Many Body Correlation Corrections to Superconducting Pairing in Two Dimensions

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Abstract

We demonstrate that in the strong coupling limit (the superconducting gap $\Delta$ is as large as the chemical potential $\mu$), which is relevant to the high-$T_c$ superconductivity, the correlation corrections to the gap and critical temperature are about 10% of the corresponding mean field approximation values. For the weak coupling ($\Delta \ll \mu$) the correlation corrections are very large: of the order of 100% of the corresponding mean field values.

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I. INTRODUCTION

In recent papers [1,2] we demonstrated the very strong d-wave pairing between dressed quasiholes in the $t-J$ model induced by spin-wave exchange. The pairing gives the critical temperature in a reasonable agreement with experimental data for Cooper Oxide Superconductors. For calculations we used BCS-like mean field approximation for dressed quasiholes. A similar approach to the pairing of dressed quasiholes has been used by Dagotto, Nazarenko, and Moreo [3]. The important difference is that in [1,2] the hole-hole interaction was derived from the parameters of the t-J model, while in [3] it was introduced ad hoc with magnitude adjusted to fit experimental data. The typical value of the gap $\Delta$ obtained in the papers [1–3] is of the order of the chemical potential $\mu$. This strong pairing inspires a natural question: how strongly the hole-hole correlations influence upon the mean field result? The purpose of the present work is to investigate this problem. We consider the simplified model instead of the sophisticated t-J one. The model under consideration describes the two-dimensional fermions...
with quadratic dispersion and contact attraction. This model inherits the main, relevant to the problem, property of the t-J model: it permits to consider strong pairing. Therefore one can suppose the obtained result to be rather general. We study the dependence of correlations on the intensity of attractive interaction. Our conclusion is that for strong coupling $\Delta \geq \mu$ the correlation corrections are not large: about 10% of the corresponding mean field value. This is a very surprising result since one could expect that strong interaction causing the pairing might make the correlation correction to be very large. Our conclusion is in agreement with the one which was recently obtained in finite-cluster numerical study of $t-J$ model by Ohta, Shimozato, Eder, and Maekawa [4].

The other surprising result of the present work is that in the weak coupling limit $\Delta \ll \mu$ the correlation corrections are very large: the renormalizations of mean field values are about 100%. This is due to the specific ultraviolet behaviour in the two dimensional theory with attraction.

We are mainly interested in the regime $\Delta \geq \mu$ when the system is close to a smooth crossover from a state with large, overlapping Cooper pairs to a Bose condensate of composite bosons. In the mean field approximation this crossover has been studied in three dimensions by Legget [5] and by Nozieres and Schmitt-Rink [6]. A similar problem for two dimensions reveals interesting features considered by Randeria, Duan, and Shieh [7]. In the present work we investigate the correlation corrections to the results presented in Ref. [7].

**II. MEAN FIELD APPROXIMATION**

Consider the Hamiltonian of fermions with spin 1/2 and contact attractive spin independent interaction

$$H = \sum_{k,\sigma} \frac{k^2}{2m} a^\dagger_{k,\sigma} a_{k,\sigma} - \frac{g}{V} \sum_{k_1, k_2, k_3} a^\dagger_{k_1+k_2-k_3,\uparrow} a^\dagger_{k_3,\downarrow} a_{k_2,\downarrow} a_{k_1,\uparrow},$$

where summation over $k$ is restricted in two-dimensional plane, $V$ is the area of the plane, $\sigma = \pm 1/2 = \uparrow, \downarrow$ is a projection of the usual 3-dimensional spin 1/2. Consider first the Schroedinger equation for a two-particle bound state with zero total momentum

$$\chi_k = -\frac{g}{V} \sum_p \frac{\chi_p}{E_a - p^2/m}.$$  

The sum here is logarithmically ultraviolet divergent, and therefore one has to introduce the ultraviolet cutoff $E_\Lambda = \Lambda^2/2m$. For the $t-J$ model the parameter $\Lambda$ is of the order of inverse lattice spacing. Solution of Eq.(2) is straightforward
\[ \chi_{p} = \text{const}, \quad E_{a} = -2E_{\Lambda}e^{-4\pi/mg}. \]

Now let us consider a many body problem with fixed number density of particles \( \delta \). The Fermi energy is: \( E_{F} = \pi \delta/m \). The BCS equation for the pairing at fixed chemical potential \( \mu \) is of the form

\[ \Delta_{k} = \frac{g}{V} \sum_{p} \frac{\Delta_{p}}{2\epsilon_{p}} \tanh \frac{\epsilon_{p}}{2T}, \]

where \( \epsilon_{p} = \sqrt{\eta_{p}^{2} + \Delta_{p}^{2}} \), and \( \eta_{p} = p^{2}/2m - \mu \). Similarly to the Schroedinger equation we have to introduce the ultraviolet cutoff \( \Lambda \). Solution of the BCS equation for zero temperature is straightforward. Assuming that \( E_{F} \ll E_{\Lambda} \) one gets

\[ \Delta = \sqrt{2E_{F}|E_{a}|} = 2\sqrt{E_{F}E_{\Lambda}}e^{-2\pi/mg}, \]

\[ \mu = E_{F} - |E_{a}|/2 = E_{F} \left( 1 - \frac{1}{4} \frac{\Delta^{2}}{E_{F}^{2}} \right), \]

where \( E_{a} \) is the binding energy \( (3) \) of the two particle bound state. This solution obtained by Randeria, Duan, and Shieh \[7\] gives a smooth crossover from the BCS limit \( (\mu \approx E_{F}) \) to the Bose condensate of composite bosons \( (\mu < 0) \). In the present work we concentrate on the case of positive \( \mu \) because, in our opinion, it is relevant to the realistic high-\( T_{c} \) superconductors.

The critical temperature \( T_{c} \) can be easily found from Eq.(4) if we define it as a point where the gap vanishes. In the weak coupling limit \( (T_{c} \ll E_{F}) \) we have the usual BCS relation: \( \Delta(T = 0)/T_{c} = 1.76 \). Numerical solution of Eq.(4) shows that even in the strong coupling limit this ratio remains very close to the BCS value. For example at \( \Delta(T = 0)/\mu(T = 0) = 3 \) which is equivalent to \( \Delta(T = 0)/E_{F} \approx 1.5 \) one finds \( \Delta(T = 0)/T_{c} = 1.68 \). We assume that the density of particles is fixed. One has to remember that under this condition the chemical potential is a function of temperature: \( \mu(T = 0) \neq \mu(T = T_{c}) \). For strong coupling the temperature dependence of the chemical potential is not negligible.

Equation (4) as well as solution (5) describe the mean field approximation. Now let us consider correlation corrections. We use the conventional Gorkov-Nambu technique, see e.g. book \[8\]. Consider first the case of zero temperature.

**III. CORRELATION CORRECTION TO THE GAP AT ZERO TEMPERATURE**

The normal \( G(1,2) = -i\langle T[\psi(1)\psi^{\dagger}(2)] \rangle \) and the anomalous \( F^{\dagger}(1,2) = -i\langle T[\psi^{\dagger}(1)\psi^{\dagger}(2)] \rangle \)

Green functions obey the usual Dyson equations \[3\]
\[
\hat{G}(p) = [1 + \hat{G}(p)\hat{\Sigma}_{11}(p) + \hat{F}^{\dagger}(p)\hat{\Sigma}_{20}(p)]\hat{G}_0(p)
\]
\[
\hat{F}^{\dagger}(p) = [\hat{F}^{\dagger}(p)\hat{\Sigma}_{11}(-p) + \hat{G}(p)\hat{\Sigma}_{02}(p)]\hat{G}_0(-p),
\]

where \( p = (\epsilon, \mathbf{p}) \), \( \hat{G}_0(p) = [\epsilon - \eta_p + i0 \cdot \text{sign} (\eta_p)]^{-1} \). In the first order of perturbation theory the normal self-energy operator \( \hat{\Sigma}_{11}^{(1)} \) is given by the diagrams presented in Fig.1. With the interaction \[\text{II}\] the self-energy \( \hat{\Sigma}_{11}^{(1)} \) is momentum independent, and therefore it gives only a correction to the chemical potential \( \mu \). The first order anomalous self-energy operator \( \hat{\Sigma}_{20}^{(1)} \) is given by the diagram Fig.2. It is equivalent to the BCS mean field approximation. Solution of Eq.(6) with \( \Sigma_{20} = \Sigma_{20}^{(1)} \) is of the form (see \[\text{III}\]): \( \hat{G}_{\alpha\beta}(p) = \delta_{\alpha\beta} G(p), \hat{F}_{\alpha\beta}^{\dagger}(p) = g_{\alpha\beta} F^{\dagger}(p), \hat{\Sigma}_{(20)\alpha\beta}(p) = g_{\alpha\beta} \Sigma_{20}(p) \), where \( \delta_{\alpha\beta} \) and \( g_{\alpha\beta} \) are standard symmetric and antisymmetric spin matrices, and

\[
G(\epsilon, \mathbf{p}) = \frac{u_p^2}{\epsilon - \epsilon_p + i0} + \frac{v_p^2}{\epsilon + \epsilon_p - i0}
\]
\[
F^{\dagger}(\epsilon, \mathbf{p}) = -u_p v_p \left( \frac{1}{\epsilon - \epsilon_p + i0} - \frac{1}{\epsilon + \epsilon_p - i0} \right)
\]
\[
\Sigma_{20}^{(1)} = \Delta
\]

with \( u_p, v_p = \sqrt{\frac{1}{2}}(1 \pm \eta_p/\epsilon_p) \). The gap \( \Delta \) is given by \[\text{III}\].

In the next order of perturbation theory the normal self-energy operator \( \Sigma_{11}^{(2)}(p) \) is represented by the diagrams in Fig.3. Note that that inside \( \Sigma^{(2)} \) we use not the bare Green functions, but the “dressed” ones. They take into account the self-energy corrections in accordance with Eq.(3). This approach is widely used in many-body problems when a correlation correction can be significant, see for example Ref. \[\text{II}\]. The self-energy corresponding to diagrams in Fig.3 can be easily evaluated.

\[
\Sigma_{11}^{(2)}(\epsilon, \mathbf{p}) = \left( \frac{g}{V} \right)^2 \sum_{\mathbf{k}_1, \mathbf{k}_2} [v_1^2 u_2^2 - (u_1 v_1)(u_2 v_2)] \left( \frac{u_3^2}{\epsilon - \epsilon_1 - \epsilon_2 - \epsilon_3} - \frac{v_3^2}{-\epsilon - \epsilon_1 - \epsilon_2 - \epsilon_3} \right)
\]  

(8)

The summation here is carried out over \( \mathbf{k}_1 \) and \( \mathbf{k}_2 \). The momentum \( \mathbf{k}_3 \) is defined as \( \mathbf{k}_3 = \mathbf{p} + \mathbf{k}_1 + \mathbf{k}_2 \). The second order anomalous self-energy operator is given by the diagrams presented in Fig.4. The calculation gives

\[
\Sigma_{20}^{(2)}(\epsilon, \mathbf{p}) = \left( \frac{g}{V} \right)^2 \sum_{\mathbf{k}_1, \mathbf{k}_2} u_3 v_3 [v_1^2 u_2^2 - (u_1 v_1)(u_2 v_2)] \left( \frac{1}{\epsilon - \epsilon_1 - \epsilon_2 - \epsilon_3} + \frac{1}{-\epsilon - \epsilon_1 - \epsilon_2 - \epsilon_3} \right)
\]  

(9)

Similar to \[\text{III}\] the momentum \( \mathbf{k}_3 \) in this sum is equal to \( \mathbf{k}_3 = \mathbf{p} + \mathbf{k}_1 + \mathbf{k}_2 \).

Note that calculating the first order anomalous self-energy operator Fig.2 we also have to use the exact Green function. Hence this operator is proportional to
\[\Xi^* = \frac{1}{V} \int \frac{d\epsilon}{2\pi i} \sum_{\mathbf{p}} F^\dagger(\epsilon, \mathbf{p}),\]  

where \(F^\dagger\) is the “dressed” anomalous Green function, not just a first order one.

Equations (10) can be rewritten as

\[
\begin{align*}
[G_0^{-1}(p) - \Sigma_{11}(p)]G(p) + (g\Xi + \Sigma_2^{(2)}(\epsilon, p))F^\dagger(p) &= 1 \\
[G_0^{-1}(-p) - \Sigma_{11}(-p)]F^\dagger(p) - (g\Xi + \Sigma_2^{(2)}(\epsilon, p))G(p) &= 0.
\end{align*}
\]

We recall that \(\Sigma_{11}^{(1)}\) is momentum independent and therefore is completely absorbed into the chemical potential. The second order self-energy (8) is logarithmically divergent at large \(k^2\).

However, the diverging part is independent of \(\epsilon\) and \(p\), and therefore can also be absorbed into the chemical potential. This freedom permits us to renormalize the self-energy imposing the condition \(\Sigma_{11}(\epsilon = 0, |p| = p_\mu) = 0\) at \(p_\mu = \sqrt{2m\mu}\). We can use also the usual linear expansion near the point \(\epsilon = 0, |p| = p_\mu\): \(\Sigma_{11}(\epsilon, p) = \frac{\partial\Sigma_{11}}{\partial\epsilon}\epsilon + \frac{\partial\Sigma_{11}}{\partial p}\eta_p\), assuming that \(\epsilon \sim \eta_p \sim \Delta\). This expansion is certainly valid for the weak coupling \(\Delta \ll E_F\). We have verified numerically using (8) that the expansion remains valid with a reasonable accuracy for the strong coupling \(\Delta \sim E_F\) as well. Now we can easily find the solution of the Eq.(11)

\[
\begin{align*}
G(\epsilon, p) &\approx Z \left( \frac{\bar{u}_p^2}{\epsilon - \bar{\epsilon}_p + i0} + \frac{\bar{v}_p^2}{\epsilon + \bar{\epsilon}_p - i0} \right) \\
F^\dagger(\epsilon, p) &\approx -Z\bar{u}_p\bar{v}_p \left( \frac{1}{\epsilon - \bar{\epsilon}_p + i0} - \frac{1}{\epsilon + \bar{\epsilon}_p - i0} \right) \\
\bar{u}_p, \bar{v}_p &= \sqrt{\frac{1}{2}(1 \pm \eta_p/\bar{\epsilon}_p)}
\end{align*}
\]

where \(Z = [1 - \frac{\partial\Sigma_{11}}{\partial\epsilon}]^{-1}\) is the quasiparticle residue, and the renormalized dispersion is \(\bar{\epsilon}_p = \sqrt{\eta_p^2 + \Delta_p^2}\) with

\[
\begin{align*}
\Delta_p &= Z \left( g\Xi + \Sigma_2^{(2)}(\epsilon, p) \right) \\
\bar{\eta}_p &= Z \left( 1 + \frac{\partial\Sigma_{11}}{\partial p}\eta_p \right).
\end{align*}
\]

Numerical computations show that the dependence of \(\Delta_p\) on \(\epsilon\) and \(p\) at \(\epsilon \sim E_F\) and \(p \sim p_\mu\) is actually rather weak. But anyway, the gap depends on energy and momentum and therefore it is convenient to introduce \(\Delta = \Delta_{\epsilon=0, p=p_\mu}\), and to represent the second order anomalous self-energy as \(\Sigma_2^{(2)}(\epsilon, p) = -\Delta \cdot \sigma(\epsilon, p)\). From Eqs.(8) and (9) one finds

\[
\begin{align*}
\left( \frac{\partial\Sigma_{11}}{\partial\epsilon} \right)_{\epsilon=0, p=p_\mu} &= -\left( \frac{gm}{2\pi} \right)^2 R_1(\Delta/\mu), \\
\left( \frac{\partial\Sigma_{11}}{\partial\eta_p} \right)_{\epsilon=0, p=p_\mu} &= \left( \frac{gm}{2\pi} \right)^2 R_2(\Delta/\mu), \\
\sigma(\epsilon = 0, p = p_\mu) &= \left( \frac{gm}{2\pi} \right)^2 R_3(\Delta/\mu).
\end{align*}
\]
The functions $R_i, i = 1, 2, 3$ depend only on the ratio $\Delta/\mu$. One can easily find that in the weak coupling limit ($\Delta \ll \mu$): $R_1(0) = \text{const}, R_2(0) = \text{const}, R_3(\Delta/\mu) \approx \ln(\mu/\Delta)$. Results of numerical computations of $R_i$ at arbitrary $\Delta/\mu$ are presented in Table I. They show that the corrections due to $\Sigma_{11}$ are negligible not only in the weak coupling limit when $gm/2\pi \ll 1$, $\Delta/\mu \ll 1$, but remain small for the strong coupling, $gm/2\pi \sim 1$, $\Delta/\mu \geq 1$, as well. Therefore we can neglect $\Sigma_{11}$ and consider only the anomalous self-energy operator $\sigma$. The smallness of $\Sigma_{11}$ corrections results in the fact that $Z \approx 1$. Therefore the exact Green functions (12) have the form similar to the Green functions (7) in the mean-field approximation. We used this fact when evaluated Eqs.(8),(9).

In order to find a relation between the gap $\Delta$ and the coupling constant $g$ we have to substitute the solution (12),(13) into the self-consistency condition (10). Then we get

$$1 = \frac{g}{V} \sum \frac{1}{2\epsilon_p} + \frac{g}{V} \sum \frac{\sigma(\epsilon, \mathbf{p})}{2\pi i (\epsilon - \epsilon_p + i0)(\epsilon + \epsilon_p - i0)}.$$  \hspace{1cm} (15)

The last term here gives the correction to the BCS mean field equation (5). Assuming that the correction to the mean field value of the gap $\Delta_{mf}$ is small ($\delta \Delta = \Delta - \Delta_{mf} \ll \Delta$) we find from (15)

$$\frac{\delta \Delta}{\Delta} = - \left(\frac{gm}{2\pi}\right)^2 L(\Delta/\mu), \hspace{1cm} (16)$$

where $L$ depends only on the ratio $\Delta/\mu$

$$L(\Delta/\mu) = - \left(\frac{2\pi}{gm}\right)^3 \frac{2}{1 + \sqrt{1 + \Delta^2/\mu^2}} \frac{g}{V} \sum \frac{\sigma(\epsilon, \mathbf{p})}{2\pi i (\epsilon - \epsilon_p + i0)(\epsilon + \epsilon_p - i0)}. \hspace{1cm} (17)$$

Numerical computation of $L$ is straightforward. It is convenient to integrate in (17) along the imaginary $\epsilon$ axis. Results are presented in Table I. In the weak coupling limit ($\Delta \ll \mu$) $L$ can be easily calculated analytically with logarithmic accuracy: $L \approx \ln^2(\mu/\Delta)$. Using (16) and (5) we find the gap in this limit

$$\Delta \approx \Delta_{mf} \left[1 - \left(\frac{gm}{2\pi} \ln \frac{E_F}{\Delta}\right)^2\right] \approx \Delta_{mf} \left[1 - \left(\frac{\ln(E_F/\Delta)}{\ln(2\sqrt{E_F E_\Lambda}/\Delta)}\right)^2\right]. \hspace{1cm} (18)$$

Thus the correction to the mean field value is very large. The reason for the large correction is simple. The gap is proportional to $\Delta \propto e^{-2\pi/mg}$. Practically we have calculated the renormalization of the coupling constant $g$. A small correction to $g$ gives a large correction to $\Delta$ when exponent $2\pi/mg$ is large. Certainly in this situation the third order self-energy can give substantial contribution as well.
For application to high-$T_c$ superconductivity we are more interested in the strong coupling limit $\Delta \geq \mu$. Surprisingly in this case the correction to the mean field approximation is small. For illustration let us set $E_\Lambda/E_F = 20$. With this ratio fixed one can easily find from (5) the value of $g m/2\pi$ as a function of $\Delta/\mu$. After substitution of this value into (16) with $L$ from Table I we find $\delta\Delta/\Delta$. Results of these calculations are presented in Table II. In our opinion the values of parameters $\Delta/\mu \sim 1-3$ and $E_\Lambda/E_F \sim 20$ correspond qualitatively to the $t-J$ model describing high-$T_c$ superconductors [1,2]. We see from Table II that in this region the correlation correction $\delta\Delta/\Delta$ is about -10%.

We conclude that the BCS-like mean field approximation for the pairing of dressed holes in the $t$-J model is justified with the accuracy $|\delta\Delta|/\Delta \sim 0.1$. It is worth to note that the situation when correlation corrections to the Hartree-Fock approximation are more important for weak coupling than for strong coupling is well known for a number of many-body problems in nuclear and atomic physics.

IV. THE CRITICAL TEMPERATURE

The correction to the critical temperature may be found in a way similar to the above developed approach for the correction to the gap at zero temperature. We have to solve Eqs.(11) and find a point where the gap vanishes. For finite temperature the energy in these equations is equal to $\epsilon = i\xi_s$, where $\xi_s = \pi T(2s + 1)$, $s = 0, \pm 1, \pm 2, \ldots$ is the Matsubara frequency. Integration over energy inside any loop should be replaced by summation over Matsubara frequencies. The mean field approximation is equivalent to the account of the diagram Fig.2 for the self-energy. The solution of Eqs.(13) is of the form: $G_{\alpha\beta}(p) = \delta_{\alpha\beta}G(p), \bar{F}_{\alpha\beta}(p) = -g_{\alpha\beta}\bar{F}(p), \Sigma_{(20)\alpha\beta}(p) = g_{\alpha\beta}\Sigma_{20}(p)$, where

$$G(i\xi_s, p) = \frac{1}{i\xi_s - \eta_p},$$

$$\bar{F}(i\xi_s, p) = \frac{\Delta}{\xi_s^2 + \eta_p^2},$$

$$\Sigma_{20}^{(1)} = \Delta.\tag{19}$$

We neglect here all powers of the gap higher than one. Self-consistency condition for the diagram Fig.2 gives the critical temperature in the mean field approximation.

The second order normal self-energy operator is given by the diagrams presented in Fig.3a,c. The contributions of the diagrams Fig.3b,d,e,f are proportional to $\Delta^2$ and therefore can be neglected. The calculation gives
\[ \Sigma_{11}^{(2)}(i \xi_s, \mathbf{p}) = \left( \frac{g}{V} \right)^2 \sum_{k_1, k_2} \frac{[n(\eta_1) - n(\eta_2)] [n(\eta_1 - \eta_2) - n(\eta_3)]}{i \xi_s + \eta_1 - \eta_2 - \eta_3}. \] (20)

Here \( n(\eta) = [1 + \exp(\eta/T)]^{-1} \) is a Fermi-Dirac function. The summation is carried out over \( k_1 \) and \( k_2 \). The momentum \( k_3 \) is equal to \( k_3 = p + k_1 + k_2 \). Similar to the case of zero temperature the detailed analysis demonstrates that the normal self-energy operator is small in the strong coupling limit as well as in the weak coupling one. Therefore below we neglect \( \Sigma_{11} \).

The second order anomalous self-energy operator is given by the diagrams Fig.4a,c,d,e. The contribution Fig.4b is proportional to \( \Delta^3 \) and therefore is neglected. The calculation gives

\[ \Sigma_{20}^{(2)}(i \xi, \mathbf{p}) = -\Delta \sigma_T(i \xi, \mathbf{p}), \]

\[ \sigma_T(i \xi_s, \mathbf{p}) = \left( \frac{g}{V} \right)^2 \sum_{k_1, k_2} \frac{[n(\eta_1) - n(\eta_2)] [n(\eta_3) - n(\eta_1 - \eta_2)] - [n(-\eta_3) - n(\eta_1 - \eta_2)]}{i \xi_s + \eta_1 - \eta_2 - \eta_3}, \] (21)

where, as above, \( k_3 = p + k_1 + k_2 \). After substitution of \( \Sigma_{20}^{(2)} \) into (20) we find

\[ \widetilde{F}(i \xi_s, \mathbf{p}) = \frac{g \Xi^* - \Delta \sigma_T}{\xi_s^2 + \eta_p^2}, \] (22)

where \( \Xi^* = T/V \sum_s \sum_p \tilde{F}(i \xi_s, \mathbf{p}) \) is given by the diagram Fig.2. Similar to (21) this gives the equation for the critical temperature

\[ 1 = \frac{g}{V} \sum_p \frac{1}{2 \eta_p} \tanh \frac{\eta_p}{2 T_c} - \frac{g}{V} T_c \sum_p \sum_s \frac{\sigma_T(i \xi_s, \mathbf{p})}{\xi_s^2 + \eta_p^2}. \] (23)

The last term here is the correction to the mean field equation (20). Assuming that the correction to the critical temperature is small we find from (23)

\[ \frac{\delta T_c}{T_c} = -\left( \frac{gm}{2 \pi} \right)^2 L_T(T_c/\mu_c). \] (24)

The function \( L_T \) depends only on the ratio \( T_c/\mu_c \), where \( \mu_c \) is the chemical potential at the critical point, and

\[ L_T(T_c/\mu_c) = \left( \frac{2 \pi}{gm} \right)^3 \frac{2}{1 + \tanh(\mu_c/2 T_c)} \frac{g}{V} T_c \sum_p \sum_s \sigma_T(i \xi_s, \mathbf{p}) \xi_s^2 + \eta_p^2. \] (25)

Numerical computation of \( L_T \) is straightforward. Results are presented in the last column of Table I. We present \( L_T \) as a function of \( \Delta(0)/\mu(0) \) (the gap at zero temperature over the chemical potential at zero temperature). It is possible to do so because \( T_c/\mu_c \) itself is a function of \( \Delta(0)/\mu(0) \). In the weak coupling limit \( L_T \) can be easily calculated analytically with logarithmic accuracy: \( L_T \approx \ln^2(\mu_c/T_c) \approx \ln^2(E_F/\Delta(0)) \). From (24) and (25) we find the critical temperature in this limit
\[ T_c \approx T_{c(mf)} \left[ 1 - \left( \frac{\ln(E_F/\Delta)}{\ln(2\sqrt{E_F E_A}/\Delta)} \right)^2 \right], \]  

where \( T_{c(mf)} \) is the critical temperature in mean field approximation, and \( \Delta = \Delta(0) \) is the gap at zero temperature. Comparing (26) with (18) we see that \( \Delta \) and \( T_c \) have the same renormalization factor. Therefore the BCS relation \( \Delta/T_c \approx 1.76 \) is preserved despite of the fact that the renormalizations of the mean field values are about 100%.

Comparing \( L \) and \( L_T \) from Table I we see that in the strong coupling limit \( (\Delta \geq \mu) \) the correction to the critical temperature is larger than the correction to the gap at zero temperature. Nevertheless the correction remains small. Consider the same example as for zero temperature: \( E_A/E_F = 20 \). The Table II gives \( \delta T_c/T_c \) as a function of \( \Delta(0)/\mu(0) \). We see that at \( \Delta(0)/\mu(0) \sim 1–3 \) the correlation correction \( \delta T_c/T_c \) is about -15%.

V. CONCLUSION

We consider the correlation corrections to the BCS mean field pairing in the two dimensional case. It is found that for the strong pairing \( (\Delta \geq \mu) \) the correlation corrections are not large: about 10% of the corresponding mean field value for the set of parameters relevant to \( t - J \) model describing high-\( T_c \) superconductors. The small values of the corrections is explained qualitatively by the fact that the energy of virtual excitations becomes higher with increase of the pairing. We conclude that the BCS mean field approximation is reasonably justified for description of the dressed quasiholes pairing in the \( t - J \) model.

Surprisingly for the weak coupling limit \( (\Delta \ll \mu) \) the correlation corrections are very large: the renormalization of the mean field values is about 100%. The large correction results from the exponential dependence of the superconducting gap on the coupling constant which makes a small correction to \( g \) to give a significant contribution for \( \Delta \).

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TABLE I. Numerical values of dimensionless functions $R_i$, $L$, $L_T$ for different values of $\Delta(T=0)/\mu(T=0)$. We present also the corresponding values of $\Delta(T=0)/E_F$. The functions $R_1, R_2$ give normal self-energy operator $\Sigma_{11}$, see Eq.(14). Their small values permit one to neglect corrections caused by $\Sigma_{11}$ and consider only corrections caused by $\Sigma_{20}$. The functions $L$ and $L_T$ describe the correlation corrections to the gap and to the critical temperature, see Eqs.(16),(17) and (24),(25).

| $\Delta/\mu$ | $\Delta/E_F$ | $R_1$   | $R_2$   | $R_3$ | $L$   | $L_T$ |
|-------------|-------------|--------|--------|------|------|------|
| 3           | 1.44        | 3.4 $\cdot$ 10$^{-2}$ | 1.9 $\cdot$ 10$^{-2}$ | 0.11 | 0.27 | 0.46 |
| 2           | 1.23        | 5.0 $\cdot$ 10$^{-2}$ | 2.9 $\cdot$ 10$^{-2}$ | 0.15 | 0.35 | 0.59 |
| 1           | 0.83        | 9.8 $\cdot$ 10$^{-2}$ | 5.8 $\cdot$ 10$^{-2}$ | 0.31 | 0.65 | 0.98 |
| 0.5         | 0.47        | 0.19   | 0.10   | 0.66 | 1.4  | 1.9  |
| 0.1         | 0.1         | 0.46   | 0.17   | 2.1  | 6.6  | 7.2  |
| 0.01        | 0.01        | 0.66   | 0.24   | 4.5  | 23   | 23   |

TABLE II. The correlation corrections to the gap at zero temperature and to the critical temperature for different values of $\Delta(T=0)/\mu(T=0)$. The ultraviolet cutoff is fixed: $E_\Lambda/E_F = 20$.

| $\Delta/\mu$ | $g m/2\pi$ | $-\delta\Delta/\Delta$ | $-\delta T_c/T_c$ |
|-------------|------------|------------------------|-------------------|
| 3           | 0.55       | 0.08                   | 0.14              |
| 2           | 0.51       | 0.09                   | 0.15              |
| 1           | 0.42       | 0.12                   | 0.18              |
| 0.5         | 0.34       | 0.16                   | 0.22              |
| 0.1         | 0.22       | 0.33                   | 0.36              |
| 0.01        | 0.15       | 0.50                   | 0.50              |

FIGURE CAPTIONS

FIG. 1. Normal self energy operator in the first order of perturbation theory $\Sigma_{11}^{(1)}$.

FIG. 2. Anomalous self energy operator in the first order of perturbation theory $\Sigma_{20}^{(1)}$.

FIG. 3. Normal self energy operator in the second order of perturbation theory $\Sigma_{11}^{(2)}$.

FIG. 4. Anomalous self energy operator in the second order of perturbation theory $\Sigma_{20}^{(2)}$. 
