Bell’s theorem, quantum mechanical non-locality and atomic cascade photons

M. Ardehali

Research Laboratories, NEC Corporation, Sagamihara, Kanagawa 229 Japan

Abstract

Bell’s theorem of 1965 is a proof that all realistic interpretations of quantum mechanics must be non-local. Bell’s theorem consists of two parts: first a correlation inequality is derived that must be satisfied by all local realistic theories; second it is demonstrated that quantum mechanical probabilities violate this inequality in certain cases. In the case of ideal experiments, Bell’s theorem has been proven. However, in the case of real experiments where polarizers and detectors are non-ideal, the theorem has not yet been proven since the proof always requires some arbitrary and \textit{ad hoc} supplementary assumptions. In this paper, we state a new and rather weak supplementary assumption for the ensemble of photons that emerge from the polarizers, and we show that the conjunction of Einstein’s locality with this assumption leads to validity of an inequality that is violated by a factor as large as 1.5 in the case of real experiments. Moreover, the present supplementary assumption is considerably weaker and more general than Clauser, Horne, Shimony, Holt supplementary assumption.
Einstein, Podolsky, and Rosen (EPR)\textsuperscript{[1]} theorem of 1935 is a proof that if local realism holds, then quantum mechanics must be an incomplete theory. In 1965, Bell\textsuperscript{[2]} showed that the assumption of local realism, as postulated by EPR, implies some constraints on the statistics of two spatially separated particles. These constraints which are collectively known as Bell inequalities are sometimes grossly violated by quantum mechanics in the case of ideal experiments. Bell’s\textsuperscript{[2]} theorem is therefore a proof that local interpretations of quantum mechanics are impossible.

Bell’s original argument, however, can not be experimentally tested because it relies on ideal polarizers and detectors. Faced with this problem, correlation inequalities have been derived for non-ideal systems\textsuperscript{[3-10]}. However, quantum mechanics does not violate any of these inequalities. In the case of real experiments, the violation of these inequalities arises only when some \textit{ad hoc} assumptions, whose correctness can never be tested, are supplemented to the original inequalities.

In this paper, we state a new and rather weak supplementary assumption for the ensemble of photons that emerge from the polarizers and we show this assumption is sufficient to make experiments which are feasible with present technology applicable as a test of locality. In particular, we deduce a correlation inequality for two-channel polarizer systems and we show that quantum mechanics violates this inequality by a maximum factor of 1.5. Since quantum mechanics violates the previous inequalities\textsuperscript{[4-10]} by a maximum factor of $\sqrt{2}$, the magnitude of violation of the present inequality is approximately 20.7\% larger than that of previous inequalities.

We start by considering the Bohm’s\textsuperscript{[12]} version of EPR experiment in which an unstable source emits pairs of photons in a cascade from state $J = 1$ to $J = 0$. The source is viewed by two apparatuses. The first (second) apparatus consists of a polarizer $P_1$ ($P_2$) set at angle $m$ ($n$), and two detectors $S_1^\pm$ ($S_2^\pm$) put along the ordinary and the extraordinary beams. We consider a particular photon that passes through the first [second] polarizer, and assign observable $A(m)$ [$B(n)$] to this photon. These observables can take the following values:

$$ A(m) = \begin{cases} 
1, & \text{the photon emerges along the ordinary axis} \\
0, & \text{the photon is absorbed by the polarizer} \\
-1, & \text{the photon emerges along the extra-ordinary axis}
\end{cases} \quad (1) $$

and

$$ B(n) = \begin{cases} 
1, & \text{the photon emerges along the ordinary axis} \\
0, & \text{the photon is absorbed by the polarizer} \\
-1, & \text{the photon emerges along the extra-ordinary axis}.
\end{cases} \quad (2) $$

During a period of time $T$ while the polarizers are set along axes $m$ and $n$, the source emits, say, $N$ pairs of photons. Let $N^{++}(m, n)$ [$N^{--}(m, n)$] be
the number of photon pairs that emerge along the ordinary [extra-ordinary] axes; $N^{+-}(m, n) [N^{-+}(m, n)]$ be the number of photon pairs in which the first photon emerges along the ordinary [extra-ordinary] axis and the second photon emerges along the extra-ordinary (ordinary) axis; $N^{+0}(m, n) [N^{0-}(m, n)]$ the number of pairs in which the first photon emerges along the ordinary [extra-ordinary] axis and the second photon is absorbed by the polarizer; $N^{0+}(m, n) [N^{0+}(m, n)]$ the number of pairs in which the first photon is absorbed by the polarizer and the second photon emerges along the ordinary [extra-ordinary] axis, and finally let $N^{00}(m, n)$ be the number of photon pairs that are emitted by the source but absorbed by the polarizers. If the time $T$ is sufficiently long, then the ensemble probabilities are defined as

$$p^{\pm \pm}(m, n) = \frac{N^{\pm \pm}(m, n)}{N}, \quad p^{\pm 0}(m, n) = \frac{N^{\pm 0}(m, n)}{N},$$

$$p^{0 \pm}(m, n) = \frac{N^{0 \pm}(m, n)}{N}, \quad p^{0 0}(m, n) = \frac{N^{0 0}(m, n)}{N}. \quad (3)$$

In terms of the ensemble probabilities, the expected value $e(a, b)$ is defined as

$$e(a, b) = p^{+-}(a, b) - p^{+-}(a, b) - p^{-+}(a, b) + p^{--}(a, b). \quad (4)$$

Now let $N^{+}(m) [N^{+}(n)]$ be the number of photons that pass through polarizer $P_1 [P_2]$ and emerge along the ordinary axis; $N^{-}(m) [N^{-}(n)]$ the number of photons that pass through polarizer $P_1 [P_2]$ and emerge along the extra-ordinary axis, and $N^{0}(m) [N^{0}(n)]$ the number of photons that are emitted by the source but absorbed by polarizers $P_1 [P_2]$. Again if the time $T$ is sufficiently long, then the ensemble probabilities are defined as

$$p^{\pm}(m) = \frac{N^{\pm}(m)}{N}, \quad p^{0}(m) = \frac{N^{0}(m)}{N},$$

$$p^{\pm}(n) = \frac{N^{\pm}(n)}{N}, \quad p^{0}(n) = \frac{N^{0}(n)}{N}. \quad (5)$$

Since $p^{00}(m, n), p^{0 \pm}(m, n), p^{00}(m, n), p^{0}(m)$, and $p^{0}(n)$ can not be measured in actual experiments, it is crucial that they do not appear in any correlation inequality that is used to test locality (it is important to emphasize that in real experiments, due to imperfection of polarizers and detectors, $p^{0+}(m, n), p^{0 \pm}(m, n)$, and $p^{00}(m, n)$ are non-zero).

Note that Eqs. 3 and 5 are quite general and follow from the standard rules of probability theory. No assumption has yet been made that is not satisfied by quantum mechanics. Hereafter, we focus our attention only on those theories that satisfy EPR criterion of locality: “Since at the time of
measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to first system \( \mathbb{I} \)’. Assuming the first polarizer \( P_1 \) may be set along the axes \( a \) or \( b \) and the second polarizer \( P_2 \) may also be set along the axes \( a' \) or \( b' \), EPR’s criterion of locality can be translated into the following relation:

\[
\text{Locality} \implies \text{There exists a four-axis probability distribution function } p(a, b, a', b').
\]  

Relation (5) is a very general form of locality that accounts for correlations subject only to the requirement that emergence of the first photon through the first polarizer does not depend on the orientation of the second polarizer. This assumption is quite natural since the two photons are spatially separated so that the orientation of the second polarizer should not influence the measurement carried out on the first photon. The extreme generality of relation 5 as the requirement for locality has been discussed in detail by Wigner [11] [see also Selleri in [3]].

In the following we show that relation (5) leads to validity of an equality that is sometimes grossly violated by the quantum mechanical predictions. First we need to prove the following algebraic theorem.

**Theorem:** If local realism holds, that is, if the four-axis probability distribution function \( p(a, b, a', b') \) exists, then the following inequality always holds:

\[
e(a, b) + e(b', a) + e(b, a') \geq 2p^{++}(a', b') + 2p^{--}(a', b') - p^{+}(a') - p^{-}(a') - p^{+}(b') - p^{-}(b') - 1,
\]  

**Proof:** Assuming \( p(a, b, a', b') \) exists, the LHS of (6) is defined as

\[
e(a, b) + e(b', a) + e(b, a') = \sum_{i, j} p_{ij}(a, b, a', b')
\]  

where \( i \) and \( j \) belong to the set \{+, −, 0\}. We now consider the following 9 cases:

(i) First assume \( A(a') = 1 \) and \( B(b') = 1 \), then

\[
A(a)B(b) + A(a)B(b') + A(a')B(b) \geq -1.
\]

(ii) Next assume \( A(a') = -1 \) and \( B(b') = -1 \), then

\[
A(a)B(b) + A(a)B(b') + A(a')B(b) \geq -1.
\]

(iii) Next assume \( A(a') = 1 \) and \( B(b') = -1 \), then

\[
A(a)B(b) + A(a)B(b') + A(a')B(b) \geq -3.
\]

(iv) Next assume \( A(a') = -1 \) and \( B(b') = 1 \), then

\[
A(a)B(b) + A(a)B(b') + A(a')B(b) \geq -3.
\]

(v) Next assume \( A(a') = 1 \) and \( B(b') = 0 \), then

\[
A(a)B(b) + A(a)B(b') + A(a')B(b) \geq -2.
\]

4
(vi) Next assume $A(a') = -1$ and $B(b') = 0$, then
\[ A(a)B(b) + A(a)B(b') + A(a')B(b) \geq -2. \]
(vii) Next assume $A(a') = 0$ and $B(b') = 1$, then
\[ A(a)B(b) + A(a)B(b') + A(a')B(b) \geq -2. \]
(viii) Next assume $A(a') = 0$ and $B(b') = -1$, then
\[ A(a)B(b) + A(a)B(b') + A(a')B(b) \geq -2. \]
(ix) Finally assume $A(a') = 0$ and $B(b') = 0$, then
\[ A(a)B(b) + A(a)B(b') + A(a')B(b) \geq -1. \]
Using relations (i-x), we obtain the following upper bound on (7).
\[ e(a, b) + e(b', a) + e(b, a') \geq -3p^+(a', b') - 3p^-(a', b') \]
\[ -2p^0(a', b') - 2p^0(a', b') - 2p^0(a', b') - 2p^0(a', b') \]
\[ -p^0(a', b') - p^+(a', b') - p^-(a', b') \]
\[ e(a, b) + e(b', a) + e(b, a') - 2p^+(a', b') - 2p^-(a', b') \]
\[ + p^+(a') + p^-(a') + p^+(b') + p^-(b') \geq -1, \]  
and the theorem is proved.

First we consider an atomic cascade experiment in which polarizers and detectors are ideal. In an ideal experiment while the polarizers are set along arbitrary axes $m$ and $n$, all emitted photons pass through the polarizers and are analyzed. Thus the probability that a photon is absorbed by the polarizer or is not analyzed is zero, i.e.,
\[ p^0(m) = p^0(n) = p^{\pm 0}(m, n) = p^{0\pm}(m, n) = p^{00}(m, n) = 0. \]

Inequality (10) may be considerably simplified if we invoke some of the symmetries that are exhibited in atomic-cascade photon experiments. For a pair of photons in cascade from state $J = 1$ to $J = 0$, the quantum mechanical detection probabilities $p^{\pm}_{QM}$ and expected value $e_{QM}$ exhibit the following symmetry
\[ p^{\pm\pm}_{QM}(a, b) = p^{\pm\pm}_{QM}(|a - b|), \quad e_{QM}(a, b) = e_{QM}(|a - b|). \]
We assume that the local theories also exhibit the same symmetry
\[ p^{\pm \pm}(a, b) = p^{\pm \pm}(|a - b|), \quad e(a, b) = e(|a - b|) \tag{13} \]

Now if we choose the following orientation \((a, b) = (b', a) = (b, a') = 120^\circ\) and \((a', b') = 0^\circ\), and using (13), inequality (10) becomes
\[ 3e(120^\circ) - 2p^{++}(0^\circ) - 2p^{--}(0^\circ) + p^+(a') + p^-(a') + p^+(b') + p^-(b') \geq -1. \tag{14} \]

For ideal polarizers and detectors, the single and joint detection probabilities for a pair of photons in a cascade from state \(J = 1\) to \(J = 0\) are given by
\[
\begin{align*}
e(m, n) &= e(\theta) = \cos 2\theta, \\
p^{++}(a', b) &= p^{--}(a') = p^{++}(b') = p^{--}(b') = \frac{1}{2}, \\
p^{++}(m, n) &= p^{++}(\theta) = \frac{\cos^2 \theta}{2}, \\
p^{--}(m, n) &= p^{--}(\theta) = \frac{\cos^2 \theta}{2}.
\end{align*}
\tag{15} \]

Substituting (15) in (14), we obtain
\[
\begin{align*}
3 \cos(240^\circ) - 2 \frac{\cos^2(0^\circ)}{2} - 2 \frac{\cos^2(0^\circ)}{2} + 1 + 1 + 1 + 1 &= 3 \times (-0.5) - 2 \times \frac{1}{2} - 2 \times \frac{1}{2} + 2 \geq -1, \\
or
-1.5 &\geq -1, \tag{16}
\end{align*}
\]
which violates inequality (10) by a factor of 1.5 in the case of ideal experiments.

It is important to emphasize that in the case of ideal experiments, the present inequality immediately reduces to Bell’s original inequality of 1965. To show this, we assume \(a'\) and \(b'\) are along the same direction, using (15), we have \(p^{++}(a', b') = p^{--}(a', b') = \frac{1}{2}, p^+(a') = p^-(a') = p^+(b') = p^-(b') = \frac{1}{2}\). Inequality (10) therefore becomes
\[
e(a, b) + e(b', a) + e(a', b) \geq -1, \tag{18}
\]
which is the same as Bell’s original inequality of 1965.

We have thus shown that in an ideal experiment where \(p^{++}(m, n) + p^{--}(m, n) + p^{++}(m, n) + p^{--}(m, n) = 1\), Bell’s original inequality (18) is sufficient and there is no need for inequality (10). However, in a real experiment where \(p^0(m), p^0(n), p^{\pm 0}(m, n), p^{0\pm}(m, n), \) and \(p^{00}(m, n)\) are
non-zero, i.e., for the case where \( p^+(m, n) + p^-(m, n) + p^+(m, n) + p^-(m, n) < 1 \), inequality (10) is a distinct and new inequality.

We now consider a real atomic cascade experiment in which polarizers and detectors are non-ideal. In the cascade experiment an atom emits two photons in a cascade from state \( J = 1 \) to \( J = 0 \). Since the pair of photons have zero angular momentum, they propagate in the form of spherical wave. Thus the joint probability for single transmission and single detection are given by

\[
D^+ (a) = D^-(a) = \eta \left( \frac{\Omega}{8\pi} \right), \quad D^+ (b) = D^- (b) = \eta \left( \frac{\Omega}{8\pi} \right). \quad (19)
\]

Similarly the joint probability for double transmission and double detection are given by

\[
D^{++} (a, b) = D^{--} (a, b) = \eta^2 \left( \frac{\Omega}{8\pi} \right)^2 g(\theta, \phi) \left[ 1 + F(\theta, \phi) \cos 2(a - b) \right],
\]

\[
D^{+-} (a, b) = D^{-+} (a, b) = \eta^2 \left( \frac{\Omega}{8\pi} \right)^2 g(\theta, \phi) \left[ 1 - F(\theta, \phi) \cos 2(a - b) \right],
\]

\[
(20)
\]

where \( \eta \) is the quantum efficiency of the detectors, \( \Omega \) is the solid angle of the detector, \( \cos \theta = a.b \), and angle \( \phi \) is related to \( \Omega \) by

\[
\Omega = 2\pi (1 - \cos \phi). \quad (21)
\]

The function \( g(\theta, \phi) \) is the angular correlation function and in the special case is given by

\[
g(\pi, \phi) = 1 + \frac{1}{8} \cos^2 \phi (1 + \cos \phi)^2. \quad (22)
\]

The function \( F(\theta, \phi) \) is the so-called depolarization factor and for the special case \( \theta = \pi \) and small \( \phi \) is given by

\[
F(\pi, \phi) \approx 1 - \frac{2}{3} (1 - \cos \phi)^2. \quad (23)
\]

The function \( F(\theta, \phi) \), in general, is very close to 1. Finally the expected values for double transmission and double detection is defined as

\[
E(a, b) = D^{++} (a, b) - D^{--} (a, b) - D^{-+} (a, b) + D^{+-} (a, b). \quad (24)
\]

Note that in an actual experiment, the measurable quantities are the joint probabilities for transmission and detection, i.e, the probabilities in Eqs. (19) and (20). However, the probabilities that appear in the inequality
that we derived in this paper, i.e., inequality (10), are not joint probabilities for transmission and detection, rather they are only the probabilities for transmission through the polarizers. One can certainly attempt to redefine the probabilities in inequality (10) by using the measurable joint probabilities for detection and transmission, but the trouble is that in experiments which are feasible with present technology \([5,13]\), because \(\frac{\Omega}{4\pi} \ll 1\), the probabilities \(D_{\pm\pm}(a, b)\) are of the order \(10^{-2}\) which are far too small to lead to violation of Bell’s inequality.

We solve this problem by means of the following supplementary assumption: Given that an ensemble of photon emerges from two (one) polarizers, the probability of their double (single) detection is equal to the sum of joint probabilities for double (single) transmission and double (single) detection. Calling \(T_0(m, n)\), \(\{t_0(m), t_0(n)\}\), the sum of joint probabilities for double [single] transmission and double [single] detection the above supplementary assumption can be translated into the following relation:

\[
D_{\pm\pm}(m, n) = T_0(m, n)p_{\pm\pm}(m, n),
\]

\[
D_{\pm}(m) = t_0(m)p_{\pm}(m),
\]

\[
D_{\pm}(n) = t_0(n)p_{\pm}(n),
\]

where

\[
T_0(m, n) = D_{++}(m, n) + D_{+-}(m, n) + D_{-+}(m, n) + D_{--}(m, n),
\]

\[
t_0(m) = D_{++}(m) + D_{-+}(m),
\]

\[
t_0(n) = D_{++}(n) + D_{--}(n).
\]

Using Eqs. (19) and (20), it can be seen that the quantum mechanical prediction for \(T_0(m, n), t_0(m)\), and \(t_0(n)\) are

\[
T_0(m, n) = \eta^2 \left(\frac{\Omega}{4\pi}\right)^2 g(\theta, \phi),
\]

\[
t_0(m) = \eta \left(\frac{\Omega}{4\pi}\right),
\]

\[
t_0(n) = \eta \left(\frac{\Omega}{4\pi}\right).
\]

The supplementary assumption (25) allows us to transform Bell’s inequality (10) into a measurable inequality. Substituting in (25) in (10), we obtain

\[
\frac{E(a, b)}{T_0(a, b)} + \frac{E(b', a)}{T_0(b', a)} + \frac{E(b, a')}{T_0(b, a')} - 2\frac{D_{++}(a', b')}{T_0(a', b')} - 2\frac{D_{--}(a', b')}{T_0(a', b')} + \frac{D_{++}(a')}{t_0(a')} + \frac{D_{-+}(a')}{t_0(a')} + \frac{D_{-+}(b')}{t_0(b')} + \frac{D_{--}(b')}{t_0(b')} \geq -1.
\]

Note that the number of emissions \(N\) from the source is eliminated from the ratio of in inequality (28). Using the quantum mechanical predictions (19) and (20), it can easily be seen that quantum mechanics violates inequality (28) in the case of real experiments where the solid angle covered by
the aperture of the apparatus, $\Omega$, is much less than $4\pi$. In particular, the magnitude of violation is maximized if the following orientations are chosen $(a, b) = (b', a) = (b, a') = 120^\circ$ and $(a', b') = 0^\circ$. Using the quantum mechanical probabilities [i.e., Eqs. (19) and (20)], inequality (28) becomes $-1.5 \geq -1$, i.e., quantum mechanics violates inequality (28) by a factor of 1.5 in the case of real experiments (here for simplicity, we have assumed $F(\theta, \phi) = 1$; this is a good approximation even in the case of real experiments. In actual experiments where the solid angle of detectors $\phi$ is usually less than $\pi/6$, from (23) it can be seen that $F(\theta, \pi/6) \approx 0.99$).

Inequality (28) may be considerably simplified if we invoke some of the symmetries that are exhibited in atomic-cascade photon experiments. For a pair of photons in cascade from state $J = 1$ to $J = 0$, the quantum mechanical detection probabilities $p_{QM}^{\pm \pm}$ and expected value $E_{QM}$ exhibit the following symmetry

$$D_{QM}^{\pm \pm} (a, b) = D_{QM}^{\pm \pm} (|a - b|), \quad E_{QM} (a, b) = E_{QM} (|a - b|). \quad (29)$$

We assume that the local theories also exhibit the same symmetry

$$D^{\pm \pm} (a, b) = D^{\pm \pm} (|a - b|), \quad E (a, b) = E (|a - b|). \quad (30)$$

Note that there is no harm in assuming Eq. (30) since it is subject to experimental test. We now take $a'$ and $b'$ to be along the same direction, and we take $a$, $b$, and $a'$ to be three coplanar axes, each making $120^\circ$ with the other two, that is we choose the following orientations, $|a - b| = |b' - a| = |b - a'| = 120^\circ$ and $|a' - b'| = 0^\circ$, then inequality (28) is simplified to

$$\frac{3E(120^\circ)}{T_0(120^\circ)} - \frac{2D^+(0^\circ)}{T_0(0^\circ)} - \frac{2D^-(0^\circ)}{T_0(0^\circ)} + \frac{2D^+(0^\circ)}{t_0(0^\circ)} + \frac{2D^-(0^\circ)}{t_0(0^\circ)} \geq -1. \quad (31)$$

Again using the quantum mechanical probabilities [i.e., Eqs. (19) and (20)], inequality (31) becomes $-1.5 \geq -1$ in the case of real experiments, i.e., Quantum mechanics violates inequality (31) by a factor of 1.5, whereas it violates previous inequalities by a factor of $\sqrt{2}$. Thus the magnitude of violation of inequality (31) is approximately 20.7% larger than the magnitude of violation of the previous inequalities [4-10].

It should be noted that the analysis that led to inequality (31) is not limited to atomic-cascade experiments and can easily be extended to experiments which use phase-momentum [14], or high energy polarized protons or $\gamma$ photons [15-16] to test Bell’s limit.

A final comment is in order about the supplementary assumption of this paper. The present assumption is considerably weaker than CHSH assumption. CHSH supplementary assumption requires that

$$\frac{D^{\pm \pm}(m, n)}{p^{\pm \pm}(m, n)} = \frac{D^{\pm \pm}(m', n')}{p^{\pm \pm}(m', n')}, \quad (32)$$
and

$$\frac{D^{\pm \pm}(m, n)}{p^{\pm \pm}(m, n)} = D(\infty, \infty),$$  \hspace{1cm} (33)

where $m, m'$ are arbitrary axes of the first polarizer and $n, n'$ are arbitrary axes of the second polarizer, and $D(\infty, \infty)$ is the probability of detection in the absence of polarizers. In contrast, the present supplementary assumption does not make any assertion about the orientation of the polarizers, i.e., according to the assumption of this paper, $\frac{D^{\pm \pm}(m, n)}{p^{\pm \pm}(m, n)}$ can be larger than or smaller than or equal to $\frac{D^{\pm \pm}(m', n')}{p^{\pm \pm}(m', n')}$. The present supplementary assumption only requires that

$$\frac{D^{\pm \pm}(m, n)}{p^{\pm \pm}(m, n)} = T_0(m, n).$$ \hspace{1cm} (34)

Since the present supplementary assumption is considerably weaker than CHSH assumption, an experiment based on inequality (31) refutes a larger family of hidden variable theories than an experiment based on CHSH inequality. Finally it is interesting to note that for an ensemble of photons, the numerical value for $T_0(m, n)$ is the same as for $D(\infty, \infty)$, i.e.,

$$T_0(m, n) = D(\infty, \infty) = \eta^2 \left( \frac{\Omega}{4\pi} \right)^2 g(\theta, \phi).$$ \hspace{1cm} (35)

In summary, we have derived a correlation inequality [inequality (10)] which can be used to test locality. In case of ideal experiments, this inequality is equivalent to Bell’s original inequality of 1965 [4]. However, in the case of real experiments where polarizers and detectors are not ideal, inequality (10) is a new and distinct inequality. We have also demonstrated that the conjunction of Einstein’s locality [relation (5)] with a supplementary assumption [Eqs. (25), which is considerably weaker than CHSH assumption, leads to validity of inequality (28) [or (31)]. Quantum mechanics violates this inequality by a maximum factor of 1.5. Thus the magnitude of violation of inequality (28) is approximately 20.7% larger than the magnitude of violation of previous inequalities [4-10].
References

[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47 (1935) 777.

[2] J. S. Bell, Physics 1 (1965) 195.

[3] For an introduction to Bell’s theorem and its experimental implications, see F. Selleri, in *Quantum Mechanics Versus Local Realism*, edited by F. Selleri (Plenum Publishing Corporation, 1988).

[4] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. 23 (1969) 880.

[5] S. J. Freedman, and J. F. Clauser, Phys. Rev. Lett. 28 (1972) 938.

[6] J. F. Clauser, and M. A. Horne, Phys. Rev. D. 10 (1974) 526.

[7] J. S. Bell, in *Foundations of Quantum Mechanics, Proceedings of the International School of Physics ‘Enrico Fermi,’ Course XLIX*, edited by B. d’Espagnat (Academic, New York, 1971).

[8] A. Garuccio, and V. Rapisarda, Nuovo Cim. A. 65 269 (1981) 269.

[9] M. Ardehali, Phys. Rev. A 47A (1993) 1633.

[10] M. Ardehali, Phys. Lett. A. 181 (1993) 187.

[11] E. P. Wigner, Am. J. Phys. 38 (1970) 1005.

[12] D. Bohm, *Quantum Theory* (Prentice-Hall, Englewood Cliffs, NJ, 1951), pp. 614-823.

[13] A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. 47 (1981) 460; A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. 49 (1982) 91; A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. 49 (1982) 1804; Z. Y. Ou, X. Y. Zou, L. J. Wang, and L. Mandel, Phys. Rev. Lett. 65 (1990) 321; Y. H. Shih, and C. O. Alley, Phys. Rev. Lett. 61 (1988) 2921.

[14] J. G. Rarity and P. R. Tapster, Phys. Rev. Lett. 64 (1990) 2921.

[15] L. R. Kasday, J. D. Ullman, and C. S. Wu, Bull. Am. Phys. Soc. 15 (1970) 586; A. R. Wilson, J. Lowe, and D. K. Butt, J. Phys. G 2 (1976) 613; M. Bruno, M. d’Agostino, and C. Maroni, Nuovo Cimento 40B (1977) 142.

[16] M. Lamehirachi and W. Mittig, Phys. Rev. D 14 (1976) 2543.