Background error statistics in the Tropics: Structures and impact in a convective-scale numerical weather prediction system

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Abstract
The background error covariance matrix plays a vital role in any data assimilation system. Proper specification, which is determined by the forecast system set-up, is often required. Previous studies have investigated its relevance in various global and regional numerical weather prediction (NWP) systems; however, very few have explored it in tropical NWP systems. Here, we present and evaluate the structures of the background error covariance matrix for a tropical convective-scale NWP system. A total of 12 background error covariance matrices are modelled using differences between pairs of forecasts of different lengths but valid at the same time, based on the application of the vertical-first and horizontal-first transform order formulations on six permutations of the training data. Through pseudo-single observation tests, we extract and test the sensitivity of their structures to the training data period (seasons), forecast lag and transform order. The structures typically exhibit more dependence on forecast lag and transform order; horizontal-first transform order covariances had structures with shorter horizontal length-scales for wind and larger wind background error standard deviations. We also note that some covariances had horizontal and vertical structures with stronger mass–wind coupling, closely resembling an equatorial Kelvin wave. To assess the performance of each of the covariances, 12 month-long data assimilation trials in May 2018 (characterised by frequent occurrences of localised thunderstorm events) are performed. We show improved short-range precipitation forecasts in trials using some of the covariances compared to the current operational covariance. These covariances generally have structures with weak mass–wind coupling, shorter horizontal length-scales for wind and larger wind background error standard deviations, compared to other covariances which led to poorer forecasts. These may be desirable factors when modelling the background error covariance matrix for tropical convective-scale data assimilation systems.

KEYWORDS
convective-scale, covariance modelling, SINGV, Tropics, variational data assimilation
1 | INTRODUCTION

Data assimilation seeks to combine the information from observations with prior information through a mathematically optimal approach. The aim is to get the best estimate of the state of the system (Lorenc, 1986). For numerical weather prediction (NWP), it is impossible to know the true full state of the system at every grid point. To best estimate the full state, a short-range forecast (background) can be used as a guess, and weather observations can be used to make small corrections to that guess. Many assumptions and complexities are involved to optimise the use of both sources of information. Various data assimilation methods have been developed, each with its own limitations and applicability. A survey of these methods is covered in Rabier (2005) and Gustafsson et al. (2018), which include variational methods (Sasaki, 1958; 1970), ensemble Kalman filters (Evensen, 1994), and particle filters (Gordon et al., 1993; Van Leeuwen, 2009).

The use of information from the background and observations can be optimised by considering a weighting based on both their error covariance matrices. The background error covariance matrix depends heavily on the forecast model set-up and needs to be properly specified. Despite this, it is computationally unfeasible to explicitly specify such a large matrix ($\sim 10^{14}$ elements: Bannister, 2008a). As the true state is not known, the background error covariance matrix is also analytically impossible to specify; it must be estimated and modelled (Bannister, 2008a).

Apart from weighting information, the background error covariance matrix also determines the spreading of information from observations. Errors in observations are typically assumed to be uncorrelated with each other; the correlation of information between grid points and variables is captured by the “off-diagonals” of the background error covariance matrix. The modelled background error covariance matrix can be understood in terms of the horizontal and vertical structures it comprises, which determine the spreading of information from observations through corrections to the background for nearby horizontal and vertical grid points. Information from observations of one model variable can also produce corrections for other variables, which should ideally be dynamically consistent.

These structures are statistically modelled and estimated from training data (Section 2.1). Assumptions of homogeneity and isotropy in the modelling process reduces the number of elements needed to specify the background error covariance matrix. The same climatological (time-averaged) structures are often used for each assimilation cycle, regardless of the varying flow conditions from different weather phenomena occurring in reality. Consequently, they may be dynamically inconsistent with the flow. Different training data will result in different error statistics (and possibly structures), so the fine-tuning would help to identify characteristics of the training data which result in modelled structures that are most consistent with the average flow conditions in an NWP system. Some of the most advanced assimilation systems include a flow-dependent estimation of the background error statistics (e.g. Bonavita et al., 2011).

While other studies have investigated the modelled structures of the background error covariance matrix in various regions (Sadiki et al., 2000; Montmerle et al., 2006; Michel and Auligné, 2010), there have been relatively few studies focused on the Tropics. Ingleby (2001) discussed some differences between the raw statistical and modelled structures in the Tropics, in the context of a global NWP system. Žagar et al. (2004) analysed the projection of the structures onto equatorially trapped waves using a limited-area numerical model framework. Chen et al. (2013) compared the balance characteristics of the background error covariance matrix in a limited-area NWP system over the Arctic and Tropics, albeit at a low horizontal resolution of 30 km. So far, no study has investigated the structures of the background error covariance matrix in a tropical convective-scale limited-area NWP system.

In this article, we present and evaluate the structures of the background error covariance matrix and their sensitivity to different training data in a tropical convective-scale limited-area NWP system (SINGV: Huang et al., 2019), known as SINGV-DA. A total of 12 different background error covariances are modelled and assessed in data assimilation trials with real observations to identify features of the training data and corresponding structures which may be desirable. Note that SINGV-DA uses the incremental three-dimensional variational first guess at appropriate time (3D-Var FGAT: Fisher and Andersson, 2001; Lorenc and Rawlins, 2005), so the modelled background error structures are static. More information on SINGV-DA can be found in Heng et al. (2020), along with a comprehensive discussion on the types of observations assimilated, their availability in the Tropics and the challenges which remain.

Section 2 provides the methodology for modelling the background error covariance matrix and details of the training data. We analyse the modelled background error structures for SINGV-DA with pseudo-single observations experiments in Sections 3 and 4. The individual performances of different background error covariance matrices in data assimilation trials with real observations are assessed in Section 5. The summary of the results and conclusions are covered in Section 6.
2 | VARIATIONAL DATA ASSIMILATION AND THE BACKGROUND ERROR COVARIANCE MATRIX

Throughout this article, we define $B$ as the background error covariance matrix which we model, $y$ as the observations vector, $R$ as the observation error covariance matrix, $H$ as the nonlinear observation operator mapping from model space into observation space, and $H$ as the observation operator linearized around a background state $x_b$. We denote the innovation as $d = y - H(x_b)$ and the analysis $(x_a)$ increments as $\delta x = x_a - x_b$ (represented using a delta, e.g. $\delta p$ for pressure increments; $\delta u$ for horizontal wind increments).

2.1 | Formulation and use of the background error covariance matrix

For variational data assimilation, we seek $x_a$ that minimizes a cost function (Kalnay, 2003). Most operational NWP centres use an incremental formulation of the cost function (Courtier et al., 1994). Here, we consider the case for 3D-Var with a non-quadratic cost function (nonlinear $H$). This requires a linearization of $H$ about a guess state $(x_g)$ and formulating the problem in terms of increments to $x_g$. We illustrate using $x_g = x_b$ for a single outer loop, the incremental cost function given by:

$$J(\delta x) = \frac{1}{2} \delta x^T B^{-1} \delta x + \frac{1}{2} (d - H \delta x)^T R^{-1} (d - H \delta x).$$

Since $B$ is symmetric and positive definite, it may be partitioned in terms of a lower triangular matrix $U$, where its outer product gives the implied $B$ (i.e. $B = U U^T$). Using this principle, we can model $B$ for an NWP system (Bannister, 2008b). By defining a set of control variable transforms (Derber and Bouttier, 1999) such that $\delta x = U \upsilon$ where $\upsilon$ is a control vector, Equation 1 becomes:

$$J(\upsilon) = \frac{1}{2} \upsilon^T v + \frac{1}{2} (d - HU \upsilon)^T R^{-1} (d - HU \upsilon).$$

In the original formulation of Lorenc et al. (2000), which is the basis for the 3D-Var scheme we are working with here, the control variable transform ($U$) can be constructed through a sequence of three transforms: a parameter transform ($U_p$), a vertical transform ($U_v$ or $U_{vs}$) and a horizontal transform ($U_h$), defined as:

$$U = U_p U_v U_h.$$  (3)

Wlasak and Cullen (2014) developed an alternate transform formulation where the order of horizontal and vertical transforms is reversed, defined as:

$$U = U_p U_h U_v.$$  (4)

The coefficients for $U$ are estimated from climatological statistics of forecast errors in an off-line calibration step (denoted $T$ with $v = Te$, where $e$ represents the forecast errors). It is easier to explain the workings of the control variable transforms starting from forecast errors, illustrated below using $T$ instead of its $U$ counterpart.

Here, we employ the lagged “National Meteorological Center (NMC)” method (Parrish and Derber, 1992) to estimate the forecast errors. The principle of the NMC method is to estimate $B$ using the differences between pairs of forecasts of different lengths which are valid at the same time. The training data comprise these pairs of forecasts. This is mathematically presented in Bannister (2008a). For limited-area models like SINGV, an additional source of error occurs from the driving model providing its lateral boundary conditions (LBCs). The lagged NMC method (Šírová et al., 2003; Sadiki and Fischer, 2005; Berre et al., 2006) is meant to avoid this source of error; this involves taking only pairs of forecasts which use the same set of LBCs. In the Met Office set-up, this means that the forecast difference fields will be zero along the boundaries for model variables and are thus bi-periodic, essential for the application of Fourier transforms. As it is a strict requirement, it would not be trivial to apply the NMC method using different sets of LBCs. In this article, all the training data are generated from the basic SINGV downscaler (SINGV-DS: Huang et al., 2019) system, run at 4.5 km horizontal resolution over the domain around Singapore, driven by global European Centre for Medium-range Weather Forecasts (ECMWF) analyses and forecasts from LBCs at 0000 and 1200 UTC.

The vertical-first transform order in Equation 3, here-after referred to as the original transform order (OTO; $T = T_y T_p T_p$) in the Met Office set-up (Lorenc et al., 2000) is outlined qualitatively here in the following steps:

1. From each of the forecast differences, the sample mean (time-mean) is removed, and referred to as $e$ (variables are denoted by a subscript $e$; strictly speaking these are approximations to the forecast errors).
2. A parameter transform ($T_p$) is performed to map model variables to control variables (stream function $\psi$, unbalanced velocity potential $\chi$, unbalanced pressure $aP$, nonlinear humidity $\mu$) which are approximately uncorrelated using knowledge of hydrostatic balance and the linear balance equation (Bannister, 2008b). Specifically for $\chi$, we have used the basic set-up; it is taken to be wholly unbalanced without computation and subtraction of a balanced component. Note that $T_p$ determines
the multivariate relationships built in the covariances. These relationships are limited to the extent to which these balances hold in the deep Tropics, especially at convective scales. For example, linear balance breaks down in the deep Tropics when geostrophy is very small. Even in the midlatitudes, the degree to which geostrophy is important also decreases approaching convective scales (Berre, 2000). Hydrostatic balance also breaks down approaching convective scales associated with non-negligible vertical accelerations. Despite these limitations, the consequences are still worthwhile exploring in an operational system.

3. For each variable of \( e (a\rho_e, \psi_e, \chi_e, \mu_e) \), the following vertical and horizontal transforms are performed. In the vertical transform \((T_v)\), the vertical modes (eigenvectors) which best approximate all vertical columns of \( e \) in the training data must be determined. A domain-averaged (with no latitudinal variation prescribed) \( K \) by \( K \) vertical covariance matrix can be computed for realisations over all horizontal grid points and samples, where \( K \) is the number of model levels. This matrix can be represented by orthogonal vertical modes with no correlations between modes, weighted by their eigenvalues. It implies that the same set of vertical modes is used throughout the model domain, as required. The approximation of each vertical column of \( e \) is given by a linear combination of this set of vertical modes. The result is a two-dimensional field of modal coefficients for each vertical mode which are only horizontally correlated.

4. In the horizontal transform \((T_h)\), the horizontal distance-based covariances of the modal coefficient fields for each vertical mode are computed and converted to horizontal correlations. A second-order autoregression (SOAR) structure function is fitted to represent the horizontal correlation power spectra, with a characteristic horizontal length-scale assigned to each vertical mode. This represents the horizontal correlations of each vertical mode solely as a function of distance; thus, isotropy is assumed. Also, since the same vertical modes are used with the same horizontal length-scale for the entire model domain, homogeneity is assumed.

For the horizontal-first transform order in Equation 4, hereafter referred to as the swapped transform order (STO; \( T = T_{vh}T_{hp} \)), the vertical covariance matrix is computed as a function of each total horizontal wave number, similar to the approach taken by Berre (2000). Most variational systems use this transform order, outlined qualitatively here in the following steps:

1. Same as for OTO.
2. Same as for OTO.
3. In the horizontal transform \((T_h)\), \( e \) is transformed from grid-point space to spectral space (computing Fourier coefficients) for each model level, for all training data. As mentioned above, \( e \) is computed from pairs of forecasts which use the same set of LBCs and are thus bi-periodic (zero along the boundaries for model variables), appropriate for each of the respective control variables in the previous step. This is important for computing the Fourier coefficients.
4. In the vertical transform \((T_v)\), it is the same as for OTO, except that the vertical covariance matrix is computed as a function of total horizontal wave number in spectral space instead of grid-point space. This means that vertical modes and respective eigenvalues are also dependent on total horizontal wave number. Consequently, no latitudinal variation is prescribed, but allows for the variation of vertical covariances as a function of horizontal scale.

In both OTO and STO formulations, we prescribe no latitudinal dependence of the covariance statistics, even though the OTO formulation allows for it. One might argue that the SINGV domain is sufficiently small to neglect latitudinal variation in the covariance statistics.

2.2 Details of training data

Table 1 shows the details of the input training data used to model \( B \). The sensitivity of the structures of \( B \) are analysed based on two criteria: the training data period (February, May or September) and the forecast lag for forecasts valid at the same time (24 or 12 hr). Six permutations of the data are generated using SINGV-DS. Both STO and OTO formulations are applied on all the six training data permutations to model 12 different \( B \).

The months for the training data periods are chosen to correspond to the end of the Northeast Monsoon (February) and Southwest Monsoon (September). Additionally, May is included since the data assimilation trials (Section 5) are performed during that period. It would be useful to investigate if the structures are more dynamically consistent when the input \( B \) is modelled from the same training data period as the data assimilation trial. The spin-up time of SINGV-DS is also known to be around 9 hr (A. Dipankar, 2019, unpublished data). Thus, we have used a longer forecast lag of 12 and 24 hr for the training data. Results from our other experiments using a forecast lag of 3 and 6 hr (6-3H and 12-6H covariances respectively) are excluded. Note that a forecast lag of 24 hr would eliminate
TABLE 1  Details of the six permutations of the training data generated using SINGV-DS

| Details of training data | February 2018 | May 2018 | September 2018 |
|--------------------------|---------------|----------|----------------|
| **SINGV-DS forecasts**   |               |          |                |
| 24-hr forecast – 12-hr forecast valid at the same time (forecast lag of 12 hr) | Sample size: 55 Abbreviation: ds_FEB24-12H_OTO ds_FEB24-12H_STO | Sample size: 61 Abbreviation: ds_MAY24-12H_OTO ds_MAY24-12H_STO | Sample size: 57 Abbreviation: ds_SEP24-12H_OTO ds_SEP24-12H_STO |
| 36-hr forecast – 12-hr forecast valid at the same time (forecast lag of 24 hr) | Sample size: 52 Abbreviation: ds_FEB36-12H_OTO ds_FEB36-12H_STO | Sample size: 58 Abbreviation: ds_MAY36-12H_OTO ds_MAY36-12H_STO | Sample size: 56 Abbreviation: ds_SEP36-12H_OTO ds_SEP36-12H_STO |

Note: Background error covariances are modelled according to training data period (columns), forecast lag (rows) and transform order. The number of pairs of forecasts used (sample size) are indicated in each cell. Between each forecast pair, the same set of LBCs valid at either 0000 or 1200 UTC is used (two forecast pairs per day). For each permutation of the training data, the original transform order (OTO) and swapped transform order (STO) are applied, resulting in the 12 background error covariances with their corresponding abbreviations describing the training data source and transform order.

potential errors in modelling the diurnal cycle (Bannister, 2008a), which is especially prominent in the Tropics. For any statistical estimation, a larger sample size is always desirable. Various operational NWP centres have used sample sizes ranging from 30 to 48 in estimating the background error statistics using the NMC method (Parrish and Derber, 1992; Derber and Bouttier, 1999; Wu et al., 2002). Other studies have used similar sample sizes (Chen et al., 2013; Ban et al., 2017).

In the following sections, we compare the old background error covariance matrix (oldDS) currently used in SINGV-DA with the 12 different B. oldDS was initially generated by the Met Office, but its full details are not documented.

3 | PSEUDO-SINGLE OBSERVATION EXPERIMENTS FOR OTO COVARIANCES

We extract the multivariate structures of B through the use of pseudo-single observation experiments. The sensitivity of the structures to different training data can then be analysed. We can write the solution for the pseudo-single observation (Gustafsson et al., 2012) as:

\[
\delta x = B ((B_u + R_u)^{-1}d_t = B_i (\sigma^2_b + \sigma^2_o)^{-1}d_i \quad (5)
\]

where \(\sigma^2_b\) is the modelled background error variance, \(\sigma^2_o\) is the observation error variance and \(d\) is the innovation at the point of observation (index \(i\)). The structures are directly related to the increments, providing a visual representation of the structures (Huang et al., 2009; Gustafsson et al., 2012).

3.1 | Comparing pressure and wind horizontal structures associated with pressure

A pseudo-single observation of pressure \((p)\) is inserted near the centre of the model domain at model level 15 (~1 km height). An innovation \((d)\) of 100 Pa (1 hPa) is specified together with an observation error of 20 Pa (0.2 hPa). Theoretically, the specified values are not crucial for the purpose of extracting the structures of \(B\).

Figure 1 shows the horizontal structures of OTO covariances associated with \(p\) at model level 15. They indicate how \(p\) errors of the modelled \(B\) are correlated spatially. This correlation structure determines how an observation of \(p\) at a point influences the \(p\) field in the domain, which is updated accordingly by \(\delta p\). For all the OTO covariances in Figure 1, \(\delta p\) decreases nonlinearly and near-isotropically from the point of observation. Note that the increments are only isotropic in control variable space (i.e. \(aP\)). The spread of \(\delta p\) is also determined by small contributions from \(\delta \psi\) (which has latitudinal variation built in \(U_p\)) and so is not fully isotropic.

Comparing between forecast lag, the horizontal length-scales for \(p\) are generally short for both ds_24-12H and ds_36-12H covariances. Correspondingly, \(\delta p\) is more localised. Comparing between training data period, the horizontal length-scales for \(p\) are also relatively similar. Consequently, there is little variation in spatial patterns of \(\delta p\); most of the variation in \(\delta p\) mainly occurs because of differing horizontal length-scales of the small contributions from \(\delta \psi\). This weak seasonal dependence is a common characteristic even across forecast lag.

It is interesting to note that longer forecast lag resulted in such short horizontal length-scales. To our knowledge,
Based on SINGV-DS training data

**FIGURE 1** A horizontal cross-section of the resulting $\delta p$ (contours) and $\delta u$ (arrows) from the insertion of a pseudo-single observation of $p$ which is 100 Pa above the background (with an observation error of 20 Pa) near the centre of the domain, at model level 15. Shown for the six OTO covariances, abbreviated and detailed in Table 1. oldDS is included for comparison.

no study has previously reported this in the Tropics. Ingleby (2001) suggested that horizontal length-scales may tend to increase as the length of forecast increases due to errors on synoptic scales. However, there is no empirical evidence for this in a limited-area model. In practice, we may instead be concerned with errors on convective scales, especially when these structures are given sufficient time to develop. Žagar et al. (2004) discussed how recovering finer horizontal structures (more detailed curvature) of the geopotential height (associated with $p$) might be desirable. For SINGV-DA, one would expect that recovering finer horizontal structures is also desirable, although the benefit is likely limited in the Tropics where $p$ is concerned.

We also diagnose the mass–wind coupling from the resulting wind increments ($\delta u$), which are sensitive to both the training data period and forecast lag. When compared between training data periods, the resulting $\delta u$ show substantial variation. The mass–wind coupling is generally weakest and strongest for May and February covariances respectively. For September covariances, the mass–wind coupling depends on forecast lag. In all cases with stronger mass–wind coupling captured by $\mathbf{B}$, there is a positive $p-u$ correlation and negligible $p-v$ correlation. The meteorological significance of this structure will be further explored in Section 3.3.

A strong mass–wind coupling in $\mathbf{B}$ may not be ideal in the Tropics; one would not expect geostrophic balance to determine the adjustment processes captured by the increments. Žagar et al. (2004) found that the mass information was not useful for recovering wind information in the Tropics, and Ingleby (2001) found only weak mass–wind coupling in the Tropics in the Met Office global
system. Any correlation between $u$ and $p$ is possibly a fallacious result of using the linear balance equation to determine multivariate covariances in the Tropics, which was demonstrated in Daley (1996). Thus, for SINGV-DA, one may expect weak mass–wind coupling to be desirable. However, we note that any seasonal dependence of the mass–wind coupling may be conflated by the dependence on forecast lag. Thus, even by taking a purely operational perspective, identifying a specific month or season to model $B$ to achieve it would not be straightforward.

### 3.2 Comparing pressure and wind vertical structures associated with pressure

Figure 2 shows the vertical structures of OTO covariances associated with $p$ using a longitudinal cross-section along the Equator (latitude of the observation). There is little variation in the vertical structure of $\delta p$ when comparing between training data periods; the vertical anticorrelation of $\delta p$ is equally robust for each period (positive and negative $\delta p$ at model levels 15 and 45 respectively). This is also noticeable across forecast lag. This is surprising since Ingleby (2001) previously showed that the dominant vertical mode in the Tropics of a global system did not indicate any anticorrelation. It is possible that this anticorrelation may not necessarily correspond to balance processes, but instead be related to the increasing height of the tropopause associated with model spin-up (M. Wlasak, personal communication, 24th June 2019). However, it would also not be appropriate to simply attribute any vertical anticorrelation of $\delta p$ as solely an artefact of model spin-up, especially since it is captured in all six covariances, suggesting that there could be some
meteorological significance involved. This is also further explored in Section 3.3.

We also perform a similar analysis for the vertical structure of the mass–wind coupling using the resulting vertical wind increments (δw) and zonal wind increments (δu). There is substantial variation comparing between both training data period and forecast lag, especially in the upper tropospheric zonal wind. For the same cases in the horizontal cross-section analysis with stronger mass–wind coupling captured by B, there is consistent lower tropospheric convergence and ascent downwind of the lower tropospheric observation at model level 15. The opposite occurs upwind of the observation.

### 3.3 Meteorological significance of the structures

Combining the analysis of the horizontal and vertical structures of both δp and the mass–wind coupling, we note that the results may show resemblance with that of an equatorial Kelvin wave (see Figure 8 of Matsuno (1966) for horizontal structures; Figure 6 of both Straub and Kiladis (2003) and Frierson (2007) for vertical structures). For high pressure in the lower troposphere, this is characterised by lower tropospheric ascent downwind of the high pressure, low pressure in the upper troposphere, and westerly winds associated with the high pressure. Further analysis of δθ also supports this; there is a negative correlation between p and θ in the mid-troposphere (not shown), consistent with the mid-tropospheric cooling above the high pressure. Previous studies have used various systems to investigate the background error covariance structures in the Tropics. Chen et al. (2013) demonstrated how mass–wind structures representing enhanced convergence/divergence could be captured in B in a tropical limited-area system, although the enhanced convergence/divergence was collocated with the mass observations. Žagar et al. (2004) and Parrish and Derber (1992) both presented structures that closely resemble an equatorial Kelvin wave, in a limited-area numerical model and global NWP system respectively. Here, we show that our results for SINGV-DA are consistent; both the horizontal and vertical structures of the mass–wind coupling resemble an equatorial Kelvin wave, especially when the coupling is strong.

Nevertheless, we note that some B did not capture this Kelvin wave structure. One may argue that this is plausible; the equatorial Kelvin wave structure should not be present because it may not be appropriate to make corrections to u given only pressure information in the deep Tropics, as discussed earlier and in Daley (1996). However, Wheeler et al. (2000) demonstrated how the variation of large-scale dynamical fields in the Tropics can be explained by various convectively coupled equatorially trapped waves of the linear shallow-water theory. Žagar et al. (2005) found that convectively coupled equatorially trapped waves can explain up to 70% of the error variances in the tropical free atmosphere, although they acknowledged that equatorial Kelvin waves only provides a small contribution. Any mass–wind coupling may not be entirely undesirable, especially since the large-scale increment response resembles one of the dominant modes of variability in the Tropics. In our case, the strength of the multivariate relation between p and u in the error structures of the NWP system could be dependent on the relative contributions of various waves, particularly the equatorial Kelvin wave. Any lack of mass–wind coupling could be due to complex and opposing contributions from other waves; such as a westward inertio-gravity wave, as suggested by Žagar (2012). We will discuss if the equatorial Kelvin wave structure is desirable for SINGV-DA based on data assimilation trial results in Section 5.2.

### 3.4 Comparing wind horizontal structures associated with zonal wind

Given that wind information is more important than mass information in the Tropics (Žagar et al., 2004), we insert a pseudo-single observation of zonal wind (u) at the same observation point and compare structures associated with zonal wind. An innovation (d) of 1 m·s⁻¹ is specified. We specify the observation error to be 0.2 m·s⁻¹.

Figure 3 shows the horizontal structures of two OTO covariances at model level 15 associated with u. They indicate how u errors of the modelled B are correlated spatially. The spatial pattern of δu is very similar between February, May and September covariances, as well as between forecast lag. Thus, only the ds_MAY36-12H covariance is compared with oldDS in Figure 3. This suggests that for OTO covariances, the horizontal length-scales for u are not sensitive to training data period or forecast lag; the wind field information is better preserved in the forecasts regardless of forecast length. These robust horizontal structures show striking resemblance to figure 2d of Undén (1989), which previously assessed the analysis response to wind background errors with the full divergent component included in the ECMWF optimum interpolation analysis scheme. It is interesting that such long length-scales for u are modelled. This may only be somewhat acceptable for recovering the large-scale δu if wind observations are sufficiently sparse, which is likely the case in the Tropics.
3.5 Zonal wind background error standard deviation

More information can be retrieved by focusing on $\delta u$ at the point of observation (index $i$). From Equation 5, we get:

$$ (\delta x)_i = B_i (\sigma_b^2 + \sigma_o^2)^{-1} d = \frac{\sigma_b^2 d}{\sigma_b^2 + \sigma_o^2}. \quad (6) $$

Rearranging for $\sigma_b$, we get:

$$ \sigma_b = \sigma_o \sqrt{\frac{\delta x}{d - \delta x}}, \quad (7) $$

where $\delta x = (\delta x)_i$ is the increment associated with the observed model variable $x$ at the point of observation (index $i$). For a pseudo-single observation of $u$, Equation 7 gives the implied zonal wind background error standard deviation ($\sigma^u_b$) at model level 15. We repeat this for all 80 model levels, inserting a pseudo-single observation of $u$ and computing the corresponding $\sigma_b^u$ at that level. As before, we focus mainly on $\sigma_b^u$ for the first 70 levels. Since we prescribe no latitudinal variation in the vertical modes (and associated eigenvalues) in $T_v$, $\sigma_b^u$ is homogeneous in the domain at each level. It is important to assess $\sigma_b^u$ because it controls the weighting of information between the background and observation. It is possible that $\sigma_b^u$ may be underestimated in SINGV-DA leading to suboptimal use of observation information in the assimilation and model initial state drifting away from observations.

Figure 4 shows the vertical profile of $\sigma_b^u$ for each of the OTO covariances. Compared to the oldDS, all the covariances have smaller $\sigma_b^u$ at each model level. This is surprising because oldDS is a 12-6H covariance modelled from training data of a similar sample size, yet our covariances did not exhibit such large $\sigma_b^u$. It could partly be related to the older version of SINGV-DS or downscaling of Met Office global analyses and forecasts used to generate the training data instead (Heng et al., 2020), but the full details are not documented.

We observe that $\sigma_b^u$ is slightly larger with longer forecast lag, regardless of training data period. The upper tropospheric peak in $\sigma_b^u$ (∼16 km tropopause in the deep Tropics) is likely associated with a peak variation in zonal wind fields associated with divergence modulated by deep convection. The vertical profiles of $\sigma_b^u$ have a similar shape to those shown in Wang et al. (2014), albeit with generally smaller magnitudes. We note that Wang et al. (2014) focused on a limited-area model over Beijing instead of the Tropics. The main difference is the height of the tropopause (with peak $\sigma_b^u$) at around 250 hPa for Beijing compared to around 100 hPa (model level 55) for our case, which is typical in the Tropics and midlatitudes. Other smaller differences are mainly related to the magnitude of $\sigma_b^u$. Comparing between training data periods, February covariances generally have less pronounced peaks in zonal wind background error standard deviation than May and September covariances, regardless of forecast lag. This could be because February is climatologically drier over the upper half of the domain (Aldrian and Susanto, 2003; Fong and Ng, 2012; As-syakur et al., 2013; Hassim and Timbal, 2019) with fewer occurrences of thunderstorms, and hence lesser variation in the background errors.

As it is impossible to know the correct magnitude of $\sigma_b^u$ (or any background error covariance quantity), it is difficult to diagnose if having larger $\sigma_b^u$ may be desirable for SINGV-DA. Here, we have used surrogate quantities (forecast differences in the NMC method) with the OTO formulation, which may be underestimating $\sigma_b^u$. Additional discussion is provided in Section 5.2.
While we have mainly focused on $\sigma_u^b$, we do not naively neglect the implied background error standard deviations for other variables ($\sigma_\theta^b$, $\sigma_p^b$). However, these do not differ substantially when compared between training data periods or forecast lag (not shown). These variables inherently have very little variation in the Tropics, therefore one would not expect the forecast differences to also have substantial variation.

4 PSEUDO-SINGLE OBSERVATION EXPERIMENTS FOR STO COVARIANCES

4.1 Comparing pressure and wind horizontal structures associated with pressure

We perform the same analysis on the STO covariances as the OTO covariances; these are presented separately for clarity, based on the same pressure observation inserted. Figure 5 shows the horizontal structures of STO covariances at model level 15. Comparing between training data periods and forecast lag, there is little variation in spatial patterns of $\delta p$. We note that STO covariances have longer horizontal length-scales for $p$ than their OTO counterparts; error correlations are relatively broader for all the STO covariances. It is possible that this will have little impact on the forecast skill compared to other factors.

As before, we diagnose the mass–wind coupling from the resulting $\delta u$. The patterns are very similar to the OTO covariances; the mass–wind coupling is somewhat sensitive to training data period and strongest for February covariances. Across all STO covariances, the mass–wind coupling is weaker (smaller $\delta u$) than their OTO counterparts. Further investigations reveal that the inner loop descent algorithm performed on the STO formulation control vector (using wave number instead of vertical mode segments in $v$) yielded smaller $\delta \chi$ but slightly larger $\delta aP$ when mapped to grid-point space. This results in smaller $\delta u$ but similar $\delta p$ after transforming the increments to model variable space with $U_p$.

4.2 Comparing pressure and wind vertical structures associated with pressure

The vertical anticorrelation of $\delta p$ is also robust regardless of training data period or forecast lag (Figure 6).
ds_MAY36-12H_STO and ds_SEP36-12H_STO covariances both have a slightly stronger anticorrelation (more negative $\delta p$ at model level 45) than their OTO counterparts. We note that the training data used in the OTO and STO formulation are the same, but it appears that the STO formulation better preserves certain features of the training data, such as the vertical anticorrelation of $\delta p$ by considering vertical covariances as a function of horizontal scale. However, it may also be possible that it is amplifying features of the training data which may not actually be realistic.

The vertical structures for STO covariances also indicate weaker mass–wind coupling for all model levels compared to OTO counterparts based on the $p$ observation at model level 15. In general, $\delta u$ and $\delta w$ are very small. The vertical structures which previously resembled the vertical cross-section of an equatorial Kelvin wave in the OTO covariances are much less pronounced in the STO covariances. From an operational perspective, one may consider using the STO formulation instead of the OTO formulation to model $B$ with weaker mass–wind coupling for SINGV-DA.

### 4.3 Comparing wind horizontal structures associated with zonal wind

We also insert the same pseudo-single observation of zonal wind as in Section 3.4. Figure 7 shows the horizontal structures of two covariances at model level 15 associated with $u$. Similar to the results for OTO covariances, the spatial pattern of $\delta u$ does not vary substantially between
training data periods and forecast lag. Thus, only the ds_MAY36-12H covariance is compared with oldDS in Figure 7. Most noticeably, when STO covariances are compared to OTO covariances, the horizontal length-scales for $u$ are extremely short; the spreading of $\delta u$ is very localised. It is important to note that the STO formulation does not impose a constraint on the spectrum, unlike the OTO formulation which scales the spectrum using a spectral representation of the SOAR structure function, which may explain the differences.

Having shorter horizontal length-scales for $u$ may recover finer details in $\delta u$ fields which are crucial for the convective scale. Previous studies have empirically tuned the horizontal length-scales of $B$ for limited-area systems, typically also resulting in a reduction of the horizontal length-scales (Xiao and Sun, 2007; Sugimoto et al., 2009; Sun et al., 2012). However, one might argue that the extent of the spread may not be sufficiently large, especially if large-scale wind corrections are required and observations are sparse. We further discuss if the shorter horizontal length-scales for $u$ are desirable for SINGV-DA in Section 5.2.

### 4.4 Zonal wind background error standard deviation

Next, we compute $\sigma_b^u$ at each model level. Figure 8 shows the vertical profile of $\sigma_b^u$ for each of the STO covariances. Compared to their OTO counterparts, some similar features are observed for STO covariances. $\sigma_b^u$ is larger with longer forecast lag for most model levels, especially the
The curious shape and noisier vertical profile of $\sigma_b^u$ in STO covariances is a result of using a substantially smaller number of points in spectral space to compute the variances for each total horizontal wave-number bin compared to the total number of grid points used to compute the domain-averaged variances for OTO covariances. Variance contributions associated with larger wave numbers are also particularly noisy in the stratosphere (not shown), a result of the small number of forecast differences used. As a comparison, 400 forecast differences are used for an experimental STO covariance for the UKV, which results in a smoother vertical profile of $\sigma_b^u$ (A. Clayton, 2019, unpublished data). Here, we used about 60 for each covariance. Unfortunately, it would be impractical to greatly increase the number of forecast differences for this sensitivity experiment; this would require more years of model simulation to produce enough training data for the same season. However, we demonstrate that the possible benefits associated with the STO formulation may outweigh these limitations in data assimilation trials.

5 | DATA ASSIMILATION TRIALS

To assess the operational impact of using the various B, 12 month-long data assimilation trials in May 2018 are performed using real observations. The trials were run at 4.5 km horizontal resolution, covering the domain around Singapore. The Singapore Parallel Suite 2 configuration for SINGV-DA is used, except that oldDS is replaced with respective modelled B in different trials. SINGV-DA is 3-hourly cycling with 16 cycles per day, so a month-long trial produces around 240 forecasts which can be analysed.

5.1 | Fraction skill scores for precipitation

Half-hourly Global Precipitation Measurement (GPM) data at $0.1^\circ \times 0.1^\circ$ resolution (Huffman, 2017) are used for comparison with precipitation forecasts because of their spatial coverage and sufficient temporal resolution. Three-hourly accumulated GPM data and precipitation forecasts are compared, up to a forecast lead time of 12 hr, where the impact of data assimilation on the initialisation is still substantial. Fraction skill scores (FSS) are computed (Roberts and Lean, 2008) using a 50 km spatial scale (neighbourhood size), as a function of eight precipitation thresholds (0.125, 0.25, 0.5, 1, 2, 4, 8 and 16 mm). FSS for the lower thresholds (less than 4 mm) capture the accuracy of the general spatial pattern of precipitation, while FSS for the larger thresholds capture the accuracy of the location of convective cores. Differences

levels which coincide with a peak in $\sigma_b^u$. However, almost all STO covariances exhibit a curious shape; there are multiple peaks at model levels 10, 40 and 55 (corresponding to ~700 m, ~8 and ~16 km respectively). There are plausible meteorological explanations for some of these. The peak at model level 10 for STO covariances can also be weakly seen for OTO covariances; it is likely related to the associated lower tropospheric convergence related to deep convection and upper tropospheric divergence. This is closely linked to the peak at model level 55, following a similar explanation as for the OTO covariances. However, the odd peak at model level 40 was not present for the OTO covariances, even though the same training data are used in the STO and OTO formulations. Note that this peculiar feature of multiple peaks was previously seen in the Met Office limited-area model (UKV: Tang et al., 2013; Gustafsson et al., 2018) covariances (A. Clayton, 2019, unpublished data).
in FSS are compared throughout the SINGV-DA domain for different experiments (data assimilation trials using the 12 different $B$) compared to the control (data assimilation trial using oldDS). Significance is determined using the non-parametric two-sided Wilcoxon signed-rank test at the 90% confidence level.

We compare the experiments according to forecast lag and training data period (Figure 9), as well as transform order (Figure 10). In general, trials using 36-12H_OTO covariances resulted in slightly larger improvement (or smaller deterioration) in FSS compared to 24-12H_OTO covariances. This may suggest that longer forecast lag should be used when modelling $B$. Trials using February covariances generally yielded a deterioration in FSS, compared to the significant improvements in FSS for respective May and September covariances. This indicates that the improved performance is also seasonally dependent; had the trial been conducted in a different month, a different choice of $B$ would possibly be more appropriate. However, the choice of $B$ is also not trivial. We note that May covariances did not necessarily give the best improved FSS even though the data assimilation trial was performed in May. One might argue that the trial using ds_SEP36-12H_OTO gave the largest improvement instead.

We also note that trials using STO covariances yielded much higher FSS than their OTO counterparts. This is clearly seen regardless of training data or forecast lag. The large improvements compared to oldDS are also statistically significant. One might argue that the trials using ds_SEP36-12H_STO and ds_SEP24-12H_STO gave the largest overall improvements.

5.2 | Factors affecting the tuning of B in SINGV-DA

Given these results, one might be able to speculate if certain features of the training data and resulting structures of $B$ discussed in Sections 3 and 4 are desirable for SINGV-DA. Since May is typically dominated by localised thunderstorm events, any improvement in precipitation forecasts is likely a result of improved localised thunderstorm forecasts.

We highlight that for the respective trials that outperformed oldDS, weak mass–wind coupling and no Kelvin wave structure were captured in their associated $B$. This suggests that any mass–wind coupling may not be desirable for SINGV-DA. Some of the reasons have been discussed in Section 3.3. Additionally, any large-scale $\delta u$ as a result of a small $p$ perturbation could contaminate the analyses. The opposite condition holds; excluding these possibly fallacious increments should better preserve the
analyses. This could partly explain why all STO covariances (with weaker mass–wind coupling) yielded a greater improvement than their OTO counterparts. Even though STO covariances had longer horizontal length-scales for $p$ than their OTO counterparts, it is not likely a crucial factor which affects precipitation forecast skill.

Also, as mentioned in Section 3.5, it is impossible to know the correct magnitude of $\sigma_u^0$. Trials using STO
covariances (with larger $\sigma_b^u$) typically showed equivalent or improved FSS compared to oldDS (which has surprisingly large $\sigma_b^u$). Given that wind information in the Tropics is limited, it is possible that the NMC method could be underestimating $\sigma_b^u$ in the Tropics, and thus reducing valuable wind information by giving too much weight to the background. We speculate that possible improvement in FSS could be gained if $\sigma_b^u$ is generally larger.

It was also shown in Section 4 that STO covariances had much shorter horizontal length-scales for $u$ compared to their OTO counterparts. The inter-monsoon period in May is typically characterised by weak and variable winds throughout the domain (Fong and Ng, 2012; As-syakur et al., 2013), due to the close proximity to the intertropical convergence zone, along with small-scale structures in the wind field caused by frequent thunderstorms. Having very short horizontal length-scales for $u$ can allow for finer structures in $\delta u$ to be recovered, and thus higher-resolution analyses, which would likely be beneficial for localised thunderstorm forecasts. This could also
explain the greater improvement in STO covariances than their OTO counterparts. Following this argument, one might argue that a larger benefit could be gleaned from having a higher-resolution observations network, as also suggested in Heng et al. (2020).

Previous studies have highlighted the benefit of capturing finer structures to get a higher-resolution analyses for precipitation forecasts, especially for limited-area convective-scale systems, such as SINGV-DA. Sun et al. (2016) showed that using different momentum control variables \((u \text{ and } v \text{ instead of } \psi \text{ and } \chi)\) better captured small-scale disturbances of \(u\) and \(w\) near convective regions which yielded better precipitation forecasts. Thiruvengadam et al. (2019) and Li et al. (2016) similarly showed that using \(u\) and \(v\) instead of \(\psi\) and \(\chi\), when assimilating Doppler weather data, resulted in shorter horizontal length-scales and larger variances for \(u\) which significantly improved the skill of high-intensity precipitation forecasts. Here, we have used \(\psi\) and \(\chi\) as momentum control variables with the STO formulation which also retrieved shorter horizontal length-scales for \(u\) and larger \(\sigma_u^2\). It is likely that the improved precipitation forecasts in our trials are also attributed to these same factors discussed in previous studies. Note that their choice of prognostic model variables as control variables inherently decouples all variables, including mass and wind. This corresponds to setting \(U_p\) as the identity matrix in the modelling of \(B\). This absence of mass–wind coupling is not discussed in their results, but could be another important factor, especially for the deep Tropics or at convective scales as seen for SINGV-DA.

As briefly mentioned previously, it would not be appropriate to simply assume that attribution of improvements to these factors listed are equally valid throughout the annual cycle, given that different regions within the domain experiences different sources of precipitation (e.g. monsoonal precipitation, squall lines or localised thunderstorms) during different monsoon seasons. Here, we have tuned \(B\) and analysed the factors desirable for the inter-monsoon months for a large part of the domain, but the results could be different for other trial months because of the different scales of the weather phenomena. For example, one might expect longer horizontal length-scales for \(u\) to be desirable in periods where capturing large-scale monsoonal flow is important with the limited or sparse wind observations. This re-emphasises the challenges and complexity of tropical data assimilation. It also illustrates the need to capture some flow dependence of the error statistics, possibly by tuning a different \(B\) suitable for each monsoon season or by incorporating information from a parallel ensemble run. Further work includes developing a hybrid ensemble-variational data assimilation system for SINGV-DA.

6 | SUMMARY AND CONCLUSIONS

In this article, we present and evaluate the structures of the background error covariance matrix for a tropical convective-scale NWP system, SINGV-DA. A total of 12 background error covariance matrices were modelled using the lagged NMC method, based on the application of the vertical-first (original) and horizontal-first (swapped) transform order formulations on six permutations of the training data. We test the sensitivity of the structures to the training data period (seasons), forecast lag and transform order.

Pseudo-single observation tests using mass variables (pressure) reveal that while there is little seasonal dependence in the horizontal and vertical autocorrelation structures for pressure, there exist substantial variation in the strength of the mass–wind coupling captured by the background error covariance matrices. Some of the covariances had stronger mass–wind coupling, which is either an artefact of the linear balance equation applied in the modelling step, or the strong projection of the structures onto the equatorial Kelvin wave mode. These covariances had structures characterised by robust vertical anticorrelation of pressure between the lower and upper troposphere, along with a positive correlation between the zonal component of wind and pressure, with a negligible meridional component. This is further supported by resulting lower tropospheric ascent downwind of the high pressure and a negative correlation with potential temperature in the mid-troposphere. It is possible that the other covariances with weak mass–wind coupling had structures that projected more strongly onto other equatorial wave modes, with opposing contributions as compared to the equatorial Kelvin wave. By using the swapped transform order formulation, with vertical covariances prescribed as a function of total horizontal wave number, modelled covariances captured weaker mass–wind coupling but slightly stronger vertical anticorrelation of pressure.

Pseudo-single observation tests using wind variables (zonal wind) reveal that there is minimal dependence on the training data period and forecast lag in the horizontal autocorrelation structures for zonal wind. However, the vertical profile of zonal wind background error standard deviation exhibits more dependence on the training data. In general, covariances based on training data with a longer forecast lag had larger zonal wind background error standard deviation. Comparing between training data periods, there are smaller differences, mainly associated with the magnitude of the peaks in zonal wind background error standard deviation. Also, using the swapped transform order formulation, we observe that the horizontal length-scales for zonal wind are greatly reduced and zonal wind background error standard deviations are
larger compared to using the original transform order formulation.

From results of month-long data assimilation trials with real observations, we have highlighted features of the training data and characteristics of their resulting modelled background error covariance matrices which led to improved precipitation forecasts compared to the current operational set-up (using the oldDS covariance). Since May is typically dominated by localised thunderstorm events, any improvement in precipitation forecasts is likely a result of improved localised thunderstorm forecasts. This could be due to three possible factors:

- **Weak mass–wind coupling**
  
  For trials that resulted in improved precipitation forecasts, weak mass–wind coupling was exhibited in the corresponding covariances. Since May is typically characterised by weak and variable winds, any large-scale wind increments as a result of a small pressure perturbation could contaminate the analyses. One would not expect to recover the wind field from a pressure observation using linear balance in the Tropics, so excluding these possibly fallacious increments should better preserve the analyses.

- **Larger wind background error standard deviation**
  
  Trials using covariances based on longer forecast lag training data and using swapped transform order formulations, with larger zonal wind background error standard deviations, resulted in equivalent or improved precipitation forecasts compared to oldDS. This agrees with previous studies which attributed improved high-intensity precipitation forecasts to larger wind background error variances. It is possible that the NMC method could be underestimating it in the Tropics, thus giving too much weight to the background and discarding whatever limited and valuable wind information there is in the Tropics.

- **Shorter horizontal length-scales for wind**
  
  Trials using swapped transform order covariances resulted in improved precipitation forecasts compared to oldDS and their original transform order counterparts, possibly attributed to comparatively shorter horizontal length-scales for wind. The argument is similar to those presented in previous studies: improved high-intensity precipitation forecasts from choosing different momentum control variables (zonal and meridional wind, instead of stream function and velocity potential) were partly attributed to shorter horizontal length-scales for wind.

The background error covariance matrices have been examined in May trials, and it would not be appropriate to simply assume that attribution of improvements to these factors are equally valid throughout the annual cycle, given that the region experiences different sources of precipitation during different monsoon seasons. It would be interesting to investigate this, although the results are likely to be different for other trial months because of the different scales of the associated weather phenomena. This illustrates the need to capture some flow dependence of the error statistics, possibly by tuning a different background error covariance matrix suitable for each monsoon season or by incorporating information from a parallel ensemble run.

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**CONFLICTS OF INTERESTS**

The authors declare no potential conflict of interests.

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