Valuation of Hong Kong REIT Based on Risk Sensitive Value Measure Method*

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\textbf{Abstract:} Utility indifference net present value (UNPV) method is a risk assessment method proposed by Miyahara \cite{8}, which is based on utility indifference pricing theory and net present value theory. Setting utility function as that of an exponential type, we formulate a special type of UNPV which is called a risk sensitive value measure (RSVM). With RSVM, the risk to investors caused by uncertainty about future cash inflow as well as scale effect which influence the valuation of investments has been investigated. In this paper, we adopt RSVM method and mean-variance (MV) approach into Hong Kong REITs to observe the advantage and effectiveness of RSVM method. In particular, we focus on the performance of risk-sensitivity parameter and scale effect parameter in RSVM and MV. Then, as a new index of risk valuation, the inner rate of risk aversion (IRRA) is defined and it is shown that the index is useful for rating REITs and measuring scale effect of REITs. Finally, we treat the rating valuation of Hong Kong REITs investment by RVSM method under consideration of default risk. Then, the IRRA also effectively works on the decision-making of investment, choosing and rating of the commodities to invest in consideration of default risk.

\textbf{Keywords:} Risk sensitive value measure; Scale effect; Inner rate of risk aversion; Default risk; Hong Kong REITs

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1 Introduction

Project valuation is one of the most important problems in finance. One of the well-known methods to this problem is the net present value method (NPV). However, it has a limitation in that random complexities of cash flow caused by various future uncertainties are not sufficiently accounted for. To overcome this limitation, Miyahara has proposed “utility indifference net present value (UNPV) method,” which is based on utility indifference pricing theory and NPV method [1, 9]. The UNPV method employs a utility function to valuate investor attitude toward risk. Miyauchi and et. al. have adopted UNPV method to assess oil thermal electric power generation projects [10]. Furthermore, Misawa, Miyauchi and et.al. have proposed some probit models to “random net present value” derived from a simplification of UNPV method [2, 3, 5, 12]. These papers verify that UNPV method is useful for a practical project assessment.

The UNPV method can be used to deal with the valuation problem of random cash flows. From setting utility function as that of an exponential type, we can derive a special type of UNPV named as “risk sensitive value measure (RSVM).” Miyahara has shown that one can overcome the shortcomings of the ordinary NPV method by using RSVM as a valuation tool for investment [9]. Furthermore, he has investigated that RSVM method also has connection with scale risk as well as optimal scale of investment which influences the valuation [10, 11].

The present paper has two aims: one is to observe the performance of RSVM method through the application of RSVM to practical investment to asset, and the other is to find out the advantageous points of RSVM method thereby. For the first aim, we adopt Hong Kong Real Estate Investment Trust (REITs) as the practical asset, because there are no articles about “financial investment problem” of UNPV or RSVM method and the close connection between Hong Kong and Mainland China motivates us to study more about Hong Kong REIT.1 For the second aim, we compare the results with those of Mean-Variance (MV) approach which is a valuation of risky prospects based on the expected value and variance of possible outcome [10]; thereby we observe the advantage and effectiveness of RSVM method. Then, we pay attention to “two parameters” which are defined by RSVM. The first one is risk-sensitivity parameter α which reflects the investor’s attitude towards risk. The investor’s preference of risk will lead to various results on his decision of an investment. Therefore, it is necessary to study the performance of the risk-sensitivity parameter α and to see how it influences the valuation of the project. In consideration of α, a special risk-sensitivity parameter α0 is defined as a new index of risk valuation of investment, which is called “inner rate of risk aversion (IRRA).” It is shown that the index together with the following scale parameter is useful for rating and measuring scale effect of the commodities to invest. The second one is the parameter λ describing the scale effect of investment. RSVM method helps us find the optimal scale λ* easily and also provides with a new way to valuate scale risk [10, 11]. When the investor enlarges the scale of investment, the risk which used to be hidden might be displayed, and even leads to bankrupt in the worst condition; this is called “the scale risk.” By changing the parameter λ, the scale risk of investment could be investigated, and hence it is important to study this parameter. We also observe the performance of these parameters in practical

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1Frequently Asked Questions for Real Estate Investment Trust. Updated 2009-08-31. (http://www.hkex.com.hk/eng/global/faq/reit.htm)
Global Real Estate Investing. Retrieved 2012-12-18. (http://www.reit.com/Investing/GlobalRealEstateInvesting.aspx)
Real Estate Investment Trusts. Retrieved 2011-03-17. (http://www.hkex.com.hk/eng/prod/secprod/reit/reit.htm)
situation mentioned above.

In Section 2, we review the background of UNPV method and introduce our RSVM method. We refer to the properties of RSVM and explain the reason why RSVM is a suitable tool for project valuation. In order to show the effectiveness of RSVM method, we also give a brief introduction to Mean-Variance (MV) approach.

In Section 3, we first give a brief introduction to Hong Kong REIT and explain the reason why we choose Hong Kong REIT as the practical asset. Next, we show the way how to process data. We focus on the annual profits of 8 REITs in the year of 2011 and regard the each profit as cash flow which is occurred with equal probability. The 60 data are collected from August 24th to November 18th in 2011 and histogram of each REIT is made to observe the feature of cash flows. The data of REITs are classified according to their mean, variance as well as the feature of their distribution.

In Section 4, we observe the performance of the risk-sensitivity parameter $\alpha$ through an application of RSVM to valuation of Hong Kong REITs. RSVM method and MV approach are applied into Hong Kong REITs’ data, and through the comparison among them, we investigate how the parameter $\alpha$ influences the valuation. Finally, we define a new concept on risk valuation which is called “inner rate of risk aversion (IRRA) $\alpha_0$” as a preparation of the application to finding an optimal investment scale of each REIT and rating the REITs in the preceding sections.

In Section 5, to indicate an advantageous property of RSVM method, we investigate the scale effect in Hong Kong REITs valuation by RSVM and MV approach. By varying the value of scale effect parameter $\lambda$, we find out the difference between the valuation results of RSVM method and those of MV, and thereby we examine the property of RSVM method for $\lambda$. Moreover, we also apply the “inner rate of risk aversion $\alpha_0$” defined in Section 4 with scale effect parameter $\lambda$ to the issue mentioned above. To do that, we define an “IRRA judging table for all the REITs” on the basis of the values of $\alpha_0$ and $\lambda$, and thereby, we may provide a suitable investment scale for an investor. To examine this, some examples on the scale effect in the investment to Hong Kong REITs are illustrated.

In the application of RSVM method to practical investment issue, it should be natural to take an account of “default risk.” From this point of view, in Section 6, we proceed to the rating valuation of Hong Kong REITs investment by RVSM method under consideration of default risk. We explain how to process the data and then give some examples on rating valuation through RSVM method by making the “IRRA judging table for all the REITs with default risk.” The result will suggest us that RSVM may become an effective rating tool in practical investment.

Finally, in Section 7, we state the results on the comparison between RSVM and MV to emphasize the desirable properties of RSVM, and next we summarize the other results and future issues related to this paper.

2 Preliminaries

2.1 UNPV Method

At the beginning of this section, we give some basic notations and a review of the UNPV method [1, 5, 9].

\[ \text{Searchina Finance. (http://stock.searchina.ne.jp/)} \]
It is well-known that NPV method is used in capital budgeting to analyze the profitability of an investment or project. However, it has a limitation in that random complexities of cash flow caused by various future uncertainties are not sufficiently accounted for. To break this limitation, we regard the cash flow in future as random variables. Let \( X = \{X_1, X_2, \ldots, X_T\} \) be a random cash flow. Then, we introduce the random present value \( \text{RPV}(X) \) as follows:

Let \( \hat{X}_t \) be a random variable satisfying

\[
\hat{X}_t = \text{RPV}(X_t) = \frac{X_t}{(1+r)^t},
\]

(1)

Then we define \( \text{RPV}(X) \) as

\[
\text{RPV}(X) = \sum_{t=1}^{T} \hat{X}_t = \sum_{t=1}^{T} \frac{X_t}{(1+r)^t},
\]

(2)

where \( t \) is the designated time and \( r \) is a discount rate. Suppose that \( I \) represents the present cost of a project/an investment. Then in an analogous way to that in the ordinary NPV method, we define “random net present value (RNPV)” by using \( \text{RPV}(X) \) and as

\[
\text{RNPV}(X) = \text{RPV}(X) - I.
\]

(3)

In our UNPV method, the RNPV is evaluated by utility indifference price. Hence we give definitions of utility function and utility indifference price.

**Definition 1 (Utility Function)** A real valued function \( u(X) \) defined on \( \mathbb{R}^1 \) is called a “utility function” if it satisfies the following conditions:

1) \( u(X) \) is continuous and non-decreasing,

2) \( u(X) \) is concave,

3) \( u(0) = 0. \)

**Definition 2 (Utility Indifference Price)** The indifference price of \( X \), denoted by \( p(X) \), is the solution of the following equation:

\[
E[u(X - p(X))] = u(0) = 0,
\]

(4)

where \( E[\cdot] \) denotes the expectation. Then we apply utility indifference price to random net present value.

**Definition 3 (Utility Indifference Net Present Value (UNPV))** Suppose that \( u(X) \) is the utility function of an investor. We call \( \nu \) satisfying the following equation (5) “utility indifference net present value (UNPV)” for the random cash flow \( X \), and denote it by UNPV(X):

\[
E[u(-\nu + \text{RNPV}(X))] = 0.
\]

(5)

The \( \nu \) satisfying (5) means a sort of worth of the RNPV(X) for the investor having the utility function \( u(X) \). Indeed, the expected return is equal to 0, if the value \( \nu \) is paid for the right to obtain the uncertain RNPV(X). In this context, \( \text{RNPV}(X) \) and \( \nu \) are balanced [5].
Remark 1 In a similar way, we name the utility indifference price of “RPV(X)” “utility indifference present value (UPV)” for X, and denote it by $UPV(X)$. It is obvious that the following equation holds:

$$UNPV(X) = -I + UPV(X).$$

Remark 2 If utility function is given as $u(x) = x$ (risk neutral type), UNPV coincides with ordinary NPV [1, 5, 9].

Remark 3 In a similar way with the ordinary NPV method, the investor with a utility function $u(x)$ may make a decision whether to carry out a project (or an investment) or not. That is,

If $UNPV > 0$, he accepts the project/investment;

If $UNPV < 0$, he rejects the project/investment.

Using UNPV method, the risk to investors caused by uncertainty about future cash inflow as well as scale effect which influence the valuation have been taken into consideration. Miyauchi and et.al. have applied UNPV method to assess oil thermal generation projects [10]. Misawa, Miyauchi and et.al have proposed some probit models for RNPV derived from a simplification of utility indifference net present value method [2, 5, 12]. These papers indicate that UNPV method is useful for a project assessment.

Remark 4 (Utility Internal Rate of Return (UIRR)) In an analogous way to that in IRR (internal rate of return) method, Misawa provides the definition of utility internal rate of return method on the basis of (5) [6] as follows:

Definition 4 (Utility Internal Rate of Return) If a discount rate $r_0$ satisfies

$$E[u(-r + RNPV(X))] = 0. \tag{6}$$

$r_0$ is called a “utility internal rate of return (UIRR),” and we denote it by UIRR.

Suppose that $r^*$ is a target rate of return for investor. Then, if $UIRR > r^*$ and UNPV is a monotone decreasing function of $r$ for a given random cash flow, the investor accepts the project or investment, and if $UIRR < r^*$, the investor rejects the project or investment. For details of UIRR with some examples, see [6]. By setting utility function as that of the following exponential type

$$u(X) = \frac{1}{\alpha} \left( 1 - e^{-\alpha X} \right), \quad \alpha > 0. \tag{7}$$

We can correlate the UNPV with a “risk sensitive value measure” which has such suitable properties as a value measure to valuate project or investment containing uncertainty risk should have. In the following subsections 2.2 and 2.3, the value measure is investigated.
2.2 Value Measures

Now we proceed to “risk sensitive value measure” defined in Miyahara [9]. To begin with, we first review a concept of value measures and some suitable conditions to valuate projects or assets which such a measure should satisfy. Let $X$ be a random variable related to the return of investment with risk. In the case of UNPV, $X$ is corresponding to RPV which is treated as a discounted sum of random cash flow. A valuation function or a value measure is a real valued function defined on a linear space $L$ of random variables $X$ as $\nu(X) \in (-\infty, \infty)$, $X \in L$ with the property that $\nu(m) = m$ if $m = \text{constant}$ [9]. To measure a value of $X$, the concept of concave monetary value measure (or concave monetary utility function) has been introduced. (See Cheridito et al. [1], for example). The properties of a concave monetary value measure are thought to be requirements for a suitable value measure.

**Definition 5 (Concave Monetary Value Measure)** A functional $\nu(X)$ defined on a space $L$ of random variables is called a “concave monetary value measure,” if it satisfies the following conditions:

1) (Normalization) 
   $\nu(0) = 0$, 

2) (Monotonicity) 
   a) If $X \leq Y$, then $\nu(X) \leq \nu(Y)$, 
   b) If $X \leq Y$ and $P(\{\omega; X(\omega) < Y(\omega)\}) > 0$, then $\nu(X) < \nu(Y)$, 

3) (Translation invariance, or Monetary property) 
   $\nu(X + m) = \nu(X) + m$, where $m$ is non-random, 

4) (Concavity) 
   $\nu(\lambda X + (1 - \lambda)Y) \leq \lambda \nu(X) + (1 - \lambda)\nu(Y)$ for $0 \leq \lambda \leq 1$, 

5) (Law invariance) 
   $\nu(X) = \nu(Y)$ whenever $\text{law}(X) = \text{law}(Y)$.

**Remark 5** If $m$ is deterministic (non-random), $\nu(m) = m$.

**Remark 6** All of the followings could happen: 

$$\nu(X + Y) > \nu(X) + \nu(Y), \nu(X + Y) = \nu(X) + \nu(Y), \nu(X + Y) < \nu(X) + \nu(Y).$$

**Remark 7** 

$$\nu(X) + \nu(-X) \leq 0.$$ 

The inequality in Remark 7 means that the investor, who obeys to the concave monetary value measure, is more sensitive to the risk rather than to the profit. (see [9], for details).

Finally, as an important fact, we should note that utility indifference price is a concave monetary value measure. This is shown in Miyahara [9].
2.3 Risk-Sensitive Value Measure Method

As mentioned above, we can make a concave monetary value measure from the class of utility indifference price $p(X)$, which has the properties to be requirements for a suitable value measure. Miyahara provides the definition of “risk-sensitive value measure (RSVM) on the basis of the utility indifference price $p(X)$ for exponential utility function (7) as follows [9]:

**Definition 6 (Risk-Sensitive Value Measure)** The utility indifference price $p(X)$ for exponential utility function (7) is called a risk-sensitive value measure, which is given by

$$U^\alpha(X) = -\frac{1}{\alpha} \log E[e^{-\alpha X}], \quad \alpha > 0.$$ (8)

The RSVM is one of concave monetary value measures, since it is a sort of utility indifference price. In what follows, we call valuation of investments by using RSVM “RSVM method.”

**Remark 8** Assume that the distribution of $X$ is Gaussian. Then, it is easy to verify that

$$U^\alpha(X) = -\frac{1}{\alpha} \log E[e^{-\alpha X}] = E[X] - \frac{1}{2}\alpha V(X),$$ (9)

where $E[X]$ and $V(X)$ are the mean and variance of $X$, respectively. Thus, RSVM for $X$ having a normal distribution represents only the mean and variance of $X$ [10]. Moreover, the right hand-side of (9) is just corresponding to the result derived from the expected utility function of investors in the famous mean-variance approach to portfolio selection theory. In consideration of this, we denote the right hand-side of (9) as MV, and call valuation of investments by using MV “MV approach” in the followings. We will use MV approach in Section 4, Section 5 and Section 6 to compare with RSVM method.

In consideration of the definition of UNPV mentioned in Section 2.1, we see that RSVM is a special case of UNPV. RSVM has the properties of UNPV as well as other advantages, which will be introduced in the following. For this purpose, we first define the concept of “independence-additivity.”

**Definition 7 (Independence-Additivity)** Suppose that a value measure $\nu(X)$ satisfies the following condition:

(Additivity): If $X$ and $Y$ are independent, then $\nu(X + Y) = \nu(X) + \nu(Y)$.

Then we call that $\nu(X)$ satisfies the independence-additivity condition.

Here, we briefly sum up the characters of RSVM according to [9]:

1) RSVM is one of concave monetary value measures.

2) RSVM is a utility indifference price on the basis of an exponential utility function (7).

3) RSVM is the only measure which satisfies the independence-additivity condition in the class of utility indifference price.

4) The investors’ attitude to risk is reflected by parameter in RSVM.

5) RSVM is connected with scale risk of investment.
These facts suggest us that RSVM is the most possible candidate for the suitable value measure to valuate investment with risk.

**Remark 9** When we take scale effect into consideration of RSVM, we set scale parameter as $\lambda$ and reset the exponential utility function as

$$u(\lambda X) = \frac{1}{\alpha}(1 - e^{-\alpha \lambda X}).$$

(10)

Then Risk-Sensitive Value Measure with scale effect becomes

$$U^\alpha(\lambda X) = -\frac{1}{\alpha} \log E[e^{-\alpha \lambda X}], \quad \alpha > 0, \quad \lambda > 0.$$  

(11)

In the followings, we will focus on the character of RSVM, particularly for character (4) and character (5), mentioned above. For this purpose, we finally provide some remarks on the investigation of the performance of RSVM and two parameters in RSVM to be studied mainly.

Firstly, we touch upon the examination of some advantageous properties of RSVM. For this purpose, we will compare the performance of RSVM together with that of “mean-variance (MV) approach” mentioned in Remark 8 and the below. The MV approach is a valuation of risky prospects based on the expected value and variance of possible outcomes [4, Ch. 6], and MV for the investor with the utility function of exponential type is given by (9). If we consider the scale parameter $\lambda$, the equation becomes

$$MV = \lambda E[X] - \frac{1}{2} \lambda^2 \alpha V(X).$$

(12)

Thus, MV approach naturally contains the scale effect in the valuation of investment. On account of this fact together with the corresponding to the ordinary mean-variance approach, we adopt MV approach as a popular valuation method to risk in order to identify the effectiveness of RSVM method in the latter sections. We are going to apply RSVM method and MV approach to investment to Hong Kong REIT and observe the difference of their results in Section 4.

Next we proceed to remark two important parameters in RSVM. The first one is risk-sensitivity parameter $\alpha$. The parameter means the investor’s attitude towards risk. The investor’s preference to risk will lead to different result in decision-making for investment, and hence, it is necessary to study the performance of the risk-sensitivity parameter $\alpha$ and to see how it influences the valuation of the project. This issue is dealt with in Section 4.

The second one is the scale effect parameter $\lambda$. As mentioned above, using RSVM we may easily find out the optimal scale $\lambda^*$, and thereby RSVM method also gives us a new way to valuate scale risk. In general, the investors may accept the risk for investment to a small scale of project or asset, since the loss of investment is not so large, even if it occurs. However, if they treat a large scale of investment, the unacceptable risk which used to be hidden might be come out, and even leads them to “default” in the worst condition. This kind of risk is called scale risk. From this point of view, Miyahaya indicates a numerical example of RSVM with scale effect [10, 11]. In a way similar to that, we will apply RSVM method to a practice investment and study the performance of parameter $\lambda$ in Section 5.
3 Hong Kong REIT and the Empirical Data

3.1 Hong Kong REIT

A Real Estate Investment Trust (REIT) is a collective investment scheme that aims to deliver a source of recurrent income to investors through focused investment in a portfolio of income-generating properties such as shopping malls, offices, hotels and service apartments. REITs have lower risk compared to stocks. REIT originates from USA, and nowadays spread worldwide. REITs did not emerge in Hong Kong until 2005 although they had variants all over the global. Even today, there is no REIT in Mainland China. However, the intimate connection between Hong Kong and Mainland China stimulates us to study more about Hong Kong REIT.

Here, we touch upon the reasons why we study Hong Kong REIT instead of other financial investment. There are two reasons. The first one is that REITs is still on its way to mainland China up to now. The close connection between Hong Kong and Mainland China strongly motivates us to study on Hong Kong REITs. The second one is that all the data about Hong Kong REITs are easily collected from the Internet, which gives us convenience to make data analysis.

Several risk factors should be taken into consideration before investing in REIT. In general, there are mainly three causes for the risk factor as follows: firstly, the total return of REIT is subject to the performance of the property market. Secondly, the unit price of a REIT may decrease if its properties decline in value. And thirdly, dividends may not be paid if the REIT reports an operating loss. Therefore, investors should think out the concentration, quality and length of its property leases, rather than stare at the expected yield only. However in this thesis, regardless of all other risk factors in Hong Kong REITs, we only pay attention to the close price of an REIT to study the distribution of returns. In what follows, we will valuate different Hong Kong REITs by RSVM method and MV approach as mentioned in the end of the previous section. We assume that all the decision-making of investment should be determined by the results calculated by RSVM and MV method.

3.2 How to Process Empirical Data

In order to study a number of Hong Kong REITs during the same period, we focus on the annual profits of REITs in the year of 2011 and regard the each profit as random cash flow which is occurred with equal probability. It is after the bankruptcy of Lehman Brothers Holdings Inc. and is the only period that 8 REITs exist at the same time. (Their REIT number are 00405, 00435, 00778, 00808, 00823, 01881, 0278 and 87001, respectively.) Note that at this period, most of the Hong Kong REITs’ price kept increasing. This means the possibility of positive returns is very high. The existence of positive returns is the basic foundation to accept an investment, and hence we adopt this period as an illustrative example. Then as “initial” close price, we collect 60 data starting from August 24th to November 18th in 2011. We denote the price of REIT i in 2011 by $P_{i,k}^{2011}$ and the same REIT i’s close price one year later (2012) by $P_{i,k}^{2012}$, where k is the data number, that is, $k = 1, \ldots, 60$ correspond to August 24th, ..., November 18th, respectively.

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3Real Estate Investment Trusts. Retrieved 2011-03-17. (http://www.hkex.com.hk/eng/prod/secprod/reit/reit.htm)
4Searchina Finance. (http://stock.searchina.ne.jp/)
Table 1: Sample-case for profits of Hong Kong REITs in 2011.

| REIT | Mean  | Variance |
|------|-------|----------|
| 405  | -0.00167 | 0.005966 |
| 2778 | 0.107157 | 0.01385  |
| 1881 | 0.145151 | 0.012321 |
| 87001| 0.189207 | 0.021227 |
| 823  | 0.383844 | 0.005996 |
| 808  | 0.436099 | 0.01219  |
| 435  | 0.449073 | 0.008874 |
| 778  | 0.663955 | 0.006803 |

Figure 1: The chart for profits of REITs in 2011.

For each $k$, we calculate rate of return $R_k$ as

$$R_k = \frac{P_{i;k}^{2012} - P_{i;k}^{2011}}{P_{i;k}^{2011}}, \quad k = 1, 2, \ldots, 60,$$

(i = 00405, 00435, 00778, 00808, 00823, 01881, 0278 and 87001).

Let $I_0$ be an initial investment (10,000 yen for one unit. Ignore most of the Hong Kong REITs is transacted in Hong Kong Dollar and other facts). For each $k$, annual profits $X_k$ are given by

$$X_k = R_k \times I_0 = \frac{P_{i;k}^{2012} - P_{i;k}^{2011}}{P_{i;k}^{2011}} \times I_0, \quad k = 1, 2, \ldots, 60,$$

respectively. As mentioned above, for each $k$, we regard $X_k$ as a random cash flow which is occurred with the probability 1/60.

Now, we classify 8 REITs by their mean and variance which give the character of their distribution:

The table and figure show that the mean of REIT 00405 is minus; this may indicate that no matter what kind of strategy, investment on this REIT should be rejected. For REITs 00823, REIT 00808, REIT 00435 and REIT 00778, all of their profits are positive. Since our RSVM method is more sensitive to negative returns, we can expect that RSVM might give a higher valuation on these 4 REITs. We are going to see the results in Section 4, Section 5 and Section 6. For REIT 02778, REIT 01881 and REIT 87001, their mean and variance are near to each other, and hence it might be interesting to make comparisons to each other.

As a result, we classify the above REITs as follows:
Group 1: REIT 00405,
Group 2: REIT 02778, REIT 01881 and REIT 87001,
Group 3: REIT 00823, REIT 00808, REIT 00435 and REIT 00778.

In order to observe the feature of cash flow (distributions of return), we make histogram of each REIT. In the followings, we are going to analyze the results of valuation by RSVM method and MV approach together with the help of histogram.

4 Risk-Sensitivity Analysis

4.1 Performance of Risk-Sensitivity Parameter $\alpha$

As mentioned in Section 2, the risk-sensitivity parameter $\alpha$ in RSVM reflects the investor’s attitude towards risk. In this section, we apply RSVM method and MV approach to Hong Kong REITs’ data to see how the parameter $\alpha$ influences the valuation of the investment to the REITs with risk.

First we note on the calculation of RSVM on the basis of the empirical data of REITs. As mentioned, RSVM with the parameter $\alpha$ and a scale parameter $\lambda$ for a random variable $X$ related to a random cash flow of investment is given by

$$RSVM = U(\lambda X) = -\frac{1}{\alpha} \log E[e^{-\alpha \lambda X}], \quad \alpha > 0, \quad \lambda > 0$$

In our example, we regard annual profits $X_k$ defined by (14) as the sample values of $X$. Then, the right hand-side of the equation (15) is reduced to

$$U(\lambda X) = -\frac{1}{\alpha} \log E[e^{-\alpha \lambda X}] = -\frac{1}{\alpha} \log \left( \frac{1}{60} \times \sum_{k=1}^{60} e^{-\alpha \lambda X_k} \right).$$

Hence we finally obtain

$$RSVM = U(\lambda X) = -\frac{1}{\alpha} \log \left( \frac{1}{60} \times \sum_{k=1}^{60} e^{-\alpha \lambda X_k} \right).$$

On the other hand, we find that the mean-variance (MV) of $X$ defined by (12) is put into

$$MV = \lambda \tilde{X} - \frac{1}{2} \alpha \lambda^2 \sigma^2,$$

where $\tilde{X}$ and $\sigma^2$ denote the sample mean and sample variance of annual profits $X_k$, respectively. Here we remark that the discounted rate $r$ is set as 0 for the convenience. We note that one may be regarded $\lambda/(1+r)$ as another scale parameter $\lambda'$, since $\lambda X/(1+r) = (\lambda/(1+r))X = \lambda'X$ holds for any cash flow $X$. Therefore, our results in the preceding sections are not essentially different from those in case that $r$ is not 0.

Now, we first compare the results of RSVM with those of MV by varying the value of $\alpha$. Note that in this section, we set $\lambda = 1$, since we observe the influence of $\alpha$. For a simplification, we only focus on the REIT 01881 as an example. Substitute the profits data $X_k$ of the REIT into (17) and (18) for a given $\alpha$, we obtain RSVM and MV of the REIT. The following figure is the histogram of
Figure 2: The histogram of REIT 01881

Figure 3: The result of valuation for REIT 01881
$X_k$ in REIT 01881 with the mean and variance: Here we assume that the initial investment $I_0$ is 100 units. If $\alpha$ changes from 0 to 1 with a small step of 0.025, then the results of RSVM (17) and MV (18) for REIT 01881 are illustrated in the following figure, respectively: This figure shows:

1) Both RSVM and MV decrease as $\alpha$ increases. Particularly, the curve to RSVM has strong concavity. This represents just a natural of concave value measure.

2) For small values of $\alpha$, RSVM decreases faster than MV.

To see the detail of 2), we reset the variation range of $\alpha$ on $[0, 0.4]$, and the following figure shows the results.

Then, from this result, we find out:

1) If $\alpha > 0.22$, $RSVM < 0$, and hence, according to RSVM, we should recommend such an investor whose risk sensitive parameter $\alpha$ is larger than 0.22, not to invest this REIT.

2) If $\alpha > 0.225$, $MV < 0$. According to MV, we should recommend such an investor as his risk sensitive parameter $\alpha$ is larger than 0.225, not to invest this REIT.

3) When $\alpha$ ranges from 0.03 and 0.31, RSVM shows lower valuation than MV.

Next, we observe the results of applying RSVM to all the REITs. They are indicated in the following figure:

Recall that we classified the REITs into the three groups in Section 3. This figure with the classification shows the followings:

1) The group 3 (REIT 00823, REIT 00808, REIT 00435 and REIT 00778) seems to be safety assets, since their values of RSVM are positive. Hence, no matter what kind of investors and no matter what attitude to risk they are, investment on these REITs should be acceptable.
2) For group 2 (REIT 02778, REIT 01881 and REIT 87001), the threshold of the investor’s decision-making to carry out the investment are 0.15 for REIT 02778, 0.2 for REIT 01881 and 0.25 for REIT 87001, respectively.

3) For group 1, as we have expected, investment would be rejected despite what attitude to risk the investors are.

4) For group 3, we find out an interesting fact related to REIT 00823 and REIT 00808.

Table 2 indicates that the difference between two mean values of these REIT is small; however, the variance of REIT 00823 is smaller than that of REIT 00808. When \( \alpha \) is so small, the RSVM value of REIT 00808 is higher than that of REIT 00823. On the contrary, when \( \alpha > 0.175 \), the result the RSVM value of REIT 00823 is higher than that of REIT 00808. This fact may show that RSVM correctly capture the value of risk for the investor under a given risk sensitive parameter.

Summing up the observations mentioned above, we have examined the RSVM is a decreasing measure function for the risk sensitive parameter \( \alpha \), and has a stronger concavity for the parameter than MV measure. In the next part, we are going to investigate the risk sensitive parameter \( \alpha \) of RSVM in the application of rating.

### 4.2 Inner Rate of Risk Aversion \( \alpha_0 \)

At the beginning of this part, we first give a definition of “inner rate of risk aversion” \( \alpha_0 \), which is proposed by Miyahara [11].
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Table 3: Inner rate of risk aversion for Group 2 (light color shows bigger number).

|    | 2778  | 1881  | 87001 |
|----|-------|-------|-------|
| mean | 10.71567 | 14.51511 | 18.92072 |
| variance | 138.5044 | 123.2079 | 212.2708 |
| \(\alpha_0\) | 0.16331 | 0.21785 | 0.26536 |

Definition 8 (Inner Rate of Risk Aversion) \(\alpha_0 = \alpha_0(X)\) is called the inner rate of risk aversion (IRRA) for \(X\), if it satisfies

\[
RSVM(\alpha, X) = U^\alpha(X) = -\frac{1}{\alpha} \log E[e^{-\alpha X}] = 0.
\]  

(19)

From the definition, we could understand that \(\alpha_0\) shows the degree of safety investment for the investors. Bigger \(\alpha_0\) shows higher level of safety. In other word, the value of \(\alpha_0\) means the minimum value of risk aversion of the investor for the asset. Therefore, the investor may invest to such an asset when the value of \(\alpha_0\) is larger, even if he has a high risk aversion. In this sense, we regard an asset with a large value of \(\alpha_0\) as a low risk asset. This is different from the concept of "risk" provided by a standard deviation \(\sigma\) in the framework of "mean-variance approach," and hence, it may be one of important characters of RSVM method.

Here we give a simple example on inner rate of risk aversion for group 2 (REIT 02778, REIT 01881 and REIT 87001). In this case, investment on REIT 87001 is much safer than investment on REIT 02778 and REIT 01881. Thus, we could suggest investors on their selection of REITs so that they could invest properly. For example, the investors whose risk sensitive parameter \(\alpha\) is 0.25, REIT 87001 would be one to choose for investment; however, it may be too large for the investors with the \(\alpha\) to invest to REIT 01881 and REIT 02778. In Section 5, we will combine scale parameter \(\lambda\) to IRRA, and apply them to the issue on ratings of REITs.

5 Scale Parameter \(\lambda\) and Scale Risk

In this section, we study on the scale effect parameter \(\lambda\) in RSVM. As mentioned in Section 2, RSVM method may give us a new way to valuate scale risk. In this section, applying the RSVM method into our empirical data of Hong Kong REITs, we will examine such an advantageous character of RSVM method.

5.1 The Character of the Parameter

First, we are going to apply RSVM method and MV approach to 3 different groups of 8 Hong Kong REITs, respectively, and compare the difference between the two results in each group.

As in Section 4, we calculate RSVM and MV on the basis of (17) and (18) for the annual profits \(X_k\), respectively. The results are given as follows:

\[
RSVM = U^\alpha(\lambda X) = -\frac{1}{\alpha} \log \left( \frac{1}{60} \times \sum_{k=1}^{60} e^{-\alpha \lambda X_k} \right),
\]  

(20)

\[
MV = \lambda \bar{X} - \frac{1}{2} \alpha \lambda^2 \sigma^2.
\]  

(21)

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Then, we choose 0.1, 0.5 and 0.8 as the value of the risk-sensitivity parameter $\alpha$ in (20) and (21), and study how the parameter $\lambda$ affects the valuation by RSVM under each value of $\alpha$.

Now, we will also treat the REIT 01881 from Group 2 as an instance, because the distribution spreads to both negative and positive values of $X_k$. Note that we have already shown the histogram, calculated mean and variance. Before observing the results of RSVM and MV, we define a concept of “maximum scale” besides optimal scale defined in Section 2:

**Definition 9 (Optimal Scale and Maximum Scale)** $\lambda_{opt}$ is called the optimal scale, if $\lambda_{opt}$ provides the maximum point of $U(\lambda) = U^\alpha(\lambda X)$, $\lambda > 0$ for given $X$, where $U^\alpha(\lambda X)$ is corresponding to RSVM defined by (15). $\lambda_m$ is called the maximum scale, if $\lambda_m$ satisfies the following three conditions:

1) $U(\lambda_m) = 0$,

2) For $\lambda < \lambda_m$, $U(\lambda)$, and

3) For $\lambda > \lambda_m$, $U(\lambda) \leq 0$.

**Remark 10** The parameters $\lambda_{opt}$ and $\lambda_m$ mean the optimal scale and the maximum scale of an investment measured by RSVM, respectively. If $\lambda_{opt}$ is found out, the investor with the RSVM may obtain the best scale on the investment. If an investment scale value $\lambda$ is larger than $\lambda_m$, the investors with the RSVM will underestimate the investment. Thus, RSVM method also gives us a new way to valuate scale risk.

For a given $\alpha$, we substitute $X_k$ of the REIT 01881 into the equations (20) and (21), and vary $\lambda$ from 1 to 40 with an interval size of 1. Here we note an initial cost $I_0$ is set as 1. The results for $\alpha = 0.8$ are as follows:

As we see, both of valuations using RSVM and MV method firstly go up and then go down with the increasing of $\lambda$. Thus, the scale risk influences the valuation of each method in a similar form. Next,
we find that RSVM decreases faster than MV except much large value of $\lambda$, although the difference between the two methods is so slight.\(^5\)

Furthermore, in case of RSVM, we also see the optimal scale $\lambda_{opt}$ is equal to 13.38234, and the maximum scale $\lambda_m$ is 27.23169. The later result suggest the investor that the scale of investment he accepts is within 27.23169, otherwise the investors should be reject the investment.

Now, we set $\alpha$ as 0.5. Then, for the same REIT 01881, we obtain the following results:

As we expected, the optimal scale $\lambda_{opt}$ increases as the risk sensitive parameter $\alpha$ decreases. In this case, we can read $\lambda_{opt}$ is 13.38234 and $\lambda_m$ is 43.5707. To verify this, we examine on the case of $\alpha = 0.1$. The result is given as follows:

Then, we can read for RSVM $\lambda_{opt}$ is 107.0587 and $\lambda_m$ is 217.8535. In each case of mentioned above, the trends of the curves of RSVM and MV graphs for $\lambda$ are similar to that in case of $\alpha = 0.8$.

Summing up the observations, we have examined the scale risk influences the valuation of RSVM method and MV approach in a way similar to each other, and the optimal and maximum scales obtained by RSVM method are consistent with the value of $\alpha$ as we expect, respectively.

In the end of this subsection, we observe the graphs of RSVM for all the REITs versus the scale parameter and their optimal and maximum scale parameters.

As $\lambda$ increases, the RSVM for group 3 also increases, since their returns are positive. The others draw the curves similar to that in the above example. Next we turn to the optimal and maximum scale. This table indicates that as $\alpha$ increases, each optimal scale as well as each maximum scale decrease except the REITs of group 3. It should be natural because our measure RSVM has concavity. However, for group 3, we could not obtain the optimal and maximum scales, because the valuation of RSVM always increases for all $\lambda$ in this case. Miyahara [10] suggests that RSVM may be sensitive to the loss. As mentioned, all of the returns for REITs in group 3 are positive, and hence there may be no scale risk

\(^5\)Under the condition above, only for REIT 01881, RSVM decreases faster than MV except much large value of $\lambda$. The difference between RSVM and MV is based on the distribution of $X_k$. See for details in (9) and (10).
Figure 8: The result of valuation for REIT 01881 ($\alpha = 0.1$)

Figure 9: Example: all REITs’ valuation with RSVM method
Table 4: Results of the optimal scale and the maximum scale on all REITs by RSVM (light color shows larger values. The “Null” in the table means the value is not found).

| Optimal scale | 0.1 | 0.5 | 0.8 |
|---------------|-----|-----|-----|
| α             | 0   | 0   | 0   |
| 405           | 74.23270365 | 14.84654041 | 9.279087959 |
| 2778          | 107.0587226 | 21.41174435 | 13.38234033 |
| 87001         | 99.09522415 | 19.81904518 | 12.38690296 |
| 823           | Null | Null | Null |
| 808           | Null | Null | Null |
| 435           | Null | Null | Null |
| 778           | Null | Null | Null |

| Maximum scale | 0.1 | 0.5 | 0.8 |
|---------------|-----|-----|-----|
| α             | 0   | 0   | 0   |
| 405           | 163.3107893 | 32.66215423 | 20.41384747 |
| 2778          | 217.8535373 | 43.57070954 | 27.23169216 |
| 87001         | 265.3623723 | 53.07248811 | 33.17030255 |
| 823           | Null | Null | Null |
| 808           | Null | Null | Null |
| 435           | Null | Null | Null |
| 778           | Null | Null | Null |

reflected by RSVM method. In the following Section 6, we will take default risk into consideration of our investment, and discuss again this issue.

5.2 Inner Rate of Risk Aversion \( \alpha_0 \) with Scale Effect

In Section 4.2, we define the inner rate of risk aversion (IRRA) \( \alpha_0 \), and give a simple example on this IRRA. Here we apply it to our examples of the investment to Hong Kong REITs with scale effect. Before proceeding to our aim, we give a proposition on the IRRA \( \alpha_0 \) which is easily derived from Definition 8:

**Proposition 1** \( \alpha_0 \) decreases with the increasing of scale \( \lambda \), that is, it satisfies

\[
\alpha_0(\lambda X) = \frac{\alpha_0(X)}{\lambda}.
\]  

**Proof:** Assume that \( \alpha_0(X) \) is an IRRA for \( X \). From the Definition 8, we see that \( E[e^{-\alpha(X)X}] = 1 \) holds. Then, note that \( E[e^{-\alpha(\lambda X)\lambda X}] = E[e^{-\alpha_0(X)\lambda X}] \). This means \( \frac{\alpha_0(X)}{\lambda} \) is an IRRA for \( \lambda X \), that is, \( \alpha_0(\lambda X) = \frac{\alpha_0(X)}{\lambda} \).  

We provide an example in order to explain how to use \( \alpha_0 \) in the practice. In this case, we assume that the initial investment \( I_0 \) is 100 units. In consideration of the existence of \( \alpha_0 \), we focus on the following 3 REITs of the group 2 at this discussion: REIT 02778, REIT 01881 and REIT 87001. All the three
Table 5: IRRA judging table for Group 2 (light color shows larger values).

| $\lambda$ | $\alpha_{02778}$ | $\alpha_{01881}$ | $\alpha_{087001}$ |
|-----------|-------------------|-------------------|-------------------|
| 1         | 0.163311          | 0.217854          | 0.265363          |
| 2         | 0.081655          | 0.108927          | 0.132681          |
| 3         | 0.054437          | 0.072618          | 0.088454          |
| 4         | 0.040828          | 0.054463          | 0.066341          |
| 5         | 0.032662          | 0.043571          | 0.053073          |
| 6         | 0.027218          | 0.036309          | 0.044227          |
| 7         | 0.02333           | 0.031122          | 0.037909          |
| 8         | 0.020414          | 0.027232          | 0.03317           |
| 9         | 0.018146          | 0.024206          | 0.029485          |
| 10        | 0.016331          | 0.021785          | 0.026536          |
| 11        | 0.014846          | 0.019805          | 0.024124          |
| 12        | 0.013609          | 0.018154          | 0.022114          |
| 13        | 0.012562          | 0.016758          | 0.020412          |
| 14        | 0.011665          | 0.015561          | 0.018954          |
| 15        | 0.010887          | 0.014524          | 0.017691          |
| 16        | 0.010207          | 0.013616          | 0.016585          |
| 17        | 0.009607          | 0.012815          | 0.01561           |
| 18        | 0.009073          | 0.012103          | 0.014742          |
| 19        | 0.008595          | 0.011466          | 0.013966          |
| 20        | 0.008166          | 0.010893          | 0.013268          |
| 21        | 0.007777          | 0.010374          | 0.012636          |
| 22        | 0.007423          | 0.009902          | 0.012062          |
| 23        | 0.0071            | 0.009472          | 0.011538          |
| 24        | 0.006805          | 0.009077          | 0.011057          |
| 25        | 0.006532          | 0.008714          | 0.010614          |

REITs have both of plus and minus returns. The next table shows the values of IRRA for the returns of the three REITs under various scale factor $\lambda$. From this list, we can observe the followings:

1) $\alpha_0$ is inversely proportion to scale $\lambda$, that is, (22) is confirmed.

2) As mentioned in Section 4.2, the larger $\alpha_0$ is corresponding to higher level of safety for investment. In the example, investment on REIT 87001 is much safer than investment on REIT 02778 and REIT 01881. Suppose that the risk sensitive parameter $\alpha$ of an investor is 0.25. Then, the values of the above table in $\lambda = 1$ suggests that REIT 87001 would be one the investor chooses for investment, since $\alpha_0 > 0.25$; however, risk of REIT 01881 and REIT 02778 may be too large for the investor.

3) We turn to consider the scale effect describing by $\lambda$. Suppose that there is an investor whose risk sensitive parameter $\alpha$ is 0.02. Then, the above table would suggest that the investor’s acceptable scale $\lambda$ of investment for REIT 02778, REIT 01881, and REIT 87001 should be smaller than $\lambda = 8$ units, 10 units and 13 units, respectively, because $\alpha_0 > \alpha$ holds in these cases. Otherwise the valuation of the investment through RSVM method will be minus which means investors
would have a loss on this investment. Thus, the above table also suggests that the investor should invest each REIT with consideration of the suitable scale. For example, for REIT 02778, the investor can invest at most for 80,000 yen (i.e. 8 units); for REIT 01881, he can invest at most for 100,000 yen (i.e. 10 units); and for REIT 87001, he can invest at most 130,000 yen (i.e. 13 units). Note that this corresponds to a sort of rating of the 3 REITs for the investor.

These results suggest us that IRRA is useful for decision-making of investment, and choosing and rating of the commodities to invest in consideration of scale risk.

6 RSVM Method to REITs Investment with Default Risk

6.1 RSVM Formula with Default Risk

In the preceding sections, we apply RSVM method to Hong Kong REITs to observe the advantages of the method. On the other hand, in the practice, the investor should always take default risk into consideration. Default risk means that companies or individuals would be unable to make the required payments on their debt obligations. In this section, we treat the RSVM valuation of the REITs investment containing the default risk. In what follows, we assume the probability of default risk is 0.01.

We first formulate RSVM under consideration of default risk. Suppose that return $X$ is a random variable (cash flow) with the following discrete distribution:

$$P(X = a_i) = P_i, \quad i = 1, 2, \ldots, n.$$  \hspace{1cm} (23)

Then, on the basis of $X$, we define a new random variable $\tilde{X}$ considering default risk as that with the following distribution:

$$P(X = a_i) = P_i \times \frac{100 - 100\beta}{100} = \tilde{P}_i, \quad i = 1, 2, \ldots, n,$$

$$P(\tilde{X} = b) = \beta, \quad \beta > 0,$$  \hspace{1cm} (24)

where $b$ is the default loss, that is, $b = -I_0$. According to the definition (15) of RSVM with the scale parameter $\lambda$, RSVM for $X$ is given by

$$U^\alpha(\lambda X) = -\frac{1}{\alpha} \log E[e^{-\alpha \lambda X}] = -\frac{1}{\alpha} \log \left( \sum_{i=1}^{n} e^{-\alpha \lambda a_i} \times P_i \right).$$  \hspace{1cm} (25)

In a way analogous to that of this, we can calculate the RSVM for $\tilde{X}$ as

$$U^\alpha(\lambda \tilde{X}) = -\frac{1}{\alpha} \log \left( \sum_{i=1}^{n} e^{-\alpha \lambda a_i} \times \tilde{P}_i + e^{\alpha \lambda b \beta} \right) = -\frac{1}{\alpha} \log \left( \sum_{i=1}^{n} e^{-\alpha \lambda a_i} \times \tilde{P}_i \times \frac{100 - 100\beta}{100} + e^{\alpha \lambda b \beta} \right),$$  \hspace{1cm} (26)

Here substitute $n = 60, P_i = \frac{1}{60}, \beta = 0.01, b = -I_0$ into (26), we obtain

$$U^\alpha(\lambda \tilde{X}) = -\frac{1}{\alpha} \log \left( \sum_{i=1}^{60} e^{-\alpha \lambda a_i} \times \frac{1}{60} \times 0.99 + e^{\alpha \lambda I_0 \times 0.01} \right),$$  \hspace{1cm} (27)
Table 6: Sample-case for profits of Hong Kong REITs with default risk in 2011.

|        | 405  | 2778 | 1881 | 87001 | 823  | 808  | 435  | 778  |
|--------|------|------|------|-------|------|------|------|------|
| variance| 0.0156 | 0.0254 | 0.0248 | 0.0343 | 0.0247 | 0.0321 | 0.0293 | 0.0339 |
| mean   | -0.0117 | 0.0961 | 0.1337 | 0.1773 | 0.37  | 0.4217 | 0.4346 | 0.6473 |

This is the formula of RSVM with consideration of default risk in our examples. In a similar way, we shows that Mean-Variance measure for \( \tilde{X} \) becomes

\[
MV = \lambda E(\tilde{X}) - \frac{1}{2}\alpha \lambda^2 V(\tilde{X})
\]  

(28)

where \( E(\tilde{X}) \) and \( V(\tilde{X}) \) are the mean and variance of \( \tilde{X} \), respectively. We use the new formula (27) and (28) for the new cash flow \( \tilde{X} \) of our 8 REITs with default risk. Their mean and variance of \( \tilde{X} \) are given as follows:

We also make histogram of REITs with default risk to observe the character of their distribution.

6.2 Risk Sensitive Parameter \( \alpha \) with Default Risk

As mentioned in Section 4, we substitute the data of \( \tilde{X} \) into formula (27) and (28). In this time, we set \( \lambda = 1 \) and \( I_0 = 1 \) or \( I_0 = 100 \) to study how the parameter \( \alpha \) influences the valuation. As in Section 4.1, we pick up REIT 00808 as an example. Note that all of the original returns, that is, annual profits \( X_k \) of 00808 are positive; on account of the default risk, a negative value of the return is added to them. The histogram of REIT 00808 with default risk is given as follows:

We substitute the data into formula (27) and (28) together with \( I_0 = 1 \) or \( I_0 = 100 \), and vary \( \alpha \) from 0 to 1 with an interval step of 0.025. The results are given as follows:
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Figure 11: The result of valuation for 00808 with default risk ($I_0 = 1$)

Figure 12: The result of valuation for 00808 with default risk ($I_0 = 100$)
Moreover, in order to examine the influence of default risk clearly, we give the result without default risk under the same condition as that mentioned above. The figure is given as follows: These figures indicate the following facts:

1) In case of consideration of default risk, valuation on both of RSVM and MV methods decrease as the risk sensitive parameter $\alpha$ increases. Particularly, the curve of RSVM has strong concavity. This is the same result as that in Section 4.1 which treats valuation without default risk.

2) RSVM with default risk decreases more deeply than MV or RSVM without default risk. Moreover, as the initial investment, that is, the default loss increases, the difference between the
results by RSVM and MV is enlarged.

These results suggest us that RSVM is more sensitively influenced by lower negative values of returns as well as default risk than MV approach, and this is consist with the consideration of Miyahara’s numerical simulation [10, 11].

To get more results, we apply RSVM method to all the REITs under \( I_0 = 1 \) and \( I_0 = 100 \) as follows:

These figures show the followings:

When, \( I_0 \) as the risk sensitive parameter \( \alpha \) increases, the RSVM values on all the REITs trend to be the same valuation. This may be occurred because we set the default risk for each REIT as the
same level. As mentioned above, RSVM method tends to be influenced by lower negative values on cash flow. Hence, if the investor becomes more and more sensitive to the risk, the influence of default risk could not be ignored, and thereby the valuation of these REIT may go to the same level. In the practical situation, default risk for different REITs should be different from each other. Such a topic should be dealt with in a future work.

6.3 The Performance of Parameter $\lambda$ with Default Risk

Next, we proceed to observe the effect on scale parameter $\lambda$ under consideration of default risk. We substitute the data of $\tilde{X}$ into formula (27) and (28) under varying the value of $\lambda$ from 1 to 40 with an interval step of 1. Moreover, we assume $I_0 = 1$ and $\alpha = 0.8$ or 0.5.

Here we show the result on REIT 00808 for $\alpha = 0.8$ as an instance. Then, we find that RSVM provides a low rating than MV on REIT 00808. Moreover, through RSVM we obtain the optimal scale $\lambda_{opt} = 3.245508$, the maximum scale $\lambda_{max} = 5.34545$, in contrast of these facts, we cannot find out such scales through MV in this range of values of $\lambda$. Hence, we next enlarge a range of the value of $\lambda$. The result is the following figure:

Then, we also find $\lambda_{opt} = 16.42284$ and $\lambda_{max} = 32.84567$ through MV approach, which are much larger than those by RSVM. Furthermore, RSVM decreases much earlier than MV. This implies that RSVM is more sensitive than MV to lower negative values of return as caused by default risk, and hence together with influence of scale risk, such results may be obtained.

Setting the value of $\alpha$ as 0.8, 0.5 and 0.1, we now observe the results of all the REITs through RSVM, respectively. They are given as follows: We observe that:

With the increasing of $\alpha$, optimal scale, optimal value as well as maximum scale go down. It is natural because the utility function is concave. In details, we summarize in the table Table 7 below. With the increasing of $\lambda$, the valuation on each REIT trend to be the same. This coincides with the result mentioned above. Thus, even if we introduce default risk into consideration, we can also calculate
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Figure 18: Valuation of REIT 00808 with default risk (long range).

Figure 19: Example: Valuation of all the REITs with default risk ($\alpha = 0.8$).
Figure 20: Example: Valuation of all the REITs with default risk ($\alpha = 0.5$).

Figure 21: Example: Valuation of all the REITs with default risk ($\alpha = 0.1$).
Table 7: Optimal and maximum scales and values with default risk through RSVM method (light color shows larger value).

| Optimal scale with default risk | 0.1 | 0.5 | 0.8 |
|---------------------------------|-----|-----|-----|
| \(\alpha\)                     |     |     |     |
| 405                             | 0   | 0   | 0   |
| 2778                            | 18.94939 | 3.789879 | 2.368674 |
| 1881                            | 21.72823 | 4.345646 | 2.716029 |
| 87001                           | 22.61632 | 4.523264 | 2.82704 |
| 823                             | 26.13703 | 5.227406 | 3.267128 |
| 808                             | 25.96406 | 5.192813 | 3.245008 |
| 435                             | 26.03266 | 5.206532 | 3.254083 |
| 778                             | 25.12699 | 5.025397 | 3.140873 |

| Optimal value with default risk  | 0.1 | 0.5 | 0.8 |
|----------------------------------|-----|-----|-----|
| \(\alpha\)                     |     |     |     |
| 405                             | 0   | 0   | 0   |
| 2778                            | 1.107352 | 0.22147 | 0.138419 |
| 1881                            | 1.846756 | 0.369351 | 0.230844 |
| 87001                           | 2.518654 | 0.503731 | 0.314832 |
| 823                             | 6.794985 | 1.358997 | 0.849373 |
| 808                             | 7.615511 | 1.523102 | 0.951939 |
| 435                             | 7.943713 | 1.588743 | 0.992964 |
| 778                             | 11.58183 | 2.316366 | 1.447729 |

| Maximum scale with default risk  | 0.1 | 0.5 | 0.8 |
|----------------------------------|-----|-----|-----|
| \(\alpha\)                     |     |     |     |
| 405                             | 0   | 0   | 0   |
| 2778                            | 31.94833 | 6.389666 | 3.993541 |
| 1881                            | 35.91773 | 7.183547 | 4.489718 |
| 87001                           | 37.74141 | 7.548282 | 4.717676 |
| 823                             | 43.89158 | 8.778317 | 5.486448 |
| 808                             | 44.27636 | 8.855273 | 5.534545 |
| 435                             | 44.46841 | 8.893683 | 5.558551 |
| 778                             | 45.52117 | 9.104237 | 5.690148 |
6.4 Inner Rate of Risk Aversion $\alpha_0$ with Default Risk

Finally, we are also going to examine the influence of default risk to inner rate of risk aversion $\alpha_0$. For each REIT, we calculate IRRA $\alpha_0$ on the data of return $\tilde{X}$ with default risk under $I_0 = 10$ and compare the result with that on the data of the original return $X$ without default risk, respectively. The results are given as follows:

For each REIT, the IRRA with default risk is smaller than that without the risk, that is, the index sensitively responds the default risk. Hence, in consideration of default risk, IRRA becomes more effective in rating as mentioned Section 5.2. Recalling the proposition 1 in Section 5.2, we can make a table of IRRA for scale effect with the default risk as Table 8. The table is given as follows:

We could use this in a practical investment in the same way as we discussed on Table 5 in Section 5.2. From the context, even in the consideration of default risk, RSVM method is useful for ratings of the investments as well as valuation of the scale risk.

Finally, in this section, we have treated a simple default risk model, because we would like to examine the characters of RSVM through an illustrative example. In a future work, we will investigate the simulation on the basis of a detailed model related to credit risk.

7 Summary and Conclusions

In this paper, we have focused on the performance and the advantageous points of RSVM method through applying to the valuation of Hong Kong Real Estate Investment Trust (REITs). This is the first time that the RSVM method is adopted to the valuation problems of financial investment. In particular, we have compared the results due to RSVM method with those derived from MV approach to examine the advantage of RSVM. To emphasize the characters of RSVM, we first state the results on the comparison between RSVM and MV, and next we summarize the other results and future issues related to this paper.
Table 9: IRRA judging table for all the REITs with default risk (light color shows bigger number. The “Null” in the table means the value is not found).

| λ  | 405  | 2778 | 1881 | 87001 | 823  | 808  | 435  | 778  |
|----|------|------|------|-------|------|------|------|------|
| 1  | Null | 3.19483 | 3.59177 | 3.77414 | 4.38916 | 4.42763 | 4.44864 | 4.55212 |
| 2  | Null | 1.597415 | 1.795885 | 1.88707 | 2.19458 | 2.213815 | 2.22342 | 2.27606 |
| 3  | Null | 0.399354 | 0.448971 | 0.471768 | 0.548645 | 0.553454 | 0.555855 | 0.569015 |
| 4  | Null | 0.212989 | 0.239451 | 0.251609 | 0.292611 | 0.295175 | 0.296456 | 0.303475 |
| 5  | Null | 0.066559 | 0.074829 | 0.078628 | 0.091441 | 0.092242 | 0.092643 | 0.094836 |
| 6  | Null | 0.033381 | 0.038011 | 0.039944 | 0.041802 | 0.042168 | 0.042351 | 0.043354 |
| 7  | Null | 0.00832 | 0.009354 | 0.009828 | 0.01143 | 0.01153 | 0.01158 | 0.011854 |
| 8  | Null | 0.319483 | 0.359177 | 0.377414 | 0.438916 | 0.442763 | 0.444684 | 0.455212 |
| 9  | Null | 0.290439 | 0.326525 | 0.343104 | 0.399015 | 0.402512 | 0.404258 | 0.413829 |
| 10 | Null | 0.133118 | 0.149657 | 0.157256 | 0.182882 | 0.184485 | 0.185285 | 0.189672 |
| 11 | Null | 0.081919 | 0.092097 | 0.096773 | 0.112543 | 0.113529 | 0.114022 | 0.116721 |
| 12 | Null | 0.028525 | 0.032069 | 0.033698 | 0.039189 | 0.039532 | 0.039704 | 0.040644 |
| 13 | Null | 0.014199 | 0.015963 | 0.016774 | 0.019507 | 0.019678 | 0.019764 | 0.020232 |
| 14 | Null | 0.00416 | 0.004677 | 0.004914 | 0.005715 | 0.005765 | 0.00579 | 0.005927 |
| 15 | Null | 0.00179 | 0.002012 | 0.002114 | 0.002459 | 0.002491 | 0.00255 | 0.002616 |
| 16 | Null | 0.000462 | 0.000546 | 0.000635 | 0.000641 | 0.000643 | 0.000659 | 0.000669 |
| 17 | Null | 0.000178 | 0.000202 | 0.000211 | 0.000249 | 0.000247 | 0.000248 | 0.000254 |
| 18 | Null | 0.0015074 | 0.017959 | 0.018871 | 0.021946 | 0.022138 | 0.022234 | 0.022761 |

7.1 RSVM vs MV

1) As mentioned in Remark 8, if the distribution of a random return \( X \) is Gaussian, then RSVM coincides with MV. The difference between RSVM and MV appears when the distribution of \( X \) is apart from Gaussian distribution.

2) In Section 5, we have taken an account of scale parameter \( \lambda \) in RSVM. As to \( \lambda \), scale risk influences the valuation of RSVM method and MV approach in a similar way to each other, especially in the case of that distribution of returns spreads both negative and positive value. On the other hand, in the case where all the returns of REITs are positive, RSVM increases as the scale parameter \( \lambda \) increases as mentioned in the observation of Fig. 9. Moreover, RSVM gives a higher valuation for the scale parameter \( \lambda \) than MV. Indeed the following figure of valuations by RSVM and MV for REIT 00808 with the scale parameter shows this. We here note that all of the original returns, that is, annual profits \( X_k \) of REIT 00808 are positive as mentioned in Section 4 and Section 6. It is natural that such a valuation as REIT 00808 is always positive and increases as \( \lambda \) increases. MV given by (12), however, does not satisfy this property, since the coefficient of the second order of \( \lambda \) in (12) is negative, and thereby the value of MV may also become negative for a large \( \lambda \). In contrast with this, we see that the value of RSVM (15) is positive if the return \( X \) is always positive. These facts suggest that RSVM is more proper for valuation of returns as like REIT 00808 than MV.
In Section 6, we introduce default risk into our investment issue. This is the key content and an innovation of this paper as well. The comparison between Fig. 22 and Fig. 17 (or Fig. 18) indicates that RSVM is more sensitive than MV to negative values of returns caused by default risk. Particularly, this character is clear, when the value of scale parameter $\lambda$ is large. This is consistent with the consideration of Miyahara’s numerical simulation [10, 11]. These facts suggests us that RSVM is more suitable for valuation of a random return influenced by very lower negative values which may occur only with a small probability as default risk.

### 7.2 Other Summary and Concluding Remarks

Finally, we summarize the other observations with respect to risk sensitive analysis and scale parameter, and give some concluding remarks.

1) As mentioned in Section 5, the optimal and maximum scales obtained by RSVM method are consistent with the value of risk sensitive parameter $\alpha$, respectively.

2) In Section 4, a new index of risk valuation of investment is defined through RSVM, which is called the inner rate of risk aversion (IRRA). Practical examples in Section 5 suggest us that IRRA together with scale parameter is useful for decision-making of investment, and choosing and rating of the commodities to invest.

3) In the consideration of default risk, the RSVM could be an effective method in ratings and valuating the scale effect. Note that we assume the default risk is the same value for the different REITs in this article. In general, it must be natural that the default risk for different REITs should be different from each other. Furthermore, it is necessary to investigate the characters of two parameters ($\alpha$ and $\lambda$) in RSVM with default risk more deeply and compare them with those without the default risk. We will treat such issues in a future work.
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