FLOW PAST A VERTICAL POROUS SURFACE UNDER FLUCTUATING THERMAL AND MASS DIFFUSION WITH MHD EFFECTS

CH. V. RAMANA MURTHY1,*, K. R. KAVITHA2, N. SWAMY KALLEPALLI3

1Department of Mathematics, Koneru Lakshmaiah Education Foundation, Vaddeshwaram-522502 (A.P), India
2Freshman Engineering Department, Lakkireddy Balireddy College of Engineering, Mylavaram-521230 (A.P), India
3Department of Mathematics, Sri Vasavi Institute of Engineering & Technology, Nandamuru-521369 (A.P), India

Copyright © 2020 the author(s). This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract: This paper is examined with reference to critical parameters in the case of free MHD convection flow on a vertical porous surface under the fluctuating thermal and mass diffusion. An attempt has been made to understand the effects of the various critical parameters that appear in field equations and their effects over velocity field. It is noticed that, the prandtl number and velocity are inversely related. Moreover, it is observed that as the prandtl number increases, there is a decrease in the velocity and also back flow is observed. Even when the pore size of the bounding surface is decreased significant change in the velocity profiles is seen. Moreover, it is noticed that the prandtl number and temperature are directly proportional to each other.

Keywords: porous media; thermal radiation; heat and mass transfer; MHD effects.

*Corresponding author
E-mail address: drchvr@gmail.com
Received September 2, 2020
Nomenclature

| Symbol | Definition                                      |
|--------|------------------------------------------------|
| C      | Dimensionless species concentration            |
| g      | Gravity                                         |
| Gc     | Modified Grashof number                        |
| k      | Thermal conductivity                            |
| Cr     | Specific heat at constant pressure             |
| $C_w^*$| Concentration at the wall                      |
| Gr     | Grashof number                                  |
| A      | Suction parameter                               |
| $C^*$  | Species concentration                           |
| $C_{\infty}^*$ | Concentration in free stream            |
| h      | Rarefaction parameter                           |
| T*     | Temperature                                     |
| $T_{\infty}^*$ | Temperature of fluid in free stream       |
| $K^*$  | Permeability parameter                         |
| $L^*$  | Constant                                       |
| M      | Magnetic intensity                              |
| $V_0^*$| Constant mean suction velocity                 |
| $q_w^*$| Heat flux at the wall                           |
| Sc     | Schmidt number                                  |
| t      | Dimensionless time parameter                    |
| $t'$   | Time                                            |
| V      | Suction velocity                                |
| $T_w^*$| Temperature at wall                             |
| $K^*$  | Dimensionless Permeability parameter            |
| u      | Dimensionless velocity component                |
| $u^*$  | Velocity component                              |
| Pr     | Prandtl number                                  |
| D      | Molecular diffusivity of the species            |
FLOW PAST A VERTICAL POROUS SURFACE

Greek Symbols

\(\rho\) : Density of the fluid
\(\nu\) : Fluid kinematic viscosity
\(\theta\) : Dimensionless temperature
\(\alpha\) : Thermal diffusivity
\(\beta\) : Coefficient of thermal expansion
\(\epsilon\) : Amplitude (<<1)
\(\mu\) : Viscosity
\(\beta_0\): Coefficient of thermal expansion with concentration
\(\eta\), \(\sigma\) : Dimensionless frequency
\(\tau\) : Stefan-Boltzmann constant
\(\tau^*\) : Dimensionless shearing stress
\(\tau^*\) : Shearing stress
\(\omega^*\) : Frequency of excitation
1. **INTRODUCTION**

In several industrial and environmental situations radiative convective flow plays an important role. The advantages are chiefly found in astrophysical flows, energy processes, cooling chambers, solar power technology, fossil fuel combustion and space vehicle re-entry. The concept of radiative convection stream has a significant role in manufacturing sectors for the optimal design of highly precision equipment. Generally, nuclear plants, gas turbines, and propulsion devises for air craft, missiles and space vehicles are some examples.

Stokes [1] first started a force moving in its own plane, studied the last glutinous incomplete fluid problem of the infinite horizontal plate. Later, Brinkman [2] tested the viscous strength of the fluid. Later, Stewartson [3] studied a viscous flow on an analytical solution on a weakly-started partial-bound horizontal plate. Later, Berman [4] investigated a two-dimensional steady state flux case of an incomplete fluid with horizontal rigid porous walls, with the flow being driven by either suction or injection. Later, the flow between the two vertical panels is electrically non-conducting and observing the concept of the wall in the direction of the wall temperature, Mori [5] has observed the influence of the heat source in the vertical channel. Subsequently, Macey [6] studied the problem which occurs in the renal tubules as viscous flow through circular tube with a permeable boundary by prescribing their radial velocity at the wall as exponentially decreasing function as axial distance. Such a similar problem by using finite differences method of a mixed explicit and implicit time for the stability of the solution studied by Hall [7]. Later, Chang examined the influence of radioactive heat transfer in free convective systems in a chamber wrapped with special applications commonly enclosed in quadrilateral and geothermal reservoirs. Mahajan et al [8] has examined the devastating effect on the nature of logarithmic currents. Later, Soundlager and Thacker [9] tested thermal radiation effects of a thin gray gas exactly by the stable vertical plate. Subsequently, Hossain et al [10] analyzed the contribution of radiation on mixed synthesis with a vertical surface with a uniform surface temperature using Rosaland's estimation. Similar such an analysis was carried out by Rapits and Perdikis [11]. Consequently, the thermal radiation effects on the flow on the partial-infinite vertical plate with uniform heat flow during the
alternate period of the magnetic field were tested by Chandrakala and Anthony Raj [12]. In an analysis by Rajakumari [13] an extensive analysis of several identified parameters on the flow entities has been made and interesting results were reported. Later, Sreedevi and Ramana Reddy etal [14] presented the influence of critical parameters in on MHD flow. Subsequently, the influence of participating parameters in the velocity field has been thoroughly examined by Vedavathi [15] in the problem of Heat transfer on MHD nanofluid flow. The influence of different parameters has been explored in detail by Ramana Reddy [16] while examining the problem of heat and mass transfer flow through a highly porous medium with radiation and other effects. Fascinating conclusions have been drawn by Dhanalakshmi and Jayarami Reddy [17] while examining the contribution of time and other parameters in the problem of MHD Convective Flow over a vertical surface through Porous Medium. While studying the characteristic features in the study of flow and heat transfer of casson fluid over an exponentially porous stretching surface, Hymavathi and Sridhar [18] highlighted the contribution of porosity and thermal radiation on the flow phenomena. The results obtained by them are notable and remarkable. In similar such situation, Raja Kumari [19] exhibited interesting results on the contribution of porosity and radiation while analysing radiation absorption on convective heat and mass transfer flow of a viscous fluid. Based on the above studies, recently Vijaya [20] illustrated interesting results while investigating the influence of radiation in an unsteady situation of flow involving casson fluid. Motivated by the above studies, recently Ramana Reddy [21] highlighted the Soret and associated influence on MHD micropolar fluid flow. In the problem stated by Chandrasekhar [22], the contribution of identified parameters on the field entities are computed and the results are found to be significant from the application point of view.

2. **Mathematical Formulation**

In this paper, the flow is assumed to be unsteady and the fluid is assumed to be viscous and incompressible. The flow is over an infinite vertical porous flat plate. In this situation, a periodic temperature and concentration with variable suction velocity distribution
\[ V^* = -V_0^* \left( 1 + \varepsilon A e^{i\omega t^*} \right) \] is considered. A rectangular Cartesian co-ordinate system is employed. The \( x^* \)-axis is considered vertically upwards along the vertical porous plate and \( y^* \)-axis is taken normal to the plate.

The boundary conditions are:

\[
\frac{\partial u^*}{\partial t^*} - V_0^* \left( 1 + \varepsilon A e^{i\omega t^*} \right) \frac{\partial u^*}{\partial y^*} = g\beta \left( T^* - T_\infty^* \right) + g\beta_0^0 (C^* - C_z^*) + \nu \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma \beta_0^2}{\rho} u^* - \frac{v}{k^*} u^* \quad (1)
\]

\[
\rho C_p \left[ \frac{\partial T^*}{\partial t^*} - V_0^* \left( 1 + \varepsilon A e^{i\omega t^*} \right) \frac{\partial T^*}{\partial y^*} \right] = k \frac{\partial^2 T^*}{\partial y^{*2}} \quad (2)
\]

\[
\frac{\partial C^*}{\partial t^*} - V_0^* \left( 1 + \varepsilon A e^{i\omega t^*} \right) \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} \quad (3)
\]

Plate is endless \( x^* \) - in the direction. So, all physical parameters are independent of \( x^* \). Under these assumptions, the physical variables are only \( y^* \) and \( t^* \). Hence, the difficulty can be controlled by the following equations equation by ignoring the viscosity of the viscosity and guessing the diversity of the body.

The boundary conditions are:

\[
 u^* = L^* \left( \frac{\partial u^*}{\partial y^*} \right), \quad T^* = T_w^* + \varepsilon (T_w^* - T_\infty^*) e^{i\omega t^*}, \quad C^* = C_w^* + \varepsilon (C_w^* - C_z^*) e^{i\omega t^*} \text{ at } y^* = 0
\]
FLOW PAST A VERTICAL POROUS SURFACE

\[ u^* \to 0, \quad T^* \to T^*_{\infty}, \quad C^* \to C^*_{\infty} \text{ as } y^* \to \infty \] (4)

The following non-dimensional quantities are introduced

\[ y = \frac{y^*V_0}{v}, \quad t = \frac{t^*V_0^2}{4v}, \quad u = \frac{4v\omega}{V_0^2}, \quad \theta = \frac{T^* - T^*_{\infty}}{T^*_{\infty} - T^*_{\infty}}, \quad C = \frac{C^* - C^*_{\infty}}{C^*_{\infty} - C^*_{\infty}}, \quad Gr = \frac{g\beta V(T^* - T^*_{\infty})}{V^*_{0}^3}, \]

\[ Gc = \frac{g\beta V(C^*_{\infty} - C^*_{\infty})}{V^*_{0}^3}, \quad Pr = \frac{\mu C_p \rho}{k}, \quad Sc = \frac{v}{D}, \quad M = \frac{\sigma \beta^2 v}{\rho V^*_{0}^2}, \quad K = \frac{K^* V^*_{0}^2}{v^2}, \quad h = \frac{V^*_{0} L^*}{v} \]

Above equations (1) to (3) reduce to dimension less form as given below:

\[ \frac{1}{4} \frac{\partial u}{\partial t} - \left( 1 + \nu \omega \right) \frac{\partial u}{\partial y} = Gr\theta + GcC + \frac{1}{M} \frac{\partial^2 u}{\partial y^2} - Mu - \frac{u}{k} \] (5)

\[ \frac{1}{4} \frac{\partial \theta}{\partial t} - \left( 1 + \nu \omega \right) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \] (6)

The conditions on the boundary in the Non-dimension form are as follows:

\[ u = h \left( \frac{\partial u}{\partial y} \right), \quad \theta = 1 + \nu \omega \right) \text{ at } y = 0\]

\[ u, \theta \to 0 \text{ and as } y \to \infty \] (7)

3. SOLUTION METHODOLOGY

Under the assumption that the oscillations are sufficiently small \( \nu \ll 1 \), We represent the speed \( \theta \) and concentration \( c \), at the bottom of this plate:

\[ u(y, t) = u_0(y) e^{i\omega t} \] (8)

\[ \theta(y, t) = \theta_0(y) e^{i\omega t} \] (9)

Substituting (8) and (9) in (5) and (6) respectively and equating the like terms on both sides

\[ u_0^* + u_0' - \left( M + \frac{1}{K} + \frac{i\omega}{4} \right) u_0 = -Gr\theta_0 - GcC_0 - Au_0' \] (10)

\[ \theta_0^* + Pr\theta_0' - \frac{i\omega Pr}{4} \theta_0 = -A\theta_0' Pr \] (11)

The respective boundary conditions will now be:

\[ u_0 = 0, \quad \frac{\partial u_0}{\partial y} = 0 \text{ at } y = 0, \quad \theta_0 = 1, \quad \frac{\partial \theta_0}{\partial y} = 0 \text{ at } y = 0 \] (12)

The solution for equations (10) and (11) satisfying the boundary conditions (12) gives rise to

\[ u_0(y) = c_1 e^{m_1 y} + c_2 e^{m_2 y} + \frac{Gr\theta_0 + GcC_0}{\left( M + \frac{1}{K} + \frac{i\omega}{4} \right)} \] (13)
\[ \theta_0(y) = c_3 e^{m_3 y} + c_4 e^{m_4 y} \]  

(14)

Wherein: 

\[ m_1 = \frac{-(1+A)+\sqrt{(1+A)^2+4(M+\frac{1}{K}+i\omega)}}{2}, \quad m_2 = \frac{-(1+A)-\sqrt{(1+A)^2+4(M+\frac{1}{K}+i\omega)}}{2}, \]

\[ c_1 = \frac{m_2}{m_1 - m_2} \cdot \frac{Gr\theta_0 + GcC_0}{(M + \frac{1}{K} + i\omega)}, \quad m_3 = \frac{-(1+A)Pr+\sqrt{(Pr (1+A))^2+i\omega Pr}}{2}, \quad m_4 = \frac{-(1+A)Pr-\sqrt{(Pr (1+A))^2+i\omega Pr}}{2} \]

\[ c_3 = \frac{m_3 - m_4 - 1}{m_3 - m_4}, \quad c_4 = \frac{1}{m_3 - m_4} \]

4. **RESULTS AND DISCUSSIONS**

1. Fig. 1 and Fig. 2 illustrates the contribution of prandtl number over velocity field. It is seen the prandtl number and velocity are directly proportional to each other. Further, in this situation, aretarded flow is noticed.

![Fig-1: Influence of Prandtl number over velocity profiles](image-url)
Fig. 2: Effect of Prandtl number over velocity profiles

Fig. 3, Fig. 4, Fig. 5 and Fig. 6 depicts the effects of prandtl number over the velocity field for the pore size of 0.4, 0.3, 0.2 and 0.1 respectively. A retarded flow is noticed in this case. It is noticed that the prandtl number and the velocity field are directly proportional to each other.

Fig. 3: Contribution of Prandtl number over velocity
Fig: 4: Contribution of Prandtl number w.r.t velocity

Fig: 5: Contribution of Prandtl number on velocity
3. Fig. 7 to Fig. 12 shows the influence of prandtl number on temperature profiles. The pore sizes considered are 0.6, 0.5, 0.4, 0.3, 0.2 and 0.1 respectively. It is observed that as the prandtl number increases, the temperature also increases.
Fig: 8: Influence of prandtl number on temperature

Fig: 9: Contribution of prandtl number on temperature
Fig: -10: Contribution of prandtl number over temperature

Fig: -11: Contribution of prandtl number on temperature
CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

REFERENCES

[1] G. G. Stokes, On the effects of Internal Friction of fluids on the Motion of Pendulums, Camb. Phil. Trans IX, 2, Pp. 8 – 106. (1851).

[2] Brinkman H. C.: A calculation of viscous force extended by flowing fluid in a dense swarm of particles. Appl. Sci. Res. A. 1 (1947), 27 – 34.

[3] K. Stewartson, On the impulsive motion of a flat plate in a viscous fluid, Part I. Quart. J. Mech. Appl. Math. 4 (1951), 182–198.

[4] A. S. Berman, Laminar flow in channel with porous walls, J. Appl. Phys. 24 (1953), 1232–1235.
FLOW PAST A VERTICAL POROUS SURFACE

[5] Y. Mori, On combined free and forced convective laminar MHD flow and heat transfer in channels with transverse magnetic field, international developments in heat transfer, ASME paper no. 124, Pp 1031 – 1037 (1961).

[6] R.I. Macey, Pressure flow patterns in a cylinder with reabsorbing walls, Bull. Math. Biophys. 25 (1963), 1–9.

[7] M.G. Hall. The Boundary layer over an impulsively started flat plate, Proc. Roy. Soc. A. 310 (1502) (1969), 401–414.

[8] R.L. Mahajan, B. gebhart, Viscous dissipation effects in buoyancy induced flows, Int. J. Heat Mass Transfer. 32 (1989), 1380–1382.

[9] V. M. Soundalgekar, H.S. Thaker. Radiation effects on free convection flow past a semi-infinite vertical plate. Model. Measure. Control B, 51 (1993), 31 – 40.

[10] M.A. Hossain, H.S. Takhar, Radiation effect on mixed convection along a vertical plate with uniform surface temperature, Heat Mass Transfer. 31 (1996), 243–248.

[11] A. Raptis, C. Perdikis, Radiation and free convection flow past a moving plate, Int. J. App. Mech. Eng. 4 (1999), 817 – 821.

[12] P. Chandrakala, S. Antony Raj. Radiation effects on MHD flow past an impulsively started vertical plate with uniform heat flux, Indian J. Math. 50 (2008), 519 – 532.

[13] P. R. Kumari. A mathematical analysis of convective heat and mass transfer pour of a non-newtonian fluid through porous medium in a rectangular duct with heat sources, J. Adv. Res. Dyn. Control Syst. 2 (2017), 84–91.

[14] G. Sreedevi, D.R.V. Prasada Rao, O.D. Makinde, Venkata Ramana Reddy, G.: Soret and Dufour effects on MHD flow with heat and mass transfer past a permeable stretching sheet in presence of thermal radiation. Indian J. Pure Appl. Phys. 55 (2017), 551–563.

[15] N. Vedavathi, G. Dharmiah, K.S. Balamurugan, J. Prakash, Heat transfer on mhd nanofluid flow over a semi infinite flat plate embedded in a porous medium with radiation absorption, heat source and diffusion thermo effect, Front. Heat Mass Transfer, 9 (38) (2017), 723–734.

[16] K. Suneetha, S.M. Ibrahim, G.V. Ramana Reddy, A study on free convective heat & mass transfer through a highly porous medium with radiation, chemical reaction and soret effects, J. Comput. Appl. Res. Mech. Eng. 8 (2) (2019), 121-132.
[17] M. Dhanalakshmi, V. Jayothi, K. Jayarami Reddy, Soret and Dufour Effects on MHD Convective Flow Past a Vertical Plate Through Porous Medium, J. Phys.: Conf. Ser. 1344 (2019), 012008.

[18] T. Hymavathi, W. Sridhar, Numerical study of flow and heat transfer of casson fluid over an exponentially porous stretching surface in presence of thermal radiation. Int. J. Mech. Product. Eng. Res. Develop. 8 (4) (2018), 1145-1154.

[19] P. R. Kumari, D. Sree Devi. Effect of radiation and radiation absorption on convective heat and mass transfer flow of a viscous electrically conducting fluid in a non-uniformly heated vertical channel, Int. J. Mech. Product. Eng. Res. Develop. 8 (Spec. Iss. 7) (2018), 1382-1390.

[20] N. Vijaya, Y. Hari Krishna, K. Kalyani, G.V.R. Reddy, Soret & radiation influence of an unsteady flow of a casson fluid through porous vertical channel with expansion and contraction, Front. Heat Mass Transfer. 11 (2018), 416 - 428.

[21] G.V.R. Reddy, Y.H. Krishna, Soret and dufour effects on MHD micropolar fluid flow over a linearly stretching sheet, through a non-darcy porous medium, Int. J. Appl. Mech. Eng. 23 (2) (2018), 485-502.

[22] V. Arundhati, K. V. Chandra Sekhar, D. R. V. Prasada Rao, G. Sreedevi, Non-darcy convective heat and mass transfer flow through a porous medium in vertical channel with soret, dufour and chemical reaction effects, JP J. Heat Mass Transfer. 15 (2) (2018), 213-240.