Quark and gluon distributions at the earliest stage of heavy ion collision

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Using the general framework of quantum field kinetics [5] we consider new principles to compute initial distribution of quarks and gluons after the first hard interaction of heavy ions. We start by rewriting the integral equations of QCD in the form which is generalizations of the familiar QCD evolution equations. These equations describe both space-time– and \((x, Q^2)\)–evolution before the collision, and allow one to use the \(ep\) DIS data without reference to parton phenomenology. New technique generate perturbation theory that avoid double count of the processes, does not contain an artificial factorization scale, and does not require low-momentum cut-offs since infrared behavior is controlled by the DIS data.

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1. Introduction.

Recently, there have been many calculations [3,2,4,1] to find the initial distribution of quarks and gluons in heavy ion collisions, which may then lead to the creation of a plasma. Contrary to other types of matter under extreme conditions, the situation in heavy ion physics is unique, since the very existence of quark–gluon plasma (QGP) is inseparable from the process of creation of the matter it consists of. Any subsequent elements of the behavior, which can be described in classical language, such as thermalization or a hydrodynamic regime for the QGP, like for any other many–particle system, is possible only after intermixing of the initial phase-space distribution. However the process of creation of quarks and gluons is essentially quantum by its origin. In the present paper only this process is considered.

Any strict formulation of a quantum–mechanical problem requires an exact definition of two main elements: the initial state of the system, and the observables in the expected final state. This reminder would hardly be necessary if the main difficulties were not associated with these points: On the one hand, it is unknown how stable nuclei of the initial state are build up from quarks and gluons; on the other, there is no clear understanding of what the final state may look like. So even the formulation of the problem unavoidably contains some uncertainty which requires special care in defining these elements. Let us begin with a discussion about what kind of problems we may anticipate.

1. The initial state: Ideally, as in, for example, an atomic collision, we would define the initial state via its wave function. The wave functions of QCD–nuclei are unknown. A natural alternative is to use the density matrix to describe each nucleus.

Here we may assume nuclei to be well shaped objects. The uncertainty of their boundaries does not exceed the typical Yukawa interaction range. In the laboratory frame both nuclei are Lorentz contracted up to a longitudinal size \(R_0/\gamma \sim 0.1\, fm\). The tail of the Yukawa potential is contracted in the same proportion. The world lines of the nuclei are two opposite generatrices of the light cone that has its vertex at the interaction point. No interaction between nuclei is possible before they overlap geometrically. For this reason the total density matrix of two nuclei is a direct product of the two individual density matrices. The spaces of states where they act do not overlap either.

Since no exact information about the initial state of the nuclei is available, it seems reasonable to rely upon the following two considerations: First, detailed information is inessential as it basically relates to the interactions which maintain every nucleus as a QCD bound state. The energy of the collision is incomparably higher. It is thus enough to require that density matrix yields the given total momentum and baryonic charge as averages of the corresponding field operators. Second, the residual dynamical information must reveal itself in the same way as in other inelastic processes at extreme energies, like deep inelastic electron-proton or muon-nucleus scattering. This statement may appear trivial, because structure functions of DIS are always used for account of this information. However, one should keep in mind that their definition – which does not refer to the parton model – is valid only for the DIS process itself. In order to apply structure functions to other interactions, using the parton model is considered unavoidable.

The first priority of this study is to avoid any intermediate phenomenology. We insist that any information taken from parallel experiment is valuable only as long as both processes can be described by the same theory and with the same initial data.

2. The final state: The final state is assumed to be some distribution of free quarks and gluons in the perturbative vacuum. This vacuum is considered to be the true ground state, and is free of QCD–condensates. It is a product of the nuclear collision and is postulated to exist in a sufficiently large volume. The spectrum of possible states forms the continuum, which is unoccupied at the beginning of the collision.
It is supposed also that the information which is most important for understanding the future evolution is concentrated in the single-particle distributions of quarks and gluons. These distributions must be calculated from their quantum-mechanical definitions, keeping in mind how they are to be measured in a hypothetical experiment. Two- and more-particle distributions should be defined as independent elements corresponding to other measurements.

In this work we rely heavily on a previous paper [5], where integral equations of QCD were derived without assuming that averaging is performed over a stationary state. To some extent these equations resemble the diagram technique of Keldysh [6], which was designed for non-equilibrium processes. They are of the same matrix form, but are not derived with a view to obtaining quasi-classical kinetic equations. Fields and their correlators remain the main objects of these equations and no phase-space distributions is introduced. For this reason the approach was named “quantum field kinetics” (QFK).

These new equations are the result of an initial resummation of the perturbation series for probabilities of inclusive processes (or any other observables). On one hand, these new equations create an approach which proves to be very effective for studying various inclusive processes. On the other, it allows one to trace the temporal evolution of a colliding system, beginning from preparation in the past right up to the moment of measurement. This latter feature makes the new approach extremely attractive for our goals: The evolution of any physical system is completely defined by the initial data, and the act of measurement only selects one or more of all the possible quantum trajectories. Therefore, one can expect that results of similar types of measurement will be similar. In this paper, we want to consider two similar processes, viz., deep inelastic electron-proton scattering and “deep inelastic pp- or AA-collisions,” in parallel.

To begin with, we must define the observables of these processes in the same way. Definitions of inclusive amplitudes and inclusive probabilities to find one quark or one gluon in the final state are given in Section 2. The DIS cross-section, viewed as an inclusive process where nothing is measured except the momentum of the scattered electron, is defined in the same terms at the beginning of Section 3. The rest of this section considers an instructive example of the lowest order calculations using a model density matrix. This calculation allows one to introduce all spinor and vector functions without extra complicated notations, and to clarify the complete calculation.

The dynamical equations in their full tensor-spinor form are derived in Section 4. It immediately becomes clear that these equations have a ladder structure. It appears that the well known ordering of ladder cells by Feynman $x$ and virtuality is a direct consequence of the retarded temporal ordering inherent to these equations in their coordinate form. Smaller $x$ and bigger virtualities correspond to the later times. Thus, as a by-product, we obtain an answer on a very old question about such correspondence [7–9].

The new equations appear to be richer than usual QCD evolution equations for DIS structure functions, as derived from the renormalization group approach. The new equations interconnect two invariant functions of the vector field, and two of the quark field. The new evolution equations do not depend upon the type of last interaction, however they may be projected onto any definite process. Specific properties of the electromagnetic interaction select only one of the spinor functions into the definition of the structure function $F_2$ of unpolarized DIS. However, both quark field functions remain in the evolution equations, along with the two functions of the vector field. The relative scale of additional terms and their possible role is examined in Appendix 1. It is also shown that among other extensions, the new equations naturally include the BFKL [10] equation and the effects of quark and gluon shadowing at small $x$. Corresponding shadowing terms appear to be parametrically larger than in previous derivations [11,12].

The meaning of the objects that obey the new evolution equation comes to light in a discussion of renormalization. The renormalization group approach can not be used here, if only because most of the terms in these equations correspond to observables (imaginary part of self-energies) are finite and may not be renormalized. Instead, we renormalize the second subgroup of the evolution equations for the real part of the self-energies using a conventional BHPZ scheme. Ultraviolet divergencies are compensated for by counter-terms from the original Lagrangian. The running coupling appears precisely from the requirement of renormalizability. For the moment, this part of the study is at the level of a basic idea.

Using some structural, rather than quantitative, assumptions, and after projecting onto specific observables of the e-p DIS, the new equations can be reduced to the system of GLAP equations [13,15]. The objects which enter the new equations are similar to self-energies, and we shall call them sources. It appears that the QCD evolution proceeds in a way such as to create a source of a field which interacts with the detector in a certain way. The evolution causes the dynamical assembly of a special wave packet which represents a bare quark or gluon. This process takes place in real time and ends at the moment of interaction.

We expect DIS to provide the dynamical information about this process. This information is valuable only as long as no measurements were done before the last interaction. All unobserved information (and providing it was not observed) is included in the definition of the sources with their full dependence upon $x$ and $Q^2$.

Here, we do not adhere to the picture of “wee partons,” and do not share the opinion that the QCD evolution equations describe how one valence quark develops a cloud of virtual quarks and gluons around it at small distances. We believe that they give (perturbatively) the “evolution” of the detector response, provided the trigger includes the...
requirement of a bare on-shell quark in the final state (the resonant condition which results in $x_{Bj} = x_F$). That they definitely do not correspond to the evolution of a single quark, is seen, for example, from possibility of including fusion.

The difference between the structure functions and the sources can be explained using an analogy: in condensed matter or scattering theory we introduce two different quantities: the density of states, $\rho(E)$, and the number of states below energy $E$, $n(E) = \int^E \rho(E) dE$. The structure functions then correspond to $n(E)$, while the sources correspond to $\rho(E)$. In an experiment we measure $n(E)$ (which is proportional to the allowed volume in the phase space), rather than $\rho(E)$. From this point of view it is not surprising that we eventually express observables, roughly speaking, via $dG(x, Q^2)/dQ^2$. Different measurements study these quantities, integrating them with their specific “upper limits.” The boundary conditions for $x$– and $Q^2$–evolution are imposed by the measurement, rather than the initial conditions which are controlled by momentum conservation, sum rules, etc. These must be set in the past without any relation to the $(x, Q)$–evolution.

Cross–sections of inclusive single-quark and single-gluon production in the lowest nonvanishing order are calculated in Section 5. All of them contain scale–dependent and –independent terms. The latter are much bigger than the former, and as a result the cross–sections are expected to exhibit only weak scale dependence. The lowest order cross–sections are strongly peaked at low rapidities and low transverse momenta.

The next order of the perturbative expansion generated by the new evolution equations is examined in Section 6. The new expansion does not lead to any diagrams which duplicate those already included in the definition of the sources (or structure functions). Any such diagrams would carry severe collinear singularities. We carefully examine the infrared finiteness of the diagrams that do occur. It is shown that they are infrared safe and that no artificial cut–offs are necessary to find the total cross-section. Higher order perturbation terms are not expected to present any difficulty, as their final state infrared behavior will be shielded by the final distributions themselves.

We are led to the approach advocated in this paper in an unavoidable manner. Ideally, one would start with a complete relativistic quantum description of the static proton and its interaction with the detector. Unfortunately, the many attempts to describe the bound states of QCD (see Refs. [16] and [17] for the reviews) have not yet met with real success. To calculate the production of particles in hadronic collisions, one is limited to reasoning along the following lines: first, an OPE-analysis of the DIS data, which gives the structure functions of DIS; next, a partonic interpretation of the structure functions; and, lastly, using the factorization technique. In the end, one still faces severe theoretical problems caused by the soft processes, the arbitrariness of the factorization scale, etc. Here, we try to avoid these problems “experimentally,” by maximizing the use of dynamic information hidden in the DIS data.

2. Single-particle distributions of the partons.

Let two nuclei $A$ and $B$, with momenta $P_A$ and $P_B$, move towards each other at almost the speed of light, and let the center-of-mass system coincide with the laboratory frame. We assume that the center-of-mass energy is very large, $s \gg M^2$, so that the laboratory frame is the infinite momentum frame for both nuclei. We intend to compute one-particle distributions of quarks and gluons created after the first interaction of these nuclei. The inclusive amplitudes leading to creation of one quark or one gluon from the initial state $|in\rangle$ are as follows,

$$\langle X | d(p, \sigma, i) S | in \rangle, \quad \langle X | c(k, \lambda, a) S | in \rangle,$$

where $d^\dagger(p, \sigma, i)$ is a creation operator for an on-mass-shell quark with momentum $p$, spin $\sigma$ and color $i$. Similarly, the operator $c^\dagger(k, \lambda, a)$ creates an on-mass-shell gluon with momentum $k$, polarization $\lambda$ and color $a$. Summing the squared moduli of these amplitudes over a complete set of uncontrolled states $|X\rangle$, and averaging over the initial ensemble, we find inclusive spectra of quarks and gluons

$$\frac{dN_q}{dp} = \sum_{\sigma, i} S_p \rho_{in} S^\dagger d^\dagger(p, \sigma, i) d(p, \sigma, i) S, \quad (2.2)$$

$$\frac{dN_g}{dk} = \sum_{\lambda, a} S_p \rho_{in} S^\dagger c^\dagger(k, \lambda, a) c(k, \lambda, a) S. \quad (2.3)$$

The initial state of the colliding system consists of two Lorentz-contracted nuclei which are causally independent, and thus the total density matrix is a direct product of two independent density matrices,

$$\rho_{in} = \rho_A \otimes \rho_B \otimes |0_{cont}\rangle\langle 0_{cont}|. \quad (2.4)$$
Matrix elements of \( \rho_{in} \) are obtained by sandwiching it between all state vectors \(|in>\) which enter definition (2.4) of inclusive amplitude. The density matrices \( \rho_A \) and \( \rho_B \) contain only bound states of quarks and gluons in the presence of vacuum condensates. The latter are assumed to be destroyed in the course of the initial hard collision and replaced by the perturbative QCD vacuum. Initially, all states in the continuum are unoccupied. It means that \( \rho_{in} \) contains a projector \(|0_{cont}>\langle 0_{cont}|\) on the vacuum state in the continuum. So we may commute the quark Fock operators with \( S \) and \( \tilde{S} \) and only commutators survive in the final result:

\[
\frac{dN_q}{dp} = \sum_{\sigma,i} \int d^4x d^4y \psi_{p,\sigma,i}^{(+)}(x) \frac{\delta S^\dagger}{\delta q_i(y)} \frac{\delta S}{\delta q_i(x)} \psi_{p,\sigma,i}^{(+)}(x). \tag{2.5}
\]

In this expression \( \psi_{p,\sigma,i}^{(+)}(x) \) is the Dirac wave function of a quark. Summing over spin and color we get

\[
\frac{dN_q}{dp} = \sum_{\sigma,i} \int d^4x d^4y \frac{e^{-ip(x-y)}}{(2\pi)^3 2p^0} \text{Tr} [\hat{\rho} \Sigma_{01}^{ij}(x, y)] , \tag{2.6}
\]

where the full \( 2 \times 2 \) matrix of the quark self-energy is given by (3).

This formula implies that both quark and gluon correlators, \( G_{AB} \) and \( D_{S\lambda,\lambda \mu}^{do} \), are averaged with the density matrix \( \rho_{in} \) given by Eq. (2.4). In the first approximation we may replace the exact \( qgq \) vertex by the bare one. Then Eq. (2.6) takes the following simple form,

\[
p^0 \frac{dN_q}{dp d^4x} = \frac{g^2}{2(2\pi)^3} \int \frac{d^4k}{(2\pi)^4} \left( \text{Tr} [\hat{\rho} \Sigma_{01}^{ij}(x, y)] + \text{Tr} [\hat{\rho} \Sigma_{01}^{ij}(x, y)] \right) \tag{2.8}
\]

where the additional superscript \( (A) \) or \( (B) \) denotes that the correlation function is averaged over the initial state of nucleus \( A \) or \( B \), respectively.

A similar procedure yields the following expression for the inclusive gluon production:

\[
\frac{dN_g}{dp} = \sum_{\lambda,\alpha} \int d^4x d^4y \frac{e^{-ip(x-y)}}{(2\pi)^3 2p^0} \gamma^\lambda \epsilon^\mu [\epsilon^\alpha] [-i \Pi_{01,\mu \nu}^{a_0}(x, y)] \tag{2.9}
\]

where the primary definition of the gluon polarization tensor \( \Pi_{01} \) is given by

\[
\Pi_{01,\mu \nu}(x, y) = i \langle \frac{\delta S^\dagger}{\delta B^\mu_\nu(y)} \frac{\delta S}{\delta B^\nu_\mu(x)} \rangle . \tag{2.10}
\]

Its general formula was derived in (5):

\[
\Pi_{AB}(x, y) = i (-1)^{A+B} g_F^2 \sum_{R,S=0} (-1)^{R+S} \int d\xi d\eta \Gamma^{\mu}_{RS,B}(\xi, \eta; y) G_{S\lambda}(\eta; x) +
\]

\[
+ \int d\xi d\eta \Gamma_{RS,B,c}(\xi, \eta; y) \left[ \epsilon^{\nu} \epsilon^{\lambda} \epsilon^{\alpha} \epsilon^{\beta} \epsilon^{\gamma} \epsilon^{\delta} \epsilon^{\epsilon} \epsilon^{\zeta} \epsilon^{\eta} \right] D_{S\lambda,\lambda \mu}(\eta, y) D_{S\lambda,\lambda \mu}(\eta, y) \right) , \tag{2.11}
\]

and postpone its further expansion because of the complexity of the emerging polarization structure. However, the main idea remains the same as for quark production: in first approximation we get a product of two quark, \( G_{01}^{(A)} G_{10}^{(B)} \), or two gluon, \( D_{01}^{(A)} D_{10}^{(B)} \) correlation functions. Each of them is averaged with the density matrix of only one of the two nuclei. This is in line with the independence of the initial states of the colliding nuclei.

To continue with the straightforward calculations, we should now specify an explicit form of the nuclear density matrix. To do this, we should solve the confinement problem – which is not our intention here. Moreover, we have already argued that most of the detailed information is unnecessary. Now we shall motivate this point by a “quantum kinematic” analysis of the extreme case when no dynamical information is required to obtain a qualitative understanding of the phenomenon.
Let the nuclei collide at an energy of about 100 TeV per nucleon, when the nuclear longitudinal size is only $10^{-4}$ fm. This size is much less than any known in nuclear interactions, and one may consider the domain where the nuclei overlap as a plane surface (or the point in the $(t, z)$-plane). All the subsequent dynamics takes place within the future light cone of this point, $t^2 - z^2 > 0$, $t > 0$. As the translation invariance in $t$- and $z$-directions is manifestly broken by the initial conditions, one should look for the appropriate quantum numbers (other than $p^0$ and $p^3$) to describe the final states of the particles. The symmetry that does survive is Lorentz invariance, and the boost defined by the operator
\[ \hat{\nu} = t[-i \frac{\partial}{\partial z}] - z[i \frac{\partial}{\partial t}] = i \frac{\partial}{\partial \phi}, \]
(2.12)
is an good quantum number. The corresponding wave functions of the free scalar particles obey the Klein-Gordon equation,
\[ \frac{1}{\tau} \frac{\partial}{\partial \tau} \left( \frac{\partial \psi}{\partial \tau} \right) = \frac{1}{\tau^2} \frac{\partial^2 \psi}{\partial \phi^2} + m^2 \psi = 0, \]
(2.13)
and are of the following form:
\[ \xi_{\nu, p_\perp} (x) = \frac{e^{-\pi \nu/2}}{2\pi^{3/2}} H_\nu^{(2)} (m_\perp \tau) e^{-i\nu \phi} e^{i\vec{p}_\perp \cdot \vec{r}_\perp}. \]
(2.14)
Here, we use coordinates $(t = \tau \cosh \phi, \ z = \tau \sinh \phi, \ r_\perp)$, and denote $m_\perp^2 = m^2 + p_\perp^2$. The eigenfunctions $\xi_{\nu, p_\perp} (x)$ are normalized on the space-like hypersurface $\tau = \text{const}$ within the future light cone of the collision point:
\[ (\xi^*_{\nu, p_\perp}, \xi_{\nu', p_\perp'}) = \int \tau d\phi d^2 \xi^*_{\nu, p_\perp} (x) i \frac{\partial}{\partial \phi} \xi_{\nu', p_\perp'} (x) = \delta (\nu - \nu') \delta (\vec{p}_\perp - \vec{p}_\perp'). \]
(2.15)

The boost of the particle is a legitimate, but seldomly used quantum number. It is more common to use the rapidity in the vicinity of the light cone. The picture looks as if the incoming nuclei carry this distribution by the finite size of the interaction domain, any high-energy collision will produce a distribution which is uniform in the $p_\perp$-plane. All the subsequent dynamics takes place within the future light cone of the collision point:
\[ \Xi_{\theta, p_\perp} (x) = \frac{-i}{(2\pi)^{1/2}} \int_{-\infty}^{+\infty} e^{-i\theta \phi} \xi_{\nu, p_\perp} (x) = \frac{1}{4\pi^{3/2}} e^{-im_\perp \tau \cosh (\phi - \theta)} e^{i\vec{p}_\perp \cdot \vec{r}_\perp}. \]
(2.16)
These wave packets represent plane waves confined to within the future light cone of the collision point, and can be rewritten as follows:
\[ \Xi_{\theta, p_\perp} (x) = \frac{1}{2\pi^{1/2}} e^{-ip^0 t + ip^z z} e^{i\vec{p}_\perp \cdot \vec{r}_\perp}, \quad p^0 = m_\perp \cosh \theta, \quad p^z = m_\perp \sinh \theta. \]
(2.17)

At large $m_\perp \tau$, the phase of the wave function $\Xi_{\theta, p_\perp}$ is stationary in a very narrow interval around $\phi = \theta$ (outside of this interval, the function reveals oscillations with exponentially increasing frequency): the wave function describes a particle with rapidity $\theta$. However, for $m_\perp \tau \ll 1$, the phase of the wave function is almost constant along the surface $\tau = \text{const}$. The smaller $\tau$, the more uniformly the particle is spread along the light cone. Up to distortions caused by the finite size of the interaction domain, any high-energy collision will produce a distribution which is uniform in rapidity in the vicinity of the light cone. The picture looks as if the incoming nuclei carry this distribution ab initio. The latter is not surprising as the same arguments can be applied to the states of particles before a strongly localized interaction. The distribution $dN \sim \text{const} \times d\theta$ corresponds to $dN \sim \text{const} \times dx_F/x_F$ in terms of the Feynman variable $x_F$. Thus, we arrive at a result which is typical for the Williams-Weizsacker approach. The full consideration for the QCD-nucleus has been recently given by McLerran and Venugopalan [3].

We assume that deviation from this ideal distribution can be studied perturbatively with increasing accuracy the more inelastic the collision is. We expect that the desired information about this deviation can be obtained from the deep inelastic electron-proton scattering data. We argue that these data do not imply exact knowledge of the density matrix of a nucleus. In order to incorporate information obtained from DIS we begin by computing the DIS cross-section in terms of the quantum fields kinetic (QFK) [3].

3. Deep inelastic scattering on the electron
The goal of this section is to find those elements of a theory which are common to two essentially different problems, viz., nucleus-nucleus (or proton-proton) collisions and deep inelastic electron-proton scattering. Our primary demand is that these elements should appear as a by-product of the two independent lines of calculations, initiated separately from first principles.

We divide this section into two parts. For the sake of completeness we begin with a brief definition of the DIS cross-section in terms of the QFK-approach and define the null-plane variables which will be used for all following calculations.

Before turning to a detailed derivation of the self-consistent equations in the next Section, we give an instructive example of the lowest order calculations. These have no direct physical value, but they allow one to overcome technical problems and avoid premature discussion of the highly nontrivial approximations.

### 3.1. Basic definitions for DIS

As was emphasized in the Introduction, it is important to have similar definitions of observables for all processes which will participate in the future information exchange. We may rewrite Eq.(2.1) for the inclusive amplitude of DIS as

\[
\langle X|a(k')Sa^\dagger(k)|in \rangle ,
\]

where \( k \) and \( k' \) are the laboratory frame momenta of the electron before and after the scattering. If \( q = k - k' \) is the space-like momentum transfer, then the DIS cross-section is given by

\[
k_0' \frac{d\sigma}{dk} = \frac{i\alpha}{(4\pi)^2} \frac{L_{\mu\nu}(k, k')}{(kP)\nu} W_{\mu\nu}(q) \frac{W_2(q^2)}{2x_{Bj}M^2} ,
\]

where \( W_{\mu\nu}(q) \) is a standard Bjorken notation for the correlator of two electromagnetic currents,

\[
W_{\mu\nu}(q) = \frac{2V_{ab}P_0}{4\pi} [-i\pi^{\mu\nu}_{10}(q)] .
\]

We accept without any discussion its standard tensor decomposition,

\[
W_{\mu\nu}(q) = \epsilon_{\mu\nu}^{\nu}W_L + \zeta_{\mu\nu}^{\nu}W_2 \frac{2x_{Bj}}{2x_{Bj}M^2} ,
\]

where \( \nu = qP, Q^2 = -q^2 > 0, x_{Bj} = Q^2/2\nu \) and

\[
\epsilon_{\mu\nu} = -g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2} ; \quad \zeta_{\mu\nu} = -g_{\mu\nu} + \frac{P_{\mu}q_{\nu} + q_{\mu}P_{\nu}}{\nu} - \frac{q^2}{\nu^2} .
\]

Hereafter we will perform all computations using the infinite momentum frame fixed by the null-plane vector \( n^\mu \),

\[
n^\mu = (1, 0_t, -1), \quad n^2 = 0.
\]

It defines the “+”-components of the Lorentz vectors,

\[
a_+ = a_+ = a_0 + a_3; \quad a_- = a_+ = a_0 - a_3
\]

In the infinite momentum frame, the 4-vector of the proton’s momentum has components

\[
P^\mu = (P^+ /2, 0_t, P^+/2), \quad P^- = P^0 - P^3 = 0.
\]

The momentum transfer has the components

\[
q^\mu = (\nu/P^+, 0_t, -\nu/P^+), \quad q^+ = 0, \quad q^- = 2\nu/P^+.
\]

Instead of the invariant \( W_2 \), we will use the mass-independent structure function \( F_2(x_{Bj}, Q^2) = \nu W_2/M^2 \), which is calculated via the equation

\[
c_2 = W_{\mu\nu}n_\mu n_\nu = \frac{(P^+)^2F_2}{\nu}.
\]
The longitudinal structure function \( F_L(x_{Bj}, Q^2) = W_L \) should be calculated in accordance with
\[
3F_L = 2x_{Bj}c_1 + 2F_2; \quad c_1 = W^{\mu\nu}g_{\mu\nu}.
\] (3.10)

### 3.2. An instructive example: the low order calculation.

In order to proceed with the calculations we must specify the density matrix. We shall begin the physical motivation of our choice by reminding the reader that a widely used approach based on the Wilson’s operator product expansion (OPE) does not utilize any information about the proton’s internal structure. Only the total momentum and the discrete quantum numbers are controlled by sum rules. Indeed, the dynamical equations of QCD contribute only to the singular coefficient functions while the regular operator functions, the averages over the proton’s state, remain unknown. We can only decide whether or not to include the high twist operators in the expansion. The twist-one operators of OPE correspond to one-particle matrix elements of the proton’s density matrix. Including the twist-two operators into OPE would correspond to irreducible two-particle correlations in the density matrix used here.

The system of integral equations which we expect to derive eventually does not require any explicit form of the density matrix either. Nevertheless it is useful to keep in mind some representation which may serve as a simple reference point. For example, we may chose an artificial exponential form which reproduces the total momentum flux of the proton and allows to derive the integral equations of the Schwinger-Dyson type [5].

The twist-one operator functions of OPE, by their structure, are binary products of quark and gluon fields and to some extent resemble occupation numbers which enter the on-mass-shell Greenians. For example, in the statistical ensemble we usually have
\[
G_{10}(p) = -2\pi i(\not{p} + m)\delta(p^2 - m^2)[\theta(p_0)(1 - n^{(+)}(p)) - \theta(-p_0)n^{(-)}(p)].
\] (3.11)

We define matrix elements of our one-particle (twist-one) density matrix by a certain set of the Greenians. We assign the superscript “#” to all states in the continuum of free on-mass-shell fields:
\[
G^{\#ij}_{10}(p) = -2\pi i\delta_{ij}(\not{p} + m)[\theta(\pm p^0)\delta(p^2 - m^2)],
\] (3.12)
\[
D^{\#ab,\mu\nu}_{10}(p) = -2\pi i\delta_{ab}d^{\mu\nu}(p)[\theta(\pm p_0)\delta(p^2)].
\] (3.13)

These states are initially empty and the vacuum correlators \( G^{\#}_{10,01}(p) \) and \( D^{\#}_{10,01}(p) \) represent only on-mass-shell particles in the final states.

The superscript “*” will label “bounded” states of “valence” quarks and gluons in the initial proton:
\[
G^{*\phantom{\#}01}(p) = 2\pi i(2\pi)^3 \frac{1}{V_{lab}}\frac{1}{3} \delta_{ij} \frac{1}{2} \not{p}\delta(p^2)\delta(p_1)\theta(p^+)^2)\mathcal{V}(p^+),
\] (3.14)
\[
G^{*\phantom{\#}10}(p) = 0.
\] (3.15)

Equation (3.14) describes the phenomenological distribution of the “valence” quarks as a function of their light-cone momenta \( p^+ \), while Eq. (3.15) means that there are no “valence” anti-quarks within the proton. (This assumes that we start at very low scale. Otherwise Eq. (3.15) will have the same form as Eq. (3.14).) The factors \( 1/2 \) and \( 1/3 \) correspond to averaging of the distribution over the quark spin and color, respectively. The factor \( (2\pi)^3/V_{lab} \) corresponds to normalization: we consider a flux with one proton in a volume \( V_{lab} \) per unit time.

In the same way we define the “initial” distribution of “valence” gluons by
\[
D^{*\phantom{\#}ab,\mu\nu}_{10}(p) = -2\pi i(2\pi)^3 \frac{1}{8} \frac{1}{V_{lab}}\frac{1}{2} d^{\mu\nu}(p)\theta(\pm p_0)\delta(p^2)\delta(p_1)\theta(p^+)^2)\mathcal{G}(p^+),
\] (3.16)
with \( 1/8 \) and \( 1/2 \) standing for the color and polarization average and where \( d^{\mu\nu} \) is a projector
\[
d^{\mu\nu}(p) = -g^{\mu\nu} + \frac{p^\mu n^\nu + n^\mu p^\nu}{(p^+)^2},
\] (3.17)
which is a sum over the physical gluon polarizations in the null-plane gauge,
\[ n^\mu B_\mu^2 = 0; \quad n^2 = 0. \] (3.18)

The reason for introducing these distributions is to give definite values to the quantum numbers (the charges and the momenta). Their densities are given by

\[ j^+(p) = \text{Tr} \gamma^+ G^{#ij}_0(p) = \frac{1}{V_{\text{lab}}} \int_0^{P^+} dp^+ V(p^+) = \frac{P^+}{V_{\text{lab}}} \int_0^1 dx V(x) \] (3.19)

for the quark’s light-cone charge flux, and by

\[ T_{qq}^{++} = -i \int \frac{d^4p}{(2\pi)^4} \text{Tr} \gamma^+ p^+ c^{#ij}_0(p) = \frac{1}{V_{\text{lab}}} \int_0^{P^+} dp^+ p^+ V(p^+) = \frac{P^+}{V_{\text{lab}}} \int_0^1 dx x V(x) \] (3.20)

for the \((++)\)-component of the quark energy-momentum tensor. In these equations we have introduced the Feynman variable, \( x = x_F = p^+/P^+ \). The momentum flux density from the gluon component is given by

\[ \text{W}^{\mu\nu}(q) \equiv \sum_\mu \sum_\nu \text{Tr} \gamma^\mu G^{#ij}_{10}(p + q) \gamma^\nu G^{#ij}_0(p) \] (3.22)

The off-diagonal quark Greenians (field correlators) in this equation obey integral equations which express them in terms of exact retarded and advanced propagators and sources \( \Sigma_{0:01} \) (the “current” correlators):

\[ G_{i0} = G_{ret}^{-1} G_{i0}^{(0)} G_{i0}^{(0)} G_{adv} - G_{ret} \Sigma_{i0} G_{adv}. \] (3.23)

The retarded and advanced Green’s functions obey more familiar equations,

\[ G_{ret} = G_{ret} + G_{ret} \Sigma_{ret} G_{ret}, \] (3.24)

\[ G_{adv} = G_{adv} + G_{adv} \Sigma_{adv} G_{adv}, \] (3.25)

which allows a symbolic solution

\[ G_{ret}^{-1} = G_{ret}^{-1} - \Sigma_{ret}, \quad G_{adv}^{-1} = G_{adv}^{-1} - \Sigma_{adv}. \] (3.26)

In the first approximation we may replace the exact retarded and advanced quark Green’s functions, \( G_{ret} \) and \( G_{adv} \), by the bare ones that carry the same leading light-cone singularity,

\[ G_{ret}(p) = \frac{p}{(p^0 + i0)^2 - p^2}, \quad G_{adv}(p) = \frac{p}{(p^0 - i0)^2 - p^2}. \] (3.27)

Then in virtue of (3.17), Eqs. (3.23) take the following form

\[ G_{01} = G_{01}^# + G_{01}^* - G_{ret} \Sigma_{01} G_{adv}, \] (3.28)

\[ G_{10} = G_{10}^# - G_{ret} \Sigma_{10} G_{adv}. \] (3.29)

In the lowest order we neglect sources and leave only correlators of initial fields:

\[ W^{\mu\nu}(q) = e^2 \frac{2V_{\text{lab}} P_{\text{lab}}}{4\pi} \int \frac{d^4p}{(2\pi)^4} \text{Tr} \gamma^\mu G^{#} G_{10}(p + q) \gamma^\nu G^{*}_{01}(p). \] (3.30)

Substituting Eqs. (3.13) and (3.14) and rewriting the delta-function of the on-mass-shell final-state quark as
we get the incredibly simple result of the “naive” parton model:

$$F_2^{(0)}(x_{Bj}) = e_f^2 \int_0^1 dx \delta(x - x_{Bj}) x \mathcal{V}(x).$$  \hfill (3.32)

In the next approximation we should include the quark fields coming from the quark and anti-quark sources, \(\Sigma_{01}\) and \(\Sigma_{10}\), i.e., the last terms in Eqs. \((3.29)\) and \((3.29)\). The general expression for the quark self-energy matrix is given by Eq. \((2.7)\). Still restricting ourselves to bare vertices and bare tree Greenians we get:

$$\Sigma_{01}^*(p) = ig^2 C_F \int \frac{d^4k}{(2\pi)^4} (Tr \gamma_\mu G_{01}(k)\gamma_\nu D_{10}^{\mu\nu}(k - p) + Tr \gamma_\mu G_{01}^{\dagger}(k)\gamma_\nu D_{10}^{*\mu\nu}(k - p)), $$  \hfill (3.33)

$$\Sigma_{10}^*(p) = ig^2 C_F \int \frac{d^4k}{(2\pi)^4} (Tr \gamma_\mu G_{10}^{\dagger}(k + p)\gamma_\nu D_{01}^{*\mu\nu}(k)). $$  \hfill (3.34)

The superscript “*#” means that one of the contributing states belongs to the set of final states, while the other originates from the initial proton.

The gluon correlators obey the following equations:

$$D_{01,10} = D_{01,10}^* + D_{ret} \Pi_{01,10} D_{adv}, $$  \hfill (3.35)

$$D_{ret} = D_{ret} + D_{ret} \Pi_{ret} D_{ret}, $$  \hfill (3.36)

$$D_{adv} = D_{adv} + D_{adv} \Pi_{adv} D_{adv}. $$  \hfill (3.37)

In the first approximation, all contributions of the gluon sources, \(\Pi_{10,01}\), should be dropped along with radiative corrections to the retarded and advanced gluon propagators. The tensor structure of the self-energy may be only of the following form

$$\Sigma(p) = \bar{\psi} \sigma_2(p) + \bar{\psi} \sigma_3(p). $$  \hfill (3.38)

In all the equations it enters in a single combination,

$$\bar{\psi} \Sigma(p) \psi = \bar{\psi} \sigma_1(p) + \bar{\psi} p^2 p^\pm \sigma_3(p); \quad \sigma_1(p) = p^2 \sigma_2(p) + 2(p^\pm)^2 \sigma_3(p). $$  \hfill (3.39)

After a long, but routine calculation we find that

$$i \sigma_1^*(p) = -\frac{\pi g^2}{V_{lab} P^+ p^2} \int_{p^+}^{P^+} \frac{dk^+}{k^+} \delta[(p^+ - k^+)p^- - p_1^+ k^+] \left\{ C_F \left[ \frac{z^2 + 1}{1 - z} P^+ \mathcal{V}(k^+) + \frac{2z^2 - 2 - 1}{2} P^+ \mathcal{G}(k^+) \right] \right\}, $$  \hfill (3.40)

$$i \sigma_2^*(p) = \frac{\pi g^2}{V_{lab} P^+} \int_{p^+}^{P^+} \frac{dk^+}{k^+} \delta[(p^+ - k^+)p^- - p_1^+ k^+] \left\{ C_F P^+ \mathcal{V}(k^+) + (1 - z) P^+ \mathcal{G}(k^+) \right\}, $$  \hfill (3.41)

where \(z = p^+/k^+\), and the invariants for the anti-quark source \(\Sigma_{10}\) do not contain terms with valence distributions \(\mathcal{V}(k^+)\).

It is now straightforward to find the first correction \(F_2^{(1)}(x_{Bj}, Q^2)\) to the DIS structure function:

$$F_2^{(1)}(x_{Bj}) = e_f^2 \int_0^1 dx \delta(x - x_{Bj}) x \Delta^{(1)} q_f(x). $$  \hfill (3.42)

It is presented in the same form as the zero-order term, \((3.32)\), with \(\mathcal{V}_f(x) = q_f(x, Q^2_0)\) replaced by

$$\Delta^{(1)} q_f(x, Q^2) = \frac{V_{lab} P^+}{(2\pi)^3} \int_{Q^2_0}^{Q^2} \frac{dP^+_f}{p^+_f} \int dp^- p^+ i \sigma_1^*(p) \bigg|_{p^2} \bigg|_{P^+}^2. $$  \hfill (3.43)
To be consistent with the resonant condition of the measurement (3.34), we must require that $Q^2$ is large enough (formally, $Q^2 \to \infty$), and that the behavior of the integrand at high $p_T$ guarantees the convergence of the integral. If we substitute (3.41) into the r.h.s. of (3.43) and perform a residual integration over $p^-$ using the delta-function:

$$
\int dp^- \theta(k^+ - p^+) \delta[(p^+ - k^+)p^- - p_T^2] = -\frac{1}{k^+p_T^2},
$$

then we easily recover the first approximation of the Altarelli-Parisi equation for the non-singlet quark structure functions of the deep inelastic electron-proton scattering.

In the next order we must iterate Eq. (3.33), including the influence of the source $\Pi_{01}^{\rho\sigma}$ on the gluon field. Cutting the accuracy of calculations in Eq. (2.11) to bare vertices, and neglecting the sources in the internal Greenians, we get the result

$$
\Pi_{01}^{\rho\sigma}(p) = i g_7^2 \int \frac{d^4k}{(2\pi)^4} \langle k|\gamma^\mu G_{01}^{\rho\sigma}(k) \gamma^\nu G_{10}^{\rho\sigma}(k-p) + \gamma^\mu G_{01}^{\rho\sigma}(k+p) \gamma^\nu G_{10}^{\rho\sigma}(k) - \gamma^\nu G_{01}^{\rho\sigma}(k) \gamma^\mu G_{10}^{\rho\sigma}(k-p) \rangle - \int \frac{d^4k}{(2\pi)^4} V_{ac}^{\rho\sigma\lambda}(p,k-p,-k) D_{01,cc'}^{\rho\sigma\lambda}(k) V_{bc'}^{\rho\sigma\lambda}(-(p,p,-k,k)) D_{10,cc'}^{\rho\sigma\lambda}(k-p) \rangle,
$$

where a sum over the quark flavor $f$ is assumed in the first term.

The polarization tensor $\Pi^{\rho\sigma}$ only appears between retarded and advanced propagators: $[D_{ret}(p)\Pi(p)D_{adv}(p)]^{\rho\sigma}$. The latter contain projectors $d^{\rho\sigma}(p)$ which are orthogonal to the 4-vector $n^\mu$. So of the general tensor, only two terms survive:

$$
\Pi^{\rho\sigma}(p) = g^{\rho\sigma} w_1(p) + \frac{p^\rho p^\sigma}{p^2} w_2(p).
$$

Others, like $p^\rho n^\nu + n^\rho p^\nu$ or $n^\rho n^\nu$, will cancel out. Introducing one more projector,

$$
\bar{d}^{\rho\sigma}(p) = -d^{\rho\sigma}(p)d_p^{\rho\sigma}(p) = -g^{\rho\sigma} + \frac{p^\rho n^\nu + n^\rho p^\nu}{(np)} - \frac{p^2 n^\rho n^\nu}{(p^+)^2},
$$

which is orthogonal to both vectors $n^\nu$ and $p^\mu$, we find that the invariants $w_1$ and $w_2$ can be found from two convolutions,

$$
-\bar{d}_{\rho\sigma}(p)\Pi^{\rho\sigma}(p) = 2w_1(p); \quad n_\rho n_\nu \Pi^{\rho\sigma}(p) = \frac{(p^+)^2}{p^2} w_2(p),
$$

independently of other invariants accompanying the missing tensor structures. The new projector, which includes only two transversal gluon modes, naturally appears in the tensor with a gluon source:

$$
[d(p)\Pi(p)d(p)]^{\rho\sigma} = -\bar{d}_{\rho\sigma}(p)w_1(p) + \frac{(p^+)^2}{p^2} w_2(p)n^\rho n^\nu.
$$

Now it is easy to find the first approximation for the invariants $w_1(p)$ and $w_2(p)$:

$$
iw_1^{\#}(p) = -\pi g_7^2 V_{lab}^{\rho\sigma} p^2 \int_{p^+} \frac{dk^+}{k^+} \delta[(p^+ - k^+)p^- - p_T^2] \times
\left\{ C_F \frac{1}{z} (1 - z)^2 P^+ V(k^+) + 2N_c [z(1 - z) + \frac{z}{1 - z}] P^+ G(k^+) \right\},
$$

$$
iw_2^{\#}(p) = -\pi g_7^2 V_{lab}^{\rho\sigma} p^2 \int_{p^+} \frac{dk^+}{k^+} \delta[(p^+ - k^+)p^- - p_T^2] \frac{C_F}{z} \left\{ 4P^+ V(k^+) + 4N_c z(1 - \frac{z}{2}) P^+ G(k^+) \right\}.
$$

In accordance with the previous convention, and as a reminder of the approximations involved, the invariants carry superscripts $^{\#}$ . These indicate that the invariants are contributed to by one proton’s “bound” state and one on-mass-shell final state of a quark or gluon. We hope that the reader is not confused by the absence of other indices like quark color, or indices indicating the type of ordering in the invariants $w_1$ and $\sigma_i$. They can easily be recovered
when it is needed. For example, \( w_i \) and \( \sigma_i \) are parts of the self-energy which are summed over the color. So if \( \Pi \) and \( \Sigma \) appear as internal elements in any formula, we must restore the color factors in the following way:

\[
\Sigma \rightarrow \Sigma_{ij} = \frac{\delta_{ij}}{3} \Sigma, \quad \Pi \rightarrow \Pi_{ab} = \frac{\delta_{ab}}{8} \Pi.
\]

Now we may reconstruct a missing element, viz., the first correction to the gluon structure function,

\[
\Delta^{(1)} G(x, Q^2) = \frac{V_{ab} P^+}{(2\pi)^3} \int_{Q^2_0}^Q d^2p_i \int d^2p^- p^- p^+ i\bar{u}_{i}^{*\#}(p) \frac{w_i}{|p^2|^2}, \quad (3.52)
\]

which is similar to the correction (3.44) to the quark structure function. In a sequence of approximations it should be added to the “valence” gluon distribution \( G(x) \).

If we substitute Eq. (3.50) into (3.52), we immediately obtain the lowest order approximation of the second Altarelli-Parisi equation for the gluon structure function of the deep inelastic electron-proton scattering.

Concluding this section, let us pay special attention to the infrared poles at \( z = 1 \), which originate from the pinch-poles of the gluon Greenians in the null-plane gauge. To cure this problem we will proceed following Altarelli and Parisi [13]. We will first shield the IR singularity by introducing the “plus-distributions,”

\[
\int_{0}^{1} \frac{f(z) dz}{(1 - z)_+} = \int_{0}^{1} \frac{f(z) - f(1)}{1 - z} dz,
\]

and then modify the end-point behavior in such a way that the first integrals could not be changed by the radiative corrections. The first integrals are the total flux of the flavor \( f \),

\[
j_f^+ = -iV_{ab} \int \frac{d^4p}{(2\pi)^4} \text{Tr} \gamma^+ [G^f_{01}(p) - G^f_{10}(p)] , \quad (3.53)
\]

and the total flux of the light-cone momentum,

\[
T^{++} = -iV_{ab} \int \frac{d^4p}{(2\pi)^4} \{ \sum_f \text{Tr} \gamma^+ p^+ [G^f_{01}(p) + G^f_{10}(p)] + (p^+)^2 g_{\mu\nu} D^{\mu\nu}_{01}(p) \} . \quad (3.54)
\]

As follows from Eqs. (3.43) and (3.52), the first radiative corrections to the flavor and momentum fluxes are

\[
\Delta j^+_f = \frac{P^+ V_{ab}}{(2\pi)^3} \int_{Q^2_0}^{p^+} dp^+ \int_{Q^2_0}^{Q^2} d^2p_i \int d^2p^- p^- p^+ \left[ \frac{i\sigma^f_{01}(p)}{|p^2|^2} - \frac{i\sigma^f_{10}(p)}{|p^2|^2} \right], \quad (3.55)
\]

and

\[
\Delta T^{++} = \frac{P^+ V_{ab}}{(2\pi)^3} \int_{Q^2_0}^{p^+} dp^+ \int_{Q^2_0}^{Q^2} d^2p_i \int d^2p^- (p^+)^2 \left[ \sum_f \frac{i\sigma^f_{01}(p)}{|p^2|^2} + \frac{i\sigma^f_{10}(p)}{|p^2|^2} + \frac{i\sigma^f_{10}(p)}{|p^2|^2} \right], \quad (3.56)
\]

respectively. The superscript “\( \#^* \)” is omitted because these equations remain valid beyond the first order approximation. The resulting conditions, \( \Delta j^+_f = 0 \) and \( \Delta T^{++} = 0 \), for the splitting kernels in the leading logarithmic approximation are obvious,

\[
\int_{0}^{1} P_{qq}(z) dz = 0, \quad (3.57)
\]

\[
\int_{0}^{1} [zP_{gg}(z) + 2n_f zP_{qg}(z)] dz = 0
\]

\[
\int_{0}^{1} [zP_{gg}(z) + zP_{qq}(z)] dz = 0.
\]

However, beyond the LLA they may change. One now readily finds an explicit form of the splitting kernels,
Greenians, and Eqs. (3.29), (3.29) and (3.35), which define the Greenians (field correlators) via the self-energies. We must also confine one of the off-diagonal field correlators to the out-states in the continuum:

\[
P_{gq}(z) = 2N_c \left[ z(1-z) + \frac{z}{(1-z)_+} + \frac{1-z}{z} + \frac{\beta_0}{4N_c} \delta(1-z) \right],
\]

(3.58)

where the factor \(\beta_0 = 11 - 2n_f/3\) coincides with the first coefficient of the Gell-Mann-Low function.

4. Evolution of the sources.

Now we are ready to derive integral equations that govern the field correlators and their sources. Actually, they have already been given above. An examination of the calculations in the previous Section shows that we did not sum any series. We were consequently performing a series expansion of the previously derived self-consistent solution of the integral Schwinger-Dyson equations. These are Eqs. (2.8) and (2.11), which define the self-energies via the Greenians, and Eqs. (3.29), (3.29) and (3.33), which define the Greenians (field correlators) via the self-energies. We shall rewrite the last equations in the form

\[
G_{10,01} = G_{10,01}^\# - G_{ret} \Sigma_{10,01} G_{adv}, \quad \text{and}
\]

\[
D_{10,01} = D_{10,01}^\# - D_{ret} \Pi_{10,01} D_{adv},
\]

(4.1)

(4.2)

omitting the \(*\)-labelled terms as they do not contribute to the differential form of evolution equations. In contrast to the integral evolution equations, the differential equations do not require any information about the initial data. At this point we do not make any approximations.

4.1. Dynamical equations in the leading logarithmic approximation.

In order to obtain the equations of the leading logarithmic approximation, which sum up the perturbation series with the leading logarithms, we must consider the vertex operators in Eqs. (2.8) and (2.11) as the bare ones. We try to set any momentum or angular ordering of the emission processes.

By inspection, these equations reveal an astonishing result - the equations which govern the dynamics of the sources \(\Sigma_{01}\) and \(\Pi_{01}\) of the field correlators \(G_{01}\) and \(D_{01}\) have a ladder structure. This result appeared though we did not try to set any momentum or angular ordering of the emission processes.

This result deserves special discussion. First, let us trace through the previous calculations once more. We started from the basic definitions (3.2) and (3.3) of the observable cross-section of the DIS. Then we expressed a specific off-diagonal polarization tensor of a proton via the off-diagonal Greenians \(G_{10,01}\) and \(D_{10,01}\) of the “kinetic” technique. Afterwards we used the Schwinger-Dyson equations for these Greenians to express them via their sources \(\Sigma_{10,01}\) and \(\Pi_{10,01}\). This resulted in a closed system of ladder-type equations for these sources.

Now we must answer two questions: (i) what was the physical input; and (ii) what follows from the application of new method to the well-known problem of DIS, for which the solution is already known.

As in any quantum-mechanic problem, the physical picture is specified by a density matrix of the initial state. Our density matrix carries the same information as the operator functions of the OPE, that is, no detailed information. Only the global quantum numbers are under its control.

A more careful analysis shows that our result is not exactly the same as that of the OPE-based approach. Though we get (up to a quantitatively inessential shift of the singularities) the leading logarithm approximation resulting in the Gribov-Lipatov-Altarelli-Parisi equations, which sums the leading logarithms of the perturbation series, an
important qualitative difference appears. The Feynman tree propagators were replaced by the retarded ones. This change reveals the causal structure of the whole process. The last interaction, which puts a single quark onto its mass shell in the perturbative vacuum, is also the latest in time. In other words, the last interaction results in a collapse of the initial wave function.

So, as a by-product, we have answered an old question about the correspondence between the evolutionary scale $Q^2$ and the temporal scale. The causal space-time structure is contained, a priori, in evolution equations like the GLAP equations. It is not necessary to impose it a posteriori. The $x$-ordering is a further consequence of the $\theta$-functions that allow only for the emission into the initially unpopulated continuum. The $Q^2$-ordering with respect to transverse momentum is not a necessary condition, and we shall discuss it shortly.

What are the practical consequences of this picture? Firstly, in order to find the structure functions of the deep inelastic scattering of a proton off an electron, one should know the intensity of the quark field source $\Sigma_{01}$ before the field interaction with the electron. That is why it does not matter before what kind of interaction. The intensities of the sources $\Sigma$ and $\Pi$ turn out to be universal functions. Unlike the structure functions of DIS they do not depend from the particular choice of the measurement procedure. We can continue this reasoning in application to pp- and AA-collisions. This leads to the conclusion that if we wish to use e-p DIS data for the description of p-p collision dynamics, we should rely on the sources $\Sigma$ and $\Pi$, rather than the structure functions $q(x, Q^2)$ and $G(x, Q^2)$. Their evolution is only a specific projection of the more complicated dynamic of the sources.

The second consequence is that after the structure functions $q(x, Q^2)$ and $G(x, Q^2)$ of the e-p DIS are found (simply by fitting data, for example), we do not need their phenomenological interpretation as a parton density in order to apply them to other types of collision.

It is well known that instead of calculating the observable e-p cross-section, the OPE method computes the imaginary part of the truncated Feynman amplitude of the auxiliary Compton process. The next step is a renormalization group analysis of this definite S-matrix amplitude. Unfortunately, the power of this method is restricted to one single problem. Any extension of this method requires a parton language.

Indeed, though the cross-section of the Drell-Yan process is defined by the same (except for kinematic region) polarization operator as in DIS, it can not be calculated via OPE. The difference is that now the operator functions should be averaged over a state with two protons. Feynman propagators which contribute to an auxiliary S-matrix amplitude do not disappear outside the light cone. For the massless partons, and especially in the infinite momentum frame, it leads to the effective interaction before the collision. The factorization theorem may be a remedy, but it requires parton language. At the same time it is clear that protons colliding at high energies are causally independent until the moment of collision.

We shall now show that the above ladder equations (1.3) and (1.4) are equivalent to the well known QCD evolution equations. Indeed, let us rewrite Eqs. (1.3) and (1.4) in terms of their tensor components. Beyond the first order calculations of Section 2.3, we must also take into account radiative corrections to the retarded and advanced Green’s functions. The spinor (tensor) structure of $\Sigma_{ret,adv}$ and $\Pi_{ret,adv}$ remains unchanged. The solution of the Schwinger-Dyson equations (3.24), (3.25) and (3.36), (3.37) for retarded and advanced Green’s functions is easily cast in the form:

$$G_{\mu\nu,adv}(p) = \frac{\hat{\rho}}{p^2 - \sigma_1^{R,A}(p)} - \frac{\hat{\rho} p^+ \sigma_2^{R,A}(p)}{(p^2 - \sigma_1^{R,A}(p))(1 - \sigma_2^{R,A}(p))},$$

$$D_{\mu\nu,adv}(p) = \frac{\hat{\rho} \mu\nu(p)}{p^2 - w_1^{R,A}(p)} + \frac{p^2}{(p^+)^2} n^\mu n^\nu.$$  

Introducing the following shorthand notations for the denominators of the propagators of different modes,

$$W_1^{R,A}(p) = p^2 - w_1^{R,A}(p), \quad W_2^{R,A}(p) = p^2 - w_2^{R,A}(p),$$

$$S_1^{R,A}(p) = p^2 - \sigma_1^{R,A}(p), \quad S_2^{R,A}(p) = 1 - \sigma_2^{R,A}(p),$$

we easily obtain

$$G_{\mu\nu}(p) = \frac{\hat{\rho}}{S^{R}(p)S_1^{R}(p)} + \frac{\hat{\rho} p^+}{2p^+ S^{R}(p)S_2^{R}(p)};$$

$$[D_{\mu\nu}(k)\Pi_{01}(k)D_{\mu\nu}(k)]^{\mu\nu} = \frac{-\hat{\rho} \mu\nu(p)w_1^{01}(p)}{W_1^{R}(p)W_1^{R}(p)} + \frac{(p^+)^2 w_2^{01}(p)n^\mu n^\nu}{W_2^{R}(p)W_2^{R}(p)}.$$  

The complete evolution equations are long, and are given in Appendix 1 along with analysis of further approximations. Here we write only the leading logarithmic terms which eventually result in the GLAP equations:
\[ \sigma_1^{01}(p) = \frac{g^2}{(2\pi)^3} \int_{p^+}^\infty \frac{dk^+}{k^+} \int d^2k_i dk^- \delta((p^+ - k^+)(p^- - k^-) - (p_t - k_t)^2) \times \]

\[ -p^2 k^+/p^+ \left[ P_{qq}(\frac{p^+}{k^+}) k^+ \sigma_1^{01}(k) + P_{gg}(\frac{p^+}{k^+}) k^+ w_1^{01}(k) \right] \]

\[ \frac{k^+ \sigma_1^{01}(k)}{S_1^R(k) S_1^A(k)} + \frac{k^+ w_1^{01}(k)}{W_1^R(k) W_1^A(k)} + \ldots \right), \] (4.9)

\[ w_1^{01}(p) = \frac{g^2}{(2\pi)^3} \int_{p^+}^\infty \frac{dk^+}{k^+} \int d^2k_i dk^- \delta((p^+ - k^+)(p^- - k^-) - (p_t - k_t)^2) \times \]

\[ -p^2 k^+/p^+ \left[ P_{qq}(\frac{p^+}{k^+}) k^+ \sigma_1^{01}(k) + P_{gg}(\frac{p^+}{k^+}) k^+ w_1^{01}(k) \right] \]

\[ \frac{k^+ \sigma_1^{01}(k)}{S_1^R(k) S_1^A(k)} + \frac{k^+ w_1^{01}(k)}{W_1^R(k) W_1^A(k)} + \ldots \right). \] (4.10)

Substituting Eqs. (4.9) and (4.10) into Eqs. (3.9) and (3.22), we easily find that the well known structure functions of the electron-proton deep inelastic scattering are defined by the equations which are similar to (3.43) and (3.52):

\[ q_f(x, Q^2) = q_f(x, Q_0^2) + \frac{V_{lab} P^+}{(2\pi)^3} \int_{Q_0^2}^{Q^2} dp_t^2 \int dp^- \frac{-i p^+ \sigma_1^{01}(p)}{S_1^R(p) S_1^A(p)}, \] (4.11)

\[ G(x, Q^2) = G(x, Q_0^2) + \frac{V_{lab} P^+}{(2\pi)^3} \int_{Q_0^2}^{Q^2} dp_t^2 \int dp^- \frac{-i p^+ w_1^{01}(p)}{W_1^R(p) W_1^A(p)}. \] (4.12)

The first of these equations is exact, it does not depend on any further approximations. The second one is approximate. It holds only in the LLA.

We observe that the structure functions of deep inelastic e-p scattering in the LLA require only one of the two invariants from each of the sources Π and Σ. This nice feature fails beyond LLA, and does not hold for processes with different polarization properties of the interaction vertex.

Unfortunately, our knowledge of the confinement dynamics is not sufficient to proceed without additional qualitative physical input. We must integrate over the null-plane momentum \( p^- \) and this is a highly nontrivial procedure, unless we postulate an ad hoc requirement that the answer should contain only logarithms. When we were doing similar calculations in the lowest order, we designed the density matrix of “valence” quarks and gluons in such a way that \( k^- = 0 \) and \( k_t = 0 \). In doing this, we kept in mind both the physics of the infinite momentum frame and the proton’s confinement before the collision.

The point is that propagation of quark and gluon fields before they reach a collision vertex is not free. Assuming the opposite, we would immediately violate the causality principle, or be in contradiction with previous calculations. We shall see shortly that the requirement of selfconsistency between emission-absorption and propagation provides a natural condition for renormalization. Suppression of the spectral patterns with \( k^- \neq 0 \) is a part of this condition. Physically, it means that the field correlators like \( \Pi(x - y) \) do not depend on the difference \( x^+ - y^+ \) of the coordinates in the direction of the light-cone propagation. We will conjecture that the exact retarded and advanced propagators indeed prevent the wave packet of a proton (or a nucleus) from a premature decay, and shall cast the requirement in two forms.

The first, weak form, of this condition can be cast in the form of the inequality: \( |k^-| \ll |p^-| \). Then we may integrate over \( p^- \) using the previous formula (4.4) as a physical approximation. The second, strong form replaces the inequality by the exact condition \( p^- = 0 \), which can be incorporated into the expression:

\[ \frac{i p^+ \sigma_1^{01}(p)}{S_1^R(p) S_1^A(p)} = \frac{(2\pi)^3}{V_{lab} P^+} (p^-) \frac{dq_f(x, p_t^2)}{dp_t^2}, \] (4.13)

\[ \frac{i p^+ w_1^{01}(p)}{W_1^R(p) W_1^A(p)} = \frac{(2\pi)^3}{V_{lab} P^+} (p^-) \frac{dG(x, p_t^2)}{dp_t^2}. \] (4.14)

These equations are a recipe on how to use DIS data in the leading logarithmic approximation. They require an additional comment about the \( p_t^2 \)-integration, which is the next step in obtaining the DIS structure functions. As we have already argued, an unambiguous definition of the structure functions is possible only in the limit of \( Q^2, \nu \rightarrow \infty \) which eventually leads to the resonant condition (3.42) of the measurement: \( x_F = x_Bj \).

When the r.h.s. of the Eqs. (4.13) and Eq. (4.14) are integrated over \( p_t^2 \) with a large upper limit \( Q^2 \), the kernel of the resulting equation depends on \( k_t^2 \) only in the combination \( k_t^2 + (k^+ / p^+) Q^2 \). The \( k_t^2 \)-behavior of the structure function
in the new integral should guarantee its convergence. Therefore, one may neglect $k_f^2$ from the very beginning by assuming that only the domain $k_f \ll p_t$ contributes in the the initial equations. This condition is known as “ordering by angles.” Unlike the ordering by Feynman $x$, it is not a fundamental requirement. Clearly, this approximation is not valid at very low $x$. Nevertheless, even for low $x$ the initial equations (3.9) and (4.10) remain unchanged. In the Appendix 1, we show that at low $x$ they may be reduced to the BFKL equation.

An explicit expression for the longitudinal structure function of the DIS follows from Eq. (3.10):

$$3Q^2 F_L(x, Q^2) = \frac{V_{AghD^+}}{(2\pi)^3} \sum \int_{Q_t^2}^{Q^2} d^2 p_t \int d^2 p \left[ \frac{p^2 \sigma_1(p)}{S_1^q(p)S_1^q(p)} - \frac{\sigma_2(p)}{S_2^q(p)S_2^q(p)} \right].$$  (4.15)

This equation is exact and it proves the Callan-Gross relation when we neglect the scaling violation ($Q^2$-dependence) in the structure functions.

Up until now, we can only trace the correspondence between our approximate equations (4.9) and (4.11) and the LLA of the OPE-based calculations or the Lipatov’s LL(1/x) approximation of the Regge calculus. The longitudinal structure function found via OPE has an extra small factor $\alpha_s$ which is not compensated for by any big logarithm. Correspondence between the two approaches in the next orders is still unclear. In what follows we are going to use standard set of structure functions derived from the data in the leading logarithm approximations. Thus, we shall neglect all terms which can be explicitly reduced to $F_L$.

### 4.2. Renormalization of the evolution equations.

Until now we have dealt only with objects which do not require renormalization. All these objects were tightly connected with observables. Corresponding Greenians and self-energies were imaginary and for this reason could not contain ultra-violet divergencies. So we could safely use a “naive” form of Schwinger-Dyson equations,

$$G_{AB} = G_{AB} + \sum_{RS} G_{AR} \Sigma_{RS} G_{SB}, \quad D_{AB} = D_{AB} + \sum_{RS} D_{AR} \Pi_{RS} D_{SB}. \quad (4.16)$$

The divergent retarded and advanced self-energies were completely neglected, and retarded and advanced Green’s functions were considered as the bare ones. Consequently, the counter terms of the Lagrangian still did not manifest themselves, and the renormalized coupling constant $\alpha_s$ still remains undefined. We must now fill this gap.

After including the counter-terms we obtain the same equations, but with the self-energies modified by the quasi-local terms:

$$D_{AB} = D_{AB} + \sum_{RS} D_{AR} [(Z_{1F} \Pi_{RS} + Z_1 \Pi''_{RS}) + (1 - Z_3)(-1)^R \delta_{RS} D_0^{-1}] D_{SB}, \quad (4.17)$$

$$G_{AB} = G_{AB} + \sum_{RS} G_{AR} [Z_{1F} \Sigma_{RS} + (1 - Z_2)(-1)^R \delta_{RS} G_0^{-1}] G_{SB}, \quad (4.18)$$

where $\Pi'$ and $\Pi''$ are the fermion and gluon loops respectively. In perturbative calculations the factors $Z_{1,1F}$ should be split further as $1 + (Z_{1F} - 1)$, with the second term assigned to the UV-renormalization of the vertex. The only changes from Eqs. (4.10) are due to additional diagonal terms in $\Pi$ and $\Sigma$. This is quite natural as the off-diagonal terms are imaginary and if they were divergent we would have no remedy to cure the problem. As the matrix structure of Eqs. (4.18) and (4.18) is literally the same as that of Eqs. (4.14), we can rotate the $2 \times 2$ basis as usual. Keeping in mind the light-cone dominance, we may rewrite equations for $T$-ordered and $T^\dagger$-ordered correlators as

$$D_{11}(p) = D_{11}^#(p) + D_{rel}(p) [(Z_{1F} \Pi'_{11} + Z_1 \Pi''_{00}) \pm (1 - Z_3) D_0^{-1}(p)] D_{adv}(p), \quad (4.19)$$

$$G_{11}(p) = G_{11}^#(p) + G_{rel}(p) [Z_{1F} \Sigma_{11}(p) \pm (1 - Z_2) G_0^{-1}(p)] G_{adv}(p), \quad (4.20)$$

where $\Pi_{11,00}(p)$ and $\Sigma_{11,00}(p)$ should be calculated using Eqs. (2.8) and (2.11) with the bare vertices and no more than one source. We omit the *-labelled terms here as we are interested in the UV-renormalization and low-$x$ effects. These terms are effectively cut off at very high momenta and do not affect the latest stages of evolution.

Eqs. (4.19) – (4.20) are too approximate to give an explicit value of running coupling constant. The questions we want to consider are: (i) does renormalization of the sources $\Pi$ and $\Sigma$ result in renormalization of the coupling
onto the normal modes, one obtains for the fermion sources.

\[
D_1 = D_{00} + D_{11} = D_{10} + D_{01}, \quad \Pi_1 = \Pi_{00} + \Pi_{11} = -\Pi_{10} - \Pi_{01},
D_0 = D_{ret} - D_{adv} = D_{10} - D_{01}, \quad \Pi_0 = \Pi_{ret} - \Pi_{adv} = -\Pi_{10} + \Pi_{01}
\]

(4.21)
define imaginary parts of \(T\)- and \(T^\dagger\)-ordered, as well as retarded and advanced, correlators in an interdependent way. The real parts are not independent either. Indeed,

\[
2D_s = D_{00} - D_{11} = D_{ret} + D_{adv} = 2\text{Re}D_{00} = -2\text{Re}D_{11} = 2\text{Re}D_{ret} = 2\text{Re}D_{adv},
2\Pi_s = \Pi_{00} - \Pi_{11} = \Pi_{ret} + \Pi_{adv} = 2\text{Re}\Pi_{00} = -2\text{Re}\Pi_{11} = 2\text{Re}\Pi_{ret} = 2\text{Re}\Pi_{adv}.
\]

(4.22)

If we recall, in addition, that the causality principle connects real and imaginary parts of the retarded and advanced correlators by means of dispersion relations, then we realise that we have practically no choice in the regularization of the divergent real functions – we must follow the BHPZ scheme [19].

Let us begin by writing down the integral equation for the gluon polarization correlators in the leading approximation:

\[
\Pi_{\mu\nu}^{(0)}(p) = \Pi_{\mu\nu}^{(0)}(p) - i\gamma^2 \int \frac{d^4k}{(2\pi)^4} Tr[\gamma^\mu G_{00}(k - p)\gamma^\nu G_{ret}(k)\{Z_{1f} \Sigma_{11}(p) + (1 - Z_2)G_0^{-1}(k)\}G_{adv}(k) - \text{Im} G_{00}(p) - \text{Re} G_{00}(p)]
\]

(4.23)
The corresponding equation for \(\Pi_{\mu\nu}^{(1)}(p)\) is the complex anti-conjugate of Eq. (4.2), and can be obtained via replacement the \(T\)-ordered functions by the minus \(T^\dagger\)-ordered ones and vice versa. Similar integral equations may be written for the fermion sources.

In complete agreement with (4.21) and (4.22), these equations have ladder structure with retarded behavior. To lowest order, gluon and fermion Green functions are given by

\[
P_{\mu\nu}^{(0)}(p) = \pm \frac{d\rho^{(0)}(p)}{p^2 \pm i0}, \quad G_{00}(p) = \pm \frac{1}{p^2 \pm i0},
\]

(4.24)
and \(\Pi_{\mu\nu}^{(0)}(p)\) is the usual ultraviolet-divergent vacuum gluon polarization tensor.

For the sake of simplicity, let us consider only the gluon sector in the leading approximation. Projecting Eq. (4.24) onto the normal modes, one obtains

\[
w^{(0)}_1(p) = w^{(0)}_0(p) - iZ_1 g_\pi^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k - p)^2 - i0} \mathcal{I}(k, z) w^{(1)}_1(k, p) + (1 - Z_3)k^2
\]

(4.25)
where we have denoted

\[
\mathcal{I}(z) = 8\{(-p^2/z + k^2)P_{gg}(z) - (k - p)^2(z + 1/z - 1/4)\}
\]

and the splitting kernel \(P_{gg}\) is the same as in the evolution equation for a gluon source \(\Pi_{01}\). It is IR-regularized in the same way, otherwise we would obtain a contradiction with equations (4.21) and (4.22). Alternating \(T\)-ordered and \(T^\dagger\)-ordered correlators in the ladder rungs is crucial for the subsequent conclusions.

The imaginary part of Eq. (4.25) is finite. It can be obtained as the sum of the two ladder equations for \(w^{(0)}_1\) and \(w^{(0)}_0\). The divergent real part of the equation in the leading approximation is an equation for \(\text{Re}w^{(0)}_1 = \text{Re}w^{(0)}_0\), i.e. the real part of the retarded self-energy. To facilitate the physical analysis, we shall derive this equation in another way, starting with the explicit expression for the retarded self-energy. Utilizing the identities \(\Pi_{ret} = \Pi_{00} + \Pi_{01} = -\Pi_{10} - \Pi_{11}\) and \(\Pi_{adv} = \Pi_{00} + \Pi_{10} = -\Pi_{10} - \Pi_{01}\), we easily obtain:

\[
\Pi_{\mu\nu}^{\dagger,\dagger}(p) = \frac{i}{2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(p - k)^2 - i0} \mathcal{I}(k, z) \Pi_{\mu\nu}^{\dagger,\dagger}(k) + \Pi^{(0)}_{\mu\nu}(p, -k - p, k)D_{ret}^{\dagger,\dagger}(k) + \Pi^{(0)}_{\mu\nu}(p, -k - p, k)D_{adv}^{\dagger,\dagger}(k) + \Pi^{(0)}_{\mu\nu}(p, -k - p, k)D_{adv}^{\dagger,\dagger}(k)
\]

(4.26)
These equations have very clear physical meaning. The propagator in the loop is retarded (advanced), and guarantees the required time direction. It is affected by the surroundings less than other correlators. The correlator $D_1$ describes the density of states which develops in course of the evolution. Thus, even the light-cone propagation is not really free – emission introduces additional phase shifts which result in the assembly of special wave packet.

The sum and the difference of the Eqs. (4.26), respectively, are the equations for the real ($\Pi_s$) and imaginary ($\Pi_0/2$) parts of the retarded self-energy:

$$\Pi_{\mu\nu}^s(p) = i g^2 \int \frac{d^4k}{(2\pi)^4} V_{\alpha\epsilon f}^{\mu\alpha}(p, -k - p, k) D_s^{\epsilon', \alpha\beta}(k + p) V_{\nu\epsilon f}^{\nu\beta}(p, -k - p, -k) D^r_{\epsilon', \sigma\lambda}(k),$$

$$\Pi_0^s(p) = i g^2 \int \frac{d^4k}{(2\pi)^4} V_{\alpha\epsilon f}^{\mu\alpha}(p, -k - p, k) D_0^{\epsilon', \alpha\beta}(k + p) V_{\nu\epsilon f}^{\nu\beta}(p, -k - p, -k) D^r_{\epsilon', \sigma\lambda}(k).$$

Using Eqs. (4.2), (4.16), and (4.19), and projecting Eq. (4.27) onto the transverse normal mode, we arrive at

$$w^s_t(p) = w^s_{(0)1}(p) + \frac{Z_1 g^2 N_c}{2(2\pi)^3} \left[ \int d^4k \delta[(k - p)^2] I_{\mu\nu}(k^+) \frac{w^s_t(k) + (1 - Z_3)k^2}{[k^2]^2} + \frac{i \alpha_s(p^2)}{2\pi} \int d^4k \frac{p^\mu}{(k - p)^2} \frac{I_{\mu\nu}^r(k^+)}{[k^2]^2} \right] w^s_t(k),$$

where $\mathcal{P}$ denotes principal value integration. Another way to derive this equation is to separate real and imaginary parts in the Eqs. (4.23). This consistency is a consequence of the dispersion relation for $\Pi_{\mu\nu}^{ret}$ which allows one to recover $\text{Re}\Pi_{\mu\nu}^{ret}$ via the already known $\text{Im}\Pi_{\mu\nu}^{ret}$. It reassures us that we are considering the propagation of the gluon field in proper environment of the pre-collision cascade.

One may easily see that the imaginary part of the self-energy, considered given, defines the free term in the inhomogeneous equation (4.29) for the real part of the gluon self-energy. The unusual feature of this equation (which is common for all ladder-type equations like (4.23)) is the counter-term which is detached from the vacuum part of the self-energy. The latter is divergent, and the corresponding counter-term is in the integrand of the equation. This immediately requires that the kernel should act on the counter-term as a

$$\frac{1}{\alpha_s(p^2)} \frac{d^4k}{(2\pi)^4} I_{\mu\nu}(k^+) \frac{w^s_t(k)}{[k^2]^2}.$$
Thus, the object we now study is a field configuration which has quite different properties that those of a free particle. This is evident, for example, from Eq. (1.14), which indicates that imaginary part of \( w^R(p) \) is strongly peaked near \( p^- = 0 \). This configuration is singled out by two requirements (boundary conditions): (i) at the end of its evolution, it produces an off-shell quark that can interact with the electron in a resonant way; (ii) this quark stays bare (on-shell) after the scattering.

In e-p deep inelastic scattering the second condition does not look too realistic. The bare quark will immediately fragment into a hadronic jet. In p-p collisions, the electron is replaced by a gluon from the second proton, but the final quark state must still propagate in the physical vacuum and decays into hadrons as well. Only in AA-collisions do we expect the creation of a volume of perturbative vacuum large enough to allow almost stable free propagation.

The most important point is that the pre-collision dynamics of the field fluctuations is the same in all three cases. This is guaranteed by the geometry of the high energy collision and the causality principle. However, one should keep in mind that the type of fluctuations studied by DIS are strictly selected by the trigger of the specific measurement. Imposing other triggers will select other types of fluctuations. It can not be ruled out a priori that the pieces \( w_2 \) and \( \sigma_2 \) that were neglected in LLA may become more significant.

To find an analogy between the pre-collision dynamics of the proton constituents and physics of continuous media, we may try associate components of gluon self-energy with electromagnetic susceptibility. If its imaginary part is peaked near \( x^0 = \) constant, it may be written as

\[
\text{Im} \Pi_\mu \sim \delta(x^-)
\]

(A significant growth of the imaginary part of the self-energy at the “resonance” \( p^- = 0 \).) A significant growth of the imaginary part of the self-energy at the “resonance” \( p^- = 0 \) should lead to anomalous dispersion – the real part must drop to zero:

\[
\text{Re} w^R(p^2, p^+, p^- = 0) - (1 - Z_3)(-p_3^2) = 0 \quad .
\]

(We can suggest the formal mathematical argument: once \( \text{Im} \Pi_\mu \sim \delta(p^-) \), then from dispersion relation \( \text{Re} \Pi_\mu \sim \mathcal{P}(1/p^-) \) which, though in a singular manner, is equal to zero at \( p^- = 0 \).) In the region of anomalous dispersion, the phase and the group velocities should have the opposite signs. This reveals one more unusual feature of the “undressing” process: the phases of the fields participating in the assembly of the wave packet representing an interacting quark (or gluon) travel in the “normal” time-direction (from the past to the future). These fields leave “holes” in the sea. Only the act of measurement (scattering) transforms the creation of these “holes” into the process of multiple emission.

5. Distributions of quarks and gluons in leading order.

In this and following Sections we present results of an explicit calculation of the single-particle distribution of light quarks and gluons produced at the earliest stage of an AA-collision. In this Section we calculate cross-sections to the lowest order. It is general practice to associate the corresponding processes with the excitation of the sea quarks and gluons. There are three processes of this type. Their graphs are shown in Fig. 1. In Section 6 we will derive a complete set of equations for the first order processes, and discuss how one avoids difficulties which accompany calculations based on the factorization theorem [18].

5.1. Production of light quarks to leading order.

We return to the initial formulae (2.4) and (2.8), and rewrite the former in the momentum representation:

\[
\frac{dN_q}{dpd^4x} = \frac{\text{Tr}[i \bar{\psi} \Sigma_{ij}(p)]}{(2\pi)^3 2p^0} 
\]

(5.1)

The lowest order of our theory assumes: (i) the exact vertex operator must be replaced by the bare one, which leads to

\[
p^0 \frac{dN_q}{dpd^4x} = \frac{g^2}{2(2\pi)^3} \int \frac{d^4k}{(2\pi)^4} \text{Tr}[\bar{\psi}t^a \gamma^\mu G_{01}(p + k) t^b \gamma^\nu D_{01,0\mu}(k)] 
\]

(5.2)

and (ii) in \( G_{01} \) and \( D_{01} \) possible contribution of the out-states of quark and gluon in the continuum are excluded. These contributions are described by the higher orders of the perturbation theory. Thus, we naturally arrive at Eq. (2.8)

\[
p^0 \frac{dN_q}{dpd^4x} = \frac{g^2}{2(2\pi)^3} \int \frac{d^4k dq}{(2\pi)^4} \delta(k + q - p) \{ \text{Tr}[\bar{\psi}t^a \gamma^\mu G_{01}(B)(q) t^b \gamma^\nu D_{01,0\mu}(k)] + (A) \leftrightarrow (B) \} 
\]

(5.3)
were the quark and gluon correlators must be taken in the following form:

\[ G_{01}^{(J)} = G_{01}^{(J)} - G_{\text{rel}}^{(J)} \Sigma_{01}^{(J)} G_{\text{adv}}^{(J)}, \quad D_{01}^{(J)} = D_{01}^{(J)} - D_{\text{rel}}^{(J)} \Pi_{01}^{(J)} D_{\text{adv}}^{(J)}, \quad (J = A, B), \]

with \( G_{01}^{(J)} \) and \( D_{01}^{(J)} \) representing the quark and gluon distributions at some arbitrary scale \( Q_0^2 \). For computations, we shall use a standard CTEQ parameterization of the nucleon structure functions [20] \( q(x, q_t^2) \) and \( G(x, k_t^2) \). These were obtained by fitting the data with the solutions of the GLAP evolution equations. For nuclei, we shall also use the semi-empirical formula fit to nuclear shadowing data.

We proceed in the laboratory frame, which is the infinite momentum frame for both proton A and proton B. The directions of their light cone propagation are fixed by the two null-plane vectors \( n_A^{\mu} \) and \( n_B^{\mu} \):

\[ n_A^{\mu} = (1, 0, -1), \quad n_B^{\mu} = (1, 0, 1), \quad n_A^2 = n_B^2 = 0. \]  

(5.5)

They define the light-cone components of the Lorentz vectors:

\[ n_A a = a^+ = a_+ = a_0 + a^3, \quad n_B b = b^- = b_+ = b_0 - b^3. \]

We split the total cross-section into the three parts:

\[ \sigma^{(0)}_q = \sigma^{(0)}_q (V, \Sigma) + \sigma^{(0)}_q (G, \Sigma) + \sigma^{(0)}_q (\Pi, \Sigma), \]  

(5.6)

The term \( \sigma^{(0)}_q (G, \Sigma) \) is naturally absent, as energy-momentum conservation prevents fusion of the two on-mass-shell particles into one on-mass-shell quark.

Unlike the case of DIS, both invariants from quark (\( \sigma_1 \) and \( \sigma_2 \)) and gluon (\( \omega_1 \) and \( \omega_2 \)) self-energies contribute to the cross-section of quark production. In order not to exceed the accuracy of the leading logarithmic approximation of the GLAP-evolution of structure functions, we shall omit \( \sigma_2 \) and \( \omega_2 \). They are of the next order by a formal count of the \( \alpha_s \)-powers, and are not under direct control of DIS data.

Any term coming from the nucleus A carries a \( \delta(k^-) \), and any term coming from the nuclei B carries a \( \delta(q^+) \). This drastically simplifies the calculation. The first two terms from Eq. (5.4) are calculated explicitly, yielding

\[ \frac{d\sigma^{(0)}_q (V, \Sigma)}{dp_t^2 dy} = \frac{16\pi^2\alpha_0}{3 s} \left[ \frac{p_t^2 e^{-y}}{s} Q_0^2 G'\left( \frac{p_t^2 e^y}{s} , p_t^2 \right) + (y \to -y) \right], \]

(5.7)

\[ \frac{d\sigma^{(0)}_q (G, \Sigma)}{dp_t^2 dy} = \frac{16\pi^2\alpha_0}{3 s} \left[ \frac{G\left( \frac{p_t^2 e^{-y}}{s} Q_0^2 \right) q'\left( \frac{p_t^2 e^y}{s} , p_t^2 \right)}{s x_A x_B} [1 + \frac{p_t^2}{s x_A x_B}] + (y \to -y) \right], \]

(5.8)

where \( y \) is the longitudinal rapidity of the final state quark, \( x_A, B = p_t e^{\pm y}/\sqrt{s} \), and the substitute term \( (y \to -y) \) accounts for the symmetry \( (A \leftrightarrow B) \). We have also defined \( G'(x, p_t^2) = dG(x, p_t^2)/dp_t^2 \) and \( q'(x, p_t^2) = dq(x, p_t^2)/dp_t^2 \). The main contribution to the cross-section from the third term in Eq. (5.4):

\[ \frac{d\sigma^{(0)}_q (\Pi, \Sigma)}{dp_t^2 dy} = \frac{8\pi\alpha_0}{3 s p_t^2} \int \frac{dk_t^2 dq_t^2 (k_t^2 + q_t^2)(1 + q_t^2/s x_A x_B)}{\sqrt{(k_t + q_t)^2 - p_t^2}[p_t^2 - (k_t - q_t)^2]} \left[ \frac{q_t}{s} G'\left( \frac{p_t e^{-y}}{s} , k_t^2 \right) + (y \to -y) \right], \]

(5.9)

where all terms which are at least as small as the longitudinal structure function \( F_L \) of the DIS were neglected. The square root in the denominator comes from the angular integration over the orientations of the transverse momenta:

\[ \int d^2 k_t d^2 q_t \delta^{(2)}(k_t + q_t - p_t) F(k_t, q_t) = \int \frac{dk_t^2 dq_t^2 F(k_t, q_t)}{4 S(k_t, q_t, p_t)} \]

(5.10)

where \( S(k_t, q_t, p_t) \) is the area of the triangle with sides \( k_t, q_t \) and \( p_t \),

\[ 4S(k_t, q_t, p_t) = \sqrt{[(k_t + q_t)^2 - p_t^2][(p_t^2 - (k_t - q_t)^2)}, \]

(5.11)

and the integration domain is restricted by the triangle inequalities,

\[ k_t + q_t \geq p_t, \quad |k_t - q_t| \leq p_t. \]

(5.12)
5.2. Production of gluons to leading order.

For the case of gluon production, we start from Eq. (5.9), and, as for Eq. (5.1), rewrite it in the momentum representation:

\[
\frac{dN_g}{dp^4} = \frac{d\mu(p,u)[-i\Pi^{aa,\mu\nu}(p)]}{(2\pi)^32p^0}
\]  

(5.13)

where the projector \( d_{\mu\nu}(p,u) \)

\[
d_{\mu\nu}(p,u) = -d_{\mu\nu} + \frac{p^\mu u^\nu + u^\mu p^\nu}{(pu)} - \frac{p^\mu p^\nu}{(pu)^2}, \quad d_{\mu\nu}(p,u)u_\nu = 0, \quad p^2 = 0 \ ,
\]  

(5.14)

is a sum over the two physical gluon polarizations in laboratory frame with time axis along the four-vector \( u^\mu = (1,0,0,0) \). For this gluon we choose the time-like axial gauge \( u^\mu B_{\mu} = 0, \quad u^2 = 1 \).

To the same approximation as in Eq. (5.2), we may write the gluon polarization tensor (temporarily omitting the fermion contribution) as

\[
\Pi^{\mu\nu}_{01}(p) = -ig_0^2\int \frac{d^4k d^4q}{(2\pi)^4} \delta(k + q - p) \{V_{\mu\nu}^{\alpha\beta}(k + q, -q, -k)D^{(A)\sigma\beta\gamma}_{01,dd'}(k)V_{\sigma\nu\lambda\beta}(q,k)D^{(B)\rho\lambda}_{10,f'f}(q) + (A \leftrightarrow B) \}
\]  

(5.15)

The gluon distribution of each nucleus consists of the two familiar terms given by Eq. (5.4). Again, we shall keep only the leading terms in the gluon sources which are under the control of the data:

\[
[d(k,n_A)\Pi(A)(k)d(k,n_A)]_{d} = \frac{\delta_{dd'}}{8}d_{\mu\nu}(k,n_A)u_{1A}^{\mu}(k) .
\]  

(5.16)

A similar expression may be written down for nucleus B. Calculation of the trace over the abundant color and vector indices results in an overall factor of \( 24 \times 32 \times T(k,q) \). The trace \( T(k,q) \) is given below. As in Eq. (5.6) we have two types of contribution to the cross-section:

\[
\sigma^{(0)}_g = \sigma^{(0)}_g(G,\Pi) + \sigma^{(0)}_g(\Pi,\Pi) , \quad (5.17)
\]

The first term comes from the interaction of the “source” with the field from the “initial” distribution:

\[
\frac{d\sigma^{(0)}_g(G,\Pi)}{dp_{\perp}^2dy} = \frac{12\pi^2g_0^2}{4s}(G'(\frac{p_t e^{-y}}{\sqrt{s}},q_0^2)G'(\frac{p_t e^{-y}}{\sqrt{s}},p_t^2) \left[ 1 + \frac{p_t^2e^{2y}}{s(x_A + x_B)^2} \right] + (y \rightarrow -y) \). 
\]  

(5.18)

The second term comes from the interaction of the two sources:

\[
\frac{d\sigma^{(0)}_g(\Pi,\Pi)}{dp_{\perp}^2dy} = \frac{24\pi^2g_0^2}{sp_{\perp}^2} \int dk_{t}^{2}dq_{t}^{2}T(k_{t},q_{t})G'(\frac{p_t e^{-y}}{\sqrt{s}},k_{t}^2)G'(\frac{p_t e^{-y}}{\sqrt{s}},q_{t}^2) . 
\]  

(5.19)

The trace \( T(k,q) \) is conveniently written in the following way:

\[
T(k,q) = k_t^2 + q_t^2 + \frac{(k_t^2 + q_t^2)^2}{s(x_A + x_B)^2} \left[ \frac{q_t^2}{s x_B^2} + \frac{k_t^2}{s x_A^2} + \frac{k_t^2 q_t^2}{s^2 x_A x_B} \right] + \frac{2k_t^2 q_t^2}{s(x_A + x_B)^2} \left[ \frac{q_t^2}{s x_A^2} + \frac{k_t^2}{s x_B^2} \right] .
\]  

(5.20)

The integrand of Eq. (5.13) is symmetric with respect to interchange of \( k_t \) and \( q_t \). Hence the contribution from the interchange term \( (A \leftrightarrow B) \) is the same, and factor 2 is already included in Eq. (5.19).

We close this Section by discussing two minor problems. The first is an apparent infrared divergence of total cross-section due to the kinematic factor \( p_t^{-2} \) in Eqs. (5.3) and (5.19). However, one can easily see that this factor disappears when the integration variables \( k_t \) and \( q_t \) are rescaled by \( p_t \). After such a rescaling, any possibly remaining issues concerning low \( p_t \) behavior are connected with the sources (or structure functions) which are controlled by the data.

The second problem arises from the positive powers of transverse momenta of the incoming quark and gluon fields. This can be seen from Eqs. (5.4), and (5.19) and (5.20), and appear in the same form in the next perturbation orders.

The formal solution of the problem is very simple: to be consistent with the condition under which the sources (or structure functions) are defined, \( \nu \rightarrow \infty \), we must drop all terms containing ratios like \( q_t^2/s x_A x_B \) and \( k_t^2/s x_A x_B \), despite the external kinematic condition \( s x_A x_B = p_t^2 \). If this is not done then the integrals (5.3) and (5.19) will
differ, as the LLA structure functions do not provide sufficient cut-off at high $k_t$ and $q_t$. In the context of the existing theory, these ratios are parametrically small and may not be switched on until the high-order twists are taken into account.

Nevertheless, the growth of the matrix element at high transverse momenta is quite physical, and may be interesting at very small $x$. The wave packets with small $x$ are smooth and extended in the longitudinal direction. If we require that large $k_t$ and $q_t$ add to form small $p_t$, then the initial geometry of momenta is almost collinear and the incoming fields effectively overlap. We thus encounter a type of collinear singularity which may be shielded only by appropriate behavior of structure functions. The latter is naturally provided by the BFKL equation (See Ref. [10] and Appendix 1.) We emphasize that this process may be taken seriously only in the domain of very low $x$, where only very preliminary data exist at the present time.

6. First order corrections to the quark distribution.

In the previous Section we have calculated the lowest (Born) contribution (of $\alpha_s^0$-order) to quark and gluon production corresponding to the “excitation” of a single quark or gluon from the sea. Now we intend to show that the next order terms of our kinetic perturbation expansion describe the more usual “creation” processes. They do it in a very special way such that no infrared divergencies appear in the calculation of the total cross-section. This may be the most important result of the present calculation. Furthermore, this requires no special proof: because the perturbation series for the probabilities was initially resummed in the new expansion, it does not generate any terms affected by the initial state collinear singularities. These are absorbed into definition of the sources (structure functions), and thus solving the problem of divergence in an experimental way.

This is in striking contrast to all known calculations based on the factorization theorem. Such calculations always start from a master formula, such as

$$\frac{d\sigma(AB \rightarrow pX)}{dp} = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \sum_{c \in X} \int_0^1 \frac{d\sigma(a,b \rightarrow p,c)}{dp} F_{aA}(x_a, Q^2)F_{bB}(x_b, Q^2),$$

(6.1)

which heavily relies on the parton model, and implicitly impart the status of an observable at least to one additional final state particle. The $2 \rightarrow 2$ processes have the lowest order in this approach, and the $Q^2$-dependence of the structure functions $F_{j/}$ defines the so-called factorization scale, rather than reflecting its full QCD-evolution. As was already mentioned in Section 4, the reason structure functions are treated mistrustfully in this approach is connected with the out-of-light-cone behavior of Feynman’s propagators for the incoming partons.

Quite naturally, apart from all other amplitudes, the $2 \rightarrow 2$ cross-section includes those corresponding to the emission of a second particle $c \in X$ from the initial state. The squared moduli of these partial amplitudes duplicate those already included in the definition of the structure functions at lower factorization scales.

The new approach does not distinguish any states absorbed into the set $X$. At the same time, it restores the proper status of the retarded propagation for the incoming fields. As a result, it excludes a priori any double counting of processes.

The first order corrections to the quark or gluon production can be divided into two major categories: the self-energy-like and the vertex-like. These names reflect only the topology of the new diagrams.

6.1. Self-energy-like terms.

The sequence of the $\alpha_s$-order diagrams in the self-energy-type term emerges from the possibility that either the quark or gluon field correlator in Eq. (5.2) represents a field in the continuum of out-states.

$$p^0 \frac{dN_q}{dp d^3x} = \frac{g^2}{2(2\pi)^3} \int \frac{d^4k}{(2\pi)^4} \{ \text{Tr}[\hat{\gamma}^\mu G_{01}(p+k)\gamma^\mu D_{10,\nu\mu}(k)] + \text{Tr}[\hat{\gamma}^\mu G_{01}(p+k)\gamma^\mu D_{10,\nu\mu}(k)] \},$$

(6.2)

This immediately means that the remaining exact correlators $G_{01}$ and $D_{10}$ carry information about both nucleus A and nucleus B. In other words, these fields were created during the course of the collision between the two nuclei. The only terms of the subsequent expansion of the quark and gluon correlator that survive in this case are

$$D_{10} = -D_{adv}^\# H_{10} D_{adv}, \quad \text{and} \quad G_{01} = -G_{adv}^\# \Sigma_{01} G_{adv}^\#,$$

(6.3)

The superscript “#” indicates that we consider propagation after collision of the nuclei, and in case we need radiative corrections to this propagation, we must consider them against the background of the distribution of quarks and gluons created in the collision itself.
The intensities $\Pi$ and $\Sigma$ of the field sources created by the two nuclei, were already calculated in the previous Section, but now these fields are off-mass-shell, and the terms with products of two $*$-labelled correlators from equations (6.4) should be added. These are just the lowest order terms of the master formula (3.3) of the QCD parton model, with a fixed factorization scale $Q_0^2$, and correspond to processes of the type $2 \rightarrow 2$, where parameters of the second emitted particle are completely integrated over. Thus the complete formula will be

$$p^0 \frac{dN_q}{dp d^3 x} = \frac{-ig_0^2}{2(2\pi)^4} \int \frac{d^4 k d^4 q}{(2\pi)^4} \left( Tr \left[ \bar{q} \gamma^\mu \gamma^\nu G_{56}^a(k) \gamma^\lambda \gamma^\rho G_{12}^b(0) \gamma^\sigma \gamma^\tau G_{34}^c(q) \right] \right) [D_{ret}(k)]_{ab}^{\#} \left[ D_{adv}(q) \right]_{ba}^{\#} \left[ \Pi_{10}^{AB}(k) \right]_{\nu \mu}^{\#} + 
\left. + Tr \left[ \bar{q} \gamma^\mu \gamma^\nu G_{56}^a(k) \gamma^\lambda \gamma^\rho G_{12}^b(0) \gamma^\sigma \gamma^\tau G_{34}^c(q) \right] \right] \left[ D_{ret}(k) \right]_{ab}^{\#} \left[ D_{adv}(q) \right]_{ba}^{\#} \left[ \Pi_{10}^{AB}(k) \right]_{\nu \mu}^{\#} \right) \right), \quad (6.4)$$

where the following chain of substitutions is supposed:

$$\Sigma_{01}^{AB}(p) = ig_0^2 \int \frac{d^4 k}{(2\pi)^4} \delta(k + q - p) \left\{ \left[ \gamma^\mu \gamma^\nu G_{56}^a(k) - G_{ret}(k) \Sigma_{01}^B(k) G_{adv}(q) \right] \gamma^\lambda \gamma^\rho \right\} \times
\left( [D_{10}^{A^*}(q) - D_{ret}(q) \Pi_{10}^A(q) D_{adv}(q)]_{ba,\nu \mu} + (A \leftrightarrow B) \right). \quad (6.5)$$

for a "collective" quark source, and

$$\Pi_{01}^{AB}(p) = -ig_0^2 \int \frac{d^4 k d^4 q}{(2\pi)^4} \left\{ V [D_{10}^{B^*}(k) - D_{ret}(k) \Pi_{10}^B(k) D_{adv}(k)] \times V [D_{10}^{A^*}(q) - D_{ret}(q) \Pi_{10}^A(q) D_{adv}(q)] \right.$$

\left. \left. + Tr \left[ G_{56}^a(k) - G_{ret}(k) \Sigma_{01}^B(k) G_{adv}(q) \right] \gamma^\lambda \gamma^\rho \left\{ [D_{10}^{A^*}(q) - D_{ret}(q) \Sigma_{01}^A(q) G_{adv}(q)] \delta(k + q - p) \right) \right), \quad (6.6)$$

for the "collective" gluon source. Here $V$-s stand for the 3-gluon vertex with all arguments dropped.

Three of the six graphs of this type are given in Fig. 3. The first corresponds to the $s$-channel part of the Compton process. The other two are of an annihilation type. They can all be represented as squared moduli of the corresponding amplitudes. The t-channel partners of these diagrams are not generated by this perturbative expansion. If they did, they would duplicate processes already included in the definition of the sources. Thus the complete formula will be

$$\text{for a "collective" quark source, and}$$

$$\text{for the "collective" gluon source. Here $V$-s stand for the 3-gluon vertex with all arguments dropped.}$$

6.2. First order terms from the "vertex-like" corrections.

The general expressions for the one-particle quark and gluon distributions (Eqs. (2.8) and (2.11)) contain dressed quark-gluon and three-gluon vertices

$$\Gamma_{SQ,R}^{d,\lambda}(x,y;\eta) = (-1)^{P+S+Q} \left[ g_{R} \delta B_{SQ}^{d}(\eta_P) \right] ,$$

$$V_{bcf,RSB}^{\nu\beta\sigma}(x,y) = (-1)^{R+S+P} \left[ g_{R} \delta B_{RS}^{f}(y_B) \right] . \quad (6.7)$$

Because of their additional matrix structure they contain some unusual elements which should be determined now. It can be done easily by using the formal solution of the matrix Schwinger-Dyson equations:

$$[G^{-1}]_{AB} = [G^{-1}]_{AB} - \Sigma_{AB}, \quad [D^{-1}]_{AB} = [D^{-1}]_{AB} - \Pi_{AB}. \quad (6.8)$$

Functional derivatives of the bare Greenians give us the bare vertices, and those of self-energies give the corrections. Using the one-loop formulae it is straightforward to find the first order corrections to the vertices:

$$(1) \Gamma_{RB,S}^{d,\lambda}(x,y;\eta) = -ig_0^2(-1)^{R+S} \left\{ \gamma^\rho \gamma^\nu G_{RS}(x,\xi) \gamma^\lambda \gamma^\sigma G_{SB}(\xi,y) \gamma^\alpha \gamma^\beta G_{RB}(x,y) \right\} \times
\left. \left. \left[ D_{SR}^{ba,\alpha \beta}(y,\xi) \right] V_{bcf,RSB}^{\nu\beta\sigma}(x,y) \right\} , \quad (6.9)$$

Eq. (6.9) contains an additional rule: besides the expected contour indices, the correction $(1) \Gamma_{RB,S}^{d,\lambda}(x,y;\eta)$ to the bare vertex with the contour index $B$ acquires an additional factor $(-1)^{R+S}$. This rule works in the same way for the three-gluon vertex which is too cumbersome to be written down separately.

The sum over the contour indices naturally divides into two groups: those with $R = S$ and $R \neq S$. We delay discussion of the first group which is responsible for those types of radiative corrections which may lead to vertex
form-factors. The second group, which we will discuss below, describes contributions of the higher-order real processes, like emission of a second uncontrolled jet.

The one-loop vertex-type correction to single light quark production contains four terms, two from the first term in Eq. (6.9) and two from the second one:

\[ p^0 \frac{dN_q}{dp^4 dx} = -\frac{ig_0^2}{2(2\pi)^3} \sum_{j=1}^4 \int \frac{d^4kd^4q}{(2\pi)^8} \text{Tr}[\not\!p\not\!I_j(p, k, q)], \tag{6.10} \]

where

\[ \mathcal{I}_1 = t^a\gamma^\mu G_{01}(k)t^d\gamma^\rho G_{10}(k + p - q)t^c\gamma^\sigma G_{01}(q)t^b\gamma^\nu D_{0\alpha,\beta\rho}(p - k)D_{1\sigma,\nu}(p - q); \]

\[ \mathcal{I}_2 = t^a\gamma^\mu G_{00}(p - k)t^d\gamma^\rho G_{01}(p - k - q)t^c\gamma^\sigma G_{11}(p - q)t^b\gamma^\nu D_{01,\alpha,\mu,\nu}(k)D_{01,\sigma,\nu}(q); \]

\[ \mathcal{I}_3 = t^a\gamma^\mu G_{00}(p - k)t^d\gamma^\rho G_{01}(q)t^b\gamma^\nu D_{0\alpha,\mu,\rho}(k)D_{01,\lambda,\sigma}(k + p)D_{11,\lambda,\sigma}(p - q)V^\rho_{\alpha\beta}(k + q - p, p - q); \]

\[ \mathcal{I}_4 = t^a\gamma^\mu G_{01}(k)t^c\gamma^\rho G_{11}(p - q)t^b\gamma^\nu D_{0\alpha,\mu,\rho}(p - k)D_{01,\lambda,\sigma}(p - k - q)D_{b/c}(p - k, p + q, p). \tag{6.11} \]

These four terms are depicted in Fig. 3. There are three cut lines in each diagram, and they can be “distributed” in six different ways between the two colliding nuclei and the additional out-state excited in the continuum. After that we can convert every term of this expansion into a product of two scattering amplitudes. This is done in Fig. 3 (up to a trivial interchange A \leftrightarrow B). It is clearly seen that amongst this group of graphs there are none which would represent the squared moduli of any amplitude, but all allowed interference terms between all the processes which produce a quark and something else are included.

Recalling the previous discussion of the self-energy-type terms, we see that our perturbative expansion does not generate any diagrams which would repeat those present in the definition of the sources (via their ladder expansion). These missing patterns are not IR-safe, and were regularized and renormalized in the course of their definition via the DIS cross-section.

We shall now show that no further infrared problems appear. We demonstrate this using the definite subprocess of a detected quark with momentum \( p \) accompanied by an uncontrolled anti-quark in the out state. They were both created in a collision of two gluons, \( g_A g_B \rightarrow q\bar{q} \).

### 6.3. Infrared safety in the \( \alpha_s \)-order

Infrared finiteness of the self-energy-type diagrams of Fig. 3 is intuitively understandable, since the intermediate s-channel gluon or quark carry a large time-like momentum. We may expect IR-problems only in the vertex-type diagrams, like (VI) of Fig. 3, where the intermediate fermion is in the \( t \)-channel. The corresponding distribution of a single quark is given by the expression:

\[
p^0 \frac{dN_q}{dp^4 dx} = \frac{-ig_0^2}{2(2\pi)^3} \int \frac{d^4kd^4q}{(2\pi)^8} \text{Tr}[\not\!p\not\!t^a\gamma^\mu G_{00}(p - k)t^d\gamma^\rho G_{01}(p - k - q)t^c\gamma^\sigma G_{11}(p - q)t^b\gamma^\nu D_{01,\alpha,\mu,\nu}(k)D_{01,\lambda,\sigma}(p - k - q)D_{b/c}(p - k, p + q, p)], \tag{6.12} \]

where retarded and advanced functions carry a superscript indicating which nucleus was the source of field. The natural variables for subsequent calculations are quark rapidity and momentum fractions defined via

\[
p^{\pm} = p_i e^{\pm y}, \quad k^+ = \sqrt{s} x_A, \quad q^- = \sqrt{s} x_B. \]

The most troublesome element in all following calculations is the trace over spinor and vector indices,

\[
T = \text{Tr}[\not\!p\gamma^\mu(\not\!p - \not\!k)\gamma^\sigma(\not\!p - \not\!k - \not\!q)\gamma^\rho(\not\!p - \not\!q)\gamma^\nu d_{\alpha\mu}(k, n_A)d_{\beta\nu}(q, n_B)]. \tag{6.13} \]

As in the lowest order, we can single out different types of terms contributing to the cross-section in the first order:

\[ \sigma^{(1)}_q = \sigma^{(1)}_q(G, G) + \sigma^{(1)}_q(G, I) + \sigma^{(1)}_q(I, I), \tag{6.14} \]

The first term corresponds to the first nonvanishing order of the master formula of the parton model, which factorizes the “hard” QCD cross-section and structure functions evaluated at some (sufficiently high) scale \( Q_0^2 \). Two mass-shell delta-functions make the calculations relatively simple, and the result reads as follows:

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We see that this term remains finite when $p_t \to 0$ and is strongly suppressed, both by the second power of $s$ in the denominator and the smallness of $\alpha_s^2$.

To facilitate the following analysis, let us trace how the structure emerges. We may expect an infrared divergence from the poles of the product of the two $t$-channel propagators in Eq. (6.12), $G_{00}(p - k)G_{11}(p - q)$. Since they are unshielded by finite masses or virtualities, the factor $(p_t^2 s x_A x_B)^{-1}$ appears. One power of $s^{-1}$ comes from the definition of cross-section, and one more from the final-state phase space. The combined spinor-vector trace in (6.14) gives a factor $p_t^2 s$. As a result, no large logarithm which could partially compensate for the smallness of the coupling constant appears appears in the total cross-section. Eventually, we may expect only a weak scale dependence of the total cross-section.

The “box diagram”, which is an unavoidable partner of $\sigma_q^{(1)}(G, G)$ in the approach based on the factorization theorem, is IR-divergent, but it has no analog in our perturbation expansion. Nevertheless, we considered it useful to calculate it in Appendix 2, in order to compare its structure with that emerging from the present calculations. The mixed term $\sigma_q^{(1)}(G, \Pi)$, and the term $\sigma_q^{(1)}(\Pi, \Pi)$ (contributed to by two “sources”) are more complicated because one or both of the incoming fields interact with their sources and hence are off the mass shell. This means that at least one of the mass-shell delta functions is no longer present, and traces become unwieldy. We will not present their explicit form here, as these terms are not expected to be large. In the box diagram has been extracted, one obtains only interference terms. These are usually small, unless they are affected by a singular infrared behavior. A sufficient qualitative analysis of this behavior can be done without explicit calculations. It is enough to notice that the trace $T(p, k, q)$ in the numerator of the integrand of Eq. (6.12) is, in general, a polynomial of fourth order with respect to $p_t/\sqrt{s}$. When the momenta of both structure functions are put on mass shell, the only powers that survive are $p_t^2/s$ and $p_t^2/s^2$. This leads to an immediate cancellation of the two unshielded infrared poles of the propagators.

In the integral for $\sigma_q^{(1)}(\Pi, \Pi)$ both poles are shielded by gluon virtualities, and infrared divergence can not appear at all. In the integral for $\sigma_q^{(1)}(G, G)$ only one pole is shielded, while the second produces an unpleasant $p_t^{-1}$ behavior. However, this does not lead to an infrared divergence of the cross-section, since the same power $p_t^{-1}$ is implicitly present in the differential $dp_t^2$ on the l.h.s.

In Appendix 2, we consider in full the $s$-channel production of light quarks from the process $gg \to q\bar{q}$. The mathematical details behind the above qualitative analysis can be found there. Here we shall discuss only the main physical issues.

The higher powers of $p_t$ in the trace (6.13) lead to an increase in the differential cross-section at high $p_t$ over the lowest order result. This is in compliance with the observation that the first order terms bring more than a simple quantitative correction to the lowest order. It is precisely the emission of two back-to-back jets which makes possible the existence of a high-$p_t$ particle in the final state.

7. Conclusion.

We have considered new principles to compute the distributions of quarks and gluons created in the first hard interaction of the two heavy ions at high energies. We essentially employed an initial resummation of the perturbation series for the probabilities $\bar{B}$. It allowed us to describe two different high energy processes, viz., $e$-p, e-nuclear interactions, in the same terms, as two versions of the same phenomenon – deeply inelastic scattering of composite systems.

It is shown that these calculations can be performed without reference to parton phenomenology. We have introduced the concept of a source as the main subject of QCD evolution, and have shown that the equations which describe the dynamics of the sources are independent of the type of high-energy process, and independent of the particular choice of the final interaction.

The additional benefit of the new approach is that it explicitly displays the causal structure of the QCD evolution equations, and their physical meaning as the spectral analysis of the composite system as performed by the interaction which results in the bare quark or gluon production. The evolution equations for the sources allow for a smooth transition between the regimes described by the GLAP and BFKL equations. The by-product of this study is a new form of the fusion term in the GLR-type equation, which might lead to the stronger low $x$ saturation of the sources than any terms considered previously.

One of the most important results of this study is the new type of perturbation expansion, which, unlike for the factorization technique, does not lead to double counting of processes. The diagrams already included in the definition...
of the sources, and controlled in aggregate by the DIS data, do not appear again in the higher orders of the new perturbative expansion. The diagrams which do appear are free from initial state infrared (collinear) singularities and do not require artificial cut-offs. The leading parts of these diagrams do not depend upon the initial factorization scale either. The price one pays for the efficiency of these calculations, is that one requires the full $x$ and $Q^2$ dependence of the sources (or structure functions) extracted from the data.

We are now in a position to calculate the single-particle distributions of the quarks and gluons after the first 0.1 fm of the heavy ions collision.

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Appendix 1. The full form of evolution equations.

In the main body of the paper, we explicitly studied only the part of evolution equations which eventually results in the GLAP equations. The latter are also known as the leading logarithmic approximation (LLA).

Corrections to Eqs. (4.3) and (4.4) are of various origin. Unfortunately, we cannot count these corrections in the traditional way, which relies on the firm hierarchy of twists in OPE-based calculations. For the moment, by “next approximation" we shall mean some kind of structural expansion based on the complexity of the processes taken into consideration. Surprisingly, it does not lead far away from the commonly used scheme.

In what follows, we study two types of corrections. First, we consider the terms missing in the analysis of the simplest (by their structure) equations, (4.3) and (4.4). In the next two subsections we study the low-$x$ region which results in BFKL equation [11] as the limit of new evolution equations. More complicated corrections lead to an equation resembling the GLR equation [12] but with some significant differences.

A1.1. Trivial corrections.

The simplest corrections arise from Eqs. (4.3) and (4.4), as a residue of the original spinor and tensor form after LLA-terms have been extracted. To make them more visible, let us split $\sigma_1(p)$ and $w_1(p)$ into a leading ( $\sigma_1'$ and $w_1'$) part, corresponding to LLA, and subleading parts ($\sigma_1''$, $w_1''$, ...): 

$$\sigma_1(p) = \sigma_1'(p) + \sigma_1''(p) + \sigma_1'''(p), \quad w_1(p) = w_1'(p) + w_1''(p) + w_1'''(p).$$

The mathematical steps just follow those of the perturbation calculations described in Section 3. Now it is only a lengthy exercise to obtain the resulting equations. Spinor components of the quark source may be expanded as follows:

$$\sigma_1'(p) = \int d^4k\Delta_{kp} \frac{-p_+ k_+}{p_+} \left[ P_{qq}(\frac{p_+}{k_+}) \frac{k_+ \sigma_1(k)}{S_1^R(k)S_1^A(k)} + P_{qg}(\frac{p_+}{k_+}) \frac{k_+ w_1(k)}{W_1^R(k)W_1^A(k)} \right]$$

(A1.2)

$$\sigma_1''(p) = \int d^4k\Delta_{kp} k^2 \left[ -P_{qq}(\frac{p_+}{k_+}) \frac{k_+ \sigma_1(k)}{S_1^R(k)S_1^A(k)} + P_{qg}(\frac{p_+}{k_+}) \frac{k_+ w_1(k)}{W_1^R(k)W_1^A(k)} \right]$$

(A1.3)

$$\sigma_1'''(p) = \int d^4k\Delta_{kp} p_+ \left[ C_F \frac{\sigma_2(k)}{S_2^R(k)S_2^A(k)} + (1 - \frac{p_+}{k_+}) \frac{k^2 w_2(k)}{W_2^R(k)W_2^A(k)} \right]$$

(A1.4)

$$\sigma_2(p) = \int d^4k\Delta_{kp} \frac{k_+}{p_+} \left[ C_F \frac{k_+ \sigma_1(k)}{S_1^R(k)S_1^A(k)} + (1 - \frac{p_+}{k_+}) \frac{k_+ w_1(k)}{W_1^R(k)W_1^A(k)} \right]$$

(A1.5)

where we have denoted

$$\Delta_{kp} = \frac{2g^2}{(2\pi)^3 k^+} \delta^3([k - p]^2) = \frac{2g^2 \theta(k^0 - p^0)}{(2\pi)^3 k^+} \delta([p_+ - k_+]^2 - (p^- - k^-)^2) - (p_t - k_t)^2]$$

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The meaning of this decomposition becomes clear if we multiply it by the kinematic factor $p_+^2$, and sandwich it between retarded and advanced propagators (see Eq. (4.13)). The joint left hand side of Eqs. (A1.2) and (A1.3) becomes the derivative with respect to $Q^2$ of the quark structure function of DIS. The r.h.s. of Eq. (A1.2) then leads to the GLAP part of the evolution. The factor $p_+^2$ in it is responsible for the logarithmic behavior. Now it is easy to see that r.h.s. of (A1.3) will result in a term with $1/Q^2$ behavior. It simulates the next twist contribution, even though the second twist is not included explicitly in the density matrix as an effect of next order correlations.

The r.h.s. of (A1.4) represents a secondary influence of the longitudinal components of the spinor and gluon sources. The components are defined by (A1.5) and (A1.8), and are of next order by a formal count of the factors $\alpha_s^2$ directly contributes to the longitudinal structure function given by Eq. (4.15). It also appears in the Born term of the quark excitation process, but we leave it aside for now since it is poorly controlled by the data.

All of the above comments apply equally to the components of the gluon source, which can be decomposed following the same principle:

$$w_1'(p) = \int d^4k \Delta k_\rho \frac{-p^2 k^+}{p^+} q \left[ P_{gg}(p_+) \frac{k^+ \sigma_1(k)}{S_1^R(k)S_1^A(k)} + P_{gg}(p_+) \frac{k^+ w_1(k)}{W_1^R(k)W_1^A(k)} \right]$$

(A1.6)

$$w_1''(p) = \int d^4k \Delta k_\rho k^2 \left[ P_{gg}(p_+) \frac{k^+ \sigma_1(k)}{S_1^R(k)S_1^A(k)} + P_{gg}(p_+) \frac{k^+ w_1(k)}{W_1^R(k)W_1^A(k)} \right]$$

(A1.7)

$$w_1'''(p) = \int d^4k \Delta k_\rho k^+ \left[ C_F (1 - \frac{p^+}{k^+}) \frac{\sigma_2(k)}{S_2^R(k)S_2^A(k)} - 2N_c \frac{p_+^2}{k^2} \frac{k^2 w_2(k)}{W_2^R(k)W_2^A(k)} \right]$$

(A1.8)

$$\frac{w_2(p)}{p^2} = \int d^4k \Delta k_\rho \frac{k^+}{p^+} \left[ -8C_F n_f (1 - \frac{p^+}{k^+}) \frac{k^+ \sigma_1(k)}{S_1^R(k)S_1^A(k)} + 4N_c \frac{k^+}{p^+} (1 - \frac{p^+}{2k^+})^2 \frac{k^+ w_1(k)}{W_1^R(k)W_1^A(k)} \right]$$

(A1.9)

A preliminary examination of the additional terms in the evolution equations reveals that they have the same singular infrared behavior as GLAP equations, and should be regularized and renormalized. The way to do this is not yet clear, since conservation of momentum seemingly fails to control all necessary subtractions. A complete study of these equations is a separate subject. In this paper, we have considered the explicit form of the DIS structure functions as granted.

### A1.2. The BFKL equation.

As has been shown in Sec.4, the new evolution equations for the sources, even in their reduced form (4.3) and (4.4), are practically the equations for the derivatives of the structure functions, rather than for the DIS structure functions themselves. The former were first introduced by Lipatov as the “unintegrated structure functions,” and they relate to the latter in the following way

$$f(x,p_+^2) = \frac{\alpha_s}{\pi} \int \frac{d x G(x,p_+^2)}{p_+^2}$$

(A1.10)

Thus, up to an insignificant normalization factor, we may write:

$$\frac{\delta(p^-) f(x, p_+^2)}{W_1^R(p) W_1^A(p)}$$

(A1.11)

In the limit of low $x$, the function $f(x, p_+^2)$ was proven to obey the so-called BFKL equation,

$$- x \frac{\partial f(x, p_+^2)}{\partial x} = 3 \frac{\alpha_s(p_+^2)}{\pi} \int_{k_0^2} \frac{d k_1^2}{k_1^2} \left[ f(x, k_1^2) - f(x, p_+^2) \right] + \frac{f(x, p_+^2)}{\sqrt{4 k_1^2 - p_+^2}}$$

(A1.12)

This equation was originally obtained by considering the amplitude of the process $2 \to 2+ n$ and summing the leading $log(1/x)$ terms.

Evolution equations like (4.4) were derived immediately as integral equations. At high $Q^2$ and not too low $x$, they allow for simplifications which lead to the GLAP equations. This means that the necessary resummation of the
leading $\log Q^2$ was done from the very beginning. Next, our intention is to determine what simplifications should be
done to obtain the BFKL equation. These simplifications are effectively equivalent to a reduction in the number of
diagrams already accounted for in the integral equation.

In the notation of Eq. (A1.10), our equation (4.11) for the transverse gluon source reads

\[
\delta(p^-) f(x, p_T^2) = \frac{\alpha_s(p_T^2)}{(2\pi)^2} \int_0^1 \frac{dy}{y} \int \frac{d^2k_t}{k_t^2} \delta((p^+ - k^+) p^- - (p_t - k_t)^2) \left[ \frac{-p^2}{W_1^2(p) W_1^4(p)} \right] P_{gg}(\frac{x}{y}) f(y, k_t^2) \tag{A1.13}
\]

We have retained all terms that contribute to the LLA. We notice that at real momenta the denominator
$W_1^2(p) W_1^4(p)$ is strictly positive both before and after any integration without additional weight. Thus the full
integral is not singular. The next steps are as follows:

**Step 1.** For $x \ll 1$, approximate the splitting kernel as $P_{gg}(x/y) \sim (y/x)$.

**Step 2.** Integrate both sides of the equation over $p^-$, considering the retarded and advanced propagators in the
integrand on the r.h.s as bare. In this way the singular infrared behavior of the integrand is unshielded. Integrate over
the azimuth angle.

**Step 3.** Expand the propagators on the l.h.s. up to the first order in radiative corrections. Retain only the vacuum
correlator and move it to the r.h.s in order to shield the singularity that resulted from the approximation.

**Step 4.** Differentiate both sides of the equation with respect to $x$.

This procedure will result in the BFKL equation (A1.12) (up to the insignificant last term in the integrand). The
asymptotic behavior of its solution is well known. Its exponential part,

\[
f(x, p_T^2) \sim \exp \left( -\frac{\ln^2(p_T^2/p_0^2)}{2N_c \ln(x_0/x)} \right) \tag{A1.14}
\]

provides a decrease of $f(x, p_T^2)$ at high $p_T^2$ that is faster than any negative power of $p_T^2$.

We postpone any discussion of the accuracy of the above approximation, and satisfy ourselves with the most
important fact that the evolution equations contain both GLAP and BFKL regimes of evolution as the limits. Thus,
we may hope to describe a smooth transition between them. From a pragmatic point of view, the above asymptotic
behavior guarantees the convergence of the integrals that appear in a calculation of the cross-sections of quark and
gluon production (see Sections 2 and 6).

### A1.3. Gluon shadowing.

More complex corrections to the GLAP evolution correspond to the replacement of the $\sigma$-labelled correlators in
Eqs. (4.3) and (4.4) by correlators with sources, viz., the second terms in Eqs. (4.3) and (4.4). This means that
instead of the final state correlators describing the emission, we include the initial state correlators. This replacement
accounts for possible fusion of the partons. Fusion is expected to be most important for gluons at low $x$. So, only the
purely gluonic component will be considered here.

Terms responsible for fusion of two gluon fields have the form

\[
\Delta_{fus} \Pi^{ab,\mu\nu}_{\alpha_1\beta_1}(p) = ig_\sigma^2 \int \frac{d^4kd^4q}{(2\pi)^4} \delta(k + q - p) V^{\mu\sigma}_{acf}(k + q, -q, -k) \times \times \left[ D_{ret}(q) \Pi_{10}(q) D_{adv}(q) \right]_{cc}^{\alpha\beta}.
\]

This equation is identical to Eq. (5.13) describing gluon fusion in a nuclear collision, except that here both gluons
are taken from the same nucleus. We shall use the standard approximation, i.e., bare tree propagators and bare
derivatives. The longitudinal gluon function $w_2$ will be neglected also. Routine calculations similar to those performed
in Section 5 then yield:

\[
\Delta_{fus} G'(x, p_T^2) = -\frac{3\pi\alpha_0}{2p_t} \frac{(2\pi)^3}{\pi R^2} x^2 \int_0^x \frac{dx_1}{x_1} \int_0^{x_1} \frac{dx_2}{x_2} \delta(x_1 + x_2 - x) \int \frac{dk_t^2 dq_t^2}{4S(k_t, q_t, p_t)} \times \times \left[ \frac{x^2}{x_1 x_2} - \frac{x_1 x_2}{x^2} \right] ^2 \left[ \frac{k_t^2}{x_1} + \frac{q_t^2}{x_2} + \frac{p_t^2}{x} \right] G'(x_1, k_t^2) G'(x_2, q_t^2), \tag{A1.16}
\]

where $G'(x, p_T^2) = dG'(x, p_T^2)/dp_T^2$ and $S(k_t, q_t, p_t)$ is the area of a triangle with the sides $k_t$, $q_t$, and $p_t$. The initial
normalization factor $(V_{ab} P^+)^{-1}$, which was convenient for calculation of cross-sections, has been replaced in Eq. (A1.16)
by $(\pi R^2)^{-1}$, which corresponds to a normalization per unit transverse area of a nucleus with radius $R$. 27
Eq. (A1.10), by its structure, is very similar to the well known Gribov-Levin-Ryskin (GLR) equation and one derived later by Mueller and Qui. It clearly reveals the same tendency to saturate the rate of the field source QCD-evolution at low $x$. However, it seems to have several differences. The most significant is that the power of $\alpha_s$ in Eq. (A1.10) is less than in Refs. [11] and [12]. The formal reason is simpler form of the vertex of the “three-ladder interaction” that is prescribed for us by the general structure of the evolution equations (4.3) and (4.4). The other differences are of dynamic origin and will be discussed elsewhere.

Appendix 2. Some estimates of the first order terms.

A2.1. The “box” diagram.

Two box-type diagrams appear in the calculation of the quark production cross-section, if we use the master formula (3.1). They are depicted in Fig. 3 and the corresponding analytic formula is:

$$2p_0^0 \frac{dN_0^{box}}{dp^4d^2x} = -ig_0^2 \int \frac{d^4k}{(2\pi)^4} \left[ \frac{d^4q}{(2\pi)^4} \right] \left[ \text{Tr} \left( \gamma^\mu t^a \gamma^\nu G_{\eta c}(p - q) t^d \gamma^\rho G_{01}^b(p - k - q) t^e \gamma^\nu \right) \times \right.
\left. G_{\eta d c}(p - q) t^b \gamma^\nu D^{(A)cd}_{01,\sigma \rho}(k) D^{(B)ab}_{01,\nu \mu}(q) + (A \leftrightarrow B) \right].$$

(App2.1)

Routine calculations result in the following expression for the differential cross-section:

$$\frac{d\sigma_{q}^{box}(G,G)}{dp^4d^2y} = -2\pi^{-2} \int_0^1 \frac{dx_Adx_B}{x_Ax_B} \theta(x_A - \frac{p_t}{\sqrt{s}} e^y) \theta(x_B - \frac{p_t}{\sqrt{s}} e^{-y}) \times$$

$$\times \left[ 1 - p_t^2 (3 - x_Ax_B) \frac{4p_t^2}{sx_Ax_B} - \frac{4p_t^4}{sx_Ax_B^2} \right] G(x_A, Q_0^2) G(x_B, Q_0^2) \delta[x_Ax_B - \frac{p_t}{\sqrt{s}} (x_A e^{-y} + x_B e^y)].$$

(App2.2)

The factor $p_t^{-2}$ appears in the same way as in Eq. (5.9) for the Born term. However, previously it could be effectively absorbed into the structure functions, which is not the case now. Indeed, Eq. (A2.2) has no additional integration over momenta which could be rescaled by $p_t$. We expect at least a logarithmic divergence of total cross-section as a result. This divergence may be strengthened by the low-$x$ behavior of structure functions because of the

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$$\times \left[ 1 - p_t^2 (3 - x_Ax_B) \frac{4p_t^2}{sx_Ax_B} - \frac{4p_t^4}{sx_Ax_B^2} \right] G(x_A, Q_0^2) G(x_B, Q_0^2) \delta[x_Ax_B - \frac{p_t}{\sqrt{s}} (x_A e^{-y} + x_B e^y)].$$

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The factor $p_t^{-2}$ appears in the same way as in Eq. (5.9) for the Born term. However, previously it could be effectively absorbed into the structure functions, which is not the case now. Indeed, Eq. (A2.2) has no additional integration over momenta which could be rescaled by $p_t$. We expect at least a logarithmic divergence of total cross-section as a result. This divergence may be strengthened by the low-$x$ behavior of structure functions because of the $p_t$ dependence of the low limits of integration over $x_A$ and $x_B$ in Eq. (A2.2).

A2.2. Inclusive production of quarks in s-channel.

The analytic expression for the diagram (S2) of Fig. 3 corresponding to the subprocess $gg \rightarrow q\bar{q}$, is:

$$2p_0^0 \frac{dN_q}{dp^4d^2x} = -ig_0^2 \int \frac{d^4k}{(2\pi)^4} \left[ \frac{d^4q}{(2\pi)^4} \right] \left[ \text{Tr} \left( \gamma^\mu t^a \gamma^\nu G_{01}^b(p - k - q) t^b \gamma^\nu D^{(#)ab}_{\eta c}(k + q) D^{(#)bd}_{\eta d}(k + q) \times \right. \right.$$

$$\left. \left\{ V_{\alpha \beta}^{(3)}(-k - q, k, q) D^{(A)cc}_{01,\rho \alpha}(k) V_{\beta \alpha}^{(3)}(k + q, -k, -q) D^{(B)dd}_{01,\sigma \beta}(q) + (A \leftrightarrow B) \right\} \right].$$

(App2.3)

The most difficult problem here is to calculate the trace over spinor and vector indices. The complete expression is very unwieldy, and we will therefore satisfy ourselves with

$$\text{Trace} = 12 \times 16p_t^2 sx_Ax_B [1 + O(p_t/\sqrt{s})],$$

where the factor 12 comes from the color algebra.

As previously, we have three contributions to this process,

$$s_0^{(2)} = s_0^{(2)}(G,G) + s_0^{(2)}(G,\Pi) + s_0^{(2)}(\Pi,\Pi).$$

(App2.4)

The first term corresponds to the factorization of the parton’s structure functions and the “hard” cross-section at a given scale,

$$\frac{d\sigma_0^{(2)}(G,G)}{dp_t^4d^2y} = \frac{3\lambda_0^2p_t^4}{2s^3} \int_0^1 \frac{dx_Adx_B}{x_Ax_B} \theta(x_A - \frac{p_t}{\sqrt{s}} e^y) \theta(x_B - \frac{p_t}{\sqrt{s}} e^{-y}) \times$$

$$\times G(x_A, Q_0^2) G(x_B, Q_0^2) \delta[x_Ax_B - \frac{p_t}{\sqrt{s}} (x_A e^{-y} + x_B e^y)].$$

(App2.5)
This term, explicitly dependent on the factorization scale, is kinematically suppressed by three powers of \( s^{-1} \). Two of these come from the kinematics of intermediate gluon, and the third from the phase volume which is confined to a line in the \((x_A, x_B)\)-plane.

The second term in Eq. \((A2.4)\) describes the interaction of a parton with the source,

\[
\frac{d\sigma_A^{(2)}(G, \Pi)}{d\vec{p}_T^2 dy} = \frac{3\alpha_s^2 p_l^2}{16\pi^2} \int_0^1 dx_A \int_0^1 dx_B \theta(x_A - \frac{p_t}{\sqrt{s}} e^y) \theta(x_B - \frac{p_t}{\sqrt{s}} e^{-y}) \times
\]

\[
\times \int_{|p_l - \xi|}^{p_l + \xi} k_l dk_l \frac{k_l^2}{4S(k_l, p_l, \xi)} \frac{G(x_B, Q^2_0)G(x_A, k_l^2)}{(s x_A^2 x_B^2 - k_l^2)^2} + (A \leftrightarrow B)
\]

where we have introduced the notation,

\[
\xi^2 = (k^+ - p^+)(k^- - p^-) = s(x_A - \frac{p_t}{\sqrt{s}} e^y)(x_B - \frac{p_t}{\sqrt{s}} e^{-y}).
\]

This second term still keeps dependence on the factorization scale. Because of finite virtuality of one of the incoming fields, the phase volume of the process is larger, and consequently one of the powers \( s^{-1} \) disappears.

The last term in Eq. \((A2.4)\) accounts for the interaction of two sources,

\[
\frac{d\sigma_A^{(2)}(\Pi, \Pi)}{d\vec{p}_T^2 dy} = \frac{3\alpha_s^2 p_l^2}{32\pi^2} \int_0^1 dx_A \int_0^1 dx_B \theta(x_A - \frac{p_t}{\sqrt{s}} e^y) \theta(x_B - \frac{p_t}{\sqrt{s}} e^{-y}) \times
\]

\[
\times \int_{|p_l - \xi|}^{p_l + \xi} l_l dl_l \frac{l_l^2}{4S(l_l, p_l, \xi)} \int d^2 f \frac{G'(x_A, (l_l + f_l)^2/4)G'(x_B, (l_l - f_l)^2/4)}{(s x_A^2 x_B^2 - l_l^2)^2}.
\]

Here, both incoming fields are virtual, and the \( s \)-channel propagators are smoothed by the total transverse momentum of initial fields. The internal integral over \( d^2 f \) represents the two-dimensional Fourier transform of the product of the densities (in coordinate space) of two sources. Hence, this term is proportional to the degree of geometrical overlap between the colliding nuclei. It does not depend on a factorization scale, and is expected to be the dominant term.

All three terms do not exhibit any problems at low \( p_t \), but a reliable knowledge of the low \( x \) behavior of the sources may be important for quantitative computations. The presence of a factor \( p_t^2 \), which is purely kinematic in its origin, guarantees that at high \( p_t \) the first order differential cross-section will be larger than the Born cross-section in this region.

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FIG. 1. Born diagrams for the one quark and one gluon production. The bold cross labels the line corresponding to the "detected" particle with momentum $p$. The dashed line crosses the Greenians representing densities of the initial (bold) or final (thin) states. Numbers near the vertices indicate the type of ordering in the Greenians. The processes: (a) $qg \rightarrow q$; (b) $gg \rightarrow g$; (a) $q\bar{q} \rightarrow g$.

FIG. 2. Self-energy-type first order diagrams for the one quark production. Notation the same as in Fig.1. Arrows label the retarded and advanced propagators and show the latest time.

FIG. 3. Four topologies of the vertex-type diagrams for the one quark production.

FIG. 4. Twelve vertex-type diagrams of for the one quark production in the first order.

FIG. 5. Two infrared-divergent “box” diagrams which are not generated by our perturbation theory.
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