From Vacuum Fluctuations to Radiation: Accelerated Detectors and Black Holes.

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Abstract The vacuum fluctuations that induce the transitions and the thermalisation of a uniformly accelerated two level atom are studied in detail. Their energy content is revealed through the weak measurement formalism of Aharonov et al. It is shown that each time the detector makes a transition it radiates a Minkowski photon. The same analysis is then applied to the conversion of vacuum fluctuations into real quanta in the context of black hole radiation. Initially these fluctuations are located around the light like geodesic that shall generate the horizon and carry zero total energy. However upon exiting from the star they break up into two pieces one of which gradually acquires positive energy and becomes a Hawking quantum, the other, its ”partner”, ends up in the singularity. As time goes by the vacuum fluctuations generating Hawking quanta have exponentially large energy densities. This implies that back reaction effects are large.

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1 Introduction

Pair creation in a strong external field is a well known aspect of quantum matter field theory. For instance, in a constant electric field, $e^+e^-$ pairs are spontaneously created. Another famous example is the Hawking flux engendered by the time dependent geometry of an incipient black hole. At present the back-reaction of these quanta on the external field which produces them is far from being understood. The semi-classical treatment alone does not give rise to difficulty. This is because the external field remains purely classical since only the mean value of the matter current operator acts on it as a source. All the quantum properties of the matter, including its fluctuations, are completely ignored by the external field. When the fluctuations become important this fluid description fails and a more quantum mechanical treatment of the back-reaction is needed.

Since a fully quantum description seems beyond the present scope of quantum field theory (this is certainly the case for the gravitational back-reaction since no renormalisable theory exists yet), an intermediate approach wherein the quantum fluctuations are at least partially taken into account is required.

In [6], such a treatment based on the weak measurement formalism of Aharonov et al. was proposed in the context of electroproduction. By isolating through a post-selection that part of the wave function which contains a specific pair of quanta, the weak value of the current operator was computed and interpreted. A clear picture of the creation act of the selected pair was obtained: It emerges out of a vacuum fluctuation which gets progressively distorted by the electric field and finally converted into a pair of asymptotic quanta. This formalism yields, in addition to an explicit evaluation of the fluctuations around the mean value, the first order quantum modification of the external field. This modification in turn governs the back reaction of the selected pair onto itself and the following ones.

The purpose of the present paper is to apply the same construction to uniformly accelerated systems and to black hole radiation. A detailed description of the vacuum fluctuations which give rise to asymptotic quanta is obtained.

In order to carry out this program a generalisation of the post-selection used in [6] is required for the following reasons. First, in the black hole problem, incomplete post-selections are needed since the ”partners” of the Hawking quanta are inaccessible to any asymptotic observer. Secondly, when a large number of quanta are produced (in the mean), it is unphysical to post-select states wherein a single pair is present because the probability that they occur is exponentially small. Thirdly, since post-selection is a rather formal and arbitrary operation, one may question the physical relevance of the resulting weak-values.

The first part of this paper (chapters 2, 3 and 4) is devoted to overcome these difficulties. We work in the context of post-selecting Rindler quanta (rindlerons) in Minkowski vacuum. In particular, in chapter 4, we show how the EPR correlations between the quantum jumps of an accelerated two-level atom and the state of the radiation field coupled to it give back, in a very natural manner, the amplitudes previously post-selected by hand. These correlations indicate that, by getting excited, the atom has selected out of Minkowski vacuum the fluctuation which contains the rindleron needed to make the transition. This vacuum fluctuation admits two complementary descrip-
tions. An inertial observer would say that it carries zero energy whereas a uniformly accelerated observer in the quadrant of the atom would assign it a positive energy. The relation between these two interpretations is at the heart of all our analysis.

In chapter 5, we reverse the strategy and analyse the particle content of the fluxes emitted by the accelerated atom in the light of the weak-values freshly obtained. We show, in accord with Unruh’s original claim \cite{10}, and contrary to more recent claims \cite{11}, \cite{12}, that, despite its being in thermal equilibrium with the Minkowski fluctuations, the two level atom emits Minkowski quanta \cite{14}. The rate of production of Minkowski quanta equals the rate of internal transitions of the atom. Note that this production of quanta would also be present in the black hole case if one puts a ”fiducial” \cite{15} detector (at fixed radius) in the vicinity of the horizon. The back reaction of the emitted quanta onto the hole cannot be neglected in view of the very high temperatures encountered.

In chapter 6 the results obtained in the uniformly accelerated case are easily mapped to the black hole problem. It is shown how a Hawking photon emerges from a vacuum fluctuation initially carrying no energy and located around the light ray that generates the horizon. In the time dependent background geometry of the collapsing star the vacuum fluctuation breaks up into two pieces, one of which escapes to infinity and gradually acquires positive energy to become the post-selected Hawking photon, its partner travels beyond the horizon and ends up in the singularity.

In addition these vacuum fluctuations very soon become located on cis-planckian distances while carrying trans-planckian energy densities \cite{16}. This fundamental aspect is presented in a separate publication \cite{17} wherein it is argued that a taming process of these trans-planckian densities is necessary in view of the nonlinearities of gravitational interactions. A model for this taming, based on a Hagedorn-type of transition \cite{18}, is also suggested. Nevertheless an essential part of black hole physics is as yet unknown, to wit whether or not they radiate and if so what is the emission process. The study of the back reaction to Hawking radiation is a necessary concomitant to understanding this problem. We refer the reader to the following recent publications: \cite{19} \cite{20} for some considerations along these lines.

2 generalised pre- and post-selection, weak measurements

Pre- and post-selection consists in specifying both the initial and the final state of a system (denoted by \( S \) in the sequel). The approach developed by Aharonov et al.\cite{7} for studying such pre- and post-selected ensembles consists in performing at an intermediate time a ”weak measurement” on \( S \). In essence one studies the first order backreaction onto an additional system taken by Aharonov et al. to be a measuring device. But the formalism is more general. In the case of pair production the additional system could be the external electric or gravitational field which now has to be described quantum mechanically. Moreover this formalism can be used to study the self interaction of the pairs without introducing the additional system. This is because, when the first order (or weak) approximation is valid, the backreaction takes a simple and universal form governed by a c-number, the ”weak value” of the operator which controls the interaction.

In this section the formalism of Aharonov et al. is generalised to post-selections that do not specify completely the initial and/or the final state of the system. Rather one
imposes only that they belong to given subspaces of the Hilbert space of the system \( \mathcal{H}_S \). In this formalism the post selection remains a rather formal operation. Therefore in the last part of this chapter we show how the post-selection may be realised operationally following the rules of quantum mechanics by coupling to \( S \) an additional system in a metastable state (the ”post selector” \( PS \)) which will make a transition only if the system is in the required final state(s). The weak value of an operator obtained in this manner changes as time goes by from an asymmetric form to an expectation value, thereby making contact with more familiar physics.

The system to be studied is in the (pre-selected) state \( |\psi_i\rangle \) at time \( t_i \) (more generally pre-selected density matrix \( \rho_i \)). The unperturbed time evolution of \( S \) can be described by the following density matrix

\[
\rho_S(t) = U_S(t, t_i)|\psi_i\rangle \langle \psi_i| U_S(t_i, t)
\]

where \( U_S = \exp(-iH_S t) \) is the time evolution operator for the system \( S \). The post-selection at time \( t_f \) consists in specifying that the system belongs to a subspace, \( \mathcal{H}'_S \), of \( \mathcal{H}_S \). The probability to find the system in this subspace at time \( t_f \) is

\[
P_{\Pi'_S} = \text{Tr}_S\left[ \Pi'_S \rho(t_f) \right] = \text{Tr}_S\left[ \Pi'_S U_S(t_f, t_i)|\psi_i\rangle \langle \psi_i| U_S(t_i, t_f) \right]
\]

where \( \Pi'_S \) is the projection operator onto \( \mathcal{H}'_S \).

Following Aharonov et al. we introduce an additional system, called the ”weak detector” \( WD \), coupled to \( S \). The backreaction of \( S \) onto \( WD \) is considered, subject to the pre- and post-selection just described. The interaction hamiltonian between \( S \) and \( WD \) is taken to be of the form \( H_{S-WD}(t) = \epsilon f(t)A_S B_{WD} \) where \( \epsilon \) is a coupling constant, \( f(t) \) is a function, \( A_S \) and \( B_{WD} \) are hermitian operators acting on \( S \) and \( WD \) respectively.

To first order in \( \epsilon \) (the coupling is weak), the evolution of the coupled system \( S \) and \( WD \) is given by

\[
\rho(t_f) = \left( U_S(t_f, t_i)U_{WD}(t_f, t_i) - i \epsilon \int_{t_i}^{t_f} dt U_S(t_f, t)U_{WD}(t_f, t)f(t)A_S B_{WD} \times \right.
\]

\[
\left. \times U_S(t, t_i)U_{WD}(t, t_i) \right)|\psi_i\rangle \langle WD| WD| \langle WD| \psi_i\rangle \left( \text{h.c.} \right)
\]

where \( U_S \) and \( U_{WD} \) are the free evolution operators for \( S \) and \( WD \) and \( |WD\rangle \) is the initial state of \( WD \). Upon post-selecting at \( t = t_f \) that \( S \) belongs to the subspace \( \mathcal{H}'_S \) and tracing over the remaining states of the system \( S \), the reduced density matrix describing the \( WD \) is obtained. In the first order approximation in which we are working it takes a very simple form

\[
\rho_{WD}(t_f) = \text{Tr}_S\left[ \Pi'_S \rho(t_f) \right] \propto \left( U_{WD}(t_f, t_i) - i \epsilon \int_{t_i}^{t_f} dt U_{WD}(t_f, t)f(t)A_{S\text{weak}}(t) B_{WD} U_{WD}(t, t_i) \right) \times \]

\[
\times |WD\rangle \langle WD| \left( \text{h.c.} \right)
\]

where

\[
A_{S\text{weak}}(t) = \frac{\text{Tr}_S\left[ \Pi'_S U_S(t_f, t_i)A_S U_S(t_i, t_i)|\psi_i\rangle \langle \psi_i| U_S(t_i, t_f) \right]}{\text{Tr}_S\left[ \Pi'_S U_S(t_f, t_i)|\psi_i\rangle \langle \psi_i| U_S(t_i, t_f) \right]}
\]
is the weak value of $A$. If one specifies completely the final state, $\Pi'_S = |\psi_f><\psi_f|$ then the result of Aharonov et al. obtains:

$$A_{\text{weak}}(t) = \frac{<\psi_f|U_S(t_f,t)A_SU_S(t,t_i)|\psi_i>}{<\psi_f|U_S(t_f,t_i)|\psi_i>}$$

(6)

The remarkable feature of the above formalism is its independence on the internal structure of the WD. The first order backreaction of $S$ onto $WD$ is universal: it is always controlled by the c-number $A_{\text{weak}}(t)$, the "weak value of $A". Therefore if $S$ is coupled to itself by an interaction hamiltonian, the backreaction will be controlled by the weak value of $H_{\text{int}}$ in first order perturbation theory. For instance the modification of the probability that the final state belongs to $\mathcal{H}'_S$ is given by the imaginary part of $H_{\text{int weak}}$.

The weak value of $A$ is complex. By performing a series of measurements on $WD$ and by varying the coupling function $f(t)$, the real and imaginary part of $A_{\text{weak}}$ could in principle be determined. Here the word "measurement" must be understood in its usual quantum sense: the average over repeated realisations of the same situation. This means that the weak value of $A_S$ should also be understood as an average. The fluctuations around $A_{\text{weak}}$ are encoded in the second order terms of eq. (3) which have been neglected.

To illustrate the role of the real and imaginary parts of $A_{\text{weak}}$, we recall the example of Aharonov et al consisting of a weak detector which has one degree of freedom $q$, with a gaussian initial state $<q|WD> = e^{-q^2/2\Delta^2}, -\infty < q < +\infty$. The unperturbed hamiltonian of $WD$ is taken to vanish (hence $U_{WD}(t_1,t_2) = 1$) and the interaction hamiltonian is $H_{S-WD}(t) = \epsilon_0(t - t_0)pA_S$ where $p$ is the momentum conjugate to $q$. Then after the post-selection the state of the WD is given to first order by

$$<q|WD(t_f)> = (1 - i\epsilon pA_{\text{weak}}(t_0))e^{-q^2/2\Delta^2}$$

$$= e^{-i\epsilon pA_{\text{weak}}(t_0)}e^{-q^2/2\Delta^2}$$

$$= e^{-(q - \epsilon A_{\text{weak}}(t_0))^2/2\Delta^2}$$

$$= e^{-(q - \epsilon \text{Re}A_{\text{weak}}(t_0))^2/2\Delta^2}e^{i\epsilon Q\text{Im}A_{\text{weak}}(t_0)/\Delta^2}$$

(7)

The real part of $A_{\text{weak}}$ induces a translation of the centre of the gaussian, the imaginary part a change in the momentum. Their effect on the WD is therefore measurable. The validity of the first order approximation requires $\epsilon A_{\text{weak}}/\Delta << 1$.

It is instructive to see how unitarity is realised in the above formalism. Take $\Pi'_S$ to be a complete orthogonal set of projectors acting on the Hilbert space of $S$. Denote by $P_i$ the probability that the final state of the system belong to the subspace spanned by $\Pi'_S$ and by $A_{\text{weak}}^i$ the corresponding weak value of $A$. Then the mean value of $A_S$ is

$$<\psi_i|A_S|\psi_i> = \sum_i P_i A_{\text{weak}}^i$$

(8)

Thus the mean backreaction if no post-selection is performed is the average over the post-selected backreactions. Notice that the imaginary parts of the weak values necessarily cancel since the l.h.s. of eq. (8) is real.

Up to now the postselection has been implemented by projecting by hand the state of the system onto a certain subspace $\mathcal{H}'_S$. Such a projection may be realised operationally by introducing an additional quantum system, a "post-selector" (PS), coupled
in such a way that it will make a transition if and only if the system $S$ is in the required final state. Then by considering only that subspace of the final states in which $PS$ has made the transition, the pre- and post-selected ensemble is recovered. This quantum description of the post-selection will turn out to be very useful when considering individual Hawking quanta. The detected Hawking photons will then be analysed using the weak measurement formalism.

We shall consider the very simple model of a $PS$ having two states, initially in the ground state, and coupled to the system by an interaction of the form

$$H_{S-PS} = \lambda g(t)(a^\dagger Q_S + aQ_S^\dagger)$$

(9)

where $\lambda$ is a coupling constant, $g(t)$ a time dependent function, $a^\dagger$ the operator that induce transitions from the ground state to the exited state of the $PS$, $Q_S$ an operator acting on the system $S$. The postselection is performed at $t = t_f$ and consists in finding the $PS$ in the exited state.

For simplicity we shall work to second order in $\lambda$ (although in principle the interaction of $PS$ with $S$ need not be weak). The wave function of the combined system $S + WD + PS$ is in interaction representation

$$\left[1 - i \int dt \left( H_{S-WD}(t) + H_{S-PS}(t) \right) - \frac{1}{2} \int dt \int dt' T \left[ H_{S-WD}(t) + H_{S-PS}(t) \right] \left( H_{S-WD}(t') + H_{S-PS}(t') \right) \right] |\psi_i\rangle > |WD\rangle |0_{PS}\rangle$$

(10)

where $|0_{PS}\rangle >$ is the ground state of $PS$ and $T$ is the time ordering operator. Upon imposing that the $PS$ be in its excited state at $t = t_f$ the resulting wave function of $S$ and $WD$ is , to order $\epsilon$,

$$\left[ -i \int dt \lambda g(t)Q_S(t) - \int dt \int dt' T \left[ \epsilon f(t)A_S(t)B_{WD}(t)\lambda g(t')Q_S(t') \right] \right] |\psi_i\rangle > |WD\rangle$$

(11)

Then tracing over the states of $S$ yields the reduced density matrix of $WD$

$$\left[1 - i\epsilon \int dt_0 f(t_0)B_{WD}(t_0)A_{S\text{weak}}(t_0) \right] |WD\rangle < WD| [h.c.]$$

(12)

where

$$A_{S\text{weak}}(t_0) = \frac{<\psi_i| \int dt g(t)Q_S^\dagger(t) \int dt' g(t')T [A_S(t_0)Q_S(t')] |\psi_i>}{<\psi_i| \int dt g(t)Q_S^\dagger(t) \int dt' g(t')Q_S(t') |\psi_i>}$$

(13)

Note that the weak value of $A_S$ results from the interference of the two states in eq. [11].

There are several cases when eq. [13] takes a simple form. If $g(t)$ is non vanishing only after $t = t_0$ the time ordering operator is very simple to implement and the resulting expression for $A_W$ takes a typical (for a weak value) asymmetric form

$$A_{S\text{weak}}(t_0) = \frac{<\psi_i| \int dt g(t)Q_S^\dagger(t) \int dt' g(t')Q_S(t')A_S(t_0) |\psi_i>}{<\psi_i| \int dt g(t)Q_S^\dagger(t) \int dt' g(t')Q_S(t') |\psi_i>}$$

(14)
If in addition \( g(t) = \delta(t-t_f) \), \( t_f > t_0 \) and \( Q_S = \Pi'_S \), eq. 5 is recovered since \( (\Pi'_S)^2 = \Pi'_S \).

If on the other hand \( g(t) \) is non vanishing only before \( t = t_0 \) then the time ordering operator becomes trivial once more and eq. 13 takes the form

\[
A_{Sweak}(t_0) = \frac{<\psi_i| \int dt g(t)Q^\dagger_S(t)A_S(t_0)\int dt' g(t')Q_S(t')|\psi_i>}{<\psi_i| \int dt g(t)Q^\dagger_S(t)\int dt' g(t')Q_S(t')|\psi_i>}
\]  

(15)

This is the expectation value of \( A_S \) if the \( PS \) has made a transition. It is necessarily real. The weak value of \( A_S \) if the \( PS \) has not made a transition can also be computed. It is related to the mean value of \( A_S \) and to eq. 13 through the unitary relation eq. 8.

In the sequel we shall use the above formalism to compute the weak value of \( T_{\mu\nu} \). The justification for considering the weak value follows from the abovementioned universality, which implies that it is the source of the first order backreaction.

3 Post-selecting Rindler quanta and the weak value of \( T_{\mu\nu} \)

3.1 Introduction

In this chapter we shall post-select the presence of Rindler quanta in Minkowski vacuum. The weak value of the energy momentum tensor is obtained in this pre- and post-selected ensemble.

This program is carried out in a mechanical way by post-selecting the presence of a pair of Rindler particles of fixed Rindler energy \( \lambda \) in Rindler vacuum. This is a straightforward calculation but it presents several unsatisfactory aspects:

1. The post-selected state is specified in both the right and left Rindler quadrant whereas in the analogous black hole problem only the region outside the horizon is accessible to measurements.

2. The state that is post-selected has probability zero of being realised \(<\text{pair of rindlerons}|0_M> \equiv 0 \) even though the weak value of \( T_{\mu\nu} \) is finite (it is a conditional measurement: a cancellation of zeros occurs between the numerator and denominator of eq. 30).

3. Post-selection is a formal procedure. Physical insight would be gained by introducing an additional system (the ”post-selector” of chapter 2) that gets correlated to the field thereby realising operationally the post-selection.

4. The energy momentum tensor obtained is singular on the horizons. As shown in ref 21, the appearance of such singularities is a generic feature when working with Rindler modes.

Most of this chapter and the next one will be devoted to solving these problems.

The first one finds its solution in the partial post-selector introduced in section 2. The state of the field is post-selected only in the right Rindler quadrant while tracing over the state in the left one.

The second problem is solved by performing an even less restrictive post-selection. One post-selects the presence of one Rindleron in the mode of interest in the right
Rindler quadrant while tracing over the other modes in that quadrant and over the entire state of the field in the left quadrant.

In order to reveal the physical meaning of post-selection we shall work in the next chapter in a more physical set-up (solving 3). We shall consider a uniformly accelerated two level atom in Minkowski vacuum initially in its ground state, coupled to the field during a proper time $T$. The post-selection shall consist in finding the two level atom in its ground or its excited state. Since the two level atom interacts with the field for a finite time it effectively post-selects the presence of a wave packet rather than a delocalised mode. We shall show that if the post-selected wave packet is sufficiently tight then the energy momentum tensor will be regular everywhere. The singularities mentioned in 4 have disappeared.

We shall restrict our analysis to a massless scalar field in 1+1 dimensions in view of its simplicity and of its relation to the emission of quanta by a black hole. The simplicity arises from the conformal invariance which implies also some very particular properties. For instance the propagator is infrared divergent and is the sum of a left and a right moving part. Hence the energy momentum tensor takes the form $T_{UU} = f(U)$, $T_{VV} = g(V)$, $T_{UV} = 0$ where $V, U = t \pm x$ are the Minkowski light like coordinates. These properties will result most notably in the eternal life of vacuum fluctuations. This is to be contrasted with the vacuum fluctuations of a massive field which exist only in a finite region of space time (typically $\Delta t \approx \Delta x \approx m^{-1}$). We thus expect that any breaking of the conformal invariance, a small mass or an interaction with another field, will change dramatically the structure at large distances of the vacuum fluctuations we shall exhibit. The same reservations apply to the black hole problem as well.

### 3.2 The kinematics of a massless scalar field in 1+1 dimensions

In this section, we review the relevant properties of the Rindler quantisation of the scalar field. The conformal invariance of this massless scalar field is best exploited by using the light like coordinates $U, V$ introduced above. Whereupon the Klein-Gordon equation takes the form $\partial_U \partial_V \phi = 0$ and any solution can be written as

$$\phi(U, V) = \phi(U) + \phi(V)$$

(16)

From now on we shall drop the right moving piece and consider the "V" term only. It is obvious that all conclusions shall be equally valid for the right movers.

The second quantised field can be decomposed into a basis of Minkowski modes

$$\phi(V) = \int_0^\infty d\omega \left( a_\omega \varphi_\omega(V) + a_\omega^\dagger \varphi_\omega^*(V) \right)$$

(17)

$$\varphi_\omega(V) = \frac{e^{-i\omega V}}{\sqrt{4\pi \omega}}$$

(18)

The Minkowski vacuum $|0_M \rangle$ is that annihilated by all the $a_\omega$’s. The propagator in Minkowski vacuum is

$$G_+(V, V') = \langle 0_M | \phi(V) \phi(V') | 0_M \rangle = -\frac{1}{4\pi} \log(V - V' - i\epsilon)$$

(19)
The (normal ordered) hamiltonian of the field is (for left movers)

\[ H_M = \int_{-\infty}^{+\infty} dV T_{VV} = \int_{0}^{+\infty} d\omega \omega (a_\omega \dagger a_\omega) \]  

(20)

The uniformly accelerated observer will be taken to be in the right Rindler quadrant \( U < 0, V > 0 \). In this quadrant one defines Rindler coordinates \( \rho, \tau \) by

\[
\begin{align*}
\rho &= \exp(\tau) \\
\sinh a \tau &= \rho \\
\cosh a \tau &= \frac{1}{\rho} \\
\end{align*}
\]

(21)

and Rindler light like coordinates \( u, v \) by

\[
\begin{align*}
U &= -a^{-1} e^{-au} \\
V &= a^{-1} e^{av} \\
\end{align*}
\]

(22)

where \( a \) is the acceleration. These coordinates may be extended to the left Rindler quadrant by the analytic continuation \( \tau \to \tau \pm i\pi/a \).

The natural basis of quantisation a uniformly accelerated observer would choose is the Rindler basis which consists of plane waves in the variables \( u, v \) (Rindler modes). But the Bogoljubov transformation from the Minkowski modes to the Rindler modes is singular \[21\] and care must be taken to define it as a limit if the Minkowski properties of the theory are to be satisfied. To this end it is useful to first introduce an alternative basis of Minkowski modes \[10\]:

\[
\varphi_{\lambda,M}(V) = \int_{0}^{\infty} d\omega \gamma_{\lambda,\omega} \varphi_{\omega}(V) = \frac{[a(e + iV)]^{-i\lambda/a}}{\sqrt{e^{\pi\lambda/a} - e^{-\pi\lambda/a}}} 4\pi \lambda
\]

\[
\simeq \frac{e^{\pi\lambda/2a}}{\sqrt{|e^{\pi\lambda/a} - e^{-\pi\lambda/a}|}} \frac{e^{-\pi\lambda/2a}}{\sqrt{4\pi|\lambda|}} + \frac{e^{-\pi\lambda/2a}}{\sqrt{|e^{\pi\lambda/a} - e^{-\pi\lambda/a}|}} \frac{|aV|^{-i\lambda/a}}{\sqrt{4\pi|\lambda|}}
\]

(23)

where

\[
\gamma_{\lambda,\omega} = \left( \frac{1}{\Gamma(\frac{i\lambda}{a})} \sqrt{\frac{\pi}{\lambda \sinh \frac{\pi\lambda}{a}}} \right) \frac{1}{\sqrt{2\pi a\omega}} \left( \frac{\omega}{a} \right)^{\frac{i\lambda}{a}} e^{-\omega\epsilon}
\]

(24)

The first factor in \( \gamma_{\lambda,\omega} \) is a pure phase introduced for convenience. The factor \( e^{-\omega\epsilon} \) is the crux of the construction. It defines the integral eq. 23 and ensures the correct Minkowski properties of the theory. For instance it gives the correct pole prescription at \( V = V' \) of the propagator eq. \[24\] when expressed in terms of the modes \( \varphi_{\lambda,M} \) as \( G_+(V, V') = \int_{-\infty}^{+\infty} d\lambda \varphi_{\lambda,M}(V) \varphi_{\lambda,M}^*(V') \). The limit \( \epsilon \to 0 \) is to be taken at the end of all calculations.

The annihilation and creation operators corresponding to the modes \( \varphi_{\lambda,M} \) are \( a_{\lambda,M} \) and \( a_{\lambda,M}^\dagger \). 

\[ \]
The right and left Rindler modes \( \varphi_{\lambda,R}(V) \) and \( \varphi_{\lambda,L}(V) \) are now defined by the Bogoljubov transformation

\[
\begin{align*}
\varphi_{\lambda,M} &= \alpha_{\lambda}\varphi_{\lambda,R} + \beta_{\lambda}\varphi_{\lambda,L}^* \\
\varphi_{-\lambda,M} &= \beta_{\lambda}\varphi_{\lambda,R}^* + \alpha_{\lambda}\varphi_{\lambda,L} \\
\lambda &> 0
\end{align*}
\]

with

\[
\alpha_{\lambda} = \frac{e^{\pi\lambda/2a}}{\sqrt{e^{2\pi\lambda/a} - e^{-2\pi\lambda/a}}} \quad \beta_{\lambda} = \frac{e^{-\pi\lambda/2a}}{\sqrt{e^{2\pi\lambda/a} - e^{-2\pi\lambda/a}}}
\]

In the limit \( \epsilon \to 0 \) the Rindler modes take the familiar form

\[
\varphi_{\lambda,R}(V) = \theta(V)\frac{1}{\sqrt{4\pi\lambda}} e^{-i\lambda V/a} = \frac{1}{\sqrt{4\pi\lambda}} e^{-i\lambda v}
\]

\[
\varphi_{\lambda,L}(V) = \theta(-V)\frac{1}{\sqrt{4\pi\lambda}}|aV|^{-i\lambda/a}
\]

For finite \( \epsilon \) they differ from these limiting forms only when \( V \leq \epsilon \). To these modes are associated the Rindler destruction and creation operators \( a_{\lambda,R}, a_{\lambda,R}^\dagger \) and \( a_{\lambda,L}, a_{\lambda,L}^\dagger \).

Using the above Bogoljubov transformation it is easy to show that

\[
|0_{RL} > = \prod_{\lambda} \frac{1}{\alpha_{\lambda}} a_{\lambda,R}^\dagger a_{\lambda,L}^\dagger |0_{RL} >
\]

where \( |0_{RL} > = |0_R > \otimes |0_L > \) is Rindler vacuum in both the right(R) and left(L) quadrants. Upon tracing over the left quanta the reduced density matrix in the right Rindler quadrant is an exact thermal distribution of right rindlerons. This proves that Rindler physics in Minkowski vacuum is identical to working in a thermal bath at temperature \( a/2\pi \).

### 3.3 Post-selecting Rindler quanta

We are now ready to evaluate the weak value of \( \phi(V)\phi(V') \) when the pre-selected state is Minkowski vacuum and the post-selected state contains Rindler quanta. The energy momentum tensor is then readily obtained by the limiting process \( T_{VV} = \lim_{V' \to V} \partial_V \phi(V)\partial_{V'} \phi(V') \). The first post-selected state considered is a pair of Rindlerons in Rindler vacuum. By availing oneself of the identity

\[
a_{\lambda,R}^\dagger a_{\lambda',L}^\dagger |0_{RL} > = \frac{1}{\alpha_{\lambda}} a_{\lambda,M}^\dagger a_{\lambda',M}^\dagger |0_{RL} > + \frac{\beta_{\lambda}}{\alpha_{\lambda}} \delta(\lambda - \lambda')|0_{RL} >
\]

it is straightforward to obtain the weak value of \( \phi(V)\phi(V') \)

\[
\frac{< 0_{RL}|a_{\lambda,R}a_{\lambda,L}\phi(V)\phi(V')|0_M >}{< 0_{RL}|a_{\lambda,R}a_{\lambda,L}|0_M >} = \frac{2}{\alpha_{\lambda}\beta_{\lambda}} \varphi_{-\lambda,M}(V)\varphi_{\lambda,M}(V') + \frac{< 0_{RL}|\phi(V)\phi(V')|0_M >}{< 0_{RL}|0_M >}
\]

\(\lambda > 0\)

It decomposes into two terms. The first depends on the quantum number \( \lambda \) and is the contribution of the post-selected pair of rindlerons. It carries an energy density equals

\[
\lim_{V' \to V} \partial_V \partial_{V'} \frac{2}{\alpha_{\lambda}\beta_{\lambda}} \varphi_{-\lambda,M}(V)\varphi_{\lambda,M}(V') = \frac{\lambda}{2\pi a^2 (V + i\epsilon)^2}
\]
The second term is independent of \( \lambda \) and appears because except for the mode \( \lambda \), Rindler vacuum has been post-selected (if Rindler vacuum is post-selected only the second term appears). It is convenient to rewrite it as the sum of the expectation value of \( T_{VV} \) in Minkowski vacuum\(^4\) plus another term

\[
\frac{<0_{RL}|T_{VV}|0_M>}{<0_{RL}|0_M>}
= \lim_{V' \to V} \frac{\partial V}{\partial V'} \int_0^\infty d\lambda \left( -2\frac{\beta_\lambda}{\alpha_\lambda} \phi_{-\lambda,M}(V)\phi_{\lambda,M}(V') + <0_M|T_{VV}|0_M> \right)
= \frac{\pi}{12} \left( \frac{a}{2\pi} \right)^2 \frac{1}{a^2(V+i\epsilon)^2} + <0_M|T_{VV}|0_M> \quad (32)
\]

These results allow for two complementary interpretations. The Rindler point of view, restricted to \( V > 0 \), is obtained by considering the Rindler energy density \( T_{VV} = (dV/dv)^2 T_{VV} \). Then eq. 31 gives the energy density of the post-selected rindleron \( \lambda/2\pi \), the jacobian being \((dV/dv)^2 = a^2 V^2 \). Since we have imposed that no other Rindleron be present in the final state, the remaining term eq. 32 yields the value of the Rindler vacuum energy which is minus the thermal energy density at a temperature \( a/2\pi \) (Minkowski vacuum contains a thermal distribution of rindlerons). The Minkowski point of view is completely different. Since the hamiltonian \( H_M \) is diagonal in \( \omega \) and annihilates Minkowski vacuum \( H_M|0_M> = 0 \), eq. 31 and eq. 32 contain zero Minkowski energy. Indeed, the pole prescription at the horizon \( V = 0 \) ensures that their integrals over the entire domain of \( V \) vanish.

We note that the energy in the left quadrant is identical to that in the right quadrant since there is a complete symmetry between the two. This symmetry is manifest by considering the total Rindler energy (the boost operator)

\[
H_R = \int_{-\infty}^{+\infty} dv T_{vv}(right) - \int_{-\infty}^{+\infty} dv T_{vv}(left) = \int_0^{+\infty} d\lambda \lambda (a^\dagger_{\lambda,M} a_{-\lambda,M} - a^\dagger_{-\lambda,M} a_{\lambda,M}) \quad (33)
\]

The minus signs arise because Rindler time runs backwards in the left Rindler quadrant. Since Minkowski vacuum is an eigenstate of \( H_R \) (it is invariant under boosts) with eigenvalue zero the total Rindler energy in the pre- and post-selected ensemble considered above must vanish. Hence, the Rindler energy in the right quadrant is equal and opposite to the energy in the left quadrant.

The post-selection that was used in the calculation above was rather crude and we shall now refine it in three successive steps.

We first consider post-selections that are performed only in the right quadrant while tracing over the state of the field in the left quadrant. Post-selecting one rindleron of energy \( \lambda \) in the right quadrant is achieved using the projector

\[
\Pi_{\lambda,R} = I_L \otimes a^\dagger_{\lambda,R}|0_R><0_R|a_{\lambda,R} \quad (34)
\]

where \( I_L(R) \) is the identity operator restricted to the left (right) quadrant and \(|0_{L(R)}> \) is Rindler vacuum in the left (right) quadrant. Using the formalism developed in the

\[^4\text{In this section and the following one we shall explicitly write the vacuum expectation value of the energy momentum tensor } <0_M|T_{VV}|0_M> \text{ even though it vanishes. It is kept only to facilitate the transcription of these results to the black hole problem where the vacuum expectation of the energy is non trivial and must be renormalised carefully.}\]
preceding chapter the corresponding weak value of $T_{VV}$ is given by

\[
\frac{\langle 0_M|\Pi_{\lambda,R}T_{VV}|0_M \rangle}{\langle 0_M|\Pi_{\lambda,R}|0_M \rangle}\quad (35)
\]

It leads back to eq. (30) because of the EPR correlation’s between the two quadrants: if there is a rindleron on the right then their necessarily also is a rindleron on the left (its partner) with the opposite Rindler energy. This partenaria follows from eq. (28) where the operators $a_{\lambda,R}^\dagger$ and $a_{\lambda,L}^\dagger$ appear in product only. In the black hole problem the equivalent EPR correlation’s will mean that to each outgoing Hawking photon their corresponds an ingoing partner on the other side of the horizon.

An even less restrictive post-selection consists in specifying only partially the state of the field in the right quadrant. One chooses that the final state contains one rindleron on the right in the mode $\lambda$ while tracing over all other right rindlerons and all left rindlerons. The resulting projector is

\[
\tilde{\Pi}_{\lambda,R} = I_L \otimes \prod_{\lambda' \neq \lambda} I_{\lambda',R} \otimes |1_{\lambda,R} \rangle \langle 1_{\lambda,R}|\quad (36)
\]

where $I_{\lambda,L(R)}$ is the identity operator restricted to the mode $\lambda$ in the left (right) quadrant and $|1_{\lambda,R} \rangle$ is the one particle state restricted to the mode $\lambda$. The corresponding weak value of $T_{VV}$ is

\[
\frac{\langle 0_M|\tilde{\Pi}_{\lambda,R}T_{VV}|0_M \rangle}{\langle 0_M|\Pi_{\lambda,R}|0_M \rangle} = \frac{\lambda}{2\pi a^2 (V+i\epsilon)^2} + \frac{1}{\langle 0_M|T_{VV}|0_M \rangle}\quad (37)
\]

The first term is the energy of the Rindleron $\lambda$ already obtained in eq. (30) and eq. (35). The second term is simply the Minkowski vacuum expectation value since no further post-specification is imposed on the final state. This is why the probability $\langle 0_M|\tilde{\Pi}_{\lambda,R}|0_M \rangle$ to be in the eigenspace of $\tilde{\Pi}_{\lambda,R}$ is finite. This is to be opposed to the probabilities encountered previously (the denominators of eq. (30) and eq. (35)) which vanish because all the Rindler modes have been specified to be in their Rindler ground state. In physically realistic situations such as considered in the next chapter only nonvanishing probabilities will occur since a finite number of modes will be coupled to the ”post-selector”. Nevertheless the weak values eq. (35) and eq. (37) can be formally related by a unitary relation similar to eq. (3) by taking a set of orthogonal projectors like $\Pi_{\lambda,R}$ whose combined eigenspace is equal to the eigenspace of $\tilde{\Pi}_{\lambda,R}$ and summing the corresponding weak values multiplied by the relative probabilities that they occur, eq. (37) is recovered. In order to realise this unitary relation one must post-select the presence of two, three, any number of rindlerons. The induced weak values of $T_{VV}$ are easily obtained and the contribution of each individual post-selected particle is found to be independent (if the particles are orthogonal) of the post-selection performed on the other particles. In other words, for a free field the vacuum fluctuations of orthogonal particles are independent of each other.

We finally consider the post-selection of wave packets. Instead of the projector eq. (34) we define:

\[
\Pi_{v_0,\lambda_0,R} = I_L \otimes a_{v_0,\lambda_0,R}^\dagger |0_R \rangle \langle 0_R|a_{v_0,\lambda_0,R}\quad (38)
\]
where $a_{v_0,\lambda_0,R} = \int_0^{+\infty} d\lambda f(\lambda) a_{\lambda,R}$ is the destruction operator of a wave packet of right rindlerons centred around $v = v_0$ and $\lambda = \lambda_0$. The state $\Pi_{v_0,\lambda_0,R}|0_M>$ is

$$\Pi_{v_0,\lambda_0,R}|0_M> = \left( \int_0^{+\infty} d\lambda f^*(\lambda) a^\dagger_{\lambda,R} \right) \left( \int_0^{+\infty} d\lambda' - \frac{\beta_{\lambda'}}{\alpha_{\lambda'}} f(\lambda') a^\dagger_{\lambda',L} \right) |0_{RL}>$$

(39)

where the EPR correlated wave packet in the left Rindler quadrant appears explicitly. Note the dissymetry of the wave packets: the induced wave packet in the left quadrant contains the factor $\beta_{\lambda'}/\alpha_{\lambda'}$ since it originates from the EPR correlations in eq. 28. This dissymetry will play a fundamental role when analysing the flux emitted by the accelerated detector and the black hole. The weak value of $T_{VV}$ is

$$\frac{<0_M|\Pi_{v_0,\lambda_0,R}T_{VV}|0_M>}{<0_M|\Pi_{v_0,\lambda_0,R}|0_M>}=2\left[\int_0^{\infty}d\lambda \int_0^{\infty}d\lambda' \frac{\beta_{\lambda'}}{\alpha_{\lambda'}} f^*(\lambda) f(\lambda') \partial_V \varphi^\dagger_{\lambda,M} \partial_V \varphi^*_{\lambda',M} \right]$$

$$\times \left[\int_0^{+\infty} d\lambda \frac{\beta^2_{\lambda}}{\alpha^2_{\lambda}} |f(\lambda)|^2 \right]^{-1} + \frac{<0_{RL}|T_{VV}|0_M>}{<0_{RL}|0_M>}$$

(40)

Notice that eq. 40 is complex due to the presence of the factor $\beta_{\lambda'}/\alpha_{\lambda'}$.

4 The uniformly accelerated two level atom.

4.1 Post-selection of wave packets by a two level atom.

The set-up consists of a two level atom initially in its ground state at $t = -\infty$, coupled to the field during a finite time. The post-selection is realised by imposing that the atom be in its ground or its excited state at $t = +\infty$. In order to reveal the structure of the vacuum fluctuation which induces the transition, we shall consider the weak value of $T_{\mu\nu}$. As discussed in chapter 2 these weak values would be the source of the linear gravitational backreaction. They could also be measured by a weak detector.

In this section, we consider for generality that the two level atom follows an arbitrary trajectory: $t = t(\tau), x = x(\tau)$ where $\tau$ is the proper time along the trajectory. The hamiltonian of the two level atom is given by

$$\int dt dx \ H_{\text{atom}}(t, x) = \int d\tau \left\{ m A^\dagger(\tau) A(\tau) + gm \left[ f(\tau) \phi(t(\tau), x(\tau)) A(\tau) + \text{h.c.} \right] \right\}$$

(41)

where $g$ is a dimensionless coupling constant that shall be taken for simplicity small enough that second order perturbation theory be valid, $m$ is the difference of energy between the ground and the excited state of the atom, $A(\tau) = Ae^{-im\tau}$ is the operator that induces a transition from the excited state to the ground state of the atom and $f(\tau)$ is a function that governs when the interaction is turned on and off. In interaction representation eq. 41 becomes

$$\int dt dx \ H_{\text{int}}(t, x) = gm \int d\tau \left[ e^{-im\tau} f(\tau) \phi(t(\tau), x(\tau)) A + \text{h.c.} \right]$$

$$= gm \left[ \phi^\dagger_m A + \text{h.c.} \right]$$

(42)

where we have introduced for convenience the field operator

$$\phi_m = \int d\tau e^{+im\tau} f^*(\tau) \phi(\tau)$$

(43)
and its hermitian conjugate $\phi_m^\dagger$.

The probability $P_e$ for the two level atom to get excited is, in second order perturbation theory

$$P_e = P_{e,v} + P_{e,u} = 2P_{e,v}$$

$$P_{e,v} = g^2m^2 <0_M|\int d\tau e^{-im\tau}f(\tau)\int d\tau' e^{+im\tau'}f^*(\tau')|0_M>$$

$$= g^2m^2 <0_M|\phi_m^\dagger\phi_m|0_M>$$  \hspace{1cm} (44)

where $P_{e,v}$ and $P_{e,u}$ are the contribution to the probability of left movers and right movers.

If $e^{-im\tau}f(\tau)$ contains no negative frequencies in its Fourier transform with respect to $\tau$ then eq. \[41\] defines a Lee model: when it is inertial it only responds to the presence of real particles. When following a non inertial trajectory it responds to the presence of local quanta. For a Lee model $f(\tau)$ may not decrease as quickly as an exponential when $\tau \to \pm \infty$. This constraint will be seen to be very strong and we shall work necessarily with non Lee models which can spontaneously excite. However by choosing $f(\tau)$ such that the negative frequency part of $e^{-im\tau}f(\tau)$ is exponentially small this spontaneous excitation is exponentially small as well.

We now consider the weak values of $T_{\mu\nu}$. As described in chapter 2 they take the form

$$<T_{\mu\nu}(t_0,x_0)>_{\text{weak e}} = \frac{g^2m^2}{2P_{e,v}} <0_M|\phi_m^\dagger T[T_{\mu\nu}(t_0,x_0)\int d\tau e^{+im\tau}f^*(\tau)\phi(\tau)]|0_M>$$  \hspace{1cm} (45)

where $T$ is the time ordering operator and the subscript e refers to the post-selection of the two level atom in its excited state at $t = +\infty$.

If the interaction lasts only from $\tau_i$ to $\tau_f$, i.e. $f(\tau) = 0$ for $\tau < \tau_i$ or $\tau > \tau_f$, then in the past of the future light cone centred on $t(\tau_i), x(\tau_i)$ (this region of space time shall be called $I_-$) or in the future of the past light cone centred on $t(\tau_f), x(\tau_f)$ (denoted $I_+$) the time ordering operator $T$ is trivial to implement and $<T_{\mu\nu}>_{\text{weak e}}$ takes the simple form

$$<T_{\mu\nu}(I_-)>_{\text{weak e}} = \frac{g^2m^2}{2P_{e,v}} <0_M|\phi_m^\dagger\phi_m T_{\mu\nu}|0_M>$$  \hspace{1cm} (46)

$$<T_{\mu\nu}(I_+)>_{\text{weak e}} = \frac{g^2m^2}{2P_{e,v}} <0_M|\phi_m^\dagger T_{\mu\nu}\phi_m|0_M>$$  \hspace{1cm} (47)

In the regions where $I_+$ and $I_-$ overlap these two expressions coincide because $T_{\mu\nu}(t_0,x_0)$ and $\phi_m$ commute since the point where $T_{\mu\nu}$ is evaluated is seperated from the trajectory by a space like distance.

The probability that the two level atom is found in its ground state, at $t = +\infty$, is $P_g = 1 - P_e$. When this occurs the weak value of $T_{\mu\nu}$ takes the form

$$<T_{\mu\nu}(I_-)>_{\text{weak g}} = \frac{1}{P_g}<0_M|T_{\mu\nu}|0_M> - g^2m^2 <0_M|\phi_m^\dagger\phi_m T_{\mu\nu}|0_M>$$

$$<T_{\mu\nu}(I_+)>_{\text{weak g}} = \frac{1}{P_g}<0_M|T_{\mu\nu}|0_M> -$$

$$2g^2m^2\text{Re} [<0_M|T_{\mu\nu}\int d\tau_2 d\tau_1 e^{-im\tau_2}f(\tau_2)\phi(\tau_2) e^{+im\tau_1} f^*(\tau_1)\phi(\tau_1)|0_M>]$$  \hspace{1cm} (48)

\[48\]
where for simplicity we have given only the expressions valid in $I_-$ and $I_+$. The first term in eq. 48 and eq. 49 comes from that part of the wave function in which the two level atom has remained for all times in its ground state whereas the second term is an interference effect between those amplitudes wherein the atom has remained in its ground state and those amplitudes wherein it got excited and deexcited successively.

The weak values eq. 46, eq. 47 and eq. 48, eq. 49 are related to the mean energy momentum by the unitary relation

$$<T_{\mu\nu}> = P_e <T_{\mu\nu}>_{weak e} + P_g <T_{\mu\nu}>_{weak g}$$  \hspace{1cm} (50)

If $(t_0, x_0)$ belongs to $I_-$, $<T_{\mu\nu}>$ is simply

$$<T_{\mu\nu}(I_-)> = <0_M|T_{\mu\nu}|0_M>$$  \hspace{1cm} (51)

since the interaction has not yet taken place. On the contrary, if $(t_0, x_0)$ belongs to $I_+$, $<T_{\mu\nu}>$ is the mean energy radiated

$$<T_{\mu\nu}(I_+)> = <0_M|T_{\mu\nu}|0_M> + g^2 m^2 <0_M|\phi_t^* T_{\mu\nu} \phi_t |0_M> - 2g^2 m^2 \text{Re} (<0_M|T_{\mu\nu} f d\tau_2 f^* d\tau_1 e^{-i\gamma \tau_2} f(\tau_2) \phi(\tau_2) e^{+i\gamma \tau_1} f^*(\tau_1) \phi(\tau_1)|0_M>)$$

$$\hspace{1cm}$$  \hspace{1cm} (52)

It is convenient to reexpress the last term in eq. 49 and eq. 52 as

$$2 \text{Re} [<0_M|T_{\mu\nu} f d\tau_2 f^* d\tau_1 e^{-i\gamma \tau_2} f(\tau_2) \phi(\tau_2) e^{+i\gamma \tau_1} f^*(\tau_1) \phi(\tau_1)|0_M>]$$

$$= \text{Re} [<0_M|\phi_t^* T_{\mu\nu} \phi_t |0_M>]$$

$$+ \text{Re} [<0_M|T_{\mu\nu} f d\tau_2 f d\tau_1 e^{-i\gamma \tau_2} f(\tau_2) \phi(\tau_2) e^{+i\gamma \tau_1} f^*(\tau_1) \phi(\tau_1)]_0 |0_M>]$$

$$\hspace{1cm}$$  \hspace{1cm} (53)

where $\epsilon(\tau) = \theta(\tau) - \theta(-\tau)$. The first term is equal to the real part of eq. 46. This stems from the fact that in both cases one is probing, through an interference effect, the structure of the vacuum fluctuations that can excite or deexcite the atom. The second term takes into account that if the atom was excited and then deexcited it necessarily occurred in that order. Being a commutator, it carries neither Minkowski energy nor Rindler energy because Minkowski vacuum is an eigenstate of $H_M$ (eq. 20) and $H_R$ (eq. 33). Furthermore it vanishes when $(t_0, x_0)$ belongs to the intersection of $I_-$ and $I_+$. In view of these properties the only effect of this term is to redistribute the flux density within the regions in causal contact.

### 4.2 The uniformly accelerated two level atom

We now evaluate the matrix elements that appear in section 4.1 when the two level atom is uniformly accelerated with acceleration $a$:

$$t_a(\tau) = a^{-1} \sinh a \tau \, , \, x_a(\tau) = a^{-1} \cosh a \tau$$  \hspace{1cm} (54)

Consider first that $f(\tau)$ is equal to 1 between $\tau_i$ and $\tau_f$ and that $\tau_f - \tau_i = T \rightarrow +\infty$ while $g^2 m T$ remains finite. Then a direct golden rule calculation shows that the probability for the uniformly accelerated atom to get excited is

$$P_{e,v} = \frac{1}{2} g^2 m T N_m$$  \hspace{1cm} (55)
where \( N_m = 1/(e^{2\pi m/a} - 1) \) is the Bose Einstein distribution.

In this limit the operator \( \phi_m^\dagger \phi_m \) appearing in eq. 46 becomes the counting operator for rindlerons of energy \( m \): \( a_m^\dagger a_m \). By getting excited, the accelerated atom has post-selected that part of the vacuum wave function \( |0_M> \) which contains a rindleron of energy \( m \). This counting operator respects the bosonic statistics of the field \( \phi \) and differs from the projector eq. 36 when acting on states with two or more rindlerons of energy \( m \). (It is only when \( m >> a \), i.e. in the Maxwell-Boltzman limit, that these two operators and their weak values eq. 37 and eq. 46 coincide. This was precisely the case studied in [6].)

We turn now to the problem of a two level atom that interacts only during a finite time with the field. First of all we notice that all the above matrix elements of \( T_{V,V}(t_0, x_0) \) when \( (t_0, x_0) \) belongs to \( I_- \) or \( I_+ \) (except the second term of eq. 53) can be expressed in terms of \( P_{e,v} \) and of the following two functions

\[
C_+(V) = <0_M|\phi(V)\phi_m^\dagger|0_M> = \int d\tau G_+(V, V_\alpha(\tau))e^{-im\tau} f(\tau)
\]

\[
C_-(V) = <0_M|\phi(V)\phi_m|0_M> = \int d\tau G_+(V, V_\alpha(\tau))e^{+im\tau} f^*(\tau)
\]

(56)

For instance, discarding \( <0_M|T_{\mu\nu}|0_M> \), eq. 46 and eq. 47 read,

\[
<T_{V,V}(I_-)>_{\text{weak } e} = \frac{g^2 m^2}{P_{e,v}} (\partial_V C_+^*)(\partial_V C_-^*)
\]

\[
<T_{V,V}(I_+)>_{\text{weak } e} = \frac{g^2 m^2}{P_{e,v}} (\partial_V C_-)(\partial_V C_+^*)
\]

(57)

For \( T_{V,V} \) not to be singular the functions \( \partial_V C_+(V) \) and \( \partial_V C_-(V) \) must be regular (The last term in eq. 53 necessarily vanishes on the horizon \( V = 0 \)). \( \partial_V C_+(V) \) is equal to

\[
\partial_V C_+(V) = -\frac{1}{4\pi} \int d\tau \frac{1}{V - a^{-1} e^{a\tau} - i\epsilon} f(\tau) e^{-im\tau}
\]

(58)

It can be singular only for \( V = 0 \) where it takes the form

\[
-\frac{1}{4\pi} \int d\tau \frac{1}{-a^{-1} e^{a\tau} - i\epsilon} f(\tau) e^{-im\tau} \simeq \frac{a}{4\pi} \int d\tau e^{-a\tau} f(\tau) e^{-im\tau}
\]

(59)

The last integral is finite if and only if \( f(\tau) \) decreases for \( \tau \to -\infty \) quicker than \( e^{a\tau} \). Similarly if we had considered right movers, the condition for finiteness on the future horizon would have been sufficient rapid decrease of \( f \) for \( \tau \to +\infty \). Putting all together the condition to not have singularities on the horizons is that

\[
\int d\tau \frac{dt}{d\tau} |f(\tau)| = \int dt |f(\tau(t))| < \infty
\]

(60)

The interaction of the atom with the field must last a finite Minkowski time. When this is satisfied \( f(\tau) \) decreases faster than \( e^{-a|\tau|} \) which implies that we are not considering a local Lee model.
4.3 The weak values

In order to obtain explicit expressions for $C_{\pm}$ we carry out the following construction. First define the fourier transform of $f(\tau)e^{-im\tau}$ by

$$f(\tau)e^{-im\tau} = \int_{-\infty}^{+\infty} d\lambda \frac{c\lambda}{2\pi} e^{-i\lambda \tau}$$

(61)

with the normalisation

$$\int d\tau |f(\tau)|^2 = \int d\lambda \frac{|c\lambda|^2}{2\pi} = T = \text{total time of interaction}$$

(62)

The field operator $\phi_m$, the probability $P_{e,v}$, the functions $C_{\pm}$ and the expectation values of $T_{V^V}$ can then be expressed in terms of $c\lambda$:

$$\phi_m = \int_0^\infty d\lambda \frac{a_{\lambda,R}}{\sqrt{4\pi|\lambda|}} c^*_{\lambda} + \int_{-\infty}^0 d\lambda \frac{a_{\lambda,R}^\dagger}{\sqrt{4\pi|\lambda|}} c_{\lambda}$$

$$C_{+}(V) = \int_{-\infty}^{+\infty} d\lambda c_{\lambda} \frac{1}{4\pi \lambda} \left(e^{\pi\lambda/2a\lambda,M} + e^{-\pi\lambda/2a\lambda,M} \varphi_{\lambda,M}(V)\right)$$

$$C_{-}(V) = \int_{-\infty}^{+\infty} d\lambda c_{\lambda} \frac{1}{4\pi \lambda} \left[(\tilde{n}_\lambda + 1)(aV)^{i\lambda/a}\theta(V) + \tilde{n}_\lambda e^{i\lambda/a}|aV|^{-i\lambda/a}\theta(-V)\right]$$

$$P_{e,v} = g^2 m^2 \int_{-\infty}^{+\infty} d\lambda \frac{|c\lambda|^2}{4\pi \lambda} \tilde{n}_\lambda$$

(63)

(65)

Where $\tilde{n}_\lambda = 1/(e^{2\pi\lambda/a} - 1)$ is equal to (see eq. 26)

$$\tilde{n}_\lambda = N_\lambda = \beta^2_\lambda \quad \text{for} \quad \lambda > 0$$

$$\tilde{n}_\lambda = -(N_{|\lambda|} + 1) = -a^2_{|\lambda|} \quad \text{for} \quad \lambda < 0.$$  

(64)

The contribution to the probability $P_{e,v}$ from left movers reads

As one picture is worth a thousand words we shall take a particular form for $c\lambda$ such that all the expressions can be evaluated explicitly.

$$c\lambda = D\frac{\lambda}{m} e^{-(\lambda-m)^2T^2/2}(1 - e^{-2\pi\lambda/a})$$

(66)

where $D$ is a normalisation constant taken such as to verify eq. 22. Then $f(\tau)$ reads

$$f(\tau) = \frac{D}{\sqrt{2\pi} T} e^{-\tau^2/2T^2} \left[(1 - i\frac{\tau}{mT^2}) - e^{-2\pi m/a} e^{2\pi^2/a^2T^2} e^{i2\pi\tau/aT^2} \left(1 - \frac{i\tau + 2\pi/a}{mT^2}\right)\right]$$

(67)
It clearly satisfies eq. (64).

In order that the behaviour of the uniformly accelerated two level atom with the coupling \( f(\tau) \) given by eq. (67) be physically unambiguous, it is necessary that \( c_\lambda \) be peaked around \( +m \) (the two level atom should be approximately a Lee model) and the golden rule probability of transition eq. (55) be recovered. If this is to be the case then \( T \) must satisfy \( T >> m^{-1} \) and \( T >> a^{-1} \). The first condition is the usual demand that a wave packet be spread over a distance at least equal to its inverse frequency. The second condition, which corresponds to \( T \) being greater than the euclidean tunneling time \( 2\pi a^{-1} \), is required for the probability \( P_{e,v} \) to be linear in time and proportional to the Bose distribution \( N_m \). Then \( D \simeq \frac{2^{1/2} \pi^{1/4}}{\sqrt{T}} (N_m + 1) \) and

\[
f(\tau) \simeq \pi^{-1/4} e^{-\tau^2/2T^2} (1 + N_m (1 - e^{i2\pi\tau/aT^2}))
\] (68)

We have indicated throughout by the symbol \( = \) the exact expressions for which no approximation has been made and by the symbol \( \simeq \) the approximate expressions valid when \( T >> m^{-1} \) and \( T >> a^{-1} \) as these last are particularly easy to read and understand.

The weak values of \( T_{wv} \) are readily obtained

\[
<T_{wv}(I_-, V > 0) >_{weak e} = \frac{g^2 m^2}{P_{e,v}} \int d\lambda \int d\lambda' c_\lambda \bar{c}_{\lambda'} \frac{1}{(4\pi)^2} \bar{n}_\lambda (\bar{n}_{\lambda'} + 1) e^{-i(\lambda - \lambda')v}
\]

\[
= \frac{m(N_m + 1)}{2\sqrt{\pi T} C_0} (1 - \frac{i}{m\pi} + \frac{2\pi}{a}) (1 + \frac{i}{mT^2}) e^{-(v-i\pi/a)^2/T^2}
\]

\[
\simeq \frac{m(N_m + 1)}{2\sqrt{\pi T}} e^{-(v-i\pi/a)^2/T^2}
\] (69)

\[
<T_{wv}(I_-, V < 0) >_{weak e} = < T_{wv}(I_+, V < 0) >_{weak e}
\]

\[
= \frac{g^2 m^2}{P_{e,v}} \int d\lambda \frac{1}{4\pi} e^{i\pi/a} - e^{-i\pi/a} e^{-i\lambda v}\]

\[
= \frac{m(N_m + 1)}{2\sqrt{\pi T} C_0} |1 - \frac{i}{m\pi} + \frac{2\pi}{a}|^2 e^{-v^2/T^2}
\]

\[
\simeq \frac{m(N_m + 1)}{2\sqrt{\pi T}} e^{-\frac{v^2}{T^2}}
\] (70)

\[
<T_{wv}(I_+, V > 0) >_{weak e} = \frac{g^2 m^2}{P_{e,v}} \int d\lambda \bar{c}_\lambda e^{-i\lambda v}\]

\[
= \frac{mN_m}{2\sqrt{\pi T} C_0} |1 - \frac{i}{m\pi} + \frac{2\pi}{a}|^2 e^{-\frac{v^2}{T^2}} e^{3\pi^2/a^2 T^2}
\]

\[
\simeq \frac{mN_m}{2\sqrt{\pi T}} e^{-\frac{v^2}{T^2}}
\] (71)

\[\text{Strictly speaking, for the model eq. (67), } I_- \text{ does not include regions where } V > 0 \text{ since the interaction lasts for an infinite Rindler time. Nevertheless, since } f(\tau) \text{ decreases very quickly for } \tau \to -\infty, \text{ the limiting value of } < T_{wv} >_{weak} \text{ for } V > 0 \text{ as } u \to -\infty \text{ coincides with our expression.}\]
where \( C_0 \) is a constant equal to

\[
C_0 = (N_m + 1)^{-1} \left[ (1 - \frac{\pi}{amT^2}) - e^{-2\pi m/a} e^{3\pi^2/a^2 T^2} (1 - \frac{2\pi}{amT^2}) \right] \\
\simeq 1
\]

We now present the complementary Rindler and Minkowski interpretations of the weak values of \( T_{vv} \).

The Rindler description is that used by a uniformly accelerated observer in the same quadrant as the two level atom. It is best understood by making appeal to the isomorphism of the state of the field in the right Rindler quadrant with an inertial thermal bath.

By getting excited the two level atom has selected that the thermal bath contain at least one particle in the mode created by \( \phi_m^\dagger \). Furthermore energy flows for a massless field along the lines \( u = \text{cst} \) and \( v = \text{cst} \). Therefore \( < T_{vv}(I-, V > 0) >_{\text{weak}} \) is centred around \( v = 0 \) with at spread \( \Delta v = T \) and carries a Rindler energy obtained by integrating eq. 69

\[
\int dv < T_{vv}(I-, V > 0) >_{\text{weak}} \approx \frac{1}{2} \int d\lambda |c_\lambda|^2 \tilde{n}_\lambda \tilde{n}_\lambda + 1) \approx \frac{1}{2} m(N_m + 1)
\]

The factor \( N_m + 1 \) takes correctly into account the Bose statistics of the field since eq. 73 corresponds to evaluating \( < n^2 > / < n > \) in a thermal distribution. The factor 1/2 arises since the atom could also have been excited by \( u \) quanta.

Since by getting excited the two level atom has absorbed one quantum the residual energy on \( I^+ \) is (see eq. 71)

\[
\int dv < T_{vv}(I+, V > 0) >_{\text{weak}} = \frac{1}{2} \int d\lambda |c_\lambda|^2 \tilde{n}_\lambda \tilde{n}_\lambda \approx \frac{1}{2} mN_m
\]

We now consider what is seen by a uniformly accelerated observer in the left Rindler quadrant. Since Minkowski vacuum is an eigenstate of the total Rindler energy (the boost operator) eq. 33, the Rindler energy in the left quadrant is equal to the energy in the right quadrant before the transition occurs. Indeed integrating eq. 70 and using the relation \( \tilde{n}_\lambda(\tilde{n}_\lambda + 1) = 1/(e^{\pi\lambda/a} - e^{-\pi\lambda/a})^2 \) yields

\[
\int dv < T_{vv}(I-, V > 0) >_{\text{weak}} = \int dv < T_{vv}(I-, V < 0) >_{\text{weak}}
\]

The symmetry between the left and the right Rindler quadrants results in \( < T_{vv}(I-, V < 0) >_{\text{weak}} \) being also centred around \( v = 0 \) with width \( \Delta v = T \). We have plotted these weak values of \( T_{vv} \) in figure 1.

The Minkowski description, i.e. that used by an inertial observer, is best understood by rewriting the weak value of \( T_{VV} \) as

\[
'er<T_{VV}(I_-)>_{\text{weak}} e = \frac{1}{a^2 V^2} \int d\lambda \int d\lambda' c_\lambda'^* c_\lambda \frac{1}{4\pi} \sqrt{\lambda' \lambda' \tilde{n}_\lambda (\tilde{n}_\lambda + 1) \varphi_{\lambda,M} \varphi_{\lambda',M}} \times (1 - \frac{i}{maT^2} \ln(-aV - i\epsilon) - \frac{\pi}{maT^2}) \times (1 - \frac{i}{maT^2} \ln(-aV - i\epsilon) - \frac{\pi}{maT^2}) e^{-[\ln(-aV - i\epsilon)]^2/a^2 T^2} (76)
\]
which we have sketched in figure 2. The \( i \epsilon \) is introduced only to define \( \ln(-(aV - i \epsilon) \) as \( \ln|aV| \) for \( V < 0 \) and as \( \ln|aV| - i \pi \) for \( V > 0 \). Upon taking the limit \( \epsilon \to 0 \) no singularity occurs. In fact \( \langle T_{VV}(I_-) \rangle_{weak e} \) vanishes for \( V = 0 \). This is an accident due to the particular form of \( c_\lambda \) chosen in eq. (66) (it has zero’s for \( \lambda = ina, n = \ldots, -1, 0, 1, \ldots \)). But, from the expression for \( C_{\pm}(V = 0) \) given in eq. (59), it results that the generic behaviour of \( T_{VV} \) is to stay finite as \( V \to 0 \). In more physical terms this corresponds to saying that the Minkowski vacuum fluctuation that induces the transition straddles the horizon with no clear cut separation between the pieces in the left and right quadrants.

In \( I_- \), the total Minkowski energy (the integral of eq. (76) with respect to \( V \)) vanishes since \( |0_M \rangle \) is an eigenstate of \( H_M \) (eq. (20)). In other words the total Minkowski energy does not fluctuate and is always equal to its eigenvalue zero: vacuum fluctuations carry no energy. The Minkowski energy in the region \( V < 0 \) is real and positive therefore the energy in the region \( V > 0 \) must integrate to an exactly compensating real and negative value. This is not in contradiction with the positivity of Rindler energy in the right quadrant since the expressions for the Rindler and the Minkowski energy differ by the jacobian \( dv/dV = 1/aV \). The oscillations of \( T_{vv} \) for \( V > 0 \) that occur in eq. (69) as \( v \to -\infty \) (which are negligible in the Rindler description) are dramatically enhanced by the jacobian in such a way that the Minkowski energy in the right quadrant becomes negative.

In \( I_+ \), after the atom has made a transition, the Minkowski energy takes the form

\[
\langle T_{VV}(I_+) \rangle_{weak e} = \frac{1}{a^2 V^2} \frac{g^2 m^2}{P_{e,v}} \int d\lambda c_\lambda \sqrt{\frac{\lambda \tilde{n}_\lambda}{4\pi}} e^{-\lambda \tilde{n}_\lambda M} \left| e^{-i a V - i \epsilon} \right|^2
\]

\[
= \frac{1}{a^2 V^2} \frac{m(N^2_m + 1)}{2 \sqrt{\pi T C_0}} \left| 1 - \frac{i}{m a T^2} \ln(-aV - i \epsilon) - \frac{\pi}{m a T^2} \right|^2 \left| e^{-[\ln(-aV - i \epsilon)]^2/a^2 T^2} \right| e^{-i m \ln(-aV - i \epsilon)/a^2} \left| e^{-im \ln(-aV - i \epsilon)/a^2} \right|^2
\]

(77)

It is manifestly real and positive. This is as it should be since we are calculating the mean value of the energy in a state that is not Minkowski vacuum. By absorbing the positive Rindler energy \( m \), the two level atom has reduced the negative Minkowski energy on the right.

These results will be used when analysing the mean fluxes emitted by an accelerated oscillator. This is the subject of the next chapter.

5 Fluxes and Particles Emitted by an Accelerated Oscillator

5.1 introduction

We analyse the mean fluxes emitted during the thermalisation period (i.e. when the initial state is the ground state) and in thermal equilibrium. The analysis is first carried out to order \( g^2 \) using the model of chapter 4. In a second stage we use the model introduced by Raine, Sciama and Grove (RSG) [12] to prove that the various properties characterising thermal equilibrium previously obtained in \( g^2 \) are recovered to all orders in \( g \).
We briefly sketch the main results. During thermalisation a steady flux of negative Rindler energy is emitted ($<T_{vv}> \simeq -g^2 m^2 N_m/2$). This is understood from the isomorphism \[11\] with the thermal bath: as the atom gets excited it absorbs energy from the thermal bath, thus the minus sign. The transcription of this flux to Minkowski quanta is more subtle. Oscillatory tails in the Rindler flux are enhanced by the jacobian that converts from Rindler to Minkowski energy with the net result that positive Minkowski energy is emitted. In the Minkowski description the origin of the steady negative flux is due to a "repolarisation" of the atom corresponding to the fact that the probability of finding the atom in its exited level decreases with time. This repolarisation is similar (CPT conjugate) with that which occurs when negative energy is absorbed by an inertial detector \[22\].

When thermal equilibrium is reached the emission of Rindler energy ceases because the aborption of Rindler energy associated with exciting the atom is exactly compensated (except for oscillatory transients) by the emission provoked by the inverse quantum jump. Nevertheless in this equilibrium situation there is a net production of Minkowski energy because both absorption and emission of Rindler quanta correspond to emission of Minkowski quanta. These Minkowski quanta interfere in such a way that their energy content is located only at the ends of the interaction period (in the oscillatory transients). The compatibility of the two descriptions arises, once more, from the time dependence of the Doppler shift relating Rindler and Minkowski frequencies. It is remarkable that this Doppler factor $dV/dv = e^{av}$ both leads to the thermalisation of the accelerated atom through the Bogoljubov transformation eq. \[25\] and allows the conciliation of the two descriptions of the emitted flux.

5.2 Fluxes and particles in $g^2$ during thermalisation

The analysis is first carried out for the adiabatic switch on and off used in chapter 4 (to reveal the oscillatory tails) and then for a sudden switch on and off (to display the stationary regime).

The mean energy radiated by the uniformly accelerated two level atom is given by (see eq. \[51\] et seq.)

\[
<T_{vv}>_e = P_e<T_{vv}(I_-)>_{weak e} + P_g<T_{vv}(I_-)>_{weak g}
\]

\[
= g^2 m^2 <0_M|\phi_m^{+} T_{vv} \phi_m|0_M> - g^2 m^2 \text{Re} \left( <0_M|\phi_m^{+} \phi_m T_{vv}|0_M> \right) \\
- g^2 m^2 \times \text{last term of eq. 53} \quad (78)
\]

The third term carries neither Rindler nor Minkowski energy, it is non vanishing only in the causal future of the atom and will be discarded. The first two terms taken separately are non vanishing outside of the causal future of the two level atom but upon taking their sum causality is restored. Their sum reads

\[
<T_{vv}(I_+)>_e = -g^2 m^2 \int d\lambda \int d\lambda' c_\lambda c_\lambda' \frac{1}{(4\pi)^2} (\tilde{n}_\lambda + \tilde{n}_{\lambda'}) e^{-i(\lambda-\lambda')v} \\
\simeq -\frac{g^2 m^2}{2} N_m \frac{e^{-v^2/T^2}}{\pi^{1/2}} \left[ (N_m + 1) \cos(2\pi v/aT^2) - N_m \right] \quad (79)
\]
As announced it carries negative Rindler energy:

\[
\int dv < T_{vv}(I_+) >_e = -\frac{g^2 m^2}{4\pi} \int d\lambda |c_\lambda|^2 \tilde{n}_\lambda
\]

\[\simeq \frac{1}{2} g^2 m^2 N_m T = -m P_{e,v} \quad (80)\]

The total Minkowski energy radiated is computed by integrating eq. 79:

\[
\int dV < T_{VV}(I_+) >_e = \int dv e^{av} < T_{vv}(I_+) >_e
\]

\[\simeq \frac{1}{2} g^2 m^2 N_m T e^{a\tau_0} (1 + 2N_m) = +m P_{e,v} e^{a\tau_0} (1 + 2N_m) \quad (81)\]

where \(e^{a\tau_0}\) is the mean Doppler effect associated with the window function \(f(\tau)\) eq. 67:

\[
\int dv e^{-av} e^{-v^2/T^2} \cos(2\pi v/aT^2) \simeq -e^{a\tau_0} \quad (82)\]

Note that the sign flip in eq. 82 of the Minkowski energy versus the Rindler energy can be conceived as arising from the imaginary part of the saddle point of eq. 82: \(v_{sp} = -aT^2/4 + i\pi/a\) and is for that reason very similar to the flip of frequency which leads through a tunnelling amplitude to a non vanishing \(\beta\) coefficient (see [9]). The additional factor \(2N_m + 1\) comes probably from the particular switch off function \(f(\tau)\).

In order to get a more precise picture of the physics involved we now analyse the case where the time dependent coupling is \(f(\tau) = \theta(\tau) \theta(T - \tau)\). With this time dependence the transients are singular (as is seen by considering the Fourier transform of the \(\theta\) function) and will not be studied. On the contrary the steady part is easily computed and corresponds exactly to the intermediate values \((-aT^2 << \tau << aT^2)\) found in the adiabatic situation described above in eq. 79. In addition it shows explicitly the relations that exist between the flux emitted and the transition rate (not only the probability as in eq. 80).

We first compute, using standard perturbation theory, the relevant formulae for the probability of transition, for the rate of transition and for the emitted flux. The probability of spontaneous emission (due to the \(v\)-modes only) is given by

\[
P_{e,v}(T) = g^2 m^2 \int_0^T d\tau_1 \int_0^T d\tau_2 e^{-im(\tau_2 - \tau_1)} < \phi(\tau_2) \phi(\tau_1) >
\]

\[\simeq \frac{1}{g^2 m N_m T} \quad (83)\]

The second line contains the golden rule result valid when \(aT \to \infty\) with \(g^2 T\) finite. It is useful for the following to introduce the rate of transition, the derivative of \(P_{e,v}(T)\):

\[
\dot{P}_{e,v}(T) = \frac{dP_{e,v}(T)}{dT} = g^2 m^2 2{\Re} \left[ \int_0^T d\tau e^{-im(T - \tau)} < \phi(T) \phi(\tau) > \right]
\]

\[\simeq \frac{1}{2} g^2 m N_m \quad (84)\]
On the left hand side of the accelerated trajectory, this rate is related to the (steady part of) the stress energy tensor:

\[
<T_{vv}(v = T)>_e = g^2 m^2 2\text{Re} \left[ \int_0^T d\tau_2 \int_0^{\tau_2} d\tau_1 e^{-im(\tau_2 - \tau_1)} <[\phi(\tau_2), T_{vv}(T)]\phi(\tau_1)> \right]
\]

\[
= g^2 m^2 2\text{Re} \left[ \int_0^T d\tau e^{-im(T-\tau)} <i\partial_v\phi(T)\phi(\tau)> \right]
\]

\[
= -m\dot{P}_{e,v}(T) + g^2 m^2 2\text{Re} \left[ie^{-imT} <\phi(T)\phi(0)> \right]
\]  

(85)

The first equality follows straightforwardly from the expansion of the evolution operator \(e^{-i\int H_{int} d\tau}\) in \(g^2\). The second equality is obtained using \([\phi(\tau_2), T_{vv}(\tau_1)] = i\partial_v\phi(\tau_1 - \tau_2)\). The third equality follows by using

\[
<\partial_{\tau_1}\phi(\tau_1)\phi(\tau_2)> = -<\phi(\tau_1)\partial_{\tau_2}\phi(\tau_2)> \quad \text{and integrating by parts.}
\]

The final result contains a steady part proportional to \(-m\dot{P}_{e,v}(T)\) which tends to \(-\frac{1}{2}g^2 m^2 N_m\) in the golden rule limit and an oscillatory term (which is exponentially damped if a slight mass is given to \(\phi\)). The steady piece simply indicates that to an increase of the probability to make a transition corresponds the absorption of the necessary Rindler energy to provoke this transition.

These expressions are now decomposed in terms of the Minkowski basis \(e^{-i\omega V}\sqrt{4\pi\omega}\).

The probability of transition eq. (83) reads

\[
P_{e,v}(T) = g^2 m^2 \int_0^\infty d\omega \left| \int_0^T d\tau e^{-imT} e^{-\frac{i\omega \tau}{4\pi\omega}} \right|^2
\]

\[
= \int_0^\infty d\omega P_{e,v,\omega}(T)
\]  

(86)

Similarly the transition rate eq. (84) becomes

\[
\dot{P}_{e,v}(T) = g^2 m^2 \int_0^\infty d\omega 2\text{Re} \left[ \int_0^T d\tau e^{-i\omega(\tau - T)} e^{-i\omega\tau(\tau + T)} \right]
\]

\[
= \int_0^\infty d\omega \dot{P}_{e,v,\omega}(T)
\]  

(87)

And the total Minkowski energy is given by

\[
H_e(T) = \int_{-\infty}^{+\infty} d\omega \omega P_{e,v,\omega}(T)
\]

(88)

Where in the first equality the integral is only over region of positive \(V\) since by causality the mean energy is unaffected in the other quadrant. The second equality follows from the diagonal character of the energy operator. The positivity of \(H_e(T)\) is manifest since all the \(P_{e,v,\omega}(T)\) are positive definite. Nevertheless the time derivative of \(H_e(T)\) is negative, within the steady regime,

\[
\frac{dH_e}{dT} = \int_0^\infty d\omega \omega \dot{P}_{e,v,\omega}(T)
\]
\[ dH/dT \text{ negative implies that, for large } \omega \text{ (since } \dot{P}_e(T) > 0), \text{ some } \dot{P}_{e,\omega} \text{ are negative. This corresponds to a "repolarisation" since all the } P_{e,\omega} \text{ are positive definite and vanish for } \tau \leq 0. \text{ This repolarisation is exactly the inverse process of the absorption of negative energy by an atom [22].} \]

5.3 Fluxes and particles in \( g^2 \) at equilibrium

Before studying the equilibrium situation it behoves us first to consider the flux emitted by an atom that makes a transition from excited to ground state.

The probability (due to the \( v \)-modes) that a uniformly accelerated two level atom initially in its exited state ends up in its ground state is

\[ P_{d,v} = g^2 m^2 <0_M|\phi_m \phi^\dagger_m|0_M> \]

\[ \simeq \frac{1}{2} g^2 m (N_m + 1) T \]

The mean energy emitted is, in the adiabatic switch off case,

\[ <T_{vv}>_d = g^2 m^2 <0_M|\phi_m T_{vv} \phi^\dagger_m|0_M> - g^2 m^2 \text{Re} \left[ <0_M|\phi_m \phi^\dagger_m T_{vv}|0_M> \right] - g^2 m^2 \times \text{last term of eq. 53} \]

The sum of the first two terms reads

\[ <T_{vv}>_d \simeq \frac{g^2 m^2}{4\pi} \int d\lambda c^* \lambda c_\lambda \frac{1}{(4\pi)^2} (\tilde{n}_\lambda + \tilde{n}_{\lambda'}) e^{-i(\lambda - \lambda')v} \]

\[ \simeq \frac{g^2 m^2}{2\sqrt{\pi}} (N_m + 1) e^{-v^2/T^2} [1 - N_m \{ \cos(2\pi v/aT^2) - 1 \}] \]

The total Rindler energy radiated is

\[ \int dv <T_{vv}(I_+)>_d = \frac{g^2 m^2}{4\pi} \int d\lambda |c_\lambda|^2 (\tilde{n}_\lambda + 1) \]

\[ \simeq \frac{1}{2} g^2 m^2 (N_m + 1) T = m P_{d,v} \]

In the example for which the time dependent coupling is \( f(\tau) = \theta(\tau)\theta(T - \tau) \), one finds the following relation between the derivative of the probability \( \dot{P}_{d,v}(T) \) and the flux \( <T_{vv}>_d \):

\[ <T_{vv}(T)>_d = +m \dot{P}_{d,v}(T) + \text{oscillatory "damped" term} \]

The sign in front of \( \dot{P}_{d,v}(T) \) is now positive (contrary to the one in eq. [85]). Deexcitation consists in emitting the energy stored in the atom.

The total Minkowski energy emitted is obtained by integrating the first term of eq. [91] only:

\[ \int dV <T_{VV}>_d = \int dV g^2 m^2 <0_M|\phi_m T_{VV} \phi^\dagger_m|0_M> \]

\[ \simeq \frac{g^2 m^2}{2} (N_m + 1) T e^{\alpha \tau_0} (2N_m + 1) = m P_{d,v} e^{\alpha \tau_0} (2N_m + 1) \]
For the deexcitation, the integrated Rindler and Minkowski energies have the same sign and are related by the mean Doppler shift $e^{\alpha \tau_0} \times (2N_m + 1)$.

We now turn to the thermal equilibrium situation. The occupation probabilities, $p_0$ for the ground state and $p_1$ for the excited one, are related to the transition rates by the Einstein relations:

$$\frac{p_1}{p_0} = \frac{\dot{P}_e}{\dot{P}_d} = \frac{N_m}{N_m + 1} = e^{-2\pi m/a} \quad (96)$$

The energy radiated is the sum of the fluxes emitted when the atom is initially in its ground state and when the atom is initially in its exited state weighted by their initial probabilities. This stems from the fact that the energy momentum operator changes the photon number by an even number and that the interaction hamiltonian changes the photon number by an odd number while changing the state of the atom. Hence one has

$$< T_{vv} >_{\text{equil}} = p_0 < T_{vv} >_e + p_1 < T_{vv} >_d$$
$$\simeq -mp_0 \dot{P}_{e,v} + mp_1 \dot{P}_{d,v} = 0 \quad (97)$$

The steady fluxes cancel exactly because of thermal equilibrium. This is Grove theorem in $g^2$ [11][13]. Only the oscillatory transients remain. They read for the smooth switch on and off

$$< T_{vv} >_{\text{equil}} = g^2 m^2 \frac{1}{2N_m + 1} \int d\lambda \int d\lambda' c_{\lambda} c_{\lambda'}^* \frac{1}{(4\pi)^2}$$
$$[N_m(n_{\lambda} + n_{\lambda'} + 2) - (N_m + 1)(n_{\lambda} + n_{\lambda'})] e^{-i(\lambda - \lambda')v} \quad (98)$$

where $1/(2N_m + 1)$ comes from the normalisation of probabilities: $p_0 + p_1 = 1$. We have sketched $< T_{VV} >_{\text{equil}}$ and $< T_{vv} >_{\text{equil}}$ in figure 3. The total Rindler energy emitted is then

$$\int dv < T_{vv} >_{\text{equil}} = g^2 m^2 \frac{1}{4\pi} \int d\lambda |c_{\lambda}|^2 (N_m - n_{\lambda}) \quad (99)$$

It tends to zero when $c_{\lambda}$ tends to a $\delta$ function as the time of interaction tends to $\infty$. However, the total Minkowski energy increases with time and is given by

$$\int dV < T_{VV} >_{\text{equil}} = p_0 \int dV < T_{VV} >_e + p_1 \int dV < T_{VV} >_d$$
$$\simeq m(p_0 \dot{P}_{e,v} + p_1 \dot{P}_{d,v}) Te^{\alpha \tau_0} (2N_m + 1) \quad (100)$$

The Minkowski energy of the two fluxes coincide and sum up. Notice that this result equals the "naive" guess which is: The total energy is the integral over the interacting period of the rate of transition times the varying Doppler shift times the energy gap $m$. We now go to all order in $g$ to prove that this emission of Minkowski quanta is not an artefact of the second order perturbation theory.

### 5.4 Particles and Fluxes to all order in $g$

We use the exactly solvable model of RSG [12][13][14] to prove that one does recover, to all order in $g$, that every quantum jump of the accelerated oscillator, in thermal equilibrium in Minkowski vacuum, leads to the emission of a Minkowski quantum. Hence
the rate of production of the Minkowski quanta is simply the rate of internal transitions of the oscillator. But, as in second order perturbation theory, these quanta interfere and their energy content is found (due to the complete neglect of the recoil) at the edges of the interacting period only. Then we give a general proof that the stationary thermal Rindler equilibrium corresponds to a production of Minkowski quanta.

We first recall the main properties of the RSG model and then analyse the particle content of the emitted fluxes.

The action of this system consisting of a massless field coupled to a harmonic oscillator maintained in constant acceleration is

\[ S = \int dt dx \left[ \frac{1}{2} \left( (\partial_t \phi)^2 - (\partial_x \phi)^2 \right) + \int d\tau \left[ \frac{1}{2} \left( (\partial_\tau q)^2 - m^2 q^2 \right) + e(\partial_\tau q)\phi \right] \right] \delta^2 \left( X^\mu - X^\mu_0(\tau) \right) \]  

(101)

where \( X^\mu(\tau) \) is the accelerated trajectory eq. 54 and \( \epsilon = g\sqrt{2m} \) is a rescaled coupling constant. Since this action is quadratic, the Heisenberg equations are identical to the classical Euler Lagrange ones. They read:

\[ \partial_u \partial_v \phi = \frac{\epsilon}{4} \theta(V) \delta(\rho - 1/a) \partial_\tau q \]  

(102)

\[ \partial^2_\tau q + m^2 q = -e\partial_\tau \phi(X^\mu(\tau)) \]  

(103)

The left part of the field (i.e. for \( V < 0 \)) is, by causality, identically free. And, for \( V > 0 \), on the left of the accelerated oscillator trajectory, the \( v \)-part of the field only is scattered. There the general solution is

\[ \tilde{\phi}(u, v) = \phi(u) + \phi(v) + \frac{\epsilon}{2}\tilde{q}(v) \]  

(104)

\[ \tilde{q}(v) = q(v) + i \int_{-\infty}^{+\infty} d\lambda \psi(\lambda) e^{-i\lambda v} \left[ \phi_{\lambda, R,v} + \phi_{\lambda, R,u} \right] \]  

(105)

where \( \phi(u) \) and \( \phi(v) \) are the homogenous free solutions of eq. 102 where the operator \( \phi_{\lambda, R,v} \) is defined by

\[ \phi_{\lambda, R,v} = \int \frac{dv}{2\pi} e^{i\lambda v} \phi(v) \]

\[ = \frac{1}{\sqrt{4\pi|\lambda|}} \left[ \theta(\lambda) a_{\lambda, R} + \theta(-\lambda) a^\dagger_{-\lambda, R} \right] \]  

(106)

(a similar equation defines \( \phi_{\lambda, R,u} \)); where \( \psi(\lambda) \) is given by

\[ \psi(\lambda) = \frac{e^\lambda}{m^2 - \lambda^2 - ie^2\lambda/2} \]  

(107)

and where \( q(v) \) is a solution of

\[ \partial^2_\tau q + m^2 q + \frac{\epsilon^2}{2} \partial_\tau q = 0 \]  

(108)
The two independent solutions of eq. [108] are exponentially damped as \( \tau \) increases. Being interested by the properties at equilibrium, we drop \( q(v) \) from now on. Then, the remaining part of \( \tilde{q}(v) \) is a function of the free field only. Hence, in Fourier transform, eq. [104] reads

\[
\tilde{\phi}_{\lambda,R,u} = \phi_{\lambda,R,u} \quad \tilde{\phi}_{\lambda,R,v} = \phi_{\lambda,R,v}(1 + ie^2 \psi_{\lambda}) + (i/2)\psi_{\lambda}\phi_{\lambda,R,u}
\]

(109)

The second term in eq. [109] mixes \( u \) and \( v \) modes. It encodes the static Rindler polarisation cloud (see [14] [13]) which accompanies the oscillator and carries neither Minkowski nor Rindler energy. In order to simplify the following equations, we drop it and multiply the other scattered term by two for unitary reason -see below. (By a simple and tedious algebra, one can explicitly verify that this modification does not affect the main properties of the emitted fluxes). Then eq. [109] becomes

\[
\tilde{\phi}_{\lambda,R,v} = \phi_{\lambda,R,v}(1 + ie\psi_{\lambda})
\]

(110)

It is useful, for future discussions, to introduce explicitly the scattered operators \( \tilde{a}_{\lambda,R} \), and the scattered modes \( \tilde{\varphi}_{\lambda,R}(v) \)

\[
\tilde{a}_{\lambda,R} =< \varphi_{\lambda,R}|\tilde{\phi} > = a_{\lambda,R}(1 + ie\psi_{\lambda})
\]

(111)

\[
\tilde{\varphi}_{\lambda,R}(v) = -\left[a_{\lambda,R}^\dagger, \tilde{\phi}(v)\right] = (1 + ie\psi_{\lambda})\varphi_{\lambda,R}(v)
\]

(112)

whereupon the scattered field operator \( \tilde{\phi}(v) \) may be written as

\[
\tilde{\phi}(v) = \int_0^\infty d\lambda \left[ \tilde{a}_{\lambda,R}\varphi_{\lambda,R} + h.c. \right]
\]

\[
= \int_0^\infty d\lambda \left[ a_{\lambda,R}\tilde{\varphi}_{\lambda,R} + h.c. \right]
\]

(113)

It is now straightforward to obtain the scattered Green function and its Rindler energy content. If the initial (Heisenberg) state is Minkowski vacuum the \( v \)-part of the scattered Green function is, for \( V,V' > 0 \),

\[
\tilde{G}_+(v,v') = <0_M|\tilde{\phi}(v)\tilde{\phi}(v')|0_M >
\]

\[
= \int_0^\infty d\lambda |1 + ie\psi_{\lambda}|^2 \left( \beta_{\lambda}^2 \varphi_{\lambda,R}(v)\varphi_{\lambda,R}(v') + \alpha_{\lambda}^2 \varphi_{\lambda,R}(v)\varphi_{\lambda,R}(v') \right)
\]

\[
= G_+(v,v')
\]

(114)

where \( G_+(v,v') \) is the unperturbed Minkowski Green function and where we have availed ourselves of the identity (see eq. [107])

\[
|1 + ie\psi_{\lambda}|^2 = 1
\]

(115)

This unitary relation expresses the conservation of the number of Rindler particles. Indeed there is no mixing of positive and negative frequencies in eq. [111] in other words, the \( \beta \)-term of the "Bogoljubov" transformation eq. [111] vanishes.

The identity of the Green functions in eq. [114] proves that, once the the steady regime is established, no flux is, in the mean, emitted. This is Grove theorem [11] [12].
We now examine how this stationary scattering of Rindler modes is perceived in Minkowski terms. The Minkowski scattered modes \( \tilde{\varphi}_{\lambda,M} \) are given by

\[
\tilde{\varphi}_{\lambda,M} = -\left[ a_{\lambda,R}^\dagger, \tilde{\phi}(V) \right] = \varphi_{\lambda,M}(1 + ie\alpha_\lambda^2 \psi_\lambda) - ie\alpha_\lambda \beta_\lambda \psi_\lambda \varphi^*_{-\lambda,M} = \tilde{\alpha}_\lambda \varphi_{\lambda,M} + \tilde{\beta}_\lambda \varphi^*_{-\lambda,M}
\]

(116)

\[
\tilde{\varphi}_{-\lambda,M} = \varphi_{-\lambda,M}(1 - ie\beta_\lambda^2 \psi_{-\lambda}) - ie\alpha_\lambda \beta_\lambda \psi_{-\lambda} \varphi^*_{\lambda,M} = \tilde{\alpha}_{-\lambda} \varphi_{-\lambda,M} + \tilde{\beta}_{-\lambda} \varphi^*_{\lambda,M}
\]

(117)

where \( 0 < \lambda < \infty \) and where we have introduced the scattered Bogoljubov coefficients:

\[
\tilde{\alpha}_\lambda = 1 + ie\alpha_\lambda^2 \psi_\lambda
\]

\[
\tilde{\beta}_\lambda = -ie\alpha_\lambda \beta_\lambda \psi^*_\lambda
\]

\[
\tilde{\alpha}_{-\lambda} = 1 + ie\beta_\lambda^2 \psi^*_\lambda
\]

\[
\tilde{\beta}_{-\lambda} = -ie\alpha_\lambda \beta_\lambda \psi_\lambda
\]

(118)

One verifies that the unitary relation is satisfied: \( |\tilde{\alpha}_\lambda|^2 - |\tilde{\beta}_\lambda|^2 = 1 \). The fact that the \( \tilde{\beta} \) are different from zero indicates that each couple of jumps of the oscillator (the absorption and subsequent emission of a Rindler quantum) leads, in Minkowski vacuum, to the production of two Minkowski quanta. The member \( \varphi_{-\lambda,M} \) is emitted when the oscillator absorbs a Rindleron and jumps into a higher level and the other one, \( \varphi_{\lambda,M} \) is emitted during the inverse process. This is manifest in the mean energy flux:

\[
\tilde{T}_{VV} = \lim_{V' \to V} \partial_V \partial_{V'} \left[ \tilde{\phi}(V)\tilde{\phi}(V') - \phi(V)\phi(V') \right]
\]

\[
= 2 \int_{-\infty}^{+\infty} d\lambda \ |\tilde{\beta}_\lambda|^2 |\partial_V \varphi_{\lambda,M}|^2 + Re \left[ \tilde{\alpha}_\lambda \tilde{\beta}^*_\lambda \partial_V \varphi_{\lambda,M} \partial_{V'} \varphi_{-\lambda,M} \right]
\]

(119)

where upon the total Minkowski energy, given by the integration of the first term only, is:

\[
\tilde{H}_M = \int_{-\infty}^{+\infty} dV \tilde{T}_{VV}
\]

\[
= \int_{-\infty}^{+\infty} d\lambda \ \lambda \ (|\tilde{\beta}_\lambda|^2 + |\tilde{\beta}_{-\lambda}|^2) \int_{-\infty}^{+\infty} dV \frac{1}{2\pi \ a^2 |V + ie|^2}
\]

(120)

Exactly as in second order perturbation theory, there is a steady regime during which all the emitted quanta interfere destructively leaving no contribution to the mean flux (see eq. 114). But all non diagonal matrix elements will be sensitive to the created pairs. This is also the case for the total energy eq. 120 since being diagonal in \( \omega \) it ignores the destructive interferences (the second term of eq. 119 whose role is to make the mean flux vanishing during the steady regime).

In order to prove that eq. 120 corresponds to a steady production of Minkowski quanta during the whole interacting period \( \Delta \tau = T \) (infinite in eq. 120) we evaluate how
many are produced. (Contrary to the energy, the total number of Minkowski quanta is a scalar under the Lorentz group)

\[
\tilde{N}(\Delta \tau) = \int_0^{+\infty} d\omega <\tilde{0}_M|a_\omega^\dagger a_\omega|\tilde{0}_M >
\]

\[
= \int_0^{+\infty} d\omega <0_M|\tilde{a}_\omega^\dagger \tilde{a}_\omega|0_M >
\]

\[
= \int_0^{+\infty} d\omega \int_{-\infty}^{+\infty} d\lambda |\gamma_{\lambda,\omega}(\Delta \tau)|^2 |\tilde{\beta}_{\lambda}|^2
\]

(121)

where \(|\tilde{0}_M >\) is the scattered (Schrödinger) state. The \(\tilde{a}_\omega\) are related to the \(\tilde{a}_{\lambda,M}\) by (see eq. 23)

\[
\tilde{a}_\omega = \int_0^{+\infty} d\omega \gamma_{\lambda,\omega}(\Delta \tau)\tilde{a}_{\lambda,M}
\]

(122)

where \(\gamma_{\lambda,\omega}(\Delta \tau)\) takes into account the time dependence of the coupling. As shown in eq. 9 \(\gamma_{\lambda,\omega}(\Delta \tau)\) is non vanishing only for the \(\omega\) which enter into resonance with the oscillator frequency \(m\) during the interaction period \(\tau_i < \tau < \tau_f = \tau_i + T\). When these frequencies belong to

\[
\omega_i = m e^{-a \tau_i} < \omega < m e^{-a \tau_f} = \omega_f
\]

(123)

\(\gamma_{\lambda,\omega}(\Delta \tau)\) may be replaced by \(\gamma_{\lambda,\omega}\) (given in eq. 24). Hence \(\tilde{N}(\Delta \tau)\) reads

\[
\tilde{N}(\Delta \tau) = \int_{\omega_i}^{\omega_f} d\omega \frac{d\omega}{2\pi a \omega} \int_{-\infty}^{+\infty} d\lambda |\tilde{\beta}_{\lambda}|^2
\]

\[
= \frac{\Delta \tau}{2\pi} \int_{-\infty}^{+\infty} d\lambda |\tilde{\beta}_{\lambda}|^2
\]

(124)

The total energy emitted obtained from eq. 124 is

\[
\tilde{H}_M(\Delta \tau) = \int_{\omega_i}^{\omega_f} d\omega \frac{d\omega}{2\pi a} \int_{-\infty}^{+\infty} d\lambda |\tilde{\beta}_{\lambda}|^2
\]

\[
= \int_{\tau_i}^{\tau_f} \frac{d\tau}{2\pi} e^{-a \tau} m \int_{-\infty}^{+\infty} d\lambda |\tilde{\beta}_{\lambda}|^2
\]

\[
= \int_{V_i}^{V_f} \frac{dV}{2\pi a^2 V^2} m \int_{-\infty}^{+\infty} d\lambda |\tilde{\beta}_{\lambda}|^2
\]

(125)

in perfect agreement with eq. 120 if the frequency width of the oscillator in small compared to \(m\). The rate of production (eq. 124 divided by \(\Delta \tau\)) is (small width limit) \(e^2 a^2 m^2 \tilde{\beta}^2\) which is the rate of jumps for an inertial oscillator in a bath at temperature \(a/2\pi\). Therefore the number of Minkowski quanta produced by the thermalised oscillator equals the number of internal jumps.

We now generalise these results to an arbitrary linear coupling. We believe that it can be generalised, using the same type of argumentation, to nonlinear couplings as well. The proof goes as follow. Any scattering of Rindler quanta by an accelerated system which leads to a thermal equilibrium during a time much larger than \(1/a\) can be described as in eq. 111 by

\[
\tilde{a}_{\lambda,R} = S_{\lambda M} a_{\lambda,R}
\]

(126)

\(^6\) The simplest way to obtain this state is to find the scattering operator \(U\) such that \(\tilde{a}_{\lambda,M} = U^\dagger a_{\lambda,M} U\). Then \(|\tilde{0}_M >= U|0_M >\).
where repeated indices are summed (or integrated) over and where the summation over $\lambda'$ includes both $u$ and $v$-modes (as in eq. 109). The matrix $S$ satisfy the unitary relation
\[ S_{\lambda\nu}S_{\lambda'\nu}^\dagger = \delta_{\lambda\lambda'} \] (127)
which express the conservation of the number of Rindler quanta since $S_{\lambda\lambda'}$ mixes positive Rindler frequencies only. It is convenient to introduce the matrix $T$ (from now on we do not write the indices)
\[ S = 1 + iT \] (128)
which satisfies
\[ 2\text{Im}T = TT^\dagger \] (129)
We introduce also the vector operator $b = \left( a_{\lambda,R}; a_{\lambda,L}; a_{\lambda,R}^\dagger; a_{\lambda,L}^\dagger \right)$. Then eq. 126 can be written as
\[ \tilde{b} = Sb \] (130)
where $S$ has the following block structure
\[ S = \begin{pmatrix}
1 + iT & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 - iT^\dagger & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix} \] (131)
since the $u$ and $v$-modes on the left quadrant are still free. On the other hand, the Bogoljubov transformation eq. 25 reads in this notation
\[ c = \mathcal{B}b \] (132)
where $c = \left( a_{\lambda,M}; a_{-\lambda,M}; a_{\lambda,M}^\dagger; a_{-\lambda,M}^\dagger \right)$ and where $\mathcal{B}$ is
\[ \mathcal{B} = \begin{pmatrix}
\alpha & 0 & 0 & -\beta \\
0 & \alpha & -\beta & 0 \\
0 & -\beta & \alpha & 0 \\
-\beta & 0 & 0 & \alpha \\
\end{pmatrix} \] (133)
the diagonal matrices (in $\lambda$) $\alpha$ and $\beta$ being taken real. The scattered Minkowski operators are given by
\[ \tilde{c} = \mathcal{B}SB^{-1}c = \left( S + \mathcal{B} \left[ S, \mathcal{B}^{-1} \right] \right) \] (134)
Since $S$ and $\mathcal{B}$ do not commute, $S_M$ has non diagonal elements which encode the production:
\[ S_M = \begin{pmatrix}
\tilde{\alpha}_1 & 0 & 0 & -\tilde{\beta}_1 \\
0 & \tilde{\alpha}_2 & \tilde{\beta}_1^\dagger & 0 \\
0 & \beta_2^\dagger & \alpha_1^\dagger & 0 \\
-\tilde{\beta}_2 & 0 & 0 & \tilde{\alpha}_2^\dagger \\
\end{pmatrix} \] (135)
where the $\tilde{\alpha}$, $\tilde{\beta}$ are given in terms of $T$ by (see eq. [138])

\[
\begin{align*}
\tilde{\alpha}_1 &= 1 + i\alpha T \alpha \\
\tilde{\beta}_1 &= -i\alpha T \beta \\
\tilde{\alpha}_2 &= 1 + i\beta T^\dagger \beta \\
\tilde{\beta}_2 &= i\beta T \alpha 
\end{align*}
\] (136)

QED

6 The Black Hole

6.1 The kinematics of the collapse and the scattered modes

We shall work in the background metric of a spherically symmetric collapsing star of mass $M$. Outside the star the geometry is described by the Schwarzschild metric

\[
ds^2 = (1 - \frac{2M}{r})dt^2 - (1 - \frac{2M}{r})^{-1}dr^2 - r^2d\Omega^2
\]

\[
= (1 - \frac{2M}{r})dudv - r^2d\Omega^2
\]

$v, u = t \pm r^*$

\[
r^* = r + 2M \ln \frac{r - 2M}{2M}
\] (137)

The specific collapse we consider is produced by a spherically symmetric shell of pressureless massless matter. Inside the shell space is flat and the metric reads

\[
ds^2 = d\tau^2 - dr^2 - r^2d\Omega^2
\]

$v, U = \tau \pm r$ (138)

where $v$ is the same coordinate in eq. [137] and eq. [138] since on $I^-$ space time is flat on both sides. The collapsing shell, taken to be thin, follows the geodesic $v = v_S$. The connection between the two metrics is obtained by imposing the continuity of $r$ along the shell's trajectory

\[
dU = du(1 - \frac{2M}{r(u, v_S)}) = du(1 - \frac{4M}{v_S - U})
\] (139)

Then by choosing $v_S = 4M$ one gets

\[
\begin{align*}
\frac{du}{U} &= \frac{dU}{(4M - U)} \\
u(U) &= U - 4M \ln \left(\frac{U}{4M}\right)
\end{align*}
\] (140)

In the static space time outside the star, the Klein-Gordon equation for a mode of the form $\varphi_{l,m} = \frac{1}{\sqrt{4\pi r^2}} Y_{lm}(\theta, \phi) \psi_l(t, r)$ reads

\[
\left[\partial_t^2 - \partial_{r^*}^2 - (1 - \frac{2M}{r}) \left[\frac{l(l + 1)}{r^2} + m^2 + \frac{2M}{r^3}\right]\right] \psi_l(t, r) = 0
\] (141)
Near the horizon $r - 2M \ll 2M$, it becomes the wave equation for a massless field in $1 + 1$ dimensions. By considering only the s-wave sector of a massless field and dropping the residual "quantum potential" $2M(r - 2M)/r^4$ the conformal invariance holds everywhere, inside as well as outside the star. (This does not depend on our specific collapse: it is also valid if one assumes, following Hawking [2], that the geometrical optics limit is valid inside the star.) From now on we shall work in this simplified context and only discuss briefly the differences with the more realistic four dimensional case.

The Heisenberg state is chosen to be the initial vacuum i.e. vacuum with respect to the modes which have positive $v$-frequency on $I^{-}$. Those modes are reflected at $r = 0$ and read

$$\varphi_{\omega,0,0}(v,u) = \frac{1}{4\pi r \sqrt{\omega}} \left( e^{-i\omega v} - e^{-i\omega U(u)} \right) (142)$$

Hence, for $u > 4M$ (or $-M < U < M$, on both sides of the horizon) the state of the field tends exponentially quickly (in $u$) to Unruh vacuum, i.e. vacuum with respect to the modes

$$\exp(-i\omega u) \quad \text{and} \quad \exp\left(\frac{i\omega}{4M} e^{-i\omega u}\right) (143)$$

The Schwarzschild $u$-modes $\chi_\lambda(u) = e^{-i\lambda u}/(4\pi r \sqrt{\lambda})$ are needed to analyse the particle content of the scattered modes $\varphi_\omega$ on $I^+$. In terms of $U$ they take the form

$$\chi_\lambda(u) = \theta(-U) \frac{1}{4\pi r \sqrt{\lambda}} \left( -\frac{U}{4M} \right)^{i4M} e^{-i\lambda U} (144)$$

The exact Bogoljubov coefficients between $\varphi_\omega$ and $\chi_\lambda$ are given by

$$\alpha_{\omega,\lambda} = \langle \varphi_\omega, \chi_\lambda \rangle = \frac{1}{4\pi} \sqrt{\frac{\omega}{\lambda}} \Gamma(1 + i4M\lambda)[4M(\omega - \lambda)]^{-i4M\lambda} e^{\pm 2\pi M\lambda} (145)$$

where the $\pm$ is to be understood as $+$ if $\omega > \lambda$ and $-$ if $\omega < \lambda$. The expression for $\beta_{\omega,\lambda}$ is obtained by taking $\lambda$ into $-\lambda$. The asymptotic Bogoljubov coefficients (relating Kruskal modes to Schwarzschild modes eq. 23 et seq.) are recovered in the limit $\omega \rightarrow +\infty$ since it corresponds to resonance at late times $u \rightarrow +\infty$ (see eq. 123). In this limit the black hole emits quanta at the Hawking temperature $1/8\pi M$ since $|\beta_{\omega,\lambda}/\alpha_{\omega,\lambda}|^2 = e^{-8\pi M\lambda}$.

Having described the kinematics of the collapse we now turn to the post-selection of the emitted quanta. The new difficulty lies in the renormalisation of the energy momentum tensor which must be carried out in curved space times. We therefore turn to this point.

### 6.2 Weak-values in curved space-time

Wald has proposed a set of eminently reasonable conditions that a renormalised energy momentum operator should satisfy [24]. By an argument similar to Wald’s (or simply by verifying that it is in accord with his axioms), it is possible to deduce that $T_{\mu\nu}(\text{ren})(x)$ can be written in the following way

$$T_{\mu\nu}(\text{ren})(x) = T_{\mu\nu}(x) - t_{\mu\nu}(S)(x)I (146)$$

where $T_{\mu\nu}(x)$ is the bare energy momentum tensor. The subtraction term $t_{\mu\nu}(S)(x)$ is an (infinite) conserved c-number function only of the geometry at $x$. It can be
understood [24] [25] as the (infinite) ground state energy of the "local inertial vacuum": that state which most resembles Minkowski vacuum at $x$. Numerous techniques have been developed to calculate $t_{\mu\nu}(S)$ and we refer the reader to [26] for a review.

In a state, say the Heisenberg vacuum $|0>$, the expectation value of $T_{\mu\nu}$ takes the form

$$<0|T_{\mu\nu}(x)|0> = <0|T_{\mu\nu}(x)|0> - t_{\mu\nu}(S)(x)$$

(147)

where both terms on the r.h.s. are infinite but their difference is finite.

In a pre- and post-selected ensemble, the weak values of $T_{\mu\nu}$ reads

$$T_{\mu\nu\text{(weak)}} = \frac{<0|\Pi T_{\mu\nu}(x)|0>}{<0|\Pi|0>} - t_{\mu\nu}(S)(x)$$

(148)

where $\Pi$ is the projector (or more generally the self adjoint operator) that realises the post-selection. Inserting eq. (148) into this expression yields

$$T_{\mu\nu\text{(weak)}}(x) = \frac{<0|\Pi T_{\mu\nu}(x)|0>}{<0|\Pi|0>} - t_{\mu\nu}(S)(x)$$

(149)

By expressing $T_{\mu\nu}(x)$ in terms of the operators which annihilate the Heisenberg vacuum one obtains

$$T_{\mu\nu\text{(weak)}}(x) = \int_0^\infty d\omega \int_0^\infty d\omega' <0|\Pi a_\omega a_\omega^\dagger|0>T_{\mu\nu}(x)|\varphi_\omega^*\varphi_\omega^*| + <0|T_{\mu\nu\text{(ren)}}(x)|0>$$

(150)

where $\hat{T}_{\mu\nu}(x)$ is the classical differential operator which acting on the waves $\varphi_\omega^*$ gives their energy density. The first term depends on the particle content of the post-selection and the second one is the energy density of the Heisenberg vacuum eq. (147).

The formula eq. (149) warrants a few additional comments. First notice that their are parts of $<T_{\mu\nu}>_{\text{weak}}$ that are entirely contained in the subtraction. Most notably there is the trace anomaly and those components of the energy momentum tensor which are related to it by energy conservation (in two dimensions they are $T_{uu,v}$ and $T_{vv,u}$). These parts are independent of the post-selection or, expressed differently, do not fluctuate.

An additional (and related) feature which has already been mentioned in chapter 3 concerns the absence of correlations between $T_{uu}$ and $T_{vv}$. Not only shall this give rise to the particular structure of vacuum fluctuations that extend back to $I^-$, but it also implies that on the horizon the in-going flow and the out-going flow fluctuate independently (for instance the post selection of an outgoing particle on $I^+$ does not affect $T_{vv}$ outside the star and in particular on the horizon $r = 2M$). This last effect disappears partially when considering the potential barrier that occurs in the wave equation eq. (141).

### 6.3 The different post-selections

Since an external observer does not have access to the region of space time beyond the horizon, the post-selections that he can perform are restricted to an incomplete ($U < 0$) region of space time and are therefore incomplete as well.

The post-selection could consist in specifying the state of the outgoing photons outside the star. For instance one could specify that the state $\Pi|0> = a_\lambda^\dagger|B>$ where
|B⟩ is Boulware vacuum and 𝑎^† 𝜆 creates a Schwarzschild photon. The corresponding weak value reads

$$T_{\mu\nu(\text{weak})} (x) = \frac{2}{\alpha_\lambda \beta_\lambda} \bar{T}_{\mu\nu}(x) \left[ \phi^*_{\lambda,K} \phi^*_{-\lambda,K} \right] + \left[ \frac{<B|T_{\mu\nu}(x)|0>}{<B|0>} - <0|T_{\mu\nu}(x)|0> \right] + <0|T_{\mu\nu(\text{ren})}(x)|0> (151)$$

where the new Kruskal modes \( \phi_{\lambda,K} \) are defined as in eq. 23. The first term is equal to the energy of the photon \( \lambda \), the second one is the difference of energy between Boulware vacuum and the Heisenberg vacuum and the third one is the Heisenberg vacuum energy eq. 147. The second term appears because one has specified that, apart from \( \lambda \), there is no other photon emitted. This is why this term is singular on the horizon. In addition, the probability to obtain the state \( a^\dagger_\lambda |B⟩ \) vanishes in the absence of backreaction and is in the semiclassical approximation of order \( e^{-M^2} \) where \( M^2 \) is approximately the total number of photons emitted.

An alternative post-selection consists in tracing over all the photons except the photon \( \lambda \) which is imposed to be present (in the Rindler problem this corresponds to the projector eq. 36). Then the weak value is simply

$$T_{\mu\nu(\text{weak})} (x) = \frac{2}{\alpha_\lambda \beta_\lambda} \bar{T}_{\mu\nu}(x) \left[ \phi^*_{\lambda,K} \phi^*_{-\lambda,K} \right] + <0|T_{\mu\nu(\text{ren})}(x)|0> (152)$$

We could post-select a wave packet rather than a mode of fixed energy \( \lambda \) in which case eq. 152 would be finite on the horizon.

Having traced over all the other photons, the second term of eq. 151 is absent in eq. 152. Nevertheless it can also be constructed as the sum of weak values that specify completely the state times the probability that they occur (in similar manner to the unitarity relation eq. 9). In this way the difference of energy between Boulware vacuum and the Heisenberg vacuum is realised as the sum over all possible radiated photons times the thermal probabilities that they occur.

Finally we consider post-selection by an inertial two level atom at large distance from the black hole. In this case the final state is partially specified, since the detector is coupled to a finite set of modes, and one therefore obtains a result similar to eq. 152 wherein the first term consists in the post-selected Hawking photon. We shall display its properties in the next section.

It is also interesting to speculate about the nature of the in-going vacuum fluctuations. These could be analysed by post-selecting the presence of ingoing quanta near the horizon. A "natural" set of modes to post-select near the horizon are Kruskal \( v \)-modes. One is therefore led to consider the Kruskal vacuum fluctuations in Schwarzschild vacuum, which is similar to considering Minkowski fluctuations in Rindler vacuum. If space time were the full Schwarzschild manifold, these would present a singularity on the past horizon that could be smoothed out using wave packets. Since space time is not the full Schwarzschild manifold (there is no past horizon) the star’s surface will play the role of past horizon and one expects large energy densities in the outermost layers of the star.
6.4 From vacuum fluctuations to black hole radiation

We now turn to that piece of the weak value which depends on the particular mode post-selected by the two level atom. This piece in completely independent of the geometry for the s-wave that we are considering since we neglect the residual potential of the dalembertian eq. [14]. Hence the mapping of the results obtained in the Rindler problem to the black hole is straightforward. Let us choose the time dependant coupling $f(t)$ of our detector such that it will be excited only by spherical photons centred around $u = u_0$ with (Schwarzshild) energy $\lambda = m$. Such an example of wave packet is offered by the Fourier components given in eq. 66

$$c_\lambda = D \lambda e^{i\lambda u_0} e^{-(\lambda - m)^2 T^2 / 2} (1 - e^{-2\pi \lambda / a})$$

(153)

The spread in time is $\Delta t = \Delta u = T$ and $u_0$ is taken well inside the region $u > 0$ where the isomorphism of the scattered waves and the Kruskal modes is achieved.

The picture that emerges, if the two level atom is found excited after the switch off, is that this photon results form a spherically symmetric vacuum fluctuation on $I^-$ which carries zero total energy and is located in a region

$$|v - v_\infty| = |\Delta U| = |\Delta u e^{-u_0/4M}| \simeq T e^{-u_0/4M}$$

(154)

where $v = v_\infty (= 0$ in our collapse) is the light ray that shall become the future horizon $U = 0$. Indeed this localisation is furnished by the $v$ dependence of the weak value on $I^-$ which reads (see equation eq. 76)

$$<T_{\mu\nu}(I^-)_{\text{weak}} \simeq \frac{1}{4\pi r^2} \frac{16M^2}{v^2} \frac{m}{2\sqrt{\pi}T} (N_m + 1) \exp \left[ -\frac{4M}{T} \left( \ln \left( \frac{-v - i\epsilon}{4M} + u_0 \right) \right) \right]$$

(155)

This results from the fact that the analysis by an inertial observer near $I^-$ is isomorphic with what was called the Minkowski interpretation in chapter 4. As in the accelerated case, the energy density is enhanced by the jacobian $du/dU = e^{u/4M}$ centered around $u = u_0$ which appears here as $1/v^2$ when the reflection at $r = 0$ is taken into account. Hence after a $u$-time of the order of $4M \ln M$, the energy density in $T_{\mu\nu}$ (rescaled by $4\pi r^2$) become "transplanckian" and located within a "cisplanckian" distance $\Delta v$ (If one does not rescale $T_{\mu\nu}$ the transplanckian energies only exist in a region of finite $r$). The analysis and the consequences of these transplanckian energies is presented in a separate paper [17]. In that article it is argued that the nonlinearity of general relativity cannot accommodate these densities and that a taming mechanism must exist if Hawking radiation does exist.

After issuing from $I^-$, the vacuum fluctuation contracts until it reaches $r = 0$ and then reexpands along $U = const$ lines. Upon crossing the surface of the star in a region $\Delta U$ centred on the horizon, it separates into a piece (the partner) that falls into the singularity, carrying a negative Schwarzshild energy equal to $-m$, and a piece carrying positive energy equal to $m$ that keeps expanding and escapes to $I^+$ to constitute the post-selected quantum that will induce the transition (see eq. 69 and figure 1). The analysis performed by an inertial asymptotic observer near the detector, on $I^+$, is isomorphic with the Rindler interpretation of chapter 4.
If the two level atom is found in its ground state after the switch off, its wave function is correlated to the absence of the Hawking photon specified by $c_\lambda$. In that case, one would find near $I^-$ a vacuum fluctuation whose energy content is exactly the opposite of the previously considered case (times $N_m$). Near $I^+$ it would contain a negative energy flux of total energy $-N_m$ encoding the fact that there is one quanta absent from the thermal flux emitted by the black hole.

If more realistically, we take a two-level atom coupled locally to the field (i.e. coupled to all the modes $l > 0$), it will post select particles coming out of the black hole in its direction. Then the picture that emerges is essentially the same as for an s-wave except that on $I^-$ the vacuum fluctuation is localised on the antipodal point of the detector. The created quantum and its partner, are on the same side and not antipodal (with respect to each other) because they have opposite energy.

We now turn to the description in the intermediate regions in order to interpolate between the descriptions between $I^-$ and $I^+$. One possible interpolation consists in using a set of static observers at constant $r$. Then the "Rindler" description would be used everywhere outside the star. However a difficulty arises in this scheme if one really considers a set of material "fiducial" detectors at constant $r$. For upon interacting with the field and thermalising at the local temperature $\sqrt{\frac{r-2M}{8\pi M}}$ the detectors will emit large amounts of ultraviolet Kruskal "real" quanta (see chapter 5 wherein it is shown how the accelerated atom transforms vacuum fluctuations into "real" quanta). The backreaction of these quanta cannot be neglected and, as already stated, cannot be evaluated owing to the transplanckian energy they carry.

An alternative interpolation consists in giving the value of $T_{\mu\nu}$ in the local inertial coordinate system (Riemann normal coordinates). This stems from the idea that local physics should be describe locally in such a coordinate system. This approach has been used in defining the subtraction necessary to renormalize the energy momentum tensor $^{[24]}$ $^{[25]}$. In the two dimensional model the local inertial coordinates are easy to construct. Since $\tilde{u} = r(u, v)$ is an affine parameter along the geodesics $v = constant$, a natural way to represent the outgoing flux outside the star is as

$$T_{\tilde{u}\tilde{u}}(\tilde{u}) = \left(\frac{du(r,v)}{dr}\right)^2 T_{uu}(u(r,v))$$

This is represented in both a Penrose diagram and Eddington-Finkelstein coordinates in figures 4 and 5.

After the Hawking photon reaches a distance $r \geq 4M$ it travels in flat space, it is no longer modified and the backreaction may be safely computed from $\langle T_{\mu\nu} \rangle_{weak}$. But a $v$ time of order $4M\ln M$ before it reached flat space the photon already carried planckian energy densities in this local description.

To obtain a first indication of the gravitational backreaction we consider the linear modification of the metric $\delta g_{\mu\nu}$ and describe it quantum mechanically. In first order perturbation theory $\delta g_{\mu\nu}$ itself could be taken to be the additional system (the weak detector) introduced in chapter 2. Then the weak value of $\delta g_{\mu\nu}$ is obtained by integrating Einstein’s equations with $\langle T_{\mu\nu} \rangle_{weak}$ as source since the weak values obey the Heisenberg equations of motion. For s-waves the constrained part of the metric only will be modified. Furthermore since the total energy carried by the weak value of $T_{\mu\nu}$ vanishes from $I^-$ till the emergence of the fluctuation from the star after reflection on $r = 0$, the
the weak value of $\delta g_{\mu\nu}$ will vanish outside the interval $\Delta v$ eq. $[154]$. Within that interval the precise shape of $\delta g_{\mu\nu \text{ weak}}$ will depend on the particular choice of post-selected wave packet. On the contrary, outside the star, for $r > 4M$ and $u > u_0$, the weak value of $\delta g_{\mu\nu}$ will encode the mass loss $\omega$ and in fact describes a new Schwarzshild space where the mass is $M - \omega$.

One can also consider the backreaction of the Hawking photon onto itself and onto the subsequent photons. This self interaction is governed by a hamiltonian of the form $H_{\text{int}} = T_{\mu\nu} D^{\mu\nu\alpha\beta} T_{\alpha\beta}$ where $D$ is the linearized gravitation propagator. In this approximation the backreaction is given by $\langle H_{\text{int}} \rangle_{\text{weak}}$. But since $\langle T_{\mu\nu} \rangle_{\text{weak}}$ becomes larger than 1 (in Planck units) when the selected photon is a Planck distance from the horizon the linear approximation invariably fails. This is compounded by the non renormalisability of the graviotional interaction which presumably does not lead to an asymptoticly free theory beyond the Planck scale. How Hawking radiation could still be realised in a consistent theory of gravity remains to be seen.

**Acknowledgements.** The authors would like to thank R. Brout, F. Englert, S. Popescu and Ph. Spindel for very helpful discussions.

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Figure Captions

Figure 1.
The weak value of $T_{vv}$ if the two level atom gets excited is represented for the values of parameters $m = 2a$ and $T = 3a^{-1}$. The $v$ axis is given in units of $a^{-1}$. On top of the figure the absolute value of the coupling function $f(\tau)$ of the two level atom to the field is represented. Underneath is a graph of $<T_{vv}(I_+, V < 0)>_{weak \ e}$ ($= <T_{vv}(I_-, V < 0)>_{weak \ e}$ by causality) and $<T_{vv}(I_+, V > 0)>_{weak \ e}$. Notice that the scale of this last drawing is different from the others since this weak value is proportional to $N_m$ whereas the others are proportional to $1 + N_m$. Underneath the real and imaginary part of $<T_{vv}(I_-, V > 0)>_{weak \ e}$ are represented. These pictures show how a Rindler observer would see the weak values.

Figure 2.
The real and imaginary parts of $<T_{VV}(I_-)>_{weak \ e}$ is represented for the same values of parameters as in figure 1. The $V$ axis is given in units of $a^{-1}$. $T_{VV}$ presents very strong oscillations near $V = 0$ which are not represented. If one considers only Re[$<T_{VV}(I_-)>_{weak \ e}$] for $V > 0$ and compares it to Re[$<T_{vv}(I_-, V > 0)>_{weak \ e}$] of figure 1, then the positive hump to the left of $V = 1$ corresponds to the central positive hump centered on $v = 0$ and the negative oscillations to the left of $V = 0.2$ correspond to the dip between $-7 < v < -2$. The tail oscillations in the Rindler description are enhanced by the jacobian that passes from Rindler to Minkowski coordinates in such a way that the integral of the graphs in figure 2 vanish.

Figure 3.
The mean energy emitted to order $g^2$ at thermal equilibrium (rescaled by the probility to emit a left photon) is represented for $m = 2a$ and $T = 3a^{-1}$ from both the Minkowski and Rindler point of vue and for $m = 2a$, $T = 10a^{-1}$ from the Rindler point of vue only. It is apparent that as $T$ increases the Rindler energy emitted per transition of the atom tends to zero. In the Minkowski description the tails of postive Rindler energy are enhanced by the jacobian $dV/dv$ to make the total Minkowski energy emitted postive.

Figure 4.
The local description of a vacuum fluctuation giving rise to a Hawking photon emitted around $u = u_0$ is represented in a Penrose diagram. The shaded areas correspond to the regions where $T_{\tilde{u}\tilde{u}}(\tilde{u})$ is non vanishing. $v = v_\Sigma$ is the trajectory of the collapsing spherically symmetric shell of massless matter.

Figure 5.
The same as in figure 4 drawn in advanced Eddington-Finkelstein coordinates $(r, v = t + r^*)$. 
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