ABSTRACT
Machine reading comprehension has aroused wide concerns, since it explores the potential of model for text understanding. To further equip the machine with the reasoning capability, the challenging task of logical reasoning is proposed. Previous works on logical reasoning have proposed some strategies to extract the logical units from different aspects. However, there still remains a challenge to model the long distance dependency among the logical units. Also, it is demanding to uncover the logical structures of the text and further fuse the discrete logic to the continuous text embedding. To tackle the above issues, we propose an end-to-end model Logiformer which utilizes a two-branch graph transformer network for logical reasoning of text. Firstly, we introduce different extraction strategies to split the text into two sets of logical units, and construct the logical graph and the syntax graph respectively. The logical graph models the causal relations for the logical branch while the syntax graph captures the co-occurrence relations for the syntax branch. Secondly, to model the long distance dependency, the node sequence from each graph is fed into the fully connected graph transformer structures. The two adjacent matrices are viewed as the attention biases for the graph transformer layers, which map the discrete logical structures to the continuous text embedding space. Thirdly, a dynamic gate mechanism and a question-aware self-attention module are introduced before the answer prediction to update the features. The reasoning process provides the interpretability by employing the logical units, which are consistent with human cognition. The experimental results show the superiority of our model, which outperforms the state-of-the-art single model on two logical reasoning benchmarks.

CCS CONCEPTS
• Information systems → Question answering; Language models; Information extraction.

KEYWORDS
logical reasoning, machine reading comprehension, graph transformer

1 INTRODUCTION
Machine reading comprehension [10, 18, 39] has been one of the major focuses in the field of Natural Language Processing (NLP) [4, 9] in recent years. A large number of models have achieved competitive performances in some famous datasets, such as SQuAD [23, 24], RACE [14]. However, these models [7, 27, 36] lack the capability of logical reasoning. To facilitate the machine for human intelligence, the task of logical reasoning MRC [17, 37] was proposed previously. Similar to the traditional MRC, the task of logical reasoning also requires the models to predict the answers depending on the given text inputs. Figure 1 illustrates an logical reasoning example from ReClor dataset [37]. The inputs include the context, question and a set of options. One of the unique characteristics of the text is the rich logical structures. As illustrated in Figure 1, the logical structure of the context can be uncovered in a certain way. We define the split short sentences as logical units (e.g., U1-U6). The logical units contain the independent and complete semantics, which are not kept in the token-level text features. The understanding of the
which sacrifices the global semantics of logical units. Take the first 
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Figure 1, the logical units U1 and U2 are connected with the con-
sonant role in covering the logical relations. For example in 
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them into the logical branch and syntax branch respectively. From 
two explicit relationships (e.g.,
completely separated. In this work, we pay much attention to the 
closer observation of the logical units in Figure 1, the units are not 
logical structures and the continuous text embedding space. Take a 
example of the context in Figure 1 as an instance, two extracted 
features limits the information update. In a word, the simple graph 
structure built for the logical text would fail to provide the efficient 
direct interactions, we tackle the issue of long distance dependency 
among the logical units. To encode the discrete logical structures to 
the continuous text embedding space, we apply the attention biases 
from both the logical and syntax branches. The whole reasoning is 
on the basis of logical units and the built graphs, which are consist- 
tent with the human cognition. The explicit relations among units 
and the weighted attention maps provide the interpretability for the 
logical reasoning. In details, firstly, Logiformer split the text into 
logical units and construct the logical graph based on the causal 
relations for the logical branch. For the syntactic branch, the split 
nodes and a syntax graph are also obtained. Secondly, we feed the 
node sequences and two graph topology to the fully connected 
graph transformers respectively. The respective adjacent matrices 
are viewed as the attention biases to encode the logical structures 
to each graph transformer. Thirdly, we combine the updated fea-
tures from two branches with a dynamic gate mechanism. With 
the additional token-level embedding, we can map the features to 
the same space. By means of the question-aware self-attention, the 
final feature can be utilized to predict the answers.

The main contributions are listed as follows:

- A two-branch graph transformer network named Logiformer 
is proposed to model the long distance dependency of the 
logical units and encode the discrete logical structure to the 
continuous text embedding. As far as we know, we are the 
first to tackle both issues in the logical reasoning task.
- In light of drawbacks of chain-type text graphs, we take the 
fully connected structures into consideration, containing the 
awareness of both logic and syntax simultaneously. Two 
graphs are constructed based on the extracted logical units 
and their topology is utilized as the attention biases.
- The extraction of the logical units and the explicit relations 
are consistent with the human cognition. The uncovered 
logical structures and the weighted attention maps of the

Figure 1: An example of the logical reasoning task and some 
detailed illustrations.

Text requires the global semantics of each logical unit, as well as 
the interactions among them based on some logical relations (e.g., 
causal and co-occurrence). Therefore, the main challenges for the 
task of logical reasoning can be summarized as the following two 
aspects.

Firstly, it remains a challenge to model the long distance depen-
dency [1] of the extracted logical units. Some previous methods, 
such as DAGN [12], have proposed to split the text into discourse 
node [52] and constructed a sequential chain graph for reason-
ing. However, it neglects the natural long-distance dependency 
among logical units. For example in Figure 1, the first and the last 
sentences share the same subject (Paula) and predicate (visit the 
dentist), though they are distant in the graph space. The chain struc-
ture limits the information update. In a word, the simple graph 
structure built for the logical text would fail to provide the efficient 
one-hop interaction [25]. Pretrained-based transformer structures 
[30] have the natural advantage of modeling the long text and 
show excellent performance on the popular tasks. To enhance the 
logical perception ability of the language models, previous works 
have attempted to employ additional segment embedding at the 
beginning. However, it is still limited to the token-level interactions, 
which sacrifices the global semantics of logical units. Take the first 
sentence of the context in Figure 1 as an example, two extracted 
units of Paula will visit the dentist tomorrow morning (U1) and Bill 
goes golfing in the morning (U2) express the causal relations within 
this sentence. The token-aware models would stress more on the 
text semantics and fail to capture such logical information.

Secondly, it is intractable to bridge the gap between the discrete 
logical structures and the continuous text embedding space. Take 
a closer observation of the logical units in Figure 1, the units are not 
completely separated. In this work, we pay much attention to the 
two explicit relationships (causal and co-occurrence). We summarize 
them into the logical branch and syntax branch respectively. From 
the logical branch, connectives (e.g., if, unless, Therefore) play a 
significant role in covering the logical relations. For example in 
Figure 1, the logical units U1 and U2 are connected with the con-
nective if while U3 and U4 share the connective unless. From the

The extraction of the logical units and the explicit relations 
are consistent with the human cognition. The uncovered 
logical structures and the weighted attention maps of the

### Question
The pattern of reasoning displayed above most closely parallels which of the following?

### Context
Paula will visit the dentist tomorrow morning: 
[if] Bill goes golfing in the morning: Bill will not go golfing. 
[unless] Damien agrees to go golfing too. 
[Therefore] Paula will not be visiting the dentist tomorrow morning.

### Options
A. If Marge goes to the bank today... Marge will wash her car and go shopping with Lauren.
B. Kevin will wash his car tomorrow only if... Kevin will not wash his car tomorrow.
C. Renee will do her homework tonight if there... Therefore, Renee will attend the party.
D. Maddie will plan a picnic only if... Therefore, Maddie will plan a picnic.

### Logical Units in Context

| Logical Units | Co-occurrence | Causal | Negation |
|---------------|---------------|--------|----------|
| Paula will visit the dentist tomorrow morning | U1 - U6 | U2 - U1 | U3 - U5 |
| Bill goes golfing in the morning | U1 - U6 | U2 - U1 | U3 - U5 |
| Bill will not go golfing | U1 - U6 | U2 - U1 | U3 - U5 |
| Damien agrees to go golfing too | U1 - U6 | U2 - U1 | U3 - U5 |
| Paula will not be visiting the dentist tomorrow morning | U1 - U6 | U2 - U1 | U3 - U5 |

### Figure 1: An example of the logical reasoning task and some detailed illustrations.
logical units provide the excellent interpretability for the logical reasoning process.

- Extensive experiments show that Logiformer outperforms the state-of-the-art (SOTA) results with single model on two logical reasoning datasets. Furthermore, ablation studies prove the effectiveness of each module in our model.

2 RELATED WORK

In this section, we will introduce the current researches on MRC and logical reasoning.

2.1 Machine Reading Comprehension

Recent years have witnessed the rapid growth of MRC[18], where the model is required to infer the answers based on the given context and a question. A variety of datasets have been proposed to check the performances of MRC models. Among them, SQuAD[23, 24] focuses on the span extractions on the factual questions. HotpotQA[34] and OpenBookQA[20] require the multi-hop reasoning capability of the models. A couple of multiple choice datasets like RACE[14] cover the examinations for middle or high school students. Some representative models achieve great success on these datasets. RetroReader [41] applies a two-state strategy to solve the questions. But it mainly investigates the overall interactions of the context and question, which fails to deal with the complex logic within the text. SG-Net [40] integrates the syntax information into the self-attention module to improve the performance, but it does not show the potential on tackling the logical information. Generally speaking, the datasets mentioned above rely much on the token-level matching, which can be well tackled with large-scale pretraining models like BERT[6] and GPT-3[2]. To make the models closer to the human intelligence, it is necessary to introduce more challenging tasks requiring logical reasoning. Previously, the task of Natural Language Inference(NLI)[1, 28] is proposed to motivate the models to infer the relations(i.e., Contradiction, Entailment and Neutral) between two sentences. Nevertheless, it is limited by the fixed inputs and outputs and fails to extend the task to more complex settings.

2.2 Logical Reasoning

To improve the reasoning ability of the models, several datasets on multiple choice have been proposed previously. ReClor[37], which is extracted from standardized graduate admission examinations and law school admission test, has aroused wide concerns. For better evaluation, it separates the biased examples into EASY set and the challenging ones into HARD set. LogiQA [17] is also one of the representatives, which also aims to improve the logical reasoning capability. It is sourced from expert-written questions and covers multiple types of deductive reasoning. Experiments show that previous SOTA models on traditional MRC perform bad on the two datasets. Under such circumstances, some of the recent works attempt to enhance logical reasoning from different perspectives. DAGN[12] proposes a reasoning network based on the discourse units extracted from the text. But it simply forms a chain-type discourse network and weakens the relations between two distant units. FocalReasoner [21] stresses that fact units in the form of subject-verb-object are significant for logical reasoning. It constructs a supergraph on top of the fact units and updates the node features relying on Graph Neural Network. However, it ignores the

relation connectives from the text and lacked the logical modeling. LReasoner [31] focuses on capturing symbolic logic from the text and puts forward a context extension framework based on logical equivalence laws. However, it relies heavily on the language models for token-level embedding and neglects the sentence-level interactions.

3 METHODOLOGY

This section will introduce the proposed end-to-end model Logiformer. The architecture of Logiformer is shown in Figure 2. The left part of the model is an example of the logical reasoning task. The understanding of text will be divided into two branches: logical branch (upper) and syntactic branch (lower). This architecture mainly includes the following three parts: a) graph construction from the text; b) logical-aware and syntax-aware graph transformers for feature updates; c) the decoder including a dynamic gate mechanism and a question-aware self-attention module.

3.1 Task Formulation

Given a dataset $\mathcal{D}$ for logical reasoning, which consists of $N$ examples totally. The inference of the $i^{th}$ question can be formulated as follows:

$$\hat{a} = \arg \max_{a_{i,j} \in A_i} (a_i, q_i, A_i; \theta),$$

where $c_i, q_i, A_i$ represent the context, question sentence and candidate set respectively. The number of options in $A_i$ is $n$, $j \in [0, n-1]$ and $a_{i,j} \in A_i$ represents the $j^{th}$ option. $\hat{a}$ denotes the predicted option. $\theta$ denotes the trainable parameters.

Since the current methods mainly focus on the token-level representation of the text with the help of PLMs, they will naturally ignore some global semantics for each sentence or phrase. To capture the global feature within each sentence, we first obtain the text fragments, which are split by connectives or punctuations. We define the text fragment that reflect the complete semantic of an event or argument as the logical unit with the symbol $U$. Take the context in Figure 2 as an instance, we split the text by both connectives and punctuations and obtain a set of logical units shown in Table 1.

| Symbol | Logical Units |
|--------|---------------|
| $U_1$  | Paula will visit the dentist tomorrow morning |
| $U_2$  | Bill goes golfing in the morning |
| $U_3$  | Bill will not go golfing |
| $U_4$  | Damien agrees to go golfing too |
| $U_5$  | Damien has decided not to go golfing |

Table 1: The set of logical units from the example context (split by connectives and punctuations).

Considering that there exist explicit causal relations between units, we further introduce the conditional connective ‘$\rightarrow$’. And in some cases, it is required to reverse logical units for the negation expression, we also employ the operation ‘$\neg$’. Combining the logical units and causal connections in the form of conjunction, we can derive the logical expression of the text:

$$(U_2 \rightarrow U_1) \land (U_4 \rightarrow \neg U_3) \land \neg U_5.$$
Figure 2: The architecture of Logiformer. The left part is an input example of the dataset. The graph construction modules (a1,a2) split the text into logical units and build two graphs from two branches respectively. The graph transformer structures (b1,b2) update the text features combined with the logical and syntactic relations. Finally, the decoder module (c) is utilized to conduct the feature fusion and predict the answers.

Obviously, there exist two key components in the logical expression: i) logical units \( U_k \); ii) logical connectives, i.e., \( \rightarrow \) and \( \neg \). The former one focuses on the syntactic information, while the latter one is more related to logical structure of the context.

3.2 Graph Construction

Given the \( i^{th} \) inputs, Logiformer first concatenates the context \( c_i \) with each option \( a_{i,j} \) respectively to form the input sequences. According to the previous analysis, Logiformer will tackle the inputs from two branches (i.e., logical branch and syntax branch) and build two graphs (i.e., logic graph and syntax graph) respectively.

3.2.1 Logical Graph. For the logical branch, Logiformer mainly concentrates on the causal relations. Considering that the causal relation often appears with explicit logical words such as \( if, unless, because \), we can leverage the explicit logical words as the basis of split. Therefore, we include 100 commonly used logical words according to PDTB 2.0 [22].

Combining the explicit logical words and punctuations, we can separate the text sequence into logical units. Each unit serves as a node for future updates. Especially, we pick out the nodes pairs connected by the explicit causal relation words and name them as condition node (orange nodes) and result node (blue nodes). Meanwhile, we classify the common nodes which do not contain causal relations into result node (blue nodes). Thus, we obtain the node set from the perspective of logic.

According to the extracted causal node pairs, we can create directed connection from each condition node \( p \) to result node \( q \). This kind of connection is reflected in the adjacent matrix \( M_{cas} \in \mathbb{R}^{K_{cas} \times K_{cas}} \) of the logical graph as \( M_{cas}[p-1, q-1] = 1 \).

Also, to avoid the semantic reverse brought by the negation, we mark the nodes with the explicit negation words (e.g., not, no). The node \( k \) with negation semantics are expressed in the adjacent matrix as \( M_{cas}[k-1, k-1] = -1 \).

Therefore, the logical graph has the perception of the causal relations and negations. And the obtained adjacent matrix \( M_{cas} \in \mathbb{R}^{K_{cas} \times K_{cas}} \) of the logical graph is asymmetric.

3.2.2 Syntax Graph. The main purpose of the syntactic understanding is to capture the inner relations between the logical units \( U_k \). Noticing that some logical units share the common words or phrases in Figure 2, e.g., Bill, Damien and go golfing. It illustrates that the text has a strong characteristics of co-occurrence. Also, co-occurrence usually exists between two complete sentences. Therefore, we consider to split the text sequence only by punctuations and obtain a set of sentence nodes with no original connection. It is required to extract the co-occurrence between the sentence nodes. As each node consists of its related tokens, we propose a simple strategy to capture the co-occurrence, shown in Algorithm 1.

Assume the total number of the nodes to be \( K_{occ} \). The input for the algorithm is the sentence node \( U_k \), corpus \( C_s \) containing redundant stop words and hyper-parameter \( \delta \). The output is an adjacent matrix \( M_{occ} \in \mathbb{R}^{K_{occ} \times K_{occ}} \), which reflects the co-occurrence relations between nodes.

As for any two nodes, we transform them into two token sets \( Set_p \) and \( Set_q \) separately, without order and duplicate elements (Line 3 & Line 5 in Algorithm 1). We define \( \text{len}(Set) \) to be the number of tokens in a set. Further, let the token overlap ratio
we take the novel architecture of graph transformer [3, 35] into account. After the extraction of nodes and the construction of two graphs, we feed them into the logical-aware and syntax-aware graph transformer structures respectively.

### 3.3 Graph Transformer

Some previous works [8, 38] point out the drawbacks of graph neural network, such as the issue of over-smooth [15]. Therefore, we take the novel architecture of graph transformer [3, 35] into account. After the extraction of nodes and the construction of two graphs, we feed them into the logical-aware and syntax-aware graph transformer structures respectively.

#### 3.3.1 Logical-aware Graph Transformer

The simple illustration of the logical-aware graph transformer is shown in Figure 3. First of all, it is necessary to get the original feature embedding for each node. Given the concatenated input sequence of the $t$th question:

$$\text{Input}(c_t, a_{ij}) = \{\text{CLS}\}c_t \{\text{SEP}\}a_{ij} \{\text{SEP}\}.$$

we employ the RoBERTa model [19] as the encoder for the token-level features. For the token sequence $\{i_1(k), i_2(k), ..., i_T(k)\}$ with the length $T$ of each node $U_k$, the obtained token embedding is represented as $\{v_1(k), v_2(k), ..., v_T(k)\}$. We take the average embedding of $T$ tokens as the original feature for node $U_k$:

$$v_k = \frac{1}{M} \sum_{i=1}^{M} v_i(k). \tag{4}$$

To keep the original order information of nodes in the text, positional embedding is added to the node representation.

$$V_i = V_o + \text{PosEmbed}(V_o), \tag{5}$$

where $V_o = [v_1; v_2; ...; v_{K_{\text{cat}}}]. V_o \in \mathbb{R}^{K_{\text{cat}} \times d}, d$ is the dimension of the hidden state, and $K_{\text{cat}}$ is the number of nodes. $\text{PosEmbed}(\cdot)$ provides a d-dimensional embedding for each node in the input sequence.

We feed the node representation $V_i$ into the logical-aware graph transformer. Firstly, $V_i$ is projected to three matrices $Q, K$ and $V$ of the self-attention module:

$$Q = V_i \cdot W^Q, \quad K = V_i \cdot W^K, \quad V = V_i \cdot W^V,$$

where $W^Q, W^K, W^V \in \mathbb{R}^{d \times d_k}$ are projection matrices, and the obtained matrices $Q, K, V \in \mathbb{R}^{K_{\text{cat}} \times d_k}$. Then, we compute the attention based on the query, key and value matrices.

$$A = \frac{QK^T}{\sqrt{d_k}} \tag{7}$$

$$\text{Att}(Q, K, V) = \text{softmax}(A) \cdot V,$$

where $A \in \mathbb{R}^{K_{\text{cat}} \times K_{\text{cat}}}$ is a weight matrix for node pairs. From the equations, the transformer structure provides a fully connected setting to all nodes, which ignores the inner causal relations. Therefore, Logiformer employs the obtained topology information $M_{\text{cat}} \in \mathbb{R}^{K_{\text{cat}} \times K_{\text{cat}}}$ of the logical graph as an attention bias. The representation of the weight matrix $A$ is adjusted as follows:

$$A' = \frac{QK^T}{\sqrt{d_k}} + M_{\text{cat}}. \tag{8}$$

To improve the robustness and capability of the attention module, we apply the multi-head attention mechanism with the head number $H$:

$$\text{Att}_{\text{MH}}(Q, K, V) = [\text{Head}_1; ...; \text{Head}_H] \cdot W^H, \tag{9}$$

where $W^H \in \mathbb{R}^{(H 	imes d_k) \times d_k}$ is the linear projection matrix, $\text{Head}_j = \text{Att}(Q, K, V)$, the input query, key and value matrices are obtained by the linear projections of $W^Q, W^K, W^V \in \mathbb{R}^{d \times d_k}$ respectively. For simplicity, we assume $d = d_k$ and omit the bias term of the linear projection.

Repeating the multi-head attention for $L$ layers, we take out the hidden states of the last two layers. To enhance the robustness of the model, we make a fusion of them as the updated node features:

$$V_{\text{cas}} = \gamma^{(L-1)} + V_{\text{cas}}^{(L)}, \quad V_{\text{cas}}^{(L)} \in \mathbb{R}^{K_{\text{cat}} \times d}, \quad V_{\text{cas}}^{(L-1)} \in \mathbb{R}^{K_{\text{cat}} \times d}, \quad V_{\text{cas}}$$

represents the hidden states of the last two layers respectively. Note that there are lots of ways of feature fusion, we only present the simple addition for illustration.
Table 2: Detailed Splits of ReClor and LogiQA.

| Dataset | #Train | #Valid | #Test | #Reason Type |
|---------|--------|--------|-------|--------------|
| ReClor  | 4,638  | 500    | 1,000 | 17           |
| LogiQA  | 7,376  | 651    | 651   | 5            |

The final feature $V$ can be represented in the following expression:

$$V = LN(V_t + \lambda \cdot V_{occ} + (1 - \lambda) \cdot V_{cas}),$$  \tag{12}

where $LN(\cdot)$ denotes the layer normalization operation. Since we do not employ the global node in the graph transformer, the global feature will not be updated. To this end, Logiformer integrates the local token-level features and gets the updated global information:

$$V_{cls} = LN(V_{t,cls} + \frac{1}{N-1} \sum_{i=1}^{N-1} (V_{occ,i} + V_{cas,i})).$$  \tag{13}

where $V_{t,cls}$ is the first token of the original token-level embedding. $V_{occ,i}$ and $V_{cas,i}$ represent the $i^{th}$ token embedding of the syntactic branch and logical branch feature respectively. We utilize the global feature $V_{cls}$ to replace the first token feature (i.e., [cls] feature) of $V$. $V$ can be expressed as the concatenation of $V_{cls}$, $V_{context}$ and $V_{option}$, that is $V = [V_{cls}; V_{context}; V_{option}]$.

To conduct the reasoning, the feature of the question $V_{question}$ is also of great significance. Logiformer applies a simple self-attention module for the global feature $V$ and question $V_{question}$. The updated question embedding is expressed as:

$$V'_{question} = \text{softmax}(\frac{V_{question} V^T}{\sqrt{d}}) \cdot V.$$  \tag{14}

For simplicity, the linear projections for the self-attention are omitted. At last, we concatenate the $V_{cls}$, $V_{context}$, $V_{option}$ and $V'_{question}$ to get the final feature $V_{final} \in \mathbb{R}^{N \times d}$:

$$V_{final} = [V_{cls}; V_{context}; V_{option}; V'_{question}].$$  \tag{15}

For each option in one example, we can get one specific final feature. They are fed into the feed forward network to obtain the scores, and we take the highest one as the predicted answer.

4 EXPERIMENTS

In this section, extensive experiments are conducted to compare our model with SOTA single model methods in both ReClor and LogiQA datasets. Ablation studies are followed to verify the effectiveness of the proposed modules.

4.1 Datasets and Baselines

4.1.1 Datasets. In this paper, we conduct the experiments on two logical reasoning datasets ReClor [37] and LogiQA [17]. ReClor consists of 6,138 examples sourced from some standardized tests, while LogiQA includes totally 8,678 questions collected from National Civil Servants Examinations of China. The detailed splits of both datasets are included in Table 2. It can be concluded that ReClor is more diverse in the number of logical reasoning types, while LogiQA contains more examples. Both of them are challenging for the task of logical reasoning.
4.2 Implementation Details

All of the experiments are conducted with a single GPU of Tesla V100. For the fair comparison, the RoBERTa-large model [19] is utilized as the encoder for text during the experiments and the hidden size is set to 1024. During the training process, the epoch number is fixed to 12 and the batch size is set to 2 for both ReClor and LogiQA datasets. We take Adam [13] with linearly-decayed learning rate and warm up and select peak learning rate as 5e-6. We select the model with best accuracy on the validation split to conduct the test. The details of important hyper-parameters and their search scopes are attached in Table 3.

4.3 Comparison Results

Logiformer is evaluated on two logical reasoning datasets. The main results on the validation split and test split of ReClor dataset are shown in Table 4. And the results on LogiQA dataset are shown in Table 5. The test split of ReClor dataset is organized into easy fold and hard fold, presented as ‘Test-E’ and ‘Test-H’ respectively in the table.

For the fair comparison, we consider the results with single model and with the encoder of RoBERTa for all the baselines. Compared with the SOTA results on two logical reasoning benchmarks, our proposed model Logiformer shows excellent improvements.

On the ReClor dataset, we witness the improvements of 2.20% and 1.10% on the validation and test split over previous SOTA model LReasoner. Since LReasoner does not make the results on Test-E and Test-H splits public, we omit the comparison. Compared with FocalReasoner on the validation split, Logiformer shows strong generalization capability with 1.6% and 4.6% improvements on the validation and test split. Especially on the Test-H split, 6.61% improvement proves our superiority for the more difficult logical reasoning questions. The most important observation is that Logiformer is the first single model with RoBERTa encoder to beat the human performance by 0.50% on the ReClor dataset. Although the machine still falls behind humans on more challenging questions, our proposed method is positively narrowing the gaps.

On the LogiQA dataset, Logiformer outperforms the previous SOTA model Logiformer by 1.13% and 2.30% on the validation and test split respectively. It proves the excellent generalization capability of Logiformer. However, we also discover the huge gap between humans and the machine. In view that the context in LogiQA dataset is organized in a more structural form, humans are easier to capture the inner logic. The deep learning based models are good at capturing the semantic changes and lack the perception of fixed logic.

4.4 Ablation Studies

Considering that the architecture of Logiformer is mainly divided into three parts: a) graph construction, b) graph transformer and c) decoder, the ablation studies are also laid out from these three aspects. The experimental results are shown in Table 6.

Firstly, in the part of graph construction, we build syntax graph and logical graph based on the different node extraction strategies. We ablate the effects of the two graphs in turn. That is to say, we only consider one of the branches each time. From the results, the logical graph contributes more to the performance on the ReClor dataset, which improves 4.80% and 3.60% on the validation and test
Table 6: Ablation Studies. The improvements on the accuracy are marked in red.

| Model               | ReClor         | LogiQA        |
|---------------------|----------------|---------------|
|                     | Valid  | Δ     | Test  | Δ     | Test-E | Δ     | Test  | Δ     |
| Logiformer          | 68.40  | -     | 63.50 | -     | 79.09  | 51.25 | 42.24 | -     |
| a) Graph Construction |       |       |       |       |        |       |       |       |
| w/o syntax graph    | 66.40  | -2.00 | 61.20 | -2.30 | 77.50  | 48.39 | 38.56 | -3.68 |
| w/o logical graph   | 63.60  | -4.80 | 59.90 | -3.60 | 75.00  | 48.04 | 38.25 | -3.99 |
| b) Graph Transformer |       |       |       |       |        |       |       |       |
| w/o co-occurrence bias | 66.80 | -1.60 | 62.80 | -0.70 | 77.05  | 51.61 | 41.94 | -0.30 |
| w/o causal bias     | 65.20  | -3.20 | 63.30 | -0.20 | 76.82  | 52.68 | 39.94 | -2.30 |
| w/o both of attention biases | 66.20 | -2.20 | 61.60 | -1.90 | 75.23  | 50.89 | 41.63 | -0.61 |
| c) Decoder          |       |       |       |       |        |       |       |       |
| w/o dynamic gates   | 67.00  | -1.40 | 61.90 | -1.60 | 76.14  | 50.71 | 41.32 | -0.92 |
| w/o question-aware attention | 66.60 | -1.80 | 60.40 | -3.10 | 76.36  | 47.86 | 41.63 | -0.61 |

Table 7: The details of ReClor Test Split on different question types. NA: Necessary Assumption, S: Strengthen, W: Weaken, CMP: Conclusion/Main Point, MSS: Most Strongly Supported, ER: Explain or Resolve, P: Principle, D: Dispute, R: Role, IF: Identify a Flaw, O: Others.

| Model               | NA  | S   | W   | I   | CMP | MSS | ER  | P   | D   | R   | IF  | O   |
|---------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Logiformer          | 74.56 | 64.89 | 55.75 | 45.65 | 75.00 | 66.07 | 61.90 | 69.23 | 70.00 | 75.00 | 58.12 | 60.27 |
| w/o syntax graph    | 70.18 | 59.57 | 55.75 | 45.65 | 66.67 | 57.14 | 67.86 | 56.92 | 56.67 | 62.39 | 57.53 |
| Δ                  | -4.38 | -5.32 | -   | -   | -8.33 | -8.93 | +5.96 | -12.31 | -13.33 | -25.00 | +4.27 | -2.74 |
| w/o logical graph   | 68.42 | 61.70 | 51.33 | 41.30 | 66.67 | 51.79 | 59.52 | 55.38 | 43.33 | 59.38 | 63.25 | 65.75 |
| Δ                  | -6.14 | -3.19 | -4.42 | -3.34 | -8.33 | -14.28 | -2.38 | -13.85 | -26.67 | -15.62 | +5.13 | +5.48 |

The syntax graph also shows 2.00% and 2.30% improvements on ReClor. As is mentioned above, we are the first to model the causal relations within the context in the task of logical reasoning. The effectiveness of logical graph also verifies our proposed method.

Secondly, we explore the impact of two attention biases on the model performance. Thus, we ablate the effects of one or both of attention bias matrices. From the results, co-occurrence bias and causal bias have different effects on the two splits of ReClor dataset, where the former one contributes more to the test split and the latter one is more helpful to the validation split. Meanwhile, positive effects are witnessed by applying both of the attention biases to the graph transformer, leading to 1.90% and 2.61% on the test split of ReClor and LogiQA respectively. Combining the ablation results for graph construction module, the fully connected structure of the logical units itself also has a positive role in the model performance.

Thirdly, we focus on the effectiveness of two important parts in the decoder. For the proposed dynamic gate mechanism, we set each element of the gate parameter vector $\lambda \in \mathbb{R}^{N \times 1}$ to 0.5 to ablate the effect of gates. The results show that dynamic gate mechanism contributes 6.1% improvement to the test split of ReClor, but does not have effects on that of LogiQA. It may result from the characteristics of LogiQA dataset, which require the equal contribution of syntax and logical information. For the question-aware attention, we remove the self-attention module and use the original token-level representation of the question to form the final vector. The ablation results illustrate that the update of the question feature contributes a lot to the model performance, especially for the ReClor dataset. Considering that the question types are various on the ReClor dataset, the awareness of the question sentence is of great help.

Additionally, we present the detailed results of ReClor test split on different question types and also list the corresponding ablation results of two graphs in Figure 7. The majority of the types witness the significant improvements, especially for Principle, Dispute and Role. It illustrates that Logiformer has the advantages of inferring the hidden fact or truth within the context. A few types, such as Explain or Resolve and Identify a Flaw, show a downward trend. We blame this issue to the lack of modeling on negation. For example, the type of Identify a Flaw requires the model to figure out the most weakness one from the options, which is sentimentally opposite to the most of the types. The feature distribution obtained from the current language model is insufficient to clearly distinguish the implicit opposite semantics. Therefore, the modeling of sentimentally negative questions is worth exploring in the future work.

4.5 Supplementary Analysis

During the experiments, hyper-parameters are utilized in many places. Limited by the space, we only select one of them to conduct the analysis, which is the overlap threshold $\delta$ for the extraction of co-occurrence. The results of different values of $\delta$ are shown in Figure 4.

Results illustrate that the best performance is achieved when $\delta$ is equal to 0.5. When the hyper-parameter $\delta$ drops, it means that more co-occurrence pairs will be extracted, leading to much extra noise. When $\delta$ reaches 0.7, the number of co-occurrence relations is limited, the performance of the model still maintains at a high level. It further proves the robustness of Logiformer.
with 9-12 logical units account for the largest proportion. Most will study the influence of the number of logical units on the accuracy. The topology of the graphs from the logical branch and the syntax branch. In this case, the ReClor dataset to illustrate the logical reasoning process in Logiformer. In Figure 6, we present a successful case on the validation split of the ReClor dataset under different $\delta$.

Also in Logiformer, logical units are important parts. Thus, we will study the influence of the number of logical units on the accuracy. We conduct the analysis on the validation split of ReClor dataset. The number of logical units are obtained based on the split of relation connectives and punctuations. The results are presented in Figure 5. The green bar represents the accuracy and the yellow bar is the number of examples. From the statistics, the examples with 9-12 logical units account for the largest proportion. Most importantly, the accuracy remains stable for the different number of logical units. On one hand, it proves the effectiveness of the split of logical units. On another hand, we attribute the results to the employment of the graph transformer. Fully connected structure tackles the issue of long distance dependency, which reduces the impact of the increase in the number of logical units.

5 CASE STUDY

In Figure 6, we present a successful case on the validation split of the ReClor dataset to illustrate the logical reasoning process in Logiformer. The basis of the reasoning process is the two constructed graphs from the logical branch and the syntax branch. In this case, Logiformer extracts 11 logical units (named from $A$ to $K$) based on punctuations and relation connectives for the logical branch. The split results are consistent with our expectation, and among them 4 pairs of causal units ($C-D, E-F, H-I, J-K$) are detected. For the syntax branch, the concatenated text is split into 7 sentence nodes (named from $a$ to $g$). Among them, 3 logical units ($a, d, f$) are detected as the co-occurrence relations. The topology of the two graphs provides the explicit understanding of the text, which is a key point to the interpretability of Logiformer. In addition, we present the attention maps in the final layer of the graph transformers from both branches (blue one for logical branch, orange one for syntax branch). The data in the attention matrices is mapped to the range of $[0,1]$ for better illustration. Darker color indicates the stronger correlations between two logical units. The weighted attention maps well reflect the relations and provide a boarder view for interpretability.

Meanwhile, we observe a drawback in this case. he gets married and his wedding are two similar expressions in semantic, indicating the similar meanings. However, they have no overlap of words and are not detected as the co-occurrence relation. This detail is worthy of studying in the future, which is beneficial to the fine-grained understanding of the logical text.

6 CONCLUSION AND FUTURE WORK

We propose a two-branch graph transformer network for logical reasoning of text, which is named as Logiformer. Firstly, we introduce two different strategies to construct the logical graph and syntax graph respectively. Especially for the logical graph, we are the first to model both causal relations and negations in the logical reasoning task. Secondly, we feed the extracted node sequences to the fully connected graph transformer for each graph. The topology of the graph is utilized to form the attention bias for the self-attention layers. Thirdly, a dynamic gate mechanism is applied to make a fusion of the features from two branches. To improve the awareness of different question types, the question feature is updated based on the self-attention module. Finally, the concatenated text sequence is passed through the feed forward layer and obtains the answer prediction. The whole reasoning process provides the interpretability, reflected by logical units with explicit relations and the visualization of the attention maps.

In the future, we will explore the role of question to further improve the interpretability [29]. Also, we are interested in extending the logical expressions based on contrastive learning, like [16].

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Context

Dennis will either purchase his mother’s house and live in San Diego, or else he will move to Miami, but Dennis will not do either of these unless he gets married. Dennis’s mother will let Dennis purchase her house only if he attends his wedding, but not otherwise. Therefore Dennis will purchase his mother’s house and live in San Diego only if his mother attends his wedding.

Question

Which one of the following, if assumed, allows the conclusion above to be properly drawn?

Options

A. Dennis will purchase his mother's house if his mother allows him to purchase the house.

*Take Option A as an example*

**Figure 6:** The illustration of an successful case. The interpretability of Logiformer lies in the logical units in text with explicit

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