC RF outperforms others.

Table 5: Summary of classification results under scenario 4 (Mild imbalanced extent with higher feature dimension and noise)

| Algorithms  | Recall | Specificity | Precision | Accuracy | F1 Score |
|------------|--------|-------------|-----------|----------|----------|
| CART       | 0.6372 | 0.9141      | 0.7554    | 0.8327   | 0.6913   |
| Weighted CART | **0.7846** | 0.8036      | 0.6245    | 0.7980   | 0.6955   |
| ROC Tree   | 0.6689 | 0.8687      | 0.6797    | 0.8100   | 0.6743   |
| PRC Tree   | 0.7347 | 0.8820      | 0.7216    | 0.8387   | 0.7281   |
| PRC-ROC Tree | 0.7143 | **0.9056** | **0.7590** |   **0.8493** | **0.7360** |
| RF         | 0.5011 | 0.8911      | 0.6097    | 0.7807   | 0.5735   |
| Weighted RF | 0.7279 | 0.9452      | 0.8470    | 0.8813   | 0.7829   |
| ROC RF     | 0.9297 | 0.8291      | 0.6937    | 0.8587   | 0.7945   |
| PRC RF     | **0.9751** | 0.8263      | 0.7003    | 0.8700   | 0.8152   |
| PRC-ROC RF | 0.9478 | **0.8565** | **0.7333** |   **0.8833** | **0.8269** |

4.1.5 Hard to classify, extreme imbalanced data set with higher feature dimension and noise

In this extreme data set, the features’ distribution is the same as scenario 4. Table 6 summarizes the comparison of all mentioned methods and the best results are denoted in bold. The PRC-ROC tree attains the best performance in the last four dimensions and makes huge improvement in F1 score. The ROC RF achieves the highest F1-score in the second block.

To sum up, the comparisons under most of those cases illustrate that two PRC trees achieve better performance than the both CART trees when the data set is imbalanced in either easy-to-differentiate or hard-to-differentiate case. Especially in the case of the extreme data set, the PRC tree is more powerful to identify the minority class and achieves higher
Figure 4: Scatterplot of data in scenario 4 (Mild imbalanced extent with low feature dimension) for hard-to-classify setting

Table 6: Summary of classification results under scenario 5 (Extreme imbalanced extent with higher feature dimension and noise)

| Algorithms       | Recall  | Specificity | Precision | Accuracy | F1 Score |
|------------------|---------|-------------|-----------|----------|----------|
| CART             | 0.2000  | 0.9966      | 0.2857    | 0.9913   | 0.2353   |
| Weighted CART    | 0.6000  | 0.9403      | 0.0632    | 0.9380   | 0.1144   |
| ROC Tree         | 0.1000  | 0.9919      | 0.0769    | 0.9860   | 0.0869   |
| PRC Tree         | 0.2000  | 0.9987      | 0.5000    | 0.9933   | 0.2857   |
| PRC-ROC Tree     | 0.2000  | 0.9993      | 0.6667    | 0.9940   | 0.3077   |
| RF               | 0.1000  | 0.9933      | 0.0909    | 0.9873   | 0.0952   |
| Weighted RF      | 0.1000  | 0.9960      | 0.1429    | 0.9900   | 0.1177   |
| ROC RF           | 0.7000  | 0.9322      | 0.0648    | 0.9307   | 0.1186   |
| PRC RF           | 0.6000  | 0.9483      | 0.0723    | 0.9460   | 0.1290   |
| PRC-ROC RF       | 0.6000  | 0.9523      | 0.0779    | 0.9500   | 0.1381   |

score in precision and recall than traditional CART. The ROC and PRC Tree have different focuses when dealing with imbalanced data. The PRC Tree is better than the ROC Tree when the main goal is to identify the rare cases and achieve the satisfactory performance in F1 score. Moreover, PRC-ROC is slightly better than PRC in our 5-dimensional evaluation in most cases since it inherits the advantage of both classification trees. For the second block, the two PRC RFs also show some advantages in classifying the minority cases. The following section moves on to the hard-to-classify setting. The next section focuses on the evaluation in the real-world data set.

4.2 Real-world data set experiment

The PRC and PRC-ROC Tree have shown their advantage in classifying the simulated data set which is severely skewed. In this section we choose several datasets from Kaggle competition and UCI Machine Learning Repository to testify the feasibility of PRC random forests. We will compare the performance with standard classification algorithms such as random forest, SVM and neural network. In addition, weighted CART and ROC random forest are all taken into account. Random forest is fit by the function in package “randomForest” [40]. SVM fits in package
“e1071” and the package “neuralnet” is for realizing neural network. Weighted Random Forest (Weighted RF) is implemented in package “wrf”.
The Table 7 summarizes the basic information of the three selected data sets. The first two data sets are about financial distress prediction and default of clients. The remaining one is about breast cancer diagnosis. For the credit data, this study employed a binary variable, default payment (Yes = 1, No = 0), as the dependent variable. Besides, this data set used 23 features as the independent variables. Similarly, the target variable in the second data is denoted by "Financial Distress" if it is greater than -0.50 the company should be considered as healthy (0). Otherwise, it would be regarded as financially distressed (1). We have 83 explanatory variables for it. The binary outcome of interest for the cancer data set that contains 30 attributes is whether a tumor is benign (0) or malignant (1).
The out-of-bag error is an error estimation technique often used to evaluate the accuracy of a random forest. We split the data into training and testing and use the OOB to estimate the error rate of the training sample. The following table shows the out-of-bag error of all the corresponding random forests and the minimum values are colored in red. Then we still use the separate testing data to further validate our RF model built with the training data and the results are given in the last three tables.

Table 7: Summary of selected real-world datasets

| Dataset Name                     | Data Sources | Number of Observations | Proportion of Minority Class | Number of Features |
|----------------------------------|--------------|------------------------|-----------------------------|-------------------|
| Default of Credit Card Clients   | UCI          | 30000                  | 3.7%                        | 23                |
| Financial Distress Prediction    | Kaggle       | 3672                   | 22%                         | 83                |
| Breast Cancer Wisconsin          | UCI          | 569                    | 37%                         | 30                |

Table 8: Summary of Out-of-bag error rate

| Dataset Name                     | RF           | Weighted RF             | ROC RF         | PRC RF         |
|----------------------------------|--------------|-------------------------|----------------|----------------|
| Default of Credit Card Clients   | 0.1900       | 0.2300                  | 0.2446         | 0.1921         |
| Financial Distress Prediction    | 0.0414       | 0.0500                  | 0.0345         | 0.0000         |
| Breast Cancer Wisconsin          | 0.1000       | 0.0912                  | 0.0714         | 0.0455         |

Table 9: Summary of performance results for Default of Credit Card Clients Data

| Algorithms          | Recall   | Specificity | Precision | Accuracy | F1 Score |
|---------------------|----------|-------------|-----------|----------|----------|
| SVM                 | 0.3343   | 0.9628      | 0.7204    | 0.8229   | 0.4567   |
| Neural Network      | 0.3947   | 0.9441      | 0.6692    | 0.8218   | 0.4965   |
| CART                | 0.3378   | 0.9627      | 0.7218    | 0.8236   | 0.4602   |
| Weighted CART       | 0.5788   | 0.8253      | 0.4870    | 0.7704   | 0.5289   |
| ROC Tree            | 0.7061   | 0.5330      | 0.3022    | 0.5716   | 0.4233   |
| PRC Tree            | 0.4187   | 0.9200      | 0.5997    | 0.8083   | 0.4931   |
| PRC-ROC Tree        | 0.3353   | 0.9634      | 0.7241    | 0.8236   | 0.4584   |
| RF                  | 0.3493   | 0.9608      | 0.7187    | 0.7839   | 0.4701   |
| Weighted RF         | 0.3558   | 0.9570      | 0.7032    | 0.8231   | 0.4968   |
| ROC RF              | 0.6307   | 0.5850      | 0.3033    | 0.5952   | 0.4096   |
| PRC RF              | 0.5744   | 0.8439      | 0.5132    | 0.8247   | 0.5432   |
| PRC-ROC RF          | 0.5813   | 0.8340      | 0.5009    | 0.7778   | 0.5381   |

Then we experiment with real-world data sets and summarize the performance comparison in Tables 9 to 11. The best performance is denoted in bold. Furthermore, we use green to highlight the best tree model and blue to color the best non-tree model. The results are quite consistent with the out-of-bag error rate table. In general, the proposed algorithm can achieve higher recall and F1-score than traditional random forest. PRC random forest achieves the largest accuracy, F1-score and PRC-ROC attains the highest precision in the first case. Despite random forest achieving a slightly higher specificity, the recall and F1-score are much lower than PRC random forest and PRC-ROC RF. This is even more confirmed in the Financial Distress Prediction Data. In this distress data, SVM tends to classify all the cases as the majority class. Weighted RF did a good job in the performance of precision and accuracy. However, PRC random
Table 10: Summary of performance results for Financial Distress Prediction Data

| Algorithms | Recall   | Specificity | Precision | Accuracy | F1 Score |
|------------|----------|-------------|-----------|----------|----------|
| SVM        | 0.0000   | 1.0000      | NaN       | 0.9600   | NaN      |
| Neural Network | 0.3100  | 0.9774      | 0.3514    | 0.9528   | 0.3294   |
| CART       | 0.1951   | 0.9830      | 0.3079    | 0.9337   | 0.2388   |
| Weighted CART | 0.8049  | 0.8727      | 0.1964    | 0.8702   | 0.3158   |
| ROC Tree   | 0.0730   | 0.9906      | 0.2308    | 0.9564   | 0.1110   |
| PRC Tree   | 0.3902   | 0.9802      | 0.4324    | 0.9583   | 0.4102   |
| PRC-ROC Tree | 0.2439  | 0.9934      | 0.5882    | 0.9655   | 0.3448   |
| RF         | 0.0244   | 0.9981      | 0.3333    | 0.9619   | 0.0455   |
| Weighted RF | 0.1951  | 0.9953      | 0.6154    | 0.9655   | 0.2963   |
| ROC RF     | 0.9512   | 0.8021      | 0.1566    | 0.8076   | 0.2689   |
| PRC RF     | 0.9024   | 0.8633      | 0.2033    | 0.8648   | 0.3138   |
| PRC-ROC RF | 0.9268   | 0.8681      | 0.2135    | 0.8702   | 0.3471   |

Table 11: Summary of performance results for Breast Cancer Data

| Algorithms | Recall   | Specificity | Precision | Accuracy | F1 Score |
|------------|----------|-------------|-----------|----------|----------|
| SVM        | 0.9672   | 0.9909      | 0.9833    | 0.9616   | 0.9751   |
| Neural Network | 0.9672  | 0.9909      | 0.9833    | 0.9616   | 0.9751   |
| CART       | 0.9016   | 0.9909      | 0.9821    | 0.9391   | 0.9401   |
| Weighted CART | 0.9344  | 0.9818      | 0.9661    | 0.9649   | 0.9500   |
| ROC Tree   | 0.8197   | 0.9909      | 0.9804    | 0.9298   | 0.8929   |
| PRC Tree   | 0.9180   | 0.9455      | 0.9032    | 0.9357   | 0.9105   |
| PRC-ROC Tree | 0.9344  | 0.9818      | 0.9661    | 0.9649   | 0.9476   |
| RF         | 0.8689   | 0.9909      | 0.9814    | 0.9473   | 0.9217   |
| Weighted RF | 0.9180  | 1.0000      | 1.0000    | 0.9707   | 0.9572   |
| ROC RF     | 0.8361   | 0.9099      | 0.9808    | 0.9357   | 0.9027   |
| PRC RF     | 0.9836   | 0.9636      | 0.9375    | 0.9708   | 0.9600   |
| PRC-ROC RF | 0.9836   | 0.9727      | 0.9524    | 0.9766   | 0.9677   |

forest and PRC-ROC RF achieve good performance in F1 score on average. To conclude, PRC Random Forest is superior to other algorithms especially traditional random forest in identifying the minority class which is exactly the goal of imbalanced classification. In the last two instances, the PRC-ROC Random Forest has higher accuracy and F1-score compared to other algorithms and PRC RF’s behavior is close to it.

5 Conclusion

In this paper, we investigate the PRC classification tree for dealing with class imbalance problem. We also extend this to the general PRC-ROC tree and random forest by setting an additional weight parameter which selects the best weighted combination of the AUPRC and the AUC based on the training data, for each problem at hand. Four new tree-based algorithms (i.e., PRC Tree, PRC RF, PRC-ROC Tree and PRC-ROC Tree RF) are studied along with several existing related algorithms. Table 31 shows the F1-score performance of the proposed classifiers against other tree-based algorithms in all datasets. The results indicate that our methods are superior to other rivals for skewed classification problems.

Several research issues are needed for in-depth investigation. Our current PRC tree/forest and PRC-ROC tree/forest are all designed for binary classifications. We shall extend them to accommodate multiple-class classifications arising naturally from many practical scenarios. In our experimental study, we do not consider the interaction term. Hence another related task is to uncover interaction effects in the framework of PRC classification tree. There are absolutely many other potential possibilities about classification of imbalanced data and it will continue to gain increasingly attention in academia and industry.
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