A Reliability-based Optimization Model for Comprehensive Lightning Protection System Design

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Abstract. Lightning strike is the main reason for the failure of overhead lines in distribution network, which seriously affects the reliability of power supply and power quality. The existing optimization research of lightning protection system (LPS) mainly focuses on a specific lightning protection method, and there are few kinds of research on comprehensive optimization methods among various lightning protection solutions. A model based on distribution network reliability index is proposed in this paper. In the model, mixed integer linear programming (MILP) is used to optimize the lightning protection measures of lightning rods, lightning wires and lightning arresters with different densities. The model considers the influence of arc over rate on the basis of related research and obtains a comprehensive optimization method of different protection measures acting on distribution lines. Taking an actual distribution network as an example, this paper applies the optimization model established to obtain the appropriate lightning protection measures for each line.

1. Introduction

Distribution network plays an important role in distributing electric energy in power system. Therefore, we put forward higher demands for power supply reliability and power quality of distribution network. However, due to the limited role of lightning protection measures, distribution network is vulnerable to lightning strikes. The electromagnetic effects of lightning strikes will cause overvoltage, and further cause overhead line trip or disconnection fault, resulting in huge economic losses. At present, lightning strikes are the most common cause of faults in distribution network, and the tripping times caused by lightning strikes account for about 70\%-80\% of the total tripping times [1]. It is necessary to study the current situation of distribution network lightning protection system (LPS), and put forward an economic comprehensive lightning protection system combined with reliability index.

The existing LPS mainly includes the installation of lightning arrestors, lightning rods and lightning wires [2]. Reference [3] proposes an optimization scheme for the installation location of the arrester based on genetic algorithm considering the economic cost. This optimization can minimize the risk of lightning strike and improve the reliability of power supply. According to the reference [4,5], optimization of LPS for distribution network is still an open research field. Among many optimization methods applied to power system, mixed integer linear programming (MILP) has good convergence. It can significantly shorten the calculation time and improve the optimization efficiency. An optimization of LPS based on MILP is proposed in reference [6], and the results are obtained by branch-and-bound method. On the basis of this research, a new optimization of LPS based on multiple power supply reliability indexes is proposed in reference [7]. By adding constraint conditions, the influence of more factors on the lightning protection system is considered.
The two optimization methods do not consider the impact of arc over rate, and believe that the insulator flashover will certainly cause the power supply interruption, which is inconsistent with the actual situation. Only when a stable power frequency arc is generated after flashover occurs, the line will fail. The two references mainly analyzed and studied the lightning rod and lightning wire, and did not mention the protection of lightning arrester with superior lightning protection performance. For the distribution system, as the most common and effective lightning protection method, arrester protection should be included in the LPS model [8].

On the basis of the research in reference [6,7], this paper considers the protection measures of lightning wire, lightning rod and different installation densities of lightning arrester. A lightning electromagnetic transient model is established to simulate the electromagnetic transient process caused by the lightning in the distribution network. The lightning failure rate of distribution line under different protection measures is obtained by considering the influence of arc over rate. Finally, the mixed integer linear programming (MILP) model is adopted in this paper. Considering the combined action of various lightning protection measures, the optimal layout scheme of LPS is obtained based on the reliability index.

2. Lightning transient model
Based on ATP-EMTP, this paper establishes a lightning transient electromagnetic transient model of the distribution network and obtains the lightning withstand level of the feeder line with different lightning protection methods.

2.1. Distribution feeder line model
At present, there are five models of feeder line in ATP-EMTP, which are the Bergeron model, π-shaped lumped parameter model, Node model, J. Marti model, and Semlyn model. Due to the advantages of the J. Marti model, such as a wide range of applications and high accuracy of calculation, it has been widely used in the lightning strike feeder line model. The main types of overhead feeders are LGJ-120, LGJ-70, and LGJ-35, among which LGJ-35 is the most widely used. In this paper, the J. Marti model is used to simulate the LGJ-35 distribution feeder line.

2.2. Pole model
There are three modeling methods of pole model, namely the concentrated parameter inductance model, the single-wave impedance model, and the multi-wave impedance model. For the distribution feeder line, the height of the pole is generally not more than 30m, and the concentrated parameter inductance model can be used.

Ground resistance is an important factor affecting the lightning resistance level of line. Because of the inductance of grounding, the lightning current can be obstructed after lightning strikes the pole or line. When the lightning current is discharged into the earth, the soil around the ground will produce a spark discharge due to the higher electric field intensity. The impact ground resistance is related to the soil resistivity of the local area, as well as the shape and depth of the ground. In the study of ATP simulation generally take 10 Ω grounding resistance [9].

2.3. Insulator model
Nowadays, the procedure method and the intersection method are generally used to judge whether the insulator flashover. The intersection method considers the process of arc development, and it is more accurate and reasonable to judge whether there is flashover through the overvoltage on the insulator string. Model flash module is used in ATP to judge and calculate the moment of flashover. Three model flash modules are set in each pole, respectively representing ABC three phases of the line. When the insulator is judged to have flashover, the right switch in the module is closed, and the voltage at both ends of the insulator changes to 0.
3. Calculation of failure rate
Distribution lines are mostly ungrounded at neutral points or through grounded through arc suppression coils. Therefore, when the overhead distribution line is affected by lightning and single-phase flashover occurs, the stable arc cannot be generated. Only when the insulator two-phase flashover will form a stable power frequency arc, leading to the failure of the line. Therefore, when calculating the failure rate of the distribution network, we should adopt the lightning current parameters when the insulator two-phase flashover occurs.

In this paper, the voltage and current parameters of distribution line during two-phase flashover are obtained through ATP simulation. Based on the above equivalent models of the distribution network, the transient process of lightning strike with different LPS are simulated in ATP respectively. The lightning withstand level is also calculated respectively. The calculation process is shown in Figure 1.

![Flow chart of calculation process](image)

**Figure 1. Flow chart of calculation process**

3.1. Direct lightning flashover rate
When lightning acts on the line, the maximum current amplitude without insulation flashover occurs is the lightning resistance level, and the unit is kA. The amplitude of two-phase flashover current under direct lightning is obtained by ATP simulation.

Direct lightning flashover rate is calculated by lightning resistance level of distribution line. Direct lightning flashover rate $P_D$ is the probability that the lightning current $I_0$ is greater than the lightning resistance level $i_0$ of the line:

$$P_D(I_0 \geq i_0) = \frac{1}{1+\left(\frac{i_0}{I_0}\right)^{2.6}} \quad (1)$$

3.2. Inductive lightning flashover rate
Because the distribution line adopts the operation mode of ungrounded neutral point, the stable arc can be formed only when two or three-phase insulators are flashed due to induced over-voltage. When one phase insulator flashover, it will affect the other insulators. Therefore, it is necessary to determine the over-voltage amplitude that causes the flashover of the second phase and the third phase.

When phase A and phase B are flashed successively, and phase A, phase B and phase Care flashed successively, the overvoltage values of flashover are respectively [10]:

$$U_{AB} = \frac{U_{50\%}(Z_{AA} + 2R_{ch})}{Z_{AA}(1-k_{AB})} \quad (2)$$

$$U_{ABC} = \frac{U_{50\%}(Z_{eq} + 2R_{ch})}{Z_{eq}(1-k_{ABC})} \quad (3)$$

where $k_{AB}$ is the coupling coefficient of A-phase conductor to B-phase conductor; $k_{ABC}$ is the coupling coefficient of A-phase conductor and B-phase conductor to C-phase conductor; $Z_{eq}$ is the equivalent self impedance of A-phase conductor and B-phase conductor, taking $(Z_{AA} + Z_{AB})/2$; $R_{ch}$ is
the earth resistance, $Z_{AA}$ is the self impedance of A-phase, $Z_{AB}$ is the mutual impedance of A-phase conductor and B-phase conductor; $U_{AB}$ is AB two-phase voltage, $U_{ABC}$ is ABC three-phase voltage, $U_{50\%}$ is 50% flashover voltage.

Take the minimum two-phase flashover voltage $U_{1,2}$ and three-phase flashover voltage $U_{1,2,3}$.

The induced overvoltage amplitudes are $U_{1,2}$ and $U_{1,2,3}$ for overhead lines with an average distance of 10 meters from the ground. The probability $P_I$ of causing two-phase and three-phase insulation flashover to exceed the overvoltage amplitude can be determined by Figure 2.

![Figure 2. Induced overvoltage amplitude and probability curve](image)

For the overhead line whose average height of the wire suspension is not 10 meters above the ground, the induced overvoltage amplitude $U_{1,2}$ and $U_{1,2,3}$ at the same probability level as that of the 10-meter overhead line can be converted by the following formula. The corresponding flashover rate $P_I$ is obtained from Figure 2[10]:

$$
U_{1,2} = \frac{U_{1,2}}{0.1h_{gt}}
$$

$$
U_{1,2,3} = \frac{U_{1,2,3}}{0.1h_{gt}}
$$

3.3. Arc over rate

The optimization of LPS in reference [9,10] did not consider the influence of arc over rate on lightning failure rate. In reality, insulator flashover does not necessarily lead to lightning failure. Only when stable power frequency arc appears, the line will fail. The probability of flashover turning into an arc is called arc over rate. The arc over rate $\eta$ is mainly related to the average electric field strength $E$. The calculation of the arc over rate $\eta$ is shown in formula (6) [8]:

$$
\eta = (4.5E^{0.75} - 14) \times 10^{-2}
$$

For the distribution network with neutral grounded,

$$
E = \frac{U_e}{2l_j + 0.5l_m}
$$

where $U_e$ is power frequency voltage, $l_m$ is the distance between cross arms of the pole, and $l_j$ is the length of insulator string. In this paper, $l_j$ is 0.3 m and $l_m$ is 11.4 m. Then,

$$
E = \frac{U_e}{2l_j + 0.5l_m} = 16.4kV/m
$$

$$
\eta = (4.5E^{0.75} - 14) \times 10^{-2} = 0.2312
$$

3.4. Failure rate

The number of lightning strikes directly affects the lightning failure rate of overhead lines. The number of lightning strikes per 100 km of the line is shown as follows:
\[ N = 0.1 \times A \times 4h \]  

where \( A \) is the lightning density and \( h \) is the average height of the upper line.

After obtaining the direct lightning flashover rate, inductive lightning flashover rate and arc over rate, the direct lightning failure rate \( n_1 \) and the direct lightning failure rate \( n_2 \) of the line are finally obtained:

\[ n_D = N \times \eta \times P_D \]  
\[ n_I = N \times \eta \times P_I \]

The failure rate \( N \) under various protective measures is shown as follows

\[ n = n_D + n_I \]

**Table 1.** Failure rate with single protection

| Number | LPS | Failure rate (times/100km/year) |
|--------|-----|---------------------------------|
| S1     | Without protection             | 2.5241                          |
| S2     | Lightning rod                   | 2.1524                          |
| S3     | Lightning wire                  | 1.8970                          |
| S4     | Lightning arrester 1            | 0.6399                          |
| S5     | Lightning arrester 2            | 1.4936                          |
| S6     | Lightning arrester 3            | 1.9910                          |

Comprehensive lightning protection measures are used in this paper. Take the combination mode of lightning wire and lightning arrester 1 as an example to calculate:

1. The lightning wire reduces the lightning overvoltage to 68%, that is, the lightning resistance level is increased to 1.47 times of the original.

2. If the lightning voltage is still a threat to the line insulator, the arrester is used. Lightning resistance multiple increased from 1.47 to 1.47 multiplied by 2.9. Where, 2.9 is the ratio of lightning arrester 1 to the flashover probability without protection.

Starting from this idea and combining with the data in Table 1 the lightning failure rate under different combinations of protective measures can be obtained. The results are shown in Table 2:

**Table 2.** Failure rate with combination protection

| Number | LPS | Failure rate (times/100km/year) |
|--------|-----|---------------------------------|
| S1     | Without protection             | 2.5241                          |
| S2     | Lightning rod                   | 2.1524                          |
| S3     | Lightning wire                  | 1.8970                          |
| S4     | Lightning arrester 1            | 0.6399                          |
| S5     | Lightning arrester 2            | 1.4936                          |
| S6     | Lightning arrester 3            | 1.9910                          |
| C1     | S2, S3                         | 1.3772                          |
| C2     | S3, S4                         | 0.2624                          |
| C3     | S3, S5                         | 0.8162                          |
| C4     | S3, S6                         | 1.2024                          |

4. Milp model

4.1. Reliability index

This paper uses SAIFI (System Average Interruption Frequency Index) in the power supply reliability index to evaluate and constrain the power supply reliability of lightning protection optimization measures. It is defined as follows:

\[ SAIFI = \frac{\text{Continued power outages affect users}}{\text{Number of users in the zone}} \]
Taking the radial distribution network shown in Figure 3 as an example, the reliability index of the distribution network is calculated.

Figure 3. The topology of a distribution network

By assuming that the line of the small system 75-80 is tripped by lightning strike, the influence of each branch fault on the power supply reliability index of the small system is analyzed. The influence of the grid structure of distribution system and the number of users on the reliability index of power supply can be clarified. The line data is shown in Table 3.

Table 3. Node user data information

| Line number | Starting node | Terminal node | Lines long (km) | Number of users |
|-------------|---------------|---------------|-----------------|----------------|
| 75          | 75            | 76            | 0.7             | 20             |
| 76          | 76            | 80            | 1               | 32             |
| 77          | 77            | 81            | 3               | 10             |
| 78          | 78            | 77            | 2               | 10             |
| 79          | 79            | 78            | 0.8             | 27             |
| 80          | 80            | 79            | 1               | 13             |

(1) When any line 76, 77 fails, the switch of line 76 will be disconnected and the power supply will be interrupted. At this time, the system SAIFI is:

\[
SAIFI = \frac{42}{112} = 0.375
\]

(2) When any line 75, 78, 80 fails, the line 75 switch is disconnected. This will result in power outages for all 75 to 80. At this time, the system SAIFI is:

\[
SAIFI = \frac{112}{112} = 1
\]

In this paper, lightning failure rate and reliability index are combined. The parameter SAIFI’ is defined as an optimization index based on the average interrupt frequency of the system:

\[
SAIFI' = n \times SAIFI \times \lambda
\]

where \( \lambda \) is the probability of continuous middle section after circuit trip, which is 20%. \( n \) is line failure rate.

4.2. Objective function

This paper selects 0-1 optimization method. Assuming that the distribution system has a total of \( i \) branches, a total of \( j \) lightning protection measures are proposed. The objective optimization result is the 0-1 optimization matrix of scale \( ij \), which is set as \( x_{ij} \).

The average interrupt frequency index of the system is taken as the reliability evaluation index:

\[
SAIFI_{ij} = \frac{n_{ij}B_i}{B_T}
\]
where $B_i$ represents the number of affected users after the failure of the $i$-th feeder line. $B_T$ represents the total number of users. $n_{ij}$ represents the failure rate of the $i$-th feeder line with $j$-th lightning protection method. The calculated $SAIFI$ should be an $i \times j$ matrix.

In this paper, the single objective optimization is adopted. Mixed integer linear programming is developed for LPS design optimization. The minimum sum of the $SAIFI$ of the entire distribution network is used as the optimization goal. The objective function and constraints are:

$$
\min SAIFI = \sum_{i=1}^{n_l} \sum_{j=1}^{n_p} SAIFI_{ij} x_{ij}
$$

subject to:

$$
\sum_{j=1}^{n_p} x_{ij} = 1
$$

$$
\sum_{i=1}^{n_l} x_{ij} = 1
$$

$$
\sum_{i=1}^{n_l} \sum_{j=1}^{n_p} cost_{ij} x_{ij} \leq \text{Investment}_{Total}
$$

where, $n_l$ is the number of feeder lines and $n_p$ is the number of lightning protection methods. $x_{ij} = 1$, if the line “$i$” adopts the lightning protection method “$j$”. Under the constraints of economic investment, the optimization method can get a reasonable choice of lightning protection method for every distribution line.

5. Case study

Taking the distribution network shown in Figure 3 as an example, the optimized LPS of the distribution network under the constraints of investment can be obtained. In this paper, 20,000 $ and 100,000 $ are applied to the distribution system. According to market research, the lightning wire usually costs 800 $ per km; the price of lightning arrester is about 1,500 $ per group; the lightning rod costs 300 $ per unit. The optimization results are shown in Table 4.

| Cost       | Protection measures | Line number |
|------------|---------------------|-------------|
| 20,000 $   | S1                  | 2,4,8,10,11,34,35 |
|            | S2                  | 17,39,41,50,59,69,77 |
|            | S3                  | All other lines|
|            | S6                  | 32,33,52,53,73,80 |
|            | C1                  | 3,12,21,37 |
|            | C4                  | 1,5,24,36,42,61 |
| 100,000 $  | S1                  | 2,4,10 |
|            | S2                  | 8,11,17,19 |
|            | S3                  | 6,25-29,30,45-50 |
|            | S4                  | 20,70,74-76,78 |
|            | C2                  | 1,21-24,36,42,61 |
|            | C3                  | All other lines |

The corresponding $SAIFI$ and actual investments are shown in Table 5.
As expected, the higher the investment amount, the more the lightning arrester will be used in lines with higher interruption numbers. For combination protection, when the budget is sufficient, the combination of lightning wire and lightning arrester is widely used.

### 6. Conclusion

Lightning strikes are the main cause of distribution network faults. This paper designs an effective lightning protection system (LPS). On the basis of the related research, the influence of arc over rate is considered, and different density of arrester protection is added into the protection method. Compared with the previous single protection mode, the combined protection mode is added in the optimization in this paper to give more choices to the distribution system. In this paper, a distribution network is optimized by LPS. The results show that the optimization model is highly sensitive to investment constraints.

In this paper, an 81 node distribution network is taken as an example, through the comparison of two different economic investment schemes, it is found that the use frequency of arrester protection is higher. At the same time, in the case of more investment, the combination of lightning wire and arrester can significantly reduce the average interruption frequency of the system, which proves the necessity of the existence of combined lightning protection measures.

This paper mainly studies three kinds of protection methods. Future research will increase the selectivity of protection measures and the accuracy of optimization methods, which will further improve the reliability of distribution network.

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