DETECTION OF SHIP WAKES IN SAR IMAGERY USING CAUCHY REGULARISATION

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Background
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• SAR image formation model:
\[ Y = CX + N. \]
where \( C = R^{-1} \) represents the inverse Radon transform.

\[ p(X|Y) = \frac{p(Y|X)p(X)}{\int p(Y|X)p(X)dX} \]

• the unnormalised posterior
\[ p(X|Y) \propto p(Y|X)p(X). \]
Using maximum a-posterior (MAP) estimator in optimization algorithms:

\[ \hat{X}_{MAP} = \arg \max_X p(X|Y) = \arg \min_X F(X) \]

where \( F(X) \) is denoted as the cost function.

\[ F(X) \propto f(x) + g(x) \]

\[
\begin{align*}
  f(x) &= \|Y - CX\|_2^2 \\
  g(x) &= -\log(p(X))
\end{align*}
\]
Methodology

- Probability density function of Cauchy distribution:

\[ p(X) \propto \frac{\gamma}{\gamma^2 + X^2} \]

- The minimization with Cauchy regularization:

\[ \hat{X}_{\text{Cauchy}} = \arg \min_{x} \|Y - CX\|_2^2 - \sum_{i,j} \log \left( \frac{\gamma}{\gamma^2 + X_{i,j}^2} \right) \]

Moreau-Yoshida unadjusted Langevin algorithm (MYULA)
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**Algorithm I** MYULA for Cauchy regularized cost function

Input: SAR image \( Y, \gamma \in [0.0001,0.1] \)

Output: Radon image \( X \)

Set: \( \delta = 1/25L, \omega = 1/4L \)

do

\[
Z^{(i+1)} \sim N(0, \mathbb{I}_d)
\]

\[
X^{(i+1)} = \left(1 - \frac{\delta}{\omega}\right)X^{(i)} - \delta \nabla f(X^{(i)}) + \frac{\delta}{\omega} \text{prox}_g^\omega(X^{(i)}) + \sqrt{2\delta}Z^{(i+1)}
\]

while \( \epsilon^{(i)} > 10^{-3} \) or \( i < MaxIter \)
Cauchy proximal operator:

\[
prox^\omega_g (x) = \arg \min_u \left[ -\log \left( \frac{\gamma}{\gamma^2 + u^2} \right) + \frac{\|u - x\|^2}{2\omega} \right]
\]

By using Cardano’s method:

\[
prox^\omega_g (x) = \frac{x}{3} + s + t
\]

\[
s = \sqrt[3]{\frac{q}{2} + \Delta}, \quad t = \sqrt[3]{\frac{p}{2} - \Delta}, \quad \Delta = \sqrt{\frac{p^3}{27} + \frac{q^2}{4}}
\]

\[
p = \gamma^2 + 2\omega - \frac{x^2}{3}
\]

\[
q = xy^2 + \frac{2x^3}{27} - \frac{x}{3}(\gamma^2 + 2\omega)
\]
Methodology
Methodology

- The confirmation of the candidate:

\[ F_l = \frac{\bar{I}_w}{\bar{I}} - 1. \]

where \( \bar{I}_w \) is the mean value over the un-confirmed wake, and \( \bar{I} \) is the mean intensity of the image.

\[
\begin{align*}
F_l &< 0 \text{ for turbulent wakes, } \\
F_l &> 0.1 \text{ for narrow-V and Kelvin wakes}
\end{align*}
\]
### Table 1. Visible wakes in used image dataset *

| Image   | Turbulent | 1<sup>st</sup> Narrow | 2<sup>nd</sup> Narrow | 1<sup>st</sup> Kelvin | 2<sup>nd</sup> Kelvin |
|---------|-----------|-------------------------|------------------------|-----------------------|-----------------------|
| CSM_1   | 1         | 1                       | 0                      | 0                     | 0                     |
| CSM_2   | 1         | 1                       | 0                      | 0                     | 0                     |
| CSM_3   | 1         | 1                       | 0                      | 1                     | 0                     |
| CSM_4   | 1         | 1                       | 0                      | 1                     | 0                     |
| CSM_5   | 1         | 1                       | 0                      | 0                     | 0                     |
| CSM_6   | 1         | 1                       | 0                      | 0                     | 0                     |

* 1 means visible and 0 represents invisible
### Table 2. Detection results over 6 COSMO-SkyMed images

| Method      | TP   | TN  | FP  | FN  | %Accuracy |
|-------------|------|-----|-----|-----|-----------|
| Cauchy      | 40.0%| 46.7%| 6.7%| 6.7%| 86.7%     |
| GMC         | 36.7%| 40%  | 20% | 3.3%| 76.7%     |
| Graziano    | 33.3%| 36.7%| 16.7%| 13.3%| 70.0%     |
CMS_3

CMS_4

CMS_5

* Yellow: Turbulent wake
Green: Narrow-V wake
Red: Kelvin wake
Summary

• The use of Cauchy distribution in ship wake detection problem.
• Realization of MYULA in image reconstruction from SAR imagery.
• Implementation of proximal Cauchy operator in solving inverse problem.
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Thank You!

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