Unidirectional surface plasmons in nonreciprocal graphene

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Abstract. We demonstrate theoretically the existence of unidirectional surface plasmons in the nonreciprocal graphene-based gyrotropic interfaces. We show that a unidirectional frequency range is raised under a static external magnetic field where only one propagating direction is allowed for the surface plasmons mode. By efficiently controlling the chemical potential of graphene, the unidirectional working frequency can be continuously tunable from THz to near-infrared and even visible. Particularly, the unidirectional frequency bandwidth can be 1–2 orders of magnitude larger than that in metal under the same magnetic field, which arises from the superiority of extremely small effective electron mass in graphene. Based on our theoretical analysis, two tunable graphene-based directional devices are proposed, showing the appealing properties of nonreciprocal graphene in the nonreciprocal optical devices design.

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Surface plasmons originate from the collective excitations of electrons coupled to the electromagnetic field at the interface between dielectric and metal [1]. Their superb capability of confining electromagnetic energy at sub-wavelength scales [1] makes it possible to squeeze down current photonic components to ultracompact nanometer dimensions [1–7]. In particular, surface plasmons have been shown theoretically and experimentally to possess nonreciprocal properties in the presence of an external static magnetic field [8–15] similar to the chiral edge states of electrons in the quantum Hall effect [16–18]. For example, the heterostructure composed of magnetized metal film and two-dimensional (2D) photonic crystal [8, 9] possesses a unidirectional frequency range where only a forward propagating mode is allowed, with neither radiation nor backward modes, being immune from defects [8–10]. However, the previously observed nonreciprocity was generally weak and required a very large magnetic field. The design of unidirectional surface plasmons with a large frequency bandwidth under a modest magnetic field, hence, is in great demand for future applications in photonic isolators, diodes and logic circuits.

Graphene [16–18], as a promising one-atom-thick photonic material where the doped electrons can be regarded as 2D electron gases (2DEGs), can support surface plasmons with widely tunable frequencies and reduced losses [19–24]. Recent advances in fabrication have enabled rapid progress in graphene plasmonics [25–34]. The photon–electron coupling in graphene is remarkably strong compared with that in metals (e.g. silver and gold) and 2DEGs in conventional semiconductors [19]. It mainly arises from both the extremely small effective electron mass at Fermi energy (i.e. cyclotron effective mass) $m_{\text{eff}}$ [16–19, 35] and the efficiently tunable electrical gating in graphene [19]. In conventional 2DEGs, the field induced 2D carrier density $n_s$ is limited to $\sim 1 \times 10^{12}\text{ cm}^{-2}$ to avoid semiconductor dielectric breakdown [19]. However, $n_s$ in graphene can be tuned by doping or gating from very low ($10^{10}\text{ cm}^{-2}$) to very high values ($10^{14}\text{ cm}^{-2}$) [19, 20, 36, 37]. Furthermore, $m_{\text{eff}}$ in graphene ($m_{\text{eff}} = 0.02m_0$ at $n_s = 1 \times 10^{12}\text{ cm}^{-2}$, $m_0$ is the mass of free electron [19]) is about two orders of magnitude smaller than that of free electron in metal, and thus the integrated plasmon oscillator strength is correspondingly much larger [19]. These appealing properties of graphene have motivated extensive investigations in the emerging research field of graphene plasmonic and metamaterial devices [25–34], including optical sensors [38], photodetectors [39, 40], modulators [24, 41–43], waveguides [21, 22], antenna [44], lens [45, 46], Mach–Zehnder interferometers [47], cloaks [48], etc. Recently, significant experimental progress on unidirectional edge magnetoplasmons in a single graphene sheet was reported [33]. However, theoretical investigations on unidirectional surface plasmons in nonreciprocal graphene have been rarely discussed.

In this paper, we present a theoretical study of nonreciprocal graphene and its applications to unidirectional surface plasmons. An externally applied static magnetic field $B$, which is assumed in the $+z$ direction and perpendicular to the graphene plane ($x$–$y$ plane), breaks the time-reversal symmetry in graphene and makes graphene gyrotropic. Two feasible waveguide structures, air/graphene single interface and air/graphene/dielectric double interfaces, are used for demonstration. The unidirectional frequency bandwidth is shown proportional to $1/m_{\text{eff}}$, where $m_{\text{eff}}$ is dependent on chemical potential $\mu_c$ in graphene. Hence, we show that even under a modest magnetic field ($|B| = 0.1\text{ T}$), graphene with small $m_{\text{eff}}$ is proven capable of making the unidirectional frequency bandwidth very large. Moreover, by controlling $\mu_c$, the unidirectional working frequency is widely tunable from terahertz (THz) to visible. Based on theoretical analysis, we proposed magnetic-field-controlled directional optical
components, indicating that nonreciprocal graphene can be a good candidate for future photonic device applications.

We start by analyzing graphene’s optical properties. The complex optical conductivity in graphene \((G = G_{\text{intra}} + G_{\text{inter}})\), where \(G_{\text{intra}}\) and \(G_{\text{inter}}\) are attributed to intra-band and inter-band transitions, respectively [49, 50], can be modeled by the Kubo formula [21, 48–50],

\[
G_{\text{intra}}(\omega, \mu_c, \tau, T) = \frac{ie^2k_BT}{\pi\hbar^2(\omega+i/\tau)} \left( \frac{\mu_c}{k_BT} + 2 \ln(e^{-\mu_c/k_BT} + 1) \right),
\]

\[
G_{\text{inter}}(\omega, \mu_c, \tau, T) = \frac{ie^2(\omega+i/\tau)}{\pi\hbar^2} \int_0^\infty \frac{f_d(-E) - f_d(E)}{(\omega+i/\tau)^2 - 4(E/\hbar)^2} dE,
\]

which relate to chemical potential \(\mu_c\), radian frequency \(\omega\), relaxation time \(\tau\) and temperature \(T\) (assuming \(T = 300\) K in this work). \(E\) is the energy, \(e\) is electron charge, \(\hbar\) is reduced Planck’s constant, \(k_B\) is Boltzmann’s constant and \(f_d(E) = (e^{(E-\mu_c)/k_BT} + 1)^{-1}\) is the Fermi–Dirac distribution. Since a DC mobility of \(\mu > 100,000\) \(\text{cm}^2\text{V}^{-1}\text{s}^{-1}\) has been experimentally achieved in high-quality suspended graphene [17, 51] and \(\mu > 60,000\) \(\text{cm}^2\text{V}^{-1}\text{s}^{-1}\) in graphene on hexagonal boron nitride (h-BN) substrate was also reported [52], we can obtain \(\tau \geq 1.8\) ps by using \(\tau = \mu_c\mu/(e\nu_F^2)\) [22] when setting \(\mu_c = 0.3\) eV, where \(\nu_F \approx 0.95 \times 10^6\) \(\text{m s}^{-1}\) is the Fermi velocity. To model one-atom-thick graphene in macroscopic electromagnetic description, the graphene can be treated as an ultrathin film with thickness \(d = 1\) nm according to the previous work [19, 21, 22, 45]. Since the graphene carriers are well localized within one-atom thick layer (\(x-y\) plane), the relative anisotropic permittivity in graphene [45] can be characterized as \(\varepsilon_{\text{eq}} = \varepsilon_{\text{eq}} + i\varepsilon_z\), where \(\varepsilon_{\text{eq}}\) and \(\varepsilon_z\) are the components that are parallel and perpendicular to the graphene plane, respectively [45]. The equivalent complex permittivity \(\varepsilon_{\text{eq}}\) parallel to the graphene plane can be cast into the Drude form [19, 21, 22, 47, 53],

\[
\varepsilon_{\text{eq}} = 1 + i\frac{G_{\text{intra}} + G_{\text{inter}}}{\omega\varepsilon_0 d} = \varepsilon_{\text{inter}} + i\frac{G_{\text{intra}}}{\omega\varepsilon_0 d} = \varepsilon_{\text{inter}} - \frac{\omega_p^2}{\omega(\omega+i/\tau)},
\]

where \(\varepsilon_{\text{inter}} = (1 + iG_{\text{inter}}/\omega\varepsilon_0d)\) and \(\varepsilon_0\) is the dielectric permittivity in vacuum. Based on the macroscopic electromagnetic theory [53], the bulk plasma frequency is defined as \(\omega_p = (Ne^2/m_{\text{eff}}\varepsilon_0)^{1/2}\), where \(N = n_s/d\) is the bulk electron density. For an isolated graphene sheet, the 2D carrier density \(n_s\) is determined by the chemical potential \(\mu_c\) [49],

\[
n_s = \frac{2}{\pi\hbar^2 v_F^2} \int_0^\infty E[f_d(E) - f_d(E + 2\mu_c)] dE.
\]

From equations (1)–(4), both the bulk plasmon frequency \(\omega_p\) and the effective electron mass \(m_{\text{eff}}\) in graphene can be re-written as a function of chemical potential \(\mu_c\),

\[
\omega_p = \left[ \frac{e^2k_BT}{\pi\hbar^2\varepsilon_0 d} \left( \frac{\mu_c}{k_BT} + 2 \ln(e^{-\mu_c/k_BT} + 1) \right) \right]^{1/2},
\]

\[
m_{\text{eff}} = \frac{2}{\pi} \int_0^\infty E[f_d(E) - f_d(E + 2\mu_c)] dE.
\]

The relations between \(|n_s|^{1/2}\), \(m_{\text{eff}}\), \(\omega_p\) and \(\mu_c\) in graphene are shown in figure 1 through equations (4)–(6). In figure 1, \(|n_s|^{1/2}\) and \(m_{\text{eff}}\) increase almost linearly with \(\mu_c\), which is mainly
due to the linear dispersion around the Dirac point in graphene [17–19]. The results shown in figure 1 are consistent with previous works such as [16–19, 35]. For instance, the effective electron mass is massless (i.e. \( m_{\text{eff}} = 0 \)) when \( \mu_c = 0 \) [17] and \( m_{\text{eff}} = 0.02m_0 \) when \( \mu_c = 0.1 \) eV (i.e. \( n_s = 1 \times 10^{12} \text{ cm}^{-2} \)) [19]. Note that when \( \mu_c \) varies in the range of 0–1 eV, \( m_{\text{eff}} \) in graphene is much smaller than that of free electron in metals.

The design of unidirectional waveguide structures (x–y plane) relies upon the magnetic-field induced nonreciprocity of surface plasmons. As shown in figure 2, we assume that there is no z-variation of the geometry and the fields. In practice, this can be achieved by sandwiching graphene or graphene with its protecting layers [54–57] (such as 2D h-BN) within two artificial magnetic conductor structures [53, 58–62], which is a synthesized perfect magnetic conductor (PMC) acting as a confining edge or waveguide similar to that in [9], where the structure can be regarded as equivalent to infinitely periodically stacked graphene with its protecting layers along the z direction [8, 41, 54–57, 63–66]. With the help of PMC boundary condition and based on the macroscopic electromagnetic theory, the three-dimensional (3D) waveguide structure can then approximately be dealt with as a 2D model. The protecting layer like h-BN is used to support the graphene and to keep the properties of the graphene, e.g. the small effective electron mass and the high electron mobility, when stacking the graphene along the z direction. The stacking periodicity in the paper is assumed to be 1 nm, which is larger than that of graphite (0.335 nm). So the interlayer interactions between the graphene sheets in our paper can be ignored. It is therefore different from the bulk material of graphite which has layer interactions but is rather equivalent to a 3D graphene. Meanwhile, since the stacking period is much smaller than a wavelength, it could be considered as a homogeneous material. Therefore, our assumption that there is no z-variation of the geometry and the fields is reasonable. For the sake of simplicity, we neglect the protecting layers’ impact and use graphene’s optical parameter to study the 2D model [34]. In the presence of a static magnetic field \( B \) along the +z direction,
Figure 2. Dispersion relations of surface plasmons propagating along three different graphene based interfaces: (a), (b) air/graphene interface; (c), (d) air/graphene/air interfaces; (e), (f) air/graphene/dielectric interfaces. The top (a), (c), (e) show the corresponding schematic structures. The horizontal axis is the wave propagation vector normalized by $k_{SP} = \omega_{SP} (\mu_0 \varepsilon_0) ^{1/2}$. In (c) and (e), graphene has a finite width of $w = 0.3 \mu m$. The regions of $\omega_{SPR} < \omega < \omega_{SPL}$ in (b) and (f) indicate the unidirectional frequency range. $B = +10T$ and $\mu_c = 0.3 \text{ eV}$ are used in order to obtain a large unidirectional frequency bandwidth for clear observation.

graphene is gyrotropic and its relative anisotropic permittivity can be described by a tensor [53]:

$$\tilde{\varepsilon} = \begin{pmatrix} \varepsilon_d & i \varepsilon_g & 0 \\ -i \varepsilon_g & \varepsilon_d & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix},$$

where $\varepsilon_d = \varepsilon_{\text{inter}} - \omega_p^2/\omega^2$, $\varepsilon_g = -\omega_p^2 \omega_B / (\omega^3 - \omega B^2)$ and $\omega_B = e B / m_{\text{eff}}$ is the cyclotron frequency. It should be noted that the linear dispersion relation in graphene band structure around the Dirac point can be quantized as a result of the external magnetic field due to Landau quantization [16–18]. Therefore, the Drude model expression for the graphene permittivity tensor in equation (7) has a validity range when an external magnetic field is applied [67, 68]. As reported in [67], as long as the following condition, $\mu_c \gg L$, where $L = (2\hbar e B v_F^2)^{1/2}$ is the Landau energy scale, is satisfied, the graphene permittivity tensor characterized in our paper is valid. From our calculation, we find that when $|B| \leq 0.1 \text{ T}$ (this range of $B$ is what we used throughout the paper except figures 2 and 3 for the purpose of clear observation of the unidirectional bandwidth), the Landau energy scale $L$ is smaller than 0.0115 eV, and therefore the condition $\mu_c \gg L$ can be satisfied. In addition, according to [67], using a larger $\mu_c$ and/or a smaller $B$ can further help to weaken the Landau quantization influence. Therefore, the Landau quantization perturbation will not qualitatively alter our main
Figure 3. $H_z$ field distribution of surface waves excited by a point source in four different cases: (a) air/graphene interface without loss ($\varepsilon_d = -1.1196$, $\varepsilon_g = 0.4059$), (b) air/graphene interface with loss ($\varepsilon_d = -1.1196 + 0.0154i$, $\varepsilon_g = 0.4059 - 0.0007i$), (c) air/graphene/air symmetric double interfaces and (d) air/graphene/dielectric ($\varepsilon_2 = 2$) asymmetric double interfaces. Graphene region is marked in yellow. The parameters are: $\mu_c = 0.3$ eV, $B = +10$ T. The working frequency $f_0 = 105$ THz is both within the unidirectional frequency range of air/graphene interface (103.3 to 108.0 THz) and that of air/graphene/dielectric interfaces (104 to 108.0 THz).

Conclusion below. For simplicity, we do not consider the Landau quantization perturbation in this paper. The $z$-component permittivity $\varepsilon_z$ will not change when graphene is under a $z$-direction external magnetic field. When neglecting the loss in graphene characterized by the relaxation time $\tau$, the single gyrotropic interface between graphene and dielectric (with $\varepsilon_1$) in figure 2(a) supports surface plasmons with the following dispersion relation:

$$\left(\varepsilon^2_d - \varepsilon^2_g\right) \sqrt{k^2_x - \omega^2 \mu_0 \varepsilon_0 \varepsilon_1 + \varepsilon_d \sqrt{k^2_x - \omega^2 \mu_0 \varepsilon_0 (\varepsilon^2_d - \varepsilon^2_g)/\varepsilon_d + \varepsilon_g k_x}} = 0,$$

where $k_x$ is the component of wave vector along propagation direction ($x$ direction) and $\mu_0$ is the permeability in vacuum. The dispersion relation is then plotted in figure 2(b) by assuming $\mu_c = 0.3$ eV and $B = +10$ T. Note that a larger external magnetic field like 10 T may influence the graphene band structures due to Landau quantization [16–18]. Here it is used only to obtain a large unidirectional frequency bandwidth for clear observation in figure 2. From figure 2(b), the linear term of $k_x$ in equation (8) breaks the left–right symmetry of the dispersion relation. In the limit of $|\omega_B| \ll \omega_p$, the frequencies of the left and right propagating modes approach their upper limits of $\omega_{SPL} = \omega_{SP} + |\omega_B|/2$ and $\omega_{SPR} = \omega_{SP} - |\omega_B|/2$ at $k_x \rightarrow \pm \infty$, respectively, where
\[ \omega_{\text{SP}} = \omega_p / (\varepsilon_1 + \varepsilon_{\text{inter}})^{1/2}. \]

Within the unidirectional frequency range \( \omega_{\text{SP}} < \omega < \omega_{\text{SPL}} \), surface plasmons can only propagate in one direction. We simulate the performance of unidirectional surface plasmons by using the finite element method (COMSOL Multiphysics). A point source at the interface is used to excite the field. Figure 3(a) shows the propagation of the surface plasmons in lossless graphene while figure 3(b) shows the results when a loss characterized by a relaxation time \( \tau = 0.9 \text{ ps} \) is included. One can see that both cases can support the unidirectional surface plasmons mode.

Now we extend our study to finite-width graphene as often used in practice. We consider the structure of dielectric (with \( \varepsilon_1 \))/graphene (with \( \bar{\varepsilon} \))/dielectric (with \( \varepsilon_2 \)) as an example. When the graphene strip has a finite width comparable with the skin depth \( d_{\text{skin}} = 0.25 \mu m \), surface plasmons in the gyrotropic interfaces can couple efficiently with each other. The dispersion relation of surface plasmons in this structure is derived as follows:

\[
e^{-2k_{y1}w} = \frac{(\kappa_d k_{y1} - \kappa_y k_x) + \frac{2\mu I}{\varepsilon_1} (\kappa_d k_{y1} + \kappa_y k_x) + \frac{2\mu I}{\varepsilon_1}}{(\kappa_d k_{y1} + \kappa_y k_x) - \frac{2\mu I}{\varepsilon_1} (\kappa_d k_{y1} - \kappa_y k_x) - \frac{2\mu I}{\varepsilon_1}},
\]

where \( \kappa_d = \varepsilon_d / (\varepsilon_1^2 - \varepsilon_d^2) \), \( \kappa_y = -\varepsilon_y / (\varepsilon_2^2 - \varepsilon_y^2) \), \( k_{y1} = \sqrt{k_1^2 - \omega^2 \mu_0 \varepsilon_0 (\varepsilon_1^2 - \varepsilon_y^2) / \varepsilon_d} \), \( k_{1y} = \sqrt{k_1^2 - \omega^2 \mu_0 \varepsilon_0 \varepsilon_1} \) and \( k_{2y} = \sqrt{k_2^2 - \omega^2 \mu_0 \varepsilon_0 \varepsilon_2} \). When \( w = 0.3 \mu m \) (slightly larger than the skin depth \( d_{\text{skin}} \), \( \mu_c = 0.3 \text{ eV}, B = +10 \text{ T} \), the dispersion relations for the air/graphene/air structure (\( \varepsilon_1 = \varepsilon_2 \), as shown in figure 2(c)) and for the air/graphene/dielectric (\( \varepsilon_1 \neq \varepsilon_2 \), as shown in figure 2(e)) are plotted in figures 2(d) and (f), respectively. When the double interfaces are symmetric (\( \varepsilon_1 = \varepsilon_2 \)), equation 9 becomes

\[
e^{-2k_{y1}w} = \frac{(\kappa_d k_{y1} + \frac{k_{1y}}{\varepsilon_1})^2 - (\kappa_y k_x)^2}{(\kappa_d k_{y1} - \frac{k_{1y}}{\varepsilon_1})^2 - (\kappa_y k_x)^2}
\]

without the linear term of \( k_x \). In this case, the left–right symmetry of the dispersion relation is preserved and the unidirectional frequency bandwidth is zero as shown in figure 2(d), indicating that the symmetric double interfaces cannot support unidirectional surface plasmons. When \( \omega < \omega_{\text{SP}} + |\omega_B| / 2 \), the numerical simulation in figure 3(c) shows that surface plasmons along the symmetric double interfaces can propagate in both directions. In contrast, equation 9 in asymmetric double interfaces (\( \varepsilon_1 \neq \varepsilon_2 \)) always has a linear term of \( k_x \), similar to that in single gyrotropic interface, indicating that it can support the propagation of unidirectional surface plasmon mode. The result shown in figure 2(f) clearly indicates that the left–right symmetry of the dispersion relation is broken. In the limit of \( |\omega_B| \ll \omega_p \), the asymmetric double interfaces have a unidirectional frequency bandwidth \( \omega_{\text{SPL}} - \omega_{\text{SP}} \approx 0.85 |\omega_B| \), which is a bit smaller than that in air/graphene single interface. When the working frequency is within the unidirectional frequency range, the numerical simulation in figure 3(d) shows that the asymmetric interfaces can support unidirectional surface plasmons. Therefore, from the comparison of the results shown in figures 3(c) and (d), one can see that the unidirectional surface plasmons can only be excited in asymmetric double interfaces.

From the above theoretical analysis, we can conclude that graphene shows the following superiorities in designing unidirectional surface plasmons compared to metal:

Firstly, the working frequency can be widely tuned by the chemical potential \( \mu_c \). Since the working frequency range \( (\omega_{\text{SP}} - |\omega_B| / 2 < \omega < \omega_{\text{SP}} + |\omega_B| / 2) \) of air/graphene interface relates.
Figure 4. Unidirectional frequency bandwidth \( \frac{(\omega_{PSL} - \omega_{PSR})}{2\pi}, \) red line) and lowest unidirectional working frequency \( \frac{\omega_{PSR}}{2\pi}, \) blue line) of surface plasmons at the air/graphene interface under a static magnetic field \( (B = +0.1 \text{ T}) \) as a function of the chemical potential \( \mu_c \) in graphene. As \( \mu_c \) increases to 1 eV, the unidirectional working frequency changes continuously from several THz to several hundred THz. Compared with that in air/metal interface \( (|eB|/2\pi m_0| = 0.0028 \text{ THz, gray dashed arrow}), \) the unidirectional frequency bandwidth of air/graphene interface can be 1–2 orders of magnitude larger under the same magnetic field.

Secondly, a unidirectional frequency range much larger than that in metals can be achieved in graphene. This is because in the limit of \( |\omega_B| \ll \omega_p, \) the unidirectional frequency bandwidth \( |\omega_B| = e|B|/m_{eff} \) is proportional to \( |B| \) and inversely proportional to \( m_{eff}, \) and it is well known that \( m_{eff} \) in graphene, which decreases linearly as \( \mu_c \) decreases, can be extremely smaller than that in metal, as shown in figure 1. When \( B = +0.1 \text{ T} \) and \( \mu_c = 0.1 \text{ eV}, \) the unidirectional frequency bandwidth is 0.12 THz in air/graphene interface, which is about 43 times wider than that in air/metal interface, i.e. 0.0028 THz. In general, from figure 4, one can see that the unidirectional frequency bandwidth, \( \frac{(\omega_{PSL} - \omega_{PSR})}{2\pi}, \) in air/graphene interface can be 1–2 orders of magnitude larger than that in air/metal interface under the same magnetic field.

The unidirectional surface plasmons can be used to design reconfigurable directional optical components. As a concrete example, a Y-shaped directional surface plasmon waveguide is designed as shown in figure 5. The Y-shaped waveguide is composed of two parts. The left part
Figure 5. $H_z$ field distribution of the Y-shaped graphene-based directional waveguide controlled by an external static magnetic field: (a) $B = 0$, (b) $B = +0.1$ T, (c) $B = -0.1$ T. Graphene region is marked in yellow. The parameters are: $\mu_c = 0.1$ eV, $f_0 = 42.7$ THz, $w = 0.8$ $\mu$m, $r = 5$ $\mu$m. The wave is incident from port 1 and flows to ports 2 and/or port 3.

of the Y-shaped waveguide (port 1) is the air/graphene/air interfaces, and the right part consists of two branches (port 2 and port 3), each composed of an air/graphene interface. The graphene width in the left part is $w = 0.8$ $\mu$m and the radius of the interface in the right part is $r = 5$ $\mu$m. The chemical potential in graphene is $\mu_c = 0.1$ eV. The working frequency is $f_0 = 42.7$ THz. The surface wave is incident from the left (port 1), propagates along the air/graphene/air interfaces, and outputs to the right down branch (port 2) and/or the right up branch (port 3). The propagating behavior of surface plasmons in the right part can be controlled by the external magnetic field $B$. Three cases with different external magnetic fields are studied. In the first case, $B = 0$ ($\varepsilon_d = -1.0196$, $\varepsilon_g = 0$), the two separate air/graphene interfaces at the output branches are reciprocal and support surface plasmons in both directions. The surface waves input from port 1 therefore couple with port 2 and port 3 equally, as shown in figure 5(a). It behaves similar to a power divider. In the second case, $B = +0.1$ T ($\varepsilon_d = -1.0198$, $\varepsilon_g = 0.052$); unidirectional surface plasmons mode will be raised in this case. The working frequency $f_0 = 42.7$ THz is right within the unidirectional frequency range of air/graphene interface (from 42.6628 to 42.7823 THz). The skin depth of unidirectional surface plasmons along the air/graphene interface is $d_{\text{skin}} = 0.3$ $\mu$m in the graphene region. Therefore, the air/graphene interface only supports the right-direction surface plasmons in port 2 and the left-direction surface plasmons in port 3. Figure 5(b) shows the simulation results, from which one can see that the power from port 1 couples with port 2 completely. In the third case, $B = -0.1$ T ($\varepsilon_d = -1.0198$, $\varepsilon_g = -0.052$). Because of that, the direction of external magnetic field is reversed; the directions of unidirectional surface modes that port 2 and port 3 can support are reversed in this case. Figure 5(c) shows that the surface waves input from port 1 completely couple with port 3. The simulations, therefore, suggest a novel directional surface plasmon waveguide tuned by external magnetic field.

With the tunability of the operating frequency by the chemical potential $\mu_c$, the graphene-based unidirectional surface plasmons can be used to realize more advanced photonic devices. As an example, we designed a tunable four-port surface plasmon waveguide coupler, which can couple the waves from one port with any of the other three ports freely. The four-port coupler
Figure 6. $H_z$ field distribution of four-port graphene-based surface plasmon coupler. The parameters are: $\mu_c = 0.1 \text{ eV}$, $f_0 = 42.7 \text{ THz}$, $w = 0.8 \mu\text{m}$ and $r = 5 \mu\text{m}$. Wave is incident from port 1 and flows to output ports 2, 3 and 4. In the left graphene part, magnetic field is $B = +0.1 \text{ T}$. In the right part, magnetic field is (a) $B = 0$, (b) $B = +0.1 \text{ T}$, (c) $B = -0.1 \text{ T}$ and (d) $B = 0$, respectively. In the right graphene part in (d), the chemical potential is tuned to $\mu_c = 0.096 \text{ eV}$ to obtain the backward coupling. The graphene region is marked in yellow.

is composed of three parts. The left and right parts are four branches (port 1, port 2, port 3 and port 4), each composed of an air/graphene interface, as shown in figure 6. The middle part, which can be considered as the couple region, is the reciprocal air/graphene/air interfaces. The parameters used are the same as in figure 5: $w = 0.8 \mu\text{m}$, $r = 5 \mu\text{m}$, $\mu_c = 0.1 \text{ eV}$ and $f_0 = 42.7 \text{ THz}$. In the designed waveguide coupler, the four ports can couple freely by tuning the magnetic field and/or chemical potential in the left and right graphene parts separately. In figure 6, we demonstrate how the surface wave that is incident from port 1 can be freely coupled to any of the other three ports by properly selecting the $B$ and $\mu_c$. The left graphene part is set with $B = +0.1 \text{ T}$ ($\varepsilon_d = -1.0198$, $\varepsilon_g = 0.052$), and thus the nonreciprocal air/graphene interface can only support right-direction surface plasmons in port 1 and left-direction surface plasmons in port 4. In the right graphene part, the following four cases with different external magnetic fields are studied. In the first case, $B = 0$ ($\varepsilon_d = -1.0196$, $\varepsilon_g = 0$), the two separate air/graphene interfaces on the right are reciprocal. The surface waves input from port 1 are therefore coupled with port 2 and port 3 equally, as shown in figure 6(a). In the second case, $B = +0.1 \text{ T}$ ($\varepsilon_d = -1.0198$, $\varepsilon_g = 0.052$), the air/graphene interface at port 2 only supports right-direction surface plasmon mode and at port 3 only supports left-direction surface plasmon mode. Hence, the power from port 1 couples with port 2 completely, as shown in figure 6(b). In the third case, $B = -0.1 \text{ T}$ ($\varepsilon_d = -1.0198$, $\varepsilon_g = -0.052$); because of the reverse of external magnetic field direction in the second case, the directions of unidirectional surface modes
supported by port 2 and port 3 are reversed. The surface waves input from port 1 completely couple with port 3, as shown in figure 6(c). In these three cases, since the right part is able to support surface plasmons, the surface plasmons input from port 1 couple completely with port 2 and/or port 3, while the coupling with port 4 is almost negligible. The device behaves as a forward waveguide coupler. In the fourth case, $B = 0$ and the chemical potential is tuned from $\mu_c = 0.1 \text{ eV}$ to $\mu_c = 0.096 \text{ eV}$ in the right graphene part ($\varepsilon_d = -0.2125$, $\varepsilon_g = 0$). In this case, the working frequency $f_0 = 42.7 \text{ THz}$ is in the stop band ($> 41.8 \text{ THz}$) of the surface plasmon mode in the air/graphene interface on the right. Therefore, the surface waves incident from port 1 have no way of propagating except port 4. Figure 6(d) shows the numerical simulation results of this case where one can see that the surface waves input from port 1 efficiently couple with port 4. In this case, the device behaves as a backward waveguide coupler, which has been achieved by using left-handed metamaterial in microwave frequencies in previous literature [69]. Here our studies suggest an alternate solution of realizing a backward waveguide coupler. In particular, we can freely control the functionality of the waveguide coupler as forward or backward by changing the external magnetic field and the chemical potential in the graphene. Our results demonstrate the superiority of nonreciprocal graphene in future applications like photonic isolators, diodes, graphene logic circuits, etc.

Although we considered the ideal lossless structures where the unidirectional surface plasmons have real wave vector $k_{xf}$ in the forward direction and purely imaginary wave vector $k_{xb}$ in the backward direction, it does not alter our conclusion when loss is introduced. In the presence of loss, both $k_{xf}$ and $k_{xb}$ become complex. Since the forward mode is significantly under-damped while the backward mode is strongly over-damped [8], the above structures can still support unidirectional surface plasmons. For example, we show in figure 3(b) that when the loss in graphene characterized by $\tau \approx 0.9 \text{ ps}$ is considered, unidirectional surface mode can still be supported. Note that since the relaxation time in graphene is larger than that in gold ($\tau \approx \sim 10^{-14} \text{ s}$) and the loss is inversely proportional to the relaxation time, surface plasmons in graphene can propagate with lower losses than that in metal, and further, its lifetime may reach hundreds of optical cycles [22].

In conclusion, we have shown that tunable unidirectional surface plasmons can be supported in nonreciprocal graphene. The unidirectional frequency bandwidth, which is proportional to the external magnetic field and the inverse of effective electron mass in graphene, can be two orders larger than that in metal under the same external magnetic field. Moreover, the working frequency is proportional to the chemical potential in graphene, and can be widely tunable from THz to even visible. A Y-shaped directional waveguide and four port couplers are proposed as examples, showing the appealing properties of nonreciprocal graphene in nonreciprocal optical devices applications. The newly gained capability of flexible graphene parameters may lead to further innovations in optics, microelectronics and material science.

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