Quasinormal Modes of Static Modified Gravity (MOG) Black Holes

Luciano Manfredi¹, Jonas Mureika¹, John Moffat²
1. Department of Physics, Loyola Marymount University, Los Angeles, CA USA
2. Perimeter Institute for Theoretical Physics, Waterloo, ON CA/N
E-mail: lmanfred@lion.lmu.edu; jmureika@lmu.edu; jmoffat@perimeterinstitute.ca

Abstract. Using an asymptotic iteration method, we calculate the gravitational and electromagnetic quasinormal mode (QNM) perturbations for a static neutral black hole described by a Scalar-Tensor-Vector Modified Gravity framework (STVG-MOG). We show that the first few harmonic modes differ from their general relativistic (GR) equivalent for a Schwarzschild black hole. Specifically, the real and imaginary components of the QNM frequencies are smaller for STVG-MOG than for GR. We posit that the differences are sufficiently large to potentially be observed in present and future black hole binary merger gravitational waveforms.

1. Introduction
The landmark detection of gravitational waves from binary black hole coalescence by LIGO [1, 2, 3, 4] has ushered in a new age in astronomy and black hole physics. Due to the unique characteristics of the waveforms in a given spacetime, this data present us with a new test of the underlying theory of gravity, be it General Relativity [5] or some other alternative theory [6, 7, 8, 9].

The observed merger signals are divided into three phases – the inspiral, merger, and ringdown, – each of which identifies key characteristics of the black holes (size, mass, spin, etc...). While much attention is paid to the “loud” inspiral phase and the direct information it can disclose about the black holes, the more subtle ringdown phase can provide addition insight into the underlying spacetime structure. During this period, the final black hole settles to an unperturbed state through the emission of additional gravitational radiation, through damped oscillations known as quasinormal modes (QNMs) [10]. These QNMs are likely to be visible in present or future LIGO data [11, 12, 13], and most definitely by LISA [14]. An array of model-dependent data from QNMs can help to identify the background spacetime structure, including the frequencies themselves, QNM onset time [15], as well as higher-order modes [16].

2. STVG-MOG Overview
In this proceedings contribution, we summarize our analysis of the ringdown phase of black hole mergers in a scalar-vector-tensor theory of gravitation also known as SVTG-MOG [17, 18, 19, 20, 21]. The theory in question is best thought of as a generalization of Einstein gravity in which the metric theory of gravitation is supplemented by two auxiliary fields, i.e. a
scalar and vector graviton. These are described by the equations

\[ G_{\mu\nu} - \Lambda g_{\mu\nu} + Q_{\mu\nu} = -8\pi G T_{\mu\nu}, \]  

\[ \frac{1}{\sqrt{-g}} \partial_\nu (\sqrt{-g} B^{\mu\nu}) + \mu^2 \phi^\mu = -J^\mu, \]  

\[ \partial_\sigma B_{\mu\nu} + \partial_\mu B_{\nu\sigma} + \partial_\nu B_{\sigma\mu} = 0, \]

with the parameters \( \Box G = K(x) \) and \( \Box \mu = L(x) \). The advantages of this framework include the elimination of dark matter in galaxy rotation curves and clusters [22, 23, 24], as well as structure formation in the early universe [25, 26, 27].

The solution to the above system yields the metric

\[ ds^2 = f(r) dt^2 - f(r)^{-1} dr^2 - r^2 d\Omega^2, \quad f(r) = \left( 1 - \frac{2G_N(1 + \alpha)M}{r} + \frac{\alpha(1 + \alpha) G_N^2 M^4}{r^2} \right)^{1/2}, \]

Algebraically, this is identical in form to the Riessner-Nordström solution, but with the electric charge term now dependent on the mass.
3. Calculating Quasinormal Modes via the Asymptotic Iteration Method

New insights to black hole physics can be obtained through the study of these objects with surrounding fields. An understanding of QNMs, characteristic to the background spacetime, can then serve as a unique fingerprint when searching for the existence of black holes. These resonant modes present a real part that describes the actual frequency, and an imaginary part that dictates the damping. Gravitational perturbations appear to be of special interest since they directly identify the black hole to their gravitational radiation.

In the following we shall use the definition that QNMs are defined as solutions of the perturbed field equations with boundary conditions:

\[ \psi (x) \rightarrow \begin{cases} e^{-i\omega x} & x \rightarrow -\infty \\ e^{i\omega x} & x \rightarrow \infty \end{cases} \] (14)

where the positive and negative solutions correspond to ingoing and outgoing waves respectively.

A number of methods have been developed to calculate QNMs for black holes, but one of the more robust semianalytic techniques is the asymptotic iteration method (AIM) [28]. This will be the algorithm of choice outlined below.

We begin by defining a second-order differential equation of the form

\[ \chi'' = \lambda_0(x) \chi' + s_0(x) \chi, \] (15)

where the functions \( \lambda_0(x) \) and \( s_0(x) \) are smooth, so we can differentiate to get

\[ \chi''' = \lambda_1(x) \chi' + s_1(x) \chi, \] (16)

whose coefficients now satisfy \( \lambda_1(x) = \lambda_0' + s_0 + \lambda_0^2 \) and \( s_1(x) = s_0' + s_0 \lambda_0 \). Iterating the differentiation approach we arrive to

\[ \chi^{(n+2)} = \lambda_n(x) \chi' + s_n(x) \chi, \] (17)

where the coefficients satisfy

\[ \lambda_n(x) = \lambda_{n-1}' + s_{n-1} + \lambda_0 \lambda_{n-1}, \quad s_n(x) = s_{n-1}' + s_0 \lambda_{n-1}. \] (18)

For \( n \) large enough, the AIM feature is introduced

\[ \frac{s_n(x)}{\lambda_n(x)} = \frac{s_{n-1}(x)}{\lambda_{n-1}(x)} \equiv \beta(x), \] (19)

and the QNMs arise from a “quantization condition” marking the end of the iterations

\[ \delta_n = s_n \lambda_{n-1} - s_{n-1} \lambda_n = 0. \] (20)

Outside the horizon, we describe the perturbations of the geometry as even- and odd-parity oscillations of a Schrödinger-like equation, i.e.

\[ \left( \frac{d^2}{dx^2} - \rho^2 - V_i^{(\pm)} \right) Z_i^{(\pm)} = 0, \] (21)

with the even and odd modes denoted by + and −, respectively [30, 31, 32]:

\[ V_i^{(-)}(r) = \Delta \left( Ar - q_j + \frac{4Q^2}{r} \right) \] (22)

\[ V_i^{(+)}(r) = V_i^{(-)}(r) + 2q_j \frac{d}{dr} \left( \frac{\Delta}{r^{(l+1)(l+2)} + q_j r} \right) \] (23)
When \( i = j = (1, 2)(i \neq j) \), we can write

\[
\frac{d^2}{dz^2} \chi + \frac{\Delta}{2} \chi = 0,
\]

\[
\Delta = (r - r_+)(r - r_-) = \frac{1}{4}[4r^2 - 4r(1 + \alpha) + \alpha(1 + \alpha)],
\]

\[
f = l(l + 1),
\]

\[
q_1 = \frac{1}{2} \left[ 3 + \sqrt{9 + 16Q_g^2(l - 1)(l + 2)} \right],
\]

\[
q_2 = \frac{1}{2} \left[ 3 - \sqrt{9 + 16Q_g^2(l - 1)(l + 2)} \right],
\]

Under an appropriate change of variables, the AIM formalism can be expressed as

\[
\chi_{\xi,\xi} = \lambda_{\xi}(\xi)\chi_{\xi,\xi} + s_{\xi}(\xi)\chi_{\xi},
\]

where

\[
\lambda_{\xi}(\xi) = \frac{\alpha(\alpha + 1) + 4\Delta}{2\Delta(1 - \xi)} - \frac{r_+(\alpha + 2\Gamma Z\Delta + 1)}{\Delta(\xi - 1)^2},
\]

\[
s_{\xi}(\xi) = \frac{r_+^6 \left( \rho^2 + V_{l=1}^{(-)} \right)}{\Delta^2(\xi - 1)^3} - \frac{\Delta r_+^2(1 + \alpha + \Delta \Gamma Z)}{\Delta(\xi - 1)^4} - \frac{\alpha(\alpha + 1)\Gamma Z r_+}{2\Delta(\xi - 1)^3} - \frac{r_+\Gamma Z \xi}{(\xi - 1)^2},
\]

\[
\Gamma Z = -\rho - \frac{1}{r_+} - \frac{\xi}{r_+} + \frac{\left( (1 - \rho)(r_+ - r_-) - \rho r_+^2 \right)}{(r_+ - r_-)(r_+ - r_- - (1 - \xi))} + \frac{\rho r_+(1 - \xi)}{(r_+ - r_-)\xi},
\]

\[
V_{l=1}^{(-)} = \frac{\Delta (1 - \xi)^5}{r_+^3} \left[ \frac{Ar_+}{1 - \xi} - q_j + \frac{4Q_g^2(1 - \xi)}{r_+} \right],
\]

\[
\Delta = \frac{r_+\xi [r_+ - r_-(1 - \xi)]}{(1 - \xi)^2} = \frac{(1 + \alpha)(\xi(2 + 2\sqrt{1 + \alpha + \alpha\xi})}{4(\xi - 1)^2},
\]

\[
r_\pm = \frac{1}{2} \left[ 1 + \alpha \pm (1 + \alpha)^{1/2} \right],
\]

\[
Q_g = \frac{\sqrt{\alpha}}{2}.
\]

Tables 1 and 2 show the real and complex QNM frequencies for the first few modes. The black hole mass has been set to unity for easy comparison with existing QNM for other gravitational theories. It is clearly shown that for \( \alpha = 0 \), the QNMs of \( V_{l=1}^{(-)} \) in Table 2 reduce to the purely gravitational Schwarzschild case, while the QNMs of \( V_{l=1}^{(-)} \) in Table 1 reduce to the purely electromagnetic case. It is important to note that for \( n \gg l \) as in Table 1 for \( l = 1 \) and \( n = 3 \), a higher number of iterations is needed to find stable solutions in the semianalytical approach.

4. Conclusions

Quasinormal modes in the ringdown phase of black hole mergers described by STVG (MOG) theory were determined for both gravitational and electromagnetic perturbations. For increasing model parameter \( \alpha > 0 \), the magnitude of both the real and imaginary components of the frequencies decreased in the gravitational \( l = 2, l = 3 \) and \( l = 4 \) modes, as well as for the \( l = 1, l = 2 \) and \( l = 3 \) electromagnetic modes. This difference between MOG and GR should be detectable for sufficiently sensitive frequency determinations in present and future black hole binary coalescence data.
Table 1. QNMs accurate to 4 decimal places for $M = 1$ scaled MOG electromagnetic perturbations $V_{i=1}^{(-)}$ for $l = 1$, $l = 2$ and $l = 3$ modes.

| $l$ | $n$ | $\alpha = 0$ | $\alpha = 1$ | $\alpha = 4$ | $\alpha = 9$ |
|-----|-----|-------------|-------------|-------------|-------------|
| 1   | 0   | 0.2483 - 0.09249i | 0.1484 - 0.04805i | 0.06343 - 0.01881i | 0.03268 - 0.009084i |
|     | 1   | 0.2145 - 0.29370i | 0.1308 - 0.1506i | 0.05882 - 0.05828i | 0.03038 - 0.02796i |
|     | 2   | 0.1748 - 0.5252i  | 0.1135 - 0.2654i | 0.05258 - 0.1014i | 0.02675 - 0.04833i |
|     | 3   | 0.1462 - 0.7719i  | 0.1090 - 0.3866i | 0.05036 - 0.1494i | 0.02434 - 0.07008i |
| 2   | 0   | 0.4576 - 0.09500i | 0.2651 - 0.04917i | 0.1164 - 0.01930i | 0.06000 - 0.009351i |
|     | 1   | 0.4365 - 0.2907i  | 0.2565 - 0.1498i | 0.1136 - 0.05854i | 0.05861 - 0.02830i |
|     | 2   | 0.4012 - 0.5016i  | 0.2420 - 0.2563i | 0.1087 - 0.09948i | 0.05607 - 0.04791i |
|     | 3   | 0.3626 - 0.7302i  | 0.2257 - 0.3699i | 0.1028 - 0.1425i | 0.05272 - 0.06840i |
| 3   | 0   | 0.6569 - 0.09562i | 0.3771 - 0.04933i | 0.1648 - 0.01936i | 0.08493 - 0.009395i |
|     | 1   | 0.6417 - 0.2897i  | 0.3709 - 0.1492i | 0.1627 - 0.05842i | 0.08391 - 0.02831i |
|     | 2   | 0.6012 - 0.4921i  | 0.3594 - 0.2522i | 0.1589 - 0.09841i | 0.08198 - 0.04760i |
|     | 3   | 0.5779 - 0.7063i  | 0.3446 - 0.3600i | 0.1537 - 0.1398i | 0.07927 - 0.06743i |

Table 2. QNMs accurate to 4 decimal places for $M = 1$ scaled MOG gravitational perturbations $V_{i=2}^{(-)}$ for $l = 2$, $l = 3$ and $l = 4$ modes.

| $l$ | $n$ | $\alpha = 0$ | $\alpha = 1$ | $\alpha = 4$ | $\alpha = 9$ |
|-----|-----|-------------|-------------|-------------|-------------|
| 2   | 0   | 0.3737 - 0.0890i | 0.2220 - 0.04650i | 0.1021 - 0.01871i | 0.05431 - 0.009171i |
|     | 1   | 0.3467 - 0.2739i | 0.2115 - 0.1423i | 0.09872 - 0.05678i | 0.05270 - 0.02781i |
|     | 2   | 0.3011 - 0.4783i | 0.1937 - 0.2457i | 0.09283 - 0.09696i | 0.04974 - 0.04725i |
|     | 3   | 0.2515 - 0.7051i | 0.1742 - 0.3579i | 0.08582 - 0.1397i | 0.04584 - 0.06776i |
| 3   | 0   | 0.5994 - 0.0927i | 0.3353 - 0.04758i | 0.1496 - 0.01891i | 0.07875 - 0.009267i |
|     | 1   | 0.5826 - 0.2731i | 0.3281 - 0.1441i | 0.1472 - 0.05711i | 0.07761 - 0.02795i |
|     | 2   | 0.5517 - 0.4791i | 0.3149 - 0.2444i | 0.1428 - 0.0964i | 0.07543 - 0.04706i |
|     | 3   | 0.5120 - 0.6903i | 0.2979 - 0.3503i | 0.1368 - 0.1373i | 0.07238 - 0.06680i |
| 4   | 0   | 0.8092 - 0.0942i | 0.4452 - 0.04804i | 0.1965 - 0.01903i | 0.1030 - 0.009311i |
|     | 1   | 0.7966 - 0.2843i | 0.4398 - 0.1449i | 0.1947 - 0.05731i | 0.1021 - 0.02802i |
|     | 2   | 0.7727 - 0.4799i | 0.4294 - 0.2441i | 0.1912 - 0.09625i | 0.1004 - 0.04699i |
|     | 3   | 0.7398 - 0.6839i | 0.4151 - 0.3468i | 0.1863 - 0.1362i | 0.09796 - 0.06636i |

Acknowledgments

L. Manfredi thanks the organizers of the KSM2017 for their generous hospitality during the meeting. LM and J. Mureika also thank the Perimeter Institute for Theoretical Physics for its generous hospitality during the initial stages of this research. J. Moffat thanks Luis Lehner and Martin Green for helpful discussions. This work was supported in part by a Summer Undergraduate Research Fellowship and Continuing Faculty Grant from the Frank R. Seaver College of Science and Engineering of Loyola Marymount University and the Perimeter Institute for Theoretical Physics. Research at Perimeter Institute is supported in part by the Government of Canada through the Department of Innovation, Science and Economic Development Canada and by the Province of Ontario through the Ministry of Research, Innovation and Science.

References

[1] Abbott B P et al. [LIGO Scientific and Virgo Collaborations], Phys. Rev. Lett. 116, 061102 (2016) Preprint 1602.03837 [gr-qc].
[2] Abbott B P et al. [LIGO Scientific and Virgo Collaborations], Phys. Rev. Lett. 116, 241103 (2016) Preprint 1606.04855 [gr-qc].
[3] Abbott B P et al. [LIGO Scientific and VIRGO Collaborations], Phys. Rev. Lett. 118, 221101 (2017) Preprint 1706.01812 [gr-qc].
[4] Abbott B P et al. [LIGO Scientific and Virgo Collaborations], Phys. Rev. Lett. 119 (2017) 141101 Preprint 1709.09660 [gr-qc].
[5] Abbott B P et al. [LIGO Scientific and Virgo Collaborations], Phys. Rev. Lett. 116, 221101 (2016) Preprint 1602.03841 [gr-qc].
[6] Konoplya R A and Zhidenko A, Rev. Mod. Phys. 83, 793 (2011) Preprint 1102.4014 [gr-qc].
[7] Sibandze D B, Goswami R, Maharaj S D and Dunsby P K S, “Echoes from the black holes: Evidence of higher order corrections to General Relativity in strong gravity regime,” Preprint 17020.4926 [gr-qc].
[8] Lasky P D and Doneva D D, Phys. Rev. D82, 124068 (2010) Preprint gr-qc/1011074.
[9] Bhattacharyya S and Shankaracharyan S, “Quasinormal modes as a distinguisher between general relativity and f(R) gravity,” Preprint gr-qc/170407044.
[10] Kokkotas K D and Schmidt B G, Living Rev. Rel. 2, 2 (1999) Preprint gr-qc/9909058.
[11] Goggin L M, “A Search For Gravitational Waves from Perturbed Black Hole Ringdowns in LIGO Data,” Preprint gr-qc/09082085.
[12] Abbott B P et al. [LIGO Scientific Collaboration], Phys. Rev. D80, 062001 (2009) Preprint gr-qc/09051654.
[13] Aasi J et al. [LIGO Scientific and VIRGO Collaborations], Phys. Rev. D89, no. 10, 102006 (2014) Preprint gr-qc/1405306.
[14] Berti E, Cardoso V and Will C M, Phys. Rev. D73, 064030 (2006) Preprint gr-qc/0512160.
[15] Sakai K, Oohara K I, Nakano H, Kaneyama M and Takahashi H, “Estimation of starting times of quasi-normal modes in ringdown gravitational waves with the Hilbert-Huang transform,” Preprint gr-qc/170504107.
[16] London L, Shoemaker D and Healy J, Phys. Rev. D90, no. 12, 124032 (2014) Erratum: [Phys. Rev. D94, no. 6, 069902 (2016)] Preprint gr-qc/14043197.
[17] Moffat J W, JCAP 0603, 004 (2006). Preprint gr-qc/0506021.
[18] Moffat J W, Phys. Lett. B763, 427 (2016), Preprint gr-qc/160305225.
[19] Moffat J W, Eur. Phys. J. C75, 175 (2015), Preprint gr-qc/14125429.
[20] Moffat J W, Eur. Phys. J. C 75, 130 (2015), Preprint gr-qc/150201677.
[21] Moffat J W, “Misaligned Spin Merging Black Holes in Modified Gravity (MOG),” Preprint gr-qc/170605035.
[22] Moffat J W and Rahvar S, MNRAS 436, 1439 (2013), Preprint astro-ph.GA/13066383.
[23] Moffat J W and Rahvar S, MNRAS 441, 3724 (2014), Preprint astro-ph.CO/13095077.
[24] Moffat J W and Toth V T, Phys. Rev. D91, 043004 (2015), Preprint astro-ph.GA/14116701.
[25] Moffat J W, Preprint astro-ph.GA/150107037.
[26] Moffat J W, Preprint gr-qc/14102464.
[27] Shojai F, Cheraghchi S and Bouzari H Nezhad, Phys. Lett. B 770, 43 (2017), Preprint gr-qc/170404161.
[28] Cho H T, Cornell A S, Doukas J, Huang T R and Naylor W, Adv. Math. Phys. 2012, 281705 (2012)
[29] Barakat T, Int. J. Mod. Phys. A21, 4127-4135 (2006).
[30] Zerilli F J, Phys. Rev. D9, 860-868 (1974).
[31] Moncrief V, Phys. Rev. D9, 2707-2709 (1974).
[32] Moncrief V, Phys. Rev. D12, 1526-1537 (1975).