SEARCH FOR THE PROTON DECAY MODE \( p \rightarrow \nu K^+ \) IN SOUDAN 2

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Abstract

We have searched for the proton decay mode \( p \rightarrow \nu K^+ \) using the one-kiloton Soudan 2 high resolution calorimeter. Contained events obtained from a 3.56 kiloton-year fiducial exposure through June 1997 are examined for occurrence of a visible \( K^+ \) track which decays at rest into \( \mu^+\nu \) or \( \pi^+\pi^0 \). We found one candidate event consistent with background, yielding a limit, \( \tau/B(p \rightarrow \nu K^+) > 4.3 \times 10^{31} \) years at 90% CL with no background subtraction.
1 Introduction

The prediction of many Grand Unified Theories, that nucleon decay occurs at accessible lifetimes, remains unverified but continues to motivate experimental searches. This expectation was foreshadowed in part by Sakharov’s early suggestion that the simultaneous effects of baryon number violation, C and CP violation, and departure from thermal equilibrium could produce the baryon-antibaryon asymmetry observed in the universe.\(^1\) It is interesting and suggestive that no fundamental symmetry is known which implies the conservation of baryon number. Currently, nucleon decay as a consequence of the minimal SU(5) GUT model is considered to be ruled out experimentally\(^2,3\). However, other unification models, both with and without supersymmetry, predict baryon number violating processes. Amplitudes for these processes involve the exchange of new particles with unknown masses so that precise nucleon lifetimes are not predicted. The expectation that these masses will be in the range between the GUT scale of \(\sim 10^{15}\) GeV and the Planck mass of \(\sim 10^{19}\) GeV leads to proton lifetimes in the range \(10^{32} - 10^{35}\) years\(^4\). Decay modes with strange particles such as \(p \to \nu K^+\), are usually favored in models which incorporate supersymmetry\(^3,5,6\).

Previous searches for \(p \to \nu K^+\) have been reported by the IMB, Kamiokande and Frejus collaborations\(^7,2,8\). The \(K^+\) track can be imaged in ionization calorimeters such as Soudan 2 and Frejus, but is usually below Cherenkov threshold in water. IMB searched for an excess of events in a region of anisotropy and energy with a large background\(^7\). Kamiokande looked for an excess of single ring mu-like events between 215 and 255 MeV/c with a muon decay, and also for three-ring events compatible with an invisible, stopped \(K^+ \to \pi^+\pi^0\) decay\(^2\). Frejus used two-track events with ranges consistent with the \(K^+\) and the \(\mu^+\)\(^3\).

In the Soudan 2 analysis, we use both the visibility of the \(K^+\) in a fine grained tracking calorimeter and the visibility of the decay electron from a stopped \(\mu^+\) to reduce backgrounds from atmospheric neutrino interactions. We searched for the proton decay mode \(p \to \nu K^+\) using two \(K^+\) decay channels, \(K^+ \to \mu^+\nu_\mu\) and \(K^+ \to \pi^+\pi^0\).

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2 The Soudan 2 Detector

The Soudan 2 detector is a time projection, modular iron tracking calorimeter with a total mass of 974 metric tons and fiducial mass of 770 tons. Details of module construction and performance may be found in References [9–11]. The detector is assembled as a close-packed rectangular stack of 224 modules; each module is made from 1.6 mm thick sheets of corrugated steel, stacked in a hexagonal “honeycomb” structure. The average density is 1.58 g/cm$^3$.

On the walls of the underground cavern surrounding the detector, there is an active “veto” shield comprised of double-layer, hexagonal cell, aluminum proportional tubes[12].

Two million Hytrel plastic drift tubes (1.0 m long by 15 mm in diameter) fill the spaces in the honeycomb stacks. Ionization electrons deposited in an Ar/CO$_2$ gas mixture drift toward either end of the tube in a 180 volt/cm electric field with a velocity 0.8 cm/µsec. Upon reaching the tube end, the electrons are detected by vertical anode wires and horizontal cathode strips. Each crossing of a tube by an ionizing particle can create an anode/cathode signal at a common drift time which we call a “hit”. The pulse area, which is proportional to the integrated charge deposited in the tube, and the drift time for each hit are recorded by both the anode and cathode electronics.

The primary trigger requires at least 7 hits, separated by at least 600 ns, in a group of 16 anode channels or at least 8 hits in a group of 16 cathode channels within a 50 µsec window. The trigger efficiency for proton decay final states considered here is $\sim 85\%$. The complete detector triggers at a rate of $\approx 0.5$ Hz from naturally occurring radioactivity and cosmic ray muons. Every 240 seconds a “pulser” trigger provides a snapshot of the random background levels in the main detector. These are used as underlying events to add detector noise to Monte Carlo events.

3 Event Analysis

3.1 Overview

The data analysis proceeds in three stages. First we identify “contained events”. Event prongs are defined by scanning as track-like ($\pi$, $\mu$ or p) or shower-like (e or $\gamma$). Contained events are defined as having no hits on tracks or the main body of showers which are less than 20cm from the outside surface of the detector and the prongs do not start or end between modules. This is the same contained event selection as was used for our atmospheric neutrino analysis [13]. Studies in reference [13] showed that the efficiency for correct identifica-
tion was 98% for tracks and 94% for showers. An absence of shield activity was required. Second, the events are required to have a topology consistent with the proton decay channel under study, based on counting the number of visible tracks and showers. Finally, kinematic selections which characterize a particular proton decay mode are applied to the data and also to event samples which monitor background processes.

The analysis procedure involves finding efficient selection criteria using our proton decay Monte Carlo program, while minimizing the backgrounds from atmospheric neutrinos and atmospheric muons. The former backgrounds are calculated using the atmospheric neutrino Monte Carlo program described in Reference [13], which incorporates the flux predictions of Barr, Gaisser and Stanev[14]. Backgrounds from atmospheric muons may result when muons inelastically scatter in the rock outside the active shield. We use the term “rock” event to describe the interactions of a resulting secondary such as a neutron or $K_L$ which goes into the Soudan 2 calorimeter and causes a contained vertex event. Most rock events have hits in the Soudan 2 shield which are time-coincident with the contained event in the calorimeter. These shield-tagged events are used to estimate any background from rock events without shield tags. A detailed analysis of the penetration depth distributions of events with and without shield hits has led to the conclusion that $91.3\% \pm 3.7\%$ of all rock events have shield tags.

Our Monte Carlo simulation program tracks the decay products through the detector geometry and generates electronic hits in the same format as real data. The generator starts with a parent (or target) nucleon within a nucleus. The nucleon is considered to have a Fermi momentum chosen from the Bodek and Ritchie parameterization[15]. The spatial location and the atomic number of the parent nucleus is chosen according to the composition and mass distribution of the detector. The rescattering of pions within a parent nucleus is generated according to a phenomenological model[16]. Parameters of the model have been set to reproduce pion production by low energy neutrinos in $\nu_\mu$-Ne and $\nu_\mu$-deuteron reactions[17]. Inelastic intranuclear rescattering for $K^+$ mesons is not expected to be significant due to the absence of low-lying $K^+N$ resonances and is not simulated. The detector response used in this Monte Carlo program was verified against calibration data from the ISIS facility using $\pi$, $e$, $\mu$ and $p$ beams at a variety of angles and energies[18].

3.2 Search for the Decay $p \rightarrow \nu K^+, K^+ \rightarrow \mu^+ \nu_\mu$

To choose the selection criteria and determine efficiencies we generated a large sample of $K^+ \rightarrow \mu^+ \nu_\mu$ Monte Carlo events from proton decay. Features of the typical Monte Carlo event shown in Figure [1] include a highly ionizing $K^+$
which (usually) comes to rest, and emits a 236 MeV/c $\mu^+$. The $\mu^+$ has an average range of 42 cm in the Soudan 2 detector. At the end of its range, the $dE/dx$ of the muon is rising. The $\mu^+$ comes to rest and decays into an $e^+$, which gives an average of 3 hits.

The results of the analysis for the proton decay simulation, the neutrino simulation, the data, and the shield-tagged rock background are given in Table 1. The simulated events are generated in the entire mass of the Soudan 2 detector. Efficiencies within the fiducial mass are calculated by dividing the fraction of events which pass a set of cuts by the ratio of the fiducial mass to the total mass (0.79).

The Monte Carlo events were first subjected to a simulated trigger, which 81 events failed. Both Monte Carlo and data events were processed through a filter program which applied containment criteria, reducing cosmic ray muons by a factor of more than $10^3$. 114 MC events, essentially all with hits outside the containment volume, were rejected. The remaining events were then scanned by physicists to remove the remaining non-contained events, mostly events starting or ending on module boundaries. At this stage, 204 $p \rightarrow \nu K^+, K^+ \rightarrow \mu^+\nu_\mu$ events remained, compared with 367 data events without shield hits, 1008 shield-tagged rock events, and 1923 events from the atmospheric neutrino Monte Carlo. The data corresponds to 3.56 fiducial kiloton-years of exposure, compared to 20.24 kiloton-years for the atmospheric neutrino MC.

The required topology is two charged tracks with a common vertex. Events in which both tracks appear to be protons based on ionization and straightness are not included. These topology features were exhibited by 95 of the remaining 204 events from the MC. We also require that the $K^+$ candidate, which is usually the shorter track, has a length less than 50 cm. The muon track length distributions for all four event samples at this stage of the analysis are shown in Figure 2. The range of muons from the $K$ decay is peaked at 43 cm whereas the background distributions are relatively flat. The muon range was therefore required to lie between 29 and 58 cm. Our final cut requires a visible muon decay electron having two or more hits. This requirement discriminates strongly against neutrino induced background, since the predominant background is $\nu n \rightarrow \mu^- p$ and in our iron detector most $\mu^-$ are absorbed rather than decay after stopping.

After all cuts, our efficiency for accepting $p \rightarrow \nu K^+, K^+ \rightarrow \mu^+\nu_\mu$ events in the fiducial volume is 14%. Two atmospheric $\nu$ Monte Carlo events pass all of the cuts and represent an expected background of 0.21 events, taking into account that we found the atmospheric neutrino flavor ratio ($\nu_\mu/\nu_e$) to be only 0.61 of the expected value[19]. The rock event background in the zero shield hit sample is calculated to be 0.19 events. This was found based on the penetration depth analysis mentioned above, which found background in the
track data equal to 9.5% of the shield-tagged track sample.

As shown in Table 1, one event in the data survives our cuts.

![Monte Carlo event of $p \rightarrow \nu K^+$](image)

Fig. 1. Monte Carlo event of $p \rightarrow \nu K^+$. The three projected views are shown. The xz/yz views correspond to the anode/time and cathode/time views. The xy view is based on matching anode and cathode hits using their time and pulse shape. For each hit, the pulse area is proportional to the recorded energy loss. The $K^+$ is the short heavily ionizing track on the left/left/right of the xz/yz/xy plot. Three hits from the $e^+$ decay of the $\mu^+$ appear at the right/right/top. Scales are in cm.

3.3 $K^+ \rightarrow \pi^+\pi^0$ analysis

We searched for the mode $p \rightarrow \nu K^+, K^+ \rightarrow \pi^+\pi^0$ by selecting events with a short heavily ionizing track ($K^+$ candidate), one other track and two showers. These features are illustrated in the Monte Carlo event shown in Figure 3. The short highly ionizing track is the $K^+$ before it decays. A $\pi^0$ is reconstructed from the two showers and the $K^+$ from the $\pi^0$ and the second track, assumed to be a $\pi^+$. At these low energies the two $\gamma$'s from the $\pi^0$ are
Fig. 2. Muon range distributions (before the muon decay cuts are imposed) for a) 236 MeV/c $\mu$'s from the $p \rightarrow \nu K^+$ Monte Carlo simulation, b) the data, c) the atmospheric neutrino Monte Carlo and d) the shield-tagged rock background. The numbers of events (including overflows) are the same as in the row labeled "K range requirement" of Table 1. The shaded events pass the muon decay cut.

usually identifiable. The $K^+$ mass is required to be in the range $100 \text{ MeV/c}^2 < m_{K^+} < 660 \text{ MeV/c}^2$. The $\pi^+$ and $\pi^0$ momenta are required to be in the ranges $80 \text{ MeV/c} < p_{\pi^+} < 400 \text{ MeV/c}$ and $40 \text{ MeV/c} < p_{\pi^0} < 390 \text{ MeV/c}$. The invariant mass of the two shower system is required to be in the range
### Event Selection

| Event Selection                                      | PDK MC | ν MC | Rock | Data |
|------------------------------------------------------|--------|------|------|------|
| MC decays in total detector                          | 493    |      |      |      |
| Triggered detector                                   | 412    |      |      |      |
| Containment filter                                   | 298    |      |      |      |
| Scanned as Contained                                 | 204    | 1923 | 1008 | 367  |
| Topology                                             | 95     | 345  | 61   | 30   |
| K range requirement                                  | 95     | 286  | 54   | 27   |
| Muon range requirement                               | 88     | 62   | 21   | 1    |
| Visible muon decay                                   | 55     | 2    | 2    | 1    |
| Exposure corrected background                        |        | 0.21 | 0.19 |      |

Table 1
Numbers of MC and data candidate events for $p \rightarrow \nu K^+, K^+ \rightarrow \mu^+\nu_\mu$ which survive the triggering, containment, topology, and kinematic cuts of this analysis. Events are generated in the full detector, while efficiencies in Table 3 are quoted for the fiducial volume.

10 MeV/c² < $m_{2\gamma}$ < 290 MeV/c². The resulting detection efficiency for this mode is 26%.

The effects of these cuts on the proton decay simulation are shown in Table 2. No data events pass these cuts. The background from atmospheric neutrino interactions is estimated to be 1.05 events based upon our MC simulated $\nu$ events. The background is from $\nu_e$ charged current and $\nu_\mu$ neutral current interactions, so the suppression factor of 0.62 is not used here. The estimated background contribution from rock events is 0.09 events, based on the one rock event which passed our cuts.

#### 3.4 Limit calculation

Using the two $K$ branching modes studied here we set a proton lifetime limit using the formula:

$$\frac{\tau}{B(p \rightarrow \nu K^+)} > \frac{N_p \times T \times [\varepsilon_1 \times B_1(K) + \varepsilon_2 \times B_2(K)]}{\mu_1 + \mu_2}$$

(1)

Here $N_p = 2.87 \times 10^{32}$ is the number of protons in a kiloton of the Soudan 2 detector, $T = 3.56$ kiloton-years is the fiducial exposure $\varepsilon_1 \times B_1(K)$ are the detection efficiencies quoted above times the appropriate $K^+$ decay branching fractions in Table 3. The $\mu_i$ are the constrained 90% CL upper limits on the
Fig. 3. Monte Carlo event of $p \rightarrow \nu K^+; K^+ \rightarrow \pi^+\pi^0$. The $K^+$ track of four heavily ionizing hits, ranges and decays at rest, yielding a pion track and two showers from the $K^+$ endpoint. The showers are the $\pi^0$ remnants; they appear in directions opposite to the pion track and are overlapping in the xz view. The xy view also shows the results of our track and shower fits. Scales are in cm.

numbers of observed events, and are found by solving the equation

$$0.10 = \frac{\sum_{n_1=0}^{n_{ev1}} \sum_{n_2=0}^{n_{ev2}} P(n_1, b_1 + \mu_1) P(n_2, b_2 + \mu_2)}{\sum_{n_1=0}^{n_{ev1}} \sum_{n_2=0}^{n_{ev2}} P(n_1, b_1) P(n_2, b_2)}$$

with the constraint

$$\frac{\varepsilon_1 \times B_1(K)}{\mu_1} = \frac{\varepsilon_2 \times B_2(K)}{\mu_2} = \frac{\sum_{i=1}^{2} \varepsilon_i \times B_i(K)}{\sum_{i=1}^{2} \mu_i}$$

(3)
### Event selection

| Event selection                        | PDK MC | ν MC | Rock | Data |
|----------------------------------------|--------|------|------|------|
| MC decays in total detector            | 493    |      |      |      |
| Triggered detector                     | 442    |      |      |      |
| Containment filter                     | 317    |      |      |      |
| Scanned as Contained                   | 229    |      |      |      |
| Topology                               | 106    | 18   | 3    | 5    |
| K range requirement                    | 106    | 15   | 3    | 4    |
| K mass requirement                     | 106    | 11   | 3    | 1    |
| π⁺ momentum cut                        | 103    | 7    | 2    | 0    |
| π⁰ momentum cut                        | 103    | 6    | 1    | 0    |
| π⁰ mass cut                            | 101    | 6    | 1    | 0    |
| Exposure corrected background          | 1.05   | 0.09 |      |      |

Table 2

Numbers of MC and data candidate events for \( p \rightarrow \nu K^+, K^+ \rightarrow \pi^+\pi^0 \) which survive the triggering, containment, topology, and kinematic cuts of this analysis.

where \( P(n, \mu) \) is the Poisson function, \( e^{-\mu} \mu^n/n! \), and the \( b_i \) are the estimated backgrounds. With the one candidate observed for \( K^+ \rightarrow \mu^+\nu_\mu \) and none found for \( K^+ \rightarrow \pi^+\pi^0 \), the values of \( \mu \) in Table 3 are obtained, and the combined limit without background subtraction is \( 4.3 \times 10^{31} \) years at 90% CL. With background subtraction, a limit \( 4.6 \times 10^{31} \) years is obtained.

### Table 3

|                  | \( K^+ \rightarrow \mu^+\nu_\mu \) | \( K^+ \rightarrow \pi^+\pi^0 \) |
|------------------|------------------------------------|----------------------------------|
| \( \varepsilon \) | 0.14                               | 0.26                             |
| \( B(K) \)       | 0.64                               | 0.21                             |
| \( \varepsilon \times B(K) \) | 0.090                         | 0.055                           |
| \( \mu(b = 0) \) | 2.1                                | 1.3                              |
| b (background)   | 0.4                                | 1.1                              |
| \( \mu(b) \)     | 2.0                                | 1.2                              |

Table 3

Summary of detection efficiencies, branching fractions, backgrounds, and combined upper limits at 90% CL with and without background subtraction for the \( p \rightarrow \nu K^+ \) channels analyzed.
4 Discussion

The Kamiokande collaboration has reported a limit for $p \rightarrow \nu K^+$ of $10.0 \times 10^{31}$ years with 9 candidate events and an estimated background of 7.3 events. Their unsubtracted limit is $5.1 \times 10^{31}$ years. IMB has reported a limit without background subtraction of $1.0 \times 10^{31}$ years with 6 candidate events and an estimated background of 4.7. More recently, they report $5.0 \times 10^{31}$ years without a background subtraction based on 14 candidates. With background subtraction of 21.4 events, they calculated their limit to be $15.1 \times 10^{31}$ years. Frejus has reported a background subtracted limit of $1.5 \times 10^{31}$ years with 1 candidate and an estimated background of 1.8. In all experiments, the largest contribution to the estimated background comes from $\nu_\mu$ quasielastic scattering. The existence of the atmospheric neutrino $\nu_\mu$ deficit then calls into question the reliability of background estimates in all of these experiments. It is not clear that rescaling the total $\nu_\mu$ rate to the observed rate leads to a correct estimate of the background for proton decay in this channel, since whatever is causing the atmospheric $\nu_\mu$ deficit may be energy dependent or different for $\nu_\mu$ and $\bar{\nu}_\mu$. We note the possibility of such differences by comparing plots 2b and 2c. We would assign a resulting systematic error of as much as 100% to the background estimates for this channel. In view of this large systematic error we distrust the background subtraction in this channel and prefer to rely on the unsubtracted limit, which is unaffected by the background uncertainly. We note that the background level in this analysis, and thus the importance of background subtraction, is an order of magnitude lower than in the water cherenkov experiments.

In summary, we have searched for the proton decay mode $p \rightarrow \nu K^+$ using two decay modes of the $K^+$, $K^+ \rightarrow \mu^+\nu_\mu$ and $K^+ \rightarrow \pi^+\pi^0$. We observe one candidate event for $K^+ \rightarrow \mu^+\nu_\mu$ and zero candidates for $K^+ \rightarrow \pi^+\pi^0$; the estimated backgrounds are 0.40 events and 1.14 events respectively. Our combined lower lifetime limit at 90% CL is $4.3 \times 10^{31}$ years. Our limit with background subtraction is $4.6 \times 10^{31}$ years.

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