Single-event Grey Target Decision Model with Vectorial Positive and Negative Bullseyes

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Abstract. Considering actual multicriteria decision-making problem can be decomposed into multiple single event grey target decisions, a novel structure of the grey target decision of single event is proposed. In order to reduce the dimensionality, index effect evaluation values are assembled as objective effect evaluation value, which simplifies the complex decision-making problem by eliminating the index layer. Vectorial positive and negative bullseyes are defined followed by the definition of comprehensive off-target distance, which integrates the positive or negative off-target distance of the decision with the projection on the line through positive and negative bullseyes. Finally, an example for selection of green product supplier is studied to verify the effectiveness and feasibility of the single-event grey target decision model based on vector-type positive and negative bullseyes.

Keywords: Grey decision; Vector bullseye; Comprehensive off-target distance.

1. Introduction
As one of the most important research domains to solve multi-index decision-making problem, studies on grey target decision have become a hot topic in social and economic activities in recent years[1-3]. And extensive researches on how to acquire sufficient decision information to solve the optimal decision with multiple attributes are made too [4-6]. However, the actual system is often a single event multi-objective multi-index decision-making system [7]. Therefore, research on the single-event grey target decision with multiple objectives and multiple indices has important theoretical and practical significance. Aiming at the single event decision with double layers indicators, i.e. objective layer and index layer, we defined a decision model based on positive bullseye and negative one [8,9]. Based on these two bullseyes, we defined the positive off-target distance and the negative one, each of which consists of effect evaluation values of objectives.

2. Methodologies
In Cartesian product of the finite event set \( A = \{a_1, a_2, \ldots\} \) and the finite countermeasure set \( B = \{b_1, b_2, \ldots\} \) is the finite decision set \( S = A \times B = \{s_1, s_2, \ldots\} \). For any decision \( s_i \in S \), written \( s_i \mapsto u^{bi}_i(\otimes), u^{ii}_i(\otimes) \in [u^{bi}_i, \bar{u}^{bi}_i] \quad (k = 1,2,\ldots, n, j = 1,2,\ldots,t_k) \) is defined as effect evaluation value.
under the \( j \)th index of objective \( k \), \( u_{kj}^{bj} \) is the lower limit of effect evaluation value, and \( \bar{u}_{kj}^{bj} \) is the upper bound of effect evaluation value.

For the single event set \( A=\{a_i\} \) the countermeasure set \( B \) has the same number, say, \( m \), of elements as the decision set \( S \). For decisions \( s_i \{i=1,2,\cdots,m\} \), effect sample matrix under indices range from 1 to \( t_k \) of the objective \( k \) \((k=1,2,\cdots,n)\) is

\[
U^{bj} = \begin{bmatrix}
\underbrace{u_1^{k1}(\ominus)} & u_1^{k2}(\ominus) & \cdots & u_1^{kt}(\ominus) \\
\underbrace{u_2^{k1}(\ominus)} & u_2^{k2}(\ominus) & \cdots & u_2^{kt}(\ominus) \\
\cdots & \cdots & \cdots & \cdots \\
\underbrace{u_m^{k1}(\ominus)} & u_m^{k2}(\ominus) & \cdots & u_m^{kt}(\ominus)
\end{bmatrix}
\]

The weight vector \( \omega=(\omega_1,\omega_2,\cdots,\omega_n) \) can be got by setting different weights for different objectives. As well, the weight vector \( \omega_k=(\omega_{k1},\omega_{k2},\cdots,\omega_{kn}) \) can be obtained by setting different weight for different index of the objective \( k \).

**Definition 1.** Given a benefit-style effect sample value \( u_{kj}^{bj}(\ominus)\in[\bar{u}_{kj}^{bj},\underbar{u}_{kj}^{bj}] \), let \( r_k^{bj}=\min_{1\leq j\leq m}u_{kj}^{bj} \) and \( \bar{r}_k^{bj}=\max_{1\leq j\leq m}u_{kj}^{bj} \). Then, \( r_k^{bj}(\ominus)\in[\bar{u}_{kj}^{bj},\underbar{u}_{kj}^{bj}] \) is called lower bound of benefit-style effect measure value; \( \bar{r}_k^{bj}=(\bar{u}_{kj}^{bj}-\underbar{u}_{kj}^{bj})/(\underbar{u}_{kj}^{bj}-\bar{r}_k^{bj}) \) is called upper bound of benefit-style effect measure value, and \( r_k^{bj}(\ominus)\in[\bar{r}_k^{bj},\underbar{r}_k^{bj}] \) is called benefit-style effect measure function. For the cost-style effect sample value.

**Definition 2.** Assume \( u_{kj}^{bj}(\ominus),\bar{r}_k^{bj} \) and \( \underbar{r}_k^{bj} \) be the same with those in Definition 1, but \( u_{kj}^{bj}(\ominus) \) is a cost-style effect sample value. Then, \( \underbar{r}_k^{bj} \) is called lower limit of cost-style effect measure value, \( \bar{r}_k^{bj} \) is called upper bound of cost-style effect measure value, and \( r_k^{bj}(\ominus)\in[\underbar{r}_k^{bj},\bar{r}_k^{bj}] \) is called cost-style effect measure function.

\[
R^{bj} = \begin{bmatrix}
\underbrace{r_1^{k1}(\ominus)} & r_1^{k2}(\ominus) & \cdots & r_1^{kt}(\ominus) \\
\underbrace{r_2^{k1}(\ominus)} & r_2^{k2}(\ominus) & \cdots & r_2^{kt}(\ominus) \\
\cdots & \cdots & \cdots & \cdots \\
\underbrace{r_m^{k1}(\ominus)} & r_m^{k2}(\ominus) & \cdots & r_m^{kt}(\ominus)
\end{bmatrix} = \begin{bmatrix}
[\underbar{r}_k^{bj},\underbar{r}_k^{bj}] & [\underbar{r}_k^{bj},\underbar{r}_k^{bj}] & \cdots & [\underbar{r}_k^{bj},\underbar{r}_k^{bj}] \\
\underbar{r}_k^{bj} & \underbar{r}_k^{bj} & \cdots & \underbar{r}_k^{bj} \\
\cdots & \cdots & \cdots & \cdots \\
\underbar{r}_k^{bj} & \underbar{r}_k^{bj} & \cdots & \underbar{r}_k^{bj}
\end{bmatrix}
\]

**Table 1.** Structure of single event decision with multiple objectives and indices

| De-   |          |          |          |          |
|------|----------|----------|----------|----------|
| _cisions | objective 1 \( \omega_1 \) | Objective 2 \( \omega_2 \) | \cdots | objective n \( \omega_n \) |
| \( s_1 \) | \( u_{11}^{(\ominus)} \) | \( u_{21}^{(\ominus)} \) | \( \cdots \) | \( u_{n1}^{(\ominus)} \) |
| \( s_2 \) | \( u_{12}^{(\ominus)} \) | \( u_{22}^{(\ominus)} \) | \( \cdots \) | \( u_{n2}^{(\ominus)} \) |
| \( \cdots \) | \( \cdots \) | \( \cdots \) | \( \cdots \) | \( \cdots \) |
| \( s_m \) | \( u_{1m}^{(\ominus)} \) | \( u_{2m}^{(\ominus)} \) | \( \cdots \) | \( u_{nm}^{(\ominus)} \) |

**Table 1.** Structure of single event decision with multiple objectives and indices
where, \( m \) is the number of decisions which include \( s_1, s_2, \cdots, s_m \), \( t_k \) is the total number of indices belonging to objective \( k \).

3. Decision Model

3.1. Vectorial Positive and Negative Bullseyes Model

The vector \( \omega_k = (\omega_{k1}, \omega_{k2}, \cdots, \omega_{kt_k}) \) is the index weight vector of objective \( k \), which satisfies \( \sum_{i=1}^{t_k} \omega_{ki} = 1, 0 \leq \omega_{ki} \leq 1 \). Then for decision \( s_i \), comprehensive effect evaluation value of objective \( k \) is \( r_i^k(\otimes) \), which meets

\[
r_i^k(\otimes) = \sum_{j=1}^{t_k} w_{kj} r_j^k(\otimes) = w_{k1} r_1^k(\otimes) + w_{k2} r_2^k(\otimes) + \cdots + w_{kt_k} r_{t_k}^k(\otimes)
\]

Further, get uniform effect measure matrix of the decision set \( S = \{s_1, s_2, \cdots, s_m\} \) as follows:

\[
R = [R^1, R^2, \cdots, R^n]^T = \begin{bmatrix}
  r_1^1(\otimes) & r_1^2(\otimes) & \cdots & r_1^n(\otimes) \\
  r_2^1(\otimes) & r_2^2(\otimes) & \cdots & r_2^n(\otimes) \\
  \vdots & \vdots & \ddots & \vdots \\
  r_n^1(\otimes) & r_n^2(\otimes) & \cdots & r_n^n(\otimes)
\end{bmatrix} = \begin{bmatrix}
  [r_1^1, r_1^2, \cdots, r_1^n] \\
  [r_2^1, r_2^2, \cdots, r_2^n] \\
  \vdots \\
  [r_n^1, r_n^2, \cdots, r_n^n]
\end{bmatrix}
\]

If index weight vector is known, the Eq. (2) can be used to calculate the uniform measure matrix, otherwise, we need to obtain weights through such methods as subjective weight method, objective weight method or combination weighting method, not repeated here.

3.2. Vectorial Bullseyes and Off-target Distance

**Definition 5.** There is \( i_0 \in \{1, 2, \cdots, m\} \) that makes \( (\bar{L}^k_{i_0} + \bar{R}^k_{i_0}) / 2 \leq \max \{(\bar{L}^k_i + \bar{R}^k_i) / 2 | 1 \leq i \leq m\} \), \( k \in \{1, 2, \cdots, n\} \). Its corresponding decision value is \( r^+_i(\otimes) = r^+_i(\otimes) = [\bar{L}^k_{i_0}, \bar{L}^k_{i_0}, \bar{R}^k_{i_0}, \bar{R}^k_{i_0}] \). Here,

\[
r^+ = \{r^+_1(\otimes), r^+_2(\otimes), \cdots, r^+_n(\otimes)\} = \{[\bar{L}^1_{i_0}, \bar{L}^1_{i_0}, \bar{R}^1_{i_0}, \bar{R}^1_{i_0}], [\bar{L}^2_{i_0}, \bar{L}^2_{i_0}, \bar{R}^2_{i_0} \cdots, [\bar{L}^n, \bar{L}^n, \bar{R}^n, \bar{R}^n]\}
\]

is called optimal effect vector, and is defined as positive bullseye.

**Definition 6.** There is \( i'_0 \in \{1, 2, \cdots, m\} \) that makes \( (\bar{L}^k_{i'_0} + \bar{R}^k_{i'_0}) / 2 = \max \{(\bar{L}^k_i + \bar{R}^k_i) / 2 | 1 \leq i \leq m\} \), \( k \in \{1, 2, \cdots, n\} \). Its corresponding decision value is \( r^-_i(\otimes) = r^-_i(\otimes) = [\bar{L}^k_{i'_0}, \bar{L}^k_{i'_0}, \bar{R}^k_{i'_0}, \bar{R}^k_{i'_0}] \). Here,

\[
r^- = \{r^-_1(\otimes), r^-_2(\otimes), \cdots, r^-_n(\otimes)\} = \{[\bar{L}^1_{i'_0}, \bar{L}^1_{i'_0}, \bar{R}^1_{i'_0}, \bar{R}^1_{i'_0}], [\bar{L}^2_{i'_0}, \bar{L}^2_{i'_0}, \bar{R}^2_{i'_0}, \bar{R}^2_{i'_0}], \cdots, [\bar{L}^n_{i'_0}, \bar{L}^n_{i'_0}, \bar{R}^n_{i'_0}, \bar{R}^n_{i'_0}]\}
\]

is called worst effect vector, and is defined as negative bullseye.

**Definition 7.** For the known objective weight vector \( \omega = (\omega_1, \omega_2, \cdots, \omega_n) \),

\[
d^+_i = \sqrt{\frac{\omega_1 ([\bar{L}^i - \bar{L}^i_0]^2 + (\bar{R}^i - \bar{R}^i_0)^2] + \cdots + \omega_n ([\bar{L}^i - \bar{L}^i_0]^2 + (\bar{R}^i - \bar{R}^i_0)^2]}{2}}
\]

is called positive off-target distance of the effect evaluation vector \( R^i = [r^+_i(\otimes), r^+_i(\otimes), \cdots, r^+_i(\otimes)] \) on \( n \) objectives of the decision \( s_i \). And likewise,

\[
d^-_i = \sqrt{\frac{\omega_1 ([\bar{L}^i - \bar{L}^i_0]^2 + (\bar{R}^i - \bar{R}^i_0)^2] + \cdots + \omega_n ([\bar{L}^i - \bar{L}^i_0]^2 + (\bar{R}^i - \bar{R}^i_0)^2]}{2}}
\]

is called
negative off-target distance of the effect evaluation vector 

\[ R^i = [r^i_1(\Diamond), r^i_2(\Diamond), \ldots r^i_n(\Diamond)] \]

on \( n \) objectives of the decision \( s_j \).

3.3. Algorithm

![Figure 1. Comprehensive off-target distance](image)

In summary, the algorithm of single-event grey target decision model based on vectorial positive and negative bullseyes is as follows:

Step 1: Construct decision set \( S = A \times B = \{s_i = (a_i, b_i) \mid \|h_i \in B\} = \{s_1, s_2, \ldots, s_m\} \).

Step 2: Determine objectives and indices of each of them.

Step 3: Calculate uniform effect measure matrix under \( t_k \) indices of objective \( k \in \{1, 2, \ldots, n\} \) for \( m \) decisions of \( s_1, s_2, \ldots, s_m \).

Step 4: Synthesizing all indices and using objective weight method[10], we can get weights \( \omega_k(k \in 1, 2, \ldots, n; j = 1, 2, \ldots, tk) \), then get uniform effect measure matrix of objectives \( k = 1, 2, \ldots, n \) for decisions \( s_1, s_2, \ldots, s_m \) can be obtained.

Step 5: Solve objective weight vector \( \omega = (\omega_1, \omega_2, \ldots, \omega_n) \) of single objective optimization.

Step 6: Work out the positive bullseye \( r^+ \) and the negative bullseye \( r^- \).

Step 7: Calculate the positive and negative off-target distances \( R^i = [r^i_1(\Diamond), r^i_2(\Diamond), \ldots r^i_n(\Diamond)] \) and work out the distance \( d_0 \) between the positive bullseye \( r^+ \) and the negative one \( r^- \).

Step 8: Obtain values of the comprehensive off-target distances \( d_i = (d_i^+ + d_i^-) / (d_i^+ + d_i^-) \)

Where, \( d_i^+ = d_0 \cos \theta = (d_0^2 + d_0^2 - (d_0^-)^2) / 2d_0 \), as shown in Figure 1.

Step 9: Solve the optimal decision \( s_o \) and the optimal countermeasure \( b_o \) by searching for \( d_0 = \min\{d_1^0, d_2^0, \ldots, d_m^0\} \).

4. Example

A coal mining enterprise plans to purchase a kind of mining equipment. It needs to select the best supplier from the 4 initially determined suppliers of \( s_1, s_2, s_3, s_4 \). The decision of supplier selection is transformed into the problem of optimal choice on the basis of full investigation of expert opinions. In papers [11], 4 objectives, associated indices and their detailed connotations are given, including product competitiveness, enterprise competitiveness, green degree and cooperation or support.

Calculate the uniform effect measure matrices of 4 objectives through Eqn. (1) in section II. Fill in Table 2 and 3 with the results. According to Eqn. (4) and Eqn. (5) in section III, the positive bullseye
\( r^+ \) = \{(0.596, 0.798), (0.878, 0.965), (0.627, 0.930), (0.702, 0.902)\}, and the negative one \( r^- \) = \{(0.067, 0.269), (0.136, 0.223), (0.000, 0.278), (0.188, 0.395)\}.

For all objectives of decision \( s_i \), determine the positive offtarget distance and the negative one of the effect evaluation vector \( R = [r^+_i(\otimes), r^-_i(\otimes)] \), then, figure out the distance between them, and work out the comprehensive off-target distances \( d^0_i(i = 1, 2, 4, 5) \) and compare the good or bad of decisions according to numerical magnitudes of \( d^0_i \). As a result, \( d^0_2 > d^0_3 > d^0_4 > d^0_1 \). It means that decision 2 is optimal, decision 3 is suboptimal and decision 4 is the worst. In contrast with the result provided by paper [12] which uses the same raw data with us, we find that its result is fully consistent with ours obtained.

### Table 2. Uniform effect measure matrix of supplier decisions under different indices in objective 1 and 2

| supplier (decision) | objective 1-product competitiveness(\( \omega_1 \)) | objective 2-enterprise competitiveness(\( \omega_2 \)) |
|---------------------|-----------------------------------------------|--------------------------------------------------|
| price/\( \omega_11=0.40 \) | quality/%\( \omega_12=0.3060 \) | performance/%\( \omega_13=0.2908 \) | quality of employees/%\( \omega_21=0.3136 \) | financial status/%\( \omega_22=0.4576 \) | technical level/point\( \omega_23=0.1569 \) | management level/point\( \omega_24=0.0719 \) |
| \( S_1(s_1) \) | \[0.500,1.00\] | 0.300 | 0.333 | 0.000 | \[0.118,0.500\] | \[0.188,0.500\] |
| \( S_2(s_2) \) | \[0.000,0.500\] | 1.000 | 0.889 | 1.000 | \[0.589,1.000\] | \[0.688,1.000\] |
| \( S_3(s_3) \) | \[0.333,0.833\] | 0.700 | 1.000 | 0.500 | \[0.529,0.765\] | \[0.250,0.563\] |
| \( S_4(s_4) \) | \[0.167,0.667\] | 0.000 | 0.000 | 0.750 | \[0.000,0.353\] | \[0.000,0.313\] |

### Table 3. Uniform effect measure matrix of supplier decisions under different indices in objective 3 and 4

| supplier (decision) | objective 3-green degree(\( \omega_3 \)) | objective 2-enterprise competitiveness(\( \omega_2 \)) |
|---------------------|-----------------------------------------------|--------------------------------------------------|
| energy consumption /point\( \omega_31=0.3663 \) | resource recovery & utilization%/\( \omega_32=0.3484 \) | environment impact /point\( \omega_33=0.2853 \) | strategic cooperation /point\( \omega_34=0.2387 \) | business reputation /point\( \omega_41=0.2321 \) | delivery%/\( \omega_42=0.4556 \) | customer satisfaction /point\( \omega_43=0.2232 \) |
| \( S_1(s_1) \) | \[0.000,0.500\] | 0.000 | \[0.000,0.333\] | \[0.000,0.353\] | \[0.071,0.5171\] | 0.500 | \[0.000,0.357\] |
| \( S_2(s_2) \) | \[0.500,0.000\] | 1.000 | \[0.333,1.000\] | \[0.294,0.588\] | \[0.786,1.000\] | 1.000 | \[0.500,1.000\] |
| \( S_3(s_3) \) | \[0.250,0.750\] | 2.500 | \[0.000,0.667\] | \[0.588,0.824\] | \[0.000,0.429\] | 0.000 | \[0.214,0.571\] |
| \( S_4(s_4) \) | \[0.500,0.750\] | 0.083 | \[0.333,0.667\] | \[0.765,1.000\] | \[0.071,0.500\] | 0.500 | \[0.143,0.500\] |

### 5. Conclusions

The paper presents a single event grey decision model with vectorial positive and negative bullseyes, and puts forward a comprehensive off-target distance containing direction information of the objective effect evaluation vector. Through the integration of the index layer, the double layer decision is transformed into single layer decision with objective layer left only and index layer disappeared. The comprehensive off-target distance takes into full account the two bullseyes’ information. Through adding the projection of positive off-target distance, discrimination accuracy of the decision-making system has been greatly improved.
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