AC DRIVEN JUMP DISTRIBUTIONS ON A PERIODIC SUBSTRATE

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A driven Brownian particle (e.g. an atom on a surface) diffusing on a low-viscosity, periodic substrate may execute multiple jumps. In the presence of an additional periodic drive, the jump lengths and time durations get statistically modulated according to a synchronization mechanism reminiscent of asymmetric stochastic resonance. Here, too, bistability plays a key role, but in a dynamical sense, inasmuch as the particle switches between locked and running states.

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The diffusion of a forced Brownian particle on one- or two-dimensional periodic substrates provides an archetypal model of relaxation in condensed phase with application to adsorbates on crystal surfaces [1], transport in superionic conductors [2], acoustoelectric currents in one-dimensional channels [3], dispersion of particles in optical traps [4], or rotation of molecules in solids [5]. The diffusive process from this category best studied in the current literature [3]–[16] is the Brownian motion in a one-dimensional washboard potential

$$\ddot{x} = -\gamma \dot{x} - \omega_0^2 \sin x + F + \xi(t)$$

(1)

(rescaled units), where the force terms on the r.h.s. represent, respectively, a viscous damping with constant $\gamma$, a spatially periodic, tilted substrate modeled by the potential

$$V(x, F) = \omega_0^2 (1 - \cos x) - Fx,$$

(2)

and a stationary Gaussian noise with zero mean $\langle \xi(t) \rangle = 0$ and autocorrelation function $\langle \xi(t)\xi(0) \rangle = 2\gamma kT \delta(t)$.

In the low damping limit, $\gamma \ll \omega_0$, (strictly speaking it would suffice that $\gamma < 1.2\omega_0$ [7]) the Brownian particle executes multiple jumps, namely over many a lattice spacing $a = 2\pi$, both in the forward and in the backward direction [13,14]; with increasing $F$ the length- and time-distributions of such multiple jumps, denoted, respectively, by $N(X)$ and $N(T_x)$, change from an almost exponential curve (locked regime, $F < F_1 \equiv (4/\pi)\gamma \omega_0$ [7]) to a kind of bimodal curve with a slow exponential tail (running regime $F > F_2 \simeq 3.36\gamma \omega_0$ [14,15], see also Fig. 1). To a good approximation $N(X)$ and $N(T_x)$ are related by the scaling law $X = (F/\gamma)T_x$ [14], where $F/\gamma$ is the average velocity of the particle in the running regime [1].

In a recent paper Talkner et al. [17] addressed the effects of a weak periodic drive on the jump statistics in the intermediate regime with $kT \ll F \ll \omega_0^2$ (exponential hopping regime [13]): An enhancement of the multiple jumping probability has been predicted in the adiabatic limit of very low modulation frequencies. Taking on their challenge to a full investigation, we have simulated numerically the Langevin equation (LE) [1] with the dependent drive

$$F \rightarrow F(t) = F_0 + \Delta F \cos(\Omega t),$$

(3)

$\Delta F > 0$. Note that the time origin has been set to zero without loss of generality and the modulation amplitude $\Delta F$ has been kept small compared to the dc term $F_0$ in all simulation runs.

In the present report we focus on the time distributions $N(T_x)$ of the multiple jumps at the crossover between the locked and the running regime: A significant synchronization mechanism takes place reminiscent of stochastic resonance (SR) in a bistable system [18].

At low temperatures $kT \ll \omega_0^2$, and in the exponential hopping regime, $\gamma \ll \omega_0^2$ and $F_0 < F_1$, jumps are thermally activated and, therefore, relatively short; their direction (either forwards or backwards) is controlled by the intensity of the dc drive $F_0$ relative to $F_1$ [14,15]. Under such conditions, the Brownian particle sits in a locked state with low mobility, namely $\gamma \mu \equiv \langle \dot{x} \rangle / F_0 \ll 1$. On raising $F_0$ above the threshold value $F_2 \simeq 3.36\gamma \omega_0$ ($F_2$ depends weakly on the temperature) $\gamma \mu$ jumps abruptly up to close to unity, thus signalling that the particle has switched into the running state with average velocity $\langle \dot{x} \rangle = F_0/\gamma$. As the particle flies over the washboard potential it executes sporadic, short stops at the bottom of some potential well. For numerical purposes the diffusing
particle is deemed as locked (or retrapped) any time it sojourns within two adjacent $V(x, F)$ barriers for a time interval longer than $(2\gamma)^{-1}$ – the characteristic relaxation time of the energy variable. In Fig. 3 we display the normalized distributions of the jump time durations, $N(T_x)$, for different values of $F_0$, ranging from the locked, well into the running regime. As expected, all of our distributions exhibit exponential tails with time constant $T_x$, termed from now on the average jump duration, that increases exponentially with $F_0$. No dramatic change in the $N(T_x)$ profile occurs in the vicinity of the thresholds $F_1$ and $F_2$.

The effect of the weak periodic modulation on the jump statistics is illustrated in Fig. 4, where $N(T_x)$ is plotted for the same ac drive, but increasing values of $F_0$ (across the threshold $F_2$). For $F_0 < F_2$ the distribution of the jump durations is well reproduced by an exponential function: for $F_0 \sim F_2$ the curve bends to form a broad hump around $T_x = T_\Omega/2$ (here $T_\Omega = 2\pi/\Omega$ is the modulation period); finally, for $F_0 > F_2$ the distribution develops a full-blown peak structure, with maxima centered at $T_n = nT_\Omega$, with $n = 1, 2, \ldots$ superimposed on a slowly decaying exponential tail. On a closer look one recognizes that even for $F_0 = F_2$ the jump distribution is multi-peaked; however, the first peak dominates apparently over the others, thus indicating a strong drive-jump synchronization. More remarkably, the positions of the $N(T_x)$ maxima shift from $T_n = (n - 1/2)T_\Omega$ at $F_0 \simeq F_2$ to $nT_\Omega$ for $F_0 \gg F_2$.

The qualitative interpretation of these results is simple. For $F_0 < F_2$ the length and, therefore, the time duration of the multiple jumps is relatively short, – we recall that $kT \ll \omega_0^2$, – anyway, shorter that the modulation period $T_\Omega$. We are dealing with an adiabatic situation, where the slowly varying external parameter $F(t)$ during the jump process can be regarded as constant; the ensuing distribution $N(T_x,t)$ is expectedly exponential at any times, so that $N(T_x) = \langle N(T_x,t) \rangle$, denoting the average over a modulation cycle. To this point, it should be remembered that the jump rate, namely the number of jumps per unit of time, is maximum at $F_0 \simeq F_2$ and decays exponentially with $F_0$ above threshold $F_2$. For $F_0 \simeq F_2$ the system operates at the unlocking threshold; a small ac drive amplitude $\Delta F$ suffices to modulate the system, alternately, from the locked to the running state and vice versa. No surprise that most jumps last the order of one half forcing period, hence the appearance of a bump in the $T_x$ distribution at $T_\Omega/2$. For $F_0 > F_2$ the Brownian particle is in the running state most of the time, so that only infrequent, very long, jumps may take place. Clearly, chances are that rare relocking events may happen mostly as $F(t)$ hits a minimum $F_0 - \Delta F$. As a consequence, we expect that $T_x$ are kind of ”multiple” of $T_\Omega$ (jump-drive phase-locking). Notice that at the threshold the average residence time in the locked state $T_r$ equals the running time $T_r$, whereas in the running regime $T_r$ becomes negligible.

The dependence of the jump synchronization mechanism on the amplitude $\Delta F$ of the ac drive is displayed in Fig. 3. After subtracting the exponential background due to the random jumps, we concluded that in the weak drive regime, $\Delta F \ll F_0$, the fraction of the synchronized jumps (of any duration $T_x$) grows quadratically with $\Delta F$. As a matter of fact, changing the ac drive amplitude affects the background decay constant, as well. At finite $\Delta F$ values the background seems to fall off faster than in the absence of modulation. This suggests that increasing $\Delta F$ has a twofold effect: It modulates more and more effectively the jump lengths and, simultaneously, diminishes the effective dc drive ($F_0$, to be defined in the following) below the input value $F_0$ by an amount that depends on the forcing period.

Finally, in Fig. 4 we show the dependence of the jump distribution $N(T_x)$ on the modulation period. Note that all distributions plotted here have been normalized to unity after rescaling $T_x$ to $T_\Omega/T_x$. The overall $\Omega$ dependence of the synchronization mechanism is apparent: For short modulation periods the jump-drive phase-locking is hardly detectable; on increasing $T_\Omega$ the $N(T_x)$ peak structure reaches first its maximum and, eventually, merges into the background for exceedingly long forcing periods. The qualitative interpretation of these results is also straightforward. The case of large $T_\Omega$ corresponds to the adiabatic situation illustrated above, when we discussed the distributions of Fig. 3 with $F_0 < F_2$; the case of small $T_\Omega$ values represents the opposite situation, where the external time-dependent parameter $F(t)$ varies so fast in time, that the diffusing particle only responds to its effective average value $F_\text{av}$ insensitive to its temporal modulation – technically speaking, also an adiabatic limit; in the intermediate regime $T_\Omega \simeq T_x$ the synchronization mechanism becomes more efficient, due to the interplay between externally driven and thermally activated locked-to-running transitions. The apparent increase of the background steepness with $T_\Omega$ is a graphic artifact introduced by rescaling the units of $T_x$; when plotted versus molecular dynamics time-units all curves of Fig. 4 decay with time constants corresponding to an effective tilt $\bar{F}_0$ such that $F_0 - \Delta F < \bar{F}_0 < F_0$.

We attempt now at interpreting the numerical results reported so far as a manifestation of stochastic resonance (SR), – an intriguing phenomenon of nonlinear physics, where an optimal amount of noise has the capability of enhancing the rate of synchronous switches between the local minima of a (possibly, asymmetric) bistable system driven by a weak periodic modulation. Stochastic resonance rests upon a noise controlled phase-locking mecha-
anism which can be vividly illustrated in term of switch-time distributions \[19\]. A convenient choice is provided by the distributions \(N_1(T_\pm)\) of the residence times \(T_\pm\) of the system in either state (denoted by \(\pm\), respectively). As the modulation forces the switch events by perturbing \(\pm\) the system in either state (denoted by \(\pm\), respectively). A convenient choice is provided by the distributions \(N_1(T_\pm)\) of the residence times \(T_\pm\) of the system in either state (denoted by \(\pm\), respectively). For a symmetric system, \(T_\pm(0) = T_\Omega/2\). An optimal input-output synchronization sets in when the first peak of \(N_1(T_\pm)\) at \(T_\pm(0)\) dominates over the remaining peaks and the exponential random switch background \(\[20\]\).

In the system at hand all ingredients of the synchronization-based (or bona-fide) SR \[19\] are easily recognizable: (i) bistability occurs at the threshold \(F_2\), where the particle mobility switches between two dynamical states, namely the locked state with \(\gamma \mu = 0\) and the running state with \(\gamma \mu = 1\). The residence times in the two states are denoted here by \(T_r\) and \(T_x\), respectively; (ii) the noise control parameter is the equilibrium temperature \(T\). Note that SR may occur even at low damping \(\[18\]\); (iii) the external periodic drive, \(\Delta F \cos(\Omega t)\), Eq. \(\[3\]\), tilts the system towards either dynamically stable state, alternately, thus determining the multi-peaked \(T_x\) distributions of Figs. \(\[4\]\).

A few well-known SR features are apparent: (1) The \(N(T_x)\) distributions develop their first (and strongest) peak for \(T_\Omega \simeq 2T_x\) (see Figs. \(\[2\]\) and \(\[3\]\)); (2) The first peak, at \(T_x = T_x(0)\), shifts from close to \(T_\Omega/2\) in the symmetric case, \(F_0 = F_2\), to \(T_\Omega\) in the running regime, \(F_0 > F_2 + \Delta F\). This is a typical asymmetry effect \(\[22\]\), as on raising \(F_0\) above \(F_2\), \(T_x\) gets smaller than \(T_\Omega\); (3) The peak structure grows sharper on increasing \(\Delta F\), according to a characteristic SR quadratic law \(\[15\]\).

Peculiar to our model are the properties following: (4) On increasing \(F_0\) at \(\Delta F\), \(T\) and \(\Omega\) constant, more and more peaks shoot up from the background, thus reducing the input-output synchronization (see Fig. \(\[2\]\). The explanation in terms of the SR phenomenology is simple: The jump times \(T_x\) coincide with the residence times in the running state, which for \(F_0 > F_2\) gets more stable than the locked state; accordingly, \(T_x\) grows much longer than \(T_\Omega\) or, equivalently, relative to the directed running-to-locked transition, the noise intensity is too low for SR to take place; hence the multi-peaked structures plotted in Figs. \(\[2\]\) \(\[15\]\) \(\[23\]\); (5) Contrary to most SR studies, here the background constant depends on the ac drive, namely on \(\Delta F\) and, though to a smaller extent, on \(T_\Omega\). A pictorial explanation of such a property is provided by Fig. 11.24 of Ref. \(\[2\]\). Consider the most symmetric case with \(F_0 = F_2\); adding \(\Delta F\) to \(F_2\) makes the running state more stable than the locked state and, vice versa, subtracting \(\Delta F\) from \(F_2\) makes the running state less stable; however, the imbalance toward the locked state at \(F_0 = F_2 - \Delta F\) is not compensated by the imbalance towards the running state at \(F_0 = F_2 + \Delta F\); therefore, an ac drive contributes to the effective static tilt \(\bar{F}_0\), with \(\bar{F}_0 < F_2\), defined implicitly through the average jump duration as \(\bar{T}_x(F_0, \Delta F, \Omega) \equiv T_x(F_0)\).

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FIG. 1. Jump distributions in the absence of modulation, \( \Delta F = 0 \), for different values of the dc drive \( F_0 \) below (a) and in the vicinity of the threshold \( F_2 \) (b). \( F_0 \) is expressed in units of \( F_2 \) with \( F_2 = 0.085 \). Other parameter values: \( \gamma/\omega_0 = 0.03 \), \( kT/\omega_0^2 = 0.3 \).

FIG. 2. Jump distributions in the presence of an ac drive with constant amplitude, \( \Delta F/F_2 = 0.12 \), and period, \( T_\Omega = 2\pi/\Omega = 6.4 \cdot 10^4 \), and for different values of the static tilt \( F_0 \). Other parameter values are as in Fig. 1.

FIG. 3. Dependence of the jump distributions on the modulation amplitude \( \Delta F \) at constant period, \( T_\Omega = 8 \cdot 10^3 \), and static tilt, \( F_0/F_2 = 1.24 \). Other parameter values are as in Fig. 1.

FIG. 4. Dependence of the jump distributions on the modulation period \( T_\Omega \) (in units of \( 10^3 \)) at constant amplitude, \( \Delta F/F_2 = 0.12 \), and static tilt, \( F_0/F_2 = 1.24 \). Other parameter values are as in Fig. 1.