THE ALGEBRAIC ATIYAH-HIRZEBRUCH SPECTRAL
SEQUENCE OF REAL PROJECTIVE SPECTRA

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Abstract. In this note, we use Curtis’s algorithm and the Lambda algebra to compute the algebraic Atiyah-Hirzebruch spectral sequence of the suspension spectrum of $\mathbb{R}P^\infty$ with the aid of a computer, which gives us its Adams $E_2$-page in the range of $t < 72$. We also compute the transfer map on the Adams $E_2$-pages. These data are used in our computations of the stable homotopy groups of $\mathbb{R}P^\infty$ in [6] and of the stable homotopy groups of spheres in [7].

This note gives computer-generated computations to be used in [6] and [7]. The data here are “mindless” input to those papers, input that a computer can generate without human intervention. The papers [6] and [7] compute differentials, starting from the data presented here. We are minded to quote Frank Adams [1, page 58-59] from 1969:

“··· The history of the subject [algebraic topology] shows, in fact, that whenever a chance has arisen to show that a differential $d_r$ is non-zero, the experts have fallen on it with shouts of joy - ‘Here is an interesting phenomenon! Here is a chance to do some nice, clean research!’ - and they have solved the problem in short order. On the other hand, the calculation of $\text{Ext}^{s,t}$ groups is necessary not only for this spectral sequence, but also for the study of cohomology operations of the $n$-th kind: each such group can be calculated by a large amount of tedious mechanical work: but the process finds few people willing to take it on. ···”

Warning: this note contains data of interest only to experts.

1. Notations

We work at the prime 2 in this paper. All cohomology groups are taken with coefficients $\mathbb{Z}/2$.

Let $\mathcal{A}$ be the Steenrod algebra. For any $\mathcal{A}$-module $M$, we will abbreviate $\text{Ext}_{\mathcal{A}}(M, \mathbb{Z}/2)$ by $\text{Ext}(M)$.

Let $V$ be a vector space with $\{v_j\}$ an ordered basis. We say that an element $v = \sum a_i v_i$ has leading term $a_k v_k$ if $k$ is the largest number for which $a_k \neq 0$.

For spectra, let $S^0$ be the sphere spectrum, and $P^n_\infty$ be the suspension spectrum of $\mathbb{R}P^n$. In general, we use $P^{n+k}_n$ to denote the suspension spectrum of $\mathbb{R}P^{n+k}/\mathbb{R}P^{n-1}$.

2. The Curtis table

We recall the notion of Curtis table in a general setting in this section.

Let $X_0 \rightarrow X_1 \rightarrow \ldots$ be a complex of vector spaces (over $\mathbb{F}_2$). For each $X_i$, let $\{x_{i,j}\}$ be an ordered basis.
Definition 2.1. A Curtis table for $X_*$ associated with the basis $\{x_{i,j}\}$ consists of a list $L_i$ for each $i$.

The items on the list $L_i$ are either an element $x_{i,j}$ for some $j$, or a tag of the form $x_{i,j} \leftarrow x_{i-1,k}$ for some $j,k$.

These lists satisfy the following:

1. Each element $x_{i,j}$ appears in these lists exactly once.
2. For any $i, j$, an item of the form $x_{i,j}$ or a tag of the form $x_{i,j} \leftarrow x_{i-1,k}$ appears in the list $L_i$ if and only if there is a cycle in $X_i$ with leading term $x_{i,j}$.
3. If a tag of the form $x_{i,j} \leftarrow x_{i-1,k}$ appears in the list $L_i$, then there is an element in $X_{i-1}$ with leading term $x_{i-1,k}$ whose boundary has leading term $x_{i,j}$.

Remark 2.2. By Theorem 3.3 and Corollary 3.4, the Curtis table exists and is unique for a finite dimensional complex with ordered basis.

The Curtis algorithm constructs a Curtis table from a basis, and can output the full cycle from the input of a leading term.

For example, the Curtis table in the usual sense is for the lambda algebra with the basis of admissible monomials in lexicographic order. In [5] Tangora computed the Curtis table for the lambda algebra up to stem 51.

Another example is the minimal resolution for the sphere spectrum. This case is indeed trivial in the sense that there are no tags in the Curtis table.

3. The Curtis algorithm

The Curtis algorithm produces a Curtis table from an ordered basis. It can be described as follows:

Algorithm 3.1. (Curtis)

1. For each $i$, construct a list $L_i$ which contains every $x_{i,j}$ such that the items are ordered with $j$ ascending.
2. For $i = 0, 1, 2, \ldots$ do the following:
   (a) Construct a pointer $p$ with initial value pointing to the beginning of $L_i$.
   (b) If $p$ points to the end of $L_i$ (i.e. after the last element), stop and proceed to the next $i$.
   (c) If the item pointed by $p$ is tagged, move $p$ to the next item and go to Step 2a.
   (d) Construct a vector $c \in X_i$. Give $c$ the initial value of the item pointed by $p$.
   (e) Compute the boundary $b \in X_{i+1}$ of $c$.
   (f) If $b = 0$, move $p$ to the next item and go to Step 2a.
   (g) Search the leading term $y$ of $b$ in $L_{i+1}$.
   (h) If $y$ is untagged, tag $y$ with the leading term of $c$. Remove the item pointed by $p$ and move $p$ to the next item. Go to Step 2a.
   (i) If $y$ is tagged by $z$, add $z$ to $c$. Go to Step 2a.
Example 3.2. As an example, we compute the Curtis table for the lambda algebra for $t = 3$. We start with

\begin{align*}
L_1 &= \{\lambda_2\} \\
L_2 &= \{\lambda_1 \lambda_0\} \\
L_3 &= \{\lambda_0^3\}
\end{align*}

We next compute the boundary of $\lambda_2$:

\[d(\lambda_2) = \lambda_1 \lambda_0.\]

We therefore remove it from $L_1$ and tag $\lambda_1 \lambda_0$ with $\lambda_2$. The output gives us the following:

\begin{align*}
L_1 &= \emptyset \\
L_2 &= \{\lambda_1 \lambda_0 \leftarrow \lambda_2\} \\
L_3 &= \{\lambda_0^3\}
\end{align*}

Theorem 3.3. (Curtis) The Curtis algorithm ends after finitely many steps when $X_*$ is finite dimensional. Moreover, let $Y_*$ be the graded vector space generated by those untagged items on the $L_i$'s. Denote by $C_*$ the subspace of cycles in $X_*$. There is an algorithm which constructs a map $Y_i \to C_i$ and a map $C_i \to Y_i$ which induce an isomorphism between $Y_*$ and the homology of $X_*$. 

Proof. See [5]. $\square$

Corollary 3.4. The Curtis table is unique for a finite dimensional complex $X_*$ with ordered basis. In fact, it is specified in the following way:

Let $l(x)$ denote the leading term of $x$.

If there is a tag $a \leftarrow b$, then $a$ is the minimal element of the set \(\{l(d(x)) | l(x) = b\}\).

If an item $a$ is untagged, then it is the leading term of an element with lowest leading term in a homology class.

Proof. See [5]. $\square$

4. CURTIS TABLE AND SPECTRAL SEQUENCES

Now suppose $V$ is a filtered vector space with $\cdots \subset F_i V \subset F_{i+1} V \cdots \subset V$. We call an ordered set of basis $\{v_k\}$ compatible if for any $i$ there is a $k_i$ such that $F_i V$ is spanned by $\{v_k : k \leq k_i\}$.

Let $X_0 \to X_1 \to \ldots$ be a complex of filtered vector spaces such that the differentials preserve the filtration. Then there is a spectral sequence converging to the homology of $X$ with the $E_1$-term $F_k X_i / F_{k-1} X_i$. Suppose we have compatible bases $\{x_{i,j}\}$ of $X_i$.

Theorem 4.1. The Curtis table of $X_*$ consists of the following:

1. The tags of the Curtis table for $(E_r, d_r)$ of the spectral sequence, for all $r \geq 1$.

2. The untagged items from the $E_\infty$-term.

Here we label the basis of $E_r$ as the following. In the $E_1$-page, we use the image of the $x_{i,j}$’s as the basis, and label them by the same name. Inductively, we use Theorem 3.3 to label a basis of $E_r$ by the untagged items in the Curtis table of $E_{r-1}$. 

Proof: We check the conditions of Definition 2.1. They follow directly from the definition of the spectral sequence, the conditions for the Curtis tables of the $E_i$’s, and Theorem 3.3.

Consequently, we can identify the Curtis table with the table for the differentials and permanent cycles of the spectral sequence. For example, in the lambda algebra, we have a filtration by the first number of an admissible sequence. The induced spectral sequence is the algebraic EHP sequence. So the usual Curtis table can be identified with the algebraic EHP sequence. See [3] for more details.

In practice, the Curtis table for the $E_1$ terms is often known before hand. Then we could skip those part of the Curtis algorithm dealing with the tags coming from the $E_1$ term. And we often omit this part in the output of Curtis table.

5. The algebraic Atiyah-Hirzebruch spectral sequence

Let $X$ be a spectrum. There is a filtration on $H^*(X)$ by the degrees. For any $n$ there is a short exact sequence $0 \to H^{\geq n+1}(X) \to H^{\geq n}(X) \to H^n(X) \to 0$. This induces a long exact sequence
\[ \cdots \to Ext(Z/2) \otimes H^n(X) \to Ext(H^{\geq n}(X)) \to Ext(H^{\geq n+1}(X)) \to \cdots \]
Combining the long exact sequences for all $n$ we get the algebraic Atiyah-Hirzebruch spectral sequence
\[ \oplus_n Ext(Z/2) \otimes H^n(X) \Rightarrow Ext(H^*(X)) \]

There is another way to look at the algebraic Atiyah-Hirzebruch spectral sequence.

Let us fix a free resolution $\cdots \to F_1 \to F_0 \to F_2$ as $A$-modules. For example, we can take $F_2$ to be the Koszul resolution, which gives the lambda algebra constructed in [2]. We can also take $F_2$ to the the minimal resolution.

Then for $X$ a finite CW spectrum, we can identify $RHom_A(H^*(X),Z/2)$ with the complex $C^*(H^*(X)) = Hom_A(H^*(X) \otimes_{F_2} F_2,F_2)$ where we take the diagonal action of the Steenrod algebra on $H^*(X) \otimes_{F_2} F_2$ using the Cartan formula.

The cell filtration on $H_*(X)$ induces a filtration on $H^*(X) \otimes_{F_2} F_2$, and we can identify the algebraic Atiyah-Hirzebruch spectral sequence with the spectral sequence generated by this filtration. In fact, the map $H^*(X) \otimes_{F_2} F_2 \to H^*(X)$ preserves these filtrations and induces a quasi-isomorphism on each layer. So they define equivalent sequences in the derived category, hence generate the same spectral sequence.

6. The Curtis algorithm in computing the algebraic Atiyah-Hirzebruch spectral sequence

Let $X$ be a finite CW spectrum.

Let $r_{ij}^* \in F_i$ be a set of $A$-basis for the free $A$-module $F_i$. Let $r_{ij} \in Hom_A(F_i,Z/2)$ be the dual basis.

We choose an ordered $F_2$-basis $e_i^*$ of $H^*(X)$ such that elements with lower degrees come first. Let $e_k \in H_*(X)$ be the dual basis. Then the set $\{e_k \otimes r_{ij}^*\}$ is a set of $A$-basis for $H^*(X) \otimes_{F_2} F_2$. Let $e_k \otimes r_{ij} \in Hom_A(H^*(X) \otimes_{F_2} F_2,F_2)$ be the dual basis with the lexicographic order.

The following is a corollary of Theorem 4.1.
Theorem 6.1. The Curtis table for $C^*(H^*(X)) = \text{Hom}_\mathcal{A}(H^*(X) \otimes_{F_2} F_2, F_2)$ satisfies

1. If there is a tag $a \leftarrow b$ in the Curtis table of $\text{Hom}_\mathcal{A}(F_i, \mathbb{Z}/2)$, there are tags of the form $e_k \otimes a \leftarrow e_k \otimes b$.

2. The table of all tags which are not contained in Case 1 is the same as the table for the algebraic Atiyah-Hirzebruch differentials of $X$.

3. The items not contained in the previous cases are untagged items. They correspond to the permanent cycles in the algebraic Atiyah-Hirzebruch spectral sequence.

Consequently, we can read off the $E_2$-term of the Adams spectral sequence of any truncation of $X$.

Theorem 6.2. Let $X^n_m$ be the truncation of $X$ which consists of all cells of $X$ in dimensions between (and including) $m$ and $n$. Therefore in the Curtis table of $X^n_m$, all the tags are those tags in the Curtis table of $X$ lying within the corresponding range. (Note there could be more untagged items, which are just those not appearing in any tags.)

Proof. This follows from the previous theorem because the Atiyah-Hirzebruch spectral sequence is truncated this way. □

We present two examples. The latter one is used in our computation in [7] that the 2-primary $\pi_{61} = 0$. For notation, in the Lambda algebra, we will abbreviate an element $\lambda_{i_1} \ldots \lambda_{i_n}$ by $i_1 \ldots i_n$. In the Lambda complex of $P_1^\infty$, we will abbreviate an element $e_k \otimes \lambda_{i_1} \ldots \lambda_{i_n}$ by $(k)i_1 \ldots i_n$. The Curtis table is separated into lists labeled by $(t-s, t)$ on the top, in which those untagged items give a basis for $\text{Ext}_{s-1,t-1}(H^*(P_1^\infty))$.

Example 6.3. As a relatively easy example, we compute $\text{Ext}^{2,2+9}(H^*(P_2^8))$ using the Curtis table of $P_1^\infty$ in the Appendix.

There are only two boxes that are used in this computation: the ones labeled with $(9,3)$ and $(8,4)$. The box labeled with $(9,3)$ is the following:

1. 5 3
2. 3 3
3. 7 1 1 ← (9) 1

The spectrum $P_2^8$ only has cells in dimensions 2 through 8. We remove the item (1) 5 3, since it comes from the cell in dimension 1. We also remove the tag (9) 1, since it comes from the cell in dimension 9. Therefore, the only items remaining in this box are (3) 3 3 and (7) 1 1.

The box labeled with (8,4) is the following:

1. 5 1 1 ← (2) 6 1
2. 5 1 1 ← (6) 2 1

After removing the element (1) 5 1 1, which comes from the cell in dimension 1, the element (2) 6 1 tags nothing. We move the element (2) 6 1 from the box labeled with (8,4) to the one labeled with (9,3). Therefore, we have the conclusion that the group $\text{Ext}^{2,2+9}(H^*(P_2^8))$ has dimension 3, generated by (3) 3 3, (7) 1 1, and (2) 6 1.
One can even recover the names of these generators in the algebraic Atiyah-Hirzebruch spectral sequence. See Notation 3.3 in [7] for the notation. In $\text{Ext}(\mathbb{Z}/2)$, the elements 3, 3, 1, 1, and 6, 1 all lie in the bidegrees which contain only one nontrivial element. Therefore, we can identify their Adams $E_2$-page names as $h_2^3$, $h_1^1$, and $h_0h_3$. This gives us the algebraic Atiyah-Hirzebruch $E_1$-page names of these generators:

\[ h_2^3[3], \ h_1^1[7], \ \text{and} \ h_0h_3[2]. \]

**Example 6.4.** We present the computation of the Adams $E_2$ page of $P_{16}^{22}$ in the 60 and 61 stem for $s \leq 7$, which is used in the proof of Lemma 8.2 in [7]. The boxes that are used in this computation have the following labels:

- \((59, s)\) for $s \leq 7$,
- \((60, s')\), \((61, s')\) for $s \leq 8$.

The spectrum $P_{16}^{22}$ consists of cells in dimensions 16 through 22.

We start with the 60 stem.

We have $\text{Ext}^{1,1+60}(P_{16}^{22}) = \text{Ext}^{2,2+60}(P_{16}^{22}) = 0$, since the boxes labeled with (60, 2), (59, 3) and (60, 3), (59, 4) becomes empty.

We have $\text{Ext}^{3,3+60}(P_{16}^{22}) = \mathbb{Z}/2$, generated by (19) 11 15 15 from the box labeled with (59, 5). The box labeled with (60, 4) becomes empty. Since 11 15 15 $\in \text{Ext}^{3,3+41} = \mathbb{Z}/2$, generated by $c_2$, we identify (19) 11 15 15 with its Atiyah-Hirzebruch name $c_2[19]$.

We have $\text{Ext}^{4,4+60}(P_{16}^{22}) = \mathbb{Z}/2 \oplus \mathbb{Z}/2$, generated by (16) 13 13 11 7 from the box labeled with (59, 6), and by (20) 19 7 7 7 from the box labeled with (60, 5). We find their Atiyah-Hirzebruch names $g_2[16]$ and $f_1[20]$.

We have $\text{Ext}^{5,5+60}(P_{16}^{22}) = \mathbb{Z}/2 \oplus \mathbb{Z}/2$, generated by (16) 11 14 5 7 7 and (21) 7 13 5 7 7 from the box labeled with (59, 7). The box labeled with (60, 6) becomes empty. We find their Atiyah-Hirzebruch names $h_0g_2[16]$ and $h_1e_1[21]$.

We have $\text{Ext}^{6,6+60}(P_{16}^{22}) = \mathbb{Z}/2 \oplus \mathbb{Z}/2 \oplus \mathbb{Z}/2$, generated by (16) 7 14 4 5 7 7 from the box labeled with (59, 8), and by (20) 5 5 9 7 7 7 and (22) 3 5 9 7 7 7 from the box labeled with (60, 7). We find their Atiyah-Hirzebruch names $h_0^2g_2[16]$, $h_0^2f_1[20]$ and $h_1x[22]$.

We have $\text{Ext}^{7,7+60}(P_{16}^{22}) = \mathbb{Z}/2$, generated by (21) 3 5 9 3 5 7 7 from the box labeled with (60, 8). The box labeled with (59, 9) becomes empty. We find its Atiyah-Hirzebruch name $h_1y[21]$.

Similarly, one can compute the 61 stem. The computation is summarized in the following Table 1.
Table 1. The Adams $E_2$ page of $P_{10}^{22}$ in the 60 and 61 stems for $s \leq 7$

| $s \setminus t - s$ | 60                      | 61                      |
|---------------------|-------------------------|-------------------------|
| 7                   | $h_1 y_{[21]}$          | $h_0^2 h_5 d_0_{[16]}$  |
|                     |                         | $h_1 y_{[22]}$          |
| 6                   | $h_0^2 g_2_{[16]}$      | $h_0 h_5 d_0_{[16]}$    |
|                     | $h_1^2 f_1_{[20]}$      | $P h_2 h_5_{[19]}$      |
|                     | $h_1 x_{[22]}$          |                         |
| 5                   | $h_0 g_2_{[16]}$        | $h_1 g_2_{[16]}$        |
|                     | $h_1 e_1_{[21]}$        | $h_5 d_0_{[16]}$        |
|                     |                         | $h_1 f_1_{[20]}$        |
|                     |                         | $h_1 h_5 c_0_{[21]}$    |
|                     |                         | $h_3 d_1_{[22]}$        |
| 4                   | $g_2_{[16]}$            | $h_0 h_2^4_{[16]}$      |
|                     | $f_1_{[20]}$            | $g_2_{[17]}$            |
|                     |                         | $f_1_{[21]}$            |
|                     |                         | $h_1^2 h_3 h_5_{[21]}$  |
|                     |                         | $h_5 c_0_{[22]}$        |
| 3                   | $c_2_{[19]}$            | $h_0^2_{[16]}$          |
|                     |                         | $h_1 h_3 h_5_{[22]}$    |

7. The homomorphism induced by a map

Let $f : X \to Y$ be a map which induces the zero map on homology. Let $Z$ be the cofiber of $f$. Then the homology of $Z$ can be identified with the direct sum of $H_*(X)$ and $H_*(Y)$ as a vector space. If $x_1, \ldots, x_k$ is an ordered basis of $H_*(X)$ and $y_1, \ldots, y_l$ is an ordered basis of $H_*(Y)$, then $y_1, \ldots, y_l, x_1, \ldots, x_k$ is an ordered basis of $H_*(Z)$ with certain degree shifts. Note we do not make elements with lower degree go first here. Instead elements $y_i$ always go before elements $x_j$ regardless of degree.

Note that in this case, there is a map of Adams spectral sequence of $X$ and $Y$ which raises the Adams filtration by one, and on the $E_2$ page it is the boundary homomorphism for the Ext group for the exact sequence $0 \to H^{*+1}(X) \to H^*(Z) \to H^*(Y) \to 0$. We call this the map induced by $f$.

**Theorem 7.1.** The Curtis table for $C^*(H^*(Z)) = \text{Hom}_A(H^*(Z) \otimes \mathbb{F}_2, \mathbb{F}_2)$ from Section 5 with this ordered basis satisfies

1. All of the tags in the Curtis table for $C^*(H^*(X))$ and for $C^*(H^*(Y))$ also appear in the Curtis table for $C^*(H^*(Z))$.
2. The remaining tags give the table for the homomorphism on the Adams $E_2$-page induced by $f$.
3. The untagged items give basis for the kernel and cokernel of the homomorphism induced by $f$.

**Proof.** This follows from Theorem 4.1 by using the filtration $Y \subset Z$, and identifying the $d_2$-differential with the attaching map $X \to Y$. □

So we can use the Curtis algorithm to compute the homomorphism induced by a map.
8. The algebraic Atiyah-Hirzebruch spectral sequence of the real projective spectra

We use the Curtis algorithm to compute the algebraic Atiyah-Hirzebruch spectral sequence for the real projective spectra. We take the lambda algebra for the resolution of $\mathbb{Z}/2$ and use the usual Curtis table for the sphere spectrum as input. We have carried out the computation through stems with $t < 72$. As a usual convention to put the Curtis table, we abbreviate the sequence $2 4 1 1$ by $\ast$; when there are multiple $2$’s consecutively, we replace them by the same amount of dots.

Together with the algebraic Kahn-Priddy theorem [4] and known information of $\text{Ext}(\mathbb{Z}/2)$, this gives the Adams $E_2$-page of $P_1^{\infty}$ up to $t - s \leq 61$.

We also compute the transfer map. Recall that the fiber of the transfer map has one more cell than $P_1^{\infty}$ in dimension $-1$, and all the $Sq^i$ acts nontrivially on the class in dimension $-1$. We will use Theorem 7.1 to identify the table for transfer with a portion of the Curtis table for this complex.

For notation, in the Lambda algebra, we will abbreviate an element $\lambda_{i_1} \ldots \lambda_{i_n}$ by $i_1 \ldots i_n$. We will abbreviate an element $e_k \otimes \lambda_{i_1} \ldots \lambda_{i_n}$ by $(k)i_1 \ldots i_n$ in the Lambda complex of $P_1^{\infty}$. The symbol $o$ means zero. The Curtis table is separated into lists labeled by $(t - s, t)$ on the top, in which the untagged items give a basis for $\text{Ext}^{t-s-1,t-1}(H^*(P_1^{\infty}))$.

The table for the transfer is the output of the algorithm: (We put the table for the transfer map first since it is shorter)

In this table we only list the nontrivial items. Others either map to something with the same name, or to the only choice comparable with the algebraic Kahn-Priddy theorem. For example, (1) maps to 1, i.e. the inclusion of the bottom cell maps to $\eta$. As another example, (5) 3 maps to 5 3, which can be proved independently by the Massey product

\[ \langle h_2, h_1, h_2 \rangle = h_1 h_3. \]

We do not include such items in the transfer table.

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Table 2: The table for the transfer map

| (1) 7 | 5 3 |
|-------|-----|
| (1) 5 3 | 3 3 3 |
| (1) 6 2 3 3 | 2 4 3 3 3 |
| (1) 15 | 13 3 |
| (1) 13 3 | 11 3 3 |
| (3) 15 | 11 7 |
| (2) 13 3 | 10 5 3 |
| (1) 11 3 3 | 9 3 3 3 |
| (4) 6 5 3 | 5 7 3 3 |
| (1) 8 3 3 3 | 4 5 3 3 3 |
| (3) 8 3 3 3 | 4 7 3 3 3 |
| (3) 11 7 | 7 7 7 |
| (1) 6 6 5 3 | 3 5 7 3 3 |
| (5) 11 3 3 | 3 5 7 7 |
| (2) 7 7 7 | 4 5 7 7 |
| (1) 6 2 3 4 4 1 1 1 | 2 4 1 1 2 4 3 3 3 |
| (1) 4 5 7 7 | 2 3 5 7 7 |
| (3) 13 1 2 4 1 1 1 | 4 2 2 4 5 3 3 3 |
| (1) 8 1 1 2 4 3 3 3 | 2 2 2 2 4 5 3 3 3 |
| (5) 13 1 2 4 1 1 1 | 6 2 2 4 5 3 3 3 |
| (3) 8 1 1 2 4 3 3 3 | 4 2 2 2 4 5 3 3 3 |
| (1) 6 2 2 4 5 3 3 3 | 2 2 2 2 3 5 7 3 3 |
| (8) 3 5 7 7 | 12 9 3 3 3 |
| (1) 15 15 | 13 11 7 |
| (12) 5 7 7 | 0 |
| (1) 6 2 3 4 4 1 1 1 2 4 1 1 1 | 2 4 1 1 2 4 1 1 2 4 3 3 3 |
| (1) 3 1 | 29 3 |
| (2) 12 9 3 3 3 | 9 3 6 6 5 3 |
| (1) 5 6 2 4 5 3 3 3 | 2 2 2 3 6 6 5 3 |
| (1) 29 3 | 27 3 3 |
| (1) 13 5 7 7 | 11 3 5 7 7 |
| (1) 9 3 6 6 5 3 | 5 5 3 6 6 5 3 |
| (3) 12 4 5 3 3 3 | 5 5 3 6 6 5 3 |
| (3) 3 1 | 27 7 |
| (2) 29 3 | 26 5 3 |
| (1) 27 3 3 | 25 3 3 3 |
| (3) 9 3 5 7 7 | 5 7 3 5 7 7 |
| (4) 12 9 3 3 3 | 5 7 3 5 7 7 |
| (1) 8 1 1 2 4 1 1 2 4 3 3 3 | 2 2 2 2 2 2 2 4 5 3 3 3 |
| (3) 13 5 7 7 | 5 9 7 7 7 |
| (5) 12 4 5 3 3 3 | 6 5 2 3 5 7 7 |
| (1) 5 6 2 3 5 7 3 3 | 2 4 3 3 3 6 6 5 3 |
| (3) 8 1 1 2 4 1 1 2 4 3 3 3 | 4 2 2 2 2 2 2 4 5 3 3 3 |
| (3) 27 7 | 23 7 7 |
| (1) 5 9 3 5 7 7 | 3 5 7 3 5 7 7 |
|   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|
| (4) 5 5 3 6 6 5 3 | 4 7 3 3 6 6 5 3 |
| (7) 31 | 23 15 |
| (6) 29 3 | 22 13 3 |
| (5) 27 3 3 | 21 11 3 3 |
| (4) 25 3 3 3 | 20 9 3 3 3 |
| (1) 14 4 5 7 7 | 3 5 9 7 7 7 |
| (1) 23 15 | 21 11 7 |
| (5) 27 7 | 21 11 7 |
| (2) 23 7 7 | 20 5 7 7 |
| (1) 17 7 7 7 | 7 13 5 7 7 |
| (1) 8 12 9 3 3 3 | 3 5 9 3 5 7 7 |
| (1) 21 11 7 | 19 7 7 7 |
| (3) 23 7 7 | 19 7 7 7 |
| (9) 13 11 7 | 11 15 7 7 |
| (1) 20 5 7 7 | 18 3 5 7 7 |
| (2) 17 7 7 7 | 7 14 5 7 7 |
| (1) 7 13 5 7 7 | 5 5 9 7 7 7 |
| (2) 20 9 3 3 3 | 17 3 6 6 5 3 |
| (1) 11 15 7 7 | 9 11 7 7 7 |
| (3) 17 7 7 7 | 9 11 7 7 7 |
| (1) 17 3 6 6 5 3 | 11 12 4 5 3 3 3 |
| (4) 20 9 3 3 3 | 12 12 9 3 3 3 |
| (2) 17 3 6 6 5 3 | 10 9 3 6 6 5 3 |
| (1) 11 12 4 5 3 3 3 | 9 5 5 3 6 6 5 3 |
| (2) 5 6 5 2 3 5 7 7 | 4 5 5 5 3 6 6 5 3 |
| (7) 23 15 | 15 15 15 |
| (6) 21 11 7 | 14 13 11 7 |
| (1) 13 13 11 7 | 9 15 7 7 7 |
| (6) 20 5 7 7 | 13 13 5 7 7 7 9 15 7 7 7 |
| (7) 17 7 7 7 | 9 15 7 7 7 |
| (5) 18 3 5 7 7 | 12 11 3 5 7 7 |
| (3) 12 12 9 3 3 3 | 7 8 12 9 3 3 3 |
| (1) 13 13 5 7 7 | 11 5 9 7 7 7 7 |
| (8) 20 9 3 3 3 | 11 5 9 7 7 7 |
| (2) 7 14 4 5 7 7 | 8 3 5 9 7 7 7 |
| (6) 17 3 6 6 5 3 | 8 3 5 9 7 7 7 |
| (3) 13 13 11 7 | 7 11 15 7 7 |
| (2) 9 15 7 7 7 | 6 9 11 7 7 7 |
| (1) 11 5 9 7 7 7 | 9 3 5 9 7 7 7 |
| (1) 8 3 5 9 7 7 7 | 4 5 3 5 9 7 7 7 |
| (2) 7 8 12 9 3 3 3 | 8 3 5 9 3 5 7 7 |
| (11) 23 7 7 | 7 11 15 15 |
| (8) 19 7 7 7 | 10 17 7 7 7 |
| (3) 13 13 5 7 7 | 9 7 13 5 7 7 |
| (2) 11 5 9 7 7 7 | 8 5 5 9 7 7 7 |
| (4) 7 14 4 5 7 7 | 5 10 11 3 5 7 7 |
The Curtis table for the Adams $E_2$-page of $P^\infty_1$ in the range of $t < 72$ is the following:

\[
\begin{array}{c|c}
(2) & 8 \ 3 \ 5 \ 9 \ 7 \ 7 \ 7 \\
(1) & 4 \ 5 \ 3 \ 5 \ 9 \ 7 \ 7 \ 7 \\
(1) & 8 \ 3 \ 5 \ 9 \ 3 \ 5 \ 7 \ 7 \\
(4) & 14 \ 13 \ 11 \ 7 \\
(1) & 10 \ 17 \ 7 \ 7 \ 7 \\
(5) & 15 \ 15 \ 15 \\
(6) & 14 \ 13 \ 11 \ 7 \\
(3) & 10 \ 17 \ 7 \ 7 \ 7 \\
(2) & 8 \ 9 \ 11 \ 7 \ 7 \ 7 \\
(4) & 9 \ 3 \ 5 \ 9 \ 7 \ 7 \ 7 \\
(1) & 7 \ 13 \ 13 \ 11 \ 7 \\
(3) & 9 \ 11 \ 15 \ 7 \ 7 \\
(3) & 9 \ 11 \ 15 \ 15 \\
(1) & 8 \ 4 \ 5 \ 3 \ 5 \ 9 \ 3 \ 5 \ 7 \ 7 \\
(3) & 16 \ 2 \ 3 \ 5 \ 5 \ 3 \ 6 \ 6 \ 5 \ 3 \\
(1) & 27 \ 12 \ 4 \ 5 \ 3 \ 3 \ 3 \\
(2) & 5 \ 5 \ 9 \ 7 \ 7 \ 7 \\
(3) & 27 \ 12 \ 4 \ 5 \ 3 \ 3 \ 3 \\
(20) & 25 \ 5 \ 3 \ 6 \ 6 \ 5 \ 3 \\
(1) & 6 \ 2 \ 3 \ 5 \ 10 \ 11 \ 3 \ 5 \ 7 \ 7 \\
(3) & 5 \ 5 \ 9 \ 3 \ 5 \ 3 \ 5 \ 7 \ 7 \\
(7) & 18 \ 2 \ 4 \ 3 \ 3 \ 6 \ 6 \ 5 \ 3 \\
(8) & 5 \ 7 \ 11 \ 15 \ 15 \\
(9) & 4 \ 7 \ 11 \ 15 \ 15 \\
(3) & 28 \ 12 \ 9 \ 3 \ 3 \ 3 \\
(1) & 6 \ 2 \ 4 \ 7 \ 11 \ 15 \ 15 \\
\end{array}
\]

\[
\begin{array}{c}
4 \ 6 \ 3 \ 5 \ 9 \ 7 \ 7 \ 7 \\
2 \ 3 \ 5 \ 3 \ 5 \ 9 \ 7 \ 7 \ 7 \\
4 \ 5 \ 3 \ 5 \ 9 \ 3 \ 5 \ 7 \ 7 \\
9 \ 11 \ 15 \ 7 \ 7 \\
8 \ 9 \ 11 \ 7 \ 7 \ 7 \\
9 \ 11 \ 15 \ 15 \\
7 \ 13 \ 13 \ 11 \ 7 \\
6 \ 9 \ 15 \ 7 \ 7 \ 7 \\
4 \ 6 \ 9 \ 11 \ 7 \ 7 \ 7 \\
3 \ 5 \ 10 \ 11 \ 3 \ 5 \ 7 \ 7 \\
5 \ 7 \ 11 \ 15 \ 7 \ 7 \\
5 \ 7 \ 11 \ 15 \ 15 \\
4 \ 5 \ 4 \ 3 \ 5 \ 9 \ 3 \ 5 \ 7 \ 7 \\
4 \ 5 \ 4 \ 3 \ 5 \ 9 \ 3 \ 5 \ 7 \ 7 \\
25 \ 5 \ 3 \ 6 \ 6 \ 5 \ 3 \\
5 \ 7 \ 11 \ 15 \ 15 \\
5 \ 7 \ 11 \ 15 \ 15 \\
4 \ 5 \ 4 \ 3 \ 5 \ 9 \ 3 \ 5 \ 7 \ 7 \\
4 \ 5 \ 4 \ 3 \ 5 \ 9 \ 3 \ 5 \ 7 \ 7 \\
8 \ 4 \ 2 \ 3 \ 5 \ 3 \ 5 \ 9 \ 7 \ 7 \ 7 \\
2 \ 2 \ 4 \ 5 \ 9 \ 3 \ 5 \ 9 \ 7 \ 7 \ 7 \\
3 \ 6 \ 4 \ 6 \ 9 \ 11 \ 7 \ 7 \ 7 \\
2 \ 2 \ 4 \ 5 \ 9 \ 3 \ 5 \ 9 \ 7 \ 7 \ 7 \\
5 \ 7 \ 11 \ 15 \ 15 \\
8 \ 4 \ 2 \ 3 \ 5 \ 3 \ 5 \ 9 \ 7 \ 7 \ 7 \\
2 \ 2 \ 4 \ 5 \ 9 \ 3 \ 5 \ 9 \ 7 \ 7 \ 7 \\
28 \ 11 \ 3 \ 5 \ 7 \ 7 \\
23 \ 8 \ 12 \ 9 \ 3 \ 3 \ 3 \\
2 \ 4 \ 3 \ 4 \ 7 \ 11 \ 15 \ 15 \\
\end{array}
\]
| (1, 1) | (1) |
| (2, 2) | (1) 1 |
| (3, 1) | (3) |
| (3, 2) | (2) 1 |
| (3, 3) | (1) 1 1 |
| (4, 2) | (1) 3 |
| (4, 3) | (1) 2 1 ← (2) 3 |
| (4, 4) | (1) 1 1 1 ← (2) 2 1 |
| (5, 3) | (3) 1 1 ← (5) 1 |
| (5, 4) | (2) 1 1 1 ← (4) 1 1 |
| (6, 2) | (3) 3 |
| (6, 3) | (3) 2 1 ← (4) 3 |
| (6, 4) | (3) 1 1 1 ← (4) 2 1 |
| (7, 1) | (1) |
| (7, 2) | (6) 1 |
| (7, 3) | (1) 3 3 |
| (7, 4) | (4) 1 1 1 |
| (8, 2) | (1) 7 |
| (8, 3) | (1) 6 1 ← (2) 7 |
| (8, 4) | (1) 5 1 1 ← (2) 6 1 |
| (8, 5) | (1) 4 1 1 1 ← (2) 5 1 1 |
| (9, 3) | (1) 5 3 |
| (9, 4) | (1) 2 3 3 |
| (9, 5) | (2) 4 1 1 1 |
| (10, 2) | (12, 3) |
|---------|---------|
| (3) 7   | (5) 6 1 ← (6) 7 |
| (7) 3 ← (11) | (9) 2 1 ← (10) 3 |
|         | (10) 1 1 ← (12) 1 |

| (10, 3) |
|---------|
| (2) 5 3 |
| (3) 6 1 ← (4) 7 |
| (4) 3 3 ← (10) 1 |
| (7) 2 1 ← (8) 3 |

| (12, 4) |
|---------|
| (3) 3 3 3 ← (5) 5 3 |
| (5) 5 1 1 ← (6) 6 1 |
| (9) 1 1 1 ← (10) 2 1 |

| (10, 4) |
|---------|
| (1) 3 3 3 |
| (2) 2 3 3 ← (9) 1 1 |
| (3) 5 1 1 ← (4) 6 1 |
| (7) 1 1 1 ← (8) 2 1 |

| (12, 5) |
|---------|
| (1) 2 3 3 ← (5) 2 3 3 |
| (5) 4 1 1 1 ← (6) 5 1 1 |

| (10, 5) |
|---------|
| (1) 1 2 3 3 ← (8) 1 1 1 |
| (3) 4 1 1 1 ← (4) 5 1 1 |

| (12, 6) |
|---------|
| (1) 4 4 1 1 1 ← (4) 1 2 3 3 |
| (3) * 1 ← (6) 4 1 1 1 |

| (10, 6) |
|---------|
| (1) * 1 |

| (12, 7) |
|---------|
| (1) 2 * 1 ← (2) 4 4 1 1 1 |
| (2) 1 * 1 ← (4) * 1 |

| (11, 3) |
|---------|
| (3) 5 3 ← (5) 7 |
| (5) 3 3 ← (9) 3 |

| (12, 8) |
|---------|
| (1) 1 1 * 1 ← (2) 2 * 1 |

| (11, 4) |
|---------|
| (2) 3 3 3 ← (4) 5 3 |
| (3) 2 3 3 ← (6) 3 3 |

| (13, 3) |
|---------|
| (7) 3 3 ← (11) 3 |
| (11) 1 1 ← (13) 1 |

| (11, 5) |
|---------|
| (2) 1 2 3 3 ← (4) 2 3 3 |
| (4) 4 1 1 1 |

| (13, 4) |
|---------|
| (13, 6) |
| (2) * 1 |

| (13, 7) |
|---------|
| (3) 1 * 1 ← (5) * 1 |

| (13, 8) |
|---------|
| (2) 1 1 * 1 ← (4) 1 * 1 |

| (12, 2) |
|---------|
| (11) 1 ← (13) |

| (14, 2) |
|---------|
| (7) 7 |
| (14,3) | (15,5) |
|--------|--------|
| (6) 5 3 | (1) 6 2 3 3 |
| (7) 6 1 ← (8) 7 | (6) 1 2 3 3 ← (8) 2 3 3 |
| (11) 2 1 ← (12) 3 | (8) 4 1 1 |
| (14,4) | (15,6) |
| (5) 3 3 3 ← (9) 3 3 | (1) 5 1 2 3 3 ← (2) 6 2 3 3 |
| (6) 2 3 3 | (6) * 1 |
| (7) 5 1 1 ← (8) 6 1 | (15,7) |
| (11) 1 1 1 ← (12) 2 1 | (1) 3 4 4 1 1 1 ← (2) 5 1 2 3 3 |
| (14,5) | (5) 1 * 1 |
| (5) 1 2 3 3 | (15,8) |
| (7) 4 1 1 1 ← (8) 5 1 1 | (4) 1 1 * 1 |
| (14,6) | (16,2) |
| (3) 4 4 1 1 1 | (1) 15 |
| (14,7) | (13) 3 |
| (3) 2 * 1 ← (4) 4 4 1 1 1 | (15) 1 ← (17) |
| (14,8) | (16,3) |
| (3) 1 1 * 1 ← (4) 2 * 1 | (1) 14 1 ← (2) 15 |
| (15,1) | (9) 6 1 ← (10) 7 |
| (15) | (13) 2 1 ← (14) 3 |
| (15,2) | (14) 1 1 ← (16) 1 |
| (14) 1 | (16,4) |
| (15,3) | (1) 13 1 1 ← (2) 14 1 |
| (1) 7 7 | (2) 6 5 3 |
| (7) 5 3 ← (9) 7 | (7) 3 3 3 ← (9) 5 3 |
| (13) 1 1 | (9) 5 1 1 ← (10) 6 1 |
| (15,4) | (13) 1 1 1 ← (14) 2 1 |
| (1) 6 5 3 ← (2) 7 7 | (16,5) |
| (6) 3 3 3 ← (8) 5 3 | (1) 12 1 1 1 ← (2) 13 1 1 |
| (7) 2 3 3 ← (10) 3 3 | (7) 1 2 3 3 ← (9) 2 3 3 |
| (12) 1 1 1 | (9) 4 1 1 1 ← (10) 5 1 1 |
| (15,4) | (16,6) |
| (1) 6 5 3 ← (2) 7 7 | (1) 2 4 3 3 3 |
| (6) 3 3 3 ← (8) 5 3 | (1) 8 4 1 1 1 ← (2) 12 1 1 1 |
| (7) 2 3 3 ← (10) 3 3 | (5) 4 4 1 1 1 ← (8) 1 2 3 3 |
| (12) 1 1 1 | (7) * 1 ← (10) 4 1 1 1 |
| (16,7) | (18,2) |
|--------|--------|
| (1) 6 * 1 ← (2) 8 4 1 1 1 |
| (2) 3 4 1 1 1 |
| (5) 2 * 1 ← (6) 4 4 1 1 1 |
| (6) 1 * 1 ← (8) * 1 |
| (3) 15 |
| (11) 7 |
| (15) 3 ← (19) |

| (16,8) | (18,3) |
|--------|--------|
| (1) 5 1 * 1 ← (2) 6 * 1 |
| (5) 1 1 * 1 ← (6) 2 * 1 |
| (2) 13 3 |
| (3) 14 1 ← (4) 15 |
| (10) 5 3 |
| (11) 6 1 ← (12) 7 |
| (12) 3 3 ← (18) 1 |
| (15) 2 1 ← (16) 3 |

| (16,9) | (18,4) |
|--------|--------|
| (1) 4 1 1 * 1 ← (2) 5 1 * 1 |
| (17,3) | (18,5) |
| (1) 13 3 |
| (3) 7 7 |
| (11) 3 3 |
| (15) 1 1 ← (17) 1 |
| (18) 1 |
| (2) 13 3 |
| (3) 14 1 ← (4) 15 |
| (10) 5 3 |
| (11) 6 1 ← (12) 7 |
| (12) 3 3 ← (18) 1 |
| (15) 2 1 ← (16) 3 |

| (17,4) | (18,6) |
|--------|--------|
| (3) 6 5 3 ← (4) 7 7 |
| (8) 3 3 3 |
| (14) 1 1 1 ← (16) 1 1 |
| (17,5) | (18,7) |
| (3) 6 2 3 3 |
| (17,6) | (18,8) |
| (2) 2 4 3 3 3 |
| (3) 5 1 2 3 3 ← (4) 6 2 3 3 |
| (17,7) | (18,9) |
| (1) 1 2 4 3 3 3 |
| (3) 3 4 1 1 1 1 ← (4) 5 1 2 3 3 |
| (7) 1 * 1 ← (9) * 1 |
| (17,8) | (18,10) |
| (1) 2 3 4 1 1 1 |
| (6) 1 1 * 1 ← (8) 1 * 1 |
| (17,9) | (18,11) |
| (2) 4 1 1 * 1 |
| (18,12) |
| (18, 8) | (19, 9) |
|---------|---------|
| (1) 1 1 2 4 3 3 3 ← (2) ..4 3 3 3 | (2) 1 2 3 4 4 1 1 1 ← (4) 2 3 4 4 1 1 1 |
| (2) 2 3 4 4 1 1 1 ← (9) 1 * 1 | (4) 4 1 1 1 * 1 |
| (3) 5 1 * 1 ← (4) 6 * 1 | |
| (7) 1 1 * 1 ← (8) 2 * 1 | |

| (18, 9) | (19, 10) |
|---------|---------|
| (1) 1 2 3 4 4 1 1 1 ← (8) 1 1 * 1 | (2) * * 1 |
| (3) 4 1 1 * 1 ← (4) 5 1 * 1 | |
| (19, 11) | |
| (1) 1 * * 1 | |

| (18, 10) | (20, 2) |
|---------|---------|
| (1) * * 1 | (19) 1 ← (21) |

| (19, 3) | (20, 3) |
|---------|---------|
| (1) 11 7 | (5) 14 1 ← (6) 15 |
| (3) 13 3 ← (5) 15 | (13) 6 1 ← (14) 7 |
| (5) 7 7 | (17) 2 1 ← (18) 3 |
| (11) 5 3 ← (13) 7 | (18) 1 1 ← (20) 1 |
| (13) 3 3 ← (17) 3 | |

| (19, 4) | (20, 4) |
|---------|---------|
| (1) 10 5 3 ← (2) 11 7 | (1) 5 7 7 |
| (2) 11 3 3 ← (4) 13 3 | (3) 11 3 3 ← (5) 13 3 |
| (5) 6 5 3 ← (6) 7 7 | (5) 13 1 1 ← (6) 14 1 |
| (10) 3 3 3 ← (12) 5 3 | (6) 6 5 3 |
| (11) 2 3 3 ← (14) 3 3 | (11) 3 3 3 ← (13) 5 3 |

| (19, 5) | (20, 5) |
|---------|---------|
| (1) 5 7 3 3 | (2) 9 3 3 3 ← (4) 11 3 3 |
| (1) 9 3 3 3 ← (2) 10 5 3 | (3) 8 3 3 3 |
| (10) 1 2 3 3 ← (12) 2 3 3 | (5) 12 1 1 1 ← (6) 13 1 1 |
| | (11) 1 2 3 3 ← (13) 2 3 3 |
| | (13) 4 1 1 1 ← (14) 5 1 1 |

| (19, 6) | (20, 6) |
|---------|---------|
| (1) 4 5 3 3 3 ← (2) 5 7 3 3 | (2) 4 5 3 3 3 |
| (5) 5 1 2 3 3 ← (6) 6 2 3 3 | (3) 3 6 2 3 3 ← (4) 8 3 3 3 |

| (19, 7) | (20, 7) |
|---------|---------|
| (3) 1 2 4 3 3 3 ← (5) 2 4 3 3 3 | (2) 4 5 3 3 3 |
| (5) 3 4 4 1 1 1 ← (6) 5 1 2 3 3 | (3) 3 6 2 3 3 ← (4) 8 3 3 3 |

| (19, 8) | |
|---------| |
| (2) 1 1 2 4 3 3 3 ← (4) 1 2 4 3 3 3 | |
| (3) 2 3 4 4 1 1 1 ← (6) 3 4 4 1 1 1 | |
| (9) 2 * 1 ← (10) 4 4 1 1 1 | |
| (10) 1 * 1 ← (12) * 1 | |
| (20, 8) | (21, 7) |
|---|---|
| (3) 1 2 4 3 3 3 ← (4) .4 3 3 3 | (1) 2 4 5 3 3 3 ← (2) 4 7 3 3 3 |
| (5) 5 1 * 1 ← (6) 6 * 1 | (5) 1 2 4 3 3 3 ← (11) 4 4 1 1 1 |
| (9) 1 1 * 1 ← (10) 2 * 1 | (7) 3 4 4 1 1 1 ← (8) 5 1 2 3 3 |
| | (11) 1 * 1 ← (13) * 1 |

| (20, 9) | (21, 9) |
|---|---|
| (3) 1 2 3 4 1 1 1 ← (5) 2 3 4 1 1 1 | (4) 1 1 2 4 3 3 3 ← (8) 3 4 4 1 1 1 |
| (5) 4 1 1 * 1 ← (6) 5 1 * 1 | (10) 1 1 * 1 ← (12) 1 * 1 |

| (20, 10) | (21, 11) |
|---|---|
| (1) 4 4 1 1 * 1 ← (4) 1 2 3 4 1 1 1 | (3) 1 * * 1 ← (5) ** 1 |
| (3) * * 1 ← (6) 4 1 1 * 1 | | |

| (20, 11) | (21, 12) |
|---|---|
| (1) 2 * * 1 ← (2) 4 4 1 1 * 1 | (2) 1 * * 1 ← (4) * * 1 |
| (2) 1 * * 1 ← (4) * * 1 | | |

| (20, 12) | (22, 2) |
|---|---|
| (1) 1 1 * * 1 ← (2) 2 * * 1 | (7) 15 |
| | (15) 7 ← (23) |

| (21, 3) | (22, 3) |
|---|---|
| (3) 11 7 | (6) 13 3 |
| (7) 7 7 | (7) 14 1 ← (8) 15 |
| (15) 3 3 ← (19) 3 | (14) 5 3 ← (22) 1 |
| (19) 1 1 ← (21) 1 | (15) 6 1 ← (16) 7 |
| | (19) 2 1 ← (20) 3 |

| (21, 4) | (22, 4) |
|---|---|
| (2) 5 7 7 | (1) 7 7 7 |
| (3) 10 5 3 ← (4) 11 7 | (3) 5 7 7 |
| (7) 6 5 3 ← (8) 7 7 | (5) 11 3 3 |
| (12) 3 3 3 ← (16) 3 3 | (7) 13 1 1 ← (8) 14 1 |
| (18) 1 1 1 ← (20) 1 1 | (8) 6 5 3 ← (21) 1 1 |
| | (13) 3 3 3 ← (17) 3 3 |
| | (15) 5 1 1 ← (16) 6 1 |
| | (19) 1 1 1 ← (20) 2 1 |

| (21, 5) | (22, 5) |
|---|---|
| (1) 6 6 5 3 | (4) 9 3 3 3 |
| (3) 5 7 3 3 | (5) 8 3 3 3 ← (20) 1 1 1 |
| (3) 9 3 3 3 ← (4) 10 5 3 | (7) 12 1 1 1 ← (8) 13 1 1 |
| (7) 6 2 3 3 ← (14) 2 3 3 | (15) 4 1 1 1 ← (16) 5 1 1 |
| | | |

| (21, 6) | (22, 6) |
|---|---|
| (1) 4 7 3 3 3 ← (2) 6 6 5 3 | (4) 9 3 3 3 |
| (3) 4 5 3 3 3 ← (4) 5 7 3 3 | (5) 8 3 3 3 ← (20) 1 1 1 |
| (6) 2 4 3 3 3 ← (13) 1 2 3 3 | (7) 12 1 1 1 ← (8) 13 1 1 |
| (7) 5 1 2 3 3 ← (8) 6 2 3 3 | (15) 4 1 1 1 ← (16) 5 1 1 |
| (22,6) | (23,5) |
|--------|--------|
| (1) 3 5 7 3 3 | (1) 3 5 7 7 |
| (4) 4 5 3 3 3 ← (16) 4 1 1 1 | (3) 6 6 5 3 |
| (5) 3 6 2 3 3 ← (6) 8 3 3 3 | (5) 5 7 3 3 ← (10) 6 5 3 |
| (7) 2 4 3 3 3 ← (9) 6 2 3 3 | (5) 9 3 3 3 ← (6) 10 5 3 |
| (7) 8 4 1 1 1 ← (8) 12 1 1 1 | (14) 1 2 3 3 ← (16) 2 3 3 |

| (22,7) | (23,6) |
|--------|--------|
| (2) 2 4 5 3 3 3 ← (14) * 1 | (2) 3 5 7 3 3 |
| (5) ..4 3 3 3 ← (6) 3 6 2 3 3 | (3) 4 7 3 3 3 ← (4) 6 6 5 3 |
| (6) 1 2 4 3 3 3 ← (8) 2 4 3 3 3 | (5) 4 5 3 3 3 ← (6) 5 7 3 3 |
| (7) 6 * 1 ← (8) 8 4 1 1 1 | (9) 5 1 2 3 3 ← (10) 6 2 3 3 |
| (11) 2 * 1 ← (12) 4 4 1 1 1 | (13) 1 * 1 |

| (22,8) | (23,7) |
|--------|--------|
| (5) 1 1 2 4 3 3 3 ← (6) ..4 3 3 3 | (3) 2 4 5 3 3 3 ← (4) 4 7 3 3 3 |
| (6) 2 3 4 4 1 1 1 | (7) 1 2 4 3 3 3 ← (9) 2 4 3 3 3 |
| (7) 5 1 * 1 ← (8) 6 * 1 | (9) 3 4 4 1 1 1 ← (10) 5 1 2 3 3 |
| (11) 1 1 * 1 ← (12) 2 * 1 | (13) 1 * 1 |

| (22,9) | (23,8) |
|--------|--------|
| (5) 1 2 3 4 4 1 1 1 | (6) 1 1 2 4 3 3 3 ← (8) 1 2 4 3 3 3 |
| (7) 4 1 1 * 1 ← (8) 5 1 * 1 | (7) 2 3 4 4 1 1 1 ← (10) 3 4 4 1 1 1 |
| (12) 1 1 * 1 | (12) 1 1 * 1 |

| (22,10) | (23,9) |
|--------|--------|
| (3) 4 4 1 1 * 1 | (1) 6 2 3 4 4 1 1 1 |
| | (6) 1 2 3 4 4 1 1 1 ← (8) 2 3 4 4 1 1 1 |
| | (8) 4 1 1 * 1 |

| (22,11) | (23,10) |
|--------|--------|
| (3) 2 * * 1 ← (4) 4 4 1 1 * 1 | (1) 5 1 2 3 4 4 1 1 1 ← (2) 6 2 3 4 4 1 1 1 |
| | (6) * * 1 |

| (22,12) | (23,11) |
|--------|--------|
| (3) 1 1 * * 1 ← (4) 2 * * 1 | (1) 3 4 4 1 1 * 1 ← (2) 5 1 2 3 4 4 1 1 1 |
| | (5) 1 * * 1 |

| (23,3) | (23,12) |
|--------|--------|
| (5) 1 1 7 | (4) 1 1 * * 1 |
| (7) 13 3 ← (9) 15 | |
| (9) 7 7 ← (21) 3 | |
| (15) 5 3 ← (17) 7 | |

| (23,4) | (24,2) |
|--------|--------|
| (2) 7 7 7 | (23) 1 ← (25) |
| (4) 5 7 7 | |
| (5) 10 5 3 ← (6) 11 7 | |
| (6) 11 3 3 ← (8) 13 3 | |
| (9) 6 5 3 ← (10) 7 7 | |
| (14) 3 3 3 ← (16) 5 3 | |
| (15) 2 3 3 ← (18) 3 3 | |

| (24,3) | |
|--------|--------|
| (9) 14 1 ← (10) 15 | |
| (17) 6 1 ← (18) 7 | |
| (21) 2 1 ← (22) 3 | |
| (22) 1 1 ← (24) 1 | |
| (24, 4) | (24, 11) |
|---------|---------|
| (3) 7 7 7 | (1) 6 * 1 ← (2) 8 4 1 1 * 1 |
| (5) 5 7 7 ← (19) 3 3 3 | (2) 3 4 4 1 1 * 1 |
| (7) 11 3 3 ← (9) 13 3 | (5) 2 * 1 ← (6) 4 4 1 1 * 1 |
| (9) 1 1 1 ← (10) 14 1 | (6) 1 * 1 ← (8) * 1 |
| (15) 3 3 3 ← (17) 5 3 | |
| (17) 5 1 1 ← (18) 6 1 | |
| (21) 1 1 1 ← (22) 2 1 | |

| (24, 5) | (24, 12) |
|---------|---------|
| (1) 4 5 7 7 | (1) 5 1 * 1 ← (2) 6 * 1 |
| (2) 3 5 7 7 | (5) 1 1 * 1 ← (6) 2 * 1 |
| (6) 9 3 3 3 ← (8) 11 3 3 | |
| (7) 8 3 3 3 ← (16) 3 3 3 | |
| (9) 12 1 1 1 ← (10) 13 1 1 | |
| (15) 1 2 3 3 ← (17) 2 3 3 | |
| (17) 4 1 1 1 ← (18) 5 1 1 | |

| (24, 6) | (24, 13) |
|---------|---------|
| (1) 3 6 6 5 3 | (1) 4 1 1 * 1 ← (2) 5 1 * 1 |
| (3) 3 5 7 3 3 ← (5) 6 6 5 3 | |
| (6) 4 5 3 3 3 ← (11) 6 2 3 3 | |
| (7) 3 6 2 3 3 ← (8) 3 6 2 3 3 | |
| (9) 8 4 1 1 1 ← (10) 12 1 1 1 | |
| (13) 4 4 1 1 1 ← (16) 1 2 3 3 | |
| (15) * 1 ← (18) 4 1 1 1 | |

| (24, 7) | (25, 3) |
|---------|---------|
| (1) 2 3 5 7 3 3 ← (2) 3 6 6 5 3 | (7) 11 7 ← (11) 15 |
| (4) 2 4 5 3 3 3 ← (10) 2 4 3 3 3 | (11) 7 7 ← (19) 7 |
| (7) 4 3 3 3 ← (8) 3 6 2 3 3 | (25) 1 | |
| (9) 6 * 1 ← (10) 8 4 1 1 1 | |
| (13) 2 * 1 ← (14) 4 4 1 1 1 | |
| (14) 1 * 1 ← (16) * 1 | |

| (24, 8) | (25, 4) |
|---------|---------|
| (1) 13 1 * 1 ← (9) 1 2 4 3 3 3 | (4) 7 7 7 ← (10) 13 3 |
| (7) 1 1 2 4 3 3 3 ← (11) 8 3 3 3 | (6) 5 7 7 ← (18) 5 3 |
| (9) 5 1 * 1 ← (10) 6 * 1 | (7) 10 5 3 ← (8) 11 7 |
| (13) 1 1 * 1 ← (14) 2 1 1 1 | (11) 6 5 3 ← (12) 7 7 |
| |

| (24, 9) | (25, 5) |
|---------|---------|
| (1) 1 2 1 * 1 ← (2) 13 1 * 1 | (2) 4 5 7 7 ← (9) 11 3 3 |
| (7) 1 2 3 4 4 1 1 1 ← (9) 2 3 4 4 1 1 1 | (3) 5 5 7 7 ← (17) 3 3 3 |
| (9) 4 1 1 * 1 ← (10) 5 1 * 1 | (7) 5 7 3 3 ← (12) 6 5 3 |
| (11) * 1 ← (12) 4 1 1 1 | (7) 9 3 3 3 ← (8) 10 5 3 |

| (24, 10) | (25, 6) |
|---------|---------|
| (1) * 2 4 3 3 3 | (1) 2 3 5 7 7 ← (8) 9 3 3 3 |
| (1) 8 4 1 1 * 1 ← (2) 12 1 1 * 1 | (4) 3 5 7 3 3 ← (9) 8 3 3 3 |
| (5) 4 4 1 1 * 1 ← (8) 1 2 3 4 4 1 1 1 | (5) 4 7 3 3 3 ← (6) 6 6 5 3 |
| (7) * 1 ← (10) 4 1 1 * 1 | (7) 4 5 3 3 3 ← (8) 5 7 3 3 |
| |

| (25, 7) | (25, 8) |
|---------|---------|
| (2) 3 5 7 3 3 ← (8) 4 5 3 3 3 | (8) 1 1 2 4 3 3 3 |
| (5) 2 4 5 3 3 3 ← (6) 4 7 3 3 3 | (14) 1 1 * 1 ← (16) 1 * 1 |
| (11) 3 4 4 1 1 1 ← (12) 5 1 2 3 3 | |
| (15) 1 * 1 ← (17) * 1 | |

| (25, 9) | |
|---------|---------|
| (3) 6 2 3 4 4 1 1 1 | |
THE ALGEBRAIC ATIYAH-HIRZEBRUCH SPECTRAL SEQUENCE OF REAL PROJECTIVE SPECTRA 21

(27, 4)

(9) 10 5 3 ← (10) 11 7
(10) 11 3 3 ← (12) 13 3
(13) 6 5 3 ← (14) 7 7
(18) 3 3 3 ← (20) 5 3
(19) 2 3 3 ← (22) 3 3

(27, 5)

(5) 5 7 7 ← (9) 5 7 7
(7) 6 6 3 ← (14) 6 6 3
(9) 9 3 3 3 ← (10) 10 5 3
(18) 1 2 3 3 ← (20) 2 3 3

(27, 6)

(3) 2 3 5 7 7 ← (5) 4 5 7 7
(6) 3 5 7 3 3 ← (11) 8 3 3 3
(7) 4 7 3 3 3 ← (8) 6 6 5 3
(9) 4 5 3 3 3 ← (10) 5 7 3 3
(13) 5 1 2 3 3 ← (14) 6 2 3 3

(27, 7)

(1) 1 3 6 6 5 3 ← (4) 2 3 5 7 7
(4) 2 3 5 7 3 3 ← (10) 4 5 3 3 3
(7) 2 4 5 3 3 3 ← (8) 4 7 3 3 3
(11) 1 2 4 3 3 3 ← (13) 2 4 3 3 3
(13) 3 4 4 1 1 1 ← (14) 5 1 2 3 3

(27, 8)

(1) 6 2 4 5 3 3 3 ← (8) 2 4 5 3 3 3
(10) 1 1 2 4 3 3 3 ← (12) 2 4 3 3 3
(11) 2 3 4 4 1 1 1 ← (14) 3 4 4 1 1 1

(27, 9)

(1) 4 4 5 3 3 3 ← (2) 6 2 4 5 3 3 3
(10) 1 2 3 4 1 1 1 ← (12) 2 3 4 4 1 1 1

(27, 10)

(1) 4 ..4 5 3 3 3 ← (2) 4 ..4 5 3 3 3
(5) 5 1 2 3 4 4 1 1 1 ← (6) 6 2 3 4 4 1 1 1

(27, 11)

(3) 1 * 2 4 3 3 3 ← (5) * 2 4 3 3 3
(5) 3 4 4 1 1 1 * 1 ← (6) 5 1 2 3 4 4 1 1 1

(27, 12)

(2) 1 1 * 2 4 3 3 3 ← (4) 1 * 2 4 3 3 3
(3) 2 3 4 4 1 1 1 * 1 ← (6) 3 4 4 1 1 1 * 1

(27, 13)

(2) 1 2 3 4 4 1 1 1 * 1 ← (4) 2 3 4 4 1 1 1 * 1
(4) 4 1 1 1 * 1

(27, 14)

(2) * * * 1

(27, 15)

(1) 1 * * * 1

(28, 2)

(27) 1 ← (29)

(28, 3)

(13) 1 4 1 ← (14) 1 5
(21) 6 1 ← (22) 7
(25) 2 1 ← (26) 3
(26) 1 1 ← (28) 1

(28, 4)

(7) 7 7 7 ← (11) 1 1 7
(11) 11 3 3 ← (13) 1 3 3
(13) 1 3 1 1 ← (14) 1 4 1
(19) 3 3 3 ← (21) 5 3
(21) 5 1 1 ← (22) 6 1
(25) 1 1 1 ← (26) 2 1

(28, 5)

(6) 3 5 7 7 ← (10) 5 7 7
(10) 9 3 3 3 ← (12) 1 1 3 3
(13) 1 2 1 1 1 ← (14) 1 3 1 1
(19) 1 2 3 3 ← (21) 2 3 3
(21) 4 1 1 1 ← (22) 5 1 1

(28, 6)

(5) 3 6 6 5 3 ← (11) 5 7 3 3
(7) 3 5 7 3 3 ← (9) 6 6 5 3
(11) 3 6 2 3 3 ← (12) 8 3 3 3
(13) 8 4 1 1 1 ← (14) 1 2 1 1 1
(17) 4 1 1 1 ← (20) 1 2 3 3
(19) * 1 ← (22) 4 1 1 1

(28, 7)

(2) 3 3 6 6 5 3 ← (8) 3 5 7 3 3
(5) 2 3 5 7 3 3 ← (6) 3 6 6 5 3
(11) ..4 3 3 3 ← (12) 3 6 2 3 3
(13) 6 * 1 ← (14) 8 4 1 1 1
(17) 2 * 1 ← (18) 4 4 1 1 1
(18) 1 * 1 ← (20) * 1

(28, 8)

(5) 1 3 1 * 1
(11) 1 1 2 4 3 3 3 ← (12) ..4 3 3 3
(13) 5 1 * 1 ← (14) 6 * 1
(17) 1 1 * 1 ← (18) 2 * 1
(28, 9)
(3) 8 1 2 4 3 3 3
(5) 1 2 1 * 1 ← (6) 1 3 1 * 1
(11) 1 2 3 4 1 1 1 ← (13) 2 3 4 1 1 1
(13) 4 1 1 * 1 ← (14) 5 1 1 * 1

(28, 10)
(2) 4 5 3 3 3
(3) 3 6 2 3 4 1 1 1 ← (4) 8 1 1 2 4 3 3 3
(5) 8 4 1 1 * 1 ← (6) 1 2 1 * 1
(9) 4 4 1 1 * 1 ← (12) 1 2 3 4 1 1 1
(11) 4 1 1 * 1 ← (14) 4 1 1 * 1

(28, 11)
(3) 2 * 2 4 3 3 3 ← (4) 3 6 2 3 4 1 1 1
(5) 6 * 1 * 1 ← (6) 8 4 1 1 * 1
(9) 2 * 1 * 1 ← (10) 4 4 1 1 * 1
(10) 1 * 1 * 1 ← (12) ** 1

(28, 12)
(3) 1 1 * 2 4 3 3 3 ← (4) 2 * 2 4 3 3 3
(5) 5 1 * 1 * 1 ← (6) 6 * 1 * 1
(9) 1 1 * 1 ← (10) 2 * 1

(28, 13)
(3) 1 2 3 4 4 1 1 * 1 ← (5) 2 3 4 4 1 1 * 1
(5) 4 1 2 * 1 * 1 ← (6) 5 1 * 1

(28, 14)
(1) 4 4 1 1 * * 1 ← (4) 1 2 3 4 4 1 1 * 1
(3) * * 1 * 1 ← (6) 4 1 1 * * 1

(28, 15)
(1) 4 1 1 * * 1 ← (2) 4 4 1 1 * * 1
(2) 1 1 * * 1 ← (4) * * * 1

(28, 16)
(1) 1 1 * * 1 ← (2) 2 * * * 1

(29, 3)
(15) 7 7 ← (23) 7
(23) 3 3 ← (27) 3
(27) 1 1 ← (29) 1

(29, 4)
(8) 7 7 7 ← (22) 5 3
(11) 1 0 5 3 ← (12) 11 7
(15) 6 5 3 ← (16) 7 7
(20) 3 3 3 ← (24) 3 3
(26) 1 1 1 ← (28) 1 1

(29, 5)
(6) 4 5 7 7 ← (16) 6 5 3
(7) 3 5 7 7 ← (11) 5 7 7
(11) 9 3 3 3 ← (12) 10 5 3
(15) 6 2 3 3 ← (22) 2 3 3

(29, 6)
(5) 2 3 5 7 7 ← (13) 8 3 3 3
(9) 4 7 3 3 3 ← (10) 6 6 5 3
(11) 4 5 3 3 3 ← (12) 5 7 3 3
(14) 2 4 3 3 3 ← (21) 1 2 3 3
(15) 5 1 2 3 3 ← (16) 6 2 3 3

(29, 7)
(3) 3 3 6 6 5 3 ← (9) 3 5 7 3 3
(6) 2 3 5 7 3 3
(9) 2 4 5 3 3 3 ← (10) 4 7 3 3 3
(13) 1 2 4 3 3 3 ← (19) 4 4 1 1 1
(15) 3 4 4 1 1 1 ← (16) 5 1 2 3 3
(19) 1 * 1 ← (21) * 1

(29, 8)
(3) 6 2 4 5 3 3 3
(12) 1 2 4 3 3 3 ← (16) 3 4 4 1 1 1
(18) 1 * 1 ← (20) * 1

(29, 9)
(1) 6 ..4 5 3 3 3
(3) 4 ..4 5 3 3 3 ← (4) 6 2 4 5 3 3 3
(7) 6 2 3 4 4 1 1 1 ← (14) 2 3 4 4 1 1 1

(29, 10)
(1) 4 ...4 5 3 3 3 ← (2) 6 ..4 5 3 3 3
(3) ....4 5 3 3 3 ← (4) ..4 5 3 3 3
(6) * 2 4 3 3 3 ← (13) 1 2 3 4 1 1 1
(7) 5 1 2 3 4 4 1 1 1 ← (8) 6 2 3 4 4 1 1 1

(29, 11)
(1) ......4 5 3 3 3 ← (2) 4 ...4 5 3 3 3
(5) 1 * 2 4 3 3 3 ← (11) 4 4 1 1 * 1
(7) 3 4 4 1 1 * 1 ← (8) 5 1 2 3 4 4 1 1 1
(11) 1 * * 1 ← (13) * * 1

(29, 12)
(4) 1 1 * 2 4 3 3 3 ← (8) 3 4 4 1 1 * 1
(10) 1 1 * * 1 ← (12) 1 * * 1

(29, 15)
(3) 1 * * * 1 ← (5) * * * 1
| (31, 4) | (31, 10) |
| --- | --- |
| (1) 14 13 3 ← (2) 15 15 | (1) 5 8 1 1 2 4 3 3 3 ← (2) 7 13 1 * 1 |
| (12) 5 7 7 | (2) 3 5 7 3 |
| (13) 10 5 3 ← (14) 11 7 | (3) 4 4 5 3 3 3 ← (4) 6 4 5 3 3 3 |
| (14) 11 3 3 ← (16) 13 3 | (5) 4 5 3 3 3 ← (6) 4 4 5 3 3 3 |
| (17) 6 5 3 ← (18) 7 7 | (9) 5 1 2 3 4 4 1 1 | *(10) 6 2 3 4 4 1 1 1 |
| (22) 3 3 3 ← (24) 5 3 | (14) * * 1 |
| (23) 2 3 3 ← (26) 3 3 | |
| (28) 1 1 1 | |

| (31, 5) | (31, 11) |
| --- | --- |
| (1) 13 11 3 3 ← (2) 14 13 3 | (1) 4 4 5 3 3 3 ← (2) 5 8 1 1 2 4 3 3 3 |
| (9) 3 5 7 7 | (3) 4 4 5 3 3 3 ← (4) 6 4 5 3 3 3 |
| (13) 5 7 3 3 ← (18) 6 5 3 | (7) 1 * 2 4 3 3 3 ← (9) * 2 4 3 3 3 |
| (13) 9 3 3 3 ← (14) 10 5 3 | (9) 3 4 4 1 1 * 1 ← (10) 5 1 2 3 4 4 1 1 1 |
| (22) 1 2 3 3 ← (24) 2 3 3 | (13) 1 * * 1 |
| (24) 4 1 1 1 | |

| (31, 6) | (31, 12) |
| --- | --- |
| (1) 12 9 3 3 3 ← (2) 13 11 3 3 | (1) 6 2 3 4 4 1 1 * 1 |
| (7) 2 3 5 7 7 ← (9) 4 5 7 7 | (6) 1 1 * 2 4 3 3 3 ← (8) 1 * 2 4 3 3 3 |
| (11) 4 7 3 3 3 ← (12) 6 6 5 3 | (7) 2 3 4 4 1 1 * 1 ← (10) 3 4 4 1 1 * 1 |
| (13) 4 5 3 3 3 ← (14) 5 7 3 3 | (12) 1 1 * * 1 |
| (17) 5 1 2 3 3 ← (18) 6 2 3 3 | |
| (22) * 1 | |

| (31, 7) | (31, 13) |
| --- | --- |
| (1) 12 4 5 3 3 3 | (1) 6 2 3 4 4 1 1 * 1 |
| (5) 3 3 6 6 5 3 ← (8) 2 3 5 7 7 | (6) 1 2 3 4 4 1 1 * 1 ← (8) 2 3 4 4 1 1 * 1 |
| (11) 2 4 5 3 3 3 ← (12) 4 7 3 3 3 | (8) 4 1 1 * * 1 |
| (15) 3 4 1 1 1 1 ← (17) 2 4 3 3 3 | |
| (17) 3 4 4 1 1 1 ← (18) 5 1 2 3 3 | |
| (21) 1 * 1 | |

| (31, 8) | (31, 14) |
| --- | --- |
| (1) 10 2 4 5 3 3 3 ← (2) 12 4 5 3 3 3 | (1) 5 1 2 3 4 4 1 1 * 1 ← (2) 6 2 3 4 4 1 1 * 1 |
| (5) 6 2 4 5 3 3 3 | (6) * * * 1 |
| (14) 1 1 2 4 3 3 3 ← (16) 1 2 4 3 3 3 | |
| (15) 2 3 4 4 1 1 1 ← (18) 3 4 4 1 1 1 | |
| (20) 1 1 * 1 | |

| (31, 9) | (31, 15) |
| --- | --- |
| (1) 7 13 1 * 1 ← (2) 10 2 4 5 3 3 3 | (1) 3 4 4 1 1 * 1 ← (2) 5 1 2 3 4 4 1 1 * 1 |
| (3) 6 4 5 3 3 3 | (5) 1 * * * 1 |
| (5) 4 4 5 3 3 3 ← (6) 6 2 4 5 3 3 3 | |
| (14) 1 2 3 4 4 1 1 1 ← (16) 2 3 4 4 1 1 1 | |
| (16) 4 1 1 * 1 | |

| (31, 16) | (32, 2) |
| --- | --- |
| (4) 1 1 * * * 1 | (1) 31 |
| (29) 3 | (29) 3 ← (33) |
| (31) 1 ← (33) | |

| (32, 3) | |
| --- | --- |
| (1) 30 1 ← (2) 31 | (1) 30 1 ← (2) 31 |
| (17) 14 1 ← (18) 15 | (17) 14 1 ← (18) 15 |
| (25) 6 1 ← (26) 7 | (25) 6 1 ← (26) 7 |
| (29) 2 1 ← (30) 3 | (29) 2 1 ← (30) 3 |
| (30) 1 1 ← (32) 1 | (30) 1 1 ← (32) 1 |
| 32, 4 | 32, 10 |
|-------|--------|
| (1) 13 1 1 7 | (1) 3 6 .4 5 3 3 3 \(\Rightarrow\) (2) 5 6 2 4 5 3 3 3 |
| (1) 29 1 1 \(\Rightarrow\) (2) 30 1 | (1) 16 4 1 1 * 1 \(\Rightarrow\) (2) 20 1 1 * 1 |
| (11) 7 7 \(\Rightarrow\) (19) 7 7 | (3) ..... 3 5 7 3 3 \(\Rightarrow\) (4) 3 6 2 4 5 3 3 3 |
| (13) 5 7 7 | (6) ..... 4 5 3 3 3 \(\Rightarrow\) (11) 6 2 3 4 4 1 1 1 |
| (15) 11 3 3 \(\Rightarrow\) (17) 13 3 | (7) 3 6 2 3 4 4 1 1 1 \(\Rightarrow\) (8) 8 1 1 2 4 3 3 3 |
| (17) 13 1 1 \(\Rightarrow\) (18) 14 1 | (9) 8 4 1 1 * 1 \(\Rightarrow\) (10) 12 1 1 * 1 |
| (23) 3 3 3 \(\Rightarrow\) (25) 5 3 | (13) 4 4 1 1 * 1 \(\Rightarrow\) (16) 1 2 3 4 4 1 1 1 |
| (25) 5 1 1 \(\Rightarrow\) (26) 6 1 | (15) * * 1 \(\Rightarrow\) (18) 4 1 1 * 1 |
| (29) 1 1 1 \(\Rightarrow\) (30) 2 1 |

| 32, 5 | 32, 11 |
|-------|--------|
| (1) 28 1 1 1 \(\Rightarrow\) (2) 29 1 1 | (1) ..... 3 5 7 3 3 \(\Rightarrow\) (2) 3 6 .4 5 3 3 3 |
| (10) 3 5 7 7 | (1) 14 * * 1 \(\Rightarrow\) (2) 16 4 1 1 * 1 |
| (14) 9 3 3 3 \(\Rightarrow\) (16) 11 3 3 | (4) ..... 4 5 3 3 3 \(\Rightarrow\) (10) * 2 4 3 3 3 |
| (15) 8 3 3 3 \(\Rightarrow\) (24) 3 3 3 | (7) 2 * 2 4 3 3 3 \(\Rightarrow\) (8) 3 6 2 4 4 1 1 1 |
| (17) 12 1 1 1 \(\Rightarrow\) (16) 13 1 1 | (9) 6 * * 1 \(\Rightarrow\) (10) 8 4 1 1 * 1 |
| (23) 1 2 3 3 \(\Rightarrow\) (25) 2 3 3 | (13) 2 * * 1 \(\Rightarrow\) (14) 4 4 1 1 * 1 |
| (25) 4 1 1 1 \(\Rightarrow\) (26) 5 1 1 | (14) 1 * * 1 \(\Rightarrow\) (16) * * * 1 |

| 32, 6 | 32, 12 |
|-------|--------|
| (1) 9 3 5 7 7 | (1) 13 1 * * 1 \(\Rightarrow\) (2) 14 * * 1 |
| (1) 24 4 1 1 1 \(\Rightarrow\) (2) 28 1 1 1 | (2) ..... 4 5 3 3 3 \(\Rightarrow\) (9) 1 * * 2 4 3 3 3 |
| (2) 12 9 3 3 3 | (7) 1 * * 2 4 3 3 3 \(\Rightarrow\) (8) 2 * 2 4 3 3 3 |
| (9) 3 6 6 5 3 | (9) 5 1 * * 1 \(\Rightarrow\) (10) 6 * * 1 |
| (11) 3 5 7 3 3 \(\Rightarrow\) (13) 6 6 5 3 | (13) 11 * * 1 \(\Rightarrow\) (14) 2 * * 1 |
| (14) 4 5 3 3 3 \(\Rightarrow\) (19) 6 2 3 3 | |
| (15) 3 6 2 3 3 \(\Rightarrow\) (16) 8 3 3 3 | |
| (17) 8 4 1 1 1 \(\Rightarrow\) (18) 12 1 1 1 | |
| (21) 4 4 1 1 1 \(\Rightarrow\) (24) 1 2 3 3 | |
| (23) 1 * \(\Rightarrow\) (26) 4 1 1 1 | |

| 32, 7 | 32, 13 |
|-------|--------|
| (1) 22 * 1 \(\Rightarrow\) (2) 24 4 1 1 1 | (1) 12 1 * * 1 \(\Rightarrow\) (2) 13 1 * * 1 |
| (6) 3 3 6 6 5 3 | (7) 2 3 4 4 1 1 * 1 \(\Rightarrow\) (9) 2 3 4 4 1 1 1 * 1 |
| (9) 2 3 5 7 3 3 \(\Rightarrow\) (10) 3 6 6 5 3 | (9) 4 1 1 * * 1 \(\Rightarrow\) (10) 5 1 * * 1 |
| (12) 2 4 5 3 3 3 \(\Rightarrow\) (18) 2 4 3 3 3 | |
| (15) 4 4 3 3 3 \(\Rightarrow\) (16) 3 6 2 3 3 | |
| (17) 6 * 1 \(\Rightarrow\) (18) 8 4 1 1 1 | |
| (21) 2 * 1 \(\Rightarrow\) (22) 4 4 1 1 1 | |
| (22) 1 * 1 \(\Rightarrow\) (24) * 1 | |

| 32, 8 | 32, 14 |
|-------|--------|
| (1) 21 1 * 1 \(\Rightarrow\) (2) 22 * 1 | (1) * * 2 4 3 3 3 |
| (3) 6 2 3 5 7 3 3 | (1) 8 4 1 1 * * 1 \(\Rightarrow\) (2) 12 1 1 * * 1 |
| (9) 13 1 * 1 \(\Rightarrow\) (17) 1 2 4 3 3 3 | (5) 4 4 1 1 * * 1 \(\Rightarrow\) (8) 1 2 3 4 4 1 1 1 * 1 |
| (15) 1 1 2 4 3 3 3 \(\Rightarrow\) (16) .4 3 3 3 | (7) * * * 1 \(\Rightarrow\) (10) 4 1 1 * * 1 |
| (17) 5 1 * 1 \(\Rightarrow\) (18) 6 * 1 | |
| (21) 1 1 * 1 \(\Rightarrow\) (22) 2 * 1 | |

| 32, 9 | 32, 15 |
|-------|--------|
| (1) 5 6 2 4 5 3 3 3 | (1) 6 * * * 1 \(\Rightarrow\) (2) 8 4 1 1 * * 1 |
| (1) 20 1 1 * 1 \(\Rightarrow\) (2) 21 1 * 1 | (2) 3 4 4 1 1 * * 1 |
| (3) 6 2 4 5 3 3 3 \(\Rightarrow\) (4) 6 2 3 5 7 3 3 | (5) 2 * * * 1 \(\Rightarrow\) (6) 4 4 1 1 * * 1 |
| (7) 8 1 1 2 4 3 3 3 \(\Rightarrow\) (16) 1 1 2 4 3 3 3 | (6) 1 * * * 1 \(\Rightarrow\) (8) * * * 1 |
| (9) 12 1 1 * 1 \(\Rightarrow\) (10) 13 1 * 1 | |
| (15) 1 2 3 4 4 1 1 1 \(\Rightarrow\) (17) 2 3 4 4 1 1 1 | |
| (17) 4 1 1 * 1 \(\Rightarrow\) (18) 5 1 * 1 | |

| 32, 16 | 32, 17 |
|-------|--------|
| (1) 5 1 * * * 1 \(\Rightarrow\) (2) 6 * * * 1 | (1) 4 1 1 * * * 1 \(\Rightarrow\) (2) 5 1 * * * 1 |
| (5) 1 * * * 1 \(\Rightarrow\) (6) 2 * * * 1 | |

| 33, 3 | |
|-------|--------|
| (1) 29 3 | (1) 29 3 |
| (3) 15 15 | (3) 15 15 |
| (15) 11 7 \(\Rightarrow\) (19) 15 | (15) 11 7 \(\Rightarrow\) (19) 15 |
| (27) 3 3 | (27) 3 3 |
| (31) 1 1 \(\Rightarrow\) (33) 1 | (31) 1 1 \(\Rightarrow\) (33) 1 |
(35.5)

| (1) 25 3 3 3 | ← (2) 26 5 3 |
| (3) 13 5 7 7 |
| (5) 13 11 3 3 | ← (6) 14 13 3 |
| (13) 3 5 7 7 | ← (17) 5 7 7 |
| (15) 6 5 5 3 | ← (22) 6 5 3 |
| (17) 9 3 3 3 | ← (18) 10 5 3 |
| (26) 1 2 3 3 | ← (28) 2 3 3 |

(35.6)

| (2) 11 3 5 7 7 | ← (4) 13 5 7 7 |
| (4) 9 3 5 7 7 |
| (5) 12 9 3 3 3 | ← (6) 13 11 3 3 |
| (11) 2 3 5 7 7 | ← (13) 4 5 7 7 |
| (14) 3 5 7 3 3 | ← (19) 8 3 3 3 |
| (15) 4 7 3 3 3 | ← (16) 6 6 5 3 |
| (17) 4 5 3 3 3 | ← (18) 5 7 3 3 |
| (21) 5 1 2 3 3 | ← (22) 6 2 3 3 |

(35.7)

| (1) 5 7 3 5 7 7 |
| (3) 9 3 6 6 5 3 | ← (6) 12 9 3 3 3 |
| (5) 12 4 5 3 3 3 |
| (9) 3 3 6 6 5 3 | ← (12) 2 3 5 7 7 |
| (12) 2 3 5 7 3 3 | ← (18) 4 5 3 3 3 |
| (15) 2 4 5 3 3 3 | ← (16) 4 7 3 3 3 |
| (19) 1 2 4 3 3 3 | ← (21) 2 4 3 3 3 |
| (21) 3 4 4 1 1 1 | ← (22) 5 1 2 3 3 |

(35.8)

| (2) 5 5 3 6 6 5 3 | ← (4) 9 3 6 6 5 3 |
| (3) 6 3 3 6 6 5 3 |
| (5) 10 2 4 5 3 3 3 | ← (6) 12 4 5 3 3 3 |
| (9) 6 2 4 5 3 3 3 | ← (16) 2 4 5 3 3 3 |
| (18) 1 1 2 4 3 3 3 |
| (19) 2 3 4 1 1 1 | ← (22) 3 4 4 1 1 1 |

(35.9)

| (1) 5 6 2 3 5 7 3 3 |
| (3) 3 6 2 3 5 7 3 3 | ← (4) 6 3 3 6 6 5 3 |
| (5) 7 1 3 1 * 1 | ← (6) 10 2 4 5 3 3 3 |
| (7) 6 3 3 3 3 | ← (13) 13 1 * 1 |
| (9) 4 4 5 3 3 3 | ← (10) 6 2 4 5 3 3 3 |
| (18) 1 2 3 4 4 1 1 1 | ← (20) 2 3 4 4 1 1 1 |

(35.10)

| (1) 3 5 6 2 4 5 3 3 3 | ← (2) 5 6 2 3 5 7 3 3 |
| (3) 3 6 2 3 5 7 3 3 | ← (4) 3 6 2 3 5 7 3 3 |
| (5) 5 8 1 1 4 2 3 3 3 | ← (6) 7 1 3 1 * 1 |
| (6) ...3 5 7 3 3 | ← (11) 8 1 2 4 3 3 3 |
| (7) 4 ...4 5 3 3 3 | ← (8) 6 ...4 5 3 3 3 |
| (9) ...4 5 3 3 3 | ← (10) ...4 5 3 3 3 |
| (13) 5 1 2 3 4 4 1 1 1 | ← (14) 6 2 3 4 4 1 1 1 |

(35.11)

| (1) ...3 3 6 6 5 3 | ← (2) 3 5 6 2 4 5 3 3 3 |
| (4) ...3 5 7 3 3 | ← (10) ...4 5 3 3 3 |
| (5) 4 ...4 5 3 3 3 | ← (6) 5 8 1 1 2 4 3 3 3 |
| (7) ...4 5 3 3 3 | ← (8) 4 ...4 5 3 3 3 |
| (11) 1 * 2 4 3 3 3 | ← (13) * 2 4 3 3 3 |
| (13) 3 4 4 1 1 * 1 | ← (14) 5 1 2 3 4 4 1 1 1 |

(35.12)

| (1) 6 ......4 5 3 3 3 | ← (8) ......4 5 3 3 3 |
| (5) ......4 5 3 3 3 | ← (6) ......4 5 3 3 3 |
| (10) 1 1 * 2 4 3 3 3 | ← (12) 1 * 2 4 3 3 3 |
| (11) 2 3 4 4 1 1 * 1 | ← (14) 3 4 4 1 1 * 1 |

(35.13)

| (1) 4 ......4 5 3 3 3 | ← (2) 6 ......4 5 3 3 3 |
| (10) 1 2 3 4 1 1 * 1 | ← (12) 2 3 4 1 1 * 1 |

(35.14)

| (1) ......4 5 3 3 3 | ← (2) ......4 5 3 3 3 |
| (5) 5 1 2 3 4 4 1 1 * 1 | ← (6) 6 2 3 4 4 1 1 * 1 |

(35.15)

| (3) 1 * * 2 4 3 3 3 | ← (5) * * 2 4 3 3 3 |
| (5) 3 4 4 1 1 * * 1 | ← (6) 5 1 2 3 4 4 1 1 * 1 |

(35.16)

| (2) 1 1 * * 2 4 3 3 3 | ← (4) 1 * * 2 4 3 3 3 |
| (3) 2 3 4 4 1 1 * * 1 | ← (6) 3 4 4 1 1 * * 1 |

(35.17)

| (2) 1 2 3 4 4 1 1 * * 1 | ← (4) 2 3 4 4 1 1 * * 1 |
| (4) 4 1 1 * * * 1 |

(35.18)

| (2) * * * * 1 |

(35.19)

| (1) 1 * * * * 1 |

(36.2)

| (35) 1 ← (37) |

(36.3)

| (5) 3 0 1 ← (6) 3 1 |
| (21) 1 4 1 ← (22) 15 |
| (29) 6 1 ← (30) 7 |
| (33) 2 1 ← (34) 3 |
| (34) 1 1 ← (36) 1 |

(36.4)

| (3) 2 7 3 3 ← (5) 29 3 |
| (5) 13 11 7 |
| (5) 29 1 1 ← (6) 30 1 |
| (15) 7 7 7 ← (19) 11 7 |
| (19) 11 3 3 ← (21) 13 3 |
| (21) 13 1 1 ← (22) 14 1 |
| (27) 3 3 3 ← (29) 5 3 |
| (29) 5 1 1 ← (30) 6 1 |
| (33) 1 1 1 ← (34) 2 1 |
### The Algebraic Atiyah-Hirzebruch Spectral Sequence of Real Projective Spectra

**The Algebraic Atiyah-Hirzebruch Spectral Sequence of Real Projective Spectra**

| (36.5) | (36.11) |
|--------|---------|
| (2) 25 3 3 3 &lt; (4) 27 3 3 |
| (3) 14 5 7 7 &lt; (16) 7 7 7 |
| (5) 28 1 1 1 &lt; (6) 29 1 1 |
| (14) 3 5 7 7 &lt; (18) 5 7 7 |
| (18) 9 3 3 3 &lt; (20) 11 3 3 |
| (21) 12 1 1 1 &lt; (22) 13 1 1 |
| (27) 1 2 3 3 &lt; (29) 2 3 3 |
| (29) 4 1 1 1 &lt; (30) 5 1 1 |
| (2) 3 3 6 6 5 3 &lt; (8) 3 5 5 3 |
| (5) 14 ** 1 &lt; (6) 16 4 1 1 ** 1 |
| (11) 2 ** 2 4 3 3 3 &lt; (12) 3 6 2 3 4 1 1 1 |
| (13) 6 ** 1 &lt; (14) 8 4 1 1 ** 1 |
| (17) 2 ** 1 &lt; (18) 4 4 1 1 ** 1 |
| (18) 1 ** 1 &lt; (20) ** 1 |

| (36.6) | (36.12) |
|--------|---------|
| (1) 5 9 7 7 7 |
| (3) 11 3 5 7 7 &lt; (4) 14 5 7 7 |
| (5) 9 3 5 7 7 |
| (5) 24 4 1 1 1 &lt; (6) 28 1 1 1 |
| (13) 3 6 6 5 3 &lt; (19) 5 7 3 3 |
| (15) 3 5 7 3 3 &lt; (17) 6 6 5 3 |
| (19) 3 6 2 3 3 &lt; (20) 8 3 3 3 |
| (21) 8 4 1 1 1 &lt; (22) 12 1 1 1 |
| (25) 4 4 1 1 1 &lt; (26) 1 2 3 3 |
| (27) 1 ** 1 &lt; (30) 4 1 1 1 |
| (5) 13 1 ** 1 &lt; (6) 14 ** 1 |
| (6) ......4 5 3 3 3 |
| (11) 11 ** 2 4 3 3 3 &lt; (12) 2 ** 2 4 3 3 3 |
| (13) 5 1 ** 1 &lt; (14) 6 ** 1 |
| (17) 1 ** 1 &lt; (18) 2 ** 1 |

| (36.7) | (36.13) |
|--------|---------|
| (2) 5 7 3 5 7 7 |
| (5) 22 ** 1 &lt; (6) 24 4 1 1 1 |
| (10) 3 3 6 6 5 3 &lt; (16) 3 5 7 3 3 |
| (13) 2 3 5 7 3 3 &lt; (14) 3 6 6 5 3 |
| (19) 4 3 3 3 &lt; (20) 3 6 2 3 3 |
| (21) 6 ** 1 &lt; (22) 8 4 1 1 1 |
| (25) 2 1 ** 1 &lt; (26) 4 4 1 1 1 |
| (26) 1 ** 1 &lt; (28) ** 1 |
| (3) 8 1 1 ** 2 4 3 3 3 |
| (5) 12 1 ** 1 &lt; (6) 13 1 ** 1 |
| (11) 1 2 3 4 4 1 1 1 ** 1 &lt; (13) 2 3 4 4 1 1 1 ** 1 |
| (13) 4 1 1 ** 1 &lt; (14) 5 1 ** 1 |

| (36.8) | (36.14) |
|--------|---------|
| (1) 6 5 2 3 5 7 7 |
| (3) 5 5 3 6 6 5 3 &lt; (5) 9 3 6 6 5 3 |
| (5) 21 1 ** 1 &lt; (6) 22 ** 1 |
| (7) 6 2 3 5 7 3 3 &lt; (14) 2 3 5 7 3 3 |
| (19) 1 2 4 3 3 3 &lt; (20) 4 3 3 3 |
| (21) 5 1 ** 1 &lt; (22) 6 ** 1 |
| (25) 1 1 ** 1 &lt; (26) 2 ** 1 |
| (2) 2 ** 2 4 3 3 3 &lt; (4) 3 6 2 3 4 1 1 ** 1 |
| (5) 6 ** 1 &lt; (6) 8 4 1 1 ** 1 |
| (9) 2 ** 1 &lt; (10) 4 4 1 1 ** 1 |
| (10) 1 ** 1 &lt; (12) ** 2 ** 1 |

| (36.9) | (36.15) |
|--------|---------|
| (1) 3 6 3 3 6 6 5 3 &lt; (2) 6 5 2 3 5 7 7 |
| (5) 6 2 4 5 3 3 3 &lt; (11) 6 2 4 5 3 3 3 |
| (5) 20 1 ** 1 &lt; (6) 21 1 ** 1 |
| (7) 3 6 2 4 5 3 3 3 &lt; (8) 6 2 3 5 7 3 3 |
| (13) 12 1 ** 1 &lt; (14) 13 1 ** 1 |
| (19) 1 2 3 4 4 1 1 1 &lt; (21) 2 3 4 4 1 1 1 |
| (21) 4 1 1 ** 1 &lt; (22) 5 1 ** 1 |
| (3) 11 ** 2 4 3 3 3 &lt; (4) 2 ** 2 4 3 3 3 |
| (5) 5 1 ** 1 &lt; (6) 6 ** 2 ** 1 |
| (9) 1 1 ** 1 &lt; (10) 2 ** 2 ** 1 |

| (36.10) | (36.16) |
|--------|---------|
| (1) 2 4 3 3 3 6 6 5 3 &lt; (2) 3 6 3 3 6 6 5 3 |
| (4) .....3 6 6 5 3 &lt; (9) 6 4 5 3 3 3 |
| (5) 3 6 4 5 3 3 3 &lt; (6) 5 6 2 4 5 3 3 3 |
| (5) 16 4 1 1 1 ** 1 &lt; (6) 20 1 1 1 ** 1 |
| (7) .....3 5 7 3 3 &lt; (8) 3 6 2 4 5 3 3 3 |
| (11) 3 6 2 3 4 4 1 1 1 &lt; (12) 8 1 1 2 4 3 3 3 |
| (13) 8 4 1 1 1 ** 1 &lt; (14) 12 1 1 ** 1 |
| (17) 4 4 1 1 1 ** 1 &lt; (20) 1 2 3 4 4 1 1 1 |
| (19) ** 1 &lt; (22) 4 1 1 1 ** 1 |
| (6) 11 11 1 &lt; (11) 2 4 3 3 3 3 |
| (14) 2 ** 1 &lt; (15) 4 1 1 1 ** 1 |
| (16) 1 ** 1 &lt; (17) ** 2 ** 1 ** 1 |

| (36.11) | (36.17) |
|--------|---------|
| (1) 4 4 1 1 1 ** 1 &lt; (4) 1 2 3 4 4 1 1 1 ** 1 |
| (3) ** 1 &lt; (6) 4 1 1 1 ** 1 |
| (3) 1 ** 1 &lt; (4) 1 ** 1 ** 1 |
| (5) 11 11 1 &lt; (6) 5 1 ** 1 ** 1 |

| (36.12) | (36.18) |
|--------|---------|
| (1) 2 ** * 1 &lt; (2) 4 4 1 1 1 ** 1 |
| (2) 2 ** 1 &lt; (3) 2 ** 1 ** 1 |
| (2) 1 ** 1 &lt; (4) 4 1 1 1 ** 1 |

| (36.13) | (36.19) |
|--------|---------|
| (1) 1 1 ** 1 &lt; (2) 2 ** 1 ** 1 |
### The Algebraic Atiyah-Hirzebruch Spectral Sequence of Real Projective Spectra

| (38, 3) | (38, 8) |
| --- | --- |
| (6) 29 | (1) 3 5 7 5 7 7 |
| (7) 30 1 ← (8) 31 | (2) 7 1 2 4 5 3 3 3 |
| (22) 13 3 | (3) 6 5 2 3 5 7 7 |
| (23) 14 1 ← (24) 15 | (5) 5 5 5 6 6 5 3 ← (9) 1 2 4 5 3 3 3 |
| (30) 3 5 3 ← (38) 1 | (7) 2 1 1 * 1 ← (8) 22 1 * 1 |
| (31) 6 1 ← (32) 7 | (9) 6 2 3 5 7 3 3 ← (16) 2 3 5 7 3 3 |
| (35) 2 1 ← (36) 3 | (15) 13 1 * 1 ← (29) 1 * 1 |

| (38, 4) | (38, 9) |
| --- | --- |
| (1) 23 7 7 | (1) 4 7 3 6 6 5 3 |
| (5) 27 3 3 | (2) 7 1 2 4 5 3 3 3 |
| (7) 13 11 7 ← (9) 1 5 1 5 | (3) 5 6 3 3 6 6 5 3 ← (4) 6 5 2 3 5 7 7 |
| (7) 29 1 1 ← (8) 3 0 1 | (5) 6 2 4 5 3 3 3 ← (13) 6 2 4 5 3 3 3 |
| (17) 7 7 7 | (6) 2 0 1 1 * 1 ← (8) 2 1 1 * 1 |
| (21) 11 3 3 | (9) 3 6 2 4 5 3 3 3 ← (10) 6 2 3 5 7 3 3 |
| (23) 13 1 1 ← (24) 1 4 1 | (13) 8 1 1 2 4 3 3 3 ← (28) 1 1 * 1 |
| (24) 6 5 3 ← (37) 1 1 | (15) 1 2 1 1 * 1 ← (16) 1 3 1 * 1 |
| (29) 3 3 3 ← (33) 3 3 | (23) 4 1 1 * 1 ← (24) 5 1 * 1 |
| (31) 5 1 1 ← (32) 6 1 | | |
| (35) 5 1 1 ← (36) 2 1 | | |

| (38, 5) | (38, 10) |
| --- | --- |
| (4) 25 3 3 3 | (1) 3 5 6 2 3 5 7 3 3 → (2) 5 6 3 3 6 6 5 3 |
| (5) 14 5 7 7 ← (8) 1 3 1 1 7 | (3) 2 4 3 3 3 6 5 3 ← (4) 3 6 3 3 6 6 5 3 |
| (6) 1 3 5 7 7 | (5) 6 3 6 6 5 3 ← (11) 6 4 5 3 3 3 |
| (7) 2 8 1 1 1 ← (8) 2 9 1 1 | (7) 3 6 4 5 3 3 3 ← (8) 5 6 2 4 5 3 3 3 |
| (15) 4 5 7 7 ← (18) 7 7 7 | (7) 1 6 4 1 1 * 1 ← (8) 2 0 1 1 * 1 |
| (20) 9 3 3 3 | (9) 3 5 7 3 3 ← (10) 3 6 2 4 5 3 3 3 |
| (21) 8 3 3 3 ← (36) 1 1 1 | (12) 4 5 3 3 3 ← (24) 4 1 1 * 1 |
| (23) 1 2 1 1 1 ← (24) 1 3 1 1 | (13) 3 6 2 4 4 1 1 1 ← (14) 8 1 1 2 4 3 3 3 |
| (31) 4 1 1 1 ← (32) 5 1 1 | (15) 2 4 3 3 3 ← (17) 6 2 3 4 4 1 1 1 |
| | (15) 8 4 1 1 1 * 1 ← (16) 1 2 1 1 * 1 |

| (38, 6) | (38, 11) |
| --- | --- |
| (1) 1 4 5 7 7 | (1) 4 3 3 6 6 6 5 3 ← (2) 3 5 6 2 3 5 7 3 3 |
| (3) 5 9 7 7 7 | (4) 3 6 3 3 6 6 5 3 ← (10) ....3 5 7 3 3 |
| (5) 1 3 5 7 7 ← (6) 1 4 5 7 7 | (7) ....3 5 7 3 3 ← (8) 3 6 4 5 3 3 3 |
| (7) 9 3 5 7 7 ← (17) 3 5 7 3 7 | (17) 1 4 * * 1 ← (8) 1 6 4 1 1 * 1 |
| (7) 2 4 4 1 1 1 ← (8) 2 8 1 1 1 | (10) ....4 5 3 3 3 ← (22) * 2 * 1 |
| (28) 1 2 9 3 3 3 | (13) 2 2 4 3 3 3 ← (14) 3 6 2 4 4 1 1 1 |
| (14) 2 5 3 3 3 ← (16) 4 5 7 7 | (14) 1 * 2 4 3 3 3 ← (16) * 2 4 3 3 3 |
| (15) 3 6 6 5 3 ← (19) 6 6 5 3 | (15) 6 * 1 ← (16) 8 4 1 1 1 |
| (20) 4 5 3 3 3 ← (32) 4 1 1 1 | (19) 2 * 1 ← (20) 4 4 1 1 1 * 1 |
| (21) 3 6 2 3 3 ← (22) 8 3 3 3 | | |
| (23) 2 4 3 3 3 ← (25) 6 2 3 3 | | |
| (23) 8 4 1 1 1 ← (24) 1 2 1 1 1 | | |

| (38, 7) | (38, 12) |
| --- | --- |
| (1) 1 3 2 3 5 7 7 ← (2) 1 4 4 5 7 7 | (1) 6 .....3 5 7 3 3 ← (8) .....3 5 7 3 3 |
| (2) 5 9 3 5 7 7 | (7) 1 3 1 * * 1 ← (8) 1 4 * * 1 |
| (4) 5 7 3 5 7 7 ← (8) 9 3 5 7 7 | (8) .....4 5 3 3 3 ← (21) * 2 * 1 |
| (6) 9 3 6 6 5 3 | (13) 1 1 * 2 4 3 3 3 ← (14) 2 * 2 4 3 3 3 |
| (7) 2 1 * 1 ← (8) 2 4 4 1 1 1 | (15) 5 1 * * 1 ← (16) 6 * 1 * 1 |
| (12) 3 6 6 5 3 ← (18) 3 5 7 3 3 | (19) 1 1 * * 1 ← (20) 2 * * 1 |
| (15) 2 3 5 7 3 3 ← (16) 3 6 6 5 3 | | |
| (18) 2 4 5 3 3 3 ← (30) * 1 | | |
| (21) 1 4 3 3 3 ← (22) 3 6 2 3 3 | | |
| (22) 1 2 4 3 3 3 ← (24) 2 4 3 3 3 | | |
| (23) 6 * 1 ← (24) 8 4 1 1 1 | | |
| (27) 2 * 1 ← (28) 4 4 1 1 1 | | |

| (38, 13) | (38, 13) |
| --- | --- |
| (1) 3 6 .....4 5 3 3 3 ← (2) 6 .....3 5 7 3 3 | (1) 3 6 .....4 5 3 3 3 ← (2) 6 .....3 5 7 3 3 |
| (5) 8 1 1 * 2 4 3 3 3 ← (20) 1 1 * 1 | (7) 1 2 1 1 * * 1 ← (8) 3 1 * * 1 |
| (7) 1 6 1 1 * 1 ← (28) 4 4 1 1 1 | (15) 4 1 1 * * 1 ← (16) 5 1 * * 1 |
| Raw Text | Natural Text |
|----------|-------------|
| (40.9)  | (40.10)     |
| (40.10) | (40.11)     |
| (40.11) | (40.12)     |
| (40.12) | (40.13)     |
| (40.13) | (40.14)     |
| (40.14) | (40.15)     |

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GUOZHEN WANG AND ZHOULI XU
The Algebraic Atiyah-Hirzebruch Spectral Sequence of Real Projective Spectra
### The Algebraic Atiyah-Hirzebruch Spectral Sequence of Real Projective Spectra

#### (42.17)

1. $8111**24333$
2. $1211**11**1 = (4) 1311**1$
3. $1234411**1 = (16) 11**1$
4. $411**1 = (12) 51**1$

#### (42.18)

1. $36234411**1 = (2) 8111**24333$
2. $**24333 = (5) 6234411**1$
3. $8411**1 = (4) 1211**1$
4. $411**1 = (12) 411**1$

#### (42.19)

1. $**24333 = (2) 36234411**1$
2. $**24333 = (4) **24333$
3. $6**1 = (4) 8411**1$
4. $34411**1 = (16) **1$
5. $2**1 = (8) 4411**1$

#### (42.20)

1. $11**24333 = (2) 2**24333$
2. $34411**1 = (9) **1$
3. $511**1 = (4) 6**1$
4. $11**1 = (8) 2**1$

#### (42.21)

1. $1234411**1 = (8) 11**1$
2. $411**1 = (4) 51**1$

#### (42.22)

1. $**1$

#### (43.3)

1. $293 = (15) 31$
2. $133 = (29) 15$
3. $53 = (37) 7$
4. $33 = (41) 3$

#### (43.4)

1. $22133 = (6) 2313$
2. $2653 = (10) 273$
3. $273 = (12) 293$
4. $14133 = (14) 1515$
5. $1053 = (26) 117$
6. $233 = (28) 133$
7. $653 = (30) 77$
8. $333 = (36) 33$
9. $233 = (38) 33$

#### (43.5)

1. $15177$
2. $19777 = (5) 2117$
3. $2117 = (6) 2213$
4. $2533 = (10) 2653$
5. $131133 = (14) 14133$
6. $2533 = (25) 357$
7. $653 = (26) 653$
8. $333 = (26) 103$
9. $1233 = (36) 233$

#### (43.6)

1. $911777 = (6) 71777$
2. $714577 = (4) 11577$
3. $183577 = (5) 20577$
4. $209333 = (6) 21133$
5. $113577 = (12) 13577$
6. $129333 = (14) 131133$
7. $23577 = (21) 4577$
8. $35733 = (27) 8333$
9. $47333 = (24) 6653$
10. $45333 = (26) 5733$
11. $51233 = (30) 6233$

#### (43.7)

1. $109366653 = (2) 12129333$
2. $111245333 = (4) 1736653$
3. $4593577 = (10) 573577$
4. $936653 = (6) 8129333$
5. $3573577 = (8) 93577$
6. $5536653 = (12) 936653$
7. $10245333 = (14) 1245333$
8. $6245333 = (24) 245333$
9. $124333 = (28) 124333$
10. $2344111 = (30) 344111$

#### (43.8)

1. $955366653 = (2) 109366653$
2. $56523577 = (9) 6523577$
3. $36525377 = (6) 6936653$
4. $36235733 = (12) 6366653$
5. $734131 = (14) 10245333$
6. $653411 = (21) 131**1$
7. $453333 = (18) 6245333$
8. $2344111 = (28) 2344111$

#### (43.9)

1. $955366653$
2. $56523577$
3. $36525377$
4. $36235733$
5. $734131$
6. $653411$
7. $453333$
8. $2344111$

#### (43.10)

1. $455536653 = (4) 6523577$
2. $235536653 = (6) 36523577$
3. $356245333 = (10) 56235733$
4. $366653 = (12) 36235733$
5. $357333 = (19) 81124333$
6. $453333 = (16) 645333$
7. $453333 = (18) 453333$
8. $51334411 = (22) 62344111$

#### (43.11)

1. $12473336653 = (5) 247336653$
2. $43336653$
3. $336653 = (10) 356245333$
4. $357333 = (18) 453333$
5. $453333 = (14) 581124333$
6. $453333 = (16) 453333$
7. $51234411 = (22) 512344111$
THE ALGEBRAIC ATIYAH-HIRZEBRUCH SPECTRAL SEQUENCE OF REAL PROJECTIVE SPECTRA

### Table 1:...

| Index | Data | Description |
|-------|------|-------------|
| 1     | 24   | 1 2 3 4 5 6 |
| 2     | 6    | 1 2 3 4 5 6 |
| 3     | 12   | 1 2 3 4 5 6 |
| 4     | 24   | 1 2 3 4 5 6 |
| 5     | 36   | 1 2 3 4 5 6 |
| 6     | 48   | 1 2 3 4 5 6 |
| 7     | 60   | 1 2 3 4 5 6 |
| 8     | 72   | 1 2 3 4 5 6 |

### Table 2:...

| Index | Data | Description |
|-------|------|-------------|
| 1     | 24   | 1 2 3 4 5 6 |
| 2     | 6    | 1 2 3 4 5 6 |
| 3     | 12   | 1 2 3 4 5 6 |
| 4     | 24   | 1 2 3 4 5 6 |
| 5     | 36   | 1 2 3 4 5 6 |
| 6     | 48   | 1 2 3 4 5 6 |
| 7     | 60   | 1 2 3 4 5 6 |
| 8     | 72   | 1 2 3 4 5 6 |

### Table 3:...

| Index | Data | Description |
|-------|------|-------------|
| 1     | 24   | 1 2 3 4 5 6 |
| 2     | 6    | 1 2 3 4 5 6 |
| 3     | 12   | 1 2 3 4 5 6 |
| 4     | 24   | 1 2 3 4 5 6 |
| 5     | 36   | 1 2 3 4 5 6 |
| 6     | 48   | 1 2 3 4 5 6 |
| 7     | 60   | 1 2 3 4 5 6 |
| 8     | 72   | 1 2 3 4 5 6 |
THE ALGEBRAIC ATIYAH-HIRZEBRUCH SPECTRAL SEQUENCE OF REAL PROJECTIVE SPECTRA

| (50.19) | (50.20) | (50.21) | (50.22) | (50.23) | (50.24) | (50.25) | (51.3) | (51.4) | (51.5) |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| (3) \ldots \ldots 3 \ 5 \ 7 \ 3 \ 3 \ \leftarrow (4) \ 3 \ 6 \ \ldots \ldots 4 \ 5 \ 3 \ 3 \ 3 \ 3 | (3) 14 * * * 1 \leftarrow (4) 16 4 1 1 * * * 1 | (3) 13 1 * * * 1 \leftarrow (4) 14 * * * 1 | (3) 6 2 3 4 4 1 1 * * * * 1 \leftarrow (5) 6 2 3 4 4 1 1 * * * * 1 | (1) 2 * * * * 1 \leftarrow (2) 6 2 3 4 4 1 1 * * * * 1 | (1) 11 * * * 2 4 3 3 3 \leftarrow (2) 2 4 3 3 3 \ 8 3 | (1) 11 * * * * 1 \leftarrow (4) 6 * * * * 1 | (19) 29 3 \leftarrow (21) 31 | (13) 22 13 3 \leftarrow (14) 23 15 | (1) 9 11 15 15 |
| (4) \ldots \ldots 4 5 3 3 3 | (4) 16 4 1 1 * * * 1 \leftarrow (17) 13 1 1 1 1 | (4) 12 11 * * * 1 \leftarrow (12) 6 * * * * 1 | (4) 3 4 1 1 * * * 1 \leftarrow (10) * * * * 1 | (4) 1 1 * * * 1 \leftarrow (12) 4 4 1 1 * * * 1 | (1) 11 * * * 2 4 3 3 3 \leftarrow (2) 2 * * * * 2 4 3 3 3 | (4) 3 4 1 1 * * * 1 \leftarrow (10) * * * * 1 | (43) 5 3 \leftarrow (45) 7 | (17) 26 5 3 \leftarrow (18) 27 7 | (3) 9 11 15 15 |
| (9) 2 * * * 2 4 3 3 3 \leftarrow (10) 3 6 2 3 4 4 1 1 * * * 1 | (10) 1 * * * 2 4 3 3 3 \leftarrow (12) * * * * 2 4 3 3 3 | (10) 1 * * * 2 4 3 3 3 \leftarrow (12) * * * * 2 4 3 3 3 | (10) 3 6 2 3 4 4 1 1 * * * 1 \leftarrow (12) 6 * * * * 1 | (10) 2 * * * * 1 \leftarrow (16) 4 4 1 1 * * * 1 | (2) 11 * * * 2 4 3 3 3 \leftarrow (22) 14 13 3 | (11) 5 1 * * * 1 \leftarrow (12) 6 * * * * 1 | (45) 3 3 \leftarrow (49) 3 | (10) 17 7 7 7 | (5) 10 11 13 5 7 7 |
| (11) 6 * * * 1 \leftarrow (12) 8 4 1 1 * * * 1 | (11) 6 * * * 1 \leftarrow (12) 8 4 1 1 * * * 1 | (11) 13 11 3 3 \leftarrow (22) 14 13 3 | (11) 13 11 3 3 \leftarrow (22) 14 13 3 | (11) 11 * * * 1 \leftarrow (16) 2 * * * * 1 | (11) 13 11 3 3 \leftarrow (22) 14 13 3 | (11) 13 11 3 3 \leftarrow (22) 14 13 3 | (3) 29 3 \leftarrow (21) 31 | (3) 3 5 7 7 \leftarrow (33) 5 7 7 | (3) 10 17 7 7 7 |
| (12) 3 4 1 1 * * * 1 \leftarrow (18) * * * * 1 | (12) 3 4 1 1 * * * 1 \leftarrow (18) * * * * 1 | (12) 3 4 1 1 * * * 1 \leftarrow (18) * * * * 1 | (12) 3 4 1 1 * * * 1 \leftarrow (18) * * * * 1 | (12) 3 4 1 1 * * * 1 \leftarrow (18) * * * * 1 | (12) 3 4 1 1 * * * 1 \leftarrow (18) * * * * 1 | (12) 3 4 1 1 * * * 1 \leftarrow (18) * * * * 1 | (34) 1 2 3 4 4 1 1 1 \leftarrow (36) 2 3 4 4 1 1 1 | (3) 3 6 5 3 \leftarrow (38) 6 5 3 | (3) 4 6 3 5 7 7 

\[ \begin{array}{cccccc}
(50.19) & (50.20) & (50.21) & (50.22) & (50.23) & (50.24) \\
(3) & 2 * * * 2 4 3 3 3 & 1 - \leftarrow (4) & 3 6 5 3 \leftarrow (10) 9 10 11 15 15 & 24 3 3 3 & 2 3 4 4 1 1 1 \\
(4) & \ldots \ldots 4 5 3 3 3 & & & & \\
(9) & 2 * * * 2 4 3 3 3 \leftarrow (10) 3 6 2 3 4 4 1 1 * * * 1 & & & & \\
(10) & 1 * * * 2 4 3 3 3 \leftarrow (12) * * * * 2 4 3 3 3 & & & & \\
(11) & 6 * * * 1 \leftarrow (12) 8 4 1 1 * * * 1 & & & & \\
(12) & 3 4 1 1 * * * 1 \leftarrow (18) * * * * 1 & & & & \\
(15) & 2 * * * * 1 \leftarrow (16) 4 4 1 1 * * * 1 & & & & \\
(50.25) & (50.26) & (50.27) & (50.28) & (50.29) & (50.30) \\
(3) & 3 6 2 3 4 4 1 1 * * * 1 \leftarrow (2) 6 2 3 4 4 1 1 * * * 1 & & & & \\
(4) & \ldots \ldots 4 5 3 3 3 & & & & \\
(3) & 13 1 * * * 1 \leftarrow (4) 14 * * * 1 & & & & \\
(9) & 11 * * * 2 4 3 3 3 & 1 - \leftarrow (10) 3 6 5 3 \leftarrow (12) 6 2 3 4 4 1 1 * * * 1 & & & \\
(10) & 2 3 4 4 1 1 * * * 1 \leftarrow (17) 1 * * * * 1 & & & & \\
(11) & 5 1 * * * 1 \leftarrow (12) 6 * * * * 1 & & & & \\
(15) & 11 * * * 1 \leftarrow (16) 2 * * * * 1 & & & & \\
(51.3) & (51.4) & (51.5) & (51.6) & (51.7) & (51.8) \\
(19) & 29 3 \leftarrow (21) 31 & & & & \\
(35) & 13 3 \leftarrow (37) 15 & & & & \\
(43) & 5 3 \leftarrow (45) 7 & & & & \\
(45) & 3 3 \leftarrow (49) 3 & & & & \\
(51.9) & & & & & \\
(13) & 22 13 3 \leftarrow (14) 23 15 & & & & \\
(17) & 26 5 3 \leftarrow (18) 27 7 & & & & \\
(18) & 27 3 \leftarrow (20) 29 3 & & & & \\
(21) & 34 13 3 \leftarrow (22) 15 15 & & & & \\
(33) & 10 5 3 \leftarrow (34) 11 7 & & & & \\
(34) & 11 3 3 \leftarrow (36) 13 3 & & & & \\
(37) & 6 5 3 \leftarrow (38) 7 7 & & & & \\
(42) & 3 3 3 \leftarrow (44) 5 3 & & & & \\
(43) & 2 3 3 3 \leftarrow (46) 3 3 & & & & \\
\end{array} \]
THE ALGEBRAIC ATIYAH-HIRZEBRUCH SPECTRAL SEQUENCE OF REAL PROJECTIVE SPECTRA
THE ALGEBRAIC ATIYAH-HIRZEBRUCH SPECTRAL SEQUENCE OF REAL PROJECTIVE SPECTRA

(57.4)

(57.5)

(57.6)

(57.7)

(57.8)

(57.9)

(57.10)

(57.11)

(57.12)

(57.13)
THE ALGEBRAIC ATIYAH-HIRZEBRUCH SPECTRAL SEQUENCE OF REAL PROJECTIVE SPECTRA 69
|  |  |  |
|---|---|---|
| (68, 4) | (69, 4) | (69, 5) |
| (3) 27 23 15 | (3) 58 5 3 ← (4) 59 7 | (1) 27 11 15 15 |
| (3) 59 3 3 ← (5) 61 3 | (6) 29 27 7 | (3) 57 3 3 ← (4) 58 5 3 |
| (5) 61 1 1 ← (6) 62 1 | (7) 30 29 3 ← (8) 31 31 | (5) 27 23 7 |
| (46) 3 5 7 7 | (30) 21 11 7 ← (38) 29 3 | (6) 29 27 3 ← (8) 30 29 3 |
| (61) 4 1 1 ← (59) 5 3 3 | (31) 22 13 3 ← (32) 23 15 | (21) 7 11 15 15 |
| (63) 7 7 ← (51) 11 7 | (35) 26 5 3 ← (36) 27 7 | (24) 14 13 11 7 ← (37) 27 3 3 |
| (37) 29 1 1 ← (38) 30 1 | (38) 13 11 7 | (29) 11 15 7 7 |
| (47) 7 7 7 ← (51) 11 7 | (39) 14 13 3 ← (40) 15 15 | (29) 19 7 7 7 ← (33) 23 7 7 |
| (51) 11 3 3 ← (53) 13 3 | (39) 13 11 3 3 ← (40) 14 13 3 | (31) 17 7 7 7 ← (49) 7 7 7 |
| (53) 13 1 1 ← (54) 14 1 | (47) 3 5 7 7 ← (52) 22 13 3 | (31) 21 11 3 3 ← (32) 22 13 3 3 |
| (59) 3 3 3 ← (64) 3 3 | (35) 25 3 3 3 ← (36) 26 5 3 | (35) 23 3 3 3 ← (36) 26 5 3 |
| (61) 5 1 1 ← (62) 6 1 | (39) 13 11 3 3 ← (40) 14 13 3 | (39) 13 11 3 3 ← (40) 14 13 3 |
| (65) 1 1 1 ← (66) 2 1 | (47) 3 5 7 7 ← (52) 22 13 3 | (47) 3 5 7 7 ← (52) 22 13 3 |

|  |  |  |
|---|---|---|
| (68, 5) | (70, 2) | (70, 3) |
| (2) 57 3 3 3 ← (4) 59 3 3 | (7) 63 | (6) 61 3 |
| (3) 26 21 11 7 ← (4) 27 23 15 | (55) 15 | (7) 62 1 ← (8) 63 |
| (5) 60 1 1 1 ← (6) 61 1 1 | (63) 7 ← (71) | (39) 30 1 ← (40) 31 31 |
| (7) 22 21 11 7 | (54) 13 3 | (54) 14 1 ← (56) 15 |
| (23) 14 13 11 7 ← (24) 15 15 15 | (62) 5 3 ← (70) 1 | (62) 5 3 ← (70) 1 |
| (27) 10 11 11 7 ← (28) 11 15 15 | (63) 6 1 ← (64) 7 | (63) 6 1 ← (64) 7 |
| (28) 19 7 7 7 ← (32) 23 7 7 | (67) 2 1 ← (68) 3 | (67) 2 1 ← (68) 3 |

|  |  |  |
|---|---|---|
| (68, 6) | (70, 4) | (70, 5) |
| (1) 19 7 11 15 15 | (1) 55 7 7 | (1) 55 7 7 |
| (1) 23 13 13 11 7 | (5) 27 23 15 | (5) 27 23 15 |
| (3) 25 19 7 7 7 ← (4) 26 21 11 7 | (3) 59 3 3 | (3) 59 3 3 |
| (4) 10 9 15 15 15 | (7) 29 27 7 ← (9) 31 31 | (7) 61 1 1 ← (8) 62 1 |
| (5) 56 4 1 1 1 ← (6) 60 1 1 1 | (5) 29 27 7 | (25) 15 15 15 ← (37) 27 7 |
| (6) 12 9 11 15 15 ← (8) 22 21 11 7 | (29) 11 15 15 | (29) 11 15 15 |
| (6) 14 7 11 15 15 | (3) 21 11 7 ← (33) 23 15 | (31) 13 11 7 ← (41) 15 15 |
| (15) 5 7 11 15 15 ← (19) 9 11 15 15 | (39) 13 11 7 ← (41) 15 15 | (39) 29 1 1 ← (40) 30 1 |
| (21) 7 11 15 7 7 | (53) 11 3 3 | (53) 11 3 3 |
| (23) 9 15 7 7 7 ← (25) 13 13 11 7 | (55) 13 1 1 ← (56) 14 1 | (55) 13 1 1 ← (56) 14 1 |
| (25) 13 13 11 7 7 ← (30) 20 5 7 7 | (56) 6 5 3 ← (69) 1 1 | (56) 6 5 3 ← (69) 1 1 |
| (27) 9 11 11 7 7 ← (28) 10 13 11 7 | (61) 3 3 3 ← (65) 3 3 | (61) 3 3 3 ← (65) 3 3 |
| (29) 7 13 5 7 7 ← (37) 13 5 7 7 | (63) 5 1 1 ← (64) 6 1 | (63) 5 1 1 ← (64) 6 1 |
| (31) 14 4 5 7 7 ← (46) 4 5 7 7 | (67) 1 1 1 ← (68) 2 1 | (67) 1 1 1 ← (68) 2 1 |
| (35) 11 3 5 7 7 ← (36) 14 5 7 7 | | |
| (37) 24 4 1 1 1 ← (38) 28 1 1 1 | | |
| (45) 3 6 6 5 3 ← (51) 5 1 3 3 | | |
| (47) 3 5 7 3 3 ← (49) 6 6 5 3 | | |
| (51) 3 6 2 3 3 ← (52) 8 3 3 3 | | |
| (55) 8 4 1 1 1 ← (54) 12 1 1 1 | | |
| (57) 4 4 1 1 1 ← (60) 1 2 3 3 | | |
| (59) * 1 ← (62) 4 1 1 1 | | |
THE ALGEBRAIC ATIYAH-HIRZEBRUCH SPECTRAL SEQUENCE OF REAL PROJECTIVE SPECTRA

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