Analysis of Heat Transfer in Thermocouples Immersed in High Temperature Gas with Short Insertion Length

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Abstract: Thermocouples are used to measure high temperature in process and steel industries. To measure high temperature gas, thermocouples should be inserted deeply enough; commercial specification documents recommend at least 5 times the diameter of sheath or more. This paper analyzes the effect of insertion length on thermocouple outputs in high temperature gas measurement. The theoretical evaluation is revised to include the radiation effect in the authors’ previous work presented in SICE AC 2015 and compared with experimental data in a heating furnace more precisely than the work presented in SICE AC 2016.

Key Words: thermocouple, high temperature gas, insertion length.

1. Introduction

Thermocouples are widely used in process and steel industries to measure high temperature. On monitoring temperature of working gases, special care to insertion length of the thermometer should be taken due to the heat conduction through thermometer itself. Commercial specification documents [1],[2] recommend the minimum insertion length of 5D for sheath thermocouples and 15D for thermocouples with protection tubes, where D stands for the corresponding diameters. This paper focuses on these empirical guidelines [3]. The authors’ previous work [4] analyzed heat conduction for fire and/or heating furnaces to evaluate the effect of insertion depth of the thermocouples under the uniform temperature distribution inside furnaces, and experimental data with a test heating furnace in a steel process validates the analysis but it was not complete yet. This study revises the analysis to include the radiation effect that was ignored in the previous study. More precise comparison than that in [5] with experimental data in a heating furnace is also given.

This subject had been introduced in the lecture, titled “Learn from failures”, by the one of the authors given in the series of lectures in process control (the SICE Process School [6]), which is for the technology transfer from senior experts to process engineers in industry.

2. Theoretical Evaluation

2.1 Sheath Temperature

We assume the relating heat conduction and radiation problem to be one-dimensional here, and consider a fire or heating furnace with the wall thickness W, the gas temperature \( T_g \) inside the furnace, and the thermocouple thermometer of the diameter \( d \) is inserted into the furnace with the insertion length \( L \). Let us take the \( x \)-coordinate along the thermometer with the origin on the inner surface of the wall directing inside the furnace; outer wall is at \( x = -W \), and the tip of the thermometer, at \( x = L \). Let the absolute temperature distribution along the thermometer sheath be \( u_s(x,t) \), that inside the wall, \( u_w(x,t) \), that inside the furnace, \( u_g(x,t) \), and the temperature of the wall inner and outer surfaces, \( T_w \) and \( T_0 \), respectively. We ignore the temperature distribution over cross-section of the thermometer, and assume constant temperature gradient inside the furnace wall, providing that the heating is homogeneous and the equilibrium is achieved. These assignment together with the experimental setup is schematically shown in Fig. 1, where in spite of the literature [7],[8] we include the radiation effect inside the uniform temperature furnace.

Using above notation, we formulate the following initial-boundary value problem describing heat conduction and radiation consisting of the partial differential equations (P.D.E.), the boundary conditions (B.C.), and the initial condition (I.C) for the temperature distribution \( u_s(x,t) \) as follows:

![Fig. 1 Coordinate systems for experimental setup and thermocouples under test.](image-url)
\[
\begin{align*}
\text{P.D.E.:} & \quad \begin{cases}
-W < x < 0, \\
\frac{\partial u_x}{\partial t} = \alpha^2 \frac{\partial^2 u_x}{\partial x^2} - \beta_w (u_x - u_w), \\
\frac{u_w}{W} = x + W, \\
\frac{u_x}{W} = x, \\
\frac{u_y}{T_w} = 1, \\
\frac{u_z}{T_0} = 1, \\
\frac{u_w}{u_x} = -W \leq x \leq 0, \\
\frac{u_y}{u_z} = T_g, \\
\frac{u_x}{u_y} = T_g, \\
\frac{u_z}{u_z} = 1 + \delta, \\
\frac{\partial u_x}{\partial x} = 1, \\
\frac{\partial u_y}{\partial x} = 0, \\
\frac{\partial u_z}{\partial x} = 1, \\
\frac{\partial u_x}{\partial y} = 0, \\
\frac{\partial u_y}{\partial y} = 0, \\
\frac{\partial u_z}{\partial y} = 0, \\
\frac{\partial u_x}{\partial z} = 0, \\
\frac{\partial u_y}{\partial z} = 0, \\
\frac{\partial u_z}{\partial z} = 0.
\end{cases}
\end{align*}
\]

with thermal conductivity \( k \) (W/m), specific heat \( c \) (J/(kg \cdot K)), density \( \rho \) (kg/m\(^3\)), heat transfer coefficients to the wall \( h_w \) (W/(m\(^2\) \cdot K)) and to the gas \( h_g \) (W/(m\(^2\) \cdot K)), area \( A \) (m\(^2\)) and perimeter \( \partial A \) (m) of sheath pipe, Stefan-Boltzmann constant \( \sigma \) (W/m\(^2\) \cdot K\(^4\)), and surface emissivity \( \varepsilon \) (1).

For simplicity, we normalize the variables with respect to \( W, T_w \), and the physical parameters to make them non-dimensional as follows:

\[
\begin{align*}
\xi = \frac{x}{W}, \quad \alpha = \frac{L}{W}, \quad \tau = \frac{\alpha^2}{W^2} t, \quad \gamma_w^2 = \frac{\beta_w}{\alpha^2} W^2, \\
\psi_w^2 = \frac{\beta_g^2}{\alpha^2} W^2, \quad U_x = \frac{u_x}{T_w}, \quad U_0 = \frac{T_0}{T_w}, \\
U_w = \frac{u_w}{U_0}, \quad \psi_z = (1 - U_0) \xi + 1, \quad U_g = \frac{T_g}{T_w} = 1 + \delta, \quad \psi_z = \Delta T \frac{T_w}{T_w},
\end{align*}
\]

Then, the problem is simplified to

\[
\begin{align*}
\text{P.D.E.:} & \quad \begin{cases}
-1 < \xi < 0, \\
\frac{\partial u_x}{\partial t} = \alpha^2 \frac{\partial^2 u_x}{\partial x^2} - \gamma_w^2 [U_x - (1 + (1 - U_0) \psi_w^2)] \\
\frac{\partial u_x}{\partial x} = \frac{\partial^2 u_x}{\partial x^2} - \gamma_z^2 [U_x - (1 + \delta)] \\
\frac{\partial u_x}{\partial z} = \frac{\partial^2 u_x}{\partial z^2} - \frac{W^2}{\alpha^2} \eta [U_g^2 - U_x^2] T_w^3,
\end{cases}
\end{align*}
\]

B.C.: \( U_x(-1, \xi) = U_0 \), \( \frac{\partial U_x}{\partial \xi}(a, \xi) = 0 \), \( \frac{\partial U_x}{\partial z}(a, \xi) = 0 \).

I.C.: \( U_x(0, \xi) = \begin{cases} U_w, & -1 \leq \xi \leq 0, \\
U_g, & 0 \leq \xi \leq a.
\end{cases} \)

The radiation term introduces non-linearity to the original linear problem of the heat conduction and closed form solutions cannot be achieved since we need to invert the integral function. We linearize the term instead using the fact, \( U_g - U_x = \delta \ll 1 \), to obtain,

\[
U_g^2 - U_x^2 = (U_x - U_g)(U_x + U_g) + U_g^2 + U_x^2
\]

By introducing the coefficient,

\[
\psi_w^2 = \gamma_w^2 + \frac{4 \psi_w^2}{\alpha^2} \eta T_w^3,
\]

we have the following linear partial differential equation:

\[
\frac{\partial U_x}{\partial t} = \frac{\partial^2 U_x}{\partial \xi^2} - \gamma_w^2 [U_x - (1 + \delta)], \quad 0 < \psi_w < a.
\]

Since this problem is almost same as the previous linear heat conduction problem [4], we trace our previous result in the following with necessary modifications.

At first, we focus on the equilibrium state solution \( U_x(\xi) \) (i.e., \( U_x(\xi, \infty) = \lim_{t \to \infty} U_x(\xi, t) \)), which is a solution of the following boundary value problem of 2nd order ordinary differential equation:

\[
\begin{align*}
\frac{d^2 U_x}{d \xi^2} &= \gamma_w^2 [U_x - (1 + (1 - U_0) \xi)], \quad -1 < \xi < 0, \\
\frac{d^2 U_x}{d \xi^2} &= \gamma_w^2 [U_x - (1 + \delta)], \quad 0 < \xi < a, \\
U_x(-1) &= U_0, \quad \frac{dU_x}{d\xi}(a) = 0.
\end{align*}
\]

A general solution to each region is obtained by inspection as follows,

\[
\begin{align*}
-1 < \xi < 0: \quad U_x(\xi) &= (1 - U_0) \xi + 1 + c_1 \cos(\gamma_w \xi) + c_2 \sinh(\gamma_w \xi), \\
0 < \xi < a: \quad U_x(\xi) &= 1 + \delta + c_3 \cos(\gamma_g \xi) + c_4 \sinh(\gamma_g \xi),
\end{align*}
\]

where we adopt a pair of hyperbolic functions as a fundamental solution instead of an ordinary pair of exponential functions.

Four coefficients in the general solutions are determined from the boundary conditions (11) and the continuity and continuous differentiability at \( \xi = 0 \),

\[
\begin{align*}
U_x(-1) &= U_0 + c_1 \cos(\gamma_w) - c_2 \sin(\gamma_w) = U_0, \\
U_x'(a) &= \gamma_w c_3 \sinh(\gamma_g a) + \gamma_g c_4 \cosh(\gamma_g a) = 0,
\end{align*}
\]

After some manipulations to solve the above linear system, we have the solution:

\[
U_x(\xi) = \begin{cases} 
(1 - U_0) \xi + 1 + \frac{\sinh(\gamma_w \xi) + 1}{\Delta} \times \left( \frac{\delta - \frac{1}{\Delta}}{\Delta} \right) \\
\times \left( (1 - U_0) \sinh(\gamma_w) + \delta \gamma_w \cosh(\gamma_w) \right), & 0 < \xi < 0,
\end{cases}
\]

\[
\Delta \equiv \gamma_w \cosh(\gamma_g a) - \frac{\sinh(\gamma_g a)}{\sinh(\gamma_w)} - \tan(\gamma_w), \\
\gamma_g = \gamma_w + 2 \gamma_w \tanh(\gamma_w).
\]

\[
\Delta \equiv \gamma_w \cosh(\gamma_g a) - \frac{\sinh(\gamma_g a)}{\sinh(\gamma_w)} - \tan(\gamma_w).
\]
Output of the thermocouple thermometer is the tip temperature:

\[ U_s(a) = 1 + \delta \frac{\tan (\gamma_w) + \delta \cdot \gamma_w}{\cosh(\gamma_w a) \left( \gamma_w + \gamma_w \tan (\gamma_w a) \right)} \]

(19)

This result shows that measurement \( U_s(a) \) indicates the exact gas temperature of the furnace with enough insertion length as the conventional specification.

2.2 Thermocouple Temperature

A thermocouple thermometer measures the temperature of its thermocouple junction inside the tip of the thermometer. Due to higher heat conductivity of the thermocouple material than that of sheath one, measurements are reduced further from the temperature described in Eq. (18).

Let \( U(\xi) \) denote a temperature distribution along the thermocouple wires. Since sheath material has higher heat capacity and lower heat conductivity, the temperature distribution of the sheath material is not influenced by the heat conduction through the sufficiently fine thermocouple wires. Hence, the problem to solve is formulated with the already obtained \( U_s \) as follows:

P.D.E.: \[ \frac{\partial U}{\partial \tau} = \frac{\partial^2 U}{\partial \xi^2} - \gamma^2 (U - U_s), \quad -1 \leq \xi \leq a, \]

(20)

B.C.: \[ \begin{cases} U(-1, \tau) = U_0, \\ \frac{\partial U}{\partial \xi}(a, \tau) = 0, \end{cases} \]

(21)

I.C.: \[ U(\xi, 0) = U_s(\xi), \]

(22)

where \( \gamma^2 = \beta_W W^2/\alpha_T^2 \), namely, physical parameters of a thermo-couple. Again, we focus on the equilibrium state, and hence, the problem reduces to the following ordinary equation:

\[ \frac{\partial^2 U}{\partial \xi^2} = \gamma^2 (U - U_s), \quad -1 \leq \xi \leq a, \]

(23)

\[ \begin{cases} U(-1) = U_0, \\ \frac{dU}{d\xi}(a) = 0. \end{cases} \]

(24)

By inspection of Eqs. (10) and (23), we seek the solution of the form,

\[ U = \begin{cases} C_1 U_s + C_2(\xi) & , \quad -1 \leq \xi \leq 0 \\ C_3 U_s + C_4(\xi) & , \quad 0 \leq \xi \leq a, \end{cases} \]

(25)

where

\[ \frac{\partial^2 C_2}{\partial \xi^2} = \frac{\partial^2 C_4}{\partial \xi^2} = 0, \]

(26)

and the substitution into Eqs. (10) and (23) gives

\[ C_1 \gamma^2 (U_s - (1 + (1 - U_0)\xi)) = \gamma^2 ((C_1 - 1)U_s + C_2), \]

(27)

\[ C_3 \gamma^2 (U_s - (1 + \delta)) = \gamma^2 ((C_4 - 1)U_s + C_4). \]

(28)

Then, we have the temperature distribution along the thermocouple wires as,

\[ -1 < \xi < 0 : \]

\[ U(\xi) = \frac{1}{1 - (\gamma_w / \gamma)^2} \left[ U_s - \frac{\gamma_w^2 (1 - U_0)\xi + 1)}{\gamma_w^2} \right]. \]

(29)

\[ 0 < \xi < a : \]

\[ U(\xi) = \frac{1}{1 - (\gamma_w / \gamma)^2} \left[ U_s - \frac{\gamma_w^2 (1 + \delta)}{\gamma_w^2} \right]. \]

(30)

Now, the tip temperature \( U(a) \) is revised and reads

\[ U(a) = \frac{1}{1 - (\gamma_w / \gamma)^2} \left[ U_s - \frac{\gamma_w^2 (1 + \delta)}{\gamma_w^2} \right] \]

\[ = (1 - \delta) \]

\[ - \frac{\gamma^2}{\gamma^2 - \gamma_w^2} \left[ (1 - U_0)\tan (\gamma_w a) + \delta \cdot \gamma_w \right], \]

(31)

When the insertion length \( a \) is large enough, the last term in Eq. (31) becomes negligibly small and the measurement is exact again.

3. Experimental Evaluation

We carried out field experiments at a burning zone in a test heating furnace of an anonymous steelworks, under the following operational condition:

- Flow rate of gas-burner: \( \sim 6200 \text{Nm}^3/\text{h} \)
- Air-fuel ratio: \( \sim 1.1 \)
- Fuel gas: mixed gas
- Heating value: \( \sim 2500 \text{kcal/Nm}^3 \)
- Gas pressure: \( \sim 700 \text{mmH}_2\text{O} \approx 7 \text{kPa} \)

As shown in Fig. 1, four pairs of R-type thermocouple were replaced and installed on the furnace; insertion lengths \( L \) were 20 mm, 70 mm, 120 mm, and 170 mm. The temperature inside the furnace was changed between 1100°C and 1300°C according to the reading of a standard thermocouple thermometer with enough insertion length.

An experimental data chart with various temperature settings is shown in Fig. 2.

A typical set of experimental results are shown in Fig. 3 with differences in the readings for various temperature setting and four insertion depths. Reading errors according to the insertion depth \( L \) are apparent in the figure. In order to compare the experimental results with the above theoretical evaluation, physical parameters should be estimated. Using the reference data for R-type thermocouple with sheath material of aluminum oxide, the parameters were estimated as in Table 1.

Among these thermo-physical properties, the heat transfer coefficient \( h_w \) between sheath and gas and the one \( h_{sw} \) between sheath and wires have approximate values in literature due to complex contact condition. Using above parameter values, the normalized tip temperature of the sheath, \( U(a) \) were evaluated with Eq. (31) as the function of normalized insertion length \( a \) and heat transfer coefficient \( h_w \). Thermocouples were inserted through furnace wall with doubly nesting protection tubes; sheathed thermocouples were not packed but free
Table 1 Physical parameters in the experiment [9]–[14].

| Parameter                      | TC Sheath [Al₂O₃] (·)₀ₙ | TC Wires [Pt] (·)wire |
|--------------------------------|--------------------------|----------------------|
| Reference Temperature $T_0$   | 31 °C                    |                      |
| Wall Thickness $W$             | 0.33 m                   |                      |
| Diameter $d$                   | 0.002 m                  | 0.0001 m             |
| Thermal Conductivity $k$      | 36 W/(m·K)               | 72 W/(m·K)           |
| Density $ρ$                    | 3.9 × 10⁶ kg/m³         | 21.45 × 10⁷ kg/m³    |
| Specific Heat $c$             | 779 J/(kg·K)             | 130 J/(kg·K)         |
| Thermal Diffusivity $α$       | 1.0 × 10⁻⁵ m²/s         | 2.5 × 10⁻⁵ m²/s     |
| Heat Transfer Coef. (still air) $h_w$ | ~ 4.64 W/(m²·K) | $h_{w-s}$ ~ 580 W/(m²·K) |
| Heat Transfer Coef. (air flow) $h_g$ | ~ 10–290 W/(m²·K) |                      |
| Thermal Emissivity $ε$        | ~ 0.25                   |                      |

with air. Consequently, $γ_w$ of the wall region is evaluated as $h_w = 4.64$ W/(m²·K), which corresponds to still air. In the experiment, type R thermocouples with the diameter of 0.5 mm and the heat transfer coefficient $h_{wire}$ between the sheath and the wires was about 500 W/(m²·K).

Figure 4 shows the 3-D surface plot of numerically evaluated $U(a; h_g)$ with $ε = 0.25$ at $T_g = 1200$ °C. This figure confirms that the longer the insertion depth and the higher the heat transfer, the less the uncertainty of the measurement. We evaluate temperature distribution $U(ξ)$ along four thermocouples at the same condition and shown in Fig. 5, where the normalized tip temperatures $U(a)$ are also plotted with circles for the corresponding thermocouples. The recommended insertion length of 5D in commercial specification documents is 0.001 m which corresponds to the non-dimensional insertion length,

$$a = 0.001 \text{ m/W} = 0.001 \text{m}/0.33 \text{ m} = 0.003.$$
Fig. 6 Measured data and corresponding compensations with $\epsilon = 0.25$ are displayed against TC1 measurements instead of $T_g$: (a) original, (b) normalized with TC1 data, (c) compensated with $U(a)$, and (d) normalized compensation.

Since $L_1 = L_{\text{max}}$ is large enough, we can estimate $T_{\text{g}} = u_1 \equiv u_{L_{\text{max}}}$, and hence, measurements $u_i$ by TC$_i$ with $L = L_i$, $i = 1, 2, 3, 4$ are compensated as

$$\bar{u}_i = U(a_i) \times u_{L_{\text{max}}}.$$ \hspace{1cm} (33)

In stead to avoid estimating unknown true gas temperature $T_{\text{g}}$, we normalize each measured and compensated data: $u_i/u_{L_{\text{max}}}$ and

$$\frac{\bar{u}_i}{u_{L_{\text{max}}}} = \frac{u_i}{u_{L_{\text{max}}}} \times \frac{U(a_i)}{U(a_{\text{max}})}.$$ \hspace{1cm} (34)

Figure 6 shows the plots of these original, compensated, and normalized data, where we adopted the emissivity $\epsilon = 0.25$. The dispersion of the compensated data in Fig. 6 (c) & (d) are reduced compared with those of the original data in Fig. 6 (a) & (b).

Figure 7 shows the rearrangement of the compensated data shown in Fig. 6 (c) with respect to non-dimensional insertion lengths.

Although this fact indicates the feasibility of the analysis, there remain significant discrepancies, which is due to violation of the homogeneous heating condition and temperature gradient existing in the vicinity of the wall along $x$ direction. And hence, dynamical and in-homogeneous changes in spatial distribution of the temperature field in the furnace should be included in the analysis and will be done in our future work.

4. Conclusion

We evaluate the effect of the insertion length on thermocou-
outputs in high temperature gas measurement. Theoretical analysis with a simple assumption was partially confirmed by the field experimental data. Further analysis, however, is required to completely analyze the experimental data.

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