Computing Convex Partitions for Point Sets in the Plane: The CG:SHOP Challenge 2020

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Abstract
We give an overview of the 2020 Computational Geometry Challenge, which targeted the problem of partitioning the convex hull of a given planar point set $P$ into the smallest number of convex faces, such that no point of $P$ is contained in the interior of a face.

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1 Introduction: Solving Hard Optimization Problems

Computational Geometry is the study of algorithms for solving problems on geometric data. Geometric problem arise in numerous applications from fields that include Robotics (motion planning and visibility problems), Operations Research (geometric location and search, route planning), Integrated Circuit Design (IC geometric design and verification), Computer-Aided Engineering (CAE), Mesh Generation, Computer Vision and Shape Reconstruction. These problems are often cast as optimization problems in which one is required to find a set of specific geometric objects that maximizes or minimizes a given objective function.

In the roughly 40 years since the beginning of Computational Geometry, a wide range of optimization problems have been proposed and investigated by computational geometers, mainly from a theoretical point of view. Many of those tasks belong to the NP-hard class of problems, for which the existence of polynomial-time algorithms implies P=NP. While Computational Geometry has considered a wide spectrum of NP-hard optimization problems, positive results typically imply polynomial-time, constant-factor approximation algorithms, without much regard for practical solution quality, realistic running times, or even exact solutions.

Even before the area of Computational Geometry was started, researchers from the mathematical field of Combinatorial Optimization were already very successful in tackling large instances of hard problems, in particular, based on Integer Linear Programming.
However, work from this field has mostly focused on discrete structures, such as graphs. This differs from geometric problems, which typically require a deeper understanding and modeling of a wide range of continuous structures: geometric structures are not automatically discrete, and additional geometric computations may be prohibitively expensive. Moreover, even aspects of geometric optimization problems that are theoretically “easy”, because they allow a polynomial-time solution, may be problematic in practice, because they need to be treated again and again during the process of solving a difficult optimization problem. This makes it important to tune algorithmic solutions, and often combine them with appropriate geometric data structures for efficient practical computation. In principle, this is the approach taken by the comparatively newer area of Algorithm Engineering; however, at this point the impact on hard problems treated in the community of Computational Geometry has been somewhat limited, as the focus has been more on streamlining computational efficiency, rather than exact methods. Combining approaches from all these different communities is highly desirable.

Scientific progress is largely triggered by considering new problems. A particularly attractive way of motivating a wide range of researchers to work on new challenges is to pose interesting competitions: These ensure that results are achieved in a timely fashion, the practical state of the art gets visibly established, and credit for success is given in a visible manner. On the theoretical side of computational geometry, these objectives have lead to The Open Problems Project (TOPP), maintained by Demaine, Mitchell and O’Rourke [3], a library of unsolved problems, rather than instances. On the more practical side of combinatorial optimization, there have been different angles to motivate practical efforts. Each benchmark library (such as the TSPLIB [16]) by itself constitutes a collection of challenges. Since 1990, the DIMACS implementation challenges have addressed questions of determining realistic algorithm performance where worst-case analysis is overly pessimistic and probabilistic models are too unrealistic. Since 1994, the Graph Drawing (GD) community has held annual contests in conjunction with its annual symposium to monitor and challenge the current state of the graph-drawing technology and to stimulate new research directions for graph layout algorithm.

Until recently, there have not been any comparable challenges in the context of Computational Geometry. The “CG:SHOP Challenge” (Computational Geometry: Solving Hard Optimization Problems) originated as a workshop at the 2019 Computational Geometry Week (CG Week) in Portland, Oregon in June, 2019. The goal was to conduct a computational challenge competition that focused attention on a specific hard geometric optimization problem, encouraging researchers to devise and implement solution methods that could be compared scientifically based on how well they performed on a database of instances. While much of computational geometry research has targeted theoretical research, often seeking provable approximation algorithms for NP-hard optimization problems, the goal of the CG Challenge was to set the metric of success based on computational results on a specific set of benchmark geometric instances. The 2019 CG Challenge focused on the problem of computing minimum-area polygons whose vertices were a given set of points in the plane. This Challenge generated a strong response from many research groups, from both the computational geometry and the combinatorial optimization communities, and resulted in a lively exchange of solution ideas.

For CG Week 2020, the second CG:SHOP Challenge became an event within the CG Week program, with top performing solutions reported in the Symposium on Computational Geometry proceedings. The schedule for the Challenge was advanced earlier, to give an opportunity for more participation, particularly among students, e.g., as part of course projects.
2 The Challenge: Minimum Convex Partitions for Planar Point Sets

2.1 The Problem

The specific problem that formed the basis of the 2020 CG Challenge was the following:

**Problem** Minimum Convex Partition (MCP) in the plane.

**Given:** A set $P$ of $n$ points in the plane.

**Goal:** A plane graph with vertex set $P$ (with each point in $P$ having positive degree) that partitions the convex hull of $P$ into the smallest possible number $u(P)$ of convex faces.

Note that collinear points are allowed on face boundaries. Each internal face angle at each point of $P$ is at most $\pi$.

2.2 Related work

The problem of computing a partition of a simple polygon, having $n$ vertices, into a minimum number of convex pieces has been well studied. If Steiner points are allowed to be added, then Chazelle and Dobkin [2] gave an algorithm, based on dynamic programming, which computes exactly an optimal solution in time $O(n + r^3)$, where $r$ is the number of reflex vertices of the given polygon. For decompositions using only diagonals (no Steiner points), Greene [7] gave an $O(r^2n^2)$ algorithm, also based on dynamic programming. Keil [10] improved the running time to $O(rn^2 \log n)$ and gave a proof of NP-hardness for polygons with holes; later, Keil and Snoeyink [11] improved the complexity to $O(n + r^2 \min\{r^2, n\})$.

The complexity of Minimum Convex Partition (MCP) in the plane was unknown when the 2020 CG Challenge began in September 2019. In November 2019, Grelier announced a proof of NP-hardness for the case of planar point sets in not necessarily general position [8]. The complexity of the MCP for points in general position is still open at the time of this writing. On the positive side, a number of positive algorithmic results have been known for a while, assuming special properties of $P$. For point sets that can be decomposed into a limited number of convex layers, Fevens, Meijer and Rappaport [5] gave a polynomial-time algorithm. Assuming that no three points are collinear, Knauer and Spillner [12] gave a 3-approximation algorithm that runs in $O(n \log n)$, and a $\frac{11}{7}$-approximation of complexity $O(n^2)$.

Worst-case bounds have also been considered; for this purpose, define $U(n)$ as the maximum number $u(P)$ over all non-degenerate point sets $P$ with $n \geq 3$. In 1998, Urrutia [18] conjectured that $U(n) \leq n + 1$. Neumann-Lara, Rivera-Campo and Urrutia [15] showed that $U(n) \leq \frac{10n-18}{7}$; Lomeli-Haro [13] showed that $U(n) \leq \frac{10}{7}n - h$, where $h$ is the number of points on the convex hull. This bound was improved by Hosono [9] to $U(n) \leq \frac{(7n-3)}{5}$, and later by Sakai and Urrutia [17] to $U(n) \leq \frac{4}{3} - 2$. Conversely, Knauer and Spillner [12] showed that $U(n) \geq n + 2$, which was improved by García-López and Nicolás [6] to $U(n) \geq \frac{12}{7}n - 2$.

Aiming for solutions to benchmark instances, Barboza, Souza and Rezende [1] gave an integer linear programming formulation of the MCP, and showed that this can be used to solve instances with up to 50 points to provable optimality.

2.3 Instances

The contest started with a total of 247 benchmark instances, as follows. Each of these instance consisted of $n$ points in the plane with integer coordinates. For $n \in \{10, 15, 20, 25, 30, 35, 40, 45, 50, 60, 70, 80, 90, 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, 10000, 20000, 30000, 40000, 50000, 60000, 70000, 80000, 90000, 100000\}$, there were six instances each. In addition, there is one instance of size $n = 100000$.
The instances were of four different types:

- **uniform**: uniformly at random from a square
- **edge**: randomly generated according to the distribution of the rate of change (the “edges”) of an image
- **illumination**: randomly generated according to the distribution of brightness of an image (such as an illumination map)
- **orthogonally collinear points**: randomly generated on an integral grid to have a lot of collinear points (similar to PCBs and distorted blueprints).

These instances were based on point sets that were originally generated for the 2019 Challenge; as such, they tended to be in general position. To account for this and the progress in complexity (which was based on instances with collinear points), a further 99 instances with larger numbers of collinear points were added in January 2020.

### 2.4 Evaluation

The comparison between different teams was based on an overall score. For each instance, this score is a number between 0 and 1, with higher scores corresponding to better solutions. The trivial solution, i.e., a triangulation, corresponds to a score of 0, and a solution without any edges, which is, of course, infeasible, corresponds to a score of 1.

For an instance, i.e., a point set \( P \) consisting of \( n \) points, let \( c \) be the number of points on the convex hull of \( P \). Observe that any triangulation of \( P \) is a convex partition with \( 2n - 2 - c \) bounded faces and \( 3(n - 1) - c \) edges. Moreover, any convex partition \( \Pi \) can be obtained by starting with a triangulation containing its edges, and removing the excess edges one by one. In this process, removing a single edge also decreases the number of bounded faces by exactly 1. Thus, any solution \( \Pi \) with \( f = 2n - 2 - c - s \) faces for some \( s \geq 0 \) has \( m = 3(n - 1) - c - s \) edges and vice versa. This allows using the number \( m(\Pi) \) of edges in a solution \( \Pi \) instead of the number of faces to determine the score of \( \Pi \) as

\[
\text{score}(\Pi) := \frac{s(\Pi)}{3(n - 1) - c},
\]

where \( s(\Pi) = 3(n - 1) - c - m(\Pi) \). In other words, the score for a solution to an instance \( P \) is the fraction of edges removed from a triangulation of \( P \).

The total score achieved by each team was the sum of all individual instance scores; only the best feasible solution submitted was used to compute the score. Participation required submitting feasible solutions. Feasibility was checked at the time of upload. Failing to submit a feasible solution for an instance resulted in a default score of 0 for that instance.

In case of ties, the tiebreaker was set to be the time a specific score was obtained. This turned out not to be necessary.

### 2.5 Categories

The contest was run in an **Open Class**, in which participants could use any computing device, any amount of computing time (within the duration of the contest) and any team composition. In the **Junior Class**, a team was required to consist exclusively of participants who were eligible according to the rules of **CG:YRF** (the Young Researchers Forum of CG Week), defined as not having defended a formal doctorate before 2018.

The demand for an additional **Limited Class** (to be run on a specific server that was to be uniform for all participants) turned out to be too low to justify the additional effort.
2.6 Server and Timeline

The contest itself was run through a dedicated server at TU Braunschweig, hosted at https://cgshop.ibr.cs.tu-bs.de/competition/cg-shop-2020/. It opened at 18:00 CEDT (noon, EDT) on September 30, 2019, and closed at 24:00 (midnight, AoE), February 14, 2020.

3 Outcomes

A total of 21 teams participated in the contest. In the end, the top 10 in the leaderboard looked as shown in Table 1; note that according to the scoring function, a higher score is better.

Table 1 The top of the final leaderboard. “Best solutions” are the best found by any participating team, which does not exclude the possibility of better solutions. “Unique best” solutions are those that were not found by any other team.

| Position | Team               | Score   | # best solutions | # unique best solutions |
|----------|--------------------|---------|------------------|------------------------|
| 1        | Team UBC           | 175.172880 | 209              | 11                     |
| 2        | OMEGA              | 175.130597 | 297              | 126                    |
| 3        | CGA-Sbg            | 175.040207 | 187              | 0                      |
| 4        | Les Shadoks        | 174.695586 | 160              | 6                      |
| 5        | G-SCOP             | 174.543068 | 138              | 0                      |
| 6        | Min2Win@Zurich     | 174.384784 | 121              | 0                      |
| 7        | TUFUnky4you        | 173.973716 | 100              | 0                      |
| 8        | Team Technion      | 173.621857 | 99               | 0                      |
| 9        | Sapucaia, de Rezende, de Souza | 170.939574 | 88               | 0                      |
| 10       | ucsbtheorylab      | 169.975180 | 76               | 0                      |

The progress over time of each team’s score can be seen in Figure 1, the best solutions for all instances (displayed by score) can be seen in Figure 2.

Clearly, the outcome was quite tight between the top teams; in particular, Team UBC and OMEGA were only separated by 0.02% in their respective scores. As can be seen from Figure 3 (which shows the normalized number of faces instead of the respective scores for the top 4 teams), OMEGA found very slightly better solutions for a large number of instances (which is also reflected by the high number of unique best solutions in Table 1), while Team UBC found significantly better solution for six of the instances. The latter was sufficient for the overall win.

The top 3 finishers in the Open Class were invited for contributions in the 2020 SoCG proceedings, as follows.

1. Team UBC: Da Wei Zheng, Jack Spalding-Jamieson and Brandon Zhang [19].
2. Team OMEGA: Laurent Moalic, Dominique Schmitt, Julien Lepagnot and Julien Kritter [14].
3. Team CGA-Sbg: Günther Eder, Martin Held, Stefan de Lorenzo and Peter Palfrader [4].

Consisting only of students, Team UBC was also the runaway winner of the Junior Class.

All three teams engineered their solutions based, broadly, on variants of local search methods, with the use of randomization and constraint programming [19], genetic approaches [14], and tailored initial decompositions [4]. Details of their methods and the engineering decisions they are made are given in their respective papers.
Figure 1 Total score over time for the best ten teams.

Figure 2 All instance scores for the best ten teams.
Figure 3 Normalized number of faces for the top 4 teams. Note how OMEGA (shown in red) achieved very slightly better solutions for many instances, while Team UBC (shown in blue) found significantly better solutions for a relatively small number of instances, which made the difference in the overall outcome.
4 Conclusions

The 2020 CG:SHOP Challenge motivated a considerable number of teams to engage in intensive optimization studies. Not only did this lead to practical developments, it also triggered theoretical progress, as demonstrated by Grelier [8]. We are confident that this will motivate further work on the problem of Minimum Convex Partitions, as well as other practical geometric optimization work.

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