SEE-SAW ENHANCEMENT OF LEPTON MIXING

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Abstract

The see-saw mechanism of neutrino mass generation may enhance lepton mixing up to maximal even if the Dirac mass matrices of leptons have structure similar to that in the quark sector. Two sets of conditions for such an enhancement are found. The first one includes the see-saw generation of heavy Majorana masses for right-handed neutrinos and a universality of Yukawa couplings which can follow from the unification of neutrinos with new superheavy neutral leptons. The second set is related to lepton number symmetry of the Yukawa interactions in the Dirac basis of neutrinos. Models which realize these conditions have strong hierarchy or strong degeneration of Majorana masses of the right-handed neutrinos.

I. Introduction

The see-saw mechanism of neutrino mass generation naturally relates a smallness of the neutrino masses with neutrality of neutrinos [1]. According to the see-saw mechanism the following mass terms are introduced in the fermion family basis:

\[ \mathcal{L}_m = -\bar{\nu}_L m_D \nu_R - \nu_R^T C^{-1} \frac{1}{2} M \nu_R + h.c. \]  

Here \( m_D \) is the Dirac mass matrix (Dirac sector) and \( M \) is the Majorana mass matrix for right-handed neutrino components (Majorana sector). At \( M \gg m_D \) the terms (1) generate the Majorana mass matrix for left (active) components (\( \approx \nu_L \)) [1]:

\[ m_{maj} = m_D M^{-1} m_D^T. \]  

An attractive feature of this mechanism is that the Dirac matrix, \( m_D \), can be similar (in scale and structure)
to that in the quark sector which is naturally implied by quark-lepton symmetry and Grand Unification. The difference in scales of neutrino masses, and probably in lepton mixing, follows from the structure of expression (2) and from the Majorana mass matrix which has no analogy in the quark sector.

Mixing of leptons in the see-saw mechanism was widely discussed before [2 - 14] (see [2] for review). For $m_D$(leptons) $\sim m_D$ (quarks) the lepton mixing turns out to be typically of the same order of magnitude as quark mixing [3 - 13], i.e. lepton mixing is relatively small at least between the first two generations. This was illustrated, in particular, by Monte-Carlo studies of different configurations of the Dirac and Majorana mass matrices [8], although some textures of matrices result in large mixing [4, 6, 8].

At the same time it is difficult to expect that lepton mixing coincides precisely with quark mixing. Mixing is related to masses but the masses of the charged leptons and down quarks from the same fermion generations are different. Neutrino masses, if they exist, are much smaller than the masses of up quarks. The results of the gallium experiments [15] with solar neutrinos show that Cabibbo mixing cannot explain the solar neutrino problem.

Moreover, there are several hints that lepton mixing might be much larger than that in the quark sector. The solar neutrino problem [15, 16] can be solved by long length vacuum oscillations (“just-so”) [17, 18] or by resonant flavor conversion, the MSW-effect [19, 20]. The former requires the values of neutrino mixing angles $\theta$ and masses squared difference $\Delta m^2$: $\sin^2 2\theta = 0.85 - 1.0$, $\Delta m^2 = (0.8 - 1.1) \cdot 10^{-10}$ eV$^2$ [18]. The latter picks up two regions of neutrino parameters, one of which involves large mixing angles: $\sin^2 2\theta = 0.6 - 0.9$ at $\Delta m^2 = (10^{-7} - 10^{-5})$ eV$^2$ [20]. The deficit of the muon neutrinos in the atmospheric neutrino flux can be explained by $\nu_\mu - \nu_e$ oscillations with parameters: $\sin^2 2\theta = 0.5 - 0.9$, $\Delta m^2 = (10^{-3} - 10^{-2})$ eV$^2$ [21].

As follows from (2) the masses of light neutrinos in these regions correspond to Majorana masses of the right-handed neutrinos of the order of $(10^{10} - 10^{12})$ GeV. This is much smaller than a possible Grand Unification scale which may testify for a strong hierarchical structure in the Majorana mass sector. Such a hierarchy (as we will show) may result in enhancement of lepton mixing.

In this paper we assume that the Dirac mass matrices in the lepton sector are similar to those in the quark sector and find conditions at which lepton mixing is enhanced by the see-saw mechanism itself. The paper is organized as follows. In Sect. II, considering the two-neutrino case, we formulate general conditions
for lepton-mixing enhancement. Two sets of conditions are found. In Sect. III and IV we consider two scenarios which realize these conditions. Sect. V contains a discussion and summary of results. Some technical details are explained in the Appendix.

II. See-saw angle. Mechanisms of enhancement.

We will first explain the mechanism of enhancement for the two-neutrino case and then generalize the results for three-neutrino mixing.

The mixing angle for two generations of leptons, $\theta_{\text{lept}}$, can be written in the following form:

$$\theta_{\text{lept}} = \theta_{D}^L - \theta_{L}^l + \theta_{ss}.$$  \hspace{1cm} (3)

The first two terms in the right-hand side of this equation are the direct analogies of mixing angles in the quark sector: $\theta_{L}^l$ is the angle of rotation of the left charged lepton components which diagonalizes the mass matrix of charged leptons; $\theta_{D}^L$ is the angle of rotation of the left neutrino components, $\nu_L$, which diagonalizes the Dirac mass matrix of neutrinos, $m_D$ (see Appendix). The last term in (3), $\theta_{ss}$, is the additional angle that specifies the effect of the see-saw mechanism itself. If there are no Majorana mass terms ($M = 0$), then $\theta_{ss} = 0$. We will call $\theta_{ss}$ the see-saw angle.

Let us suggest that the Dirac matrix contribution to $\theta_{\text{lept}}$ is of the order of quark mixing angle: $\theta_{L}^D - \theta_{L}^l \lesssim \theta_c \approx 13^0$ ($\theta_c$ is the Cabibbo angle), and that $\theta_{L}^D$ and $\theta_{L}^l$ are separately also small $\approx \sqrt{\frac{m_e}{m_\mu}}$. We will find the conditions at which $\theta_{ss}$ is appreciably larger than $\theta_{L}^D$ and $\theta_{L}^l$, so that $\theta_{\text{lept}} \sim \theta_{ss}$ and the latter may be close to $45^0$.

The angle $\theta_{ss}$ is determined by the properties of both the Majorana, $M$, as well as of the Dirac, $m_D$, mass matrices, and it is convenient to write down $\theta_{ss}$ in terms of mixing angles and mass hierarchies that characterize $M$ and $m_D$. Namely, let $\theta^M$ and $\theta^D_R$ be the angles of rotation which diagonalize $M$ and $m_D$ correspondingly (see Appendix). Let us define the mass hierarchies of $M$ and $m_D$ as

$$\epsilon^D \equiv \frac{m_1}{m_2}, \quad \epsilon^M \equiv \frac{M_1}{M_2},$$  \hspace{1cm} (4)

where $m_i$ and $M_i$ ($i = 1, 2$) are the eigenvalues of $m_D$ and $M$ respectively. Then $\theta_{ss}$ is determined from (see
Appendix)

\[
\tan 2 \theta_{ss} = -2 \tan (\theta^M - \theta^D_R) \cdot \frac{\epsilon^D \cdot (1 - \epsilon^M)}{\tan^2 (\theta^M - \theta^D_R) + \epsilon^M - \delta}, \tag{5}
\]

where \(\delta\) is a small term of second order in the Dirac mass hierarchy:

\[
\delta \equiv (\epsilon^D)^2 \left[ 1 + \epsilon^M \tan^2 (\theta^M - \theta^D_R) \right]. \tag{6}
\]

Two remarks are in order. As follows from (5, 6), the angle \(\theta_{ss}\) is precisely zero at \(\theta^M = \theta^D_R\) (i.e. when the rotations of right-handed components that diagonalize \(M\) and \(m_D\) are the same), or at \(\epsilon^M = 1\), (when the masses of Majorana sector are degenerate). The angle \(\theta_{ss}\) is proportional to the mass hierarchy in the Dirac sector (rather than \(\sqrt{\epsilon^D}\)), and since \(\epsilon^D\) is small one obtains typically small values of \(\theta_{ss}\), unless the denominator in the RHS of Eq.(5) is strongly suppressed.

Let us consider the conditions for \(\theta_{ss}\) enhancement. According to (5) there are two possibilities: (i) all terms in the denominator of the RHS of the Eq.(5) are very small, (ii) there is a strong cancellation among two first terms.

(i) The first case corresponds to a strong mass hierarchy in the Majorana sector. Indeed, according to Eq.(5), one obtains \(\tan 2 \theta_{ss} \gtrsim 1\), when

\[
\tan (\theta^D_R - \theta^M) \lesssim \epsilon^D, \tag{7}
\]

and

\[
\epsilon^M \lesssim (\epsilon^D)^2. \tag{8}
\]

Note, if \(\epsilon^M > 0\), then the maximal value of \(\theta_{ss}\),

\[
\tan 2 \theta_{ss}^{max} \approx -\frac{\epsilon^D}{\sqrt{\epsilon^M - (\epsilon^D)^2}}, \tag{9}
\]

is achieved at \(\tan (\theta^D_R - \theta^M) = \sqrt{\epsilon^M - (\epsilon^D)^2}\). Consequently, the equality \(\tan 2 \theta_{ss}^{max} \approx 1\) implies \(\epsilon^M \sim (\epsilon^D)^2\); the mass hierarchy in the Majorana sector should be of the order of the Dirac mass hierarchy squared.

The condition (7) means that the angles \(\theta^M\) and \(\theta^D_R\) are close to each other: since \(\theta^D_R \sim \sqrt{\epsilon^D}\), we get from (7) \(\theta^D_R - \theta^M \ll \theta^D_R\).

(ii). The condition of strong cancellation in the denominator of the RHS of Eq.(5) reads

\[
\tan^2 (\theta^M - \theta^D_R) \approx -\epsilon^M. \tag{10}
\]
Now a strong mass hierarchy is not needed and, moreover (as we will show), an approximate mass degeneration in the Majorana sector naturally results in cancellation. If the equality in Eq.(10) is exact, then according to (5)

$$\tan 2\theta_{ss} \approx \frac{2\sqrt{|\epsilon^M|}}{\epsilon^D}.$$  \hspace{1cm} (11)

Consequently, at $\epsilon^M \approx \epsilon^D$, one has $\tan 2\theta_{ss} \sim 1 / \sqrt{\epsilon^D} \gg 1$; i.e. the angle $\theta_{ss}$ can be close to the maximal mixing value. The weaker the hierarchy $\epsilon^M$, the larger $\theta_{ss}$ and, consequently, the larger the lepton mixing, in contrast to the previous case.

Since $\theta^M$ is determined by the Majorana matrix, $M$, whereas $\theta^D_R$ is determined by the Dirac matrix, $m_D$, the equality (10) implies, in general, the relation between the structures of these matrices. It is instructive to express this relation in the following form. Let us define the Dirac basis of neutrinos, $\nu^D_R$, as the basis of the neutrino states in which $\theta^D_R = 0$, i.e. in which the Dirac mass matrix, $m_D$, is diagonal. Then the condition (10) means that in the Dirac basis the Majorana mass matrix should satisfy the following condition:

$$\tan^2 \theta^M \approx -\epsilon^M, \quad (\theta^D_R = 0).$$  \hspace{1cm} (12)

In turn, Eq. (12) implies the following inequality of the matrix elements of $M$:

$$M_{11} \cdot M_{22} \ll M^2_{12}.$$  \hspace{1cm} (13)

If $M_{11} \cdot M_{22} = 0$, the equality (12) is exact.

Let us comment on some special cases.

1) $M_{11} = 0, M_{22} \neq 0$. (Inequality $M_{22} \gg M_{12}$ corresponds to the Fritzsch ansatz [22] for $M$). One gets near to maximal mixing according to Eq. (11), and an inverse mass hierarchy in the light neutrino sector.

2) $M_{11} \neq 0, M_{22} = 0$. These conditions result in an inverse mass hierarchy of the heavy Majorana neutrinos: $\epsilon^M > 1$, and in an inequality $\tan^2 \theta^M = -1/\epsilon^M$. Therefore, a strong cancellation will take place only at strong mass degeneration, $\epsilon^M \approx 1$ ($M_{11} \ll M_{12}$). In this case, the expression for the angle $\theta_{ss}$ can be written as

$$\tan 2\theta_{ss} \approx \frac{2\epsilon^D}{1 - |\epsilon^M|} = -\frac{2\epsilon^D M}{\Delta M},$$  \hspace{1cm} (14)

where $\Delta M$ is the difference in absolute values of Majorana masses. (At $M_{11} = M_{22} = 0$ there are no mass splitting and no mixing; the light components form a ZKM-neutrino.)
3) In the more natural case, $M_{11} \sim M_{22} \ll M_{12}$, one can get both mixing enhancement and a normal mass hierarchy of light neutrinos (see Sect. IV).

Mixing is enhanced also if the condition (12) is fulfilled at nonzero $\theta^D_R$, but this angle should be small. Indeed substituting (12) in (5) we get:

$$\tan 2\theta_{ss} \approx \epsilon^D \tan \theta^D_R,$$

i.e. for $\tan 2\theta_{ss}$ to be of the order of 1, one needs $\tan \theta^D_R \leq \epsilon^D \approx 10^{-2}$, which is much smaller than the typical rotation angle in the Dirac sector. Consequently, a strong non-diagonality (13) of the Majorana mass matrix of right-handed neutrinos alone is not enough to get strong enhancement of mixing.

The above conditions can be immediately generalized for three-neutrino mixing. In the first case one should suggest a strong mass hierarchy in the Majorana sector: $M_i \propto (m^D_i)^2$ and that the Majorana and the Dirac mass matrices are diagonalized by approximately the same transformations of right-handed neutrino components $S_R$, ($S_R \equiv S(\theta^D_R)$ for two neutrinos). In the second case, at least some of the elements of the Majorana mass matrix in the Dirac basis of neutrinos should satisfy the condition $M^2_{ij} \gg |M_{ii} \cdot M_{jj}|$. Note that such a non-diagonality of the Majorana mass matrix is a generic feature of models where neutrino components have nonzero lepton numbers and a symmetry related to this lepton number is preserved (or approximately preserved) by the Yukawa couplings and possible bare mass terms.

III. Enhancement of mixing in the case of strong mass hierarchy

in the Majorana sector.

Both conditions of the enhancement—strong hierarchy in the Majorana sector (8), and approximate equality of the rotation angles (7), are satisfied simultaneously if

$$M \sim \frac{1}{\mu} \cdot m^T_D m_D \sim \frac{1}{\mu} \cdot S_R (m^\text{diag}_D)^2 S^T_R.$$

(15)

Here $\mu$ is some mass parameter and $S_R$ is the transformation which diagonalizes $m_D$. (Strictly speaking, the last equality in (15) is true for the CP-conserving $m_D$). In case of exact equality (15), the matrix $M$ is also diagonalized by $S_R$, i.e. $\theta^M = \theta^D_R$, and $\epsilon^M = (\epsilon^D)^2$ for the two-neutrino mixing. In fact, the equality (15) should be slightly broken because at $\theta^M = \theta^D_R$ one gets $\theta_{ss} \equiv 0$. 


The relation (15) can be obtained if the right-handed neutrino components acquire Majorana masses via the see-saw mechanism too. This implies the existence of new heavy neutral leptons and the “cascade”, or extended see-saw (see e.g. [6], where the extended see-saw was used for another purpose). The first (high-mass scale) see-saw induced by the interactions of $\nu_R$ with new leptons gives the Majorana masses to $\nu_R$’s, and the second one generates the masses to the light neutrinos.

Let us introduce three (one for each fermion family) neutral leptons $N_L = (N_{1L}, N_{2L}, N_{3L})$—singlets of the electroweak symmetry—with bare Majorana masses, $m_N$, at some large scale (e.g. $m_N$ can coincide with Grand Unification scale, $M_{GU}$). Let us introduce also the scalar field, $\sigma$, singlet of $SU(2) \times U(1)$, which acquires a vacuum expectation value $\sigma_0 \sim 10^{12}$ GeV. Then the Yukawa interactions and the bare mass terms of the model are

$$\bar{\nu}_L h_{\nu} \nu R \phi + \bar{N}_L h_N \nu R \sigma + N^T L C \frac{1}{2} m_N N_L + h.c.$$ (16)

Here $\phi$ is the usual Higgs doublet, and $h_{\nu}$ and $h_N$ are the matrices of the Yukawa couplings. We suggest that the bare Majorana masses of $\nu_R$’s are small or forbidden by some symmetry $G$. For example, one can introduce the global $G = U(1)$ and prescribe the following G-charges: $G(\nu_R) = G(\nu_L) = G(\sigma) = 1$, $G(N_L) = G(\phi) = 0$. Another possibility is that the $\nu_R$ enters the doublet of $SU(2)_R$ symmetry which is broken at the scale $V_R \ll M_{GU}$. In this case $\nu_R$ may acquire a mass $\ll V_R$ [1].

To reproduce the relation (15) one should suggest that the matrices of the Yukawa interactions, $h_{\nu}$ and $h_N$, are correlated. The simplest possibility, $h_{\nu} = h_N$, means universality of Yukawa interactions: the couplings of $\nu_R$ with $\nu_L$ and $N_L$ are the same. Such a universality implies unification; it can be realized if $\nu_L$ and $N_L$ as well as $\phi$ and $\sigma$ enter the same fermion and scalar multiplets correspondingly. In fact, the equality of $h_{\nu}$ and $h_N$ is not necessary; the enhancement takes place if the matrices of the Yukawa couplings have the following form:

$$h_{\nu} \equiv S_L \frac{\text{diag} m_{\nu}}{v} S^T_R, \quad h_N = S_N \frac{\text{diag} m_{N}}{v} S^T_R.$$ (17)

Here $v$ is the vacuum expectation value of $\phi$ and $S_N$ is some almost-arbitrary matrix which may be related to features of $N_L$ interactions. In particular, it can be equal to $I$ or $S_L$.

When the fields $\phi$ and $\sigma$ acquire nonzero vacuum expectation values, neutrino masses are generated and
the mass matrix in the block basis \((\nu_L, \nu_c, N_L)\) can be written as

\[
\begin{pmatrix}
0 & m_D & 0 \\
m_D^T & 0 & \frac{\sigma_0}{v} m_D^T S_N^T m_N \\
0 & \frac{\sigma_0}{v} S_N m_D & 0
\end{pmatrix}.
\] (18)

It gives the Majorana mass matrix for right-handed neutrino components:

\[
M = - \left(\frac{\sigma_0}{v}\right)^2 m_D^T S_N^T m_N^{-1} S_N m_D,
\] (19)

and then the Majorana mass matrix for light components:

\[
m_{\text{maj}} = \left(\frac{v}{\sigma_0}\right)^2 S_N^T m_N S_N
\] (20)

(for real \(S_N\)). As follows from Eq. (20), at \(S_N = I\), the matrix \(m_{\text{maj}}\) is proportional to the mass matrix of superheavy leptons, \(N_L\). We may suggest that the Yukawa couplings or/bare mass terms at the highest mass scale have no hierarchy; all elements of the matrix \(m_N\) are of the same order. The matrix \(S_N\) with small nondiagonal elements gives only small corrections to the above picture.

Note that at \(\sigma_0 \sim m_N \sim 10^{16} \text{ GeV}\), the typical mass scale for the lightest components is about \(m \sim v^2 m_N / \sigma_0^2 \approx 10^{-2} \text{ eV}\). Therefore a small spread of parameters in \(m_N\) allows us to explain the scales of both the solar \((m \sim 0.3 \cdot 10^{-2} \text{ eV})\) and atmospheric \((m \sim (3 - 10) \cdot 10^{-2}\text{eV}^2)\) neutrino problems. The Majorana masses of right-handed neutrinos are naturally in the intermediate mass scale region \((10^{10} - 10^{14})\) GeV.

Two remarks are in order. A Goldstone boson which could appear due to spontaneous violation of \(G\) symmetry by the vacuum expectation of \(\sigma\) has negligibly small interactions with active neutrinos. Moreover, it can acquire nonzero mass due to explicit violation of this symmetry at low scales. Additional scalars can be introduced to generate \(m_N\) spontaneously.

The above mixing enhancement based on the extended see-saw mechanism and the universality of the Yukawa couplings allows us to systematically compensate for the smallness in the lepton mixing related to the hierarchical structure of \(m_D\). In the extreme case the lepton mixing is defined by the structure of the mass matrix of superheavy leptons and does not depend on the Dirac matrix at all.

IV. Enhancement of mixing due to lepton symmetry in the Dirac basis. Degeneration of heavy Majorana masses.
The enhancement of mixing takes place due to the cancellation in the denominator of the RHS of Eq.(5). The condition of cancellation (the relation (12) for the Majorana mass matrix in Dirac basis of neutrinos) implies a correlation between structures of the Dirac and the Majorana matrices. Possible deviation from this correlation quantified by the angle $\theta_{D}^{R}$ should be small. These features again imply the universality of the Yukawa couplings. The neutrino family states, at least $\nu_{R}$, should enter both the Yukawa interactions and the bare mass terms in the same combinations, which coincide with eigenstates of the Dirac mass matrix: $S_{R}^{T}\nu_{R} \equiv \nu_{R}^{D}$. However, in contrast with the previous case, the couplings of $\nu_{R}^{D}$ need not be proportional to $m_{D}^{diag}/v$. To naturally satisfy the condition (12), the Yukawa interaction of $\nu_{R}^{D}$ should obey definite symmetry.

It is possible to formulate these conditions in another form. Usually lepton symmetry and lepton numbers are introduced in the Dirac basis of the charged leptons (i.e. in the basis where the mass matrix of charged leptons is diagonal; see e.g. [7] in context of the see-saw mechanism). Lepton number symmetry introduced in such a way is violated by neutrino mass terms. In contrast, one can impose the lepton symmetry in the neutrino Dirac basis and suggest that this symmetry is violated by mass terms of charged leptons. These two cases are physically equivalent if there are only Dirac masses, but they give different results in presence of the Majorana mass terms. In the neutrino Dirac basis, the lepton mixing matrix can be written as

$$S_{\text{lept}}^{\text{lept}} = (S_{L}^{T})^{+} \cdot S_{ss},$$

(21)

where $S_{L}^{T}$ is the transformation of the left components which diagonalizes the charged lepton mass matrix and $S_{ss}$ is the see-saw transformation which diagonalizes the mass matrix for light components in the Dirac basis (see Appendix). In the two-neutrino case, Eq. (21) corresponds to Eq. (3) at $\theta_{D}^{L} = 0$; by definition, $S_{L}^{T}$ relates the Dirac and the flavor bases: $\nu_{f} = S_{L}^{T} \nu_{D}$; $S_{L}^{T}$ is similar to the Cabibbo-Kobayashi-Maskawa matrix of the quark mixing.

Consider neutrino mixing in the Dirac basis, $\nu_{D}$. Let us suppose that the Yukawa interactions as well as the bare mass terms obey a global $G = U(1)$ symmetry. We prescribe zero G-charge for the usual Higgs doublet, $G(\phi) = 0$, and different G-charges for different pairs of left and right-handed neutrino components: $G(\nu_{L}) = G(\nu_{R}) = G_{i}$, $G_{i} \neq G_{j}$ (i, j = 1, 2, 3). Then the Yukawa couplings of neutrinos with $\phi$ and consequently the Dirac mass matrix are diagonal (as is demanded by definition of the Dirac basis). We will
also introduce a scalar field \( \sigma \) with nonzero charge \( G_\sigma \), which acquires nonzero vacuum expectation value \( \sigma_0 \) and thus breaks G-symmetry. Now Yukawa interactions and bare mass terms of the model can be written as:

\[
\bar{\nu}_L m_D^{\text{diag}} \nu_R^T \phi + \nu_R^T C^{-1} \frac{1}{2} \nu_R \sigma + \nu_R^T C^{-1} \frac{1}{2} M_b \nu_R + \text{h.c.}.
\] (22)

Matrices of the Yukawa couplings, \( h \), and bare masses, \( M_b \), are determined by the G-charge prescription and in general are nondiagonal. The neutrino mass matrix which results from the spontaneous breaking of the electroweak and G- symmetries can be written in the basis \( (\nu_L^c, \nu_R) \) as

\[
\begin{pmatrix}
0 & m_D^{\text{diag}} \\
M & 0
\end{pmatrix}.
\] (23)

The matrix of right-handed neutrino components, \( M \), should satisfy the cancellation condition (see Eqs. (12) and (13) in two neutrino case). According to the see-saw mechanism the mass matrix for light neutrinos is

\[
m_{ss} = -m_D^{\text{diag}} M^{-1} m_D^{\text{diag}}.
\] (24)

Depending on \( G_\sigma \) and on the charge prescription for \( \nu_R^D \): \( G_\nu \equiv (G_1, G_2, G_3) \), one gets different realizations of \( M \) needed for mixing enhancement. Let us comment on the simplest possibilities.

1) For the charge prescription \( G_\nu = (1, 0, -1) \), \( G_\sigma = 2 \) the Majorana mass matrix is equal to:

\[
M = \begin{pmatrix}
h_{11} \sigma_0 & 0 & M_{13} \\
0 & M_{22} & 0 \\
M_{13} & 0 & h_{33} \sigma_0
\end{pmatrix}.
\] (25)

The state \( \nu_{2R} \) decouples and the task is reduced to the two-neutrino case. At \( h_{11} \sigma_0 \sim h_{33} \sigma_0 \ll M_{13} \), one naturally obtains a direct mass hierarchy in the light sector and a strong cancellation in the denominator.

The see-saw transformation diagonalizing matrix (24) is

\[
S_{ss} = \begin{pmatrix}
\cos \theta_{ss} & 0 & \sin \theta_{ss} \\
0 & 1 & 0 \\
-\sin \theta_{ss} & 0 & \cos \theta_{ss}
\end{pmatrix},
\]

where according to (14)

\[
\tan 2\theta_{ss} = \frac{2 \epsilon^D \cdot M_{13}}{[(\epsilon^D)^2 h_{33} - h_{11}] \sigma_0} \approx \frac{2 \epsilon^D M_{13}}{h_{11} \sigma_0}.
\] (26)

Here \( \epsilon^D \equiv m_1/m_3 \). The \( \nu_e - \nu_\tau \) mixing turns out to be enhanced, whereas \( \nu_e - \nu_\mu \) and \( \nu_\mu - \nu_\tau \) mixings induced only by Dirac matrices are small.
For the charge prescription \( G_\nu = (1, -1, 0) \), \( G_\sigma = 2 \) one gets similarly an enhancement of 1 - 2 and, consequently, \( \nu_e - \nu_\mu \) - mixing. For \( G_\nu = (0, 1, -1) \), \( \nu_\mu - \nu_\tau \) mixing is enhanced.

2) For \( G_\nu = (1, 0, -1) \), \( G_\sigma = 1 \) the Majorana mass matrix has a form

\[
M = \begin{pmatrix}
0 & h_{12}\sigma_0 & M_{13} \\
h_{12}\sigma_0 & M_{22} & h_{23}\sigma_0 \\
M_{13} & h_{23}\sigma_0 & 0
\end{pmatrix}.
\]

A straightforward calculation of the \( m_{ss} \) according to (24) gives that at \( h_{23}\sigma_0 \gg M_{13} \) the enhancement of 1 - 2 \( (\nu_e - \nu_\mu) \) mixing takes place:

\[
\tan 2\theta_{ss} \approx \frac{2\epsilon^D \cdot \frac{M_{13}}{h_{23}\sigma_0}}{(\epsilon^D)^2 - \left(\frac{M_{13}}{h_{23}\sigma_0}\right)^2},
\]

where \( \epsilon^D \equiv \frac{m_1}{m_2} \).

3) For \( G_\nu = (0, 1, 2) \), \( G_\sigma = 2 \), the Majorana mass matrix is

\[
M = \begin{pmatrix}
M_{11} & 0 & h_{13}\sigma_0 \\
0 & M_{22} & 0 \\
h_{13}\sigma_0 & 0 & 0
\end{pmatrix}.
\]

The state with charge \( G = 1 \) decouples and the task is reduced again to the two-neutrino case. In contrast to version 1), here mixing is induced by the Yukawa couplings with \( \sigma \). The enhancement of mixing implies \( h_{13}\sigma_0 \gg M_{11} \) and according to (14) one has

\[
\tan 2\theta_{ss} = 2\epsilon^D \frac{h_{13}\sigma_0}{M_{11}}.
\]

The opposite order of charges \( G_\nu = (2, 1, 0) \) gives strong enhancement of 1 - 3 mixing, but an inverse mass hierarchy of light neutrinos. In this case, large mixing angles, i.e. \( \sin^2 2\theta > 0.6 \), are disfavored by SN1987A data [23].

Changing the neutrino component which has charge \( G = 1 \) and, consequently, decouples, one can enhance mixings of other pairs of neutrino states.

4) For \( G_\nu = (2, 1, 0) \), \( G_\sigma = 1 \), the Majorana mass matrix,

\[
M = \begin{pmatrix}
0 & h_{12}\sigma_0 & 0 \\
h_{12}\sigma_0 & 0 & h_{23}\sigma_0 \\
0 & h_{23}\sigma_0 & M_{33}
\end{pmatrix},
\]

results in strong enhancement of 1 - 2 \( (\nu_e - \nu_\mu) \) mixing at \( M_{33} \gtrsim h_{12}\sigma_0 \):

\[
\tan 2\theta_{ss} = -\frac{2}{\epsilon_{12}^D} \frac{M_{33}h_{12}}{\sigma_0 h_{23}},
\]
and in an inverse mass hierarchy of light neutrinos. At \( h_{12}/h_{23} \lesssim \epsilon^D \) one can get simultaneously the (1 - 3) - mixing enhancement:

\[
\tan 2\theta_{ss} \approx \frac{2\epsilon^D \cdot \frac{h_{23}}{h_{12}}}{1 - (\epsilon^D)^2 \left(\frac{h_{23}}{h_{12}}\right)^2}.
\]  

(33)

For \( G_\nu = (0, 1, 2) \), and \( G_\sigma = 1 \), \( \nu_\mu - \nu_\tau \) - mixing is enhanced at \( M \gtrsim h_\sigma_0 \).

All elements of the Majorana mass matrix \( M \) can be generated spontaneously by introducing additional scalar bosons, so that the hierarchy of masses in the Majorana sector is related to the hierarchy of the vacuum expectation values of different scalars. Moreover, the extended global symmetry and/or discrete symmetry can be imposed to obtain the needed texture of mass matrices.

To explain the universality of the Yukawa interactions one can also develop the scenario of Sect. III. Namely, it is possible to generate the the Majorana mass matrix of \( \nu_R \) via interactions with additional superheavy leptons.

V. Discussion and conclusions

1. The most natural scenario of lepton mass and mixing generation implies that the Dirac mass matrices in the lepton sector are similar to those in the quark sector. The difference in the neutrino masses and probably in mixing follows from the Majorana mass matrix of the right-handed neutrinos.

2. The effect of the see-saw mechanism itself in lepton mixing can be described by the see-saw matrix or the see-saw angle (when the task is reduced to the two-neutrino case). The see-saw angle is proportional to the mass hierarchy in the Dirac sector \( \epsilon^D \), rather than \( \sqrt{\epsilon^D} \). Therefore, usually the see-saw corrections to lepton mixing are small, or are of the same order as mixing induced by the Dirac matrices. However the smallness of the see-saw angle related to such a proportionality can be systematically compensated. At definite conditions, the see-saw angle can be larger than the angles induced by Dirac matrices and the lepton mixing is determined mainly by the see-saw angle.

3. There are two sets of conditions at which the see-saw angle dominates and the lepton mixing is large. The first set demands a strong mass hierarchy in the Majorana sector as well as approximately the same transformations of the right-handed neutrino components which diagonalize the Dirac and Majorana mass matrices. The latter implies the universality of the Yukawa couplings of the right-handed components.
These conditions can be realized in models with additional superheavy leptons, $N_L$, where right-handed neutrinos acquire Majorana masses via the see-saw mechanism induced by interactions with $N_L$. The universality of Yukawa couplings of $\nu_R$ could be explained by the unification of $\nu_L$ and $N_L$ in one multiplet. As a result of the extended (cascade) see-saw, the mixing of the light neutrinos is determined essentially by mixing of $N_L$. The typical mass scale of light neutrinos, $(10^{-3} - 10^{-1})$ eV, allows us to explain both the solar and atmospheric neutrino deficits.

4. The second set of conditions demands definite form of the Majorana mass matrix of right-handed components in the Dirac basis of neutrinos (where Dirac mass matrix is diagonal). Namely, dominance of the nondiagonal elements is needed and in most natural cases the Majorana masses of right-handed neutrinos turn out to be degenerate. Such a nondiagonality of $M$ (or suppression of the diagonal elements) is a generic feature of models in which neutrinos have nonzero charges of some symmetry, $G$. For this scenario an enhancement of mixing for two neutrinos is typical. Different $G$-charge prescriptions result in enhancement of mixing of different components.

5. Future solar neutrino experiments will be able to confirm (or reject) the solutions based on large lepton mixing and therefore check (at least partly) the possibilities considered in this paper.

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Appendix. See-saw angle and see-saw matrix.

In the two-neutrino case, the mass matrix is diagonalized by the unitary transformation $S(\theta_i)$ which is determined by a rotation angle $\theta_i$:

$$S(\theta_i) \equiv \begin{pmatrix} \cos \theta_i & \sin \theta_i \\ -\sin \theta_i & \cos \theta_i \end{pmatrix}.$$  \hfill (A.1)

Let $\theta_{L}^{D}$ and $\theta_{R}^{D}$ be the angles of rotations of the left and right-handed neutrino components, $\nu_L = S(\theta_L^{D})\nu_L^{D}$.
and \( \nu_R = S(\theta^D_R)\nu^D_R \), which diagonalize the Dirac mass matrix:

\[
S^T(\theta^D_L)m_DS(\theta^D_R) = m^\text{diag}_D \equiv \text{diag}(m_1, m_2) .
\]  

(A.2)

Here \( \nu^D_L, \nu^D_R \) and \( m_1, m_2 \) are the eigenstates and the eigenvalues of the Dirac mass matrix correspondingly (subscript \( T \) means transponent). Let \( \theta^M \) be the angle of rotation of the right-handed neutrino components which diagonalizes the Majorana mass matrix of right-handed neutrino components, \( \nu_R = S(\theta^M)\nu^M_R \):

\[
S^T(\theta^M)MS(\theta^M) = M^\text{diag} \equiv \text{diag}(M_1, M_2) ,
\]

(A.3)

where \( M_i \) are the eigenvalues of the Majorana mass matrix. Then taking into account that the inverse matrix \( M^{-1} \) is diagonalized by the same transformation (A.3) we can write the Majorana mass matrix for light neutrino components in terms of diagonalized matrices and rotations in the following form:

\[
m^\text{maj} = S(\theta^D_L)m^\text{diag}_DS(\theta^D_R)(M^\text{diag})^{-1}S^T(\theta^M-\theta^D_R)m^\text{diag}_D S(\theta^D_L)^T .
\]

(A.4)

This matrix can be diagonalized by transformation (A.1) with the angle

\[
\theta_\nu \equiv \theta^D_L + \theta_{ss} ,
\]

(A.5)

where \( \theta_{ss} \) is the angle of rotation which diagonalizes according to (A.4) the matrix

\[
m_{ss} = m^\text{diag}_D S(\theta^M-\theta^D_R)(M^\text{diag})^{-1}S^T(\theta^M-\theta^D_R)m^\text{diag}_D .
\]

(A.6)

Now the lepton mixing angle can be written as \( \theta^{\text{lept}} = \theta^D_L + \theta_{ss} - \theta^D_L \); here \( \theta^D_L \) is the angle of rotation which diagonalizes the Dirac mass matrix of charged leptons. At \( \theta_{ss} = 0 \) the form of lepton mixing coincides with that in quark sector. So, \( \theta_{ss} \) specifies the features of the see-saw mechanism and we will call it the see-saw angle.

In the general case (three-neutrino mixing), one can introduce the see-saw matrix, \( S_{ss} \), so that the lepton mixing matrix can be written in the form

\[
S^{\text{lept}} = (S^L_L)^+ S^D_L S_{ss} ,
\]

(A.7)

where the transformation \( S_{ss} \) diagonalizes the matrix

\[
m_{ss} = m^\text{diag}_D S^T_R M^{-1} S_R m^\text{diag}_D .
\]
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