Exciting gauge unstable modes of the quark-gluon plasma by relativistic jets

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Abstract. We present a study of the properties of the collective modes of a system composed by a thermalized quark-gluon plasma traversed by a relativistic jet of partons. We find that when the jet traverses the system unstable gauge field modes are excited and grow on very short time scales. The aim is to provide a novel mechanism for the description of the jet quenching phenomenon, where the jet crossing the plasma loses energy exciting colored unstable modes. In order to simplify the analysis we employ a linear response approximation, valid for short time scales. We assume that the partons in the jet can be described with a tsunami-like distribution function, whereas we treat the quark-gluon plasma employing two different approaches. In the first approach we adopt a Vlasov approximation for the kinetic equations, in the second approach we solve a set of fluid equations. In both cases we derive the expressions of the dispersion law of the collective unstable modes and compare the results obtained.

1. Introduction
One of the methods for unveiling the properties of matter produced in ultrarelativistic heavy-ion collisions is to study the propagation properties of high $p_T$ partons generated by hard scatterings in the initial stage of the collision. When the jet of partons travels across the medium it loses energy and degrades, mainly by radiative processes (see [1] for reviews). The energy and momentum of the jet are absorbed by the plasma and result in an increased production of soft hadrons in the direction of propagation of the partons.

We propose a novel mechanism [2, 3] for describing how the jet loses energy and momentum while traveling in a thermally equilibrated quark-gluon plasma (QGP). Since the jet of particles is not in thermal equilibrium with the QGP it perturbs and destabilizes the system inducing the generation of gauge fields. Some of these gauge modes are unstable and grow exponentially fast in time absorbing the kinetic energy of the jet.

The study of the interaction of a relativistic stream of particles with a plasma is a topic of interest in different fields of physics, ranging from inertial confinement fusion, astrophysics and cosmology. When the particles of the stream are charged, plasma instabilities develop, leading to an initial stage of fast growth of the gauge fields. The study of chromo instabilities is a very active field of research (see [4] for a review).

In order to study the system composed by the jet and the plasma we have employed two different methods. In [2] both the plasma and the jet are described using a fluid approach. This approach developed in [5] has been derived from kinetic theory expanding the transport equations in moments of momenta and truncating the expansion at the second moment level.
The system of equations is then closed with an equation of state relating pressure and energy density. The fluid approach has several advantages with respect to the underlying kinetic theory. The most remarkable one is that one has to deal with a set of equations much simpler than those of kinetic theory. Then one can easily generalize the fluid equations to deal with more complicated systems. This is a strategy that has been successfully followed in the study of different dynamical aspects of non-relativistic electromagnetic plasmas [6].

In the second approach we consider the same setting, i.e. a static plasma traversed by a relativistic jet, but we use kinetic theory instead of the fluid approach. Transport theory provides a well controlled framework for studying the properties of the quark-gluon plasma in the weak coupling regime, $g \ll 1$. Indeed it is well known that the physics of long distance scales in an equilibrated weakly coupled QGP can be described within semiclassical transport equations [7, 8, 9]. In this approach the hard modes, with typical energy scales of order $T$, are treated as (quasi-)particles which propagate in the background of the soft modes, whose energies are equal or less than $gT$, which are treated as classical gauge fields. This program has been very successful for understanding some dynamical aspects of the soft gauge fields in an almost equilibrated QGP [10, 11].

We conclude comparing the results for the growing rates of the unstable modes obtained with kinetic theory in Ref. [3] with the analogous results we obtained with the fluid approach in Ref. [2].

2. Kinetic theory approach

We consider a system composed by a quark-gluon plasma traversed by a jet of partons. We assume that the system is initially in a colorless and thermally equilibrated state and we will study the behavior of small deviation from equilibrium. The distribution functions of quarks, antiquarks and gluons belonging to the quark-gluon plasma are, respectively

$$ Q(p, x) = f^\text{eq}_{FD}(p_0) + \delta Q(p, x), \quad Q(p, x) = f^\text{eq}_{FD}(p_0) + \delta \bar{Q}(p, x), \quad G(p, x) = f^\text{eq}_{BE}(p_0) + \delta G(p, x) $$

(1)

where the various quantities are hermitian matrices in color space (we have suppressed color indices) and where $f^\text{eq}_{FD/BE}(p_0) = \frac{1}{e^{p_0/T} + 1}$ are the (colorless) Fermi-Dirac and Bose-Einstein equilibrium distribution functions.

Also the particles of the jet are assumed to be initially in equilibrium and that small color fluctuations are present:

$$ W_{\text{jet}}(p, x) = f_{\text{jet}}(p) + \delta W_{\text{jet}}(p, x). $$

(2)

For the initial jet distribution function we will not consider a thermal distribution function. We will indeed approximate the distribution function of the jet with a colorless tsunami-like form [12]

$$ f_{\text{jet}}(p) = \bar{n} \bar{u}^0 \delta^{(3)}(p - \Lambda \bar{u}), $$

(3)

that describes a system of particles of constant density $\bar{n}$, all moving with the same velocity $\bar{u} = (\bar{u}^0, \bar{u}) = \gamma (1, \mathbf{v})$, where $\gamma$ is the Lorentz factor and $\Lambda$ fixes the scale of the energy of the particles.

Although this distribution function is adequate for describing a uniform and sufficiently dilute system of particles, it would be very interesting to extend our analysis to more complicated forms. Using more involved distribution functions on the one hand would lead to a more accurate description of the jets relevant for heavy ion phenomenology, when the density of particles composing the jet is not uniform and there is a spread in momentum. However, on the other hand this would complicate the study of the collective modes.

The distribution function of quarks satisfy the following transport equation

$$ p^\mu D_\mu Q(p, x) + \frac{g}{2} p^\mu \{ F_{\mu\nu}(x), \partial_\nu Q(p, x) \} = C, $$

(4)
whereas the distribution functions of antiquarks, gluons and of the particles of the jet satisfy similar equations. With \{...,\} we denote the anticommutator, \( \partial_\mu^\nu \) is the four-momentum derivative and \( g \) is the QCD coupling constant. The covariant derivatives \( D_\mu \) and \( \mathcal{D}_\mu \) act as

\[
D_\mu = \partial_\mu - ig[A_\mu(x), \ldots] , \quad \mathcal{D}_\mu = \partial_\mu - ig[A_\mu(x), \ldots] ,
\]

with \( A_\mu = A_\mu^a(x)\tau^a \) and \( A_\mu = A_\mu^a(x)T^a \), and \( \tau^a \) and \( T^a \) are \( SU(3) \) generators in the fundamental and adjoint representations, respectively. The strength tensor in the fundamental representation is \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \), while \( \mathcal{F}_{\mu\nu} \) denotes the field strength tensor in the adjoint representation.

In Eq.(4) \( C \) represents the collision term. For time scales shorter than the mean free path time the collision terms can be neglected, as typically done in the so-called Vlasov approximation. The knowledge of the distribution function allows one to compute the associated color current, which in a self-consistent treatment enters as a source term in the Yang-Mills equation. Therefore in this approximation the different components of the system formed by the plasma and the jet interact with each other only through the generated average gauge fields.

In the Vlasov approximation one can compute the contribution to the polarization tensor of the particles of species \( \alpha \) (where \( \alpha \) refers to quarks, antiquarks, gluons or the partons of the jet) as

\[
\Pi_{\mu\nu}^{ab,\alpha}(k) = -g^2C_P\delta_{ab}\int \frac{d^4p}{(2\pi)^3} \frac{(p \cdot k)(k^\mu p^\nu + k^\nu p^\mu) - k^2 p^\mu p^\nu - (p \cdot k)^2 g^{\mu\nu}}{(p^2)^2} , \tag{5}
\]

where \( a, b \) are color indices and \( C_P \) is the value of the quadratic Casimir associated with the particle specie \( \alpha \) which takes values 1/2 and 3 for the fundamental and adjoint representations, respectively. The momenta measure is defined as

\[
\int_p \cdots \equiv \int \frac{d^4p}{(2\pi)^3} 2\Theta(p_0)\delta(p^2 - m_\alpha^2) , \tag{6}
\]

where \( m_\alpha \) is the mass of the particle of specie \( \alpha \). For simplicity we assume that the particles belonging to the plasma are massless. Instead the particles of the jet have a non-vanishing mass.

When \( f_\alpha(p) \) is a thermal equilibrated distribution function, Eq. (5) reduces to the form of the hard thermal loop (HTL) polarization tensor [7, 8, 9]. However for the tsunami-like distribution function, Eq. (3), the polarization tensor obviously takes a different form.

The gauge fields obey the Yang-Mills equation

\[
D_\mu F^{\mu\nu}(x) = \delta j^\nu_\text{pl}(x) = \delta j^\nu_\text{pl}(x) + \delta j^\nu_\text{jet}(x) , \tag{7}
\]

where we have defined

\[
\delta j^\mu_\text{pl}(x) = -\frac{g}{2} \int_p p^\mu \left[ \delta Q(p, x) - \delta \bar{Q}(p, x) + 2\tau^a \text{Tr}[T^a \delta G(p, x)] \right] , \tag{8}
\]

which describes the plasma color current, and

\[
\delta j^\mu_\text{jet}(x) = -\frac{g}{2} \int_p p^\mu \delta W_\text{jet}(p, x) , \tag{9}
\]

which describes the fluctuations of the current associated with the jet.

Equation (7) together with Eq. (4) form a set of equations that has to be solved self-consistently. Indeed the gauge fields which are solutions of the Yang-Mills equation enter into the transport equations of every particle species and, in turn, affect the evolution of the distribution functions.
3. Liquid approach

Hydrodynamical equations are the expressions of the conservation laws of a system when it is in local equilibrium. In Ref. [13] the local equilibrium state for the quark-gluon plasma has been determined. It is in general described by one singlet four velocity, a baryon density, singlet energy and pressure, and in principle, a non-vanishing color density. However, dynamical processes associated to the existence of Ohmic currents tend to whiten the plasma quickly, on time scales much shorter than momentum equilibration processes [14]. For this reason one can expect that only colorless (singlet) fluctuations are relevant at large time and space scales. However, there are situations, as the one considered here, when color fluctuations grow on short time scales instead of being damped. Therefore in order to describe the short time evolution of the plasma, one needs to include color hydrodynamical fluctuations in the equations. In Ref. [5] such a chromohydrodynamical approach for the short time evolution of the system has been formulated. The fluid equations can be obtained expanding the collisionless transport Equation (4) (and the analogous equations for antiquarks and gluons) in moments of momenta and truncating the expansion at the second order level. For simplicity, as in Ref. [5], we will only consider the contribution of quarks in the fundamental representation. The inclusion of antiquarks and gluons is straightforward. The fluid approach consist of the covariant continuity equation for the fluid four-flow

$$D_\mu n^\mu = 0$$  \hspace{1cm} (10)

and of the equation that couples the energy-momentum tensor $T^{\mu\nu}$ to the gauge fields

$$D_\mu T^{\mu\nu} - \frac{g}{2} \{ F_{\mu\nu}, n^\mu \} = 0.$$ \hspace{1cm} (11)

We further assume that the four-flow and the energy-momentum tensor have the expression valid for an ideal fluid, i.e.

$$n^\mu(x) = n(x) u^\mu(x) \quad \text{and} \quad T^{\mu\nu}(x) = \frac{1}{2} \{ \epsilon(x) + p(x) \} \{ u^\mu(x), u^\nu(x) \} - p(x) g^{\mu\nu},$$ \hspace{1cm} (12)

where the hydrodynamic velocity $u^\mu$, the particle density $n$, the energy density $\epsilon$ and the pressure $p$ are $3 \times 3$ matrices in color space.

The color current due to the flow of the fluid can be expressed in terms of the hydrodynamic velocity and the particle density as

$$j^\mu(x) = -\frac{g}{2} \left( n u^\mu - \frac{1}{3} \text{Tr}[n u^\mu] \right).$$ \hspace{1cm} (13)

As in the kinetic theory approach, the color current acts as a source term for the gauge fields in the Yang Mills equation

$$D_\mu F^{\mu\nu}(x) = j^\nu(x).$$ \hspace{1cm} (14)

Thus, we will assume that all the gauge fields that appear in the fluid equations are only due to the presence of a colored current in the medium. The fluctuations of the current induced by the fluctuation of the density and of the hydrodynamic velocity are related in linear response theory to fluctuations of the gauge fields via

$$\delta j^\mu_a(k) = -\Pi^{\mu\nu}_{ab}(k) A_{\nu,b}(k),$$ \hspace{1cm} (15)

which defines the polarization tensor in the fluid approach.

Notice that Eqs. (10), (11) and (14) do not form a closed set and one more relation has to be provided. In hydrodynamical treatments, one usually imposes a relation between pressure and energy density. We will assume that pressure and energy density fluctuations are related by

$$\delta p^a(x) = (c_s^a)^2 \delta \epsilon^a(x)$$ where $c_s^a$ is a parameter.

Also notice that since Eqs. (10) and (11) were derived from the collisionless transport Eq. (4) obeyed by the particle distribution function, the conservation laws expressed by the Eqs. (10) and (11) are strictly valid on time scales shorter than the mean free path time.
4. Collective modes in the system composed by the QGP and jet

We now consider the collective modes of the system composed by an equilibrated QGP traversed by a jet of particles. We are interested in very short time scales when the Vlasov approximation can be employed. The effect of the beam of particles is to induce a color current, which provides a contribution to the polarization tensor. The polarization tensor of the whole system is additive in this short time regime, meaning that

\[ \Pi_{\mu\nu}^t(k) = \Pi_{\mu\nu}^p(k) + \Pi_{\mu\nu}^{\text{jet}}(k), \]  

(16)

where \( \Pi_{\mu\nu}^p(k) \) and \( \Pi_{\mu\nu}^{\text{jet}}(k) \) are the polarization tensor of the plasma and of the jet respectively.

The total dielectric tensor is given by

\[ \varepsilon^{ij}_t(\omega, k) = \delta^{ij} + \frac{\Pi^{ij}_t}{\omega^2}, \]  

(17)

and the dispersion laws of the collective modes of the whole system can be determined solving the equation

\[ \det \left[ k^2 \delta^{ij} - k^i k^j - \omega^2 \varepsilon^{ij}_t(k) \right] = 0. \]  

(18)

The solutions of this equation depend on \(|k|, |v|, \cos \theta = \hat{k} \cdot \hat{v}, \omega^2 = \omega^2_p + \omega^2_{\text{jet}}\) and on \(b = \frac{\omega^2_{\text{jet}}}{\omega^2_p}\). Clearly, when the plasma and the jet do not interact (or equivalently \(b = 0\)), they have stable collective modes. However, once we consider the composed system of plasma and jet interacting via mean gauge field interactions, unstable gauge modes may appear.

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**Figure 1.** Imaginary part of the dispersion law of the unstable mode for the system composed by a plasma and a jet with the kinetic theory approach for \(k \perp v\) as a function of the momentum of the mode. Left panel refers to \(b = 0.1\) and right panel refers to \(b = 0.02\). In both case results for four different values of the velocity of the jet, \(|v|\), are shown.

In Fig. 1 we report the results for the unstable mode obtained with the kinetic theory approach for the case where \(k \perp v\). The case \(k \perp v\) corresponds to the most unstable mode in the kinetic theory approach. Estimating the maximum growing rate in the weak coupling limit at \(T \sim 350\) MeV, we find that instabilities develop on time scales \(t \sim 1 - 2\) fm/c.

In Fig. 2 we compare the results for the transverse unstable modes obtained with the kinetic theory with the analogous results obtained within the fluid approach. The agreement between the two approaches is quite remarkable. However, it depends on the value of the parameter \(c_\alpha^a\). On the left panel we have chosen the “conformal” value \(c_\alpha^a = 1/\sqrt{3}\). On the right panel we have employed \(c_\alpha^a = 1/2\), that is the value that minimizes the difference between the results of the two approaches in this case [3].
Figure 2. Comparison between the imaginary part of the dispersion law of the unstable mode in the two approaches in the case $k \perp v$ as a function of the momentum of the mode at $b = 0.1$ for $v = 0.9$ (lower curves) and $v = 1.0$ (upper curves) in the conformal limit, $c_s^a = 1/\sqrt{3}$, (left panel) and for $c_s^a = 1/2$ (right panel). Dashed (red) lines correspond to the results obtained with the fluid approach; full (black) lines correspond to results obtained with kinetic theory.

There are two qualitative differences between the results obtained within the kinetic theory method and the fluid approach [3]. First, in the fluid approach the instabilities develop for velocities $v > c_s^a$ whereas in kinetic theory the threshold value of the velocity for the development of the instability is not related to the parameter $c_s^a$. This is due to the fact that the equation of state does not enter the kinetic theory picture and therefore $c_s^a$ does not play any role.

Second, in the Vlasov approximation for sufficiently small velocities, $v < 0.6$, there is no preferred unstable direction, whereas for larger velocities the most unstable modes correspond to large angles between $k$ and $v$. This has to be contrasted with the results obtained using fluid equations, where one finds that for velocities $v \sim c_s^a$ the most unstable mode correspond to momenta $k$ collinear with the velocity of the jet, whereas for ultrarelativistic velocities $v \sim 1$ the unstable modes corresponding to angles $\theta > \pi/8$ are dominant and the most unstable mode corresponds to $\theta \approx \pi/4$.

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