The Spin Alignment of Vector Mesons in High Energy $pp$ Collisions

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The spin alignment of vector meson produced in high energy reactions is determined by the spin-dependent fragmentation function $D_{1LL}(z, \mu_F^2)$ that is shown to be independent of the polarization of the fragmenting quark. In this paper, we extract the spin-dependent fragmentation function $D_{1LL}(z, \mu_F^2)$ from data on the spin alignment of $K^{*0}$ in $e^+e^-$ annihilation at LEP in two different scenarios and apply them to make predictions in $pp$ collisions. We make detailed analysis of contributions from different sub-processes and show that the spin alignment should be quite significant also in high energy $pp$ collisions.

I. INTRODUCTION

The spin dependence of fragmentation functions (FFs) is one of the important aspects in high energy spin physics and plays an important role in studying the properties of Quantum Chromodynamics (QCD) in general and the hadronization mechanism in particular. So far as the polarization of produced hadrons is concerned, two classes of polarizations have been often studied, the vector and the tensor polarization. The former can be studied by measuring the polarization of hyperons via their spin self analyzing weak decays, and the latter are studied via strong decays of vector mesons into two pseudo-scalar mesons. The tensor polarization is usually decomposed into five components. Among them, the $S_{LL}$-component is directly related to the probability for the third component of spin to take zero that is called the spin alignment. The spin alignment of vector meson has been measured in $e^+e^-$ annihilations and other high energy reactions [1–8].

Compared with parton distribution functions (PDFs), we know even less about the spin dependence of FFs. Among different aspects, hyperon polarizations are best studied both experimentally [9–26] and phenomenologically [27–47]. Parameterizations of the corresponding spin dependent FF have been proposed [48].

For the tensor polarization of vector mesons, the study has in fact advantages: there is little contamination from decay process, and no decay parameter is involved in the two-body strong decay of the vector meson so that there is no uncertainty caused by the decay parameter [26] and the measurement efficiency is high. Measurements have been carried out on the spin alignment and also the off-diagonal components in high energy reactions [1–8]. We have in particular data on the spin alignment with relatively high accuracy from experiments at LEP [1–4]. The data show an evident spin alignment of vector mesons produced in $e^+e^-$ annihilations and triggered many phenomenological studies [49–56]. Since the collision energy is at the $Z^0$ pole, the fragmenting quark and anti-quark are highly polarized. Therefore, it was quite natural to attribute the spin alignment to the polarization of the parent quark and/or anti-quark. Most of the phenomenological efforts have been accomplished following such a perception [53–56].

Recently, progresses in the theoretical study have been made in particular in the formal QCD description of the spin dependence of FFs [57–67]. In QCD field theory, FFs are defined via Lorentz decompositions of the quark-quark correlator. A systematic study of such a decomposition has been accomplished [63, 64] and the results show in particular that the spin alignment is determined solely by the $S_{LL}$-dependent FF $D_{1LL}$ and $D_{1LL}$ is independent of the spin of the fragmenting quark. Correspondingly the first attempt to extract $D_{1LL}(z)$ from the LEP data [1–3] has been carried out in [65].

Although it might be counter-intuitive, this conclusion is actually expected by the parity invariance. This can be seen clearly in the helicity base. As a component of the polarization tensor, $S_{LL}$ is a scalar that is invariant under space inversion. Hence, one cannot establish a connection between $S_{LL}$ and the helicity of the quark in a parity conserved manner. This is quite different from the case for the longitudinal polarization of $\Lambda$, where $\lambda_Q\lambda_{\Lambda}$ is a parity-invariant structure that should be included in the decomposition of fragmentation function, where $\lambda_Q$ and $\lambda_{\Lambda}$ are helicities of the quark and $\Lambda$ respectively.

Though the prediction is very solid, it is however quite difficult to understand why the fragmentation of an unpolarized quark leads to vector meson with a larger probability at the helicity zero state. Experimental check of the quark polarization independence of the vector meson spin alignment should be a very basic test of the fragmentation picture and deep studies in this direction should lead to new insights on the hadronization mechanism. In this connection, it might be also interesting to mention that spin effects have also attracted much attention recently in heavy ion collisions. Here, a very special state of hadronic matter – the quark gluon plasma (QGP) is formed and the hadronization mechanism is different. Both hyperon polarization and vector meson...
spin alignment have been studied at RHIC as well as at LHC in this connection. The studies have been inspired by the theoretical predictions [68, 69] and the experimental confirmation [70] on the global polarization of QGP with respect to the reaction plane. The vector meson spin alignment was predicted [69] to be strongly dependent on the global polarization of quarks and anti-quarks because they are produced via the quark combination rather than the quark fragmentation mechanism.

Currently, both RHIC and LHC provide good opportunities in experiments to study vector meson spin alignment in $pp$ collisions. In particular at RHIC the quark polarization independence can easily be tested since RHIC is also a polarized $pp$ collider. It is thus timely and important to make predictions for such measurements.

In this paper, we study the spin alignment of vector meson in $pp \to VX$. We extract the $S_{LL}$-dependent FF $D_{1LL}$ from the LEP data and make predictions for $pp$ collisions. In Sec. II, we present the basic formulae needed for such numerical calculations. In Sec. III, we present parameterizations of $D_{1LL}$ and numerical results in Sec. IV. A short summary is given in Sec. V.

II. THE FORMALISM

In this section, we present the differential cross section of vector meson production in $pp$ collisions needed to calculate the spin alignment. We do the calculations up to the order where the first order of pQCD evolution to calculate the spin alignment. We do the calculations of vector meson production in $D_{1LL}$ from the LEP data and make predictions for $pp$ collisions. In Sec. II, we present the basic formulae needed for such numerical calculations. In Sec. III, we present parameterizations of $D_{1LL}$ and numerical results in Sec. IV. A short summary is given in Sec. V.

A. The differential cross section

We consider $pp \to VX$ in the high $p_T$ region where collinear factorization is applicable and study the spin alignment of produced vector meson $V$. Since the spin alignment is independent of the polarization of the fragmenting quark, the calculations are the same in the polarized or unpolarized collisions. We simply take unpolarized $pp$ as the example.

To calculate the spin alignment of $V$, we need to consider the spin dependent differential cross section. We recall that the polarization of spin-1 particles is described by a $3 \times 3$ spin density matrix $\rho$. In the rest frame of the particle, $\rho$ is usually decomposed as [63, 64, 71],

$$\rho = \frac{1}{3} (1 + 3 S_i \Sigma^i + 3 T^{ij} \Sigma^{ij}),$$

where $\Sigma^i$ is the spin operator of a spin-1 particle, and $\Sigma^{ij} = \frac{1}{2} (\Sigma^i \Sigma^j + \Sigma^j \Sigma^i) - \frac{1}{2} \delta^{ij}$. $T^{ij} = \text{Tr}(\rho \Sigma^{ij})$ is the polarization tensor and is parameterized as,

$$T = \frac{1}{2} \begin{pmatrix} -\frac{2}{3} S_{LL} + S_{xx} & S_{xy} & \frac{1}{3} S_{TT} \\ S_{yx} & -\frac{2}{3} S_{LL} + S_{xx} & \frac{1}{3} S_{TT} \\ \frac{1}{3} S_{LT} & \frac{1}{3} S_{LT} & \frac{2}{3} S_{LL} \end{pmatrix}.$$  \hspace{1cm} (2)

Here, the polarization vector $S$ is similar to that for spin-1/2 hadrons. The polarization tensor $T$ is further decomposed into a Lorentz scalar $S_{LL}$, a Lorentz vector $S_{LT} = (S_{xT}, S_{yT}, S_{zT}, 0)$, and a Lorentz tensor $S_{TT}^{\mu\nu}$, that has two nonzero independent components $S_{TT}^{xx} = -S_{TT}^{yy}$ and $S_{TT}^{zz} = S_{TT}^{yy}$. It has in total five independent components. The spin alignment $\rho_{00}$ is directly related to $S_{LL}$ by $\rho_{00} = (1 - 2 S_{LL})/3$, where $\rho_{00}$ takes the physical meaning of the probability for the third component $m$ of spin of $V$ to take zero while $S_{LL} = (\rho_{++} + \rho_{--})/2 - \rho_{00}$ is the difference of $m$ to take $\pm 1$ and 0. In the helicity basis, $m$ is just the helicity $\lambda_V$ of the vector meson $V$.

To calculate the spin alignment $\rho_{00}$ of the produced vector meson $V$, we need to consider the $S_{LL}$-dependent part of the cross section and sum over all other components of polarization. Since $S_{LL}$ is a Lorentz scalar, the $S_{LL}$-dependent part takes the same form as that of the unpolarized part. In this way, we obtain the differential cross section in the collinear factorization form as [72],

$$\frac{d\sigma_{pp \to VX}}{dy d^2p_T} = \sum_{abcd} \int dy_2 \int \frac{d^2z}{z^2} x_1 f_a(x_1, \mu_f) x_2 f_b(x_2, \mu_f) \times \frac{1}{\pi} \frac{d \bar{\sigma}_{ab \to cd}}{dt} \left[ D_{LL}^V(z, \mu_f) + S_{LL} D_{1LLc}^V(z, \mu_f) \right], \hspace{1cm} (3)$$

where $f_{a,b}(x, \mu_f)$ is the parton distribution function [73] with $x$, the longitudinal momentum fraction and $\mu_f$ the factorization scale, $D_{LL}^V(z, \mu_f)$ and $D_{1LLc}^V(z, \mu_f)$ are the spin-averaged and $S_{LL}$-dependent FFs of $c \to VX$ respectively; $y$ and $p_T$ denote the rapidity and transverse momentum of $V$ and they are related to $x_1, x_2$ and $z$ by $x_1 = p_T (e^y + e^{-y})/z \sqrt{s}$, $x_2 = p_T (e^{-y} + e^y)/z \sqrt{s}$; $y_2$ is the rapidity of parton $d$ after the scattering; $d \bar{\sigma}_{ab \to cd}/dt$ is the cross section of the partonic process $ab \to cd$ at the leading order. The partonic process includes all different elementary processes at the parton level such as $q_1 q_2 \to q_1 q_2$, $q_1 q_2 \to q_1 q_2$, $q_1 q_1 \to q_1 q_1$, $q_1 q_1 \to q_1 q_1$, $q_1 g \to q_1 g$, $g g \to g g$, $q_1 q_1 \to q_1 q_1$, $q_1 q_1 \to q_1 q_1$, $q_1 q_2 \to q_2 q_2$, $q q \to g g$, and $g g \to q q$. We consider the unpolarized reaction and the cross sections for these elementary processes are available in literature [72]. Here, we note in particular that in Eq. (3), FFs are defined for a given polarization state following the same convention as that in [61] where $D_{LL}^V(z, \mu_f)$ is the spin-averaged FF and is related to the spin-summed FF $D_V^V(z, \mu_f)$ by $D_V^V(z, \mu_f) = 3 D_{LL}^V(z, \mu_f)$.

Besides presenting the differential cross section in terms of $y$ and $p_T$, we can also make predictions in terms of other variables such as $(x_F, p_T)$ where $x_F \equiv 2 p_T / \sqrt{s} = 2 m_T \sinh y / \sqrt{s}$, $m_T = \sqrt{m^2 + p_T^2}$ and

$$dy d^2p_T = dx_F d^2p_T / \sqrt{x_F^2 + 4 m_T^2 / s}. \hspace{1cm} (4)$$
B. The spin alignment

The spin-alignment of $V$ is then given by,

$$
\rho^V_{00} = \frac{d\sigma^{\lambda_V=0}}{\sum_{\lambda_V=\pm1,0} d\sigma^{\lambda_V}}.
$$

(5)

For the helicity $\lambda_V = \pm 1$ state, $S_{LL} = 1/2$, while for $\lambda_V = 0$ state, $S_{LL} = -1$. Hence, we obtain,

$$
\rho^V_{00}(y,\rho_T) = \frac{1}{3} \frac{d\sigma_{pp\rightarrow VX}^{\text{spin-summed}}}{dy d^2\rho_T} / \frac{d\sigma_{pp\rightarrow VX}}{dy d^2\rho_T},
$$

(6)

where the spin-summed cross section is given by,

$$
\frac{d\sigma_{pp\rightarrow VX}^{\text{spin-summed}}}{dy d^2\rho_T} = 3 \sum_{abcd} \int dy_2 \int \frac{dz}{z^2} x_1 f_a(x_1, \mu_f) \times x_2 f_b(x_2, \mu_f) \frac{1}{\pi} \frac{d\hat{s}_{ab\rightarrow cd}}{dt} D_{1c} (z, \mu_f),
$$

(7)

while the $S_{LL}$-dependent part is,

$$
\frac{d\sigma_{pp\rightarrow VX}^{S_{LL}}}{dy d^2\rho_T} = \sum_{abcd} \int dy_2 \int \frac{dz}{z^2} x_1 f_a(x_1, \mu_f) \times x_2 f_b(x_2, \mu_f) \frac{1}{\pi} \frac{d\hat{s}_{ab\rightarrow cd}}{dt} D_{1LLc} (z, \mu_f).
$$

(8)

From the definition of $S_{LL}$ in particular its relation to $\rho_{00}$ we see that its value range is $-1 \leq S_{LL} \leq 1/2$ so that $-2 \leq D_{1LL}(z, \mu_f)/D_1(z, \mu_f) \leq 1$. In this way $0 \leq \rho_{00} \leq 1$ is guaranteed.

C. The QCD evolution of $D_{1LL}$

The QCD evolution of collinear FFs is given by corresponding DGLAP equations [74-77] with time-like splitting functions [78-80]. The evolution equation of the $S_{LL}$-dependent FF $D_{1LL}$ is the same as that for unpolarized FF $D_1$, i.e.,

$$
\frac{\partial}{\partial \ln Q^2} D_{1LLa} (z, Q^2) = \alpha_s(Q^2) \frac{\alpha_s}{2\pi} \sum_{j} \int dz \frac{d\xi}{\xi} D_{1LLb} (\frac{z}{\xi}, Q^2) P_{ja} (\xi, \alpha_s),
$$

(9)

where $a$ or $b$ denotes different types of partons including different flavors of quarks, anti-quarks and gluon. The splitting functions that we need in the numerical calculations are given by,

$$
P_{qq}(\xi) = C_F \left[ \frac{1 + \xi^2}{(1 - \xi)^2} + \frac{3}{2} \delta(1 - \xi) \right],
$$

(10)

$$
P_{qg}(\xi) = C_F [1 + (1 - \xi)^2] / \xi,
$$

(11)

$$
P_{gq}(\xi) = -[\xi^2 + (1 - \xi)^2] / 2, \xi,
$$

(12)

$$
P_{gg}(\xi) = N_c \left[ \frac{2\xi}{(1 - \xi)^2} - 2(\xi^2 - \xi - \frac{1}{\xi} + 1) \right] + \frac{1}{6} (11 N_c - 2 N_f) \delta(1 - \xi).
$$

(13)

where $C_F$ and $N_c$ are color factors and $N_f$ is the number of flavors.

III. THE PARAMETERIZATION OF THE FRAGMENTATION FUNCTION

Even in the unpolarized case, we do not have an appropriate parameterization for the fragmentation function of vector mesons. Hence, we take the form of parameterizations based on symmetry properties, models and conjunctions and fix the free parameters using data available.

A. The unpolarized fragmentation function

Currently, there is no parameterization of the fragmentation function of vector meson production available in the market even for the unpolarized case. However, we have parameterizations of production of the pseudoscalar meson $K^\pm$ e.g. AKK08 in [81]. Also a simple relationship between the yields of $K^{*0}/\bar{K}^{*0}$ and $K^{\pm}$ has been observed [82] that leads to a linear dependence of $z$ for the ratio $D_{1u}^{K^{*0}}/D_{1u}^{K^+}$ approximately [65], i.e.,

$$
D_{1u}^{K^+} (z, \mu_0) = A(2z + 1) D_{1u}^{K^{*0}} (z, \mu_0),
$$

(14)

where $\mu_0 = 2$ GeV is the initial scale and $A \approx 0.3$ is the overall normalization factor. We extend this relationship to FFs of all different kaons, i.e.,

$$
D_{1u}^{K^*} (z, \mu_0) = A(2z + 1) D_{1u}^{K^{*0}} (z, \mu_0),
$$

(15)

where $a$ stands for $u, d, s, \bar{u}, \bar{d}, \bar{s}$ and gluon $g$: $K^*$ stands for $K^{*\pm,0}$ and $\bar{K}^{*0}$ and $K$ for the corresponding pseudoscalar mesons.

For FFs of pseudoscalar mesons, we use isospin and charge conjugation symmetries and take,

$$
D_{1u}^{K^0} = D_{1u}^{K^0} = D_{1d}^{K^+} = D_{1d}^{K^-},
$$

(16)

$$
D_{1d}^{K^0} = D_{1d}^{K^0} = D_{1u}^{K^+} = D_{1u}^{K^-},
$$

(17)

$$
D_{1s}^{K^0} = D_{1s}^{K^0} = D_{1s}^{K^+} = D_{1s}^{K^-},
$$

(18)

$$
D_{1u}^{K^0} = D_{1u}^{K^0} = D_{1d}^{K^+} = D_{1d}^{K^-},
$$

(19)

$$
D_{1d}^{K^0} = D_{1d}^{K^0} = D_{1d}^{K^+} = D_{1d}^{K^-},
$$

(20)

$$
D_{1s}^{K^0} = D_{1s}^{K^0} = D_{1s}^{K^+} = D_{1s}^{K^-}.
$$

(21)

Here, for clarity, we omit arguments of fragmentation functions in Eqs. (16-21).

For the unpolarized FF of $\rho$ meson, we take it similar to that of $K^*$ besides the strangeness suppression factor in the fragmentation process. As usual, we differentiate between the favored and unfavored fragmentation. For the favored FF, we divide it into the leading and non-leading parts. The leading part is for hadron that contains the
fragmenting quark and the non-leading part is the rest, i.e., we take,
\[ D_{1a}^{\text{favored}}(z, \mu_0) = D_{1a}^{\text{favored, leading}}(z, \mu_0) + D_{1a}^{\text{favored, nonleading}}(z, \mu_0), \]
\[ D_{1a}^{\text{unfavored}}(z, \mu_0) = D_{1b}^{\text{unfavored}}(z, \mu_0). \]

Where \( \lambda_s \) is the strangeness suppression factor and is simply taken as \( \lambda_s = 1/3 \) in the numerical calculations presented in the following of this paper. In this way, we obtain, e.g.,
\[ D_{1d}^{LL}(z, \mu_0) = D_{1d}^{\text{unfavored}}(z, \mu_0) + \frac{1 - \lambda_s}{\lambda_s} D_{1d}^{\text{K}^0}(z, \mu_0), \]
\[ D_{1d}^{\text{K}^+}(z, \mu_0) = \frac{1}{2} D_{1d}^{\text{K}^0}(z, \mu_0) + \frac{2 - \lambda_s}{2\lambda_s} D_{1d}^{\text{K}^0}(z, \mu_0), \]
\[ D_{1d}^{\text{K}^0}(z, \mu_0) = D_{1d}^{\text{unfavored}}(z, \mu_0) = D_{1d}^{\text{LL}}(z, \mu_0). \]

B. The \( S_{LL} \)-dependent fragmentation function

We take two different scenarios for the parameterizations of \( S_{LL} \)-dependent FFs. In the first scenario, we follow the same strategy employed in [65] and differentiate between favored and unfavored fragmentations, i.e.,
\[ D_{1LL}^{\text{unfavored}}(z, \mu_0) = c_1 D_{1LL}^{\text{unfavored}}(z, \mu_0), \]
\[ D_{1LL}^{\text{favored}}(z, \mu_0) = c_1(a_2z + 1) D_{1LL}^{\text{favored}}(z, \mu_0), \]
where \( c_1 \) and \( a_2 \) are two free parameters.

In the second scenario, we adopt the same form of parameterizations for both favored and unfavored fragmentations. In this case, we find that the linear factor \( a_2z + 1 \) does not provide a good fit to the data available [2] and we change the power of \( z \) to 1/2, i.e.,
\[ D_{1LL}(z, \mu_0) = c_2(a_2z^{1/2} + 1) D_{1}(z, \mu_0), \]
where \( c_2 \) and \( a_2 \) are free parameters.

From the condition that \( -2 \leq D_{1LL}/D_1 \leq 1 \), we obtain constraints for the parameters in the parameterizations given by Eqs. (30-32). They should be taken in the range \( -2 \leq c_i \leq 1 \) and \( \text{min}\{1/c_i, -2/c_i\} \leq a_i + 1 \leq \text{max}\{1/c_i, -2/c_i\} \).

C. Fits to the LEP data and results of \( D_{1LL} \)

We fix the parameters in the parameterizations given by Eqs. (30-32) using the data available. Since the amount of data available for such a fit is not large, we take only the data on the spin alignment of \( K^{*0} \) from LEP [1, 2] and simply perform an eye-fit instead of applying the \( \chi^2 \) analysis. We choose the values of parameters to obtain FFs at the initial scale \( \mu_0 \) that is taken as \( \mu_0 = 2 \) GeV, evolve them using DGLAP given by Eq. (9) to the corresponding \( Q \) values and compare the results with the data [1, 2] for \( K^{*0} \) to fix the parameters. In this way, we fix \( c_1 = 0.15 \) and \( a_1 = -8.0 \) for the parameterizations in scenario I. The obtained results for the spin alignment of \( K^{*0} \) compared with the LEP data are shown in the left panel of Fig. 1. Here, as well as in the following of this paper, to be consistent with the LEP data [1, 2], \( K^{*0} \) denotes contributions to both \( K^{+} \) and its anti-particle \( \bar{K}^{*0} \).

Having fixed the parameters, we calculate the spin alignment also for \( \rho^0 \) and obtain the results in the right panel of Fig. 1.

![FIG. 1.](image-url) The spin alignment of \( K^{*0} \) (left panel) and that of \( \rho^0 \) (right panel) in \( e^+e^- \rightarrow VX \) at the Z-pole calculated with Scenario I compared with experimental data [1, 2]. In the calculations, we chose the center of mass energy of \( e^+e^- \) as the factorization scale, i.e., \( \mu_f = \sqrt{s} \).

From Fig. 1, we see that the scale dependence is more obvious in the small \( z \) region but quite small at large \( z \). It is also more obvious for \( K^{*0} \) than that for \( \rho^0 \). To see where this difference comes from, we look at the corresponding results for FFs.

In Fig. 2, we show the ratios \( D_{1LL}^{K^{*0}}/D_{1e}^{K^{*0}} \) for different flavors of quarks and that of gluon. The corresponding \( S_{LL} \)-dependent FFs \( D_{1LL}^{K^{*0}} \) are shown in Fig. 3.

We note that for the production of \( K^{*0} \), \( u \)-quark fragmentation is unfavored while \( d \) and \( s \) fragmentations are favored. From Fig. 2, we see that, in scenario I, the ratio \( D_{1LL}/D_1 \) is almost the same for favored fragmentations of different flavors of quarks but very different from that for the unfavored quark fragmentation. It is negative and relatively larger in magnitude in most of the \( z \) region in the favored case, but is positive and relatively smaller in the unfavored case. The scale dependence in the favored case is quite weak but seems much stronger in the unfavored case. We see also that, though starting from the same ratio at the initial scale, the gluon fragmentation function behaves quite different from the unfavored quark fragmentation function after the QCD evolution. It becomes even negative at large \( z \). This is
Here we see explicitly that favored FFs dominate at larger $z$ while unfavored and gluon fragmentations play important roles at small $z$. We also see that because of the strangeness suppression in fragmentation, the leading contributions from $s$-quark fragmentation is much larger than that from $d$-quark.

From the results shown in Figs. 2 and 3, we can now understand why there is a slight difference between the scale dependence of the spin alignment of $K^{*0}$ and $\rho^0$ as shown in Fig. 1. Because of the strangeness suppression in the favored $d$-fragmentation, contributions from unfavored quark and gluon fragmentations are relatively larger for the production of $K^{*0}$ than that of $\rho^0$. The stronger scale dependence of $D_{11LL}/D_1$ for the unfavored and gluon fragmentation leads to a slightly stronger scale dependence of the spin alignment of $K^{*0}$ than that of $\rho^0$.

The calculation in the second scenario is similar. By fitting the LEP data for $K^{*0}$ [1, 2], we fix the parameters as $c_2 = 1.0$ and $a_2 = -2.0$. The obtained results for the spin alignment of $K^{*0}$ are shown in the left panel of Fig. 4. With the same parameters, we obtain that for the spin alignment of $\rho^0$ in the right panel of Fig. 4. The obtained results of the ratios $D_{11LL}/D_1$ and those for the corresponding $S_{1LL}$-dependent FFs $D_{11LL}^{0*}$ are shown in Figs. 5 and 6 respectively.

From Fig. 5, we see that the ratios $D_{11LL}/D_1$ in this scenario for favored, unfavored and gluon fragmentations are quite similar with each other. By starting with the same parameterization at the initial scale, we obtain similar results after the QCD evolution. The tiny differences are resulted from the differences in the corresponding unpolarized FFs. Also because there is no large difference in the ratios $D_{11LL}/D_1$ between the favored and unfavored fragmentations in this scenario, we do not see similar difference in Fig. 4 between the spin alignment of $K^{*0}$ and that of $\rho^0$ in this scenario as that shown in Fig. 1 in scenario I.

Because of the differences in the corresponding unpolarized FFs, the obtained $D_{11LL}(z, \mu_f)$ shown in Fig. 6 exhibits also quite large differences between the favored and unfavored quark fragmentation and that of gluon. Here we see that, similar to those in scenario I, the favored FFs also dominate at larger $z$ but the unfavored and gluon FFs may have large contribution in the small $z$-region. The gluon FF $D_{11Lg}(z, \mu_f)$ is negative and
quite large in magnitude for small \( z \) and should play an important role in this region.

Comparing the FFs obtained in the two different scenarios, we see quite large differences. Nevertheless the obtained spin alignments in both cases can fit the LEP data [1, 2]. This is because the freedom to choose different parameterizations is quite large, the LEP data [1, 2] alone cannot fix them to high accuracy. In this connection, we note that we have not considered the flavor dependence of the ratio between the unpolarized and the spin parameterizations is quite large, the LEP data [1, 2] because the freedom to choose different scenarios, we see quite large differences. Nevertheless the obtained spin alignments in both cases can fit the LEP data [1, 2]. This is because the freedom to choose different parameterizations is quite large, the LEP data [1, 2] alone cannot fix them to high accuracy. In this connection, we note that we have not considered the flavor dependence of the ratio between the unpolarized and the spin aligned FFs besides different choices for the favored and unfavored fragmentation in scenario I. It is clear that more data in different reactions are necessary in order to determine these FFs to high precisions.

IV. NUMERICAL RESULTS FOR \( pp \to VX \)

In this section, we apply the FFs obtained in Sec. III to \( pp \to VX \) and calculate the spin alignment of vector meson numerically. To have a better understanding of the results in such a complicated process, we first present the fractional production rate of different flavor of partons. After that, we show our predictions on the spin alignment of \( K^{*0} \) and \( \rho^0 \) mesons in both scenarios.

A. Contributions of different flavors

From Eq. (3), we can calculate contributions from different subprocesses to the cross section separately. The fractional contribution from a given type of parton \( c \) to jet production is given by,

\[
R_c^{\text{jet}}(y, p_T) = \frac{d\sigma_{pp\to cX}}{dyd^2p_T} / \sum_c d\sigma_{pp\to cX} / dyd^2p_T, \tag{33}
\]

\[
\frac{d\sigma_{pp\to cX}}{dyd^2p_T} = \sum_{abcd} \int dy_2 x_1 f_a(x_1, \mu_f) \times x_2 f_b(x_2, \mu_f) \frac{1}{\pi} \frac{d\hat{\sigma}_{ab\to cd}}{dt}. \tag{34}
\]

Similarly, the fractional contribution to vector meson production is given by,

\[
R_c^V(y, p_T) = \frac{d\sigma_{pp\to cX\to VX}}{dyd^2p_T} / \sum_c d\sigma_{pp\to cX\to VX} / dyd^2p_T, \tag{35}
\]

\[
\frac{d\sigma_{pp\to cX\to VX}}{dyd^2p_T} = \sum_{abcd} \int dy_2 \int \frac{dz}{z^2} x_1 f_a(x_1, \mu_f) \times x_2 f_b(x_2, \mu_f) \frac{1}{\pi} \frac{d\hat{\sigma}_{ab\to cd}}{dt} D_{1c}^V(z, \mu_f). \tag{36}
\]

In Fig. 7, we show the results of \( R_c^{\text{jet}}(y, p_T) \) calculated using Eqs. (33) and (34) at the RHIC and LHC energies in the middle rapidity region as functions of \( p_T \). Take \( K^{*0} \) as an example, we show the corresponding results of \( R_c^V(y, p_T) \) calculated using Eqs. (35) and (36) in Fig. 8.

From Fig. 7, we see that in the presented \( p_T \) regions, the gluon contribution dominates at both RHIC and LHC energies for jet productions. The \( u/\bar{u} \) contribution is the largest among the three flavors of quarks while \( s/\bar{s} \) is the smallest. This results from the differences in PDFs [73] for different flavors of partons.

However, when FFs are taken into account, from Fig. 8, we see that the gluon contribution becomes less dominate. The \( d/\bar{d} \) contribution is even larger than the gluon contribution at the RHIC energy for \( p_T > 12 \) GeV while \( u/\bar{u} \) contribution becomes the smallest one. This is because the differential cross section for the production of parton \( c \) decreases very fast with increasing \( p_T \), much faster than the FF of \( c \to VX \) decreases with increasing \( z \). Usually the \( z \)-dependence of FF is much smoother compared with the \( p_T \)-dependence of the cross section.
As a result, in the large $p_T$ region for hadron production, contributions from relatively large $z$ (say $z > 0.3$) dominate. The leading contributions from favored quark fragmentations play more and more important roles with increasing $p_T$.

From Fig. 8, we also see that, by studying the $p_T$ dependence in the central rapidity region, we can study the interplay of contributions of gluon and favored quark fragmentation, while at LHC, we mainly study the contribution from gluon fragmentation. Quark fragmentations should dominate the fragmentation regions in the collisions processes.

To see the behaviors at the fragmentation regions explicitly, in Figs. 9 and 10, we show the corresponding results at the RHIC energy with $p_T > 5$ GeV and those at the LHC energy with $p_T > 10$ GeV as functions of $x_F$.

From Fig. 9 and 10, we see clearly that in the large $x_F$ region quark contribution dominates. For jet production, $u/\bar{u}$ plays the most important role. Taking the FFs into account, for $K^{*0}$ production, the favored fragmentation from $d/\bar{d}$ dominates. Hence, by studying hadron production at larger $x_F$, we study predominately the favored quark fragmentation.

At the end of this part, we emphasize that, by studying vector meson production in $pp \rightarrow VX$ for large $p_T$ at RHIC and LHC energies, even in the central rapidity regions, contributions from FFs at relatively large $z$ dominate. From the results for FFs obtained in Sec. III B, we see also that $D_{1LL}$ is significantly different from zero also in the relatively large $z$ region. This leads us to the expectation that the vector meson spin alignment should be quite significant in $pp$ collisions.

### B. The spin alignment in $pp \rightarrow VX$

Using the spin-dependent FFs obtained in Sec. III B by fitting the LEP data on $e^+e^-$ annihilations [1, 2], we calculate the spin alignment of vector meson in $pp \rightarrow VX$ using Eqs. (6) and (8). We present the results obtained in the following.

In Fig. 11, we show the spin alignments for $K^{*0}$ and $\rho^0$ at the RHIC energy in two rapidity regions as functions of $p_T$.

From Fig. 11, we see the following distinct features for the spin alignment in $pp \rightarrow VX$ at RHIC energy.

First, both the results for $K^{*0}$ and those for $\rho^0$ are significantly different from $1/3$, i.e., they show quite significant spin alignments in both cases. The deviations
of \(\rho_0\) from 1/3 increase monotonically with increasing \(p_T\). This is just consistent with the qualitative expectation mentioned at the end of Sec. IV A. The increases with increasing \(p_T\) are mainly due to increasing relative contributions from the quark fragmentation in particular those in the large \(z\) region where \(D_{1LL}/D_1\) is more significant.

Second, there is a significant difference between the results obtained in scenario I and those in scenario II. This is mainly due the difference in gluon fragmentation functions in the two scenarios.

Third, there is also a quite significant difference between the results obtained in the two different rapidity regions. This is mainly because of the relative contributions from quark fragmentations to the gluon fragmentation. In the \(1 < |y| < 2\) region, the relative contributions from quark fragmentations are larger than those in \(|y| < 0.5\) and they lead to larger vector meson spin alignments.

Forth, there is no distinct difference between the results for \(K^{*0}\) and those for \(\rho^0\). This is because that we have not considered the flavor dependence in our parameterizations of \(D_{1LL}/D_1\). The small difference comes mainly from strangeness suppressions in the unpolarized fragmentation functions.

Here, we have the advantage to study a much wider \(p_T\) range so that we can study the \(p_T\)-dependence more intensively. As mentioned above, the increase with \(p_T\) of the spin alignment is caused by the increasing contributions from quarks fragmentations relative to the gluon fragmentation. It is also because the gluon contribution becomes more dominate at LHC energy in the relative small \(p_T\) region in Fig. 12, the spin alignment in that region is closer to 1/3 and the differences between scenario I and II are also more significant.

In Figs. 13 and 14, we show results for \(p_T\)-integrated spin alignments of \(K^{*0}\) and \(\rho^0\) as functions of \(x_F\) at RHIC and LHC energies respectively.

In Fig. 12, we show the results obtained at the LHC energy. From Fig. 12, we see quite similar qualitative features as those seen from Fig. 11 at the RHIC energy.

FIG. 11. (Color online) Spin alignments of vector mesons in \(pp\) collisions at RHIC energy \(\sqrt{s} = 200\) GeV for \(K^{*0}\) and \(\rho^0\) in two rapidity regions as functions of \(p_T\).

FIG. 12. (Color online) Spin alignments of vector mesons in \(pp\) collisions at the LHC energy \(\sqrt{s} = 5.02\) TeV for \(K^{*0}\) and \(\rho^0\) in two rapidity regions as functions of \(p_T\).

FIG. 13. (Color online) Spin alignments of vector mesons in \(pp\) collisions at the RHIC energy \(\sqrt{s} = 200\) GeV for \(K^{*0}\) and \(\rho^0\) at \(p_T > 5\) GeV as functions of \(x_F\).

FIG. 14. (Color online) Spin alignments of vector mesons in \(pp\) collisions at the LHC energy \(\sqrt{s} = 5.02\) TeV for \(K^{*0}\) and \(\rho^0\) at \(p_T > 10\) GeV as functions of \(x_F\).

Here, from Figs. 13 and 14, we see rapid increases of the spin alignment with increasing \(x_F\), quite similar to that observed in \(e^+e^-\) shown in Fig. 1 and such a behavior is more obvious in scenario I. The increase reflects again the increasing relative contributions from favored quark fragmentations to the gluon fragmentation and also \(z\)-dependence of the favored \(S_{1LL}\)-dependent FF \(D_{1LL}\) relative to the corresponding unpolarized FF \(D_1\). The relative larger values in the small \(x_F\) region in scenario II are due to the quite large \(D_{1LL}\) of gluon fragmentation in the small \(z\) region. We recall that gluon fragmentation is even less known in the unpolarized case, this provides also a good opportunity to study gluon fragmentation mechanism.

From all the results shown in Figs. 11-14, we see clearly
that spin alignments of vector mesons are in general quite significant \( pp \to VX \) at high energies. Studying these spin alignments should provide a good test to QCD fragmentation mechanism in general and differentiate between different parameterizations scenarios, provide precise information on quark or gluon fragmentation in different kinematic regions in particular.

V. SUMMARY

In the QCD description of high energy reactions, the spin alignment of vector meson in a fragmentation process is described by the \( S_{LL} \)-dependent fragmentation function \( D_{1LL} \) defined via the Lorentz decomposition of the quark-quark correlator. A systematic study of the Lorentz decomposition show that \( D_{1LL} \) is independent of the polarization of the fragmenting quark. The first attempt to extract \( D_{1LL} \) for \( K^{*0} \) from the LEP data \([1, 2]\) on e\(^+\)e\(^-\)-annihilations has been made in \([65]\).

In this paper, we follow the same procedure of \([65]\) and make parameterizations of \( D_{1LL} \) in two different scenarios for \( K^{*0} \) and \( \rho^0 \) from different flavors of quarks, antiquarks and gluon and evolve them using DGLAP equations.

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