The two-Pomeron eikonal approximation for the high-energy EDS of nucleons

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Abstract

It is demonstrated that the elastic diffractive scattering of nucleons at collision energies higher than 540 GeV and transferred momenta lower than 2 GeV, including the Coulomb-nuclear interference region, can be described in the framework of a very simple Regge-eikonal model where the eikonal is just a sum of two supercritical Regge pole terms. The predictive efficiency of the proposed approximation is verified.

1. Introduction

During the last several decades, perturbative quantum chromodynamics (pQCD) confirmed many times its usefulness as a powerful theoretical tool in the sector of high energies and high transferred momenta of strongly interacting particles. However, at present, a very large part of hadron physics cannot be treated in the framework of this quantum-field model. Particularly, to describe quantitatively the elastic diffractive scattering (EDS) of nucleons at high values of the collision energy and low values of the transferred momentum, we have to use phenomenological models which are not based on any analytic approximations within QCD. The absence of direct connection between these models and the fundamental theory of strong interaction very often reduces their predictive power. As a result, in 2011, many hadron diffraction models nicely described the available experimental data on the proton-(anti)proton EDS in the energy range from the ISR to Tevatron (with the collision energy increase in several tens of times), but failed completely to reproduce the behavior of the pp angular distribution in the region of the diffraction dip and non-forward peak at the LHC [1]. The discrepancy between the model predictions and the experimental data was so huge that it could lead to a justified conclusion about our total misunderstanding of the physical mechanisms of hadron diffraction (for detailed discussion, see mini-review [2]).

Nonetheless, the fraction of EDS events in the total number of events at high-energy hadron colliders is so significant (~ 25% at the LHC) that we still strongly need to develop reliable phenomenological approaches which could help to interpret properly the results of measurements in the high-energy hadron diffraction region. Of course, any model should be verifiable (and, certainly, discriminatable) on the available and forthcoming experimental data.

All the modern models of EDS could be divided into two groups: those ones based on Regge theory [3] and the non-Reggeon models. The references to various Reggeon and non-Reggeon phenomenological schemes can be found in a recent review [4] and in the RPP [5]. The aim

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of this paper is to verify the reliability of the two-Pomeron eikonal approximation proposed earlier in [6]. It will be done via application to the new EDS data sets produced recently by the TOTEM Collaboration [7].

2. The model

2.1. The strong interaction subamplitude

The physical content of the two-Pomeron eikonal approximation is very simple and transparent. In the kinematic range wherein the Coulomb interaction may be considered negligible, the eikonal for the high-energy EDS of nucleons can be represented as a sum of two supercritical Regge pole terms. The first term corresponds to exchange by the so-called soft Pomeron (SP). This interaction is the basic cause of the visible growth of the proton-proton total and elastic cross-sections at high energies. The second term corresponds to exchange by the hard Pomeron (HP), also known as the BFKL Pomeron. A detailed discussion of the considered model can be found in [6]. Below we just give a recipe for calculation of the EDS angular distributions in the framework of this approximation:

\[
\frac{d\sigma}{dt} = \frac{|T_N(s,t)|^2}{16\pi s^2}, \quad T_N(s,t) = 4\pi s \int_0^\infty d(b^2) J_0(b\sqrt{-t}) \frac{e^{2i\delta_N(s,b)} - 1}{2i},
\]

\[
\delta_N(s,b) = \frac{1}{16\pi s} \int_0^\infty d(-t) J_0(b\sqrt{-t}) \left[ \left[ \delta_{SP}(s,t) + \delta_{HP}(s,t) \right] \right] = \frac{1}{16\pi s} \int_0^\infty d(-t) J_0(b\sqrt{-t}) \times \left[ \xi_+ \left( \alpha_{SP}(t) \right) g_{SP}^2(t) \pi \alpha_{SP}'(t) \left( \frac{s}{2s_0} \right)^{\alpha_{SP}'(t)} + \xi_+ \left( \alpha_{HP}(0) \right) \beta_{HP}(t) \left( \frac{s}{2s_0} \right)^{\alpha_{HP}(0)} \right],
\]

where \(s\) and \(t\) are the Mandelstam variables, \(b\) is the impact parameter, \(s_0 = 1\) GeV\(^2\), \(J_0(x)\) is the Bessel function, \(\alpha_{SP}(t)\) is the SP Regge trajectory, \(g_{SP}(t)\) is the SP coupling to nucleon, \(\alpha_{HP}(0)\) is the intercept of the HP Regge trajectory (it was argued in [6] why the \(t\)-dependence of \(\alpha_{HP}(t)\) is negligible in the region of EDS), \(\beta_{HP}(t) \equiv g_{HP}^2(t) \pi \alpha_{HP}'(t)\) is the HP Regge residue, \(\xi_+ (\alpha) = (i + \tan \frac{\pi (\alpha - 1)}{2})\) are the signature factors for even Reggeons, \(\delta_N\) is the eikonal (Born amplitude), and \(T_N\) is the full amplitude related to strong interaction.

| Parameter | Value |
|-----------|-------|
| \(\alpha_{SP}(0) - 1\) | 0.109 |
| \(\tau_a\) | 0.535 GeV\(^2\) |
| \(g_{SP}(0)\) | 13.8 GeV |
| \(\alpha_g\) | 0.23 GeV\(^{-2}\) |
| \(\beta_{HP}(0)\) | 0.08 |
| \(b\) | 1.5 GeV\(^{-2}\) |
| \(\alpha_{HP}(0) - 1\) | 0.32 (FIXED) |

Table 1: The parameter values for (2) obtained earlier [10, 6] via fitting to the EDS data in the collision energy range 546 GeV \(\leq \sqrt{s} \leq 7\) TeV.

The HP intercept can be extracted from the data on the proton unpolarized structure function \(F_2^p(x, Q^2)\) [8] at high values of the incoming photon virtuality \(Q^2\) and low values of the Bjorken scaling variable \(x\): \(\alpha_{HP}(0) \approx 1.32\) [9]. In the region of low negative \(t\), the unknown
functions $\alpha_{SP}(t)$, $g_{SP}(t)$, and $\beta_{HP}(t)$ may be approximated with the help of the following simple test parametrizations [10, 6]:

$$
\alpha_{SP}(t) = 1 + \frac{\alpha_{SP}(0) - 1}{1 - \frac{t}{\tau_a}}, \quad g_{SP}(t) = \frac{g_{SP}(0)}{(1 - a_g t)^2}, \quad \beta_{HP}(t) = \beta_{HP}(0) e^{bt},
$$

(2)

where the free parameters take on the values presented in Table 1.

2.2. The impact of electromagnetic interaction

To describe the EDS of protons in the region of Coulomb-nuclear interference we need to take account of electromagnetic interaction.

In the framework of the eikonal approach, the full amplitude of the proton-(anti)proton EDS in the coordinate representation has the following structure:

$$
T(s, b) = e^{2i(\delta_C(s, b) + \delta_N(s, b))} - 1 = T_N(s, b) + \delta_C(s, b) + 2i T_N(s, b) \delta_C(s, b) + O(\alpha_e^2),
$$

(3)

where $\delta_C(s, b) \sim \alpha_e$ is the tree level subamplitude of electromagnetic interaction, and $\alpha_e$ is the fine structure constant.

At large enough values of the impact parameter, electromagnetic interaction dominates: $|\delta_C(s, b)| \gg |T_N(s, b)|$ and, hence, $T(s, b) \approx \delta_C(s, b)$. In its turn, at small values of $b$ the electromagnetic interaction of protons can be ignored: $T(s, b) \approx T_N(s, b)$. In the range $3 \text{ fm} < b < 10 \text{ fm}$, wherein $|\delta_C(s, b)| \sim |T_N(s, b)|$, we may ignore the third term in (3), because $|T_N(s, b)| \ll 1$ in this region (the fast decreasing of $|T_N(s, b)|$ can be seen in Fig. 1), and, consequently, this term is negligible as compared with the first two ones.

Thus, finally, we come to the leading approximation of the full (electromagnetic + strong) amplitude in the entire kinematic range of EDS:

$$
T(s, b) \approx \delta_C(s, b) + T_N(s, b), \quad \Rightarrow \quad T(s, t) \approx \delta_C(s, t) + T_N(s, t).
$$

(4)
In other words, we neglect the Coulomb-nuclear interference in the amplitude level.

It should be noted here that such a negligibility is a model-dependent effect. In the framework of many models, the corresponding terms are expected to yield a significant contribution into the full amplitude (for detailed discussion, see, say, [11] and references therein).

In the transferred momentum range $0 < \sqrt{-t} < 2$ GeV, the Coulomb term can be approximated by a simple expression

$$\delta_C(s, t) = \pm \frac{8 \pi s \alpha_e}{t} F_E^2(t),$$

where $F_E(t) = \left(1 - \frac{t}{0.71 \text{GeV}^2}\right)^{-2}$ is the dipole electric form-factor of proton.

3. Verification of the model

3.1. The model predictions versus the newest experimental data

To check the model efficiency, we need to compare the model predictions with the new data [7] on the proton-proton EDS at $\sqrt{s} = 13$ TeV. The results of such a verification (without refitting) are presented in Fig. 2. The data description quality in terms of the method of least squares is $\chi^2 = 1796$ over 428 points (the description quality in the range $\sqrt{-t} < 0.1$ GeV is $\chi^2 = 18$ over 25 points). Hereby, we observe a rather weak deviation of the model curves from the experimental data.

The model predictions for the $pp$ total cross-section and for $\rho = \frac{\text{Re} T_N(s,0)}{\text{Im} T_N(s,0)}$ at $\sqrt{s} = 13$ TeV are $\sigma_{\text{tot}}^{\text{model}}(13 \text{ TeV}) \approx 109.4$ mb and $\rho^{\text{model}}(13 \text{ TeV}) \approx 0.125$, while the corresponding measured values are $\sigma_{\text{tot}}(13 \text{ TeV}) = (110.5 \pm 2.4)$ mb and $\rho(13 \text{ TeV}) = 0.10 \pm 0.01$ [7]. It should be noted here that extraction of these quantities from the experimental angular distributions is a strongly model-dependent procedure.

![Figure 2: The model predictions [6] versus the TOTEM data at $\sqrt{s} = 13$ TeV [7]. The dashed line corresponds to the case $\delta_C(s, t) = 0.$](image)
3.2. The results of refitting

The next step is to refit the model parameters to the enlarged set of data. The results are presented in Tables 2 and 3 and Fig. 3.

The main cause of the observed slight discrepancy between the model issues and the experimental data is, certainly, the stiffness of the used parametrizations for the SP Regge trajectory and the SP coupling to proton. More flexible parametrizations could result in a better description of the data, though one should keep in mind that the true analytic behavior of \( \alpha_{SP}(t) \), \( g_{SP}(t) \), and \( \beta_{HP}(t) \) remains unknown. Nonetheless, the simplicity of test functions (2) makes them very attractive for usage in the region of low negative \( t \), while the achieved quality of description makes the model quite suitable for rough estimations and predictions of the nucleon-nucleon EDS observables at ultrahigh energies.

| Parameter | Value |
|-----------|-------|
| \( \alpha_{SP}(0) - 1 \) | 0.1055 |
| \( \tau_a \) | 0.572 GeV\(^2\) |
| \( g_{SP}(0) \) | 14.7 GeV |
| \( a_g \) | 0.32 GeV\(^{-2}\) |
| \( \beta_{HP}(0) \) | 0.108 |
| \( b \) | 1.55 GeV\(^{-2}\) |
| \( \alpha_{HP}(0) - 1 \) | 0.32 (FIXED) |

Table 2: The parameter values for (2) obtained via fitting to the EDS data in the entire kinematic range \( \{ 546 \text{ GeV} \leq \sqrt{s} \leq 13 \text{ TeV}, \sqrt{-t} < 2 \text{ GeV} \} \).

Figure 3: The differential cross-sections of nucleon-nucleon EDS at ultrahigh energies. The dashed lines correspond to the one-Pomeron eikonal approximation wherein the HP exchange contribution to the eikonal is ignored.
| $\sqrt{s}$, GeV | Number of points | $\chi^2$ |
|----------------|------------------|--------|
| $546$ ($\bar{p}p$; UA4) | 187 | 264 |
| $630$ ($\bar{p}p$; UA4) | 19 | 34 |
| $1800$ ($\bar{p}p$; E710) | 51 | 19 |
| $1960$ ($\bar{p}p$; D0) | 17 | 24 |
| $2760$ ($pp$; TOTEM) | 63 | 180 |
| $7000$ ($pp$; TOTEM, ATLAS) | 205 | 323 |
| $8000$ ($pp$; TOTEM, ATLAS) | 69 | 159 |
| $13000$ ($pp$; TOTEM) | 428 | 982 |
| **Total** | **1039** | **1985** |

Table 3: The quality of description of the data [1, 7, 12] on the EDS angular distributions.

### 4. Conclusion

In the light of the aforesaid, we may conclude that the two-Pomeron eikonal approximation successfully confirmed its predictive efficiency and, thus, it can be considered as a simple and reliable phenomenological tool for qualitative description of the nucleon-nucleon EDS at ultrahigh energies. The test functions $\alpha_{SP}(t)$ and $g_{SP}(t)$ with the free parameter values fitted to the available data can be used in the framework of Reggeon models for more compound reactions, such as high-missing-mass single diffractive dissociation (SDD) or central exclusive production (CEP) of low-mass neutral mesons.

### References

[1] The TOTEM Collaboration, Europhys. Lett. **95** (2011) 41001
The TOTEM Collaboration, Europhys. Lett. **101** (2013) 21002

[2] A.A. Godizov, *arXiv*: 1203.6013v3 [hep-ph]

[3] P.D.B. Collins, *An Introduction to Regge Theory & High Energy Physics*. Cambridge University Press 1977

[4] L. Jenkovszky, R. Schicker, and I. Szanyi, Int. J. Mod. Phys. E **27** (2018) 1830005

[5] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98** (2018) 030001 (page 596)

[6] A.A. Godizov, Phys. Rev. D **96** (2017) 034023

[7] The TOTEM Collaboration, *arXiv*: 1812.04732 [hep-ex]
The TOTEM Collaboration, *arXiv*: 1812.08283 [hep-ex]

[8] www.desy.de/h1zeus/combined_results
The H1 and ZEUS Collaborations, JHEP **1001** (2010) 109

[9] A.A. Godizov, Nucl. Phys. A **927** (2014) 36

[10] A.A. Godizov, Eur. Phys. J. C **75** (2015) 224

[11] V.A. Petrov, Eur. Phys. J. C **78** (2018) 221
[12] UA4 Collaboration (M. Bozzo et al.), Phys. Lett. B 147 (1984) 385
UA4 Collaboration (M. Bozzo et al.), Phys. Lett. B 155 (1985) 197
UA4 Collaboration (D. Bernard et al.), Phys. Lett. B 198 (1987) 583
UA4 Collaboration (D. Bernard et al.), Phys. Lett. B 171 (1986) 142
E-710 Collaboration (N. Amos et al.), Phys. Lett. B 247 (1990) 127
D0 Collaboration, Phys. Rev. D 86 (2012) 012009
ATLAS Collaboration, Nucl. Phys. B 889 (2014) 486
The TOTEM Collaboration, Nucl. Phys. B 899 (2015) 527
ATLAS Collaboration, Phys. Lett. B 761 (2016) 158
The TOTEM Collaboration, arXiv: 1812.08610 [hep-ex]