Prospective teachers' difficulties in second order linear differential equation: a case of constructing methods in solving a non-homogeneous problem

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Abstract. This study aims to explore students' difficulty in constructing a method used to solve non-homogeneous problems in 2nd order ordinary differential equations. The research method used is descriptive-explorative research. The data collected were in the form of responses of 46 prospective teachers of the class of ordinary differential equations semester 6 studying at Universitas Negeri Surabaya in Indonesia. Data obtained using a student work sheet in which consist of several steps in order to construct a method can be used to find the solution of non-homogeneous in 2nd order ordinary differential equations. The results show that prospective teachers have some difficulties in making relations between two or more equations they had found. We found that 69.5% of students had most difficulty in emerging an equation consists of \( u_1'(t) \) and \( u_2'(t) \) so that they didn't really understand the concept of solutions of equations.

1. Introduction

Ordinary differential equation is a course that often scores average less than of other courses in mathematics education courses. Especially, on the topic of 2nd order non-homogeneous ordinary differential equations (ODE), prospective teachers often experience difficult in using the method of variation of parameter. This is caused by several procedures that require students to recall the concepts that have been obtained before, mainly derivative and integral [1].

Learning differential equations is difficult [2]. According to Finizio & Ladaz [3] that differential equations are courses that describe equations that contain one or more unknown functions. Mallet [4] also explains that differential equations focus on algorithmic techniques in determining the solutions of some specific types of differential equations. There are two points of concern, namely to determine the type of differential equation and choose the right method to determine the solution of the differential equation. The tendency of students in constructing methods on differential equations often has difficulty. This is supported by data of student test results in 2017 which shows that 70% of students have difficulty determining the solution of the differential equation.

Based on some research results show that students who take the courses of differential equations can’t apply the concept of derivative functions correctly [5]. In addition, Kristayulita & Nurhardiani [6] said that students in solving the problem of 1st order differential equation have difficulties, due to the lack of the concept of integral and logarithmic properties. In line with this study, Budiyono & Guspriati [1] also explained that students in following the courses of differential equations also often have difficulty in determining the results of roots m1 and m2 on the characteristic equations of 2nd order differential equations, determining of general solutions, derived functions and simplify form of equation on the integral form of function.

One way to know the difficulties of students in following this course is by identifying student errors in constructing the method when solving the problem of ordinary 2nd order differential equations. According to Dubinsky & McDonald [7] also said that to know a student concept has been obtained, even though the student had a failure or not, it can be done by looking at the results of
construction methods undertaken by the student. Errors made by students need to be identified, in order to know what kind of difficulties made by students in this course. These difficulties can later be reduced when solving the problem with the same method. It shows that when solving a problem of ordinary differential equations, we can analyse the difficulty in the problem or the construct method in solving the problem.

2. Method
This research is a descriptive explorative research that to explore students’ difficulty in constructing a method used to solve non-homogeneous problems in 2nd order ordinary differential equations.

The subjects of this research are 6th semester students, Mathematics Education Program, Surabaya State University who are taking a course of Differential Equal Equations. This research was conducted from February to May 2018. Research subjects consisted of 46 students, 23 students of class 2015U and 23 students of class 2015A.

The data of this study were collected using question about the non-homogeneous problems in 2nd order ordinary differential equations. The questions used are as follows:

Consider the following 2nd order ODE
\[ y'' + p(t)y' + q(t)y = G(t) \]
where \( p, q, \) and \( G \) are given continuous functions.

The general solution of the corresponding homogeneous equation is
\[ y_h(t) = c_1y_1(t) + c_2y_2(t) \]
where \( y_1 \) and \( y_2 \) are the solutions of \( y'' + p(t)y' + q(t)y = 0 \).

The idea is to replace the constants \( c_1 \) and \( c_2 \) by functions \( u_1(t) \) and \( u_2(t) \), respectively; thus we have
\[ y(t) = u_1(t)y_1(t) + u_2(t)y_2(t). \]
Equation (3) is the particular solution of equation (1).
Then we try to determine \( u_1(t) \) and \( u_2(t) \) so that the expression in equation (3) is a solution of the non-homogeneous equation (1).

In the worksheet is given several stages, so it can be known at what stage students have difficulty in constructing methods of parameter variation.

3. Results
Data obtained from research subjects are analysed and discussed then the results are presented in tabular form. The next will be presented in the form of a related table of prospective teachers’ answers on the stages in constructing the method of parameter variation, as follows:

| Type of Difficulty                  | Total | Percentage |
|-------------------------------------|-------|------------|
| Derivating \( y(t) \) to obtain \( y'(t) \) | 5     | 10.9%      |
| Substituting of a derivative \( y(t) \) to ODE | 8     | 17.4%      |
| Finding \( u_1'(t) \) and \( u_2'(t) \) | 32    | 69.5%      |
| Integrating \( u_1'(t) \) and \( u_2'(t) \) | 1     | 2.2%       |

Based on the above table, the results of the answers from the prospective teachers show that 10.9% of students have difficulty in determining \( y''(t) \) from \( y'(t) \), the difficulty in substituting the \( y(t) \) to differential equation is 17.4%. In addition, 69.5% of students experiencing the greatest difficulty lie in finding \( U_1'(t) \) and \( U_2'(t) \). While, the student having the least difficulty in integrating \( u_1'(t) \) and \( u_2'(t) \) is 2.2%.

Then will be explained and shown sample of student work on type difficulty in constructing method, as follows:

a. Difficulty in determining the derivative of \( y \)
Figure 1 shows that students have difficulties in determining the derivative of $y$, $y'$. This is seen in the results of their work that concerned the result $y''(t)$ is wrong in deriving the multiplication form at function $y'(t)$. Based on the data obtained as much as 10.9% of 46 students who have difficulty at this stage.

|Figure 1. Difficulty in determining the derivative of $y$|

b. Difficulty in substituting the derivatives from $y(t)$ to ODE

Figure 2 shows that students have difficulty in substituting $y(t)$, $y'(t)$, and $y''(t)$ into ODE. This is seen in the results of their work related to substitution $p(t)$ and $q(t)$. Based on the data obtained as much as 17.4% of 46 students who have difficulty at this stage.

|Figure 2. Difficulty in substituting the derivatives from $y(t)$ to ODE|

c. Difficulty in finding $u_1'(t)$ and $u_2'(t)$

Figure 3 shows that the student difficulties in finding $u_1'(t)$ and $u_2'(t)$ after $y(t), y'(t), y''(t)$ is substituted to ODE. The student is unable to recall the equation $u_1'(t), y_1(t) + u_2'(t), y_2(t) = 0$ and has not been able to make the relationship between $y_1(t)$ and $y_1(t)$ which is the solution of equation (1) so they cannot find the final result $u_1'(t)$ and $u_2'(t)$. Based on the data obtained as much as 69.5% of 46 students who have difficulty at this stage.
Figure 3. Difficulty in finding $u_1'(t)$ and $u_2'(t)$

d. Difficulty in integrating $u_1'(t)$ and $u_2'(t)$

Figure 4 shows that students have difficulty in integrating $u_1'(t)$ and $u_2'(t)$ after $y(t), y'(t),$ and $y''(t)$ are substituted to ODE. It is seen that the student does not continue the integral of $u_1'(t)$. Based on data obtained as much as 2.2% of 46 students who have difficulty at this stage.

Figure 4. Difficulty in integrating $u_1'(t)$ and $u_2'(t)$
4. Discussion and Conclusion

The findings of the answers from the prospective teachers showed that 69.5% of students having the greatest difficulty lie in finding \( u_1'(t) \) and \( u_2'(t) \). The difficulties occur because students are less understood in understanding and bring up the functions known in the problem. The students cannot continue to search \( u_1'(t) \) and \( u_2'(t) \). This is shown in Figure 3, the student cannot substitute \( y'' + p(t)y' + q(t)y = 0 \) to the ODE. As a result many students cannot continue to solve the non-homogeneous problems in 2\textsuperscript{nd} order ordinary differential equations.

In addition, the least difficulty lies in integrating \( u_1'(t) \) and \( u_2'(t) \) by 2.2%. Students are very difficult to change the form \( u_1'(t) \) to the form \( u_1(t) \) and \( u_2'(t) \) to \( u_2(t) \). This is seen in the student’s answer Figure 4 which shows the student’s difficulty to continue the integral process from \( u_1'(t) \) to \( u_1(t) \). In line with that, Kristayulita & Nurhardiani [6] said that the students are difficulty in integrating on the problem of ODE because students’ concepts about integral are very lack when they have been obtained in the previous course.

Another difficulty is also seen that 10.9% of students have difficulty in derivating \( y(t) \). Students are less conscientious in finding for the derivatives of the functions \( f'(t) \), seen in Figure 1 that the student is still wrong in finding \( y'(t) \) where there are some symbols still not appropriate. According to Budiyono & Guspriati [1], the most error in solving the problem of ordinary differential equations is in derivating function. While other difficulties, also seen in Figure 2 that students are also less thorough in substituting the results of the derivative \( y(t) \) to ODE. As many as 17.4% of students have difficulty in substituting \( y''(t) \) to PD, this is because students have difficulties from scratch in finding \( y''(t) \) and if we substituting it is also inappropriate.

Based on the result of the research, it can be concluded that the students’ understanding on the concept of non-homogeneous problems in 2\textsuperscript{nd} order ordinary differential equations is less because students are still difficulty in derivating, substituting, integrating and calculating a solution function \( y(t) \). The results also show the greatest difficulty in finding \( u_1'(t) \) and \( u_2'(t) \) and the least difficulty lies in integrating \( u_1'(t) \) and \( u_2'(t) \) into \( u_1(t) \) and \( u_2(t) \). Further research, need to find a method of learning where students will easily understand the material non-homogeneous problems in 2\textsuperscript{nd} order ordinary differential equations so that the difficulties that have been obtained in this study will be reduced.

5. Acknowledgement

We would like to thank Universitas Negeri Surabaya, Indonesia, which give us a grant and chance to disseminate an intellectual output of our research.

6. References

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