Research article

Study on impact dynamic characteristics of semi-autogenous mill liner based on improved multivariable method

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ABSTRACT

The impact resistance of semi-autogenous mill liner has an important influence on the reliable operation of the system. The complex dynamic characteristics in the process of collision with abrasive medium are studied by the improved multivariable method proposed in this paper. Firstly, based on manifold element method, the deformation of abrasive medium and lining plate in the contact area is expressed by the coordinates of first-order manifold element basis function, while the deformation of the non-contact region is described by modal co-ordinates to improve the traditional multivariable method, and the displacement constraint equations as well as the acceleration constraint equations of the collision system are derived, then the Lagrange method is used to establish the impact dynamic equation of abrasive medium and lining plate system. Secondly, a collision test rig between abrasive medium and lining plate was built to verify the correctness and accuracy of the proposed method. Finally, the influence of structural parameters and motion parameters of abrasive medium and lining plate on the dynamic characteristics of collision system is analyzed by parameter analysis method. It is found that when the Poisson’s ratio of abrasive medium to lining plate material exceeds 1.0, the increase amplitude of stress becomes smaller. The increase amplitude of displacement, velocity, acceleration and stress at the collision contact point decreases when the initial velocity of abrasive medium exceeds 3.0 m/s. The stress at the collision contact point changes greatly, and the maximum acceleration occurs in advance when the radius of abrasive medium exceeds 20 mm. When the thickness of liner exceeds 40 mm, the time of maximum acceleration to reach is almost unchanged. The results show that compared with the traditional multivariable method, the improved multivariable method can increase the modeling efficiency and solving accuracy, and lay a theoretical foundation for optimizing the structure of the liner and raising the service life of the system.

1. Introduction

With the rapid development of semi-autogenous grinding technology, the semi-autogenous grinding equipment is gradually developing towards large-scale and automation. On the one hand, the grinding efficiency and quality can be improved, but on the other hand, the lining plate of the mill will be greatly impacted, resulting in lining plate wear, shortening service life and increasing grinding cost [1, 2]. Although other factors such as aging of material properties and chemical corrosion also have a certain impact on the wear, the main reason is that the abrasive strikes the lining plate at a certain speed during long-term high-speed operation.

The main research methods of lining plate wear at home and abroad are experimental method, numerical calculation method based on discrete element and finite element method. The test method is the earliest structural wear research method, but the wear of structures under impact is a strong nonlinear problem. The test results of proportional model usually cannot directly reflect the collision process between structural systems. Many scholars coupled EDEM and ANSYS to study the equivalent stress and its distribution of corrugated liner from the perspective of statics [3, 4, 5, 6]. The discrete element method was used to describe the material movement and the tangential collision energy wear model to predict the lining wall wear [7]. Millsoft software was used to study the energy distribution on the lining plate of semi autogenous mill [8]. Kalala et al [9] studied the wear on the liner based on the results of 2D discrete element simulation. The discrete element method was used to study the effect of liner design on wear behavior in the mill [10, 11, 12, 13, 14, 15]. Banisi et al [16] used the discrete element method...
2. Improved multivariable method

2.1. Multivariable method of deformed body

As shown in Figure 1, the point \( P \) is an arbitrary point on the deformed body \( B_j \), and its radial diameter in the inertial basis \( e'_r \) is \( r^f \), and its radial diameter in the floating basis \( e^f_r \) is \( r^i \). Then the generalized displacement matrix and velocity matrix of the deformed body \( B_j \) are obtained as shown by Eqs. (1) and (2):

\[
\mathbf{q} = \left[ r^f \ \mathbf{\psi}^f \ \mathbf{u}^f \right]^T
\]

\[
\mathbf{v} = \left[ \mathbf{\psi}^f \ \mathbf{u}^f \right]^T
\]

where, \( r \) and \( \psi \) are the displacement of the floating base of the deformed body relative to the inertial base respectively, and \( u \) is the displacement array of all nodes on the deformed body.

The deformed body \( B_j \) is divided into non-contact region \( I \) and contact region \( II \). In order to reduce the degree of freedom of the system, modal coordinates are used to replace node coordinates of non-contact region \( I \), as shown by Eq. (3):

\[
\mathbf{u}_i = \mathbf{\psi}_i \ \mathbf{\eta}_i
\]

In which, \( \mathbf{\psi}_i \) and \( \mathbf{\eta}_i \) are modal coordinates of non-contact region.

Generalized velocity matrix is shown by Eq. (4):

\[
\mathbf{v} = \mathbf{T} \mathbf{\bar{v}}
\]

\[
\mathbf{T} = \begin{bmatrix} \mathbf{I}_I & \mathbf{\eta}_I \end{bmatrix}
\]

Generalized mass matrix and force matrix are obtained as shown by Eqs. (7) and (8):

\[
\mathbf{\hat{M}} = \mathbf{T}^T \mathbf{M} \mathbf{T}
\]

\[
\mathbf{\hat{F}} = \mathbf{T}^T \mathbf{F}
\]

Then, the dynamic equation of the multivariable method is obtained as shown by Eq. (9):

\[
\delta \mathbf{\bar{v}}^T \left( - \mathbf{\hat{M}} \mathbf{\bar{v}} + \mathbf{\hat{F}} \right) = \mathbf{0}
\]

2.2. Contact restriction between the lining plate of mill barrel and abrasive

Based on the manifold element method, the contact between the abrasive and the liner of the mill barrel is point-surface contact, as shown in Figure 2. Point \( M \) on abrasive \( B_i \) and point \( N \) on the lining plate \( B_j \) form contact manifold element pairs. The grid in Figure 2 is a manifold element mathematical cover, which intersects with the actual structure of the liner and the sphere to form a physical cover.

During the collision, the positions of the contact manifold pairs on the abrasive and the liner remain equal from the assumption of

![Figure 1](image1.png)

Figure 1. Kinematic description of deformable body system.

![Figure 2](image2.png)

Figure 2. Contact manifold element pairs based on first order manifold element method.
non-penetration at the moment of impact of the deformed body in the global coordinate system \( o \zeta \eta \). Then, there is the position constraint equation:

\[
\Delta = r_j^N - r_j^M = 0 \tag{10}
\]

where, \( r_j^M \) is the position of abrasive medium in floating coordinate \( i \), can be obtained by interpolating the position of the zero-order basis function of the surrounding manifold element, \( r_j^M \) is the position of the contact manifold elements on the lining plate and in the \( M \)-direction. In order to reduce the total number of elements and ensure the solution accuracy, the first-order basis function can be used for the mathematical coverage of the adjacent area by the contact manifold elements (manifold elements 1, 2, 3 and 4) and then positions were obtained by interpolation:

\[
\begin{align*}
    r_j^N &= \sum_{i=1}^{6} \begin{bmatrix} f_i & 0 \\ f_i & 0 \end{bmatrix} \begin{bmatrix} d_{2i-1} \\ d_{2i} \end{bmatrix} r_i' \\
    r_j^M &= \sum_{i=1}^{6} \begin{bmatrix} f_i & 0 \\ f_i & 0 \end{bmatrix} \begin{bmatrix} d_{2i-1} \\ d_{2i} \end{bmatrix} r_i'
\end{align*} \tag{11}
\tag{12}
\]

where, \( d_{2i-1}, d_{2i}, d_{2i-1}, \text{ and } d_{2i} \) are intermediate variables, \( f_i \) and \( f_i' \) are first-order and zero-order basis functions respectively, and their values are:

\[
\begin{bmatrix} f_i \\ f_i' \end{bmatrix} = \begin{bmatrix} 1 & 0 & x & 0 & y & 0 \\ 0 & f_i & 0 & 1 & 0 & x \end{bmatrix}
\]

\[
\begin{bmatrix} f_i \\ f_i' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]
Figure 5. The process of sphere impacting block at high speed.

(a) 0 ms  (b) 20 ms  (c) 40 ms

(d) 48 ms  (e) 60 ms  (f) 80 ms

Figure 6. Comparison of results.

(a) Stress of point A  (b) Stress of point B  (c) Stress of point C
By substituting Eqs. (11) and (12) into Eq. (10), constraint equations of each set of contact manifold elements on the form of acceleration can be obtained:

$$\Phi = \left[ \frac{\Delta}{\partial q_1}, \frac{\Delta}{\partial q_2}, \frac{\Delta}{\partial q_3}, \frac{\Delta}{\partial q_4}, \frac{\Delta}{\partial q_5} \right]$$

(13)

In which

$$\Phi = \left[ \frac{\Delta}{\partial q_1}, \frac{\Delta}{\partial q_2}, \frac{\Delta}{\partial q_3}, \frac{\Delta}{\partial q_4}, \frac{\Delta}{\partial q_5} \right].$$

$$\gamma = -\left( \Delta q, \frac{\Delta}{\partial q_1}, \Delta q, \frac{\Delta}{\partial q_2}, \Delta q, \frac{\Delta}{\partial q_3} \right)$$

$$\Delta q = \frac{\Delta}{\partial q_1}, \Delta q = \frac{\Delta}{\partial q_2}, \Delta q = \frac{\Delta}{\partial q_3}$$

Grouping all pairs of contact manifold elements, then according to Lagrange equation with multiplier \[27\] and parallel Eqs. (7), (8), and (13), the dynamic equation of the collision stage between abrasive medium and cylinder liner can be obtained as shown by Eq. (14):

$$\begin{bmatrix} M & \Phi_{q}^F & \Phi_{q}^C \end{bmatrix} \begin{bmatrix} \sigma^F \\ \sigma^C \end{bmatrix} = \begin{bmatrix} F \\ \gamma \end{bmatrix}$$

(14)

In which \(\sigma^F\) represents Lagrangian multiplier with fixed constraints, \(\sigma^C\) represents contact constrained Lagrange multipliers and \(\Phi_{q}^F\) represents the fixed constraint Jacobin matrix of the deformed body.

Where, superscript \(F\) represents fixed constraint and superscript \(C\) represents contact constraint.

3. Experiments to verification

In order to verify the correctness and accuracy of the method proposed in this paper, the sphere and the rectangular block are taken as abrasive medium and lining plate for field collision experimental study as shown in Figure 3. The sphere obtains the initial horizontal velocity and initial vertical velocity through the slope slide. The structural parameters and position parameters of the rectangular block and the collision sphere are shown in Table 1. The mechanical properties of rectangular blocks and spheres are shown in Table 2. The geometric structure diagram of the block and sphere collision system is shown in Figure 4. The rectangular block is fixed on the base by a fixed cylinder.

During the field experiment, a high-speed camera was used to track the velocity and position of the sphere. Three strain gauges were pasted near the possible collision area of the block, and the dynamic signal testing system and computer equipment were connected to obtain the test results of each strain gauge group.

The initial position of the sphere is the dashed line at the upper end of the slide in Figure 4, and it will hit the block at the lower end of the slide at the initial speed \(v\). Patch positions of strain gauges A, B and C are shown in Figure 4. \(\alpha\) is the global coordinate of the collision system, and \(\gamma, \eta\) is the fixed state. The process of the ball impacting the block from the exit of the inclined slide at an initial speed of 3.0 m/s observed by the high-speed camera as shown in Figure 5.

The corresponding stress \(\sigma\) can be obtained by multiplying the strain \(\varepsilon\) measured by the strain gauge and the elastic modulus of the rectangular block material \(E\), as shown in Eq. (15). The results

![Figure 7. Influence of Poisson’s ratio on collision effect.](image-url)
obtained by the method in this paper, the results of the multivariable method and the test results are compared, as shown in Figure 6.

\[ \sigma = E\varepsilon \]  

It can be seen from Figure 6 that compared with the test results, the maximum errors of stress at point A, B and C are 3.56%, 4.68% and 4.12% using the method proposed in this paper, respectively, and the maximum errors of stress at point A, B and C are 8.55%, 9.49% and 10.13% using the traditional multivariable method, respectively, indicating that compared with the traditional multivariable method, the improved multivariable method proposed in this paper is more correct and accurate, because it can be more close to the experimental results when the total number of system elements is the same.

4. Parameter analyses

In order to further analyze the impact of the structural parameters and motion parameters of abrasive medium and liner on the impact effect, the sphere and rectangular block collision test system built in Section 2 is taken as the research object. In this paper, the influence of Poisson’s ratio \( u = \frac{\mu_1}{\mu_2} \), the initial velocity \( v \) and radius \( r \) of abrasive medium and the thickness of liner \( b \) on the impact effect are studied by parametric analysis. To simplify the calculation, the simulation time was set at 40 ms on the premise of ensuring the analysis effect. When one class of parameters changed during the analysis, other parameters remained unchanged, and the position parameters of the collision system were consistent with Table 1.

4.1. Influence of Poisson’s ratio between abrasive medium and liner on impact dynamic characteristics

In order to analyze the impact of the change of the ratio of the Poisson’s ratio between the abrasive medium and the lining plate on the collision, Poisson’s ratio \( u \) is set as 0.5, 1.0 and 1.5, the initial velocity \( v \) of the abrasive medium is set as 3.0 m/s, the radius \( r \) of the abrasive medium is set as 20 mm, and the section thickness \( b \) of the lining plate is set as 40 mm. Other parameters remain unchanged. The curves of displacement, velocity, acceleration and stress at the collision contact point are obtained through detailed analysis, as shown in Figure 7.

As shown in Figure 7, with the increase of Poisson’s ratio \( u \) between abrasive medium and liner, the periodicity and amplitude of displacement, velocity and acceleration at the collision contact point remain unchanged, but the periodicity of stress at the contact point remains

![Figure 8. Influence of initial velocity of abrasive medium on collision effect.](image-url)
unchanged and the amplitude increases. It can also be seen from Figure 7(d) that when the ratio $u$ exceeds 1.0, the contact stress of the collision point changes little. The analysis shows that the Poisson’s ratio of abrasive medium and liner material has a great influence on the stress during the collision.

4.2. Influence of initial velocity of abrasive medium on impact dynamic characteristics

In order to analyze the influence of the initial velocity of abrasive media on the impact effect, the initial velocity $v$ of abrasive media was set as 1.0 m/s, 3.0 m/s and 5.0 m/s, the ratio $u$ of the Poisson’s ratio between abrasive media and the lining plate was set as 1.0, the radius $r$ of abrasive media was set as 20 mm, and the section thickness $b$ of the lining plate was set as 40 mm. Other parameters remained unchanged. The curves of displacement, velocity, acceleration and stress at the collision contact point are obtained through detailed analysis, as shown in Figure 8.

It can be seen from Figure 8 that the change of the initial velocity $v$ of abrasive medium has a great influence on the impact effect. With the increase of the initial velocity, the displacement, velocity, acceleration and stress amplitude of the collision contact point increase, and the collision time gradually advance. When the initial velocity $v$ of the abrasive medium exceeds 3.0 m/s, the displacement, velocity, acceleration and stress increase of the collision contact point decrease. It can also be seen from Figure 8(a) that the larger the initial velocity $v$ of abrasive medium is, the increase of deformation amplitude generated at the collision contact point.

4.3. Influence of abrasive medium radius on impact dynamic characteristics

In order to analyze the influence of the radius of the abrasive medium on the impact effect, the radius $r$ of the abrasive medium was set as 15 mm, 20 mm and 25 mm, the ratio $u$ of the Poisson’s ratio between the abrasive medium and the lining plate was set as 1.0, the initial velocity $v$ of the abrasive medium was set as 3.0 m/s, and the section thickness $b$ of the lining plate was set as 40 mm. Other parameters remained unchanged. The curves of displacement, velocity, acceleration and stress at the collision contact point are obtained through detailed analysis, as shown in Figure 9.

It can be seen from Figure 9 that as the radius of abrasive medium increases, the displacement, velocity and stress amplitude of collision contact point gradually increase, while the amplitude of acceleration changes little. It can also be seen from Figure 9 that when the radius $r$ of abrasive medium exceeds 20 mm, the stress at the collision contact point

![Graphs](image-url)
changes greatly and the maximum acceleration occurs in advance, indicating that the radius of abrasive medium has a small influence on the impact acceleration and a great influence on the stress.

4.4. Influence of section thickness of liner on impact dynamic characteristics

In order to analyze the impact of the section thickness of the lining plate on the impact effect, the section thickness $b$ of the abrasive lining plate was set as 30 mm, 40 mm and 50 mm, the ratio $u$ of the Poisson’s ratio between the abrasive medium and the lining plate was set as 1.0, the initial velocity $v$ of the abrasive medium was set as 3.0 m/s, and the radius $r$ of the abrasive medium was set as 20 mm. Other parameters remained unchanged. The curves of displacement, velocity, acceleration and stress at the collision contact point are obtained through detailed analysis, as shown in Figure 10.

It can be seen from Figure 10 that with the increase of the section thickness $b$ of the lining plate, the displacement and acceleration of the collision contact point increase, while the stress amplitude decreases, and the velocity does not increase significantly. It can also be seen from Figure 10(c) that the maximum acceleration of impacting occurs in advance, and when the thickness $b$ of the lining plate exceeds 40 mm, the maximum acceleration occurs almost unchanged.

5. Conclusions

The dynamic characteristics of the impact between the liner of semi-autogenous mill and abrasive medium are studied by the improved multivariable method proposed in this paper. Based on the traditional multivariable method, firstly, the deformation of abrasive medium and liner in the contact area is represented by manifold element node coordinates, and the deformation of non-contact area is described by modal coordinates. Then, the whole process of collision contact is simulated. Secondly, the results of this paper, the results of the traditional multivariable method and the experimental results are compared to verify the correctness and accuracy of the proposed method. Finally, the influences of Poisson’s ratio between abrasive medium and liner, initial velocity and radius of abrasive medium and section thickness of liner on dynamic characteristics of collision system were studied by parametric analysis method. The specific conclusions are as follows:

1. Compared with the traditional multivariable method, the improved multivariable method adopts different basis functions to form element stiffness matrices in the contact region and non-contact region respectively and then assemble them, which improves the modeling efficiency and solving accuracy.
2. The Poisson’s ratio of abrasive medium and liner material has little effect on the displacement, velocity and acceleration of the
contact point, but has a great change on the stress of the contact point. When the ratio exceeds 1.0, the increase amplitude of stress becomes smaller.

(3) With the velocity of abrasive medium increases, the displacement, velocity, acceleration and stress of collision contact point increase. But when the initial velocity exceeds 3.0 m/s, the increase amplitude of displacement, velocity, acceleration and stress of collision contact point decreases.

(4) With the radius of abrasive medium increases, the displacement, velocity and stress amplitude of collision contact point gradually increase, while the amplitude of acceleration changes little. But when the radius of abrasive medium exceeds 20 mm, the stress of collision contact point changes greatly and the maximum acceleration occurs in advance.

(5) With the thickness of the lining section increases, the displacement and acceleration of the collision contact point increase, the stress amplitude decreases, and the speed increase is not obvious. When the thickness exceeds 40 mm, the time for the maximum acceleration is almost unchanged.

Declarations

Author contribution statement

Wenwu LIU: conceived and designed the experiments; performed the experiments; analyzed and interpreted the data; wrote the paper.
Hongxin LIU: supported by key technologies research and development program [2017YFD0700105-2].
Changsheng HU: analyzed and interpreted the data; wrote the paper.
Hongxin LIU: contributed reagents, materials, analysis tools or data.
Wenwu LIU: conceived and designed the experiments; performed the experiments; analyzed and interpreted the data; wrote the paper.

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Data availability statement

Data included in article/supporting material/referenced in article.

Declaration of interest’s statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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