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The macroeconomic impacts of the COVID-19 pandemic: A SIR-DSGE model approach

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1. Introduction

The recent outbreak of coronavirus (COVID-19) disease had already infected more than 150 million people and caused approximately 3 million deaths worldwide as of the end of May 2021. Although most of the major economic data covering the relevant period have not been released, it is expected that the pandemic has substantially disrupted economic activity. To address this, quarantine measures and fiscal and monetary policies have been implemented in many countries. However, the economic framework for studying transmission mechanisms and the effectiveness of these policies is still under-researched. In this paper, we develop a susceptible-infected-removed dynamic stochastic general equilibrium (SIR-DSGE) model that combines the SIR model (Kermack & McKendrick, 1927) from mathematical biology that depicts the transmission of infectious disease with a canonical new Keynesian DSGE model to characterize the event. In particular, we use the model to characterize the macroeconomic effects and transmission mechanisms of the COVID-19 disease outbreak. Furthermore, we use the model to analyze the economic cost and benefits of containment policy and random testing. We also consider a Ramsey social planner problem to solve for the optimal monetary policy in response to the outbreak.

To bridge the SIR and DSGE models, we assume that the severity of the disease, which is measured by the number of infected individuals, affects the economy through both the demand and supply sides. For the demand side, as in Faria-e Castro (2020), we assume that households’ marginal utility of consumption decreases with the severity of the disease. We make this assumption because, for example, the risk of being infected when staying in crowded areas would reduce households’ desire to consume. On the other hand, the economic impact of the pandemic could be on the supply side since firms might not be able to assess the supply of intermediate
goods due to the immigration restrictions imposed by many countries. We characterize this effect as a reduction of the total factor productivity (TFP) level. We show that both of these effects would result in reductions in consumption and output. Moreover, households that consume less would have less incentive to work and, therefore, supply less labor. Although consumption converges back to the steady-state level once the pandemic is over, we find that considerably more time is required for output to recover due to the prior declines in labor and capital investment.

We show that both containment policies and random testing are effective means to curb the infected number. The former works through reducing the number of susceptible, while the latter works through discovering asymptomatic individuals. In particular, randomly testing 10% of the uninfected could reduce the peaked value of the infected share by two-third. Furthermore, it is more effective in curbing the infected share if one implements both policies simultaneously. Our result suggests that economic impacts of the two policies differs: containment policy prevents output loss more effectively in the short run, while can lead to slower output recovery if persistently implemented. A random testing policy, in contrast, has minimal short-term impact on output, but can accelerate output recovery.

By comparing the centralized and decentralized equilibria, we find that although the centralized output decreases more substantially during the pandemic, the decentralized output is recovered at a slower rate than the centralized output. Due to the presence of the price stickiness, the decrease in the TFP level leads to a higher wage rate in the short run. The households would, therefore, increase their labor supply in the short run and decrease it in the long run. With this decision, the decentralized output slowly recovers in the long run after the pandemic. However, the social planner allows both the labor supply and investment to decrease in the short run. To this end, the centralized output and employment recover at a much faster rate.

The remainder of the paper proceeds as follows. Section 2 discusses the related literature. Section 3 discusses the background of the standard SIR model. We also explain how this model can be linked to the standard DSGE model. Then, Section 4 presents the calibration procedure and the results of the numerical exercises. Finally, Section 5 concludes the paper.

2. Related literature

Since the COVID-19 outbreak began, macroeconomists have begun to develop frameworks for policy analysis. Berger, Herkenhoff, and Mongey (2020) examine the effects of conditional quarantine policy and testing using a susceptible-exposed-infectious-recovered (SEIR) model. Since there are asymptomatic individuals, one cannot single out and quarantine only the infected individuals. Hence, a pure quarantine policy that applies to everyone, which would entail a massive economic cost, seems to be the only possible measure to curb the spread of the disease. Remarkably, they show that combining a looser quarantine policy with the random testing of asymptomatic individuals is as effective as a simple quarantine policy in terms of the numbers of deaths and symptomatic infections. Using a SEIR model, Piguillem and Shi (2020) also conclude that a combination of quarantine policy and testing of the asymptomatic individuals is better than imposing a simple, indiscriminate quarantine. The authors find that 87% and 12% of the asymptomatic individuals should be tested if the social welfare function is linear and logarithmic, respectively, where the latter results in a double consumption-equivalent gain compared to the pure quarantine policy. We perform a related numerical analysis and reach a similar conclusion.

Faria-e Castro (2020) constructs a DSGE model with two households (borrowers and savers) and two sectors (non-service and service). Using the model, the author compares different types of fiscal policies. The author characterizes the pandemic as a 60% decrease in the marginal utility of consumption in the service sector and emphasizes unemployment insurance as the most effective policy in the presence of the shock. In line with Faria-e Castro (2020) we also find that it is important to preserve labor supply in the long run. In contrast to our model, the multi-sector and multi-household model of Faria-e Castro (2020) should be better at modeling the distributional impacts of the pandemic. On the other hand, endogenous labor, monetary policy, and more important, the epidemiology component in our model are not considered in Faria-e Castro (2020) but are necessary for our findings. In addition, the author also does not consider the Ramsey planner solution. Alternatively, Guerrieri, Lorenzoni, Straub, and Werning (2020) characterize the COVID-19 outbreak as a negative supply shock. In a general equilibrium model, a decrease in firm productivity would reduce household income, which would in turn decrease the household goods demand. Guerrieri et al. (2020) show that such a reduction in aggregate goods demand would not induce a significant effect in a one-sector model, regardless of the existence of a household liquidity constraint. They construct a multi-sector model with firm exit and job destruction and show that a negative supply shock in one sector (the contact-intensive sector) could result in recession by triggering the Keynesian multiplier effect. To this end, they suggest that monetary policy and shutting down the contact-intensive sector can mitigate the impact of the shock. In this paper, we follow Faria-e Castro (2020) to consider the pandemic’s effect on the demand side. Inspired by Guerrieri et al. (2020), we also consider the pandemic’s effect as a negative supply shock.

Our work is closest to Eichenbaum, Rebelo, and Trabandt (2020), who also merge the SIR model with a general equilibrium model. They modify the SIR model such that the infection rate endogenously depends on individuals’ consumption and working decisions. During the pandemic, household reductions in consumption and labor supply would, on the one hand, mitigate the spread of the disease and, on the other hand, exacerbate the economic slowdown. Using the model, the authors show that the optimal containment rate should move in parallel with the number of infected. Under this policy, the peak percentage of the infected population could be reduced from 5.3% to 3.2%. We differ from Eichenbaum et al. (2020) in the following respects. First, while Eichenbaum et al. (2020) solve for the optimal containment policy, we consider a Ramsey problem for computing the optimal monetary policy. Moreover, apart from Eichenbaum et al. (2020), who focus only on containment policy, we also examine the effectiveness of a combination of random testing and containment policy, as in Berger et al. (2020) and Piguillem and Shi (2020). Second, our model differs from Eichenbaum et al. (2020) by including more components, such as capital, nominal rigidity, and monetary policy. Third, we consider a stochastic
3. Model

3.1. An introduction of to the SIR model

The susceptible-infected-removed (SIR) model originated in the early 20th century and is a popular toy model in epidemiology for depicting the transmission of infectious disease (Anderson, 1991; Kermack & McKendrick, 1927). As its name suggests, the model consists of three components, namely, the number of susceptibles $S_t$, the number of infected people $I_t$, and the number of recovered people $R_t$ at time $t$. All three variables are time varying. In particular, we refer to a person as susceptible (at time $t$) if he/she is uninfected and could catch the disease during the period. A person is infected if he/she is confirmed to have the disease at time $t$ and has not yet recovered or died. A person is recovered if he/she caught the disease before time $t$, has recovered and is immune to the disease from period $t$ onward.

As explained below, since we can only obtain the COVID-19 data in Hubei province after the lockdown policy is implemented, it is not possible to estimate the underlying model in the SIR model in which no quarantine policy is assumed. In this regard, unlike the SIR model, we assume that with the lockdown policy, there are $1 - a_0 \in [0, 1]$ share of the population is protected by the lockdown policy or is cautious enough that it is unlikely to be infected. As a result, the total population in the states of $\tilde{S}_t$, $\tilde{I}_t$, and $\tilde{R}_t$ only share $a_0$ of the total population. The value of $a_0$ depends on the strength of the lockdown policy.\(^1\)

Furthermore, we focus only on the transmission of the disease in a location or a closed economy. The standard SIR model assumes that no deaths or births occur and that no people enter or exit the location. Hence, the population $N$ is constant over time. For any time $t$, we have:

$$\tilde{S}_t + \tilde{I}_t + \tilde{R}_t = a_0 N$$

which implies that, excluding the non-susceptible, the individuals in the location must be in one of the three states. It is worth noting that the model assumes that there is no latent period for infected people.\(^3\) In other words, all infected people show symptoms of the disease and, thus, can be observed. One usually normalizes the population number to 1, and we have the following equation:

$$S_t + I_t + R_t = 1$$

(2)

where $S_t \equiv \tilde{S}_t/(a_0 N_t)$, $I_t \equiv \tilde{I}_t/(a_0 N_t)$, and $R_t \equiv \tilde{R}_t/(a_0 N_t)$. As discussed in Section 4.3, with the estimate $a_0$, we will counterfactually change the value of $a_0$ to examine the disease transmission process with different degrees of containment policy. The discrete-time version of the SIR model is a system of nonlinear difference equations as follows:

$$S_{t+1} = S_t - a_1 S_t I_t$$

(3)

$$I_{t+1} = I_t + a_1 S_t I_t - \kappa I_t$$

and

$$R_{t+1} = R_t + a_1 S_t I_t - \kappa R_t$$

(4)

where

$$a_1 = \frac{\tilde{I}_t + \tilde{R}_t}{N}$$

according to Eq. (1).

\(^1\) Note that $a_0$ cannot be 0 if there are individuals who have already in the states of $\tilde{I}_t$ or $\tilde{R}_t$. Precisely, in the extreme case where all the susceptible are perfectly protected by the lockdown policy, we have $\tilde{S}_t = 0$ and $a_0 = \frac{\tilde{I}_t + \tilde{R}_t}{N}$ according to Eq. (1).

\(^2\) It is worth noting that Maier and Brockmann (2020) also propose a SIR-X model that incorporates a containment policy such that not all of the population will eventually become infected. In their model, they introduce an additional state $X_t$ that denotes the individuals who are protected by the containment policy. Then, Eqs. (3) and (4) below are modified into:

$$S_{t+1} = S_t - a_1 S_t I_t - \kappa S_t$$

and

$$I_{t+1} = I_t + a_1 S_t I_t - \kappa I_t$$

respectively. $\kappa_0$ and $\kappa_1$ are the rate of transmission from susceptible and infected to the protected state, respectively. That is,

$$X_t = \kappa_0 S_t + \kappa_1 I_t$$

In contrast to this model, we do not model the transitional dynamics between the susceptible and protected state. Instead, we assume that a fixed proportion of the population is protected by the containment policy throughout all the periods. We believe that this is a more appropriate assumption in our case since the containment policy is already implemented in China on the first day of our dataset.

\(^3\) We will remove this assumption in Section 4.5.
In the future would reduce the household where

3.2. Linkage between the SIR and DSGE models

To link the above SIR model with the DSGE model, we assume that the disease can affect the economy through both the demand and supply sides. Assume that in the economy, there are infinitely many identical households. The representative household has the following expected lifetime utility:

\[ V_t = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^s \left( H_{C,s} \frac{C^{1-\gamma_C}}{1-\sigma_C} - \mu_s L^{1+\varphi} \right) \]  

(6)

where \( \beta \) is a discount factor, \( \sigma_C \) is a risk-averse parameter, \( C_t \) and \( L_t \) are consumption and labor supply at time \( t \), respectively. \( \mu_s \) and \( \varphi \) are the scale of labor disutility and the inverse of Frisch elasticity, respectively. The key difference between our model and the literature lies in the introduction of the term \( H_{C,t} \), which is a health-related variable that affects consumption. For the demand side, we assume that the household’s health condition affects its desire to consume. An increase in \( H_{C,t} \) increases the desire to consume. To this end, assume that \( H_{C,t} \) is determined by the following equation:

\[ \frac{H_{C,t}}{H_{C}} - 1 = -\gamma_C a_0 I_t \]  

(7)

where \( H_C \) is the steady-state value of \( H_{C,t} \), \( a_0 I_t \) is the share of infected individuals at time \( t \), and \( \gamma_C > 0 \) denotes the percentage decrease in \( H_{C,t} \) for a 1 unit increase in the number of infected. The setting in Eq. (7) is to capture the fact that the fear of being infected or unemployed in the future would reduce the household’s desire to consume. To align ours with the standard DSGE model, we set \( H_C = 1 \), so that household utility Eq. (6) coincides with that of the standard model in steady state.

For the supply side, we assume that the TFP level \( A_t \) is time-varying and satisfy the equation:

\[ \frac{A_t}{A} - 1 = -\gamma_A a_0 I_t \]  

(8)

for some parameter \( \gamma_A > 0 \) and steady-state value \( A \). It is believed that the pandemic should affect relatively more on the demand side rather than the supply side. It is because with the same capital and labor inputs, the firms should be able to produce the same amount of output regardless of the disease. However, if one considers the fact that some intermediate inputs are imported from other countries, whose supply might be affected by the foreign containment policy or immigration restriction, the pandemic could have an impact on the good supply side. In our closed economy model, this can be translated to a reduction in TFP level as specified in Eq. (8).

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4 Note that if one assumes that individuals can die from the disease, Eq. (4) becomes:

\[ I_{t+1} = I_t + a_1 S_t I_t - a_2 I_t - \delta I_t \]

where \( \delta > 0 \) is the death rate. The terminal values would still be the same in this setting.

5 Please refer to Atkeson (2020) for a more detailed introduction to the SIR model.
We adopt the standard model settings for the rest of the new Keynesian DSGE model. The model details should be familiar to the economic community and are therefore only presented in Appendix A.

4. Numerical exercise

4.1. Calibration

In this section, we discuss the procedure for calibrating the parameter values. Assume that one period in the model represents a day. Since the standard model usually assumes yearly or quarterly model frequencies, our parameter values are correspondingly converted from those in the literature.

First, as in the literature, we set the capital share of production \(\sigma\) to 1/3 (e.g., Basu and Bundick (2017)) and the degree of risk aversion \(\sigma_C\) to 2 (e.g., Heutel (2012)). Since the economic components of our models are similar to most of the related literature, we choose the values of the remaining parameters based on Li and Liu (2017) who estimate a standard new Keynesian model using Chinese data. In particular, we set the inverse Frisch elasticity to 0.2004. Li and Liu (2017) estimate that the quarterly discount factor \(\beta\) equals 0.99. This value translates to a 1% \((1/(1 + 1\%) \approx 0.99)\) quarterly discount rate, which implies a 0.011% \(((1 + 1\%)^{1/(90)} - 1)\) daily discount rate, and thus, the daily discount factor is 0.999889\(/(1 + 0.011\%)\). The scale parameter of labor disutility \(\mu_L\) is calibrated such that the steady-state value of labor supply \(L\) equals 1/3. We find that \(\mu_L\) equals 0.4631.

On the firm side, as in Li and Liu (2017), the degree of substitution between intermediate goods \(\theta\) is set to 10. Li and Liu (2017) estimate that the quarterly capital depreciation rate equals 3.5%. This implies that daily depreciation rate \(\delta_L\) equals 0.0382\% \((1 + 3.5\%)^{1/(90)} - 1\). For the price stickiness parameter \(\nu\), the quarterly rate estimated by Li and Liu (2017) is too large (0.9454) to convert to a daily frequency. Kim (2010) also points out that increasing the frequency of a DSGE model would lead to an upward bias on the Calvo parameter \(\nu\). Hence, we follow Christiano, Trabandt, and Walentin (2010) and assume that the quarterly rate equals 0.75 which implies that 25% of the firms can adjust their prices every quarter. Hence, \(\nu\) is equal to 0.9975 \((1 - ((1 + 0.25)^{1/(90)} - 1))\). Finally, the TFP level \(A\) is normalized to 1. For the Taylor rule Eq. (39), we calibrate the parameters to match the process in Li and Liu (2017) in quarterly frequency. The detailed calibration procedure is presented in Appendix B. We estimate that the degree of interest rate smoothing \(\rho_1\) is 0.9417. The elasticity parameters \(\tau_x\) and \(\tau_y\) are set to 1.724 and 3.944, respectively.

For the parameters \(\{a_0, a_1, a_2, I_0\}\) in the SIR model, we choose them to match COVID-19 data from China. While coronavirus is still growing in many countries around the world, the number has already peaked in China, and so we believe that the estimated parameters will be stable even after more data are released in the future. The data are obtained from an online repository held by the Johns Hopkins University Center for Systems Science and Engineering (JHU CSSE). The raw dataset contains the cases of confirmed infections, deaths, and recovered individuals in the provinces every day starting on 22 January 2020. We obtained the data on 12 April 2020, so three series of 82 days are obtained. Recall that the variable \(I_0\) denotes the number of those who are infected but have not yet recovered or died. Hence, we measure the number of infected by using the number of confirmed cases less the number of recovered and dead.

It is worth noting that there are limitations to using the SIR model to fit the data. First, the SIR model assumes that all infected individuals can be discovered. However, many of the infected do not show any symptoms, especially in the first few days of infection. Furthermore, some of the recovered individuals lose their immunity and are reinfected again. Such a possibility is also not considered by the SIR model. To this end, we will consider extensions of the SIR model in Section 4.5 below.

In sum, we choose \(a_0, a_1, a_2, I_0\), and \(N\) to minimize the sum of squares of the difference between the infected data and the series generated by the system Eqs. (3)-(5). The total population \(N\) is set to 1,435,000,000, which is approximately the population size in China in 2019. We find that \(\{a_1, a_2\} = \{0.3047, 0.1049\}\), \(I_0 = 0.011487\), and \(a_0 = 1.318e^{-4}\). The small value of \(a_0\) indicates that the lockdown policy implemented is very effective in containing the disease. We estimate that the number of susceptible is only \(a_0N = 189, 117\). As of 12 April, 2020, the total number of infected was 83,134, which is approximately 44% of the estimated susceptible. From the estimates, the basic reproduction number \(R_0 \equiv a_1/a_2 = 2.9\), which is within the range discussed in the literature (e.g., Wang et al. (2020)). Wang et al. (2020) only focus on Wuhan, and estimate that \(R_0\) could be as high as 3.1 in the first phase (1 December 2019 to 23 January 2020) and reach 0.9 or 0.5 in the last phrase (15 February 2020 onward).

Fig. 1 compares the series of data and fitted values of the number of infected during the full period. There are two subperiods that the model does not fit very well, namely, the sudden upward jump in the data on day 23 (13 Feb) and the two tails. The former is because Chinese officials changed the definition of a confirmed case that day. The latter is not fitted well partly because the data series does not reach zero at the end date.

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6 The data can be directly downloaded from the webpage: https://github.com/CSSEGISandData/COVID-19. The dataset includes the 22 provinces (excluding Taiwan), 5 autonomous regions, 4 municipalities, and 2 special administrative regions in China.
According to the National Bureau of Statistics of China, the reductions of the Chinese GDP and consumption are 6.80% and 5.13% in 2020Q1, respectively, compared to the values in the same quarter last year. Furthermore, the registered unemployment in the urban area in the 2020Q2 is 10.05 million persons, which is 6.20% lower than that of the previous year. We use these three targets to calibrate $\gamma_C$, $\gamma_A$, and $\gamma_I$, where $\gamma_I$ is the scale of the investment adjustment cost. In particular, we find the values of $\gamma_C$, $\gamma_A$, and $\gamma_I$ such that on day 90 after the pandemic outbreak, output, consumption, and labor supply have decreased by 6.80%, 5.13%, and 6.20%, respectively, from their steady-state values. We find that $\{\gamma_C, \gamma_A, \gamma_I\} = \{1.9700, 0.5834, 0.0350\}$. The calibration procedure is plotted in Fig. 1.

Note that we use the unemployment data in 2020Q2 instead of 2020Q1 because the number of unemployed in 2020Q1 is even less than that in the previous year. This could be explained by models with, for example, search friction, firing cost, and firms’ cost of exiting the market. Since our model does not have these components, the equilibrium labor would respond instantly to the pandemic shock. To this end, we choose a quarter after to allow the pandemic effect on employment to be clearly revealed.

Note that the parameter $\gamma_I$ is also in Li and Liu (2017) and is estimated to be 0.022. We do not adopt this value for two reasons. First, the estimated value of $\gamma_I$ varies largely (from 0.022 to 8.280) in Li and Liu (2017) in the models with different specifications of the Taylor rules. As mentioned above, the parameter values of the Taylor rule are largely affected by the model frequency assumed. Since we assume a model with different frequencies as Li and Liu (2017), this value of $\gamma_I$ is not used here. Second, in our model, such a value of $\gamma_I$ is too small such that the investment would increase throughout the pandemic, which is not consistent with the data.

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We do not use the quarter-on-quarter percentage change because the Chinese GDP is usually lower in the first quarter even after deseasonalization. Here we want to measure the GDP reduction from its steady-state level, so we use the year-on-year percentage change.

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Fig. 1. The solid and dashed lines are, respectively, the data and fitted values of the number of infected individuals in China. 22nd January 2020 is set to be Day 0. The data values are divided by $N$. Fig. 2. Flowchart.

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(a) Standard SIR model

(b) SIR model with partial immunity

(c) SEIR model
explained in Appendix B.

The model is solved using Dynare software using the first-order perturbation method.\textsuperscript{10} Table 1 reports the choice of the parameters for the numerical exercise below.

4.2. Macroeconomic effects of infection shock

In this section, we explain the transmission mechanism for an increase in infected individuals in the economy. Fig. 3 reports the dynamic responses of the endogenous variables to the disease outbreak.\textsuperscript{11} The primary effect of the shock is an increase in the share of infected \( I_0 \), which a day later, gradually begins to be reflected by an increase in the share of recovered individuals. A larger share of infected individuals also implies fewer susceptibles.

One direct consequence of the increase in the infected share is the decline in output. As mentioned, we calibrate the parameters such that the accumulated reduction in output is as much as 6.8% on the first 90 days. Note that output is driven down through both the supply and demand sides. On the one hand, the output demand declines due to the reduction in marginal utility to consume. On the other hand, the output supply declines due to the reduction in the TFP level.

In particular, for the demand side, the increase in the share of infected would decrease the marginal utility of consumption. Thus, as shown, the consumption path moves in the opposite direction as the path of the share of infected. Household consumption converges rapidly back to the steady state once the number of infected falls back to zero. The reduction in consumption would reduce the output demand. For the supply side, the TFP level is decreased during the pandemic, which would directly lead to the decline in output through reducing the output supply.

Note that the effects from the supply and demand sides on labor supply are opposite. For the supply side, as explained in Galí (2015) and Galí and Rabanal (2004), the negative TFP shock is associated with more labor supply. It is because, first, the shock would reduce the supply of output, which should increase the product price. However, the nominal rigidity restricts the price readjustment for some of the firms. Hence, there would be an excess demand for intermediate goods. In this regard, the firms would increase their demand for labor to scale up their production. This would result in positive responses in labor and wage rate in the short run. For the demand side, households who demand less consumption are less reliant on labor income and so have less incentive to work. With a higher wage rate in the short run, the forward-looking households would choose to reduce their labor supply in the long run. These two opposite forces explain why the labor supply increases in the first 28 days and decreases in the long run.

Similar arguments can be applied in explaining the responses of investment and capital. In the short run, households that spend less on consumption would shift their spending to investment. As a result, investment \( I_{K,0} \) capital \( K_0 \) and the capital price \( q_i \) increase. Moreover, in the short run, the wage rate \( w_t \) and the rate of capital return \( r_{K,i} \) increase, due to the increase in the demand for labor and capital induced by the reduction in TFP level.

In the long run, the decline in labor supply decreases the marginal product of capital and, therefore, the rate of capital return \( r_{K,i} \), which would substantially discourage households’ investment in capital. As a result, investment \( I_{K,i} \) decreases. Moreover, firms reduce their production scale by cutting production \( Y_t \) and hiring less labor, which drives down the wage rate \( w_t \). With a lower TFP level, the profits earned by the firms and, thereby, the household income decrease. The lower demand for capital by households also contributes to the reduction in capital stock \( K_t \) in the long run. Finally, according to the Taylor rule Eq. (39), the negative output growth also results in a fall in the nominal interest rate.

As shown, output and labor are slowly recovering after day 100. However, capital continues to decrease after day 100, owing to the very low depreciation rate specified in the daily frequency. Interestingly, many variables continue to deviate from their steady-state values even after the share of infected has already dropped to zero. In other words, in line with existing DSGE models estimated at a quarterly frequency, several quarters must pass for the economy to absorb the temporary decrease in consumption.

4.3. The effect of containment policy

Note that in the above, we calibrate the model using the data obtained when the lockdown policy is already implemented. What are the economic consequences if the government implements stricter or looser lockdown policies during the time? In China, the lockdown policy effectively forces people to stay at home. People cannot interact with each other and hence the possibility of being infected is substantially reduced. In this sense, we characterize a looser (stricter) lockdown policy by an increase (decrease) in the value of \( a_0 \) in Eq. (1) decreases. In this sense, we characterize a looser (stricter) lockdown policy by an increase (decrease) in the value of \( a_0 \). In particular, when a different containment policy is implemented in day \( t_0 \), the time-varying \( a_{0,t} \) should satisfy:

\[
a_{0,t} = \begin{cases} 
\tilde{a}_0 & \text{for } t \geq t_0 \\
 a_0 & \text{for } t < t_0
\end{cases}
\]

where \( \tilde{a}_0 \) is the share of the population in the three states under the new containment policy. \( a_0 = 1.318 \text{e}^{-4} \) is the original share calibrated above. The new policy is stricter (looser) than the original one if \( \tilde{a}_0 < (>) a_0 \). When the new policy is implemented at time

\textsuperscript{10} Note that Eqs. (3)-(5) are not proceeded by the first-order perturbation, and so their nonlinearity will still be preserved.

\textsuperscript{11} The dynamic responses where only the demand- and supply-side effects exist are presented in the online Appendix.
Table 1
Parameter values.

| Parameters | Value | Description | Sources/ Targets |
|------------|-------|-------------|------------------|
| $a_0$      | $1.318e^{-4}$ | Share of the initial susceptible | COVID-19 data |
| $a_1$      | 0.3047 | Transmission rate | COVID-19 data |
| $a_2$      | 0.1049 | Recovery rate | COVID-19 data |
| $N$        | 189117 | Affected population | COVID-19 data |
| $I_0$      | 0.011487 | Initial infected | COVID-19 data |
| $\sigma$  | 1/3 | Share of capital in production | Basu and Bundick (2017) |
| $\delta_k$ | 0.000382 | Capital depreciation rate | Li and Liu (2017) |
| $\beta$   | 0.99988 | Discount factor | Li and Liu (2017) |
| $\sigma_C$ | 2 | Risk aversion | Heutel (2012) |
| $\varphi$ | 0.2004 | Inverse of Frisch elasticity | Li and Liu (2017) |
| $\mu_C$   | 0.4631 | Parameter of labor disutility | Christiano et al. (2010) |
| $\nu$     | 0.9975 | Parameter of Calvo pricing adjustment | Li and Liu (2017) |
| $\theta$  | 10 | Elasticity of substitution | Li and Liu (2017) |
| $\lambda$ | 1 | Steady state TFP level | normalization |
| $\tau_t$  | 0.035 | Investment adjustment cost scale | The growth rate of output, consumption, and labor |
| $\gamma_C$ | 1.97 | Demand-side effect | Match the quarterly-frequency |
| $\gamma_s$ | 0.5834 | Supply-side effect | Taylor rule in Li and Liu (2017) |
| $\tau_L$  | 1.724 | Taylor rule parameter | Taylor rule in Li and Liu (2017) |
| $\tau_Y$  | 3.944 | Taylor rule parameter | Taylor rule in Li and Liu (2017) |
| $\rho_1$  | 0.9417 | Taylor rule parameter | Taylor rule in Li and Liu (2017) |

$t = t_0$, the share of susceptible would jump up or down. In particular, we have:

$$S_{new}^{S} - S_{old}^{S} = \frac{a_{0,t} - a_{0}}{a_{0}}$$

(10)

where $S_{new}^{S}$ and $S_{old}^{S}$ are the shares of susceptible with and without the policy change, respectively.\(^{12}\)

Although implementing the containment policy could reduce the share of susceptible, it comes with an economic cost. In our model, we assume that households’ disutility from working increases once a stricter lockdown policy is implemented. Specifically, the households’ utility Eq. (6) is replaced by:

$$E_t \sum_{i=1}^{\infty} \beta^i \left( H_{L,t} C_{i-1}^{1-\alpha_c} - H_{L,t} \mu L_{1+\gamma}^{1+\gamma} \right)$$

(11)

where the additional term $H_{L,t}$ is referred to as the health-related variable of labor supply. Assume that $H_{L,t}$ satisfies the following equation:

$$H_{L,t} = 1 - \gamma_L a_{0,t} - a_{0}$$

(12)

where $\gamma_L > 0$ is a sensitivity parameter that measures the response of $H_{L,t}$ to the change of $a_{0,t}$. The equation states that $H_{L,t}$ is negatively associated with the value of $a_{0,t}$. Note that when $a_{0,t} = a_{0}$, $H_{L} = 1$ and household utility Eq. (11) reduces to the original function Eq. (6).

Once a stricter (looser) policy is implemented, we have $a_{0,t} \leq (>)a_{0}$, and so $H_{L} \geq (<)1$. Using this rule, we examine how the macroeconomic variables respond to different containment policies. Although the choices of $a_{0}$ and $\gamma_L$ are subjective, their values do not affect the qualitative findings reached in this section. Recall that $a_{0,0}$ denotes the share of the population that is not protected by the containment policy in period $t$. Hence, $\gamma_L$ measures the percentage change in the labor disutility parameter $H_{L,t}$ when there is one percent more population protected by the containment policy. Below, we set $\gamma_L = 100$. We also consider different values of $\gamma_L$ in the online Appendix. We also consider two other containment policies where $a_{0} = 2a_{0}$ and $0.5a_{0}$.

Fig. 4 displays the time paths of the endogenous variables under the standard containment policy (solid lines), and the containment

---

\(^{12}\) Note that when $t \geq t_0$, Eq. (1) becomes:

$$S_{t} + \bar{I}_{t} + \bar{R}_{t} = \bar{a}_{0} N$$

when $t = t_0$, $a_{0,t}$ changes from $a_{0}$ to $\bar{a}_{0}$, but the values of $\bar{I}_{t}$ and $\bar{R}_{t}$ remain constant. Hence, compare with the model without the policy change, we have:

$$S_{new}^{S} - S_{old}^{S} = N(\bar{a}_{0} - a_{0})$$

where $S_{new}^{S}$ and $S_{old}^{S}$ are the variables with and without the policy change, respectively. Dividing both sides of the above equation by $a_{0} N$ yields Eq. (10).
policy with $\tilde{a}_0 = 1.5a_0$ (dashed lines) and $\tilde{a}_0 = 0.5a_0$ (dotted lines). The new policies are implemented on day $t_0 = 30$. Note that the new policies would directly lead to a jump of the susceptible, which would transform into different time paths of the infected and recovered shares. The stricter containment policy could effectively flatten the curve representing the share of infected. Even if the new policy is implemented around the peak of the curve, the share of infected drops significantly faster than in the benchmark. In contrast, with a looser policy, the infected share would further increase from its original peak. As can be told from the terminal state of the recovered share, more (less) people have eventually been infected under a looser (stricter) policy.

Concerning the economic impacts of the policy, it is shown that a stricter containment policy is effective in preventing substantial consumption and output declines. In contrast, the looser containment policy exacerbates the decreases in output and consumption. In particular, on day 100, output decreases from 17% to over 25% under the new policy when $\tilde{a}_0 = 1.5a_0$. This result is intuitive because we link the SIR and DSGE blocks by the infected share $I_t$ according to Eqs. (7) and (8). A higher (lower) infected share under the new policy would negatively affect output and consumption to a greater (lesser) extent.\footnote{The result that consumption decreases to a lesser extent under a stricter lockdown might not be realistic in reality. It is because we assume that the containment policy has no impact on the marginal utility of consumption, and one can easily modify this assumption by including the value of $a_{0,t}$ in the variable $H_{C,t}$ specified in Eq. (7). This is not assumed here to single out the labor disutility effect.}

Fig. 3. Dynamic responses of the endogenous variables to the changes in the SIR variables. The responses are in percent.
With a greater decrease in consumption, households would switch their spending to investment. Hence, a looser containment policy is associated with more increases in investment and capital in the short run. In addition, by Eq. (12), the labor disutility also increases under a stricter containment policy. The setting explains why the labor supply would decrease at a faster rate in the short run (from day 30 to 50) right after the new policy is implemented. In contrast, labor supply increases in this period under the looser policy. However, from day 50 to day 200, since the stricter policy is effective in curbing the disease outspread, labor supply reduces to a lesser extent.

Moreover, one major impact of the policy is on the wage rate $w_t$, which increases substantially immediately after the stricter policy is implemented. This is because the policy induces a higher disutility of labor supply, which would reduce the labor supply in the market. Finally, the smaller output decline in the short run under the stricter policy also induces a minor drop in the nominal interest rate. In addition to the impulse response functions, in the online Appendix, we also examine the effects of the containment policy on the deterministic steady state.

4.4. Ramsey problem

To understand the optimal monetary policy and the movement of macroeconomic variables in the first-best scenario, we consider a
Ramsey problem in this section. The Ramsey planner problem assumes that there is a benevolent government in the economy that is subject to the first-order conditions of the decentralized equilibrium and can choose all the endogenous variables to maximize the expected lifetime utility of the representative household. The detailed formation of the problem and the FOCs are reported in the online Appendix.

Fig. 5 compares the dynamic responses of the endogenous variables in the centralized (dotted lines) and decentralized (solid lines) equilibria. First of all, the shapes of the consumption responses under the two equilibria are similar. Both the decentralized and centralized consumption decrease by 30% from its steady state when the infected share reached the peak. However, the output responses are very different: compared to the decentralized output, the centralized output decreases more in the short run but recovers more rapidly to the steady state.

In the short run, the centralized investment $I^c_t$ and wage rate $w_t$ decrease, in contrast to the increases in the decentralized equilibrium. As explained above, the short-run increases of the labor and wage rate in the decentralized equilibrium are due to the presence of price stickiness. Although the households that are less desirable to consumption want to reduce their labor supply, the higher wage rate in the short run would postpone the households’ labor supply reduction to the long run, which would result in a slower recovery of the output. In contrast, the social planner is flexible enough to sacrifice the short-run investment and labor and allow them to return to the steady-state values earlier.

The higher centralized labor supply and greater capital accumulation in the long run speed up the process of economic recovery. As a result, although the centralized output experiences a decline in the short run, it rapidly converges back to the steady state. In particular, the centralized output already returns to its steady-state level on day 100, while the decentralized output remains 15% below its steady-state level on the same day.

In addition, we find that the nominal interest rates in the two equilibria move in different directions. The difference in interest rate responses in the two equilibria is due to the presence of the Taylor rule. Note that the nominal interest rate in the decentralized equilibrium is determined by the Taylor rule. In the decentralized equilibrium, output decreases, and the Taylor rule Eq. (39) indicates a large fall in the nominal interest rate. In contrast, the social planner is not constrained by the Taylor rule. It allows the interest rate to increase to let the firms increase their product prices and decrease their output so as to mitigate the impacts of the negative TFP shock. Note that when the firms can scale down their production, the demand for capital and labor decreases. This explains why there are declines in the centralized labor and capital in the short run. In sum, the social planner chooses to allow output to decrease substantially during the pandemic, in exchange for a faster process of economic recovery. Although the output would decrease to a greater extent in the decentralized equilibrium, in the online Appendix, we show that one can use a fiscal policy to reduce this output reduction.

4.5. Alternative SIR models

The SIR model considered in the previous section might have deficiencies in modeling the spread of COVID-19 in two respects. First, it assumes that recovered individuals are immune to the disease. However, as mentioned by Berger et al. (2020), there is thus far no evidence that immunity necessarily develops among the recovered. How would the economic impact of the pandemic be different if the recovered could be re-infected? Second, the SIR model assumes that all the infected individuals present symptoms and can be discovered, which is not the case for COVID-19. We follow Berger et al. (2020) and Piguillem and Shi (2020) and consider the asymptomatic individuals in our model and examine the effectiveness of random testing of uninfected individuals in conjunction with containment policy.

4.5.1. SIR model with partial immunity

We modify the SIR model Eqs. (3) to (5) as follows:

\[
S_{t+1} = S_t - a_1 S_t I_t + a_2 R_t \tag{13}
\]

\[
I_{t+1} = I_t + a_1 S_t I_t - a_2 I_t \tag{14}
\]

\[
R_{t+1} = R_t + a_3 I_t - a_3 R_t \tag{15}
\]

The major difference between the two systems of equations lies in the term $a_3 R_t$ in Eqs. (13) and (15) that denote the number of individuals who have recovered but lose their immunity at time $t$. There has been news showing that up to 3 – 10% of recovered patients in Wuhan (a city in Hubei province) test positive again.\(^{14}\) As pointed out by Amesh Adalja, an infectious disease expert at the Johns Hopkins Center for Health Security, the coronavirus could eventually turn endemic, akin to epidemic typhus, and circulate seasonally and never disappear, which would be in contrast to the predictions of the standard SIR model.\(^{15}\)

Fig. 2 displays the flowcharts between the standard SIR model Eqs. (3)–(5) (panel a) and the modified SIR model Eqs. (13)–(15) (panel b). As shown, in the standard SIR model, the susceptibles keep transitioning to become the infected who then continue the transition to recovered. In the modified SIR model, the introduction of a community that loses immunity (the red arrow) allows inflow and outflow in each of the three states.

\(^{14}\) See, for example, [https://www.scmp.com/news/china/society/article/3076989/coronavirus-10pc-recovered-patients-test-positive-later-say](https://www.scmp.com/news/china/society/article/3076989/coronavirus-10pc-recovered-patients-test-positive-later-say).

\(^{15}\) Please refer to [https://www.sciencealert.com/the-new-coronavirus-could-circulate-forever-says-experts](https://www.sciencealert.com/the-new-coronavirus-could-circulate-forever-says-experts).
Fig. 6 plots the dynamic responses of the endogenous variables with \(a_3 = 0\) (solid lines), \(a_3 = 0.03\) (dashed lines), and \(a_3 = 0.1\) (dotted lines). As shown, the primary difference among the three lines is on the SIR variables: an increase in the rate of immunity loss \(a_3\) would reduce the number of recovered people over time. This would in turn lead to fewer susceptibles and more infected individuals. More important, the number of infected would not fall back to zero over time. From the above system, one can show that the number of infected would converge to a terminal value \(a_2/(a_1 + a_2/a_3)\). It is expected that a high and persistent number of infected would increase the damage to the economy. In particular, output, consumption, and labor face a greater reduction during the disease outbreak. Moreover, in the standard SIR model under which output, labor, and investment recover on approximately day 100, all of them continue decreasing in the cases where \(a_3 > 0\). This numerical exercise reveals that the possibility of reinfection, even at a rate as low as 3%, could entail considerable costs for the economy in terms of consumption, output, and labor. It is worth noting that in this model, output would keep decreasing over time until a vaccine is developed.

4.5.2. SEIR model

The susceptible-exposed-infected-removed (SEIR) model is an extension of the SIR model that incorporates the exposed population. An individual is exposed if he/she is infected but does not yet show any symptoms. Denote by \(E_t\) the number of exposed individuals in period \(t\); the SEIR model is characterized by the following equations:
Furthermore, the accounting Eq. (2) is replaced by the following:

\[ S_{t+1} = S_t - a_1 S_t E_t \quad (16) \]
\[ E_{t+1} = E_t + a_1 S_t E_t - a_4 E_t - \kappa E_t \quad (17) \]
\[ I_{t+1} = I_t + a_1 E_t + \kappa E_t - a_2 I_t \quad (18) \]
\[ R_{t+1} = R_t + a_2 I_t \quad (19) \]

Furthermore, the accounting Eq. (2) is replaced by the following:

\[ S_t + E_t + I_t + R_t = 1 \quad (20) \]

As shown above, the main difference between the SEIR and SIR models lies in the additional Eq. (17) which characterizes the evolution of the exposed population. In period \( t \), a \( a_1 S_t E_t \) share of the population that is originally susceptible becomes exposed. Furthermore, a \( a_4 E_t \) share of the population that is originally asymptomatic becomes infectious. For simplicity, the values of \( a_1 \) and \( a_2 \) are set to be the same as above. As mentioned by Berger et al. (2020), the latent phase of COVID-19 lasts for approximately 7 days, so

Fig. 6. Dynamic responses of the endogenous variables to the changes in the SIR variables. The solid, dashed, and dotted lines represent the paths when \( a_3 \) equals 0, 0.03, 0.1, respectively. The responses are in percent.
we set the value of $a_4$ to 1/7. The flow dynamics among different groups are shown in panel c of Fig. 2 above.

Indeed, simply extending the SIR model to the SEIR model would have no impact on the results discussed above. This is because, in contrast to the exposed population $E_t$, the infected population $I_t$ that is observable in both models. If both models are fitted to the data, the dynamics of $I_t$ would be the same in the two models, and hence, the economic impact, induced through Eq. (7), would also be the same. In this regard, instead of comparing the predictions of the SIR and SEIR models, we make use of the SEIR model to examine the effectiveness through virus testing. It is worth noting that exposed individuals can be discovered through virus testing. For simplicity, we rule out the possibility of false positives and false negatives, and assume that the test results are 100% accurate.

Suppose that policymaker randomly tests a share $\kappa$ of the susceptibles and exposed population every period. From the testing, a $\kappa E_t$ share of the total population tests positive in period $t$. In other words, $\kappa$ represents the testing intensity set by the policymaker. As shown in Eqs. (17) and (18), the group of individuals who test positive would transition from the exposed to the infected population. Our focus in this section is on examining the effect of random testing by varying the value of $\kappa$.

In both of our main references, Berger et al. (2020) and Piguillem and Shi (2020), the economic impacts of the testing policy are discussed. The testing cost is not taken into account by Berger et al. (2020), so we follow Piguillem and Shi (2020) to model it as a term subtracted from the production function:

$$Y_t = A_{L_t}^{1-\kappa}K_t^\kappa - \Psi(\kappa(E_t + S_t))$$  \hspace{1cm} (21)

where $\Psi(.)$ is a testing cost function that depends on the share of individuals tested. As mentioned, $\kappa$ is the parameter of testing intensity. Both susceptibles and exposed individuals were tested. If the susceptible is tested, a negative result is shown and he/she remains susceptible. If the exposed is tested, a positive result is shown and he/she becomes infected. In Eq. (21), $Y_t$ is interpreted as the economy’s output less the testing cost.\footnote{Another approach to model the testing cost is to assume it is a part of the government expenditure. In particular, we can introduce a lump-sum tax to the household’s budget constraint Eq. (23), and assume that the tax is spent as the testing cost. In this case, the GDP accounting Eq. (45) would become:}

$$Y_t = C_t + I_{k_t} + \Omega \left( \frac{I_{k_{t-1}}}{I_{k_{t-1}}} \right) I_{k_t} + \Psi(\kappa(E_t + S_t))$$

where $c_0$ is the testing cost per person. To calibrate the test cost parameter $c_0$, we find that a COVID nucleic acid test fee in China is approximately 100 RMB.\footnote{See for example: http://ybj.beijing.gov.cn/zwgk/2020_zcwj/202101/t20210127_2234493.html} When local governments conduct massive testing campaigns, however, they usually test pools of 5–10 different donor samples, thereby drastically reducing costs per test. For example, Beijing municipal medical insurance Bureau announced that individuals should not be charged more than 30 RMB if the test is performed on pools of 5 donor samples.\footnote{The announcement is available at the official webpage: http://ybj.beijing.gov.cn/zwgk/2020_zcwj/202101/t20210127_2234493.html} Since the mixed-testing strategy is not always implemented, we estimate the test cost per person to be approximately (100 + 30)/2 = 65 RMB. The GDP per capita for China in 2020 is 10500 USD, which is approximately 200 RMB per day, using the exchange rate of 7 RMB/USD in 2020. Hence, we set the test cost $c_0 = 65/200 = 0.325$.

Panel a of Fig. 7 reveals the dynamic responses of the share of infected and output under the no-policy (solid lines), pure containment policy (dashed lines), containment and random testing (dotted lines), and pure testing (dashed-dotted lines) scenarios. As shown, randomly testing the asymptomatic individuals is very effective in reducing the share of infected. In particular, testing 10% of asymptomatic individuals can reduce the peak value of the infected population to approximately 30% of its original value. Although in the long run after day 100, the infected paths with $\kappa > 0$ are slightly above their untreated counterpart, the benefits of testing in terms of curbing the share of infected are evident. Economically, a daily test of 10% of the population can result in substantial output reductions and high costs.

### 4.5.3. Random testing and containment policy

How effective would random testing be compared to containment policy? We first assume that the containment rule Eq. (9) with $\bar{a}_0 = 0.5a_0$ is implemented. Then, complementing the containment rule Eq. (9), we assume that the policymaker simultaneously provides random testing to 5% of the asymptomatic individuals every period. Assume that both of the policies are implemented on day 30.

Panel b of Fig. 7 displays the dynamic responses of the share of infected and output under the no-policy (solid lines), pure containment policy (dashed lines), containment and random testing (dotted lines), and pure testing (dashed-dotted lines) scenarios. As above, the responses of other endogenous variables can be found in Fig. 8 in the Appendix. Concerning the policy impact of the number of infected, we find that (i) all three policies are effective in speeding up the converging rate of the infected to zero. (ii) Random testing increases the share of infected in the short run since many more asymptomatic individuals are discovered in the first few days of testing. (iii) Among the three policies, the combination of containment and random testing is the most effective in terms of curbing the share of
Concerning the economic impact, we find that, in the short run, output decreases to a lesser extent under a stricter containment policy, owing to its lower infected share. However, the economy with a containment policy implemented sees its output decrease over time, whereas the economy without it is already recovering. In contrast, although random testing does not prevent output reductions in the short run, the resulting output is consistently higher than that of the no-policy benchmark in the long run.

In sum, in line with Berger et al. (2020) and Piguillem and Shi (2020), we show that random testing is an effective policy to complement containment policy in curbing the infected number. Compared with the containment policy, the disadvantage of the random testing is that it leads to a short-term increase in the infected share right after its implementation, and so in the short run, output would still be greatly reduced with this policy. The advantage of it is that it reduces the extent of the output reduction in the long run.

5. Conclusion

This paper proposes a SIR-DSGE that incorporates the classical SIR model into a canonical new Keynesian DSGE model. We use COVID-19 data from China to calibrate the pandemic component of the model. Using the model, first, we examine how the increase in the share of infected individuals could result in an economic slowdown. Second, we study the effectiveness of various policies in curbing the disease and mitigating economic loss.
Assume that the disease would negatively affect the economy by simultaneously reducing the household marginal utility of consumption and the TFP level. We show that an increase in the share of infected could result in a substantial decline in output, consumption, and labor during the pandemic. Moreover, in the long run, right after the pandemic ends, output, labor, and investment continue decreasing, and the economy starts recovering only on day 100 after the pandemic began.

We find that a stricter containment policy, that reduces the share of susceptible, is effective in “flattening the curve” from its peak in a shorter time. The advantage of this policy is that during the outbreak, the output and consumption loss could be reduced. Apart from the containment policy, we find that if there is an incubation period of the disease, randomly testing susceptibles and asymptomatic individuals could effectively decrease the number of infected. In particular, randomly testing 10% of the uninfected individuals each period could reduce the peaked value of the infected share to one-third of its original value, even without any containment policy. Indeed, a combination of containment and testing outperforms the single-policy options, in terms of reducing the share of infected. Moreover, while the standard SIR model assumes that recovered individuals are immune to the disease, we also show that the possibility of losing immunity could result in a substantial loss of output and consumption in the long run.

In the centralized equilibrium, we show that the centralized output decreases more substantially, but converges back to the steady-state level at a much faster rate. In the decentralized equilibrium, the households increase their labor supply in the short run and decrease it in the long run, which is not socially optimal as indicated by the social planner problem. Instead, the social planner allows
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The model presented in this paper is undoubtedly a toy model with many simplified settings. Future work should merge pandemic and economic models in a more realistic setting, such as an open economy (Gali & Monacelli, 2005), global supply chain (Wei & Xie, 2020), and search-and-matching (Mortensen & Pissarides, 1994) elements to more completely capture the impact of the outbreak. The numerical exercises presented here are brief and elementary. A more detailed analysis concerning monetary and fiscal policies should be performed in similar frameworks.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
Appendix A. Model detail

In this section, we present the details of the rest of the model setting.

A.1. Household

The budget constraint faced by the household is:

\[ P_i C_i + P_i I_{K_i} + Q^0_t B_t = W_i I_t + R_{K,i} K_t + B_{i-1} + P_t D_t \]  (23)

where \( B_t \) is the amount of one-period-riskless government bond held by the household. \( Q^0_t \) is the price of the government bond. Denote as \( i_t \) the gross nominal interest rate at time \( t \). Note that the bond price is reciprocal of the gross interest rate. That is, \( Q^0_t \equiv 1/i_t \). Again, \( C_t \), \( I_{K,i} \), and \( K_t \) are consumption, investment, and capital at time \( t \), respectively. The representative household is the owner of all the firms in the economy. Let \( D_t \) be the dividend earned by the household at time \( t \). In addition, let \( W_t \) and \( R_{K,i} \), be the nominal wage rate and the rate of capital return, respectively. Finally, \( P_t \) is the aggregate price level.

The law of motion of capital is:

\[ K_{t+1} = (1 - \delta_t) K_t + \left[ 1 - \Omega \left( \frac{I_{K,t}}{I_{K,t-1}} \right) \right] I_{K,t} \]  (24)

where \( \delta_t \) is a capital depreciation rate. \( \Omega(I_{K,t}/I_{K,t-1}) \equiv \gamma_t(I_{K,t}/I_{K,t-1} - 1)^2/2 \) is the investment adjustment cost with the scale parameter \( \gamma_t > 0 \). The functional form of the adjustment cost follows Christoffel, Coenen, and Warne (2008).

The representative household maximizes the lifetime utility Eq. (6), subject to the constraints Eqs. (23) and (24). The first-order conditions (FOCs) for \( C_t \), \( B_t \), \( L_t \), \( K_t \), and \( I_{K,i} \) are

\[ H_i C_{t-\infty} = \lambda_t \]  (25)

\[ R_t = \theta Q_t = \beta \frac{\lambda_{t+1}}{\Pi_{t+1} \lambda_t} \]  (26)

\[ w_t \lambda_t = H_{I_t} \lambda_t I_t \]  (27)

\[ q_t = \frac{\Pi_{t+1} \gamma_{t+1} q_{t+1}}{\lambda_t} + (1 - \delta_t) \frac{\beta_{t+1}}{\lambda_t} q_{t+1} \]  (28)

\[ 1 = q_t \left[ 1 - \Omega \left( \frac{I_{K,t}}{I_{K,t-1}} \right) - \Omega \left( \frac{I_{K,t}}{I_{K,t-1}} \right) - \Omega \left( \frac{I_{K,t+1}}{I_{K,t}} \right) \frac{I_{K,t+1}}{I_{K,t}} \right] + \Pi_{t+1} \Omega_{t+1} \Omega \left( \frac{I_{K,t+1}}{I_{K,t+1}} \right) \frac{I_{K,t+1}}{I_{K,t}} \]  (29)

, respectively. \( \Pi_t = P_t/P_{t-1} \) is the gross inflation rate. \( \lambda_t \) is a Lagrange multiplier. \( r_{K,t} \equiv R_{K,t}/P_t \) is the real rate of capital return. \( q_t \) is the real capital price. As shown in Eq. (25), \( \lambda_t \) is the marginal utility to consume. Moreover, Eq. (27) is the labor supply curve of the economy. The health-related variables \( H_{C,t} \) in Eq. (25) and \( H_{I,t} \) in Eq. (27) are determined by Eq. (7) and (12), respectively.

A.2. Firms

It is assumed that the final good market is perfectly competitive. In addition, the capital and labor markets are also perfectly competitive. The final good is composed by a continuum of intermediate good, indexed by \( i \in [0, 1] \), according to a constant elasticity of substitution (CES) function as:

\[ Y_t = \left( \int_0^1 Y_t(i) \beta^i di \right)^{1/\theta} \]  (30)

where \( \theta > 1 \) is the degree of substitutability of any two intermediate goods. Let \( P_t(i) \) be the price level of intermediate good \( i \) at time \( t \). From Eq. (30), one can derive the demand function for the intermediate good \( i \) as:

\[ Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} Y_t \]  (31)

which also implies that the final good price and the price of intermediate goods are related by the formula: \( P_t = \left( \int_0^1 P_t(i) \beta^i di \right)^{1/(1-\theta)} \).

The production function of intermediate good firm \( i \) is Cobb-Douglas as:

\[ Y_t(i) = A_i K_t(i) L_t(i)^{1-a} \]  (32)

where \( K_t(i) \) and \( L_t(i) \) are the capital and labor employed by firm \( i \), respectively. \( A \) is the total factor productivity (TFP) level of the
economy, which is shared by all the intermediate good firms and is assumed to be constant. We allow $A$ to be time-varying in Appendix ???. The firms’ maximization problem gives the following FOCs for $K_t$ and $L_t$:

$$
\frac{\alpha Y_t}{K_t} = r_X (33)
$$

$$
(1 - \alpha) \frac{Y_t}{K_t} = w_t (34)
$$

where $mc_i$ is a real marginal cost.

A.2.1. Calvo pricing

The nominal rigidity is imposed in the manner of Calvo (1983). Assume that every period, a $\nu \in [0, 1]$ portion of firms are not allowed to adjust their prices level. Put differently, there is only $1 - \nu$ of firms can re-adjust their prices. With this restriction, firm $i$ chooses a product price $P_t^i(i)$ to maximize the expected lifetime profit as:

$$
\Pi_i = \max_{P_t^i(i)} \sum_{t=1}^{\infty} \nu^t \mathcal{M}_{t,i} \left( \frac{P_t^i(i)}{\bar{P}_t} Y_t^i(i) - \text{mc}_t Y_t^i(i) \right) (35)
$$

subject to the demand function

$$
Y_t^i(i) = \left( \frac{P_t^i(i)}{\bar{P}_t} \right)^{-\theta} Y_t (36)
$$

where $\mathcal{M}_{t,i} \equiv \beta \lambda_i / \lambda_t$ is a stochastic discount factor. The FOC of $P_t^i$ satisfies:

$$
P_t^i = \frac{\theta}{\theta - 1} X_{1,t} (37)
$$

$$
P_t^i = \frac{\theta}{\theta - 1} \frac{X_{2,t}}{L_{2,t}} (38)
$$

where $X_{1,t} \equiv \sum_{t=1}^{\infty} X_{1,t}^i \text{mc}_t + \rho \bar{P}_t Y_t + X_{2,t}$ and $X_{2,t} \equiv \sum_{t=1}^{\infty} X_{2,t}^i \text{mc}_t + \rho \bar{P}_t Y_t$. One can express $X_{1,t}$ and $X_{2,t}$ recursively as:

$$
X_{1,t} = \text{mc}_t Y_t + \nu \mathcal{M}_{t+1,i} \sum_{t=1}^{\infty} \nu^t \mathcal{M}_{t+1,i} \bar{P}_t \rho Y_t (37)
$$

$$
X_{2,t} = Y_t + \nu \mathcal{M}_{t+1,i} \sum_{t=1}^{\infty} \nu^t \mathcal{M}_{t+1,i} \bar{P}_t \rho Y_t (38)
$$

A.3. Monetary policy

Following Li and Liu (2017), the central bank determines the nominal interest rate $i_t$ according to the Taylor rule:

$$
\frac{i_t}{\bar{i}} = \left( \frac{i_t}{\bar{i}} \right)^{\rho_1} \left[ 1 + \frac{\Pi_t}{\Pi_t} \delta_t \left( \frac{Y_t}{\bar{Y}_t} \right) \right]^{1 - \rho_1} (39)
$$

where $\Pi_t$ is the steady-state values of the gross inflation rate. $\tau_\rho$ and $\tau_\gamma$ denote the elasticity of nominal interest rate to the inflation rate and output, respectively. $\rho_1$ is a parameter that determines the interest rate persistence. For simplicity, the public sector is abstracted from the model.

A.4. Equilibrium

In the decentralized equilibrium, recall that the aggregate price level $P_t = \left( \int_0^t P_t(i)^{1-\theta} \text{di} \right)^{1/(1-\theta)}$. In period $t$, since $1 - \nu$ of firms choose a new price $P_t^i(i)$ and the remaining $\nu$ firms have price level $P_{t-1}$, we have $P_t = \nu P_{t-1}^{1-\theta} + (1 - \nu) P_t^{1-\theta} \left(1/(1-\theta) \right)$. Dividing both sides of the equation by $P_t$, we have

$$
1 = \nu P_t^{-\theta} + (1 - \nu) P_t^{1-\theta} \left(1/(1-\theta) \right) (40)
$$

Let $K_t \equiv \int_0^t K_t(i) \text{di}$ and $L_t \equiv \int_0^t L_t(i) \text{di}$ be the aggregate capital and labor supply, respectively. Integrating both sides of the


production function with respect to $i$ gives $I_n^i Y_i(i)di = \int_0^1 (P_i(i)/P_i)^{-\gamma}Y_i di = D_{\rho i}Y_i$ with $D_{\rho i} \equiv \int_0^1 (P_i(i)/P_i)^{-\gamma}di$ be the price dispersion which would evolve according to the equation:

$$D_{\rho i} = (1 - \nu)\rho_i^{1-\theta} + \nu\Pi^{\phi}D_{\rho i-1}$$  \hspace{1cm} (41)

Similarly, the aggregate labor and capital demand function can be obtained by integrating both sides of Eqs. (33) and (34) with respect to $i$:

$$(1 - \alpha)AK^{-1}_{t-1}L^{-\alpha_{mc}_{t-1}} = r_{K,t}$$  \hspace{1cm} (42)

$$(1 - \alpha)AK^{-\alpha}L^{-\alpha_{mc}_{t-1}} = w_{t}$$  \hspace{1cm} (43)

Similarly, the aggregate production function is:

$$Y_t = AK^{-\alpha}L^{-\alpha_{mc}_{t}}$$  \hspace{1cm} (44)

The final good market is closed by the GDP accounting equation:

$$Y_t = C_t + I_{K,t} + \Omega\left(\frac{J_{K,t}}{I_{K,t-1}}\right)I_{K,t}$$  \hspace{1cm} (45)

Appendix B. Calibration details

B.1. Calibrating the Taylor rule parameters

Taking logarithm on both sides of the Taylor rule Eq. (39), we have

$$\ln i_t = \rho_i \ln i_{t-1} + (1 - \rho_i)\ln i + (1 - \rho_i)\tau_{\pi} \ln \left(\frac{\Pi_t}{\Pi_{t-1}}\right) + (1 - \rho_i)\tau_{\pi} \ln \left(\frac{Y_t}{Y_{t-1}}\right)$$

Denote as $i_t \equiv \ln(\Pi_t), y_t \equiv \ln Y_t$. We have in $\Pi = 0$. The above process reduces to:

$$\ln i_t = (1 - \rho_i)\ln i + \rho_i \ln i_{t-1} + (1 - \rho_i)\ln i + (1 - \rho_i)\tau_{\pi} \ln i + (1 - \rho_i)\tau_{\pi} (y_t - y_{t-1})$$  \hspace{1cm} (46)

Li and Liu (2017) assume that the above Taylor rule is in quarterly frequency with the estimated parameters $\{\rho_*^o, \tau_{\pi}^o, \pi_t^o\} = \{0.3296, 2.9352, 11.262\}$, where we added the Q superscript to indicate that they are from the Taylor rule with quarterly frequency. Below, we will also add a superscript $D$ to denote the variable (or the parameter of the corresponding process) that is in daily frequency.

Note that we also assume the same functional form as Li and Liu (2017). The only difference is that we assume the data is in daily frequency. Hence, our objective is to find the parameters $\{\rho_*^o, \tau_{\pi}^o, \pi_t^o\}$ such that the daily data generated by the above process, when converted to the quarterly data, are equivalent to the data directly generated from the data generating process in Li and Liu (2017).

Firstly, the three variables in quarterly and daily frequencies are related by:

$$\ln i_{t-1} = \sum_{i=0}^{T-1} \ln i_{t-1-i}$$  \hspace{1cm} (47)

$$\ln(\Pi_t) = \sum_{i=0}^{T-1} \ln(\Pi_{t-1-i})$$  \hspace{1cm} (48)

$$Y_t = \left(Y_t^o + \frac{T}{\sum_{i=0}^{T-1} \Pi_{t-1-i}^o} \Pi_{t-1-i}^o\right)$$  \hspace{1cm} (49)

where $T = 89$. The last equation above is derived using the fact that the quarterly nominal GDP equals the sum of the daily nominal GDP. A similar equation (in logarithm form) is also used in Kim (2010). Note that the quarterly data $\{\ln i_{t}^o, \Pi_t^o, Y_t^o\}$ is only obtained in every 90 days (or at the end-day of each quarter). The following is the procedure of calibrating the parameters $\{\rho_*^o, \tau_{\pi}^o, \pi_t^o\}$:

Given any value of $\{\rho_*^o, \tau_{\pi}^o, \pi_t^o\}$:

1. Randomly generate 100 replications of the infected shock $\{i_{t}^o\}_{t=1}^{27000}$ from a normal distribution $N(0, \sigma_t)$, where $\sigma_t$ is the standard deviation of the actual infected shock from the calibrated SIR model in Section 4.

2. For each replication $\{i_{t}^o\}_{t=1}^{27000}$, perform the following:
   (a) Fit the shock to our SIR-DSGE model and obtain the series of data $\{i_{t}^o\}_{t=1}^{27000}$, and $\{\Pi_t^o\}_{t=1}^{27000}$.
   (b) Generate $\{y_t^o\}_{t=1}^{27000}$ according to the Taylor rule Eq. (39).
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(c) Convert \((\hat{\rho}_t^Q)_{t=1}^{27000}, (\hat{\sigma}_t^Q)_{t=1}^{27000}\), and \((\hat{\gamma}_t^Q)_{t=1}^{27000}\) into quarterly data \((\hat{\rho}_t^Q)_{t=1}^{300}, (\hat{\sigma}_t^Q)_{t=1}^{300}\), and \((\hat{\gamma}_t^Q)_{t=1}^{300}\) using Eqs. (47), (48), and (49).

(d) Run a linear regression of \((\hat{Y}_t^Q)_{t=1}^{27000}\) on \((\hat{\rho}_t^Q)_{t=1}^{27000}, (\hat{\sigma}_t^Q)_{t=1}^{27000}\), and \((\hat{\gamma}_t^Q)_{t=1}^{27000}\) in order to obtain the estimates \((\hat{\rho}_t^Q, \hat{\gamma}_t^Q, \hat{\sigma}_t^Q)\) according to the Taylor rule Eq. (46).

3. Find the average of the estimates \((\hat{\rho}_t^Q, \hat{\gamma}_t^Q, \hat{\sigma}_t^Q)\) in the 100 replications.

4. Compute the discrepancy error = \((\hat{\rho}_t^Q - 0.3296)^2 + (\hat{\gamma}_t^Q - 2.9352)^2 + (\hat{\sigma}_t^Q - 11.262)^2\), which is the difference between the estimated parameters and parameter choice in Li and Liu (2017).

5. Repeat Steps 1 to 4 with different parameter values \((\hat{\rho}_t^Q, \hat{\gamma}_t^Q, \hat{\sigma}_t^Q)\) until the discrepancy is minimized.

B.2. Calibrating \((\gamma_G, \gamma_A, \gamma_I)\)

As mentioned in the calibration section, we search for the parameters \(\gamma_G, \gamma_A\), and \(\gamma_I\) to match the percentage changes of output, consumption, and labor supply. The detailed procedure is presented in this section.

Note that our model is in daily frequency, but the calibration targets are obtained using the quarterly data. Hence, we convert the data simulated from the model to quarterly frequency using the equations:

\[
Y_t^Q = \left( Y_t^D + \sum_{i=1}^{T} \frac{Y_{i+1}^D - Y_i^D}{\prod_{j=0}^{i-1} Y_j^D} \right)
\]

\[
C_t^Q = \left( C_t^D + \sum_{i=1}^{T} \frac{C_{i+1}^D - C_i^D}{\prod_{j=0}^{i-1} C_j^D} \right)
\]

\[
L_t^Q = \sum_{i=1}^{T} L_{i+1}^D
\]

Similar to Section B.1, the variables with superscripts \(Q\) and \(D\) are in quarterly and daily frequency, respectively. From these equations, the steady-state values of the variables are simply: \(Y^Q = 90Y^D\), \(C^Q = 90C^D\), and \(L^Q = 90L^D\).

We search for \((\gamma_G, \gamma_A, \gamma_I)\) to match the following three conditions:

\[
\frac{Y_t^Q}{Y_t^D} - 1 = -0.068
\]

\[
\frac{C_t^Q}{C_t^D} (\frac{C_t^Q}{C_t^D} - 1) = -0.0513
\]

\[
\frac{L_t^Q}{L_t^D} - 1 = -0.062
\]

Targets Eqs. (53) and (55) state that the real quarterly GDP and labor supply decrease from their steady-state values by 6.8% and 6.2%, respectively.

Then, we explain how do we derive target Eq. (54). From the data obtained from the National Bureau of Statistics of China, the 6.8% reduction in output is contributed by three factors, namely, consumption, capital formation, and net export. The shares of them are -4.32%, -1.41%, and -1.07%, respectively. Since we do not model net export, the contribution by consumption is converted to -4.32% / (-4.32% - 1.41%) = 5.13%.

Log-linearizing the GDP accounting Eq. (45) yields:

\[
\hat{Y}_t = \frac{C_t}{Y} + \frac{I_t}{Y} K_t + \frac{\text{cost}}{Y} t_{\text{cost}},
\]

where \(t_{\text{cost}} = \Omega (K_{t-1}/I_{K-1})L_{K,t-1}^2\) is the investment adjustment cost. Hence, we search for the parameters so that the value of the first term (in quarterly frequency) on their h.s. decreases by 5.13%. This yields Eq. (54).

Note that parameters \((\gamma_G, \gamma_A, \gamma_I)\) and those in Taylor rule \((\hat{\rho}_t^D, \hat{\gamma}_t^D, \hat{\sigma}_t^D)\) are interrelated. Hence, these six parameters are searched such that simultaneously we have the targets Eqs. (53), (54), and (55) are matched and the discrepancy mentioned in Section B.1 is minimized.

Appendix C. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.chieco.2021.101725.
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