ABSTRACT New applications for mobile networks are driving up data rates, requiring new solutions to meet the demand for throughput. In urban and densely populated areas, massive multiple-input multiple-output, high bands in millimeter waves, and ultra-dense networks based on small cells are being used to achieve high data rates. However, these solutions do not apply to the enhanced remote areas communications scenario, where large cells must be deployed using vacant VHF or UHF bands with an opportunistic spectrum allocation approach. In this case, a reasonable solution to increase the overall system data rate is to enhance the waveform spectral efficiency. This paper proposes a method to improve the spectral efficiency of the generalized frequency division multiplexing scheme by exploiting the faster-than-Nyquist principle in the time and frequency domains without introducing any penalties in terms of bit error rate. This achievement is obtained by under-sampling the time-frequency grid, which results in more data symbols being transmitted per waveform sample while introducing a controlled amount of interference into the transmitted waveform. On the receiver side, low complexity detectors, such as sphere decoder, successive symbol-by-symbol sequence estimation, successive symbol-by-symbol with go-back K sequence estimator, and frequency-domain equalization, can be used to reconstruct the transmitted data. This paper presents the details of adapting the mentioned detectors to the proposed waveform and analyzes their performance in terms of bit error rate under different channel models and implementation complexity. Among all the detectors considered in this paper, the sphere decoder stands out as it presents an interesting compromise between complexity and bit error rate, even in scenarios with high interference. Other detection techniques, originally proposed for multiple-input multiple-output systems, can be adapted to handle the interference introduced by the faster-than-Nyquist generalized frequency division multiplexing with reasonable complexity and acceptable bit error rate performance. Hence, one may conclude that the faster-than-Nyquist generalized frequency division multiplexing is a suitable candidate waveform for the enhanced remote areas communications scenario when high spectrum efficiency is required.

INDEX TERMS Faster-than-nyquist, frequency-domain equalization, generalized frequency division multiplexing, sphere detector, sequence estimation, symbol-by-symbol estimation.

I. INTRODUCTION

The advent of the fifth-generation (5G) of mobile networks is introducing applications that require high throughput and spectrum efficiency [1]. Reduced coverage radius cells operating at wide bandwidths above 20 GHz are being considered to support these applications. 5G new radio (NR) [2], for example, increases overall system efficiency by using massive multiple-input multiple-output (MIMO) schemes operating at a wide bandwidth in the millimeter-wave frequency range and by using small cells in an ultra-dense network (UDN). The latency reduction enforced by ultra-reliable low-latency communications (URLLC) applications also limits the coverage area since the propagation time in this scenario is...
on the order of microseconds [3]. However, these approaches cannot be applied to all the scenarios envisioned for future mobile networks. An important example is enhanced remote area communications (eRAC), where broadband and Internet of Things (IoT) services must be provided in low-populated areas, i.e., rural regions.

One of the challenges of bringing connectivity to remote areas is the limited communication infrastructure. Because of the high cost and difficulty of installing and maintaining equipment in these locations, base stations must be sparsely distributed. Therefore, the cells in remote areas must have a wide coverage radius. High-frequency bands and small cells are an interesting approach for mobile networks in densely populated urban areas but are not an appropriate solution for the eRAC scenario.

Sub-gigahertz frequencies should be considered in rural and remote areas because the propagation characteristics of these bands are better suited for operating over long distances. As the antennas are large in this frequency range, massive MIMO is not a realistic approach to improving spectral efficiency in this case. Spectrum at lower frequencies is scarce, so opportunistic use of TV white spaces (TVWSs) for mobile services in this band is often considered to reduce deployment costs by avoiding spectrum licenses and sharing spectrum with incumbents and other secondary networks. Thus, throughput is increased by optimizing the use of the available bandwidth.

For these reasons, in this particular use case, massive MIMO, UDN, and millimeter waves, proposed in 5G NR, are not applicable because sub 1 GHz bands in TVWS must be used opportunistically to provide large cells to farms and remote locations [3]. One possible solution to achieve higher data rates in these cases is to increase the spectral efficiency of the waveform. Therefore, the research presented in this paper aims to increase the spectral efficiency of a waveform suitable for eRAC applications.

The exploitation of TVWSs requires a flexible frame structure and a very low out-of-band emission (OOB-E) waveform to coexist with primary users. Generalized frequency division multiplexing (GFDM) [4] is a configurable multi-carrier waveform with good frequency localization and the ability to cover conventional waveforms such as corner cases [3].

Slight modifications in the definition of the time-frequency grid allow GFDM to include faster-than-Nyquist (FTN) signaling [5] as a new corner case, improving the system’s spectral efficiency. The problem with the FTN-GFDM waveform lies in the high complexity required to detect the signal, since linear receivers, such as the zero-forcing (ZF) and minimum mean square error (MMSE) [4], perform poorly. To reduce the bit error rate (BER) performance loss, the receiver must mitigate the inherent interference caused by the FTN signaling. The maximum likelihood sequence estimation (MLSE) detector [6] can resolve the interference without BER performance degradation. However, it requires an exhaustive search for all possible combinations of transmitted symbols, leading to prohibitive complexity in practical applications.

Over the years, several works have exploited the sequence structure of FTN signaling to propose techniques based on sequence estimation. In [7], the authors proposed a sequence detector based on the sphere detector (SD). In this detector, the search space is limited to a multidimensional sphere, whose initial radius is defined by pre-estimate data provided by a low-complexity detector. Moreover, it was shown in [7] that when SD is applied to conventional binary phase-shift keying (BPSK) FTN signaling under the additive white Gaussian noise (AWGN) channel, it can achieve near-optimal performance with reduced complexity. However, it is interesting to observe that in the worst case, when the initial radius covers all possible candidate sequences, the SD complexity can be as high as that of the MLSE detector. In [8], the authors proposed two low-complexity sequence detectors based on a successive symbol-to-symbol interference cancellation algorithm. For these detectors to work perfectly in a noise-free transmission, an operating region is defined based on the prototype filter and the FTN compression factor. In a noisy channel, the successive symbol-by-symbol sequence estimator (SSSSE) performance is degraded by error propagation. To solve this problem, the authors proposed the successive symbol-by-symbol with go-back K sequence estimator (SSSgbKSE) in which the previous K symbols are re-estimated to avoid error propagation. In [9], the authors proposed a sequence detection technique based on frequency-domain equalization (FDE). In this case, a cyclic prefix (CP) is inserted in each transmission block, making the signal circular and allowing low-complexity MMSE demodulation based on fast Fourier transform (FFT) at the receiver. This proposal becomes interesting in scenarios where the block size is much larger than the CP. In [10], the authors proposed an iterative detector in the frequency domain based on the decision feedback equalizer (DFE). The FTN receiver using iterative block DFE (IB-DFE) has better BER performance than FDE. In contrast, this type of receiver requires matrix inversions, and the computational complexity increases dramatically with the number of iterations. With the exception of [11], which proposes integrating FTN signaling into GFDM, these works consider the conventional FTN system under the AWGN channel, with compression in one domain only.

The main objective of this paper is to propose low computational complexity detectors based on sequence estimation to retrieve data bits in the FTN-GFDM receiver. The MLSE is the optimal detector for the FTN-GFDM system but has a prohibitive computational cost. The SD [7], SSSSE [8], SSSgbKSE [8], and FDE [9] detectors are extended to the scenario of non-orthogonal waveforms with compression in the time and frequency domains. The IB-DFE performance is not evaluated in this work because the other detectors achieve similar performance and do not require matrix inversions. Moreover, this work does not aim to provide an exhaustive analysis and adequacy of all state-of-the-art detection techniques for the proposed system but rather shows that detectors originally derived for MIMO systems can be adapted to mitigate the self-interference introduced in FTN-GFDM systems.
The results presented in this paper show that SD maintains good performance even in high spectral efficiency scenarios, with simultaneous compression in the time and frequency domains, while requiring low computational cost compared to the MLSE detector. Therefore, the SD is considered a benchmark in this work to compare the performance of other solutions with lower computational complexity.

The remainder of this paper is organized as follows. In Section II, the FTN-GFDM system model is presented. Section III reviews the SD concepts proposed in [7]. Sections IV and V present the concepts of SSSSE and SSSgKSE [8], respectively, and how these estimators can be adapted to the FTN-GFDM system. Section VI reviews the concepts of FDE based on MMSE [9]. Section VII compares the BER performance of the techniques presented in the previous sections for FTN-GFDM systems under different compression factors and channels, and it also brings a complexity analysis between the SD and MLSE since these techniques outperformed the others. Finally, Section VIII concludes this paper.

II. FTN-GFDM SYSTEM MODEL

GFDM [4] is an innovative waveform developed to meet the conflicting requirements of future mobile networks. In this waveform, \( N = KM \) data symbols are transmitted over \( K \) subcarriers, each of which carries \( M \) sub symbols. A prototype filter \( g \) is circularly shifted in time and frequency to transmit each data symbol. The GFDM signal is given by

\[
x[n] = \sum_{k} \sum_{m} d_{k,m} g_{k,m}[n],
\]

where \( d_{k,m} \) is the data symbol transmitted in the \( n \)th subcarrier and the \( m \)th sub symbol, and

\[
g_{k,m}[n] = g[(n - mK)N] \exp\left(j2\pi \frac{k}{K} n\right),
\]

where \( n = 0, \ldots, N-1 \) is the time index, \( m = 0, \ldots, M-1 \) is the sub symbol index, and \( k = 0, \ldots, K-1 \) is the subcarrier index.

For GFDM to cover FTN signaling [11], [12], the prototype filter with \( \tilde{S} \) samples must be divided into \( \tilde{P} \) periods, each with \( \tilde{S} \) samples. Each subsequent sub symbol is circularly shifted by \( \tilde{K} \) samples, and the sub carriers are shifted by \( M \) samples in the frequency domain. Hence, the GFDM signal can be rewritten as

\[
x[v] = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} d_{k,m} g[(v - m\tilde{K})\tilde{S}] \exp\left(j2\pi \frac{kM}{\tilde{N}} v\right),
\]

where \( v = 0, \ldots, \tilde{N} - 1 \) is the new time index.

FTN signaling can be introduced in this notation by making \( \tilde{K} < \tilde{S} \) and \( \tilde{M} < \tilde{P} \), i.e., making the spacing between the sub symbols in the time domain, \( \tilde{K} \), and the spacing between the sub carriers in the frequency domain, \( \tilde{M} \), less than the number of samples per period and the number of periods of the prototype filter, respectively. Consequently, the squeezing factors in time and frequency are given by

\[
\nu_f = \tilde{K}/\tilde{S} \quad \nu_f = \tilde{M}/\tilde{P}.
\]

The number of sub carriers and sub symbols are defined as

\[
K = \frac{\tilde{P}\tilde{S}}{\tilde{M}} = \frac{\tilde{S}}{\nu_f} = \begin{bmatrix} \tilde{N} \\ \tilde{M} \end{bmatrix} \quad \tilde{M} = \frac{\tilde{P}\tilde{S}}{K} = \frac{\tilde{S}}{\nu_f} = \begin{bmatrix} \tilde{N} \\ K \end{bmatrix}.
\]

If \( \tilde{N}/\tilde{M} \) and \( \tilde{S}/\tilde{K} \) are not integers, the squeezing factors must be adjusted by \( \tilde{\nu}_f = \tilde{P}/\tilde{M} \) and \( \tilde{\nu}_f = \tilde{S}/\tilde{K} \), and the GFDM amplitude must be properly scaled, resulting in

\[
\tilde{x}[v] = \sqrt{\nu_f \nu_f} \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} d_{k,m} g[(v - m\tilde{K})\tilde{S}] \exp\left(j2\pi \frac{\tilde{K}M}{\tilde{N}} v\right).
\]

Here, the constant \( \sqrt{\nu_f \nu_f} \) normalizes the power of the FTN-GFDM signal. When \( \nu_f < 1 \), intersymbol interference (ISI) is intentionally introduced into the transmitted symbol. Similarly, when \( \nu_f < 1 \), intercarrier interference (ICI) occurs.

The FTN-GFDM signal can be represented by a matrix operation as

\[
\tilde{x} = Ad,
\]

where \( \tilde{x} \) is the vector containing the FTN-GFDM samples, \( d \) is the complex data vector, and

\[
A = \begin{bmatrix} \tilde{g}_{0,0} & \tilde{g}_{1,0} & \cdots & \tilde{g}_{K-1,0} & \tilde{g}_{0,1} & \cdots & \tilde{g}_{K-1,M-1} \end{bmatrix},
\]

where

\[
[\tilde{g}_{k,m}]_{v} = \sqrt{\nu_f \nu_f} g[(v - m\tilde{K})\tilde{S}] \exp\left(j2\pi \frac{\tilde{K}M}{\tilde{N}} v\right).
\]

A CP is added to each FTN-GFDM prior transmission generating the transmit symbol \( \tilde{x} \).

Assuming that the FTN-GFDM signal is transmitted over a multipath channel, the received vector after removing the CP is given by

\[
y = H\tilde{x} + w,
\]

where \( H \) is the circulant matrix based on the channel impulse response \( h \) and \( w \) is the AWGN noise vector.

On the receiver side, a detector retrieves an estimate of the transmitted data symbols by processing the received vector \( y \). Figure 1 depicts the simplified block diagram of the FTN-GFDM communication chain. After CP removal, the signal is equalized with a simple ZF or FDE [13]. Then, a matched filter (MF) is used to maximize the signal-to-noise ratio (SNR). Although MF can extract some information from the received symbol, it cannot handle the ISI and ICI introduced during the modulation process. Therefore, a proper detection technique must be used to retrieve the data symbols. Assuming the
perfect equalization of the channel, the discrete signal at the MF output is given by

\[ r = A^H H^{-1} H A d + A^H H^{-1} w = G d + \tilde{w}, \]  

(11) where \( G = A^H A \) is the correlation coefficient matrix, \((.)^H\) is the Hermitian operation, and \( w = A^H H^{-1} w \) is the noise vector with correlated samples and covariance matrix \( \sigma^2 G \). Hence, the noise samples are not white after MF [7]. The optimal detector for this case has to accommodate the colored noise.

The ideal detector estimates the transmitted symbol vectors using the maximum a posteriori (MAP) criterion [14]. The posterior probability can be expressed as

\[ p(d|r) = \frac{p(r|d)p(d)}{p(r)}. \]  

(12) Assume that the data symbols are independent and identically distributed (i.i.d.) samples with uniform distribution, i.e. \( p(d) = \frac{1}{N^J} \), where \( J \) is the modulation order. To maximize \( p(d|r) \), it is necessary to maximize the likelihood function \( p(r|d) \) [14], which is given by

\[ p(r|d) = \frac{1}{(2\pi)^{\frac{N}{2}} \det(\Sigma_\tilde{N})^{\frac{1}{2}}} e^{-\frac{1}{2}(r-\hat{G}d)^H \Sigma_\tilde{N}^{-1} (r-\hat{G}d)}, \]  

(13) where \( \Sigma_\tilde{N} \) is a diagonal matrix of \( \tilde{N} \) singular values of \( G \). To maximize the likelihood function, the term

\[ (r-\hat{G}d)^H (\sigma^2 G)^{-1} (r-\hat{G}d) \]  

must be minimized. The optimal detector minimizes the Euclidean norm to maximize the likelihood function. In systems with ISI and ICI, the noise coloration and the interference matrix should be considered. One solution to this problem is to employ the MLSE estimator for the FTN-GFDM scheme with a whitening filter, leading to

\[ \arg \min_{d\in\mathbb{C}^N} ||(A^H)^{-1}(r - \hat{G}d)||^2, \]  

(15) where \( ||.|| \) is the Euclidean norm and \( \xi^N \) is the set of all possible data symbol sequences.

The MLSE detector achieves good performance and can be used to demonstrate that FTN-GFDM can increase spectrum efficiency without sacrificing the symbol error rate (SER). However, its complexity is prohibitive, and more feasible detectors are needed for practical implementation.

### III. SD ALGORITHM FOR FTN-GFDM DETECTION

Since MLSE is too complex for practical applications, this paper considers SD [15] as an efficient method to solve (15). The principle of the SD algorithm is to limit the search space in a multidimensional sphere, seeking only candidate solutions that are within the sphere of radius \( \rho \), whose center is the received vector \( r \). Since the closest candidate within the sphere is also the closest candidate in the entire search space, SD achieves the optimal solution with lower complexity than MLSE [16]. For a candidate \( \hat{d} \) to be within the sphere of radius \( \rho \), the following condition must be satisfied:

\[ \arg \min_{d\in\mathbb{C}^N} ||(A^H)^{-1}(r - \hat{G}d)||^2 \leq \rho^2. \]  

(16) Since the unconstrained maximum likelihood estimate is \( \zeta = (A^H A)^{-1} A^H r \), the SD radius can be rewritten as

\[ (\zeta - \hat{d})^H G^H A^{-1} (A^{-1})^H G (\zeta - \hat{d}) = (\zeta - \hat{d})^H G^H (\zeta - \hat{d}) = \rho^2. \]  

(17) Using the Cholesky decomposition [17], the matrix \( G^H \) can be represented by

\[ \text{chol}(G^H) = L^H L, \]  

(18) where \( L \) is an upper triangular matrix. Then, (16) can be rewritten as

\[ \arg \min_{d\in\mathbb{C}^N} ||L(\zeta - \hat{d})||^2 \leq \rho^2. \]  

(19) The upper triangular matrix \( L \) allows the inequality (19) to be expressed as

\[ \rho^2 \geq \sum_{i=1}^{N} \left| \sum_{j=i}^{N} L_{i,j}(\xi_j - d_j) \right|^2. \]  

(20) Thus, the necessary condition for the \( N \)th data symbol to be within the sphere is

\[ \rho^2 \geq |L_{N,N}(\xi_N - d_N)|^2. \]  

(21) Since the SD algorithm is iterative, every \( d_N \) that satisfies (21) defines

\[ \rho^2_{N-1} = \rho^2 - |L_{N,N}(\xi_N - d_N)|^2. \]  

(22) and a new search condition must be evaluated by

\[ \rho^2_{N-1} \geq |L_{N-1,N-1}(\xi_{N-1} - d_{N-1}) + L_{N-1,N}(\xi_N - d_N)|^2. \]  

(23)
Therefore, the condition for the $i$th data symbol to be inside the sphere is expressed by
\[
\rho^2 \geq \sum_{l=1}^{N} \left| \sum_{j=l}^{N} L_{l,j} (\xi_j - d_j) \right|^2.
\] (24)

The algorithm can be seen as a search tree with a depth of $N+1$ levels, where each node has $J$ branches [14]. The SD algorithm begins searching from the $N$th symbol to the first symbol. Figure 2 shows an example of a search tree for BPSK and $N = 3$. The transition from a node of level $i+1$ to a node of level $i$ determines the decision for the $i$th data symbol. For a given symbol of level $i$, if the inequality in (24) is not satisfied, the node and all nodes below it must be discarded. Every time a node is found in the first level, a point is found inside the sphere. Hence, the initial search radius and interval values should be updated. The number of candidates within the sphere depends on the initial estimate. For a better initial estimate, fewer candidates will be in the sphere. In Fig. 2, there is only one candidate within the sphere.

![FIGURE 2. Search tree for $N = 3$ and BPSK symbols.](image)

The steps to implement the SD detector are summarized in Algorithm 1.

**Algorithm 1 SD Detector**

1. **Input:** Upper triangular matrix $L$, unconstrained maximum likelihood estimate $\hat{\xi}$, initial sphere radius $\rho_{\text{initial}}$, modulation order $J$, and constellation $D$
2. $\rho^2 = \rho_{\text{initial}}^2$, $\rho_{N+1}^2 = 0$; $i = N$; $\xi_{N+1} = 0$; $\xi_N = 1$
3. while $\xi_{N+1} < 1$
   4. $\rho_{i+1}^2 = \rho_{i+1}^2 + \sum_{j=i}^N L_{i,j} (\xi_j - D(\xi_j))^2$
   5. if $i > 1$
      6. if $\rho^2 \geq \rho_i^2$
         7. $i = i - 1$; $\xi_i = \xi_i + 1$
      8. else
         9. if $\xi_i \geq J$
            10. $\xi_{i+1} = 0$; $i = i + 1$; $\xi_i = \xi_i + 1$
            11. while $\xi_i > J$
               12. $\xi_{i+1} = 0$; $i = i + 1$; $\xi_i = \xi_i + 1$
            13. end while
            14. $\xi_i = \xi_i + 1$
      15. end if
   16. end if
   17. else
      18. if $\rho^2 \geq \rho_i^2$
         19. $d = D(\xi_{i+1})$; $\rho^2 = \rho_i^2$
      20. end if
   21. if $\xi_i \geq J$
      22. $\xi_{i+1} = 0$; $i = i + 1$; $\xi_i = \xi_i + 1$
      23. while $\xi_i > J$
         24. $\xi_{i+1} = 0$; $i = i + 1$; $\xi_i = \xi_i + 1$
      25. end while
   26. else
      27. $\xi_i = \xi_i + 1$
   28. end if
   30. end while
31. **Output:** Estimated data symbols vector $\hat{d}$

where $J(N+1-i)$ is the number of nodes in the $i$th level of the search tree. Applying some sum identities, (26) can be rewritten as
\[
\frac{2J((5N+1)J^{N+1} - (5N+6)J^N - J + 6)}{(J-1)^2}.
\] (27)

Assuming the best-case scenario where only a single path is traversed from the root to the first leaf node and the remaining nodes are discarded, the lower bound for the SD complexity in terms of FLOPs is given by
\[
\sum_{i=1}^{N} J \mathcal{O}_i.
\] (28)

In this case, $J$ nodes are visited at each level of the search tree, where one visited node corresponds to the path with the minimum Euclidean distance (leading to the leaf), and the other nodes represent the pruning process (back to the root). By applying the sum identities in (28), the lower bound of the
SD complexity can be defined as

\[ J(5N^2 + 7N). \]  (29)

Thus, the SD complexity depends on the initial search radius and, therefore, on the initial estimate. Since the initial estimate is based on ZF detection, the complexity and performance will depend on SNR and system conditioning. From the search tree perspective, the complexity also depends on the modulation order and \( N \). In the worst case, SD can reach exponential complexity.

### IV. SSSS FOR FTN-GFDM DETECTION

SSSSE is a symbol-by-symbol successive detection technique originally proposed in [8]. SSSSE exploits the inherent ISI and ICI of FTN signaling to find estimation conditions that guarantee perfect symbol-by-symbol estimation. In a noise-free communication scenario, these estimation conditions depend on the pulse shape and compression factors. In this scenario, the received FTN-GFDM samples are given by

\[ r = Gd. \]  (30)

As shown in (31), at the bottom of the page, the \( G \) matrix brings the correlation between \( g_{x,m} \) and \( g_{x',m'} \). Fig. 3 shows an example of \( |G| \), assuming a Dirichlet pulse, \( K = 5, M = 5, v_r = 0.8, \) and \( v_f = 1 \). The main diagonal of \( G \) represents the gains resulting from the modulation and demodulation processes at the desired data symbol positions, and the other values represent ISI and ICI. The vector representation of the received samples in (30) can be rewritten as shown in (32), at the bottom of the page.

Each received sample in (32) can be considered as a weighted sum of \( N \) data symbols. Thus, the \( n \)th received sample \( r_n \) in a noise-free scenario can be expressed as

\[ r_n = \sum_{i=0}^{n-1} G_{0,n-i}d_i + \sum_{i=n+1}^{N-1} G_{0,n-i}d_i, \]  (33)

where term a represents the ISI and ICI of the previous \( n \) symbols, term b is the desired symbol, and term c represents the ISI and ICI of the subsequent \( N - n - 1 \) symbols. To estimate \( d_n \), one must remove the interference caused by the other symbols. Thus, the interference caused by the \( n \) previous symbols, already estimated, and by the \( N - n - 1 \) future symbols, not estimated yet, must be removed. As proposed in [8], the condition for perfect estimation for noise-free transmission considering quadrature amplitude modulation (QAM) modulation can be described by

\[ |G_{0,0}|/|d_n| > \sum_{i=n+1}^{N-1} |G_{0,n-i}|/|d_i|, \]  (34)
and
\[ |G_{0,0}| > \sum_{i=n+1}^{N-1} |G_{0,n-i}|. \quad (35) \]

These conditions do not depend on the value of \( d_0 \) and the subsequent \( N-n-1 \) data symbols, but on the pulse shape and compression factors. To ensure a perfect estimation in a noise-free transmission, the conditions in (34) and (35) must hold even for the worst ISI and ICI scenarios. The maximization of ISI and ICI in the transmitted sample occurs when the subsequent \( N-n-1 \) data symbols weighted by the respective elements of the \( n \)th row of the matrix \( G \) have the opposite sign to \( G_{0,0}d_n \). Since \( G \) has complex elements in the FTN-GFDM system, the estimation condition is given by
\[ |G_{0,0}| > \sum_{i=n+1}^{N-1} |G_{0,n-i}|. \quad (36) \]

Thus, for SSSSE to operate in the FTN-GFDM system, the condition in (36) must be satisfied. However, as the estimation condition is based on a noise-free scenario, the performance is expected to degrade in noisy scenarios since the noise may violate the estimation condition and (36) cannot be satisfied [8].

Finally, considering the estimation condition, \( d_n \) is estimated by the SSSSE detector in the presence of AWGN and ISI/ICI caused by the other symbols, such as
\[ \hat{d}_n = \text{quantize} \left( r_n - \sum_{i=0}^{n-1} G_{0,n-i}d_i \right), \quad (37) \]

where quantize rounds to the nearest BPSK symbol [8] and term a represents the ISI and ICI from the previous \( n \) symbols. It is important to note that SSSSE suffers from the effect of error propagation. When a data symbol is incorrectly estimated, the estimation accuracy of the previous symbols is degraded.

The steps to implement the SSSSE detector can be seen in Algorithm 2.

**Algorithm 2 SSSSE Detector**

1: Input: ISI matrix \( G \), and received samples \( r \)
2: for \( n = 0 : N-1 \) do
3: \( \text{if } |G_{0,0}| > \sum_{i=n+1}^{N-1} |G_{0,n-i}| \text{ then} \)
4: \( \hat{d}_n = \text{quantize} \left( r_n - \sum_{i=0}^{n-1} G_{0,n-i}d_i \right) \);
5: \( \text{end if} \)
6: \( \text{end for} \)
7: Output: Estimated data symbols \( \hat{d} \)

Since SSSSE subtracts the interference caused by the previously estimated symbols from each estimated symbol, the computational complexity of SSSSE as a function of the number of FLOPs is \( 4N(N-1) \). Thus, to estimate the last symbol of the FTN-GFDM block, \( N-1 \) multiplication operations must be performed. In contrast, no additional multiplication operation is required to estimate the first data symbol.

This means that the SSSSE complexity is lower compared to the SD detector proposed previously in Section III.

**V. SSSgbKSE FOR FTN-GFDM DETECTION**

SSSgbKSE is a symbol-by-symbol detection technique proposed in [8] to mitigate the influence of error propagation that occurs in the SSSSE detector and increase the estimation accuracy. The idea behind SSSgbKSE is to increase the estimation accuracy of the previous \( K \) symbols based on the knowledge of the desired symbol \( \hat{d}_n \) and then re-estimate \( \hat{d}_n \) considering the improved estimates of the \( K \) previous symbols. Consequently, (33) can be rewritten as
\[ r_n = \sum_{i=0}^{n-K-1} G_{0,n-i}d_i + \sum_{i=0}^{n-K} G_{0,n-i}d_i + G_{0,0}d_n + \sum_{i=n+1}^{N-1} G_{0,n-i}d_i, \quad (38) \]

where term a represents the ISI and ICI from the previous \( n \) symbols, term b is the previous \( K \) symbols to be re-estimated, term c is the desired data symbol to be re-estimated, and term d is the ISI and ICI from the subsequent \( N-n-1 \) symbols.

Similar to SSSSE, for satisfactory performance of SSSgbKSE, it is necessary to satisfy the estimation condition. First, the desired data symbol \( \hat{d}_n \) is estimated using the SSSSE algorithm, and then, based on \( \hat{d}_n \), the previous \( K \) data symbols must be updated. For this purpose, \( \hat{d}_{n-K} \) is re-estimated by
\[ \hat{\hat{d}}_{n-K} = \text{quantize} \left( r_{n-K} - \sum_{i=0}^{n-K-1} G_{0,n-K-i}d_i - \sum_{i=n-K+1}^{n} G_{0,n-K-i}d_i \right), \quad (39) \]

where term a is the ISI and ICI from the previous \( n-K \) data symbols and term b is the ISI and ICI from the subsequent \( K \) data symbols.

After the previous \( K \) symbols have been re-estimated according to (39), the desired data symbol \( d_n \) must be re-estimated as follows:
\[ \hat{d}_n = \text{quantize} \left( r_n - \sum_{i=0}^{n-K-1} G_{0,n-i}d_i - \sum_{i=n-K}^{n-1} G_{0,n-i}d_i \right), \quad (40) \]
where term a represents the ISI and ICI from the previous \( n - K \) data symbols and term b represents the ISI and ICI from the previous \( K \) re-estimated data symbols.

Algorithm 3 provides the steps necessary to perform the SSSgbKSE detection.

**Algorithm 3 SSSgbKSE Detector**

1: **Input:** ISI matrix \( G \), received samples \( r \), and \( K \)
2: for \( n = 0 : N - 1 \) do
3: if \( |G_{0,0}| > \sum_{i=n+1}^{N} |G_{0,n-i}| \) then
4: if \( n=0 \) then
5: \( \hat{d}_n = \text{quantize}(r_n); \)
6: else if \( n > K \) then
7: \( \hat{d}_n = \text{quantize}(r_n - \sum_{i=0}^{n-1} G_{0,n-i} \hat{d}_i); \)
8: for \( j = 1 : K \) do
9: \( \hat{d}_{n-j} = \text{quantize}(r_{n-j} - \sum_{i=0}^{n-j-1} G_{0,n-j-i} \hat{d}_i) - \sum_{i=0}^{n-j+1} G_{0,n-j-i} \hat{d}_i; \)
10: end for
11: \( \hat{d}_n = \text{quantize}(r_n - \sum_{i=0}^{n-K-1} G_{0,n-i} \hat{d}_i) - \sum_{i=0}^{n-K} G_{0,n-i} \hat{d}_i; \)
12: else
13: \( \hat{d}_n = \text{quantize}(r_n - \sum_{i=0}^{n-1} G_{0,n-i} \hat{d}_i); \)
14: for \( j = 1 : n - 1 \) do
15: \( \hat{d}_{n-j} = \text{quantize}(r_{n-j} - \sum_{i=0}^{n-j-1} G_{0,n-j-i} \hat{d}_i) - \sum_{i=0}^{n-j+1} G_{0,n-j-i} \hat{d}_i; \)
16: end for
17: \( \hat{d}_n = \text{quantize}(r_n - \sum_{i=0}^{n-K-1} G_{0,n-i} \hat{d}_i) - \sum_{i=0}^{n-K} G_{0,n-i} \hat{d}_i; \)
18: end if
19: end if
20: end for
21: **Output:** Estimated data symbols \( \hat{d} \)

The computational complexity for SSSgbKSE, applied to the FTN-GFDM system, can be defined in steps. First, the desired symbol is detected using SSSSE. The number of FLOPs in this step is \( 8N(N - 1)/2 \). To re-estimate the \( K \) symbols before the desired symbol \( d_n \), for \( n < K \), \( 4K(2K - 1)/3 \) FLOPs are performed. In this case, it is impossible to go back and re-estimate the previous \( K \) symbols, and then only the previous \( n - 1 \) symbols are re-estimated. For \( n \geq K \), \( 4K(K - N)(K + N - 1) \) FLOPs are performed. Finally, the desired symbol must be re-estimated considering the updated estimates of the \( K \) previous symbols, which requires \( 8N(N - 1)/2 \) FLOPs. Thus, the total complexity of the SSSgbKSE algorithm in terms of FLOPs is

\[
8N(N - 1) + \frac{4K(2K - 1)(2K - 1)}{3} - 4K(K - N)(K + N - 1). \tag{41}
\]

Therefore, the SSSgbKSE detector has a higher complexity than the SSSSE detector described in Section IV.

**VI. FREQUENCY DOMAIN EQUALIZATION**

The FDE receiver was proposed in [9] as a solution for the FTN system with low computational cost. In the proposed FDE-assisted structure, a short CP is added in each transmission block, which enables low-complexity MMSE demodulation based on FFT on the receive side [9]. Figure 4 shows the block diagram of this scheme.

Since the matrix \( G \) has a circulating structure, the eigenvalues decomposition is given by

\[
G = F^H \Lambda F,
\]

where \( \Lambda = \text{diag}(\lambda_1, \ldots, \lambda_N) \) is the matrix containing the eigenvalues of \( G \), and \( F \) is the matrix of eigenvectors represented by the Fourier matrix whose elements are given by

\[
F_{ij} = \frac{1}{\sqrt{N}} \exp(-j2\pi i j/N), \quad i = 1, \ldots, N - 1 \quad \text{and} \quad j = 1, \ldots, N - 1.
\]

In FDE, the received samples, \( r \), are represented in the frequency domain as

\[
Fr = r_f = FGd + Fw.
\]

Substituting (42) in (43) leads to

\[
r_f = F^H \Lambda d_f + \hat{w}_f = \Lambda d_f + \hat{w}_f,
\]

where \( r_f, d_f, \) and \( \hat{w}_f \) represent the vectors of received samples, transmitted data, and noise in the frequency domain, respectively. Considering the MMSE, the FDE matrix, \( W \), is given by

\[
W = \Lambda^H (\Lambda \Lambda^H + \frac{N_0}{E_s} I_N)^{-1}.
\]

Since \( W \) is a diagonal matrix, the estimator operates symbol-by-symbol [13]. Thus, the estimated symbol in the time domain is given by

\[
\hat{d} = F^H W Fr = F^H W (\Lambda d_f + \hat{w}_f).
\]

In addition, to facilitate the detection of the FTN signal, the diagonal structure of the MMSE weight matrix reduces the complexity required to calculate the inverse in (45) and allows symbol-by-symbol detection. Thus, the computational complexity of MMSE-based FDE is \( N \log N \) [9], [13]. However, the weight matrix proposed in [9] ignores the effects of FTN noise correlation.

Under these considerations and after the analysis of the weight matrix, it is possible to conclude that the eigenvalues of \( G \) play an important role in the performance of FDE. Depending on the values of \( v_t, v_f, N, \) and the pulse shape, the \( G \) matrix may contain eigenvalues close to zero, which makes it difficult to detect the associated symbols, causing BER penalties [13], [19] [20]. Therefore, the FDE detector is expected to underperform in real FTN-GFDM applications.

**VII. PERFORMANCE EVALUATION**

The BER performance of the detectors described in the previous sections is evaluated under AWGN, time-invariant frequency-selective (TIFS), and time-variant frequency-selective (TVFS) channels. Table 1 shows the channel parameters used in the simulations. The TVFS channel has four
TABLE 1. Channel models.

| Channel                  | Impulse response                  | Reference BER |
|--------------------------|-----------------------------------|---------------|
| AWGN                     | $h_{AWGN} = 1$                     | $10^{-6}$     |
| Time-invariant frequency-selective | $h_{TIFS} = [1 0.4 0.2 0.08]^T$  | $10^{-5}$     |
| Time-variant frequency-selective | $[e^{0} h_{0} e^{-4} h_{1} e^{-8} h_{2} e^{-12} h_{3}]^T$ | $3 \cdot 10^{-3}$ |

Table 2 contains the system parameters used for the performance evaluation. A reference BER is used to evaluate the performance of the detectors. The reference BER value for each channel is listed in Table 1.

TABLE 2. System parameters. BPSK, $\hat{p}=4$ and $\hat{s}=5$.

| FTN-GFDM Parameter        | Time   | Frequency | Time-Frequency |
|---------------------------|--------|-----------|----------------|
| Prototype filter          | Ditrichlet | rect | RRC            |
| Subsymbol distance factor ($v_1$) | 0.5 | 1         | 0.8            |
| Subcarrier distance factor ($v_2$) | 1    | 0.8      | 0.75           |
| Number of subsymbols ($M$) | 5      | 4        | 5              |
| Number of subcarriers ($K$) | 5      | 6        | 6              |

The GFDM framework makes it easy to assign resource blocks (RBs) to users since each FTN-GFDM symbol can be allocated as an RB. With this approach, the FTN-GFDM symbol can be configured according to the condition of each user, providing robustness and flexibility to guarantee quality of service (QoS). If a guard carrier is inserted between the RBs [21], each RB can be processed as an independent FTN-GFDM block.

Fig. 5 presents the BER performance of the detectors for time-compressed FTN-GFDM for different $\frac{E_b}{N_0}$ values over the three channels: (a) AWGN, (b) TIFS, and (c) TVFS. Due to $v_1 = 0.8$, ISI is inserted into the signal. The data rate is increased by 25% in this case. The results show that SD can achieve the same performance as MLSE but with less complexity. Since the estimation condition is satisfied, SSSgbKSE achieves BER performance very close to that of the optimal detector for values of $K$ close to $N$. However, SSSSE exhibits a performance loss of about 5 dB for the AWGN channel, 4.3 dB for the TIFS channel, and 0.8 dB for the TVFS channel compared to the MLSE detector. For the parameters shown in Table 2, the $G$ matrix is circulating and, therefore, the FDE detector can be used. However, in Fig. 5 (a), its BER performance is severely affected by $(N - \hat{N})$ eigenvalues close to zero, resulting in a loss of about 5.8 dB for the AWGN channel in the reference BER.

Fig. 6 shows the BER performance for the frequency-compressed FTN-GFDM assuming the channels proposed in Table 1: (a) AWGN, (b) TIFS, and (c) TVFS. Since the real compression factor $\gamma = \frac{\hat{M}}{M} = \frac{5}{6} = 0.8333$ is slightly higher than the compression factor in Fig. 5, the SSSSE and SSSgbKSE detectors should perform better than in the time squeezing scenario. In this case, the data rate has increased by 20%. While the estimation condition is maintained, SSSgbKSE can achieve performance very close to that of SD, even with low values of $K$. The SSSSE detector achieved a BER performance loss of approximately 3.5 dB for the AWGN channel, 2.4 dB for the TIFS channel, and 0.5 dB for the TVFS channel, in relation to the optimal detector. In Fig. 6 (a), it can also be seen that FDE has worse performance than in Fig. 5 (a). For the AWGN channel, the FDE detector achieves a BER performance loss of approximately 7.5 dB. This is because the FTN-GFDM matrix for the parameters presented in Table 2 is not circulating. Therefore, this detector should not be used in this case.

Fig. 7 shows the simulation results for time-frequency compressed FTN-GFDM. Figs. 7 (a), (b), and (c) show the BER performance over AWGN, TIFS, and TVFS channels, respectively. In this case, a root-raised cosine (RRC) prototype filter with a roll-off factor of $\beta = 0.5$ was used. The SD performance shows that it is possible to increase the data rate by 50% without BER performance degradation using $\nu_j = 0.75$ and $\nu_s = 0.8$. The squeezing factors $\nu_j$ and $\nu_s$ have independent gains that can be combined to achieve higher spectral efficiency. Since the estimation condition is violated in this scenario, the SSSSE and SSSgbKSE detectors perform poorly, while SD performance is still close to MLSE.

A. COMPUTATIONAL COMPLEXITY ANALYSIS

Assuming that the maximum likelihood (ML) detection is performed as

$$\arg \min_{d \in \mathbb{N}} \left| r - Gd \right|^2,$$

the computational complexity of ML as a function of the number of FLOPs is $(16N^2 + 8N - 2)J^N$. The ML complexity

FIGURE 4. Block diagram of the FDE scheme.
is constant and grows exponentially with the increase in the size of the transmission matrix and the modulation order. Unlike ML, SD complexity is variable and depends on the \( G \) matrix conditioning, noise level, transmission matrix size, and modulation order.

Fig. 8 shows the probability density function of the number of FLOPs of the SD detector for different SNR values: (a) SNR = 0 dB, (b) SNR = 5 dB, and (c) SNR = 10 dB. These results were obtained using the frequency squeeze parameters shown in Table 2 and the AWGN channel. As expected, the SD complexity decreases as the SNR of the signal increases. It can also be observed that the variance of the number of FLOPs for SD is smaller at high SNR. For SNR = 0 dB, the complexity of SD is similar to that of MLSE. However, the average value of the number of FLOPs for SD is smaller than that observed for MLSE.

The complexity statistics for MLSE, SD, SSSSE, and SSSgbKSE detectors at different SNRs are shown in Table 3. The number of FLOPs is calculated for the FTN-GFDM system.
Unlike SD, the complexity of SSSgbKSE, SSSSE, and MLSE detectors does not change with SNR. Table 3 also shows the average percentage reduction in the complexity of the proposed detectors compared to MLSE. In the proposed scenario, SSSSE reduces the number of FLOPs by $1.3992 \times 10^{-6}\%$ compared to MLSE, SSSgbKSE by $1.1016 \times 10^{-5}\%$, and SD by $3.9850 \times 10^{-4}\%$, considering SNR of 0 dB. With a SNR of 10 dB, SD has lower complexity than SSSgbKSE. However, for $K = 1$, the complexity is quadratic, and the BER performance approaches that of SSSSE. Finally, the FDE detector has logarithmic complexity and, therefore, lower computational cost. However, as mentioned earlier, its performance is worse than the other detectors, and its applicability in the proposed FTN-GFDM system is limited.

### Table 4. Big-O complexity of MLSE, SD, SSSgbKSE, SSSSE, and FDE.

| Detector | Big-O Analysis of Detectors |
|----------|-----------------------------|
| MLSE     | $O(N^{2J})$ | $O(N^{2J} + K^2 J)$ | $O(J^{2J})$ | $O(J^{2J} + N J + g J)$ |
| SD       | $O(N^{2J})$ | $O(N^{2J} + K^2 J)$ | $O(J^{2J})$ | $O(J^{2J} + N J + g J)$ |
| SSSgbKSE | $O(N^{2J})$ | $O(N^{2J} + K^2 J)$ | $O(J^{2J})$ | $O(J^{2J} + N J + g J)$ |
| SSSSE    | $O(N^{2J})$ | $O(N^{2J} + K^2 J)$ | $O(J^{2J})$ | $O(J^{2J} + N J + g J)$ |
| FDE      | $O(N^{2J})$ | $O(N^{2J} + K^2 J)$ | $O(J^{2J})$ | $O(J^{2J} + N J + g J)$ |

Table 4 shows the big-O complexities of the detectors studied in the previous sections of this paper. Note that the complexities are in descending order, i.e., from highest to lowest complexity. The SSSSE complexity is determined only by $N$, the SSSgbKSE complexity by $N$ and $K$, and the MLSE complexity by $N$ and $J$. Considering the asymptotic notation, the MLSE has higher complexity, followed by the upper bound of the SD, i.e., the worst-case scenario. Both detectors have exponential complexity. However, in the lower bound, i.e., the best-case scenario, the SD has quadratic complexity close to that of SSSSE. For the SSSgbKSE detector, the worst-case scenario is when $K = N$ and the complexity becomes cubic. However, for $K = 1$, the complexity is quadratic, and the BER performance approaches that of SSSSE. Finally, the FDE detector has logarithmic complexity and, therefore, lower computational cost. However, as mentioned earlier, its performance is worse than the other detectors, and its applicability in the proposed FTN-GFDM system is limited.

### B. POLAR CODED SCENARIOS

Channel coding is a tool to improve the reliability and performance of wireless communication systems. Channel coding aims to correct transmission errors through error detection and error correction codes. These codes must have good error correction capability, high flexibility, and low complexity. Polar codes are flexible linear block codes with a low computational cost that can achieve channel capacity in discrete memoryless channels. The polar code family was introduced by Arikan [22], [23] and is based on channel polarization to transmit information bits on reliable channels and frozen bits on noisy channels. The modest coding and decoding complexity make these codes attractive for many applications. Moreover, polar codes and low-density parity-check (LDPC) were compared in [24], and it was found that polar codes offer higher flexibility in terms of code rate compared to LDPC.
codes. For these reasons, the polar coding was chosen for integration into the FTN-GFDM scheme.

Fig. 9 shows the BER performance of SD, SSSSE, and SSSgbKSE detectors for the FTN-GFDM scheme with polar coding and time compression. Figs. 9 (a), (b), and (c) show the BER performance over the AWGN, TIFS, and TVFS channels, respectively. The coded FTN-GFDM performance is evaluated for the three channels listed in Table 1. The polar code parameters are coding rate $1/2$, block size $1024$, 24 shortened bits, and a decoding algorithm based on successive cancellations [22]. The results show that polar coding provides performance gains for the three hard-output detectors. The gain is defined as the reduction in $E_b/N_0$ in dB compared to the uncoded scenario. Table 5 shows the approximate gain achieved by each detector in the coded system.

Fig. 10 shows the BER performance of the detectors in a polar-coded FTN-GFDM system with frequency compression over (a) AWGN, (b) TIFS, and (c) TVFS channels. The polar code uses a coding rate of $1/2$, a block size of 2048 with 8 bits shortened, and successive cancellation decoding. The BER performance is evaluated for hard-output SD, SSSSE, and SSSgbKSE detectors over the channels in Table 1. The performance gains achieved by the detectors in the coded system with frequency compression can also be seen in Table 5.

In addition, Fig. 11 presents the BER results for the FTN-GFDM encoded with time-frequency compression over the channel models in Table 1: (a) AWGN, (b) TIFS, and (c) TVFS. In this scenario, the polar code also employs coding rate $1/2$ and successive cancellation decoding, but uses blocks of size 1024 and 4 bits shortened. As expected, polar coding increases the BER performance. The performance gains achieved by the coding scenario for each detector are shown in Table 5.

From the results presented in Figs. 9, 10, 11, and Table 5, we can conclude that polar coding can provide performance and reliability gains even in high ISI and ICI scenarios.
Moreover, SD continues to outperform the other detectors analyzed, but at the cost of higher complexity.

VIII. CONCLUSION
Higher spectrum efficiency will be mandatory for all application scenarios envisioned for future mobile networks, and increasing the data rate using efficient waveforms will be necessary where UDNs and massive MIMO cannot be employed. FTN-GFDM is a very flexible waveform that can increase the data rate by compressing the subcarriers and subcarriers. However, the detection of the received signal is not trivial. MLSE can retrieve the data symbols without imposing any BER performance loss, but its high complexity hinders its application in real-world conditions. SD can guarantee a BER performance very close to the optimal detector while achieving near-optimal BER performance. The results presented in this paper show that the SD can guarantee a BER performance very close to the optimal detector, even in scenarios with high ISI and ICI. The complexity analysis showed that the SD complexity is significantly lower than the MLSE complexity even for low SNR values, but higher than the other nonlinear detectors analyzed in this paper. FDE had poor BER performance in all scenarios, and one can consider that it is not a feasible detection technique for FTN-GFDM, mainly when the interference matrix is not circulant. The SSSSE receiver has the lowest computational cost among the nonlinear receivers, but its performance is suboptimal and depends on the estimation condition. SSSSE can perform very closely to MLSE, as long as the operating condition is respected and \( K \) is large enough. Thus, this technique can be an interesting solution when the system parameters lead to appropriate estimation conditions and the SD complexity cannot be afforded. Nevertheless, the results evaluated in this paper show that FTN-GFDM can significantly increase the spectral efficiency of the system with controllable BER performance degradation and reasonable complexity, indicating that this waveform is an interesting candidate for future mobile communication scenarios for which other spectrally efficient techniques are not suitable.

Future research includes extending other detectors, such as IB-DFE and message passing algorithm (MPA), to non-orthogonal waveforms with time and frequency compression. Furthermore, future work should investigate the performance gain of soft-output detectors in FTN-GFDM coded scenarios and tree-search algorithms to find the log-likelihood ratios (LLRs).

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