Particle production in $p - p$ collisions at $\sqrt{s} = 17$ GeV within the statistical model

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I. INTRODUCTION

The statistical model has been used to describe particle production in high-energy collisions for more than half a century [1]. In this period it has evolved into a very useful and successful model describing a large variety of data, in particular, hadron yields in central heavy-ion collisions [2, 3] have been described in a very systematic and appealing way unmatched by any other model. It has also provided a very useful framework for the centrality [4] and system-size dependence [5, 6] of particle production. The applicability of the model in small systems like $p - p$ [7] and $e^+ - e^-$ annihilation [8] has been the subject of several recent publications [9, 10, 11].

The statistical-model analysis of elementary particle interactions can be summarized by the statement that the thermal parameters show almost no energy dependence in the range of $\sqrt{s} = 14 - 900$ GeV with the temperature being about 165 MeV and the strangeness undersaturation factor $\gamma_S$ being in the range between 0.5 and 0.7.

In the context of the system-size dependence of particle production, the $p - p$ collisions at $\sqrt{s} = 17$ GeV have been analyzed in detail recently. Based on similar data sets, the extracted parameters in different publications deviated significantly from each other: in a previous analysis (Ref. [12]) we derived $T = 164 \pm 9$ MeV and $\gamma_S = 0.67 \pm 0.07$, with $\chi^2/n = 1.7/3$, while the authors in Ref. [6] obtained $T = 178 \pm 6$ MeV, $\gamma_S = 0.45 \pm 0.02$ with $\chi^2/n = 11/7$. These findings motivated different conclusions: In Ref. [6] no system size dependence of the thermal parameters was found, except for $\gamma_S$ which tends to increase when more nucleons participe in the collisions but this rise is weaker than the errors on the strangeness suppression parameter. In Ref. [5] it was therefore concluded that the hadron gas produced in central collisions at $\sqrt{s} = 17$ GeV reaches its limiting temperature. Based on Ref. [6] on the other hand, it was argued in Ref. [12] that, decreasing $\gamma_S$ and, in particular, increasing temperature towards smaller systems allow for probing QCD matter beyond the freeze-out curve established in Pb-Pb and Au-Au collisions [13, 14].

The goal of this paper is to understand the origin of these rather different thermal-model results obtained in the analysis of $p - p$ data. We use an up-to-date complete set of data and discuss the sensitivity of the thermal model parameters on their values. We present systematic studies of data used as inputs and the methods applied in their thermal model analysis.

The paper is organised as follows: In Section II we discuss the experimental data on which different analysis are based. In Section III we summarize the main features of the statistical model and present the analysis of the SPS data obtained in $p - p$ collisions. In the final section we present our conclusions and summarize our results.

II. DATA

The data used throughout this paper for hadron yields in $p - p$ collisions at $\sqrt{s}=17.3$ GeV are summarized in Table I. Data in column Set A were exploit in our previous analysis [11] and the corresponding references are given in the table. If the numerical values deviate in the analysis of Ref. [12], they are listed in column Set B. The relative differences between the particle yields from sets A and B are also indicated in Table I. The commonly used data in the statistical model description of particles production in $p - p$ collisions at the SPS are displayed below the horizontal line in Table II.

In Table III the experimental data are grouped in sets which are used in Section III to perform the Statistical
Model analysis. In the following we motivate the particular choice of data in these sets and discuss how they can influence the model predictions on thermal conditions in $p - p$ collisions.

The data set $A I$ is most restricted. Firstly, the production yields of $\Xi$ and $\Omega$ are not included because their numerical values are only preliminary (Ref. [21]). Secondly, the $\Lambda^*$ resonance is also not included so as to restrict the analysis to stable hadrons. Finally, the $\phi$ meson is omitted in $A I$ since this particle is difficult to address in the statistical model due to its hidden strangeness as discussed in Ref. [8].

The lower yields of charged kaons in $B$ of Table II are taken from results published in conference proceedings [18]. Such kaon yields are in disagreement with trends from data measured at lower and higher energies as seen in Fig. 1.

The left panels of Fig. 1 show the charged kaon multiplicities from $p - p$ interactions at lower and higher beam momenta [22] together with data from Table II. The lines in this figure are simple parametrizations interpolating to SPS energies. The $K^-/h^-$ yield from [16] is seen to be 7% below the expected value from the above parametrization, however agrees within errors. The $K^-$ abundance from [16] is by 24% lower and its error is only 10%. As we discuss below, such a low value for the multiplicity of charged kaons influences the statistical model fit in an essential way.

The upper right panel of Fig. 1 shows the negatively charged pions from $h^-$ and then by rescaling the expected result for the $p_{lab}$ dependence of the negatively charged pions. The yields of $\pi^-$ at SPS from Table II agree quite well with that expected from an interpolation line shown in Fig. 1. They are only slightly higher, by 1% for yields taken from [16] and by 5% for yields used in Ref. [6].

The lower right panel in Fig. 1 shows the $K^-/\pi^-$ ratio at SPS compared to the interpolated data from other beam momenta. The mean value of the $K^-/\pi^-$ used in [11] is 8% below the interpolated line but agrees within errors, while the corresponding value used in [6] is 28%
smaller and exhibits an error of only 11%. Clearly, the above differences in the $K^-/\pi^-$ ratios influence the thermal model fits.

In general, a smaller kaon yield implies a stronger suppression of the strange-particle phase space resulting in a smaller value for the strangeness undersaturation factor $\gamma_S$. If other strange particles are included, then the strong suppression caused by $\gamma_S$ has to be compensated by a higher temperature. This might be one of the origins for the different thermal fit parameters obtained in Refs. [11] and [6]. In order to quantify this we have selected a data set $A4$ which is equivalent to the Set $A1$ but with the kaon yields of Ref. [12] being replaced by the values from Ref. [18].

The Set $B1$ is (besides the $\Lambda^*$) equivalent to $A1$ but with numerical values for particle yields from column $B$ in Table I. The Set $B4$ is used to demonstrate the influence of the $\Lambda^*$ resonance on thermal fit parameters. The Sets $A3$, $B3$ and $A2$, $B2$ are chosen to study the influence of the $\phi$ meson and the multistrange hyperons on thermal fit parameters.

III. STATISTICAL MODEL ANALYSIS

The usual form of the statistical model formulated in the grand-canonical ensemble cannot be used when either the temperature or the volume or both are small. As a rule of thumb one needs $VT^3 > 1$ for a grand-canonical description to hold [24, 25]. Furthermore, even if this condition is matched but the abundance of a subset of particles carrying a conserved charge is small, the canonical suppression still appears even though the grand-canonical description is valid for the bulk of the produced hadrons. There exists a vast literature on the subject of canonical suppression and we refer to several articles (see e.g. [3, 26, 27]).

The effect of canonical suppression in $p-p$ collisions at ultra-relativistic energies is relevant for hadrons carrying strangeness. The larger the strangeness content of the particle, the stronger is the suppression of the hadron yield. This has been discussed in detail in [28].

In line with the previous statistical model studies of heavy-ion scattering at lower energies, the collisions of small ions at SPS revealed [3] that the experimental data show stronger suppression of strange-particle yields than what was expected in the canonical model [3, 24, 30]. Consequently, an additional suppression effect had to be included in order to quantify the observed yields. Here we introduce the off-equilibrium factor $\gamma_S \leq 1$ which reduces densities $n_s$ of hadrons carrying strangeness $s$ by $n_s \rightarrow n_s \cdot \gamma_S^{|s|}$ [25].

We investigate whether or not all quantum numbers have to be conserved exactly in $p-p$ collisions within a canonical approach by comparing data with two model settings:

- Canonical (C) Model: all conserved charges, i.e. strangeness, electric charge and baryon number are conserved exactly within a canonical ensemble.

- Strangeness Canonical (SC) Model: only strangeness is conserved exactly whereas the baryon number and electric charge are conserved on the average and their densities are controlled by the corresponding chemical potentials.

The parameters of these models are listed in Table III. In the following we compare predictions of the above statistical models with $p-p$ data summarized in different sets discussed above.

A. Comparative study of $p-p$ data at SPS

We start from the analysis of data set $A1$ and modify it stepwise to find out in which way one matches the conclusion of larger temperature in $p-p$ than in central $A-A$ collisions at SPS as indicated in Ref. [6]. All numerical values of model parameters are listed in Table IV. A detailed discussion on their choice and correlations is presented in the Appendix based on the $\chi^2/n$ systematics.

The fit to data set $A1$ in the SC model complies with our previous analysis from Ref. [11], see also Fig. 2 (top). The SC model fit to these data does not change when including $\Lambda^*$ hyperons resulting in the same values of thermal parameters and their errors as summarized in Table IV.

The most striking effect on thermal parameters is expected when replacing the kaon yields in Set $A1$ (Fig. 3 top) by those from Ref. [18]. Set $A1$, Fig. 2 (bottom). Indeed, smaller kaon yields cause an increase of temperature and a decrease of $\gamma_S$. These changes come along with a reduced volume and in case of the SC fit with increase of the baryon chemical potential. The kaons from Ref. [18] dominate the fit because their errors are 10%
while the uncertainties of the $K^+$ and $K^-$ yields, taken from Ref. [16], are 21% and 31%, respectively. Consequently, the smaller errors dominate the statistical model fit.

In the next step we add $Λ^*$, $Ξ$ and $Ω$ hyperons resulting in Set A2, see Fig. 3 middle panel. The measured hyperon multiplicities coincide with the model results obtained before, thus within errors the statistical model parameters remain unchanged. We focus on Set A3 and add the $ϕ$ meson, Fig. 3 bottom. In this case the temperature is indeed much higher. In the SC model the temperature is very high and $γ_s$ drops below 0.5. We can conclude that in the case where the $ϕ$ meson is included in the fit, one needs to apply the C analysis to get lower temperatures, however with very small values of $γ_s$ and a large $χ^2/n$. For Set B3 the numerical results for $T$ and $γ_s$ summarized in Table IV coincide with that obtained in Ref [6], however with a larger $χ^2/n$ [31].

**IV. DISCUSSION AND SUMMARY**

The statistical-model analysis of hadron yields for $p − p$ collisions at $\sqrt{s} = 17$ GeV from Refs. [5] and [6], yield different results and lead to different conclusions on the system-size dependence of thermal parameters [2,12]. In this paper we have reanalyzed the $p − p$ data and studied the sensitivity of the thermal fit to data selection and on model assumptions. We have shown that different

| SC model results | $T$ (MeV) | $γ_s$ | $R$ (fm) | $μ_B$ (MeV) | $χ^2/n$ |
|------------------|-----------|-------|---------|-------------|---------|
| A1 163 ± 5  0.68 ± 0.05 1.50 ± 0.11 208 ± 14 1.7/4 |
| A2 168 ± 1  0.66 ± 0.02 1.37 ± 0.03 221 ± 8  8.6/9 |
| A3 174 ± 1  0.59 ± 0.03 1.23 ± 0.10 233 ± 16 5.1/4 |
| A4 175 ± 5  0.56 ± 0.01 1.24 ± 0.01 240 ± 12 7.7/7 |
| A1 176 ± 1  0.61 ± 0.02 1.19 ± 0.09 242 ± 18 16/9 |
| A2 179 ± 5  0.51 ± 0.03 1.30 ± 0.09 250 ± 14 0.5/3 |
| A3 180 ± 4  0.52 ± 0.03 1.22 ± 0.07 253 ± 16 3.5/3 |
| A4 177 ± 5  0.50 ± 0.03 1.31 ± 0.09 258 ± 14 4.8/3 |

| C model results | $T$ (MeV) | $γ_s$ | $R$ (fm) | $χ^2/n$ |
|------------------|-----------|-------|---------|---------|
| A1 175 ± 5  0.57 ± 0.04 1.33 ± 0.09 – 0.5/3 |
| A2 174 ± 4  0.59 ± 0.02 1.34 ± 0.08 – 6.6/8 |
| A3 189 ± 5  0.46 ± 0.02 1.12 ± 0.09 – 23/5 |
| A4 181 ± 4  0.52 ± 0.03 1.22 ± 0.07 – 3.5/3 |
| A1 177 ± 5  0.51 ± 0.03 1.30 ± 0.09 – 6.8/4 |
| A2 180 ± 4  0.56 ± 0.02 1.23 ± 0.08 – 18/8 |
| A3 178 ± 5  0.45 ± 0.02 1.30 ± 0.10 – 19/5 |
| A4 177 ± 5  0.50 ± 0.03 1.31 ± 0.09 – 4.8/3 |

FIG. 2: The $χ^2$ scan in the ($T$–$γ_s$)-plane. Starting from its minimum, $χ^2$ increases by 2 for each contour line. Upper figure: fit to data set A1 in the model where only strangeness is conserved exactly (SC). Lower figure: fit to data set A4 in the canonical (C) model. The minima are indicated by the crosses.
conclusions from Refs. 3 and 6 are mostly due to differences in data selections.

Slightly different numerical values for charged pions and Λ hyperons used in Refs. 3 and 6 as well as the contribution of the Λ* resonance altered thermal parameters only within errors. However, the used charged kaon yields in both approaches differ substantially. We have argued that data of kaon yields in Ref. 6 deviate from trends seen in data at different energies resulting in a higher temperature.

We have shown that higher kaon yields expected from the systematics in the energy dependence in p − p collisions are in line with data on multi-strange baryons. Unlike the hyperons, when adding the φ meson the thermal model fit leaves a reasonable range of parameters resulting in a very high temperature exceeding 190 MeV and large χ²/n. We have quantified the modifications of these results when including an exact conservation of all quantum numbers in the canonical statistical model. We have shown that in the absence of φ meson the thermal fits are rather weakly influenced by canonical effects due to an exact conservation of the baryon number and an electric charge leading in some cases to a systematic increase of the freezeout temperature. Fits including the φ meson are sensitive to an exact conservation of all quantum numbers resulting in lower temperatures. However, the thermal model analysis of data sets with hidden strangeness has the largest χ²/n indicating that this particle cannot be addressed properly in this model.

From our analysis, we conclude that within the presently available data on p − p collisions at SPS energy and uncertainties on thermal parameters obtained from fits within the statistical model, it is rather unlikely that the temperature in p − p collisions exceeds significantly that expected in central collisions of heavy ions at the same energy.

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APPENDIX A: THE χ² CONTOURS OF THERMAL FITS

In this appendix we quantify the choice of thermal parameters within the statistical model through χ²-contours in the parameter space. Since the temperature T and γS are of particular interest here the quality of the fits are shown in Figs. 2 and 3 in the (T − γS)-plane. In these figures for fixed (T, γS)-pair the remaining model parameters were fitted and the corresponding χ² was calculated.

Figure 2 (top) shows the analysis of the data Set A1 within the strangeness canonical (SC) model. The analysis in the model with canonical treatment of all conserved charges (C) is shown in Fig. 3 for all data sets besides the Set A4 which is presented in Fig. 2 (bottom).

In the C model description of data Set A1 there is a large region of a very low χ² which manifests the expected
anti-correlation of $T$ and $\gamma_S$. Reasonable fits are possible over a large range of parameters. For the Set A2 the minimum is located at the same temperature and slightly higher $\gamma_S$. The contributions of $\Xi$ and $\Omega$ baryons disfavor small values of $\gamma_S$.

The $\phi$, as seen in Fig. 3, directs fits towards very high temperatures and very strong strangeness suppression. Also the pattern of $(T - \gamma_S)$ anti-correlations shows decreasing $\chi^2$ with increasing temperature at fixed $\gamma_S$.

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