Gibbons-Maeda-de Sitter Black Holes

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Abstract

The metric of Gibbons-Maeda black hole in the presence of a cosmological constant is constructed and verified. The dilaton potential with respect to the cosmological constant is obtained. It is found that the cosmological constant is coupled to the dilaton field.

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I. INTRODUCTION

Much interest has been devoted to the study of the dilaton black holes in recent years. Exact solutions of charged dilaton black holes have been constructed by many authors [1-7]. It is found that the properties of black holes are greatly modified when the dilaton field is present. These black holes are all asymptotically flat. In the presence of one Liouville-type potential which is regarded as the generalization of the cosmological constant, a class of charged black hole solutions have been discovered [8, 9]. Unfortunately, these solutions are asymptotically neither flat nor (anti)-de Sitter.

In fact, Poletti, Wiltshire and Okai [10, 11, 12] have shown that with the exception of a pure cosmological constant, no asymptotically flat, asymptotically de Sitter or asymptotically ant-de Sitter static spherically symmetric solutions to the field equations associated with only one Liouville-type potential exist. In a recent work, we obtained the asymptotically de Sitter and asymptotically anti-de Sitter dilaton solutions in four dimensions [13]. It is found that the dilaton potential which is regarded as an extension of the cosmological constant have three Liouville-type potentials. In this short paper, we would report the Gibbons-Maeda-de Sitter black hole solution.

II. GIBBONS-MAEDA-DE SITTER BLACK HOLES IN COSMIC COORDINATE SYSTEM

The metric of a Gibbons-Maeda black hole in the Schwarzschild coordinate system is given by [1]

\[
ds^2 = - \left(1 - \frac{r_+}{x}\right) \left(1 - \frac{r_-}{x}\right) \left(1 - \frac{\Sigma^2}{x^2}\right)^{-1} dt^2 \\
+ \left(1 - \frac{r_+}{x}\right)^{-1} \left(1 - \frac{r_-}{x}\right)^{-1} \left(1 - \frac{\Sigma^2}{x^2}\right) dx^2 + \left(x^2 - \Sigma^2\right) d\Omega_2^2,
\]

where the constants \(r_+, r_-\) and \(\Sigma\) are related to the mass, electric charge, magnetic charge and dilaton charge of the black hole.
In order to obtain the de Sitter version of the metric, we should rewrite the metric Eq.(1) in the cosmic coordinate system. So make variable transformation

$$x = \frac{(r + r_+ + r_-)^2 - 4r_+ r_-}{4r},$$

then we can rewrite Eq.(1) as follows

$$ds^2 = -\left(1 - \frac{r_+ + r_-}{r}\right)^2\left(1 + \frac{r_+ - r_-}{r}\right)^2 \left(1 + \frac{4\Sigma^2}{r^2}\right)^2 dt^2$$

$$+ \frac{1}{16}\left[\left(1 + \frac{r_+ + r_-}{r}\right)^2 - \frac{4r_+ r_-}{r^2}\right]^2 \left(1 - \frac{4\Sigma^2}{r^2}\right)^2 \left(1 + \frac{4\Sigma^2}{r^2}\right)^2 (dr^2 + r^2 d\Omega_2^2).$$

(3)

Re-scale the variables $t$ and $s$, we have

$$ds^2 = -\left(1 - \frac{r_+ + r_-}{r}\right)^2\left(1 + \frac{r_+ - r_-}{r}\right)^2 \left(1 + \frac{4\Sigma^2}{a^2 r^2}\right)^2 dt^2$$

$$+ \frac{1}{16}\left[\left(1 + \frac{r_+ + r_-}{r}\right)^2 - \frac{4r_+ r_-}{r^2}\right]^2 \left(1 - \frac{4\Sigma^2}{a^2 r^2}\right)^2 \left(1 + \frac{4\Sigma^2}{a^2 r^2}\right)^2 (dr^2 + r^2 d\Omega_2^2).$$

(4)

Following the method we developed recently [13], we make the following replacements

$$1 \rightarrow a^\frac{4}{7}, \quad \frac{r_+}{r} \rightarrow \frac{r_+}{r a^\frac{1}{7}}, \quad \frac{r_-}{r} \rightarrow \frac{r_-}{r a^\frac{1}{7}}, \quad \frac{\Sigma}{r} \rightarrow \frac{\Sigma}{r a^\frac{1}{7}},$$

(5)

where $a \equiv e^{Ht}$, $H$ is a constant which has the meaning of Hubble constant. Then the Gibbons-Maeda-de Sitter metric is achieved

$$ds^2 = -\left(1 - \frac{r_+ + r_-}{ar}\right)^2\left(1 + \frac{r_+ - r_-}{ar}\right)^2 \left(1 + \frac{4\Sigma^2}{a^2 r^2}\right)^2 dt^2$$

$$+ a^2\left[\left(1 + \frac{r_+ + r_-}{ar}\right)^2 - \frac{4r_+ r_-}{a^2 r^2}\right]^2 \left(1 - \frac{4\Sigma^2}{a^2 r^2}\right)^2 \left(1 + \frac{4\Sigma^2}{a^2 r^2}\right)^2 (dr^2 + r^2 d\Omega_2^2).$$

(6)

Eq.(6) is just the Gibbons-Maeda black hole solution in the background of de Sitter universe in the cosmic coordinate system. When $r_+ = r_- = \Sigma = 0$, it recovers the well-known de Sitter metric. On the other hand, when $H = 0$, it recovers the Gibbons-Maeda metric.
Eq.(4). Furthermore, the metric Eq.(6) can be rewritten in the Schwarzschild coordinate system via coordinates transformations
\[
ds^2 = - \left[ \left(1 - \frac{r_+}{r}\right) \left(1 - \frac{r}{r}\right) \left(1 - \frac{\Sigma^2}{r^2}\right)^{-1} - \frac{1}{3} \lambda \left(r^2 - \Sigma^2\right) \right] dt^2
+ \left[ \left(1 - \frac{r_+}{r}\right) \left(1 - \frac{r}{r}\right) \left(1 - \frac{\Sigma^2}{r^2}\right)^{-1} - \frac{1}{3} \lambda \left(r^2 - \Sigma^2\right) \right]^{-1} dr^2 + \left(r^2 - \Sigma^2\right) d\Omega_2^2, \tag{7}
\]
where \( \lambda = 3H^2 \) which is just the cosmological constant. When \( \Sigma = 0 \), it restores to the well-known Reissner-Nordstrom-de Sitter metric. On the other hand, when \( \lambda = 0 \), it restores to the Gibbons-Maeda metric. In the following section, we would derive the potential of the dilaton with respect to the cosmological constant and the solution is concomitantly verified.

### III. DILATON POTENTIAL FOR THE COSMOLOGICAL CONSTANT

We consider the four-dimensional theory in which gravity is coupled to dilaton and Maxwell field with an action
\[
S = \int d^4x \sqrt{-g} \left[ R - 2 \partial_{\mu} \phi \partial^{\mu} \phi - V (\phi) - e^{-2\phi} F^2 - e^{-2\phi} H^2 \right], \tag{8}
\]
where \( R \) is the scalar curvature, \( F^2 = F_{\mu \nu} F^{\mu \nu} \) and \( H^2 = H_{\mu \nu} H^{\mu \nu} \) are the Maxwell electric and magnetic contributions, respectively, and \( V (\phi) \) in which the cosmological constant is included is a potential for \( \phi \).

Varying the action with respect to the metric, Maxwell fields, and dilaton field, respectively, yields
\[
R_{\mu \nu} = 2 \partial_{\mu} \phi \partial^{\nu} \phi + \frac{1}{2} g_{\mu \nu} V + 2 e^{-2\phi} \left( F_{\mu \alpha} F^{\alpha \nu} - \frac{1}{4} g_{\mu \nu} F^2 \right) + 2 e^{-2\phi} \left( H_{\mu \alpha} H^{\alpha \nu} - \frac{1}{4} g_{\mu \nu} H^2 \right), \tag{9}
\]
\[
\partial_\mu \left( \sqrt{-g} e^{-2\phi} F^{\mu \nu} \right) = 0, \tag{10}
\]
\[
\partial_\mu \left( \sqrt{-g} e^{-2\phi} H^{\mu \nu} \right) = 0, \tag{11}
\]
\[ \partial_{\mu} \partial^{\mu} \phi = \frac{1}{4} \frac{\partial V}{\partial \phi} - \frac{1}{2} e^{-2\phi} F^2 - \frac{1}{2} e^{-2\phi} H^2. \]  

(12)

The most general form of the metric for the static space-time can be written as

\[ ds^2 = -U(r) \, dt^2 + \frac{1}{U(r)} \, dr^2 + f(r)^2 \, d\Omega_2^2. \]  

(13)

Then the Maxwell equations Eqs.(10-11) can be integrated to give

\[ F = \frac{Q e^{2\phi}}{f^2} \, dt \wedge dr, \quad H = P e^{2\phi} \, d\theta \wedge d\varphi, \]  

(14)

where \( Q \) and \( P \) are the electric and magnetic charge, respectively. With the metric Eq.(13) and the Maxwell fields Eq.(14), the equations of motion Eqs.(9-12) reduce to four equations

\[ \frac{1}{f^2} \frac{d}{dr} \left( f^2 U \frac{d\phi}{dr} \right) = \frac{1}{f^4} \left( Q^2 e^{2\phi} - P^2 e^{-2\phi} \right), \]  

(15)

\[ \frac{1}{f^2} \frac{d^2 f}{dr^2} = - \left( \frac{d\phi}{dr} \right)^2, \]  

(16)

\[ \frac{1}{f^2} \frac{d}{dr} \left( 2U f \frac{df}{dr} \right) = \frac{2}{f^2} - V - \frac{2}{f^4} \left( Q^2 e^{2\phi} + P^2 e^{-2\phi} \right), \]  

(17)

\[ \frac{1}{f^2} \frac{d}{dr} \left( f^2 \frac{dU}{dr} \right) = -V + \frac{2}{f^4} \left( Q^2 e^{2\phi} + P^2 e^{-2\phi} \right). \]  

(18)

Substituted

\[ f = \sqrt{r^2 - \Sigma^2}, \quad U = \left( 1 - \frac{r_+}{r} \right) \left( 1 - \frac{r_-}{r} \right) \left( 1 - \frac{\Sigma^2}{r^2} \right)^{-1} - \frac{1}{3} \lambda \left( r^2 - \Sigma^2 \right), \]  

(19)

into Eqs.(15-18), we obtain the dilaton field \( \phi \), the electric charge \( Q \), the magnetic charge \( P \) and the dilaton potential \( V \) as follows

\[ e^{2\phi} = e^{2\phi_0} \left( 1 + \frac{2\Sigma}{r - \Sigma} \right), \]  

\[ 2Q^2 = e^{-2\phi_0} \left[ \Sigma^2 + r_+ r_- - \Sigma (r_+ + r_-) \right], \]  

\[ 2P^2 = e^{2\phi_0} \left[ \Sigma^2 + r_+ r_- + \Sigma (r_+ + r_-) \right], \]  

\[ V(\phi) = \frac{\lambda}{3} \left[ e^{2(\phi - \phi_0)} + e^{-2(\phi - \phi_0)} + 4 \right], \]  

(20)
where $\phi_0$ is the asymptotic value of the dilaton field. It is apparent the potential of the dilaton is the combination of a constant and two Liouville-type potentials. It is the same as our previous result [13]. When $\phi = \phi_0 = 0$, the potential is just the Einstein cosmological constant.

So far, we obtained the action of the Gibbons-Maeda-de Sitter black hole

$$S = \int d^4x \sqrt{-g} \left\{ R - 2\partial_\mu \phi \partial^\mu \phi - \frac{4}{3} \lambda - \frac{1}{3} \lambda \left[ e^{2(\phi - \phi_0)} + e^{-2(\phi - \phi_0)} \right] - e^{-2\phi} F^2 - e^{-2\phi} H^2 \right\}. \quad (21)$$

When $\phi = \phi_0 = 0$, it reduces to the action of Reissner-Nordström-de Sitter black hole.

In conclusion, we have constructed the Gibbons-Maeda black hole solution in the presence of the cosmological constant. So far no fully satisfactory de Sitter version of black hole solutions in string theory has been found [14]. Our solution is asymptotically both flat and (anti)-de Sitter. Thus it may be a fully satisfactory solution of string theory. We found that the dilaton potential with respect to the cosmological constant is not of the form of a pure cosmological constant but the form of three-Liouville-type potential. This is different from the Einstein-Maxwell theory.

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