Energy accumulation in relativistic sub-Bohr orbitals, Franklin’s relativistic rotation of quarks and gravitational field bounceback as processes relevant to explosion of supernovae

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Abstract. We discuss exploding stars exceeding $8 \, M_{\text{Sun}}$. Supernova processes start with implosion before the explosion. The processes causing implosion are relatively well understood. Not so the processes that reverse the implosion into explosion. We propose and discuss three possible processes that can reverse implosion of a star into an explosion, causing supernova: a) energy accumulation in sub-Bohr electron orbitals populated by an electron moving at high relativistic velocities [1], b) Franklin’s relativistic rotation of quarks [2], [3] and c) Einstein’s gravitational field bounceback [4]. The relativistic sub-Bohr orbitals are derived by using an analogy to exponential gravitation (recent refs [1], [4], [5], [6], [7] and [8]).

1. Supernova

Supernova type II is a kind of exploding star, in which the whole star first collapses and then bounces back into an explosion in a short time. A supernova can outshine its entire galaxy during the first days after the explosion. Matter is ejected at velocities up to thousands of km/sec [9], [10], creating a propagating cloud of supernova remnants, and usually also a neutron star remnant is inferred. The remnants are not investigated here. One of the possibilities that initiate a supernova explosion is core collapse when the core exceeds Chandrasekhar mass of $1.4 \, M_{\text{Sun}}$. A neutron star may be thus created.

Thousands of observations of supernovae led to a classification of many types of supernovae. See detailed reviews [11] and [12] Chapter 8. We are dealing here with type II supernovae caused by explosion of stars exceeding $8 \, M_{\text{Sun}}$.

In a star the gravitational attraction, which tends to cause the collapse of the star, is balanced by the pressure caused by exothermic nuclear reactions, in this case fusion. When the exothermic nuclear fuel in the core of the star is exhausted, the star core collapses causing collapse of the outer shells of the star as well.

Main heat processes in stars usually considered in simulations of supernovae include fusion of:
(i) $^4\text{H}$, which join to form $^4\text{He}$ (by what is called CNO process), while releasing energy that works against gravitational implosion,

(ii) $^3\text{He}$, which join to $^{12}\text{C}$ while releasing energy that works against gravitational implosion.

Other exothermic reactions produce nuclei up to $^{56}\text{Fe}$ if the star is sufficiently massive and if processes that initiate earlier implosion are avoided, namely

(i) Electron-positron pair production,

(ii) Electron capture (relevant to nuclei like $^{16}\text{O}$, $^{32}\text{Si}$ or few isotopes of Mg or Fe [12], [13], [14]).

If one of these processes occurs before $^{56}\text{Fe}$ production, a supernova can occur earlier. The conditions for these two processes to occur and trigger supernova are known ([13], [14]; [12] for review).

If the evolution of the star avoids these two processes then at temperature approaching $10^{10}\text{K}$, corresponding to 0.85 MeV, $^{56}\text{Fe}$ disintegrates to $^{13}\text{He} + 4$ neutrons, which is an endothermic process that contributes to collapse and implosion because it demands 2.22 MeV per nucleon.

At temperature approaching $2\times10^{10}\text{K}$, corresponding to 1.7 MeV, $^4\text{He}$ disintegrates endothermically to 4 nucleons, enhancing implosion because this demands 8 MeV per nucleon.

Computer simulations under many different assumptions (star rotation, neutrino pressure, different state equations, magnetic field, different nuclear reactions, electron capture, pair production, general relativity effects, convection) show implosion, followed by weak bounceback, but not bounce back to an explosion so violent that it blows up almost the entire star, causing the supernova [11], [12], [15], [16]. Podsiadlowski [11] wrote: "But... this energy [of the outward propagating shock] is quickly consumed; a prompt shock is always found to stall and is unable to drive an explosion." and "It has remained one of the most enduring unsolved problems in supernova physics, how a fraction of the implosion energy can be deposited just below the accretion shock [during implosion] and be allowed to accumulate till enough energy is available to drive an explosion."

Shaviv [12] notes that "once the collapse starts, the basic problem is how the infall of the outer layers onto the core is transformed into an explosion which ejects the envelope at speeds of thousands of kilometer per second. This problem is very complex, probably involves not yet known physics, and has not yet been completely solved."

Woosley and Heger [16] concluded that "to this day, a first-principle model of how the explosion is energized is lacking."

A "List of unsolved problems in physics" [17] poses the problem: What is the exact mechanism by which an implosion of a dying star becomes an explosion?

Figure 1 illustrates the problem schematically.

Without sufficient storage of energy for explosion, the result is implosion to what is considered as black hole, but we observe supernova explosions!

Following are three proposals for energy accumulation and release. One of them includes also deployment of significant additional energy.

2. **Exponential gravitation and exponential electric charge: Sub-Bohr orbitals**

Our first proposal is based on variable electric charge [1], analogous to variable mass gravitation [1], [4], [5], [6], [7] and [8], which we summarize in short below.

First about variable mass gravitation.

The force between a small and a large mass $m$ and $M$ interacting gravitationally is

$$ F = -m(r)MG/r^2 $$

(1)
We let $m$ to vary with $r$ (the distance between point masses $m$ and $M$). The variation of $r$ is quasistatic. The force is the negative of the derivative of the energy $E(r)$:

$$F = -\frac{d[E(r)]}{dr} = -\frac{d[m(r)c^2]}{dr} = -c^2\frac{d[m(r)]}{dr}$$

(2)

Equating (1) to (2):

$$m(r)MG/r^2 = c^2\frac{d[m(r)]}{dr}$$

(3)

Solving, we obtain variable exponential mass

$$m(r) = m_\infty \exp(-MG/c^2r)$$

(4)

for masses $m(r)$ and $M$ with zero velocity. $m_\infty$ is the value of the mass $m(r)$ at a distance (from $M$) $r \rightarrow \infty$. Others have also arrived at the same solution (4) (Milne [18], [19], [20], [21] and [22], Majerník [23], Yilmaz [24], [25], Kiesslinger [26], Marmet [27], and more authors [28], [29], [30], [31], [32], [33], [34] and [35], and recently Majerník [5], [6] and Walker [7], [8]). This variable mass theory lays beyond the Einstein’s Equations.

Because of analogy between the law of gravitational attraction force and Coulomb forces, we can consider a variable electric charge analogous to the variable rest mass of equation (4) (We deal with only one electron orbiting a proton):

$$q(r) = q_\infty \exp(-QY/r)$$

(5)

where $Q$ is the electric charge of the central mass and $Y$ is a constant (we skip here its explanation). (In contrast, usually it is assumed that the elementary electric charge is invariable).

Ben-Amots [1] used variable $q(r)$ of equation (5) in the Bohr atom model and found an additional set of sub-Bohr circular orbitals of the electron orbiting around a proton with relativistic velocity and with radiiuses orders of magnitude smaller than the Bohr radius. The sub-Bohr orbitals found by Ben-Amots have positive orbital energy, the first orbital energy being 51 MeV.
By means of a different theory Milne ([19], [20], [21] and [22]) found just one additional sub-Bohr orbital of the electron (and not a set of sub-Bohr orbitals) and calculated its energy as 50 MeV. Later Corben (see Schild[36]) independently arrived at the same sub-Bohr orbital as Milne’s sub-Bohr orbital, with the same orbital energy. Except for Ben-Amots ([1] and here), variable electric charge was not used for the calculation of sub-Bohr orbitals, and in fact exponential electric charge was never used for elementary charge.

The radiuses of the sub-Bohr set of electron orbitals $r_{sub-Bohr}$ are much smaller than the radiuses of the Bohr electron orbitals $r_{Bohr}$. They are of the order of $10^{-13}$ cm, but larger than the accepted proton radius. Our derivation (Ben-Amots [1]) leads to a second order algebraic equation for $r$ that gives approximations to both solutions:

(i) The Bohr set of circular electron orbitals, and
(ii) A set of sub-Bohr orbitals, which have radius of about $2.8 \times 10^{-13}$ cm, which is slightly more than twice of the accepted proton radius of about $1.2 \times 10^{-13}$ cm.

The two solutions of the second order algebraic equations are:

$$[1/2 + (1/2)\sqrt{1 - (4q_{e\infty}^2 \pi / nch)^2}](nh/2\pi q_{e\infty})^2/m_{e\infty} \approx r_{Bohr}$$

and

$$[1/2 - (1/2)\sqrt{1 - (4q_{e\infty}^2 \pi / nch)^2}](nh/2\pi q_{e\infty})^2/m_{e\infty} = r_{sub-Bohr}$$

where $n = 1, 2, 3$ . . .

Notice that (6) and (7) of the same $n$ are two solutions of the same second order algebraic equation, the difference being the plus or minus sign in the two solutions.

In Equation (6), noting that $(4q_{e\infty}^2 \pi / nch)^2 << 1$ we get for $r_{Bohr}$ the value

$$r_{Bohr} = (nh/2\pi q_{e\infty})^2/m_{e\infty}, \quad n = 1, 2, 3..., \quad (8)$$

which is the well known Bohr set of radiuses.

(In Equation (7) we cannot neglect $(4q_{e\infty}^2 \pi / nch)^2$: Such neglect leads to absence of sub-Bohr orbital(s), characteristic to the assumption of constant rest electric charge).

From Equation (7) by using the expansion $\sqrt{1 - x^2} = 1 - x^2/2 - ...$ for small $x$ we get in first approximation of sub-Bohr radius:

$$r_{sub-Bohr} \approx (2q_{e\infty}^2 \pi / nch)^2(nh/2\pi q_{e\infty})^2/m_{e\infty} = q_{e\infty}^2/m_{e\infty} c^2$$

equal for all $n$ (found by Milne [20], [21], [22] by other means).

By second order approximation using the expansion $\sqrt{1 - x^2} = 1 - x^2/2 - x^4/8 - ...$ for small $x$ we get a set of very close sub-Bohr radiuses for different $n = 1, 2, 3, \ldots$

$$r_{sub-Bohr} = q_{e\infty}^2/m_{e\infty} c^2 + [2\pi q_{e\infty}^3/(nc^2\hbar)]^2/m_{e\infty} \quad (9)$$

or,

$$r_{sub-Bohr} = [(q_{e\infty}/c)^2/m_{e\infty}][1 + 1/(137.036n)^2] \quad (10)$$

So, for larger $n$ we get smaller $r_{sub-Bohr}$.

The electron in our sub-Bohr orbitals has positive energy, which for small quantum number $n$ is about $n \times 51.5$ MeV. See Figure 2. This high positive energy is a consequence of the centrifugal forces that in combination with the exponential forces create a narrow but high barrier peaked at about $2.8 \times 10^{-13}$ cm, which prevents electron capture by the proton. Because of the high positive energy, a sub-Bohr orbital is unstable at pressures lower than in supernova core during implosion and may be populated by an electron only under these enormous pressures.
For the corresponding sub-Bohr orbitals of hydrogen our theory has the lowest orbital energy of 51 MeV implying relativistic electron velocity of approximately $v = c - 8\text{ km/sec}$ or

$$v = c \cos (1/137.036)$$

and in general orbitals of energy

$$E_n = [51+51.5(n-1)] \text{ MeV}, \quad n=1, 2, 3...n_{\text{max}}$$

(12)

after subtracting the rest mass of the electron 0.5 MeV. The radiuses approach a limit close to $2.8 \times 10^{-13}\text{ cm}$. In other words this is a whole set of orbitals with energies given by (12)

$$E_n[\text{MeV}] = 51 ; 102.5 ; 154 ; \ldots E_{\text{max}}, \quad n = 1, 2, 3 \ldots n_{\text{max}}$$

(13)

(while Milne [20], [21], [22] obtained only one 50 MeV orbital).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Energy dependence on the distance $r$ between a proton and an electron orbiting the proton in sub-Bohr and Bohr orbits with quantum number $n=1, 2, 3$ (not to scale)}
\end{figure}
3. Sub-Bohr orbitals as accumulator of energy in supernova

When the core of the star collapses its temperature increases. When the temperature approaches \(6 \times 10^{11} \text{K}\) (corresponding to 51 MeV) electrons will start to populate the first sub-Bohr orbital around protons in the core.

An electron that populates the first sub-Bohr orbital

(i) takes an energy of 51.5 MeV from the available thermal energy, and
(ii) does not contribute anymore to the total number of free particles in the core of the star.

Both 1 and 2 enhance implosion. But populated sub-Bohr orbitals accumulate significant energy. As the pressure increases further with collapse, the temperature rises and the same electron can jump to inner, higher, more energetic orbitals, accumulating additional 51.5 MeV for each ascending step.

We think that in reality there is a limit to reachable sub-Bohr orbital, the highest having an energy \(E_{\text{max}}\), beyond which no higher-energy sub-Bohr orbital can be populated, mainly because the narrow barrier separating the sub-Bohr energy branch from the neutron energy (Figure 2) becomes easily penetrable by the electron at high \(E_n\) (sub-Bohr). Thus, if an electron in the \(n_{\text{max}}\) orbital gains more energy because of increased pressure, its probability to populate a higher orbital becomes small. So, when this electron goes inside to be captured by the proton to form a neutron, it suddenly releases accumulated energy of \([50.22+(n_{\text{max}}-1)\times51.5]\) MeV. This is actually a new type of electron capture (from a sub-Bohr orbital).

In the electron capture by the proton from a sub-Bohr orbital a neutrino is emitted too.

To get an idea of the released energy, for example if \(n_{\text{max}}\geq19\) then the maximum accumulated energy is larger than the rest energy of the proton, i.e. \(M\times c^2\)!

This can be part of the bounceback energy we are searching for to solve the enigma of the energy needed for the supernova explosion. As the star collapses the pressure in the core increases, so the energy of the core increases at the expense of gravitational energy. Then, there is a sudden release of energy when the sub-Bohr electrons in the core of the star are captured by the protons to form neutrons, or separate from the protons, releasing the sub-Bohr orbital huge accumulated energy, which stops the implosion and causes explosion of the star.

Note that models based on relativity alone, in which constant electric charge is assumed, miss the set of sub-Bohr electron orbitals.

Ordinary electron capture is a weak interaction reaction that happens slowly at ordinary pressures, but faster with increased pressure. During implosion of the core the process of populating the sub-Bohr orbitals may or may not occur faster than ordinary electron capture, depending on how fast the pressure rises during the implosion of the core. The conditions in stars under pressure at which ordinary electron capture occurs are known [13], [14]. Under certain conditions during the implosion ordinary electron capture may occur before the electron populates a sub-Bohr orbital, while under other conditions populating sub-Bohr orbitals happens before ordinary electron capture rate increases, depending on the rate of the raise of pressure during implosion of the star. Apparently this determines the final result of an implosion of a star, that is the star's fate - supernova explosion or no external explosion, (leading to what we think could be yet unknown type of star of high density and mass above \(5\, M_{\odot}\)).

In section 1 above we presented the yet unsolved problem, of where and how the kinetic energy of implosion of the star is accumulated and reused to be transformed into kinetic energy of bounceback explosion.

We showed a first possible way to accumulate energy sufficient to create a significantly strong bounceback, but not yet sufficient for a full supernova explosion.

The sub-Bohr electrons, while being energy accumulators, do not supply more energy than they accumulate. Alone, they enable a significant bounceback, but a limited one, with no
larger kinetic energy than the gravitational energy given by the infalling matter to the sub-Bohr electrons.

What supplies the additional energy that results in so strong explosion that increases the velocity of the ejected matter of the entire or almost the entire star up to thousands km/sec?

In the next section we suggest another bounceback process that may be involved too and is capable of producing the additional necessary energy.

4. Quenching the rotation of quarks as source of energy in supernova

A possible mechanism that can explain the source of energy during collapse of the supernova involves quarks. The high pressure in the collapsed core of the star produced by sudden stopping of the fast collapse may cause sudden close contact between the spinning quarks of neighbor nucleons. The friction between these close contact quarks causes fast quenching of their rotation (we presume that after the quarks of neighbor nucleons touch each other, the circumferential layers of the quarks interpenetrate, because rolling of all touching neighboring quarks is not possible where four or more quarks are involved) (Ben-Amots [3]). (Rolling here means revolving of touching spheres without slipping at the point of the touching surfaces).

The high speed rotation of the quark means highest circumferential velocity and highest density at the equator of the quark [3]). The quark density at its equator where velocity is at maximum is about \(4.3 \times 10^{15}\) gram/cm\(^3\). As with known dense matter the velocity of light in it should be significantly smaller than \(c\). Yet the relative velocity between two quarks in the interpenetrating layers is about \(0.99999999999984 c\) when two quarks are revolving with parallel angular momentum vectors (calculated according to Ben-Amots [3]).

The penetration causes intense Cherenkov radiation in this case, which, at stellar scale, becomes important. The energy needed for Cherenkov radiation within the star is taken from the kinetic energy of the rotation of the quarks. The rotation energy of the quark spinning with Franklin’s relativistic rotation [2] constitutes more than 99% of the quark mass (Ben-Amots [3]). High energy Cherenkov photons are created. Quenching quarks rotation between compressed neutrons supplies energy.

The kinetic energy of a mass \(m_0\) moving at a tenth of the speed of light is

\[
E_k = m_0c^2\left(1/\sqrt{1-0.1^2} - 1\right) \approx 0.005 m_0c^2
\]

In the extreme case of a supernova model where the entire mass of the star is ejected at a tenth of the speed of light \(c\), the kinetic energy is 0.5% of \(M_\ast c^2\). The necessary mass of quarks whose rotation should be fully quenched for supplying this maximal kinetic energy is only about 0.5% of the mass of the star. Additional energy is necessary for thermal heating and mostly for neutrino ejection. In all, quenching of the rotation energy of the quarks of a few percents of the mass of the star is necessary for all forms of energy of supernova explosion.

The quenching of the quark rotation causing associated photon pressure may be the much looked after source of additional energy needed in models of supernovae. Together with the energy accumulated in the sub-Bohr orbitals it may stop the implosion of a star and reverse it to an explosion with energy beyond the gravitational potential energy gained during the implosion.

5. Gravitational field as accumulator of energy in supernova

Einstein and Infeld in their book “The evolution of physics” [37] argued: ”The mass of the matter, and mass of the gravitational field, can be indistinguishable” and later Einstein [38] wrote: ”Matter cannot be concentrated arbitrarily.”

We deduce that gravitational field also cannot be concentrated infinitely. Ben-Amots [4] went into some details:
The gravitational field of a spherically symmetric body has mass by itself and is attracted inward towards the central mass. The central attraction creates an inward flux of mass of the gravitational field. Yet this cannot be the whole picture. The [pressure of] gravitational field fall should be almost balanced by radiation outward by the central mass.

The residue of these two (the inward flux of "falling field" and the radiation outward by the central mass) is the origin of the gravitational field ([4] and section 2 above).

In the event of implosion of a celestial spherical body of large mass (above $8 M_{\text{Sun}}$) in the earlier stage of a supernova, the inner core is compressed, causing an increase of the gravitational radiation outward, and with it an increase of the repelling pressure of this radiation, in accord with the above mentioned Einstein’s claim [38] that “matter cannot be concentrated arbitrarily.” The increase of the repelling pressure of the radiation together with the pressure of the neutrinos, stop the implosion of upper layers and contributes to an eventual bounceback outward in the process of supernovae explosion.

Under the present assumptions, the mass/field increased radiation outward by the inner core during the implosion (of the outer layers) stage of a supernova, causes the core to expand outward. It stops the outer layers implosion inward, and redirects them to bounce outward. The falling outer layers collide with the expanding core and are bounced back outward. The sharply increased pressure caused by collision, raises the temperature that ignites the fusion of the helium, carbon, oxygen etc. of the falling outer layers, and causes the final explosion of the supernova.

Summing up: the implosion compresses the gravitational field beyond a maximal balanced density. The overcompressed gravitational field causes outward pressure, which stops the implosion and triggers outward explosion.

6. Remarks

(i) In some cases the implosion might be stopped without producing supernova explosion, but instead by forming a massive celestial object stabilized by continuous energy production of quenching the rotation of the quarks, which produces electromagnetic radiation pressure. This radiation opposes gravitational collapse and supplies radiation energy of the massive celestial body as suggested in Ben-Amots [3]).

(ii) We are aware of the drawbacks of the approximations in our derivation of the sub-Bohr orbitals, but still our results are similar to Milne’s result [18], [19], [20], [21] and [22], derived by independent method not involving variable electric charge. We expect that in the future more precise solutions of Milne’s type calculations will give his sub-Bohr orbital, and in addition our sub-Bohr orbitals of 102.5, 154...[MeV] orbitals.

7. Summary

We suggested various processes that alone or together may stop implosion of a star and cause its explosion,

(i) First possibility: The $[51+(n−1)\times51.5]$ MeV set of sub-Bohr relativistic electron orbitals around a proton accumulates and then releases energy. The accumulated energy is significant at temperature greater than $6\times10^{11}$K° (corresponding to 51 MeV.)

(ii) The compressed massive gravitational field during implosion is an additional accumulator of energy, which is then released causing bounceback.

(iii) Quenching of quark rotation of nucleons as a source of energy additional to the accumulated energies of 1 and 2.
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