Analysis of Slab Culvert Bridges using Conventional Method and Grillage Method

Amit Kumar Pandey\textsuperscript{1} and Praveen Nagarajan\textsuperscript{2}

\textsuperscript{1} PG Student, Department of Civil Engineering, NITC, Kerala, India
\textsuperscript{2} Associate Professor, Department of Civil Engineering, NITC, Kerala, India

\textsuperscript{*}amitpandey1715@gmail.com

Abstract. IRC 112:2011, the Indian standard pertaining to concrete bridge design is based on the limit state design principle. It suggests the use of effective width method for the analysis of the structure. For many years, the analysis part of concrete bridge structures design has been based on two-dimensional (2D) analysis and the results obtained from longitudinal analysis were supposed to be valid over the whole bridge width. In this paper, the grillage method has been used to model and analyse the superstructure to get the three-dimensional effect, and a comparison of results between effective width method and grillage method is presented.

1. Introduction
The superstructure in a bridge is the portion of the structure that directly receives the live load. While the part of the bridge below superstructure as pier, abutment, and other support structures are known as the substructure. If the length of the superstructure is less than 6m, it is known as culvert. The design of a civil engineering structure includes two major steps: analysis part and detailing part. Analysis part of the structure includes the evaluation of forces at different locations for possible number of load cases and in detailing part we have to find the required area of steel and cross-section of the structure considering different design philosophy e.g. ultimate load condition and serviceability load condition. Slab culvert is designed as one-way slab for unit width. In IS 456 two-way slab design, analysis can be done by moment coefficient method. In the case of bridge two-way slab analysis can be done using Pigeaud’s curve and shell element method for finite element method.

The grillage analogy includes the description of a three-dimensional (3-D) concrete structure by a two-dimensional collection of discrete one-dimensional interconnected beams in bending and as well as torsion. Chithra et al (2019) \cite{1} did a comparison among three conventional method i.e. sandwich model (IRC 112-2011), Wood Armner method (EN1992-1-1:2004) and the moment coefficient method as per IS 456-2000. This paper gives a comparison of reinforcement using effective width method and grillage analogy.

2. Design Method

2.1. Effective width method
This method is used to find the effective width of slab for resisting the bending moment due to concentrated and patch load when the slab is spanning in the longitudinal direction. The calculation of shears and moments for these loads is a statically indeterminate problem and to find an accurate solution in real practice is very time consuming and difficult because the deflection of slab is bi-directional. So, under a vertical load, a slab will have curvature in both directions i.e. along the span and perpendicular
to the span. A semi-empirical method can be used when a slab is supported on two opposite edges only. Thus, bending moments developed in the slab will be in span direction and normal to it.

Therefore, the effective width method considers the distribution of load on a width, i.e. width of load and both side of this width. So, it can be assumed that the load is carried by a certain width of the slab, known as the effective width. If this effective width is known, the forces developed along the span can be analysed using the line beam analysis i.e. effective unit width method.

![Figure 1. Effective width of load.](image)

a) For a single concentrated load, the effective width, \( B_{ef} = K x (1 - x/L) + b_w \)
b) Solid cantilever slab, \( b_e = 1.2 x + b_w \)
c) Dispersion of loads along the span, \( l_e = x + 2(D + H) \)

Note: Notations according to IRC112: 2011.

2.2. M. Pigeaud method
When bridge behaves like a two-way slab i.e. in case of Tee beam and slab bridges, the slab forces have to be analysed by moments coefficients that will depend upon the aspect ratio. In the case of bridge, most common method to find the moment coefficients is Pigeaud’s curve. This method is relevant to rectangular slabs supported freely on all the four sides and slab should be symmetrically loaded.

2.3. Grillage analysis
A finite number of grillage of interconnected beams can be modelled to give a satisfactory dispersion of loads and deformations inside the concrete structure. For any structure to be modelled by an equivalent grillage, the elastic rigidities of the given grillage members must be equal to the elastic rigidities of the structure so that the grillage member has to deform or rotate in the same amount as that of the actual structure under similar loading conditions. Another main design principle that must be taken into consideration while modelling is that, every grillage member position should coincide with the centre line of the actual member. Once all the members are modelled into a grillage structure, all the boundary conditions i.e. support condition should be provided to best suited in reality. These boundary conditions are to be applied at the centre line. Then, the loads should be idealized as line or point loads that can be applied to the grillage members and then the analysis is performed to get the final output of force.

2.3.1. Grillage of geometry. Grid of longitudinal and transverse beam is known as Grillage. Grillage analogy is most widely used to analyse slab and beam-slab type bridge decks. The spacing of transverse beam should be about 1.5 times the spacing of the main longitudinal members but may extend up to 2.1. Transverse beams are provided at the diaphragm and intermediate cross girder positions. It is necessary to be an odd number of transverse beams to get an intermediate member at mid-span. When the deck is overhanging from the edge of the outer main girder, the grillage will be extended to the parapet beam. In the case of skew-slabs, the transverse members should be modelled perpendicular to the main members to get the correct value of forces and deflections.
2.3.2. **Grillage for loading.** Self-weight of the bridge can be achieved by giving a density to the cross-sectional area of the main longitudinal members. While modelling for self-weight, designers should be careful to avoid double accounting for the self-weight of the deck slab at the junction of beams. Superimposed dead load (wearing coat, footpath load and parapet loads) are idealised as uniformly distributed loads along the length of the longitudinal grillage members. For live loads, the hypothetical vehicle specified in IRC 6: 2017 can be used.

3. **Analysis of solid slab bridge:**

The following data were used for the analysis of slab bridge:

- Clear carriage width = 7.5m
- Width of footpath = 0.6m on both side
- Width of bearing = 0.4m
- Live load = IRC CLASS-AA Track and CLASS-A
- Grade of steel Fe415
- Grade of concrete M25
- Exposure condition: Moderate (Kozhikode)

3.1. **Calculation of forces**

A bridge is designed for the different partial factor of safety for different load conditions and for different design purposes e.g. ultimate design and design for serviceability. Table 1 shows the maximum bending moment without any factor of safety using effective width method and using grillage method (STAAD software). Self-weight of the member, super imposed dead load and live load is applied in this case.

| Length of bridge (m) | Bending moment (KN-m) |
|----------------------|-----------------------|
|                      | Effective width method | Grillage STAAD result |
|                      | CLASS-AA track | CLASS-A wheel | CLASS-AA track | CLASS-A wheel |
| 5.00                 | 100.00          | 78.54         | 90.67          | 63.21          |
| 6.00                 | 122.98          | 96.06         | 114.24         | 77.16          |
| 7.00                 | 142.36          | 120.55        | 134.66         | 90.00          |
| 8.00                 | 159.97          | 141.44        | 153.49         | 103.60         |
| 9.00                 | 175.27          | 162.66        | 170.23         | 117.64         |
3.2. Design for ultimate condition
To design the member for ultimate condition the partial factor of safety for different load can be taken from the Table B3 from IRC:6 loads.

Table 3. Ultimate design moment (DL+SIDL+LL).

| Span (m) | Class-AA track | Class-A wheel | Class-AA track | Class-A wheel | Class-AA track | Class-A wheel | Class-AA track | Class-A wheel |
|---------|----------------|---------------|----------------|---------------|----------------|---------------|----------------|---------------|
| 5m      | 5.37           | 0.96          | 2.69           | 1.62          | 2.88           | 6.69          | 28.42          |
| 6m      | 6.45           | 0.96          | 3.23           | 1.35          | 2.74           | 8.44          | 31.30          |
| 7m      | 7.55           | 0.96          | 3.78           | 1.15          | 2.62           | 7.23          | 22.76          |
| 8m      | 8.62           | 0.96          | 4.31           | 1.01          | 2.48           | 7.43          | 20.78          |
| 9m      | 9.70           | 0.96          | 4.85           | 0.90          | 2.35           | 9.41          | 16.74          |

Figure 3. Grillage of 5m span bridge.

Figure 4. BMD for Class -AA track.

Table 2. Different parameter to calculate bending moment by effective width method.

| Span (m) | Load | $l_{eff}(m)$ | $b_1(m)$ | a | B/L | $b_{eff}(m)$ | Load intensity (kN/m²) |
|----------|------|--------------|----------|---|-----|--------------|------------------------|
|          | Class-AA track | 5.37 | 0.96 | 2.69 | 1.62 | 2.88 | 6.69 | 28.42 |
| 5m       | Class-A wheel   | 5.37 | 0.61 | 1.96 | 1.62 | 2.88 | 8.44 | 31.30 |
|          | Class-AA track | 6.45 | 0.96 | 3.23 | 1.35 | 2.74 | 6.97 | 25.50 |
| 6m       | Class-A wheel   | 6.45 | 0.61 | 2.63 | 1.35 | 2.74 | 8.74 | 26.08 |
|          | Class-AA track | 7.55 | 0.96 | 3.78 | 1.15 | 2.62 | 7.23 | 22.76 |
| 7m       | Class-A wheel   | 7.55 | 0.61 | 3.18 | 1.15 | 2.62 | 9.02 | 21.57 |
|          | Class-AA track | 8.62 | 0.96 | 4.31 | 1.01 | 2.48 | 7.43 | 20.78 |
| 8m       | Class-A wheel   | 8.62 | 0.61 | 3.71 | 1.01 | 2.48 | 9.23 | 18.99 |
|          | Class-AA track | 9.70 | 0.96 | 4.85 | 0.90 | 2.35 | 7.61 | 18.96 |
| 9m       | Class-A wheel   | 9.70 | 0.61 | 4.25 | 0.90 | 2.35 | 9.41 | 16.74 |
Table 4. Required tension reinforcement ($mm^2$).

| Length of bridge (m) | Area of steel ($mm^2$) |
|----------------------|------------------------|
|                      | Effective width method | Grillage STAAD result |
|                      | CLASS-AA track | CLASS-A wheel | CLASS-AA track | CLASS-A wheel |
| 5.00                 | 1691            | 1412          | 1570            | 1218          |
| 6.00                 | 2375            | 2000          | 2252            | 1744          |
| 7.00                 | 3275            | 2936          | 3152            | 2482          |
| 8.00                 | 4379            | 4045          | 4260            | 3406          |
| 9.00                 | 6004            | 5705          | 5883            | 4753          |

Figure 5. Comparison of required area of reinforcement for Class- AA track loading.

Figure 6. Comparison of required area of reinforcement for Class- A wheel loading.

The comparison of required main reinforcement for Class-AA track and Class -A wheel using effective width method and grillage method (STAAD) can be find from Figure 5 and Figure 6 and it can be observed that effective width method gives higher value of area of steel for the same type of load.
Table 5. Total designed torsional moment and its comparison with total design BM.

| Length of bridge (m) | Design Torsion (KN-m) | % Torsional of total design BM |
|---------------------|-----------------------|-------------------------------|
|                     | CLASS-AA track        | CLASS-A wheel                  | CLASS-AA track | CLASS-A wheel |
| 5                   | 51                    | 22.5                          | 26.2%          | 14.6%         |
| 6                   | 64.5                  | 26.55                         | 23.9%          | 12.4%         |
| 7                   | 75                    | 29.28                         | 20.7%          | 9.9%          |
| 8                   | 82.5                  | 31.8                          | 17.9%          | 8.3%          |
| 9                   | 87                    | 35.1                          | 15.0%          | 7.0%          |

It can be observed from Table 5 that as the span of bridge increases the torsional moment increases but the % of torsional moment with respect to design bending moment decreases.

4. Conclusion

Effective width method, which is recommended in IRC:112 to analyse the solid slab type bridges, leads to safer design as it gives more bending moments than STAAD using grillage method which is most widely adopted by bridge designers. The variation of bending moment for IRC class -AA track loads decreases from 9.33% for 5 m of bridge span to 2.8% for 9 m of bridge span but for IRC class -A wheel loads the variation increases from 19.5 % for 5 m to 27.6 % for 9 m of span. IRC:112-2011 recommends the used of 20% of main steel reinforcement as distribution reinforcement to counter torsional moment. However, from grillage analysis it is seen that 20% reinforcement will not be sufficient in some cases. So, it is needed to check the torsional moment while using the effective width method.

It can also be observed that the effective width method leads the structure to be safe but sometimes it is uneconomical. So some factor is necessary to limit this variation.

5. References

[1] Chithra J, Nagarajan P, Sajith A S and Roshan R A 2019 Critical review of design procedures for reinforced concrete slabs as per IRC 112-2011 Materials Science Forum 969 349–54
[2] Jaeger L G and Bakht B 1982 The grillage analogy in bridge analysis Canadian Journal of Civil Engineering 9(2) 224–35
[3] IRC 112:2011 Code of Practice for Concrete Road Bridges
[4] IRC 6:2017 Standard Specifications and Code of Practice for Road Bridges, Section II –Loads and Stresses (Fourth Revision)
[5] IRC 5: 2015 Standard Specifications and Code of Practice for Road Bridges, Section I (General Features of Design) (Seventh Revision).
[6] Hambly E C 1991 Bridge Deck Behaviour (CRC Press)
[7] Surana C S and Agrawal R 1998 Grillage Analogy in Bridge Deck Analysis (Alpha Science Int’l Ltd.)
[8] Victor D J 2017 Essentials of Bridge Engineering (Oxford and IBH Publishing)