Epidemic spreading in an expanded parameter space: the supercritical scaling laws and subcritical metastable phases

Gaetano Campi$^{1,2}$, Antonio Valletta$^3$, Andrea Perali$^{2,4}$, Augusto Marcelli$^{2,5}$ and Antonio Bianconi$^{1,2,6}$

$^1$ Institute of Crystallography, CNR, via Salaria Km 29. 300, Monterotondo Stazione, Roma I-00015, Italy
$^2$ Rome International Centre Materials Science Superstripes RICMASS via dei Sabelli 119A, 00185 Rome, Italy
$^3$ Institute for Microelectronics and Microsystems, IMM, Consiglio Nazionale delle Ricerche CNR Via del Fosso del Cavaliere 100, 00133 Roma, Italy
$^4$ School of Pharmacy, Physics Unit, University of Camerino, 62032 Camerino (MC), Italy
$^5$ INFN—Laboratori Nazionali di Frascati, 00044 Frascati (RM), Italy
$^6$ National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), 115409 Moscow, Russia

E-mail: gaetano.campi@ic.cnr.it, antonio.bianconi@ricmass.eu, antonio.valletta@artov.imm.cnr.it, andrea.perali@unicam.it and augusto.marcelli@lnf.infn.it

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Abstract
While the mathematical laws of uncontrolled epidemic spreading are well known, the statistical physics of coronavirus epidemics with containment measures is currently lacking. The modelling of available data of the first wave of the Covid-19 pandemic in 2020 over 230 days, in different countries representative of different containment policies is relevant to quantify the efficiency of these policies to face the containment of any successive wave. At this aim we have built a 3D phase diagram tracking the simultaneous evolution and the interplay of the doubling time, $T_d$, and the reproductive number, $R_t$, measured using the methodological definition used by the Robert Koch Institute. In this expanded parameter space three different main phases, supercritical, critical and subcritical are identified. Moreover, we have found that in the supercritical regime with $R_t > 1$ the doubling time is smaller than 40 days. In this phase we have established the power law relation between $T_d$ and $(R_t - 1)^{-\nu}$ with the exponent $\nu$ depending on the definition of reproductive number. In the subcritical regime where $R_t < 1$ and $T_d > 100$ days, we have identified arrested metastable phases where $T_d$ is nearly constant.

1. Introduction
While the mean-field theory of intrinsic dynamics of uncontrolled epidemics is well known [1, 2], nowadays the scientific discussion is focusing on the dynamics of epidemics with containment measures [2–5], addressing the role of extrinsic effects due to the spatio-temporal evolution of contact networks [6–9]. In this work we verify proposed mathematical laws driving coronavirus 2020 epidemics with containment measures (called here CEwCM). Different epidemiology protocols such as lockdown, case finding, mobile tracing (LFT) [10–16] and lockdown stop and go (LSG) [17–19] has been applied by different countries.

While many works [20–30] have analyzed short-time intervals of the CEwCM, here we analyse the full-time window of the first Covid-19 wave in: (i) South Korea, which applied the LFT policy compared with (ii) Italy and (iii) United States of America which applied the LSG policy with strict and loose rules, respectively. We have verified the physical laws of the time evolution of the CEwCM [24–30] using a new 3D expanded parameter space $T_d(t, R_t)$: where $T_d$ and $R_t$ are the time-dependent doubling and reproductive number.

In CEwCM time evolution three main regimes are clearly identified: supercritical, critical and subcritical. The regimes supercritical, with $R_t > 1$ and subcritical with $R_t < 1$ are defined in epidemic spreading theories in references [4, 5]. In a previous work [10] we have considered the critical regime occurring when $1 < R_t < 1.1$. Correspondingly, it was found that the time dependent $T_d$ values are $T_d < 40$ days in the
supercritical phase, \( T_d > 100 \) in the subcritical phase and \([40 < T_d < 100]\) in the critical phase.

We have verified here that in the first wave the supercritical phase is characterized by the \( T_d = C_k(R_t - 1)^{-\nu} \) power law function of the variable doubling time \( T_d \) versus the reproductive number \( R_t \). The key point of this work is the use of the log–log plots of \( T_d \) versus \( (R_t - 1) \) to understand the time evolution of coronavirus in different countries adopting different containment policies. Finally, we provide a quantitative comparison of the Covid-19 first and second wave evolution in Italy compared with South Korea and USA.

2. Methods

The data for each country have been taken from the recognized public data base OurWorldInData [31]. We have extracted, first, the time-dependent doubling time \( T_d \) from the curve of total infected cases, \( Z(t) \), and, second, the time-dependent reproductive number \( R_t \) from the curve of active infected cases, \( X(t) \), using the methodological definition provided by the Robert Koch Institute [33]. Thus, the strategy in our methodological approach has been the extraction of two time dependent parameters (1) the doubling time, \( T_d \), and (2) the reproduction number, \( R_t \), from two independent datasets: \( T_d \) has been extracted from the total cases while \( R_t \) has been obtained from the active cases.

2.1. The basic doubling time \( T_{d0} \) and basic reproductive number \( R_0 \)

Figure 1(a) shows a pictorial view of the viral epidemic spreading starting with the uncontrolled epidemic in the early days of the outbreak with a basic reproductive number \( R_0 = 2 \) and the basic doubling time \( T_{d0} = 2 \) days (which separates successive \( n \)th generations with \( 2^n \) infected persons). In the early days the cumulative curve of the total number of cases \( Z(t) \) increases exponentially with a characteristic rate \( \alpha \)

\[
Z(t) = Z(0) e^{\alpha t} = Z(0) 2^{\frac{t}{T_{d0}}}
\]

therefore, the basic doubling time

\[
T_{d0} = \frac{\log(2)}{\alpha}
\]

has been quickly extracted by several groups showing that it is in the range \( 2 < T_{d0} < 2.8 \) days with \( 2 < R_0 < 3 \), as reported in several works. [19]

2.2. Time evolution law of the time-dependent doubling time \( T_d \) and reproduction number \( R_t \) in the 3D phase diagram \( T_d(t, R_t) \)

As the epidemic goes forward, the total number of cases \( Z(t) \) will include both the active cases and the removed (recovered) individuals. The time evolution of the epidemics modified by extrinsic effects of the containment measures is tracked by the time-dependent doubling time, \( T_d \). The time-dependent \( T_d \) of coronavirus 2020 epidemics is obtained by fitting the cumulative curve \( Z(t) \) over a five days period centered at \( t \pm \Delta t \) with \( \Delta t = 2 \) days. \( T_d \) has been evaluated by taking the time derivative of the cumulative curve \( Z(t) \) over five days of the logarithm of the cumulative infection curve \( \frac{d}{dt} \log(Z(t)) = \alpha(t) = \frac{\log(Z(t))}{T_d(t)} \) that leads to

\[
T_d(t) = \frac{\log(2)}{\frac{d}{dt} \log(Z(t))}, \quad (3)
\]

The time-dependent variable reproductive number \( R_t \) can be measured by different methods [2–7, 32–34]. In this work we have used the model independent procedure proposed by the Robert Koch Institute [33], where \( R_t \) is the ratio of the actually infected individuals \( X(t) \) at time \( t \) divided by the actually infected individuals at the time \( (t - \Delta t) \):

\[
R_t(t) = \frac{X(t)}{X(t - \Delta t)}, \quad (4)
\]

The relationship between the time-dependent doubling time \( T_d \) and the reproductive number \( R_t \) as defined by the Robert Koch Institute [33] can be derived by considering that

\[
Z(t + \Delta t) = 2 \frac{\Delta t}{T_d} Z(t) = \exp \left( \log(2) \frac{\Delta t}{T_d} \right) Z(t)
\]

\[
\approx \left( 1 + \log(2) \frac{\Delta t}{T_d} \right) Z(t)
\]

,where the equality holds true if \( \Delta t < T_d \).

Hence, we find that

\[
R_t(t) - 1 = \log(2) \frac{\Delta t}{T_d(t)} \quad (6)
\]

and

\[
T_d(t) = C_k(R_t(t) - 1)^{-1} \quad (7)
\]

with \( C_k = \Delta t \log(2) \).

3. Results and discussion

3.1. The three-dimensional parameter space \( T_d(t, R_t) \)

Figure 1(b) shows the expanded three-dimensional parameter space \( T_d(t', R_t) \) proposed in this work. The joint plot of \( T_d \) and \( R_t \) calculated as a function of time provides an exhaustive description of the epidemic spreading during the time interval \( t' = t - t_0 = 230 \) days, where \( t_0 \) is the day onset of the first wave of Covid-19 outbreak in the three studied countries. The grey slab indicates the critical regime where \( 40 < T_d < 100 \) days and \( R_t \sim 1 \). Around the time \( t^* \) the curve of the active infected cases \( X(t) \) of the Covid-19 epidemic wave reaches the maximum of
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T of figure 1(b) characterized by a shorter doubling time of the active infected cases $X_{\text{critical}}$. Regime occurs in the upper part of figure 1(b) where $T_0$ regime from the subcritical supercritical a dome. The critical zone separates the supercritical regime from the subcritical regime.

The supercritical regimes occurs in the lower part of figure 1(b) characterized by a shorter doubling time $T_d < T_d(t^*) = 40$ days, for $t' < t^*$ and a larger reproductive number $R_t > 1$ which occurs where the curve of the active infected cases $X(t)$ is growing. The subcritical regime occurs in the upper part of figure 1(b) where $T_d > 100$ days and $R_t < 1$, i.e. where the curve $X(t)$ is decreasing. Figure 1(c) shows the projection of the 3D plot in the $T_d$–$R_t$ plane for the three considered countries. This figure allows us to verify that in the supercritical regime (orange area) $T_d(R_t)$ follows the universal power law of equation (8) characterized by the divergence of $T_d(R_t)$ while approaching $R_t = 1$ (thick dotted line).

Figure 1(c) shows that in the orange supercritical regime all curves of the three countries are fitted by the function $T_d(t) = C(R_t(t) − 1)^{-\nu}$ with $\nu = 0.7$.

In the subcritical phase we see a random distribution of the data pairs $(T_d, R_t)$ in the green area. The critical phase is indicated by the full gray slab as in the 3D plot of figure 1(b), which corresponds to the range [40–100] days of the doubling time $T_d$ and the reproductive number $R_t$ in the range $1 < R_t < 1.1$. The supercritical regime is indicated by the yellow areas, ranging from the outbreak threshold time $t_0$ and the day $t^*$, where $T_d(t^*)$ reaches the value of 40 days. The straight line $T_d$ in the semi-logarithm scale in the yellow region shows that in the supercritical regime, the doubling time $T_d$ follows the universal exponential law [10, 27]

$$T_d(t) = A e^{(t-t_0)/\beta}$$ (8)

Figure 1(d) shows that in the orange supercritical regime all curves of the three countries are fitted by the function $T_d(t) = C(R_t(t) − 1)^{-\nu}$ with $\nu = 0.7$.

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where \( t_0 \) is the time of the onset of the outbreak and \( s \) (called the s-factor) is the characteristic time of the applied containment policy [10, 27]. In some countries, particularly those where the mitigation measures were not strict, we do not observe an initial exponential growth [35, 36]. This is not the case of Covid-19 pandemic in the considered countries by using the SIR model with estimates extracted from clinical studies have failed to reproduce the experimental data because of the negligible variation of the susceptible population during the first wave in the three countries investigated in this work. The variation of the susceptible population \( S(t) \) at the end of the first wave normalized to the maximum value \( S(0) \) at the threshold of epidemic break out are given by

\[
1 - \frac{S(t)}{S(0)} = 1 - \frac{60\,360\,000 - 317\,000}{60\,360\,000} = 0.0053 \text{ in Italy;}
\]

\[
1 - \frac{S(t)}{S(0)} = 1 - \frac{330\,775\,889 - 758\,957}{330\,775\,889} = 0.0229 \text{ in USA;}
\]

\[
1 - \frac{S(t)}{S(0)} = 1 - \frac{50\,599\,528 - 92\,817}{50\,599\,528} = 0.0018 \text{ in South Korea.}
\]

Therefore, the maximum variation of the susceptible population reported by the data bases are 0.53%, 2.3% and 0.18% for Italy, USA and South Korea, respectively.

In this scenario several authors have discusses the Covid-19 epidemics with containment measures using the susceptible–infected model [4]. Barabasi in his book [7] has considered the Susceptible–Infected–Susceptible model where the number of infected cases in the endemic state is

\[
1(\infty) = 1 - \frac{\mu}{\beta(k)},
\]

and he found the relation between the characteristic time of a pathogen \( \tau = 1/\mu(\beta(0) - 1) \) where \( \beta(0) = 0 \). How is the basic reproductive number which is similar to our equation (7).

Some authors have proposed an unconventional inverted SIR model [25, 26] where the susceptible population \( S \) is assumed to be constant. In this non-conventional epidemiological model the values of the extracted infection \( \tau_i \) and recovery time \( \tau_r \) do not agree with clinical data, but have to be considered only as the extracted effective values of \( \tau_i \) and \( \tau_r \) by solv (‘inverting’) the SIR model. Therefore the effective \( R_e \) number extracted by the ratio between \( \tau_i \) and \( \tau_r \) parameters, given by the inverted SIR model, deviates from the standard epidemiological definitions.

The values of \( \tau_i \) and \( \tau_r \) extracted using the inverted SIR model show a very large variation with time which quantify the effect of the containment measures that have been adopted and are different from the ‘clinical’ \( \tau_i \) and \( \tau_r \) [37].

The \( T_d(t, R_e) \) phase diagram, where the effective reproduction time, namely \( R_e \), is extracted by the inverted SIR theory [25, 26] is shown in figure 2. The panel (a) in figure 2 shows that the transition from the supercritical to the subcritical regime is driven by the joint increase of \( \tau_i \) and decrease of \( \tau_r \). The opposite occurs when the subcritical to the supercritical transition happens in metastable phases at end of the first wave and the threshold of the second wave. In figures 2(b) and (c) a similar analysis is performed on the epidemic data of Italy and USA respectively. The comparison among countries shown in figures 2(a)
and (b), 1(c) indicates different efforts in testing and tracing. The cumulative curve $Z(t)$ of the total number of cases of the epidemics in the supercritical phase, slows down approaching the critical regime, where it has been fitted by the complex Ostwald growth law [10, 17, 38–40]

$$Z(t-t_0) = C \left\{ 1 - e^{-\frac{t-t_0}{\tau}} \right\} \cdot (t-t_0)^\gamma$$  \hspace{1cm} (9)

which is a mixed exponential and power-law behavior determined by nucleation of different phases and ordering phenomena in complex multiphase systems out of equilibrium [38–40]. The onset of this behavior, obtained by best curve fitting, is reported by the dashed lines in the central panels of figure 2.

The time-dependent doubling time given by

$$T_d(t) = \frac{\log(2)}{\frac{d}{dt} \log(Z(t))} = \frac{\log(2)}{\frac{dZ(t)}{Z(t)}}$$  \hspace{1cm} (10)

can be obtained from the equation for $\tau_1(t)$. In the framework of the SIR model, we obtain the following expression for $T_d$

$$T_d(t) = \log(2) \tau_1(t) \frac{Z(t)}{X(t)}$$  \hspace{1cm} (11)

$$T_d(t) = \log(2) \tau_1(t) \frac{Z(t)}{X(t)} \frac{1}{R_e(t)}$$  \hspace{1cm} (12)

The doubling time $T_d$ calculated using equation (12), has been plotted in figure 2 with the effective reproduction number $R_e$, computed within the inverted SIR model. The $T_d(t)$ curve estimated with the inverted SIR model agrees with the $T_d$ curve estimated from the epidemic data (shown in the lower panels of figure 2).

At variance the effective $R_e$ curves exhibit differences from the reproductive number $R_t$ obtained in this work using the Robert Koch Institute method shown in figure 1. Nevertheless, the time where the critical regime occurs, i.e. when both $R_t$ and $R_e$ are equal is the same in the predictions by the inverted SIR approach.

The large value of $\tau_1 \approx 100$ days during the early days or during the whole period have to be interpreted as the lack of testing and tracing cases, not certainly as the presence of individuals that are capable to transfer the infections for a such long time.

### 3.3. The universal power law relation between time-dependent doubling time, $T_d$, and reproduction number, $R_t$, in the supercritical phase

Log–log plot of the doubling time $T_d$ as a function of $R_t - 1$ and $R_e - 1$ in the three countries are compared in the upper and lower panels of figure 3. The $R_t$ values are higher than $R_e$ calculated with inverted SIR method. Although the difference, the $T_d$ vs $R_t - 1$ behavior in the two approach appears qualitatively similar. We get the critical exponent $\nu = 1$, using $R_e - 1$ extracted by the mean-field inverted SIR model.

We have used $R_t$ as defined by the Robert Koch Institute method with $\Delta t = 5$ days which gives a values of $R_t$ in qualitative agreement with other methods. [34] Fitting the data we have found

$$T_d(t) = C_k (R_t(t) - 1)^{-\nu}$$  \hspace{1cm} (13)
with the exponent $\nu \approx 0.7 \pm 0.05$ in the three studied countries. We have estimated $R_t$ with the Robert Koch Institute method [33] using different time intervals $\Delta t = 3, 5, 7, 9, 11, \text{and } 21$ days. Afterwards we have fitted the data $T_d(R_t)$ using equation (13) in the three selected countries using $R_t$ measured with the different time intervals $\Delta t$. We have found that the exponent $\nu$ decreases by increasing the time interval $\Delta t$ following the stretched exponential function $\nu = e^{-(\Delta t/\tau)^\beta}$ with $\tau = 24.5$ days and $\beta = 0.8$ shown in figure 4. Therefore, $\nu \to 1$ for $\Delta t \to 0$ as predicted by equation (7) in agreement with the mathematical limit for the exponent extracted by the Robert Koch Institute method for the time interval $\Delta t \to 0$.

This result is particularly relevant to predict the onset of new epidemic waves. The onset of the supercritical phase of successive wave will be easily detected in the $[T_d, R_t \geq 1]$ or $[T_d, R_t < 1]$ diagram when dots move towards the yellow area, where the supercritical regime will be following the universal power law of equation (7). Figure 3 shows that in USA the epidemic spreading never entered in the subcritical phase and reversible oscillations ($R_t$ decreasing is induced by red dots) and are observed in the critical regime.

3.4. Metastable phases and the second wave in Italy

In figure 3 the phase diagram of epidemic spreading in Italy during the first wave shows the gray strip corresponding to the critical regime above the supercritical orange area, and below the subcritical regime. As previously mentioned, in this subcritical phase the $T_d$ vs $R_t$ values show a non-analytical disordered distribution (see also figure 1(c)), while in the supercritical regime it is described by the analytical universal power law of equation (7). The green dots in the $T_d$ vs $R_t$ plots in figure 3 show that the subcritical phase occurs also for cases where $T_d > 100$ although $R_t > 1$. When put on a lattice, the SIR model depends on the geometry of contacts, and is in the same universality class as ordinary percolation. $R_t$ is essentially the percolation threshold for the SIR mean-field model, in fact infection can grow without bound where $R_t$ is greater than 1 on a Erdős–Rényi network or a Bethe lattice [41], while there is no percolation for $R_t$ less than 1. However, making the network composed of closed loops which attenuate the probability of an epidemic, different complex geometries [42], and in presence of long range interactions the percolating epidemic
Figure 4. The figure shows the exponent $\nu$ in the curve $T_d(t) = C_\nu (R_t(t) - 1)^{-\nu}$ of the doubling time $T_d$ versus $R_t$ in the supercritical regime, as a function of the time lapse interval $\Delta t$, used to extract $R_t$ using the Koch Institut phenomenological method. Different time lapse intervals $\Delta t$ in the range $3 < \Delta t < 30$ days have been used from the epidemiological data for South Korea USA, and Italy to calculate $R_t$. The exponent $\nu$ as a function of the time lapse interval $\Delta t$ (used to calculate $R_t$) follows a stretched exponential law $\nu = e^{-\frac{\Delta t}{\tau}}$ with $\tau = 24.8$ days and $\beta = 0.8$ showing that $\nu = -1$ in the limit $\Delta t \to 0$ days.

Figure 5. Panel (a) the curve of the active infected cases and fatalities per million populations as a function of time during the second wave in Italy. The time is measured by the day of the year 2020 in the lower scale, and the time $t' = t - t_0$ from the onset of outbreak in Italy in the upper x scale. Panel (b) shows $T_d$ and $R_t$ for the second wave in Italy with the same scale as in figure 1 for the first wave in Italy. The yellow area indicates the explosive percolating epidemic regime of the second wave in Italy. Panel (c) shows the $T_d$ versus $R_t$ plot of the second wave (red dots) compared with the first wave (black dots) panel (d) shows the log–log plot of $T_d$ versus $(R_t - 1)$. The red dots in the second wave shows the first uncontrolled increase of $R_t$ and decrease of $T_d$ up to its peak in the time lapse interval $274 < \text{DOY} < 300$ which is followed by the coronavirus epidemics controlled by containment measures in the time range $300 < \text{DOY} < 330$ which follows the power law given by equation $T_d(t) = C_\nu (R_t(t) - 1)^{-\nu}$ with $\nu = 0.7$.

The plot of the Italian case unveils the unexpected arrested metastable phases for $1.06 < R_t < R_t^* = 1.2$ with a decreasing reproduction number at constant doubling-time ($R_t$ decreases while $T_d$ remains constant) as indicated by horizontal arrows in the curve (green dots) of the ($T_d$ vs $R_t$) plot.

After the flat metastable state in the subcritical phase at the end of the first wave on October 7, 2020 the doubling time decreased toward the supercritical regime at the end of the zig-zag behavior where the arrows indicate the time direction. In figure 3 the data of South Korea at the end of the flat regime show that the efficient mobile contact tracing elevated the doubling time. On the contrary in Italy the poor contact tracing method here enforced caused the decreasing of the doubling time with the onset of supercritical regime of the second Covid-19 epidemic wave.

The evolution of the second epidemic wave in Italy between September 1 and December 13, is plotted in figure 5. The data in panel (b) of figure 3 and the data in figure 5 overlap in the time measured in days of year (DOY) for the period September 1 (245 DOY)
and October 7 (281 DOY) where we have observed the metastable arrested phase discussed above which can be considered either as the end of the first wave as well as the onset of the second wave. Panel (a) of figure 5 shows the time lapse interval where the curve of active cases has shown its maximum increasing rate due to the second wave in Italy. In the three panels it is possible to see that from DOY 275 (October 1) to DOY = 300 (October 26) the reproductive number increased from 1.05 to 1.4 while the doubling-time decreased from $T_d = 102$ days in the subcritical regime to $T_d = 19$ days in the supercritical regime. The rate of the growth rate of active cases started to decrease only after DOY = 300 (October 26). It is remarkable that in the time period 300 < DOY < 330 days, when some strict containment rules have been enforced, the CEwCM developed again as in the same regime of the first wave following both equation (7) $T_d(t) = C_k(R_e(t) - 1)^{-\nu}$ and equation (8) $T_d(t) = A e^{t/\nu}$ where the $s$ factor is about two times larger than in the South Korea second wave shown in figure 3.

4. Conclusions

The results of this work provide an original quantitative approach for understanding the time evolution of the Covid-19 pandemic. We show that it is necessary to expand the parameter space, monitoring the evolution of the pair of relevant variables ($T_d, R_e$). By expanding the parameter space it became possible to analyze in a more precise and complete manner the data of the epidemics, probing and tuning at the same time containment measures. This work sheds light and provides new quantitative experimental tools for the quantitative statistical physics of this Covid-19 pandemic, but certainly also to face future epidemic events thanks to its predicting power.

The results of our work can be summarized as follows: the joint analysis of the doubling time $T_d$, i.e. the time it takes for the number of infected individuals to double in value extracted from the cumulative curves of total (infected plus removed) cases, combined with the reproductive number $R_e$, i.e. the average number of infected persons by a single positive case, extracted from the cumulative curves of the number of active infected cases, provide complementary information on the efficiency of the applied containment policies. Therefore, the proposed approach could be used to dynamically control and to improve the effects of mitigation policies.

Author contributions

The authors equally contributed to this work.

Competing interest statement

Authors declare that they have no competing interests.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

ORCID iDs

Gaetano Campi  https://orcid.org/0000-0001-9845-9394
Antonio Valletta  https://orcid.org/0000-0002-3901-9230
Andrea Perali  https://orcid.org/0000-0002-4914-4975
Augusto Marcelli  https://orcid.org/0000-0002-8138-7547
Antonio Bianconi  https://orcid.org/0000-0001-9795-3913

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