THE INTRINSIC SHAPES OF LOW SURFACE BRIGHTNESS DWARF IRREGULAR GALAXIES AND COMPARISON TO OTHER TYPES OF DWARF GALAXIES

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ABSTRACT

In this paper, we measure the ellipticities of 30 low surface brightness (LSB) dwarf irregular (dI) galaxies and compare the ellipticity distribution with that of 80 dwarf elliptical (dEs) and 62 blue-compact dwarfs (BCDs). We find that the ellipticity distribution of LSB dIs is very similar to that of BCDs, and marginally different from that of dEs. We then determine the distribution of intrinsic shapes of dI galaxies and compare this to the distributions of other types of dwarf galaxies under various assumptions. First, we assume that LSB dIs are either all oblate or all prolate, and use a nonparametric analysis to find the best-fitting distribution of intrinsic shapes. With this assumption, we find that the scarcity of nearly circular LSB dIs implies, at the 99% confidence level, that they cannot be a population of randomly oriented oblate or prolate objects, implying that LSB dIs are highly unlikely to be disk-shaped systems. Next, we assume that dIs are triaxial, and use a parametric analysis to find permissible distributions of intrinsic shapes. We find that if the intrinsic axis ratios \( \beta \) and \( \gamma \) are distributed according to a Gaussian with means \( \beta_0 \) and \( \gamma_0 \) and a common standard deviation of \( \sigma \), the best-fitting set of parameters for LSB dIs is \( (\beta_0, \gamma_0, \sigma) = (0.66, 0.50, 0.15) \), and the best fit for BCDs is \( (\beta_0, \gamma_0, \sigma) = (0.66, 0.55, 0.16) \), while the best fit for dEs is \( (\beta_0, \gamma_0, \sigma) = (0.78, 0.69, 0.24) \). The dIs and BCDs thus have very similar shape distributions, given this triaxial hypothesis, while the dEs peak at a somewhat more spherical shape. Therefore, our results provide strong observational evidence to support the evolutionary scenario in which the three types of dwarf galaxy have a close relation with each other.

Subject headings: galaxies: elliptical and lenticular, cD — galaxies: irregular — galaxies: photometry — galaxies: structure

1. INTRODUCTION

Recent studies indicate that dwarf galaxies are by far the most numerous type of galaxy, and contribute a significant fraction of the mass of the universe (Reaves 1983; Phillipps et al. 1987). Morphologically, dwarf galaxies, like their counterpart bright galaxies, are classified into several types. The most common type of dwarf galaxy (~ 80% of the total) is the dwarf elliptical (dE). These galaxies have regular elliptical isophotes, with relatively high central surface brightnesses and roughly exponential profiles. The second type of dwarf galaxy is the blue-compact dwarf (BCD) galaxy. In contrast to gas-poor dEs, BCDs contain centrally concentrated giant H II regions surrounding O and B stars within a massive H I reservoir; BCDs exhibit spectra slowly rising toward the blue, implying that they are undergoing intense star formation (du Puy 1970; Searle & Sargent 1972). Most BCDs have regular isophotes in the outer region, like dEs, but the inner isophotes are frequently distorted from ellipses, due to the presence of bright H II regions (Loose & Thuan 1986).

The final type of dwarf galaxy is the low surface brightness (LSB) dwarf galaxy. LSB dwarfs include both irregular (dI) and more regular spiral (dS) galaxies. Like BCDs, LSB dIs contain a large amount of H I, often with small OB associations, and have blue colors \( (B - V \sim 0.5 \text{ mag}) \), indicating a significant level of recent star formation (Staveley-Smith, Davies, & Kinman 1992). However, they are distinguished from BCD galaxies by having amorphous shapes even in their outer regions, and by having star-forming regions that are not centrally concentrated. In addition, they have higher total masses within a given surface brightness and larger linear diameters than BCDs.

The evolutionary connections among the three different types of dwarf galaxies remain both elusive and confusing. A possible close link between dEs and LSB dIs is hinted at by their similar structural parameters; both types of dwarfs...
have nearly exponential surface-density profiles and similar ranges in luminosity, scale length, and central surface brightness (Kormendy 1985; Davies et al. 1988).

A popular scenario for the evolutionary link between dEs and LSB dIs is that dEs were originally dIs, but have since lost all their gas. The gas may have been depleted by a number of mechanisms: by supernova-driven galactic winds (Saito 1979; Vader 1986; Dekel & Silk 1986), by external ram-pressure stripping (Lin & Faber 1983), or by the conversion of gas into stars (Kormendy 1985; Davies & Phillipps 1988).

The evolutionary connection, if any, between low surface brightness dwarfs (dEs and LSB dIs) and the higher surface brightness BCDs remains a puzzle at present. Dekel & Silk (1986) proposed that LSB dIs and dEs represent a fundamentally different evolutionary sequence from BCDs, the lower surface brightness dwarfs never being converted into BCDs or vice versa. An alternate hypothesis regards BCDs as dI galaxies that happen to be observed during a brief burst of star formation (Searle & Sargent 1972; Gerola, Seiden, & Schulman 1980; Thuan 1985; Drinkwater & Hardy 1991). Davies & Phillips (1988) have combined the different hypotheses for evolutionary linkages into an integrated scenario in which gas-rich LSB dIs evolve through a starbursting BCD phase, during which bursts of star formation are fed by the infall of gas, then fade into dEs when the gas is no longer replenished.

One important constraint on the evolutionary relation among different types of dwarf galaxies is provided by a comparison of their apparent shapes. (A galaxy’s apparent shape is customarily expressed, to lowest order, as its axis ratio $q$ or ellipticity $e = 1 - q$.) If the three types of dwarfs simply represent different stages of a galaxy’s evolution, we may naively expect the different types to have similar distributions of apparent flattening. There are subtleties involved, however; if LSB dIs evolve to dEs by the loss of gas by winds or ram-pressure stripping, the gas loss would cause the galaxy to “puff up” and become both larger and more nearly spherical (Saito 1979; Vader 1986). There have been several previous attempts to measure and compare the flattening distributions of different types of dwarf galaxies. For instance, Caldwell (1983) compared the axis ratios of a small sample of dEs to the shapes of Magellanic irregular galaxies observed by Fisher & Tully (1977), finding that their shape distributions, given the small sample size, were not statistically distinguishable. Ichikawa, Wakamatsu, & Okamura (1986) compared a sample of 69 Virgo dEs to a sample of galaxies with de Vaucouleurs types 8–10 (late-type spiral and Magellanic irregular galaxies), and also found that their shape distributions were not statistically distinguishable. Staveley-Smith et al. (1992) constructed the axis ratio distribution for 438 Uppsala Catalogue (hereafter UGC; Nilson 1973) LSB galaxies and compared it to that of 99 BCDs whose ellipticities were measured from Palomar Observatory Sky Survey (POSS) plates by Gorden & Gottesman (1981), and also found that the distributions were indistinguishable. Bingelli & Popescu (1995) examined the photographically measured axis ratios of a sample of 260 Virgo dwarfs, breaking them down into subsamples of dEs, BCDs, Im’s, and late spirals (types Sdm through Sm). Although the dEs were rounder on average than the other subsamples, Kolmogorov-Smirnov tests did not reveal a significant difference among the ellipticity distributions of the subsamples.

Unfortunately, these previous studies of axis ratio distributions suffer from large uncertainties for several reasons. First, owing to the small dimensions and low surface brightnesses of dwarf galaxies, estimating their axis ratio is difficult and leads to large uncertainties. Second, the analyses of Caldwell (1983) and Ichikawa et al. (1986) were based on a comparison of dEs to Magellanic irregulars, not LSB dIs. Third, the UGC sample used by Staveley-Smith et al. (1992) is known to be inhomogeneous, containing galaxies ranging from true dwarf galaxies to more luminous very low surface brightness systems (Thuan & Seitzer 1979; McGaugh, Schombert, & Bothun 1995). Finally, previous determinations of LSB dI axis ratios have been based on photographic plates; for comparison with recent CCD observations of other types of dwarf galaxies, it is essential to have measurements of the axis ratios of a homogeneous sample of LSB dIs based on modern CCD observations.

In this paper, we measure the ellipticities of 30 LSB dI galaxies and compare the ellipticity distribution with that of 80 dEs (Ryen & Terndrup 1994; Ryden et al. 1998) and 62 BCDs (Sung et al. 1998, hereafter Paper I). We find that the ellipticity distribution of LSB dIs is very similar to that of BCDs, and marginally different from that of dEs. We then determine, under various assumptions, the distribution of intrinsic shapes of dI galaxies and compare it to those of other types of dwarfs. First, we assume that LSB dIs are either all oblate or all prolate, and use a nonparametric analysis to find the best-fitting distribution of intrinsic shapes. With this assumption, we find that the scarcity of nearly circular LSB dIs implies, at the 99% confidence level, that they cannot be a population of randomly oriented oblate or prolate objects, implying that LSB dIs are highly unlikely to be disk-shaped systems. Next, we assume that dIs are triaxial, and use a parametric analysis to find permissible distributions of intrinsic shapes. We find that if the intrinsic axis ratios, $\beta$ and $\gamma$, are distributed according to a Gaussian fit with means $\beta_0$ and $\gamma_0$ and a common standard deviation of $\sigma$, the best-fitting set of parameters for LSB dIs is $(\beta_0, \gamma_0, \sigma) = (0.66, 0.50, 0.15)$, for BCDs $(\beta_0, \gamma_0, \sigma) = (0.66, 0.55, 0.16)$, and for dEs $(\beta_0, \gamma_0, \sigma) = (0.78, 0.69, 0.24)$. The LSB dIs and BCDs thus have a very similar shape distribution, given this triaxial hypothesis, while the dEs peak at a somewhat more spherical shape. Therefore, our results provide strong observational evidence to support the evolutionary scenario in which the three types of dwarf galaxies have a close relation with each other.

2. OBSERVATIONS

Our sample consists of 30 LSB dI galaxies drawn from the list of UGC dwarfs and LSB galaxies detected in H I by Schneider et al. (1990, 1992). In the UGC catalogue, “dwarfs” are categorized as “objects with very low surface brightness and little or no concentration of light on the red prints,” with Hubble types of Sc-Irr or later (Nilson 1973). Among these galaxies, we select only galaxies with small 21 cm H I line widths ($\Delta v_{21} \leq 100 \, \text{km} \, \text{s}^{-1}$), small redshifts ($v_z \leq 1500 \, \text{km} \, \text{s}^{-1}$), and faint B-band luminosities ($M_B \geq -16$). For galaxies not observed by Schneider et al. (1992), H I data were taken from Huchtmeier & Richter (1989). In addition, we exclude galaxies with noticeable spiral patterns, so that the sample is composed of pure dwarf irregular galaxies. In POSS prints, most galaxies in our sample are found to be of generally low surface brightness, with superimposed irregular patches of star formation.
Photometric observations of the sample galaxies were carried out during several observing runs from 1985 to 1993 with different CCD chip and telescope combinations, using the KPNO 4 m with a 320 × 512 RCA1 chip during 1985 May 22–23, the KPNO 2.1 m with a 512 × 512 TSHA chip during 1990 October 19–22 and 1991 April 17–21, the 2.1 m with a 1024 × 1024 T1KA chip during 1993 January 23–24, the KPNO 0.9 m with a 1024 × 1024 ST1K chip during 1991 September 13–16, and the 0.9 m with a 1024 × 1024 T2KA chip during 1993 April 18–20. The galaxy names and observed bands are listed in Table 1; images of individual galaxies can be seen in Figure 2 of Patterson & Thuan (1996).

All steps of the data reduction and analysis were carried out using a standard CCD reduction process with IRAF. First, a bias offset was subtracted from each raw frame. To compensate for pixel-to-pixel variations in the bias level, we constructed a composite zero-frame of 20 individual bias frames (with the overscan already subtracted), and subtracted it from each frame. Next, images were divided by the combined flat images constructed from high-S/N level dome and sky flats for individual nights to remove the pixel-to-pixel variation in the detector sensitivity. Then, blank-dark-sky exposures were used to remove the interference pattern produced by night-sky emission lines. Finally, after individual object frames were processed through the flat-fielding correction, separate exposures were aligned and combined, followed by sky subtraction. Further details of the data reduction can be found in Patterson & Thuan (1996).

3. AXIS RATIO DETERMINATION

We determine the apparent axis ratios of individual LSB dEs by fitting ellipses to the isophotes of obtained images. For this process, the most widely used program is the Space Telescope Science Data Analysis System (STSDAS) routine ISOHOTEX, which is based on an iterative least-squares fit to a Fourier expansion. However, since the routine is designed for fitting the surface brightness distributions of stellar systems with uniform profiles, such as elliptical galaxies, it often produces an unstable fit when it is used for spirals and irregular galaxies. Therefore, we take a different approach, which uses a fully two-dimensional linear fit of the harmonics to the image. In this approach, the intensity of a galaxy is parameterized as a series of harmonic terms by

\[ I(a, \psi) = \sum_{n=0}^{k} I_n(a) \cos \{n[\psi - \psi_n(a)]\}, \]

where \( \psi \) is the position angle with respect to the major axis of the ellipse (Franx, Illingworth, & Heckman 1989). The results of harmonic fits are then used as the initial values for the usual Fourier series expansion ellipse-fitting routine, ISOHOTEX. Because of the small angular size of our images, we allowed the program to fit ellipses up to the radius at which only 60% of the points on the ellipse lay within the image. The center and position angle of the isophotes were generally allowed to vary. During the fitting process we excluded H ii regions within the galaxy from each image, along with foreground stars and cosmic rays. The measured ellipticity profiles of individual galaxies can be seen in Figures 64–71 of Patterson (1995).

The axis ratios of the LSB dEs in our sample, like those of many stellar systems, vary as a function of semimajor axis. We find that the variation is most severe in the inner parts, and is predominantly caused by the existence of irregular structures. In the outer parts, on the other hand, the ellipse-fitting process fails when the surface brightness falls far below the sky brightness. Therefore, as a representative axis ratio, we determine the intensity-weighted axis ratio averaged over the intermediate region where we could obtain stable values of axis ratios with successful ellipse-fitting processes. The intensity-weighted mean axis ratio is computed by

\[ \bar{q} = \frac{\int q(a) dL}{\int dL}, \quad dL = 2\pi qa \left[ 1 + \frac{d \ln q(a)}{d \ln a} \right] \Sigma(a) da, \]

where \( q = b/a, a \) is the semimajor axis of the isophote, \( b \) is the semiminor axis, and \( \Sigma(a) \) is the surface brightness of the isophote with semimajor axis \( a \). In Table 1 we present the mean ellipticities, \( \bar{\epsilon} \equiv 1 - \bar{q} \), and their uncertainties. The errors are estimated by computing the variance of \( q(a) \).

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1 IRAF is distributed by National Optical Astronomy Observatories, which is operated by the Association of Universities for Research in Astronomy, Inc., under cooperative agreement with the National Science Foundation.
within the range of semimajor axes where ellipticities are measured.

Because of the variation of axis ratios within a galaxy, it is important to apply a consistent method of ellipticity determination for the comparison between different types of galaxies. Since the mean ellipticities for the dE samples of Terndrup and et al. (1998) and for the BCDs in Paper I were determined by adopting the same method for similarly obtained CCD data, we can directly compare the axis ratio distribution of LSB dIs with those for other types of dwarf galaxies. In the upper panel of Figure 1, we present the cumulative function of for 30 LSB dIs in our sample (solid line), and compare it with those of 80 dE and 62 BCD samples. From this comparison, we find that the axis ratio distribution of LSB dIs is very similar to that of BCDs; the Kolmogorov-Smirnov (hereafter K-S) probability for comparing these two samples is $P_{\text{KS}} = 0.70$. Compared to dEs, LSB dIs are slightly flatter, on average. For the sample of LSB dIs, the mean and standard deviations of $q$ are $0.64 \pm 0.15$, while those for the dE sample are $0.70 \pm 0.16$. However, the difference in the axis ratio distributions between these two samples is marginal, with a K-S probability of $P_{\text{KS}} = 0.06$. The results of comparing the apparent axis ratio distributions between the different types of dwarf galaxies are summarized in Table 2.

### 4. Determining Intrinsic Shapes

To determine the intrinsic shape of LSB dIs and compare it to those of other types of dwarf galaxies, we apply two different methods: nonparametric and parametric. The nonparametric method assumes that the galaxies in a sample are either all oblate or all prolate, with intrinsic axis ratio $\gamma$, and are randomly oriented relative to us. With these assumptions, the distribution $f(q)$ of intrinsic shapes can be uniquely determined from the distribution $f(q)$ of apparent shapes. The parametric method, by contrast, assumes that the galaxies are triaxial, with axis lengths in the ratio $1:b:c$, where $1 \geq b \geq c$. In this case, there is no longer a unique inversion from the observed distribution $f(q)$ to the intrinsic distribution $f(\beta, \gamma)$. However, using a parametric model for $f(\beta, \gamma)$, we can find the model distribution of intrinsic axis ratios that best fits the observed distribution of apparent axis ratios. In our analysis, we model the distribution of

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**TABLE 2**

| Galaxy Type 1 | $\langle q \rangle^*$ | Galaxy Type 2 | $P_{\text{KS}}$ |
|---------------|----------------|---------------|----------------|
| LSB dIs ...... | $0.64 \pm 0.15$ | BCDs .......... | 0.701          |
| BCDs .......... | $0.67 \pm 0.15$ | dEs .......... | 0.057          |
| dEs .......... | $0.72 \pm 0.16$ | LSB dIs ...... | 0.060          |

* Mean axis ratios $\langle q \rangle$ are based on the samples of 30 LSB dIs from this paper, 62 BCDs from Paper I, and 80 dEs from Ryden & Terndrup (1994) and Ryden et al. (1998).
intrinsic axis ratios as a Gaussian, with means $\beta_0$ and $\gamma_0$ and a common width $\sigma_0$, i.e.,

$$f(\beta, \gamma) \propto \exp \left[ -\frac{(\beta - \beta_0)^2 + (\gamma - \gamma_0)^2}{2\sigma_0^2} \right].$$  (4.1)

Compared to the first method, the parametric method has the disadvantage that one must assume a functional form (Gaussian in our case) for the model axis ratio distribution, which is actually poorly known. Nevertheless, parametric fits are useful because they show us how statistics such as the K-S and $\chi^2$ scores vary as the parameters are changed. Details of the nonparametric and parametric shape analyses are given in Paper I.

4.1. Nonparametric Method

To test the hypothesis that galaxies in the sample are all randomly oriented oblate (or prolate) spheroids, we made a nonparametric kernel estimate of the distribution $f(q)$ of apparent axis ratios, using bootstrap resampling to place confidence intervals on our estimate (Tremlay & Merritt 1995; Ryden 1996). We then numerically inverted the estimate for $f(q)$ to find estimates for $f_\alpha(\gamma)$ and $f_\beta(\gamma)$, the distributions of the intrinsic axis ratios given the oblate and prolate hypotheses, respectively.

In the upper panel of Figure 2, we present the nonparametric kernel estimate of the distribution of the apparent axis ratios $q$ for our sample of 30 LSB dIs. In the middle panel, we show the distribution of intrinsic axis ratios assuming that the LSB dIs are oblate ($f_\alpha$); in the lower panel, we show the distribution of intrinsic axis ratios assuming that they are prolate ($f_\beta$). In each panel, the solid line is the best estimate, the dashed lines show the 80% confidence band, and the dotted lines show the 98% confidence band. (That is, at a given value of $q$, 1% of the bootstrap estimates fall above the upper boundary of the 98% confidence band, and 1% fall below the lower boundary of the 98% confidence band.) A Gaussian kernel is used to ensure a smooth, differentiable estimate of $f$, with a width $h = 0.069$. Because we impose a more or less arbitrary reflective boundary condition at $q = 1$, we do not believe our estimates for $f_\alpha$, $f_\beta$, and $f_\gamma$ within a distance $\sim h$ of the right-hand edge of Figure 2.

Note that there is a decided lack of nearly circular LSB dIs in our sample; the roundest galaxy we observed has $q = 0.873$. This scarcity of nearly circular galaxies is the characteristic sign that the galaxies cannot be a population of oblate spheroids. Looking at the middle panel, we see that the oblate hypothesis can be ruled out at the 99% (one-sided) confidence level. The 98% confidence band for $f_\alpha$ drops below zero for axis ratios $\gamma > 0.85$. Indeed, so pronounced is the lack of nearly circular galaxies that even the prolate hypothesis can be ruled out at the 99% (one-sided) confidence level. The 98% confidence band for $f_\beta$ drops below zero for $\gamma > 0.90$. Thus, even with our relatively small sample of galaxies, we can reject at a high confidence level the hypothesis that the LSB dI galaxies are a population of randomly oriented spheroids, either oblate or prolate. This leads us to consider, in the next section, possible distributions of triaxial shapes for the LSB dI galaxies in our sample.

4.2. Parametric Method

In our parametric analysis, we model the distribution of intrinsic axis ratios $(\beta, \gamma)$ as a Gaussian with a peak at $(\beta_0, \gamma_0)$ and a standard deviation $\sigma$, then search the parameter space for the best fit. In Figures 3a and 3b, we present the isoprobability contours on six slices through the $(\beta_0, \gamma_0, \sigma)$ parameter space, as measured by K-S and $\chi^2$ tests, respectively, for our LSB dI sample. When measured by a K-S test, the best-fitting distribution has parameters $(\beta_0, \gamma_0, \sigma) = (0.66, 0.50, 0.15)$ with K-S probability $P_{KS} = 0.98$, implying that the intrinsic shape of LSB dIs can be well fitted by a population of triaxial ellipsoids. We obtain consistent results when measured by $\chi^2$ tests; the best-fitting distribution has parameters of $(\beta_0, \gamma_0, \sigma) = (0.80, 0.42, 0.20)$, with $\chi^2$ probability $P_{\chi^2} = 0.91$.

For comparison with other types of dwarf galaxies, we list the parameters of the best-fitting distributions for BCDs and dEs in Table 3. The isoprobability contours from which these parameters are drawn were presented in Figures 6a, 6b, and 7 of Paper I. In addition, in the lower panel of Figure 1 we present the computed cumulative distribution (smooth curve) for the best-fitting triaxial model, as measured by the K-S test. The best-fitting model is overlaid on the measured cumulative distribution (step function) for each type of dwarf galaxy.
5. SUMMARY

We measure the ellipticities for a sample of 30 LSB dIs and compare the distribution of ellipticities with those for the samples of 62 BCDs and 80 dEs. From this comparison, we find that the axis ratio distribution of LSB dIs is statistically indistinguishable from that of BCDs. We also find that LSB dIs are slightly flatter, on average, than dEs, although the difference is marginal ($P_{KS} = 0.06$). The hint that dEs are rounder than LSB dIs and BCDs is intriguing in the context of the evolutionary scenario of Davies & Phillips (1988), in which dEs are the end state of dwarf evolution; if this scenario is correct, dwarfs must become rounder as they evolve from BCDs to dEs. We also determine the intrinsic shape of LSB dIs from the distribution of apparent axis ratios. From the nonparametric analysis, we find that the hypothesis that our sample LSB dIs are randomly oriented oblate or prolate objects is rejected at a high confidence level. On the other hand, the shapes of LBS dI galaxies are well described by triaxial spheroids if their axis ratios, $\beta$ and $\gamma$, have a Gaussian distribution. From the

TABLE 3

| Galaxy Type | Statistical Test | $\beta_0$ | $\gamma_0$ | $\sigma$ | $P_{KS}$ | $P^{\chi^2}$ |
|-------------|------------------|----------|-----------|----------|----------|-------------|
| LSB dIs ....... | K-S | 0.66 | 0.50 | 0.15 | $P_{KS} = 0.98$ | $P^{\chi^2} = 0.99$ |
| BCDs ............ | $\chi^2$ | 0.80 | 0.42 | 0.20 | $P_{KS} = 0.91$ | $P^{\chi^2} = 0.99$ |
| dEs .......... | $\chi^2$ | 0.77 | 0.51 | 0.16 | $P_{KS} = 0.96$ | $P^{\chi^2} = 0.96$ |

FIG. 3a

FIG. 3a - (a) Isoprobability contours, as measured by K-S tests, for 30 LSB dIs, on six slices through ($\beta_0$, $\gamma_0$, $\sigma$) parameter space. Contours are drawn at the levels of $P_{KS} = 0.01$, 0.1, 0.5, and 0.9, starting from the outside. (b) Isoprobability contours, as measured by $\chi^2$ tests for 30 LSB dIs, on six slices through ($\beta_0$, $\gamma_0$, $\sigma$) parameter space. Contours are drawn at the levels of $P_{KS} = 0.01$, 0.1, 0.5, and 0.9, starting from the outside.
parametric analysis, we determine that the best-fitting parameters are $(\beta_0, \gamma_0, \sigma) = (0.66, 0.50, 0.15)$. These results directly contradict the long-standing belief that LSB dIs have very flattened disky shapes, quite different from the spheroidal shapes of dEs and BCDs. Therefore, our results are consistent with the scenario in which the three major types of dwarf galaxies have very close evolutionary connections.

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