Basics of F-theory from the Type IIB Perspective

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These short lecture notes provide an introduction to some basic notions of F-theory with some special emphasis on its relation to Type IIB orientifolds with O7/O3-planes.

1 Introduction

Historically outstripping heterotic string compactifications and intersecting D-brane models, the last two years have seen the main activity in the field of string phenomenology shifting towards F-theory models. In this framework some of the model building shortcomings of D-brane realizations of grand unified theories (GUTs) can be overcome.

To appreciate this, let us recall some issues on D-brane constructions. The gauge theories are supported on D-branes, which in general can have a dimension smaller than the bulk. Completely occupying our observable large scale four-dimensional world, they wrap certain sub-manifolds of the internal geometry. The matter fields are localized on the intersections of such D-branes. Since the early years of so-called intersecting D-brane models, it was clear that this set-up naturally allows for semi-simple gauge groups with matter fields in the bifundamental representations. Therefore, here one directly engineers the $SU(3) \times SU(2) \times U(1)$ MSSM, while leaving the unification of gauge couplings at the GUT scale essentially unexplained. Indeed, the gauge couplings depend on the generally different volumes of the internal cycles wrapped by the D-branes supporting each gauge factor.

It was quickly realized that the construction of GUT groups $SO(10)$ and $SU(5)$ was obstructed by the perturbative absence of matter fields in the 16 representation of $SO(10)$ respectively by the absence of the top Yukawa coupling $10 \times 10 \times 5$ for the $SU(5)$ case. These two latter features are of non-perturbative origin for orientifold models (see [1] for a review). It was realized two years ago that the aforementioned problems with realizing simple GUT groups in orientifold constructions are nicely reconciled in F-theory models on elliptically fibered Calabi-Yau four-folds [2–5] (see [6] for a more phenomenological review).

One can think of F-theory as Type IIB compactifications on compact complex n-dimensional manifolds $B_n$ with general $(p, q)$ 7-branes wrapping $2(n-1)$ cycles of $B_n$. Since the 7-branes are of real co-dimension two, the solutions to the Laplace equations are of logarithmic type. Therefore, the backreaction of the 7-branes on the geometry and the dilaton is always substantial and has to be taken into account. By identifying the strong-weak $SL(2, \mathbb{Z})$ duality of the Type IIB superstring with the modular group of a torus, C. Vafa [7] showed that the backreaction can geometrically be taken into account by an elliptic fibration over the base $B_n$, where the modular parameter of the fiber is identified with the axio-dilaton field of Type IIB. The location of the 7-branes correspond to the degeneration loci of the elliptic fibration and for supersymmetry the fibrations have to be of Calabi-Yau type.

Due to the strong backreaction, only in a global $g_s \to 0$ limit a general F-theory model is expected to correspond to an orientifold. F-theory inherently contains some features which are non-perturbative from the orientifold point of view. This is the reason for the appearance of exceptional groups in F-theory, which by a further breaking also realize the spinor representation of a GUT $SO(10)$ as well as the top-quark Yukawa couplings $10 \times 10 \times 5^H$ in GUT $SU(5)$. For four-dimensional models, the basis $B_3$ is a Fano three-fold and the 7-branes wrap complex surfaces, i.e. four-cycle.

Thus, F-theory is a non-perturbative completion of Type IIB orientifolds where the 7-branes are completely encoded in the geometry of the elliptic fibration. The aim of this lecture is to give an introduction
D7-branes and $SL(2, \mathbb{Z})$ self-duality

As a starting point, we consider Type IIB orientifolds compactified on a Calabi-Yau three-fold $X$ and an orientifold projection $\Omega \sigma (-1)^F$ (see [9] for a review). Here $\sigma$ denotes a holomorphic involution of $X$ acting as

$$\sigma^*(J) = J, \quad \sigma^*(\Omega_3) = -\Omega_3$$

(1)

on the Kähler respectively holomorphic $(3,0)$-form of $X$. This orientifold quotient introduces an $O7$-plane into the theory, whose tadpole is canceled by the introduction of stacks of D7-branes wrapping various holomorphic four-cycles of $X$, whose total homology class in $H_3(X, \mathbb{Z})$ is equal to the one of the $O7$-plane. One can now compute the (chiral) massless spectrum coming from the lowest excitations of open strings stretched between various pairs of D7-branes. This gives rise to gauge bosons of only unitary or orthogonal/symplectic gauge groups and in addition to matter fields transforming solely in bifundamental or (anti)-symmetric representations of the gauge group. This is simply a consequence of the fact that an open string has two ends. Clearly, such open string excitations can never give rise to exceptional gauge groups and, as a group theoretic consequence, to matter in the spinor representation of an $SO(10)$ gauge group.

What we have just briefly described is the construction of perturbative Type IIB orientifold string vacua and its shortcomings when it comes to GUT like structures. Naively, this seems to be the end of the story.

However, taking the perturbative string limit $g_s \ll 1$ is, to say the least, quite questionable, if branes of (real) co-dimension two, such as D7-branes, are present. This becomes evident by studying the D7-brane solution in Type IIB supergravity [10], which is magnetically charged under the R-R scalar field

$$c_0, \Phi$$

and its short-comings when it comes to GUT like structures. Naively, this seems to be the end of the story. However, taking the perturbative string limit

$$\frac{g_s}{\alpha'} \ll \frac{1}{16 \pi^2}, \quad \alpha' \ll 1$$

into some basis notions of F-theory, which essentially addresses those students, who are already familiar with D-brane constructions. It is explained why F-theory is inevitable for the correct study of Type IIB compactifications with D7-branes, in which sense it goes beyond the perturbative Type IIB superstring and how this leads to a solution of the above mentioned problems with GUT models. Note that this was the first of a series of two lectures on F-theory GUTs held at the 9th Hellenic School on Elementary Particle Physics and Gravity, Corfu 2009. The second lecture focused more on the specifics of realizing four-dimensional $SU(5)$ GUTs from F-theory. Please consult [8] for more detailed lecture notes on F-theory.
weak-coupling regime. Once the string coupling is non-zero and maybe small somewhere, it necessarily becomes large in other regions of the transverse space. However, eq. (4) implies that \( g_s = \exp(\Phi) \) is small close to the D7-brane so that for the gauge theory on the D7-brane we expect a weak coupling description.

As will now review, also the backreaction on the metric is strong. Consider the 10-dimensional space-time metric of the D7-brane solution: \( ds^2 = -dt^2 + \sum_{i=1}^{7} dx_i^2 + e^{B(z, \bar{z})} dz d\bar{z} \). Then the Einstein equation connects the warp factor \( B \) with the dilaton field, and one obtains the following simple relation \( \partial \bar{\partial} B = \partial \bar{\partial} \log(\text{Im} \tau) \). It turns out that the solution with the correct modular properties and the right asymptotic behavior far away from the D7-brane is given by

\[
e^{B(z, \bar{z})} = (\text{Im} \tau) \frac{\eta^2(\tau) \bar{\eta}^2(\bar{\tau})}{|\prod_{i=1}^{N}(z - z_i)|^2}.
\]

Here \( N \) is the number of 7-branes and the \( z_i \) denote their positions in the two-dimensional transverse space. Expanding this function for large \(|z|\) one gets that \( B(z, \bar{z}) \sim -\frac{N}{12} \cdot \log |z| \). Using this asymptotic behavior one realizes that the metric goes like \(|z|^{-N/12} \cdot |dz|^2\) far away from the 7-branes. This means that each 7-brane leaves a deficit angle of \( 2\pi/12 \) in the transverse space. In fact precisely 24 7-branes are required to get the deficit angle of \( 2\pi \) of the compact two-dimensional sphere \( \mathbb{CP}^1 \). Therefore, there exists a supersymmetric compactification of the Type IIB superstring on \( \mathbb{CP}^1 \) with precisely 24 7-branes and a varying dilaton.

So far we have only been talking about D7-branes. However, due to the \( SL(2, \mathbb{Z}) \) duality symmetry there exist infinitely many different kinds of 7-branes. Indeed, since there exists a doublet of two-forms \((B_2, C_2)\) in the ten-dimensional Type IIB string theory, there are not only fundamental strings and D1-branes, but also string carrying electric charges \((p, q)\), where in this notation a fundamental string is \((1, 0)\) string. Acting with an \( SL(2, \mathbb{Z}) \) transformation on the fundamental string gives

\[
\begin{pmatrix} p \\ q \end{pmatrix} \rightarrow \begin{pmatrix} p & r \\ q & s \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{with } ps - qr = 1.
\]

Similarly, one has a doublet of ten-dimensional scalars \((\Phi_0, \Phi)\) leading to magnetically charged \((p, q)\) 7-branes, where a D7-brane (charged only under \( \Phi_0 \)) is a \((1, 0)\) 7-brane. Since a fundamental string can end on a D7-brane, \( SL(2, \mathbb{Z}) \) implies that a \((p, q)\) string can end on a \((p, q)\) 7-brane. In eq. (3) we have seen that the solitonic solution of a D7-brane induces an \( SL(2, \mathbb{Z}) \) monodromy \( M_{D7} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \). Applying the \( SL(2, \mathbb{Z}) \) symmetry, a general \((p, q)\) 7-brane induces a monodromy

\[
M_{(p, q)} = \begin{pmatrix} p & r \\ q & s \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} p & r \\ q & s \end{pmatrix}^{-1} = \begin{pmatrix} 1 - pq & p^2 \\ -q^2 & 1 + pq \end{pmatrix}.
\]

For later purpose we consider the three \( 7 \)-branes \( A = (1, 0), B = (1, -1) \) and \( C = (1, 1) \). It is now straightforward to compute the monodromy matrices for the combinations of these three \( 7 \)-branes listed in table I. To understand the massless modes between such more general \((p, q)\) 7-branes, one notices that the \((p, q)\)-strings can form so-called string junctions. For instance a \((1, 1)\) string can split into a \((1, 0)\) and \((0, 1)\) string. Similar to open \((1, 0)\) strings ending on D7-branes, for more general 7-brane there can exist so-called string junctions ending on them. For instance there can be a string junction with four external strings of type \((-1, 0) \rightarrow (1, 0) \rightarrow (1, 1) \rightarrow (1, -1)\), which can end on the respective 7-branes \( A \rightarrow A - B - C \). Clearly, such objects can give qualitatively new massless states beyond what is possible with perturbative fundamental open strings. We will come back to this in the next section.

### 3 F-theory

The observations made in the previous section led C. Vafa in 1996 to the idea of F-theory. This is a hypothetical or rather auxiliary twelve dimensional theory which, when compactified on a two-dimensional torus, gives the Type IIB superstring. The modular group of the torus is identified with the \( SL(2, \mathbb{Z}) \) symmetry of Type IIB. However, this twelve-dimensional interpretation is not meant in the sense of a standard
Kaluza-Klein reduction, as first there does not exist a twelve dimensional supergravity theory with signature $(1,11)$ in the first place and second in ten dimensions there is no scalar field corresponding to the volume modulus of this $T^2$. Hence the 12-dimensional interpretation serves just to provide a geometrization of the Type IIB $SL(2,\mathbb{Z})$ duality symmetry rather than to correspond to a real compactification from twelve to ten dimensions. What makes true sense though is to start with the eleven-dimensional M-theory compactified on $T^2$ and define F-theory as the volume modulus of this $T^2$.

The true power of this F-theory picture reveals itself when compactifying the Type IIB superstring to lower dimensions. We have just recalled that there should exist a compactification of Type IIB on $\mathbb{CP}^1$ with 24 7-branes preserving half the supersymmetry, i.e. 16 supercharges. Observing that M-theory compactified on a K3 surface breaks half the supersymmetry, one finds that F-theory compactified on an elliptically fibered K3 with base $\mathbb{CP}^1$ is the Type IIB string compactified on the base $\mathbb{CP}^1$ with 24 7-branes.

For this to make sense, we have to find the 7-branes in this purely geometric description, where we recall that in Type IIB there exist not just ordinary D7-branes but also these $(p,q)$ 7-branes introduced in the previous section. We have seen that close to a D7-brane at position $u_1 \in \mathbb{CP}^1$ the complexified dilaton behaves like $j(\tau) \simeq 1/(u - u_1)$. Now $\tau$ is really the modular parameter of a geometric elliptic curve and it is known from mathematics that the $j$-function naturally appears in this context. For this purpose we explicitly write the elliptic curve as the hypersurface $\mathbb{F}_{1,2,3}[6]$ in the homogeneous coordinates $(z,x,y)$. The fibration over the base $B = \mathbb{CP}^1$ can then be written as the hypersurface constraint

$$y^2 + a_1 xyz + a_3 yz^3 = x^3 + a_2 x^2 z^2 + a_4 xyz + a_6 z^6,$$

where the coefficients $a_n$ are homogeneous polynomials of degree $2n$ of the two homogeneous coordinates $(u_1,u_2)$ on $\mathbb{CP}^1$. More correctly stated, the $a_n$ are sections of $K_B^n$, where $K_B$ denotes the canonical bundle of the base $B = \mathbb{CP}^1$. Note that $z = 0$ defines a section of the elliptic fibration, i.e. the divisor $z = 0$ is the base $\mathbb{CP}^1$. Completing the square and the cubic term, this so-called Tate form can be written in the so-called Weierstraß form

$$y^2 = x^3 + f_4 x z^4 + g_6 z^6.$$  

Table 1    Monodromies around stacks of 7-branes of types $A,B,C$. 

| 7-branes | number | monodromies   |
|----------|--------|---------------|
| $A$      | 1      | $M_A = (1,1)$ |
| $B$      | 1      | $M_B = (2,1)$ |
| $C$      | 1      | $M_C = (0,1)$ |
| $A^n$    | $n$    | $M^A_n = (1,n)$ |
| $AB$     | 2      | $M_A M_B = (1,0)$ |
| $A^2 B$  | 3      | $M^2_A M_B = (0,1)$ |
| $A^2 B A$| 4      | $M_A^2 M_B M_A = (0,1)$ |
| $A^n B C$| $n+2$  | $M^n_A M_B M_C = (-1^{n+4})$ |
| $A^3 B C B$| 8 | $M^3_A M_B M_C M_B = (-1,1)$ |
| $A^3 B C B$| 9 | $M^3_A M_B M_C M_B = (0,1)$ |
| $A^3 B C B A$| 10 | $M^3_A M_B M_C M_B M_A = (0,1)$ |

$^1$ For base $\mathbb{CP}^1$ the Weierstraßform is sufficient, but for compactifications to six and four dimensions the Tate is very convenient.
To express $f_4$ and $g_6$ in terms of the $a_i$, it is convenient to introduce the objects $b_2 = a_1^2 + 4a_2$, $b_4 = a_1a_3 + 2a_4$ and $b_6 = a_1^2 + 4a_6$ so that

$$f_4 = \frac{1}{48} (24b_4 - b_2^2), \quad g_6 = \frac{1}{864} (216b_6 - 36b_4b_2 + b_2^3).$$

(9)

Given the Weierstraß form with sections $f_4$ and $g_6$, the complex structure $\tau$ of the elliptic fiber over a point $(u_1,u_2)$ is implicitly given by

$$j(\tau) = \frac{4(24f_4)^3}{4f_4^3 + 27g_6^2},$$

(10)

where indeed the $j$-function appears. Now, the location of the 7-branes should be at the zeros of the denominator

$$\Delta = 4f_4^3 + 27g_6^2 = -\frac{1}{4}b_2^5(b_2b_6 - b_2^2) - 8b_4^3 - 27b_6^2 + 9b_2b_4b_6.$$

(11)

This is the so-called discriminant of the elliptic fibration and its zeros are mathematically precisely the points where the torus degenerates. Note that $\Delta$ is a polynomial of degree 24 in $(u_1,u_2)$, and thus has 24 zeros. These points are the positions of the 24 7-branes on $\mathbb{CP}^1$.

So far we assumed that the discriminant has 24 different zeros. However, when some of these zeros coincide the elliptic fibration further degenerates, i.e. certain 2-cycles shrink to zero size. In the M-theory description, M2-branes wrapped on these shrunken 2-cycles provide new massless states, which give rise to non-abelian gauge symmetries. In fact there exists a classification by Kodaira [12] of the different types such an elliptic fibration over $\mathbb{CP}^1$ can degenerate. As shown in table 2, this classification is of the A-D-E type expected for singularities on $K3$ respectively enhanced gauge symmetries.

| ord($f$) | ord($g$) | ord($\Delta$) | fiber | singularity | comp. | local geometry | monod. |
|---------|---------|-------|-------|-------------|-------|---------------|-------|
| $\geq 0$ | $\geq 0$ | 0     | $I_0$ | smooth     | 1     |                 | $(\frac{1}{0}, 0)$ |
| 0       | 0       | 1     | $I_1$ | dbl. point | 1     | $y^2 = x^2 + z$ | $(\frac{1}{0}, 1)$ |
| 0       | 0       | $n$   | $I_n$ | $A_{n-1}$ | $n$   | $y^2 = x^2 + z^n$ | $(\frac{1}{0}, n)$ |
| $\geq 1$ | 1       | 2     | $II$  | cusp       | 1     |                 | $(\frac{1}{1}, 0)$ |
| $\geq 1$ | $\geq 2$ | 3     | $III$ | $A_1$     | 2     | $y^2 = x^2 + z^2$ | $(\frac{0}{0}, 1)$ |
| $\geq 2$ | 2       | 4     | $IV$  | $A_2$     | 3     | $y^2 = x^2 + z^3$ | $(\frac{0}{1}, 1)$ |
| 2       | 3       | 6     | $D_4$ | $D_4$     | 5     | $y^2 = x^2z + z^3$ | $(\frac{-1}{0}, 0)$ |
| $\geq 2$ | $\geq 3$ | $3$   | $I_n$ | $D_{n+4}$ | $n + 5$ | $y^2 = x^2z + z^{n+3}$ | $(\frac{-n}{0}, 0)$ |
| $\geq 3$ | 4       | 8     | $IV^*$ | $E_6$     | 7     | $y^2 = x^3 + z^4$ | $(\frac{1}{0}, 1)$ |
| 3       | $\geq 5$ | 9     | $III^*$ | $E_7$     | 8     | $y^2 = x^3 + xz^3$ | $(\frac{0}{1}, 1)$ |
| $\geq 4$ | 5       | 10    | $II^*$ | $E_8$     | 9     | $y^2 = x^3 + z^5$ | $(\frac{0}{1}, 1)$ |

Table 2 The Kodaira classification of singular fibers in elliptic surfaces. The local geometry of the elliptic surface around such an A-D-E singularity is modeled in terms of coordinates $(x,y,z) \in \mathbb{CP}^2$. In the last column the elliptic monodromy of the singular fiber is given in terms of a $SL(2,\mathbb{Z})$-matrix.

Note that in particular the exceptional gauge groups $E_6$, $E_7$ and $E_8$ can be realized as enhanced gauge symmetries in F-theory. Clearly they cannot be realized by fundamental open strings of the perturbative
Type IIB string, i.e. not just with (1,0) strings and D7-branes. These enhancements must involve more general (p,q) seven branes and the corresponding string-junctions between them.

To get an idea how this works, we compare the geometric monodromy matrices in table with those listed for stacks of A,B,C branes in table. It is evident that for the A-D-E series and the three fiber types II, III, IV we have a perfect match. Moreover, the number of 7-branes is in all cases identical to the vanishing order of the discriminant. One can also show that the string junctions ending on the stacks of branes provide precisely the massless states to fill out the adjoint representation of A-D-E gauge groups. Note that the Dp-series is realized by the AqBC 7-branes, which indicates that the BC pair can be considered as the non-perturbative description of an O7 plane. Therefore, F-theory goes beyond the perturbative Type IIB orientifolds in that it allows for general (p,q) 7-branes and their corresponding (p,q)-strings. It is precisely this more general structure which realizes the exceptional gauge groups and as a consequence all their group theoretic consequences, such as matter in spinor representations of SO(10) or the 10105H Yukawa coupling for SU(5) GUT models.

4 F-theory compactifications and the Sen limit

Finally, to connect to the second lecture on F-theory, let us briefly comment on lower dimensional compactifications of F-theory. Instead of fibering the torus over a complex one-dimensional base, one can consider fibrations over surfaces S2 or three-dimensional bases B3. Supersymmetry then implies that the total space should either be a Calabi-Yau three-fold (for B2) or Calabi-Yau four-fold (for B3). One can still write down a Weierstraß model, where f4 and g6 are sections of K2 and K3. The zeros of the discriminant define complex co-dimension one curves in B2 respectively surfaces in B3 and give the location of 7-branes. In these cases, it is more convenient to use the Tate form of the elliptic fibration, as there exists a refinement of the Kodaira classification, the so-called Tate algorithm, which allows to determine the gauge group essentially from the vanishing order of the discriminant and the sections a_n (see for more details). Now it can however happen that the singularity enhances further where this co-dimension one objects intersect. Similar to intersecting D-branes, this is where additional matter fields are localized.

In the case of an F-theory compactification on a smooth Calabi-Yau four-fold Y a couple of new issues need to be considered. First, one can show that chiral matter only arises on the intersection curve between two 7-branes, if there exists a non-trivial G2-form background (M-theory point of view). Second, one finds a non-trivial D3-brane tadpole cancellation condition, which in this case reads

\[ N_{D3} + \frac{1}{2} \int_Y G_4 \wedge G_4 = \frac{\chi(Y)}{24} \]  

where \(\chi(Y)\) denotes the Euler characteristic of the smooth (appropriately resolved) four-fold Y.

One can define a limit in which the string coupling goes to zero almost everywhere on the base. This is the so-called Sen-limit, defined by rescaling \(a_3 \to \varepsilon a_3\), \(a_4 = \varepsilon a_4\), \(a_6 = \varepsilon^2 a_6\) and sending \(\varepsilon \to 0\). In this parameterization one finds

\[ f_4 = \frac{1}{48} (24 \varepsilon b_4 - b_4^2) , \quad g_6 = \frac{1}{864} (216 \varepsilon^2 b_6 - 36 \varepsilon b_4 b_2 + b_2^2) \]  

so that the discriminant becomes

\[ \Delta = - \frac{\varepsilon^2 b_2^2 (b_2 b_6 - b_2^2)}{4} + O(\varepsilon^3) \quad \Rightarrow \quad j(\tau) \simeq \frac{b_2^2}{4 \varepsilon^2 (b_2 b_6 - b_2^2)} . \]  

Therefore, for \(\varepsilon \to 0\) the Type IIB string coupling constant \(g_s\) goes to zero almost everywhere except on the locus where \(b_2\) vanishes. Studying the monodromies one finds a D7-brane on the locus \((b_2 b_6 - b_2^2) = 0\) and an O7-plane where \(b_2 = 0\). Therefore, the Sen-limit defines the region in the complex structure moduli space, where F-theory is (almost everywhere) weakly coupled and a perturbative Type IIB orientifield description is justified.
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