Trimaximal mixing and extended magic symmetry in a model of neutrino mass matrix

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Abstract – The trimaximal mixing scheme (TM\textsubscript{2}) results in “magic” neutrino mass matrix \((M_\nu)\) which is known to accommodate neutrino oscillation data. In this paper, we propose a phenomenological ansatz for \(M_\nu\) by extending the magic symmetry that leads to further reduction in the number of free parameters, thereby increasing the predictability of the model. The neutrino mixing parameters, effective Majorana mass \(m_{ee}\) and \(CP\) invariants \((J_{CP}, I_1, I_2)\) are found to exhibit strong correlations for TM\textsubscript{2} mixing paradigm. One of the generic feature of the model is the requirement of non-maximal \(\theta_{23}\) for possible \(CP\) violation measurable in neutrino oscillation experiments. The observables \(m_{ee}\) and sum of neutrino masses \((\sum m_i)\) have imperative implications for yet unknown neutrino mass hierarchy. For inverted hierarchy, the lower bound on \(m_{ee} > 0.02\,\text{eV}\), predicted by the model, is found to be within the sensitivity reach of the \(0\nu\beta\beta\) decay experiments. Also, cosmological bound of 0.12\,eV on \(\sum m_i\), at 95\% CL, refutes inverted hierarchy implying TM\textsubscript{2} with normal hierarchy as the only viable possibility in the model. We have, also, illustrated a scenario wherein such a construction of the neutrino mass matrix can be realized using \(\Delta(54)\) symmetry in the framework of Type-I+II seesaw mechanism.

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Introduction. – Within the three-neutrino oscillation paradigm, the neutrino mixing matrix can be parameterized in terms of three mixing angles \((\theta_{12}, \theta_{23}, \theta_{13})\) and one \(CP\) phase \(\delta\). The mixing angles have been measured by neutrino oscillation experiments with impressive precision. The long baseline experiments Deep Underground Neutrino Experiment (DUNE) and Tokai to Hyper-Kamiokande (T2HK) aim at measuring the yet unknown \(CP\) phase \(\delta\). Further, the mixing matrix is rendered more non-deterministic by the existence of two additional \(CP\) phases due to the Majorana nature of neutrino. In light of the incomplete information about the mixing parameters, the phenomenological approaches play a pivotal role in elucidating the nature of neutrino mass matrix and underlying symmetry. For review of various approaches see ref. [1] and references therein. The tri-bimaximal (TBM) mixing [2–5] is one such scenario extensively studied in the literature. TBM predicts maximal atmospheric mixing angle \(\theta_{23}\) and vanishing reactor angle \(\theta_{13}\). In light of observation of non-zero \(\theta_{13}\) [6–10] several extensions of TBM ansatz have been proposed to accommodate observed pattern of neutrino mixing [11–33]. One such possibility is the “trimaximal mixing (TM)” pattern of neutrino mixing matrix [34–53] wherein if the second (first) eigenvector remains the same while the first (second) and third (third) columns deviate from their TBM values it is called TM\textsubscript{2} (TM\textsubscript{1}) mixing, \textit{viz.},

\[
U_{TM_2} = \begin{pmatrix}
\sqrt{\frac{2}{3}} & \frac{\cos \theta}{\sqrt{3}} & \frac{\sin \theta}{\sqrt{3}} \\
\frac{1}{\sqrt{6}} & \frac{\cos \theta - e^{i\phi} \sin \theta}{\sqrt{2}} & \frac{\sin \theta}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & \frac{\cos \theta + e^{i\phi} \sin \theta}{\sqrt{2}} & \frac{\sin \theta}{\sqrt{2}}
\end{pmatrix}, \quad (1)
\]
with row/column sum equal to “magic sum”. In general, the magic neutrino mass matrix resulting from eq. (3) is given by [54–56]

\[ M_\nu = U^*_{\text{T}M_2} M^\dagger_\nu U^T_{\text{T}M_2}, \]

where \( \theta \) and \( \phi \) are two free parameters. It is well known that the neutrino mass matrix obtained using the transformation

\[ M_\nu = \begin{pmatrix} 1 & \sqrt{\frac{2}{3}} \cos \theta & \sqrt{\frac{2}{3}} \sin \theta \\ -\cos \theta + e^{-i\phi} \sin \theta & 1 & -\sin \theta + e^{-i\phi} \cos \theta \\ \cos \theta + e^{i\phi} \sin \theta & 1 & -\sin \theta + e^{i\phi} \cos \theta \end{pmatrix}, \]

where \( a, b, c \) and \( d \) are, in general, complex parameters. It is evident from eq. (4) that row/column sum is equal to \( a + b + c \) (“magic sum”). In this work, given eq. (4), we explore a unique possibility for extension of the magic symmetry which decreases the number of free parameters and, thus, increases the predictability of the model while keeping the magical nature of the neutrino mass matrix intact. We write eq. (4) as

\[ M_\nu = \begin{pmatrix} a & b & c \\ b & a + b + c & -b \\ c & b & a + c - d \end{pmatrix}, \]

assuming

\[ d = a + b + c, \]

with row/column sum equal to “magic sum”, as before. In the fourth section, we shall discuss the dynamical origin of this scenario based on \( \Delta(54) \) symmetry.

**Formalism.** We consider eq. (6) as additional constraint under the ambit of TM\(_2\) mixing to ameliorate the allowed parameter space of the model. Using eqs. (2) and (3), eq. (6) can be written as

\[ -4e^{2i\alpha}m_2 + \sin^2 \theta (e^{2i\beta}m_3 + 3m_1e^{2i\phi}) \\
-3\sqrt{3}\sin^2 \theta (m_1 - e^{2i\beta}m_3) \\
+ \cos^2 \theta (m_1 + 3m_3e^{2i(\beta+\phi)}) = 0. \]

Equation (7) yields two real equations, viz.,

\[ -\frac{2}{3}m_2 \cos 2\alpha + \frac{1}{6}m_3 \left( 3 \cos^2 \theta \cos 2(\beta + \phi) \\
+ 3\sqrt{3}\sin^2 \theta \cos 2(\beta + \phi) + \cos 2\beta \sin^2 \theta \right) \\
+ \frac{1}{6}m_1 \left( 3 \sin^2 \theta \cos 2\phi - 3\sqrt{3}\sin 2\theta \cos \phi + \cos^2 \theta \right) = 0, \]

and

\[ -\frac{2}{3}m_2 \sin 2\alpha + \frac{1}{6}m_3 \left( 3\sqrt{3}\sin 2\theta \sin(2\beta + \phi) \\
+ 3\cos^2 \theta \sin 2(\beta + \phi) + \sin 2\beta \sin^2 \theta \right) \\
+ \frac{1}{6}m_1 \left( 3 \sin^2 \theta \sin 2\phi - 3\sqrt{3}\sin 2\theta \sin \phi \right) = 0, \]

which are, further, solved to obtain two mass ratios, \( R_{21} \equiv \frac{m_2}{m_1} \) and \( R_{31} \equiv \frac{m_3}{m_1} \) given by

\[ \text{see eq. (10) above} \]

and

\[ \text{see eq. (11) above} \]

respectively, with

\[ A = 16 \left( 3\sqrt{3}\sin 2\theta \sin(2\alpha - 2\beta - \phi) \\
+ 3\cos^2 \theta \sin 2(\alpha - \beta - \phi) + \sin^2 \theta \sin 2(\alpha - \beta) \right), \]

\[ m_2 = \sqrt{m_1^2 + \Delta m^2_{21}}, \quad m_3 = \sqrt{m_1^2 + \Delta m^2_{31}}. \]

The two mass-squared differences \( \Delta m^2_{21} = m^2_2 - m^2_1 \) and \( \Delta m^2_{31} = m^2_3 - m^2_1 \) along with mass ratios \( \frac{m_1}{m_2} = R_{21}, \)
\( \frac{m_1}{m_3} \equiv R_{31} \) yield two values of neutrino mass \( m_1 \) given by

\[
m_1^a = \sqrt{\frac{\Delta m_{21}^2}{(R_{21})^2 - 1}}, \quad m_1^b = \sqrt{\frac{\Delta m_{31}^2}{(R_{31})^2 - 1}},
\]

respectively. The mass ratios in eqs. (10) and (11) are functions of four parameters, viz., \( \theta, \phi, \alpha \) and \( \beta \). The consistency of the formalism requires that two values of \( m_1 \), viz. \( m_1^a \), \( m_1^b \) must be equal which can, further, be translated to the condition

\[
R_{21} \equiv \frac{\Delta m_{21}^2}{\Delta m_{31}^2} = \frac{R_{31}^2 - 1}{|R_{31}^4 - 1|}. \tag{12}
\]

Also, in terms of elements of the TM2 mixing matrix, the neutrino mixing angles can be written as

\[
\begin{align*}
\sin^2 \theta_{12} &= \frac{|\langle U_{TM2} \rangle_{12}|^2}{1 - |\langle U_{TM2} \rangle_{12}|^2} = \frac{1}{\cos 2\theta + 2}, \\
\sin^2 \theta_{13} &= |\langle U_{TM2} \rangle_{13}|^2 = \frac{2\sin^2 \theta}{3}, \\
\sin^2 \theta_{23} &= \frac{|\langle U_{TM2} \rangle_{23}|^2}{1 - |\langle U_{TM2} \rangle_{23}|^2} = \frac{1}{2} \left( \frac{\sqrt{3}\sin 2\theta \cos \phi}{\cos 2\theta + 2} + 1 \right).
\end{align*}
\tag{13}
\]

The Jarlskog \( CP \) invariant \([57–59]\), and other two invariants corresponding to Majorana phases \( \alpha \) and \( \beta \) are given by

\[
J_{CP} = \Im[\langle U_{TM2} \rangle_{12}^\dagger \langle U_{TM2} \rangle_{12}^\dagger \langle U_{TM2} \rangle_{21} \langle U_{TM2} \rangle_{22}^\dagger] = \frac{1}{6\sqrt{3}} \sin 2\theta \sin \phi,
\]

\[
I_1 = \Im[\langle U_{TM2} \rangle_{11}^\dagger \langle U_{TM2} \rangle_{12} e^{2\iota \alpha}] = \frac{\sqrt{2}}{3} \cos \theta \sin 2\alpha,
\]

\[
I_2 = \Im[\langle U_{TM2} \rangle_{11}^\dagger \langle U_{TM2} \rangle_{13} e^{2\iota \beta}] = \frac{1}{3} \sin 2\theta \sin 2\beta.
\]

The effective Majorana mass \( m_{ee} = |\langle M_{ee} \rangle_{11}| = |\sum_{i=1}^3 \langle U_{TM2} \rangle_{1i}^2 | m_1 | \) is an important physical observable in 0\( v/\beta/\beta \) decay experiments establishing the Majorana nature of neutrinos. In this model, \( m_{ee} \) is given by

\[
m_{ee} = \frac{1}{3} \left| 2m_1 \cos^2 \theta + m_2 e^{2\iota \alpha} + 2m_3 \sin^2 \theta e^{2\iota \beta} \right|.
\]

The current and forthcoming 0\( v/\beta/\beta \) decay experiments such as SuperNEMO [60], KamLAND-Zen [61], NEXT [62,63] and nEXO [64] have impressive sensitivity \( O(10^{-2}) \) eV to probe this, yet elusive, decay process. Furthermore, the cosmological bound on sum of neutrino masses \( \sum_{i=1}^3 m_i \) is another physical observable which can have imperative implication for the viability of the model. With an increasing statistics, the Planck data P11[T, TE, EE+lowE+tensing] combined with BAO put a very strong upper bound of 0.12 eV at 95% CL [65]. However, we consider a relatively stable and conservative bound of 1 eV in the numerical analysis.

**Table 1:** The neutrino oscillation data used in the numerical analysis [69].

| Parameter | Best fit \( \pm 1\sigma \) range | \( 3\sigma \) range |
|-----------|----------------------------------|------------------|
| \( \sin^2 \theta_{12}/10^{-1} \) | 3.18 \( \pm 0.16 \) | 2.71–3.69 |
| \( \theta_{12}/^\circ \) | 34.3 \( \pm 1.0 \) | 31.4–37.4 |
| \( \sin^2 \theta_{13}/10^{-2} \) | 2.200 \( ^{+0.069}_{-0.062} \) | 2.000–2.405 |
| \( \theta_{13}/^\circ \) | 8.53 \( ^{+0.13}_{-0.12} \) | 8.13–8.92 |
| \( \sin^2 \theta_{23}/10^{-1} \) | 5.74 \( \pm 0.14 \) | 4.34–6.10 |
| \( \theta_{23}/^\circ \) | 49.26 \( \pm 0.79 \) | 41.20–51.33 |
| \( \Delta m_{21}^2/10^{-5} \text{ eV}^2 \) | 7.50 \( ^{+0.22}_{-0.20} \) | 6.94–8.14 |
| \( |\Delta m_{31}^2|/10^{-3} \text{ eV}^2 \) | 2.55 \( ^{+0.02}_{-0.03} \) | 2.47–2.63 |

**Inverted mass hierarchy (IH) \( m_3 < m_1 < m_2 \):**

| Parameter | Best fit \( \pm 1\sigma \) range | \( 3\sigma \) range |
|-----------|----------------------------------|------------------|
| \( \sin^2 \theta_{12}/10^{-1} \) | 3.18 \( \pm 0.16 \) | 2.71–3.69 |
| \( \theta_{12}/^\circ \) | 34.3 \( \pm 1.0 \) | 31.4–37.4 |
| \( \sin^2 \theta_{13}/10^{-2} \) | 2.255 \( ^{+0.064}_{-0.070} \) | 2.018–2.424 |
| \( \theta_{13}/^\circ \) | 8.58 \( ^{+0.12}_{-0.11} \) | 8.17–8.96 |
| \( \sin^2 \theta_{23}/10^{-1} \) | 5.78 \( ^{+0.10}_{-0.07} \) | 4.33–6.08 |
| \( \theta_{23}/^\circ \) | 49.46 \( ^{+0.22}_{-0.19} \) | 41.16–51.25 |
| \( \Delta m_{21}^2/10^{-5} \text{ eV}^2 \) | 7.50 \( ^{+0.22}_{-0.20} \) | 6.94–8.14 |
| \( |\Delta m_{31}^2|/10^{-3} \text{ eV}^2 \) | 2.45 \( ^{+0.02}_{-0.03} \) | 2.37–2.53 |

**Numerical analysis and discussion.** – In the numerical analysis, we have used neutrino oscillation data given in table 1 and \( 3\sigma \) experimental range of \( R_{\nu} \) \( (0.025 \leq R_{\nu} \leq 0.036) \) to constrain the parameter space predicted by the model. In order to obtain mass ratios (eqs. (10) and (11)), the values of free parameters \( \theta, \phi, \alpha \) and \( \beta \) have been generated randomly with uniform distribution within their physical ranges. Out of \( 10^9 \) random samples of the possible solutions only those parameters sets are allowed for which \( R_{\nu} \) (eq. (12)) lies within its \( 3\sigma \) experimental range.

The predictions are depicted as correlation plots amongst different observables in figs. 1 and 2, at \( 3\sigma \). In fig. 1 we have shown predictions for TM2 mixing with normal hierarchy (NH) of neutrino masses. One of the generic features of TM2 mixing is that there exists a slight tension in \( \theta_{12} \) prediction at \( 1\sigma \) which is, also, exhibited in fig. 1(a) (experimental range of \( \theta_{12} \), at \( 1\sigma \), is \( 33.3^\circ < \theta_{12} < 35.3^\circ \)). \( \theta_{23} \), in general, can be in the upper \( \theta_{23} > 45^\circ \) or lower \( \theta_{23} < 45^\circ \) octant including its maximal value \( \theta_{23} = 45^\circ \). Furthermore, it is evident from fig. 1(b) that \( CP \) violating phase \( \delta \) lies in the first and fourth quadrants. The \( CP \) conserving solutions (with \( \delta = 0^\circ \) or \( 360^\circ \)) require a maximal value of \( \theta_{23} \). Thus, in order to have \( CP \) violation measurable in neutrino oscillation experiments \( \theta_{23} \) must deviate from allowed maximal value. It is evident from fig. 1(c) that there exists an allowed region of effective Majorana mass parameter \( m_{ee} \) which is beyond the sensitivity reach of 0\( v/\beta/\beta \) decay experiments. The predicted lower bound on \( m_{ee} \) is 0.004 eV, at \( 3\sigma \).
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Fig. 1: The predictions for $T_{2}$ mixing with normal hierarchy of neutrino masses. The horizontal lines in (c) are sensitivities of the respective $\nu_{\beta\beta}$ decay experiments.

The model is, in general, consistent with both $CP$ conserving and $CP$ violating solutions (figs. 1(d)–(f)). The allowed ranges of the parameters, at $3\sigma$, are given in table 2.

In fig. 2, we depict the correlation plots of the parameters assuming inverted hierarchical (IH) spectrum of neutrino masses. As in fig. 1(a), fig. 2(a), also, shows a positive linear correlation between $\theta_{13}$ and $\theta_{12}$ with mild tension in predicted values of $\theta_{12}$ at $1\sigma$. For $\theta_{23}$ below maximality ($\theta_{23} < 45^\circ$), the Dirac $CP$ phase $\delta$ can be in the first ($0^\circ < \delta < 90^\circ$) and fourth ($270^\circ < \delta < 360^\circ$) quadrants, however, above maximality ($\theta_{23} > 45^\circ$), $\delta$ remains unconstrained, i.e., allowed in the whole physical range ($0^\circ < \delta < 360^\circ$). Also, in order to have $CP$ violation in the leptonic sector $\theta_{23}$ must not have maximal value, i.e., $\theta_{23} \neq 45^\circ$. The effective Majorana mass $m_{ee}$ is bounded from below $m_{ee} > 0.02$ eV which can be probed in $\nu_{\beta\beta}$ decay experiments. The non-observation of $\nu_{\beta\beta}$ decay or invoking the cosmological bound of 0.12 eV on $\sum m_{i}$, at 95% CL, shall refutes inverted hierarchy implying $T_{2}$ mixing with normal hierarchy as the only viable possibility in the model. The $CP$ invariants $J_{CP}$, $I_{1}$ and $I_{2}$ are found to be in the ranges ($-0.035$–$0.035$), ($-0.460$–$0.460$) and ($-0.130$–$0.130$) at $3\sigma$, respectively (fig. 2(d)–(f)). In the following, we have constructed a flavor model which realizes the extended magic symmetry ansatz considered in this work.

Symmetry realization of extended magic symmetry based on $\Delta(54)$ group. – In the effective field
Table 2: The numerical predictions of the model at 3σ.

| Parameter | TM$_2$ mixing with NH | TM$_2$ mixing with IH |
|-----------|-----------------------|----------------------|
| $m_{ee}$ (eV) | >0.004 | >0.02 |
| $J_{CP}$ | (0.024-0.035) $\oplus$ (-0.035-0.018) | (-0.035-0.035) |
| $I_1$ | (-0.470-0.470) | (-0.460-0.460) |
| $I_2$ | (-0.130-0.130) | (-0.130-0.130) |
| $\delta$ (°) | (0-90) $\oplus$ (270-360) | for $\theta_{23} < 45°$ |
| $\sum m_i$ (eV) | (0.08-0.35) | (0.15-0.80), in light of cosmological bound $\sum m_i < 0.12$ eV, this scenario is disallowed |

The degeneracy leading to non-degenerate charged lepton masses. The charged lepton mass matrix is given by

$$M_L = \frac{v_H v_2}{\sqrt{2}} \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{pmatrix}$$

$$+ \frac{\beta v_H}{\sqrt{2}} \begin{pmatrix} \omega^{v_{11}} - v_{12} & 0 & 0 \\ 0 & \omega^{2v_{11}} - \omega^2 v_{12} & 0 \\ v_{11} - \omega v_{12} & 0 & 0 \end{pmatrix}.$$  \hspace{1cm} (17)

where $v_H/\sqrt{2}$ is vev of the neutral component of Higgs field ($H$).

**Type-I seesaw:** At dimension 5, using the flavon field ($\Phi_2$) and one right-handed neutrino ($N_1$), Type-I seesaw mechanism is implemented via Dirac mass matrix ($M_D$) and right-handed neutrino mass matrix ($M_R$). The relevant Lagrangian is given by

$$\mathcal{L}_I = y_D (D_L \bar{H} N_1) \Phi_2 + M_N^2 N_1 + \text{h.c.},$$  \hspace{1cm} (18)

where $\bar{H} = i \tau_2 H$, $\tau_2$ is the Pauli matrix and $M$ is the bare mass term for $N_1$. The mass matrices $M_D$ and $M_R$ are given by

$$M_D = \begin{pmatrix} y_D \Phi_2(1) \\ y_D \Phi_2(2) \\ y_D \Phi_2(3) \end{pmatrix} , M_R = M.$$  \hspace{1cm} (19)

With vacuum alignment $v_{\phi_2} (0, -1, 1)$ of the flavon field $\Phi_2$, Type-I seesaw contribution to $M_{\nu}$ is

$$M_{\nu}^I = -M_D M_R^{-1} M_D^T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & b & -b \\ 0 & -b & b \end{pmatrix}.$$  \hspace{1cm} (20)

where $b = y_D^2 v_{\phi_2}/M$.

**Type-II seesaw:** Furthermore, the addition of scalar triplet field (‡) leads to Type-II seesaw contribution to $M_{\nu}$. The Yukawa Lagrangian responsible for this contribution is given by

$$\mathcal{L}_{II} = y_{\Delta 1} (D_L i \tau_2 \Delta D_L) \Phi_1 + y_{\Delta 2} (D_L i \tau_2 \Delta D_L) \rho_1 \Phi_1 + y_{\Delta 3} (D_L i \tau_2 \Delta D_L) \rho_3 + \text{h.c.},$$

$$+ y_{\Delta 1} (D_L \epsilon \Delta D_L + D_{\mu L} \rho \Delta D_{\mu L}) v_{\Delta} v_{\phi_1} + y_{\Delta 2} (D_L \epsilon \Delta D_L + D_{\mu L} \rho \Delta D_{\mu L}) v_{\Delta} v_{\phi_1} + y_{\Delta 3} (D_L \epsilon \Delta D_L + D_{\mu L} \rho \Delta D_{\mu L}) v_{\Delta} v_{\phi_1} + \text{h.c.},$$  \hspace{1cm} (21)

with the flavon alignments $v_{\phi_1} (0, 1, 0)$ and $v_{\phi_3} (0, 0, 1)$ for the fields $\Phi_1$ and $\Phi_3$, respectively; $v_\Delta$ is the vev of scalar triplet field, $\Delta$. The Type-II seesaw contribution to neutrino mass matrix is given by

$$M_{\nu}^{II} = \begin{pmatrix} a & d & c \\ d & a + c & 0 \\ c & 0 & a + d \end{pmatrix}.$$  \hspace{1cm} (22)

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Table 3: The field content of the model and respective charge assignments under $SU(2)_L$, $\Delta(54)$, $Z_4$ and $Z_2$ symmetries, where $\eta = e^{i \frac{\pi}{2}}$.

| Symmetry | $D_{1L}$ | $l_R$ | $H$ | $\Delta$ | $N_1$ | $\rho_1$ | $\rho_2$ | $\chi$ | $\Phi_1$ | $\Phi_2$ | $\Phi_3$ |
|----------|----------|-------|-----|---------|-------|---------|---------|-----|---------|---------|---------|
| $SU(2)_L$ | 2 | 1 | 2 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\Delta(54)$ | $3_{2(1)}$ | $3_{1(2)}$ | 1 | 1 | 1 | 2 | 1 | 2 | 2 | 1 | 3 | 1 | $3_{2(2)}$ | $3_{2(1)}$ |
| $Z_4$ | 1 | $\eta^2$ | 1 | $\eta^1$ | 1 | $\eta^1$ | $\eta^2$ | $\eta$ | 1 | $\eta$ |
| $Z_2$ | 1 | $\eta^2$ | 1 | $\eta^1$ | 1 | $\eta^1$ | $\eta^2$ | $\eta$ | 1 | $\eta$ |

where $a = y_{\Delta_1} v_{\phi_1} v_{\Delta}$, $c = y_{\Delta_3} v_{\phi_1} v_{\rho_1}$ and $d = y_{\Delta_3} v_{\phi_1} v_{\Delta}$. Within Type-I+II seesaw setting, with $d = b$, the total effective Majorana neutrino mass matrix is given by

$$M_\nu = M_\nu^{I} + M_\nu^{II},$$

$$= \begin{pmatrix} a & b & c \\ b & a + b + c & -b \\ c & -b & 2b + a \end{pmatrix},$$

which is the magic neutrino mass matrix with (2,2) element equal to “magic sum”, i.e., $a + b + c$ ($\text{eq. (5)}$).

This is a representative realization of the phenomenological ansatz considered in this work. For completeness of the model, the flavon alignments of fields $\Phi_i (i = 1, 2, 3)$ are required to be tested under the minimization of scalar potential [1,67,68].

Conclusions. – In conclusion, we have considered an extension of the magic symmetry ansatz within the paradigm of TM$_2$ mixing scheme wherein the (2,2) element of $M_\nu$ is, also, equal to the “magic sum”. This phenomenological ansatz exhibits strong correlations and homoscedasticity amongst the physical observables signifying the high predictability of the model. Some interesting phenomenological consequences of the model are:

- In order to have $CP$ violation measurable in the neutrino oscillation experiments, the atmospheric mixing angle $\theta_{23}$ is predicted to be non-maximal, i.e., for $\theta_{23} \neq 45^\circ$.

- The observables $m_{ee}$ and $\sum m_i$ have imperative implication for the model. There exists a lower bound on $m_{ee} > 0.02$ eV for TM$_2$ (with IH) which can be probed in $0\nu\beta\beta$ decay experiments. The non-observation of $0\nu\beta\beta$ decay shall refute IH for TM$_2$ mixing.

- We have considered a relatively stable and conservative bound, on $\sum m_i$, of 1 eV in the numerical analysis but the current bound from Planck data combined with WMAP and BAO of 0.12 eV at 95%CL refutes IH in the model (fig. 2(c)) implying TM$_2$ with normal hierarchy as the only viable possibility in the model. The numerical predictions of the model, at 3$\sigma$, are given in table 2.

- The model parameter space is, further, constrained if we take into consideration the recent global reassessment of the neutrino oscillation parameters [69] which have reported Dirac type $CP$ phase $\delta$ in the ranges (128$^\circ$–359$^\circ$) and (200$^\circ$–353$^\circ$), at 3$\sigma$, for normal and inverted hierarchical neutrino masses, respectively. For example, the region of the parameter space for which $\delta$ lies in the first quadrant is disallowed (see figs. 1(b) and 2(b)).

We have, also, proposed a scenario, based on $\Delta(54)$ symmetry within Type-I+II seesaw setting to realize the extended magic symmetry ansatz, considered in this work.

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