Amelioration of Little Hierarchy Problem in

$SU(4)_c \times SU(2)_L \times SU(2)_R$

Ilia Gogoladze 1, Mansoor Ur Rehman and Qaisar Shafi

Bartol Research Institute, Department of Physics and Astronomy,
University of Delaware, Newark, DE 19716, USA

Abstract

The little hierarchy problem encountered in the constrained minimal supersymmetric model (CMSSM) can be ameliorated in supersymmetric models based on the gauge symmetry $G_{422} \equiv SU(4)_c \times SU(2)_L \times SU(2)_R$. The standard assumption in CMSSM (and in SU(5) and SO(10)) of universal gaugino masses can be relaxed in $G_{422}$, and this leads to a significant improvement in the degree of fine tuning required to implement radiative electroweak breaking in the presence of a characteristic supersymmetry breaking scale of around a TeV. Examples of Higgs and sparticle mass spectra realized with 10% fine tuning are presented.

1On leave of absence from: Andronikashvili Institute of Physics, Tbilisi, Georgia.
1 Introduction

The constrained minimal supersymmetric standard model (CMSSM) based on supergravity [1] with R-parity conservation is a well motivated extension of physics beyond the Standard Model (SM). Among other things it predicts that the lightest CP-even Higgs boson mass, after including radiative corrections, is \( m_h \lesssim 125 \text{ GeV} \) [2, 3]. This is to be compared with the LEP2 lower bound \( m_h \geq 114.4 \text{ GeV} \) [4]. Values of \( m_h \) around 125 GeV or so require that the soft supersymmetry (susy) breaking parameters are of order a TeV scale. Such values, in turn, lead to the so-called little hierarchy problem [5] because in implementing radiative electroweak symmetry breaking, TeV scale quantities must conspire to yield the electroweak mass scale \( M_Z \).

A variety of scenarios have been proposed [6, 7] to solve this so-called little hierarchy problem. Many of them extend the gauge and/or matter sector of the CMSSM in order to increase \( m_h \) while keeping the SUSY particle mass spectrum as light as possible. It has been shown in [8] that non-universal gaugino masses at the GUT scale \( M_G \) can help resolve the little hierarchy problem. To do this the authors have studied a variety of gaugino mass ratios obtained from some underlying theories. Following up on this, in this paper we investigate \( SU(5) \) and \( SO(10) \) GUTs in which non-universal gaugino masses are realized via dimension five operators. We focus, in particular, on the \( SU(4)_c \times SU(2)_L \times SU(2)_R \) (G422) model [9] which provides a natural setup for non-universal gaugino masses. In the G422 model the little hierarchy problem can be largely resolved if \( SU(2)_L \) and \( SU(3)_c \) gaugino masses satisfy the asymptotic relation \( M_2/M_3 \approx 4 \).

The plan of the paper is as follows. In Section 2 we review the little hierarchy problem in the CMSSM with universal gaugino masses. In Section 3 we compare the fine tuning in \( SO(10) \) and \( SU(5) \) GUT models with non-universal gaugino masses induced by suitable dimension five operators. In Section 4 we discuss the G422 model where we present the solution of the little hierarchy problem as well as its implications for the sparticle and Higgs spectrum. Our conclusions are summarized in Section 5.

2 Little Hierarchy Problem in MSSM

At tree level the lightest CP even Higgs boson mass \( m_h \) in MSSM is bounded from above by the mass of the \( Z \) boson

\[
m_h \leq M_Z.
\]

Thus, significant radiative corrections are needed in order to push lightest CP even Higgs boson mass above the LEP2 limit \( m_h \geq 114.4 \text{ GeV} \). One finds [2, 3]

\[
m_h^2 \approx M_Z^2 \cos^2 2\beta \left( 1 - \frac{3}{8\pi^2} \frac{m_t^2}{v^2} \right) + \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \left[ t + \frac{1}{2} X_t + \frac{1}{(4\pi)^2} \left( 3 \frac{m_t^2}{2 v^2} - 32\pi\alpha_s \right) (X_t t + t^2) \right],
\]

where

\[
t = \log \left( \frac{M_S^2}{M_t^2} \right), \quad X_t = \frac{2\tilde{A}_t^2}{M_S^2} \left( 1 - \frac{\tilde{A}_t^2}{12 M_S^2} \right).
\]

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Figure 1: Higgs mass $m_h$ vs $M_S$ for different maximal and minimal values of $A_t/M_S$, with $\tan \beta = 10$ and $M_t = 172.6$ GeV. Solid, dashed and dotted lines correspond to $A_t = 0$, $M_S$ and $\sqrt{6} M_S$ respectively. The horizontal line denotes the LEP2 bound $m_h \geq 114.4$ GeV.

$\tilde{A}_t = A_t - \mu \cot \beta$, where $A_t$ denotes the stop left and stop right soft mixing parameter, $\mu$ is the MSSM Higgs bilinear mixing term, and $M_S^2 = \sqrt{m_Q^2 m_U^2}$ is the geometric mean of left and right stop masses squared. As seen from Figure 1, if the Higgs mass $m_h$ turns out to be significantly larger than 114.4 GeV, one would require $|A_t| \gtrsim M_S$, or relatively heavy stop quarks ($M_S \gtrsim 1$ TeV), or a suitable combination of the two.

The $Z$ boson mass is obtained from the minimization of the Higgs scalar potential, and for $\tan \beta \gtrsim 10$, we have the standard approximate relation

$$\frac{1}{2} M_Z^2 \approx -\mu(M_Z)^2 + \left( \frac{m_{H_d}^2(M_Z) - m_{H_u}^2(M_Z) \tan^2 \beta}{\tan^2 \beta - 1} \right) \approx -\mu(M_Z)^2 - m_{H_u}^2(M_Z). \quad (4)$$

In order to see the explicit dependence of $m_{H_u}^2(M_Z)$ on $A_t$ and $M_S$ we employ a semi-analytic expression for $m_{H_u}^2(M_Z)$ using one loop renormalization group equations (RGEs) [10] in the $\overline{DR}$ regularization scheme for the soft supersymmetry breaking terms and the MSSM gauge couplings. We take the GUT scale $M_G \approx 2.0 \times 10^{16}$ GeV, and $\alpha_2 = \alpha_1 = \alpha_G = 1/24.32$. We do not enforce exact unification $\alpha_3 = \alpha_2 = \alpha_1$ at $M_G$, since a few percent deviation from the unification condition can be expected due to unknown GUT scale threshold corrections [11]. Including only the dominant terms, the semi-analytic expression for $m_{H_u}^2(M_Z)$ in CMSSM can be written as

$$-m_{H_u}^2(M_Z) \approx 1.05 A_t^2 + 1.39 A_t M_S + 0.87 M_S^2 = \left( 1.05 \frac{A_t^2}{M_S^2} + 1.39 \frac{A_t}{M_S} + 0.87 \right) M_S^2. \quad (5)$$
Figure 2: $\Delta_i$ vs $m_0$ for $M_S = 500$ GeV, $m_h = 119$ GeV, $\tan \beta = 10$ and $M_t = 172.6$ GeV.

For $m_h \gtrsim 114.4$ GeV, which is realized with either $M_S \gtrsim 1$ TeV and/or $|A_t| \gtrsim M_S$, $|m_{H_u}|$ is much larger than $M_Z$ which, in turn, implies that $\mu$ should be fine tuned in order to achieve the correct EW symmetry breaking in Eq.(4). This fine tuning is referred to as the little hierarchy problem. Following the analysis in [12, 8], we consider a quantitative measure of fine tuning

$$\Delta_X = \frac{1}{2} \frac{X}{M_Z^2} \frac{\partial M_Z^2}{\partial X}, \quad (X = \mu_0, M_1, M_2, M_3, A_{t_0}, m_0), \quad (6)$$

where $M_{1,2,3}$, $A_{t_0}$ and $m_0$ are respectively the gaugino masses, trilinear coupling and the universal soft scalar mass at $M_G$. In the CMSSM the dominant contribution is given as

$$M_Z^2 \simeq -2.04 \mu_o^2 + 5.44 m_{1/2}^2 + 0.183 m_0^2 + 0.2 A_{t_0}^2 - 0.87 m_{1/2} A_{t_0}, \quad (7)$$

with $M_1 = M_2 = M_3 = m_{1/2}$. The definition of $\Delta_X$ is consistent with the normalization,

$$\sum_X \Delta_X = 1. \quad (8)$$

The quantity $\Delta_X$ measures the sensitivity of the $Z$-boson mass to the parameter $X$ (at $M_G$), such that $\left( \frac{100}{\max(\Delta_X)} \right) \%$ is the degree of fine tuning. Normally, 10% or somewhat large value of fine tuning is regarded as the acceptable, and therefore we require $\max(\Delta_X) \lesssim 10$. For instance, with $m_h = 119$ GeV, $\tan \beta = 10$, $M_S = 500$ GeV and $m_0 = 0$, as shown in the Figure 2, the degree of fine tuning in CMSSM $\sim 1.5\%$, and it gets worse with an increase in $M_S$. It is also interesting to view the dependence of the sensitivity parameters on $m_0$, as shown in Figure 2. For lower values of $m_0$, $\max(\Delta_X) = \Delta_{m_{1/2}}$, while for higher values of $m_0$,
max(|ΔX|) = |ΔA_t0|. We see from Eq.(7) and Figure 2 that both $M_Z^2$ and max(|ΔX|) have a weak dependence on the universal scalar mass $m_0$, provided $m_0 < \text{TeV}$. Unless explicitly stated otherwise, we set $m_0 = 0$ in our numerical analysis. As suggested in ref. [8], we can reduce the degree of fine tuning to an acceptable level by employing suitable non-universal asymptotic gaugino masses. This non-universality can be generated in GUTs from dimension five operators, but it can be present in the $G_{422}$ model by a straightforward assumption. In the next two sections, after discussing the origin of non-universality, we study the status of little hierarchy problem in $SU(5)$, $SO(10)$ and especially in the $G_{422}$ model.

3 Little Hierarchy Problem in $SU(5)$ and $SO(10)$

A chiral superfield whose scalar component spontaneously breaks a GUT symmetry (such as $SU(5)$ or $SO(10)$) may also possess a non-vanishing $F$-component, in which case supersymmetry can also be broken [13]. In this section we briefly summarize how this may lead to non-universal gaugino masses at $M_G$. In turn, this can help ameliorate the little hierarchy problem. Consider the following dimension five operator for generating the soft gaugino mass terms:

$$\int d^2\theta W^a W^b \left( \frac{\Phi_{ab}}{m_P} \right) + h.c. \supset \langle F_\Phi \rangle_{ab} \lambda^a \lambda^b + h.c. . \quad (9)$$

Here $W^a$ is the supersymmetric gauge field strength, $F_\Phi$ is the auxiliary field component of the chiral superfield $\Phi$, and $\lambda_a$ denotes the gaugino field. The field $F_\Phi$, whose vacuum expectation value (VEV) breaks supersymmetry, is a singlet under the MSSM gauge group. In $SU(5)$, for example, $\Phi$ can belong to one of the following representations:

$$\begin{align*}
(24 \times 24)_{sym} &= 1 + 24 + 75 + 200 , \quad (10) \\
(45 \times 45)_{sym} &= 1 + 54 + 210 + 770 . \quad (11)
\end{align*}$$

In this paper we will employ the widely used 24 and 75 dimensional representations of $SU(5)$, and the 54 dimensional representation of $SO(10)$.

In Table 1 we display the ratios among gaugino masses at scales $M_G$ and $M_Z$. In 54-plet the $SO(10)$ breaking proceeds via $G_{422}$ [14]. As seen from Table 1, the 24-plet of $SU(5)$ and the 54-plet of $SO(10)$ yield identical gaugino mass ratios at $M_G$ [15], and therefore the results, as far as the little hierarchy problem is concerned, are the same for these two cases.

Before discussing the effects of specific choices of non-universal gaugino masses on the little hierarchy problem we present a more general analysis for arbitrary gaugino masses at $M_G$. For this purpose we perform a semi-analytic calculation for the MSSM sparticle spectra with the following boundary conditions

$$\left\{ \alpha_G, \, M_G, \, y_t(M_G) \right\} \approx \{ 1/24.32, \, 2.0 \times 10^{16}, \, 0.512 \} . \quad (12)$$

By integrating the one loop RGEs [10], we express the MSSM sparticle masses at scale $M_Z$ in terms of the GUT scale fundamental parameters $(m_0, M_{1,2,3}, A_t)$ and the Higgs bi-linear
mixing term $\mu$. For example, the gaugino masses at $M_Z$ are given by

$$\{M_1(M_Z), M_2(M_Z), M_3(M_Z)\} \approx \{0.412 M_1, 0.822 M_2, 2.844 M_3\}. \quad (13)$$

Similarly,

$$-m_{H_u}^2(M_Z) \approx 2.67 M_2^2 - 0.2 M_2^2 - 0.003 M_1^2 + 0.091 m_0^2 + 0.099 A_t^2$$
$$-0.345 M_3 A_{t_0} - 0.077 M_2 A_{t_0} - 0.012 M_1 A_{t_0}$$
$$+ 0.22 M_3 M_2 + 0.031 M_3 M_1 + 0.006 M_2 M_1, \quad (14)$$

$$A_t(M_Z) \approx -2.012 M_3 - 0.252 M_2 - 0.0316 M_1 + 0.273 A_{t_0}, \quad (15)$$

$$m_{Q_t}^2(M_Z) \approx 5.41 M_3^2 + 0.392 M_2^2 - 0.007 M_1^2 + 0.64 m_0^2 - 0.033 A_t^2$$
$$+ 0.115 M_3 A_{t_0} + 0.026 M_2 A_{t_0} + 0.004 M_1 A_{t_0}$$
$$- 0.072 M_3 M_2 - 0.01 M_3 M_1 - 0.002 M_2 M_1, \quad (16)$$

$$m_{\tilde{u}_t}^2(M_Z) \approx 4.52 M_3^2 - 0.188 M_2^2 + 0.044 M_1^2 + 0.273 m_0^2 - 0.066 A_t^2$$
$$+ 0.23 M_3 A_{t_0} + 0.051 M_2 A_{t_0} + 0.008 M_1 A_{t_0}$$
$$- 0.145 M_3 M_2 - 0.0208 M_3 M_1 - 0.004 M_2 M_1. \quad (17)$$

| SU(5) Representation | $M_1 : M_2 : M_3$ at $M_G$ | $M_1 : M_2 : M_3$ at $M_Z$ |
|----------------------|----------------------------|----------------------------|
| 1                    | 1:1:1                      | 1:2:7                      |
| 24                   | (-1):(-3):2                | (-1):(-6):13.8             |
| 75                   | (-5):3:1                   | (-1):1:2:1.4               |
| SO(10) Representation | $M_1 : M_2 : M_3$ at $M_G$ | $M_1 : M_2 : M_3$ at $M_Z$ |
| 1                    | 1:1:1                      | 1:2:7                      |
| 54: $SU(4)_c \times SU(2)_L \times SU(2)_R$ | (-1):(-3):2 | (-1):(-6):13.8 |

Table 1: Gaugino mass ratios for $SU(5)$ and $SO(10)$ [15] at GUT and electroweak scales.

We observe from Eq.(14) that in order to reduce the absolute value of $m_{H_u}^2(M_Z)$ (which, for our purpose, can be regarded as a measure of fine tuning), it is useful to have comparable values for the first two terms. This suggests the need for non-universal gaugino masses at $M_G$, with $|M_2| > |M_3|$. It turns out that the relative sign between $M_3$ and $M_2$ plays an important role. Since $M_1$ appears with relatively small coefficients in Eq.(14)–(17), we can neglect terms containing it in the qualitative discussion. However, all terms are included in the numerical analysis.

From Eq.(15) it follows that opposite signs for $M_2$ and $M_3$ help reduce the absolute value of $A_t(M_Z)$. However, this reduces $m_{H_u}$, and to compensate for this one should increase the value of $M_3$ at $M_G$. This, in turn, increases the absolute value of $m_{H_u}^2(M_Z)$. In order to show how $m_{H_u}^2(M_Z)$ depends on the sign accompanying the ratio $M_3/M_2$ we present in Figure 3, the
RGE running of $A_t$ and $m_{H_u}^2$ by taking $|M_2/M_3| = 1.5$ (as an example), with $m_h = 119$ GeV, $M_S = 250$ GeV and $A_t(M_Z)/M_S = -\sqrt{6}$. In order to generate the same $A_t$ value at low scale, for $M_S = 250$ GeV we obtain $M_3 = 167.4$ GeV and $A_{t_0} = -825$ GeV with $M_2/M_3 = +1.5$, while $M_3 = 183.3$ GeV and $A_{t_0} = -1440$ GeV for $M_2/M_3 = -1.5$. Next, with the same asymptotic value for $m_{H_u}^2$ and using the above results in its RGE running (right panel in Figure 3), smaller values of $m_{H_u}^2$ are realized for $M_2/M_3 = +1.5$. Thus, in order to reduce the amount of fine tuning, the following two conditions should be met, namely $|M_2/M_3| > 1$ and the ratio $M_2/M_3$ has to be positive.

This observation helps us to explain the results presented in Figure 4 for various values of the gaugino mass ratios and $M_S$ given in Table 1. With non-universal gaugino masses the little hierarchy problem is somewhat less severe compared to the CMSSM if the 75 dimensional representation of $SU(5)$ is employed. This case has $M_2 > M_3$ at $M_G$, with the ratio $M_2/M_3$ positive. Note that the 75-plet of $SU(5)$ can also lead to a natural solution of the doublet-triplet splitting problem [16]. Note, however, that the little hierarchy problem becomes more severe compared to CMSSM if we employ the 24-plet of $SU(5)$ or 54-plet of $SO(10)$.

We will compare the fine tuning measures in the CMSSM with the $SU(5)$ model containing the 75 representation. Employing the semi-analytic result for $M_Z^2$ and Eq.(6) with $(M_1, M_2, M_3) = (-5, 3, 1) M$, we can express the $\Delta_X$ parameters for the $SU(5)$ model as follows:

$$\Delta_M = 2.32 \hat{M}_Z^2 - 0.51 \hat{M} \hat{A}_{t_0},$$

$$\Delta_{A_{t_0}} = 0.2 \hat{A}_{t_0}^2 - 0.51 \hat{M} \hat{A}_{t_0},$$

$$\Delta_{\mu_{t_0}} = -2.04 \hat{\mu}_{t_0}^2,$$

where

$$\hat{M}_Z^2 = 2.32 \hat{M}_Z^2 + 0.2 \hat{A}_{t_0}^2 - 1.02 \hat{M} \hat{A}_{t_0} - 2.04 \hat{\mu}_{t_0}^2,$$

and $\hat{M}_a = M_a/M_Z$, $\hat{A}_{t_0} = A_{t_0}/M_Z$ and $\hat{\mu}_0 = \mu_0/M_Z$. 

Figure 3: Evolution of $A_t$ and $|m_{H_u}|$ for $|M_2/M_3| = 1.5$ at $M_G$, $M_S = 250$ GeV and $A_t/M_S = -\sqrt{6}$ at EW scale, which corresponds to $m_h = 119$ GeV.
Using Eqs. (18)-(21) and the semi-analytic relations in Eqs. (13)-(17), we calculate the degree of fine tuning and the SUSY parameters $M_1, M_2, M_3, m_Q(M_Z), m_U(M_Z), \mu(M_Z)$ and $A_t(M_Z)$ in the CMSSM and $SU(5)$ with 75-plet, for different values of $M_S$. We choose $m_h = 119$ GeV as an example. The results presented in Table 2 show a marginal improvement in fine tuning in $SU(5)$ relative to the CMSSM case.

4 Little Hierarchy Problem in $G_{422}$

In this section we will show that the little hierarchy problem is largely resolved if the MSSM is embedded in $G_{422}$ [9]. It seems natural to assume that in $G_{422}$ the asymptotic gaugino masses associated with $SU(4)_c, SU(2)_L$, and $SU(2)_R$ are three independent parameters. This number can be reduced from three to two in the presence of C (or D) parity [17]. (C parity interchanges left and right and simultaneously conjugates the representations). For instance, C invariance requires that the $SU(2)_L$ and $SU(2)_R$ gauge couplings are equal at the $SU(2)_R$
Table 2: Fine tuning in CMSSM with universal gaugino masses and in $SU(5)$ with non-universal gaugino masses. We set $m_h = 119$ GeV and $\tan\beta = 10$. All masses are in GeV.

| $r$ | 1 | 1 | 1 | 1 | 3 | 3 | 3 | 3 |
|-----|---|---|---|---|---|---|---|---|
| $M_S$ | 250 | 500 | 750 | 1000 | 250 | 500 | 750 | 1000 |
| $\left(\frac{100}{\text{max}(|\Delta X|)}\right)$% | 4% | 1.5% | 0.8% | 0.5% | 5.4% | 3% | 1.6% | 1% |
| $-A_c(M_Z)$ | 2.4 | 1.8 | 1.7 | 1.6 | 2.4 | 1.8 | 1.7 | 1.6 |
| $M_3$ | 167 | 279 | 398 | 518 | 159 | 262 | 372 | 477 |
| $M_2$ | 167 | 279 | 398 | 518 | 477 | 785 | 1117 | 1432 |
| $\mu(M_Z)$ | 443 | 664 | 898 | 1121 | 362 | 522 | 687 | 840 |
| $m_{Q_L}(M_Z)$ | 342 | 605 | 879 | 1153 | 426 | 724 | 1040 | 1354 |
| $m_{t_L}(M_Z)$ | 183 | 413 | 640 | 867 | 147 | 345 | 541 | 739 |

Table 2: Fine tuning in CMSSM with universal gaugino masses and in $SU(5)$ with non-universal gaugino masses. We set $m_h = 119$ GeV and $\tan\beta = 10$. All masses are in GeV.

Breaking scale, which we identify with the GUT scale $M_G$. Applying C-parity to the gaugino sector we can realize $SU(2)_L$ and $SU(2)_R$ gaugino mass unification at $M_G$, but the $SU(3)_c$ asymptotic gaugino mass is still independent. $G_{422}$ symmetry and C-parity imply the following asymptotic relation among the MSSM gaugino masses:

$$M_1 = \frac{2}{5} M_3 + \frac{3}{5} M_2,$$

(22)

where $M_3, M_2, M_1$ denote the $SU(3)_c, SU(2)_L$ and $U(1)_Y$ asymptotic gaugino masses respectively. In deriving Eq.(22), we used the relation $Y = \sqrt{\frac{2}{5}} (B - L) + \sqrt{\frac{3}{5}} I_{3R}$, where $B - L$ and $I_{3R}$ are the diagonal generators of $SU(4)_c$ and $SU(2)_R$ respectively.

Following our earlier discussion about the fine tuning measure (defined in Eq. (6)), the three relevant $\Delta X$ parameters are given by

$$\Delta_M = 0.4 (4.3 - r) (3.1 + r) \hat{M}^2 - 0.5 (0.7 + 0.17 r) \hat{M} \hat{A}_t,$$

(23)

$$\Delta_{A_t} = 0.2 \hat{A}^2_{t_0} - 0.5 (0.7 + 0.17 r) \hat{M} \hat{A}_t,$$

(24)

$$\Delta_{\mu} = -2.04 \hat{\mu}_0^2,$$

(25)

with

$$\hat{M}_Z^2 = 0.4 (4.3 - r) (3.1 + r) \hat{M}^2 + 0.2 \hat{A}^2_{t_0} - 0.7 + 0.17 r) \hat{M} \hat{A}_t - 2.04 \hat{\mu}_0^2,$$

(26)

where $r \equiv M_2/M_3$ and $M \equiv M_3$. Thus, $(M_1, M_2, M_3) = (\frac{2+3r}{5}, r, 1) M$. We expect to reduce $|m_{H_u}|$ for larger values of $r$. This can be seen from a comparison between $G_{422}$ model and CMSSM shown in Table 3. For $M_S \sim$ TeV the problem is largely overcome with 10% fine tuning in $G_{422}$ versus a fine tuning of 0.5% in the CMSSM. This is further exemplified in Figure 5. With $r$ in the interval $3.5 < r < 4.5$, and $M_S = 250$ GeV (top-left panel in Figure 5), the EW symmetry breaking condition requires only 10% cancellation, which may be regarded as modest amount of fine tuning.
Table 3: Fine tuning measure in CMSSM with universal gaugino masses and in $G_{422}$ with non-universal gaugino masses. We set $m_h = 119$ GeV and $\tan\beta = 10$. All masses are in GeV.

Before concluding, we present two examples of the Higgs and sparticle mass spectra which is predicted from the $G_{422}$ model with $\tan\beta \sim 10$. The data presented in Table 4 is generated using the software program SuSpect [18], and for these two examples the MSSM parameter $\mu$ is close to 200 GeV, only a factor of two larger than $M_Z$. The data is consistent with the low energy constraints such as $m_h \geq 114.4$ GeV, lightest chargino mass $m_{\tilde{\chi}_1^\pm} > 103.5$ GeV, and $2.85 \times 10^{-4} \leq BR(B \to X_s\gamma) \leq 4.24 \times 10^{-4} (2\sigma)$. The lightest neutralino (LSP) mass in this table has the right magnitude to account for the recent results reported by the PAMELA experiment [19], provided one assumes that the LSP is not absolutely stable but decays primarily into leptons with a lifetime $\sim 10^{26}$ sec [20]. Finally, it was recently shown in [21] that third family Yukawa unification and neutralino dark matter are fully consistent in a framework with $G_{422}$ compatible non-universal gaugino masses.
Figure 5: $|m_{H_u}|$ vs $r$ for $M_S = 250$ GeV, 500 GeV, 750 GeV and 1000 GeV, with $M_1 = \frac{2}{5}M_3 + \frac{3}{5}M_2$ and $M_2 = rM_3$. In all four plots the lower (dashed red) curve corresponds to $m_h = 114.4$ GeV, while the upper (dotted blue) curve corresponds to the maximum Higgs mass $m_h = 119$ GeV, 122 GeV, 123 GeV and 123 GeV respectively.

5 Conclusion

We have argued that the little hierarchy problem is ameliorated in supersymmetric models based on the gauge symmetry $G_{422} \equiv SU(4)_c \times SU(2)_L \times SU(2)_R$ supplemented by a discrete left-right symmetry (C-parity). We have also investigated $SU(5)$ and $SO(10)$ models in which non-universal gaugino masses can arise from dimension five operators. Based on these considerations some benchmark points depicting the Higgs and sparticle masses in $G_{422}$ are highlighted.
|               | Point 1 | Point 2 |
|---------------|---------|---------|
| $M_1$         | 891     | 1486    |
| $M_2$         | 1241    | 2071    |
| $M_3$         | 365     | 609     |
| $m_0$         | 100     | 100     |
| $\tan \beta$ | 10      | 10      |
| $A_0$         | -260    | -485    |
| $\mu$         | 190     | 190     |
| $m_h$         | 115.5   | 119.0   |
| $m_H$         | 834.2   | 1351.1  |
| $m_A$         | 834.0   | 1350.9  |
| $m_{H\pm}$    | 838.1   | 1353.6  |
| $m_{\tilde{\chi}^\pm_{1,2}}$ | 189.9, 999.6 | 193.2, 1673 |
| $m_{\tilde{\chi}^0_{1,2,3,4}}$ | 181.9, 195.8, 386.1, 999.6 | 189.4, 196.7, 649.3, 1673 |
| $m_{\tilde{g}}$ | 866.2   | 1390    |
| $m_{\tilde{u}_{L,R}}$ | 1081, 772.4 | 1752, 1228 |
| $m_{\tilde{t}_{1,2}}$ | 339.1, 995.6 | 510.4, 1588 |
| $m_{\tilde{d}_{L,R}}$ | 1083, 571.1 | 1753, 1190 |
| $m_{\tilde{b}_{1,2}}$ | 739.9, 973.2 | 1172, 1576 |
| $m_{\tilde{\nu}_{1,2,3}}$ | 823.3, 823.3, 820.8 | 1355, 1355, 1351 |
| $m_{\tilde{e}_{L,R}}$ | 826.9, 346.3 | 1357, 557.1 |
| $m_{\tilde{\tau}_{1,2}}$ | 333.8, 824.5 | 536.4, 1353 |

Table 4: Sparticle and Higgs masses (in GeV), with $M_t = 172.6$ GeV, $r = 3.4$ and 10% fine tuning.

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