 Observable Gravity Waves From U(1)$_{B-L}$ Higgs and Coleman-Weinberg Inflation

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We present a realistic non-supersymmetric inflation model based on a gauged U(1)$_{B-L}$ symmetry and a tree-level Higgs potential. The inflaton is identified with the scalar field which spontaneously breaks U(1)$_{B-L}$, and we include radiative corrections à la Coleman-Weinberg in the inflaton potential. If the scalar spectral index $n_s$ lies close to 0.96, as indicated by the recent Planck and WMAP 9-yr measurements, the tensor-to-scalar ratio $r$, a canonical measure for gravity waves, exceeds 0.01. Thus, according to this model, gravity waves should be found in the near future. In this case, the quantity $|dn_s/d\ln k|$ lies in the range $0.004 - 0.005$. Successful baryogenesis can be realized in this class of models either via thermal or non-thermal leptogenesis.

The highly successful Standard Model (SM) of the strong, weak and electromagnetic interactions possesses an accidental global U(1)$_{B-L}$ symmetry at the renormalizable level. This symmetry can be upgraded to an anomaly free local gauge symmetry by introducing three SM singlet (right-handed) neutrinos.

Within the framework of supersymmetric hybrid inflation $[1, 2]$, the symmetry breaking scale of U(1)$_{B-L}$ in a scenario with a renormalizable superpotential and minimal Kahler potential is estimated to be around $1 - 2 \times 10^{15}$ GeV $[3, 4]$. This version of supersymmetric U(1)$_{B-L}$ inflation also predicts that the tensor-to-scalar ratio $r$, a canonical measure of primordial gravity waves, lies many orders of magnitudes below the observable capabilities ($r \lesssim 0.02$) of Planck and other contemporary measurements. On the other hand, within a more elaborate framework with a non-minimal Kahler potential, the quantity $r$ is found to lie in the observable range with an appropriate choice of parameters $[5]$.

In this paper we implement inflation within the context of the minimal $B-L$ extension of the SM, which consists of the SM supplemented by three right-handed (RH) neutrinos and a complex scalar carrying two units of the minimal U(1)$_{B-L}$ symmetry and a tree-level Higgs potential. The inflaton is identified with the scalar field which spontaneously breaks U(1)$_{B-L}$ to $Z_2$. Associated with the $B - L$ gauge symmetry breaking, the RH neutrinos acquire their Majorana mass and the seesaw mechanism $[6]$ is automatically implemented to yield the tiny neutrino mass.

The inflationary phase is linked to U(1)$_{B-L}$ breaking and is driven by an appropriate Higgs potential $[7, 8]$, with radiative corrections arising from the inflaton couplings to RH neutrinos, SM Higgs doublet and U(1)$_{B-L}$ vector gauge boson also taken into account $[9]$. By requiring that the scalar spectral index $n_s$ lies close to 0.96, as determined by the recent Planck $[10]$ and WMAP 9-years $[11]$ measurements, we are able to provide a lower bound $r \gtrsim 0.01$. Thus, according to this model of inflation, primordial gravity waves should be found in the near future. Note that the U(1)$_{B-L}$ symmetry breaking scale has transPlanckian value, but the cosmic strings associated with the spontaneous breaking of U(1)$_{B-L}$ are inflated away. For a somewhat different model of U(1)$_{B-L}$ inflation involving non-minimal coupling to gravity, see Ref. $[12]$.

| $\Phi$ | 1 | 1 | 0 | +2 |

TABLE I: Particle content. In addition to the SM particle content, there are three right-handed neutrinos $\nu_R^i (i = 1, 2, 3)$ denotes the generation index) and a complex scalar $\Phi$.

The presence of RH neutrinos with direct couplings to the inflaton is naturally compatible with either thermal $[13]$ or non-thermal $[14]$ leptogenesis.

We consider a minimal $B - L$ extension of the SM based on the gauge group SU(3)$_c \times$SU(2)$_L \times U(1)_{Y-B-L}$, and the particle content are listed in Table 1. Here, three generations of right-handed neutrinos ($\nu_R^i$) are introduced in order to make the model free from all gauge and gravitational anomalies. The VEV of the SM singlet scalar ($\Phi$) breaks the U(1)$_{B-L}$ gauge symmetry, and at the same time generates masses for the right-handed neutrinos.

The Lagrangian relevant for the seesaw mechanism is given as

$$L \supset -Y_{\nu D}^i \bar{\nu}_R^i H^\dagger \nu_L^i - \frac{1}{2} \frac{v^2}{2} |\Phi|^2 |\Phi|^2 + h.c.,$$

where the first term generates the Dirac neutrino mass term after electroweak symmetry breaking, while the right-handed neutrino Majorana mass term is generated through the second term associated with the $B - L$ gauge symmetry breaking. Without loss of generality, we work in a basis where the second term is diagonalized and $Y_{\nu R}^i$ are real and positive.

The tree level potential of the model is given by

$$V_{\text{tree}} = \lambda (\Phi^\dagger \Phi - \frac{v^2}{2})^2 + \lambda_{\text{mix}} (\Phi^\dagger \Phi)(H^\dagger H) + V_H,$$
\( \lambda_{\text{mix}} \) is sufficiently small (this is justified later), and so it can be ignored in our analysis for inflation.

In our numerical analysis, we employ the renormalization group improved effective potential at the 1-loop level. We identify the inflaton with the real part of the \( B - L \) Higgs field \( (\phi = \sqrt{2} v H) \), and parameterize the effective potential in the leading-log approximation as

\[
V = \lambda \left[ \frac{1}{4} (\phi^2 - v_{\text{BL}}^2) + a \log \left( \frac{\phi}{v_{\text{BL}}} \right) \phi^4 + V_0 \right],
\]

where \( a = \frac{\beta_\lambda}{16 \pi^2} \) with

\[
\beta_\lambda = 20 \lambda^2 + 2 \lambda_{\text{mix}}^2 + 2 \lambda \left( \sum_i (Y_i')^2 - 24 g_{\text{BL}}^2 \right) + 96 g_{\text{BL}}^4 - \sum_i (Y_i')^4.
\]

Here we have fixed the renormalization scale to be \( v_{\text{BL}} \). In the presence of quantum corrections at the 1-loop level \( (a \neq 0) \), the potential minimum is shifted from its tree level location \( (v_{\text{BL}}) \), and a constant potential energy density \( (V_0) \) is suitably chosen so as to reproduce the observed (almost vanishing) cosmological constant at the potential minimum. In our analysis, only three free parameters \( \{ \lambda, v_{\text{BL}}, a \} \) are involved.

The inflation takes place as the inflaton slowly rolls down to the potential minimum from an initial VEV smaller than the VEV at the potential minimum. The inflationary slow-roll parameters are given by

\[
\epsilon(\phi) = \frac{1}{2} m_P^2 \left( \frac{V'}{V} \right)^2, \quad \eta(\phi) = m_P^2 \left( \frac{V''}{V} \right),
\]

\[
\zeta^2(\phi) = m_P^4 \left( \frac{V V'''}{V^2} \right),
\]

where \( m_P = 2.4 \times 10^{18} \) GeV is the reduced Planck mass, and a prime denotes a derivative with respect to \( \phi \). The slow-roll approximation is valid as long as the conditions \( \epsilon \ll 1, |\eta| \ll 1 \) and \( \zeta^2 \ll 1 \) hold. In this case, the scalar spectral index \( n_s \), the tensor-to-scalar ratio \( r \), and the running of the spectral index \( \alpha = \frac{d n_s}{d \ln k} \) are given by

\[
n_s \simeq 1 - 6 \epsilon + 2 \eta, \quad r \simeq 16 \epsilon,
\]

\[
\alpha = \frac{d n_s}{d \ln k} \simeq 16 \epsilon \eta - 24 \epsilon^2 - 2 \zeta^2.
\]

The number of e-folds after the comoving scale \( k \) has crossed the horizon is given by

\[
N_k = \frac{1}{\sqrt{2} m_P} \int_{\phi_k}^{\phi_c} \frac{d \phi}{\sqrt{\epsilon(\phi)}},
\]

where \( \phi_k \) is the field value at the comoving scale \( k \), and \( \phi_c \) denotes the value of \( \phi \) at the end of inflation, defined by \( \max(\epsilon(\phi_c), |\eta(\phi_c)|, \zeta^2(\phi_c)) = 1 \). The amplitude of the curvature perturbation \( \Delta_R \) is given by

\[
\Delta_R^2 = \frac{V}{24 \pi^2 m_P^2 \epsilon} \bigg|_{\phi_k},
\]

which should satisfy \( \Delta_R^2 = 2.215 \times 10^{-9} \) from the Planck measurement \([10]\) with the pivot scale chosen at \( k_0 = 0.05 \) Mpc\(^{-1}\).

In our parameterization of the effective potential of Eq. (3), the slow roll parameters as well as the number of e-foldings only depend on two parameters, \( v_{\text{BL}} \) and \( a \). Thus, the predictions for \( n_s \), \( r \) and \( d n_s/d \ln k \) are given for fixed values of \( v_{\text{BL}} \) and \( a \), while the quartic coupling constant \( \lambda \) is determined so as to satisfy \( \Delta_R^2 = 2.215 \times 10^{-9} \). Fig. 1 shows the predicted values of \( n_s \) and \( r \) for various values of \( v_{\text{BL}} \) and \( a \) with the number of e-foldings \( N_e = 60 \), along with the 68% and 95% CL contours from the Planck measurement \([10]\). Each thick red contour from left to right corresponds to the results with \( v_{\text{BL}}/m_P = 10, 11, 12, 13, 14, 15, 17, 20, 30, 50 \) and 500, respectively. Along each contour for a fixed \( v_{\text{BL}} \), the predicted values of \( n_s \) and \( r \) change from top to bottom with \( a \) being varied from \( a = -0.2 \) to \( a = 1000 \). The dashed line denotes the prediction with \( a = 0 \) for various \( v_{\text{BL}} \) values in the range of \( 10 m_P \leq v_{\text{BL}} \leq 500 m_P \). It is known \([8, 15]\) that in the limit \( v_{\text{BL}} \rightarrow \infty \), the predictions coincide with those of the \( m^2 \phi^2 \) chaotic inflation model, \( (n_s, r) \simeq (0.967, 0.132) \).

The corresponding results for the predicted values of the running of the spectral index \( \alpha \) are depicted in Fig. 2. For the best fit value of \( n_s \simeq 0.96, r \) is predicted to exceed 0.01, so that primordial gravity waves should be found in the near future. In this case, \( -0.005 \leq \alpha \leq -0.004 \), which is consistent with the Planck measurement \([10]\), \( \alpha = -0.0134 \pm 0.018 \).

\[
\text{FIG. 1: Predicted values of } n_s \text{ and } r \text{ with various values of } v_{\text{BL}}, \text{ and } a, \text{ for } N_e = 60, \text{ along with the 68\% and 95\% CL contours from the Planck measurement} \([10]\) (Planck+WP: gray, Planck+WP+BAO: red, Planck+WP+highL: blue). Each thick red contour from left to right corresponds to \( v_{\text{BL}}/m_P = 10, 11, 12, 13, 14, 15, 17, 20, 30, 50 \) and 500, respectively. On each contour for a fixed \( v_{\text{BL}} \), the predicted values change from top to bottom as \( a \) varies in the range of \( -0.2 \leq a \leq 1000 \). The dashed line denotes the prediction with \( a = 0 \) for various \( v_{\text{BL}} \) values, from \( v_{\text{BL}} = 10 m_P \) to 500 \( m_P \).
\]

For \( a \gg 1 \), the effective potential is dominated by the 1-loop corrections, which corresponds to the Coleman-Weinberg potential \([16, 17]\) and the B-L symmetry breaking occurs radiatively. In this case, we assume that the \( B - L \) gauge coupling \( (g_{\text{BL}}^4 \text{ term}) \) dominates in Eq. (3). On the other
hand, for \( a < 0 \), we assume that the Yukawa coupling \((Y_i^N)^4\) term dominates in Eq. (4). These assumptions will be justified later. For \( a < 0 \), the effective potential is unbounded from below for \( \phi \to \infty \), and thus we implicitly assume that our universe sits at a local minimum. We find a lower bound \( a \gtrsim -0.2 \) for the local minimum to exist.

After inflation is over, the inflaton decays to the SM particles with subsequent thermalization of the universe. To discuss the post inflationary scenario, we first calculate the mass spectrum of the model. In the Coleman-Weinberg limit \((a \gg 1)\), we have found that for \( v_{BL} \gg m_P \), the inflaton mass (calculated by the second derivative of the effective potential at the minimum) and the \( B - L \) gauge boson mass are almost independent of \( v_{BL} \):

\[
\begin{align*}
m_{\phi} &\simeq 10^{13} \text{ GeV}, \\
m_{Z'} &\simeq 2v_{BL}(\phi) \simeq 10^{17} \text{ GeV},
\end{align*}
\]

where \( \langle \phi \rangle \simeq v_{BL} \) is the inflaton VEV at the potential minimum. The mass spectrum is determined by the constraint on \( \Delta R \), almost independently of \( v_{BL} \). Since \( m_{\phi} \ll m_{Z'} \), and \( \lambda \ll g_{BL}^2 \), the assumption that the \( B - L \) gauge coupling dominates in Eq. (4) is justified. As is well-known, this condition is necessary for the Coleman-Weinberg mechanism for radiative symmetry breaking to work [10].

Next we estimate the reheat temperature from the inflaton decay to a pair of SM Higgs bosons through the coupling \( \lambda_{mix} \) in Eq. (4). The decay width is given by

\[
\Gamma(\phi \to hh) = \frac{\lambda_{mix}^2 \langle \phi \rangle^2}{32\pi m_{\phi}}.
\]

In order for our calculation with the small decay width approximation to be reliable, we have a constraint on \( \lambda_{mix} \) from the requirement that \( \Gamma(\phi \to hh) \ll m_{\phi} \). The maximum value of \( \lambda_{mix} \) is obtained via \( \Gamma(\phi \to hh) = m_{\phi} \), so that \( \lambda_{mix}^{Max} = \sqrt{2\pi m_{\phi}/\langle \phi \rangle} \). A numerical analysis results in the relation \( \lambda_{mix}^{Max} \sim g_{BL}^2 \) for \( 10 \leq m_P \leq 500 \text{ m}_P \). Thus, \( \lambda_{mix} \ll \lambda_{mix}^{Max} \) is negligible in Eq. (4), justifying the assumption. We estimate the reheat temperature by

\[
\Gamma(\phi \to hh) = H = \sqrt{\frac{\pi^2}{90}} \frac{T_{RH}^2}{g_* m_P},
\]

and find \( T_{RH}^{Max} \simeq 10^{15} \text{ GeV} \), almost independently of \( v_{BL} \) in the limit of \( \lambda_{mix} = \lambda_{mix}^{Max} \). This is the maximum value of the reheat temperature.

If the reheat temperature is sufficiently high for the right-handed Majorana neutrinos to be in thermal equilibrium, the thermal leptogenesis scenario [13] works to reproduce the observed baryon asymmetry. We can choose appropriate values for \( Y_{N}^2 \) to realize the condition for thermal leptogenesis to work [18].

\[
10^{10} \text{ GeV} \lesssim M_{N_i} < T_{RH} < T_{RH}^{Max},
\]

where \( M_{N_i} = Y_{N}^2 \langle \phi \rangle / \sqrt{2} \) is the Majorana mass of the RH neutrinos. Therefore, in our inflation scenario, baryogenesis via (thermal) leptogenesis is successful.

When \( \lambda_{mix} \) is negligibly small, the inflaton dominantly decays to a pair of RH neutrinos with a decay width

\[
\Gamma(\phi \to \nu_R \nu_R) = \frac{Y_{N}^2}{64\pi} m_{\phi},
\]
This reheat temperature is much smaller than the RH neutrino mass $M_{N_i} = Y_N^i (\phi) / \sqrt{2}$ with a transPlanckian value of $\langle \phi \rangle$. In this case, we apply non-thermal leptogenesis [14] to realize baryogenesis, where the RH neutrinos produced by the inflation subsequently decay and create the observed baryon asymmetry in the universe.

For the case with $a = 0.2$, we perform the same analysis with the assumption that the Yukawa coupling ($(Y_N^3)^2$ term) dominates in Eq. (4). For simplicity, we assume $Y_N^1, Y_N^2 \ll Y_N^3$ and then calculate the third generation RH neutrino mass as a function of $\nu_{BL}$. We find the mass spectrum

$$m_\phi \simeq 10^{13} \text{GeV},$$

$$M_{N_3} = \frac{Y_N^3 (\phi)}{\sqrt{2}} \simeq 10^{17} \text{GeV},$$

(15)

almost independently of $\nu_{BL}$. This mass spectrum justifies the assumption that the Yukawa coupling ($(Y_N^3)^2$ term) dominates in Eq. (4).

As in the Coleman-Weinberg limit, we assume the inflaton dominantly decays to a pair of the SM Higgs bosons and find $\lambda_{mix}^{max} \sim (Y_N^3)^2$, so that $\lambda_{mix} \ll \lambda_{mix}^{max}$ in Eq. (13) is negligible, justifying the assumption again. Since we have $T_{RH}^{max} \simeq 10^{15}$ GeV, the same as in the Coleman-Weinberg limit, we can suitably arrange the value for $Y_N^i (i = 1, 2)$ to realize the condition for thermal leptogenesis to work,

$$10^{10} \text{GeV} \lesssim M_{N_1}, M_{N_2} < T_{RH} \ll T_{RH}^{max} \lesssim M_{N_3},$$

(16)

As previously discussed for the Coleman-Weinberg limit, we may consider non-thermal leptogenesis (for a successful baryogenesis) if $\lambda_{mix}$ is negligibly small and the inflaton dominantly decays to the RH neutrinos of the first and second generations.

For comparison, we also present the results for the number of e-foldings $N_e = 50$ in Figs.3 and 4. The predicted $r$ values are larger than those for the $N_e = 60$ case and a small portion of the parameter space lies inside the 68% CL contours from the Planck measurement. We find that the resultant mass spectrum is quite similar to the case for $N_e = 60$ and the post inflationary scenario is also essentially the same.

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