Chemical Abundances Of Open Clusters From High-Resolution Infrared Spectra. II. NGC 752

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ABSTRACT

We present a detailed near-infrared chemical abundance analysis of 10 red giant members of the Galactic open cluster NGC 752. High-resolution (R = 45000) near-infrared spectral data were gathered with the Immersion Grating Infrared Spectrograph (IGRINS), providing simultaneous coverage of the complete H and K bands. We derived the abundances of H-burning (C, N, O), α (Mg, Si, S, Ca), light odd-Z (Na, Al, P, K), Fe-group (Sc, Ti, Cr, Fe, Co, Ni) and neutron-capture (Ce, Nd, Yb) elements. We report the abundances of S, P, K, Ce, and Yb in NGC 752 for the first time. Our analysis yields solar metallicity and solar abundance ratios for almost all of the elements heavier than the CNO group in NGC 752. O and N abundances were measured from a number of OH and CN features in the H band, and C abundances were determined mainly from CO molecular lines in the K band. High excitation C i lines present in both near-infrared and optical spectra were also included in the C abundance determinations. Carbon isotopic ratios were derived from the R-branch band heads of first overtone (2−0) and (3−1) 12CO and (2−0) 13CO lines near 23440 Å and (3−1) 13CO lines at about 23730 Å. The CNO abundances and 12C/13C ratios are all consistent with our giants having completed “first dredge-up” envelope mixing of CN-cycle products. We independently assessed NGC 752 stellar membership from Gaia astrometry, leading to a new color-magnitude diagram for this cluster. Applications of Victoria isochrones and MESA models to these data yield an updated NGC 752 cluster age (1.52 Gyr) and evolutionary stage indications for the program stars. The photometric evidence and spectroscopic light element abundances all suggest that the most, perhaps all of the program stars are members of the helium-burning red clump in this cluster.

Key words: stars: abundances – stars: atmospheres. Galaxy: open clusters and associations: individual: NGC 752

1 INTRODUCTION

Open star clusters provide important snapshots of the chemistry of the Galactic disk with time because they can be photometrically tagged with ages, which are difficult to assess for individual field stars. Most open clusters (OCs) are young, t ≤ 2 Gyr. Very few of them are truly old, t > 7 Gyr; only Be 17 and NGC 6791 appear to have ages approaching 10 Gyr (e.g., Salaris et al. 2004, Brogaard et al. 2012). Fortunately there are many intermediate-age clusters close enough to the Sun that chemical compositions of their brighter members can be studied at high spectroscopic resolution. The recent Gaia DR2 catalog now provides opportunities for more accurate membership and evolutionary state data for OC red giant members. Several groups are conducting extensive OC abundance studies using echelle spectrographs in the optical spectral region.

Optical spectroscopy of OCs has a fundamental observational...
limit caused by Galactic disk dust extinction. While more than 1000 OCs have been cataloged, most are too obscured to yield detailed information at optical wavelengths. This problem is especially acute for clusters at small Galactocentric radii. High-resolution spectroscopy at more transparent infrared wavelengths (IR, $\lambda \geq 1 \mu m$) is essential for further progress in OC chemical composition studies.

We have begun a program to use $H$ and $K$ band high-resolution spectroscopy to determine reliable metallicities and elemental abundance ratios for OCs spanning a large range of Galactocentric distances. Special emphasis is put on determining accurate values for the CNO group and other light elements, which have many transitions in these IR bands. As steps toward this goal we first are performing combined optical/infrared spectroscopic analyses of three relatively nearby and well-studied intermediate-age OCs that suffer only small amounts of interstellar dust extinction: NGC 6940, NGC 752, and M67. Our IR spectra are obtained with the Immersion Grating Infrared Spectrograph (IGRINS), which offers complete $H$ and $K$ spectral coverage (1.45–2.5 μm), with a band gap of only 0.01 μm lost instrumentally between the two bands (the gap due to telluric absorption is ~0.2 μm). IGRINS delivers high spectral resolution similar to those of most optical spectrographs used for abundance analysis.

For NGC 6940, Böcek Topcu et al. (2019), hereafter Paper 1, reported analyses of IGRINS spectra of 12 red giant members, the same stars with atmospheric parameters and detailed abundance ratios derived from high-resolution optical spectra by Böcek Topcu et al. (2016) (hereafter BT16). Among their principal results were: (a) good agreement in all cases of optical and IR abundances; (b) determination for the first time in NGC 6940 the abundances of S, P, K, Ce, and Yb, and much strengthened abundances of Mg, Al, Si, and Ca; (c) derivation of much more reliable abundances of the CNO group; (d) discovery of one star with evidence of high-temperature proton fusion products; and (e) improved assessment of the NGC 6940 color-magnitude diagram, with clear assignment of most of the program stars to the He-burning red clump.

In this paper we report results of a similar optical/IR study for 10 red giant (RG) members of NGC 752. This relatively nearby OC has been the subject of several abundance studies (e.g., Piachowski & Hobbs 1988; Carrera & Pancino 2011; Reddy et al. 2012). Recently Lum & Boesgaard (2019) have presented an extensive large-sample analysis of 6 giant and 23 main-sequence stars. Our optical spectroscopic investigation was published in Böcek Topcu et al. (2015) (hereafter BT15). In this study we focus especially on the CNO abundances and $^{12}C/^{13}C$ ratios, interpreting the results within a more complete wavelength window. This larger spectral coverage leads us to a better analysis of the evolutionary status of the RG members. In addition to the red giant abundances derived from IGRINS spectra, we have revisited the questions of the distance and age of NGC 752 with Gaia DR2 data, and have re-considered the evolutionary states of these stars via new stellar isochrone computations. The Gaia kinematic and photometric data leading to NGC 752 membership, distance, and age are discussed in §2. In §3 we summarize the IGRINS observations and reductions. The methods used to derive the elemental abundances and the temperatures of the target stars are described in §4 and §5, respectively, while the results of the abundance determinations are given in section §6. The fitting of isochrones to the CMD of NGC 752, resulting in our best estimate of the cluster age, is discussed in §7. We compare stellar model predictions to the observed cluster RC stars in §8, and summarize the main results of this investigation in §9.

Figure 1. Vector point diagram of our data set, where each dot shows the proper motion components of a star in right ascension ($\mu_\alpha$) and declination ($\mu_\delta$). The 95% confidence ellipse in the lower right corner encircles most of the cluster stars and represents the intrinsic cluster center and dispersion. The colors, defined in the side bar, signify NGC 752 membership probabilities for each star. The total data set consisted of more stars more widely separated in proper motion from the field and cluster groups; they have been omitted for clarity in this plot.

2 MEMBERSHIP ASSIGNMENT USING GAIA DR2

For determination of our NGC 752 member set, we created a Gaussian mixture model. This model was built using proper motion data from the Gaia (Gaia Collaboration et al. 2016) Data Release 2 (Gaia Collaboration et al. 2018). All stars that had Gaia DR2 proper motions and resided within 75′ of the approximate NGC 752 cluster center, $\varpi_{2000} = 41.0''$ and $\varpi_{2000} = 37.4''$, were considered for membership. A vector point diagram for these stars is shown in Figure 1. Each dot represents the proper motion components of a single candidate star.

Fitting mixture models is an applied statistical method that allows for the Bayesian determination of membership probabilities for individual stars. Open cluster applications of membership models like ours date at least back to Sanders (1971), whose model is fundamentally similar to ours: the sum of two normal probability densities – bivariate in the case of proper motion data – is fit to observed right ascension and declination proper motion components. Our probability density function is of the form

$$
\Phi(\mu_\alpha, \mu_\delta, \epsilon_\alpha, \epsilon_\delta) = \phi_\epsilon + \phi_f,
$$

where $\mu_\alpha$ and $\mu_\delta$ are the proper motion components for the $i^{th}$ star in our data set, and $\epsilon_\alpha$ and $\epsilon_\delta$ are their respective errors. $\phi_\epsilon$ and $\phi_f$ are Gaussians and represent the cluster and field star distributions, respectively. Both Gaussians are symmetrical and elliptical. It is common to make the assumption of a circular cluster distribution, but we have left it as elliptical for increased accuracy. In addition, we have adopted a method derived by Zhao & He (1990) that takes into account not only the intrinsic dispersions of $\phi_\epsilon$ and $\phi_f$ but also the observed Gaia DR2 errors for each individual star. In total, there are 11 parameters needed to characterize the distributions $\phi_\epsilon$ and $\phi_f$ in our model. After we solved for these, cluster membership probabilities were calculated. Figure 1 presents the final probabilities for stars within the proper motion ranges...
Table 1. Kinematics and radial velocities.

| Star   | RA (Gaia)       | DEC (Gaia)       | \( V \) | \( H \) | \( K \) | \( G \) | \( G_{BP} - G_{RP} \) | \( (B - V)_0 \) | \( (V - K)_0 \) | Date (UT) | S/N |
|--------|----------------|----------------|--------|-------|-------|-------|---------------------|---------------|---------------|------------|-----|
| MMU 1  | 01 55 12.62    | 37 50 14.55    | 9.50   | 7.37  | 7.23  | 9.23  | 1.13                | 0.92          | 2.17          | 02 12 2015 | 108 |
| MMU 3  | 01 55 15.29    | 37 50 31.30    | 9.37   | 7.32  | 7.20  | 9.28  | 1.16                | 0.96          | 2.27          | 02 12 2015 | 106 |
| MMU 11 | 01 55 27.67    | 37 59 55.24    | 9.29   | 7.16  | 7.04  | 9.03  | 1.12                | 0.93          | 2.15          | 02 12 2015 | 120 |
| MMU 24 | 01 55 39.37    | 37 52 52.51    | 8.92   | 6.67  | 6.55  | 8.65  | 1.16                | 0.98          | 2.28          | 03 12 2015 | 109 |
| MMU 27 | 01 55 42.39    | 37 37 54.57    | 9.16   | 6.90  | 6.80  | 8.88  | 1.17                | 0.98          | 2.27          | 03 12 2015 | 117 |
| MMU 77 | 01 56 21.64    | 37 36 08.43    | 9.38   | 7.05  | 6.92  | 9.09  | 1.21                | 0.99          | 2.36          | 03 12 2015 | 117 |
| MMU 137| 01 57 03.11    | 38 08 02.65    | 8.93   | 6.66  | 6.54  | 8.64  | 1.18                | 0.99          | 2.29          | 04 12 2015 | 119 |
| MMU 295| 01 58 29.82    | 37 51 37.57    | 9.30   | 7.17  | 7.04  | 9.05  | 1.13                | 0.93          | 2.17          | 04 12 2015 | 141 |
| MMU 311| 01 58 52.90    | 37 48 57.23    | 9.06   | 6.80  | 6.64  | 8.77  | 1.20                | 1.00          | 2.33          | 04 12 2015 | 113 |
| MMU 1367| 01 59 14.80   | 38 00 55.29    | 9.01   | 6.77  | 6.65  | 8.72  | 1.18                | 0.98          | 2.26          | 26 11 2018 | 113 |

a Gaia Collaboration et al. (2018)

b Daniel et al. (1994)

c Cutri et al. (2003)

d Cantat-Gaudin et al. (2018)

plotted. See the appendix for more detail on the exact forms of \( \phi_c \) and \( \phi_\delta \) and the techniques used to solve for the parameters.

In a mixture model such as ours, it is inevitable that some field stars will pass the cluster membership probability cutoff simply due to random chance. To mitigate this effect, we imposed parallax bounds on the remaining stars. By examination of the parallax density of the stars remaining after the proper motion cutoff, we found that the parallax distribution for NGC 752 peaked at \( \pm 2.235 \) mas with a baseline of \( \pm 0.8 \) mas. We therefore set parallax bounds of 1.735 mas to 2.735 mas. All stars that passed a 50% proper motion model cutoff and fell within these parallax bounds were considered to be physical cluster members.

Our membership study was performed independently of any other NGC 752 membership analysis. Our computations yield cluster parameters of \( \mu_\alpha = 9.827 \pm 0.017 \) mas/yr, \( \mu_\delta = -11.713 \pm 0.019 \) mas/yr, and parallax \( p = 2.229 \pm 0.009 \) mas. This corresponds to a distance of 448 pc and a true distance modulus \( m - M_H = 8.26 \), which has been adopted in the fitting of isochrones to the observed CMD in [7]. We caution the reader that our parallax uncertainty for NGC 752 is purely statistical; a more realistic estimate would take into account possible Gaia uncertainties, which Arenou et al. (2018) suggest can be as large as 0.03 mas. Recently Cantat-Gaudin et al. (2018) have performed a membership study of NGC 752 using very different methods. They suggest cluster parameters \( \mu_\alpha = 9.810 \pm 0.019 \) mas/yr, \( \mu_\delta = -11.713 \pm 0.019 \) mas/yr, and \( p = 2.239 \pm 0.005 \) mas. Although our study focused on proper motions and we do not claim to have determined precise parallax estimates, our results are essentially in agreement with their work.

In Table 1 we have listed the program stars with their Gaia DR2 identifications, parallaxes and proper motions. Table 1 also contains radial velocities (RVs) from Gaia (Gaia Collaboration et al. 2018), optical (BT15) and IR spectra. The IR RVs were measured applying a similar method described in Paper 1 using at least 10 spectral orders that are less affected by the atmospheric telluric lines. The mean cluster RVs from these measurements agree well within the mutual uncertainties: \( \langle RV \rangle_{opt} = 4.82 \pm 0.20 \) kms\(^{-1} \) (\( \sigma = 0.71 \)), \( \langle RV \rangle_{IR} = 4.97 \pm 0.24 \) kms\(^{-1} \) (\( \sigma = 0.45 \)), and \( \langle RV \rangle_{Gaia} = 5.35 \pm 0.31 \) kms\(^{-1} \) (\( \sigma = 0.33 \)) (from 9 RGs).

3 OBSERVATIONS AND DATA REDUCTION

We gathered IGRINS H- and K-band high resolution spectra for the 10 NGC 752 RG members studied in the optical spectral region by BT15. The stars chosen for that paper were selected from the radial velocity membership catalog of Mammiolid et al. (2008), before the release of Gaia DR2 astrometric data. The membership analysis...
presented here confirms that our targets belong to NGC 752. Additionally, several stars not included here appear to be RG members (this study and B. Twarog, private communication). Future spectroscopic study of these stars would be welcome. The log of the IGRINS observations is given in Table 2 along with the basic parameters of program stars. These stars are all red giants with similar parameters, as indicated by spectroscopic analyses \((T_{\text{eff}} \sim 4900 \, \text{K}, \log g \sim 2.7; \text{BT15})\) and by photometric data \((V \approx 9.2, M_V \approx 1.0, (B-V)_{\odot} \approx 0.97)\). Three other stars with similar photometric characteristics satisfy our NGC 752 membership criteria: BD+37 404 (MMU 2054), BD+36 328 (MMU 1533), and BD+37 422 (MMU 1367). The derived distance for BD+37 422 is \(30 \, \text{pc}\) (Mace et al. 2016). One object, MMU 1367, was observed with IGRINS on Lowell Observatory’s 4.3m Discovery Channel Telescope (Mace et al. 2018). Typical exposure times were 300s and used ABBDA nod sequences along the spectrograph slit length. We also observed telluric standards with spectral types of B9IV to A0V. They were observed right after each science exposures at very close airmasses to the ones at program stars were observed. The spectra used in this analysis were reduced using the IGRINS pipeline (Lee et al. 2017). The pipeline performs flatfield correction, A-B frame subtractions to remove skyline emission, wavelength correction using OH emission and telluric absorption, and optimal spectral extraction. Due to their high rotational velocities \((\sim 150 \, \text{km s}^{-1})\), telluric stars come with extremely broadened absorption features that can be easily distinguished from the atmospheric telluric lines. After removing the extremely broadened features from the spectra of telluric standards, we used the telluric task of IRAF\(^1\) to remove the contamination of atmospheric absorption lines from the spectra of our program stars.

**4 MODEL ATMOSPHERES AND ABUNDANCES FROM THE OPTICAL REGION**

Model atmospheric parameters (Table 3) of the 10 RG members of NGC 752 were previously presented in BT15, along with the abundances for 26 species of 23 elements present in the optical spectral region (see Table 10 in BT15). In this study, we newly report abundances of species \(\text{Si}, \text{K}, \text{Ca}\), and \(\text{Ce}\) from those spectra. We present optical sulfur abundances for our targets for the first time in NGC 752 using the \(\text{S}\) triplet centered at 6757.17 Å. Takeda et al. (2016) reported non-local thermodynamic equilibrium (non-LTE) corrections for \(\text{S}\), estimated them to be \(\lesssim 0.1\) dex for \(\text{G-K}\) giants for this blended \(\text{S}\) feature. We have also repeated the analyses for optical \(\text{Sc, Fe}\) lines, adopting new \(\text{log g}\) values from Lawler et al. (2019). Detailed description of the elemental abundance analysis methods in the optical region were provided in BT15, in which we also derived the solar photospheric abundances following the same procedure applied for the program stars, to obtain the differential values of stellar abundances relative to the Sun. Here we used the same method as in Paper 1, adopting the Asplund et al. (2009) solar photospheric abundances for both regions in order to achieve consistency between the optical and IR data sets. These slightly revised relative optical abundances are listed in the upper part of Table 4. Mean abundances of the species and their standard deviations are given in columns 12 and 13 of this table. For the [X/Fe] calculations, we took star-by-star species differences using both \(\text{Fe}\) and \(\text{Fe}\) abundances as appropriate. We will discuss the differences between the NGC 752 and NGC 6940 abundance sets in §6.7.

**5 TEMPERATURE DETERMINATION USING IGRINS DATA**

Accurate effective temperatures, gravities, and microturbulent velocities are required for abundance analyses. In our OC studies we have adopted traditional line-by-line equivalent width (EW) analyses to derive atmospheric parameters \(T_{\text{eff}}, \log g, \xi_t,\) and \([M/H]\). For NGC 752 these parameters derived in BT15 are listed in Table 3 and we have used them for all of the abundances reported in this paper. However, for heavily dust-obscured clusters optical parameter determinations will not be possible, and IR-based methods will be needed. Here we explore \(I_R T_{\text{eff}}\) estimates.

Line-depth ratios (LDR) have proven to be good temperature indicators in several studies (e.g. Gray & Johanson 1991; Kovyukh et al. 2006; Biazzo et al. 2007a,b). The LDR method is based on depth ratios of high-excitation atomic lines (relatively sensitive to

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\(^1\) http://iraf.noao.edu/
The lack of agreement between spectroscopic and IR temperatures for the whole sample. By inspection, Gaia temperatures along with the spectroscopic \( T_{\text{eff}} \)’s of the NGC 752 (full red circles) and NGC 6940 (open grey symbols). The dashed line represents equality of the temperatures for both panels.

For most of the species with a few exceptions: O, Sc, Ti, Cr, Fe, Co, Ni, and neutron-capture (n-capture) (Ce, Nd, Yb) elements, and also determined \( ^{125}\text{C}^{13}\text{C} \) ratios. The relative IR abundances for our NGC 752 RGs are listed in the second part of Table 4. In Figure 3, we plot the mean IR abundances along with the optical ones from BT15, updated as described in §4. The figure shows general agreement between IR and optical abundances. Defining \( \Delta \text{IR} = [A/\text{B}]_{\text{IR}} - [A/\text{B}]_{\text{opt}} \), we find \( \Delta \text{IR}[\text{X/Fe}] = 0.07 \pm 0.04 \) for 18 species with both optical and IR abundances. Figure 4 shows optical and IR abundances of each species for all program RGs vs. effective temperature. This figure shows that in a small temperature range (~175 K) that is covered by our RG sample, abundances do not show significant changes with temperature. Both figure 3 and 4 indicate the optical/IR agreement for most of the species with a few exceptions: O, Sc, Ti and K. The IR abundances of these species deviate more than 0.15 dex from their optical counterparts. We will discuss these deviations in the subsections below.

### 6.1 Fe-group elements:

We have investigated NGC 752 Fe abundances from about 20 Fe I transitions. As noted in our previous studies, there are no known useful Fe II transitions in the IGRINS spectral range. The Fe I...
### Table 4. Relative abundances and $^{12}$C/$^{13}$C ratios of NGC 752 RGs and their cluster means.

| Species | [X/Fe] | MMU | Optical Spectral Region | IGRINS H & K Spectral Region |
|---------|--------|-----|-------------------------|-------------------------------|
|       |        |     | Na I                   | Na I                          |
|       |        |     | Mg I                   | Mg I                          |
|       |        |     | Al I                   | Al I                          |
|       |        |     | Si I                   | Si I                          |
|       |        |     | Si II                  | Si II                         |
|       |        |     | K I                    | K I                           |
|       |        |     | Ca II                  | Ca II                         |
|       |        |     | Sc II                  | Sc II                         |
|       |        |     | Cr I                   | Cr I                          |
|       |        |     | Ti I                   | Ti I                          |
|       |        |     | V I                    | V I                           |
|       |        |     | Ni II                  | Ni II                         |
|       |        |     | Cu I                   | Cu I                          |
|       |        |     | Zn I                   | Zn I                          |
|       |        |     | Y II                   | Y II                          |
|       |        |     | La II                  | La II                         |
|       |        |     | Ce III                 | Ce III                        |
|       |        |     | Nd II                  | Nd II                         |
|       |        |     | Eu II                  | Eu II                         |
|       |        |     | Gd II                  | Gd II                         |
|       |        |     | Tb II                  | Tb II                         |
|       |        |     | Dy II                  | Dy II                         |
|       |        |     | Ho II                  | Ho II                         |
|       |        |     | Er II                  | Er II                         |
|       |        |     | Tm II                  | Tm II                         |
|       |        |     | Yb II                  | Yb II                         |
|       |        |     | Lu II                  | Lu II                         |
|       |        |     | Optica Spectral Region | IGRINS H & K Spectral Region |

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\frac{\text{Relative abundances and }^{12}\text{C/}^{13}\text{C ratios of NGC 752 RGs and their cluster means.}}{\text{Relative abundances and }^{12}\text{C/}^{13}\text{C ratios of NGC 752 RGs and their cluster means.}}
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**Notes:**
- For other Fe-group elements, we derived abundances from \(\text{Fe I/Fe II}\) and \(\text{Mg I/Fe II}\) and \(\text{Al I/Fe II}\) abundances were derived using the EW method (BT15).
- The optical cluster means are: \(\langle\text{Fe I/Fe II}\rangle = 0.01 (\sigma = 0.07)\) and \(\langle\text{Fe I/Fe II}\rangle_{\text{opt}} = -0.06 (\sigma = 0.04)\). The 0.07 dex difference between the neutral and ionized iron abundances stays, in general, within the uncertainty limits. The cluster mean from the \(\text{Fe I/Fe II}\) line is \(\langle\text{Fe I/Fe II}\rangle_{\text{IR}} = 0.00 (\sigma = 0.06)\), clearly in agreement with the optical values within the mutual uncertainties. The metallicity of NGC 752 from the neutral- and ionized-species Fe and Ti lines, \(\langle\text{M/H}\rangle = -0.07 \pm 0.04\) (BT15), also agrees well with these values and indicates a solar metallicity for NGC 752.
- For other Fe-group elements, we derived abundances from species Sc I, Ti I, Ti II, Cr I, and Ni I. For Sc we used two weak transitions were adopted from Afşar et al. (2018). In Table 3 we list the optical and IR Fe abundances for each RG. Optical Fe I and Fe II abundances were derived using the EW method (BT15). The optical cluster means are: \(\langle\text{Fe I/Fe II}\rangle = 0.01 (\sigma = 0.07)\) and \(\langle\text{Fe II/Fe II}\rangle_{\text{opt}} = -0.06 (\sigma = 0.04)\). The 0.07 dex difference between the neutral and ionized iron abundances stays, in general, within the uncertainty limits. The cluster mean from the \(\text{Fe I/Fe II}\) lines is \(\langle\text{Fe I/Fe II}\rangle_{\text{IR}} = 0.00 (\sigma = 0.06)\), clearly in agreement with the optical values within the mutual uncertainties. The metallicity of NGC 752 from the neutral- and ionized-species Fe and Ti lines, \(\langle\text{M/H}\rangle = -0.07 \pm 0.04\) (BT15), also agrees well with these values and indicates a solar metallicity for NGC 752.

For other Fe-group elements, we derived abundances from species Sc I, Ti I, Ti II, Cr I, and Ni I. For Sc we used two weak
Figure 4. Mean abundances of the species for all NGC 752 program stars plotted as functions of their $T_{\text{eff}}$ values. The panels labeled simply C, N, and O are based on multiple abundance indicators that are discussed in §6.5. Optical and IR abundances are shown with blue crosses and red dots, respectively. In the Sc panel, Sc i (red dots) and Sc ii (blue crosses) represent the measurements from IR and optical, respectively.

$K$ band transitions, taking into account their hyperfine structures. We applied synthetic spectrum analysis to these absorption lines and the difference from the optical is $[\text{Sc ii}/\text{Fe}]_{\text{opt}} - [\text{Sc i}/\text{Fe}]_{\text{IR}} = 0.12$ dex. The difference between two spectral regions resembles the difference between neutral and ionized species of Cr in the optical and Ti both in the optical and IR. We calculated Ti abundances from 10 Ti i lines and the one Ti ii line at 15783 Å. Although optical and IR Ti ii abundances are in agreement, for Ti i the difference is $[\text{Ti i}/\text{Fe}]_{\text{opt}} - [\text{Ti i}/\text{Fe}]_{\text{IR}} = 0.20$ dex. This situation was also discussed in Paper I and Afşar et al. (2018). For 12 RGs of NGC 6940, the difference between the optical and IR Ti ii abundances was 0.16 dex, and for the three RHB stars presented in Afşar et al. (2018) was also 0.16 dex. The $H$-band Ti ii line comes with a CO blend but for the temperature range for our stars its contamination of Ti ii feature is negligible. Further investigation of the IR Ti ii line is needed to better understand the discrepancy between optical and IR Ti ii abundances. The other Fe-group elements have agreement between optical and IR transitions. The mean $[X/Fe]$ abundance from Table 4 for Fe-group elements is $\langle [X/Fe] \rangle_{\text{IR}} = -0.05$ ($\sigma = 0.05$) for six
species and optical mean is \(\langle \alpha / \text{Fe} \rangle_{\text{opt}} = -0.01 (\sigma = 0.14)\) for 11 species including V ii, Cr ii, Mn i, Cu i and Zn i.

6.2 Alpha elements

We derived the abundances of Mg, Si, S and Ca in NGC 752 from their neutral species using the lines in both H and K bands (Table 4). For the IR Si abundance, we made use of about ten transitions in the H and K bands. In the top panels of Figure 5 observed and synthetic spectra of two S lines in the K band and the combined absorption of three closely-spaced S lines in the optical domain for MMU 77 are shown. Sulfur abundances are above solar for both optical and IR spectral regions; \(\langle \text{S} / \text{Fe} \rangle_{\text{opt}} = 0.05 (\sigma = 0.06)\) and \(\langle \text{S} / \text{Fe} \rangle_{\text{IR}} = 0.02 (\sigma = 0.04)\). Optical Ca and Si abundances from BT15 have a small line-to-line scatter about 0.03 dex, but Mg, on the other hand, obtained from the spectrum synthesis of two strong Mg lines at 5528 and 5711 Å resulted in \(-0.20\) dex difference. Mg abundances from the IR region were derived from about ten absorption lines with a mean standard deviation of about 0.08 for 10 RGs, which suggests greater reliability for IR-based Mg abundances. Mean abundances for \(\alpha\) elements \(\langle \alpha / \text{Fe} \rangle = \langle \text{Mg}, \text{Si}, \text{Ca}/\text{Fe} \rangle\), for both optical and IR regions are \(\langle \alpha / \text{Fe} \rangle_{\text{opt}} = 0.10 (\sigma = 0.10)\) and \(\langle \alpha / \text{Fe} \rangle_{\text{IR}} = 0.06 (\sigma = 0.06)\), which are well in agreement and slightly above solar.

6.3 Odd-Z light elements

The odd-Z light elements investigated in this study are Na, Al and rarely-studied P and K. Their abundances are in Table 4. Na abundances were derived from four neutral K band transitions: 22086.4, 22083.7, 23348.4 and 23348.1 Å. To our knowledge, possible non-LTE effects on these transitions have not yet been investigated. The optical and IR Na abundances are both above the solar values. The IR mean for NGC 752 (Table 4) is \(\langle \text{Na} / \text{Fe} \rangle_{\text{IR}} = 0.12 (\sigma = 0.05)\), while the optical mean is \(\langle \text{Na} / \text{Fe} \rangle_{\text{opt}} = 0.20 (\sigma = 0.05)\). Al abundances were obtained from two lines in the H and four lines in the K band. H band abundances are always \(-0.1\) dex lower than the K band abundances and the lower H band abundances are more in accord with the optical ones: \(\langle \text{Al} / \text{Fe} \rangle_{\text{IR}} = 0.02 (\sigma = 0.03)\) and \(\langle \text{Al} / \text{Fe} \rangle_{\text{opt}} = -0.04 (\sigma = 0.04)\).

Phosphorus abundances were determined from two weak H-band transitions at 15711.5 and 16482.9 Å. As illustrated in Figure 6 for MMU 77, the P i lines always have central depths \(\lesssim 5\%\) for the members studied here. The P abundance difference obtained from these two lines is 0.13 dex, but this is an extreme case; for other program stars the agreement is much better, usually \(<0.1\) dex. The mean phosphorus abundance, \(\langle \text{P} / \text{Fe} \rangle_{\text{IR}} = 0.04 (\sigma = 0.09)\) is consistent with the solar value.

As noted in §4 we derived NGC 752 optical-region K abundances, using the very strong K i resonance line at 7698.97 Å (lower right panel of Figure 5). Our derived mean K abundance for the cluster is large, \(\langle \text{K} / \text{Fe} \rangle_{\text{opt}} = 0.50 (\sigma = 0.05)\), but this resonance line is subject to significant non-LTE effects. Takeda et al. (2002) and Mucciarelli et al. (2017) have computed non-LTE corrections between 0.2 and 0.7 dex for disk/halo stars of various \(T_\text{eff} - \log g\) combinations. Afar et al. (2018) found \(-0.6\) dex higher abundances for the 7699 Å line in three RHB stars. Taking into account the non-LTE corrections suggested for 7698.97 Å K i line leads to a conclusion of solar K abundance for our targets. But since this is not based on our own calculations, we have chosen to keep the LTE abundance in Table 4.

Potassium abundances from the IR region were derived from two K i lines at 15163.1 and 15168.4 Å which are affected by CN contamination. We illustrate this with observed/synthetic spectrum comparisons in the lower left panel of Figure 5. Unlike the optical resonance line, the H band K i lines yield approximately solar abundances: \(\langle \text{K} / \text{Fe} \rangle_{\text{IR}} = -0.06 (\sigma = 0.08)\). This consistency suggests that at most very small (or no) non-LTE corrections may be needed for these K i lines. Non-LTE studies of all detectable K i lines in cool stars will be welcome.

Following the detailed description of HF analyses in Pilachowski & Pace (2015), we have also inspected the unblended HF feature located at 23358.3 Å in the K band region. Unfortunately no obvious absorption of fluorine is detectable in our targets.

6.4 \(n\)-capture Elements

In this study we have obtained abundances of three \(n\)-capture elements from their ionized species transitions: Ce and Nd (mostly due to the \(s\)-process in the solar-system), and Yb (mostly from the \(r\)-process). Ce abundances were derived from about four transitions, Nd from one weak transition at 16262.04 Å, and Yb also from one weak line at 16498.4 Å. Yb ii is blended with CO but that contamination is weak enough to be neglected for the temperature/gravity range for our stars. Mean abundances of all three \(n\)-capture ele-
ments are about/above solar, $[(\text{Ce}/\text{Fe})]_{\text{IR}} = 0.11$ ($\sigma = 0.06$), $[(\text{Nd}/\text{Fe})]_{\text{IR}} = 0.24$ ($\sigma = 0.12$) and $[(\text{Yb}/\text{Fe})]_{\text{IR}} = 0.04$ ($\sigma = 0.05$) (Table 4). We have also analyzed optical Ce abundances from 5274.23, 5330.56, 5975.82 and 6043.37 Å transitions. The mean value for the NGC 752 RGs is $[(\text{Ce}/\text{Fe})]_{\text{opt}} = 0.11$ ($\sigma = 0.03$), which is in harmony with the IR abundance with smaller star-to-star scatter. In BT15 we derived the Nd abundances from two lines at 5255.5 and 5319.8 Å; the overabundance is similar what we found from the $H$ band transition. $[(\text{Nd}/\text{Fe})]_{\text{opt}} = 0.22$ ($\sigma = 0.08$). In Paper 1, the RGs of NGC 6940, which were analyzed in the same manner with the NGC 752 RGs in this study, showed slightly over-abundance in $r$-process and more in the $s$-process elements. We observe a similar behavior; the simple mean of the optical and IR La, Ce and Nd abundances is $⟨(s - \text{process}/\text{Fe})⟩ ≃ 0.18$, while mean of Eu and Yb is $⟨(r - \text{process}/\text{Fe})⟩ = 0.10$.

6.5 The CNO Group

The IGRINS spectral range contains many useful OH, CN and CO molecular bands that can be used to obtain CNO abundances. We have followed the same iterative scheme used in Paper 1 to obtain the CNO abundances. We have also determined carbon abundances from its neutral transitions in both optical and IR spectral regions.

There are many OH molecular lines in the $H$ band but most of them are very weak for our temperature and metallicity range and also blended with other lines and/or molecular bands. We were able to use about 10 OH molecular lines located between 15200–17700 Å; the abundances for each star in Table 4 are simple means of the abundances derived from individual OH features. The resulting cluster mean is $⟨(\text{O}/\text{Fe})]_{\text{IR}} = 0.02$ ($\sigma = 0.09$) (Table 4). In BT15, we were able use only the $[\text{O} \, i]$ line at 6300.3 Å to determine optical O abundances, and noted that this feature is plagued with Ni i and CN contamination. The calculated mean for the cluster from this line is $⟨(\text{O}/\text{Fe})]_{\text{opt}} = -0.13$ ($\sigma = 0.02$). Having the advantage of obtaining O abundances from many OH features, we rely more on the O abundance we determine from the IR region.

Carbon abundances were derived from multiple optical and IR species. The summary of the results for each star are given in Table 5. In the IR, we used primarily the CO molecular features in the $K$ band. $^{12}\text{CO}$ first overtones, $Δv = 2$, (2-0) and (3-1) bands at 23400 and 23700 Å. The scatter based on different abundance measurements for a single RG is very small, about $-0.03$ dex. The mean C abundance from the CO molecular lines is $⟨[(\text{C}/\text{Fe})]_{\text{CO}}⟩ = -0.32$ ($\sigma = 0.06$), a value which would be expected after first dredge-up and envelope mixing in metal-rich disk stars. There are second overtone $^{12}\text{CO}$ band heads also in the $H$ band but due to relatively high temperatures of our programme stars they are too weak for detection. In BT15 we obtained the carbon abundances from the CH $G$ band, the Swan band heads of $C_2$ (0-0) at 5155 Å and the (0-1) at 5635 Å (Figure 9 in BT15). These molecular bands are heavily blended with other atomic transitions and the $C_2$ bands are weak in strength, which makes the spectral analysis challenging in these regions. But from those features BT15 derived $⟨[(\text{C}/\text{Fe})]_{\text{CH}, C_2}⟩ = -0.41$ ($\sigma = 0.03$). Considering the analytical difficulties for CH and C$_2$, the $-0.1$ dex difference from the IR CO result indicates reasonable accord.

We obtained the carbon abundances also from the high-excitation $Cl$ lines. Carbon abundances derived from the $C_1$ transitions agree very well with CO results, $⟨[(\text{C}/\text{Fe})]_{\text{IR}}⟩ = -0.34$ ($\sigma = 0.04$). As a further check we also determined the C abundances from synthetic spectrum analyses of three high-excitation $C_1$ lines located in the optical at 5052.1, 5380.3 and 8335.1 Å. The line-to-line C abundance scatter from these transitions is about 0.1 dex, and the mean abundance for the cluster is $⟨[(\text{C}/\text{Fe})]_{\text{opt}}⟩ = -0.22$ ($\sigma = 0.10$), on average only $-0.14$ dex higher compared to the mean C abundance obtained from other features mentioned above.

In Table 5 we have listed the individual and mean carbon abundances. The quoted carbon abundances in this table are the average of the molecular and high-excitation carbon abundances and they are in relatively good agreement; $⟨[(\text{C}/\text{Fe})]_{\text{IR}}⟩ = -0.33$ ($\sigma = 0.04$), $⟨[(\text{C}/\text{Fe})]_{\text{opt}}⟩ = -0.31$ ($\sigma = 0.06$).

We obtained nitrogen abundances from the CN molecular transitions in the $H$-band. We used about 18 CN features between 15000 and 15500 Å, and calculated N abundances. The mean $IR$ N abundance is $⟨[(\text{N}/\text{Fe})]_{\text{IR}}⟩ = 0.44$ ($\sigma = 0.08$). Optical nitrogen abundances were obtained from $^{12}\text{CN}$ and $^{13}\text{CN}$ red system lines in the 7995–8040 Å region in BT15, and the means are in accord with those from the IR, $⟨[(\text{N}/\text{Fe})]_{\text{opt}}⟩ = 0.48$ ($\sigma = 0.02$).

Finally, we measured the $^{12}\text{C}/^{13}\text{C}$ ratios from the first overtone $^{12}\text{CO}$ ($\Delta v = 2$) (2-0) and (3-1) band lines, which are accompanied by the $^{13}\text{CO}$ band heads near 23440 and 23730 Å. These are more robust features for $^{12}\text{C}/^{13}\text{C}$ ratio determination than the standard optical $^{13}\text{CN}$ feature near 8003 Å used by BT15. The top panel of Figure 7 shows that the $^{13}\text{CN}$ triplet, the strongest feature of this band system, is barely detectable in MMU 77 (nor is it much

Table 5. $\log e$ Abundances of Carbon in optical and infrared regions.

| Star   | $C_1$ | CH  | $C_2$ | $C_1$ | CO  | mean | mean |
|--------|-------|-----|-------|-------|-----|------|------|
|        | opt   | opt | opt   | $IR$  | $IR$|      |      |
| MMU 1  | 8.04  | 7.90| 8.03  | 8.10  | 8.11| 7.99 | 8.11 |
| MMU 3  | 8.16  | 7.78| 7.98  | 8.00  | 7.98| 7.97 | 7.99 |
| MMU 11 | 8.20  | 7.98| 8.08  | 8.10  | 8.11| 8.08 | 8.11 |
| MMU 24 | 8.16  | 7.80| 7.95  | 8.00  | 7.97| 7.91 | 8.01 |
| MMU 27 | 8.14  | 7.93| 8.05  | 8.13  | 8.20| 8.04 | 8.17 |
| MMU 77 | 8.05  | 7.88| 8.05  | 8.11  | 8.21| 7.99 | 8.16 |
| MMU 137| 8.21  | 7.83| 8.00  | 8.07  | 8.07| 8.01 | 8.07 |
| MMU 295| 8.18  | 7.95| 8.08  | 8.11  | 8.18| 8.07 | 8.15 |
| MMU 311| 8.14  | 7.90| 8.04  | 8.17  | 8.18| 8.03 | 8.18 |
| MMU 1367| 8.18 | 7.83| 8.03  | 8.06  | 8.03| 8.01 | 8.05 |

Figure 7. Observed and synthetic spectra illustrating the carbon isotopic ratio of NGC 752 MMU 77 in both optical and IR regions. The upper panel is centred on the triplet or $^{13}\text{CN}$ red system (2-0) lines, and the bottom panel shows the $^{13}\text{CO}$ (3-1) R-branch band head region. The blue, red, and green synthesis represent $^{12}\text{C}^{13}\text{C} = 100, 25,$ and 5 in the upper panel, and $^{12}\text{C}/^{13}\text{C} = 100, 30,$ and 10 in the bottom panel, respectively.
stronger in any NGC 752 star). In contrast, the $^{13}$CO features shown in the bottom panel of this figure are much stronger. We compare the optical and IR $^{12}$C/$^{13}$C values in Table 6. They are in reasonable accord, given the extreme weakness of the CN bands.

### 6.6 Abundance Uncertainties

Detailed investigation of the internal and external uncertainty levels of the atmospheric parameters and their effects on the elemental abundances were provided in BT15, in which we calculated an average uncertainty limit of about 150 K by comparing spectroscopically derived $T_{\text{eff}}$ values with the literature, photometric and LDR temperatures. In Table 8 of BT15 we list the sensitivity of derived abundances to the model atmosphere changes within uncertainty limits for the star MMU 77. Additional investigation of LDR temperatures from the IR data has shown that our temperature uncertainty limit has remained almost the same as determined in BT15, considering the highest LDR and spectral temperature difference of 154 K for MMU 1367 (see §5). Therefore, in Table 7, we present the sensitivity of derived abundances in the elements only newly studied in this work adopting the same atmospheric parameter uncertainties in BT15. The uncertainties were determined using the IR spectra of some additional stars, MMU 77, as applied in BT15. In general, abundance changes are mostly well within 1σ level of the [X/Fe] values (Table 4). However, the sensitivity level of Sc abundance to the change in temperature stands out. The temperature sensitivity of some IR Sc lines has been previously noticed by Thorsbro et al. (2018), based on Sc i lines identified in K band of cool M giants observed with NIRSPEC/Keck II. They reported up to 0.2 dex uncertainties in Sc abundances mostly originated from the temperature sensitivity for stars $T_{\text{eff}} < 3800$ K. Although our stars have higher temperatures and the Sc i lines we used are different than those that Thorsbro et al. discussed, caution should be taken in interpreting the IR Sc abundances for our stars until the underlying physical process for the temperature sensitivity of Sc lines are better understood.

### 6.7 Comparison with NGC 6940

We have now derived metallicities and relative abundance ratios for OCs NGC 6940 and NGC 752 with high resolution spectra in the optical spectral region (BT15, BT16) and infrared (BT19, this study). The NGC 6940 optical data were obtained with the Hobby-Eberly Telescope and its high-resolution echelle spectrometer (Tull 1998), and those for NGC 752 with the 2.7m Smith Telescope and Tull echelle spectrometer (Tull et al. 1995); both data sets have high

### Table 6. Carbon isotopic ratios of optical and infrared regions.

| Stars | $^{13}$CN (8004 Å) | $^{13}$CO (23440 Å) | $^{13}$CO (23730 Å) |
|-------|-------------------|-------------------|-------------------|
| MMU 1 | 25                | 25                | 30                |
| MMU 3 | 25                | 25                | 30                |
| MMU 11| 25                | 25                | 30                |
| MMU 24| 13                | 20                | 20                |
| MMU 27| 17                | 19                | 25                |
| MMU 77| 25                | 20                | 30                |
| MMU 137| 15              | 20                | 20                |
| MMU 295| 20               | 25                | 25                |
| MMU 311| 15              | 23                | 22                |
| MMU 1367| 17             | 15                | 16                |

| Species | $\Delta T_{\text{eff}}$ (K) | $\Delta \log g$ | $\Delta V$ (km s$^{-1}$) |
|---------|----------------------|----------------|------------------|
| P1      | $-0.01 / +0.01$      | $0.09 / -0.09$ | $0.01 / 0.00$    |
| Si      | $-0.06 / 0.09$       | $0.09 / -0.05$ | $0.03 / 0.03$    |
| K i     | $0.10 / -0.11$       | $0.02 / -0.01$ | $0.01 / 0.05$    |
| Sc i    | $0.18 / -0.18$       | $0.01 / 0.01$  | $0.01 / 0.00$    |
| C e     | $0.08 / -0.06$       | $0.12 / -0.11$ | $0.03 / 0.03$    |
| Yb i    | $0.05 / -0.05$       | $0.10 / -0.10$ | $0.02 / 0.03$    |

### Figure 8. Differences between relative abundances [X/Fe] in NGC752 and NGC6940 in the optical and IR spectral regions. The dotted line at $\Delta$[X/Fe] = 0.00 indicates equality between [X/Fe] values in the two clusters. The dashed line at $\Delta$[X/Fe] = 0.04 represents the mean value for all abundances, excluding the aberrant values for K i in the IR and Cu i in the optical spectral region.

Our derived metallicities for the two clusters suggest that NGC 752 is slightly more metal-rich than NGC 6940. Defining $\Delta$[X/Fe] $\equiv$ [X/Fe]$_{\text{NGC752}}$ − [X/Fe]$_{\text{NGC6940}}$, from optical data $\Delta$[Fe/Fe]$_{\text{opt}}$ = +0.05 and $\Delta$[Fe/Fe]$_{\text{opt}}$ = +0.08, but these differences are well within the observational/analytical uncertainties. The IR metallicities are essentially identical: $\Delta$[Fe/Fe]$_{\text{IR}}$ = +0.02. We conclude, in agreement with past studies, that both NGC 6940 and NGC 752 have solar metallicities.

The general accord between the two clusters extends to the abundance ratios of individual elements. In Figure 8 we show abundance differences for all species studied in the optical and IR regions. The uncertainties shown in the figure are approximate, being averages of the $\sigma$ values of the abundances in each cluster. Excluding the aberrant points for optical Cu i and IR K i, we derive $\Delta$[Fe/Fe] = +0.045 (+0.06 in the optical, +0.03 in the IR). The Cu difference is not well determined, as the NGC 752 optical spectra permitted use of only one Cu i feature. At present we lack an explanation for the 0.2 dex abundance difference between the IR-based K i lines in NGC 6940 and NGC 752. This issue will be considered again in our future studies of M67 and other OCs. In Ta-
In Paper I stellar evolutionary models with a solar abundance set were fitted to the CMD of NGC6940, yielding an age of 1.15 Gyr. To well within the uncertainties, the metallicity that we have determined for NGC752 is the same as that of NGC6940. Both clusters appear to have the same helium abundance as well, given that (as discussed below) models for Y = 0.270 provide equally good fits to the luminosities of the core He-burning red clump (RC) stars if (a) distance moduli based on Gaia parallaxes are adopted, and (b) the observed RGs in the clusters are mostly in the RC evolutionary stage. Inspection of the CMD for NGC 6940 (Figure 1 of Paper I) suggests that most RGs in that cluster are not associated with the RGB evolutionary tracks, and thus truly are RC stars. For NGC 752 we discuss this issue below, but for the moment simply assume that our program stars are mostly RCs. Then to derive our best estimate of the age of NGC 752, it is simply a matter of interpolating in the same model grids that were used in Paper I to identify which isochrone provides the best fit to the cluster turnoff stars.2

However, this process involves the cluster reddening, for which most estimates fall in the range 0.03 ≤ E(B − V) ≤ 0.05 (Daniel et al. 1994; Taylor 2007; Schlafly & Finkbeiner 2011; Twarog et al. 2015), and the adopted color–T eff relations (from Casagrande & VandenBerg 2018; hereafter CV18). Fortunately, the Sun provides a valuable constraint on both the predicted T eff and color scales. According to CV18, their determinations of M G⊙ = 4.67 and (G − G R P⊙) = 0.49 from reference solar spectra are accurate to within ± 0.01 mag. Encouragingly, the bolometric corrections (BCs) derived from the MARCS library of theoretical stellar fluxes (Gustafsson et al. 2008), yield the same value of M G on the assumption of [Fe/H] = 0.0 and the solar values of T eff and log g, but a bluer G − G R P color by ≈ 0.01 mag. We have therefore applied a +0.01 mag zero-point correction to the synthetic colors in order that our solar model reproduces the “observed” G − G R P color of the Sun.

In Figure 9 we show the best model fits to the observed (G − G R P, G) color-magnitude diagram for NGC 752. The upper MS and turnoff stars are fit quite well by a 1.52 Gyr isochrone for solar abundances if the the adopted reddening is E(B − V) = 0.035 mag. The isochrone begins to deviate slightly to the blue of the observed MS at G ∼ 14, with the offset in color at a given magnitude rising to as much as 0.1 mag at G ∼ 18. Inadequacies in the CV18 color–T eff relations for cooler stars are likely responsible for this problem given that the transmission function of the G filter extends well into the ultraviolet.

The tables of BCs presented by CV18 take into account the dependence of the extinction on spectral type in a fully consistent way; i.e., these transformations enable one to convert predicted luminosities and temperatures directly to absolute G and G R P magnitudes that have been suitably corrected for an assumed reddening. In order for the resultant models to appear on the observed (G − G R P, G) CMD, they must then be shifted in the vertical direction by an amount corresponding to the true distance modulus, (m − M)⊙. In Figure 9 the solar symbol indicates where the Sun would be located if it was as distant as NGC 752 and subject to the same reddening. The red filled circle represents a model at an age of 1.52 Gyr along an evolutionary track that has been calculated for a Standard Solar Model, and similarly adjusted by the adopted reddening and

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2 See Paper I for a fairly detailed description of the evolutionary codes and stellar models that are used in the present series of papers — including, in particular, a discussion of the treatment of convective core overshooting during the main-sequence (MS) phase.

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### Table 8. Abundance differences.

| Species | N6940-N752 | Lum - us | Carrera-us | Reddy-us |
|---------|------------|---------|------------|----------|
|         | Optical Spectral Region |         |            |          |
| C       | 0.07       | 0.09    | 0.04       | 0.02     |
| N       | 0.02       | 0.20    | 0.04       | 0.02     |
| O       | 0.04       | 0.13    | 0.00       | 0.02     |
| Na i    | 0.07       | 0.23    | 0.08       | 0.08     |
| Mg i    | 0.08       | 0.03    | 0.00       | 0.00     |
| Al i    | 0.00       | 0.02    | 0.19       | 0.19     |
| Si i    | 0.00       | 0.11    | 0.09       | 0.11     |
| S i     | 0.11       | 0.01    | 0.11       | 0.11     |
| Ca ii   | 0.00       | 0.04    | 0.09       | 0.09     |
| Sc ii   | 0.00       | 0.02    | 0.12       | 0.12     |
| Ti ii   | 0.03       | 0.15    | 0.00       | 0.00     |
| V i     | 0.01       | 0.08    | 0.08       | 0.08     |
| Cr i    | 0.01       | 0.00    | 0.00       | 0.00     |
| Cr ii   | 0.04       | 0.22    | 0.04       | 0.22     |
| Mn ii   | 0.12       | 0.07    | 0.16       | 0.16     |
| Co i    | 0.05       | 0.04    | 0.04       | 0.04     |
| Ni i    | 0.00       | 0.09    | 0.09       | 0.09     |
| Cu i    | 0.28       | 0.16    | 0.16       | 0.16     |
| Zn ii   | 0.07       | 0.12    | 0.12       | 0.12     |
| Y ii    | 0.11       | 0.04    | 0.04       | 0.04     |
| La ii   | 0.00       | 0.09    | 0.09       | 0.09     |
| Ce ii   | 0.00       | 0.04    | 0.04       | 0.04     |
| Eu ii   | 0.04       | 0.16    | 0.16       | 0.16     |
| average | 0.04       | 0.09    | 0.09       | 0.09     |
| sigma   | 0.07       | 0.15    | 0.14       | 0.10     |

| Species | IGRINS H & K Spectral Region |         |            |          |
|---------|--------------------------------|---------|------------|----------|
| C       | 0.05                           | 0.11    | 0.11       | 0.11     |
| N       | 0.08                           | 0.16    | 0.16       | 0.16     |
| O       | 0.03                           | 0.17    | 0.17       | 0.17     |
| Na i    | 0.16                           | 0.15    | 0.15       | 0.15     |
| Mg i    | 0.03                           | 0.03    | 0.03       | 0.03     |
| Al i    | 0.05                           | 0.08    | 0.08       | 0.08     |
| Si i    | 0.04                           | 0.17    | 0.17       | 0.17     |
| P i     | 0.06                           | 0.10    | 0.10       | 0.10     |
| S i     | 0.05                           | 0.09    | 0.09       | 0.09     |
| K i     | 0.23                           | 0.12    | 0.12       | 0.12     |
| Ca i    | 0.05                           | 0.12    | 0.12       | 0.12     |
| Sc i    | 0.04                           | 0.11    | 0.11       | 0.11     |
| Ti i    | 0.07                           | 0.12    | 0.12       | 0.12     |
| Ti ii   | 0.09                           | 0.10    | 0.10       | 0.10     |
| Cr i    | 0.06                           | 0.07    | 0.07       | 0.07     |
| Co i    | 0.01                           | 0.04    | 0.04       | 0.04     |
| Ni i    | 0.05                           | 0.04    | 0.04       | 0.04     |
| Cu i    | 0.13                           | 0.02    | 0.02       | 0.02     |
| Nd ii   | 0.10                           | 0.18    | 0.18       | 0.18     |
| Yb ii   | 0.03                           | 0.07    | 0.07       | 0.07     |
| average | 0.07                           | 0.08    | 0.08       | 0.08     |
| sigma   | 0.06                           | 0.13    | 0.13       | 0.13     |
isochrone (solid black curve; dotted and dashed curves show 10% reddening of NGC 752 must be quite close to distance modulus. Thus, in order to satisfy the solar constraint, the tracks computed for \( t \) et al. (2018) obtained an age of \( t \) Gyr isochrones). The blue and red curves are the MESA evolutionary tracks computed for \( t \) = 1.82 \( M_\odot \), [Fe/H] = 0, \( Y = 0.27 \), assuming that \( f_{ov} = 0.035 \) in Equation 1 from Paper I. The red track has the mixing length increased by 10% compared to the solar-calibrated value of \( \alpha_{\text{MLT}} = 2 \) adopted for the blue track. See the text for descriptions of the solar symbol and the red dot.

Figure 9. The CMD of NGC 752 (black dots) and its fit with a new 1.52 Gyr isochrone (solid black curve; dotted and dashed curves show 1.62 Gyr and 1.42 Gyr isochrones). The blue and red curves are the MESA evolutionary tracks computed for \( t \) = 1.82 \( M_\odot \), [Fe/H] = 0, \( Y = 0.27 \), assuming that \( f_{ov} = 0.035 \) in Equation 1 from Paper I. The red track has the mixing length increased by 10% compared to the solar-calibrated value of \( \alpha_{\text{MLT}} = 2 \) adopted for the blue track. See the text for descriptions of the solar symbol and the red dot.

distance modulus. Thus, in order to satisfy the solar constraint, the reddening of NGC 752 must be quite close to \( E(B-V) = 0.035 \) mag if it has \( (m-M)_0 = 8.26 \). The inferred reddening would be larger than this if the cluster is less distant, and vice versa.

Ages in the range of 1.7−2.0 Gyr were typically found for NGC 752 in the mid-1990s (e.g. Daniel et al. 1994; Dinescu et al. 1995), but subsequent determinations have generally favored ages closer to 1.5 Gyr (e.g. Anthony-Twarog & Twarog 2006; Twarog et al. 2015). The earlier age determinations are especially uncertain because the stellar models used in those studies assumed little or no overshooting from convective cores during the MS phase. Because such isochrones are incapable of reproducing the observed turnoff morphology, the ages derived from them are highly questionable.

In contrast, later investigations employed models that allowed for significant amounts of core overshooting, resulting in fits to the NGC 752 CMD that are quite similar to that shown in Fig. 9, and they yield ages that have little ambiguity.

Recently, a similar color-magnitude diagram study by Agüeros et al. (2018) obtained an age of 1.34 ± 0.06 Gyr for NGC 752, which is inconsistent with our determination by more than 2 \( \sigma \). Although those researchers used a sophisticated Bayesian approach to determine the cluster parameters (including such observational properties as the distance, metallicity, and extinction) from fits of isochrones to the photometric data, their results will be subject to systematic errors that are very difficult to quantify. In particular, the predicted \( T_{\text{eff}} \) scale is quite sensitive to, e.g., the adopted atmospheric boundary condition and the treatment of super-adiabatic convection. Errors in the adopted color transformations can further impact how well stellar models are able to reproduce an observed CMD. Consequently, one cannot rely on such isochrone predictions as the location of the giant branch relative to the turnoff to provide a useful constraint on absolute cluster ages (see, e.g., Vandenberg et al. (1990), who show that this diagnostic may be used to obtain accurate relative ages of star clusters.) We suspect that the derivation by Agüeros et al. of \( A_V = 0.198 \pm 0.0085 \) mag, which is appreciably higher than most determinations, including the line-of-sight Galactic extinction (Schlafly & Finkbeiner 2011), can be attributed, in part, to errors in the model \( T_{\text{eff}} \) and/or color scales.

Isochrones appropriate to young and intermediate-age clusters are also very dependent on how much overshooting from convective cores during the MS phase is assumed. In fact, the MESA models (Choi et al. 2016) that were used by Agüeros et al. assume a value of the overshooting parameter that is, according to our analysis (see Paper I and the next section) too low by about a factor of two. This appears to be the main reason (see below) why they obtained a significantly younger age than our determination. Unfortunately, Agüeros et al. do not include a figure that compares their best-fit isochrone with the CMD of NGC 752; hence it is not possible to make a visual assessment of how well the data are fitted. Our age determination should be particularly robust because we have used the Sun to calibrate the predicted \( T_{\text{eff}} \) and color scales, and have adopted the \( \text{Gaia} \) distance and a spectroscopically derived metallicity, from which we have deduced the \( E(B-V) \) ≈ 0.035 in order to achieve consistency with the solar constraint. Thus, nearly all of the cluster parameters are derived independently of our stellar models and the age is effectively obtained from an overlay of the isochrone that provides the best fit to the turnoff stars.

8 STELLAR EVOLUTION MODELING OF NGC 752

In Paper I we emphasized the importance of calibrating the efficiency of convective overshooting beyond the Schwarzschild boundary of the hydrogen convective core in MS stars with \( 1 \leq M/\text{M}_\odot \leq 2 \).

In the Victoria stellar evolution code employed here to generate isochrones, the convective overshooting is estimated using the integral equations of Roxburgh (1989) as described in VandenBerg et al. (2006). In particular, Figure 1 in the latter paper shows the variation of the free parameter \( F_{\text{over}} \) in Roxburgh’s equations calibrated by comparing the predicted and observed CMDs for a number of open clusters with different ages. This parameter starts to increase from \( F_{\text{over}} = 0 \) at \( M = 1.14 \text{M}_\odot \), reaches a maximum value of \( F_{\text{over}} = 0.55 \) at \( M = 1.7 \text{M}_\odot \), and then remains constant. In the MESA code, that we use to model the evolution of MS turn-off (MSTO) stars up to the red-clump (RC) phase, the convective overshooting is approximated by a diffusion coefficient that is exponentially decreasing outside the convective boundary on a lengthscale of 0.5 \( f_{ov} \overline{H}_P \), where \( H_P \) is a local pressure scale height. In Paper I we showed that the MESA code with \( f_{ov} = 0.035 \) produces a stellar evolution track for an initial mass \( M = 2 \text{M}_\odot \) that is approximately equal to the MSTO mass of stars in the open cluster NGC 6940, in the excellent agreement with the Victoria 1.15 Gyr isochrone generated for this cluster.

The MSTO mass for the estimated 1.52 Gyr age of NGC 752 is \( M \approx 1.8 \text{M}_\odot \). This mass is high enough that the maximum value of \( F_{\text{over}} = 0.55 \) should still be used according to Figure 1 in VandenBerg et al. (2006). Therefore we have used the same value of \( f_{ov} = 0.035 \) in the MESA code to model the evolution of MSTO stars in NGC 752. Figure 9 demonstrates that in this case the evolu-
Figure 10. Upper panel: the MESA evolutionary tracks of the solar-metallicity $1.82 \, M_\odot$ model with the convective overshooting parameter $f_{\text{ov}} = 0.035$ (red) and of the $1.85 \, M_\odot$ model with $f_{\text{ov}} = 0.016$ (green) that both fit the luminosity of the stars leaving the MS in NGC752. Lower panel: the RGB and RC evolutionary timescales of these models.

increase the initial mass to $1.85 \, M_\odot$ to keep the same MSTO luminosity, the morphology of their corresponding evolutionary track becomes inconsistent with the observed CMD of NGC752 (green curves in Figure 10). There are multiple problems: (a) the effective temperature at the end of the core H-burning phase is too high; (b) the track produced by core He-burning is too narrow in color and it does not reach the minimum luminosity of the observed RC stars (top panel); and (c) the RGB and RC evolutionary timescales are now comparable (bottom panel), which would lead us to expect comparable numbers of RGB and RC stars in NGC752, which is not observed. The last inconsistency arises because the reduced efficiency of convective H-core overshooting leads to an extended RGB evolution with the He core becoming electron degenerate and experiencing a flash at the end, while in the models with $f_{\text{ov}} = 0.035$ the He core remains non-degenerate, and He in the core is ignited quiescently.

Applying these evolutionary computations to C and N abundances, in the upper panel of Figure 11 we compare the predicted and observed $[\text{C/Fe}]$ abundance ratios for the RC stars in NGC752. The observed C abundances can be reproduced by our models only if we assume that they were already slightly reduced initially by $\approx 0.1$ dex, compared to the solar-scaled $[\text{C/Fe}]$ ratio (the dashed black curve), because without this assumption the predicted RC abundance is $[\text{C/Fe}] = -0.19$, while our mean observed value is $[\text{C/Fe}] = -0.31 \pm 0.06$. Lum & Boesgaard (2019) support a slightly subsolar initial C abundance in NGC752, deriving $[\text{C/Fe}] = -0.10$ ($\sigma = 0.13, 21$ stars). However, the Lum & Boesgaard red giant abundance, $[\text{C/Fe}] = -0.22$ ($\sigma = 0.08, 6$ stars) is consistent with our predictions with solar or slightly subsolar initial C abundances.

In the lower panel of Figure 11 we make the same kind of comparison for N. The observed optical and IR values, $[\text{N/Fe}] = 0.48 \pm 0.02$ and $0.44 \pm 0.08$, respectively, are slightly larger than the predictions, $[\text{N/Fe}] = 0.41$ (red curve) and 0.37 (black dashed curve). The initial N abundance has been assumed to be solar. However, Lum & Boesgaard (2019) derives $[\text{N/Fe}] = 0.12$ (no stated $\sigma$), and that would raise the predicted red giant N abundance to be nearly comparable to the observed one.

Finally, for carbon isotopic ratios our model predicts $^{12}\text{C}/^{13}\text{C} = 22.2$ and 20.9 for the red and black dashed tracks. They are comparable with the mean C isotopic ratios measured in the RC stars in NGC752: $^{12}\text{C}/^{13}\text{C} = 22$ (optical) and 16 (IR). Note that our predicted C and N abundances are in a good agreement with those obtained for a non-rotating $1.8 \, M_\odot$ star by Charbonnel & Lagarde (2010) ($^{12}\text{C}/^{13}\text{C} = 19.9, [\text{C/Fe}] = -0.18, [\text{N/Fe}] = 0.37$), who did not consider any convective overshooting, but did include thermal-haline mixing on the RGB. In our models, the enhanced convective overshooting significantly decreases the RGB evolution time (the lower panel in Figure 10), therefore if we included thermal-haline mixing on the RGB its effect on the surface abundances of C and N would be even less pronounced and our assumption on the reduced initial abundance of C would still be required. According to Charbonnel & Lagarde (2010), rotation with a ZAMS velocity of $110 \, \text{km} \, \text{s}^{-1}$ only slightly changes these abundances to $^{12}\text{C}/^{13}\text{C} = 15.2, [\text{C/Fe}] = -0.19$ and $[\text{N/Fe}] = 0.31$.

3 The Lum & Boesgaard (2019) red giant abundance mean is $[\text{N/Fe}] = 0.28$ ($\sigma = 0.07$), somewhat lower than our predicted and observed N values.
Figure 11. The predicted changes of the surface C (top panel) and N (bottom panel) abundances for the solar-metallicity 1.82 $M_\odot$ stellar evolutionary tracks computed with $f_\text{MD} = 0.035$ and $\delta M_\text{MT} = 2.2$ are compared with the C and N abundances determined for the red-clump stars in NGC 752 using optical (black circles) and infrared (magenta squares) spectra. The red and black dashed curves are obtained assuming that the initial C abundance is $[C/Fe] = 0$ and $[C/Fe] = -0.1$, respectively.

9 SUMMARY

This is the second of three papers that report analyses of high-resolution optical and IR spectra RG members of prominent OCs. In this study, we have performed the detailed chemical abundance analysis for 10 RGs in the NGC 752 open cluster using the high-resolution near-IR $H$ and $K$ band spectral data obtained with the IGRINS spectrograph. BT15 investigated the same RG members in the optical region from their high-resolution optical spectra, and here we combine data from both regions and explore the NGC 752 from a more complete wavelength window.

We revisited the CMD of NGC 752, investigating the membership assignments using Gaia DR2 (Gaia Collaboration et al. 2018). We applied a Gaussian mixture model to set the parallax bounds, leading to an estimated cluster distance of 448 pc with a true distance modulus of $(m - M)_0 = 8.26$. We also remeasured the radial velocities of our targets from the $H$ and $K$ band spectra, finding a cluster mean of $4.97 \pm 0.24$ km s$^{-1}$ (Table 1), which is in general agreement with both our previous optical RV (Paper 1) and Gaia.

We applied LDR relations reported by Fukue et al. (2015) and estimated the IR-LDR effective temperatures for our targets. LDR temperatures obtained from both optical and IR line depth ratios are in good agreement with the spectral temperatures within $\sim 150$ K, indicating that this method provides reliable temperature estimations in the cases of lack of information from the optical region. This encouraging result paves the way for dust-obscured open cluster chemical composition studies.

Adopting the model atmospheric parameters from Paper 1, we performed detailed abundance analysis for 20 elements in the $H$ and $K$ band spectral regions of our targets. The abundances for 18 of these elements were determined both in the optical and IR regions. In general, we derived the abundances of H-burning, $\alpha$, light odd-Z, Fe-group, and $n$-capture elements, and also determined $^{12}$C/$^{13}$C ratios from both regions. In general, they are in accord with their optical counterparts and have abundances similar to their solar-system values. IR abundances of CNO and some $\alpha$ elements (such as Mg and S) were found to be more reliable compare to their optical counterparts due to more number of lines and regions used during the IR spectral analysis.

In some cases, small abundance differences were seen between neutral and ionized species of the same element. In particular there is only one Ti II line present in the $H$ band and, compare to its optical counterparts, it seems to yield sub-solar abundances for our stars in general. Further investigation is needed to better understand this issue. For Sc abundances, Sc II and Sc I lines were used in the optical and IR regions, respectively. Their mean Sc cluster abundance differs by 0.12 dex. But only two weak Sc I lines in the $K$ band with hyperfine structures were used to determine the IR Sc abundances, so we do not regard this as a significant discrepancy.

To the best of our knowledge, P, S and K abundances have been derived here for the first time for our targets, and all are consistent with solar abundances in NGC 752. Potassium abundances obtained from two lines in the $H$ band indicate that these lines are likely to be less affected by non-LTE line formation problems than is the strong K i 7698.7 Å resonance line. A similar suggestion could be made for the IR Na lines. They provide Na abundances $\sim 0.1$ dex lower than the optical ones, which might indicate that they are also less affected by non-LTE conditions.

Five $n$-capture elements were identified in the spectra of NGC 752 RGs. The $n$-capture abundances in our stars resulted in somewhat overabundances both in the $s$-process and $r$-process elements, later being less slightly overabundant. Encouragingly, the abundances of Ce and Nd, show agreement between their optical and IR values. Detection of a Yb II line at 16498.4 Å in the $H$ band provides a unique opportunity to study this element, since the strong resonance Yb II 3694 Å line occurs in a very crowded low-flux region of cool stars, essentially useless for abundance studies in most solar-metallicity stars.

Analyzing CNO abundances using the many available IR CO, CN, and OH molecular features, we found cluster mean abundances from optical and IR regions to be in reasonable agreement. We suggest that IR O abundances may provide more robust O abundances than does the [O i] 6300.3 Å optical line. Our study multiple $^{12}$CO and $^{13}$CO first overtone band lines yields a similar endorsement: these regions provide more robust measurements of $^{12}$C/$^{13}$C values than ones possible from the weak CN optical features near 8004 Å. Our CNO results indicate that all NGC 752 RC stars have abundances consistent with those predicted from first dredge-up predictions, e.g., Charbonnel & Lagarde (2010).

We used the NGC 752 CMD to investigate the evolutionary states of 10 RG members, first concluding that they are at least mostly red clump members. The best evolutionary model for solar metallicity yielded core He-burning RC stars consistent with our stars if a helium abundance $Y = 0.270$ is adopted. Isochrones fitted to the cluster turnoff yield an age of 1.52 Gyr for the reddening $E(B-V) = 0.035$ mag and the turnoff mass $M = 1.82 M_\odot$ of NGC 752. Our light element abundance values, $\langle [C/Fe] \rangle = -0.32$, $\langle [N/Fe] \rangle = +0.46$, $\langle [C/Fe] \rangle = -0.05$, and $\langle^{12}$C$/^{13}$C$\rangle = 22$, are in reasonable accord with those predicted by our MESA evolutionary models.
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10 APPENDIX

The Gaussian mixture model is of the form:

$$\Phi(\mu_x, \mu_y, \epsilon_x, \epsilon_y) = \phi_c + \phi_f$$

where

$$\phi_c = \frac{1 - N_f}{2\pi \sigma_{x,c} \sigma_{y,c}} \left[ \frac{1}{\sqrt{\sigma_{x,c}^2 + \epsilon_x^2 + \epsilon_y^2}} \exp\left[ -\frac{\alpha}{2(1 - \rho_c^2)} \right] \right]$$

$$\phi_f = \frac{N_f}{2\pi \sigma_{x,f} \sigma_{y,f}} \left[ \frac{1}{\sqrt{\sigma_{x,f}^2 + \epsilon_x^2 + \epsilon_y^2}} \exp\left[ -\frac{\beta}{2(1 - \rho_f^2)} \right] \right]$$

and where

$$\alpha = \frac{(\mu_x - \mu_{x,c})^2}{\sigma_{x,c}^2 + \epsilon_x^2} + \frac{(\mu_y - \mu_{y,c})^2}{\sigma_{y,c}^2 + \epsilon_y^2} + \frac{(\mu_x - \mu_{x,y})^2}{\sigma_{x,y}^2 + \epsilon_x^2}$$

$$\beta = \frac{(\mu_x - \mu_{x,f})^2}{\sigma_{x,f}^2 + \epsilon_x^2} + \frac{(\mu_y - \mu_{y,f})^2}{\sigma_{y,f}^2 + \epsilon_y^2} + \frac{(\mu_x - \mu_{x,f})^2}{\sigma_{x,f}^2 + \epsilon_x^2}$$

The notation for the Gaia DR2 proper motion data and the 11 model parameters is as follows:

$$\mu_{x_i}, \mu_{y_i} = \text{proper motion components for } i^{th} \text{ star}$$

$$\epsilon_{x_i}, \epsilon_{y_i} = \text{proper motion component errors for } i^{th} \text{ star}$$

$$N_f = \text{field scaling parameter}$$

$$\mu_{c,x}, \mu_{c,y} = \text{cluster center}$$

$$\mu_{f,x}, \mu_{f,y} = \text{field center}$$

$$\sigma_{c,x}, \sigma_{c,y} = \text{cluster std. deviations}$$

$$\sigma_{f,x}, \sigma_{f,y} = \text{field std. deviations}$$

$$\rho_c, \rho_f = \text{cluster and field correlation coefficients}$$

While it is common to use ordinary maximum likelihood estimation to determine the parameters defining mixture models, as Sanders (1971) did, we used an expectation-maximization (EM) machine-learning algorithm for finite mixtures as derived by Dempster et al. (1977). We found that convergence of the model parameters using EM was more reliable than when applying MLE to our model. Central to the EM algorithm, the expectation of our complete-data log-likelihood function is of the form

$$Q = \sum_{i=1}^{NST} T_{ci} \log(\phi_{ci}) + T_{fi} \log(\phi_{fi})$$

where \(NST\) is the number of total stars in our data set, and \(\phi_{ci}\) and \(\phi_{fi}\) are simply \(\phi_c\) and \(\phi_f\) evaluated at the \(i^{th}\) star using the current parameter guesses. \(T_{ci}\) and \(T_{fi}\) are the conditional probabilities that the \(i^{th}\) star belongs to the cluster or field distribution, respectively. They are calculated with \(T_{ci} = \phi_{ci} / (\phi_{ci} + \phi_{fi})\) and \(T_{fi} = \phi_{fi} / (\phi_{ci} + \phi_{fi})\). In our EM algorithm, \(9\) was maximized with respect to each of the 11 parameters numerically and the probabilities, which feed into it, were in turn updated. This process was iterated until convergence of the parameters. While the conditional probabilities \(T_{ci}\) and \(T_{fi}\) changed during the process of running the EM algorithm, the final \(T_{ci}\) probabilities after parameter convergence were the probabilities that we used for cluster membership determination.

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