Electroweak baryogenesis by primordial black holes in Brans-Dicke modified gravity

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Abstract

A successful baryogenesis mechanism is proposed in the cosmological framework of Brans-Dicke modified gravity. Primordial black holes with small mass are produced at the end of the Brans-Dicke field domination era. The Hawking radiation reheats a spherical region around every black hole to a high temperature and the electroweak symmetry is restored there. A domain wall is formed separating the region with the symmetric vacuum from the asymmetric region where electroweak baryogenesis takes place. First order phase transition is not needed. In Brans-Dicke cosmologies black hole accretion can be strong enough to lead to black holes domination which extends the lifetime of black holes and therefore baryogenesis. The analysis of the whole scenario, finally, results in the observed baryon number which can be achieved for a CP-violating angle that is predicted by observationally accepted Two-Higgs Doublet Models. The advantage of our proposed scenario is that naturally provides both black hole domination and more efficient baryogenesis for smaller CP violating angles compared to the same mechanism applied in a FRW cosmological background.

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1 Introduction

An important open issue for cosmology is baryogenesis. For baryon number to be produced, three criteria must be satisfied, as stated by Sakharov [1]:
1. Baryon number non-conservation.
2. C and CP symmetry violation.
3. Conditions out of thermal equilibrium.

The baryon number violation can occur both at grand unification [2] and electroweak energy scale [3].

Quite a lot of baryogenesis models have been produced over the last decades (reviews [4] - [9]). One scenario ([1], [10] - [12]) is the baryon asymmetry to be produced by heavy particles decay in an expanding universe, with C and CP symmetry broken. These heavy particles can be gauge bosons of a grand unified theory. A problem with these models is that the baryon number produced can be wiped out in some later process, as sphaleron processes at $\sim 100\text{GeV}$.

Electroweak baryogenesis is another possibility [13], [14]. Chiral anomaly is a cause for baryon number violation [3]. The phase transition of the electroweak breaking could be of first or second order. However, in the standard model, the transition proved to be second order; the large value of Higgs mass killed any hopes for first order transition and thus the baryon number produced is destroyed by sphalerons. Another problem for standard model electroweak baryogenesis is that it predicts CP - violation angles smaller than required [15]. The electroweak baryogenesis can also be combined with some variation of general relativity, like TeV scale gravity [16], [17].

Baryo-through-leptogenesis [18] refers to lepton number production by heavy Majorana particles decay, at energies high as $10^{10}\text{GeV}$. The lepton asymmetry then leads to baryon asymmetry through electroweak processes that violate the (B+L) symmetry [19]. Some other possibilities are Affleck - Dine [20] and spontaneous [21] baryogenesis.

Baryon asymmetry can also be produced by primordial black holes (PBH) [22]. PBHs could be created at the beginning of the universe [23]. Initially, it was considered that PBHs can produce baryon excess by GUT processes [24]. The problem with this, as with other GUT baryon number producing models, is that the baryon number created can be washed out later by sphaleron processes [25], as we have explained. A model of electroweak baryogenesis by PBHs was proposed by Nagatani [26]. According to this, the baryon excess is produced in a thermal domain wall that separates a reheated, by Hawking radiation, area around the PBHs from the outer regions, where $T < 100\text{GeV}$. 
Another interesting recent model, which incorporates electroweak baryogenesis around PBHs is \[27\] by B. Carr et al. Electroweak baryogenesis by PBHs becomes very efficient \[28\] in the case of high energy modifications in the early universe, as in Randall - Sundrum cosmology \[29\].

In the present paper, we propose a novel model of electroweak baryogenesis by PBHs in Brans - Dicke cosmology. We assume that the early Universe starts from a small enough primordial black hole dominated era or a mixture of radiation and small enough primordial black holes. Brans-Dicke theories can realize such a scenario. While universe temperature has been lowered below electroweak symmetry breaking point ($\sim 100\, GeV$), a region around each PBH is reheated by Hawking radiation to $T > 100\, GeV$. A domain wall is formed between the symmetric and asymmetric regions and this is where baryogenesis takes place, by sphaleron processes. Its key characteristics are:

1. The phase transition at the domain wall can be of second order. The baryon over anti-baryon excess is created by sphalerons, not destroyed.
2. In order to produce the observed baryon number ($b/s \approx 6 \times 10^{-10}$), the universe needs to become PBH dominated. In BD - cosmology this may happen naturally, because of accretion by the PBHs. In standard cosmology, on the contrary, it is accepted that accretion cannot be significant \[30\].
3. The CP-violating angle must be quite large for adequate baryogenesis. This can be satisfied incorporating a realistic two Higgs doublet, instead of a single Higgs, in our model \[31\].

Brans - Dicke (BD) \[32\] is a modified gravity theory \[33\]. Its difference from general relativity (GR) is that the gravitational constant G is not constant. Instead, its value is the inverse of a time-dependent scalar field $\phi$. $\phi$ couples to gravity with a coupling parameter $\omega$. When $\omega \to \infty$ BD becomes GR. Solar system measurements require $\omega \gtrsim 10^4$. In BD $\omega$ is constant and so this present time limit holds also for the very early universe. Nevertheless, there are generalizations of the BD theory where $\omega$ varies with time. Its present value may obey the above limit, but may be much smaller during the early universe. Another class of generalised BD theories is that of the complete BD theories \[34\]. They incorporate energy exchange between the scalar field and ordinary matter.

We assume that PBHs are created at the end of the BD - field ($\phi$) domination era, although the model is not dependent on how they were created. Accretion can lead to BHs mass increase only when there is enough radiation for BHs to accrete. This may happen during radiation domination or even BH - domination, if there is enough radiation

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density, as we’ll show. We examine two cases: the first is that the universe becomes BH dominated immediately after PBHs creation, with BH density $\rho_{BH} = 0.7\rho$ and radiation density $\rho_{rad} = 0.3\rho$. The second is the case that PBHs are initially, immediately after their formation, only a small part of the universe but then, because of intense accretion, become dominant. It will be shown that for both cases there is a range of initial PBHs masses for which accretion leads the universe to become completely BH dominated ($\rho_{BH} \simeq 100\%$).

The advantage of the proposed scenario is that Brans-Dicke gravity, due to enhanced accretion, can naturally provide black holes domination in the early Universe and at the same time, as we are going to show, efficient baryogenesis for smaller CP-violating angles compared to the case of the same scenario but with the gravity of General Relativity.

In the following section, the baryon asymmetry mechanism is described. In section 3 we analyse the fist of the two cases of the proposed scenario, a black hole dominated Universe, while in section 4 we study a Universe that initially is radiation dominated but then becomes black hole dominated. Next a section with various bounds is given and finally the last section provides a conclusive summary.

2 Baryon number created by a single primordial black hole

The PBHs of our proposed mechanism are surrounded by radiation colder than the electroweak breaking point ($T_W \sim 100GeV$). They are very small and thus Hawking temperature $T_{BH}$ is much greater than this temperature. Then all kinds of Standard Model (SM) particles are emitted and they are in symmetric phase. So, the Hawking emission causes the thermalization of the black hole surrounding region. A local temperature $T(r)$ can be defined for a region with size greater than the mean free path (MFP) of the emitted particles. The MFP of a particle $f$ is $\lambda_f(T) = \frac{\beta_f}{T}$, where $\beta_f$ is a constant that depends on the particle species. Quarks and gluons have a strong interaction and they have the shortest MFP with $\beta_s \simeq 10$. Because of the high, $> T_{EW}$, reheating temperature, all SM particles contribute to the massless degrees of freedom ($g_{SM} \equiv \sum_f g_{*f} = 106.75$). So, the radiation density is $\rho = \frac{\pi^2}{30} g_{SM} T^4(r)$.

Yet the area closest to the PBH horizon externally, with depth the quarks and gluons MFP, is not thermalized. For this reason, the emitted particles move freely there and most of them don’t drop back to the black hole. Thus, the black hole radiation obeys the law of Stefan-Boltzmann with no corrections. Now let $r_o$ be the minimum thermalized radius and $T_o$ the local temperature there: $T_o = \frac{\beta_s}{r_o}$. We consider then the transfer equation of
the energy in the thermalized region to determine the temperature distribution \( T(r) \) \cite{26}. We assume diffusion approximation of photon transfer at the deep light-depth region is valid \cite{35}. The diffusion current of energy in Local Temperature Equilibrium (LTE) is \( J_\mu = -\frac{\beta}{3T^2} \partial_\mu \rho \). \( \beta/T \) is the effective MFP of all particles by all interactions with \( \beta \simeq 100 \). The diffusion equation is

\[
\frac{\partial}{\partial t} \rho = -\nabla_\mu J^\mu.
\]

A stationary spherical-symmetric solution \cite{35} is

\[
T(r)^3 = T_{bg}^3 + \frac{r_o}{r} \left( T_o^3 - T_{bg}^3 \right).
\]

where \( T_{bg} \) is the background temperature. It can be as high as somewhat lower than \( T_{EW} \), where sphaleron rate is suppressed.

The quantities \( r_o \) and \( T_o \) can be written as functions of black holes temperature \( T_{BH} \) by equalizing the outgoing diffusion flux \( 4\pi r^2 J(r) \simeq \frac{8\pi^3}{135} \beta_s \beta g_{SM} \left[ 1 - (T_{bg}/T_o)^3 \right] T_o^2 \) with the Hawking radiation flux \( 4\pi r_{BH}^2 \times \frac{\pi^2}{120} g_{SM} T_{BH}^4 \):

\[
r_o = \frac{16\pi}{3} \frac{1}{T_{BH}} \sqrt{\beta_s^3 \beta \left[ 1 - \left( T_{bg}/T_o \right)^3 \right]} \tag{2}
\]

and

\[
T_o = \frac{3}{16\pi \sqrt{\beta_s \beta}} \frac{T_{BH}}{\sqrt{1 - \left( T_{bg}/T_o \right)^3}}. \tag{3}
\]

\( T_{bg} \ll T_o \) and so the spherical thermal distribution surrounding the black hole is

\[
T(r)^3 = T_{bg}^3 + \frac{9}{256\pi^2} \frac{1}{\beta} \frac{T_{BH}^2}{r} \tag{4}
\]

for \( r > r_o \).

As mentioned before, the region around PBHs is reheated to temperatures higher than the electroweak breaking point and so symmetry is restored there. The background temperature, at the same time, remains below the electroweak breaking point and the symmetry broken. That means that an electroweak domain wall forms around the black hole and it starts at \( r_{DW} \). The phase transition at the domain wall does not have to be of first order. It can be a second order transition. This enlarges the parameter space of the validity of our proposed scenario.

Instead of a single Higgs SU(2) doublet, we incorporate a two Higgs doublet model (2HDM) in our model, since it can accommodate both a large CP-violation angle and CP-violation in the Higgs sector. In our study in order to be able to validate our scenario and provide bounds, we use the 2HDM of \cite{31} which is not only a serious model but also a typical example of the models that could comply with our idea. Our scenario needs
a non-negligible CP violation in the Higgs finite temperature corrected potential. The tree-level, CP-breaking scalar potential in \[31\] is
\[
V_{\text{tree}} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.},
\]
where
\[
\Phi_1 = \left( \begin{array}{c} \phi_1^+ \\ \phi_1^- \end{array} \right), \quad \Phi_2 = \left( \begin{array}{c} \phi_2^+ \\ \phi_2^- \end{array} \right)
\]
are the two SU(2) scalar field doublets. One can see that a Z_2 discrete symmetry holds, under which \( \Phi_1 \to \Phi_1 \) and \( \Phi_2 \to -\Phi_2 \). Because of this symmetry there are no flavour changing neutral currents. The symmetry is softly broken only by \( m_{12} \). The parameters of the potential are real, because of its hermiticity, except from the mass parameter \( m_{12} \) and the quartic coupling \( \lambda_5 \). With this scalar potential it is possible the doublets VEVs to be complex and this CP-violation cannot be gauged away due to the complex values of \( m_{12} \) and \( \lambda_5 \). Note that in our previous work \[28\] a different two Higgs model had been adopted.

We can simplify the form of the doublets with an SU(2) rotation. \( \partial V / \partial \phi_i = 0 \) solutions give stationary points, including the asymmetric minimum that respects the \( U(1) \) of electromagnetism: \( \Phi_1 = \frac{1}{\sqrt{2}} \left( 0, u \right)^T \), \( \Phi_2 = \frac{1}{\sqrt{2}} \left( 0, v e^{i\varphi} \right)^T \). where \( u, v, \varphi \) are real and \( \varphi \) is the CP-violating angle. This tree-level CP-violating phase depends on \( m_{12} \) and \( \lambda_5 \) and cannot be shifted by an SU(2) rotation. However, in this case, we need this CP angle to be very small due to Electron Dipole Moment constraints (EDM) \[36\]. To achieve strong CP-violations one can hope the loop finite temperature corrected potential to result to big CP-violating cases \[37,38,39,40\]. In this case, the constraints from EDM do not apply if at zero temperature the CP angle goes to very small values.

Regarding the cosmological consequences, anyway, the finite temperature effective potential is this that should be used. The temperature loop corrections incorporate for the larger range of the parameters space only small cubic resulting to a second order phase transition (in \[31\] the case of first order transition is also studied, something that is not needed in our scenario). We shift the scalar fields about their expectation values and the second doublet asymmetric minimum becomes
\[
\Phi_1 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ u(T) \end{array} \right), \quad \Phi_2 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v(T) e^{i\varphi(T)} \end{array} \right),
\]
with
\[
v(T) = v f(r),
\]
where \( f(r) \) is a form-function of the wall and has a value from zero to one; \( f(r) = 0 \) for \( r \leq r_{\text{DW}} \) and \( f(r) = \sqrt{1 - \left( \frac{T(r)}{T_W} \right)^2} \) for \( r > r_{\text{DW}} \).

At the limit between the thermalized sphere and the domain wall, the temperature is \( T(r_{\text{DW}}) = T_W \). Setting this in Eq. (4), we find the radius of the thermalized region \( r_{\text{DW}} \). The width of the domain wall \( d_{\text{DW}} \) is about of the order of \( r_{\text{DW}} \).

\[
d_{\text{DW}} \simeq r_{\text{DW}} = \frac{9}{256\pi^2 \beta_{br}} \frac{1}{1 - (T_{bg}/T_W)^3} \frac{T_{BH}^2}{T_W^3}. \tag{9}
\]

The structure of the electroweak domain wall is determined only by the thermal structure of the black hole and not by the dynamics of the phase transition as in the ordinary electroweak baryogenesis scenario (the CKN model).

3 First case: Black hole domination from the moment of creation

In our model, the universe at the beginning of its life is dominated by the BD - field. We assume that the PBHs creation happens at about the end of this period. Then the universe becomes a mixture of radiation and black holes. A first scenario we examine is that universe is BH dominated immediately after \( t_i \), that is \( \rho_{BH} > \rho_{\text{rad}} \). It becomes completely BH dominated because of accretion, if their initial masses are above the limit that accretion exceeds evaporation and the radiation is dense enough, as it will be shown. What follows is that having no more radiation to accrete, they only evaporate. \( t_{\text{ev}} \) is the time of complete evaporation. The universe then turns radiation dominated, with the observed baryon number already produced. Later the universe turns from radiation to dust dominated at \( t_{\text{eq}} \). It remains dust dominated until now \((t_0)\).

Barrow and Carr at [42] have obtained solutions for \( G \) for the three different eras of a model where the universe is initially dominated by the BD - field, then it turns radiation dominated and finally dust dominated:

\[
G(t) \simeq G_0\left(\frac{t_1}{t}\right)^\sqrt{n}\left(\frac{t_0}{t_{eq}}\right)^n, \text{ if } t < t_1 : \text{BD - field dominated}
\]
\[
G_0\left(\frac{t_0}{t_{eq}}\right)^n, \text{ if } t_1 < t < t_{eq} : \text{radiation dominated}
\]
\[
G_0\left(\frac{t_0}{t}\right)^n, \text{ if } t_{eq} < t : \text{dust dominated} \tag{10}
\]
where \( t_1 \) is the time of transition from BD - field dominated to radiation and \( n = \frac{2}{4+3\omega} \). To avoid confusion it is worth mentioning that there is no PBHs - domination era at Barrow - Carr work.

The modified solutions for our model are:

\[
G(t) \approx G_0 \left( \frac{t}{t_0} \right) \left( \frac{t_0 t_{eq}}{t_{eq} t_i} \right)^n, \text{ if } t < t_i : \text{BD – field dominated}
\]

\[
G_0 \left( \frac{t_0 t_{eq}}{t_{eq} t_i} \right)^n, \text{ if } t_i < t < t_{ev} : \text{PBHs dominated}
\]

\[
G_0 \left( \frac{t_0}{t_{eq}} \right)^n, \text{ if } t_{ev} < t < t_{eq} : \text{radiation dominated}
\]

\[
G_0 \left( \frac{t_0}{t} \right)^n, \text{ if } t_{eq} < t : \text{dust dominated}
\] (11)

Then we need to write formulas for universe density due to PBHs \( \rho_{BH} \) and scale factor \( \alpha \). The number density of PBHs at the time of their creation is:

\[
n_{BH}(t_i) = \frac{\rho_{BH}(t_i)}{m_{BH}(t_i)} \] (12)

BHs can be treated as dust, regarding the universe 's density due to them. Because of the fact that their mass changes due to accretion and evaporation, it is their number density \( n_{BH}(t) \), not density, that is inversely proportional to scale factor 3rd power, and so:

\[
\rho_{BH}(t) = n_{BH}(t)m_{BH}(t) = n_{BH}(t_i)\frac{\alpha^3(t_i)}{\alpha^3(t)}m_{BH}(t) = \rho_{BH}(t_i)\frac{m_{BH}(t)}{m_{BH}(t_i)}\frac{\alpha^3(t_i)}{\alpha^3(t)}
\] (13)

We assume that the number of black holes after their primordial creation and till their evaporation remains the same. Thus, we assume that these PBHs do not "eat" each other in a considerable rate during the accretion. Accretion concerns the surrounding radiation mainly.

In [42] can be found the scale factor’s time evolution:

\[
\alpha(t) \propto t^{(1-\sqrt{n})/3}, \text{ BD – field dominated}
\]

\[
t^{1/2}, \text{ radiation dominated}
\]

\[
t^{(2-n)/3}, \text{ dust dominated}
\] (14)

and so
The radiation density at the same time will be \( \rho_{\text{rad}}(t) = \rho(t) - \rho_{\text{BH}}(t) \).

Now we can have a formula for \( \rho(t) \) solving the first Friedmann equation. Friedmann equations for \( k = 0 \) (flat universe) and including the BD-field \( \phi \) are:

\[
\frac{\ddot{a}}{a} + \frac{\dot{a} \dot{\phi}}{a \phi} - \frac{\omega \dot{\phi}^2}{6 \phi^2} = \frac{8 \pi \rho}{3 \phi}
\]

\[
2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + 2 \frac{\dot{a} \dot{\phi}}{a \phi} + \frac{\omega \dot{\phi}^2 + \ddot{\phi}}{2 \phi^2} = \frac{8 \pi \rho}{\phi}
\]

\[
\frac{\dot{\phi}}{8 \pi} + 3 \frac{\dot{a} \dot{\phi}}{a 8 \pi} = \frac{\rho - 3 \rho}{2 \omega + 3}
\]

Then we can use Eq. (14) for dust domination:

\[
\frac{\dot{a}}{a} = \frac{2 - n}{3} \ t^{-1}
\]

and also Eq. (11) for BH domination:

\[
\phi = \frac{1}{G(t)} = \frac{1}{G_0} \left( \frac{t_{\text{eq}}}{t_0 t_{\text{ev}}} \right)^n t^n \Rightarrow \frac{\dot{\phi}}{\phi} = \frac{2 - n}{3} \ t^{-1}
\]

Substituting these and also \( \omega = \frac{2}{3n} - \frac{4}{3} \) to the first Friedmann equation, it becomes:

\[
\rho(t) = \frac{n + 4}{8 \pi G_0} \left( \frac{t_{\text{eq}}}{t_0 t_{\text{ev}}} \right)^n t^{n-2}
\]

To calculate the baryon number produced by each one PBH, we have to know how their mass evolves with time due to accretion and evaporation.

\[
m_{\text{acc}} = 4 \pi f R^2_{\text{BH}} \rho_{\text{rad}}
\]

where \( f \) is accretion efficiency \( \sim O(1) \). We set \( f = 2/3 \), as in [44]. \( R_{\text{BH}} = 2 G m_{\text{BH}} \) is the radius of the BH.

\[
m_{\text{ev}} = -4 \pi R^2_{\text{BH}} a_H T^3_{\text{BH}} = -\frac{a_H}{256 \pi^3 G^2 m^2_{\text{BH}}}
\]

where \( a_H \) is the Stefan-Boltzmann constant. Combining accretion and evaporation and using G(t) from Eq. (11) for BH-domination, we get:

\[
m_{\text{BH}} = 16 \pi f G_0^2 \left( \frac{t_0 t_{\text{ev}}}{t_{\text{eq}}} \right)^{2n} t^{-2n} m^2_{\text{BH}} \rho_{\text{rad}} - \frac{a_H}{256 \pi^3 G_0^2} \left( \frac{t_0 t_{\text{ev}}}{t_{\text{eq}}} \right)^{-2n} t^{2n} m^2_{\text{BH}}
\]
At this point in order to analyze the whole scenario, we have to set some indicative values to our free parameters. Since we want to study a black hole dominated Universe from the moment of PBHs domination we select $\rho_{BH} = 2 \rho_{rad}$.

In order to have a feeling about the black hole masses that are relevant for our scenario we demand $\dot{m}_{BH} = 0$ and we find the initial BH mass for which accretion equals evaporation, $m_i \simeq 10^{25} GeV$ (or $\simeq 1 gr$). Thus, we further choose $t_i = 10^{-30} sec$ and $m_i = 10^{27} GeV$. As shown in Fig. (1), accretion is able to increase the mass of the PBH, but only a little at the beginning. This is so because the radiation that was to be eaten becomes rapidly less dense, due to the universe expansion. The same, almost, happens for an initial mass even up to $10^{31} GeV$ (Fig. (2)). Only $10^{32} GeV$ or greater values accretion lead BHs to accumulate almost the entire universe mass (Fig. 3). We note here, though, that the value $m_{BH} = 10^{32} GeV$ is an upper limit, as shown in the bounds section.

Things are different in the case that PBH creation takes place earlier: $t_i \simeq 10^{-35} sec$. Initial accretion now equals evaporation for $m_i = 2.7 \times 10^{22} GeV$. Denser radiation makes accretion strong enough to lead to almost complete (95%) BH domination, for smaller initial masses (Fig. (4)). Then there is no more radiation for accretion to proceed.

The time that evaporation becomes stronger than accretion is given from Eq. (22), for $\dot{m}_{BH} = 0$ and for $m_{BH} = m_{max}$. For the $m_i = 10^{27} GeV$, $t_i = 10^{-35} sec$ case it is $m_{max} \sim 1.45 \times 10^{27} GeV$ and $t_{ev=acc} \sim 10^{-26} sec$. Universe will turn to radiation dominated with the evaporation of the PBHs. The evaporation and the result for the values in regard is shown in Fig. (5). The time of complete evaporation is for $m(t) = 0$ and it is $t_{ev} \simeq 2.7 \times 10^{-17} sec$.

### 3.1 Baryogenesis

In the following, we calculate the baryon number generated by a single black hole and then the baryon to entropy ratio $b/s$ of the universe.

Although sphaleron process takes place both in the symmetric region around a black hole and the domain wall, the required CP - violation and non-equilibrium conditions coexist only in the domain wall. So, it is there that the baryon assymetry is created. In addition, $f(r) = |\langle \phi_2(r) \rangle|/v \leq \epsilon = 1/100$ is needed, so as the order of the sphaleron process exponential factor to be one and the baryon asymmetry not to be suppressed. In other words, baryon generation happens in the region of the domain wall that Higgs scalar value is small and this is from $r_{DW}$ to $r_{DW}+d_{sph}$, where $d_{sph}$ is defined from $f(r_{DW}+d_{sph}) = \epsilon$. Then, it is $\int_{r_{DW}}^{r_{DW}+d_{sph}} dr \frac{d}{dr} \varphi(r) = \epsilon \Delta \varphi_{CP}$, where $\varphi(r, T) = [f(r) - 1] \Delta \varphi_{CP}$. Thus,
\[
\dot{B} = V \frac{\Gamma_{\text{sph}}}{T_W} N \dot{\varphi} \\
= 4\pi N \kappa \alpha_W^5 T_W^3 r_{\text{DW}}^2 v_{\text{DW}} \int_{r_{\text{RDW}}}^{r_{\text{RDW}}+d_{\text{sph}}} dr \frac{d}{dr} \varphi(r) \\
= \frac{1}{16\pi} N \kappa \alpha_W^5 \epsilon \Delta \varphi_{\text{CP}} T_{\text{BH}}^2 \frac{T_W}{T_W}
\]

where \(\Gamma_{\text{sph}}\) is the sphaleron transition rate, \(\Delta \varphi_{\text{CP}}\) the net CP phase. \(\mathcal{N} \approx O(1)\) is a model dependent constant which is determined by the type of spontaneous electroweak baryogenesis scenario and the fermion content, \(\kappa \approx O(30)\) is a numerical constant expressing the strength of the sphaleron process. This is the same as in 4-d GR.

Integrating numerically through the BHs lifetime, we calculate the total baryon number by a single BH.

\[
B = \int_{t_i}^{t_{\text{ev}}} \dot{B} \, dt
\]

The baryon number produced during accretion is orders of magnitude smaller than during evaporation.

After the BHs have gained their maximum mass, they only evaporate at a very slow rate until the very last moments before their complete annihilation (Fig. (5)). Thus, we can use an approximation where BHs mass remains constant until the time of evaporation when it turns to radiation completely.

The total baryon number density produced is:

\[
b = B \frac{\rho_{\text{BH}}(t_{\text{ev}}^-)}{m_{\text{max}}} \]

where

\[
\rho_{\text{BH}}(t_{\text{ev}}^-) = \rho_{\text{rad}}(t_{\text{ev}}^+) = \frac{\pi^2}{30} g_{\text{reh}} T_{\text{reh}}^4.
\]

\(T_{\text{reh}}\) is the temperature that the universe is reheated as BHs evaporate. We set \(T_{\text{reh}} = 95\,\text{GeV}\) so as to be below \(T_W\). \(m_{\text{max}}\) is \(1.5m_i\), as shown before. The entropy density is

\[
s = \frac{2\pi^2}{45} g_{\text{reh}} T_{\text{reh}}^3
\]

where \(g_{\text{reh}}\) is the massless degrees of freedom of the reheated plasma in the asymmetric phase.

Finally \(b/s \sim 8.7 \times 10^{-10} \Delta \theta_{\text{CP}}\). In order to have the observed \(6 \times 10^{-10}\) we need \(\Delta \theta_{\text{CP}} \sim 0.7\,\text{rad}\). Such a CP-violation phase is very possible for a 2HDM [31].
4 Second case: Primordial black holes domination because of accretion

Another case, even more interesting, is the one where the PBHs, at the time of their creation, are only a small fraction of the total universe density and the universe is radiation dominated. As it will be shown, accretion can be strong enough to lead to PBH domination and the production of the observed baryon number.

In this scenario, PBHs form also around the end of the $\phi$-domination era, but they consist of only a portion of $\rho$. That means a radiation domination period begins after the $\phi$-domination era. If accretion is able to lead most radiation inside the BHs, a PBH domination epoch follows, after $t_{eq1}$. $t_{eq1}$ is the time BHs density becomes equal to radiation density. The universe turns radiation dominated for the second time with BHs evaporation.

The evolution of $G(t)$ now is (if accretion lead from radiation to PBH domination):

$$ G(t) \simeq G_0 \frac{(t_i)^n}{t_{eq} t_{eq1}} \left( \frac{t_0 t_{ev}}{t_{eq1} t_{eq}} \right), \text{ if } t < t_i : \text{BD - field dominated} $$

$$ G_0 \left( \frac{t_0 t_{ev}}{t_{eq1} t_{eq}} \right)^n, \text{ if } t_i < t < t_{eq1} : \text{radiation dominated} $$

$$ G_0 \left( \frac{t_0 t_{ev}}{t_{eq1} t_{eq}} \right)^n, \text{ if } t_{eq1} < t < t_{eq} : \text{PBHs dominated} $$

$$ G_0 \left( \frac{t_0}{t_{eq}} \right)^n, \text{ if } t_{eq} < t < t_{ev} : \text{radiation dominated} $$

$$ G_0 \left( \frac{t_0}{t_{eq}} \right)^n, \text{ if } t_{ev} < t < t_{eq} : \text{dust dominated} $$

Equation (28)

For the period $t_i < t < t_{eq1}$, $G = \text{constant}$, as one can see at Eq. (28) for radiation domination, and so $\dot{\phi} = 0$. Then, the first Friedmann equation (Eq. (16)) becomes:

$$ \frac{\dot{a}^2}{a^2} = \frac{8\pi \rho}{3\phi} $$

Equation (29)

where $a \propto t^{1/2}$ and thus we have the total density $\rho(t)$. Eq. (13) for the PBHs energy density holds. Substituting the corresponding $a$:

$$ \rho_{BH}(t) = \rho_{BH}(t_i) \frac{m_{BH}(t)}{m_{BH}(t_i)} \left( \frac{t_i}{t} \right)^{3/2}, \text{ if } t_i < t < t_{eq1} $$

$$ = \rho_{BH}(t_{eq1}) \frac{m_{BH}(t)}{m_{BH}(t_{eq1})} \left( \frac{t_{eq1}}{t} \right)^2, \text{ if } t_{eq1} < t < t_{ev} $$

Equation (30)
For radiation it is still \( \rho_{\text{rad}}(t) = \rho(t) - \rho_{BH}(t) \).

The mass evolution of the PBHs is determined, as in the previous case, by accretion (Eq. (20)) and evaporation (Eq. (21)). The limit for accretion to be stronger than evaporation is now \( m_{BH}(t_i) \sim 2 \times 10^{22} \text{GeV} \). In Fig. (6), (7) is shown the mass evolution during the accretion period. One can see that accretion is very effective for \( m_i \geq 10^{27} \text{GeV} \) and as a consequence the universe becomes almost completely PBH dominated. With 95% of the density inside the BHs, there is nothing else to accrete. Evaporation follows (Fig. (8)).

So, the mass of a single PBH can increase up to 100,000 times (from \( 10^{27} \) to \( 10^{32} \) GeV) because of accretion. The black hole lifetime also increases. The mechanism of baryon number production is the same as in the previous case and thus the baryonic asymmetry created by a single PBH is considerably enhanced. The total baryon number to entropy density is calculated \( b/s \sim 8.7 \times 10^{-10} \Delta \theta_{CP} \), compatible with the observed one. This is the same with the previous case because the baryon number density \( b \) is proportional to the PBHs density, which in both cases is almost the total density of the universe.

5 Bounds

One limit for PBHs mass is posed by the fact that the size of the domain wall \( d_{DW} \) must be greater than the mean free path (MFP) \( \lambda \).

\[
d_{DW} = \frac{9}{256\pi^2} \frac{1}{\beta_{SM} c_W} \frac{T_{BH}^2}{T_W^3} \quad (31)
\]

\[
\lambda = \beta_s / T_W \quad (32)
\]

\[
d_{DW} > \lambda \Rightarrow T_{BH} > 53 \text{TeV} \Rightarrow m_{BH} < 10^{32} \text{GeV}. \quad (33)
\]

Another limit appears because the black hole lifetime \( \tau_{BH} \) should be quite greater than the time for the stable weak domain wall to form. The evaporation equation (Eq. (21)) is integrated analytically:

\[
m(t) = \frac{3^{\frac{1}{2}}(( f - 1) a_H t_0^{-2n} t_{eq}^{2n} t_{ev}^{-2n} t^{1+2n} + 256 G_0^2 \pi^3 (1 + 2n) m_{max}/3)^{1/3}}{4 \times 2^{2/3} G_0^{2/3} (1 + 2n)^{1/3} \pi} \quad (34)
\]

We use the formula for BH lifetime without accretion (that is from \( m_{max} \) till complete evaporation) because the evaporation period of BH life is orders of magnitude greater
than the accretion period. We take the evaporation period as the time duration from the moment that the evaporation becomes stronger than accretion, until the moment of PBH complete annihilation.

\[ \tau_{BH} \sim \frac{256 G_0^2 m_{\text{max}}^3 (1 + 2n) \pi^3 t_0^{2n} t_{eq}^{2n}}{3 a_H (1 - f)}. \]  

(35)

The domain wall formation time is

\[ \tau_{DW} = \frac{d_{DW}}{u_{DW}} = \frac{27 T_{BH}^4}{4096 \pi^4 \beta_{SM}^3 c_W^3 T_W^5}. \]  

(36)

Solving for \( m_{\text{max}} \) we find that it should be, approximately \( m_{\text{max}} > 10^{28} \text{ GeV} \). The masses that provide successful baryogenesis in our model are within these limits. To avoid confusion, this second constraint provides a lower bound on masses. The parameter \( m_{\text{max}} \) refers to the maximum value after accretion finishes to be dominant.

6 Primordial black holes mass spectrum

In the previous sections we worked with the assumption that all the black holes have the same mass. Thus, it was possible to have some analytical solutions, to check if the model produces the observed baryon number and to set bounds on black holes’ mass. Yet, it is more natural to assume that there is a spectrum of the initial masses. So, we are going to examine how this affects our model.

The two limits set in the previous section are still valid in the case of mass spectrum, since they refer to each one black hole’s mass. PBHs with mass greater than the upper bound are not hot enough to thermalize their neighbourhood. If they have, on the other hand, mass less than the lower bound, then their lifetime is not long enough to form the domain wall where the baryogenesis would take place. Only the part of PBHs mass spectrum in the range between the two bounds contributes to the baryon number generation.

Eqs. (23), (24) for the baryon number created by a single PBH are still valid, but the total baryon asymmetry created by all PBHs is

\[ b = \int_{0}^{\infty} B N(m, t) \, dm, \]  

(37)

where \( N \) is the number density of PBHs with masses from \( m \) to \( m + dm \). As a general conclusion, it suffices to state that the very efficient baryogenesis due to accretion remains unaffected from the presence of mass spectrum. Based on a certain cosmological scenario
of the creation of PBHs one can estimate the exact baryon asymmetry straightforwardly. More details will follow concerning the relation of the black hole mass spectrum and the time evolution of the scale factor and the cosmic densities.

Next, we derive the equations governing the evolution of the spectrum of PBHs. We assume that the initial number density of the black hole spectrum is described by a power-law form, as in [46] and [47]. Thus, the initial number density of the PBHs with masses between $m_0$ and $m_0 + dm_0$ is

$$N(m_0)dm_0 = A m_0^n \Theta(m_0 - m_c) dm_0,$$

(38)

where $m_0 = m(t = 0)$ is the initial PBH mass. For the analytic calculations not to become unnecessarily complicated, we accept that all PBHs form at the same initial time $t_0$. We use $\Theta$ to introduce a cut-off mass $m_c$. This protects from divergences at the low-masses limit. Thus, we set $\Theta = 1$ for $m > m_c$ and $\Theta = 0$ for $m \leq m_c$. We assume that $m_c$ is proportional to the Planck mass, $m_c = km_{pl}$, where the constant $k$ is arbitrary and has no dimensions. For the total energy density not to diverge at large masses, it has to be $n > 2$. According to Carr [46], initial density perturbations that produce PBHs in standard cosmology, indicate that $n$ is between 2 and 3. $A$ is the amplitude of the spectrum. Its units are such that $N(m_0)dm_0$ is number density.

The total number density of the black holes, as a function of time, is

$$N(t) = \int_0^\infty N(m, t) dm,$$

(39)

and their total energy

$$\rho_{BH}(t) = \int_0^\infty N(m, t) m dm.$$

(40)

Treating analytically the evolution of the mass spectrum considering both accretion and evaporation, was not possible. Yet, in our model the epoch when accretion is dominant is succeeded very quickly by an epoch when evaporation prevails, resulting in a reheated, radiation dominated universe. Thus, one can treat the two epochs separately.

### 6.1 Dominant accretion time period

Here we will analyze the time period after primordial black hole creation where the accretion is important. Our aim is to calculate the evolution of black holes and radiation densities and the scale factor.

The factors that determine the PBHs mass spectrum evolution are not only the universe’s expansion, but accretion also. The rate of gain, because of accretion, for a single
black hole is given by Eq. (20). Solving it we get

\[ m_0 = \frac{1}{m(t)^{-1} + 16\pi f I_\rho}, \]

(41)

where \( I_\rho = \int_0^t G^2 \rho_{rad} \).

Differentiating Eq. (41) with respect to \( m_0 \), we can have an expression for the evolution of the number density of PBHs with masses from \( m \) to \( m + dm \) at time \( t \), combining it with Eq. (38) (special care must be given for the jacobian factor). So, the evolution of the mass spectrum with time is

\[ N(m, t) dm = N(m_0, t) dm_0 = A \left( \frac{1}{m(t)^{-1} + 16\pi f I_\rho} \right)^{2-n} m^{-2} \Theta(m - m_{ca}(t)) dm, \]

(42)

where \( m_{ca} \) is the cut-off mass that is evolved from \( m_c \):

\[ m_{ca}(t) = \left( \frac{1}{k m_{pl}} - 16\pi f I_\rho \right)^{-1}. \]

(43)

One can see that, contrary to the evaporation epoch, the cut-off mass never becomes 0.

The energy density rate is determined using the identity

\[ \frac{d}{dx} \int_{f(x)}^{g(x)} h(x, y) dy = \int_{f(x)}^{g(x)} \frac{\partial h(x, y)}{\partial x} dy + h(x, f(x)) \frac{df(x)}{dx} - h(x, g(x)) \frac{dg(x)}{dx}, \]

(44)

The energy density of the radiation that is eaten by the PBHs and so is added to the black hole density \( \rho_{bh} \) (not the comoving), from time \( t \) to \( t + dt \), is \( dE = \rho_{bh}(t + dt) - \rho_{bh}(t) = \frac{\partial \rho_{bh}}{\partial t} dt \). The energy density rate, then, is calculated using also Eq. (44):

\[ \frac{dE}{dt} = \frac{d}{dt} \int_0^\infty N(m, t) m dm = \int_0^\infty A(n - 2) (m^{-1} + \xi I_\rho)^{n-3} \xi \frac{dI_\rho}{dt} m^{-1} dm \]

\[ -A \left( m_{ca}^{-1} + \xi I_\rho \right)^{n-2} m_{ca}^{-1} \frac{dm_{ca}}{dt} \Theta(m - m_{ca}(t)). \]

(45)

where \( \xi = 16\pi f \) and

\[ \frac{dm_{ca}}{dt} = \left( \frac{1}{k m_{pl}} - \xi I_\rho \right)^{-2} \xi \frac{dI_\rho}{dt}. \]

(46)

The first term of Eq. (45) is actually the evolution of the spectrum. The second term is present because of the mass cut-off evolution. In the accretion era this term does not vanish, since evaporation is insignificant, compared to accretion. Then, we can have the full equations that determine the expansion, where the densities must be multiplied by \( a^{-3} \), in order to become comoving.
The resulting set of equations is

\[ \rho_{BH} = \frac{1}{a^3} \int_0^{\infty} N(m, t) \, m \, dm, \quad \rho = \rho_{BH} + \rho_{rad} \quad (47) \]

\[ \frac{dE_{com}}{dt} = \frac{1}{a^3} \frac{dE}{dt} \quad (48) \]

\[ \dot{\rho}_{rad} = -4\frac{\dot{a}}{a} \rho_{rad} - |\frac{dE_{com}}{dt}|, \quad (49) \]

since the kinetic pressure by the black holes is not important.

This set is supplemented by Eqs. \((16)\) and either \((11)\) for the first case or \((28)\) for the second case. They are an integro-differential system, which is solved only numerically for various ranges of the parameters.

### 6.2 Dominant evaporation time period

At some point in time accretion becomes less important than evaporation. This happens due to the ongoing expansion of the universe and, mainly, because the whole of the universe’s radiation ends inside the PBHs, as we explained in the previous sections. From that time on, evaporation dominates the evolution of the black hole mass. The significance of this analysis lies in finding the modifications to the expansion rate, allowing the emergence of the conventional radiation expansion law. We aim to determine the deviations of the PBHs and of radiation densities and the evolution of the scale factor.

The evolution of the PBHs mass spectrum depends on the expansion of the universe and, more importantly, on the evaporation of the PBHs. The rate of mass loss of a single black hole, because of evaporation, is given by Eq.(21). Integrating it we get Eq.(34).

Note that now the initial value \(m_0 = m_{max}\) is the maximum value of the black hole mass after the end of the dominant accretion time period.

\[ m^3 = m_0^3 - \frac{3 a_H}{256\pi^3} I_g \quad (50) \]

where \(I_g = \int_0^t G^{-2} dt\). Then, we solve with respect to \(m_0\) and differentiate. Thus, we can have the evolution of the black holes number density from \(m\) to \(m + dm\) at time \(t\) from Eq.(38). The evolved mass spectrum is given by

\[ N(m, t)dm = A \left( m^3 + \frac{3 a_H}{256\pi^3} I_g \right)^{-(n+2)/3} m^2 \Theta(m - m_{cr}(t)) \, dm, \quad (51) \]

where the cut-off mass is evolved, too:

\[ m_{cr}(t) = [(k m_{pl})^3 - \frac{3 a_H}{256\pi^3} I_g]^{1/3}. \quad (52) \]
We can see that there is a time \( t_{\text{lim}} \) that the cut-off mass becomes 0.

The energy density that is emitted by the black hole as radiation from \( t \) to \( t + dt \) is estimated from Eq. (40). It is

\[
dE = \varrho_{bh}(t) - \varrho_{bh}(t + dt) = -\left(\frac{\partial \varrho_{bh}}{\partial t}\right) dt.
\]

where the quantities are not comoving. We can have the energy density rate using the identity Eq. (44). So, we find

\[
-\frac{dE}{dt} = \frac{d}{dt} \int_0^\infty N(m, t) m \, dm
\]

where

\[
dm_{\text{cr}} = \frac{-aH}{256\pi^3} \left( k m_{\text{pl}} \right)^3 \left[ \frac{3aH}{256\pi^3} I_g \right]^{-2/3} \frac{dI_g}{dt},
\]

and

\[
m_{c,\text{max}}(t) = \text{max}[0, m_{\text{cr}}(t)].
\]

The first term in Eq. (55) expresses the evolution of the spectrum and is the only non-zero term at late times. The second term of Eq. (55) is present because of the time evolution of the mass cut-off. It is apparent that for times larger than \( t_{\text{lim}} \), the lightest black holes completely evaporate and the \( \Theta \) function causes this term to vanish.

In all the quantities calculated so far, the dilution from the expansion will have to be added; the comoving density is \( \rho_{BH} = \varrho_{bh} a^{-3} \), and the comoving energy is \( E_{\text{com}} = Ea^{-3} \).

Finally, the set of equations is the following

\[
\rho_{BH} = \frac{1}{a^3} \int_0^\infty N(m, t) m \, dm, \quad \rho = \rho_{BH} + \rho_{\text{rad}}
\]

\[
\frac{dE_{\text{com}}}{dt} = \frac{1}{a^3} \frac{dE}{dt}
\]

\[
\dot{\rho}_{\text{rad}} = -\frac{4}{a} \rho_{\text{rad}} + \left| \frac{dE_{\text{com}}}{dt} \right|,
\]

since black holes exert unimportant kinetic pressure.

Eqs. (58), (59), (60), (16) and either (11) for the first case or (28) for the second case are an integro-differential system. Like in the dominant accretion time period, the equations system can be solved only numerically for various ranges of the parameters.
7 Conclusions

Primordial black holes born in a BD - universe, at the end of the BD - field domination era (∼ 10^{-35} sec), with initial mass \( m_i \geq 10^{27} GeV \), accrete radiation from their surroundings intensively, leading to almost complete PBH domination, even if PBHs density was initially only 1/100,000 of the universe density. However, the maximum of PBH mass should not exceed ∼ 10^{32} GeV.

The produced baryon to entropy is \( b/s \sim 8.7 \times 10^{-10} \Delta \theta_{CP} \) and can match the observed \( 6 \times 10^{-10} \) value with a loop corrected \( \Delta \theta_{CP} \sim 0.7 rad \), which is within the limits of phenomenologically accepted two Higgs doublet models.

We proved that BD gravity, due to enhanced accretion, can naturally provide black holes domination in the early Universe and at the same time, efficient baryogenesis for smaller CP violating angles compared to the case of the conventional gravity of General Relativity.

It is worth studying the ideas presented in this work for Asymptotic Safe Gravity [48], since it shares some similar properties to Brans-Dicke models. Another interesting question is to analyse how initial anisotropic or inhomogeneous backgrounds (with small anisotropies/inhomogeneities that smooth out later) affect the mechanism [49].

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Figure 1: $t_i = 10^{-30}\text{sec}$: initial $\rho_{BH} \sim 67\% \rho_{universe}$, $\rho_{rad} \sim 33\% \rho_{un}$, $M_i = 10^{27}\text{GeV}$: accretion is insufficient to increase BH mass significantly.

Figure 2: $t_i = 10^{-30}\text{sec}$: initial $\rho_{BH} \sim 67\% \rho_{un}$, $\rho_{rad} \sim 33\% \rho_{un}$, $M_i = 10^{31}\text{GeV}$: accretion is insufficient to increase BH mass significantly.
Figure 3: for $t_i = 10^{-30}$ sec, $M_i = 10^{32}$ GeV: $\rho_{rad} \sim 33\% \rho_{un}$, for $t \sim 10^{-27}$ sec, $M_{BH} \sim 1.45 \times 10^{32}$ GeV: $\rho_{rad} \sim 3\% \rho_{un}$: accretion is sufficient to lead to almost total BH domination.

Figure 4: for $t_i = 10^{-35}$ sec, $M_i = 10^{27}$ GeV: $\rho_{rad} \sim 33\% \rho_{un}$, for $t \sim 10^{-32}$ sec, $M_{BH} \sim 1.45 \times 10^{27}$ GeV: $\rho_{rad} \sim 3\% \rho_{un}$: accretion is sufficient to lead to almost total BH domination.
Figure 5: $\rho_{BH} = 2\rho_{rad}$, $M_i = 10^{27}\text{GeV}$, $t_i = 10^{-35}\text{sec}$, evaporation

Figure 6: $t_i = 10^{-35}\text{sec}$: initial $\rho_{BH} \sim 10^{-6}\rho_{un}$, $\rho_{rad} \sim \rho_{un}$, $M_i = 10^{26}\text{GeV}$: accretion is insufficient to increase BH mass significantly
Figure 7: For $t_i = 10^{-35}\text{sec}$: $M_i = 10^{27}\text{GeV}, \rho_{BH} \sim 10^{-3}\rho_{\text{un}}, \rho_{\text{rad}} \sim \rho_{\text{un}}$. For $t \sim 10^{-32}\text{sec}$: $M_{BH} \sim 10^{30}\text{GeV}, \rho_{BH} \sim \rho_{\text{un}}, \rho_{\text{rad}} \sim 0$: accretion is sufficient to lead to almost total BH domination.

Figure 8: $t_i = 10^{-35}\text{sec}, \rho_{BH} = 10^{-3}\rho_{\text{rad}}, M_i = 10^{27}\text{GeV}, M_{BH(\text{max})} \sim 10^{30}\text{GeV}$, evaporation
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