Complete fusion of $^9$Be with spherical targets

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(Dated: March 2, 2010)

The complete fusion of $^9$Be with $^{144}$Sm and $^{208}$Pb targets is calculated in the coupled-channels approach. The calculations include couplings between the $3/2^-, 5/2^-$, and $7/2^-$ states in the $K=3/2$ ground state rotational band of $^9$Be. It is shown that the $B(E2)$ values for the excitation of these states are accurately described in the rotor model. The interaction of the strongly deformed $^9$Be nucleus with a spherical target is calculated using the double-folding technique and the effective M3Y interaction, which is supplemented with a repulsive term that is adjusted to optimize the fit to the data for the $^{144}$Sm target. The complete fusion is described by in-going-wave boundary conditions. The decay of the unbound excited states in $^9$Be is considered explicitly in the calculations by using complex excitation energies. The model gives an excellent account of the complete fusion (CF) data for $^9$Be+$^{144}$Sm, and the cross sections for the decay of the excited states are in surprisingly good agreement with the incomplete fusion (ICF) data. Similar calculations for $^9$Be+$^{208}$Pb explain the total fusion data at high energies but fail to explain the CF data, which are suppressed by 20%, and the calculated cross section for the decay of excited states is a factor of three smaller than the ICF data at high energies. Possible reasons for these discrepancies are discussed.

PACS numbers: 24.10.Eq, 25.60.Pj, 25.70.-z

I. INTRODUCTION

The influence of breakup on the complete and incomplete fusion of weakly bound nuclei with stable targets is currently being studied at many experimental facilities around the world. Experiments with unstable nuclei are particularly challenging because of weak beam currents and poor statistics. Fortunately, there are several light elements that are both stable and weakly bound and they provide the opportunity to study the influence of breakup on fusion with good statistics.

A good example of a stable and weakly bound nucleus is $^9$Be and its fusion has been measured with $^{208}$Pb and $^{144}$Sm targets [1-3]. The simplest view of $^9$Be is a strongly deformed three-body system consisting of two $\alpha$ particles held together by a weakly bound neutron. The Q-value for the $\alpha + \alpha + n$ three-body breakup is -1.574 MeV. In both experiments it was possible to separate the complete from the incomplete fusion.

One of the most dominant features in coupled-channels calculations of the fusion of the strongly deformed $^9$Be nucleus with a stable target is the excitation of states in the $K=3/2$ ground state rotational band of $^9$Be. The large quadrupole deformation of $^9$Be (derived from the measured quadrupole moment of the ground state) implies that conventional calculations that are based on a deformed Woods-Saxon potential may become unrealistic, for example, if the curvature corrections to the ion potential [4], which are due to the deformation of the reacting nuclei, are ignored. These problems are overcome in the following by using the double-folding technique [5,6] to calculate the Coulomb plus nuclear interaction between the deformed $^9$Be nucleus and a spherical target.

Another interesting feature of $^9$Be in the non-zero spin of the ground state. This feature was pointed out in Ref. [7], and it was recommended that the spin of the $3/2^-$ ground state and the $5/2^-$ and $7/2^-$ excited states of $^9$Be should be treated explicitly in the calculations. In particular, the fusion cross section should be calculated for each of the initial magnetic quantum numbers, $m = \pm 1/2$ and $\pm 3/2$, of the $3/2^-$ ground state of $^9$Be, and the average cross section should be compared to measurements. In the rotor model one can easily calculate the necessary matrix elements from the multipole expansion of the total interaction between projectile and target.

The structure and the parametrization of the one-body density of $^9$Be is discussed in the next section. The calculation of the double-folding interaction between $^9$Be and a spherical target is presented in Sect. III. A model for calculating the complete fusion cross section is presented in Sect. IV. The results of coupled-channels calculations of the fusion of $^9$Be+$^{144}$Sm and $^9$Be+$^{208}$Pb are presented in Sect. V, and Sect. VI contains the conclusions.

II. STRUCTURE OF $^9$BE

The nucleus $^9$Be behaves like an almost perfect rotor with respect to quadrupole excitations of the $K = 3/2$ ground state rotational band with spins $I^\pi = 3/2^-$, $5/2^-$, and $7/2^-$. This can be seen by comparing the measured, reduced transition probabilities for quadrupole transitions to the results obtained from the expression, Eq. (4.68a) of Ref. [2],

$$B(E2, KI \rightarrow K'I') = \frac{5Q_0^2e^2}{16\pi} \langle IK 20|I'K\rangle^2,$$  \hspace{1cm} (1)

which applies to a perfect rotor. Here $Q_0$ is the intrinsic quadrupole moment. The measured quadrupole moment of the $3/2^-$ ground state of $^9$Be is 5.29(4) fm$^2$ [8] which translates into the intrinsic quadrupole moment $Q_0 =
TABLE I: The measured reduced transition probabilities $B(E2, 3/2^- \rightarrow I^-)$ for exciting the $K=3/2$ ground state rotational band of $^{9}$Be [8] are compared to predictions of the rotor model. Also shown are the excitation energies $E_I$ and decay widths $\Gamma_I$.

| $I^e$ | $E_I$ (MeV) | $\Gamma_I$ (keV) | Experiment | Rotor model |
|-------|-------------|-----------------|------------|-------------|
| $3/2^-$ | 0.54 | 70 | 14.0(2) | 14 |
| $5/2^-$ | 4.29 | 0.78 | 40.7(6) | 36 |
| $7/2^-$ | 4.38 | 1120 | 18.9(3) | 20 |
| sum | | | 73.6(11) | 70 |

26.45(20) fm$^2$, according to Eq. (4-69) of Ref. [7]. Inserting this value into Eq. (11) one obtains the reduced transition probabilities that are shown in the last column of Table I. They are in good agreement with the measured values shown in the 4th column of Table I. The sum of the B-values is shown in the last row of Table I. They are in good agreement with the measured experimental value by about 5%.

A great advantage of the rotor model is that it can be applied to calculate the transition matrix elements in cases where they have not been measured, for example, for the $5/2^-$ to $7/2^-$ transition, and the quadrupole moments of the excited states. One can also calculate matrix elements of the total interaction between $^{9}$Be and a spherical target from a multipole expansion of this interaction as discussed in Sect. III.

The excitation energies and widths of the three states are shown in the 2nd and 3rd columns. It is noted that the width of the $7/2^-$ state is very large which implies that the state, if excited, may decay by particle emission before the fusion with the target takes place. This possibility is investigated in Sect. IV and V.

There are other low-lying states in $^{9}$Be but they are not expected to play any significant role in heavy-ion collisions. Thus the spin-orbit partners of the ground state rotational band, i.e., the $1/2^-$, $3/2^-$, and $5/2^-$ states, will be ignored because spin-excitations are weak. The excitation of positive parity states, starting with the lowest $1/2^+$ state, is also weak and will be ignored.

A. Density parametrization

The densities of the deformed projectile and spherical target nuclei will be parametrized by the expression

$$\rho(r, \theta') = C \frac{1 + \cosh(R/a)}{\cosh(r/a) + \cosh(R/a)},$$  \hspace{1cm} (2)

where $C$ is a normalization constant, $R$ is the radius, and $a$ is the diffuseness. The radius of the deformed, axially symmetric projectile depends on the direction with respect to the symmetry axis. It is parametrized as

$$R(\theta') = R_0 [1 + \beta_2 Y_{20}(\theta')],$$ \hspace{1cm} (3)

where $\theta'$ is the angle between the position vector $r$ and the direction $e$ of the symmetry axis.

The advantages of the parametrization (2) are that it is similar to a Fermi function at larger values of $r$ and it is well behaved as a function of $\theta'$ for $r \rightarrow 0$, where it approaches an orientation independent constant. Another advantage of Eq. (2) is the analytic properties it has for spherical nuclei [10]. For example, the Fourier transform is an analytic function, and the expression for the root-mean-square (RMS) radius,

$$\langle r^2 \rangle = \frac{3}{5} (R^2 + \frac{7}{3}(a\pi)^2),$$ \hspace{1cm} (4)

is an exact relation (see the appendix of Ref. [10].) These features will be utilized for the spherical target nuclei, $^{144}$Sm and $^{208}$Pb. The parameters that have been chosen are shown in Table II. They were adjusted so that the measured RMS charge radii [11] were reproduced. The parameters for the neutron ($\nu$) densities were assumed to be the same as for protons ($\pi$), except in the case of $^{208}$Pb, where a slightly larger radius is used to accommodate for the neutron skin of this nucleus. The adopted skin thickness: $\delta_{np} = < r^2 >_n - < r^2 >_p \approx 0.16(6) \text{ fm}$ was chosen because it falls in the midst of values predicted by Skyrme Hartree-Fock (HF) calculations [12]. Moreover, it is also consistent with the skin thickness $\delta_{np} = 0.16 \pm (0.02)_{\text{stat}} \pm (0.04)_{\text{sys}} \text{ fm}$ that has been extracted from antiprotonic $^{208}$Pb atoms [13]. The parameters for $^{9}$Be are determined below.

B. Multipole expansion of density

The density of the deformed nucleus is expanded on Legendre polynomials,

$$\rho(r, \theta') = \sum_{\lambda} \rho_{\lambda}(r) P_{\lambda}(\cos(\theta')),$$ \hspace{1cm} (5)

where $P_{\lambda}$ is the $\lambda$th Legendre polynomial. The parameters for $^{9}$Be are determined below.
where $\rho_\lambda(r)$ is calculated numerically,

$$\rho_\lambda(r) = \frac{2\lambda + 1}{2} \int_{-1}^{1} dx \, P_\lambda(x) \rho(x).$$

The multipole expansion of the Fourier transform of the density is

$$\rho(k) = \sum_\lambda i^{-\lambda} \rho_\lambda(k) \, P_\lambda(\cos(\theta'_k)),\quad (7)$$

where $\theta'_k$ is the angle between $k$ and the direction $e$ of the symmetry axis, and

$$\rho_\lambda(k) = 4\pi \int_0^\infty dr \, r^2 \, \rho_\lambda(r) \, j_\lambda(kr).$$

The above expressions are used in the next section to calculate the double-folding potential. They are also used to calculate the electric multipole moments of the deformed charge density $\rho_c(r,\theta')$,

$$M(E\lambda) = M(E\lambda) \, Y_{\lambda\mu}(e),$$

where

$$M(E\lambda) = \frac{4\pi}{2\lambda + 1} \int_0^\infty dr \, r^{\lambda+2} \rho_c(r).$$

The intrinsic quadrupole moment $Q_0$ is traditionally defined as $2M(E2)$.

**C. Calibration of the density of $^9$Be**

The parameters of the density of the deformed $^9$Be nucleus were adjusted so that both the intrinsic quadrupole moment and the RMS charge radius agree with experiments. That was achieved as follows. The mean square charge radius of $^9$Be,

$$\langle r^2 \rangle = \frac{4\pi}{Z} \int_0^\infty dr \, r^4 \rho_{c,\lambda=0}(r),$$

and the intrinsic quadrupole moment,

$$Q_0 = 2M(E2) = \frac{4\pi}{5} \int_0^\infty dr \, r^4 \rho_{c,\lambda=2}(r).$$

were calculated as functions of the deformation parameter $\beta_2$ for a fixed radius $R = 2.08$ fm and for three values of the diffuseness. The results are shown in Fig. 1 as a correlation between the RMS charge radius and $Q_0$. It is seen that the curve which is based on the diffuseness $a = 0.375$ fm passes through the experimental values, and agreement with both values is achieved for $\beta_2 = 1.183$. This is the value that will be used in the following, and the shape it produces according to Eq. 3 looks almost like two touching $\alpha$ particles as illustrated in Fig. 2. In fact, the intrinsic quadrupole moment of $^9$Be, $Q_0 = 26.45(20)$ fm$^2$, is almost identical to the calculated quadrupole moment of the unbound nucleus $^9$Be. The published value obtained in Variational Monte Carlo (VMC) calculations is $26.6(3)$ fm$^2$ [14]. The intrinsic quadrupole moment of $^{10}$Be, on the other hand, is slightly smaller; the value one obtains from the measured $B(E2)$ value of the lowest $2^+$ excitation [8] is $Q_0 = 22.9$ fm$^2$.

**III. DOUBLE-FOLDING POTENTIAL**

Having adopted the rotor model for $^9$Be and determined the densities of the projectile and the spherical $^{144}$Sm and $^{208}$Pb targets, one can now use the double-folding technique to calculate the potential that will be used in the coupled-channels calculations. The double-folding potential is defined by

$$U(r) = \int dr_1 \int dr_2 \, \rho_1(r_1, e) \, \rho_T(r_2) \, v(|r_2 + r - r_1|),$$

where $v$ is the effective nucleon-nucleon interaction and $r$ is the relative distance between projectile and target. The target density $\rho_T$ is assumed to be spherical whereas the density of the projectile $^9$Be is deformed and parametrized as described in the previous section.
The double-folding potential is calculated most conveniently from the Fourier transforms of the densities according to the expression \[ U(r) = \int \frac{dk}{(2\pi)^3} \rho(k) \rho_T(-k) v(k) e^{ikr}. \] (14)

Inserting the expression \[ \frac{1}{4\pi e^2} \] for the deformed projectile and spherical target densities into Eq. (14) one obtains

\[ U(r) = U(r, \theta') = \sum_{\lambda} U_{\lambda}(r) P_{\lambda}(\cos(\theta')), \] (15)

where \( \theta' \) is the angle between \( r \) and \( e \) and

\[ U_{\lambda}(r) = \frac{1}{2\pi^2} \int dk \ k^2 \rho_\lambda(k) \rho_T(k \ v(k) \ j_\lambda(kr)). \] (16)

The double-folding calculation of the ion-ion potential and its multipole expansion, Eq. (16), will be based on the M3Y effective interaction, supplemented with a repulsive term that simulates the effect of nuclear incompressibility. This method has been applied previously by Mišicu and Greiner \[ \text{[15]} \] to calculate the fusion between spherical and deformed nuclei. It was also used in Ref. \[ \text{[15]} \] to explain the hindrance in the fusion of spherical nuclei at extreme subbarrier energies.

The repulsive term in the effective \( NN \) interaction is parametrized as a contact interaction,

\[ v^{\text{rep}}_{NN}(r) = v_r \delta(r), \] (17)

and the densities that are used in the associated double-folding calculation have the same radius as shown in Table I but the diffuseness \( a_r \) is usually chosen much smaller \[ \text{[16]} \]. The value chosen here is \( a_r = 0.3 \) fm.

The Coulomb interaction can also be generated from Eq. (16) simply by replacing \( v(k) \) by the Fourier transform \( 4\pi e^2/k^2 \) of the proton-proton Coulomb interaction, and replacing the nuclear densities with the charge densities \( \rho_c \) of projectile and target. The results have the same form as Eq. (16),

\[ U_C(r, \theta') = \sum_{\lambda} U^{C}_{\lambda}(r) P_{\lambda}(\cos(\theta')), \] (18)

where

\[ U^{C}_{\lambda}(r) = \frac{1}{2\pi^2} \int dk \ k^2 \rho_c(k) \rho_cT(k) \ 4\pi e^2 \ j_\lambda(kr). \] (19)

For large separations of projectile and target this interaction approaches the usual monopole-multipole interaction,

\[ U^{C}_{\lambda}(r) = \frac{Z_T e^2 M(E\lambda)}{r^{\lambda+1}}. \] (20)

A. Matrix elements

Having expressed the total interaction \( U(r, \theta') \) (Coulomb + Nuclear) in terms of the multipole expansion Eq. (15), one can now easily calculate the diagonal as well as off-diagonal couplings between states in the ground state rotational band of \( ^9\text{Be} \). All one needs to calculate is matrix elements of the Legendre polynomials,

\[ P_{\lambda}(\cos(\theta')) = \sum_{\mu} D^{\lambda}_{\mu0}(\hat{r}) \ D^{\lambda}_{\mu0}(\hat{e}). \]

The matrix elements between different states are

\[ \langle K'I'M'|P_{\lambda}(\cos(\theta'))|KIM\rangle = \sqrt\frac{2I+1}{2I'+1} \sum_{\mu} D^{\lambda}_{\mu0}(\hat{r}) \langle IM \lambda \mu |I'M'\rangle \langle IK \lambda0 |I'K\rangle. \] (21)

The calculation is even simpler in the rotating frame approximation, which is used in the coupled-channels calculations described in the next section. In this approximation one assumes that \( r \) (the relative distance between projectile and target) defines the \( z \)-axis. The angle \( \theta' \) is then identical to the angle \( \theta_e \) of the symmetry axis with respect to the \( z \)-axis. Since \( D^{\lambda}_{\mu0}(\hat{r}) = \delta_{\mu0} \) this implies that \( \mu=0 \) is the only non-zero term in Eq. (21).

The total potentials one obtains for the two systems \( ^9\text{Be}+^{144}\text{Sm} \) and \( ^9\text{Be}+^{208}\text{Pb} \) are shown in Fig. 3. Shown are for each system the monopole potential (solid line) and the entrance channel potentials for the magnetic quantum numbers \( m=1/2 \) and \( 3/2 \), of the 3/2\(^-\), \( K=3/2 \) ground state of \( ^9\text{Be} \). All three potentials were obtained with the strength \( v_r = 410 \text{ MeV fm}^3 \) of the repulsive effective \( NN \) interaction (which is determined in subsection V.A.)

The magnetic quantum numbers \( m=1/2 \) and \( 3/2 \) used in Fig. 3 refer to a \( z \)-axis that points in the direction of the relative position of projectile and target. The \( m=3/2 \) channel therefore corresponds to an orientation where the tip of the deformed \( ^9\text{Be} \) points towards the target, whereas the \( m=1/2 \) corresponds to the belly pointing towards the target. Consequently, the Coulomb barrier for the \( m=3/2 \) entrance channel is lower than the barrier for \( m=1/2 \). Another observation is that the potential pocket is deeper for \( m=1/2 \) than for \( m=3/2 \). This is a consequence of a larger radius of curvature and a stronger nuclear attraction for the \( m=1/2 \) belly configuration.

IV. MODEL OF \( ^9\text{Be} \) INDUCED FUSION

The cross sections for the complete fusion of \( ^9\text{Be} \) with a heavy target are calculated in the coupled-channels approach. The complete fusion is simulated by ingoing wave boundary conditions that are imposed in all channels at
The coupled equations are solved in the rotating frame approximation [17–19], where the potential is the same in all channels and equal to the isocentrifugal approximation [17] because the centrifugal potential in the entrance channel. The total width is the lower dotted curve is for $m = 1/2$, and the average cross section (solid curve).

The strength of the repulsive interaction was set to $v_r = 410$ MeV fm$^3$. The dominant decay mode of the $7/2^-$ state is neutron emission, and 55% of it populates the $2^+$ excited state of $^9$Be [8]. The latter state has an excitation energy of 3.03 MeV and a large width of 1.51 MeV, with an exclusive decay into two $\alpha$ particles [8]. The two $\alpha$ particles are emitted back to back in the $^8$Be rest frame, so if one of them is emitted towards the target, the other $\alpha$ partner will recoil away from the target nucleus and will most likely escape. It is therefore assumed that the decay of the $7/2^-$ state will not lead to complete fusion (CF), and the CF will be calculated as described above from the ingoing flux, whereas the decay is registered in the absorption cross section.

The data for the complete fusion of $^9$Be+$^{144}$Sm [3] are compared to calculations for $m = 1/2$ and $3/2$, and the average cross section (solid curve). The strength of the repulsive interaction was set to $v_r = 410$ MeV fm$^3$.

The data for the complete fusion of $^9$Be+$^{144}$Sm [3] are compared to calculations for $m = 1/2$ and $3/2$, and the average cross section (solid curve).

The data for the complete fusion of $^9$Be+$^{144}$Sm [3] are compared to calculations for $m = 1/2$ and $3/2$, and the average cross section (solid curve). The calculated CF cross sections were derived from the ingoing flux as described above. The top dashed curve is the cross section for the $m = 3/2$ magnetic substate, the lower dotted curve is for $m = 1/2$. It is seen that the curve for $m = 3/2$ dominates the CF at all energies, consistent with the lower Coulomb barrier for this magnetic substate (see Fig. 3). The data should be compared to the solid curve which is the average of the CF cross sections for $m = 1/2$ and $3/2$. The comparison is discussed in more detail in subsection V.A.

### A. Incomplete fusion

The decay of the $7/2^-$ state will end up in the breakup of $^9$Be. The precise outcome of the decay in terms of incomplete fusion or breakup is not so clear. It would require a multi-cluster description to follow the two $\alpha$ particles after the decay. As mentioned earlier, it is unlikely that both $\alpha$ particles fuse with the target because they are emitted back to back. However, it is possible that one of them will fuse with the target nucleus and

![Figure 3](image3.png)

**FIG. 3:** (Color online) Entrance channel potentials for $^9$Be+$^{144}$Sm and $^9$Be+$^{208}$Pb, respectively, obtained for the repulsive strength $v_r = 410$ MeV fm$^3$. The solid curve is the monopole potential; the dashed curves are the entrance channel potentials for the magnetic quantum numbers $m = 1/2$ and $3/2$. The ground state energies of the two compound nuclei, $^{153}$Dy and $^{217}$Rn, are indicated.

![Figure 4](image4.png)

**FIG. 4:** (Color online) Measured cross sections for the complete fusion of $^9$Be+$^{144}$Sm [3] are compared to calculations for $m = 1/2$ and $3/2$, and the average cross section (solid curve). The strength of the repulsive interaction was set to $v_r = 410$ MeV fm$^3$. The data for the complete fusion of $^9$Be+$^{144}$Sm [3] are compared to calculations for $m = 1/2$ and $3/2$, and the average cross section (solid curve). The calculated CF cross sections were derived from the ingoing flux as described above. The top dashed curve is the cross section for the $m = 3/2$ magnetic substate, the lower dotted curve is for $m = 1/2$. It is seen that the curve for $m = 3/2$ dominates the CF at all energies, consistent with the lower Coulomb barrier for this magnetic substate (see Fig. 3). The data should be compared to the solid curve which is the average of the CF cross sections for $m = 1/2$ and $3/2$. The comparison is discussed in more detail in subsection V.A.
lead to incomplete fusion (ICF). It should also be emphasized that there are other sources of ICF, for example, the neutron transfer from $^9$Be, which are not included in the coupled-channels calculations presented here. The calculated cross section for the decay of the excited states will in the following be referred to as the absorption cross section. In view of the above discussion one should not expect that the absorption would account for the measured ICF cross sections but it is clearly of interest to compare the two cross sections. The experimental total fusion (TF) cross section is the sum of the CF and the absorption cross sections. The measured and calculated fusion cross sections are compared in a linear plot in Fig. 5B. It is seen that the different components of the measured and calculated fusion cross sections are in good agreement. In particular, the CF is suppressed by about 10% compared to the total fusion (TF) at high energies, both experimentally

![Graph showing measured complete (CF) and incomplete (ICF) fusion cross sections for $^9$Be+$^{144}$Sm and calculated cross sections for CF (solid) and absorption (dashed curve).](image1)

![Graph showing $\chi^2$ per data point for the complete (CF) and total fusion (TF) of $^9$Be+$^{144}$Sm as functions of the strength $v_r$ of the repulsive interaction, Eq. 15.](image2)

FIG. 5: (Color online) Measured complete (CF) and incomplete (ICF) fusion cross sections for $^9$Be+$^{144}$Sm are compared in (A) to the calculated cross sections for CF (solid) and absorption (dashed curve). The linear plot in (B) also shows the total fusion (TF) cross section.

FIG. 6: (Color online) The $\chi^2$ per data point for the complete (CF) and total fusion (TF) of $^9$Be+$^{144}$Sm are shown as functions of the strength $v_r$ of the repulsive interaction, Eq. 15.

is discussed first because the couplings are weaker in this case and the adopted model is therefore expected to be more successful. This case will also provide the opportunity to calibrate the repulsive part of the effective $NN$ interaction.

A. Fusion of $^9$Be+$^{144}$Sm.

The calculated cross sections for the fusion of $^9$Be with $^{144}$Sm are compared in Fig. 5A to the data of Ref. [3]. The measured and calculated cross sections for complete fusion (CF) are seen to be in good agreement. This was achieved by adjusting the repulsive part of the effective $NN$ interaction which is used in the calculation of the double-folding nuclear potential. The best fit to the data is obtained for the strength $v_r \approx 410$ MeV fm$^3$, and that is the value that will be used in the following. The $\chi^2$ per data point is shown in Fig. 6A as a function of the strength of the repulsive interaction. There is another solution with a small $\chi^2$ for $v_r \leq 300$ MeV fm$^3$ but it is unphysical because it produces a pocket for $^9$Be+$^{208}$Pb that is deeper than the energy of the compound nucleus.

The dashed curve in Fig. 5A shows the calculated cross sections for the decay of the excited states of $^9$Be; it is in surprisingly good agreement with the ICF data. The good agreement may be accidental but it could also indicate that the decay of the excited states of $^9$Be is the main source of ICF for the $^{144}$Sm target. The best fit to the data for total fusion (TF) is also achieved for $v_r \approx 410$ MeV fm$^3$. This consistency of the CF and TF is illustrated in Fig. 6A in terms of the $\chi^2$ per data point.

The measured and calculated fusion cross sections are compared in a linear plot in Fig. 5B. It is seen that the different components of the measured and calculated fusion cross sections are in good agreement. In particular, the CF is suppressed by about 10% compared to the total fusion (TF) at high energies, both experimentally

V. COMPARISON TO MEASUREMENTS

The results of coupled-channels calculations that are based on the model presented in the previous sections are compared in the following to the data for the complete (CF) and incomplete fusion (ICF) of $^9$Be+$^{208}$Pb and $^9$Be+$^{144}$Sm. Both targets are spherical, closed shell nuclei and the excitation of these nuclei is relatively weak compared to the excitation of $^9$Be and they will be ignored. The fusion with the $^{144}$Sm target...
and in the calculations. This suppression is caused in the coupled-channels calculations by the decay of the excited states of $^9$Be. Without any decay in the coupled-channels calculations, the fusion cross section obtained from the ingoing-wave boundary conditions would be close to the measured TF cross section. In other words, the suppression of the CF compared to the TF requires some sort of absorption mechanism, and the decay mechanism suggested here seems to provide a natural explanation. Let us now investigate whether this mechanism can explain the data for the Pb target.

### B. Fusion of $^9$Be+$^{208}$Pb.

The results of the coupled-channels calculations of the fusion of $^9$Be+$^{208}$Pb are compared in Fig. 7 to the data of Ref. [2]. The calculations are similar to those presented above for $^9$Be+$^{144}$Sm. It is seen in Fig. 7B that the calculated absorption cross section (due to the decay of excited states) can only explain 1/3 of the measured ICF cross section at high energies. The suppression of the CF compared to the TF cross section is about 30% in the experiment [2], whereas the calculations only show a 10% suppression. There are evidently other sources of ICF in collisions of $^9$Be with a $^{208}$Pb target, besides the decay of excited states considered here.

A 30% suppression of the CF data was observed in Ref. [2] by comparing to coupled-channels calculations. The calculations were based on a deformed Woods-Saxon potential but did not consider the effects of incomplete fusion nor the decay of excited states. It was shown that a scaling of the calculated fusion cross section by a factor of 0.7 leads to a very good agreement with the CF data at all energies. In Fig. 7 it is sufficient to multiply the CF calculation by a factor 0.8 in order to match the CF data at high energies. The reason for the smaller scaling factor is that the decay of the excited states has already taken care of a 10% reduction.

It is often necessary to employ a weak, short-ranged imaginary potential in order to be able to reproduce the fusion data of stable nuclei at high energies by coupled-channels calculations. This is particularly the case when the calculations are based on a shallow entrance channel potential [10]. Since the potentials shown in Fig. 3 are relatively shallow it is of interest to see what is the effect of a weak imaginary potential on the fusion of $^9$Be+$^{208}$Pb. Let us therefore choose the potential

$$W(r) = W_0 \left[1 + \exp((r - R_w)/a_w)\right]^{-1}. \quad (22)$$

with $a_w = 0.2$ fm and $R_w = 9.5$ fm so that it acts near the minimum of the potential pockets shown in Fig. 3. The strength $W_0$ was adjusted to optimize the fit to the CF data. The best fit is shown in Fig. 8A: it was achieved for $W_0 = -2.5$ MeV and has a $\chi^2/N=1.4$.

The absorption cross section shown in Fig. 8 due to the combined effect of the imaginary potential, Eq. (22), and the decay of the excited states, is in good agreement with the ICF data at high energies but the discrepancy is large at low energies. The discrepancy indicates that the breakup leading to ICF must take place at larger separations of projectile and target than assumed in the potential, Eq. (22). It may be possible to construct a more realistic imaginary potential (of the volume plus surface type) but that idea will not be pursued here.

There are other reaction channels that could be a source of ICF. Examples are the dissociation of $^9$Be induced by the neutron emission into the continuum and into bound states of the target nucleus. Both processes produce $^8$Be which decays into two $\alpha$-particles, and one of them could end up as incomplete fusion. These reaction mechanisms should be studied theoretically in detail in order to develop a better understanding and description of the breakup, complete and incomplete fusion. In fact, a recent experiments [21] show that the $^9$Be breakup following neutron transfer dominates the total breakup yield.

### VI. CONCLUSIONS

The complete fusion of $^9$Be with spherical target nuclei was calculated in the coupled-channels approach. It was
shown that the $B(E2)$ values for the excitation of the ground state rotational band of the $^9$Be nucleus can be described quite well in the rotor model. This feature was exploited in the calculation of matrix elements of the interaction between the deformed projectile and a spherical target.

The interaction of the deformed $^9$Be projectile with a spherical target was calculated using the double-folding technique and an effective M3Y $NN$ interaction, which was supplemented with an adjustable repulsive term. The deformed density of $^9$Be was determined so the measured quadrupole moment and RMS charge radius were reproduced. The densities of the spherical targets were calibrated to reproduce the measured charge radii; the radius of the neutron density in $^{208}$Pb was calibrated to be consistent with the neutron skin thickness predicted by Skyrme Hartree-Fock calculations and with the value extracted from measurements of antiprotonic $^{208}$Pb atoms.

The double-folding potential was applied in coupled-channels calculations of the fusion of $^9$Be with $^{144}$Sm. The decay of the excited states of $^9$Be was included explicitly in terms of complex excitation energies, whereas excitations of the target were ignored for simplicity. The repulsive part of the effective $NN$ interaction, which essentially is the only parameter that remains to be determined, was adjusted to produce an optimum fit to the complete fusion data. The calculated cross sections for the decay of the excited states in $^9$Be turned out to be in very good agreement with the incomplete fusion data.

Having determined all of the parameters of the theory, coupled-channels calculations were performed for the fusion of $^9$Be+$^{208}$Pb. The complete fusion data were suppressed at high energies by 20% compared to the predicted cross sections, and the incomplete fusion data were a factor of three larger than the calculated cross section for the decay of the excited states. There are obviously other reaction mechanisms, besides the decay of excited states, that are responsible for the large incomplete fusion cross sections that have been measured for the lead target. A likely candidate is the neutron transfer from $^9$Be to bound states in the target and to continuum states.

This is also the conclusion of a recent experimental investigation by Rafiei et al.[21].

**Acknowledgments** This work was supported by the U.S. Department of Energy, Office of Nuclear Physics, under Contract No. DE-AC02-06CH11357.

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