Optimal Dynamic Multicast Scheduling for Cache-Enabled Content-Centric Wireless Networks

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Abstract—Caching and multicasting at base stations are two promising approaches to support massive content delivery over wireless networks. However, existing scheduling designs do not make full use of the advantages of the two approaches. In this paper, we consider the optimal dynamic multicast scheduling to jointly minimize the average delay, power and fetching costs for cache-enabled content-centric wireless networks. We formulate this stochastic optimization problem as an infinite horizon average cost Markov decision process (MDP). It is well-known to be a difficult problem and there generally only exist numerical solutions. By using relative value iteration algorithm and the special structures of the request queue dynamics, we analyze the properties of the value function and the state-action cost function of the MDP for both the uniform and nonuniform channel cases. Based on these properties, we show that the optimal policy, which is adaptive to the request queue state, has a switch structure in the uniform case and a partial switch structure in the nonuniform case. Moreover, in the uniform case with two contents, we show that the switch curve is monotonically non-decreasing. The optimality properties obtained in this paper can provide design insights for practical networks.

I. INTRODUCTION

The demand for wireless communication services has been shifting from connection-oriented services such as, traditional voice telephony and messaging to content-oriented services such as digital media, social networking and smartphone applications. This phenomenon propels the development of content-centric wireless networks [1]. Recently, to support the dramatic growth of the wireless data traffic, caching at base stations (BSs) has been proposed as a promising approach for massive content delivery [2]. Moreover, enabling multicast service at BSs is an efficient way to deliver contents to multiple requesters simultaneously [3].

Many content-centric applications are delay-sensitive and it is critical to consider delay performance in cache-enabled content-centric wireless networks. To support delay-sensitive services, multicast scheduling has been considered in the literature. For example, in [4], the authors consider multicasting for inelastic services (with strict deadlines) in cache-enabled small-cell networks. A heuristic caching algorithm is proposed to reduce the service cost. In [5], the authors consider multicasting for inelastic services in cache-enabled multi-cell networks. A joint throughput-optimal caching and scheduling algorithm is proposed to maximize the service rates of inelastic services. However, [4] and [5] assume that the users have uniform channel conditions, and hence all the users can be served simultaneously by a single multicast transmission. It remains unclear how to design multicast scheduling to make full use of the broadcast nature of the wireless medium when users have nonuniform channel conditions. Moreover, for delay-sensitive services without strict deadlines, it is unknown how to design optimal multicast scheduling by exploiting the tradeoff between the delay cost and the service cost.

In this paper, we shall address the above issues. We consider a cache-enabled content-centric wireless network with one BS, K users (with possibly different channel conditions) and M contents (with possibly different content sizes). The BS stores a certain number of contents in its cache and can fetch any uncached content from the core network through a backhaul link, with a fetching cost depending on the content size. In each slot, the BS schedules one content for multicasting to serve the users’ pending requests, with a power cost depending on both the content size and the channel conditions of the users being served. We consider the optimal dynamic multicast scheduling to jointly minimize the average delay, power and fetching costs. We formulate this stochastic optimization problem as an infinite horizon average cost Markov decision process (MDP) [6]. There are several technical challenges involved.

• Optimality analysis: The infinite horizon average cost MDP is well-known to be a difficult problem. While dynamic programming represents a systematic approach for MDPs, there generally exist only numerical solutions, which do not typically offer many design insights, and are usually not practical due to the curse of dimensionality [6]. Therefore, it is desirable to analyze the structures of the optimal policies. However, existing structural analysis is mainly for simple queueing systems, e.g., two-queue systems [7], [8] or queueing systems with symmetric arrivals [9]. In this work, we consider a multiple-queue system with general request arrivals, channel conditions and content sizes. Therefore, the structural analysis in our problem is more challenging.

• Optimal scheduling for parallel queues: The considered problem in this work can be treated as the problem of scheduling a broadcast server to parallel queues with general arrivals and switching costs. Several existing works have studied the related problems [7]–[9]. In particular, [7] and [9] consider the problems of scheduling a broadcast server to a two-queue system with general arrivals and a multiple-queue system with symmetric arrivals, respectively. Reference [8] studies the problem of scheduling a single server (without broadcast capability) to two queues with switching costs. Note that, the switching costs, which relate to the fetching costs in
our problem, are not considered in [7], [9], and the broadcast capability is not considered for the server in [8]. To the best of our knowledge, the optimal scheduling of a broadcast server to parallel queues with general arrivals and switching costs remains unknown and is highly nontrivial.

In this paper, we consider the uniform and nonuniform channel cases. By using relative value iteration algorithm (RVIA) [6] and the special structures of the request queue dynamics, as well as the power and fetching costs, we analyze the properties of the value function and the state-action cost function of the MDP for both the uniform and nonuniform cases. Based on these properties, for the uniform case, we show that the optimal policy has a switch structure. In particular, the request queue state space is divided into $M$ regions corresponding to the $M$ contents. The optimal policy schedules a content for multicasting when the request queue state falls in the region corresponding to the content. For the uniform case with two contents, we further show that the switch curve is monotonically non-decreasing. Next, for the nonuniform case, we show that the optimal policy has a partial switch structure, which is similar to the switch structure in the uniform case. The difference reflects the channel asymmetry among the users. The analytical results obtained in this paper can provide insights for designing computationally efficient multicast scheduling algorithms in practical cache-enabled content-centric wireless networks.

II. NETWORK MODEL

As illustrated in Fig. 1 we consider a cache-enabled content-centric wireless network with one BS, $K$ users and $M$ contents. Let $K = \{1, 2, \cdots, K\}$ denote the set of users, where each user $k \in K$ can represent a group of users in the same location. Let $M = \{1, 2, \cdots, M\}$ denote the set of contents, where content $m \in M$ has the size of $l_m$ (in bits). Consider time slots of unit length (without loss of generality), and indexed by $t = 1, 2, \cdots$. In each slot, each user submits content requests to the BS according to a general distribution. The BS maintains request queues for different contents, which are implemented using counters. The BS is equipped with a cache storing a certain number of contents, depending on the cache size and the sizes of the cached contents. We assume the contents stored in the cache are given. Let $C \subseteq M$ denote the set of cached contents. The BS can fetch any uncached content from the core network through the backhaul link, with a fetching cost depending on the content size. In each slot, the BS schedules one content for multicasting to serve the users’ pending requests, with a power cost depending on both the content size and the channel conditions of the users being served. In the following, we elaborate on the physical layer model, the service model and the request model.

A. Physical Layer Model

We assume that the duration of the scheduling slot is long enough to average the small-scale channel fading process, and hence the ergodic capacity can be achieved using channel coding. Let $h_k$ denote the average channel gain between user $k$ and the BS. Assume that only one content is delivered in each slot. Let $p(m, k)$ denote the minimum transmission power required for delivering content $m$ to user $k$ within a scheduling slot. Assume $p(m, k)$ satisfies $p(m, k) = y(h_k, l_m)$, where $y(h, l)$ is monotonically non-increasing with $h$ for all $l \geq 0$. Without loss of generality, we assume that $h_1 \geq h_2 \geq \cdots \geq h_K$, which implies $p(m, 1) \leq p(m, 2) \leq \cdots \leq p(m, K)$ for all $m$. In this paper, we consider the uniform and nonuniform channel cases. In the uniform case, the channel gains of different users are the same, and hence, we have $p(m, 1) = p(m, 2) = \cdots = p(m, K) \equiv p(m)$ for each $m$. In the nonuniform case, the channel gains of different users can be different, and hence for each $m$, $p(m, k)$ can be different for different users.

B. Service Model

We consider multicast service for content delivery. In each slot, the BS schedules one content for multicasting to serve the users’ pending requests. Let $K(m, t) \in K$ denote the set of users who have pending requests for content $m$ at slot $t$. Let $u(t) \in M$ denote the content scheduled for multicasting at slot $t$. If content $u(t)$ is cached (i.e., $u(t) \in C$), the BS transmits it to all the users in $K(u(t), t)$ directly; otherwise, the BS first downloads $u(t)$ from the core network through the backhaul link, then multicasts it to the users in $K(u(t), t)$ and finally discards it after the transmission.

Next, we illustrate the fetching and power costs. Let $c(m)$ denote the cost for fetching content $m$ via the backhaul link, depending on the content size. Then, the fetching cost is given by

$$f(m) \equiv 1(m \notin C)c(m), \quad (1)$$

where $1(\cdot)$ denotes the indicator function. Let $k^*(m, t) \in K(m, t)$ denote the user who requires the highest transmission power among the users in $K(m, t)$, i.e., $k^*(m, t) \equiv \max_{k \in K(m, t)} p(m, k)$. Then, to deliver content $m$ to all the users in $K(m, t)$ within a slot, the power cost $p(m, t)$ is given by

$$p(m, t) \equiv p(m, k^*(m, t)) = \max_{k \in K(m, t)} p(m, k). \quad (2)$$

C. Request Model

In each slot, each user submits content requests to the BS. Let $A_{m, k}(t)$ denote the number of the new request arrivals for
content $m$ from user $k$ at the end of slot $t$, where $m \in M$ and $k \in K$. We assume that $A_{m,k}(t)$ is i.i.d. over slots according to a general distribution. Let $A(t) = (A_{m,k}(t))_{m \in M, k \in K}$ denote the request arrival matrix at slot $t$. The BS maintains request queues for different contents. The request queues are implemented using counters and no data is stored in these request queues. In the following, we introduce two request queue models for the uniform and nonuniform cases, respectively.

1) Uniform Case: In the uniform case, once content $m$ is multicasted using transmission power $p(m)$, all the users can receive content $m$. Therefore, we do not differentiate the requests for each content at the user level. Specifically, the BS maintains a separate request queue for each content $m \in M$. Let $Q_{m}(t) \in Q_{m} \triangleq \{0, 1, \cdots, N_{m}\}$ denote the request queue length for content $m$ at the beginning of slot $t$. As illustrated in Section II-B, if content $m$ is scheduled for transmission at slot $t$ (i.e., $u(t) = m$), all the pending requests for content $m$ are satisfied, i.e., the request queue for content $m$ is emptied. Thus, the request queue dynamics for content $m$ is as follows:

$$ Q_{m}(t+1) = \min\{1(u(t) \neq m)Q_{m}(t) + A_{m}(t), N_{m}\}, \quad (3) $$

where $A_{m}(t) \triangleq \sum_{k} A_{m,k}(t)$ denotes the total number of the request arrivals for content $m$ at the end of slot $t$. Let $Q(t) = (Q(t))_{m \in M} \in \mathcal{Q}$ denote the request queue state vector at the beginning of slot $t$ in the uniform case, where $\mathcal{Q} \triangleq \prod_{m \in M} Q_{m}$ denotes the request queue state space in the nonuniform case. By (1) and (2), the average fetching and power costs are given by:

$$ \bar{f}(\mu) \triangleq \limsup_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[f(u(t))], \quad (6) $$

$$ \bar{p}(\mu) \triangleq \limsup_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[p(Q(t), u(t))]. \quad (7) $$

Here, with abuse of notation, we also use $p(Q(t), u(t))$ to represent $p(u(t), t)$ given in (3), as $K(u(t), t) = \{k|Q_{u(t),k}(t) > 0\}$. Therefore, under a given stationary unichain policy $\mu$, the average system cost (weighted sum cost) is defined as:

$$ \bar{g}(\mu) \triangleq \bar{d}(\mu) + w_{f}\bar{f}(\mu) + w_{p}\bar{p}(\mu) $$

$$ = \limsup_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[g(Q(t), u(t))], \quad (8) $$

where $w_{f}$ and $w_{p}$ are the associated weights for the fetching and power costs, respectively, and $g(Q, u) \triangleq d(Q) + w_{f}f(u) + w_{p}p(Q, u)$ is the per-stage cost.

We wish to find an optimal multicast scheduling policy to minimize the average system cost $\bar{g}(\mu)$ in (3).

**Problem 1 (System Cost Minimization Problem):**

$$ \bar{g}^{*} \triangleq \min_{\mu} \limsup_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[g(Q(t), u(t))], \quad (9) $$

where $\mu$ is a stationary unichain multicast scheduling policy and $\bar{g}^{*}$ denotes the minimum system cost achieved by the optimal policy $\mu^{*}$.

Note that, Problem 1 is an infinite horizon average cost MDP, which is well-known to be a difficult problem.

**B. Optimality Equation**

The optimal multicast scheduling policy $\mu^{*}$ can be obtained by solving the following Bellman equation.

A unichain policy is a policy, under which the induced Markov chain has a single recurrent class (and possibly some transient states) [4].

By Little’s law, (3) is related to the average delay.
In Sections IV and V, we shall analyze the structures of the request queue dynamics, as well as the power and detailed discussion). First, by RVIA and the special structures of the request queue, we have the following property of \( V(Q) \).

**Lemma 1 (Bellman Equation):** Suppose a scalar \( \theta \) and a real-value function \( V(\cdot) \) satisfy

\[
\theta + V(Q) = \min_{u \in M} \{ g(Q, u) + E[V(Q')] \}, \quad \forall Q \in Q. \tag{10}
\]

where \( Q' = (Q_{m})_{m \in M} \) with \( Q_{m}' = \min(1(u \neq m)Q_{m} + A_{m}, N_{m}) \) in the uniform case, and \( Q' = (Q_{m,k})_{m \in M, k \in E} \) with \( Q_{m,k}' = \min(1(u \neq m)Q_{m,k} + A_{m,k}, N_{m,k}) \) in the nonuniform case. Then, \( \theta = \hat{g}^* \) is the optimal value to Problem 1 for all initial state \( Q(1) \in Q \) and \( V(\cdot) \) is called the value function. Furthermore, if \( \mu^*(Q) \) attains the minimum in the R.H.S. of (10) for each \( Q \), then the stationary unchain policy \( \mu^* \) is the optimal policy achieving the optimal value \( \hat{g}^* \).

**Proof:** Please see Appendix A.

From the Bellman equation in (10), we can see that \( \mu^* \) depends on the state \( Q \) through the value function \( V(\cdot) \). Obtaining \( V(\cdot) \) involves solving the Bellman equation for all \( Q \), for which there is no closed-form solution in general [6]. Brute force numerical solutions such as value iteration and policy iteration do not typically offer many design insights, and are usually impractical for implementation in practical systems due to the curse of dimensionality [6]. Therefore, it is desirable to study the structure of \( \mu^* \).

To analyze the structure of \( \mu^* \), we also introduce the state-action cost function:

\[
J(Q, u) \triangleq g(Q, u) + E[V(Q')] \tag{11}
\]

Note that \( J(Q, u) \) is related to the R.H.S. of the Bellman equation in (10). In particular, based on Lemma 1 the optimal policy \( \mu^* \) can be expressed in terms of \( J(Q, u) \), i.e.,

\[
\mu^*(Q) = \arg \min_{u \in M} J(Q, u), \quad \forall Q \in Q. \tag{12}
\]

In Sections IV and V, we shall analyze the structures of the optimal policies for the uniform and nonuniform cases, respectively, based on the properties of the value function \( V(Q) \) and the state-action cost function \( J(Q, u) \).

**IV. Optimality Properties in Uniform Case**

In this section, we consider the uniform case. We first show that the optimal policy has a switch structure. Then, we show that the switch curve is monotonically non-decreasing for the uniform case with two contents.

**A. Structure of Optimal Policy**

Problem 1 can be treated as the problem of scheduling a broadcast server to parallel queues with general random arrivals, channel conditions and content sizes. Therefore, the structural analysis is more challenging than the existing structural analysis for simple queuing systems (see Section I for the detailed discussion). First, by RVIA and the special structures of the request queue dynamics, as well as the power and fetching costs, we have the following property of \( V(Q) \).

**Lemma 2 (Monotonicity of Value Function):** In the uniform case, for any \( Q^{1}, Q^{2} \in Q \) such that \( Q^{2} \geq Q^{1} \), we have \( V(Q^{2}) \geq V(Q^{1}) \).

**Proof:** Please see Appendix B.

Then, based on Lemma 2 and the special properties of multicasting, we have the following property of \( J(Q, u) \).

**Lemma 3 (Monotonicity of State-Action Cost Function):**

In the uniform case, for any \( u, v \in M \) and \( v \neq u \), \( J(Q, u) - J(Q, v) \) is monotonically non-increasing with \( Q_{u} \), i.e.,

\[
J(Q + e_{u}, u) - J(Q + e_{u}, v) \leq J(Q, u) - J(Q, v), \tag{13}
\]

where \( e_{u} \) denotes the \( 1 \times M \) vector with all entries 0 except for a 1 in its \( u \)-th entry.

**Proof:** Please see Appendix C.

Note that, the property of \( J(Q, u) \) in Lemma 3 is similar to the diminishing-return property of submodular functions used in the existing structural analysis. Lemma 3 comes from the special structure introduced by multicasting and is key to analyze the optimality properties. Lemma 3 indicates that, if it is better to multicast content \( u \) than content \( v \) for some state \( Q \) (i.e., \( J(Q, u) \leq J(Q, v) \)), then it is also better to multicast content \( u \) than \( v \) for state \( Q + e_{u} \) (i.e., \( J(Q + e_{u}, u) \leq J(Q + e_{u}, v) \)). This leads to the following switch structure of \( \mu^* \).

**Theorem 1 (Switch Structure of Optimal Policy):** The optimal policy \( \mu^* \) in the uniform case has a switch structure, i.e., for all \( u \in M \), we have

\[
\mu^*(Q) = u, \quad \text{if } Q_{u} \geq s_{u}(Q_{-u}), \tag{14}
\]

where the switch curve for content \( u \) is given by

\[
s_{u}(Q_{-u}) \triangleq \begin{cases} 
\min S_{u}(Q_{-u}), & \text{if } S_{u}(Q_{-u}) \neq \emptyset, \\
\infty, & \text{otherwise}
\end{cases} \tag{15}
\]

with \( S_{u}(Q_{-u}) \triangleq \{ Q_{u} | J(Q, u) \leq J(Q, v) \forall v \in M, v \neq u \} \). Here, \( Q_{-u} \triangleq (Q_{m})_{m \in M, m \neq u} \) denotes the request queue state vector corresponding to all other contents except content \( u \).

**Proof:** Please see Appendix D.

**Remark 1:** Theorem 1 indicates that, the request queue state space is divided into \( M \) regions corresponding to the \( M \) contents, and the optimal policy schedules a content for multicasting when the request queue state falls in the region corresponding to the content, as illustrated in Fig. 2(a). In addition, given \( Q_{-u} \), the scheduling for content \( u \) is of the threshold type, i.e., content \( u \) is scheduled if \( Q_{u} \geq s_{u}(Q_{-u}) \), as illustrated in Fig. 2(b). This indicates that, it is not efficient to schedule content \( u \) when \( Q_{u} \) is small (i.e., the delay cost is small), as a higher power cost (and a higher fetching cost if \( u \notin C \)) is consumed per request for content \( u \). This reveals the tradeoff between the delay cost and the power cost (and the fetching cost if \( u \notin C \)) for content \( u \).

**B. Special Case: Two Contents**

Now, consider the special uniform case with two contents, i.e., \( M = 2 \). By Theorem 1 we can see that, for \( M = 2 \), either one of the two switch curves, i.e., \( s_{1}(Q_{2}) \) and \( s_{2}(Q_{1}) \), is sufficient to characterize the optimal policy. Moreover,
Fig. 2: Switch structure of optimal scheduling in the uniform case.

Theorem 2 (Partial Switch Structure of Optimal Policy):

The optimal policy \( \mu^* \) in the nonuniform case has a partial switch structure, i.e., for all \( u \in \mathcal{M} \) and \( k \in \mathcal{K} \), we have

\[
\mu^*(Q) = u, \quad \text{if } Q_{u,k} \geq s_{u,k}(Q_{u,-k}) \text{ and condition (a) or (b) holds},
\]

where condition (a) is \( k < k^*(Q_{k,u}) \), condition (b) is \( k > k^*(Q_{k,u}) \) and \( s_{u,k}(Q_{u,-k}) > 0 \), and the switch curve for content-user pair \((u, k)\) is given by

\[
s_{u,k}(Q_{u,-k}) = \begin{cases} 
\min_{\mathcal{M}} \mathcal{S}_{u,k}(Q_{u,-k}), & \text{if } \mathcal{S}_{u,k}(Q_{u,-k}) \neq \emptyset \\
\infty, & \text{otherwise}
\end{cases}
\]

where \( \mathcal{S}_{u,k}(Q_{u,-k}) = \{Q_{u,k} \mid J(Q, u) \leq J(Q, v) \quad \forall v \in \mathcal{M}, v \neq u \} \). Here, \( Q_{u,-k} = \{ Q_{m,i} \mid m \in \mathcal{M}, m \neq u, i \in \mathcal{K}, i \neq k \} \) denotes the request queue state matrix corresponding to all the other content-user pairs except the content-user pair \((u, k)\), and \( k^*(Q_{k,u}) = \max\{i \mid Q_{u,i} > 0, i \neq k\} \).

Proof: Please see Appendix H.

From Theorem 2, we can see that, the structure of the optimal policy in the nonuniform case is very similar to the one in the uniform case, as illustrated in Fig. 3. The only difference is that, the structural property for \( k > k^*(Q_{k,u}) \) and \( s(Q_{u,-k}) = 0 \) depends on the specific channel asymmetry among the users and is still not known in general, as illustrated in the dashed box in Fig. 3(b). Insights from Theorem 2 are similar to those from Theorem 1.

VI. CONCLUSION

In this paper, we consider the optimal dynamic multicast scheduling to jointly minimize the average delay, power and fetching costs for cache-enabled content-centric wireless networks. We formulate this stochastic optimization problem as an infinite horizon average cost MDP. We show that the optimal policy has a switch structure in the uniform case and a partial switch structure in the nonuniform case. Moreover,
in the uniform case with two contents, we show that the switch curve is monotonically non-decreasing. The optimality properties obtained in this paper can provide design insights for multicast scheduling in practical cache-enabled content-centric wireless networks.

APPENDIX A: PROOF OF LEMMA 1

By Proposition 4.2.5 in [6], the Weak Accessibly (WA) condition holds for unchain policies. Thus, by Proposition 4.2.3 and Proposition 4.2.1 in [6], the optimal system cost of the MDN in Problem 1 is the same for all initial states and the solution \((\theta, V(Q))\) to the following Bellman equation exists.

\[
\theta + V(Q) = \min_{u \in M} \left\{ g(Q, u) + \sum_{Q' \in Q} \Pr[Q'|Q, u]V(Q') \right\}
\]

where \(g(Q, u_n) = \sum_m Q_m + w_p p(u_n) + w_ff(u_n)\) and \(Q' = (Q'_m)_{m \in M}\) with \(Q'_m = \min\{1, u_n \neq m\}Q_m + A_m, N_m\) in the uniform case; \(g(Q, u_n) = \sum_{m,k} Q_{m,k} + w_p p(u_n, k^1(Q, u_n)) + w_ff(u_n)\) with \(k^1(Q, u_n) = \max\{k|Q_{u_n,k} > 0\}\), \(Q' = (Q'_m, k)_{m \in M, k \in Q}\) and \(Q_{m,k} = \min\{1, u_n \neq m\}Q_{m,k} + A_{m,k}, N_{m,k}\) in the nonuniform case.

Note that \(J_{n+1}(Q, u_n)\) is related to the R.H.S of the Bellman equation in (10). We refer to \(J_{n+1}(Q, u_n)\) as the state-action cost function in the \(n\)th iteration. For each \(Q\), the relative value iteration algorithm calculates \(V_{n+1}(Q)\) according to

\[
V_{n+1}(Q) = \min_{u_n} J_{n+1}(Q, u_n) - \min_{u_n} J_{n+1}(Q^5, u_n), \forall n
\]

where \(J_{n+1}(Q, u_n)\) is given by (21) and \(Q^5 \in \Omega\) is some fixed state. Under any initialization of \(V_0(Q)\), the generated sequence \(\{V_n(Q)\}\) converges to \(V(Q)\), i.e.,

\[
\lim_{n \to \infty} V_n(Q) = V(Q), \forall Q \in \Omega
\]

where \(V(Q)\) satisfies the Bellman equation in (10). Let \(\mu^*_n(Q)\) denote the control that attains the minimum of the first term in (22) in the \(n\)th iteration for all \(Q\), i.e.,

\[
\mu^*_n(Q) = \arg \min_{u_n} J_{n+1}(Q, u_n), \forall Q \in \Omega
\]

We refer to \(\mu^*_n\) as the optimal policy for the \(n\)th iteration.

Next, we prove Lemma 2 through mathematical induction using the aforementioned relative value iteration algorithm. Denote \(Q^1 \triangleq (Q^1_m)_{m \in M}\) and \(Q^2 \triangleq (Q^2_m)_{m \in M}\). To prove Lemma 2, it is equivalent to show that for any \(Q^1, Q^2 \in \Omega\) such that \(Q^2 \geq Q^1\),

\[
V_n(Q^2) \geq V_n(Q^1),
\]

holds for all \(n = 0, 1, \cdots\). First, we initialize \(V_0(Q) = 0\) for all \(Q \in \Omega\). Thus, we have \(V_0(Q^1) = V_0(Q^2) = 0\), i.e., (25) holds for \(n = 0\). Assume that (25) holds for some \(n > 0\). We
will prove that (25) also holds for \( n + 1 \). By (22), we have
\[
V_{n+1}(Q^1) = J_{n+1}(Q^1, \mu^*_n(Q^1)) - \min_{u_n} J_{n+1}(Q^\delta, u_n)
\]
\[
= J_{n+1}(Q^1, \mu^*_n(Q^2)) - \min_{u_n} J_{n+1}(Q^\delta, u_n)
\]
\[
\leq \mathbb{E}[V_n(Q^{\delta})] + \sum_m Q_m + w_p(p(\mu^*_n(Q^2))) + w_f(\mu^*_n(Q^2))
\]
\[
- \min_{u_n} J_{n+1}(Q^\delta, u_n),
\]
where (a) follows from the optimality of \( \mu^*_n(Q^1) \) for \( Q_1 \) in the \( n \)th iteration, (b) directly follows from (21) and
\[
Q^{\delta'} = (Q_m)_{m \in M} \text{ with } Q^{\delta'}_m = \min\{\|\mu^*_n(Q^2)\| \neq m\}Q^1_m + A_m, N_m \}. \]
Then, we compare (25) and (27) term by term. Due to \( Q^{\delta'} \geq Q^1 \), we have \( \sum_m Q_m \geq \sum_m Q_m \) and \( Q^{\delta'} \geq Q^1 \). Thus, we have \( \mathbb{E}[V_n(Q^{\delta'})] \geq \mathbb{E}[V_n(Q^1)] \) by the induction hypothesis. Therefore, we have \( V_{n+1}(Q^2) \geq V_{n+1}(Q^1) \).

**APPENDIX C: PROOF OF LEMMA 3**

By (11), we have
\[
J(Q, u) - J(Q, v) = \mathbb{E}[V(Q^{\delta'})] + g(Q, u) - g(Q, v)
\]
\[
= \mathbb{E}[V(Q^{\delta'})] + g(Q, u) - g(Q, v)
\]
\[
- \mathbb{E}[V(Q^{\delta'})] - g(Q + e_u, u) + \mathbb{E}[V(Q^4)] + g(Q + e_u, v)
\]
\[
= \mathbb{E}[V(Q^{\delta'})] - \mathbb{E}[V(Q^4)] + \mathbb{E}[V(Q^4)], \quad (28)
\]
where \( Q = (Q_m)_{m \in M}, Q^{\delta'} = (Q_m)_{m \in M}, i = 1, 2, 3, 4 \) with
\[
\begin{align*}
Q^{\delta'}_m &= \min\{\|u \neq m\}Q_m + A_m, N_m \}, \quad (29a) \\
Q^{\delta'}_m &= \min\{|v \neq m\}Q_m + A_m, N_m \}, \quad (29b) \\
Q^4_m &= \begin{cases} 
\min\{A_m, N_u\} & \text{if } m = u \\
\min\{Q_m + A_m, N_m\} & \text{otherwise} 
\end{cases}, \quad (29c) \\
Q^4_m &= \begin{cases} 
\min\{Q_m^u + 1 + A_u, N_u\} & \text{if } m = u \\
\min\{\|v \neq m\}Q_m + A_m, N_m\} & \text{otherwise} \end{cases}, \quad (29d)
\end{align*}
\]
and (c) is due to
\[
g(Q, u) - g(Q, v) - g(Q + e_u, u) + g(Q + e_u, v)
\]
\[
= \left( \sum_m Q_m + w_p(p(u) + w_f(u) \right)
\]
\[
- \left( \sum_m Q_m + w_p(p(v) + w_f(v) \right)
\]
\[
= \left( \sum_m Q_m + 1 + w_p(p(u) + w_f(u) \right)
\]
\[
+ \left( \sum_m Q_m + 1 + w_p(p(v) + w_f(v) \right)
\]
\[
= 0. \quad (30)
\]
To prove Lemma 3 it remains to show that the R.H.S. of (28) is nonnegative. By comparing (29a) with (29c), we can see that \( Q^{\delta'}_m = Q^1_m \) for all \( m \), i.e., \( Q^{\delta'} = Q^1 \). Thus, we have \( \mathbb{E}[V(Q^{\delta'})] = \mathbb{E}[V(Q^1)] \). By comparing (29a) with (29b), we can see that \( Q^{\delta'}_m = Q^{\delta'}_m = Q^2_m \) for all \( m \neq u \), i.e., \( Q^{\delta'} \geq Q^2 \). Thus, by Lemma 2 we have \( \mathbb{E}[V(Q^{\delta'})] \geq \mathbb{E}[V(Q^2)] \). Therefore, by (28), we have
\[
J(Q + e_u, u) - J(Q + e_u, v) \leq J(Q, u) - J(Q, v).
\]
We complete the proof of Lemma 3.

**APPENDIX D: PROOF OF THEOREM 1**

Consider content \( u \in \mathcal{M} \) and state \( Q = (Q_m)_{m \in M} \) where \( Q_u = s_u(Q_{-u}) \). Note that, if \( s_u(Q_{-u}) = \infty \), (14) always holds. Therefore, in the following, we only consider that \( s_u(Q_{-u}) < \infty \). According to the definition of \( s_u(Q_{-u}) \) in Theorem 1 we can see that \( J(Q, u) \leq J(Q, v) \) for all \( v \in \mathcal{M} \) and \( v \neq u \). Thus, it is optimal to multicast content \( u \) for state \( Q \), i.e., \( \mu^*(Q) = u \). Consider another state \( Q' = (Q'_m)_{m \in M} \) where \( Q'_u \geq Q_u \) and \( Q'_m = Q_m \) for all \( m \neq u \). To prove Theorem 1 it is equivalent to show that \( \mu^*(Q') = u \). By Lemma 3 for all \( v \in \mathcal{M} \) and \( v \neq u \), we have
\[
J(Q', u) - J(Q', v) \leq J(Q, u) - J(Q, v) \leq 0. \quad (31)
\]
Thus, it is optimal to multicast content \( u \) for \( Q' \). We complete the proof of Theorem 1.

**APPENDIX E: PROOF OF LEMMA 1**

To prove the monotonically non-decreasing property of \( s_2(Q_1) \) with respect to \( Q_1 \), it is equivalent to show that, if \( \mu^*(Q + e_1) = 2 \), then \( \mu^*(Q) = 2 \). This is sufficient to show that
\[
J(Q, 2) - J(Q, 1) \leq J(Q + e_1, 2) - J(Q + e_1, 1), \quad (32)
\]
where \( Q = (Q_1, Q_2) \) and \( e_1 = (1, 0) \).

By (11), we have
\[
J(Q, 2) - J(Q, 1) - J(Q + e_1, 2) + J(Q + e_1, 1)
\]
\[
= \mathbb{E}[V(Q^{\delta'})] + g(Q, 2) - \mathbb{E}[V(Q^4)] - g(Q, 1)
\]
\[
- \mathbb{E}[V(Q^4)] - g(Q + e_1, 2) + \mathbb{E}[V(Q^4)] + g(Q + e_1, 1)
\]
\[
= \mathbb{E}[V(Q^4)] - \mathbb{E}[V(Q^4)] \leq \mathbb{E}[V(Q^4)], \quad (33)
\]
where

\[ Q^{1'} = \min\{Q_1 + A_1, N_1\}, \min\{A_2, N_2\} \}, \quad (34a) \]
\[ Q^{2'} = \min\{A_1, N_1\}, \min\{Q_2 + A_2, N_2\} \}, \quad (34b) \]
\[ Q^{3'} = \min\{Q_1 + A_1 + 1, N_1\}, \min\{A_2, N_2\} \}, \quad (34c) \]
\[ Q^{4'} = \min\{A_1, N_1\}, \min\{Q_2 + A_2, N_2\} \}, \quad (34d) \]

and (d) is due to

\[ g(Q, 2) - g(Q, 2) - g(Q + e_1, 2) + g(Q + e_1, 1) \]
\[ = (Q_1 + Q_2 + w_p p(1) + w_f f(1)) \]
\[ - (Q_1 + Q_2 + 1 + w_p p(2) + w_f f(2)) \]
\[ + (Q_1 + Q_2 + 1 + w_p p(1) + w_f f(1)) = 0. \quad (35) \]

To prove Lemma 4, it remains to show that the R.H.S. of (32) is nonpositive. By comparing (34a) with (34c), we have \( Q^{3'} \geq Q^1 \), implying that \( E[V(Q^1)] \geq E[V(Q^3)] \) by Lemma 2. By comparing (34b) with (34d), we have \( Q^{4'} = Q^2 \), implying that \( E[V(Q^1)] = E[V(Q^2)] \). Thus, by (35), we can show that (32) holds.

Similarly, we can show that the following inequality holds:

\[ J(Q, 1) - J(Q, 2) \leq J(Q + e_2, 1) - J(Q + e_2, 2), \quad (36) \]

where \( Q = (Q_1, Q_2) \) and \( e_2 = (0, 1) \). Thus, if \( \mu^*(Q + e_2) = 1 \), then \( \mu^*(Q) = 1 \). This implies the monotonic non-decreasing property of \( s_i(Q_2) \) with respect to \( Q_2 \). We complete the proof of Lemma 4.

**APPENDIX F: PROOF OF LEMMA 5**

We prove Lemma 5 through mathematical induction using the relative value iteration algorithm in Appendix B. Denote \( Q^1 = (Q^1_{m,k})_{m \in M, k \in K} \) and \( Q^2 = (Q^2_{m,k})_{m \in M, k \in K} \). To prove Lemma 5 by (23), it is equivalent to show that for any \( Q^1, Q^2 \in \mathcal{Q} \) such that \( Q^2 \succeq Q^1 \),

\[ V_n(Q^2) \geq V_n(Q^1), \quad (37) \]

holds for all \( n = 0, 1, \ldots \). We initialize \( V_0(Q) = 0 \) for all \( Q \in \mathcal{Q} \). Thus, we have \( V_0(Q^1) = V_0(Q^2) = 0 \), i.e., (37) holds for \( n = 0 \). Assume that (37) holds for some \( n > 0 \). We will prove that (37) also holds for \( n + 1 \). By (23), we have

\[ V_{n+1}(Q^1) = J_{n+1}(Q^{1'}_1, \mu^*_n(Q^1)) \leq \min_{u_n} \left( J_{n+1}(Q^{1'}_1, u_n) \right) \]
\[ = \left( E[V_n(Q^{1'})] + \sum_{m,k} Q^1_{m,k} + w_p p(\mu^*_n(Q^2), k^1(Q^1, \mu^*_n(Q^2))) \right) \]
\[ + w_f f(\mu^*_n(Q^2)) - \min_{u_n} \left( J_{n+1}(Q^{1'}_1, u_n) \right), \quad (38) \]

where (e) follows from the optimality of \( \mu^*_n(Q^1) \) for \( Q^1 \) in the \( n \)th iteration, (f) directly follows from (21). \( k^1(Q^1, \mu^*_n(Q^2)) = \max \{ \{ k \mid Q^1_{m,k} \geq 0 \} \} \) and \( Q^{1'} = (Q^1_{m,k})_{m \in M, k \in K} \) with \( Q^1_{m,k} = \min \{ \{ \mu^*_n(Q^2) \neq m \} Q^1_{m,k} + A_{m,k}N_{m,k} \} \). By (21) and (23), we also have

\[ V_{n+1}(Q^2) = J_{n+1}(Q^2, \mu^*_n(Q^2)) \leq \min_{u_n} \left( J_{n+1}(Q^2, u_n) \right) \]
\[ = \left( E[V_n(Q^2)] + \sum_{m,k} Q^2_{m,k} + w_p p(\mu^*_n(Q^2), k^1(Q^2, \mu^*_n(Q^2))) \right) \]
\[ + w_f f(\mu^*_n(Q^2)) - \min_{u_n} \left( J_{n+1}(Q^2, u_n) \right), \quad (39) \]

where \( k^1(Q^1, \mu^*_n(Q^2)) = \max \{ k \mid Q^1_{m,k} = 0 \} \) and \( Q^{2'} = (Q^{2'}_{m,k})_{m \in M, k \in K} \) with \( Q^{2'}_{m,k} = \min \{ \{ \mu^*_n(Q^2) \neq m \} Q^2_{m,k} + A_{m,k}N_{m,k} \} \).

Next, we compare (38) and (39) term by term. Due to \( Q^2 \succeq Q^1 \), we have \( Q^{2'} \succeq Q^1 \). Thus, by induction hypothesis, we have \( E[V_n(Q^1)] \geq E[V_n(Q^2)] \).

Due to \( Q^2 \succeq Q^1 \), we have \( \sum_{m,k} Q^1_{m,k} \geq \sum_{m,k} Q^1_{m,k} \) and \( k^1(Q^2, \mu^*_n(Q^2)) = k^1(Q^1, \mu^*_n(Q^2)) \), implying that \( p(\mu^*_n(Q^2), k^1(Q^2, \mu^*_n(Q^2))) = p(\mu^*_n(Q^2), k^1(Q^1, \mu^*_n(Q^2))) \).

Thus, we have \( V_{n+1}(Q^1) \geq V_{n+1}(Q^2) \), i.e., (37) holds for \( n + 1 \). Therefore, by induction, we can show that (37) holds for any \( n \). By taking limits on both sides of (37) and by (23), we complete the proof of Lemma 5.

**APPENDIX G: PROOF OF LEMMA 6**

By (11), we have

\[ J(Q, u) - J(Q, v) - J(Q + E_{u,k}, u) + J(Q + E_{u,k}, v) = E[V(Q')] + g(Q, u) - E[V(Q')] - g(Q, v) - E[V(Q')] \]
\[ = E[V(Q')] + E[V(Q')] + E[V(Q')] - E[V(Q')] \]
\[ = E[V(Q')] - E[V(Q')] + E[V(Q')] - E[V(Q')] - \]
\[ w_p p(u, k^1(Q, u)) - p(u, k^1(Q + E_{u,k}, u)) \]
\[ = E[V(Q')] - E[V(Q')] + E[V(Q')] - E[V(Q')] - \]
\[ w_p p(v, k^1(Q, v)) - p(v, k^1(Q + E_{u,k}, v)) \] \quad \quad (40)

where \( Q = (Q_{m,i})_{m \in M, i \in K} \) and \( Q' = (Q'_{m,i})_{m \in M, i \in K} \), \( j = 1, 2, 3, 4 \) with

\[ Q_{m,i}^1 = \min \{ I(u \neq m)Q_{m,i} + A_{m,i}, N_{m,i} \} \]
\[ Q_{m,i}^2 = \min \{ I(v \neq m)Q_{m,i} + A_{m,i}, N_{m,i} \} \]
\[ Q_{m,i}^3 = \begin{cases} \min \{ A_{u,k}, N_{u,k} \} & \text{if } m = u, k = i \\ \min \{ Q_{m,i} + A_{m,i}, N_{m,i} \} & \text{otherwise} \end{cases} \]
\[ Q_{m,i}^4 = \begin{cases} \min \{ Q_{u,k} + 1 + A_{u,k}, N_{u,k} \} & \text{if } m = u, k = i \\ \min \{ I(v \neq m)Q_{m,i} + A_{m,i}, N_{m,i} \} & \text{otherwise} \end{cases} \]

To prove Lemma 6, it remains to show that the R.H.S. of (40) is nonnegative. By comparing (41a) with (41c), we can see that \( Q_{m,i}^3 = Q_{m,i}^3 \) for all \( m, i \), i.e., \( Q_{m,i}^3 = Q_{m,i}^3 \). Thus, we have \( E[V(Q')] = E[V(Q')] \).

By comparing (41b) with (41d), we can see that \( Q_{m,i}^3 \geq Q_{m,i}^3 \geq Q_{m,i}^3 > 0 \) and \( Q_{m,i}^3 = Q_{m,i}^3 \) for all \( m \neq u, k \neq i \), i.e., \( Q'_{m,i} = Q'_{m,i} \).

Thus, by Lemma 5, we have \( E[V(Q')] \geq E[V(Q')] \).

In addition, due to \( k \leq \min \{ k \mid Q_{u,k} > 0 \} \), we have \( k^1(Q + E_{u,k}, u) = k^1(Q, u) \) and \( k^1(Q + E_{u,k}, v) = k^1(Q, v) \), implying that \( p(u, k^1(Q, u)) = p(u, k^1(Q + E_{u,k}, u)) \) and
\[ p(u, k^+(Q, v)) = p(u, k^+(Q + E_{u,k}, v)) \]. Therefore, by (40), we have
\[ J(Q + E_{u,k}, u) = J(Q + E_{u,k}, v) \leq J(Q, u) - J(Q, v) \].
We complete the proof of Lemma 6.

APPENDIX H: PROOF OF THEOREM 2

Consider content \( u \in M \), user \( k \in K \) and state \( Q = (Q_{m,i})_{m \in M, i \in K} \) where \( Q_{u,k} = s_{u,k}(Q_{u,-k}) \). Note that, if \( s_{u,k}(Q_{u,-k}) = \infty \), (17) always holds. Therefore, in the following, we only consider that \( s_{u,k}(Q_{u,-k}) < \infty \). According to the definition of \( s_{u,k}(Q_{u,-k}) \) in Theorem 1, we can see that \( J(Q, u) \leq J(Q, v) \) for all \( v \in M, v \neq u \). Thus, it is optimal to multicast content \( u \) for state \( Q \), \( \mu^*(Q) = u \). Consider another state \( Q' = (Q'_{m,i})_{m \in M, i \in K} \) where \( Q'_{u,k} \geq Q_{u,k} \) and \( Q'_{m,i} = Q_{m,i} \) for all \( m \neq u, i \neq k \). To prove Theorem 2 it is equivalent to show that it is also optimal to multicast content \( u \) for state \( Q' \), i.e.,
\[ J(Q', u) \leq J(Q', v), \forall v \in M, v \neq u \].

According to the relationship between \( k \) and \( k^+(Q, u) \) as well as the value of \( s_{u,k}(Q_{u,-k}) \), we have the following three cases.

1) If \( k < k^+(Q, u) \), i.e., condition (a) holds, we have \( k < \max\{k|Q_{u,k} > 0\} \). By Lemma 6, for any \( v \in M \) and \( v \neq u \), we have
\[ J(Q', u) - J(Q', v) \leq J(Q, u) - J(Q, v) \leq 0 \].
Thus, it is optimal to multicast content \( u \) for state \( Q' \).

2) If \( k > k^+(Q, u) \) and \( s_{u,k}(Q_{u,-k}) > 0 \), i.e., condition (b) holds, we have \( k = \max\{k|Q_{u,k} > 0\} \). By Lemma 6, for any \( v \in M \) and \( v \neq u \), also holds. Thus, it is optimal to multicast content \( u \) for state \( Q' \).

3) If \( k > k^+(Q, u) \) and \( s_{u,k}(Q_{u,-k}) = 0 \), implying that \( k > \max\{k|Q_{u,k} > 0\} \), then Lemma 6 does not apply and it is unknown whether \( \mu^*(Q) \) holds. Therefore, it is unclear whether it is optimal to multicast content \( u \) for state \( Q' \).

We complete the proof of Theorem 2.

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