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The hadronic $k_t$-spectrum inside one jet is determined including corrections of relative magnitude $O(\sqrt{\alpha_s})$ with respect to the Modified Leading Logarithmic Approximation (MLLA), at and beyond the limiting spectrum (assuming an infrared cut-off $Q_0 = \Lambda_{QCD}$ and $Q_0 \neq \Lambda_{QCD}$). The agreement between our results and preliminary measurements by the CDF collaboration is impressive, much better than at MLLA, pointing out very small overall non-perturbative contributions.

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Hadronic single inclusive $k_t$ distributions inside one jet beyond MLLA

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with \( \psi(\ell, y) = G(\ell, y)/G(\ell, y) \). The MLLA coefficients \( \hat{a}_1 = 3/4 \) and \( \hat{a}_2 \approx 0.935 \) are computed in [8] while at NMLLA, we get [9]:

\[
\hat{a}_2 = \frac{7}{8} + \frac{C_F}{N_c} \left( \frac{5}{8} - \frac{\pi^2}{6} \right) \approx 0.42, \\
(6)
\]

\[
\hat{a}_2 = \frac{67}{36} - \frac{\pi^2}{6} - \frac{13 n_c T R C_F}{18 N_c N_c} \approx 0.06 . 
(7)
\]

Computing the NMLLA partonic distributions inside a quark and gluon jet, \( Q(z) \) and \( G(z) \), is the first step to determine the double differential spectrum \( d^2 N/dx d\Theta \) of a hadron produced with energy \( xE \) and at angle \( \Theta \) with respect to the jet axis identified with the direction of the energy flow (see [8]). As shown in [9], it is given by

\[
\frac{d^2N}{dx d\ln \Theta} = \frac{d}{d\ln \Theta} F_{A_0}^h(x, \Theta, E, \Theta_0), 
(8)
\]

where \( F_{A_0}^h \) is given by the convolution of two fragmentation functions

\[
F_{A_0}^h = \sum_A \int_1^1 du D_{A_0}^A(u, E \Theta_0, u E \Theta) D_{A}^h \left( \frac{x}{u} , u E \Theta , Q_0 \right), 
(9)
\]

\( u \) being the energy fraction of the intermediate parton \( A \). \( D_{A_0}^A \) describes the probability to emit \( A \) with energy \( u E \) off the parton \( A_0 \) (which initiates the jet), taking into account the evolution of the jet between \( \Theta_0 \) and \( \Theta \). \( D_{A}^h \) describes the probability to produce the hadron \( h \) off \( A \) with energy fraction \( x/u \) and transverse momentum \( k_\perp \approx u E \Theta \geq Q_0 \) (see Fig. 1). As discussed in [9], the convolution [9] is dominated by \( u \approx 1 \) and therefore \( D_{A_0}^A(u, E \Theta_0, u E \Theta) \) is given by DGLAP evolution [10]. On the contrary, the distribution \( \tilde{D}_{A}^h(\ell + \ln u, y) \) low \( x \ll u \) reduces to the hump-backed plateau,

\[
\tilde{D}_{A}^h(\ell + \ln u, y) \approx u^{\hat{a}_1}(\ell + \ln u, Y_{\Theta} + \ln u), 
(10)
\]

with \( Y_{\Theta} = \ell + y = \ln E \Theta / Q_0 \). Performing the Taylor expansion of \( \tilde{D} \) to the second order in \( \ln u \) and plugging it into Eq. (9) leads to

\[
xF_{A_0}^h \approx \sum_A \int_0^1 du u D_{A_0}^A(u, E \Theta_0, u E \Theta) \tilde{D}_{A}^h(\ell, y) + \frac{1}{2} \int_0^1 du u \ln^2 u D_{A_0}^A(u, E \Theta_0, u E \Theta) \frac{d^2 \tilde{D}_{A}^h(\ell, y)}{d\ell^2}. 
(11)
\]

The first two terms in Eq. (11) correspond to the MLLA distribution calculated in [8] when \( \tilde{D}_{A}^h \) is evaluated at NLO and its derivative at LO. NMLLA corrections arise from their respective calculation at NNLO and NLO, and, mainly in practice, from the third line, which is new. Indeed, since \( x/u \) is small, the inclusive spectrum \( \tilde{D}_{A}^h(\ell, y) \) is the solution of the next-to-MLLA evolution equations [11] and [12]. However, because of the smallness of the coefficient \( \hat{a}_2 \) (see (7)), \( G(\ell, y) \) shows no significant difference from MLLA to NMLLA. As a consequence, we use the MLLA expression for \( G \). It is determined here from a representation in terms of a single Mellin transform of confluent hypergeometric functions (see Eq. (24) or (10)), well suited for numerical studies [13]. The NMLLA quark distribution \( Q(\ell, y) \) can then be deduced from \( G(\ell, y) \) using (11) and (9), which yields

\[
Q(\ell, y) = \frac{C_F}{N_c} \left[ G(\ell, y) + \left( a_1 - \hat{a}_1 \right) G(\ell, y) + \left( a_1 - \hat{a}_1 + a_2 - \hat{a}_2 \right) G(\ell, y) \right] + \mathcal{O}(\gamma^2). 
(12)
\]

The functions \( F_{A_0}^h \) and \( F_{A}^h \) are related to the gluon distribution via the color currents \( \langle C \rangle \) defined as:

\[
x F_{A_0}^h = \frac{\langle C \rangle}{N_c} G(\ell, y). 
(13)
\]

\( \langle C \rangle \) can be seen as the average color charge carried by the parton \( A \) due to the DGLAP evolution from \( A_0 \) to \( A \). Introducing the first and second logarithmic derivatives of \( \tilde{D}_{A}^h \),

\[
\psi_{A, \ell}(\ell, y) = \frac{1}{\tilde{D}_{A}^h(\ell, y)} \frac{d\tilde{D}_{A}^h(\ell, y)}{d\ell} = \mathcal{O}(\alpha_s), \\
(\psi_{A, \ell} + \psi_{A, u})(\ell, y) = \frac{1}{\tilde{D}_{A}^h(\ell, y)} \frac{d^2 \tilde{D}_{A}^h(\ell, y)}{d\ell^2} = \mathcal{O}(\alpha_s),
(14)
\]

Eq. (11) can now be written as

\[
x F_{A_0}^h \approx \sum_A \left[ \langle u \rangle_{A_0} A^A + \langle u \ln u \rangle_{A_0}^A \psi_{A, u}(\ell, y) \\
+ \frac{1}{2} \langle u \ln^2 u \rangle_{A_0}^A (\psi_{A, \ell} + \psi_{A, u})(\ell, y) \right] \tilde{D}_{A}^h, 
(14)
\]

with the notation

\[
\langle u \ln^i u \rangle_{A_0}^A = \int_0^1 du \left( u \ln^i u \right) D_{A_0}^A(u, E \Theta_0, u E \Theta).
(14)
\]
limiting spectrum, 

\[
\delta(C)_{\text{MLLA} - \text{LO}} = N_c \left( \langle u \ln u \rangle_{A_0}^g \psi_{g,\ell} + C_F \langle u \ln u \rangle_{A_0}^q \psi_{q,\ell} \right),
\]

\[
\delta(C)_{\text{NMLLA} - \text{MLLA}} = N_c \left( \langle u \ln^2 u \rangle_{A_0}^g \left( \psi_{g,\ell}^2 + \psi_{g,\ell} \right) \right) + C_F \left( \langle u \ln^2 u \rangle_{A_0}^q \left( \psi_{q,\ell}^2 + \psi_{q,\ell} \right) \right).
\]

The MLLA correction, \( O(\sqrt{\alpha_s}) \), was determined in [8] and the NMLLA contribution, \( O(\alpha_s) \), to the average color current is new. The latter can be obtained from the Mellin moments of the DGLAP fragmentation functions

\[
D_{A_0}^A(j, \xi) = \int_0^1 du \ u^{-1} \ D_{A_0}^A(u, \xi),
\]

leading to

\[
\langle u \ln^2 u \rangle_{A_0}^A = \left. \frac{d^2}{dj^2} D_{A_0}^A(j, \xi(E\Theta_0) - \xi(E\Theta)) \right|_{j=2}.
\]

Plugging (17) into (16), the NMLLA color currents for gluon and quark jets are determined analytically [8]. For illustrative purposes, the LO, MLLA, and NMLLA average color current of a quark jet with \( Y_{\Theta_0} = 6.4 \) – corresponding roughly to Tevatron energies – is plotted in Fig. 2 as a function of \( y \), at fixed \( \ell = 2 \). As discussed in [8], the MLLA corrections to the LO color current are found to be large and negative. As expected, the correction \( O(\alpha_s) \) from MLLA to NMLLA proves much smaller; it is negative (positive) at small (large) \( y \).

This calculation has also been extended beyond the limiting spectrum, \( \lambda \neq 0 \), to take into account hadronization effects in the production of “massive” hadrons, \( m = O(Q_0) \). The NMLLA (normalized) corrections to the MLLA result are displayed in Fig. 3 for different values \( \lambda = 0, 0.5, 1 \). It clearly indicates that the larger \( \lambda \), the smaller the NMLLA corrections. In particular, they can be as large as 30% at the limiting spectrum (\( \lambda = 0 \)) but no more than 10% for \( \lambda = 0.5 \). This is not surprising since \( \lambda \neq 0 \) (\( Q_0 \neq \Lambda_{\text{QCD}} \)) reduces the parton emission in the infrared sector and, thus, higher-order corrections.

The double differential spectrum \( d^2N/dy\ell d\ell \), Eq. (8), can now be determined from the NMLLA color currents using the MLLA quark and gluon distributions integrating it over \( \ell \) leads to the single inclusive \( y \)-distribution (or \( k_\perp \)-distribution) of hadrons inside a quark or a gluon jet:

\[
\left( \frac{dN}{dy} \right)_{g,q} = \left( k_{\perp} \frac{dN}{dk_{\perp}} \right)_{g,q} = \int_{\ell_{\text{min}}}^{Y_{\Theta_0} - y} d\ell \left( \frac{d^2N}{d\ell dy} \right)_{g,q}.
\]

The MLLA framework does not specify down to which values of \( \ell \) (up to which values of \( x \)) the double differential spectrum \( d^2N/dy\ell d\ell \) should be integrated over. Since \( d^2N/dy\ell d\ell \) becomes negative (non-physical) at small values of \( \ell \) (see e.g. [9]), we chose the lower bound \( \ell_{\text{min}} \) so as to guarantee the positiveness of \( d^2N/dy\ell d\ell \) over the whole \( \ell_{\text{min}} \leq \ell \leq Y_{\Theta_0} \) range (in practice, \( \ell_{\text{min}} \sim 1 \) and \( \ell_{\text{min}} \sim 2 \)).

Having successfully computed the single \( k_\perp \)-spectra including NMLLA corrections, we now compare the result with existing data. The CDF collaboration at the Tevatron recently reported on preliminary measurements over a wide range of jet hardness, \( Q = E\Theta_0 \), in \( p\bar{p} \) collisions at \( \sqrt{s} = 1.96 \text{ TeV} \). CDF data, including systematic errors, are plotted in Fig. 4 together with the MLLA predictions of [9] and the present NMLLA calculations, both at the limiting spectrum (\( \lambda = 0 \)) and taking \( \Lambda_{\text{QCD}} = 250 \text{ MeV} \); the experimental distributions suffering from large normalization errors, data and theory are normalized to the same bin, \( \ln(k_\perp / 1 \text{ GeV}) = -0.1 \). The agreement between the CDF results and the NMLLA dis-
The question of the matching of these two definitions at the limiting spectrum, or at least for small values of $k_\perp$, is reached at the limiting spectrum, or at least for small $k_\perp$ are cut for the sake of clarity).

In contrast, the MLLA predictions prove reliable in a much larger $k_\perp$-interval. At fixed jet hardness (and thus $Y_0$), NMLLA calculations prove accordingly trustable in a much larger $x$ interval.

Despite this encouraging agreement with data, the present calculation still suffers from various theoretical uncertainties, discussed in detail in [8]. Among them, the variation of $\Lambda_{QCD}$ -- giving NMLLA corrections -- from the default value $\Lambda_{QCD} = 250$ MeV to 150 MeV and 400 MeV affects the normalized $k_\perp$-distributions by roughly 20% in the largest $\ln(k_\perp/1\,\text{GeV}) = 3$ GeV-bin at $Q = 100$ GeV. Also, cutting the integral $[15]$ at small values of $\ell$ is somewhat arbitrary. However, we checked that changing $\ell_{\text{min}}^0$ from 1 to 1.5 modifies the NMLLA spectra at large $k_\perp$ by $\sim 20\%$ only [10]. Finally, the $k_\perp$-distribution is determined with respect to the jet energy flow from 2-particle correlations (which includes a summation over secondary hadrons), while experimentally the jet axis is determined exclusively from all particles inside the jet. The question of the matching of these two definitions at $O(\alpha_s)$ accuracy goes beyond the scope of this Letter.

The NMLLA $k_\perp$-spectrum has also been calculated beyond the limiting spectrum, as illustrated in Fig. [5]. However, the best description of CDF preliminary data is reached at the limiting spectrum, or at least for small values of $\lambda \lesssim 0.5$, which is not too surprising since these inclusive measurements mostly involve pions. Identifying produced hadrons would offer the interesting possibility to check a dependence of the shape of $k_\perp$-distributions on the hadron species, such as the one predicted in Fig. [5].

To summarize, single inclusive $k_\perp$-spectra inside a jet are determined including higher-order $O(\alpha_s)$ (i.e. NMLLA) corrections from the Taylor expansion of the MLLA evolution equations and beyond the limiting spectrum, $\lambda \neq 0$. The agreement between NMLLA predictions and CDF preliminary data in $p\bar{p}$ collisions at the Tevatron is very good, indicating very small overall non-perturbative corrections. The MLLA evolution equations for inclusive enough variables prove once more (see e.g. [4]) to include reliable information at a higher precision than the one at which they have been deduced.

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[15] It was also given in [5] a compact Mellin representation from which an analytic approximated expression was found using the steepest descent method [11].
[16] The effect of varying $\ell_{\text{min}}$ is more dramatic at MLLA.