Brownian motion of a charged test particle in vacuum between two conducting plates

Hongwei Yu
CCAST(World Lab.), P. O. Box 8730, Beijing, 100080, P. R. China and Department of Physics and Institute of Physics, Hunan Normal University, Changsha, Hunan 410081, China*

Jun Chen
Department of Physics and Institute of Physics, Hunan Normal University, Changsha, Hunan 410081, China

Abstract
The Brownian motion of a charged test particle caused by quantum electromagnetic vacuum fluctuations between two perfectly conducting plates is examined and the mean squared fluctuations in the velocity and position of the test particle are calculated. Our results show that the Brownian motion in the direction normal to the plates is reinforced in comparison to that in the single-plate case. The effective temperature associated with this normal Brownian motion could be three times as large as that in the single-plate case. However, the negative dispersions for the velocity and position in the longitudinal directions, which could be interpreted as reducing the quantum uncertainties of the particle, acquire positive corrections due to the presence of the second plate, and are thus weakened.

PACS numbers: 05.40.Jc, 12.20.Ds, 03.70.+k

* Mailing address
I. INTRODUCTION

Quantum vacuum fluctuations have been subjected to extensive studies since the emergence of quantum theory which has profoundly changed our conception of empty space or vacuum. Now we believe that vacuum is not just a synonym of nothingness. Vacuum fluctuates all the time and may have rich structures. Although some physical quantities in quantum field theory, such as energy, are not well-defined in vacuum and we have to use certain renormalization scheme to make them finite, changes in the vacuum fluctuations, however, usually exhibit normal behavior and can produce observable effects. The Lamb shift and the Casimir effect are two examples of this.

Recently, the Brownian motion of a charged particle caused by changes in the electromagnetic vacuum near a perfectly reflecting plane boundary has been investigated [1] and the effects have been calculated of the modified electromagnetic vacuum fluctuations due to the presence of the boundary upon the motion of a charged test particle. In particular, it has been shown that the mean squared fluctuations in the normal velocity and position of the test particle can be associated with an effective temperature of

\[ T_{\text{eff}} = \frac{\omega}{\kappa k_B z^2} = 1.7 \times 10^{-6} \left( \frac{1 \text{m}}{z} \right)^2 K = 1.7 \times 10^2 \left( \frac{1 \text{Å}}{z} \right)^2 K, \]

where \( k_B \) is Boltzmann’s constant and \( z \) is the distance from the boundary. This result seems to be experimentally accessible.

However, in addition to the fluctuating quantum field-theoretic force, a charged particle also feels a classical image charge force in the single-plate geometry. This force tends to pull the particle toward the plate and thus makes it difficult to observe the random motion of the particle caused by quantum vacuum fluctuations studied in Ref. [1]. One way to minimize the influence of the image charge is to add another parallel plate. Then, a charged particle moving exactly at midway between the plates feels no net classical force. Note, however, that this trajectory is unstable. It can be shown that adding a second plate could increase the characteristic falling time of the particle to a plate by approximately an order of magnitude, making it experimentally more feasible to observe the effects of the quantum vacuum fluctuations. So, we wish to examine, in the present paper, the Brownian motion of a charged test particle coupled to the electromagnetic vacuum fluctuations between two conducting plates.

Finally, it is worth noting that the present problem bears some analogy to the problem of lightcone fluctuations, where photons undergo Brownian motion due to modified quantum fluctuations of the quantized gravitational field [2, 3, 4, 5].
II. VACUUM FLUCTUATION AND THE BROWNIAN MOTION OF THE TEST PARTICLE

Let us consider the motion of a charged test particle subject to quantum electromagnetic vacuum fluctuations in the vacuum between two conducting plates. In the limit of small velocities and assuming that the particle is initially at rest and has a charge to mass ratio of $e/m$, the mean squared speed in the $i$-direction can be written as (no sum on $i$)

$$
\langle \Delta v_i^2 \rangle = \frac{e^2}{m^2} \int_0^t \int_0^t \langle E_i(x, t_1) E_i(x, t_2) \rangle_R dt,
$$

(2)

where $\alpha$ is the fine-structure constant and $\langle E_i(x, t_1) E_i(x, t_2) \rangle_R$ is the renormalized electric field two-point function obtained by subtracting the boundary-independent Minkowski vacuum term. We have, for simplicity, assumed that the distance does not change significantly in a time $t$, so that it can be treated approximately as a constant.

If there is a classical, nonfluctuating field in addition to the fluctuating quantum field, then Eq. (2) describes the velocity fluctuations around the mean trajectory caused by the classical field.

Let us now calculate the vacuum expectation of the squared electric field in the case of two parallel conducting plates. The two point function for the photon field may be expressed as

$$
D_{\mu\nu}(x, x') = \langle 0 | A^\mu(x) A^\nu(x') | 0 \rangle = D_{0\mu\nu}(x - x') + D_{R\mu\nu}(x, x'),
$$

(3)

where $D_{0\mu\nu}(x - x')$ is the two point function in the usual Minkowski vacuum, and the renormalized two point function, $D_{R\mu\nu}(x, x')$, is the correction induced by the presence of boundaries which can be obtained by the method of images [6]. In the Feynman gauge, we have, assuming one plate is at $z = 0$ and the other at $z = a$,

$$
D_{0\mu\nu}(x - x') = \frac{\eta_{\mu\nu}}{4\pi^2(\Delta t^2 - \Delta x^2)}
$$

(4)

and

$$
D_{R\mu\nu}(x, x') = - \sum_{n=-\infty}^{\infty} \frac{\eta_{\mu\nu} + 2n^\mu n^\nu}{4\pi^2(\Delta t^2 - \Delta x^2 - \Delta y^2 - (z + z' + 2na)^2)} + \sum_{n=-\infty}^{\infty} \frac{\eta_{\mu\nu}}{4\pi^2(\Delta t^2 - \Delta x^2 - \Delta y^2 - (z - z' + 2na)^2)},
$$

(5)

Here and after a prime means that the $n = 0$ term is omitted in the summation, $\eta_{\mu\nu} = \text{diag}(1,-1,-1,-1)$ and the unit normal vector $n^\mu = (0, 0, 0, 1)$. Note that the two-point function Eq. (5) is constructed in such a way that the tangential components of the
electric field two-point function vanish on the conducting plates \[6\], i.e., whenever \(z = 0\) or \(z = a\). At a point a distance \(z\) from the plate at \(z = 0\), the components of the renormalized electric field two-point function, \(\langle E(x, t_1) E(x, t_2) \rangle_R\), are\(^1\)

\[
\langle E_x(x, t') E_x(x, t'') \rangle_R = \langle E_y(x, t') E_y(x, t'') \rangle_R = \frac{1}{\pi^2} \left\{ \sum_{n=-\infty}^{\infty} \frac{\Delta t^2 + 4n^2a^2}{(\Delta t^2 - 4n^2a^2)^2} \right\},
\]

and

\[
\langle E_z(x, t') E_z(x, t'') \rangle_R = \frac{1}{\pi^2} \left\{ \sum_{n=-\infty}^{\infty} \frac{1}{(\Delta t^2 - 4n^2a^2)^2} + \sum_{n=-\infty}^{\infty} \frac{1}{[\Delta t^2 - 4(na + z)^2]^2} \right\}.
\]

Substituting the above results into Eq. (2) and carrying out the integration, we find that the velocity dispersions in the parallel directions are given by

\[
\langle \Delta v_z \rangle = \langle \Delta v_a \rangle = \frac{e^2}{m^2} \int_0^t \int_0^{t_1} \int_0^{t_2} \int_0^{t_3} \langle E_x(x, t') E_x(x, t'') \rangle_R dt' dt''
\]

\[
= \frac{e^2}{\pi^2m^2} \left\{ \sum_{n=-\infty}^{\infty} \left[ \langle E_x(x, t') \rangle - \langle E_x(x, t'') \rangle \right] - \sum_{n=-\infty}^{\infty} f_L(na + z, t) \right\}.
\]

Here we have defined

\[
f_L(x, t) = \frac{t^2}{8a^2(t^2 - 4x^2)} - \frac{t}{64|x|^2} \ln \left( \frac{t + 2|x|}{t - 2|x|} \right)^2.
\]

The mean squared position fluctuations can be calculated as follows

\[
\langle \Delta x^2 \rangle = \langle \Delta y^2 \rangle = \int_0^t dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt'' \langle E_x(x, t') E_x(x, t'') \rangle_R
\]

\[
= \frac{e^2}{\pi^2m^2} \left\{ \sum_{n=-\infty}^{\infty} \left[ g_L(na, t) - \sum_{n=-\infty}^{\infty} g_L(na + z, t) \right] \right\},
\]

with

\[
g_L(x, t) = \frac{e^2}{\pi^2m^2} \left[ -\frac{t^3}{192|x|^3} \ln \left( \frac{t + 2|x|}{t - 2|x|} \right)^2 + \frac{t^2}{24x^2} + \frac{1}{6} \ln \left( \frac{t^2 - 4x^2}{4x^2} \right) \right].
\]

In the transverse direction, the velocity dispersion is

\[
\langle \Delta v_z^2 \rangle = \frac{e^2}{\pi^2m^2} \int_0^t \int_0^{t} \langle E_z(x, t') E_z(x, t'') \rangle_R dt' dt''
\]

\[
= \frac{e^2}{\pi^2m^2} \left\{ \sum_{n=-\infty}^{\infty} f_T(na, t) + \sum_{n=-\infty}^{\infty} f_T(na + z, t) \right\},
\]

\(^1\) Lorentz-Heaviside units with \(c = \hbar = 1\) will be used here.
where \( f_T(x, t) \) is defined as
\[
f_T(x, t) = \frac{t}{32|x|^3} \ln \left( \frac{2|x| + t}{2|x| - t} \right)^2.
\] (13)

The corresponding position fluctuation is
\[
\langle \Delta z^2 \rangle = \int_0^t dt_1 \int_0^{t_1} dt' \int_0^t dt_2 \int_0^{t_2} dt'' (E_z(x, t') E_z(x, t'')) R
\]
\[= \frac{e^2}{\pi^2 m^2} \left\{ \sum_{n=-\infty}^{\infty} \left[ g_L(na, t) - g_L(na + z, t) \right] \right\},
\] (14)
with
\[
g_L(x, t) = \frac{e^2}{\pi^2 m^2} \left[ \frac{t^2}{24x^4} + \frac{t^3}{96|x|^3} \ln \left( \frac{t + 2|x|}{t - 2|x|} \right)^2 + \frac{1}{6} \ln \left( \frac{t^2 - 4x^2}{4x^2} \right) \right].
\] (15)

It does not seem to be an easy task to get a closed-form result for the above summations. Now we wish to discuss two special cases of interest, i.e., the cases of \( a \gg t \) and \( a \ll t \).

A. The \( a \gg t \) case

In the \( a \gg t \) case, the mean squared fluctuations in the velocity of the particle can be approximately evaluated as follows
\[
\langle \Delta v_x^2 \rangle = \langle \Delta v_y^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left[ \frac{\pi^4 t^2}{720a^4} + \frac{\pi^4 t^2 (2 + \cos \frac{2\pi x}{a}) \csc^4 \frac{\pi x}{a}}{48a^4} \right.
\]
\[- \frac{t^2}{16(a - z)^4} - \frac{t^2}{16z^4} - f_T(z, t) - f_T(a - z, t) \left. \right],
\] (16)
and
\[
\langle \Delta v_z^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left[ \frac{\pi^4 t^2}{720a^4} + \frac{\pi^4 t^2 (2 + \cos \frac{2\pi z}{a}) \csc^4 \frac{\pi z}{a}}{48a^4} \right.
\]
\[- \frac{t^2}{16(a - z)^4} + \frac{t^2}{16z^4} + f_T(z, t) + f_T(a - z, t) \left. \right],
\] (17)
where we have used
\[
\sum_{n=-\infty}^{\infty} \frac{1}{(n a + z)^4} = \frac{\pi^4 (2 + \cos \frac{2\pi z}{a}) \csc^4 \frac{\pi z}{a}}{3a^4}.
\] (18)

It should be pointed out that the above expressions are singular at \( t = 2z \) or \( t = 2(a - z) \). This corresponds to a time interval equal to the round-trip light travel time
between the particle and the plate at \( z = 0 \) or \( z = a \). Presumably, this might be a result of our assumption of a rigid perfectly reflecting plane boundary, and would thus be smeared out in a more realistic treatment, where fluctuations in the position of the plates are taken into account. Note that the above results are symmetric under \( z \leftrightarrow a - z \) as they should be by the symmetry of the system and furthermore \( \langle \Delta v^2 \rangle \) is regular in limits of both \( z \to 0 \) and \( z \to a \), which can be seen by expanding \( \langle \Delta v^2 \rangle \) around \( z = 0 \). This may come as a surprise at the first glance. It can, however, be understood as a result of the fact that the tangential components of the electric field two-point function vanish at the conducting boundaries. \( \langle \Delta v^2 \rangle \), on the other hand, diverges as the boundaries are approached and it is a reflection of the divergence of the normal electric field two-point function at the boundaries. If we further assume that \( a \gg z \), then

\[
\langle \Delta v^2 \rangle = \langle \Delta v^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left[ -f_L(z, t) + \frac{1}{8} \left( \frac{t}{a} \right)^4 \left( \frac{z}{a} \right)^8 \frac{1}{t^2} \right],
\]

and

\[
\langle \Delta v^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left[ f_T(z, t) + \frac{\pi^4 t^2}{360a^4} \right].
\]

A comparison of the above results with Eqs. (9, 11) of Ref. [1] shows that the two last terms in the above equations are the corrections introduced by the presence of the second plate which, in the approximation being considered, can be regarded as being very far away from the other plate. For \( t \gg z \), Eq. (19) and Eq. (20) become

\[
\langle \Delta v^2 \rangle = \langle \Delta v^2 \rangle \approx -\frac{e^2}{3\pi^2 m^2} \frac{1}{t^2} + \frac{e^2}{8\pi^2 m^2} \left( \frac{t}{a} \right)^4 \left( \frac{z}{a} \right)^8 \frac{1}{t^2},
\]

and

\[
\langle \Delta v^2 \rangle \approx \frac{e^2}{4\pi^2 m^2} \frac{1}{z^2} + \frac{\pi^2 e^2}{360m^2 a^2} \left( \frac{t}{a} \right)^2 + \frac{e^2}{3\pi^2 m^2} \frac{1}{t^2}.\]

The correction term has an opposite sign in the longitudinal directions and it makes the dispersion less negative than what it would be with just a single plate. While the dispersion in the transverse direction acquires an positive correction and is thus greater than that in the single-plate case.

Similarly, we have for the dispersions in position

\[
\langle \Delta x^2 \rangle = \langle \Delta y^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left[ -\frac{\pi^4 t^4}{2880a^4} + \frac{\pi^4 t^4(2 + \cos \frac{2\pi z}{a}) \csc^4 \frac{\pi z}{a}}{192a^4} \right.
\]

\[
-\frac{t^4}{64(a - z)^4} - \frac{t^4}{64z^4} - g_L(z, t) - g_L(a - z, t),
\]

and

\[
\langle \Delta z^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left[ \frac{\pi^4 t^4}{2880a^4} - \frac{\pi^4 t^4(2 + \cos \frac{2\pi z}{a}) \csc^4 \frac{\pi z}{a}}{192a^4} \right.
\]

\[
-\frac{t^4}{64(a - z)^4} - \frac{t^4}{64z^4} + g_L(z, t) + g_L(a - z, t) \right].
\]
Their limiting forms for \( a \gg z \) and \( t \gg z \) are
\[
\langle \Delta x^2 \rangle = \langle \Delta y^2 \rangle \approx -\frac{e^2}{3\pi^2 m^2} \ln(t/2z) + \frac{e^2}{32\pi^2 m^2} \left( \frac{t}{a} \right)^4 \left( \frac{z}{a} \right)^8 ,
\]
and
\[
\langle \Delta z^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left[ \frac{t^2}{8z^2} + \frac{1}{3} \ln \left( \frac{t}{2z} \right) + \frac{\pi^4}{1440} \left( \frac{t}{a} \right)^4 \right] .
\]

B. The \( t \gg a \) case

We now turn to the \( t \gg a \) case, which is of more significance as far as the experimental measurement of the effects are concerned. Let us first write the velocity dispersions in the parallel directions as
\[
\langle \Delta v_x^2 \rangle = \langle \Delta v_y^2 \rangle = \frac{e^2}{\pi^2 m^2} \left[ -f_L(z, t) - f_L(z - a, t) + h(z, t) \right] ,
\]
where
\[
h(z, t) \equiv \sum_{n=1}^{+\infty} \left[ 2f_L(na, t) - f_L(z + na, t) \right] - \sum_{n=2}^{+\infty} f_L(z - na, t) .
\]

Now the summation can be estimated by integration. Defining \( \gamma = t/2a \), we have, for example,
\[
\sum_{n=1}^{+\infty} 2f_L(2na, t) \approx \frac{\gamma}{t^2} \int_1^{1/\gamma} \frac{1}{x^2(1 - x^2)} dx - \frac{\gamma}{2t^2} \int_1^{1/\gamma} \frac{1}{x^3} \ln \left( \frac{1 + x}{1 - x} \right) dx
\approx \gamma w(1/\gamma) .
\]
It then follows that
\[
h(z, t) \approx \gamma \left[ w(1/\gamma) - \frac{1}{2} w(1/\gamma + 2z/t) - \frac{1}{2} w(2/\gamma - 2z/t) \right] .
\]
Performing the integration and expanding the result as a power series of \( 1/\gamma \), we obtain
\[
\langle \Delta v_x^2 \rangle = \langle \Delta v_y^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left[ -\frac{1}{3t^2} + \frac{32a^2 - 24z(a - z)}{15t^4} \right] .
\]

Similarly, we find
\[
\langle \Delta v_z^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left[ \frac{a^2 - 2z(a - z)}{4z^2(a - z)^2} + \frac{1}{2a^2} + \frac{3}{4(a + z)(2a - z)} - \frac{1}{t^2} \right] .
\]

The above results have been written in such a way that the symmetry under \( z \leftrightarrow a - z \) is manifest. In the further limit \( a \gg z \), these velocity dispersions become
\[
\langle \Delta v_x^2 \rangle = \langle \Delta v_y^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left[ -\frac{1}{3t^2} + \frac{32a^2}{15t^4} \right] ,
\]
\[ \langle \Delta v_z^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left( \frac{1}{4z^2} + \frac{7}{8a^2} - \frac{1}{t^2} \right). \] (34)

It is interesting to note that the mean squared velocity fluctuation in the directions parallel to the plates dies off quickly as time progresses and it is basically a transient effect. Unlike the velocity dispersion in the parallel directions, that in the direction perpendicular to the plate approaches a nonzero constant value at late times. It is interesting to note that here no dissipation is needed for \( \langle \Delta v_i^2 \rangle \) to be bounded at late times in contrast to the Brownian motion due to thermal noise.

The mean squared fluctuations in position in both parallel and perpendicular directions can also be found in the same way as follows

\[ \langle \Delta x^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left[ -\frac{1}{3} \ln \frac{t}{2z} - \frac{1}{3} \ln \frac{t}{2(a-z)} + \frac{2a-z}{3a} \ln \frac{t}{2(a+z)} \right], \] (35)

and

\[ \langle \Delta z^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left[ \frac{t^2(a^2-2z(a-z))}{8z^2(a-z)^2} + \frac{t^2}{4a^2} + \frac{3t^2}{8(a+z)(2a-z)} + \frac{1}{3} \ln \frac{t}{4z(a-z)} - \frac{2}{3} \ln \frac{t}{2a} - \frac{a+z}{3a} \ln \frac{t}{2(a+z)} - \frac{2a-z}{3a} \ln \frac{t}{2(a-z)} \right]. \] (36)

Their limiting forms for \( a \gg z \) are

\[ \langle \Delta x^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left( -\frac{1}{3} \ln \frac{t}{2z} - \frac{2}{3} \ln 2 \right), \] (37)

and

\[ \langle \Delta z^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left( \frac{t^2}{8z^2} + \frac{7t^2}{16a^2} + \frac{1}{3} \ln \frac{t}{2z} - \frac{2}{3} \ln \frac{t}{2a} - \frac{2}{3} \ln \frac{t}{4a} \right). \] (38)

Finally, let us note that all dispersions attain their extremal values at \( z = a/2 \) as listed below

\[ \langle \Delta v_x^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left[ -\frac{1}{3t^2} + \frac{26a^2}{15t^4} \right], \] (39)

\[ \langle \Delta v_z^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left( \frac{17}{6a^2} - \frac{1}{t^2} \right), \] (40)
\[ \langle \Delta x^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left( -\frac{2}{3} \ln \frac{t^2}{2a^2} + \ln \frac{t}{3a} \right), \]  

(41)

\[ \langle \Delta z^2 \rangle \approx \frac{e^2}{\pi^2 m^2} \left( \frac{17t^2}{12a^2} + \frac{2}{3} \ln 2 - \ln \frac{t}{3a} \right). \]  

(42)

If we compare the above results with those in the case of a single plate when \( z = a/2 \), we can see that the dispersions in the normal direction to the plates get amplified roughly by a factor of \( 17/6 \approx 2.8 \), i.e., the dispersions in the present case are approximately three times that of the single-plate case.

One may wonder if the \( t \gg a \) case is meaningful, since the particle will ultimately fall to one of the plates due to the classical image force. There is no exception to this destiny even if it is placed at the midway, when quantum fluctuations are considered.

It can be shown that characteristic falling time, \( t_c \), is given approximately by \( \sqrt{\frac{mz_0^3}{e^2}} \). Here \( z_0 \) is the initial distance to a plate. For the calculations in the \( t \gg a \) case to make sense, we must have \( t_c \gg a \). Taking an electron initial at the midway as an example, this leads to the constraint on \( a \gg 2.4 \times 10^{-10} \text{cm} = 2.4 \times 10^{-4} \mu \text{m} \). Another issue is the condition for our assumption in Eq. (2) to be valid that the particle does not significantly change its position, i.e., \( \langle \Delta z^2 \rangle \ll z \). It can be shown that this requirement yields \( t \ll (mz)_z \). For an electron initially at \( z = a/2 \) at \( t = 0 \), this means \( t \ll 10^{10}(a/1 \text{cm})a \). So, there is much room for both \( t \gg a \) and our assumption to be fulfilled.

III. SUMMARY

In summary, we have examined the Brownian motion of a charged test particle caused by quantum electromagnetic vacuum fluctuations between two perfectly conducting plates and calculated the mean squared fluctuations in the velocity and position of the test particle. Our results show that the Brownian motion in the transverse direction to the plates is reinforced in comparison to that in the single-plate case. The effective temperature associated with this transverse Brownian motion could be three times as large as that in the case of the single-plate. And this, together with the fact that adding the second plate could increase the characteristic falling time of the particle by roughly an order of magnitude, suggests that the Brownian motion caused by the vacuum fluctuations would be easier to be observed in the two-plate case. Our calculations also reveal that the negative dispersions for the velocity and position in the longitudinal directions, which could be interpreted as reducing the quantum uncertainties of the particle, acquire positive corrections due to the presence of the second plate, and are hence weakened.
Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant No. 10375023, the National Basic Research Program of China under Grant No. 2003CB71630.

[1] H. Yu and L.H. Ford, Phys. Rev. D 70, 065009 (2004).
[2] L.H. Ford, Phys. Rev. D 51, 1692 (1995).
[3] H. Yu and L.H. Ford, Phys. Rev. D 60, 084023 (1999).
[4] H. Yu and L.H. Ford, Phys. Lett. B 496, 107 (2000); gr-qc/0004063.
[5] H. Yu and P.X. Wu, Phys. Rev. D 68, 084019 (2003).
[6] L.S. Brown and G.J. Maclay, Phys. Rev. D 184, 1272 (1969).