Nonreciprocal Pumping of Phonon Spin by Magnetization Dynamics

Xiang Zhang, Gerrit E. W. Bauer, and Tao Yu

1 Kavli Institute of NanoScience, Delft University of Technology, 2628 CJ Delft, The Netherlands
2 Institute for Materials Research & WPI-AIMR & CSRN, Tohoku University, Sendai 980-8577, Japan
3 Max Planck Institute for the Structure and Dynamics of Matter, 22761 Hamburg, Germany

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We propose a method to control surface phonon transport by weak magnetic fields based on the theory of nonreciprocal pumping of surface acoustic waves (SAWs) by magnetostriction. We predict that the magnetization dynamics of a nanowire on top of a dielectric films pumps SAWs with opposite angular momenta into opposite directions, thus generating unidirectional phonon spin transport. Two parallel nanowires form a phononic cavity that at geometrical resonances pump a unidirectional SAW current into half of the substrate.

Introduction.—Surface acoustic waves (SAWs) on the surface of high-quality piezoelectric crystals are frequently employed for traditional signal processing [1, 2], but are also excellent mediators for coherent information exchange between distant quantum systems such as superconducting qubits and/or nitrogen-vacancy centers [3–6]. Coherent SAWs excited by the piezoelectric effect are known as good sources to drive the ferromagnetic resonance (FMR) by magnetostriction [7–13] and to mechanically generate electron spin currents by rotation-skin coupling [14, 15]. Conventional insulators have excellent acoustic quality but only tiny piezoelectric effects, rendering the excitation, manipulation and detection of the coherent SAWs challenging. A solution may be provided by phonon pumping [16], i.e., the excitation of bulk sound waves in a high-quality acoustic insulator by the dynamics of a proximity magnetic layer via the magnetoelastic coupling [16–18]. As reported, the pumped bulk phonons in the acoustic insulator—gadolinium gallium garnet (GGG)—can couple two yttrium iron garnet (YIG) magnetic layers over millimeters [19, 20].

Here we address the coherent excitation and manipulation of Rayleigh-SAWs by magnetization dynamics, which is possible in a lateral planar configuration with ferromagnetic nanowires on top of a high-quality non-magnetic insulator, as illustrated in Fig. 1. Similar configurations but with magnetic film substrates led to electrical detection of diffuse magnon transport [21, 22] or coherent magnon propagation by microwave spectroscopy [23]. The latter set-up showed chiral spin pumping—generation of a unidirectional spin current in half space—of magnons in thin films [21, 25]. The magnetic stray fields of the magnetization dynamics also generate coherent electron [26] and waveguide photon [27] transport. The incoherent counterpart, i.e. the chiral spin Seebeck effect [25], promises unidirectional spin current driven by temperature differences [28].

We focus on the chiral excitation of SAWs via magnetic nanostructures on top of a dielectric substrate that are brought into FMR by external microwaves. We find effects that are very different from the non-reciprocity, i.e. a sound velocity that depends on direction in the absence of time- and space-reversal symmetry [29, 30], which can be strongly enhanced in magnetic multilayers on top of a piezoelectric substrate [31, 32]. We find that the magnetic order of, e.g., a wire on top of a dielectric, does not couple chirally to the surface phonons, but excites both left- and right-propagating phonons, which carry an angular momentum current as they propagate also rotate in opposite directions, as illustrated in Fig. 1. Chiral excitation of SAW phonons therefore becomes possible in a phononic cavity formed by two parallel wires. The SAWs actuated by the first wire interact with the second one (which does not see the microwaves) and excite its magnetization which in turn emits phonons. When the phonons from both sources interfere destructively, the net phonon pumping becomes chiral—a unidirectional phonon current on half of the surface. Constructive interference, on the other hand, induces standing SAWs between the two nanowires as in a Fabry-Pérot cavity.

Model.—We consider a rectangular magnetic nanowire (YIG) on top of the surface of a thick dielectric (GGG)

*tao.yu@mpsd.mpg.de

FIG. 1. Surface-phonon pumping by one magnetic nanowire (brown) on top of the acoustic insulator (blue). A static magnetic field $H_0$ applied in the $\hat{x}$-direction saturates the magnetization. The pumped Rayleigh SAWs by the nanowire FMR propagate and rotate in opposite directions at the two sides of the nanowire as indicated by the green and black arrows, respectively.
that spans the $x, y$ plane. It extends along the $y$-direction with $z \in [0, \ell]$ and $x \in x_1 + [-w/2, w/2]$, as shown in Fig. 1. For an analytical treatment, $d$ is assumed to be much smaller than the skin depth of the SAWs, such that the displacement field in the wire is nearly uniform in the $z$-dependence. The lattice and elastic parameters at the YIG/GGG interface match well [16] [19] [20] [34] and are assumed equal. A uniform and sufficiently large static magnetic field $\mathbf{H}_0$ along $\hat{x}$ saturates the equilibrium magnetization $M_0 = M_0\hat{x}$ normal to the wire. We can modulate the magnon-phonon coupling simply by rotating $\mathbf{H}_0$.

The system Hamiltonian consists of the elastic energy $H_e$, the magnetoelastic coupling $H_c$, and the magnetic energy of the Kittel mode [35],

$$\hat{H}_m = \int d\mathbf{r} \left( -M_z H_0 + \frac{1}{2}N_{xx}M_z^2 + \frac{1}{2}N_{zz}M_z^2 \right),$$

where $\mathbf{M} = (M_x, M_y, M_z)^T$ is the magnetization vector and the demagnetization constants are taken as $N_{xx} \simeq d/(d + w)$ and $N_{zz} \simeq w/(d + w)$ [24]. Although the predicted effects are classical, we use a quantum description for convenience and future applications in quantum phononics [36] [37]; we can always recover the classical picture by replacing operators by amplitudes.

The transverse magnetization is then quantized by the Kittel-magnon operator $\hat{\beta}(t)$ with normalized wavefunction $m_{y,z}$ [37] [38], see Supplemental Material [39]:

$$\hat{\mathbf{M}}_{y,z} = -\sqrt{2\hbar} m_{y,z}(m_{y,z}^* \hat{\beta}(t) + m_{y,z}^\dagger \hat{\beta}^\dagger(t)),$$

leading to $\hat{H}_m = \hbar \omega_p \hat{\beta}^\dagger \hat{\beta}$ with frequency

$$\omega_p = \mu_0 \gamma \sqrt{(H_0 - N_{xx}M_s)(H_0 - N_{xx}M_s + N_{zz}M_s)}.$$  

Here, $-\gamma$ and $\mu_0$ are the gyromagnetic ratio and vacuum permeability.

In our configuration, only the Rayleigh SAWs couple efficiently with the magnet which by their surface nature and long decay length are well suited to exchange information with spatially remote magnets (see Supplemental Material [39]). Sufficiently thin and narrow wires do not affect the substrate strongly, so we may treat them perturbatively. The surface eigenmodes of an isotropic elastic half space read [40]

$$\psi_x = i k \varphi_k \left( e^{i s z} - \frac{2q s}{k^2 + s^2} e^{i 2 z} \right) e^{i k x},$$

$$\psi_z = q \varphi_k \left( e^{i s z} - \frac{2k^2}{k^2 + s^2} e^{i s z} \right) e^{i k x},$$

where $q = \sqrt{k^2 - k_l^2}$ and $s = \sqrt{k^2 - k_l^2}$ with $k_l = \omega_k / \sqrt{\mu / \rho}$ and $k_l = \omega_k / \sqrt{\mu} / \lambda$ are the wave vectors for longitudinal and transverse bulk waves, respectively. Here, $\rho$ is the material density, $\mu$ and $\lambda$ are the elastic Lamé constants, and $\varphi_k$ is a normalization constant.

$$\omega_k = |k| \sqrt{\mu / \rho} = c_v |k|$$ represents the eigenfrequency of Rayleigh SAWs with velocity $c_v$ and $\eta < 1$ is the root of the SAW characteristic equation [40] that depends only on the Lamé constants. The relative phase of the displacement field $\text{Arg}(\psi_x/\psi_z)|_{z=0} = \pm \lambda$ is opposite for right- and left-propagating waves, which reflects the rotation-momentum locking [40].

The quantized displacement field $(\hat{u}_x, \hat{u}_z)$ can be expanded into the eigenmodes $\psi(x, z, k)$ and phonon operators $\hat{b}_k(t)$ [35]

$$\hat{u}(x, z, t) = \sum_k \left[ \psi(x, z, k) \hat{b}_k(t) + \psi^*(x, z, k) \hat{b}_k^\dagger(t) \right].$$

We normalize the mode amplitudes $\psi$ to recover the elastic Hamiltonian for Rayleigh SAWs [39] such that

$$H_e = \rho \int d\mathbf{r} \hat{u}^\dagger \hat{u} = \sum_k \hbar \omega_k \hat{b}_k^\dagger \hat{b}_k.$$  

In YIG films the magnetocrystalline anisotropy, which is important in CoFeB [12] [41], is relatively weak [16] [19] [34] and the (linearized) magnon-phonon coupling energy is dominated by

$$\tilde{H}_e^m = (B_z / M_s) \oint \mathbf{M}_s \partial_z u_y + \mathbf{M}_z \partial_z u_x + \mathbf{M}_z \partial_z u_z) d\mathbf{r}$$ with magnetoelastic coupling constant $B_z$ [35] [42]. The magnonic and non-magnetic substrate are coupled by the dynamics of the surface strain. We require the interaction between a given SAW and the Kittel mode. By the translational symmetry along the nanowire $y$-direction, the displacement field excited by the Kittel magnon does not depend on $y$, and SAWs propagating along $x$ do not contribute to $u_y$. The magnetoelastic energy contributed by the magnetic wire with length $L$ then becomes

$$\tilde{H}_e^m = \frac{B_z L}{M_s} \int_{-w/2+x_i}^{w/2+x_i} \mathbf{M}_z (u_x|_{z=d} - u_x|_{z=0}) \, dx$$

$$+ \frac{B_z L}{M_s} \int_0^d \mathbf{M}_z (u_x|_{z=w/2+x_i} - u_x|_{z=-w/2+x_i}) \, dz.$$  

We limit attention to the realistic situation in which the wire thickness $d$ is much smaller than the decay length of the SAWs into the bulk. The strain in the magnet then mirrors that of the SAW at $z = 0$ of the dielectric and the first term in Eq. (7) vanishes [12]. We then arrive at

$$\tilde{H}_e^m \to \frac{B_z L d}{M_s} \mathbf{M}_z (u_x|_{z=w/2+x_i} - u_x|_{z=-w/2+x_i}),$$

that corresponds to an oscillating surface force $F|_{z=\pm w/2+x_i} = \mp B_z L d \mathbf{M}_z / M_s$ in the $z$-direction that excites SAWs traveling outwards in both directions [39]. Substituting Eqs. (2) and (5) into Eq. (8), we arrive at the interaction Hamiltonian

$$\hat{H}_c = \hbar \sum_k g_k \hat{b}_k^\dagger \hat{b}_k + \text{H.c.},$$

where $g_k = \sqrt{\frac{\rho}{\mu}} k / \lambda$.
in which the coupling constant \( qd \ll 1, sd \ll 1 \)

\[
g_k \simeq -B \sqrt{\frac{\gamma}{M_\psi \rho c_r}} \frac{d}{w} \sin \left( \frac{kw}{2} \right) \xi_M \xi_P e^{ikx_k},
\]

(10)

with factors \( \xi_M \) and \( \xi_P \) that are governed by the magnetic and acoustic material parameters \[ 39 \]. The form factor oscillates and decreases algebraically as a function of nanowire width and phonon wavelength and vanishes when \( d, w \to 0 \). The coupling is not chiral since \( |g_k| = |g_{-k}| \).

**SAW pumping.**—We now calculate the phonon pumping by a single magnetic nanowire transducer centered at \( x_0 \) and excited by microwave photons represented by the (annihilation) operator \( \hat{p}_m \). The Hamiltonian \( \hat{H} = \hat{H}_h + \hat{H}_m + \hat{H}_c \) leads to the Heisenberg equation of motion \[ 43,44 \]

\[
\frac{1}{\hbar} \frac{d\hat{\beta}}{dt} = -i \omega_F \hat{\beta} - i \sum_k |g_k|^2 e^{ikx_k} \hat{b}_k - \left( \kappa_m + \frac{\kappa_\omega}{2} \right) \hat{\beta} - \sqrt{\kappa_\omega} \hat{p}_m, \quad \frac{1}{\hbar} \frac{d\hat{b}_k}{dt} = -i\omega_F \hat{b}_k - i|g_k|^2 e^{-ikx_k} \hat{\beta} - \frac{\delta_k}{2} \hat{b}_k,
\]

(11)

where \( \kappa_m \) and \( \delta_k \) are the intrinsic damping rates for the nanowire magnon and surface phonon, while \( \kappa_\omega \) is the radiative damping induced by the microwave field. In the frequency domain, \( \hat{\beta}(\omega) = \int dt \hat{\beta}(t) e^{i\omega t} \),

\[
\hat{\beta}(\omega) = -\frac{i}{\sqrt{\kappa_\omega}} \frac{\hat{p}_m(\omega)}{\omega - \omega_F + i(\kappa_m + \kappa_\omega)/2 - \sum_k |g_k|^2 G_k(\omega)}, \quad \hat{b}_k(\omega) = G_k(\omega)|g_k|^2 e^{-ikx_k} \hat{\beta}(\omega),
\]

(12)

where \( G_k(\omega) = 1/(\omega - \omega_k + i\delta_k/2) \) is the phonon Green function. The additional magnetic damping by the phonon pumping at the FMR \[ 10,45 \] is given by the imaginary part of the magnon self-energy

\[
\sigma_k(\omega) = -\text{Im} \left( \sum_k |g_k|^2 G_k(\omega) \right) = \frac{|g_k|^2}{c_r},
\]

(13)

where we use the on-shell approximation \[ 46,47 \] with \( \omega \to \omega_F \) and \( \kappa_r = \omega_F/c_r \). The real part of the self-energy shifts a small frequency shift that is absorbed into \( \omega_F \) in the following.

The displacement field given by Eq. \[ 5 \] is a superposition of coherent phonons \( \{b_k\} \) that are excited by the microwave input \( \{\hat{p}_m(\omega)\} \). At resonance \( \omega \to \omega_F \), the contour of the \( k \) integral must be closed in the upper (lower) half of the complex plane for \( x > x_0 \) (\( x < x_0 \)), selecting the poles \( k_r + i\epsilon \) (\( k_r - i\epsilon \)) in the denominator, where \( \epsilon \) is the inverse of the phonon propagation length that is limited by surface imperfections and/or leakage into bulk modes. The low ultrasonic attenuation in GGG at room temperature corresponds to characteristic SAW decay lengths of up to 6 mm \[ 48 \]. We can therefore safely disregard the phonon damping \( (\epsilon \to 0_+) \), which leads to displacement fields

\[
\mathbf{u}(x,t) = -\frac{2}{c_r} \text{Re} \left( i\psi_k(k_r, z) g_k(\beta(t)) \right), \quad x > x_0 \quad \text{and} \quad i\psi_k(-k_r, z) g_{-k}(\beta(t)) \to 0_+, \quad x < x_0.
\]

(14)

On the right (left) side of the nanowire \( x > x_0 \) (\( x < x_0 \)), the right- and left-propagating waves with opposite rotations, whose directions depend on \( z \), are pumped as illustrated in Fig. 1. A classical treatment leads to the same result \[ 39 \].

These phonons carry a constant mechanical angular momentum density \( \hat{I}_{DC}(x, z) = \rho \langle \mathbf{u} \times \dot{\mathbf{u}} \rangle_t \), where the subscript \( t \) indicates time average, which is often referred to as phonon spin \[ 17,49,50 \]:

\[
\hat{I}_{DC}(x,z) = (4\rho \omega_F/c_r^2)|\langle \beta \rangle|^2 |g_m|^2 \hat{y} \times \text{Im} \left( \psi_x(k_r, z) \psi^{*}_x(k_r, z), \quad x > x_0 \right. \quad \text{and} \quad \left. \psi_x(-k_r, z) \psi^{*}_x(-k_r, z), \quad x < x_0 \right).
\]

(15)

\( \hat{I}_{DC} \) is proportional to the excited magnon population, parallel to the wire, and opposite on both sides of the nanowire since \( \psi_x(-k) \psi^{*}_x(-k) = -\psi_x(k) \psi^{*}_x(k) \). Into the substrate (\( z \)-direction), the SAW eigenmodes have a node at which \( \hat{I}_{DC} \) changes sign \[ 39 \] as sketched in Fig. 1.

The phonon pumping does not remove angular momentum from the ferromagnet, since only the \( z \)-component of the magnetic precession is damped. The force on the interface is a superposition of opposite angular momenta \( 2\mathbf{a} = (z + i\mathbf{x}) + (z - i\mathbf{x}) \) that by the spin-momentum locking couple to phonons moving in opposite direction. Angular momentum is therefore conserved, but the decay of the phonon spin current at the edges of the sample should generate an observable bending stress.

The efficiency of phonon spin pumping depends on the nanowire and substrate. For GGG at room temperature, \( \rho = 7080 \text{ kg/m}^3, \quad c_l = 6545 \text{ m/s} \) and \( c_t = 3531 \text{ m/s} \) \[ 51 \], leading to \[ 40 \] \( \eta = 0.927, \quad c_r = \eta c_t = 3271.8 \text{ m/s} \), and \( \xi_F = 0.537 \). For YIG \[ 50 \], \( \gamma = 1.82 \times 10^{11} \text{ s}^{-1} \text{T}^{-1}, \quad \mu_0 M_s = 0.177 \text{ T} \) \[ 52 \], \( B_\perp = 6.96 \times 10^5 \text{ J/m}^3 \) \[ 10 \] and \( \xi_M \approx 1 \) when \( H_0 \) is comparable to \( M_s \). We plot the pumped phonon spin density at different \( z \) in Fig. 2a with \( \omega_F = 3 \text{ GHz} \), \( d = 200 \text{ nm} \) and \( w = 2.5 \mu \text{m} \). We use a small precession cone angle \( 10^{-3} \) degrees and phonon diffusion length \( \sim 6 \text{ mm} \). The spin density is opposite at the two sides of the nanowire and changes sign at larger \( z \). Figure 2b is a plot of the additional magnon damping coefficient \( \alpha = \sigma_k/\omega_F \) in the dependence of FMR frequency \( \omega_F \) and nanowire width \( w \). We observe geometric resonances \( \sim 1/w \) with \( \alpha \lesssim 6 \times 10^{-5} \), which is of the order of the intrinsic Gilbert damping of YIG single crystals \( \alpha_0 \sim 4 \times 10^{-5} \) \[ 53 \] and films \( 8 \times 10^{-5} \) \[ 54 \]. In the thin YIG film, the additional damping \( \alpha \sim d \).

**Chiral phonon pumping.**—The single wire emits spin-momentum locked SAWs into two directions. We propose truly chiral phonon pumping by a device of two
parallel and identical nanowires located at \( \mathbf{r}_1 = R_1 \hat{x} \) and \( \mathbf{r}_2 = R_2 \hat{x} \), of which only the left one is addressed by a local microwave stripline [53]. The excited phonons below propagate to and are absorbed by the second nanowire. Its dynamics re-emits phonons that subsequently interfere with the original ones [20]. Denoting the magnon operators in the left and right nanowires as \( \hat{\beta}_L \) and \( \hat{\beta}_R \) [39],

\[
\hat{\beta}_R(\omega) = \frac{\sum_k |g_k|^2 G_k(\omega) e^{ik(R_2 - R_1)}}{\omega - \omega_F + i \kappa_m/2} \hat{\beta}_L(\omega),
\]

\[
\hat{b}_k(\omega) = |g_k| G_k(\omega) \left( e^{-ikR_1} \hat{\beta}_L(\omega) + e^{-ikR_2} \hat{\beta}_R(\omega) \right)
\]

(16)

At the FMR \( \omega \to \omega_F \),

\[
\hat{\beta}_R(\omega_F) = \chi(k_r) e^{i\pi + i k_r (R_2 - R_1)} \hat{\beta}_L(\omega_F),
\]

(17)

where \( \chi(k_r) = \sigma(k_r)/(\kappa_m/2 + \sigma(k_r)) \) modulates the magnetization amplitude in the second wire and \( k_r (R_2 - R_1) \) is the phase delay by the phonon transmission. The phase shift \( \pi \) reflects the phase relation between magnons and phonons that is the key for the chirality addressed below. This relation can be detected inductively in the microwave transmission [39].

By substituting Eq. (17) into (16) at the FMR:

\[
\begin{align*}
\hat{b}_{k_r} &= |g_{k_r}| G_{k_r} e^{-ik_r R_1} \hat{\beta}_L(\omega_F) \left( 1 - \chi(k_r) \right) , \\
\hat{b}_{-k_r} &= |g_{k_r}| G_{k_r} e^{ik_r R_1} \hat{\beta}_L(\omega_F) \left( 1 - \chi(k_r) e^{2ik_r (R_2 - R_1)} \right).
\end{align*}
\]

(18)

In the strong magnon-phonon coupling limit \( \sigma(k_r) \gg \kappa_m/2 \), \( \chi(k_r) \to 1 \), thus the right-going phonon \( k_r > 0 \) is not excited by the double-wire configuration. Finite \( \langle \hat{b}_{-k_r} \rangle \) but vanishing \( \langle \hat{b}_{k_r} \rangle \) implies a unidirectional (chiral) phonon current. Such chirality vanishes when the second wire is weakly coupled to the SAW, i.e. \( \sigma(k_r) \ll \kappa_m/2 \), i.e. phonons transmit without interacting with the magnet.

By Eqs. (17) and (16), the displacement fields of frequency \( \omega_F \) read

\[
\mathbf{u}(x, t) = \frac{2|g_{k_r}|}{\epsilon_r \omega} \text{Im} \left\{ e^{ik_r (R_2 - R_1)} \frac{\chi(k_r) e^{-ik_r R_1}}{\chi(k_r) e^{-ik_r R_1} - \chi(k_r)} \langle \hat{\beta}_L(t) \rangle \left( 1 - \chi(k_r) e^{2ik_r (R_2 - R_1)} \right) \right\} \psi(-k_r) e^{ik_r R_1} \left( 1 - \chi(k_r) \right)
\]

When \( \chi(k_r) \to 1 \), the displacement field vanishes in the region \( x > R_2 \), but is a traveling wave for \( x < R_1 \). Between the two nanowires with \( R_1 < x < R_2 \), the SAWs form standing waves with \( u_x \sim \sin k_r (x - R_2) \) and \( u_x \sim \cos k_r (x - R_2) \). The chirality is not complete when \( \chi(k_r) < 1 \), however. Figure 3a) is a plot of the magnitude of the displacement field at the GGG surface \( |\mathbf{u}(x, z = 0)| = \sqrt{u_x^2 + u_z^2} \) at center at \( R_1 = 0 \) and \( R_2 = 30 \mu m \). The asymmetry between the two sides of the phonon cavity reflects the chirality which is not perfect for realistic coupling parameters. In Fig. 3b) we plot the phonon (DC) spin density at the GGG surface for a precession cone angle \( 10^{-3} \) degrees in the left wire. The asymmetry of the pumped phonon spin at the two sides of YIG cavity is clearly observable also for larger damping. In the YIG—GGG system chiral phonon pumping should therefore be measurable, but magnets with large magnetoelasticity may be more favorable.

The pumped phonon (spin) can propagate coherently over millimeters on the substrate surface, which is very promising for classical and quantum transport of spin information. It can be measured by Brillouin light scattering [17], the spin-rotation coupling by fabricating a conductor on top of the acoustic medium [14][15], and other techniques [56]. The generation of chirality by interference does not require a chiral coupling mechanism [24][27].
but only an out-of-phase relation of the two fields at resonance. It therefore appears to be universal for many field propagation phenomena, such as exchange coupled magnetic nanowires and films \cite{24} and non-chirally coupled magnons and waveguide photons \cite{27}.

**Discussion.**—In conclusion, we developed a theory for pumping SAWs and proposed a phonon cavity device that realizes unidirectional phonon current in a non-chiral system. When exciting a single magnetic nanowire transducer by microwaves, the rotation-momentum locking of SAWs generates a phonon spin current in the underlying acoustic medium. When adding a second, passive wire we predict emergence of a unidirectional phonon current and formation of standing waves in the region between two magnetic nanowires. This mechanism should also lead to a chiral spin Seebeck effect generated by a temperature gradient between the magnetic and acoustic insulators. The non-reciprocal phonon propagation discussed in this letter generalizes the chiral pumping and open intriguing perspectives for application in magnonics \cite{24} \cite{25}, spintronics, plasmonics \cite{57} \cite{58} and nano-optics \cite{65}.

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\[ |u(x,z=0)| = 10^{14} / m^2 \]

\[ \alpha_0 = 4 \times 10^{-5} \]

\[ \alpha_0 = 4 \times 10^{-5} \]

\[ 1 \times 10^{-4} \]

\[ 0 \times 10^{-4} \]

\[ -3 \]

\[ -2 \]

\[ -1 \]

\[ 0 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ -300 \]

\[ -200 \]

\[ -100 \]

\[ 0 \]

\[ 100 \]

\[ 200 \]

\[ 300 \]

\[ (a) \]

\[ (b) \]

\[ \text{FIG. 3. Snapshot of the displacement field at the GGG surface [u (a)] and phonon spin density [(b)], pumped by a YIG wire at the origin under FMR and modulated by a second YIG wire at 30 \mu m (parameters in text).} \]
and Y. Otani, arXiv:2001.05135.

[34] L. D. Landau and E. M. Lifshitz, Theory of Elasticity (Pergamon, Oxford, 1970).

[35] L. D. Landau and E. M. Lifshitz, Electrodynamics of Continuous Media, 2nd ed. (Butterworth-Heinemann, Oxford, 1984).

[36] M. J. A. Schuetz, E. M. Kessler, G. Giedke, L. M. K. Vandersypen, M. D. Lukin, and J. I. Cirac, Phys. Rev. X 5, 031031 (2015).

[37] T. Yu, S. Sharma, Y. M. Blanter, and G. E. W. Bauer, Phys. Rev. B 99, 174402 (2019).

[38] L. R. Walker, Phys. Rev. 105, 390 (1957).

[39] See Supplemental Material [...] for the derivation of magnon/phonon wavefunction, classical description of SAW pumping, and microwave transmission.

[40] I. A. Viktorov. Rayleigh and Lamb waves: Physical theory and applications. (Plenum Press, New York, 1967).

[41] R. Sasaki, Y. Nii, Y. Iguchi, and Y. Onose, Phys. Rev. B 95, 020407(R) (2017).

[42] A. Rückriegel, P. Kopietz, D. A. Bozhko, A. A. Serga, and B. Hillebrands, Phys. Rev. B 89, 184413 (2014).

[43] C. W. Gardiner and M. J. Collett, Phys. Rev. A 31, 3761 (1985).

[44] A. A. Clerk, M. H. Devoret, S. M. Girvin, F. Marquardt, and R. J. Schoelkopf, Rev. Mod. Phys. 82, 1155 (2010).

[45] S. Streib, N. V. Silva, K. Shen, and G. E. W. Bauer, Phys. Rev. B 99, 184442 (2019).

[46] G. D. Mahan, Many Particle Physics (Plenum, New York, 1990).

[47] G. F. Giuliani and G. Vignale, Quantum Theory of the Electron Liquid (Cambridge University Press, Cambridge, 2005).

[48] M. Dutoit, J. Appl. Phys. 45, 2836 (1974).

[49] Y. Long, J. Ren, and H. Chen, PNAS. 115, 9951 (2018).

[50] Q. Wang, B. Heinz, R. Verba, M. Kewening, P. Pirro, M. Schneider, T. Meyer, B. Lägel, C. Dubs, T. Brächer, and A. V. Chumak, Phys. Rev. Lett. 122, 247202 (2019).

[51] M. Schreier, A. Kamra, M. Weiler, J. Xiao, G. E. W. Bauer, R. Gross, and S. T. B. Goennenwein, Phys. Rev. B 88, 094410 (2013).

[52] A. A. Serga, A. V. Chumak, and B. Hillebrands, J. Phys. D: Appl. Phys. 43, 264002 (2010).

[53] B. M. Yao, T. Yu, Y. S. Gui, J. W. Rao, Y. T. Zhao, W. Lu, and C.-M. Hu, Commun. Phys. 2, 161 (2019).

[54] H. Chang, P. Li, W. Zhang, T. Liu, A. Hoffmann, L. Deng, and M. Wu, IEEE Magn. Lett. 5, 6700104 (2014).

[55] H. Yu, G. Duerre, R. Huber, M. Bahr, T. Schwarze, F. Brandl, and D. Grundler, Nat. Commun. 4, 2702 (2013).

[56] C. Shi, R. Zhao, Y. Long, S. Yang, Y. Wang, H. Chen, J. Ren, and X. Zhang, Nat. Sci. Rev. 6, 4 (2019).

[57] K. Y. Bliokh, D. Smirnova, and F. Nori, Science 348, 1448 (2015).

[58] K. Y. Bliokh and F. Nori, Phys. Rep. 592, 1 (2015).

[59] L. Novotny and B. Hecht, Principles of Nano-Optics (Cambridge University Press, Cambridge, England, 2006).
MAGNON AND PHONON MODES

In this part, we derive the Kittel magnon and surface phonon normalized eigenmodes used in the main text.

From the Landau-Lifshitz equation, the transverse magnetizations in our configuration with saturated magnetization along $\hat{x}$ direction obey the equation of motion

$$\frac{dm_y}{dt} = -\mu_0\gamma(H_{\text{app}} - N_{xx}M_s + N_{zz}M_s)m_z,$$
$$\frac{dm_z}{dt} = \mu_0\gamma(H_{\text{app}} - N_{xx}M_s)m_y.$$  \hfill (S1)

which are solved by

$$m_z = i\xi_M^2m_y,$$  \hfill (S2)

with dimensionless parameter

$$\xi_M = \left(\frac{H_0 - N_{xx}M_s}{H_0 - N_{xx}M_s + N_{zz}M_s}\right)^{1/4}. \hfill (S3)$$

When $H_0$ is large, $\xi_M \to 1$. The magnon amplitudes $m_y, z$ satisfy the normalization condition $\int dV (m_y m_z^* - m_y^* m_z) = -\frac{i}{2}$, \hfill (S4)

which leads to the normalized magnon wave function

$$m_y = -\frac{1}{2\sqrt{Lwd}} \frac{1}{\xi_M}, \quad m_z = -\frac{i}{2\sqrt{Lwd}} \xi_M. \hfill (S5)$$

We treat the thin YIG nanowire as small perturbation of the elastic GGG substrate. The surface modes are found by the elastic equation of motion \cite{2, 40}

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sum_j \frac{\partial \sigma_{ij}}{\partial x_j}, \hfill (S6)$$

with free boundary condition $\sigma_{ij}|S = 0$ at the surface “S”. We focus on an isotropic material in the linear regime of Hooke’s Law $\sigma_{ij} = \sum_{kl} C_{ijkl} \epsilon_{kl}$ with stiffness coefficient $C_{ijkl}$ and strain tensor $\epsilon_{ij} = (\partial u_i/\partial x_j + \partial u_j/\partial x_i)/2$ \cite{2}. The displacement field can be written in terms of the scalar and vector potentials $V$ and $A$ as $u = \nabla V + \nabla \times A$, with $A = A_y$ because the displacement field of an acoustic wave propagating in the $\hat{x}$ direction does not depend on $y$. With $u_x = \partial V/\partial x - \partial A/\partial z$ and $u_z = \partial V/\partial z + \partial A/\partial x$ we arrive at

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial z^2} = \frac{\rho}{\lambda + 2\mu} \frac{\partial^2 V}{\partial t^2},$$
$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial z^2} = \frac{\rho}{\mu} \frac{\partial^2 A}{\partial t^2}. \hfill (S7)$$

A sufficiently thin YIG nanowire follows the displacement field of the substrate. By a plane wave ansatz for $V$ and $A$, we obtain the eigenmodes of surface phonon

$$\psi_x = i\kappa \varphi_k \left(e^{iqz} - \frac{2qs}{k^2 + s^2} e^{sz}\right) e^{ikx},$$
$$\psi_z = q\varphi_k \left(e^{qz} - \frac{2k^2}{k^2 + s^2} e^{sz}\right) e^{ikx}, \hfill (S8)$$

where $q = \sqrt{k^2 - k_0^2}$ and $s = \sqrt{k^2 - k_1^2}$ are introduced in the main text. The decay of the surface modes is determined by two exponential functions $e^{iqz}$ and $e^{sz}$, with a node in the $z$-dependence of $\psi_x(z)$. 

The Rayleigh surface acoustic wave (SAW) eigenmodes $\psi$ can be normalized by

$$
\int_{-\infty}^{0} dz \left( |\psi_x|^2 + |\psi_z|^2 \right) = \frac{\hbar}{2 \rho L \omega_k},
$$

leading to the normalization factor

$$
\varphi_k = \frac{1}{|k|} \frac{1 + b^2}{2a(1 - b^2)} \sqrt{\frac{2\hbar}{\rho L c_r}} \xi_p,
$$

where

$$
\xi_p = \frac{a(1 - b^2)}{1 + b^2} \left( \frac{1 + a^2}{2a} + \frac{2a(a - 2b)}{b(1 + b^2)} \right)^{-1/2}
$$

with dimensionless material constants

$$
a = q/|k| = \sqrt{1 - (c_r/c_l)^2},
$$

$$
b = s/|k| = \sqrt{1 - \eta^2}.
$$

Here $c_r = \eta \sqrt{\mu/\rho}$ and $c_l = \sqrt{(\lambda + 2\mu)/\rho}$ are the sound velocities of the surface and longitudinal bulk waves, respectively.

The decay behavior of the surface eigenmodes leads to a node in the $z$-dependence of the excited DC spin density (parameters in the main text), as shown in Fig. S1 for the typical parameters.

![FIG. S1. (Color online) Spatial $z$-dependence of the excited phonon spin density, normalized by the maximal magnitude at the surface (parameters in the main text).](image)

**CLASSICAL FORMALISM OF SAWS PUMPING**

In this section, we formulate the excitation of Rayleigh SAWs by the oscillating surface force

$$
F|\mathbf{x} = \pm w + x_i = \frac{\delta H_c}{\delta u_z(x, t)} = \frac{B_{\perp} L d M_s}{M_s} \mathbf{M}_z \mathbf{z},
$$

that arises from the magnetoelastic coupling [16, 40], which is consistent with the matrix elements in the (quantum) Hamiltonian approach used in the text.

The boundary condition for the stress tensor (line load)

$$
\sigma_{zz}|_{z = 0} = \frac{1}{L} \int_{-\infty}^{0} dz \frac{dF(x, z)}{dx} \approx - \frac{B_{\perp} d M_s}{M_s} M_z \left[ \delta \left( x - \left( \frac{w}{2} + x_i \right) \right) - \delta \left( x - \left( -\frac{w}{2} + x_i \right) \right) \right],
$$

$$
\sigma_{xz}|_{z = 0} = 0,
$$

(S14)
conserve momentum. We solve the elastic equation of motion [Eq. (S7)] with this boundary condition by a plane-wave ansatz (suppressing the time harmonic)

\[ V = \int_{-\infty}^{\infty} \mathcal{V}(k)e^{i(kx + \sqrt{k^2 - k_z^2} z)} dk, \]

\[ A = \int_{-\infty}^{\infty} \mathcal{A}(k)e^{i(kx + \sqrt{k^2 - k_z^2} z)} dk. \] (S15)

The Fourier transformation of the boundary condition \( \sigma_z(k) \)_{z=0} = i f_z/(/\mu) \sin (kw/2)e^{-ikx}, \) with \( f_z = (B_z d/M_s) M_z \) and \( \sigma_z(k) \)_{z=0} = 0 leads to the excited elastic potentials in k-space

\[ \mathcal{V}(k) = i f_z \frac{2k^2 - k_z^2}{\pi \mu F(k)} \sin \left( \frac{kw}{2} \right) e^{-ikx}, \]

\[ \mathcal{A}(k) = -\frac{f_z}{\pi \mu} \frac{2k\sqrt{k_z^2 - k_z^2}}{F(k)} \sin \left( \frac{kw}{2} \right) e^{-ikx}, \] (S16)

with

\[ F(k) = (2k^2 - k_z^2)^2 - 4k^2 \sqrt{k^2 - k_z^2} \sqrt{k^2 - k_z^2}. \] (S17)

\( F(k) = 0 \) is known in acoustics as the Rayleigh equation. The displacement field on the substrate surface \((z = 0)\) is then expressed by

\[ u_x \mid_{z=0} = -\frac{f_z}{\pi \mu} \int_{-\infty}^{\infty} k \sin (kw/2) \left[ \frac{(2k^2 - k_z^2) - 2\sqrt{k^2 - k_z^2} \sqrt{k^2 - k_z^2}}{F(k)} \right] e^{ik(x-x_i)} dk, \]

\[ u_z \mid_{z=0} = -\frac{f_z k_z^2}{\pi \mu} \int_{-\infty}^{\infty} \sqrt{k^2 - k_z^2} \sin (kw/2) e^{ik(x-x_i)} dk. \] (S18)

Far away from the nanowire, i.e. \(|x| > x_i\), this integral can be carried out analytically by integrating along the contour in Fig. S2. The roots of the Rayleigh equation \( F(k_r) = 0 \) are poles while \( \pm k_l \) and \( \pm k_t \) are branching points. We perform the contour integral by closing contours depending on \( x > x_i \) or \( x < x_i \). We apply the Watson’s lemma for the asymptotic treatment of the integral along the branch cuts and find the displacement field on the substrate surface \((z = 0)\)

\[ u_x^\pm \mid_{z=0} = \pm \frac{f_z}{2\mu} \int_{-\infty}^{\infty} \frac{(1 - b^2)}{4 + (1 + b^2)} \left[ \frac{1}{\pi} \left( \frac{1}{\sqrt{1 - b^2}} - \frac{1}{2} \frac{1}{\sqrt{1 + b^2}} \right) \right] \sin \left( \frac{k_r w}{2} \right) e^{\pm ik_r(x-x_i)} \]

\[ + \mathcal{O}(|k x|)^{-2}, \]

\[ u_z^\pm \mid_{z=0} = \frac{f_z}{2\mu} \left[ \frac{-4 + (1 + b^2)\left[ 1 + \frac{1}{2} \left( \frac{1}{\sqrt{1 - b^2}} \right) \right]}{4 + (1 + b^2)\left[ 1 + \frac{1}{2} \left( \frac{1}{\sqrt{1 + b^2}} \right) \right]} \right] \sin \left( \frac{k_r w}{2} \right) e^{\pm ik_r(x-x_i)} \]

\[ + \frac{f_z}{2\mu} \left[ \frac{1}{\sqrt{1 - b^2}} \right] \left[ \frac{1}{\sqrt{1 + b^2}} \right] \sin \left( \frac{k_r w}{2} \right) e^{\pm ik_r(x-x_i)} \]

\[ + \mathcal{O}(|k x|)^{-2}, \] (S19)

where the “±” sign stands for the right- and left-going waves on the right and left sides of the wire, respectively. The second terms decay far from the nanowire as \( 1/(k_l x)^{3/2} \) and \( 1/(k_t x)^{3/2} \) and can be attributed to the evanescent contributions from longitudinal \( (e^{ik_l x}) \) and transverse \( (e^{ik_t x}) \) bulk acoustic waves. These contributions can be disregarded far from the nanowire, where only the Rayleigh SAWs \( (e^{ik_r x}) \) survive.

Retaining only the SAWs, we can find the displacement field from the integral for the general \( z \),

\[ u_x = -\frac{f_z}{\pi \mu} \int_{-\infty}^{\infty} \frac{k \sin (kw/2) \left[ (2k^2 - k_z^2) e^{\sqrt{k^2 - k_z^2} z} - 2\sqrt{k^2 - k_z^2} \sqrt{k^2 - k_z^2} e^{\sqrt{k^2 - k_z^2} z} \right]}{F(k)} e^{ik(x-x_i)} dk, \]

\[ u_z = -\frac{f_z}{\pi \mu} \int_{-\infty}^{\infty} \frac{\sqrt{k^2 - k_z^2} \sin (kw/2) \left[ (2k^2 - k_z^2) e^{\sqrt{k^2 - k_z^2} z} - 2k^2 e^{\sqrt{k^2 - k_z^2} z} \right]}{F(k)} e^{ik(x-x_i)} dk, \] (S21)
FIG. S2. Contour in the complex $k$ plane. The upper half plane is used for the integral when $x > x_i$ and lower half plane when $x < x_i$. Branch cuts at $\pm k_l$ and $\pm k_t$ are indicated. The Rayleigh point $\pm k_r$ acquires a small imaginary part by causality or sound attenuation.

we obtain

$$u_{x}^\pm = \pm \frac{f_x}{\mu} \frac{1}{-4 + (1 + b^2)} \sin \left( \frac{k_rw}{2} \right) \left[ e^{qz - \frac{2ab}{1 + b^2} e^{sz}} \right] e^{\pm ik_r(x-x_i)} ,$$

$$u_{z}^\pm = f_z \frac{\alpha}{\mu} \frac{1}{-4 + (1 + b^2)} \sin \left( \frac{k_rw}{2} \right) \left[ e^{qz - \frac{2}{1 + b^2} e^{sz}} \right] e^{\pm ik_r(x-x_i)}. $$

(S22)

Inserting the force term $f_x = B_\perp \hat{M}_x/M_s$ and the expression for $M_z$ in the main text, we arrive at the pumped displacement fields,

$$u_{x}^\pm = \pm \frac{B_\perp \xi M_s}{\rho c^2} \sqrt{\frac{2h\gamma d}{LwM_s}} \frac{\xi_p}{1 - b^2} \sin \left( \frac{k_rw}{2} \right) \left( e^{qz - \frac{2ab}{1 + b^2} e^{sz}} \right) e^{\pm ik_r(x-x_i)} \langle \hat{\beta}(t) \rangle$$

$$= - \frac{1}{c_r} i \psi \xi g_{\pm k_r} \langle \hat{\beta}(t) \rangle,$$

$$u_{z}^\pm = \pm \frac{B_\perp \xi M_s \xi_p}{\rho c^2} \sqrt{\frac{2h\gamma d}{LwM_s}} \frac{\xi_p}{1 - b^2} \sin \left( \frac{k_rw}{2} \right) \left( e^{qz - \frac{2}{1 + b^2} e^{sz}} \right) e^{\pm ik_r(x-x_i)} \langle \hat{\beta}(t) \rangle$$

(S23)

in which we used the Rayleigh relation $4ab = (1 + b^2)^2$. These results agree with those derived in the main text by the quantum formalism.

MICROWAVE TRANSMISSION SPECTRA

Here we address the excitation of magnons and phonons for two parallel magnetic nanowires on a dielectric substrate of which one is excited by microwaves while the other coupled inductively to another stripline, and calculate the microwave scattering matrix. The Kittel-magnon operators in the two magnetic nanowires at $R_1$ and $R_2$ are expressed by $\hat{\beta}_L$ and $\hat{\beta}_R$, respectively. Augmented by the magnetic and elastic damping and microwave input $\hat{p}_\text{in}$ acting on the
left nanowire, the Heisenberg equations of motion for the magnon-phonon coupled system read
\[
\begin{align*}
\frac{d\hat{\beta}_L}{dt} &= -i\hbar\omega_F\hat{\beta}_L(t) - i\hbar\sum_k |g_k|e^{ikR_1}\hat{b}_k(t) - \left(\frac{\kappa_L + \kappa_{\omega,L}}{2}\right)\hat{\beta}_L(t) - \sqrt{\kappa_{\omega,L}}\hat{p}_m^L(t), \\
\frac{d\hat{\beta}_R}{dt} &= -i\hbar\omega_F\hat{\beta}_R(t) - i\hbar\sum_k |g_k|e^{ikR_2}\hat{b}_k(t) - \frac{\kappa_R}{2}\hat{\beta}_R(t), \\
\frac{db_k}{dt} &= -i\hbar\omega_k\hat{b}_k(t) - i\hbar|g_k|e^{-ikR_1}\hat{\beta}_L(t) - i\hbar|g_k|e^{-ikR_2}\hat{\beta}_R(t) - \frac{\delta_k}{2}\hat{b}_k(t).
\end{align*}
\] (S24)

Here \(\kappa_L\) and \(\kappa_R\) denote the (Gilbert) damping of the Kittel modes in the left and right nanowires, and \(\kappa_{\omega,L}\) is the radiative coupling of the left nanowire to the microwave source. For sufficiently larger \(|R_2 - R_1|\) (tens of micrometers for the present system), the excited magnon and phonon operators in frequency space read
\[
\begin{align*}
\hat{\beta}_L(\omega) &= \frac{-i\sqrt{\kappa_{\omega,L}}}{\omega - \omega_F + i\kappa_{\omega,L}/2 - \sum_k |g_k|^2G_k(\omega) - f(\omega)}\hat{p}_m^L(\omega), \\
\hat{\beta}_R(\omega) &= \frac{\sum_k |g_k|^2G_k(\omega)e^{i(kR_2-R_1)}}{\omega - \omega_F + i\kappa_{\omega,R}/2 - \sum_k |g_k|^2G_k(\omega)}\hat{\beta}_L(\omega), \\
\hat{b}_k(\omega) &= |g_k|G_k(\omega)\left(e^{-ikR_1}\hat{\beta}_L(\omega) + e^{-ikR_2}\hat{\beta}_R(\omega)\right),
\end{align*}
\] (S25)

where
\[
f(\omega) = \frac{(|g_k|^2/\mathcal{C}_r)^2e^{i2\omega(R_2-R_1)}/\mathcal{C}_r}{\omega - \omega_F + i\kappa_{\omega,R}/2 - \sum_k |g_k|^2G_k(\omega)}.
\] (S26)

The phonon operator \(\hat{b}_k\) governs the displacement field in the main text.

The microwave output of the left and right nanowires is inductively detected by the striplines represented by photon operators \(\hat{p}_m^L\) and \(\hat{p}_m^R\) that are connected by the input-output relations [43, 44]
\[
\begin{align*}
\hat{p}_m^L(\omega) &= \hat{p}_m^L(\omega) + \sqrt{\kappa_{\omega,L}}\hat{\beta}_L(\omega), \\
\hat{p}_m^R(\omega) &= \sqrt{\kappa_{\omega,R}}\hat{\beta}_R(\omega).
\end{align*}
\] (S27)

The microwave reflection \((S_{11})\) and transmission \((S_{21})\) spectra become
\[
\begin{align*}
S_{11}(\omega) &\equiv \frac{\hat{p}^L_{\text{out}}}{\hat{p}^L_{\text{in}}} = 1 - \frac{i\kappa_{\omega,L}}{\omega - \omega_F + i(\kappa_L + \kappa_{\omega,L})/2 - \sum_k |g_k|^2G_k(\omega) - f(\omega)}' \\
S_{21}(\omega) &\equiv \frac{\hat{p}^R_{\text{out}}}{\hat{p}^L_{\text{in}}} = (S_{11}(\omega) - 1)\frac{\sqrt{\kappa_{\omega,R}}}{\kappa_{\omega,L}}\frac{\sum_k |g_k|^2G_k(\omega)e^{i(kR_2-R_1)}}{\omega - \omega_F + i\kappa_{\omega,R}/2 - \sum_k |g_k|^2G_k(\omega)}.
\end{align*}
\] (S28)

When the two magnetic nanowires are identical, the microwave transmission with excitation (input) at \(R_1\) and detection (output) at \(R_2\),
\[
S_{21}(\omega) = (S_{11}(\omega) - 1)\langle\hat{\beta}_R(\omega)\rangle/\langle\hat{\beta}_L(\omega)\rangle,
\] (S29)
can measure the phase relation between the Kittel modes at FMR in the two wires:
\[
S_{21}(\omega_F) = (1 - S_{11}(\omega_F))\chi(k_r)e^{ik_r(R_2-R_1)}.
\]

Here \(\chi(k_r)\) stands for the ratio of magnetizations in the right and left nanowires which is defined in the main text. At the special microwave frequencies \(\omega_n = \omega_F = \pi c_r(n + 1/2)/(R_2 - R_1)\), where \(n\) is a non-negative integer, \(S_{11}(\omega_F)\) is real while the phase factor \(e^{ik_r(R_2-R_1)}\) is \((-1)^n\) becomes purely imaginary and \(\text{Re}S_{12}(\omega_F = \omega_n) = 0\) develops minima. However, the microwave transmission alone does not show the chirality of the excited displacement field, which must therefore be detected by other means.

We plot the real part of the transmission amplitude in Fig. S3 as a function of microwave frequency close to \(\omega_F = 3\) GHz and static magnetic field \(H_0\). Here, with \(w = 2.5\) μm and thickness \(d = 200\) nm for the YIG nanowires, the additional damping coefficient \(\alpha = 5.2 \times 10^{-5}\). The intrinsic magnetic damping is chosen as \(\kappa_{\text{m}} = 5 \times 10^{-5}\) for
0.15 MHz and the radiative damping $\kappa_\omega = 1$ MHz. The dips in the transmission are traced by the black contour in Fig. S3(a), while the horizontal dash lines correspond to the special frequencies $\omega_n = \pi c_r (n + 1/2)/(R_2 - R_1)$, where $n$ is a non-negative integer. Due to the magnon-phonon coupling, these horizontal lines are deformed to the anticrossings when $\omega_F \to \omega_n$. Indeed, we can understand these features by $S_{12}(\omega_F) \to (1 - S_{11}(\omega_F)) e^{ik_r (R_2 - R_1)}$, where $S_{11}(\omega_F)$ is real while the phase factor $e^{ik_r (R_2 - R_1)} = i (-1)^n$ becomes purely imaginary and then $\text{Re} S_{12}(\omega_F = \omega_n) = 0$. The dips in the transmission on the FMR resonance line are shown in Fig. S3(b). We note that the transmission does not vanish at dips as it becomes purely imaginary. For $R_2 - R_1 = 300 \mu m$ the frequency spacing between these dips $\Delta \omega = \pi c_r /(R_2 - R_1) = 34.26$ MHz.

FIG. S3. (Color online) Microwave transmission ($|\text{Re}(S_{21})|$) between two YIG nanowire transducers on top of a GGG substrate. (a) The minima in the transmission (black contour) anticross with the FMR at regularly spaced frequencies $\omega_n$. (b) Transmission spectra at the FMR frequency. The transmission dips originate from the thin black contour hidden within the FMR peaks (yellow region).