An Insight into the Dynamics of a Dual Active Bridge

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Abstract—This paper aims at analyzing the effect of the zero dynamics of the Dual Active Bridge Isolated Bidirectional dc-dc converter (DAB) on the dynamics of the complete DAB system. It also explains its influence on controller design for the DAB system. In carrying out these analyses, the state space model of the DAB, as well as the first harmonic approximation (FHA) of the model are derived. The ZVS and the stability analysis of the system are undertaken based on the FHA model of the system. The system is shown to be stable for a constant output voltage operation for the entire power range while it is unstable for the constant power load (CPL) operation for load demands close to the system maximum power. It is also shown that the transformer winding currents are part of the zero dynamic states and are always stable regardless of the operating conditions of the system.

Keywords—DAB, zero dynamics, FHA modeling, stability analysis.

I. INTRODUCTION

The dual active bridge isolated bidirectional dc-dc converter (DAB) was first mentioned in 1988[1]. It consists of two single-phase VSCs (Voltage Source Converters) connected through a high frequency transformer for galvanic isolation and flexible voltage applications. It is a buck and boost converter and allows the control and bidirectional flow of power. One of the merits of the DAB is its ease of implementation of the soft-switching techniques[2]. This enables the employment of high frequency operation which allows for a significant reduction in sizes of electronic and magnetic components of the converter, thereby improving its power density. In the analysis of the converter dynamics and controller design of the DAB, the reduced-order and full-order models are usually employed [3]. In the full-order model, the complete model equations of the DAB are employed in analyzing the dynamics of the DAB. For the reduced order model, the transformer currents which are purely ac are neglected. The negligible effect of the neglect of the transformer current is possible because the current has a much faster dynamics than the other states, and as will be shown in this paper, the currents are part of the stable zero dynamics of the system. According to [4], zero dynamics are the worst case of the internal dynamics of a system achieved by constraining the non-zero dynamic states of the system to zero.

The performance of the controller designed for a dual active bridge converter greatly depends on the accuracy of the converter’s model. Qin et al studied the dynamics of the DAB using its full order model. The authors however noted that the transformer current of the converter makes continuous-time modeling difficult. Hence, average modeling techniques are deployed to derive the input-output transfer functions of the system for its controller design [3]. Bai et al presented a reduced order model of a DAB by neglecting the transformer current in the full order continuous-time system. From results presented, the dynamic response of the system was close to the response from the simulated full order continuous-time model under steady-state and transient conditions [5]. It is a common practice in the control of the DAB to neglect the dynamics of the inductor/transformer current as seen in several papers. [6],[7],[8]. To the best of our knowledge, no paper has explicitly stated the reason for the negligible effect of the neglect of the transformer current dynamics on the steady-state operation of closed loop control system of the dual active bridge converters. This paper therefore aims to use the phenomenon of zero dynamics to analytically show the reason for stable operation of the DAB closed loop system, even when the inductor/transformer dynamics are neglected. The paper also carries out further dynamic and steady-state analysis of the DAB, including observability and controllability analysis as well as the stability and ZVS analysis of the dual active bridge converters. In Section 1, a general introduction of dual active bridge and zero dynamics is done. In Section 2, a description of the system under consideration is presented. The physical system model and FHA of the continuous-time model is carried out. The steady state analysis, ZVS and stability analysis based on the FHA of the model are presented. In Section 3, the controllability, observability and zero dynamics of the system are addressed.

II. DESCRIPTION, MODELING AND STEADY-STATE ANALYSIS OF SYSTEM

A. System Description

The DAB in Figure 1 consists of two full active bridges connected by a high frequency transformer with a turns ratio n. A diode-bridge-rectified ac voltage \( V_1 \) is applied to the primary bridge through an LC filter of inductor \( L_4 \) and capacitor \( C_4 \).
The output of the secondary bridge is connected through a capacitor \( C_2 \) to a current source (representing the load current) with current \( I_L \). The DAB is operated by triple phase shift (TPS) control method.

As shown in Figure 2, \( d_1 \) and \( d_2 \) represent respectively, the primary and secondary side dc voltages, and \( d_3 \) represents the outer phase shift ratio between the primary and secondary bridges. The phase shift angle between the primary and secondary bridges which has more significance in practical terms is defined in terms of \( d_3 \) as:

\[
\delta = \left( d_3 + \frac{(d_1 - d_2)}{2} \right) \pi
\]

**B. Mathematical Model of DAB**

The dynamic equation of the DAB is derived by first writing out the KVL equations for the dc and ac circuit of the system in Figure 1. The dynamic equations of the system are as written below:

\[
p I_d = \frac{v_1}{L_d} - \frac{v_2}{L_d} - \frac{r}{L_d} I_d
\]

\[
p V_{c1} = \frac{1}{c_1} I_d - \frac{1}{c_1} s_1 i_1
\]

\[
p I_1 = \frac{L_1 h_1}{h} i_1 + \frac{L_m h_2}{h} i_2 - \frac{L_2}{h} s_1 V_{c1} + \frac{L_m}{h} s_2 V_{c2}
\]

\[
p I_2 = \frac{L_m h_2}{h} i_1 + \frac{L_2}{h} s_1 V_{c1} + \frac{L_m}{h} s_2 V_{c2}
\]

\[
p V_{c2} = \frac{v_2}{c_2} i_2 - \frac{1}{c_2} I_d
\]

\[
y_1 = V_{c1}, \quad y_2 = V_{c2}
\]

\[
x = [I_d \quad V_{c1} \quad i_1 \quad i_2 \quad V_{c2}], u = [s_1 \quad s_2]
\]

\( V_{c1} \) and \( V_{c2} \) are the primary and secondary side dc capacitor voltages, \( s_1 \) and \( s_2 \) are the primary and secondary full bridges converters’ switching functions, \( i_1 \) and \( i_2 \) are primary side referred primary and secondary transformer winding current, \( I_d \) is the input inductor dc current, \( L_1 \) and \( L_2 \) are the primary-side referred transformer winding self-inductances, \( L_m \) is the transformers’ magnetizing inductance, \( R_1 \) and \( R_2 \) are the transformer’s winding resistances and \( p = \frac{d}{dt} \) is a time-derivative operator.

Again:

\[
s_1 = s_{a1p} - s_{bp} \quad \text{and} \quad s_2 = s_{a2p} - s_{bp}
\]

\( s_{a1p} \) and \( s_{a2p} \) are the switching signals of the upper switches \( T_{a1p} \) and \( T_{a2p} \) of the primary bridge while \( s_{bp} \) is the switching signals of the upper switches \( T_{bp2} \) of the secondary bridge.

**C. Steady-State Analysis**

To conduct the steady-state analysis of the system, the nonlinear time-periodic dynamic equations of the transformer winding currents are simplified, using the first harmonic approximations of the currents and the converter’s switching functions. The switching functions which are quasi-square waves are expressed by their Fourier series as:

\[
s_1 = \sum_{k=1,3,5,...}^{\infty} \frac{4}{k\pi} \sin \left( k \frac{d_1 \pi}{2} \right) \sin (k \omega t)
\]

\[
s_2 = \sum_{k=1,3,5,...}^{\infty} \frac{4}{k\pi} \sin \left( k \frac{d_2 \pi}{2} \right) \sin (k \omega t - k \delta)
\]

Using the first harmonic approximation, (7) and (8) reduce to:

\[
s_1 = \text{real} \left( S_{qd1} e^{j \omega_1 t} \right) \quad \text{and} \quad s_2 = \text{real} \left( S_{qd2} e^{j \omega_2 t} \right)
\]

\( S_{qd1} \) and \( S_{qd2} \) are complex variables expressed by:

\[
S_{qd1} = \frac{1}{\pi} \sin \left( \frac{d_1 \pi}{2} \right) \angle -\frac{\pi}{2}, \quad S_{qd2} = \frac{1}{\pi} \sin \left( \frac{d_2 \pi}{2} \right) \angle -\frac{\pi}{2} - \delta
\]

The transformer winding currents which are responses to switching functions in (9) are given by:

\[
i_1 = \text{real} \left( i_{qd1} e^{j \omega_1 t} \right), \quad i_2 = \text{real} \left( i_{qd} e^{j \omega_2 t} \right)
\]

\( i_{qd1} \) and \( i_{qd2} \) are the primary and secondary complex-valued currents of the high frequency transformer winding.

Using (9) and (10) and the harmonic balance technique (HB) (3)-(6) are re-written as:

\[
p V_{c1} = \frac{1}{c_1} I_d - \frac{1}{2c_1} \text{real} \left( s_{qd1} i_{qd} \right)
\]

\[
p i_{qd} = \frac{L_1 h_1}{h} i_{qd1} + \frac{L_m h_2}{h} i_{qd2} - \frac{L_2}{h} s_{qd} V_{c1} + \frac{L_m}{h} s_{qd} V_{c2} - j \omega s_{qd1}
\]
\[ p I_{qd2} = \frac{L_m r_1}{h} i_{qd} + \frac{L_m r_2}{h} i_{qd} - \frac{L_m}{h} s_{qd} V_{c1} + \frac{L_m}{h} s_{qd} V_{c2} - f \omega_i I_{qd} \]

\[ p V_{c2} = \frac{n}{2 c_2} \text{real}(s_{qd} i_{qd}^*) - \frac{L_o}{c_2} \]

\[ f_{qd} = f_a + j f_d \]

\[ f_{qd} = f_a + j f_d \]

The primary-side-referred peak primary and secondary current of transformer windings. Figure 5 depicts the total conduction loss with respect to the DAB’s output power.

\[ \text{TABLE I. DAB PARAMETERS} \]

| Parameters                          | Values                                           |
|-------------------------------------|--------------------------------------------------|
| Input voltage                       | \( V_i = 100 \text{V} \)                         |
| Transformer winding self and mutual inductances | \( L_1 = 4.2134 \text{mH}, \) \( L_2 = 4.2158 \text{mH}, \) \( L_m = 4.205 \text{mH} \) |
| Input Inductance                    | \( L_d = 10 \text{mH} \)                         |
| Transformer ,winding resistances    | \( R_1 = 0.45 \Omega, \) \( R_2 = 0.45 \Omega \) |
| Turns ratio                         | \( n = 0.5 \)                                   |
| Desired output voltage              | \( V_{c2} = 200 \text{V} \)                     |
| Desired Power                       | \(-1 \text{kW to 1kW}\)                         |
| Switching frequency                 | \( f_s = 25 \text{kH} \)                        |
D. Zero Voltage Switching Analysis

Based on the convention for the flow of current adopted for this paper, the conditions for ZVS are expressed in terms of the transformer winding currents at switching instants. The conditions are given as:

- Half-Bridge 1 \((T_{ap1}, T_{an1})\): \(i_1(\omega_s t = -\frac{\alpha_1}{2}) < -i_{1\text{min}}\)
- Half-Bridge 2 \((T_{bp1}, T_{bn1})\): \(i_1(\omega_s t = \frac{\alpha_1}{2}) < -i_{1\text{min}}\)
- Half-Bridge 3 \((T_{ap2}, T_{an2})\): \(i_2(\omega_s t = \delta - \frac{\alpha_2}{2}) > i_{2\text{min}}\)
- Half-Bridge 4 \((T_{bp2}, T_{bn2})\): \(i_2(\omega_s t = \delta + \frac{\alpha_2}{2}) > i_{2\text{min}}\)

Where \(\alpha_1 = \pi - d_1, \alpha_2 = \pi - d_2, \) and \(i_{1\text{min}}\) and \(i_{2\text{min}}\) are the minimum absolute current required to charge the device output capacitance completely within the deadtime. Figures 6 and 7 show the results of the ZVS analyses. The results show that the first half bridge on the primary side does not meet ZVS requirements for the entire power range while, the second half bridge meets ZVS requirements from -1kW up to -100Watts. The secondary bridge meets the ZVS requirements throughout the entire power range except between -0.5kW and 0.25kW for the first half bridge and -0.5kW- 0.4kW for the second half bridge.

E. Stability Analysis

Based on the steady-state analysis results obtained, the stability of the system is analyzed using the FHA model of the system for a constant voltage operation and a constant power operation. From the eigenvalues of the DAB system for the constant voltage operation, as shown in Figure 8, it is observed that the system is stable for the entire power range considered as all the eigenvalues of the system are on the left-hand side of the complex plane. Also, from the eigenvalues obtained for the constant power operation, shown in Figure 9, the DAB system is observed to be stable for low output active power, but unstable for power demand close to the rated value for the considered system. There are positive eigenvalues for power ranges equal to or greater than 700W.

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**Fig. 6.** Transformer winding currents at switching instants. Instantaneous current at \(I_1(\omega_s t = -\frac{\alpha_1}{2})\) in blue, and instantaneous current at \(I_1(\omega_s t = \frac{\alpha_1}{2})\) in red

**Fig. 7.** Transformer winding currents at switching instants. Instantaneous current at \(I_2(\omega_s t = \delta - \frac{\alpha_2}{2})\) in blue, and instantaneous current at \(I_2(\omega_s t = \delta + \frac{\alpha_2}{2})\) in red

**Fig. 8.** Scatter plot of the eigenvalues for FHA model of the DAB for a constant output voltage operation

**Fig. 9.** Scatter plot of the eigenvalues for FHA model of the DAB for a constant power load operation
III. CONTROLLABILITY, OBSERVABILITY AND ZERO DYNAMICS OF THE SYSTEM

A. Controllability and Observability Analysis

For a nonlinear system given as:
\[
\dot{x} = f(x) + g(x)u \tag{22}
\]
\[
y = h(x) \tag{23}
\]
and \( h(x) = [h_1(x) \ h_2(x) \ \ldots \ h_j(x)] \),
\[ g(x) = [g_1(x) \ g_2(x) \ g_m(x)] \],

The controllability matrix is given by:
\[
M = \begin{bmatrix} g_1 & \ldots & g_m & a d_f^{k-1} g_1 & \ldots & a d_f^{k-1} g_m \end{bmatrix}
\]

\( m \) is the number of control inputs, and \( k \) is the dimension of the system. The rank of \( M \) based on state space model equations ((2) – (6)) of the DAB system is 5 which is equal to the dimension of the system. This means the system has a full rank for the controllability matrix and the system is completely controllable. Again, for a nonlinear system, given a matrix \( N \)
\[
N = \begin{bmatrix} h_1 & L_f^{k-1} h_1 & \ldots & h_m & L_f^{k-1} h_m \end{bmatrix}
\]

The observability matrix is given by:
\[
J = \frac{dN}{dx} \text{ at } x = x_0 \tag{24}
\]

The system is locally weakly observable at \( x = x_o \) if any \( k \) row combination in \( J \) has full rank [9]. For the system considered, the maximum rank is three (3), which is less than the dimension of the system. This means the system is not fully observable. The rank of the observability and controllability matrix helps to determine if a system is fully observable/controllable or not fully observable/controllable. It does not show the degree of observability or controllability of the system’s states. For the DAB system, since it is fully controllable, it means all the states are controllable. However, it is important to identify the unobservable or least observable states since the system is not fully observable. The Popov–Belevitch–Hautus (PBH) observability test is usually used to determine the influence of the measured variable on the eigenvalues of the system. However, the more important measurement is the influence the measure state on the estimation of the system’s state variables. Since the participation factor \( P \) gives the contribution of each state to the system’s eigenvalues, the combination of the observability PBH test and participation factor is used to determine the degree of observability of each state with respect to the measured/controlled variable. The observability sensitivity is given as[10], [11]:
\[
K_o = P \times \delta_{o,\min} \tag{25}
\]

The participation factor, \( P \) which depends on the system’s parameters and steady-operating points is given as:
\[
P_{kj} = \frac{|V_{kj}| |W_{kj}|}{W_f V_f} \]

\( V_{kj} \) and \( W_{kj} \) are the left and right eigenvector of the system’s matrix.

B. Zero Dynamics

Zero dynamics is the dynamics of a nonlinear system when the output states and reachable states of the system are constrained to zero [12]. It is the worst case of the internal dynamics of a nonlinear system. Thus, if the zero dynamics of a nonlinear system is stable, then the system can be stabilized by the input-output feedback control law. The relative degree of the system is determined by continuously differentiating the outputs of the system described by (11) – (14) until an input variable appears.

Zero dynamics equations are given
\[
\dot{y}_1 = p V_{c1} = \frac{1}{C_1} i_d - \frac{1}{C_1} \text{real}(s_{qad}^* i_{q1}) \tag{27}
\]
\[
\dot{y}_2 = p V_{c2} = \frac{n}{C_2} \text{real}(s_{qad}^* i_{q2}) - \frac{L_m}{C} r_2 \tag{28}
\]

Since \( V_{c1} \) and \( V_{c2} \) are the outputs, the zero dynamic states are \( i_d \), \( i_{q1} \) and \( i_{q2} \). The zero dynamics equations are given by:
\[
p i_d = \frac{V_{c1}}{L_d} - \frac{V_{c2}}{L_d} - r_1 \tag{29}
\]
\[
p i_{q1} = \frac{l_{mR_1}}{h} i_{q1} + \frac{l_{mR_2}}{h} i_{q2} - \frac{i_d}{h} s_{qad} V_{c1} + \frac{l_m}{h} s_{qad} V_{c2} - j \omega s_{i_{q1}} \tag{30}
\]
\[
p i_{q2} = \frac{L_m R_2}{h} i_{q1} + \frac{l_{mR_2}}{h} i_{q2} - \frac{l_m}{h} s_{qad} V_{c1} + \frac{l_m}{h} s_{qad} V_{c2} - j \omega s_{i_{q2}} \tag{31}
\]

By (29) is decoupled from (30) and (31),
\[
s_{i_d} + \frac{r}{L_d} i_d = 0 \rightarrow s = - \frac{r}{L_d} \tag{32}
\]

This means the input inductor current is stable.

At zero dynamics, \( V_{c1} = 0 \), \( V_{c2} = 0 \) and (12) and (13) reduce to:
\[
p i_{q1} = \frac{l_{mR_1}}{h} i_{q1} + \frac{l_{mR_2}}{h} i_{q2} - j \omega s_{i_{q1}} \tag{33}
\]
\[
p i_{q2} = \frac{L_m R_2}{h} i_{q1} + \frac{l_{mR_2}}{h} i_{q2} - j \omega s_{i_{q2}} \tag{34}
\]

(27) and (28) can also be written as:

### Table II: Degree of Observability of System State

| states | \( V_{c1} \) | \( i_d \) | \( i_{q3} \) \( i_{q4} \) | \( i_{q5} \) \( i_{q6} \) | \( V_{c2} \) |
|--------|--------------|-------------|-----------------|-----------------|-------------|
| degree | 0.4225       | 0.3967      | 0.0016          | 0.0022          | 0.0017       | 0.8406      |

\[
\delta_{o,\min}(\lambda_i) = \min(\sqrt{|\lambda_i e_s^T e_o|}) \tag{26}
\]

The degree of observability of all the states is determined as shown in Table 2. From the results the transformer currents are the least observable and hence the unobservable states of the system when the output voltage of the system is measured and regulated.
\[ pI_{qd} = AI_{qd} \quad (29) \]

\[
A = \begin{bmatrix}
\frac{R_1L_1 - j\omega_s}{h} & \frac{L_mR_2}{h} \\
\frac{R_1L_m}{h} & \frac{R_2L_1 - j\omega_s}{h}
\end{bmatrix}
\]

It the characteristic equation is generally written as:

\[
a_\omega \lambda^2 + (a_1 + jb_2)\lambda + a_2 + jb_2 = 0
\quad (30)
\]

\[
a_\omega = 1, \quad a_1 = -\frac{L_m}{R_1 + R_2}, \quad b_1 = 2\omega_s,
\]

\[
a_2 = \frac{L_2^2R_1R_2}{h^2} - \frac{L_m^2R_1R_2}{h^2} - \omega_s^2, \quad b_2 = -\frac{\omega_sL_1}{h}(R_1 + R_2)
\]

Routh-Hurwitz Stability criterion for the second order system is given as [13]:

\[
\Delta_1 > 0; \quad \Delta_2 > 0
\]

\[
\Delta_1 = a_1, \quad \Delta_2 = \det \begin{bmatrix}
a_2 & -b_2 \\
0 & b_2
\end{bmatrix}
\]

\[
\Delta_1 = a_1 > 0; \quad \Delta_2 = \frac{R_1R_2L_1^2}{h^2}(R_1 + R_2)^2(L_1^2 - L_m^2) > 0
\]

\[
L_1^2 - L_m^2 > 0,
\]

The primary winding self-inductance will always be greater than magnetizing inductance. This means regardless of the operating conditions of the system; the zero dynamics of the system will always be stable. The transformer winding currents have been identified as the internal dynamics of the DAB system and have been shown to always be stable regardless of the DAB’s operating conditions. The stability of the internal dynamics also explains the reason the neglect of the dynamics of the transformer current does not affect the design of a stable controller for the DAB using the input-output feedback method.

IV. CONCLUSION

In this paper, the continuous time as well as the FHA model of the DAB are derived. Steady state analysis of the system is done by constructing it as an optimization problem where the sum of the squares of the transformer peak current was minimized. The stability analysis of the system is conducted, and results obtained shows that the system is always stable for a constant output voltage operation and unstable for a constant power load demand which is close to the DAB system’s rated output power. It is established that the zero dynamics which is the worst case of the system’s internal dynamics are the dynamics of the transformer currents. The transformer current dynamics are shown to be stable regardless of the DAB’s operating conditions. The zero dynamics analysis confirms that the DAB can be stabilized through the now popular input-output feedback control laws, neglecting the influence of the transformer currents, since they are stable, even though they are unobservable and may not be possibly estimated by an observer.

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