Dissipative nonlinear waves in a gravitating quantum fluid

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Abstract – Nonlinear wave propagation is studied analytically in a dissipative, self-gravitating Bose Einstein condensate, in the framework of Gross-Pitaevskii model. The linear dispersion relation shows that the effect of dissipation is to suppress dynamical instabilities that destabilize the system. The small amplitude analysis using reductive perturbation technique is found to yield a modified form of KdV equation. The soliton energy, amplitude and velocity are found to decay with time, whereas the soliton width increases, such that the soliton exists for a finite time only.

Keywords: Self-gravitating Bose Einstein Condensate; Gross-Pitaevskii equation; Nonlinear waves

1 Introduction. – Theoretically predicted by Bose and Einstein in 1925 [1] and first produced experimentally by the Cornell and Wiemann group some seven decades later in 1995 [2], Bose Einstein Condensates (BECs) have fascinated scientists for their intriguing properties [3–9]. The major role they play in condensed matter physics is pretty well known [10]. Fairly recent studies suggest that self gravitating BECs could play an important role in astrophysics and cosmology, in the study of neutron stars and dark matter halos [11][12]. It is anticipated that a substantial amount of matter in neutron stars might exist as BECs due to their superfluid quantum core. However, such compact astronomical objects have very strong magnetic fields and will be considered in a future work. The existence of dark matter might be linked to the observed flat rotation curves of galaxies. Dark matter halos could be gigantic quantum objects, interpreted as self-gravitating BECs at \( T = 0 \). At large scales, quantum effects are negligible and the classical hydrodynamic equations are good enough in explaining the large-scale structure of the universe. However, at small-scales, the pressure arising from the Heisenberg uncertainty principle or from the repulsive scattering of the bosons may stabilize dark matter halos against gravitational collapse. In case the bosons are assumed to be non self-interacting, gravitational collapse is prevented by the Heisenberg Uncertainty Principle, which is equivalent to a quantum pressure. In such cases the mass of the individual bosons \( \approx 10^{-24} \text{ eV}/c^2 \). If the bosons are assumed to have a repulsive self interaction, gravitational collapse is prevented by the pressure arising from the scattering. For scattering lengths \( \sim 10^6 \) fm, the mass of such bosons \( \approx 1 \text{ eV}/c^2 \). This stabilizing of dark matter halos against gravitational collapse — either due to pressure arising from Heisenberg Uncertainty Principle, or the repulsive scattering of bosons — could lead to smooth core densities in agreement with observations [13], instead of cuspy density profiles [14] predicted by the cold dark matter model [15][16]. In [17], the authors studied Jeans instability in a self-gravitating dusty plasma, considering an attractive force between two equally charged dust particles. For attractive self interacting BECs with negative scattering length, the ultimate collapse cannot be prevented, and the BEC is not stable. This leads to the formation of supermassive black holes at the centre of galaxies. Additionally, in the astrophysical context, boson stars are conceptualized as self-gravitating bosons, exclusively trapped in their own gravitational potential [18]. Theoretically, boson stars are objects of highly diverse
sizes and masses, depending on the assumed particle mass and the strength of self-interaction. For example, minibo-son stars are very compact objects with radii $\approx 10^{-13} m$ [19], whereas those of the size of the sun, but having mass $\sim 10^6$ solar masses, could mimic supermassive black holes [20]. However, for such compact bodies, we must use general relativity and couple the Klein-Gordon equation to the Einstein field equations; hence these are exempted from the present study.

With the experimental verification of BECs in magnetically trapped dilute vapors of alkali metals [2,21], interest in this particular field has been revived in recent times [22–25]. It has been observed that dissipation plays a crucial role in the stabilization of a BEC droplet by suppressing the dynamical instabilities [20] — observation of collective damped oscillations of the condensate shows the presence of dissipation [27, 28]. Additionally, the observation of vortex lattices when the trapping potential is rotated fast enough, implies the presence of dissipation [29]. In the present study, we shall investigate the nonlinear wave propagation in a self gravitating BEC, in the framework of Gross-Pitaevskii equation, in the presence of dissipation. It is worth mentioning here that Ghosh and Chakrabarti studied this problem in [30], but without the dissipative term. They showed that the small amplitude analysis is governed by the KdV equation. Our aim in this work is to emphasize on the effect of the dissipative term on the propagation of nonlinear structures.

This article is organized as follows. To make the paper self contained, the basic equations are introduced in Section 2. Section 3 is kept for the derivation and discussion of the linear dispersion relation. The small amplitude analysis is carried out in Section 4, using the standard reductive perturbation technique. The nonlinear evolution is also derived analytically in Section 4. Finally, Section 5 is kept for Discussions and Conclusions.

2 Basic Governing Equations. – Self gravitating quantum systems, proposed by Penrose in his discussion of quantum state reduction by gravity [31], are a dilute quantum gas with short-range dipole interactions between atoms. At very low temperatures ($T \to 0$), all particles (bosons) in a dilute Bose gas confined in a trapping potential $V_{ext}(r)$, condense to the same quantum ground state and form a Bose Einstein condensate. The condensation of the bosons takes place when their thermal (de Broglie) wavelength $\lambda_{DB} = \left(2\pi h^2/mk_B T\right)^{1/2}$ exceeds their mean separation. The system is described by the one parameter condensate wave function $\psi(r, t)$. We consider a system of $n$ bosons, with mass $m$ in interaction. For ultra cold temperatures, the dynamical behaviour of a weakly interacting Bose gas in the presence of dissipation, considering the mean field analysis, is described by the Gross-Pitaevskii (GP) equation [32]

$$\left(i - \nu\right) \frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{ext}(r) + g|\psi|^2\right] \psi$$  \hspace{1cm} (1)$$

where $g = 4\pi a_s \hbar^2/m$ is the interacting constant, $a_s$ is the s-wave scattering length ($a_s > 0$ for repulsive self-interactions), and $\nu$ is a phenomenological dissipation constant, the dissipation being caused by the interaction between a BEC and thermal cloud [33]. Here $g|\psi|^2$ is the self-interaction term and $n = \int d^3r|\psi(r, t)|^2$ denotes the total number of bosons in the condensate. It is worth mentioning here that for BEC to take place, the s-wave scattering length must be very small as compared to the de Broglie wavelength : $\lambda_{DB} > a_s$. However, in the presence of a gravitational trap, no additional trap is required, and the external trapping potential $V_{ext}$ may be replaced by $V_G$, where $V_G = m\Phi$, $\Phi$ being the gravitational potential obeying the Poisson equation

$$\nabla^2 \Phi = 4\pi G (\rho - \rho_0)$$  \hspace{1cm} (2)$$

Here, $\rho = m n |\psi(r, t)|^2$ is the mass density, $\rho_0$ is its equilibrium value, and $n$ is the number density. In such a situation, the condition for stable BEC is : $a_s > \lambda_{DB} > a_s$. $a_s = 4\pi \hbar^2/2mu$, $a_s$ being the Bohr radius associated with gravitational coupling $u$

Typical characteristic values of the above parameters in a BEC are : $a_s \sim 10$ cm, $\lambda_{DB} \sim 10^{-5} - 10^{-3}$ m, $a_s \sim 3$ nm, $n = 10^{21}$ m$^{-3}$, and $m = 35.2 \times 10^{-27}$ kg [30–34].

To rewrite the GP equation in the form of hydrodynamic equations, we apply the Madelung transformation

$$\psi(r, t) = \sqrt{\frac{\rho(r, t)}{m}} \exp[i S(r, t)/\hbar]$$  \hspace{1cm} (3)$$

where $S(r, t)$ has the dimension of action, and define the irrotational flow velocity as

$$\mathbf{v} = \nabla S/m$$  \hspace{1cm} (4)$$

The flow is irrotational since $\nabla \times \mathbf{v} = 0$.

Substituting eq. (3) in eq. (1), and separating real and imaginary parts, we obtain

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) - \frac{2\nu \rho}{\hbar} \frac{\partial S}{\partial t} = 0$$  \hspace{1cm} (5)$$

which is equivalent to the continuity equation, and

$$\rho \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla P - \rho \nabla V_G$$  \hspace{1cm} (6)$$

which is equivalent to the momentum equation of hydrodynamics. Here $P$ is the pressure term, and the term containing $\hbar^2$ on the rhs is the quantum potential.
In BEC the non thermal pressure term \( P \) arises from the short-range interactions between the bosons, and is related to the mass density through \[10\]
\[
P = \frac{1}{2} g \left( \frac{\rho}{m} \right)^2
\]

The continuity equation \[5\] can be written as
\[
\frac{\partial \rho}{\partial t} = \frac{\hbar}{2m} \left( \nabla \cdot (\rho \mathbf{v}) \right) - \frac{\hbar^2}{2m} \nabla^2 \left( \frac{1}{\rho} \right)
\]

Taking gradient of equation \[8\], and using equation \[4\], we obtain
\[
\nabla \cdot \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) + \frac{\hbar^2}{m} \nabla^2 \left( \frac{1}{\rho} \right)
\]
\[
= -4\pi G (\rho - \rho_0)
\]

where \( c_s = \sqrt{g\rho_0/m} = \sqrt{gn/m} \) is the velocity of sound in the medium. For BEC comprising of sodium atoms, \( c_s \approx 10^{-3} \text{ m/s} \) \[34\]. Equations \[9\] and \[10\] are the final set of equations which need to be analyzed simultaneously. In the absence of an exact analytical solution, numerical results and approximate methods like the linear dispersion relation, give a good estimate of the system behaviour.

3 Linear Dispersion Relation. – The linear dispersion relation is a powerful tool to give a qualitative picture of dissipation in the system. To derive its form, we shall start with the equations \[9\] and \[10\], and linearize them about equilibrium values \( \nu = 0 \) and \( \rho(\mathbf{r}, t) = \rho_0 \), with the perturbations varying as \( e^{i(k \cdot \mathbf{r} - \omega t)} \). For brevity, we just quote the final result, without going into the mathematical details. The linear dispersion relation is obtained as a quadratic equation in \( \omega \):
\[
(1 + \nu^2) k^2 \omega^2 + \i \nu \hbar \left( 2mc^2 s (k^2 - k_J^2) + \frac{\hbar^2 k^4}{m} \right) \omega
\]
\[- \left( c_s^2 k^2 (k^2 - k_J^2) + \frac{\hbar^2 k^4}{4m^2} \right) = 0
\]

where \( k = |k| \), \( \omega_J = \sqrt{4\pi G\rho_0} \) is the Jeans frequency and \( k_J = \omega_J/c_s \) is the Jeans wave number. Hence
\[
\omega = \pm \omega_R + i\omega_I
\]

with
\[
\omega_I = -\nu \frac{m \left( c_s^2 (k^2 - k_J^2) + \frac{\hbar^2 k^4}{2m^2} \right)}{(1 + \nu^2) \hbar k^2}
\]

and
\[
\omega_R = \sqrt{\frac{\hbar^2 k^8}{m^2} + 4c_s^2 k^4 (k^2 - k_J^2) - \frac{4m^2 \nu^2}{\hbar^2} c_s^4 (k^2 - k_J^2)^2}{2 \left( 1 + \nu^2 \right) k^2}
\]

In the absence of dissipation (\( \nu = 0 \)), we get back the dispersion relation given in \[30\]
\[
\omega = \pm c_s \sqrt{(k^2 - k_J^2) + H_Q^2 \lambda^2 k^4}
\]

where \( H_Q = \hbar/(2mc_s \lambda) \) is a dimensionless quantum parameter, and \( \lambda = 2\pi/k \) is the perturbation wavelength. Thus, in the absence of dissipation, the system is stable only if the perturbation wave number is greater than the Jeans wave number : \( k > k_J \) or \( \lambda < \lambda_J \). However, in the presence of dissipation, \( \omega \) may have an imaginary part even for \( k < k_J \) as observed from equation \[14\]. But, it is worth noting here that dissipation alone is responsible for the imaginary part in \( \omega \). For positive dissipation constant \( \nu \), \( \omega_I \) is negative, showing a decaying wave. Thus, analogous to the observation made in ref. \[29\], applying numerical simulation, our analytical approach arrives at a similar result — the effect of dissipation is to resist dynamical instabilities that destabilize the system. We plot the linear dispersion relation for a dissipative gravitating fluid.
fluid in Fig. 1. For fixed value of the quantum parameter \( H_Q \), the real part of \( \omega \) is more or less unaffected by the dissipation term \( \nu \), but the magnitude of its imaginary part increases with increasing dissipation, and hence the system gets more and more damped. This is depicted in Fig. 1. It is to be noted that \( \omega \) has an imaginary part only in the presence of dissipative term \( \nu \).

4 Evolution of Small Nonlinear Structures. – We now consider a 3-dimensional Bose gas, confined tightly in two spatial dimensions, and weakly propagating in the \( x \) direction only. Reduced dimensionality leads to strong quantum fluctuations which tend to destroy macroscopic coherence. However, since we are considering extremely low temperatures (\( T \approx 0 \)), this effect can be neglected. Thus, in the limit of one spatial dimension, the longitudinal dynamics dominate the system, and the transverse modes are heavily suppressed. In this approximation, the wave function \( \psi(x,t) \) can be decoupled into the product of a time-independent transverse component (which does not contribute to the dynamics of the system in the present case) and a time dependent axial component \( \psi(x,t) \). At temperatures close to zero where phase fluctuations are negligible such weakly-interacting 1D and 2D condensates are possible, and have been studied theoretically as well as experimentally [35]. To explore the evolution of weak nonlinear structures in one spatial dimension, for \( k_j \ll k \), we shall apply the reductive perturbation technique [36]. Before proceeding further, we write down the normalizations used:

\[
\begin{align*}
\bar{x} &= \frac{x}{\lambda}, \quad \bar{t} = \frac{ct}{\lambda}, \quad \bar{\rho} = \frac{\rho}{\rho_0}, \quad \bar{v} = \frac{v}{c_s} \quad (16)
\end{align*}
\]

In terms of the new normalized variables, equations (11) and (12) take the forms

\[
\frac{\partial \bar{v}}{\partial \bar{t}} = \frac{H_Q}{\bar{v}} \frac{\partial}{\partial \bar{x}} \left[ \frac{1}{\bar{\rho}} \left( \frac{\partial \bar{\rho}}{\partial \bar{x}} + \frac{\partial}{\partial \bar{x}} (\bar{\rho} v) \right) \right] \quad (17)
\]

and

\[
\frac{\partial}{\partial \bar{x}} \left[ \frac{\partial \bar{v}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{x}} + \nu H_Q \frac{\partial^2}{\partial \bar{x}^2} \left( \frac{1}{\bar{\rho}} \frac{\partial \bar{\rho}}{\partial \bar{t}} \right) - 2H_Q^2 \frac{\partial^2}{\partial \bar{x}^2} \left( \frac{1}{\sqrt{\bar{\rho}}} \right) \right] \quad (18)
\]

\[
-2H_Q^2 \frac{\partial^2}{\partial \bar{x}^2} \left( \frac{1}{\sqrt{\bar{\rho}}} \right) + \frac{\partial^2 \bar{\rho}}{\partial \bar{x}^2} \quad (18)
\]

where we have dropped all the bars for the sake of simplicity.

At this stage we introduce the stretched coordinates for reductive perturbation technique [36]:

\[
\xi = \sqrt{\epsilon} (x - Vt) \quad , \quad \tau = \epsilon^{3/2} t \quad (19)
\]

where \( \epsilon \) is a small non zero parameter proportional to the amplitude of perturbation, and \( V \) is the Mach number. The dynamical variables \( \rho \) and \( v \) are expanded about their equilibrium value, in a power series of \( \epsilon \), as

\[
\begin{align*}
\rho &= 1 + \epsilon \rho^{(1)} + \epsilon^2 \rho^{(2)} + \cdots \quad (20)
\end{align*}
\]

subject to the boundary conditions that all perturbed values and their derivatives vanish at \( \xi = -\infty \):

\[
\rho^{(1)}, \quad v^{(1)} \to 0 \quad \text{as} \quad \xi \to -\infty \quad (21)
\]

Instead of going into detailed algebraic calculations, we shall just quote the results here. The zeroth order terms of equations (17) and (18) yield \( v^{(1)} = V \rho^{(1)} \) and \( \rho^{(1)} = V v^{(1)} \) so that \( V = 1 \) and

\[
\begin{align*}
v^{(1)} &= \rho^{(1)} \quad (22)
\end{align*}
\]

For the perturbation expansion to be valid with the inclusion of gravitational and dissipative terms, we must have the following scaling:

\[
\nu = \nu_0 \epsilon^{3/2}, \quad \frac{\lambda}{\lambda_J} \sim 0(\epsilon) \Rightarrow \left( \frac{\lambda}{\lambda_J} \right)^2 \sim 0(\epsilon^2)
\]

This is in agreement with our assumption for the stable case \( k_j \ll k \); i.e., perturbation wavelength \( \lambda \ll \) Jeans wavelength \( \lambda_J \). After some lengthy but straightforward algebra, we obtain the final equation as

\[
\begin{align*}
\frac{\partial}{\partial \xi} \left( \frac{\partial \rho^{(1)}}{\partial \tau} + 3 \frac{3}{2} \frac{\rho^{(1)}}{\partial \rho^{(1)}} - \frac{H_Q^2}{2} \frac{\partial^3 \rho^{(1)}}{\partial \xi^3} + \frac{\nu_0}{2H_Q} \rho^{(1)} \right) + \mu \rho^{(1)} &= 0 \quad (23)
\end{align*}
\]

where \( \mu = 2\pi^2 \left( \frac{\lambda}{\lambda_J} \right)^2 \).

Integrating once wrt \( \xi \), equation (23) takes the form

\[
\begin{align*}
\frac{\partial}{\partial \xi} \left( \frac{3}{2} \rho^{(1)} \frac{\partial \rho^{(1)}}{\partial \xi} - \frac{H_Q^2}{2} \frac{\partial^3 \rho^{(1)}}{\partial \xi^3} + \frac{\nu_0}{2H_Q} \rho^{(1)} \right) + \mu \int_{-\infty}^{\xi} \rho^{(1)} d\xi' &= 0 \quad (24)
\end{align*}
\]

Thus the final equation is obtained as a modified form of KdV equation, with the quantum term (proportional to \( H_Q \)) playing the role of dispersion. Our next step would be to study the effect of dissipation on the nonlinear structures.

In the absence of both gravitational and dissipative effects (\( \mu = 0, \nu_0 = 0 \)), we get back the KdV equation, with negative dispersion. Incidentally, the sign of dispersion (positive or negative) is important in determining only the direction of propagation of the wave. To obtain the soliton energy, we multiply eq. (24) by \( \rho^{(1)} \), and integrate the same, with proper boundary conditions; viz., \( \psi \) and its
derivatives vanish at \( \xi \to \pm \infty \). In the absence of dissipative and gravitational effects \((\nu_0 = 0, \mu = 0)\), the soliton energy, given by

\[
I = \frac{1}{2} \int_{-\infty}^{\infty} \rho^{(1)}(\xi, \tau) d\xi
\]

is conserved: \( \partial I / \partial \tau = 0 \). The soliton solution is obtained as

\[
\rho^{(1)}(\xi, \tau) = -N \text{ sech}^2 \left( \frac{\xi + U \tau}{W} \right)
\]

where \( U \) denotes the soliton velocity, \( N \) is the dimensionless soliton amplitude, and \( W \) is its dimensionless spatial width, inter-related through the relation

\[
U = \frac{N}{2}, \quad W = \frac{2H_Q}{\sqrt{N}}
\]

(27)

Thus as the velocity of the soliton increases, its width decreases. These observations are consistent with those mentioned in [30]. Substituting (26) in (25), the soliton energy in the absence of gravitational and dissipative effects is obtained as

\[
I = \frac{2}{3} N^2 W = \frac{4}{3} N^{3/2} H_Q.
\]

However, in the presence of gravitational and dissipative effects, eq. (24) does not represent a completely integrable Hamiltonian. The soliton energy is no longer conserved: \( \partial I / \partial \tau \neq 0 \). In fact,

\[
\frac{\partial I}{\partial \tau} = -\frac{\nu_0}{H_Q} I - \mu \int_{-\infty}^{\xi} \rho^{(1)}(\xi', \tau) \left( \int_{-\infty}^{\xi} \rho^{(1)}(\xi', \tau') d\xi' \right) d\xi
\]

(28)

Since \( \nu_0 \) and \( \mu \) are very small parameters compared to unity, we can assume a slowly varying time-dependent form of the solution in (26):

\[
\rho^{(1)}(\xi, \tau) = -N(\tau) \text{ sech}^2 \left( \frac{\xi + U(\tau) \tau}{W(\tau)} \right)
\]

(29)

Using (29) in (25), the soliton energy turns out to be

\[
I(\tau) = \frac{4}{3} [N(\tau)]^{3/2} H_Q
\]

(30)

Now, the integration in equation (28) can be performed analytically, using equations (29) and (30). Finally, equation (28) reduces to

\[
\frac{\partial [N(\tau)]^{3/2}}{\partial \tau} + \frac{\nu_0}{H_Q} [N(\tau)]^{3/2} = -6 \mu H_Q N(\tau)
\]

(31)

where we have used eq. (27). This last equation (31) can be solved analytically. The final solution is obtained as

\[
\sqrt{N(\tau)} = \sqrt{N_0} \left\{ \frac{1 + \tau_0}{\tau_0} \right\} e^{-\frac{\nu_0}{3H_Q} \tau}
\]

(32)

where \( N_0 \) is the initial soliton amplitude, and

\[
\tau_0 = \frac{3 \mu}{\nu_0} H_Q W(0)
\]

(33)

Fig. 2: Plot of Energy \( I \) against \( \tau \), for different values of the parameters \( \mu \) and \( \nu_0 \).

Fig. 3: Plots showing variation of the amplitude \( N(\tau) \) w.r.t. various parameters — viz., \( \mu \), \( \nu_0 \) and \( \tau \).
The effect of gravitation. Figures 2 to 5 consolidate our observations. To the best of our knowledge, equations (32) and (34) are new results, not reported elsewhere.

Thus the soliton exists for a finite time only — say $\tau_c$, given by

$$\tau_c = \frac{3HQ}{\nu_0} \ln \left[ \frac{1 + \tau_0}{\tau_0} \right]$$

To the best of our knowledge, equations (32) and (34) are new results, not reported elsewhere.

Thus the soliton exists for a finite time only — so long as the soliton energy is positive. We plot the variation of the soliton energy in Fig. 2 and the soliton amplitude in Fig. 3, with $\tau$, for different values of the dissipation factor $\nu$ and the gravitational parameter $\mu$. Both figures indicate that the effect of dissipation is far more pronounced than the effect of gravitation.

Fig. 4 gives the 3-dim. plot of the soliton solution $\rho(1)$ against $\xi$ and $\tau$, for $\mu = 0.1$, $\nu_0 = 0.04$. The amplitude diminishes with progressing $\tau$.

$W(0) = 2HQ/\sqrt{N_0}$ being the initial soliton width.

Thus the amplitude of the soliton decreases with time $\tau$. This indicates that the soliton exists for a finite time only — say $\tau_c$, given by

$$\tau_c = \frac{3HQ}{\nu_0} \ln \left[ \frac{1 + \tau_0}{\tau_0} \right]$$

Fig. 5: Plot of the solution $\rho(1)$ against $\xi$, at $\tau = 1$, for $\mu = 0.1$. The amplitude diminishes and width increases with increasing $\nu_0$, for $\tau \neq 0$. Inset plot shows the solution has same amplitude and width for different $\nu_0$ at $\tau = 0$.

To investigate the evolution of small nonlinear structures, we considered the 3-dimensional Bose gas to be confined tightly in two spatial dimensions, and weakly propagating in the $x$ direction only. This is possible at very low temperatures, when longitudinal dynamics dominate over the transverse one. The analysis was carried out using reductive perturbation technique, for the stable case — perturbation wavelength $\lambda$ less than the Jeans wavelength $J$. The final form was obtained as a modified form of the KdV equation, giving soliton solution in some finite time interval. The soliton energy, velocity and amplitude were observed to decay with time. However, the width of the soliton increases such that the product of the soliton amplitude and square of the width remains constant. Though gravitational effect $\mu$ and dissipative term $\nu$ have similar effects on the propagation of nonlinear structures, the effect of dissipation is far more pronounced than that of gravitation. Figures 2 to 5 consolidate our observations.

The graphs were plotted for physically relevant parameters mentioned in Section 2 [94]. We propose to extend the evolution of small amplitude weak nonlinear structures, in two and three spatial dimensions in the near future.

With a surge of activity in theoretical and experimental studies related to Bose Einstein condensates in recent times, our present work may be of significant relevance in this field.

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REFERENCES

[1] P.H.Chavanis, Self-gravitating Bose-Einstein Condensates,
in Quantum Aspects of Black Holes, edited by X. Calmet
Dissipative nonlinear waves in a gravitating quantum fluid

[2] M.H.Anderson, J.R.Ensher, M.R.Matthews, C.E.Wieman and E.A.Cornell, Science 269, 198 (1995).
[3] M.Colpi, S.L.Shapiro and I.Wasserman, Phys. Rev. Lett. 57 2485 (1986).
[4] S.J.Shin, Phys. Rev. D 50, 3650 (1994).
[5] S.U.Ji and S.J.Sin, Phys. Rev. D 50, 3655 (1994).
[6] P.J.E.Pebbles and B.Ratra, Rev. Mod. Phys. 75, 559 (2003).
[7] A.K.Nekrasov, Phys. Rev. E 73, 026310 (2006).
[8] W-P.Zhong, M.R.Belic, Y.Lu and T.Huang, Phys. Rev. E 81, 016605 (2010).
[9] R.Britoa, V.Cardosoa, C.A.R.Herdeiroc and E.Raduc, Phys. Lett. B 752 291 (2016).
[10] C.J.Pethik and H.Smith, *Bose-Einstein Condensation in dilute gases*, Cambridge University Press, (Cambridge, UK, 2008).
[11] P.H.Chavanis, Phys. Rev. D 92 103004 (2015).
[12] S.L.Shapiro and S.A.Teulolsky, *Black Holes, White Dwarfs and Neutron Stars*, Wiley, New York, N.Y. (1983).
[13] A.Burkert, Astrophys. J. 447, L25 (1995).
[14] J.F.Navarro, C.S.Frenk, S.D.M.White, Mon. Not. R. Astron. Soc. 462, 563 (1996).
[15] J.F.Navarro, C.S.Frenk and S.D.M.White, Astrophys. J. 490, 493 (1997).
[16] T.Harko, J. Cosmol. Astropart. Phys. 2011, 022 (2011).
[17] P.K.Shukla and L.Stenflo, Proc. R. Soc. A 462, 403(2006).
[18] R. Ruffini and S. Bonazzola, Phys. Rev. 187, 1767 (1969).
[19] T.Lee and Y.Pang, Nucl. Phys. B 315, 477 (1989).
[20] F. Schunck and E.Milke, Class. Quan. Grav. 20 R301 (2003).
[21] K.B.Davis, M.O.Mewes, M.R.Andrews, N.J. van Druten, D.S.Durfee, D.M.Kurn and W.Ketterle, Phys. Rev. Lett. 75, 3969 (1995).
[22] C.J.Kennedy, W.C.Burton, W.C.Chung and W.Ketterle, Nature Phys. 11 859 (2015).
[23] J.Struck, M.Weinberg, C.Olschlager, P.Windpassinger, J.Simonet, K.Sengstock, R.Hoppner, P.Hauke, A.Eckardt, M.Lewenstein and L.Mathey, Nature Phys. 9, 738 (2013).
[24] M. Atala, M.Aidelsburger, M.Lohse, J.T.Barreiro, B.Paredes and I.Bloch, Nature Physics 10, 588 (2014).
[25] M.Aidelsburger, M.Atala, S.Nascimbne, S.Trotzky, Y.-A.Chen and I.Bloch, Phys. Rev. lett. 107, 255301 (2011).
[26] H.Saito and M.Ueda, Phys. Rev. A 70, 053610 (2004).
[27] D.S.Jin, J.R.Ensher, M.R.Matthews, C.E.Wieman and E.A.Cornell, Phys. Rev. Lett. 77, 420 (1996).
[28] M.-O. Mewes, M.R.Andrews, N.J. van Druten, D.M.Kurn, D.S.Durfee, C.G.Townsend and W.Ketterle Phys. Rev. Lett. 77, 988 (1996).
[29] M.Tsubota, K.Kasamatsu and M.Ueda, Phys. Rev. A 65, 023603 (2002).
[30] S.Ghosh and N.Chakrabarti, Phys. Rev. E 84 046601 (2011).
[31] R.Penrose, Gen. Relativ. Gravit. 28, 581 (1996).
[32] F.Dalfovo, S.Giorgini, L.P.Pitaevskii and S.Stringari, Rev. Mod. Phys. 71, 463 (1999).
[33] Y. Xiao-Xian, S. Yu-Ren and D. Wen-Shan, Commun. Theor. Phys. (Beijing, China) 49, 119 (2008).
[34] A.Camacho, *Speed of sound in a Bose-Einstein condensate*, arXiv:1205.4774v1 [cond-mat.quant-gas] (2012).
[35] Emergent Nonlinear Phenomena in Bose-Einstein Condensates : Theory and Experiment, Springer Series on Atomic, Optical and Plasma Physics, Vol. 45 (2008), Edited by P.G.Kevrekidis, D.J.Frantiçekakis, R.Carretero-Gonzlez, Chapter 1.
[36] H.Washimi, T.Taniuti, Phys. Rev. Lett. 17, 996 (1996).