Modelling of resistance to destruction of multi-storey frame-connected buildings at sudden loss of bearing elements stability

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Abstract. Authors modelled frame-bracing structural system resistance against progressive collapse at sudden buckling one of the compressed bearing elements of system in the paper. In the foundation of problem algorithmization it is put principle of initial structure dividing to substructures, combination of numerical and analytical methods at multistage calculation. Determination of critical force in compressed reinforced concrete elements of chosen substructure is carried out combining the initial parameters method and the displacement method. On the energy basis using works values at initial and end section it is evaluated nearby elements influence level on buckling of the most compressed rod. The numerical results of developed algorithm realization for substructure of reinforced concrete frame-bracing structural system of multi-storey building made of panel-frame elements are presented in the paper.

1. Introduction
Issues of buildings and structures protection against progressive collapse became actively discussing in scientific papers after accidents occurred in the world last years, which were caused by non-proportional failures of structures at sudden emergency damage or destruction one of the bearing constructions. In the significant number of world publications by this problem [1–10] and national building standards of some countries [11–14] alternate path method (APM) to analyze structural transformations of constructive system after initial local damage is applied, for example destruction of a column in framing building. It is used also kinematic variant of the ultimate equilibrium method [5, 15] in Russian and world engineering computational practice. Prevailing number of papers deal with sudden mechanical removal of structures elements at vehicle impact, high temperature and explosive impacts as situation for structural analysis.

At the same time, the character of impact significantly determines strategy of structural system modeling at local damage and predicting of destructure character after such impact. One of the reasons leads to sudden destruction of compressed structural elements; it can be the loss of stability at the accumulation of corrosion damages in it. For example, it can be occurred in columns of building frame. As it is noted by A V Alexandrov, V I Travush, V I Kolchunov, V A Gordon, N V Fedorova and others [17–20], the buckling character of some structural elements may be different: active or passive in accordance positive or negative value of integral work of moments and shear forces at the edges of rod element.
New procedure of non-linear static analysis to predicting critical load value and structural response of building frame constructive system at sudden loss of compressed bearing element stability caused by accumulation of corrosion damages is presented in this paper.

2. Method for dividing of initial structural system to substructures (secondary computational models)

Procedure of constructively and physically nonlinear analysis applying to solve described problems is the most accuracy among other ones to predict structural system behavior. At the same time, as it is shown by using experience of such strategy [5, 15, 21], it inevitably deals with significant volume of computation for complex structures, that requires long time and makes difficult interpretation of computational results. In addition, applying of PC programs to solve such problem leads to situation when physical meaning of a process modeling becomes 'blurred', in particular for modeling of degradation phenomena at corrosion of structures.

It is proposed multilevel iterative creating and using of resistance against progressive collapse computational models for building frames at buckling of structural element to develop simple and effective for engineering practice algorithm.

Such algorithm based on the combination of numerical and analytical iterative methods to realize it.

At the first stage, space rod model of building frames or space rod-plate model is used, which called as first level computational model presented in Figure 1. It should be noted that choice of reinforced concrete panel-frame structural system [22] is determined by possibility to use thin-walled columns made of high strength concrete as it is shown in Figure 1 (a) that can lead to removal of compressed rod element by stability criterion instead strength criterion.

![Figure 1](image1.png)

**Figure 1.** Plan of panel-frame elements placing for typical floor (a) and initial finite element computational model (b) of 17-storey building made of panel-frame elements.

Computational results for first level model gives information about internal forces in all elements of structural system, particular in studied the most compressed columns of first storey.

Experimental studies of reinforced concrete flat and space structural system survivability, which were conducted in Russia [17, 23] and in the world [16, 24, 25, 26], shows that local destruction of experimental structures at situation of sudden loss of a bearing element ends stability relatively small zone adjacent to destructured element. For described above structural system it is obtained secondary computational model where such zone is modeled by fragment "A" of the first storey as it is shown in Figure 2, the columns, which are the most loaded by compressive forces. In first approximation, loading of column only longitudinal compressive force for analytical analysis and visualization of
computational algorithm is considered. The general meaning of the secondary computational model in the form of fragment “A” is determination of critical forces for compressed elements of structural system at sudden removal of connectors between panel-frame elements columns, caused by accumulation of corrosion damages in it.

Figure 2. Scheme of excluded substructure (fragment “A”) placing in building frame on the first storey (a) and fragment “A” of second level computational model (b).

In accordance with constructive scheme of fragment “A” presented in Figure 2 (b) the computational model of second level is one-storey frame compressed by forces $\alpha P$, which legs are connected by linear connectors (supports) 1 and 2 between itself as it is shown in Figure 3 (a).

Determination of critical force in such elements of second level computational model is carried out by the displacement method. Stability equation for arbitrary rod element of reinforced concrete frame that is modeling such fragment can be written in the following form [19]:

$$\frac{\partial^4 \bar{w}}{\partial \xi^4} + k^2 \frac{\partial^2 \bar{w}}{\partial \xi^2} = 0. \quad (1)$$

In accordance with the paper [20], equation (1) is written in dimensionless coordinates and parameters, where $\xi = x/l$ is dimensionless coordinate along the rod element of $l$ length, that is counted from clamped end, $w = w/l$ is dimensionless deflection of rod from its initial axis.

Parameter $k$ in formula (1) is determined by the formula
\[ k = \left( \frac{Nl^2}{B_{red}} \right)^{1/2}, \]  

where \( B_{red} \) is reduced bending stiffness of considered reinforced concrete element with cracks for considered iteration; \( N \) is longitudinal compressive force.

Solution of the equation (1) can be written in the form of the initial parameters method:

\[
\bar{w} = \bar{w}_0 + \bar{w}'_0 \xi + \bar{w}''_0 \left(1 - \cos k\xi\right)/k^2 + \bar{w}_0'' \left(k\xi - \sin k\xi\right)/k^3,
\]  

where \( \bar{w}_0, \bar{w}'_0, \bar{w}''_0 \) are dimensionless deflection, rotational angle, bending moment and shear force in initial section respectively, which are determined from boundary condition of the problem.

Dimensionless deflection, rotation angle, bending moment and shear force in arbitrary section can be found differentiating by \( \xi \) of the equation (3). In this case, the dimensionless shear force corresponds to deformable axis of rod element.

\[ \frac{\partial \bar{w}}{\partial \xi} = \bar{Q}_n + N \frac{\partial \bar{w}}{\partial \xi} = -C_1 \bar{w}' + k^2 \bar{w}'' = -\bar{w}'''; \]  

\[ \bar{M} = -C_2 \bar{w}' = -\bar{w}''', \]  

where index ‘n’ corresponds to internal forces related with initial non-deforming axis of a rod element.

Substituting in the formula (4) the relationships for dimensionless forces and deformations written through initial parameters (3) for general case in end section (\( \xi = 1 \)) we obtain system of two equations with four variables. Additional two equations to solve this problem for considered leg of the building frame can be obtained using boundary conditions.

In particular, in the papers [19, 20] the solution for compressed rod element with clamped initial section and arbitrary boundary conditions on the second end of rod was carried out.

Putting single values to end sections displacements and rotational angles or forces in initial and end sections. Functions of parameter \( k \) are obtained. These functions are used to calculate considered rod-frame systems by the displacement method.

Figure 3. Computational model of the substructure (fragment “A”).

Let us consider one leg of the frame fragment presented in Figure 3, the initial section, which is clamped and another edge has flexible supporting against rotation (S1) and transverse displacement (S2) respectively as it is shown in Figure 4 (a).

Assuming that rotational angles for rod element are small the relationships for dimensionless shear force \( \bar{Q} \) and bending moment \( \bar{M} \) can be written in the form:

\[ \bar{Q} = \bar{Q}_n + N \frac{\partial \bar{w}}{\partial \xi} = -C_1 \bar{w}' + k^2 \bar{w}'' = -\bar{w}'''; \]  

\[ \bar{M} = -C_2 \bar{w}' = -\bar{w}''', \]  

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For considered building frame, the matrix system of this method takes the form:

$$ r_{ji}(k)z_j = 0, $$

(6)

where $r_{ji}(k)$ is matrix of single reactions caused by displacement or rotation of $j$-th nodes; $z_j$ is matrix of unknown displacement.

Parameter of critical force $k_{cr}$ can be found by equaling of $r_{ji}(k)$ determinant to zero.

Compressed rods working as elements of structural system are under influence from nearby elements and take it in common bifurcation of system. In this case, in accordance with [18] each rod of system is buckling by active or passive bifurcation scenario. Evaluation of bifurcation type in accordance with [18] can be provided by energy basis considering deformation energy $U_i$ that appear at the initial moment of compressed element bifurcation. In general, for $i$-th compressed rod the deformation energy equals to work of the longitudinal force $N_i$ and bending moments $M_i$ and shear forces $Q_i$ at the ends of this element:

$$ U_i = A_i(N_i) + A_i(M_i; Q_i), $$

(7)

where $A_i(N_i)$ is a work of longitudinal force at convergence of its end sections "A" and "B" caused by longitudinal bending; $A_i(M_i; Q_i)$ is a work of bending moment and shear forces at the end section of $i$-th element at transverse bending with $z_{ij}$ displacement.

Forces and displacements are carried out from stability problem solution for rod-frame system by (6). Since $U_i$ is always positive if the work sum is positive too. However, some components of equation (7) may be negative. The work of compressive force is always positive therefore, it stimulates bifurcation.

The work acting at the ends of rod element bending moments and shear forces $A_i(M_i; Q_i)$ may be more, less or equal to zero. In the first case, the rod is subjected to active type of bifurcation and involves structural system in this process. In the second case the elements, which are situated nearby to the considered rod, constrain it from bifurcation and occurs constrained bifurcation of these elements. Finally at $A_i(M_i; Q_i) = 0$ it is considered the simultaneous buckling of the rod element and loss of the system stability.
Calculation of the works by formula (7) for considered fragment of building frame is provided using reactions and nodal displacement obtained by formula (6) at stability analysis and calculation of critical force and buckling frame form.

When critical force is determined, calculation of initial structural system against progressive collapse is providing for the most compressed element of the considered building frame fragment. In this case, it is necessary to use second level computational model [6] without buckling element.

3. Analysis of computational results
Using considered algorithm, out calculation of critical force in compressed thin-walled reinforced columns of the first storey of 17-storey building with panel-frame elements is carried as it is shown in Figure 1 (a). According to basis of building frame calculation by first level computational model, the second level computational model for the most loaded fragment “A” with the most compressed column of the first storey is obtained as it is shown in Figure 2. Herewith in numerical calculation it is variated bending reduced stiffness of the column and distance between connection element linking nearby panel-frame elements along the height of it columns.

The presented analysis of computational results (Figure 5) shows that values of critical force parameter \( k_{cr} \) determined in assumption of the nearby elements influence on the most loaded leg of the frame at variating of it reduced stiffness. The distance between connection elements along height, practically are not depends on reduced rigidity of the leg and changes in small range from 5.2 to 10.1%.

Based on the results, the important conclusion follows, if frame leg is rigidly connected with beams at the both edges of leg, then installation additional connectors along the frame leg is not required. At the same time, bending stiffness changing of the leg cross section significantly influence on the critical force value for this element of the building frame fragment. Nearby element's influence on account shows, that dimensionless values of critical force in compressed rod, which deforming together with connected rod elements, is more than two times larger of critical force for single rod with clamping in initial section and elastic supports in another end section. Herewith, reduced bending stiffness of reinforced concrete element decreasing at cracks formation leads to decreasing of this influence.

![Figure 5](image-url) Figure 5. Change of dimensionless critical force value for fragment “A” of building first storey at various values of reduced bending stiffness \( B_{red} \): 1 – when there are two connection details through the length of the frame legs; 2, 3 – at removal of support 1 or 2 respectively; 4 – at removal of two supports simultaneously.

4. Conclusions
It is proposed algorithm of physically and constructively nonlinear analysis of reinforced concrete building frame against progressive collapse at stability sudden loss one of the compressed bearing elements. In the foundation of this algorithm, it is put principle dividing of initial space building frame to substructures and combination of numerical and analytical methods at multistage realization of calculation.
Determination of critical force for compressed reinforced concrete elements of substructures is carried out by using combination of the initial parameters method and the displacement method. On the energy basis using works values at initial and end section it is evaluated nearby elements influence level on buckling of considered compressed rod.

Numerical realization of developed algorithm for substructure of reinforced concrete building frames shows that critical force value significantly depends on reduced bending stiffness of panel-frame legs, character of connection with beams, influence of nearby elements.

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