Student's Concepts of Subtraction in Mathematics Classroom Using Lesson Study and Open Approach

S Phukongchana¹,², S Thinwiangthong¹,²

¹ Mathematics Education, Faculty of Education, Khon Kaen University, Thailand
² Centre of Excellence in Mathematics, CHE, Bangkok 10400, Thailand

Author’s e-mail address: sawitree_pokky@hotmail.com, sampan@kku.ac.th

Abstract. The purpose of this research was to analyze mathematical concept of students through mathematical representation in the classroom of using Lesson Study and Open Approach. The target group of this research was 25 Grade 1 students at Bansomkob (Ratradbamrung) School, Kaset Sombun District, Chaiyaphum Province in the academic year 2016. The school has been participating in the Project of Development of student’s Mathematics Higher-Order Thinking at North East of Thailand, and also the professional development process through Lesson Study and Open Approach of Inprasitha's framework. The collected data included: 1) protocol data transcribed from voice and video recorders during classroom instruction 2) students' writings and 3) protocol from interviews with students. The data were analyzed by using Vergnaud’s conceptual framework. The results of this research indicated that student’s concepts of subtraction in mathematics classroom using Lesson Study and Open Approach consisted of 1) set of situations: students were involved to the real world situations which was meaningful to them so they could describe the situations while ‘take away’ and the number of objects in the situations decreased and ‘Take different’ they could describe situations that there was one more thing than one. 2) set of operational invariant: students’ concept of subtraction ‘take away’ students presented the problem solving by removing objects from the situations so the number was decreased, while the similar or analogous operation occurred in each situation repeatedly in relation to other situations until become operational invariant and students’ concept of subtraction ‘take different’ students presented the problem solving by operation of matching for comparison was also found before figuring out the emergent number of the concept on subtraction of taking different. and 3) set of symbolic representations: students used symbols as sentences or words through the forms of natural language such as being ‘expelled, ‘scooped out’, ‘disappeared’, ‘none’, ‘given’, in concept of ‘take away’ and ‘over’ in concept of ‘take different’ as well as the sentences or words in mathematical language and other symbols which included number sentence, mathematical sign drawing a line, circle, and arrow for representing the operation of being ‘took out’, ‘eaten’, ‘gone, and ‘took away’ whereas concept of ‘take different’ student drew line to matching one to one thing and drawing a line, circle, and arrow for representing the operation of ‘over’.
1. Introduction
The method of mathematics instruction that involves the component of only teacher, students, and content, as teacher only plays a role of transferring the content to students by teaching them to remember and focusing on informing, explaining, and learning achievement through the test rather than emphasizing on learning process, does not promote students to think reasonably, and there is no mathematical learning process among them. As a result, there is no context to promote mathematical learning process for students. In addition, student’s thinking is obstructed. So traditional classroom is required to change as a new classroom, which emphasizes on both process and result [4] Mathematics classroom that employs learning management using Lesson Study and Open Approach innovations is considered as a problem solving classroom through the Open Approach as a teaching approach [5]. These innovations are adapted to use in Thailand by the Center for Research in Mathematics Education, Khon Kaen University. Takahashi [12] stated that the Lesson Study is a process that helps improving mathematics instruction and significantly affects learning process of both teacher and student: it helps promoting the ability of adapting knowledge and understanding gaining from classroom. Inprasitha [7] indicated the clear characteristics of Lesson Study system in which teachers shall jointly discuss and develop learning management tools and plans, and mainly focuses on the use of open-ended questions. This will be based on the contexts of student’s learning and thinking in association with Open Approach. The aim of teaching through Open Approach is to emphasize on encouraging all students to be able to think and solve mathematical problems by self-learning, and expand or increase the quality of mathematical process [9].

One of the significant mathematical processes, according to the National Council of Teachers of Mathematics [11], is mathematical representation which is needed for learning mathematics through problem solving. This is consistent with Despina [2] who mentioned that representation is considered as a key of solving mathematical problems. The ability of dealing with mathematical representation is a significant component of mathematical writing and reading abilities in which students must be promoted to use representation in order for figuring out the ways of problem solving [17].

In terms of classroom contexts emphasizing on problem solving, Isoda and Nakamura [8] indicated the significant representations on expressing the ideas in the classroom of problem solving, such as sentence, diagram, arrow. These representations are categorized in the symbol representation, which can be the facilitator for communicating the linkage and relation of operational invariant through the concept [14]. Vergnaud [15] and Inprasitha [6] also stated that the integration of this concept can be initiated in the Open Approach classroom while students are trying to solve the problems throughout searching, collecting, designing and presenting various approaches.

As a result, the mentioned discussion represents a significance of representation which affects the communication of student’s idea and problem solving. The representation is related to operational invariant resulting into the concept. This can be happened in the classroom that emphasizes on solving problem using Open Approach for instructional management as it widely opens for problem solving and promotes students towards various thinking. Student also deals with problem solving, searching, collecting, and designing by oneself. Thus, the researcher is interested in analysing student’s concepts on emergent representations towards mathematical problem solving on subtraction in the classroom using Lesson Study and Open Approach.

2. Research Objectives
This research aimed to analyze students’ concepts on subtraction through mathematical representations in the classroom using Lesson Study and Open Approach.
3. Research Methodology

3.1. Target Group

The target group of this research was 25 Grade 1 students at Bansomkob (Ratradbamrung), Sraponthong Sub-district, Kaset Sombun District, Chaiyaphum Province during the first semester of academic year 2016.

3.2. Research Instruments

- Mathematics textbook for Grade 1 students translated from Japanese to Thai by Center for Research in Mathematics Education.
- The 7 lesson plans for unit 5 of mathematics learning management for Grade 1 students entitled Subtraction 1;
- Video recorder for recording classroom instructional activities, which will be used as a key data for analysis;
- Audio recorder for recording voices of teacher and students during instructional activities, which will be used as a key data for analysis. Voices were recorded by researcher before starting classroom;
- Camera for capturing students’ pieces of work and classroom instructional activities, which will be used as a part of data analysis;

4. Data Collection

4.1 Researcher spent a month to study general background of the school and target classroom of data collection. Learning behaviors of students were observed. Mathematics teacher and homeroom teacher were asked for additional information. School director was also asked for the use of Lesson Study and Open Approach innovations of the classroom.

4.2 Researcher employed the Lesson Study to create the lesson plan of Open Approach. Each class involved 5 persons to jointly create the plan, including the teacher of Grade 1 student, the school director who had jointly observed Grade 1 classroom throughout academic year, and 3 master degree students majoring in Mathematics Education from the Faculty of Education, Khon Kaen University.

4.3 The meeting between researcher and research assistants was held for understanding the framework of data analysis, as well as clarifying the roles of researcher and research assistants on recording voices, still photographs, and videos, as a preparation before conducting data analysis.

After that, the created lesson plans were implemented to instructional management at the target classroom. The classroom was collaboratively observed. There were also comprehensive data record and collection according to the plans mentioned above. The reflection was then conducted by considering students’ ideas during classroom activities and their pieces of work, and the issues related to representations raised by students in the class. During the lesson plan that was being created, there was a discussion about students’ ideas that came out during instructional activities.

5. Data Analysis

This research employed qualitative research methodology. Data was the behaviors during teaching and learning mathematics on subtraction that were recorded by video recorder and audio recorder. The wordings were transcribed as the item of wordings or dialogue. Students’ behaviors were also described as shown in the brackets, which called protocol transcription. In addition, pieces of writing of target students written during classroom activities were brought to analyze towards mathematical concepts of students on subtraction based on conceptual framework of Vergnaud [15] and Inprasitha [6].
6. Results
Components of students’ mathematical concepts were as follows.

6.1. Set of Situations:
Problem situations meant to students, which related to students’ real world. Students could explain those situations with the concept of subtraction on taking away as it represents the existence and disappearance, or number of the objects in that situation decreased (See Figure 1). And the concept of subtraction on taking different as the problem situations was related to real world and meant to students towards overviewing number of a thing that was much more than the other (See Figure 2).

6.2. Operational Invariant:
Students presented the problem solving by removing the objects from the situation so the number decreased (See Figure 3 and table 1). The operation of each situation occurred similarly or analogously, as well as occurred repeatedly with other operation until become invariant. The operation of matching for comparison was also found before figuring out the emergent number of the concept on subtraction of taking different (See Figure 4).

![Figure 1](image1.png)

**Figure 1.** An example of situation in concept ‘take away’ that means to students

![Figure 2](image2.png)

**Figure 2.** An example of situation in concept ‘take different’ that means to students

![Figure 3](image3.png)

**Figure 3:** An example of students’ operations in concept ‘Take away’
Table 1. An example of students’ operational protocol

| Item   | Teacher/Meen | Question/Response |
|--------|--------------|-------------------|
| 164    | Teacher      | How many sheets of paper did you fold? |
| 165    | Meen         | Four sheets.       |
| 166    | Meen         | Draw the pictures of nine blocks on the paper sheet. |
| 167    | Teacher      | How many folded sheets? How many in total? |
| 168    | Meen         | Nine.              |
| 169    | Teacher      | So there are nine in total. |
| 170    | Meen         | Take away, draw the circle covering four blocks then draw the arrow pointing out. |
| 171    | Meen         | Four, Five, Six, Seven. |

Figure 4. An example of students’ operations in concept ‘Take different’

Table 2. An example of students’ operational protocol

| Item   | Teacher/Pam/Fam/Student | Question/Response |
|--------|-------------------------|-------------------|
| 208    | Teacher                 | How to prove that? |
| 209    | Pam                     | Matching          |
| 210    | Teacher                 | Ah matching and then, which’s it over. |
| 211    | Fam                     | Second (He pointed to two blocks in blackboard.) |
| 212    | Teacher                 | Was two pieces that you took away. |
| 213    | Fam                     | Yes.              |
| 214    | Teacher                 | What is that arrow mean? |
| 215    | Fam                     | Take away         |
| 216    | Teacher                 | Take away. So, are their equal? |
| 217    | Student                 | Equal             |
| 218    | Teacher                 | Why you take two this block away? |
| 219    | Student                 | It’s over         |

6.2. Set of Symbolic Representations:
The representations were made for explaining and communicating the ideas of solving operational problem in that problem situation through symbolic representations of operational invariant. This was conducted for solving the set of open-ended problem situation towards subtraction, which included 1) sentences of students’ natural language: ‘expel’, ‘take out’, ‘scoop out’, ‘fly away’ for concept of ‘take away’ (See Figure 5 and table 3) and ‘over’ for concept of ‘take different’(See Figure 8 and table 6) 2) numeral symbol and mathematical symbol: subtraction (See Figure 4 and table 7 for ‘take away’ and Figure 10 and table 8 for ‘take different’) and 3) diagram: ‘drawing a circle’ ‘arrow’ as a part of operation in order for representing the meaning of that operation for concept of ‘take away’ (See Figure 6 and table 4) and ‘drawing line matching’ for comparison was also found before figuring out the emergent number of the concept on subtraction of taking different and ‘drawing a circle’ ‘arrow’ for representing. (See Figure 9 and table 7)
Figure 5. An example of a diagram as a symbolic representation in concept ‘take away’

Table 3. An example of a diagram protocol as a symbolic representation

| Item   | Participant | Action |
|--------|-------------|--------|
| 166    | Meen        | draw the pictures of nine blocks on the paper sheet |
| 167    | Teacher     | How many sheets that were folded? How many in total? |
| 168    | Meen        | Nine |
| 169    | Cheppe      | So there are nine in total |
| 170    | Meen        | Take away, draw the circle covering four blocks then draw the arrow pointing out |

Figure 6. An example of a natural language as a symbolic representation in concept ‘take away’

Table 4. An example of a natural language protocol as a symbolic representation

| Item   | Participant | Action |
|--------|-------------|--------|
| 166    | Meen        | draw the pictures of nine blocks on the paper sheet |
| 167    | Teacher     | How many sheets that were folded? How many in total? |
| 168    | Meen        | Nine |
| 169    | Cheppe      | So there are nine in total |
| 170    | Meen        | Take away, draw the circle covering four blocks then draw the arrow pointing out |
| 171    | Teacher     | Look at the circle drawn by your friend. What does it mean? |
| 172    | Meen        | Take away |
| 173    | Chon        | Leave out |

Figure 7. An example of number sentence and mathematics symbol as a symbolic representation
Table 5. An example of a protocol of number sentence and mathematics symbol as a symbolic representation

| Item | Teacher | Note |
|------|---------|------|
| 222  | Teacher | Let’s see the work of other group. Pam, who wrote this number sentence? Tell your friends about it. |
| 223  | Pam     | Nine minus four equals the squares |
| 224  | Teacher | Well, listen to your friend. Look at the number sentence of your friend. So what is the number sentence of this group? Pam, try to say it again. |
| 225  | Pam     | Nine minus four equals the squares |
| 226  | Teacher | Well, nine minus four equals the squares. So, what is this? right here? What is this? Let’s look at your friend’s picture. Pam, you drew this one for me. What does it mean? What does it represent? |
| 227  | Students| Represent the block. |
| 228  | Teacher | Represent the block. Let’s see Pam’s picture. What is it? |
| 229  | Pam     | It’s block. |
| 230  | Teacher | So, what does block represent? |
| 231  | Pam     | Paper. |

Figure 8: An example of a diagram as a symbolic representation in concept ‘Take different’

Table 6. An example of students’ operational protocol

| Item | Teacher | Note |
|------|---------|------|
| 208  | Teacher | How to prove that? |
| 209  | Pam     | matching |
| 210  | Teacher | Ah matching and then, which’s it over. |
| 211  | Fam     | second (He pointed to two blocks in blackboard.) |
| 212  | Teacher | Was two pieces that you took away. |
| 213  | Fam     | Yes. |
| 214  | Teacher | What is that arrow mean? |
| 215  | Fam     | Take away |
| 216  | Teacher | Take away. So, are their equal? |
| 217  | Student | equal |
| 218  | Teacher | Why do you take two this block away? |
| 219  | Student | It’s over |

Figure 9. An example of a natural language as a symbolic representation in concept ‘Take different’
Table 7. An example of students’ operational protocol

| Item  | Role   | Statement                                                                 |
|-------|--------|---------------------------------------------------------------------------|
| 218   | Teacher| Why do you take two this block away?                                      |
| 219   | Student| It’s over                                                                  |
| 220   | Teacher| Two blocks are over. What does it mean?                                   |
| 221   | Pam    | It’s over, more than 5                                                    |
| 222   | Teacher| Can you write?                                                            |
| 223   | Pam    | (She wrote 7 more than 5 is 2)                                             |

![Image](image.png)

‘Mathematical language or Mathematical Signal’

Figure 10: An example of number sentence and mathematics symbol as a symbolic representation

| Item  | Role   | Statement                                                                 |
|-------|--------|---------------------------------------------------------------------------|
| 224   | Teacher| Seven more than five is two. Can we write in mathematical signal form?    |
| 225   | Phai   | Seven minus two                                                           |
| 226   | Teacher| Why can you describe                                                      |
| 227   | Pam    | No no it’s seven minus five                                               |
| 228   | Teacher| Can you write in blackboard.                                              |
| 229   | Pam    | )wrote 7-5 = 2)                                                           |
| 230   | Teacher| What is result.                                                           |
| 231   | Meen   | Two                                                                       |

Table 8: An example of students’ operational protocol

7. Conclusion
According to the findings of this research, the mathematical concepts were found in the classroom of using Lesson Study and Open Approach, which consisted of 1) Set of Situations: problem situations that were meaningful to students. Students were involved to the real world situations which was meaningful to them so they could describe the situations while ‘take away’ and the number of objects in the situations decreased and ‘Take different’ they could describe situations that there was one more thing than the one. 2) Operational Invariant: students’ concept of subtraction ‘take away’ students presented the problem solving by removing objects from the situations so the number was decreased, while the similar or analogous operation occurred in each situation repeatedly in relation to other situations until become operational invariant and students’ concept of subtraction ‘take different’ students presented the problem solving by operation of matching for comparison was also found before figuring out the emergent number of the concept on subtraction of taking different and 3) set of symbolic representations: set of symbolic representations: students used symbols as sentences or words through the forms of natural language such as being ‘expelled’, ‘eaten’, ‘scooped out’, ‘disappeared’, ‘none’, ‘given’, in concept of ‘take away’ and ‘over’ in concept of ‘take different’ as well as the sentences or words in mathematical language and other symbols which included number sentence, mathematical sign drawing a line, circle, and arrow for representing the operation of being ‘took out’, ‘eaten’, ‘gone, and ‘took away’ whereas concept of ‘take different’ student drew line to matching one to one thing and drawing a line, circle, and arrow for representing the operation of
‘over’. These represented the connected relation and were used to communicate about it in order for representing the meaning of operation at that time. In the contexts of Lesson Study and Open Approach as a teaching approach, teachers collaboratively develop the teaching as a team according to the steps of Lesson Study and Open Approach. The team collaboratively create lesson plan, observe the classroom activities, and reflect the teaching after instruction. These cooperative works facilitate students in the classroom to learn through expressing ideas, self-learning, discussing the results and summarizing them together. So these students could construct mathematical concepts.

Isoda [8] stated that the basic principle of mathematics instruction emphasizing on problem solving is teaching students to learn mathematics by oneself. This means the teacher must develop students to be able to think and solve the problems by themselves. And the representation is necessary for Grade 1 students as their reading and writing skills are at basic level, which is difficult for them to communicate their ideas. So the representation is necessary as a facilitator for communicating ideas of kids. Inprasitha [6] claimed that the use of Open Approach as a guideline for teaching can have teachers to considerably change their roles. During the second stage of teaching through the Open Approach, teachers have to change the role of lecturing or giving knowledge to be an observer in order to make a note of students’ ideas during solving problems by themselves. This research finding is consistent with [7] who introduced and adapted the Lesson Study and the model of problem solving to use in Thailand. The findings indicated the meaningful understanding in the classroom of Lesson Study and Open Approach towards integral number. It was also observed that each student had own style to use diagram and arrow. These represent the development of mathematics classroom and the change in students’ active learning. In other words, teaching mathematics in the classroom of Open Approach aims at preparing students to reach the challenge situations by using potential open-ended problems, which will respond to individual difference both in terms of abilities and understanding, and in order for supporting the development of the way of students’ mathematics thinking [6].

8. Recommendation

8.1. Recommendation for Implementation of Research Findings

8.1.1. This research can be a guideline for teachers to understand the students’ concepts in problem solving. And it can be used to predict students’ ideas and concepts in order for planning next classes’ learning management.

8.1.2. This information can facilitate teachers to use the meaningful situation, invariant feature, and the meaningful symbolic representations and to promote students to understand more and be able to better solve the problems.

8.2. Recommendation for Further Research

8.2.1. There should be other studies on students’ mathematical concepts through representations of mathematical operation apart from subtraction. As this research employed the mathematics textbook translated from Japanese textbook, it helped promoting various representations among students. Further research should study other concepts of mathematical skills apart from representations, e.g., communication.

8.2.2. There should be a study on the use of representations for promoting the concept construction among students with special need in order for help promoting mathematical thinking for students with special need.
9. Acknowledgments
This research is (partially) supported by The Centre of Excellence in Mathematics, The Commission on Higher Education, Thailand and Centre for Research in Mathematics Education, Faculty of Education, Khon Kaen University and The Project of Development of student’s Mathematics Higher-Order Thinking at North East of Thailand, Faculty of Education, Khon Kaen University.

References
[1] Changsri N 2015 First Grade Students’ Mathematical Ideas of Addition in The Context of Lesson Study and Open Approach (Conference Proceedings of ICMI Study 23. June) pp 3-7 2015 Macau, China.
[2] Despina A 2010 An examination of middle school students’ representation practices in mathematical problem solving through the lens of expert work: towards an organizing scheme (Springer Science Business Media B.V. 2010)
[3] Hitotsumatsu S 2010 Mathematics Textbook for Grade 1 Students (Translated by Maitree Inprasitha) Khon Kaen: Klang Nana Wittaya.
[4] Inprasitha M 2003 A Reform of Learning Process of Mathematics Subject in Schools Emphasizing on Mathematical Process (Khon Kaen: Khon Kaen Printing)
[5] Inprasitha M 2011 One Feature of Adaptive Lesson Study in Thailand: Designing a Learning Unit (Journal of Science and Mathematics Education in Southeast Asia 2011 vol 34) No 1 pp 47 – 66
[6] Inprasitha M 2015 An Open Approach Incorporating Lesson Study: An Innovation for Teaching Whole Number Arithmetic (Conference Proceedings of ICMI Study 23 June) pp 3-7 2015. Macau, China pp 315-322
[7] Inprasitha M 2015 Preparing Ground for the Introduction of Lesson Study in Thailand (Lesson Study Challenges in Mathematics Education) pp 109-122 Singapore. World Scientific Publishing Co. Pte.
[8] Isoda and Katagiri 2012 Mathematical Thinking, How to develop it in the classroom (Singapore: World Scientific Publishing Co Pte Ltd)
[9] Loifa S and Inorasitha M 2004 Development of New Teaching Profession Promoting Mathematics Learning (KKU Journal of Mathematics Education vol 1) chapter 1 pp 18-28
[10] Ministry of Education 2009 Basic Education Core Curriculum B.E. 2008 (Bangkok: The Agricultural Federative Co-operation of Thailand)
[11] National Council of Teacher of Mathematics 2000 Principles and standards for school mathematics (Reston: The National Council of Teachers of Mathematics, Inc)
[12] Takahashi A 2015 Lesson Study: An Essential Process for Improving Mathematics Teaching and Learning (Lesson Study Challenges in Mathematics Education) pp 51-58 Singapore. World Scientific Publishing Co. Pte.
[13] Vergnaud G 1982 Cognitive and Developmental Psychology and Research in Mathematics Education; some theoretical and methodological issues (For the Learning Mathematics vol 3) chapter 2 (November 1982)
[14] Vergnaud 1996 The Theory of Conceptual Fields Theories of Mathematical Learning (New Jersey: Lawrence eribaum associates Inc Publisher, 1996)
[15] Vergnaud 1997 The Nature of Mathematical Concepts. Learning and teaching mathematics (England: Psychology Press/Erlbaum (UK) Taylor & Francis)
[16] Vergnaud 2013 Conceptual Development and Learning (Revista Quurriculum vol 26) marzo 2013, pp 39-59
[17] Zhe L 2012 Survey of Primary Students’ Mathematical Representation Status and Study on the Teaching Model of Mathematical Representation (Journal of Mathematics Education, August 2012, Vol 5) No 1 pp 63-76