STELLAR GROWTH BY DISK ACCRETION: THE EFFECT OF DISK IRRADIATION ON THE PROTOSTELLAR EVOLUTION

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ABSTRACT

Young stars are expected to gain most of their mass by accretion from a disk that forms around them as a result of angular momentum conservation in the collapsing protostellar cloud. Accretion initially proceeds at high rates of $10^{-5} - 10^{-3} M_{\odot}$ yr$^{-1}$, resulting in strong irradiation of the stellar surface by the hot inner portion of the disk and leading to the suppression of the intrinsic stellar luminosity. Here we investigate how this luminosity suppression affects evolution of the protostellar properties. Using a simple model based on the energy balance of accreting star, we demonstrate that disk irradiation causes only a slight increase of the protostellar radius, at the level of several percent. Such a weak effect is explained by a minor role played by the intrinsic stellar luminosity (at the time when it is significantly altered by irradiation) in the protostellar energy budget compared to the stellar deuterium burning luminosity and the inflow of the gravitational potential energy brought in by the freshly accreted material. Our results justify the neglect of irradiation effects in previous studies of the protostellar growth via disk accretion. Evolution of some other actively accreting objects such as young brown dwarfs and planets should also be only weakly sensitive to the effects of disk irradiation.

Subject headings: accretion, accretion disks — planets and satellites: formation — solar system: formation — stars: formation

1. INTRODUCTION

It is currently established that circumstellar disks are quite ubiquitous around young stellar objects of all masses (Muzerolle et al. 2003; Cesaroni et al. 2007). They represent an important ingredient of the star formation, since initially protostars must be growing predominantly by accretion through the disk: angular momentum conservation forces the infalling protostellar cloud material to form a centrifugally supported disk which then accretes onto a star. It is quite likely that only a small fraction of the final stellar mass gets acquired by the direct infall onto the protostellar surface, so that almost all of the stellar mass gets processed through the disk.

This picture of star formation implies very high initial mass accretion rates in the disk, at the level of $10^{-6} - 10^{-3} M_{\odot}$ yr$^{-1}$ as the solar-type stars are thought to gain most of their mass during the first several $10^5$ yr. The presence of such a high-$\dot{M}$ accretion flow just outside the protostar immediately raises an issue of its possible effect on the protostellar properties.

There are several ways in which disk accretion affects the protostar. First, the star gains mass from the disk which increases its binding energy and tends to make the star more compact. Second, accreting gas brings in some amount of thermal energy with it, which contributes to the pressure support in the star. The exact amount of heat advected into the star with the accreted material is unknown but it seems likely that because of the disk geometry, the accreted gas would have enough time to radiate away most of its thermal energy and would join the convective interior of the star with temperature much smaller than the stellar virial temperature. Third, intense energy dissipation taking place in the innermost parts of the accretion disk leads to strong irradiation of the stellar surface by the disk (Adams & Shu 1986; Popham 1997).

Quite naturally, first attempts to understand effects of gas accretion on protostellar evolution employed the assumption of spherical symmetry. In this simplified framework Winkler & Newman (1980), Stahler et al. (1980a, 1980b), and Stahler (1988) have been able to clarify the important role of the radiative accretion shock that forms at the protostellar surface, where the material infalling at almost free-fall velocity suddenly comes to rest: substantial energy release taking place in the shock suppresses convection in the surface layers of the protostar leading to the appearance of a subsurface radiative region which regulates the entropy of the inner convective regions. Stahler et al. (1980a) and Stahler (1988) have also clarified an important role of the deuterium burning in driving the convection inside the protostar and setting the protoplanetary mass-radius relation (the so-called thermostatic effect of D burning).

In the framework of disk accretion it has been recently realized (Rafikov 2008) that irradiation produced by the local viscous dissipation in the inner regions of accretion disk can affect protostar in ways reminiscent of the effects of accretion shock described by Winkler & Newman (1980) and Stahler et al. (1980a). In particular, irradiation suppresses convection in the outer layers of the star resulting in the formation of an external radiative zone, similar to that forming in the atmospheres of close-in extrasolar giant planets irradiated by their parent stars. This acts to suppress the internal luminosity of the protostar analogous to the reduction of the cooling rate of hot Jupiters driven by their intense irradiation (Guillot et al. 1996; Burrows et al. 2000; Baraffe et al. 2003; Chabrier et al. 2004). It was shown in Rafikov (2008) that disk accretion at rates $\dot{M} \sim 10^{-6} - 10^{-5} M_{\odot}$ yr$^{-1}$ can easily reduce internal stellar luminosity by a factor of several which may have important implications for the early stellar evolution.

It is generally thought that in classical T Tauri systems which have typical ages on the order of Myr accretion from the circumstellar disk is magnetically regulated: strong (reaching kG levels in the T Tauri systems; see Johns-Krull et al. 1999; Bouvier et al. 2007) stellar magnetic field disrupts accretion flow with $\dot{M} \sim 10^{-7} - 10^{-9} M_{\odot}$ yr$^{-1}$ (typical value in T Tauri systems; see Gullbring et al. 1998) outside the star, at a distance of several$^2$

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\footnote{$^2$ In the case of nondipolar stellar field for which there is emerging evidence (Johns-Krull 2007; Lamzin et al. 2007) disk truncation occurs even closer to the stellar surface.}
stellar radii from the surface. Accreting material then gets funneled toward the magnetic poles and deposited in magnetospheric columns.

However, the situation must be quite different early on when the protostar still actively accumulates its mass, and one expects $M$ to be higher by 2–3 orders of magnitude than in the T Tauri disks. The magnetospheric truncation radius decreases as $M$ increases (in the simple picture of Königl [1991] this radius scales as $\propto M^{-2/3}$), and one can easily show that for $M \sim 10^{-6} - 10^{-5} M_\odot$ yr$^{-1}$, typical for the protostellar mass assembly stage, even a KG magnetic field is not capable of disrupting the accretion flow outside the star. Given that we have neither observational nor theoretical evidence of magnetic fields stronger than KG in such young objects it appears reasonable to assume that accretion in these systems is not magnetically channeled and circumstellar disks instead extend all the way to the stellar surface. Thus, throughout this study we assume that the inner edge of the disk touches the stellar surface at the equator, where a boundary layer forms (Lynden-Bell & Pringle 1974; Popham et al. 1993) in which the speed of accreting gas is reduced from the Keplerian velocity to the velocity of the stellar surface.

The goal of the present paper is to assess the implications of a protostar being irradiated by its own accretion disk by following the evolution of accreting protostar, properly accounting for the effects of disk irradiation. We present our calculations in §§2 and 3, and the interpretation of the results in § 4.

## 2. METHOD OF CALCULATION

We consider a protostar of mass $M$ and radius $R$ growing by accretion of gas at rate $\dot{M}$ from the circumstellar disk which extends all the way to the stellar surface. In this work $M$ is specified as an explicit function of time so that $M(t)$ is also known.

To evaluate the effect of disk irradiation on the protostellar evolution we use an approach based on the energy conservation which was developed in Hartmann et al. (1997). In this approach the convective part of the star comprising most of its mass is assumed to behave like a polytrope with index $n = 3/2$, so that inside the star pressure $P$ is related to the density $\rho$ via $P \propto \rho^{5/3}$. This approximation works very well in highly ionized, dense, and fully convective interiors of young stars. The total energy of such a star (a sum of its thermal and gravitational energies) is $E_{\text{tot}} = -(3/7)G M^2/R$ (Kippenhahn & Weigert 1994). Evolution of protostellar properties—luminosity $L$, radius $R$—as a function of time [or, equivalently, stellar mass $M(t)$] is then governed by the following equation:

$$
\frac{d}{dt} \left( - \frac{3}{7} \frac{G M^2}{R} \right) = - \frac{G M \dot{M}}{R} + E_{\text{th}} + L_D - L.
$$

(1)

The left-hand side of this equation represents the change in the total stellar energy; the first and second terms on the right-hand side are the gravitational potential energy and the thermal energy brought in with the accreted material, while $L_D(M, R)$ is a deuterium luminosity of a protostar. Stellar luminosity $L$ is the luminosity carried toward the photosphere by the convective motions in the stellar interior. It is different from the integrated emissivity of the stellar surface, since the star also intercepts and reradiates a fraction of energy released in the accretion disk.

The rate at which thermal energy gets accreted by the star is

$$
E_{\text{th}} = \frac{\dot{M}}{\gamma - 1} \frac{k_B T}{\mu} = \frac{\alpha_T G M \dot{M}}{R},
$$

(2)

where $\gamma$ is the ratio of specific heats of accreted gas (which can be different from $\gamma = 5/3$, characteristic for the stellar interior), $k_B$ is the Boltzmann constant, and $\mu$ and $T$ are the mean molecular weight and the temperature of the accreted gas. The dimensionless parameter $\alpha_T$ can be written as

$$
\alpha_T = \frac{1}{\gamma - 1} \frac{T}{T_{\text{vir}}}, \quad T_{\text{vir}} = \frac{\mu GM}{k_B R},
$$

(3)

where $T_{\text{vir}}$ is the stellar virial temperature. Gas accreting from the disk experiences strong dissipation in the boundary layer near the stellar surface. In this layer, the gas temperature can become an appreciable fraction of $T_{\text{vir}}$. However, the cooling time in the boundary layer and the outermost layers of the star is very short, so that the accreted gas cools efficiently and should ultimately join stellar interior with temperature $T$, which is much lower than $T_{\text{vir}}$ (unless $M$ is extremely high, in excess of $10^{-4} M_\odot$ yr$^{-1}$, see Popham 1997). Thus, under the conditions considered in this work, one expects $\alpha_T \ll 1$, but the actual value of this parameter is not well known (it depends on the gas thermodynamics, radiative transport, and the viscosity prescription in the boundary layer which are poorly constrained). As here we are primarily interested in the effects of disk irradiation, we set $\alpha_T = 0$ for simplicity. In this case equation (1) can be rewritten as

$$
\frac{d}{dt} \left( - \frac{3}{7} \frac{G M^2}{R} \right) = - \frac{G M \dot{M}}{R} + E_{\text{th}} - \frac{1}{3} \frac{L_D}{M}.
$$

(4)

This is an evolution equation for $R$ and can be easily integrated numerically once the dependencies of $L_D$ and $L$ on stellar parameters are known.

For $L_D$ we adopt the expression obtained in Stahler (1988) by integrating the rate of energy released due to D burning within the $n = 3/2$ polytrope:

$$
L_D = f_0 [D/H] L_{D,0} \left( \frac{M}{M_\odot} \right)^{13.8} \left( \frac{R}{R_\odot} \right)^{-14.8},
$$

(5)

where $L_{D,0} = 1.92 \times 10^{17} L_\odot$ and $f_0$ is the fractional D abundance relative to the initial D number abundance [D/H] taken to be $2 \times 10^{-5}$. The parameter $f_0$ is not constant in time; it evolves, since D burns in the stellar interior while the new D is being brought in with the accreting material (with the initial abundance [D/H]). As a result, one finds (Stahler 1988; Hartmann et al. 1997)

$$
\frac{d}{dt} (f_0 M) = \dot{M} - \frac{L_D}{\beta_D},
$$

(6)

where $\beta_D = 9.2 \times 10^{13}$ ergs g$^{-1}$ is the energy released by D fusion per gram of stellar material (assuming [D/H] = $2 \times 10^{-5}$). The most important aspect of this work which distinguishes it from Hartmann et al. (1997) is the calculation of $L$. In the approximation adopted by Hartmann et al. (1997) $L$ is a function of $R$ and $M$ only. In our case the situation is different: irradiation of the stellar surface gives rise to an outer convectively stable layer below the stellar photosphere (Rafikov 2008), similar to the radiative layer that forms in the atmospheres of the close-in giant planets irradiated by their parent stars (Guillot et al. 1996; Burrows et al. 2000). This external radiative zone suppresses the local

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4. Prialnik & Livio (1985) and Hartmann et al. (1997) have previously investigated the effect of the variation of $\alpha_T$ on the protostellar evolution.
radiative flux coming from the stellar interior and this changes the integrated stellar luminosity, which becomes a function of irradiation intensity. As a result, in the irradiated case \( L \) depends not only on \( R \) and \( M \) but also on \( \dot{M} \) (which determines the strength of the irradiation flux).

Rafikov (2008) has demonstrated that for a given opacity behavior at the stellar surface (parameterized in his case to be a power-law function of gas pressure \( P \) and temperature \( T, \kappa \propto P^{\alpha}T^3 \)) the degree of luminosity suppression depends only on the so-called irradiation parameter

\[
L = L_0 \chi(\Lambda), \quad \Lambda = 4\pi \frac{GM}{RL_0}, \tag{7}
\]

which is (up to a constant factor) the ratio of the accretion luminosity of the disk \( GM/R \) to the luminosity \( L_0 \) that a star would have had in the absence of irradiation. The suppression factor \( \chi(\Lambda) \to 1 \) as \( \Lambda \ll 1 \), while \( \chi(\Lambda) \leq 1 \) when \( \Lambda \gg 1 \). In a simple case considered by Rafikov (2008) the dependence of \( \chi \) on the opacity behavior comes only through the parameter \( \xi \)

\[
\xi = \beta + (1 + \alpha)/\nabla_{\text{ad}}, \tag{8}
\]

where \( \nabla_{\text{ad}} \) is the adiabatic temperature gradient near the stellar surface.

In the strongly irradiated case the temperature of the stellar surface varies as a function of latitude: the equatorial belt is strongly heated by the hot inner parts of the disk, while the polar regions of the star are virtually unaffected by irradiation and preserve their temperature at the level of \( T_0 = (L_0/4\pi R^2 \sigma)^{1/4} \). In this situation one may wonder whether it is reasonable to assume a fixed opacity law (as was done in Rafikov 2008) for the calculation of luminosity suppression, given that the behavior of \( \kappa \) can be different between the polar and the equatorial regions of the star. However, it was shown in Rafikov (2008) that for the opacity behavior typical for stellar photospheres in the temperature interval from \( \sim 2.5 \times 10^3 \) to \( 10^5 \) K the cooling of irradiated stars occurs mainly through their polar caps\(^5\) (even though the opacity scaling with temperature and pressure changes quite drastically within this temperature interval at around 5000 K). As a result, no matter how hot the equatorial parts of the star become and how complicated the opacity behavior is in this portion of the stellar surface, the integrated stellar luminosity \( L \) does not depend on these details very strongly, but is rather determined by the properties of the polar regions of the star: the size of the cool polar caps in which the photospheric temperature is preserved at the level of \( T_0 \), and the opacity behavior in the adjacent parts of the stellar surface. This provides motivation for using the stellar luminosity prescription represented by equation (7). In this paper we adopt \( \kappa \) characteristic for the temperature interval \( 2.5 \times 10^3 \leq T \leq 5 \times 10^3 \) K (Bell & Lin 1994):

\[
\kappa \approx 6 \times 10^{-14} P^{2/3} T^{-7/3}. \tag{9}
\]

This scaling should be reasonable in the polar regions of irradiated stars, where the photospheric temperature \( T_0 \) is not strongly affected by irradiation and is close to the photospheric temperature that an isolated nonaccreting star would have possessed in the Hayashi phase. The opacity law (eq. [9]) corresponds to \( \xi = 6.5 \), assuming \( \nabla_{\text{ad}} = 2/5 \) as appropriate for the fully ionized stellar interior (\( \gamma = 5/3 \)). Having specified the opacity law we have thus fully determined the behavior of \( \chi(\Lambda) \) (which we take from Rafikov [2008]; see the curve corresponding to \( \xi = 6.5 \) in Fig. 4 of that paper), which is necessary to compute \( L \).

One remaining ingredient of the calculation is the choice of \( L_0(M, R) \) — the luminosity of a nonirradiated star. Hartmann et al. (1997) have adopted the following fit to the stellar evolution tracks of D’Antona & Mazzitelli (1994):

\[
L_0(M, R) = 1.0 \bigg( \frac{M}{0.5 M_\odot} \bigg)^{0.9} \bigg( \frac{R}{2 R_\odot} \bigg)^{2.34}. \tag{10}
\]

The evolution tracks in D’Antona & Mazzitelli (1994) have been calculated using the equation of state from Magni & Mazzitelli (1979) that has been superseded by the more refined treatments (Saumon et al. 1995). Also, D’Antona & Mazzitelli’s treatment of convection is based on Canuto & Mazzitelli (1991) which has previously raised some concerns (Demarque et al. 1999; Nordlund & Stein 1999). Despite these deficiencies, we have chosen to adopt prescription (10) in our work because of its simplicity and also to allow direct comparison with the results of Hartmann et al. (1997).

Equations (4), (5), (6), (7), and (10) supplemented with \( dM/dt = \dot{M}(t) \) and the dependence \( \chi(\Lambda) \) from Rafikov (2008) fully determine the evolution of an accreting protostar irradiated by its own disk. This system of equations is then evolved numerically assuming that the prescription for \( \dot{M}(t) \) is given.

3. RESULTS

Here we present the results of our calculations. As initial conditions we choose \( M = 0.1 M_\odot \) and \( f_0 = 1 \). We vary \( R \) and the prescription for \( \dot{M}(t) \) to see their effect on the evolution of stellar properties.

In Figure 1 we display protostellar evolution for initial \( R = 1.5 R_\odot \) and a uniform \( M = 4 \times 10^{-6} M_\odot \) yr\(^{-1} \) with and without the effects of disk irradiation included. One can see that in both irradiated and nonirradiated cases evolution is pretty much the same: the star initially contracts until its central density and temperature become high enough for the deuterium to ignite. Right after that, D burning strongly dominates over the stellar luminosity and the D abundance \( f_0 \) starts going down. The resulting energy release in the stellar interior causes the star to expand out to 2.5 \( R_\odot \), and D luminosity \( L_D \) decreases appreciably (but still exceeds \( L_0 \) by a factor of several). These results are in full agreement with the calculations of Hartmann et al. (1997).

In Figures 1a and 1b we show quantities unique for the irradiated case: the run of the irradiation parameter \( \Lambda \) and the suppression factor \( \chi(\Lambda) \). As \( R \) initially decreases, \( \Lambda \) goes up to \( \approx 1.5 \times 10^3 \), since \( \Lambda \propto M M_{\text{f}}^0 R^{-3.34} \) for \( L_0 \) given by equation (10). As a result, the internal stellar luminosity is appreciably suppressed, and \( \chi \) reaches \( \approx 0.55 \), demonstrating the importance of disk irradiation in regulating \( L \).

At the same time, although the effect of irradiation on \( L \) is of order unity, Figure 1 clearly demonstrates that irradiation affects other stellar properties such as \( R \) and \( f_0 \) only very weakly. As expected, \( R \) in the irradiated case is larger than in the nonirradiated case.

\footnote{5 For convection to set in at some depth below the outer radiative zone a condition \( \xi > 4 \) must be fulfilled, see Rafikov (2008) for details.}

\footnote{6 Situation is different in the case of giant planets irradiated by the circumplanetary disks, see Rafikov (2008) for details.}

\footnote{7 We do not require perfect knowledge of \( L_0 \) since our primary goal is to evaluate the importance of disk irradiation.}
The luminosity given by eq. (10), \( M \) (parameter irradiated). We display the corresponding quantities both in irradiated (solid line) and non-irradiated (dashed line) cases.

Fig. 1.—Evolution of the protostellar properties for \( M = 4 \times 10^{-6} M_\odot \) yr\(^{-1}\). Initially \( M = 0.1 M_\odot \) and \( R = 1.5 R_\odot \). We display the runs of (a) irradiation parameter \( \Lambda \), (b) suppression factor \( \chi \), (c) ratio \( L_D/L_0 \) of the D burning luminosity to the luminosity given by eq. (10), (d) stellar radius \( R \), (e) and the D abundance \( f_0 \) relative to its initial value, as functions of stellar mass \( M \). In the last three panels we display the runs of (dashed line) \( M = 5 \times 10^{-6} M_\odot \) yr\(^{-1}\) and (solid line) \( M = 10^{-5} M_\odot \) yr\(^{-1}\).

4. DISCUSSION

Weak sensitivity of \( R \) to disk irradiation, given the strong effect that irradiation has on the internal luminosity \( L \) of a protostar, may seem surprising. However, one should bear in mind that besides \( L \) there are other contributors to the stellar energy budget, namely the D burning luminosity \( L_D \) and the gravitational potential energy.

FIG. 2.—(a) Relative difference between the stellar radii in the irradiated \( R \) and nonirradiated \( R_\odot \) cases for a protostar accreting at a uniform rate \( M = 2.0 M_\odot \) yr\(^{-1}\). Different curves correspond to different initial radius: 1.0 \( R_\odot \) (solid curve), 1.5 \( R_\odot \) (dashed curve), and 2.0 \( R_\odot \) (dotted curve). (a, b) Runs of irradiation parameter \( \Lambda \) and suppression factor \( \chi \) as a function of stellar mass.

Fig. 2.—(c) Relative difference between the stellar radii in the irradiated \( R \) and nonirradiated \( R_\odot \) cases for a protostar accreting at a uniform rate \( M = 2 \times 10^{-6} M_\odot \) yr\(^{-1}\). Different curves correspond to different initial radius: 1.0 \( R_\odot \) (solid curve), 1.5 \( R_\odot \) (dashed curve), and 2.0 \( R_\odot \) (dotted curve). (a, b) Runs of irradiation parameter \( \Lambda \) and suppression factor \( \chi \) as a function of stellar mass.

Fig. 3.—Same as Fig. 2, but for a uniform \( M = 10^{-5} M_\odot \) yr\(^{-1}\).

To check that this result is not an artifact of our initial conditions and assumed \( M \), we have additionally calculated protostellar evolution for uniform \( M = 2 \times 10^{-6} \) and \( 10^{-5} M_\odot \) yr\(^{-1}\) and different initial \( R \). The results are presented in Figures 2 and 3 in which we display the evolution of \( \Delta R/\Delta R = (R - R_\odot)/R_\odot \), where \( R \) and \( R_\odot \) are the values of the protostellar radius with and without disk irradiation taken into account. It is quite clear from these plots that despite the rather severe luminosity suppression (\( \chi \) reaching 0.3 in the high \( M \) case for the initial \( R = 1 R_\odot \) the relative stellar radius increase due to irradiation is always rather small. Note that at \( M = M_\odot \) we find \( \Delta R/\Delta R \approx 4% \) when \( M = 2 \times 10^{-6} M_\odot \) yr\(^{-1}\) (Fig. 2) which is larger than in the higher \( M \) case shown in Figure 3 (when \( \Delta R/\Delta R \approx 3% \)).

A very similar outcome has been found in the case of non-uniform \( M \). In Figure 4 we plot \( \Delta R/\Delta R \) for different initial \( R \) and \( M = aM \) with \( a \) chosen in such a way that a protostar grows to \( 1 M_\odot \) in \( 10^5 \) yr. This growth time is close to the time needed to reach \( 1 M_\odot \) in the case of constant \( M = 10^{-5} M_\odot \) yr\(^{-1}\), see Figure 3. For nonuniform \( M \) one again finds that irradiated protostar differs in radius from the nonirradiated star by only a few percent (\( \Delta R/\Delta R \approx 3% \) at \( M = M_\odot \)). It is also obvious from the comparison of Figures 3c and 4c that roughly the same growth time translates into very similar behavior of \( \Delta R/\Delta R \) in the two cases, independent of how \( M(t) \) evolves. We thus conclude that irradiation by the circumstellar accretion disk has a rather small eff-
gained with the accreted material. It turns out that these contributions dominate the energy budget over $L$. 

To see this we rewrite equation (4) using definition (7) in the following form:

$$\frac{\dot{R}}{R} = \frac{2\pi}{3} \Lambda^{-1} \frac{\dot{M}}{M} \left[ \frac{L_D}{L_0} - \Lambda \right] - \chi(\Lambda) - \frac{\Lambda}{28\pi}.$$ 

(11)

In the right-hand side of this equation the first term in brackets describes the relative role of D burning in the total energy budget, the second term represents intrinsic stellar luminosity, while the third term is due to the inflow of the gravitational potential energy $G M^2/R$ — the ratio of this energy inflow to $L_0$ differs from $\Lambda$ only by a constant factor.

Suppression of $L$ is largest when $\Lambda$ is highest which is easy to see by inspecting Figures 1–4. But this automatically means that the maximum deviation of the suppression factor $\chi$ from unity occurs precisely when the inflow of the gravitational potential energy far exceeds the stellar luminosity. Apparently, under these circumstances $L$ is a subdominant contribution to the stellar energy budget and thus even a significant reduction of $L$ compared to $L_0$ is going to be negligible compared to the gravitational energy influx.

Moreover, Figure 1 demonstrates that $\Lambda$ reaches its maximum when $\dot{R}$ is at its minimum, while $f_D$ is still very close to unity. At this point vigorous D burning commences inside the star giving rise to very high $L_D/L_0$. As a result, at the evolutionary stage when $\chi$ is minimal $L$ is subdominant in comparison to not only $G M^2/R$ but also $L_D$. This additionally downplays the role of the luminosity suppression by irradiation in the early protostellar evolution.

This line of reasoning also explains why at $M \sim 1 M_\odot$ we have found $\Delta R/R$ to be larger for lower $M$ (see §3). First, smaller $M$ means lower $\Lambda$ so that the ratio of $L$ to the gravitational energy inflow rate $G M^2/R$ in the low $M$ case is larger than in the high $M$ case. Also, at $M \sim M_\odot$ one generally finds $f_D \ll 1$ (see Fig. 1e) so that $L_D$ is mainly due to the burning of the freshly accreted D (rather than the D that remained in the protostar from previous accretion). Since in the low $M$ case less fresh D is supplied to the protostar $L_D$ must also be lower than in the high $M$ case. As a result, in the lower $M$ case $L$ plays a more significant role compared to $L_D$ (in which case $\Delta R/R$ should be more sensitive to changes in $L$ caused by irradiation) than in the high $M$ case.

This conclusion immediately raises the following question: since $\Delta R/R$ increases as $M$ decreases would one find $\Delta R/R \sim 1$ at low enough $M$? For example, one may expect this situation to be realized when $\Lambda \sim 1$ and $L$ is comparable to the accretion luminosity of the protostar, so that the suppression of $L$ by irradiation is more significant compared to $G M^2/R$ than in the high $M$ case. The answer to this question is no, and it has to do with the nontrivial fact that $\chi$ appreciably differs from unity (obviously, a necessary condition for getting $\Delta R/R \sim 1$) only at rather large $\Lambda$. This is a generic feature of disk irradiation which is illustrated in Figure 5 where we display $\Lambda_{\odot}$—the value of $\Lambda$ at which $\chi(\Lambda_{\odot}) = 0.5$ — as a function of the assumed opacity law represented by the parameter $\xi$ (see eq. [8]). One can see that $\Lambda_{\odot} \gtrsim 10^2$ for all $\xi > 4$, meaning that significant luminosity suppression requires rather high $M$. This inefficiency of irradiation in suppressing $L$ is caused by the specific geometry of disk irradiation in which the irradiation flux is a very sensitive function ($\propto \theta^5$) of the latitude at the stellar surface $\theta$ (see Rafikov 2008). Because of this fact stellar polar caps can stay cool even at rather high $M$, allowing unsuppressed flux to be emitted over a significant portion of the stellar surface.

In the case of $\xi = 6.5$ as appropriate for cold, low-mass protostars one finds that $\Lambda_{\odot} = 2.2 \times 10^3$, which according to equation (7) immediately implies that $G M^2/R \approx 175 L_0$ when $\chi = 0.5$. Clearly, in this case stellar luminosity should have small effect on the protostellar evolution. If $M$ is reduced so that $G M^2/R \sim L_0$ (and $\Lambda \sim 1$) stellar luminosity would be playing a significant role in the stellar energy budget; however, $\chi$ would be very close to unity (see Rafikov 2008) and the $L$ suppression by irradiation

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*Fig. 4.—Same as Fig. 2, but for $M \propto M$ and the growth time $10^5$ yr to reach $M = 1 M_\odot$.*

*Fig. 5.—Plot of $\Lambda_{\odot}$ (the value of $\Lambda$ at which the suppression factor $\chi = 0.5$) as a function of parameter $\xi$ characterizing the opacity law in the outer layers of irradiated protostar. Note that $\Lambda_{\odot}$ never decreases below several hundred.*
would be negligible. Thus, under no circumstances should one expect \( \Delta R/R > 6 \) larger than several percent, meaning that quite generally the irradiation by accretion disk is unlikely to play a significant role in the evolution of the protostellar properties.

Looking at this conclusion at a slightly different angle, our results also imply that when considering evolution of the protostars accreting at \( M \sim 10^{-6} - 10^{-3} M_\odot \) yr\(^{-1}\) one may completely neglect \( L \) in the stellar energy budget and still get rather decent description of the protostellar evolution.

Our results are obtained assuming a specific value of \( \xi \) typical for the low-mass stars. However, one expects our major conclusions to also remain valid in the case of other accreting objects, such as young brown dwarfs and giant planets. Although these objects are likely to be characterized by values of \( \xi \) different from 6.5, our Figure 5 clearly demonstrates that the efficient suppression of stellar luminosity requires very large \( \xi \) for any value of \( \xi \).

As a result, we expect \( L \) to be a subdominant energy contributor for any actively accreting object meaning that its radius (and other properties) should be different from the radius of its nonirradiated counterpart by at most a few percent.

Previous evolutionary studies of the protostellar assembly by accretion (Mercer-Smith et al. 1984; Palla & Stahler 1992; Siess & Forestini 1996; Hartmann et al. 1997; Siess et al. 1997, 1999) have always neglected the effect of disk irradiation on stellar properties. Results of this work demonstrate that this simplification should not have led to major deviations from reality since disk irradiation affects stellar properties only at the few percent level. Protostellar evolution may more sensibly depend\(^{10}\) (Prialnik & Livio 1985; Hartmann et al. 1997) on the exact amount of thermal energy that gets accreted with the disk material, which is parameterized by the parameter \( \alpha_T \) (see eq. [3]). Disk irradiation is likely to play a very important role in setting the value of \( \alpha_T \), since the conditions in the external radiative zone near the stellar surface regulate the thermodynamic properties of the gas at the convective-radiative boundary of the protostar, which is likely to be crucial in determining \( \alpha_T \).

5. SUMMARY

We considered evolution of the properties of a growing protostar accreting gas from the circumstellar disk with the goal of assessing the impact of irradiation by the inner regions of the disk. We find that disk irradiation plays minor role in the radius evolution of a protostar so that the radius of an irradiated star is larger by only a few percent compared to the nonirradiated case. This result is largely independent of the specific behavior of the protostellar mass accretion rate as long as \( M \) is high enough, at the level of \( 10^{-6} - 10^{-3} M_\odot \) yr\(^{-1}\). The weak sensitivity of the stellar properties to the disk irradiation is explained by the minor role played by the stellar luminosity \( L \) in the energy budget of the star at the time when \( L \) is significantly altered by irradiation. At these stages of protostellar evolution the inflow of the negative gravitational potential energy brought in by the accreting material and the deuterium burning luminosity are the much more significant contributors to the stellar energy budget. Our results imply that the previous studies of the protostellar accretion which neglected disk irradiation may have underpredicted stellar radius only at the level of several percent. Our major conclusions should be directly applicable to the evolution of other actively accreting objects such young brown dwarfs and planets.

\(^{10}\) As long as \( \alpha_T \) is not very small. Whether this is true is debatable (Popham 1997), and we assume \( \alpha_T = 0 \) in this work.

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