Dark Matter that can form Dark Stars

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ABSTRACT: The first stars to form in the Universe may be powered by the annihilation of weakly interacting dark matter particles. These so-called dark stars, if observed, may give us a clue about the nature of dark matter. Here we examine which models for particle dark matter satisfy the conditions for the formation of dark stars. We find that in general models with thermal dark matter lead to the formation of dark stars, with few notable exceptions: heavy neutralinos in the presence of coannihilations, annihilations that are resonant at dark matter freeze-out but not in dark stars, some models of neutrinoophilic dark matter annihilating into neutrinos only and lighter than about 50 GeV. In particular, we find that a thermal DM candidate in standard Cosmology always forms a dark star as long as its mass is heavier than $\simeq 50$ GeV and the thermal average of its annihilation cross section is the same at the decoupling temperature and during the dark star formation, as for instance in the case of an annihilation cross section with a non–vanishing $s$-wave contribution.

KEYWORDS: Dark Star, Thermal Dark Matter, MSSM, leptophilic, neutrinoophilic.
1. Introduction

The first stars, also referred to as Population III stars, are the first luminous objects in the Universe. They contribute to the reionization of the interstellar medium, they provide the heavy elements (metals) that eventually become part of the later generations of stars, and they may be the seeds of the very massive black holes observed in quasars.

It was shown in [1, 2, 3] that the first stars to form in the Universe may be powered by the annihilation of dark matter particles instead of nuclear fusion. These dark-matter powered stars, or dark stars for short, constitute a new phase of stellar evolution. Besides the assumption that dark matter is made of weakly interacting massive particles (WIMPs) that can self-annihilate into ordinary particles, three conditions are necessary for the formation of a dark star.

The first condition is that the density of dark matter at the location of the (proto)star must be high enough for dark matter to efficiently and rapidly annihilate into ordinary particles, releasing a large amount of energy. The first stars are believed to form at the center of dark matter halos when the Universe was young (redshift $z \sim 10-50$) and denser than today. Not only the dark matter density at the center of those early halos was high, but as the baryonic gas contracted into the first protostars, more dark matter was gathered around the forming object by the deepening of the gravitational potential (gravitational contraction). Cosmological parameters and the evolution of the gas density completely determine the resulting density of dark matter at the location of the first protostars. Analytic and numerical evaluations [1, 2, 3] lead to a resulting density which is high enough to satisfy the first condition for the formation of a dark star.
The second condition is that a large fraction of the energy released in the dark matter annihilation must be absorbed in the gas that constitutes the (proto)star. The fraction $f_Q$ of annihilation energy deposited into the gas depends on the nature of the annihilation products. Typical products of WIMP annihilation are charged leptons, neutrinos, hadrons, photons, $W$ and/or $Z$ bosons, and Higgs bosons. The latter ($W$, $Z$, and Higgs) decay rapidly into leptons and hadrons. The hadrons themselves, which are mostly charged and neutral pions, decay rapidly into charged leptons, neutrinos, and photons (although a small number of stable particles like protons can also be produced). After $\sim 10^{-8}$ seconds, all unstable elementary particles, including the muon, have decayed away, and only protons, electrons, photons and neutrinos survive. Protons have a large scattering cross section with the protostar medium and are quickly absorbed. Electrons and photons can ionize the medium and/or generate electromagnetic showers. For WIMPs with mass $m \gtrsim 0.5$ GeV, electromagnetic showers are the dominant process. At the time when the following third condition for a dark star is satisfied, the protostar has a diameter of more than 40 radiation lengths, implying that all the energy released in protons, electrons, and photons is absorbed inside the protostar. Only the fraction of energy carried away by the neutrinos is lost for what concerns a dark star.

The third condition for the formation of a dark star is that the heating of the (proto)star gas arising from the dark matter annihilation energy must dominate over any cooling mechanism that affects the evolution of the (proto)star. In Ref. [1], it was shown that the dark matter heating rate $Q_{DM}$, in energy deposited per unit time and unit volume, is given by the expression

$$Q_{DM} = f_Q \frac{\langle \sigma v \rangle_{ds}}{m} \rho^2,$$

where $\rho$ is the dark matter density inside the (proto)star, which is determined by the cosmological model, and $\langle \sigma v \rangle_{ds}$ is the average value of the dark matter annihilation cross section $\sigma$ times WIMP relative velocity $v$ inside a dark star. To the extent that electromagnetic showers are generated, i.e. $m \gtrsim 0.5$ GeV, all dark star properties depend on the particle physics model only through the quantity

$$f_Q \frac{\langle \sigma v \rangle_{ds}}{m}.$$  \hfill (1.2)

Ref. [1] fixed the annihilation cross section to $\langle \sigma v \rangle_{ds} = 3 \times 10^{-26}$ cm$^3$/s and examined a range of WIMP masses $m$ from 1 GeV to 10 TeV. In addition, Ref. [1] assumed $f_Q = 2/3$, based on simulations of neutralino dark matter annihilation in the Minimal Supersymmetric Standard Model (MSSM). For this range of $Q_{DM}$, they compared the heating and cooling rates along protostar evolution tracks from [5], and concluded that there is a time during the evolution of the protostar in which the dark matter heating dominates over all cooling rates. This finding lead to the realization that dark stars may be possible.

In this paper, we examine the possible values of $Q_{DM}$ for a large selection of particle physics models, and verify if the third condition above is satisfied in these models. We find that not all particle dark matter models lead to the formation of dark stars, although the models that do not form dark stars are either tuned to resonant annihilation or rather artificial.
The restriction imposed by the third condition for dark star formation is best expressed in terms of a condition on the quantity $f_Q \langle \sigma v \rangle_{ds}/m$. Following [1], we have computed the critical lines in the gas temperature-density plane at which the heating rate from dark matter annihilation equals the total cooling rate. These lines are shown in Figure 1 for a wide range of values of $f_Q \langle \sigma v \rangle_{ds}/m$, from $10^{-18}$ cm$^3$ s$^{-1}$ GeV$^{-1}$ to $10^{-32}$ cm$^3$ s$^{-1}$ GeV$^{-1}$. Below the latter value, the heating-cooling critical line no longer intersects the thermodynamic track of the protostellar gas, indicated by the gray band obtained through numerical simulations of the formation of the first stars in a ΛCDM cosmology [5]. In other words, for $f_Q \langle \sigma v \rangle_{ds}/m < 10^{-32}$ cm$^3$ s$^{-1}$ GeV$^{-1}$, the protostar is expected to contract to a regular Population III star powered by nuclear fusion without passing through the dark star phase. At the other side of the $f_Q \langle \sigma v \rangle_{ds}/m$ range, the critical line reaches a limiting curve given by the vertical line labeled $f_Q \langle \sigma v \rangle_{ds}/m = 10^{-18}$ cm$^3$ s$^{-1}$ GeV$^{-1}$. Larger values of $f_Q \langle \sigma v \rangle_{ds}/m$ give the same vertical line. Thus, as expected, if the annihilation rate is large, a protostar passes through the dark star phase. Therefore the third condition for the formation of a dark star is

$$f_Q \langle \sigma v \rangle_{ds}/m > 1 \times 10^{-32} \text{ cm}^3 \text{ s}^{-1} \text{ GeV}^{-1}. \quad (1.3)$$

The choice $\langle \sigma v \rangle_{ds} = 3 \times 10^{-26}$ cm$^3$/s in [1] was motivated by the assumption that the dark matter WIMPs are produced thermally in the early Universe. That is, that the WIMPs are generated in matter-antimatter collisions at temperatures higher than $T_{fo} \sim m/20$, which is the temperature after which WIMP production “freezes out” and...
the comoving WIMP number density remains (approximately) constant. Ref. \[1\] used the following simple inverse-proportionality relation between the present WIMP density $\Omega_\chi$ and the annihilation cross section $\langle \sigma v \rangle_{f_0}$ at the time of WIMP freeze-out,

$$\Omega_\chi h^2 = \frac{3 \times 10^{-27} \text{cm}^3/\text{s}}{\langle \sigma v \rangle_{f_0}}. \quad (1.4)$$

Furthermore, ref. \[1\] simply assumed that the velocity-averaged annihilation cross section times relative velocities at the time of freeze-out and in a dark star have the same value, $\langle \sigma v \rangle_{ds} = \langle \sigma v \rangle_{f_0}$. In reality, the relation between $\Omega_\chi$ and $\langle \sigma v \rangle_{f_0}$ is more complex, and in addition $\langle \sigma v \rangle_{ds}$ may differ from $\langle \sigma v \rangle_{f_0}$ because $\sigma v$ may depend sensitively on the WIMP velocity $v$. In this regard, we notice that the average WIMP speed at freeze-out is of the order of

$$v_{f_0} \sim \sqrt{\frac{T_{f_0}}{m}} \sim \frac{c}{\sqrt{20}} \sim 7 \times 10^4 \text{ km/s}, \quad (1.5)$$

while the typical speed of WIMPs in a dark star can be estimated from the orbital velocity

$$v_{ds} \sim \sqrt{\frac{GM}{r}} \sim 30 \text{ to } 300 \text{ km/s}, \quad (1.6)$$

namely $\sim 30 \text{ km/s}$ for a newly-born $1-M_\odot$ dark star of 1 AU radius or $\sim 300 \text{ km/s}$ for a mature $600-M_\odot$ dark star of 5 AU radius.

A neutralino in the MSSM provides an example of a more complex relation between $\Omega_\chi$ and $\langle \sigma v \rangle_{f_0}$. At the same time, it allows the direct evaluation of both $\langle \sigma v \rangle_{ds}$ and $f_Q$, and in general it has $\langle \sigma v \rangle_{ds} \neq \langle \sigma v \rangle_{f_0}$. Section 2 explores this case.

Kaluza-Klein dark matter is examined in Section 3, where it is concluded that generically in these models $\langle \sigma v \rangle_{ds}$ tends to be larger or comparable to $\langle \sigma v \rangle_{f_0}$.

Leptophilic models of dark matter proposed to explain the PAMELA positron excess and the Fermi and HESS cosmic-ray electron-positron data provide another example in which $\langle \sigma v \rangle_{ds}$ may not be the same as $\langle \sigma v \rangle_{f_0}$. They are examined in Section 4.

Finally, we push $f_Q$ down using dark matter particles that annihilate exclusively into neutrinos ("neutrinophilic" models). In these models, even if annihilation produces predominantly neutrinos that escape the forming star, W- and Z-bremsstrahlung processes may generate enough charged leptons to actually form a dark star. We examine this case in Section 5.

2. MSSM

Because of its many free parameters (more than 100), the MSSM provides a variety of examples in which the annihilation cross section in the dark star differs from the annihilation cross section at the time of freeze-out, or more precisely $\langle \sigma v \rangle_{ds} \neq \langle \sigma v \rangle_{f_0}$.

There are several ways in which the equality $\langle \sigma v \rangle_{ds} = \langle \sigma v \rangle_{f_0}$ can be violated in the MSSM. First, the quantity $\sigma v$ may depend on the relative velocity $v$. This includes three cases: (i) $p$-wave annihilation in which $\sigma v = a + bv^2$ is dominated by the $bv^2$ term at freeze-out (here $a$ and $b$ are constants); (ii) resonant annihilation in which $\sigma v$ follows
a Breit-Wigner function \((1 + v^2)^c/[(v^2 + \delta)^2 + \gamma^2]\), where \(\delta, \gamma\) and \(c\) are constants; and

(iii) threshold annihilation in which an annihilation channel is kinematically accessible at freeze-out but not in a dark star thanks to the higher particle kinetic energies at freeze-out. Second, the annihilation reactions that determine the freeze-out time may be unrelated to the neutralino-neutralino annihilation that occurs inside a dark star, in that the freeze-out temperature may be high enough to convert neutralinos into heavier supersymmetric particles that annihilate much faster (a phenomenon called coannihilation). It is then the annihilation cross section of the heavier supersymmetric particles that determines the neutralino relic density, and this cross section is in general not the same as the neutralino-neutralino annihilation cross section, thus \(\langle \sigma v \rangle_{\text{ds}} \neq \langle \sigma v \rangle_{\text{fo}}\). We remark in passing that resonant annihilation and coannihilations are not rare phenomena in the MSSM, and are actually essential to obtain a neutralino dark matter in the minimal supergravity or constrained MSSM models.

To illustrate these four cases (\(p\)-wave annihilation, resonant annihilation, threshold annihilation, and coannihilation), it is sufficient to consider a so-called effective MSSM (effMSSM) with eight free parameters fixed at the electroweak scale [21]. These parameters are: the CP-odd Higgs boson mass \(m_A\), the ratio of neutral Higgs vacuum expectation values \(\tan\beta\), the Higgs mass parameter \(\mu\), the gaugino mass parameters \(M_1\) and \(M_2\), the slepton mass parameter \(m\tilde{\ell}\), the squark mass parameter \(m\tilde{q}\), the ratios \(A_{\tilde{\tau}}/m\tilde{\ell}, A_{\tilde{t}}/m\tilde{q}\) and \(A_{\tilde{b}}/m\tilde{q}\) involving the trilinear couplings \(A_{\tilde{\tau}}, A_{\tilde{t}}\) and \(A_{\tilde{b}}\) of the third generation of sleptons and squarks (the three ratios are assumed to be equal).

We consider the parameter region of the effMSSM in which the lightest neutralino is the lightest supersymmetric particle and its relic density \(\Omega_\chi\) is within the cosmological range \(0.098 < \Omega_\chi h^2 < 0.122\). In this region, we compute \(\langle \sigma v \rangle_{\text{ds}}\) as the value of \(\sigma v\) at \(v = 0\). For each point in this region we also compute \(f_Q\) as the fraction of annihilation energy that does not go into neutrinos. In obtaining \(f_Q\), it is safe to assume that the particle cascades after annihilation develop in vacuum, since muons, taus and light mesons produced in the annihilation decay to neutrinos before being stopped in the dark star medium.

Figure 2 shows the values of the combination \(f_Q \langle \sigma v \rangle_{\text{ds}}/m_\chi\) obtained in the way just described as a function of the neutralino mass \(m_\chi\). There are four classes of points: (i) the spread of points along the direction sloping down to the right is due to \(p\)-wave annihilation; (ii) the V-shaped feature at \(m_\chi \sim 45\) GeV is due to resonant annihilation through the Z boson (other resonant annihilations, through the lightest Higgs boson of mass varying from 115 GeV to 120 GeV, are visible at \(m_\chi \sim 60\) GeV), (iii) the “fingers” of points dropping from the \(p\)-wave band of points arise from threshold coannihilation, and (iv) the shaded region on the right of \(m_\chi \sim 100\) GeV corresponds to possible coannihilation with staus (the dashed line shows the similar boundary for sneutrino coannihilations). These four cases are described in the following.

In \(p\)-wave annihilation, the dominant contribution at freeze-out comes from the \(p\)-wave term \(bv^2\) in \(\sigma v\). The \(p\)-wave contribution to \(\sigma v\), which is instrumental to provide the correct neutralino relic density, is suppressed as far as the evolution of the dark star is concerned. In fact, for a newly-born 1-\(M_\odot\) dark star, one has \((v_{\text{ds}}/v_{\text{fo}})^2 \sim 2 \times 10^{-9}\). Thus in this case, \(\langle \sigma v \rangle_{\text{ds}} \sim a\) while \(\langle \sigma v \rangle_{\text{fo}} \sim b(v^2) \sim b/20\), and in general they differ. Their exact ratio
Figure 2: Scatter plot of the combination $f_Q \langle \sigma v \rangle_{\text{d}s}/m_\chi$ plotted as a function of the neutralino mass $m_\chi$ for an effective eight-parameter MSSM model. Below the horizontal line a dark star cannot form. For resonant annihilation (the V-shaped feature at $m_\chi \sim 45$ GeV and the descending points around $m_\chi \sim 60$ GeV) and coannihilations (the shaded region and the dashed line on the right of $m_\chi \sim 100$ GeV), the quantity $f_Q \langle \sigma v \rangle_{\text{d}s}/m_\chi$ may be too small for a dark star to form. For $p$-wave annihilation (the band sloping down to the right) and threshold annihilation (the various “fingers” of points dropping from the $p$-wave annihilation band), dark stars would be able to form.

depends on the particle physics parameters contained in the coefficients $a$ and $b$. In our effMSSM scan, $p$-wave annihilation gives rise to a spread in $f_Q \langle \sigma v \rangle_{\text{d}s}/m_\chi$ of about one order of magnitude (band of points sloping down to the right in Figure 2).

The $Z$ resonance at $m_\chi \sim 45$ GeV provides an example of resonant annihilation. The resonant part of the neutralino-neutralino annihilation cross section is given by

$$\langle \sigma v \rangle_Z = \beta_f g^4_{\text{eff}} \frac{(s - m_Z^2)^2}{m_\chi^2 (s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2},$$

(2.1)

where $\beta_f$ is the speed of the final products in units of the speed of light, and $g_{\text{eff}}$ contains the coupling constants and the mixing angles of the neutralinos and of the final particles involved. The velocity dependence of $\langle \sigma v \rangle_Z$ can be obtained by writing $s = 4m_\chi^2(1 + v^2)$, from which one finds, neglecting the mass of the final products,

$$\langle \sigma v \rangle_Z = g^4_{\text{eff}} \frac{(v^2 + \delta)^2}{m_\chi^2 (v^2 + \delta)^2 + \gamma^2},$$

(2.2)

where $\delta = 1 - m_Z^2/(4m_\chi^2)$ and $\gamma = \Gamma_Z m_Z/(4m_\chi^2)$. On resonance, that is for $2m_\chi = m_Z$ or $\delta = 0$ and $\gamma = \Gamma_Z/m_Z = 0.0273$, the velocity-averaged $\langle (\sigma v)_Z \rangle$ has very different values.
at freeze-out ($v \approx 0.2c$) and in a dark star ($v \approx 0$). At freeze-out, the thermal average of \((\sigma v)_Z\) on resonance is, using \(T_{\text{fo}} = m_\chi/20\), \(\langle (\sigma v)_Z \rangle_{\text{fo}} = 0.97 g_{\text{eff}}^4/m_\chi^2\). On the other hand, in a dark star, one has on resonance, for \(v = 30 \text{ km/s}\), \(\langle (\sigma v)_Z \rangle_{\text{ds}} = 1.3 \times 10^{-13} g_{\text{eff}}^4/m_\chi^2\). While \(\langle (\sigma v)_Z \rangle_{\text{fo}} \sim 3 \times 10^{-26} \text{ cm}^3/\text{s}\) to provide the correct relic density, \(\langle (\sigma v)_Z \rangle_{\text{ds}}\) is thirteen orders of magnitude smaller. These very different values of \(\langle (\sigma v) \rangle_{\text{fo}}\) and \(\langle (\sigma v) \rangle_{\text{ds}}\) give rise to the V-shaped feature in Figure 2 around \(m_\chi = m_Z/2 \sim 45 \text{ GeV}\). (Similar resonant features through the lightest Higgs boson appear superposed at \(m_\chi = m_H/2 \sim 60 \text{ GeV}\).)

Threshold annihilation occurs when the neutralino mass is slightly smaller than half the total mass of the final annihilation products in a specific channel (for example, \(\chi \chi \to WW\)). In this case, kinetic energy is required for the reaction to occur. This kinetic energy is available at the time of freeze-out thanks to the relatively high temperature of the neutralinos, but is not available at the lower velocities of neutralinos in a dark star. Therefore, the annihilation into the specific channel \((\chi \chi \to WW\) in the example) occurs at freeze-out but not in a dark star. The cross section \(\langle (\sigma v) \rangle_{\text{ds}}\) is thus smaller than \(\langle (\sigma v) \rangle_{\text{fo}}\).

In Figure 2 this is illustrated by the “fingers” of points dropping from the \(p\)-wave band at \(m_\chi \sim 80 \text{ GeV}\) (the \(WW\) channel) and \(m_\chi \sim 190 \text{ GeV}\) (the \(tt\) channel). In neither case the suppression of \(\langle (\sigma v) \rangle_{\text{ds}}\) is severe enough to bring the points outside the parameter region in which dark stars can form.

For coannihilations, the relic density is determined by an effective annihilation cross section \(\langle (\sigma v) \rangle_{\text{eff}}\), which is an average of the annihilation cross sections of all reactions between the neutralino and the coannihilating particles. In minimal supergravity models, which are a subset of MSSM models, coannihilations occur in specific regions of the parameter space in which the stau \(\tilde{\tau}\) is very close in mass to the neutralino \(\chi\), and with more tuning of the parameters when the stop \(\tilde{t}\) is very close in mass to \(\chi\). In the general MSSM, coannihilations may also occur between the lightest and second lightest neutralino, and between the neutralino and the chargino.

For the sake of illustration in the context of dark stars, we focus on stau coannihilations, because the experimental lower bound on the stau mass \((m_{\tilde{\tau}} \gtrsim 98 \text{ GeV})\) is smaller than the lower bound on squark masses and thus the coannihilation region in parameter space is larger. In the case of stau coannihilations, the effective annihilation cross section is (approximately)

\[
\langle (\sigma v) \rangle_{\text{eff}} = \frac{\langle (\sigma v) \rangle_{\chi\chi} + \langle (\sigma v) \rangle_{\chi\tilde{\tau}} e^{-(m_{\tilde{\tau}}-m_\chi)/T_{\text{fo}}} + \langle (\sigma v) \rangle_{\tilde{\tau}\tilde{\tau}} e^{-2(m_{\tilde{\tau}}-m_\chi)/T_{\text{fo}}} \left[ 1 + e^{-(m_{\tilde{\tau}}-m_\chi)/T_{\text{fo}}} + e^{-2(m_{\tilde{\tau}}-m_\chi)/T_{\text{fo}}} \right]}{1 + e^{-(m_{\tilde{\tau}}-m_\chi)/T_{\text{fo}}} + e^{-2(m_{\tilde{\tau}}-m_\chi)/T_{\text{fo}}}}.
\]

(2.3)

Here \(\langle (\sigma v) \rangle_{\chi\chi}\), \(\langle (\sigma v) \rangle_{\chi\tilde{\tau}}\) and \(\langle (\sigma v) \rangle_{\tilde{\tau}\tilde{\tau}}\) are the total annihilation cross sections for \(\chi\chi \to \text{anything}, \chi\tilde{\tau} \to \text{anything},\) and \(\tilde{\tau}\tilde{\tau} \to \text{anything},\) respectively. Since reactions like \(\tilde{\tau}\tilde{\tau} \to \tau\tau\) are electromagnetic processes, \(\langle (\sigma v) \rangle_{\tilde{\tau}\tilde{\tau}} \sim \alpha^2/m_{\tilde{\tau}}^2\), which is much larger than the cross section for \(\chi\chi \to \tau\tau\), \(\langle (\sigma v) \rangle_{\chi\chi} \sim \alpha^2 m_{\tilde{\tau}}^2/m_\chi^4\). In fact, for \(m_{\tilde{\tau}} = 100 \text{ GeV (1 TeV)}\), their ratio is approximately \(\langle (\sigma v) \rangle_{\tilde{\tau}\tilde{\tau}}/\langle (\sigma v) \rangle_{\chi\chi} \sim m_{\tilde{\tau}}^2/m_\chi^2 > 3 \times 10^3 \times (3 \times 10^5)\). Thus with an appropriate choice of the mass difference \(m_{\tilde{\tau}} - m_\chi\), one can obtain an effective annihilation cross section three or more orders of magnitude larger than the \(\chi\chi\) annihilation cross section, and a relic density three or more orders of magnitude smaller than without coannihilations.

This argument allows us to estimate a lower limit on \(\langle (\sigma v) \rangle_{\text{ds}}\) in a dark star using just the
annihilation cross section for $\chi \chi \to \tau \tau$ without having to compute the relic density in the presence of coannihilations. The annihilation cross section for $\chi \chi \to \tau \tau$ can be computed analytically and can be limited from below by keeping only the diagram with $\tilde{\tau}$ exchange and choosing appropriate neutralino and stau mixings. In this way, we obtain

$$\langle \sigma(\chi \chi \to \tau \tau) v \rangle_{ds} \geq \frac{\pi \alpha^2}{32 \cos^2 \theta_W} \frac{m_{\tilde{\tau}}^2}{m_{\chi}^4}.$$  \hspace{1cm} (2.4)$$

Then we set $m_{\tilde{\tau}} = m_{\chi}$ as appropriate for stau coannihilations. Moreover, bremsstrahlung ($\chi \chi \to \tau \tau \gamma$) gives a contribution to $\langle \sigma v \rangle_{ds}$ that exceeds the lower limit just computed in Eq. (2.4) at large neutralino masses. For $m_{\chi} = m_{\tilde{\tau}}$ we estimate

$$\langle \sigma(\chi \chi \to \tau \tau \gamma) v \rangle_{ds} \simeq \frac{\alpha^3}{m_{\chi}}.$$  \hspace{1cm} (2.5)$$

In addition, we compute $f_Q$ by examining the fraction of energy that escapes into neutrinos in the decay chains of the $\tau$ lepton. Eqs. (2.4) and (2.5) are used to plot the shaded region to the right of $m_{\chi} \sim 100$ GeV in Figure 2, namely

$$f_Q \frac{\langle \sigma v \rangle_{ds}}{m_{\chi}} \geq \begin{cases} 1.2 \times 10^{-28} \text{ cm}^3 \text{ s GeV} \left( \frac{100 \text{ GeV}}{m_{\chi}} \right)^5, & \text{for } m_{\chi} \lesssim 800 \text{ GeV} \\ 2 \times 10^{-30} \text{ cm}^3 \text{ s GeV} \left( \frac{100 \text{ GeV}}{m_{\chi}} \right)^3, & \text{for } m_{\chi} \gtrsim 800 \text{ GeV}. \end{cases}$$  \hspace{1cm} (2.6)$$

For the bremsstrahlung of gamma rays, we take $f_Q = 1$. Dark stars can form for $m_{\chi} \leq 880$ GeV. If $f_Q \leq 0.86$, the bremsstrahlung does not play any role in determining what is the largest possible mass of neutralino forming dark stars, and dark stars can form for $m_{\chi} \leq 830$ GeV. The annihilation cross section in dark stars can be as low as the lower edge of this shaded region, while the correct relic density is obtained through a much larger effective annihilation cross section. We notice that in most of the shaded region dark stars can still form, except at the higher masses where the shaded region crosses the boundary of the area marked ‘no dark star.’

Other coannihilations may arise in the MSSM. For instance, one might have coannihilations between the neutralino and a sneutrino or a selectron or a smuon. These may lead to even smaller $\langle \sigma v \rangle_{ds}$ than the case of stau coannihilations we use as an example, and so lead to a situation in which dark stars do not form. The worst case for dark stars is coannihilation with sneutrinos, in that neutrinos are generated in the final state and neutrinos escape from the forming protostar without depositing energy. Similarly to the neutrinophilic case discussed below, three-body annihilation channels need to be considered, in particular internal and final state bremsstrahlung of charged leptons. A simple estimate of $Z$ bremsstrahlung in the final state gives us

$$f_Q \frac{\langle \sigma(\chi \chi \to \nu \nu Z) v \rangle}{m_{\chi}} \simeq 3 \times 10^{-30} \text{ cm}^3 \text{ s GeV} \left( \frac{100 \text{ GeV}}{m_{\chi}} \right)^3.$$  \hspace{1cm} (2.7)$$
Here we took $f_Q = 1/2$ as a representative value. In the process of virtual internal bremsstrahlung, $Z$ can take away a sizable fraction of the energy and $f_Q$ may be sizable. Eq. (2.7) is plotted in Figure 2 as the dashed line near the edge of the shaded coannihilation region. In terms of dark star formation, this case is similar to coannihilation with the $\tilde{\tau}$. Dark stars can form up to $m_\chi = 1$ TeV. For different choice of $f_Q$, the cross point is at $m_\chi = (2f_Q)^{1/3}$ TeV ($m_\chi = 800$ GeV for $f_Q = 1/4$.)

We therefore conclude that except in very special cases, namely on top of the $Z$ resonance or for coannihilations of heavy sleptons or sneutrinos ($m_\chi \gtrsim 800$ GeV), dark stars can form in the MSSM.

3. Kaluza-Klein Dark Matter

If the Standard Model lives in five or six dimensions and the extra dimensions are compactified at a radius $\sim 1$/TeV, the Kaluza-Klein (KK) number can be preserved in a consistent way with all the interactions involving an even number of odd KK number particles. In this setup, the lightest KK particle (LKP) is stable and can be a good dark matter candidate [7]. In particular, there are two interesting KK candidates for the dark matter: the KK photon (more precisely the KK modes of the $U(1)_Y$ gauge boson) and the KK neutrino.

The KK photon annihilates to quarks and leptons through the $t$-channel exchange of KK fermions, and its relic abundance is compatible to observation for its mass around 1 TeV. If the right-handed KK electron, muon, and tau are nearly degenerate (i.e if the mass difference is $\lesssim 1\%$), the KK mass needed for the right relic density can drop to 700 GeV, since in this case the coannihilation cross section is very small compared to the self annihilation cross section, leading to a smaller effective cross section.

As far as the KK neutrino is concerned, its annihilation cross section to quarks and leptons proceeds via $t$- or $s$-channel exchange of gauge bosons, while annihilations to gauge bosons are mediated by $t$-channel KK lepton exchange or $s$-channel gauge bosons. If one flavor of the KK neutrino is considered, the correct relic density is obtained for a mass around 1.5 TeV. Including three flavors the effective cross section becomes smaller due to coannihilations between different flavors and the mass leading to the correct relic density is around 1 TeV. An additional coannihilation process with the KK left-handed electron is also possible when the latter has a smaller mass splitting with the KK neutrino, but this effect is almost negligible.

In the case of KK dark matter the $s$-wave annihilation cross section is always sizable both for the KK photon and for the KK neutrino, so that there is little difference between $\langle \sigma v \rangle_{ds}$ and $\langle \sigma v \rangle_{fo}$. Moreover, the temperature in the dark star is very low compared to the freeze out temperature, so coannihilations with other particles give no contributions to the effective cross section with the exception of exact degeneracy of the masses. Anyway, since for KK dark matter the coannihilation cross section is either smaller than the one without coannihilation or the difference between the two is negligible, $\langle \sigma v \rangle_{ds}$ is expected to be always larger or comparable to $\langle \sigma v \rangle_{fo}$.

Two interesting exceptions to the scenario described above are resonant annihilation with level–2 KK particles [8] and coannihilation with the KK gluon [9]. In principle these
effects can enhance the effective cross section at the freeze out temperature, so that, if the latter is normalized to that of a thermal relic, the cross section in the dark star can be suppressed. In particular, the s-channel annihilation at one loop through the exchange of the second KK Higgs with mass $m_{h(2)}$ is discussed in [8]. The canonical value for a thermal relic $\langle \sigma v \rangle_{fo} = 3 \times 10^{-26} \text{cm}^3\text{s}^{-1}$ is obtained for a KK photon with mass $m_{KK} \simeq 800 \text{ GeV}$ if the relation $m_{h(2)} = 2m_{KK}$ holds up to 5%. This new enhancement changes the cross section only by 10 to 20%. Therefore, the largest possible difference between $\langle \sigma v \rangle_{fo}$ and $\langle \sigma v \rangle_{ds}$ can be at most 10 to 20%. Unlike the MSSM, there is no p-wave suppression for the KK photon annihilation and the cross section at freeze out temperature is large enough due to the t-channel exchange of KK fermions. The same is true for KK neutrinos through the t-channel exchange of KK Z and KK W bosons. The KK Z and KK W bosons have similar masses, and in this case $f_Q \ll 1$ is not possible. Thus $f_Q \langle \sigma v \rangle_{ds}/m_{KK} \geq 10^{-32} \text{cm}^3\text{s}^{-1}/\text{GeV}$ is safely satisfied. Coannihilation with the KK gluon is a last interesting possibility [9]. If the KK gluon and the KK photon are degenerate with an accuracy much less than 1%, the correct relic density is obtained for $m_{KK} \simeq 5 \text{ TeV}$. By comparing it to the case without coannihilation ($m_{KK} = 700 \text{ GeV}$), one can see that coannihilation with the KK gluon can enhance the effective cross section by a factor of 50. Even in the worst scenario in which coannihilation with the KK gluon is effective at the freeze out temperature but is absent in the dark star, $f_Q \langle \sigma v \rangle_{ds}/m_{KK} \geq 4 \times 10^{-32} \text{cm}^3\text{s}^{-1}/\text{GeV}$ (assuming a conservative value $f_Q = 1/3$) and the dark star can form.

As a consequence, if $\langle \sigma v \rangle_{fo} = 3 \times 10^{-26} \text{cm}^3\text{s}^{-1}$ is imposed in order to explain the observed relic density, the condition for the dark star formation is always satisfied. We can conclude that KK dark matter that explains the observed relic density can always form dark stars.

4. Leptophilic Models

Leptophilic DM models [10] have recently become popular in order to explain simultaneously the excess in PAMELA positrons [11], the excess in Fermi-LAT electrons [12], as well as the excellent agreement between the observed antiproton spectrum and the corresponding standard expectation [13]. In order to explain the excesses, the mass of the DM is also constrained to be larger than about 100 GeV. This constraint might be more stringent if the electron FERMI-Lat data are taken into account, $m > 400 \text{ GeV}$.

In leptophilic models the DM particles generically annihilate exclusively to charged leptons, either of only one type (electrons, taus or muons) or democratically to all the three families. It is also possible to consider decays to neutrinos, but for simplicity we will not consider this case which would simply imply a straightforward generalization (see Section 5 for annihilation into neutrinos only). In particular, in this section we discuss the case of democratic annihilation to the three lepton families. In this case, using PYTHIA [20] one gets $f_Q \simeq 0.56$ almost constant in the range $200 \text{ GeV} \lesssim m \lesssim 2 \text{ TeV}$.

In order to explain the PAMELA and Fermi-LAT excesses, large annihilation cross sections $10^{-25} \lesssim \langle \sigma v \rangle_{gal}/\text{cm}^3\text{s}^{-1} \lesssim 10^{-23}$ are needed at the velocity of DM particles in our Galaxy, $v_{gal} \simeq 300 \text{ km/s}$. Assuming $\langle \sigma v \rangle_{gal} = \langle \sigma v \rangle_{fo}$ (s-wave annihilation) these
values are up to two orders of magnitude larger than the value $\langle \sigma v \rangle_{\text{fo}} \simeq 3 \times 10^{-26} \text{ cm}^3 \text{s}^{-1}$ compatible with a standard thermal relic abundance in agreement with observations. Clearly, the case of $p$-wave suppression of $\langle \sigma v \rangle_{\text{gal}}$ would be even worse. Several mechanisms have been devised in order to explain this discrepancy, such as a non-thermal production of the DM particles, a non-standard evolution history of the Universe or an enhancement of the annihilation cross section at low velocities (Sommerfeld effect). In this sense leptophilic models represent another interesting possibility in connection with the formation of a dark star: a very large annihilation cross section throughout the history of the Universe and/or at the low temperatures where dark stars are formed.

Assuming $\langle \sigma v \rangle_{\text{gal}} = \langle \sigma v \rangle_{\text{ds}}$, the previous discussion implies that in a dark star:

$$7 \times 10^{-28} \text{ cm}^3 \text{s}^{-1} \text{GeV}^{-1} \lesssim f_Q \frac{\langle \sigma v \rangle_{\text{ds}}}{m} \lesssim 8 \times 10^{-27} \text{ cm}^3 \text{s}^{-1} \text{GeV}^{-1},$$

for $100 \text{ GeV} < m < 2 \text{ TeV}$. This is shown in Fig. 3, where along with the intervals required to explain the PAMELA and Fermi/LAT excesses the present constraints for the combination $f_Q \langle \sigma v \rangle_{\text{ds}}/m$ are summarized as a function of $m$. In this plot we have assumed that $\langle \sigma v \rangle$ does not depend on the temperature, in order to directly compare constraints relative to different epochs.

In particular, the thick and thin solid line closed contours show the range of values compatible with the PAMELA positron excess [11] and the Fermi–LAT $e^+ + e^-$ data [12], respectively. On the other hand, the two solid open lines represent conservative $2 \sigma$ C.L. upper bounds for $f_Q \langle \sigma v \rangle_{\text{ds}}/M_\chi$ obtained from the flux of $e^+ + e^-$ observed by Fermi [12] (thin line) and the $e^+ + e^-$ flux measured by HESS [15] (thick line).

In the same Figure, we also plot with the dotted line the upper bounds on $f_Q \langle \sigma v \rangle_{\text{ds}}/m$ obtained by comparing the expected gamma–ray flux produced by Inverse Compton (IC) scattering of the final state leptons to the diffuse flux of gamma–rays measured by Fermi at intermediate Galactic latitudes [17].

Finally the long and short dashed line shows the upper bound on $f_Q \langle \sigma v \rangle_{\text{ds}}/m$ obtained by considering the imprint on the Cosmic Microwave Background Radiation (CMB) from the injection of charged leptons from DM annihilations at the recombination epoch [16].

It is clear from Fig. 3 that the sizable values of the combination $f_Q \langle \sigma v \rangle_{\text{ds}}/M_\chi$ are compatible to the formation of a dark star (Eq. 1.3) when $\langle \sigma v \rangle_{\text{ds}} = \langle \sigma v \rangle_{\text{gal}}$, with the range $m \gtrsim 1 \text{ TeV}$ disfavored [14] by several constraints. On the other hand, by assuming a Sommerfeld enhancement of the annihilation cross section one may have $\langle \sigma v \rangle_{\text{ds}} \gtrsim \langle \sigma v \rangle_{\text{gal}}$ depending on whether the enhancement effect is already saturated at the velocity $v \gtrsim 10 \text{ km s}^{-1}$ inside the dark star, possibly implying in this case an even more favorable situation for the dark star formation. However, in presence of a non–saturated Sommerfeld enhancement at the recombination epoch the CMB constraint could be stronger, since at $z \simeq 1100$ DM particles are slower ($v \simeq 10^{-8} c \simeq 10^{-3} \text{ km s}^{-1}$) than inside a dark star. In this case leptophilic DM could explain the PAMELA and Fermi/LAT excesses only by assuming a boost factor of astrophysical origin such as clumpiness. In any case, barring some specific cases such as the presence of resonances in the annihilation cross section associated with
bound states [18] even in this circumstance the bound in Eq. (1.3) would be easily verified and a dark star would be formed.

Thus we can also conclude that leptophilic models also satisfy the condition to form a dark star.

5. Neutrinophilic Models

As discussed in the previous sections the most popular examples of thermal dark matter candidates, namely the neutralino in the MSSM and the KK photon or KK neutrino in Kaluza–Klein DM, can easily produce a dark star, provided that the annihilation cross section at the temperature of the dark star is similar to that at freeze out and is not suppressed by mechanisms such as $p$–wave annihilation or by the fact that the annihilation
cross section is resonant at the freeze out temperature but not inside the dark star. In this Section we wish to generalize this statement to the general case of a thermal DM candidate, discussing what are the minimal conditions to form a dark star once \( \langle \sigma v \rangle_{ds} \) is normalized to the canonical value \( \langle \sigma v \rangle_{fo} = 3 \times 10^{-26} \text{cm}^3/\text{s} \). Moreover, we will also briefly comment on the case \( \langle \sigma v \rangle_{ds} < \langle \sigma v \rangle_{fo} \).

For this purpose, we need to give a general discussion of the energy fraction \( f_Q \) released by DM annihilation into the gas, a quantity that is in general model dependent. Our approach to this problem is to consider in this Section the most conservative case of a DM candidate annihilating exclusively into neutrinos, i.e. “neutrinophilic” Dark Matter.

Naively one would expect that the energy fraction of neutrinophilic DM going into visible particles vanishes: DM annihilation would generate only neutrinos that would escape from the collapsing gas freely. If this were indeed the case, neutrinophilic dark matter annihilations would not be able to support a dark star phase. However Z and W bosons are expected to be produced from bremsstrahlung radiation of the final state neutrinos, so some visible energy, increasing with the mass of the DM particle, is expected to be produced by the decay of the Z and/or W. Electroweak bremsstrahlung in the annihilation of neutrinophilic DM has already been considered in the context of DM indirect detection [19].

Let’s first assume that neutrinophilic dark matter has an annihilation cross section in the dark star equal to the cross section that provides a thermal relic density, namely \( \langle \sigma v \rangle_{ds} = 3 \times 10^{-26} \text{cm}^3/\text{s} \). At the tree level the branching ratio to \( \nu \bar{\nu} \) is 1 if bremsstrahlung radiation is neglected. However, when \( 2m > m_W \) (or \( m_Z \)), on-shell production of W-bosons (or Z-bosons) dominates the bremsstrahlung process, which can be viewed as a three body decay followed by the subsequent decay of the W or Z gauge bosons. Since the visible energy fraction of the W- and Z-boson decays is of order 1, one finds

\[
f_Q \sim \frac{g^2}{16\pi^2} \frac{E_W}{2m} > \frac{g^2}{16\pi^2} \frac{m_W}{2m}.
\]  

(5.1)

Here \( E_W \) is the energy of the W boson. In this case, as can be easily checked numerically, the condition for forming a dark star is always fulfilled. On the other hand, when \( 2m < m_W \), off-shell bremsstrahlung occurs, which for \( 2m \ll m_W \) can be treated as a four-body decay in the limit of a 4-Fermi interaction.

The visible energy fraction \( f_Q \) is plotted as a function of the DM mass \( m \) in Fig. 4. In this Figure we have used PYTHIA [20] to calculate the subsequent decay of the final state particles in the radiative correction, since the value of \( f_Q \) critical for the formation of the dark star is near the threshold for the production of an on-shell W-boson, where a narrow width approximation or a 4-Fermi interaction are not reliable. From Fig. 4 we can conclude that a dark star can be formed if the mass of the neutrinophilic dark matter is larger than \( \sim 50 \text{ GeV} \).

If the DM particle is a scalar or a Majorana fermion, it is possible that the annihilation cross section in the dark star is significantly different from the cross section at decoupling. A simple example is a scalar neutrinophilic dark matter particle \( \phi \) annihilating by \( t/u \)-channels through the exchange of a heavy fermion. In particular, the s-wave contribution...
Figure 4: Energy fraction $f_Q$ released by neutrinophilic dark matter annihilation inside a dark star as a function of the DM particle mass $m$. The black curve represents $f_Q$ for an ideal neutrinophilic DM model where the $W$ and $Z$ bremsstrahlung effects are calculated using PYTHIA [20]. The blue dashed line shows the constraint given in Eq. (1.3). Only models above the blue line can form a dark star.

vanishes if the heavy fermion mediating the annihilation is a Dirac fermion and if it interacts chirally with the dark matter $\phi$ and the neutrinos, i.e.

$$\mathcal{L} \sim g\phi \Psi_L \nu + M \Psi_L \Psi_R + \text{h.c.} + m^2 |\phi|^2,$$

where $M$ is the mass of the heavy fermion and $m$ is the mass of the dark matter scalar. In this case, the annihilation cross section is purely $p$-wave and is given by

$$\sigma v = v^2 \frac{|g|^4 m^2 ((M^2 + m^2)^2 + 2m^2 M^2)}{16\pi (m^2 + M^2)^4}. \quad (5.3)$$

In this case, since the average velocity of the dark matter in the dark star is $\sim 30$ km/s, one has $\langle \sigma v \rangle_{ds} = \langle \sigma v \rangle_{fo} (v_{ds}/v_{fo})^2 \sim \langle \sigma v \rangle_{fo} v_{ds}^2 (m/T_{fo}) \sim 10^{-32}$ cm$^3$/s. So $p$-wave annihilating neutrinophilic DM cannot make a dark star.

Notice that in our evaluation of neutrinophilic DM annihilation we have not included internal bremsstrahlung of charged particles. Internal bremsstrahlung would increase the annihilation cross section of neutrinophilic dark matter, and extend the mass limit for the formation of dark stars to values below $\sim 50$ GeV, but would depend on the specific particle physics model.

Strictly speaking a purely neutrinophilic DM model is not natural, since in most specific scenarios other tree–level decay channels contributing to the visible energy are usually expected, and a contribution of the latter at the level of $\sim 10^{-5}$ level or larger would be sufficient to form a dark star. Thus neutrinophilic DM represents a limiting case, allowing to show that, as long as $\langle \sigma v \rangle_{ds} = \langle \sigma v \rangle_{fo}$, any thermal DM candidate heavier than $m \gtrsim 50$ GeV can lead to the formation of a dark star.

6. Conclusions

The first stars to form in the Universe may be powered by the annihilation of weakly interacting dark matter particles [1]. In this paper we explored several popular examples of
thermal dark matter models in order to discuss whether they can satisfy the conditions for the formation of a dark star: the neutralino in an effective MSSM scenario; leptophilic models that might explain recent observations in cosmic rays; the KK-photon and the KK-neutrino in UED models; a conservative neutrinoophilic model where the dark matter particles annihilate exclusively to neutrinos. We find that in general models with thermal dark matter lead to the formation of dark stars, with few notable exceptions: heavy neutralinos in the presence of coannihilations; annihilations that are resonant at dark matter freeze-out but not in dark stars; neutrinoophilic dark matter lighter than about 50 GeV. In particular the discussion of the latter conservative scenario allows us to conclude that a thermal DM candidate in standard Cosmology always forms a dark star as long as its mass is heavier than $\simeq 50$ GeV and the thermal average of its annihilation cross section is the same at the decoupling temperature and during the dark star formation, as for instance in the case of a cross section with a non–vanishing $s$-wave contribution.

Therefore, we can conclude that the formation of a first generation of stars powered by dark matter annihilation is an almost inevitable consequence of thermal dark matter when a standard thermal history of the Universe is assumed and if the mechanism of Ref. [1] is at work. So a dark star is always there whenever there is thermal dark matter.

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