Time Crystals Protected by Floquet Dynamical Symmetry in Hubbard Models

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We investigate an unconventional symmetry in time-periodically driven systems, the Floquet dynamical symmetry (FDS). Unlike the usual symmetries, the FDS does not give conserved quantities in the strict sense, but preserves some quantum coherence and leads to two kinds of time crystals: the commensurate and incommensurate time crystals that have different periodicity in time. We show that these time crystals appear in the Bose- and Fermi-Hubbard models under ac fields and their commensurability can be tuned only by adjusting the strength of the field. These time crystals arise only from the FDS and thus appear in both dissipative and isolated systems and in the presence of disorder as long as the FDS is respected. We discuss their experimental realizations in coldatom experiments and generalization to the SU(N)-symmetric Hubbard models.

Introduction.— Symmetry is a key concept in physics, presenting us with various information such as conserved quantities, phase transitions and critical phenomena [1], and topological nature [2]. Even out of equilibrium, dynamics and nonequilibrium properties are governed by symmetries. In particular, (time-)periodically driven (Floquet) systems [3,5] involve novel symmetries without equilibrium counterparts due to the additional discrete time-translation symmetry, and these symmetries give rise to, for example, the Floquet (discrete) time crystals [6,20]. Floquet symmetry protected topological phases [21–24], selection rules of high-harmonic generation in solids [25,26], and so on.

Unlike the usual symmetries leading to the conserved quantities, there exist unconventional symmetries characterizing the nonequilibrium dynamics. The dynamical symmetry [27,31] in time-independent systems is one of them, ensuring the existence of a physical quantity that keeps oscillating periodically in time. This means the breaking of the continuous time-translation symmetry $\mathbb{R}$ down to the discrete one characterized by integers $\mathbb{Z}$, leading to a time-crystalline state [27,29,52,37]. The mechanism of this time crystal is different from the conventional Floquet time crystals, which occur in periodically driven systems. The Floquet time crystals are characterized by the breaking of the discrete time-translation symmetry $\mathbb{Z}$ down to its subgroup such as $\mathbb{Z}/2$.

In this Letter, we investigate an unconventional symmetry in periodically driven systems, the Floquet dynamical symmetry (FDS), showing that the FDS governs the long-time behavior of the system. We show that the FDS protects quantum coherence between energetically-separated subspaces and leads to two kinds of time crystals: the commensurate and incommensurate time crystals (CTC and ITC) (the latter is also called as time quasicrystals [19]). These time crystals both break the discrete time-translation symmetry, but are different in that a perfect periodicity is retained or not (see Fig. 1). We show that these time crystals appear in various Hubbard models under an ac field and their commensurability can be tuned only by adjusting the strength of the field.

Floquet dynamical symmetry and time crystals.— We begin by considering periodically-driven dissipative systems for a well-defined formulation, and will discuss isolated systems later. For this purpose, we focus on the Floquet-Lindblad master equation [38–41] ($\hbar = 1$ throughout this Letter):

$$\frac{d\rho}{dt} = \mathcal{L}_t(\rho) = -i[H(t), \rho] + \gamma \sum_k \left( L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\} \right),$$

which describes trace-preserving nonunitary dynamics for the density matrix $\rho$. Here, $H(t) = H(t + T)$ is a time-periodic Hamiltonian describing the unitary part of the evolution, and $L_k$’s are the Lindblad operators representing the Markovian dissipation by an environment coupling to the system.

The solution of Eq. (1) is formally given by $\rho(t) = \mathcal{V}(t, 0) \rho(0)$, where $\mathcal{V}(t, t') = \text{Tr}(\int_{t'}^{t} \mathcal{L}_s ds)$ is the time evolution superoperator from $t'$ to $t$. Due to the periodicity of $\mathcal{L}_t = \mathcal{L}_{t + T}$, we can decompose $\mathcal{V}(t, 0)$ into a stroboscopic evolution and a micromotion: $\mathcal{V}(t, 0) = \mathcal{V}(\tilde{t}, 0) \mathcal{U}_F$, where $\tilde{t} = t + \ell T$ (0 $\leq \ell < T$, $\ell \in \mathbb{N}$), and $\mathcal{U}_F = \mathcal{V}(T, 0)$.
is the one-cycle time evolution superoperator. The long-
time behavior (\( t \to \infty \)) is characterized by \( \mathcal{U}_F \). If \( \mathcal{U}_F \) is diagonalizable \( \mathcal{U}_F(\rho_j) = z_j \rho_j \) \( (j = 1, \cdots, D^2 \) and \( D \) is the Hilbert-space dimension), the formal solution of Eq. (1) is given by \( \rho(t) = \mathcal{V}(t, 0) \sum_j a_j z_j^\ell \rho_j \), where \( a_j \) is an expansion coefficient of \( \rho(0) = \sum_j a_j \rho_j \). Note that the trace-preserving nature guarantees \( |z_j| \leq 1 \) and there exists at least one eigenvalue \( z_1 = 1 \) [42].

The unique nonequilibrium steady state (NESS) appears if all the other eigenvalues except \( z_1 = 1 \) satisfy \( |z_j| < 1 \), meaning that \( \rho_j \neq 1 \) are all decaying modes with relaxation time \( 1/|z_j| \) [43]. In fact, any initial state asymptotically relax to a unique NESS, \( \rho(t) = \mathcal{V}(t, 0) \sum_j a_j z_j^\ell \rho_j \overset{t \to \infty}{\longrightarrow} \mathcal{V}(0) \rho_1 \) \( (a_1 = 1 \) due to the trace preservation). This NESS has time-period \( T \) by definition of \( t = \hat{t} + c T \), which means that the long-time behavior of Eq. (1) typically has the discrete time-transformation symmetry \( Z \).

The conventional discrete time crystals [12,13] appear if there exist several eigenstates with eigenvalues \( e^{i\theta} \) \( (\theta = 2\pi n/n, n \in \mathbb{N}) \). For example, when \( z_1 = 1, z_2 = -1, \) and \( |z_j| < 1 \), any initial state asymptotically relax to the following NESS: \( \rho(t) = \mathcal{V}(t, 0) \sum_j a_j z_j^\ell \rho_j \overset{t \to \infty}{\longrightarrow} \mathcal{V}(0) \rho_1 - \rho_2 \). When \( a_2 \neq 0 \), the NESS is a time-crystalline state with time-period \( 2T \), which implies \( Z \to Z/2 \) symmetry breaking. From now on, we call the eigenstate with eigenvalue 1 as a steady state, and that with eigenvalue \( e^{i\theta} \) \( (\theta \in \mathbb{R}) \) as a coherent state.

Now we introduce the Floquet dynamical symmetry (FDS), which leads to unconventional time crystals. First, we define the FDS for \( \mathcal{U}_F \) by

\[
\mathcal{U}_F(A \rho) = e^{-i\lambda T} A \mathcal{U}_F(\rho), \quad \forall A
\]

\[
\mathcal{U}_F(\rho A^\dagger) = e^{i\lambda T} \mathcal{U}_F(\rho) A^\dagger, \quad \forall \rho
\]

where \( A \) is a dynamical symmetry operator, and \( \lambda \) is a real number. This definition is a natural extension of the strong dynamical symmetry in time-independent systems [27,29], which is defined as \([H, A] = \lambda A \) and \([L_k, A] = [L_k^\dagger, A] = 0 \) \( (e.g., \) the Zeeman Hamiltonian \( H = \lambda S^z \) and the raising operator \( A = S^+ \) satisfy this relation, \([H, A] = \lambda A \). One can easily show that, in the Floquet systems, when the strong dynamical symmetry holds at any time, there exists the FDS, but the converse is not true. We note that the dissipation \( L_k \) is not required for the FDS, and the time crystals can appear even in isolated systems as we will show later.

The FDS protects some quantum coherence, and prevents the quantum state from relaxing to the unique NESS. Given that \( \rho_s \) is a steady state satisfying \( L \rho_s = \rho_s \), \( \rho_{mn} = A^n \rho_s (A^\dagger)^m \) are the steady \((m = n)\) and coherent \((m \neq n)\) states,

\[
\mathcal{U}_F(\rho_{mn}) = e^{i(n-m)\lambda T} \rho_{mn}.
\]  

If there are no other steady and coherent states except \( \rho_{mn} \), we obtain the long-time behavior from them,

\[
\rho(t) \overset{t \to \infty}{\longrightarrow} \sum_{mn} \mathcal{V}(t, 0) c_{mn} e^{i(n-m)\lambda T} \rho_{mn} \equiv \rho_\infty(t), \quad (4)
\]

where \( c_{mn} \) is expansion coefficients of \( \rho_{mn} \). There exist two typical energy (time) scales in \( \rho_\infty(t) \). One is the Floquet frequency \( \omega = 2\pi/T \) stemming from the periodicity of \( \mathcal{V}(t, 0) \), and the other is \( \lambda \) characterized by the FDS.

Depending on whether the ratio \( \lambda/\omega \) is a rational number or not, the long-time behavior (4) represents the CTC or ITC, respectively. If \( \lambda/\omega \in \mathbb{Q} \), i.e., \( \lambda/\omega = q/p \) for some coprime integers \( p \geq 2 \) and \( q \), the CTC emerges:

\[
\rho_\infty(t + T) = \rho_\infty(t), \quad \text{but} \quad \rho_\infty(t + pT) = \rho_\infty(t) \text{ holds true.}
\]

This means the discrete time-translation symmetry breaking, \( Z \to \mathbb{Z}/p \). On the other hand, if \( \lambda/\omega \not\in \mathbb{Q} \), the discrete time-translation symmetry \( Z \) is broken, but there is no integer \( p \) such that \( \rho_\infty(t + pT) = \rho_\infty(t) \). Nevertheless, there exist an infinite number of times \( s \) such that \( \rho_\infty(t + s) \) is arbitrarily close to \( \rho_\infty(t) \). Thus, the dynamics is quasiperiodic, and we call it the ITC.

**Tunable time crystals in dissipative Hubbard models.—** Here we present simple models exhibiting the time crystals protected by the FDS: spin-\( S \) Bose- or Fermi-Hubbard models driven by a circularly polarized ac field in \( d \) dimensions. The Hamiltonian \( H(t) = H_0 + V(t) \) is given by

\[
H_0 = -J \sum_{(i,j),\sigma} (c^\dagger_{i,\sigma} c_{j,\sigma} + \text{h.c.}) + \frac{U}{2} \sum_j n_j^2 + \frac{K}{2} \sum_{(i,j)} n_i n_j,
\]

\[
V(t) = B \left( S^x \cos \omega t + S^y \sin \omega t \right),
\]

where \( c_{i,\sigma} (c^\dagger_{i,\sigma}) \) is the annihilation (creation) operator for the boson or fermion with spin \( \sigma \in \{ -S, \cdots, S \} \) on the site \( j \), and \( n_j = \sum_{\sigma} c^\dagger_{i,\sigma} c_{i,\sigma} \) is the particle number operator. The operators for the spin at site \( j \) and for the total spin are denoted by \( S^\mu_j \) and \( S^\mu = \sum_j S^\mu_j (\mu = x, y, z) \). The strength of the hopping amplitude, the onsite interaction, and the nearest neighbor interaction are denoted by \( J, U, \) and \( K \), respectively. The nearest neighbor interaction is added to elucidate below that our results do not rely on the integrability. In the ac Zeeman coupling term \( V(t) \), \( B \) and \( \omega \) are the strength and frequency of the magnetic field, respectively [44].

We assume that the dissipation is described by local dephasing Lindblad operators acting on each site, \( L_j = n_j \), which suppresses the particle number fluctuation. We will discuss later how to realize them experimentally.

Our model has the FDS in one cycle of the ac field. To show this, we consider a unitary transformation to the rotating frame [25,47], \( \rho_{RF}(t) = R(t) \rho(t) R(t)^\dagger \) \( (R(t) = e^{i\omega t S^z}). \) Then, the Hamiltonian is transformed to the Hubbard model in an effective static magnetic field, \( H_{RF} = R(t)[H(t) - i\partial_t]R(t)^\dagger = H_0 + \mathbf{h} \cdot \mathbf{S}, \)
where \( h = (B, 0, \omega) \), while the Lindblad operators are invariant. Therefore, in the rotating frame, the system has the strong dynamical symmetry \([27, 29]\), or the FDS \( \mathcal{U}_F^R(\rho) = e^{-i\lambda T} \mathcal{A} \mathcal{U}_F^R(\rho) \mathcal{A}^\dagger \) and \( \mathcal{U}_F^R(\rho A^\dagger) = e^{i\lambda T} \mathcal{U}_F^R(\rho A) A^\dagger \), where \( \mathcal{U}_F^R \) is the one-cycle time evolution superoperator in the rotating frame, \( A = S_h^+ \) is the total spin raising operator along the effective magnetic field \( \mathbf{h} \), and

\[
\lambda = |\mathbf{h}| = \sqrt{\omega^2 + B^2} \quad (\text{mod } \omega). \tag{6}
\]

Going back to the original frame, we have \( \mathcal{U}_F(\rho) = R(\omega) \mathcal{U}_F^R(\rho) R(\omega) \) due to \( R(0) = 1 \) and \( R(T) = (-1)^{2SV} \) (\( V \) is the total number of sites), obtaining the FDS

\[
\begin{align*}
\mathcal{U}_F(S_h^+ \rho) &= e^{-i\lambda T} S_h^+ \mathcal{U}_F(\rho), \quad \forall \rho \\
\mathcal{U}_F(\rho S_h^-) &= e^{i\lambda T} \mathcal{U}_F(\rho) S_h^- \quad \forall \rho
\end{align*} \tag{7}
\]

We note that the symmetry operator \( A = S_h^+ \) depends on the time origin \( t_0 \), which has been set to 0 above, and is generally given by \( A(t_0) = S_h^+ \) with \( \mathbf{h}(t_0) = (B \cos \omega t_0, B \sin \omega t_0, \omega) \).

This FDS gives the steady and coherent states \( \rho_{mn} = (S_h^+)^m \rho_s (S_h^-)^n \) with eigenvalue \( e^{i(n-m)\lambda T} \) in Eq. (3). This implies that the FDS protects the coherence between the different spin sectors labelled by \( S_h = h \cdot \mathbf{S}/|\mathbf{h}| \), which are Zeeman-split by \( \lambda \) (see Fig. 2 (a)). Furthermore, the system has another intrinsic energy scale \( \omega \) of the drive frequency. These two scales, \( \omega \) and \( \lambda \), interfere with each other and give rise to the CTC and ITC.

We emphasize the tunability of our time crystals. As shown before, the commensurability and the period of these time crystals depend on the ratio \( \lambda/\omega = \sqrt{1+(B/\omega)^2} \), which can be tuned only by varying the field strength \( B \) with \( \omega \) fixed. If \( \lambda/\omega = q/p \) (\( \leftrightarrow B = \omega \sqrt{(q/p)^2-1} \)) with coprime integers \( p \) and \( q \), the CTC appears with period \( pT \), whereas, if \( \lambda/\omega \notin \mathbb{Q} \), the ITC appears.

**Synchronized CTC and ITC in the Hubbard model.**—Let us numerically demonstrate the CTC and ITC by taking the spin-1/2 Fermi-Hubbard model in one dimension with \( L \) sites. We assume the periodic boundary condition. Throughout this Letter, the initial state is a 1/4-filled state where the \( j \)-th site \((j = 1, 2, \ldots, L/2) \) is occupied by one fermion, and every third fermions are polarized along \( x \) and all others are polarized along \( x \) (we assume \( L \) is a multiple of 6).

Figure 2 (b) shows the time evolution of \( \langle S_y^0(t) \rangle \) in the CTC phase for \( \omega = \pi \) and \( \lambda = 2\pi/3 \). After the initial relaxation dynamics, \( \langle S_y^0(t) \rangle \) oscillates with period \( T = 2\pi/\omega = 2 \) and \( 2\pi/\lambda = 3 \). This means the discrete time-translation symmetry breaking \( \mathbb{Z} \to \mathbb{Z}/3 \). Moreover, the dynamics of \( \langle S_y^0(t) \rangle \) and \( \langle S_y^1(t) \rangle \) are synchronized after relaxation, which implies all the steady and coherent states are translationally symmetric \([27, 29]\). The Fourier component of \( \langle S_y^0(t) \rangle \) is shown in Fig. 2 (c), where there exist two sharp peaks at \( f = \omega \) and \( \lambda \). The rationality of the two peaks, \( f = \omega \) and \( \lambda \), leads to the commensurability of the time crystal.

On the other hand, Fig. 2 (d) shows the time evolution in the ITC phase for \( \omega = \pi \) and \( \lambda = (\sqrt{2} - 1)\pi \). As shown in the figure, the synchronized spin dynamics oscillate aperiodically. The irrationality of two peaks, \( f = \omega \) and \( \lambda \), in the Fourier space (Fig. 2 (e)) shows the incommensurability of the time crystal and no perfect periodicity.

The trajectories of the CTC and ITC dynamics in the \( \langle S_y^0, S_y^1 \rangle \)-plane are shown in Fig. 2 (f). The trajectory of the CTC dynamics (red) behaves as the limit cycle, and gradually converges to the star-shaped closed curve.
On the other hand, the trajectory of the ITC dynamics (blue) never converges to a closed curve, and keep rotating aperiodically on the plane. This aperiodic dynamics highlights the difference from the CTC.

**Quantum-thermalization-induced time crystals without dissipation.**—Until now, we have considered the dissipative systems to clarify the role of the FDS and the mechanism of the time crystals. However, the time crystals do not necessarily require the dissipation. According to the recent studies [48–50], an isolated quantum system without dissipation exhibits thermalization if nonintegrable. Here we show that the CTC and ITC protected by the FDS also occur in the isolated Hubbard model due to this quantum thermalization.

Figures 3 (a) and (b) shows the results without the dissipation for $\omega = \pi$ and $\lambda = 2\pi/3$. In contrast to the non-interacting (integrable) case (Fig. 3 (b)), in the interacting (nonintegrable) case (Fig. 3 (a)), the spin dynamics at the different sites are synchronized, and the time crystal appears. The quantum thermalization effectively plays the role of dissipation and eliminates quantum coherence except those protected by the FDS, bringing about the time crystals.

These synchronized time crystals in isolated systems are interpreted by the maximum entropy principle under multiple conserved quantities [51] that is also known as the generalized Gibbs ensemble (GGE) [52]. The key to this interpretation is the unconventional conserved quantities that are derived from the dynamical symmetry and conserved only stroboscopically [28]. In the rotating frame, one can easily show that $S^\pm$ are such quantities: $S^\pm_H(s + nT_\lambda) = S^\pm_H(s)$ in the Heisenberg picture for each $s \in [0, T_\lambda)$ with $T_\lambda = 2\pi/\lambda$ and $n \in \mathbb{N}$. Thus, each stroboscopic evolution from $t = s$ leads, after a long time, to the GGE characterized by $S^\pm_H$: $\rho_{RF}(t) = \exp(-\sum_j \beta_j Q_j - \mu(t) S^+_H - \mu(t)^* S^-_H)/Z_t$. Here $Z_t = Z_{t+T_\lambda}$ is the periodic partition function, $\mu(t) = \mu(t + T_\lambda)$ is the periodic chemical potential for $S^\pm_H$, and $Q_j$ and $\beta_j$ are the conventional (not stroboscopic) local conserved quantities and their generalized inverse temperatures. The noncommutativity between the conserved quantities does not affect the foundation of the GGE [51]. Going back to the original frame, we obtain a time-dependent GGE $\rho_{TC}(t) = R^\dagger(t) \rho_{RF} R(t)$ as follows:

$$\rho_{TC}(t) = \frac{\exp\left(-\sum_j \beta_j Q_j(t) - \mu(t) S^+_H(t) - \mu(t)^* S^-_H(t)\right)}{Z_t},$$

where $Q_j(t) = R^\dagger(t) Q_j R(t)$ and $S^\pm_H(t) = R^\dagger(t) S^\pm_H R(t)$ (see also below Eq. (7)).

The time-dependent GGE (8) is a two-color generalization of the previous ones [28, 57]. Whereas $Q_j(t)$ and $S^\pm_H(t)$ have the period $T = 2\pi/\omega$ of the external field, $\mu(t)$ has a different one $T_\lambda = 2\pi/\lambda$ of the FDS. These two periods, depending on their ratio, give the CTC and ITC in isolated systems. The synchronization derives from the translation symmetry of $Q_j(t)$ and $S^\pm_H(t)$.

**Robustness against disorder.**—Finally, we show that the time-crystalline nature is robust against perturbations as long as the FDS is preserved, although the synchronization may become imperfect. To illustrate this, we consider the disordered onsite potential $V_d = \sum_{j,n} \epsilon_{j,n}$, where $\epsilon_{j,n}$'s are independent random variables following the uniform distribution over $(-\delta, \delta)$. This disorder respects the FDS due to the spin-SU(2) symmetry, but breaks the spatial translation symmetry.

In the presence of the dissipation $L_j = n_j$, not only the time-crystalline nature but also the synchronization are robust against the disorder. Figure 3 (c) shows the dynamics of $\langle S^\gamma_j \rangle$ ($j = 1, 3$) in the disordered Hubbard model with dissipation for the CTC phase, $\omega = \pi$ and $\lambda = 2\pi/3$, and thus the period is $3T$. As in the case without disorder (Fig. 3 (b)), the two spin dynamics become synchronized to oscillate with period $3T$. This synchronization is caused by the dissipation $L_j = n_j$ suppressing the particle number fluctuations to realize the translationally symmetric state.

On the other hand, in the absence of dissipation, the perfect synchronization does not occur, but the time-
crystalline nature persists. Figure 3(d) shows the dynamics of $\langle S_j^y \rangle$ ($j = 1, 3$) in the disordered Hubbard model without dissipation for the CTC phase with period $3T$. Unlike the dissipative system, the amplitudes of the two spin dynamics are different and have fluctuations in time. However, the rhythms of the oscillations are synchronized, and the time crystal with period $3T$ appears. This spatial inhomogeneity can be understood by the quantum thermalization. Since a state thermalizes to an equilibrium state of the disordered Hamiltonian, the realized state is not translationally symmetric.

Discussions and conclusions.— In this Letter, we have introduced the FDS and proposed a new class of time crystals protected by the FDS. In this class, the time-crystalline nature is determined only by the FDS and robust against perturbations respecting the symmetry. We also have demonstrated that in the Hubbard model driven by the circularly polarized ac field.

The time crystals also appear in the Hubbard model with a static and linearly polarized ac fields both along the $z$-direction. This is because a trivial dynamical symmetry holds at any time, $[H(t), S^+] \propto S^+$. This setup is known as the electron spin resonance (ESR) in the condensed matter physics [58].

Our model (5) can be realized in ultracold atoms on an optical lattice [59, 60]. The hyperfine states of the atom behave as a pseudospin, and the coupling between the states, or the Zeeman effect on the pseudospin, is manipulated by radio waves [60]. State-of-the-art laser technology enables us to control the strength and frequency of the coupling, realizing the highly tunable time crystals. Also, the particle number dephasing $L_1 = n_1$ is achieved by immersing the optical lattice into a Bose-Einstein condensate [61, 62].

Finally, we note that our model (5) actually has the SU($N$) symmetry with $N = 2S + 1$. [63, 65]. Such an SU($N$)-symmetric Hubbard model has been realized in ultracold atoms [66–68], and has attracted much attention recently. Generalizing our rotating-frame arguments to the SU($N$) algebra is an interesting open question, where time crystals should meet large Lie algebras together with coldatom experiments.

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Note added.— In preparing this manuscript, we have become aware of an independent work [30], where the FDS and its consequences are briefly discussed.

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