From Chirps to Random-FM Excitations in Pulse Compression Ultrasound Systems

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Abstract—Pulse compression is often practiced in ultrasound Non Destructive Testing (NDT) systems using chirps. However, chirps are inadequate for setups where multiple probes need to operate concurrently in Multiple Input Multiple Output (MIMO) arrangements. Conversely, many coded excitation systems designed for MIMO miss some chirp advantages (constant envelope excitation, easiness of bandwidth control, etc.) and may not be easily implemented on hardware originally conceived for chirp excitations. Here, we propose a system based on random-FM excitations, capable of enabling MIMO with minimal changes with respect to a chirp-based setup. Following recent results, we show that random-FM excitations retain many advantages of chirps and provide the ability to frequency-shape the excitations matching the transducers features.

I. INTRODUCTION

Many ultrasonic frameworks for Non Destructive Testing (NDT) rely on pulse compression to cope with highly dissipative materials or with setups where large dissipation is encountered at the probe interfaces [1]. The approach extends the excitation time-bandwidth product to enhance the echo SNR (eSNR) without sacrificing resolution at the cost of some signal processing typically practiced by matched filters [2].

In order to maximize the eSNR one should feed the Material Under Test (MUT) with a large acoustic energy. At the same time, loading the probes or exceeding acoustic pressure limits, even transiently, could result in distortion. These requirements make constant envelope excitations an attractive feature. Consequently, chirps are a widely adopted large-bandwidth excitation. Chirps also have the advantage of easiness of bandwidth control. For a linear, slow chirp, the Power Density Spectrum (PDS) is almost uniform within the two extreme frequencies and almost null outside. This makes it straightforward to carefully match the probes bandwidth.

Nonetheless, chirps cannot be considered a universal solution. In some setups, it is desirable to rely on multiple transmitting probes that are simultaneously operated (e.g., to reduce test time, or to deal with situations where the MUT moves as in some automated testing facilities.). Wide-band excitations such as those needed for pulse compression offer a natural opportunity to create families of excitations where the members are highly orthogonal and thus suited for Multiple Input Multiple Output (MIMO) operation. This code-division approach sets a parallel with some tele-communication schemes [3]. Yet, chirps are clearly not immediately suited for this approach.

Thus, different types of excitations have been developed. Coded excitation are often based on Pseudo Noise (PN) sequences (m-sequences, Kasami, Gold code families, etc.) which have proved quite fruitful since the seminal work of Golomb [4]. Discrete-valued (often binary) sequences are either used directly or upconverted/modulated into ultrasound excitations [5]. Good δ-like autocorrelation properties guarantee good time resolution. Extension in time assures that a sufficient amount of energy is pushed into the MUT. Finally, low cross-correlation assures reduced mutual interference. In spite of these advantages, pitfalls exist. For instance, modulated sequences often have a non-constant envelope. Thus, to fit inside the maximum power range of the probes, one ends up with an average power lower than the maximum one, wasting power conversion ability. Secondly, difficulties may emerge in bandwidth control and in containing energy inside the probe bandwidth. Finally, this excitations may require a rather different hardware than chirps, where an FM block fed by a ramp is often all that is needed at the transmitters.

Here, we propose a solution that may be seen as a bridge between chirp-based excitations and coded excitations. It is based on the FM modulation of Pulse Amplitude Modulated (PAM) sequences so that hardware changes with respect to chirp based systems can be minimal and constant envelope operation can be guaranteed. It offers sufficiently good cross-correlation properties for MIMO. It can be extended to differentiate excitations based on discrete codes.

It provides an easy adjustment to the probe bandwidth. Furthermore, it has advantages of its own, like the possibility to carefully shape the PDS, following [6]. Similarly to nonlinear chirps [7] this lets one equalize the probe response, to enhance the actual bandwidth (and thus resolution) or transferred power (and thus eSNR) [8].

II. RANDOM-FM EXCITATIONS

A. Background on chirps and random-FM signals

A chirp is defined by the following modulation

\[ s(t) = \Re \left( e^{2\pi i (f_0 t + \Delta f \int_{-\infty}^{t} x(r) \, dr)} \right) \]  

where \( A \) is the envelope amplitude, \( f_0 \) is the central frequency, \( \Delta f \) is the maximum frequency deviation from \( f_0 \), and \( x(t) \) is a

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monotonically increasing, smooth modulating signal taking values in \([-1, 1]\). The simplest case is evidently that of a linear chirp in which \(x(t) = -1 + 2t/T_c\) where \(T_c\) is the chirp length. As long as \(T_c\) is sufficiently large with respect to \(1/\Delta f\) (i.e., the modulation is slow enough), the energy of the linear chirp is almost uniformly distributed in \([f_0 - \Delta f, f_0 + \Delta f]\) and almost null elsewhere, which makes it an effective excitation.

Fig. 1a shows a linear chirp (top), together with the modulus of its acyclic auto-correlation (middle), that illustrates the suitability for matched filter schemes, having a well localized central pulse and being rapidly vanishes elsewhere. Clearly, with this excitation it is impossible to have arbitrary many transmitters simultaneously operated. The only family of quasi-orthogonal functions that can be derived from it is made just of the chirp and its reversed waveform. The modulus of their cross-correlation (accounting for the mutual interference in a matched filter based receiver) is shown at the bottom of Fig. 1a.

Random-FM excitations can be generated with the same modulator as chirps, given in Eqn. (1), yet using a more articulated PAM modulating waveform

\[
x(t) = \sum_{k=-\infty}^{+\infty} x_k g(t - kT_c)
\]

where \(g(t)\) is a unit pulse of duration \(T_c\) and the values \(x_k \in [-1, +1]\) make up a random modulating sequence.

Fig. 1b makes the parallel of Fig. 1a for a sample random-FM excitation. Any detailed comparison must be postponed until all the parameters influencing the modulation have been introduced (also due to the rather artificial values used for representation). Nonetheless, it is already possible to see that arbitrary couples of random-FM excitations have relatively low cross-correlations, since the PAM sequence acts as a signature differentiating them. This makes random-FM suited for MIMO.

A thorough theory of random-FM signals has been developed in [6]. Particularly, it is proved that the PDS of an (infinitely long) modulated signal depends on the Probability Density Function (PDF) of the modulating sequence according to

\[
\Phi_{ss}(f) = \int_{-1}^{1} K_1(x, f - f_0)\rho(x) dx + \text{Re} \left( \frac{(\int_{-1}^{1} K_2(x, f - f_0)\rho(x) dx)^2}{1 - \int_{-1}^{1} K_3(x, f - f_0)\rho(x) dx} \right)
\]

(3)

where \(\Phi_{ss}(f)\) is the PDS, the modulating sequence is assumed to be made of independent samples, and \(\rho(x)\) is its PDF. Kernels \(K_1(x, f), K_2(x, f), K_3(x, f)\) are defined as

\[
K_1(x, f) = \frac{1}{2} T_c \sin^2(\pi T_c (f - \Delta f x))
\]

\[
K_2(x, f) = e^{-i2\pi T_c(f - \Delta f x)} - 1
\]

\[
K_3(x, f) = e^{-i2\pi T_c(f - \Delta f x)}
\]

(4)

Complicated as it may seem, the relationship can be interpreted as a (nonlinear) smoothing and leaking operator so that the \(\Phi_{ss}(f)\) tends to be shaped as \(\rho(x)\) with some distortion. For certain parameter values, the relationship can be greatly simplified since the following asymptotic tie holds:

\[
\lim_{T_c \to \infty} \Phi_{ss}(f) = \frac{1}{2\Delta f} \rho \left( \frac{f - f_0}{\Delta f} \right)
\]

(5)

Such tie, where the PDS accurately copies the PDF, is also approximately valid for finite but large \(T_c\), namely when the PAM signal has a slow update frequency \(f_s = 1/T_c\). This case will be indicated as a slow modulation. Whether \(T_c\) makes the modulation slow or fast depends on its magnitude relative to \(1/\Delta f\). Thus, it is convenient to define a modulation index \(m = T_c/\Delta f\) as long as \(m\) is a few units or more, (5) provides a satisfactory approximation. Interestingly, using the tie in Eqn. (3) one can design a modulating sequence PDF capable of producing a desired PDS. The inversion can be practiced directly through (5) for the slow case or by iterative methods (and reduced accuracy) for the fast case [9].

### B. Applicability of random-FM signals to ultrasound NDT

Considering the applicability of random-FM signal to pulse compression, a few items deserve attention.

- **Auto-correlation away from the peak.** These correlation entries contribute to the noise floor at the receiving probes as self-interference, which may hide the detection of secondary echoes. From Fig. 1, it is evident that the auto-correlation is higher for the random-FM excitation than for the chirp, which may appear problematic. However, a few aspects are worth considering. First, the auto-correlation of the chirp keeps decreasing at a significant rate even at large lags. Conversely, the random-FM one only decreases rapidly for small lags, then it flattens. Thus, the best auto-correlation entries (large lags) are certainly much better for the chirp, yet this does not necessarily mean that the random-FM excitation is much worse in the worst case (small lags). Secondly, the higher auto-correlation profile is the price that one pays for the MIMO abilities. Indeed, other coded excitations suitable for MIMO also pay a price in auto-correlation with respect to the chirp [5]. Furthermore, in a MIMO setup, there is not just self-interference,
but also mutual-interference that can be visualized through the cross-correlation entries. Having a self-interference much lower than the mutual-interference would only bring marginal advantages since the latter would dominate. Consequently, a fair evaluation of the random-FM auto-correlation curve also requires a comparison to the cross-correlation curves. Since the auto-correlation floor of the random-FM excitation is no worse than its cross-correlation floor or the cross-correlation floor of the chirp, one may conclude that the auto-correlation floor is actually good enough since any improvement would only bring marginal advantages in a MIMO setup.

- **Correlation dependency on system parameters.** Even if a complete discussion of this aspect is out of the scope of this paper, it is worth noticing that (as expectable) the correlation floors of random-FM modulations scale (in amplitude) with $1/\sqrt{TB}$ where $TB$ is the time-bandwidth product of the excitation, approximately $2T_\Delta f$. Incidentally, the same relationship holds for the cross-correlation floor of PN sequences such as Gold or Kasami. This provides some ability to tune the noise levels due to self- and mutual-interference to the application needs.

- **Secondary peaks in the correlation curves.** In Fig. 1, the correlation plots of the random-FM excitation appear irregular and peaky compared to those of the chirp. Secondary peaks are dangerous as they might be misinterpreted for weak echos from other reflectors at the receiving probes. Focusing on the auto-correlation curve, an intuitive investigation of their origin is possible. Auto-correlation is tied to PDS by the Fourier transform. The PDS of random-FM signals, as returned by Eqn. (3) is, by the very properties of the involved operators, quite regular. The corresponding auto-correlation should necessarily be smooth. Thus, the peakiness in Fig. 1b (middle) is not intrinsic in the random-FM waveforms. Its origin gets evident considering that Eqn. (3) holds for infinitely long signals. Peakiness emerges as a short-length effect since the number of PAM pulses used to build the excitation is limited, so that the value distribution of the modulating sequence can significantly differ from the prescribed PDF. Similar considerations could hold for the peaks in the cross-correlations. Thus, the secondary peaks can be reduced by enlarging $T_c/\Delta f$. Being $T_c = m/\Delta f$, peaks can also be seen as a consequence of low $(T_c/\Delta f)/m$. Therefore, large $m$ values, that from Section II-A may appear convenient for spectral control, can actually be undesirable. Note that in Fig. 1b the peakiness is accentuated by the choice of very short excitations (25 µs) and relatively large $m$ (1), used for representation purposes.

### C. Specific advantages of random-FM excitations

Established the applicability of random-FM excitations to NDT (together with the main limits in the choice of parameters), the specific pros of random-FM excitation can finally be considered. Obviously, there is a first advantage in using the same modulation type as chirps, since the implementation can thus be based on quite similar hardware. Yet, the major asset of random-FM modulations comes from the fine spectral shaping that can be practiced through Eqns. (3) and (5).

To appreciate this point, one should consider that the excitations are passed to/from the MUT through probes and amplifiers that are typically characterized by non-flat frequency responses. For instance, Fig. 2a illustrates the overall amplitude response of a 5 MHz probe system, as measured on field. If an excitation with a PDS uniform in the 1 to 9 MHz range is adopted (shown in 2b), about 20% of the excitation bandwidth and about 43% of the excitation power is lost in the probes.

![Figure 2](image_url)

**Figure 2.** Overall normalized magnitude response of the probe system (a) and sample excitations power distributions: uniform (b); seconding the probe response (c); and equalizing the probe response (d). Data on ordinate $\times 10^{-7}$.

Depending on the particular application setup, one may want a different trade-off between the lost power and the lost bandwidth:

1) If the eSNR is critical and the noise is dominated by components that do not scale with the excitation power (i.e., components that are not self/mutual interference due to the transmitters, nor dispersion noise due to the intrinsic structure of the material, rather effects like thermal or quantization noise), reducing the power loss on the probes may be beneficial. Having the possibility to shape the excitation PDS, this result can be achieved by picking a PDS seconding the probe response, as shown in Fig. 2c. This choice reduces the power loss to just 23% with a minimal reduction in bandwidth.

2) Conversely, if the eSNR is non critical, one may want to maximize the effective bandwidth. This can be done by picking a PDS equalizing the probe response, as shown in Fig. 2d. This choice brings the actual excitation bandwidth up to the original 8 MHz, but augments the power loss to 77%.

It is worth recalling that a spectral shaping such as that in plots (c) and (d) cannot be practiced starting with a uniform-PDS excitation and applying a linear filter, because the filter would make the envelope of the excitation non-constant. Conversely, random-FM signals can obtain the desired power distribution quite simply by picking a modulating PAM sequence with a PDF shaped as the desired spectrum. Indeed, being the required spectrum (be it seconding or equalizing the probe response) typically smooth, either slow or fast modulations can generally achieve it reasonably well. Incidentally, note that in a recent past, a similar type of spectrum shaping has been attempted (following the scenario at point 1), with non-linear chirps [8].

As a last consideration, note that for the case of sufficiently fast modulations, the smoothing properties of the operator in Eqn. (3) let one approximate reasonably well the desired spectra using discrete-valued PAM sequences. As an example, Fig. 3 shows the probability distribution required for a 16-levels PAM modulation in order to obtain, at $m = 4$, a random-FM signal with a PDS seconding the probe response. Pane (a) shows the probabilities, while pane (b) shows the achieved PDS together with the desired one.
with the desired one (dashed), identical to that in Fig. 2c.

III. EXPERIMENTAL EVALUATION AND CONCLUSIONS

The correlation properties and spectrum-shaping capability of random-FM excitation have been experimentally validated by using a measurement set-up composed by: (i) two broadband ultrasonic probes Olympus V109-RB centered at 5 MHz; (ii) a National Instrument PXI-5412 Arbitrary Waveform Generator with generation rate up to $100 \text{Msamples/s}$ and 14-bit of resolution; (iii) a 14-bit National Instrument PXIe-5122 Analog-to-Digital Converter with maximum sampling rate of $100 \text{Msamples/s}$. The two probes, one used as transmitter, the other as receiver, have been arranged in through-transmission on a thin aluminum plate in order to characterized the overall transmission channel. The transmitting probe has been excited with a random-FM signal in compliance with Eqn. (1) and (2) and characterized by the following parameters $T_e=1\text{ms}$; $f_0=5\text{MHz}$; $\Delta f=4\text{MHz}$; amplitude $A=6\text{V}$; $m=8$. Signals have been digitally generated and acquired at a sampling rate of $100 \text{Msamples/s}$. Experimental data is summarized in Fig. 4.

Acquired signals at the receiving probes (top), corresponding spectra (middle) and retrieved auto-correlation functions (bottom) are illustrated for the two cases of random-FM modulation finalized at equalizing the probe response or seconding it. The correspondence between numerical simulations and experimental data is quite good and supports the possibility of exploiting random-FM signals in NDT ultrasonic MIMO systems. Specifically, the ability of the excitation seconding the probe response to enhance the received power is quite visible. Conversely, at least in the tested setup, the equalization of the probe response does not appear particularly advantageous. The features of the auto- and cross- correlation functions, with specific regard to the noise floor level, make this approach suitable in applications where single echoes have to be detected by each transmitter-receiver pair, such as ultrasonic localization, time of flight measurement, etc., and several pairs are needed. On the other hand, in those applications that require a high ratio between the main lobe of the auto-correlation and the noise floor, such as certain defect-detection systems, chirp signals may still be preferable. Strategies to reduce the noise floor are currently under investigation, involving a deep analysis of the relations between modulating function features, time-frequency behavior of signals and noise floor levels.

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