Sensor Fault Diagnosis and Fault Tolerant Control for Forklift Based on Sliding Mode Theory

ZHILU ZHANG AND BENXIAN XIAO
School of Electrical Engineering and Automation, Hefei University of Technology, Hefei 230009, China
Corresponding author: Benxian Xiao (xiaobenxian@126.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 51577046, and in part by Hefei Banyitong Science and Technology Development Company Ltd.

ABSTRACT For the forklift equipped with electrical sensors, a fault tolerant control (FTC) strategy is proposed. First, considering the uncertainty of forklift cargo and the external output disturbance, the equivalent sensor fault model of forklift is constructed. Then, a sliding mode observer (SMO) with adaptive regulation law is proposed to solve the problem that some fault reconstruction methods demand the upper bound of faults. Based on the fault value reconstructed by SMO, a sliding mode fault-tolerant controller is designed. It can realize active FTC of typical sensor faults of forklift in the presence of uncertainties and output disturbance. Finally, experiment is given to verify the effectiveness of the proposed FTC strategy.

INDEX TERMS Forklift, sensor fault diagnosis, multi-sensor fault reconstruction, fault tolerant control.

I. INTRODUCTION
With the rapid development of forklift, a growing number of electronic components have been adopted. It improves the flexibility of control system, but impairs the robustness because the reliability of electronic components is generally lower than that of traditional mechanical and hydraulic components [1]. The electronic components are likely to fail without warning and a small malfunction could be passed down and amplified to result in a fatal systematic failure [2], [3]. Generally, sensor faults will bring more serious consequences than actuator faults or some other failures because incorrect signals obtained from sensors will drive the control system to lose control [4], [5]. Therefore, there is an urgent demand for fault-tolerant control (FTC) strategy to ensure the reliable operation of modern vehicles in the presence of sensor faults.

In recent years, several methods for sensor fault diagnosis and FTC have been proposed. In [6], an approach of active fault-tolerant control based on signal reconfiguration is proposed for the fault of missile attitude control systems caused by failed inertial sensors. A fault-tolerant control strategy for the longitudinal dynamics of an autonomous vehicle is presented to detect potential failures of the vehicle’s speed sensor and then to keep the vehicle in a safe state [7]. Reference [8] proposes a fault detection and isolation strategy for torque safety of automobile which is caused by the pedal mechanical stiction fault and pedal sensor faults. The aforementioned references could achieve the desired performances, but they are proposed for the specific sensor faults in the system. On the other side, as a forklift, the changing of load should also be taken into consideration. It means that it is necessary to design FTC controller in case of the system model uncertainty and disturbances.

Sliding mode observer (SMO) is always regarded as an effective robust control strategy [9], [10]. It holds the superi- orities of fast response, excellent transient performance, and insensitiveness to perturbations and disturbances. Enlightened by these, SMO was extended to be utilized for FTC purposes. In [11], an improved second-order SMO is designed to cope with the fault reconstruction problem for autonomous underwater vehicle (AUV) subject to ocean current disturbances and modelling uncertainty. Wang Y.Y. et.al present a fault-tolerant tracking control strategy for Takagi–Sugeno fuzzy model-based nonlinear systems. It combines integral sliding mode control with adaptive control technique and has been applied to the dynamic positioning control of unmanned marine vehicles [12]. To solve the fault-tolerant control problem of Markov jump systems (MJS) with Itô stochastic process and output disturbances, a proportional-derivative SMO and an observer-based controller are designed and synthesized in [13].

The associate editor coordinating the review of this manuscript and approving it for publication was Sing Kiong Nguang. This work is licensed under a Creative Commons Attribution 4.0 License. For more information, see https://creativecommons.org/licenses/by/4.0/
Motivated by above discussions, a FTC method is proposed to cope with the key sensor faults of forklift. It can be divided into two parts. One is the SMO with adaptive regulation law which reconstructs the fault value of sensors. Another is the FTC controller. Based on the fault value produced by the SMO, it can realize fault-tolerant control of uncertain linear system of forklift with output disturbances.

The rest of this paper is organized as follows. In Section 2, a fault model of forklift is established. In Section 3, the sliding mode FTC system is demonstrated. Experimental results are discussed in Section 4 and the conclusion is presented in Section 5.

II. FAULT MODEL OF FORKLIFT

This paper takes an electric forklift with front wheel steering as an example, and its dynamics model is shown in Figure 1. It can be simplified as: roll motion around X-axis, lateral motion along Y-axis and yaw motion around Z-axis. The frame origin is at the original vehicle center of mass, with the X-axis along the longitudinal vehicle direction pointing forward, the Y-axis pointing to the left vehicle side, and the Z-axis pointing upward. Assuming that the cornering characteristics of wheels are in linear range and the effect of load transfer caused by forklift motion on the wheel is neglected.

The dynamic model of forklift is given as [14]

\[
\begin{align*}
I_x \dot{\psi} - I_{xz} \dot{\gamma} &= M_\psi h \dot{\psi} - K_\psi \dot{\psi} - C_\psi \dot{\psi} + M_{xy} \dot{\nu} \\
M_{Vx} (\ddot{\theta} + \gamma) - M_\psi h \dot{\psi} &= F_{y/yf} + F_{y/yf} + F_{y/yr} + F_{y/yr} \\
I_{xz} \ddot{\gamma} - I_{xz} \ddot{\psi} &= -I_r (F_{y/yf} + F_{y/yf} + F_{y/yr} + F_{y/yr}) \\
F_y &= k \alpha
\end{align*}
\]

where \( M \) is the total mass of the vehicle, \( M_i \) is the sprung mass of the vehicle, \( h_i \) is the distance between the original center of the forklift and the roll center, \( l_i \) is the distance between center of gravity and the center line of the front axle, \( l_c \) is the distance between center of gravity and the center line of the rear axle, \( \delta_i \) is the steering angle of the front wheel, \( \beta \) is the sideslip angle, \( \gamma \) is the yaw rate about the z axis, \( \nu_i \) and \( \nu_c \) are the longitudinal and lateral velocities at the center of gravity of vehicle, \( \psi \) is the body roll angle, \( \nu \) is the roll velocity, \( K_\psi \) is the total roll rate, \( C_\psi \) is the roll damping, \( I_z \) and \( I_l \) are the moment of inertia around x-axis and z-axis respectively, \( I_{xz} \) is the product of \( I_z \) and \( I_l \), and \( k_1 \) and \( k_2 \) are the front and rear tire cornering stiffness, respectively. Additionally, \( F_{y/yf}, F_{y/yr}, F_{y/yf}, F_{y/yr} \), respectively, denotes the left and right lateral tire forces, and \( t \) represents front and rear. \( \alpha \), which is directly proportional to \( F \), \( F_{y/yf}, F_{y/yr}, F_{y/yf}, F_{y/yr} \), denotes the sideslip angle of tire. The state space of forklift can be derived as follows by considering \( x(t) = [\alpha, \beta, \psi, \nu]^T \) as the state vectors and \( u(t) = \delta_i \) as the input vector:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \quad y(t) = Cx(t)
\end{align*}
\]

where

\[
A = M_1^{-1} M_2, B = M_1^{-1} M_3, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
\]

\[
M_1 = \begin{bmatrix} 0 & M_{Vx} & 0 & -M_{h1l1} \\ I_z & 0 & 0 & -I_{xz} \\ -I_{xz} & M_{h1v} & 0 & I_z \\ 0 & 0 & 1 & 0 \end{bmatrix},
M_3 = \begin{bmatrix} k_1 \\ k_1 a \end{bmatrix},
\]

\[
M_2 = \begin{bmatrix} -\frac{k_1 a + k_2 b}{v_i} M_{Vx} - k_1 - k_2 & (k_1 R_f + k_2 R_r) \nu_i \\ -\frac{k_1 a^2 + k_2 b^2}{v_i} & -ak_1 + bk_2 (k_1 a R_f - k_2 b R_r) \\ M_i h_1 v & 0 \end{bmatrix} - \begin{bmatrix} K_\psi & -M_\psi h \dot{\psi} \end{bmatrix}
\]

In the SBW system, the sensor faults include gain fault, stuck fault and constant deviation fault. Yaw rate sensor, roll rate sensor and front wheel angle sensor are considered in this manuscript. When the \( i \)th sensor occur faults, its output is

\[
Y_{yi} = \Delta_i Y_i + a_i
\]

where \( Y_i \) and \( Y_{yi} \) represent the true output and fault output respectively. \( \Delta_i \) reflects the fault degree when sensor occurs the gain fault. \( a_i \) represents the value of stuck fault and constant deviation fault.

Assume that the sensor occur faults as follows:

1) Yaw rate sensor fault: \( Y_{yi} = \Delta_i Y_i (\Delta_i \in (0, 1)) \). It means that the sensor occurs gain fault.
2) Roll rate sensor fault: \( Y_{yi} = a_i \). It means that the sensor occurs stuck fault.
3) Front wheel angle sensor fault: \( Y_{yi} = Y_i + \begin{cases} a_i, & t_1 \leq t < t_2 \\ a_i + \frac{\Delta_i}{t} & t_2 \leq t \leq t_4 \end{cases} \). It means that the sensor occurs constant deviation fault intermittently.
The fault output is summarized as follows

\[ \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} \Delta_1 Y_1 + a_1 \\ \Delta_2 Y_2 + a_2 \\ \Delta_3 Y_3 + a_3 \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} + \begin{bmatrix} -(\Delta_1 - 1)Y_1 \\ -Y_2 + a_2 \\ a_3 \end{bmatrix} \]

(4)

Considering the uncertainty of cargo and the external output disturbance, the equivalent sensor fault model of forklift can be defined as

\[ \begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y_1(t) &= C_1x(t) + H_1\theta(t) + F_1f_{si} \\
&\vdots \\
y_m(t) &= C_mx(t) + H_m\theta(t) + F_mf_{sm} 
\end{align*} \]

(5)

where \( x(t) \in \mathbb{R}^n \) denotes the unmeasurable state vector, \( u(t) \in \mathbb{R}^d \) denotes the measurable input vector, \( y(t) \in \mathbb{R}^m \) denotes the measurable output vector, \( \theta(t) \) denotes the external uncertainties and nonlinear term, \( f_{si} \in \mathbb{R}^m \) denotes the sensor fault, \( H \) denotes the known external interference matrix, \( F \) denotes the known distribution matrix of sensor fault, \( A, B, \) and \( C \) are known constant matrices.

Assume that multiple sensor faults, the number of which is at most \( m \), may occur at any time, the model can be described as:

\[ \begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y_1(t) &= C_1x(t) + H_1\theta(t) + F_1f_{si} \\
&\vdots \\
y_m(t) &= C_mx(t) + H_m\theta(t) + F_mf_{sm} 
\end{align*} \]

(6)

The above sensor faults model can be simplified into the following equation (7), and it can be divided into multiple single sensor fault modules for the fault detection. When multiple sensors fail at the same time, the faults of the corresponding parts can be detected to realize fault detection and reconstruction.

\[ \begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y_1(t) &= C_1x(t) + H_1\theta(t) + F_1f_{si} \\
&\vdots \\
y_m(t) &= C_mx(t) + H_m\theta(t) + F_mf_{sm} 
\end{align*} \]

(7)

The above sensor faults model can be simplified into the following equation (7), and it can be divided into multiple single sensor fault modules for the fault detection. When multiple sensors fail at the same time, the faults of the corresponding parts can be detected to realize fault detection and reconstruction.

The above sensor faults model can be simplified into the following equation (7), and it can be divided into multiple single sensor fault modules for the fault detection. When multiple sensors fail at the same time, the faults of the corresponding parts can be detected to realize fault detection and reconstruction.

The above sensor faults model can be simplified into the following equation (7), and it can be divided into multiple single sensor fault modules for the fault detection. When multiple sensors fail at the same time, the faults of the corresponding parts can be detected to realize fault detection and reconstruction.

The above sensor faults model can be simplified into the following equation (7), and it can be divided into multiple single sensor fault modules for the fault detection. When multiple sensors fail at the same time, the faults of the corresponding parts can be detected to realize fault detection and reconstruction.

The above sensor faults model can be simplified into the following equation (7), and it can be divided into multiple single sensor fault modules for the fault detection. When multiple sensors fail at the same time, the faults of the corresponding parts can be detected to realize fault detection and reconstruction.

The above sensor faults model can be simplified into the following equation (7), and it can be divided into multiple single sensor fault modules for the fault detection. When multiple sensors fail at the same time, the faults of the corresponding parts can be detected to realize fault detection and reconstruction.

The above sensor faults model can be simplified into the following equation (7), and it can be divided into multiple single sensor fault modules for the fault detection. When multiple sensors fail at the same time, the faults of the corresponding parts can be detected to realize fault detection and reconstruction.

The above sensor faults model can be simplified into the following equation (7), and it can be divided into multiple single sensor fault modules for the fault detection. When multiple sensors fail at the same time, the faults of the corresponding parts can be detected to realize fault detection and reconstruction.

The above sensor faults model can be simplified into the following equation (7), and it can be divided into multiple single sensor fault modules for the fault detection. When multiple sensors fail at the same time, the faults of the corresponding parts can be detected to realize fault detection and reconstruction.

The above sensor faults model can be simplified into the following equation (7), and it can be divided into multiple single sensor fault modules for the fault detection. When multiple sensors fail at the same time, the faults of the corresponding parts can be detected to realize fault detection and reconstruction. }
From (14), when \( \text{sgn}(D_{i} \tilde{e}_{yi} - \lambda_{i}) = 1 \), the adjustable gain parameter \( \rho_{i} (t) \) will increase and its rate is proportional to \( D_{i} \tilde{e}_{yi} \). Also, when \( \text{sgn}(D_{i} \tilde{e}_{yi} - \lambda_{i}) = -1 \), \( \rho_{i} (t) \) will decrease. It means that once sliding mode motion occurs, which means \( D_{i} \tilde{e}_{yi} \) is close to zero, \( D_{i} \tilde{e}_{yi} \leq \lambda_{i} \) when \( \lambda_{i} \) takes a minor constant. Then the adjustable gain parameter changes a little, which ensures that the sliding mode adjustable parameter is not too large, so that the chattering phenomenon caused by the excessive gain parameter can be weakened to some extent.

**Lemma 1**: The adaptive adjustment law of the adjustable gain parameter in (14) should observe the following condition: the upper limit of \( \rho_{i} (t) \) is \( \rho^{+}_{i} \) and \( \rho^{-}_{i} > \gamma_{i} \) [19].

Define the state estimation error as:

\[
\tilde{e}_{i} = \tilde{x}_{i} - \hat{x}_{i}
\]  

(15)

By subtracting equation (12) from equation (11), when the \( i \)th sensor fails, the deviation equation of the \( i \)th observer is:

\[
\dot{\tilde{e}}_{i} = (\bar{A}_{i} - L_{i} \bar{C}_{i}) \tilde{e}_{i} + \bar{H}_{i} \theta (t) + \bar{F}_{i} (f_{si} - v_{i})
\]  

(16)

**Theorem 1**: Based on the above definitions, when the \( i \)th sensor fails and the following conditions satisfy:

1) \( \| \tilde{e}_{i} \| > 2 \| P_{i} \| \| \bar{H}_{i} \| \gamma_{i} \min(Q_{i}) \|

2) LMI:

\[
\begin{bmatrix}
(A_{i} - L_{i} \bar{C}_{i})^{T} P_{i} + P_{i} (A_{i} - L_{i} \bar{C}_{i}) & 0 \\
0 & -P_{i}
\end{bmatrix} \leq 0,
\]

the dynamic systems of state estimation error are asymptotically stable.

**Proof**: Consider the following Lyapunov function:

\[
V(\tilde{e}_{i}) = \tilde{e}_{i}^{T} P_{i} \tilde{e}_{i} + \frac{1}{2} (\rho_{i} - \rho_{i}^{-})^{2}
\]  

(17)

and its time derivative is calculated as follows:

\[
\dot{V} (\tilde{e}_{i}) = \tilde{e}_{i}^{T} \left[ (A_{i} - L_{i} \bar{C}_{i})^{T} P_{i} + P_{i} (A_{i} - L_{i} \bar{C}_{i}) \right] \tilde{e}_{i}
\]

\[
+ 2 \tilde{e}_{i}^{T} P_{i} \bar{F}_{i} (f_{si} - v_{i}) + \rho_{i} (\rho_{i} - \rho_{i}^{+}) + 2 \tilde{e}_{i}^{T} P_{i} \bar{H}_{i} \theta (t)
\]

\[
= -\| \tilde{e}_{i} \| \gamma_{i} \min(Q_{i}) \| \tilde{e}_{i} \| - 2 \| P_{i} \| \| \bar{H}_{i} \| \| \omega \| \| \tilde{e}_{i} \| \|
\]

\[
+ 2 \tilde{e}_{i}^{T} P_{i} \bar{F}_{i} (f_{si} - v_{i}) + \eta_{i} (\rho_{i} - \rho_{i}^{+}) \| D_{i} \tilde{e}_{yi} \|
\]

\[
\times \text{sgn} \left( \| D_{i} \tilde{e}_{yi} \| - \lambda_{i} \right)
\]

\[
\leq -\| \tilde{e}_{i} \| \gamma_{i} \min(Q_{i}) \| \tilde{e}_{i} \| - 2 \| P_{i} \| \| \bar{H}_{i} \| \| \omega \| \| \tilde{e}_{i} \|
\]

\[
+ (2 \gamma_{i} - 2 \rho_{i} + \eta_{i} (\rho_{i} - \rho_{i}^{+})) \| D_{i} \tilde{e}_{yi} \|
\]

\[
= -\| \tilde{e}_{i} \| \gamma_{i} \min(Q_{i}) \| \tilde{e}_{i} \| - 2 \| P_{i} \| \| \bar{H}_{i} \| \| \omega \|
\]

\[
+ (2 \gamma_{i} - 2 \rho_{i} + 2 \rho_{i}^{+} - 2 \rho_{i}^{+} + \eta_{i} (\rho_{i} - \rho_{i}^{+})) \| D_{i} \tilde{e}_{yi} \|
\]

\[
= -\| \tilde{e}_{i} \| \gamma_{i} \min(Q_{i}) \| \tilde{e}_{i} \| - 2 \| P_{i} \| \| \bar{H}_{i} \| \| \omega \|
\]

\[
+ [2 \gamma_{i} - \rho_{i}^{+} - (\rho_{i} - \rho_{i}^{+}) (\eta_{i} - 2)] \| D_{i} \tilde{e}_{yi} \|
\]

Combine with the condition in Theorem 1, \( V(\tilde{e}_{i}) < 0 \) and \( \| \tilde{e}_{i} \| \) converges in the following domain:

\[
U (\tilde{e}_{i}) = \left\{ \| \tilde{e}_{i} \| \leq 2 \| P_{i} \| \| \bar{H}_{i} \| \| \omega \| / \gamma_{i} \min(Q_{i}) \right\}
\]

To reduce the chattering of the sliding mode motion, add an appropriate scalar and the sliding mode control input (13) can be adjusted to:

\[
v_{yi} = \begin{cases} 
\rho_{i} (t) \frac{D_{i} \tilde{e}_{yi}}{\| D_{i} \tilde{e}_{yi} \| + \delta_{i}} & \tilde{e}_{yi} \neq 0 \\
0 & \tilde{e}_{yi} = 0
\end{cases}
\]  

(18)

where \( \delta_{i} \) is a fully small positive constant. Thus, the value of the fault can be approximated as:

\[
\hat{f}_{si} (t) \rightarrow \rho_{i} \left[ (\bar{C}_{i} \bar{F}_{i})^{+} (\bar{C}_{i} \bar{F}_{i}) \right] \frac{D_{i} \tilde{e}_{yi}}{\| D_{i} \tilde{e}_{yi} \| + \delta_{i}} - (\bar{C}_{i} \bar{F}_{i})^{+} \bar{C}_{i} \bar{H}_{i} \theta (t)
\]  

(19)

**B. FTC Controller Design**

Considering the model uncertainties, the sensor fault model of forklift is described as:

\[
\begin{aligned}
\dot{x}(t) &= (\bar{A} + \Delta \bar{A}) x(t) + (\bar{B} + \Delta \bar{B}) u(t) + \bar{H}_{i} \theta (t) + \bar{F}_{fs} \\
\dot{\gamma}(t) &= \bar{C}_{i} x(t)
\end{aligned}
\]  

(20)

where \( \Delta \bar{A} \) and \( \Delta \bar{B} \) represent the uncertainty of the model, they can be expressed by

\[
\begin{aligned}
\Delta \bar{A} &= \sum_{i=1}^{N} a_{i} \bar{A}_{i} \\
\Delta \bar{B} &= \sum_{i=1}^{N} b_{i} \bar{B}_{i}
\end{aligned}
\]

(21)

where \( \bar{A}_{i} \) and \( \bar{B}_{i} \) are the known matrices which have the same dimensions as \( \Delta \bar{A} \) and \( \Delta \bar{B} \). \( a_{i} \) and \( b_{i} \) are the corresponding scalar. Then the uncertainty of the model can be described as

\[
W_{i} \omega_{i} (t) = \Delta \bar{A} \tilde{x}(t) + \Delta \bar{B} u(t)
\]

(22)

To simplify the design of matrix with model uncertainties and external disturbances, the external disturbances and model uncertainties can be integrated as

\[
\hat{W}_{i} \omega (t) = W_{i} \omega_{i} (t) + \bar{H}_{i} \theta (t)
\]

(23)

Substituting (23) into (20), it can be transformed to

\[
\dot{x}(t) = \bar{A} \bar{x}(t) + \bar{B} u(t) + \hat{W}_{i} \omega (t) + \bar{F}_{fs}
\]

(24)

\[
\dot{\gamma}(t) = \bar{C} \bar{x}(t)
\]

(25)

The above process converts the model uncertainty to external disturbances, so that the model uncertain parameters and external disturbances can be uniformly processed to simplify the calculation.

To design a fault-tolerant controller for the corresponding model of fault system, make the following assumptions:

**Hypothesis 1**: \( (A, B) \) is controllable, \( (A, C) \) is observable and \( (A, F, C) \) remains unchanged.

**Hypothesis 2**: The faults have been reconstructed precisely. It means \( f_{si} (t) \rightarrow f_{si} (t) \).

The active FTC for the system (23) can be described as: when no fault occurs, \( f_{si} (t) = 0 \), there exists the appropriate
state feedback control law to achieve asymptotic stability of the system. When faults occur, \( f_s (t) \neq 0 \), the control law \( U (t) \) can be designed to achieve adjustment and control of faults.

Hypothesis 3: \((\hat{A}, \hat{C})\) is observable and there exists appropriate matrix \( L, D, K \), positive definite matrix \( P \) and \( Q \) such that \((\hat{A} - L\hat{C})\) remains stable, \( P\hat{F} = \hat{C}^T D^T \), \( P\hat{H} = \hat{C}^T K^T \) and \((\hat{A} - L\hat{C})^TP + P(\hat{A} - L\hat{C}) = -Q \).

Hypothesis 4: \( \Delta A \) and \( \Delta B \) are bounded matrices, there exists \( \xi_1, \xi_2 > 0 \) such that
\[
\begin{align*}
\|\Delta A\| &\leq \xi_1, \\
\|\Delta B\| &\leq \xi_2.
\end{align*}
\]

Hypothesis 5: On the basis of hypothesis (4), \( \omega(t) \) is a bounded function, then there exists \( \omega_0 > 0 \) such that \( \|\omega(t)\| \leq \omega_0 \).

Hypothesis 6: The upper bound of the adjustable gain parameter \( \rho(t) \) satisfies
\[
\rho^* \geq \left[ \begin{array}{c}
\tilde{C}\tilde{F}^{-1} \\
\bar{F}_{x2} \\
\bar{W}_2 \end{array} \right] \gamma + \left[ \begin{array}{c}
\tilde{W} \\
\omega_0 + \kappa
\end{array} \right],
\]
where \( \kappa \) is a positive scalar, \( \gamma \) is the upper bound of the norm of sensor faults \( \|f_s\| \) and its value is unknown.

For the sensor fault model (23), the design of a sliding mode fault-tolerant controller is designed by the following steps.

1) CONTROLLER MODEL
On the basis of (23), consider the following reference model according to the law of sliding mode motion:
\[
\dot{x}_s = A_s x_s + B_s r_s
\]
(24)
where \( x_s \in R^{6 \times 1} \) is the state parameter of the model, \( r_s \in R^{1 \times 1} \) is the input vector of the model, \( A_s \) and \( B_s \) are matrices with appropriate dimensions and the model remains stable.

Define the state estimation error as:
\[
e_s = \bar{x} - x_s
\]
(25)
From (23) and (24), the dynamic equation of state tracking error system can be derived as
\[
\dot{e}_s = A_s e_s + (\hat{A} - A_s) \bar{x} + \tilde{B} \bar{u} - B_s r_s + \tilde{F}_{s2} + \tilde{W} \omega(t)
\]
(26)

2) CONSTRUCTION OF INTEGRAL SLIDING SURFACE
The robustness of the system can be enhanced by designing the integral sliding surface. Construct the function of integral sliding surface as follows
\[
\delta_s = S_x e_s - S_x e_s(0) - S_x \int_0^t (\hat{A}_s - \tilde{B}K) e_s dt
\]
(27)
where \( S_x \in R^{1 \times 6} \) is a constant matrix with full rank so that the square matrix \( S_x \tilde{B} \) is nonsingular. \( K \in R^{1 \times 6} \) meets the following inequality conditions:
\[
\max (Re[\lambda(\hat{A}_s - \tilde{B}K)]) < 0
\]
(28)
where \( Re[\lambda(*)] \) is the real part of \( \lambda(*) \), \( \lambda(*) \) is the eigenvalue of matrix.

Correspondingly, the convergence property can be calculated as
\[
\delta_s = 0 \rightarrow S_x e_s - S_x e_s(0) - S_x \int_0^t (\hat{A}_s - \tilde{B}K) e_s dt = 0
\]
(29)
and its derivative is
\[
\dot{e}_s = (\hat{A}_s - \tilde{B}K) e_s
\]
(30)
It means that the greater state estimation error \( e_s \) is, the faster the convergence speed is. And the speed can be controlled by changing the value of \( A_s - \tilde{B}K \).

3) SLIDING MODE FTC LAW DESIGN
On the basis of upper discussion, the sliding mode FTC law is defined as
\[
U(t) = u(t) + U_s(t)
\]
(31)
where \( u(t) \) is the linear part of the sliding mode control law. It can ensure that the system performs ideal sliding mode motion on the sliding surface and according to equivalent control, it can be constructed as
\[
u = - (S_x \tilde{B})^{-1} \left[ S_x \tilde{B}K e_s + S_x (\hat{A}_s - A_s) \bar{x} - S_x B_s r_s \right]
\]
(32)

As a non-linear part, \( U_s \) can generate the discontinuous signal which forces the system trajectory to reach and maintain on the predetermined sliding surface, thus ensure the robustness to external disturbances. Combining with \( \tilde{F}_s \) and the inherent robustness of sliding mode control, \( U_s \) can be constructed as
\[
U_s = - (S_x \tilde{B})^{-1} \frac{\delta_s (\tau_f + \tau)}{\|\delta_s\| + \sigma} (\delta_s \neq 0)
\]
(33)
where \( \sigma \) is a fully small positive constant. By using fault estimation \( \tilde{F}_s \) provided by SMO, the nonlinear gain \( \tau_f \), which handles with sensor faults in the system, can be constructed as
\[
\tau_f \geq \left[ S_x \tilde{F} \right] \left[ \tilde{F}_s \right] \left[ \tilde{C}_3 \right]^{-1} \left[ \tilde{F}_{s2} \right]^{-1} + k_f
\]
(34)
where \( k_f \) is a known positive integer. By introducing non-linear gain \( \tau_f \), the impact of unknown information in the fault reconstruction value can be processed online, which can improve the fault tolerance of the system to handle more complex fault information.

The nonlinear gain \( \tau \) is used to deal with the effects of external disturbances. By using the boundary conditions, it can be designed as:
\[
\tau \geq \left[ S_x \tilde{W} \right] \omega_0 + k_{\theta}
\]
(35)
where \( k_{\theta} \) is a known positive integer.
4) VERIFICATION OF SYSTEM STABILITY

Theorem 2: When the conditions of the state tracking error system (26) are satisfied, design the sliding mode surface according to (27) and FTC law according to (29)-(31), then the error system (26) is asymptotically stable.

Proof: Consider a candidate of the Lyapunov function as $V_s = \frac{1}{2}\delta_s^T \delta_s$ and its derivative is

$$\dot{V}_s = \delta_s^T \dot{\delta}_s$$

Substituting the time derivative of (27) into (29)-(31), we can get

$$\dot{\delta}_s = S_s \dot{e}_s - S_s (A_s - \tilde{B}K) e_s$$

$$= S_s \left[ A_s e_s + (\tilde{A} - A_s) \tilde{x} + \tilde{B} \dot{e}_s + \tilde{F}_s + \tilde{W} \omega(t) \right] - S_s A_s e_s + S_s \tilde{B} \dot{e}_s$$

$$= -\tau \frac{\delta_s}{||\delta_s|| + \sigma} - \tau \frac{\delta_s}{||\delta_s|| + \sigma} + S_s \tilde{F}_s + S_s \tilde{W} \omega(t)$$

(37)

Then (34) can be calculated as

$$\dot{V}_s = \delta_s^T \left( -\tau \frac{\delta_s}{||\delta_s|| + \sigma} - \tau \frac{\delta_s}{||\delta_s|| + \sigma} + s \tilde{F}_s + s \tilde{W} \omega(t) \right)$$

$$= -\tau ||\delta_s|| + \delta_s^T S_s \tilde{W} \omega(t) - \tau ||\delta_s|| + \delta_s^T S_s \tilde{F}_s$$

$$\leq -\tau ||\delta_s|| + ||S_s|| \left[ ||S_s \tilde{W} \omega(t)|| - \tau ||\delta_s|| \right] + ||S_s|| \left[ ||S_s \tilde{F}_s|| + ||\delta_s|| \right] + ||S_s \tilde{F}|| \gamma ||\delta_s||$$

$$= - (\kappa_f + \kappa_f) ||\delta_s|| - ||S_s \tilde{F}|| \left( \tilde{F}_s^{-1} ||G_3^{-1} N_2||^{-1} \tilde{F}_s^{-1} + \gamma \right)$$

(38)

From hypothesis 6 and (33),

$$\tilde{F}_s^{-1} \left[ G_3^{-1} N_2 \right]^{-1} \tilde{F}_s \geq \left[ \tilde{F}_s^{-1} \right] \gamma + \left[ \tilde{W}_2 \omega + \kappa \right] > \gamma$$

(39)

then

$$\dot{V}_s \leq - (\kappa_f + \kappa_f) ||\delta_s|| < 0$$

(40)

In summary, under the sliding mode FTC law (29)-(33), $V_s$ will decrease and stop changing when $||\delta_s|| = 0$. Therefore, the systematic error will also be driven to zero and the error system will be asymptotically stable [20].

IV. EXPERIMENT RESULTS

The experimental forklift is TFC35 electric forklift with SBW system. The experiment is carried out in a warehouse with noise and 20% indoor relative humidity. The experiment environment is shown in Fig.3.

The external output is $\theta(t) = \sin(2t)$ and the uncertainty of system ($\Delta A$ and $\Delta B$) is bounded matrix. There exists $\sigma_1, \sigma_2 > 0$ such that $\|\Delta A\| \leq \sigma_1$, $\|\Delta B\| \leq \sigma_2$, $\sigma_1 = \sigma_2 = 0.1$. According to (2), the state parameters of forklift are $A_s, B$, as shown at the bottom of the next page.

The external interference matrix is $H = \begin{bmatrix} 0.2 & 0.02 & 0.04 & 0.3 \end{bmatrix}^T$, $F = C$. The parameters of SMO are set as $\rho_1(0) = \rho_2(0) = \rho_3(0) = \rho_4(0) = 10$, $\eta_1 = \eta_2 = \eta_3 = \eta_4 = -100$, $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0.5$. The parameters of sliding mode FTC law is set as $S_s = [0 0 0 1 1 1]$, $K = [-6 20 5 7 -24 9]$. In the case of yaw rate sensor, roll rate sensor and front wheel angle sensor occurring faults respectively, the types of faults are shown in TABLE 1 correspondingly [21]. In order to compare the results before and after FTC, the FTC introducing time is set to $T_r$.

Take the current of steering motor as the input. During experiment, input the step signal at 2s and return back the steering handle at 16s.

When yaw rate sensor, roll rate sensor and front wheel angle sensor occur the corresponding faults, the output is reconstructed according to (19). The experimental results are shown as follows.

In Fig.4, before the time of fault-occurring ($T_f = 4s$), there is no fault in yaw rate sensor and the output $Y_{1f}$ is similar as the fault-free state. The fault output with uncertainty and
interference \((Y_{1f}^f)\) differs considerably from \(Y_{1f}\) and \(Y_1\). The adaptive SMO can estimate the status of sensor accurately. After the fault, the adaptive SMO can estimate the output and fault value of sensor accurately in the case of fault type 1 and the fault reconstruction value is basically consistent with the true value of the fault. It will not be affected by uncertainty and external interference.

As shown in Fig. 5, Fig. 6 and Fig. 7, before and after roll rate sensor and the front wheel angle sensor occurring the faults of type 2, 3 respectively, the output with no uncertainty and disturbance is almost the same as the fault-free state. And the fault output with uncertainty and disturbance differs greatly from the fault-free state. But the adaptive SMO and the fault reconstruction system can track the fault signal and reconstruct fault value. The fault reconstruction value is also consistent with the true value of the fault.

The experiments of FTC are carried out with the fault conditions listed in Table 1. To verify the benefits of the developed method proposed in this manuscript, the regular sliding mode controller is also adopted to make a comparison and the control law is defined as \(U(t) = u(t) = - (S_1B^e_5 + S_1 (\bar{A} - A) \bar{x} - S_1B^e_5 \bar{r} \bar{s})\). When yaw rate sensor, roll rate sensor and front wheel angle sensor occur the faults respectively, the comparison results of each sensor are shown in Fig. 7-Fig. 9.

As shown in Fig. 7-Fig. 9, the outputs of yaw rate sensor, roll rate sensor and front wheel angle sensor begin to vibrate from the fault time \((T_f = 4s)\). And from FTC time \((T_r = 8s)\), the sliding mode FTC methods are adopted for the fault system of forklift based on sensor signal reconstruction. We can observe that the methods can all make the system recover gradually after a short period of 1~3s.
But the developed method ($\bar{Y}_3^f$) has a better performance than the regular method ($\bar{Y}_3^f$). When there is output disturbance, the proposed method can suppress it and reduce the vibration. The outputs under the developed method with and without uncertainty and interference are both similar with the output in the fault-free state which can reflect the driving condition of the electric forklift.

V. CONCLUSION

Forklift is one of the most widely used vehicles for short-distance transportation of cargo and its safety is very important. As electronic components have been used in modern forklift widely, the fault diagnosis, fault reconstruction and FTC technology are particularly important to ensure safety.

For the problem of multi-sensor fault detection and reconstruction of forklift, a SMO with adaptive regulation law is adopted to detect and reconstruct sensor faults. It can separate the fault from the external disturbance and realize robust fault estimation. Specifically, without knowing the upper bounds of the unknown fault, the adaptive algorithm can also make the observer effective. The comparison experiment between the real fault value and the reconstructed result shows that this method can achieve a good estimation of sensor fault.

Based on the SMO proposed, a sliding mode fault-tolerant controller is adopted to deal with key sensor faults of forklift system. It realizes the robust processing of system parameter uncertainties and external output disturbances, and can make the forklift return to normal.

Experiments on TFC35 SBW electric forklift show that the adaptive SMO can reconstruct sensor fault value effectively, and the sliding mode fault-tolerant controller designed is also effective when sensor faults occur. And the performance of each sensor in the fault system can gradually restore to the similar level as that of fault-free SBW forklift. But in this research, the FTC method relies on the fault model of forklift. To a certain extent, the sliding mode control strategy can overcome the influence of model uncertainty. In future research activities, we shall investigate FTC controller without knowing the complete information about the state of the forklift.

ACKNOWLEDGMENT

The authors thank senior engineer Junliang Guo, Zijian Fang, Pengfei Li, Xingzhi Fang, and Weilin Lv for their cooperation and helpful suggestions. They also thank graduate students Chenggong Sun, Junjie Huang, Xu Zhang, and Zhizheng Jiang in the same study room.
REFERENCES

[1] F. Chen, J. Niu, and G. Jiang, “Nonlinear fault-tolerant control for hypersonic flight vehicle with multi-sensor faults,” IEEE Access, vol. 6, pp. 25427–25436, 2018.

[2] Y. Jiang and S. Yin, “Recursive total principle component regression based fault detection and its application to vehicular cyber-physical systems,” IEEE Trans. Ind. Informat., vol. 14, no. 4, pp. 1415–1423, Apr. 2018.

[3] S. Yin, A. Rodriguez, J. Juan, and Y. C. Jiang, “Real-time monitoring and control of industrial cyberphysical systems,” IEEE Ind. Electron. Mag., vol. 13, no. 4, pp. 38–47, Dec. 2019.

[4] Y. Wang, W. Zhou, J. Luo, H. Yan, H. Pu, and Y. Peng, “Reliable intelligent path following control for a robotic airship against sensor faults,” IEEE/ASME Trans. Mechatronics, vol. 24, no. 6, pp. 2572–2582, Dec. 2019.

[5] Y. Wang, H. R. Karimi, H.-K. Lam, and H. Yan, “Fuzzy output tracking control and filtering for nonlinear discrete-time descriptor systems under unreliable communication links,” IEEE Trans. Cybern., early access, Jun. 17, 2019.

[6] X. Y. Cao, C. H. Hu, and Q. L. Ma, “Research on active fault-tolerant control for sensor failures of missile attitude control systems,” Control Dec., vol. 27, no. 3, pp. 379–382, Mar. 2012.

[7] M. Boukhari, A. Chaibet, M. Boukhnifer, and S. Glaser, “Proprioceptive Sensors’ fault tolerant control strategy for an autonomous vehicle,” Sensors, vol. 18, no. 6, p. 1893, 2018.

[8] J. Zhang, A. Amodio, T. Li, B. Aksun-Guvenc, and G. Rizzoni, “Fault diagnosis and fault mitigation for torque safety of drive-by-wire systems,” IEEE Trans. Veh. Technol., vol. 67, no. 9, pp. 8041–8054, Sep. 2018.

[9] X. Chang, J. Huang, and F. Lu, “Robust in-flight sensor fault diagnostics for aircraft engine based on sliding mode observers,” Sensors, vol. 17, no. 4, p. 835, 2017.

[10] F. Chen, R. Jiang, K. Zhang, B. Jiang, and G. Tao, “Robust backstepping sliding-mode control and observer-based fault estimation for a quadrotor UAV,” IEEE Trans. Ind. Electron., vol. 63, no. 8, pp. 5044–5056, Aug. 2016.

[11] X. Liu, M. Zhang, and F. Yao, “Adaptive fault tolerant control and thruster fault reconstruction for autonomous underwater vehicle,” Ocean Eng., vol. 155, pp. 10–23, May 2018.

[12] Y. Wang, B. Jiang, Z.-G. Wu, S. Xie, and Y. Peng, “Adaptive sliding mode fault-tolerant fuzzy tracking control with application to unmanned marine vehicles,” IEEE Trans. Syst., Man, Cybern. Syst., early access, Jun. 24, 2020, doi: 10.1109/TSMC.2020.2964808.

[13] H. Yang, Y. Jiang, and S. Yin, “Fault-tolerant control of time-delay Markov jump systems with Itô stochastic process and output disturbance based on sliding mode observer,” IEEE Trans. Ind. Informat., vol. 14, no. 12, pp. 5299–5307, Dec. 2018.

[14] Z. Jiang and B. Xiao, “LQR optimal control research for four-wheel steering forklift based on state feedback,” J. Mech. Sci. Technol., vol. 32, no. 6, pp. 2789–2801, Jun. 2018.

[15] X. Li and W. Zhang, “An adaptive fault-tolerant multisensor navigation strategy for automated vehicles,” IEEE Trans. Veh. Technol., vol. 59, no. 6, pp. 2815–2829, Jul. 2010.

[16] Z. Mao, Y. Zhan, G. Tao, B. Jiang, and X.-G. Yan, “Sensor fault detection for rail vehicle suspension systems with disturbances and stochastic noises,” IEEE Trans. Veh. Technol., vol. 66, no. 6, pp. 4691–4705, Jun. 2017.

[17] W. Guan and G. Yang, “Adaptive fault-tolerant control of linear systems with actuator saturation and 1–2 disturbances,” J. Control Theory Appl., vol. 7, no. 2, pp. 119–126, Feb. 2009.

[18] X.-G. Yan and C. Edwards, “Robust sliding mode observer-based actuator fault detection and isolation for a class of nonlinear systems,” Int. J. Syst. Sci., vol. 39, no. 4, pp. 349–359, Apr. 2008.

[19] F. Plestan, S. Shtessel, V. Brégeaut, and A. Poznyak, “New methodologies for adaptive sliding mode control,” Int. J. Control, vol. 83, no. 9, pp. 1907–1919, Sep. 2010.

[20] J. Park, L. Wang, J. M. Dawson, L. A. Hornak, and P. Famouri, “Sliding mode-based microstructure torque and force estimations using MEMS optical monitoring,” IEEE Sensors J., vol. 5, no. 3, pp. 546–552, Jun. 2005.

[21] S. U. Jan, Y.-D. Lee, J. Shin, and I. Koo, “Sensor fault classification based on support vector machine and statistical time-domain features,” IEEE Access, vol. 5, pp. 8682–8690, 2017.