Filter and correction for a hybrid sound field analysis of geometrical and wave-based acoustics

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Abstract: Statistical, geometrical, or wave-based acoustics are used as the basis for sound field analysis of buildings. As each technique has its own merits and limitations, it is imperative to select a suitable method for the target space and frequency. However, different techniques can be combined to obtain a precise wideband solution. Herein a hybrid technique, which combines the calculated results using the back-tracing method and the finite-difference time-domain (FDTD) method, is proposed to realize a wideband impulse response. Additionally, the required filters and corrections to properly combine the results are discussed. Then the effectiveness of the hybrid technique is evaluated from the viewpoints of required memory and computational time. Compared to the back-tracing method alone, the hybrid technique significantly improves the time and frequency response results, especially at low frequencies. Finally, the appropriate crossover frequency for hybrid analysis is discussed.

Keywords: Hybrid analysis, Impulse response, Back-tracing method, FDTD method, Digital filter

1. INTRODUCTION

Statistical, geometrical, or wave-based acoustics can be used as the basis for sound field analysis of buildings. Statistical acoustics assumes a diffuse sound field with a uniform energy density where sound waves propagate in all directions with equiprobability to derive analytical prediction formulas such as reverberation time equations. Geometrical acoustics ignores the wave nature such as diffraction and scattering, but sound propagations are treated geometrically to obtain the energy distribution in a sound field and its transient behavior. By contrast, wave-based acoustics pursues a solution that satisfies the governing equation, boundary condition, and initial condition. Hence, it fully considers the wave nature. Due to the different assumptions, each technique has its own prediction accuracy and computational cost. The computational cost of the statistical acoustics is really low, but the real field satisfying the diffuse sound field assumption is limited. Although the assumptions for wave-based acoustics are close to the real ones, the computational cost is huge. The range where wave-based acoustics provides a reliable solution is limited to low frequencies. On the other hand, geometrical acoustics has intermediate features and gives a wideband prediction at a reasonable cost. However, in the low-frequency range where the wave nature is predominant, the prediction accuracy is unreliable. As each technique has its advantages, it is imperative to select a suitable method for the target space and frequency. A straightforward approach to obtain a precise wideband solution is to effectively combine different techniques. The wideband solution can give important insights of time-dependent acoustical disturbance such as long-pass echo and flutter echo. Furthermore, an auralization can be carried out with the wideband impulse response to evaluate the auditory impression.

In their pioneering work, Granier et al. [1] reported a broadband impulse response using a hybrid technique. Specifically, they auralized a sound in a car cabin with a hybrid technique using wave-based acoustics for the low-frequency range and geometrical acoustics for the mid- and high-frequency ranges, where the Schroeder frequency [2] was employed as the crossover frequency. After investigating a combination of the finite element method (FEM) and the cone-tracing method in the frequency domain and the time domain, they adopted a time-domain approach because the frequency-domain approach generated an unnatural sound for listening. However, their paper did not detail the required filters or the necessary amplitude and phase corrections. Furthermore, in the Fourier inverse transform of the FEM results, they implemented an
arbitrary process for the frequency characteristics to avoid a causality problem. Thus, their approach is not well suited as a prediction method.

Summers et al. [3] employed commercial software (CATT-Acoustic [4] and SYSNOISE boundary element method (BEM) [5]) as geometrical and wave-based methods. The results obtained by each software were filtered with lowpass and highpass filters with a 100-Hz cutoff frequency. For the time-domain synthesis, finite impulse response (FIR) filters were used to avoid phase distortions. However, the amplitude correction was not investigated in detail. The impulse response in a 2,400-m³ room was calculated and evaluated. In addition, an evaluation method for auditory experiments was proposed. It was reported that the wave-based results affect audible variations at low frequencies.

Aretz et al. [6] adopted FEM as a wave-based technique, whereas the image source method for early reflections and the stochastic ray-tracing method for late reflections were employed as geometrical techniques. The sound pressure at the receiver located 1 m from the sound source in a free field was corrected such that it was equal in each method. Multiple combining methods were investigated. One was the linear combination of lowpass and highpass filters. Although the Butterworth filter and the minimum phase approach were considered, the results were unsatisfactory. The other was the separate addition of the magnitude and phase, where the Linkwitz-Riley filter was employed. This nonlinear synthesis was expected to overcome problems of the linear method, but the auditory experiments did not show any significant difference between these methods. Similar to Granier et al. [1], this study adopted the Schroeder frequency for the crossover frequency.

Southern et al. [7] adopted the finite-difference time-domain (FDTD) method for the wave-based acoustics and beam tracing (BT), and acoustic radiance transfer (ART) method for the geometrical acoustics. Although the BT method was basically used in the geometrical simulation, the ART method was selected for late reflections, resulting in a high computational cost for the BT method. They discussed the correction ratios of the amplitude for the appropriate combination of geometrical and wave-based acoustics. The correction ratio was derived from the sound energy received at an arbitrary distance from a sound source. Then a formula, which depended on the distance and sampling frequency, was proposed to obtain the absolute correction ratio. They showed that it is not necessary to calculate the correction ratio for each hybrid analysis, which is very useful from the viewpoint of computational cost. The obtained correction ratio was validated by case studies and the applicability of the proposed method to both simple and complex geometries was illustrated by comparing the obtained standard physical metrics. However, the correction ratio was specific to the source configuration in the FDTD method. Moreover, a phase correction and details of the necessary bandpass filter were not described. Furthermore, although they showed that the amplitudes of sound pressure can be calculated with sufficient accuracy in the low-frequency range for a simple chamber without diffraction and scattering, the results were not directly compared with the references to investigate whether the impulse response in the time domain was synthesized with an appropriate phase correction.

Oxnard et al. [8,9] followed the combining method by Southern et al. [7] using commercial software ODEON [10] as the geometrical simulation software and the two-dimensional FDTD method for orthogonal planes as the wave-based simulation. They proposed a correction process related to the attenuation in order to obtain three-dimensional results from the two-dimensional ones. The results implied that although some oblique modes can be omitted, the proposed combination significantly reduced the computational cost compared to the three-dimensional FDTD method. They employed the Butterworth filter, but the correlation related to the phase was not described.

Mourik et al. [11] adopted the FDTD method as the wave-based technique and the ray-tracing method as the geometrical one. The method proposed by Southern et al. [7] was employed for the correction, and they used the Butterworth filter. The reverberation time and the early decay time in a church obtained by the hybrid analysis were compared with the measured data. Although a certain correlation was achieved, the results were not sufficiently accurate.

As demonstrated above, several hybrid methods of geometrical and wave-based acoustics have been studied. However, most of the results were evaluated either by standard physical metrics related to architectural acoustics or by auditory experiments. Furthermore, detailed phase corrections are seldom considered. Both the waveform of the impulse response and its frequency response must be compared with the results that are believed to be sufficiently correct to evaluate whether the amplitude and phase are corrected appropriately. Herein a hybrid technique is proposed to obtain a wideband impulse response by combining the calculated results using the back-tracing method [12] based on geometrical acoustics and the FDTD method [13] based on wave-based acoustics. The required filters and corrections to combine the results properly both in the time and frequency domains are described in detail. As a validation, the solutions obtained by the hybrid technique are compared with those by the fine FDTD method tuned for the high-frequency range. Moreover, the effectiveness of the hybrid technique, required memories, and computational times are discussed.
Section 2 describes the configuration of the target spaces, sound sources, and receiving points. Sections 3 and 4 overview the back-tracing method and the FDTD method, respectively. Section 5 discusses the filters for the proposed hybrid technique. Section 6 describes the correction process to combine the two calculated results. Section 7 compares the results and the computational costs of the hybrid technique and the fine FDTD method. Section 8 discusses the appropriate crossover frequency, and Sect. 9 summarizes this study.

2. CONFIGURATION

This study validates a combining method of a hybrid technique. Herein four simple and small spaces, which can be analyzed precisely by the fine FDTD method, are considered (Fig. 1). Model 1 is a semi-infinite sound field with one infinite wall. Model 2 is a closed box field surrounded by finite walls. Models 3 and 4 are respectively the same spaces as Models 1 and 2 but a finite rigid wall with dimensions of 0.5 m × 1.0 m is added at x = 0.25 m. In Fig. 1, the red color illustrates the finite rigid wall, and the gray walls are located at x = 0 m for Models 1 and 3 and x, y, and z are between 0 m and 1 m for Models 2 and 4. All gray walls have a normal-incidence absorption coefficient $\alpha_n = 0.2$. For simplicity, the phase delay due to the wall absorption is ignored. Therefore, the absorption coefficient is expressed as the real impedance of 7,321 Pa·s/m with a speed of sound in air of 340 m/s and an air density of 1.2 kg/m$^3$. In three-dimensional space, the receiving point is located at (0.050 m, 0.050 m, 0.050 m) and the sound source is located at (0.954 m, 0.942 m, 0.938 m).

3. BACK-TRACING METHOD

The back-tracing method [12] is employed as a geometrical technique because it uses a simple assumption while providing solutions similar to the analytical ones for spaces without diffraction and scattering such as Models 1 and 2. This method first seeks effective reflection paths by the ray-tracing approach. Then based on the detected paths, an impulse response is obtained by the image-source approach.

When applying the back-tracing method to the problem considered here, two points should be noted. The first is about wall absorption. Although the absorption considered here is the normal-incidence absorption coefficient, sound rays generally impinge to walls with an oblique angle. Therefore, the reflected energy should be calculated as

$$E_R = E_i \left( \frac{(1 + \sqrt{1 - \alpha_n}) \cos \theta - (1 - \sqrt{1 - \alpha_n})^2}{(1 + \sqrt{1 - \alpha_n}) \cos \theta + (1 - \sqrt{1 - \alpha_n})^2} \right),$$

(1)

where $E_R$ is the reflected energy, $E_i$ is the incidence energy, and $\theta$ is the angle between the incidence vector and the normal vector of a wall ($0 \leq \theta \leq \pi/2$). At the same time as the reflection paths are detected, the energy decay by Eq. (1) is calculated, and the strength of the related image source can be determined.

The second point is in regard to the superposition of waves from image sources. In the classical image-source method, after a group of image sources is detected, the impulse response is determined by calculating the received
energy in the time domain considering the arrival time and the distance decay of each pulse. However, when a digital impulse response is necessary, the arrived energies between the sampling intervals must be superimposed and shifted to the sampling time. This leads to an error, and an impulse response of the sound energy is given instead of the sound pressure. Therefore, the obtained impulse response cannot be directly compared with that from wave-based acoustics. Herein the superposition of the waves from the image sources is performed in the frequency domain as

\[ p = \sum_{n=1}^{N} \sqrt{E_n} e^{ikr_n}, \]  

where \( p \) is the sound pressure, \( N \) is the number of effective image sources, \( e \) is the base of the natural logarithm, \( i \) is the imaginary unit, and \( k \) is the wave number. \( E_n \) and \( r_n \) are the strength and the distance of \( n \)th image source, respectively. The impulse response can be obtained by the Fourier inverse transform of the sound pressure calculated in the frequency domain.

4. FDTD METHOD

The FDTD method [13] is employed as a wave-based technique because it can directly give a time-domain solution and seems to be appropriate for combination with geometrical techniques. However, as described in Sect. 3, the results obtained by the geometrical technique are eventually processed in the frequency domain. Furthermore, the FDTD method cannot directly provide an ideal impulse response due to the dispersion relation. Because the results need to be processed by digital filters, a frequency domain process is inevitable even if the FDTD method is employed. Consequently, the FDTD method is not superior to frequency domain analysis such as FEM and BEM. Depending on the target problem, any method can be used because an outstanding wave-based method does not exist for hybrid analysis.

If the FDTD method is employed, a band-limited wave like a Gaussian pulse is generally given as a sound source. The initial condition can be expressed either as the appropriate spatial distribution of physical values [14] or the physical values sequentially at a point [15]. In either case, controlling the sound pressure created in the field to exhibit flat frequency characteristics is challenging. To obtain the impulse response, the sound pressure response must be corrected by an inverse filter [14,15]. Here, the volume velocity of the sound source given by the Gaussian pulse, which is given sequentially to the field, is used. Then the results are processed by an inverse filter to obtain the impulse response. It should be noted that Perfectly Matched Layers (PMLs) are employed to model the open spaces for Models 1 and 3 [16].

5. FILTER SUMMATION

This section discusses an appropriate filter for a hybrid process. A linear-phase FIR filter designed by the window method and a 3rd-order Butterworth and Chebyshev infinite impulse response (IIR) filters [17] are considered here. For each type, two bandpass filters are designed. One spans 15.625 Hz to 800 Hz and the other from 800 Hz to 4,000 Hz. All the filters have the same length and are simply summed in the time domain. Figure 2 shows the frequency characteristics of each filter and the combined filters. The combined FIR filter shows ideal characteristics. On the other hand, the combined Butterworth and Chebyshev IIR filters show some fluctuations, especially around the crossover frequency. These fluctuations are due to the nonlinearity of the phase of the IIR filters. When IIR filters are employed for a hybrid process, the phase mismatch should be corrected, which is a difficult process.

Hybrid process in this study adopts FIR filters. FIR filters have a long delay, which is a serious problem for real-time simulations. However, this study aims to identify an appropriate filter combination from the viewpoint of time and frequency responses. Hence, the time delay is acceptable.

6. HYBRID TECHNIQUE

Figure 3 illustrates a flowchart of the hybrid process proposed here. The sound pressure response in the frequency domain is obtained by the back-tracing method, while the sound pressure response in the time domain for a band-limited wave is obtained by the FDTD method. These responses are combined at the crossover frequency \( f_c \). To give the impulse responses, each result must undergo a Fourier inverse transform and inverse filtering. The first step is to determine the sampling frequency \( f_s \) of a wideband impulse response.

The second step is to conduct FDTD analysis for a free field with the same spatial interval, time interval, time duration, and sound source as those used in the FDTD analysis for the target field. As an example, the configuration in Fig. 4 is adopted. To precisely calculate the propagation time in the target field, the sound source and receiving point should be located at the same position. If they are co-located, then they can be positioned anywhere. This free-field analysis provides the relationship between the volume velocity of the sound source and the sound pressure created in the field. Therefore, the filter with inverse characteristics of the results for the free field becomes the inverse filter. Although the inverse filter can be approximated by Tsuru’s method [15], the free field analysis provides a more precise inverse filter.

Before designing the inverse filter, the results of the target field and the free field are resampled with the
sampling frequency $f_s$. To create a stable inverse filter, the inverse characteristics of the results for the free field must be bandpass-filtered. Here, the minimum target frequency of the wideband impulse response is set to $f_{\text{min}}$. Then a bandpass filter, which passes from $f_{\text{min}}$ to $f_c$, is designed with a certain time delay that considers the causality and the preservation of the waveform. Herein a linear-phase FIR filter which is designed by the window method using a Hamming window is adopted [14,17]. A sufficiently long filter length gives reasonable results in both the time and frequency domains. Typically, the time length is two or three times that of the FDTD analysis. Consequently, the filter length becomes the length of the wideband impulse response. After designing the inverse filter, it is processed using the results of the target field to obtain the band-limited impulse response.

For the back-tracing method, the frequency characteristics are calculated for the discrete frequencies by Eq. (2) considering the sampling frequency and the filter length used for FDTD analysis. To simultaneously conduct the Fourier inverse transform and cutoff information below $f_c$, the bandpass filter is designed to pass from $f_c$ to $f_{\text{max}}$ ($\leq f_s/2$). The same window function and filter length as those employed for the FDTD analysis must be selected. This filter is transformed to the frequency domain and

**Fig. 3** Flow chart of the proposed hybrid process.

Sound source and receiving point
(0.954, 0.942, 0.938)

**Fig. 4** Free-field model where the sound source and receiving point are at the same position to obtain the inverse filter for the FDTD method (FF' in Fig. 3).
multiplied by the results obtained by the back-tracing method. Conducting a Fourier inverse transform of the band-limited frequency characteristics yields the band-limited impulse response.

Finally, the impulse responses obtained by the FDTD method and the back-tracing method are combined with corrections. Again, free-field FDTD analysis with the same spatial interval, time interval, time duration, and sound source should be conducted. Figure 5 shows the configuration adopted in this study. The receiving point can be located anywhere except the source position. The obtained results are resampled with the sampling frequency \( f_s \). Then the same configuration is analyzed by the back-tracing method. Because there is not a wall, only the direct wave is detected. Therefore, \( N=1 \), \( E_n=1 \), and \( r_n \), which is the distance between the sound source and the receiving point, can be substituted into Eq. (2). The results obtained by each method are filtered using the same procedures as described above to obtain band-limited impulse responses.

Afterward, the amplitude frequency characteristics are calculated by the Fourier transform. The ratio of mathematically averaged amplitudes over the frequencies from \( f_{\text{min}} \) to \( f_c \) for the FDTD method and from \( f_c \) to \( f_{\text{max}} \) for the back-tracing method can be used to determine the ratio of the results of the back-tracing method to those of the FDTD method. In the hybrid method, the band-limited impulse response of the FDTD method for the target field is multiplied by this ratio. Then the result is added to the band-limited impulse response of the back-tracing method for the target field in the time domain to give the wideband impulse response. As discussed in Sect. 5, a phase correction is unnecessary because linear-phase FIR filters with the same time delay are employed.

7. RESULTS AND COMPUTATIONAL COSTS

Figures 6–9 show the results obtained by the hybrid analyses for each configuration in Sect. 2 under the conditions listed in Table 1. As a reference, the results by the back-tracing method alone and the fine FDTD method are shown. As shown in Table 1, the sampling frequency and the filter length of the hybrid process are 12,000 Hz and 32,768 points, respectively. Therefore, the time duration of the wideband impulse responses is 2.730 s (from \( \frac{1}{c_0} \times 365 \) s to 1.365 s). It should be noted that 0 s means the generation time of impulse and minus value of the time denotes the delay due to the filtering. Although the frequency responses are obtained by the Fourier transform of these long impulse responses, Figs. 6–9(a) show a close up of the early time responses from 0 ms to 20 ms to discuss how accurately hybrid analyses can predict the waveforms. The time responses are normalized by the peak value of impulse response in a free field (FF in Fig. 3). Therefore, 0 dB in the frequency responses denotes the sound pressure level of the direct sound.

Model 1, which only has one infinite wall, gives the results as a superposition of the direct pulse and one reflected pulse. Figure 6(a) exhibits one pulse because the direct and reflected pulses have some time width due to the frequency band limitation, and they are superimposed in a
very short time duration. However, the interference effect of the direct and reflected pulses is apparent in the frequency domain, and a steep dip is observed around 2,800 Hz (Fig. 6(b)). Model 2 has more reflections than Model 1. Both the time response and frequency characteristics become more complicated (Fig. 7). The floor of the time response is below zero, which is due to the remaining causality problem of the filters. Although slight discrepancies are observed in both the time and frequency domains for these two models, the values and the positions of the peaks and dips obtained by the hybrid analyses both in the time and frequency domains agree well with those by the fine FDTD method. It should be noted that the results obtained by the back-tracing method alone also agree well with those by the fine FDTD method because these two models are not affected by diffraction or scattering. Considering the hybrid process, it may be natural that the results below the crossover frequency of 800 Hz agree well because both are calculated by the FDTD method. However, the good agreement above 800 Hz validates the processes, including the proposed correction and calculations by Eqs. (1) and (2).

Model 3 has a finite wall where the direct wave is interrupted by the finite wall and the received wave is affected by diffractions from the edges of the finite wall. Therefore, the back-tracing method cannot provide a correct solution, especially at low frequencies. Figure 8 highlights the relatively large discrepancies in both the time and frequency characteristics between the back-tracing method alone and the fine FDTD method. The hybrid method shows a good agreement below 800 Hz and above 2 kHz (Fig. 8(b)). Although discrepancies remain from 800 Hz to 2 kHz, the improvement below 800 Hz is significant. Additionally, an improvement is confirmed in the time response (Fig. 8(a)). The first peak of the reflected sound which cannot be observed by the back-tracing method can be obtained by the hybrid method. Model 4 also has a finite wall. However, diffraction has a smaller influence than Model 3 because only one edge affects the received wave. Thus, although some discrepancies espe-
cially from 100 Hz to 500 Hz can be seen between the results by the back-tracing method and the fine FDTD method, the frequency characteristics agree better than Model 3 (Fig. 9(b)). Consequently, the results by the hybrid method agree well with those by the fine FDTD method. This implies that closed spaces can be predicted better with the hybrid technique than open spaces. Hence, the hybrid technique is appropriate for room acoustics. However, relatively strong vibration of about 800 Hz can be seen in Figs. 8(a) and 9(a) before the arrival of first reflected sound. The cause of this vibration might be an undesirable positive interference of bandpass filters but the detailed discussion cannot be given from the results obtained here. The further study is necessary to reduce the vibration.

Table 2 shows the required memories and central processing unit (CPU) times for the calculations in all four models. The CPUs are dual Intel Xeon processors E5-2687W v3 (3.10 GHz, 10 cores/processor). The required CPU times of the back-tracing method are about 1 s for Models 1 and 3 but about 11 s for Models 2 and 4. The required memories of the back-tracing method are larger than those of the FDTD method for hybrid analysis but are much less than the fine FDTD method. It should be noted that to obtain more precise results, geometrical techniques sometimes use more memory and CPU time than wave-based techniques when considering large numbers of rays and reflections. The required memories and CPU times for the hybrid processes themselves (filtering, correction, and summation) are negligible compared to the other calculations. As discussed in Sect. 6, the proposed hybrid process requires free-field calculations. Figures 4 and 5 show that the FDTD calculations for the free-field models can be simultaneously carried out in one program. For example, the total memory and CPU time of the hybrid analysis for Model 2 become 239 + 253 = 492 MB and 3,227,400 = 3,669,160 s, respectively. On the other hand, the total memory and CPU time of the fine FDTD method for Model 2 are 847 + 6,970 = 7,817 MB and 441,760 + 3,227,400 = 3,669,160 s, respectively. Consequently, under the configurations and conditions considered here, hybrid analyses can reduce the total memory to less than 1/15 and the total CPU time to about 1/370 compared to the fine FDTD method.

8. CROSSOVER FREQUENCY

This section briefly discusses the appropriate crossover
frequency for hybrid analysis. Consider the configuration shown in Fig. 10, where there is a finite wall located at \( x = 0 \), which is similar to Model 1. The center of the wall is fixed at \((0.000 \text{ m}, 0.500 \text{ m}, 0.500 \text{ m})\) and is assumed to be rigid. The wall size is set to (a) \( 3 \text{ m} \times 3 \text{ m} \), (b) \( 2 \text{ m} \times 2 \text{ m} \), and (c) \( 1 \text{ m} \times 1 \text{ m} \). Figure 11 shows the results of the frequency characteristics obtained by the hybrid analyses. Unlike Model 1, the back-tracing method cannot provide a perfect prediction because the scattered waves from the edges of the finite wall arrive at the receiving point. The discrepancies between the back-tracing method and the fine FDTD method are generally less than 2 dB above 204 Hz (Fig. 11(a)) and 368 Hz (Fig. 11(b)). On the other hand, Fig. 11(c) shows large differences even at high frequencies, indicating that the prediction accuracy is insufficient using the back-tracing method.

Taking the source and receiver positions into account, Fig. 12 shows the Fresnel zones drawn on the \( x = 0 \) plane in Fig. 10 at 800 Hz [18], which is the crossover frequency employed in this study. The ellipses near the center represent the Fresnel zones in order from the first, second, \ldots, and tenth zones, while the squares with white lines represent the wall positions of (a), (b), and (c). The number of Fresnel zones included in the walls is (a) 8, (b) 4, and (c) 0. It can be inferred that if the wall contains four or more Fresnel zones, the discrepancies between the back-tracing method and the fine FDTD method will be sufficiently small. The minimum frequencies where two Fresnel zones are included are (a) 198 Hz, (b) 407 Hz, and (c) 5,940 Hz. These frequencies approximately correspond to the minimum frequencies where the difference between the back-tracing method and the fine FDTD method is less than 2 dB. Therefore, the Fresnel zones for the walls should be calculated to yield diffraction and scattering waves while considering the source and receiver positions, and then setting the crossover frequency to include at least two Fresnel zones.

9. CONCLUSION

Herein a hybrid technique to obtain a precise wideband impulse response is proposed. The technique combines the results of the back-tracing method and the FDTD method. It has two important features: (1) amplitude correction using the free-field results and (2) phase correction is unnecessary because the linear-phase FIR filters have the same length and delay. These two features can give the same frequency characteristics by each method as those by
the fine FDTD method. The hybrid analysis results agree well with those by the fine FDTD method for spaces without diffraction and scattering. Additionally, the hybrid analysis improves the low-frequency characteristics compared with those by the back-tracing method alone for spaces with diffraction and scattering, while significantly reducing the computational cost.

If the development of computational resources follows Moore’s law [19], the computational performance will reach $10^6$ times in 30 years. However, this is impractical because a $6 \times 10^6$ times performance of the fine FDTD method employed in the present paper is necessary to analyze a 10,000-m$^3$ concert hall up to 20,000 Hz. Therefore, the hybrid analysis proposed here should be effective in the next few decades. Since this study focuses on the combining process of the hybrid method, the target spaces are assumed to be small and simple. Sufficient agreement may not be obtained in a real large and complex field where the wave nature strongly affects the results. In the future, the applicability of the hybrid technique to a large-scale hall should be investigated.

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