Odd Decays from Even Anomalies: Gauge Mediation Signatures Without SUSY

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Abstract

We analyze the theory and phenomenology of anomalous global chiral symmetries in the presence of an extra dimension. We propose a simple extension of the Standard Model in 5D whose signatures closely resemble those of supersymmetry with gauge mediation, and we suggest a novel scalar dark matter candidate.
# 1 Introduction

Anomalies and the interactions they imply proved crucial in identifying the ultraviolet physics underlying the chiral Lagrangian, playing an important role in the formulation of the dynamical $SU(3)_C$ theory of quarks and gluons [1–5]. From the decay rate of $\pi^0 \rightarrow \gamma \gamma$, for example, one can infer the number of colors in the UV theory. This is due to the fact that, in the $SU(3)_C$ model, anomaly cancellation occurs non-trivially with the left and right-handed sectors contributing in equal but opposite non-zero amounts to the anomaly.

In the effective field theory at low energies, this non-trivial anomaly cancellation of the UV theory is manifest non-locally in the $SU(3)_L \times SU(3)_R / SU(3)_V$ theory space of the chiral symmetry breaking Lagrangian, and emerges as a topological (and thus quantized) “Wess-Zumino-Witten” term labelled by a winding number that corresponds to the number of colors in the UV theory [6–8]. Additionally, the $U(1)$ problem of QCD, the unexpectedly large masses of the $\eta$ and $\eta'$ mesons have been resolved through non-perturbative instanton contributions through $U(1)$ global anomalies [9].

As we enter the LHC era, we have identified numerous theories which may play some role in stabilizing the weak scale. The most well studied of these physics scenarios is TeV scale supersymmetry [10], however, in recent years, enormous progress has been made on TeV scale extra-dimensional theories and effective field theories such as little Higgs models. As was the case with the chiral Lagrangian, these theories may be supplanted at still higher energies by some confining UV dynamics, and anomalies may again play an important role.

The study of anomalies in such contexts is in its infancy, but has already produced some important results for the phenomenology of extensions of the Standard Model (SM). To date, most studies have focused on scenarios where all anomalies vanish in the IR. In these models, anomaly cancellation occurs non-locally in an extra dimension [11,12], or, as happens in the chiral Lagrangian, non-locally in theory space [13]. For consistency, such theories require a Chern-Simons flux or Wess-Zumino-Witten term, respectively. These terms encapsulate the integrated out UV dynamics through which anomaly cancellation occurs locally as well as globally.

In this paper, we study the implications of extra dimensional classical symmetries which contain non-vanishing anomalies in the low energy 4D effective theory. Earlier work on such theories (with some overlapping results) has been performed in [14]. The Peccei-Quinn (PQ) symmetry [15] which has been originally proposed as a solution to the strong CP problem is a popular and well-motivated example of such a theory, and thus we consider a $U(1)_{\text{PQ}}$ extension of the 5D Universal Extra Dimension model (UED) [16–18]. In standard UED, the usual 4D SM fields are extended so that they all propagate in the bulk of a compactified extra dimension. This results in a tower of massive Kaluza-Klein (KK) partners of each SM field.

We note that this is only one application of the techniques we develop, and that other constructions are possible that may have novel phenomenology. Examples include warped extra dimensions, or even little Higgs theories, which in certain cases can be related to
extra dimensional theories through the language of deconstruction [27].

Even though UED does not explain stability of the weak scale against radiative corrections, there are several compelling reasons to consider such theories. In UED there is remnant of 5D translation invariance known as KK-parity which stabilizes the lightest KK-mode. Due to KK-parity tree-level electroweak precision corrections will be absent (at least from the lightest states), and so these particles can be quite a bit lighter than the TeV scale. The stability of the lightest KK mode (LKP) also results in a realistic dark matter candidate [19]. What makes the theory particularly interesting however is that the UED particle spectrum and collider phenomenology may be very similar to that of a generic SUSY theory, and thus UED is a good “straw-man” to pit against supersymmetry [20]. As in SUSY, the collider signatures consist of decay chains that contain high \( p_T \) jets in association with large amounts of missing energy. As such, the models may be difficult to differentiate without resorting to observables that are sensitive to spin correlations [21–24], although techniques are being developed which may be able to discriminate models in early stages of LHC running [25, 26].

In our study of this \( U(1)_{PQ} \) extension of UED (PQ-UED), we find that anomalies can mediate decays of the KK-odd partners of the hypercharge gauge boson which is often the lightest KK-odd particle (LKP), to SM photons and Z’s in association with a new KK-odd scalar field that lives in the 5-component of an extra-dimensional gauge field. This \( B_5 \) is both stable and neutral, and thus presents as missing energy at colliders. The signal event topologies at a hadron collider generically contain high \( p_T \) jets and a pair of neutral SM gauge bosons (either photon or \( Z \)). Final state leptons may also make up a portion of the event topology, depending on the spectrum of KK-modes. Such events are also characteristic of gauge mediated SUSY breaking [28–30], where a bino NLSP decays through a Goldstino coupling to the gravitino plus either a photon or \( Z \). We thus overturn the lore that such signatures are a “smoking gun” for supersymmetry.

This paper is organized as follows. In Section 2, we describe the basic setup of the PQ-UED model. In Section 3, we describe in detail the physics underlying anomalies which persist in the 4D effective theory. In particular, we discuss a gauged \( U(1)_{PQ} \) symmetry which is broken by boundary conditions on an \( S_1/Z_2 \) orbifold. In Section 3.1, we discuss gauge fixing and the residual gauge transformations, showing that a massless Goldstone boson results from this choice of boundary conditions. In 3.2, we discuss the tree-level interactions of the Goldstone boson. In 3.3, we analyze the physics of additional spontaneous and explicit breaking of the \( U(1)_{PQ} \) symmetry, identifying the spectrum and the wave functions of the physical scalar modes. In 3.4, we discuss quantum mechanical violation of the \( U(1)_{PQ} \) symmetry, and the interactions of the Goldstone modes that are generated by the anomalies. In Section 4, we study the phenomenology of this scenario including collider physics, discussions about dark matter, and the existing constraints on the model (which turn out not to be stringent in the parameter space that is most interesting from the perspective of collider physics).
2 Basic Setup

The model is in 5D Minkowski space, with the flat distance element:

\[ ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - dz^2, \]  

(2.1)

where \( \eta_{\mu\nu} \) is the metric for 4D Minkowski space. The extra dimensional coordinate \( z \) is compactified on an \( S_1/Z_2 \) orbifold, and the \( z \)-coordinate is taken to range from \( z = [0, L] \). All SM fields are taken to propagate in the bulk, and the Lagrangian is constructed to obey a discrete \( Z_2 \) symmetry known as KK-parity, a remnant of full 5D translation invariance which is broken by the presence of the branes at \( z = 0, L \) [16]. At the Lagrangian level, KK-parity forbids bulk Dirac masses for the fermions, requires that brane localized interactions be identical on the branes at \( z = 0, L \), and constrains boundary conditions for bulk fields to be the same on each brane. Orbifold boundary conditions for the fermions and gauge fields are chosen such that the fermion and gauge boson zero mode spectrum reproduces that of the Standard Model. The bulk Higgs sector then gives masses to these modes in the usual way.

In our setup, we slightly extend UED to incorporate a new bulk gauge symmetry. This gauge symmetry is chosen to be chiral in the zero mode spectrum, with the charges matching those of a Peccei-Quinn global symmetry [15] in Weinberg-Wilczek and DFSZ type axion models [31–34]. In order to do this consistently we must also have up and down-type Higgs doublets, since the SM with one Higgs does not have any such symmetry, even at the global level. In Table 2.2 we list the charges of the SM fields under hypercharge and the new gauged PQ symmetry.

|     | \( H_u \) | \( H_d \) | \( Q \) | \( \bar{u} \) | \( d \) | \( L \) | \( \bar{e} \) |
|-----|---------|---------|-------|-------|-------|-------|-------|
| \( Y \) | 1/2 | -1/2 | 1/6 | -2/3 | 1/3 | -1/2 | 1 |
| \( \text{PQ} \) | 1 | 1 | -1/2 | -1/2 | -1/2 | -1/2 | -1/2 |

(2.2)

Note that a bulk \( \mu \) term, \( \mu H_u^T(i\tau_2)H_d \), is forbidden with these charge assignments. On the boundaries, we fix the 4D components of the PQ gauge field, \( B_M \) to zero: \( B_\mu|_{z=0,L} = 0 \). In the absence of other symmetry breaking effects, this leads to a single physical zero mode for the 5-component of the gauge field, \( B_5 \) [35,36]. As is normally the case, the remaining KK tower of \( B_5 \) modes can be gauged out of the spectrum as they are Goldstone bosons eaten by the KK tower of massive \( B_\mu \) fields. We discuss this in further detail in Sections 3.1 and 3.3 where we also take into account bulk breaking of the gauge symmetry due to the Higgs vacuum expectation values. Additional explicit breaking of the \( U(1)_{\text{PQ}} \) symmetry is added in the form of brane localized \( \mu \)-terms. This is done in order to lift a potential electroweak-scale axion which is ruled out by experiment [37].

In this theory, all gauge anomalies (cubic anomalies for gauge fields with zero modes) vanish as required for consistency. However global anomalies (e.g. PQ anomalies quadratic in the SM gauge fields) localized on the branes at \( z = 0, L \) persist in the theory [11]. These
anomalies lead to couplings of the $B_5$ scalar zero mode to the 5D field strengths and their duals, $G\tilde{G}$, $W\tilde{W}$ and $F\tilde{F}$. These couplings allow a decay of the lightest KK-mode in UED, which is often the first KK mode of the hypercharge gauge boson, down to a photon (or $Z$), and a PQ $B_5$ field. This is surprising at first glance, since the $B_5$ has a flat profile, and is thus naively even under KK-parity. However, we show in Section 3.2 that the zero mode $B_5$ is in fact a KK-odd field in all of its interactions at both the classical and quantum levels.

3 The Gauged Peccei-Quinn Symmetry

In this section, we illustrate the physics underlying a gauge symmetry which is broken by boundary conditions at both branes in an extra dimension constructed on an $S_1/Z_2$ orbifold. First we perform gauge fixing, identifying the residual gauge symmetries. Then we study the interactions of the lowest lying mode, a scalar field arising from the 5-component of the gauge field, and look at the implications of additional spontaneous breaking of the gauge symmetry via a Higgs mechanism. We end with an analysis of anomalies of this symmetry and the interactions they imply.

3.1 Residual Gauge Transformations

As described in the previous section, we gauge a $U(1)_{PQ}$ symmetry in the bulk, and break this symmetry via boundary conditions on the branes at $z = 0$ and $z = L$. In this section, we analyze this theory, identifying the residual gauge symmetries after imposing the boundary conditions on the branes, and adding gauge fixing terms in the bulk which decouple the unphysical modes.

Requiring preservation of the boundary conditions by the gauge transformations, $B_M \rightarrow B_M + \partial_M \beta(x, z)$, gives:

$$B_\mu|_{z=0,L} = 0 \implies \partial_\mu \beta(x, z)|_{z=0,L} = 0. \quad (3.3)$$

This condition requires that the gauge transformation on the branes is a constant function of the 4D coordinates, or is a global symmetry from the perspective of the 4D theory at $z = 0, L$.

We now turn to gauge fixing the $U(1)_{PQ}$ in the bulk. The 5D Lagrangian for a free
\( U(1) \) gauge field is given by:

\[
\mathcal{L}_{U(1)_{PQ}} = -\frac{1}{4g_{PQ}^2} \int dz B_{MN} B^{MN}
\]

\[
= -\frac{1}{4g_{PQ}^2} \int dz \left[ B_{\mu\nu} B^{\mu\nu} - 2(\partial_5 B_\mu)^2 - 2(\partial_\mu B_5)^2 + 4(\partial_\mu B^\mu)(\partial_5 B_5) \right]
\]

\[
= -\frac{1}{4g_{PQ}^2} \int dz \left[ B_{\mu\nu} B^{\mu\nu} - 2(\partial_5 B_\mu)^2 - 2(\partial_\mu B_5)^2 + 4(\partial_\mu B^\mu)(\partial_5 B_5) \right] - \frac{1}{2g_{PQ}^2} \left. B^\mu \partial_\mu B_5 \right|_0^L,
\]

(3.4)

where we have rearranged the interaction that mixes \( B_\mu \) and \( B_5 \) through integration by parts in the last step. Note that the boundary localized term vanishes for the boundary conditions that we have chosen, \( B_\mu |_{L,0} = 0 \), so there is no brane localized mixing between \( B_5 \) and \( B_\mu \).

As we gauge fix, it is convenient to remove the terms that mix \( B_5 \) and \( B_\mu \) in the bulk. This is achieved by adding a gauge fixing term to the Lagrangian given by [38]:

\[
\mathcal{L}_{GF} = -\frac{1}{2} \int dz G^2 \equiv -\frac{1}{2g_{PQ}^2 \xi_B} \int dz \left[ \partial_\mu B^\mu - \xi_B \partial_5 B_5 \right]^2.
\]

(3.5)

Note that there is a residual gauge symmetry where the gauge transformation parameter obeys the following equation:

\[
\partial_\mu \partial^\mu \beta(x,y) - \xi_B \partial_5^2 \beta(x,y) = 0.
\]

(3.6)

We choose to go to unitary gauge, \( \xi_B \to \infty \) where the eaten \( B_5 \) modes are projected out of the spectrum. In this limit, the solutions are:

\[
\beta(x,z) = \beta^+(x) + \left( \frac{2z - L}{2L} \right) \beta^-(x) \Rightarrow \beta_{res}(x,z) = \beta^+ + \beta^- \left( \frac{2z - L}{2L} \right),
\]

(3.7)

where we have imposed the boundary conditions in Eq. (3.3) for the gauge transformation in the second step.

Under this residual transformation, the PQ gauge fields transform as:

\[
B_\mu \to B_\mu
\]

\[
B_5 \to B_5 + \frac{\beta^-}{L}.
\]

(3.8)

Thus the remaining physical \( B_5 \) zero mode behaves as a Goldstone boson, undergoing a constant shift under the KK-odd part of the residual gauge transformation. This implies that the choice of these boundary conditions is equivalent to having spontaneously broken a global symmetry. As we will show explicitly in Section 3.4, the effective scale of this
symmetry breaking is given by \( f_{\text{PQ}} = (g_{\text{5D}}^\text{PQ} \sqrt{L})^{-1} \). For the remainder of our analysis, we replace the gauge coupling with this effective breaking scale using this relation.

Note that the constant transformations \( \beta^+ \) correspond to a true (unbroken) PQ global symmetry in terms of the transformation properties of the light SM fields. This residual transformation is unbroken at this stage, and thus the \( B_5 \) cannot play the role of a usual axion in resolving the strong CP problem (the \( B_5 \) is not a traditional PQ axion).

Before discussing the interactions of the light \( B_5 \), it is useful to understand this pattern of symmetry breaking in the language of deconstruction [27]. This model can be deconstructed as a chain of \( U(1) \) symmetries linked by scalar fields which each transform under two neighboring \( U(1) \) sites. To mimic the choice of boundary conditions we have chosen, we only gauge the internal sites, and the endpoints of the chain are taken to be global symmetries. In total, we have \( N \) sites, and \( N - 2 \) of the sites are gauged. There are \( N - 1 \) scalar fields breaking this set of symmetries, so there remains one unbroken \( U(1) \) symmetry, corresponding to \( \beta^+ \) in the continuum theory. There are \( N - 1 \) Goldstone bosons, and \( N - 2 \) are eaten since \( N - 2 \) of the sites were gauged. The remaining physical Goldstone mode corresponds to a non-trivial linear combination of \( U(1) \)'s and becomes a Wilson line for \( B_5 \) in the continuum limit.

### 3.2 Tree level interactions of the \( B_5 \) zero mode

In this section, we study the interactions of the PQ \( B_5 \) with the KK-modes and SM fields. In doing so we dispel the notion that the KK-parity transformation properties of a KK mode are determined solely by the transformation properties of the wave function.

This can be seen in a simple way. First we note that 5D gauge invariance associates every \( \partial_5 \) with a \( B_5 \) and vice versa through the covariant derivative:

\[
D_5 = \partial_5 - iqB_5. \tag{3.9}
\]

The form of the 5D flat space metric requires that any index must be repeated an even number of times in any single term in the Lagrangian. This is because everything must be contracted through the metric tensor (or through the vielbeins). This means that for interactions with an odd number of \( B_5 \)'s, there must be an odd number of \( \partial_5 \)'s (or a \( \gamma^5 \equiv \epsilon_5^a \gamma^a \)). Since both of these pick up a sign under the transformation \( z \rightarrow L - z \), the parity transform of the tower of \( B_5 \)'s is effectively the opposite of how the wavefunctions transform. In short, the internal KK parity of the 4D \( B_5 \) zero mode is \( - \).

As a concrete example, we consider the tree level interactions with a 5D fermion. The interactions arise from the 5D gauge covariant kinetic term:

\[
L_{\text{eff}} = \int dz \bar{\Psi} iD_M \gamma^a \Psi \supset q \int dz \bar{\Psi} B_5 \epsilon^a \gamma^a \Psi \tag{3.10}
\]

*Except in the case of contraction through the 5D Levi-Civita tensor, however such terms explicitly violate KK parity as they correspond to a net \( U(1)_{\text{PQ}} \) flux along the extra dimensional coordinate.*
The 5D Dirac fermion can be expanded in solutions of the 4D Dirac equation with masses $m_n$:

$$\Psi = \sum_n \begin{pmatrix} g_n(z) \chi_n(x) \\ f_n(z) \bar{\psi}_n(x) \end{pmatrix}$$

(3.11)

The boundary conditions that produce a $\chi_0$ massless mode are $f_n(z = 0, L) = 0$. Choosing these boundary conditions, the solutions for $f_n$ and $g_n$ are given by:

$$g_n = A_n \cos \frac{n\pi z}{L}$$

$$f_n = -A_n \sin \frac{n\pi z}{L}$$

(3.12)

with $A_0 = 1/\sqrt{L}$, and $A_n = \sqrt{2}/L$ for $n \neq 0$. This choice reproduces canonically normalized fields in the 4D effective theory.

We now expand Eq. (3.10) in KK modes and integrate over $z$, finding

$$\mathcal{L}_{\text{eff}} = -\frac{1}{f_{\text{PQ}}L} q \sum_{m,n} c_{nm} B_5(x) \left[ \psi_n \chi_m - \bar{\chi}_m \bar{\psi}_n \right]$$

$$c_{nm} = \begin{cases} \frac{4}{\pi} \frac{n^2}{m^2 - n^2} & m + n \text{ odd, } m \neq 0 \\ \frac{2\sqrt{2}}{\pi n} & m + n \text{ odd, } m = 0 \\ 0 & m + n \text{ even} \end{cases}$$

(3.13)

The $B_5$ is thus a KK-odd field in its interactions with fermions. The tree-level interactions with scalars are simpler to calculate, and the result is similar. At tree level, the massless $B_5$ is KK-odd in all of its interactions.

### 3.3 Spontaneous Breaking in the Bulk

When the SM Higgs fields obtain vacuum expectation values, the $U(1)_{\text{PQ}}$ symmetry undergoes additional spontaneous breaking in the bulk. We show that, in the absence of additional explicit breaking, the Higgsing along with the choice of boundary conditions produces two massless modes. One of these is a KK-even would-be electroweak scale axion that must be lifted, as such a scalar has interactions that are too strong to remain consistent with bounds from nuclear and astro-particle physics [37]. The other is the KK-odd zero mode whose phenomenology we are most interested in. Both modes will now be partly contained in $B_5$ and in the Goldstone field $\pi$ in the bulk Higgs. In this subsection we first identify these two modes, and then show that an explicit symmetry breaking term (which is allowed on the boundaries) will give a mass to both of these states. First we use a simplified version with a single bulk Higgs, and then show that it is easy to find the full answer for the two Higgs doublet case relevant for the bulk $U(1)_{\text{PQ}}$ model.
The two Goldstone zero modes

The Lagrangian, before gauge fixing, in our toy model is given by

\[ \mathcal{L} = \int dz \left[ \frac{1}{L} |D_M H|^2 - V(H) - \frac{f_{PQ}^2 L}{4} B_{MN} B^{MN} \right] - V_{\text{bound}}(H_{|0}) - V_{\text{bound}}(H_{|L}). \quad (3.14) \]

With the assumption that there are no brane localized scalar potential terms, the Higgs develops a \( z \)-independent vev profile. For now, we assume that this is the case, and add brane localized interactions later, treating them perturbatively in the low-energy 4D effective theory.

First, as in Section 3.1, we identify the interactions which kinetically mix the gauge bosons with the Goldstone bosons, so that we can remove them with a suitable gauge fixing term. Taking \( H \equiv \frac{v}{\sqrt{2}} e^{i\pi/v} \), keeping only the Goldstone fluctuations \( \pi \), we have:

\[ \mathcal{L}_{\text{mix}} = -f_{PQ}^2 L (\partial_5 B^\mu)(\partial_\mu B_5) - \frac{1}{L} v \partial_\mu \pi B^\mu \]  

(3.15)

A gauge fixing term that removes the 4D kinetic mixing is:

\[ \mathcal{L}_{\text{GF}} = -\frac{1}{2} G^2 = -\frac{f_{PQ}^2 L}{2\xi} \left[ \partial_\mu B^\mu - \xi \left( \partial_5 B_5 - \frac{1}{f_{PQ}^2 L^2} v \pi \right) \right]^2. \quad (3.16) \]

The residual gauge symmetry obeys the following boundary conditions:

\[ \partial_\mu \partial^\mu \beta - \xi \left( \partial_5^2 \beta - \frac{v^2}{f_{PQ}^2 L^2} \beta \right) = 0 \]  

(3.17)

In the \( \xi \to \infty \) limit, with constant \( v \), the solutions to the equation with appropriate boundary conditions are:

\[ \beta(x, y) = \beta^+ \cosh(\kappa(z - L/2)) + \beta^- \sinh(\kappa(z - L/2)), \]  

(3.18)

where we have introduced an expansion parameter \( \kappa \equiv v/(f_{PQ} L) \). We can find the Goldstone-like zero modes which are shifting under \( \beta \) by carefully analyzing the bulk EOM’s and the BC’s, which is performed in detail in Appendix A. The resulting zero modes can be written in terms of KK even and odd combinations. In the case where the \( B_5 \) part has a KK-even wave-function (but remembering that the interactions are KK-odd) the \( B_5 \) and \( \pi \) zero modes given by

\[ B_5^{(0)\text{odd}} = A'_B \cosh \kappa(z - L/2) \zeta_+(x) \]

\[ \pi^{(0)\text{odd}} = A'_B \frac{v}{\kappa} \sinh \kappa(z - L/2) \zeta_+(x) \]  

(3.19)

The subtlety about the KK-parity quantum numbers of the \( B_5 \) plays out here, as a single zero mode KK-eigenstate has simultaneous KK-even and KK-odd wavefunctions (although the interactions are all consistent, as they must be).
The KK-even modes are given by:

\[ B^{(0)\text{even}}_5 = B'_B \sinh \kappa(z - L/2) \zeta_+(x) \]
\[ \pi^{(0)\text{even}} = B'_B \frac{v}{\kappa} \cosh \kappa(z - L/2) \zeta_+(x) \] (3.20)

Imposing canonical normalization for the 4D fields then fixes the overall coefficients \( A'_B \) and \( B'_B \). Note that the residual symmetries in Eq. (3.18) are consistent with the profiles of these zero modes: the residual gauge transformations are shift symmetries for the 4D massless modes, \( \zeta_- \) and \( \zeta_+ \).

**Explicit brane localized \( U(1)_{\text{PQ}} \) breaking**

We now analyze what happens when we add explicit symmetry breaking on the boundaries. We add PQ breaking \( \mu \) terms of the form \( V_{\text{bound}} = -\frac{\mu}{2} (H^2 + H'^2) \) on each boundary. This is allowed, since the symmetry is only global on the endpoints. Expanding in the Goldstone fluctuations, this leads to brane localized mass terms for the 5D field \( \pi \):

\[ V_{\text{bound}} \bigg|_{z=0,L} = \mu \pi^2 \bigg|_{z=0,L}. \] (3.21)

Keeping track of only the (now approximate) zero modes, this becomes:

\[ V_{\text{bound}} \bigg|_{0,L} = \mu \left[ A'_B \frac{v}{\kappa} \sinh \kappa(z - L/2) \zeta_-(x) + B'_B \frac{v}{\kappa} \cosh \kappa(z - L/2) \zeta_+(x) \right]^2 \bigg|_{0,L}. \] (3.22)

The effective 4D potential is obtained by summing over the two boundary contributions, which gives:

\[ V_{\text{eff}} = 2\mu A'^2_B \left( \frac{v}{\kappa} \right)^2 \sinh^2 \frac{\kappa L}{2} \zeta_-(x) + 2\mu B'^2_B \left( \frac{v}{\kappa} \right)^2 \cosh^2 \frac{\kappa L}{2} \zeta_+(x) \] (3.23)

Expanding in small \( \kappa \) and imposing canonical normalization on the scalar zero modes in the 4D effective theory takes this to:

\[ V_{\text{eff}} = 2\mu A'^2_B \left( \frac{v}{\kappa} \right)^2 \sinh^2 \frac{\kappa L}{2} \zeta_-(x) + 2\mu B'^2_B \left( \frac{v}{\kappa} \right)^2 \cosh^2 \frac{\kappa L}{2} \zeta_+(x) \] (3.24)

The masses of the KK-even and KK-odd modes are then \( m^2_+ = 4\mu \), and \( m^2_- = \mu v^2 / f_{\text{PQ}}^2 \). A full numerical evaluation of the equations of motion, including deformation of the VEV due to the \( \mu \)-terms, confirms that these approximations hold at the level of 2% for the KK-odd mode, and < 1% for the KK-even mode for \( \mu \) as large as \((300 \text{ GeV})^2\).
Pseudo-Goldstones in the full 2-Higgs doublet model

The generalization of this model to the two Higgs doublet model (2HDM) of our construction is quite simple. We first write the two Higgs doublets keeping only the Goldstone fluctuations along the $U(1)_{PQ}$ flat direction, ignoring the 2 neutral Higgses, and the charged Higgs fields. The Goldstone fluctuation $\pi$ is the neutral pseudoscalar often referred to as $A_0$ in 2HDMs.

$$H_u = \frac{v_u}{\sqrt{2}} e^{i\pi/V}, \quad H_d = \frac{v_d}{\sqrt{2}} e^{i\pi/V}, \quad \text{with} \quad V \equiv \sqrt{v_u^2 + v_d^2}$$

(3.25)

In this case, the entire analysis above follows through the same way with the replacements

$$v \to V = \sqrt{v_u^2 + v_d^2}$$

and

$$\mu \to \frac{\mu}{2} \sin 2\beta,$$

(3.26)

where the angle $\beta$ is defined in the usual way for a 2HDM, $v_u/v_d \equiv \tan \beta$. The explicit symmetry breaking terms in this case are given by

$$L_{\text{mix}} = \frac{\mu}{2} H_u^T (i \tau_2) H_d \bigg|_{z=0,L}$$

(3.27)

The final masses are:

$$m^2_+ = 2\mu \sin 2\beta$$

$$m^2_- = \frac{\mu V^2}{2 f_{PQ}^2} \sin 2\beta.$$  

(3.28)

Taking $\mu_{\text{eff}} \equiv \frac{\mu \sin 2\beta}{2}$, the numerical expression for the mass of the light pseudo-Goldstone boson is:

$$m_- = (f_{PQ} L)^{-1} \left( \frac{\sqrt{\mu_{\text{eff}}}}{300 \text{ GeV}} \right) \left( L \cdot 10^3 \text{ GeV} \right) \cdot 74 \text{ GeV}$$

$$= \left( \frac{\sqrt{\mu_{\text{eff}}}}{300 \text{ GeV}} \right) \left( \frac{10^9 \text{ GeV}}{f_{PQ}} \right) \cdot 74 \text{ keV}.$$  

(3.29)

For perturbative values of the coupling $(f_{PQ} L)^{-1}$, and for weak scale $\mu$, the mass of $\zeta_-$ is less than the mass of any level one KK-mode, whose masses are generally $m^{(1)} \sim \pi/L$. So for most choices of parameters, this pseudo-Goldstone is the LKP. The reference value of $10^9$ GeV in the second expression is chosen to match the point at which the decay length of the NLKP is of order tens of centimeters, as we show in Section 4.
3.4 $U(1)_{\text{PQ}}$ Anomalies

With the fermion charges given in Table 2.2, the $U(1)_{\text{PQ}}$ symmetry is anomalous. However, as we have shown in Section 3.1, the residual symmetry after imposing Dirichlet boundary conditions on the 4D components of the PQ gauge field is global on the endpoints of the extra dimension. In this section we calculate the chiral anomalies in this model, emphasizing that the chiral anomalies are localized on the branes [11], where the gauge transformation is global rather than local. As a result, the theory is consistent at the quantum level. However, as is crucial in our model, the anomalies imply effective interactions between the $U(1)_{\text{PQ}} B_5$ and the SM gauge fields. We focus on anomalies of the form $U(1)_{\text{PQ}} \times \text{SM} \times \text{SM}$, since these lead to the interactions we are most interested in.

An intuitive argument for the localized anomaly terms

First we present an intuitive argument that suggests the required form of the localized anomaly terms based on the shift properties of the action and the Goldstone bosons under the anomalous symmetries. Later we will give a more rigorous derivation based on the anomalous transformations of the path integral measure.

Under an anomalous $U(1)_{\text{PQ}}$ transformation $B_M \rightarrow B_M + \partial_M \beta(x, z)$, the action shifts by:

$$\delta S = \int d^4x \int_0^L dz \beta \partial_M J^M - \int d^4x \beta J^5 \equiv \int d^5 \beta A,$$

where $J^M$ is the classically conserved PQ current, and $A$ is the anomalous divergence. The boundary term vanishes by construction, through the assignment of the orbifold boundary conditions which produce the chiral spectrum in Table 2.2. The anomaly is itself purely localized on the branes, and has been calculated in [11] to be:

$$A(x, z) = \frac{1}{2} [\delta(z) + \delta(z - L)] \sum_f q_{\text{PQ}}^f \left( \frac{g^2}{16\pi^2} F \cdot \tilde{F} + \frac{Tr \tau_3 \tau_3}{16\pi^2} W \cdot \tilde{W} + \frac{Tr \tau_1 \tau_1}{16\pi^2} G \cdot \tilde{G} \right)$$

$$\equiv \frac{1}{2} [\delta(z) + \delta(z - L)] Q_{\text{PQ}}(x, z)$$

(3.31)

where $F$, $W$, and $G$ are the hypercharge, $SU(2)_L$, and QCD field strengths, and $F \cdot \tilde{F}$ is given by $\frac{1}{2} \epsilon^{\mu
u\rho\sigma} F_{\mu\nu}(x, z) F_{\rho\sigma}(x, z)$ (with similar expressions for $W \cdot \tilde{W}$ and $G \cdot \tilde{G}$).

To reproduce the above shift in the action, the Lagrangian has to contain a coupling involving the Goldstone bosons, whose shifts will exactly correspond to the above change in the action. Remembering that the decomposition of $\beta$ is

$$\beta = \beta^+ \cosh[\kappa(z - \frac{L}{2})] + \beta^- \sinh[\kappa(z - \frac{L}{2})]$$

(3.32)

and the fact that under this shift $B_5 \rightarrow B_5 + \partial_5 \beta$, we can identify the shifts of the fields $\zeta_{\pm}$. We find, that

$$\zeta_{\pm} \rightarrow \zeta_{\pm} + v \sqrt{\frac{\sinh \kappa L}{\kappa L}} \beta^\pm.$$

(3.33)
Therefore the shift in the action is reproduced if the following couplings are added to the Lagrangian:

\[
\mathcal{L}^\text{anomaly}_{\text{eff}} = \frac{1}{2v} \zeta - \sqrt{\frac{kL}{\sinh kL}} \sinh \frac{kL}{2} \left[ \mathcal{Q}_{\text{PQ}}(x, L) - \mathcal{Q}_{\text{PQ}}(x, 0) \right] + \frac{1}{2v} \zeta^+ \sqrt{\frac{kL}{\sinh kL}} \cosh \frac{kL}{2} \left[ \mathcal{Q}_{\text{PQ}}(x, L) + \mathcal{Q}_{\text{PQ}}(x, 0) \right].
\] (3.34)

To lowest order in the bulk PQ gauge coupling, this becomes:

\[
\mathcal{L}^\text{anomaly}_{\text{eff}} = \frac{1}{4f_{\text{PQ}}} \zeta^- \left( \mathcal{Q}_{\text{PQ}}(x, L) - \mathcal{Q}_{\text{PQ}}(x, 0) \right) + \frac{1}{2v} \zeta^+ \left( \mathcal{Q}_{\text{PQ}}(x, L) + \mathcal{Q}_{\text{PQ}}(x, 0) \right). \quad (3.35)
\]

Anomalous interactions from the path integral measure

Above we have seen a simple argument for the existence of the brane localized anomalous interactions, motivated by the shifts of the various Goldstone fields. We now present the full derivation of these terms through the shift in the path integral measure as first identified by Fujikawa [4, 5]. For this we add two fermions to the single Higgs toy model described by the effective Lagrangian in Eq. [3.14] These fermions have \((\pm, \pm)\) and \((\mp, \mp)\) boundary conditions respectively, such that one fermion has a left handed zero mode, and the other has a right handed zero mode. Additionally, they each carry opposite charge under the \(U(1)\) symmetry, \(q_{L,R} = \pm 1/2\). The additional terms in the classical effective Lagrangian are:

\[
\mathcal{L}_{\text{fermion}}^\text{eff} = \int dz \left\{ \bar{\Psi}_L \gamma^5 \left( \partial_5 - iq_L B_5 \right) \psi_L + \bar{\Psi}_R \gamma^5 \left( \partial_5 - iq_R B_5 \right) \psi_R + \left( \lambda H \bar{\Psi}_L \psi_R + \text{h.c.} \right) \right\}. \quad (3.36)
\]

We now restrict ourselves to the terms in this Lagrangian that involve the Goldstone bosons \(\pi\) and \(B_5\):

\[
\mathcal{L}_{\text{fermion}}^\text{eff} \supset \int dz \left\{ \bar{\Psi}_L \gamma^5 \left( \partial_5 - iq_L B_5 \right) \psi_L + \bar{\Psi}_R \gamma^5 \left( \partial_5 - iq_R B_5 \right) \psi_R + \left( \frac{\lambda v}{\sqrt{2}} e^{i(q_L - q_R) \pi} \bar{\Psi}_L \psi_R + \text{h.c.} \right) \right\}. \quad (3.37)
\]

We now perform a redefinition of the fermion fields such that the new fermion degrees of freedom do not transform under the broken \(U(1)\) symmetry. After this is done, the path integral measure itself no longer transforms under rotations, and all interactions of the Goldstone bosons through the anomaly are manifest. The redefinition is given by:

\[
\psi_j = e^{i\delta_j} f(\pi, B_5) \psi'_j, \quad (3.38)
\]

with \(f\) transforming as \(f \rightarrow f + \beta(x, z)\), and \(\psi'_j \rightarrow \psi'_j\). The most general choice of \(f\) that satisfies this property is a linear combination of a Wilson line and the 5D field \(\pi\) from the bulk Higgs:

\[
f(\pi, B_5) = a \left[ \int_{z_0}^{z} dz' B_5(x, z') + \frac{\pi(z_0, x)}{v(z_0)} \right] + (1 - a) \frac{\pi(z, x)}{v(z)}, \quad (3.39)
\]
where $a$ is an arbitrary c-number.

In terms of the two physical Goldstone modes, $\zeta_+$ and $\zeta_-$, the function $f(\pi, B_5)$ is given by:

$$f(\pi, B_5) = \frac{1}{v} \sqrt{\frac{\kappa L}{\sinh \kappa L}} \left[ \sinh \kappa (z - L/2) \zeta_- (x) + \cosh \kappa (z - L/2) \zeta_+ (x) \right]$$  \hspace{1cm} (3.40)

It is reassuring that this result is completely independent of the two undetermined parameters $z_0$ and $a$. These parameters are thus unphysical, and do not affect any interactions after performing the redefinition.

The redefinition does, however, reorganize other interactions in the theory. The 5D fermion kinetic terms are modified in the following way at the classical level:

$$\bar{\Psi}_j D_\mu \gamma^\mu \Psi_j = \bar{\Psi}'_j D_\mu \gamma^\mu \Psi'_j - q_j (\partial_\mu f(\pi, B_5)) \bar{\Psi}'_j \gamma^\mu \Psi'_j + \bar{\Psi}'_j i \partial_5 \gamma^5 \Psi'_j$$

$$- q_j (\partial_5 f(\pi, B_5) - B_5) \bar{\Psi}'_j i \gamma^5 \Psi'_j.$$  \hspace{1cm} (3.41)

Note that this expression is completely gauge invariant under $U(1)_{PQ}$. In addition, the Goldstone interactions from the Yukawa term in the Lagrangian become:

$$\frac{\lambda v}{\sqrt{2}} \exp \left[ i (q_L - q_R) \left( \frac{\pi(z, x)}{v(z)} - f(\pi, B_5) \right) \right] \bar{\Psi}'_{L5} \Psi'_{R5}.$$  \hspace{1cm} (3.42)

The argument of this exponential and the coefficient of the 5D pseudoscalar current in Eq. (3.41) are both invariant under all $U(1)_{PQ}$ gauge transformations, and thus these expressions do not involve either of the physical Goldstone bosons. This can be verified using the wave functions derived in the previous section.

It is instructive to compute the effective 4D currents corresponding to the broken symmetries associated with the KK-even and KK-odd pseudo-Goldstone bosons. At lowest order in the 5D PQ gauge coupling, the $\zeta_+$ couples diagonally due to wave function orthogonality, and the current corresponding to this symmetry is

$$j^\mu_+ = \sum_{j,n} q_j \bar{\Psi}_{j,n}^{4D} \gamma^\mu \Psi_{j,n}^{4D}, \quad \Psi_{j,n \neq 0}^{4D} = \begin{pmatrix} \chi_{j,n}(x) \\ \bar{\psi}_{j,n}(x) \end{pmatrix}, \quad \Psi_{j,0}^{4D} = P_j \begin{pmatrix} \chi_{j,0}(x) \\ \bar{\psi}_{j,0}(x) \end{pmatrix}.$$  \hspace{1cm} (3.43)

which can be determined by reading off the coupling of the $\zeta_+$ in the 4D effective theory (arising from the second term in 3.41):

$$\mathcal{L}_+ = -\frac{1}{v} \left( \partial_\mu \zeta_+ (x) \right) j^\mu_+,$$  \hspace{1cm} (3.44)

where $j$ labels the species of fermion, and $n$ labels the KK-level. The projector, $P_j$, is either $P_+$, or $P_-$, depending on whether $\bar{\Psi}_j$ contains a right- or left-handed zero mode. With
the charge assignments we have chosen, from the perspective of the zero modes, this is an axial-vector current. The KK-odd current is more involved:

\[
\begin{align*}
    j_\mu^- &= \sum_{m,n,j} q_{j} c_{mn} \bar{\Psi}_4 D_{j,m} \gamma^\mu \left[ (m - n)^2 + (m + n)^2 \right] \Psi_4 D_{j,n}, \\
    c_{mn} &= \begin{cases} 
    0 & m + n \text{ even} \\
    2/\pi^2 & m + n \text{ odd, } m, n \neq 0 \\
    \sqrt{2}/\pi^2 & m + n \text{ odd, } m \cdot n = 0
    \end{cases}
\end{align*}
\]

where the coupling is

\[
\mathcal{L}_- = -\frac{1}{f_{\text{PQ}}} (\partial_\mu \zeta_-(x)) j_\mu^-. \tag{3.46}
\]

Note that we have finally explicitly identified the effective symmetry breaking scale associated with the $B_5$ Goldstone boson, justifying our identification $g_{5D} \sqrt{L} \equiv f_{\text{PQ}}^{-1}$.

Due to the anomaly, the redefinition \[3.38\] produces a non-trivial Jacobian in the path integral measure \[4,5\]. The couplings of the Goldstone bosons due to the anomaly can then be found by expanding

\[
\mathcal{L}_{\text{anomaly}}^\text{eff} = \int dz f(\pi, B_5) A \tag{3.47}
\]

in terms of the scalar zero modes. Using again the expression of the anomaly from [11] in \[3.31\] we reproduce the expressions \[3.34\]-\[3.35\] for the brane localized anomalous couplings of the Goldstone bosons.

**The interactions of $\zeta_-$**

We now turn our focus to the interactions of the KK-odd Goldstone, $\zeta_-$, in the effective action Eq. \[3.35\]. Using the KK decomposition of the 5D hypercharge gauge boson (in the absence of electroweak symmetry breaking), we get

\[
F_{\mu \nu}(x, z) = g_{5D} \sqrt{\frac{1}{L}} F_{\mu \nu}^{(0)}(x) + g_{5D}' \sum_{n \geq 1} \sqrt{\frac{2}{L}} \cos \left( \frac{n\pi z}{L} \right) F_{\mu \nu}^{(n)}(x), \tag{3.48}
\]

with similar expansions for the $SU(2)_L$ and $SU(3)_C$ field strengths. The normalization coefficients are chosen to produce a canonically normalized 4D effective theory. This yields

\[
\mathcal{L}_{B_5 AA}^\text{eff} = \frac{1}{16\pi^2} \frac{1}{f_{\text{PQ}}^2} \frac{g_{5D}^2}{L} \zeta_-(x) \sum_{m \geq n \geq 0} c_{mn} F^{(n)} \cdot \tilde{F}^{(m)} \tag{3.49}
\]

\[
= \frac{\alpha_1}{4\pi f_{\text{PQ}}} \zeta_-(x) \sum_{m \geq n \geq 0} c_{mn} F^{(n)} \cdot \tilde{F}^{(m)},
\]
where $\alpha_1 = \frac{g^2}{4\pi}$, $g' = g'_{5D}/\sqrt{L}$ is the usual 4D effective hypercharge gauge coupling, $f_{\text{PQ}} \equiv 1/(g_{5D}^P\sqrt{L})$ is the effective PQ decay constant, and the coefficients $c_{nm}$ are given by

$$c_{nm} = \begin{cases} 
0 & n + m \text{ even} \\
2\sum_{f} q_{\text{PQ}f} q_{Y}^{f2} & n + m \text{ odd, } n, m \geq 1 \\
\sqrt{2}\sum_{f} q_{\text{PQ}f} q_{Y}^{f2} & n + m \text{ odd, } n \cdot m = 0.
\end{cases} \quad (3.50)$$

### 4 $B_5$ Phenomenology

In this section, we perform a study of the basic phenomenology of this new model. The collider signatures are quite dramatic: nearly all final state signal events contain high $p_T$ photons or $Z$ bosons along with large amounts of missing energy. Even more remarkable is that for some ranges of the extra dimensional $U(1)_{\text{PQ}}$ gauge coupling, the photons or $Z$’s do not generally point back to the original interaction vertex (that is, the photons or $Z$’s are “delayed”). Such signatures have long been considered a smoking gun for supersymmetry broken by low scale gauge mediation, and so our analysis suggests that more detailed experimental analyses may be necessary to distinguish supersymmetry from this model. We calculate the lifetime of the lightest KK-mode and the displacement of the decay vertex from the interaction point. We assume here that the lightest KK-mode is the level-1 partner of the hypercharge gauge boson. We also consider the possibility that the $\zeta_-$ Goldstone boson may constitute a large fraction of the observed relic abundance of dark matter, calculating the relic abundance over a range of free parameters in the model.

#### 4.1 Decays of the NLKP

We presume that the NLKP is the first KK-mode of the hypercharge gauge boson. This is often the case in UED, since mass splittings in the level 1 KK sector are achieved at the quantum level through brane localized kinetic terms. The small value of $\alpha_1$ implies a smaller contribution to the mass of the level-1 hypercharge gauge boson.\footnote{The level 1-KK mode of the PQ gauge boson may be lighter, however this mode is even under KK-parity, and additionally has a very small coupling to SM fields. This particle is thus rarely produced, and does not appear substantially in the decay products of the KK-modes of SM fields.} Using the effective Lagrangian in Eq. (3.49), we evaluate the matrix element between the level one hypercharge gauge boson, the $\zeta_-$, and a SM photon or $Z$. The final polarization averaged and summed amplitude squared for the decay of the level-1 KK-mode of the hypercharge gauge boson is given by:

$$\frac{1}{3} \sum_{\text{pol}} |i M_{\gamma,Z}|^2 = \frac{8}{3} \lambda_{\gamma,Z}^2 \left[ (p^{(0)} \cdot p^{(1)})^2 - p^{(0)2}p^{(1)2} \right] = \frac{2}{3} \lambda_{\gamma,Z}^2 m^{(1)4} \left[ 1 - \left( \frac{m^{(0)}}{m^{(1)}} \right)^2 \right]^2 \quad (4.51)$$
where $p^{(0)}$ is the momentum of the photon or $Z$, and $\lambda_{\gamma,Z}$ is given by

$$\lambda_{\gamma,Z} = \frac{\alpha_1^2}{4\pi f_{\text{PQ}}} \sqrt{2} \sum_f q_f^2 q_Y^2 (c_w, s_w).$$  \hspace{1cm} (4.52)

In the last step, we have evaluated the products of momenta in the rest frame of the decaying KK-mode, and we have neglected the mass of the $B_5$.

For $1/L \gg v$, we can ignore the mass of the $Z$ boson, and the partial widths in this limit are given by:

$$\Gamma_{\gamma,Z} \approx \frac{\alpha^2}{192 \pi^3 c_w^4 f_{\text{PQ}}^2} m^{(1)3} \left( \sum_f q_f^2 q_Y^2 \right)^2 (c_w^2, s_w^2).$$  \hspace{1cm} (4.53)

The sum over charges as can be read in Table 2.2 is $\sum_f q_f^2 q_Y^2 = -5$. We express the final width numerically for reference values of the free parameters as:

$$\Gamma_{\text{tot}} \approx 4.3 \times 10^{-7} \text{ eV} \left( \frac{m^{(1)}}{10^3 \text{ GeV}} \right)^3 \left( \frac{10^9 \text{ GeV}}{f_{\text{PQ}}} \right)^2,$$

with branching fractions given by

$$R_\gamma \approx c_w^2 \quad R_Z \approx s_w^2$$  \hspace{1cm} (4.55)

up to terms of order $m_Z^2/m^{(1)2}$. The total width corresponds to a lifetime for the NLKP equal to

$$\tau = 1.5 \times 10^{-9} \text{ s} \left( \frac{10^3 \text{ GeV}}{m^{(1)}} \right)^3 \left( \frac{f_{\text{PQ}}}{10^9 \text{ GeV}} \right)^2.$$  \hspace{1cm} (4.56)

The NLKP is at the bottom of a decay chain of exotica produced at a collider experiment, and the NLKP may travel some measurable distance before decaying, producing a rather spectacular signature of high energy photons or $Z$’s which decay to jets or leptons that do not point back to a central interaction vertex. The distance traveled by the NLKP is given by:

$$\Delta x = \gamma v \tau \approx 46 \text{ cm} \left( \frac{10^3 \text{ GeV}}{m^{(1)}} \right)^3 \left( \frac{f_{\text{PQ}}}{10^9 \text{ GeV}} \right)^2 \sqrt{\left( \frac{E}{m^{(1)}} \right)^2 - 1}.$$  \hspace{1cm} (4.57)

Where $\gamma$ is the relativistic time-dilation factor, and $v$ is the velocity. The typical range for the energy $E$ of the NLKP in a collider experiment is both model and analysis dependent. For larger mass splittings between the different members of the level-1 KK sector, $E$ will typically be larger, as a greater portion of the parent exotica is converted to kinetic energy. Also the analyses performed at collider experiments require specific cuts on the sample. For example, an analysis may focus on a trigger sample in which events are required to contain large amounts of missing transverse energy. Such requirements again bias towards larger $E$ for the NLKP, and thus longer decay lengths.
4.2 $B_5$ Dark Matter

In the scenario we study, the $B_5$ is most likely the LKP for all perturbative choices of the 5D PQ coupling, and is thus a dark matter candidate when KK-parity is preserved. In this section, we discuss the constraints on parameter space based on over-closure considerations, and the potential of the $B_5$ to make up a significant fraction of the dark matter relic abundance. We vary the scale $f_{\text{PQ}}$ over a large range, from a standard $\mathcal{O}(1)$ weak coupling to a very high suppression. An excellent review that describes the analysis in these different cases can be found in [40].

The case with weak scale $m_-$

The gauge coupling may not be very small, in which case the decays will be prompt, and the $\zeta_-$ may be a more standard dark matter candidate, being in thermal equilibrium prior to decoupling. In this case, one can evaluate the annihilation cross section, and follow the usual prescription to evaluate the relic abundance. The annihilation to SM particles primarily takes place via s-channel Higgs exchange. For our calculation, we assume large $\tan\beta = v_u/v_d$, and that the heavy neutral Higgs is much more massive than the light neutral Higgs: $m_{H_0} \gg m_{h_0}$.

The thermally averaged non-relativistic annihilation cross section to massive SM gauge fields is given in this limit by:

$$\langle \sigma v \rangle_{W^\pm, Z} = \frac{2m_\zeta^6}{\pi v_{\text{eff}}^4} \frac{1}{(4m_\zeta^2 - m_H^2)^2 + m_H^2 \Gamma_H^2} \left( 1 - \frac{m_\zeta^2}{m_\zeta^2} + \frac{3m_\zeta^4}{4m_\zeta^4} \right) \sqrt{1 - \frac{m_\zeta^2}{m_\zeta^2}},$$

where $v_{\text{eff}} = 246$ GeV is the effective electroweak symmetry breaking scale and $m_V = m_{W,Z}$ is the mass of the massive SM gauge bosons into which the $\zeta_-$ annihilates. The annihilation cross section into fermions via the s-channel Higgs in the large $\tan\beta$ and $m_{H_0} \gg m_{h_0}$ limit is given by:

$$\langle \sigma v \rangle_{\bar{f}f} = \frac{m_{\zeta}^8 m_f^2}{\pi v_{\text{eff}}^4} \frac{1}{(4m_\zeta^2 - m_H^2)^2 + m_H^2 \Gamma_H^2} \left( 1 - \frac{m_f^2}{m_\zeta^2} \right)^{3/2}.$$  

The annihilation into vectors is rather efficient, even relatively far off of the light Higgs resonance. Thus the preferred band in which the $\zeta_-$ relic abundance saturates the WMAP bound in this mass range is close to the threshold for annihilation into $W$ bosons. For the annihilation into light fermions, the cross section is suppressed by the fermion mass, and the WMAP window is saturated on the tails of the Higgs resonance.

There are additional channels where the $\zeta_-$ annihilates to photons or gluons, however these are essentially two loop diagrams, since each vertex arises through the anomaly. These annihilation channels can thus be ignored. The results for the relic abundance calculation are shown in Figure 1. We plot contours for when the WMAP result for the
Figure 1: In this Figure, we plot contours of the relic abundance, $\Omega_{dm} h^2$, of the $\zeta_-$ dark matter candidate in the case that the mass of the $\zeta_-$ is near the electroweak scale. The narrow gray band corresponds to the WMAP 2$\sigma$ band, where we take the density of non-baryonic dark matter to be $\Omega_{nbdm} = 0.106 \pm 0.008$ [37]. The white area corresponds to an under-density of $\zeta_-$ dark matter where it annihilates efficiently, and the dark area corresponds to an over-density.

relic abundance is saturated (within the 2$\sigma$ band), as well as contours where there is less or more dark matter.

**The case with low $T_{\text{reheat}}$, small $(f_{\text{PQ}} L)^{-1}$**

In the case that the reheating temperature is very low (on order the mass of the level 1 KK-modes), and the PQ gauge coupling is small, the KK-odd Goldstone boson is never in equilibrium with the thermal bath, and the relic abundance of the $B_5$ in this case originates primarily from decays of the NLKP. The final relic abundance is then given by:

$$\Omega_{B_5} h^2 = \frac{m_{B_5}}{m_{\text{NLKP}}} \Omega_{\text{NLKP}} h^2.$$  \hspace{1cm} (4.60)

The NLKP abundance has been calculated as a function of mass, and splittings between KK-modes [19,41,42]. Unless the the relic abundance of the NLKP is anomalously large, this is clearly not enough dark matter to saturate the measured relic abundance. Of course, in such scenarios, there may be another dark matter candidate (such as a standard pseudo-scalar axion) which can make up the remainder. We note that baryogenesis and
leptogenesis are very problematic in such scenarios, as they must also occur at this low scale of reheating.

**The case with larger $T_{\text{reheat}}$, small $(f_{\text{PQ}}L)^{-1}$**

In the case where the gauge coupling is small, the universe is overclosed if the $B_5$ was in thermal equilibrium. This implies that some intervening era of inflation must dilute the initial relic abundance, and that post-reheating, the dark matter never reached thermal equilibrium with the bath. The reheat temperature is likely significantly higher than the mass of the level-1 KK-modes, as is necessary for generating a baryon asymmetry. In this case the situation is considerably more complicated than the previous ones. The relic abundance in such a scenario can be found as a function of the reheating temperature and the couplings to the species which are in equilibrium. The relic abundance in this case primarily arises through thermal production via scattering processes that occur in the bath.

This has been calculated to leading order in the QCD gauge coupling for the scenario of a supersymmetric axino DM candidate [43] in supersymmetric extensions of the SM [44], and the calculation is quite involved. In the PQ-UED model, the situation is even more complicated due to the fact that not only are level-1 KK modes present in the thermal bath, but the entire tower of KK-modes contributes at a given reheat temperature. Additionally, the 5D theory is non-renormalizable, and perturbative unitarity is lost at energies of order $4\pi/L$. The 5D theory must be UV completed at some relatively low scale, and the characteristics of this UV completion will likely play a crucial role in the final relic abundance. These complications do not by themselves rule out the potential of the KK-odd Goldstone as a DM candidate in this region of parameter space, but the calculation is clearly beyond the scope of this analysis. We note that it is quite easy to construct a model that is very similar to that of the MSSM by deconstructing the extra dimension into a simple 2-site model. If the symmetry breaking in this scenario is achieved by a linear sigma model, then the results would likely be very similar to those in [44], with differences arising only from spin statistics in the production matrix elements, and an extended scalar sector.

In the case of very small $(f_{\text{PQ}}L)^{-1}$, one might also worry about constraints from big-bang nucleosynthesis, or perturbations in the cosmic microwave background due to the late injection of electromagnetic energy from NLKP decays. Neither of these are relevant for the range of couplings we are most interested in. BBN is safe so long as the lifetime of the NLSP is less than 1 second, the time at which BBN takes place. This limit on the lifetime, for weak scale $\mu$, corresponds to a limit on the PQ scale of $f_{\text{PQ}} < 10^{14}$ GeV. The CMB constraints are even more relaxed, requiring a lifetime of not more than $10^{1-5}$s, conservatively. For these large values of the PQ scale, the NLKP decays far outside of the detector, and does not play a role in collider physics.
4.3 Electroweak precision and direct collider constraints

We estimate the size of shifts in electroweak precision observables due to the variation in the vev due to the localized $\mu$ terms. The terms in the 5D Lagrangian relevant to EWP are:

$$\int dz \frac{g^2 v^2(z)}{8} \left[ W^{(1)2}_\mu + W^{(3)2}_\mu - 2 \frac{g'}{g} W^{(3)} B^\mu \right]$$ (4.61)

We expand the Lagrangian in terms of the KK-modes, examining the terms which give mass mixing between the lowest lying modes and the higher KK-modes. We treat the vev perturbatively, expanding it as

$$v(z) = v_0 + \delta v(z).$$

$$\sum_n \int dz \frac{g^2 v_0 \delta v(z)}{2} \left[ W^{(1)} W^{(1)\mu}_n + W^{(3)} W^{(3)\mu}_n - \frac{g'}{g} W^{(3) B^\mu}_n \right]$$ (4.62)

The diagrams involving heavy $W$ exchange cancel in calculating $\Pi_{11} - \Pi_{33}$, so we need only calculate the diagrams mixing the heavy $B$ with $W^{(3)}$, the last term in Eq. (4.62).

We Taylor expand the vev about the midpoint of the extra dimension, $\delta v(z) = 1/2 v''(z) = 1/2 v''(z = L/2)(z - L/2)^2$, and input the canonically normalized gauge boson wave functions to find the relevant overlap integrals for the mixing terms:

$$\frac{g g' v_0 v''}{2 \sqrt{2L}} \int dz (z - L/2)^2 \cos \frac{n\pi z}{L} = \frac{g g' L^2 v_0 v''_{L/2}}{\sqrt{2n^2\pi^2}} \cdot \left\{ \begin{array}{l} 1 \ n \text{ even} \\ 0 \ n \text{ odd} \end{array} \right. \quad (4.63)$$

The diagrams then evaluate to:

$$g^2 (\Pi_{11} - \Pi_{33}) = \sum_{n \text{ even}} \frac{g^2 g'^2 L^6 v_0^2 (v''_{L/2})^2}{2n^6\pi^6} = \frac{g^2 g'^2 L^6 v_0^2 (v''_{L/2})^2}{120, 960} \quad (4.64)$$

where we have used the fact that the masses of the hypercharge gauge boson KK-modes are approximately given by $m_n = \frac{n\pi}{L}$. $\Delta \rho$ is then given by:

$$\Delta \rho = \alpha T = \frac{4}{\alpha T} (\Pi_{11} - \Pi_{33}) = \frac{g^2 L^6 (v''_{L/2})^2}{240, 30} \approx 8 \cdot 10^{-9} \left( \frac{L}{1 \text{ TeV}} \right)^6 \left( \frac{\mu}{300^2 \text{ GeV}^2} \right)^2, \quad (4.65)$$

well within current experimental limits. To understand the overall scaling with $L$ and $\mu$, remember that $v''|_{0,L} = \pm \mu L v|_{0,L}$, and thus $v'' \approx \frac{v''|_0 - v''|_L}{L} \approx 2 \mu v$.

Regarding direct collider constraints, it is unlikely that the Tevatron experiments searching for GMSB-like scenarios [45, 46] place any limits on this scenario. This is due to the fact that there are indirect electroweak precision constraints on the extra dimensional model in addition to the ones calculated above. These arise from higher dimensional operators in the non-renomalizable 5D theory that are suppressed by the cutoff scale. Electroweak precision constraints require that this cutoff scale must be at least 5 TeV. These limit the size of the extra dimension to be about $L \lesssim (400 \text{ GeV})^{-1}$. Searches for parity
odd quarks in the acoplanar dijet topology at the Tevatron do not yet probe this region of parameter space [47], and the searches for GMSB like scenarios place even less stringent limits. The upcoming LHC experiments will have much greater kinematic access to the region which is allowed by electroweak precision constraints. However, distinction between GMSB scenarios and this extra dimensional model may be difficult given a discovery of an excess of this type of signal.

5 Conclusions

We have performed an analysis of spontaneously broken anomalous global symmetries in the context of one universal extra dimension compactified on an $S_1/Z_2$ orbifold. A light pseudo-Goldstone scalar field arises from a 5D gauge symmetry that is broken by orbifold boundary conditions. Anomalous couplings to the unbroken gauge field strengths emerge after performing a 5D field redefinition that produces a non-trivial Jacobian. Over a large range of couplings and explicit symmetry breaking terms, the resulting effective action permits decays of the lightest level one SM KK-mode (of the hypercharge gauge boson) to a scalar field associated with the 5-component of an extra dimensional gauge field along with either a photon or $Z$-boson. In particularly interesting regions of parameter space, the decays occur on detector sized length scales with sizable displaced vertices. Such signals were long thought to be a smoking gun signature of SUSY models in which the soft masses are generated through gauge mediation, and in which the NLSP decays to a light gravitino in association with a photon or $Z$-boson. We have calculated constraints on this extra dimensional scenario, finding these to be minimal, and irrelevant for the range of couplings most interesting from the perspective of collider phenomenology. This pseudo-Goldstone scalar field is a potential dark matter candidate, and it may be possible for it to saturate the relic abundance observed by WMAP and numerous other astrophysical experiments. We have performed a standard relic abundance calculation for the case in which the extra dimensional gauge coupling is $O(1)$. For small values of the gauge coupling, the relic abundance calculation is intensive, model dependent, and depends on unknown details of early cosmology such as the reheat temperature. It is unlikely that this region of parameter space is ruled out by overclosure of the universe, however the calculation is beyond the scope of this analysis. BBN and the CMB spectrum do not place any constraints on the parameter space most relevant for collider physics.

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Appendix

A The Goldstone Wave Functions with a Bulk Higgs VEV

The full classical equations of motion for $B_5$ and $\pi$ are given by:

\[ \square \pi - \pi'' + \xi \kappa^2 \pi + \frac{v''}{v} \pi + (1 - \xi) v B_5' - 2 v' B_5 = 0 \]
\[ \square B_5 - \xi B_5'' + \kappa^2 B_5 - (1 - \xi) \frac{\kappa^2}{v} \pi' + (1 + \xi) \frac{\kappa^2}{v^2} v' \pi = 0 \]

where we have kept the terms containing the derivatives of $v$ for completeness. After enforcing $B_\mu|_{z=0,L} = 0$, the boundary conditions for $\pi$ and $B_5$ are given by:

\[ \pi' - v B_5 - \frac{v'}{v} \pi \pm L \frac{\delta V_{\text{bound}}}{\delta \pi} \bigg|_{z=0,L} = 0 \]
\[ B_5' - \frac{\kappa^2}{v} \pi \bigg|_{z=0,L} = 0. \]

(A.67)

In the cases where $v' = 0$, we can decouple the second order bulk equations by taking the first equation, solving for $B_5'$,

\[ B_5' = \frac{1}{v(\xi - 1)} \left[ \square \pi - \pi'' + \xi \kappa^2 \pi \right], \]

taking the $z$-derivative of the second equation, and substituting using the above formula. The result is a 4-th order equation for $\pi$:

\[ \pi''' - 2 \kappa^2 \pi'' + \kappa^4 \pi + m^2 \left\{ (1 + 1/\xi) \pi'' + [m^2/\xi - \kappa^2 (1 + 1/\xi)] \pi \right\} = 0 \]

(A.69)

The same 4-th order equation can be obtained for $B_5$. Note that the only dependence on $\xi$ is in the mass terms. One can immediately find the physical states (those that don’t depend on $\xi$). For solutions to the second order equation

\[ \pi'' + (m^2 - \kappa^2) \pi = 0, \]

(A.70)
there is no $\xi$ dependence in the second half of the equation, and the bulk eom is also automatically satisfied. This means that the remaining two solutions to the full fourth order equation must be the ones that are eaten/unphysical.

For zero modes, there is trivially no $\xi$ dependence, since $\xi$ appears only in the mass terms. The most general solutions for the massless case are:

$$\pi = A_\pi e^{\kappa z} + B_\pi e^{-\kappa z} + C_\pi z e^{\kappa z} + D_\pi z e^{-\kappa z},$$

$$B_5 = A_B e^{\kappa z} + B_B e^{-\kappa z} + C_B z e^{\kappa z} + D_B z e^{-\kappa z}.$$  (A.71)

We first eliminate 4 of these 8 coefficients by requiring that the original second order coupled equations are satisfied. Satisfying the boundary conditions further requires that there are no solutions of the form $z e^{\pm \kappa z}$. Two undetermined coefficients remain, implying that there are two physical scalar zero modes in the spectrum. The full massless solution is given by:

$$B_5 = A_B e^{\kappa z} + B_B e^{-\kappa z}$$
$$\pi = -\frac{v}{\kappa} \left[A_B e^{\kappa z} - B_B e^{-\kappa z}\right].$$  (A.72)

By rewriting these in KK even and odd combinations we obtain the final Goldstone wave functions in eqns. (3.19) and (3.20).

References

[1] S. L. Adler, Phys. Rev. 177, 2426 (1969).
[2] J. S. Bell and R. Jackiw, Nuovo Cim. A 60, 47 (1969).
[3] W. A. Bardeen, Phys. Rev. 184, 1848 (1969).
[4] K. Fujikawa, Phys. Rev. Lett. 42, 1195 (1979).
[5] K. Fujikawa, Phys. Rev. D 21, 2848 (1980) [Erratum-ibid. D 22, 1499 (1980)].
[6] E. Witten, Nucl. Phys. B 223, 422 (1983).
[7] J. Wess and B. Zumino, Phys. Rev. 163, 1727 (1967).
[8] O. Kaymakcalan, S. Rajeev and J. Schechter, Phys. Rev. D 30, 594 (1984).
[9] G. ’t Hooft, Phys. Rev. D 14, 3432 (1976) [Erratum-ibid. D 18, 2199 (1978)].
[10] For an introduction, see e.g. S. P. Martin, arXiv:hep-ph/9709356.
[11] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B 516, 395 (2001) arXiv:hep-th/0103135.
[12] C. T. Hill, Phys. Rev. D 73, 085001 (2006) [arXiv:hep-th/0601154].

[13] C. T. Hill and R. J. Hill, Phys. Rev. D 76, 115014 (2007) [arXiv:0705.0697 [hep-ph]].

[14] B. Gripaios, Phys. Lett. B 663, 419 (2008) [arXiv:0803.0497 [hep-ph]]; B. Gripaios and S. M. West, Nucl. Phys. B 789, 362 (2008) [arXiv:0704.3981 [hep-ph]]; T. Flacke, B. Gripaios, J. March-Russell and D. Maybury, JHEP 0701, 061 (2007) [arXiv:hep-ph/0611278].

[15] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38, 1440 (1977).

[16] T. Appelquist, H. C. Cheng and B. A. Dobrescu, Phys. Rev. D 64, 035002 (2001) [arXiv:hep-ph/0012100].

[17] H. C. Cheng, K. T. Matchev and M. Schmaltz, Phys. Rev. D 66, 036005 (2002) [arXiv:hep-ph/0204342].

[18] K. w. Choi, Phys. Rev. Lett. 92, 101602 (2004) [arXiv:hep-ph/0308024].

[19] G. Servant and T. M. P. Tait, Nucl. Phys. B 650, 391 (2003) [arXiv:hep-ph/0206071]; H. C. Cheng, J. L. Feng and K. T. Matchev, Phys. Rev. Lett. 89, 211301 (2002) [arXiv:hep-ph/0207125].

[20] H. C. Cheng, K. T. Matchev and M. Schmaltz, Phys. Rev. D 66, 056006 (2002) [arXiv:hep-ph/0205314].

[21] A. J. Barr, Phys. Lett. B 596, 205 (2004) [arXiv:hep-ph/0405052]; J. M. Smillie and B. R. Webber, JHEP 0510, 069 (2005) [arXiv:hep-ph/0507170].

[22] P. Meade and M. Reece, Phys. Rev. D 74, 015010 (2006) [arXiv:hep-ph/0601124].

[23] L. T. Wang and I. Yavin, JHEP 0704, 032 (2007) [arXiv:hep-ph/0605296]; for a review see L. T. Wang and I. Yavin, [arXiv:0802.2726 [hep-ph]].

[24] C. Csáki, J. Heinonen and M. Perelstein, JHEP 0710, 107 (2007) [arXiv:0707.0014 [hep-ph]].

[25] J. Hubisz, J. Lykken, M. Pierini and M. Spiropulu, Phys. Rev. D 78, 075008 (2008) [arXiv:0805.2398 [hep-ph]].

[26] G. Hallenbeck, M. Perelstein, C. Spethmann, J. Thom and J. Vaughan, [arXiv:0812.3135 [hep-ph]].

[27] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Rev. Lett. 86, 4757 (2001) [arXiv:hep-th/0104005]; C. T. Hill, S. Pokorski and J. Wang, Phys. Rev. D 64, 105005 (2001) [arXiv:hep-th/0104035].
[28] M. Dine and W. Fischler, Phys. Lett. B 110, 227 (1982); M. Dine and W. Fischler, Nucl. Phys. B 204, 346 (1982); L. Alvarez-Gaume, M. Claudson and M. B. Wise, Nucl. Phys. B 207, 96 (1982); S. Dimopoulos and S. Raby, Nucl. Phys. B 219, 479 (1983).

[29] M. Dine, A. E. Nelson and Y. Shirman, Phys. Rev. D 51, 1362 (1995) [arXiv:hep-ph/9408384]; M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D 53, 2658 (1996) [arXiv:hep-ph/9507378].

[30] G. F. Giudice and R. Rattazzi, Phys. Rept. 322, 419 (1999) [arXiv:hep-ph/9801271].

[31] S. Weinberg, Phys. Rev. Lett. 40, 223 (1978).

[32] F. Wilczek, Phys. Rev. Lett. 40, 279 (1978).

[33] M. Dine, W. Fischler and M. Srednicki, Phys. Lett. B 104, 199 (1981).

[34] A. R. Zhitnitsky, Sov. J. Nucl. Phys. 31, 260 (1980) [Yad. Fiz. 31, 497 (1980)].

[35] Y. Kawamura, Prog. Theor. Phys. 105, 999 (2001) [arXiv:hep-ph/0012125].

[36] R. Contino, Y. Nomura and A. Pomarol, Nucl. Phys. B 671, 148 (2003) [arXiv:hep-ph/0306259].

[37] C. Amsler et al. [Particle Data Group], Phys. Lett. B 667, 1 (2008).

[38] L. Randall and M. D. Schwartz, JHEP 0111, 003 (2001) [arXiv:hep-th/0108114]; A. Muck, A. Pilaftsis and R. Ruckl, Phys. Rev. D 65, 085037 (2002) [arXiv:hep-ph/0110391]; G. Cacciapaglia, C. Csáki, C. Grojean, M. Reece and J. Terning, Phys. Rev. D 72, 095018 (2005) [arXiv:hep-ph/0505001].

[39] M. Bolz, A. Brandenburg and W. Buchmuller, Nucl. Phys. B 606, 518 (2001) [Erratum-bid. B 790, 336 (2008)] [arXiv:hep-ph/0109001].

[40] J. L. Feng, [arXiv:hep-ph/0405215].

[41] H. C. Cheng, J. L. Feng and K. T. Matchev, Phys. Rev. Lett. 89, 211301 (2002) [arXiv:hep-ph/0207125].

[42] K. Kong and K. T. Matchev, JHEP 0601, 038 (2006) [arXiv:hep-ph/0509119].

[43] T. Goto and M. Yamaguchi, Phys. Lett. B 276, 103 (1992).

[44] A. Brandenburg and F. D. Steffen, JCAP 0408, 008 (2004) [arXiv:hep-ph/0405158].

[45] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 101, 111802 (2008) [arXiv:0806.2223 [hep-ex]].

[46] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. D 78, 032015 (2008) [arXiv:0804.1043 [hep-ex]].
[47] V. M. Abazov et al. [D0 Collaboration], Phys. Lett. B 668, 357 (2008) [arXiv:0808.0446 [hep-ex]]; M. S. Carena, J. Hubisz, M. Perelstein and P. Verdier, Phys. Rev. D 75, 091701 (2007) [arXiv:hep-ph/0610156].