Research on Lateral Positioning Algorithm of Fixed Radiation Source by Mobile Single Station

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Abstract. Wireless communication and satellite navigation technology to promote passive positioning technology has become a hot topic in the field of electronic countermeasures. The passive positioning technology can determine the position of the radiation source under the condition that it does not generate radiation, and has the characteristics of long distance of action and strong anti-interference ability. In this paper, the theory of passive lateral positioning is discussed. In the three-dimensional space, a new algorithm for moving the single station to the lateral positioning and geometric combination of fixed radiation source is established, and the error analysis and simulation are carried out. The distance of the observation station is 300km, \( \Delta \beta = 0.06^\circ \), \( \Delta \epsilon = 0.01^\circ \), \( r = 30 \)m, and \( r' = 320 \)m can be calculated. That is, when the distance between the measurement target and the ground observation station is 300km, the system space positioning error is up to 320m. When the distance between the measurement target and the ground observation station is 100km, the system space positioning error is up to 115m. Therefore, the positioning accuracy of the calculation method can reach 0.12% of the distance. The experimental verification and result analysis show that the algorithm has good applicability in the field of lateral positioning of fixed radiation sources.

Keywords: Mobile single station; Fixed radiation source; Passive lateral positioning; Error analysis.

1. Introduction
The passive positioning system refers to passively receiving electromagnetic waves (including visible light and infrared) signals of the radiation source under the condition that it does not radiate electromagnetic waves, measuring various parameters thereof, and convincingly determining the target position and the motion state. Passive positioning technology can be divided into single station passive positioning technology and Lamentations passive positioning technology from the number of observation platforms. Traditional Lamentations systems require simultaneous work between stations and a large amount of data transmission. In addition, centralized data is required to be processed. This not only complicates the positioning system, but also limits the independence and mobility of the system. The single-station system just overcomes these shortcomings, and its good maneuverability and independence can ensure the task of positioning and tracking in a hidden manner. Single-station passive
positioning refers to the state of the target (including position, velocity, etc.) obtained by measuring the radiation information of the target radiation source through a single observation station. According to the conventional radar positioning principle, a single passive observer cannot measure the position of the radiation source, so the position of the radiation source is observable [1]. If the relative geometric position changes between the single station observer and the radiation source, the information data of the target can be obtained by multiple sequential measurements during the position change, and the position and motion state of the target can be estimated. In the three-dimensional space, the author introduces another single-station localization algorithm, which uses the lateral device to receive the angle of arrival of the electromagnetic wave emitted by the radiation source to be tested and the rate of change of the angle of arrival to achieve positioning. Because the algorithm also utilizes the angle of arrival change rate parameter compared with the POD angle parameter direction finding in the past, it is called the “single station passive location algorithm for the change angle of the angle of arrival of the fixed radiation source; At the same time, the error analysis and error simulation of the algorithm are carried out to verify the feasibility of the algorithm.

2. DOA change rate positioning principle
It can be known from the mass kinematics that in the case of single-station direction finding, in the case of relative motion between the direction-finding terminal and the radiation source to be tested, in addition to the magnitude of the direction of arrival, the motion of the direction finding platform can also be measured. The parameter obtains the rate of change of the angle of arrival, including the rate of change of azimuth and the rate of change of pitch. When performing aerial direction-finding reconnaissance, the reconnaissance machine measures faster, so that other fixed radiation sources have a relative change with respect to the direction-finding equipment over time, especially when the radiation source is at the same level as the direction-finding equipment. The change in the relative direction between the two is more obvious, and the rate of change in direction is also significantly increased [2]. The direction-finding device can utilize the characteristic of the relative direction change rate, that is, the degree of change of the angle of arrival, with the target radiation source, and combines the measured angle of arrival parameters and the position parameters of the target to achieve rapid target positioning. However, when the target radiation source and the on-board direction-finding device are in the same motion trajectory, the relative direction and angle between the two are not changed, that is, the change rate of the angle of arrival is zero. It can be seen that the method is not applicable at this time. Therefore, the method of locating the angle of change of the angle of arrival is not to locate the target at 360°, but to locate the target outside the trajectory of the airborne equipment.

3. Positioning model
As shown in Fig.1, the three-dimensional spatial arrival angle change rate positioning principle map, the initial position of the direction finding platform \( P(x_p, y_p, z_p) \) is at the coordinate origin \( O \), and the uniform linear motion is performed, and the speed is \( V(v_x, v_y, v_z) \) and \( E(x_e, y_e, z_e) \) is the passionless radiation source to be tested. The distance between the radiation source and the direction-finding platform is expressed as \( r = |P| = \sqrt{(x_e - x_p)^2 + (y_e - y_p)^2 + (z_e - z_p)^2} \), the sitting mark of the radiation source relative to the direction finding station is \( (x, y, z) = (x_e - x_p, y_e - y_p, z_e - z_p) \) due to the azimuth angle \( \beta \) and the pitch angle \( \varepsilon \) of the target relative to the direction finding platform. As the constant motion of the direction-finding platform changes, the rate of change of the two can be used to achieve the problem of the location of the radiation source to be measured [3-4].
As shown in Figure 1, the expression of azimuth $\beta$ is derived from the triangular relationship:

$$\beta = \arctan \frac{x - x_p}{y - y_p}$$

(1)

Derived on both sides of the above formula, and after finishing, get the expression of the azimuths rate of change:

$$\dot{\beta} = \frac{-\cos \beta + y \sin \beta}{r \cos \varepsilon}$$

(2)

Simplify the distance expression under the azimuths rate of change:

$$r = \frac{-\dot{x} \cos \beta + \dot{y} \sin \beta}{\dot{\beta} \cos \varepsilon}$$

(3)

Therefore, using the azimuth rate change rate, the coordinates of the target E at any time t are:

$$\begin{bmatrix}
  x_{et} = x_{pt} + r \sin \beta_i \cos \varepsilon_i = x_{pt} + \left(-\dot{x}_{pt} \cos \beta_i + \dot{y}_{pt} \sin \beta_i\right) \sin \beta_i / \dot{\beta}_i \\
y_{et} = y_{pt} + r \cos \beta_i \cos \varepsilon_i = y_{pt} + \left(-\dot{x}_{pt} \cos \beta_i + \dot{y}_{pt} \sin \beta_i\right) \cos \beta_i / \dot{\beta}_i
\end{bmatrix}$$

(4)

Similarly, the expression of the elevation angle $\varepsilon$ is derived from the triangular relationship:

$$\varepsilon = \arctan \frac{z_e - z_p}{d} = \arctan \frac{z_e - z_p}{\sqrt{(y_e - y_p)^2 + (x_e - x_p)^2}}$$

(5)

The two sides of the above formula are derived, and the expression of the rate of change of the pitch angle is obtained after finishing:
After simplification, the distance expression at the rate of change of the pitch angle is obtained:

\[
 r = -\frac{\dot{z}\cos + (\dot{x}\sin + \dot{y}\cos)\sin \varepsilon}{\dot{\varepsilon}}
\]

(7)

Therefore, using the pitch angle change rate to locate, the coordinates of the target E at any time are:

\[
 \begin{cases}
 x_{et} = x_{pt} + r \sin \beta \cos \varepsilon \\
 y_{et} = y_{pt} + r \cos \beta \cos \varepsilon \\
 z_{et} = z_{pt} + r \sin \beta \sin \varepsilon
\end{cases}
\]

(8)

4. Error analysis

4.1. Positioning error analysis

It is assumed that there is an error in the observation \(\dot{x}, \dot{y}, \dot{z}, \dot{\beta}, \dot{\varepsilon}, \dot{\varepsilon}\), in which the mean value of each measurement error of \(\dot{x}=-v_x, \dot{y}=-v_y, \dot{z}=-v_z\) is zero and independent of each other, and the error is \(\sigma_{v_x}, \sigma_{v_y}, \sigma_{v_z}, \sigma_{\dot{\beta}}, \sigma_{\dot{\varepsilon}}, \sigma_{\dot{\varepsilon}}\). The equations (3) and (7) are all deferentially differentiated by the above observations to obtain the following formula [5].

\[
 \sigma_r^2 = \left(\frac{\cos \beta}{\beta \cos \varepsilon}\right)^2 \sigma_{v_x}^2 + \left(\frac{\sin \beta}{\beta \cos \varepsilon}\right)^2 \sigma_{v_y}^2 + \left(\frac{\dot{x}\sin \beta + \dot{y}\cos \beta}{\dot{\beta}\cos \varepsilon}\right)^2 \sigma_{\dot{\beta}}^2
\]

\[
 + \left(-\frac{\dot{x}\cos \beta + \dot{y}\sin \beta}{\dot{\beta}\sin \varepsilon}\right)^2 \sigma_{\dot{\varepsilon}}^2 + \left(-\frac{\dot{x}\cos \beta + \dot{y}\sin \beta}{\dot{\beta}\cos \varepsilon}\right)^2 \sigma_{\dot{\varepsilon}}^2
\]

(9)

\[
 \sigma_r^2 = \left(\frac{\sin \beta \sin \varepsilon}{\dot{\varepsilon}}\right)^2 \sigma_{v_x}^2 + \left(\frac{\cos \beta \sin \varepsilon}{\dot{\varepsilon}}\right)^2 \sigma_{v_y}^2 + \left(\frac{\cos \varepsilon}{\dot{\varepsilon}}\right)^2 \sigma_{v_z}^2
\]

\[
 + \left(\frac{\dot{x}\cos \beta - \dot{y}\cos \beta}{\dot{\varepsilon}}\sin \varepsilon\right)^2 \sigma_{\dot{\beta}}^2 + \left(\frac{\dot{z}\sin \varepsilon + (\dot{x}\sin \beta + \dot{y}\cos \beta)\cos \varepsilon}{\dot{\varepsilon}}\right)^2 \sigma_{\dot{\varepsilon}}^2
\]

\[
 + \left(\frac{\dot{z}\cos \beta + (\dot{x}\sin \beta + \dot{y}\cos \beta)\sin \varepsilon}{\dot{\varepsilon}^2}\right)^2 \sigma_{\dot{\varepsilon}}^2
\]

(10)

It can be seen from equation (9) that when the denominator \(\dot{\beta}=0\) and \(\cos \varepsilon=0\) are \(\varepsilon=90^\circ\), the left side of the equation is infinite. It is indicated that when the azimuth rate is used for positioning, if the trajectory of the direction finding platform is in the direction of the radiation source to be tested and
the line connecting the direction finding platform and the radiation source is perpendicular to the horizontal plane, the positioning error is infinite. Large, this method cannot be used for positioning at this time.

It can be seen from equation (10) that the denominator \( \hat{e} = 0 \) has an infinity on the left side of the equation. There are three cases of \( \hat{e} = 0 \). One is that the trajectory of the direction finding platform is in the direction of the radiation source to be tested; the second is that the trajectory of the direction finding platform is on the horizontal plane where the radiation source is to be measured; The motion trajectory is on a tapered surface formed by connecting the two wires as a bus bar. It is indicated that when positioning with the rate of change of the pitch angle, the positioning error of the above three cases is infinite, which is not applicable to the method [6-7].

4.2. Positioning Error Simulation

Figure 2 shows the equivalent error simulation curve obtained by the direction finding platform using azimuth change rate and pitch rate change rate, the unit of value is km. The simulation condition is: the direction finding platform performs uniform linear motion along the x-axis, \( v = 200 \text{m/s} \), \( \sigma_p = \sigma_z = 10 \text{mrad} \), \( \sigma_\varphi = 0.1 \text{mrad/s} \), \( \sigma_\psi = \sigma_\chi = \sigma_\theta = 5 \text{m} \). The height of the direction finding device is \( z_e = 50 \text{km} \), and the observation area is -90km-90km in the x and y directions. The simulation condition of Fig. 2 is that the direction finding platform is located at the coordinate origin when positioning. The simulation condition of Fig. 2 is that the direction finding platform moves uniformly from the coordinate origin in the x-axis direction at the time of positioning by \( t = 100 \text{s} \) [8]. The simulation results of the error angle of the direction of change of the direction of arrival of the wave are as follows.

![Fig.2 Simulation diagram of the error curve of the direction change rate of the direction of arrival](image)

By comparing the above equal error curves obtained by using the azimuth rate change rate parameter and the pitch angle change rate parameter for the direction finding and positioning, the following conclusions are obtained: (1) the equal error under the influence of the azimuth rate change rate and the pitch angle change rate The graphs are all asymmetrical with respect to the position of the direction finding station, and the error changes using the two parameters are relatively stable and the positioning effect is good. (2) When performing a single measurement, the positioning error increases as the distance between the direction finding station and the target radiation source increases. (3) In comparison, the error of the direction finding change according to the azimuth change rate is smaller than the error of the direction change rate based on the rate of change of the pitch angle, and the positioning accuracy is high.
4.3. System space positioning error estimation

Using the radio ranging and direction finding system to obtain the distance value and the azimuth of the incoming wave at the same time and the ground observation station, and the elevation angle calculated by the altimeter to calculate the elevation angle, the mathematical model can be used for positioning. Calculate, the maximum value of the spatial positioning error caused by the error is $r$. Calculated according to the distance between the measurement target and the ground observation station of 300km, $\Delta \beta = 0.06^\circ$, $\Delta \gamma = 0.01^\circ$, $r = 30m$, $r = 320m$ can be calculated. That is, when the distance between the measurement target and the ground observation station is 300km, the system space positioning error is up to 320m. When the distance between the measurement target and the ground observation station is 100km, the system space positioning error is up to 115m. Therefore, the positioning accuracy of the calculation method can reach 0.12% of the distance.

5. Conclusion

In this paper, the positioning of mobile single station must meet the requirements of fast and stable tracking. A fast single-station to three-dimensional motion radio is proposed by rapidly measuring the azimuth, elevation angle of the target and the distance between the target and the fixed observation station. The cooperative high-precision positioning method of the radiation source adds a new solution to the engineering application for high-precision positioning. Through the detailed analysis of various factors affecting the measurement error in the commonly used radio ranging and angle measuring methods, the positioning accuracy that can be achieved by this method is calculated. According to the measurement test results of the actual positioning system, it can be seen that the accuracy of the high-speed moving target in the air can reach about 0.1% of the distance, which is far superior to the positioning accuracy achieved by other common positioning methods. The method can be applied to systems such as position measurement, track correction or landing guidance for moving targets in the air.

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