Log-periodic self-similarity: an emerging financial law?

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Abstract

A hypothesis that the financial log-periodicity, cascading self-similarity through various time scales, carries signatures of a law is pursued. It is shown that the most significant historical financial events can be classified amazingly well using a single and unique value of the preferred scaling factor $\lambda = 2$, which indicates that its real value should be close to this number. This applies even to a declining decelerating log-periodic phase. Crucial in this connection is identification of a “super-bubble” (bubble on bubble) phenomenon. Identifying a potential “universal” preferred scaling factor, as undertaken here, may significantly improve the predictive power of the corresponding methodology. Several more specific related results include evidence that:

(i) the real end of the high technology bubble on the stock market started (with a decelerating log-periodic draw down) in the beginning of September 2000;

(ii) a parallel 2000-2002 decline seen in the Standard & Poor’s 500 from the log-periodic perspective is already of the same significance as the one of the early 1930s and of the late 1970s;

(iii) all this points to a much more serious global crash in around 2025, of course from a level much higher (at least one order of magnitude) than in 2000.

Key words: Complex systems, financial markets, fundamental laws of Nature

PACS: 89.20.-a, 89.65.Gh, 89.75.-k

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1 Introduction

The suggestion that financial dynamics may be governed by phenomena analogous to criticality in the statistical physics sense and, especially, the related subtle concept of log-periodicity [1–4] proves exciting and at the same time somewhat controversial [5–7]. In its conventional form criticality implies a scale invariance which, for a properly defined function $F(x)$ characterizing the system, means that

$$F(\lambda x) = \gamma F(x). \quad (1)$$

A positive constant $\gamma$ in this equation describes how the properties of the system change when it is rescaled by the factor $\lambda$. One obvious solution to this equation is:

$$F_0(x) = x^\alpha, \quad (2)$$

where $\alpha = \ln(\gamma)/\ln(\lambda)$. It represents a standard power-law that is characteristic of continuous scale-invariance and $\alpha$ is the corresponding critical exponent.

The zig-zag character of financial dynamics attracts attention to the general solution [8] of Eq. (1):

$$F(x) = x^\alpha P(\ln(x)/\ln(\lambda)). \quad (3)$$

$P$ denotes a periodic function of period one. The dominating scaling (2) thus acquires a correction that is periodic in $\ln(x)$. This solution can be interpreted in terms of discrete scale-invariance [9] and a complex critical exponent [10]. A functional form of $P$ is not determined at this level. It only demands that if

$$x = |T - T_c|, \quad (4)$$

where $T$ denotes the ordinary time labeling the original price time series, represents a distance to the critical point $T_c$, the resulting spacings between the corresponding consecutive repeatable structures at $x_n$ (i.e., minima or maxima) of the log-periodic oscillations seen in the linear scale follow a geometric contraction according to the relation

$$\frac{x_{n+1} - x_n}{x_{n+2} - x_{n+1}} = \lambda. \quad (5)$$
The critical points coincide with the accumulation of such oscillations and, in the context of the financial dynamics, it is this effect that potentially can be used for prediction provided $\lambda$ is really well defined and constant. Our previous contribution [4] provides two related elements that turn out to be essential for a proper interpretation and handling of the financial patterns. One is the suggestion that consistency of the theory requires that, if applicable, the log-periodic scenario is to manifest its action self-similarly through various time scales. Imprints of such effects have also been found [4] in the real stock markets and further confirmed in Ref. [13]. Second is identification [4] that $\lambda \approx 2$ is the most appropriate preferred scaling factor through various time scales, in amazing consistency with those found for a whole variety of other complex systems [9–12]. Below we present an attempt to classify all the significant historical events on the world’s leading stock market, including the 2000–2002 declining and log-periodically decelerating phase, within such a scheme.

2 Log-periodic S&P500 in 1970 - 2002

The above period includes essentially the whole spectrum of effects of interest from the present perspective. It seems that the best scalar representation of the world global economic development during this period is in terms of the Standard & Poor’s 500 index. Keeping in mind that there exists some freedom in choosing a specific form of the periodic function $P$ in Eq.(3), which imposes a serious restriction on the mathematical rigour of the corresponding methodology, we take the first term of its Fourier expansion,

$$P(\ln(x)/\ln(\lambda)) = A + B \cos(\frac{\omega}{2\pi}\ln(x) + \phi).$$

This of course implies that $\omega = 2\pi/\ln(\lambda)$. A unit used to measure $x$ (equivalently $T$) can be absorbed into $\phi$. With our $\lambda = 2$ we then try to obtain the best representation of the oscillatory structure seem in the real market during this period. The other parameters of Eqs. (3) and (6) are not relevant for the present discussion and, since they are also nonuniversal [4], are not listed here. The result for the S&P500 is shown in Fig. 1a and can be seen to remain systematically in phase with the corresponding market trends approximately pointing to September 1, 2000 as the date of the reversal of the almost 20 years global increasing trend. It is interesting to see that the famous Black Monday of October 19, 1987, fits perfectly and constitutes one of the prominent log-periodic precursors of a more serious global crash that started in September 2000. A closer inspection of the vicinity of this date, obtained by the magnification presented in Fig. 1b, shows two other relevant elements. It clearly indicates that the modulus of the cosine in Eq. (6) provides a better representation for the log-periodic modulation. Secondly, it provides indepen-
Fig. 1. (a) Logarithm of the Standard & Poor's 500 over the period 1970–2002 versus its corresponding log-periodic representation (solid line) in terms of Eqs. (3) and (6). (b) S&P500 from 1997 till end of August 2002. The solid lines serve as a reference illustrating the log-periodically accelerating and decelerating scenarios with a common time of crash $T_c = 1.9.2000$. The modulus of the cosine in Eq. (6) is found here to constitute a better representation. The preferred scaling factor $\lambda = 2$ is used everywhere.
dent evidence that the real date $T_c$ marking reverse of the upward global trend is the beginning of September 2000, as it is exactly at this time that the decelerating log-periodic oscillations accompanying the decline start. Such an impressive synchronization of the end of a log-periodically accelerating bubble phase with the beginning of log-periodically decelerating “anti-bubble” [14] phase is spectacular, and can indeed be considered as an extra argument in favour of the consistency of the log-periodic scenario may offer. Furthermore, the same preferred scaling factor $\lambda = 2$ has been used in the analytical representation for the decelerating phase shown in Fig. 1b, and it looks optimal.

3 Phenomenon of a “super-bubble”

The last few years of the stock market development during the period discussed above was driven by the high-technology sector, whose appropriate measure is provided by the Nasdaq. How its specific time-dependence relates to the S&P500 of Fig. 1b, especially in the context of the log-periodic phase transition seen there, is thus a natural and intriguing question. Since the high technology sector has been the leader in dictating the global trend, one expects the same scenario to apply. While this is true during the decline starting in September 2000, as can be seen from Fig. 2a, the Nasdaq development does not, however, parallel exactly that of the S&P500 during the bubble phase. The Nasdaq value in March 2000 is significantly higher than at $T_c$ of September 1. This may tempt one to view [15] late March 2000 as the time marking the end of the high-technology speculative bubble. This, to some extent, may be considered a matter of taste, although the Nasdaq clearly follows its long term trend precisely until September. Its oscillation patterns in time also coincide with accumulation of oscillations pointing to September, as prescribed by our postulated universal value of $\lambda$. Reconciliation within the spirit of the hierarchy of self-similar log-periodic patterns can be obtained by the following additional postulate, which allows one to better understand the subtleties of the underlying dynamics: The substructure in the period November 1999 - March 2000, as one of the consecutive increases in the sequence of long term log-periodic pattern, gets boosted into a local bubble on top of a long term bubble, and therefore we term it a “super-bubble”. This local “super-bubble” then crashes (as at the end of March 2000) and the system returns to a normal bubble state that eventually crashes at the time (here September 2000) determined by the long term patterns. In fact, a trace of the same frequency log-periodic oscillations can even be seen to accompany the dynamics of this “super-bubble”, as shown in Fig. 2b.

As far as such exotic effects are concerned, the Nasdaq is not an exception. In this regard even more spectacular, as illustrated in Fig. 3a and Fig. 3b, was the gold price development around 1980. Its extremely sharp log-periodic
Fig. 2. (a) The Nasdaq Composite from 1997 through August 2002 and its log-periodic representation (solid line) in terms of Eqs. (3) and (6), with the modulus of the cosine. (b) The Nasdaq “super-bubble” of November 1999 – March 2000. The solid line illustrates its own short term log-periodic development. The preferred scaling factor $\lambda = 2$ is used throughout.
Fig. 3. The gold price over the period 1978–1982 and the corresponding log-periodic representations in terms of Eqs. (3) and (6), on both sides of $T_c = 15.9.1980$ with the same $\lambda = 2$. (b) The gold price "super-bubble" compared to the optimal short term log-periodic scenarios with the same preferred scaling factor.
(\(\lambda = 2\)) bubble also boosts one of its upgoing substructures into a “super-bubble”, which itself develops its own log-periodic oscillations with the same scaling factor, and the global long term log-periodic bubble eventually starts decaying, also log-periodically with \(\lambda = 2\). Such a scenario also resolves the difficulty encountered in Ref. [15] of filling the gap between the gold price maximum and the onset of the decelerating log-periodic phase. Our general remark in this context is that overlooking such effects of the “super-bubbles” may lead to a whole spectrum of \(\lambda\)’s, which is both unaesthetic and misleading.

4 Looking into future

Having collected from several time scales quite interesting evidence of universality of the financial log-periodicity, it is now natural to look from this perspective at the most extended period of the recorded stock market activity as dated since 1800 [16]. A nearly optimal corresponding log-periodic representation versus the S&P500 data is shown in Fig. 4 using the usual \(\lambda = 2\). It well reproduces the two obvious dips of the 1930s and late 1970s, and even the broad one in the mid of the 19th century, and it also points to the one that started in September 2000, as discussed above, as another of the same order. The year 2002 made already clear that it can be considered as such. The significance of this last draw down indicates that it may not fully recover before 2004. A more detailed likely intermediate development can be estimated by extrapolating a decelerating structure of Fig. 1b, which allows some vital increase starting late in 2002, possibly accompanied by accelerating log-periodic sub-patterns on smaller time scales [4]. It, however, also indicates that in the year 2010 the S&P500 is very likely to assume values factor of a few larger than in 2002. Extrapolating this development even further ahead in time, one also sees that it tends around 2025 to a much larger decline than anything we have experienced so far. That such a scenario deserves to be seriously taken into account we also conclude from the fact that we had it provisionally already in 1999 at the time of extreme euphoria, and it was signaling a large dip exactly at the beginning of 21st century, which indeed occurred.

5 Summary

The analysis presented above provides not only further arguments in favour of the existence of the log-periodic component in financial dynamics, self-similarly on various time scales, but also indicates that the corresponding central parameter - the preferred scaling factor - may very well be a constant close to 2. In this way it is possible to obtain a consistent relation between
Fig. 4. Logarithm of the Standard & Poor’s 500 index illustrating its development since 1800 [16]. The values prior to its official introduction are reconstructed from historical data [16]. The solid line represents the optimal log-periodic representation appropriate to this time scale with the usual preferred scaling factor. The fact that the third minimum in the solid line comes a little bit later than in the real data may seem somewhat disturbing. In this connection, however, it is interesting to notice that when correcting for a huge inflation, especially at that time, the minimum in the real data gets shifted to the early 1980s.

the patterns and it allows more reliable extrapolations into the future. It also allows the log-periodicity to pretend to the status of a law. Of course, on short time scales it is a fragile one, as the real financial market is exposed to many “external” factors, such as unexpected wars or other political events, which may distort its internal hierarchical structure on the organizational as well as on the dynamical level. In this connection it is worth remembering that the functional form of the log-periodic modulation so far is not supplied by theoretical arguments and this opens room for some mathematically unrigorous assignments of patterns, as is often needed in order to properly interpret them. Identifying a hierarchy of time scales and a universal preferred scaling ratio is crucial in this connection and very helpful for real predictions. Strict fitting of the lowest order term in the Fourier expansion of the periodic function in Eq. (3) is typically not an optimal procedure. Here it serves basically as a convenient representation to guide the eye.
6 Acknowledgement

We thank J. Kwapień for very fruitful exchanges. S.D. acknowledges support from Deutsche Forschungsgemeinschaft under contract Bo 56/160-1.

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