Masses and Mixing of $cqqq$ Tetraquarks
Using Glozman-Riska Hyperfine Interaction

V. Borka Jovanović
Laboratory of Physics (010), Vinča Institute of Nuclear Sciences, P.O. Box 522
11001 Belgrade, Serbia
(Dated: February 5, 2008)

In this paper we perform a detailed study of the masses and mixing of the single charmed scalar tetraquarks: $cqqq$. We also give a systematic analysis of these tetraquark states by weight diagrams, quantum numbers and flavor wave functions. Tetraquark masses are calculated using four different fits. The following SU(3)$_F$ representations are discussed: $T_5$, $3_S$, $6_A$ and $3_A$. We use the flavor-spin Glozman-Riska interaction Hamiltonian with SU(3) flavor symmetry breaking. There are 27 different tetraquarks composed of a charm quark $c$ and of the three light flavors $u, d, s$: 11 cryptoexotic (3 $D_{1}^{+}, 4 D^{+}, 4 D^{0}$) and 16 explicit exotic states. We discuss $D_0$ and its isospin partners in the same multiplet, as well as all the other four-quark states. Some explicit exotic states appear in the spectrum with the same masses as $D_{1}^{+}(2632)$ in $T_5$ and with the same masses as $D_{1}^{+}(2317)$ in $6_A$ representation, which confirm the tetraquark nature of these states.

PACS numbers: 12.38.-x, 12.39.Pn, 12.40.Yx, 14.65.Bt
Keywords: phenomenological quark models; potential models; hadron mass models and calculations; light quarks.

I. INTRODUCTION

The meson $D_{1}^{+}(2317)$, discovered in 2003, year in high energy electron-positron collisions at SLAC (Stanford Linear Accelerator Center) by the BABAR group [1] and confirmed by BELLE experiments [2], possesses a mass of 2317 MeV, some 170 MeV lighter than expected, at least according to prevalent theories of quark interactions. Hence physicists need a new explanation of how a charm quark attached to an antistrange quark should have this particular mass. In general, $D_s$ and $D$ mesons are a class of particles, each consisting of a charm quark attached to a light antiquark. The BABAR detection group at SLAC [1] responsible for the experimental discovery suggests that the $D_{1}^{+}(2317)$ might be a novel particle made of four quarks. Meson $D_{0}^{0}(2308)$ was discovered by BELLE group [3]. The mass difference between strange $D_{1}^{+}(2317)$ and nonstrange $D_{0}^{0}(2308)$ meson (9 MeV) is at least ten times below the expected value of $m_c - m_u$ mass difference. Also experimentally state $D_{1}^{+}(2632)$ (discovered in SELEX experiments [4]) does not fit into former theoretical predictions because it is too light to be an (radial) excitation of the $D_{1}^{+}(2317)$.

Jaffe [5] suggested the possible existence of four-quark states for light flavor dimesons and made predictions for tetraquark spectroscopy. In Ref. [6] it is also provided a framework for a quark-model classification of the many two-quark-two-antiquark states.

In Refs. [2, 3, 4] the $D_{1}^{+}(2317)$ meson is explained as a scalar $c\bar{s}$ system: van Beveren et al. claim that in their model, assuming that the meson is indeed a charm-antistrange combination, the mass comes out in the right range if the strong-nuclear-force interactions responsible for the creation and annihilation of extra quark-antiquark pairs are taken into account. According to van Beveren and Rupp [10] and Barnes et al. [11], the $D_{1}^{+}(2632)$ resonance, being 0.52 GeV heavier than the $D_{0}$ ground state, could turn out to be the first radial excitation of the $D_{1}(2112)$ meson. On the other hand, Terasaki and Hayashigaki [12, 13, 14] have assigned the $D_{1}^{+}(2317)$ to the $T_{3} = 0$ member of the isoscalar which belong to the lighter class of four-quark $cqqq$ mesons and have investigated the decay rates of the members of the same multiplet. Also in Refs. [15, 16, 17] it is shown that it might be expected that the measured $D_{1}^{+}(2317)$ is an exotic state with the structures of a four-quark. Liu et al. [18] argue that the $D_{1}^{+}(2632)$ resonance may be a member of a scalar tetraquark multiplet. The possible tetraquark nature of the three mentioned mesons is discussed e.g. in Refs. [15, 20, 21, 22, 23].

The three charmed scalar mesons: $D_{1}^{+}(2317)$, $D_{0}^{0}(2308)$ and $D_{1}^{+}(2632)$ does not fit well into predictions of the quark model because of these three reasons:

(i) absolute mass of the $D_{1}^{+}(2317)$ is 170 MeV below mass predicted from the quark model for the scalar $c\bar{s}$ meson,
(ii) the small mass gap between $D_{1}^{+}(2317)$ and $D_{0}^{0}(2308)$ is puzzling and leads to a new model for these states.
(iii) the state $D_{1}^{+}(2632)$ does not fit into former theoretical predictions because it is too light to be an (radial) excitation of the $D_{1}^{+}(2317)$.

These three dissimilarities influenced giving some theoretical proposals about the possible structure of the mesons $D_{1}^{+}(2317)$, $D_{0}^{0}(2308)$ and $D_{1}^{+}(2632)$. According to this, we analyze the possibility that these three states (or some of them) are tetraquarks.

In this work we perform a schematic study of the mass

*Electronic address: vborka@vin.bg.ac.yu*
splitting of the single charmed $cqar{q}$ tetraquarks in the SU(3) flavor representations. In Section II we construct the wave functions of mentioned tetraquarks. Then we present the flavor-spin Glozman-Riska interaction Hamiltonian. The formalism of calculating SU(3) flavor symmetry breaking corrections to the flavor-spin interaction energy is presented in Section III. Also it discusses meson and baryon fit and numerical analysis. The light and heavy meson and baryon experimental masses are fitted with aim to calculate the constituent quark masses and then to calculate tetraquark masses from our theoretical model. We discuss masses with Glozman-Riska (GR) hyperfine interaction (HFI). Equations that correspond to our theoretically predicted masses are given for all 27 $cqar{q}ar{q}$ states, as well as their numerical values. The quark model of confinement cannot reproduce the spin-dependent hyperfine splitting in the hadron spectra without additional contributions from a hyperfine interaction. That is why we take into account GR hyperfine interaction. We include mass mixing effects for particles with the same quantum numbers and show it in mass spectra. The last section is a short summary.

II. ANALYSIS AND METHOD

Tetraquarks with charm quantum number $C = 1$ and with three light flavors are grouped by the same properties, into multiplets with the same baryon number, spin and intrinsic parity. If a particle belongs to a given multiplet, all of its isospin partners (the same isotopic spin and intrinsic parity). If a particle belongs to a given multiplet, all of its isospin partners (the same isotopic spin and intrinsic parity). They also have similar masses.

Weight diagrams which represent the following product: $3 \otimes (3 \otimes 3) = 3 \otimes (6 + 3) = (3 \otimes 6) + (3 \otimes 3) = (15S + 3A) + (6A + 3A)$ are given in Figure 2. The subscripts $S$ and $A$ on the multiplets indicate that the flavor states are symmetric ($15S$ and $3S$) or antisymmetric ($6A$ and $3A$) under interchange of the last two antiquarks.

Knowing quantum numbers for the set of 27 scalar tetraquarks, they are classified in groups as shown in Figure 3. We denote the states with strangeness $S = 2$ as $\Xi$, with $S = 1$ as $\Sigma_a$ ($T = 1$) and $D_s$ ($T = 0$), with $S = 0$ as $\Delta$ ($T = 3/2)$ and $D$ ($T = 1/2$) and with $S = -1$ as $\Sigma$.

We plot the eigenvalues of $T_3$ and Y that occur for the quarks in a representation as points in the $T_3 - Y$ plane. We first combine two of the antiquarks. The quantum numbers $Y$ and $T_3$ are additive and thus their values for a $qq$ state are obtained by simply adding the values for $\bar{q}$ and $\bar{q}$. The points in the weight diagram for the $3 \otimes 3$ representation are thus obtained by taking every point of one antiquark diagram to be the origin of another antiquark diagram. Figure 2 shows that the nine $q\bar{q}$ combinations arrange themselves into two SU(3) multiplets, where the 3 is symmetric and the 6 is antisymmetric under interchange of the two antiquarks. Then we add the third quark triplet. The final decomposition is displayed in Figures 3 and 4. The subscripts $S$ and $A$ on the multiplets indicate that the flavor states are symmetric ($15S$ and $3S$) or antisymmetric ($6A$ and $3A$) under interchange of the last two antiquarks.

Knowing quantum numbers for the set of 27 scalar tetraquarks, they are classified in groups as shown in Figure 3. We denote the states with strangeness $S = 2$ as $\Xi$, with $S = 1$ as $\Sigma_a$ ($T = 1$) and $D_s$ ($T = 0$), with $S = 0$ as $\Delta$ ($T = 3/2)$ and $D$ ($T = 1/2$) and with $S = -1$ as $\Sigma$.

We plot the eigenvalues of $T_3$ and Y that occur for the quarks in a representation as points in the $T_3 - Y$ plane. We first combine two of the antiquarks. The quantum numbers $Y$ and $T_3$ are additive and thus their values for a $qq$ state are obtained by simply adding the values for $\bar{q}$ and $\bar{q}$. The points in the weight diagram for the $3 \otimes 3$ representation are thus obtained by taking every point of one antiquark diagram to be the origin of another antiquark diagram. Figure 2 shows that the nine $q\bar{q}$ combinations arrange themselves into two SU(3) multiplets, where the 3 is symmetric and the 6 is antisymmetric under interchange of the two antiquarks. Then we add the third quark triplet. The final decomposition is displayed in Figures 3 and 4. The subscripts $S$ and $A$ on the multiplets indicate that the flavor states are symmetric ($15S$ and $3S$) or antisymmetric ($6A$ and $3A$) under interchange of the last two antiquarks.

Knowing quantum numbers for the set of 27 scalar tetraquarks, they are classified in groups as shown in Figure 3. We denote the states with strangeness $S = 2$ as $\Xi$, with $S = 1$ as $\Sigma_a$ ($T = 1$) and $D_s$ ($T = 0$), with $S = 0$ as $\Delta$ ($T = 3/2)$ and $D$ ($T = 1/2$) and with $S = -1$ as $\Sigma$.

We plot the eigenvalues of $T_3$ and Y that occur for the quarks in a representation as points in the $T_3 - Y$ plane. We first combine two of the antiquarks. The quantum numbers $Y$ and $T_3$ are additive and thus their values for a $qq$ state are obtained by simply adding the values for $\bar{q}$ and $\bar{q}$. The points in the weight diagram for the $3 \otimes 3$ representation are thus obtained by taking every point of one antiquark diagram to be the origin of another antiquark diagram. Figure 2 shows that the nine $q\bar{q}$ combinations arrange themselves into two SU(3) multiplets, where the 3 is symmetric and the 6 is antisymmetric under interchange of the two antiquarks. Then we add the third quark triplet. The final decomposition is displayed in Figures 3 and 4. The subscripts $S$ and $A$ on the multiplets indicate that the flavor states are symmetric ($15S$ and $3S$) or antisymmetric ($6A$ and $3A$) under interchange of the last two antiquarks.

Knowing quantum numbers for the set of 27 scalar tetraquarks, they are classified in groups as shown in Figure 3. We denote the states with strangeness $S = 2$ as $\Xi$, with $S = 1$ as $\Sigma_a$ ($T = 1$) and $D_s$ ($T = 0$), with $S = 0$ as $\Delta$ ($T = 3/2)$ and $D$ ($T = 1/2$) and with $S = -1$ as $\Sigma$.

We plot the eigenvalues of $T_3$ and Y that occur for the quarks in a representation as points in the $T_3 - Y$ plane. We first combine two of the antiquarks. The quantum numbers $Y$ and $T_3$ are additive and thus their values for a $qq$ state are obtained by simply adding the values for $\bar{q}$ and $\bar{q}$. The points in the weight diagram for the $3 \otimes 3$ representation are thus obtained by taking every point of one antiquark diagram to be the origin of another antiquark diagram. Figure 2 shows that the nine $q\bar{q}$ combinations arrange themselves into two SU(3) multiplets, where the 3 is symmetric and the 6 is antisymmetric under interchange of the two antiquarks. Then we add the third quark triplet. The final decomposition is displayed in Figures 3 and 4. The subscripts $S$ and $A$ on the multiplets indicate that the flavor states are symmetric ($15S$ and $3S$) or antisymmetric ($6A$ and $3A$) under interchange of the last two antiquarks.
\[ 3 \otimes \overline{3} \otimes \overline{3} = \square \otimes (\square \otimes \square) = \square \otimes (\square + \square) = \square \otimes \square + \square \]

\[ = (\square + \square) + (\square + \square) = (\square + \square) + (\square + \square) \]

FIG. 1: Young diagrams for SU(3)_F multiplets according to \( 3 \otimes \overline{3} \otimes \overline{3} = (\overline{15}_S + \overline{3}_S) + (\overline{3}_A + 6_A) \). Tetraquarks with quark content \( c\bar{q}\bar{q} \) form four multiplets: two anti-triplets, one anti-15-plet and one sextet.

FIG. 2: Weight diagrams for the product \( \overline{3} \otimes \overline{3} = \bar{6} + 3 \). The ordinate shows hypercharge \( Y \) and abscissa 3-component \( T_3 \) of isotopic spin magnitude.

FIG. 3: The same as in Figure 2 but for the product \( 3 \otimes \bar{6} = \overline{15}_S + \overline{3}_S \).

FIG. 4: The same as in Figure 2 but for the product \( 3 \otimes 3 = 6_A + \overline{3}_A \).
Using the obtained flavor wave functions of scalar $cq\bar{q}\bar{q}$ tetraquarks (see Table II), the tetraquark masses $m_{\nu,0}$ without influence of GR HFI are determined. GR HFI contributions to the tetraquark masses $m_{\nu,GR}$ are calculated according to the relation (4) and the total tetraquark masses by relation (5). The corresponding results are given in Table III.

The $\chi^2$ fit of hadron masses is used to determine masses of constituent quarks. We performed mass fit for: light mesons ($\pi$, $K$, $\eta$, $\eta'$, $\rho$, $K^*$, $\omega$, $\varphi$), heavy mesons ($D^+$, $D^0$, $D^{*0}$, $D^{*+}$, $D_s^+$, $D_s^{*+}$, $\eta_c$, $J/\psi$), light baryons ($N$, $\Sigma$, $\Xi$, $\Lambda$, $\Delta$, $\Sigma^*$, $\Xi^*$, $\Omega$) and heavy baryons ($\Sigma_c$, $\Xi_c^+$, $\Xi_c^0$, $\Lambda_c$, $\Sigma_c^*$, $\Omega_c$). Applying the Hamiltonian (3) to the constituent quarks, we obtained the theoretical meson and baryon masses with the GR contribution included. Consequently, we have the set of equations (6)–(13) for theoretical masses of light pseudoscalar mesons, light vector mesons, charmed mesons, strange charmed mesons, double charmed mesons, light baryons - octet, light baryons - decuplet and heavy baryons, respectively. The corresponding experimental masses, taken from "Particle Data Group" site: http://pdg.lbl.gov [25], are appended to the right side of each equation.
| multiplet | tetraquark label | quark content | electrical charge | isospin projection | isospin strangeness |
|-----------|----------------|---------------|------------------|-------------------|---------------------|
| Ξ^++ | cuśś | 2 | 1/2 | 1/2 | 2 |
| Ξ^+ | cdśś | 1 | -1/2 | 1/2 | 2 |
| Σ^++ | cuđś | 2 | 1 | 1 |
| Σ^+ | cqqś | 1 | 0 | 1 |
| Σ^0 | cdăś | 0 | -1 | 1 |
| D^+ | cqqś | 1 | 0 | 0 |
| Δ^++ | cudđ | 2 | 3/2 | 3/2 | 0 |
| Δ^+ | cqqđ | 1 | 1/2 | 3/2 | 0 |
| Δ^0 | cqqū | 0 | -1/2 | 3/2 | 0 |
| Δ^- | cdăū | -1 | -3/2 | 3/2 | 0 |
| D^+ | cqqđ | 1 | 1/2 | 1/2 | 0 |
| D^0 | cqqū | 0 | -1/2 | 1/2 | 0 |
| Σ^+ | csdd | 1 | 1 | 1 |
| Σ^0 | csūđ | 0 | 0 | 1 |
| Σ^- | csīū | -1 | -1 | 1 |

| multiplet | tetraquark label | quark content | electrical charge | isospin projection | isospin strangeness |
|-----------|----------------|---------------|------------------|-------------------|---------------------|
| D^+ | cqqś | 1 | 0 | 0 | 1 |
| D^+ | cqqđ | 1 | 1/2 | 1/2 | 0 |
| D^0 | cqqū | 0 | -1/2 | 1/2 | 0 |

| multiplet | tetraquark label | quark content | electrical charge | isospin projection | isospin strangeness |
|-----------|----------------|---------------|------------------|-------------------|---------------------|
| Ξ^++ | cuśś | 2 | 1/2 | 1/2 | 2 |
| Ξ^+ | cdśś | 1 | -1/2 | 1/2 | 2 |
| Σ^++ | cuđś | 2 | 1 | 1 |
| Σ^+ | cqqś | 1 | 0 | 1 |
| Σ^0 | cdăś | 0 | -1 | 1 |
| D^+ | cqqś | 1 | 0 | 0 |
| Δ^++ | cudđ | 2 | 3/2 | 3/2 | 0 |
| Δ^+ | cqqđ | 1 | 1/2 | 3/2 | 0 |
| Δ^0 | cqqū | 0 | -1/2 | 3/2 | 0 |
| Δ^- | cdăū | -1 | -3/2 | 3/2 | 0 |
| D^+ | cqqđ | 1 | 1/2 | 1/2 | 0 |
| D^0 | cqqū | 0 | -1/2 | 1/2 | 0 |
| Σ^+ | csdd | 1 | 1 | 1 |
| Σ^0 | csūđ | 0 | 0 | 1 |
| Σ^- | csīū | -1 | -1 | 1 |

\[
\begin{align*}
(m_{\Xi}^+) &= 2m_u - 2C_x \frac{1}{m_c^2} = 140 \text{ MeV} \\
(m_K^+) &= m_u + m_s - 2C_x \frac{1}{m_c^2} = 494 \text{ MeV} \\
(m_{\eta}^+) &= 2m_u - 2C_x \frac{1}{m_c^2} = 548 \text{ MeV} \\
(m_{\eta'}^+) &= 2m_s + 16C_x \frac{1}{m_c^2} = 958 \text{ MeV} \\
(m_{\rho}^+) &= 2m_u + 2C_x \frac{1}{m_c^2} = 776 \text{ MeV} \\
(m_{K^*}^+) &= m_u + m_s + 2C_x \frac{1}{m_c^2} = 892 \text{ MeV} \\
(m_{\omega}^+) &= 2m_u + 2C_x \frac{1}{m_c^2} = 783 \text{ MeV} \\
(m_{\varphi}^+) &= 2m_s - 16C_x \frac{1}{m_c^2} = 1020 \text{ MeV} \\
(m_{D^+,\pm}^+) &= m_u + m_c - 2C_x \frac{1}{m_c^2} = 1869 \text{ MeV} \\
(m_{D^+,0}^+) &= m_u + m_c - 2C_x \frac{1}{m_c^2} = 1865 \text{ MeV} \\
(m_{D^+,\pm}^0) &= m_u + m_c + 2C_x \frac{1}{m_c^2} = 2010 \text{ MeV} \\
(m_{D^+,0}^0) &= m_u + m_c + 2C_x \frac{1}{m_c^2} = 2007 \text{ MeV} \\
(m_{D^+,\pm}) &= m_u + m_c + 2C_x \frac{1}{m_c^2} = 1968 \text{ MeV} \\
(m_{D^+,0}) &= m_u + m_c + 2C_x \frac{1}{m_c^2} = 2112 \text{ MeV}
\end{align*}
\]

(6) \((m_{D^+,\pm}^+) = m_u + m_c + 2C_x \frac{1}{m_c^2} = 1968 \text{ MeV} \) (9) \((m_{D^+,\pm}^0) = m_u + m_c + 2C_x \frac{1}{m_c^2} = 2010 \text{ MeV} \)

(7) \((m_{D^+,\pm}) = m_u + m_c + 2C_x \frac{1}{m_c^2} = 1968 \text{ MeV} \) (10) \((m_{D^+,0}^+) = m_u + m_c + 2C_x \frac{1}{m_c^2} = 2112 \text{ MeV} \)

(11) \((m_{D^+,0}) = m_u + m_c + 2C_x \frac{1}{m_c^2} = 2007 \text{ MeV} \)

\[
\begin{align*}
(m_{\Xi}^0) &= 3m_u - 8C_x \frac{1}{m_c^2} = 940 \text{ MeV} \\
(m_{\Xi^+}) &= 2m_u + m_s - C_x \frac{1}{m_c^2} \left(1 + 7 \frac{m_u}{m_s}\right) = 1190 \text{ MeV} \\
(m_{\Xi^0}) &= m_u + 2m_s - C_x \frac{1}{m_c^2} \left(1 + 7 \frac{m_u}{m_s}\right) = 1315 \text{ MeV} \\
(m_{\Lambda}) &= 2m_u + m_s - C_x \frac{1}{m_c^2} \left(13 + 11 \frac{m_u}{m_s}\right) = 1116 \text{ MeV}
\end{align*}
\]
TABLE II: The flavor wave functions of scalar $cqar{q}ar{q}$ tetraquarks distributed in SU(3)$_F$ multiplets, with mixing between states with the same quantum numbers.

| multiplet | tetraquark | flavor wave function |
|-----------|------------|---------------------|
| $15_S$    | $\Xi^{++}$ | $cu\bar{s}d$        |
|           | $\Xi^+$    | $cd\bar{s}s$        |
|           | $\Sigma^{++}$ | $\frac{1}{\sqrt{2}}c(u(\bar{s}\bar{u} + \bar{u}s) - d(\bar{s}\bar{d} + \bar{d}s))$ |
|           | $\Sigma^+$  | $\frac{1}{2}cd(\bar{s}\bar{u})$ |
|           | $\Sigma^0$ | $cudd$              |
|           | $\Delta^{++}$ | $\frac{1}{\sqrt{2}}c(-u(\bar{u}\bar{d} + \bar{d}u) + u\bar{d}\bar{d})$ |
|           | $\Delta^+$  | $\frac{1}{\sqrt{2}}c(-d(\bar{u}\bar{d} + \bar{d}u) + u\bar{u}\bar{u})$ |
|           | $\Delta^0$  | $c\bar{d}\bar{u}\bar{u}$ |
|           | $\Sigma^0$ | $cs\bar{d}d$        |
|           | $\Sigma^-$  | $-\frac{1}{\sqrt{2}}cs(\bar{u}\bar{d} + \bar{d}u)$ |

| $15_S$ and $3_S$ mixed states | $D^+_s(15_S - 3_S)$ | $\frac{1}{\sqrt{2}}c(u(\bar{s}\bar{u} + \bar{u}s) + d(\bar{s}\bar{d} + \bar{d}s) - 2s\bar{s}s)$ |
|                               | $D^+_s(15_S - 3_S)$ | $\frac{1}{2\sqrt{2}}c(u(\bar{u}\bar{d} + \bar{d}u) - 3s(\bar{s}\bar{d} + \bar{d}s) + 2d\bar{d}d)$ |
|                               | $D^0_s(15_S - 3_S)$ | $\frac{1}{2\sqrt{2}}c(-d(\bar{u}\bar{d} + \bar{d}u) + 3s(\bar{s}\bar{u} + \bar{u}s) - 2u\bar{u}\bar{u})$ |

| $6_A$ | | $\Xi^{++}$ | $\frac{1}{\sqrt{2}}cu(\bar{d}s - \bar{s}d)$ |
|       | $\Xi^+$     | $\frac{1}{2}c(u(\bar{s}\bar{u} + \bar{u}s) + d(\bar{s}\bar{d} + \bar{d}s) + 2s\bar{s}s)$ |
|       | $\Sigma^+$  | $\frac{1}{\sqrt{2}}cd(\bar{s}\bar{u})$ |
|       | $\Sigma^0$ | $cudd$              |
|       | $\Omega^0$ | $\frac{1}{\sqrt{2}}cs(\bar{u}\bar{d} - \bar{d}u)$ |

| $3_A$ | | $D^+_s$ | $\frac{1}{2}c(d(\bar{s}\bar{d} - \bar{d}s) - u(\bar{s}\bar{u} - \bar{u}s))$ |

| $6_A$ and $3_A$ mixed states | $D^+_s(6_A - 3_A)$ | $\frac{1}{2}c(\bar{u}(\bar{u}\bar{d} - \bar{d}u) + s(\bar{d}s - \bar{s}d) - s(\bar{d}s - \bar{s}d))$ |
|                               | $D^0_s(6_A - 3_A)$ | $\frac{1}{2}c(s(\bar{s}\bar{u} - \bar{u}s) + d(\bar{u}\bar{d} + \bar{d}u))$ |

\[
(m_\Delta = 3m_u - 4C_\chi \frac{1}{m^2} = 1232 \text{ MeV}
\]
\[
(m_{\Sigma^+=} = 2m_u + m_s - 8C_\chi \frac{1}{3m_s} \left( \frac{1}{2} + \frac{m_u}{m_s} \right) ) = 1385 \text{ MeV}
\]
\[
(m_{\Sigma^0} = m_u + 2m_s - 8C_\chi \frac{1}{3m_s} \left( \frac{1}{2} + \frac{m_u}{m_s} \right) ) = 1530 \text{ MeV}
\]
\[
(m_{\Omega} = 3m_s - 4C_\chi \frac{1}{m^2} = 1672 \text{ MeV}
\]
\[
(m_{\Sigma^c} = 2m_u + m_c - C_\chi \frac{1}{m^2} \left( 1 + 7 \frac{m_s}{m_u} \right) ) = 2455 \text{ MeV}
\]
\[
(m_{\Xi_{c,=}} = m_u + 2m_c - C_\chi \frac{1}{m^2} \left( 1 + 7 \frac{m_s}{m_u} \right) ) = 2470 \text{ MeV}
\]
\[
(m_{\Xi_{c,0}} = m_u + 2m_c - C_\chi \frac{1}{m^2} \left( 1 + 7 \frac{m_s}{m_u} \right) ) = 2475 \text{ MeV}
\]
\[
(m_{\Lambda_c} = 2m_u + m_c - C_\chi \frac{1}{m^2} \left( 13 + 11 \frac{m_s}{m_u} \right) ) = 2285 \text{ MeV}
\]
\[
(m_{\Sigma_{c,=}} = 2m_u + m_c - 8C_\chi \frac{1}{3m_s} \left( \frac{1}{2} + \frac{m_u}{m_s} \right) ) = 2520 \text{ MeV}
\]
\[
(m_{\Omega_c} = 2m_s + m_c - 8C_\chi \frac{1}{3m_s} \left( \frac{1}{2} + \frac{m_u}{m_s} \right) ) = 2698 \text{ MeV}
\]

\[
(m = m_{u,d}) \text{, } m_s \text{ and } m_c \text{ are the results of the hadron fit. The constant } C_\chi \text{ is set so that the lightest tetraquark from } 3_A \text{ multiplet has equal mass as } D^+_s(2317). \text{ We performed these calculations using theoretical and experimental masses of all particles listed in equations } 6-13, \text{ except for mixed states } \eta - \eta' \text{ and } \omega - \phi \text{ because the meson octet and singlet mix and the flavor functions of mixed states are given only in a first approximation (see } 3). \text{ The } \chi^2 \text{ values for each set of equations for masses are evaluated as:}
\]
\[
\chi^2 = \sum_{i=1}^{N} \frac{(T_i - E_i)^2}{\sigma_i^2},
\]

where $T_i$ is the model prediction for the hadron mass, $E_i$ is the experimental hadron mass and $\sigma_i$ is the uncertainty of the mass. After the values of parameters $m_{u,d}$, $m_s$, $m_c$ were obtained by fitting meson and baryon experimental masses, they were used for calculation of the
TABLE III: Masses of scalar $cqqq$ tetraquarks distributed in SU(3)$_F$ multiplets, with mixing between states with the same quantum numbers. $m_{i,0}$ are tetraquark masses without influence of GR HFI and $m_{i,GR}$ are GR HFI contributions to tetraquark masses.

| multiplet       | tetraquark | $m_{i,0}$ ($m_u = m_d$) | $m_{i,GR}$ ($m_u = m_d$)       |
|-----------------|------------|--------------------------|--------------------------------|
| $\bar{T}^5_S$   | $\Xi$      | $m_u + 2m_s + m_c$       | $\frac{-4C_X}{3} \left( \frac{1}{m_u^2} + \frac{1}{2m_u m_s} \right)$  |
|                 | $\Sigma$   | $2m_u + m_s + m_c$       | $\frac{-4C_X}{3} \left( \frac{1}{m_u^2} + \frac{1}{2m_u m_s} \right)$  |
|                 | $\Delta$   | $3m_u + m_c$             | $-4C_X \frac{1}{m_u^2}$       |
|                 | $\Sigma$   | $2m_u + m_s + m_c$       | $-4C_X \frac{1}{m_u^2}$       |
| $\bar{T}^5_S$ and $3_S$ mixed states | $D_s(\bar{T}^5_S - 3_S)$ | $2m_u + m_s + m_c; 3m_u + m_c$ | $\frac{-4C_X}{3} \left( \frac{1}{m_u^2} + \frac{1}{m_s^2} \right) + 4C_X \left( \frac{1}{m_u^2} + \frac{1}{m_c^2} + \frac{1}{m_u m_c} \right)$ |
|                 | $D(\bar{T}^5_S - 3_S)$ | $3m_u + m_c; m_u + 2m_s + m_c$ | $-4C_X \frac{1}{m_u^2} - \frac{4C_X}{6} \left( \frac{1}{m_u^2} + \frac{4}{m_c^2} + \frac{1}{m_u m_c} \right)$ |
| $6_A$           | $\Sigma$   | $2m_u + m_s + m_c$       | $-8C_X \frac{1}{m_u m_s}$     |
|                 | $\Omega$   | $2m_u + m_s + m_c$       | $-8C_X \frac{1}{m_u m_s}$     |
| $3_A$           | $D_s$      | $2m_u + m_s + m_c$       | $-8C_X \frac{1}{m_u m_s}$     |
| $6_A$ and $3_A$ mixed states | $D(6_A - 3_A)$ | $3m_u + m_c; m_u + 2m_s + m_c$ | $-8C_X \frac{1}{m_u m_s}$  |

Tetraquark masses.

The hadron mass fits resulted in the parameter values given in the Table I. From the meson fit, calculated masses for $u$ and $s$ quarks are smaller ($m_u \approx 311$ MeV, $m_s \approx 487$ MeV) than those from the baryon fit ($m_u \approx 388$ MeV, $m_s \approx 556$ MeV). Due to smaller value of $\chi^2$ the masses obtained from the meson fit are more reliable. Both fits gave the similar values for constant $C_X$ ($\sim 7 \times 10^3$ MeV$^3$).

With parameters from Table I we calculated tetraquark masses. The tetraquark masses calculated from meson fit parameters are given in Table IV. The results from Table IV show that the isoscalar from $\bar{T}^5_S$ has the same mass as $D^*_s$ (2632) and that the isoscalar from $6_A$ has the same mass as $D^+_s$ (2317).

GR contribution is positive or negative due to signs of the $(\bar{s}i\bar{q}_j)$ and $(\lambda^F_i\lambda^F_j)$ products. It is negative in $\bar{T}^5_S$, $6_A$ and $3_A$-plets, and for $\bar{T}^5_S - 3_S$ mixed states one of the mixed states has negative and the other one has positive GR contribution. The positive GR contribution for two mixed states (see Table III) comes out because of the mixing of the states: it changes the properties and shifts masses from the theoretical predictions.

For experimentally detected states ($D^+_s$ (2317), $D^0$ (2308) and $D^*_s$ (2632)), hadron fits resulted in theoretical masses with relatively significant statistical uncertainties. These uncertainties are mainly due to inaccuracies in constitutive quark masses obtained using hadron fits (see Table IV). For example, experimental masses for $D_s$ and $D^*_s$ mesons are 1968 MeV and 2112 MeV, but they have the same quark content (see eq. (9)) and therefore their constituent quarks have different theoretical masses. Besides, $D^0(2308)$ and $D^*_s(2632)$ are mixed states (see Table III) and therefore their flavor wave functions are given only in a first approximation. In spite of the uncertainties, none of the found states with strangeness equal to zero have mass around $m = 2405$ MeV, which agrees with the conclusion obtained in Refs. 20 and 26 that the state $D^+_s(2405)$ (found by the FOCUS collaboration [27]) is not a tetraquark, but a normal $cq$ state.

Tetraquark mass spectrum from the meson fit, without and with GR HFI influence and with SU(3)$_F$ symmetry breaking is presented in Figure 6. The general conclusion is that tetraquarks are arranged in the same way in both spectra: from meson and baryon fits. The spectra obtained from different fits have a similar arrangement of particles and if the values of parameters are changed, the whole spectrum could be shifted towards higher or lower masses and it could be shrunk or broadened. In both spectra it is possible to identify $D^*_s$ (2317) as the lowest state in multiplet $3_A$ and $D^*_s$ (2632) as a mixed state from mixing of multiplets $\bar{T}^5_S$ and $3_S$. Also, in both spectra (for example see Table V), GR HFI mostly reduces the obtained masses except for one of $D_s$ mixed states and one of $D$ mixed states from $\bar{T}^5_S - 3_S$ mixing.

Figure 6 may be compared with Figure 3 from Ref. 23 which was plotted for Fermi-Breit (FB) color-spin HFI.
TABLE IV: The results for masses (in MeV) of constituent quarks, obtained from hadron masses by $\chi^2$ fit. Constant $C_\chi$ (in $10^6$ MeV$^3$) is set so that the lightest tetraquark from the $\bar{3}_A$ multiplet has equal mass as $D_s^+(2317)$.

| hadrons | $m_u$ (MeV) | $m_s$ (MeV) | $m_c$ (MeV) | $\chi^2$ | $C_\chi$ ($10^6$ MeV$^3$) |
|---------|-------------|-------------|-------------|---------|--------------------------|
| mesons  | 311         | 487         | 1592        | 1.04 x 10$^{-2}$ | 7.30         |
| baryons | 388         | 556         | 1267        | 2.29 x 10$^{-1}$ | 7.60         |

TABLE V: The results for masses (in MeV) of scalar $c\bar{q}\bar{q}$ tetraquarks distributed in SU(3)$_F$ multiplets, with mixing between states with the same quantum numbers, obtained from meson fit. $m_{\nu,0}$ (MeV) are tetraquark masses without influence of GR HFI, $m_{\nu,GR}$ (MeV) are GR HFI contributions to tetraquark masses and $m_{\nu}$ (MeV) are the total tetraquark masses.

| multiplet | tetraquark | $m_{\nu,0}$ (MeV) | $m_{\nu,GR}$ (MeV) | $m_{\nu}$ (MeV) |
|-----------|------------|-------------------|--------------------|-----------------|
| $\bar{15}_S$ | $\Xi$      | 2877              | -146               | 2731            |
|           | $\Sigma_s$ | 2701              | -228               | 2475            |
|           | $\Delta$   | 2525              | -301               | 2224            |
|           | $\Sigma$   | 2701              | -228               | 2473            |
| $\bar{15}_S$ and $3_S$ mixed states | $D_s(\bar{15}_S - 3_S)$ | 2701; 3053 | -228; 615 | 2473; 3668 |
|           | $D(\bar{15}_S - 3_S)$ | 2525; 2877 | -301; 877 | 2224; 3754 |
| $6_A$     | $\Sigma_s$ | 2701              | -384               | 2317            |
|           | $\Omega$   | 2701              | -601               | 2100            |
| $3_A$     | $D_s$      | 2701              | -384               | 2317            |
| $6_A$ and $3_A$ mixed states | $D(6_A - 3_A)$ | 2525; 2877 | -601; -384 | 1924; 2493 |

from the constituent quark masses obtained in this way: $m_u$ was the result of a light meson fit, the mass difference $m_s - m_u$ was taken to be 100 MeV and $m_c$ is fitted so that the lightest tetraquark from the $3_A$ multiplet had equal mass as $D_s^+(2317)$. This comparison shows that obtained tetraquark masses for both GR and FB HFI are similar, except for $\bar{15}_S - 3_S$ mixed states. Besides, FB HFI is causing additional splitting between $\Sigma_s$ ($D_s$) and $\Sigma$. These dissimilarities are due to term $\left(\lambda_1^F \lambda_1^F\right)$ in GR and $\left(\lambda_2^C \lambda_2^C\right)$ in FB interaction (F - flavor, C - color). From this comparison one can see that the forms of tetraquark spectra with FB and GR interaction are similar, only they are shifted for some value. This is an important result which was not expected because FB is a color-spin and GR is a flavor-spin interaction. The results obtained from these two interactions confirm that both HFIs give similar results for tetraquark masses.

The lightest $qq\bar{q}\bar{q}$ scalars that are experimentally known are the $\sigma$ (500), $f_0$ (980), $\kappa$ (800) and the $a_0$ (980). They form an SU(3) flavor nonet. Already in the seventies Jaffe [23] suggested the tetraquark structure of this scalar nonet and proposed a four quark bag model. Their quark content is given in Refs. [3] and [28]. It is shown [28] that these mesons fit well in the tetraquark scheme. We considered them as four quark states and calculated their masses, which are given in Table [VI]. As can be seen from

![FIG. 6: Tetraquark mass spectrum from meson fit without (left column) and with (right column) GR HFI, both with SU(3)$_F$ symmetry breaking.](image-url)
Table VI: The results for masses (in MeV) of light scalar tetraquark nonet, obtained from meson masses by $\chi^2$ fit. Constant $C_\chi = 6.0 \times 10^6$ MeV$^3$ is obtained so the lightest tetraquark $\sigma$ has equal mass as experimentally state.

| tetraquark | $m_{\nu,0}$ ($m_u = m_s$) | $m_{\nu,GR}$ ($m_u = m_s$) | $m_{\nu,0}$ (MeV) | $m_{\nu,GR}$ (MeV) | $m_\nu$ (MeV) |
|------------|-----------------------------|----------------------------|-------------------|-------------------|---------------|
| $\sigma$   | $4m_u$                      | $-12C_\chi \frac{1}{m_u^2}$ | 1244              | -744              | 500           |
| $f_0$      | $2m_u + 2m_s$               | $-\frac{4}{3}C_\chi \left( \frac{1}{m_u^2} + \frac{1}{m_s^2} + \frac{1}{m_u m_s} \right)$ | 1596              | -481              | 1115          |
| $\kappa$   | $3m_u + m_s$                | $-6C_\chi \left( \frac{1}{m_u^2} + \frac{1}{m_s^2} + \frac{1}{m_u m_s} \right)$ | 1420              | -610              | 810           |
| $a_0$      | $2m_u + 2m_s$               | $-\frac{4}{3}C_\chi \left( \frac{1}{m_u^2} + \frac{1}{m_s^2} + \frac{1}{m_u m_s} \right)$ | 1596              | -481              | 1115          |

Table VI. GR HFI significantly reduces the theoretical masses of the light scalar tetraquarks and brings them closer to their experimental masses. This fact confirms the conclusion from Ref. 28 about tetraquark nature of these light scalars.

IV. CONCLUSIONS

We have made a systematic analysis of the charm tetraquark states. Weight diagrams, irreducible representations and flavor wave functions are shown and analyzed. Detailed method of calculation is described.

The mass spectrum with mixing of particles with the same quantum numbers is shown. The discussion how results depend on parameters is also given.

There are 27 different tetraquarks with $C = 1$ and with three light flavors. We calculated mass spectra and wave functions for all 27 states using GR HFI. Among these states there are 11 cryptoexotic (3 $D^+_c$, 4 $D^+_s$, 4 $D^0$) and 16 explicit exotic states. In the tetraquark model it is possible to identify $D^+_c(2317)$ and $D^+_s(2632)$ with two cryptoexotic states in tetraquark spectrum when Glozman-Riska hyperfine interaction is included. Namely, explicit exotic states, for instance isotriplet $\Sigma_u (\Sigma^+_u, \Sigma^0_u)$, appear in the spectrum with the same masses (i.e. at 2317 MeV and at 2632 MeV). One intriguing possibility is that $D^0(2308)$ is a tetraquark. Then, there should be the other tetraquark partner of $D^0(2308)$ to form an isospin doublet with isospin $T = 1/2$ (see Table 1). We suggest that the recently discovered charm-strange meson $D^0(2308)$, with unusual properties, could be a cryptoexotic tetraquark state $cq\bar{q}u$. The tetraquark nature for scalar charmed mesons $D^+_c(2317)$, $D^0(2308)$ and $D^+_s(2632)$ is confirmed by showing existence of the tetraquark component in their wave functions.

We also gave estimates for masses of experimentally detected light scalars $\sigma (500)$, $f_0 (980)$, $\kappa (800)$ and $a_0 (980)$ and confirmed that they satisfactorily fit in the tetraquark scheme when GR HFI is included.

If we compare masses with and without hyperfine interactions we can conclude that mass arrangement of tetraquark flavor multiplets depends almost entirely on the strong hyperfine interaction. We show that in both cases of hyperfine interaction (FB and GR) the lowest lying multiplet is $6_A$-plet, and the mixing and ordering of other states is similar in the two models. Maybe, FB and GR are not the complete effective two-quark interactions, and because of that theoretical prediction is not the same as the experiment.

We also showed for the first time wave functions and quark content for all predicted 27 quark states of $cq\bar{q}u$ combination. We obtained all masses using GR interaction with two fits and we showed that GR HFI gave similar results as FB interaction. More experimental searches for detection of other $cq\bar{q}u$ members especially those exotic ones are needed in the future.

Acknowledgments

The author acknowledges support by the Ministry of Science of Serbia. Also, the author would like to thank V. Dmitrašinović for useful suggestions and support.

[1] BABAR Collab. (B. Aubert et al.), Phys. Rev. Lett. 90, 242001 (2003)
[2] BELLE Collab. (Y. Mikami et al.), Phys. Rev. Lett. 92, 012002 (2004)
[3] BELLE Collab. (K. Abe et al.), Phys. Rev. D 69, 112002 (2004)
[4] SELEX Collab. (A. V. Evdokimov et al.), Phys. Rev. Lett. 93, 242001 (2004)
[5] R. L. Jaffe, Phys. Rev. D 15, 267 (1977)
[6] R. L. Jaffe, Phys. Rev. D 15, 281 (1977)
[7] E. van Beveren and G. Rupp, AIP Conf. Proc. 687, 86 (2003)
[8] E. van Beveren and G. Rupp, Phys. Rev. Lett. 91, 012003 (2003)
[9] E. van Beveren, J. E. G. N. Costa, F. Kleefeld and G. Rupp, Phys. Rev. D 74, 037501 (2006)
[10] E. van Beveren and G. Rupp, Phys. Rev. Lett. 93, 202001 (2004)
[11] T. Barnes, F. E. Close, J. J. Dudek, S. Godfrey and E. S. Swanson, Phys. Lett. B 600, 223 (2004)
[12] A. Hayashigaki and K. Terasaki, Prog. Theor. Phys. 114, 1191 (2005)
[13] K. Terasaki, arXiv:hep-ph/0405146 (2004)
[14] K. Terasaki, Prog. Theor. Phys. 116, 435 (2006)
[15] K. Terasaki, AIP Conf. Proc. 717, 556 (2004)
[16] K. Terasaki, Prog. Theor. Phys. 114, 205 (2005)
[17] A. Hayashigaki and K. Terasaki, arXiv:hep-ph/0411285 (2004)
[18] Y.-R. Liu, S.-L. Zhu, Y.-B. Dai and C. Liu, Phys. Rev. D 70, 094009 (2004)
[19] B. Nicolescu and J. P. B. C. de Melo, arXiv:hep-ph/0407088 (2004)
[20] M. Nielsen, R. D. Matheus, F. S. Navarra, M. E. Bracco and A. Lozea, Nucl. Phys. B, Proc. Suppl. 161, 193 (2006)
[21] V. Dmitrašinović, Phys. Rev. Lett. 94, 162002 (2005)
[22] V. Dmitrašinović, Modern Phys. Lett. A 21, 533 (2006)
[23] V. Dmitrašinović, Int. J. Mod. Phys. A 21, 5625 (2006)
[24] L. Ya. Glozman and D. O. Riska, Phys. Rep. 268, 263 (1996)
[25] W.-M. Yao et al., J. Phys. G 33, 1 (2006)
[26] M. E. Bracco, A. Lozea, R. D. Matheus, F. S. Navarra and M. Nielsen, Phys. Lett. B 624, 217 (2005)
[27] FOCUS Collab. (J. M. Link et al.), Phys. Lett. B 586, 11 (2004)
[28] T. V. Brito, F. S. Navarra, M. Nielsen and M. E. Bracco, Phys. Lett. B 608, 69 (2005)