Nambu mass hierarchies in strings.

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Abstract

We show that a recent proposal by Nambu to generate a hierarchy among Yukawa couplings in the standard model may be easily implemented in superstring models. In such models, two of the main ingredients of the Nambu proposal find a natural explanation: minimising with respect to the Yukawas amounts to a minimisation with respect to the underlying moduli fields and a constraint on the Yukawas of the type of the Veltman condition may be attributed to the relaxation process to a phenomenologically viable string vacuum.
1 Introduction.

The hierarchy observed between the different masses of the known particles (5 to 6 orders of magnitude between the electron and the top) which translates into a hierarchy of Yukawa couplings is certainly one of the most challenging issues at stake in the standard model. Now that we know that the top quark is heavy and that its mass lies in the range of the electroweak unification scale, the problem has been recently rephrased as to why the other quarks and leptons are so much lighter than this scale. This does not seem however to have led to any obvious solution yet and there is a definite need for new ideas to tackle this issue.

In this situation, an interesting proposal has been put forward by Nambu [1]. The idea consists in minimizing the vacuum energy density with respect to the Yukawa couplings – keeping all the other parameters fixed, in particular the field vacuum expectation values (vevs) and the other couplings – under the condition of vanishing quartic and quadratic divergences in the Higgs sector (the latter being the Veltman condition [2]).

In the toy model chosen by Nambu, the Veltman condition reads, in the case of two Yukawa couplings $\lambda_1$ and $\lambda_2$:

$$\lambda_1^2 + \lambda_2^2 = a^2,$$  \hfill (1.1)

with $a$ a constant. This constrains both Yukawa couplings to the region $[0, a]$. Such a condition cancels the order $\Lambda^2$ contributions to the vacuum energy, where $\Lambda$ is the cut-off. One is left in the scalar potential with the $O(\ln \Lambda)$ contributions which read:

$$V_1 = -A(\lambda_1^4 + \lambda_2^4) + O(\lambda_1^2 \ln \lambda_2^2)$$  \hfill (1.2)

Such a potential favors, in the case of $A > 0$, large hierarchies of couplings: $(\lambda_1, \lambda_2) = (a, 0)$ or $(0, a)$ when one disregards the logarithmic corrections. The generic effect of these logarithmic corrections is to generate a non-zero value for the smaller Yukawa coupling. The key ingredient for this mechanism to work is the sign of $A$: we will see in what follows that it is naturally negative in the case of supersymmetric models. Similarly, the sign of the logarithmic corrections is important since it may otherwise destabilize the hierarchy [3].

An interesting feature of this mechanism is that it easily extends to the case of an arbitrary number of Yukawa couplings. In such a situation, one of
the Yukawa couplings is naturally much larger than all the other ones. This
of course would provide a very simple explanation as to why the top quark is
much heavier than the remaining quarks and leptons of the standard model.

The Nambu proposal however raises several questions as to its range of
applicability. In the first place, is it valid to treat the Yukawa couplings as
dynamical variables? Also, what about the stability of the Veltman condi-
tion under renormalisation? One might also wonder whether it is licit to
minimize with respect to the Yukawa couplings while keeping the fields at
their vacuum expectation values: this procedure could hide some dangerous
instabilities of the model.

Some of these potential problems might be cured by making the model
supersymmetric. For example, the contribution of the scalar fields neces-
sarily present in supersymmetric models tends to give the right sign for the
constant $A$ introduced above and for the logarithmic radiative corrections.
Moreover, the question of the dynamical nature of the Yukawa couplings
finds a natural answer in superstring models. In these models, it is well-
known that Yukawa couplings have a non-trivial dependence on the moduli
fields which characterize the Kähler (and complex) structure of the compact
manifold. In some instances [4, 5], such couplings appear through non-
perturbative effects on the string world-sheet and are typically of order $e^{-T}$
where $T$ is the modulus whose vev determines the sigma model coupling.
In the case of $(2,2)$ vacua, the couplings are determined by holomorphic
functions of the moduli which also generate the Kähler metrics [6]. Also, it
has been recently noted [7] that through wave function renormalisation due
to massive modes, Yukawa couplings receive moduli dependent corrections
at one loop: these corrections can be understood as threshold effects at the
string scale and they make the boundary conditions for Yukawa couplings
moduli dependent.

In this context, minimization with respect to the Yukawa couplings in
the low energy theory amounts to a minimization with respect to the moduli
fields of the underlying string theory. Two cases may arise:

i) when one reaches low energies, enough moduli remain undetermined
(i.e. they correspond to a flat direction of the potential) so that one can
minimize with respect to all Yukawa couplings.

ii) at low energies, in particular after supersymmetry breaking has taken
place, there remains fewer moduli than Yukawa couplings. This situation
would typically yield new constraints on the Yukawas which might prove to
be useful by further constraining the Yukawa parameter space.

In what follows, we will suppose that we are in the first case and will
minimize with respect to all the Yukawa couplings. We will first discuss in Section 2 to what extent string models have properties which allow to easily implement the Nambu mechanism. In Section 3 we study in some details a toy model which we find representative of the general features of these models.

2 Nambu hierarchies in string models

In what follows we will use the formulation of the effective potential \[ V \] . To one loop order, this reads:

\[
V = V_{\text{tree}} + \frac{1}{32\pi^2} \left( \Lambda^2 \text{Str} M^2 - \frac{1}{2} \text{Str} M^4 \left[ \ln \frac{\Lambda^2}{M^2} + \frac{1}{2} + O\left(\frac{M^2}{\Lambda^2}\right) \right] \right),
\]

where

\[
\text{Str} F(M^2) = \sum_J (-1)^{2J}(2J + 1)F(M^2_J).
\]

We will assume that supersymmetry is broken in a hidden sector. This induces soft supersymmetry breaking terms of the order of the gravitino mass \( m_{3/2} \) in the observable sector of quarks and leptons. In the general case, \( \text{Str} M^2 \) is of the order of \( m_{3/2}^2 \). This yields dangerous terms in the potential of order \( m_{3/2}^2 \Lambda^2 \) which tend to destabilize the hierarchy \( m_{3/2}/M_{\text{Pl}} \).

In view of this, we will consider only the more realistic models where this leading contribution vanishes and therefore where \[ \text{Str} M^2 = O \left( \frac{m_{3/2}^4}{M_{\text{Pl}}^2} \right). \]

This is in some sense a weaker form of the Veltman condition for the models that we study. It yields quite generally a quadratic constraint on the Yukawa couplings.

This last statement requires some explanation since it is known that supersymmetry (possibly softly broken) gives mass relations which give rise to cancellations in \( \text{Str} M^2 \). We will take for the sake of illustration the following model, which is somewhat representative of what is found in string models. The field content is a dilaton \( S \), a modulus \( T \) and a set of \( n \) chiral superfields \( \Phi_i \). The couplings are described by a Kähler potential:

\[
K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T} - |\Phi|^2)
\]
where $|\Phi|^2 = \sum_i |\Phi_i|^2$, and a superpotential $W(\Phi_i)$ which is cubic in the matter fields $\Phi_i$.

Then global supersymmetry ensures automatic cancellation in $\text{Str} \, M^2$ of some of the terms depending on Yukawas but not of all of them. Indeed, let us write the terms of dimension 2 and 4 as respectively

$$W = \frac{1}{9} \frac{1}{(S + \bar{S})(T + T - |\Phi|^2)} \sum_{i,j=1}^N \left| \frac{d^2 W}{d\Phi_i d\Phi_j} \right|^2,$$

$$\mathring{V} = \frac{1}{3} \frac{1}{(S + \bar{S})(T + T - |\Phi|^2)^2} \sum_{i=1}^N \left| \frac{dW}{d\Phi_i} \right|^2.$$  \hfill (2.7)

They contribute to the scalar trace $\text{Tr} M^2_S$ and spinor trace $\text{Tr} M^2_F$ as follows:

$$\text{Tr} M^2_S = 2(5 + \frac{2N}{3}) \mathring{V} + 2W + \cdots$$

$$\text{Tr} M^2_F = \frac{14}{3} \mathring{V} + W + \cdots.$$  \hfill (2.8)

We thus check that the terms of order 2 cancel in the supertrace, as required by global supersymmetry:

$$\text{Str} \, M^2 = \frac{2}{3} (1 + 2N) \mathring{V} + \cdots$$  \hfill (2.9)

But there remain some terms of dimension 4 which depend on the Yukawa couplings: these terms are obviously of order $1/M_{Pl}^2$ and are therefore not constrained by global supersymmetry. For instance, in the case of the toy model that we will consider below,

$$W = \frac{1}{3} \lambda_1 |\Phi_1|^3 + \frac{1}{3} \lambda_2 |\Phi_2|^3$$

$$\mathring{V} = \hat{\lambda}_1^2 |\Phi_1|^4 + \hat{\lambda}_2^2 |\Phi_2|^4,$$

and $\mathring{V} = \hat{\lambda}_1^2 |\Phi_1|^4 + \hat{\lambda}_2^2 |\Phi_2|^4$, where $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are the low-energy Yukawa parameters:

$$\hat{\lambda}_i^2 = \frac{1}{2T} \frac{1}{S + \bar{S}} \lambda_i^2.$$  \hfill (2.10)

and the $\Phi_i$ fields have been rescaled to $\hat{\Phi}_i$ in order to have normalized kinetic terms.

There are of course other terms in $\text{Str} \, M^2$ which come from a hidden sector that we have not completely specified. Following our assumptions,
the contribution of these terms is at most of order \( m^{4/3}/M^{2} \). Any condition of the type of the Veltman condition will therefore yield

\[
\lambda_{1}^{2} |\Phi_{1}|^{4} + \lambda_{2}^{2} |\Phi_{2}|^{4} \sim m^{4/3}_{3/2}
\]  

(2.12)

and assuming \( <\Phi_{1}^{2}> \) and \( <\Phi_{2}^{2}> \) of the order of \( m^{2}_{3/2} \), we obtain a constraint of the form (1.1).

Assuming a cancellation of the leading terms in \( StrM^{2} \), one is left with the terms of order \( M^{4} \) in the effective potential (2.3) which we can write, up to a constant additive term:

\[
V = -\frac{1}{64\pi^{2}} StrM^{4} \left[ \log \frac{\Lambda^{2}}{M^{2}} + \frac{1}{2} + O \left( \frac{M^{2}}{\Lambda^{2}} \right) \right]
\]

\[
= -\frac{1}{64\pi^{2}} StrM^{4} \log \frac{\Lambda^{2}}{m^{2}_{3/2}} + \frac{1}{64\pi^{2}} StrM^{4} \left[ \log \frac{M^{2}}{m^{2}_{3/2}} - \frac{1}{2} + O \left( \frac{M^{2}}{\Lambda^{2}} \right) \right]
\]

\[
= -A StrM^{4} + O(\lambda_{n}^{i} \log \lambda_{i})
\]  

(2.13)

where \( A = \frac{1}{64\pi^{2}} \log(\Lambda^{2}/m^{2}_{3/2}) \) is by construction positive (\( \Lambda \gg m^{3/2} \)). Because of the quadratic constraint (2.12), \( StrM^{4} \) is quartic in the Yukawa couplings and we thus obtain a potential which has a behaviour very similar to (1.1), as advocated by Nambu in his toy model.

### 3 A toy model

We consider in this section a simple toy model which, we believe, includes the basic features of more realistic string models, in order to see how the Nambu mechanism works in these models. It is beyond the scope of this work to present realistic scenarios explaining for example why the top quark is much heavier than the bottom quark and the tau lepton or why the third family is heavier than the remaining two. But we will see from the study of our model to what extent supersymmetry and radiative corrections are important ingredients for generating a hierarchy of the Nambu type.

Our model contains two observable chiral superfields \( \Phi_{1} \) and \( \Phi_{2} \) whose low energy supersymmetric couplings are described by the superpotential:

\[
W = \frac{1}{3} \lambda_{1} \Phi_{1}^{3} + \frac{1}{3} \lambda_{2} \Phi_{2}^{3}
\]  

(3.14)
and an unspecified hidden sector where we assume that supersymmetry is broken spontaneously. We will avoid discussing the issue of supersymmetry breaking although this might play an important role in the realization of the Veltman condition.

In the observable sector under study here, supersymmetry breaking induces soft supersymmetry breaking terms whose scale is determined by the gravitino mass $m_{3/2}$. We will include here scalar mass terms of the form

$$V_{soft} = \mu_1^2 |z_1|^2 + \mu_2^2 |z_2|^2$$

where $z_1, z_2$ are the scalar components of $\Phi_1, \Phi_2$ respectively. In the usual scenarios of supersymmetry breaking, the scalar masses are universal in order to prevent flavour changing neutral currents and they are of the order of the gravitino mass. We will therefore assume

$$\mu_1 = \mu_2 = \mu = O(m_{3/2}).$$

The procedure we adopt goes as follows:

i) we compute the one-loop effective potential $V(z_1, z_2)$ and the corresponding saddle point equations which determine $v_1$ and $v_2$.

ii) we then obtain the vacuum energy $E_0(\mu, \lambda_1, \lambda_2, \Lambda)$ in this one-loop approximation.

iii) we write a Veltman type condition for the cancellation of quadratic divergences up to order $m_{3/2}^4 \Lambda^2 / M_P^2$. As discussed above and as we will return below, this might not be necessary for the mechanism to work. The condition acts as a quadratic constraint on the couplings $\lambda_1$ and $\lambda_2$.

iv) we minimize $E_0$ with respect to $\lambda_1$ and $\lambda_2$ using the constraint. Consequently, the couplings $\lambda_1$ and $\lambda_2$ are dynamically determined, and so are the fermion masses $m_1$ and $m_2$.

In all generality the soft supersymmetry breaking terms compatible with the $Z_3 \times Z_3$ symmetry of the observable sector present in our model include $\Lambda$-terms: $V' = \Lambda_1 \lambda_1 z_1^3 + \Lambda_2 \lambda_2 z_2^3 + h.c.$ For the sake of simplicity and in order to be able to present analytic computations, we will not include them in the discussion.
Assuming the presence of a dilaton $S$ and at least one modulus $T$ coupled to the matter superfields $\Phi_i$ through a Kähler potential of the type \(2.6\), we find at the ground state a supertrace of $M^2$ which reads, in the absence of soft supersymmetry breaking terms,

\[
\text{Str}M^2 = \frac{\alpha}{M_{Pl}^2}(\lambda_1^2|v_1|^4 + \lambda_2^2|v_2|^4) + \beta R + m_{3/2}^2 \sum_{n=1}^{\infty} \gamma_n \left(\frac{m_{3/2}}{M_{Pl}}\right)^{2n}.
\]

(3.18)

The origin of the first term was discussed in the previous section.\(^2\)

The second term appears when one considers a background with nonzero curvature $R$. Finally, the last term corresponds to the contribution of the hidden sector, proportional to $m_{3/2}^2$. As discussed above, the remaining fields in the model are chosen in order that the dangerous contribution of order $m_{3/2}^4$ in Str$M^2$ automatically cancels.\(^3\)

We will assume that the supertrace cancels to order $m_{3/2}^4$, i.e.

\[
\frac{\alpha}{M_{Pl}^2}(\lambda_1^2|v_1|^4 + \lambda_2^2|v_2|^4) + \beta R + \gamma_1 \frac{m_{3/2}^4}{M_{Pl}^2} = 0
\]

(3.19)

where the possibly non-vanishing background curvature is automatically of order $|v_i|^4 \sim m_{3/2}^4$ ($R = 4 < V > /M_{Pl}^2$). This yields a constraint of the type (1.1) for the Yukawa couplings. As will become clear in what follows, a strict cancellation as in (3.19) is not necessary: it is sufficient for the hierarchy to be generated that the left-hand side be bounded by a quantity of order $m_{3/2}^4/M_{Pl}^2$.

The tree level scalar potential in the observable sector simply reads:

\[
V_0 = \mu^2(|\Phi_1|^2 + |\Phi_2|^2) + \lambda_1^2|\Phi_1|^4 + \lambda_2^2|\Phi_2|^4
\]

(3.20)

and the field vevs are given by (3.17). It follows that the vacuum energy $E_0$, using the condition (3.19), is proportional to $m_{3/2}^4$. As any scale in a string model, $m_{3/2}$ is determined dynamically through the vev of some scalar field, say the dilaton $S$. It is an important aspect of the strategy that we adopt here to assume that the minimisation procedure that leads to the determination of $m_{3/2}$ is independent of the minimisation with respect

\(^2\)The couplings $\lambda_i$ used here are actually the rescaled couplings $\hat{\lambda}_i$ of eq.(2.11). Similarly, the fields $\Phi_i$ whose vev is $v_i$ are in fact the rescaled fields $\hat{\Phi}_i$ with normalized kinetic terms.

\(^3\)Ferrara, Kounnas and Zwirner are presently constructing such string models where the moduli sector of the theory is chosen precisely in order to cancel this term [12].
to the Yukawa couplings. We therefore consider $m_{3/2}$ as an independent constant. There is then no dependence at tree level of $\mathcal{E}_0$ on the couplings $\lambda_1$ and $\lambda_2$ and we need to go to the one-loop level.

At one loop, we use the effective potential of eq.(2.3) where the cut-off $\Lambda$ is taken to be of order $M_{Pl}$. The supertraces of $M^2$ and $M^4$ read:

$$
\text{Str}M^2 = 4\mu^2 + \frac{1}{M_{Pl}^2}(\lambda_1^2|\Phi_1|^4 + \lambda_2^2|\Phi_2|^4) + \gamma m_{3/2}^2 + O\left(\frac{m_{3/2}^6}{M_{Pl}^4}\right),
$$

$$
\text{Str}M^4 = 4\mu^4 + 16\mu^2(\lambda_1^2|\Phi_1|^2 + \lambda_2^2|\Phi_2|^2) + 8(\lambda_1^4|\Phi_1|^4 + \lambda_2^4|\Phi_2|^4)
+ O\left(\Phi^6/M_{Pl}^2\right),
$$

(3.21)

Due to the presence of the quadratic divergence $\Lambda^2$, the terms of order $1/M_{Pl}^2$ in $\text{Str}M^2$, and only them in leading order, will contribute to the scalar potential at low energy. Neglecting for the moment terms logarithmic in the Yukawa couplings, we obtain for the scalar potential to order one-loop

$$
V = C + \mu^2 \left(1 - \frac{\lambda_1^2}{4\pi^2} \log \frac{\Lambda^2}{\mu_0^2}\right)|\Phi_1|^2 + \mu^2 \left(1 - \frac{\lambda_2^2}{4\pi^2} \log \frac{\Lambda^2}{\mu_0^2}\right)|\Phi_2|^2
$$

$$
+ \lambda_1^2 \left(\rho - \frac{\lambda_1^2}{8\pi^2} \log \frac{\Lambda^2}{\mu_0^2}\right)|\Phi_1|^4 + \lambda_2^2 \left(\rho - \frac{\lambda_2^2}{8\pi^2} \log \frac{\Lambda^2}{\mu_0^2}\right)|\Phi_2|^4
$$

$$
+ O(\lambda_i^4 \log \lambda_i)
$$

(3.22)

where $C$ is a constant – i.e. independent of $\lambda_i$ and $\Phi_i - \mu_0$ is some low energy renormalisation scale and $\rho = 1 + (\Lambda^2/M_{Pl}^2)/(32\pi^2) > 1$. The expression (3.22) is symmetric under the exchange $(\lambda_1, \Phi_1 \leftrightarrow \lambda_2, \Phi_2)$.

We will use the notations:

$$
x_i = \frac{\lambda_i^2}{8\pi^2} \log \frac{\Lambda^2}{\mu_0^2}, \quad \xi_i = \frac{1}{x_i} \left(1 - \frac{2x_i}{\rho - x_i}\right)^2, \quad i = 1, 2.
$$

(3.23)

In this case the vevs $v_1$ and $v_2$ are given by

$$
v_i^2 = -\frac{\mu^2 \log(\Lambda^2/\mu_0^2)}{16\pi^2 x_i} \frac{1 - 2x_i}{\rho - x_i} = -\frac{\mu^2 \log(\Lambda^2/\mu_0^2)}{16\pi^2} \left(\frac{\xi_i}{x_i}\right)^{1/2}
$$

(3.24)

and the vacuum energy dependence reads (neglecting an additive numerical constant)

$$
\mathcal{E}_0 = -M^4[\xi_1(\rho - x_1) + \xi_2(\rho - x_2)]
$$

(3.25)
where $M^4 = \mu^4 \log(\Lambda^2/\mu_0^2)/(32\pi^2)$.

In these notations, the condition (2.5) reads

$$\xi_1 + \xi_2 = \tilde{a}. \quad (3.26)$$

This condition can be used to eliminate one variable and write the effective potential as a function of $\xi_1$ only ($x_1$ and $x_2$ being understood as implicit functions of $\xi_1$ through (3.23). We will then show that $E_0(\xi_1)$ has a single extremum corresponding to the symmetric solution $\xi_1 = \xi_2$. This extremum being a maximum as we will see, the minimum value of $E_0$ is rejected to the boundary values for $\xi_1$, i.e. $\xi_1 = \tilde{a}$ ($\xi_2 = 0$) or $\xi_1 = 0$ ($\xi_2 = \tilde{a}$).

Starting with

$$\frac{dE_0}{d\xi_1} = -M^4 \left( x_2 - x_1 - \xi_1 \frac{\partial x_1}{\partial \xi_1} - \xi_2 \frac{\partial x_2}{\partial \xi_1} \right) \quad (3.27)$$

and computing $\partial \xi_1/\partial x_i$ ($i = 1, 2$), one finds that

$$\frac{dE_0}{d\xi_1} = -(x_2 - x_1) f(x_1, x_2) \quad (3.28)$$

where $f(x_1, x_2)$ is strictly positive for $x_i \leq 1/2$. If $x_i \geq 1/2$ the only solution of the saddle point equations is $v_1 = v_2 = 0$ so the equations above no longer hold.

Having proved that the only extremum of the effective potential corresponds to $x_1 = x_2$, i.e. to $\xi_1 = \xi_2 = \tilde{a}/2$, the simplest way to proceed is to compare the energy of this solution with the energy of another solution of the quadratic constraint (3.26), for example $\xi_1 = 0, \xi_2 = \tilde{a}$ (or vice-versa).

In the symmetric case ($\xi_1 = \xi_2 = \tilde{a}/2$ hence $x_1 = x_2 \equiv x_{\text{sym}}$), one finds from (3.23)

$$E_0|_{\text{sym}} = -M^4 \tilde{a}(\rho - x_{\text{sym}}). \quad (3.29)$$

In the asymmetric case (say $\xi_1 = 0, \xi_2 = \tilde{a}$), one finds

$$E_0|_{\text{asym}} = -M^4 \tilde{a}(\rho - x_{\text{asym}}) \quad (3.30)$$

where $x_{\text{asym}}$ is the corresponding value of $x_2$. It is easy to prove that $x_{\text{sym}} > x_{\text{asym}}$. Hence $E_0|_{\text{asym}} < E_0|_{\text{sym}}$ and the single extremum is a maximum. It follows that $E_0$ is ever decreasing from its maximum value $E_0|_{\text{sym}}$: its minimum is reached at the boundary values for $\xi_1$, which corresponds precisely to $E_0|_{\text{asym}}$.
The last step consists in including the terms of order $\lambda_i^n \log \lambda_i^2$ which were discarded until now. In the toy model considered by Nambu, it is precisely these terms with $n = 2$ which generate a non-zero value for the Yukawa coupling which was zero in the previous approximation. It is easy to check that, keeping these logarithmic terms as in the second of eqs. (2.13), one generates terms of order $\xi_i \log \xi_i$, for $\xi_i \ll 1$, which are precisely the ones needed to generate a large hierarchy among the two Yukawa couplings.

This toy model can easily be generated to the case of $N$ Yukawa couplings. An interesting property of the Nambu mechanism is that in this more general situation, one Yukawa coupling is naturally much larger than all the remaining ones. In the simplest version of the model, the latter Yukawas are equal but this degeneracy can easily be lifted with intergenerational couplings.

4 Conclusion

We have studied how the mechanism proposed by Nambu to generate a hierarchy between Yukawa couplings may be naturally implemented in superstring low energy models.

Unlike the case of the standard model, it is natural in these models to consider that Yukawa couplings are dynamical variables and the corresponding minimisation amounts to a minimisation with respect to the moduli of the underlying theory. Also, already at one loop, the framework of softly broken supersymmetric theories makes it easier to generate the right terms with the right signs. We stressed above that the key ingredient for the success of the Nambu proposal in these models is the cancellation of leading terms of order $m_{3/2}^2$ in $StrM^2$. Then this supertrace is necessarily of order $m_{3/2}^4/M_{Pl}^2$. It remains of course to be seen how such a constraint which is crucial for the success of the mechanism may be dynamically generated.

In our work, we have refrained from applying these ideas to a realistic situation. Our goal was to study how the Nambu proposal can be realized in a string low energy model. This is also the reason why we have considered solely the generation of a hierarchy among the Yukawa couplings. However, we have seen above several times that this leads to some other important issues in these models: supersymmetry breaking, the generation of a hierarchy between $M_W$ and $M_{Pl}$ (or almost equivalently $m_{3/2}$ and $M_{Pl}$), the
problem of a cosmological constant. For instance, our constraint (2.12) most certainly arises from a minimisation process that we have avoided to discuss here. In the same line of thought, our potential depends on an overall scale – say $m_{3/2}$ – which has to be determined dynamically. It must in particular be checked that such a determination, in conjunction with our minimisation procedure, does not lead to an instability of the theory. The whole approach might for example be included into the ambitious program undertaken by Kounnas, Pavel and Zwirner [13] who try to determine dynamically $m_{3/2}$ and the top mass in a class of string models where precisely $StrM^2$ cancels to leading order in $m_{3/2}^2$.

Our own work is intended as a first look at the Nambu mechanism by itself but it is clear that such issues as the ones listed above will now have to be addressed in order to make it a candidate for explaining hierarchies in the observed spectrum. For example, in the realisation of the Nambu idea, the key ingredient is the realisation of the quadratic constraint among the Yukawa couplings. We have only shown here that such a constraint is consistent with what is known of the string models which are compatible with the low energy phenomenology. We certainly do not feel that we have a definite answer as to the origin of such a constraint. And much progress remains to be done before a realistic scenario can be proposed.

Acknowledgments.

We wish to thank T. Gherghetta, M. Peskin and F. Zwirner for valuable discussions.
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