I examines the applicability and possible need for a re-formulation of pQCD, as we know it, in the DIS limit of small $Q^2$ and $x$. Gluon saturation, implied by s-channel unitarity, and its possible experimental signatures are critically assessed.

1 Introduction

The physics of DIS small $Q^2$ and small $x$ is associated with the search for gluon saturation signatures implied by s-channel unitarity \(^1\). We expect this transition to be preceded by s-channel unitarity screening corrections (SC) which should be experimentally visible even though the relevant amplitude has not yet reached its unitarity (black disk) bound. We also know from our experience with soft Pomeron physics that unitarity SC become appreciable at different energies for different channels. Specifically, the SC associated with soft diffractive channels are considerably bigger than those associated with the elastic channel \(^2\).

In the context of DIS and its pQCD interpretation, we recall that while the global analysis of $F_2(x, Q^2)$ (or $\sigma_{\gamma p}^\gamma (W, Q^2)$) shows no significant deviations from DGLAP, there are dedicated HERA investigations of $\frac{\partial F_2}{\partial \ln Q^2}$ which may suggest a departure from the DGLAP expectations when $Q^2$ and $x$ are sufficiently small. In my opinion, the question if these observations conclusively imply gluon saturation is not settled. To this end I shall summarize in the following a detailed recent study \(^3\) of $\frac{\partial F_2}{\partial \ln Q^2}$ coupled to an analysis of $J/\Psi$ photo and DIS production.

2 The s-channel unitarity bound

$\frac{\partial F_2}{\partial \ln Q^2}$ and the cross section for $J/\Psi$ photo and DIS production are linearly dependent, in the LLA of pQCD, on $xG(x, Q^2)$ and $[xG(x, Q^2)]^2$ respectively. Even though this simple dependence is not maintained at higher perturbative orders, we can benefit from the comparison of $xG$, which is obtained from a global p.d.f. fit to the inclusive DIS data, with its s-channel unitarity bound.
Since the bound is calculated within the framework of the dipole approximation, it does not depend on our p.d.f. input. In the dipole approximation we consider the time sequence of small $x$ DIS in the proton target rest frame. In this frame the virtual photon fluctuates into a hadronic system ($q\bar{q}$ to the lowest order) well before the interaction with the target. Accordingly we have

$$\sigma_{tot}(x,Q^2) = \int_0^1 dz \int d^2r_\perp |\psi^{\gamma^*}(Q^2,r_\perp,z)|^2 \hat{\sigma}(r_\perp,x,Q^2),$$

where $\psi^{\gamma^*}$ is the well known QED wave function of the $q\bar{q}$ system within the photon and $\hat{\sigma} = 4\pi Im a_{el}$ is the unknown hadronic total cross section of the $q\bar{q}$ system with the target. $a_{el} \leq 1$ is the elastic amplitude of a dipole, with a size $r_\perp = 2/Q$ and energy $W^2 = Q^2$ scattering off the proton target. The s-channel black unitarity bound on $\hat{\sigma}$ is obtained by fixing $a_{el}$ to equal its bound of unity. In a similar procedure we can also calculate the unitarity bound for $xG$ where $q\bar{q}$ is replaced by $gg$. 

The results of this comparison are shown in Fig.1, for fixed $x = 10^{-5}$, compared with $xG$ as obtained from the commonly used p.d.f. parameterizations. We observe that $xG$ obtained from GRV’94 significantly exceeds the unitarity bound for $Q^2 < 5 GeV^2$. With GRV’98 we have a slight excess over the unitarity bound, whereas both CTEQ5 and MRS’99 are well below the bound. Similar results are obtained when we compare $\frac{\partial F_2}{\partial \ln Q^2}$ with the corresponding unitarity bound.

In the following I shall examine the data on the logarithmic $Q^2$ slope of $F_2$ against NLO DGLAP without and with SC. The SC were calculated in the eikonal formalism for which a LLA of the dipole approximation was utilized to calculate the input opacity. In this procedure the SC diminish once $Q^2$ and/or $x$ are large enough. As we did not succeed to obtain a consistently reasonable reproduction of the analyzed data with MRS’99, this parameterization will be omitted in the following discussion.

3 The small $Q^2$ and $x$ behavior of $\frac{\partial F_2}{\partial \ln Q^2}$

Detailed HERA data on $\frac{\partial F_2}{\partial \ln Q^2}$ have recently became available. As I shall show, the corresponding pQCD calculation reproduces the data well with either CTEQ5HQ with no SC or with GRV’98(MS) corrected for SC. SC were calculated in both the quark sector, to account for the percolation of a $q\bar{q}$ through the target, and the gluon sector, to account for the screening
of $xG(x,Q^2)$. The factorizeable result obtained is

$$\frac{\partial F_{2}^{SC}(x,Q^2)}{\partial \ln Q^2} = D_q(x,Q^2) D_g(x,Q^2) \frac{\partial F_{2}^{DGLAP}(x,Q^2)}{\partial \ln Q^2},$$

(2)

where the SC damping factors have been calculated from the opacities:

$$\kappa_q = \frac{2\pi \alpha_s}{3Q^2} xG^{DGLAP}(x,Q^2) \Gamma(b^2)$$

and

$$\kappa_g = \frac{4}{9} \kappa_q.$$  

The calculation is significantly simplified if we assume a Gaussian parameterization for the two gluon non perturbative form factor, $\Gamma(b^2) = \frac{1}{R^2} e^{-b^2/R^2}$, where $R^2 = 8 - 10 GeV^2$ is determined directly from the experimental forward slope of $J/\Psi$ photoproduction. Our results are presented in Fig.2. We conclude that CTEQ5NSC and GRV'98SC provide a very good reproduction of the data and are almost identical.

4 Photo and DIS production of $J/\Psi$

We try to corroborate our conclusions by examining the exclusive channel of $J/\Psi$ photo and DIS production. Note that the hardness of the $J/\Psi$ channel is comparable to the hardness investigated in our $F_2$ study. The $J/\Psi$ t=0 differential cross section is calculated in the dipole LLA. When using the static non relativistic approximation of the vector meson wave function, the
cross section has a very simple form

\[
\left( \frac{d\sigma(\gamma^* p \to V p)}{dt} \right)_0 = \frac{\pi^3 \Gamma_{ee} M_V^3}{48\alpha} \frac{\alpha_s^2(\bar{Q}^2)}{Q^8} \left( xG(x, \bar{Q}^2) \right)^2 \left( 1 + \frac{Q^2}{M_V^2} \right),
\]

where in the non relativistic limit we have \( \bar{Q}^2 = M_V^2 + Q^2 \) and \( x = \frac{4\bar{Q}^2}{W^2} \).

The integrated cross section data available for this channel span a relatively wide energy range \( 4 \). To relate the integrated cross section to Eq. (3) we need to know \( B_{J/\Psi} \), the \( J/\Psi \) forward differential cross section slope. For this we may use the experimental values for which \( B_{J/\Psi} \approx 2 \).

The main problem with the theoretical analysis of \( J/\Psi \) is the realization that the simple pQCD calculation needs to be corrected for the following reasons:

1) The contribution of the real part of the production amplitude denoted.
2) The contribution of the skewed (off diagonal) gluon distributions.
3) Relativistic effects produced by the Fermi motion of the bound quarks. In our calculation we have considered this correction as a free parameter.

Our calculation of the SC for \( J/\Psi \) photo and DIS production is very similar to the \( Q^2 \) logarithmic slope calculation presented earlier.

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Figure 2. \( x \) dependence of \( \partial F_2/\partial \ln Q^2 \) data at fixed \( Q^2 \).
Our calculations compared with the experimental photoproduction data are presented in Fig.3. GRV’98SC provides an excellent reproduction of the data. CTEQ5NSC gives a lesser quality reproduction, in particular at the lower energies, but it cannot be ruled out.

Thus far we were unable to detect conclusive signatures of gluon saturation manifested by SC. Very recent HERA data on the forward x dependence of $B_{J/\Psi}$ may provide this clue. The hard Pomeron manifested by either DGLAP or the dipole model is not expected to yield any shrinkage of the $J/\Psi$ forward photoproduction slope. SC can reproduce this phenomena. The two gluon form factor we have assumed, which corresponds to $F_{\text{exp}} = e^{\frac{1}{2}R^2}$, provides a very mild shrinkage. However an electromagnetic form factor, $F_{\text{dipole}} = \frac{1}{(1 - \frac{1}{2}R^2)x}$, provides an excellent reproduction of the data while maintaining the quality of our previous results. This is shown in Fig.4.

5 Conclusions

At this stage I do not consider the analysis presented in this review to provide conclusive support for the observation of gluon saturation, even though the
Figure 4. $J/\Psi$ photoproduction forward slope.

$B_{J/\Psi}$ data is promising.

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4. References to the data are given in the previous reference.