Tilted black hole accretion disc models of Sagittarius A*: time-variable millimetre to near-infrared emission

Jason Dexter1* and P. Chris Fragile2,3

1Theoretical Astrophysics Center and Department of Astronomy, University of California, Berkeley, CA 94720-3411, USA
2Department of Physics & Astronomy, College of Charleston, Charleston, SC 29424, USA
3KITP Visiting Scholar, Kavli Institute for Theoretical Physics, Santa Barbara, CA 93106, USA

Accepted 2013 April 4. Received 2013 April 3; in original form 2012 April 17

ABSTRACT

High-resolution, multiwavelength and time-domain observations of the Galactic Centre black hole candidate, Sgr A*, allow for a direct test of contemporary accretion theory. Most models assume alignment between the accretion disc and black hole angular momentum axes, but this is not necessarily the case for geometrically thick accretion flows like that on to Sgr A*. Instead, we calculate images and spectra from a set of numerical simulations of accretion flows misaligned (‘tilted’) by 15° from the black hole spin axis and compare them with millimetre (mm) to near-infrared (NIR) observations. Non-axisymmetric standing shocks from eccentric fluid orbits dominate the emission, leading to a wide range of possible image morphologies. The strong effects of disc tilt lead to poorly constrained model parameters. These results suggest that previous parameter estimates from fitting aligned models, including estimates of the dimensionless black hole spin, likely only apply for small values of spin or tilt (upper limits of \(a < 0.3\) or \(\beta < 15°\)). At 1.3 mm, the black hole images have crescent morphologies as in the aligned case, and the black hole shadow may still be accessible to future very long baseline interferometry (mm-VLBI) observations. Shock heating leads to multiple populations of electrons, some at high energies (\(T_e > 10^{12}\) K). These electrons can naturally produce the observed NIR flux, spectral index and rapid variability (‘flaring’). This NIR emission is uncorrelated with that in the mm, which also agrees with observations.

These are the first numerical models to explain the time-variable mm to NIR emission of Sgr A*. Predictions of the model include significant structural changes observable with mm-VLBI on both the dynamical (hour) and Lense–Thirring precession (day–year) time-scales, and \(\sim 30–50 \mu\text{as}\) changes in centroid position from extreme gravitational lensing events during NIR flares, detectable with the future VLT instrument GRAVITY. We further predict that multiwavelength monitoring should find no significant correlations between mm and NIR/X-ray light curves. The weak correlations reported to date are shown to be consistent with our model, where they are artefacts of the short light-curve durations. If the observed NIR emission is caused by shock heating in a tilted accretion disc, then this would require the Galactic Centre black hole to have a positive, non-zero spin parameter (\(a > 0\)) and may rule out a magnetically arrested state.

Key words: accretion, accretion discs – black hole physics – radiative transfer – Galaxy: centre.

1 INTRODUCTION

Due to its proximity, Sgr A* is the most intensively studied supermassive black hole candidate and the largest in angular size. Recent very long baseline interferometry observations at 1.3 mm (mm-VLBI; Doeleman et al. 2008; Fish et al. 2011) have detected source structure on event-horizon scales (tens of microarcseconds, \(\mu\text{as}\)). Future measurements with this Event Horizon Telescope may detect the black hole shadow, providing the first direct evidence for an event horizon (Bardeen 1973; Falcke, Melia & Agol 2000; Bromley, Melia & Liu 2001; Dexter et al. 2010).

The spectral energy distribution (SED) of Sgr A* arises from the radio to a peak in the submillimetre (mm ‘bump’, Zylka et al. 1995).
The radio emission is synchrotron radiation from non-thermal electrons far from the black hole, either in the accretion flow (Melia 1992; Narayan & Mahadevan 1995; Yuan, Quataert & Narayan 2003) or in a mildly relativistic jet (Falcke & Markoff 2000). The mm bump is well described by synchrotron radiation from hot \((\sim 10^{10} - 10^{11} \text{ K})\), mostly thermal electrons in the immediate vicinity of the black hole \((r \approx 5-10M \odot)\); Narayan et al. 1995; Yuan et al. 2003; Mościbrodzka et al. 2009; Dexter et al. 2010).

In addition to 30–50 per cent variability in the mm (Zhao et al. 2003; Eckart et al. 2006; Yusuf-Zadeh et al. 2006; Marrone et al. 2008), Sgr A* is observed to ‘flare’ rapidly in the near-infrared (NIR; Genzel et al. 2003; Ghez et al. 2004) and the X-ray (Baganoff et al. 2001) with order-of-magnitude amplitudes. The NIR/X-ray flares occur simultaneously (Eckart et al. 2004), and weak correlations in simultaneous light curves have suggested possible lags between NIR/X-ray and mm variability (Marrone et al. 2008; Yusuf-Zadeh et al. 2008; Kunneriath et al. 2010). The NIR emission is likely synchrotron radiation from energetic electrons, while the X-rays are produced by either direct synchrotron radiation or inverse Compton scattering of mm or NIR seed photons by the NIR emitting electrons (Markoff et al. 2001).

No theoretical model of Sgr A* self-consistently accounts for the observed time-variable multiwavelength emission. Radiatively inefficient accretion flow (RIAF; Yuan et al. 2003) and jet (Falcke & Markoff 2000) models can describe the multiwavelength ‘quietest’ spectrum. RIAF models also provide excellent fits to existing mm-VLBI data (Broderick et al. 2009, 2011).

Models for the NIR flares include coherently orbiting inhomogeneities (‘hotspots’; Broderick & Loeb 2006) in the inner radii of the accretion flow. Recently, correlated multiwavelength flares have instead favoured an adiabatically expanding blob model (Yusuf-Zadeh et al. 2009; Eckart et al. 2012), which can explain the reported time lags between the NIR/X-ray and mm flares. The blobs could be formed from non-thermal events such as magnetic reconnection in the accretion flow (Dodds-Eden et al. 2010).

Many of these models are non-relativistic, and all employ ad hoc prescriptions for the magnetic fields responsible for angular momentum transport and accretion via the magnetorotational instability (MRI; Balbus & Hawley 1991). Magnetohydrodynamic (MHD) simulations can provide a more physical description of the accretion flow. They have been used to model the synchrotron emission from Sgr A*, either in three spatial dimensions with a pseudo-Newtonian potential (Goldston, Quataert & Igumenshchev 2005; Ohsuga, Kato & Mineshige 2005; Chan et al. 2009; Huang et al. 2009) or in full general relativity (GRMHD) in two (Noble et al. 2007; Mościbrodzka et al. 2009; Ilbourn et al. 2010) or three (Dexter, Agol & Fragile 2009; Dexter et al. 2010; Shcherbakov, Penna & McKinney 2010; Dolence et al. 2012; Shiokawa et al. 2012) dimensions. Non-relativistic simulations are especially inappropriate for modelling the mm/NIR/X-ray emission, which originates in the innermost portion of the accretion flow where relativistic effects are strongest. Axisymmetric simulations cannot sustain the MRI, and cannot accurately model variability.

Existing models from 3D simulations provide an excellent description of mm observations including polarization (Shcherbakov et al. 2010), variability and mm-VLBI (Dexter et al. 2009, 2010). These models also produce large-amplitude NIR flares (Dolence et al. 2012), but with much lower fluxes and steeper spectral indices than observed. Although most current global GRMHD simulations are factors of a few in resolution from convergence (Hawley, Guan & Krolik 2011), the resulting spectra of Sgr A* are fairly insensitive to resolution (Shiokawa et al. 2012).

There are two major limitations to current models. First, the electron distribution function is highly uncertain. Most theoretical models adopt a thermal distribution for the mm bump, and some add a non-thermal power-law tail. Models from simulations use a thermal distribution with a constant electron–ion temperature ratio, \(T_e/T_i\), allowing the electron temperature to be calculated from the output density and pressure (Goldston et al. 2005; Mościbrodzka et al. 2009). Simple prescriptions for allowing this ratio to vary spatially lead to only minor changes in the images/spectra. This parameter can be calibrated either using physically motivated subgrid models for electron heating (Sharma et al. 2007) or through direct particle-in-cell calculations of MRI turbulence (Riquelme et al. 2012). The second major limitation to current models of Sgr A* emission, and the subject of this article, is the assumption that the accretion flow angular momentum axis is aligned with the black hole spin axis.

Geometrically thick accretion flows like that on to Sgr A* should not align with a central black hole as is expected for geometrically thin discs (Bardeen & Petterson 1975). For alignment to occur, the black hole must accrete an amount of material with constant angular momentum orientation comparable to its mass. For this to occur in a Hubble time requires an accretion rate \(\sim 10^{-4} M_{\odot} \text{yr}^{-1}\), an order of magnitude larger than the estimated accretion rate at the Bondi radius (Yuan et al. 2003) and several orders of magnitude larger than that on to Sgr A* \((10^{-9} - 10^{-8} M_{\odot} \text{yr}^{-1})\); Aitken et al. 2000; Bower et al. 2003; Marrone et al. 2007). Alternatively, McKinney, Tchekhovskoy & Blandford (2013) showed that alignment of thick discs can occur if a sufficiently coherent magnetic field accretes on to the black hole, forming a dynamically important jet. However, this process requires a jet with \(> 100\) per cent efficiency, and there is no evidence for such a powerful jet in Sgr A*. Therefore, it is likely that the accretion flow is misaligned (‘tilted’).

The expected disc misalignment has extraordinary consequences for the dynamics and observable properties of a black hole accretion flow. Lense–Thirring precession leads to tilts and warps that are oscillatory functions of radius (Ivanov & Illarionov 1997), and gravitational torques cause the entire disc to precess essentially as a solid body (Pringle 1992; Rockefeller, Fryer & Melia 2005; Fragile et al. 2007). Eccentric orbits in the disc converge, leading to non-axisymmetric standing shocks (Fragile & Blaes 2008). Excess angular momentum transport from the shocks truncates the accretion flow outside of the marginally stable orbit (Fragile 2009; Dexter & Fragile 2011). The asymmetry of the accretion flow leads to images and spectra that depend strongly on the observer azimuthal viewing angle (Dexter & Fragile 2011).

In this work we calculate time-dependent images and spectra from the only published GRMHD simulations of tilted black hole accretion discs (Fragile et al. 2007, 2009; Fragile 2009), and compare them with mm to NIR observations of Sgr A*. We review the physics of tilted accretion discs in Section 2 and describe the numerical simulations in Section 3. The methods used to construct radiative models from the simulations are outlined in Section 4. These radiative models are then fitted to existing mm-VLBI and spectral observations in Section 5, and the mm images, and mm to NIR spectral and variable properties of the best-fitting models are analysed in Section 6. The implications and limitations of this study are discussed in Section 7, and the major results are summarized in Section 8.

\(^1\) We frequently use units with \(G = c = 1\); in these units for Sgr A*, \(1 M\) is approximately \(6 \times 10^{11} \text{ cm and 20s}\).
2 TILTED BLACK HOLE ACCRETION DISCS

Tilted black hole accretion discs experience differential Lense–Thirring precession owing to the frame dragging of the rotating black hole. In a disc, Lense–Thirring precession is able to build up over many orbital periods, so it can affect the structure well beyond radii normally associated with relativistic effects.

The effect of Lense–Thirring precession in a disc is to cause it to warp, and this warping can propagate through the disc in either a diffusive or wave-like manner. In the diffusive case, the warping is limited by ‘viscous’ responses within the disc, and the Lense–Thirring precession dominates out to a unique transition radius $r_{BP}$ (Bardeen & Petterson 1975; Kumar & Pringle 1985), inside of which the disc is expected to be flat and aligned with the black hole mid-plane, and outside of which it is also expected to be flat but in a plane determined by the angular momentum of the gas reservoir. This ‘Bardeen–Petterson’ configuration is expected for Keplerian discs whenever the dimensionless stress parameter $\alpha$ is larger than the ratio $H/r$, where $H$ is the disc semi-thickness (Papaloizou & Pringle 1983). Given that current calculations find $\alpha$ to be significantly less than one (e.g., Hawley et al. 2011), this requires geometrically thin discs.

In thicker discs, as would be expected in low-luminosity sources such as Sgr A*, warps propagate in a wave-like manner, and the disc does not go into the Bardeen–Petterson configuration. Instead, the tilt of the disc can be an oscillatory function of radius, with the amplitude of the oscillations growing as one approaches the black hole (Ivanov & Illarionov 1997). These oscillations are stationary, however, so they cannot act to absorb the torque of the black hole. For thick discs, this torque instead causes the disc to precess as if it were a solid body.

The warping of tilted thick discs creates unbalanced pressure gradients that can drive significant latitude-dependent epicyclic motion within the disc. The induced motion of the gas can be coherent over the entire scale of the disc, with magnitudes that are substantial fractions of the orbital velocity. Radial variations in the eccentricity of the associated fluid-element trajectories can lead to a crowding of orbit trajectories at certain locations within the disc. This results in local density enhancements akin to compressions. These compressions can be sufficiently strong to produce standing shocks within the disc, particularly in the vicinity of the black hole (Fragile & Blaes 2008).

Tilted accretion disc models can be constructed semi-analytically using height-integrated equations in the so-called twisted frame (Ivanov & Illarionov 1997; Zhuravlev & Ivanov 2011). However, these models lack the magnetic fields that lead to angular momentum transport in the disc and give rise to the observed synchrotron emission from Sgr A*. They also have an unconstrained vertical structure, leading to additional free parameters. Finally, they are stationary, and so cannot be used to study variability self-consistently. Instead, we employ 3D GRMHD simulations of misaligned black hole accretion discs. Simulations have fewer free parameters and higher physical fidelity. A few exist, however, preventing anything like a thorough exploration of the parameter space in dimensionless black hole spin, disc tilt and geometric disc thickness.

3 SIMULATIONS

The numerical data used in our analysis were taken from simulations presented in Fragile et al. (2007, 2009) and Fragile (2009). All of the simulations used the CO3MOS++ GRMHD code (Aminos, Fragile & Salmonson 2005). CO3MOS++ includes several schemes for solving the GRMHD equations; in these simulations, the internal energy, artificial viscosity formulation was used. The magnetic fields were evolved in an advection-split form, while using a hyperbolic divergence cleanser to maintain an approximately divergence-free magnetic field. The GRMHD equations were evolved in a ‘tilted’ Kerr–Schild polar coordinate system $(t, r, \vartheta, \varphi)$. This coordinate system is related to the usual (untilted) Kerr–Schild coordinates $(t, r, \vartheta, \varphi)$ through a simple rotation about the y-axis by an angle $\beta$, as described in Fragile & Aminos (2005).

The simulations were carried out on a spherical polar mesh with nested resolution layers. The base grid contained $12^3$ mesh zones and covered the full $4\pi$ steradians. Two levels of refinement were added on top of the base layer, each refinement level doubling the resolution relative to the previous layer, thus achieving peak resolutions equivalent to a $128^3$ simulation. In the radial direction, a logarithmic coordinate of the form $\eta \equiv 1.0 + \ln (r/r_{BH})$ was used, where $r_{BH}$ is the black hole horizon radius. In the angular direction, in addition to the nested grids, a concentrated latitude coordinate $x_2$ of the form $\vartheta = x_2 + \frac{1}{2}(1 - h) \sin(2x_2)$ was used with $h = 0.5$ to concentrate resolution towards the mid-plane of the disc.

The simulations started from the analytic solution for an axisymmetric torus around a rotating black hole (Chakrabarti 1985). The inner radius of the torus was $r_m = 15M$, the radius of the initial pressure maximum was $r_{\text{centre}} = 25M$ and the power-law exponent used in defining the initial specific angular momentum distribution was $q = 1.68$. An adiabatic equation of state was assumed, with $\Gamma = 5/3$. The torus was seeded with a weak dipole magnetic field in the form of poloidal loops along the isobaric contours within the torus. The field was normalized such that initially $B_{\text{mag}} = P/P_B \geq B_{\text{mag,0}} = 10$ throughout the torus. The black hole was inclined by an angle $\beta = 0^\circ$ or $15^\circ$ relative to the disc (and the grid). From this starting point, the simulations were allowed to evolve for a time equivalent to 10 orbits at the initial pressure maximum, $r_{\text{centre}}$, corresponding to hundreds of orbits at the innermost stable circular orbit. Table 1 summarizes the simulation parameters: the top four are presented here, while the bottom four were analysed in Dexter et al. (2010) and are used here for comparison. There is no evidence for Bardeen–Petterson alignment during the simulations, as expected from the measured time-averaged values of $H/R \simeq 0.1$–$0.2$ and $\alpha \sim 0.01$.

The radiative models described below use the last $\approx 4000 M$ of simulation time, after the initial transients have died down. The radial accretion rate profile for the 915 h simulation averaged over this time span is constant out to $\approx 15 M$, suggesting that infall equilibrium has been established (Penna et al. 2010) over the range of radii where the emission is produced ($r \lesssim 10 M$). The MRI is also sufficiently resolved within the disc, with typical resolution parameters $Q_{t, \lambda} \gtrsim 10, 25$ using the definitions from McKinney, Tchekhovskoy & Blandford (2012). Similar values have been found from many

Table 1. Simulation parameters.

| Simulation | $a$   | $\beta$ | Reference       |
|------------|-------|---------|-----------------|
| 315 h      | 0.30  | 15°     | Fragile (2009) |
| 515 h      | 0.50  | 15°     | Fragile et al. (2009) |
| 715 h      | 0.70  | 15°     | Fragile (2009) |
| 915 h      | 0.90  | 15°     | Fragile et al. (2007) |
| 50 h       | 0.50  | 0°      | Fragile et al. (2009) |
| 90 h       | 0.90  | 0°      | Fragile et al. (2007) |
| MBD        | 0.92  | 0°      | McKinney & Blandford (2009) |
| MBQ        | 0.94  | 0°      | McKinney & Blandford (2009) |
untitled GRMHD simulations (Hawley et al. 2011; Shiokawa et al. 2012), and they importantly suggest convergence in the emitted spectrum (Shiokawa et al. 2012).

4 RADIATIVE MODELLING

A radiative model is specified by three components: a dynamical model for fluid variables as functions of space and time, a model for the electron distribution function and an emission model. In this work, the GRMHD simulations described above are used as the dynamical model. As in previous radiative models of Sgr A* from simulations (Goldston et al. 2005; Moscibrodzka et al. 2009; Dexter et al. 2010), we employ a thermal Maxwell–Juttner distribution for the electrons, assuming a constant ion–electron temperature ratio, $T_e/T_i$, as a crude approximation to the electron distribution function in a collisionless plasma. This ratio is a free parameter in the models. Finally, we assume that the emission is entirely from synchrotron radiation and use the approximate unpolarized emission coefficient from Leung, Gammie & Noble (2011).

With these assumptions, observables can be calculated from the simulations using ray tracing. Starting from an observer’s camera, rays are traced backwards in time towards the black hole (assuming that they are null geodesics) using the public code GEOKERR (Dexter & Agol 2009). In the region where rays intersect the accretion flow, the radiative transfer equation is solved along the geodesic using the code GRTRANS (Dexter 2011), which then represents a pixel of the image. This procedure is repeated for many rays to produce an image, and at many time steps of the simulation to produce time-dependent images (movies). Light curves are computed by integrating over the individual images. Repeating the procedure over observed wavelengths gives a time-dependent spectrum.

To calculate fluid properties at each point on a ray, the space–time coordinates of the geodesic are transformed from Boyer–Lindquist to the tilted Kerr–Schild coordinates used in the simulations. Since the accretion flow is dynamic, light travel time delays along the geodesic are taken into account. Data from the sixteen nearest zone centres (eight on the simulation grid over two time steps) were interpolated to each point on the geodesic. The simulations are scale free, and must be scaled to cgs units in order to calculate emission and absorption coefficients. Fixing the black hole mass ($M_\odot \approx 4 \times 10^6 M_\odot$ for Sgr A*) sets the length and time-scales. The total mass in the accretion flow is a free parameter, and its choice is equivalent to fixing the time-averaged accretion rate. To convert image intensities to flux, we adopt a distance $D = 8 \, \text{kpc}$ to the Galactic Centre.

4.1 Model fitting

For each simulation, images are produced over a grid of parameters at 0.4 and 1.3 mm: dimensionless black hole spin, $a$ (or equivalently simulation), time-averaged accretion rate, $\dot{M}$, ion–electron temperature ratio, $T_i/T_e$, observer inclination, $i$, azimuth, $\phi_0$, and sky orientation (position angle measured E of N), $\xi$. These images (including the effects of interstellar scattering; Bower et al. 2006) are fitted jointly to mm-VLBI observations (Doeleman et al. 2008; Fish et al. 2011) and spectral measurements (Marrone et al. 2008).

The mm-VLBI and spectral measurements are averaged over different time periods (10 min and 2.5 h, respectively), and both encompass many different epochs (observing days). We average the theoretical images over these same integration periods, but fit each image to all of the data from all integrations and epochs. We could instead preserve the time dependence within each epoch, and fit observations 30 min apart to images with the same time separation. This would likely reduce the fit quality in all models (see Appendix A), and should be tried in the future.

This fitting procedure gives the probability of observing the measured values from a particular model, which is converted to the probability distribution as a function of the model parameters using the method described in Broderick et al. (2009). The result is a joint posterior probability distribution as a function of the above parameters. To estimate individual parameters, we marginalize over the others using uniform priors on $a$, $T_i/T_e$, $\phi_0$ and $\xi$, a logarithmic prior on $M$ and a sin $i$ prior on the inclination angle. Although we only fit to the mm data to estimate parameters, we do veto models which greatly exceed existing mid-infrared upper limits (e.g. Melia & Falcke 2001). Only models from the 515 h simulation with large $T_i/T_e \simeq 20–200$ are ruled out in this way.

In practice, the range of $M$ is used to bracket the range of observed mm (230 GHz) fluxes from Sgr A*: $\approx 2–5 \, \text{Jy}$. Achieving convergence requires a fine enough grid so that multiple images have mm fluxes close to those observed by mm-VLBI ($\approx 2.4 \, \text{Jy}$) and spectral observations ($\approx 3.5–4.25 \, \text{Jy}$). In untilted simulations, this is possible with a fairly coarse grid of 10M values, since the image structure is nearly unchanged as the flux varies by 30–50 per cent (see Section 6.1). Variability then acts to sample the total flux more finely. This is not the case in tilted simulations, where the image structure and flux can change significantly on short time-scales. In probability distributions over both the observer time and position angle ($\xi$), reasonable convergence is obtained when using 20 values of $M$.

The observer azimuth, $\phi_0$, is a new parameter for tilted simulations, since the accretion flow is no longer azimuthally symmetric on times much longer than the orbital time. Instead, the non-axisymmetric structure in these simulations averages out only on the much longer precession time-scale. Also unlike untilted simulations, the non-axisymmetric structure dominates images from tilted discs (Dexter & Fragile 2011). We find that eight values of $\phi_0$ are sufficient for convergence in the resulting probability distributions and parameter estimates. For details about the fitting procedure and convergence of the resulting probability distributions, see Appendix A.

4.2 Sgr A* millimetre emission region

The mm emission in Sgr A* is at the peak of the SED (Melia 1992; Narayan et al. 1995; Zylka et al. 1995) and marks the transition from optically thick to thin (Falcke et al. 1998). It is primarily produced by thermal electrons (Yuan et al. 2003), radiating from a photosphere where the optical depth $\tau \approx 1$, so that the temperature of the emitting electrons is given by the brightness temperature, $T_b$:

$$T_b = \frac{c^2 I_{\nu_0}}{2k\nu_0^2} \approx 6 \times 10^{10} \left( \frac{F_\nu}{3 \, \text{Jy}} \right) \left( \frac{\nu_0}{230 \, \text{GHz}} \right)^2 \left( \frac{\Delta \theta}{40 \, \mu\text{as}} \right)^{-2} \text{K},$$

where $F_\nu (I_{\nu_0})$ is the observed flux (specific intensity) at frequency $\nu_0$ and $\Delta \theta$ is the observed linear angular size of the emitting region scaled to the value found recently from mm-VLBI observations (Doeleman et al. 2008; Fish et al. 2011). This is effectively a lower

\footnote{The simulations considered here have durations of $\approx 0.75 \, \text{Gyr}$.}
Figure 1. Images as a function of $T_i/T_e$ for the 90, 315 and 515 h simulations. The colours are scaled linearly with a dynamic range of 60, and the images are $160 \times 160$ μas in all cases. Images from untilted simulations such as 90 h become optically thick and emit from a growing photosphere with increasing $T_i/T_e$. Images from tilted simulations do not become uniformly optically thick and can even become smaller at large $T_i/T_e$, as in the case of 515 h, due to the presence of multiple electron populations with varying electron temperatures at distinct spatial locations in the accretion flow.

In untilted simulations, the accretion flow effectively consists of one population of electrons – those in the inner radii near the mid-plane, where temperatures, field strengths and particle densities are highest. The best-fitting value of $T_i/T_e$ in untilted simulations is then that for which this electron population has $T_e \approx T_b$. At higher values of $T_i/T_e$, a larger accretion rate is required to match the observed flux ($j_\nu \propto M^2 T_e^2 / M^3$). The absorption coefficient is less strongly dependent on $T_e$ than the emission coefficient ($\alpha_\nu \propto j_\nu / B_\nu \propto j_\nu / T_e$ in local thermodynamic equilibrium), so that absorption becomes increasingly important for higher $T_i/T_e$. The accretion flow then develops a photosphere which grows outwards in radius with increasing $T_i/T_e$. This effect is shown in the top row of Fig. 1 for the 90 h simulation. Other untilted simulations behave similarly, except that in conservative simulations (e.g. MBD or MBQ in Dexter et al. 2010, from McKinney & Blandford 2009) the $T_i$ values are larger and hence so are the favoured values of $T_i/T_e$.

The brightness temperature argument holds for the tilted simulations as well (see Section 5). However, heating from the standing shocks effectively leads to additional electron populations at varying densities, field strengths and ion temperatures, depending on latitude. The densest material with the lowest ion temperature is near the (tilted) mid-plane and is brightest for small $T_i/T_e$. The densities and field strengths decrease away from the mid-plane, and these populations of electrons become more important at higher $T_i/T_e$. The material closer to the mid-plane becomes optically thick as $T_i/T_e$ increases, but the emissivity of hotter electrons also increases, and their emission is largely unobscured by optically thick material.

Increasing $T_i/T_e$ then does not lead to a uniformly growing photosphere, since it can simply increase the emissivity of electrons that previously had $T_e > T_b$. This effect can lead to a complicated dependence of the image on $T_i/T_e$, as shown in the bottom two rows of Fig. 1 for the 315 and 515 h simulations. While the 315 h image becomes increasingly uniformly optically thick with increasing $T_i/T_e$, it does so much more slowly than the untilted simulations. The 515 h simulation begins to grow a uniform photosphere until hotter electrons dominate the image at $T_i/T_e \approx 3$, after which the photosphere begins to recede. These images remain qualitatively similar from $T_i/T_e \approx 20–200$ due to the presence of very hot electrons.

5 SGR A* PARAMETER CONSTRAINTS

Even with the limited coverage and sensitivity of current mm-VLBI data, it is possible to estimate parameters of both the black hole and accretion flow in Sgr A* in the context of either semi-analytic
Tilted disc models of Sgr A* (Broderick et al. 2009, 2011) or GRMHD (Dexter et al. 2010, 2012) RIAF models. The inclination and position angles are particularly well constrained, and are in excellent agreement between the two types of models. Both models assume alignment between the disc and black hole, which is unlikely to be the case for Sgr A*.

For the tilted disc simulations with a misalignment of 15°, there are many possible models which provide excellent fits to current mm observations (reduced $\chi^2$ from a single epoch of mm-VLBI of $\lesssim 0.6$). The wide range of images is mostly due to the strong changes with $T_i/T_e$, observer time and azimuth. The first has a very narrow allowed range in untilted models, while the last two have minimal effects on the structure of untilted images. Since the parameter space is larger for tilted disc images, we should expect better fitting models to be present than in the untilted case. Using the priors listed above, we can calculate the Bayesian odds ratio for the tilted models from this work (four simulations, $\approx 97\,000$ total images) relative to the untilted models from Dexter et al. (2010) (four simulations, $\approx 84\,000$ total images), marginalizing over all parameters in each case. This ratio is $p(\beta = 0°)/p(\beta = 15°) = 0.11$, indicating that the tilted models provide marginally better fits to the data even accounting for their additional free parameters. Other tests give similar marginal significance for better fit quality from tilted models ($1-2\sigma$ depending on the test and how it is formulated; see Appendix A for details).

Because of the wide range of acceptable tilted disc models that provide excellent fits, the parameters are essentially unconstrained with current observations (Figs 2 and 3). We can formally estimate the parameter values as $i = 80^{+42}_{-40}$, $\xi = -78^{+130}_{-11}$ $M = 165^{+284}_{-161} \times 10^{-9} M_{\odot}\,\text{yr}^{-1}$ and $T_e = 4^{+7}_{-5} \times 10^{10} \text{K}$, all to 90 percent confidence. All values of $\phi_0$ are allowed at this confidence. More data and a more complete set of simulations will be required to meaningfully constrain the parameters.

As in untilted models, face-on inclinations are disfavoured, primarily because they are too optically thin to explain the observed spectral indices (Mościbrodzka et al. 2009). A wide range of position angles are viable, but the location of the broad peak in probability density corresponds to the range found for untilted models (Dexter et al. 2010; Broderick et al. 2011). In each model, an observer azimuth is favoured where velocities downstream of one of the non-axisymmetric shocks are approaching the observer. For 915 h, both such orientations are favoured, while in the other simulations the probability distribution is asymmetric with one standing shock favoured.

As discussed above (Section 4), it is non-trivial to find all viable values of $T_i/T_e$ for the tilted simulations. Since a higher accretion rate is required at higher $T_i/T_e$ to match the observed flux of Sgr A*, this leads to a wide range of possible accretion rates, unlike the untilted case. However, the electron temperature is still well constrained. Lowering $T_i/T_e$ and increasing the accretion rate serves to pick out different electrons, but does not change their temperature by more than a factor of 2.

All previous models fitted to mm data have assumed a disc tilt of zero and have found much stronger constraints on Sgr A* parameters than we find for a tilt of 15°. Although both the sample of simulations here and in previous studies are highly incomplete in their sampling of black hole spin and initial conditions, the difference between tilts of 0° and 15° is extremely significant: individual tilted models are less constrained than their untilted counterparts.

Figure 2. Normalized probability distributions as functions of black hole spin (top left), observer inclination (top right), sky orientation (bottom left) and observer azimuth (bottom right) for each simulation separately and combined (solid lines). Unlike in aligned models (Broderick et al. 2009, 2011; Dexter et al. 2010), the parameters of the viewing geometry are essentially unconstrained.
and the probability distributions over various parameters in the tilted models differ significantly between spins, $T_i/T_e$ and $\phi_0$. In contrast, different models from untilted simulations tend to give similar probability distributions (figs 2 and 3 of Dexter et al. 2010).

For all these reasons, previously reported parameter estimates from GRMHD simulations likely only apply for either very low values of tilt or spin ($\alpha < 0.3$ or $\beta < 15^\circ$ are the upper limits from these simulations). In addition, the viewing geometry constraints from untilted GRMHD simulations in Dexter et al. (2010) are nearly identical to those from semi-analytic RIAF models in Broderick et al. (2009, 2011). Therefore, we expect that the constraints found there likely only apply in the same limits.

One explanation for the wide range of allowed viewing geometries is that we are measuring inclination relative to the black hole spin axis, rather than to the disc itself. So even if the tilted models were identical to the untilted ones at a given $i_{\text{disc}}$, we would expect broader probability distributions over $i$ to the different values of $i_{\text{disc}}$ contributing to each value of $i$ depending on the observer azimuth. This hypothesis can easily be tested by (i) looking at $p(i)$ at fixed $\phi_0$, exactly as done for the untilted simulations and (ii) rebinning $p(i)$ into $p(i_{\text{disc}})$ by calculating $i_{\text{disc}}(i, \phi_0)$. The results from both procedures are shown in Fig. 4. The probability densities are similarly plotted versus $i$ or $i_{\text{disc}}$, while the models at fixed $\phi_0$ are still much more poorly constrained in the tilted case. So the probability distributions for the tilted disc models are intrinsically broader (more poorly constrained), and this effect is not an artefact of how we measure the inclination (relative to the disc or black hole).

6 MODEL PROPERTIES

Despite the lack of robust or meaningful parameter constraints, there are many generic properties of the images, spectra and light curves of tilted accretion disc models with important observational implications for Sgr A*. The best-fitting models from each simulation are representative of the range of viable possibilities from our fitting, and in this section these models are studied in detail. Their parameters are given in Table 2.

6.1 Images

The importance of non-axisymmetric standing shocks for Sgr A* images is shown in Fig. 5. Images from a tilted simulation (715 h, left-hand panel) are dominated by material heated by the standing shocks, which appear as non-axisymmetric, roughly $m = 2$ structures. Any non-axisymmetric structure in untilted images (right-hand panel of Fig. 5) travels at the orbital speed, while the standing shocks move at the much slower precession speed. In addition to the choice of $T_i/T_e$ described above, images from tilted simulations are then primarily set by the observer azimuth, which determines the viewing orientation of the standing shocks.

The asymmetry of the tilted disc images leads to a wide range of complex image morphologies, which depending on the model and observer time can be dominated either by the post-shock fluid, dense material near the mid-plane of the tilted disc, or other hot electrons. This is the primary reason that the model parameters cannot be well constrained thus far. Despite the additional complexity, viable images tend to be similar. This is demonstrated using the best-fitting models from all simulations, whose 1.3 mm images (top row of Fig. 6) are all ‘crescents’, as found previously for images from untilted simulations (Noble et al. 2007; Mościbrodzka et al. 2009; Dexter et al. 2010) and semi-analytic models (Bromley et al. 2001; Broderick et al. 2009). The crescent morphology results from a combination of strong Doppler beaming from orbital disc motion, which causes asymmetry between approaching and receding material, and gravitational light bending, which causes the back of the accretion flow to appear above and below the black hole in the image. The crescent morphology is especially apparent when the images are convolved with interstellar scattering (second row of Fig. 6), which at 1.3 mm blurs most of the small-scale image structure. The black hole shadow is apparent as a central minimum in all images, at the orientation corresponding to light coming from the back of the accretion flow, which is bent above and below the black hole. The shadow appears in the visibility amplitude (third row of Fig. 6) as a local minimum on the same orientation at a baseline whose length corresponds to the size scale of the shadow in the image. It also appears at the same location as a region of rapidly varying visibility phase (fourth row of Fig. 6).

We can make predictions for future mm-VLBI observations by interpolating visibilities to the locations of future baselines between ALMA/APEX in Chile, LMT in Mexico and current telescopes: SMTO in Arizona, James Clerk Maxwell Telescope/SMA in Hawaii and CARMA in California. The results are shown in Fig. 7 for the visibility amplitudes for the best-fitting models. The models make a wide variety of predictions for future baselines, and the black hole
Tilted disc models of Sgr A*

Figure 4. Normalized probability density as a function of inclination relative to the accretion disc and black hole (top), and as a function of inclination for individual simulations relative to the black hole for models with fixed $\phi_0$ in the tilted (middle) and untilted (bottom) case. There is no significant difference between $p(i)_{\text{disc}}$ and $p(i)_{\text{BH}}$, and individual tilted models are much more poorly constrained than their untilted counterparts. Both cases show that the poor parameter constraints in tilted models are due to their inherent complexity, rather than the dependence of $i_{\text{disc}}$ on both $i$ and $\phi_0$.

shadow appears as a local minimum in the visibility amplitude near 3000M$\lambda$ in the 515 and 715 h models.

In addition to visibility amplitudes, phases also carry important information about the image structure. The closure phase is formed by summing phases over a triangle of baselines and is more robust to instrumental errors than individual phase measurements. For favoured (and most) position angles for all best-fitting models, the closure phase is near zero, in agreement with recent mm-VLBI observations (Fish et al. 2011). In addition, the black hole shadow shows up as a highly variable closure phase either versus position angle or time for position angles where the shadow is accessible. For nearly co-linear baselines such as those including current mm-VLBI telescopes, the closure phase tends to be near zero except for viewing orientations corresponding to the black hole shadow. However, phase information is more interesting for other baseline triangles. Depending on viewing orientation and model, a wide range of possible closure phases are possible, many of which differ significantly from zero. Measuring closure phase in future mm-VLBI campaigns will therefore provide significant additional constraints on the models, and should be feasible in future observations (e.g. Doeleman et al. 2009).

At 0.87 mm (Fig. 8), the other potential wavelength for future mm-VLBI observations, the effects of interstellar scattering are greatly reduced. These images have intrinsically steeper emissivities, however, so that the emission is more concentrated and the black hole shadow is less apparent. For these reasons, shorter wavelengths may be better suited for constraining parameters and testing accretion models than for detecting the black hole shadow. The structure and total flux of the accretion flow at this wavelength are poorly constrained, and the different simulations predict a wide range of both possible image structures and fluxes. Even a simultaneous spectral constraint at 0.87 mm simultaneous with 1.3 mm VLBI could distinguish between many of the possibilities.

6.1.1 Implications for mm-VLBI observations

While a crescent morphology is often found for mm tilted disc images of Sgr A* (especially at 1.3 mm), similar to previous models of untilted discs, their time-dependent structures are completely different. Untilted disc images are basically static, as shown in the Table 2. Best-fitting model parameters.

| Name  | Spin | $M(10^{-9} M_{\odot} \text{yr}^{-1})$ | $i$ | $\xi$ | $\phi_0$ | $T_i/T_2$ |
|-------|------|----------------------------------|-----|------|---------|-----------|
| 315 h | 0.3  | 130                              | 70° | −13° | π/4     | 3         |
| 515 h | 0.5  | 18                               | 40° | −81° | −π/4    | 1         |
| 715 h | 0.7  | 10                               | 90° | −70° | −π/2    | 1         |
| 915 h | 0.9  | 6                                | 80° | −38° | 3π/4    | 1         |

Figure 5. Nearly face-on images from the 715 (left) and 90 h (right) simulations scaled as in Fig. 1. The structure in images from untilted simulations such as 90 h is mostly azimuthally symmetric (circularly symmetric when viewed face-on), while that in the tilted simulations such as 715 h is highly non-axisymmetric due to the presence of standing shocks which dominate the mm emission in models of Sgr A*. In addition, asymmetries in untilted images travel at the orbital speed, whereas the standing shocks travel at the much slower Lense-Thirring precession speed.
Figure 6. Best-fitting images from each simulation (columns) with (second row) and without (top row) the effects of interstellar scattering, as well as the corresponding visibility amplitudes (third row) and phases (fourth row) for $v_0 = 1.3$ mm (230 GHz). Image scales are the same as in Fig. 1. The visibility amplitudes also have a dynamic range of 60, and both the visibility amplitude and phase images are $1200 \times 1200 M_\odot$ in size. The phase colours range from blue (negative) to green (zero) to red (positive). The triangles show the locations of existing mm-VLBI observations. Once convolved with interstellar scattering, the best-fitting images are crescents with significant but non-dominant Doppler beaming. The black hole shadow appears as a local minimum in the visibility amplitude and a region of rapidly varying phase.

Unfortunately, the relevant time-scale is highly uncertain. It depends sensitively on the effective ‘outer’ radius of the accretion flow, its radial density profile and the black hole spin. The outer radius is especially problematic, since both its location and interpretation are uncertain for the collisionless plasma around Sgr A*. In terms of these parameters, the precession time is given by (Fragile et al. 2007)

$$t_{\text{prec}} \approx \frac{1.3}{a} \left( \frac{r_{\text{out}}}{1000 M_\odot} \right)^2 \text{yr},$$

where $r_{\text{out}}$ is the effective outer radius. We have assumed $\rho \propto r^{-3/2}$ as expected for Bondi accretion and as found by hydrodynamical simulations of large-scale accretion on to Sgr A* (Cuadra et al. 2006). The precession time is shown in Fig. 11 as a function of outer radius for both the Bondi and RIAF solutions. In both cases, for large ranges of outer radius, the precession time-scale is likely to be longer than the length of individual mm-VLBI campaigns.
Visibility amplitude versus baseline length for the best-fitting models interpolated to the $uv$ locations of current and future baselines. The local minimum in the visibility amplitudes for 515 and 715 h near 3000 $\lambda$ is a signature of the black hole shadow. Current data are insufficient to meaningfully test for changes in observer azimuth (and precession) between or within epochs. The prospects for detecting Lense–Thirring precession with Event Horizon Telescope observations will be considered in more detail in future work.

6.2 Spectra
Spectra from the best-fitting tilted disc models are shown in Fig. 12. The black lines show the average over the observer time, while the dark grey envelope indicates the range. These models cannot explain the radio emission outside the sub-mm bump due to their limited spatial extent and lack of non-thermal emission. The IR tilted disc spectra, however, are much different from any from previous MHD simulations. With purely thermal electrons and no additional free parameters, all best-fitting models to the mm data are consistent with upper limits in the far- and mid-IR while producing the observed flux range in the NIR [although the 915 h model marginally violates the mid-IR upper limit from Schödel et al. (2011)].

The mean spectra from the 515, 715 and 915 h models are in remarkable agreement with reported mean values from Schödel et al. (2011) and Witzel et al. (2012), and with the median value from Dodds-Eden et al. (2011). Further, the NIR spectral indices (defined as $F_\nu \propto \nu^{-\alpha_{\text{NIR}}}$ and listed in Table 3) from these models are within the range reported for Sgr A* (Ghez et al. 2005; Gillessen et al. 2006; Hornstein et al. 2007). The spectral index from the 515 h model is in remarkable agreement with the result found by Hornstein et al. (2007), while those from the 715 and 915 h models agree with the redder reported spectral index of Gillessen et al. (2006). The spectral evolution is in agreement with observations in both cases as well (see Section 6.3). The 315 h spectrum underproduces the average NIR emission, and its spectral index is steeper than observed. Other model spectra from these simulations may provide excellent fits to the observed NIR emission in Sgr A* as well, and the NIR data could be included in the joint fitting in future studies.

Spectra from untilted disc simulations have similar shapes to that shown for 90 h in the left-hand panel of Fig. 13 (e.g. Mościbrodzka et al. 2009). The NIR emission is on the exponential tail of the synchrotron spectrum. For this reason, the untilted models underestimate the observed emission by orders of magnitude and have much redder spectral indices than observed. Comparing the 90 h spectrum with that from 515 h (right panel of Fig. 13) shows that distinct electron populations (and/or a non-thermal distribution function) are crucial for simultaneously producing the mm to NIR spectrum.
The NIR emission in the tilted models is produced by extremely hot electrons ($T_e \approx 1-4 \times 10^{12}$ K) near the black hole ($r \approx 3M$). Heating from the standing shocks in the tilted disc models allows for multiple electron populations, some of which can be at very high temperatures. These can effectively mimic a power-law tail to the synchrotron emission, especially since they are at significantly lower particle densities than the electrons producing the mm bump (Section 6.4). This physical heating mimics the effect of adding a non-thermal tail to the electron distribution function (e.g. Yuan et al. 2003).

These hot electrons are also the reason we discard models with $T_i/T_e \gtrsim 3$ for the 515 and 715 h simulations, despite the fact that some of them can fit the mm-VLBI and spectral observations. Larger accretion rates are required to match the observed mm flux for larger $T_i/T_e$. The peak synchrotron frequency scales as

$$\nu_c \sim BT_e^2 \sim \sqrt{MT_e^2},$$  

(3)

while the peak flux scales roughly as

$$F_{\nu_c} \sim nB^2 T_e^2 \sim M^2 T_e^2,$$  

(4)

so that $\nu_c$ decreases for larger $T_i/T_e$ at fixed flux. This scaling causes the high-frequency parts of the spectra to shift up in normalization and down in frequency. The approximate power-law tail can extend into the optical/UV when $T_i/T_e \approx 1$, and increasing its normalization leads to the violation of many IR upper limits.
Tilted disc models of Sgr A* 2263

Figure 12. Best-fitting model spectra compared to Sgr A* data. The solid curves denote the median values at each frequency over the observer time, while the dark grey envelope shows the range. Radio data are from Falcke et al. (1998, open circles) and An et al. (2005, filled diamonds). The models are fitted to the mm data (filled squares; Marrone 2006), but are in good agreement with the mean/median (orange and purple, Schödel et al. 2011; Dodds-Eden et al. 2011, respectively) and flaring (pink and light blue, Genzel et al. 2003; Dodds-Eden et al. 2011, respectively) NIR data with no additional free parameters. They also satisfy observational upper limits in the mid-IR (blue upper limits; Melia & Falcke 2001, and references therein). At lower frequencies the non-thermal emission is produced outside of the simulation domain, either in the accretion flow (Yuan et al. 2003) or in a short jet (Falcke & Markoff 2000).

Table 3. NIR variability parameters.

| $\alpha_{\text{NIR}}$ | $\exp \mu$ (mJy) | $\sigma$ |
|----------------------|------------------|--------|
| 315 h                | $-4.1 \pm 1.0$   | $0.7-1.9 \times 10^{-2}$ | 11–18 |
| 515 h                | $-0.5 \pm 0.1$   | 1.1–1.3 | 1.8–2.0 |
| 715 h                | $-1.6 \pm 0.7$   | 0.5–0.7 | 3.0–3.4 |
| 915 h                | $-2.2 \pm 0.4$   | 1.0–1.3 | 1.6–1.7 |

6.3 Millimetre to infrared variability

Radiative models of Sgr A* based on simulations are thus far the only to include variability information from first physical principles (Goldston et al. 2005; Chan et al. 2009; Dexter et al. 2009, 2010; Dolence et al. 2012). Best-fitting models to mm-VLBI and spectral information from untilted GRMHD simulations naturally produce millimetre variability qualitatively consistent with that observed (Dexter et al. 2009, 2010). This variability is due to relatively global ($m = 0, 1$) fluctuations in the particle density and magnetic field strength, and is strongly correlated with the accretion rate. We find similar mm light curves for the best-fitting tilted disc models, as shown in Fig. 14. All of the best-fitting light curves exhibit variability on hour time-scales with $\lesssim 50$ per cent amplitudes, roughly in agreement with observations. Millimetre ‘flares’ would be identified as two per light curve, consistent with their observed frequency in Sgr A*. However, tilted disc models produce larger amplitude and longer duration flares than their untilted counterparts (cf. fig. 6 of Dexter et al. 2010). The variability is still strongly correlated with the accretion rate, which also varies more in tilted disc simulations.

As discussed above, tilted disc models naturally produce roughly the correct mean levels of IR emission, very different from previous

Figure 13. Spectra from best-fitting untilted (90 h, left) and tilted (515 h, right) simulations. Sgr A* data are the same as in Fig. 12. In both cases, the spectra are fitted to the green sub-mm points. Multiple electron populations from shock heating in tilted discs can naturally produce the observed NIR emission, which is underproduced by approximately two orders of magnitude in untilted simulations. Other untilted simulations exhibit larger amplitude variability than 90 h, but its spectral shape is representative.
Figure 14. Light curves from the best-fitting models at 1.3 (230 GHz, solid lines) and 0.4 mm (690 GHz, dotted lines). The times of best fit to the visibility data are shown as upward triangles, while the ranges of times averaged for the best fit to the spectral data are bracketed by left- and right-facing arrows. The simulated light curves exhibit 50 per cent variability on hour time-scales, similar to the untilted case and in qualitative agreement with observations. The mm spectral index increases with flux, because the optical depth is lower at a higher frequency.

Figure 15. NIR light curves from the best-fitting models in the H, K, and L bands. All of the simulated NIR light curves are highly variable on short (≲ hour) time-scales). The 315 h simulation underproduces the observed NIR flux and its variability. The 515, 715 and 915 h models produce the observed flux range and exhibit ‘flares’ with roughly the right amplitudes (factors ≲ 30) and time-scales (≳ 1 h). The 315 and 715 h spectral slopes are time variable, while those from the 515 and 915 h models are nearly constant with NIR flux.

Dexter et al. (2012) found a broken power-law power spectrum in a model of Sgr A* from an untilted GRMHD simulation with a similar break frequency. However, we find no significant quasi-periodic oscillations similar to what they reported near the frequency of the marginally stable orbit. Using many cameras spaced in observer azimuth, they decomposed the power spectrum into azimuthal modes (their equation 1 and fig. 1). A comparison is shown in Fig. 16. Unlike in their models, the $m = 0$ mode is dominant, with $m = 1, 2$ only contributing on the shortest time-scales. These discrepancies could be due to differences in the simulations (either numerical algorithms or duration), or due to physical differences between tilted and untilted accretion discs.

6.4 Physical origin of time-variable mm and NIR emission

The mm emission in the tilted disc models arises from hot, dense electrons in the inner radii of the simulations. Typical parameters
field strengths ($B \sim 50$ G) and high temperatures ($T_e \gtrsim 10^{12}$ K). This location is fairly insensitive to spin, and is inside the marginally stable orbit for all but the 915 h simulation. These are lower particle densities and higher electron temperatures than previously reported for IR-emitting electrons from axisymmetric, untilted GRMHD simulations (Moscibrodzka et al. 2009). In their simulations, the NIR emission was dominated by heating from numerical magnetic reconnection in current sheets, which is thought to be an artefact of axisymmetry. In our model, the NIR-emitting electrons are heated by the non-axisymmetric standing shocks. This is the only source of entropy generation in these simulations, since heat from numerical reconnection is lost from the grid. For example, the maximum temperature obtained anywhere in the 90 h simulation is $\lesssim 10^{12}$ K.

The NIR ‘flares’ in our models are due to fluctuations in the particle density, field strength and electron temperature of these hot post-shock electrons. The 915 h $K$-band light curve can be recovered approximately with an emissivity $j_e \propto nB^2$, as long as only hot electrons are included ($T_e > 5 \times 10^{11}$ K). In the 515 and 715 h simulations (with hotter electrons), the emissivity can be approximated as $j_e \propto nBT_e^2$. Temperature fluctuations are important in the 515 and 715 h simulations, but not in 915 h. The approximate forms can be understood by the variations in emissivity expected from fluctuations in the individual quantities at the appropriate ratio of $c/\nu$. This ratio is near 1 for the hotter 515 and 715 h simulations, and $\approx 10$–20 for 915 h. Closer to $c/\nu$, the emissivity is less strongly dependent on $B$ and $T_e$.

General relativistic effects are also important for the NIR light curves from 515 and 715 h. Many of the flares in these models correspond to extreme gravitational lensing events, where emission from hot electrons behind the black hole is lensed into a bright ring. An example from the 515 h model is shown in Fig. 17. A flare occurs between the first and second panels when hot gas behind the black hole is strongly lensed, concentrating its emission in a bright ring at the circular photon orbit. In the subsequent frames, this hot material passes by the black hole and moves to a larger radius, leading to a decay of the flare. The lensing acts as a filter, enhancing the emission from electrons at larger radius ($r \gtrsim 3M$) and can produce order unity modulations in the light curves. These types of extreme gravitational lensing events are important in orbiting hotspot models as well (Broderick & Loeb 2006), but there is no clear signature of orbital motion from the NIR light curves in the simulations.

The NIR image structure is highly variable, and so is its centroid. The $K$-band centroid position is shown in Fig. 18. The typical excursions are $\approx 30$–$50$ μas in all best-fitting models except that from the 915 h simulation. The position wander is only weakly correlated with the NIR flux. All periods of rapid variability and very high NIR fluxes also exhibit significant centroid motions, but not all periods of large position wander correspond to NIR flares.

The NIR spectral index is set by the critical frequency for synchrotron emission,

$$\nu_c = 6 \times 10^{12} \left( \frac{T_e}{10^{12} \text{K}} \right)^2 \left( \frac{B}{50 \text{G}} \right)^{1/2} \text{Hz.}$$

(6)

This primarily depends on the electron temperature, which varies between simulations. The NIR electrons are coldest in the 315 h simulation, and hottest in the 515 h simulation. Somewhat surprisingly, the shock heating is less effective in the 715 and 915 h simulations. Dexter & Fragile (2011) argued that the non-axisymmetric standing shocks are stronger in higher spin simulations. Colder NIR electrons and lower critical frequencies lead to more negative spectral indices. These also tend to be more variable, since the emissivity depends strongly on frequency when $c/\nu_c \gg 1$. When $c/\nu_c \approx 1$, as for the
515 h simulation, the emissivity is fairly flat and the spectral index is nearly constant.

The mm and NIR light curves are uncorrelated in this model, since they arise from different populations of electrons. The full 690 GHz and K-band light curves and their cross-correlation are shown in Fig. 19. Although the full duration light curves are uncorrelated, 6 h segments can produce spurious peaks in the cross-correlation with hour lags, similar to those reported for Sgr A* (e.g. Yusef-Zadeh et al. 2008).

7 DISCUSSION

Geometrically thick, low-luminosity accretion flows are not expected to align with their central black holes. Both the current accretion rate on to Sgr A* at the Bondi radius and that at the inner radii are orders of magnitude smaller than required for sufficient mass accretion to have changed the black hole spin orientation over a Hubble time. Therefore, it is unlikely that the black hole spin axis in Sgr A* aligns with the angular momentum of the accreting gas. We have constructed the first radiative models of Sgr A* from misaligned (tilted) black hole accretion flows by performing radiative transfer calculations on data from GRMHD simulations (Table 1). These models provide an excellent description of existing observations, and are significantly different from any theoretical model previously considered. Even for the modest 15° misalignment

Figure 17. $H$-band (1.6 μm) images from the 515 h model spaced ≃ 5 min apart during an NIR ‘flare’ with a factor of ≃4 flux variation between the first and second images. The images are scaled logarithmically with a dynamic range of 2048, and the panel size is 85 × 85 μas. The flare emission is clearly lensed and concentrated at the photon ring, corresponding to the unstable photon orbit in the Kerr space–time.

Figure 18. X (solid) and Y (dotted) centroid positions for the $K$-band (2.2 μm) images. Periods of rapid centroid movement of ≃30–50 μas occur every few hours in all models but 915 h, and should be detectable by the future VLT instrument GRAVITY.

Figure 19. Top: light curves from the 515 h simulation at 0.4 mm (red) and 3.6 μm (blue). Bottom: cross-correlation of the full light curves (solid curve) and four evenly spaced segments (dashed curves). For these short (several hour) durations, lags can be found at correlations of ≃0.4–0.6 as reported in the literature, despite the fact that the full light curves are uncorrelated.
considered between black hole spin and accretion angular momentum axes, the (thermo)dynamics of the accretion flow change significantly. Non-axisymmetric shocks from the convergence of eccentric orbits in the warped disc give rise to asymmetric image morphologies, leading to the breakdown of many simplifying assumptions in the untilted models. The observer azimuthal viewing angle becomes an important new parameter, and the dependence of observables on both the viewing geometry and the ion–electron temperature ratio becomes non-trivial. The combination of these factors greatly expands the possible parameter space for tilted disc models relative to those assumed in all previous models of Sgr A*.

There are numerous implications. The parameters of both the black hole and the viewing geometry are poorly constrained (Figs 2 and 3) by mm-VLBI and spectral observations. Previously reported parameter constraints (Broderick et al. 2009, 2011; Dexter et al. 2009, 2010; Mościbrodzka et al. 2009; Shcherbakov et al. 2010) likely only apply if the black hole spin or tilt values are extremely small. Upper limits from this work are $a < 0.3$ or $\beta < 15^\circ$, although recent simulations with smaller tilt ($\beta = 10^\circ$) suggest that the limits may be even tighter (Henisey et al. 2012). The best-fitting models (Table 2) to the mm data sort into two groups at low ($10^{-10} - 10^{-8}$ $M_\odot$ yr$^{-1}$) and high ($\gtrsim 10^{-7}$ $M_\odot$ yr$^{-1}$) time-averaged mass accretion rates.

The ‘crescent’ morphology frequently found in untilted black hole images (e.g. Bromley et al. 2001; Broderick et al. 2009; Dexter et al. 2010) is also apparent in our best-fitting tilted images at 1.3 mm, especially when including the effects of interstellar scattering. For this reason, the black hole shadow is still expected to be accessible, although large uncertainties in the viewing geometry for these models prevent a robust prediction for baselines where it should be seen. In two of the best-fitting models (Fig. 7), the shadow is visible on baselines between telescopes in Chile (ALMA/APEX) and Mexico (LMT), but a wide variety of other geometries are nearly as likely. The mm variability properties reported previously (Dexter et al. 2009, 2010) qualitatively hold for the tilted disc models, although their variability amplitudes and characteristic time-scales seem somewhat larger (Fig. 14).

Although images from tilted discs are crescents at 1.3 mm, at 0.87 mm the images can become double peaked due to emission from near the standing shocks (915 h model in Fig. 8). The structure of the tilted images is also highly time variable, in stark contrast to the static morphology in the untilted case (Fig. 9). Tilted and untilted models can then be distinguished either with higher frequency mm-VLBI or simply with many epochs of observations using the current array. Polarized mm-VLBI observations may also help to distinguish between models. Shcherbakov et al. (2010) found that spectropolarimetric data contain comparable constraining power to current mm-VLBI observations.

Perhaps the most important difference is that the low-accretion-rate best-fitting models (those from the 515, 715 and 915 h simulations) naturally reproduce IR spectral (Fig. 12) and variability (Fig. 15) properties consistent with observations, while those at high mass accretion rates (from the 315 h simulation) underproduce the observed flux. All models satisfy the observed upper limits on mid-infrared emission (Telesco, Davidson & Werner 1996; Cotera et al. 1999; Schödel et al. 2011). All three low-accretion-rate models are in excellent agreement with the reported mean NIR fluxes from Sgr A* (Dodds-Eden et al. 2011; Schödel et al. 2011). Flux distributions from the 515 and 915 h models are well described by a log-normal, and the inferred parameters (Table 3) are in excellent agreement with those found by Dodds-Eden et al. (2011). Both the mean and time dependence of the NIR spectral index vary considerably between models. The 515 h spectral index is constant in time with a value of $\alpha_{\text{NIR}} = -0.5 \pm 0.1$, nearly identical to the observations of Ghez et al. (2005) and Hornstein et al. (2007). The 715 and 915 h light curves have much redder spectral indices, and the 715 h spectral index increases with flux. These values are in good agreement with Gillessen et al. (2006). Because of the short duration of our light curves ($\approx 25$ h), it is unclear whether the 515, 715 and 915 h light curves are best interpreted as different states of the same theoretical model or as a means to distinguish between the models. This agreement is found with a purely thermal electron distribution function, in contrast to previous work (e.g. Yuan et al. 2003). Thus, a significant non-thermal component to the distribution function is not necessary to produce the observed NIR emission, although it is likely present and may be necessary to fit the low-frequency radio spectrum.

In any event, this is the first model of Sgr A* that produces the observed time-variable mm to NIR emission from a first principles physical time-dependent model for the dynamics of the accretion flow. The two spectral components are completely uncorrelated. The mm variations are due to magnetic turbulence and are strongly correlated with the accretion rate. The NIR variations are caused by particle density, field strength and temperature variations in tenuous, hot gas heated by the non-axisymmetric standing shocks, and can be significantly modulated by extreme gravitational lensing. Short segments of these light curves can, however, still lead to the reported weak correlations (Fig. 19). We caution therefore that cross-correlation coefficients of $\lesssim 0.6$ between wavebands in single epoch observations do not necessarily imply a common physical origin.

This NIR emission model is novel. It is not due to orbiting ‘hotspots’ (Broderick & Loeb 2006; Eckart et al. 2006) or expanding blobs of gas (Yusef-Zadeh et al. 2008; Eckart et al. 2012). However, significant structural changes can still be seen during large changes in NIR flux (Fig. 18). It is similar in spirit to models that invoke magnetic reconnection events (Dodds-Eden et al. 2010) to accelerate electrons to the observed high energies. But magnetic reconnection cannot heat particles in these simulations. Instead, the heating is from the non-axisymmetric standing shocks in the tilted accretion flow. Shock heating should lead to a non-thermal component to the electron distribution function, which we have ignored in order to avoid additional free parameters in the models. Broderick et al. (2011) find that $\approx 50$ per cent of the mm flux is produced by a power-law component, which could significantly change the resulting mm images and the NIR light curves. This should be studied in the future.

This emission is also completely different from that described by Dolence et al. (2012), who detected quasi-periodic oscillations in NIR/X-ray light curves of Sgr A* from an untilted GRMHD simulation. Their model underproduces the observed NIR flux by an order of magnitude, similar to previous models from untilted simulations (Dexter et al. 2010). Including non-thermal electrons would likely allow their model to produce the observed NIR flux. However, the NIR/X-ray variability in their model is strongly correlated with that in the mm, in conflict with observations. Avoiding this correlation would require some new physics in the simulations, which is present in the form of shock heating at even the modest ($15^\circ$) tilt angle considered here.

These radiative calculations ignore the effects of cooling. However, the cooling time-scale for the NIR-emitting electrons is short:

$$t_{\text{cool}} \approx 10 \left( \frac{B}{50 \text{G}} \right)^{-3/2} \left( \frac{v}{10^{14} \text{Hz}} \right)^{-2} \text{min.}$$

(7)
At \( r \approx 3M \), where the emission is produced, this time-scale is still larger than the dynamical time \((t_{\text{dyn}} \approx 2 \text{ min})\). If this is where the particle heating occurs, ignoring the effects of cooling on the electron temperature may be a safe approximation. Both radiative cooling and a more realistic electron distribution function can be included in future calculations by following particle trajectories through the standing shocks and self-consistently accounting for heating and cooling.

We have only considered unpolarized synchrotron radiation. Including inverse Compton scattering via Monte Carlo calculations (e.g. Dolence et al. 2012) would allow the calculation of X-ray light curves, and in particular the X-ray luminosity during NIR flares. We can crudely estimate this using typical values of fluid variables during rapid changes in NIR flux: \( T_e = 2 \times 10^{12} \text{ K} \), \( R = 3M \) and \( n = 10^5 \text{ cm}^{-3} \) (low, H-band flux of 1 mJy) to \( R = 3M \) and \( n = 6 \times 10^7 \text{ cm}^{-3} \) (high, H-band flux of 10 mJy). For this temperature, the Compton peak is at \( \sim 1 \text{ keV} \). The ratios of Compton to synchrotron bolometric luminosities estimated using the formulae in Esin et al. (1996) are \( L_c / L_s \approx 0.1 \) (low) and \( \approx 2 \) (high). At those H-band fluxes, the estimated X-ray luminosities are \( L_X \sim 10^{33} \text{ low} \) and \( \sim 10^{35} \text{ erg s}^{-1} \) (high). These numbers are in rough agreement with observations of X-ray flares (e.g. Baganoff et al. 2001, 2003). Using an approximate unpolarized emissivity typically leads to only modest errors \( \lesssim 10 \% \) (Shcherbakov et al. 2010; Leung et al. 2011), and this is supported by our own preliminary calculations of polarized radiative transfer from untilted simulations. However, previous work (Bromley et al. 2001) has found that depending on the strength of Faraday conversion and rotation, significant changes in total intensity images of Sgr A* can occur when polarization is included. Fully polarized calculations from tilted discs should be carried out in the future.

The effects of radiative cooling on multiwavelength spectra from axisymmetric (aligned) GRMHD simulations have been studied by Dibi et al. (2012) and Drappeau et al. (2013). They find that adding radiation in post-processing, as done here, is a good approximation for accretion rates \( M \lesssim 10^{-7} \text{M}_\odot \text{ yr}^{-1} \). All of our best-fitting models satisfy this criterion (Table 2) except for that from 315 h, where the accretion rate is slightly larger than this value. Radiative cooling may be important for this and similar models, although it is unknown whether this accretion rate limit applies to tilted discs.

We have only considered four simulations, all with a tilt angle of \( \beta = 15^\circ \), and all with the same poloidal initial magnetic field configuration. The non-conservative GRMHD algorithm used in these simulations leads to a scaleheight \( H/r \approx 0.1-0.2 \), whereas the accretion flow on to Sgr A* may have \( H/r \approx 1 \). We are unable to probe the dependence of our results on either tilt angle or scale-height, both of which may change the formation and strength of the standing shocks. It is unclear whether the models considered here are representative of the range of possibilities from tilted black hole accretion discs. In untilted models, the initial magnetic field configuration can greatly affect the properties of the outflows (Beckwith, Hawley & Krolik 2008; McKinney & Blandford 2009), but does not significantly change the disc emission (Beckwith et al. 2008; Dexter et al. 2010; Drappeau et al. 2013). Since the outflows in our models produce negligible emission due to numerical limitations, the results may not depend strongly on the initial magnetic field. However, jet emission should also be considered in future work.

McKinney et al. (2012) have recently carried out the first GRMHD simulations with \( H/r \approx 1 \), and used what they argue is a more appropriate initial condition for the Galactic Centre. Both of these aspects may lead to important changes in the dynamics of the accretion flow. In particular, for high spin, McKinney et al. (2013) have discovered a mechanism for aligning geometrically thick \( (H/R > \alpha) \) accretion discs that applies when the magnetic field threading the black hole becomes dynamically important. The typical jet power in these simulations is extremely high (efficiency \( > 100 \% \)) (Tchekhovskoy, Narayan & McKinney 2011), and there is no evidence for powerful jets in Sgr A*.

For accretion flows in such a state, it is unclear whether these misaligned or traditional aligned models would serve as a better approximation.

None of the simulations used here conserve total energy. We previously found that magnetic energy lost to grid-scale numerical reconnection leads to a radiatively efficient effective luminosity, \( \sim 0.1 \text{M}_\odot \text{c}^2 \) (Dexter et al. 2009). Although untilted models based on these simulations give very similar results to those that recapture this energy as heat (Dexter et al. 2010), it is unclear whether that is the case for tilted discs. The lack of energy conservation means that the only form of entropy generation is through shock heating. Numerical dissipation of magnetic fields would provide another heat source, and could potentially lessen the dramatic effects seen here that are caused by the standing shocks. This issue can be addressed by future energy-conserving tilted simulations (Shiokawa et al., in preparation).

If standing shocks forming from eccentric orbits in a tilted, warped disc provide the heating mechanism for the observed NIR emission, then the inner disc must be tilted, which, in turn, means the Galactic Centre black hole must have a non-zero spin. Furthermore, as recent general relativistic, semi-analytic models of tilted accretion discs find that the inner disc aligns with the black hole for \( a < 0 \) (Zhuravlev & Ivanov 2011), this picture would require the black hole in Sgr A* to have a positive spin \( (a > 0) \).

8 SUMMARY

There is little reason to expect alignment between the black hole spin axis and the angular momentum axis of accreting gas in radiatively inefficient sources without powerful jets. We have calculated time-variable mm to NIR emission from the only published GRMHD simulations of misaligned (‘tilted’) black hole accretion discs and compared them to observations of Sgr A*.

Our main results are as follows.

(i) Tilted disc models provide an excellent description of existing mm-VLBI and spectral observations of Sgr A*.

(ii) Previously reported parameter constraints from fitting models to mm-VLBI, spectral and/or polarization measurements likely only apply if the spin or tilt value is very small (upper limits of \( a < 0.3 \), \( \beta < 15^\circ \)).

(iii) Predicted images for mm-VLBI are crescents as in the aligned case, and the black hole shadow, direct evidence for the existence of an event horizon, is still potentially visible on baselines between Mexico and Chile.

(iv) Heating from non-axisymmetric standing shocks can naturally produce the observed NIR emission. This emission is uncorrelated with the mm variability, which also agrees with observations and is strongly correlated with the mass accretion rate. The X-ray flares in this picture could arise from Compton upscattering of NIR seed photons or from direct synchrotron radiation from non-thermal electrons.

(v) We predict several observational signatures of disc tilt: (a) structural changes observable with mm-VLBI from strong asymmetry in the emission region on hour (dynamical) and/or day–year (Lense–Thirring precession) time-scales; (b) bimodal structures in 345 GHz mm-VLBI images from the presence of standing shocks.
in the accretion flow; and (c) NIR centroid motions of 30–50 μas from extreme gravitational lensing during flares. (vi) If the NIR emission is caused by heating from standing shocks in a tilted accretion flow, then the spin parameter of Sgr A* should satisfy $a > 0$.

ACKNOWLEDGEMENTS

JD thanks E. Agol, O. Blaes, K. Dodds-Eden, J. Dolence, J. McKinney, and E. Quataert for useful discussions, and the anonymous referee for detailed comments which improved this manuscript. This work was partially supported by NSF grants AST-0807385 and PHY11-25915, NASA grant 05-ATP05-96 and NASA Earth and Space Science Fellowship NNX08AX59H (JD).

REFERENCES

Aitken D. K., Greaves J., Chrysostomou A., Jenness T., Holland W., Hough J. H., Pierce-Price D., Richer J., 2000, ApJ, 534, L173
An T., Goss W. M., Zhao J., Hong X. Y., Roy S., Rao A. P., Shen Z., 2005, ApJ, 634, L49
Anninos P., Fragile P. C., Salmonson J. D., 2005, ApJ, 635, 723
Baganoff F. K. et al., 2001, Nat, 413, 45
Baganoff F. K. et al., 2003, ApJ, 591, 891
Ballbus S. A., Hawley J. F., 1991, ApJ, 376, 214
Bardeen J. M., 1973, in DeWitt B. S., DeWitt C., eds, Black holes (Les astres occlus). Gordon and Breach, New York, p. 215
Bardeen J. M., Petterson J. A., 1975, ApJ, 195, L65
Beckwith K., Hawley J., Kroll J., 2008, ApJ, 678, 1180
Bower G. C., Wright M. C. H., Falcke H., Backer D. C., 2003, ApJ, 588, 331
Bower G. C., Goss W. M., Falcke H., Backer D. C., Lithwick Y., 2006, ApJ, 648, L127
Broderick A. E., Loeb A., 2006, ApJ, 636, L109
Broderick A. E., Fish V. L., Doeleman S. S., Loeb A., 2009, ApJ, 697, 45
Broderick A. E., Fish V. L., Doeleman S. S., Loeb A., 2011, ApJ, 735, 110
Bromley B. C., Goss W. M., Falcke H., Matsuo H., Teuben P., Zhao J., Zylka R., 1998, ApJ, 504, 3083
Cotera A. S., Simpson J. P., Erickson E. F., Colgan S. W. J., Burton M. G., 2009, ApJ, 691, 1021
Dexter J., Agol E., Fragile P. C., McKinney J. C., 2011, MNRAS, 415, 1228
Dexter J., Agol E., Fragile P. C., McKinney J. C., 2010, ApJ, 717, 1092
Dexter J., Agol E., Fragile P. C., McKinney J. C., 2012, J. Phys.: Conf. Ser., 372, 012023
Dibi S., Drappeau S., Fragile P. C., Markoff S., Dexter J., 2012, MNRAS, 426, 1928
Do T., Ghez A. M., Morris M. R., Yelda S., Meyer L., Lu J. R., Hornstein S. D., Matthews K., 2009, ApJ, 691, 1021
Dodos-Eden K., Sharma P., Quataert E., Genzel R., Gillessen S., Eisenhauer F., Porquet D., 2010, ApJ, 725, 450
Dodos-Eden K. et al., 2011, ApJ, 728, 37
Doeleman S. S. et al., 2008, Nat, 455, 78
Doeleman S. S., Fish V. L., Broderick A. E., Loeb A., Rogers A. E. E., 2009, ApJ, 695, 59
Dolence J. C., Gammie C. F., Shiokawa H., Noble S. C., 2012, ApJ, 746, L10
Dexter J., Agol E., Fragile P. C., Markoff S., Dexter J., 2013, MNRAS
Eckart A. et al., 2004, A&A, 427, 1
Eckart A. et al., 2006, A&A, 450, 535
Eckart A. et al., 2012, A&A, 537, A52
Esin A. A., Narayan R., Ostriker E. J., Yi I., 1996, ApJ, 465, 312
Falcke H., Markoff S., 2000, A&A, 362, 113
Falcke H., Goss W. M., Matsuo H., Teuben P., Zhao J., Zylka R., 1998, ApJ, 499, 731
Falcke H., Melia F., Agol E., 2000, ApJ, 528, L13
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L. et al., 2011, ApJ, 727, L36
Fish V. L. et al., 2011, ApJ, 727, L36
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L. et al., 2011, ApJ, 727, L36
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L. et al., 2011, ApJ, 727, L36
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
Fish V. L., Broderick A. E., Doeleman S. S., Loeb A., 2009, ApJ, 692, L14
The fluxes at 0.4 and 1.3 mm are time averaged over 2.5 h intervals to mimic the observations from Marrone (2006). We average the observed fluxes at 1.3 mm and spectral indices between 0.4 and 1.3 mm over the four observational epochs, and fit the theoretical values to the observational values. Note that the 1.3 mm flux during these observations was ∼50 per cent higher than during the VLBI measurements from Doeleman et al. (2008). Image intensities are converted to fluxes using a black hole mass of 4 × 10^6 M⊙ and a distance of 8 kpc.

In order to ensure convergence of these probability distributions, a sufficient number of images must be calculated varying each parameter. We calculate images over a regular grid with a fixed number of samples per parameter. The number of sampled observer times per simulation was ∼150. Using 1/2 or 1/4 of the samples from a given simulation, chosen randomly or in segments, does not change the resulting parameter estimates. The untilted models in Dexter et al. (2010) used 10 values of i spaced evenly from 0–90°, 50 values of χ spaced evenly between ±90°, 10 values of M which spanned a range such that the 1.3 mm flux varied from ∼2 to 5 Jy at each observer time and values of T/Tc that depended on the simulation, essentially so that average electron temperatures in the mm emission region varied from 2 × 10^10 to 2 × 10^11 K. This is roughly the range required by the observed brightness temperature of Sgr A*. Depending on the ion temperature of the simulation, this required 3 to 10 separate values of T/Tc. Only one observer azimuth was used, because with only one preferred direction (the black hole spin axis), the simulation is axisymmetric on time-scales longer than the orbital time. This means that many different samples in time are equivalent to using multiple observer azimuths.

In the current case with a disc tilt of 15°, the required number of samples in M and ϕ0 increases. Because of the second preferred direction (disc angular momentum axis), the resulting images are non-axisymmetric on time-scales shorter than the precession time. Since the simulation duration spans only ∼1/8 of the precession time, we must now sample the images at multiple values of ϕ0. Secondly, because of the complicated time-variable structure of the emission region (described in Section 4.2), more values of M are required to accurately capture the range of viable images at each observer time. This is basically because our previous modelling exploited the stability of images over time, so that nearby time samples gave images with different total fluxes, but roughly identical structures. With time-variable structures in the tilted images, this is no longer possible and more samples at each observer time are required.

Due to the computational expense involved in the increased size of the parameter space, we use the minimum samples of each parameter required for convergence in the probability distributions. As shown in Fig. A1, for the 315 h simulation with i = 90° this turns out to be 20 samples of M at each observer time, with a range chosen carefully to maximize the number of samples with 1.3 mm fluxes in the observed range (2–5 Jy). Similarly, Fig. A2 shows probability distributions over ϕ0 for the 315 h simulation with i = 90° and T/Tc = 3 marginalized over the observer time. Evidently, convergence requires a minimum of eight values of ϕ0. This particular example is chosen because it is the most time-variable best-fitting model, and therefore presents the most difficult test for achieving convergence. Although using more samples may lead to smoother resulting marginalized probability distributions, it is unlikely to make a major difference in the resulting parameter estimates. The error in these distributions can be estimated as ∼10 per cent from the scatter in nearby ϕ0 values in Fig. A2.

Finally, we can compare the overall goodness of fit for the tilted and untilted models, e.g. p(β = 0°)/p(β = 15°), in a few different
Tilted disc models of Sgr A* 2271

Figure A1. Probability distributions for model 315 h with $T_i/T_e = 3$ and $i = 90^\circ$ as functions of $\xi$ (top) and $t_{\text{obs}}$ (bottom) using 40 (solid line) and 20 (dashed and dotted lines) samples of $\dot{M}$, marginalized over $\dot{M}$ and observer time (top) or sky orientation (bottom). Reasonable convergence is obtained at 20 samples.

Figure A2. Probability distribution for model 315 h with $T_i/T_e = 3$ and $i = 90^\circ$ as a function of $\phi_0$ marginalized over $\dot{M}$, $t_{\text{obs}}$ and $\xi$ using 16 (solid) or 8 (dashed or dotted) values of $\phi_0$. Reasonable convergence to $\sim 20$ per cent is obtained using eight values.

The models considered are from the four untilted simulations used by Dexter et al. (2010) and the four tilted simulations presented here. For both methods, we can either use the minimum $\chi^2$ obtained across all samples of each model fitted to the mm-VLBI data or by calculating the equivalent $\chi^2$ for the total joint probability marginalized over all parameters. Table A1 shows the results for AIC/BIC/F-test for all combinations of $\chi^2$ values and extra degrees of freedom. The values indicate the likelihood that the untilted models are statistically the same as the tilted ones. In all cases except the BIC with two extra degrees of freedom for the untilted models, the tilted models are marginally favoured ($\sim 95$ per cent significance) in this procedure. All of these results indicate that there is no statistical basis for ruling out models with disc tilt of $15^\circ$. If anything, they are marginally favoured in our current fitting procedure.

However, it would be more appropriate to consider fully time-dependent fitting in the future, especially for models with significant structural variation. From the lower panel of Fig. A1, we can estimate the duty cycle of quality (reduced $\chi^2 \leq 1$ for a single epoch of data) fits in this model to be $\sim 25$ per cent. This is the most extreme case, however, and it is unlikely that taken as a whole the tilted models would be statistically less likely than untilted ones.

### Table A1. F-test null hypothesis probability and Bayesian odds ratios for tilted versus untilted models.

|                | $\Delta \chi^2$ | F-test | AIC  | BIC  |
|----------------|-----------------|-------|------|------|
| Marginalized   | 8.4             | 0.04  | 0.05 | 0.13 |
| Best model     | 5.9             | 0.08  | 0.18 | 0.44 |
| Marginalized $\Delta \text{d.o.f.} = 2$ | 8.4 | 0.18 | 0.16 | 1.0  |
| Best model $\Delta \text{d.o.f.} = 2$ | 5.9 | 0.31 | 0.60 | 3.7  |
with current data. For the best-fitting tilted (untilted) models, the 'duty cycle' of good fits to the mm-VLBI data defined in this way is \( \simeq 40\text{–}60\text{ per cent} \) (\( \simeq 40\text{–}85\text{ per cent} \)). The 90 h model from Dexter et al. (2010) is the only tilted or untilted model with a duty cycle higher than 60 per cent. Using completely time-dependent fitting therefore could increase the relative probability of this model, which would likely increase the overall relative probabilities of the untilted models. With current data, this is unlikely to make a large difference in our results, since while the data are constraining as a whole, even breaking them down by day (let alone within a given day) greatly decreases their constraining power (see Broderick et al. 2011). This is due both to the small number of data points and the limited \( uv \)-coverage on all days except for the second day of the 2009 campaign (Fish et al. 2011). Still, as noted in the main text, looking for structural variations in future mm-VLBI observations is a powerful means to distinguish between tilted and untilted models.

This paper has been typeset from a \TeX/\LaTeX\ file prepared by the author.