Investigation of the Unsteady External and Underhood Airflow of the DrivAer model by Dynamic Mode Decomposition Methods

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ABSTRACT: In this research, we conduct unsteady CFD to investigate the effect of engine bay flow on the steady and unsteady aerodynamics of the extended DrivAer model reproducing engine bay flow. Dynamic Mode Decomposition (DMD) is performed to analyze unsteady aerodynamics. As a result, it is revealed that different engine bay setups result in not only differences of steady aerodynamics but also the differences of unsteady aerodynamic characteristics. Furthermore, we perform an on-the-fly algorithm of DMD called Streaming Total DMD (STDMD) which can be conducted with much less memory than conventional DMD to investigate the relevancy and applicability of STDMD on the analysis of unsteady aerodynamics of a road vehicle.

KEY WORDS: heat • fluid, computational fluid dynamics, aerodynamic performance, unsteady aerodynamics, Dynamic Mode Decomposition [D1]

1. Introduction

When road vehicles are designed, basic aerodynamic characteristics have been evaluated by time averaged, steady aerodynamics such as C\(_D\) and C\(_L\) values which are measured through wind tunnel experiments or Reynolds Averaged Navier-Stokes Simulations (RANS). In contrast, recent research has indicated that the unsteady aerodynamics caused by vehicle motion or fluctuations in incoming flows is important, especially for estimating running stability or ride comfort of a vehicle at high speed. These unsteady aerodynamics are estimated by unsteady CFD such as Large Eddy Simulation (LES) and Detached Eddy Simulation (DES) which are widely used for aerodynamic design of vehicles in recent years.

Unsteady CFD is useful with respect to the evaluation of the time dependent, complex flow field around a vehicle with high accuracy, however, the flow field around a vehicle is very complex and consists of various time and length scale flow structures. Therefore, it is very difficult to identify the flow structures causing a certain aerodynamic effect directly from the results of unsteady CFD, in many cases. It seems that Dynamic Mode Decomposition (DMD) \(^{1,2}\), which is a type of modal analysis method, is useful for this task. DMD extracts the coherent flow structures from the complex, unsteady flow field by decomposing the flow field into dynamic modes which describe the coherent flow structure fluctuating with a single frequency and stability of the flow structure. For example by DMD analysis Rowley, et al. \(^2\) extracted flow structures having a specific frequency by applying DMD on a Jet flow. In the field of applied aerodynamics, Muld, et al. \(^3\) analyzed the unsteady flow field around a high-speed train by using DMD, and identified the flow structure related to slip-stream. Peichl, et al. \(^4\) applied DMD on the flow field around a road vehicle and discussed the coherent flow structures which are dominant in the unsteady flow field around the vehicle through the results of DMD. In these researches, DMD was applied on the velocity field, but DMD can be applied on any multi-dimensional data. Hence, in this research, we perform DMD on the wall force vectors acting on the road vehicle to analyze the unsteady aerodynamic characteristics.

Even though DMD is believed to be useful for the analysis of unsteady flow field as mentioned above, there are still some problems in practical use of DMD for aerodynamic design. One problem is that massive memory is required for DMD computation. When executing DMD, all snapshots of the flow field analyzed by DMD have to be stored on the RAM, however, the size of flow field data computed by unsteady CFD is usually very big. Furthermore, this flow field data has to be saved on the disk space during CFD and loaded when performing DMD, which results in the requirement of a lot of disk space and time for loading and saving data. Streaming DMD (SDMD) \(^5\) and Streaming Total DMD (STDMD) \(^6\), which are on-the-fly algorithms of DMD, seem to be useful with respect to these problems. In the algorithm of STDMD, matrices used for DMD computation are incrementally updated when a new snapshot of the flow field is obtained. Therefore, STDMD can be performed in parallel to CFD, and it is not necessary to save flow field data to disk. In addition, matrices for STDMD can be compressed, resulting in much less memory requirement than conventional DMD. However, applicability of STDMD on the very complex flow field such as the one around a road vehicle hasn’t been investigated yet, even though it was validated with a somehow simple flow field. Therefore, we perform STDMD and compare the results of STDMD with those by conventional DMD \(^1\), \(^2\) to investigate the relevancy and applicability of the STDMD on the analysis of unsteady vehicle aerodynamics.

As a target model of a road vehicle in this research, we adopt the DrivAer model \(^7\) which is a generic car model developed in 2011 at the Institute of Aerodynamics and Fluid Mechanics of the TU Munich. The geometry is a combination of an Audi A4 and a BMW 3 series vehicle. It is meant to close the gap between low detail geometries, e.g. the Ahmed body, that have mostly been investigated in academic research and real production car geometries which are developed and investigated in industry but to which academia has only limited access \(^7\). The geometry data is publicly available and the DrivAer has therefore been well respected and is the subject of multiple investigations and
publications both numerically and experimentally \((4), (8) - (10)\). Since the model comes with three different rear end types, a notchback, an estate back and a fastback type, it offers a variety of possible insights into the external flow around a vehicle. However, external flow is generally not the only field of interest for aerodynamicists in academia and especially in industry. Cooling flow through the engine bay has a considerable influence on the aerodynamic performance of a vehicle due to pressure loss in the radiator and engine bay and influence of the cooling mass flow on the external flow field \((11)\). This effect could so far not be covered with the publicly available DrivAer geometry. Due to this gap and on the initiative of the European Car Aerodynamic Research Association (ECARA) an engine and engine bay geometry was developed at our institute in close collaboration with the Research Institute of Automotive Engineering and Vehicle Engines Stuttgart (FKFS). In this research, we adopt this extended DrivAer model reproducing engine bay flow.

In this research, we conduct unsteady CFD on the DrivAer model to analyze the effect of engine bay flow on the steady and unsteady aerodynamics of the vehicle. We also conduct DMD analysis to investigate the difference in the characteristics of unsteady aerodynamics caused by different engine bay flow resulting from different configurations of the engine compartment. We additionally perform STDMD and compare results with those of conventional DMD to study the relevance and applicability of streaming algorithm of DMD on the vehicle aerodynamics.

2. Numerical Methods and Setups

2.1. Numerical Methods

The unsteady flow simulations are carried out with the CFD Toolbox OpenFOAM® which is produced by ESI-OpenCFD. Since the finite volume CFD code is open source, this program is widely used in the aerodynamics community. The numerical setup incorporated unsteady simulations where turbulence is modeled with the Delayed Detached Eddy Simulation (DDES) model \((12)\). This zonal Detached Eddy turbulence model is opted to combine the superior accuracy of Large Eddy Simulations (LES) with the lower computational demands of Reynolds Averaged Navier-Stokes (RANS) turbulence models and has been validated for automotive applications \((13)\). In the RANS regions the one equation Spalart-Allmaras (SA) turbulence model is used. Time marching is done implicitly.

2.2. Target model

We adopt extended DrivAer model reproducing engine bay flow as a target model of analysis. The extension of the DrivAer geometry can be seen as an evolution of a generic low detail engine bay geometry which was already present in our institutes wind tunnel model for internal use \((10)\). The design changes were kept to a minimum in order to assure that researchers who have already built the hardware model of the DrivAer geometry can reuse this model with a minor amount of adjustments.

The new geometry features a generic inline four cylinder engine block. This engine type was chosen since it represents the most frequently sold engine type for the two production models, the Audi A4 and the BMW 3 series in Germany \((14)\). Furthermore, a realistic exhaust system is included which exits into the gearbox tunnel where it is attached to the exhaust pipe of the detailed underbody of the original DrivAer geometry without engine bay flow. The underbody remains mostly unchanged except for the generic gearbox which is now replaced by a more realistic gearbox and gearbox tunnel. The new geometry also includes a cooler package consisting of an upper and a lower grille, a cooling air duct leading to the radiator model and a cooling fan housing. There are two variants of the cooling air duct. One which allows leakage flow around the model radiator and one which prevents leakage. The engine with the gearbox and the cooler package are mounted to two longitudinal rails which are the exact resemblance of the support frame of the institutes 40% scaled wind tunnel model \((9)\) (see Fig. 2). The new components are shown in Fig. 1. The whole engine bay is encapsulated by an engine bay lining (Fig. 2) to provide a defined flow region. Exit openings for the cooling air are located in the two front wheel houses and in the gearbox tunnel, illustrated in Fig. 3. In terms of detail, the whole engine bay geometry is designed to fit the level of detail of the original model.

Fig. 1: Isometric exploded view of the newly designed components for the DrivAer with engine bay flow

Fig. 2: DrivAer model with ducted cooling air inlets and encapsulated engine bay

Fig. 3: Modified gearbox tunnel of the new DrivAer with engine bay flow; cooling air can exit in the anterior wheel arches and the gearbox tunnel

The extended geometry is available on the institute homepage \((15)\). The known nomenclature for the configuration of the geometry \((7)\) is extended by the abbreviation EB instead of the underbody type since here, engine bay flow is only possible with a detailed underbody configuration. Additionally, a fifth letter set will be added for this configuration, where “wL” represents the configuration with leakage flow around the radiator, “woL” represents the configuration without leakage around the radiator respectively and “MU” (Mock Up) represents a closed cooling inlet. Following this amended nomenclature a configuration for a estate back with mirrors, wheels and flow through the engine compartment but without leak around the radiator will have the short name E_EB_wM_wW_woL.

Due to the cooperation on the development of the new open grill DrivAer geometry with FKFS, first experimental and computational results were already presented by Wittmeier, et al. \((16)\). However, the used geometry slightly differs from the
downloadable model due to experimental setup specifications. In the following sections, first computational results with the downloadable open grille DrivAer geometry are presented for a notchback type body with and without leak around the radiator. The obtained results shall be compared to results of a baseline configuration with closed grilles comparable to the original DrivAer body for external flow and a detailed underbody configuration.

2.3. Computational grid and Boundary conditions

The used computational meshes are hexa-dominant meshes with a size of approximately 66 million cells. The sizes of the smallest cells are set to be approximately 3mm and with prism layers on the car surface an averaged dimensionless wall distance y+ of 31.9 is achieved.

The simulations are performed in full scale with a wind speed of u ≈ 40 m/s (144 km/h) which gives a Reynolds number of 7.1 × 10^6 with respect to 4.61 m of reference length of the vehicle.

The rims are closed in order to avoid the necessity of additional wheel rotation models other than a rotating wall boundary condition and tire tread and grooves are omitted. The cooler is modeled with a pressure loss approach following the Darcy-Forchheimer equation.

3. Dynamic Mode Decomposition

In this section, DMD algorithms adopted for this research are introduced. Firstly, the conventional DMD algorithm is introduced briefly, following descriptions of Schmid (15) and Tu, et al. (16). Scaling of DMD modes is also described, following descriptions of Rowley, et al. (17). In this paper, we call conventional DMD ‘projected DMD’ such as in Tu, et al. (17) to distinguish it from other alternative DMD algorithms.

Additionally, on-the-fly algorithm of DMD called Streaming Total DMD (STDMD) developed by Hemati, et al. (18), is also introduced. STDMD is combined algorithm of Streaming DMD (SDMD) (19) and Total DMD (TDMD) (20) resulting in a more noise-robust algorithm of DMD. As the STDMD algorithm is based on the SDMD algorithm, we introduce the SDMD algorithm before introducing the STDMD algorithm. The MATLAB codes of SDMD and STDMD are published by Rowley, et al. (21), however, we performed STDMD by our self-developed code in order to apply on large datasets.

3.1. Dynamic Mode Decomposition

3.1.1. Decomposition

The objective of DMD is extracting dynamic information and its structure extracted by eigenvectors and eigenvalues of DMD operator A which is described as follows,

\[ Y = AX \] (1)

where \( X = \{x_0, x_1, \ldots, x_{m-1}\} \in \mathbb{R}^{n \times m} \) and \( Y = \{x_1, x_2, \ldots, x_m\} \in \mathbb{R}^{n \times m} \) are snapshot matrices consisting of the snapshot of the flow field \( x_i \in \mathbb{R}^n \) which is recorded at a constant time interval \( \Delta t_{DMD} \). One solution to compute the DMD operator A is solving the following equation. \( (X^T \text{represents pseudo-inverse of } X) \)

\[ A = Y (X^T) \] (2)

However, it is usually difficult to solve Eq.(2) and the eigendecomposition of A, because the number of states n is much bigger than the number of snapshots m in the case of unsteady CFD on a road vehicle. Therefore, a “projected” DMD operator is defined by projecting the DMD operator onto the orthogonal basis shown as follows,

\[ \tilde{A} \equiv U^T A U \] (3)

where orthogonal basis U is the left singular vector computed by Singular Value Decomposition (SVD) of X, as follows,

\[ X = U \Sigma W^T \] (4)

Please note that the left singular vectors \( \mathbf{U} \) represents the POD modes of X. Therefore, truncating the higher ranks of POD modes when projecting (Eq.(3)), can work as a kind of filtering. It is because the higher ranks of POD modes have less contribution to the original flow field and expected to consist of high frequency flow structures.

Substituting Eq.(3) in Eq.(2), the projected DMD operator can be rewritten as follows,

\[ \tilde{A} = U^T Y X^T U = U^T Y W \Sigma^{-1} \] (5)

where pseudo-inverse of X is expressed by using matrices derived by SVD.

\[ X^T = W \Sigma^{-1} U^T \] (6)

Then, eigendecomposition of projected DMD operator \( \tilde{A} \) is performed as follows,

\[ \tilde{A} V = \mathbf{VA} \] (7)

Finally, projected DMD modes \( \tilde{\Phi} = \{\tilde{\phi}_1, \tilde{\phi}_2, \ldots, \tilde{\phi}_r\} \) are computed by projecting the eigenvectors \( V = \{v_1, v_2, \ldots, v_r\} \) on the left singular vectors \( \mathbf{U} \).

\[ \tilde{\Phi} = U V \] (8)

DMD modes describe the spatial structure of the flow field extracted by DMD. Eigenvalues \( \Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_r) \) describe the amplification and damping rate and frequency of the flow structures extracted by the corresponding DMD modes. Especially, frequency of the k-th DMD mode is computed as follows.

\[ f_k = \frac{\text{Im}(\log(\lambda_k))}{2 \pi \Delta t_{DMD}} \] (9)

DMD modes and their eigenvalues, which are computed through projected DMD operator, are regarded as a low dimensional approximations of eigenvectors and eigenvalues of DMD operator \( \Lambda \).

3.1.2. Scaling of DMD modes and Reconstruction

Considering that DMD modes are an approximation of Koopman modes, the i-th flow field \( x_i \) can be approximated by scaled DMD modes \( \Phi_k \) and corresponding eigenvalues \( \lambda_k \), as follows.

\[ x_i = \sum_{k=1}^{r} \lambda_k \Phi_k, \quad i = 0, \ldots, m - 1 \] (10)

where k-th DMD mode is scaled by scaling factor \( d_k \)

\[ \Phi_k = d_k \Phi_k \] (11)

Then, the first snapshot of the flow field \( x_0 \) can be expressed as a linear combination of scaled DMD modes.
\[ x_0 = \sum_{k=1}^{r} \phi_k \]  

(12)

Scaling factor \( d = [d_1, d_2, \ldots, d_r] \) is derived by solving the following linear equation.

\[ \Phi d = x_0 \]  

(13)

As such, the \( i \)-th flow field of \( x_i \) is reconstructed from scaled DMD modes and corresponding eigenvalue by Eq.(10).

### 3.2. Streaming algorithms of DMD

#### 3.2.1. Streaming DMD

In the Streaming DMD algorithm, DMD operator \( \text{A} \) (Eq. (2)) is projected not on the left singular vector as Eq.(3), but on the Gram-Schmidt orthogonal basis \( Q_x \) of snapshot matrix \( X \). Then, the projected DMD operator \( \tilde{A} \) is defined as follows.

\[ \tilde{A} \equiv Q_x^T \text{A} Q_x \]  

(14)

This equation can be rewritten as follows.

\[ \tilde{A} = Q_x^T Q_Y K G_x^T \]  

(15)

where \( G_X = X X^T \) and \( K = \bar{Y} \bar{X}^T \) which are expressed with projected snapshot matrices \( \bar{X} = Q_X^T X \) and \( \bar{Y} = Q_Y^T Y \) on their orthogonal basis \( Q_X \) and \( Q_Y \).

DMD modes are computed by projecting eigenvectors of \( \tilde{A} \) which are computed in the same way as Eq.(7), on \( Q_X \), as follows.

\[ \tilde{\Phi} = Q_X V \]  

(16)

Advantage of Eq.(15) is that all matrices used for computing \( \tilde{A} \) can be updated incrementally. Orthogonal basis \( Q_X \) of \( X \) and \( Q_Y \) of \( Y \) are updated when new snapshot is computed. In addition, \( K \) and \( G_X \) can be updated as those matrices are rewritten as linear combination of projected snapshot vectors \( \bar{x}_j = Q_X^T x_j \) and \( \bar{y}_j = Q_Y^T y_j \), as follows. (Please note that \( y_i \) corresponds to \( x_{i+1} \) in the case of sequential datasets.)

\[ K = \sum_{j=1}^{i} \bar{y}_j \bar{x}_j^T \]  

(17)

\[ G_X = \sum_{j=1}^{i} \bar{x}_j \bar{x}_j^T \]  

(18)

\( K \) and \( G_X \) can be updated as \( K \leftarrow K + \bar{y}_i \bar{x}_i^T \) and \( G_X \leftarrow G_X + \bar{x}_i \bar{x}_i^T \), when a new snapshot vector is computed. After the update of these matrices is finished, DMD modes are computed from the updated and projected DMD operator \( \tilde{A} \).

However, as the flow field has a high dimensional state of space, the rank \( r_X \) and \( r_Y \) of orthogonal basis \( Q_X \) and \( Q_Y \) are close to the number of snapshots \( m \), in the case of very complex flow field. Therefore, massive memory is occupied to save huge orthogonal basis \( Q_X \) and \( Q_Y \).

Therefore, compression of matrices when the rank \( r_X \) or \( r_Y \) of orthogonal basis exceed prescribed maximum rank \( r_0 \) during update is considered. For compression of matrices, the additional matrix \( G_Y = \bar{Y} \bar{Y}^T \) is prepared, which is also described as follows.

\[ G_Y = \sum_{j=1}^{i} \bar{y}_j \bar{y}_j^T \]  

(19)

When the rank of orthogonal basis \( r_X \) exceeds the prescribed rank \( r_0 \), the orthogonal basis \( Q_X \) is replaced as \( Q_X \leftarrow Q_X V_X \) where \( V_X \) is computed by eigendecomposition of \( G_X \) shown as follows.

\[ G_X V_X = V_X S_X \]  

(20)

Then, \( Q_X V_X \) represents POD mode of \( X \), because substituting Eq.(20) in \( G_X = X X^T \), the following equation is derived, which is analogous to the definition of POD mode of \( X \).

\[ (X X^T)(Q_X V_X) = (Q_X V_X) S_X \]  

(21)

As the higher ranks of POD mode have less contribution to the flow field, ranks higher than \( r_0 \) are truncated. In the same way, \( Q_Y \) is also replaced as \( Q_Y \leftarrow Q_Y V_Y \) where \( V_Y \) is eigenvector of \( G_Y \), and truncated higher rank of \( Q_Y V_Y \) than \( r_0 \), when \( r_Y > r_0 \).

Then, \( Q_X , G_Y \) and \( K \) are also replaced as follows by projecting them on \( V_X \) and \( V_Y \) which are truncated at ranks higher than \( r_0 \).

\[ G_X \leftarrow V_X^T G_X V_X \]  

(22)

\[ G_Y \leftarrow V_Y^T G_Y V_Y \]  

(23)

\[ K \leftarrow V_Y^T K V_Y \]  

(24)

It is regarded that matrices are projected not on \( Q_X \) or \( Q_Y \), but on \( Q_X V_X \) or \( Q_Y V_Y \).

#### 3.2.2. Streaming Total DMD

In the algorithm of Streaming Total DMD, orthogonal basis is computed through the augmented snapshot matrix \( Z \) to solve a total least-square problem. At first, QR-factorization is performed on the augmented snapshot matrix \( Z \).

\[ Z = \begin{bmatrix} X \\ Y \end{bmatrix} = Q Z R_Z \]  

(25)

When \( Q_Z \) represents orthogonal basis of \( Z \). Snapshot matrices \( X \) and \( Y \) are represented by \( Q_Z \) and \( R_Z \) computed by Eq.(25), as follows.

\[ X = [1 \ 0] Q_Z R_Z \]  

(26)

\[ Y = [0 \ 1] Q_Z R_Z \]  

(27)

In addition, QR-factorization is performed on \([1 \ 0] Q_Z \) to compute orthogonal basis \( Q_X \)

\[ [1 \ 0] Q_Z = Q_X R_X \]  

(28)

In the algorithm of Total Streaming DMD, the DMD operator is projected on \( Q_X \) which is computed by Eq.(28), but the expression is equal to Eq.(14). Substituting Eq. (26) - (28) in Eq. (14) considering \( X^T = X^T (X X^T)^{-1} \), the projected DMD operator \( \tilde{A} \) is derived as follows.

\[ \tilde{A} = Q_X^T [1 \ 0] Q_Z G_X Q_X^T [1 \ 0] Q_X G_X^+ \]  

(29)

where,

\[ G_X = R_X G_Z R_X^T \]  

(30)

\[ G_Z = R_Z R_Z^T = Q_Z^T Z^T Q_Z \]  

(31)

DMD modes are computed by projecting eigenvectors of \( \tilde{A} \) on \( Q_X \) in the same way as Eq.(16) with Eq.(7).

From Eq.(31), \( G_Z \) is regarded as analogous to \( G_X \) and \( G_Y \) which are defined in Streaming DMD. \( G_Z \) can be represented by linear
combination of projected augmented snapshot vector \( \bar{z}_j = Q_{Z}^T z_i \), as follows

\[
G_{Z} = \sum_{j=1}^{r_z} \bar{z}_j \bar{z}_j^T
\]

Please note that augmented snapshot vector is represented as

\[
z_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix} = Q_{Z} R_{Z}
\]

Therefore, \( G_{Z} \) and \( Q_{Z} \) can be updated as \( G_{Z} \leftarrow G_{Z} + \bar{z}_j \bar{z}_j^T \), when new snapshot vector is computed.

\( G_{Z} \) and \( Q_{Z} \) are also compressed by POD modes as well as \( G_{X} \) and \( Q_{X} \) which were defined in the Streaming DMD algorithm. When the rank \( r_z \) of orthogonal basis \( Q_{Z} \) exceeds the prescribed rank \( r_0 \), \( Q_{Z} \) and \( G_{Z} \) are replaced as follows using \( V_{Z} \) which is truncated at ranks higher than \( r_0 \) considering \( Q_{Z} G_{Z} \) represents POD mode.

\[
Q_{Z} \leftarrow Q_{Z} V_{Z}
\]

\[
G_{Z} \leftarrow V_{Z}^T G_{Z} V_{Z}
\]

where eigendecomposition of \( G_{Z} \) is performed as follows.

\[
G_{Z} V_{Z} = V_{Z} S_{Z}
\]

4. Results

In this section, results of unsteady CFD are discussed. The computational time for a parallel run on 512 intel cores of the SuperMUC high performance computer (Sandy Bridge-EP Xeon E5-2680 8C) has taken about 40 hours.

4.1. Steady Aerodynamics

As a baseline case the model was simulated with closed cooling air inlets but with the outlet openings and the engine bay geometry being present \( (E_{EB\_wM\_wW\_MU}) \). This is in industry often referred to as the Mock Up configuration. The Mock Up configuration was chosen as a baseline setup, since it allows the deduction of the cooling air drag \( C_{DC} \) which is the drag difference between the setups with closed and open cooling air inlets. Furthermore, the results for the Mock Up case can be compared to earlier experimental results for the DrivAer without engine bay flow and can therefore serve as a validation for the numerical setup.

When comparing the calculated drag coefficient of \( C_D = 0.275 \) with experimental results for the original DrivAer geometry in the corresponding configuration e.g. by Mack et al. (8) \( (C_D = 0.275) \) the coincidence of the values hints firstly to a validity of the used numerical model and secondly to no major changes of the flow field due to outlet openings and recirculation within the engine bay. This minor recirculation can also be seen in Fig. 6.A. Furthermore, the drag value for the Mock Up configuration agrees well with the one presented by Wittmeier et al. (10) \( (C_D = 0.277) \).

From Fig. 4 it can be seen that opening the grilles gives a drag increase of \( C_{DC} = 0.010 \) in the case of no leakage around the cooler and \( C_{DC} = 0.019 \) in the case of existing leakage around the cooler. The drag increase for the configuration without leakage flow corresponds remarkably exact to the measured cooling drag value \( C_{DC} = 0.010 \) by Wittmeier et al. (10). The difference in cooling drag for the configurations with and without leakage around the cooling package indicates a considerable dependence of the flow field on this feature. This impression is emphasized by the lift distribution compared to the baseline case in Fig. 5. While both configurations increase overall lift and front lift, rear lift is decreased. However, the increase in front lift is almost doubled for the configuration with leakage around the radiator.
Fig. 7: Pressure coefficient distribution of the three simulated setups. A: Mock Up. B: Without leakage around the cooler. C: With leakage around the cooler.

4.2. DMD on the wall force

In order to analyze characteristics of unsteady aerodynamics caused by different configurations of the engine compartment, DMD analysis by projected DMD algorithm is performed on the wall force vector acting on the vehicle. In this research, we perform DMD analysis only on the DrivAer models reproducing engine bay flow, without leakage and with leakage around the cooler. DMD results from these models are compared to analyze the effect of different engine bay flow on the unsteady aerodynamics.

Before performing DMD analysis, a low-pass filter with 10Hz of cut off frequency is applied on the wall force for the temporal direction by forward-backward filtering with 2nd order Butterworth filter, which results in 4th order zero-phase filter, in addition to filtering performed by SVD. These preprocessing techniques are conducted according to Peichl, et al. (4). (Please also refer to Schmid (1), and Duke, et al. (20) regarding sensitivity of dataset on DMD.) Total time length of the dataset used for DMD is 3.5 sec in total and snapshots of the flow field are sampled with $\Delta t_{DMD} = 0.005$sec. Total amount of memory used for DMD computation is about 500GB with respect to approximately 6 million cells of surface mesh of the vehicle.

Firstly, amplitudes of DMD modes, which are normalized by the highest amplitude, are shown in Fig.8 with corresponding frequencies, but only modes with positive frequencies are shown. The amplitude, which is an absolute value of the scaling factor $d_i$ in Eq. (11), represents the magnitude of fluctuation of the flow field described by the DMD mode. The numbering of the DMD mode is also decided according to its amplitude, as starting from Mode 1. (Please note that the most energetic mode (Mode 1) exists at 0Hz.) According to Fig.8, the most fluctuating DMD mode (Mode 2) exists at similar but different frequencies between the model with and without leakage around the cooler, which is at 3.50Hz without leakage and at 4.13Hz with leakage.

To discuss the characteristics of mode 2, distribution of $z$-component of scaled DMD mode 2 ($\phi_2$) is shown in Fig. 9. The intensive region of DMD mode distribution are almost similar regardless of leakage around the cooler, comparing the model with and without leakage in Fig.9. However, focusing on the floor just behind front wheels (surrounded by dashed line in Fig.9), mode distributions of the model without leakage at the left and right side of the vehicle are oriented to the same direction (Fig.9 (A)), while those of the model with leakage are oriented to the opposite direction (Fig.9 (B)).

Fig. 9: Distribution of $z$-component of scaled DMD mode 2

For further discussion of these different phenomena, the $z$-component of the wall force is reconstructed from DMD mode 2, which are shown in Fig.10 with the time presented with the period $T$ of one cycle. In the case of the model without leakage, $z$-direction of the reconstructed wall force on the floor behind front wheels seems to fluctuate with the same phase between the left and right of the vehicle (Fig.10 (A)). On the other hand, the wall force there seems to fluctuate in antiphase between the left and right, in the case of the model with leakage, comparing Fig.10 (B) at $t=0$ and at $t=T/2$. As such, it is believed that the different configuration of the fan housing in the engine compartment results in differences in dominant fluctuation of the wall force, such as illustrated in Fig.11.

Fig. 10: The $z$-component of reconstructed wall force from mode 2

Fig. 11: Different phenomenon of wall force found by DMD
4.3. Streaming Total DMD on the wall force

In this section, Streaming Total DMD (STDMD) is additionally performed on the wall force of the vehicle to study the applicability of the STDMD on the aerodynamic design of a road vehicle, as a first attempt. We investigate whether relevant results can be computed by streaming algorithm of DMD or not, by comparing with results of projected DMD introduced in section 4.2. Please note that STDMD in this research is performed on the datasets saved on the disk as a post processing, not performed in parallel with CFD.

2nd order Butterworth filter is applied with 10Hz of cut off frequency on the streaming dataset of the wall force incrementally to realize similar pre-processing on the dataset as projected DMD, even though 4th order zero-phase filter is adopted for the preprocessing of projected DMD computation in section 4.2. It is because forward-backward filtering which realizes zero-phase filter cannot not be applied on streaming dataset. The prescribed maximum rank $r_0$ is set as $r_0 = 21$ for the compression of the matrices and total amount of memory used for DMD computation is about 30GB.

Firstly, normalized amplitudes of DMD modes by STDMD are shown in Fig.12 with corresponding frequencies, but only modes of positive frequencies are shown. In Fig.12, mode 2, which is the most dominant, of both models is at around 1Hz, which doesn’t appear in the result of projected DMD. In addition, mode 3 of both models are almost static mode existing at close to 0Hz frequency. However, mode 4 of the model without leakage was at 3.64Hz which is close to the frequency of mode 2 computed by projected DMD (3.50Hz). On the other hand, mode 4 of the model with leakage is at 4.45Hz which is slightly different from the frequency of mode 2 by projected DMD (4.13Hz), but seems to be not greatly different. Therefore, we focus on the mode 4 by STDMD whose mode distribution $\phi_4$ is shown in Fig.13, and compared with results of projected DMD discussed in section 4.2.

According to Fig. 13, mode distribution on the floor just behind front wheels (surrounded by dashed line in Fig.13) shows the slight difference between 2 models which seems to be the same variation found in the result of projected DMD shown in Fig.9, but not so obviously. Therefore, the $z$-component of the reconstructed wall force from DMD mode 4 is investigated, which is shown in Fig.14, for further discussion of these different phenomena between models. From Fig.14, it seems that $z$-direction of the wall force on the floor by DMD mode 4 fluctuating with the same phase between the left and right seems to be dominant in the case of the model with leakage. (Fig.14. (A)) On the other hand, in the case of the model without leakage, it seems that that $z$-direction of wall force fluctuating in antiphase between left and right seems to be dominant, especially focusing on at $t = T/4$ (Fig.14. (b)). Hence it seems that STDMD successfully extracted the same variation in wall force fluctuation between 2 models as found in projected DMD.

![Fig 14: z-component of reconstructed wall force from mode 4](image)

As such, there is a possibility that STDMD can also compute somehow relevant DMD modes compared to projected DMD with smaller memory. STDMD is certainly useful with respect to less memory usage and on-the-fly algorithm calculation, especially for large datasets, however there are still some problems to be considered in order to use STDMD for detailed analysis of unsteady aerodynamics, for example STDMD mode 2 at around 1Hz isn’t found in projected DMD.

5. Conclusions

Unsteady CFD on the DrivAer models reproducing engine bay flow is conducted to investigate the effect of engine bay flow on the steady and unsteady aerodynamic characteristics. Unsteady aerodynamics are analyzed by using DMD methods. The following conclusions are drawn from this research.

(1) Steady aerodynamic drag on the vehicle is increased with engine bay flow, which is counted as cooling drag. Cooling drag with leakage around the cooler is higher than without leakage.

Overall lift is increased by engine bay flow. Change in aerodynamic balance between closed grilles and open grilles is caused by the pressure distribution on the underbody between front wheels and gearbox tunnel. Higher front lift with leakage around the cooler than without leakage is caused by the increased momentum towards the roof of the engine bay lining due to the leakage flow.

(2) Projected DMD is performed on the wall force of the vehicle to analyze unsteady aerodynamic characteristics caused by engine bay flow. As a result, different engine bay flow results in the difference of the most fluctuating DMD mode (mode 2) with respect to the frequency and $z$-component of mode distribution behind front wheels. Furthermore, reconstructing wall force from...
mode 2, the z-component of the wall force on the floor behind the front wheels fluctuates with the same phase between the left and right of the vehicle without leakage around the cooler, on the other hand the wall force there fluctuates in antiphase between the left and right of the vehicle with leakage.

(3) In order to study the applicability of the streaming algorithm of DMD on vehicle aerodynamics, Streaming Total DMD (STDMD) is performed on the wall force. STDMD mode 4 is found at a similar frequency as the most fluctuating projected DMD mode. In addition, the different phenomenon between the models without and with leakage which is found by the most fluctuating mode of projected DMD is also found by STDMD. Hence, it seems that there is a possibility for STDMD to compute relevant DMD results as projected DMD, however, further validation is needed to be used for detailed analysis of the flow field.

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