Mathematical modeling of MHD processes in the casting of aluminum alloys in electromagnetic mold

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Abstract. The article contains the overview of mathematical modeling of electromagnetic field and magnetohydrodynamic process of the ingot moulding during casting into the electromagnetic mold. We have formulated the system of equations describing the molding process. The authors describe the principle of casting into the electromagnetic mould to produce small diameter ingots, as well as the results obtained. This article gives the analysis of magnetohydrodynamic flows in the area of molding and their impact on the resulting ingot. It is shown that the intensity of the MHD processes is influenced by the frequency of the supply voltage and the shape of the crystallization front. The calculations are based on favorable conditions for the formation of a homogeneous, fine-grained structure. The experimental studies were conducted using samples of aluminum alloy 01417.

1. Introduction
In modern industry, high-alloyed aluminum alloys are becoming more and more used, the production of which is impossible without use of special casting methods. The combination of high cooling rates and methods of active influence on the crystallizing melt allows to obtain structural materials based on aluminum, which have a significant increase in special properties [1, 2]. The prospective direction in the use of electromagnetic technologies in casting is ingots contactless casting into electromagnetic mold (EMM) [3-8].

2. Formulation of the problem
MHD processes occurring in the ingot depend on design and electromagnetic parameters of the electromagnetic mold [9]. An effective method for studying the effect of the parameters of an electromagnetic crystallizer on the quality of an ingot is mathematical modeling in combination with experimental studies [10].

3. Mathematical model
The system of equations describing coupled electromagnetic, hydrodynamic and thermal processes in ingot formed by an electromagnetic field consists of the vector potential equation, the continuity equation, the equation of motion, the continuity equation, the energy conservation equation:
\[ \nabla^2 \mathbf{A} = -\mu_0 \mathbf{\delta} \]  
\[ \nabla \mathbf{\delta} = 0 \]  
\[ \rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \nabla) \mathbf{v} \right) = -\nabla p + \rho \mathbf{g} + \nabla \mathbf{\tau} + \mathbf{f}_{\text{em}} \]  
\[ \nabla \mathbf{v} = 0 \]  
\[ \mathbf{A} = \bar{e}_x A_x + \bar{e}_y A_y + \bar{e}_z A_z \]  
where: \( \mathbf{A} \) – vector potential; \( \mathbf{\delta} \) – current density vector; \( \mu_0 \) – magnetic constant; \( \mathbf{v} \) - liquid metal velocity vector; \( \mathbf{\tau} \) – viscous stress tensor; \( \mathbf{f}_{\text{em}} \) – bulk electromagnetic forces; \( \rho \) – pressure in the liquid metal; \( t \) – time, \( c \) – specific heat; \( T \) – temperature; \( H \) – specific enthalpy; \( \lambda \) – thermal conductivity; \( Q \) – heat sources; \( \mathbf{g} \) – gravity acceleration vector.

The solution of system (1) - (5) in general is a difficult task. Therefore, based on the features of the flow of physical processes, when constructing mathematical models, a number of assumptions are introduced, which make it possible to simplify the calculation process without a significant loss of accuracy. The form of writing equations (1) - (3) is determined on the basis of the analysis of the similarity criteria: reynolds number \( Re \approx 4000 \) (mathematical model must take into account the turbulent flow), hartmann number \( Ha \approx 100 \) (external magnetic field will have a significant impact on the movement of the melt in the liquid phase of the ingot), magnetic reynolds number \( Re_m \approx 0,1 \) (calculation of magnetohydrodynamic processes can be carried out in a non-inductive approximation), weber number \( We \approx 0,001 \) (the effect of surface tension plays a significant role in the process of forming the liquid phase of the ingot).

\[ \nabla^2 \mathbf{A} = -\mu_0 \mathbf{\delta} \]  
\[ \nabla \mathbf{\delta} = 0 \]

**Figure 1.** The calculated model of the crystallizing ingot at time (a) and (b).
The complex current density will be:

$$\mathbf{\dot{J}} = -j\gamma \omega \mathbf{A}$$  \hspace{1cm} (8)

where: \(\gamma\) – electrical conductivity; \(\omega = 2\pi f\) – cyclic frequency of the electromagnetic field.

On the boundary of the computational domain for the tangent and the normal component of the magnetic vector potential, the boundary conditions are set [6]:

$$\frac{\partial \mathbf{A}_t}{\partial \mathbf{n}} = 0 \quad \mathbf{A}_n = 0$$

where: \(n\) – normal to the surface of the computational domain.

The initial data for the calculation are the geometrical dimensions of the “inductor-ingot” system, the electrophysical properties of the materials of the computational domain, the value of the frequency of the supply voltage. As a source of electromagnetic field, the current in the inductor is set, which is selected iteratively, until the condition

$$p_{sm} = p_z - \frac{\sigma_{non}}{r}$$  \hspace{1cm} (9)

where: \(p_{sm}\) – electromagnetic pressure; \(p_z\) - hydrostatic pressure; \(\sigma_{non}\) – surface tension coefficient; \(r\) – ingot radius.

The analysis of thermal and hydrodynamic processes in the liquid phase of an ingot crystallizing in an electromagnetic field is reduced to solving the system of equations (3) - (5) in a three-dimensional formulation in a cylindrical coordinate system. Due to the turbulence of the considered currents, thermal and hydrodynamic processes, in the liquid phase of the ingot, are three-dimensional. The design model (Figure 1) contains: 1 — the liquid phase of the ingot, 2 — seed, 3 — solid ingot, 4 — two-phase region.

In accordance with the accepted “LES” model of turbulence and the agreed assumptions, the “filtered” system of Navier – Stokes equations has the form:

$$\mathbf{\nabla} \mathbf{\dot{v}} = 0$$  \hspace{1cm} (10)

$$\rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \mathbf{\nabla}) \mathbf{v} \right) = -\nabla p + \nu \nabla^2 \mathbf{v} - \nabla \tau + \mathbf{f}_{\text{ext}}$$  \hspace{1cm} (11)

where: \(\tau\) – subgrid stress tensor.

In equations (10) and (11), all quantities with the “\(\hat{\ }\)" sign are considered filtered on the computational grid.

The energy equation (5) taking into account the phase transition, takes the form

$$\frac{\partial \rho H}{\partial t} + \nabla (\rho \mathbf{v} \cdot H) = \nabla (\lambda \nabla T) - \left( \frac{\partial \rho \Delta H}{\partial t} + \nabla (\rho \mathbf{v} \Delta H) \right) + Q_{\text{cond}}$$  \hspace{1cm} (12)

In expression (12), \(\Delta H\) is the latent heat, which is expressed through the latent heat of the phase transition of the alloy \(L\):

$$\Delta H = L f_L$$

where: \(f_L\) - volume fraction of fluid in the cell.

The solution of equations (10) and (11) is carried out for the three components of the velocity vector:

$$\mathbf{v} = \mathbf{\hat{e}}_r v_r + \mathbf{\hat{e}}_\phi v_\phi + \mathbf{\hat{e}}_z v_z.$$

The following conditions are set at the boundaries of the computational domain (Table 1).
On the input and output boundaries, all quantities, with the exception of pressure, satisfy the Neumann condition.

**Table 1. Boundary condition of computational domain.**

| Condition | Description |
|-----------|-------------|
| $\frac{\partial \Phi}{\partial z} = 0$ | on the input and output boundaries, all quantities, with the exception of pressure, satisfy the Neumann condition, where $\Phi$ – the set of all quantities used in solving the problem |
| $P_w = 0$ | for pressure on the inlet and outlet boundaries, Dirichlet conditions are fulfilled, imitating the condition of infinite removal of walls |
| $v_n = 0$ | “sticking” and “non-flowing” conditions are set on solid walls, i.e. normal and tangential velocity components on the walls are zero |
| $\frac{\partial F}{\partial n} = 0$ | for the volume fraction of the liquid phase on the wall, the drift conditions are set, where $n$ is the normal vector to the surface of crystallization |
| $-\lambda \frac{\partial T}{\partial r} = \alpha (T_{\text{ros}} - T_{\text{oc}})$ | on the outer surface of the ingot, heat exchange with the environment occurs according to the Newton-Richmann law, where $\alpha$ – heat transfer coefficient; $T_{\text{ros}}$ – ingot surface temperature; $T_{\text{oc}}$ – ambient temperature |
| $T_M = T_{\text{01}}$ | The initial conditions are given: $T_z$ - initial seed temperature 1, $T_M$ - initial metal temperature 2 and the initial velocity in the liquid phase of the ingot 1 |

Electromagnetic forces and Joule dissipation, in expression (11) and (12) are imported from electromagnetic calculation. Equations (10) - (18) completely describe the process of heat and mass transfer in an ingot, which crystallizes in an electromagnetic field.

**4. Analysis of the results of mathematical modeling**

As a result of calculation on a mathematical model, a picture of the velocity distribution in the liquid phase was obtained for an ingot with a diameter of 10 mm, cast at a frequency of 60 kHz and a current in the inductor of 3500 A (Figure 2). The velocity field of the liquid metal has two circulation contours: circuit 1, located near the phase transition boundary and having a direct thermal and mechanical effect on the crystallization front and circuit 2 located higher and providing only heat transfer in the liquid phase of the ingot.
Figure 2. The velocity field in the liquid phase of the ingot.

Figure 3. Dependence of the average metal flow rate in the layer $\delta$ on the inductor current.

Increasing the frequency of the supply voltage from 20 to 60 kHz leads to a decrease in the average velocity of the metal along the crystallization front (Figure 3). So, at a current of 3500 A, $v_{cp}$ decreases 1.5 times from 0.2 to 0.13 m/s. This is due to a decrease in the depth of penetration of the electromagnetic field into the metal and, accordingly, a decrease in the area of the liquid phase covered by the vortex 1. Increasing the casting speed leads to a “deformation” of the temperature field in the ingot in the direction of its extrusion (Figure 4, 5), which leads to an increase in the depth of the liquid phase of the ingot and a change in the shape of the crystallization front. The shape of the crystallization front has a strong influence on the distribution of flows in the hydrodynamic layer.

Figure 4. Picture of the distribution of thermal and hydrodynamic fields in the liquid phase of the ingot at pulling speed 5 mm/s.

Figure 5. Picture of the distribution of thermal and hydrodynamic fields in the liquid phase of the ingot at pulling speed 15 mm/s.

The picture of the numerical distribution of velocity values along the crystallization front as a function of the speed of the ingot extrusion is presented in Figure 6. As can be seen from the graph, the maximum rates of metal circulation along the crystallization front occur when the plane shape of the front corresponds to a casting speed of 5-7 mm/s. With an increase in casting speed, the intensity of MHD processes decreases, from which it can be judged that the effectiveness of the effect of metal circulation on the crystallization process decreases.
5. Experimental study
The effect of electromagnetic stirring on the physical and mechanical properties of the ingot, consider the example of samples of aluminum alloy, which consist Al – 93%, rare earth metals – 7%. Samples with a diameter of 10 mm were obtained by continuous casting into an electromagnetic mold at frequencies of the supply voltage in the range from 20 to 60 kHz. The temperature of the melt before casting was 720 - 730 °C, the inductor current - 3500 A.

The microstructure was investigated according to the diameter of thin section from the edge of the sample to the center at three points. The cooling rate in the considered area of the ingot is 700 K / s. From the considered frequency range, the most coarse-grained structure is formed at a frequency of 60 kHz (Figure 8), which corresponds to the average design velocity of the metal in the hydrodynamic layer equal to 0.12 m / s. At a frequency of 20 kHz (= 0.21 m / s), a uniform fine structure is formed (Figure 7).

![Figure 6](image1.png)

**Figure 6.** Dependence of the average metal velocity at the crystallization front on the ingot pull rate

![Figure 7](image2.png)  ![Figure 8](image3.png)

**Figure 7.** The microstructure of the samples obtained at the frequencies of the supply voltage 20kHz.  **Figure 8.** The microstructure of the samples obtained at the frequencies of the supply voltage 60kHz.

In articles [13,14] also wide shown mane advantages of EMM aluminum casting in such cases like purity, fine dispersion, and most metallurgical, casting and pressing defects free. Mechanisms of clearing alloy from non-metallic particles in such type of induction system, with special metal flow, shown in articles [15].

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