Constraints on dark energy from the observed density fluctuations spectrum and supernova data

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Abstract

One of the greatest challenges in cosmology today is to determine the nature of dark energy, the source of the observed present acceleration of the universe. High precision experiments are being developed to reduce the uncertainties in the observations. Recently, we showed that the agreement to an accuracy of 10% of measurements of the present density fluctuations $(\delta \rho / \rho)^2$, derived from galaxy distribution (GD) data and cosmic microwave background (CMB) anisotropies in the $\Lambda$CDM model, puts very strong limits on the possible decay of the vacuum energy into cold dark matter. Using this agreement, combined with the evidence that the matter density $\Omega^0_M = 0.28 \pm 0.02$ and that the universe is approximately flat, we show that the vacuum metamorphosis model (VMM) and the popular brane-world model (BWM), both used to explain dark energy, can be discarded. When we relax the $\Omega^0_M$ requirement, we find that an agreement within 10% can be obtained only with $\Omega^0_M \simeq 0.36$ for the VMM and $\Omega^0_M \simeq 0.73$ for the BWM, both of which are not consistent with observations. The agreement of the CMB and GD data and previous constraints from SNIa data exclude, or put strong limits on, other dark energy models, which have been suggested, that can be described by the parametrized equation of state (EOS) $w = p / \rho = w_0 + w_a (1 - a)$, where $w_0$ and $w_a$ are constants, $a$ is the cosmological scale factor and $p (\rho)$ is the pressure (energy density) of the dark energy. We find that the supergravity (SUGRA) model with $w_0 = -0.82$ and $w_a = 0.58$ can be discarded. In general, we find best values $-1.86 < w_0 < -1.72$ with $1.53 < w_a < 2.0$. For redshifts $z \sim 0.5 - 1$, where the supernova data is sensitive, $w \sim -1$ for this parametrized EOS.

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I. INTRODUCTION

Many dark energy models have been suggested to explain the recent acceleration of the universe, first indicated by SNIa observations \cite{1, 2}. The nature of dark energy is one of the major problems in cosmology. Theories in which gravity is modified as well as parametrizations of the dark energy equation of state (EOS) \( w(z) = p/\rho \), where \( p(\rho) \) is the pressure (energy density) of the dark energy, have been suggested \cite{3, 4, 5}. Based on observations, various constraints have been put on the EOS for a variety of models (e.g., \cite{6, 7, 8, 9, 10}).

We begin by analyzing two interesting models that have been suggested for modifying gravity, which allow for a description of dark energy in terms of an effective EOS. The first is the five-dimensional brane-world model (BWM) of Defayet et al. \cite{11}, where gravity is modified by adding a five-dimensional Einstein-Hilbert action that dominates at distances which are greater than the crossover distance \( r_c = M_{Pl}^2/(M_5^3) \), where \( M_{Pl} \) is the Planck mass and \( M_5 \) is a five-dimensional Planck mass. The second model is the vacuum metamorphosis model (VMM) of Parker and Raval \cite{12}, which assumes the existence of a quantized non-interacting scalar field coupled to the Ricci scalar curvature. In the VMM, the quantum vacuum undergoes a phase transition at a redshift \( z_j \): from a zero value at \( z > z_j \) to a non-zero value for \( z < z_j \).

Linder \cite{13} analyzed the linear growth of a density perturbation \( (\delta \rho/\rho)^2 \) and the gravitational potential for both of these models, comparing them with the simplest linear parametrization of the dark energy EOS as a function of \( a \): \( w(a) = w_0 + w_a (1 - a) \), where \( w_0 \) and \( w_a \) are constants and \( a \) is the cosmological scale factor.

Measurements of the present density fluctuations \( (\delta \rho/\rho)^2 \), derived from the cosmic microwave background (CMB) anisotropies in the \( \Lambda \)CDM model, have been compared with those derived from the 2dF Galaxy Redshift Survey (2dFGRS) \cite{14, 15}. It was found that their difference \( (F) \) is no more than 10\% \cite{14}. We recently showed that an agreement within 10\% (i.e., \( F_{\text{max}} = 0.1 \)) of the two sets of \( (\delta \rho/\rho)^2 \) puts strong limits on a possible decay of the vacuum energy into CDM \cite{16}. Here we make a similar analysis to show that the BWM and the VMM as dark energy candidates can be discarded, in the face of the present evidence that \( F_{\text{max}} = 0.1 \), the matter density \( \Omega_M^0 = 0.28 \pm 0.02 \) and that the universe is flat, indicated by recent CMB data.

We use the agreement to within 10\% of the present observed \( (\delta \rho/\rho)^2 \) between the CMB
and galaxy distribution (GD) data and the constraints from the Gold SNIa data [17] to restrict the parameters of the EOS, \( w(a) = w_0 + w_0(1 - a) \). In particular, we analyze a model suggested by supergravity (SUGRA) [18], studied by Linder [13] and recently by Solevi et al. [19]).

In § II, we discuss the effect of dark energy on the linear growth of \( (\delta \rho/\rho) \) in the models: 1) the vacuum energy decay into CDM; 2) the BWM and VMM; 3) dark energy models parametrized by \( w(a) = w_0 + w_0(1 - a) \); and 4) the SUGRA model. Our conclusions are presented in section III.

II. DARK ENERGY AND THE GROWTH OF DENSITY FLUCTUATIONS

The nature of dark energy is still unknown and there are many alternative models to explain it. One possibility is that instead of a constant vacuum energy, described by a cosmological constant, we have a vacuum energy which is decaying. Other possibilities are models that modify gravity and phenomenological models that parametrize the dark energy EOS in the form \( w(a) = w_0 + w_0(1 - a) \), setting values for \( w_0 \) and \( w_a \). In this section, all of the above models will be analyzed. For these models, the Friedmann equation can be written in a general form in terms of an effective EOS [20]. Modeling the dark energy as an ideal fluid in a flat universe, we can write the Friedmann equation as

\[
\frac{H^2(z)}{H_0^2} = \Omega_M^0(1 + z)^3 + (1 - \Omega_M^0) e^{3 \int_0^1 d \ln (1 + x') [1 + w(x')]} ,
\]

or

\[
\frac{H^2(z)}{H_0^2} = \Omega_M^0 (1 + z)^3 + \frac{\delta H^2}{H_0^2} ,
\]

where \( H_0 \) is the present value for the Hubble parameter, \( \Omega_M^0 \) is the present normalized matter density and \( \delta H^2/H_0^2 \) depends on the phenomenological model [20]. The EOS \( w(z) \) for the dark energy can be written as

\[
w(z) \equiv -1 + \frac{1}{3} \frac{d \ln \delta H^2/H_0^2}{d \ln (1 + z)} .
\]

The linear growth of a density fluctuation, \( D = \delta \rho/\rho \), depends on the EOS. We define the growth factor \( G \equiv D/a \), where \( a \equiv 1/(1 + z) \), the cosmological scale factor, and \( G \) is normalized to unity at \( z \sim 1100 \), the recombination epoch. In terms of \( G \), we have

\[
G''(a) + \left[ 7 - \frac{3}{2} \frac{w(a)}{1 + X(a)} \right] \frac{G'(a)}{a} + \frac{3}{2} \frac{1 - w(a) G(a)}{1 + X(a)} \frac{G(a)}{a^2} = 0 ,
\]

or

\[
G''(a) + \left[ 7 - \frac{3}{2} \frac{w(a)}{1 + X(a)} \right] \frac{G'(a)}{a} + \frac{3}{2} \frac{1 - w(a) G(a)}{1 + X(a)} \frac{G(a)}{a^2} = 0 ,
\]
where \( X(a) \) is defined as

\[
X(a) = \frac{\Omega_M^0 a^{-3}}{\delta H^2/H_0^2}.
\]

The linear mass power spectrum is proportional to \( D^2 \) and we define the deviation from the standard \( \Lambda \text{CDM} \) model by

\[
F = \left| \frac{D^2 - D_L^2}{D_L^2} \right| \bigg|_{z=0},
\]

where \( D_L^2 \) is the density fluctuation in the standard \( \Lambda \text{CDM} \) model. Since \( D^2 \) derived from GD data differs from \( D_L^2 \) derived from the CMB anisotropies by no more than 10 per cent, the maximum value of \( F \) is \( F_{\text{max}} = 0.1 \).

We apply the above description to the following models used to explain the recent acceleration of the universe, suggested in the literature: a phenomenological vacuum energy decay model; two models in which gravity is modified, BWM and VMM; and the SUGRA model.

### A. Vacuum energy decay into CDM

In a previous paper, we put limits on the rate of a possible decay of the vacuum energy (i.e., the decay of the cosmological constant) into CDM from the observed agreement, to within 10\%, of the \((\delta \rho/\rho)^2\) derived from the CMB and the galaxy survey data [16].

Let us consider the vacuum energy decay model described, in a flat universe, by a power law dependence

\[
\Omega_\Lambda(z) = \Omega_\Lambda^0 (1 + z)^n,
\]

where \( \Omega_\Lambda^0 = 1 - \Omega_M^0 \). From conservation of energy and Eq.(6), we have

\[
\Omega_M(z) = \Omega_M^0 (1 + z)^3 - \frac{n \Omega_\Lambda^0}{3 - n} [ (1 + z)^3 - (1 + z)^n ] ,
\]

where \( \Omega_M(z = 0) = \Omega_M^0 \). Eq.(7) modifies the Friedmann equation [Eq.(2)] by a factor

\[
\frac{\delta H^2}{H_0^2} = \Omega_\Lambda^0 \left[ \frac{3}{3 - n} a^{-n} - \frac{n}{3 - n} a^{-3} \right].
\]

Comparing the vacuum energy decay model with the \( \Lambda \text{CDM} \) model, the deviation \( F_{\text{decay}} \) is shown in Table| assuming \( \Omega_M^0 = 0.28 \), the observed value. It is to be noted that \( F \lesssim 0.1 \) occurs only for \( n < 0.02 \).
We evaluated Eq. (4) numerically and show $G$ as a function of $a$ for the vacuum energy decay model in Fig. 1. A value of $n = 0.03$ for the vacuum energy decay model and $\Omega_{M}^0 = 0.28 \pm 0.02$ are shown. Larger $(\delta \rho/\rho)^2$ are predicted by the vacuum energy decay model than by the $\Lambda$CDM model. $G$ increases with $n$ for all values of $n < 3$. Allowing $\Omega_{M}^0$ to vary for $n \geq 0.02$, it is not possible to obtain an agreement with the $\Lambda$CDM within 10%.

The effective EOS as a function of $a$ [Eq. (3)] for the $\Lambda$-decay model with $n = 0.02$ is shown in Fig. 2.
FIG. 2: The effective EOS, $w$, for the $\Lambda$-decay model with $n = 0.02$.

**B. Brane-world and vacuum metamorphosis models**

In the BWM [11], gravity is modified by adding a five-dimensional Einstein-Hilbert action that dominates at distances which are larger than the crossover length $r_c$ that defines an effective energy density $\Omega_{bw} = (1 - \Omega_M^0)^2/4 = 1/(4H_0^2r_c^2)$ for a flat universe. The factor $\delta H^2/H_0^2$ in Eq. (2) then becomes

$$\delta H^2/H_0^2 = 2\Omega_{bw} + 2\sqrt{\Omega_{bw}}\sqrt{\Omega_M^0(1 + z)^3 + \Omega_{bw}}. \quad (9)$$

In the VMM [12], the vacuum contributions are due to a quantized massive scalar field, which is coupled to gravity. For $z < z_j$, the $\delta H^2/H_0^2$ in Eq. (2) is

$$\delta H^2/H_0^2 = (1 - m^2/12)(1 + z)^4 + m^2/12 - \Omega_M^0(1 + z)^3, \quad (10)$$

where $z_j = [m^2/(3\Omega_M^0)]^{1/3} - 1$ and $m^2 = 3\Omega_M^0[(4/m^2) - (1/3)]^{-3/4}$. Both the BWM and the VMM can be described by the EOS,

$$w(a) = w_0 + w_a(1 - a), \quad (11)$$

with $(w_0, w_a) = (-0.78, 0.32)$ and $(w_0, w_a) = (-1, -3)$, respectively [13].

The growth of the density fluctuation $G = [(\delta \rho/\rho)/a$ as a function of $a$ for the BWM,
the VMM, and the ΛCDM model is shown in Fig. 3. We note that $F$ is greater than the maximum allowed value, 0.1, for the BWM and the VMM.

![Graph showing the growth of the density fluctuation $G = [(\delta \rho/\rho)/a]$ for the VMM (top curve), ΛCDM model (middle curve), and the BWM (bottom curve). The dashed lines show the deviation for the matter density $\Omega_M^0 = 0.28 \pm 0.02$.](image)

**TABLE II:** The deviation $F$ for the BWM and the VMM, respectively, as a function of the matter density $\Omega_M^0$. The values $H_0 r_0$, $m^2$ and $z_j$ are defined in § II-B.

| $\Omega_M^0$ | $F_{\text{BWM}}$ | $H_0 r_0$ | $F_{\text{VMM}}$ | $m^2$ | $z_j$ |
|-------------|-----------------|-----------|-----------------|-------|-----|
| 0.26        | 0.27            | 1.4       | 0.29            | 11    | 1.4 |
| 0.28        | 0.27            | 1.4       | 0.24            | 11    | 1.4 |
| 0.30        | 0.26            | 1.4       | 0.20            | 11    | 1.5 |
| 0.32        | 0.25            | 1.5       | 0.16            | 11    | 1.2 |
| 0.34        | 0.25            | 1.5       | 0.13            | 11    | 1.2 |
| 0.36        | 0.24            | 1.6       | 0.095           | 10    | 1.1 |
| 0.72        | 0.11            | 3.6       | 0.24            | 8     | 0.5 |
It can be seen that an agreement within 10% between the VMM and the ΛCDM is possible only for the matter density \( \Omega^0_M \approx 0.36 \). For the BWM, an agreement with the ΛCDM within 10% is possible only if the matter density \( \Omega^0_M \approx 0.72 \). Both of these values for \( \Omega^0_M \) are greater than the observational estimate \( \Omega^0_M = 0.28 \pm 0.02 \).

C. Dark energy models described by a parametrized EOS

We now discuss the simplest parametrization of the EOS, that has been widely used for dark energy models, since it is well-behaved at high redshifts (unlike \( w(z) = w_0 + w_1 z \) which diverges at high \( z \)). This parametrization [Eq.(11)], was introduced by Linder [21]. The best fit parameters \( w_0 \) and \( w_a \) that are consistent with the Gold SNIa dataset were found to be in the intervals \(-1.91 \leq w_0 \leq -1.25 \) and \( 1.53 \leq w_a \leq 5.05 \) [17]. Assuming \( F = 0.10 \pm 0.02 \) and \( \Omega^0_M = 0.28 \pm 0.02 \), we further restrain the best fit values of \( w_a \) and \( w_0 \). These values, whose ranges are \(-1.91 \leq w_0 \leq -1.72 \) and \( 1.53 \leq w_a \leq 2.9 \), are shown in Table III.

| \( w_a \) | \( w_0 \)          |
|---------|------------------|
| 1.53    | -1.72 \pm 0.02   |
| 1.63    | -1.77 \pm 0.02   |
| 1.73    | -1.82 \pm 0.02   |
| 1.83    | -1.86 \pm 0.02   |
| 1.93    | -1.82-0.01       |
| 2.03    | -1.89-0.01       |
| 2.9     | -1.86-0.02       |

TABLE III: Our best fit values of \( w_0 \) and \( w_a \), for the EOS \( w(a) = w_0 + w_a(1-a) \), for Gold SNIa dataset [17] with the deviation \( F = 0.10 \pm 0.02 \) and the matter density \( \Omega^0_M = 0.28 \pm 0.02 \).

D. Supergravity model

The SUGRA model [18] is an attractive model to possibly explain the acceleration of the universe. This model can be described by the EOS of § II-C with \( w_0 = -0.82 \) and \( w_a = 0.58 \).
This equation of state is in agreement with observations for the low redshift SNIa dataset [18] and GD data [19].

![FIG. 4: The growth of the density fluctuation $G = [(\delta \rho / \rho) / a]$, for the $\Lambda$CDM model (top curve) and the SUGRA model (bottom curve) with the matter density $\Omega_M^0 = 0.28 \pm 0.02$ (dashed lines).](image)

Fig. 4 shows that the growth of $\delta \rho / \rho$ is smaller for the SUGRA model than for the $\Lambda$CDM model. The $F$ for the SUGRA model is $F_{\text{SUGRA}} \approx 0.38^{+0.04}_{-0.04}$ for $\Omega_M^0 = 0.30^{+0.04}_{-0.04}$, which is appreciably greater than the maximum allowed value $F_{\text{max}} = 0.1$.

### III. CONCLUSIONS

We calculate the value of $F$ numerically for well-known dark energy models from the growth equation for $\delta \rho / \rho$. From observations, the maximum value of $F$ is $F_{\text{max}} = 0.1$. A $\Lambda$-decay into CDM model, described by a power law dependence (studied in our previous paper [16]), was first considered. It was found that the factor $F$ increases as the exponent $n$ increases. The maximum possible value of $n$ was found to be ($n < 0.02$).

The BWM and VMM were then analyzed. We showed that these models as dark energy candidates can be discarded, assuming that $F_{\text{max}} = 0.1$, $\Omega_M^0 = 0.28 \pm 0.02$ and that the universe is flat.

We combined the constraints from the Gold SNIa data [17] and the condition that $F_{\text{max}} = 0.1$.
0.1 to restrict the values of the parameters of the linear EOS for dark energy, \( w(a) = w_0 + w_a(1 - a) \). It was found that the best fit values of \( w_0 \) and \( w_a \) are \(-1.86 < w_0 < -1.72\) with \(1.53 < w_a < 2.9\). For \( z \sim 0.5 - 1 \), where the supernova data is sensitive, \( w \sim -1 \) for this parametrized EOS.

Finally, we also analyzed the SUGRA model for the above parametrized EOS with \( w_0 = -0.82 \) and \( w_a = 0.58 \). \( F \) was found to be very large: \( F_{\text{SUGRA}} \approx 0.38^{+0.04}_{-0.02} \) for \( \Omega_M^0 = 0.30^{-0.04}_{-0.02} \), which is appreciably greater than the maximum value \( F_{\text{max}} = 0.1 \).

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[1] A. G. Riess, Astron. J. 116, 1009 (1998).
[2] S. Perlmutter, Astrophys. J. 517, 565 (1999).
[3] R.V. Wagoner, unpublished lecture notes (1986).
[4] E.V. Linder, Astron. Astrophys. 206, 175 (1988).
[5] E.V. Linder, First Principles of Cosmology (Addison-Wesley, 1997).
[6] J.A. Frieman, D. Huterer, E.V. Linder, and M.S. Turner, Phys. Rev. D 67, 083505 (2003).
[7] D. Huterer and M.S. Turner, Phys. Rev. D 64, 123527 (2001).
[8] D. Huterer and G. Starkman, Phys. Rev. Lett. 90, 031301 (2003).
[9] W. Hu, Phys. Rev. D 65, 023003 (2002).
[10] E.V. Linder, Phys. Rev. D 70, 043534 (2004).
[11] C. Deffayet, G. Dvali, and G. Gabadadze, Phys. Rev. D 65, 044023 (2002).
[12] L. Parker and A. Raval, Phys. Rev. D 62, 083501 (2000).
[13] E.V. Linder, Phys. Rev. D 70, 023511 (2004).
[14] O. Lahav et al., MNRAS 333, 961 (2002).
[15] W.J. Percival et al., MNRAS 337, 1068 (2002).
[16] R. Opher and A. Pelinson, Phys. Rev. D 70, 063529 (2004).
[17] S. Nesseris and L. Perivolaropoulos, Phys. Rev. D 70 043531 (2004).
[18] P. Brax and J. Martin, Phys. Lett. B 468, 40 (1999).
[19] P. Solevi et al., astro-ph/0504124.

[20] E.V. Linder and A. Jenkins, MNRAS 346, 573 (2003).

[21] E.V. Linder, Phys. Rev. Lett. 90, 091301 (2003).

[22] S. Nesseris and L. Perivolaropoulos, Phys. Rev. D 70, 043531 (2004).

[23] A.G. Riess et al., Astrophys. J. 607, 665 (2004).