Formation of Neutrino Stars from Cosmological Background

Neutrinos

M. H. Chan, M.-C. Chu

Department of Physics, The Chinese University of Hong Kong,
Shatin, New Territories, Hong Kong, China

Abstract

We study the hydrodynamic evolution of cosmological background neutrinos. By using a spherically symmetric Newtonian hydrodynamic code, we calculate the time evolution of the density profiles of neutrino matter in cluster and galactic scales. We discuss the possible observational consequences of such evolution and the resulting density profiles of the degenerate neutrino ‘stars’ in galaxies and clusters.

PACS Numbers: 95.35.+d, 98.62.Gq, 98.65.Cw
I. INTRODUCTION

The observations of neutrino oscillations indicate that at least one flavour of neutrinos have non-zero rest mass, and at least one type of sterile neutrinos with mass of eV order must be taken into account for the three scales of mass squared differences if the LSND experiment is confirmed [1]. A huge number of primordial neutrinos were produced in the early universe [2], [3], and they were cooled down as the universe expanded, decoupling from baryons much earlier than the photon decoupling epoch. The temperature of the neutrinos dropped to below 0.1 meV after several billions of years, and so if their rest mass $m_\nu$ is an eV or so, the neutrinos have become non-relativistic. Sterile neutrinos may be produced in the early universe by the oscillations of the active neutrinos [4]. These sterile neutrinos may have rest mass greater than 10 eV and may play a role in structure formation. Also, a neutrino rest mass $m_\nu$ of the eV to keV order can account for most of the dark matter, which dominates the mass in galaxies and clusters [5]. In this article, we study the gravitational collapse of such non-relativistic cosmic background active and sterile neutrinos, and we discuss the formation of degenerate neutrino dark halos in cluster and galactic scales. It has been proposed that such degenerate neutrino ‘stars’ can be found in centers of galaxies [6], [7].

II. COSMIC BACKGROUND NEUTRINOS

In standard cosmology, the neutrinos decoupled at around 1 MeV, and the present value of their number density $n_\nu$ is about 100 cm$^{-3}$ for each type of neutrinos [8]. Therefore, we can write the number density of the primordial neutrinos as

$$n_\nu \approx \frac{100}{a^3} \text{ cm}^{-3}, \quad (1)$$

where $a$ is the cosmic scale factor, set to be 1 at present. If the neutrinos were ultra-relativistic, the neutrino temperature would be

$$T_\nu = \frac{T_0}{a}, \quad (2)$$

2
where $T_0 \approx 1.95 \text{ K}$ is a constant [8]. When the neutrinos cooled down, they became non-relativistic and $T_\nu \sim a^{-2}$. Suppose at a certain $a = a_c$ and $T_\nu = T_c$, $kT_c = m_\nu c^2$, and the neutrinos become non-relativistic. We have

$$T_c \approx \frac{T_0}{a_c} \approx \frac{T_1}{a_c^2},$$

(3)

where $T_1$ is the present neutrino temperature. Therefore, we can express $T_\nu$ in terms of $m_\nu$(in eV):

$$T_\nu \approx \frac{3.3 \times 10^{-4}}{m_\nu a^2} \text{K}. \quad (4)$$

for the non-relativistic regime. We will use this relation to check the degeneracy of the neutrinos at the structure formation epoch in the next section.

In order to estimate the time for the beginning of structure formation, we should consider the Jeans mass of the primordial neutrinos, which is defined as

$$M_J = \frac{4\pi}{3} \rho_\nu \lambda_J^3, \quad (5)$$

where $\rho_\nu$ is the mass density of the neutrinos and $\lambda_J$ is called the Jeans length which is defined by

$$\lambda_J = \left(\frac{\pi c^2}{G \bar{\epsilon}} \frac{\partial P_\nu}{\partial \rho_\nu}\right)^{1/2}, \quad (6)$$

with $\bar{\epsilon}$ the mean energy density and $P_\nu$ the pressure of the neutrinos. When the mass of an object is larger than its Jeans mass, the object will collapse gravitationally to form structure. The mass density of the neutrinos in the zero $T_\nu$ limit is given by

$$\rho_\nu = \frac{K}{c^2} \left[ x_F (2x_F^2 + 1) \sqrt{1 + x_F^2} - \ln \left( x_F + \sqrt{1 + x_F^2} \right) \right], \quad (7)$$

where $K$ is a constant and $x_F$ is defined as

$$x_F = \frac{2\pi h}{m_\nu c^2} \left[ \frac{3n_\nu}{8\pi} \right]^{1/3}. \quad (8)$$

$x_F \ll 1$ and $x_F \gg 1$ represent the non-relativistic and ultra-relativistic regimes respectively for the neutrinos. By combining Eqs. (5)-(8), we obtain a relation between $M_J$ and $a$ for a
given $m_\nu$ (see Fig. 1). If the Jeans mass is of galactic scale (about $10^{12} M_\odot$), the value of $a$ is about 0.0585 for $m_\nu = 10 \text{ eV}$ but greater than one for $m_\nu = 1 \text{ eV}$. For cluster scale (about $10^{15} M_\odot$), the corresponding values of $a$ are $6 \times 10^{-4}$ and 0.0125 for $m_\nu$ being 10 eV and 1 eV respectively.

By looking at the Jeans mass only, we find that clusters will start to form first and then the galaxies, which is known as the top-down scenario. This disagrees with the observation, which indicates that the formation of smaller structures occurred earlier in time [5]. However, we should compare not only the time that the structures start to form, but also the duration of the formation of structures in order to determine whether neutrinos can still form structures and agree with the observation.

We believe that different masses of neutrinos may correspond to different structures—the larger the neutrino mass is the smaller the structure and vice versa. The radius of a hydrostatic neutrino star is given by [9]

$$R = 20.7 \text{ pc} \left( \frac{M}{10^6 M_\odot} \right)^{-1/3} \left( \frac{m_\nu}{1 \text{ keV}} \right)^{-8/3},$$

where $M$ is the total mass of the neutrino star. The corresponding $R$ are Mpc, kpc and 0.01 pc for $m_\nu = 1 \text{ eV}$, 10 eV and 1 keV respectively if $M = 10^{15} M_\odot$. Therefore, we assume that neutrinos with rest mass of 10 eV or above will dominate in galaxy scale and those with rest mass 1 eV or below will form structures in cluster scale.

### III. HYDRODYNAMICS OF NEUTRINO STAR FORMATION

The formation of degenerate heavy neutrino stars was discussed in Ref. 10 using the analogy with an interacting self-gravitating Bose condensate. Here, we use a hydrodynamical code to simulate the formation of neutrino stars in galaxies and clusters. We use the Lagrangian mass coordinate to write the spherically symmetric hydrodynamic equations [11]:

$$\frac{\partial u}{\partial t} = -4\pi r^2 \left[ \frac{\partial P}{\partial m} + \frac{\partial Q}{\partial m} \right] - \frac{Gm(r)}{r^2},$$

(10)
\[ \frac{1}{\rho} = 4\pi r^2 \frac{\partial r}{\partial m}. \] (11)

Here, \( u \) is the fluid velocity, \( \rho \) the mass density, \( m \) the enclosed mass, and \( Q \) is the artificial viscous stress. The above equations indicate that the neutrinos are under gravitational attraction and influence of the pressure \( P \). There exists neutrino degeneracy pressure as the neutrinos become degenerate after they have gravitationally collapsed. The criterion of degeneracy is given by

\[ \rho_{\text{deg}} \geq m_\nu \left( \frac{m_\nu kT_\nu}{2\pi \hbar^2} \right)^{3/2}. \] (12)

We can use Eq. (4) to approximate \( T_\nu \) and set a lower limit for the mass density in degenerate state. The neutrino degeneracy pressure is given by

\[ P_\nu = \frac{4\pi^2 \hbar^2}{5m_\nu} \left( \frac{3}{4\pi} \right)^{2/3} n_\nu^{5/3}. \] (13)

In fact, the neutrinos are collisionless except for the degeneracy pressure. Therefore, we set the effective temperature of the neutrinos to be zero because the neutrino temperature is much less than their Fermi energy. Here, we consider non-relativistic degenerate neutrinos, the Fermi momentum for which is much smaller than their rest mass. In the hydrodynamic code, we use an artificial viscous stress to smooth out possible shock waves. The artificial viscous stress is defined as:

\[ Q = -\alpha \left[ \frac{\partial u}{\partial r} - \frac{1}{3r^2} \frac{\partial}{\partial r} (r^2u) \right], \] (14)

where \( \alpha \) is the scalar artificial viscosity defined by

\[ \alpha = l^2 \left| \frac{\partial \rho}{\partial t} \right|, \] (15)

with \( l \) being a constant length which will be defined later. In the hydrodynamic simulation, we use a finite difference scheme to evaluate the above equations [12]. We represent all the variables as: \( u \rightarrow u^{n+1/2}_k, r \rightarrow r^{n+1/2}_k, \rho \rightarrow \rho^{n}_{k+1/2}, P \rightarrow P^{n}_{k+1/2} \), where the upper index represents time step and the lower index labels mass shell, and we let
where $\Delta r_{k+1/2}$ is the thickness of the $k^{th}$ mass shell and $C_Q$ is an adjustable parameter set to be between 2 to 3, to spread the shock front into several zones. The difference between two consecutive Lagrangian coordinates is the mass of a zone, $\Delta m \rightarrow \Delta m_{k+1/2} = m_{k+1} - m_k$. In addition we use the Courant stability condition and test for convergence to make sure that the computer code works effectively.

In our simulations, we set the initial density profiles to be uniform. We use $10^{12} M_\odot$ and $10^{15} M_\odot$ as the Jeans mass to find the values of $a$ at which gravitational collapse begins for different $m_\nu$. The initial density $\rho_0$ can be obtained from the standard cosmology:

$$
\rho_0 = \frac{\Omega_\nu \rho_c}{a^3},
$$

where $\Omega_\nu$ and $\rho_c$ are the cosmological density parameter of the neutrinos and critical density of the universe respectively. All the parameters used in the simulations are shown in Table 1. Also, for simplicity, the initial hydrodynamic velocity of the neutrinos is set to be zero.

In fact, after the epoch of decoupling, the neutrinos were not fully degenerate and had no interaction with other neutrinos, baryons, or photons. Therefore, the pressure is very small and not enough to prevent gravitational collapse. However, when the neutrinos continue to collapse, their density will increase until they become fully degenerate. Further increase of the number density will boost up their degeneracy pressure until the pressure gradient balances the gravitational attraction and the collapse is stopped. If the neutrino mass is too large, the degeneracy pressure will be small and the neutrinos will collapse to a high density within a very small region with a high speed. Therefore, the neutrinos will rebound back with a high velocity. If the neutrino mass is too small, the neutrinos would remain relativistic and the gravitational attraction is not enough to bind them together. In other words, there only exists a range of neutrino mass to form a stable neutrino star. In Fig. 2-5, we demonstrate three cases with the parameters in Table 1. In Fig. 2, the resulting density becomes smaller as time evolves, which means that the neutrinos cannot form structures by themselves if $m_\nu$ is 1 eV. In Fig. 3, we still use $m_\nu = 1$ eV but we impose a constant density
core \((10^{-24} \text{ gcm}^{-3} \text{ within 500 kpc})\) inside the cluster. The neutrinos can form structure with the help of additional mass. In Fig. 4 and 5, we use \(m_\nu = 10 \text{ eV and 1 keV}\), and the neutrinos can form structures by themselves. The total mass of the structures we used are \(10^{15} M_\odot\) for \(m_\nu = 1 \text{ eV}\) and \(10^{12} M_\odot\) for \(m_\nu = 10\) eV or 1 keV. These few figures show the possibility of neutrinos forming structures in both galaxies and clusters. Also, the time scale of formation is smaller as \(m_\nu\) is larger. The time scales are about \(10^{12} \text{ s}, 10^{16} \text{ s and } 10^{17} \text{ s}\) if \(m_\nu\) are 1 keV, 10 eV and 1 eV respectively. Therefore, when one includes the formation time, the sequence of neutrino structure formation agrees with the observation, i.e. smaller scales form first.

In Fig. 6, we compare the density profile at hydrostatic equilibrium with the late time profile of the dynamical evolution. They appear almost identical except at the outermost region, where the difference arises because of some undamped oscillations in the outer part of the neutrino star. The density profile is nearly constant at small radius and drops greatly at large radius.

In cluster formation, most of the baryons become hot gas particles and stars in galaxies. Their mass is about one-tenth of the total mass of the cluster. We have ignored the effect of baryons here in the simulation. Nevertheless, the constant density profile at small radius agrees with some observational data of cluster dark matter [13], [14].

When the neutrinos begin to collapse by gravitational attraction, they gain kinetic energy and the density of the inner part of the neutrino star increases. When the central density is high enough, the pressure gradient prevents the neutrinos to collapse further and some of the neutrinos begin to rebound back. We find that there are oscillations in the neutrino star. In Fig. 3, we can see oscillations of the central density. Also in Fig. 7, we plot the density against time at \(r = 50 \text{ kpc}\) with the same parameters as those used in Fig. 4. The period of oscillation \(\tau\) is almost constant and the amplitude varies with time. The value \(\tau\) in the central region can be found by Taylor expansion of the hydrodynamic equation with small displacement, which is roughly given by the following relation [15]:

\[7\]
\[ \tau \approx \frac{2\pi}{\sqrt{4\pi G\rho_f/3}} \]  

(18)

From Fig. 7, the period of oscillation is about 1.6 billion years. The density at \( r = 50 \) kpc is about \( 6 \times 10^{-26} \) g cm\(^{-3} \). Therefore, by Eq. (18), \( \tau \approx 1.5 \) billion years, which is roughly consistent with the result. Because of the weak coupling among the neutrinos, the oscillations probably would not be damped out for a long time.

**IV. DISCUSSION**

We have demonstrated the possibility that the neutrinos can form structures in galaxies and clusters. Including the formation time, our model is consistent with the observation but not the top-down scenario. Our hydrodynamical simulation results (see Fig. 4) agree with recent observed data indicating that the central density of dark matter profile is constant [13].

Light neutrinos (1 eV) cannot form structure by their own gravitational attraction. However they can still form structures under the gravitational field of a heavier component (see Fig. 2 and 3). It is well known that light primordial neutrinos (eV order) cannot account for all the dark matter as \( \Omega_\nu \) is much less than 0.3 for \( m_\nu = 1 \) eV [16]. There must exist heavier particles such as sterile neutrinos which dominate the mass of the dark matter and provide such a gravitational field in the clusters. These sterile neutrinos (10 eV order) may also form smaller structures in galaxies (about 10-20 kpc), and all these smaller structures link up together to form a large structure in clusters. Therefore, neutrinos with rest mass 10 eV or above are important in structure formation, and we should not neglect their contribution. The range of possible masses of active neutrinos and sterile neutrinos can both be constrained by more data from the distribution of hot gas in clusters and the rotation curves of galaxies.

From Fig. 3 and 7, we can see that the central density of a neutrino star oscillates, which may affect the hot gas in clusters. It may force oscillate the hot gas, which may be
observable. Also, the oscillations in the hot gas may convert the kinetic energy into internal energy of the hot gas and thus heat them up, which may at least partially account for the heat source of the hot gas.

ACKNOWLEDGMENTS

The work described in this paper was substantially supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. 400803).
REFERENCES

[1] S. M. Bilenky, C. Giunti and W. Grimus, Eur. Phys. J. C 1, 247 (1998).

[2] See for example, P. J. E. Peebles, Principles of Physical Cosmology (Princeton University Press, Princeton, 1993).

[3] S. Dodelson and L. M. Widrow, Phys. Rev. Lett., 72, 17 (1994).

[4] A. D. Dolgov and S. H. Hansen, hep-ph/0009083 v3 (2001).

[5] B. S. Ryden, Introduction to Cosmology (Addison-Wesley, 2003).

[6] R. D. Viollier, N. Bilic and F. Munynaneza, Phys. Rev. D 59, 1 (1998).

[7] K. M. V. Apparao, astro-ph/0204375 v1 (2002).

[8] M. Roos, Introduction to Cosmology (John Wiley and Sons, 1994).

[9] R. Bowers and T. Deeming, Astrophysics I Stars (Jones and Bartlett, 1984).

[10] N. Bilic, R. J. Lindebaum, G. B. Tupper and R. D. Viollier, astro-ph/0106209 v2 (2001).

[11] R. L. Bowers and J. R. Wilson, Numerical Modelling in Applied Physics and Astrophysics, Chapter 4 (Jones and Bartlett, 1991).

[12] T. W. Lee, Computational Study of Type II Supernova Explosion (M.Phil. Thesis, CUHK, 1999).

[13] J. A. Tyson, G. P. Kochanski and I. P. Dell’Antonio, ApJ. 498, L107 (1998).

[14] M. H. Chan and M.-C. Chu, astro-ph/0401329 (2004).

[15] M. H. Chan, Properties and Formation of Neutrino Star (M.Phil. Thesis, CUHK, 2004).

[16] W. Hu, D. J. Eisenstein and M. Tegmark, Phys. Rev. Lett. 80, 5255 (1998).
TABLES

TABLE I. The parameters used in the simulations.

| $m_\nu$  | $\rho_0 (\text{g cm}^{-3})$ | Cut off radius (kpc) |
|---------|-----------------------------|----------------------|
| 1 eV    | $1.02 \times 10^{-25}$     | 539                  |
| 10 eV   | $9.97 \times 10^{-27}$     | 11.7                 |
| 1 keV   | $1.41 \times 10^{-18}$     | 0.022                |
 FIG. 1. Jeans mass of primordial neutrinos with mass of 1 eV (solid line) and 10 eV (dashed line) respectively vs. cosmic scale factor. Also, indicated are typical cluster and galactic mass scales.

 FIG. 2. Time evolution of the density profile of 1 eV neutrinos, from initially uniform and stationary distribution. Total mass = $10^{15} M_\odot$ and the time steps are in $10^9$ years.
FIG. 3. Time evolution of the density profile of 1 eV neutrinos with a constant density core ($\sim 10^{-24} \text{ g cm}^{-3}$ within 500 kpc) of dark matter and total mass $10^{15} M_\odot$ in the background. The time steps are in $10^9$ years.

FIG. 4. Time evolution of the density profile of 10 eV neutrinos. Total mass = $10^{12} M_\odot$. The time steps are in $10^9$ years.
FIG. 5. Time evolution of the density profile of 1 keV neutrinos. Total mass $= 10^{12} M_\odot$. The time steps are in $10^4$ years.

FIG. 6. Comparison between the final density profile of the simulation and hydrostatic density profile with the same parameters as those used in Fig. 4. The solid line represents the final density profile of the hydrodynamic simulation. The circles represent the density profile of the hydrostatic neutrino star.
FIG. 7. Oscillation of the mass density of a neutrino star at $r = 50$ kpc with the same parameters as those used in Fig. 4.