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Magnetic moments in a helical edge can make weak correlations seem strong

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We study the effect of localized magnetic moments on the conductance of a helical edge. Interaction with a local moment is an effective backscattering mechanism for the edge electrons. We evaluate the resulting differential conductance as a function of temperature T and applied bias V for any value of V/T. Backscattering off magnetic moments, combined with the weak repulsion between the edge electrons results in a power-law temperature and voltage dependence of the conductance; the corresponding small positive exponent is indicative of insulating behavior. Local moments may naturally appear due to charge disorder in a narrow-gap semiconductor. Our results provide an alternative interpretation of the recent experiment by Li et al. 5 where a power-law suppression of the conductance was attributed to strong electron repulsion within the edge, with the value of Luttinger liquid parameter K fine-tuned close to 1/4.

Introduction - In search for topological insulators, the III-V semiconductor structures with band inversion appeared as a viable option. 2 The band inversion does occur in the type-2 heterostructure, InAs/GaSb. If the layers forming the well are narrow enough, the hybridization of states across the interface results in a formation of a gap; in the “topological” phase, the gap is accompanied by edge states free from elastic backscattering. These putative states became a target of an extensive set of measurements. 1 11 12 First, a surprisingly robust conductance quantization was found. 1 A later experiment explained the temperature-independent quantized conductance G as an inadvertent deviation from the linear-response regime. The observed power-law temperature and bias voltage dependence of the differential conductance was suggestive of insulating behavior. Assuming topologically protected edge states, it can be interpreted as a manifestation of strong-interaction physics: at low energies, even a single impurity can “cut” the edge, suppressing charge transport if the Luttinger parameter is very small, $K < 1/8$ ($K = 1$ corresponds to non-interacting electrons). Measurements yield $K \approx 0.22$ (with a 5% error), which is very close to the critical value of 1/4; an increase of $K$ by mere 12% would change the sign of $dG/dT$. Fine-tuning $K$ to such a stable value seems improbable, given the dependence of the edge state velocity on the gate voltages, varied in the experiment. The reliance on fine-tuning in the current explanation of the setup and qualitative description of the main results - We start by considering a single spin-1/2 magnetic moment $S$ coupled to a helical edge. The isolated edge is described by a Luttinger liquid Hamiltonian $H_0$; the local moment is coupled to the edge electrons by, generally, anisotropic exchange interaction. Separating out its isotropic part, the full time-reversal symmetric Hamiltonian of the coupled edge-impurity system can be written as

$$H = H_{iso} + \sum_{ij} \delta J_{ij} S_i s_j(x_0)$$  (1)

with $H_{iso}$ being the Hamiltonian with isotropic exchange:

$$H_{iso} = H_0 + J_0 S \cdot s(x_0)$$  (2)

Here, $S$ is the spin-1/2 impurity spin operator, and $s(x_0) = \frac{1}{2} \sum_{\alpha\beta} \psi_{\alpha}^{\dagger}(x_0) \sigma_{\alpha\beta} \psi_{\beta}(x_0)$ is the edge electron spin density at the position $x_0$ of the contact interaction with the local moment. (From hereon we will omit the position arguments.) We shall assume $\delta J_{ij} \ll J_0$ so that the exchange is almost isotropic. Thus we can treat the second term in Eq. (1) as a perturbation.

The first term in $H_{iso}$, Eq. (2) is the bosonized Luttinger-liquid Hamiltonian describing the interacting edge electrons, $H_{0} = (2\pi)^{-1} \int dx [\Pi^{2} + (\partial_{x} \varphi)^{2}]$; we assume the dimensionless exchange coupling parameter to be small, $\rho J_0 \ll 1$ (here $\rho$ is the electron density of states per spin per unit edge length). The bosonic fields commute as $[\varphi(x), \Pi(y)] = i\pi \delta(x-y)$. We have rescaled the fields by appropriate factors of $\sqrt{K}$; the bosonization identity is $\psi_{\beta} = (2\pi a)^{-1/2} e^{-i(2\sqrt{K} \varphi - \frac{\pi}{\sqrt{K}} \int_{-\infty}^{x} dx'' \Pi)}$ with $\beta = +/-$ for right/left movers (or spin up/down; we take z-axis to be the spin quantization axis of helical electrons at Fermi energy); $a$ is the short-distance cutoff. In bosonic representation, the spin density takes form $s_x = \pm s_y = \pm i(2\pi a)^{-1} e^{\pm 2i\sqrt{K} \varphi}$, $s_z = \frac{1}{2\pi \sqrt{K}} \Pi$. Using it,
we re-write the exchange interaction Hamiltonian as

$$J_0 s_x \rightarrow J_{\perp \Sigma \rightarrow SU(2)} + J_{\perp \Sigma \rightarrow U(1)} \frac{1}{2\pi s_x S_z \Pi}.$$  

Even though the bare Hamiltonian \( J_{\perp \Sigma \rightarrow SU(2)} \) is isotropic, \( J_{\perp \Sigma \rightarrow SU(2)} \) becomes anisotropic under renormalization group (RG) flow, as the scaling dimensions of the corresponding spin densities in Eq. \((3)\), \( \Delta_\perp = K \) and \( \Delta_z = 1 \), differ from each other, see also Eqs. \((4)\)–\((6)\) below. The isotropy breaking is not an artefact: anisotropy is already present in the bare Hamiltonian even at \( K = 1 \) due to the spin-orbit interaction; the Hamiltonian has no SU(2) symmetry but only a smaller U(1) symmetry (spin rotations about z-axis).

The weak-coupling \( \rho J_0 \ll 1 \) and \( 1 - K \ll 1 \) RG equations for \( J_{\perp \Sigma \rightarrow SU(2)} \) and \( J_{\perp \Sigma \rightarrow U(1)} \) (here \( E \) is the running cutoff)

$$\frac{dJ_{\perp \Sigma \rightarrow SU(2)}}{d\ln E} = -(1 - K)J_{\perp \Sigma \rightarrow SU(2)} - \rho J_{\perp \Sigma \rightarrow SU(2)}J_{\perp \Sigma \rightarrow SU(2)},$$  

$$\frac{dJ_{\perp \Sigma \rightarrow U(1)}}{d\ln E} = -\rho J_{\perp \Sigma \rightarrow U(1)}.$$  

The right-hand-side of the first equation starts at tree level with a coefficient \( 1 - \Delta_\perp = 1 - K \); the second equation does not have such a term since \( \Delta_z = 1 \). The terms second-order in \( J \) are due to the Kondo effect and can be derived from poor man scaling \( 1^2 \), or from an operator product expansion \( 1^2 \)

Starting from isotropic initial condition, \( J_0 > 0 \), Eq. \((1)\) shows that there are two regimes of parameters: \( \rho J_0 \ll 1 - K \) and \( \rho J_0 \gg 1 - K \). In the latter case \( 1 - K \) can be dropped from Eq. \((1)\), and the physics is similar to that of the case \( K = 1 \).

In this paper we focus on the opposite limit, \( \rho J_0 \ll 1 - K \). (Note, such initial condition can be satisfied even if the electron-electron interaction is weak, \( 1 - K \ll 1 \).) In this case the RG flow governed by Eqs. \((1)\)–\((5)\) can be divided into two regimes separated by an energy scale \( T^* \) (we use units \( k_B = \hbar = 1 \)) defined by the crossover conditions \( 1^1 \)

$$T^* = D \frac{1}{\sqrt{2} \left( 1 \left( 1 - K \right) \right)}.$$  

Here \( D \sim E_g \) is the bare cutoff which we take to be the bulk gap \( 1^2 \). At energies \( E \gg T^* \) one can ignore \( \rho J_{\perp \Sigma \rightarrow SU(2)}(E) \) in \((1)\), whereas at \( E \ll T^* \) one can ignore \( 1 - K \). Next, we discuss electron backscattering in the high energy limit, \( E \gg T^* \) where interaction \( K \neq 1 \) is important.

The backscattering current above \( T^* \) - The isotropic exchange Hamiltonian \( 2 \) alone does not backscatter edge electrons in steady state (DC bias) since each backscattering event is accompanied by an action of the nilpotent operator \( S_+ \) on the impurity spin polarized along \( z \)-axis \( 1^2 \). The presence of anisotropy in the exchange, Eq. \((1)\), gives rise to backscattering. This perturbation in Eq. \((1)\) can be treated using Fermi Golden Rule, assuming equilibrium impurity polarization \( S = z \cdot \tanh \left( \frac{1}{2} \right) \) \( 2^1 \). Integration over electron phase space volume leads to a backscattering current \( \delta I \sim e^2 V (\rho J_0 J)^2 \). We can find the full temperature and bias voltage dependence by solving for the renormalized coupling \( \delta J \). Since the pertinent constant \( \delta J \) couples to the spin-flip operators \( e^{\pm 2 i \vec{K} \cdot \vec{r}} \), it acquires a power-law energy dependence \( \delta J(E) = \left( \frac{D}{E} \right)^{1 - K} \delta J(D) \) for \( E > T^* \). Taking \( E \sim \max(T, eV) \), the \( T \) and \( V \) dependent backscattering becomes (valid at \( \max(T, eV) \gg T^* \))

$$\langle \delta I \rangle = e^2 \frac{h}{e} c T^{-2(1 - K)} \left[ \max(1, eV/T) \right]^{2(1 - K)},$$  

where constant \( c \) depends on the bare exchange tensor. Equation \((7)\) is a simplified version of our main result. Its detailed version, see Eq. \((7)\), reveals, in addition to \( eV/T \sim 1 \), yet another crossover in the current-voltage characteristic occurring at \( T^* \sim \rho J_0 \ll 1 \); it is associated with the details of impurity spin torque and relaxation, ignored in Eq. \((7)\).

Long edge conductance at energies above \( T^* \) - Let us now consider a long sample which may host many impurities near the edge. A single impurity contributes an amount \( \delta R \approx \delta G/G_0^2 \) to the edge resistance (here \( G_0 = e^2/h \) and \( \delta G = \delta (\delta I)/dV \)). In a long sample with \( N \) impurities we can simply add resistances if the impurities are dilute enough \( 2^1 \). The impurities dominate the edge resistance if \( N \delta G \gg G_0 \), where the same typical value \( \delta G \) for each impurity is used. In this case one finds \( G \approx G_0^2/N \delta G \) for the conductance of a single edge. Here \( \delta G \) is evaluated with the help of Eq. \((1)\) or its simplified version, Eq. \((7)\), both valid at \( \max(T, eV) > T^* \).

Using Eq. \((7)\), one finds a power-law dependence \( G(V, T) \sim (G_0/c N) \left[ \max(T, eV) \right]^{2(1 - K)} \). In Ref.\( \square \) the authors found a fit \( G \propto V^{0.37} \) in the regime \( eV > T \) for a sample of length \( L = 1.2 \mu m \) (see inset in Fig. 4 of Ref.\( \square \)). Matching with our theory of many impurities leads to \( 2(1 - K) \approx 0.37 \), or \( K \approx 0.82 \). Thus, in presence of many impurities, even moderately weak interactions can give rise to the power law seen in Ref.\( \square \). The two possible explanations (many impurities and weak interaction vs. single impurity and strong interaction) of the observed conductance predict different dependencies of \( G \) on the edge length: for many impurities one expects \( N \propto L \) and hence resistive behavior \( G \propto L^{-1} \). Although \( G(L) \) dependence is not reported in Ref.\( \square \), the earlier work found it to be linear at \( L > 10 \mu m \) \( 2^2 \). The presence of magnetic impurities may also be identified from their subtle effect on the non-linear I-V characteristics, which we discuss next.

Refinement of Eq. \((7)\) - The simplified form Eq. \((7)\) of the current-voltage characteristic misses several fine points relevant for the future analysis of experiments: (1) it does not provide the accurate form of the crossover at \( eV/T \sim 1 \), and (2) it does not reveal an additional crossover at smaller bias, \( eV/T \sim \rho J \). The latter crossover is associated with the precession of the local magnetic moment in the exchange field \( h \sim eV \rho J \).
FIG. 1. (Color online) Log-log plot of the scaled conductance, \( G(T) \), in the presence of many impurities, \( G \propto 1/\delta G \). Here \( \delta G = d\langle \delta I \rangle/dV \) and \( \alpha = 2 - 2K > 0 \) are taken from Eq. (14), valid at intermediate energies \( \max(T, eV) > T^* \). The conductance has two crossover scales in its \( V \)-dependence. The higher crossover is at \( eV \sim T \); above it, conductance increases (upon increasing \( V \)) asymptotically as a power law with exponent \( \alpha > 0 \) (dashed line). Below it, \( G \) stays roughly constant until the lower crossover scale, \( eV \sim \rho J_{\text{eff}}(T)T \), is reached. Below it, the conductance changes by a factor \( 1/b(T) > 1 \) that depends weakly on temperature, see discussion below Eq. (16). The inset shows \( G(V) \) at three different temperatures \( T \) (increasing from the lowest to highest curve).

duced by the spins of itinerant edge electrons under a finite bias.\(^{21}\) The crossover occurs once the precession frequency \( \propto h \) becomes comparable to the Korkringa relaxation rate\(^{22}\) \( 1/\tau_K \sim (\rho J)^2T \), as we will see in a detailed derivation of backscattering current.

The current operator of backscattered electrons is given by\(^{22}\) \( \delta I = -e\partial_t \delta N \) where \( \delta N = (N_L - N_R) \) is the difference between the number of left and right movers on the edge; it obeys \( [\delta N, s_i(x_0)] = i\varepsilon_{i,lm}s_m(x_0) \) and commutes with \( H_0 \). The decomposition\(^{23}\) of the Hamiltonian is useful because at zero frequency the Hamiltonian \( H_{\text{soa}} \), Eq. (2), does not lead to backscattering of helical edge electrons.\(^{22}\) It can be seen by noticing that: (i) \( \partial_t \langle S_z \rangle = 0 \) in a steady state, because \( S_z \) is bounded; this allows one to write the average backscattering current as\(^{22}\) \( \langle \delta I \rangle = -e\partial_t \langle S^\text{tot}_z \rangle = \delta J_{\text{xx}} \) with \( S^\text{tot}_z = \delta N + S_z \), and (ii) the operator \( S^\text{tot}_z \) commutes with \( H_{\text{soa}} \) and therefore is a conserved quantity in absence of \( \delta J_{\text{xy}} \). Hence \( \partial_t \langle S^\text{tot}_z \rangle \big|_{\delta J_{\text{xy}} = 0} = 0 \) and \( \langle \delta I \rangle \big|_{\delta J_{\text{xy}} = 0} = 0 \). We focus here on the case of a single magnetic moment; in the presence of many moments, we can define \( S^\text{tot}_z = \delta N + \sum_n s^{(n)}_z \) where the sum is over the localized spins \( S^{(n)} \). In this work, we ignore the effects of correlations between the localized spins and coherent backscattering, allowing us to simply add up single-moment contributions to the edge resistance. This is justified for dilute spins, as discussed in more detail in Ref.\(^{22}\)

From hereon, we consider scattering off a single spin, and express the average steady-state backscattered current as \( \langle \delta I \rangle = -e\partial_t \langle S^\text{tot}_z \rangle \). Commuting with the Hamiltonian\(^{24}\) leads to [we denote \( \delta J_{\text{xx}} = \delta J_{xy} - \delta J_{yy} + i(\delta J_{yz} + \delta J_{zy}) \), \( S_\pm = S_x \pm iS_y \) for brevity]

\[
\langle \delta I \rangle = e \sum_{i,j=x,y} \epsilon_{ijz} \left( \langle S_i : s_j : \rangle + \delta J_{ij} \langle S_i : s_j : \rangle \right) + e\text{Im}\delta J_{+} \langle s_+ : \rangle + \frac{1}{2} eV \langle \delta J_{yz} \langle S_x : \rangle - \delta J_{xz} \langle S_y : \rangle \rangle .
\]

(8)

In agreement with the presence of an integral of motion, the average current vanishes when \( \delta J \to 0 \). The averaging above is done with respect to the density matrix \( \rho \) with Hamiltonian\(^{22}\) in presence of a finite bias voltage, \( \rho \sim e^{-\beta(\hat{H} - eV S^\text{tot}_z)} \). We denote: \( s_j = s_j - \langle s_j \rangle \) with \( \langle \rangle \) being the thermal average in absence of exchange interaction, \( \langle \rangle \). The last term in (8) comes from the reducible part \( \langle s_j \rangle = \frac{1}{2} \delta J_{zz}\rho eV \).

Equation (8) is evaluated at time \( t \) long enough so that the steady-state value of \( \langle S \rangle \) has been reached. The averages \( \langle S_k : s_i : \rangle \) can be evaluated approximately in the exchange interaction assuming a separation of time scales for the itinerant electron and spin dynamics.\(^{25}\) The approximation results in

\[
\langle S_k : s_i : \rangle(t) \approx - \sum_j \langle \delta_{kj} J_{0} + \delta J_{kj} \rangle \frac{1}{2} e\text{Im}C_{ij} t
\]

\[
- \sum_{ij,n} \langle \delta_{ij} J_{0} + \delta J_{ij} \rangle \epsilon_{ikn} \langle s_n \rangle \text{Re}C_{ij} t .
\]

(9)

Here \( \langle S_n \rangle \) is the steady-state impurity spin polarization created by the current passing on the edge. The integrated correlation function \( \langle s_n \rangle = \int_0^\infty dt \langle s_n(t) : s_i(t') : \rangle \) depends on temperature and bias voltage (through the average \( \langle \ldots \rangle \)). The only non-zero components of the matrix of \( C_{ij} \) are the diagonals and \( C_{xy} = -C_{yx} \neq 0 \), the latter being due to finite bias voltage. The temperature and bias dependence of \( C_{ij} \) appearing in Eq. (9) can be moved into the \( T \) and \( V \) dependence of running couplings \( J_{ij}(T,V) \).\(^{22}\) Inserting Eq. (9) into Eq. (8) allows us to express the backscattering current in terms of the running couplings and steady-state values of the local-moment spin polarization \( \langle S \rangle \), see Ref.\(^{22}\) The last is found from the Bloch equation.\(^{22}\) At \( \delta J = 0 \), its only finite component is \( \langle S_z \rangle = \frac{1}{2} \text{tanh} \left( \frac{eV}{2T} \right) \) due to the \( U(1) \) symmetry. At the lowest-order in \( \delta J \) result for \( \langle \delta I \rangle \), we need to find \( \langle S_{x,y} \rangle \) to the first order in \( \delta J \). Unlike \( \langle S_z \rangle \), which is a function of \( eV/T \) given by thermodynamics, the components \( \langle S_{x,y} \rangle \) depend\(^{22}\) on both the effective field \( h_z = \frac{1}{2} eV \rho J_z \) generated by the bias voltage, and on the local-moment Korkringa relaxation rate \( \tau_K^{-1} = \frac{\pi^2}{2} \rho^2 \left( J_{z}^2 + \frac{\pi^2}{2} + J_{y}^2 \right) T \). (We use here the running couplings with their implicit dependence on \( V \) and \( T \).) The backscattering current is

\[
\langle \delta I \rangle = e \frac{\pi}{4} eV \rho^2 \left| \delta J_{+} (T,V) \right|^2 \left[ 1 + \frac{\pi^2}{4} eV \rho^2 \left| \delta J_{+} (T,V) \right|^2 \right] \times \sum_{i=z,y} \rho^2 \left( \delta J_{iz} (T,V) + \frac{J_{+} (T,V)}{J_{z} (T,V)} \delta J_{zz} (T,V) \right)^2 .
\]

(10)
Here the first term arises from non-zero $\langle S_z \rangle$ and can be derived simply from Fermi Golden Rule by assuming $\langle S \rangle = z^2_2 \tanh \frac{V}{2T}$. In the second term, function
\[ R(T, V) \approx \frac{J_x(T, 0)}{J_x(T)} + \frac{x^2}{1 + x^2}, \quad x = \frac{eV}{2T} \frac{2/\pi}{\rho J_{\text{eff}}(T)}, \] (11)
comes from $\langle S_x, y \rangle \neq 0$ and therefore depends on the ratio $h_T/K_T = x$. Here we abbreviated $\rho J_{\text{eff}}(T) = \rho J_L(T, 0)^2 + J_z(T, 0)^2 / J_x(T, 0) \ll 1$. In Eq. (11) the term $J_z/J_{\text{eff}} \lesssim 1$ only matters at very small bias $eV \ll T \rho J_{\text{eff}} \ll T$; thus we have neglected the $V$-dependence in it.

In Eq. (10) the current is written in terms of the running couplings $J_{ij}(T, V)$. Next, we will write it in terms of the bare couplings, which allows us to see explicitly the $T, V$-dependence of $\langle \delta I \rangle$. At $T^* < \max(eV, T) < D$ one has\(^2\)
\[ X(T, V) \approx X(D) \left( \frac{D}{2\pi T} \right)^{1-K} \sqrt{F\left( \frac{eV}{2T} \right)} \] (12)
with a function
\[ F(y) = KB(K+i\frac{y}{\pi}, K-i\frac{y}{\pi}) \sinh y \approx \frac{B(K, K)}{1 + A(K)y^2} \frac{1 - K}{1 - K}. \] (13)
Here $A(K) = \pi^{-2} \Gamma(K) \frac{2}{\pi^2}$ and $B$ is the Euler Beta function; $X$ stands for any of the quantities, Re$J_{++,}$, Im$J_{++}$, and $\delta J_{z1} + \frac{1}{2} \delta J_{z2}$ ($i = x, y$), which appear in Eq. (10).

Using Eqs. (10)–(13) we arrive at the central result of this paper: the temperature and bias dependence of the current can be lumped in a product of several simple terms,
\[ \langle \delta I \rangle = \delta G_0 \left[ \frac{D}{2\pi T} \right]^{2-2K} \frac{V}{[1 + A(K)\left( \frac{eV}{2T} \right)^2]^{1-K}} f(x, T), \]
\[ f(x, T) = \frac{b(T) + x^2}{1 + x^2}, \quad x = \frac{eV}{2T} \frac{2/\pi}{\rho J_{\text{eff}}(T)} \] (14)
Here the $T$-independent factor is $\delta G_0 = \frac{e^2}{\hbar} \frac{\pi}{4} \rho J_{\text{tot}}^2(D)$, $\delta J_{z+}^2(D) = [\delta J_{z+}(D)]^2 + \frac{1}{2} \sum_{i=x,y} [\delta J_{z1}(D) + \delta J_{z2}(D)]^2$, while $J_{\text{eff}}(T)$ and
\[ b(T) = 1 – \frac{1 - J_x(T, 0)}{J_x(T, 0)} \frac{1}{2} \sum_{i=x,y} [\delta J_{z1}(D) + \delta J_{z2}(D)]^2 \]
\[ \delta J_{z+}(D)^2 + \frac{1}{2} \sum_{i=x,y} [\delta J_{z1}(D) + \delta J_{z2}(D)]^2 \] (16)
display a weak temperature dependence. For typical values of exchange couplings $\delta J_{ij}(D)$ function $b(T)$ can be well approximated by a constant of order 1: $0.67 \leq b(T) \leq 0.83$ in the interval $T^* \leq T \leq T^{22}$. At a fixed temperature $T$, the current dependence on bias $V$ has two well-separated crossover scales described by the last two factors in (14). The smaller scale, $V \sim T \rho J_{\text{eff}}(T)$, is associated with the impurity spin dynamics. The crossover at the higher scale, $V \sim T$, occurs between the linear and weakly-nonlinear ($\delta I$) vs. $V$ dependencies. Near this crossover one may set $f \rightarrow 1$ in Eq. (14), reproducing the result of Eq. (17) with, however, accurate crossover behavior near $eV \sim T$.

**The backscattering current at energies below $T^*$ -** At energies $E \lesssim T^*$, one may neglect the small term $\propto (1 - K)$ in (14) and consider the result backscattering Kondo RG with the initial condition $p_L(T^*) = \sqrt{2} (1 - K)^{22}$. For small $1 - K$, it yields the Kondo temperature $T_K \sim T^* e^{-1/\sqrt{2} (1 - K)} \ll T^*$. The RG flow erases the uniaxial anisotropy created by $K \neq 1$, and $J_z \approx J_1$ at energies below $T^*$. As a result, $J_{\text{eff}} = 2 J_1$ in Eq. (11) and $R = (1/4 + x^2)/(1 + x^2)$. Similarly, the anisotropic perturbation in Eq. (1) becomes RG-irrelevant, and Eq. (12) is replaced\(^{11}\) by $X(E) \approx X(T) \ln E/T_K$. Hence, the backscattering current becomes
\[ \langle \delta I \rangle = \delta G_0 V \left[ \frac{\ln \max(T, eV)}{T_K T_K} \right]^2 b(T) + x^2 \frac{1}{1 + x^2} \] (17)
valid for $T_K < \max(eV, T) < T^*$. Here $b$ is given by Eq. (16) which becomes independent of $T$ upon setting $J_{\text{eff}} \approx 2 J_2$. Similarly, $\delta G_0$ was introduced below Eq. (14) but now one must use $\delta J_{z+}(T^*)$ in it with the “new” bare cutoff.

The coupling constant $\rho J_1(E) \sim [\ln(E/T_K)]^{-1}$ grows in the course of RG, and below the Kondo temperature, $\max(T, eV) < T_K$, Eqs. (11), (14) are no longer valid. In this regime one can use the phenomenological local-interaction Hamiltonian\(^{27,28}\) to obtain $\delta G(V, T) \propto T^4 g(V/T)$; the crossover function $g(x)$ has asymptotes $g(x \rightarrow 0) = \text{const}$ and $g(x \gg 1) \sim x^4$. Details can be found in Ref.\(^{29}\) upon setting $K = 1$ therein. Note that $\delta G$ decreases when reducing $T, eV$ and thus leads to $G = e^2/h$ in the limit of zero temperature and bias. This behavior is opposite from Eq. (14) which indicated an insulating edge at low energies.

**Conclusions -** We analyzed the joint effect of two weak interactions on the edge conduction in a 2D topological insulator. These interactions are: the repulsion between itinerant electrons of an edge state, and their exchange with the local magnetic moments. This joint effect may result in a seemingly insulating behavior of the edge conduction down to a low temperature scale $T^*$, see Eq. (10): at $T, eV \gtrsim T^*$, the single-impurity backscattering current $\langle \delta I \rangle$ grows as a power law upon lowering temperature or bias, see Eq. (14), or Fig. 1 for the conductance in presence of many moments. Localized magnetic moments may appear in a narrow-gap semiconductor as a consequence of charge disorder.\(^{11}\) Scattering off magnetic moments provides an alternative explanation of the recent experiment\(^{11}\), assuming $T^*$ is below the temperature range explored in.\(^{11}\) [None of the considered interactions break the time-reversal symmetry\(^{28}\) so at low energies, $\max(T, eV) \ll T^*$, backscattering is suppressed,]
see Eq. (17). The developed theory is also applicable to magnetically-dopedQ heterostructures. Finally, we find two crossovers in the I-V characteristics: the main one occurs at $eV \sim T$; a more subtle one occurs at lower bias, $eV \sim \rho JT$, see Fig. 1. Its observation in future experiments may provide evidence for the considered mechanism of the edge state excess resistance.

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References:

1. T. Li, P. Wang, H. Fu, L. Du, K. A. Schreiber, X. Mu, X. Liu, G. Csathy, X. Lin, and R.-R. Du, Phys. Rev. Lett. 115, 136804 (2015).

2. C. Liu, T. L. Hughes, X.-L. Qi, K. Wang, and S.-C. Zhang, Phys. Rev. Lett. 100, 236601 (2008).

3. I. Knez, C. T. Rettner, S.-H. Yang, S. S. P. Parkin, L. Du, R.-R. Du, and G. Sullivan, Phys. Rev. Lett. 112, 026602 (2014).

4. E. M. Spanton, K. C. Nowack, L. Du, G. Sullivan, R.-R. Du, and K. A. Moler, Phys. Rev. Lett. 113, 026804 (2014).

5. L. Du, I. Knez, G. Sullivan, and R.-R. Du, Phys. Rev. Lett. 114, 096802 (2015).

6. F. Nichele, H. J. Suominen, M. Kjaergaard, C. M. Marcus, E. Sajadi, J. A. Folk, F. Qu, A. J. A. Beukman, F. K. de Vries, J. van Veen, S. Nadj-Perge, L. P. Kouwenhoven, B.-M. Nguyen, A. A. Kiselev, W. Yi, M. Sokolich, M. J. Manfra, E. M. Spanton, and K. A. Moler, ArXiv e-prints (2015), arXiv:1511.01728 [cond-mat.mes-hall].

7. J. C. Xu and J. E. Moore, Phys. Rev. B 90, 035414 (2014).

8. C. Xu and J. E. Moore, Phys. Rev. B 73, 045322 (2006).

9. J. I. Väyrynen, M. Goldstein, and L. I. Glazman, Phys. Rev. Lett. 110, 216402 (2013).

10. J. I. Väyrynen, M. Goldstein, Y. Gefen, and L. I. Glazman, Phys. Rev. B 90, 115309 (2014).

11. D. Sénéchal, in *Theoretical Methods for Strongly Correlated Electrons*, CRM Series in Mathematical Physics, edited by D. Sénéchal, A.-M. Tremblay, and C. Bourbonnais (Springer New York, 2004), pp. 139–186.

12. D.-H. Lee and J. Toner, Phys. Rev. Lett. 69, 3378 (1992).

13. A. Furusaki and N. Nagaosa, Phys. Rev. Lett. 72, 892 (1994).

14. J. Maciejko, C. Liu, Y. Oreg, X.-L. Qi, C. Wu, and S.-C. Zhang, Phys. Rev. Lett. 102, 256803 (2009).

15. J. Cardy, *Scaling and renormalization in statistical physics*, Vol. 5 (Cambridge university press, 1996).

16. P. Anderson, Journal of Physics C: Solid State Physics 3, 2436 (1970).

17. T. Giamarchi, *Quantum Physics in One Dimension*, International Series of Monographs on Physics (Clarendon Press, 2003).

18. In general, the cutoff depends on the microscopic characteristics: the main one occurs at $eV \sim T$; a more subtle one occurs at lower bias, $eV \sim \rho JT$, see Fig. 1. Its observation in future experiments may provide evidence for the considered mechanism of the edge state excess resistance.

19. The bias voltage creates an imbalance between left and right movers and a non-zero $\langle S_z \rangle$ through the exchange interaction (2).

20. See Supplemental Material for details. The supplement includes references to 21–38.

21. Recently, in the topologically trivial regime (but still edge-dominated) $G \propto L^{-1}$ has been observed even at sub-micron length (13).

22. J. Korringa, Physica 16, 601 (1950).

23. C. L. Kane and M. P. A. Fisher, Phys. Rev. Lett. 72, 724 (1994).

24. F. Bloch, Phys. Rev. 70, 460 (1946).

25. T. L. Schmidt, S. Rachel, F. von Oppen, and L. I. Glazman, Phys. Rev. Lett. 108, 156402 (2012).

26. N. Lezmy, Y. Oreg, and M. Berkooz, Phys. Rev. B 85, 235304 (2012).

27. We disregard here the possibility of spontaneous symmetry breaking (19).

28. T. Jungwirth, J. Sinova, J. Mašek, J. Kučera, and A. H. MacDonald, Rev. Mod. Phys. 78, 809 (2006).

29. We expect $T^* \sim 0.9mK$ for a Mn-doped InAs-based heterostructure (11). To arrive at this estimate we used the following parameters: exchange coupling $J/a_0^3 \sim 1eV$ per unit cell volume, exchange constant $a_0 \approx 0.6nm$, quantum well thickness $d \approx 11.5nm$, edge state velocity $v \approx 5.7 \times 10^{6}m/s$, penetration depth $\xi \approx 16nm$, $E_g \approx 54K$, and $K \approx 0.8$.

30. H.-P. Breuer and F. Petruccione, *The theory of open quantum systems* (Oxford University Press on Demand, 2002).

31. I. Gradshteyn, I. Ryzhik, and A. Jeffrey, *Table of Integrals, Series and Products 5th edn* (New York: Academic) (1994).

32. T. Giamarchi and H. J. Schulz, Phys. Rev. B 37, 325 (1988).

33. I. V. Gornyi, A. D. Mirlin, and D. G. Polyakov, Phys. Rev. Lett. 95, 046404 (2005).

34. M. Dyakonov, Solid State Communications 92, 711 (1994).

35. J. Maciejko, Phys. Rev. B 85, 245108 (2012).

36. E. Eriksson, Phys. Rev. B 87, 235414 (2013).

37. B. L. Altshuler, I. L. Aleiner, and V. I. Yudson, Phys. Rev. Lett. 111, 086401 (2013).

38. O. M. Yevtushenko, A. Wugalter, V. I. Yudson, and B. L. Altshuler, EPL (Europhysics Letters) 112, 57003 (2015).

39. Q.-Z. Wang, X. Liu, H.-J. Zhang, N. Samarth, S.-C. Zhang, and C.-X. Liu, Phys. Rev. Lett. 113, 147201 (2014).