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Dynamical quintessence fields Press-Schechter mass function: detectability and effect on dark haloes

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Abstract. We present an investigations on the influence of dynamical quintessence field models on the formation of non-linear structures. In particular, we focus on the structure traced by the mass function. Our contribution builds on previous studies by considering two potentials not treated previously and investigating the impact of the free parameters of two other models. Our approach emphasises the physical insight into the key role of the evolution of the equation of state: we use the variations in the spherical non-linear collapse caused by quintessence as a function of the model’s potential in a Press Schechter scheme to obtain the differences in halo mass functions. The comparison is also done with the more usual scenario ΛCDM. We conclude that the key role lies within the evolution of the equation of state and that the method displays promises for discrimination using cluster mass determination by upcoming surveys.

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1. Introduction

Recent evidence is piling up in favour of a dark energy component in the dynamics of the universe forming up to 70% of the energy density: from SN1a magnitudes [Riess et al. 1998, Perlmutter et al. 1998]; and more recently [Wang & Mukherjee 2004, Nesseris & Perivolaropoulos 2004, Riess et al. 2004, Daly & Djorgovski 2004, Biesiada et al. 2005]; from the CMB (Cosmic Microwave Background) measured by WMAP (Wilkinson Microwave Anisotropy Probe) coupled with complementary inputs [Bennett et al. 2003, Page et al. 2003] (for reviews, see [Padmanabhan 2003, Sahni 2004]). The nature of this dark energy is still unknown but could lie within two major families: it is either coming from a static cosmological constant or developing from a dynamical scalar field just emerging from subdominance called quintessence [Ratra & Peebles 1988]. The interest of quintessence lies in the possibility it holds to solve the four major problems raised by the cosmological constant model: the problem of a need for fine-tuning the cosmological constant to unnatural values of vacuum energy, the problem of coincidence between its present energy density and the order of magnitude of the critical density, the problem of the equation of state of the cosmological fluid, which can only take one value for a cosmological constant, and the problem of building a model that would naturally come up with solutions to these problems while coming from fundamental physics.

The drive that lead to quintessence models was based on the attempt to address those four problems of the observed behaviour of dark energy. The first problem requires a theory which yields the present value for the dark energy **without the need to tune initial conditions at unnaturally small values compared with** the natural energy scales of the **early universe**. This addresses the huge discrepancy (typically $M_{Pl}/\rho_{DE} \sim 10^{120}$; $M_{Pl}$: Planck mass; $\rho_{DE}$: Dark Energy density) commonly found between classical quantum vacuum energy calculations and its cosmological measurements. The coincidence problem deals with the current transition status of the universe between dominations by matter and dark energy. It requires a theory that yields such a tuned peculiar equilibrium without strong constraints on its initial conditions. it deals with the value of the dark energy component $P/\rho$ ratio, which is constrained to lie within $[-1; -0.6]$ [Wang et al. 2000] or even within $[-1; -0.8]$ [Efstathiou 2000, Hannestad & Mörtsell 2004] (for dark energy equation of state supposed to respect the weak energy condition, thus $P/\rho > -1$). Finally the model building problem express the need to have a theoretically motivated dark energy potential.

The fine tuning problem has been the main drive for quintessence models, hence it is solved by almost all proposed quintessence potentials, involving high energy physics natural energy scales. Various models have been proposed but few are deeply grounded in high energy physics. The main examples of motivated potentials are the original Ratra-Peebles’ potentials [Ratra & Peebles 1988]
that have been linked with the context of global SUSY [Binetruy 1999] and the so-called SUGRA potentials derived from supergravity arguments [Brax & Martin 1999, Brax & Martin 2000]. The tracking property is attracting much interest because it solves the coincidence problem naturally. It defines classes of potentials which Klein-Gordon equation admits a time varying solution that acts as an attractor for wide ranges of field initial conditions. In [Steinhardt et al. 1999], general conditions to obtain a tracking potential are given.

In front of the wealth of scalar fields available from physics beyond the standard model, the need for discrimination between different quintessential models proposed in the literature is a strong drive to confront their predictions with observable features. Since the advent of the COBE satellite results and the pursuit of refined Cosmic Microwave Background Radiation (CMBR) measurements, the attention of cosmologists dealing with quintessence had been focused on CMBR anisotropy measurement as primary probes for the various quintessence models (e.g. Brax et al. 2000). That has foreshadowed the possibility offered by quintessential cosmic dynamics to alter in turn the formation of large scale structures, that are more readily available to observations. Some of the earlier attempts in this direction that have been started out have restricted themselves to linear or perturbation theory of structure formation [Benabed & Bernardeau 2001], to pseudo-quintessence models (approximation of a constant equation of state parameter different than the Λ term; Lokas & Hoffman 2001, Lokas et al. 2003, Kuhlen et al. 2003) or have not pushed the envelope beyond the study of the spherical collapse model [Mainini et al. 2003b, Mota & van de Bruck 2004, Nunes & Mota 2004] or the impact on halo concentrations [Dolag et al. 2004, Kuhlen et al. 2005].

The aim of this paper is to compute the non-linear mass function of collapsed structures in the presence of a quintessence field. One approach is to use direct numerical simulation [Klypin et al. 2003, Linder & Jenkins 2003, Dolag et al. 2004, Macciò et al. 2004, Kuhlen et al. 2003, Solevi et al. 2003], but this is only practical for a few models with well defined parameters and gives little physical insight into the influence of the quintessence potential. Our approach takes into account the fully dynamical nature of the field using the methodology developed by Press & Schechter [Press & Schechter 1974, hereafter PS]. This method uses a spherically symmetric dynamical model to relate the collapse of massive structures to a density threshold in the linearly extrapolated density field. In this way, it is possible to apply Gaussian statistics to the initial density field in order to count the numbers of collapsed structures above a given mass threshold at a particular epoch. Because the method is nearly analytic, it can be applied to rapidly explore the parameter space of possible quintessence models and inverted to constrain quintessence models using observational data. Some approaches using semi-analytical methods have been published in the course of the present work with a restricted range of quintessence potentials [Mammi et al. 2003a, Mammi et al. 2003b, Solevi et al. 2004, Nunes & Mota 2004], some restricting to very indirect observables [Mainini et al. 2003b, Nunes & Mota 2004]. None have yet looked at the same time
at observables like the mass function with many different potentials and an approach aimed at understanding the dominant physics involved, using the full potential of semi-analytical methods.

The paper is laid out as follows. In section 2, homogeneous universe models with quintessence are explored and familiarity with the physics of quintessence is developed. Some results on the behaviour of the universe’s radial scale in various such models are presented, showing the possibility offered by large scale structures to probe the quintessence potentials. In section 3, the collapse of a top hat spherical model for a single primordial inhomogeneity is computed. This provides a simple way of linking perturbations in the initial density field with collapsed structures. In section 4, we combined this model with information from the statistics of the initial inhomogeneities. Using a PS type scheme we obtain predictions for the mass function in the presence of various quintessential models, and compare the evolution of different models. Finally, in section 5 we summarise the results, and explore the constraints that future astronomical surveys will be able to set on the form of the quintessence potential.

2. Homogeneous quintessence evolution

In this section we will review the features in the homogeneous models of dynamical dark energy and emphasize that which points toward an effect on structure formation. This roadmap will later be useful for interpreting our results. The first step is to establish the behaviour of various quintessence models within FLRW-type solutions.

2.1. The dynamical system

In a homogeneous model, the system evolution is entirely defined by the evolution of its scale factor and its quintessence field with its time derivative. The Einstein’s Field Equations governing the Friedman Lemaitre Robertson Walker universe and the Klein-Gordon Equation for the quintessence homogeneous, time dependent, scalar field can be written as evolutions for the scale factor $a$ and the scalar field $Q$.

The first Friedman equation of our model can be expressed as

$$\dot{a}^2 = a^{-1} \left( \Omega_{m_0} + \Omega_{r_0} a^{-1} + \frac{\rho_Q}{\rho_c} a^3 + \Omega_{\Lambda_0} a^3 + \Omega_{k_0} a \right), \quad (2.1)$$

the dots refer to cosmic time derivatives and $'$ will refer (e.g. in Eq.2.2 below) to the total derivative with respect to the field $Q$. Conventionally, subscript 0 refers to the present epoch. $\Omega_X = \rho_X/\rho_c$ represents the density parameter of species $X$, $\rho_c$ is the critical total density required by a flat universe model. We introduce here the density factor $\Omega^\dagger_X = \rho_X/\rho_{c_0}$; note $\Omega^\dagger_{X_0} = \Omega_{X_0}$.

Each term corresponds respectively to the contribution in matter (dark and luminous: $\Omega_{m_0}$), radiation (neutrinos and photons: $\Omega_{r_0}$), cosmological constant ($\Omega_{\Lambda_0}$), either time dependent ($\Omega_Q^\dagger = \frac{\rho_Q}{\rho_{c_0}}$) or present epoch ($\Omega_{Q_0} = \rho_{Q_0}/\rho_{c_0}$, as follows) quintessence density factor and the corresponding FLRW curvature $\Omega_{k_0}$, with the
Dynamical quintessence fields PS mass function

In this work we have assumed throughout a globally flat geometry of the universe \(\Omega_k = 0\) and a fully dynamical dark energy \(\Omega_\Lambda = 0\).

It should be noted that the very non-linear Klein-Gordon equation requires forward integration to reach the tracking solution while the boundary conditions in the density parameters are set for a backward time integration.

The natural units for a primordial quantum field like quintessence are Planck units (Planck time: \(t_{Pl}\)). In those units, expressing the characteristic time, which is the Hubble time: \(t_{H_0}^{-1} = H_0 \simeq 65\, km\cdot s^{-1}\, Mpc^{-1} \simeq 2.10610^{-18}s^{-1} = (8.65 \times 10^{60}t_{Pl})^{-1}\), shows a discrepancy between the two timescales of the problem of more than 60 orders of magnitudes. Since the Planck time should govern the evolution of the field, this could have revealed problematic if we were not focusing on cosmic evolution of the quintessence field: neglecting the effects very of rapid fluctuations, we can observe the evolution of the field as following a regular evolution of its energy density if we adopt the Hubble time as characterizing our system.

We also assumed spatial homogeneity of the field. \[\text{Ma et al. 1999, Lokas & Hoffman 2001}\] argue for homogeneous \(Q\), given that they found the smallest scale of \(Q\) fluctuations to be larger than the clusters scale. Nevertheless it has to be mentioned that their results hold their validity from their studies in the mildly non-linear regime. \[\text{Mota & van de Bruck 2004, Nunes & Mota 2004, Macci` o et al. 2004}\] argue that the highly non-linear regime involved might require some quintessence clustering or coupling to the dark matter. We will concentrate here on effects in non-clustering minimally coupled models as a threshold model to ascertain the impact of quintessence on non-linear clustering.

In Eq. (2.1), where we also assume no curvature nor cosmological constant contribution, the effect of quintessence is focused in the density factor, involving the energy density of the field \(Q\) defined with its potential: in our units it reads

\[
\Omega_Q^1 = \frac{8\pi}{3} \left( \frac{\dot{Q}^2}{2} + \frac{V(Q)}{\mathcal{H}_0^2} \right)
\]

following notations defined above and with \(\dot{Q}\), the quintessence field (Hubble) time derivative, \(V(Q)\) its potential energy and \(\mathcal{H}_0\) the Hubble constant in Planck mass units. Its dynamics is governed by the Klein-Gordon equation in the case of homogeneity:

\[
\ddot{Q} = -\frac{3}{a} \dot{a} \dot{Q} - \mathcal{H}_0^{-2} V'(Q).
\]

Though the pressure of the field is not involved in its homogeneous evolution, it is crucial to the effect of quintessence on non-linear collapse: for a scalar field recall that (with cosmic time) \(P_Q = \left(\frac{1}{2} \dot{Q}^2 - V(Q)\right)\), and together with the density, they define the acceleration state of the universe. One can characterise it using the equation of state

\[
\omega_Q = P_Q / \rho_Q
\]

(see figure 1’s upper panels). In terms of the energy momentum tensor of the field,
Table 1. Our choice of various potentials satisfying the Steinhardt et al. 1999 test. They are discussed in section 2.2. They are all either of inverse power, gaussian or exponential types. $\alpha_Q$ and $\lambda$ are slope parameters, $\Lambda_Q$ characterises the potential’s energy scale and the values of the field $Q$ are in (dimensionless) units of Planck mass. $\kappa = 8\pi G = 8\pi/m_{Pl}^2$ is the gravitational coupling ($\kappa = 8\pi$ in units of $m_{Pl}$).

| Name                        | Potential $V$                                    |
|-----------------------------|--------------------------------------------------|
| R.P. [Ratra & Peebles 1988] | $\frac{\Lambda_{4+\alpha_Q}}{Q^{\alpha_Q}}$    |
| SUGRA [Brax & Martin 2000]  | $\frac{\Lambda_{4+\alpha_Q}}{Q^{\alpha_Q}}e^{-\frac{Q^2}{2}}$ |
| Ferreira & Joyce 1998       | $\Lambda_{4}^{1}e^{-\lambda Q}$                  |
| Steinhardt et al. 1999      | $\Lambda_{4}^{1}e^{-\frac{Q^2}{2}}$              |

recall that for scalar fields with these definitions of density and pressure, we have $\rho_Q \equiv T_{00}; T_{ij} \equiv P_Q \delta_{ij}$.

Now, we will discuss a variety of potentials proposed for quintessence models and their previously known homogeneous properties, emphasizing their impact on matter domination.

2.2. Explorations with several tracking potentials

In this section we restrict our choice upon a set of potentials and recall their previously studied equation of state (Eq. 2.3) and density parameter ($\Omega_Q$) homogeneous evolution, emphasizing behaviours that can affect the formation of large scale structures.

We have narrowed our study on such potentials from the literature that we found to agree (at least marginally) with the Steinhardt et al. 1999 general conditions to obtain a tracking potential. We therefore selected several forms of potential (the list is not exhaustive) [Ratra & Peebles 1988, Brax & Martin 1999, Brax & Martin 2000, Ferreira & Joyce 1998, Steinhardt et al. 1999] for the rest of this study. The Ratra-Peebles potential [Ratra & Peebles 1988, hereafter RP or Ratra-Peebles], first discussed potential in the literature has retained its interest with its more recent discussion within the context of global SUSY [Binetruy 1999]. The [Brax & Martin 2000, hereafter SUGRA] potential has been motivated in the framework of supergravity low energy approximation. The simple exponential [Ferreira & Joyce 1998, hereafter FJ or Ferreira & Joyce] potential displays a generic form for moduli fields from extradimensional theories flat directions while the [Steinhardt et al. 1999, hereafter Steinhardt et al.] potential has been proposed as an infinite sum of Ratra-Peebles-type potentials. This set of potential has been chosen
Dynamical quintessence fields PS mass function

as a well motivated starting point. It also spans the main types of potentials, power laws, gaussian and exponentials and display very different behaviours.

Using those potentials, spelled out in table 1, we can explore the impact of different models on the timescale of the quintessence dominated epoch and the apparent strength of matter domination between radiation and quintessence eras.

We also can compare the observable effects of various potentials at the homogeneous level through the equation of state evolution. The equation of state evolution should be attained by SNIa measurements which constrains directly the integrated luminosity distance evolution, although it relies on SNIa to be good standard candles without systematic errors. In this paper, we are even more interested in the density parameters: since, on the rough, dark energy with its negative pressure has a freezing effect on clustering of dark matter, the differences between potentials in the equivalence epoch for matter-quintessence and in the strength of the matter dominance phase are signs respectively of differing inhibiting times for structure formation and of overall matter clustering activity.

We thus emphasize those features in the otherwise known homogeneous evolution with Ratra-Peebles potentials for the slope $\alpha_Q = 6$ and 11, with the SUGRA potentials for the slope $\alpha_Q = 6$ and 11, Ferreira & Joyce potential for $\lambda = 10$ ‡ and the Steinhardt et al. 1999 potential (the choices for parameter values follow the authors). Comparison of the density parameters evolutions allows to conclude on the fact that we expect a stronger inhibition of structure formation with the Ratra-Peebles potential than with the Steinhardt et al. 1999, than with the SUGRA, than with the Ferreira & Joyce potentials. Within models (i.e. for Ratra-Peebles and SUGRA), variations of their respective evolution points towards the degree of freedom inherent to each potential and thus towards the falsifiability of structure formation tests on each model. Another remark drawn from the density parameters concerns the scales of the structures affected: in the hierarchical CDM scenario, clusters and superclusters scales being formed last, we expect them to be most affected by the changes in inhibition epochs from the various potentials because those epochs occur during the most recent periods and inhibition is expected to act most on them. Eventually it should be stressed that the observational constraints being applied nowadays, differences between models are expected to increase as we look back in time. It should be noted that we do not expect subdominant quintessence to alter the matter radiation equivalence, and in relation, the recombination, thus the power spectrum to remain essentially unchanged for the purpose of structure formation.

The lower panels of figure 1 show some changes for the Ratra-Peebles model that are much less pronounced in the case of the SUGRA model, and also illustrate, together with the FJ/Steinhardt et al. panel, the variations in equivalence epochs and strength of matter domination. The upper panels all illustrate the fact that the

‡ This choice is historical and does not allow the marginal tracking behaviour of FJ to be reached. It allows nevertheless to explore a very different behaviour of the equation of state that yields crucial insights (see section 4.3).
Dynamical quintessence fields PS mass function

**Figure 1.** Calculations using the Ratra-Peebles [Ratra & Peebles 1988] and SUGRA [Brax & Martin 2000] models for the shape parameter values $\alpha_Q = 6$ and 11, the Ferreira & Joyce [Ferreira & Joyce 1998] and Steinhardt et al. [1999] models. The equations of state $\omega_Q$ of all models are definitely not constant over structure formation era. The [Ferreira & Joyce 1998] model, although constant for a while, also varies strongly at recent epochs. The density parameters show (i) matter-radiation equivalence is unaffected by subdominant quintessence, (ii) changes between models are backwards since they are jointed by present density conditions, (iii) models differ in matter quintessence equivalence (which late epoch poses the so called coincidence problem) and (iv) models differ in the strength of matter dominance. For both Ratra-Peebles models, these discrepancy illustrate the variability of this potential and relativize its testability. For both SUGRA models, they illustrate the narrow variability of this potential and emphasize its testability, although distinguishing between the shape parameters may prove difficult. The equations of state shows that the models are well accommodating the measurements [Wang et al. 2000, Efstathiou 2000], except for the Ratra-Peebles models which is their know pitfall.

The equation of state cannot be considered constant during the epoch of structure formation (contrary to Lokas & Hoffman 2001).

The solving of Eqs.(2.1, 2.2) was effected with a second order Runge-Kutta integration method. As previously mentioned, giving initial conditions for each potential shown in table 1 is not constrained with input observations. The search for tracking
solutions excludes reverse integration methods: the selection of the tracker solution which leads to observed quintessence density was obtained with a simple iteration on the characteristic energy scale \( \Lambda_Q \) with respect to the target final quintessential density.

The cosmic time numerical increment is chosen so as to keep a constant logarithmic scale factor increment: \( dt = \alpha \frac{\dot{a}}{a} \), where \( \alpha \) is fixed (we usually take \( \alpha = 10^{-2} \), a value of \( 10^{-3} \) changing the outcome by less than 1%).

Given the roadmap that we now have on variations between models in the influence of quintessence on large scale structure formation. We are going to effectively test them in the mass functions of cosmological haloes within quintessential context as computed by the PS scheme [Press & Schechter 1974]. First, in order to get the mass function, we need to study the spherical collapse model. This is done in the next section.

3. Spherical collapse in the presence of quintessence

The influence of quintessence models on structure formation comes from the repulsive gravitational effect of its negative pressure on the bulk of spacetime during its phase of less than \(-1/3\) equation of state. Collapsing structures are then slowed down in their build up by this relative aggravated expansion. An elementary model of this phenomenon can be used to get simple quantitative results that can then be inputted in a PS-type scheme that will be discussed in section 4: the spherical collapse model (pioneered in [Larson 1969, Penston 1969, Gunn & Gott 1972, Fillmore & Goldreich 1984, Bertschinger 1985] and summarised in [Peebles 1980]) in the presence of quintessence holds the key to this exploration from the beginning of the collapse phase after recombination down to shell crossing. After shell crossing, mass conservation does not allow to follow the system with just its outer shell but a prescription can be used for the virialization of the model. We will now describe the dynamics of the cosmological spherical collapse.

3.1. The dynamical system

The cosmological spherical collapse is embedded into an FLRW-type universe for which all the component of energy density are supplemented with a spherical overdense region. Birkhoff’s theorem holds the key to the spherical non linear collapse model: any spherical region embedded in a spherically symmetric universe behaves shell by shell as a patch of FLRW universe with each characteristics modified to match the corresponding average inner ones. In this case, any given shell behaves according to the average of its inner density. With a flat background this leads to positive curvature inside the overdensity. However, because this inner curvature may change with time, one has to be cautious in using the Friedmann’s equations [Mota & van de Bruck 2004].

For simplicity reasons and because it is a building block of the original PS scheme, we will use the spherical top hat model (sphere of constant overdensity). In this case, the averages are equal to the local values and every shell reach the center of the overdensity.
Dynamical quintessence fields PS mass function

at the same time, when shell crossing occurs. Problematic for the virialization, this feature allows one to follow only the evolution of the outermost shell representing the whole system. Virialization is then assumed to take place soon after shell crossing and a halo of mass given by the extent of the initial overdensity is formed.

Thus the radius \( r_{od} = r \) of the overdensity can be written as a function of its initial value \( r_{od} = r_i as r_{od}(t) = r_{od}a_{k>0}(t) = r_i s(t) \), where we note the scale factor of the positive curvature patch \( a_k > 0 \) as a fiducial or rescaled radius of the overdensity region \( s = r/r_i \). Thus the radius of the overdensity follows Friedman’s equation for the modified overdensity patch of universe. For this shifted FLRW model, the Einstein’s Equations yield the evolution (recall \( \kappa = 8\pi G \), the gravitational coupling):

\[
\dot{s}^2 = \frac{\kappa s^2}{3} \sum \rho - k(t), \tag{3.1}
\]

\[
\ddot{s} = -s \left[ \frac{\kappa}{6} \left( \sum (3P + \rho) \right) \right] \tag{3.2}
\]

where the sums on energy density and pressure concern each cosmic species inside the patch radius. Again note that the curvature \( k \) inside the overdensity is not constant in general, so that we prefer to use Eq.\( (3.2) \). Since we assumed homogeneous behaviour for the scalar field (i.e. no clustering), that means that there is no conservation inside the spherical patch for the quintessence field \[\text{Mota & van de Bruck 2004}\] and its pressure is that for the background universe.

The acceleration follows the shifted second Friedman equation (Eq. \( 3.3 \)), which contains as well the pressure terms. The density term for the matter follows the same pattern as for Eq. \( 3.1 \):

\[
-\frac{\kappa r}{6} \dot{\rho} = -\frac{4\pi Gr}{3}\rho = -\frac{GM}{r^2} ,
\]

with the total mass \( M \) conserved and contained initially inside the spherical patch. The pressure terms depend on the respective state equations. Hence the acceleration equation reads, with our time and radius units,

\[
\ddot{s} = \left[ \lambda_0 - \left( \Omega_m a^{-4} + \frac{4\pi G}{3H_0^2} (3P + \rho_Q) \right) \right] s - \frac{\Omega_m a_i^{-3} (1 + \Delta_i)}{2s^2} . \tag{3.3}
\]

Initial conditions of the homogeneous evolution are taken after inflation \((a = 10^{-30})\), the field is taken in a reasonable range allowing for the tracking solution to establish. For the collapse evolution, initial time is chosen in the relevant overdensity linear regime (we usually take the arbitrary cut \( a_i = 10^{-5} \)), the overdensity is set in a Hubble flow, that is following the general expansion of the universe at the initial onset of the overdensity, (the definition of \( s \) sets its initial condition to be \( s_i = 1 \) at initial time) so

\[
\dot{s}_i = \frac{\dot{a}_i}{a_i} . \tag{3.4}
\]

In the model of the spherical collapse, the non-linear density at the border of the overdense region can be monitored using the Lagrangian mass
For the calculation of the overdensity, one just follows the canonical definition \( \delta = \frac{\rho_m - \rho_b}{\rho_b} \), then
\[
\delta = (1 + \Delta_i) \left( \frac{a}{a_i s} \right)^3 - 1. \tag{3.5}
\]

The corresponding initial overdensity value thus proceeds from Eq.(3.3) at \( a = a_i \) and \( s = 1 \) and using Eq.(3.4):
\[
\begin{align*}
\delta_i &= \Delta_i, \tag{3.6} \\
\dot{\delta}_i &= 3(\delta_i + 1) \left( \frac{\dot{a}}{a} - \frac{\dot{s}}{s} \right) = 0. \tag{3.7}
\end{align*}
\]

Here we have used a fiducial initial time from which the collapse was evolved. This choice is of course arbitrary and in the PS scheme, this arbitrariness is resolved by the extrapolating to present time (redshift \( z = 0 \)) the initial overdensity \( \delta_i = \Delta_i \) using the linear evolution theory
\[
\ddot{\delta}_L + 2\frac{\dot{a}}{a} \dot{\delta}_L = \frac{3}{2} H_0^2 \Omega_m a_i^{-3} \delta_L. \tag{3.8}
\]
This theory is only valid for overdensities in the linear regime (\( \delta \ll 1 \)) but the non-linear collapse leads to infinite density at a finite time given by \( a = a_c \) while the linear theory allows the density to remain finite up to \( z = 0 \).

The immediate interest of solving this model lies in the possible comparison between different collapse times. The relevant quantity extracted was the function \( \delta_{c0} = f(a_c) \),
the linearly extrapolated overdensity as a function of collapse epoch expressed in terms of the scale factor. Given a potential, we construct its characteristic extrapolated density contrast function of collapse scale: each initial overdensity yields both the collapse scale and extrapolated linear scale using the non linear and linear collapse respectively (see figure 2). The construction of the function involved scanning a whole range of initial overdensities as illustrated in figure 2. The critical density contrast function of collapse scale factor $a_c$ is constructed by the linear extrapolation to the present from the arbitrarily chosen initial epoch’s starting overdensity. Therefore $\delta_c$ yields an object that collapses at an epoch given by the value of scale factor $a_c$. Thus $\delta_c$ is found by evolving our initial conditions with Eq.(3.8)’s linear theory, and the corresponding collapse scale is found by the non-linear collapse of Eq.(3.3) – given by the vertical asymptote at $a_c$, in the homogeneous background provided by Eqs.(2.1, 2.2).

For implementing this scheme, we first determine the correct QCDM potential energy scale, as described in section 2.2, then we bring the model to the field’s tracking regime and produce initial parameters for the spherical collapse. Eventually the coupled evolution of the background FLRW, the quintessence field, the non-linear spherical collapse and the linear overdensity evolution obtains the values of $\delta_c(a_c)$.

A fourth order Runge-Kutta integration method was implemented over the whole Eqs.(2.1, 2.2, 3.3, 3.8) system. We still used the logarithmic increment $dt = \alpha_s \dot{s}$, where we usually take $\alpha = 10^{-2}$, as described in section 2.2, but we had to limit the lower increment value of $dt$ to avoid inflation of numerical expenses when integration approaches the overdensity turnaround point.

We are now ready to use the Top Hat Spherical collapse model to decide when objects are considered to have collapsed, that is when their non-linear overdensity diverges. The following section discuss those first results.

### 3.2. Critical densities

Once we have computed a series of $\delta_c(a_c)$, for a range of initial conditions, set in the linear regime, at our arbitrary starting epoch, we can display for each model their extrapolated critical overdensity as a function of collapse scale factor and compare them among models, which is done in figures 3 (comparing pseudo-quintessence models) and 4 (for dynamical quintessence).

Since we are dealing with density contrasts over the background cosmological matter density, the main evolution effects are expected to come from the lower homogeneous matter density yielded by the quintessence models during their matter dominated era. Also the observation setting the models corresponding to $z=0$ ($a = \frac{1}{1+z} = 1$; present epoch), we expect the differentiation between models to increase backwards in time.

A comparison between our various potentials, the $\Lambda$CDM model§ and three pseudo-§ It should be noted that for a standard CDM ($\Omega_m = 1$) model, the spherical collapse yields a linear evolution $\delta = \delta_i a/a_i$ [Peebles 1980] so, extrapolating the collapse value nowadays, $\delta_c = \delta_c a/a_c$ which gives the straight line asymptote to each models in these log-log coordinates and the well known
quintessential models (i.e. with constant equation of state) have been performed. The overdensity as a function of collapse shows clear distinctions between models are possible. The results obtained are plotted on figures 3 for the pseudo quintessences and two common models (Ratra-Peebles and SUGRA), and 4 for all our studied models.

The pseudo-quintessence models are shown for reference and comparison to other works even though our homogeneous study has pointed that such an assumption is highly unlikely to hold compared to real quintessence models (e.g. figure 1’s upper panels). However figure 3 reveals that even though the region scanned by the pseudo-quintessence models is covering most of the values taken by our sample of models (except for Ratra-Peebles), there is still a strong distinction if measurements are confronted at different epoch since no pseudo-quintessence model is matching quintessence evolution; for instance, the SUGRA11 (notation defined in the figure caption) model cannot be fitted with only one wQCDM model.

The main point is that there are distinctions in this type of representations between the various models when examined at earlier epochs and between successive era. Whilst evaluation of variability for the SUGRA model yields very little separation, that of the Ratra-Peebles are so large that it allows for an observational selection of the free power index. Whereas the FJ model crosses the curves from various values of the free power index in the Ratra-Peebles model, the [Steinhardt et al. 1991] model is not very $\delta_c = 1.686$. 

**Figure 3.** Comparison between quintessence and pseudo-quintessence models through their influence on non-linear collapse scale, function for the linearly extrapolated overdensity. The notations are following table 1 with the number referring to the index value used in the power of the potential (e.g. RP11: Ratra-Peebles with $\alpha_Q = 11$). Pseudoquintessence models are referred to by the value of their ad hoc equation of state. The spread of curves shows possible distinctions between models. The crossing of pseudo-quintessence models by SUGRA11 shows none can mimic its evolution. LCDM, or ACDM, the boundary cosmological constant model, is shown for reference.
Figure 4. Comparison between quintessence models through their influence on non-linear collapse scale as a function of linearly extrapolated overdensity. The notations are following table 1 with the number referring to the free index value used in the power of the potential. The spread of curves shows possible distinctions between models (e.g. SUGRA6: SUGRA with $\alpha_Q = 6$). LCDM, or ΛCDM, the boundary cosmological constant model, is shown for reference.

distinguishable from the SUGRA6. However we will see that the sensitivity of the mass function to the overdensity allows for some distinction.

To summarise, it is the departure, in quintessential spherical collapse, from the constant value of the collapse critical overdensity of the standard CDM collapse that will affect the mass function. On figures 3 and 4, this is translated into the departure from the simple linear log-log relation of the critical overdensity with the scale factor. Thus the linear density contrast indexed by the non-linear scale of collapse sets the redshift evolution of the PS mass function observable as we will see in the next section.

4. The mass function in presence of a quintessence field

In this section we will lay down our assumptions for constructing the mass dispersion of our models which imprints initial conditions in the PS scheme, we will then recall it before discussing the integrated mass functions for the quintessential models studied.

4.1. Mass dispersion

Initial conditions of large scale structure formation are condensed in the PS scheme into their mass dispersion. To do this for gaussian fluctuations the basic information lies in the density field power spectrum.

In general, the Fourier power spectrum $P(k)$ for the density fluctuations of scale $\frac{1}{k}$ can be decomposed into a primordial part and a linear evolution part, written as follows
Dynamical quintessence fields PS mass function

\[ P(k, a) = B k^n T_Q(k, a)^2 = B k^n T_\Lambda(k, a)^2 T_{Q/\Lambda}(k, a)^2 \]  

where B is a normalization factor and \( T_Q \) the quintessence transfer function written as \( T_Q = T_\Lambda \). \( T_{Q/\Lambda} \), \( T_\Lambda \) being the \( \Lambda \) dark energy transfer function. The transfer function is itself written as: \( T(k, a) = T(k, a = a_i) D(a)/D(a_i) \) where the linear growth D gives the time evolution.

In the following section, we briefly decompose the construction of the structure formation initial spectrum between a primordial part and a linear evolution, we discuss our assumed spectrum and show the mass dispersion and its normalisation to observations.

4.1.1. Primordial Power Spectrum

In the context of inflation, for commonly used potentials, one gets a Gaussian primordial spectrum \( P(k, a_{init}) = |\delta(k, a_{init})|^2 = \lambda_{inf.} k \) (where \( a_{init} \) is the epoch at which a perturbation of size \( l=1/k \) is reentering inside the horizon): i.e the scale invariant Harrison-Zeldovich spectrum, with \( \lambda_{inf.} \) and A constants depending the inflationary potential and expansion period.

4.1.2. Linear Power Spectrum

To describe the growth of the fluctuations and the evolution of the initial power spectrum \( P(k, a_{init}) \) one usually evolves linearly fluctuations up to a given epoch \( a \) (typically after recombination when \( 1/a - 1 = z < 10^3 \)) as \( P(k, t) = P(k, a_{init}) T^2(k, a) \), where T is the above transfer function.

Previous studies have shown that \( P(k) \) depends only weakly on the equation of state \( w_Q(a) = P(a)/\rho(a) \) since the Q field is sub-dominant at epochs where z is large (e.g., at recombination, \( z \sim 10^3 \)). In the case of a universe dominated by cold dark matter (CDM) the initial T(k) function is known to be well fitted \[ \text{Bardeen et al. 1986, Sugiyama 1995} \]. The \( \Lambda \text{CDM} \) (case \( w_Q=-1 \)) transfer function \( T_\Lambda \) fit is very close to the CDM function with a slightly smaller cutoff scale \[ \text{Efstathiou et al. 1992} \] and identical power laws. Flat cosmologies transfer functions fits from N-body simulations of pseudoQuintessence models \[ \text{models with constant equation of state } w_Q \] have been proposed in the form \( T_Q = T_\Lambda \). \( T_{Q/\Lambda} \) \[ \text{Ma et al. 1999} \]. They show only modifications to \( T_\Lambda \) of order unity in the observationally relevant range of \( w_Q \).

Following previous work, we therefore apply the same initial density fluctuation power spectrum to all of the models. This ensures that the differences that we see result from the dynamical evolution of the density fluctuations rather than differences in the initial power spectrum. In practice, CMBR experiments will establish the fluctuation power spectrum independently of the mass functions that we consider here and a successful model will have to successfully reproduce both datasets. We thus consider that the \( T_\Lambda \) function given for a model with a cosmological constant can describe suitably the fluctuations and do not allow the Q field to influence the initial power spectrum of density fluctuations. We therefore adopt the \[ \text{Bond & Efstathiou 1984} \] form for the...
Dynamical quintessence fields PS mass function

4.1.3. mass dispersion

The mass dispersion $\sigma^2(M, a)$ is computed using a top-hat filter in real space with a radius of filtering $R$, corresponding to $M = 4\pi \rho_b(a) R^3 / 3$, $\rho_b(a)$ being the density of mass of the Universe at a given epoch marked with the scale factor $a$ (thus we can express $R(M) = (3M/4\pi \rho_b(a))^{1/3}$).

One has, for the filtered mass dispersion, the relation:

$$\sigma_Q^2(R, a) = D^2(a) \int d^3k |W(R, k)|^2 P(k) / 8\pi^3$$

with the filter Fourier transform $W$ (which is here the top hat filter), indices in $Q$ denotes the model dependence. Thus we get the relation:

$$\sigma_Q^2(R, a) = \sigma_Q^2(f(a) M^{1/3}, a) = D^2(a) \int dk |W(R, k)|^2 P_{BondiEfstathion}(k) / 8\pi^3$$

with $f(a) = (3/4\pi \rho_b(a))^{1/3}$ and $D$, the linear growing mode for fluctuations.

4.1.4. spectrum normalization

To get the value of the constant in $P(k, a)$ one usually normalizes $\sigma$ to the value $\sigma_{sh^{-1}}$ which is observed today for a typical radius $R = 8h^{-1}$Mpc (here we take $h=0.65$) and to reproduce the present amplitude of the density contrast one gets that $\sigma_{sh^{-1}}$ is of order 1 (see for example reference [Eke et al. 1996] that yields the product $\sigma_{sh^{-1}} \Omega_0^{1/2} = 0.6$, or $\sigma_{sh^{-1}} = 1.1$ for $\Omega_0 = 0.3$).

4.2. The mass function evaluation

To compute the mass function in the non-linear regime without using fits to N-body simulations we restrict here to the popular PS prescription.

This semi-analytical tool turns out to be the simplest way to construct a mass function from the assumed gaussian statistics of the initial density. It neglects a large number of effects that tend to compensate each other yielding often accurate fits to the numerical simulations.

It is yet an empirical recipe yielding the number of (collapsed) objects with a mass greater than a given one [Press & Schechter 1974].

Hence, we obtain the fraction of collapsed mass linearly extrapolated at a selected present time $t_0$ defined by $\delta_{c0}(t_c)$ as in section (3.1), and account for Eq.(4.3) for the variance $\sigma$ itself linearly extrapolated (and normalised with $\sigma_8$) at the same time $t_0$. The mass fraction thus writes:

$$F(m > M, a) = 1 - Erf(\delta_{cr}(a)/2\sigma(M, a)),$$

and its derivative with respect to the mass leads to the density of collapsed objects

\[\text{note their code was built to encompass } \Lambda\text{CDM dark energy only}\]
Dynamical quintessence fields PS mass function

\[ n_{PS}(m, a) = -\rho_b(a) (dF(m > M, a)/dM) /M \]

\[ = (2/\pi)^{1/2} (\rho_b(a)\delta_{c_0}(a)) (d\sigma(M, a)/dM) / (M\sigma^2(M, a)) \]

\[ \cdot \exp \left( -\frac{\delta_{c_0}^2}{2\sigma^2(M, a)} \right) \]  \hspace{1cm} (4.5)

\[ = (-\rho_b/\sigma M) (d\sigma(M, a)/dM) F_{PS}(M) \] \hspace{1cm} (4.6)

with the characteristic for the PS scheme condensed into the function

\[ F_{PS}(M) = (2/\pi)^{1/2} (\delta_{c_0}(a)/\sigma(M, a)) \exp \left( -\frac{\delta_{c_0}^2(a)}{2\sigma^2(M, a)} \right) \] \hspace{1cm} (4.7)

that can be replaced in later studies with more complex schemes.

We now have set in place the machinery for computing mass functions for non-linear structures in the presence of non-clustering dynamical quintessence. We will then present comparisons of integrated mass functions that can be obtained for the potentials we selected in the following section.

4.3. Integrated mass functions

One of the main interests of this study is to have shown the sensitivity of \( \delta_{c_0}(a) \) as function of the chosen \( Q \) field potential \( V(Q) \). This can be seen above (section 3.2) with figures 3 and 4 when dealing with the collapse of an initial overdensity. Placing these results within the PS scheme, we have found that some differences among the various models can emerge from the mass functions. In figure 5, the upper panel shows as a reference the mass functions for all the selected models at present time \((z=0)\). The models there are all very close to each others, which reflects our \( z=0 \) normalisation and confirms the usual hypothesis taken of a normalization of the mass functions to present days observations. This is why figure 5’s lower panel only recalls the LCDM (i.e. \( \Lambda \)CDM) mass function as a reference.

When we examine the mass function at larger \( z \), as shown on figure 5’s upper panel for \( z=0.5 \) and lower panel for \( z=1 \), discrimination can occur between the various models. All models’ mass functions are lying above the LCDM (i.e. \( \Lambda \)CDM) curve for the same epoch. The distinctions remain nevertheless mild at \( z=0.5 \), as shown by the relative proximity between the curves; they, however, become more pronounced at \( z=1 \), with the following observations: the FJ model displays the most structure suppression, that is the least evolution of mass function, i.e. the least structure formation during recent times. Then, in the same decreasing hierarchy, one encounters the Ratra-Peebles models, in decreasing order of their power law, the Steinhardt et al. model, the SUGRA models, also in decreasing order of their power law, before reaching the \( \Lambda \)CDM model (the Steinhardt et al. and the SUGRA11 models’ mass functions are almost on top of each other). Surprisingly, those results are not completely agreeing with the naive interpretation given in section 2.2, that would have yielded the same hierarchy of structure inhibition and matter dominance except for the FJ model, that would be expected to have a lower mass function than all
Figure 5. Computation of the integrated mass function for various quintessence potentials for past epochs, $z=0.5$ ($a=2/3$) and $z=1$ ($a=1/2$), compared with the present epoch ($z=0$). The upper panels lack of dispersion among mass functions for all models at $z=0$ illustrates our normalisation and the accuracy of our method. The spreads for past epochs, even more so at the earliest epoch, allows for confrontation with cluster mass measurements.
other quintessence models (see figure 1’s lower panels). Confronted with their lookback effects on the number of structures present at a certain epoch, these are both agreeing with the mass function hierarchy except for the ordering of the FJ model. We are thus compelled to consider the only qualitative difference that springs to the eye between the FJ model and the rest of them: the evolution of the equation of state (figure 1’s upper panels). The FJ model appears as the only one with \( \omega_Q \) evolving towards higher values at its latest stages, during structure formation. It seems thus that the effect of its negative pressure in the acceleration equation of the spherical collapse (Eq. 3.3) is strong enough to counter the strength of matter domination earlier than expected in the natural first approach.

This can be seen from the ratio of quintessence- and matter-induced accelerations of the overdensity, from Eq.(3.3), so we get \((3|\omega_Q| - 1)\frac{\Omega_Q}{\Omega_m} > \frac{(1+\delta_i)}{2(a_i s/a)}\) despite having \( \Omega_Q \) small at a given stage of collapse and cosmic evolution. These results cast very interesting light into the intimate mechanisms of quintessence that would escape to non semi-analytic approaches.

The evaluation of model’s variability is expected after section 3.2’s more or less pronounced differences among each class of model and their respective parameter variations (Ratra-Peebles and SUGRA). In the light of integrated mass functions, each model evaluated displays little variability so discrimination among power law values may prove difficult with this method.

5. Conclusions

In this paper we have extended the evidence that several different recent models of quintessence predict significant difference in the evolution of the halo mass function, in general agreement with previously more restricted works (Mainini et al. 2003a, Mainini et al. 2003b, Nunes & Mota 2004, Solevi et al. 2004), and used our wider range of potentials to emphasize the impact of the equation of state in the formation of dark haloes. Although the best evidence seems to come from the largest mass structures, as expected from the homogeneous exploration (section 2.2), the models studied do not behave simply according to matter and quintessence dominance epochs. Nevertheless, the sensitivity of mass functions to the value of \( \delta_{c_0} \) at the limits of accuracy cautions us against strong assertions.

The models can be distinguished if observational measurements of the cluster mass function at \( z \sim 1 \) achieve a precision of better than \( \sim 10\% \) at \( 10^{14} h^{-1} M_\odot \). Because the models are normalised to give the same mass function at \( z = 0 \), a higher level of precision, approaching 1%, is required to distinguish between the models at \( z = 0.5 \). Several experiments are planned that will exceed this level of accuracy and will have good control of the systematic errors.

One of the most promising is the South Pole Telescope (SPT) survey - a cosmic microwave background experiment aimed at detecting the Sunyaev-Zeldovich effect (SZE) due to clusters of galaxies (Ruhl et al. 2004). The effect arises because inverse Compton scattering by the hot intra-cluster plasma distorts the spectrum of the cosmic
microwave background. Importantly, the amplitude of the decrement is independent of the cluster redshift, and the integrated signal decreases only with the clusters angular diameter. The aim of the South Pole Telescope experiment is to map a region of 4000 deg.$^2$, detecting essentially all clusters with masses greater than $2 \times 10^{14} h^{-1} M_\odot$ and with redshift less than 2 [Carlstrom et al. 2002]. The survey is expected to detect around 30,000 clusters in the ΛCDM cosmology, with 30% expected to have redshifts greater than 0.8. The amplitude of the SZ decrement is determined by the integrated pressure of the intra-cluster plasma, and can thus be used to determine the cluster mass. Random errors in the normalisation of the mass function will be $\sim 1\%$ at both $z \sim 1$ and $z \sim 0.5$. Systematic errors in the determination of mass and the scatter between cluster mass and the observed SZ decrement are a significant source of concern, but these can be controlled using the “self calibration” techniques discussed in [Hu 2003] and [Majumdar & Mohr 2003]. An other technique based on the optical detection of clusters may also yield promising results when combined with similar self-calibration methods [Gladder & Yee 2005].

These observational programmes will map the development of the mass function from $z = 1$ to $z = 0$ at the level precision that is required to distinguish between the quintessence models considered in this paper. The techniques we have developed allow this map to be directly related to the form and parameterisation of the quintessence potential.

Although this work focuses on the actual mass functions of studied models, that is on the real volume number densities predicted, some authors have argued that detection of such density should be hindered by the global geometric effect of dark energy [Solevi et al. 2004, Solevi et al. 2005]. However, their treatment of observed cluster samples with a unique mass cut-off forgets about the bias-geometry dependence, discussed by [Kaiser 1984] and retained as mass-observable relation in dark energy studies [Mohr 2004], that should certainly call for further scrutiny. Such a study is in progress that should test whether a break of degeneracy is induced by this bias-geometry dependence and that will be at the core of a companion paper.

More potentials have attracted attention [Barreiro et al. 2000, Albrecht & Skordis 2000] and although they combine our studied exponential and power laws, they should be confronted using our method. Confirmation could be sought using parallel semi-analytical methods like the [Jenkins et al 2001] one in the context of quintessence [Linder & Jenkins 2003]. Eventually, the homogeneity and minimal coupling of the field lacks proof in the highly non-linear regime [Mota & van de Bruck 2004, Nunes & Mota 2004] and calls for some wider explorations in the line of [Mota & van de Bruck 2004, Nunes & Mota 2004, Macciò et al. 2004]. This will be the subject of a follow up paper.
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