Heuristics to Determine Radial Topology for Distribution System Restoration

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Abstract—A radial topology is needed for distribution system restoration after extreme events. However, determining a good topology in real-time for online use is a challenge. In this letter, two heuristics considering power flow state are proposed to fast determine the radial topology. Case studies show the validity and effectiveness of the proposed methods.

Index Terms—distribution system restoration, radial topology, heuristic, resilience.

I. INTRODUCTION

AFTER outages due to extreme events, the distribution system is usually disconnected from the upstream transmission system. In this case, coordinating multiple sources for service restoration to critical loads is a way to enhance resilience and reduce the loss of outages [1].

The distribution system restoration (DSR) problem a hard-to-solve mixed-integer non-convex problem, due to the non-convex power flow constraint and the integer variables including line and load status. Scholars seek mathematical programming methods to solve the DSR problem. However, even though the nonconvex power flow can be formulated and relaxed as convex constraints, the 0-1 integer variables are not easy to handle. The computation burden will be heavy when the number of integer variables is huge, which cannot satisfy the online requirement. In [1], the DSR problem is solved in two stages, where the radial topology (line status variable) is determined in the first stage and load status variables are handled in the second. However, the heuristic for radial topology determination in [1] may reduce the restoration capacity of the system. Radial topology is also needed in the feeder reconfiguration problem [2] [3]. In [2] and [3], the radial topology is determined by heuristic methods, by which heuristics in this letter are inspired.

A good radial topology is needed to maximize the restoration capability of the system. The restoration capability can be affected by the topology as power flow distribution will be different for systems with different topologies. Therefore, the power flow state should be considered when determining the topology. In this letter, two heuristics considering power flow state are proposed to fast determine the radial topology for distribution system restoration.

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II. PROPOSED HEURISTICS

A. Problem Description

Assume that after an extreme event, the distributed power sources in the distribution network are utilized for service restoration to critical loads. The distribution network can be deemed a connected meshed graph \( \mathcal{G} = (\mathcal{N}, \mathcal{E}) \), where \( \mathcal{N} \) is the set of all buses and \( \mathcal{E} \) the set of all available lines. We need to find a tree \( \mathcal{G}' = (\mathcal{N}, \mathcal{E}') \) by cutting \( |\mathcal{E}| - |\mathcal{N}| + 1 \) lines.

B. The Iterative Heuristic

The iterative heuristic is to cut one loop line in each iteration based on the power flow state, which converges when no loop exists in the graph. In each iteration, two basic steps are involved:

Step 1: Solve a critical load restoration optimization model for meshed network \( \mathcal{G} \), which will be presented in detail in the next part.

Step 2: Open loop-lines to eliminate loops based on the value of the active power of lines.

The iteration times is equal to the number of meshes. A diagram of the iteration is shown in Fig. 1.

Fig. 1. A diagram of iterative heuristic. (a) A sample of distribution network; (b) The iterative process.

We cut the line with the minimum active power in the loop lines because it carries the minimum active power, indicating that the line is the least important for transmitting the active power with the restoration objective. This idea is inspired by the feeder reconfiguration method in [2] [3] where the line who receives positive real power from both sides in the path connecting two feeders is opened. The line receiving positive real power from both sides is exactly the line with the minimum active power.

The pseudo-code of the iterative heuristic is as follows. It returns the radial topology \( \mathcal{E}' \) and the set of cut lines \( \mathcal{E}_c \).

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The performance of the algorithm relies on the algorithms solving the restoration model in line 5 and finding the set of loop lines in line 6. The restoration model is a quadratic program (QP) as shown in next part. To solve QP, one can find the optimum using algorithms such as the interior algorithm in polynomial time [5]. The depth-first search can also terminate in polynomial time [4]. Therefore, the iteration algorithm can also obtain the solution in polynomial time.

C. Critical Load Restoration Model for Meshed Network

As mentioned, the topology will affect the power flow distribution. The number of loads restored is usually mainly yielded by power capacity constraints rather than voltage constraints, as the sources are geographically dispersed and they can support the voltage. The bus voltage does not monotonically decrease along with the lines from substations to loads, so support the voltage. The bus voltage does not monotonically as the sources are geographically dispersed and they can be found readily by the off-the-shelf solvers. The maximum computation time of mixed-integer optimization solver is set as 100s and the relative optimality gap is 10−4. Tests are conducted on an Intel Core I7 CPU at 3.6GHz with 32GB of RAM.

The modified 32-node [6] and IEEE 123-Node systems [1] with distributed generations (DGs) added and critical load classified are used. The numbers of switchable lines and loads for two systems are 36, 32, 124, and 85, respectively. The algorithms are programmed in Python 3 with CVXPY package and the optimization models are solved using the solvers in MOSEK. The pseudo-code of MST-based heuristic is as follows. It also returns the radial topology and the set of cut lines. It can also find the solution in polynomial time as the and MST can be found in polynomial time [4].

D. MST-based Heuristic

The number of loops in the original graph will affect the efficiency of the iterative heuristic as it determines the iteration times. To improve the efficiency, based on the idea of iterative heuristic, we propose a heuristic based on maximum spanning tree (MST) weighted by \( |P_{ij}| \) to eliminate loops at once. The pseudo-code of MST-based heuristic is as follows. It also returns the radial topology and the set of cut lines. It can also find the solution in polynomial time as the and MST can be found in polynomial time [4].

III. Case Studies

The proposed two heuristics are used to obtain radial topologies of the two systems for restoration. To measure the quality of the restoration results, the relative optimality gap is 10−4.
of the obtained topologies, the critical load restoration model CLR-misocp with objective function (1) in the Appendix of [1] is conducted to obtain the maximum weighted number of loads the system can restored $N_{load}$ and the power loss $P_{loss}$.

The mixed-integer second-order conic program (MISOCP) in [6] determining both line and load status is conducted, whose objective value is deemed the global optimum $f^*$. For comparison, the minimum diameter spanning tree (MDST)-based heuristic in [1] is also tested to obtain the radial topologies of two systems.

With different locations of DGs and critical loads, 300 scenarios for both systems are generated to test the performance of two heuristics. An error factor $R_f$ is defined to measure the solution quality of different methods with the global solution:

$$R_f = \frac{|f^* - f_x|}{f^*}$$

where $f_x$ means the objective value obtained by topology determined by heuristic $x$. The solution is deemed a near-optimum when $R_f \leq 10^{-4}$. The results of the solution quality are shown in Table I.

| Table I |
|-----------------|-----------------|-----------------|-----------------|
| Case            | Meshes          | No. of scenarios | IH   | MST  | MDST |
| 32-Node         | 5               | Same Topo.       | 20   | 43   | 0    |
|                 |                 | Near-optimum     | 300  | 300  | 234  |
|                 |                 | $N_{load} < N_{load}^*$ | 0   | 0     | 51   |
| 123-Node        | 2               | Same Topo.       | 20   | 19   | 1    |
|                 |                 | Near-optimum     | 300  | 300  | 298  |
|                 |                 | $N_{load} < N_{load}^*$ | 0   | 0     | 2    |

The entry “Same Topo.” means the scenarios that heuristic $x$ obtains the same topology with MISOCP, “Near-optimum” the scenarios that heuristic $x$ obtains near-optimal solution, and “$N_{load} < N_{load}^*$” the scenarios that heuristic $x$ obtains less weighted number of restored loads indicating the restoration capacity is reduced by the topology.

From the results, several conclusions can be made:

1) The proposed heuristics can obtain the near-optimal topology to ensure the restoration capability of the system.

2) Heuristic MDST cannot guarantee the restoration capability of the system as the weighted number of loads may decrease using the topology determined. It is also concluded that inappropriate topology can reduce restoration capability.

3) The topologies obtained by MISOCP and the proposed heuristics may not be exactly the same but the restoration capability can also be retained with minor difference in power loss. Near-optimal topologies that can retain the restoration capability are acceptable in practice.

The computation time results are illustrated in Table II.

The average computation time for IH is about 5 and 2 times longer than MST-based one for 32-Node and 123-Node system, as the numbers of iterations are 5 and 2, respectively. Generally, the proposed heuristics are faster than MISOCP with acceptable near-optimal solutions. After topology determined, one can use the algorithm in [1] to determine load status within 30s for the 123-Node system.

B. Discussions

To analyze the applicability of two heuristics, cases with large loads, low thermal limits, high impedance indicating different types of distribution systems are tested. The results show that MST-based heuristic is faster than IH one with similar solution quality except for the conditions with low thermal limits. The results are illustrated in Table III.

| Table II |
|-----------------|-----------------|-----------------|-----------------|
| Case            | Meshes          | Time(s)         | IH   | MST  | MISOCP |
| 32-Node         | 5               | $t_{min}$       | 1.91 | 0.36 | 0.70   |
|                 |                 | $t_{max}$       | 2.57 | 0.55 | 100.00 |
|                 |                 | $t_{ave}$       | 2.02 | 0.40 | 19.90  |
| 123-Node        | 2               | $t_{min}$       | 2.75 | 1.17 | 0.89   |
|                 |                 | $t_{max}$       | 3.43 | 1.70 | 100.00 |
|                 |                 | $t_{ave}$       | 2.92 | 1.43 | 74.87  |

It is concluded that for distribution systems with low thermal limits, IH is recommended, and MST-based one can be applied for other types of distribution systems with higher efficiency.

IV. CONCLUSION

This letter proposes two heuristics considering power flow state to fast determine radial topology for service restoration. The case study shows that the proposed heuristics can obtain near-optimal radial topology in seconds for online use.

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