Autonomous Navigation on Modified AOR Iterative Method in Static Indoor Environment

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Abstract. One of the main issues when dealing with mobile robot navigation is that we have to resolve the obstacle avoidance problem, where when moving from a starting point to the goal point, the mobile robot will have to generate a collision-free path in order ensure that it can move efficiently in the environment. In this study, we attempt to resolve the issue by solving it iteratively via numerical technique. This solution is based on the potential field method that utilizes the Laplace’s equation to constrain the generation of potential functions over the regions in the configuration space where the mobile robot operates in. This paper proposed Modified Accelerated Over-Relaxation (MAOR) iterative method for solving robot path planning problem. Through the application of finite-difference technique in it, the experiment shows that the mobile robot is able to generate a smooth path from starting point to goal point. Furthermore, the results obtained from the simulation has shown that this numerical method was able to perform faster solution and generated smoother path comparing the previous works on the similar problem.

1. Introduction
The path planning applications such as robot navigation and autonomous agents has recently became a common subject of studies. In general, an essential matter in the mobile robot path planning is to generate a smooth path while moving in the designated environment. That means the mobile robot should be able to plan a path to avoid any obstacles along its way in order to reach its goal point. Implementing the theory of heat transfer into it, this paper will devise a mobile robot path planning method through numerical potential function in the configuration space. The problem of heat transfer is modelled by using Laplace’s equation and its solutions are known as the harmonic functions. In the designated environment, temperature values are used to simulate paths derived from harmonic functions. With the availability of high-performance devices, which enables efficient resolution for the problem, we were able to employ numerical techniques to achieve harmonic functions. This research has performed several experiments to examine the efficiency of Modified Accelerated Over-Relaxation (MAOR) iterative method in creating mobile robot paths in multiple configuration space sizes with various obstacles along the way.
2. Related Work
The application of potential functions on robot path planning was first introduced by Khatib, where in the configuration space setup, the goal point was set to exert attraction force while the obstacles exert repelling force on an end effector [1]. Meanwhile Connolly et al. [2] and Akishita et al. [3] have used Laplace’s equation to develop a global method to enable smooth collision-free path for robot path planning. The results obtained from the works show that harmonic functions provide a quick way to create routes in a configuration space of robot and prevent the formation of local minima spontaneously. Sasaki [4] then prove the use of numerical technique to resolve the problem of path finding, where it argues that by simulating complex labyrinth problems, the new computational approach is efficient in motion planning as it worked well with each other. Karova et al. [5] made use the technology of image processing to present the Dijkstra’s algorithm for mobile robot path planning in a labyrinth, where it was discovered that algorithm was able to produce the shortest path in the minimum time to reach the goal point in a larger size of labyrinth. Whereas Hachour [6] suggested by using hybrid intelligent, an independent mobile approach was able to be navigated in the form of grid-map with unknown environment and static unknown obstacles. The characteristics which made the study appealing was that it utilized the best path of biological genetic principle, in combination with networks within the task of fuzzy reasoning and inference which captures human expert knowledge in order to determine the best possible way of avoiding obstacles. The simulation was shown by using two programming languages: visual basic language, where the robot reaches the target by avoiding all obstacles, and Delphi language, where the robot takes the shortest path in to reach the target.

In other past researches, few standard numerical techniques, such as Gauss-Seidel (GS) and Successive Over-Relaxation (SOR) iterative methods has been applied for computation of harmonic potentials [2,4,7,8]. Block variants of the SOR method have been used in recent times for faster computing [9,10]. Other applications such as space exploration [11], trajectory control [12], ship navigation [13], naval vessel path planning [14], UAV motion planning [15,16], and so on have been implementing harmonic potentials in relation to autonomous mobile robot motion planning. Therefore, this studies proposed a Modified Accelerated Over-Relaxation (MAOR) iterative method in solving robot path planning problem by employing Laplace’s equation through finite-difference technique.

3. Materials and Methods
The Laplace’s equation in the study of heat conduction, is also known as the steady-state heat equation [17]. For the purpose of this study, the heat transfer problem represents the mobile robot path planning problem while the Laplace’s equation solution as the harmonic functions. The harmonic potentials are calculated throughout the region containing obstacles and mobile robot path line from starting point to goal point was determined using the harmonic solutions. In the configuration space, the goal point was represented as a sink which pulls the heat in while other elements including outer boundaries, inner walls and obstacles which we want to prevent the mobile robot from having a contact with was treated as heat sources that were fixed at a constant temperature value. This setup resulted in the occurrence of heat conduction process in the configuration space where the environment was filled in with the process causing in the flow of heat flux lines to the sink, representing the Laplacian potential values or temperature distribution. By observing the heat flux lines produced, the path can then be readily discovered. This setup also ensures that the path line produced is smooth while preventing the occurrence of local minima and avoiding all the obstacles presented along the way as shown by Connolly et al. [2].

In explaining the process mathematically, a harmonic function on a domain $\Omega \subset R^n$ satisfies the Laplace’s equation, where $x_i$ is the $i$-th Cartesian coordinate and $n$ is the dimension. In the case of robot path construction, the domain $\Omega$ consists of the outer boundaries, inner walls, obstacles, starting points and goal point.
\[ \nabla^2 \phi = \sum_{i=1}^{n} \frac{\partial^2 \phi}{\partial x_i^2} = 0 \]  

(1)

The equation of Laplacian (1) can be effectively solved through numerical method. Many block iteration techniques producing impressive performance were executed \[18,19\]. The robot is represented as a point in the configuration space in this setup. The grid form configuration space iteratively computes the coordinates and function values related with each node using numerical technique to meet equation (1). A high fixed potential value is assigned to the starting point, while the destination point is the lowest fixed potential value, whereas the outer wall boundaries and obstacles are given various initial temperature values. In this work, the Laplace’s equation solution was subjected to Dirichlet boundary conditions, \( \Phi | \partial \Omega = c \), with constant \( c \). After the harmonic function is established under the limits, the mobile robot path line can be obtained by observing the heat flux line of gradient descent approach on calculated potential values. A gradient descent search was conducted as the mobile robot follows the path that generally moves from a point of higher potential value to a point of lower potential value, and eventually reaching the goal point which has the lowest potential value in the configuration space. The coordinates and nodal temperature gradients acquired from the analysis of finite difference can be used to draw the path.

4. Formulation of Modified Accelerated Over Relaxation (MAOR) Iterative Method

The standard GS \[2\] and SOR \[8-10\] were used to resolve equation (1) in the robotics literature. This study suggested quicker numerical solver by engaging MAOR iterative method to compute the solutions of Laplace’s equation (1). Basically, MAOR method is the generalization of the AOR method. It is observed that the MAOR method decreases the Jacobi and Modified SOR method extrapolation with distinct parameters corresponding to the row blocks of matrices for specific choices of the acceleration and relaxation matrices.

The fact that the formulation for MAOR iterative method are similar to AOR method. However, the MAOR iteration schemes involves the implementation of red-black ordering strategy through the use of three different weighted parameters, \( r, \omega \) and \( \omega' \). The formulation of family of Point FSMAOR methods can be expressed generally

\[ U_{i,j}^{(k+1)} = \frac{r\omega}{4} \left[ U_{i,j-1}^{(k)} + U_{i+1,j}^{(k)} + U_{i,j+1}^{(k)} + U_{i-1,j}^{(k)} \right] + (1 - \omega) U_{i,j}^{(k)} , \]

\[ U_{i+1,j}^{(k+1)} = \frac{\omega'}{4} \left[ U_{i+1,j+1}^{(k)} + U_{i+1,j-1}^{(k)} + U_{i+1,j}^{(k)} + U_{i+1,j}^{(k)} \right] + (1 - \omega') U_{i+1,j}^{(k)} , \]

in red nodes, whereas black nodes as

\[ U_{i,j}^{(k+1)} = \frac{r}{4} \left[ U_{i-1,j}^{(k)} - U_{i-1,j}^{(k)} + U_{i,j-1}^{(k)} + U_{i,j+1}^{(k)} - U_{i,j}^{(k)} \right] + \frac{\omega'}{4} \left[ U_{i,j-1}^{(k)} + U_{i,j+1}^{(k)} + U_{i,j}^{(k)} + U_{i,j}^{(k)} \right] + (1 - \omega') U_{i,j}^{(k)} , \]

\[ U_{i+1,j+1}^{(k+1)} = \frac{r'}{4} \left[ U_{i+1,j+1}^{(k)} - U_{i+1,j+1}^{(k)} + U_{i+1,j+1}^{(k)} - U_{i+1,j+1}^{(k)} \right] + \frac{\omega'}{4} \left[ U_{i+1,j+1}^{(k)} + U_{i+1,j+1}^{(k)} + U_{i+1,j+1}^{(k)} + U_{i+1,j+1}^{(k)} \right] + (1 - \omega') U_{i+1,j+1}^{(k)} , \]

where \( r, \omega \) and \( \omega' \) are the optimum relaxation parameters and defined in the range of \([1,2]\). In determining the optimum values of \( r, \omega \) and \( \omega' \) for a number of iterations, there are no general formula needed. The value of \( r \) and \( \omega' \) are usually selected, according to Hadjidimos \[20\], to be close to the value \( \omega \) of the corresponding SOR.
5. Results and Discussion

The experiment took into account three different static environment sizes, composed of multiple obstacles in four distinct environments. Higher temperature values are fixed to the walls and obstacles in the initial setting. The goal point was set to have the lowest potential value in the configuration space, while the starting point had no initial value. The free space in the configuration space was set at zero temperature value. The computation was carried out with a 2.50GHz speed PC with 8GB of RAM. At all points, the calculation of the temperature values remained numerically until the stop conditions were met. The loop would be terminated if the temperature values were no longer changed, where there was a very small distinction between harmonic potentials at the iterations k and k+1, i.e. $1.0 \times 10^{-10}$. This elevated accuracy was needed in order to prevent the occurrence of path generation failure, and to prevent flat areas that are also known as saddle points in the configuration space. Tables 1 and 2 show the iteration numbers and execution time (in seconds) needed to calculate all temperature values in the environment for every numerical technique compared in the experiment. As mentioned previously, the optimum values of weighted parameters, $\omega_r$ and $\omega'$, are chosen in the range of (1,2). Clearly, in terms of iteration number, MAOR iterative method provided high performance compared to other considered methods. Meanwhile, execution times for modified families proved to be slightly faster than the standard methods.

### Table 1. Performance of the considered methods in terms of number of iteration

| Case | Methods | $N \times N$ | $300 \times 300$ | $600 \times 600$ | $900 \times 900$ |
|------|---------|--------------|----------------|----------------|----------------|
| 1    | SOR     | 1728         | 8117           | 17831          |
|      | AOR     | 1591         | 7529           | 16594          |
|      | MSOR    | 1583         | 7557           | 16697          |
|      | MAOR    | 1524         | 7311           | 16069          |
| 2    | SOR     | 2228         | 8776           | 19254          |
|      | AOR     | 2006         | 7973           | 17538          |
|      | MSOR    | 2097         | 8323           | 18307          |
|      | MAOR    | 1872         | 7542           | 16617          |
| 3    | SOR     | 3624         | 14624          | 33004          |
|      | AOR     | 3236         | 13165          | 29680          |
|      | MSOR    | 3402         | 13814          | 31194          |
|      | MAOR    | 3023         | 12395          | 28037          |
| 4    | SOR     | 2507         | 9868           | 21654          |
|      | AOR     | 2288         | 9025           | 19840          |
|      | MSOR    | 2395         | 9411           | 20667          |
|      | MAOR    | 2169         | 8623           | 18949          |

### Table 2. Performance of the considered methods in terms of CPU time (in seconds)

| Case | Methods | $N \times N$ | $300 \times 300$ | $600 \times 600$ | $900 \times 900$ |
|------|---------|--------------|----------------|----------------|----------------|
| 1    | SOR     | 8.13         | 227.95         | 1134.25        |
|      | AOR     | 8.61         | 230.17         | 1148.87        |
|      | MSOR    | 6.72         | 240.99         | 1227.39        |
|      | MAOR    | 7.44         | 247.99         | 1295.65        |
| 2    | SOR     | 10.69        | 251.72         | 1270.23        |
|      | AOR     | 10.27        | 248.24         | 1226.66        |
|      | MSOR    | 9.36         | 269.68         | 1355.34        |
|      | MAOR    | 9.30         | 267.18         | 1360.64        |
| 3    | SOR     | 16.22        | 427.27         | 2190.45        |
|      | AOR     | 18.66        | 418.45         | 2073.25        |
|      | MSOR    | 14.40        | 462.03         | 2361.08        |
|      | MAOR    | 15.35        | 450.60         | 2420.88        |
| 4    | SOR     | 11.02        | 281.85         | 1441.47        |
|      | AOR     | 12.52        | 281.78         | 1423.54        |
|      | MSOR    | 9.78         | 309.74         | 1576.44        |
|      | MAOR    | 9.83         | 309.98         | 1581.29        |

The required path was created by execution of steepest descent search from starting points to target point when temperature values are acquired. In all experiments, figure 1 displays the paths that were successfully generated in a known static environment through numerical computation on the basis of gained Laplacian potential. All starting points (green dot/square dot) were efficiently ended at the given target point (red dot/circle dot), avverting different forms of obstacles in various environment. Since no interpolation is carried out, in certain cases there are some jagged nature of the paths. The fact that gradient interpolation would provide smoother paths. So as to accelerate the convergence rate of the proposed iterative method, in addition to the full-sweep iteration, an advance investigation into half-[8,21-23] and quarter-sweep [24-28] iterations can also be measured.
6. Conclusions and Future Work
Due to the recent advanced numerical techniques and the availability of faster machine nowadays, this experiment study demonstrates that solving Laplace’s equation (1) numerically in resolving the mobile robot path finding problem was certainly very appealing and attainable. By comparing to the prior existing MSOR method, the MAOR iterative method proved to have high performance in terms of iterations counting, as shown in Table 1 and 2. Increased obstacles number and multiple obstacles shapes does not adversely influence the efficiency, instead, the computation gets quicker as the obstacles regions are ignored during calculation.

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