H\textsc{i} CONTENT AND OPTICAL PROPERTIES OF FIELD GALAXIES FROM THE ALFALFA SURVEY. II. 
MULTIVARIATE ANALYSIS OF A GALAXY SAMPLE IN LOW-DENSITY ENVIRONMENTS

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ABSTRACT

This is the second paper of two reporting results from a study of the H\textsc{i} content and stellar properties of nearby galaxies detected by the Arecibo Legacy Fast ALFA (ALFALFA) Survey blind 21 cm line survey and the Sloan Digital Sky Survey in a 2160 deg\textsuperscript{2} region of high galactic latitude sky covered by both surveys, in the general Virgo direction. Here, we analyze a complete H\textsc{i} flux-limited subset of 1624 objects with homogeneously measured 21 cm and multi-wavelength optical attributes extracted from the control sample of H\textsc{i} emitters in environments of low local galactic density assembled by Toribio et al. (Paper I). Strategies of multivariate data analysis are applied to this data set in order to (i) investigate the correlation structure of the space defined by an extensive set of potentially independent observables describing gas-rich systems, (ii) identify the intrinsic parameters that best define their neutral gas content, and (iii) explore the scaling relations arising from the joint distributions of the quantities most strongly correlated with the H\textsc{i} mass. The principal component analysis performed over a set of five galaxy properties reveals that they are strongly interrelated, supporting previous claims that nearby H\textsc{i} emitters show a high degree of correlation. The best predictors for the expected value of $M_{\text{H}\text{i}}$ are the diameter of the stellar disk, $D_{25,\text{r}}$, followed by the total luminosity (both in the $r$ band), and the maximum rotation speed, while morphological proxies such as color show only a moderately strong correlation with the gaseous content attenuated by observational error. Among the various inferred prescriptions, the simplest and most accurate is $\log(M_{\text{H}\text{i}}/M_{\odot}) = 8.72 + 1.25 \log(D_{25,\text{r}}/\text{kpc})$. We find a slope of $-8.2 \pm 0.5$ for the relation between optical magnitude and log rotation speed, in good agreement with Tully–Fisher studies, as well as a log slope of $1.55 \pm 0.06$ for the H\textsc{i} mass–optical galaxy size relation. Given the homogeneity of the measurements and the completeness of our data set, the latter outcome suggests that the constancy of the average (hybrid) H\textsc{i} surface density advocated by some authors for the spiral population is just a crude approximation.

Key words: galaxies: fundamental parameters – galaxies: spiral – methods: statistical – radio lines: galaxies – surveys

Online-only material: color figures

1. INTRODUCTION

While the literature abounds with attempts of improving our knowledge about the formation and evolution of galaxies from the cross-correlation of the main properties of their baryonic components (e.g., Gavazzi et al. 1996; Rosenberg et al. 2005; Garcia-Appadoo et al. 2009, to name a few representative examples), the possibility of using this sort of relationships to set reference standards for the H\textsc{i} content has received comparatively less attention. Early comparisons of the neutral hydrogen abundance between Virgo cluster and field galaxies put the emphasis on using distance-independent measures, such as the $M_{\text{H}\text{i}}/L$ and $M_{\text{H}\text{i}}/D^2$ ratios (e.g., Davies & Lewis 1973; Chamaaux et al. 1980), with $L$ and $D$ being, respectively, the optical luminosity and intrinsic linear diameter at a certain wavelength. Haynes & Giovanelli (1984; hereafter HG84) were the first both to carry out an objective evaluation of the performance of different diagnostic tools for the H\textsc{i} content and to provide a rigorous operational definition of this quantity. With the help of a control sample of 288 galaxies with 21 cm line emission belonging to the Catalog of Isolated Galaxies (CIG; Karachentseva 1973), these authors demonstrated that, whatever the Hubble type, the optical linear diameter is the most important diagnostic tool for the H\textsc{i} mass of galaxies. New expressions for the standards of H\textsc{i} content according to HG84’s definition were later derived in an unbiased way by Solanes et al. (1996) from a larger, integrated H\textsc{i} flux-limited sample of 532 galaxies from the Catalog of Galaxies and Clusters of Galaxies (CGCG; Zwicky et al. 1961–1968) located in the lowest density environments of the Pisces–Perseus supercluster region.

One serious limitation of these studies, carried out at a time when wide-field redshift surveys were still in their infancy, is that they had to deal with heterogeneous data sets of optically selected targets assembled from incomplete catalogs and affected by complex sampling biases that undermined the validity of the results. In this paper and the accompanying one (Toribio et al. 2011; hereafter Paper I), we conduct a systematic analysis of the main structural properties of galaxies selected according to their H\textsc{i}-line emission, which in terms of both sampling quality and statistics represents a significant improvement with respect to earlier studies of this kind. Our study grows out from the combination of data from two large surveys that homogeneously map the distribution of extragalactic sources over a significant fraction of the local universe. These are a compilation of all the data from the ongoing Arecibo Legacy Fast ALFA (ALFALFA) Survey blind 21 cm line survey (Giovanelli et al. 2005) gathered so far in the northern Galactic hemisphere, which contains H\textsc{i} measurements distributed in two separate regions of the
high Galactic latitude sky that cover a z-space volume of about 2160 deg$^2 \times 18,000$ km s$^{-1}$, and the Sloan Digital Sky Survey Data Release Seventh (SDSS DR7; Abazajian et al. 2009), which is complemented with additional data from the New York University Value-Added Galaxy Catalog (NYU-VAGC; Blanton et al. 2005).

In Paper I, we deal with the assembly of control samples of ALFALFA galaxies that are expected to show little or no evidence of interaction with their surroundings and, therefore, that are suitable for providing absolute measures of the H$\text{I}$ mass. According to the results of this study, the optimal data set to set up standards for the neutral gas content of galaxies is a sample of 5647 H$\text{I}$ emitters found in regions of low local galactic density, as defined by a nearest-neighbor approach. A complete 21 cm flux-limited subset of this control sample will be used here (Section 2) with the aim of exploring inter-variable linear correlations and determining the combinations of intrinsic properties that best define the H$\text{I}$ mass of gas-rich objects. In the remainder of the present paper, we will show first that the galactic stellar size, luminosity, rotation speed, and to a lesser extent the color are the intrinsic factors most closely related to the H$\text{I}$ mass (Section 3). Then, we will apply strategies of non-parametric multivariate data analysis to these variables in order to determine their correlation structure and the number and orientation of the statistically significant principal components (Section 4.1), establish standards of normalcy for the H$\text{I}$ content of galaxies (Section 4.2), and examine the constraints that can be inferred from these relationships on the most firmly established empirical scaling laws for disk galaxies (Section 4.3). Appendix A briefly discusses two statistical tests implemented in order to assess the completeness limits of the ALFALFA data as a function of the integrated H$\text{I}$ flux.

The luminosity distances needed to calculate the various distance-dependent structural properties used in this investigation have been inferred within the framework of the standard concordant flat ΛCDM cosmology with a reduced Hubble constant $h = H_0/(100$ km s$^{-1}$ Mpc$^{-1}) = 0.7$.

2. SAMPLE SELECTION

We use a trimmed version of the low-density environment (LDE) H$\text{I}$ galaxy sample assembled in Paper I. The original LDE sample consists of 5647 reliable ALFALFA detections with a signal-to-noise ratio (S/N) $> 4.5$ that have an optical counterpart in the SDSS catalog and that inhabit regions of low local galactic density ($\rho_0 \leq 0.5$ h$^3$ galaxies Mpc$^{-3}$; see Paper I for details). In order to deal with data of the highest completeness and quality, the present analysis has been restricted, however, to those H$\text{I}$ sources designated code 1, i.e., with an S/N $> 6.5$, a clean spectral profile, and a good match between the two polarizations independently observed by ALFALFA, for which it can be assumed that the completeness limit is well represented by the detection limit of the survey. Tests on the performance of the signal detection pipeline made by Saintonge (2007) estimate a reliability close to 100% for ALFALFA objects above the prescribed S/N threshold and an overall completeness approaching $\sim 90\%$ for those with a narrow observed velocity width $(< 150$ km s$^{-1})$. In contrast, for the few code 2 sources originally included in the LDE data set, which have $4 < S/N < 6.5$, the reliability for detections of narrow signal is reduced to values near 60% and the overall completeness to $\sim 70\%$.

Likewise, we have removed from the parent LDE sample those galaxies located beyond 15,000 km s$^{-1}$, where the ALFALFA survey detection ability drops significantly due to a strong radio frequency interference (RFI) signal from the San Juan airport’s radar. (We remind the reader that the original LDE data set is already confined to objects beyond 2000 km s$^{-1}$ in order to eliminate the sources with the most uncertain distances and, in particular, a great deal of the kinematic influence of the Virgo cluster.)

The large size of the LDE sample has allowed us to be also demanding when selecting galaxies with favorable inclination angles. Thus, we have taken into account that at low apparent inclinations, orientation estimates become more uncertain and are skewed toward larger values by nonaxisymmetric features in the images, ultimately leading to deprojected properties with divergent errors as a face-on orientation is approached, an issue well known in Tully–Fisher (TF) studies (e.g., Toribio & Solanes 2009). Furthermore, the extinction and reddening corrections that must be applied to optical observables are stronger and less reliable for nearly edge-on objects. Accordingly, we have chosen to restrict the present analysis to the 3032 LDE galaxies that, in addition to verifying the above conditions, have also an isophotal r-band axial ratio $0.85 \geq (b/a) \geq 0.2$ (equivalent to $30^\circ \leq i \leq 80^\circ$ for disks of negligible thickness). A detailed analysis of the inclination dependence of the different relationships examined in this work corroborates that all the correlation coefficients vary little within this range of $b/a$. Furthermore, we have attempted to reduce as much as possible the impact of possible outliers on the inferred relationships, which could be significant especially for low-H$\text{I}$-mass objects, by excluding galaxies with highly inaccurate SDSS measurements. Although the restrictions on the axial ratio already discard most of the potential optical outliers, an additional $\sim 7\%$ of the remaining sources were eliminated for this reason.

Last but not least is the fact that ALFALFA is a noise-limited survey, with a sensitivity that depends on the observed source’s H$\text{I}$ line width, while the correlations that we want to study ideally ought to be inferred from a volume-limited sample. To sidestep the natural bias of blind 21 cm surveys against sources with low fluxes and large velocity widths, we will be dealing with a subset of the LDE sample that includes objects brighter than a stringent integrated H$\text{I}$ flux $\geq 1.3$ Jy km s$^{-1}$, for which the survey can be considered complete in a statistical sense regardless of line width (see Appendix A). Each galaxy in this restricted data set will be weighted by the inverse of the maximum effective volume, $V_{\text{max}}$, in which it should, on average, have been detected. To calculate the latter, we have taken into account not just the observed integrated flux of the source, its distance, and the adopted sensitivity limit, but also the presence of large-scale structure in the surveyed volume and the loss of signal that result from man-made RFI, which alter locally the survey completeness and therefore the detection probability of the sources. In the remainder of this paper, we will refer to this selection as our H$\text{I}$ flux-limited LDE “high-quality” (LDE-HQ) galaxy sample, which totals 1624 gas-rich objects.

3. PARAMETER SELECTION

Central to this work is the search of dependences between the neutral gas mass content and other intrinsic properties of H$\text{I}$ emitters. We have selected the largest possible number of observational parameters that, besides being suitable for characterizing gas-rich objects, are inferred from good quality measurements and either publicly available or easy to compute.
3.1. Available Parameters

In our quest for the most relevant galaxian properties that define the H\textsc{i} content, we first review the extensive set of radio and optical parameters that can be inferred from the observables listed by the ALFALFA and SDSS DR7 and NYU-VAGC catalogs, which in some cases provide different estimates of the same attribute (e.g., Petrosian and model magnitudes to measure brightness, isophotal diameter, and Petrosian radius to measure the angular size, and so on). After a first screening of all the possible variables available, paying attention to factors such as the relative size of the observational errors, as well as the suitability and robustness of the measurements for the kind of galaxies under scrutiny (i.e., blue, late-type, and star-forming systems), we select the following properties as the most convenient for our study.

1. The two main measures that can be obtained from ALFALFA observations: the 21 cm line width of the source at the 50\% level of the two peaks, $W_{50}$, and the H\textsc{i} mass, which is estimated from the equation

$$M_{H\textsc{i}} = 2.356 \times 10^5 d^2 F_{H\textsc{i}},$$  

where $F_{H\textsc{i}}$ is the 21 cm line flux integral expressed in Jy km s$^{-1}$ and $d$ is the cosmological distance to the source in Mpc calculated from the multiattractor flow model of local peculiar velocities developed by Masters (2005).

Examination of the variation of H\textsc{i} surface density with axial ratio for ALFALFA galaxies has led us to neglect the effects of internal H\textsc{i} self-absorption on $F_{H\textsc{i}}$. Note that in this paper, $W_{50}$ represents the intrinsic width corrected not only for inclination, but also for the effects of redshift broadening and turbulence (see Springob et al. 2005).

Due to the absence of reliable estimates of the intrinsic axial ratio $q$ of the targets, we choose to use the $r$-band isophotal axial ratio $(b/a)_r$ from the SDSS as a proxy for inclination in the calculation of the intrinsic line widths. The error on inclination corrections arising from taking $q = 0$ is in general negligible, except for nearly edge-on objects, which have been removed from our data set (see Section 2).

2. The luminosities (and their corresponding absolute magnitudes) in the five SDSS bands from Petrosian apparent magnitudes. The latter, which are especially suited for bright, extended objects, lead us to recover almost all the light from late-type galaxies and around 80\% for early types (Blanton et al. 2001). Absolute magnitudes have been corrected to face-on values following Shao et al. (2007). They are not corrected for the seeing effect.

3. The 25 mag arcsec$^{-2}$ isophotal major-axis diameter, $D_{25}$, in the five SDSS bands, which provide a more continuous measure of the scale of galaxies than the Petrosian radii $R_{50}$ and $R_{90}$. The minimum velocity cutoff of 2000 km s$^{-1}$ adopted in Paper I when defining the LDE sample leaves out of the present analysis more than half of the mostly nearby blue faint objects having unrealistically small isophotal angular radii in the SDSS database.

Isophotal linear diameters have been corrected for inclination by using transformations of the form

$$\log D_{25} = \log D_{25}^{obs} + \beta \log (b/a),$$  

where $D_{25}$ and $D_{25}^{obs}$ are, respectively, the intrinsic and observed values of this variable in a given band, whereas the coefficient $\beta$ measures the strength of the corresponding attenuation. We note that our corrections are somewhat stronger than those estimated by Maller et al. (2009) from Equation (2) for $R_{50}$. For instance, we obtain $\beta_r = 0.35$ in the $r$ band, while the attenuation for $R_{50,r}$ calculated by the latter authors is 0.20.

4. The colors from model magnitudes. Here, we explore the combinations $(u - g)$, $(g - r)$, $(r - i)$, $(i - z)$, $(u - r)$, $(u - i)$, $(u - z)$, $(g - i)$, $(g - z)$, and $(r - z)$.

The effect of inclination on the colors of our galaxies has been corrected by using the variation of color with the axial ratio quoted in Masters et al. (2010). For galaxies with $\log (b/a) < 0.7$, we use the straight-line fits listed in their Equation (3), inferred for Galaxy Zoo (GZ) spirals with $R_{50} > 10''$ and a constant extension of these fits for objects with $\log (b/a) > 0.7$, as measured in the $g$ band.

We have not applied the strongest color corrections derived by Masters et al. for systems with smaller apparent sizes, because we have verified that the adopted transformation is enough to make the average intrinsic $(g - r)$ color of ALFALFA sources independent of viewing angle.

5. The Sérsic index, $n$, from the NYU-VAGC catalog, which measures the shape of the observed $r$-band luminosity profile of a galaxy fitted using the Sérsic $R^{1/n}$ formula with elliptical isophotes. Available only for galaxies with $r < 18$ mag.

6. The (inverse) index of light concentration, $C_{59} = R_{50}/R_{90}$, in the five SDSS bands, which ranges from 0 to 1 and is available for the full SDSS DR7 data set. Not corrected for seeing.

Along with these variables, we also include the isophotal $(b/a)_r$, which given its apparent nature should be uncorrelated with any of the former quantities and can act therefore as a control parameter. The values of this axial ratio quoted in the SDSS are not corrected for seeing, which makes the most inclined galaxies appear rounder than they should be. We note, however, that the possible impact of seeing in our data is likely to be insignificant since its effects are only noticeable for small, highly inclined galaxies, which are mostly excluded from our sample due to the adopted integrated flux cut (the members of the LDE-HQ data set are regular gas-rich galaxies with $M_{H\textsc{i}} \gtrsim 10^9 M_\odot$). Besides, blind H\textsc{i} surveys in which the detection probability depends on the observed line width like ALFALFA are naturally biased against this kind of targets (compare, for instance, the data point densities in our Figure 1 and those in Figure 1 from Masters et al. 2010 derived for GZ spirals).

Note also that in the present analysis, the color, as well as the Sérsic and light concentration indexes, is used as objective proxies of morphology. Attempts to work with “classical” indicators of morphological type, such as the continuous de Vaucouleurs numerical code listed in the HyperLeda database, have been thwarted by lack of completeness: about half of the SDSS galaxies do not have information on their Hubble types, while the same is true for $\sim 16\%$ of the ALFALFA sources.

In order to examine the correlation structure, we feed in the logarithms of all these basic variables except $C_{59}$ and $(b/a)_r$. In this manner, we should be able to find any scaling law that might exist among them (this is also a must when variables have a lognormal distribution). This means, in particular, that it is not necessary to explicitly include in the analysis interesting composite parameters, such as the stellar mass given by Gavazzi et al. (2008), $\log (M_*/M_\odot) = -0.152 + 0.518 (g - i) + \log L_i$ (a similar formula based on the $g - r$ color is provided by Bell et al.
Pearson’s correlation coefficients, size of our sample results in very low critical values for the measurements listed above. This has to do with the fact that on a surface brightness, isophotal, or Petrosian, more single input variables. This is also the case for the mean values of the 25th percentile best seeing in the SDSS (see Masters et al. 2010). (A color version of this figure is available in the online journal.)

Figure 1. r-band axial ratio vs. the radius enclosing 90% of the r-band light for galaxies in the ALFALFA LDE sample. Vertical lines correspond to the limits in galaxy in the ALFALFA LDE sample. Vertical lines correspond to the limits in

2003), that are linear combinations of (the logarithm of) two or more single input variables. This is also the case for the mean surface brightness, isophotal, or Petrosian, \( \mu \equiv M + 5 \log R \), which can likewise be expressed as a function of two of the measurements listed above.

3.2. Relevant Parameters

Before we proceed with the present study, there is, however, an important consideration that must be taken into account. It has to do with the fact that on a t-test of significance, the large size of our sample results in very low critical values for the Pearson’s correlation coefficients, \( r_p \), i.e., for the elements of the various correlation matrices that are inferred in this section. For instance, \( r_{p}^{\text{crit}} \sim 0.06 \) for a level of significance of 0.01 on a two-tailed test with about 1600 degrees of freedom. This implies that variables that ideally should be essentially independent of each other might end up exhibiting a very weak, but nonetheless statistically significant, linear relationship according to this test, even in the presence of attenuation from measurement errors. For this reason, as well as to avoid working with an excessive number of parameters, we have decided to consider that two given properties are truly connected in practice only if they have a Pearson’s \( r \) that, aside from being statistically significant, is also minimally strong (\( |r_p| > 0.3 \)).

The correlation matrices inferred from different subsets of the attributes selected in Section 3.1 demonstrate that colors and luminosities (absolute magnitudes) are strongly correlated among themselves, as was to be expected. This allows us to discard all but one of the colors and all but one of the luminosities. Among these photometric variables, the ones showing the largest correlation coefficients with the H\(_{\text{I}}\) mass are the \( g \), \( r \), and \( i \)-band luminosities, as well as all the optical colors that can be obtained from their combinations. Given that the photometric errors in these three bands are rather similar (Strateva et al. 2001), we have taken into account both the dynamic ranges of the different colors and the economy in the number of involved bands to finally select the \( r \)-band luminosity and the \( (g - r) \) color as the most adequate representatives of these two fundamental stellar properties.

The correlation analysis has also shown evidence that the available measurements of the isophotal diameter and the Sérsic index in the five SDSS bands are degenerate. As before, the superior quality of the SDSS photometry in the central \( g \), \( r \), and \( i \) bands results in somewhat stronger correlations of these variables with H\(_{\text{I}}\) mass at such wavelengths. Consistency with the adopted bandpass for measuring the amount of light has led us to select \( r \)-band estimates of \( D_{25} \) and \( n \) to represent these quantities, too.

Overall, we find that the isophotal \( r \)-band diameter, the \( r \)-band luminosity, and the \( W_{50} \) line width are tightly related to the \( M_{\text{HI}} \). A second group of attributes, the \( (g - r) \) color and the \( r \)-band Sérsic index, are moderately aligned with this quantity (\( |r_P| \sim 0.3–0.5 \); see Disney et al. 2008 for a similar conclusion regarding color). While we have decided to retain \( (g - r) \) in the set of parameters that may be needed to describe the cold gas content of a galaxy, we have chosen to omit from this list the Sérsic index, as it has a somewhat smaller \( |r_P| \) and is available only to galaxies in the NYU-VAGC catalog (i.e., with \( r \leq 18 \) mag). Finally, the third morphological separator selected, the index of light concentration, shows little or no indication of a major dependence on the H\(_{\text{I}}\) mass, exhibiting a Pearson’s \( r \) of similar strength (\( \lesssim 0.2 \)), for instance, that correlations involving the apparent inclination, so it will also be discarded in the remainder of this study.

4. RELATIONSHIPS AMONG H\(_{\text{I}}\) AND OPTICAL PROPERTIES

We now complete the characterization of gas-rich galaxies by circumscribing ourselves to the final set of five non-degenerate, intrinsic properties selected in the previous section.

To begin with, we will examine in detail the correlation structure of this multivariate space to identify both its most statistically significant principal components and their degree of alignment with the selected observed parameters. We will then use the correlations among the measured variables to derive formulae for predicting the neutral hydrogen content according to two different approaches. On the one hand, we will seek for the most probable value of \( M_{\text{HI}} \) assuming that the other four properties are precisely known. This involves solving a multivariate regression problem in order to obtain equations useful to establish standards of normalcy for the H\(_{\text{I}}\) content of galaxies—we shall define as “normal” the typical values of our LDE-HQ galaxy sample members—from a set of diagnostic parameters easily accessible to observation. On the other hand, we will also determine the best-fitting axes of all the different pairs that can be built from these five basic attributes and the constraints they impose on the scaling laws connecting fundamental properties of galaxies.

4.1. PCA Results

Here we deal with the correlation matrix of the variables \( \log(M_{\text{HI}}, (M_{\odot})), \log(W_{50} (\text{km s}^{-1})), \log(D_{25}, (\text{kpc})), M_{\text{r}} (\text{mag}), \) and \( (g - r) (\text{mag}) \) in order to perform a principal component analysis (PCA) in this parameter space. Unlike the covariant matrix, the use of the correlation matrix entails the standardization of the original variables putting them on an equal footing: all samples of variables are scaled to have zero mean and unit standard deviation. This scaling of the measurements avoids the creation of spurious interrelations arising from the
Table 1

Principal Component Analysis of H I and Optical Properties for Weighted Data and Disattenuated Correlations

| Quantity   | log \( M_{\text{HI}} \) | log \( W_{\text{50}} \) | log \( D_{25,r} \) | \( M_r \) | \( (g-r) \) | \( C_{90,r} \) | \( (b/a)_r \) |
|------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| \( \log M_{\text{HI}} \) | 1.00 | 0.65 | 0.80 | −0.74 | 0.58 | −0.18 | 0.11 |
| \( \log W_{\text{50}} \) | 1.00 | 0.71 | −0.76 | 0.88 | −0.22 | 0.08 |
| \( \log D_{25,r} \) | 1.00 | −0.96 | 0.85 | −0.09 | 0.02 |
| \( M_r \) | 1.00 | −0.87 | 0.25 | −0.00 |
| \( (g-r) \) | 1.00 | −0.19 | 0.20 |
| \( C_{90,r} \) | 1.00 | 0.10 |
| \( (b/a)_r \) | 1.00 |

Mean \( \left( N_{\text{gal}} = 1624 \right) \) | 8.82 | 2.22 | 0.42 | −17.43 | 0.33 | 0.44 | 0.50 |
Standard deviation \( \) | 0.36 | 0.18 | 0.23 | 1.48 | 0.12 | 0.05 | 0.16 |
Observational error \( \) | 0.02 | 0.03 | 0.05 | 0.10 | 0.10 | 0.01 | 0.07 |
\( \sqrt{R_{XX}} \) | 1.00 | 0.99 | 0.98 | 1.00 | 0.79 | 0.98 | 0.92 |

Eigenvectors

| \( \hat{e}_1 \) | 0.40 | 0.43 | 0.47 | −0.47 | 0.46 | \( \lambda_1 = 4.17 \) (83.31) |
| \( \hat{e}_2 \) | 0.73 | −0.45 | 0.21 | −0.03 | −0.46 | \( \lambda_2 = 0.49 \) (93.02) |
| \( \hat{e}_3 \) | 0.46 | 0.63 | −0.43 | 0.43 | −0.12 | \( \lambda_3 = 0.30 \) (99.07) |
| \( \hat{e}_4 \) | −0.08 | 0.21 | −0.41 | −0.75 | −0.47 | \( \lambda_4 = 0.05 \) (100) |
| \( \hat{e}_5 \) | −0.29 | 0.40 | 0.62 | 0.17 | −0.58 | \( \lambda_5 = 0.00 \) (100) |

rms residuals

| Principal axis | 0.22 | 0.09 | 0.09 | 0.51 | 0.09 |
| Principal plane | 0.11 | 0.09 | 0.07 | 0.50 | 0.06 |
| Principal 3-plane | 0.06 | 0.05 | 0.05 | 0.42 | 0.06 |
| Principal 4-plane | 0.05 | 0.04 | 0.07 | 0.12 | 0.04 |
| Principal 5-plane | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Note. PCA is carried on the first five properties only: \( \log(M_{\text{HI}} (M_\odot)), \log(W_{\text{50}} \text{ km s}^{-1}), \log(D_{25,r} \text{ (kpc)}), M_r \text{ (mag)}, \) and \( (g-r) \text{ (mag)} \).

The LDE-HQ data set represents a volume-limited sample of more than 29,000 galaxies.

The preponderance (i.e., larger dynamic range) of certain properties, for the PCA calculations, we employ the IDL procedure pca.pro5 implemented following the description by Murtagh & Heck (1987). This algorithm has been modified in order to account for the flux limitation of our sample and the measurement error of the estimates.

Table 1, which presents the results for disattenuated correlations, also lists the adopted estimate of the typical measurement error and the square root of the reliability coefficient for the observables. In general, the selected parameters have a near perfect reliability, except the color, for which the attenuation correction is significant.

As stated in Section 2 (see also Appendix A), the integrated flux cutoff of the LDE-HQ sample has been compensated by weighting each of its member galaxies by the inverse of the maximum volume, \( V_{\text{max}} \), in which it could have been observed corrected for the systematic effects of large-scale structure and loss of signal due to RFI. This correction is larger for the lowest HI-mass objects since they are only detected at a short distance.6 We have also taken into account that the correlations between variables are weakened in the presence of measurement error. Given two sets of estimates \( \hat{X} \) and \( \hat{Y} \) with independent measurement errors, the disattenuated correlation between the underlying variables \( X \) and \( Y \) can be obtained from the formula (cf. Spearman 1904)

\[
\hat{r}_p(X, Y) = \frac{r_p(\hat{X}, \hat{Y})}{\sqrt{R_{XX} R_{YY}}} ,
\]

where the reliability coefficients \( R_{XX} \) and \( R_{YY} \) are defined as one minus the ratio between the variance on the corresponding measurement error and the total observed variance. In practice, this means that variables are standardized using an estimate of their true standard deviation, instead of the observed one. We have followed Fuller & Hidiroglou (1978; see also Bock & Petersen 1975) and extended this correction to the multivariate case by imposing the constraint of producing a valid correlation matrix for our disattenuated variables, i.e., a matrix that is at least positive semidefinite, after checking that their associated measured error scores are essentially independent.

The results of our PCA analysis are summarized in Tables 1 and 2 in the form of the \( 1/V_{\text{max}} \)-weighted correlation matrix, all its eigenvectors and eigenvalues, i.e., the variances of the data in the directions of the principal axes as well as the rms residuals between the five-dimensional space (manifold) of the observations and the different \( p \)-dimensional subspaces \( (p = 1, \ldots, 5) \) that best describe them. Table 1, which presents the results for disattenuated correlations, also lists the adopted estimate of the typical measurement error and the square root of the reliability coefficient for the observables. In general, the selected parameters have a near perfect reliability, except the color, for which the attenuation correction is significant.

The correlation matrices indicate the existence of large linear correlations \( (|r_p| > 0.60) \) for all pairs of variables except for the HI mass and color that exhibit correlation coefficients of medium size \( (|r_p| \sim 0.45–0.60; \text{note, in contrast, the substantially higher coefficients of the correlations between luminosity and color}) \). Table 1 shows that the five galactic attributes selected are all well correlated with the first principal component. The latter is endowed with direction cosines of nearly equal absolute value \( (\approx 1/\sqrt{5} \approx 0.45) \) and has an associated eigenvalue \( \lambda_1 \) of \( \sim 4.2 \), out of a maximum possible of 5.0, implying that about 83% of the total variance in the adopted five-parameter space can be explained by a single principal axis. The second, and already statistically insignificant, principal component, draws mostly from \( M_{\text{HI}} \), and accounts for an additional \( \sim 10\% \) of the global variance, while a linear subspace of three dimensions is enough to explain the practical totality \( (\sim 99\%) \) of it (these numbers are reduced somewhat

5 http://idlastro.gsfc.nasa.gov/ftp/pro/math/pca.pro.

6 The LDE-HQ data set represents a volume-limited sample of more than 29,000 galaxies.
when we do not use disattenuated correlation estimates; see Table 2). The principal four-plane brings the rms residuals down to the level of the adopted observational errors with the exception of \( M_* \), whose variance cannot be fully accounted for. This might indicate either that there are still hidden parameters controlling the structure of disk galaxies, such as perhaps the mass-normalized star formation rate, which is tightly related to the light concentration index (e.g., Nikolic et al. 2004), or that the observational error adopted for this variable has been too optimistic. Note also that the correlation matrix reveals a second significant parameter, we offer a different explanation matrix, we obtain a second principal component, PC2, well aligned with color. Nevertheless, once the measurements are corrected from observational error, the color contribution for PC2 significantly weakens and is no longer the dominant one (compare the corresponding eigenvectors in Tables 2 and 1). This leads us to conclude that the possible statistically significant second degree of freedom found by Disney et al. (2008) is actually an artifact produced by the substantial attenuation exerted by the measurement error of the latter observable on the moderately strong intrinsic \( \text{H}_i \) mass–color correlation (see also Paper I).

### 4.2. Standards of \( \text{H}_i \) Content

We have just shown that the five observables we are dealing with are actually interconnected. In this situation, any multiple regression model which attempts to describe the \( \text{H}_i \) mass in terms of interrelated predictor variables (multicollinearity) would be associated with an ill-conditioned correlation matrix, i.e., a matrix whose inversion is numerically unstable (if there were one or more exact linear relationships among the variables the matrix would not be invertible). Thus, in the presence of multicollinearity, the impact of the individual predictors on the response variable tends to be less precise than if the predictors were uncorrelated with one another. To remedy this problem, we have adopted a two-stage procedure known as Principal Component Regression (PCR; e.g., Cook 2007) that first carries out a PCA of all the predictor variables, and then uses the resulting principal components—which are correlated with the response variable tends to be less precise than if the predictors were uncorrelated with one another. To remedy this problem, we have adopted a two-stage procedure known as Principal Component Regression (PCR; e.g., Cook 2007) that first carries out a PCA of all the predictor variables, and then uses the resulting principal components—which are independent, and hence associated with a correlation matrix of full rank—together with the dependent variable in an ordinary least-squares regression fit. Besides, one can take advantage of the initial PCA transformation to reduce the dimensionality of the data by keeping only those new variables most correlated with the \( \text{H}_i \) mass (e.g., Jolliffe 1982).

We have applied the above procedure to subspaces of increasing dimension, starting by finding the regression relations

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### Table 2
Principal Component Analysis of \( \text{H}_i \) and Optical Properties for Weighted Data

| Quantity          | \( \log M_{\text{HI}} \) | \( \log W_{50} \) | \( \log D_{25,r} \) | \( M_* \) | \( (g-r) \) | \( C_{59,r} \) | \( (b/a)_h \) |
|-------------------|----------------------|----------------|----------------|--------|----------|----------|-------------|
| Mean (\( N_{gal} = 1624 \)) | 8.82                 | 2.22          | 0.42           | -17.43 | 0.33     | 0.44     | 0.50        |
| Standard deviation | 0.36                 | 0.18          | 0.23           | 1.48   | 0.12     | 0.05     | 0.16        |

| Eigenvectors | Eigenvalues (%) |
|--------------|------------------|
| \( \hat{e}_1 \) | 0.42 0.44 0.48 \( -0.48 0.41 \) \( \lambda_1 = 3.83 \) (76.57) |
| \( \hat{e}_2 \) | 0.61 -0.25 0.21 \( -0.09 -0.72 \) \( \lambda_2 = 0.57 \) (88.06) |
| \( \hat{e}_3 \) | -0.25 -0.78 0.43 \( -0.31 0.23 \) \( \lambda_3 = 0.33 \) (94.60) |
| \( \hat{e}_4 \) | -0.61 0.34 0.20 \( -0.45 -0.52 \) \( \lambda_4 = 0.22 \) (98.98) |
| \( \hat{e}_5 \) | -0.14 0.15 0.71 \( 0.67 -0.05 \) \( \lambda_5 = 0.05 \) (100) |

Note. PCA is carried on the first five properties only: \( \log(M_{\text{HI}} (M_\odot)) \), \( \log(W_{50} \text{ km s}^{-1}) \), \( \log(D_{25,r} \text{ kpc}) \), \( M_* \) (mag), and \( (g-r) \) (mag).
between \( M_{\text{HI}} \) and each one of the four remaining properties (see the plots above the diagonal of Figure 2), and then adding input variables progressively to seek for the combinations of regressors that best predict the \( \text{H}_\text{I} \) content. This process is stopped when, after adding a new predictor variable, the rms residual of the multiple regression model increases or does not get reduced in an amount comparable to or larger than the typical observational error in \( M_{\text{HI}} \) quoted in Table 1.

In agreement with the results of the previous section, we find that the best predictions for the \( \text{H}_\text{I} \) mass are those depending on a single regressor variable. Table 3 lists, ordered according to decreasing accuracy as given by the size of the rms residuals, the central values and associated errors of the coefficients \( a_i \) of the correlations

\[
\log \left( \frac{M_{\text{HI}}}{M_\odot} \right) = a_0 + a_1 X
\]

for fits with and without \( 1/V_{\text{max}}' \)-weighting carried on disattenuated data. It can be seen that the most precise predictor of the \( \text{H}_\text{I} \) mass is \( D_{25,r} \), just the property most strongly correlated with it, followed by \( M_r \). Among the distance-independent observables,
the best predictor of the H\textsubscript{i} content is the rotational width of the disk, whereas the regression model using the ($g-r$) color is the least accurate, as expected. Of course, the use of the H\textsubscript{i} rotation speed to estimate $M_{\text{H}i}$ only makes sense when the 21 cm line flux integral is not available and this predictor can be replaced by proxies like, for instance, the estimators defined in Catinella et al. (2007) from optical rotation curves.

The uncertainties of the correlation coefficients quoted in Table 3 include two contributions. The first one is the random error, calculated by adding in quadrature the statistical error of the parameter estimates, based on 5000 non-parametric bootstrap trials of the correlations, and the spread due to observational errors, which we have calculated through the creation of 10,000 realizations of the correlations after assigning Gaussian random measurement and distance errors to each galaxy.

The second error term is included to show the impact that voids (and overdensities of comparable size present in the LDE-HQ sample) can have on the inferred relationships by systematically undercounting (overcounting) galaxies in those regions. Remember that we are dealing with galaxies in LDE and that the weighting scheme depicted in Appendix A only corrects for the redshift-averaged density of galaxies in the surveyed volume. This systematic error has been evaluated by dividing the sample into eight equal area sky regions of $\sim 15^\circ$ and calculating the correlations eight times, leaving out a different section each time. At the median survey redshift of $\sim 8000$ km s$^{-1}$, the adopted angular size corresponds to about 30 Mpc, a scale comparable to the typical diameter of voids in the local universe (Hoyle & Vogele 2004). The contribution of galaxy under- and overdensities to the variance in any correlation coefficient $a_i$ is measured through the jackknife error estimator (Lupton 1993)

$$a_i^2 = \frac{N-1}{N} \sum_{j=1}^{N} (a_{i,j} - \bar{a})^2 ,$$

where $N = 8$.

Finally, we have also verified that combinations of two distance-dependent input variables, which obey equations of the form

$$\log \left( \frac{M_{\text{H}i}}{M_\odot} \right) = a_0 + a_1 X_1 + a_2 X_2 ,$$

do not contribute to reduce the spread that may arise from distance uncertainties. We report in Table 3 the coefficients obtained using the optical size and luminosity as diagnostic variables, which is also the only multilinear regression model with an rms residual as good as that of the best linear model (but at the cost of using two predictors). Looking at the coefficients of this multiple regression for weighted and unweighted data, it is clear that the right-hand side of the equations shows a dependence with distance that neither is null (the ratio $a_1/a_2 \neq -5$), as would be expected if they were defining a surface magnitude, nor does it compensate the $d^2$-dependence of the H\textsubscript{i} mass. As done previously for the single regression models, we account for this breach of distance independence when calculating the random error of the correlation coefficients.

### 4.3. Planar Correlation Diagrams

In the previous section, we have been interested in obtaining predictions for one integral property, the H\textsubscript{i} mass, from the observed values of other four ones: size, total magnitude, velocity width, and color. In this section, we turn our attention to the constraints that these intrinsic quantities put on the scaling relations among the fundamental attributes of individual galaxies. This means that we now consider the five compiled variables on an equal footing and focus on the correlations arising from the same PCA technique as used in Section 4.1 applied to pairs of them. The involved quantities are therefore treated symmetrically, thus minimizing the inconsistencies that may arise from the possible 'non-commutativity' of the inferred relationships. The coefficients of the $1/V_{\text{max}}$-weighted and unweighted orthogonal fits of the form $Y = a_0 + a_1 X$ between all 10 possible pairs of galaxy properties are presented in Table 4 for disattenuated data. (For this exercise, the quoted uncertainties depict only the random error estimates calculated as in Section 4.2.) The scatter plots and their best linear fits can be visualized in the boxes below the diagonal in Figure 2.

| Weighting | $Y$ | $X$ |
|-----------|-----|-----|
|           | $\log D_{25,r}$ | $M_r$ | $\log W_{50}$ | $(g-r)$ |
|           | $a_0$ | $a_1$ | $a_0$ | $a_1$ | $a_0$ | $a_1$ |
| $1/V_{\text{max}}$ | $\log M_{\text{H}i}$ | $8.55 \pm 0.05$ | $1.55 \pm 0.06$ | $5.36 \pm 0.19$ | $-0.24 \pm 0.010$ | $5.01 \pm 0.30$ | $1.99 \pm 0.13$ | $8.45 \pm 0.12$ | $2.99 \pm 0.29$ |
|           | $\log D_{25,r}$ | $-2.05 \pm 0.09$ | $-0.16 \pm 0.004$ | $-2.28 \pm 0.21$ | $1.29 \pm 0.14$ | $-6.06 \pm 0.06$ | $1.93 \pm 0.15$ | $-12.8 \pm 0.40$ | $-12.2 \pm 0.98$ |
|           | $M_r$ | $1.46 \pm 1.14$ | $-8.15 \pm 0.48$ | $1.73 \pm 0.06$ | $1.50 \pm 0.13$ | $9.06 \pm 0.04$ | $2.28 \pm 0.07$ | $0.29 \pm 0.02$ | $1.38 \pm 0.03$ |
|           | $\log W_{50}$ | $0.24 \pm 0.42$ | $-7.57 \pm 0.16$ | $-14.6 \pm 0.13$ | $-8.74 \pm 0.26$ | $1.90 \pm 0.01$ | $1.15 \pm 0.04$ | $5.24 \pm 0.08$ | $1.66 \pm 0.03$ | $5.24 \pm 0.08$ | $-0.26 \pm 0.004$ | $-1.98 \pm 0.08$ | $1.19 \pm 0.03$ | $-0.24 \pm 0.42$ | $-7.57 \pm 0.16$ | $-14.6 \pm 0.13$ | $-8.74 \pm 0.26$ |

The study of the scalings of the most basic properties of galaxies is central for constraining theories of their formation and evolution. It has generated an abundant literature, whose detailed revision far exceeds the scope of the present work. Instead, we have decided to focus on the comparison between the values for the mean slopes of the strongest correlations we predict, which are those in the $L(M)RV$ subspace of luminosity (mass), size, and rotation speed, and those reported in other studies that also combine H\textsubscript{i} and optical observations (HG84; Salpeter & Hoffman 1996), or that specifically study the scalings among the above fundamental properties in late-type objects (Courteau et al. 2007). When comparing results allowance should be made not just for differences in sample size, but also for other factors such as the waveband of the optical observations, the specific observables chosen to estimate the above attributes, their dynamic ranges, or the fitting method.
employed. Another complication that distorts the comparison among the different outcomes is the incompleteness of the data sets that, with the exception of ours, are all affected by intractable selection biases.

With all these caveats in mind, the agreement between the central values of the slopes (rounded to the two most significant digits) reported by the different studies listed above (see Table 5) can be classified as generally satisfactory. The largest discrepancies correspond to the correlation involving the luminosity versus the rotation speed, i.e., the TF relation, reflecting the fact that it is always problematic to accurately find the slope of this empirical law due to the low dynamic range in log $V$. The mean values of the log slope $\gamma$ listed in Table 5 range from $\sim 2.6$ ($-6.5$ in magnitude units; HG84) to $\sim 3.4$ (Courteau et al. 2007) and $\sim 3.7$ (Salpeter & Hoffman 1996; the first and the last ones being measured at blue wavelengths and the Courteau et al.’s for the $I$ band). From our weighted data, we get a slope of $3.3 \pm 0.2$ (random error) or, equivalently, $-8.2 \pm 0.5$ mag, which roughly falls in the middle of this range and is fully consistent with estimates reported in TF-specific literature for bright spirals in the blue/near-IR band (e.g., Willick et al. 1997; Giovanelli et al. 1997; Courteau 1997; Masters et al. 2006).

Among all the results obtained, the most striking is perhaps the relationship between the H I mass and the characteristic size of the stellar distribution of gaseous galaxies, represented here by the isophotal diameter in the $r$ band, $D_{25,r}$. Our finding that it has a central slope $\alpha = 1.55 \pm 0.06$ does not support the idea that all H I-rich galaxies have roughly the same global H I column density, as recently advocated by Garcia-Appadoo et al. (2009; see also references therein) from a sample of HIPASS galaxies and implied by correlations such as the one found by Salpeter & Hoffman (1996), listed in Table 5, from optical size measurements in the $B$ band. (Implicit in this conclusion is the assumption that H I and stellar disk sizes are roughly proportional as shown by Broeils & Rhee 1997.) We remind the reader once again that none of these previous studies based their conclusions on correlation analyses performed on complete multivariate data sets, as we have done here. In particular, we suspect that the use of samples largely dominated by late-type spirals, which are more prone to exhibit a nearly constant mean H I surface density, may contribute to exacerbating the tendency to find such an aesthetically appealing result (see also Solanes et al. 1996). In this regard, we wish to emphasize that the claimed constancy of the mean H I column density by Garcia-Appadoo et al. (2009), who deal with a sample that, morphologically speaking, is representative of the entire population of H I emitters, emanates from an (unweighted) H I mass–$R_{25}$ correlation with an actual central slope of $\sim 1.7$, in pretty good agreement with our outcome.

5. SUMMARY AND CONCLUDING REMARKS

We have sought for correlations among a large set of extensive 21 cm and optical homogeneous measures available for the 1624 members of a complete, H I flux-limited sample of non-clustered, gas-rich galaxies, not influenced by their environment. Our main aim has been identifying the combinations of intrinsic variables directly arising from observable quantities that make up the best diagnostic tools for the H I content. The sources used for this research have been selected from the LDE H I galaxy sample of the ALFALFA blind H I survey defined in Paper I. The size of this primary database has been reduced to include only high-quality ALFALFA detections with an S/N $> 6.5$ found up to 15,000 km s$^{-1}$ and with moderate inclinations $0.85 \geq b/a \geq 0.2$ ($30^\circ \leq i \leq 80^\circ$ for $q = 0$). Furthermore, we have selected only those H I emitters with an integrated flux $\geq 1.3$ Jy km s$^{-1}$ from which the sensitivity of the survey becomes essentially independent of profile width.

The examination of the correlation structure of these data, conveniently weighted to compensate for the flux limitation, as well as for the systematic effects of large-scale structure and loss of signal due to man-made RFI in the surveyed volume, has produced the following interesting results.

1. In (bright) gas-rich galaxies, the isophotal $r$-band linear diameter, total $r$-band luminosity, $W_{50}$ line width, and $(g-r)$ color are the galaxian properties most tightly correlated with the total H I mass. The PCA of the manifold defined by these variables has revealed relationships with large correlation coefficients that are suggestive of a high degree of organization in the LDE-HQ sample. This is consistent with the idea that H I emitters behave essentially as a one-parameter family. As previously noted by Disney et al. (2008; see also van den Bergh 2008 and references therein), the observed structural simplicity of disk galaxies is difficult to reconcile with the prevailing theory of hierarchical galaxy formation, which holds that the physical properties of these objects are determined by the interplay of several potentially independent factors, such as mass, spin, and (the chaotic at high redshift) merger history of galactic halos.

2. In accordance with the output of the PCA, we have found that the best predictions for the most probable value of $M_{HI}$, assuming that the regressor variables are precisely known, are those depending on a single parameter. Our fits carried on $1/V_\text{max}$-weighted and disattenuated data show that the most accurate predictor for the H I mass of a galaxy is its optical diameter, through the equation

$$\log \left( \frac{M_{HI}}{M_\odot} \right) = 8.72 \pm 0.06 \pm 0.06 + 1.25 \pm 0.06$$

$$\pm 0.07 \log \left( \frac{D_{25,r}}{\text{kpc}} \right),$$

where the first error term is statistical and the second is the systematic uncertainty arising from the large-scale structure present in the surveyed volume. In Table 3, we provide alternative prescriptions to calculate $M_{HI}$ from $M_r$, $W_{50}$, and $(g-r)$, as well as from a combination of $D_{25,r}$ and $M_r$.

| Reference                | $M_{HI} \sim R^a$ | $M_{HI} \sim L^b$ | $L \sim V^c$ | $R \sim L^d$ | $R \sim V^e$ |
|-------------------------|-------------------|-------------------|-------------|-------------|-------------|
| HG84 (1984)             | 1.8               | 0.66              | 2.6         | ...         | ...         |
| Salpeter & Hoffman (1996)| 2.0               | 0.74              | 3.7         | 0.37        | 1.4         |
| Courteau et al. (2007)  | ...               | ...               | 3.4         | 0.32        | 1.1         |
| This work               | 1.6               | 0.60              | 3.3         | 0.40        | 1.3         |
The fact that the models based on the crude morphological indicator \((g - r)\) yield rms residuals comparable to the global standard deviation associated with \(M_{\text{HI}}\) hints that the morphology of \(\text{H}\) emitters plays a secondary role in determining their neutral gas content, as already inferred by HBG4 and Solanes et al. (1996).

3. From the joint distributions of the quantities most strongly correlated with the \(\text{H}\) mass, we derive the mean relationships

\[
M_{\text{HI}} \propto R^{1.6}, \quad M_{\text{HI}} \propto L^{0.60}, \quad L \propto V^{3.3}, \quad R \propto L^{0.40}, \quad R \propto V^{1.3},
\]

where \(L\) represents the total \(r\)-band luminosity of the (old) stellar disk of a galaxy, and \(R\) and \(V\) its size and rotation speed, respectively. Among the scaling relations that involve \(\text{HI}\) measurements, the most interesting ones are the well-known \(LV\) or \(TR\) relation and the ratio between the total neutral gas mass and optical radius. For the former, it is noteworthy that we find a central slope fully consistent with the typical values reported in \(LV\) studies at optical/near-IR wavelengths from data that have not been specifically selected for this task. On the other hand, the slope inferred for the second scaling implies that the hybrid surface density of neutral hydrogen \((\propto M_{\text{HI}}/R^2)\) is not constant, but moderately decreasing with galaxy size. Claims in favor of the near universality of the global \(\text{HI}\) surface density for the entire spiral population rely on incomplete data sets biased toward galaxies of the latest Hubble subtypes (mostly Sc and Irr), for which the constancy of this intensive property is a relatively acceptable approximation.

To date, most multidimensional statistical studies focusing on the interrelations among the main properties of galaxies have had to contend with largely incomplete, heterogeneous samples of modest size and affected by important selection artifacts. It is therefore evident that disregarding any of these factors when they are in fact present may result in inconsistent estimation. In this respect, efforts like the present one based on the interrelations among the main properties of galaxies for which \(M_{\text{HI}}\) and \(\text{H}\) are in fact present may result in inconsistent estimation. In this respect, efforts like the present one based on the interrelations among the main properties of galaxies for which \(M_{\text{HI}}\) and \(\text{H}\) are in fact present may result in inconsistent estimation. In this respect, efforts like the present one based on the interrelations among the main properties of galaxies for which \(M_{\text{HI}}\) and \(\text{H}\) are in fact present may result in inconsistent estimation. In this respect, efforts like the present one based on the interrelations among the main properties of galaxies for which \(M_{\text{HI}}\) and \(\text{H}\) are in fact present may result in inconsistent estimation. In this respect, efforts like the present one based on the interrelations among the main properties of galaxies for which \(M_{\text{HI}}\) and \(\text{H}\) are in fact present may result in inconsistent estimation. In this respect, efforts like the present one based on the interrelations among the main properties of galaxies for which \(M_{\text{HI}}\) and \(\text{H}\) are in fact present may result in inconsistent estimation. In this respect, efforts like the present one based on the interrelations among the main properties of galaxies for which \(M_{\text{HI}}\) and \(\text{H}\) are in fact present may result in inconsistent estimation. In this respect, efforts like the present one based on the interrelations among the main properties of galaxies for which \(M_{\text{HI}}\) and \(\text{H}\) are in fact present may result in inconsistent estimation. In this respect, efforts like the present one based on the interrelations among the main properties of galaxies for which \(M_{\text{HI}}\) and \(\text{H}\) are in fact present may result in inconsistent estimation. In this respect, efforts like the present one based on the interrelations among the main properties of galaxies for which \(M_{\text{HI}}\) and \(\text{H}\) are in fact present may result in inconsistent estimation. In this respect, efforts like the present one based on the interrelations among the main properties of galaxies for which \(M_{\text{HI}}\) and \(\text{H}\) are in fact present may result in inconsistent estimation. In this respect, efforts like the present one based on the interrelations among the main properties of galaxies for which \(M_{\text{HI}}\) and \(\text{H}\) are in fact present may result in inconsistent estimation. In this respect, efforts like the present one based on the interrelations among the main properties of galaxies for which \(M_{\text{HI}}\) and \(\text{H}\) are in fact present may result in inconsistent estimation. In this respect, efforts like the present one based on the interrelations among the main properties of galaxies for which \(M_{\text{HI}}\) and \(\text{H}\) are in fact present may result in inconsistent estimation. In this respect, efforts like the present one based on the interrelations among the main properties of galaxies for which \(M_{\text{HI}}\) and \(\text{H}\) are in fact present may result in inconsistent estimation. In this respect, efforts like the present one based on the interrelations among the main properties of galaxies for which \(M_{\text{HI}}\) and \(\text{H}\) are in fact present may result in inconsistent estimation. In this respect, efforts like the present one based on the interrelations among the main properties of galaxies for which \(M_{\text{HI}}\) and \(\text{H}\) are in fact present may result in inconsistent estimation.
Figure 3. Rauzy’s test for completeness applied to data from the ALFALFA survey (top) and from its LDE subset (bottom). We only consider code 1 sources with redshifts between 2000 and 15,000 km s\(^{-1}\). In both cases, the HI flux completeness limit, \(F_{HI, lim}\), (determined by \(T_C = -3\)) falls somewhat below 1.3 Jy km s\(^{-1}\) (vertical dotted line).

Figure 4. Same as in Figure 3, but for the \(\langle V/V_{max} \rangle\) completeness test.

the average \(V/V_{max}\) of the data, which should be equal to \(1/2\) provided that the effective search volume of each galaxy, or equivalently its detection probability, is accurately established. The \(\langle V/V_{max} \rangle\) test presupposes that the galaxies are on average homogeneously distributed and is therefore sensitive to selection and large-scale structure effects. Therefore, the correct application of this second technique to the above HI data sets requires that in the calculation of the individual search volumes we account for the true density of targets as a function of redshift, as well as for the artificial annular underdensities that arise from man-made RFIs.

Accordingly, we have modified \(V\) and \(V_{max}\) in order to include weighting by the average density interior to the corresponding radial distances normalized to the average density within the surveyed volume. The weights have been calculated from a volume-limited subsample of the spectroscopic SDSS DR7 data set, which is complete to a Petrosian r-band magnitude of 17.77, selected to include only galaxies obeying Maller et al.’s (2009) disk criterion, which is very effective in identifying objects structurally similar to HI emitters (see Paper I). In addition, we have corrected for RFI by using the same average relative weight as a function of the observed heliocentric velocity depicted in Figure 6 by Martin et al. (2010).

In Figure 4, we depict the expectation value and dispersion of the average ratio of weighted volumes, \(\langle V'/V'_{max}\rangle\), as a function of the integrated HI flux. As we have done previously for Rauzy’s test, we show results for the ALFALFA data (top panel) and its LDE subset (bottom panel; in this case, the radial run of the density weighting is estimated using Maller et al.’s disks in LDE). Galaxies not belonging to the redshift range 2000–15,000 km s\(^{-1}\) or classified as code 2 detections have been discarded. In very good agreement with the \(T_C\) method, we find that above \(F_{HI, lim} \approx 1.3\) Jy km s\(^{-1}\) the values of this statistic remain, for the two data sets, practically constant around 0.50. Note that the observed rough invariance of \(\langle V'/V'_{max}\rangle\) above this integrated-flux limit supports the assertion that the two samples are statistically complete, while the fact that the observed value of the statistic is so close to its expectation indicates that the effects of large-scale structure and RFI have been correctly averaged out with the adopted weighting strategy and, therefore, that we are dealing with homogeneous data sets.

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ERRATUM: HI CONTENT AND OPTICAL PROPERTIES OF FIELD GALAXIES FROM THE ALFALFA SURVEY. II. MULTIVARIATE ANALYSIS OF A GALAXY SAMPLE IN LOW DENSITY ENVIRONMENTS (2011, ApJ, 732, 93)

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The 25 mag arcsec$^{-2}$ isophotal major-axis parameter provided in the SDSS DR7 (Abazajian et al. 2009) galaxy catalog corresponds to a measure of the radius. Accordingly, the linear diameter $D_{25}$ defined in Equation (2), and all its appearances in the text and tables should be substituted by a radius $R_{25}$.

REFERENCES

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