Probability Optimal Portfolio of Jakarta Islamic Index using Stochastic Dominance and Multi Index Models Indicator

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Abstract: Investment is a process of purchasing asset with the purpose that the asset will be sold in the future at a higher price. This generates stable growth over the long term and provides pension fund. In investing activities, the risk-return trade-off needs to be concerned.

To reduce risk, investment diversification can be done by splitting the investment into several companies in the form of portfolios. This research is aimed to determine the optimal portfolio through Stochastic Dominance (SD) and Multi Index Model (MIM). The data used in this research were stocks from Jakarta Islamic Index (JII) for the period of January 2012 to December 2014. The result shows that using SD and MIM, there are 15 optimal stocks and 11 optimal stocks, respectively. Furthermore, it can be stated that portfolio obtained based on MIM is considered to be the optimal portfolio with 2.25% expected returns and 3.99% risk portfolio.

1. Introduction

Stock, as an instrument of investment, is commonly chosen by the investor as it provides interesting profit [1]. The investors try to minimize risk while striving for the possible highest return. Return is the expected profit after refinancing, while risk is the amount of equity which must be hold to guarantee a certain given level of the bank [2][3][4]. To reduce the risk, diversification can be done by splitting the investment into several stocks in the form of portfolio or combining investment in the asset. Portfolio can span a wide range of real asset and finance asset. A portfolio is considered to be optimal when the expected income and the risk are at the possibly highest potential return at a given level of risk[3][4][5].

Most common problem that the investors face is to choose stocks in order to establish optimal portfolio. There are several methods to analyze the stocks in order to determine optimal portfolio. We may consider Stochastic Dominance (SD) [3][4][5], Single Index Model (SIM) [6], and Multi Index Model (MIM)[6]. SD is the criteria which explains the relationship between two distribution functions in which one function ranks higher than the other distribution function[7].

Single index model is a model which considers the stocks movement which is only influenced by the market. When the stocks movement is also influenced by non-market factors then one may use MIM [8].
Research in finding optimal portfolio have been carried out in last decade. For examples we refer the reader to [3][4][9]. Dentcheva and Ruszczynski[9] who applied SD to choose optimal portfolio while Slamet et al[4], applied Almost Marginal Conditional Stochastic Dominance (AMCSD) on forming efficient portfolios. Kwan[6] analyzed portfolio using SIM, MIM and Constant Correlation Models. On the other hand, Laksono[10] determined optimal portfolio using SD and SIM. In this research, we determined optimal portfolio of Jakarta Islamic Index stocks using SD and MIM. This is first attempt on finding the optimal portfolios based on data used.

2. Methodology

The data used in this research is monthly data of closing stock prices Jakarta Islamic Index (JII) for the period of 2012-2014[11]. Purposive sampling was used to determine samples. Monthly return is measured before applying the model to get optimal portfolio. Optimal portfolio is derived by measuring the highest expected return and the lowest risk.

3. Return and Risk

Return is the expected profit, while risk is an uncertainty which causes unexpected deviation[2]. Based on Brigham and Houston[5], return at time t, \( R_t \), is defined as

\[
R_t = \frac{P_t - P_{t-1}}{P_{t-1}}
\]

where \( P_t \) is stock price index at time t and \( P_{t-1} \) is closing stock price index at time \( t - 1 \).

The measurement of distribution is used to know the probability of a deviation compared to the expectation. According to Hartono[12], risk is measured using standard deviation. Referring to Husnan[7], expected return of portfolio is denoted by \( E[R_p] \), where

\[
E[R_p] = \sum_{i=1}^{n} X_i E[R_i],
\]

and portfolio risk is denoted by \( \sigma_p \), where

\[
\sigma_p = \left( \sum_{i=1}^{n} X_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \sigma_{ij} \right)^{\frac{1}{2}}
\]

with \( X_i \) is proportion of stock \( i \), \( E[R_i] \) is expected return of stock \( i \), and \( \sigma_{ij} \) is covariance of stocks \( i \) and \( j \).

3.1. Stochastic Dominance.

Stochastic Dominance is a term referring to the relationship between two distribution functions, whether a distribution function is more dominant than the other distribution function. In other words, SD is a technique to choose risky investment without considering the distribution of the return[12]. Followings are steps of SD in determining stocks.

1. Determining the probability of every return. This probability is \( P(Y = y) \) which is \( \frac{m}{M} \) where \( m \) is frequency of occur of particular return \( y \) and \( M \) is number of sample.
2. Listing in ascending order (the lowest to the highest) of all returns and their respective probabilities for each stock.
3. Measuring first order SD (FSD) by adding every probability of each return of stocks ascendingly. FSD is a cumulative probability of a return.
4. Measuring second order SD (SSD) is done if there is no dominance in FSD. SSD is an addition of cumulative probability of every return ascendingly.
5. Measuring third order SD (TSD) is done if there is no dominance in SSD. TSD is an addition of cumulative probability. It is done by adding the value of cumulative probability addition of every return ascendingly.
6. Conclusion. If there is no dominance in TSD, then there is no SD.

3.2. Multi Index Model.
Multi Index Model is a multi linear regression in which the level of effect $i$ is influenced by the number of independent variables $F_1, F_2, ..., F_n$. According to Hartono[12], independent variables used in MIM are Return on Equity (ROE), Return on Investment (ROI), Sertifikat Bank Indonesia (SBI), and inflation. The stocks chosen to be a portfolio in MIM are stocks having excess return to beta ratio (ERB) bigger than the value of cut off point ($C^*$). The ERB is equal to $\frac{E[R_i] - R_{BR}}{\beta_i}$ with $R_{BR} = \frac{\sum \text{Return SBI}}{N}$ and $\beta_i = \frac{\sum (R_i - E[R_i])(R_m - E[R_m])}{\sum (E[R_m] - R_m)^2}$ where $E[R_i]$ is expected return of stock $i$, $E[R_m]$ is expected return of market, $R_i$ is return of stock $i$, $R_m$ is return of stock market, and $R_{BR}$ is level of risk return. Meanwhile, $C^*$ is the maximum value of $C$. The $C$ can be calculated using

$$C_i = \frac{R_{BR} \sum_1^N (E[R_i] - R_{BR})\beta_i}{\sigma_{ei}^2} \cdot \frac{1 + R_{BR} \sum_1^N \beta_i^2}{\sigma_{ei}^2}$$

Calculation of $X_i$ in MIM according Hartono[12] is

$$X_i = \frac{Z_i}{\sum Z_j}$$

where $z_i = \frac{\beta_i}{\sigma_{ei}^2} (ERB_i - C^*)$. As the results, there 12, 11, 9, 8, 6, 4, 3, 2, 1 domination(s) under stocks ASRI, LSIP, ADRO, PTBA, UNTR, (AALI, ASII, ITMG, LPKR), (AKRA, CPIN, SMGR), INTP, TLKM, PGAS, respectively. There are 4 stocks i.e ICBP, INDF, KLBF, UNVR without domination.

4. Results and Discussion
4.1. Sampling.
In this research, the samples were stocks from Jakarta Islamic Index for the period of 2012-2014. The samples were obtained by purposive sampling method. The stocks was constantly recorded in Jakarta Islamic Index and with closing stock prices for period of January 2012 - December 2014. The stocks were be certified by Bank Indonesia, the Central Bank of Republic Indonesia. We obtained 19 samples i.e AALI, ADRO, AKRA, ASII, ASRI, CPIN, ICBP, INDF, INTP, ITMG, KLBF, LPKR, LSIP, PGAS, PTBA, SMGR, TLKM, UNTR, and UNVR.

4.2. Data Analysis using Stochastic Dominance.
Analysis of SD results 3 dominance of FSD, 78 dominance of SSD, 13 dominance of TSD, and 77 without dominance. Expected return of portfolio is calculated by equation (2) resulting 0.005485. Meanwhile, risk of portfolio is calculated by equation (3) resulting 0.048035.

4.3. Data analysis using Multi Index Model.
Based on Halim[4], SIM is the returns between two effects or more which will converge into one reaction towards one factor or single index included in the model called Indeks Harga Saham Gabungan (IHSG). MIM is another factor to affect the converging effects. Therefore, MIM uses
multiple linear regressions to know the influence of independent variable to dependent variable. The dependent variable is the return of monthly closing stock price and independent variable is ROE, ROI, SBI, and inflation level.

1. Calculating the market returns ($R_m$). This is executed by subtracting the JII closing stock price with the closing stock price in the previous month then divided by the closing stock price in the previous month.

2. Producing the regression model. The regression model of AALI stock uses AALI return ($Y$) as dependent variables, ROE AALI ($X_1$), ROI AALI ($X_2$), inflation($X_3$) and SBI ($X_4$) as the independent variable, and $e_i$ as the error variable. The regression model obtained is

$$
\hat{Y}_i = -0.0308 - 0.3251X_1 + 0.3738X_2 + 0.0076X_3 + 0.0196X_4 + e_i.
$$

3. Calculating the returns of return $R_{BR}$, $E[R_i]$, $ERB_i$, and $C^*$. Based on the calculation on $R_{BR}$, $E[R_i]$, $ERB_i$, $\sigma^2_{ei}$, and $C_i$, $A_i$, $B_i$, and $C_i$ then we obtained $C^*$ which is equal to 0.0114.

4. Producing Portfolio. The incoming stocks as multi-index model is the stocks that have ERBi value more than $C_i$. The incoming stocks are AALI, ADRO, ASII, CPIN, ICBP, INDF, INTP, KLBF, LSIP, PTBA, and UNTR with the proportion of each stock is 0.033850915, 0.010806657, 0.005649208, 0.153550989, 0.186850141, 0.300031187, 0.108855571, 0.074030591, 0.108855571, 0.074030591, 0.017457588, 0.065962357, respectively. Using (2) the expectation return is 0.022518 and using (3) the portfolio risk is 0.039897.

4.4. The Selection of Optimal Portofolio.

Based on Hartono[12], the optimal portfolio is taken by the efficient portfolio that has the highest returns expectation and the lowest risk. The portfolio resulted by SD and MIM produces the returns expectation 0.005485 and 0.022518, respectively with risk 0.002307 and 0.039897, respectively. Therefore, the optimal portfolio is resulted by MIM.

5. Conclusion

Based on the discussion, it can be concluded that SD and MIM can be used to form optimal portfolio. For JII data used, SD produces portfolio of 15 stocks while MIM produces portfolio of 11 stocks. The optimal portfolio is a portfolio resulted by MIM with expected returns 0.022518 and risk 0.001592, where the stocks proportion of optimal portfolio (that is %) is AALI 3.39%, ADRO 1.08%, ASII 0.56%, CPIN 15.36%, ICBP 18.69%, INDF 30.00%, INTP 4.30%, KLBF 10.89%, LSIP 7.40%, PTBA 1.75%, and UNTR 6.60%.

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