MAESTRO: A Low Mach Number Stellar Hydrodynamics Code

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Abstract.
Convective astrophysical flows, such as those leading up to Type Ia supernovae, can be characterized by low Mach numbers ($M \ll 1$). One can neglect acoustic waves in the modeling of these flows, but must retain compressibility effects due to nuclear reactions and background stratification in order to accurately model the convection and initial burning. We present here a new algorithm, MAESTRO, that is specifically designed for efficient long-time integration leading to ignition. Here, we discuss the state of this new algorithm and its application to Type Ia supernovae.

1. Introduction
Type Ia supernovae are characterized by a centuries-long period of subsonic convection [1] followed by a seconds-long explosion (see [2] for a good review). It is of critical importance to understand the convection, as it sets the initial conditions for the explosion. Simulations of the convective phase are difficult because of the requirement to follow $M \sim 0.01 - 0.1$ convective eddies over many turnover times. Nevertheless, progress has been made using implicit [3] and anelastic [4] hydrodynamics methods. We have developed a new algorithm, based on the low Mach number approximation, that allows for large time steps, finite-amplitude density and temperature perturbations, and the hydrostatic evolution of the base state of the star. We have demonstrated [5, 6, 7] that this new code, MAESTRO, accurately and efficiently simulates low Mach number astrophysical flows. Here, we summarize the basic features of the algorithm and discuss the initial three-dimensional results.

In the low Mach number approximation, we exploit the asymptotic behavior of the equations of fluid dynamics to decouple the pressure and density perturbations and thereby filter out sound waves. This is accomplished by expressing the pressure as the sum of a thermodynamic pressure, $p_0$, and a dynamic pressure, $\pi$, such that $\pi/p_0 \sim O(M^2)$. The thermodynamic pressure is used to construct a base state in hydrostatic balance, i.e. we define a density, $\rho_0$, such that $\partial p_0/\partial r = -\rho_0 g$. Here $g$ is the gravitational acceleration. The total pressure is replaced by the thermodynamic pressure everywhere but in the momentum equation; we exploit the hydrostatic balance of the base state to write the buoyancy term in the momentum equation in perturbational form, but no approximation to the exact momentum equation is made. The following system of equations for the fluid state results:
\[
\frac{\partial (\rho X_k)}{\partial t} = -\nabla \cdot (U \rho X_k) + \rho \dot{\omega}_k ,
\]
\[
\frac{\partial (\rho h)}{\partial t} = -\nabla \cdot (U \rho h) + \frac{Dp_0}{Dt} - \sum_k \rho q_k \dot{\omega}_k + \rho H_{\text{ext}} ,
\]
\[
\frac{\partial U}{\partial t} = -U \cdot \nabla U - \frac{1}{\rho} \nabla \pi - \frac{(\rho - \rho_0)}{\rho} \rho g e_r .
\]

The nuclear species are described by their mass fraction, \( X_k \), binding energy, \( q_k \), and production rates, \( \dot{\omega}_k \), with the total mass density defined as
\[
\rho = \sum_k \rho X_k ,
\]

We evolve the specific enthalpy, \( h \), rather than energy, and allow for external heating sources, \( H_{\text{ext}} \) in addition to the energy release from a reaction network. \( U \) is the velocity field.

The general astrophysical equation of state is recast as a constraint on the velocity field,
\[
\nabla \cdot (\beta_0 U) = \beta_0 \left( S - \frac{1}{\Gamma_1 p_0} \frac{\partial p_0}{\partial t} \right) ,
\]

where \( \Gamma_1 = \partial \log p / \partial \log \rho \)|s,
\[
\beta_0(r, t) = \beta(0, t) \exp \left( \int_0^r \frac{1}{(\Gamma_1 p_0) \partial r} \partial r \right)
\]

and
\[
S = -\sigma \sum_k (\xi_k + q_k) \dot{\omega}_k + \frac{1}{\rho p_0} \sum_k p X_k \dot{\omega}_k + \sigma H_{\text{ext}} .
\]

Here, \( \sigma = p_T / (\rho c_p p_0) \) and the species derivatives, \( p X_k = \partial p / \partial X_k \) and \( \xi_k = \partial h / \partial X_k \) are computed as described in [7]. This constraint incorporates compressibility effects due to the background stratification through the presence of \( \beta_0 \) on the left hand side in (3), and accounts for compressibility effects due to heat release and compositional changes through the presence of a nonzero right hand side in (3).

This system of equations is solved using a fractional step approach, consisting of advection, projection, and reaction steps. The overall algorithm is designed to be second-order accurate, using Strang splitting to couple the reactions into the hydrodynamics. This method was compared without reactions to fully compressible hydrodynamics and to the anelastic approximation in [5], and shown to be in excellent agreement. In [7] we demonstrated that we can accurately couple reactions into the overall algorithm.

An essential requirement for the low Mach number approximation to remain valid is that the total pressure remain close to the base state pressure, \( p_0 \), at all time. As the star expands in response to large-scale heating, the base state must capture this global expansion in order for the low Mach number asymptotics to remain valid. Thus we decompose the velocity field, \( U \), and derive a radial velocity that is used to evolve the base state. We have demonstrated that we accurately capture this base state expansion for plane-parallel geometries in [6] and for spherical, self-gravitating geometries in [7].
2. Scaling

The convection leading up to Type Ia supernovae occurs at enormous Reynolds numbers, \( \text{Re} \sim O(10^{14}) \) [1]. Even with the largest computers we cannot reach this limit, but it is still important to push the resolution of our simulations and explore the dependence of the flow on Reynolds number. This requires algorithms that scale well to large number of processors. Currently, MAESTRO decomposes a uniform grid of constant resolution into a specified number of non-overlapping grid patches. For the scaling studies presented below, the decomposition is such that each processor owns a single grid patch with each patch having the same number of unknowns. The interprocessor communication is handled with MPI. To obtain an initial assessment of the scaling behavior of MAESTRO, we measured the wallclock time for runs with a fixed problem size on a range of processor numbers. We began each run from a checkpoint file at \( t = 0 \) and ran for 20 time steps. Plotfiles, composed of the primary variables as well as a number of diagnostic quantities, were written every 10 time steps and a checkpoint file, composed of the primary variables needed for restart, was written at the end. Since the amount of work per processor drops quickly as we increase the number of processors, we ran two different problem sizes. The first was a \( 512^3 \) grid, with 128, 256, or 512 processors; the second case was a \( 768^3 \) grid, with 512, 1024, or 2048 processors. The simulations were carried out on the Livermore Computing Open Computing Facility Atlas machine, a Linux cluster consisting of 1152 nodes, each with 8 2.4 GHz AMD Opteron CPUs and 16 GB of memory. The nodes are connected via InfiniBand and the parallel file-system is Lustre. The default Intel 9.1 compilers were used. Each run was performed 4 times and the average was taken, to account for variability on the machine. We note that the time reported includes the I/O time.

Figure 1 shows the observed scaling behavior. We see that for the \( 512^3 \) problem size, we remain close to the ideal scaling curve. For the \( 768^3 \) run, we see good scaling behavior from 512 to 1024 processors, but deviate from ideal scaling quite a bit at 2048 processors. The large variability in the observed timings for a given number of processors most likely results from variations in the load of the machine, as the runs were done at different times with the cluster under different loads. We suspect that the deviation from ideal scaling is also most likely due to I/O issues; further studies will be needed to examine this premise.
3. Initial Three-Dimensional Results
As an initial test of the algorithm in three dimensions with a Cartesian representation of a spherical star, we initialized the star with a one-dimensional hydrostatic white dwarf model. The model was created by specifying a density of $2.6 \times 10^9$ g cm$^{-3}$ and temperature of $7 \times 10^8$ K at the center of the star and integrating the equation of hydrostatic equilibrium outward while constraining the model to be isentropic. The composition was held constant at 0.3 $^{12}$C and 0.7 $^{16}$O, and the gravitational acceleration was computed as a monopole. This procedure provides a reasonable approximation of the state of the white dwarf just before runaway. In addition, we introduced a sponge layer at the outer edge of the star, in which the velocities are damped before the projection according to a specified damping profile. The specification of the sponge is given in more detail in [7].

The star was centered in a domain $5.76 \times 10^8$ cm on a side, with a total of $256^3$ zones, giving a resolution of $2.25 \times 10^6$ cm. The resolution of the radial base state was $1.625 \times 10^5$ cm.

As a first test, not presented here, we specified no external heating or perturbations from the base state, turned off the reactions, and verified that the algorithm was able to maintain the star in hydrostatic equilibrium with no spurious motion.

We then perturbed the star with eight Gaussian heat sources, one in each octant near the center of the domain. The evolution of the star is shown in Figure 2. Initially we see plumes rising out of the center of the star and forming well-defined vortex rings as the top of the plume begins to roll up. Later we see the vortex rings begin to break down into finer structures as they rise through the star. At the final time shown, although the central plumes near the center of the star are still visible, the vortex rings have broken down further as the onset of turbulent convection begins.

4. Conclusions
We have developed a new code, MAESTRO, that is able to model stellar evolution using a low Mach number formulation. This approach can take time steps based on the advective CFL limit while including physical effects due to finite amplitude perturbations in temperature and density, reactions and expansion of the background hydrostatic state. We have demonstrated that the algorithm scales well up to 1000 processors and is

Figure 2. Time sequence of vorticity for a 3D stellar evolution calculation with eight heat sources near the center of the star.
able to model convective behavior in a spherical star using a three-dimensional Cartesian mesh. Future work will include development of an adaptive version of the algorithm and incorporation of additional physical models such as thermal radiation and turbulent flame models. With these additional developments we will be able to use MAESTRO to study the ignition of Type Ia supernovae as well as other low Mach number phenomena in astrophysics.

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