Parameter identification of fractional-order chaotic system with time delay via multi-selection differential evolution

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ABSTRACT
In this paper, the parameter identification issue of fractional-order chaotic system with time delay is studied, which is important for its modelling and controlling. A numerical algorithm for fractional differential equation with time delay is given. Time delay and fractional order along with other ordinary parameters are estimated together, which is rarely considered before. The identification issue is converted to an optimization problem, which is nonlinear, multivariable and multimodal. To solve this complex optimization problem effectively, a multi-selection differential evolution (MS-DE) is proposed. In MS-DE, multiple vectors are generated as candidate for selection, which can avoid the local extremum and speed up the convergence speed. The simulation results illustrate the effectiveness of the proposed MS-DE method.

ARTICLE HISTORY
Received 6 December 2016
Accepted 16 December 2016

KEYWORDS
Nonlinear system parameter identification; chaotic system; fractional-order; time delay; multi-selection differential evolution

1. Introduction
Chaos is a complex nonlinear phenomenon. It is generalized by determined nonlinear equations and possesses stochastic behaviour (Hilborn, 2000). Chaos has showed great application value and attracted lots of researches in different scientific fields such as power system (Zhao, Ma, and Liu, 2011), data encryption (Yuan and Huang, 2012), gross domestic product analysis (Kříž, 2014), chemical system (Xu and Wu, 2014), optimization algorithm (Wang, E, and Deng, 2012) and so on.

Recently, chaotic behaviour has been observed in numerous fractional-order and time-delayed systems, for example fractional-order Chua’s circuit (Hartley, Lorenzo, and Qammer, 1995), fractional-order Jerk model (Ahmad and Sprott, 2003), fractional-order Wien bridge oscillator (Ahmad, El-khazali, and Elwakil, 2001) and so on. Bhalekar and Daftardar-Gejji (2010) firstly studied the effect of time delay on the fractional-order Liu system and given different fractional orders corresponding to delay times under which system is chaotic. Wang, Huang, and Shi (2011) analysed nonlinear dynamics in a fractional-order financial system with time delay and presented the corresponding delay time and order for chaos to exist. A fractional differential model of HIV infection of CD4+ T-cells with time delay was introduced in Yan and Kou (2012) and fractional-order time-delay system stability analysis was presented.

In various application fields of chaos such as chaos control and synchronization, system parameters are supposed to be known. Unfortunately, it is difficult to obtain the exact values of parameters for a practical chaotic system. In these cases, parameter identification is important. The parameter identification of chaotic system is essentially a complex optimization problem. Conventional gradient-based optimization methods cannot solve this problem effectively. Many researchers resort to swarm intelligence-based optimization methods to identify the parameters of chaotic system. Modares, Alft, and Fateh (2010) discussed parameter identification issue of integer-order chaotic systems and proposed an improved particle swarm optimization (PSO) algorithm. In Tien and Li (2012), a hybrid Taguchi chaos of multilevel immune and artificial bee colony algorithm was adopted to identify the parameters of chaotic system. In Wang, Xu, and Li (2011), parameter identification of chaotic system was realized through a hybrid Nelder–Mead simplex search and differential evolution algorithm. Tang and Guan (2009) used PSO to identify the parameters and the delay time of chaotic system with time delay. Dai, Chen, and Li (2011) adopted a seeker optimization algorithm to identify parameters of chaotic system with time delay. Tang, Zhang, Hua, Li, and Yang (2012) studied the parameter identification of commensurate fractional-order chaotic system via differential evolution algorithm.
In Zhu, Fang, Tang, Zhang, and Xu (2012), a switching DE (SDE) was employed to estimate the orders and parameters in incommensurate fractional-order chaotic systems.

In the literature, there is little work about parameter identification of fractional-order chaotic system with time delay. It is well known that parameter identification of fractional-order chaotic system with time delay is more complex than integer-order chaotic systems, which is nonlinear, multivariable, and multimodal. So, an efficient global optimization technique is a must. In this paper, we focus on this issue using an improved differential evolution, multi-selection differential evolution (MS-DE). Like other evolutionary algorithms (EAs), the performance of DE is affected by many factors. To obtain better identification results, we proposed an improved scheme to enhance the performance of DE for parameter identification of fractional chaotic system with time delay.

The rest of the paper is organized as follows: Section 2 introduces the basic knowledge of fractional calculus. In Section 3, the formulation of parameter estimation issue is described. Section 4 gives a detailed description of MS-DE. The simulation results are given in Section 5. Finally, the conclusion remarks are presented in Section 6.

2. The basis of fractional calculus

2.1. Definitions of fractional calculus

Fractional calculus is a generalization of integration and differentiation to non-integer-order. The integro-differential operator $\mathcal{D}_t^\alpha$ is defined as (Petrá, 2011)

$$\mathcal{D}_t^\alpha = \begin{cases} \frac{d^n}{dt^n} & R(\alpha) > 0, \\ 1 & R(\alpha) = 0, \\ \int_t^\infty (t-s)^{-\alpha} ds & R(\alpha) < 0, \end{cases}$$

where $a$ and $t$ are the limits of the operator, $R(\alpha)$ is the real part of $\alpha$. There are several definitions of fractional derivatives: Grünwald–Letnikov (G–L) definition, Riemann–Liouville (R–L) definition and Caputo definition.

2.2. Numerical algorithm for fractional differential equation with time delay

A predictor–corrector method is a numerical algorithm for solving fractional-order differential equations proposed by Diethelm, Ford, and Freed (2002). Based on it, a numerical algorithm for fractional-order differential equation with time delay is obtained in this subsection.

Consider the following fractional differential equation with time delay:

$$D^\alpha x(t) = f(t, x(t), x(t - \tau)), \quad 0 \leq t \leq T,$$  \hspace{1cm} (2)

with the initial conditions

$$x^{(k)}(0) = x_0^{(k)}, \quad k = 0, 1, \ldots, m - 1$$

and

$$x(t - \tau) = 0 \quad \text{if} \ t < \tau,$$  \hspace{1cm} (3)

where $\alpha$ and $\tau$ are positive real constants, $m = [\alpha]$. Equations (2) and (3) are equivalent to the Volterra integral equation

$$x(t) = \sum_{k=0}^{[\alpha]-1} x_0^{(k)} \frac{t^k}{k!} + \frac{1}{\Gamma(\alpha)} \int_0^t (t - z)^{\alpha-1} f(z, x(z), x(z - \tau)) dz.$$  \hspace{1cm} (4)

Set $h = T/N$, $t_n = nh$ ($n = 0, 1, \ldots, N$), according to the one-step Adams–Bashforth–Moulton method, we can get the corrector formula

$$x_h(t_{n+1}) = \sum_{k=0}^{[\alpha]-1} x_0^{(k)} \frac{t_{n+1}^k}{k!} + \frac{h^\alpha}{\Gamma(\alpha + 2)} f(t_{n+1}, x_h(t_{n+1}), x_h(t_{n+1} - \tau))$$

$$+ \frac{h^\alpha}{\Gamma(\alpha + 2)} \sum_{j=0}^n a_{j,n+1} f(t_j, x_h(t_j), x_h(t_j - \tau)),$$  \hspace{1cm} (5)

and the predicted value $x_h^p(t_{n+1})$

$$x_h^p(t_{n+1}) = \sum_{k=0}^{[\alpha]-1} x_0^{(k)} \frac{t_{n+1}^k}{k!} + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^n b_{j,n+1} f(t_j, x_h(t_j), x_h(t_j - \tau)).$$  \hspace{1cm} (6)

Now the problem is how to evaluate $x_h(t_j - \tau)$. Assume that $\tau > h$, we adopt linear interpolation scheme, that is

$$x_h(t_j - \tau) = x_h(t_{j-d})$$

$$+ \frac{t_j - \tau - t_{j-d}}{t_{j-d+1} - t_{j-d}} (x_h(t_{j-d+1}) - x_h(t_{j-d})).$$  \hspace{1cm} (9)

for $d \leq j \leq n + 1$, where $d = [\tau/h]$. And for $0 \leq j < d$, it is obvious that the value of $x_h(t_j - \tau)$ is equal to the initial condition $x_0$. 
3. Problem formulation

Considering the following $n$-dimensional fractional-order chaotic system with time delay:

$$D^\alpha X(t) = f(X(t), X(t - \tau), X_0, \theta),$$  \hspace{1cm} (10)

where $X(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is the state vector with the initial state $X_0 = (x_{10}, x_{20}, \ldots, x_{n0})^T$, $\theta = (\theta_1, \theta_2, \ldots, \theta_m)^T$ is a set of original parameters, $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T$ represents the orders of fractional derivative. In this paper, the time delay $\tau$ and the orders $\alpha$ are also treated as parameters to be estimated. Suppose that the structure of system (10) is known, then the estimated system can be written as

$$D^\hat{\alpha} \hat{X}(t) = f(\hat{X}(t), \hat{X}(t - \hat{\tau}), \hat{X}_0, \hat{\theta}),$$ \hspace{1cm} (11)

where $\hat{X}(t) = (\hat{x}_1(t), \hat{x}_2(t), \ldots, \hat{x}_n(t))^T$ is the $n$-dimensional state vector of the estimated system with same initial state $\hat{X}_0$ as original system (10), $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_m)^T$ is the estimated parameters, $\hat{\alpha} = (\hat{\alpha}_1, \hat{\alpha}_2, \ldots, \hat{\alpha}_n)^T$ is the estimated fractional orders and $\hat{\tau}$ is the estimated time delay.

The objective of this paper is to estimate the parameters of system (10). To this end, a cost function is defined as

$$J(\alpha, \theta, \tau) = \sum_{k=1}^{N} \| X(k) - \hat{X}(k) \|^2,$$ \hspace{1cm} (12)

where $k = 1, 2, \ldots, N$ is the sampling time point and $N$ denotes the number of data points used for identification. $X(k)$ and $\hat{X}(k)$ represent the state vector of the original and estimated system at sampling time $t = t_0 + kh$ ($h$ is the sampling step), respectively. The optimal estimated parameters $((\alpha^*, \tau^*, \theta^*))$ of system (10) are those that minimize the objective function (12), that is,

$$(\alpha^*, \tau^*, \theta^*) = \arg \min_{(\alpha, \tau, \theta) \in \Gamma} J(\alpha, \theta, \tau),$$ \hspace{1cm} (13)

where $\Gamma$ is the search space admitted for orders $\alpha$, delay time $\tau$ and parameters $\theta$, which is determined empirically.

The principle of parameter identification for a fractional-order time-delay chaotic system can be described as in Figure 1.

4. Multi-selection differential evolution

4.1. Standard differential evolution

The DE algorithm is a kind of EAs introduced by Stron and Price (1995). It possesses the advantage of simple structure, ease of use and speed of convergence. In the original DE algorithm, there are three operators: mutation, crossover and selection, which can be briefly expressed as following equations

$$u_i(l + 1) = p_{1 i}(l) + F(p_{2 i}(l) - p_{3 i}(l)),$$ \hspace{1cm} (14)

crossover:

$$v_i(l + 1) = \begin{cases} u_i(l + 1) & \text{if rand} < \text{CR}, \\ p_i(l) & \text{else}, \end{cases}$$ \hspace{1cm} (15)

selection:

$$p_i(l + 1) = \begin{cases} v_i(l + 1) & \text{if fit}(v_i(l + 1)) < \text{fit}(p_i(l)), \\ p_i(l) & \text{else}, \end{cases}$$ \hspace{1cm} (16)

where $p_i(l) = [p_{1 i}(l), p_{2 i}(l), \ldots, p_{m i}(l)]$ denotes the $i$th individual at the $l$th generation for an $n$-dimensional optimization problem, $r_1$, $r_2$ and $r_3$ are integers randomly selected with that $i \neq r_1 \neq r_2 \neq r_3, F$ is the mutation factor ($F \in [0, 2]$), CR is the crossover rate ($CR \in [0, 1]$), fit(·) denotes the fitness of individual. Here, DE/rand/1/bin is used. For other schemes, refer to [22]. With the advantage of simple structure, less control parameters and fast convergence, DE is successfully used in optimization problems in many different practical applications such as system identification (Subudhi and Jena, 2009), chemical processes optimization (Badu and Angira, 2006) and structural optimization (Satoshi and Arakawa, 2011).

4.2. Multi-selection differential evolution

It is shown that the performance of DE is affected by many factors such as the control parameters $F$, CR and the evolving operators. In standard DE, selection operator is the final operator, it is a very important operator because it selects a better individual into next generation between the parent individual and trial individual generated by a crossover operator. However, if the trial individual generated by crossover operator is not better than its corresponding parent individual, DE loses a chance to improve the population. This shortcoming is raised by the fact that

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**Figure 1.** Block diagram of parameter identification.
the selection is only made between the parent individual and trial individual generated by crossover operators. To improve the performance of DE, it is reasonable to increase the number of candidates in the stage of selection. Motivated by this idea, an improved DE algorithm is proposed in this paper by adding an individual as a candidate in the selection stage.

In the stage of mutation, two mutated trial individuals are generated, one generated according to Equation (14), another is generated as follows:

\[ w_i(l+1) = p_i(l) + F(p_{\text{best}}(l) - p_{r4}(l)), \]  

(17)

where \( p_{\text{best}}(l) \) is the best individual up to the \( l \)th generation and \( r4 \) is different from \( l \), \( r1 \), \( r2 \) and \( r3 \). The trial individual \( w_i(l+1) \) does not undergo crossover operation, it is only used in the selection stage for generating better offspring. The new selection operator is defined as

\[ p_i(l+1) = \begin{cases} v_i(l+1) & \text{if fit}(v_i(l+1)) = \min, \\ w_i(l+1) & \text{if fit}(w_i(l+1)) = \min, \\ p_i(l) & \text{else}. \end{cases} \]  

(18)

The newly defined selection operator adds an individual to constitute candidates, thus, it increases chances that better offspring can be selected as next generation. On the other hand, the newly defined selection makes full use of the information of parent individual.

5. Numerical example

In this section, one typical fractional-order chaotic systems with time delay is used as a simulation example. The identification results using MS-DE are compared with DE in terms of accuracy, reliability and convergence speed. For DE algorithms, the choice of the control parameters \( F \) and \( CR \) has a significant impact on the algorithm performance. To make comparison more convincing, we choose two sets of control parameters: \( F = 0.5 \), \( CR = 0.9 \) and \( F = 0.6 \), \( CR = 0.7 \). The other control parameters of DE and MS-DE are set as the number of individuals \( M = 30 \) and the maximum generation number \( G = 400 \). Since DE is stochastic, each DE algorithm runs independently 20 times in each example.

Considering the following time-delayed fractional-order Mackey–Glass chaotic system:

\[ D^\alpha x(t) = \frac{-ax(t-r) + bx(t-\tau)}{1 + x(t-\tau)^{10}}, \]  

(19)

where \( \alpha, \tau, a \) and \( b \) are unknown parameters to be estimated. The parameters of original system are set as \( \alpha = 0.9, \tau = 10, a = -2, b = 1 \) and \( x_0 = 0.6 \). The chaotic behaviour of the system is shown in Figure 2. The search ranges of parameters are set as \( 0.01 \leq \alpha \leq 1.5, 5 \leq \tau \leq 15, -10 \leq a \leq 0 \) and \( 0 \leq b \leq 10 \).

In the identification process, a maximum generation number 400 is set as the stopping condition. The identification results are listed in Table 1. Figures 3 and 4 show the evolution process of each parameter under two different parameter settings.

It can be seen from Table 1 that the identification results of MS-DE are always better than those of DE. That is to say, MS-DE is more accurate than DE no matter what the control parameters \( CR \) and \( F \) are.

To evaluate the reliability of the algorithm, successful rate (SR) of the algorithm is defined. For each run, if the optimal objective function value is lower than \( 10^{-5} \), the algorithm is defined as successful in this run. SR is defined as the ratio of the successful times over the total runs.

| Parameter Identification Results | Best | Worst | Average |
|---------------------------------|------|-------|---------|
|                                 | DE   | MS-DE | DE      | MS-DE | DE        | MS-DE |
| \( j \)                         | 1.0083e-17 | 0.0000s | 3.1427  | 1.8919  | 0.2599    | 0.0946 |
| \( F = 0.5 \)                   |       |       |         |        |           |       |
| \( \alpha \)                    | 0.9000 | 0.9000 | 0.5851  | 0.5544  | 0.8638    | 0.8827 |
| \( \tau \)                      | 10.0000 | 10.0000 | 10.6000 | 10.5513 | 10.0711   | 10.0276 |
| \( CR = 0.9 \)                  |       |       |         |        |           |       |
| \( a \)                         | -2.0000 | -2.0000 | -10.0000 | -9.2480 | -2.6237   | -2.3624 |
| \( b \)                         | 1.0000 | 1.0000 | 5.1124  | 4.6882  | 1.4155    | 1.1844 |
| \( F = 0.6 \)                   |       |       |         |        |           |       |
| \( \alpha \)                    | 0.9000 | 0.9000 | 0.9021  | 0.9000  | 0.9001    | 0.9000 |
| \( \tau \)                      | 10.0000 | 10.0000 | 9.9794  | 10.0000 | 9.987     | 10.0000 |
| \( CR = 0.7 \)                  |       |       |         |        |           |       |
| \( a \)                         | -2.0000 | -2.0000 | -2.0000 | -2.0000 | -2.0000   | -2.0000 |
| \( b \)                         | 1.0000 | 1.0000 | 1.0018  | 1.0000  | 1.0001    | 1.0000 |
Figure 3. The evolution process of each parameter ($F = 0.9, CR = 0.5$).

Figure 4. The evolution process of each parameter ($F = 0.7, CR = 0.6$).
The SR of MS-DE is 95% under parameter setting $F = 0.5$, $CR = 0.9$ and 100% under parameter setting $F = 0.6$, $CR = 0.7$, while DE is 80% and 100%. So, MS-DE is more reliable than DE.

It can be seen from Figures 3 and 4 that the convergence speed of MS-DE is faster than DE. To further verify this, we record the elapsed time and the number of iterations when the objective function value is lower than $10^{-5}$. If the objective function is never lower than $10^{-5}$ within 400 iterations, then the algorithm stops. The detailed data are listed in Table 2. Obviously, the convergence speed of MS-DE is faster than DE.

### 6. Conclusions

This paper studies parameter identification issue of fractional-order chaotic system with time delay, which is rarely considered before. To solve the issue efficiently, an improved differential evolution algorithm, MS-DE algorithm, is proposed in this paper. Numerical simulations and comparisons with DE demonstrate the feasibility and superior performance of the proposed MS-DE method. The main contribution of this paper are: Firstly, a numerical algorithm for fractional differential equation with time delay is given; Secondly, time delay, fractional order and other parameters of fractional-order time-delayed chaotic system are identified together, which is rarely considered before; Thirdly, a new efficient DE method is proposed. The further work is to research real application of fractional-order chaotic system with time delay.

### Disclosure statement

No potential conflict of interest was reported by the authors.

### Funding

This work was supported by the National Natural Science Foundation of China [grant number 61273260], the Natural Science Foundation of Hebei Province [grant number F2015203362].

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### Table 2. Iterations and elapsed time of DE and MS-DE.

|                | Best      | Worst     | Average |
|----------------|-----------|-----------|---------|
|                | DE        | MS-DE     | DE      | MS-DE     | DE      | MS-DE     |
| $F = 0.5$      |Iterations | 69        | 36      | ×         | ×        | 88       | 48      |
| $CR = 0.9$     |Time (s)   | 40.87     | 30.87   | ×         | ×        | 51.91    | 40.49   |
| $F = 0.6$      |Iterations | 100       | 60      | 218       | 113      | 144      | 76      |
| $CR = 0.7$     |Time (s)   | 103.77    | 52.00   | 124.38    | 94.71    | 81.76    | 59.80   |

Note: × Donates the algorithm did not reach $10^{-5}$ within 400 iterations.
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