PARTIAL LEVEL DENSITIES FOR NUCLEAR DATA CALCULATIONS

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Abstract

The main formalisms of partial level densities (PLD) used in preequilibrium nuclear reaction models, based on the equidistant spacing model (ESM), are considered. A collection of FORTRAN77 functions for PLD calculation by using 14 formalisms for the related partial-state densities is provided and 28 sample cases (73 versions) are described. The results are given in graphic form too. Composite (recommended) formulas, which include the optional use of various corrections, i.e. the advanced pairing and shell correction in addition to the Pauli effect, and average energy-dependent single-particle level densities for the excited particles and holes, are also given. The formalism comprises the density of particle-hole bound states, and the effects of an exact correction for the Pauli-exclusion principle are considered.

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PROGRAM SUMMARY

Title of program: PLD
Catalogue identifier: 
Program obtainable from: CPC Program Library, Queen’s University of Belfast, N. Ireland
Licensing provisions: none
Computer for which the program is designed and others on which it is operable: PCs (486 and Pentium)
Operating systems under which the program has been tested: DOS
Programming language used: FORTRAN 77 (MS-FORTRAN v.5.0)
Memory required to execute with typical data: 491 Kbytes
No. of bytes in distributed program, including test data, etc.: 1,256,659
Distribution format: ASCII
Keywords: Partial nuclear level density, nuclear level density, single-particle level density, equidistant-spacing model, preequilibrium emission, nuclear reactions

Nature of physical problem:
This Fortran code is a collection of subroutines for calculation of the partial nuclear level densities (PLD) mainly used in preequilibrium nuclear reaction models, by using 14 formalisms for the related partial state densities (PSD).

Method of solution:
The main approaches to the calculation of the partial state density, based on the equidistant spacing model (ESM), are used. Composite (recommended) formulas including optionally various corrections, i.e. the advanced pairing and shell correction in addition to the Pauli effect, and average energy-dependent single-particle level (s.p.l.) densities for the excited particles and holes, are also involved. The density of the particle-hole bound states is moreover comprised, and the effects of an exact correction for the Pauli-exclusion principle are considered.

Restrictions on the complexity of the problem:
Although the quantum-mechanical s.p.l. density and the continuum effect can also be reproduced by a corresponding Fermi-gas formula, to be used accordingly within the average energy-dependent PSD in multistep reaction models, this effect is not included. The calculation of PLD with linear momentum, of first interest for modelling preequilibrium-emission angular distributions, is not available either.

Typical running time:
The execution time is strongly problem-dependent: it is roughly proportional to both the number of the excitation energies and the exciton configurations considered in the calculation, while consistent differences arise when various PSD formalisms are used. Twenty-eight sample cases with 73 versions which require from 0.1 to 1661 s on a PC Pentium/166MHz are provided.

Unusual features of the program:
The PSD functions have been optimized for their independent use, in order to provide tools for PSD/PLD users (see [5]). The related drawback is the increase in execution time, while a proper use would involve the calculation of some coefficients only once in the main program. Second, the PLD.FOR has been organized so that various formulas and versions may be tried as well as the comparison between their predictions.
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LONG WRITE-UP

1. Introduction

The particle-hole excitations caused by the nuclear reactions which proceed through a number of nucleon-nucleon interactions are described within either the semiclassical models or the quantum-statistical theories of the preequilibrium emission (PE) [1,2] by means of the particle-hole state densities. Basic approaches to the partial state density (PSD) consist in combinatorial calculations performed in the space of realistic shell-model single-particle levels (s.p.l.) [3]. Lenske et al. [4] have already used them in order to connect in a consistent way the quantum-statistical theories of the multistep-direct (MSD) and multistep-compound (MSC) processes [5]. However, the strong dependence on the s.p.l. basic set is the main of several shortcomings inherent in the method (e.g., [6–9] and references therein).

The equidistant spacing model (ESM) state density [10] including the effect of the Pauli-exclusion principle [11] is still widely used, as well as the phenomenological s.p.l. density value \( g \sim A/14 \text{ MeV}^{-1} \). Basic developments of the Williams formula [11] are due to Běták and Dobeš [12] including the nuclear-potential finite depth correction, Stankiewicz et al. [13] and Obložinský [14] who added the bound-state condition, and Fu [15] and Kalbach [16] who included an advanced pairing correction. Additional studies along this line have involved exact Pauli-correction calculation [17–19]. Kalbach [20–22] also discussed different energy dependences of the s.p.l. spacings and pointed out the necessity to study this subject closely related to PE surface effects, due to the interdependence of the corresponding assumptions. PSD including different energy-dependences of the excited-particle and hole state densities has recently been used [23] in the geometry-dependent hybrid (GDH) model [24,25]. A similar attempt [26,27] has focused on the MSD and MSC processes in the framework of the Feshbach-Kerman-Koonin (FKK) theory [3]. An independent semiclassical analysis [28,29] has additionally justified the surface localization of the most important first nucleon-nucleon interaction within PE processes, and provided average quantities useful for the corresponding PSD calculation [30].

The various PLD formalisms based on the ESM Williams-type formula, which are still extensively used in nuclear-reaction calculations, determined the need for an appropriate subroutine collection. It is on the request of a project concerning a reference input-parameter library for nuclear model calculations [31] that the present work is based. Thus, the program PLD.FOR is a collection of algorithms developed until now and widely used for PSD/PLD calculations. The one- and two-fermion system versions of six different approaches and one composite (recommended) formula including various corrections are available as FORTRAN77 functions.

The main points of the PSD and PLD formalisms are presented in Section 2. At the same time, the sample cases (Table I) for the program PLD.FOR are described. The structure of the
program and the input-data description is given in Section 3. Finally, a worked example is presented in Section 4.

2. Formalism

2.1. Partial state density in the uniform spacing model

The state density of a system of $p$ excited particles above the Fermi level and $h$ holes below it, considered within the uniform spacing model (based on a constant spacing $d=1/g$ between the non-degenerate single-particle levels) at the last occupied level in the ground state of the nucleus, was obtained by Williams [11]

$$\omega(p, h, E) = \frac{g^n (E - A)^{n-1}}{p!h!(n-1)!}, \quad (1)$$

by decreasing the excitation energy $E$ with the correction for the Pauli blocking

$$A = \frac{p(p + 1) + h(h - 1)}{4g} - \frac{h}{2g} \quad (2)$$

with respect to Ericson early formula [10] for $n=p+h$ total number of excitons. The sum of the partial state densities for all allowed particle-hole numbers $p=h$ is consistent with the total nuclear state density formula obtained in the frame of the ESM of the one-component Fermi gas [10]

$$\omega_1(E) = \frac{\exp[2(aE)^{1/2}]}{\sqrt{48E}}. \quad (3)$$

The asymptotic equality

$$\omega_1(E) \simeq \sum_{p=h} \omega(p, h, E), \quad (4)$$

is illustrated in Fig. 1(a) (the sample cases 1/1A) as proved by Williams for the generic value $g=1$ MeV$^{-1}$ of the s.p.l. density.

Similarly, in the case of the two kinds of fermions considered, with the $g_\pi$ and $g_\nu$ being the single-proton and single-neutron state densities, respectively, the PSD for $p_\pi$ and $h_\pi$ proton particle and hole numbers, respectively, and $p_\nu$ and $h_\nu$ neutron particle and hole numbers ($n = p_\pi + h_\pi + p_\nu + h_\nu$) is

$$\omega(p_\pi, p_\nu, h_\pi, h_\nu, E) = \frac{g_{p_\pi + h_\pi}^p g_{p_\nu + h_\nu}^h (E - B)^{n-1}}{p_\pi!p_\nu!h_\pi!h_\nu!(n-1)!}, \quad (5)$$

where the Pauli effect correction has now the form

$$B = \frac{1}{4} \left[ \frac{p_\pi(p_\pi + 1) + h_\pi(h_\pi - 1)}{g_\pi} + \frac{p_\nu(p_\nu + 1) + h_\nu(h_\nu - 1)}{g_\nu} \right] - \frac{1}{2} \left( \frac{h_\pi}{g_\pi} + \frac{h_\nu}{g_\nu} \right). \quad (6)$$

The corresponding ESM total nuclear state density for a two-fermion system with an average total single-particle state density $g=g_\pi+g_\nu$ and related level density parameter $a=(\pi^2/6)g$ [10]

$$\omega_2(E) = \frac{\sqrt{\pi} \exp[2(aE)^{1/2}]}{12 a^{1/4} E^{5/4}}. \quad (7)$$
is also consistent with the sum of the partial state densities (4) for all allowed pairs of particle-hole numbers \( p_\pi = h_\pi \) and \( p_\nu = h_\nu \), i.e.

\[
\omega_2(E) \simeq \sum_{p_\pi = h_\pi, p_\nu = h_\nu} \omega(p_\pi, p_\nu, h_\pi, h_\nu, E),
\]

shown in Fig. 1(b) (Cases 2/2A) for the similar generic values \( g_\pi = g_\nu = g/2 = 1 \text{ MeV}^{-1} \). It results that Eq. (8) is true within 4\% for \( E > 3 \text{ MeV} \). A comparison of the PSD given by the Williams one- and two-fermion formulas for the real case of the nucleus \(^{93}\text{Nb}\) is carried out in Fig. 1(c) (Cases 3/3A) by using the phenomenological value \( g = A/13 \text{ MeV}^{-1} \) and the derived quantities

\[
g_\pi = \frac{Z}{A} g, \quad (9a)
\]

\[
g_\nu = \frac{A - Z}{A} g. \quad (9b)
\]

The renormalization of the PSDs given by one-component Williams-type formula can be done by using the ratio between the total state densities given by the two- and respectively one-component Fermi gas formulas of the general form \([33,34]\)

\[
\omega_1(E) = \frac{\exp[2(aU)^{1/2}]}{\sqrt{48} U}, \quad (10)
\]

\[
\omega_2(E) = \frac{\sqrt{\pi} \exp[2(aU)^{1/2}]}{12 \ a^{1/4} (U + t)^{5/4}}, \quad (11)
\]

where the nuclear temperature \( t \) corresponding to the effective excitation energy \( U \) (see below) is defined by \([35]\)

\[
U = at^2 - t. \quad (12)
\]

Thus, the renormalized one-fermion PSD has been defined by

\[
\omega(n, E) = f(U) \omega_1(p, h, E), \quad (13)
\]

where the renormalization factor \( f(U) \) (two-fermion system correction – TFC)

\[
f(U) = \left( \frac{\pi}{3} \right)^{1/2} U \frac{a^{1/4} (U + t)^{5/4}}{a^{1/4} (U + t)^{5/4}}, \quad (14)
\]

has only a weak energy dependence approximately equal to \( U^{-1/4} \).

Since each of the closed formulas \([11]\) and \([11]\) are asymptotically equal to the sum of the one- and respectively two-component PSDs over all allowed particle-hole numbers \( p = h \), the corresponding sum of the renormalized PSD \([13]\) is consistent with the closed formula for a two-fermion system, as shown in Fig. 1(d) (Cases 3/3A/3B).

2.2. Bound-state and finite depth corrections

The limitation on the hole maximum energy due to the finite depth of the nuclear potential \([12]\) as well as on the maximum particle excitation by the nucleon binding energy \( B \) in the case of the bound states, yielded the approximate one-fermion ESM formula \([14]\)
\[ \omega(p, h, E) = \frac{g^n}{p!h!(n-1)!} \sum_{i=0}^{p} \sum_{j=0}^{h} (-1)^{i+j} C_p^i C_h^j (E - A_{ph} - iB - jF)^{n-1} \Theta(E - \alpha_{ph} - iB - jF), \]  

(15)

where \( F \) is the Fermi energy which is now considered to be halfway between the last filled and the first free s.p.l. in the nucleus ground state \([14,20]\) in order to have a PSD form symmetrical in \( p \) and \( h \). Under these circumstances, the Pauli correction term in Eq. (15) is

\[ A_{ph} = \frac{p(p-1) + h(h-1)}{4g}, \]  

(16)

while

\[ \alpha_{ph} = \frac{p^2 + h^2}{2g} \]  

(17)

is the minimum energy of the \((p, h)\) state due to the Pauli blocking. \( \Theta \) in Eq. (15) is the unit step function, i.e. 1 for a positive argument and 0 otherwise. The effect of the nuclear-potential finite depth on the PSD as firstly pointed out by Běták and Dobeš \([12]\), and the additional one due to the condition of bound states, included by Obložinský \([14]\), are shown in Fig. 2 (Case 4) and Fig. 3(a) (Cases 5/5A/5B). The changes obtained by releasing consecutively the finite-depth potential and bound state conditions, by means of large \( F \)- and \( B \)-values, are shown in the later case for the basic exciton configuration \( 1_p 1_h \). The consequences of these conditions on the total nuclear state density, as well as the corresponding results obtained by using the asymptotic formula \([3]\), are illustrated in Fig. 3(b) (Cases 6/6A/6B).

The same method \([14]\) applied within the two-fermion ESM led to the density of partial two-fermion bound states of the form

\[ \omega(p_\pi, h_\pi, p_\nu, h_\nu, E) = \frac{g_{p_\pi}^{p_\pi + h_\pi} g_{p_\nu}^{p_\nu + h_\nu}}{p_\pi! h_\pi! p_\nu! h_\nu! (n-1)!} \sum_{i_\pi=0}^{p_\pi} \sum_{i_\nu=0}^{p_\nu} \sum_{j_\pi=0}^{h_\pi} \sum_{j_\nu=0}^{h_\nu} (-1)^{i_\pi+i_\nu+j_\pi+j_\nu} C_{p_\pi}^{i_\pi} C_{p_\nu}^{i_\nu} C_{h_\pi}^{j_\pi} C_{h_\nu}^{j_\nu} \]

\[ \times (E - A_{p_\pi h_\pi, p_\nu h_\nu} - i_\pi B_\pi - i_\nu B_\nu - j_\pi F_\pi - j_\nu F_\nu)^{n-1} \]

\[ \times \Theta(E - \alpha_{p_\pi h_\pi, p_\nu h_\nu} - i_\pi B_\pi - i_\nu B_\nu - j_\pi F_\pi - j_\nu F_\nu), \]  

(18)

where

\[ A_{p_\pi h_\pi, p_\nu h_\nu} = \frac{1}{4} \left[ p_\pi(p_\pi - 1) + h_\pi(h_\pi - 1) \right] \frac{g_\pi}{g_\pi} + \frac{p_\nu(p_\nu - 1) + h_\nu(h_\nu - 1)}{g_\nu} \]  

(19)

and

\[ \alpha_{p_\pi h_\pi, p_\nu h_\nu} = \frac{p_\pi^2 + h_\pi^2}{2g_\pi} + \frac{p_\nu^2 + h_\nu^2}{2g_\nu} \]  

(20)

are symmetrical in the respective particle and hole numbers. The total nuclear state density, i.e. the sum of all PSD given by Eq.(3) for allowed pairs of proton and neutron particle-hole numbers, calculated with/without the potential finite-depth and bound-state conditions are shown in Fig. 3(c) (Cases 7/7A/7B).
2.3. The advanced pairing correction

2.3.1. One-component Fermi gas formula

(a) In order to take into account the pairing interaction between nucleons a correction was included into the formula derived by Williams in the frame of the free Fermi-gas model (FGM), function of the particle and hole numbers as well as excitation energy of the configuration. The Pauli correction was also modified to be consistent with the pairing correction, so that the PSD formula became

$$\omega(p, h, E, P + B) = \frac{g^n(E - P - B)^{n-1}}{p!h!(n-1)!},$$

where $B$ is the modified Pauli correction following the Williams term $A$:

$$B = A[1 + (2g\Delta/n)^2]^{1/2},$$

and the pairing correction term

$$P = \frac{1}{4}g(\Delta_0^2 - \Delta^2)$$

is determined by the ground- and excited-state gaps $\Delta_0$ and $\Delta(p, h, E)$. The former is related to the condensation energy $C = g\Delta_0^2/4$. On the other hand, the constant pairing correction $U_p$ for the total state density, based on the odd-even mass differences (e.g., $[35]$), may be rather well related to the value $P(\hat{n})$ $[15]$ where $\hat{n}$ is the most probable exciton number. Since $\Delta=0$ if $n \geq \hat{n}$, it results that $\Delta_0$ can be derived from the relation $U_p = g\Delta_0^2/4$. Then, Fu obtained the following parametrizations for $\Delta$ $[15,16]$:

$$\frac{\Delta}{\Delta_0} = 0.996 - 1.76(n/n_c)^{1.60}(E/C)^{-0.68} \quad \text{if } E \geq E_{\text{phase}}$$

$$= 0 \quad \text{if } E < E_{\text{phase}},$$

where $n_c = 0.792g\Delta_0$ is the critical number of excitons and $E_{\text{phase}}$ is the energy of the pairing phase transition given by

$$E_{\text{phase}} = C[0.716 + 2.44(n/n_c)^{2.17}] \quad \text{if } n/n_c \geq 0.446$$

$$= 0 \quad \text{if } n/n_c < 0.446.$$  

Actually, the lower limit in Eq. (23) was adopted in order to take into account explicitly the lack of a phase transition for small $n$. The original comparison of the calculated PSD with and without pairing correction is shown in Fig. 4(a) (Case 8, with the threshold-condition released). One should note that the respective equations provide PSD-values even below the minimum excitation energies (thresholds) characteristic of each configuration

$$U_{th} = C[3.23(n/n_c) - 1.57(n/n_c)^2] \quad \text{if } n/n_c \leq 0.446$$

$$= C[1 + 0.627(n/n_c)^2] \quad \text{if } n/n_c > 0.446.$$  

The correct densities, i.e. above the respective thresholds, are shown in Fig. 4(b) (Case 8).

The total state density obtained as the sum of all PSD for allowed pairs of particle-hole numbers $p = h$ are compared with the closed formula of the one-component Fermi gas used at an effective excitation energy decreased by the constant pairing correction $U_p$ $[15]$. 


\( \omega_1(E, U_p) = \frac{\exp\{2[a(E - U_p)^{1/2}]\}}{\sqrt{48} (E - U_p)}. \)  
(27)

A distinct underestimation by this formula of the sum of PSD for \( E < 8-10 \text{ MeV} \) also results from the comparison shown in Fig. 4(b) (Case 8) for the constant pairing correction \( U_p = 3.5 \text{ MeV} \). The analysis performed with respect to the effect of various \( U_p \) values on this agreement \([13]\) is shown in Fig. 4(c) (Case 9) for the generic values \( g = 4 \text{ MeV}^{-1} \) and 0, 2, and 4 MeV for \( U_p \).

The relationship between the pairing corrections for the PSD and the ESM total state density was extended to considering the nuclear-shell effects by using an additional back-shift \( S \) of the effective excitation energy \([15]\]

\[ \omega(p, h, E, P + B + S) = \frac{g^n(E - P - B - S)^{n-1}}{p!h!(n-1)!}. \]  
(28)

By analogy with the BSFG model \([13]\) for the two-fermion system total state density

\[ \omega_2(E) = \frac{\sqrt{\pi}}{12} \frac{\exp\{2[a(E - \Delta)^{1/2}]\}}{a^{1/4} (E - \Delta - t)^{5/4}}, \]  
(29)

the back-shift parameter is connected to the BSFG virtual ground-state shift parameter \( \Delta \) through the relation

\[ \Delta = U_p + S \]  
(30)

Thus, the predictions of the closed formula of the one-component Fermi gas \([27]\) for the effective excitation energy \( U = E - \Delta \), and the total state density standing for the PSDs given by Eq. \((28)\), should be asymptotically equal as shown by the upper curves in Fig. 4(d) (Cases 10/10A). The lack of consistency between the sum of the one-component PSDs and the two-fermion BSFG state density formula has been illustrated by means of the various predictions of one- and two-component closed formulas with the same parameters for the excited odd-\( A \) nucleus \( ^{41}\text{Ca} \) \([13]\), also shown in Fig. 4(d).

(b). An improved implementation of the advanced pairing correction within the Williams formula

\[ \omega(p, h, E, A_K) = \frac{g^n[E - A_K(p, h)]^{n-1}}{p!h!(n-1)!}. \]  
(31)

adopted a Pauli correction function symmetric in particles and holes, which also included the effects of a passive hole \([16]\]

\[ A_K(p, h) = E_{\text{thresh}}(p, h) - \frac{p(p + 1) + h(h + 1)}{4g}, \]  
(32)

where \( E_{\text{thresh}}(p, h) = \frac{p^2}{m} / g \) and \( p_m = \text{maximum}(p, h) \). The inclusion of the pairing interaction led to the modified form of the threshold energy for a given exciton configuration

\[ E_{\text{thresh}}(p, h) = \frac{g(\Delta^2 - \Delta^2)}{4} + p_m \left[ \left( \frac{p_m}{g} \right)^2 + \Delta^2 \right]^{1/2} \]  
(33)

A third term was added to the modified Pauli-and-pairing correction

\[ A_K(p, h) = E_{\text{thresh}}(p, h) - \frac{p(p + 1) + h(h + 1)}{4g} + \frac{(p - 1)^2 + (h - 1)^2}{gF(p, h)}, \]  
(34)
where

\[ F(p, h) = 12 + 4g[E - E_{\text{thresh}}(p, h)]/p_m, \tag{35} \]

in order to obtain the PSD values of \( g \) and \( 2g \) for \( E = E_{\text{thresh}}(p, h) \) and \( E = E_{\text{thresh}}(p, h) + 1/g \), respectively. The comparison of the values given by the formulas of Fu \[15\] and Kalbach \[16\] above \( E_{\text{thresh}}(p, h) \) for each exciton configuration, respectively, is reproduced in Fig. 5(a) (Case 12).

The pairing effects were also included in the total state-density formula by using an effective excitation energy decreased by the effective pairing shift \[16\]

\[ P_{\text{eff}}(E, C) = \max(E, C/\{1 + \exp[4(0.625 - E/C)]\}) \tag{36} \]

where the quantity \( E_2 \) is written as

\[ E_2 = C \left[ 1 + 2.508/(n_c)^2 \right] \quad \text{if} \quad n_c \leq 4.48 \]
\[ = C \left[ 6.46/n_c - 6.28/(n_c)^2 \right] \quad \text{if} \quad n_c \geq 4.48. \tag{37} \]

The sum of the PSD provided by Eq. (36) is now consistent with the Fermi-gas formula (37) provided that the constant \( U_p \) is replaced by the effective-energy shift \( P_{\text{eff}}(E, C) \), as shown in Fig. 5(b) (Case 13).

(c). The PSD formula also within the ESM and based on an exact calculation of the Pauli correction term \[18\], extended to the case of the finite well depth and bound states, and including the Kalbach \[16\] pairing correction, is \[19\]

\[ \omega(p, h, E) = g^n/p!h! \sum_{i=0}^{p} \sum_{j=0}^{h} (-1)^{i+j} C_p^i C_h^j \sum_{\lambda=0}^{n-1} (E - E_{\text{thresh}} - iB - jF)^{n-1-\lambda} \]
\[ \times \Theta(E - E_{\text{thresh}} - iB - jF)B(p, h, \lambda) \frac{1}{(n-1-\lambda)!}, \tag{38} \]

where the coefficients \( B(p, h, \lambda) \) have the expression

\[ B(p, h, \lambda) = \sum_{\lambda_1}^{\lambda} C(p, \lambda_1)C(h, \lambda - \lambda_1), \tag{39} \]

and the coefficients \( C(m, \lambda) \) are determined by the recursive relation

\[ C(m, \lambda) = \sum_{i=0}^{\lambda} \frac{1}{i!} b_i (-m/g)^i C(m-1, \lambda - i), \tag{40} \]

with

\[ C(0, \lambda) = 1 \quad \text{for} \quad \lambda = 0 \]
\[ = 0 \quad \text{for} \quad \lambda \neq 0, \tag{41} \]

while the Bernoulli numbers \( b_i \) can be found in mathematical tables. Eq. (38) becomes the PSD expression of Baguer \[18\] in the limiting case of large \( B \) and \( F \). The exact coefficient \( B(p, h, \lambda) \) corresponds to the factor

\[ \frac{1}{\lambda!} \left( \frac{p^2 + p + h^2 + h}{4g} \right)^\lambda, \tag{42} \]
which follows from the approximate formula of Obložinský [14] if the energy-term power is expanded by means of the binomial theorem. The comparison with the PSD values obtained by means of Kalbach formula and parameters $g=14 \text{ MeV}^{-1}$ and $\Delta_0=1 \text{ MeV}$, shown in Fig. 5(c) (Case 17A), proves the suitability of the Pauli-correction approximation. At the same time, the total state density given by the sum of the corresponding PSDs calculated by using the parameter global values $g=8 \text{ MeV}^{-1}$ and $\Delta_0=1 \text{ MeV}$ [14,19] is in good agreement with the one-fermion closed formula (42) by using the effective-energy shift $P_{eff}(E,C)$ [Fig. 5(d) and Case 17].

The comparison of Eq. (38) with the Obložinský formula can be carried out by neglecting the pairing effect, i.e. through substitution of the threshold energy $E_{thresh}$ by the Pauli energy $\alpha_{ph}$. While Obložinský results for small exciton numbers are well represented (Case 15), a relative deviation of about 10-40% exists between the two sets of calculated PSDs when larger 2$p3h$ and $3p2h$ exciton numbers are involved [Fig. 6(a) and Case 15A]. On the other hand, the inclusion of the pairing effect [19] decreases the partial bound-state densities for small $E$ and enhances them for large $E$ [Fig. 6(b) and Case 16].

2.3.2. Two-component Fermi gas formula

(a). The PSD formula derived by Williams in the frame of the two-component free Fermi-gas model, became by inclusion of the pairing correction $P_2$ [36]

$$
\omega(p_\pi, h_\pi, p_\nu, h_\nu, E, P_2 + B_2 + S) = \left(\frac{g}{2}\right)^n \frac{(E - P_2 - B_2 - S)^{n-1}}{p_\pi^1! h_\pi^1! p_\nu^1! h_\nu^1!(n - 1)!},
$$

(43)

where $B_2$ is the simple extension of the one-fermion system correction factor for the Pauli-exclusion principle modified by the pairing effects. The pairing correction term has been adopted under the assumption of no pairing interaction between the protons and neutrons so that

$$P_2(E, n_\pi, n_\nu) = P_1(E_\pi, n_\pi) + P_1(E_\nu, n_\nu),$$

(44)

where

$$P_1(E_\pi, n_\pi) = \frac{1}{4} g_\pi [\Delta^2_{0\pi} - \Delta^2_{\pi}(E_\pi, n_\pi)],$$

(45a)

$$P_1(E_\nu, n_\nu) = \frac{1}{4} g_\nu [\Delta^2_{0\nu} - \Delta^2_{\nu}(E_\nu, n_\nu)],$$

(45b)

and $E = E_\pi + E_\nu$, $n_\pi = p_\pi + h_\pi$, and $n_\nu = p_\nu + h_\nu$. Based on the pairing theory for two kinds of fermions, an approximate solution was adopted for the gaps $\Delta_{\pi}$ and $\Delta_{\nu}$. Actually, by using the mean gap-approximation, i.e. $g_\pi=g_\nu$ and $\Delta_{0\pi}=\Delta_{0\nu}$, it was shown that the results obtained for the one-fermion system can be used for each of the two systems, while the pairing theory yields that the proton system and the neutron system are excited isothermally [36]. Next, the following simple procedure was chosen to define $E_\pi$ and $E_\nu$ to approximately 10% of the exact values except for energies near the threshold

$$E_\pi = E n_\pi / n,$$

(46a)

$$E_\nu = E n_\nu / n,$$

(46b)

with the overall error in $P_2$ estimated to be about 2% except for energies near the threshold. Moreover, similar to the one-fermion case, the pairing correction $U_p$ for the total state density is related to $P_2(E, \hat{n}_\pi, \hat{n}_\nu)$ [30]

$$U_p = P_2(E, \hat{n}_\pi, \hat{n}_\nu),$$

(47)
so that
\[ \Delta_{0\pi}^2 = \Delta_{0\nu}^2 = 4U_p/g , \] (48)
i.e. the ground-state pairing gaps in the proton system and the neutron system are both equal to that of the one-fermion system if the same values of \(g\) and \(U_p\) are used.

(b). The improved implementation of the pairing correction given by Kalbach [16] has the two-component version
\[ \omega(p_{\pi}, h_{\pi}, p_{\nu}, h_{\nu}, E, A_K) = \frac{g_{\pi}^{n_{\pi}} g_{\nu}^{n_{\nu}} [E - A_K(p_{\pi}, h_{\pi}, p_{\nu}, h_{\nu})]^{n-1}}{p_{\pi}! h_{\pi}! p_{\nu}! h_{\nu}! (n-1)!} , \] (49)
where
\[ A_K(p_{\pi}, h_{\pi}, p_{\nu}, h_{\nu}) = A_K(p_{\pi}, h_{\pi}) + A_K(p_{\nu}, h_{\nu}) , \] (50)
and the respective one-fermion expressions are used except the function \(F(p, h)\) which now has the form [16]
\[ F(p_i, h_i) = 12n_i/n + 4g_i [E_i - E_{\text{thresh}}(p_i, h_i)]/p_{\text{im}} , \] (51)
where \(i\) can be either \(\pi\) or \(\nu\). The average excitation energies of the two kinds of nucleons have also been assumed to be proportional to the number of degrees of freedom of each type.

Additional comments should concern Kalbach [17] energy-dependent pairing corrections for the consistency between the total state-density closed formula of the two-component Fermi gas at the effective excitation energy decreased by the effective-energy shift \(P_{\text{eff}}(E, C)\), and the sum of all PSD provided by Eq. (49) for allowed pairs of particle-hole numbers \(p_{\pi}=h_{\pi}\) and \(p_{\nu}=h_{\nu}\), as shown in Fig. 7(a) (Case 13A). It was expected that the two-fermion PSDs sum should be well approximated by the Fermi gas formula when \(P=C_{\pi}+C_{\nu}\), at excitation energies not too close to threshold, under the assumption of the same gap parameters \(\Delta_{0\pi}\) and \(\Delta_{0\nu}\). However, the use of the sum \(P_{\text{eff}}(E, C_{\pi})\) and \(P_{\text{eff}}(E, C_{\nu})\) as well as of the constant \(P=U_p\) is shown in Fig. 7(b) (Case 13A) to be correct only for \(E/C \geq 2\). The PSDs sum is underestimated, i.e. the pairing correction is overestimated at the lowest energies where the exciton configurations of only one kind of fermions are significant [Fig. 7(a)]. This suggests the use at these energies of only one correction term out of the two \(P_{\text{eff}}(E, C_{\pi})\) and \(P_{\text{eff}}(E, C_{\nu})\). The gradual inclusion of the second one, by means of the sum \(P_{\text{eff}}(E, C_{\pi})+xP_{\text{eff}}(E, C_{\nu})\) with \(x\) between 0 and 1 for \(E/C\) varying from 1 to 2 as also shown in Fig. 7(c), is correct just above the first threshold. Therefore, we found the following form was necessary to obtain the closed-formula predictions rather consistent with the PSDs sum as in Fig. 7(b)
\[ P_{\pi+\nu,\text{eff}}(E, C) = P_{\text{eff}}(E, C_i) \left( \frac{E - E_2}{2C - E_2} + 1 \right) , \] (52)
where \(i\) can be either \(\pi\), or \(\nu\) or an average of the two terms, while the quantity \(E_2\)
\[ E_2 = C_i [1 + 2.508/(n_{ci})^2] \quad \text{if} \quad n_{ci} \leq 4.48 \]
\[ E_2 = C_i [6.46/n_{ci} - 6.28/(n_{ci})^2] \quad \text{if} \quad n_{ci} \geq 4.48 , \] (53)
is the ground-state threshold energy for the \(n_i=2\) states, which Kalbach [13] also involved in the definition of \(P_{\text{eff}}(E, C)\). The reduced consistency of the two curves just above \(E=C\) could be shifted to the region above the threshold by replacing the energy \(E_2\) with the zero value, i.e. by
carrying out the transition described by Eq. (52) in the energy range between 0 and C. The corresponding pairing correction is denoted by $P_{\pi+\nu,\text{eff}}$ in Fig. 7(b).

The comparison of the two-fermion PSDs sum with the sum of the one-fermion PSDs [16] multiplied by the two-fermion system correction [32,34] for all allowed particle-hole numbers $p=h$, as well as of the corresponding closed-formula values, is shown in Fig. 7(d) (Cases 13A/13B). The agreement of the related quantities is quite good but only at $E/C \geq 2$. The use of $P_{\text{eff}}(E,C)$ within the two-component Fermi-gas total state density, which is the case when the two-fermion system correction is used, leads once again to an underestimation around the condensation energy $C$. This could be the main limit of the TFC method, which reveals the need for a different pairing correction at energies $E/C \leq 2$ within the complete two-fermion system approach.

(c). The PSD formula within the ESM and exact calculation of the Pauli correction term [19], extended to the case of the finite well depth and bound states and including the Kalbach [16] pairing correction, has the following form in the two-fermion system case

$$\omega(p_\pi, h_\pi, p_\nu, h_\nu, E) = \frac{g^{n_\pi}_{p_\pi} g^{n_\nu}_{p_\nu}}{p_\pi! h_\pi! p_\nu! h_\nu!} \sum_{i_\pi=0}^{p_\pi} \sum_{j_\pi=0}^{h_\pi} \sum_{i_\nu=0}^{p_\nu} \sum_{j_\nu=0}^{h_\nu} (-1)^{i_\pi+j_\pi+i_\nu+j_\nu} C^{i_\pi}_{p_\pi} C^{j_\pi}_{h_\pi} C^{i_\nu}_{p_\nu} C^{j_\nu}_{h_\nu}$$

$$\frac{n-1}{\lambda=0} t^{n-1-\lambda} \Theta(t) A(p_\pi, h_\pi, p_\nu, h_\nu, \lambda) \frac{1}{(n-1-\lambda)!} , \quad (54)$$

where

$$t = E - E_{\text{thresh}} - i_\pi B_\pi - j_\pi F_\pi - i_\nu B_\nu - j_\nu F_\nu , \quad (55)$$

$$A(p_\pi, h_\pi, p_\nu, h_\nu, \lambda) = \sum_{\lambda_\nu=0}^{\lambda} B(p_\nu, h_\nu, \lambda_\nu) B(p_\pi, h_\pi, \lambda - \lambda_\nu) , \quad (56)$$

while the coefficients $B(p, h, \lambda)$ are determined by the Eqs. (33-41). The total state density obtained as the sum of the PSD for all allowed particle-hole numbers is shown in Fig. 8(a) (Case 17B), as well as the comparison with Kalbach closed formula including the above-discussed effective pairing correction, the only constant pairing-correction $U_p$ [13], and the energy-dependent correction $P_{\text{eff}}$, shown in Figs. 8(b) (Cases 17B/17C) and 8(c) (Cases 17A/17D). Fig. 8(d) (Cases 17A/17D) also gives the comparison with the corresponding PSD values obtained by means of the Kalbach formula [16], which supports the correctness of the approximation for the Pauli-correction term.

2.4. Partial state density with surface effects and energy-dependent s.p.l. densities

The surface effects which may be considered within the initial target-projectile interaction [22] by means of the PSD formula introduced by Kalbach [21,24]

$$\omega(p, h, E, F) = \omega(p, h, E, \infty) f(p, h, E, F) , \quad (57)$$

where the finite-depth correction in addition to $\omega(p, h, E, \infty)$ given by, e.g., Eq. (27), is brought off by the function

$$f(p, h, E, F) = \sum_{j=0}^{h} (-1)^j C^j \left( \frac{E - jF(h)}{E} \right)^{n-1} \Theta(E - jF(h)) \quad (58)$$
where the Fermi energy $F = 38 \text{ MeV}$ corresponds to the central nuclear well. The shallower potential in the region of the nuclear surface, where the first target-projectile interaction in PE reactions is most probably localized (see also [37,29]), is taken into account by using an average effective well depth and the related Fermi energy $F_1$ for the hole number $h \leq 2$

$$F(h) = F_1 \quad \text{for } h \leq 2 \quad (59a)$$

$$= F_0 = 38 \text{ MeV} \quad \text{for } h > 2, \quad (59b)$$

so that $F_1$-value is taken into account for only the initial configurations, following the assumption of the surface localization of PE two-body interactions exciting them.

The smaller effective well depth may also determine an increased significance of another effect, namely the s.p.l. energy-dependence of $g(\varepsilon)$. First, by taking into account this dependence, separate excited-particle state density $g_p$ and single-hole state density $g_h$ are involved. Second, the increase in $g_p$ has generally compensated for the decrease in $g_h$ except when the reduced potential-well depth makes the latter much closer to the value at the Fermi energy, $g_0 = g(F)$; since the excited particles may well be excited above the Fermi level in PE reactions, opposite to the case of the statistical equilibrium, the $g_p$ could increase significantly.

The interdependence of the PE surface effects and the energy dependence of the s.p.l. density makes the functional form of the latter to be yet an open question [22]. Thus, the first-order effects of the FGM dependence have been adopted [22,23,28] for the time being

$$g(\varepsilon) = g_0 \left( \frac{\varepsilon}{F} \right)^{1/2} = \frac{3A}{2F} \left( \frac{\varepsilon}{F} \right)^{1/2} \quad (60)$$

Next, the actual basic point consists in the use of the average excitation energies $\bar{u}_p = \varepsilon_p - F$ for excited particles, and $\bar{u}_h = F - \varepsilon_h$ for holes [22]. The former has been estimated to first order from the ESM Eq. (57), with the result

$$\bar{u}_p = \frac{E}{n} \frac{f(p + 1, h, E, F)}{f(p, h, E, F)} \quad (61)$$

while the related one for holes is

$$\bar{u}_h = \frac{E - p\bar{u}_p}{h} \quad (62)$$

both of them reducing to $E/n$ if the finite depth of the potential well is not considered. The corresponding average FGM s.p.l. densities for the excited particles and holes

$$g_p(\bar{u}_p) = g_0 \left( \frac{F_0 + \bar{u}_p}{F_0} \right)^{1/2} = g_p(p, h) \quad (63a)$$

$$g_h(\bar{u}_h) = g_0 \left( \frac{F_0 - \bar{u}_h}{F_0} \right)^{1/2} = g_h(p, h) \quad (63b)$$

are used within the ESM partial state-density (57) which becomes

$$\omega(p, h, E, F) = \frac{[g_p(p, h)]^p [g_h(p, h)]^h [E - A_{ph}]^{n-1}}{p|h|!(n-1)!} f(p, h, E, F) \quad (64)$$

It should be underlined that the effective well depth $F_1$ is involved in evaluating only the quantities $\bar{u}_p$, $\bar{u}_h$ and $f(p, h, E, F)$ while the central well depth $F_0$ is assumed in calculating the effective s.p.l. densities. The factor $g_p^p g_h^h / g_0^n$ shown in Fig. 2 of Ref. [22] gives just the ratio between the
PSD including the FGM energy dependence of these effective s.p.l. densities, and the ESM state densities with only the finite-well depth correction. Both kinds of PLDs are shown in Figs. 9(a) (Cases 18/18A) and 9(b) (Cases 18B/18C) for either the normal Fermi energy $F_0$ or the average effective value $\bar{F}_1=14$ MeV. The analysis concerns the exciton configurations corresponding to the first two-body interaction in nucleon-induced PE reactions, which are the most important for PE-reaction calculations.

The two-component PSD formula taking into account the surface effects for the initial target-projectile interaction was given as [39]

$$\omega(p_\pi, h_\pi, p_\nu, h_\nu, E) = \left[ g_{\pi}(p) \right]^{p_\pi} \left[ g_{\pi}(h) \right]^{h_\pi} \left[ g_{\nu}(p) \right]^{p_\nu} \left[ g_{\nu}(h) \right]^{h_\nu} \frac{(E - A(p_\pi, h_\pi, p_\nu, h_\nu))^{n-1}}{p_\pi! h_\pi! p_\nu! h_\nu! (n-1)!} f(p, h, E, F),$$

where

$$A(p_\pi, h_\pi, p_\nu, h_\nu) = \frac{p_\pi^2}{g_\pi} + \frac{p_\nu^2}{g_\nu} - \frac{p_\pi^2 + h_\pi^2 + n_\pi}{4g_\pi} - \frac{p_\nu^2 + h_\nu^2 + n_\nu}{4g_\nu}$$

(66)

is the simpler form of the Pauli correction for the Fermi level placed halfway between the last occupied s.p.l. in the ground state of the nucleus and the first vacant s.p.l. The same formalism as in the one-fermion system case has been used for the average-excitation energies for excited particles and holes. The only additional assumption (46) concerns the definition of the average excitation energies $E_\pi$ and $E_\nu$, for protons and neutrons respectively.

The PLDs for the two-fermion system exciton configurations $(p_\pi, h_\pi, p_\nu, h_\nu) =$ (2100), (1110), (1100), and (1001) are also compared in Figs. 9(c) (Cases 18A/19/19A) and 9(d) (Cases 18C/20/20A) with the one-fermion PSDs of the related configurations $(p, h) =$ (21) and (11), as well as with the results obtained by using the method involving the two-fermion system correction. A behavior specific of the particle-hole bound state density arises for the two-fermion-system exciton configurations with only one hole, due to the use of Eq. (46).

2.5. Composite (recommended) PSD formulas

The completeness of a composite PSD formula can be obtained by the inclusion and optional use of the various ESM corrections:

- extension to the case of the finite well depth and bound states [14,18,22];
- advanced pairing correction by Fu [15] and Kalbach [16] added to the Pauli correction of Williams, proved to be still in good agreement with exact calculations [17–19];
- inclusion of the shell effects together with the pairing correction [15] and use of the usual level-density parameters also for the PSD/PLD calculation;
- energy dependence of the single-particle state densities as well as inclusion of the surface effects in the case of the first two steps of multistep processes [22], which appear to be most significant for the exciton configurations mainly involved in the description of PE reactions.

Therefore, in the one-fermion system case, a composite (recommended) PSD formula is

$$\omega(p, h, E) = \frac{[g_p(p, h)]^p [g_h(p, h)]^h E^{n-1}}{p! h! (n-1)!} f_K(p, h, E, F),$$

where
\[ g_p(p, h) = g(F + \mathbf{\pi}_p) \tag{68a} \]

\[ g_h(p, h) = g(F - \mathbf{\pi}_h) , \tag{68b} \]

with the average excitation energies for particles and holes

\[ \mathbf{\pi}_p = \frac{E f^K_p(p, h, E, F)}{n f_K(p, h, E, F)} \tag{69a} \]

\[ \mathbf{\pi}_h = \frac{E - p \mathbf{\pi}_p}{h} , \tag{69b} \]

where

\[ f_K(p, h, E, F) = \sum_{i=0}^{p} \sum_{j=0}^{h} (-1)^{i+j} C^i_p C^j_h \left( \frac{E - A_K(p, h) - S - iB - jF}{E} \right)^{n-1} \]

\[ \times \theta(E - E_{\text{thresh}} - S - iB - jF) , \tag{70} \]

and

\[ f^+_K(p, h, E, F) = \sum_{i=0}^{p} \sum_{j=0}^{h} (-1)^{i+j} C^i_p C^j_h \left( \frac{E - A_K(p, h) - S - iB - jF}{E} \right)^{n} \]

\[ \times \left( 1 + \frac{n}{p E - A_K(p, h) - S - iB - jF} \right) \theta(E - E_{\text{thresh}} - S - iB - jF) . \tag{71} \]

On the other hand, the two-component partial state density has the general expression

\[ \omega_{\pi, \pi, \nu, \nu, E} = \frac{[g_\pi(p)]_{p=0} [g_\pi(h)]_{h=0} [g_\nu(p)]_{p=0} [g_\nu(h)]_{h=0} \sum_{i=0}^{p} \sum_{j=0}^{h} \sum_{i=0}^{p} \sum_{j=0}^{h} (-1)^{i+j+i\pi+j\pi} }{p! h! p! h! (n-1)!} \]

\[ \times C^{i\pi}_{p\pi} C^{j\pi}_{h\pi} C^{i\nu}_{p\nu} C^{j\nu}_{h\nu} \left[ E - A_K(p, h, p, h) - S - i\pi B_{\pi} - j\pi F_{\pi} - i\nu B_{\nu} - j\nu F_{\nu} \right]^{n-1} \]

\[ \times \theta(E - E_{\text{thresh}} - S - i\pi B_{\pi} - j\pi F_{\pi} - i\nu B_{\nu} - j\nu F_{\nu}) , \tag{72} \]

where

\[ A_K(p, h, p, h) = A_K(p, h) + A_K(p, h) , \tag{73} \]

the two-fermion systems being considered at the average excitation energies \[A_K\], respectively. The average s.p.l. densities in Eq. \[72\] are evaluated within the same distinct fermion systems. One may observe some difference between the one- and two-component formulas concerning the calculation of the nuclear potential finite-depth correction, following the original formalisms. However, a two-fermion PSD formula quite similar to the one-component case and consistent with the average excitation energies \[A_K\] may be obtained with minor changes.
The optional provision of the advanced corrections to the ESM formulas by Eqs. (67) and (72), respectively, is illustrated in Fig. 10. Thus, no correction to the Williams formulas for either the one-fermion system, Fig. 10(a) (Cases 21/21A), or two-fermion system, Fig. 10(b) (Cases 22/22A), determines PLD values obtained with the composite formulas which are so close or even identical to the original ones, for the simplest configurations \( 1p1h \). The same is true for the bound-state case with only the finite-well depth correction, the comparison with the predictions of the Obložinský formulas [14] being shown for specific configurations as well as the total state density given by the corresponding PLDs sum. These results are given in Figs. 10(c) (Cases 23/23A) and 10(e) (Cases 23A/24A), respectively, for the one-fermion system, and in Figs. 11(a) (Cases 23B/24B/25B) and 11(c) (Cases 23B/24B) for the two-fermion system formulas.

The specific behavior following the inclusion of the average energy-dependent formalism is pointed out by comparison with the predictions of the Obložinský formulas [14] for particular configurations and the total state density given by the PLDs sum. This is shown in Figs. 10(d) (Cases 23/23C) and 10(f) (Cases 23C/24C), respectively, for the one-fermion system, and Figs. 11(b) (Cases 23C/24C/25B) and 11(d) (Cases 23D/24D) for the two-fermion system formulas. The enhancement of the "bound-state effect" (i.e. reaching a PSD maximum value followed by vanishing at a certain higher-energy limit) due to the addition of the energy-dependent s.p.l. densities is obvious in Figs. 10(d) and 11(b). A particular feature is shown comparatively in Figs. 11(a),(b) for the one- and two-fermion system configurations with \( 1p1h \), namely the effects of the sequential releasing of the bound-state and finite-well depth conditions.

The comparison between the predictions of the composite formula, using the FGM energy-dependent s.p.l. densities, and the ESM formula including the advanced pairing correction [13,10] is shown in Fig. 12(a) (Cases 26/26A). The same but for the ESM one- and two-fermion system formulas including the exact calculation of the Pauli correction [17–19] are shown in Figs. 12(c) (Case 28) and 12(d) (Case 28B). The corresponding bound-state PLDs are identical as long as the average excitation energies of the two kinds of excitons are lower than the respective limits, while next the "bound-state effect" is well increased within the average energy-dependent formalism.

Finally, the changes due to the inclusion of the surface effects for exciton configurations with one and two holes, are illustrated in the case of both fermion systems in Figs. 12(a) and Fig. 12(b) (Cases 27/27B). These changes are higher than, and increasing, the ones due to the energy-dependent s.p.l. densities. The comparison with the original results [22] not including the pairing correction in Fig. 12(b) points out also that taking into account the pairing effects is insignificant at higher energies but dominant in the threshold region.

### 2.6. The partial level density

#### 2.6.1. One-component Fermi gas formula

The level density of \( p \)-particle–\( h \)-hole configurations of excitation energy \( E \) and nuclear spin \( J \), in the one-fermion formulation, is

\[
\rho(p, h, E, J) = \omega(p, h, E) R(n, E, J),
\]

where the spin-distribution formula has the general expression [5]

\[
R(n, E, J) = \frac{2J + 1}{2(2\pi)^{3/2}} \exp \left[ \frac{-(J + 1/2)^2}{2\sigma^2(E, n)} \right],
\]

and the spin-cutoff factor \( \sigma^2(E, n) \) may have the simplified energy- and exciton-configuration dependence [32].
\[ \sigma^2(E, n) = (\ln 4) \left( \frac{n}{n_c} \right) \left( \frac{E - U_{th}}{E} \right)^x \sigma_c^2, \]  

(76)

where, in addition to the quantities from the previous section,

\[ x = -0.413 + 1.08(n/n_c)^{1/2} - 0.226(n/n_c), \]  

(77)

and \( \sigma_c^2 \) is the spin-cutoff factor at the critical thermodynamic temperature \( T_c = 2\Delta_0/3.5 \), namely \( \sigma_c^2 = gT_c \langle m^2 \rangle \). The differences between Eq. (76) and the spin-cutoff factor \( \sigma_F^2(n) \approx 0.16nA^{2/3} \) initially introduced by Feshbach et al. concern firstly the threshold existing in the former case. Secondly, the result of Feshbach et al. is more appropriate for exciton configurations around the most-probable exciton number \( \hat{n} \), which are important to the compound-nucleus contribution; at the same time the larger value of about \( 0.28nA^{2/3} \) obtained by Reffo and Hermann [40] is better for the \( n=2 \) component – the PE-dominant contribution. However, \( \sigma^2(E, n) \) can be used for all exciton numbers since the respective values are closer to the results of Reffo and Hermann around \( n=2 \), and also in good agreement with \( \sigma_F^2(\hat{n}) \).

The sum over the nuclear spins, of the PLDs given by Eq. (74), corresponds to the partial level density \( \rho(p, h, E) \) related to the PSD of the same configuration by means of the closed formula

\[ \rho(p, h, E) = \frac{\omega(p, h, E)}{(2\pi)^{1/2} \sigma(E, n)}. \]  

(78)

On the other hand, the sum of the PLDs over all allowed particle-hole numbers \( p = h \) should provide the usual nuclear level density involved in the statistical model calculations of compound-nucleus processes, which has also the one-fermion ESM formula

\[ \rho_1(E, J) = \omega_1(E) R(E, J), \]  

(79)

with the spin-distribution

\[ R(E, J) = \frac{2J + 1}{2(2\pi)^{1/2} \sigma^3(E)} \exp \left[ \frac{-J(J + 1/2)^2}{2\sigma^2(E)} \right]. \]  

(80)

For the spin-cutoff factor \( \sigma^2(E) \) in the above equation, which should be consistent with the value \( \sigma^2_H(E) \) used in compound-nucleus (Hauser-Feshbach) calculations, the average of \( \sigma^2(E, n) \) over \( n \) was adopted [32]

\[ \sigma^2(E) = \frac{\sum_{p=h=1}^n \omega(p, h, E) \sigma^2(E, n)}{\sum_{p=h=1}^n \omega(p, h, E)}, \]  

(81)

where the PSD \( \omega(p, h, E) \) may be given by, e.g., Eqs. (21) or (28). This average was also proved to be consistent with the general form of \( \sigma^2_H(E) \) [33]

\[ \sigma^2_H(E) = g\langle m^2 \rangle t, \]  

(82)

where the value \( \langle m^2 \rangle = 0.24A^{2/3} \) was assumed [32].

Furthermore, the sum of the PLDs over both \( p \) (with the restriction \( p=h \)) and spins gives the total level density

\[ \rho_1(E) = \sum_{p=h=1}^n \sum_J \rho(p, h, E, J) \]  

(83)

which is related to the one-fermion ESM total state density by
\[
\rho_1(E) = \frac{\omega_1(E)}{(2\pi)^{1/2} \sigma(E)}.
\] (84)

The comparison of the values given by the above closed formula and the sum \((83)\) (i.e. Eqs. \((52)\) and \((53)\) respectively of Ref. \([32]\)) for the excited nucleus \(^{41}\)Ca is shown in Fig. 13(a) (Case 11).

The partial and total state and level densities corresponding to the two-component ESM formulas can be obtained by using either the renormalization method \([32,34]\) or the adjustment method of the two-fermion formula parameters \([15,32]\). The former, illustrated in Fig. 4(d), offers the advantage of using only one set of e.g. BSFG model parameters for both the PLDs involved in PE calculations and the nuclear level densities for Hauser-Feshbach calculations.

2.6.2. Two-component Fermi gas formula

The two-fermion level density formula is \([36]\)

\[
\rho(p_\pi, h_\pi, p_\nu, h_\nu, E, J) = \omega(p_\pi, h_\pi, p_\nu, h_\nu, E, J) R(n_\pi, n_\nu, E, J),
\] (85)

where the spin-distribution formula has the similar general expression \([4]\)

\[
R(n_\pi, n_\nu, E, J) = \frac{2J + 1}{2(2\pi)^{1/2} \sigma^3(E, n_\pi, n_\nu)} \exp \left[ -\frac{(J + 1/2)^2}{2\sigma^2(E, n_\pi, n_\nu)} \right],
\] (86)

and the spin-cutoff factor \(\sigma^2(E, n_\pi, n_\nu)\) for two kinds of fermions is defined as the sum of the two one-fermion components

\[
\sigma^2(E, n_\pi, n_\nu) = \sigma^2(E_\pi, n_\pi) + \sigma^2(E_\nu, n_\nu).
\] (87)

The mean-gap approximation and the approximations \([40]\) for \(E_\pi\) and \(E_\nu\) as well as the parameterized function for the one-fermion system \([32]\) are used in this respect.

The sum of the two-fermion PLDs over spins and both \(p_\pi\) and \(p_\nu\) (with the restriction \(p_\pi = h_\pi\) and \(p_\nu = h_\nu\)) provides the total level density

\[
\rho_2(E) = \sum_{p_\pi = h_\pi, p_\nu = h_\nu} \sum_I \rho(p_\pi, h_\pi, p_\nu, h_\nu, E, J)
\] (88)

which is related to the two-fermion ESM total state density by

\[
\rho_2(E) = \frac{\omega_2(E)}{(2\pi)^{1/2} \sigma(E)}.
\] (89)

The same averages of \(\sigma^2(E, n)\) over \(n\) as for the one-fermion formulas \([32]\) are used for the above spin-cutoff factor

\[
\sigma^2(E) = \frac{\sum_{p_\pi = h_\pi, p_\nu = h_\nu} \omega(p_\pi, h_\pi, p_\nu, h_\nu, E) \sigma^2(E, n)}{\sum_{p_\pi = h_\pi, p_\nu = h_\nu} \omega(p_\pi, h_\pi, p_\nu, h_\nu, E)},
\] (90)

where \(\omega(p_\pi, h_\pi, p_\nu, h_\nu, E)\) is given by Eq. \([18]\).

The comparison of the values given by the above closed formula and the sum \((88)\) for the excited nucleus \(^{41}\)Ca is shown in Fig. 13(b) (Case 11B). Fig. 14 (Cases 11/11B) shows the comparison between the average spin-cutoff values \(\sigma^2_2(n=2) = [\sigma^2_2(E, n_\pi=2, n_\nu=0) + \sigma^2_2(E, n_\pi=0, n_\nu=2)]/2\) \([36]\) and \(\sigma^2_1(n = 2)\) corresponding to the one-fermion system, as well as of the average values \([40]\) noted \(\sigma^2_2\) and the corresponding \(\sigma^2_1\) (both of them noted ”all n” in Figs. 13 and 14). The related
PLD and total level density values are shown in Fig. 13(c) (Cases 11/11B) while the comparison between the two-fermion system PLD values and the ones obtained by using the two-fermion correction \[32,34\] is given in Fig. 13(d) (Cases 11B/11E).

3. Program organization

The program PLD.FOR is the collection of the above-described algorithms developed until now and widely used for PSD/PLD within nuclear model calculations. It also includes the recommended (combined) PLD formulas given in this work. Fourteen different PSD formulas are available as FORTRAN77 functions to be used as they are in various applications or codes. The one method involved in calculating the PLD is given within a subroutine only for the tabular printing of the results, while the replacement of the SUBROUTINE statement by the FUNCTION one is immediate.

Second, the PLD.FOR has been organized so that various formulas and versions may be tried as well as the comparison between their predictions. This could also be useful for further development of the PSD calculation methods.

Since the first aim of this work has been to provide tools for users of PSD/PLD, the optimization of the respective procedures is made firstly in this respect. The possibility of using these functions independently has the related drawback of increasing the execution time. A proper use, e.g., of the PLD.FOR for PSD calculation with exact Pauli-correction term of Mao Ming De and Guo Hua \[19\] would involve the calculation of the exact coefficients \(B(p, h, \lambda)\) only once in the main program. One should keep this aspect in mind if PLD.FOR will be used on low-speed PCs.

3.1. Subprograms

The MAIN program reads the input data which are listed and described in Table 1 in reading order (including the names of variables which are also used below and stand for various quantities given in the previous section). The formatted read is used for all data just to make possible the input of only few of them, while by-default values may be involved for the rest. Then, the partial state densities \(w(p, h, E)\) are calculated by using the specified formula, as well as eventually the related partial level densities \(D(p, h, E, J)\). In the general case when the calculation is carried on for all pairs of the exciton numbers \(p=h\), it is also done for (i) the nuclear state density \(w(E)\), as the PSD-sum over all allowed exciton numbers for which the PSD is higher than the value \(W_{\text{MINACC}}=0.1\) MeV\(^{-1}\), or (ii) the total level density \(D(E)\) as the PLD-sum over the exciton numbers and the nuclear angular momentum \(J\). At the same time the corresponding values \(W_{\text{sym}}(E)\) or \(D_{\text{sym}}(E)\), respectively, are calculated by means of the ESM closed formulas, in order to make possible a test of the overall consistency. The two-fermion system correction \[34\] is involved optionally when the one-component Fermi gas formulas are used.

The subroutine PRINTIN prints the type of the PSD/PLD formula and the parameters used in calculation.

The subroutine PRINTWN tabulates the calculated values \(w(p, h, E)\) or \(D(p, h, E)\) (the latter being the sum over \(J\) of \(D(p, h, E, J)\), i.e. the total level density for a given exciton configuration), for either (i) some given particle-hole configurations, or (ii) all pairs of equal numbers of excited particles and holes. In the latter case the values \(w(E)\) or \(D(E)\) are also printed within a first table including the PSD/PLDs for \(p=h\) from 1 to 7, while the corresponding closed-formula values \(W_{\text{sym}}(E)\) or \(D_{\text{sym}}(E)\) are given within the second table including the PSD/PLDs for \(p=h\) from 8 to 16. A table of the values \(D(p, h, E, J)\), but only for the last excitation energy \(E\) involved in one calculation, is also printed in the case of the PLD calculation for a particular
exciton configuration.

The functions \texttt{WIL1} and \texttt{WIL2} calculate the partial state density \( w(p, h, E) \) for a given exciton configuration, by using theWilliams [11] one- and two-fermion system formulas, respectively.

The functions \texttt{WOB1} and \texttt{WOB2} calculate the partial state density for a given particle-hole configuration by means of the Běták-Dobeš one- and two-fermion system formulas, respectively [12], with the nuclear potential finite-depth correction. The calculation of the bound-state density according to the Obložinský formulas [14] is carried out if a value is specified for the nucleon binding energy.

The functions \texttt{WFU1} and \texttt{WFU2} calculate the PSD for a given exciton configuration by means of the one- and two-fermion system formulas, respectively, including the advanced pairing correction by Fu [15].

The functions \texttt{WK1} and \texttt{WK2} calculate the PSD for a given exciton configuration by using the improved implementation of the pairing correction by Kalbach [16] within the one- and two-fermion system formulas, respectively.

The functions \texttt{WK3} and \texttt{WK4} calculate the PSD for a given exciton configuration by using the FGM energy-dependence of the single-excited particle and single-hole state densities, and/or the finite-depth correction including the nuclear-surface effects introduced by Kalbach [22] within the one- and two-fermion system formulas, respectively.

The functions \texttt{WM1} and \texttt{WM2} calculate the PSD for a given exciton configuration by using the exact calculation of the Pauli-exclusion effect [19] and the pairing correction by Kalbach [16] within the one- and two-fermion system formulas, respectively.

The function \texttt{PFU} calculates the advanced pairing correction by Fu [13], for the one-component Fermi-gas.

The function \texttt{AK} calculates the advanced pairing correction by Kalbach [16], for the one-component Fermi-gas.

The function \texttt{FDC0} calculates the nuclear potential finite-depth correction factor \( f(p, h, E, F) \) [22] for the one-component Fermi-gas.

The function \texttt{FDC} calculates the nuclear potential finite-depth correction factor \( f(p + 1, h, E, F) \) [22] but for the case of bound states.

The subroutine \texttt{SUBPLD} calculates the partial level density \( D(p, h, E, J) \) for a given particle-hole configuration, excitation energy \( E \) and nuclear spin \( J \), as well as the respective total level density \( D(p, h, E) \) as their sum over \( J \), by using the partial state density \( w(p, h, E) \) and the formalism of Fu [32,36].

The function \texttt{SIG2FU} calculates the spin-cutoff factor for a given excited particle-hole configuration [32].

The function \texttt{FCTR} calculates the factorial of natural numbers.

4. An illustrative test run

The sample case 23C (see also Fig. 10) is here discussed since it documents few of the specific features of the program PLD at once. Thus, the particle-hole bound state densities for some of the few-exciton configurations analyzed by Obložinský (Fig. 1 of [14]) are calculated by using both
the composite formula and the one of Obložinský. In the former case the FGM energy dependence for the s.p.l. density is additionally taken into account, while the input data correspond to the reference work [14]. The comparison of the results obtained with the two formulas is possible within the same table with the columns of results given in the input-data reading order (the table in the attached output copy below is reduced to the first half of the involved excitation energies). The corresponding curves are shown in Fig. 10(d). One note should concern the printed type of the PSD formula used: it denotes only the last one when the option parameter ICONT=2 is used and more than one formula are involved in calculation.

An additional calculation is included in this case with respect to its title. To make possible the comparison between the PSD for a given exciton configuration with equal numbers of holes and excited particles which is calculated both specifically and within the general case for all pairs, the latter calculation is also made. Moreover, in the attached copy of the reduced output only the first two tables for this calculation are shown (which would correspond to the use of the option parameter value ICONT=-1). One may thus find the printed values of the total state density $w(E)$ obtained as the PSD-sum, in the first table, and the related $W_{\text{sym}}(E)$ closed-formula values, in the second table. The corresponding curves are shown in Fig. 10(e). However, this part of the case output consists of four tables which correspond to the maximum numbers $p=h=25$ for which the PSD value at higher limit of excitation energies is yet higher than WMINACC.

Actually, the PSD/PLD values for exciton numbers higher than $\hat{n}$ are usually of less interest, while the PLD-program output includes them just for the sake of completeness. In the possible case that an output table would include only zero values, it is omitted from print. We may add that questions may arise for calculations for all pairs $p=h$ when quite large excitation energies and the two-fermion system formulas are involved, due to the limits of vectors. However, the use of the respective FUNCTION subprograms for given exciton numbers and energy, i.e. the usual case of nuclear reaction cross-section calculations, is always straightforward.

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TEST RUN (REDUCED) OUTPUT

PARTIAL STATE/LEVEL DENSITIES CALCULATED FROM THE FOLLOWING PARAMETERS
**********************************************************************
Case 23C: C-formula(g-FGM)/Oblozinsky(1986): one-f. bound PSD for configs.

NUCLEUS: CHARGE NO. Z = 0. MASS NO. A = 0. PAIRING Up = .000 MEV

ONE-FERMION FORMULA:
P. OBLOZINSKY, NUCL. PHYS. A453,127(1986), Eqs.(7,9)

SINGLE-PARTICLE STATE DENSITY: G = 8.000 /MEV
FERMI ENERGY: F = 32.000 MEV
NUCLEON BINDING ENERGY: B = 8.000 MEV

| ENERGY (MeV) | w(p,h,E) (1/MeV) |
|--------------|------------------|
| (p,h)= 1 1 0 2 2 1 1 2 1 1 | 0 2 2 1 1 2 |
| 1.00 | 64.0 | 21.2 | 70.8 | 70.1 | 64.0 | 30.0 | 113. | 113. |
| 2.00 | 128. | 51.9 | 393. | 385. | 128. | 62.0 | 481. | 481. |
| 3.00 | 192. | 81.6 | 976. | 947. | 192. | 94.0 | .110E+04 | .110E+04 |
| 4.00 | 255. | 110. | .183E+04 | .175E+04 | 256. | 126. | .198E+04 | .198E+04 |
| 5.00 | 319. | 138. | .294E+04 | .280E+04 | 320. | 158. | .312E+04 | .312E+04 |
| 6.00 | 382. | 164. | .433E+04 | .407E+04 | 384. | 190. | .451E+04 | .451E+04 |
| 7.00 | 445. | 190. | .600E+04 | .558E+04 | 448. | 222. | .616E+04 | .616E+04 |
| 8.00 | 508. | 215. | .794E+04 | .731E+04 | 512. | 254. | .806E+04 | .806E+04 |
| 9.00 | 499. | 238. | .999E+04 | .917E+04 | 512. | 286. | .100E+05 | .101E+05 |
| 10.00 | 490. | 261. | .118E+05 | .110E+05 | 512. | 318. | .117E+05 | .122E+05 |
| 11.00 | 480. | 282. | .132E+05 | .127E+05 | 512. | 350. | .131E+05 | .142E+05 |
| 12.00 | 470. | 303. | .144E+05 | .144E+05 | 512. | 382. | .143E+05 | .163E+05 |
| 13.00 | 460. | 323. | .153E+05 | .160E+05 | 512. | 414. | .152E+05 | .183E+05 |
| 14.00 | 450. | 341. | .158E+05 | .175E+05 | 512. | 446. | .158E+05 | .204E+05 |
| 15.00 | 440. | 359. | .160E+05 | .190E+05 | 512. | 478. | .162E+05 | .224E+05 |
| 16.00 | 429. | 375. | .160E+05 | .204E+05 | 512. | 510. | .164E+05 | .244E+05 |
| 17.00 | 418. | 391. | .156E+05 | .217E+05 | 512. | 542. | .164E+05 | .265E+05 |
| 18.00 | 407. | 405. | .153E+05 | .230E+05 | 512. | 574. | .164E+05 | .285E+05 |
| 19.00 | 396. | 419. | .149E+05 | .242E+05 | 512. | 606. | .164E+05 | .306E+05 |
| 20.00 | 384. | 432. | .146E+05 | .253E+05 | 512. | 638. | .164E+05 | .326E+05 |
| 21.00 | 372. | 443. | .142E+05 | .264E+05 | 512. | 670. | .164E+05 | .347E+05 |
| 22.00 | 359. | 454. | .138E+05 | .274E+05 | 512. | 702. | .164E+05 | .367E+05 |
| 23.00 | 346. | 463. | .134E+05 | .283E+05 | 512. | 734. | .164E+05 | .388E+05 |
| 24.00 | 333. | 472. | .130E+05 | .292E+05 | 512. | 766. | .164E+05 | .408E+05 |
| 25.00 | 318. | 480. | .126E+05 | .300E+05 | 512. | 798. | .164E+05 | .429E+05 |
| 26.00 | 304. | 486. | .122E+05 | .307E+05 | 512. | 830. | .164E+05 | .449E+05 |
| 27.00 | 288. | 492. | .117E+05 | .313E+05 | 512. | 862. | .164E+05 | .470E+05 |
| 28.00 | 272. | 496. | .113E+05 | .319E+05 | 512. | 894. | .164E+05 | .490E+05 |
PARTIAL STATE/LEVEL DENSITIES CALCULATED FROM THE FOLLOWING PARAMETERS

Case 23C: C-formula(g-FGM)/Oblozinsky(1986): one-f. bound PSD for configs.

NUCLEUS: CHARGE NO. Z= 0. MASS NO. A= 0. PAIRING Up= .000 MEV

ONE-FERMION FORMULA:

COMPOSITE FORMULA

SINGLE-PARTICLE STATE DENSITY: G= 8.000 /MEV
FERMI ENERGY: F= 32.000 MEV
AV. EFF. FERMI ENERGY/1ST-2ND STEPS: F1= 32.000MEV
NUCLEON BINDING ENERGY: B= 8.000 MEV

| ENERGY (MeV) | w(E) (1/MeV) | w(p,h,E) (1/MeV) |
|-------------|--------------|------------------|
| p=h=1       |              |                  |
| 1.00        | 173.         | 64.0             |
| 2.00        | .183E+04     | 128.             |
| 3.00        | .124E+05     | 192.             |
| 4.00        | .657E+05     | 255.             |
| 5.00        | .293E+06     | 319.             |
| 6.00        | .115E+07     | 382.             |
| 7.00        | .411E+07     | 445.             |
| 8.00        | .136E+08     | 508.             |
| 9.00        | .419E+08     | 499.             |
| 10.00       | .122E+09     | 490.             |
| 11.00       | .341E+09     | 480.             |
| 12.00       | .912E+09     | 470.             |
| 13.00       | .235E+10     | 460.             |

1998- 1-10 10:15:42:99
EXEC TIME= .2 S
| E (MeV) | E+06 | E+06 | E+06 | E+06 | E+06 | E+06 | E+06 | \\
|-------|------|------|------|------|------|------|------|
| 14.00 | 0.585 | 0.269 | 0.399 | 0.172 | 0.242 | 0.113 | 0.242 |
| 15.00 | 0.425 | 0.373 | 0.660 | 0.340 | 0.588 | 0.349 | 0.660 |
| 16.00 | 0.335 | 0.503 | 0.105 | 0.364 | 0.133 | 0.986 | 0.133 |
| 17.00 | 0.765 | 0.660 | 0.115 | 0.285 | 0.257 | 0.115 | 0.285 |
| 18.00 | 0.269 | 0.845 | 0.240 | 0.578 | 0.627 | 0.578 | 0.627 |
| 19.00 | 0.377 | 0.444 | 0.100 | 0.344 | 0.144 | 0.144 | 0.344 |
| 20.00 | 0.813 | 0.348 | 0.112 | 0.194 | 0.315 | 0.194 | 0.315 |
| 21.00 | 0.333 | 0.157 | 0.340 | 0.131 | 0.131 | 0.131 | 0.131 |
| 22.00 | 0.765 | 0.187 | 0.658 | 0.131 | 0.131 | 0.131 | 0.131 |
| 23.00 | 0.857 | 0.915 | 0.284 | 0.183 | 0.472 | 0.183 | 0.472 |
| 24.00 | 0.172 | 0.130 | 0.569 | 0.459 | 0.150 | 0.459 | 0.150 |
| 25.00 | 0.377 | 0.157 | 0.784 | 0.703 | 0.256 | 0.703 | 0.256 |
| 26.00 | 0.733 | 0.187 | 0.106 | 0.320 | 0.853 | 0.320 | 0.853 |
| 27.00 | 0.148 | 0.219 | 0.240 | 0.111 | 0.111 | 0.111 | 0.111 |
| 28.00 | 0.358 | 0.254 | 0.133 | 0.285 | 0.268 | 0.285 | 0.268 |
| 29.00 | 0.765 | 0.254 | 0.986 | 0.627 | 0.131 | 0.627 | 0.131 |
| 30.00 | 0.172 | 0.294 | 0.735 | 0.853 | 0.315 | 0.853 | 0.315 |
| 31.00 | 0.377 | 0.396 | 0.690 | 0.293 | 0.472 | 0.293 | 0.472 |
| 32.00 | 0.765 | 0.407 | 0.334 | 0.144 | 0.144 | 0.144 | 0.144 |
| 33.00 | 1.000 | 0.418 | 0.113 | 0.113 | 0.113 | 0.113 | 0.113 |
| 34.00 | 0.857 | 0.430 | 0.242 | 0.242 | 0.242 | 0.242 | 0.242 |
| 35.00 | 0.172 | 0.440 | 0.340 | 0.340 | 0.340 | 0.340 | 0.340 |
| 36.00 | 0.377 | 0.450 | 0.399 | 0.399 | 0.399 | 0.399 | 0.399 |
| 37.00 | 0.765 | 0.460 | 0.172 | 0.172 | 0.172 | 0.172 | 0.172 |
| 38.00 | 1.000 | 0.470 | 0.242 | 0.242 | 0.242 | 0.242 | 0.242 |
| 39.00 | 0.857 | 0.480 | 0.113 | 0.113 | 0.113 | 0.113 | 0.113 |
| 40.00 | 0.172 | 0.490 | 0.340 | 0.340 | 0.340 | 0.340 | 0.340 |

ENERGY Wasym(E) w(p,h,E)
(MeV) (1/MeV) (1/MeV)

| E (MeV) | E+06 | E+06 | E+06 | E+06 | E+06 | E+06 | E+06 | E+06 | E+06 |
|-------|------|------|------|------|------|------|------|------|------|
| 1.00  | 204. | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 2.00  | .206 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 3.00  | .138 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 4.00  | .723 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 5.00  | .321 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 6.00  | .126 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 7.00  | .447 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 8.00  | .147 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 9.00  | .455 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 10.00 | .133 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 11.00 | .370 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 12.00 | .989 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 13.00 | .255 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 14.00 | .635 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 15.00 | .154 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| Time (sec) | Value 1   | Value 2   | Value 3   | Value 4   | Value 5   | Value 6   | Value 7   | Value 8   |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 16.00     | 0.36E+11  | 0.247E+10 | 0.176E+09 | 0.225E+07 | 0.112E+04 | 0.000     | 0.000     | 0.000     |
| 17.00     | 0.833E+11 | 0.826E+10 | 0.827E+09 | 0.185E+08 | 0.348E+05 | 0.000     | 0.000     | 0.000     |
| 18.00     | 0.187E+12 | 0.252E+11 | 0.338E+10 | 0.118E+09 | 0.554E+06 | 0.000     | 0.000     | 0.000     |
| 19.00     | 0.412E+12 | 0.707E+11 | 0.123E+11 | 0.628E+09 | 0.577E+07 | 0.191E+04 | 0.000     | 0.000     |
| 20.00     | 0.890E+12 | 0.186E+12 | 0.407E+11 | 0.286E+10 | 0.446E+08 | 0.538E+05 | 0.000     | 0.000     |
| 21.00     | 0.189E+13 | 0.458E+12 | 0.124E+12 | 0.115E+11 | 0.276E+09 | 0.822E+06 | 0.000     | 0.000     |
| 22.00     | 0.394E+13 | 0.107E+13 | 0.351E+12 | 0.417E+11 | 0.144E+10 | 0.843E+07 | 0.126E+04 | 0.000     |
| 23.00     | 0.811E+13 | 0.240E+13 | 0.933E+12 | 0.138E+12 | 0.654E+10 | 0.652E+08 | 0.390E+05 | 0.000     |
| 24.00     | 0.164E+14 | 0.514E+13 | 0.235E+13 | 0.426E+12 | 0.265E+11 | 0.408E+09 | 0.644E+06 | 0.000     |
| 25.00     | 0.328E+14 | 0.106E+14 | 0.562E+13 | 0.122E+13 | 0.971E+11 | 0.216E+10 | 0.705E+07 | 0.287      |
| 26.00     | 0.647E+14 | 0.210E+14 | 0.129E+14 | 0.332E+13 | 0.328E+12 | 0.100E+11 | 0.577E+08 | 0.124E+05 |
| 27.00     | 0.126E+15 | 0.405E+14 | 0.283E+14 | 0.855E+13 | 0.103E+13 | 0.416E+11 | 0.380E+09 | 0.253E+06 |
| 28.00     | 0.243E+15 | 0.757E+14 | 0.601E+14 | 0.303E+13 | 0.157E+12 | 0.211E+10 | 0.324E+07 | 0.000     |
| 29.00     | 0.462E+15 | 0.138E+15 | 0.123E+15 | 0.494E+14 | 0.845E+13 | 0.545E+12 | 0.102E+11 | 0.299E+08 |
| 30.00     | 0.872E+15 | 0.244E+15 | 0.245E+15 | 0.112E+15 | 0.224E+14 | 0.176E+13 | 0.443E+11 | 0.217E+09 |
| 31.00     | 0.163E+16 | 0.423E+15 | 0.473E+15 | 0.244E+15 | 0.565E+14 | 0.536E+13 | 0.174E+12 | 0.131E+10 |
| 32.00     | 0.301E+16 | 0.716E+15 | 0.889E+15 | 0.517E+15 | 0.137E+15 | 0.154E+14 | 0.631E+12 | 0.683E+10 |
| 33.00     | 0.551E+16 | 0.119E+16 | 0.163E+16 | 0.106E+16 | 0.320E+15 | 0.421E+14 | 0.213E+13 | 0.315E+11 |
| 34.00     | 0.100E+17 | 0.193E+16 | 0.292E+16 | 0.211E+16 | 0.721E+15 | 0.110E+15 | 0.672E+13 | 0.132E+12 |
| 35.00     | 0.180E+17 | 0.308E+16 | 0.512E+16 | 0.411E+16 | 0.157E+16 | 0.276E+15 | 0.201E+14 | 0.504E+12 |
| 36.00     | 0.322E+17 | 0.483E+16 | 0.879E+16 | 0.778E+16 | 0.333E+16 | 0.665E+15 | 0.571E+14 | 0.179E+13 |
| 37.00     | 0.572E+17 | 0.745E+16 | 0.148E+17 | 0.144E+17 | 0.687E+16 | 0.155E+16 | 0.155E+15 | 0.594E+13 |
| 38.00     | 0.101E+18 | 0.113E+17 | 0.245E+17 | 0.261E+17 | 0.138E+17 | 0.350E+16 | 0.403E+15 | 0.186E+14 |
| 39.00     | 0.176E+18 | 0.169E+17 | 0.398E+17 | 0.465E+17 | 0.270E+17 | 0.767E+16 | 0.101E+16 | 0.553E+14 |
| 40.00     | 0.306E+18 | 0.250E+17 | 0.637E+17 | 0.809E+17 | 0.517E+17 | 0.163E+17 | 0.244E+16 | 0.157E+15 |

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**Figure Captions**

FIG. 1. Particle-hole state densities for the given \( ph \) configurations, and the sum of the PSD for all allowed exciton numbers \( p=h \) (crosses), obtained with (a) the Williams’ one- and (b)-(d) two-fermion formulas for (a) the generic value \( g=1 \text{ MeV}^{-1} \), (b) \( g_\pi=g_\nu=g/2=1 \text{ MeV}^{-1} \), and (c),(d) the nucleus \(^{93}\text{Nb} \) and the phenomenological value \( g=A/13 \text{ MeV}^{-1} \). There are also shown the total nuclear state-densities given by the ESM formulas for (a) the one- and (b)-(d) two-fermion systems \([10]\) (solid curves), compared with (c),(d) the one-fermion system closed formula (dotted curves) and (d) the sum of the renormalized one-fermion PSDs \([34]\) (dashed curve).

FIG. 2. Particle-hole state densities for the \( 2p1h \) and \( 3p2h \) configurations, obtained with the ESM one-fermion formula with (dashed and dotted curves, for various \( F \) values) and without (solid curve) the potential finite-depth correction \([12]\), for the value \( g=1 \text{ MeV}^{-1} \).

FIG. 3. Particle-hole bound state densities obtained with the Williams’ one-fermion formula including the nuclear-potential finite depth \([14]\) for (a) the given \( ph \) configurations (solid curves), and (a),(b) the configuration \( 1p1h \), releasing consecutively the finite-depth potential (dashed curve) and the bound state conditions (dotted curve) by means of large \( F- \) and \( B- \) values, (b),(c) their sum for all allowed exciton numbers \( p=h \) without/with the two corrections (upper/lower crosses), and (c) the similar sum of the two-fermion PSDs (upper/lower x) compared with the one- and two-fermion system closed formulas (dashed and solid curves, respectively), for the values \( g=8 \text{ MeV}^{-1} \), \( F=32 \text{ MeV} \), and \( B=8 \text{ MeV} \).

FIG. 4. PSDs obtained with the Williams one-fermion formula (a) without/with the advanced pairing correction \([15]\) (dashed/solid curves) and (b) above the threshold energy for each configuration (dotted curves), leading to the sum of the PSD for all allowed exciton numbers \( p=h \) (solid curve), compared with the closed formula (dashed curve) for the values \( g=14 \text{ MeV}^{-1} \) and \( \Delta_0=1 \text{ MeV} \), as well as (c) for \( g=4 \text{ MeV}^{-1} \) and constant pairing correction \( U_p \) of 0, 2, and 4 MeV, and (d) for the one/two-fermion system formulas (upper/lower respective curves) and the BSFG parameters \( a=4.12 \text{ MeV}^{-1} \) and \( \Delta=-1.07 \text{ MeV} \) for the excited nucleus \(^{41}\text{Ca} \) \([15]\).

FIG. 5. Comparison of the PSDs obtained with the Williams one-fermion formula, for the given exciton configurations, and the advanced pairing correction of (a) Fu \([15]\) (dashed curves) and Kalbach \([16]\) above the threshold energy for each configuration (solid curves), as well as of (c) the latter and the PSDs with exact calculation of the Pauli-principle correction \([19]\) (dotted curves), (b) the sum of the Kalbach PSDs (shown also below the threshold) for all allowed exciton numbers \( p=h \) (solid curve) compared with the closed formula (dashed curve), for the values \( g=14 \text{ MeV}^{-1} \) and \( \Delta_0=1 \text{ MeV} \), and (d) the same but for the exact formula and the value \( g=8 \text{ MeV}^{-1} \) \([15]\).

FIG. 6. Comparison of the particle-hole bound state densities for the given \( ph \) configurations and the values \( g=8 \text{ MeV}^{-1} \), \( F=32 \text{ MeV} \), and \( B=8 \text{ MeV} \), obtained with the Williams one-fermion formula (a) including the nuclear-potential finite depth \([14]\) (dotted curves) and the exact calculation of the Pauli-principle correction \([19]\) (solid curves), as well as with (b) the latter form but without/with the advanced pairing correction and parameter value \( \Delta_0=1 \text{ MeV} \) (dotted/solid curves).

FIG. 7. The sum (solid curves) of (a) the two-fermion system PSDs \([16]\) (dotted curves) for allowed exciton numbers \( p_\pi=h_\pi \) and \( p_\nu=h_\nu \), compared with (b) the closed formula of the two-component ESM total state density for the excitation energy decreased by the effective pairing correction given by the term \( P_{\pi+\nu,\text{eff}} \) (dotted curve), and (b),(d) the term \( P_{\pi+\nu,\text{eff}} \) (dashed curve), (c) the constant \( U_p \) (long-dashed curve), the sum \( P_{\text{eff}}(E,C_\pi)+xP_{\text{eff}}(E,C_\nu) \) with \( x=1 \) (dashed curve) as well as between 0 and 1 for \( E/C \) varying from 1 to 2 (dotted...
curve), and (d) the Kalbach term $P_{\text{eff}}(E, C)$, also compared with the sum of the renormalized one-fermion PSDs $[34]$ (including the advanced pairing correction $[10]$) for all allowed exciton numbers $p=h$ (long-dashed curve), for the values $g=14$ MeV$^{-1}$ and $\Delta_0=1$ MeV.

FIG. 8. The sum (solid curves) (a) of the two-fermion system PSDs $[19]$ (dotted curves) for allowed exciton numbers $p_\pi=h_\pi$ and $p_\nu=h_\nu$, compared with the closed formula of the two-component ESM total state density for the excitation energy decreased by the effective pairing correction given by (b),(c) the Kalbach effective-energy shift $P_{\text{eff}}(E, C)$ (dotted curve), the term $P_{\pi+\nu,\text{eff}}$ (dashed curve), and (c) the constant shift $U_p$. (d) The comparison of the PSD values of Refs. $[19]$ (dashed curves) and $[16]$, for the given exciton configurations. The global parameter values used are $\Delta_0=1$ MeV, and (a),(b) $g=8$ MeV$^{-1}$ $[14,19]$ and (c),(d) $g=14$ MeV$^{-1}$ $[15,16]$.

FIG. 9. PSDs for (a),(b) the given $ph$ configurations, obtained with the ESM one-fermion system formula with (solid dotted) and without (dotted curves) inclusion of the FGM energy dependence of the s.p.l. density, and (c),(d) the given $p_\pi h_\pi p_\nu h_\nu$ configurations, obtained with the two-fermion system formula including the FGM energy-dependent s.p.l. density (solid curves) compared with the related results of the one-fermion system formula (dashed curves) and including the two-fermion system correction (dotted curves), as well as (a),(c) without and (b),(d) with the surface effect taken into account by using the average effective well depth $F_1=14$ MeV $[22]$, for the s.p.l. density value $g_0=14$ MeV$^{-1}$ and Fermi energy $F_0=38$ MeV.

FIG. 10. Comparison of the PSD values for the given $ph$ configurations and the sum providing the total state density, obtained by means of the given parameter values and (a),(b) the Williams (dotted curves) and the composite formula with no additional corrections (solid curves), for the one- and two-fermion systems respectively, and (c),(e) the Obložinský one-fermion formula $[14]$ (dotted curves) and the composite formula including the finite-well depth and bound-state conditions (solid curves) as well as (d),(f) the average energy-dependent formalism.

FIG. 11. Comparison of the PSD values for the given $ph$ configurations and the sum providing the total state density, obtained by means of the given parameter values and (a),(c) the Obložinský one- and two-fermion system formulas $[14]$ (dotted curves) and the composite formula including the finite-well depth and bound-state conditions (solid curves) as well as additionally (b),(d) the energy-dependent s.p.l. densities within the composite formula. The PSDs for configurations including $1p1h$ which are obtained as noted in (a),(b) by releasing consecutively the finite-depth potential and bound-state conditions by means of large $F$- and $B$-values are also shown.

FIG. 12. Comparison of the PSD values for the given exciton configurations, obtained by means of the given parameter values and the composite formula including the finite-well depth, the energy-dependent s.p.l. densities (solid curves) and the surface effects taken into account by using the effective well depth $F_1=14$ MeV (dashed curves), and (a) the Kalbach one-fermion system formula $[14]$ (dotted curves), (b) the Kalbach two-fermion system formula $[22]$ (dotted curves), as well as (c),(d) for the bound states and the exact calculation of the Pauli-principle correction $[19]$ (dotted curves).

FIG. 13. Comparison of the total level density given by the sum of the ESM (a) one-fermion and (b) two-fermion system PLDs over both excited-particle number(s), equal to hole number(s), and spins (solid curves) and the related closed-formulas (dashed curves), as well as of the closed-formula values obtained by using (c) the two-fermion system average spin-cutoff values $\sigma_2^2(n=2)$ $[36]$ and the correspondent $\sigma_1^2(n=2)$ for the one-fermion PLD (dotted curves) and the average values $\sigma_2^2$ and $\sigma_1^2$ for the related total level densities (solid curves noted “all
n”), and (d) the two-fermion system formula (solid curve) and the one-fermion formula with the two-fermion correction [32,34] (long-dashed curve), for the excited nucleus $^{41}$Ca and the given parameter values. The corresponding PLD values are shown for configurations with two excitons (dotted curves).

FIG. 14. Comparison of the two-fermion system average spin-cutoff values $\sigma_2^2(n = 2)$ [36] and the corresponding $\sigma_1^2(n = 2)$ for the one-fermion PLD (dotted curves), and the average values $\sigma_2^2$ and $\sigma_1^2$ (solid curves noted "all n"), for the excited nucleus $^{41}$Ca and the given parameter values.
| Case No. | TITLE (variable on first input record) | \(E_{\text{max}}\) (MeV) | Execution time\(^a\) (s) |
|----------|----------------------------------------|----------------|--------------------------|
| 1        | Williams(1971)/Fig.2(one-fermion, asymptotical form in 2nd table!) | 60            | 0.2                      |
| 1A       | Williams(1971) one-fermion formula, for distinct configurations | 40            | 0.1                      |
| 2        | Williams(1971), as Fig.2 but two-fermion formula/asymptotical form | 40            | 0.2                      |
| 2A       | Williams(1971) two-fermion formula, for distinct configurations | 40            | 0.1                      |
| 3        | Williams(1971) one-fermion formula/asympt. form for Nb, g=A/13 | 35            | 0.1                      |
| 3A       | Williams(1971) two-fermion formula/asympt. form for Nb, g=A/13 | 35            | 0.3                      |
| 3B       | Williams(1971) one-fermion + TFC\(^b\) formula/asympt. form for Nb, g=A/13 | 35            | 0.1                      |
| 4        | Betak+(1976)/Fig.1: one-fermion state density + finite depth corr. | 400           | 0.3                      |
| 5        | Oblozinsky(1986)/Fig.1: one-fermion bound state dens. for configs. | 80            | 0.1                      |
| 5A       | Oblozinsky(1986)/Fig.1: one-fermion state density + finite depth | 80            | 0.1                      |
| 5B       | Oblozinsky(1986)/Fig.1: one-fermion state density formula | 80            | 0.1                      |
| 6        | Oblozinsky(1986) one-fermion bound-state densities: g=8 | 80            | 0.4                      |
| 6A       | Oblozinsky(1986) one-fermion state density + finite depth corr. | 80            | 0.4                      |
| 6B       | Oblozinsky(1986) one-fermion formula/asymptotical form: g=8 | 80            | 9.2                      |
| 7        | Oblozinsky(1986) two-fermion bound-state densities: g=8 | 80            | 0.3                      |
| 7A       | Oblozinsky(1986) two-fermion state density + finite depth corr., g=8 | 80            | 7.0                      |
| 7B       | Oblozinsky(1986) two-fermion formula/asymptotical form: g=8 | 80            | 6.8                      |
| 8        | Fu(1984)/Fig.3: one-fermion state density with/without pairing | 15            | 0.3                      |
| 9        | Fu(1984)/Fig.6: one-f. closed-formula values for various pairings | 20            | 0.3                      |
| 10       | Fu(1984)/Fig.7: one/two-f. closed-formula with various BSFG data | 40            | 0.2                      |
| 10A      | Fu(1984)/Fig.7: two-f. closed-formula with two-f. correction | 40            | 0.1                      |
| 11       | Fu(1986)/Fig.6: one-fermion partial level dens. sum/closed-formula | 11            | 0.1                      |
| 11A      | Fu(1986) one-fermion partial level density, total PLD for config. | 11            | 0.1                      |
| 11B      | Fu(1986)/Fig.1: two-fermion PLD for 41Ca: sum/closed-formula | 25            | 0.2                      |
| 11C      | Fu(1989)/Fig.1: two-fermion PLD for 41Ca configuration | 25            | 0.1                      |
| 11D      | Fu(1989)/Fig.1: two-fermion PLD for 41Ca /sum/closed-formula | 25            | 0.1                      |
| 11E      | Fu(1989)/Fig.1: one-fermion PLD + TFC: sum/closed-formula | 25            | 0.1                      |
| 12       | Kalbach(1987)/Fig.3: one-fermion partial state dens. for configs. | 15            | 0.1                      |
| 13       | Kalbach(1987) one-fermion partial state density/sum/closed form | 15            | 0.1                      |
| 13A      | Kalbach(1987) two-fermion PSD sum/closed formula, for g=8/Up=3.5 | 15            | 0.3                      |
| 13B      | Kalbach(1987) one-fermion PSD + TFC /sum/closed form: g=8/Up=3.5 | 15            | 0.1                      |
| 14       | Kalbach(1989)/Tab.1: one-fermion partial state dens. for configs. | 32            | 0.1                      |
| 14A      | Kalbach(1989)/Tab.1b: one-fermion partial state dens. for configs. | 128           | 0.1                      |
| 14B      | Kalbach(1989)/Tab.1c: one-fermion partial state dens. for configs. | 256           | 0.1                      |
| 15       | Mao Ming De(1993)/Fig.1: one-fermion bound PSD + exact Pauli corr. | 100           | 106.7                    |
| 15A      | Mao Ming De(1993)/Fig.2/Oblozinsky(86): bound PSD+exact Pauli corr. | 100           | 42.9                     |
| 16       | Mao Ming De(1993)/Fig.3: bound PSD+exact Pauli corr.+pairing corr. | 100           | 64.1                     |
| 17       | Mao Ming De(1993): one-fermion PSDs+pairing corr./sum/closed form | 15            | 123.4                    |
| 17A      | Mao Ming De(1993)/Kalbach(87): one-f. PSDs: exact Pauli corr. effect | 15            | 134.6                    |
| 17B      | Mao Ming De(1993): two-fermion PSDs+pairing corr./sum/closed form | 15            | 351.5                    |
| 17C      | Mao Ming De(1993): 1-f.PSDs+pair corr.(2-3.5)+TFC/sum/closed form | 15            | 257.4                    |
| 17D      | Mao Ming De(1993)/Kalbach(87): two-f. PSDs: exact Pauli corr. effect | 15            | 427.1                    |
| Case No. | TITLE (variable on first input record) | $E_{\text{max}}$ (MeV) | Execution time$^a$ (s) |
|---------|----------------------------------------|-------------------------|-------------------------|
| 18      | Kalbach(1985)/(Fig.2): one-fermion PSD with Finite Depth Cor.: $F_1=38$ | 100                     | 0.1                     |
| 18A     | Kalbach(1985)/(Fig.2): one-f. Energy-dep. s.p.s. density + FDC$^c$: $F=38$ | 100                     | 0.1                     |
| 18B     | Kalbach(1985)/(Fig.2): one-f. PSD with FDC + surface effects: $F_1=14$ | 100                     | 0.1                     |
| 18C     | Kalbach(1985)/(Fig.2): E-dep. s.p.s. density + FDC: $F=38$ + surf. ef.: $F_1=14$ | 100                     | 0.1                     |
| 19      | Kalbach(1985)/(Fig.2): two-fermion PSD + FDC ($F=38$), g-FGM for | 100                     | 0.2                     |
| 19A     | Kalbach(1985): one-fermion PSD + FDC ($F=38$) + TFC, for configs. | 100                     | 0.1                     |
| 20      | Kalbach(1985)/(Fig.2): two-fermion PLD + FDC ($F_1=14$), g-FGM, for | 100                     | 0.2                     |
| 20A     | Kalbach(1985): one-fermion PLD + FDC ($F_1=14$) + TFC | 100                     | 0.1                     |
| 21      | C-formula/Williams(1971)/(Fig.2): one-fermion PSD /sum/closed form | 80                      | 0.3                     |
| 21A     | C-formula/Williams(1971)/(Fig.2): one-fermion PSD for configs. | 60                      | 0.1                     |
| 22      | C-formula/Williams(1971): two-fermion PSD /sum/closed formula | 40                      | 0.5                     |
| 22A     | C-formula/Williams(1971): two-fermion PSD for distinct configs. | 40                      | 0.1                     |
| 23      | C-formula/Oblozinsky(1986)/(Fig.1): one-f. bound PSD for configs. | 80                      | 0.1                     |
| 23A     | C-formula/Oblozinsky(1986): one-fermion bound PSD /sum/closed form | 80                      | 0.11                    |
| 23B     | C-formula/Oblozinsky(1986): two-fermion bound PSD /sum/closed form | 80                      | 24.8                    |
| 23C     | C-formula(g-FGM)/Oblozinsky(1986): one-f. bound PSD for configs. | 80                      | 3.5                     |
| 23D     | C-formula(g-FGM)/Oblozinsky(1986): two-f. bound PSD /sum/closed form | 80                      | 28.1                    |
| 24      | C-formula/Oblozinsky(1986): one-fermion PSD + FDC for configs. | 80                      | 0.1                     |
| 24A     | C-formula/Oblozinsky(1986): one-f. PSD + FDC /sum/closed form | 80                      | 1.0                     |
| 24B     | C-formula/Oblozinsky(1986): two-f. PSD + FDC /sum/closed form | 80                      | 17.3                    |
| 24C     | C-formula(g-FGM)/Oblozinsky(1986): one-f. PSD + FDC /sum/closed form | 80                      | 1.5                     |
| 24D     | C-formula(g-FGM)/Oblozinsky(1986): two-f. PSD + FDC /sum/closed form | 80                      | 19.7                    |
| 25      | C-formula/Oblozinsky(1986): one-fermion ESM PSD for configuration | 80                      | 0.1                     |
| 25A     | C-formula/Oblozinsky(1986): one-fermion ESM PSD /sum/closed form | 80                      | 1.0                     |
| 25B     | C-formula/Oblozinsky(1986): two-fermion ESM PSD /sum/closed form | 80                      | 16.7                    |
| 26      | C-form(g-FGM)/Kalbach(1987): one-f. PSD + FDC ($F=38$ MeV) /sum/closed form | 80                      | 2.3                     |
| 26A     | C-form(g-FGM)/Kalbach(1987): one-f. PSD + FDC ($F_1=14$ MeV) /sum/closed form | 80                      | 2.5                     |
| 27      | C-form(g-FGM)/Kalbach(1985): two-f. PSD + FDC ($F=38$ MeV) /sum/closed form | 80                      | 46.7                    |
| 27A     | C-form(g-FGM)/Kalbach(1985): two-f. PSD + FDC ($F_1=14$ MeV) /sum/closed form | 80                      | 46.9                    |
| 28      | C-form(g-FGM)/Mao Ming(1993): one-f. PSD + FDC ($F=32$ MeV) /sum/closed form | 80                      | 276.6                   |
| 28A     | C-form(g-FGM)/Mao Ming(1993): two-f. PSD + FDC ($F=32$ MeV) /sum/closed form | 80                      | 1679.3                  |

$^a$ For calculation carried out on PC Pentium/166MHZ Intel.

$^b$ Two-fermion system correction (TFC).

$^c$ Nuclear potential finite-depth correction (FDC).
| Record | Variable | FORMAT | Meaning |
|--------|----------|--------|---------|
| 1      | NE       | 2I3,74A1 | if NE<0, |NE| is the number of excitation energies to be read (≤200); if NE>0, the excitation energies are (FLOAT(I),I=1,NE) (MeV) |
|        | IOPTJ    |        | spin distribution (PLD calculation) option: 0, ω(p, h, E) or ω(pπ, hπ, pν, hν, E) are calculated 1, ρ(p, h, E, J) or ρ(pπ, hπ, pν, hν, E, J) are calculated |
| 2      | TITLE    |        | title of the problem |
|        | I       |        |excitation energies (MeV) for PSD/PLDs calculation |
| 3      | IMOD     | 2I3,7F10.5 | option for PSD formula (odd/even: one/two-fermion system formulas): =-1, 0: composite (recommended) formula = 1, 2: F.C. Williams, Nucl. Phys. A166, 231 (1971) = 3, 4: P.Oblozinsky, Nucl. Phys. A453, 127 (1986), Eqs.(7,9) = 5, 6: C.Y. Fu, Nucl. Sci. Eng. 86, 344 (1984) = 7, 8: C. Kalbach, Nucl.Sci.Eng.95,70(1987), Z.Phys.A 332,157(1989) = 9,10: C. Kalbach, Phys. Rev. C 32, 1157 (1985) =11,12: Mao Ming De Gua Hua, J. Phys. G 19, 421 (1993) |
|        | ITFC     |        | option for the two-fermion system correction: = 0: no two-fermion system correction = 1: J.M.Akkermans, H.Gruppelaar, Z. Phys. A 321,605(1985), Eq. (9) |
|        | A        |        |excited-nucleus mass number (it may be omitted if GIN≥0.) |
|        | Z        |        |excited-nucleus atomic number |
|        | UP       |        |pairing correction based on the odd-even mass differences (if UP=-1, and A≥0, Eq.(9) of 33 is adopted) |
| 4      | NP0      | 2I3,7F10.5 | [record to be read only for odd IMOD] number p of excited particles |
|        | NH0      |        |number h of holes (if NP0=NH0=0, calculation is done for all pairs p=h=1, 2,... for which PSD/PLD-value is ≥0.1 MeV⁻¹) |
|        | GIN      |        |single-particle state density G (MeV⁻¹): if GIN≤0, then G=(6/3.14²2)*DR(1) is adopted; if GIN=0 and A=0, then G=1.0 is adopted; if GIN=0 and A≥0, then G=A/13.0 is adopted |
|        | FIN      |        |Fermi energy F (MeV); if FIN=0, then F=10⁶ is adopted |
|        | BIN      |        |nucleon binding energy B (MeV); if BIN=0, then B=10⁶ is adopted |
|        | F1N      |        |average effective Fermi energy F1 (MeV); if FIN=0, then F1=10⁶; if FIN≤0, then constant G is used within the WR1 and WR2 functions |
| Record Variable | FORMAT | Meaning |
|-----------------|--------|---------|
| **5** NP0       | 4I3,6F10.5 | [record to be read only for even IMOD] number \( p_\pi \) of proton excited particles |
|                 |        | number \( h_\pi \) of proton holes |
|                 |        | number \( p_\nu \) of neutron excited particles |
|                 |        | number \( h_\nu \) of neutron holes |
| (**5** NH0)     |        | (if NP0=NH0=NPN0=NHN0=0, all configurations \((p_\pi=h_\pi, p_\nu=h_\nu)\) with \( N=2, 4, .. \) and PSD/PLD sum >0.1 MeV\(^{-1}\) are considered) |
| (**5** NPN0)    |        | |
| (**5** NHN0)    |        | |
| **GIN**         |        | single-proton state density \( G \) (MeV\(^{-1}\)); |
| **GN**          |        | single-neutron state density \( G_N \) (MeV\(^{-1}\)); |
|                 |        | if \( GIN\leq0 \), then \( G=Z/A*(6/3.14^2)*DR(1) \) (see record 6) |
|                 |        | and \( G_N=(A-Z)/A*(6/3.14^2)*DR(1) \) are adopted; |
|                 |        | if \( GIN=0 \) and \( A=0 \), then \( G=GN=1.0 \) are adopted; |
|                 |        | if \( GIN=0 \) and \( A\geq0 \), then \( G=Z/13.0 \) and \( G_N=(A-Z)/13.0 \) are adopted |
| **FIN**         |        | proton Fermi energy \( F \) (MeV); |
| **FN**          |        | neutron Fermi energy \( F_N \) (MeV); |
|                 |        | if \( FIN=0 \), then \( F=FN=10^6 \) are adopted |
| **BIN**         |        | proton binding energy \( B \) (MeV); if \( BIN=0 \), then \( B=10^6 \) is adopted |
| **BN**          |        | neutron binding energy \( B_N \) (MeV); |
|                 |        | if \( BIN=0 \), then \( B=BN=10^6 \) are adopted |
| **F1IN**        |        | average effective proton Fermi energy \( F_1 \) (MeV); |
|                 |        | if \( F1IN=0 \), it is adopted the value \( F_1=10^6 \); |
|                 |        | if \( F1IN\leq0 \), constant \( G \) is used within WR1 and WR2 functions |
| **F1N**         |        | average effective neutron Fermi energy \( F_1N \) (MeV); |
|                 |        | if \( F1N=0 \), it is adopted the value \( F_1N=10^6 \); |
|                 |        | if \( F1N<0 \), constant \( GN \) is used within WR1 and WR2 functions |
| **6** DR(I)     | 8F10.5 | [record to be read only if \( GIN\leq0 \)] BSFG level density parameters [35]: |
|                 |        | \( DR(1) \) - nuclear level density parameter \( a \); |
|                 |        | \( DR(2) \) - ratio of effective nuclear moment of inertia to rigid-body value; |
|                 |        | \( DR(3) \) - shift of the fictive nuclear ground state |
| **7** ICONT     | 2I3    | output and recycle option: |
|                 |        | \(-1\), print first 2 tables of calculated PSD/PLDs (see PRINTWN description); |
|                 |        | resumption according to IEND; |
|                 |        | \( =0 \), print calculated PSD/PLDs; resumption according to IEND; |
|                 |        | \( =1 \), new case by input-data record 1; results printed at once with previous case; |
|                 |        | \( =2 \), new case by input-data record 3 (same energy grid); |
|                 |        | \( =3 \), calculation for other exciton configuration by input-data record 4 or 5; |
|                 |        | \( =4 \), calculation for other BSFG parameter set by input-data record 6 |
| **IEND**        |        | recycle option: |
|                 |        | \( =0 \), end; |
|                 |        | \( =1 \), new complete case by input-data record 1; |
|                 |        | \( =2 \), new case by input-data record 3; |
|                 |        | \( =3 \), new case for exciton configuration by record 4 or 5; |
|                 |        | \( =4 \), new case for BSFG parameters by record 6 |
FIG. 8

FIG. 9
Fig. 13

Fig. 14