Electron-hole asymmetry and superconductivity

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In a solid, transport of electricity can occur via electrons or via holes. In the normal state no experiment can determine unambiguously whether the elementary mobile carriers have positive or negative charge. This is no longer true in the superconducting state: superconductors know the difference between electrons and holes. This indicates that electron-hole asymmetry plays a fundamental role in superconductivity, as proposed by the theory of hole superconductivity.

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I. INTRODUCTION

Imagine a parallel universe where the heavy proton is negatively charged and the light electron is positively charged. Solids in such a world will look the same as in ours. Solid state experimentalists would measure a positive Hall coefficient for Li and Au, and a negative Hall coefficient for Pb and Nb, a positive thermoelectric power for Na and a negative one for In. Imagine furthermore that in this world they are unable to do experiments with cathode ray tubes where the mobile charge carriers would escape from the solid and propagate in free space. How would experimentalists in this world be able to tell whether the elementary charge carriers that are actually moving when electricity flows through a metal have positive or negative charge?

The answer is, they would not if they can only do experiments in the normal state of the metals and the carriers never leave the solid. There is no experiment in the normal state that can determine unambiguously that the actual elementary particles that are moving in the metal in this parallel universe have positive charge, and that the heavy ions that are only rattling around their equilibrium positions have negative charge. All measurements would be equally consistent with the assumption that in some materials (Pb, Nb, In, V, MgB$_2$, YBa$_2$Cu$_3$O$_7$), and that the best conductors with positive Hall coefficient never become superconducting (eg Cu, Ag, Na, Ca). Furthermore, that the superconductors with the highest critical temperatures are usually very bad conductors in the normal state. This could be taken to suggest that the negative carriers in the normal state of these superconducting materials have the ‘wrong’ sign, and that by going superconducting the material somehow manages to avoid this situation and instead have the real mobile positive carriers do the conduction as in the ‘good’ normal state conductors Cu or Na. This hypothesis would be supported by the observation that as the temperature is raised and approaches $T_c$ many superconductors do show a positive Hall coefficient, that becomes negative again as $T$ is raised above $T_c$. However, theorists in this world would argue against this that first, the sign of the Hall coefficient has little to do with the sign of the charge carriers, and second, that the sign reversal of the Hall coefficient in the mixed state has to do with the complicated dynamics of vortex motion rather than with the sign of the charge carriers.

In summary, without cathode ray tubes or the photoelectric effect or thermionic emission or STM’s or any other experiment where the charge carrier is extracted from the solid it is impossible to determine unambiguously the sign of the mobile charge carriers in solids from experiments in the normal state, as well as from many experiments in the superconducting state. However, the observations discussed above suggest, even if they don’t prove, that in this parallel world the sign of the elementary charge carrier in metals is positive, as they suggest that in our world it is negative.

II. THE EXPERIMENTS

Remarkably, there are experiments in the superconducting state that can determine the sign of the elementary charge carriers in solids, as well as their mass, without the charge carriers ever leaving the solid.
A. The magnetic field of a rotating superconductor

Consider a cylindrical superconductor with its axis along the $z$ direction. If this superconductor is rotated with angular velocity $\vec{\omega}$ around its symmetry axis a magnetic field develops in its interior, given by:

$$\vec{B} = -\frac{2mc}{e} \vec{\omega}$$  \hspace{1cm} (1)

where $e$ is the charge of the mobile charge carrier, with its sign, $m$ its mass and $c$ the speed of light. For our world, where $e < 0$, it means that $\vec{B}$ is in the same direction as $\vec{\omega}$. In the parallel universe, $\vec{B}$ would point in direction opposite to $\vec{\omega}$.

Just in case in the parallel world they may use a different sign convention to define magnetic fields, let’s clarify operationally what this means: a test charge $q$ moving above the top surface of the rotating superconductor in the same direction as the surface right below it will feel a Lorenz force that points in if $q$ has the same sign as the mobile charge carriers in the superconductor, and points out if $q$ has opposite sign as the mobile charge carriers in the superconductor. In Figure 1, the force on the test charge is pointing our, so the test charge is positive if Figure 1 depicts our world, and negative if it depicts the parallel world. This experiment determines unambiguously the sign of the mobile charge carriers in the solid.

The magnetic field of a rotating superconductor has been measured for both conventional (Pb, Nb, Sn), and Hg) and high $T_c$ superconductors (Y Ba$_2$Cu$_3$O$_7$) superconductors. The relation Eq. (1) is found to hold, with the sign discussed above and $m$ the bare electron mass. The reason the magnetic field points in the direction discussed is understood qualitatively as follows: when the solid starts rotating the ions move and the superfluid electrons ‘lag behind’ in the region within a penetration depth of the surface; the result is a magnetic field pointing in the direction given by the sign of the charge of the rotating ions, i.e. parallel to the angular velocity of the solid in our world.

B. The gyromagnetic effect

A related effect results if a magnetic field is suddenly applied to a superconductor. Consider again a cylindrical superconductor with axis along the $z$ direction, at rest initially, and apply a magnetic field in the positive $z$ direction. The supercurrent that develops to nullify the magnetic field in the interior (Meissner effect) will have the negative electrons rotating with angular momentum pointing in the +z direction. For the total angular momentum of the cylinder to be unchanged, the cylinder will start rotating with angular momentum in the −z direction, as shown in Figure 2. Once again the result would be opposite if the elementary charge carriers giving rise to the supercurrent had positive charge. This experiment has been done in conventional superconductors and not, to our knowledge, in high $T_c$ superconductors.
is little doubt however that the result would be the same.

In summary, superconductors know for a fact that the elementary charges that carry the supercurrent are negative in our world. Normal metals instead exhibit properties consistent with both negative and positive charges being the charge carriers.

III. SUPERCONDUCTIVITY AND ELECTRON-HOLE ASYMMETRY

The existence of the (conventionally called) London field Eq. (1) in the interior of a rotating superconductor can be derived from the London equation

$$\vec{\nabla} \times \vec{v}_s = -\frac{e_s}{m_s c} \vec{B} \tag{2}$$

where $e_s$, $m_s$, and $\vec{v}_s$ are the charge, mass and velocity of the superfluid carriers, assuming that in the interior of the superconductor the superfluid is rotating at the same velocity as the lattice,

$$\vec{v}_s = \vec{\omega} \times \vec{r} \tag{3}$$

hence $\vec{\nabla} \times \vec{v}_s = 2\vec{\omega}$. The London penetration depth

$$\lambda = \left( \frac{m_s c^2}{4\pi n_s e_s^2} \right)^{1/2} \tag{4}$$

is obtained from Eq. (2) using the expression for the supercurrent $\vec{j}_s = n_s e_s \vec{v}_s$ together with Ampere’s law $\vec{\nabla} \times \vec{B} = (4\pi/c)\vec{j}_s$.

Conventional descriptions of superconductivity use electron-hole symmetric models, hence they cannot predict an effect as profoundly electron-hole asymmetric as the London field. In a model superconductor with hole carriers in the normal state, eg an attractive Hubbard model with the band almost full, in the London penetration depth Eq. (4) $n_s$, $e_s$ and $m_s$ refer to the density, charge and mass of superfluid hole carriers. Of course Eq. (4) is independent of the sign of $e_s$, but Eq. (2) from where it was derived is not.

So where is electron-hole symmetry broken in the London equations? Eq. (2) would still be valid in a hole description in terms of

$$\vec{v}_{s, \text{hole}} = -\vec{v}_{s, \text{electron}} \tag{5a}$$

$$e_{s, \text{hole}} = -e_{s, \text{electron}} \tag{5b}$$

however Eq. (3) would not hold for $\vec{v}_{s, \text{hole}}$. This is because Eq. (5a) is valid for the relative velocity of electrons and holes with respect to the crystal structure; instead, in writing Eq. (3) one is stating that $\vec{v}_s$ is the absolute superfluid velocity, independent of the velocity of the crystal structure. For the absolute velocity of electrons and holes, Eq. (5a) does not hold when the crystal is moving and hence Eq. (2) for holes does not hold.

![FIG. 3: Phenomenology of hole superconductivity. As the Fermi level goes up in the band, bare electrons become dressed holes. When holes pair it is as if locally the band becomes less full, hence holes undress and turn into electrons.](image)

It is also important to note that the mass entering the London field Eq. (1) is experimentally determined to be the bare electron mass. Instead in the expression for the London penetration depth Eq. (4), $m_s$ is conventionally interpreted to be the effective mass of the hole carrier in the normal state, which is usually larger than the bare electron mass due to ‘dressing’ effects from electron-electron and electron-lattice interactions.

So the puzzle is then: how do dressed hole carriers in the normal state with a positive charge and a large effective mass become undressed electron carriers in the superconducting state with a small (bare) mass? An answer is provided by the theory of hole superconductivity.

IV. RELATION WITH THE THEORY OF HOLE SUPERCONDUCTIVITY

The theory of hole superconductivity proposes that superconductivity originates in the fundamental asymmetry between electrons and holes in condensed matter. It points out that in electronic energy bands carriers are undressed and light when the Fermi level is close to the top of the band (electrons) and high kinetic energy when they are at the bottom of the band (holes), as shown schematically in Figure 3. Furthermore the carriers have low kinetic energy when they are at the top of the band (electrons) and high kinetic energy when they are at the top of the band (holes). The energetics that drives superconductivity is kinetic energy lowering, or equivalently effective mass reduction, of the hole carriers when they pair. When holes pair, the local density of holes increases, hence the band becomes locally less full and the carriers become more electron-like and less dressed. In calculating the London penetration depth in the theory, it is found that the effective mass that enters is smaller than the large effective mass of the hole carriers in the normal state, hence the London penetration depth is smaller than expected from the...
normal state effective mass. This in turn gives rise to low frequency optical sum rule violation\cite{18} and to color change\cite{19}: as the effective mass in the superconducting state is smaller optical spectral weight is transferred down from high frequencies to the $\delta$–function that determines the London penetration depth. These effects have recently been seen experimentally\cite{20,21}. The fact that the carriers that carry the supercurrent near the surface are electrons rather than holes implied by Eq. (1) is also predicted by the theory\cite{22}.

Thus the theory of hole superconductivity in its present formulation conservatively asserts that dressed hole carriers partially undress, their effective mass decreases and they partially turn into electrons when they become superconducting. Nature is bolder than that: experiments that measure Eq. (1) tell us that this happens all the way, the dressed hole carriers become undressed free electrons, with the free electron mass and charge, in the superfluid state.

V. DISCUSSION

In contemporary solid state physics there is a common practice, especially among many-body theorists, to regard electrons and holes as equivalent quasiparticles and hence to use electron-hole symmetric Hamiltonians to describe physical systems. When a particular model or a particular situation does not accommodate that expectation it is often stated that such model or situation ‘violates’ electron-hole symmetry, as if that would be a bad or unphysical or at least an unusual thing to do. The experiments and observations discussed above however underline the fact that there is a profound asymmetry between electrons and holes in condensed matter. The facts that superconductors in the normal state exhibit almost always dominantly hole carrier transport\cite{23,24,25}, and that electron-hole asymmetry shows up clearly and unambiguously in the superconducting state, suggest that there is an intimate relationship between electron-hole asymmetry and superconductivity. The theory of hole superconductivity rests on the proposition that electron-hole asymmetry is the key to superconductivity\cite{26}.

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