Prediction of Mechanical Properties of Glass Fiber Reinforced Epoxy Resin Matrix Composites Based on Homogenization Method

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Abstract. A cell model of glass fiber reinforced epoxy resin matrix composite was established under the environment of finite element software Abaqus. Unidirectional deformation in six directions applied to the cell, and the average stress and average strain of the cell was obtained. According to the homogenization idea of the composite, the material for cell was supposed to three-dimensional orthogonal anisotropy homogeneous material. According to the relationship between elastic deformation stress and strain, the elastic modulus of homogeneous material were obtained as the elastic modulus of composite material. Compared with elastic modulus obtained by the experimental measurement or theoretical calculation, the effectiveness of the method was verified.

1. INTRODUCTION
Glass fiber reinforced epoxy resin matrix composite is lightweight and high strength material. It is widely used in aviation, aerospace, transportation, national defense and other fields. Its mechanical property parameters are the most basic use basis, however at present, the mechanical properties of composite are generally obtained by experimental measurement or theoretical calculation. The cost of experimental measurement is high, and the composite that have been prepared must be exist. The composite that is still in the design stage cannot be measured experimentally, the experimental measurement also cannot understand the relationship between the property parameters and the microstructure arrangement of the constituent materials. The theoretical calculation ignores the microscopic shape and arrangement of the composition material, thus resulting in large errors. At present, the common methods of composite design cannot accurately determine the material property parameters according to the shape, size and arrangement of the composition materials, and cannot meet the requirements of optimal design.

In this paper, glass fiber reinforced epoxy resin matrix composite was used as a sample to solve this problem. Because the mechanical properties of the composite are not uniform in microstructure, and the mechanical properties measured by experiments are actually the average values of the microscopic properties. The homogenization method of composite is to assume the existence of a homogeneous material. If its stress and strain relationship and elastic strain energy are the same as those of the actual material, then its elastic modulus are the elastic modulus of the actual material.

According to the composition of glass fiber and epoxy resin in the composite, a periodic cell model was established under the Abaqus software environment to reflect the microstructure and arrangement of the composition materials. The boundary conditions were set and the given unidirectional deformations were carried out. The average stresses of the cell were taken as the stresses of the
hypothesized homogeneous material. According to the known values of stresses and strains and their relationships, the elastic modulus of hypothetical homogeneous material were obtained. These values were the elastic modulus of the actual composite material. Compared with the elastic modulus values in the manual, the errors were very small, which verified the reliability of the method. This method is not only suitable for glass fiber reinforced epoxy resin matrix composite, but also suitable for most similar composites. This method has a wide range of application.

2. MATERIALS AND METHODS

2.1. Mathematical model

The homogenization method should first establish a representative cell model according to the actual composition of the composite, and then calculate the average stresses and the average strains of the cell as the stresses and strains of the hypothetical homogeneous material in the finite element software. Finally, the elastic modulus of homogeneous material are calculated according to the relationship between stresses and strains. If the volume of the representative cell is \( V \), the average stress of the representative cell is as follows:

\[
\overline{\sigma}_{ij} = \frac{1}{V} \int_{V} \sigma_{ij} dV = \frac{\sum \sigma_{ij} v^i v^j}{V} \quad (i,j = 1, 2, 3)
\]

(1)

The average strain of the representative cell is as follows:

\[
\overline{\varepsilon}_{kl} = \frac{1}{V} \int_{V} \varepsilon_{kl} dV = \frac{\sum \varepsilon_{kl} v^k v^l}{V} \quad (k,l = 1, 2, 3)
\]

(2)

Because of elastic deformation, the relationship between average stresses and average strains is as follows:

\[
\overline{\sigma}_{ij} = D_{ijkl} \overline{\varepsilon}_{kl}
\]

(3)

The average stress and average strain here are the stress and strain of the hypothetical homogeneous material. Because the composites of reinforced fiber are orthotropic materials, the elastic matrix of glass fiber reinforced epoxy resin matrix composites is as follows:

\[
D = 
\begin{bmatrix}
D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\
D_{12} & D_{22} & D_{23} & 0 & 0 & 0 \\
D_{13} & D_{23} & D_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & D_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & D_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & D_{66}
\end{bmatrix}
\]

(4)

The constitutive equation of elastic deformation is as follows:

\[
\begin{bmatrix}
\sigma_{1} \\
\sigma_{2} \\
\sigma_{3} \\
\sigma_{23} \\
\sigma_{31} \\
\sigma_{12}
\end{bmatrix} = 
\begin{bmatrix}
D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\
D_{12} & D_{22} & D_{23} & 0 & 0 & 0 \\
D_{13} & D_{23} & D_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & D_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & D_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & D_{66}
\end{bmatrix} 
\begin{bmatrix}
\varepsilon_{1} \\
\varepsilon_{2} \\
\varepsilon_{3} \\
2\varepsilon_{23} \\
2\varepsilon_{31} \\
2\varepsilon_{12}
\end{bmatrix}
\]

(5)

According to the constitutive equation, the following results can be deduced as follows:

Where \( \varepsilon^{(1)} = (1, 0, 0, 0, 0, 0), D_{11} = \sigma_{11}, D_{12} = \sigma_{21}, D_{13} = \sigma_{31} \), where \( \varepsilon^{(2)} = (0, 1, 0, 0, 0, 0), D_{12} = \sigma_{12}, D_{22} = \sigma_{22}, D_{23} = \sigma_{32} \), where \( \varepsilon^{(3)} = (0, 0, 1, 0, 0, 0), D_{13} = \sigma_{13}, D_{23} = \sigma_{23}, D_{33} = \sigma_{33}, D_{44} = \sigma_{23} / (2\varepsilon_{23}), D_{55} = \sigma_{13} / (2\varepsilon_{13}), D_{66} = \sigma_{12} / (2\varepsilon_{12}). \)
The stiffness matrix $D$ can be obtained from the above values. By calculating the inverse $D$, the flexibility matrix of the material can be obtained. The relationship between the coefficients of flexibility matrix and elastic modulus is as follows:

$$
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\
S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\
S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & S_{44} & 0 & 0 \\
0 & 0 & 0 & S_{55} & 0 & 0 \\
0 & 0 & 0 & 0 & S_{66} & 0
\end{bmatrix}
= \begin{bmatrix}
1/E_1 & -D_{12}/E_2 & -D_{13}/E_3 & 0 & 0 & 0 \\
-D_{21}/E_1 & 1/E_2 & -D_{23}/E_3 & 0 & 0 & 0 \\
-D_{31}/E_1 & -D_{32}/E_2 & 1/E_3 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/G_{12} & 0 & 0 \\
0 & 0 & 0 & 0 & 1/G_{13} & 0 \\
0 & 0 & 0 & 0 & 0 & 1/G_{23}
\end{bmatrix}
$$

According to the above formula, the elastic modulus of the composite can be easily obtained.

2.2. A subsection Numerical simulation Geometric model and parameters

2.3. Numerical results.

2.3.1. Z axis tension deformation
To ensure that deformation occurred only in the direction of the Z axis, the boundary conditions of the model were set such that two end faces perpendicular to Z axis moved 3.54 $\mu$m in the opposite direction and two end faces parallel to the X axis as well as two end faces parallel to the Y axis moved 0 $\mu$m in all directions except Z axis direction. A stress cloud diagram of the cell after tensile deformation along the Z axis was shown in figure 2.

Mean strain of the cell could be concluded as: $\varepsilon^{(3)}=(0, 0, 1, 0, 0, 0)$, and mean stresses of the cell could be concluded as: $\sigma_1=8.5$ GPA, $\sigma_2=9.3$ GPA, $\sigma_3=46$ GPA. According to the above derivation, it can be concluded as: $D_{33}=\sigma_3=46$.

2.3.2. X axis tension deformation
To ensure that deformation occurred only in the direction of the X axis, the boundary conditions of the model were set such that two end faces perpendicular to X axis moved 3.54$\mu$m in the opposite direction and two end faces parallel to the Y axis as well as two end faces parallel to the Z axis moved 0$\mu$m in all directions except X axis direction. A stress cloud diagram of the cell after tensile deformation along the X axis was shown in figure 3.
Mean strain of the cell could be concluded as: $\varepsilon^{(1)}=(1, 0, 0, 0, 0, 0)$, and mean stresses of the cell could be concluded as: $\sigma_1=12$ GPA, $\sigma_2 = 6.8$ GPA, $\sigma_3=6.2$ GPA. According to the above derivation, it can be concluded as: $D_{11}=\sigma_1=12$, $D_{12}=\sigma_2 = 6.8$, $D_{13}=\sigma_3=6.2$.

2.3.3. Y axis tension deformation

To ensure that deformation occurred only in the direction of the Y axis, the boundary conditions of the model were set such that two end faces perpendicular to Y axis moved 3.54 $\mu$m in the opposite direction and two end faces parallel to the X axis as well as two end faces parallel to the Z axis moved 0 $\mu$m in all directions except X axis direction. A stress cloud diagram of the cell after tensile deformation along the Y axis was shown in figure 4. Mean strain of the cell could be concluded as: $\varepsilon^{(2)}=(0, 1, 0, 0, 0, 0)$, and mean stresses of the cell could be concluded as: $\sigma_1=6.8$ GPA, $\sigma_2 = 12$ GPA, $\sigma_3=6.2$ GPA. According to the above derivation, it can be concluded as: $D_{22}=\sigma_2 = 12$, $D_{23}=\sigma_3=6.2$.

2.3.4. X-Y plane shear deformation

For ensuring shear deformation in X-Y plane, the loads of the model were set such that four ends parallel to the Z axis were subjected to 100N surface shear. The X-Y plane Surface shear loads was shown in figure 5. A stress cloud diagram of the cell after X-Y plane shear deformation was shown in figure 6.

Mean strain of the cell could be concluded as $\varepsilon_{12}=8.5\times10^{-9}$, and mean stress of the cell could be concluded as $\sigma_{12}=103\times10^{-9}$ GPA. According to the above derivation, it can be concluded as: $D_{66}=\sigma_{12}/(2\varepsilon_{12}) = 6.1$. 
2.3.5. X-Z plane shear deformation
For ensuring shear deformation in X-Z plane, the loads of the model were set such that four ends parallel to the Y axis were subjected to 100N surface shear. The X-Y plane Surface shear loads was shown in figure 7. A stress cloud diagram of the cell after X-Z plane shear deformation was shown in figure 8. Mean strain of the cell could be concluded as $\varepsilon_{13}=14\times10^{-9}$, and mean stress of the cell could be concluded as $\sigma_{13}=102\times10^{-9}$ GPA. According to the above derivation, it can be concluded as: $D_{55}=\sigma_{13}/(2\varepsilon_{13})=3$.

2.3.6. Y-Z plane shear deformation
For ensuring shear deformation in Y-Z plane, the loads of the model were set such that four ends parallel to the X axis were subjected to 100N surface shear. The Y-Z plane Surface shear loads is shown in figure 9. A stress cloud diagram of the cell after Y-Z plane shear deformation is shown in figure 10.

Mean strain of the cell could be concluded as $\varepsilon_{23}=14\times10^{-9}$, and mean stress of the cell could be concluded as $\sigma_{23}=102\times10^{-9}$ GPA. According to the above derivation, it can be concluded as: $D_{44}=\sigma_{23}/(2\varepsilon_{23})=3.6$. As described above, the resulting stiffness matrix $D$ and flexibility matrix $S$ were shown as below respectively.
According to the relationship between the coefficient of flexibility matrix and elastic modulus, elastic modulus of 50% glass fiber reinforced epoxy resin matrix composites can be obtained as follows:

$$E_1 = 8 \text{ GPA} \quad E_2 = 8 \text{ GPA} \quad E_3 = 42 \text{ GPA} \quad G_{12} = 6 \text{ GPA} \quad G_{13} = 4 \text{ GPA} \quad G_{23} = 4 \text{ GPA} \quad \nu_{12} = 0.5 \quad \nu_{13} = 0.3.$$

### 3. RESULTS AND DISCUSSION

CONCLUSIONS

By theoretical calculation and prediction of elastic modulus of composites, the most widely used model is Halpin-Tsai model. Halpin-Tsai formula for calculating the elastic modulus of the model is as follows:

$$E_c = E_m \left(1 + \frac{2l}{d} \right) \left(1 - \eta V_f \right) \left(1 - \frac{E_f}{E_m} \right)$$

$$\xi = \frac{2l}{d} \quad \eta = \frac{E_f}{E_m} - 1$$

$$E_c$$ is elastic modulus of composite, $$E_m$$ is elastic modulus of matrix, and $$V_f$$ is fiber percentage of composite volume. The $$l$$ is fiber length, $$d$$ is the diameter of the fiber, and $$E_f$$ is the elastic modulus of the fiber. According to the above formula, the elastic modulus of the composite material along the fiber direction is 37 GPA. The result of this paper is 42 GPA, which is similar to the theoretical calculation. When the proportion of fiber is small and the precision is high, the error is great when the proportion increases. The accuracy of the method used in this paper is independent of the proportion of fibers. The theoretical calculation can only calculate one elastic modulus parameter. In the paper, all elastic moduli and poisson ratios can be calculated by this method. For this material which with 50% glass of epoxy resin matrix composites, the measurement of the modulus of elasticity along the direction of the fiber is 42 GPA.

### 4. CONCLUSIONS

The method in this paper is not only suitable for FRP but also for most composites. The obtained performance parameters are not only complete in data but also high in accuracy. By this method, the relationship between the microstructure of the composite and its macroscopic performance parameters can be clearly obtained. It has a good guiding effect on the optimal design of composite materials and is worth popularizing.

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