The electroweak phase structure and baryon number violation for large Higgs mass.

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The classical transitions between topologically distinct vacua in a SU(2)–Higgs model, using a Higgs field of mass approximately 120 GeV, is examined to probe the crossover region between the symmetric and broken phase. For the volumes used, this crossover is approximately 10 GeV wide.

1 Introduction

It is clear now from a number of numerical simulations in 3 and 4 dimensions that the first order phase transition from the broken to symmetric phase of the electroweak sector of the Standard Model ends for a Higgs mass of approximately 80 GeV. For larger Higgs masses, there is analytic crossover or very possibly a phase transition of third or lower order (whether numerical studies could ever distinguish between these two scenarios remains to be seen). While the present quoted lower bound of 65 GeV for the Higgs mass and an estimate for the Higgs mass from electroweak precision data is $m_H = 145^{+164}_{-77}$ GeV is still consistent with an explanation of baryogenesis via this phase transition, the amount of parameter space is rapidly shrinking.

It seems logical to ask what is occurring to the real time process of baryon number violation for larger Higgs masses. Does the crossover become increasingly broad as the Higgs mass is increased or does it remain reasonably sharp (changing from the symmetric to broken phase over a few GeV)? An interesting scenario, discussed elsewhere in these proceedings, suggests that for a different cosmology at electroweak temperatures one can still generate sufficient levels of baryon number violation, even with crossover. At the very least, understanding baryon number violation for this model will be important if we wish to employ a similar mechanism for more complicated field theories such as SUSY or Grand Unified Theories.

2 Real time transitions

Many of the details of this calculation have been covered extensively in these proceedings and so I will not go into detail here. Suffice it to say that we wish to examine the real time process of the rate of change of the Chern-Simons number, which is related to baryon number violation. While a full quantum
field theory calculation is almost impossible numerically, a classical treatment of the problem may give a reasonable estimate for the rate.

2.1 Classical analysis

The following lattice $SU(2)$-Higgs Hamiltonian was constructed

$$H = \beta \left[ -\sum_{\square} \left( 1 - \frac{1}{2} Tr U_{\square} \right) - \frac{1}{2} \sum_{x,i} (\Delta_i \Phi)^\dagger (\Delta_i \Phi) \right]$$

$$-\sum_x \left( \frac{M_H^2}{2} \Phi^\dagger \Phi + \frac{\lambda_L}{4} (\Phi^\dagger \Phi)^2 \right) + \frac{1}{(\Delta t)^2} \left( z_E \sum_{i,x} E_i E_i + \frac{z_\pi}{2} \sum_x \pi^\dagger \pi \right)$$

The sums are defined over spatial sites only and the sum $\sum_{\square}$ corresponds to a sum over spatial plaquettes of the gauge field $U_i$. The first two terms in the Hamiltonian are discretized versions of $F_{a ij}^a F_{a ij}^a$ and $(D_i \Phi)^\dagger D_i \Phi$ respectively. The fields $E_i$ and $\pi$ are the conjugate momenta to the coordinate fields $U_i$ and $\Phi$. The lattice spacing, $a$ has been taken to be 1. Formally, Eqn. (1) results from the dimensional reduction of the 4-dimensional $SU(2)$-Higgs Lagrangian with kinetic terms reintroduced. One would therefore expect in the Hamiltonian a Debye mass term and a renormalisation of the kinetic terms. As noted by Moore and Turok, the Debye mass term is effectively introduced by the U.V. cutoff of the lattice. The renormalisation of the kinetic terms is expressed in the coefficient $z_E$ and $z_\pi$. It is believed that they take the form $1 + O(g^2)$ and for this calculation they are assumed to be 1. The time step factor $\Delta t$ is taken to be 0.05, which has been used in similar simulations.

A set of initial configurations are generated which satisfy Gauss constraints and then evolved classically using a leapfrog algorithm. Instead of evaluating the Chern-Simons number directly, which have a number of inherent problems, it is indirectly measured using a slave-field method, proposed by Moore and Turok. This method has several advantages, being computationally quite cheap and does not require a finite renormalisation.

One can set the Higgs mass by fixing the self-coupling term $\lambda_L$ since at tree level

$$\frac{m_H^2}{m_W^2} \approx 2 \lambda_L ,$$

for $m_H \approx 120 \text{ GeV}$, $\lambda_L$ was set to 1.125. In order to get an estimate of the temperatures, the relationships between the 4-dimensional and 3-dimensional parameters can be used to determine the ratio $m_H/T$ as a function of the
remaining bare parameters, $\beta$ and $M_{H_0}$. For $m_H \approx 120$ GeV the temperature of crossover, $T_{\text{cross}}$ is around 200 GeV. Hence, $M_{H_0}$ was set to -0.596 so that $aT_{\text{cross}}$ corresponded to $\beta \approx 7.5$ (which is inversely proportional to $aT$).

2.2 Results

For a lattice size of $24^4$, $\beta$ was varied from 6.8 to 7.8. The number of initial configurations and number of sweeps are shown in table 1. As can be seen from figure 1, one could determine the diffusion coefficient directly from the data by evaluating the slope. However, in order to maximise the sample size, a cosine transform was applied to the data, and the first 200 coefficients evaluated. The diffusive contribution to the coefficient will be proportional to $1/m^2$, where $m$ is the number of the coefficient. An example of the resulting coefficients is shown in figure 2. The error for each coefficient is evaluated via jack-knife. Correlations between different coefficients appear to drop significantly as the statistics is improved with the percentage of correlation coefficients greater than 0.9 dropping to less than 5% for the larger data sets. The coefficients were averaged in bins of 50, which reduces the error significantly. The errors were summed quadratically. (One could also evaluate the error for the binned data by treating the central values of the coefficients as independent points and estimating the error as a standard deviation. This approach varied the statistical error by a factor of 0.5 to 2. Ideally, one should perform a correlated

![Figure 1: Diffusion rate of winding number at $\beta = 7.1$ and $\beta = 7.6$. Note the difference in the vertical scales.](image-url)
| $\beta$ | #configurations | #time steps | $\kappa$          |
|---------|-----------------|-------------|-------------------|
| 6.8     | 16              | 7000        | 0.973 ± 0.057 ± 0.161 |
| 6.9     | 16              | 7000        | 0.984 ± 0.061 ± 0.107 |
| 7.0     | 16              | 10000       | 1.013 ± 0.083 ± 0.187 |
| 7.1     | 8               | 20000       | 0.770 ± 0.059      |
| 7.2     | 8               | 10000       | 0.388 ± 0.034 ± 0.038 |
| 7.3     | 8               | 10000       | 0.092 ± 0.008      |
| 7.4     | 8               | 6000        | 0.018 ± 0.002 ± 0.003 |
| 7.5     | 8               | 9000        | 0.00787 ± 0.00068  |
| 7.6     | 16              | 6000        | 0.00088 ± 0.00006  |
| 7.7     | 8               | 6000        | 0.00130 ± 0.00016  |
| 7.8     | 8               | 9000        | 0.00055 ± 0.00005  |

Table 1: Numbers of configurations and the time steps iterated forward for each $\beta$. The second error quoted for $\kappa$ is the difference between the first and second bins of 50 coefficients when the difference was greater than 1 standard deviation.

For some values of $\beta$, differences between different regions of binning varied by more than one standard deviation. A conservative estimate of the systematic error due to fitting took this into account.

3 Conclusions

From figure 3 one can see that the diffusion rate vanishes for $\beta > 7.4$. The diffusion rate is decreasing from $\beta = 7.0$. This corresponds to a temperature variation of around 5%, which, assuming the crossover temperature is around 200 GeV implies a variation of about 10 GeV. The rate for $\beta = 7.0$ is of the same order as similar simulations done for $m_H \approx m_W$, 7, 10. The calculation of $\kappa$ for $\beta < 7.0$ is consistent with $\kappa(\beta = 7.0)$; however the Moore-Turok definition of the winding number will very often change by more than 1 between measurements which introduces a systematic error. The next step is to adjust $M_H$ so that $aT_{cross}$ is much smaller and thereby larger $\beta$’s (smaller lattice spacings) are required. A study of the effect of the finite volume is also necessary. Ultimately this should be repeated for a larger Higgs mass (150 GeV, for example) to determine if the width is increasing with the Higgs mass.
Figure 2: The mean square of the cosine coefficients at $\beta = 7.4$ and the average of the coefficients with a bin size of 50.

Figure 3: $\kappa$ as a function of $\beta$ using a $24^3$ lattice.
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