Research on Autonomous Orbit Determination of Navigation Satellite Based on Crosslink Range and Orientation Parameters Constraining

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ABSTRACT
Autonomous navigation of navigation satellite is discussed. The method of auto-orbit determination using the crosslink range and orientation parameters constraining is put forward. On the basis of the analysis of its feasibility, some useful conclusions are given.

KEY WORDS
satellite navigation; autonomous navigation; orbit determination

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Introduction

The potential vulnerability of satellite navigation system that relies on ground stations is that the system would break down if ground stations were destroyed, which can not meet the requirement of navigation warfare\(^{(1)}\). With the development of such space-based systems as ground-oriented observation and space-borne remote sensing, basic services of space and time for those systems provided by the autonomous navigation satellite system will bring great advantages\(^{(2,4)}\). Therefore, under the urge of navigation warfare and space-based system applications, autonomous navigation of navigation constellation is becoming more and more imperative.

The autonomous navigation of satellite navigation system refers to that navigation satellites can perform orbit determination, time maintenance, and broadcast navigation message to users uninterruptedly by themselves under the circumstances of no support of ground stations. One important characteristic of the GPS new satellites(e.g., Block IIF) is that the function of inter-satellite link was added to implement autonomous navigation. As a result, the navigation system can run 180 days autonomously and keep navigation accuracy equivalent to that under the support of ground stations\(^{(5,6)}\), which greatly enhances the capability of navigation warfare.

In addition, the modernization of Russia’s Glonass and the Europe’s Galileo system are also considering adding the function of inter-satellite links.

1 Autonomous orbit determination method based on orbit orientation parameter constraints and inter-satellite distance measurement

The purpose of intersatellite distance measuring and communication is to acquire observations of intersatellite distance and relative clock error, and to implement interchange of such data as distances between different satellites, clock errors and a priori orbit information. In the intersatellite distance measuring and communication a bidirectional and dual-frequency mode is adopted. The reason for adopting the bidirectional
mode is to obtain the deviation between satellite clocks and to eliminate most systematic and correlational errors that influence intersatellite distance measuring. As for the dual-frequency mode, the purpose is to eliminate the influence of the ionosphere delay on distance measuring. The intersatellite communication is through microwave in the manner of time division multiple address (TDMA). The principle of intersatellite distance measuring and communication can be described as follows.

Firstly, let us suppose at the predetermined epoch, navigation satellites are performing intersatellite distance measuring and communication. Assuming there are 24 navigation satellites, the length of each frame of TDMA is 48 seconds, and each satellite occupies 2 seconds. Intersatellite communication includes distance measuring frame and data frame. In the distance measuring frame, each satellite broadcasts dual-frequency distance measuring information within UKF band in the 2 seconds that belongs to itself and in the data frame, each satellite broadcasts data information measured with respect to itself. In the distance measuring frame, all satellites in common view measure the pseudoranges between themselves and the satellite emitting signals. Information sent from the data frame contains the computed pseudorange measurements, the estimated orbit and clock error of each satellite as well as their corresponding covariance. Through the pseudorange processing two kinds of observables, intersatellite distance and relative clock error can be obtained, from which a lot of correlational and systematic errors will be eliminated, and they can be used as the inputs of satellite ephemeris and clock error filtering. The autonomous navigation of navigation satellites is implemented when the above-mentioned steps repeat continuously.

The data processing of inter-satellite distance observables includes reduction, combination, etc. Assuming we have two navigation satellites in common view, i-th and j-th, then there are 4 P-code pseudo-range observables. Ignoring stochastic and systematic errors, the observation equation can be simply formulated as:

\[
\begin{align*}
\rho_{1i} &= \rho + I_{ij} + \delta_{ij} \\
\rho_{2i} &= \rho + I_{ji} + \delta_{ji} \\
\rho_{1j} &= \rho + I_{ij} + \delta_{ij} \\
\rho_{2j} &= \rho + I_{ji} + \delta_{ji}
\end{align*}
\]

where subscripts 1 and 2 stand for frequency number; and \(\rho\) is the distance between the position of the j-th satellite that is sending signal and the position of the i-th satellite that is receiving signal; and \(I_{ij}\) is the distance between the position of the i-th satellite that is sending signal and the position of the j-th satellite that is receiving signal; and \(\delta_{ij}\) is the clock deviation between satellite i and satellite j.

It must be pointed out that, if the inter-satellite distance measurement is performed in the mode of TDMA, then the measurement from satellite i to satellite j and that from satellite j to satellite i are not done synchronously, so the observables must be converted into the same epoch to perform orbit determination computations. Then we have \(\rho_{ij} = \rho_{ji}\) and \(\delta_{ij} = -\delta_{ji}\), and Eq. (1) plus Eq. (3) and Eq. (1) minus Eq. (3) with frequency number omitted will lead to two derived observables:

\[
z_r = (P^g + P^s)/2 = \rho + (I^g + I^s)/2
\]
\[ z_0 = \frac{P_i - P_j}{2} = \alpha x_0 + \frac{I_i - I_j}{2} \]  \hspace{1cm} (6) 

where Eq. (5) being the observation model of autonomous orbit determination and Eq. (6) being that of autonomous time conservation. Furthermore, supposing in the inertial coordinate framework, the rectangular coordinates of satellite \( i \) is \((x^i, y^i, z^i)\), and satellite \( j \), \((x^j, y^j, z^j)\), and then Eq. (5) can be written as:

\[ z_t = \sqrt{(x^i - x^j)^2 + (y^i - y^j)^2 + (z^i - z^j)^2 + \left(\frac{P_i - P_j}{2}\right)^2} \]  \hspace{1cm} (7)

As in Eq. (7) relative position information of the two satellites in common view is included, therefore, the relative position of navigation satellite can be determined by distance measurements. However, inter-satellite distance measurement is insensitive to such orientation parameters of the orbit plane as \( i, \Omega \) and \( \omega \), so the normal equations become ill-conditioned when only inter-satellite distance measurement is used to determine the satellite orbit. An effective technique to overcome the ill-condition is to exert proper constraints onto the orientation parameters \( i, \Omega \) and \( \omega \).

2 Simulation computation and result analysis

2.1 Variation characteristic analysis for predicted orbit elements of navigation satellites

The information of orbit prediction from ground stations is indispensable for the orbit determination using inter-satellite observations. Here we take GPS satellites for example and make an analysis of the long-term prediction characteristics of orbit elements of middle and high Earth orbit satellites.

Equations of satellite orbit perturbation in terms of orbit elements can be written as follows:

\[ \dot{a} = \frac{\sqrt{P}}{GM} \cdot 2a \cdot \frac{e \cdot \sin v \cdot R + \frac{p}{r} \cdot S}{1 - e^2} \]  \hspace{1cm} (8) 

\[ \dot{e} = \frac{\sqrt{P}}{GM} \cdot \left(\frac{\sin v \cdot R + (\cos v + \cos E) \cdot S}{1 - e^2}\right) \]  \hspace{1cm} (9) 

\[ \dot{i} = \frac{r \cdot \cos(\omega + \nu)}{n \cdot a^2 \cdot (1 - e^2)^{1/2}} \cdot W \]  \hspace{1cm} (10)

\[ \dot{\Omega} = \frac{r \cdot \sin(\omega + \nu)}{n \cdot a^2 \cdot (1 - e^2)^{1/2} \cdot \sin i} \cdot W \]  \hspace{1cm} (11) 

\[ \dot{\omega} = \frac{1}{e} \cdot \sqrt{\frac{p}{GM}} \cdot \left[ -\cos v \cdot R + \left(1 + \frac{r}{p}\right) \cdot \sin v \cdot S\right] - \cos i \cdot \dot{\Omega} \]  \hspace{1cm} (12) 

\[ M = \frac{1}{na} \cdot \frac{1 - e^2}{e} \cdot \left(\cos v - 2 \cdot e \cdot \frac{r}{p}\right) \cdot R - \left(1 + \frac{r}{p}\right) \cdot \sin v \cdot S + \frac{3}{2} \cdot \frac{n}{a} \cdot (t - t_0) \cdot \dot{a} \]  \hspace{1cm} (13)

where \( a, e, i, \Omega, \omega, M \) are the six orbit elements describing satellite motion; and \( p = a \cdot (1 - e^2) \) is one ellipse parameter; and \( v \) is the true anomaly; and \( r \) is the distance from satellite to geocenter; and \( GM \) is the gravitational constant; and \( n \) is the average angular velocity; and \( R, S \) and \( W \) are radial, transverse and normal acceleration of the satellite respectively.

Eqs. (8), (9) and (13) indicate that the perturbation of orbit elements \( a, e \) and \( M \) is caused by radial and tangential perturbing forces, which lies in the orbit plane. Eqs. (10) and (11) show that only the normal perturbing force of the orbit causes the change of \( i \) and \( \Omega \). Eq. (13) shows that the change of \( \omega \) results from the radial, tangential and normal perturbing forces together.

For middle and high orbit satellite constellation like GPS, the major parts of perturbing forces are lower order of the earth’s gravitational force (up to 12 degrees), gravitational forces of the sun and the moon, and the sun’s radiation pressure. The earth’s gravitation \( f_{\omega} \) amounts to \( 5 \times 10^{-6} \text{ m/s}^2 \), and other higher parts adds up to \( 3 \times 10^{-7} \text{ m/s}^2 \), the sun and the moon’s gravitation is \( 5 \times 10^{-4} \text{ m/s}^2 \), and the sun’s radiation pressure is \( 9 \times 10^{-1} \text{ m/s}^2 \). Since orbit elements \( a, e \) and \( M \) can be determined through inter-satellite distance measurement, here we only discuss the other three elements \( i, \Omega \) and \( \omega \), which determine the orientation of the orbit plane.

Let us make an analysis of the perturbing forces. The earth’s non-spheric gravitation force, together with the sun’s and the moon’s gravitational forces are conservative and periodic, and can be accurately modeled. The pertur-
The earth’s flattening term $J_{20}$ causes a periodic variation of the orbit plane and if it is treated as 0 approximately, then the perturbing equation containing the term $J_{20}$ can be written as:

$$
\dot{\Omega} = -\frac{3}{8} \cdot \left(\frac{a_E}{a}\right)^2 \cdot J_{20} \cdot \cos i \cdot n \cdot (1 - \cos(2\omega))
$$

(14)

$$
i(t) = \frac{3}{8} \cdot \left(\frac{a_E}{a}\right)^2 \cdot J_{20} \cdot \sin(2i) \cos(2\omega)
$$

(15)

Eq. (14) indicates that $J_{20}$ influences $\Omega$ to have long term and short period variation, and Eq. (15) shows that $J_{20}$ causes $i$ to become short periodically with a short period of half that of GPS, about 6 hours, so prediction errors of the orbit plane would occur with an inaccurate $J_{20}$. Similarly, the sun’s and the moon’s perturbing forces are along the direction from the sun and the moon respectively to the satellite and from their decomposition we can obtain forces along the normal direction of the orbit plane, which are conservative forces and can be accurately modeled. From the above analysis we can know that the perturbing forces, both radiation pressure and the sun’s and the moon’s gravitational forces, or the earth’s oblateness term, do not have big influence on the prediction error of $\Omega$ and $i$, which means that high accuracy in prediction of $\Omega$ and $i$ should be achieved.

Set the initial epoch as the GPS precise ephemeris at 12: 00, Jan 1, 2002. Considering the earth’s non-spheric perturbing force up to 12 degrees, the sun’s and the moon’s gravitational forces and the sun’s radiation pressure, using numerical integration, we obtain 180-day predicted ephemerides with a sampling interval of 15 min and then compare them with the IGS precise ephemerides whose accuracy is better than $\pm 5$ cm to check what an accuracy our long term ephemerides prediction can gain. Fig. 1 gives the prediction errors of the right ascension $\Omega$ of ascending nodes of each GPS satellite. Fig. 2 provides the prediction errors of the inclination $\omega$ of GPS satellites. Fig. 3 shows the prediction errors of $\Omega$. However in this period some satellites make orbital manoeuvres. Although orbital manoeuvres are made within orbital planes, the influence on the prediction of $\Omega$ and $i$ is inevitable, so in our figures satellites with early orbital manoeuvres are excluded. Satellite PRN27 (manoeuvre on the 160th day), satellite PRN13 (manoeuvre on the 172nd day), and PRN29 and PRN31 (manoeuvre on the 178th day) are also provided since their manoeuvre dates are close to our final date.

Fig. 1 shows that in a 180-day prediction, the error of the right ascension $\Omega$ of the ascending node accumulates and increases, and on the 180th day, the prediction accuracy of each satellite ranges from $-0.729$ to $1.508$ arc seconds. And 17 out of 21 satellites ranges from $-0.4$ to $0.4$ arc seconds. Fig. 2 indicates that the accuracy of inclination $i$ is much higher and ranges from $-0.383$ to $0.383$ arc seconds except for very few satellites. From Fig. 1 and Fig. 2 we can see that the prediction error discloses a phenomenon of grouping, whose reason must be further analyzed. Fig. 3 shows that the accuracy of $\omega$ is much worse comparatively, which is caused by orbital radial and transverse perturbing force model errors and its correlation with $M$. Fig. 4 presents mean values of errors of $\Omega$ and $i$ of each epoch, which reflects the rotation of the entire constellation.

When there are only inter-satellite distance observations, the normal equations of orbit determination is ill-conditioned due to rotation and translation problems, and thus it is necessary to exert constraints onto some parameters. As it has been shown, the prediction accuracy of $\Omega$ and $i$ is very high in long term orbit predictions, for example, the prediction error of $\Omega$ and $i$ is
only ±0.4 arc seconds, which is equivalent to a rotation error of 12 m on the ground. Therefore, it is fully justified to exert constraints on the two parameters to eliminate the ill-condition in orbit determination.

Fig. 1 Predict error of ascending node of GPS

Fig. 2 Predict error of inclination of GPS

Fig. 3 Predict error of argument of perigee of GPS

Fig. 4 Rotation of GPS constellation

2.2 Analysis of autonomous orbit determination using inter-satellite distance Measurements

Supposing we have a walker constellation with 27/3/1 satellite distribution, orbit height being 28,378.140 km, eccentricity being 0.01, inclination being 56°. Considering a two-body problem, the accuracy of time synchronization being 2 ns, distance measuring noise being 0.2 m, through batch processing method using inter-satellite distance measurements of a 24-hour orbit arc with sampling rate of 5 min, results of orbit determination are given in Table 1 and Table 2. Table 1 shows the results using only inter-satellite distance, and Table 2 shows the results with ±0.5 arc second constraints on exerted $\Omega$ and $i$ based on the scheme used by Table 1. For the sake of the paper's length, in tables results of only 3 satellites in each orbit plane are given, and data from column 2 to 7 are corrections to initial orbit elements, which reflect the accurateness of solutions, and column 8 gives statistical results of the difference of reference orbit and computed orbit in terms of user range error (URE).

From Table 1, a distinct correlation exists between the orbital orientation parameters $\Omega$, $i$ and $\omega$, which shows that the entire constellation rotation cannot be determined using only inter-satellite distance measurements. From Table 2, there is no grouping phenomenon for $\Omega$ and $i$ while for the near-circular orbit ($e \approx 0$), $\omega$ is strongly correlated with $M$, so the addition of $\Delta\omega$ and $\Delta M$ decreases distinctly which means that the correlation of orientation parameters no longer exists. That is to say, orbit determination of navigation satellites can be done using inter-satellite distance measuring and orientation parameter constraints of predicted orbit, on the basis of dynamic information of satellites.

3 Conclusions

The purpose of satellite navigation systems is
Table 1  Orbit determination results only based on crosslink range

| PRN | Δa/m | Δr | Δi/ΔΩ | Δao/ΔM | URE/m |
|-----|------|----|--------|--------|-------|
| 1   | 0.00144 | 0.00000 | -25.3 | -0.6 | 21.5 | -3.1 | 0.847 |
| 2   | 0.00190 | 0.00000 | -25.4 | -0.5 | 23.4 | -5.0 | 0.794 |
| 3   | 0.00144 | 0.00000 | -25.4 | -0.6 | 18.5 | -0.1 | 0.699 |
| 10  | 0.00018 | 0.00000 | -0.4  | 29.4  | -36.2 | 0.5  | 0.709 |
| 11  | -0.00027 | 0.00000 | -0.5  | 29.5  | -37.2 | 1.6  | 0.799 |
| 12  | 0.00019 | 0.00000 | -0.5  | 29.4  | -38.3 | 2.7  | 0.848 |
| 19  | -0.00126 | 0.00000 | 26.0  | -0.1  | 17.9  | -0.5 | 0.815 |
| 20  | -0.00161 | 0.00000 | 26.0  | -0.1  | 18.2  | -1.0 | 0.712 |
| 21  | 0.00005 | 0.00000 | 25.7  | -0.2  | 16.4  | 1.0  | 0.705 |

Table 2  Orbit determination results based on crosslink range and orientation parameters constraining

| PRN | Δa/m | Δr | Δi/ΔΩ | Δao/ΔM | URE/m |
|-----|------|----|--------|--------|-------|
| 1   | 0.02  | 0.000 | 0.0    | -0.3   | 11.4  | -9.7  | 0.561 |
| 2   | 0.03  | 0.000 | 0.1    | -0.2   | 29.7  | -27.8 | 0.581 |
| 3   | 0.02  | 0.000 | -0.2   | 0.5    | 2.4   | 3.4   | 0.573 |
| 10  | 0.03  | 0.000 | 0.2    | -1.0   | 2.8   | 2.8   | 0.557 |
| 11  | 0.02  | 0.000 | 0.2    | -0.6   | 3.4   | 4.4   | 0.574 |
| 12  | 0.03  | 0.000 | -0.1   | 1.3    | -11.4 | 11.4  | 0.588 |
| 19  | 0.03  | 0.000 | 0.2    | -0.5   | 19.5  | 21.7  | 0.551 |
| 20  | 0.02  | 0.000 | 0.0    | 0.2    | -18.9 | 20.2  | 0.552 |
| 21  | 0.02  | 0.000 | 0.2    | -0.3   | -2.8  | 4.5   | 0.564 |

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...to provide time and space origin with high accuracy for users, whose key problem is autonomous orbit determination and autonomous timing of navigation satellites.

Coordinate system connected with the earth cannot be set up only using inter-satellite observations, and therefore, dynamic equations of satellites and information on orbit prediction must be combined to determine the satellite orbits. Inter-satellite distance measurements cannot reflect the rotation of the entire constellation within inertial space, which is represented as the correlation of orientation parameters of the orbit plane, and the problem can be solved by exerting constraints to orientation parameters of the orbit plane. Studies on such middle and high orbit constellation as GPS show that the error just amounts to ±0.4 arc second on a 180-day term of orbit prediction, which demonstrates that the constraints is reasonable and can satisfy the demand for accuracy.