Distinguishing Between CDM and MOND:
Predictions for the Microwave Background

Stacy S. McGaugh
Department of Astronomy, University of Maryland, College Park, MD 20742-2421
e-mail: ssm@astro.umd.edu

ABSTRACT

Two hypothesized solutions of the mass discrepancy problem are cold dark matter (CDM) and modified Newtonian dynamics (MOND). The virtues and vices of these very different hypotheses are largely disjoint, making the process of distinguishing between them very dependent on how we weigh disparate lines of evidence. One clear difference is the nature of the principal mass constituent of the universe (CDM or baryons). This difference in the baryon fraction ($f_b \approx 0.1$ vs. 1) should leave a distinctive signature in the spectrum of fluctuations in the cosmic microwave background. Here I discuss some of the signatures which should be detectable in the near future. The most promising appears to be the ratio of the amplitudes of the first two peaks relative to the intervening trough.

Subject headings: cosmic microwave background — cosmology: theory — early universe

1. Introduction

Central to cosmology is the resolution of the mass discrepancy problem. In the current standard picture, the discrepancy between observed luminous mass and inferred dynamical mass in extragalactic systems is attributed to the presence of nonbaryonic cold dark matter (CDM). However, the predictions of CDM (e.g., Navarro, Frenk, & White 1997) fail the precision tests afforded by the rotation curves of low surface brightness galaxies (McGaugh & de Blok 1998a; Moore et al. 1999). In contrast, the modified Newtonian dynamics (MOND) introduced by Milgrom (1983) as an alternative to dark matter accurately predicted the behavior of these systems well in advance of the observations (McGaugh & de Blok 1998b; de Blok & McGaugh 1998; see also Begeman, Broeils, & Sanders 1991; Sanders 1996; Sanders & Verheijen 1998).

In conventional cosmology, CDM is required for two fundamental reasons. One is that the dynamically inferred mass density of the universe greatly exceeds that appropriate for baryons as determined from primordial nucleosynthesis ($\Omega_m \gg \Omega_b$; e.g., Copi, Schramm, & Turner 1995). The other is that gravitational formation of large scale structure proceeds slowly (as $t^{2/3}$) in an expanding universe. It is only possible to reach the rich amount of structure observed at $z = 0$ from the smooth ($\Delta T/T \sim 10^{-5}$) microwave background at $z \approx 1400$ if there is a nonbaryonic component whose density fluctuations can grow unimpeded by radiation pressure.

It does appear possible to explain these points with MOND. MOND is an alteration of the force law at very small acceleration scales, $a < a_0 = 1.2 \times 10^{-8}$ cm s$^{-2}$. The low acceleration scale applies in most disk
galaxies and the universe as a whole. In the context of MOND, conventional measures of the dynamical mass density of the universe are overestimated by a factor which depends on the typical acceleration. Accounting for this leads to a very low density universe with $\Omega_m \approx \Omega_b$ (McGaugh & de Blok 1998b; Sanders 1998).

Perhaps the simplest possible MOND universe one can consider is one in which $a_0$ remains constant in time (Felten 1984; Sanders 1998). In this case, the universe does not enter the MOND regime of very low acceleration until $1 + z \approx 2.33(\Omega_m h/0.02)^{-1/3}$, or $z \approx 1.6$. Everything is normal at higher redshift, so conventional results like primordial nucleosynthesis and recombination are retained. However, small regions can enter the MOND regime at early times as the phase transition begins (Sanders 1998). Once radiation releases its hold on the baryons ($z \approx 200$ in a low density universe), these regions will behave as if they possess a large quantity of dark matter. Consequently, structure forms very rapidly. Indeed, early structure formation is another promising way to distinguish between CDM and MOND (Sanders 1998). For the present purpose, it suffices to realize that if the MOND force law is operative, structure forms much more rapidly than the Newtonian $t^{2/3}$. It is not necessary to have CDM for a rich amount of large scale structure to grow from an initially smooth cosmic microwave background.

In this paper I begin to explore how anisotropies in the microwave background might help to distinguish between CDM and MOND. As a starting point, for MOND I make two basic assumptions: that $a_0$ is constant, and the background metric is flat in the usual Robertson-Walker sense. This is not the only possibility for a MOND universe (Milgrom 1989). The acceleration constant may vary with time, and the nature of the background geometry is unclear. Still, this seems like the most obvious point of departure. In essence, I am examining the microwave background anisotropy properties of a conventional cosmology in which all the matter is baryonic in the amount specified by primordial nucleosynthesis, but the amplitude of the anisotropies is not constrained to be large by the slow growth of structure. Failure of the assumptions should result in a more pronounced effect on the microwave background than what I discuss below, making it easier to distinguish between CDM and MOND.

2. The Baryon Fraction Test

The most obvious difference between CDM and MOND in the context of the microwave background anisotropies is the baryon fraction. CDM is thought to outweigh ordinary baryonic matter by a factor of $\sim 10$ (Evrard, Metzler, & Navarro 1996). The precise value depends on the Hubble constant and on the type of system examined (McGaugh & de Blok 1998a). If instead MOND is the cause of the observed mass discrepancies, there is no CDM. The difference between a baryon fraction $f_b \approx 0.1$ and unity should leave a distinctive imprint on the microwave background.

The main impact of varying the baryon fraction is on the relative amplitude of the peaks in the angular power spectrum of the microwave background (as expanded in spherical harmonics). In general, increasing $f_b$ increases the baryon drag, which enhances the amplitude of compressional (odd numbered) peaks while suppressing rarefaction (even numbered) peaks (Hu, Sugiyama, & Silk 1997). The precise shape of the power spectrum is thus very sensitive to $f_b$.

In order to investigate this aspect of the problem, I have used CMBFAST (Seljak & Zaldarriaga 1996).

\footnote{Note that the modification is not on some length scale. The predictions of MOND therefore do not vary by many orders of magnitude from the scales of galaxies to that of the entire universe.}
to compute the expected microwave background power spectrum in several representative cases. These have reasonable baryon fractions for each case and baryon-to-photon ratios consistent with primordial nucleosynthesis. Several specific cases are illustrated in Figure 1. These have \( \Omega_b = 0.01, 0.02, \) and 0.03 with \( \Omega_{CDM} = 0.2 \) or 0 (so \( f_b = 0.05, 0.1, 0.15 \) or 1). Other model parameters are held fixed (\( h = 0.75, T_{CMB} = 2.726 \) K, \( Y_p = 0.24, N_\nu = 3 \)), and adiabatic initial conditions are assumed. As a check, models with \( \Omega_{CDM} = 0.3 \) and 0.4 were also run with the same baryon fractions and \( H_0 \) scaled to maintain the same baryon-to-photon ratio. As expected, these resulted in power spectra which are indistinguishable in shape.

I am interested in the shape of the power spectrum, not the absolute positions of the peaks. The latter depends mostly on the scale and geometry of the universe. For purposes of computation, I assume the universe is flat, with \( \Omega_A = 1 - \Omega_{CDM} - \Omega_b \). This results in a CDM universe close to the current “concordant” model (e.g., Ostriker & Steinhardt 1995). In the case of MOND, the resulting model is very close to the de Sitter case. This is a plausible case for a MOND universe (indeed, the relation of inertial mass to a finite vacuum energy density has been suggested as a possible physical basis for MOND: Milgrom 1999), but is by no means the only possibility. A model with no cosmological constant and \( \Omega_m = \Omega_b \approx 0.02 \) is plausible, but would be very open if the geometry were Robertson-Walker. The position of the first peak in the power spectrum moves to smaller angular scales in open universes because of the dependence of the angular diameter distance on \( \Omega_m \). For such low \( \Omega_m \) with \( \Omega_A = 0 \), the position of the first peak occurs at \( \ell_1 > 1000 \). This is inconsistent with recent observations which constrain \( \ell_1 \) to be near 200 (Miller et al. 1999). However, the geometry in MOND might not be Robertson-Walker, so the position of the first peak is not uniquely specified. It is important to realize that while the position of the first peak provides an empirical constraint on the geometry traversed by the microwave background photons, in the context of MOND this does not necessarily translate into a measure of \( \Omega_m \).

The test is therefore not in the absolute positions of the peaks, but in the shape of the spectrum. As the baryon fraction becomes very high \( f_b \rightarrow 1 \), the even numbered peaks are suppressed to the point of disappearing. One is left with a spectrum that looks rather like a stretched version of the standard CDM case.

The difference between the CDM and MOND cases is obvious by inspection (Figure 1). However, from an observer’s perspective, it is not so easy to distinguish them. The second peak has disappeared in the MOND case, so what would have been the third peak we would now count as the second peak. The absolute positions of the peaks are not specified \emph{a priori} by either theory. The absolute amplitude in the CDM case is constrained by the need to match large scale structure at \( z = 0 \). The mechanics to do a similar exercise with MOND do not currently exist, so the absolute amplitude is also not specified \emph{a priori}. We must therefore rely on the relative amplitudes and positions of the peaks to measure the difference. Since the third peak becomes the second peak in MOND, the observable difference is rather more difficult to perceive than one might have expected, at least for the assumptions made here.

The ratios of the positions and amplitudes of the peaks are given in Table 1. The peak position ratios depend on the sound horizon at recombination, which should not depend on MOND (for constant \( a_0 \)) because this is well before the universe approaches the low acceleration regime. Other parameters do matter a bit, which can complicate matters.

One difference we could hope to distinguish is in the ratio of the positions of the first and second peaks.

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\footnote{Since it is possible that neutrinos have significant mass, I also consider a model with \( \Omega_{CDM} = 0 \) and \( \Omega_\nu = \Omega_b = 0.02 \). This sharpens the peaks noticeably, but is otherwise similar to the pure baryon models.}
In the CDM models, $\ell_2/\ell_1 \approx 2.35$, while in the case of MOND $\ell_2/\ell_1 \approx 2.66$. This requires a positional accuracy determination of $\sim 5\%$ beyond $\ell > 500$, no small feat.

If we can recognize that second peak is actually missing, so that what we called the second peak in MOND actually corresponds to the third peak in CDM, then the distinction is greater: for CDM, $\ell_3/\ell_1 \approx 3.6$, which should be compared to MOND’s 2.66. It is not clear how to do this observationally. Once the position of the first peak is tied down, the given ratio predicts the expected position of the second observable peak (under the assumptions made here). This is not very different in the two cases.

The ratios of the positions of the next observable peaks help not at all. For CDM, $\ell_3/\ell_2 = 1.54$. For MOND, $\ell_3/\ell_2 = 1.57$.

The ratio of the absolute amplitudes of the peaks can also distinguish the two cases, but require comparable accuracy. In CDM, $(C_{\ell,1}/C_{\ell,2})_{\text{abs}} \approx 1.7$, while in MOND $(C_{\ell,1}/C_{\ell,2})_{\text{abs}} \approx 2.4$. This may appear to be a substantial difference, but recall that what is measured is the temperature anisotropy. Since $\Delta T \propto \sqrt{C_\ell}$, one requires $\sim 7\%$ accuracy to distinguish the two cases at the $2\sigma$ level. The amplitude ratios of the second and third peaks have a bit more power to distinguish between CDM and MOND, but are more difficult to measure. The precise value of this ratio is very sensitive to $f_b$ in the CDM case. In CDM, $(C_{\ell,2}/C_{\ell,3})_{\text{abs}} < 1.6$ for $f_b > 0.05$, while in MOND $(C_{\ell,2}/C_{\ell,3})_{\text{abs}} \approx 1.9$.

Using the absolute amplitude of the peak heights does not utilize all the information available. In the purely baryonic MOND cases, there is a longer drop from the first peak to the first trough, and a shorter rise to the second peak than in the CDM cases. Therefore, measuring the peak heights relative to the bottom of the intervening trough may be a better approach. To do this, we define $(C_{\ell,n}/C_{\ell,n+1})_{\text{rel}} = (C_{\ell,n} - C_{\ell,\text{min}})/(C_{\ell,n+1} - C_{\ell,\text{min}})$ to be the ratio of the amplitudes at maxima $n$ and $n + 1$ less the amplitude of the intervening minimum. This does indeed appear more promising. The purely baryonic MOND cases all have $(C_{\ell,1}/C_{\ell,2})_{\text{rel}} > 5$, while the CDM cases have $(C_{\ell,1}/C_{\ell,2})_{\text{rel}} < 4$ (Table 1). This is a nice test, for in most cases this ratio falls well on one side or the other (for $\Omega_b = 0.02$, $(C_{\ell,1}/C_{\ell,2})_{\text{rel}}^\text{MOND}/(C_{\ell,1}/C_{\ell,2})_{\text{rel}}^\text{CDM} = 2$).

By inspection of Figure 1, one might also think that the width of the first peak could be a discriminant, as measured at the amplitude of the first minimum. This is a bit more sensitive to how other parameters shift or stretch the power spectrum. It is also very sensitive to the neutrino mass. Baryonic models with zero neutrino mass have perceptibly broader peaks than the equivalent CDM model, but zero CDM models with finite neutrino mass have peaks which are similar in width to those in the CDM models.

### 3. Assumptions and Caveats

I have made some predictions for the microwave background temperature anisotropies which should, with sufficiently accurate measurements, distinguish between CDM and MOND dominated universes. The predictions are based on some simple assumptions, most notably that the MOND acceleration constant $a_0$ does not vary substantially with time, and the geometry of the universe is flat in the Robertson-Walker sense. Neither of these need hold in MOND, but plausibly may (Sanders 1998), making this the obvious point of departure for this discussion. I have endeavored to make the most conservative assumptions in the sense that failures of these assumptions should lead to microwave background anisotropies more deviant from the standard CDM case, and hence more readily perceptible, than the cases I have discussed.

It should be noted that the signature of a purely baryonic universe is not necessarily reflected in the
usual way in the power spectrum of large scale structure at $z = 0$ [$P(k)$ instead of $C_\ell$]. The calculation for the microwave background power spectrum can be made under the assumption that MOND is not yet important at the epoch of recombination. It certainly is relevant by $z = 0$. The scale which is nonlinear now is much larger in MOND than in CDM. The rapid nonlinear growth of structure seems likely to wash out the bumps and wiggles that would otherwise be imprinted on and preserved in the power spectrum of large scale structure in the standard framework. So while one expects a definite signature of baryon domination in the microwave background, one does not necessarily expect this to be reflected in $P(k)$.

A conventional effect which may be different in the CDM and MOND cases is reionization. I have assumed that the background radiation encounters effectively zero optical depth along the way to us. However, the optical depth can be nonzero if the universe is reionized early enough, thus perturbing the signal in the microwave background (cf. Peebles & Juszkiewicz 1998). Structure forms faster in MOND than in CDM, so this is a greater concern. However, the degree to which it happens depends on the details of how stars and other potential ionizing sources actually form, which is not understood in either case. The main effect of a significant optical depth is to wash out the anisotropy signal. This should not much perturb the observational signatures I have discussed, which focus on the detailed structure of the peaks relative to one another. In purely baryonic models, it is conceivable that the amplitude of the second peak will be amplified by this process, which formally would invalidate the test based on the ratio of the peak-to-trough amplitudes. However, such a microwave background power spectrum would be clearly distinct from the standard CDM case.

The integrated Sachs-Wolfe effect is another matter which may be affected by the rapid growth of structure in MOND. How much depends on the unknown details of the timing. Matter domination does not occur in MOND until $z \approx 200$ because of the low mass density of a baryon-only universe. Growing potentials vary rapidly, but there is not a tremendous amount of time between then and $z \approx 10$ when $L^*$ galaxy mass objects have collapsed (Sanders 1998). So it is not obvious how strong this effect will be, though it can potentially have a significant impact.

It seems unlikely that there are any effects which will cause CDM and MOND universes to be indistinguishable once sufficiently accurate observations of the microwave background are obtained. For the simple assumptions I have made, the distinction is surprisingly subtle, but certainly present. Any breakdown of these assumptions should lead to a greater distinction between the two. However, it remains a substantial challenge to understand some of basic effects which can impact the microwave background in the context of MOND.

4. Conclusions

Modern cosmological models require copious amounts of nonbaryonic cold dark matter for well established reasons. Yet the existence of CDM has yet to be confirmed. The alternative to dark matter postulated by Milgrom (1983), MOND, has long had considerable success in describing the rotation curves of spiral galaxies (Begeman et al. 1991; Sanders 1996; Sanders & Verheijen 1998), a fact which has no explanation in the standard framework. Moreover, MOND successfully predicted, \textit{a priori}, the behavior of low surface brightness galaxies (McGaugh & de Blok 1998b; de Blok & McGaugh 1998), a test which CDM models fail (McGaugh & de Blok 1998a; Moore et al. 1999). Yet MOND has no clear cosmology.

In this paper, I have attempted to make some predictions for the temperature anisotropies in the microwave background which might potentially discriminate between CDM and MOND dominated
cosmologies. In this context, the essential difference between the two is the baryon fraction ($f_b \approx 0.1$ for CDM and $f_b = 1$ for MOND). I have used this fact to examine the differences expected for microwave background observations in as conservative and model independent a way as possible.

Upcoming experiments to measure the anisotropies of the microwave background to high precision should be able to distinguish between CDM and MOND. For the simple assumptions investigated here, the observational signatures are surprisingly subtle, requiring high accuracy (i.e., peak position or amplitude to $\sim 5\%$ at $\ell > 500$. ) Perhaps the most promising test is the ratio of peak-to-trough amplitudes of the first two peaks, with $(C_{\ell,1}/C_{\ell,2})_{rel} < 4$ in plausible CDM models and $(C_{\ell,1}/C_{\ell,2})_{rel} > 5$ in MOND.

These predictions are offered in the hope of clearly distinguishing between CDM and MOND in the near future.

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Table 1. Microwave Background Anisotropy Measures

| $\Omega_{CDM}$ | $\Omega_b$ | peak $n$ | $\ell_{n+1}/\ell_n$ | $(C_{\ell,n}/C_{\ell,n+1})_{abs}$ | $(C_{\ell,n}/C_{\ell,n+1})_{rel}$ |
|---------------|-----------|----------|-------------------|-------------------------------|---------------------------------|
| 0.2           | 0.01      | 1        | 2.31              | 1.54                          | 2.99                            |
|               |           | 2        | 1.55              | 1.56                          | 4.80                            |
|               |           | 3        | 1.37              | 2.36                          | ...                             |
| 0.02          |           | 1        | 2.36              | 1.60                          | 2.90                            |
|               |           | 2        | 1.54              | 1.25                          | 2.07                            |
|               |           | 3        | 1.38              | 2.02                          | ...                             |
|               |           | 4        | 1.26              | 1.80                          | ...                             |
| 0.03          |           | 1        | 2.40              | 1.83                          | 3.48                            |
|               |           | 2        | 1.53              | 1.10                          | 1.37                            |
|               |           | 3        | 1.38              | 1.97                          | ...                             |
|               |           | 4        | 1.26              | 1.61                          | ...                             |
| 0.0           | 0.01      | 1        | 2.74              | 2.57                          | 7.61                            |
| 0.02          |           | 1        | 2.65              | 2.37                          | 5.72                            |
|               |           | 2        | 1.58              | 1.96                          | 5.84                            |
| 0.03          |           | 1        | 2.62              | 2.40                          | 5.41                            |
|               |           | 2        | 1.57              | 1.88                          | 4.68                            |
| $\Omega_\nu = 0.02$ | 0.02  | 1        | 2.57              | 2.22                          | 5.13                            |

Note that in the $\Omega_{CDM} = 0$ models, what would have been the even numbered peaks are completely suppressed. The peak we call $n = 2$ for MOND corresponds to $n = 3$ in CDM, and $n = 3$ to $n = 5$. 
Fig. 1.— The power spectrum of temperature anisotropies in the microwave background with (a) and without (b) CDM. Three choices for the baryon density are illustrated in each case. The highest (lowest) baryon content corresponds to the highest (lowest) curve. CDM models with $\Omega_{\text{CDM}} = 0.2$, 0.3, and 0.4 all gave indistinguishable results provided the baryon fraction was the same and $H_0$ was scaled to maintain the same baryon-to-photon ratio. CDM models have several distinct peaks before $\ell = 1000$ while in the pure baryon cases representing MOND the even numbered peaks have disappeared. Also shown are current measurements with errors $\Delta T < 40 \mu K$ from the compilation of Tegmark (http://www.sns.ias.edu/~max/main.html#CMB) as of March 1999.