Study of geological processes of absorption of pressure waves from injected fluids in the rock structure

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Abstract. The paper presents a detailed study of the propagation of oscillations in an infinite absorbing medium in order to discuss the behavior of a material under various conditions during pulsed injection of fluids into an injection well. The dependence of deformation and stress was solved mathematically on the basis of the modified Hooke's law, which includes the rate of change in deformation. As a result, graphical dependences of the phase velocity of propagation of oscillations and the absorption coefficient on the oscillation frequency were obtained for the most common types of rocks. Analysis of the obtained results showed that the absorption coefficient attains its maximum values in the region of resonance frequencies, and further increase in the oscillation frequency leads to a slight decrease in the absorption coefficient. In our opinion, the most optimal frequency range for the propagation of oscillations in the rock structure varies from 100 to 800 Hz with regard to the natural frequencies of the rocks studied.

1. Introduction
The analysis of theoretical studies into the propagation of pressure waves of fluid in the layers showed that the studies were focused on the propagation of the fluid itself in the rock pores in terms of the frequency of pressure fluctuation and its motion amplitude. The study of this process, in our opinion, does not take into account all factors and, therefore, the results obtained cannot be considered objective [1–5]. When exposed to hydraulic impulses, a decisive factor is the transfer of fluid oscillations to the rock, considering that the rock located at a certain distance from the source returns oscillations to fluid, since the absorption of oscillations of the injected fluid itself occurs catastrophically. Thus, it is of interest to study the propagation of oscillations in an infinite absorbing medium, which is the basis for discussing the behavior of a material under various conditions.

2. Materials and methods
The stresses acting on the elementary volume of a solid can be expressed as a linear combination of deformation using Hooke’s law. For an isotropic solid, all proportionality constants can be expressed in terms of two elastic moduli. Although Young’s modulus and Poisson’s ratio are generally accepted elastic constants, Lamé parameter $\lambda$ and shear modulus $\mu$ are used in the study. For an isotropic body, according to [6] and taking into account the elastic constants, the stress–strain relationship has the form
\[
\begin{align*}
\sigma_{xx} &= (\lambda + 2\mu)l_{xx} + \lambda l_{yy} + \lambda l_{zz} \\
\sigma_{yy} &= \lambda l_{xx} + (\lambda + 2\mu)l_{yy} + \lambda l_{zz} \\
\sigma_{zz} &= \lambda l_{xx} + \lambda l_{yy} + (\lambda + 2\mu)l_{zz} \\
\sigma_{xy} &= \mu l_{xy}, \quad \sigma_{yz} = \mu l_{yz}, \quad \sigma_{zx} = \mu l_{zx}
\end{align*}
\]

where \( E \) – elastic modulus; \( \nu \) – Poisson’s ratio.

\[\mu = \frac{E}{2(1+\nu)}\]  \hfill (3)

To express the change in stress between adjacent points of an elementary volume, it is sufficient to consider the value that linearly depends on the distance between the points. Obviously, in this approximation, the surface forces acting on the faces of the elementary volume are not exactly balanced. To achieve equilibrium, it is essential to add mass forces that cause the elementary volume acceleration. In addition to surface forces, the medium exhibits both gravitational and volumetric forces. Equating the sum of all forces to the product of mass and acceleration along each of the three axes, we obtain the equations of an isotropic medium:

\[
\begin{align*}
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + G_x &= \rho \frac{\partial^2 U_x}{\partial t^2} \\
\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + G_y &= \rho \frac{\partial^2 U_y}{\partial t^2} \\
\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + G_z &= \rho \frac{\partial^2 U_z}{\partial t^2}
\end{align*}
\]

where \( G_x, G_y, G_z \) – volumetric force components.

Equations (1) can be differentiated and substituted into the equations of an isotropic medium to eliminate stresses. If the mass forces are equal to zero, the equations of motion expressed through the displacements of particles have the form:

\[
\begin{align*}
(\lambda + 2\mu)\frac{\partial^2 U_x}{\partial x^2} + \mu \left( \frac{\partial^2 U_y}{\partial y^2} + \frac{\partial^2 U_z}{\partial z^2} \right) + (\lambda + \mu) \left( \frac{\partial^2 U_y}{\partial x \partial y} + \frac{\partial^2 U_z}{\partial x \partial z} \right) &= \rho \frac{\partial^2 U_x}{\partial t^2}, \\
(\lambda + 2\mu)\frac{\partial^2 U_y}{\partial y^2} + \mu \left( \frac{\partial^2 U_x}{\partial x^2} + \frac{\partial^2 U_z}{\partial z^2} \right) + (\lambda + \mu) \left( \frac{\partial^2 U_x}{\partial x \partial y} + \frac{\partial^2 U_z}{\partial y \partial z} \right) &= \rho \frac{\partial^2 U_y}{\partial t^2}, \\
(\lambda + 2\mu)\frac{\partial^2 U_z}{\partial z^2} + \mu \left( \frac{\partial^2 U_x}{\partial x^2} + \frac{\partial^2 U_y}{\partial y^2} \right) + (\lambda + \mu) \left( \frac{\partial^2 U_x}{\partial x \partial z} + \frac{\partial^2 U_y}{\partial y \partial z} \right) &= \rho \frac{\partial^2 U_z}{\partial t^2}.
\end{align*}
\]

A rigorous mathematical solution of the obtained equations of motion with regard to the entire complex of real boundary and initial conditions in heterogeneous layers is apparently very difficult. To consider the main characteristics of elastic waves in an infinite medium, we assume that the displacement is parallel to the horizontal \( X \) axis (then \( U_z = 0 \)) and that \( U_z \) is independent of \( y \) and \( z \). Then system (5) is reduced to one equation
\[ \frac{\partial^2 U_x}{\partial t^2} = \frac{\lambda + 2\mu}{\rho} \frac{\partial^2 U_x}{\partial x^2}, \] (6)

which corresponds to the wave equation. Considering oscillations propagating in the positive direction, the dependence of the particle displacement can be represented in the form of an exponential law of change

\[ U(x) = U_0 e^{i\omega t - \frac{x}{a}}, \] (7)

where \( a = \sqrt{\frac{\lambda + 2\mu}{\rho}}. \)

Applying the method of separation of variables, a solution to equation (6) was obtained:

\[ U(x,t) = \sqrt{\frac{2}{l}} \sum_{n=1}^{\infty} \left[ a_n \cos \left( \frac{n\pi}{l} x + b_n \sin \frac{n\pi}{l} x \right) \right. \]

\[ \left. \sin \left( \frac{n\pi}{l} x \right) \right], \] (8)

where

\[ a_n = \sqrt{\frac{2}{l}} \int_0^l U_0(x) \sin \frac{n\pi x}{l} \, dx, \quad b_n = \sqrt{\frac{2l}{n\pi a_0}} \int_0^l U_1(x) \sin \frac{n\pi x}{l} \, dx, \]

and it satisfies the following initial and boundary conditions:

\[ U_{i=0} = U_0(x) = U_0 e^{-i\omega \frac{x}{a}}; \]
\[ U'_{i=0} = U_1(x) = -U_0 \frac{i\omega}{a} e^{-i\omega \frac{x}{a}}; \]
\[ U|_{x=0} = 0; \]
\[ U|_{x=l} = 0. \] (9)

The obtained equation (8) was analyzed for two types of rocks. For limestones, the following parameters [7] were taken: Young's modulus 2.45\times10^4 \text{ MPa}, Poisson's ratio 0.32, density 2.67 \text{ g/cm}^3. For sandstones, the parameters were as follows: Young's modulus 6.7\times10^3 \text{ MPa}, Poisson's ratio 0.33, density 2.84 \text{ g/cm}^3. The change in the oscillation amplitude was studied in arbitrary units, depending on the spatial characteristics and with no regard to the time dependence. Due to the fact that the analysis of this equation has two assumptions that do not allow a more qualitative description of the change in the oscillation parameters in the rock, it should be noted that an increase in the values of the pulse oscillation frequency leads to a significant decrease in the wave displacement amplitude. This trend was observed when exceeding the value \( \omega = 1 \text{ kHz}. \)

3. Results and Discussion

However, a more detailed analysis of the wave propagation showed that an infinite absorbing medium exhibits a number of related quantities characterizing the energy loss appear, such as the phase shift between stress and strain, the relative energy loss per period, the absorption coefficient, and the logarithmic decrement. All these quantities can be referred to as absorption parameters. The given absorption parameter requires two independent quantities to describe energy losses in an isotropic medium, which is similar to two elastic constants required to describe an ideally elastic isotropic medium. Two absorption parameters characterizing the plane wave propagation help interpret the behavior of waves in a thin rod or in a thin-layered medium with absorption, as well as in simple resonators [8]. The quantities obtained in different experiments can be compared by bringing them to the equivalent absorption parameters for plane waves. The behavior of waves in the rock structure,
which are caused by the injected fluid, reveals the relationship between absorption and the phase velocity of propagation of oscillations, which is valid if the absorbing medium satisfies the principle of causality. This enables calculating the average elastic constants and absorption parameters for the medium with absorbing linear inhomogeneities.

The need for a good physical explanation of wave damping gave rise to numerous works that reported various absorption mechanisms. In 1848, Stokes suggested that the compression of the absorbed material is purely elastic, while its displacement is accompanied by a viscosity similar to that of a liquid. This assumption leads to a quadratic dependence of the absorption coefficient on frequency in the low frequency range. However, many changes indicated a linear dependence of the absorption coefficient on frequency. Numerous researchers report that absorption is associated with dry friction, which, for instance, can accompany sliding in the contact area between grains. The concept of internal friction was proposed to characterize the property of a solid, which is expressed in the fact that the stress–strain diagram contains hysteresis. This model shows a linear dependence of absorption on frequency. A number of researchers show that the measured absorption can be explained by thermoelasticity, and an appropriately selected inhomogeneity in the medium can provide a satisfactory agreement with the experimental data on the dependence of absorption on frequency.

One of the well-known methods for accounting for absorption based on a linear wave equation was considered above. The stresses are assumed to be directly proportional to the rate of change in deformation and to the components of deformation itself. With regard to the initial statement, and the results of laboratory experiments confirm this with an acceptable error, we present the dependence of deformation and stress on the basis of the modified Hooke's law, which includes the rate of change in deformation [9].

\[
\begin{align*}
\sigma_\alpha &= \left(\lambda + 2\mu\right)\frac{\partial U_x}{\partial z} + \lambda \frac{\partial U_y}{\partial z} + \left(\lambda' + 2\mu'\right)\frac{\partial U_z}{\partial t} + \lambda' \frac{\partial U_x}{\partial t}, \\
\sigma_\beta &= \lambda' \frac{\partial U_x}{\partial t} + \left(\lambda + 2\mu\right)\frac{\partial U_y}{\partial t} + \lambda \frac{\partial U_z}{\partial t} + \left(\lambda' + 2\mu'\right)\frac{\partial U_x}{\partial t}, \\
\sigma_\gamma &= \lambda \frac{\partial U_x}{\partial t} + \lambda' \frac{\partial U_y}{\partial t} + \left(\lambda + 2\mu\right)\frac{\partial U_z}{\partial t} + \left(\lambda' + 2\mu'\right)\frac{\partial U_z}{\partial t}, \\
\sigma_\mu &= \mu \frac{\partial U_y}{\partial t} + \mu' \frac{\partial U_z}{\partial t}, \\
\sigma_\nu &= \mu \frac{\partial U_x}{\partial t} + \mu' \frac{\partial U_x}{\partial t}, \\
\sigma_\zeta &= \mu \frac{\partial U_x}{\partial t} + \mu' \frac{\partial U_x}{\partial t},
\end{align*}
\]

(10)

where \(\lambda', \mu'\) – parameters that characterize energy losses.

This system of equations corresponds to the system of equations (1) for an ideally elastic medium. Equations (4) that describe the equilibrium condition for an isotropic medium are valid. Consequently, we can make the corresponding equation of motion, which differs from (5) by the term depending on the deformation rate. The behavior of the longitudinal wave can be obtained using the corresponding analogue of equation (6), where \(U_x\) represents the only displacement component and the motion does not depend on the coordinates \(y\) and \(z\):

\[
(\lambda + 2\mu)\frac{\partial^2 U_x}{\partial x^2} + (\lambda' + 2\mu')\frac{\partial^2 U_x}{\partial t^2} = \rho \frac{\partial^2 U_x}{\partial t^2}.
\]

(11)

Since \(\lambda + 2\mu\) is the modulus of flat compression, it is necessary to introduce the following:
\[ M = \lambda + 2\mu, \quad M' = \lambda' + 2\mu'. \]  

Then, the transformed equation (11) has the form:

\[ M \frac{\partial^2 U}{\partial x^2} + M' \frac{\partial^2 U}{\partial t^2} = \rho \frac{\partial^2 U}{\partial t^2}. \]  

Considering that the time dependence of displacement has an exponential law of change, we can write:

\[ U(t) = U_0 e^{i\omega t}. \]  

Solve equation (13) by the method of separation of variables, and for each value of frequency \( \omega \) the function that depends on the spatial coordinate \( U(x, \omega) \) must satisfy the following equation [10]:

\[ (M + i\omega M') \frac{d^2 U}{dx^2} = -\rho \omega^2 U. \]  

The dependence on the spatial \( X \) coordinate can be represented as exponential:

\[ U(x, \omega) = U_0 e^{i\beta x}. \]  

Substitute equation (16) into equation (15), and after transformations we obtain:

\[ G = \sqrt{-\frac{\rho \omega^2}{M + i\omega M'}}. \]  

Taking into account the condition of causality, namely, that the absorption coefficient \( k \) and the phase velocity of propagation of oscillations \( c \) are related to each other, that is \( c \) should depend on frequency if \( k \) is not equal to zero, and so the absorbing medium must be dispersive, the complex value \( G \) can be expressed through the absorption coefficient and phase velocity:

\[ G = k + i\omega \frac{1}{c}. \]  

Solving equations (17) and (18) and taking into account the real and imaginary terms, we obtain equations that characterize the wave penetration parameters in the rock structure:

\[ k = \frac{\omega^2}{\omega_0 \sqrt{2 \left( \frac{M}{\rho} \right) \left( 1 + \frac{\omega^2}{\omega_0^2} \right) \sqrt{1 + \frac{\omega^2}{\omega_0^2} + 1}}}, \]  

\[ c = \sqrt{\frac{2 \left( \frac{M}{\rho} \right) \left( 1 + \frac{\omega^2}{\omega_0^2} \right)}{1 + \frac{\omega^2}{\omega_0^2} + 1}}, \]  

where \( \omega_0 = M/M' \).

4. Conclusion

Figures 1 and 2 present the dependencies of the phase velocity of propagation of oscillations and the absorption coefficient on the oscillation frequency calculated for the most common types of rocks [11]. These rocks are characterized by the following parameters [11]: shale (Young's modulus 2.1·10^3
MPa; Poisson's ratio 0.15; density 1.96 g/cm$^3$), limestone (2.5-10$^{-4}$ MPa; 0.32; 2.67 g/cm$^3$), sandstone (6.7-10$^{-4}$ MPa; 0.33; 2.84 g/cm$^3$).

**Figure 1.** Dependence of the phase velocity on the oscillation frequency (in dimensionless quantities)

**Figure 2.** Dependence of the absorption coefficient on the oscillation frequency (in dimensionless quantities)
As can be seen, the absorption coefficient attains its maximum values in the region of resonance frequencies, and further increase in the vibration frequency causes a slight decrease in the absorption coefficient. This pattern is characteristic of the observed types of rocks. In our opinion, the most optimal frequency range for the propagation of oscillations in the rock structure is 100–800 Hz with regard to the natural frequencies of the studied rocks, which is consistent with the conclusions reported in [12].

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