AUCTION AND CONTRACTING MECHANISMS FOR CHANNEL
COORDINATION WITH CONSIDERATION OF PARTICIPANTS’
RISK ATTITUDES

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Abstract. This paper considers a two-supplier one-retailer coordinated supply
chain system with auction and contracting mechanism incorporating participants’ risk attitudes. The risk attitude is quantified using the value-at-risk (VaR) measure and the retailer faces a stochastic linear price-dependent demand function. In the supply chain, the suppliers (providing identical products) compete with each other in order to win the ordering contract of the retailer. Several auction and contracting mechanisms are developed and compared. It can be analytically shown that the retail price of the risk-averse system is higher than that of the risk-neutral system, but the order quantity is lower than that of the risk-neutral system.

1. Introduction. Coordinating supply chains has been a major issue in supply chain management research. The importance of effective supply chain coordination is widely recognized since it enables all parties in a supply chain to work together so as to maximize the total profit of the entire supply chain system. Supply chain contracts have been used as an effective means to coordinate the participants in a supply chain to achieve higher system efficiency. Examples of effective contracts are buy-back contract, revenue-sharing contract, wholesale price contract and two-part tariff contract, etc.

An auction\footnote{An auction is a process of buying and selling goods or services by offering them up for bid, taking bids, and then selling the item to the profitable bidder for the seller (Krishna [19]).} provides an ideal mechanism for competitive establishment of the market price for any commodity, good, or service. As a market leader within the mass merchandise industry, Wal-Mart adopts reverse auctions mechanism to select competitive manufacturers whose representatives are brought into a room with a
Wal-Mart buyer and they bid for the lowest price to sell their product at. In the electronic marketplace, online auctions are one of the most successful business models. In 2008, eBay, which is the largest consumer-oriented auction site, posted that their gross merchandise volume was US$59.65 billion (United States Securities and Exchange Commission (SEC) [27]). In the same year, the Organization for Economic Development (OECD) report [25] indicated that the Business-to-Business (B2B) online market generated US$3.1 trillion in sales, accounting for over 27% of total B2B transactions. Much of this business would be conducted through auctions, which are rapidly replacing requests for quotation as the preferred means of business procurement (The Organization for Economic Development Background Report [25]). Another typical example is SUN company. Earlier in 2000, SUN began to utilize reverse auctions to award supply contracts to a selected group of suppliers for a given component. Today, SUN uses Dynamic Bidding extensively both for direct and indirect MRO and procurement. This channel amounts to 20%-25% of its total spending. Compared to previous procurement spending, Dynamic Bidding provided double-digit incremental savings totaling to more than US$300 Million annually (Tunca and Wu [26]). Additional examples include Compaq Computer Co., Staples Inc., The Limited Inc. and Kmart Corporation, who implemented combinatorial auctions for procurement with the aid of Logistics.com (Elmaghraby and Keskinocak [11]).

Today’s advanced information technology, communication and transportation should have provided unprecedented opportunities for supply and delivery successes in the whole supply chain. As reported, the number of cases about supply failures, however, has gradually increased. There are considerable causes to induce supply failures such as the unstable climate environment, earthquake, flood, tsunami, regional violent conflicts etc.. Economic losses from supply chain disruptions increased 465% between 2009 and 2011. The Allianz experts rate business and supply chain interruptions as the biggest business risk (46 percent of responses). Choosing to run lean global supply chains to reduce costs, many companies lack alternative suppliers. The flooding disaster in Thailand showed that business interruption at a key supplier can cause a ripple effect felt across an entire industry (Risk Barometer 2013). Another well known case study is Ericsson. Nearly a decade ago, lightning struck a Philips microchip plant in New Mexico, causing a fire that contaminated millions of mobile phone chips. Among Philips’ biggest customers were Nokia and Ericsson, the mobile phone manufacturers, but each reacted differently to the disaster. Nokia’s supply-chain management strategy allowed it to switch suppliers quickly; it even re-engineered some of its phones to accept both American and Japanese chips, which meant its production line was relatively unaffected. Ericsson, however, accepted Philips word that production at the plant would be back on track in a week and it took no action.

Therefore, it is vitally necessary to take supply chain risk into account and develop a practical method for determining supply chain risk-return trade-offs. Yet there is very little work in the literature that combines the supply chain risk with coordination. Motivated by these considerations, the purpose of this work is to examine whether auction and contracting mechanisms could also serve as a coordination mechanism for a supply chain in the presence of participants’ risk preferences. We also wish to investigate how different participants’ risk preferences impact on their own optimal decisions.

http://www.agcs.allianz.com/about-us/news/risk-barometer-2013/
In this paper, we consider a supply chain with two suppliers providing identical products to a single risk-averse retailer, which inevitably causes competition between the two suppliers. We first consider a simple situation where no complicated auction or contract mechanism is involved and business is done between the retailer and the low cost supplier only. We first derive analytically the optimal order quantity, the optimal retail price and the optimal wholesale price, all set at a value-at-risk measure, for the supply chain. There are two methods for consideration, namely, the channel coordination method and the independent policy method. For the channel coordination method, the retailer and the low cost supplier work as a team to determine the optimal values of the above-mentioned decision variables so that the profit of the entire supply chain is maximized (The optimal values of the above decision variables for this channel coordination method are called the channel coordination optimums.), whereas for the independent policy method, the low-cost supplier independently determines the wholesale price to maximize his own profit. (As mentioned earlier, the high-cost supplier cannot obtain any contracts from the retailer.) Analytical results show that the total profit of the entire supply chain obtained by the channel coordination method is higher than that obtained by the independent policy method; however, 100% of this higher profit goes to the retailer, leaving no profit at all to the low cost suppliers.

In order to encourage the suppliers to cooperate fully with the retailer, appropriate auction and contracting mechanisms are established so that the profit of the supply chain can be shared between the retailer and the suppliers. Analytical results show that in certain circumstances, the optimal values of the decision variables obtained by the auction and contracting mechanisms are exactly the same as the channel coordination optimums. In addition to the establishment of the above auction and contracting mechanisms, to obtain more insights into the relationship between the retailers risk attitude and his decision policy, we perform comprehensive comparison of the channel coordination optimums obtained by the risk-neutral system and the risk-averse system.

Thus, our work contributes to the procurement and auction mechanism literature by incorporating participants’ risk preferences, a growing popular element of concern in the supply chain. Our specific contribution is to design different schemes of procurement auction and contracting mechanisms to provide new insights on the bidding and auction strategies with both complete information and asymmetric information, respectively.

The rest of the paper is organized as follows. In Section 2, we present the literature review. We then formulate the two-supplier one-retailer supply chain system (risk-averse agents system) with complete information in Section 3 and with asymmetric information in Section 4. In these two sections, several auction and contracting mechanisms are also developed. In Section 5, a risk-neutral agent system is formulated. The system optimums of the risk-neutral agent system and the risk-averse agent system are established. Section 6 provides a numerical example illustrating the mechanisms. Section 7 presents the conclusions.

2. Literature review. There is a considerable literature devoted to contracts that coordinate a supply chain involving risk-neutral agents (Cachon [2]). There are an enormous amount of papers discussing supply chain contracting with asymmetric cost information ([10], [9], [8], [16]). Lau et al. [20] considered a contracting problem with asymmetric cost information and price dependent demand.
Auctions have successful applications in the procurement market, especially in the electricity procurement market and attract an increasing number of researchers to make further investigation in the market mechanism researches. Van Ryzin and Vulcano [28] studied the inventory management for items sold through forward (selling) auctions and Chen et al. [4] investigated keyword auctions for advertising. A review of the electronic business auctions literature was presented in [15]. Another recent stream of work in procurement auctions analyzed auctions in a more complicated supply chain setting. Chen et al. [6] compared the auction mechanism with the audit-based approach to obtain supplier information. Iyengar and Kumar [17] proposed an optimal auction mechanism for a buyer that sources from suppliers with different capacity limits and production cost. Chen and Vulcano [7] compared first- and second-price auctions as mechanisms for two competing retailers to procure capacity from a supplier. Li and Scheller-Wolf [21] analyzed and compared the outcomes of auctions for push and (enhanced) pull contracts as well as established an auction design and contract design in procurement auctions. Multi-attribute procurement using auction has also been extensively studied. Che [3] studied a two-attribute auction in which price and quality are the two attributes under consideration. Branco [1] extended Che’s model to allow a correlation in the suppliers’ cost. Parkes and Kalagnanam [23] provided an iterative auction design for an important special case of the multi-attribute allocation problem with special preferential independent additive structure on the buyer value and seller costs.

Most of the literature on decision making in a supply chain assumed that the participants in the supply chain are risk neutral. Recently, there has been an emerging group of literature on supply chain coordination in the presence of participants’ different risk preferences. Zhang et al. [29] designed a new scheme to coordinate the supply chain system with loss-averse retailer. Gan et al. [12] defined a new concept for supply chain coordination with risk averse agents and developed efficient methods for coordinating the supply chain under their definition of coordination. Shi and Xiao [24] presented two types of contracts, buy-back contract and markdown price contract in order to coordinate a supply chain with a loss-averse retailer. Gan et al. [13] investigated the coordination of a supply chain consisting of a risk-neutral supplier and a retailer who wishes to maximize his expected profit, but with a stipulation that his downside risk is limited.

There are also some papers on coordination mechanism for auction supply chain procurement. Jin and Wu [18] considered that a two-part contract can be served as a coordination mechanism for supply chain procurement. Chen [5] investigated the coordination mechanism for supply chain with one manufacturer and multiple competing suppliers in the electronic market. There are main distinctions between our research and theirs, which can be summarized as follows. Their papers did not consider participants’ risk preferences. Our paper not only incorporates participants’ risk attitudes, but also identifies how the quantile for the retailer’s risk measure impacts on his optimal decisions and compares the retailer’s optimal decisions between the risk-averse and risk-neutral cases.

3. **Two-suppliers, one retailer system with complete information.** In this section, we consider a supply chain with two suppliers providing identical products to a single risk-averse retailer. We focus on the following simple situation:

(i) Full information is shared between the retailer and the two suppliers; this includes the retailers handling cost and the suppliers costs.
(ii) The business is done between the low cost supplier and the retailer only, as
the high cost supplier cannot obtain any contract from the retailer.

(iii) There are two methods for consideration, the channel coordinated method
and the independent policy method. For both methods, the optimal retail
price, the optimal order quantity as well as the optimal profit of the retailer
are first expressed in terms of the wholesale price of the low cost supplier.
For the coordinated policy method, the optimal wholesale price is obtained
by maximizing the profit of the entire supply chain. (The optimal values of
the above decision variables for this channel coordination method are called
the channel coordination optimums.) For the independent policy method, this
wholesale price is obtained by maximizing the profit of the low cost supplier
only.

We use the following notation throughout the paper.

- $r$: retail price determined by the retailer;
- $q$: ordering quantity from the suppliers determined by the retailer;
- $c$: the retailer’s handling cost per unit product;
- $w_i$: the supplier $i$’s wholesale price, $i = 1, 2$;
- $\Pi_i$: the retailer’s profit when $i = R$; the total supply chain profit when
  $i = SC$; the suppliers’ profit when $i = S$;
- $s_i$: the supplier $i$’s cost, for example, production cost and transportation cost
  etc, $i = 1, 2$;
- $\alpha$: confidence level for retailer’s risk measure.

We first derive analytically the optimal order quantity, the optimal retail price
and the optimal profit of the retailer, all set at a value-at-risk measure, for the
entire supply chain and all in term of the low cost supplier’s wholesale price.

The retailer’s problem, which is equally applicable for the independent policy
method and the channel coordinated method, can be described as follows:

The linear price-dependent demand function is denoted by

$$D_d = k - ar + x, \text{ for } 0 < r \leq \frac{k}{a},$$

where $k > 0, a > 0$ and $k - ac > 0$. Here, $x$ is a nonnegative random variable with
a finite mean $\mu$, that follows a density function $f(\cdot)$ and a cumulative distribution
function $F(\cdot)$. We assume that $F(\cdot)$ is strictly increasing. Denote the unique inverse
function of $F(\cdot)$ by $F^{-1}(\cdot)$. Therefore, the profit of the retailer can be expressed as

$$\Pi_R(r, q) = \begin{cases} r(k - ar + x) - (c + w)q, & \text{if } x < q - k + ar, \\ (r - c - w)q, & \text{if } x \geq q - k + ar. \end{cases} \quad (1)$$

Consider the case that the retailer follows the value-at-risk criterion to determine
the optimal quantity and optimal retail price. Therefore, for a given confidence level
$0 < \alpha < 1$, the VaR at $\alpha$ of the retailer’s profit is defined as

$$\max_{r, q \geq 0} \max_{T} T \text{ s.t. } P(\Pi_R(r, q) \leq T) \leq 1 - \alpha. \quad (2)$$

The objective function of the above optimization problem maximizes the profit
of retailer’s price. The VaR constraint turns out to be the main challenge in the
above formulation since it is expressed in terms of the probability distribution of
the profit.
By simple calculation, (2) can be transformed into a simple mathematical programming problem as follows:

\[
\begin{align*}
\max_{r,q \geq 0} & \quad T \\
\text{s.t.} & \quad P(\Pi_R(r,q) \leq T) \leq 1 - \alpha \\
\Leftrightarrow \quad & \max_{r,q \geq 0} (r - c - w)q \\
\text{s.t.} & \quad P(x \geq q - k + ar) \leq 1 - \alpha \\
\Leftrightarrow \quad & \max_{r,q \geq 0} (r - c - w)q \\
\text{s.t.} & \quad q - k + ar = \Phi_a \\
\Leftrightarrow \quad & \max_{r \geq 0} T = (\Phi_a + k - ar)(r - c - w) \\
\end{align*}
\]

where \(\Phi_a = F^{-1}(1 - \alpha)\). We then derive the optimal retail price and the optimal order quantity. Since

\[
\frac{dT}{dr} = -a(r - c - w) + \Phi_a + k - ar, \quad \frac{d^2T}{dr^2} = -2a < 0,
\]

\(T\) is a concave function of \(r\). Thus, there exists a unique optimal retail price.

When \(r = c\),

\[
\frac{dT}{dr} = aw + \Phi_a + k - ac > 0. \quad (\text{from } k - ac > 0)
\]

When \(r = \frac{k}{a}\),

\[
\frac{dT}{dr} = \Phi_a - k + ac + aw.
\]

Since \(0 < r \leq \frac{k}{a}\), \(\frac{dT}{dr}|_{r=\frac{k}{a}}\) is closely related to the optimal retail price.

Let \(r^*, q^*\) and \(T^*\) be the optimal retail price, the optimal order quantity and the optimal profit of the retailer respectively. There are two cases for consideration.

(i) If \(k - ac - aw \geq \Phi_a\), we have

\[
\begin{align*}
& r^* = \frac{\Phi_a + k + ac + aw}{2a}, \\
& T^* = \frac{\Phi_a + k - ar^*}(r^* - c - w), \\
& = \frac{\Phi_a + k - \frac{\Phi_a + k + ac + aw}{2} - 2ac - 2aw}{2a}, \\
& = \frac{\Phi_a + k - ac - aw}{2}, \\
& q^* = \frac{T^*}{r^* - c - w}. \\
\end{align*}
\]

(ii) If \(k - ac - aw < \Phi_a\), we have

\[
\begin{align*}
& r^* = \frac{k}{a}, \quad q^* = \Phi_a, \quad T^* = \Phi_a(\frac{k}{a} - c - w). \\
\end{align*}
\]

As mentioned earlier, we consider the complete information situation where \(s_1, s_2\) as well as the retailer’s handling cost \(c\) per unit product are suppliers’ costs known by all the parties. Without loss of generality, we assume that \(s_1 < s_2\). Moreover, we assume that the market imposes a competitive requirement \(s_2 \leq \frac{\Phi_a + k - ac + as_1}{2a}\) on the costs of the two suppliers, which is a mild assumption. The main reason for imposing this assumption is that supplier 2’s cost should not be too high, when
compared to supplier 1’s cost, in order to make the market more competitive. On the other hand, another reason for imposing this requirement is to make the retailer and the two suppliers have incentives to participate in the two-part contract auction described in Section 3.2.

Since the above optimal retail price, the optimal order quantity and the optimal profit are all expressed in terms of the suppliers wholesale price, we need to find the optimal wholesale price. For this purpose, we consider the coordinated system method and the independent policy method separately. For the coordinated system method, we need to maximize the total profit of the whole supply chain (with respect to the low cost supplier 1’s wholesale price \( w \)) which is given by

\[
\begin{align*}
\text{(i) When } k - ac - aw &\geq \Phi \alpha, \\
\text{total profit of the supply chain} &= \text{profit of the retailer} + \text{profit of the supplier 1} \\
&= \frac{(\Phi \alpha + k - ac - aw)^2}{4a} + (w - s_1)q^* \\
&= \frac{(\Phi \alpha + k - ac - aw)^2}{4a} + (w - s_1)\Phi \alpha + k - ac - aw.
\end{align*}
\]

\[
\begin{align*}
\text{(ii) When } k - ac - aw < \Phi \alpha, \\
\text{total profit of the supply chain} &= \text{profit of the retailer} + \text{profit of the supplier 1} \\
&= \Phi \alpha (\frac{k}{a} - c - w) + (w - s_1)q^* \\
&= \Phi \alpha (\frac{k}{a} - c - w) + (w - s_1)\Phi \alpha \\
&= \Phi \alpha (\frac{k}{a} - c - s_1).
\end{align*}
\]

By maximizing the above total profits with respect to \( w \), it is not difficult to see that the channel coordinated method always generate an optimal wholesale price \( w^* \), where \( w^* = s_1 \). (Note that when \( k - ac - aw < \Phi \alpha \), the wholesale price \( w^* = s_1 \) will maximize the profit of the entire supply chain as well as the profit of the retailer.)

Thus, the channel coordinated method generates a unique optimal order quantity \( q^*_{SC,\alpha} \), a unique optimal retail price \( r^*_{SC,\alpha} \) and a unique optimal profit \( \Pi^*_{SC,\alpha} \) as follows:

\[
\begin{align*}
\text{(i) When } k - ac - as_1 &\geq \Phi \alpha, \\
q^*_{SC,\alpha} &= \frac{\Phi \alpha + k - ac - as_1}{2}, \\
r^*_{SC,\alpha} &= \frac{\Phi \alpha + k + ac + as_1}{2a}, \\
\Pi^*_{SC,\alpha} &= \frac{(\Phi \alpha + k - ac - as_1)^2}{4a}.
\end{align*}
\]

\[
\begin{align*}
\text{(ii) When } k - ac - as_1 < \Phi \alpha, \\
q^*_{SC,\alpha} &= \frac{k}{a}, r^*_{SC,\alpha} = \frac{k}{a}, \\
\Pi^*_{SC,\alpha} &= \Phi \alpha (\frac{k}{a} - c - s_1).
\end{align*}
\]

(The optimal values of the above decision variables for this channel coordination method are called the channel coordination optima.)
Note that if the suppliers and the retailer are independent, the suppliers will solve their decision problems to determine the wholesale price \( w^\star \) and accordingly, the retailer sets the retail price and ordering quantity so that his profit is maximized. However, the obtained total supply chain profit cannot achieve the system optimum as mentioned. From the above analysis, when the retailer chooses the low-cost supplier announcing the wholesale price \( w \), the retailer’s optimal retail price and ordering quantity are \( r^\star = \frac{\Phi_a + k + ac + aw}{2a} \) and \( q^\star = \Phi_a + k + ac + aw \), respectively. For the independent policy method, the low-cost supplier solves the following optimization problem to determine the optimal wholesale price \( w^\star \) to maximize his own profit:

\[
\max_w (w - s_1)q^\star \iff \max_w (w - s_1)\left(\frac{\Phi_a + k - ac - aw}{2}\right).
\]

It is not difficult to obtain that \( w^\star = \frac{\Phi_a + k - ac - aw}{2a} \). Thus, the retailer’s profit is \( \frac{3(\Phi_a + k - ac - aw)}{4a} \), and the supplier 1, with cost \( s_1 \), has the revenue of \( \frac{(\Phi_a + k - ac - aw)^2}{4a} \). As well known, the total supply chain profit is the sum of the profits of the retailer and the suppliers. Thus, the total supply chain profit is \( \frac{3(\Phi_a + k - ac - aw)}{4a} \), which is less than the channel coordination system profit \( \frac{(\Phi_a + k - ac - aw)^2}{4a} \). Accordingly, the supply chain coordination becomes vitally important. On the other hand, in the channel coordinated system, the retailer’s profit is \( \frac{(\Phi_a + k - ac - aw)^2}{4a} \), which is equal to the whole system profit. In other words, the suppliers have no profit. Thus, though the supply chain system optimum has been achieved, the supplier has no incentive to participate in this supply chain because of the zero profit. Thus, more practical and effective supply chain coordination mechanisms need to be devised in order to ensure the efficient management of the entire supply chain. In this paper, we present the auction and contracting mechanism which can play a role as a coordinated mechanism for the supply chain.

3.1. Catalog auction without contract coordination. We first define catalog auction as follows.

(i) Each supplier announces his wholesale price \( w_i, i = 1, 2 \) for general product categories. A supplier may revise his posted wholesale price in real-time if any other suppliers offer a lower price, so long as it is still higher than his own cost.

(ii) The retailer has direct access to the catalogs and will choose the supplier offering the lowest wholesale price. The retailer determines the ordering quantity \( q \) and retail price \( r \).

Considering the retailer’s risk attitude, we utilize value-at-risk measure to deal with the risk-return trade-offs. Thus, we consider the following problem:

\[
\max_{r, q \geq 0} T \quad \text{s.t. } P(\Pi_R(r, q) \leq T) \leq 1 - \alpha.
\]

Let \( r^\star_{R, \alpha}, \Pi^\star_{R, \alpha} \) be, respectively, the optimal retail price, the optimal order quantity and the optimal profit of the retailer obtained by this catalog auction, corresponding to the value at risk equal to \( \alpha \). Let \( w_{c_1} \) be the wholesale price set by the low cost supplier (i.e., supplier 1). The optimal solution of the above problem is as follows:

(i) When \( k - ac - aw_{c_1} \geq \Phi_a \),

\[
q^\star_{R, \alpha} = \frac{\Phi_a + k - ac - aw_{c_1}}{2}, \quad r^\star_{R, \alpha} = \frac{\Phi_a + k + ac + aw_{c_1}}{2a},
\]
obtained by this catalog auction are as follows:

Thus, the optimal ordering quantity \( q^*_{R,\alpha} \), the optimal retail price \( r^*_{R,\alpha} \), the profit for retailer \( \Pi^*_{R,\alpha} \) and the profit of the supplier \( \Pi^*_{S,\alpha} \) are as follows:

\[
\begin{array}{c|c|c}
   & k - ac - as_2 \geq \Phi_\alpha & k - ac - as_2 < \Phi_\alpha \\
\hline
q^*_{R,\alpha} & \frac{\Phi_\alpha + k - ac - aw_{c_1}}{2a} & \Phi_\alpha \\
r^*_{R,\alpha} & \frac{\Phi_\alpha + k - ac + as_2}{2a} & \frac{k}{a} \\
\Pi^*_{R,\alpha} & \frac{(\Phi_\alpha + k - ac - as_2)^2}{4a} & \Phi_\alpha(k - ac - as_2) \\
\Pi^*_{S,\alpha} & \frac{(\Phi_\alpha + k - ac - as_2)(s_2 - s_1)}{2} & (s_2 - s_1)\Phi_\alpha \\
\end{array}
\]

Combining the profits of the suppliers and the retailer, the system optimums obtained by this catalog auction are as follows:

\[
\begin{array}{c|c|c}
   & k - ac - as_2 \geq \Phi_\alpha & k - ac - as_2 < \Phi_\alpha \\
\hline
q^*_{SC,\alpha} & \Phi_\alpha & \Phi_\alpha \\
r^*_{SC,\alpha} & \frac{\Phi_\alpha + k - ac + as_2}{2a} & \frac{k}{a} \\
\Pi^*_{SC,\alpha} & \Phi_\alpha(k - ac - as_1) & \frac{\Phi_\alpha + k - ac + as_2}{2a} - c - s_1 \\
\end{array}
\]

When \( k - ac - as_1 \leq \Phi_\alpha \), then \( k - ac - as_2 < \Phi_\alpha \), the optimal retail price, optimal quantity and the total supply chain profit are the same as those obtained from the channel coordination system optimums. When \( k - ac - as_2 > \Phi_\alpha \), then \( k - ac - as_1 > \Phi_\alpha \), the retail price under the catalog auction is higher than the optimal retail price in the channel coordinated system. The order quantity and the total supply chain’s profit are lower than those obtained from the channel coordinated system optimums. Moreover, when \( k - ac - as_1 \geq \Phi_\alpha \) and \( k - ac - as_2 < \Phi_\alpha \), we have

\[
\begin{align*}
q^*_{SC,\alpha} &= \frac{\Phi_\alpha + k - ac - as_1}{2} \geq \Phi_\alpha = q^*_{SC,\alpha}, \\
r^*_{SC,\alpha} &= \frac{\Phi_\alpha + k - ac + as_1}{2a} \leq \frac{2k}{2a} = \frac{k}{a} = r^*_{SC,\alpha}, \\
\Pi^*_{SC,\alpha} &= \frac{(\Phi_\alpha + k - ac - as_1)^2}{4a} \geq \Phi_\alpha(k - c - s_1) = \Pi^*_{SC,\alpha},
\end{align*}
\]

where \( q^*_{SC,\alpha} \), \( r^*_{SC,\alpha} \) and \( \Pi^*_{SC,\alpha} \) are, respectively, the optimal order quantity, the optimal retail price and the optimal profit of the entire supply chain in the channel coordinated optimums. Thus, the channel coordinated optimums can be achieved if \( k - ac - as_1 = \Phi_\alpha \).

3.2. Two-part contract auction with ordering quantity and retail price functions.

(i) The retailer announces an ordering quantity function as defined in \([4]\) and \([5]\), which are the best response settings and a retail price function as \( r = \min\{c + w^e, \frac{k}{a}\} \).

(ii) The suppliers compete in a two-part contract auction where they each proposes a wholesale price \( w^* \) and a side payment \( L \) to the retailer.
(iii) The retailer first chooses the more favorable of the two contracts, then determines his ordering quantity and retail price on $w^*$ using the announced ordering quantity and retail price functions.

In this contract, we note that both the retailer’s ordering quantity and retail price are based on the supplier’s wholesale price, which gives the retailer more flexibility to make decisions. We first study the case $q = \frac{\Phi_0 + k - ac - aw}{2}$. From the definition of $q$ in (4) and (5), $r = c + w$ holds. For the low-cost supplier 1, he determines his optimal wholesale price through solving the following optimization problem,

$$\max_w \left( w - s_1 \right) \frac{\Phi_0 + k - ac - aw}{2}. $$

It can be shown that the optimal wholesale price is $w^* = \frac{\Phi_0 + k - ac + as_1}{2a}$. Thus, from the retail price function and contract auction rule, we know that if the low-cost supplier 1 wins the auction, he must submit a side payment no less than $(\Phi_0 + k - ac - as_2)^2$, which is the maximal net profit obtained by the retailer if the contract has been given to supplier 2, instead of supplier 1. Thus, the best strategy for supplier 1 is to set $w^* = \frac{\Phi_0 + k - ac + as_1}{2a}$ and the side payment $L = \frac{(\Phi_0 + k - ac - as_2)^2}{4a}$. By this scheme, the retail price $r_{R,a}$ is $\frac{\Phi_0 + k - ac + as_1}{2a}$ and the ordering quantity $q_{R,a}$ is $\frac{\Phi_0 + k - ac - as_1}{2a}$. The retailer can extract the profit from the side payment $\frac{(\Phi_0 + k - ac - as_2)^2}{4a}$, thus, in this two-part contract auction, the supplier 1’s profit is $\frac{(\Phi_0 + k - ac - as_2)^2}{4a}$ and the retailer’s profit is $\frac{\Phi_0 + k - ac - as_1}{2a}$. However, when there is no catalog auction and no contract auction contracts (that is, the situation of the independent policy method mentioned at the beginning of Section 3 where the low-cost supplier and the retailer work independently to determine the wholesale price, the retail price and the order quantity to maximize their own profits), the supplier 1 and the retailer’s profit are $\frac{(\Phi_0 + k - ac - as_2)^2}{4a}$ and $\frac{(\Phi_0 + k - ac - as_1)^2}{8a}$, respectively. In order to stimulate both the retailer and the supplier 1 to participate into this two-part contract auction, we need

$$\frac{(\Phi_0 + k - ac - as_1)^2}{4a} - \frac{(\Phi_0 + k - ac - as_2)^2}{4a} \geq \frac{(\Phi_0 + k - ac - as_1)^2}{8a}$$

and

$$\frac{(\Phi_0 + k - ac - as_2)^2}{4a} \geq \frac{(\Phi_0 + k - ac - as_1)^2}{16a}.$$
Thus, when
\[
\frac{(\sqrt{2} - 1)(\Phi_s + k - ac) + as_1}{\sqrt{2}a} \leq s_2 \leq \frac{\Phi_s + k - ac + as_1}{2a},
\]
both the supplier 1’s profit and the retailer’s profit can be improved under this two-part contract auction plan.

Next, we consider the case that \( k - ac - as_1 < \Phi_s \). For this case, we have \( q = \Phi_s \) and \( r = \frac{k}{a} \). In order to win the auction, supplier 1 must set the pair \((w, L)\) so that the retailer’s profit obtained is not less than \( \Phi_s (\frac{k}{a} - c - s_2) \), which is the maximal net profit obtained by the retailer if the contract has been given to supplier 2, instead of supplier 1. Thus, in this two-part contract auction, the supplier 1’s profit and the retailer’s profit are \((s_2 - s_1)\Phi_s\) and \( \Phi_s (\frac{k}{a} - c - s_2) \), respectively. Thus, the profit of the entire supply chain for this two-part contract auction is \( \Phi_s (\frac{k}{a} - c - s_1) \).

But, when \( k - ac - as_1 < \Phi_s \),
\[
q_{SC,\alpha}^* = \Phi_s,
\]
\[
r_{SC,\alpha}^* = \frac{k}{a},
\]
\[
\Pi_{SC,\alpha}^* = \Phi_s (\frac{k}{a} - c - s_1)
\]
= profit of the entire supply chain for this two-part contract auction,

where \( q_{SC,\alpha}^* \), \( r_{SC,\alpha}^* \) and \( \Pi_{SC,\alpha}^* \) are, respectively, the optimal order quantity, the optimal retail price and the optimal profit in the channel coordinated optimums. Thus, the channel coordinated optimums can be achieved if \( k - ac - as_1 = \Phi_s \).

4. Two-suppliers, one retailer system with asymmetric information. We focus our attention to the more general case where the suppliers hold asymmetric cost \( s_i \) \((i = 1, 2)\) and the retailer knows his own unit handling cost \( c \). The retailer assumes that the two suppliers’ marginal costs both have a prior probability density function \( g(s) \) and a cumulative distribution function \( G(s) \) with supports \( \underline{s} \) and \( \overline{s} \). Supplier \( i \) \((i = 1, 2)\) also assumes that the other suppliers’ marginal costs have a prior probability density function \( g(s) \) and a cumulative distribution function \( G(s) \) with supports \( \underline{s} \) and \( \overline{s} \).

Similar to Section 3, we again let \( q_{SC,\alpha}^* \), \( r_{SC,\alpha}^* \) and \( \Pi_{SC,\alpha}^* \) be the optimal order quantity, optimal retail price and the optimal expected profit of this asymmetric information system.

Similar to the previous discussion, the channel coordinated optimums of this asymmetric information system are given in the following table:

+-----------------+-----------------+-----------------+
| \( q_{SC,\alpha}^* \) (\( s_1, s_2 \)) | \( k - ac - a \min(s_1, s_2) \geq \Phi_s \) | \( k - ac - a \min(s_1, s_2) < \Phi_s \) |
|-----------------+-----------------+-----------------|
| \( r_{SC,\alpha}^* \) (\( s_1, s_2 \)) | \( \frac{\Phi_s + k - ac + a \min(s_1, s_2)}{2a} \) | \( \Phi_s \) |
|-----------------+-----------------+-----------------|
| \( \Pi_{SC,\alpha}^* \) (\( s_1, s_2 \)) | \( \frac{[\Phi_s + k - ac + a \min(s_1, s_2)]^2}{4a} \) | \( \frac{\Phi_s (k - ac - a \min(s_1, s_2))}{a} \) |
+-----------------+-----------------+-----------------+

Let \( E(s) = \int_{\underline{s}}^{\overline{s}} sg(s)ds \) and \( \eta = \int_{\underline{s}}^{\overline{s}} sg(s)G(s)ds \). Then the expected optimal retail price, the expected optimal order quantity and the expected profit of the
whole supply chain are

\[
E[q^*_{SC,\alpha}] = \frac{\Phi_\alpha + k - ac - aE(\min(s_1, s_2))}{2a} = \frac{\Phi_\alpha + k - ac + a(2E(s) - 2\eta)}{2a},
\]

\[
E[r^*_{SC,\alpha}] = \frac{\Phi_\alpha + k + ac + aE(\min(s_1, s_2))}{2} = \frac{\Phi_\alpha + k + ac + a(2E(s) - 2\eta)}{2},
\]

and

\[
E[\Pi^*_{SC,\alpha}] = \frac{E(\Phi_\alpha + k - ac - a(\min(s_1, s_2)))^2}{4a} = \frac{(\Phi_\alpha + k - ac)^2 - 2a(\Phi_\alpha + k - ac)E(\min(s_1, s_2)) + a^2E(\min(s_1, s_2))^2}{4a} = \frac{(\Phi_\alpha + k - ac)^2 - 2a(\Phi_\alpha + k - ac)(E(s) - \eta)}{4a} + \frac{2a^2 \int_{s}^{\eta} s^2 g(s)(1 - G(s))ds}{4a}
\]

respectively. (See Theorem A.1 in the Appendix A.)

The channel coordinated optimum can then be updated as follows:

| $E[q^*_{SC,\alpha}(s_1, s_2)]$ | $k - ac - a \min(s_1, s_2) \geq \Phi_\alpha$ | $k - ac - a \min(s_1, s_2) < \Phi_\alpha$ |
|-------------------------------|---------------------------------|---------------------------------|
| $E[r^*_{SC,\alpha}(s_1, s_2)]$ | $\frac{\Phi_\alpha + k + ac + a(2E(s) - 2\eta)}{2a}$ | $\frac{k}{a}$ |
| $E[\Pi^*_{SC,\alpha}(s_1, s_2)]$ | $\frac{\Phi_\alpha + k - ac - a(2E(s) - 2\eta)}{4a} + \frac{2a^2 \int_{s}^{\eta} s^2 g(s)(1 - G(s))ds}{4a}$ | $\frac{\Phi_\alpha (k - ac - 2a(E(s) - \eta))}{4a}$ |

Furthermore, as before, we also impose the following competitiveness requirement $\frac{E(s)}{2a} \leq \eta \leq \frac{\Phi_\alpha + k - ac + 2aE(s)}{4a}$ on the market. The reason for imposing this competitive requirement is to ensure that both the retailer and the suppliers have incentives to participate in the two-part contract described in Section 4.1.

4.1. Two-part contract auction with ordering quantity and retail price functions with one parameter.

(i) The retailer announces that the order quantity $q$ and retail price $r$ are functions of the retail price $w$ as follows:

\[
q = \max\{a(\hat{k} - w), \Phi_\alpha\}
\]

and

\[
r = \min\{\frac{\Phi_\alpha + k - a(\hat{k} - w)}{a}, \frac{k}{a}\},
\]

where $\hat{k}$ is a parameter determined by the channel system.

(ii) The suppliers compete in a two-part contract auction where they each proposes a wholesale price $w^*$ and a side payment $L$ to the retailer.

(iii) The retailer first chooses the more favorable one from the two contracts, then determines his ordering quantity and retail price based on announced functions and the wholesale price $w^*$. 
Here, the retailer’s ordering quantity and retail price are closely related to the supplier’s wholesale price. Compared to previous settings, introducing a new parameter $\hat{k}$ enable the retailer more flexibility. To improve the system efficiency, a specified parameter $\hat{k}$ on the retail price and ordering quantity is imposed.

(i) When $q = a(\hat{k} - w)$, the low-cost supplier must solve the following optimization problem to determine his own optimal wholesale price:

$$\max_w \Pi_{S,\alpha} = a(w - \min(s_1, s_2))(\hat{k} - w) - L$$
$$= -aw^2 + a(\min(s_1, s_2) + \hat{k})w - a\min(s_1, s_2)\hat{k} - L.$$

Since

$$\frac{d\Pi_{S,\alpha}}{dw} = -2w + (\min(s_1, s_2) + \hat{k}),$$

and

$$\frac{d^2\Pi_{S,\alpha}}{dw^2} = -2 < 0,$$

the supplier’s profit function is unimodal with respect to $w$. Thus, there exists a unique optimal wholesale price $w^*$, where $w^* = \frac{\min(s_1, s_2) + \hat{k}}{2}$. Furthermore, the retailer’s optimal order quantity and the optimal retail price functions are given by

$$q^* = \frac{a(\hat{k} - \min(s_1, s_2))}{2}$$

and

$$r^* = \frac{2\Phi_\alpha + 2k - a(\hat{k} - \min(s_1, s_2))}{2a}.$$

Now we verify that there exists a unique parameter setting $\hat{k}$ so that the channel coordination system optimum can be achieved. Since

$$\frac{d\Pi_{S,\alpha}}{dk} = -\frac{a(\hat{k} - \min(s_1, s_2))}{4} + \frac{2\Phi_\alpha + 2k - a(\hat{k} - \min(s_1, s_2))}{4} - \frac{a(c + \min(s_1, s_2))}{2}$$

and

$$\frac{d^2\Pi_{S,\alpha}}{dk^2} = -a < 0,$$

there exists a unique optimal parameter setting $\hat{k}^*$ so that the total supply chain profit value is maximized, where $\hat{k}^* = \frac{\Phi_\alpha + k - ac}{a}$. The reason that this conclusion is worth noting is because there may not exist any other values of $k$ which can improve the total supply chain profit. Therefore, the optimal retail price is $r^* = \frac{\Phi_\alpha + k + ac + a\min(s_1, s_2)}{2a}$ and the optimal order quantity is $q^* = \frac{\Phi_\alpha + k - ac - a\min(s_1, s_2)}{2a}$. The total supply chain profit can be derived as
The largest profit the retailer can get from the high-cost supplier is \((r^* - c - w^*)q^* + L^* = \frac{(\Phi_o + k - ac - a \max(s_1,s_2))^2}{4a}\). So the low-cost supplier must offer the side payment to the retailer no less than \(\frac{(\Phi_o + k - ac - a \max(s_1,s_2))^2}{4a}\). Let \(q^*_{SC,o}\), \(r^*_{SC,o}\) and \(\Pi^*_{SC,o}\) be the optimal order quantity, optimal retail price and the optimal profit of the supply chain of this two-part asymmetric information contract system.

Therefore, when \(k - ac - a \min(s_1,s_2) \geq \Phi_o\), the expected optimal retail price, the expected optimal order quantity and the expected optimal profit of the supply chain of this two-part asymmetric information contract system are

\[
E[r^*_{SC,o}] = \frac{\Phi_o + k + ac + a(2E(s) - 2\eta)}{2a},
\]

\[
E[q^*_{SC,o}] = \frac{\Phi_o + k - ac - a(2E(s) - 2\eta)}{2a},
\]

and

\[
E[\Pi^*_{SC,o}] = \frac{(\Phi_o + k - ac)^2}{4a} - 4a(\Phi_o + k - ac)(E(s) - \eta)
\]

\[
+ a^2 \int_0^s s^2 g(s)(1 - G(s))ds.
\]

respectively.

Hence \(E[r^*_{SC,o}] = E[r^*_{SC,o}]\), \(E[q^*_{SC,o}] = E[q^*_{SC,o}]\) and \(E[\Pi^*_{SC,o}] = E[\Pi^*_{SC,o}]\).

Thus, from the above analysis, the channel coordination system optimums can be achieved.

Furthermore, under this two-part asymmetric information contract system, the supplier 1’s profit is

\[
\frac{(\Phi_o + k - ac - aE(\min(s_1,s_2)))^2}{4a} - \frac{(\Phi_o + k - ac - aE(\max(s_1,s_2)))^2}{4a},
\]

and the retailer’s profit is \(\frac{(\Phi_o + k - ac - aE(\min(s_1,s_2)))^2}{4a}\). However, when there is no auction contract, supplier 1’s profit and the retailer’s profits are \(\frac{(\Phi_o + k - ac - aE(\min(s_1,s_2)))^2}{8a}\) and \(\frac{(\Phi_o + k - ac - aE(\max(s_1,s_2)))^2}{16a}\) respectively.
In order to stimulate both the retailer and the supplier 1 to participate into this two-part contract auction, we need

\[
\frac{(\Phi_\alpha + k - ac - aE(\min(s_1, s_2))^2}{4a} \geq \frac{(\Phi_\alpha + k - ac - aE(\max(s_1, s_2))^2}{8a}
\]

and

\[
\frac{(\Phi_\alpha + k - ac - aE(\max(s_1, s_2))^2}{4a} \geq \frac{(\Phi_\alpha + k - ac - aE(\max(s_1, s_2))^2}{16a}.
\]

Solving the first inequality, we have

\[
\Phi_\alpha + k - ac - aE(\min(s_1, s_2)) \geq \sqrt{2}(\Phi_\alpha + k - ac - aE(\max(s_1, s_2)))
\]

\[
(\Phi_\alpha + k - ac) - a(2E(s) - 2\eta) \geq \sqrt{2}(\Phi_\alpha + k - ac - 2a\eta)
\]

\[
\eta \geq \frac{(\sqrt{2} - 1)(\Phi_\alpha + k - ac) + 2aE(s)}{(2\sqrt{2} + 2)a}.
\]

(See Theorems A.1 and A.2 in the Appendix A)

Solving the second inequality, we have

\[
4(\Phi_\alpha + k - ac - aE(\max(s_1, s_2))^2 \geq (\Phi_\alpha + k - ac - aE(\min(s_1, s_2))^2
\]

\[
2(\Phi_\alpha + k - ac - 2a\eta) \geq \Phi_\alpha + k - ac - a(2E(s) - 2\eta)
\]

\[
\eta \leq \frac{\Phi_\alpha + k - ac + 2aE(s)}{6a}.
\]

Thus, when

\[
\frac{(\sqrt{2} - 1)(\Phi_\alpha + k - ac) + 2aE(s)}{(2\sqrt{2} + 2)a} \leq \eta \leq \frac{\Phi_\alpha + k - ac + 2aE(s)}{6a},
\]

both the supplier 1’s profit and the retailer’s profit can be improved under this two-part contract auction plan. Thus, both the retailer and the supplier 1 have an incentive to cooperate with each other in this auction and contracting mechanism.

(ii) From the above discussion, when \(k - ac - a\min(s_1, s_2) < \Phi_\alpha\), \(q^* = \Phi_\alpha\) and \(r^* = \frac{k}{a}\).

The largest profit brought from the high-cost supplier to the retailer is

\[
\Pi_{R,\alpha} = \Phi_\alpha(\frac{k}{\alpha} - c - w^*) + (w^* - \max(s_1, s_2))\Phi_\alpha
\]

\[
= \Phi_\alpha(\frac{k}{\alpha} - c - \max(s_1, s_2)).
\]

Therefore, the low-cost supplier must offer the profit to the retailer no less than \(\Phi_\alpha(\frac{k}{\alpha} - c - \max(s_1, s_2))\), and thus the low-cost supplier’s profit is \(\Phi_\alpha(\max(s_1, s_2) - \min(s_1, s_2))\).

Therefore, when \(k - ac - a\min(s_1, s_2) < \Phi_\alpha\), the expected optimal retail price, the expected optimal order quantity and the expected profit of the whole supply chain are

\[
E(r^*_{SC,\alpha}) = \frac{k}{\alpha} = E(r^*_{SC,\alpha}),
\]

\[
E(q^*_{SC,\alpha}) = \Phi_\alpha(\frac{k}{\alpha} - c - \max(s_1, s_2)).
\]

\[
\Pi_{SC,\alpha} = \Phi_\alpha(\frac{k}{\alpha} - c - \max(s_1, s_2)) + (\max(s_1, s_2) - \min(s_1, s_2))\Phi_\alpha
\]

\[
= \Phi_\alpha(\max(s_1, s_2) - \min(s_1, s_2)).
\]
\[ E(q_{SC,\alpha}^*) = \Phi_\alpha = E(q_{SC,\alpha}^*), \]

and
\[ E(\Pi_{SC,\alpha}^*) = \Phi_\alpha \frac{k - ac - 2a(E(s) - \eta)}{a} = E(\Pi_{SC,\alpha}^*), \]

respectively. Thus, for the case \( k - ac - a \min(s_1, s_2) < \Phi_\alpha \), the channel coordinated system optimum can also be achieved.

In summary, for this contract auction system, the channel coordinated system optima can always be achieved.

Remark 1. For the retail price and ordering quantity settings in this section, if \( \hat{k} \neq \frac{\Phi_\alpha + k - ac}{a} \), then the system optimums cannot be achieved. The findings are consistent with Jin and Wu [18] and Chen [5] that an additional restrictive condition may be necessary to eliminate system inefficiency.

5. Two-supplier, one retailer supply chain with risk-neutral attitude. In the previous sections, we discuss the optimal retail price, the optimal ordering quantity and the maximal profit under several different kinds of contract auctions when at least one agent participate in the supply chain system is risk-averse with VaR performance measure. In this section, we consider the optimal retail price, the optimal ordering quantity and the maximized supply chain profit when the participants are all risk-neutral. The relationships of system optimums between all risk-neutral agents case and at least one risk-averse agent case will be analyzed in detail.

In the risk-neutral agents system, facing the announced wholesale price, the retailer needs to solve the following optimization to determine his retail price and ordering quantity
\[
\max_{r > c, q \geq 0} E[\Pi_R(r, q)]
\]
where,
\[
E[\Pi_R(r, q)] = r E[\min(k - ar + x, q)] - (c + w)q
= r[q(1 - F(q - k + ar)) + \int_0^{q - k + ar} (k - ar + x)f(x)dx] - (c + w)q
= r[q(1 - F(q - k + ar)) + (k - ar)F(q - k + ar)]
+ (q - k + ar)F(q - k + ar) - \int_0^{q - k + ar} F(x)dx - (c + w)q
= r[q - \int_0^{q - k + ar} F(x)dx] - (c + w)q.
\]
Thus
\[
E[\Pi_{SC}(r, q)] = r[q - \int_0^{q - k + ar} F(x)dx] - (c + s)q.
\]
Differentiate \( E[\Pi_{SC}(r, q)] \) w.r.t. \( q \) and \( r \), and let \( \frac{dE[\Pi_{SC}(r, q)]}{dq} = \frac{dE[\Pi_{SC}(r, q)]}{dr} = 0 \), the optimal values \( q_N^* \) and \( r_N^* \) can be obtained as follows.
\[
q_N^* = k - ar^*_N + F^{-1}\left(\frac{r_N^* - c - s}{r_N^*}\right) \tag{6}
\]
and
\[
\int_0^{r_N^*}\left(\frac{r_N^* - c - s}{r_N^*}\right)^{-1} (1 - F(x))dx + k + ac + as - 2ar_N^* = 0. \tag{7}
\]
Let \( SL(r) = \frac{c - s}{r - a}, \alpha = 1 - SL(r^*_N) \). If \( SL(r^*_N) \geq F(k - ac - as) \), then

\[
 k - ac - as \leq F^{-1}(\frac{r^*_N - c - s}{r^*_N}) = F^{-1}(1 - \alpha) = \Phi_\alpha.
\]

Therefore, if \( r^*_N = \frac{k}{\alpha} \),

\[
 q^*_a = q^*_N = \Phi_\alpha, \quad \text{and} \quad r^*_a = r^*_N = \frac{k}{a}.
\]

Although a closed-form expression of the optimal ordering quantity and optimal retail price in the risk-neutral agents case is not available, we are able to show in the next proposition that a risk-averse retailer tends to set a retail price higher than that set by a risk-neutral retailer and induce a lower ordering quantity than that induced by a risk-neutral retailer.

**Proposition 1.** For any given \( 0 < \alpha < 1 \): If \( k - ac - as > \Phi_\alpha \), then \( r^*_a > r^*_N \); (b) when \( SL(r^*_N) > 1 - \alpha \), then \( q^*_a < q^*_N \).

**Proof.** (a) If \( SL(r^*_N) < 1 - \alpha \), then \( F^{-1}(SL(r^*_N)) < \Phi_\alpha \). Since \( E[\Pi_{SC}(r,q)] \) is unimodal in \( r \) for \( r > c \), if \( r^*_a \leq r^*_N \), then \( \frac{dE[\Pi_{SC}(r,q)]}{dr} |_{r=r^*_a} > 0 \). Therefore, we have

\[
 \frac{dE[\Pi_{SC}(r,q)]}{dr} |_{r=r^*_a} = k - a(2r^*_a - c - s) + \int_0^{F^{-1}(SL(r^*_a))} (1 - F(x))dx > 0.
\]

Since \( r^*_a = \frac{\Phi_\alpha + k + ac + as}{2a} \), we get from the above inequality that

\[
 k - a(2 \times \frac{\Phi_\alpha + k + ac + as}{2a} - c - s) + \int_0^{F^{-1}(SL(r^*_a))} (1 - F(x))dx > 0
\]

\[
 \Rightarrow \int_0^{F^{-1}(SL(r^*_a))} (1 - F(x))dx > \Phi_\alpha.
\]

Furthermore, since \( 0 \leq 1 - F(x) \leq 1 \), we have

\[
 F^{-1}(SL(r^*_a)) = \int_0^{F^{-1}(SL(r^*_a))} 1dx \geq \int_0^{F^{-1}(SL(r^*_a))} (1 - F(x))dx > \Phi_\alpha.
\]

Therefore \( \Phi_\alpha < F^{-1}(SL(r^*_a)) \) if \( k - ac - as > \Phi_\alpha \). However, if \( r^*_a \leq r^*_N \), then \( F^{-1}(SL(r^*_a)) \leq F^{-1}(SL(r^*_N)) \). We then have

\[
 F^{-1}(SL(r^*_N)) < \Phi_\alpha < F^{-1}(SL(r^*_a)) \leq F^{-1}(SL(r^*_N)).
\]

This leads to a contradiction. Therefore \( r^*_a > r^*_N \).

(b) If \( SL(r^*_N) > 1 - \alpha \), then \( F^{-1}(SL(r^*_N)) > \Phi_\alpha \). Since \( E[\Pi_{SC}(r,q)] \) is unimodal in \( r \) for \( r > c \), we have

\[
 \frac{dE[\Pi_{SC}(r,q)]}{dr} |_{r=r^*_N} = k - a(2r^*_N - c - s) + \int_0^{F^{-1}(SL(r^*_N))} (1 - F(x))dx = 0.
\]

Hence,

\[
 ar^*_N = \frac{k + ac + as + \int_0^{F^{-1}(SL(r^*_N))} (1 - F(x))dx}{2}.
\]

Furthermore, from

\[
 q^*_N = F^{-1}(SL(r^*_N)) + k - ar^*_N,
\]

\[
 q^*_a = \Phi_\alpha + k - ar^*_a,
\]
we have
\[ q_N^*-q_\alpha^* = \frac{F^{-1}(SL(r_N^*))}{2} + k - ar_N^* - \Phi_{\alpha+k-ac-as} \]
\[ = \frac{F^{-1}(SL(r_N^*))}{2} + k - ar_N^* - \Phi_{\alpha+k-ac-as} - \int_0^{F^{-1}(SL(r_N^*))} \frac{(1-F(x))dx}{2} \]
\[ = \frac{F^{-1}(SL(r_N^*))}{2} - \Phi_{\alpha+k-ac-as} - \int_0^{F^{-1}(SL(r_N^*))} \frac{(1-F(x))dx}{2} \]
\[ = \frac{F^{-1}(SL(r_N^*))}{2} - \Phi_{\alpha+k-ac-as} + \int_0^{F^{-1}(SL(r_N^*))} F(x)dx \]
\[ > \frac{F^{-1}(SL(r_N^*))}{2} - \Phi_{\alpha+k-ac-as} + \int_0^{F^{-1}(SL(r_N^*))} F(x)dx \]
\[ \geq 0. \]
Therefore, \( q_\alpha^* < q_N^* \). The second last inequality is based on the fact \( F^{-1}(SL(r_N^*)) > \Phi_{\alpha+k-ac-as} \).

6. Numerical illustration. In order to illustrate our results, we assume that Foxconn and Quanta are two competing manufacturing service suppliers for Apple Company, who plays the role of a retailer. As stated in Mathewson and Winter [22] and Gans and King [14], buy power endows the retailer Apple company first mover advantage during negotiations with suppliers, which can lead to lower wholesale price and some other benefits such as side payments. We assume that the demand faced by Apple is price dependent. That is,
\[ D_d = k - ar + x \quad \text{for} \quad 0 < r \leq \frac{k}{a}. \] (8)
We set the parameters as follows: \( k = 100, a = 4 \), Apple company’s handling cost \( c = 4 \). We also assume that Foxconn is the low cost supplier and Quanta computer is the high cost supplier whose manufacturing costs are \( s_1 \) and \( s_2 \) respectively, where \( 10 \leq s_1, s_2 \leq 20 \) and \( s_1 < s_2 \). We wish to study how these manufacturing costs can affect the ordering quantities, the retail prices of the retailer and the optimal profits of the supply chain under different situations, as well as the decisions of both the retailer and the low cost supplier to participate in the two part contract auction mentioned in Section 3.2.

Firstly, we assume that the variable \( x \) representing the disturbance in the demand \( D_d \) follows an exponential distribution with \( \lambda = 1 \). In other words, the probability density function and cumulative distribution function of \( x \) are
\[ f_1(x) = e^{-x}, \quad x \geq 0 \] (9)
and
\[ F_1(x) = 1 - e^{-x}, \quad x \geq 0 \] (10)
respectively. Hence,
\[ F_1^{-1}(x) = -\ln(1-x). \] (11)
Thus, we have
\[ \Phi_{1,\alpha} = F_1^{-1}(1-\alpha) = -\ln(\alpha) \]
Next, we assume that the variable \( x \) representing the disturbance in the demand \( D_d \) follows a normal distribution with mean \( \mu = 4 \) and variance \( \sigma^2 = 1 \). In other
words, the probability density function and the cumulative distribution function of 

\[ f_2(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-4)^2}{2}} \]  

(12)

and

\[ F_2(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-4)^2}{2}} dy = \int_{-\infty}^{x-4} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \]  

(13)

respectively. Hence, \( \Phi_{2,\alpha} \) is defined by

\[ \Phi_{2,\alpha} = F_2^{-1}(1 - \alpha) \]

That is,

\[ \int_{-\infty}^{\Phi_{2,\alpha}-4} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy = 1 - \alpha. \]

(14)

From the normal distribution table, we have \( \Pr(x \leq 0) \leq 10^{-5} \), which implies that \( x \) is almost a non-negative random variable. Thus, the above normal distribution density function could also be an appropriate choice for the distribution of \( x \). We divide into three cases according to Apple company’s risk attitudes as follows: (i) \( \alpha = 1 \), (ii) \( \alpha = 0.05 \) and (iii) \( \alpha = 0.01 \).

6.1. **Comparison of optimal decisions under the complete information and asymmetric information situations.** For the case of complete information, we assume that manufacturing cost of the low cost supplier Foxconn \( s_1 \) is either 10 or 15. For the case of symmetric information, we assume that Apply knows that the costs of the suppliers follow a uniform distribution on the interval \([10, 20]\) and thus the expected cost \( E(s) \) is 15. The probability density function is

\[ g(s) = \begin{cases} 
0.1, & 10 \leq s \leq 20, \\
0, & \text{otherwise}.
\end{cases} \]

(15)

The cumulative distribution function is

\[ G(s) = \begin{cases} 
0, & 0 \leq s < 10, \\
0.1(s - 10), & 10 \leq s \leq 20, \\
1, & s \geq 20.
\end{cases} \]

(16)

Let \( q_{i,\alpha}^* \) and \( r_{i,\alpha}^* \) (respectively, \( \bar{q}_{i,\alpha}^* \) and \( \bar{r}_{i,\alpha}^* \)) denote, respectively, the optimal ordering quantity and the optimal retail price under the complete information situation, (respectively, under the asymmetric information situation) with risk measure \( \alpha \) and the demand disturbance follows the exponential distribution described earlier in this section. Let \( q_{2,\alpha}^*, r_{2,\alpha}^*, \bar{q}_{2,\alpha}^* \) and \( \bar{r}_{2,\alpha}^* \) be defined from \( q_{1,\alpha}^*, r_{1,\alpha}^*, \bar{q}_{1,\alpha}^* \) and \( \bar{r}_{1,\alpha}^* \) respectively, by replacing ‘the demand disturbance follows the exponential distribution’ by ‘the demand disturbance follows the normal distribution’. We fix the parameter as follows: \( k = 10, a = 4, c = 4 \). We then calculate the optimal ordering quantities and the optimal retail prices under complete information and asymmetric information using two density functions (9) and (12) to represent the demand disturbances. Using the formulae

\[ q_{i,\alpha}^* = \frac{\Phi_{i,\alpha} + k - ac - as_1}{2}, \quad r_{i,\alpha}^* = \frac{\Phi_{i,\alpha} + k + ac + as_1}{2a}, \]

\[ \bar{q}_{i,\alpha}^* = \frac{\Phi_{i,\alpha} + k - ac - a(2E(s) - 2\eta)}{2}, \quad \bar{r}_{i,\alpha}^* = \frac{\Phi_{i,\alpha} + k + ac + a(2E(s) - 2\eta)}{2a}, \]

where \( \eta = \int_{10}^{20} sg(s)G(s)ds \), we obtain the following results as given in Table 1.
### Table 1. Comparison of the optimal ordering quantities and optimal retail prices obtained under the complete information and asymmetric information situations

| α   | s₁ | q₁,α | r₁,α | q₁,α | r₁,α | q₂,α | r₂,α | q₂,α | r₂,α |
|-----|----|------|------|------|------|------|------|------|------|
| 0.1 | 10 | 23.15| 19.79| 16.47| 21.46| 24.64| 20.16| 17.94| 21.83|
|     | 15 | 13.15| 22.29|      |      | 14.64| 22.26|      |      |
| 0.05| 10 | 23.50| 19.87| 16.82| 21.54| 24.82| 20.21| 18.14| 21.88|
|     | 15 | 13.50| 22.37|      |      | 14.82| 22.71|      |      |
| 0.01| 10 | 24.30| 20.08| 17.62| 21.75| 25.17| 20.29| 18.49| 21.96|
|     | 15 | 14.30| 22.58|      |      | 15.17| 22.79|      |      |

From Table 1, we observe the following phenomena.

Regardless of the risk measure and the probability density function of the demand disturbances, when the low cost suppliers actual manufacturing cost is at the lowest possible value from its probability density function, the optimal ordering quantities and the optimal retail prices under the asymmetric information situation are, respectively, more than 25% higher and slightly lower than those under the complete information situation. On the other hand, when the low cost supplier's actual manufacturing cost is equal to the average cost from its probability density function, the optimal ordering quantities and the optimal retail prices under the asymmetric information situation are, respectively, more than 20% lower and slightly higher than those under the complete information situation. Intuitively, these results indicate that when the Apple company with risk-averse attitude does not know the suppliers' actual manufacturing costs, it adopts a strategy such that its ordering quantities and retail prices are calculated in accordance with the assumption that the actual manufacturing cost of the low cost manufacturer equals to $\hat{s}_1$, where $\hat{s}_1$ is midway between the lowest possible manufacturing cost and the average manufacturing cost from its probability density function.

6.2. **Comparison of the performance of the simple model (using the coordinated system method and the independent policy method) with that of the two-part contract auction model.** In this section, we use numerical examples to compare the performances of the simple model using the coordinated policy method, the simple model using the independent policy method and the two-part contract model, in terms of maximizing the optimal profits of the retailer, the low cost supplier and the entire supply chain. Similar to Section 6.1, we fix the parameters as follows: $k = 100$, $a = 4$, $c = 4$. From the discussion in the previous sections, although the profit of the supply chain in the simple model depends only on the manufacturing cost of the low cost supplier Foxconn $s_1$, the profit of the supply chain in the two-part auction depends on the manufacturing costs of both the high cost supplier Quanta computer $s_2$ and the low cost supplier Foxconn $s_1$. Thus, to compare the performances of all the models in this section, we use two different density functions (6) and (12) to represent the demand disturbances, together with three different sets of manufacturing costs $(s_1, s_2)$, namely, $(s_1, s_2) = (10, 13), (10, 15)$ and $(10, 17)$ respectively. By fixing the value of $s_1$ and varying the value of $s_2$ only, we can test how the difference in manufacturing costs $s_2 - s_1$ can affect
the decisions of both the retailer and the low cost supplier to participate in the mechanism of the two-part auction mechanism.

Let \( \Pi_{1,r,\alpha}^*, \Pi_{1,s1,\alpha}^* \) and \( \Pi_{1,SC,\alpha}^* \) (respectively, \( \Pi_{2,r,\alpha}^*, \Pi_{2,s1,\alpha}^* \) and \( \Pi_{2,SC,\alpha}^* \) ) denote, respectively, the profit of the retailer, the profit of the low cost supplier and the profit of the supply chain of our different models, with risk measure \( \alpha \) and the demand disturbance follows the exponential distribution (respectively, the normal distribution) described earlier in this section. From the discussion in Section 2 and Section 3.2, we can obtain the following results as given in Table 2 and Table 3.

| \( \alpha \) | \( s_1 \) | \( \Pi_{1,r,\alpha}^* \) | \( \Pi_{1,s1,\alpha}^* \) | \( \Pi_{1,SC,\alpha}^* \) | \( \Pi_{1,r,\alpha}^* \) | \( \Pi_{1,s1,\alpha}^* \) | \( \Pi_{1,SC,\alpha}^* \) |
|---|---|---|---|---|---|---|---|
| 0.1 | 13 | 134.00 | 0.00 | 134.00 | 33.50 | 67.00 | 100.50 |
| | 15 | 134.00 | 0.00 | 134.00 | 33.50 | 67.00 | 100.50 |
| | 17 | 134.00 | 0.00 | 134.00 | 33.50 | 67.00 | 100.50 |
| 0.05 | 13 | 138.04 | 0.00 | 138.04 | 34.51 | 69.02 | 103.53 |
| | 15 | 138.04 | 0.00 | 138.04 | 34.51 | 69.02 | 103.53 |
| | 17 | 138.04 | 0.00 | 138.04 | 34.51 | 69.02 | 103.53 |
| 0.01 | 13 | 147.65 | 0.00 | 147.65 | 36.91 | 73.83 | 110.74 |
| | 15 | 147.65 | 0.00 | 147.65 | 36.91 | 73.83 | 110.74 |
| | 17 | 147.65 | 0.00 | 147.65 | 36.91 | 73.83 | 110.74 |

Table 2. Comparison of the performance of the simple model with the two-part contract model when the demand disturbance follows an exponential distribution

| \( \alpha \) | \( s_1 \) | \( \Pi_{1,r,\alpha}^* \) | \( \Pi_{1,s1,\alpha}^* \) | \( \Pi_{1,SC,\alpha}^* \) | \( \Pi_{1,r,\alpha}^* \) | \( \Pi_{1,s1,\alpha}^* \) | \( \Pi_{1,SC,\alpha}^* \) |
|---|---|---|---|---|---|---|---|
| 0.1 | 13 | 151.78 | 0.00 | 151.78 | 37.95 | 75.89 | 113.84 |
| | 15 | 151.78 | 0.00 | 151.78 | 37.95 | 75.89 | 113.84 |
| | 17 | 151.78 | 0.00 | 151.78 | 37.95 | 75.89 | 113.84 |
| 0.05 | 13 | 154.04 | 0.00 | 154.04 | 38.51 | 77.02 | 115.53 |
| | 15 | 154.04 | 0.00 | 154.04 | 38.51 | 77.02 | 115.53 |
| | 17 | 154.04 | 0.00 | 154.04 | 38.51 | 77.02 | 115.53 |
| 0.01 | 13 | 158.32 | 0.00 | 158.32 | 39.58 | 79.16 | 118.74 |
| | 15 | 158.32 | 0.00 | 158.32 | 39.58 | 79.16 | 118.74 |
| | 17 | 158.32 | 0.00 | 158.32 | 39.58 | 79.16 | 118.74 |

Table 3. Comparison of the performance of the simple model with the two-part contract model when the demand disturbance follows a normal distribution

From Table 2 and Table 3, we observe the following phenomenon:

(i) Regardless of the risk measure and the probability density function of the demand disturbances, for the simple model using the coordinated system method, 100% of the profit goes to the retailer and the low cost supplier gets no profit; for the simple model using the independent policy method, the optimal profit of the low cost supplier is 100% higher than that of the retailer. However, the optimal profit of the supply chain of the simple model using the independent policy method is 25% lower than that using the coordinated system method.
(ii) Regardless of the risk measure and the probability density function of the demand disturbances, whenever the difference in manufacturing costs between the two suppliers is sufficiently small (such as the case \(s_1 = 10\) and \(s_2 = 13\)), the optimal profits of the retailer and the low cost supplier in the two part auction model are, respectively, higher and lower than those of the simple model, using the independently policy method. On the other hand, whenever the difference in manufacturing costs between the two suppliers is sufficiently large (such as the case \(s_1 = 10\) and \(s_2 = 17\)), the optimal profits of the retailer and the low cost supplier in the two part auction model are, respectively, lower and higher than those of the simple model, using the independently policy method. Thus, when the difference in manufacturing costs between the two suppliers is neither too large nor too small (such as the case \(s_1 = 10\) and \(s_2 = 15\)), the optimal profits of the retailer and the low cost supplier in the two part auction model are both higher than those in the simple model, using the independently policy method. Under this situation, both parties are willing to participate in the mechanism of the two part auction model.

6.3. Comparison of the optimal ordering quantity and the optimal retail price of a risk-averse and a risk neutral supply chain. In this section, we use numerical examples to compare the optimal ordering quantities and the optimal retail prices of a risk-averse and a risk neutral supply chain, under the risk measures \(\alpha = 0.1, 0.05\) and \(0.01\) respectively. Since the above comparison only makes sense if \(0.9 < SL(r^*_N) = 1 - \frac{c + s_1}{r^*_N} \approx 0.99\), where \(r^*_N\) is the optimal retail price of the risk neutral supply chain, (See Proposition 1 for details.) we need to use another set of parameters different from those in Section 6.1 and 6.2 to achieve the comparison purpose. Thus, we use the following set of data: \(k = 10, a = 0.5, c = 0.1\) and \(s_1 = 1\).

Let \(q^*_{i,N}\) and \(r^*_{i,N}\) (respectively, \(q^*_{2,N}\) and \(r^*_{2,N}\)) be the optimal ordering quantity and optimal retail price of the risk neutral supply chain, when the demand disturbance follows the exponential distribution (respectively, the normal distribution) described earlier in this section. From (6) and (7), we have

\[
\int_0^{F_i^{-1}\left(\frac{F^*_N - c - s_1}{r^*_N}\right)} (1 - F_i(x)) dx + k + ac + as_1 - 2ar^*_i,N = 0, i = 1, 2 \tag{17}
\]

and

\[
q^*_{i,N} = k - ar^*_i,N + F_i^{-1}\left(\frac{r^*_i,N - c - s_1}{r^*_i,N}\right), i = 1, 2. \tag{18}
\]

We first consider the case that the demand disturbance follows an exponential distribution whose probability density and probability distribution functions are as given in (9) and (10). By substituting (11) into (17), we get

\[
\int_0^{-\ln\left(1 - \frac{r^*_i,N - c - s_1}{r^*_1,N}\right)} e^{-x} dx + k + ac + as_1 - 2ar^*_1,N = 0
\]

Solving this equation, we get

\[
r^*_1,N = \frac{k + ac + as_1 + 1 + \sqrt{(k + ac + as_1 + 1)^2 - 8a(c + s_1)}}{4a} = 11.454.
\]
Hence, \( SL(r_{1,N}^*) = \frac{r_{1,N}^* - c - s_1}{r_{1,N}^*} = 0.905 \). Thus, from (18) and (11), we get \( q_{1,N}^* = k - ar_{1,N}^* \ln \left( \frac{c + s_1}{r_{1,N}^*} \right) = 6.616. \)

We next consider the case that the demand disturbance follows a normal disturbance as defined earlier in this section. We need to transform (17) into a simpler form so that it can be solved by a numerical method. In view of (12) and (13), equation (17) can be simplified as follows:

\[
\int_0^{F_{\alpha}^{-1} \left( \frac{r_{2,N}^* - c - s_1}{r_{2,N}^*} \right)} (1 - F(x)) dx + k + ac + as_1 - 2ar_{2,N}^* = 0
\]

\[
\Rightarrow [x - xF(x)]_0^{F_{\alpha}^{-1} \left( \frac{r_{2,N}^* - c - s_1}{r_{2,N}^*} \right)} + \int_0^{F_{\alpha}^{-1} \left( \frac{r_{2,N}^* - c - s_1}{r_{2,N}^*} \right)} \frac{x}{\sqrt{2\pi}} e^{-\frac{(x - \mu)^2}{2\sigma^2}} dx
\]

\[
+ k + ac + as_1 - 2ar_{2,N}^* = 0
\]

\[
\Rightarrow F_{\alpha}^{-1} \left( \frac{r_{2,N}^* - c - s_1}{r_{2,N}^*} \right) (1 - \frac{r_{2,N}^* - c - s_1}{r_{2,N}^*}) - \frac{1}{\sqrt{2\pi}} e^{-\frac{(x - \mu)^2}{2\sigma^2}} f(x) dx = 0
\]

\[
\Rightarrow F_{\alpha}^{-1} \left( \frac{r_{2,N}^* - c - s_1}{r_{2,N}^*} \right) (1 - \frac{r_{2,N}^* - c - s_1}{r_{2,N}^*}) - \frac{1}{\sqrt{2\pi}} e^{-\frac{(x - \mu)^2}{2\sigma^2}} f(x) dx = 0
\]

as \( e^{-8} \) is a very small number.

\[
\Rightarrow F_{\alpha}^{-1} \left( \frac{r_{2,N}^* - c - s_1}{r_{2,N}^*} \right) (1 - \frac{r_{2,N}^* - c - s_1}{r_{2,N}^*}) - \frac{1}{\sqrt{2\pi}} e^{-\frac{(x - \mu)^2}{2\sigma^2}} f(x) dx = 0
\]

\[
+ 4[F_{\alpha}^{-1}(x)]_0^0 + k + ac + as_1 - 2ar_{2,N}^* = 0
\]

as \( F(0) = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2\sigma^2}} dy \) is a very small number. Thus, (17) is transformed into a much simpler form (19), which can be solved by the bisection method. Solving (19), we get \( r_{2,N}^* = 14.516 \). Hence, \( SL(r_{2,N}^*) = \frac{r_{2,N}^* - c - s_1}{r_{2,N}^*} = 0.9242 \). Substituting \( r_{2,N}^* = 14.516 \) into (11), we get

\[
q_{2,N}^* = k - ar_{2,N}^* + F_{\alpha}^{-1} \left( \frac{r_{2,N}^* - c - s_1}{r_{2,N}^*} \right) = 8.17
\]

Using the formulae

\[
q_{i,\alpha}^* = \frac{\Phi_{i,\alpha} + k - ac + as_1}{2}, \quad r_{i,\alpha}^* = \frac{\Phi_{i,\alpha} + k + ac + as_1}{2a}, \quad i = 1, 2
\]

we obtain the following result as given in Table 4.

From Table 4, we observe that

\[
1 - 0.1 < SL(r_{1,N}^*) < 1 - 0.05 < 1 - 0.01, \quad (i = 1, 2)
\]
Demand Disturbance \sim Exponential
\begin{tabular}{|c|c|c|c|c|}
\hline
Risk Averse & Risk Neutral & Risk Averse & Risk Neutral \\
\hline
\alpha q^*_1 & r^*_1 & q^*_1 N & SL(r^*_1,N) & q^*_2 & r^*_2 & q^*_2 N & SL(r^*_2,N) \\
\hline
0.1 & 5.88 & 12.85 & 11.45 & 0.91 & 7.37 & 15.83 & 8.17 & 14.52 & 0.92 \\
0.05 & 6.22 & 13.55 & 6.62 & 0.1 & 7.55 & 16.19 & 8.17 & 14.52 & 0.92 \\
0.01 & 7.03 & 15.16 & 7.37 & 0.1 & 7.89 & 16.88 & 8.17 & 14.52 & 0.92 \\
\hline
\end{tabular}
\begin{flushleft}
Table 4. Comparison of the optimal ordering quantity and the optimal retail price of a risk-averse and a risk neutral supply chain
\end{flushleft}

Thus,
\begin{align*}
5.88 = q^*_{1,0.1} & < q^*_1 N = 6.62 \quad \text{and} \quad 7.37 = q^*_{2,0.1} < q^*_2 N = 8.17 \\
13.55 = r^*_{1,0.05} & > r^*_1 N = 11.45 \quad \text{and} \quad 16.19 = r^*_{2,0.05} < r^*_2 N = 14.52 \\
15.16 = r^*_{1,0.01} & > r^*_1 N = 11.45 \quad \text{and} \quad 16.88 = r^*_{2,0.01} < r^*_2 N = 14.52.
\end{align*}

Thus, for both situations that the demand disturbance follows an exponential distribution or a normal distribution, our numerical results are consistent with the statement of Proposition 1. Intuitively, these results indicate that when the risk percentile \( 1 - \alpha \) is small, the optimal ordering quantity of a risk adverse retailer is less than that of a risk neutral retailer. Similarly, when the risk percentile \( 1 - \alpha \) is large, the optimal retail price of a risk adverse retailer is greater than that of a risk neutral retailer. Thus when Apple Company is risk adverse, it needs to order less electronic equipments at a higher retail price. On the other hand, when Apple Company is risk neutral, it needs to order more electronic equipments at a lower retail price.

7. Conclusions. This paper considers a supply chain system with two suppliers providing identical products to a single risk-adverse retailer. We first consider a simple situation where no complicated auction is involved and business is done between the retailer and the low-cost supplier only. For this simple situation, whenever the retailer and the low-cost supplier are cooperating together to maximize the total profit of the supply chain, 100% of the profit will go to the retailer, leaving no profit at all to the low-cost supplier. The total minimum system cost of this coordinated method is called the channel coordination optimum.

To overcome the above weakness, we develop several auction mechanisms so that the profit can be shared between the retailer and the low-cost supplier and at the same time, the total minimum system costs obtained by using these auction mechanisms are equal to the channel coordination optimum. Then we extend the auction mechanism for coordination to cater for the situation that only asymmetric information (not full information) can be obtained by the retailer. Similar to the full-information situation, the total minimum system costs obtained by using these auction mechanisms are equal to the channel coordination optimum.

Lastly, we consider the situation where the two suppliers are providing identical products to a single risk-neutral (instead of a risk-adverse) retailer. Analytical and numerical results show that the optimal ordering quantity of the risk-neutral retailer is lower than that of the risk-adverse retailer (unless the retailer is too risk-averse) and the optimal retail price of the risk-adverse supplier is higher than that of the risk-neutral retailer, irrespective of whether the retailer is receiving full or asymmetric information from the suppliers.
In the near future, we would like to construct a new auction mechanism so that profits can be shared between the retailer, the low-cost supplier and the high-cost supplier. Thus, all the three agents will be interested in participating in the mechanism of this new auction.

Appendix A. Theorems.

**Theorem A.1.** Let $S$ be a random variable having prior probability density function $g$ and cumulative density function $G$ with support $s$ and $\bar{s}$. Let $S_1$ and $S_2$ be two independent random variables having the same prior probability density function as $S$. Let $\bar{S} = \min(S_1, S_2)$. Then the expected value of $\bar{S}$ is given by

$$E(\bar{S}) = 2E(S) - 2\eta,$$

where

$$\eta = \int_s^\bar{s} sg(s)G(s)ds.$$

Moreover, the expected value of $S^2$ is given by

$$E(S^2) = \int_s^\bar{s} s^2 g(s)ds = 2 \int_s^\bar{s} s^2 g(s)ds - 2 \int_s^\bar{s} s^2 g(s)G(s)ds.$$

**Proof.** Since $S_1$ and $S_2$ are two independent random variables with the same prior density function $g$, we have

$$P(\bar{S} > s) = P(\min (S_1, S_2) > s) = P(S_1 > s) \times P(S_2 > s) = [1 - \int_s^\bar{s} g(t)dt]^2.$$

Thus, the cumulative density function of $\bar{S}$, denoted by $G_s$, is as follows:

$$G(s) = P(\bar{S} \leq s) = 1 - [1 - \int_s^\bar{s} g(t)dt]^2.$$

Thus, the probability density function of $\bar{S}$, denoted by $g_s$, is as follows:

$$g(s) = \frac{dG(s)}{ds} = 2g(s)[1 - \int_s^\bar{s} g(t)dt].$$

Thus,

$$E(\bar{S}) = \int_s^\bar{s} sg(s)ds = 2 \int_s^\bar{s} sg(s)ds - \int_s^\bar{s} sg(s)G(s)ds = 2E(S) - 2\eta.$$

Moreover,

$$E(S^2) = \int_s^\bar{s} s^2 g(s)ds = 2 \int_s^\bar{s} s^2 g(s)ds - 2 \int_s^\bar{s} s^2 g(s)G(s)ds.$$

\[\square\]

**Theorem A.2.** Let $S$ be a random variable having prior probability density function $g$ and cumulative density function $G$ with support $s$ and $\bar{s}$. Let $S_1$ and $S_2$ be two independent random variables having the same prior probability density function as $S$. Let $\bar{S} = \max(S_1, S_2)$. Then the expected value of $\bar{S}$ is given by $E(\bar{S}) = 2\eta$, where $\eta = \int_s^\bar{s} sg(s)G(s)ds$. 


Proof.

\[ P(S < s) = P(\max(S_1, S_2) \leq s) = P(S_1 \leq s) \times P(S_2 \leq s) = \left[ \int_{\frac{s}{2}}^{s} g(t) dt \right]^2. \]

Thus, the cumulative density function of \( S \), denoted by \( G \), is as follows:

\[ G(s) = P(S \leq s) = \left[ \int_{\frac{s}{2}}^{s} g(t) dt \right]^2. \]

Thus, the probability density function of \( S \), denoted by \( g \), is as follows:

\[ g(s) = \frac{dG(s)}{ds} = 2g(s) \int_{\frac{s}{2}}^{s} g(t) dt. \]

Thus,

\[ E(S) = \int_{\frac{s}{2}}^{s} s g(s) ds = 2 \int_{\frac{s}{2}}^{s} s g(s) G(s) ds = 2\eta. \]

\[ \square \]

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