Abstract. We discuss the double-spin asymmetries in transversely polarized Drell-Yan process, calculating all-order gluon resummation corrections up to the next-to-leading logarithmic accuracy. This resummation is relevant when the transverse-momentum $Q_T$ of the produced lepton pair is small, and reproduces the (fixed-order) next-to-leading QCD corrections upon integrating over $Q_T$. The resummation corrections in $p\bar{p}$-collision behave differently compared with $pp$-collision cases, and are small at the kinematics in the proposed GSI experiments. This fact allows us to predict large value of the double asymmetries at GSI, using recent empirical information on the transversity.

Keywords: Transversity, Drell-Yan process, Antiprotons, Soft gluon resummation

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The double-spin asymmetry in Drell-Yan (DY) process with transversely-polarized protons and antiprotons, $p^1\bar{p}^1 \rightarrow l^+l^- X$, for azimuthal angle $\phi$ of a lepton measured in the rest frame of the dilepton $l^+l^-$ with invariant mass $Q$ and rapidity $y$, is given by

$$A_{TT} = \frac{d\sigma^{\uparrow\uparrow}/d\omega - d\sigma^{\uparrow\downarrow}/d\omega}{d\sigma^{\uparrow\uparrow}/d\omega + d\sigma^{\downarrow\downarrow}/d\omega} = \frac{\Delta_T d\sigma/d\omega}{d\sigma/d\omega} = \cos(2\phi) \frac{\sum_q e_q^2 \delta q(x_1, Q^2) \delta q(x_2, Q^2) \cdots}{\sum_q e_q^2 q(x_1, Q^2) q(x_2, Q^2) \cdots}$$

(1)

where $d\omega \equiv dQ^2 dy d\phi$, the summation is over all quark and antiquark flavors with $\delta q(x, Q^2)$ and $q(x, Q^2)$ being the transversity and unpolarized quark-distributions inside a proton, and the ellipses stand for the corrections of next-to-leading order (NLO) and higher in QCD perturbation theory. The scaling variables $x_{1,2}$ represent the momentum fractions associated with the partons annihilating via the DY mechanism, such that $Q^2 = (x_1 P_1 + x_2 P_2)^2 = x_1 x_2 S$ and $y = (1/2) \ln(x_1/x_2)$, where $S = (P_1 + P_2)^2$ is the CM energy squared of $p^1\bar{p}^1$. In the proposed polarization experiments at GSI [1], moderate energies, $30 \lesssim S \lesssim 200$ GeV$^2$, allow us to measure (1) for $0.2 \lesssim Q/\sqrt{S} \lesssim 0.7$, and probe the product of the two quark-transversities in the “valence region”. Recently, QCD corrections in (1) at GSI kinematics have been studied: the NLO ($O(\alpha_s)$) corrections [2] as well as the higher order ones in the framework of threshold resummation [3] are rather small, so that the LO value of $A_{TT}$, which turns out to be large, is rather robust.

When the transverse momentum $Q_T$ of the final $l^+l^-$ is also observed, we obtain the new double-spin asymmetry as the ratio of the $Q_T$-differential cross sections, $A_{TT}(Q_T) \equiv [\Delta_T d\sigma/d\omega dQ_T]/[d\sigma/d\omega dQ_T]$. The bulk of $l^+l^-$ pair is produced at small $Q_T \ll Q$, where the cross sections $(\Delta_T) d\sigma/d\omega dQ_T$ receive large perturbative corrections with logarithms $\ln(Q^2/Q_T^2)$ multiplying $\alpha_s$ at each order, by the recoil from gluon
radiations, and those have to be treated in an all-order resummation in QCD \cite{4}. The corresponding “$Q_T$-resummation” has formally some resemblance to the threshold resummation \cite{3}, but embodies contributions from different “edge region” of phase space. The $Q_T$-resummation for $\Delta_T d\sigma / d\omega dQ_T$ has been derived recently \cite{5}, summing the corresponding large logarithms up to next-to-leading logarithmic (NLL) accuracy. Combined with that for $d\sigma / d\omega dQ_T$ \cite{4}, we get \cite{6} ($b_0 \equiv 2e^{-\gamma_E}$ with $\gamma_E$ the Euler constant)

$$\mathcal{A}_{TT}(Q_T) = \frac{\cos(2\phi)}{2} \int d^2b \, e^{bQ_T} e^{S(b,Q)} \sum_q e_q^2 q^0 \delta q(x_1, b_0^2/b^2) \delta q(x_2, b_0^2/b^2) + \cdots,$$

(2)

where the numerator and denominator are, respectively, reorganized in the impact parameter $b$ space in terms of the Sudakov factor $e^{S(b,Q)}$ resumming soft and flavor-conserving collinear radiation, while the ellipses involve the remaining contributions of the $O(\alpha_s)$ collinear radiation, which can be absorbed into the exhibited terms as $\delta q \to \Delta_T C_{qg} \otimes \delta q$, $q \to C_{qg} \otimes q + C_{gq} \otimes g$ using the corresponding coefficient functions $(\Delta_T)C_{ij}$; note that there is no “chiral-odd” gluon distribution to participate in the numerator of \cite{2} as well as \cite{1}. Using universal Sudakov exponent $S(b,Q)$ with the first nonleading anomalous dimensions in \cite{2}, the first three towers of large logarithmic contributions to the cross sections, $\alpha_s^m \ln^n(Q^2 / Q_T^2) / Q_T^2$ ($m = 2n - 1, 2n - 2, 2n - 3$), are resummed to all orders in $\alpha_s$, yielding the NLL resummation. In addition to these resummed components relevant for small $Q_T$, the ellipses in \cite{2} also involve the other terms of the fixed-order $\alpha_s$, which treat the LO processes in the large $Q_T$ region, so that \cite{2} is the ratio of the NLL+LO polarized and unpolarized cross sections. We include a Gaussian smearing as usually as $S(b,Q) \to S(b,Q) - g_{NP} b^2$, corresponding to intrinsic transverse momentum of partons inside proton. The integrations of the NLL+LO cross sections $\Delta_T d\sigma / d\omega dQ_T$ and $d\sigma / d\omega dQ_T$ over $Q_T$ coincide, respectively, with the (fixed-order) NLO cross sections $\Delta_T d\sigma / d\omega$ and $d\sigma / d\omega$, associated with $A_{TT}$ of \cite{1} \cite{5, 6}.

The resummation makes $1/b \sim Q_T$ the relevant scale in \cite{2}, in contrast to $Q$ in \cite{1}. Figure \cite{1} shows \cite{2} the numerical evaluation of \cite{2}, as well as of $\Delta_T d\sigma / d\omega dQ_T$ associated with its numerator, at GSI kinematics using the NLO transversities that saturate the Soffer bound, $2Q \delta(x, \mu^2) \leq q(x, \mu^2) + \Delta q(x, \mu^2)$, at a low scale $\mu$ with $\Delta q$ the helicity distribution. The NLL resumsed component dominates $\Delta_T d\sigma / d\omega dQ_T$ in small and moderate $Q_T$ region, and similarly for $d\sigma / d\omega dQ_T$ reflecting universality of the large Sudakov effects, which leads to almost constant $\mathcal{A}_{TT}(Q_T)$, in particular, with even flatter behavior than the corresponding asymmetry \cite{5} for the $pp$-collision case. Remarkably, $\mathcal{A}_{TT}(Q_T)$ at NLL+LO has almost the same value as that at LL; this is in contrast to the $pp$ case where the resummation at higher level enhances the asymmetry \cite{5}. We note that $\mathcal{A}_{TT}(Q_T)$ at LL is given by \cite{2} omitting all nonleading corrections, i.e., omitting the ellipses, replacing $S(b,Q)$ by that at the LL level, and replacing the scale of the parton distributions as $b_0^2/b^2 \to Q^2$, so that the result coincides with $A_{TT}$ at LO (see \cite{1}). Therefore, at GSI, both $\mathcal{A}_{TT}(Q_T)$ and $A_{TT}$ are quite stable when including the QCD (resummation and fixed-order) corrections, with $\mathcal{A}_{TT}(Q_T) \simeq \mathcal{A}_{TT}(0)$, and

$$\mathcal{A}_{TT}(Q_T) \simeq A_{TT}.$$  

(3)

To clarify the relevant mechanism, $\mathcal{A}_{TT}(Q_T) \simeq \mathcal{A}_{TT}(0)$ allows us to consider the $Q_T \to 0$ limit of \cite{2}: for $Q_T \approx 0$, the $b$ integral is controlled by a saddle point $b = b_{SP}$,
which has the same value between the numerator and denominator in (2) such that [5,6]

$$\mathcal{A}_{TT}(0) \simeq \frac{\cos(2\phi)}{2} \frac{\sum_{q} e_{q}^{2} \delta q(x_{1}, b_{0}^{2}/b_{SP}^{2}) \delta q(x_{2}, b_{0}^{2}/b_{SP}^{2})}{\sum_{q} e_{q}^{2} q(x_{1}, b_{0}^{2}/b_{SP}^{2})q(x_{2}, b_{0}^{2}/b_{SP}^{2})},$$

(4)

omitting the small corrections from the LO components involved in the ellipses in (2). This saddle-point evaluation is exact at NLL accuracy; in particular, the $O(\alpha)$ contributions from the coefficients $(\Delta T)C_{ij}$, e.g., $C_{qg} \otimes g$ associated with gluon distribution, completely decouple as $Q_{T} \to 0$ (see [4,5]). The simple form of (4) is reminiscent of $A_{TT}$ of (1) at LO, but is different from the latter in the unconventional scale $b_{0}^{2}/b_{SP}^{2}$; the actual position of the saddle point implies $b_{0}/b_{SP} \simeq 1$ GeV, irrespective of the values of $Q$ and $g_{NP}$ [5,6]. In the valence region relevant for GSI kinematics, the $u$-quark contribution dominates in (4) and (1), so that these asymmetries are controlled by the ratio, $\delta u(x_{1,2}, \mu^{2})/u(x_{1,2}, \mu^{2})$ with $\mu^{2} = b_{0}^{2}/b_{SP}^{2}$ and $Q^{2}$, respectively. Actually the scale dependence in this ratio almost cancels between the numerator and denominator as $\delta u(x, b_{0}^{2}/b_{SP}^{2})/u(x, b_{0}^{2}/b_{SP}^{2}) \simeq \delta u(x, Q^{2})/u(x, Q^{2})$ (see Fig. 3 in [6]); this implies (3) at GSI. Note that this is not the case for $pp$ collisions because of very different behavior of the sea-quark components under the evolution between transversity and unpolarized distributions [5]; indeed $\mathcal{A}_{TT}(Q_{T}) > A_{TT}$ at RHIC and J-PARC [5]. A similar logic applied to (2) also explains why $\mathcal{A}_{TT}(Q_{T})$ at GSI are flatter than in $pp$ collisions.

Another consequence of the similar logic is that $\delta u(x_{1,2}, 1$ GeV$^{2})/u(x_{1,2}, 1$ GeV$^{2})$ as a function of $x_{1,2}$ directly determines the $Q$- as well as $S$-dependence of the value of (3) at GSI, through $x_{1,2} = (Q/\sqrt{S})e^{\pm y}$. In Fig. 2 using the NLO transversity distributions corresponding to the Soffer bound, the symbols “triangle” plot $\mathcal{A}_{TT}(Q_{T} \simeq 1$ GeV) of (2) at NLL+LO [6]. The dashed curve draws the result using (4); this simple formula indeed works well. Also plotted by the two-dot-dashed curve is $A_{TT}$ of (1) at LO with the transversities corresponding to the Soffer bound at LO level, to demonstrate (3). The $Q$- and $S$-dependence of these results reflect that the ratio $\delta u(x, 1$ GeV$^{2})/u(x, 1$ GeV$^{2})$ is an increasing function of $x$. These results using the Soffer bound show the “maximally possible” asymmetry, i.e., optimistic estimate. A more realistic estimate of (2) with $Q_{T} \simeq 1$ GeV and (4) is shown [6] in Fig. 2 by the symbols “n” and the dot-dashed curve, respectively, with the NLO transversity distributions assuming $\delta q(x, \mu^{2}) = \Delta q(x, \mu^{2})$ at

FIGURE 1. (a) $\Delta T d\sigma/dQ_{T}$ (b) $\mathcal{A}_{TT}(Q_{T})$ with GSI kinematics, $S = 210$ GeV$^{2}$, $Q = 4$ GeV, $y = \phi = 0$, and with $g_{NP} = 0.5$ GeV$^{2}$, using the NLO transversity distributions that correspond to the Soffer bound.
a low scale $\mu$, as suggested by nucleon models and favored by the results of empirical fit for transversity [7]. The new estimate gives smaller asymmetries compared with the Soffer bound results, but still yields rather large asymmetries [6]. Based on (3), these results in Fig. 2 may be considered as estimate of $A_{TT}$ of (1).

At present, empirical information of transversity is based on the LO global fit, using the semi-inclusive DIS data and assuming that the antiquark transversities vanish, $\delta \bar{q}(x) = 0$, so that the corresponding LO parameterization is available only for $u$ and $d$ quarks [7]. Fortunately, however, the dominance of the $u$-quark contribution in the GSI kinematics allows quantitative evaluation of $A_{TT}$ at LO using only this empirical information [6]; the upper limit of the error band for the $u$- and $d$-quark transversities obtained by the global fit [7] yields the “upper bound” of $A_{TT}$ shown by the dotted curve in Fig. 2 using (3), this result may be considered also as estimate of $A_{TT}(Q_T)$. In the small $Q$ region, our full NLL+LO result of $A_{TT}(Q_T \approx 1 \text{ GeV})$, shown by “▽”, can be consistent with estimate using the empirical LO transversity, but these results have rather different behavior for increasing $Q$, because the $u$-quark transversity for the former lies slightly outside the error band of the global fit for $x \gtrsim 0.3$ [6]. Thus, in the large asymmetries to be observed at GSI, the behavior of $A_{TT}(Q_T)$, $A_{TT}$ as functions of $Q$ will allow us to determine the detailed shape of transversity distributions.

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REFERENCES

1. V. Barone et al. [PAX Collaboration], arXiv:hep-ex/0505054.
2. V. Barone, A. Caferella, C. Coriano, M. Guzzi and P. G. Ratcliffe, Phys. Lett. B639, 483 (2006).
3. H. Shimizu, G. Sterman, W. Vogelsang and H. Yokoya, Phys. Rev. D71, 114007 (2005).
4. J. C. Collins, D. E. Soper and G. Sterman, Nucl. Phys. B250, 199 (1985).
5. H. Kawamura, J. Kodaira, H. Shimizu and K. Tanaka, Prog. Theor. Phys. 115, 667 (2006); H. Kawamura, J. Kodaira and K. Tanaka, Nucl. Phys. B777, 203 (2007); H. Kawamura, J. Kodaira and K. Tanaka, Prog. Theor. Phys. 118, 581 (2007).
6. H. Kawamura, J. Kodaira and K. Tanaka, Phys. Lett. B662, 139 (2008).
7. M. Anselmino et al., Phys. Rev. D75, 054032 (2007); arXiv:0812.4366 [hep-ph].