A CLUSTER MERGER AND THE ORIGIN OF THE EXTENDED RADIO EMISSION IN ABELL 3667

KURT ROETTGER

Department of Physics and Astronomy, University of Missouri-Columbia, Columbia, MO 65211; kroett@hades.physics.missouri.edu

JACK O. BURNS

Office of Research and Department of Physics and Astronomy, University of Missouri-Columbia, Columbia, MO 65211; burnsj@missouri.edu

AND

JAMES M. STONE

Department of Astronomy, University of Maryland, College Park, MD 20742-2421; jstone@astro.umd.edu

Received 1998 December 3; accepted 1999 January 26

ABSTRACT

We present a numerical model for the extended steep-spectrum radio sources and the elongated X-ray structure in A3667 based on new three-dimensional MHD/N-body simulations. The X-ray and optical analyses of A3667 indicate that it has undergone a recent subcluster merger event. We believe that the Mpc-scale radio sources identified in A3667 are also a consequence of the merger. Our previous numerical simulations show that mergers often produce large-scale shocks and turbulence capable of both magnetic field amplification and in situ reacceleration of relativistic particles. Our model suggests that these radio structures, separated by \( \sim 2.6 \, h^{-1}_{100} \) Mpc, are in fact causally linked via a slightly off-axis merger that occurred nearly in the plane of the sky approximately 1 Gyr ago with a subcluster having a total mass equal to \( \sim 20\% \) of the primary cluster.

Subject headings: galaxies: clusters: individual (Abell 3667) — intergalactic medium — methods: numerical — MHD — X-rays: galaxies

1. INTRODUCTION

Abell 3667 (see Fig. 1) at \( z = 0.055 \) (Sodré et al. 1992) is a massive (\( > 10^{15} \, M_\odot \); Sodré et al. 1992; Knopp, Henry, & Briel 1996), X-ray-luminous (\( \sim 10^{45} \, h^{-2} \) ergs s\(^{-1} \); Knopp et al. 1996) cluster of galaxies that exhibits a variety of the observational signatures of a recent cluster merger (Roettiger, Burns, & Loken 1996). Here, we present a self-consistent numerical MHD/N-body model suggesting that the most unique features of A3667, the Mpc-scale radio sources (Röttgering et al. 1997), are similarly a signature of a recent merger.

Characterized variously as a halo or a relic, we believe that the extended radio emission in A3667 represents a stage in the evolution of a radio halo. Radio halos are generally described as large (\( > 0.5 \) Mpc), amorphous steep-spectrum structures that do not appear to be associated with any particular galaxy and, as such, are believed to be intrinsic to the cluster itself (Hanisch 1980). By contrast, a radio relic was once associated with a particular active galactic nucleus (AGN) that has turned off and possibly drifted away leaving the radio source to diffuse and fade away. It is possible that radio relics supply the seed relativistic particles necessary to produce radio halos, but the connection is unclear at this time and we do not address this issue here.

Theoretically, the difficulty in understanding radio halos is related to their large spatial extent. If the relativistic electrons responsible for the radio synchrotron emission originated from a single source (e.g., AGN), they would be required to diffuse over Mpc scales during their relatively short radiative lifetimes (of order \( 10^8 \) yr) requiring velocities well in excess of the limiting Alfvén speed. (Holman, Ionson, & Scott 1979 present an alternate view.) This has led several researchers to suggest that radio halos require a means by which relativistic particles can be accelerated in situ, such as via galactic wakes (Roland 1981) or shocks and MHD turbulence (e.g., Eilek & Henriksen 1984) generated during a cluster merger (De Young 1992; Tribble 1993b). Whereas galactic wakes appear to be insufficient to power the radio halo (Goldman & Rephaeli 1991; De Young 1992), cluster mergers, which have a kinetic energy comparable to the total thermal energy of the cluster, are more than sufficient to power the radio source (Tribble 1993b; Burns et al. 1994; Burns 1998). Mergers may also overcome part of the electron transport problem by supplying bulk flows (> 1500 km s\(^{-1} \); Roettiger et al. 1997a) capable of transporting the relativistic electrons over large distances during their radiative lifetime (Röttgering et al. 1994).

Observationally, mergers have been linked to radio halos by both a correlation with substructure (X-ray and optical) and by an anticorrelation with cooling flows (Edge, Stewart, & Fabian 1992). For example, Coma, A2255, and A2256 all contain radio halos of one type or another and exhibit substructure that is indicative of recent dynamical evolution. It has also been suggested that cooling flows (e.g., see Fabian 1994 for a review of cooling flows) will be disrupted by massive mergers (McGlynn & Fabian 1984; Edge et al. 1992; Burns et al. 1997). Gómez et al. (1999), using numerical simulations, have shown that mergers can disrupt cooling flows under various initial conditions. None of the above mentioned clusters contain a cooling flow, nor for that matter does A3667.

The purpose of this paper is to demonstrate using fully three-dimensional, numerical MHD/N-body simulations that the X-ray, optical, and radio observations of A3667 are consistent with a single merger model in which shocks formed during the merger provide sites for diffusive shock acceleration of cosmic-ray (CR) electrons (see Longair 1994, and references therein, for a review of particle acceleration in shocks). We do not attempt to model the CR acceleration, diffusion, advection, and subsequent “aging” self-consistently with the MHD. When done in detail, this is a
difficult problem and can be computationally expensive. Recently, Jun & Jones (1999) and Jones, Ryu, & Engel (1999) presented a promising approach to this problem (applied to supernova remnants), which we will look to incorporate into future work. At this time, we present a model in which we introduce a population of CR electrons based on the shock characteristics during an epoch believed to be representative of A3667's current dynamical state, as inferred from the X-ray and optical data. The CR diffusion, advection, and aging are modeled by assuming that the MHD properties (i.e., shock strength, structure and velocity, bulk flows, etc.) do not evolve significantly during the radiative lifetime of the particles ($\sim 10^8$ yr) and that particle reacceleration behind the shock is unimportant. The modeling presented here is similar to that of Tribble (1991, 1993a, 1994) but with two important exceptions. His models were purely static, while ours are based on the self-consistent evolution of shocks and magnetic fields within the merger model, and we are attempting to specifically address the characteristics of the extended radio emission in A3667.
Although others have suggested that A3667 has recently undergone a merger (Knopp et al. 1996), and still others have suggested a possible shock origin for the extended radio emission (Röttgering et al. 1997; Ensslin et al. 1998), this is the first study that presents a quantitative numerical model that attempts to simultaneously explain the radio, optical, and X-ray morphology of this cluster within the context of a single merger.

Section 2 is a review of recent observations and evidence for substructure in A3667. In § 3 we discuss our numerical method. Our initial conditions are presented in § 4. Section 5 summarizes our model constraints. We discuss the radio halo model and its limitations in § 6. Finally, we summarize our results in § 7. The model is scaled assuming $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$. The observational data (see § 2) are parameterized by $h = H_0/100$.

2. A REVIEW OF THE OBSERVATIONS

2.1. The X-Ray Data

Knopp et al. (1996) provide a detailed analysis of the ROSAT X-ray data that we summarize here. The X-ray surface brightness (XSB; contours, Fig. 1) is seen to be elongated from the southeast to the northwest. The ellipticity is greatest in the cluster core and decreases with increasing radius. There is an isophotal twisting of $\sim 15^\circ$ from the inner to the outer cluster. The XSB extends an extension to the northwest of the X-ray core while it is seen to be flattened to the southeast. This results in a significant centroid shift ($\sim 16.6$ to the northwest but still within the observed XSB distribution). Figure 2 shows the positions of galaxies having redshifts supplied by Sodrê et al. (1992) and the results of the $\delta$ test for substructure (Dressler & Shectman 1988). The circles, marking individual galaxy positions, are scaled in radius according to their respective $\delta$ parameter. The value of $\delta$ indicates the degree to which a galaxy and its 10 nearest neighbors deviate from the global velocity properties. In Figure 2, we have filled the circles corresponding to galaxies having the lowest $\delta$ parameters, thereby accentuating the location of the two groups noted by Sodrê et al. (1992). The degree of substructure demonstrated by the $\delta$ test gives some indication as to why the observed range of velocity dispersions, and therefore cluster virial masses, is so large. Sodrê et al. (1992) estimate a total mass of $2.6 \times 10^{15} h^{-1} M_\odot$ for the entire cluster while Biviano et al. (1993) find $1.6 \times 10^{15} h^{-1} M_\odot$ within 0.75 $h^{-1}$ Mpc. Both values are larger than the X-ray–derived values (see § 2.1).

2.3. The Radio Data

There are many radio point sources located within the spatial extent of A3667, but for our purposes, we are concerned only with the large extended radio sources located to the southeast and northwest of the X-ray core (see Fig. 1, gray scale). The uniqueness of the extended radio sources in A3667 has been known for many years (Schilizzi & McAdam 1975; Goss et al. 1982; Jones & McAdam 1992). Here, we summarize the most detailed analysis to date, Röttgering et al. (1997). Approximately $1.7 h^{-1}$ Mpc north-
west of the cluster’s X-ray core (and beyond the extent of the XSB) is an extremely large (~1.4 h⁻¹ Mpc) arc-shaped source that does not appear to be associated with any particular galaxy in the cluster. Along the northwest rim of this source, there is a sharp edge to the synchrotron distribution. The source becomes more diffuse to the southeast. The spectral index appears to be relatively flat (α = 0.5; Sₜ ∝ ν⁻α) along the northwest edge and exhibits a gradient toward the southeast steepening to α = 1.5. Overall, α ≈ 1.1.

To the southeast of the X-ray core there are two large, linear sources situated perpendicular to the X-ray major axis (parallel to each other). The largest and farthest southeast extends ~0.7 h⁻¹ Mpc and is ~0.7 h⁻¹ Mpc from the core. The source closest to the core has been identified by R. W. Hunstead & M. Wieringa (1998, private communication) as a narrow-angle tailed source (NAT) associated with galaxy 157 from Sodré et al. (1992) and is therefore not of interest to this study. The northwest and southeast halo sources are separated by nearly 2.6 h⁻¹ Mpc. Both sources are located near or beyond the extent of ROSAT-observed X-ray emission.

3. NUMERICAL METHOD

The numerical method is the same as that used in Roettiger, Stone, & Burns (1999). The intracluster medium (ICM) and magnetic fields are evolved using ZEUS (Stone & Norman 1992a, 1992b), an Eulerian, finite-difference code that solves, self-consistently, the equations of ideal MHD. The numerical evolution of the magnetic field components is performed by the constrained transport (CT) algorithm (Evans & Hawley 1988), which guarantees preservation of the divergence-free constraint at all times. The method of characteristics (MOC) is used for computing the electromagnetic force (Hawley & Stone 1995). An extensive series of MHD test problems has demonstrated that the MOC-CT method provides for the accurate evolution of all modes of MHD wave families (Stone et al. 1992). We employ outflow boundary conditions on the MHD.

The collisionless dark matter is evolved using an N-body code based on a standard particle-mesh algorithm (PM, Hockney & Eastwood 1988). The particles and gas are evolved on the same grid using the same time step. The time step is determined by applying the Courant condition simultaneously to both the dark matter and the magnetohydrodynamics. The only interaction between the collisionless particles and the gas is gravitational. Since we are modeling an isolated region, the boundary conditions for Poisson’s equation are determined by a multipole expansion (Jackson 1975) of the total mass distribution (dark matter and gas) contained within the computational grid. Particles that leave the grid are lost to the simulation. Typically, less than a few percent of the particles leave the grid.

The hybrid ZEUS/PM code was parallelized using the message-passing interface (MPI; Gropp, Lusk, & Skjellum 1994). These simulations were run on the Cray T3E in the Earth and Space Data Computing Division of the NASA Goddard Space Flight Center.

The simulation whose results are presented here had an effective resolution scaled to 18 kpc, or, more significantly, there are 16 zones per primary cluster core radius. The computation was performed on a grid having dimensions 512 × 256². The grid is uniform from zone 100 to 412 along the merger axis and from zone 90 to 166 perpendicular to the merger axis. Outside of this central region, we gradually increase the zone size by ~3% from one zone to the next.

4. INITIAL CONDITIONS

4.1. Dark Matter and Gas

Our initial conditions are similar to those used in our previous studies (e.g., Roettiger, Burns, & Loken 1993; Roettiger, Loken, & Burns 1997a; Roettiger, Stone, & Mushotzky 1997b, 1998). We begin with two clusters whose gas distributions are consistent with observations of relaxed systems. Our cluster dark matter distributions may be less consistent with observations in that they do not have the central density cusp implied by recent strong lensing observations and numerical simulations of large-scale structure formation (Navarro, Frenk, & White 1997). We do not believe that this difference will significantly alter the merger dynamics. The mass distributions in our simulations are based on the lowered isothermal King model described in Binney & Tremaine (1987). The lowered isothermal King model is a family of mass distributions characterized by the quantity ψ/σ², which essentially defines the concentration of matter. As ψ/σ² increases, the core radius (rₜ) decreases with respect to the tidal radius (rₜ). We have chosen a model with ψ/σ² = 12 in which we have truncated the density distribution at 15 rₜ. Near the half-mass radius, the total mass density follows a power-law distribution, ρ ~ r⁻α, where α ~ 2.6. This model is consistent with mass distributions produced by cosmological N-body simulations that show α ~ 2.4 in a high-density universe (Ω = 1) and ~2.9 in a low-density universe (Ω = 0.2) (Crone, Evrard, & Richstone 1994). Initially the gas distribution (ICM) is in hydrostatic equilibrium within the gravitational potential defined by both the gas and dark matter components. The gas distribution is isothermal within the central 6 rₜ. At larger radii, the temperature drops gradually. The density peaks are initially separated by 6.5 Mpc. Each cluster was given a small initial velocity in order to speed up the merger process and thus conserve computational resources. The final impact velocity (~2750 km s⁻¹) is not affected significantly by the initial velocity that has components of ~275 km s⁻¹ parallel to the line of centers and ~30 km s⁻¹ perpendicular to the line of centers, resulting in a slightly off-axis merger. The physical scaling of the premerger clusters can be found in Table 1.

4.2. Magnetic Field

The only galaxy clusters certain to have large-scale or intrinsic magnetic fields are those with radio halos, and these sources are rare (Hanisch 1982). Of those containing halos, Coma has been studied in the most detail (e.g., Jaffe 1977; Kim et al. 1990 among others). Deiss et al. (1997) show that the synchrotron halo in Coma, after subtracting point sources, traces the X-ray surface brightness distribution indicating that the magnetic pressure gradient is similar to that of the thermal pressure. Of course Coma is only one cluster and may not be representative of halos in general. In addition to the spatial distribution of the magnetic pressure, we must also be concerned with the spectral power distribution. Cluster magnetic fields are believed to exist as flux ropes that are tangled on a variety of physical scales (e.g., Ruzmaikin, Sokoloff, & Shukurov 1989). Observations of polarization in cluster radio sources indicate that tangling occurs on scales ranging from a kiloparsec (e.g., Feretti & Giovannini 1997) to tens or even hundreds of
kiloparsecs (Kim et al. 1990; Kim, Kronberg, & Tribble 1991; Feretti et al. 1995). Studies of cluster radio halos using a variety of methods (see Miley 1980) indicate that typical mean field strengths are of order 1 \( \mu \)G (e.g., Coma, Kim et al. 1990, Ensslin & Biermann 1998; A2255, Burns et al. 1995; A2256, Röttgering et al. 1994, Bagchi, Pislar, & Lima Neto 1998).

In order to model a random field tangled on a variety of scales, we begin by defining a magnetic vector potential, \( A(k) = A_0 \, k^{-d} \), where the amplitudes \( A_0 \) for each Cartesian coordinate are drawn from a Gaussian distribution. The vector potential is then transformed via a three-dimensional fast Fourier transform (FFT) into physical space where it is scaled spatially by the gas density distribution. Assuming a uniform spherical collapse (likely a gross oversimplification, e.g., Evrard 1990; Bryan et al. 1994; among many others) and flux freezing, it can be shown that the magnetic field will scale as \( \rho_{\text{gas}}^{2/3} \) or \( B \sim r^{-1.7} \). Finally, we initialize a tangled divergence-free magnetic field from \( A \) via \( B = \nabla \times A \). This results in \( B^2 \propto k^{2(1-d)} \). Here we adopt \( d = 5/3 \). Unfortunately, neither observation or theory are capable of constraining our choice of power-law index. We have scaled the magnetic pressure such that the mean field is a dynamically insignificant \( 0.3 \mu \)G within \( 2r_c \). Because of numerical resolution limitations, the minimum resolved magnetic field scale is about four zones or nearly 80 kpc.

5. THE MODEL

Based on our numerical model and limited survey of merger parameter space (cluster mass ratios, impact parameters, relative core densities, etc.), we believe that A3667 underwent a merger approximately 1 Gyr ago with a subcluster having a mass 20% of the primary cluster. The merger was slightly off-axis having an impact parameter of

| Cluster ID | \( M_{\text{tot}} \) \( (10^{14} M_\odot) \) | \( T_e \) \( (\text{keV}) \) | \( \sigma_v \) \( (\text{km s}^{-1}) \) | \( r_c \) \( (\text{kpc}) \) | \( f_g \) | \( v_{\text{impact}} \) \( (\text{km s}^{-1}) \) |
|-----------|----------------------------------|------------------|------------------|------------------|------|------------------|
| 1         | 13.0                             | 6.5              | 943              | 288              | 0.045 | 2750             |
| 2         | 2.6                              | 3.6              | 540              | 140              | 0.045 | . . .            |

* Total mass \( R < 3 \) Mpc.
* Temperature.
* One-dimensional velocity dispersion \( R < 1.5 \) Mpc.
* Core radius.
* Global gas fraction, by mass.
* Impact velocity (dark matter).

Fig. 3.—Left: Simulated X-ray surface brightness (dashed contours) and projected emission-weighted temperature (solid contours). We have added noise to the synthetic X-ray image that has been contoured at the same levels (relative to peak) as the ROSAT image (see Fig. 1). The temperature contours are labeled with the corresponding temperature in keV. The image is \( 3.15 \times 3.85 \) Mpc. Right: ASCA temperature profile (Markevitch et al. 1998, open diamonds) and the simulated temperature profile (asterisks) within the same spatial bins (as indicated by the horizontal bar, assuming \( H_0 = 70 \) km s\(^{-1}\) Mpc\(^{-1}\)). The profiles have been slightly displaced for clarity. The vertical bars on the ASCA data indicate the uncertainty in the temperature while the vertical bars on the simulated temperatures represent the dispersion in temperature within the volume. Note that although the cluster is nonisothermal, it appears to be quite isothermal in profile.
The merger occurred largely in the plane of the sky with the subcluster moving from the southeast to the northwest passing on the southern side of the more massive or primary cluster. Figure 3 (left) shows the synthetic XSB (dashed contours) overlaid onto the projected, emission-weighted temperature (solid contours) at the epoch determined to most closely resemble A3667. The sharp edges in the temperature distribution indicate the locations of shocks that we associate with sites of particle acceleration for the purposes of modeling the radio emission (see § 6). Figure 3 (right) is a comparison of the simulated and ASCA-observed temperature profiles (Markevitch et al. 1998). Figure 4 (left) shows the synthetic XSB (solid contours) overlaid on the simulated synchrotron emission (gray scale; see § 6). The plus signs indicate the location of dark matter/galaxy centroids. The primary cluster is largely coincident with the XSB peak while the subcluster remnant is located to the northwest. The future evolution of this system would be similar to that of A754 as described in detail by Roettiger et al. (1998).

In producing our model of A3667, we have made some effort to match as many of A3667’s observational characteristics (see § 2, Figs. 1 and 2) as closely as possible, while keeping in mind that our primary concern for this work is the merger-induced shock structures that, as our previous simulations show (e.g., Roettiger et al. 1997a) are a rather ubiquitous feature of major merger events. Regarding the XSB distribution, we are able to reproduce the overall elongation that defines the merger axis, the isophotal twisting that indicates a slightly off-axis merger, and the steepness of the XSB distribution to the southeast relative to the northwest. We believe that the subcluster first impacted from the southeast moving to the northwest causing a compression of the X-ray-emitting gas on the southeast side. As the subcluster dark matter remnant exits the primary cluster core, it draws out the ICM, creating the northwest extension. With regard to the galaxy distribution, we are able to reproduce the relative positions of the galaxy concentrations (dark matter particles in our simulations, plus signs in Fig. 4, left) with respect to the XSB and the radio sources. We describe radio source modeling in greater detail below.

6. DETAILS OF THE RADIO HALO MODEL

6.1. Basic Assumptions

There are several simplifying assumptions inherent in this model. First, we assume that all particle acceleration occurs at the shocks. Once the particles leave the region of the shock, there is no further acceleration. Second, we assume a simple diffusive shock model in which the strength of the shock (as measured by the density compression) determines the power-law slope of the injected particle energy spectrum (e.g., Drury 1983; Longair 1994). The electrons are assumed to be accelerated within a single zone. That is, their scattering length is small compared to a computational zone, and the time required to accelerate them is small compared to a time step. Third, we assume that the basic shock structure (strength, shape, velocity, etc.) does not change significantly during the radiative lifetime of the relativistic electrons. Finally, we assume that some small fixed fraction...
of the thermal particle flux is accelerated to its equilibrium distribution as it passes through the shock.

6.2. Method

Having chosen the epoch most representative of A3667 based on the X-ray and optical data, we begin modeling the radio emission by identifying the locations of the shocks. In ZEUS, an artificial viscosity is used to smooth shocks over four to five zones. We locate the shock by calculating the gas compression ratio \((r = \rho_2/\rho_1)\) at two points separated by five zones throughout the computational volume. The compression ratio can be related directly to the power-law index \((\gamma)\) of the injected relativistic electron energy spectrum, \(N(E) \propto E^{-\gamma}\) such that \(\gamma = 3.0/(r - 1) + 1\). Once identified, each point along the shock is assigned a value of \(\gamma\) based on the local strength of the shock. For the most part, the shock strength is relatively uniform across the face of the shock; however, it does weaken somewhat near its edges. The shock structure present within the plane of the merger can be seen in Figure 5. Note that there are multiple shocks to the southeast of the X-ray core, each presenting a potential site for particle acceleration.

The next step is to associate each zone behind (or upstream from) the shock with the nearest point on the shock along the fluid flow lines. We then assume that the particles in a given zone behind the shock are derived from that location on the shock and that their distance \((d)\) from the shock is directly proportional to the velocity of the shock relative to the bulk fluid flow \((V_s)\) and the time since acceleration \((t_a)\) such that \(d = \kappa V_s t_a\). The scaling factor \(\kappa\) is used to parameterize the diffusion/advection rate. If the electrons diffuse freely in straight-line motion, unaffected by the magnetic field or turbulence, \(\kappa = 1\). This would represent the limiting case in which the magnetic field is weak and/or aligned parallel to the flow, and accelerated particles are simply deposited in the wake of the shock. Of course, the situation is almost certainly more complicated. At the other extreme (strong, tangled field), Tao (1995) suggests that the maximum inhibition of electron diffusion by magnetic fields is of the order of a factor of 10. That is, the effective path length of an electron traveling through a tangled magnetic field environment will be at most 10 times longer than the straight-line path \((\kappa = 0.1)\). Here, we adopt the weak-field/high-diffusion limit where \(\kappa \sim 1.0\).

In order to quantify the rate at which particles diffuse/advec from the shock in the absence of a magnetic field \((i.e., V_s)\), we have incorporated a passive-scalar field into ZEUS. The passive-scalar is a dynamically insignificant quantity \((\text{in that sense, it is likely similar to the radio plasma})\), which is advected with the thermal gas density. In this implementation, we inject the scalar field at the locations of shocks and simply allow it to evolve. As expected, the passive-scalar field \((\text{or relativistic electrons})\) is seen to trail the shocks as they propagate through the ambient medium. This is consistent with observations of shocks within the solar system (Kennel et al. 1986). By evolving the scalar field over a limited time interval near the A3667 epoch, we can estimate the effective shock velocity for each point along the shock front. In this manner, we find effective shock velocities of 700–1000 km s\(^{-1}\) while the southeast shocks have an effective velocity of \(\sim 250\) km s\(^{-1}\). In fact, the shock nearest the X-ray core is effectively stationary. This analysis shows little evidence of mixing between the relativistic and thermal plasmas behind the northwest shock. Of course this result could be strongly resolution dependent. It may also depend on the initial conditions. The shocks in this model, particularly to the northwest, propagate into a largely non-turbulent, homogeneous medium, which is certainly not the case.

6.3. Synchrotron Aging, Emissivity, and Spectral Index Distribution

We use the formalism of Myers & Spangler (1985) to age our synchrotron spectrum. Their work is based on the models of Kardashev (1962) and Paczynski (1970) and Jaffe & Perola (1973). Here, we specifically employ the modification introduced by Jaffe & Perola, which assumes that the electrons become isotropized on timescales short compared to their radiative lifetimes, and therefore can be represented by a time-averaged distribution of pitch angles. Adiabatic loses are not included in this model. Based on our analysis of the simulation dynamics and the Röttgering et al. (1997) analysis showing the radio source to be greatly underpressurized with respect to the thermal plasma, we believe that expansion losses can be neglected.

Given the initial electron energy spectrum power-law index \((\gamma)\), the magnetic field strength \((B)\), and the time since acceleration, we can determine the spectral index and relative intensity of emission between 1.4 and 4.9 GHz. The emissivity within a given zone is then simply \(\epsilon_v \propto (B \sin \theta)^{\gamma + 1} v^{-x}\), where \(\theta\) is the angle between the magnetic field and the observer's line of sight. The Myers & Spangler (1985) model does not account for varying magnetic field strengths. Therefore, the field used to age the particles is the
mean field experienced during the electron’s lifetime (i.e., the mean magnetic field along the line connecting the particle’s current location and its presumed point of origin on the shock). The magnetic field used to calculate the emissivity within a given zone is simply the field within that zone. Having calculated the synchrotron spectrum steepens to $\sim 1.5$ at about $0.7$ Mpc ($H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$) upstream from the shock. Assuming the Jaffe-Perola model and high diffusion ($\kappa = 1$; see § 6.2), we can reproduce the distribution of spectral index if the mean magnetic field within the region of the source is $\sim 0.6$ $\mu$G. If we restrict the rate of diffusion (i.e., $\kappa < 1$), the magnetic field can be decreased accordingly because particles will take longer to diffuse/advect a distance of $0.7$ Mpc and will have aged spectrally in the meantime. It is not possible to use this method to constrain the field strength in the southeast radio sources. We do not have spectral index data, and the shocks have not moved significantly during the particles’ lifetimes. Therefore, we scale the global field according to our analysis of the northwest source. Assuming we had the correct initial magnetic pressure distribution (see § 4.2), the mean field in the region of the southeast radio source is $\sim 1.5$ $\mu$G. The greater field strength results from the southeast source being closer to the X-ray core than the northwest source (remember $B \sim r^{-1/2}$, initially) and from the greater compression (and amplification) of the field on the southeast side of the cluster as a result of the merger. Again, if we have overestimated the electron diffusion/advective rate, as is likely, these field estimates will decrease.

### 6.4. Limitations of the Model

Of course the model has its limitations, which are owing primarily to the large number of free and not necessarily independent parameters such as relative cluster masses, the shape of their initial mass distribution, impact parameter, initial velocities, magnetic field distribution, projection with respect to the observer, etc. It is computationally expensive and physically impossible to explore all of parameter space. Consequently, we present the best fit to the current data given our limited survey of the many free parameters. Even so, the largest discrepancies between our model and the observations appear to be the strength of the northwest shock and the location of the most northerly of the southeast shocks. The northwest shock in our model, based on the observed spectral index distribution, appears to be too weak. If the spectral index along the northwest rim of the northwest radio source ($x = 0.5$) is correct, we need a shock with a density compression near 4 (i.e., a strong shock). At the chosen epoch, it is typically less than 3, which produces a somewhat steeper spectrum ($x \sim 0.95$). This may imply that we need to consider an earlier epoch since the shock weakens as it expands outward. The shock could also be strengthened if the impact velocity of the subcluster were greater (i.e., more massive clusters or a greater initial velocity) or if the exiting subcluster remnant were interacting with higher density gas at large radii or with material falling into the cluster from a large-scale filament. This highlights another limitation of the model. These numerical simulations are noncosmological. We do not attempt to model the external cluster environment such as filaments or spherical accretion. The structure of galaxy clusters at large radii ($>1.5$ Mpc) is uncertain from an observational standpoint, and, consequently, our initial conditions become potentially less accurate at larger radii.

As mentioned above, our simulation produces essentially two shocks located southeast of the X-ray core, whereas only one extended radio source has been observed that does not appear to be associated with a particular galaxy. Both shocks appear to have a morphology, strength, and evolution consistent with producing the southeast radio source. That is, they are both smaller, more linear, and slower moving than the northwest shock while still being strong enough to produce significant particle acceleration. In terms of its location with respect to the XSB, the southeast shock farthest from the core is more consistent with the observed radio source. However, its properties suffer from the same sensitivity to initial conditions as we described above regarding the northwest shock. That said, the important point is that the single merger does produce shocks on either side of the cluster core that are capable of generating the observed radio morphology. Why some shocks within a cluster may produce extended radio sources and others do not will likely depend on the specific details of the local magnetic field structure and relativistic particle populations. These are the same issues that likely determine why some clusters have radio halos and others do not (see § 6.5).

#### 6.5. Why Are There Not More Halos of This Type?

If the large shocks are ubiquitous features of major merger events and mergers appear to be relatively common, why do we not see more radio halos of this type? Specifically, according to our models, A754 is in a stage of evolution similar to (though somewhat earlier than) A3667, but it does not have the same type of radio halo (Roettiger et al. 1998). Andernach et al. (1988) claim the existence of a radio halo in A754. However, higher resolution images indicate that this source is composed of three discrete sources, one of which is a NAT (Owen & Ledlow 1997). The other two are likely background objects. In any case, there does not appear to be a halo in A754 that is comparable to A3667. There are several possible explanations for this apparent inconsistency. First, the shock structures, although long lived relative to the canonical radio source age, are still relatively short lived compared to the period between major mergers ($2–4$ Gyr; Edge et al. 1992). Second, although the shocks may represent a necessary condition of halo formation, by themselves, they are not sufficient. Certainly, a large-scale magnetic field is required, as well as possibly a preexisting population of relativistic electrons that are accelerated by the shock. At this time, very little is known about either the large-scale distribution of magnetic field or relativistic particles as a general property of clusters. Some clusters may simply not have them or may have them in insufficient quantities. Third, cluster richness appears to be a factor, since only the richest appear to have halos.
Richer (i.e., more massive) clusters have deeper gravitational potentials that provide for more violent, higher velocity mergers and stronger shocks. They also have a more extended, higher pressure ICM and more galaxies that may supply seed relativistic electrons and magnetic fields (Okoye & Onuora 1997). Finally, projection effects may also play a role.

7. SUMMARY

We have presented a plausible model for the extended radio emission in A3667. In this model, A3667 has undergone a recent (∼1 Gyr) merger event with a subcluster having a total mass of 20% of the primary cluster. We believe that the subcluster impacted slightly off-axis while moving from the southeast to the northwest. The merger generates multiple shocks that provide sites for diffusive shock acceleration of relativistic electrons. The spatial extent of the radio sources is determined by the shock dimensions, as well as the assumed strength and structure of the magnetic field that ultimately determines the rate at which the relativistic particles age spectrally and the rate at which they diffuse/advection from the shock. This model is capable of reproducing several features of the basic X-ray morphology including the twisted isophotes (although the twisting is more extreme in our model), the steep X-ray surface brightness gradient to the southeast (indicating a compression of the ICM), and the X-ray extension to the northwest (indicating that the ICM has been drawn out of the cluster by the subcluster remnant). By modeling the spectral aging of the electron distribution, we estimate a mean magnetic field of order less than $1 \mu G$.

This model also explains many features of the A3667 radio morphology including (1) the locations of the northwest and southeast radio sources with respect to the X-ray surface brightness (these are where the shocks are located), (2) the sharp edge of the northwest radio source (the site of particle acceleration), (3) the steepening of the spectral index toward the southeast in the northwestern source (particles age as they move away from the shock), (4) the relative sizes of the radio sources (the northwest shock has a much greater lateral extent and is moving much faster than the southeast shocks that are virtually stationary at this epoch), and (5) the shapes of the radio components (the southeast shocks are more linear than the northwest shock).

Although we have been able to reproduce the basic radio morphology, there are limitations to this model. The northwest shock, in our model, is not strong enough to produce the flat ($\alpha = 0.5$) spectrum observed on the leading edge of the source. This shock could be strengthened if there is infall along an external filament (which we do not model) as suggested by Ensslin et al. (1998). The southeast shocks are stronger than the northwest shock largely because of residual infall from the subcluster that impinges on gas expelled from the cluster core by oscillations in the gravitational potential. It is also important to note that we do not evolve the relativistic particles self-consistently within the MHD simulation. Modeling the diffusion of particles from the shock is very complicated. For simplicity, we have assumed a weak-field (high-diffusion) limit in which particles are immediately left behind the shock. The true situation is undoubtedly more complex. Future work will entail evolving the relativistic particles in a more self-consistent manner as a second fluid (e.g., Jun & Jones 1999; Jones et al. 1999).

At this time we can only speculate as to why this type of radio source is so rare. It is most likely that very specific initial conditions are required that may not exist in all clusters. It is possible that not all clusters contain a cluster-wide magnetic field or a seed population of relativistic electrons. It is also likely that the relatively short radiative lifetime of the relativistic particles combined with the short lifetime of the shocks contributes to the rarity of these sources.

We thank the Earth and Space Data Computing Division at the NASA Goddard Space Flight Center for use of the Cray T3E supercomputer. We acknowledge support from NSF grant AST 98-96039. We also thank H. Röttgering for use of Figure 1.

REFERENCES

Andernach, H., Tie, H., Sievers, A., Reuter, H.-P., Junkes, N., & Wielebinski, R. 1988, A&AS, 73, 265
Bageh, J., Pizlar, V., & Lima Neto, G. B. 1998, MNras, 296, 23
Binney, J., & Tremaine, S. 1987, Galactic Dynamics (Princeton: Princeton Univ. Press)
Biviano, A., Girardi, M., Giuricin, G., Mardirossian, F., & Mezzetti, M. 1993, ApJ, 411, L13
Bryan, G. L., Klypin, A., Loken, C., Norman, M. L., & Burns, J. O. 1994, ApJ, 437, L5
Burns, J. O. 1998, Science, 280, 345
Burns, J. O., Roettiger, K., Ledlow, M., & Klypin, A. 1994, ApJ, 427, L87
Burns, J. O., Roettiger, K., Pinkney, J., Perley, R. A., Owen, F. N., & Voges, W. 1995, ApJ, 446, 583
Burns, J. O., et al. 1997, in ASP Conf. Ser. 115, Galactic and Cluster Cooling Flows, ed. N. Soker (San Francisco: ASP), 21
Crone, M., Evrard, A., & Richstone, D. 1994, ApJ, 434, 402
Deiss, B. M., Reich, W., Lesch, H., & Wielebinski, R. 1997, A&A, 321, 55
De Young, D. S. 1992, ApJ, 386, 464
Dressler, A., & Shectman, S. A. 1988, AJ, 95, 985
Drury, L. 1983, Rep. Prog. Phys., 50, 1425
Edge, A. C., Stewart, G. C., & Fabian, A. C. 1992, MNras, 258, 177
Eilek, J. A., & Henriksen, R. N. 1984, ApJ, 277, 820
Ensslin, T. A., & Biermann, P. L. 1998, A&A, 330, 90
Ensslin, T. A., Biermann, P. L., Klein, U., & Kohle, S. 1998, A&A, 332, 395
Evans, C. R., & Hawley, J. F. 1988, ApJ, 332, 659
Evrard, A. E. 1990, ApJ, 363, 249
Fabian, A. C. 1994, ARA&A, 32, 277
Fadda, D., Girardi, M., Giuricin, G., Mardirossian, F., & Mezzetti, M. 1996, ApJ, 473, 670
Feretti, L., Dallacasa, D., Giovannini, G., & Tagliani, A. 1995, A&A, 302, 680
Feretti, L., & Giovannini, G. 1997, in Proc. Untangling Coma Berenices: A New Vision of an Old Cluster, ed. A. Maurice, F. Casoli, F. Durret, & D. Gerbel (Singapore: World Scientific), 123
Gallo, L., & Rephaeli, Y. 1991, ApJ, 379, 80
González, P. L., Loken, C., Roettiger, K., & Burns, J. O. 1999, in preparation
Goldman, L., Ekers, R., Kellner, D. J., & Smith, R. M. 1982, MNras, 198, 259
Groopp, W., Lusk, E., & Skjellum, A. 1994, Using MPI: Portable Parallel Programming with the Message-Passing Interface (Cambridge: MIT Press)
Hainrichs, R. J. 1980, AIJ, 85, 1565
Hawley, J. F., & Stone, J. M. 1995, Comp. Phys. Comm., 89, 127
Hockney, R., & Eastwood, J. 1988, Computer Simulation Using Particles (Philadelphia: IOP)
Holman, G. D., Ionson, J. A., & Scott, J. S. 1979, ApJ, 228, 176
Jackson, J. D. 1975, Classical Electrodynamics (New York: Wiley)
Jaffe, W. J. 1977, ApJ, 212, 1
Jaffe, W. J., & Perola, G. C. 1973, A&A, 26, 423
Jones, P. A., & McAdam, W. B. 1992, ApJS, 80, 137
Jones, T. W., Ryu, D., & Engel, A. 1999, ApJ, 512, 105
Jun, B.-I., & Jones, T. W. 1999, ApJ, 511, 774
Kardashev, N. S. 1962, Soviet Astron. 7.
Kasahara, H., Tie, H., Sievers, A., Reuter, H.-P., Junkes, N., & Wielebinski, R. 1997.
Kim, K.-T., Kronberg, P. P., Dewdney, P. E., & Landecker, T. L. 1990, ApJ, 355, 29
Kim, K.-T., Kronberg, P. P., & Triffle, P. 1991, ApJ, 379, 80
Knopp, G. P., Henry, J. P., & Briel, U. G. 1996, ApJ, 472, 125
Longair, M. S. 1994, High Energy Astrophysics (Cambridge: Cambridge Univ. Press)
Markevitch, M., Forman, W. R., Sarazin, C. L., & Vikhlinin, A. 1998, ApJ, 503, 77
McGlynn, T. A., & Fabian, A. C. 1984, ApJ, 208, 709
Miley, G. 1980, ARA&A, 18, 165
Mohr, J. J., Evrard, A. E., Fabricant, D. G., & Geller, M. J. 1995, ApJ, 447, 8
Myers, S. T., & Spangler, S. R. 1985, ApJ, 291, 52
Navarro, J. F., Frenk, C. S., & White, S. D. 1995, MNRAS, 275, 720
———. 1997, ApJ, 490, 493
Okoye, S. E., & Onuora, L. I. 1997, MNRAS, 283, 1047
Owen, F. N., & Ledlow, M. J. 1997, ApJS, 108, 41
Pacholczyk, A. G. 1970, Radio Astrophysics (San Francisco: Freeman)
Roettiger, K., Burns, J. O., & Loken, C. 1993, ApJ, 407, L53
———. 1996, ApJ, 473, 651
Roettiger, K., Loken, C., & Burns, J. O. 1997a, ApJS, 109, 307
Roettiger, K., Stone, J. M., & Burns, J. O. 1999, ApJ, 518, 594
Roettiger, K., Stone, J. M., & Mushotzky, R. F. 1997b, ApJ, 482, 588
———. 1998, ApJ, 493, 62
Roland, J. 1981, A&A, 93, 407

Note added in proof.—M. Markevitch, C. L. Sarazin, & A. Vikhlinin (1999, ApJ Letters, in press) recently produced a temperature map of A3667 using ASCA data. Our model is largely consistent with this map over the limited extent of the X-ray emission. The greatest inconsistency is the very hot gas southeast of the X-ray core which is not apparent in the data. This would seem to indicate that our subcluster gas may be penetrating too deep into the primary cluster.