Electroweak Axion as the Fuzzy Dark Matter
— A Proposal for the Mixed Fuzzy and Cold Dark Matters —

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The electroweak axion is identified with the fuzzy dark matter of a mass $m \simeq 10^{-20} - 10^{-19}$ eV. The model predicts two components of dark matter, one is ultralight and the other is WIMP-like. The Chern-Simons-type interaction between the fuzzy dark matter and photon and the $B + L$ breaking proton decays are predicted.

I. INTRODUCTION

The presence of axion-like Nambu-Goldstone bosons is a quite natural prediction of string theories [1]. The QCD axion is one of them which couples to QCD instantons, solving the CP problem in QCD. This success encourages us to consider another axion which couples to weak $SU(2)$ instantons. The weak $SU(2)$ instantons generate a very small mass for the axion. We call it as the electroweak (EW) axion.

The EW axion was originally introduced to explain the observed extremely small cosmological constant by its potential energy density [2]. In this paper, we propose to identify the EW axion with the Fuzzy dark matter (DM) [3] instead of explaining the cosmological constant.

II. A BRIEF REVIEW ON THE ORIGINAL ELECTROWEAK AXION

In this section we give a brief review on the original EW axion introduced to explain the observed very small cosmological constant by the axion potential energy density [2]. The EW axion $A$ couples to the $SU(2)$ gauge fields as

$$\mathcal{L} \supset \frac{g_2^2}{32\pi^2} A^\mu W_\mu^{i\nu} \tilde{W}^{i\nu},$$

where $W_\mu^i$ (with $i = 1, 2, 3$) is the weak $SU(2)$ gauge field strength tensor, $\tilde{W}^{i\mu}$ its dual tensor and $g_2$ is the weak $SU(2)$ gauge coupling constant and $F_A$ is the decay constant of the EW axion. Here, we explain why the EW axion can explain the observed cosmological constant by its potential energy $^1$.

We have 18 fermion zero modes around an instanton as shown in Fig. 1 and we have to contract all fermion zero modes by higher dimensional operators to generate the axion potential [2]. In particular, the higher dimensional operators necessarily contain the $B + L$ breaking operators, which generate too fast proton decays [6, 7]. We need some flavor symmetry acting on the quarks and leptons to suppress such fast proton decays, but on the other hand, the flavor symmetry must be broken to generate the observed mass matrices of quarks and leptons. We adopt, in this section, the global Froggatt-Nielsen $U(1)_{FN}$ flavor symmetry [8]. We introduce a symmetry breaking spurion $\epsilon \simeq 1/17$ [9] to generate the realistic mass matrices for quarks and leptons.

With this spurion parameter $\epsilon$ the instanton calculus gives us the axion potential as [2]

$$V_A = \frac{A_A^4}{2} \left[ 1 - \cos \left( \frac{A}{F_A} \right) \right],$$

with

$$A_A^4 \simeq 2 e^{-\frac{2\pi^2 m_{3/2}^2}{M_{Pl}}} \left( \frac{1}{\pi^2} \right)^4 \epsilon^{10} m_{3/2}^3 M_{Pl}$$

$$\simeq (1.4 \times 10^{-3} \text{ eV})^4 \epsilon \left( \frac{1}{\pi^2} \right)^4 \left( \frac{\epsilon}{1/17} \right)^{10} \left( \frac{m_{3/2}}{1 \text{ TeV}} \right)^3,$$

where $c$ is a dimensionless constant of $O(1)$, $m_{3/2}$ is the gravitino mass, and $\alpha_2(M_{Pl})$ is the weak $SU(2)$ gauge

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$^1$ It is also shown to be able to explain the recently observed cosmic birefringence [4, 3].
coupling constant at the Planck scale \(^2\). An example of
the diagrams generating the above EW axion potential is
also shown in Fig. I. One might think that the instanton
contributions are very much suppressed by loop factors.
However, the meaning of loops is not trivial in the
instanton calculus. The correct \(\pi^2\) factor is given in the
appendix in \([2]\), which gives us the factor \((\frac{\pi^2}{17})^4\) in Eq. (3) in
the present case. However, there are many contributions
from all instanton diagrams and ambiguities coming from
the effective coupling constants of the higher dimensional
operators and the effective cutoff scale of the instanton-
size integrations. The constant \(c\) represents such am-
biguities. We take \(c = O(1)\) as a representative value
throughout this paper.

The above EW axion potential Eq. (2) generates the
correct observed dark energy at around the hilltop of the
\(\cosin\) potential in Eq. (2) for \(c = O(1)\) and \(m_{3/2} =
O(10)\) TeV. Here, we have considered that the Planck
scale \(M_{\text{Pl}} \simeq 2.4 \times 10^{18}\) GeV is the cut-off scale of the
theory as explained in \([4]\). From the axion potential Eq. (2)
we obtain the EW axion mass \(m_A\) around the potential minimum as

\[
m_A = \frac{A^3}{\sqrt{2} F_A} \simeq 6 \times 10^{-34} \text{eV}
\]

\[
\times \left( \frac{1}{\pi^2} \right)^2 \left( \frac{\epsilon}{1/17} \right)^5 \left( \frac{m_{3/2}}{1 \text{ TeV}} \right)^{3/2} \left( \frac{M_{\text{Pl}}}{F_A} \right),
\]

(4)

taking \(c = 1\).

III. ELECTROWEAK AXION POTENTIAL
WITH THE ANOMALY-FREE DISCRETE
FROGGATT-NIELSEN SYMMETRY

In the previous section, we have assumed the Froggatt-
Nielsen \(U(1)_{\text{FN}}\) flavor symmetry. In this section, how-
ever, we consider the anomaly-free flavor symmetry,
that is, the Froggatt-Nielsen discrete \(Z_{10}\) symmetry \([10]\).
There is a big difference in the axion potential from the
estimation in the original paper since there is no sup-
pression factor, \(\epsilon^{10}\). Notice that the dangerous \(B + L\)
breaking operators for proton decays are still suppressed
by powers of the \(\epsilon\) as in the case of the \(U(1)_{\text{FN}}\). An
example of the diagrams generating the EW axion potential is
shown in Fig. 2.

Now, we obtain the EW axion mass as

\[
m_A \simeq 1.2 \times 10^{-21} \sqrt{\epsilon} \left( \frac{1}{\pi^2} \right)^{5/2} \left( \frac{m_{3/2}}{1 \text{ TeV}} \right)^{3/2} \left( \frac{M_{\text{Pl}}}{F_A} \right) \text{ eV}.
\]

(5)

Notice that the \(\frac{1}{\pi^2}\) term becomes \((\frac{\pi^2}{17})^5\) in the axion
potential because of the exchange of a new massive particle

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\(^2\) This result does not change even if some \(SU(2)\) charged
particles exist at the intermediate energy scale owing to the SUSY
miracle \([2]\).

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\(^3\) If we take the decay constant \(F_A \simeq 10^6 - 10^9\) GeV we
may explain the recently reported high energy photon from
GRB221009A \([13]\).
IV. THE FUZZY DARK MATTER

The Fuzzy DM of mass $10^{-20} - 10^{-19}$ eV [17, 18] is very attractive, since we may naively understand the size of galaxies by its de Broglie wavelength. Furthermore, it may not have small-scale problems including the cuspy-core problem. Interestingly, the required initial value of the Fuzzy DM field to explain the DM density by its coherent oscillation is close to the string inspired decay constant $F_A \approx 10^{16}$ GeV for the axion as shown below. Such a coincidence [11] inspired many people to investigate the dynamics and astrophysical consequences of the Fuzzy DM. However, the origin of the mass was not known. We consider in this paper that the Fuzzy axion DM mass is provided by the weak SU(3) electroweak (EM) field after the EW symmetry breaking, and in fact, we show that in the previous section the correct mass $10^{-20} - 10^{-19}$ eV is generated by the weak SU(2) instantons with the decay constant $F_A \approx 10^{16}$ GeV.

The abundance of Fuzzy DM is obtained from the misalignment mechanism, by considering that such ultralight boson form a condensate state $A(t)$ in the background, whose cosmic evolution follows the equation,

$$ \ddot{A} + 3H \dot{A} + \frac{\partial V_A}{\partial A} = 0, \quad (9) $$

where $H(t)$ is the Hubble constant of the universe and $V_A$ is the potential in Eq. (2). Here, notice that the leading contribution from the potential is the mass term, $\partial V_A/\partial A \approx m_A^2 A$. When $H \gtrsim m_A$, the axion state $A(t)$ is frozen and behaves like dark energy. After $H \lesssim m_A$, the axion $A$ starts to oscillate around its VEV with an exponentially decaying amplitude. Then, its abundance could be calculated as [12]

$$ \Omega_A \approx 0.08 \left( \frac{m_A}{10^{-20} \text{eV}} \right)^{1/2} \left( \frac{F_A \theta_0}{10^{16} \text{GeV}} \right)^2, \quad (10) $$

where $\theta_0 = A_0/F_A$ labels the initial misalignment angle and $A_0$ is the initial field value. If one takes $\theta_0 \approx 1$, the DM density $\Omega_{CDM} h^2 \approx 0.12$ [19] could be fitted by $\Omega_A$ for $m_A \approx 10^{-19}$ eV and $F_A \approx 10^{16}$ GeV.

From the coupling (1), one would obtain a Chern-Simons(CS)-type coupling of $A$ with electromagnetic (EM) field after the EW symmetry breaking,

$$ \mathcal{L} \supset \frac{e^2}{32 \pi^2} A F_{\mu \nu} \tilde{F}^{\mu \nu}, \quad (11) $$

where $F_{\mu \nu}$ is the electromagnetic field tensor and $\tilde{F}^{\mu \nu}$ is its dual. $e$ is the $U(1)_Y$ gauge coupling constant defined as $e = g_3 \sin \theta_W$ and $\theta_W$ is the EW mixing angle [4].

Note that the existence of the intergalactic magnetic field may have backreactions to the Fuzzy DM due to Eq. (11). The cosmic axion field will produce magnetic helicity during cosmic evolution, which shall in turn affect the axion dynamics. As shown in Ref. [20], the magnetic helicity acts like a dissipation channel during the cosmic evolution of Fuzzy DM. This is described by letting $3H \rightarrow 3H + \epsilon$ in Eq. (9), where $\epsilon/H \sim 10^{-20} b^2 T/\text{GeV}$ for $F_A \approx 10^{16}$ GeV and $b$ [20] is the parameter describing the initial magnetic field. For high temperatures, $A$ behaves like dark energy in the early stage, the backreaction from the magnetic field does not change the evolution of the axion. When temperature drop below 1 GeV, even for a strong magnetic field, $b \sim O(1)$, the ratio $\epsilon/H$ is extremely small, which means that the $\epsilon$ is just a small correction to Hubble friction during the cosmic evolution of $A(t)$. This indicates that Eq. (9) is valid for the axion’s cosmological evolution even in the presence of strong magnetic fields.

V. DISCUSSION AND CONCLUSIONS

In this paper, we have shown that the EW axion acquires the mass of the order $10^{-20} - 10^{-19}$ eV through the electroweak instantons with the string inspired decay constant $F_A \approx 10^{16}$ GeV. Therefore, it is natural to consider that the EW axion is nothing but the Fuzzy DM. The correct DM density is also obtained with the same decay constant $F_A \approx 10^{16}$ GeV for the EW axion as shown in the section IV.

Eq. (11) is a prediction of our model, which encourages observational searches for the coupling. The CS-type coupling would induce phase rotations for two helicity modes of EM waves, which would lead to a rotation on polarization angle for a linear polarized EM wave propagating through the Fuzzy DM. Different from isotropic cosmic birefringence in the CMB, this birefringence effect is local, which could be probed in the recently proposed Pulsar Polarization Arrays [21].

If the $B + L$ symmetry is exact, the rotation of the $B + L$ shifts the EW axion field, and hence the axion is massless as pointed out in section II. Thus, if the EW axion is indeed the Fuzzy DM, the $B + L$ must be broken and the proton must decay through the dimension five operators [6, 7]. This is also a prediction of our model. As investigated in Ref. [22], the proton decays are predicted within the reach of JUNO and Hyper-Kamiokande.

As we have stressed in section III the SUSY plays a crucial role in generating the EW axion mass of the order $10^{-20}$ eV. Since the SUSY standard model has a DM candidate, it is natural to have a mixed DM in the present scenario. However, the SUSY DM density depends on details of the SUSY breaking model, and here we should take the ratio of the each DM density to be a free parameter, which can be investigated in astrophysical analysis and/or cosmology. In this case, the EW axion should contribute partially to the DM density, from which one could obtain a bound in parameter space by letting $\Omega_A < \Omega_{DM}$.

As long as $m_{3/2} \geq 10 - 100$ GeV we have no gravitino

\footnote{The coupling constant in Eq. (11) may have an $O(1)$ modification since there could be CS-type coupling between the axion $A$ and gauge field tensors of hyper-charge $U(1)_Y$.}
problem [14] and as long as the axino and the scalar partner of the axion (called saxion) are heavier than the lightest SUSY particle, they decay before the BBN. However, we need special care about the saxion, since its coherently oscillation may dominate the energy density of the early universe if the initial value of the saxion is $O(F_A)$, and its decay produces a large entropy in the late time. This late-time entropy production dilutes the preexisting baryon number in the universe. However, this problem can be easily solved by imposing a “parity” as the axion chiral multiplet $\bar{A} \rightarrow -\bar{A}$ and $W^{i\mu\nu}_\mu W_{\mu\nu} \rightarrow -W^{i\mu\nu}_\mu W_{\mu\nu}$.

Then the saxion field value is naturally set vanishing during the inflation and its coherent oscillation never occurs. The other solution is given by assuming the coupling between the axion multiplet and inflaton is relatively stronger than usual Planck suppressed operators (called the adiabatic solution) [23, 24].

We have another cosmological problem, that is, the isocurvature problem. Since the EW axion is massless during the inflation the axion has quantum fluctuations $\delta A \simeq H_{\text{inf}}/2\pi$ which causes too much isocurvature fluctuations if the inflation scale $H_{\text{inf}}$ is too large. We obtain a constraint as $H_{\text{inf}} < 10^{14}$ GeV [25] from the Planck data [26]. However, this constraint is not necessarily serious, since we have many consistent inflation models satisfying this condition [27–29]. However, if the tensor mode $r$ in CMB is observed as $r > 10^{-7}$, our model will be excluded [30, 31].

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