Design optimization of joint parameters using Frequency Based Substructuring

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Dynamic Substructuring (DS) is a research field that has gained high attention in both science and industry. The aim of DS techniques is to provide engineers in structural vibrations and sound practical solutions to analyze the dynamic behavior of complex systems. This paper addresses the design optimization problem of joint parameters to reduce the magnitude of the total system response in the context of the Lagrange Multiplier - Frequency Based Substructuring (LM-FBS) coupling process. For illustration, we use rubber bushings from an automotive application in the field of noise, vibration and harshness (NVH). Depending on which parameter is changed on the respective rubber bushing, it has an influence on the sound pressure in the total vehicle system. By grouping the joints into engine mounts, front suspension and rear suspension mounts, a method is proposed to implement an optimization function within the LM-FBS approach. For this purpose, the rubber bushings are considered as a substructure using the pseudoinverse method, so that in order to determine the optimum parameters, an optimization function only has to be established for the substructure of the bushings in relation to the total system result.

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1 LM-FBS and design optimization approach

![Substructure A](substructure_a.png) ![Substructure Bush](substructure_bush.png) ![Substructure C](substructure_c.png)

Fig. 1: Assembly of substructures with bushings implemented by pseudoinverse method.

The classical Lagrange Multiplier - Frequency Based Substructuring (LM-FBS) method [1] allows to assemble an arbitrary number of substructures in a systematic fashion. However, this method is limited to substructures having mass. In the case of massless substructures, as it occurs in simple joints (Kelvin-Voigt elements), the dynamic stiffness matrix is singular. In order to take this into account a generalized LM-FBS formulation is given in [2], which allows to integrate actual singular matrices into the process. Fig. 1 shows a system consisting of substructures A and C, with a rubber bushing arranged between them which is also considered as a substructure. The dynamic stiffness $Z^{(Bush)}$ for the rubber bushing element from the illustrated model can thus be set up as

$$
Z^{(Bush)} = \begin{bmatrix}
(1 + i\gamma)k & -(1 + i\gamma)k \\
-(1 + i\gamma)k & (1 + i\gamma)k
\end{bmatrix}. \tag{1}
$$

Here, we are considering hysteretic damping, an assumption often made to model linear damping in the frequency domain. It is assumed that the damping forces are independent of the frequency but proportional to the motion amplitude and are in phase with the velocity [3]. In order to get an admittance $Y^{(Bush)}$ of the substructure Bush depicted in Fig. 1, the pseudoinverse of the dynamic stiffness matrix (Eq. 2) has to be considered. It can be computed either by singular value decomposition (SVD) or, more efficiently, by an incomplete factorization as described in [3].

$$
Y^{(Bush)} = (Z^{(Bush)})^+ \tag{2}
$$

The assembled system response $u_{assembled}$ is obtained through the extended generalized formulation of the LM-FBS approach (Eq. 3), which considers also singular substructures, as it is the case for rubber bushings. Matrix $B$ contain the rigid body modes and $B$ are signed Boolean matrix describing the interface compatibility.

$$
u_{assembled} = Y - \begin{bmatrix} Y B^T & R \end{bmatrix} \begin{bmatrix} BY B^T \ R^T B^T \ R^{-1} BY \end{bmatrix} f = Y_{assembled} f \tag{3}$$

As design optimization approach, the Multi Level Single Linkage (MLSL) algorithm from the NLopt Python library [4] was selected. It is one of the best known and efficient stochastic algorithm for global optimization problems of moderate size. For a detailed explanation of the following algorithm 1, see [5].
Algorithm 1 The Multi Level Single Linkage (MLSL) algorithm [5]

1: \( X^* \leftarrow \emptyset; k \leftarrow 0; 0 < \gamma < 1 \)
2: \textbf{repeat}
3: \( k \leftarrow k + 1 \)
4: Generate \( N \) points \( x_{(k-1)N+1}, \ldots, x_{kN} \) with uniform distribution on \( X \).
5: Determine the reduced sample \( (X_r) \) consisting of the \( \gamma kN \) best points from the cumulated sample \( x_1, \ldots, x_{kN} \).
6: \textbf{for} \( i \leftarrow 1 \text{ to } \text{length}(X_r) \) \textbf{do}
7: \textbf{if} NOT (there is a \( j \) such that \( f(x_j) < f(x_i) \) and \( \|x_j - x_i\| < r_k \) \textbf{then}
8: Start a local search method (LS) from \( x_i \).
9: \( x^* \leftarrow \text{LS}(x_i) \)
10: \( X^* \leftarrow X^* \cup \{x^*\} \)
11: \textbf{until} Some global stopping rule is satisfied.
12: \textbf{return} The smallest local minimum value found.

2 Case study and conclusion

This section provides an insight into the practical application of the generalized LM-FBS method combined with the MLSL optimization algorithm. Simulated FRF data for individual components of a vehicle system was provided for coupling. The substructures were four tires, a front suspension, a rear suspension, the rubber bushings and the body in white (BIW). The main focus here was on the rubber mounts, which were coupled to the corresponding DOFs between front suspension and BIW, and rear suspension and BIW. Fig. 2 shows the response \( u_{\text{assembled}}(\omega) \) of the sound pressure level at drivers ear before and after the optimization. This procedure should be performed in future work with different massless joint elements, because in this case the optimizer for the design variable only aims at the maximum possible value for the stiffness of the bushings.

![Diagram showing design optimization of joint parameters through generalized LM-FBS method.](image)

Objecive function:
\[
\minimize(u_{\text{assembled}}(\omega))
\]

Design variables:
Static stiffness of bushings \( k \)

Algorithm:
Multi Level Single Linkage (MLSL)

Fig. 2: Design optimization of joint parameters through generalized LM-FBS method.

References
[1] D. de Klerk, D. J. Rixen, and J. de Jong, The Frequency-Based Substructuring (FBS) method reformulated according to the dual domain decomposition method, in: 24th International Modal Analysis Conference, St. Louis, MO, (2006).
[2] A. El Mahmoudi, D. Rixen, and C. Meyer, Comparison of different approaches to include rubber bushings into Frequency-Based Substructuring (was completed).
[3] M. Géradin and D. J. Rixen, Mechanical vibrations: Theory and application to structural dynamics (John Wiley & Sons, 2015).
[4] S. G. Johnson, The NLopt nonlinear-optimization package, http://github.com/stevengj/nlopt, Accessed: 2019-05-15.
[5] A. H. G. Rinnooy Kan and G. T. Timmer, Stochastic global optimization methods part II: Multi level methods, in: Mathematical Programming, (1987).