The Measurement of Solar Differential Rotation from Proper Motion of Individual Sunspots

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Abstract The differential rotation is the result of the interaction between rotation and convection and causes dynamo circulation that affect the cycle of solar activity. Tracer method using features in the photosphere such as sunspot is a simple method to measure the differential rotation. In this study, 98 individual sunspots on January 8 - 22, 2013 and August 25 - September 7, 2013 were used to measure the differential rotation of the Sun. The Sun's continuum images were obtained from HMI (Helioseismic Magnetic Imager) instrument at SDO (Solar Dynamic Observatory). Coordinate and area of sunspot were measured using ImageJ software and converted to Carrington coordinates. From the measurement, we derived the differential rotation equation, the relation of velocity and area of sunspot, and the relation of sunspot’s velocity and Zurich classification. The differential rotation equation obtained in this study is \( \omega (B) = (14.376 \pm 0.04) + (0.6 \pm 0.38) \sin^2 B \) \(^{(\circ/\text{day})}\). For the relation between velocity and area of sunspot, we got the sidereal rotation rate of sunspots with area < 100 pixels\(^2\) is about 0.8% higher than sunspots with area > 100 pixels\(^2\).

Keywords: Solar Differential Rotation and Individual Sunspots

1. Introduction
In 1630, Christoph Scheiner discovered that sunspots on the equator had shorter rotation periods and moved faster than sunspots at higher latitudes[1]. Scheiner has many observational data and he derived differential rotation equations in the function of latitude. However, the idea of differential rotation has not been received and appreciated at that time. In 1860, Richard Christopher Carrington observed the Sun and determined the rotation period of the Sun based on latitude. Sunspots around the equator have 25-day rotation period, while sunspots at 45° latitude have 28-day rotation period.

Differential rotation shows that the Sun is not a solid body that has different rotational rate of each latitude. Differential rotation is the result of the interaction between rotation, magnetic fields, and convection flow that causes dynamo circulation that affect the cycle of solar activity. The factors in the interior in the Sun that causes differential rotation are still a big question. Knowledge of the activity in Sun’s interior is still limited. Therefore, rotation studies are important to get more information about the interior of the Sun.

There are several methods of measuring differential rotation, the tracer method and spectroscopic method. In the tracer method, photosphere features such as sunspots and magnetic fields are used to determine the differential rotation. In spectroscopic method, spectral measurements and the Doppler equation is used to obtain differential rotation values [2]. This method is very precise but very sensitive because it has a small wavelength order. A slight error in positioning will cause a large error in getting the velocity value. In this research, we used tracer method with sunspot as a tracer. Sunspots...
have advantages for easier to identify due to their high contrast intensity to the surroundings, and have long life time. Differential rotation equations are expressed in the following equation [2].

\[ \omega (B) = a - b \sin 2B - c \sin 4B \]  

(1)

For the calculation of the mean longitudinal drift based on the sunspot tracer, the third term is small, so the equation that used is \( \omega (B) = a - b \sin 2B \) where \( \omega (B) \) is the sidereal rotation rate (°/day), a and b are the constant, and B is the average latitude. For the calculation of latitudinal drift, the equation is

\[ \nu_m (B) = c \sin 2B \]  

(2)

where \( \nu_m \) is the latitudinal drift (°/day), c is the constant, and B is the average latitude.

The Carrington heliographic coordinates are coordinate systems to determine the position of features such as sunspot on the surface of the Sun. To find heliographic longitude and latitude, the equation that used is [3]

\[ \sin B = \sin Bo \cos \rho + \cos Bo \sin \rho \cos (P - \beta) \]  

(3)

\[ \sin (L - Lo) = \sin \rho \sin (P - \beta) \sec B \]  

(4)

\[ \sin (\rho + \rho_1) = \rho_1 \]  

(5)

\[ \rho_1 = \sqrt{x^2 + y^2} \]  

(6)

where \((L, B)\) is the heliographic longitude and latitude, \((Lo, Bo)\) is the heliographic coordinate at the centre of the disk, \(\rho\) is the heliocentric angular distance, \(\rho_1\) is the distance of the radial angle, \(\beta\) is the angle of position measured from the north pole of the Sun (N) to eastward, D is the angular semi-diameter of the Sun, and P is the position angle of the Sun's north pole (N) relative to the direction of the celestial pole (No).

Figure 1. Sketch of the solar disk to illustrate the heliographic coordinates [3]

2. Objectives
The objectives of this research is measuring the proper motion of individual sunspots to determine the differential rotational of the Sun using continuum image data from the HMI instrument (Helioseismic and Magnetic Imager) onboard the SDO (Solar Dynamic Observatory) satellite. We determine the relation of proper motion’s velocity of individual sunspot with area of the sunspot, and determine the relation of the proper motion’s velocity of individual sunspot with the Zurich classification.

3. Method
The continuum image used in SDO is the image of the Sun on the continuum wavelength which has been corrected with the limb darkening. Image resolution is 4096x4096 pixels (resolution 0.5 arcsecond degree) with image size 4 – 5 MB. The data are downloaded on the https://sdo.gsfc.nasa.gov. We used ImageJ (https://imagej.nih.gov/ij/) to measure the area and coordinates of sunspot. ImageJ is a Java-based software that able to process and analyse images. ImageJ can read various image formats, measure distance, angle, area, pixel value. The HelioViewer software is used to find the values of Bo, Lo, P, and semi-diameter of the Sun. These parameters are needed to convert the coordinate in pixel \((x, y)\) obtained from ImageJ to Carrington's heliographic coordinates in degrees.
The data in this research are sunspots on August 25 to September 7, 2013 and January 8 - 22, 2013. On those dates, the Sun's activity is high so there are many sunspots from various groups based on Zurich classification. From the data on August 25 to September 7, 2013, 27 individual sunspots from 7 groups of sunspots were observed and from the data on January 8 - 22, 2013, 71 individual sunspots of 10 sunspots groups were observed. For each date, 5 images are measured at 00.30 UT, 01.30 UT, 02.30 UT, 03.30 UT, and 04.30 UT. Sunspots that we used for the measurements are sunspots that have life time more than two days. To avoid the appearance of asymmetrical sunspot on the limb of the Sun or the Wilson effect, measurements were taken for the group of sunspots with positions less than 70° from the meridian. We checked the sunspot group’s position from https://solarmonitor.org.

4. Result and Discussion
4.1 Differential Rotation Equations
To find the equation of differential rotation, we used linear regression between the sidereal rotation rate denoted by \( \omega \) (B) and average latitude for each individual sunspots denoted by sin2(B). The values of a and b from the linear regression entered into the equation \( \omega \) (B) = a + b sin2 (B). The Carrington velocity (Ve) is 14.184 °/day[3]. The value of the sidereal rotation rate is obtained from the following formula.

\[
\omega \ (B) = \frac{(14.184°/\text{day} \times T) + (dL/dt)}{T}
\]

with T is the duration or age of individual sunspot and longitudinal drift (dL/dt) is the longitude velocity of each individual sunspot. The differential rotation equation from 98 sunspots is

\[
\omega \ (B) = (14.376 \pm 0.04) + (0.6 \pm 0.38) \sin 2B \ (°/\text{day}).
\]

However, the constant b is positive. This is caused by small sample of individual sunspots. If the number of samples is multiplied, the constant b will have a negative value, according to the differential rotation phenomenon that the rotation rate will be slower in the higher latitude. In addition, the majority of sunspots measured in this study have small area. The existence of small sunspots with high velocity and short life periods causes the value of differential rotation equations to still be seen uniformly at various latitudes.

| Ref. | a (μrad/s) | b (μrad/s) | Period | Note |
|------|------------|------------|--------|------|
| [4]  | 2.905      | -0.598     | 1934-1944 | 136 sunspots with lifetime > 1 solar rotation |
| [5]  | 2.939 ±0.001 | -0.5796 ±0.01 | 1921-1982 | White-light images of sunspot with area < 5 (mHem) |
| [5]  | 2.917 ±0.002 | -0.5284 ±0.018 | 1921-1982 | White-light images of sunspot with 5 < area < 10 (mHem) |
| [5]  | 2.885 ±0.0036 | -0.5325 ±0.034 | 1921-1982 | White-light images of sunspot with area > 10 (mHem) |
| This study (2018) | 2.9118 ±0.008 | 0.12 ± 0.07 | 2013 | 98 individual sunspots |

In table 1, there are differential rotation measurements using different sunspots. The constant a obtained in these studies is near with the references (2.885 to 2.939) μrad/s. For the constant b, the value is far from the references (-0.436 to -0.732) μrad/s. The difference in the value of b in this study with references was caused by a small sample of sunspots, compared to other 96,283 sunspots [5], 276
sunspots [3], and 136 spots [4]. Actually, the number of sunspots used by the latter [4] was not very different from the number of sunspots used in this study. However, they used selected sunspots that large, almost symmetrical, and have lifetime more than one solar rotation. The majority of the sunspots measured in this study are less than 15° in latitude, so they are less representative for higher latitudes. For better differential rotation equation results, more sample sunspots in various latitudes are needed. It is also estimated that an error occurred because many individual sunspots have very small area. The movement of small sunspots are fluctuating, shrinking, enlarging, appearing, and disappearing in a short time. This causes the measured centre coordinates to be different and not constant.

Figure 2 shows the sidereal rotation rate for 98 individual sunspots. There is one sunspot at high latitude that has high velocity 15.38°/day. This high velocity may be due to the small size of the sunspot at 58.7 pixels. This sunspot showed a short duration and evolved from one small sunspot and enlarged. The shifting of the centre point of a sunspot from one small sunspot to the larger one cause a difference in the position, so the value of sidereal rotation rate is high.

Figure 2. Sidereal rotation rate for 98 sunspots

Sunspots around the equator (<5°) have average velocity 14.449°/day. Sunspots at moderate latitudes (10°-20°) have average velocity 14.392°/day, lower than the velocity at the equator. By removing a sunspot that has highest velocity than other sunspots, the velocity at high latitudes (> 25°) is 14.337°/day. Velocity at high latitudes is lower than velocity at the equator and lower latitude.

4.2 Latitudinal Drift

To find the latitudinal drift, we used linear regression between latitude and sin (2B). The latitudinal drift equation obtained is as follows.

\[ v_m(B) = (-0.72 \pm 0.32) \sin(2B) \text{ (°/day)} \] (9)

The latitudinal drift equation obtained with 276 individual sunspots as follows [3].

\[ v_m(B) = (-0.05 \pm 0.03) \sin(2B) \text{ (°/day)} \] (10)

The small area of sunspots in this study led to a greater latitudinal drift value because small sunspots have high velocity. There is also a difference in latitudinal drift values caused by the number of different sunspots. More sunspots lead to the smaller errors. Possibility of misidentifying small sunspots also causes a large error in the latitudinal drift equation obtained.

4.3 Relation between Velocity and Area of Sunspots

Figure 3 shows relation between longitudinal drift and latitudinal drift with area of sunspot. The biggest sunspot on this measurement has average area 1061.7 pixels² and the average area of smallest sunspot is 3.5 pixels². Using linear regression, the relation between longitudinal drift and area of sun spots is

\[ \text{Longitudinal Drift} = -0.0005 \times \text{Area} + 0.01 \text{ (°/day/pixel²)} \] (11)

\[ \text{Latitudinal Drift} = -0.72 \times \sin(2B) + 0.05 \text{ (°/day)} \] (12)
\[ \omega = -1.9 \times 10^{-4} A \pm 0.46 \text{ (°/day)} \]  \hspace{1cm} (11)

with \( \omega \) (B) is the longitudinal drift and A is the area of individual sunspot. The relation between latitudinal drift with area of sunspot obtained from linear regression is

\[ v_m = -0.001 A \pm 1.74 \text{ (°/day)} \]  \hspace{1cm} (12)

with \( v_m \) is latitudinal drift and A is the area of individual sunspot.

**Figure 3.** Relation between longitudinal drift and latitudinal drift with area of sunspots

These two equations show that smaller sunspots have a greater velocity. Small sunspots with area less than 100 pixels\(^2\) has average sidereal rotation rate 14.451 °/day, while large sunspots with area more than 100 pixels\(^2\) has average sidereal rotation rate 14.333 °/day. Sunspots with area less than 5 mHem rotate about 0.8% faster than large sunspots with area 15 mHem [2]. In this study, it was found that sunspots with area <5 mHem was 0.9% faster than sunspots with area >5 mHem. Sunspots with area <5 mHem rotate faster around 1.5% than sunspots with area > 15 mHem. This value is almost two times greater than the previous results [2]. However, there are only 7 sunspots with area > 15 mHem in this study, less representative of the velocity of sunspots which had a very large size.

For the value of speed, with the formula

\[ v = \sqrt{\omega^2 + v_m^2} \]  \hspace{1cm} (13)

We get the speed value as follows.

\[ v = -0.001A \pm 1.8 \text{ (°/day)} \]

Small sunspot area that has a weaker magnetic field tends to move faster than large spots that has greater magnetic field, which show similar results from the above relation. The difference in velocity is caused by the different depths of the magnetic flux. Large sunspots may also have greater shift in viscosity [2, 7].

**Figure 4.** Relation between speed and area of sunspots.
4.4 Relation between Velocity and Zurich Classification

The velocity of longitude and latitude obtained is the result of the average velocity of individual sunspots in each class. Velocity for high classes (F and H) is smaller than the velocity of sunspots in the earlier class. Class H is a class in the sunspot group that is at the end of its life and will disappear on the west. The sunspot group also has an old age, so the movement is slow.

Figure 5. Relation between velocity and Zurich classification

Class F is the largest group compared to other classes. Class F also has a complex magnetic structure and large magnetic field strength, so the movement of sunspots is slower. Early classes like class B have high velocity because the area of the sunspots is small, and the magnetic field is weaker. The sunspot group in class B also has a young age so the velocity is high. The high latitude velocity is caused by the large number of small sunspots with high velocity measured in this study.

5. Conclusions

We have measured 98 individual sunspots on August 25 - September 7, 2013 and January 8 – 22, 2013 to find differential rotation equations, relation between velocity and area of sunspots, and relation between velocity and Zurich classification. For the differential rotation equation, we get \( \omega (B) = (14.376 \pm 0.04) + (0.6 \pm 0.38) \sin 2B \) °/day. The constant a of \( 14.376 \pm 0.04 \) has a value that near to the references. However, the constant b of \( 0.6 \pm 0.38 \) has a different value with the references and shows the uniformity of velocity in all measured sunspots. In fact, the sunspots have higher velocity for the higher latitude, this is caused by less number of sunspots measured. In addition, 93 sunspots are measured, have high velocity and short life periods so at any latitude, the differential rotation equation shows almost the same value. In this study, the majority of the sunspots measured were at latitude less than 15° so there was less diversity of sunspots to show differential rotation. By using more diverse sunspot’s position and greater number of sunspots, the value of the differential rotation equation obtained will be much better. Sunspots with area less than 100 pixels\(^2\) have average sidereal rotation rate 14.451°/day, while large sunspots with the size of more than 100 pixels\(^2\) have sidereal rotation rate 14.333°/day. Sidereal rotation rate of sunspots with area < 100 pixels\(^2\) is about 0.8% higher than sunspots with area > 100 pixels\(^2\). Small sunspots have a weaker magnetic field and move faster than large sunspots that have greater magnetic field strength and move more slowly. Sunspots in high classes (F and H) have low velocity, sunspots in class B and D have high velocity, sunspots in class C and E have medium velocity. Sunspots in class F have the lowest velocity because the magnetic field structure is the most complex and the size is the largest. Class B have the highest velocity because they are young, have the smallest size, and the magnetic field structure is still not complex. However, there is only 1 sunspot in class B, while class F has 47 sunspots. More number of sunspots are needed on this measurement to get a better relation between velocity of sunspot and Zurich classification.

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