On the Moduli Problem and Baryogenesis in Gauge-mediated SUSY Breaking Models

S. Kasuya, M. Kawasaki, and Fuminobu Takahashi

Research Center for the Early Universe, School of Science, University of Tokyo, Bunkyo-ku, Tokyo 113-0033, Japan

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We investigate whether the Affleck-Dine mechanism can produce sufficient baryon number of the universe in the gauge-mediated SUSY breaking models, while evading the cosmological moduli problem by late-time entropy production. We find that the Q-ball formation renders the scenario very difficult to work, irrespective of the detail mechanism of the entropy production.

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I. INTRODUCTION

In superstring theories, there generally exist various dilaton and modulus fields. These fields (we call them "moduli" in this paper) are expected to acquire masses of the order of the gravitino mass $m_{3/2}$ through some non-perturbative effects of supersymmetry (SUSY) breaking. It is well known that the moduli cause serious cosmological problem because they have only gravitationally suppressed interactions with other particles and hence have long lifetimes. The moduli with mass $O(100)$ GeV decay at the Big Bang Nucleosynthesis (BBN) epoch and spoil the success of the BBN by destroying the synthesized light elements, while moduli with lighter mass ($\lesssim 1$ GeV) may overclose the universe or emit X($\gamma$)-rays giving too many contributions to the cosmic background radiation.

The mass of moduli (which is assumed to be the same as the gravitino mass $m_{3/2}$) depends on models of SUSY breaking. In hidden sector models the SUSY breaking in the hidden sector is mediated by gravitation and SUSY particles (squarks, sleptons, etc.) in the observable sector as well as gravitino obtain the mass of weak scale $\sim O(100)$ GeV. On the other hand, in gauge-mediated SUSY breaking models, the gauge interactions mediate the SUSY breaking effects. In gauge-mediated SUSY breaking, the gravitino cannot acquire mass by the gauge interactions but only through gravitation. Thus, the mass of gravitino is much lighter ($\lesssim 1$ GeV) than that in the hidden sector models. The gauge mediation models have attractive points that are absent in the hidden sector models; they can avoid the flavor problem and the mass pattern of the particles in the observable sector is predictable. Therefore, in this paper we consider the moduli problem in the gauge-mediated SUSY breaking models.

In order to avoid the moduli problem, we need some huge entropy production process by which the moduli density is diluted. So far the most successful mechanism for entropy production is "thermal inflation" proposed by Lyth and Stewart. The thermal inflation model in the gauge-mediated SUSY breaking models was intensively investigated in Refs. where it was shown that the thermal inflation can solve the moduli problem. However, any process that dilutes the moduli also dilutes primordial baryon asymmetry of the universe. Since the entropy production should take place after the start of the moduli oscillation, the reheating temperature is generally very low, which makes regeneration of the baryon asymmetry almost impossible. Thus, we must produce sufficiently large baryon asymmetry before the entropy production occurs.

Although it is known that the GUT baryogenesis or the leptogenesis could work for the mechanism for the baryogenesis, they could produce the baryon numbers $\eta_B \sim 10^{-10}$ at most before the dilution. The only promising candidate for mechanism of such efficient baryon number generation is the Affleck-Dine (AD) baryogenesis. In fact, it was shown in Ref. that both the present baryon asymmetry and small moduli density can be explained by the thermal inflation and the Affleck-Dine mechanism for the gauge-mediated SUSY breaking models.

However, it has been found that the dynamics of the Affleck-Dine baryogenesis is complicated by the existence of Q balls. In the gauge-mediated SUSY breaking models the potential for the Affleck-Dine field becomes flat at large amplitudes. For such a flat potential the Q-ball formation is inevitable. In order to produce a large baryon number the initial amplitude of the AD field should be large, which also leads to the formation of Q balls with huge baryon(=Q) number. Since large Q balls are stable, the baryon number may be confined in the form of Q balls and there may exist very small baryon asymmetry in the cosmic plasma, which means that the baryogenesis does not work.

Unstable Q balls can provide all the charges created before the charge trapping by the produced Q balls, but rather small amplitudes of the AD field are necessary for the Q balls to decay into the ordinary baryons, nucleons. Thus, sufficient baryon number is difficult to be created from the beginning.

In this paper we study the cosmological moduli problem and baryogenesis in the gauge-mediated SUSY breaking models taking into account the Q-ball formation. It is found that the Q balls seriously affect the Affleck-Dine baryogenesis and lower its efficiency. As result we show...
that the AD baryogenesis hardly works in the presence of the entropy production which is necessary to dilute the dangerous moduli.

II. MODULI PROBLEM

Here we briefly discuss the moduli problem. The modulus field $\eta$ obtain a mass of the order of the gravitino mass $m_{3/2}$. During the primordial inflation the modulus field is expected to sit at some minimum of the effective potential determined by the Kähler potential and the Hubble parameter. In general, the minimum during the inflation deviates from the true minimum of the moduli potential at low energies and the difference of the two minimum is considered to be of the order of the gravitational scale $M (= 2.4 \times 10^{18}\text{GeV})$. After the inflation, the Hubble parameter becomes comparable to the mass of the modulus, the modulus field begins to roll down toward the true minimum and oscillates. Then, the modulus density (= oscillation energy) is estimated as

$$ \rho_{\text{mod}} \simeq \frac{1}{8} T_{RH} \left( \frac{\eta_0}{M} \right)^2, $$

where $s$ is the entropy density, $T_{RH}$ is the reheating temperature and $\eta_0$ is the initial amplitude of the modulus oscillation ($\eta_0 \sim M$). In deriving Eq.(1), we have assumed that the modulus mass is equal to $m_{3/2}$ and the reheating takes after the modulus field starts the oscillation. (When we estimate the baryon-to-entropy ratio later, the opposite case is also considered. See below.) Since $T_{RH}$ should be higher than about 10 MeV to keep the success of the BBN, the modulus-to-entropy ratio is bounded from below,

$$ \frac{\rho_{\text{mod}}}{s} \gtrsim 1.25 \times 10^{-3}\text{GeV}. $$(2)

The decay rate of the modulus is very small because it has only gravitationally suppressed interaction. The lifetime is roughly estimated as

$$ \tau_\eta \sim 10^{18}\text{sec} \left( \frac{m_{3/2}}{100\text{MeV}} \right)^{-3}. $$

Thus, for $m_{3/2} \lesssim 100$ MeV, the lifetime is longer than the age of the universe and its present density much larger than the critical density which is given by

$$ \rho_c = 3.6 \times 10^{-9} h^2\text{GeV}, $$

where $h$ is the present Hubble parameter in units of 100 km/sec/Mpc and $s_0 (\simeq 2.8 \times 10^{9} \text{cm}^{-3})$ is the present entropy density. The modulus with larger mass ($100$ MeV $\lesssim m_{3/2} \lesssim 1$ GeV) decays into photons whose flux exceeds the observed background X(or $\gamma$)-rays. Therefore the modulus is cosmological disaster and should be diluted by some entropy production process.

![Graph](image)

FIG. 1. Observational upper limit of the density parameter of the modulus from observations. It is determined by the facts the modulus density should not exceed the dark matter density for $m_{3/2} \lesssim 100$ keV, and the observed background X(or $\gamma$)-rays for $100$ keV $\lesssim m_{3/2} \lesssim 1$ GeV.

III. AFFLECK-DINE MECHANISM AND Q-BALL FORMATION

In the Minimal Supersymmetric Standard Model (MSSM), there exist flat directions, along which there are no classical potentials. Since flat directions consist of squarks and/or sleptons, they carry baryon and/or lepton numbers, and can be identified as the Affleck-Dine flat direction. These flat directions are lifted by SUSY breaking effects. In the gauge-mediated SUSY breaking models, the potential of a flat direction is parabolic at the origin, and almost flat beyond the messenger scale $M_S$.

$$ V_{\text{gauge}} \sim \begin{cases} m_{\phi}^2 |\Phi|^2 & (\Phi \ll M_S) \\ M_F^2 \log \frac{|\Phi|^2}{M_S^2} & (\Phi \gg M_S) \end{cases}, $$

where $M_S$ is the messenger mass scale.

Since the gravity always exists, flat directions are also lifted by the gravity-mediated SUSY breaking effects $V_{\text{grav}}$:

$$ V_{\text{grav}} \simeq m_{3/2}^2 \left[ 1 + K \log \left( \frac{|\Phi|^2}{M} \right) \right] |\Phi|^2, $$

where $K$ is the numerical coefficient of the one-loop corrections. This term can be dominant only at high energy scales because of the small gravitino mass $\lesssim O(1\text{GeV})$. 

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If the AD field directly couples with fields $\psi$ in the thermal bath, it acquires a thermal mass term in the effective potential at one-loop order:

$$V_T^{(1)} \sim f^2 T^2 |\Phi|^2,$$

(7)

where $f$ is a Yukawa, or gauge coupling constant between the AD field and the thermal particles which directly interact with it, and larger than $10^{-5}$. Note that this effect is exponentially suppressed when the effective mass of the thermal particle, $f|\Phi|$, is larger than the temperature. This term can make the AD field oscillate earlier than the case without this term, since the high reheating temperature is possible in the absence of the gravitino problem.\(^1\)

In addition to this term, there is another thermal effect on the potential, which appears at two-loop order, as pointed out in Ref. \([13]\). This comes from the fact that the running of the gauge coupling $g(T)$ is modified by integrating out heavy particles which directly couples with the AD field. This contribution is given by

$$V_T^{(2)} \sim T^4 \log \frac{|\Phi|^2}{T^2}. \qquad (8)$$

The baryon number is usually created just after the AD field starts coherent rotation in the potential, and its number density $n_B$ is estimated as

$$n_B(t_{osc}) \simeq \varepsilon \omega \phi_{osc}^2,$$

(9)

where $\varepsilon (< 1)$ is the ellipticity parameter, which represents the strength of the $A$-term, and $\omega$ and $\phi_{osc}$ are the angular velocity and the amplitude of the AD field at the beginning of the oscillation (rotation) in its effective potential.

Actually, however, the AD field feels spatial instabilities during its coherent oscillation, and deforms into non-topological solitons, Q balls \([13]\). In the case that the zero-temperature potential $V_{grav}$ dominates, the gauge-mediation type Q balls are formed, whose properties are as follows \([19]\):

$$M_Q \sim M_F Q^{3/4}, \quad R_Q \sim M_F^{-1} Q^{1/4},$$

(10)

where $M_Q$ and $R_Q$ are the mass and the size of the Q ball, respectively. If the mass per unit charge, $M_F Q^{-1/4}$, is smaller than the proton mass $\sim 1\text{GeV}$, the Q ball is stable against the decays into nucleons, which follows that Q balls with very large Q can be stable.

From numerical calculations \([12,15]\), Q balls absorb almost all the baryon charges which the AD field obtains, and the typical charge is estimated as \([13]\):

$$Q \simeq \beta \left( \frac{\phi_{osc}}{M_F} \right)^4,$$

(11)

where $\beta \approx 6 \times 10^{-4}$. Consequently, the present baryon asymmetry should be explained by the charges which come out of the Q balls through the evaporation, diffusion, and decay of Q balls. In the case that $V_T^{(2)}$ dominates, one must replace $M_F$ with $T_*$, where $T_*$ is the temperature at the Q-ball formation. Notice that the shape of the Q ball reconfigures as the temperature drops, releasing energy, but not charge. Finally, it will become the zero-temperature configuration, as in Eq. \([10]\).

In the case of the unstable Q balls, they decay into nucleons and light scalar particles. Since the temperature at the BBN time is very low ($\sim 1\text{MeV}$), Q balls cannot decay into light scalars. The decay rate is thus given by

$$\frac{dQ}{dt} \lesssim \frac{\omega^3 A}{192\pi^2},$$

(12)

where $A$ is a surface area of the Q ball.\(^2\)

In the case of the stable Q balls, the evaporation is the only way to extract the baryon charges from Q balls. The total evaporated charge from the Q ball is estimated as \([21,22,15]\),

$$\Delta Q \sim 10^{15} \left( \frac{m_\phi}{\text{TeV}} \right)^{-2/3} \left( \frac{M_F}{10^6\text{GeV}} \right)^{-1/3} Q^{1/12}. \qquad (13)$$

Hence the baryon number density is suppressed by the factor $\Delta Q/Q$, in comparison with the case of no stable Q-ball production.

On the other hand, where $V_{grav}$ dominates the potential at larger scales, the gravity-mediation type Q balls (‘new’ type) are produced \([14]\), if $K$ is negative, while, if $K$ is positive, it is not until the AD field enters $V_{gaug}$ dominant region that it feels instabilities, and the gauge-mediation type Q balls are produced (the delayed Q balls) \([13]\). Notice that the sign of $K$ is in general indefinite in the gauge-mediated SUSY breaking models. We will thus consider both cases later.

When the AD field starts to oscillate in the $V_{grav}$-dominant region, where $H_{osc} \sim \omega \sim m_{3/2}$, the baryon number is produced as $n_B \simeq \varepsilon m_{3/2}$. For the negative $K$ case, the ‘new-type’ Q balls are created, and its charge is written as

$$Q \approx \tilde{\beta} \left( \frac{\phi_{osc}}{m_{3/2}} \right)^2,$$

(14)

where $\tilde{\beta} \simeq 6 \times 10^{-3}$. This type of the Q ball is also stable against the decay into nucleons, and the amount

\(^1\)Since we consider the late-time entropy production for diluting the modulus field, gravitino is also diluted. Thus, there is no cosmological gravitino problem even if the reheating temperature is high.

\(^2\)For conservative estimation of the baryon density, we use the maximal decay rate, i.e., $dt/dQ \simeq \omega^3 A/(192\pi^2)$.
of the baryons in the present universe is explained by the charge evaporation from the Q balls. The charge evaporated from the Q ball is estimated as \[ \Delta Q \sim 2.2 \times 10^{20} \left( \frac{m_{3/2}}{100 \text{keV}} \right)^{-1/3} \left( \frac{m_\phi}{\text{TeV}} \right)^{-2/3}. \] (15)

Since the gauge-mediation type of the delayed Q ball are formed only after the AD field enters the $V_{\text{gauge}}$-dominant region for the positive $K$, the charge of the Q ball is given by

\[ Q \sim \beta \left( \frac{\phi_{eq}}{M_F} \right)^4 \sim \beta \left( \frac{M_F}{m_{3/2}} \right)^4, \] (16)

where $\phi_{eq} \sim M_F^2/m_{3/2}$ is used. If $V_T^{(2)}$ dominates over the zero-temperature potential $V_{\text{gauge}}$, one must only replace $M_F$ in the Eq. (16) with $T_{eq}$. Here subscript ‘eq’ denotes the values when the gauge- (or thermal logarithmic) and gravity-mediation potential are the same.

IV. BARYOGENESIS AND THE MODULI PROBLEM

As we mentioned in the Introduction, the late-time entropy production necessary for the dilution of the moduli also dilutes the baryon numbers created earlier very seriously, but the sufficient numbers could remain, if the Q-ball production is not taken into account. We will see that the Q-ball formation puts very serious restriction on the efficiency of the AD baryogenesis, and makes it useless, whether the produced Q balls are stable or not.

A. Stable Q balls

Since the baryon number is supplied only by the evaporation from stable Q balls, it should be suppressed by the factor $\Delta Q/Q$, compared with no Q-ball formation. We will show that this fact considerably reduces the power of the AD baryogenesis.

1. Gauge-mediation type Q balls when the zero-temperature potential is dominated

The AD field starts to oscillate when $H_{\text{osc}} \sim M_F^2/\phi_{\text{osc}}$. This is earlier than the beginning of the moduli oscillation $H_{\text{mod}} \sim m_{3/2}$, since $M_F^2 \gtrsim m_{3/2}^3 \phi_{\text{osc}}^2$. There are two situations when the moduli fields start the oscillation: before and after the reheating. In the former case, the reheating temperature should be lower than $T_{RH}^{(c)}$, which is defined as

\[ T_{RH}^{(c)} = \left( \frac{90 \pi^2 g(T_{RH})}{m_{3/2}^3 M} \right)^{1/4} \sqrt{m_{3/2} M}, \]

\[ \sim 7.2 \times 10^6 \text{GeV} \left( \frac{m_{3/2}}{100 \text{keV}} \right)^{1/2}, \] (17)

where $g(T_{RH}) \sim 200$ counts the effective degrees of freedom of the radiation. Since the ratio between $n_B$ and the energy of the inflaton $\rho_{\text{infl}}$ stays constant until the reheating, we have

\[ \frac{n_B}{\rho_{\text{mod}}} = \frac{n_B}{\rho_{\text{infl}} H_{\text{osc}} \rho_{\text{mod}}}, \]

\[ \simeq \frac{n_B}{3H_{\text{osc}}^2 M^2} \frac{3m_{3/2}^2 M^2}{m_{3/2}^2 M^2} \times \frac{T_{RH}}{T_{RH}^{(c)}}, \]

\[ = \frac{2n_B}{H_{\text{osc}}^2 M^2} \times \frac{T_{RH}}{T_{RH}^{(c)}}, \] (18)

On the other hand, if the moduli start to oscillate after the reheating, the baryon-to-moduli ratio at the beginning of the moduli oscillation becomes larger by the factor $a_{\text{mod}}/a_{RH} \simeq T_{RH}/T_{mod}$, where subscript ‘mod’ denotes the values at the moduli oscillation time, since the universe is radiation-dominated after the reheating until the moduli start to oscillate. We thus have

\[ \frac{n_B}{\rho_{\text{mod}}} \simeq \frac{n_B}{\rho_{\text{infl}} H_{\text{osc}} \rho_{\text{mod}}}, \]

\[ \simeq \frac{n_B}{3H_{\text{osc}}^2 M^2} \frac{3m_{3/2}^2 M^2}{m_{3/2}^2 M^2} \times \frac{T_{RH}}{T_{RH}^{(c)}}, \]

\[ = \frac{2n_B}{H_{\text{osc}}^2 M^2} \times \frac{T_{RH}}{T_{RH}^{(c)}}, \] (19)

where $\rho_{\text{rad}}$ is the energy density of radiation.

Therefore the ratio of the baryon number and the energy density of the moduli can be written as

\[ \frac{n_B}{\rho_{\text{mod}}} = \frac{2n_B}{H_{\text{osc}}^2 M^2} \times T_{RH}, \] (20)

where

\[ T_{RH} = \begin{cases} 1 & \text{for } T_{RH} < T_{RH}^{(c)} \\ \frac{T_{RH}}{T_{RH}^{(c)}} & \text{for } T_{RH} > T_{RH}^{(c)} \end{cases}. \] (21)

If the $V_{\text{gauge}}$ dominates over the thermal logarithmic potential $V_T^{(2)}$, $M_F \gtrsim T_{osc}$, so that the initial amplitude is constrained as

\[ \phi_{\text{osc}} \simeq \left( \frac{T_{RH}}{M_F} \right)^2 M. \] (22)

$V_{\text{gauge}}$ also dominates over $V_{\text{grav}}$, which leads to the condition

\[ \phi_{\text{osc}} \simeq \frac{M_F^2}{m_{3/2}}. \] (23)

Combining these two equations, we have

\[ T_{RH} \lesssim \frac{M_F^2}{\sqrt{m_{3/2}^2}}. \] (24)
Since the scale of the SUSY breaking sector $\Lambda_{\text{DSSB}}^{1/2}$ is larger than $M_F$, the following condition should be hold:

$$M_F \lesssim \Lambda_{\text{DSSB}}^{1/2} \sim (m_{3/2}M)^{1/2},$$

where the vanishing cosmological constant is assumed in the last equality. From Eqs.(24) and (25), the condition on the reheating temperature becomes $T_{\text{RH}} \lesssim (m_{3/2}M)^{1/2}$. It corresponds to the case that the modulus field starts its oscillation before the reheating, hence $r_H = 1$.

Since $n_B \simeq \varepsilon \omega \phi_{\text{osc}}^2 \Delta Q/Q$, and $H_{\text{osc}} \simeq \omega \simeq M_F/\phi_{\text{osc}}$, the baryon-to-entropy ratio can be written as

$$Y_B = \frac{n_B}{\rho_{\text{mod}}} \frac{\rho_c}{\rho_c^{80}} = 1.2 \times 10^{-30} \varepsilon \left(\frac{\Omega_{\text{mod}} h^2}{0.2}\right) \left(\frac{m_\phi}{\text{TeV}}\right)^{-2/3} \times \left(\frac{M_F}{10^6\text{GeV}}\right)^{4/3} \left(\frac{\phi_{\text{osc}}}{M}\right)^{-2/3},$$

$$\lesssim 6.2 \times 10^{-27} \varepsilon \left(\frac{\Omega_{\text{mod}} h^2}{0.2}\right) \left(\frac{m_\phi}{\text{TeV}}\right)^{-2/3},$$

where we put into the lower limit of $\phi_{\text{osc}}$ derived from the stability condition: $M_Q/Q \lesssim 1$ GeV, which is expressed as

$$\frac{\phi_{\text{osc}}}{M} \gtrsim 2.6 \times 10^{-6} \left(\frac{M_F}{10^6\text{GeV}}\right)^2.$$  

When we estimate the upper bound of $Y_B$, we can also put the survival condition $\Delta Q \lesssim Q$, which reads as

$$\frac{\phi_{\text{osc}}}{M} \gtrsim 3.2 \times 10^{-8} \left(\frac{m_\phi}{\text{TeV}}\right)^{2/11} \left(\frac{M_F}{10^6\text{GeV}}\right)^{10/11},$$

for $M_F \lesssim M_F^*$, where $M_F^*$ is obtained by equating the RHSs of Eqs.(27) and (28):

$$M_F^* \simeq 1.8 \times 10^4\text{GeV} \left(\frac{m_\phi}{\text{TeV}}\right)^{-1/6}.$$  

This leads to the upper bound on $Y_B$ as

$$Y_B \lesssim 1.2 \times 10^{-25} \varepsilon \left(\frac{\Omega_{\text{mod}} h^2}{0.2}\right) \times \left(\frac{m_\phi}{\text{TeV}}\right)^{-6/11} \left(\frac{M_F}{10^6\text{GeV}}\right)^{8/11},$$

$$\lesssim 6.2 \times 10^{-27} \varepsilon \left(\frac{\Omega_{\text{mod}} h^2}{0.2}\right) \left(\frac{m_\phi}{\text{TeV}}\right)^{-2/3},$$

where we put $M_F = M_F^*$ in the last line, since it make $Y_B$ be maximum. Notice that this value is exactly the same as that constrained by the stability condition, since the maximum $Y_B$ is achieved at the boundary $M_F = M_F^*$ for the estimation derived using the survival condition.

In addition to the above case, complete evaporation of the Q ball can be considered for appropriate parameters, and its condition is $\Delta Q \gtrsim Q$, which is the opposite condition to Eq.(28). The baryon-to-entropy ratio is

$$Y_B \simeq \frac{2 \varepsilon \omega \phi_{\text{osc}}^2 \Omega_{\text{mod}} \rho_c}{\varepsilon \rho_c^{80}},$$

$$\simeq 3.5 \times 10^{-3} \varepsilon \left(\frac{\Omega_{\text{mod}} h^2}{0.2}\right) \left(\frac{M_F}{10^6\text{GeV}}\right)^{-2} \left(\frac{\phi_{\text{osc}}}{M}\right)^3,$$

$$\lesssim 6.2 \times 10^{-27} \varepsilon \left(\frac{\Omega_{\text{mod}} h^2}{0.2}\right) \left(\frac{m_\phi}{\text{TeV}}\right)^{-2/3},$$  

where $M_F = M_F^*$ is inserted in the last line. In either case, the baryon-to-entropy ratio is too small to explain the present value of the order $10^{-10}$.

2. Gauge-mediation type Q balls when the thermal logarithmic potential is dominated

In this case, $V_T^{(2)}$ is dominant over $V_{\text{gauge}}$, and the AD field starts to oscillate when $H_{\text{osc}} \sim T_{\text{osc}}^2/\phi_{\text{osc}}$. We should also use the charge of the formed Q ball as $Q \simeq 3(\phi_{\text{osc}}/T_{\text{osc}})^4$. Therefore, the fraction of the evaporated charge is written as

$$\frac{\Delta Q}{Q} \sim 10^{15} \left(\frac{m_\phi}{\text{TeV}}\right)^{-2/3} \left(\frac{M_F}{10^6\text{GeV}}\right)^{-1/3} Q^{-11/12},$$

$$\sim 8.9 \times 10^{17} \left(\frac{m_\phi}{\text{TeV}}\right)^{-2/3} \left(\frac{M_F}{10^6\text{GeV}}\right)^{-1/3} \times \left(\frac{T_{\text{RH}}}{M}\right)^{11/3} \left(\frac{\phi_{\text{osc}}}{M}\right)^{-11/2}.$$  

Then the baryon-to-entropy ratio becomes

$$Y_B \simeq 5.3 \times 10^{-10} \varepsilon T_{\text{RH}} \left(\frac{\Omega_{\text{mod}} h^2}{0.2}\right) \left(\frac{m_\phi}{\text{TeV}}\right)^{-2/3} \times \left(\frac{M_F}{10^6\text{GeV}}\right)^{-1/3} \left(\frac{T_{\text{RH}}}{M}\right)^{5/3} \left(\frac{\phi_{\text{osc}}}{M}\right)^{-3/2}.$$  

Stability condition constrains the initial amplitude of the AD field as

$$\frac{\phi_{\text{osc}}}{M} \gtrsim 3.4 \times 10^4 \left(\frac{M_F}{10^6\text{GeV}}\right)^{2/3} \left(\frac{T_{\text{RH}}}{M}\right)^{2/3}.$$  

This relation can be applied only to the situation when the following condition holds:

$$\frac{T_{\text{RH}}}{M} \lesssim 1.6 \times 10^{-7} \left(\frac{M_F}{10^6\text{GeV}}\right)^{-1},$$

which comes from $\phi_{\text{osc}} \lesssim M$. On the other hand, there is another constraint on $\phi_{\text{osc}}$, which comes from the survival condition. It reads as
\[ \frac{\phi_{osc}}{M} \gtrsim 1.8 \times 10^3 \left( \frac{m_\phi}{\text{TeV}} \right)^{-4/33} \times \left( \frac{M_F}{10^6 \text{GeV}} \right)^{-2/33} \left( \frac{T_{RH}}{M} \right)^{2/3} \]

where, if this condition is applied, the RHS should be less than unity, which is expressed as

\[ T_{RH}^{-1} \lesssim 1.3 \times 10^{-5} \left( \frac{m_\phi}{\text{TeV}} \right)^{2/11} \left( \frac{M_F}{10^6 \text{GeV}} \right)^{1/11}. \]

Notice that the survival condition constrains more strictly on \( \phi_{osc} \) for \( M_F \lesssim M_F^\ast \simeq 1.8 \times 10^4 (m_\phi/\text{TeV})^{-1/6} \) GeV.

It is easily seen that the largest upper limit on \( Y_B \) comes from the survival condition, and we have

\[ Y_B \lesssim 6.7 \times 10^{-15} \varepsilon r_H \left( \frac{\Omega_{mod h^2}}{0.2} \right) \left( \frac{m_\phi}{\text{TeV}} \right)^{-16/33} \times \left( \frac{M_F}{10^6 \text{GeV}} \right)^{-8/33} \left( \frac{T_{RH}}{M} \right)^{2/3}. \]

For the reheating temperature lower than \( T_{RH}^{(c)} \), \( r_H = 1 \), and \( T_{RH} \lesssim T_{RH}^{(c)} \) should be used for the upper bound on \( T_{RH} \), not Eq. \((37)\). Therefore, the upper limit on the baryon-to-entropy ratio becomes

\[ Y_B \lesssim 1.4 \times 10^{-22} \varepsilon \left( \frac{\Omega_{mod h^2}}{0.2} \right) \left( \frac{m_\phi}{\text{TeV}} \right)^{-16/33} \times \left( \frac{m_{3/2}}{100 \text{keV}} \right)^{1/3} \left( \frac{M_F}{10^6 \text{GeV}} \right)^{-8/33} \]

\[ \lesssim 7.4 \times 10^{-22} \varepsilon \left( \frac{\Omega_{mod h^2}}{0.2} \right) \left( \frac{m_\phi}{\text{TeV}} \right)^{-16/33} \times \left( \frac{m_{3/2}}{100 \text{keV}} \right)^{1/3}, \]

where we put \( M_F = 1 \) TeV, since it leads to the possible maximum limit. Thus, it is too small to explain the present value.

On the other hand, when the reheating temperature is higher than \( T_{RH}^{(c)} \), Eq. \((37)\) gives the upper bound on \( T_{RH} \), and the baryon-to-entropy ratio is estimated as

\[ Y_B \lesssim 1.6 \times 10^{-11} \varepsilon \left( \frac{\Omega_{mod h^2}}{0.2} \right) \left( \frac{m_\phi}{\text{TeV}} \right)^{-2/11} \times \left( \frac{m_{3/2}}{100 \text{keV}} \right)^{-1/2} \left( \frac{M_F}{10^6 \text{GeV}} \right)^{-1/11}, \]

where we use Eq. \((37)\) and

\[ r_H \simeq 3.4 \times 10^{11} \left( \frac{m_{3/2}}{100 \text{keV}} \right)^{-1/2} \left( \frac{T_{RH}}{M} \right). \]

Thus, the largest possible upper limit is achieved at \( M_F = 1 \) TeV as

\[ \frac{\phi_{osc}}{M} \gtrsim 1.8 \times 10^3 \left( \frac{m_\phi}{\text{TeV}} \right)^{-4/33} \times \left( \frac{M_F}{10^6 \text{GeV}} \right)^{-2/33} \left( \frac{T_{RH}}{M} \right)^{2/3}, \]

where the condition on the present baryon-to-entropy ratio, although the reheating temperature should be very high in order for this value to be achieved. Notice that it is the same as for the case that the Q balls survive from the evaporation, since \( Y_B \) is maximized at \( \Delta Q \sim Q \) in both cases.
3. Delayed Q balls when the zero-temperature potential is dominated

Since both the AD field and modulus field start to oscillate when $H_{\text{osc}} \sim m_{3/2}$, the ratio of the baryon number and the energy density of the modulus stays constant to the present. The baryon number is given by $n_B \sim \varepsilon m_{3/2} \phi_{\text{osc}}^2 \Delta Q/Q$, and the baryon-to-entropy ratio becomes

$$Y_B \sim 2.8 \times 10^{-24} \varepsilon \left( \frac{\Omega_{\text{mod}}h^2}{0.2} \right) \left( \frac{m_{\phi}}{\text{TeV}} \right)^{-2/3} \quad \times \left( \frac{m_{3/2}}{100\text{keV}} \right)^{8/3} \left( \frac{M_F}{10^6\text{GeV}} \right)^{-4} \left( \frac{\phi_{\text{osc}}}{M} \right)^2,$$

where Eqs. (13) and (16) are used. The gravitino mass is restricted by the stability condition, which can be expressed as $m_{3/2} \lesssim 0.16$ GeV.

Since survival condition sets the upper limit on $Y_B$, we will consider only this condition. It reads as

$$\frac{M_F}{10^6\text{GeV}} \gtrsim 2.1 \times 10^{-5} \left( \frac{m_{\phi}}{\text{TeV}} \right)^{-1/6} \left( \frac{m_{3/2}}{100\text{keV}} \right)^{11/12}.$$  (46)

This can be applied if

$$m_{3/2} \gtrsim 6.8 \left( \frac{m_{\phi}}{\text{TeV}} \right)^{2/11} \text{MeV}. \quad (47)$$

Otherwise, we must use $M_F \gtrsim 1$ TeV, when we estimate the upper bound on $Y_B$. Thus, we have

$$Y_B \lesssim 1.1 \times 10^{-12} \varepsilon \left( \frac{\Omega_{\text{mod}}h^2}{10^{-6}} \right) \left( \frac{m_{3/2}}{6.8\text{MeV}} \right)^{-1} \left( \frac{\phi_{\text{osc}}}{M} \right)^2,$$

(48)

for $m_{3/2} \gtrsim 6.8(m_{\phi}/\text{TeV})^{2/11}$ MeV, where Eqs. (13) and (16) are used. We also take $\Omega_{\text{mod}}h^2 \sim 10^{-6}$ (See Fig. 1). Notice that this limit is same as the case for the complete evaporation of the Q ball. On the other hand, when the gravitino mass is smaller than $6.8(m_{\phi}/\text{TeV})^{2/11}$ MeV,

$$Y_B \lesssim 2.8 \times 10^{-24} \varepsilon \left( \frac{\Omega_{\text{mod}}h^2}{0.2} \right) \left( \frac{m_{\phi}}{\text{TeV}} \right)^{-2/3} \quad \times \left( \frac{m_{3/2}}{100\text{keV}} \right)^{8/3} \left( \frac{M_F}{10^6\text{GeV}} \right)^{-4} \left( \frac{\phi_{\text{osc}}}{M} \right)^2.$$  (49)

Figure 3 shows the maximum value of the baryon-to-entropy ratio. As can be seen, this scenario is marginally successful ($Y_B \sim 10^{-11}$) only for $m_{3/2} \sim 200$ keV. Notice that, the maximal value of $Y_B$ is the same as that in the thermal logarithmic potential, to be considered in the next subsection, for $m_{3/2} \lesssim 0.16$ GeV.

4. Delayed Q balls when the thermal logarithmic potential is dominated

We consider the case that $V^{(2)}_T$ is dominant over $V_{\text{gauge}}$. This is the case if $T_{\text{eq}} \gtrsim M_F$, which is satisfied when

$$\frac{\phi_{\text{osc}}}{M} \lesssim \left( \frac{T_{\text{RH}}}{M_F} \right)^2.$$  (50)

This condition can be applied when $T_{\text{RH}} \lesssim M_F$. Otherwise, $\phi_{\text{osc}} \lesssim M$ should be used. $\phi_{\text{osc}}$ must be larger than $\phi_{\text{eq}}$ for the delayed Q-ball formation. It leads to

$$\frac{\phi_{\text{osc}}}{M} \gtrsim 1.6 \times 10^{11} \left( \frac{m_{3/2}}{100\text{keV}} \right)^{-1/2} \left( \frac{T_{\text{RH}}}{M_F} \right).$$  (51)

This condition holds only if $T_{\text{RH}} \lesssim (m_{3/2}M)^{1/2}$. Combining Eqs. (50) and (51), we have

$$\frac{T_{\text{RH}}}{M} \gtrsim 2.6 \times 10^{-14} \left( \frac{m_{3/2}}{100\text{keV}} \right)^{-1/2} \left( \frac{M_F}{10^6\text{GeV}} \right)^2.$$  (52)

Since the charge of the Q ball is written as

$$Q = \beta \left( \frac{\phi_{\text{eq}}}{T_{\text{eq}}} \right)^4 \simeq \beta \left( \frac{T_{\text{eq}}}{m_{3/2}} \right)^4,$$  (53)

the stability condition, given by $\omega \sim M_F Q^{-1/4} \lesssim 1$ GeV, is expressed as

$$\frac{\phi_{\text{osc}}}{M} \lesssim 1.4 \times 10^{31} \left( \frac{m_{3/2}}{100\text{keV}} \right)^{-2} \left( \frac{M_F}{10^6\text{GeV}} \right)^{-2} \left( \frac{T_{\text{RH}}}{M} \right)^2,$$  (54)
where this condition is effective only when the RHS is less than unity, which leads to a constraint on the reheating temperature as

$$\frac{T_{RH}}{M} \lesssim 2.6 \times 10^{-16} \left( \frac{m_{3/2}}{100\text{keV}} \right) \left( \frac{M_F}{10^9\text{GeV}} \right). \quad (55)$$

Otherwise, stability condition only implies that $\phi_{\text{osc}} \lesssim M$. In addition, the survival condition holds when

$$\frac{\phi_{\text{osc}}}{M} \lesssim 9.6 \times 10^{34} \left( \frac{m_\phi}{\text{TeV}} \right)^{4/11} \left( \frac{m_{3/2}}{100\text{keV}} \right)^{-2} \times \left( \frac{M_F}{10^9\text{GeV}} \right)^{2/11} \left( \frac{T_{RH}}{M} \right)^2, \quad (56)$$

where this condition can be applied if

$$\frac{T_{RH}}{M} \lesssim 3.2 \times 10^{-18} \left( \frac{m_\phi}{\text{TeV}} \right)^{-2/11} \times \left( \frac{m_{3/2}}{100\text{keV}} \right) \left( \frac{M_F}{10^9\text{GeV}} \right)^{-1/11}, \quad (57)$$

Otherwise, survival condition only implies that $\phi_{\text{osc}} \lesssim M$.

We must find the largest possible value of the baryon-to-entropy ratio,

$$Y_B \approx 1.1 \times 10^{-69} \varepsilon$$

$$\times \left( \frac{\Omega_{mod}h^2}{0.2} \right) \left( \frac{m_\phi}{\text{TeV}} \right)^{-2/3} \left( \frac{m_{3/2}}{100\text{keV}} \right)^{8/3} \times \left( \frac{M_F}{10^9\text{GeV}} \right)^{-1/3} \left( \frac{T_{RH}}{M} \right)^{-11/3} \left( \frac{\phi_{\text{osc}}}{M} \right)^{23/6}, \quad (58)$$

in the parameter space constrained by the above five conditions: thermal potential dominance, stability, survival, and delayed Q-ball formation conditions, and $\phi_{\text{osc}} \lesssim M$. The maximum value of $Y_B$ is achieved from the survival condition for $m_{3/2} \gtrsim 6.8(m_\phi/\text{TeV})^{2/11}$ MeV, which is written as

$$Y_B \lesssim 1.1 \times 10^{-12} \varepsilon \left( \frac{\Omega_{mod}h^2}{10^{-6}} \right) \left( \frac{m_{3/2}}{6.8\text{MeV}} \right)^{1}, \quad (59)$$

and from the thermal potential dominance condition for $m_{3/2} \gtrsim 6.8(m_\phi/\text{TeV})^{2/11}$ MeV, which can be expressed as

$$Y_B \lesssim 2.8 \times 10^{-12} \varepsilon \left( \frac{\Omega_{mod}h^2}{0.2} \right) \left( \frac{m_{3/2}}{100\text{keV}} \right)^{8/3} \left( \frac{m_\phi}{\text{TeV}} \right)^{-2/3}, \quad (60)$$

where we take $M_F = 1$ TeV. These are plotted in Fig. 3, and we can see that $Y_B$ is marginally enough ($Y_B \sim 10^{-11}$) only for $m_{3/2} \approx 200$ keV. Notice that the largest possible value of $Y_B$ is the same as Eq. (58) for the case that the charge of the Q ball evaporates completely, since the largest value is achieved when $\Delta Q \sim Q$.

5. New type Q balls

From Eqs. (54) and (55), we have

$$\Delta Q \approx 6.1 \times 10^{-23} \left( \frac{m_\phi}{\text{TeV}} \right)^{-2/3} \left( \frac{m_{3/2}}{100\text{keV}} \right)^{5/3} \left( \frac{\phi_{\text{osc}}}{M} \right)^{-2}. \quad (61)$$

This leads to the baryon-to-entropy ratio as

$$Y_B \lesssim 8.8 \times 10^{-28} \varepsilon \left( \frac{\Omega_{mod}h^2}{0.2} \right)^{-2/3} \left( \frac{m_{3/2}}{100\text{keV}} \right)^{2/3}, \quad (62)$$

which is too small to explain the present value. Notice that this is also true for the case of the complete evaporation, which condition can be written as

$$\frac{\phi_{\text{osc}}}{M} \lesssim 7.8 \times 10^{-12} \left( \frac{m_\phi}{\text{TeV}} \right)^{-1/3} \left( \frac{m_{3/2}}{100\text{keV}} \right)^{5/6}. \quad (63)$$

B. Unstable Q balls

In the case of the unstable Q balls which decay into nucleons, it may destroy light elements synthesized at the BBN. Thus, a new constraint which the Q ball should decay before the BBN ($\sim 1$ sec), must be imposed.

1. Gauge-mediation type Q balls when the zero-temperature potential is dominated

There are several condition to be imposed. The first is the condition that the Q ball is unstable, given by

$$\frac{\phi_{\text{osc}}}{M} \lesssim 2.6 \times 10^{-6} \left( \frac{M_F}{10^9\text{GeV}} \right)^2. \quad (64)$$

Second, the decay of the Q ball must be completed until the BBN, otherwise it would spoil the success of the BBN, so that the life time $\tau_Q$ should be

$$\tau_Q \equiv \left( \frac{1}{Q} \frac{dQ}{dt} \right)^{-1} \approx \frac{48\pi}{M_F} Q^{5/4} \lesssim 1\text{sec}, \quad (65)$$

hence the following constraint:

$$\frac{\phi_{\text{osc}}}{M} \lesssim 1.0 \times 10^{-6} \left( \frac{M_F}{10^9\text{GeV}} \right)^{6/5}. \quad (66)$$

In addition, we have conditions Eqs. (22) – (25), which leads to $r_H \approx 1$. Notice that the inequality (26) is stronger than the inequality (64), only for $m_{3/2}$ is larger than 0.16 GeV. In either case, the largest possible value
of $Y_B$ is obtained by the conditions Eq. (60) and $M_F \lesssim (m_{3/2} M)^{1/2}$, and will be written as

$$Y_B \simeq 3.5 \times 10^{-3} \varepsilon \left( \frac{\Omega_{m0} h^2}{0.2} \right) \left( \frac{M_F}{10^6 \text{GeV}} \right)^{-2} \left( \frac{\phi_{\text{osc}}}{M} \right)^3 \lesssim 2.8 \times 10^{-19} \varepsilon \left( \frac{\Omega_{m0} h^2}{0.2} \right) \left( \frac{m_{3/2}}{100 \text{ keV}} \right)^{4/5}.$$  \hspace{1cm} (67)

This is thus too small to explain the present value $\sim 10^{-10}$.

2. Gauge-mediation type Q balls when the thermal logarithmic potential is dominated

Since the Q-ball charge is expressed as

$$Q \simeq \beta \left( \frac{\phi_{\text{osc}}}{M} \right)^6 \left( \frac{T_{RH}}{M} \right)^{-4},$$  \hspace{1cm} (68)

the unstable condition, $M_F Q^{-1/4} \gtrsim 1$ GeV, is given by

$$\frac{\phi_{\text{osc}}}{M} \lesssim 3.4 \times 10^4 \left( \frac{M_F}{10^6 \text{GeV}} \right)^{2/3} \left( \frac{T_{RH}}{M} \right)^{2/3},$$  \hspace{1cm} (69)

while the lifetime condition that the Q ball decays before the BBN time ($\sim 1$ sec), is written as

$$\frac{\phi_{\text{osc}}}{M} \lesssim 1.9 \times 10^4 \left( \frac{T_{RH}}{M} \right)^{2/3}.$$  \hspace{1cm} (70)

As will be seen, $Y_B$ becomes larger for larger $M_F$, so that the lifetime condition determines the upper limit on $Y_B$.

Let us first consider the case $r_H \approx 1$, which sets the upper bound on the reheating temperature. In general, the SUSY breaking scenario sets the upper bound on $M_F$, such as $\lesssim (m_{3/2} M)^{1/2}$. In this case, the RHS of Eq. (70) is less than unity, and the lifetime condition can be directly applied for estimating the baryon-to-entropy ratio. Therefore, it will be

$$Y_B \simeq 6.0 \times 10^{-28} \varepsilon r_H \left( \frac{\Omega_{m0} h^2}{0.2} \right) \left( \frac{T_{RH}}{M} \right)^{-2} \left( \frac{\phi_{\text{osc}}}{M} \right)^4 \lesssim 5.6 \times 10^{-18} \varepsilon \left( \frac{\Omega_{m0} h^2}{0.2} \right) \left( \frac{m_{3/2}}{100 \text{ keV}} \right)^{3/5},$$  \hspace{1cm} (71)

where we use $T_{RH} \lesssim T^{(c)}_{RH}$ and $M_F \lesssim (m_{3/2} M)^{1/2}$ in the last line. It is thus much smaller than the present value.

On the other hand, when the reheating temperature is higher than $T^{(c)}_{RH}$, the lifetime condition again puts the upper limit on $Y_B$. The condition can be applied if

$$\frac{T_{RH}}{M} \lesssim 4.1 \times 10^{-7} \left( \frac{M_F}{10^6 \text{GeV}} \right)^{-1/5}.$$  \hspace{1cm} (72)

Taking this constraint into account, we have the baryon-to-entropy ratio as

\[ \text{FIG. 4. Largest possible baryon-to-entropy ratio in the unstable Q-ball scenario in the thermal logarithmic potential.} \]

$$Y_B \lesssim 8.4 \times 10^{-10} \varepsilon \left( \frac{\Omega_{m0} h^2}{0.2} \right) \left( \frac{m_{3/2}}{100 \text{ keV}} \right)^{-2/5},$$  \hspace{1cm} (73)

where $M_F \lesssim (m_{3/2} M)^{1/2}$ is again used. We plot the largest possible value of $Y_B$ [Eq. (73)] in the function of $m_{3/2}$ in Fig. 4. As can be seen, we can explain the present value for $m_{3/2} \lesssim 500$ keV. However, the reheating temperature should be as high as $5.6 \times 10^{11}$ GeV, which may be rather too high for the actual inflation models.

3. Delayed Q balls

When the zero-temperature potential, $V_{\text{gauge}}$, dominates over the thermal logarithmic one, $V^{(2)}_T$, the unstable condition can be expressed as $m_{3/2} \gtrsim 0.16$ GeV. On the other hand, when $V^{(2)}_T \gtrsim V_{\text{gauge}}$, the same condition can be given by

$$\frac{\phi_{\text{osc}}}{M} \gtrsim 1.4 \times 10^{31} \left( \frac{m_{3/2}}{100 \text{ keV}} \right)^{-2} \left( \frac{M_F}{10^6 \text{GeV}} \right)^{-2} \left( \frac{T_{RH}}{M} \right)^2.$$  \hspace{1cm} (74)

In addition, the thermal potential dominance, $M_F \lesssim T_{\text{osc}}$, is rewritten as

$$\frac{\phi_{\text{osc}}}{M} \lesssim \left( \frac{T_{RH}}{M_F} \right)^2.$$  \hspace{1cm} (75)

Combining these two, we obtain the constraint on the gravitino mass as $m_{3/2} \gtrsim 0.16$ GeV. This is exactly the same as the former case that the zero-temperature potential is dominant. Therefore, in either case, we can estimate the baryon-to-entropy ratio as
\[ Y_B \lesssim 4.5 \times 10^{-17} \varepsilon \left( \frac{\Omega_{\text{mod}} h^2}{10^{-9}} \right) \left( \frac{m_{3/2}}{0.16 \text{GeV}} \right)^{-1} \left( \frac{\phi_{\text{osc}}}{M} \right)^2, \]  
(76)

where \( \Omega_{\text{mod}} h^2 \lesssim 10^{-9} \) for \( m_{3/2} \gtrsim 0.16 \text{ GeV} \) (See Fig. 2). This is too small to explain the present value, even if \( \phi_{\text{osc}} \sim M \).

C. Early oscillation due to the thermal mass term

Since there is no cosmological gravitino problem because of the late-time entropy production, the reheating temperature can be as large as \( \sim 10^{16} \text{ GeV} \). We take this value, because the COBE data implies that \( V_{\text{inf}} / \epsilon_s^4 \sim 6.7 \times 10^{16} \text{ GeV} \) [2], where \( \epsilon_s \) is the slow-roll parameter, and the instantaneous reheating is assumed for conservative discussion. Therefore, the thermal mass term \( V_T^{(1)} \sim f^2 T^2 \phi^2 \) can extend towards as large as the Planck scale if the coupling constant is not so large \( (f \lesssim 4.1 \times 10^{-3}) \). Here we consider the case that the thermal mass term causes the early oscillation of the AD field. If the particles coupled to the AD field are in the thermal bath, their mass should be less than the temperature, so that

\[ f \phi_{\text{osc}} \lesssim T_{\text{osc}}, \]  
(77)

where \( T_{\text{osc}} \) is the temperature at the beginning of the oscillation of the AD field. The oscillation starts when \( H_{\text{osc}} \sim f T_{\text{osc}} \), which reads as

\[ \omega \sim H_{\text{osc}} \sim \left( f^2 T_{RH} M^{1/2} \right)^{2/3}. \]  
(78)

Substituting this into Eq. (77), we have

\[ \frac{\phi_{\text{osc}}}{M} \lesssim f^{-2/3} \left( \frac{T_{RH}}{M} \right)^{2/3}. \]  
(79)

This constraint can be used if its RHS is less than unity:

\[ T_{RH} \lesssim T_{RH} \equiv 10^{-5} \left( \frac{f}{10^{-5}} \right) M. \]  
(80)

Otherwise, \( \phi_{\text{osc}} \lesssim M \) and \( T_{RH} \lesssim 10^{16} \text{ GeV} \) should be the only constraints.

The baryon-to-entropy ratio is given by

\[ Y_B \approx \frac{2 \omega^2 \phi_{\text{osc}}^2 r_{RH} \Omega_{\text{mod}} \rho_c s_0}{M^2} \]  
\[ \sim 6.0 \times 10^{-28} \varepsilon_{\text{RH}} f^{-4/3} \left( \frac{\Omega_{\text{mod}} h^2}{0.2} \right) \]  
\[ \times \left( \frac{T_{RH}}{M} \right)^{-2/3} \left( \frac{\phi_{\text{osc}}}{M} \right)^2, \]  
\[ \lesssim 6.0 \times 10^{-28} \varepsilon_{\text{RH}} f^{-8/3} \left( \frac{\Omega_{\text{mod}} h^2}{0.2} \right) \left( \frac{T_{RH}}{M} \right)^{2/3}. \]  
(81)

We show the upper limit on the baryon-to-entropy ratio in the function of \( m_{3/2}, \) Eq. (81) in Fig. 2. We can see that the present baryon number can be explained for \( m = 3/2 \lesssim 100 \text{ keV} \). However, this reheating temperature is unrealistically high. Moreover, since the Q-ball production will diminish the efficiency of the baryon number creation, this value will be much smaller, and cannot explain the present value.
V. CONCLUSION

We have investigate the possibility of the AD baryogenesis in the gauge-mediated SUSY breaking scenario, while evading the cosmological moduli problem by the late-time entropy production. In all the cases, the Q-ball formation makes the efficiency of the baryon number production considerably diminish. For the zero-temperature potential $V_{\text{gauge}}$-dominated case, whether the produced Q balls are stable or not, the largest possible baryon-to-entropy ratio is too small to explain the present value. This completely kills the successful situations considered in Ref. [24].

We have also found that there are some marginally successful situations when we take into account of the thermal effects on the effective potential of the AD field. However, these successful situations require very high reheating temperatures such as $10^{12} - 10^{16}$ GeV, which might be impossible to achieve in the actual inflation models. Furthermore, Q balls must decay at the maximal decay rate for these situations to be successful. However, it is questionable whether such a fast decay process exists. Therefore, we can conclude that the AD baryogenesis here (i.e., the delayed Q-ball scenario) will not work.

In the delayed Q-ball formation case, we have found that enough baryon-to-entropy ratio can be created, even in the zero-temperature potential $V_{\text{gauge}}$-dominated case, whether the produced Q balls are stable or not, the largest possible baryon-to-entropy ratio is too small to explain the present value. This completely kills the successful situations considered in Ref. [24].

We have also found that there are some marginally successful situations when we take into account of the thermal effects on the effective potential of the AD field. However, these successful situations require very high reheating temperatures such as $10^{12} - 10^{16}$ GeV, which might be impossible to achieve in the actual inflation models. Furthermore, Q balls must decay at the maximal decay rate for these situations to be successful. However, it is questionable whether such a fast decay process exists in the gauge-mediated SUSY breaking models [24].

In the delayed Q-ball formation case, we have found that enough baryon-to-entropy ratio can be created, even in the zero-temperature potential $V_{\text{gauge}}$ is dominant over the thermal logarithmic potential $V_T^{(2)}$. It might be the unique solution for the AD baryogenesis with solving the cosmological moduli problem, although the scale of $M_F$ is rather low.

In addition, successful situations above need the following conditions: the large initial amplitude such as $\phi_{\text{osc}} \simeq M$, and $\epsilon \simeq 1$. This is realized if the $A$-terms, which make the AD field rotate in the effective potential, originate from some Kähler potential with vanishing superpotential. Then, $\epsilon \sim (\phi/M)^\gamma \sim 1$ for $\phi \sim M$, where $\gamma > 2$. If the $A$-terms are determined by the nonrenormalizable superpotential $W \sim \phi^n/M^{n-3}$, $\epsilon \sim 1$ can be obtained in some parameter region, but the amplitude of the AD field becomes much less than the Plank scale, i.e., $\phi_{\text{osc}} < M$, which makes the baryon-to-entropy ratio much smaller than the present value.

If we consider the large late-time entropy production, the only candidate we know is the thermal inflation models. In order for the entropy production to be enough to dilute the dangerous moduli fields, the thermal inflation must last for long enough. This constrains the scale of $M_F$ to be larger than $\sim 10^9$ GeV. If so, all the successful scenario with not too high reheating temperatures found here (i.e., the delayed Q-ball scenario) will not work. Therefore, we can conclude that the AD baryogenesis is not compatible with the late-time entropy production evading the cosmological moduli problem.

Since the baryogenesis before the late-time entropy production does not account for enough amount of the baryons in the present universe, there should be some mechanism worked after the late-time entropy production. If the reheating temperature after the late-time entropy production is higher than the electroweak scale, the electroweak baryogenesis might work. However, it is generally very difficult to have strong first-order phase transition for the Higgs mass of $\sim 114$ GeV or larger.

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