Supernova electron capture rates on odd-odd nuclei

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At densities between $10^9$ and $10^{10}$ g/cm$^3$ electron capture in a presupernova collapse is believed to mainly occur on odd-odd nuclei. We have derived the rates for six of the most important electron capturing nuclei, $^{54,56,58}$Mn and $^{56,58,60}$Co, based on calculations of the Gamow-Teller strength distributions for the ground states and first excited states. These calculations have been performed by shell model diagonalization in the $pf$ shell using a recently modified version of the KB3 interaction. The shell model rates are noticeably smaller than the presently adopted rates as the latter have been derived by placing the Gamow-Teller (GT) resonance at too low excitation energies.

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If the core of a massive star exceeds the appropriate Chandrasekhar mass, electron degeneracy pressure cannot longer stabilize the center and it collapses. In this early stage of the collapse electron capture plays an essential role. At first, it reduces the number of leptons per baryons $Y_e$ and hence the pressure which the electron gas can stem against the collapse. Secondly, the densities are still low enough for the neutrinos, produced by the electron capture process, to leave the star and thus to carry some energy away and cool the core. Thus, both effects conspire to accelerate the collapse. The importance of electron capture for the presupernova collapse is for example discussed in [4].

Core collapse models employ the electron capture rates by Fuller, Fowler and Newman (FFN) [4] who have systematically estimated the rates for nuclei in the mass range $A = 45 - 60$. The FFN rates are derived from two distinct contributions. At first the authors estimated the Gamow-Teller (GT) contribution to the rate by a parametrization on the basis of the independent particle model. The rate estimate has then been completed by an empirical contribution placed at zero excitation energy simulating low-lying transitions. After experimental evidence suggested that the GT strength is quenched with respect to the independent particle model, the FFN rates have been updated by Aufderheide et al. [6] by quenching of the Gamow-Teller strength by an overall factor of two. Furthermore these authors simulated the low-lying transitions by the same $ft$-value for all nuclei, while FFN adopted specific values for individual nuclei.

Using their own rate estimates, Aufderheide et al. have ranked the most important electron capturing nuclei – defined by the product of the abundance of a given nucleus and its electron capture rate – along a stellar trajectory for core collapse densities $\rho = 10^7 - 10^{10}$ g/cm$^3$. They find that for densities $\rho > 10$ ($\rho_T$ measures the density in $10^7$ g/cm$^3$) electrons are most effectively captured by odd-odd nuclei. In particular, with increasing density, $^{54}$Mn, $^{60}$Co and $^{58}$Mn are subsequently the top-ranked nuclei, which decrease $Y_e$ by electron capture most effectively. It is important to note that the FFN rates agree with those of Ref. [4] within a factor of two, thus also showing the dominance of electron capture by odd-odd nuclei in this stage of the collapse. It is the aim of this paper to show that this finding results from a misplacement of the Gamow-Teller resonance position in the parametrizations used by FFN [4] and Aufderheide et al. [6].

Unfortunately there exists no experimental information about the Gamow-Teller strength distribution for odd-odd nuclei in the $pf$-shell. Therefore our conclusions have to be entirely based on theory. As our theoretical model of choice we adopt the interacting shell model. Recent progress allows now for virtually converged calculations of the Gamow-Teller strength for all nuclei in the $pf$ shell [6]. In fact, it has been shown that the shell model studies reproduce all measured GT distributions for nuclei in the mass range $A = 50 - 64$, which is important for the core collapse phase we are concerned with here [6] (also see [7]). The nuclei, for which GT data exist, comprise both even-even ones (e.g. $^{54,56,58}$Fe, $^{58,60,64}$Ni) and odd-A nuclei ($^{51,52}$V, $^{55}$Mn, $^{59}$Co). For the following discussion it is important to note that the calculations [6,7], in concordance with data [2,3], showed systematic misplacements of the GT resonance strength in the parametrizations used by FFN [4] and subsequently by Aufderheide et al. [6]. These authors placed the centroid of the GT strength at too low excitation energies in the daughter nuclei for electron capture on odd-A nuclei, while they assumed too high excitation energies for capture on even-even nuclei.

Motivated by the successful application to even-even and odd-A nuclei [6,7,12] we assume that the interacting shell model will also describe the GT distribution for odd-odd nuclei well. Thus we have calculated the GT strength distribution for the six odd-odd nuclei $^{54,56,58}$Mn and $^{56,58,60}$Co on the basis of a shell model diagonalization approach in the $pf$ shell. As residual interaction we adopted the recently modified version of the KB3 interaction which corrects the slight inefficiencies in the KB3 in-
teraction around the $N = 28$ subshell closure \cite{4}. In fact the modified KB3 interaction i) reproduces all measured GT strength distributions very well and ii) describes the experimental level spectrum of the nuclei studied here quite accurately \cite{3,4}. Due to the very large m-scheme dimensions involved, the GT strength distributions have been calculated in truncated model spaces which fulfills the Ikeda sum rule and allowed a maximum of 4 particles from the lowest independent particle model configuration to be excited from the $f_{7/2}$ shell to the rest of the $pf$ shell in the final nucleus. At this level of truncation the GT strength distribution is virtually converged and the total GT strength agrees with the exact value typically within 10%.

As $0\hbar \omega$ shell model calculations, i.e. calculations performed in one major shell, overestimate the experimental GT strength by a universal factor \cite{13,14}, we have scaled our GT strength distribution by this factor, $(0.74)^2$.

We have performed 33 Lanczos iterations for each final angular momentum, which are usually sufficient to converge in the states at excitation energies below $E = 3$ MeV. At higher excitation energies, $E > 3$ MeV, the calculated GT strength represents centroids of strengths, which in reality are splitted over many states. For calculating the electron capture rate, however, a resolution of this strength at higher energies is unimportant.

Once the GT distributions are known the electron capture rate can be calculated as outlined in \cite{3,4}. We note, however, that in the core collapse environment the capture process occurs at finite temperature ($T \approx (4-7) \cdot 10^9$ K at the densities we are concerned with here \cite{1}). Thus we have included in our rate calculations also the capture from thermally excited states in the parent nucleus at excitation energies below 1 MeV. As the ground state spin of the even-even daughter nuclei is $I = 0$ (which is often strongly mismatched with the spins of the low-lying states in the odd-odd parent), we have included for all parent nuclei capture from at least one $1^+$ state. In $^{56}$Co the lowest excited $J = 1$ state is at 1.71 MeV. For the excitation energies we have used the experimental values rather than the shell model results, although they usually agree within 100 keV. Further, if the energy of a specific final state is known experimentally we have used this value. For the mass splittings between daughter and parent nucleus we adopt the experimental values.

In Fig. 1 we have plotted the GT strength distributions for the ground states of the odd-odd nuclei $^{54,56,58}$Mn and $^{56,58,60}$Co. As the ground states of these nuclei have spin $J \neq 0$, GT transitions can lead to final states with angular momentum $J - 1, J$ and $J + 1$; the figure shows the three individual strength distributions. In Fig. 2 we compare the ground state GT distribution with those of the excited states, adopting $^{54}$Mn as a typical example. Several observations can be derived from the two figures.

As the most striking feature we find that the shell model places the centroid of the GT distribution at higher energies than adopted in the parametrizations of FFN \cite{3} and Aufderheide et al. \cite{1}. To be more quantitative we have calculated the GT centroids $E_{GT}$ for the various ground states, averaged over the three possible final states, and find $E_{GT} = 7.15$ MeV ($^{54}$Mn), 5.9 MeV ($^{56}$Mn), 5.5 MeV ($^{58}$Mn), 8.2 MeV ($^{56}$Co), 7.35 MeV ($^{58}$Co) and 6.35 MeV ($^{60}$Co). These values are typically more than 2 MeV higher than the parametrizations used in \cite{3,4} (see Fig. 1). Only for $^{56}$Mn the difference is only around 0.6 MeV. For the total $B(GT)$ values (in units of $g_A^2$, where $g_A$ is the axialvector coupling constant) we calculate 4.4 (8.6), 2.7 (8.6), 1.5 (7.2) for $^{54,56,58}$Mn and 7.7 (12.0), 5.9 (12.0), 3.7 (10.0) for $^{56,58,60}$Co, where the numbers in parentheses are the independent particle values. Considering the additional reduction of the total $B(GT)$ strength related to the universal renormalization factor $(0.74)^2$ we conclude that the GT strength is stronger quenched than even assumed in \cite{3}. As a consequence of the differences in the total strength and in the position of the centroid one expects that the bulk of the GT transition will contribute less to the electron capture rates than assumed previously.

The shell model gives very weak transition strengths to low-lying states in the daughter nucleus. In particular, the rather large ground state spins of the odd-odd nuclei (except for $^{58}$Mn) allow only transitions to excited states in the daughter; this is particularly drastic for $^{56}$Co ($J = 4$) and $^{60}$Co ($J = 5$). Viceversa, transitions to the daughter ground state are only possible from $J = 1$ states in the parent which are usually (except for $^{58}$Mn) suppressed by the thermal Boltzmann factor.

In previous compilations \cite{3,4} the electron capture rate at finite temperature has been calculated employing the so-called Brink hypothesis. This assumes that the GT strength distribution on excited states is the same as for the ground state, only shifted by the excitation energy of the state \cite{8}. As can be seen in Fig. 2, this assumption is valid for the bulk of the GT strength, but is not applicable for the individual transitions to states at low-excitation energy in the daughter. To be more quantitative, we have calculated the GT centroids ($E_{GT}$) for various states in $^{56}$Co: the ground state, the $J = 3$ state at 0.22 MeV (0.16 MeV), the $J = 5$ state at 0.60 MeV (0.57 MeV), the $J = 2$ state at 1.04 MeV (0.97 MeV) and the $J = 1$ state at 1.94 MeV (1.72 MeV) where we have compared our shell model excitation energies $E_x$ with the experimental values given in parentheses. A measure for the validity of the Brink hypothesis is then given by the difference $E_{GT} - E_x$ and we find within 60 keV the same values for this quantity for the lowest $J = 2$-5 states ($\approx 8.2$ MeV); the difference for the $J = 1$ state is smaller (7.4 MeV) mainly caused by the strong transition to the ground state of $^{56}$Mn which exhaust about 25% of the total $B(GT)$ strength found in the transition to final $J = 0$ states. The applicability of Brink’s hypothesis has already been discussed in \cite{8,13}.

The shell model results for the low-lying strength indicate that the empirical ft value adopted in \cite{1} ($B(GT) = 0.1$) to simulate low-lying transitions is too large.

The calculated electron capture rates for the six nuclei
are shown in Fig. 3 as function of temperature ($T_9$ measures the temperature in $10^9$ K) and at those densities at which the individual nuclei have been identified in Ref. [3] as most important for the electron capture process. For the chemical potential we use the approximation [20]

$$\mu_e = 1.11(\rho T Y_e)^{1/3} \left[ 1 + \left( \frac{\pi}{1.11} \right)^2 \frac{T^2}{(\rho T Y_e)^{2/3}} \right]^{-1/3}. \quad (1)$$

The present shell model rates are compared to the FFN rates at the same densities. Additionally the figure indicates the rate of Ref. [4] taken from their Tables 15-17.

As expected from the discussion above, the shell model rates are significantly smaller than the rates of Ref. [4] and typically also than the FFN rates. The only exception here is the capture rate on $^{58}$Mn, where the FFN rates are smaller than ours at low temperatures. This, however, is due to the fact that the mass of $^{58}$Cr, the daughter of $^{58}$Mn, has not been known experimentally at the time the FFN rates have been derived and these authors used the mass systematics from Seeger and Howard [21]. As discussed in [4], this resulted in a quite different $Q$-value for this reaction. We note that it is particularly interesting to compare the rates in Fig. 3 at the density and temperature combinations quoted for the results from Ref. [4] as they are along the stellar trajectory at which the collapse is expected to occur if the FFN rates are employed. For the Mn isotopes the shell model rates are smaller by factors 4 ($^{50}$Mn) to 12 ($^{54}$Mn). The rather small ratio for $^{56}$Mn reflects the fact that this is the case among the nuclei studied here where the FFN and shell model centroids of the Gamow-Teller distributions agree best. In passing we note that for $^{56}$Mn the low-lying strength assumed in Ref. [4] strongly exceeds the shell model value and the one estimated by FFN. For the Co isotopes the reduction of the rates compared to FFN is drastic ranging from a factor 30 ($^{58}$Co) to 400 ($^{60}$Co), mainly caused by the misplacement of the GT centroid in the previous parametrizations. The shell model calculations certainly do not substantiate the large amount of the capture rate attributed to the GT resonance in Ref. [4]. The reductions for $^{54}$Mn and $^{60}$Co are quite relevant as Aufderheide et al. state that these nuclei contribute about 20% and 50% to the change of $Y_e$ at certain stages of the collapse [4].

We do believe that the odd-odd nuclei studied here reflect a typical, rather than an exceptional sample. Accepting this point of view one is lead to the conclusion that the current compilations of electron capture rates are based on a parametrization which places the GT centroid for odd-odd parent nuclei at too low excitation energies. Consequently the electron capture rates on odd-odd nuclei, as recommended in [4] and [3], are too large. Judging the overestimation of the rates from the six nuclei studied here, a reduction of the rates by about an order of magnitude might be anticipated. Previous shell model studies indicate that the recommended capture rates for odd-A nuclei are also likely too large due to a similar misplacing of the GT centroid, while the FFN rates are strongly confirmed for capture on even-even nuclei [13, 22]. Summarizing these indications, one expects that the total electron capture rate relevant for the presupernova collapse at densities $\rho T \lesssim 1000$ is smaller than currently believed. As a consequence of a slower electron capture rate, the core radiates less energy away by neutrino emission, keeping the core on a trajectory with higher temperature and entropy. However, drawing conclusions about possible effects which lower electron capture rates might have on the collapse mechanism, in particular on the size of the homologous core, are premature and prohibited at this stage. First, one has to compile a complete set of shell model based capture rates for all relevant nuclei. Secondly, during the collapse electron capture has to compete with $\beta$-decay and preliminary results indicate that the shell model roughly confirms the total FFN rates [23].

If true, electron capture and $\beta$ decay rates balance during the stellar collapse and might lead to a cooling of the star without changing its $Y_e$ value. This possibility has already been suggested in Ref. [23] on the basis of a few experimental GT distributions.

In summary, we have performed state-of-the-art large-scale shell model diagonalization calculations to determine the presupernova electron capture rates on selected odd-odd nuclei $^{54,56,58}$Mn and $^{56,58,60}$Co, which are believed to be the most important “electron poisons” during the stellar collapse. Our calculations suggest that the previous compilations of these rates placed the GT centroids for odd-odd parent nuclei at too low excitation energies in the daughter. As a consequence we calculate significantly smaller electron capture rates for all studied odd-odd nuclei than given in the standard compilation of Fuller, Fowler and Newman [3].

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FIG. 1. Gamow-Teller strength distributions for the ground states of $^{54,56,58}$Mn and $^{56,58,60}$Co, as calculated in the present shell model approach. The distributions for the various final angular momenta are given separately. The arrows indicate the energies at which the compilations of Fuller, Fowler and Newman placed the centroid of the GT strength. The energy scale refers to excitation energies in the daughter nucleus.

FIG. 2. Gamow-Teller strength distribution for the $^{56}$Co ground state ($J = 4$) and the first excited states with angular momentum $J = 3, 5$ and $J = 1$. Experimentally these states are found at an excitation energy of 0.158 MeV, 0.576 MeV, and 1.72 MeV, respectively.
FIG. 3. Electron capture rates on $^{54,56,58}$Mn and $^{56,58,60}$Co as a function of temperature and at selected densities at which these nuclei are most important for electron capture in the presupernova core collapse as suggested by Ref. [4]. The solid line shows the present shell model results, the dots give the FFN rates [3], while the triangles are rates taken from Tables 15-17 in [4].