Control chart analysis for interferometric length measurement

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Abstract. INTI is the national metrology institute of Argentina and the responsible for maintenance and the dissemination of the national length standard. The practical realization of this reference is performed according to the optical wavelength standards established and published by the International Committee of Weights and Measures (CIPM). The length unit is materialized by artefacts known as gauge blocks which length is measured by optical interferometric methods. For this purpose INTI has a commercial interferometer based on a Twyman-Green configuration. By this system INTI provides length traceability to national calibration laboratories and others national metrology institutes of the region. Even though the equipment that forms the interferometric system is calibrated periodically, it is convenient to establish quality control procedures for monitoring not only the system as a whole but also the calibration process. We report on a 5-year study of two control steel gauge block sets, the statistical techniques applied on control chart and its selection criteria. The pool of data allows long term stability analysis of these standards as well as the operating examination of the interferometer.

1. Introduction
According to the International System of Units (SI), the metre is defined as “the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 second”[1]. In Argentina, INTI maintains and disseminates the national length standard. The practical realization of this unit is performed according to the optical wavelength standards established and published by the International Committee of Weights and Measures (CIPM) [2]. The materialization of the metre is carried out by measuring gauge blocks of different nominal lengths applying interferometric techniques which use optical radiations of known wavelength traceable to the realization of the metre. This is really the first link in the length traceability transfer for the national and regional industry [3], impacting the quality of length measurements associated with industrial processes where demanding mechanical tolerance values are required, such as the automotive, air and nuclear industries, among others.

INTI offers an interferometric gauge block calibration service by operating a commercial interferometer based on a Twyman-Green configuration. This NPL-TESA automatic gauge block

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interferometer (AGI 1/300) uses two stabilized He-Ne lasers of wavelengths of 632 nm and 543 nm. To measure the central length of gauge blocks by this technique one of the gauge face has to be wrung on an auxiliary platen [4]. The temperature has a measurable effect on the gauge block length, so in order to guarantee thermal stabilization at 20 ºC of this assembly inside the interferometer a temperature control system is used in laboratory. The expanded uncertainties considered for a single measurement of a gauge block correspond to that published in Appendix C database at BIPM website (http://kcdb.bipm.org/appendixC/). The measurement uncertainty is evaluated taking into account components attributed to factors which influence the measurement [5].

Interferometric techniques are highly sensitive to environmental conditions since light beams propagate in air. For this reason it is necessary to use high precision instruments that allow monitoring relevant environmental conditions inside the interferometer. Even though these equipments are periodically calibrated, it is convenient to have certain quality control procedures for monitoring and detecting anomalies in the measurement system immediately [6]. We report on a 5-year study of two control steel gauge block sets. The control chart analysis is based on the warning and control limits determined by statistics. The pool of data allows long term stability analysis of the control gauge block sets as well as the operating examination of the interferometer, i.e. whether the measurement system is in-control or out-of-control state.

2. Control gauge block sets
Two control steel gauge block sets were implemented in order to carry out the control chart analysis. High accuracy gauge blocks in grade K were used for the selection of the sets and a series of tests for evaluating the wringability were performed. These sets of gauge blocks are measured periodically and the results are analyzed for establishing the warning and control limits corresponding to each block in the sets.

2.1. Fixed wringing control gauge blocks (FWCGB)
It is a set of ten gauge blocks in the range of nominal length from 0.5 mm to 100 mm that are wrung to a specific auxiliary reference platten which is permanently left inside the calibration cabin of the interferometer. In this way the calibration face is always the same assuming also a constant wringing film.

2.2. Variable wringing control gauge blocks (VWCGB)
Initially it was a set of two gauge blocks with 100 mm-nominal length and three with 300 mm-nominal length. Nowadays we have included five gauge blocks more in the range from 100 mm to 300 mm. Unlike the previous set, these gauge blocks are not permanently wrung to a reference auxiliary platten. Due to their aspect ratio most of these gauge blocks are more susceptible to possible turning over than short gauge blocks, therefore they are wrung to the reference surface with the platen localized into the interferometer measurement cabin every time they are measured. The calibration of each of the faces is also performed periodically and alternating the measuring faces between calibrations.

3. Control charts, criteria and results
Statistical treatment of the measurements is carried out in an analogous way for both control gauge block sets. Most of the measurements were made by the same operators and each of them was measured only once in each series. In addition, dimensional stability limits established by ISO 3650 are taken into account. The specification for relative stability, for K-grade gauge block, is $0.25 \times 10^{-6}$ per year, so it takes a long time to measure dimensional stability in this kind of gauge blocks.

The following criteria for the control charts analysis are considered:
1. If the measured value falls outside the established control limits, it would be an outlier so the calibration is repeated. If the outlier is obtained again, others gauge blocks of the same nominal
length are calibrated. If the outlier does not repeat, a problem intrinsic to the first block measured is assumed and the first measurement then is discarded. If the same behaviour is observed then proceed according to 2.

2. If the results of the whole set of control gauge blocks fall outside the established control limits, i.e. all the calibrations show outliers, the service is suspended and the operation of the interferometric system as a whole is evaluated.

3.1. General control charts
Assuming a normal distribution, the warning (WL) and control limits (CL) are determined by $\sigma$ and $2\sigma$, respectively, i.e. the probability of the measured value falling within the upper and lower limits is 68 % and 95 % [6-7]. The centreline (CL) is determined by the mean value. Figure 1 shows the measurement history in the deviation of nominal length for the 0.5 mm steel gauge block between 2012 and 2017.

![Figure 1](image)

This fixed wringing control gauge block was manufactured several years before this control measurement was started hence the data show the 0.5 mm control gauge block is very stable, as it was expected.

3.2. Control charts with drift
In some cases the analysed control charts show temporal drift. In this case the criteria of 3.1 will not be effective to detect anomalies in the measurement system hence the control chart is determined by standard regression analysis. All the data with drifts are analysed assuming that the distribution of the $y$-variable at each value of $x$ is normal with the same variance, but no assumptions are made about the distribution of the $x$-variable. The drift is not included in the measurement uncertainty of the deviation from the nominal length. The estimated standard errors of the slope, $\hat{\beta}_1$, and the fitted intercept coefficient, $\hat{\beta}_0$, are calculated by:

$$
\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}
$$

$$
\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}
$$
\[ SE(\beta_1) = \sqrt{\frac{s^2}{\sum_{i=1}^{n} (X_i - \bar{X})^2}} \quad \text{and} \quad SE(\beta_0) = \sqrt{s^2 \left[ \frac{1}{n} + \frac{\bar{X}^2}{\sum_{i=1}^{n} (X_i - \bar{X})^2} \right]} \]  

(1)

Two types of intervals are determined:

- **Confidence bands.** They combine the confidence intervals of the slope and intercept of the regression function. It determines how precisely the data define the best-fit line. Thus the confidence interval bounds at \( X = X_0 \) are given by:

\[ Y_0 \pm t_c \sqrt{s^2 \left[ \frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum_{i=1}^{n} (X_i - \bar{X})^2} \right]} \]  

where the square root term is the standard error that accounts for the uncertainty in estimating the true mean of \( Y \) at \( X = X_0 \), and \( t_c \) is the appropriate value from the \( t \) distribution with \( n - 2 \) degrees of freedom.

- **Prediction bands.** This interval is wider than the associated confidence interval because the scatter of data about the regression line is more important. These intervals represent the uncertainty of predicting the value of a single future observation, the new observation, and the uncertainty associated with the regression parameters as well. The prediction interval bounds at \( X = X_0 \) are given by:

\[ \hat{Y}_0 \pm t_c \sqrt{s^2 \left[ 1 + \frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum_{i=1}^{n} (X_i - \bar{X})^2} \right]} \]  

(3)

The most cases which present drifts correspond to gauge blocks of nominal lengths below of 10 mm. Figure 2 shows the measurement history in the deviation of nominal length for the 5 mm steel gauge block between 2011 and 2017.

![Figure 2](image)
There appears to be a long-term increase in length since this gauge block exhibits lengthening of approximately +3 nm year\(^{-1}\).

Length instabilities in gauge block measurement could be attributed to some details on the manufacturing and hardening process of the gauge block, in which case the gauge blocks would exhibit lengthening [8]. Contrary to the observed on previous figure, Figure 3 shows a contraction of around -3 nm year\(^{-1}\). This long-term decrease in length corresponds to the 9 mm steel gauge block between 2011 and 2017.

Drifts associated with contractions of length are more uncertain. The drifts observed in the fixed wringing control gauge blocks vary between -1 nm year\(^{-1}\) and +3 nm year\(^{-1}\). Since all the FWCGB were manufactured several years before this control measurement was started, drifts are suspected to be caused by variation of the wringing film over the time. In order to verify this hypothesis some FWCGB with temporal drift will be removed and re-wrung on the platen to be measured again.

3.3. Other control charts
Some VWCGB were very new when they were included as control gauge blocks. In these cases the results show how stability length improves with age. Figure 4 shows the measurement history in the deviation of nominal length for one of the 300 mm steel gauge block between 2011 and 2017. The first data point corresponds to manufacturing certificate and it is included only as a reference value. Between the end of 2012 and the end of 2014 the results evidence a negative variation in length of approximately -50 nm year\(^{-1}\). In the last years the length seems to be reaching temporal stability. Both criteria 3.1 and 3.2 are applied in this case as the control limits show in Figure 4. In this case, the confidence and prediction bands coincide, i.e. due to the few data points considered for the regression, scatter is the main source of uncertainty hence the width of the confidence limits enlarges.
On the other hand the analysis of this particular control chart allowed anomaly detection on the temperature sensors in 2012. The event was verified by outliers measured for all the VWCGB and proportional to nominal lengths.

4. Conclusion
A quality control procedure was effectively established for monitoring interferometric length measurement. The performance of two control gauge block sets over 5 years has been analysed and selection criteria to accept or reject a measured value based on historical measurements have been established. The pool of data also allows for quantitative examination of temporal stability of the gauge blocks in the set analysed as well as the long-term performance of the interferometer as a global system.

5. References
[1] Documents concerning the new definition of the metre 1984 *Metrologia* vol 19 pp 163-177
[2] Quinn T J 1992 *Mise en pratique* of the definition of the metre *Metrologia* vol 30 pp 523-541
[3] BIPM, IEC, IFCCC, ILAC, ISO., IUPAC, IUPAP and OIML 2007 International Vocabulary of Metrology-Basic and General Concepts and Associated Terms (VIM) ISO/IEC Guide 99:2007 (Geneva, Switzerland: International Organization for Standardization)
[4] ISO3650 1998 Geometrical Product Specifications (GPS)-Length Standards-Gauge Blocks (Geneva, Switzerland: International Organization for Standardization)
[5] Decker J E and Pekelsky J R 1996 *Metrologia* vol 34 pp 479-493
[6] Doiron T and Beers J S 1995 The Gage Handbook US Department of Commerce, Technology Administration, National Institute of Standards and Technology
[7] Chen-Yun H, Gwo-Sheng P and Kam-Wa L 2014 How to Select the Appropriate Type of Control Chart in Metrology NCSL International Workshop & Symposium
[8] Lewis A J, Hughes B, and Aldred P J 2010 Long-term study of gauge block interferometer performance and gauge block stability *Metrologia* vol 47 pp 473-486