Star Cluster Formation and Disruption Time-Scales – I. 
An empirical determination of the disruption time of star clusters in four galaxies

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ABSTRACT

We derived the disruption times of star clusters from cluster samples of four galaxies: M51, M33, SMC and the solar neighbourhood. If the disruption time of clusters in a galaxy depends only on their initial mass as \( t_{\text{dis}}^{4} \text{(yrs)} = (M_{\text{cl}}/10^{4}M_{\odot})^{\gamma} \), and if the cluster formation rate is constant, then the mass and age distributions of the observed clusters, will each be given by double powerlaw relations. For clusters of low mass or young age the powerlaw depends on the fading of the clusters below the detection limit due to the evolution of the stars. For clusters of high mass and old age the powerlaw depends on the disruption time of the clusters. The samples of clusters in M51 and M33, observed with \textit{HST} – \textit{WFPC2} indeed show the predicted double powerlaw relations in both their mass and age distributions. The values of \( t_{\text{dis}}^{4} \text{ and } \gamma \) can be derived from these relations. For the cluster samples of the SMC and the solar neighbourhood, taken from the literature, only the age distribution is known. This also shows the characteristic double powerlaw behaviour, which allows the determination of \( t_{\text{dis}}^{4} \text{ and } \gamma \) in these galaxies. The values of \( \gamma \) are the same in the four galaxies within the uncertainty and the mean value is \( \gamma = 0.62 \pm 0.06 \). However the disruption time \( t_{\text{dis}}^{4} \) of a cluster of \( 10^{4} M_{\odot} \) is very different in the different galaxies. The clusters in the SMC have the longest disruption time, \( t_{\text{dis}}^{4} \approx 8 \times 10^{9} \text{yrs} \), and the clusters at 1 to 3 kpc from the nucleus of M51 have the shortest disruption time of \( t_{\text{dis}}^{4} \approx 4 \times 10^{7} \text{yrs} \). The disruption time of clusters 1 to 5 kpc from the nucleus of M33 is \( t_{\text{dis}}^{4} \approx 1.3 \times 10^{8} \text{yrs} \) and for clusters within 1 kpc from the Sun we find \( t_{\text{dis}}^{4} \approx 1.0 \times 10^{9} \text{yrs} \).

Key words: Galaxy: open clusters – Galaxy: solar neighbourhood – Galaxies: individual: M33 – Galaxies: individual: M51 – Galaxies: individual: SMC – Galaxies: star clusters

1 INTRODUCTION

In this paper we derive an empirical relation between the disruption time and the initial mass of clusters in limited regions of the spiral galaxies M51 and M33. We also derive the disruption times of clusters in the solar neighbourhood of the Milky Way and in the Small Magellanic Cloud. The determination of such an empirical relation is important for several reasons: (a) to compare the disruption of clusters in different conditions of different galaxies, (b) to explore and describe the evolution of the population of cluster systems in different galaxies using the simplest reasonable model, (c) to predict and explain the evolution of field star populations that result from cluster disruption.

Stellar clusters in the Milky Way show a bimodal age distribution with peaks at the youngest age (open clusters) and at the oldest age (globular clusters). This is because most clusters disrupt over time, due to the decrease of mass by stellar evolution and due to tidal interactions. Only the
most massive and most concentrated clusters, such as the globular clusters, survived after about 10 Gyrs. This was first pointed out by Oort (1958), who noticed the lack of old clusters with ages longer than about 1 Gyr in the Galaxy. He suggested a time scale for the disruption of Galactic clusters of $5 \times 10^8$ yrs.

In fact, the dynamical evolution of clusters is determined by at least two timescales:

(a) the relaxation time $t_{\text{rel}}$, which is the time for a cluster to reach equilibrium between the kinetic energy distribution of the stars and the potential well. Since the stars of a newborn cluster have a wide range in velocities and masses, the interactions between the stars cause the massive ones to accelerate the lower mass stars. As a result the low mass stars continuously gain energy and may leave the cluster whereas the massive stars move deeper into the potential well. This effect includes mass segregation and core collapse.

(b) the disruption time, $t_{\text{dis}}$, for interactions with the surroundings. If the density of the cluster is not high enough external factors like tidal forces and interactions with the surrounding medium may cause a cluster to disrupt. The disruption is accelerated by stellar evolution effects such as mass loss by winds and by supernova explosions, which decrease the density and the total amount of mass of the cluster.

Relaxation and disruption are two independent mechanisms which both occur in a cluster, albeit at different time scales, and both determine the dynamical evolution of a cluster.

The first calculations of these timescales were made by Spitzer (1957). He studied the dynamical effects of stellar clusters and derived an expression for the time needed for interstellar clouds to disrupt a cluster. He found a disruption age of

$$t_{\text{dis}} = 1.9 \times 10^8 \rho \text{ yrs},$$

where $\rho$ is the mean density of a cluster in $M_\odot$ pc$^{-3}$, for clusters with a density $2.2 < \rho < 22 M_\odot$ pc$^{-3}$. In a second paper Spitzer (1958) studied the relaxation time of clusters. He found that low mass clusters with a small radius have a short relaxation time, whereas the relaxation time is long for massive and extended clusters. Therefore, massive and extended clusters may disrupt before they relax. For a review see Wielen (1988).

Numerical simulations of the dynamical evolution of clusters have been made by several groups, e.g. Chernoff & Weinberg (1990), de la Fuente Marcos (1997) and Portegies Zwart et al. (1998). Many of these studies concentrated on the dynamics of globular clusters. For this study we are most interested in the evolution of lower mass clusters.

Chernoff & Weinberg (1990) made numerical simulations of clusters in the mass range of $3 \times 10^4 < M_{\text{cl}} < 3 \times 10^7 M_\odot$, with different density distributions and different stellar initial mass functions (IMF). They found that most of their model clusters disrupt on a very long timescale of $9 < t_{\text{dis}} < 40$ Gyrs. Only the less massive, highly concentrated clusters with a steep stellar IMF, survived disruption.

The disruption of lower mass clusters with $100 < M_{\text{cl}} < 750 M_\odot$ was studied by de la Fuente Marcos (1997), who used a single density distribution for the numerical simulations and took the presence of binaries into account. He found that all his model clusters disrupt. The time at which this occurs increases with the cluster mass and depends on the adopted stellar IMF.

In this paper we derive the empirical relation between the disruption time and the mass of clusters in different locations of galaxies, based on the mass and/or age distributions of magnitude limited cluster samples. For M51 we use a cluster sample in the inner spiral arms observed with $HST - WFPC2$ and studied by Bik et al. (2002). For M33 we use the sample at the galactocentric distance range of about 0.8 to 5 kpc, studied by Chandar et al. (1999a, 1999b, 1999c) based on observations with $HST - WFPC2$. For the LMC we use the sample of clusters with ages derived by Hodge (1987) based on photographic plates, and for the solar neighbourhood we use the sample of clusters with ages derived by Wielen (1971).

In Sect. 2 we describe the method to derive the disruption time from the observed distribution of clusters over time and mass. In Sect. 3 we apply this method to the study of the clusters in M51 that were observed with the $HST - WFPC2$ camera. In Sect. 4 we derive the disruption time of clusters in M33, observed with $HST$ by Chandar et al. (1999a, 1999b, 1999c). In Sect. 5 we study the disruption time of clusters in the SMC, observed by Hodge (1983, 1986, 1987) and in Sect. 6 we study the disruption time of Galactic clusters within a distance of 1 kpc from the Sun, from the sample studied by Wielen et al. (1971). We compare the results of the different galaxies in Sect. 7. Section 8 contains a critical discussion of our method and assumptions. The results are summarized in Sect. 9.

## 2 The Expected Distribution of Age and Mass of Clusters

In this paper we derive an empirical relation between the disruption time and the mass of clusters, from the observed distribution of clusters over age and mass. In this section we describe the concept and we discuss the expected age and mass distributions.

### 2.1 The concept and the assumptions

We assume a constant cluster formation rate, i.e. the same number of clusters are formed per unit time. We also assume a cluster initial mass function (CIMF). Clusters are formed in the mass range of $M_{\text{min}} < M < M_{\text{max}}$, with an assumed powerlaw probability $N(M) \sim M^{-\alpha}$ with $\alpha > 0$. Because of this CIMF more clusters with low mass are formed than with high mass.

Due to the evolution of the stars, clusters fade as they age. Consequently at a certain age a cluster will fade below the detection limit. For massive clusters this will occur at a higher age than for low mass clusters. This implies that

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$\S$ At specific wavelength bands, especially in the infrared, clusters may actually show peaks in brightness at certain ages, e.g. due to the formation of the first red supergiants at an age of about 10 Myr, or the development of an asymptotic giant branch at a later phase.
there is a “fading limit” in the mass versus age distribution of the clusters: for every cluster age there is a mass limit below which the clusters are too faint to be detected. This limiting mass is higher for old clusters than for young clusters. The fading limit depends on the detection limit of the instrument, the extinction and the distance of the parent galaxy of the cluster. For sensitive instruments or nearby galaxies, the fading limit may be at lower mass than the lower mass limit $M_{\text{min}}$ of the clusters.

Suppose as a first order approximation that clusters keep their initial mass until they are suddenly disrupted. We assume that clusters are disrupted at a certain age and that this disruption age depends on their initial mass $M_i$ in such a way that massive clusters survive longer than low mass clusters, for instance a powerlaw of the type $t_{\text{dis}}(M_i) \sim M_i^\alpha$ with $\alpha > 0$. This also implies the existence of the inverse relation $M_{\text{dis}}(t) \sim t^{1/\alpha}$. Clusters with an initial mass $M_i$ which are older than their disruption time $t_{\text{dis}}(M_i)$ will be disrupted. Similarly, clusters with an age $t$ and with an initial mass below $M_{\text{dis}}(t)$ will also be disrupted. We will show that the disruption relation $t_{\text{dis}}(M_i)$ can be derived from the mass and age histograms of clusters.

We summarize the assumptions:

(i) the cluster formation rate is constant,

(ii) the clusters are formed with a fixed cluster initial mass function (CIMF),

(iii) the clusters have the same stellar initial mass function and the same lower mass cutoff,

(iv) the clusters fade as they get older due to the evolution of their stars,

(v) the clusters keep their initial mass until they are suddenly disrupted,

(vi) the disruption time of a cluster depends on its initial mass.

We realize that the assumption of sudden disruption is a severe simplification of the real disruption. Theory and dynamical models suggest that the mass of a cluster will decrease approximately linearly with time (e.g. Spitzer 1957; Portegies Zwart et al. 1998). We have opted for this simplification for three reasons:

(i) it results in a very simple and easy to understand method to derive the cluster disruption time from the mass and age distributions of observed cluster samples,

(ii) the results can be used to quantitatively compare the cluster disruption times in different galaxies or different locations in the same galaxy,

(iii) the disruption times derived from the age and mass distributions of clusters with sudden disruption are quite similar to those derived from the distributions of clusters with gradual disruption. This is shown in Appendix A.

We also realize that the disruption timescale of a cluster does not only depend on its mass but also on its density or radius and on its environment. However, since we are interested in the dependence of $t_{\text{dis}}$ on mass, for the reasons mentioned in the Introduction, we do not take the density into account in this study. If clusters are approximately in pressure equilibrium with their environment, we can expect the density of all clusters in a limited volume of a galaxy to be about the same, so the disruption time of clusters in a limited volume of a galaxy will depend on the mass (apart from clusters in highly eccentric orbits). On the other hand, if the initial density of clusters depends on their mass, e.g. as $\rho \sim M^{\gamma}$, and $t_{\text{dis}}$ depends on mass and density as $t_{\text{dis}} \sim M^{\alpha} \rho^\beta$ then the empirical dependence of $t_{\text{dis}}$ on mass will be $t_{\text{dis}} \sim M^\gamma$ with $\gamma = a + bx$. If $t_{\text{dis}}$ is proportional to the relaxation time $t_{\text{rel}} \sim M^{1/2}$ (Spitzer 1987) we expect $a = 1$ and $b = -1/2$, so $\gamma = 1 - x/2$.

### 2.2 The distribution in case of evolutionary fading only

Even if there would be no disruption or evaporation of clusters, they would still get fainter with time due to the evolution of their stars. This effect results in a decrease of the number of observable clusters as a function of age for a given magnitude limit, described in this section.

Assume a constant cluster formation rate $S$ and a constant CIMF of slope $-\alpha$. Then the number of clusters formed per unit time and per unit mass in a certain area of a galaxy will be

$$dN(M_\odot, t) = S M_\odot^{-\alpha} dM_\odot dt \tag{2}$$

with $S$ in $M_\odot^{-\alpha-1}$ yr$^{-1}$, where $M_\odot$ is the initial stellar mass of the cluster. This equation is valid for the mass range between the minimum and maximum cluster mass. The constant $S$ is related to the cluster formation rate in the considered part of the galaxy. Observations and theory show that $\alpha \approx 3$ (Zhang & Fall 1999; Whitmore et al. 1999; Bik et al. 2002). The total number of clusters formed per unit time in the mass range of $M_{\text{min}}$ to $M_{\text{max}}$ is

$$dN dt = \frac{S}{\alpha - 1} (M_{\text{min}}^{1-\alpha} - M_{\text{max}}^{1-\alpha}) \approx \frac{S}{M_{\text{min}}} \tag{3}$$

if $\alpha = 2$ and $M_{\text{max}} > M_{\text{min}}$. The total mass of the stars formed per unit time in all clusters is

$$\frac{dM_{\text{tot}}}{dt} = \frac{S}{\alpha - 2} (M_{\text{min}}^{2-\alpha} - M_{\text{max}}^{2-\alpha}) \quad \text{if } \alpha \neq 2$$

$$= S \ln(M_{\text{max}}/M_{\text{min}}) \quad \text{if } \alpha = 2 \tag{4}$$

Suppose that the brightness of a cluster in a particular wavelength band, for instance in the V-band, decreases with time as a powerlaw, $F_V \sim t^{-\zeta}$ with $\zeta > 0$ for $t \gtrsim 10^7$ years, due to the evolution of the stars in the cluster (Leithner et al. 1999, Bruzual & Charlot 1993). In that case the magnitude of a cluster varies with time and mass as

$$V = m_{\text{ext}} - 2.5 \log(M_{\odot}/10^4 M_{\odot}) + 2.5 \zeta \log(t/10^8 \text{yrs}) \tag{5}$$

with $t$ in yrs and $M_{\odot}$ in $M_{\odot}$. We have used the fact that for a given stellar IMF the flux of the cluster scales linearly

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\* The value of the adopted lower mass cutoff does not affect the empirical determination of the dependence of $t_{\text{dis}}$ on $M_i$ directly. However, it may have affected the derived masses of the clusters, if they are determined from integrated cluster photometry.

\| The empirical determination of the density dependence of $t_{\text{dis}}$ is the topic of a study in progress.
with the total mass of the cluster stars, $M_{cl}$. The constant $m_{ref}$ depends on the distance and the mean extinction of the clusters and on the stellar IMF and metalicity of the cluster stars.

$$m_{ref} = M_{cl}^{\text{ref}} - 5 + 5 \times \log d(\text{pc}) + A_V$$

(6)

where $M_{cl}^{\text{ref}}$ is the absolute visual magnitude of a cluster with an initial mass of $10^4 M_\odot$ and an age of $10^8$ yr, which can be derived from cluster evolution models, and $A_V$ is the mean extinction in the observed region. (For instance, $M_{cl}^{\text{ref}} = -11.89$ magn. for Starburst99 models of solar metallicity, a Salpeter IMF and a lower mass cutoff of 1 $M_\odot$) Let $V_{lim}$ be the limiting visual magnitude of the clusters that can be detected. This implies that a cluster with a given mass $M_{cl}$ is detectable at an age shorter than $t_{lim}(M_{cl})$ with

$$\log \left( \frac{t_{lim}(M_{cl})}{10^8} \right) = -0.4 \times (m_{ref} - V_{lim}) + \log M_{cl}/10^4$$

(7)

Similarly, at an age $t$ only clusters with a mass larger than $M_{lim}(t)$ can be detected, with

$$\log \left( \frac{M_{lim}(t)}{10^4} \right) = 0.4 \times (m_{ref} - V_{lim}) + \frac{\log t}{10^8} \zeta$$

(8)

If there was no disruption of the clusters, the age distribution of the observable clusters at any time would simply be

$$\frac{dN_{\text{obs}}}{dt} = \int_{M_{lim}(t)}^{M_{cl}(\text{max})} S M_{cl}^{-\alpha} dM_{cl}$$

$$\simeq 10^{-4.0-0.4(m_{ref} - V_{lim})(\alpha-1)} \frac{S}{\alpha-1} \left( \frac{t}{10^8} \right)^{-\zeta(\alpha-1)}$$

(9)

for $t \geq 10^7$ years. Here we have assumed that the maximum cluster mass, $M_{cl}(\text{max})$, is much larger than the limiting mass at $t = 10^7$ years, and we assumed explicitly that the cluster formation rate $S$ is constant. Similarly, the mass distribution of the observable clusters is

$$\frac{dN_{\text{obs}}}{dM_{cl}} = \int_0^{t_{lim}(M_{cl})} S M_{cl}^{-\alpha} \, dt$$

$$\simeq 10^{-4(\alpha-2)-0.4(m_{ref} - V_{lim})/\zeta} S \left( M_{cl}10^4 \right)^{-\alpha + \gamma(1/\gamma)}$$

(10)

We see that under these assumptions both the mass distribution and the age distribution of clusters are powerlaws of $M_{cl}$ and $t$ respectively. These powerlaws can be determined observationally.

2.3 The distribution in case of cluster disruption

Let us now assume that clusters disrupt at a time that depends only on their initial mass, as

$$t_{\text{dis}}(M_{cl}) = t_{\text{dis}}^{\text{cl}} \left( M_{cl}/10^3 M_\odot \right)^{1/\gamma}$$

(11)

where $t_{\text{dis}}^{\text{cl}}$ is the disruption time (in yrs) of a cluster with an initial mass of $10^3 M_\odot$. The positive sign with $\gamma > 0$ indicates that the disruption time is expected to be larger for the more massive clusters. Clusters with an age $t$ will only be observable if they have a mass larger than $M_{\text{dis}} = 10^4 (t/t_{\text{dis}}^{\text{cl}})^{1/\gamma}$ and larger than the limiting mass given by Eq. (8). Alternatively, clusters with a mass $M_{cl}$ will only be observable up to an age that is given by the minimum of the ages in Eqs. (7) and (11).

If disruption is important, the observed age distribution of the clusters will be

$$\frac{dN_{\text{obs}}}{dt} = \int_{M_{\text{dis}}(t)}^{M_{cl}(\text{max})} S M_{cl}^{-\alpha} dM_{cl}$$

$$\simeq \frac{S}{\alpha-1} 10^{-4(\alpha-1)} \left( \frac{t}{t_{\text{dis}}^{\text{cl}}} \right)^{(1-\alpha)/\gamma}$$

(12)

and the observed mass distribution will be

$$\frac{dN_{\text{obs}}}{dM_{cl}} = \int_0^{t_{\text{dis}}(M_{cl})} S M_{cl}^{-\alpha} \, dt$$

$$\simeq S t_{\text{dis}}^{\text{cl}} 10^{-4(\alpha)} \left( M_{cl}/10^4 \right)^{\gamma-\alpha}$$

(13)

We see that the slope of the mass distribution is less steep than that of the CIMF, because disruption removes the low mass clusters first.

If both the disruption time and the observation limit are important, the mass and age distribution of clusters will both consist of two powerlaws, as shown in Fig. (1). This figure shows the expected relations of log($dN_{\text{obs}}/dt$) as a function of log $t$ and of log($dN_{\text{obs}}/dM_{cl}$) as a function of log $M_{cl}$. The slopes are:

$$\frac{1}{\zeta} - \alpha$$ for log($dN_{\text{obs}}/dM_{cl}$) = $f(\log M_{cl})$ fading

$$\gamma - \alpha$$ for log($dN_{\text{obs}}/dM_{cl}$) = $f(\log M_{cl})$ disruption

$$\zeta(1-\alpha)$$ for log($dN_{\text{obs}}/dt$) = $f(\log t)$ fading

$$\frac{1}{\gamma} - \alpha$$ for log($dN_{\text{obs}}/dt$) = $f(\log t)$ disruption

(14)

So for a given value of the slope $\alpha$ of the CIMF, and a given value of the slope $\zeta$ (from cluster evolution models) we can derive the value of $\gamma$ from the observed slope of either the mass and or the age distribution of the clusters. If both the mass and age distributions are known, the values of both $\alpha$ and $\gamma$ can be derived.

If fadind line is not important (because the detection limit is so good that all surviving clusters can be detected) then the fading part of the age distribution will be horizontal. This follows from Eq. (9) with a lower limit of the integral of $M_{\text{min}}$. If fadind is not important and if the disruption time $t_{\text{dis}}(M_{\text{min}})$ for clusters with the minimum mass would be longer than the age of the galaxy, the mass distribution would reflect the cluster IMF. Notice that the shapes of the mass and age distributions in Fig. 1 are independent of the cluster formation rate and only depends on the CIMF and on the disruption timescale. The vertical shift of these distributions depend on the cluster formation rate.

The crossing points between the powerlaws in the two figures 1 are indicated by $M_{\text{cross}}$ and $t_{\text{cross}}$. They give the cluster mass and the cluster age at which disruption becomes important. This depends on the value of $t_{\text{dis}}^{\text{cl}}$, i.e. on the constant of the expression for the disruption time (Eq. 11) as
log\left(\frac{t_{\text{cross}}}{10^8}\right) = \frac{1}{1 - 7\zeta} \left\{ \log\left(\frac{t_{\text{dis}}}{10^8}\right) + 0.4 \gamma (m_{\text{ref}} - V_{\text{lim}}) \right\} \tag{15}

and

log\left(\frac{M_{\text{cross}}}{10^4}\right) = \frac{1}{1 - 7\zeta} \left( \zeta \log\left(\frac{t_{\text{dis}}}{10^8}\right) + 0.4 (m_{\text{ref}} - V_{\text{lim}}) \right) \tag{16}

These two values of $M_{\text{cross}}$ and $t_{\text{cross}}$ are related via Eq. (8) as

log\left(\frac{M_{\text{cross}}}{10^4}\right) = 0.4 \times (m_{\text{ref}} - V_{\text{lim}}) + \zeta \times \log\left(\frac{t_{\text{cross}}}{10^8}\right) \tag{17}

This implies that we can derive the value of $0.4 \times (m_{\text{ref}} - V_{\text{lim}})$ by comparing the empirical values $M_{\text{cross}}$ and $t_{\text{cross}}$, and then we can derive the value of $t_{\text{dis}}^\text{lim}$ from $M_{\text{cross}}$ or $t_{\text{cross}}$ (Eqs. (16) and (15)).

We will show below that the observed mass and age distributions of cluster samples in the different galaxies indeed show two powerlaw slopes. From the empirical slopes we can test the assumptions of this simple model and derive the dependence of the disruption time on the cluster mass, given by $t_{\text{dis}}^\text{lim}$ and $\gamma$ (Eq. 11).

In this section we have assumed that the cluster disruption occurs instantaneously when the cluster reaches an age $t_{\text{dis}}(M_{\text{cl}})$. That means, a cluster keeps its mass (apart from stellar evolution effects) until it suddenly disrupts. This is a severe assumption, that allows us to determine from the observed mass and age distributions how the disruption time of clusters depends on their initial mass. In Appendix A we show that the mass and age distributions of gradually disrupting clusters are very similar to those of instantaneously disrupting clusters, if the expressions for the disruption time are the same in both cases.

3 A STUDY OF THE DISRUPTION OF CLUSTERS IN M51

M51 or the Whirlpool galaxy (NGC 5194) is a grand design Sc spiral galaxy that interacts with the dwarf galaxy NGC 5195. Their close encounter is estimated to have occurred about 250 – 400 Myr ago (Hernquist 1990). Salo & Laurikainen (2000) argued that there may even have been a more recent encounter about 50 to 100 Myrs ago. The whole system lies in a distance of approximately 8.4 Mpc (Feldmeier et al. 1997). The Whirlpool galaxy has an almost face-on orientation, with an inclination angle of 23 to 35 degrees (Monnet et al. 1981). This makes it ideal for the study of its cluster formation history.

Scuderi et al. (2002) found that the interaction with its companion produced a huge starburst in the nucleus. Lamers et al. (2002) suggested that the formation of primarily massive stars in the bulge of M51 is also due to the interaction. Bik et al. (2002), hereafter referred to as Bik02, studied clusters in the inner spiral arms and derived the initial mass function of the clusters, as well as the cluster formation rate. They found no evidence for an increased cluster formation rate due to the interaction in this region.

** We have derived the expressions of the mass and age distributions under the assumption that the cluster sample is magnitude limited in the $V$-band. The expressions can easily be modified for a sample that is magnitude limited in any other band. In that case only the values of $M_{\text{lim}}^V$, $\zeta$ and $m_{\text{ref}}$ have to be adapted.

Figure 1. Schematic pictures of the predicted mass distribution (top figure) and age distribution (bottom figure) of clusters in case of no disruption and fading only (full lines) and with disruption (dotted lines). The y-axes have arbitrary units. The relation between the crossing points $M_{\text{cross}}$ and $t_{\text{cross}}$ is given by Eq. (17). The slopes of the different line sectors are indicated.

The ages and masses of the M51 clusters derived by Bik02 will be adopted in this study.

3.1 The observations

The M51 system was observed with the HST–WFPC2 in five wideband filters ($U$, $B$, $V$, $R$ and $I$) as part of the HST Supernova Intensive Study program. The observations have been described by Scuderi et al. (2002) and Bik02. Bik02 detected 877 clusters in an area of about 3.2 $\times$ 3.2 kpc about 0.8 to 3.1 kpc from the nucleus, observed with the WFC camera. The area studied by Bik02 is shown in Figs. (2) and (3). All clusters were detected in at least three bands, including $V$ and $R$. If a cluster is detected in only three or four bands, the empirically determined lower magnitude limits were adopted for the other bands.

Bik02 determined the ages, the initial masses and the values of $E(B-V)$ of these clusters using the 3/2DEF-method, which is a least-square fit comparison between the observed energy distribution and the predicted energy dis-
The cluster models for instantaneous starformation, with age from the Starburst99 models of Leitherer et al. (1999). The cluster parameters derived from the models with solar metallicity and with an upperlimit for the stellar mass of 30 \( M_\odot \) and a lower limit of 1 \( M_\odot \). These models were calculated using the Starburst99 program by N. Bastian (private communication). The Starburst99 models do not extend beyond an age of 1 Gyr. For older clusters, in the range of 1 to 10 Gyr, Bik02 adopted the energy distributions for solar metallicity clusters with a stellar IMF of \( \alpha = 2.35 \) and an upper mass limit of 25 \( M_\odot \), using the Frascati evolutionary tracks (Romaniello, private communication).

The least square fit of the model energy distributions to the observed ones resulted in estimates of the mass, age and \( E(B-V) \) of the clusters and their uncertainties. The quality of the fit is given by the value of the reduced \( \chi^2 \) for each cluster. (For the details of these determinations and their accuracy, the reader is referred to Bik02.) We will use the ages and the initial masses of the clusters determined by Bik02 to derive the disruption time of clusters in M51, with the method explained in Sect. 2.

We point out that the cluster mass, determined in this way is the total initial mass of the stars that are still in the cluster. This means that it is corrected for the stellar evolution effects. However it is not corrected for stars that may have left the cluster. Stars that have left the cluster and are at a distant of more than about 0.4 arcsec, corresponding to about 16 pc, do not contribute to the photometry and their mass is not counted. (A radius of 4 pixels or 0.4 arcsec was used by Bik02 for measuring the photometry of the clusters.)

Figure 3. The relative radius of 16 pc used for measuring the magnitudes implies that our sample is basically magnitude limited, rather than surface brightness limited as the vast majority of the clusters is expected to have a radius less than 16 pc. For instance, Chandar et al. 1999b found that the bright clusters in M33, which is about ten times closer than M51, have a core radius of only 0.1 to a few pc and a FWHM radius about twice as large.

### 3.2 The detection limit

The observational detection limit of clusters in M51 depends on the accuracy of the observations, in this case the \( HST - WFPC2 \) photometry. For the determination of the detection limit, we will use the \( R \)-magnitude, because it is less sensitive to extinction than the photometry in the \( V \) band, and because all clusters were detected in the \( R \)-band. Figure (4) shows the histogram of the \( R \)-magnitude of the sample of 877 clusters studied by Bik02. The figure shows three samples: the full sample, and the two samples of clusters for which the observed energy distribution could be fitted with a model with an accuracy of \( \chi^2 < 3.0 \) and 1.0 respectively. For all three samples the number of clusters increases towards fainter clusters from \( R \approx 18 \) to \( R \approx 22 \), as expected for a cluster IMF that predicts many more low mass clusters than high mass ones. For all three samples the turnover occurs near \( R \approx 22 \) and the distribution drops steeply to fainter clusters. The magnitude halfway down this steep drop is \( R \approx 22.5 \), and the faintest clusters have \( R \approx 23.7 \).
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Figure 4. The histogram of the $R$-magnitudes of the M51 clusters studied by Bik et al. (2002). The lightest distribution is for all 877 clusters measured. The other two distributions are for subsets of clusters whose energy distribution could be fitted to cluster models with an accuracy of $\chi^2 < 1.0$ and 3.0. The turnover is at $R \approx 22.0$. We adopt this value as the magnitude limit of our sample.

Figure 5. The relation between the limiting age and initial mass of instantaneously formed clusters of solar metallicity with $\alpha = 2.35$ (from Leitherer et al. 1999) at a distance of 8.4 Mpc with a mean extinction of $A_R = 0.36$ and for a given observational magnitude limit of $R_{lim} = 22.0$. The straight full line gives a powerlaw fit to the data for $\log t \geq 7.3$ with a slope of $d \log(M_{lim})/d \log(t) = 0.648$.

Figure 6. The relation between the initial mass $M_0$ (in $M_\odot$) and age (in yrs) of 392 M51 clusters with $M_V < -7.5$, whose energy distribution could be fitted to a cluster model with an accuracy of $\chi^2 < 3.0$. The full line is the detection limit for a limiting magnitude of $R_{lim} = 22.0$ at a distance of 8.4 Mpc without extinction. The cross indicates the size of typical errorbars (see Bik et al. 2002).

We will adopt the conservative upperlimit of $R = 22.0$ for the completeness limit of our sample.

The detection limit in $R$-magnitude corresponds to a certain lower limit in mass and age of the clusters. This relation can be derived from the Starburst99 cluster models of Leitherer et al. (1999). These models show that a cluster with instantaneous starformation at solar metallicity and with a stellar IMF of $\alpha = 2.35$ for stars in the range of $1 < M_\star < 30 M_\odot$, with a total mass of $10^4 M_\odot$ and an age of $10^7$ yrs has an HST $R$-band magnitude of $M_R = -8.37$. This implies that $m_{ref} = 21.61$ for a distance of 8.4 Mpc and a mean extinction of $A_R = 0.36$ (Bik02).

The detection limit as a function of time is shown in Fig. (5). The figure shows a perfect linear relation for clusters with age $\log t \geq 7.3$, which is given by

$$\log M_{lim} \simeq -1.350 + 0.648 \times \log t (\text{yr})$$

(18)

For smaller ages the relation is irregular with dips at $\log t \simeq 6.9$ and $\log t \simeq 7.2$, due to the presence of red supergiants. In this study we will use the exact dependence of $M_{lim}$ on age, shown by the dashed line in Fig. (5). The linear approximation is only used for checking the results and for easy comparison with the predicted age and mass distribution, derived in Sect. 2. Comparing Eq. (18) with Eq. (8) we see that $\zeta = 0.648$ and $0.4 \times (m_{ref} - R_{lim}) = -0.16$ for $\log M_0 \gtrsim 3.4$ and $\log t \gtrsim 7.3$.

3.3 The clusters

The initial mass versus age distribution of the clusters studied by Bik02 is shown in Fig. (6). We only show this distribution for clusters with $M_V < -7.5$, for which the fit of the energy distribution has $\chi^2 \lesssim 3.0$. The distributions of the full sample and of the sample of clusters with $\chi^2 \lesssim 1.0$ show the same characteristics as the one shown here. The distribution shows several features:

(a) The lower limit of the data, agrees with the predicted limit for instantaneously formed clusters and for a detection limit of $R_{lim} = 22.0$, derived above. Even the predicted dips near $\log t = 6.9$ and 7.2 may be present.

(b) The clusters older than about $10^8$ years all have initial masses in excess of $\log(M_0) \simeq 3.8$ whereas the younger clusters have masses down to about 500 $M_\odot$.

(c) There are concentrations in the distribution at $\log(t) = 6.70$ and 7.45 and possibly also near 7.2. These are due to statistical effects in the age range of $6.5 < \log(t) < 7.5$, where the colours of the cluster models change rapidly with time (see Bik02).
(d) There is a drop in the density of the points at log(t) > 7.5. Since the age scale is logarithmic, we might have expected an increasing density towards higher age if the cluster formation rate was constant and if all formed clusters would have survived disruption.

(e) The presence of massive clusters with an age of about 5 Gyr shows that either the disruption time in M51 is very long, or that massive clusters disrupt much slower than low mass clusters. We will show that the first possibility disagrees with the observed mass and age distributions.

(f) The general shape of the observed mass versus age distribution agrees with that expected for gradual disruption with a mass dependent disruption time (See Appendix 1).

Bik02 determined the cluster initial mass function (CIMF) from the mass distribution of clusters younger than 10 Myr. These clusters are not yet affected by disruption. They derived a powerlaw with a slope of

$$d\log(N_{\text{obs}})/d\log(M_{cl}) = -2.1 \pm 0.3$$

(19)

in the mass range of $3.4 < \log(M_{cl}) < 5.0$. Bik02 also studied the cluster formation history from the age distribution of the observed clusters. They used clusters with an initial mass in excess of $10^4 M_\odot$. They found that the formation rate of these clusters has been increasing continuously with time from about 10 Gyr up to the present. They argue that this apparent increase in the cluster formation rate over such a long time is due to the disappearance of clusters with increasing age due to disruption. So we assume for this study that the real cluster formation rate in M51 has been about constant. We return to this assumption in the discussion.

3.4 The determination of the cluster disruption time

The mass distribution of the 512 clusters with an accurate mass determination, $\chi^2 < 3.0$, is shown in Fig. (7a) and the age distribution of these clusters is shown in Fig. (7b). In this sample we only include clusters with an initial mass in excess of $10^3 M_\odot$, in order to avoid the sample being “polluted” by the brightest stars. The vertical axes give respectively $dN_{\text{obs}}/dM_{cl}$ in number per $M_\odot$ and $dN_{\text{obs}}/dt$ in number per Myr. The vertical error bars indicate the 1σ Poisson errors. The horizontal error bars indicate the width of the adopted intervals. Both distributions clearly show a trend of decreasing number with increasing age. The full sample of all 877 clusters, not shown here, has the same distribution, but with a larger scatter, due to the larger uncertainties in the ages and masses.

In Sect. 2 we have predicted that if evolutionary fading and disruption both result in a limiting detectable mass that depends as a powerlaw on the age, then both the mass and the age distributions of the observed clusters should show a double powerlaw relation, given by Eqs. (9), (10), (12) and (13). Here we adopt the $M(t)$ relation of the detection limit as shown in Fig. (5) in case of no disruption and the relation Eq. (11) for higher ages and masses with disruption.

3.4.1 Fitting the mass and age distributions

The mass distribution of the clusters, shown in Fig. (7a), decreases to higher masses much faster than expected for evolutionary fading only. We have numerically calculated the expected mass distribution in case of no disruption, using the first part of Eq. (10) for a CIMF of $\alpha = 2$. This is the value that was found by Zang & Fall (1999) and Whitmore et al. (1999) for the Antennae galaxies and it agrees with the value of 2.1 ± 0.3 derived for M51 by Bik02.

The predicted relation for evolutionary fading (full line) has been vertically adjusted to fit the data points in the smallest mass bins. This shift is not well determined, because it depends on only two points. This implies that the crossing point of the two lines for evolutionary fading and for disruption of the mass distribution is not well determined.

We have fitted a powerlaw of the type

$$\log(dN_{\text{obs}}/dM_{cl}) = a_0 + a_1 \times \log(M_{cl})$$

(20)

to the data in the mass range of $3.5 \leq \log(M_{cl}) \leq 5.25$ by
Empirical determination of the disruption times of star clusters in four galaxies

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means of a weighted linear regression. The data point at log(Mcl) = 5.45 was not taken into account because this appears to be close to the mass upper limit. The powerlaw has a slope in of −1.62 ± 0.09. We have done the same analysis for the full cluster sample and for the sample of clusters with χ2 < 1.0. The resulting parameters of the fits are listed in Table (1). We see that the slopes of the powerlaw fits are very similar for the three samples, but that the value of the constant α0 scales approximately with the log of the number of clusters in the three samples, as expected. Taking the weighted mean of the three determinations, we find a mean value of < a1 > = −1.54 ± 0.07.

The age distribution is shown in Fig. (7b). The predicted relation due to fading (full line) has been fitted through the four youngest age bins. The observed distribution for log(t) > 7.5 is far below the fading line and decreases steeper with age than the fading line. We have fitted a powerlaw expression of the type

\[ \log(dN_{\text{obs}}/dt) = b_0 + b_1 \times \log(t) \]  

(21)
to the data. The relation was forced to extend to the last bins on the fading line, i.e. at log(t) = 7.1 and 7.4. The parameters of the powerlaw fit are given in Table (1), for the sample shown here as well as for the other two cluster samples. We find that < b1 > = −1.49 ± 0.07.

The points where the linear extensions of the fading lines and the powerlaw fits in the mass and age distributions of Fig. (7) cross is

\[ \log(M_{\text{cross}}) = 3.4 \pm 0.5 \text{ in } M_\odot \]

\[ \log(t_{\text{cross}}) = 7.3 \pm 0.3 \text{ in yrs} \]  

(22)

The empirical values of < a1 > and < b1 >, together with the values of M_{cross} and t_{cross}, will be used to derive the cluster disruption time in Sect. 3.4.3.

3.4.2 The cluster formation rate

The predicted distributions of the fading lines were fitted through the first two mass bins and the first four age bins in Fig. (7). This vertical adjustment depends on the detection limit and on the cluster formation rate in the observed region of M51, as given by Eqs. (10) and (9). Adopting the value of \( \zeta = 0.648 \) and \( 0.4 \times (m_{\text{rot}} - R_{\text{lim}}) = -0.16 \) (see Sect. 3.2) we find that log S = −1.70 ± 0.20 and −1.66 ± 0.10 (in Nr M_\odot/Myr) from the mass and the age distribution respectively. The value derived from the mass distribution is more uncertain than that from the age distribution, because only two data-bins were used for fitting the fading line to the mass distribution. Nevertheless, we see that the value of S derived in two independent ways is about the same.

The cluster formation rate in the observed region of M51 is about 10^{-1.66} M_\odot yr^{-1} in Nr M_\odot yr^{-1} (see Eq. (2)). We conclude that the mean cluster formation rate in the observed area of M51 is (see Eq. (3) with M_{min} = 10^3 M_\odot )

\[ \log(dN_{\text{obs}}/dt)_{\text{form}} = -4.7 \pm 0.10 \text{ in Nr clusters yr}^{-1} \]  

(23)

This is for clusters with M_{cl} > 10^3 M_\odot. The mass of the stars formed in the clusters per unit time is (see Eq. (4))

\[ \log(dM_{\text{tot}}/dt)_{\text{form}} = -0.9 \pm 0.10 \text{ in } M_\odot \text{ yr}^{-1} \]  

(24)

where we adopted M_{min} = 10^3 and M_{max} = 3 \times 10^7 M_\odot. This is the cluster formation rate in an area of 3.25x 3.25 kpc about 1 to 3 kpc from the nucleus (see Fig. (3)).

3.4.3 The cluster disruption time

We compare the empirical slope of the powerlaws of < a1 >= −1.54 ± 0.07 and < b1 >= −1.49 ± 0.07, derived from the distributions in Fig. (7), with the predicted slopes of γ - α and (1 − α)/γ respectively (Eqs. (13) and (12)). This gives the following two estimates of γ = 0.46 ± 0.07 and γ = 0.67 ± 0.03 if α = 2.0. We see that the two determinations of γ from the mass and age distributions differ significantly. We adopt the mean value of γ ≃ 0.57 ± 0.10.

The constant t_{4\text{dis}} in the expression for the disruption time (Eq. (11)) can be derived from the values of t_{cross} and M_{cross} where the fading and the disruption lines in Fig. (7) cross. Their values are given in Eq. (22). These values were derived independently from one another from the empirical age and mass distributions of Fig. (7). We have predicted that the values of t_{cross} and M_{cross} should be related to one another by means of the relation of Eq. (17). From the values of log t_{cross} = 7.3 ± 0.3, 0.4 (m_{rot} - R_{lim}) = −0.16 (Sect. 3.2) and \( \zeta = 0.648 \) we predict that log M_{cross} = 3.38 ± 0.19. The observed value of 3.4 ± 0.5 agrees excellently. So the values of M_{cross} and t_{cross} are consistent with one another.

The value of t_{4\text{dis}} can be derived from a comparison between the predicted mass or age distributions (Eqs. 13 or 12) and the observed fits of Table 1, or from the values of t_{cross} or M_{cross} (Eq. 16 or 15). Since t_{cross} and M_{cross} are consistent with each other, they give the same value of t_{4\text{dis}}^\text{22} ; but since t_{cross} could be determined with a higher accuracy than M_{cross} we use the expression for t_{cross} to calculate t_{4\text{dis}}^\text{22}.

Taking into account the uncertainty in t_{cross} and in γ we find from Eq. (15) that log(t_{4\text{dis}}^\text{22}) = 7.64 ± 0.22 in yrs. Combining this with the derived value of γ we conclude that the disruption time of clusters in the observed area of M51, i.e. the inner spiral arms, is

\[ \log(t_{\text{dis}}) = 7.64(0.22) + 0.57 (0.10) \times \log(M_{cl}/10^4) \]  

(25)

with t_{dis} in years and the initial cluster mass in M_\odot. This relation is valid for clusters in the mass range of 10^2 to 10^5 M_\odot. We see that clusters with an initial mass of M_{cl} = 10^4 M_\odot disrupt in about 40 Myr and those of 10^5 M_\odot in 160 Myr. We will show below that this is much faster than the disruption times of clusters in the other galaxies that we studied.

4 THE DISRUPTION OF CLUSTERS IN M33

The spiral galaxy M33 is at a distance of 840 kpc (Madore et al. 1991) and it has an inclination angle of 56 degrees (Regan & Vogal, 1994). Chandar et al. (1999a, 1999b, 1999c) have studied 60 clusters in 20 HST - WFCPC2 fields of M33 at galactocentric distances between 0.8 and 5 kpc (with one exception at 0.19 kpc). The clusters were observed in the HST UBV and F170W filters. The limiting magnitude of the sample is V_{lim} ≃ 19.3.

Chandar et al. (1999b) determined the ages and masses of the clusters from a study of the energy distributions, by comparing the colours with those predicted by Bertelli et al.
(1994) for instantaneous star formation with Salpeter’s IMF, for three metallicities. Since there is a gradient in metallicity in M33, the metallicities of the clusters were derived from optical spectroscopy, using metal dependent indices defined by Huchra et al. (1996). The extinction of the clusters was derived from colour-colour diagrams and the masses and ages were derived by comparison of the energy distribution with the predicted ones. Chandar et al. (1999c) redetermined the ages and masses from the study of those clusters that also have a detectable flux in the F170W filter. The combination of the data of Chandar et al. (1999b, 1999c) provides a list of 49 clusters with an age determination and 45 clusters with a mass determination. The authors quote an uncertainty in age of about 0.15 to 0.30 dex for clusters older than 10^8 years and 0.1 dex for younger clusters. The derived masses have a quoted accuracy better than 0.1 dex in most cases and better than 0.3 dex in the worst case. Chandar et al. (1999b, 1999c) argued that the cluster formation rate in M33 was about constant.

We use the data from Chandar et al. (1999b, 1999c) to study the disruption of clusters in M33, in the same way as done for M51. Metallicity differences will influence the evolution of the stars, which is taken into account in the fitting of the energy distributions by Chandar et al. Metallicity is not expected to affect the disruption of the clusters directly. Therefore we can use the statistics of the clusters, independent of their metallicity.

Figure (8) shows the mass versus age relation of the 45 clusters with known age and mass. The full line is the magnitude limit for clusters with solar metallicity. The shape is the same as in Fig. (6), but the curve is shifted vertically by 0.92 dex to allow for the fact that the distance of M33 is ten times smaller than the distance of M51, and for the fact that the limiting magnitude of the M33 clusters is 2.7 magn. brighter than for the M51 clusters. We see the similar effects as in the distribution of the M51 clusters, discussed in Sect. 3.3:

(a) the observed lower limit shows the expected increase of mass with age, as predicted for clusters that fade as they get older.
(b) the number of clusters drops at ages higher than about 10^8 years. The effect is even stronger than it appears in the figure, because of the logarithmic age scale of the plot.

Figures (9a) and (9b) show the age (top) and mass (bottom) distribution of 49 and 45 clusters respectively. The full line is the prediction due to the fading of the clusters. The mass distribution follows the prediction for fading clusters up to a mass of about 2×10^4 M⊙ and a steep drop to higher values. The drop is even steeper than suggested by the figure, because several mass bins contain no clusters at all. The data for higher masses are too scarce to derive a disruption relation from the mass distribution.

The age distribution of 49 clusters (Fig. 9a) is rather flat for clusters with ages below 10^8 years. The slope of this distribution agrees roughly with the predictions for fading clusters, as shown by the full line. The high point at log t = 6.6 and the low point at 6.9 (compared to the predicted relation) are probably spurious. They depend on the accuracy of the age determination of very young clusters, and on the limits of the age-bins used for this plot. At log t > 8 the observed distribution drops below the predicted relation. This shows that older clusters must have been disrupted. A least square fit through the data yields the empirical relation

\[
\log dN_{\text{obs}} / dt = -7.05 - 1.38 (\pm 0.18) \times \log (t/10^8) \quad (26)
\]

in clusters per yr. This relation is shown by a dashed line.

The crossing point is at \( \log(t_{\text{cross}}) = 7.90 \pm 0.2 \). Adopting a cluster IMF of \( \alpha = 2 \) we find that the disruption time of the clusters scales with mass as \( t_{\text{dis}} \sim M_\odot^{\gamma} \) with \( \gamma = 0.72 \pm 0.12 \) (see Eq. (12)). Using this slope we can predict the corresponding relation for the mass distribution (see Eq. (13)), which results in a predicted slope of \(-1.28 \pm 0.12\). This relation is shown in Fig. (9b) by a dashed line. The vertical shift of the line is fitted through the last three data points.

### Table 1. Powerlaw fits to the mass and age distributions of M51 clusters

| Sample | Nr  | \( a_0 \)         | \( a_1 \)         | \( \log(M_{\text{cross}}) \) | \( b_0 \)         | \( b_1 \)         | \( \log(t_{\text{cross}}) \) |
|--------|-----|-------------------|-------------------|-----------------|-------------------|-------------------|-----------------|
| All    | 744 | -1.87 ± 0.06      | 3.4 ± 0.5         | -6.04           | -1.43 ± 0.05      | 7.3 ± 0.3         |
| \( \chi^2 \leq 3.0 \) | 512 | -2.05 ± 0.09      | 3.4 ± 0.5         | -6.22           | -1.55 ± 0.07      | 7.2 ± 0.3         |
| \( \chi^2 \leq 1.0 \) | 380 | -2.18 ± 0.11      | 3.4 ± 0.5         | -6.34           | -1.57 ± 0.09      | 7.2 ± 0.3         |

\( M_{\odot} \) is the solar mass; \( M_{\text{cross}} \) is the age limit for clusters with solar metallicity. The smallest \( t \) (in yrs) of the age-bins used for this plot. At \( \log(t) > 8 \) the observed lower limit shows the expected increase of mass with age, as predicted for clusters that fade as they get older.
Figure 9. The age (upper=Fig a.) and mass (lower=Fig b.) distribution of the observed clusters in M33. The distributions are fitted with two lines: the full lines show the expected decrease due to the detection limit only (fading). The dashed line in the upper figure is the least square powerlaw fit for the clusters of $t > 10^3$ years. The expression of the fit is given in the figure. This line indicates the dependence of the disruption time on mass. The dashed line in the lower figure was derived from the dashed line in the upper figure (see text).

We see that this relation fits the data quite poorly. This is due to the very small numbers of clusters with initial masses above $10^3 M_\odot$ in the sample studied by Chandar et al. (1999b, 1999c). The crossing point of the (dashed) disruption line and the (full) fading line in this figure is at $\log(M_{\text{cross}}) = 3.70 \pm 0.3$. If we had adopted the lower limit of $\gamma = 0.60$ the disruption line would have been slightly steeper, fitting the data slightly better, but the value of $M_{\text{cross}}$ would have changed very little.

Converting Eq. (26) into a relation for the cluster disruption time, by using the fitting of the fading lines to determine the values of $\log(S) = -3.24$ and $0.4(m_v - V_{\text{lim}}) = -1.47$ (Eqs. (10) and (9)) similar to the method applied in Sect. 3.4.3, we find

$$\log t_{\text{dis}} = 8.12 (\pm 0.30) + 0.72 (\pm 0.12) \times \log(M_\odot/10^4)$$

for M33 clusters. Comparing this expression with the one derived for M51 (Eq. (25)) we see that the slope is slightly shallower in M51, but within the errors of the rather uncertain slope derived for M33. The constant $t_d^{\text{dis}}$ for the disruption time is about a factor 3 times larger for M33 than for M51.

5 THE DISRUPTION OF CLUSTERS IN THE SMC

Hodge (1987) has determined the ages of 326 clusters in the SMC from the apparent B-magnitude of the brightest star in each cluster. The accuracy of the age determination is about 40 %. We will use this sample, rather than the more homogeneous sample of cluster ages from isochrone fitting by Pietrzynski & Udalski (1999), based on the OGLE – II photometry, because that sample contains only 7 clusters older than 300 Myrs, 4 of which have an uncertain age.

The sample of clusters with age determinations by Hodge (1987), was determined from photographic plates taken with the CTIO 4-m telescope, down to a limiting magnitude of $B \simeq 22$ to 23 (Hodge 1983). This corresponds to an absolute magnitude limit of $M_B \simeq +4$ to +3, or a turnoff age of about 10 to 15 Gyr. This implies that clusters of all initial masses, down to a few hundred $M_\odot$ should be detectable. In other words, the age distribution of the observed clusters is not affected by fading effects described in Sect. 2.2, so the value of $\xi$ in the age distribution is expected to be zero.

There is, however, another selection effect of the sample. The clusters were identified by Hodge (1983) on the basis of visual inspection of photographic plates. These clusters have a radius between 2 and 15 pc, with a mean radius of 7 pc. Dense clusters are more easily recognized than less dense clusters, and massive clusters are more easily recognized than low mass clusters. So, although the magnitude limit of the brightest star in the cluster does not produce a bias in the age distribution, the selection of the clusters may be biased to the denser and more massive clusters. This bias is described in Appendix B, where we will show that it is not important for the SMC clusters studied by Hodge, except for the oldest ones. For the moment we ignore this possible bias in the detection of the SMC clusters.

Figure (10) shows the age distribution of 314 SMC clusters with age determinations by Hodge (1987). The figure shows a distribution that is flat for ages less than about 1 Gyr and decreases steeply to higher ages. The horizontal part supports the assumption that selection effects and fading play a minor role. The least square fit to the decreasing part gives

$$\log dN_{\text{obs}}/dt = -5.32 - (1.65 \pm 0.22) \times \log(t/10^8)$$

in clusters per year. The crossing point of the two relations occurs at $\log t = 8.8 \pm 0.2$, in yrs. The slope of the powerlaw fit implies that $\gamma = 0.61 \pm 0.08$ if $\alpha = 2.0$.

We can estimate $t_d^{\text{dis}}$ from the crossing point between the horizontal part and the sloped part of the observed $\log dN_{\text{obs}}/dt$-relation in Fig. (10). The horizontal part of the distribution for ages less than about 1 Gyr, shows that the fading of clusters has played no role in the selection of the clusters, so the constant $\xi$ in Eq. (5) is about zero. In that case the predicted age distribution for clusters without disruption is
Figure 10. The age distribution of the clusters in the SMC with age determinations by Hodge (1987). The distribution is fitted with two lines: the horizontal full line indicates a constant cluster formation rate. The dashed line is the least square power-law fit for the distribution of older clusters. The expression of the fit is given in the figure.

\[
\frac{dN_{\text{obs}}}{dt} = \int_{M_{\text{lim}}(t)}^{M_{\text{cl(max)}}} S \ M_{\text{cl}}^{-\alpha} \ dM_{\text{cl}} \simeq \frac{S}{\alpha - 1} \ M_{\text{min}}^{1-\alpha} \tag{29}
\]

where \(M_{\text{lim}} = M_{\text{min}}\) is the minimum mass of the detected clusters. We see that \(dN_{\text{obs}}/dt\) is predicted to be a constant, as is indeed observed for the younger clusters in the SMC. In case of disruption, the age distribution is given by Eq. (12). The crossing point of the two equations is located where

\[
\frac{t_{\text{cross}}}{t_4} = (M_{\text{min}}/10^4)^\gamma \tag{30}
\]

The minimum initial mass of the clusters detected by Hodge (1983) is estimated to be about \(\log(M_{\text{min}}) \simeq 2.3 \pm 0.3\) for the SMC (see Appendix B). The value of \(t_4\) can now be estimated from the observed crossing point of the two relations of the age distribution of the SMC in Fig. (10). This results in the disruption relation

\[
\log t_{\text{dis}} = 9.9 (+0.2) + 0.61 (+0.08) \times \log(M_{\text{cl}}/10^4) \tag{31}
\]

We see that the disruption time of clusters in the SMC is considerably larger than for M51 and M33, but that the slopes of the disruption relations are very similar.

6 THE DISRUPTION OF CLUSTERS IN THE SOLAR NEIGHBOURHOOD

Wielen (1971) studied the age distribution of galactic clusters from the catalogues of Becker & Fenkart (1971) and Lindoff (1968). His samples contain respectively 70 and 59 clusters within a projected distance of 1 kpc from the Sun. The ages of the clusters studied by Becker & Fenkart were determined from the colour-magnitude diagrams, with the age calibration by Barbaro et al. (1969). The ages of clusters from the Lindoff catalogue were also derived using the Barbaro et al. isochrones by Wielen. We will use the clusters from the Becker & Fenkart catalogue and from the Lindoff catalogue with the ages listed by Wielen. Wielen (1971) has shown that the above mentioned lists of clusters, within a galactic distance of 1 kpc from the Sun, is not complete, but that they provide representative samples of the age distribution of the clusters. Therefore we will use these two samples to derive the disruption of galactic clusters from a study of their age distribution.

Figures (11a) and (11b) show the age distributions of the clusters, expressed in \(dN_{\text{obs}}/dt\), of the clusters from the Becker and Fenkart (1971) catalogue (hereafter called BF) and from the Lindoff (1968) catalogue with the age calibration by Barbaro et al. (1969), hereafter called LB. Both distributions show rather similar characteristics: a flat part at \(\log t < 8.0\), and a steeply declining part for higher ages. The flat part corresponds to a cluster formation rate of \(10^{-0.58} \text{ clusters Myr}^{-1}\) for the BF sample and \(10^{-0.77} \text{ clusters Myr}^{-1}\) for the LB sample. The steep parts are fitted with linear least square fit relations

\[
\log dN_{\text{obs}}/dt = -6.37 - (1.963 \pm 0.43) \times \log(t/10^8) \tag{32}
\]

\[
\log dN_{\text{obs}}/dt = -6.77 - (1.592 \pm 0.42) \times \log(t/10^8) \tag{32}
\]

in clusters per yr for the BF and LB samples respectively. The resulting values of the mass dependence of the disruption law (Eq. (11)) are respectively \(\gamma = 0.59 \pm 0.12\) and \(\gamma = 0.63 \pm 0.18\) if the CIMF has a slope of \(\alpha = 2\).

The almost horizontal distribution for young clusters suggests that there is no bias towards the more massive ones in the samples of Galactic clusters used here, as was already argued by Wielen (1971). Therefore we can estimate the cluster disruption time from the crossing points, \(\log t_{\text{cross}} = 8.1\), of the age distributions shown in Fig. (11), in the same way as applied for SMC sample.

For galactic clusters within 1 kpc we adopt the minimum mass of about \(\log M_{\text{min}} \approx 2.5 \pm 0.5 M_\odot\). This is about as high as for the SMC, because the higher brightness of the Galactic cluster stars, compared to the SMC stars, is partly compensated by the larger angular diameter of the Galactic clusters, which will make the detection of sparse clusters more difficult. The crossing point then indicates a disruption time of

\[
\log t_{\text{dis}} \simeq 9.0 (+0.3) + 0.60 (+0.12) \times \log(M_{\text{cl}}/10^4) \tag{33}
\]

This disruption time is longer than for the M51 clusters, but shorter than for the SMC clusters.

7 COMPARING THE DISRUPTION TIMES OF DIFFERENT GALAXIES

We have derived estimates of the parameters \(\gamma_{\text{cl}}\) and \(\gamma\) that describe the dependence of the cluster disruption time on the \(\text{initial}\) cluster mass, Eq. (11), for different galaxies. These parameters are summarized in Table (2). Columns 2 and 3 give the number of clusters used for the determination of the disruption times. Columns 4 and 5 give the age range and the mass range of the clusters. Column 6 gives the disruption time \(t_{\text{dis}}\) of a cluster with initial mass of \(10^4 M_\odot\). Column 7 gives the derived slopes of the age distribution (expressed in \(\log dN_{\text{obs}}/dt\)). Column 8 gives the resulting slope of the age dependence of the disruption law, under the assumption that the cluster IMF has a slope of \(\alpha = 2\).

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Table 2. The parameters of the disruption time: $t_{\text{dis}} = t_4^{\text{dis}} \times (M_{\text{cl}}/10^4)^\gamma$

| Galaxy   | $N_t$ (log $dN_{\text{obs}}/dt$) | $N_t$ (log $dN_{\text{obs}}/dM$) | Age range (log yrs) | Mass range (log $M_\odot$) | log $t_{\text{dis}}^4$ (yrs) | $(1 - \alpha)/\gamma$ | $\gamma$ |
|----------|---------------------------------|---------------------------------|--------------------|-----------------|-----------------|-----------------|--------|
| M51      | 380/512/744                    | 380/512/744                    | 6.0 – 9.7          | 3.0 – 5.2       | 7.64 ± 0.22     | 1.75 ± 0.32     | 0.57 ± 0.10 |
| M33      | 49                             | 45                             | 6.5 – 10.0         | 3.6 – 5.6       | 8.12 ± 0.30     | 1.38 ± 0.18     | 0.72 ± 0.12 |
| Milky Way | 72/59                          | –                              | 7.2 – 10.0         | –               | 9.0 ± 0.3       | 1.77 ± 0.40     | 0.60 ± 0.12 |
| SMC      | 314                            | –                              | 7.6 – 10.0         | –               | 9.9 ± 0.2       | 1.65 ± 0.22     | 0.61 ± 0.08 |
| Mean     |                                |                                |                    |                 | 0.62 ± 0.06     |                 |        |

The values of $t_{\text{dis}}^4$, however, are very different for the different galaxies: M51 has the shortest cluster disruption time and the SMC has the longest disruption time. The difference amounts to about a factor $10^2$.

8 DISCUSSION

8.1 The description of the disruption time

In our simple description of the disruption time as a function of the initial mass of the cluster (Eq. (11)) we have assumed that for each initial mass the cluster stays intact up to a certain time $t_{\text{dis}}$ and then suddenly disappears. This is of course a severe simplification of the true situation.

Theoretical and dynamical simulations (e.g. Spitzer 1957; Portegies Zwart et al. 1999; Fall & Zhang 2001) both suggest that disruption results in a linear decrease of the mass of a cluster with time, rather than the sudden disappearance that we adopted in this study. Therefore our assumption of sudden disruption is a severe simplification.

However, it can be shown that, even for gradually disrupting clusters, the logarithmic age and mass histograms of the surviving clusters above the detection limit will show a distribution that can be fitted with two powerlaws. One of these powerlaws will depend on the fading of the clusters and the other one will depend on the cluster decay time, very similar to the histograms predicted for sudden disruption, shown in Fig. (1). This is shown in Appendix A, where we also show that the predicted mass versus age distributions for gradual disruption are very similar to the observed distributions of clusters in M51 and M33, Figs. (6) and (8).

Theoretical arguments and studies of cluster disruption, e.g Spitzer (1957), Chernoff & Weinberg (1990), de la Fuente Marcos (1997), Portegies-Zwart et al. (1998) and Fall & Zhang (2001), show that the disruption time is expected to depend on their initial density and internal velocity disruption, rather than on their mass, as was adopted in this study. However, since the radius of most of the clusters is not known, we have used their initial mass as the only parameter for the description of the disruption time. Basically this implies that we have included the density dependence of the disruption time in the mass dependence. If the density of a cluster in a particular sample depends on its initial mass, e.g. $\rho \sim M^\alpha$ then the derived value of $\gamma$ reflects the dependence of $t_{\text{dis}}$ on a combination of the mass and density dependence, as described in Sect 2.1. In particular, if
the disruption time is proportional to the relaxation time \( t_{\text{dis}} \sim t_{\text{ext}} \sim M \rho^{-1/2} \), then the derived value of \( \gamma \approx 0.62 \) is consistent with a mean density-versus-mass relation of \( \rho \sim M^{0.76} \). The empirical determination of the dependence of the disruption time on both the mass and radius for clusters in M81 is described in a forthcoming paper.

### 8.2 Continuous cluster formation?

In the determination of the disruption rates based on the mass and age distributions of the clusters, we have assumed that the cluster formation rate was constant. This is probably a reasonable assumption for the solar neighbourhood, and for M33. However this is not a trivial assumption for the interacting galaxy M51 with its companion NGC 5195, nor for the SMC which interacts with the LMC and the Galaxy.

Scuderi et al. (2002) and Lamers et al. (2002) found that the nucleus of M51 has a strong star burst with an age of several 10^8 years, in agreement with the time of closest approach of the companion. Bik02 has studied the age distribution of the M51 clusters in the inner spiral arms and found only a faint indication (less than 2 \( \sigma \)) of an increased cluster formation rate in M51 a few 10^8 years in this region. This indication can also be seen in the possibly small bump in the age distribution of Fig. 7b near log \( t \approx 8.7 \). If indeed present, this bump might be responsible for the difference between the two values of \( \gamma \) derived from the mass and the age distributions. Ignoring this bump would have resulted in a slightly smaller value of \( \gamma \) derived from the age distribution. The effect on the determination of \( t_{\text{dis}} \) from the mass and age distributions together is negligible.

The problem might also exist for the SMC clusters. Several authors, e.g. Mateo (1988), Bica et al. (1996), Pietrzyński & Udalski (1999), (see the review by Da Costa (2002) and references therein), have shown that the age distribution of clusters in the LMC shows peaks due to the interaction with the SMC and the Galaxy. However, the age distribution of the SMC clusters is much more homogeneous (Da Costa 2002). Therefore we did not study the cluster disruption in the LMC and we adopted a constant cluster formation rate for the SMC as a first approximation to derive the disruption time of SMC clusters. When larger samples of clusters with more accurate ages and masses become available, it is possible to verify the influence of non-constant cluster formation rates on the determination of the disruption times by considering both the mass and the age distributions. In case of a non-constant cluster formation rate, the determination of the disruption time can then easily be corrected by a numerical calculation of the integrals of Eqs. (9) and (12) for an age dependent \( \tau \).

### 8.3 Comparison with other studies

Our resulting values for the parameters of the cluster disruption relations, summarized in Table (2), can be compared with the results of other studies, e.g. by Elson & Fall (1985, 1988), Hodge (1987) and Mateo (1988). In these studies the age distributions for clusters in the LMC, SMC and solar neighbourhood, normalized to the same value at \( t \approx 100 \) Myrs, are compared. The comparisons show clearly that the age distribution is steeper for the solar neighbourhood than for the LMC and SMC. The difference was expressed in terms of a "mean age", \( \tau_m \), of the sample of clusters above the detection limit. Elson and Fall (1985, 1988) found \( \tau_m = 2 \times 10^8 \) yrs for the solar neighbourhood and \( \tau_m = 4 \times 10^9 \) yrs for the LMC, which indicates that the disruption of clusters in the LMC is slower than in the solar neighbourhood.

To compare the results of Elson and Fall (1985, 1988) with our results, let us assume for the moment that the disruption time of clusters in the LMC is about the same as the disruption time of clusters in the SMC, which we have measured. The factor 20 difference in the mean ages of the LMC and Galactic clusters, found by Elson & Fall appears to be much larger than the factor 4 or so in our values of \( t_{\text{dis}} \) between the SMC and the Milky Way. However, one should bear in mind that even a small difference in \( t_{\text{dis}} \) can produce a large difference in the mean age of the observed clusters. This is because the mean age depends on the mass and age range of the studied clusters, and their brightness above the detection limit. In fact, we used the same sample of SMC and Galactic clusters as Hodge (1988) and almost the same sample as used by Elson & Fall (1985). So the difference of only a factor 4 in \( t_{\text{dis}} \) between the SMC and the Galaxy is consistent with the factor 20 difference in mean cluster age. Our analysis has the advantage that it results in a description of the mass dependence of the disruption times in both galaxies.

For the solar neighbourhood we compare our result with that of van den Bergh (1981). The disruption time of 1 Gyr for a 10^4 \( M_\odot \) cluster can be compared with the e-folding time 0.15 Gyr for the disruption of clusters within 0.75 kpc derived by van den Bergh (1981), who used the sample of 63 clusters from the catalogue by Mermilliod (1980). These clusters have a mean \( M_V \) of about -3 to -4, which corresponds to a total mass of \( \log(M/M_\odot) \approx 2.5 \pm 0.2 \) if the clusters have a mean age of 100 Myrs and 0.3 dex lower if their mean age is 30 Myrs (Leitherer et al. 1999). From Eq. (33) we see that our analysis suggests a disruption time of 0.13 Gyr for clusters of \( M \approx 300 M_\odot \). This agrees surprisingly well with the mean value of 0.15 Gyr derived derived by van den Bergh (1981), considering the uncertainties involved.

### 8.4 Possible biases

The low extinction values of the clusters in M51 studied by Bik et al. (2002) and in M33 studied by Chandar et al. (1999b, 1999c), due to the almost face-on orientations of M51 and M33, together with the fact that the clusters were detected with one instrument under constant conditions, results in a minimum bias in the statistics of these cluster samples. The fading of the clusters due to the aging of the stars, makes the older low mass clusters drop below the detection limit. However the resulting bias can very well be taken into account because the cluster models predict the fading of the clusters in the different wavelength bands as a function of age and cluster mass. The predicted bias in the age and mass statistics of the clusters is nicely confirmed by the statistics of the observed clusters with ages younger than the disruption times (see Figs. (6) and (8)).

For the studies of clusters in the SMC this method of
correcting for the expected fading does not apply, because the clusters were selected by visual inspection of photographic plates (Hodge 1986, 1988). For the SMC clusters we have assumed that the sample is incomplete but relatively unbiased down to some minimum cluster mass, because the stars above a certain brightness limit could be detected individually (see Sect. 5 and Appendix B). The nearly flat part in the age distribution of the SMC cluster sample (Fig. (10)) confirms that this was a reasonable assumption.

For the clusters in the solar neighbourhood the main bias is due to the variable large extinction. However, Wielen (1971) has described several tests that show that, although the sample within a distance of 1 kpc may be incomplete, it is relatively free of bias. Also in this case we have estimated the lower mass limit of the detected clusters in an approximate way, similar to the method applied to the SMC clusters. Again the flat part of the age distribution for ages less than the disruption time supports this assumption. Therefore we feel confident that the disruption time can indeed be derived from the cluster samples of the SMC and solar neighborhood.

The situation can be improved when the detection of clusters can be made in an automatic and unbiased way (or rather, with a bias that is known and can be corrected for) from large homogeneous data bases.

9 SUMMARY

Under the assumptions that:
(a) clusters are formed at a constant rate with the same cluster IMF and the same stellar IMF, and
(b) that clusters disrupt suddenly after a certain time $t_{\text{dis}}(M_\ell)$ that depends as a powerlaw on their initial mass $M_\ell$,
we derived the following results.

(i) Both the mass distribution and the age distribution of clusters in M51 and M33 show a double powerlaw distribution, with a small slope for young and low mass clusters and a steeper slope for older and more massive clusters.

(ii) Such a distribution is predicted for clusters that are formed with a single CIMF, but fade due to stellar evolution below the detection limit and disrupt suddenly at a time that depends as a powerlaw on their initial mass, $t_{\text{dis}} = t_{\text{dis}}^4 \times (M_\ell/10^4 M_\odot)^\gamma$.

(iii) The age distribution of clusters in the SMC and the solar neighbourhood also shows this characteristic double powerlaw distribution, with a flat age distribution for small ages. This is in agreement with the fact that evolutionary fading below the detection limit is not important for the young observed clusters in these galaxies. For these galaxies we do not know the mass distribution of the used cluster samples.

(iv) From the slopes and the crossing points of the powerlaws of the age and mass distributions the values of $t_{\text{dis}}^4$ and $\gamma$ are derived. For M51 the results can be checked because the slopes and crossing points of the age distribution and the mass distribution should be consistent with one another. For the M33, SMC and the solar neighbourhood, the lack of a reliable mass distribution prevents this check.

(v) The value of $\gamma$ is the same for the four galaxies within the uncertainty of the observations, with a mean value of $\gamma = 0.62 \pm 0.06$ if the CIMF has a slope of $\alpha = 2$, i.e. $dN_{\ell}/dM_\ell \sim M_\ell^{-\alpha}$. If $\alpha > 2$ then $\gamma < 0.60$. For other values of $\alpha$ the value of $\gamma$ has to be corrected by $\Delta \gamma \simeq -0.62 \alpha$.

(vi) If the disruption time of the cluster samples that we have studied here is proportional to their relaxation timescale, $t_{\text{dis}} \sim t_{\text{ext}} \sim M^{\alpha-1/2}$, then the derived value of $\gamma \simeq 0.62 \pm 0.06$ suggests a mass-density relation of the type $\rho \sim M^x$ with $x \simeq 0.75 \pm 0.10$.

(vii) The constant $t_{\text{dis}}^4$, which is the disruption time of a cluster of $10^4 M_\odot$, differs drastically from galaxy to galaxy, between 40 Myrs for clusters in M51 at galactocentric distances of $0.8 < r < 3.1$ kpc, and 8 Gyr for the SMC. The disruption time $t_{\text{dis}}^4$ is about 130 Myrs for the region at galactocentric distances of about 0.8 to 5 kpc in M33 and 1.0 Gyr for the solar neighbourhood.

(viii) The masses of the clusters were derived from a comparison of their observed energy distributions with Starburst99 models (Leitherer et al. 1999). These cluster models have a stellar lower mass limit of 1 $M_\odot$. If the true lower mass limit $M_{\text{low}}$ is smaller, the cluster masses are underestimated by a factor 2.09 if $M_{\text{low}} = 0.2 M_\odot$. In that case the value of $t_{\text{dis}}^4$ has to be decreased by about 0.18 dex.

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REFERENCES

Barbary G., Dellaporta N., Fabris G., 1969, Ap&SS, 3, 123
Bastian N., Lammers H.J.G.L.M., 2002 (in preparation)
Becker W., Fenkart, R., 1971, A&AS, 4, 241
Bertelli G., Bressan A., Chiosi C., Fagotto F., Nasi, E., 1994, A&AS, 106, 275
Bica E.L.D., Claria J.J., Dottori H., Santos J.F.C., Piatti A.E., 1996, ApJS, 102, 57
Bik A., Lammers H.J.G.L.M., Bastian N., Panagia N., Rioniello M., Kirsher, R., 2002, ApJ, submitted
Bruzual G., Charlot, S., 1993, ApJ, 405, 538
Chandar R., Bianchi L., Ford, H.C., 1999a, ApJS, 122, 431
Chandar R., Bianchi L., Ford, H.C., 1999b, ApJ, 517, 668
Chandar R., Bianchi L., Ford H.C., Salasnich, B., 1999c, PASP, 111, 794
Chernoff P., Weinberg M., 1990, ApJ, 351, 121
Da Costa G.S., 1999, in Chu Y.-H., Suntzeff N.B., Hesser J.E., Bohlender D.A., eds, Proc.IAU Symp. 190, New Views of the Magellanic Clouds, Kluwer Acad. Publ., Dordrecht, p.397
Da Costa G.S., 2002, in Grebel E., Geisler D., Minniti D., eds,
The initial mass function given by Eq. (2). Suppose that the disruption time scales with mass as given by Eq. (11). Then the mass of a cluster decreases with time as $dM/dt \approx -M/t_{\text{dis}}$ and so

$$M(t) = M_i \gamma \left( \frac{t}{t_{\text{dis}}} \right) - \frac{t}{t_{\text{dis}}} M_i - B t$$

where $M_i$ is the initial cluster mass (in $M_\odot$) and $B = \gamma 10^{4.5} t_{\text{dis}}^{-4.5}$. Suppose that the detection limit of clusters with a present mass $M$ and an age $t$ is given by

$$M_{\text{lim}} = M_i (t/t_L)^\gamma$$

This expression is equivalent to Eq. (8). Then it is easy to show that clusters with a present mass $M$ can be observed if their age is in the range of

$$t_{\text{lower}} = \frac{M_{\text{min}} - M}{B} < t < \min \left[ \left( \frac{M_{\text{max}}}{M} - M \right)^{1/\gamma} t_L \left( \frac{M}{M_L} \right)^{1/\gamma} \right] = t_{\text{upper}}(37)$$

Similarly, clusters of age $t$ can be observed if their present mass is in the range of

$$M_{\text{lower}} = \max \left[ M_L \left( t/t_L \right)^\gamma ; (M_{\text{min}} - B t)^{1/\gamma} \right] < M < (M_{\text{max}} - B t)^{1/\gamma} = M_{\text{upper}}$$

(38)

The age distribution of the clusters above the detection limit is

$$\frac{dN_{\text{obs}}}{dt} = \int_{t_{\text{lower}}(t)}^{t_{\text{upper}}} S M_i (M,t)^{-\alpha} dM = \frac{S}{\alpha - 1} \left[ (M_{\text{lower}} + B t)^{(1-\alpha)/\gamma} - (M_{\text{upper}} + B t)^{(1-\alpha)/\gamma} \right]$$

and the mass distribution is

$$\frac{dN_{\text{obs}}}{dM_{\text{cl}}} = \int_{t_{\text{lower}}}^{t_{\text{upper}}} S M_i (M,t)^{-\alpha} dt = \frac{S t_0}{(\alpha - 1)M_0} \left[ \frac{M}{M_0} \right]^{-\gamma - 1} \times \left[ (M_{\text{upper}} + B t^{\text{lower}})^{(1-\alpha)/\gamma} - (M_{\text{lower}} + B t_{\text{upper}})^{(1-\alpha)/\gamma} \right]$$

with the values of $M_{\text{lower}}, M_{\text{upper}}, t_{\text{lower}}$ and $t_{\text{upper}}$ given above.

We have calculated the location of the observable clusters in the $(M,t)$-diagram and the predicted mass and age distributions of the observable clusters for a cluster IMF of $\alpha = 2$ with $M_{\text{min}} = 10^2 M_\odot$ and $M_{\text{max}} = 10^6 M_\odot$, for a disruption time with $\gamma = 0.50$ and $t_{\text{dis}} = 3 \times 10^8$ yrs. Figure (12)a shows the location of the clusters in the $(M,t)$-diagram. We see that the clusters occupy an approximately triangular region with the upper limit given by the decrease of $M_{\text{max}}$ with age. The lower limit consists of the detection limit, which is the lower mass limit that increases with age due to the evolutionary fading of the cluster. The observed mass versus age distributions of the clusters observed in M51 (Fig. 6) and M33 (Fig. 8) show the same overall structure.

Figures (12)b and c show the age and mass distributions of the clusters, normalized to the total number of clusters.

10 APPENDIX A: COMPARISON BETWEEN INSTANTANEOUS AND GRADUAL DISRUPTION

In the analysis of the mass and age distributions we assumed that clusters disrupt instantaneously. Theory predicts that clusters are in fact disrupted gradually. In this appendix we will show that gradual disruption produces almost the same mass and age distributions as instantaneous disruption.

Suppose that clusters form at a rate and with a cluster initial mass function given by Eq. (2). Suppose that the disruption time scales with mass as given by Eq. (11). Then the
Empirical determination of the disruption times of star clusters in four galaxies

The full line is the distribution in case of gradual disruption. The dash-dotted lines give the expected distributions if only evolutionary fading of the clusters below the detection limit was important, i.e. no disruption. The dotted lines give the distributions in case of instantaneous disruption. In both diagrams we see that the distributions for gradual disruption approaches the linear relation of the evolutionary fading at the low mass or age end and the steeper linear relation for instantaneous disruption at the high mass or age end, with a smooth transition in between these linear parts. This demonstrates that, if the observed mass and age distributions are fitted by a combination of two linear relations (as was done in this paper) the correct values of the disruption time $t_{\text{dis}}$ and the exponent $\gamma$ can be derived.

11 APPENDIX B: THE MINIMUM INITIAL MASS OF DETECTED SMC CLUSTERS

The estimate of the cluster disruption time for the SMC depends on the minimum initial mass of the clusters detected by Hodge (1983, 1986, 1987). In this appendix we derive a crude estimate of this initial mass.

The clusters were detected on blue photographic plates of the 4m CTIO telescope (Hodge, 1986). Stars down to $B \simeq 22$ to 23 could be detected in the SMC. Adopting a distance modulus of 18.85 (van den Bergh 1999) and a mean $E(B-V) = 0.08$ or $A_B = 0.33$ (Pietrzynski & Udalski 2000) we find $M_B \simeq +3.7 \pm 0.5$ corresponding to a main sequence type between F6 with a mass of about $1.3 M_\odot$ and F1 with a mass of $1.7 M_\odot$ (Lang, 1992). So the minimum mass limit is $1.5 \pm 0.2 M_\odot$.

Hodge (Private Communication) estimates that he could detect a cluster if it contained at least between about 10 and 30 stars above the detecting limit, although some more compact clusters were detected with even fewer stars. Suppose that the clusters could be detected if they contained at least 20 stars above these detection limits, and that the stellar IMF of the clusters can be written as $N(M) dM = C M^{-2.35}$, i.e. with Salpeters' exponent. The initial mass of the cluster is

$$M_{\text{cl}} = \int_{M_{\text{min}}}^{M_{\text{max}}} C_{\text{rich}} M^{-1.35} dM \simeq \frac{C_{\text{rich}}}{0.35} M_{\text{min}}^{-0.35} \quad (41)$$

where $C_{\text{rich}}$ is a parameter that describes the richness of the cluster. For the minimum mass of the cluster stars we adopt a value of $0.5 M_\odot$. We assumed that the maximum mass of the cluster stars, $M_{\text{max}} \geq 50 M_\odot$, is much higher than $M_{\text{min}}$. The requirement that the detectable clusters should contain at least $N = 20 \pm 10$ stars brighter than a main sequence star of $1.5 \pm 0.2 M_\odot$ implies that

$$N = \int_{M_{\text{lim}}}^{M_{\text{max}}} C_{\text{rich}} M^{-2.35} dM \simeq \frac{C_{\text{rich}}}{1.35} M_{\text{lim}}^{-1.35} > 20 \quad (42)$$

where $M_{\text{lim}} = 1.5 \pm 0.2 M_\odot$. We find that $C_{\text{rich}} \simeq 50 \pm 30$, and the minimum initial mass of the detectable clusters is $M_{\text{cl}} \simeq 170 \pm 100 M_\odot$. For an adopted minimum stellar mass of $0.25 M_\odot$ the initial mass is 27 percent larger or $220 \pm 120 M_\odot$ for the detectable clusters.

In this estimate we have assumed that the fraction of the stars that have disappeared as supernovae or dropped...
below the detection limit as white dwarfs is small compared to the observable number of stars with masses above 1.4 $M_{\odot}$.

This is a reasonable assumption for clusters with the turn-off point above a few $M_{\odot}$ and turn-off ages shorter than about $10^9$ years. These were the clusters that were used to derive the value of $t^\text{dis}_4$ in Sect 6. For a conservative estimate we adopt a minimum mass of the detectable clusters of about $\log(M_{\odot}) = 2.3 \pm 0.3 ~M_{\odot}$. 