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Study on Three Dimensional Dual Peg-in-hole in Robot Automatic Assembly

Fei Yanqiong¹, Wan Jianfeng², Zhao Xifang¹

¹Research Institute of Robotics, Shanghai Jiao Tong University, Huashan Road 1954, Shanghai 200030 (P. R. of China)
²School of Materials Science and Engineering, Shanghai Jiao Tong University, Huashan Road 1954, Shanghai 200030 (P. R. of China)

1. Introduction and Importance

The analyses of the physics of robot assembly tasks are important in developing a flexible assembly system. Assembly analyses involve force/moment analysis due to contacts and jamming analysis due to improper force/moment. Because the uncertainty of geometry and control, contact states exist during the assembly process. Xiao presented the notion of contact formation (CF) in terms of principal contacts (PCs) to characterize contact states, and defined contact states as CF-connected regions of contact configurations (Xiao, 1993). They also examined the neighboring relations between contact states and characterized the contact state space as a contact state graph. Xiao developed an algorithm that automatically generated a high-level contact state graph and planned contact motions between two known adjacent contact states (Xiao, 2001).

Force/moment analysis is an important element in robot assembly analyses. Lots of research has been done on single peg-in-hole. Simunivic analyzed a round peg insertion problem, and derived two dimensional single peg-in-hole jamming conditions (Simunivic, 1972). Whitney adopted Simunovic’s approach to determine the force analysis of the same problem (Whitney, 1982). He identified wedging and jamming conditions and recommended ways to avoid insertion failure. A compliant mechanism, RCC (Remote-Center-Compliance) was designed for axisymmetric part insertions. Sturges and Laowattana analyzed the wedging condition in rectangular peg insertion and developed a Spatial Remote-Center-Compliance (SRCC) (Sturges, 1988, 1996, 1996). Non-axisymmetric part insertions can be performed. Tsaprounis focused on the determination of forces at contact points between a round peg and a hole (Tsaprounis, 1998). Considerable theories have also been studied over the past years that define two dimensional mating between double rigid cylindrical parts (Arai, 1997, McCarragher, 1995 and Sathirakul, 1998). Arai described error models and analyzed the search strategy of a two dimensional dual peg-in-hole (Arai, 1997). McCarragher modeled assembly as a discrete event dynamic system using Petri nets and developed a discrete event controller.

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He applied his method to a two dimensional dual peg-in-hole insertion problems, enumerated possible contact states, and derived geometric conditions and force-moment equations for static-equilibrium states of two dimensional dual peg insertions (Sathirakul and Sturges, 1998). The jamming diagrams of a two dimensional dual peg-in-hole were obtained.

At present, research activities in the area are concentrated on:

1. Three dimensional multiple peg-in-hole analyses
2. Modeling of contact
3. Design of assembly devices

Multiple peg-in-hole insertions represent a class of practical and complicated tasks in the field of robotic automatic assembly. Because of the difficulty and complexity in analyzing three dimensional multiple peg-in-hole insertions, most of the analyses of this class of assembly tasks to date were done by simplifying the problems into two dimensions (Arai, 1997, McCarragher, 1995 and Sathirakul, 1998). The analyses of the dual peg-in-hole in three dimensions are few (Rossitza, 1998). Rossitza considered the geometric model and quasi-static force model of a three dimensional dual peg-in-hole and presented a method for 3D simulation of the dual peg insertion. The assembly process, considered as a sequence of discrete contact events, is modeled as a transition from one state to another. Sturges’ work on the three dimensional analysis of multiple-peg insertion gave insight into the behavior of the pegs during the chamfer-crossing phase of the problem (Sturges, 1996). The net forces and torque during chamfer-crossing of a triple-peg insertion were obtained. The force/torque prediction gave a better insight into the physics of multiple peg-in-hole insertion with the aid of compliance mechanisms (SRCC) (Sturges, 1996). Fei’s geometric analysis of a triple peg-in-hole mechanism gives the base of three dimensional assembly problems (Fei, 2003). The geometric features of three-dimensional assembly objects are represented by two elements. The geometric conditions for each contact state are derived with the transformation matrix (Fei, 2003).

However, for three dimensional multiple peg-in-hole insertion tasks, there are not yet perfect assembly theory and experimental analyses at present. After analyzing multiple peg-in-hole in three dimensions, the whole assembly process and its complexity and difficulty can be known in detail. Thus, some simple and special machines can be designed to finish this type of tasks.

In this article, three dimensional dual peg-in-hole insertion problems are analyzed. Firstly, all possible contact states are enumerated in three dimensions. Secondly, contact forces can be described by the screw theory. The screw theory gives a very compact and efficient formulation. Applying the static equilibrium equations, the conditions on applied forces and moments for maintaining each contact state are formulated. Thirdly, jamming diagrams are obtained. 33 kinds of possible jamming diagrams are analyzed. The force/moment conditions to guarantee successful insertion are obtained from all possible jamming diagrams. Then, some contact states are given, which finish a dual peg-in-hole process. Their geometric conditions are derived to verify the analyses.
2. Contact states classification of dual peg-in-hole

The geometric model of a dual peg-in-hole problem is shown in Fig. 1. Left peg has a radius of \( r_{P1} \), whereas right peg has a radius of \( r_{P2} \). The radii of left hole and right hole are \( r_{H1} \) and \( r_{H2} \), respectively. \( D \) (or \( D_H \)) represents the distance between the peg axes, and \( D_H \) represents the distance between the hole axes. The clearances between left peg and left hole, and between right peg and right hole are \( c_1 \) and \( c_2 \), respectively. To simplify our analysis, we only consider multiple pegs which are round, parallel and have the same length. Besides these, we make the following assumptions:

1) In the assembly process, the angle \( \theta \) between the axes of the pegs and those of the holes is small and can be neglected.
2) Insertion direction is only vertically downward.
3) Quasi-static motion.
4) All pegs have the same length and are parallel.
5) All axes of the holes are parallel to each other, and perpendicular to the horizontal plane.
6) The clearances between pegs and holes are positive.

Expect three kinds of line contact states (Sathirakul and Sturges, 1998), there are ten kinds of point contact states when pegs tilt to the left, as shown in Table 1. Table 1 shows all possible point contact states of a dual peg-in-hole in three dimensions. The vectors of contact points, and the angular ranges between the vectors of contact forces and the x-axis can be seen in Table 1.
When pegs tilt to the left, $k$ stands for $l$. When pegs tilt to the right $k$ stands for $r$.

| Dual peg-in-hole | Three dimensional contact states | $\alpha$ | $r_i$ |
|------------------|----------------------------------|---------|-------|
| (k-1)            | $\alpha_i \in [0^\circ, 90^\circ) \cup [270^\circ, 360^\circ]$ | $r_i = (x_i, y_i, -j_ih_i)$ |
| (k-2)            | $\alpha_i \in [90^\circ, 270^\circ]$ | $r_i = (x_i, y_i, -\mu h_i)$ |
| (k-3)            | $\alpha_i \in [0^\circ, 90^\circ) \cup [270^\circ, 360^\circ]$ | $r_i = (x_i, y_i, -j_ih_i)$ |
| (k-4)            | $\alpha_i \in [90^\circ, 270^\circ]$ | $r_i = (x_i, y_i, -\mu h_i)$ |
| (k-5)            | $\alpha_i \in [0^\circ, 90^\circ) \cup [270^\circ, 360^\circ]$, $\alpha_i \in [90^\circ, 270^\circ]$ | $r_i = (x_i, y_i, -j_ih_i)$, $r_i = (x_i, y_i, -\mu h_i)$ |
| (k-6)            | $\alpha_i \in [0^\circ, 90^\circ) \cup [270^\circ, 360^\circ]$, $\alpha_i \in [90^\circ, 270^\circ]$ | $r_i = (x_i, y_i, -j_ih_i)$, $r_i = (x_i, y_i, -\mu h_i)$ |
| (k-7)            | $\alpha_i \in [0^\circ, 90^\circ) \cup [270^\circ, 360^\circ]$, $\alpha_i \in [90^\circ, 270^\circ]$ | $r_i = (x_i, y_i, -j_ih_i)$, $r_i = (x_i, y_i, -\mu h_i)$ |
| (k-8)            | $\alpha_i \in [0^\circ, 90^\circ) \cup [270^\circ, 360^\circ]$, $\alpha_i \in [90^\circ, 270^\circ]$ | $r_i = (x_i, y_i, -j_ih_i)$, $r_i = (x_i, y_i, -\mu h_i)$ |
| (k-9)            | $\alpha_i \in [0^\circ, 90^\circ) \cup [270^\circ, 360^\circ]$, $\alpha_i \in [90^\circ, 270^\circ]$ | $r_i = (x_i, y_i, -j_ih_i)$, $r_i = (x_i, y_i, -\mu h_i)$ |
| (k-10)           | $\alpha_i \in [0^\circ, 90^\circ) \cup [270^\circ, 360^\circ]$, $\alpha_i \in [90^\circ, 270^\circ]$ | $r_i = (x_i, y_i, -j_ih_i)$, $r_i = (x_i, y_i, -\mu h_i)$ |

When pegs tilt to the left $j=1$. When pegs tilt to the right $j=0$.

Table 1. Assembly contact states of a dual peg-in-hole in three dimensions.

![Fig. 2. $\alpha_i$ between the vector $f_i^\ast$ and the x-axis.](image-url)
For example, let pegs tilt to the left, if \( i=1 \), one-point contact state \((l-1)\) exists. If \( i=1 \) and \( i=4 \), two-point contact state \((l-8)\) exists (Fig. 3). Let pegs tilt to the right, If \( i=1 \), one-point contact state \((r-4)\) exists; if \( i=1 \) and \( i=4 \), two-point contact state \((r-8)\) exists (Fig. 3). During the insertion, the dual-peg may undergo several contact states. In the paper, some states can be chosen, such as, pegs tilting to the left: state \((l-1)\), state \((l-5)\), state \((l-8)\), a possible three-point contact state and a mating state or pegs tilting to the right: state \((r-4)\), state \((r-6)\), state \((r-8)\), a possible three-point contact state and a mating state to finish a dual peg-in-hole assembly process. Their three dimensional charts and two dimensional charts are shown in Fig. 3 and Fig. 4.

2.2 Contact states of a dual peg-in-hole in two dimensions

When \( \alpha = 0^\circ \) or \( 180^\circ \), three dimensional problems can be simplified to two dimensional problems. The two dimensional chart of each state in Fig. 3 is shown in Fig. 4.

\[
\alpha_1 \in [0^\circ, 90^\circ] \cup [270^\circ, 360^\circ], \quad r_1 : (x_1, y_1, -h_1)
\]

\[
\alpha_2 \in [90^\circ, 270^\circ], \quad r_2 : (x_2, y_2, 0)
\]

\[
\alpha_3 \in [0^\circ, 90^\circ] \cup [270^\circ, 360^\circ], \quad r_3 : (x_3, y_3, -h_3)
\]

\[
\alpha_4 \in [90^\circ, 270^\circ], \quad r_4 : (x_4, y_4, 0)
\]
Fig. 3. Assembly states of a dual peg-in-hole in three dimensions.

(b) pegs tilt to the right
3. Geometric constraints and contact states analysis

According to the above chosen states (pegs tilting to the left: state \( l-1 \), state \( l-5 \), state \( l-8 \), a possible three-point contact state and a mating state or pegs tilting to the right: state \( r-4 \), state \( r-6 \), state \( r-8 \), a possible three-point contact state and a mating state), the relations between these states and their geometric constraints are analyzed. Thus, the complexity of the three dimensional dual peg-in-hole can be known. The dimensions of a dual peg and a dual hole are given as follows (Fig. 1): 

\[
2r_{p1} = 2r_{p2} = 9.9\, \text{mm}, \quad 2r_{h1} = 2r_{h2} = 10.0\, \text{mm}, \quad D_p = 90.2\, \text{mm}, \quad D_H = 90.25\, \text{mm}, \quad C_1 = C_2 = 0.075\, \text{mm}, \quad \mu = 0.1.
\]

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3.1 Two dimensions

The boundary state of state l-1 or state r-4 is shown in Fig. 5. Its geometric constraint is

\[
\begin{align*}
&h_0 s \theta_0 + 2 r_p c \theta_0 = 2 r_H, \\
&h_0 c \theta_0 = 2 r_p s \theta_0
\end{align*}
\]

(1)

Thus, \( \theta_0 = 8.1^\circ, \ h_0 = 1.397 \text{mm} \) can be obtained. When \( \theta < \theta_0 \), state l-1 or state r-4 can be reached.

According to Fig. 3, for pegs tilting to the left, the geometric constraint of state l-5 can be expressed as,

\[
h_1 s \theta + 2 r_p c \theta = 2 r_H
\]

(2a)

The geometric constraint of state l-8 can be expressed as,

\[
h_3 s \theta = (D_p + r_p + r_H) \theta = (D_H + r_H + r_H)
\]

(3a)

According to Fig. 3, for pegs tilting to the right, the geometric constraint condition of state r-6 in two dimensions can be expressed as,

\[
h_4 s \theta + 2 r_p c \theta = 2 r_H
\]

(2b)

The geometric constraint condition of state r-8 in two dimensions can be expressed as,

\[
h_5 s \theta + (D_p + r_p + r_H) \theta = (D_H + r_H + r_H)
\]

(3b)

If Eqs. (2a) and (3a) or Eqs. (2b) and (3b) have one solution at least, a three-point contact state of two dimensional dual peg-in-hole can be reached. Fig. 6 suggests that a three-point contact state is relative to the geometry of assembly objects. At the point where two curves of plots intersect, the three-point contact states can exist. From Fig. 6, we can see that three-point contact states are transient. Here \( \theta_0 \) and \( \theta \) are angles between the axes of a peg and a hole, \( h_0 \) and \( h_t \) are insertion depths.

![Fig. 5. Boundary state.](image)

3.2 Three dimensions

Three dimensional multiple peg-in-hole problems are more complex than two dimensional peg-in-hole problems. But they have similar geometric constraint conditions. For the boundary state of state l-1 or state r-4 in three dimensions, we can obtain:

\[
\begin{align*}
&h_0 s \theta_0 + 2 r_p c \theta_0 = 2 r_H', \\
&h_0 c \theta_0 = 2 r_p s \theta_0, \quad r_p' \leq r_p, \quad r_H' \leq r_H
\end{align*}
\]

(4)
If \( \theta < \theta^*, \) state \( l-1 \) or state \( r-4 \) can be reached.

For state \( l-5 \) in three dimensions, we can obtain the following formulas:

\[
h s \theta + 2r P c \theta = 2r_H c \theta, \quad r_P' \leq r_P, \quad r_H' \leq r_H
\]  
(5a)

For state \( l-8 \) in three dimensions, we can obtain the following formulas:

\[
h s \theta + (D_P' + r_P' + r_H') \cos \theta = (D_H' + r_H'),
\]

\[
r_P' \leq r_P, \quad r_H' \leq r_H, \quad D_P' \leq D_P, \quad r_H' \leq r_H, \quad D_H' \leq D_H
\]  
(6a)

For state \( r-6 \) in three dimensions, we can obtain the following constraint formulas:

\[
h s \theta + 2r_P c \theta = 2r_H c \theta, \quad r_P' \leq r_P, \quad r_H' \leq r_H
\]  
(5b)

For state \( r-8 \) in three dimensions, we can obtain the following formulas:

\[
h s \theta + (D_P' + r_P' + r_H') \cos \theta = (D_H' + r_H'),
\]

\[
r_P' \leq r_P, \quad r_H' \leq r_H, \quad D_P' \leq D_P, \quad r_H' \leq r_H, \quad D_H' \leq D_H
\]  
(6b)

If constraint conditions are formulas (5a) and (6a) or (5b) and (6b) at the same time, three-point contact states of the three dimensional peg-in-hole can be obtained.

Mapping two contact points on the peg to the plane \( xoy \), we can obtain two corresponding points. The distance between these two contact points on the left hole or on the right hole, respectively. Mapping one contact point on the left peg and the other contact point on the right peg to the plane \( xoy \), we can obtain two corresponding points. The distance between these two points is \( D_P' \), \( D_H' \) represents the distance between one contact points on the left hole and the other contact point on the right hole.

According to the analysis of the two dimensional dual peg-in-hole, the following results can be inferred in three dimensions. For different contact geometric dimensions (such as \( r_P', r_H', D_P', r_P', r_H', \) and \( D_H' \) ) of the three dimensional dual peg-in-hole, state \( l-5 \) and state \( l-8 \) show different curves, but their change trends are similar. At the point where two curves of state \( l-5 \) and state \( l-8 \) intersect, three-point contact states can exist, which are relative to the geometry of assembly objects. It means that three-point contact states, in 3D cases, can exist only when a certain set of conditions are satisfied. They are transient and considered trivial.

Fig. 6. \( \theta \) vs \( h \) of state \( l-5 \) and state \( l-8 \) in two dimensions.

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4. Analyzing contact force/moment of three dimensional dual peg-in-hole based on the screw theory

By understanding the physics of assembly processes, one can plan fine motion strategies and guarantee successful operations. An analysis of forces and moments during the three dimensional dual-peg insertion is important in planning fine motions. In order to insert dual pegs into their holes, insertion forces and moments \(|F_x, F_y, F_z, M_x, M_y, M_z|\) applied to the pegs at point 0 have to satisfy a number of conditions. In this section, the conditions on applied forces and moments for maintaining each contact state are derived. Then jamming conditions are obtained. These are basic conditions of fine motions and can avoid unsuccessful insertions. The conditions which guarantee successful insertions can be obtained from the jamming diagrams.

As mentioned previously, there are 20 kinds of point contact states expect 6 kinds of line contact states. The contact states that have more than 2 contacts are ignored. According to a series of chosen contact states in Fig. 3, forces and moments can be described by screw theory (Tsaprounis, 1998). Because a quasi-static process is assumed, each of the contact states is analyzed from static equilibrium conditions. With the static equilibrium conditions and geometric constraints, the relationship between moments and forces is analyzed. The coordinate system (oxyz) is set up at the center of dual pegs and contact forces are shown in Fig. 3,4. The dual-peg is about to move down or is moving down. Examples of one-point contact state l-1 or state r-4 and two-point contact state l-8 or state r-8 are analyzed in detail. The relationship between forces and moments of other contact states can be analyzed similarly (Table 2). Table 2 shows the relationship between forces and moments of all possible point contact states about the three dimensional dual peg-in-hole.

The wrenches of the contact cases are described as follows

\[ \mathbf{F}_c = \mathbf{F}_f + \epsilon (\mathbf{F}_f \times \mathbf{r}_f) \]  
(7)

\[ \mathbf{f}_f = \mathbf{f}_f + \epsilon (\mathbf{f}_f \times \mathbf{r}_f) \]  
(8)

\[ f_i = \mu f_i \]

\[ \mathbf{F}_c : \mathbf{F}_f : \mathbf{F}_n : \mathbf{F}_c \times \mathbf{r}_f \]

\[ \mathbf{f}_f : \mathbf{f}_f \times \mathbf{r}_f \]

(1) One-point contact state l-1 and state r-4:

State l-1: the compact forms of the wrenches of contact forces in contact points are described as follows,

\[ \begin{pmatrix} f_x \alpha_1 & f_y \alpha_1 & f_z \alpha_1 & -h_f \alpha_1 & x \alpha_1 & -y_f \alpha_1 & 0 \\ 0 & -\mu f_1 & -y_1 \mu f_1 & x_1 \mu f_1 & 0 \end{pmatrix} \]

The wrenches of the assembly forces are described as follows \(|F_x, F_y, M_x, M_y, M_z|\).

According to the static equilibrium equations, we can obtain
\[ \begin{align*}
F_x + f_1 c \alpha_4 &= 0 \\
F_y + f_3 s \alpha_4 &= 0 \\
F_z - \mu f_4 &= 0 \\
M_x + h_1 f_3 s \alpha_4 - y_1 \mu f_4 &= 0 \\
M_y - h_1 f_1 c \alpha_4 + x_1 \mu f_4 &= 0 \\
M_z + x_1 f_3 s \alpha_4 - y_1 f_1 c \alpha_4 &= 0
\end{align*} \]  
(9)

From Eq. (9), we can obtain the following relationship equations,
\[ \begin{align*}
F_x &= -\frac{c \alpha_4}{\mu} \\
F_y &= -\frac{s \alpha_4}{\mu} \\
M_x &= \frac{h_1 c \alpha_4 - x_1}{\mu} \\
M_y &= \frac{h_1 s \alpha_4 + y_1}{\mu}
\end{align*} \]  
(10)

\[ \begin{align*}
F_x &= -\frac{c \alpha_4}{\mu} \\
F_y &= -\frac{s \alpha_4}{\mu} \\
M_x &= \frac{h_1 c \alpha_4 - x_1}{\mu} \\
M_y &= \frac{h_1 s \alpha_4 + y_1}{\mu}
\end{align*} \]  
(11)

where \[\alpha_4 \in [0^\circ, 90^\circ] \cup [270^\circ, 360^\circ]\].

State \(r-4\): the wrenches of contact forces in contact points are as follows,
\[ \begin{align*}
(f_1 c \alpha_4, f_3 s \alpha_4, 0, h_3 f_3 s \alpha_4 - h_3 f_1 c \alpha_4, x_4 f_3 s \alpha_4 - y_4 f_1 c \alpha_4) \\
(0, 0, -\mu f_4, -y_4 \mu f_4, x_4 \mu f_4)
\end{align*} \]

The wrenches of the assembly forces are as follows \((F_x, F_y, F_z, M_x, M_y, M_z)\).

According to the static equilibrium equations, we can obtain
\[ \begin{align*}
F_x + f_4 c \alpha_4 &= 0 \\
F_y + f_4 s \alpha_4 &= 0 \\
F_z - \mu f_4 &= 0 \\
M_x + h_4 f_3 s \alpha_4 - y_4 \mu f_4 &= 0 \\
M_y - h_4 f_1 c \alpha_4 + x_4 \mu f_4 &= 0 \\
M_z + x_4 f_3 s \alpha_4 - y_4 f_1 c \alpha_4 &= 0
\end{align*} \]  
(12)

Thus, we can obtain the following relationship equations,
\[ \begin{align*}
F_x &= -\frac{c \alpha_4}{\mu} \\
F_y &= -\frac{s \alpha_4}{\mu} \\
M_x &= \frac{h_4 c \alpha_4 - x_4}{\mu} \\
M_y &= \frac{h_4 s \alpha_4 + y_4}{\mu}
\end{align*} \]  
(13)

\[ \begin{align*}
F_x &= -\frac{c \alpha_4}{\mu} \\
F_y &= -\frac{s \alpha_4}{\mu} \\
M_x &= \frac{h_4 c \alpha_4 - x_4}{\mu} \\
M_y &= \frac{h_4 s \alpha_4 + y_4}{\mu}
\end{align*} \]  
(14)

where \[\alpha_4 \in [90^\circ, 270^\circ]\], \(r_4 : (x_4, y_4, -h_4)\).

(2) two-point contact state \(l-8\) and state \(r-8\):
State l-8: the compact forms of the wrenches of contact forces in contact points are described as follows,
\[
\begin{pmatrix}
  f_x c \alpha_1 & f_y c \alpha_1 & 0 & h_y f_x c \alpha_1 & -h_x f_y c \alpha_1 & x_4 f_x c \alpha_1 - y_1 f_y c \alpha_1 \\
  0 & 0 & -\mu f_1 & -y_1 \mu f_1 & x_1 \mu f_1 & 0
\end{pmatrix}
\]
\[
\begin{pmatrix}
  f_x c \alpha_4 & f_y c \alpha_4 & 0 & 0 & 0 & x_4 f_x c \alpha_4 - y_4 f_y c \alpha_4 \\
  0 & 0 & -\mu f_4 & -y_4 \mu f_4 & x_4 \mu f_4 & 0
\end{pmatrix}
\]
The wrenches of the assembly forces are as follows,
\[
\begin{align*}
M_x & = -\frac{\mu}{\alpha_1 - \alpha_4} \left[ \frac{x_4 - x_1}{D} + \frac{h_y c \alpha_4}{\mu D} \right] F_y + \frac{\mu \nu c \alpha_4 - \mu \nu c \alpha_1 - h_y c \alpha_4 c \alpha_4}{\mu D(\alpha_1 - c \alpha_4)} F_z \\
M_y & = -\frac{\mu}{\alpha_4 - \alpha_1} \left[ -\frac{y_4 - y_1}{D} + \frac{h_y c \alpha_1}{\mu D} \right] F_y + \frac{\mu \nu c \alpha_4 + \mu \nu c \alpha_1 + h_y c \alpha_4 c \alpha_4}{\mu D(\alpha_4 - c \alpha_1)} F_z
\end{align*}
\]
Eqs. (15) and (16) show linear relations. For Eq. (15), when \( \frac{F_z}{F_y} = -\frac{\alpha_1}{\mu} \), this equation reduces to Eq.(10). We can write
\[
M_y = \frac{\mu}{\alpha_1 - \alpha_4} \left[ \frac{x_4 - x_1}{D} \right] F_y,
\]
when \( \frac{F_y}{F_z} = -\frac{\alpha_4}{\mu} \), we can write
\[
M_x = \frac{\mu}{\alpha_4 - \alpha_1} \left[ \frac{y_4 - y_1}{D} \right] F_y.
\]
State r-8: the wrenches of contact forces in contact points are described as follows,
\[
\begin{pmatrix}
  f_x c \alpha_1 & f_y c \alpha_1 & 0 & 0 & 0 & x_1 f_x c \alpha_1 - y_1 f_y c \alpha_1 \\
  0 & 0 & -\mu f_1 & -y_1 \mu f_1 & x_1 \mu f_1 & 0
\end{pmatrix}
\]
\[
\begin{pmatrix}
  f_x c \alpha_4 & f_y c \alpha_4 & 0 & 0 & 0 & x_4 f_x c \alpha_4 - y_4 f_y c \alpha_4 \\
  0 & 0 & -\mu f_4 & -y_4 \mu f_4 & x_4 \mu f_4 & 0
\end{pmatrix}
\]
The wrenches of the assembly forces are as follows,
\[
\begin{align*}
M_x & = -\frac{\mu}{\alpha_1 - \alpha_4} \left[ \frac{x_4 - x_1}{D} + \frac{h_y f_x c \alpha_1}{\mu D} \right] F_y + \frac{\mu \nu c \alpha_4 - \mu \nu c \alpha_1 - h_y f_x c \alpha_4 c \alpha_4}{\mu D(\alpha_1 - c \alpha_4)} F_z \\
M_y & = -\frac{\mu}{\alpha_4 - \alpha_1} \left[ -\frac{y_4 - y_1}{D} + \frac{h_y f_y c \alpha_1}{\mu D} \right] F_y + \frac{\mu \nu c \alpha_4 + \mu \nu c \alpha_1 + h_y f_y c \alpha_4 c \alpha_4}{\mu D(\alpha_4 - c \alpha_1)} F_z
\end{align*}
\]
Eqs. (17) and (18) show linear relations. For Eq. (17), when \( \frac{F_z}{F_y} = -\frac{\alpha_1}{\mu} \), we can write
\[
M_y = \frac{\mu}{\alpha_1 - \alpha_4} \left[ \frac{x_4 - x_1}{D} \right] F_y,
\]
when \( \frac{F_y}{F_z} = -\frac{\alpha_4}{\mu} \), we can write
\[
M_x = \frac{\mu}{\alpha_4 - \alpha_1} \left[ \frac{y_4 - y_1}{D} \right] F_y.
\]
Three-point contact states are transitory and considered trivial. Their analysis is ignored.

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The compact forms of the wrenches of three-point contact can be described as follows.

\[
\begin{align*}
F_1 &= \frac{m}{\mu} - 2 \frac{m}{\mu} \\
F_2 &= \frac{m}{\mu} - 2 \frac{m}{\mu} \\
F_3 &= \frac{m}{\mu} - 2 \frac{m}{\mu}
\end{align*}
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F_3 &= \frac{m}{\mu} - 2 \frac{m}{\mu}
\end{align*}
\]

where \( \alpha \in [90, 270] \) and \( \alpha \in [90, 270] \).
When pegs tilt to the left, k stands for l and \( j = 1 \). When pegs tilt to the right, k stands for r and \( j = 0 \).

Table 2. Force/moment of contact states about the three dimensional dual peg-in-hole.

5. Jamming diagrams of three dimensional dual peg-in-hole

Jamming is a condition in which the peg will not move because the forces and moments applied to the peg through the support are in the wrong proportions (Whitney, 1982).

5.1 Jamming diagrams

Because the relation between forces and moments is not proper, pegs are stuck in the holes and jamming appears. In this section, according the relations between \( \frac{M_y}{F_z} \) and \( \frac{M_x}{F_z} \), \( \frac{M_y}{F_z} \) and \( \frac{F_y}{F_z} \), \( \frac{M_x}{F_z} \) and \( \frac{F_y}{F_z} \), we can obtain three kinds of jamming diagrams about three dimensional dual peg-in-hole problems (Fig.7):

(a) \( \frac{M_y}{DF_z} \propto \frac{F_x}{F_z} \); (b) \( \frac{M_x}{DF_z} \propto \frac{F_y}{F_z} \), \( \alpha_1 \in [0^\circ \ 90^\circ] \), \( \alpha_3 \in [0^\circ \ 90^\circ] \), \( \alpha_2 \in [180^\circ \ 270^\circ] \), \( \alpha_4 \in [180^\circ \ 270^\circ] \); (c) \( \frac{M_y}{DF_z} \propto \frac{F_x}{F_z} \), \( \alpha_1 \in [270^\circ \ 360^\circ] \), \( \alpha_3 \in [270^\circ \ 360^\circ] \), \( \alpha_2 \in [90^\circ \ 180^\circ] \), \( \alpha_4 \in [90^\circ \ 180^\circ] \). Here \( \frac{M_y}{F_z} \) represents the linear relation between \( \frac{M_y}{DF_z} \) and \( \frac{F_x}{F_z} \), and \( \frac{M_x}{DF_z} \propto \frac{F_y}{F_z} \) represents the linear relation between \( \frac{M_x}{DF_z} \) and \( \frac{F_y}{F_z} \). There are some differences between the jamming diagram of a two dimensional dual peg-in-hole and that of a three dimensional dual peg-in-hole. In two dimensions, the jamming diagram only shows the linear relation between \( \frac{M_x}{F_z} \) and \( \frac{F_y}{F_z} \). The jamming diagram is not
relative to $\alpha_i$ and contact point positions $(x_i, y_i)$. It is only relative to $\mu$, the insertion depth and the radii of peg and hole. In three dimensions, the linear relations between $\frac{M_y}{F_z}$ and $\frac{F_x}{F_z}$, $\frac{M_x}{F_z}$, and $\frac{F_y}{F_z}$ must be considered. The jamming diagrams are not only relative to $\mu$, the insertion depth and the radii of peg and hole, but also relative to $\alpha_i$ and contact point positions $(x_i, y_i)$. Because the relation between $\frac{M_x}{F_z}$ and $\frac{F_y}{F_z}$ is relative to $\sin \alpha_i$, according to the different range of $\alpha_i$, the jamming diagrams can be discussed in Fig. 7(b) and Fig. 7(c).

In this section, the jamming diagrams of a three dimensional dual-peg-in-hole are analyzed. The range of the jamming analysis is extended. Thus, the assembly process is analyzed in detail. We can know the whole assembly process of the three dimensional dual-peg-in-hole clearly.

Table 2 lists the force/moment conditions for 20 possible contact states. Fig. 7 shows the jamming diagrams of a three dimensional dual-peg problem with these contact states. When the force/moment conditions of all possible contact states of the three dimensional dual-peg insertion problem are plotted in force/moment space, the plots form a closed boundary which separates the force/moment space into two spaces; statically unstable space, and jamming space. The unstable space is the area enclosed by the boundary, and represents the conditions on the applied forces and moments that cause the pegs to unstably fall into the holes. The jamming space, on the other hand, is the area outside the boundary, and represents the force/moment conditions in which the pegs appear stuck in the holes because the applied forces and moments cannot overcome the net forces and moments due to frictional contact forces (Fig. 7). The jamming diagrams can supply us some successful insertion information. If the conditions on the applied forces and moments fall into the unstable space, the pegs can unstably fall into the holes and the continuity of insertion can be guaranteed. Since all 20 contact states cannot exist at a particular geometric condition, each jamming diagram representing a particular insertion problem is constructed from only some of the straight lines shown in Fig. 7. When the geometric dimensions and the insertion depth are known, some possible contact states can be formed and some possible jamming diagrams can be obtained (Fig. 8).

5.2 33 kinds of possible jamming diagrams

When the exact dimensions of the pegs and the insertion depth are known, possible contact states can be derived, and the jamming diagrams can be constructed. These jamming diagrams provide us with the information on how to apply forces and moments to the pegs in order to yield successful insertions. For three kinds of jamming diagrams in Fig. 7, each has 11 kinds of possible jamming diagrams. For example, for some particular insertion condition and geometric condition, the possible jamming diagrams can be constructed by joining four corresponding lines in Fig. 7 (a) forming an enclosed region (Fig. 8 a). There are several possible jamming diagrams that can be constructed in this way. In Fig. 8 a, vertexes of the quadrangle represent one-point contact states, such as $(l-1)$, $(l-2)$, $(r-1)$, $(r-2)$ in Fig. 8 (a-1). Lines of the quadrangle represent two-point contact states, such as $(l-5)$, $(r-5)$ in Fig. 8 (a-1).
Fig. 7. Jamming diagrams.

(a) \[ \frac{M_x}{DF_z} \propto \frac{F_x}{F_z} \]

(b) \[ \frac{M_x}{DF_z} \propto \frac{F_x}{F_z}, \quad \alpha_1 \in [0° \ 90°], \quad \alpha_3 \in [0° \ 90°], \quad \alpha_2 \in [180° \ 270°], \quad \alpha_4 \in [180° \ 270°] \]

(c) \[ \frac{M_x}{DF_z} \propto \frac{F_x}{F_z}, \quad \alpha_1 \in [270° \ 360°], \quad \alpha_3 \in [270° \ 360°], \quad \alpha_2 \in [90° \ 180°], \quad \alpha_4 \in [90° \ 180°] \]

Fig. 7. Jamming diagrams.
Possible jamming diagrams ($\frac{M_y}{DF_z}$).

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(b-1)

(b-2)

(b-3)

(b-4)

(b-5)

(b-6)
(b) Possible jamming diagrams: \[ \frac{M_x}{DF_z} \propto \frac{F_y}{F_z}, \quad \alpha_1 \in [0^\circ, 90^\circ], \quad \alpha_3 \in [0^\circ, 90^\circ], \quad \alpha_2 \in [180^\circ, 270^\circ], \quad \alpha_4 \in [180^\circ, 270^\circ] \]
(c) Possible jamming diagrams: 
\[
\frac{M_y}{DF_z} \propto \frac{F_x}{F_z}, \quad \alpha_1 \in [270^\circ, 360^\circ], \quad \alpha_3 \in [270^\circ, 360^\circ], \quad \alpha_2 \in [90^\circ, 180^\circ], \quad \alpha_4 \in [90^\circ, 180^\circ]
\]

Fig. 8. Possible jamming diagrams.
In two dimensions, there are only 9 kinds of jamming diagrams, which are static and relative to $\mu$, the insertion depth and the radii of peg and hole. In three dimensions, there are 33 kinds of jamming diagrams, which are relative to not only $\mu$, the insertion depth and the radii of peg and hole, but also $\alpha$ and contact point positions $(x_i, y_i)$. They are dynamic. The quadrangles are more complicated than those in two dimensions. In three dimensions, the possible jamming diagrams, such as Fig. 8(a-10) and Fig. 8(a-11), may appear. Fig. 8(a-10) shows the possible jamming diagram of left-contact. Fig. 8(a-11) shows the possible jamming diagram of right-contact.

The 33 kinds of possible jamming diagrams in Fig. 8 may be interpreted as follows. The vertical lines in the diagrams describe line contact states. Combinations of $F_x$, $F_z$, and $M_y$ falling on the parallelograms’ edges describe equilibrium sliding in. Outside the parallelogram lie combinations which jam the dual-peg, either in one- or two-point contact. Inside, the dual-peg is in disequilibrium sliding or falling in.

6. Conclusions

In order to know the assembly process of a dual peg-in-hole and plan an insertion strategy in detail, it is important to analyze the dual peg-in-hole in three dimensions. In this paper, the assembly contact and jamming of a dual peg-in-hole in three dimensions are analyzed in detail. Firstly, 20 contact states of the three dimensional dual-peg are enumerated and geometric conditions are derived. Secondly, the contact forces are described by the screw theory in three dimensions. With the static equilibrium equations, the relationship between forces and moments for maintaining each contact state is derived. Thirdly, the jamming diagrams of the three dimensional dual peg-in-hole are presented. Different peg and hole geometry, and different insertion depth may yield different jamming diagrams. 33 different types of possible jamming diagrams for each jamming diagram are identified. These jamming diagrams can be used to plan fine motion strategies for successful insertions. In addition, they also show that the assembly process in six degrees of freedom is more complicated than that for planar motion. The above analyses are the theoretical bases for multiple peg-in-hole insertion. It is one most significant advance in robotic assembly.

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The authors and editor of this book wish to inspire people, especially young ones, to get involved with robotic and mechatronic engineering technology and to develop new and exciting practical applications, perhaps using the ideas and concepts presented herein.

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InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
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Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821
