Transient vibration of wind turbine blades

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Abstract. This article aims to the transient vibration of wind turbine blades. We firstly introduce transient vibration and previous studies in this area. The report then shows the fundamental equations and derivation of Euler Equation. A 3-D beam is created to compare the analytical and numerical result. In addition we operate the existing result and Patran result of a truncation wedge beam, especially the frequencies of free vibration and transient vibration. Transient vibration cannot be vanished but in some case it can be reduced.

1. Introduction

1.1. Transient vibration
Transient vibration is the vibration which is defined as a temporarily sustained one in a mechanical system, consisting of one of forced or free vibration, or both of them. Transient loading, also known as impact or mechanical shock, is a non-periodic excitation, which is characterized by a sudden and severe application.

In many cases, transient vibration can be ignored in steady state vibration. However, large margins of safety, in which ignoring of transient conditions can destroy, are not always possible. Today the study about transient phenomena is one of the largest areas of concern in mechanical vibration. The analysis of transient motion requires extensive mathematical treatment and modern computational methods.

1.2. Study of Method
In our article twisting structure is ignored. Blade is considered as a wedge cantilever beam for simplicity. Studying dynamic characteristics is a significant part of the design and control processes of a wind turbine blade. It provides necessary modal information to do aeroelastic dynamics and stability analysis [1].

A wind turbine rotor system containing blades has its own characteristics. The vibration characteristics of blades with special airfoil can influence the vibration of the rotor system [2]. The Finite Element Method (FEM) can be applied to establish the rotor model. Meanwhile we choose a finite element software named MSC Patran to create model and calculate.

2. Fundamental Equations
As considering the situation of a uniform beam, it is assumed that the density, area, Young’s modulus and second moment of area are all constant. The free vibration of the beam is defined as well. As a result, rotary inertia and shear deformation can be ignored.

Consider an arbitrary element of a beam as shown in figure 1 below:
Figure 1. Element beam.

Where: \( v = \) shear force; \( m = \) bending moment; \( w = \) displacement of a beam; the mass of the element is given as: \( \rho A \delta x \). Since shear force is applied:

\[
v - (v + \delta v) = \rho A \delta x \frac{\partial^2 w}{\partial t^2}
\]

(1)

Simplifying equation above gives:

\[
- \frac{\partial v}{\partial x} = \rho A \frac{\partial^2 w}{\partial t^2}
\]

(2)

It is now necessary to sum the moments from the right hand side of the element of the beam:

\[
m + \delta m = m + v \delta x
\]

(3)

Therefore:

\[
\frac{\partial v}{\partial x} = \frac{\partial^2 m}{\partial x^2}
\]

(4)

With simple bending theory:

\[
m = E I \frac{\partial^4 w}{\partial x^4}
\]

(5)

Substitute and rearrange equations above, we have:

\[
E I \frac{\partial^4 w}{\partial x^4} + \rho A \frac{\partial^2 w}{\partial t^2} = 0
\]

(6)

The equation above is the Bernoulli-Euler Equation.

3. Early Work for free Vibration

To start with the software, a 3-D beam is created. The original choice was 2-D beams while the software require a thickness, so we use 3-D element instead. And also because of the beginning of the study, we consider that the beam is flat and assumed no twist.

3.1. Analytical Results

For a cantilever beam is subjected to free vibration, and the system is considered as continuous system in which the beam mass is considered as distributed along with the stiffness of the shaft, the equation of motion can be written as:

\[
\frac{d^2}{dx^2} \left( E I(x) \frac{d^2 y(x)}{dx^2} \right) = \omega^2 m(x) Y(x)
\]

(7)

Where \( E \) is the modulus of rigidity of beam material, \( I \) is the moment of inertia of the beam cross-section, \( Y(x) \) is displacement in \( y \) direction at distance \( x \) from fixed end, \( \omega \) is the circular natural frequency, \( m \) is the mass per unit length, \( m = \rho A(x) \), \( \rho \) is the material density, \( x \) is the distance measured from the fixed end.
For a uniform beam under free vibration as in equation (7), we have:

$$\frac{d^4Y(x)}{dx^4} - \beta^4Y(x) = 0 \quad \beta^4 = \frac{\omega^2m}{EI} \quad (8)$$

The mode shape for a continuous cantilever beam is given as:

$$f(x) = A_n\{(\sin \beta_n x - \sin \beta_n L) + (\cos \beta_n x - \cos \beta_n L)\} \quad (n = 1,2,3,...) \quad (9)$$

A closed form of the circular natural frequency $\omega_{nf}$ from above equation of motion and boundary conditions can be written as:

$$\omega_{nf} = a_n^2 \left(\frac{EI}{ml^4}\right)^{\frac{1}{2}} \quad (10)$$

We chose steel to calculate. Young’s Modulus is $E = 2.1 \times 10^{11}$ N/m², the density is $\rho = 7850$ kg/m³, the geometries are defined that the undamped natural frequency of a steel beam with $L = 0.45$ m, $d = 0.003$ m, and $b = 0.02$ m.

First and second natural frequency:

$$\omega_{nf1} = 1.875^2 \left[\frac{E(bh^3/12)}{\rho(bh)L^4}\right]^{\frac{1}{2}} = 1.875^2 \left[\frac{2.1 \times 10^{11} \times 0.003^2}{12 \times 7850 \times 0.45^4}\right]^{\frac{1}{2}} = 77.76 \text{ rad/ sec} \quad f_{m1} = 12.37 \text{ Hz} \quad (11)$$

$$\omega_{nf2} = 4.694^2 \left[\frac{E(bh^3/12)}{\rho(bh)L^4}\right]^{\frac{1}{2}} = 4.694^2 \left[\frac{2.1 \times 10^{11} \times 0.003^2}{12 \times 7850 \times 0.45^4}\right]^{\frac{1}{2}} = 487.34 \text{ rad/ sec} \quad f_{m2} = 77.53 \text{ Hz} \quad (12)$$

The frequencies above have to be modified since there is a mass in the form of an accelerometer at the free end of the continuous beam. By continuous approach the solution is difficult since with tip mass the boundary condition at free end is now time dependent. Now we can explain a simpler procedure by which corrections to the natural frequency could be made so as to get closer to the measured natural frequency. In case of non-contacting sensors, there will not be any correction required since there will not be any additional tip mass on the beam.

### 3.2 Numerical Result

We type the data of steel into Patran, in which we have built a FEM model of 3-D beam with specific shape and dimensions.

After operating we can work out the data:

| Table 1. Patran result of 3-D beam. |
|-------------------------------------|
| First | Second |
|-------|--------|
| $\omega$ (rad/s) | 77.8 | 475 |
| $f$ (Hz) | 12.4 | 75.5 |

Comparing with equation (11) and equation (12), we shall confirm that the numerical and analytical results are quite similar.
4. Truncated Beam for Wedge

4.1. Existing Results

However it is discovered that very little researches have been published with regards to finite element modelling in this area.

The need for accurate numerical results in this area is discussed in Downs’ paper [3]. Therefore we put forward a method for determining these frequencies and produced results for reference. In order to allow simple comparisons between beams of different dimensions, Downs has produced results in a dimensionless form. These dimensionless Euler frequencies are produced using the following equation:

$$\lambda = \omega \left( \frac{\rho A l^4}{EI_0} \right)^{1/2}$$  \hspace{1cm} (13)

In the equation above the subscript zero denotes the conditions at the fixed end of the beam. The equation for calculating of natural frequencies of the truncated wedged cantilever is as follows:

$$\begin{bmatrix} I_2(B) & Y_2(B) & I_2(B) & K_2(B) \\ I_3(B) & Y_3(B) & -I_3(B) & K_3(B) \\ I_4(A) & Y_4(A) & I_4(A) & -K_4(A) \\ I_5(A) & Y_5(A) & I_5(A) & K_5(A) \end{bmatrix} = 0$$  \hspace{1cm} (14)

Where: $r = \frac{1}{1-r}$; $B = \frac{1}{1-r}$; $r$ = (Beam diameter at tip/Beam diameter at root).

4.2. Project Results

From equation (13) it can be derivate that:

$$\omega = \lambda \left( \frac{Et_0}{\rho A l^4} \right)^{1/2}$$  \hspace{1cm} (15)

Where: $\rho$ = density, $A$ = area at abutment, $l$ = length of beam, $E$ = Young’s Modulus, $I$ = moment of inertia.

The only condition we work about is truncated wedged cantilever, because it is the most similar shape of model with real wind turbine blade. The difference is the wind turbine blade is twisted but the truncated wedge is not.

As we can figure out a real frequency with assuming the real property, it is assumed that: $l = 40$ m, $b = 4$ m, $d = 0.5$ m. The material is aluminium so the density is 2700 kg/m$^3$, $E = 0.7 \times 10^{11}$ N/m$^2$.

Determine the truncation ratio is 0.5, then the frequency of vibration of the beam, $\omega$, can be calculated.

The result as in table 2:

| $\lambda$ | 3.82381 | 18.3172 | 47.2648 | 90.4505 | 148.002 |
| $\omega$  | 0.01756 | 0.08413 | 0.2171 | 0.41547 | 0.67982 |

4.3. Patran Results

After creating the wedged beam in Patran using the value of real aluminum and the dimension assumed before, figure 3 and figure 4 are worked out to show the mode shapes of the beam. The mode shapes belong to natural frequency of free vibration.
The mode shapes of beam’s vibration take the form of sine waves but in reality they are not exact sine waves. The beam is not free to rotate around the fixed end due to the clamped cantilever. So the beam cannot form into an exact sine wave. Moreover, there is a small amount of elements that do not deflect. As shown in figure 4. The elements on the left hand side are not deflecting.

5. Transient Vibration of 3D Wedge

A wedged beam is created in Patran in order to determine the transient frequency for truncated wedged cantilevers. As we use the seamless beam for free vibration of truncated wedged beam because of all the properties and boundary conditions are the same. The only thing we need is to add pressure on the surface of the beam.

Assuming the pressure of the load case is 100 N/m². Figure 5 is worked out to show four nodes on the surface of the beam which are assumed to endure the pressure from wind. The line of four nodes are parallel to the cantilevers.

6. Discussion

In part 4 we focus on the non-dimension result of truncated wedge especially with the truncation ratio 0.5. As the result of the limitation of computing power in the ages of Downs, he was not able to determine some of the results for beams with part of truncation ratio. However it is now possible to calculate all of the ratio and to go through a good result omitted from Downs’ study. The results coming from Matlab and Patran agree with Downs’ theory. Also as assuming a real condition, the natural frequencies can be figured out in ratio of 0.5.

The first natural frequency of the wedge is a kind of banding in Mode 1. While in Mode 2 it is also bended but there is one cross point to the initial location. The data of truncated cantilever also show pairs of results caused by the deflections in the y and z axis, which are complicated types of frequency. As decided before, the condition of twist would not be considered in the analysis of transient results.
In the study of the transient vibration of truncated wedge, we load the pressure with 100 N/m². And then chose four nodes on the surface with loads. Figure 5 shows that as the time goes by, the displacements of the nodes change irregularly and there is no relevance between these four nodes.

Furthermore, another two nodes, 117 and 258, which distributing along the length of beam a chosen.

![Figure 6. Node 117.](image)

![Figure 7. Node 285.](image)

Node 285 is further from the cantilever fixed end than Node 117. A result can be come out that the transient vibration of the node or element further from the cantilever fixed end is more rapid (figure 6 and figure 7). Anyway, transient vibration is determined as a vibration with no constant period or amplitude. We can only analyze the situation and the frequency in a specific time period with specific load condition.

7. Conclusion

Free vibration can be regarded as the basic version of transient vibration, as they may have same boundary conditions. Transient vibration happens in random directions and goes irregularly. However transient motion cannot be ignored in a developing study area such as wind turbine. Most of time transient is harmful. In general, any change of transient motion accompanies forms of energy storing in the vibrating system. Too much transient vibration may damage the internal structure of elements of a subject. When it happens to a mechanical structure, it will be much more difficult to control the condition of the structure. Transient vibration cannot be vanished but in some case it can be reduced. It can be developed in future works.

However, the wind turbine blades are still twisted and much more complex. A future step with adding twisting, more reality load and boundary conditions should be expected.

References

[1] Wei L, Yin J C, Pu C, et al. 2011 Dynamic analysis and aeroelastic stability analysis of large composite wind turbine blades Acta Aerodynamica Sinica. 29 391–395
[2] Liu C, Jiang D, Chen J 2010 Vibration characteristics on a wind turbine rotor using modal and harmonic analysis of FEM World Non-Grid-Connected Wind Power and Energy Conf. (Nanjing: IEEE) pp 1–5
[3] Downs, B 1978 Reference frequencies for the validation of numerical solutions of transverse vibrations of non-uniform beams Journal of Sound & Vibration 61 71–78
[4] Hansen M O L 2000 Aerodynamics of wind turbines Rotors Loads & Structure James 5 141–167
[5] Rahimi M, Parniani M 2009 Dynamic behavior and transient stability analysis of fixed speed wind turbines Renewable Energy 34 2613–2624
[6] Staino A, Basu B 2013 Dynamics and control of vibrations in wind turbines with variable rotor speed Engineering Structures 56 58–67
[7] Kaczmarczyk S, Ostachowicz W 2003 Transient vibration phenomena in deep mine hoisting cables Part 1: Mathematical model Journal of Sound & Vibration 262 219–244