Magnetization curve modelling of soft magnetic alloys

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Abstract. In this paper we present an application of the so called hyperbolic model of magnetization. The model was modified and it was applied for nine different soft magnetic alloys. The tested samples were electro-technical steels (FeSi alloys) and a permalloy (FeNi alloy) with strongly different magnetic properties. Among them there are top, medium and definitely poor quality soft magnetic materials as well. Their minor hysteresis loops and normal magnetization curves were measured by alternating current measurement. The hyperbolic model of magnetization was applied for the experimental normal magnetization curves. It was proved that the applied model is excellent for describing mathematically the experimental magnetization curves.

1. Introduction
In this work a novel phenomenological model of magnetization was used and validated. The applied model is called hyperbolic model of magnetization (HMM). The predecessors of this model were the Flatley-Henretty model [1] and the so called T(x) model developed by J. Takacs in [2]. The hyperbolic model of magnetization provides an easy analytical solution to the complex phenomena of magnetic hysteresis. It is known that the thermodynamically equilibrium state of a magnetic material can be characterized by its anhysteretic magnetization curve [3]. In the hyperbolic model the following equation (1) was used for describing the anhysteretic magnetization curve:

\[
\frac{M}{M_0} = \sum_{i=1}^{N} \frac{A_i \tanh(N_i H)}{\sum_{i=1}^{N} A_i},
\]

where: \(N_i\) are scaling parameters, \(A_i\) are the amplitudes of magnetization components (\(A_i = 1\)).

As it can be seen the anhysteretic magnetization curve is composed as a linear combination of \(N\) pieces of tangent hyperbolic functions. They will be referred as hyperbolic components of magnetization in the following.

Following the way of Takacs's calculation [2] the whole hysteresis loop can be constructed. The components of the anhysteretic curve (1) shifted horizontally by different \(\alpha_i\) as well as vertically in symmetrical way. The loci of the crossover points of the up-going and down-going parts of the
symmetrical minor loops is referred as normal magnetization curve [4]. From the hyperbolic model the following mathematical function can be obtained for the normal magnetization curve:

\[
\frac{M}{M_0} = \sum_{i=1}^{N} \left[ A_i \tanh \left( N_i (H + \alpha_i) \right) + A_i \tanh \left( N_i (H - \alpha_i) \right) \right] - 2 \sum_{i=1}^{N} A_i,
\]

where: \( \alpha_i \) is the shifting factor which is associated with the coercivity.

In this approach the model was applied for \( N = 2 \) therefore two hyperbolic components were supposed. The theoretical normal magnetization curve (2) was fitted to the measured normal magnetization curves. As it can be seen the applied form of the hyperbolic model has five fitting parameters.

In this work eight different so-called electro-technical steels and a permalloy were taken as test samples. They were chosen intentionally to have strongly different magnetic properties. Among them there are top, medium and definitely poor quality products as well. Their minor hysteresis loops and normal magnetization curves were measured by alternating current measurement.

### 2. Experimental arrangement and samples

A permeameter-type magnetic property analyzer developed at our department was used for measuring the magnetization curves. The applied measuring yoke contains a robust "U" shaped laminated Fe-Si iron core with a magnetizing coil. The excitation current was produced by a digital function generator and a power amplifier used in voltage regulated current generator mode. The pick-up coil was around the middle of the sample. The permeameter was completely controlled by a personal computer in which a 16 bit input-output data acquisition card accomplished the measurements. The applied maximal excitation field strength was 2150 A/m. In each case 200 minor hysteresis loops were measured. Each minor hysteresis loops were recorded by measuring 1000 points of them. The excitation magnetic field was increased in steps and there was 5 seconds delay between the increase of the excitation and the data acquisition for hysteresis loops to ensure the sample’s perfect magnetic accommodation.

All the magnetic measurements were completed by using 5 Hz sinusoidal excitation frequency. Because of the relatively low excitation frequency and small thickness of the samples (0.35 - 0.5 mm) the completed magnetic measurements can be considered as pseudo-static and the effect of eddy-currents to the magnetization curves is supposed to be not significant.

The permeameter allowed us to derive all the traditional magnetic properties from the hysteresis loop like maximal induction, remanent induction, coercivity, relative permeability and hysteresis loss values.

The measured samples were soft magnetic alloys with strongly different magnetic properties. Samples E1, E2, E3, E4, E5A, E5B, E6, E7 and E8 are iron-silicon, S11 is an iron-nickel alloy (permalloy). The chemical composition of the samples was measured by energy dispersive X-ray analyzer (EDS). The obtained compositions can be found in table 1. Samples E5A and E5B were grain-oriented with cube texture the others were non-oriented.

From the metal sheets 150x30 mm specimens were cut. The thickness of each FeSi and FeNi samples was 0.35 mm and 0.5 mm respectively. The samples were longitudinally magnetized according to their rolling direction.

As an illustration the measured hysteresis loops of three of the investigated samples are shown in figure 1.

The relative initial (\( \mu_0 \)) and maximal permeability (\( \mu_m \)) and the maximal induction (\( B_m \)) values were determined from the measured magnetization curves. These results are summarized in table 2.
This set of specimens was used as model materials for testing the usability of the proposed form of the hyperbolic model of magnetization.

### Table 1. Chemical composition of the tested samples (figures in volume %).
The rest is Fe in all cases.

| Sample | Al  | Si  | Mn  | P  | Ni  |
|--------|-----|-----|-----|----|-----|
| E1     | -   | 2.41| 1.12| -  | -   |
| E2     | 1.61| 2.14| -   | -  | -   |
| E3     | -   | 2.10| 0.93| -  | -   |
| E4     | -   | 4.82| -   | 0.86| -   |
| E5A    | 1.11| 3.46| -   | -  | -   |
| E5B    | 1.61| 3.41| -   | -  | -   |
| E6     | -   | 1.45| -   | -  | -   |
| E7     | -   | 0.43| 0.91| -  | -   |
| E8     | 1.53| 0.51| 0.96| -  | -   |
| S11    | -   | 1.29| 0.87| -  | 76.24|

![Figure 1. Illustrative hysteresis loops of three tested samples.](image)

### 3. Results and discussion

The experimental normal curves were generated as the loci of the crossover points of the up-going and down-going parts of the measured minor hysteresis loops.

The theoretical normal magnetization curve derived from the hyperbolic model of magnetization (2) was fitted to the measured normal curves. The five fitting parameters \( N_1, \alpha_1, A_2, N_2, \alpha_2 \) of the hyperbolic model were determined by applying the Levenberg-Marquardt iteration method. Some of the measured and the fitted normal magnetization curves are presented in Figure 2 and 3.

As it can be seen the fitted normal magnetization curves are practically perfectly fit to the experimental points from zero fields to the near saturation range. The calculated deterministic coefficients \( R^2 \) were always better than 0.999 in all tested cases (table 3).

It can be noted that the fitting is always significantly better than in the previous form of the model in which only one irreversible magnetization part was supposed [5].
Table 2. Saturation induction and permeability values of the tested samples.

| Sample | \( B_m \) (T) | \( \mu_0 \) | \( \mu_m \) |
|--------|--------------|------------|------------|
| E1     | 1.71         | 2730       | 9303       |
| E2     | 1.88         | 3942       | 10117      |
| E3     | 1.46         | 2297       | 9568       |
| E4     | 1.89         | 9264       | 16330      |
| E5A    | 1.91         | 12351      | 22024      |
| E5B    | 2.03         | 13239      | 23231      |
| E6     | 1.86         | 583        | 4546       |
| E7     | 1.89         | 450        | 2834       |
| E8     | 1.76         | 400        | 2596       |
| S11    | 1.10         | 17657      | 28809      |

Figure 2. The experimental and fitted normal magnetization curves of the E2, E5 samples.

Figure 3. The experimental and fitted normal magnetization curves of the E7, S11 samples.

The applied hyperbolic model can perfectly describe the near saturation range. Therefore it can be used for determining the real saturation state of magnetic materials. Earlier several another phenomenological approaches were attempted to use for determination the saturation induction value
(3). One of the most commonly used equations for describing the high-field region of the normal magnetization curves is the so called ‘law of approach to saturation’ (3) [6].

\[ B = B_S \left( 1 - \frac{a}{H} - \frac{b}{H^2} \right), \]  

(3)

The saturation induction values of the tested samples were determined by the hyperbolic model \((B_S\text{-Hip})\) and by the law of approach \((B_S\text{-LA})\) (3) as well. The obtained results are summarized in table 3. As it can be seen the two applied extrapolation methods resulted practically the same saturation induction values.

However the law of approach can describe only the near saturation region while the hyperbolic model can model the whole normal magnetization curve from zero to the saturation field.

| Sample | \(R^2\) | \(B_S\text{-Hip}\) (T) | \(B_S\text{-LA}\) (T) |
|--------|-------|----------------|----------------|
| E1     | 0.99971 | 1.80           | 1.82           |
| E2     | 0.99971 | 1.99           | 1.97           |
| E3     | 0.99964 | 1.57           | 1.57           |
| E4     | 0.99985 | 1.89           | 1.89           |
| E5A    | 0.99986 | 1.91           | 1.91           |
| E5B    | 0.99977 | 2.03           | 2.04           |
| E6     | 0.99995 | 1.97           | 1.99           |
| E7     | 0.99986 | 2.08           | 2.10           |
| E8     | 0.99983 | 2.04           | 2.07           |
| S11    | 0.99931 | 1.11           | 1.12           |

4. Summary
The hyperbolic model of magnetization was applied for eight different iron-silicon alloys and for an iron–nickel alloy. The tested samples were soft magnetic materials with intentionally strongly different magnetic properties.

The hyperbolic model of magnetization presents a closed form mathematical description of magnetic hysteresis loops and normal magnetization curves partially based on physical principles, rather than simply on the mathematical curve-fitting of observed data. This model can be applied for a lot of magnetic materials using only some fitting parameters. In the applied form of the model two hyperbolic components were supposed.

The normal magnetization curves of the samples were measured and the theoretical normal magnetization curves calculated from the hyperbolic model were fitted to them. It can be concluded that the calculated curves nearly perfectly fit onto the measured normal magnetization curves from the low field (Rayleigh region) to the near saturation range.

The calculated deterministic coefficients \((R^2)\) were always better than 0.999 in cases of the tested nine samples. It demonstrates that the application of two hyperbolic components is sufficient and necessary for describing the normal magnetization curves of soft magnetic alloys.
The application of the model can have several additional advantages. Among others it can be used for determination of the real magnetic saturation state of the material and it can help to understand the physical sub processes of the magnetization.

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