Spectral Albedo of Dusty Martian H$_2$O Snow and Ice

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Abstract  Recent evidence of exposed H$_2$O ice on Mars suggests that this ice was deposited as dusty ($<1\%$ dust) snow. This dusty snow is thought to have been deposited and subsequently buried over the last few million years. On Earth, freshly fallen snow metamorphoses with time into firn and, if deep enough, into glacier ice. While spectral measurements of martian ice have been made, no model of the spectral albedo of dusty martian firn or glacier ice exists at present. Accounting for dust and snow metamorphism is important because both factors reduce the albedo of snow and ice by large amounts. However, the dust content and physical properties of martian H$_2$O ice are poorly constrained. Here, we present a model of the spectral albedo of H$_2$O snow and ice on Mars, which is based on validated terrestrial models. We find that small amounts ($<1\%$) of martian dust can lower the albedo of H$_2$O ice at visible wavelengths from $\sim 1.0$ to $\sim 0.1$. Additionally, our model indicates that dusty ($>0.01\%$ dust) firm and glacier ice have a lower albedo than pure dust, making them difficult to distinguish in visible or near-infrared images commonly used to detect H$_2$O ice on Mars. Observations of excess ice at the Phoenix landing site are matched by 350-μm snow grains with 0.015% dust, indicating that the snow has not yet metamorphosed into glacier ice. Our model results can be used to characterize orbital observations of martian H$_2$O ice and refine climate-model predictions of ice stability.

Plain Language Summary  There is widespread evidence of water ice on Mars, with numerous locations where it is currently exposed at the surface. However, Mars is a dusty planet, and small amounts of dust are usually present within the ice. These small amounts of dust drastically alter the brightness of the ice, and therefore affect how much solar energy the ice absorbs. We present a model of the effect of dust content on the brightness of martian snow and glacier ice, to help determine the dust content and physical properties of snow and ice on Mars. Characterizing these properties can significantly improve models of ice stability on Mars and inform us about its age and origin.

1. Introduction

The presence, stability, and physical nature of H$_2$O ice on Mars has major implications for understanding martian history, evolution, and future robotic and manned exploration. However, the pervasive presence of dust on Mars causes the ice to contain typically $<1\%$ dust, especially when exposed at the mid-latitudes (Dundas et al., 2018; Khuller & Christensen, 2021). The ice is thought to have been deposited as snow during periods of high obliquity that occurred numerous times over the last few million years (Christensen, 2003; Jakosky & Carr, 1985; Madeleine et al., 2014). At visible wavelengths, dust is far more absorbing than H$_2$O ice (Figure 1; Warren & Wiscombe, 1980; Wolff et al., 2009), so small amounts of dust can lower the albedo at these wavelengths (e.g., Dozier et al., 2009; Painter et al., 2013). Lower albedos lead to enhanced radiative heating, which affects the ice's energy balance and its stability and evolution over time.

Radiative heating is also enhanced by snow metamorphism. On Earth, fresh snow (with grain radius 50–100 μm) quickly metamorphoses due to vapor diffusion and and grain-boundary diffusion (Kaempfer & Schneebeli, 2007); surface snow grains can grow to radii of several hundred μm. The density also increases, in three stages by three different mechanisms (grain-boundary sliding, mechanical deformation, and bubble shrinkage): (a) In snow, densification proceeds by grain-boundary sliding, up to 550 kg/m$^3$. This is the maximum density obtainable for snow at the surface. (b) In subsurface firn, density can increase by mechanical deformation up to 830 kg/m$^3$; at this density, the air becomes closed off into bubbles, becoming “glacier ice.” (c) Overburden pressure in glacier ice causes the air bubbles to shrink, further increasing the density to approach that of pure ice, 917 kg/m$^3$ (Cuffey & Paterson, 2010). Density per se has no effect on the albedo of opaque media, but in snow and ice the density does often correlate with grain size, which is the dominant
determinant of albedo. Coarser snow grains reduce the number of scattering events per unit pathlength, thereby increasing the likelihood of photon absorption, and reducing the albedo. Snow that has metamorphosed into glacier ice absorbs up to five times more solar radiation than snow, which can lead to subsurface melting within the ice, as is seen on Earth (Brandt & Warren, 1993; Liston & Winther, 2005).

Recent observations have indicated that dusty H₂O ice is currently being exposed at martian mid-latitudes (e.g., Byrne et al., 2009; Dundas et al., 2018; Khuller & Christensen, 2021). The precise nature (grain size and dust content) of this exposed ice is currently uncertain, and estimates of the time scale of martian snow metamorphism range from decades/centuries (Clow, 1987; Kieffer, 1990) to millions of years (Branson et al., 2017). While the spectral albedo of dusty martian H₂O snow has been calculated for some cases (Clow, 1987; Cull et al., 2010; Gyalay et al., 2019; Kieffer, 1990; Singh et al., 2018), no model of dusty martian firn or glacier ice (i.e., H₂O ice with small air content) exists at present. Modeling the spectral albedo of dusty snow, firn, and glacier ice will allow for detailed characterization of the observed exposures of dusty martian ice, and help improve global climate models (GCMs) that are currently unable to completely replicate the spatial distribution of ice deposits on Mars (Haberle et al., 2017; Madeleine et al., 2014; Naar et al., 2020).

In this study, we model the spectral albedo of dusty martian H₂O snow and ice across the solar spectrum (0.3–3 μm), and compare our results with in-situ measurements of martian ice at the Phoenix landing site (Blaney et al., 2009; Smith et al., 2009).

2. Methods

We incorporate the spectral absorption properties of martian dust (Wolff et al., 2009) into models that have been used for mixtures of dust with terrestrial snow and glacier ice. Refractive index data for water-ice are taken from Warren and Brandt (2008). The densities of pure H₂O ice and martian dust are assumed to be 917 and 1,300 kg/m³, respectively (Arvidson et al., 2004; Moore & Jakosky, 1989; Moore et al., 1999).

To calculate the albedos of dusty martian snow, we use Mie theory and the delta-Eddington method (Dang et al., 2015; Joseph et al., 1976; Warren & Wiscombe, 1980). For dusty martian firn or glacier ice, we use a “specular delta-Eddington” model that was initially developed by Mullen and Warren (1988) for modeling the albedo of terrestrial lake ice. Subsequently, it was modified for firn or glacier ice, and validated against spectral measurements of terrestrial firn and glacier ice, some of which contained volcanic ash (Dadic et al., 2013). This model incorporates an optional specular flat surface overlying a scattering layer whose properties are calculated using the delta-Eddington method. The overlying specular surface occurs in glacier ice, and to some extent in firn, but not in snow, leading to the authors using a “specularity parameter” s, which varies from 0 for an optically rough, scattering medium such as snow, to 1 for a flat, smooth surface such as ice (Dadic et al., 2013). Details are provided in Appendix A.

We model a variety of snow and ice grain sizes (Table 1) because martian ice has been observed in-situ only at the Phoenix landing site, where no direct measurements of grain size, dust content or density were made (Smith et al., 2009). In order to distinguish between snow and ice, all snow densities greater than 550 kg/m³ were modeled as firn/glacier ice rather than snow because the primary scattering mechanism within dense ice is caused by the bubbles present within the ice rather than the snow grains (Warren, 2019). The “grain sizes” listed in Table 1 for glacier ice are derived from the Specific Surface Area (SSA; the area of air-ice interfaces per unit mass of ice), following Bohren (1983), who showed that bubbly ice can be modeled as very-coarse-grained snow. Unless specified, all calculations are performed assuming a solar zenith angle of 49.5° (the effective zenith angle for diffuse incidence), and assuming that the snow/ice is thick enough to obscure any underlying material (>~1 m; termed semi-infinite). The grain radius for dust is set to 1.8 μm for all calculations shown in Section 3, based on orbital and in-situ observations of martian dust that indicate
Model results are compared with measured albedo of observed in-situ excess ice (ice exceeding the soil pore space) at the Phoenix landing site (68.22°N, 234.25°E; Blaney et al., 2009; Smith et al., 2009). Note that ice within soil pore spaces was also observed at the Phoenix landing site (Blaney et al., 2009; Mellon et al., 2009; Smith et al., 2009), but in this work we are focusing on albedos of excess ice.

### 3. Results

#### 3.1. Grain Size Effects

Figure 2 illustrates how larger snow grains increase the likelihood of photon absorption (Dang et al., 2015), which leads to a reduction in albedo. There is a noticeable transition between snow albedos and firn/glacier ice albedos. In particular, the absorption features at 1.5 and 2 μm caused by overtones and combinations of fundamental vibrational modes become damped. At these two wavelengths the albedo is very small; the glacier ice has higher albedo than coarse-grained snow because of specular reflection.

#### 3.2. Martian Dust Effects

Martian dust is 3–9 orders of magnitude more strongly absorbing than H₂O ice at wavelengths 0.2–1.0 μm (Figure 1). Thus, small amounts of dust significantly reduce the single-scattering albedo \( \bar{\omega} \) of H₂O snow and ice. \( \bar{\omega} \) is a dimensionless parameter that ranges from 0 to 1: \( \bar{\omega} = 0 \) for pure absorption and \( \bar{\omega} = 1 \) for pure scattering.

The size of a dust grain (set to 1.8 μm in Figures 3 and 4) affects both its absorption and its scattering. A small dust grain has more surface area per unit mass than a large grain, so its scattering-to-mass ratio is higher. But the absorption-to-mass ratio is also higher in smaller grains, because the interior mass of a large dust grain is not exposed to radiation and is thus “wasted” for absorption (because incident photons are quickly absorbed in the outer shell of the grain and do not reach the interior). Which of these two effects dominates depends on the wavelength, and the size of the ice and dust grains, as will be discussed below.

Figure 3 illustrates how the addition of small amounts of dust (10⁻⁴%–0.1%, i.e., 1–1,000 ppm by weight) drastically alters the single-scattering albedo \( (1 - \bar{\omega}) \). Addition of dust to snow or ice dramatically increases \( (1 - \bar{\omega}) \) for \( \lambda < 1.2 \) μm, where the imaginary index of refraction for martian dust is several orders of magnitude larger than that of H₂O ice, causing greater absorption. Also note the general increase in \( (1 - \bar{\omega}) \) with ice grain size for the pure cases (black curves in Figure 3), because larger ice grains are more absorptive.
At longer wavelengths, the imaginary index of ice is more similar to that of dust, and the greater volume fraction of ice in the mixture causes the effect of even 0.1% dust to be almost negligible for snow (Figures 3a and 3b). For firm and glacier ice, the addition of dust >0.01% causes the coalbedo \((1 - \bar{\omega})\) to decrease at longer wavelengths, indicating an increase in scattering within the ice-dust mixture. The coalbedo is approximately proportional to \(r_n\); that is, the product of grain radius \(r\) and imaginary index \(n_i\) (Equation 10 of Bohren & Barkstrom, 1974). Although the imaginary index of dust is 1–2 orders of magnitude larger than that of ice at these wavelengths, the size of dust particles is 3–4 orders of magnitude smaller than the ice grains, allowing the addition of dust to reduce the coalbedo.

Figure 4 shows the effects of small amounts of martian dust on snow and glacier ice albedo. In fine-grained snow, dust reduces the albedo for \(\lambda < \sim 1.4 \mu m\) (Figure 4a). As the ice grain radius increases, dust causes greater reductions of albedo (Figures 4b–4d), because radiation penetrates more deeply in coarser grained snow and ice, where it encounters more absorbing material before reemerging at the surface. However, adding dust to coarser grained snow and ice causes an increase in albedo for wavelengths larger than 1.2, 1.0, and 0.9 \(\mu m\) in Figures 4b–4d, respectively. At visible wavelengths, successively adding more dust to firm or ice sequentially decreases the albedo, but eventually the absorption by dust saturates, and the scattering by dust then becomes dominant, so the firm or ice albedo begins to increase toward the spectrum of pure dust. This reversal happens at \(\sim 0.1%\) dust (Figures 4c and 4d).
4. Comparison With Phoenix Ice Data

At present, the only available in-situ observations of excess martian ice are from the Phoenix landing site, taken by the Phoenix Surface Stereo Imager (Lemmon et al., 2008). Orbital measurements of martian ice over the entire solar spectrum are available from CRISM and Observatoire pour la Minéralogie, l’Eau, les Glaces et l’Activité (OMEGA; Bibring et al., 2005), but atmospheric effects make quantitative analysis of dust content and ice grain size challenging. Thus, while the Phoenix measurements cover only part of the solar spectrum (\(\sim 0.45–1 \mu m\)), they can be compared with our model results for dusty snow and ice.

What was measured at the Phoenix landing site was the radiance reflected by the target (ice or soil), ratioed to the radiance reflected by a calibration surface (a “perfectly diffusive reflective white surface”), and corrected for the incidence angle of the sunlight (Drube et al., 2010; Zamani et al., 2009). The Phoenix team’s conversion from measured radiance to the albedo plotted in Figure 5 thus implicitly makes the default assumption that the soil or ice target is an isotropic reflector, since its actual bidirectional reflectance distribution function (BRDF) is unknown.

The soil albedo observed at the Phoenix landing site is well matched by the model result for 1.65 \(\mu m\) martian dust, suggesting that small amounts of dust may be present on the surfaces near the ice (Figure 5). The
amount of dust needed to completely obscure underlying soil and ice is 0.6 mm (equivalent to 10 e-folding depths), based on the absorption coefficient of martian dust at these wavelengths. Scattering by dust would make an even thinner layer opaque, so 0.6 mm is an upper limit.

From Figure 4, it is apparent that the grain size of the Phoenix ice must be less than 2.5 mm, because the addition of small amounts of dust does not increase firn or glacier ice albedo at wavelengths near 1 μm enough to match the Phoenix ice albedo of ∼0.6 (Figure 5). Although the solar incidence angle for the Phoenix observations (~45°) is slightly smaller than the angle used in Figure 4 (49.5°), this difference in angle causes a negligible effect at these wavelengths (Wiscombe & Warren, 1980).

We varied the snow grain size to produce an agreement between modeled and observed albedo near wavelengths of 1 μm, where dust amounts up to 0.1% have minimal effect. This procedure led to us obtaining a snow grain radius of 350 μm (SSA = 9.3 m²/kg). However, at shorter wavelengths, all grain sizes for clean snow produce albedos that exceed the Phoenix observations. To match the Phoenix observations, the inclusion of 0.015% dust (1.3 μm grain size) was required (Figure 5). This result constrains the output from previous modeling work by Cull et al. (2010), who reported that this relatively friable ice contained up to 1% dust. Unfortunately, Cull et al. (2010) were unable to derive grain-size estimates because the bidirectional reflectance model they used would have required observationally determining the scattering properties (asymmetry parameter) of the ice to model the ice-dust mixture, which was not done at the Phoenix site. More recently, Gyalay et al. (2019) used a third, separate technique to estimate that the ice content was up to 93% by volume, or 87% by weight, based on their assumed densities for soil and ice. That result was obtained for a modeled ice grain size of ∼1 mm, with 60 μm soil grains (Gyalay et al., 2019). However, their estimated ice content is likely to be too low, because dust contents greater than 1% make dusty snow and ice indistinguishable from pure dust at visible wavelengths (see Figure S2 in Khuller & Christensen, 2021).

5. Discussion

5.1. Detecting H₂O Ice Using HiRISE Images

Data with high spectral resolution but relatively coarse spatial resolution (~18 m/pixel) are available from the Compact Reconnaissance Imaging Spectrometer for Mars (CRISM; Murchie et al., 2007). For finer spatial resolution, recent attempts to identify H₂O ice on Mars (e.g., Byrne et al., 2009; Dundas et al., 2018; Dundas et al., 2021) have used relative color variations within images formed from three bands in the visible and near-infrared (VNIR), with pixel size ~25 cm, from the High Resolution Imaging Science Experiment (HiRISE; McEwen et al., 2007). The mapping of wavelength to false color is 0.874 μm (red), 0.694 μm (green), and 0.536 μm (blue). In stretched false-color images of likely ice-hosting features on Mars, materials considered to be ice are those that appear relatively “blue” (i.e., having relatively higher values in HiRISE’s 0.536 μm filter).

However, our modeling results indicate that snow metamorphism and the addition of small amounts of dust drastically alter the albedo of H₂O ice at the HiRISE filter wavelengths. Thus, it is possible for ice to appear relatively yellow/white in stretched, false-color HiRISE images even though it contains less than 1% dust (Khuller & Christensen, 2021). Dusty ice can also resemble nearby lithic material at visible wavelengths, when present as firn or glacier ice with 0.01%–0.1% dust (Figures 4c and 4d), potentially like some of the recently discovered scarps thought to be ice-rich (e.g., Figure 2 of Dundas et al., 2021). Greater amounts of dust (>1%) will cause snow and ice to be indistinguishable at these wavelengths.
5.2. Origin of Ice at the Phoenix Landing Site

A dusty-snow origin for the excess ice at the Phoenix landing site is questioned primarily because of the presence of a few pebbles overlying the ice (Mellan et al., 2009; Sizemore et al., 2015). While a few models have been proposed to explain the presence of excess ice in the near-surface (Fisher, 2005; Sizemore et al., 2015), those models are difficult to reconcile with decameter-thick ice deposits being discovered throughout the mid-latitudes of Mars (Byrne et al., 2009; Dundas et al., 2018; Khuller & Christensen, 2021). These recent discoveries indicate that it is more likely that the majority of shallow ice on Mars was deposited as dusty snow, which was subsequently buried through lag buildup and aeolian activity. Snow burial likely caused dust, pebbles and boulders to be incorporated within and on top of the ice, as is seen on Earth (Boulton, 1978). Alternatively, lithics within and on top of the ice might also have been emplaced by impact ejecta and/or meteorites.

The optical properties of the Phoenix ice are also consistent with formation by dusty snowfall. Ice formed through vapor diffusion into soil pore spaces would imply a far greater soil:ice ratio (>~50% soil), which is not consistent with the light-toned ice’s high visible albedo. Additionally, ice in the form of lenses (Fisher, 2005; Sizemore et al., 2015) is typically dense, and therefore will not have enough scatterers (bubbles) to explain the Phoenix ice’s relatively high albedo. Thus, the Phoenix ice likely formed by snowfall with small amounts of dust in the snow, which has metamorphosed over time into 350 μm-sized snow grains.

A 350-μm grain size for the ice exposed at the Phoenix landing site is smaller than the 700–800 μm grain size derived for H2O ice at the northern residual polar cap (Langevin et al., 2005). However, the H2O ice at the cap experiences seasonal deposition and sublimation of H2O and CO2 frost, which can mobilize dust and alter local thermal and atmospheric conditions (Kieffer, 1990; Langevin et al., 2005). These frost processes make it difficult to constrain the age and rate of metamorphism of the cap H2O ice. In contrast, the Phoenix ice was buried under a few centimeters of soil and dust. Radar detections suggest that the Phoenix ice might extend to depths of 9–66 m (Putzig et al., 2014). Assuming an average depth (or equivalently, subsurface ice thickness) of 40 m, this amount of ice could have been deposited as dusty snow over the last 1 Myr (Madeleine et al., 2014).

If the Phoenix ice originated as snow long ago, how did it evolve over this time? The cross-sectional area of snow grains (or equivalently, the snow grain size squared) grows linearly with time (Gow, 1969; Linow et al., 2012; Stephenson, 1967). Thus, if the age of the Phoenix ice is 1 Myr and the initial snow grain size was 50 μm, the rate of snow metamorphosis for the Phoenix ice is 0.12 μm2/yr, which is an order of magnitude smaller than the rate of snow metamorphosis we derive using estimates from Bramson et al. (2017). Bramson et al. (2017) obtained a value of 86 Myr for 550 kg/m3 firn to metamorphose into 917 kg/m3 density glacier ice. Based on Table 1, these density values correspond to grain sizes of 500 μm and 16 mm. Assuming these grain sizes implies that the rate of snow metamorphosis is \((16 \text{ mm})^2 - (0.5 \text{ mm})^2)/(86 \times 10^6 \text{ years}) = 3 \mu m^2/\text{ yr}.

Note that these predicted rates of snow metamorphosis can vary greatly with local atmospheric and thermal conditions. For example, ice that might have experienced melting (Khuller & Christensen, 2021) is likely to have experienced far greater rates of metamorphism, because even slight melting results in a rapid increase in grain size (Warren, 2019).

6. Summary

We have modeled the effects of dust and snow metamorphism on the albedo of martian H2O ice. Both effects generally cause a reduction in albedo, thereby increasing the energy absorbed by the ice. For example, dust can reduce the albedo of fine-grained snow (100 μm) from its pure-snow value of 0.9–1.0, down to 0.4–0.7, for wavelengths short of 1 μm. Similarly, the addition of small amounts of dust (10−6%–1%) reduces the visible albedo of bubbly H2O ice to as low as ~0.1, depending on the wavelength and the grain size of the ice. In some cases, the albedo of dusty firn and glacier ice can be lower than that of pure dust alone. At long wavelengths, 0.1% dust can increase the albedo of coarse-grained snow, firn and glacier ice (Figure 4).

Accounting for these radiative effects is crucial to our understanding and detection of exposed H2O ice on Mars. 0.01%–0.1% dust can render firn and glacier ice indistinguishable from pure dust at HiRISE filter.
wavelengths. Thus, dusty snow and ice might not necessarily appear relatively blue in stretched, false-color images (as is commonly assumed for martian H$_2$O ice).

Our model matches measurements of the excess ice observed at the Phoenix landing site, using 350 μm snow with 0.015% dust. This result suggests that the snow has not yet metamorphosed into glacier ice. If the results of our modeling are incorporated into GCMs, simulations of the distribution of martian H$_2$O ice may be improved. In addition, results can be compared with currently ongoing studies of atmospherically corrected orbital measurements of H$_2$O ice using hyperspectral CRISM data (e.g., Pascuzzo et al., 2019).

Appendix A: Calculating Albedos for Dusty Martian Snow and Ice

A1. Albedo of Pure Snow and Dust

For snow and dust, we use a Mie scattering code (Mätzler, 2002) to calculate single-scattering quantities separately, based on terrestrial models for snow (Dang et al., 2015; Wiscombe & Warren, 1980). Scattering by snow and dust grains is assumed to be similar to spheres in their far fields, with radii $r_{\text{ice}}$ and $r_{\text{dust}}$, respectively, in meters. Each particle has volume $V = \frac{4}{3}\pi r^3$ and cross-sectional area $2\pi r^2$ (i.e., $V_{\text{ice}}, V_{\text{dust}}, A_{\text{ice}}, A_{\text{dust}}$). Based on the formulas for scattering by spheres, the complex refractive index and the dimensionless size parameter $x$ are required as inputs to the Mie scattering code:

$$x = \frac{2\pi r}{\lambda}. \quad (A1)$$

where $\lambda$ is the wavelength, and $r$ is the average grain radius of the material. We then calculate the following quantities for ice and dust, separately:

$$\sigma_{\text{ext}} = \pi r^2 Q_{\text{ext}}, \quad (A2)$$

$$\bar{\omega} = \sigma_{\text{scat}} / \sigma_{\text{ext}}, \quad (A3)$$

$$g = \text{mean} \left( \cos(\theta) \right). \quad (A4)$$

where $\sigma_{\text{ext}}$ is the extinction cross section, $Q_{\text{ext}}$ is the dimensionless extinction efficiency, $\bar{\omega}$ is the single-scattering albedo, $\sigma_{\text{scat}}$ is the scattering cross section, $g$ is the asymmetry factor and $\theta$ is the scattering angle (see van de Hulst (1957) for a review of these terms). The absorption and scattering efficiencies $Q_{\text{abs}}$ and $Q_{\text{scat}}$ are also calculated for each material. Note that we average over a small range of grain radii ($r \pm r/10$) to eliminate the ripple generated by Mie theory for a monodispersion (Wiscombe & Warren, 1980).

Using the delta-Eddington approximation for a semi-infinite material, we obtain the following transformations for $g$ and $\bar{\omega}$:

$$\bar{\omega}_g = \frac{(1 - g)^2 \bar{\omega}}{1 - g^2 \bar{\omega}}. \quad (A5)$$

$$g_* = \frac{g}{1 + g}. \quad (A6)$$

The spectral albedo, $A$ of pure snow and dust is given by (for the semi-infinite case):

$$A \left( \mu_0 \right) = \frac{\bar{\omega}_g - b_* \xi \mu_0}{1 + P \left( 1 + \xi \mu_0 \right),} \quad (A7)$$

where $\mu_0$ is the zenith cosine ($\mu_0 = \cos(\theta_{\text{zenith}})$ for a given zenith or solar incidence angle $\theta_{\text{zenith}}$, $b_* = g_* / \left( 1 - \bar{\omega}_g g_\ast \right)$, $\xi = \left[ 3 (1 - \bar{\omega}_g g_\ast) (1 - \bar{\omega}_g) \right]^{1/2}$ and $P = 2 \xi / \left( 3 (1 - \bar{\omega}_g g_\ast) \right)$).
A2. Albedo of Pure Firn and Glacier Ice

The spectral albedo of firn and glacier ice accounts for specular reflection at the surface. Based on terrestrial
models for firn and glacier ice, the albedo $A$ is given by:

$$A = sR_i + \frac{(1 - sR_i)(sA_{trans} + (1 - s)A_{inc})(1 - sR_2)}{1 - sR_2A_d}, \quad (A8)$$

where $s$ is the “specularity parameter,” which specifies the degree to which the upper surface behaves spec-
ularly; $s$ ranges from 0 to 1. $R_i$ is the external reflection coefficient (air to ice), whereas $R_2$ is the internal
reflection coefficient (ice to air). $A_{inc}$ and $A_{trans}$ are the delta-Eddington albedos calculated for incident and
transmitted light at angles $\theta_i = \theta_{zenith}$ and $\theta_i$, respectively. Scattering is assumed to take place only due to air
bubbles within the ice. Snell’s law gives:

$$\frac{n_2}{n_1} = \frac{\sin(\theta_i)}{\sin(\theta_t)} = m_{rat}, \quad (A9)$$

where $m_{rat}$ is the ratio of the real components of the refractive indices of the transmitted media to the inci-
dent media (i.e., air: 1 and ice: $n_{ice}$).

Squaring the cosine of the transmitted angle $\left(\cos(\theta_t)\right)^2 = \mu_{trans}^2$ gives

$$\mu_{trans}^2 = 1 - \frac{1 - \mu_0^2}{m_{rat}}. \quad (A10)$$

If $\mu_{trans}^2 \leq 0$, the light undergoes total internal reflection and the net transmittance $T = 0$. In all other cases,
from Fresnel laws, the polarized reflectances $R_s$ and $R_p$ are given by:

$$R_s = \left(\frac{\mu_0 - m_{rat}\mu_{trans}}{\mu_0 + m_{rat}\mu_{trans}}\right)^2, \quad (A11)$$

$$R_p = \left(\frac{\mu_{trans} - m_{rat}\mu_0}{\mu_{trans} + m_{rat}\mu_0}\right)^2. \quad (A12)$$

These reflectances are used to calculate the transmittance $T$:

$$T = 1 - \left(R_s + R_p\right)/2. \quad (A13)$$

$T$ is then used to calculate $R_2$:

$$R_2 = 2\int_0^1 f(R_2)d\mu, \quad (A14)$$

where $f(R_2) = 1 - T_s\mu$. $T_s$ can be calculated using the transmittance equations above, using the refractive
index for air-to-ice $(m_{rat} = 1/n_{ice})$, and $\mu$ (which is integrated from 0 to 1 for the entire hemisphere).

To calculate $A_d$,

$$A_d = 2\int_0^1 f(A_d)/R_2 d\mu, \quad (A15)$$

where $f(A_d) = A(\mu)(1 - T_s)\mu$. $A(\mu)$ is the delta-Eddington albedo for a direct beam at incidence angle
$\cos^{-1} \mu$.

To compute the delta-Eddington albedos ($A_{inc}, A_{trans}, A(\mu),$ and $A_d$), the specific surface area (SSA) is
used as an input, along with firn or glacier ice density $\rho_{ice}$. The porosity $\phi$ is then related to the density by
$\rho_{ice} = (1 - \phi)917$.

For absorption by firn/glacier ice, the absorption coefficient $\beta_{abs}$ can be calculated from the imaginary part
of the ice refractive index $k_{ice}$ and the ice porosity.
The scattering coefficient \( \beta_{\text{sca}} \) can be calculated using the SSA and the porosity,

\[
\beta_{\text{sca}} = \frac{\text{SSA}(1 - \phi)917}{2} = \frac{(\text{SSA})\rho}{2}.
\]  

(A17)

This is the same as Equation 5 of Dadic et al. (2013). Adding \( \beta_{\text{sca}} \) and \( \beta_{\text{abs}} \) gives the extinction coefficient \( \beta_{\text{ext}} = \beta_{\text{abs}} + \beta_{\text{sca}} \). The asymmetry parameter \( g \) is taken from Mullen and Warren (1988). The single-scattering albedo \( \bar{\omega} \) and the optical depth \( \tau \) are then given by \( \bar{\omega} = \beta_{\text{sca}} / \beta_{\text{ext}} \) and \( \tau = z / \beta_{\text{ext}} \), where \( z \) is the thickness of the ice. \( z \geq 20 \) m is assumed to be optically semi-infinite, so \( z = 20 \) m is assumed for all cases.

### A3. Albedo of Dusty Snow, Firn, and Glacier Ice

For snow and dust, using the absorption and scattering efficiencies \( Q_{\text{abs}} \) and \( Q_{\text{sca}} \) and the areas of the particles, the absorption and scattering cross-sections can be calculated:

\[
\sigma_{\text{abs}} = Q_{\text{abs}} A.
\]

(A18)

\[
\sigma_{\text{sca}} = Q_{\text{sca}} A.
\]

(A19)

The mass absorption and scattering coefficients can then be derived for snow and dust:

\[
k_{\text{abs}} = \frac{\sigma_{\text{abs}}}{m} = \frac{\sigma_{\text{abs}}}{\rho V}.
\]

(A20)

\[
k_{\text{sca}} = \frac{\sigma_{\text{sca}}}{m} = \frac{\sigma_{\text{sca}}}{\rho V}.
\]

(A21)

For firn and glacier ice, mass absorption and scattering coefficients can be derived using \( \beta_{\text{abs}} \) and \( \beta_{\text{sca}} \):

\[
k_{\text{abs}} = \frac{\beta_{\text{abs}}}{\rho_{\text{ice}}},
\]

(A22)

\[
k_{\text{sca}} = \frac{\beta_{\text{sca}}}{\rho_{\text{ice}}},
\]

(A23)

Let the mass fractions of ice and dust be given by:

\[
C_{\text{ice}} + C_{\text{dust}} = 1.
\]

(A24)

The absorption, scattering, and extinction coefficients of the mixture (\( \beta_{\text{abs,net}}, \beta_{\text{sca,net}}, \) and \( \beta_{\text{ext,net}} \)) can be calculated by weighting the previously derived properties of ice and dust by their respective mass fractions,

\[
\beta_{\text{abs,net}} = C_{\text{ice}} \rho_{\text{ice}} k_{\text{abs,ice}} + C_{\text{dust}} \rho_{\text{dust}} k_{\text{abs,dust}},
\]

(A25)

\[
\beta_{\text{sca,net}} = C_{\text{ice}} \rho_{\text{ice}} k_{\text{sca,ice}} + C_{\text{dust}} \rho_{\text{dust}} k_{\text{sca,dust}},
\]

(A26)

\[
\beta_{\text{ext,net}} = \beta_{\text{abs,net}} + \beta_{\text{sca,net}}.
\]

(A27)

note that \( \rho_{\text{ice}} \) is the density of the pure snowpack, firn or glacier ice being modeled. Using these coefficients, the single-scattering albedo, optical depth, and the asymmetry parameter of the mixture (\( \bar{\omega}_{\text{net}}, \tau_{\text{net}}, \) and \( g_{\text{net}} \)) can be found:

\[
\bar{\omega}_{\text{net}} = \beta_{\text{sca,net}} / \beta_{\text{ext,net}},
\]

(A28)

\[
\tau_{\text{net}} = z / \beta_{\text{ext,net}}.
\]

(A29)
\[ g_{\text{net}} = \frac{C_{\text{ice}} R_{\text{ice}} k_{\text{ice, ice}} + C_{\text{dust}} R_{\text{dust}} k_{\text{dust, ice}}}{C_{\text{ice}} R_{\text{ice}} k_{\text{ice, ice}} + C_{\text{dust}} R_{\text{dust}} k_{\text{dust, dust}}}. \]  

(A30)

The spectral albedo of dusty snow and glacier ice can then be found using Equation A7 and A8, respectively, with the mixture optical properties derived in Equations A28–A30.

Data Availability Statement

Phoenix Surface Stereo Imager (SSI) data are available at https://pds-imaging.jpl.nasa.gov/data/phoenix/phxssi_1xxx/. Results and data from the snow or ice-dust spectral model are available in Khuller et al., (2021) (repository: https://doi.org/10.5281/zenodo.4653768).

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Data Availability Statement

Phoenix Surface Stereo Imager (SSI) data are available at https://pds-imaging.jpl.nasa.gov/data/phoenix/phxssi_1xxx/. Results and data from the snow or ice-dust spectral model are available in Khuller et al., (2021) (repository: https://doi.org/10.5281/zenodo.4653768).
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