Background-dependent Lorentz violation:
natural solutions to the theoretical challenges
of the superluminal neutrino propagation

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Abstract To explain both the possible superluminal neutrino propagation and all the known phenomenological constraints/observations on Lorentz violation, the Background-Dependent Lorentz Violation (BDLV) has been proposed. We study the BDLV in a model-independent way. Assuming that the Lorentz violation on the Earth is much larger than those on the interstellar scale, we automatically escape all the astrophysical constraints on Lorentz violation. For the BDLV from the effective field theory, we present a simple model and discuss the possible solutions to the theoretical challenges of the superluminal neutrino propagation such as the Bremsstrahlung effects for muon neutrinos and the pion decays. Also, we address the Lorentz violation constraints from the LEP and KamLAMD experiments. For the BDLV from the Type IIB string theory with D3-branes and D7-branes, we point out that the $D3$-branes are flavour blind, and all the SM particles are the conventional particles as in the traditional SM when they do not interact with the $D3$-branes. Thus, we not only can naturally avoid all the known phenomenological constraints on Lorentz violation, but also can naturally explain all the theoretical challenges. Interestingly, the energy-dependent photon velocities may be tested at the experiments.

1 Introduction

The ICARUS neutrino experiment at the underground Gran Sasso Laboratory (LNGS) has recently determined the muon neutrino ($\nu_\mu$) velocity with high accuracy through the measurement of the flight time and the distance (730 km) between the source of the CNGS neutrino beam at CERN (CERN Neutrino beam to Gran Sasso) and the ICARUS detector at the LNGS \cite{1}. The result is

\begin{equation}
\delta t = 0.3 \pm 4.9(\text{stat.}) \pm 9.0(\text{syst.}) \text{ ns},
\end{equation}

which is compatible with the simultaneous arrival of all events with speed equal to that of light.

In this paper, we would like to study the possible superluminal neutrino propagation. Within the uncertainties of the ICARUS experiment, we can still assume $\delta t \sim 10 \text{ ns}$. This implies a superluminal propagation velocity for neutrinos by a relative amount

\begin{equation}
\delta v_\nu \equiv \frac{v_\nu - c}{c} = 4.1 \times 10^{-6}.
\end{equation}

Interestingly, the above possible results are compatible with the MINOS results \cite{2}. Although not statistically significant, the MINOS Collaboration has found \cite{2}

\begin{equation}
\delta v_\nu = (5.1 \pm 2.9) \times 10^{-5} \quad \text{(MINOS)},
\end{equation}

for muon neutrino with a spectrum peaking at about 3 GeV, and a tail extending above 100 GeV. Moreover, the earlier short-baseline experiments have set the upper bounds on $|\delta v_\nu|$ around $4 \times 10^{-5}$ in the energy range from 30 GeV to 200 GeV \cite{3}. From theoretical point of view, many groups have already studied the possible solutions or pointed out the challenges to the superluminal neutrino propagation \cite{4–25}. For an early similar study, see Ref. \cite{26}.

The major challenges to the superluminal neutrino propagation are the following: (1) Bremsstrahlung effects \cite{14}. The superluminal muon neutrinos with $\delta v_\nu$ given in Eq. (2) would lose energy rapidly via Cherenkov-like processes on their ways from CERN to LNGS, and the most important
process is $\nu_\mu \rightarrow \nu_\mu + e^+ + e^-$. Thus, the ICARUS experiment cannot observe the muon neutrinos with energy in excess of 36.6 GeV [14]; (2) Pion decays [15, 18, 21]. The superluminal muon neutrinos with $\delta v_\nu$ in Eq. (2) can not have energy larger than about 24.5 GeV from pion decays, for example, $\pi^+ \rightarrow \mu^+ \nu_\mu$ and $\mu \rightarrow \nu_\mu + e + \bar{\nu}_e$ [18]. One natural solution to these challenges is the deformed Lorentz symmetry, which preserves the relativity of inertial frames [27]. These deformations add non-linear terms to the energy-momentum relations, conservation laws, and Lorentz transformations in a way which is consistent with the relativity of inertial observers [27]. Thus, the above anomalous processes are forbidden [23]. However, the studied model is a toy model, which cannot explain the above superluminal neutrino propagation [23].

To explain both the possible superluminal neutrino propagation and all the known phenomenological constraints/on observations on Lorentz violation, the Background-Dependent Lorentz Violation (BDLV) has been proposed by considering the spin-2, spin-1, spin-0 particles, and Type IIB string theory, respectively, in Refs. [7, 13, 17, 22]. In this paper, we briefly review the relevant phenomenological constraints and observations on Lorentz violation [28–47], and we study the BDLV in a model-independent way. In particular, we conjecture that there may exist a “Dream Special Relativity Theory”, where all the Standard Model (SM) particles can be subluminal due to the background effects. We also suggest that the ICARUS experiment can measure the velocities of the muon neutrinos with energies around a few GeV, which may test whether the muon neutrino velocities depend on their energies or not. Assuming that the Lorentz violation on the Earth is much larger than those on the interstellar scale, we automatically escape all the astrophysical constraints on Lorentz violation. To explain the superluminal neutrino propagation in the effective field theory approach, we present a simple model with a triplet Higgs field as in the Type II seesaw mechanism [48–53], where we introduce the non-renormalizable operators which violate the Lorentz symmetry. For the BDLV from the effective field theory, we discuss the possible solutions to the above theoretical challenges, and we address the Lorentz violation constraints from the LEP [45] and KamLAMD [46, 47] experiments. For the BDLV from the Type IIB string theory with D3-branes and D7-branes [22], we point out that the D3-branes are flavour blind, and all the SM particles are the conventional particles as in the traditional SM when they do not interact with the D3-branes. Thus, we not only can naturally avoid all the known phenomenological constraints on Lorentz violation, but also can naturally explain all the above theoretical challenges [14, 15, 18, 21]. Moreover, we can explain the time delays for the high-energy photons compared to the low-energy photons in the MAGIC [31], HESS [32], and FERMI [33, 34] experiments. Such kind of model predicts that the photon velocities linearly depend on their energies. For a photon with energy around a few GeV we find that $\delta v_\gamma$ is around $4 \times 10^{-6}$ on the Earth. Thus, we can test our models at the experiments in principle.

2 The relevant phenomenological constraints and observations on Lorentz violation

In this section, we will briefly review the relevant phenomenological constraints and observations on Lorentz violation.

First, the detection of neutrinos emitted from SN1987a gave us a lot of information not only on the process of supernova explosion, but also on neutrino properties. The Irvine–Michigan–Brookhaven (IMB) [28], Baksan [29], and Kamiokande II [30] experiments collected $8 + 5 + 11$ neutrino events (presumably mainly $\bar{\nu}_e$) with energies between 7.55 MeV and 395 MeV within 12.4 seconds. In particular, the neutrinos arrived on the Earth about 4 hours before the corresponding light. Although this is compatible with the supernova explosion models, we can still obtain the upper limit on $\delta v_\nu$: 

$$|\delta v_\nu(15 \text{ MeV})| \leq 2 \times 10^{-9}. \quad (4)$$

This limit should be understood with an order-one uncertainty since the precise time delay between light and neutrinos is unknown. Also, we can employ the time coincidence of these events to constrain the velocity differences for neutrinos with various energies. Rescaling a statistical analysis for the case with quadratic energy dependence, $\delta v_\nu \propto E^2$ [26], we obtain the bound

$$|\delta v_\nu(30 \text{ MeV}) - \delta v_\nu(10 \text{ MeV})| < 5 \times 10^{-13} \quad \text{at 95 \% CL.} \quad (5)$$

However, there exists a larger uncertainty if the average neutrino energy changes with time during the detection interval around 10 seconds.

Second, the MAGIC [31], HESS [32], and FERMI [33, 34] Collaborations have reported time-lags in the arrival times of high-energy photons, as compared with photons of lower energies. The most conservative interpretations of such time-lags are that they are due to emission mechanisms at the sources, which are still largely unknown at present. However, such delays might also be hints for the energy-dependent vacuum refractive index, as first proposed 14 years ago in Ref. [54]. Assuming that the refractive index $n$ depends linearly on the $\gamma$-ray energy $E_\gamma$, i.e., $n_\gamma \sim 1 + E_\gamma/M_{\text{OG}}$ where $M_{\text{OG}}$ is the effective quantum gravity scale, it was shown that the time delays observed by the MAGIC [31], HESS [32], and FERMI [33, 34] Collaborations are compatible with each other for $M_{\text{OG}}$ around...
$0.98 \times 10^{18}$ GeV [55]. Also, there are the stringent constraints coming from synchrotron radiation of the Crab Nebula [37–41]. The D-particle models of space-time foam have been proposed to explain all these effects within the framework of string/brane theory, based on a stringy analogue of the interaction of a photon with internal degrees of freedom in a conventional medium [38–40, 56–61]. However, FERMI observation of GRB 090510 seems to allow only a much smaller value for time delay and then requires $M_{QG} > 1.22 \times 10^{19}$ GeV [35]. Because these data probe different redshift ranges, they may be compatible with each other by considering a redshift-dependent D-particle density [61].

Third, the superluminal $\pi^+$ will lose energy quickly via the radiative emission process $\pi^+ \rightarrow \pi^+ \gamma$. The IceCube experiment has measured the atmospheric neutrino spectrum up to $\sim 400$ TeV, which agrees pretty well with the model calculations [36]. Thus, the high-energy charged pions do not lose much energy before they decay to neutrinos, which gives a strong constraint on the process $\pi^+ \rightarrow \pi^+ \gamma$. Note that the threshold of this process is $E_{\pi} > m_{\pi}/\sqrt{2\delta v_\mu}$, and the maximal pion energy is about 2 PeV, we obtain an upper bound on $\delta v_\mu < 2 \times 10^{-14}$ [36].

Fourth, there are astrophysical constraints on Lorentz violation for charged leptons, which are given as follows.

- The constraint from the Crab Nebula synchrotron radiation observations on the electron dispersion relation [41]. For $\delta v_\nu$ linearly dependent on its energy, we obtain the electron dispersion relation
  \[ E^2 = p^2 + m_e^2 - \frac{p^3}{M_{QG}}, \]  
  where the corresponding effective quantum gravity scale $M_{QG}$ is larger than about $1 \times 10^{24}$ GeV [41].
- The electron vacuum Cherenkov radiation via the decay process $e \rightarrow e\gamma$ becomes kinematically allowed for $E_\gamma > m/\sqrt{2\delta v_\gamma}$. Note that the cosmic ray electrons have been detected up to 2 TeV, we have $\delta v_e < 10^{-13}$ from cosmic ray experiments [42].
- The high-energy photons are absorbed by CMB photons and annihilate into the electron-positron pairs. The process $\gamma\gamma \rightarrow e^+e^-$ becomes kinematically possible for $E_{\gamma, \gamma} > m/\sqrt{2\delta v_\gamma}$. Because it has been observed to occur for photons with energy about $E_\gamma = 20$ TeV, we have $\delta v_\gamma < 2 m_e^2/E_\gamma \sim 10^{-15}$ from cosmic ray experiments [43].
- The process, where a photon decays into $e^+e^-$, becomes kinematically allowed at energies $E_\gamma > m_e\sqrt{-2\delta v_\gamma}$. As the photons have been observed up to 50 TeV, we have $-\delta v_\gamma < 2 \times 10^{-16}$ from cosmic ray experiments [42]. The analogous bound for the muon is $-\delta v_\mu < 10^{-11}$ [44].

Fifth, there are the relevant constraints on Lorentz violation from the experiments done on the Earth. Let us list them in the following.

- The agreement between the observation and the theoretical expectation of electron synchrotron radiation as measured at LEP [45] gives the stringent bound on isotropic Lorentz violation $|\delta v_e| < 5 \times 10^{-15}$. Here, we emphasize that the electrons and positrons propagated in the vacuum tunnel at the LEP experiment.
- For the electron neutrinos at the KamLAND experiment, the non-trivial energy dependence of the neutrino survival probability implies that the Lorentz violating off-diagonal elements of the $\delta v_\nu$ matrix in the flavour space are smaller than about $10^{-20}$ [46, 47]. Thus, if the Lorentz violation cannot realize the flavour-independent couplings naturally, we do need to fine-tune the relevant couplings.

3 Background-dependent Lorentz violation

The main assumption for the background-dependent Lorentz violation theories is: the Lorentz violation for all the SM particles is not constant in the space time. In particular, the Lorentz violation for all the SM particles on the Earth is much larger than those on the interstellar scale or in the vacuum. For example, to explain the results from the OPERA experiment and the SN1987a observations, we will show that the Lorentz violation on the Earth is at least four orders larger than those on the interstellar scale or in the vacuum.

We would like to discuss the background-dependent Lorentz violation in a model-independent way. Considering the effective field theory or string theory, we can parametrize the generic $\delta v$ for a particle $\phi$ as follows:

\[ \delta v_\phi = - \frac{m_\phi^2}{2p^2} + \sum_{n\geq 0} a_\phi^n \frac{p^n}{M_\phi^n}, \]  
where $m_\phi$ and $p$ are, respectively, the mass and momentum of the particle, $a_\phi^n$ are the coefficients, and $M_\phi$ is the relevant effective scale. In the Type IIB string theory, we can obtain the $a_1^i$ term naturally by calculating the four-point function [22, 60], while in the effective field theory approach, we can realize all the terms in the above equation. By the way, for a massless particle like the photon or particle with tiny mass like the neutrinos, we can neglect the mass term and change $p$ to $E$.

We denote the couplings $a_n^i$ on the Earth as $[a_n^i]^E$, and the couplings $a_n^i$ on the interstellar scale as $[a_n^i]^IS$. As a concrete example, we employ the possible superluminal neutrino propagation and the SN1987a results to calculate the constraints on the ratios $[a_n^\nu]^IS/[a_n^\nu]^E$ for muon neutrinos. For simplicity, we consider $n = 0, 1, 2$, and assume that only one $[a_0^\nu]^E$ term generates OPERA $\delta v_\nu$ in Eq. (2) and satisfies the SN1987a constraints in Eqs. (4) and (5). Thus,
we obtain
\[
\frac{[a_0^1]^{IS}}{[a_0^1]^{E}} \leq 4.86 \times 10^{-4}, \quad \frac{[a_1^1]^{IS}}{[a_1^1]^{E}} \leq 1.38 \times 10^{-4},
\]
\[
\frac{[a_2^1]^{IS}}{[a_2^1]^{E}} \leq 5.16 \times 10^{-2}.
\]

Interestingly, considering the uncertainties in the SN1987a observations, we find that the ratio \[\frac{[a_0^1]^{IS}}{[a_0^1]^{E}}\] is similar to the ratio \[\frac{[a_1^1]^{IS}}{[a_1^1]^{E}}\]. Notice that both the \[\frac{[a_2^1]^{IS}}{[a_2^1]^{E}}\] term and the \[\frac{[a_1^1]^{E}}{[a_1^1]^{E}}\] term can be consistent with the superluminal neutrino propagation on weak energy dependence for \(\delta v_\nu\) or \(\delta t\), thus, these results may be generated by both terms. In particular, we propose that the ICARUS experiment can test the \(\delta v_\nu\) energy dependence by lowering their muon neutrino energies to about a few GeV. If the \(\delta v_\nu\) energy dependence is confirmed, it will be a big discovery for sure.

In addition, the speed of light in the vacuum has been measured on the Earth. We conjecture that there may exist a “Dream Special Relativity Theory”. In particular, the photon velocity on the Earth is not the maximal photon velocity in the “Dream Vacuum” due to the background effects. Thus, in principle, all the SM particles including both the photons and neutrinos are subluminal after Lorentz violation.

We can explain the superluminal neutrino propagation and MINOS results as well. As a concrete example, we can consider that all the SM particles have the same \(a_0^\phi\), \(a_1^\phi\), and \(a_2^\phi\), and we require \(a_0^\phi < 0\) and \(a_2^\phi < 0\) while \(a_2^\phi > 0\). The point is that the speed of light has been measured on the Earth for the photons with very low energies. The more detailed study will be given elsewhere. Interestingly, although we cannot measure the \([a_0^\phi]^{E}\) term in the Laboratories on the Earth, we can try to measure the \([a_1^\phi]^{E}\) terms in the Laboratories on the Earth for \(n \geq 1\) by varying the photon energies if the photon energies are not very small.

Let us subscribe to Eddington’s dictum: “Never believe an experiment until it has been confirmed by theory”. Thus, we need to understand the \(a_n^\phi\) terms in Eq. (7) from the theoretical point of view. First, we consider the effective field theory, and will generate the \(a_n^\phi\) terms for \(n = 0, 1, 2\) simultaneously, which has not been studied yet. As we know, to explain the neutrino masses and mixings in the Type II seesaw mechanism, we need to introduce a triplet Higgs field \(\Phi\) whose quantum number under \(SU(3)_C \times SU(2)_L \times U(1)_Y\) gauge symmetries is \((1, 3, 1)\) [48–53]. To break the Lorentz symmetry, we choose a unit vector \(u^\mu = (1, 0, 0, 0)\), which does not break the \(SO(3)\) space rotation symmetry. This can be realized by giving Vacuum Expectation Value (VEV) to a vector field or \(\partial_\mu \Phi\). Therefore, we can have the following non-renormalizable terms in the Lagrangian:

\[
L = \frac{1}{1 + \delta_{ij}} \left( iy_{jk} \frac{\Phi}{M_s} u^\mu L_j (\partial_\mu L_k) + y_{jk} \frac{\Phi}{M_s} (u^\nu \partial_\mu L_j) (u^\nu \partial_\mu L_k) \right).
\]

where \(y_{jk}\) and \(y'_{jk}\) are coupling constants, and \(L_i\) denote the lepton doublets. For simplicity, we assume \(y_{jk} = 0\) and \(y'_{jk} = 0\) for \(j \neq k\). Thus, we obtain

\[
\delta v_\nu = \gamma_{jk} \frac{\Phi}{M_s} \left( \frac{\gamma_{jk}^2 V_\Phi^2}{2 M_s^2} + \frac{\gamma_{jk}^2 V_\Phi^2 E}{M_s^2} + \frac{3 \gamma_{jk}^2 V_\Phi^2 E^2}{2 M_s^4} \right),
\]

where \(V_\Phi\) is the VEV of \(\Phi\). In short, we can instead have the \(a_n^\phi\) terms for \(n = 0, 1, 2\) in the mean time from the effective field theory. Similar discussions hold for the \(a_n^\phi\) terms with \(n > 2\).

Furthermore, the \(a_1^\phi\) terms in Eq. (7) can be generated in the Type IIB string theory [22, 60]. Let us briefly review the results. We consider the Type IIB string theory with D3-branes and D7-branes where the D3-branes are inside the D7-branes [22, 60]. The D3-branes wrap a three-cycle, and the D7-branes wrap a four-cycle. Thus, the D3-branes can be considered as point particles in the Universe, while the SM particles are on the world-volume of the D7-branes. We assume that the \(V_{A3}\) is the average three-dimensional volume which has a D3-brane locally in the Minkowski space dimensions, and \(R^\prime\) is the radius for the fourth space dimension transverse to the D3-branes in the D7-branes. Especially, \(V_{A3}\) is the inverse of the D3-brane number density and can vary in the space time. Also, we define the dimensionless parameters \(\eta\) and \(\xi\) as follows [22]:

\[
\eta \equiv \frac{(1.55)^4}{V_{A3} R^\prime M_s^{1/3}}, \quad \xi \equiv M_s \times V_{A3}^{1/3},
\]

where \(M_s\) is the string scale. Especially, \(\eta\) is a small number around 0.1 or smaller for a perturbative theory.

For the \(SU(3)_C \times SU(2)_L \times U(1)_Y\) gauge fields and their corresponding gauginos, we have [22]

\[
\delta v \simeq \pm \frac{(2n + 1) \eta E}{\xi M_s}.
\]

For all the other SM particles, we have [22]

\[
\delta v \simeq \pm \frac{(2n + 1) \eta E}{\xi M_s}.
\]

Let us consider the lowest order term, \(n = 1\). As shown in Ref. [22], we can explain the superluminal neutrino propagation and the MAGIC, HESS, FERMI, and MINOS experiments simultaneously. Moreover, if we do not consider the time delays in the MAGIC, HESS, and FERMI experiments, we can choose the positive signs in both Eq. (12) and Eq. (13). Note that the speed of light has been measured for the very low-energy photons, we can still explain the superluminal neutrino propagation, which is similar to the discussions in Ref. [22]. The point is that \(\delta v_\nu\) for a very
low-energy photon can be much smaller than $\delta v_\nu$ for a muon neutrino with energy about 17 GeV. Interestingly, note that $\delta v_\nu/\delta v_\gamma = \eta$ if the photon and neutrino have the same energies, we see that for a photon with energy around a few GeV $\delta v_\gamma$ can be around $4.1 \times 10^{-6}$ on the Earth. Thus, we would like to propose new experiments to determine whether the photon velocities are dependent on their energies or not.

4 The solutions to the theoretical challenges of the superluminal neutrino propagation

With background-dependent Lorentz violation, we assume that the Lorentz violation on the Earth is much larger than those on the interstellar scale. So we can automatically escape the stringent astrophysical constraints on Lorentz violation. However, we still need to address the constraints on Lorentz violation from the LEP and KamLAND experiments. In particular, there are two theoretical challenges to the superluminal neutrino propagation: the Bremsstrahlung effects for muon neutrinos [14] and the pion decays [15, 18, 21]. Thus, we will study these issues in background-dependent Lorentz violation scenarios from the effective field theory and Type IIB string theory.

4.1 Background-dependent Lorentz violation from the effective field theory

To explain the superluminal neutrino propagation and the astrophysical constraints/observations on Lorentz violation, we must assume that the relevant effective operators for Lorentz violation are confined to the Earth. For example, the triplet Higgs field $\Phi$ in our model must be on the Earth.

To avoid the constraints from the KamLAND experiment, we do need to fine-tune the relevant couplings since the Lorentz violation is not flavour blind in general. Moreover, the constraint on $\delta v_\eta$ arises from the astrophysical observations, and then $\delta v_\eta$ can be comparable to $\delta v_\nu$ on the Earth. Thus, the pion decays are not a problem. The Bremsstrahlung effects for muon neutrinos and the LEP constraint on $\delta v_\nu$ are subtle. If the vacuum in the LEP tunnel is similar to the vacuum on the interstellar scale, we can indeed explain them simultaneously: $\delta v_\nu$ can be smaller than $5 \times 10^{-15}$ in the LEP tunnel, while it can be very close to $\delta v_\nu$ on the Earth. Thus, the threshold energy of muon neutrinos for the process $\nu_\mu \rightarrow \nu_\mu + e^+ + e^-$ can be larger than 100 GeV or even higher.

However, if $\delta v_\nu$ in the LEP tunnel is similar to that on the Earth, we may have a problem here. One possible solution is that we consider the scenarios where the Lorentz symmetry is deformed. The other possible solution that we can imagine is that both photons and neutrinos are subluminal. In short, the detailed calculations on the Bremsstrahlung effects for muon neutrinos definitely deserve further careful study.

4.2 Background-dependent Lorentz violation from the type IIB string theory

For the background-dependent Lorentz violation in the Type IIB string theory with D3-branes and D7-branes, we can simultaneously explain the superluminal neutrino propagation and the MINOS results, avoid all the astrophysical constraints on Lorentz violation, and explain the time delays in the MAGIC [31], HESS [32], and FERMI [33, 34] experiments simultaneously [22].

Because our D3-branes are flavour blind, we naturally explain the KamLAND experiments. The time delays or advances for the SM particles arise from their interactions with the D3-branes. Thus, when they do not interact with the D3-branes, for example, when they are away from the D3-branes, all the SM particles are just the conventional particles in the traditional SM without any Lorentz violation at all. Thus, we not only naturally avoid the LEP constraint on $\delta v_\nu$, but also naturally explain the theoretical challenges such as the Bremsstrahlung effects for muon neutrinos and the pion decays. In fact, all the astrophysical Lorentz violation constraints on charged leptons are automatically escaped by the same reason.

The only constraints on our models arise from the time delays in the MAGIC, HESS, and FERMI experiments and the Lorentz violation constraint from the SN1987a observations. For a photon with energy around a few GeV, we obtain $\delta v_\gamma \sim 4 \times 10^{-6}$ on the Earth. Note that the photon velocities are linearly dependent on their energies in our model, we can test such proposal by doing experiment to measure the velocities of the photons with different energies. We shall study it further in the future.

5 Conclusions

To explain both the possible superluminal neutrino propagation and all the known phenomenological constraints/observations on Lorentz violation, the background-dependent Lorentz violation has been proposed. We studied the BDLV in a model-independent way, and conjectured that there may exist a “Dream Special Relativity Theory”, where all the SM particles can be subluminal due to the background effects. We also suggested that the ICARUS experiment can measure the velocities of the muon neutrinos with energies around a few GeV, which may test whether the muon neutrino velocities depend on their energies or not. Assuming that the Lorentz violation on the Earth is much larger than those on the interstellar scale, we automatically escaped all the astrophysical constraints on Lorentz violation. For the BDLV from the effective field theory, we considered a simple model with a triplet Higgs field as in the Type II seesaw mechanism, where we introduced the non-renormalizable
operators which violate the Lorentz symmetry. We discussed the possible solutions to the theoretical challenges such as the Bremsstrahlung effects for muon neutrinos and the pion decays. Also, we addressed the Lorentz violation constraints from the LEP and KamLAND experiments. For the BDLV from the Type IIB string theory with D3-branes and D7-branes, we point out that the D3-branes are flavour blind, and all the SM particles are the conventional particles as in the traditional SM when they do not interact with the D3-branes. Thus, we not only can naturally avoid all the known phenomenological constraints on Lorentz violation, but also can naturally explain all the theoretical challenges. Moreover, we can explain the time delays for the high-energy photons compared to the low-energy photons in the MAGIC, HESS, and FERMI experiments simultaneously. It was predicted that the photon velocities linearly depend on their energies. For a photon with energy around a few GeV we found that $\delta v_\gamma$ is around $4 \times 10^{-6}$ on the Earth. Thus, we can test our models at the experiments in principle.

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