How strong is the evidence for accelerated expansion?

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Abstract. We test the present expansion of the universe using supernova type Ia data without making any assumptions about the matter and energy content of the universe or about the parametrization of the deceleration parameter. We assume the cosmological principle to apply in a strict sense. The result strongly depends on the data set, the light curve fitting method and the calibration of the absolute magnitude used for the test, indicating strong systematic errors. Nevertheless, in a spatially flat universe there is at least 5σ evidence for acceleration which drops to 1.8σ in an open universe.

Keywords: dark energy theory, supernova type Ia, classical tests of cosmology

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1. Introduction

The first evidence for an accelerated expansion of the universe at the present epoch was presented in the late 1990s by Riess et al [1] and Perlmutter et al [2]. Fitting a cosmological model including a cosmological constant matched the type Ia supernovae of the data sets considered significantly better than a model without such a constant. In the following years, a lot of effort was put into getting larger supernova (SN) sets of increasing quality [3]–[6]. On the other hand, a variety of cosmological models have been developed that could be fitted to the newest data sets. These models are characterized by certain parameters which are used to derive a redshift–luminosity relation that can be compared to the observed values. The problem with this so-called dynamical approach is that it is impossible to test without assumptions on the matter and energy content of the universe whether there really is a phase of acceleration in the expansion history of the universe.

Some authors tried to avoid this problem by taking a kinematical approach, i.e. they only considered the scale factor $a$ and its derivatives, such as the deceleration parameter $q(z)$, without using any model specific density parameters or a dark energy equation of state. The first kinematical analysis of SN data was done by Turner and Riess [7] who considered averaged values $q_1$ for redshift $z < z_1$ and $q_2$ for $z > z_1$ concluding that a present acceleration and a past deceleration are favoured by the data. Other authors tested a variety of special parametrizations of $q(z)$ [8, 9] or used $a(t)$ [10] or the Hubble rate $H(z)$ [11]. Rather than considering a special parametrization, Shapiro and Turner [12] took a more general approach: they expanded the deceleration parameter $q$ in principal components. Rapetti et al [13] expanded the jerk parameter $j$ in a series of orthonormal functions. However, one has to be careful when doing a series expansion in $z$ as SNe with a redshift larger than 1 are not within the radius of convergence. This problem can be solved by reparametrizing the redshift [14].

In the present work we will not make any assumptions about the content of the universe, or about the parametrization of $q(z)$ or another kinematical quantity. Moreover,
we do not need to assume the validity of Einstein’s equations. Instead, we will only ask the question of whether the hypothesis holds that the universe never expanded in an accelerated way between the time when the light was emitted from a SN and today. Our assumption for the test is that the universe is isotropic and homogeneous. This is certainly not true for the real universe in a strict sense. However, we follow the standard approach in assuming that the cosmic structure does not modify the observed SN magnitudes and redshifts apart from as regards random peculiar motion. The basic idea of this analysis has already been presented by Visser [15] and has been applied to SN data by different groups [16, 17]. In contrast to those previous works, ours considers calibration effects, quantifies the evidence for accelerated expansion and studies systematic effects.

For the fit one usually marginalizes over a function of the absolute magnitude $M$ and the Hubble constant $H_0$ because these two values cannot be determined independently by only considering SNe. Marginalization is not suitable for our analysis because in order to do that a special cosmological model or at least a parametrization of $q(z)$ has to be inserted, which is what we want to avoid. But the SNe can be calibrated using cepheid measurements and thus the values of $M$ and $H_0$ are determined. Using this additional information, no marginalization is needed. As there is still controversy [18] as regards the appropriate calibration method, we will take two quite different results of calibration into account [19, 20].

In order to test the robustness of our analysis, we consider two different SN data sets (the 2007 Gold sample [5] and the ESSENCE set [6]), where the data of the ESSENCE set are once obtained by means of the multicolour light curve shape (MLCS2k2) [3] fitting method and once by using the spectral adaptive light curve template (SALT) [21]. Both calibration methods are applied to each set. The analysis shows that in all cases the data indicate an accelerated expansion. But the confidence level at which acceleration can be stated strongly depends on the data set, the fitting method and the calibration. We begin our analysis by assuming a flat universe, but later also consider the cases of open and closed universes.

2. Method

We want to keep our test as model independent as possible, but still a few assumptions have to be made. We consider inflationary cosmology to be correct. This implies large scale homogeneity and isotropy of the universe as well as spatial flatness. (Yet, we will give up the assumption of a flat universe later in section 5.) We also assume that cosmological averaging (here along the line of sight) does not modify the result obtained in a Friedmann–Lemaître model. In such a universe, the luminosity distance $d_L(z)$ is given by

$$d_L(z) = (1 + z) \int_0^z \frac{dz}{H(z)}.$$  

As we are interested in the question of whether the universe accelerates or decelerates at the present epoch, we need to examine the deceleration parameter $q(z)$: if $q$ is positive, the universe expands in a decelerated way; for a negative $q$ it accelerates. $q(z)$ can be expressed in terms of the Hubble parameter $H(z)$:

$$q(z) = \frac{H'(z)}{H(z)} (1 + z) - 1,$$  

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where the prime denotes the derivative with respect to $z$. Integrating this equation yields

$$\ln \frac{H(z)}{H_0} = \int_0^z \frac{1 + q(\tilde{z})}{1 + \tilde{z}} d\tilde{z}. \quad (3)$$

Our null hypothesis is that the universe never expanded in an accelerated way, i.e. $q(z) \geq 0$ for all $z$. Under this assumption, the above equation turns into the inequality

$$\ln \frac{H(z)}{H_0} \geq \int_0^z \frac{1}{1 + \tilde{z}} d\tilde{z} = \ln(1 + z) \quad (4)$$

or $H(z) \geq H_0(1 + z)$. Thus, for the luminosity distance we have

$$d_L(z) \leq (1 + z) \frac{1}{H_0} \int_0^z \frac{d\tilde{z}}{1 + \tilde{z}} = \left(1 + z\right) \frac{1}{H_0} \ln(1 + z). \quad (5)$$

In order to test the null hypothesis, we consider different data sets of SNe type Ia. If the observed luminosity distance is significantly larger than the luminosity distance of a universe with a constant $q = 0$, the hypothesis can be rejected at a high confidence level. Note that the rejection of the hypothesis does not mean that there was no deceleration between the time the light was emitted from a SN and now; it only gives evidence that there was at some time a phase of acceleration. Thus, we cannot determine the transition redshift between deceleration and acceleration. This restriction to our analysis comes from the integral over redshift in the calculation of $d_L(z)$. On the other hand, fulfilling inequality (5) does not imply the null hypothesis.

As a first step, the data in the sets considered have to be calibrated consistently. The distance modulus $\mu$ is related to the luminosity distance by

$$\mu = m - M = 5 \log d_L + 25, \quad (6)$$

where $d_L$ is given in units of Mpc. The distance modulus of the SNe Ia in those sets is always given in arbitrary units because the absolute magnitude $M$ cannot be measured independently of the Hubble constant $H_0$. Only the apparent magnitude $m$ and thus the relative distance moduli are measured at high precision. In order to determine the absolute magnitude, the SNe have to be calibrated by measuring the distance of cepheids in the host galaxies. Then also the Hubble constant can be determined. But there is still disagreement between different groups about the correct calibration analysis. In this work we will consider two results for $M$ and $H_0$, namely that of Riess et al [19] and that of Sandage et al [20]. In the following we will refer to those results as Riess calibration and Sandage calibration, respectively. The difference in calibration comes mainly from the different assumptions on the evolution of the cepheid period-luminosity relation with host metallicity [18]. So for preparing the data for our analysis, first the distance moduli $\mu_i$ have to be adjusted to the assumed absolute magnitude.

We define the magnitude $\Delta \mu_i$ as the observed distance modulus $\mu_i$ of the $i$th SN minus the distance modulus in a universe with constant deceleration parameter $q = 0$ at the same redshift $z_i$:

$$\Delta \mu_i = \mu_i - \mu(q = 0) = \mu_i - 5 \log \left[ \frac{1}{H_0} (1 + z_i) \ln(1 + z_i) \right] - 25. \quad (7)$$
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If the error in redshift and the peculiar velocities of the SNe are already included in the error \( \sigma_{\mu_i} \) given in a certain data set, then the error \( \sigma_i \) of \( \Delta \mu_i \) equals \( \sigma_{\mu_i} \). Otherwise, the resulting error of \( \Delta \mu_i \) is calculated using

\[
\sigma_i = \sqrt{\frac{5}{(1 + z_i) \ln(1 + z_i) \ln 10}} \left( \sigma_{\mu_i}^2 + \sigma_v^2 \right)^{1/2}.
\]  

(8)

Let \( \mu_{\text{th}}(z) \) be the theoretical distance modulus of a cosmological model, which describes the expansion of the universe correctly. Then

\[
\Delta \mu_{\text{th}}(z_i) = \mu_{\text{th}}(z_i) - \mu(q = 0)
\]

would be the ‘true’ value corresponding to the measured SN value \( \Delta \mu_i \). The null hypothesis for our test is that the universe never expanded in an accelerated manner, i.e. \( \Delta \mu_{\text{th}} \leq 0 \) for each SN. We reject that hypothesis if the measured value \( \Delta \mu_i \) lies above a certain action limit \( A_a \); otherwise we accept it. We want to keep the risk low that we conclude a late time acceleration of the universe when there is in fact no acceleration at all. Therefore the action limit must be relatively high. If we want the confidence level for concluding an accelerated expansion to be 99%, the action limit must be \( A_a(99\%) = 2.326 \sigma_i \). For a confidence level of 95%, the limit is \( A_a(99\%) = 1.645 \sigma_i \). Here we assume that the measured values \( \Delta \mu_i \) at a given redshift follow a normal distribution.

On the other hand, we can test the hypothesis that the universe expanded in an accelerated manner all the time from light emission of a SN until today. This hypothesis can be rejected at a 99% CL if \( \Delta \mu_i \) is below the action limit \( A_d(99\%) = -2.326 \sigma_i \). That would mean that there must have been a phase of deceleration in the late time universe, but it would not exclude a phase of acceleration.

3. Data sets

The important parameters of the SNe are obtained by fitting their light curves. The results depend on the fitter that is used. The most common fitter is MLCS2k2 [3]. Here we will also consider the SALT fitting method [21]. The main difference between the two methods is in how the peak luminosity corrections are determined by the colour of the SNe [22]. While MLCS2k2 assumes that the corrections that have to be made are only due to dust, SALT takes an empirical approach to determining the relation between the colour and the luminosity correction.

For the analysis we take the following SN Ia data sets:

Gold 2007 (MLCS2k2): the Gold sample of Riess et al [5], which was obtained by using the MLCS2k2 fitting method;

ESSENCE (MLCS2k2): the set given by Wood-Vasey et al [6], which includes data from ESSENCE, SNLS [4] and nearby SNe [3], fitted with MLCS2k2;

ESSENCE (SALT): the same set fitted with SALT.

As suggested by Riess et al [5], we discarded all SNe with a redshift of \( z < 0.0233 \) from the Gold sample, which leaves us 182 SNe with \( 0.0233 < z < 1.755 \). In ESSENCE (MLCS2k2) and ESSENCE (SALT), the SNe with bad light curve fits were rejected for each fitting method separately. This leaves 162 SNe for ESSENCE (MLCS2k2) and 178
for ESSENCE (SALT) with $0.015 < z < 1.01$. 153 of the SNe are contained in both sets. Due to the differences of the SNe contained in ESSENCE (MLCS2k2) and ESSENCE (SALT) we will refer to them as different sets in the following. Nevertheless, you should keep in mind that they share a large number of SNe and thus are not independent sets.

The Riess calibration yields a value for the V-band magnitude of $M_V(t_0) = -19.17 \pm 0.07$ mag, where $t_0$ is the time of the B-band maximum. For the Gold 2007 set $M_V(t_0)$ was considered to be $-19.44$ mag. Therefore, in order to get the appropriate distance modulus $\mu = m - M$, we need to subtract 0.27 mag from the value given in the Gold sample. From the distance modulus values in the ESSENCE sets 0.22 mag have to be subtracted because in these sets a B-band magnitude of $M_B = -19.5$ mag is assumed and Riess et al [19] give the relation $M_B - M_V = -0.11$. For this calibration one gets a Hubble constant of $H_0 = 73 \pm 4$(statistical) $\pm 5$(systematic).

The results of Sandage et al [20] for the SNe absolute magnitudes are $M_V = -19.46$ mag and $M_B = -19.49$ mag and for the Hubble constant $H_0 = 62.3 \pm 1.3$(statistical) $\pm 5.0$(systematic). So considering the Sandage calibration, we have to add 0.02 mag to distance moduli of the Gold sample and subtract 0.01 mag from the values given in the ESSENCE sets.

4. Results for a flat universe

4.1. Single SNe

The relevant parameter for our analysis is $\Delta \mu_i$ given by equation (7). Its values are plotted in figure 1 for the SNe of the three data sets and the two different calibration methods. The curve for a flat $\Lambda$CDM model (with $\Omega_m = 0.3$) is shown in the plots in order to give the reader a notion of how the redshift dependence of $\Delta \mu$ could look. Most of the data values are positive which indicates an accelerated expansion. A similar diagram has been presented by Gong et al [17] for single SNe as well as for binned SN data where they used a combined Gold and ESSENCE data set. They kept the arbitrary value of the absolute magnitude $M$ given in the set and then determined the Hubble constant by using SNe with $z \leq 0.1$ to be $H_0 = 66.04$. At this point they stopped their analysis by concluding that due to the large number of SNe that lie above the curve of a universe with $q = 0$, accelerated expansion is evident. But the fact that there are also several data points below that curve and that the errors $\sigma_i$ (with a typical value of 0.23 mag for MLCS2k2 and 0.18 mag for SALT) are approximately of the same size as the values $\Delta \mu_i$ gives rise to the question of how certain acceleration really is. Thus, we will provide a more quantitative analysis in this work.

We counted the SNe of each set whose values of $\Delta \mu_i$ were above the action limit $A_a(95\%) = 1.645\sigma_i$ and those above $A_a(99\%) = 2.326\sigma_i$ (which indicates acceleration at a 95\% CL and at a 99\% CL, respectively) and those with values below $A_d(95\%) = -1.645\sigma_i$ and below $A_d(99\%) = -2.326\sigma_i$ (which indicates deceleration). The results are shown in table 1. The clearest evidence for an accelerated expansion is given by the ESSENCE (SALT) set with the Riess calibration with 64 SNe indicating acceleration at a 99\% confidence level and none indicating deceleration. Also for the other sets accelerated expansion is preferred—with the exception of ESSENCE (MLCS2k2) for the Sandage calibration where neither of the two expansion histories is preferred in the test using the action limit $A(99\%)$. 

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Figure 1. Magnitude $\Delta \mu$, as defined in equation (7), for the three data sets and the two calibrations. SNe that indicate acceleration at 99% CL are plotted in red ($\times$), those that indicate deceleration in blue ($\times$). Also shown is the curve for a flat $\Lambda$CDM model with $\Omega_m = 0.3$ (which is not a fit to the data). Note that the SALT method leads to a larger spread in $\Delta \mu$, whereas the Gold set extends to higher redshifts.
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Figure 2. Differences of apparent magnitudes obtained by the SALT and the MLCS2k2 fitting methods (a) and these differences divided by $\sigma_i$ (b).

Table 1. Number of SNe indicating acceleration or deceleration at 95% and 99% CL for the different data sets and calibrations. Also given is the total number of SNe in each set. The most different results are highlighted.

|               | Gold 2007 (MLCS2k2) | ESSENCE (MLCS2k2) | ESSENCE (SALT) |
|---------------|---------------------|-------------------|----------------|
|               | Riess Sandage       | Riess Sandage     | Riess Sandage  |
| Acceleration (95% CL) | 37 27               | 37 14             | 96 53          |
| Acceleration (99% CL) | 13 7                | 11 3              | 64 30          |
| Deceleration (95% CL) | 2 4                 | 3 7               | 1 7            |
| Deceleration (99% CL) | 0 1                 | 2 3               | 0 2            |
| Number of SNe  | 182 182             | 162 162           | 178 178        |

It is noticeable that the two light curve fitting methods for the ESSENCE set yield very different results. (For a discussion on these differences at low redshifts, see [22].) This could be an indication that at least one of the two fitting methods does not give the correct result. In order to consider this possibility we take a look at the differences between $\Delta \mu_i$ obtained by SALT and $\Delta \mu_i$ obtained by MLCS2k2. They are shown in figure 2(a) for the 153 SNe contained in both ESSENCE sets. For some SNe the difference is very large (namely up to one magnitude). In order to see whether the two sets are consistent, we need to know how large the difference is in terms of the error $\sigma_i = [\sigma_i^2(\text{SALT}) + \sigma_i^2(\text{MLCS2k2})]^{1/2}$. The result is shown in figure 2(b). We want the systematical error due to the different fitting methods to be smaller than the statistical error of the observational data. Thus, we discard all SNe with a difference in $\Delta \mu_i$ larger than $1\sigma_i$ and those that are only contained in one of the two ESSENCE sets, which leaves 129 SNe in the sets. Again we counted the SNe indicating acceleration or deceleration for each set and each calibration method. The result is shown in table 2. The outcome of this test did not change qualitatively on discarding suspicious SNe from the sets. Thus, we will use all the SNe of the ESSENCE sets in the following.
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Table 2. Number of SNe indicating acceleration or deceleration for SNe of the ESSENCE sets with $|\Delta \mu_i(\text{SALT}) - \Delta \mu_i(\text{MLCS2k2})|/\sigma_i \leq 1$. The total number of SNe in each set is 129. Again, the most discrepant results are highlighted.

|                   | ESSENCE (MLCS2k2) | ESSENCE (SALT) |
|-------------------|-------------------|----------------|
|                   | Riess  | Sandage | Riess  | Sandage |
| Acceleration (95% CL) | 33     | 11     | 69     | 30     |
| Acceleration (99% CL) | 8      | 2      | 40     | 14     |
| Deceleration (95% CL) | 1      | 2      | 0      | 6      |
| Deceleration (99% CL) | 0      | 1      | 0      | 1      |

Table 3. Number of SNe indicating acceleration or deceleration for SNe of the ESSENCE sets with different peculiar velocity dispersions $\sigma_v$.

| $\sigma_v$ (km s$^{-1}$) | ESSENCE (MLCS2k2) | ESSENCE (SALT) |
|--------------------------|-------------------|----------------|
|                          | Riess  | Sandage | Riess  | Sandage |
| 300 Acceleration (95% CL) | 37     | 14     | 99     | 53     |
| 300 Acceleration (99% CL) | 11     | 3      | 64     | 30     |
| 300 Deceleration (95% CL) | 3      | 11     | 1      | 7      |
| 300 Deceleration (99% CL) | 2      | 3      | 0      | 2      |
| 400 Acceleration (95% CL) | 37     | 14     | 96     | 53     |
| 400 Acceleration (99% CL) | 11     | 3      | 64     | 30     |
| 400 Deceleration (95% CL) | 3      | 7      | 1      | 7      |
| 400 Deceleration (99% CL) | 2      | 3      | 0      | 2      |
| 500 Acceleration (95% CL) | 36     | 13     | 95     | 53     |
| 500 Acceleration (99% CL) | 10     | 3      | 63     | 30     |
| 500 Deceleration (95% CL) | 3      | 7      | 1      | 5      |
| 500 Deceleration (99% CL) | 2      | 3      | 0      | 2      |
| Number of SNe            | 162    | 162    | 178    | 178    |

The error of the distance modulus contains the peculiar velocities $v_{pec}$ of the SNe. In the ESSENCE sets $\sigma_v$ is assumed to be 400 km s$^{-1}$ for all SNe. We wanted to know how sensitive our test is with respect to changes of the peculiar velocity. Table 3 shows that varying $\sigma_v$ almost does not change the result—with the exception of for ESSENCE (MLCS2k2) for the Sandage calibration, where the number of SNe indicating deceleration at a 95% CL is 7 for $\sigma_v = 400$ and 500 km s$^{-1}$, but 11 for $\sigma_v = 300$ km s$^{-1}$.

4.2. Averaging over SN data

Until now, we have only made tests for single SNe. The problem in combining these data is that the quantity $\Delta \mu$ depends on the redshift $z$. Thus the data from different redshifts do not have the same mean value. Nevertheless it is possible to average over $\Delta \mu_i$. Then the result of course depends on how many SNe from a certain redshift are used to calculate this mean value. Therefore, the average over all SNe of a set does not characterize the
function $\Delta \mu(z)$. But still, when combined with its standard deviation, the mean value can give evidence for acceleration or deceleration. Thus, we define

$$\overline{\Delta \mu} = \frac{\sum_{i=1}^{N} g_i \Delta \mu_i}{\sum_{i=1}^{N} g_i},$$

where $g_i = 1/\sigma_i^2$. The mean value is calculated in such a way that data points with a small error are weighted more than those with large errors. The standard deviation of the mean value is calculated by

$$\sigma_{\Delta \mu} = \left[ \frac{\sum_{i=1}^{N} g_i (\Delta \mu_i - \overline{\Delta \mu})^2}{(N-1) \sum_{i=1}^{N} g_i} \right]^{1/2}.$$  

We start with averaging $\Delta \mu_i$ over redshift bins of width 0.2. For all data sets the averaged value of $\Delta \mu$ increases with redshift (see figure 3) which could be expected for an accelerated expansion. The curve for a flat $\Lambda$CDM with $\Omega_m = 0.3$ has its maximum at $z = 1.2$ and then decreases again. Unfortunately, there are not enough SNe at high redshifts to state a possible decrease of $\Delta \mu$ at some point. The differences in data points of the two ESSENCE sets again seem to be too large. MLCS2k2 gives smaller values than the SALT fitter for all redshift bins. As $\Delta \mu(z)$ should be close to zero at low redshifts in a homogeneous and isotropic universe, the MLCS2k2 data look more consistent with our assumptions for the Riess calibration, whereas SALT gives the better results for the Sandage calibration. The Gold sample has sensible values for both calibrations. $\overline{\Delta \mu}$ divided by the error $\sigma_{\Delta \mu}$ gives evidence for acceleration in each redshift bin. As can be seen in table 4, the strongest evidence is given for redshifts between 0.4 and 0.8.

Next we average over all SNe of each data set with a redshift $z \geq 0.2$. We discard SNe with a smaller redshift for the following reasons:

(a) $\Delta \mu(z)$ is expected to be relatively close to zero for small redshifts. Therefore, nearby SNe do not contribute to the evidence for acceleration.
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Table 4. Statistical evidence $\Delta \mu / \sigma_{\Delta \mu}$ within the given redshift range for a flat and an open universe.

| $z$       | Gold 2007 (MLCS2k2) | ESSENCE (MLCS2k2) | ESSENCE (SALT) |
|-----------|---------------------|-------------------|---------------|
|           | Riess    | Sandage | Riess     | Sandage     | Riess    | Sandage |
|           |          |         |           |             |          |         |
| Flat universe |         |         |           |             |          |         |
| 0.0–0.2   | 2.1      | 0.4     | 1.6       | −2.8        | 5.7      | 0.8     |
| 0.2–0.4   | 4.2      | 2.9     | 3.9       | 0.7         | 7.3      | 4.3     |
| 0.4–0.6   | 9.8      | 7.6     | 7.1       | 3.1         | 12.1     | 7.7     |
| 0.6–0.8   | 5.4      | 4.0     | 7.7       | 3.7         | 8.5      | 4.8     |
| 0.8–1.0   | 4.8      | 3.5     | 6.8       | 4.1         | 6.0      | 4.2     |
| 1.0–1.2   | 2.8      | 2.2     |           |             |          |         |
| 1.2–1.4   | 2.3      | 1.8     |           |             |          |         |
| Open universe |         |         |           |             |          |         |
| 0.0–0.2   | 2.1      | 0.4     | 1.5       | −2.8        | 5.7      | 0.7     |
| 0.2–0.4   | 3.5      | 2.2     | 3.2       | −0.0        | 6.6      | 3.6     |
| 0.4–0.6   | 7.4      | 5.2     | 5.5       | 1.4         | 10.2     | 5.8     |
| 0.6–0.8   | 2.8      | 1.4     | 4.9       | 0.9         | 5.9      | 2.2     |
| 0.8–1.0   | 1.4      | 0.2     | 3.9       | 1.2         | 4.0      | 2.2     |
| 1.0–1.2   | 0.9      | 0.4     |           |             |          |         |
| 1.2–1.4   | −0.2     | −0.7    |           |             |          |         |

Table 5. Mean values and standard deviations of $\Delta \mu$ obtained by using only SNe with $z \geq 0.2$ for a flat universe.

|          | Gold 2007 (MLCS2k2) | ESSENCE (MLCS2k2) | ESSENCE (SALT) |
|----------|---------------------|-------------------|---------------|
|          | Riess    | Sandage | Riess     | Sandage | Riess    | Sandage |
|          |          |         |           |         |          |         |
| $z$      | 0.63     | 0.63    | 0.54      | 0.54    | 0.51     | 0.51    |
| $\Delta \mu$ | 0.2196 | 0.1655  | 0.2398    | 0.1056  | 0.3457   | 0.2115  |
| $\sigma_{\Delta \mu}$ | 0.0167 | 0.0167  | 0.0201    | 0.0201  | 0.0203   | 0.0203  |
| $\Delta \mu / \sigma_{\Delta \mu}$ | 13.1  | 9.9     | 11.9      | 5.2     | 17.0     | 10.4    |

(b) Considering only the nearby universe, local effects can modify the results for the distance modulus as the cosmological principle is not valid on small scales.

(c) Another disadvantage of nearby SNe is that they were observed with many different telescopes and thus the systematic error obtained is potentially higher.

Table 5 shows the mean values $\overline{\Delta \mu}$ and their standard deviations $\sigma_{\overline{\Delta \mu}}$ evaluated by using only SN data with $z \geq 0.2$. $\overline{\Delta \mu}$ is positive for all data sets. $\overline{\Delta \mu}$ divided by $\sigma_{\overline{\Delta \mu}}$ indicates the confidence level at which an accelerated expansion can be stated. The weakest evidence for acceleration is given for ESSENCE (MLCS2k2) for the Sandage calibration. Here the mean value lies $5.2\sigma$ above 0, i.e. above the value for a universe that neither accelerates nor decelerates. In the other cases the confidence level is even larger, up to $17.0\sigma$ for ESSENCE (SALT) for the Riess calibration.
5. Open and closed universes

Although there are good reasons to believe that the universe is flat we give up this assumption in the following section. In an open universe the luminosity distance of a universe that neither accelerates nor decelerates is given by

\[ d_L(z) = \frac{1 + z}{H_0 \sqrt{\Omega_k}} \sinh \left( \sqrt{\Omega_k} \ln(1 + z) \right) \]  

(12)

and thus depends on the density parameter of the scalar curvature \( \Omega_k \). \( d_L \) increases with increasing \( \Omega_k \) and thus the evidence for acceleration becomes weaker. As we are interested in the lower limit of this evidence we have to take the highest possible value for the scalar curvature, i.e. \( \Omega_k = 1 \), which corresponds to an empty universe. (Here we allow ourselves to make use of the Einstein equation.) Then equation (7) for \( \Delta \mu_i \) changes to

\[ \Delta \mu_i = \mu_i - \mu(q = 0) = \mu_i - 5 \log \left[ \frac{1}{H_0} (1 + z_i) \sinh[\ln(1 + z_i)] \right] - 25. \]  

(13)

The evidence for accelerated expansion is then calculated in the same way as for a flat universe. The result obtained by averaging over redshift bins is shown in table 4. For the overall average we only used SNe between redshifts 0.2 and 1.2 because including higher redshifts would weaken the evidence. This is due to the fact that the values of \( \Delta \mu_i \) become negative when the phase of deceleration within the redshift range over which it is integrated is large enough. The result is shown in table 6. As could be expected, the evidence is now much weaker than for a flat universe. But we still find a hint of acceleration at 1.8\( \sigma \).

For a closed universe we have a different situation: here \( d_L \) decreases with increasing spatial curvature. Thus the lower limit of the evidence for acceleration is given in the case of the lowest possible curvature which corresponds to a flat universe.

6. Conclusion

We tested the cosmic expansion without specifying any density parameters or parametrizing kinematical quantities. This was not possible without assuming a certain calibration of \( M \) and \( H_0 \). Therefore, we considered two very different calibrations. We also considered two different data sets and two different light curve fitters and varied the peculiar velocity dispersion. Using single SNe for the test has already given a...
clear indication of acceleration in a flat universe although a few SNe strongly favour deceleration. We find large systematic effects already at the level of individual SNe.

Similar analyses have already been done by other groups [16, 17]. But note that the work of Santos et al [16] exhibits two major shortcomings. First, they did not calibrate the SN data consistently. They took a certain value for $H_0$ but kept the arbitrary value of the absolute magnitude $M$ given in the set which led to distance moduli that are too high. Thus, they concluded erroneously that there was a recent phase of super-acceleration. The second problem is that they did not realize that due to the integration over redshift this method is not suitable for determining the transition redshift $z_t$ between deceleration and acceleration. Thus, their value of $z_t$ does not hold. The shortcoming of the work of Gong et al [17] is that they have not applied any statistics at all.

Instead of only considering single SNe, we obtained a more significant result by averaging over the SN data of each set. Here it is justified to discard nearby SNe in order to decrease systematics from using different telescopes or from local effects. We argue that we reject the null hypothesis of no acceleration for a spatially flat, homogeneous and isotropic universe at high confidence ($>5\sigma$). However, the results show large differences for each data set, calibration and light curve fitter. For example, changing the calibration from Sandage to Riess calibration in a flat universe using ESSENCE(MLCS2k2) increases the evidence for acceleration from $5.2\sigma$ to $11.9\sigma$. For the two different light curve fitters applied to the ESSENCE SNe using the Riess calibration, we get values of $11.9\sigma$ and $17.0\sigma$.

Wood-Vasey et al argue that the differences due to different light curve fitters are not important as they disappear when marginalization is applied [6]. This is true if the data are only used to determine the parameter values of a certain cosmological model, as in this case no calibration of $M$ and $H_0$ is needed. But nevertheless, the fitters should give the same result for a given absolute magnitude $M$. If this is not the case, a systematic error in the determination of the apparent magnitude $m$ is introduced by at least one of the fitters. Systematics due to different fitters have also been described in [22] and those within the Gold sample in [23]–[25].

The different results obtained with the two calibrations are very surprising. Our test only depends on the reduced distance modulus $\mathcal{M} = M - 5\log H_0 + 25$ and not on the absolute magnitude or the Hubble constant individually. (Note that this is the quantity which is marginalized when fitting cosmological parameters.) Starting from different calibrations $M_1$ and $M_2$ leads to different values of $H_0$, determined by observation of SNe, in such a way that $\mathcal{M}$ should in principle be the same for both values of $M$. Thus our test should not depend on the calibration. However, the fact that the calibration changes our result significantly can be explained as follows: $H_0$ is determined by observing SNe. Riess et al and Sandage et al use different sets of SNe and different fitting methods for this determination. The systematic errors due to different sets and fitters then of course also influence the result for the Hubble constant leading to different values $\mathcal{M}_1$ and $\mathcal{M}_2$.

For a conservative conclusion, we need to take the set that gives the weakest evidence for acceleration, namely ESSENCE (MLCS2k2) for the Sandage calibration. Using this set, accelerated expansion can be stated at $5\sigma$ if we assume a spatially flat or closed universe. For an open universe the evidence is much weaker, namely $1.8\sigma$. These results only hold if the correct analysis of SNe is somewhere in the range of the cases we considered here. But as we observe enormous systematic effects in our test, we cannot be sure of this.
assumption. Thus, it is a major issue to better understand how SNe have to be observed and analysed.

Remember that the results of this work are only valid if the universe is homogeneous and isotropic. The situation changes dramatically if we give up those assumptions. There exist enormously large structures in the universe; an example is the Sloan Great Wall with an extent of $\sim 400$ Mpc \cite{26}. Besides, there are also big voids and superclusters on a 100 Mpc scale. This roughness of the universe could spoil the validity of equation (1). Indeed it is possible to construct inhomogeneous models that can describe the observational data without the need of an average acceleration \cite{27}–\cite{30}.

Some observational evidence for inhomogeneity and anisotropy in SN Hubble diagrams has recently been presented by \cite{31,32}, probably due to large scale bulk motion and perhaps systematic effects. A non-trivial scale dependence of the Hubble rate due to the so-called cosmological backreaction \cite{33}–\cite{38} has been shown in \cite{39}. This effect of averaging can also mimic curvature effects \cite{40} and thus strongly influence the reconstruction of the dark energy equation of state, as has been shown in \cite{41}.

Thus we think a major task must be to establish the acceleration of the universe independently of the assumption of strict homogeneity and isotropy.

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References

\[1\] Riess A et al, Observational evidence from supernovae for an accelerating universe and a cosmological constant, 1998 Astron. J. 116 1009 [SPIRES] [astro-ph/9805201]

\[2\] Perlmutter S et al, Measurements of $\Omega$ and $\Lambda$ from 42 high-redshift supernovae, 1999 Astrophys. J. 517 565 [SPIRES] [astro-ph/9812133]

\[3\] Jha S, Riess A and Kirshner R, Improved distances to type Ia supernovae with multicolor light curve shapes: MLC2k2, 2007 Astrophys. J. 659 122 [SPIRES] [astro-ph/0612666]

\[4\] Astier P et al, The Supernova Legacy Survey: measurement of $\Omega_M$, $\Omega_\Lambda$ and $w$ from the first year data set, 2006 Astron. Astrophys. 447 31 [SPIRES] [astro-ph/0510447]

\[5\] Riess A et al, New Hubble space telescope discoveries of type Ia supernovae at $z > 1$: narrowing constraints on the early behavior of dark energy, 2007 Astrophys. J. 659 98 [SPIRES] [astro-ph/0611572]

\[6\] Wood-Vasey W et al, Observational constraints on the nature of the dark energy: first cosmological results from the ESSENCE supernova survey, 2007 Astrophys. J. 666 694 [SPIRES] [astro-ph/0701041]

\[7\] Turner M and Riess A, Do SNe Ia provide direct evidence for past deceleration of the universe?, 2002 Astrophys. J. 569 18 [SPIRES] [astro-ph/0106051]

\[8\] Riess A et al, Type Ia supernova discoveries at $z > 1$ from the Hubble space telescope: evidence for past deceleration and constraints on dark energy evolution, 2004 Astrophys. J. 607 665 [SPIRES] [astro-ph/0402512]

\[9\] Elgarøy O and Multamäki T, Bayesian analysis of Friedmannless cosmologies, 2006 J. Cosmol. Astropart. Phys. JCAP09(2006)002 [SPIRES] [astro-ph/0603053]

\[10\] Wang Y and Tegmark M, Uncorrelated measurements of the cosmic expansion history and dark energy from supernovae, 2005 Phys. Rev. D 71 103513 [SPIRES] [astro-ph/0501351]

\[11\] John M, Cosmography, decelerating past, and cosmological models: learning the bayesian way, 2005 Astrophys. J. 630 667 [SPIRES] [astro-ph/0506284]

\[12\] Shapiro C and Turner M, What do we really know about cosmic acceleration?, 2006 Astrophys. J. 649 563 [SPIRES] [astro-ph/0512586]

\[13\] Rapetti D, Allen S, Amin M and Blandford R, A kinematical approach to dark energy studies, 2007 Mon. Not. R. Astron. Soc. 375 1510 [astro-ph/0605683]
How strong is the evidence for accelerated expansion?

[14] Cattoën C and Visser M, The Hubble series: convergence properties and redshift variables, 2007 Preprint 0710.1887
[15] Visser M, General relativistic energy conditions: the Hubble expansion in the epoch of galaxy formation, 1997 Phys. Rev. D 56 7578 [SPIRES]
[16] Santos J, Alcaniz J, Pires N and Rebouças M, Energy conditions and cosmic acceleration, 2007 Phys. Rev. D 75 083523 [SPIRES] [astro-ph/0702728]
[17] Gong Y, Wang A, Wu Q and Zhang Y-Z, Direct evidence of acceleration from distance modulus redshift graph, 2007 J. Cosmol. Astropart. Phys. JCAP08(2007)018 [SPIRES] [astro-ph/0703583]
[18] Jackson N, The Hubble constant, 2007 Liv. Rev. Rel. 10 4 [0709.3924]
[19] Riess A et al, Cepheid calibrations from the Hubble space telescope of the luminosity of two recent type Ia supernovae and a re-determination of the Hubble constant, 2005 Astrophys. J. 627 579 [SPIRES] [astro-ph/0503159]
[20] Sandage A, Tamman G, Saha A, Reinell B, Macchetto F and Panagia N, The Hubble constant: a summary of the HST program for the luminosity calibration of type Ia supernovae by means of cephids, 2006 Astrophys. J. 653 843 [astro-ph/0603647]
[21] Guy J, Astier P, Nobili S, Regnault N and Pain R, SALT: a spectral adaptive light curve template for type Ia supernovae, 2005 Astron. Astrophys. 443 781 [SPIRES] [astro-ph/0506583]
[22] Conley A, Carlberg R, Guy J, Howell D, Jha S, Riess A and Sullivan M, Is there evidence for a Hubble bubble? The nature of type Ia supernova colors and dust in external galaxies, 2007 Astrophys. J. 664 L13 [SPIRES] [0705.0367]
[23] Jassal H K, Bagla J S and Padmanabhan T, Observational constraints on low redshift evolution of dark energy: how consistent are different observations?, 2005 Phys. Rev. D 72 103503 [SPIRES]
[24] Jassal H K, Bagla J S and Padmanabhan T, The vanishing phantom menace, 2006 Preprint astro-ph/0601389
[25] Nesseris S and Perivolaropoulos L, Tension and systematics in the Gold06 SNeIa dataset, 2007 J. Cosmol. Astropart. Phys. JCAP02(2007)025 [SPIRES] [astro-ph/0610331]
[26] Enqvist K and Mattsson T, The effect of inhomogeneous expansion on the supernova observations, 2007 J. Cosmol. Astropart. Phys. JCAP02(2007)019 [SPIRES] [astro-ph/0609120]
[27] Ishak M, Richardson J, Whittington D and Garred D, Dark energy or apparent acceleration due to a relativistic cosmological model more complex than FLRW?, 2007 Preprint 0708.2943
[28] McClure M L and Dyer C C, Anisotropy in the Hubble constant as observed in the HST extragalactic bubble? The nature of type Ia supernova colors and dust in external galaxies, 2007 Astrophys. J. 664 L13 [SPIRES] [0705.0367]
[29] Alnes H and Amarzguioui M, The supernova Hubble diagram for off-center observers in a spherically symmetric inhomogeneous universe, 2006 Phys. Rev. D 75 023506 [SPIRES] [astro-ph/0510331]
[30] Enqvist K and Mattsson T, The effect of inhomogeneous expansion on the supernova observations, 2007 J. Cosmol. Astropart. Phys. JCAP02(2007)019 [SPIRES] [astro-ph/0609120]
[31] Basseì B A et al, Is the dynamics of tracking dark energy detectable?, 2007 Preprint 0709.0526