The $pp \rightarrow pp\pi^0$ Reaction near Threshold: A Chiral Power Counting Approach

Thomas D. Cohen\textsuperscript{a}, James L. Friar\textsuperscript{b}, Gerald A. Miller\textsuperscript{c}, and Ubirajara van Kolck\textsuperscript{c}

\textsuperscript{a} Department of Physics  
University of Maryland  
College Park, MD 20742

\textsuperscript{b} Theoretical Division  
Los Alamos National Laboratory  
Los Alamos, NM 87545

\textsuperscript{c} Department of Physics  
University of Washington, Box 351560  
Seattle, WA 98195-1560

Abstract

We use power-counting arguments as an organizing principle to apply chiral perturbation theory, including an explicit $\Delta$, to the $pp \rightarrow pp\pi^0$ reaction near threshold. There are two lowest-order leading mechanisms expected to contribute to the amplitude with similar magnitudes: an impulse term, and a $\Delta$-excitation mechanism. We examine formally sub-leading but potentially large mechanisms, including pion-rescattering and short-ranged contributions. We show that the pion-rescattering contribution is enhanced by off-shell effects and has a sign opposite to that of a recent estimate based on a PCAC pion interpolating field. Our result is that the impulse term interferes destructively with the pion rescattering and $\Delta$-excitation terms. In addition, we have modeled the short-ranged interaction using $\sigma$ and $\omega$ exchange mechanisms. A recoil correction to the impulse approximation is small. The total amplitude obtained including all of these processes is found to yield cross sections substantially smaller than the measured ones.
I. INTRODUCTION

The theory of the threshold behavior of the reaction $pp \rightarrow pp\pi^0$ has been studied for some time [1–8]. The numerical results of the early analyses [1–4] were that this behavior is dominated by the impulse term (Fig. (1a)). In this process, a single pion is emitted from a nucleon. Effects of initial- and final-state interactions are taken into account by using scattering wave functions that solve the Schrödinger equation incorporating the full nucleon-nucleon potential. In such a process at threshold, the pion will emerge in an s-wave. In the early calculations the main competitor process was thought to be “pion rescattering” from a pion seagull term in which the nucleon emits two pions and one of them is re-absorbed by the other nucleon, as illustrated in Fig. (1b). The early estimate of this pion-rescattering term gave a small contribution [1] and it was believed that the impulse term alone could account for the essential features of the data.

Good data for the $pp \rightarrow pp\pi^0$ reaction near threshold became available far later than these original theoretical estimates [9,10]. These data clearly show that the early theoretical calculations were inadequate. While the impulse term can account for the energy dependence of the cross section (once the phase space and Coulomb effects are treated properly) [3,4], the total cross-section is about a factor of five below the data. Clearly something is missing in the original theoretical analysis. To the best of our knowledge there have been two proposed explanations for the missing physics.

The first explanation, due to Lee and Riska [5], is that the original analysis is lacking important effects due to exchanges of shorter range. In particular, they suggest that an effect due to $\sigma$ and $\omega$ meson exchanges such as that shown in Fig. (1c) can account for the discrepancy. They find that if they use meson-nucleon coupling constants and meson masses obtained from meson-exchange potentials this process plus the impulse approximation reproduces the data within good accuracy. Their analysis was confirmed in the work of Horowitz, Griegel and Meyer [6]. This explanation is interesting in several ways: it is perhaps the first example of a shorter than pion-ranged meson-exchange effect playing such a critical role in an observable and it is based on a “z-graph” effect of scattering into and out of a negative energy state. Hence, the latter could provide further evidence for the validity of the Dirac phenomenology used in intermediate-energy $p$-nucleus collisions [11].

A second explanation has recently been proposed by Hernández and Oset [7]. They observed that the early estimates of the pion-rescattering term were based on the on-shell $\pi N$ scattering amplitudes, whereas in the contribution to the $pp \rightarrow pp\pi^0$ reaction one of the pions is well off-shell. Using models based on a pion interpolating field satisfying the partially conserved axial current (PCAC) requirement, they estimate that the effect of the off-shell behavior of the amplitude in the pion-rescattering term, together with the impulse term, is large enough to account for the data, although they note that at present the uncertainties are large.

Which of these explanations, if either, is the correct one? Clearly both effects cannot simultaneously be as large as the authors of the papers suggest unless there are some compensating effects. For example, if the amplitude of the impulse term were added to the amplitude for both of these effects, the resulting cross section would be approximately 2–2.5 times larger than the experimental cross section. A fundamental difficulty in trying to assess whether either of these explanations is correct is that almost all of the work on the
subject to date has been somewhat *ad hoc* in an important sense. The analyses have not
been based on a systematic expansion for which one can account for all processes without
double counting, and for which one has some *a priori* method of deciding which processes
should be small. In short, there has been no simple organizing principle.

This lack of an organizing principle has led to a reliance on intuition in deciding which
processes are likely to be important. There are an infinite number of processes one can
consider and the question is which processes will make significant contributions. Unfortu-
nately, intuition can be faulty. For example, consider the process in Fig.(1d) in which the
interaction generates a $\Delta$ that emits a pion and becomes a nucleon. This process has not
been included in any of the analyses to date. There is a simple intuitive reason why such an
effect should be small \[9\]: the $\pi N \Delta$ vertex proceeds only through p-waves in the center of
mass of the $\Delta$. At threshold, the pion is in an s-wave relative to the center of mass of the
entire system. Thus, one can get a nonvanishing amplitude for this process only because the
center of mass of the $\Delta$ may differ from the center of mass of the system, and one expects
such differences to be small \[4\]. Indeed, we estimate this effect below and find it small
compared to typical hadronic amplitudes. However, all of the effects discussed so far are
small in this sense. When we evaluate this effect numerically its amplitude is found to be
sensitive to the choice of the potential used to produce the scattering wave functions, being
in some cases similar in magnitude to these other effects.

The fact that all of these effects are small is no coincidence. Indeed, it is a straightforward
consequence of approximate chiral symmetry. Suppose, hypothetically, that we lived in a
chiral world in which the up- and down-quark masses were exactly zero. It is easy to establish
the theorem that the amplitude for emission of a pion at threshold must vanish. Of course,
in the real world the pion mass is not zero—but it also is not large compared to typical
hadronic scales. This suggests that the amplitude can be described in a systematic manner
as an expansion in $(Q/M)$, where $Q$ is a typical (small) momentum or energy scale and $M$
is a typical (large-mass) hadronic scale $\sim 1$ GeV, such as $m_N$, $m_\rho$ or $4\pi f_\pi$. This systematic
description is called chiral perturbation theory (\[\chi PT\]).

For the example just described, $Q \sim m_\pi$ characterizes the behavior of the outgoing pion
and one has $m_\pi/M \sim 0.1 - 0.2$. Our problem has an additional scale that is less obvious
(although certainly well known). The internal nuclear momentum is quite large, since the
pion mass must be generated entirely from the nuclear kinetic energy. We will show below
that this corresponds to $Q \sim (m_\pi M)^{1/2}$. This results in a larger than desirable expansion
parameter, $(m_\pi/M)^{1/2} \sim 0.4$, but one that nevertheless will allow a systematic expansion.

The simplest way to implement \[\chi PT\] is via effective Lagrangians aided by power-counting
arguments. The seminal idea was contained in a paper by Weinberg \[12\]. This idea was
developed systematically for interactions of mesons \[13\] and for interactions of mesons with
a baryon \[14,15\]. The generalization of these techniques to describe properties of more than
one baryon was also due to Weinberg \[16\] and was carried out in detail in Refs. \[17,18\]. Here
we will use Weinberg’s power-counting arguments to organize a calculation of the threshold
production of a $\pi^0$ in a $pp$ collision.

The general idea of power counting in effective Lagrangians is, in fact, far more powerful
than \[\chi PT\]. In principle, one can include in the Lagrangian and the power counting any
light degree of freedom (and not simply the Goldstone excitations). Indeed, the failure to
treat all relevant degrees of freedom explicitly may well lead to very slow convergence for
the expansion and may limit its usefulness. In the case of baryons an obvious light degree of freedom is the $\Delta$, since $m_\Delta - m_N$ is small, being about $2 \, m_\pi$, and the $\Delta$ is strongly coupled to the $\pi N$ channel. There remains some controversy over whether or not one should include the $\Delta$ explicitly. We strongly advocate the position that the $\Delta$ should be included explicitly. Moreover, we will show by explicit calculation that the leading-order amplitude associated with an explicit $\Delta$ is similar in magnitude to the impulse approximation and makes significant contributions. Hence, in our counting we will consider $m_\Delta - m_N$ to be of order $m_\pi$. We note that alternative approaches assume that this quantity is of order $M$. For pion production reactions a virtual $\Delta$ can have a small energy denominator ($\sim m_\pi$), and it makes little practical sense to treat that particle as far off shell.

The power counting has considerable power in organizing the calculation. It tells us which processes are expected to dominate. But it also provides strong constraints on the form of the low-energy Lagrangian, which in turn is reflected in constraints on effects of the off-shell dependence of the various sub-processes, in particular the seagull or pion rescattering. In principle, one ought to be able to apply the power counting in a completely systematic manner and work consistently to some given order. The effective theory has a finite number of free parameters at a given order that one fits from some set of observables. Once determined one can predict other observables. This approach has been used with great success in describing the properties of the pseudoscalar mesons [13]. In the present case, for technical reasons which we will discuss later, this procedure cannot be implemented in a practical way. Instead, we will use power counting as a guide to which processes we expect to be large.

This article is organized as follows: In the following section we briefly review Weinberg’s power counting scheme in general. Next, we discuss special features of this scheme for the present problem. Various terms are considered. We then derive the operators associated with the various leading-order processes in $\chi$PT: the impulse, $\Delta$-excitation, pion-rescattering, recoil correction, and short-range mechanism as modeled by $\sigma$ and $\omega$ exchanges. In the same section these amplitudes are evaluated using three different potentials to describe the initial and final state $pp$ interactions. The impulse term interferes destructively with the $\Delta$-excitation, and pion-rescattering amplitudes. The resulting cross sections that we compute are found to be far smaller than the measured ones, even when the $\sigma, \omega$-exchange term is included. Next these results are discussed, and the difference between the off-shell behavior of our seagull term and that seen in Ref. [7] is explained. The paper ends with a discussion of a possible direction for future research using $\chi$PT for pion production reactions.

A preliminary report of these results was presented elsewhere [19]. Before this manuscript was completed, a paper appeared [20] that also uses $\chi$PT to reach the same conclusion regarding the interference between impulse and seagull terms. Our work differs from theirs in that we modify Weinberg’s power counting to the particular kinematics of this problem, and that we consider the $\Delta$ explicitly.

II. REVIEW OF WEINBERG’S POWER COUNTING

Three-momenta $Q$ exchanged in typical nuclear systems are on the order of the pion mass, $Q \sim m_\pi$, which is small compared to the characteristic QCD mass scale: $M \sim 1$
GeV. Whenever we face such a two-scale problem it is useful to separate the corresponding physics by considering an effective, low-energy theory that involves only the relevant degrees of freedom, all with small three-momenta $Q$. The pion and the nucleon obviously play this role, since they are the lightest stable (with respect to strong interactions) hadrons. Low-lying resonances also ought to be relevant. Prominent among them is the $\Delta$ isobar of mass $m_\Delta = m_N + \delta$, with $\delta \simeq 2m_\pi$, which is comparable to $Q$ \cite{14}. Moreover, consistency of the chiral expansion with a large-$N_c$ expansion requires an explicit $\Delta$ \cite{21–23}. The case for higher-mass baryon states is less clear, since their mass differences with respect to the nucleon are larger, and they couple more weakly to nucleons and pions. Higher-mass meson states also have masses comparable to $M$. In the following we will, therefore, take the view that the degrees of freedom that must be accounted for explicitly are the pion, the nucleon and the isobar, while effects of higher-mass states are included indirectly, as contributions to the several parameters of the effective theory.

The low-energy parameters are not necessarily small, so the only potential expansion parameter is $Q/M$. ($Q$ stands not only for typical three-momenta, but also for factors of $m_\pi$ and $\delta$.) Note that, because we are restricted to small three-momenta, nucleons and isobars are non-relativistic and act very much like static sources of pions. Corrections to the static limit can be accounted for by an expansion in $Q/m_N$.

The next task is to count powers of $Q$ in an arbitrary diagram. This is simple to do in the case of diagrams with only pions \cite{12}, and can be straightforwardly extended to diagrams with one nucleon. However, systems with several nucleons require more care, due to the appearance of infrared enhancements in reducible graphs \cite{16}. Diagrams where all energy denominators are of order $Q$ are called irreducible. In this case the same power counting applies: an irreducible diagram with $V_i$ vertices of type $i$, $L$ loops, $C$ separately connected pieces, and $E_f = 2A$ external fermion lines, will be proportional to $Q^\nu$, with

$$\nu = 4 - A + 2L - 2C + \sum_{i=1}^{\infty} V_i \Delta_i. \quad (1)$$

Here the so-called index of a type $i$ vertex is defined as

$$\Delta_i = d_i + \frac{f_i}{2} - 2, \quad (2)$$

in terms of $d_i$ (which is the sum of the number of derivatives, the number of powers of $m_\pi$, and the number of powers of $\delta$) and of $f_i$ (which is the number of fermion field operators).

However, because of the non-relativistic character of nucleons, there also exist graphs with intermediate states that differ in energy from initial or final states by only a small amount of order $Q^2/2m_N$. They are larger than irreducible diagrams by factors of $m_N/Q$. We call these diagrams reducible; they can be split into irreducible sub-diagrams by cutting only lines corresponding to initial or final particles. The sum of irreducible diagrams is just what is usually called the potential. Reducible diagrams can be obtained by iteration of the potential, generating wave functions corresponding to scattering or bound states. In processes such as the ones in which we are interested, the amplitude can be organized into an irreducible part (to which the external pions are attached) sandwiched between wave functions of the initial and final nuclear states, which now contain all of the reducible parts.
Now, if \( \Delta_i \geq 0 \) for all \( i \), then a perturbative expansion in \( Q/M \) exists for irreducible diagrams. Chiral symmetry provides exactly this constraint. All evidence suggests that QCD has an approximate \( SU(2) \times SU(2) \sim SO(4) \) symmetry that is spontaneously broken to \( SU(2) \sim SO(3) \). The pions are the Goldstone bosons associated with this breaking; as such, there is at least one choice of fields for which in the chiral limit \( (m_\pi \rightarrow 0) \) they couple via derivatives to other particles and themselves \[24\], and this is sufficient to guarantee \( \Delta_i \geq 0 \).

The interactions of the effective low-energy Lagrangian can thus be ordered according to the index in Eq. (2). Below we present only those terms that are directly relevant to our subsequent calculation; in particular we subsume in “…” those interactions with additional pion fields that are necessary to construct the correct (nonlinear) realizations of chiral symmetry, but are not required in the lower-order calculations that we will perform. The lowest-order Lagrangian is the one where \( \Delta_i = 0 \) for each interaction \[12\,18\],

\[
\mathcal{L}^{(0)} = \frac{1}{2}(\pi^2 - (\overline{\nabla}\pi)^2) - \frac{1}{2}m_\pi^2
+ N^\dagger[i\partial_0 - \frac{1}{4f_\pi^2}\tau \cdot (\pi \times \pi)]N + \frac{g_A}{2f_\pi}N^\dagger(\tau \cdot \overline{\sigma} \cdot \overline{\nabla}\pi)N
+ \Delta^\dagger[i\partial_0 - \delta]\Delta + \frac{h_A}{2f_\pi}[N^\dagger(\tau \cdot \overline{S} \cdot \overline{\nabla}\pi)\Delta + h.c.] + \cdots, \tag{3}
\]

where \( f_\pi = 93 \text{ MeV} \) is the pion decay constant, \( \delta = m_\Delta - m_N \) is the isobar-nucleon mass difference, \( g_A \) is the axial-vector coupling of the nucleon, \( h_A \) is the \( \Delta N \pi \) coupling, and \( \overline{S} \) and \( \mathbf{T} \) are the transition spin and isospin matrices, normalized such that

\[
S_iS_j^+ = \frac{1}{3}(2\delta_{ij} - i\varepsilon_{ijk}\sigma_k) \tag{4}
\]

\[
T_aT_b^+ = \frac{1}{3}(2\delta_{ab} - i\varepsilon_{abc}\tau_c). \tag{5}
\]

Notice that we defined the fields \( N \) and \( \Delta \) in such a way that there is no factor of \( \exp(-im_N t) \) in their evolution. Hence \( m_N \) does not appear explicitly to this order: the baryons are static.

We also wrote \( \mathcal{L}^{(0)} \) in the rest frame of the baryons, which is the natural choice. (Galilean invariance will be assured by including terms with additional derivatives.) Chiral symmetry determines the coefficient of the so-called Weinberg-Tomozawa term \( (N^\dagger\tau \cdot (\pi \times \pi)N) \) but not of the single-pion interactions \((g_A, h_A)\).

The first-order Lagrangian has \( \Delta_i = 1 \) \[14\,15\,17\,18\],

\[
\mathcal{L}^{(1)} = \frac{1}{2m_N}[N^\dagger \overline{\nabla}^2 N + \frac{1}{4f_\pi^2}(iN^\dagger\tau \cdot (\pi \times \overline{\nabla}\pi) \cdot \overline{\nabla} N + h.c.)]
+ \frac{1}{f_\pi^2}N^\dagger[(c_2 + c_3 - \frac{g_A^2}{8m_N})\pi^2 - c_3(\overline{\nabla}\pi)^2 - 2c_1m_\pi^2\pi^2 - \frac{1}{2}(c_4 + \frac{1}{4m_N})\varepsilon_{ijk}\varepsilon_{abc}\sigma_k\tau_c\partial_i\pi_a\partial_j\pi_b]N
+ \frac{\delta m_N}{2}N^\dagger[\tau_3 - \frac{1}{2f_\pi^2}\pi_3\pi \cdot \tau]N + \frac{1}{2m_N}\Delta^\dagger[\overline{\nabla}^2 + \cdots]\Delta
- \frac{g_A}{4m_N f_\pi}[iN^\dagger\mathbf{T} \cdot \overline{\pi}\overline{S} \cdot \overline{\nabla}\Delta + h.c.] - \frac{h_A}{2m_N f_\pi}[iN^\dagger\mathbf{T} \cdot \overline{\tau}\overline{S} \cdot \overline{\nabla} + h.c.]
- \frac{d_1}{f_\pi}N^\dagger(\tau \cdot \overline{\sigma} \cdot \overline{\nabla}\pi)N N^\dagger - \frac{d_2}{2f_\pi}\varepsilon_{ijk}\varepsilon_{abc}\partial_i\pi_a N^\dagger\sigma_j\tau_b N N^\dagger\sigma_k\tau_c N + \cdots, \tag{6}
\]
where the $c_i$'s are coefficients of $O(1/M)$, $\delta m_N \sim m_d - m_u$ is the quark mass difference contribution to the neutron-proton mass difference, and the $d_i$'s are coefficients of $O(1/f_{\pi}^2 M)$. These seven numbers are not fixed by chiral symmetry, but it is important to point out that Galilean invariance requires that the other coefficients explicitly shown above be related to those appearing in $L^{(0)}$. This in particular fixes the strength of the single-pion interactions in terms of the lowest-order coefficients $g_A$ and $h_A$, and of the common mass $m_N$.

The second-order Lagrangian, with $\Delta_i = 2$, is

$$L^{(2)} = \frac{d_1}{2m_N f_{\pi}} [iN^{\dagger} \tau \cdot \vec{\pi} \cdot \vec{\nabla} N^{\dagger} N + h.c.] - \frac{e_1}{2m_N f_{\pi}} [iN^{\dagger} \tau \cdot \vec{\pi} \sigma N \cdot N^{\dagger} \vec{\nabla} N + h.c.] + \frac{e_2}{2m_N f_{\pi}} [N^{\dagger} \tau \cdot \vec{\pi} \sigma \times \vec{\nabla} N \cdot N^{\dagger} \bar{\sigma} N + h.c.] + \cdots,$$

(7)

where the $e_i$'s are other coefficients of $O(1/f_{\pi}^2 M)$.

### III. POWER COUNTING AND THE PP $\rightarrow$ PP$\pi^0$ REACTION

The power-counting arguments can be extended to the $pp \rightarrow pp\pi^0$ reaction. However, there is one fundamental difference relative to Weinberg's standard power counting due to the kinematics of the present problem. In the standard power-counting arguments it is assumed that typical momenta carried by nucleons are $\sim m_\pi$. However, this condition cannot be satisfied for the $pp \rightarrow pp\pi^0$ reaction. Instead one finds:

$$p_{typ} \sim \sqrt{m_Nm_\pi}.$$

(8)

The reason for this is quite simple. Consider the initial state in the center of mass frame. At threshold the total energy is $2m_N + m_\pi$, so that the initial kinetic energy of each nucleon is $m_\pi/2$. Since the energy is small the nonrelativistic kinetic energy formula should apply, $p_{typ}^2/(2m_N) = m_\pi/2$, and Eq.(8) follows.

While having typical momenta of order $(m_Nm_\pi)^{1/2}$ can alter the details of the power counting, it should not spoil the scheme. Indeed, the scale of the typical momenta still goes to zero as we approach the chiral limit and hence a chiral expansion remains sensible. On the other hand these momenta are characteristically larger than what one usually encounters while chiral power counting and thus the expansion can be expected to be more slowly convergent than a more typical case. In particular the expansion parameter becomes $\sqrt{m_\pi}/M$ rather than $m_\pi/M$. The point is that since a typical momentum of the nucleons is $(m_Nm_\pi)^{1/2}$ nothing prevents a momentum transfer of order $(m_Nm_\pi)^{1/2}$ in interactions. Consider for example processes involving a single-pion exchange. Chiral symmetry requires that (to leading order), each vertex contains a derivative and hence is proportional to the momentum transfer. In the traditional Weinberg power counting a pion-nucleon vertex contributes $(m_\pi/M)$ to the total power counting of an irreducible graph. With our kinematics, however, those vertices would contribute $(m_\pi m_N)^{1/2}/M \sim (m_\pi/M)^{1/2}$, where the second form follows from $m_N \sim M$. Similarly, in the traditional power counting a meson propagator $1/(q^2 - m_\pi^2)$ contributes as $1/m_\pi^2$ while with our kinematics it goes as $1/(Mm_\pi)$. In
short, with these kinematics whenever a three momentum enters into the power counting it contributes to the power counting as \((m_\pi M)^{1/2}\).

There is another subtlety in the present power-counting scheme and this concerns the notion of irreducibility. It should be recalled that a sub-diagram is reducible in Weinberg’s sense if it includes a small energy denominator \(\sim m_\pi^2/m_N\) and is irreducible otherwise. This leads to the possibility that a sub-diagram may look topologically as though it were reducible, in the sense that one could cut the sub-diagram into smaller sub-diagrams with only nucleon external legs, while in fact the diagram is irreducible. This happens if the kinematics requires that the energy denominator associated with cutting the sub-diagram is order \(m_\pi\) and is not \(\sim m_\pi^2/m_N\). This is precisely what happens for the impulse term.

There is finally a caveat. In principle, in order to implement the power counting in the irreducible piece, one ought to use a potential obtained from the same chiral Lagrangian according to the power-counting rules. While such a potential has been developed to third order \([17]\) it is only successful at energies well below the threshold for pion production. Accordingly, we will follow the strategy advocated by Weinberg in his treatment of the three-body problem \([16]\) and use semi-phenomenological potentials that incorporate experimental information into the nucleon-nucleon interaction. This strategy has a conceptual cost: there may be a mismatch between the nucleon field used in the \(NN\) potential and the one based on the chiral Lagrangian used in our calculation of the operators. To attempt to estimate the scale of the uncertainty due to this we will use a number of different \(NN\) potentials. These effects are not negligible.

Let us now detail how this modified power counting works for the irreducible diagrams close to threshold, where we can restrict ourselves to s-waves. We examine each of the various contributions.

Consider first the impulse term, Fig.(1a). At first glance this appears to be order \(m_\pi^{3/2}/(f_\pi M^{1/2})\). It has been recognized that the s-wave amplitude is small because in Eq.(3) a pion of momentum \(q\) couples to baryons only via \(\vec{\sigma} \cdot \vec{q}\). Close to threshold, the interaction consequently proceeds via the Galilean term \(\partial_0 \pi\) in Eq.(6). This yields an explicit factor of \(m_\pi/f_\pi\), but also involves \(1/m_N\) times a gradient that contributes \(p_{\text{typ}}\). Thus, the net contribution at the vertex is \(m_\pi^{3/2}/(f_\pi M^{1/2})\). However, this is not yet the correct order for this process because of the subtlety associated with the concept of irreducibility discussed above. Since the outgoing pion carries an energy of the order of the pion mass, the energies of the \(NN\) intermediate state before and after pion emission differ by \(\sim m_\pi\). Therefore both of the intermediate states cannot simultaneously be within \(\sim m_\pi^2/m_N\) of being on-shell: at least one intermediate state, before or after emission, is off shell by \(\sim m_\pi\).

This single, relatively-high-momentum \((\sim \sqrt{m_\pi m_N})\) pion exchange must therefore be included in the irreducible class of operators for our process (unlike the usual case). All other initial- and final-state interactions will be considered reducible and included in the wave functions. Thus the irreducible sub-diagrams of Fig.(1) should in lowest order be drawn as in Fig.(2). The two–nucleon interaction itself provides a factor \(1/f_\pi^2\). Indeed, if it originates from virtual (static) pion exchange (Fig.(2a,b)), it results from two factors of \(\sqrt{m_\pi m_N}/f_\pi\) from each vertex and one of \((m_\pi m_N)^{-1}\) from the propagator; if it arises from exchange of a heavier meson \(h\) (Fig.(2c,d)), it is of order \(g_{NNh}^2/m_h^2\) which is typically \(\sim (4\pi/1\text{GeV})^2 \sim 1/f_\pi^2\). The inclusion of the interaction in the irreducible part also produces an energy denominator between the pion exchange and the pion emission. This energy
denominator is \((E_{\text{intermediate}} - E_{\text{initial}})^{-1} \sim \text{kinetic energy} \sim m_\pi\). So, along with the explicit factor of order \(m_\pi^{3/2}/(f_\pi M^{1/2})\) from the pion emission vertex, we have an energy denominator of order \(1/m_\pi\) and a factor of \(1/f_\pi^2\) from the potential. This gives an overall contribution \(\mathcal{O}(f_\pi^{-3}\sqrt{m_\pi/M})\).

One can use similar power-counting arguments to estimate the relative contribution of other processes. In studying other processes we have found that they all depend on a higher power of the pion mass than the \(\sqrt{m_\pi}\) dependence of the impulse process. Thus, power counting verifies the intuition that had we lived in a world with sufficiently light quarks the impulse term would have been dominant. Thus, the failure of the impulse term to explain the data by itself is interesting in that it highlights the role of chiral-symmetry breaking. That is, it quantifies the sense in which the up and down quark masses cannot be regarded as sufficiently light.

For example, consider the \(\Delta\) terms of Fig.(3). The power counting here is completely analogous to the case just considered; the only difference is that now the intermediate state energy is \(1/(m_\pi - \delta)\), the rest-mass difference between intermediate and initial state being \(\delta\), while the typical kinetic energy is, as before, of order \(m_\pi\). Thus, this process is \(\mathcal{O}(f_\pi^{-3}\frac{m_\pi}{(m_\pi - \delta)}\sqrt{m_\pi/M})\) or, equivalently, \(\mathcal{O}(\frac{m_\pi}{(m_\pi - \delta)})\) relative to the impulse approximation. In the limit-world where the quark masses \(m_q \to 0\) implying \(m_\pi \to 0\), while \(m_\Delta - m_N\) remains finite, \(\Delta\) excitation would be greatly suppressed relative to the impulse term. However, in the real (i.e., physical) world, we have \(\frac{m_\pi}{(m_\pi - \delta)} \sim 1\), and this process has to be considered as of the same order as the impulse term.

All other processes are suppressed compared to the leading impulse and \(\Delta\)-excitation processes by powers of \(\sqrt{m_\pi/M}\). For example, the recoil corrections to the impulse approximation (Fig.(4)) and to the static \(\Delta\) excitation are down by relative order \(m_\pi/M\), since they involve two extra factors of the ratio between transferred energy (\(\sim m_\pi\)) and transferred momentum (\(\sim \sqrt{m_\pi M}\)).

The pion-rescattering or seagull process of Fig.(4) is also down by relative order \(m_\pi/M\) compared to the leading terms. In order to see this, we first need to understand how the chiral power counting goes for the \(\pi\pi N^1N\) vertex. Because of its isospin structure, the Weinberg-Tomozawa term in Eq.(3) does not contribute. From \(\mathcal{L}^{(1)}\) in Eq.(3), we see that i) the \(c_1\) term and \(\delta m_N\), being proportional to the \(\sigma\)-term, are proportional to \(m_q\) and hence \(m_\pi^2\); ii) the \(c_2 + c_3\) term yields an interaction proportional to \(q_0 E_\pi\), where \(q_0\) is the energy of the exchanged pion, so it is also \(\propto m_\pi^2\); iii) the other terms, while potentially big because they are proportional to the momentum of the exchanged pion, contribute only to p-waves. The \(\pi\pi NN\) vertex thus contributes a factor \(m_\pi^2/(f_\pi^2 M)\). To complete the power counting for the process, note that the pion propagator is of order \(1/(m_\pi M)\) and the coupling of the exchanged pion to the second nucleon is \(\sqrt{m_\pi M}/f_\pi\). Combined with the seagull vertex, this yields a total amplitude of order \(\mathcal{O}(f_\pi^{-3}(m_\pi/M)^{3/2})\).

A few comments are in order about the seagull contribution. The first is that although it is formally sub-leading, it is by no means obvious that it must be very small. Dimensionless spin and isospin factors occur in most amplitudes. They can significantly enhance some partial waves, and these dimensionless factors are not “counted” in power-counting arguments. It is also clear why this contribution is likely to be much larger than estimates based on the on-shell vertex. The on-shell s-wave scattering amplitude is anomalously small.
in a power-counting sense. The basic point is that on-shell the \( \sigma \) term contribution is nearly equal and opposite to the \( c_2 \) and \( c_3 \) contributions, yielding a total much smaller than any of the individual terms. However, with the kinematics of pion production the terms do not cancel so efficiently. Thus, one expects the on-shell estimates to be low.

Unfortunately, the recoil and the seagull processes are not the only ones of this order. One-loop diagrams with and without \( \Delta \)'s (e.g., those in Fig. 6) also appear to this order. Ideally, of course, we should simply calculate these processes and include them in our analysis. There are two difficulties with this. The first is that we are using phenomenological potentials and we do not how to implement the traditional \( \chi \)PT scheme of regulating the loop and absorbing the regulator dependence into the coefficient of a contact term in a way consistent with the potential. The second is that even if were we able to do this in a consistent way, we would not be able to make a prediction until we fit the contact term from some other process. Accordingly we have not included these loop diagrams in our analysis. We realize, of course, that these graphs could be quite significant, especially because there is considerable cancellation among the other terms.

The most relevant contact terms in question are the ones in Eq. (7), and shown in Fig. 7. Their finite parts are expected to be \( \mathcal{O}(\sqrt{m_{\pi}M/m_{\pi}}/(f_\pi^3Mm_N)) = \mathcal{O}(f_\pi^{-3}(m_{\pi}/M)^{3/2}) \), so it is also of the same order as the rescattering mechanism. In order to gauge the relevance of this term, we adopt here a phenomenological strategy: following Ref. [6], we assume it to be saturated by \( \sigma \) and \( \omega \) exchanges. If this exchange model is correct, this short-range physics is quite significant—the amplitude is of the same magnitude as the impulse term. On the other hand, it is not totally obvious that the model is correct. It depends on using large coupling constants and a light \( \sigma \). At least in part, this is believed to be a phenomenological parameterization of some two-pion-exchange physics in the potential. It is not obvious that this phenomenology should be used in a context other than the potential. Accordingly we will first consider calculations omitting this contribution and subsequently add it in assuming the strength given in Ref. [3].

We can continue with higher-order examples, but it should be clear now how this can be generalized. (For simplicity we drop here the overall \( f_\pi^{-3} \) factor.) In the Weinberg scheme the amplitude associated with an irreducible diagram is of order \( (Q/M)^\nu \) where \( \nu \) is given in Eq. (11) and \( Q \) is the typical momentum in the problem or \( m_\pi \) or \( \delta \), which are taken to be of the same order. With the present kinematics the order of an irreducible diagram can be written as

\[
A \sim \left( \frac{m_\pi}{M} \right)^{\nu_d/2} \left( \frac{m_\pi}{M} \right)^{\nu_e+\nu_{ex}} = \left( \frac{m_\pi}{M} \right)^{\nu_d/2+\nu_e+\nu_{ex}},
\]

where \( \nu_{ex} \) counts the order of the explicit chiral symmetry breaking arising directly from the effective Lagrangian, \( \nu_e \) counts the order of the energy transfer in the external legs (i.e. the total number of time derivatives on external legs) and \( \nu_d \) is the dynamical order given by \( \nu - \nu_{ex} - \nu_e \). Thus, we can introduce an effective index \( \nu_{eff} \) defined by

\[
\nu_{eff} = \frac{\nu_d}{2} + \nu_e + \nu_{ex} = \frac{(\nu + \nu_e + \nu_{ex})}{2},
\]

and the irreducible diagram will be of order \( (m_\pi/M)^{\nu_{eff}} \).
IV. EXPLICIT FORMS OF OPERATORS

We now obtain the explicit forms of the various contributions by evaluating the most important irreducible diagrams in momentum space. Our notation is as follows: \( \omega^2_q = \vec{q}^2 + m^2_\pi \) is the energy of the (on-shell) pion produced with momentum \( \vec{q} \) in the center of mass; \( \vec{p}' (\vec{p}) \) is the center-of-mass momentum of the incoming (outgoing) proton labelled “1” (those of proton “2” are opposite); \( \vec{k} = \vec{p}' - \vec{p} (k^0 = (\vec{p}^2 - \vec{p}'^2)/2m_N) \) is the momentum (energy) transferred; \( \omega^2_k = \vec{k}^2 + m^2_\pi; \vec{P} = \vec{p} + \vec{p}' \); \( \vec{\sigma}(i) \) is the spin of proton \( i \); \( \vec{\Sigma} = \vec{\sigma}^{(1)} - \vec{\sigma}^{(2)} \); and \( T(\vec{k}) \equiv \vec{\sigma}^{(1)} \cdot \vec{k} \vec{\sigma}^{(2)} \cdot \vec{k} \). We define the T-matrix in terms of the S-matrix via \( S=1+iT \).

According to the previous discussion, we expect the leading contributions to arise from Fig.(2a,b) where the virtual pion is exchanged between static baryons. In the case of pion exchange with a nucleus in the intermediate state (Fig.(2a,b)), we get

\[
T^{IA,p} = \frac{ig_A^3}{8m_N f^3_\pi \omega_k^2} \left[ \vec{\Sigma} \cdot \vec{p}' T(\vec{k}) - T(\vec{k}) \vec{\Sigma} \cdot \vec{p} \right],
\]

which is listed for comparative purposes only. We will actually calculate the impulse approximation directly from Eq.(6). Recoil corrections, as in Fig.(4), are expected to be smaller by a factor of \( m/N \), and are also included:

\[
T^{recoil} = \frac{ig_A^3 (k^0)^2}{8m_N f^3_\pi \omega_k^2} \frac{1}{(k^0)^2 \delta^2 - \omega_q^2} \left[ \vec{\Sigma} \cdot \vec{p}' T(\vec{k}) - T(\vec{k}) \vec{\Sigma} \cdot \vec{p} \right].
\]

In the case of pion exchange with a delta intermediate state (Fig.(3a,b)),

\[
T^\Delta = \frac{-ig_A h^2}{18m_N f^3_\pi \omega_k^2 - (k^0)^2} \frac{\omega_q}{\delta^2 - \omega_q^2} \left[ \vec{k}^2 \omega_q - \vec{k} \cdot \vec{P} \delta \vec{\Sigma} \cdot \vec{k} + i \omega_q (\vec{\sigma}(1) \cdot \vec{k} \vec{\sigma}(2) \cdot \vec{P} \times \vec{k} - \vec{\sigma}(1) \cdot \vec{P} \times \vec{k} \vec{\sigma}(2) \cdot \vec{k}) \right].
\]

Results similar to Eqs.(11),(13) follow for the shorter-range terms (Fig.(2c,d), Fig.(3c,d)), and we do not write them explicitly here. The diagrams in Fig.(2) will all be included in Fig.(3) as far as the explicit calculation goes. Diagrams in Fig.(3c,d) have to be accounted for explicitly as well as diagrams in Fig.(3a,b). In any reasonable model, however, they turn out to be smaller than those in diagrams of Fig.(3a,b). For example, they could arise from \( a_1 \) exchange, but then the relatively high \( a_1 \) mass suppresses this contribution. For the purpose of estimating the effect of the \( \Delta \), we use Eq.(13).

There are two other corrections of order \( m/N \) compared to the leading diagrams of Figs.(2) and (3). Fig.(5) represents the s-wave rescattering:

\[
T^{ST} = \frac{ig_A}{f^3_\pi \omega_k^2 - (k^0)^2} \left[ (c_2 + c_3 - \frac{g_A^2}{8m_N}) k^0 \omega_q - 2c_1 m^2_\pi - \frac{\delta m_N}{4} \right] \vec{\Sigma} \cdot \vec{k}.
\]

Fig.(7) is a short-range mechanism provided by Eq.(7):

\[
T^{sr} = \frac{i}{2f_\pi m_N} \omega_q \left[ (d' + 2e_1) \vec{\Sigma} \cdot \vec{P} + 2ie_2 \vec{\sigma}(1) \times \vec{\sigma}(2) \cdot \vec{k} \right] + \ldots .
\]
Chiral symmetry tell us nothing about the strength of these terms. We can use data to determine the coefficients $d'_1$, $e_1$, and $e_2$. Alternatively, we can use a model to determine these coefficients and then try to explain the experimental results. Here we use the mechanism first proposed by Lee and Riska \cite{5} and by Horowitz et al. \cite{4}, where the short-range interaction is supposed to originate from $z$-graphs with $\sigma$ and $\omega$ exchanges. In this case,

$$
T^{\sigma,\omega} = -\frac{i g_A}{4 f_\pi m_N^2} \omega_0 \left( \frac{g_\sigma^2}{k^2 + m_\sigma^2} + \frac{g_\omega^2}{k^2 + m_\omega^2} \right) \vec{\Sigma} \cdot \vec{P}
-2i g_\omega^2 \frac{(1 + C_\omega)}{k^2 + m_\omega^2} \vec{\Sigma}(1) \times \vec{\Sigma}(2) \cdot \vec{k} + \cdots,
$$

where $m_\sigma$ ($m_\omega$) and $g_\sigma$ ($g_\omega$) are the mass and the (vector) coupling to nucleons of the $\sigma$ ($\omega$) meson, and $C_\omega$ denotes the ratio of tensor to vector coupling for the $\omega$ meson. (Note that there is a misprint in the sign of the second term of Eq. (7b) of Ref. \cite{3}; the $i$ should be replaced by $-i$.) Comparison between Eqs.(15) and (16) yields the model-dependent estimates: $d'_1 + e_1 = -g_A g_\sigma^2 / 2 m_\sigma m_N \approx -1.5 (1/f_\pi^2 M)$, $e_1 = -g_A g_\omega^2 / 2 m_\omega m_N \approx -2 (1/f_\pi^2 M)$, and $e_2 = g_A g_\omega^2 (1 + C_\omega) / 2 m_\omega^2 m_N \approx 2 (1/f_\pi^2 M)$. The numerical factors are obtained from the Bonn OBEP A potential, Table A.2 of reference \cite{25}.

We are concerned with evaluating the matrix elements of the above operators between the initial $^3P_0$ and final $^1S_0$ $pp$ wave functions. It is convenient to use the Reid \cite{20}, Reid93 \cite{27} and Argonne V18 \cite{28} potentials which, for a given $pp$ channel, are local potentials. Thus we evaluate the operators between coordinate space initial $(i)$ and final $(f)$ wave functions expressed by

$$
\langle \vec{r} | i \rangle = \frac{\sqrt{2}}{pr} i u_{1,0}(r) e^{i \delta_{i,0}} \sqrt{4 \pi} \ | ^3P_0 \rangle,
$$

and

$$
\langle \vec{r} | f \rangle = \frac{1}{pr} u_{0}(r) e^{-i \delta_0} \sqrt{4 \pi} \ | ^1S_0 \rangle.
$$

We convert the operators of Eqs.(12)-(14) to configuration space by inverting the Fourier transforms. The resulting operators can then be used in configuration-space matrix elements. Hermiticity and $[\vec{\sigma}(1) \cdot \vec{\sigma}(2), \vec{\Sigma} \cdot \vec{r}] = 4 i \vec{\sigma}(1) \times \vec{\sigma}(2) \cdot \vec{r}$ lead to

$$
\langle ^1S_0 \mid i \vec{\sigma}(1) \times \vec{\sigma}(2) \cdot \vec{r} \mid ^3P_0 \rangle = -i \langle ^1S_0 \mid \vec{\Sigma} \cdot \vec{r} \mid ^3P_0 \rangle,
$$

$$
\langle ^1S_0 \mid \vec{\Sigma} \cdot \vec{p} \mid ^3P_0 \rangle = -i \langle ^1S_0 \mid \vec{\Sigma} \cdot \vec{r} \mid ^3P_0 \rangle \left( \frac{\partial}{\partial r} + \frac{2}{r} \right),
$$

while direct evaluation leads to

$$
\langle ^1S_0 \mid \vec{\Sigma} \cdot \vec{r} \mid ^3P_0 \rangle = -2.
$$

We may then begin to tabulate the results. We define the matrix elements of the operators of Eqs.(12)-(16) as
\[ M^X = \langle f|T^X|i \rangle \]

\[ = - \frac{4\pi \sqrt{2}}{pp'} i e^{i(\delta_0 + \delta_{1,0})} \int_0^\infty dr \, u_0(r) \, H^X(r) \, u_{1,0}(r), \quad (21) \]

where \( X \) represents \( IA \), recoil, etc, and \( H^X(r) \) is the corresponding operator, to be given shortly. We have extracted all of the constant factors from Eqns.(17) and (18) together with a sign from Eqns.(19) and (20) and placed them in front of the integral. The integral can be broken up into a part from zero to \( r_0 \) where \( r_0 \) is any distance greater than the range of the nucleon-nucleon force. For \( r > r_0 \), the wave functions \( u_0, u_{1,0} \) are linear combinations of regular and irregular Coulomb wave functions, so that the integral from \( r_0 \) to \( \infty \) can be done analytically. Note that we can also define \( J \)-matrix elements \( J^X \), so that \( J^{IA} = J_1 \) of Ref. [1]. The relationship is given by

\[ M^X \equiv i \sqrt{2} \frac{4\pi \, g_A}{pp'} \frac{1}{m_N m_\pi} J^X. \quad (22) \]

The impulse approximation is given by

\[ H^{IA}(r) = - \frac{g_A \, m_\pi}{f_\pi \, m_N} \left( \frac{\partial}{\partial r} + \frac{1}{r} \right). \quad (23) \]

The recoil term is evaluated from

\[ H^{recoil}(r) = \frac{g_A}{f_\pi \, m_N} \left[ 4 \frac{\partial}{\partial r} (V_T(\tilde{m}_\pi, r) - V_T(m_\pi, r)) - \frac{\partial}{\partial r} (V_S(\tilde{m}_\pi, r) - V_S(m_\pi, r)) \right], \quad (24) \]

with

\[ \tilde{m}_\pi \equiv \sqrt{\frac{3}{4} m_\pi}, \quad (25) \]

and

\[ V_S(\mu, r) = \left( \frac{g_A}{2 f_\pi} \right)^2 \frac{\mu^3}{4 \pi} e^{-\mu r}, \quad (26) \]

\[ V_T(\mu, r) = \left( \frac{g_A}{2 f_\pi} \right)^2 \frac{\mu^3}{4 \pi} e^{-\mu r} \left( \frac{1}{\mu^2 r^2} + \frac{1}{\mu r} + \frac{1}{3} \right). \quad (27) \]

The dependence on \( \tilde{m}_\pi \) arises from the restriction occurring at threshold that the total momentum of the two final nucleons be zero. In that case, we have \( k^0 \to m_\pi/2 \).

The seagull term \( (ST) \) is evaluated in a similar manner from

\[ H^{ST}(r) = - \frac{g_A m_\pi^2}{f_\pi} \left[ 4c_1 + \frac{\delta m_N}{2 m_\pi^2} - \left( c_2 + c_3 - \frac{g_A^2}{8 m_N} \right) \right] \frac{1}{4 \pi} \frac{1}{r^2} e^{-\tilde{m}_\pi r}. \quad (28) \]

One may also evaluate the graph of Fig.(5) by treating the pion-nucleon vertex in an on-shell approximation. In this case the term \( k^0 \) is replaced by \( m_\pi \). The result is that one obtains an on-shell form of the seagull term \( (ST_{on}) \) with
\[ H^{ST\text{on}}(r) = -\frac{g_A m_N^2}{f_\pi^3} \left[ 4c_1 + \frac{\delta m_N}{2m^2_\pi} - 2 \left( c_2 + c_3 - \frac{g_A^2}{8m_N} \right) \right] \frac{(1 + \bar{m}_\pi r)}{4\pi r^2} e^{-\bar{m}_\pi r}. \] (29)

The term involving the intermediate \( \Delta \) is found to take the form
\[ H^\Delta(r) = \frac{g_A^3}{18m_N f_\pi^3} \frac{h_A}{g_A} \left( \frac{m^2_\pi}{m_N^2} \right)^2 \left[ -A_\Delta(r) + \left( 1 - \frac{2\delta}{m_\pi} \right) B_\Delta(r) + C_\Delta(r) \right], \] (30)
where
\[ A_\Delta(r) = \frac{3(1 + \bar{m}_\pi r)}{4\pi r^2} e^{-\bar{m}_\pi r}, \]
\[ B_\Delta(r) = \frac{e^{-\bar{m}_\pi r}}{4\pi r} \left( 1 + \bar{m}_\pi r \right) \frac{2}{r} \left( 1 + \frac{4}{m_\pi r} + \frac{4}{m_\pi^2 r^2} \right) - 2 \left( 1 + \frac{2}{m_\pi r} + \frac{2}{m_\pi^2 r^2} \right) \frac{\partial}{\partial r}, \]
\[ C_\Delta(r) = \frac{2}{4\pi} \frac{e^{-\bar{m}_\pi r}}{r} \left[ \frac{\partial}{\partial r} + \frac{1}{r} \right]. \]

We also include the effects of the sigma and omega exchange terms (\( \sigma, \omega \)). The result is
\[ H^{\sigma,\omega}(r) = -\frac{g_A m_\pi}{f_\pi m_N^2} \left[ \left( f_\sigma(r) + f_\omega(r) \right) \left( \frac{\partial}{\partial r} + \frac{1}{r} \right) + \frac{1}{2} \frac{\partial f_\sigma}{\partial r} - \frac{1 + 2C_\omega}{2} \frac{\partial f_\omega}{\partial r} \right], \] (32)
where the function \( f_h(r) \) accounts for exchange of the meson \( h \) between nucleons,
\[ f_h(r) = \frac{g_h^2}{4\pi} \frac{e^{-m_h r}}{r}. \] (33)

We follow Ref. [6] by including the effects of form factors as defined in the Bonn potential [25]. For completeness, we repeat that monopole form factors are used at each meson vertex according to the replacement
\[ g_h \to g_h \frac{\Lambda_h^2 - m_h^2}{\Lambda_h^2 - k_\mu k_\nu}, \] (34)
where \( k_\mu \) is the transferred momentum and \( \Lambda_h \) is the cutoff mass.

These expressions for \( H^{\sigma,\omega} \) differ from the corresponding ones of Ref. [6] because we use \( \vec{p}' = \vec{p} - \vec{k} \) instead of approximating \( \vec{p}' \) by \( \vec{p} \) as in the earlier work. The failure of this approximation has very recently been pointed out by Niskanen [29]. Keeping the term proportional to \( \vec{k} \) yields terms proportional to \( \frac{dp}{dr} \) that are not included in Ref. [6]. However, this change does not affect the conclusions of Ref. [6], because a decrease in the contribution of the sigma exchange is compensated by an increase of that of the omega exchange.

The final steps consist of computing the total matrix element \( \mathcal{M} \):
\[ \mathcal{M} = \mathcal{M}^{IA} + \mathcal{M}^{ST} + \mathcal{M}^{\text{recoil}} + \mathcal{M}^{\Delta} + \mathcal{M}^{\sigma,\omega}, \] (35)
squaring that sum, and integrating over the available phase space. We find
\[ \sigma = \frac{1}{v} \int_0^{p_{\text{max}}'} \frac{dp'p'^2}{(2\pi)^3} q' |\mathcal{M}|^2 \frac{m_N}{2m_N + \omega(q')}, \] (36)
where \( v \) is the laboratory velocity of the incident proton, \( q' \) is the pion momentum, \( \omega(q') = \sqrt{q'^2 + m_\pi^2} \) and \( p'_{\text{max}} = \sqrt{p^2 - m_N m_\pi} \).
V. INPUT PARAMETERS AND RESULTS

The various amplitudes considered in the last section depend on several parameters that we can determine from other processes. The impulse-approximation operator of Eq.(23) and the recoil operator of Eq.(24) depend on the pion mass, \( m_\pi = 135 \text{ MeV} \) \(^{[30]}\), and on 

\[
\frac{g_A}{f_\pi} = \frac{g_{\pi NN}}{m_N};
\] 

we use the value of \( g_{\pi NN} \) appropriate for each potential. The \( \Delta \) operator of Eq.(30) further depends on the \( \Delta - N \) mass splitting \( \delta = 294 \text{ MeV} \) \(^{[30]}\) and on the \( \pi N\Delta \) coupling constant, \( h_A \). This has been fixed from p-wave \( \pi N \) scattering (see, e.g. Ref. \(^{[31]}\)),

\[
\frac{h_A}{g_A} \approx 2.1.
\] (38)

The seagull operator of Eq.(28) depends on four parameters \( c_1, c_2, c_3 \) and \( \delta m_N \). The \( c_i \)'s can be obtained by fitting s-wave \( \pi N \) scattering. In Ref. \(^{[15]}\) they were found to be

\[
c_1 = -1.63/2m_N \\
c_2 = 6.20/2m_N \\
c_3 = -9.86/2m_N,
\] (39)

from the \( \sigma \)-term, the isospin-even scattering length, and the axial polarizability. Note that the analysis of Ref. \(^{[15]}\) does not include the isobar explicitly. Since the inclusion of the \( \pi N\Delta \) interaction only affects s-waves at one order higher than the \( c_i \)'s, the above values can still be used to estimate the effect of s-wave rescattering. The remaining parameter, \( \delta m_N \), can in principle also be determined from s-wave \( \pi N \) scattering, but would require a careful analysis of other isospin-violating effects. Chiral symmetry relates it to the strong interaction contribution to the nucleon mass splitting, which is also difficult to determine directly. Estimates of the electromagnetic contribution \( \delta m_N \) are more reliable, \( \delta m_N \sim -1.5 \text{ MeV} \) \(^{[32]}\), and give \( \delta m_N \sim 3 \text{ MeV} \). To be definite, we use

\[
\delta m_N = 3 \text{ MeV}.
\] (40)

Finally, the \( \sigma, \omega \) operator of Eq.(32) involves \( g_h, \Lambda_h, m_h, \) and \( C_\omega \), parameters listed in Table A.3 of Ref. \(^{[25]}\).

We shall consider results of individual terms before presenting complete calculations using \( \mathcal{M} \) of Eq.(35). We shall often compare our results with the IUCF \(^{[9]}\) and Uppsala \(^{[10]}\) data. This allows us to understand whether or not the size of a particular term is relevant. The values of the cross sections divided by \( \eta^2 \) will be displayed, where \( \eta \) is the maximum value of the pion momentum divided by the mass of the \( \pi^0 \). The three points at the lowest values of \( \eta \) are from Ref. \(^{[10]}\), the remainder are from Ref. \(^{[9]}\). We restrict our calculations to values of \( \eta \) such that \( \eta \leq 0.4 \), because the \( \vec{\sigma} \cdot \vec{\nabla} \pi \) term of the Lagrangian (which we ignore) is important for higher values \(^{[8]}\).

We start by considering the effects of the impulse-approximation term of Eq.(23). We compute the cross section using \( \mathcal{M} = \mathcal{M}^{IA} \). The results are shown in Fig.(8), where the
Coulomb distortion of the proton-proton initial and final wave functions is neglected, and in Fig.(9), where the Coulomb effects are included. We observe that the impulse approximation provides cross sections much smaller than the measured ones, and that including Coulomb effects is important in describing the \( \eta \) dependence of the new accurate cross section data. Henceforth, the only results we shall present include the Coulomb effects.

The observations about the small size of the impulse term and the importance of the Coulomb distortion have been made before \[3,9,6\]. Furthermore, the small values of the cross sections can be traced to cancellations in the integrands. This is shown in Fig.(10). The oscillations arise from the high momentum \( (p \sim \sqrt{m_N m_\pi}) \) of the initial state.

Thus terms other than the impulse approximation must be included. This is also the case in the \( pp \rightarrow d\pi^+ \) reaction and in pion production on nuclear targets (e.g., see the reviews \[33,34\]). The computation of the terms of Eqs.(21) and (24)-(34) is straightforward. These matrix elements are evaluated as a function of \( p' \) and for \( \eta = 0.3 \) using the Reid potential (Fig.(11)), the Reid93 potential (Fig.(12)) and the V18 potential (Fig.(13)). The \( pp \rightarrow d\pi^+ \) cross sections can be understood in terms of rescattering mechanisms, so it is natural to include the effects of the rescattering via the seagull term. We see that including this term in a manner dictated by the chiral Lagrangian (Eq.(28)) leads to a matrix element with a sign opposite to that of the impulse-approximation term. These two terms cancel to a large extent, so that one is forced to examine other terms. If one uses on-shell kinematics to evaluate the influence of the seagull term (Eq.(29)), one finds a contribution of the same sign as the impulse term.

The reason the seagull amplitude changes its sign when going from on-shell to half-off-shell kinematics can be seen immediately by comparing the square brackets of Eqs.(28) and (29) using the values of \( c_i \) of Eq.(39) and \( \delta m_N \) Eq.(40). One finds for the quantity governing off-shell rescattering

\[
\left[ 4c_1 + \frac{\delta m_N}{2m_\pi^2} - \left( c_2 + c_3 - \frac{g_A^2}{8m_N} \right) \right] = -\frac{2.31}{2 m_N}
\]

and for the quantity governing the on-shell rescattering

\[
\left[ 4c_1 + \frac{\delta m_N}{2m_\pi^2} - 2 \left( c_2 + c_3 - \frac{g_A^2}{8m_N} \right) \right] = +\frac{1.75}{2 m_N}
\]

Note that the contribution from isospin violation \( -\frac{\delta m_N}{2m_\pi^2} \) is about 10% of the total on-shell contribution, which is relatively large but insufficient to play a significant role in the current stage of understanding of the \( pp \rightarrow pp\pi^0 \) reaction.

We also discuss the other terms. The recoil term should be of order \( m_\pi/M \) relative to the impulse term, and it is indeed quite small, as indicated in the figures. The term involving the intermediate \( \Delta \) is also smaller than impulse, but depending on the potential is about 2–4 times bigger than the recoil term. The smaller nature of the term results from cancellations in the integrand, as shown in Fig.(14). The sensitivity of the value of the matrix element to the choice of potential is also due to this cancellation. Qualitatively the integrands shown in the figure are very similar, but the net values shown in Figs.(11) and (13) are very different.

The importance of the interference between the impulse and seagull terms is underscored in Fig.(15), which shows the result of the sum of the impulse, recoil, seagull and \( \Delta \) terms for...
the three different potentials that we use. The correct evaluation of our Lagrangian leads to cross sections that are very nearly zero! Only the lowest-energy data point can be shown on this figure with a linear scale. Including the effects of the $\Delta$ in the intermediate state is not sufficient to provide a description of the data. None of the calculated cross sections is near the data, except at the lowest energy.

This failure to account for the measured cross sections causes us to examine the effects of also including the effects of $\sigma$ and $\omega$ exchange. As shown in Fig. (16) we can not account for the cross sections with the heavy meson exchange effects. This is because the $\Delta$ and pion seagull terms interfere destructively with the other terms.

VI. DISCUSSION

The cross sections we compute are far smaller than the measured ones, so we cannot account for this process in terms of the physics we have included. However, we have learned something significant about the reaction. We have found processes that contribute amplitudes of a similar size to the impulse process and which should be included in any reasonable description: the $\Delta$ process, and the seagull or pion-rescattering process. The $\Delta$ process has not been considered previously for the $pp \rightarrow pp\pi^0$ reaction at threshold, but it is important when the newest $pp$ potentials are used to describe the scattering wave functions. We have also seen that the seagull process is highly dependent on its off-shell behavior.

The validity of the previous conclusions is independent of chiral perturbation theory itself (i.e., of the convergence of the expansion in momenta). The relevance of the $\Delta$ isobar depends mostly on the observations i) that it is a relatively light degree of freedom and ii) that its coupling to a nucleon and an s-wave pion is essentially fixed by Galilean invariance in terms of its (large) coupling to a p-wave pion. The importance of the off-shell behavior of the s-wave $\pi^0N$ (re)scattering results from the existence of momentum-dependent $\pi\pi NN$ interactions of a size similar to the momentum-independent one (which is proportional to the quark masses). This in turn is a consequence of chiral symmetry.

We have also seen that these two processes interfere destructively with the impulse term. This is in contrast with the recent calculation of Ref. [7], where constructive interference was found between impulse and seagull terms. It is useful to understand the origin of this discrepancy.

The key point is that the magnitude and sign of the seagull process do depend on our power-counting arguments. So far as the power counting is concerned, we have identified an interesting problem: how one proceeds when the typical momenta are $\sim (m_\pi m_N)^{1/2}$. This is relatively straightforward with the choices of pion and nucleon fields in which the pseudo-Goldstone-boson nature of the pion is manifest; in any such basis, as used above, these fields provide non-linear realizations of chiral symmetry. As is well-known, it is possible to redefine these fields without changing the physics, provided we use our field definition consistently throughout the calculation [24]. Different choices of interpolating fields amount, after all, to no more than different bookkeeping of where various bits of the physics reside. For example [35], one can transform to a linear basis at the expense of producing extra seagull vertices that are momentum independent but not proportional to the quark masses. The sole purpose of such large terms is to ensure a cancellation of large pole terms and to
produce a small $\pi N$ scattering amplitude. Such a cancellation would seem magical were it not understood as a consequence of a concealed chiral symmetry; power counting in this basis becomes highly non-trivial. Moreover, failure to include such seagull vertices in a calculation would spoil chiral symmetry.

Now, the pion field in Ref. [7] and in $\chi$PT are not equivalent. The pion interpolating field in Ref. [7] is based on models exploiting PCAC whereas the pion field in the chiral Lagrangian is not. Thus, a priori there is nothing wrong with getting a different result for the seagull term contribution in the two calculations. If both calculations consistently included all other processes, then the differences between the present calculation and Ref. [7] in the off-shell seagull contributions would be compensated for by differences in the contributions of other mechanisms. Indeed, starting from our general chiral Lagrangian and performing a simple pion-field redefinition, we can reproduce the off-shell behavior of the PCAC $\pi N$ amplitude of Ref. [7]; however, in doing so an extra, anomalously large contact term is generated, whose sole function is to cancel the effect of the change in off-shell behavior of the $\pi N$ amplitude brought up by the field redefinition. This is in complete analogy to the change to a linear basis mentioned above. Since the calculation of Ref. [7] does not include other mechanisms besides the impulse and seagull terms, it is rather difficult to interpret its results.

We are not able to reproduce the data with the processes we have included, even if the short-ranged process is modeled by $\sigma$ and $\omega$ exchange using the large coupling constants of one boson exchange potentials. Perhaps, in face of the preceding remarks, this should come as no surprise. To the extent that pieces in the potential other than static OPE are important, we might expect similarly important contributions from one loop diagrams (such as the ones in Fig.(6)) in the irreducible sub-diagram, again to be evaluated consistently with the potential. It might be no accident, then, that without such contributions we can not explain the data.

The threshold behavior of $pp \rightarrow pp\pi^0$ in principle provides us with considerable information about the nature of low-energy strong-interaction physics and in particular about the dynamics associated with approximate chiral symmetry and its spontaneous breaking. The amplitude for this process would be zero in the chiral limit of $m_q = 0$. Moreover, as demonstrated by the power-counting analysis in this paper, we see that for sufficiently small $m_q$ the amplitude must be dominated by the impulse term. However, we have also shown that the chiral expansion for a process with these kinematics is essentially an expansion in $(m_\pi/M)^{1/2}$. In the real world, this parameter is rather large, $\sim 4$, and it is by no means surprising that the expansion either converges slowly or fails as an asymptotic series after some low order. Given this large effective expansion parameter, it is not surprising that many processes contribute non-negligible amounts to the final amplitude. Moreover, there is significant cancellation between amplitudes from the various mechanisms which makes reliable calculations very difficult. However, even given the large uncertainties present in our calculations due to these cancellations, we cannot explain the data given the processes we have included. It is reasonably plausible that the terms we have excluded could provide a large enough contribution to bring our calculations into line with the observations. Clearly, however, developing a reliable systematic description of this processes which explains the data represents a large challenge.
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FIGURES

FIG. 1. Various contributions to the $pp \to pp\pi^0$ reaction. In this and the following figures a single (double) solid line stands for a nucleon (Delta) and a single (double) dashed line represents a pion (sigma, omega); $\Psi_i$ ($\Psi_f$) is the wave function for the initial (final) state.

FIG. 2. Diagrams contributing to the impulse term that are irreducible in the context of chiral power counting for the $pp \to pp\pi^0$ reaction.

FIG. 3. Irreducible diagrams contributing to the leading $\Delta$-excitation mechanism.

FIG. 4. Irreducible diagrams contributing to the recoil correction of the impulse term.

FIG. 5. The seagull or pion rescattering irreducible diagram.

FIG. 6. Some one loop irreducible diagrams.

FIG. 7. Short-ranged irreducible diagram.

FIG. 8. Impulse approximation. Three different potentials are used. The Coulomb distortion is neglected. The solid curve is obtained using the Reid potential, the dashed curve using the Reid93 potential and the dot-dashed curve using the V18 potential.

FIG. 9. Impulse approximation. Three different potentials are used. The Coulomb distortion is included. The solid curve is obtained using the Reid potential, the dashed curve using the Reid93 potential and the dot-dashed curve using the V18 potential.

FIG. 10. Typical integrand for the impulse term. The dashed curve is obtained using the Reid potential and the solid with the V18 potential.

FIG. 11. Matrix elements as a function of $p'$ for $\eta = 0.3$. See Eq.(22). The Reid potential is used.

FIG. 12. Matrix elements as a function of $p'$ for $\eta = 0.3$. See Eq.(22). The Reid93 potential is used.

FIG. 13. Matrix elements as a function of $p'$ for $\eta = 0.3$. See Eq.(22). The V18 potential is used.
FIG. 14. Integrands for the $\Delta$ contribution. The solid curve is obtained with the Reid and the dashed with the V18 potential.

\[ \mathcal{M} \approx \mathcal{M}^{IA} + \mathcal{M}^{\text{recoil}} + \mathcal{M}^{ST} + \mathcal{M}^{\Delta}. \]

FIG. 15. \[ \mathcal{M} \approx \mathcal{M}^{IA} + \mathcal{M}^{\text{recoil}} + \mathcal{M}^{ST} + \mathcal{M}^{\Delta}. \]

FIG. 16. Calculations with all of our mechanisms for the Reid, Reid93, and V18 potentials, using eq. (32) for the heavy meson exchange effects.
Figure 1
Figure 3
Figure 5
Fig. 8

$\frac{\sigma}{\eta^2}$ (\(\mu b/sr\))

- Impulse, without Coulomb
- Reid - solid
- Reid93 - dash
- V18 - dot - dash
Impulse, with Coulomb

Reid—solid

Reid93—dash

V18—dot—dash

\[ \frac{\sigma}{\eta^2} \; (\mu b/sr) \]

\( \eta \)  Fig. 9
\[ d_{IA}^{33} \]

\[ \eta = 0.3, \quad p' = 0.16 \text{ fm}^{-1} \]

Reid dash

V18 solid

Fig. 10
\[ \eta = 0.3, \ p' = 0.16 \text{ fm}^{-1} \]
All terms

$\frac{\sigma}{\eta^2}$ (mb/sr)

$\eta$

Fig. 16

Reid

Reid93

V18