The electron and neutron EDM from supersymmetric see-saw thresholds

Gian F. Giudice\textsuperscript{a}, Paride Paradisi\textsuperscript{b} and Alessandro Strumia\textsuperscript{ac}

\textsuperscript{a} CERN, PH-TH, CH-1211, Geneva 23, Switzerland
\textsuperscript{b} Physik-Department, Technische Universität München, D-85748 Garching, Germany
\textsuperscript{c} Dipartimento di Fisica dell’Università di Pisa and INFN, Italia

Abstract

We consider the corrections that arise at one loop when integrating out heavy fields in supersymmetric models. We show that, in type-I see-saw models, complex $A_N$- and $B_N$-terms of the heavy right-handed neutrino give radiative contributions to the neutron EDM, as well as new dominant contributions to the electron EDM. Type-II and type-III see-saw also predict a pure gauge correction that makes complex the masses of the weak gauginos. All the see-saw models can predict observable EDM for the electron and for the neutron in a peculiar ratio.

1 Introduction

The observed neutrino masses give solid indications for the existence of some new physics at a superheavy scale $M \lesssim 10^{14}$ GeV. Identifying the nature of the dynamics at the scale $M$ is a very difficult task because experiments measuring neutrino masses and mixings probe only a dimension-five operator with coefficient suppressed by $1/M$. Given the largeness of the scale $M$, any other effect is essentially invisible to every experimental search, and the only possible handle is some indirect information from the requirement of a successful leptogenesis [1].

Supersymmetric theories offer opportunities to probe experimentally some of the features of the physics at the scale $M$ — which will be called here “see-saw sector” — because such dynamics could leave indelible traces on the soft terms of the light sparticles. The vestiges of
the see-saw sector left on the soft terms correspond to interactions not suppressed by inverse powers of \(M\), and therefore can lead to sizable and measurable effects. In particular, since the dynamics at the scale \(M\) is responsible for neutrino masses and — possibly — for leptogenesis, violations of individual lepton number and of CP are expected in the soft terms.

The information about individual lepton number and CP violation is communicated from the see-saw sector to the supersymmetric Standard Model (SM) by renormalization-group (RG) effects above the scale \(M\) \[2, 3\] and by finite threshold effects at the scale \(M\) \[4\]. In this paper we revisit these effects using the method of analytic continuation into superspace \[5, 6\]. Besides simplifying the calculation, this method allows for a more transparent interpretation of the contributions to the soft terms obtained from integrating out the dynamics at the scale \(M\).

But the most important result of our study is to reveal new effects that were not noticed in previous analyses. Finite threshold corrections at the scale \(M\) affect not only the soft terms for the charged sleptons, but also the \(A\) term for up-type quarks as well as the \(B_{\mu}\) term of the Higgs mixing. This latter term is also affected by the usual RGE running above \(M\): going to the standard phase convention where \(B_{\mu\mu}\) is real, the \(\mu\) term has a phase opposite to \(B_{\mu}\). This means that, if the \(B\) and/or \(A\)-term of the right-handed neutrinos is complex, the see-saw sector gives comparable contributions to both the electron and the neutron electric dipole moments (EDM).

In section 2 we describe the calculation of the contributions to the soft terms from integrating out the see-saw sector at the scale \(M\) with the formalism of analytic continuation into superspace. We will consider the usual see-saw with singlet right-handed neutrinos (type-I) as well as type-II and type-III see-saw mechanisms involving weak triplets \[1\]. The integration of weak triplets at the scale \(M\) gives pure gauge threshold effects (namely independent of the unknown neutrino Yukawa couplings) contributing to the gaugino masses and possibly inducing new CP violating effects. The phenomenological implications of our results for the electron and neutron EDM are discussed in section 3. In section 4 we conclude summarizing our results.

## 2 Threshold effects from superspace analyticity

### 2.1 Type-I supersymmetric see-saw

The method of analytic continuation into superspace \[5, 6\] allows us to extract information on the supersymmetry-breaking soft terms from calculations in the exactly supersymmetric theory, where we can fully exploit the power of the non-renormalization theorem.

We start by considering the simplest see-saw sector, which contains one singlet right-handed neutrino \(N\) for each generation. In the limit of exact supersymmetry, the theory is described by the Lagrangian

\[
\mathcal{L} = \int d^4 \theta \left( N^\dagger Z_N N + L^\dagger Z_L L + H_u^\dagger Z_{H_u} H_u \right) + \left[ \int d^2 \theta \left( N^T \frac{M}{2} N + N^T \lambda_N L H_u \right) + \text{h.c.} \right].
\]  (1)
Here the wave functions $Z_N$ and $Z_L$, the coupling constant $\lambda_N$, and the right-handed neutrino mass $M$ are $3 \times 3$ matrices in generation space. In this scheme, the neutrino mass matrix is given by the matrix product $m_\nu = -(H_u)^2 \lambda_N^2 M^{-1} \lambda_N$. For the moment we neglect gauge interactions, which will be introduced only when necessary.

Supersymmetry breaking is described by the terms in the Lagrangian

$$L_{\text{soft}} = -N^\dagger m^2_N N - L^\dagger m^2_L L - H_u^\dagger m^2_{H_u} H_u + \left( N^T B_N M \frac{M}{2} N + N^T A_N \lambda_N L H_u + \text{h.c.} \right),$$

where we use the same symbol to identify a chiral superfield or its scalar component. The soft masses $m^2_{N,L}$, the trilinear ($A_N$) and bilinear ($B_N$) parameters are also $3 \times 3$ matrices in generation space and all flavor indices are contracted with the usual rules of matrix product. All soft terms can be incorporated in the Lagrangian in eq. (1) by analytically continuing the parameters into superspace, with the replacement

$$\lambda_N \rightarrow (1 + \theta^2 A_N) \lambda_N, \quad M \rightarrow (1 + \theta^2 B_N) M, \quad Z_a \rightarrow 1 - \theta^2 \theta^2 m^2_a a = N, L, H_u.$$  

The non-renormalization theorem of supersymmetry guarantees that the superpotential parameters $\lambda_N$ and $M$ are not modified by quantum corrections, while all the renormalization is contained only in the wave functions $Z_a$. Therefore the expression in eq. (3c) is not stable under renormalization flow. Quantum effects generate corrections to $Z_a$ for all of its $\theta$ components, affecting the soft terms. In particular, the procedure of integrating out the right-handed neutrinos at the scale $M$, in the limit of exact supersymmetry, generates one-loop corrections to the wave functions of the $L$ and $H_u$ superfields which, in the $\overline{\text{MS}}$ (or, equivalently, in the $\text{DR}$) subtraction scheme, are given by

$$\delta Z_L = \frac{\lambda^R_N}{16\pi^2} \left( 1 - \ln \frac{M^R N^R}{\Lambda^2} \right) \lambda^R_N, \quad \delta Z_{H_u} = \text{Tr} \delta Z_L, \quad \lambda^R_N \equiv Z^{-1/2}_N \lambda_N Z^{-1/2}_L Z^{-1/2}_{H_u}, \quad M^R \equiv Z^{-1/2}_N M Z^{-1/2}_N.$$

Here $\Lambda$ is the ultraviolet cutoff and $\lambda^R_N$ and $M^R$ are the usual renormalized parameters defined in a superfield basis in which the kinetic terms are canonical. The trace in eq. (4) is taken over flavor indices.

We can use the supersymmetric expression for the wave-function renormalization in eq. (4) to obtain the quantum corrections to the soft terms by promoting the parameters into superspace. Since the quantum corrections $\delta Z$ generate $\theta^2$ terms, it is convenient to redefine the superfields in order to bring the one-loop corrected wave functions back into their tree-level form, given in eq. (3c). This can be done with the superfield redefinitions

$$L \rightarrow \left( 1 - \frac{\delta Z_L}{2} \right) \left( 1 - \theta^2 \delta Z_L |_{\theta^2} \right) L, \quad H_u \rightarrow \left( 1 - \frac{\delta Z_{H_u}}{2} \right) \left( 1 - \theta^2 \delta Z_{H_u} |_{\theta^2} \right) H_u,$$
where we have used the expansions
\[ \delta Z_{L,H_u} = \delta Z_{L,H_u} |_{\theta^2} + \theta^2 \delta Z_{L,H_u} |_{\theta^2} + \theta^2 \bar{\theta}^2 \delta Z_{L,H_u} |_{\theta^2 \bar{\theta}^2} . \]  

Note that the transformation in eq. (6) preserves the chiral properties of the superfields \( L \) and \( H_u \). This superfield rescaling transforms the kinetic terms in the following way
\[ L^\dagger (1 + \delta Z_L) L \rightarrow L^\dagger \left( 1 + \theta^2 \bar{\theta}^2 \delta Z_L |_{\theta^2 \bar{\theta}^2} \right) L , \]
working in the one-loop approximation. Comparing this expression (and the analogue for the \( H_u \) superfield) with eq. (3c), we obtain the one-loop corrections to the soft masses
\[ \delta m^2_{L,H_u} = - \delta Z_{L,H_u} |_{\theta^2 \bar{\theta}^2} . \]

Once we apply the superfield redefinitions in eq. (6) to the superpotential, we obtain two effects. First, the term \( 1 - \delta Z |_{\theta^2} \) gives the usual renormalization of the superpotential coupling constants. Second, the term \( 1 - \theta^2 \bar{\theta}^2 |_{\theta^2 \bar{\theta}^2} \) generates trilinear and bilinear soft terms. Starting from the superpotential
\[ \mathcal{W} = E \lambda_E L H_d + U \lambda_U QH_u + D \lambda_D QH_d + \mu H_u H_d , \]
the superfield redefinition induces one-loop corrections to the soft terms
\[ \mathcal{L}_{\text{soft}} = E \lambda_E A_E LH_d + U \lambda_U A_U QH_u + D \lambda_D A_D QH_d + B_{\mu} H_u H_d + \text{h.c.,} \]
\[ \delta A_E = - \delta Z_L |_{\theta^2} \quad \delta A_U = - \| \delta Z_{H_u} |_{\theta^2} \quad \delta A_D = 0 \quad \delta B_{\mu} = - \delta Z_{H_u} |_{\theta^2} . \]

Here \( \| \) is the identity matrix in flavor space.

Equations (9) and (12) give the quantum corrections to the soft terms induced by integrating out the right-handed neutrinos in terms of the supersymmetric corrections to the wave function \( \delta Z \). Since we are working in the one-loop approximation, we can obtain the explicit result by simply promoting the expression of \( \delta Z \) given in eq. (4) into superspace according to the tree-level superspace continuation of eq. (3). In the basis in which the right-handed neutrino mass matrix \( M \) is diagonal, real and positive, we obtain
\[ (\delta A_E)_{ij} = \frac{\lambda^* E_{ijkl} \lambda^* N_{jkl}}{16 \pi^2} \left[ B_{N k} F \left( \frac{M_k}{M_f} \right) - \left( \ln \frac{\Lambda^2}{M_k^2} + 1 \right) \right] A_{N k} \]
\[ (\delta m^2_{L})_{ij} = \frac{1}{16 \pi^2} \left\{ - \left( \lambda^* E_{ijk} \lambda^* N_{jkl} \frac{m^2_{N k}^2}{2} + \lambda^* E_{ijkl} m^2_{N k} \right) + \lambda^* N_{jkl} \lambda^* N_{jkl} \frac{m^2_{L i}}{2} \right\} + \lambda^* N_{jkl} \lambda^* N_{jkl} \frac{m^2_{L i}}{2} \]
\[ + \lambda^* N_{jkl} \lambda^* N_{jkl} \frac{m^2_{L i}}{2} + \lambda^* N_{jkl} \lambda^* N_{jkl} m^2_{N k} + \lambda^* N_{jkl} \lambda^* N_{jkl} A^*_{N k} A_{N s} \left[ \ln \frac{\Lambda^2}{M_k^2} + 1 \right] \]
\[ + \lambda^* N_{jkl} \lambda^* N_{jkl} \left( \frac{m^2_{N k}}{2} + \frac{m^2_{N k}}{2} \left( \frac{M_k}{M_f} + \frac{M_f}{M_k} \right) \right) F \left( \frac{M_k}{M_f} \right) \]
\[ + \left( \lambda^* N_{jkl} \lambda^* N_{jkl} B_{N k} A_{N s} + \lambda^* N_{jkl} \lambda^* N_{jkl} A^*_{N k} B_{N k} F \left( \frac{M_k}{M_f} \right) \right) \]
\[ + \lambda^* N_{jkl} \lambda^* N_{jkl} \left[ B_{N k} B_{N s} F \left( \frac{M_k}{M_f} \right) - \left( B_{N k} B_{N s} + B_{N s} B_{N k} \right) \right] G \left( \frac{M_k}{M_f}, \frac{M_k}{M_f} \right) \]
\[ \delta m_{H_u}^2 = \text{Tr} \delta m_L^2 \]  

(16)

where the functions \( F \) and \( G \) are normalized such that \( F(1) = G(1, 1) = 1 \):

\[ F(x) = \frac{2x \ln x}{x^2 - 1}, \quad G(x, y) = \frac{4xy}{x^2 - y^2} \left( \frac{\ln x}{1 - x} - \frac{\ln y}{1 - y} \right). \]

(17)

It is interesting to note how the loop functions emerge in the method of analytic continuation not as the result of integrals (like in ordinary Feynman diagram calculations), but rather as the expansion of a logarithmic function in superspace.

To make the result more transparent, we consider the degenerate case in which \( m_{N,L,H_u}^2 \) and \( M \) are proportional to the identity (but \( \lambda_N, A_N \) and \( B_N \) are general matrices in flavor space).

In this case, the expressions for \( \delta A_E \) and for \( \delta m_L^2 \) become (in matrix notation)

\[ \delta A_E = \frac{\lambda_N^\dagger}{16\pi^2} \left[ B_N - A_N \left( \ln \frac{\Lambda^2}{M^2} + 1 \right) \right] \lambda_N, \]

(18a)

\[ \delta m_L^2 = \frac{\lambda_N^\dagger}{16\pi^2} \left[ 2m_N^2 - \left( m_N^2 + m_L^2 + m_H_u^2 + A_N^\dagger A_N \right) \left( \ln \frac{\Lambda^2}{M^2} + 1 \right) \right. 
\]

\[ \left. + B_N^\dagger A_N + A_N^\dagger B_N + \frac{\left( B_N^\dagger B_N - B_N B_N^\dagger \right)}{2} \right] \lambda_N, \]

(18b)

while the other soft terms are still given by eqs. (14) and (16). In the case in which the matrix \( B_N \) is universal or hermitian, the last term vanishes.

The logarithmic terms in eqs. (18) reproduce the renormalization-group evolution of the soft terms from \( \Lambda \) to the mass scale \( M \) of the right-handed neutrinos, once the leading logs are resummed with the usual technique. Our expressions also give the finite threshold corrections, confirming and extending the results in ref. [7].

The terms proportional to \( B_N \) in eqs. (18a) and (18b) are (scheme-independent) finite threshold corrections at the scale \( M \). They are completely analogous to the gauge-mediation effects in presence of superpotential couplings between messengers and matter, since \( N \) is playing the role of a messenger field coupled to a spurion superfield \( \mathcal{X} = M + \theta^2 F \) with \( F = B_N M \). Just as in that case, as long as \( \left[ X, X^\dagger \right] = 0 \), there are no one-loop contributions to the square scalar masses proportional to \( F^2/M^2 = B_N^2 \) [8]. This is because the one-loop

\[ \delta m_{H_u}^2 = \text{Tr} \delta m_L^2 \]

\[ F(x) = \frac{2x \ln x}{x^2 - 1}, \quad G(x, y) = \frac{4xy}{x^2 - y^2} \left( \frac{\ln x}{1 - x} - \frac{\ln y}{1 - y} \right). \]
correction to the wave function is proportional to $\ln X^\dagger X$, which can be written as $\ln X + \ln X^\dagger$, if $X$ and $X^\dagger$ commute; no $\theta^2 \bar{\theta}^2$ term can then be present. For this reason, in the basis where $M$ is diagonal, the contribution in eq. (18b) is proportional to $[B, B^\dagger]$. Our result for the threshold correction to the soft masses proportional to $\text{Re}(A_N B_N^\dagger)$ disagrees with ref. [4] in the sign, and the term proportional to $m^2$ agrees with ref. [7].

The novelty of our results with respect to previous analyses lies primarily in the new contributions to $A_U$ and $B_\mu$. Note that, differently than in the case of $\delta A_E$, the corrections to $A_U$ and $B_\mu$ are proportional to a trace involving $\lambda_N^\dagger \lambda_N$. Therefore, while $A_E$ is sensitive to the flavor structure, the corrections to $A_U$ and $B_\mu$ are sizable whenever any of the entries of $\lambda_N$ is large, irrespectively of its flavor index.

### 2.2 Type-III supersymmetric see-saw

An alternative see-saw sector is described by weak SU(2) triplet chiral superfields $N^a$ ($a = 1, 2, 3$ is the SU(2) index) with zero hypercharge and with superpotential

$$\mathcal{W} = N^a T^a M_{2N}^a + N^a T^a \lambda_N L^a H_u$$

such that the formula for neutrino mass matrix remains the same as in type-I. As in the previous case, there are 3 generations of $N^a$ and the parameters $M$ and $\lambda_N$ are $3 \times 3$ matrices in flavor space, with flavor indices omitted. The soft terms are given by

$$\mathcal{L}_{\text{soft}} = -N^a \bar{m}_N^2 N^a - L^a \bar{m}_L^2 L - H_u^\dagger m_{H_u}^2 H_u + \left( N^a T^a B_N M_{2N}^a N^a + N^a T^a A_N \lambda_N L^a H_u + \text{h.c.} \right)$$

The calculation of the corrections to the low-energy soft terms is completely analogous to the previous case, and the supersymmetric wave function renormalization is still given by eq. (4), after multiplying the right-hand side by a factor of 3. This extra factor of 3 counts the states circulating in the loop of the wave-function diagram. Hence, all soft terms are given by the expressions derived for type-I see-saw, multiplied by a factor of 3.

There is one important novelty of type-III see-saw, related to the fact that the heavy fields $N^a$ now have gauge interactions, such that also the vectors supermultiplet gets a wave-function renormalization that depends on $M$. This is the well known correction to the gauge $\beta$ function, which, after analytical continuation into superspace, also implies a finite one-loop corrections at the threshold $M$ to the gaugino mass. This contribution is completely analogous to the familiar case of gauge mediation, and it is proportional to the discontinuity of the corresponding $\beta$-function at the scale $M$. The wino mass $M_2$ receives a correction

$$\delta M_2 = \frac{g^2}{8\pi^2} \text{Tr} B_N,$$

while the gluino and bino masses, $M_3$ and $M_1$, are unaffected because the superfield $N^a$ carries neither color nor hypercharge. Since the effect is proportional to the complex parameter $B_N$, this correction introduces a new CP-violating phase in the gaugino sector.
2.3 Type-II supersymmetric see-saw

Another possible see-saw sector is obtained by introducing weak triplets $T$ and $\bar{T}$ with hypercharge $Y_T = -Y_{\bar{T}} = -2Y_L$, such that the relevant superpotential is

$$
\mathcal{W} = MT\bar{T} + \frac{1}{2} \left( \lambda_T L^T \epsilon \tau^a L^a + \lambda_{H_u} H_u^T \epsilon \tau^a H_u T^a + \lambda_{H_d} H_d^T \epsilon \tau^a H_d T^a \right). 
$$

(22)

Here $\lambda_T$ is a $3 \times 3$ matrix in flavor space, and $\epsilon = i\tau^2$ is the anti-symmetric tensor. One single $T$ is enough to induce generic neutrino masses. Since $\bar{T}$ does not couple to leptons, the neutrino mass matrix is $m_{\nu}^{ij} = \lambda_T^{ij} \lambda_{H_u} (H_u)^2 / M$ and its flavor structure is identical to the one of the Yukawa coupling $\lambda_T$.

The soft terms can be introduced by analytically continuing the parameter into superspace according to the rules

$$
Z_a \rightarrow 1 - \theta^2 \bar{\theta} m_a^2, \quad a = T, \bar{T}, L, H_u, H_d
$$

(23a)

$$
\lambda_T \rightarrow (1 + \theta^2 A_{\lambda_T}) \lambda_T, \quad \lambda_{H_u,d} \rightarrow (1 + \theta^2 A_{\lambda_{H_u,d}}) \lambda_{H_u,d}, \quad M \rightarrow (1 + \theta^2 B_T) M.
$$

(23b)

For simplicity, we assume flavor universality for the soft terms and thus $m_L^2$ and $A_{\lambda_T}$ in eq. (23) do not carry flavor indices.

In this case we find the following one-loop corrections for the supersymmetric wave functions of the fields $L$, $H_u$, and $H_d$

$$
\delta Z_L = 3 \frac{\lambda_T^{ij} \lambda_T}{16\pi^2} \left( \ln \frac{\Lambda^2}{M^2} + 1 \right),
$$

(24a)

$$
\delta Z_{H_u} = 3 \frac{\lambda_{H_u,\tau}^i \lambda_{H_u}}{16\pi^2} \left( \ln \frac{\Lambda^2}{M^2} + 1 \right),
$$

(24b)

$$
\delta Z_{H_d} = 3 \frac{\lambda_{H_d,\tau}^i \lambda_{H_d}}{16\pi^2} \left( \ln \frac{\Lambda^2}{M^2} + 1 \right).
$$

(24c)

Promoting the wave functions into superspace with the prescription given in eq. (23), we find

$$
\delta A_E = 3 \frac{\lambda_T^{ij} \lambda_T}{16\pi^2} \left[ B_T - A_{\lambda_T} \left( \ln \frac{\Lambda^2}{M^2} + 1 \right) \right] + \| F_D, 
$$

(25)

$$
\delta A_U = \| F_U, \quad \delta A_D = \| F_D, \quad \delta B_{\mu} = F_U + F_D, \quad
$$

(26)

$$
F_{U,D} = 3 \frac{\lambda_{H_u,d}}{16\pi^2} \left[ B_T - A_{\lambda_{H_u,d}} \left( \ln \frac{\Lambda^2}{M^2} + 1 \right) \right],
$$

(27)

$$
\delta m_L^2 = 3 \frac{\lambda_T^{ij} \lambda_T}{16\pi^2} \left[ m_T^2 + m_{\bar{T}}^2 - \left( 2m_L^2 + m_{\bar{T}}^2 + |A_{\lambda_T}|^2 \right) \left( \ln \frac{\Lambda^2}{M^2} + 1 \right) + 2 \text{Re} \left( A_{\lambda_T}^* B_T \right) \right],
$$

(28)

$$
\delta m_{H_u}^2 = 3 \frac{\lambda_{H_u}}{16\pi^2} \left[ m_T^2 + m_{\bar{T}}^2 - \left( 2m_{H_u}^2 + m_{\bar{T}}^2 + |A_{\lambda_{H_u}}|^2 \right) \left( \ln \frac{\Lambda^2}{M^2} + 1 \right) + 2 \text{Re} \left( A_{\lambda_{H_u}}^* B_T \right) \right],
$$

(29)
$$\delta m_{H_u}^2 = 3 \frac{\left| \lambda_{H_u} \right|^2}{16\pi^2} \left[ m_T^2 + m_{\tilde{T}}^2 - \left( 2m_{H_u}^2 + m_{\tilde{T}}^2 + a_{\lambda_{H_u}, A_{\lambda_{H_u}}^2} \right) \left( \ln \frac{\Lambda^2}{M^2} + 1 \right) + 2 \text{Re} \left( A_{\lambda_{H_u}}^* B_T \right) \right].$$ \hspace{1cm} (30)

In type-II see-saw the threshold corrections at the scale $M$ contribute, in the squark sector, to both $A_U$ and $A_D$. Moreover, since the fields $T$ and $\tilde{T}$ carry both weak and hypercharge quantum numbers, we find corrections to both wino and bino masses [9]:

$$\delta M_2 = \frac{g^2}{4\pi^2} B_T, \quad \delta M_1 = \frac{3g'^2}{8\pi^2} B_T.$$ \hspace{1cm} (31)

All formulæ are trivially extended to the cases of more $T + \tilde{T}$ repetitions or hybrids between type-I and/or type-II and/or type-III see-saws. RGE-improving and RGE running from $M$ down to the weak scale is well known.

### 3 Electric dipole moments

In the MSSM, the potential sources of CP violation are the $\mu$ term, the gaugino masses $M_i$ (with $i = 1, 2, 3$), $B_\mu$ and the $A$ terms $A_F$ (with $F = E, U, D$) as well as the sfermion masses. As well known, only some of their combinations provide physical CP violating phases [10]. As shown in the previous section, complex $A_N$- and $B_N$-terms of the see-saw heavy fields give corrections to the $B_\mu$ term and to some $A_F$-terms of the light MSSM fields. To reach the usual phase convention $B_\mu > 0$ (i.e. $\tan \beta > 0$) a complex $B_\mu$ needs an opposite phase in $\mu$.

Specializing the well known generic formulæ for EDM from sparticle loops to this case, and assuming sparticles degenerate at a common mass $m_{\text{SUSY}}$, the electron EDM is

$$d_e = -\frac{m_{e} e}{4\pi m_{\text{SUSY}}^2} \left[ \left( \frac{5\alpha_3}{27} \sin \phi_3 + \frac{\alpha_Y}{24} \sin \phi_1 \right) \tan \beta + \frac{\alpha_Y}{12} \sin \phi_{A_e} \right],$$ \hspace{1cm} (32a)

the quark EDM are (neglecting the $\mathcal{O}(\alpha_Y)$ contribution):

$$d_d = -\frac{m_{d} e}{4\pi m_{\text{SUSY}}^2} \left[ \left( \frac{2\alpha_3}{27} \sin \phi_3 + \frac{7\alpha_2}{24} \sin \phi_2 \right) \tan \beta + \frac{\alpha_3}{27} \sin \phi_{A_d} \right],$$ \hspace{1cm} (32b)

$$d_u = +\frac{m_{u} e}{4\pi m_{\text{SUSY}}^2} \left[ \left( \frac{4\alpha_3}{27} \sin \phi_3 + \frac{\alpha_2}{4} \sin \phi_2 \right) \tan^{-1} \beta + \frac{4\alpha_3}{27} \sin \phi_{A_u} \right],$$ \hspace{1cm} (32c)

and their chromo-electric dipoles are:

$$d_{c_d} = -\frac{m_{d} e}{4\pi m_{\text{SUSY}}^2} \left[ \left( \frac{5\alpha_3}{18} \sin \phi_3 + \frac{\alpha_2}{8} \sin \phi_2 \right) \tan \beta + \frac{5\alpha_3}{18} \sin \phi_{A_d} \right],$$ \hspace{1cm} (32d)

$$d_{c_u} = -\frac{m_{u} e}{4\pi m_{\text{SUSY}}^2} \left[ \left( \frac{5\alpha_3}{18} \sin \phi_3 + \frac{\alpha_2}{8} \sin \phi_2 \right) \tan^{-1} \beta + \frac{5\alpha_3}{18} \sin \phi_{A_u} \right],$$ \hspace{1cm} (32e)

\footnote{We here neglect CP violation in the neutrino Yukawa couplings. We remind that, if the neutrino Yukawa couplings $\lambda_N$ are the only source of CP violation, the induced leptonic EDM are small [11, 7], below the future planned experimental sensitivities [12]. The situation is different in SUSY GUT see-saw models [13, 14].}
where the quark masses and all parameters are renormalized at the scale $m_{\text{SUSY}}$ and

$$\phi_i = \arg(\mu M_i), \quad \phi_{A_u} = \arg(M_1 A_u^*), \quad \phi_{A_d} = \arg(M_3 A_d^*)$$ (33)

in the convention of [15]. The neutron EDM $d_n$ can be estimated from the naive quark model as $d_n \approx \frac{2}{3} d_d - \frac{1}{3} d_u$. A better estimate is obtained from QCD sum rules [16, 17] taking also into account QCD renormalization of $d_q$ and $d_c$ from $m_{\text{SUSY}} \approx m_t$ down to $m_n$ [18]:

$$d_n = 1.2 d_d - 0.3 d_u + 0.2 e d_q^c + 0.8 e d_u^c$$ (34)

up to an uncertainty of about $\pm 50\%$. Given the current experimental bounds $|d_n| < 2.9 \times 10^{-26} \text{ e cm (90\%CL)}$ [19] and $|d_{\text{Hg}}| < 3.1 \times 10^{-29} \text{ e cm (95\%CL)}$ [20], it might be that $|d_{\text{Hg}}|$ is more sensitive than $d_n$ to SUSY effects [16, 17]. Given the large theoretical uncertainties affecting the prediction of $d_{\text{Hg}}$, we prefer here to be conservative and focus only on $d_n$.

Before proceeding, we recall the correlation between $d_e$ and $a_\mu = (g-2)_\mu/2$. Recent analyses of $a_\mu$ point towards a $3\sigma$ discrepancy [21, 22]: $\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \approx (3 \pm 1) \times 10^{-9}$. Therefore, it is interesting to evaluate the expected value for $d_e$ within SUSY see-saw models assuming that the above discrepancy is due to supersymmetry. Assuming again sparticles degenerate at a common mass $m_{\text{SUSY}}$, we find

$$|d_e| \approx 10^{-27} \text{ e cm} \times \frac{\Delta a_\mu}{3 \times 10^{-9}} \frac{\phi_2}{10^{-3}}.$$ (35)

We recall that $|\phi_2| \equiv |\arg(\mu M_2)| \approx 10^{-3}$ is the natural size of a loop induced CP violating phase, as arising in the context of type-II and III SUSY see-saw models.

3.1 Type-I SUSY-see-saw

Complex $A_N$ and $B_N$ terms induce two main effects:

i) contributions to the $A$-terms of the electron and of the up-quark: $\delta A_e \sim (\lambda_N^+ \lambda_N)_{ee}(A_N, B_N)$ and $\delta A_u \sim \text{Tr}(\lambda_N^+ \lambda_N)(A_N, B_N)$;

ii) a contribution to $B_\mu$: $\delta B_\mu \sim \text{Tr}(\lambda_N^+ \lambda_N)(A_N, B_N)$. Going to the standard phase convention where $B_\mu\mu$ is real and positive, this needs a complex $\mu$ term.\footnote{Even within the CMSSM a complex $A_0$ indirectly induces a complex $\mu$, and thus studies of EDM from complex $A$-terms and real $\mu$ term do not seem motivated.} A qualitatively similar result holds in the next-to-minimal MSSM, where the $\mu$ term is replaced by the vev of a singlet.

Therefore, type-I see-saw with complex $A_N$ or $B_N$ predict contributions to the EDM typical of a complex $\mu$-term. In fact, the effects driven by a complex $\mu$ to the electron EDM are parametrically enhanced by a factor of $(\alpha_2/\alpha_Y) \tan \beta$ compared to the effects from a complex
In the quark sector, the enhancement is only of order \( \tan \beta \) as the gluino effects are dominant both in case of a complex \( A_u \) or \( \mu \) terms, see eq. (32e).

A characteristic prediction of this scenario is the ratio \( d_e/d_n \), that can be computed knowing the sparticle spectrum. The \( \mu \) term with a small complex phase gives \( \phi_3 = \phi_2 = \phi_1 \equiv \phi_i \), so that

\[
|d_n| \approx 20|d_e| \approx 2 \times 10^{-27} \text{cm} \tan \beta \left( \frac{200 \text{ GeV}}{m_{\text{SUSY}}} \right)^2 |\phi_i| \times 10^{-4} .
\]

(36)

The ratio \( |d_n/d_e| \approx 20 \) gets reduced if squarks are heavier than sleptons, and can be computed once sparticles are discovered and their masses measured.

As an illustrative example of a full computation, we assume now a constrained-MSSM (CMSSM) spectrum. Following [23], we perform a ‘naturalness’ scan of the parameter space, i.e. with density inversely proportional to the fine-tuning, such that regions with higher points density are proportionally more likely. Operatively, this is obtained by sampling the CMSSM parameters at the GUT scale as

\[
m_0 = [0...1]\tilde{m}, \quad M_{1/2} = [0...1]\tilde{m}, \quad A_0 = [-1...1]\tilde{m}, \quad \mu = [-1...1]\tilde{m}
\]

(37)

where \([a...b]\) denotes a random number between \( a \) and \( b \), and fixing the overall scale \( \tilde{m} \) by requiring the correct scale of electro-weak symmetry breaking, \( M_Z^2 \approx -2m_{\tilde{H}_u}^2 - 2|\mu|^2 \) at tree level for large \( \tan \beta \). Most of the obtained sparticle spectra have sparticle masses below \( M_Z \) and are thereby excluded by the current constraints which make the CMSSM fine-tuned, so we discard them.

As an illustrative example, in our numerical analysis we consider the degenerate case in which \( m_{N,L,H_u}, A_{N,T}, B_{N,T} \) and \( M \) are proportional to the identity. Keeping the parameter \( \tan \beta \) fixed to \( \tan \beta = 10 \), and assuming \( \text{Tr}(\lambda^\dagger_N \lambda_N) = \text{Tr}(\lambda^\dagger_T \lambda_T) = |\lambda_{H_u}|^2 = |\lambda_{H_d}|^2 = 10^{-3} \), in fig. 1 on the left (right) we show the predictions for \( |d_n| \) vs. \( |d_e| \) in case of \( \text{Im} A_{N,T} = M_{1/2} \) (\( \text{Im} B_{N,T} = M_{1/2} \)), where \( M_{1/2} \) is the unified gaugino mass parameter. Both EDM roughly scale as \( \text{Im}(A,B) \times \tan \beta \times \text{Tr}(\lambda^\dagger_{N,T} \lambda_{N,T}) \). As shown in fig. 1, the case where \( \text{Im} A_{N,T} \neq 0 \) predicts EDM enhanced by a large log factor \( \log \Lambda^2/M^2 \) compared to the case where \( \text{Im} B_{N,T} \neq 0 \) (see also eq. (13))\(^4\). As we can see, \( |d_n| \) and \( |d_e| \) are highly correlated such that \( |d_n| \approx 10|d_e| \) in most of the CMSSM parameter space. The ratio \( d_n/d_e \) depends on the specific sparticle mass spectrum considered.

Our results show that contributions to the EDMs require phases in \( A_{N,T} \) or \( B_{N,T} \) but not necessarily in both. Therefore even type-I models with vanishingly small \( B_N \), like those motivated by soft leptogenesis [24, 25], can lead to observable effects.

\(^4\)If we assume a fixed \( \text{Im}(A_{N,T}, B_{N,T}) \) independent of \( \tan \beta \), \( d_e \) and \( d_n \) have an extra \( \tan \beta \) enhancement, because a large \( \tan \beta \) is obtained from a small \( B_\mu \sim m^2_\lambda/\mu \tan \beta \), consequently CP-violating phases would scale as \( \phi_2 \propto \tan \beta \).
Figure 1: Predictions of SUSY-see-saw models for $|d_n|$ versus $|d_e|$. We assume a CMSSM-like spectrum making a ‘naturalness’ scan of the parameter space as described in eq. (37), such that higher density of points means higher probability. We assume the degenerate case, in which $m^2_{N,L,H_{u,v}, A_{N,T}, B_{N,T}}$ and $M$ are proportional to the identity, and we set $\tan \beta = 10$, $\text{Tr}(\lambda_N^\dagger \lambda_N) = \text{Tr}(\lambda_T^\dagger \lambda_T) = |\lambda_{H_u}|^2 = |\lambda_{H_d}|^2 = 10^{-3}$ and $M = 10^{12}$ GeV such that observed neutrino masses are typically reproduced and leptogenesis is possible in all see-saw models. For definiteness we assumed $\text{Im}(A_{N,T}) = M_{1/2}$ (left) and $\text{Im}(B_{N,T}) = M_{1/2}$ (right).

3.2 Type-III SUSY-see-saw

The effects driven by $A_N$ are similar to type-I see-saw up to order one factors, as shown by fig. 1 on the left. The qualitatively new feature is the correction to $M_2 \sim g_2^2 B M$. This effect is dominant as long as $g_2 \gg \lambda$, i.e. $M_{N_{1,2,3}} \lesssim 10^{14}$ GeV, and we consider this new scenario.

We recall that a common phase of the gaugino masses $M_{1,2,3}$ is usually turned into a common phase of the $A$-terms via an $R$-transformation. This is not possible if $M_{1,2,3}$ have different complex phases. Thereby it is more convenient to keep $M_2$ complex.

Under the usual RGE running a complex $M_2$ induces complex $A$-terms and a complex $B_\mu$ (thus a complex $\mu$ in the basis where $B_\mu \mu$ is real) that we systematically take into account in our numerical analysis.

Fig. 1 shows the typical range of $d_e$ and $d_n$ again assuming $\text{Tr}(\lambda_N^\dagger \lambda_N) = \text{Tr}(\lambda_T^\dagger \lambda_T) = |\lambda_{H_u}|^2 = |\lambda_{H_d}|^2 = 10^{-3}$ and $\text{Im}(A_{N,T}, B_{N,T}) = M_{1/2}$. As in type-I SUSY see-saw, it turns out that $|d_n| \approx 10|d_e|$. The effect of $A_{N,T}$ is quantitatively similar to the type-I see-saw case, while the effect of $B_{N,T}$ can now be much larger, in view of the gauge threshold effect.
3.3 Type-II SUSY-see-saw

Type II see-saw allows to predict the flavor of the Yukawa matrix $\lambda_T$ in terms of the neutrino mass matrix $m_\nu$, but the overall size of $\lambda_T$ is not fixed in terms of $m_\nu$ and $M$, as the ratio between $\lambda_T$ and $\lambda_{Hu}$ is unknown. It is possible to make arbitrarily small either $\delta A_E$ (that contributes to the electron EDM) or $\delta A_U$ (that contributes to the neutron EDM) but not both.

As in type-III see-saw there is an extra effect due to the gauge couplings, see eq. (31). All electroweak gauginos are now affected, but the new effect due to the bino is not dominant. Thereby the gauge-induced EDM are qualitatively similar to type-III and it turns out that $|d_{e,n}|_{\text{II}} \approx 2|d_{e,n}|_{\text{III}}$ as shown analytically by eqs. (21), (31) and numerically in fig. 1 on the right.

If CP violation is driven by $A_T$, the predictions for the EDM highly depend on the values of the relevant Yukawa couplings. Assuming $\text{Tr}(\lambda_N^\dagger \lambda_N) = \text{Tr}(\lambda_T^\dagger \lambda_T) = |\lambda_{Hu}|^2 = |\lambda_{Hd}|^2 = 10^{-3}$, all the types of see-saw models provide the same predictions for the EDM up to order one factors, as shown in fig. 1 on the left.

4 Conclusions

We considered the corrections to soft terms that arise at one loop in supersymmetric models when integrating out heavy field related to the see-saw sector. The use of analytic continuation in superspace makes clear that the possibly complex $B$-term of the heavy field propagates to the $A$-terms and $B$-terms of all light fields coupled to it. Furthermore, the possibly complex $A$-terms of heavy fields have a similar effect, already from the well-known RGE.

In particular, the heavy fields present in see-saw models to mediate the neutrino mass operator $(LH_u)^2$ necessarily couple to $L$ and to $H_u$, making complex not only the $A$-terms that involve the leptons $L$, but also those involving $H_u$ (coupled to up-quarks), as well as its $B_\mu$ term. In the standard phase convention where $B_{\mu\mu}$ is real, the $\mu$-term has a phase opposite to $B_\mu$. Consequently see-saw models give contributions to the electron EDM and to the quark EDM that were previously neglected. Such effects are proportional to the unknown neutrino Yukawa couplings squared.

Furthermore, within type-II and type-III see-saw models, where neutrino masses are mediated by heavy triplets under SU(2)$_L$, there are new effects proportional to their well-known gauge couplings squared: at one loop the $B_N$-term makes complex the weak gaugino masses $M_2$ and (in type-II only) $M_1$. Again this induces electric dipole moments for all fermions, not only leptons. A order unity complex phase in the see-saw $B$ term gives a small phase, of order $\alpha_2/4\pi$, to light soft terms, and consequently EDM just below the present bounds, if sparticles exist around the weak scale.

Lepton EDM are predicted to be proportional to the mass of the corresponding lepton, but nevertheless $d_e$ is presently a better probe than $d_\mu$ and especially $d_\tau$. Moreover, there is a loose connection between $d_e$ and lepton flavor violating processes like $\mu \rightarrow e\gamma$: in type-I see-saw $d_e$
and $\mu \to e\gamma$ are both generated by the neutrino Yukawa couplings, whose flavor structure is however unknown, while in type-II and III the presumably dominant gauge threshold effect only gives rise to electric dipoles.

In conclusion, there are previously unnoticed signals of SUSY-see-saw models which are not confined to leptons. On the contrary, SUSY see-saw models can induce observable electric-dipole moments for the electron and for the neutron, in a characteristic ratio.

Acknowledgments: We thank Michele Frigerio, Riccardo Rattazzi, Andrea Romanino for useful discussions. P.P. thanks the CERN where part of his work was carried out. The work of P.P. was supported in part by the Cluster of Excellence “Origin and Structure of the Universe” and by the German Bundesministerium für Bildung und Forschung under contract 05H09WOE.

References

[1] For a review on neutrino masses, leptogenesis, see-saw models, their supersymmetric signals, see A. Strumia, F. Vissani, arXiv:hep-ph/0606054.

[2] L. J. Hall, V. A. Kostelecky and S. Raby, Nucl. Phys. B 267 (1986) 415.

[3] F. Borzumati and A. Masiero, Phys. Rev. Lett. 57 (1986) 961.

[4] Y. Farzan, Phys. Rev. D 69 (2004) 073009 [arXiv:hep-ph/0310055]. See also E. J. Chun, A. Masiero, A. Rossi and S. K. Vempati, Phys. Lett. B 622 (2005) 112 [arXiv:hep-ph/0502022]. S. K. Kang, A. Kato, T. Morozumi and N. Yokozaki, Phys. Rev. D 81 (2010) 016011 [arXiv:0909.2484]. F. R. Joaquim and A. Rossi, Phys. Rev. Lett. 97 (2006) 181801 [arXiv:hep-ph/0604083].

[5] G. F. Giudice and R. Rattazzi, Nucl. Phys. B 511 (1998) 25 [arXiv:hep-ph/9706540].

[6] N. Arkani-Hamed, G. F. Giudice, M. A. Luty and R. Rattazzi, Phys. Rev. D 58 (1998) 115005 [arXiv:hep-ph/9803290].

[7] Y. Farzan and M. E. Peskin, Phys. Rev. D 70, 095001 (2004) [arXiv:hep-ph/0405214].

[8] G. F. Giudice and R. Rattazzi, Phys. Rept. 322 (1999) 419 [arXiv:hep-ph/9801271].

[9] T. Goto, T. Kubo and Y. Okada, arXiv:1001.1417.

[10] M. Dugan, B. Grinstein, and L. Hall, Nucl. Phys. B 255 (1985) 413.

[11] A. Romanino and A. Strumia, Nucl. Phys. B 622 (2002) 73 [arXiv:hep-ph/0108275]. J. R. Ellis, J. Hisano, M. Raidal and Y. Shimizu, Phys. Lett. B 528, 86 (2002) [arXiv:hep-ph/0111324]; I. Masina, Nucl. Phys. B 671, 432 (2003) [arXiv:hep-ph/0304299].

[12] M. Raidal et al., Eur. Phys. J. C 57, 13 (2008) [arXiv:0801.1826].
[13] S. Dimopoulos and L. J. Hall, Phys. Lett. B 344, 185 (1995); R. Barbieri, A. Romanino and A. Strumia, Phys. Lett. B 369, 283 (1996); A. Romanino and A. Strumia, Nucl. Phys. B 490, 3 (1997).

[14] J. Hisano, M. Nagai and P. Paradisi, Phys. Rev. D 80, 095014 (2009) [arXiv:0812.4283].

[15] S. Pokorski, J. Rosiek and C. A. Savoy, Nucl. Phys. B 570, 81 (2000) [arXiv:hep-ph/9906206].

[16] M. Pospelov and A. Ritz, Phys. Rev. Lett. 83, 2526 (1999) [arXiv:hep-ph/9904483].

[17] M. Pospelov and A. Ritz, Phys. Rev. D 63, 073015 (2001) [arXiv:hep-ph/0010037].

[18] G. Degrassi, E. Franco, S. Marchetti and L. Silvestrini, JHEP 0511 (2005) 044 [arXiv:hep-ph/0510137].

[19] C. A. Baker et al., Phys. Rev. Lett. 97 (2006) 131801.

[20] W. C. Griffith, M. D. Swallows, T. H. Loftus, M. V. Romalis, B. R. Heckel and E. N. Fortson, Phys. Rev. Lett. 102 (2009) 101601.

[21] M. Passera, J. Phys. G 31 (2005) R75; Nucl. Phys. Proc. Suppl. 155 (2006) 365; M. Davier, Nucl. Phys. Proc. Suppl. 169, 288 (2007); K. Hagiwara et al., Phys. Lett. B 649, 173 (2007).

[22] M. Passera, W. J. Marciano and A. Sirlin, Phys. Rev. D 78, 013009 (2008).

[23] L. Giusti, A. Romanino and A. Strumia, Nucl. Phys. B 550 (1999) 3 [arXiv:hep-ph/9811386].

[24] Y. Grossman, T. Kashti, Y. Nir and E. Roulet, Phys. Rev. Lett. 91, 251801 (2003) [arXiv:hep-ph/0307081].

[25] G. D’Ambrosio, G. F. Giudice and M. Raidal, Phys. Lett. B 575 (2003) 75 [arXiv:hep-ph/0308031].