The sound velocity in an equilibrium hadron gas

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We calculate the velocity of sound in an ideal gas of massive hadrons with non-vanishing baryon number. The gas is in thermal and chemical equilibrium. Also we show that the temperature dependence $T(\tau) \approx T_0 \cdot \left(\frac{\tau_0}{\tau}\right)^{c_s^2}$ is approximately valid, when the gas expands longitudinally according to the Bjorken law.

I. INTRODUCTION

The main subject of the following paper is the calculation of the sound velocity $c_s$ in the multi-component hadron gas with nonzero baryon number density. The sound velocity appears in the hydrodynamical description of the matter which is created in the central rapidity region (CRR) of a heavy-ion collision. It turned out [1,2] that the evolution of the matter (whatever it is: a quark-gluon plasma or the hadron gas) proceeds as longitudinal and superimposed transverse expansions. And the latter has the form of the rarefaction wave moving radially inward with the velocity of sound. The second place where the sound velocity appears is the temperature equation for an ideal baryonless gas (it can be a gas of quarks and gluons as well), which cools as the result only of the longitudinal expansion (see e.g. [3])

$$T(\tau) = T_0 \cdot \left(\frac{\tau_0}{\tau}\right)^{c_s^2},$$

where $\tau$ is a local proper time, $\tau_0$ an initial moment of time and $c_s$ is assumed constant. In the following, we show that the above equation, now as some approximation, is also valid in some range of temperature for the hadron gas with non-vanishing baryon number.

In our previous paper [4] we showed that taking into account also the transverse expansion (in the the form of the rarefaction wave) changes the $J/\Psi$ theoretical suppression pattern qualitatively. This was done for the scenario without the quark-gluon plasma appearance in the CRR. Of course, this is very important because $J/\Psi$ suppression observed in NA38 [3] and NA50 [3] heavy-ion collision data is treated as the main signature of the creation of the quark-gluon plasma (QGP) during these collisions [3]. Since the study of $J/\Psi$ suppression patterns was the main purpose of our paper [4], we left aside the more complete presentation of the behaviour of the sound velocity in the hadron gas. Now, we would like to present our results in the form of a separate paper.

II. THE EXPANDING MULTI-COMPONENT HADRON GAS

For an ideal hadron gas in thermal and chemical equilibrium, which consists of $l$ species of particles, energy density $\epsilon$, baryon number density $n_B$, strangeness density $n_S$, entropy density $s$ and pressure $P$ read ($\hbar = c = 1$ always)

$$\epsilon = \frac{1}{2\pi^2} \sum_{i=1}^{l} (2s_i + 1) \int_0^\infty \frac{dp p^2 E_i}{\exp \left\{ \frac{E_i - \mu_i}{T} \right\} + g_i},$$

$$n_B = \frac{1}{2\pi^2} \sum_{i=1}^{l} (2s_i + 1) \int_0^\infty \frac{dp p^2 B_i}{\exp \left\{ \frac{E_i - \mu_i}{T} \right\} + g_i},$$

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\[ n_S = \frac{1}{2\pi^2} \sum_{i=1}^{l} (2s_i + 1) \int_0^\infty \frac{dpp^2S_i}{\exp \left\{ \frac{E_i - \mu_i}{T} \right\} + g_i}, \quad (2c) \]

\[ s = \frac{1}{6\pi^2T^2} \sum_{i=1}^{l} (2s_i + 1) \int_0^\infty \frac{dpp^4}{E_i} \frac{E_i - \mu_i}{\left( \exp \left\{ \frac{E_i - \mu_i}{T} \right\} + g_i \right)^2}, \quad (2d) \]

\[ P = \frac{1}{6\pi^2} \sum_{i=1}^{l} (2s_i + 1) \int_0^\infty \frac{dp}{E_i} \frac{1}{\exp \left\{ \frac{E_i - \mu_i}{T} \right\} + g_i}, \quad (2e) \]

where \( E_i = (m_i^2 + p^2)^{1/2} \) and \( m_i, B_i, S_i, \mu_i, s_i \) and \( g_i \) are the mass, baryon number, strangeness, chemical potential, spin and a statistical factor of specie \( i \) respectively (we treat an antiparticle as a different specie). And \( \mu_i = B_i\mu_B + S_i\mu_S \), where \( \mu_B \) and \( \mu_S \) are overall baryon number and strangeness chemical potentials respectively.

We shall work here within the usual timetable of the events in the CRR of a given ion collision (for more details see e.g. [1,2]). We fix \( t = 0 \) at the moment of the maximal overlap of the nuclei and assume that it takes place at \( z = 0 \), where \( z \) is the coordinate of a collision axis. After half of the time the nuclei need to cross each other, matter appears in the CRR. We assume that soon thereafter matter thermalizes and this moment, \( \tau_0 \) (\( \tau = \sqrt{t^2 - z^2} \)), is estimated at about 1 fm [3,4]. Then matter starts to expand and cool and after reaching the freeze-out temperature \( T_{f.o.} \) it is no longer a thermodynamical system. As we have already mentioned in the introduction, we assume that this matter is the hadron gas, which consists of all hadrons up to \( \Omega^- \) baryon. The expansion proceeds according to the relativistic hydrodynamics equations and for the longitudinal component we have the following solution (for details see e.g. [1,4])

\[ s(\tau) = \frac{s_0\tau_0}{\tau}, \quad n_B(\tau) = \frac{n_0\tau_0}{\tau}, \quad (3) \]

where \( s_0 \) and \( n_0 \) are initial densities of the entropy and the baryon number respectively. The transverse expansion has the form of the rarefaction wave moving radially inward with a sound velocity \( c_s \) [1,2].

To obtain the time dependence of temperature and baryon number and strangeness chemical potentials one has to solve numerically equations \((2a) - (2e)\) with \( s, n_B \) and \( n_S \) given as time dependent quantities. For \( s(\tau), n_B(\tau) \) we have expressions \((3)\) and \( n_S = 0 \) since we put the overall strangeness equal to zero during all the evolution (for more details see [3]).

### III. The Sound Velocity in the Multi-Component Hadron Gas

In the hadron gas the sound velocity squared is given by the standard expression

\[ c_s^2 = \frac{\partial P}{\partial \epsilon} \quad (4) \]

Since the experimental data for heavy-ion collisions suggests that the baryon number density is non-zero in the CRR at AGS and SPS energies [10,12], we calculate the above derivative for various values of \( n_B \).

To estimate initial baryon number density \( n_B^0 \) we can use experimental results for S-S [10] or Au-Au [11,12] collisions. In the first approximation we can assume that the baryon multiplicity per unit rapidity in the CRR is proportional to the number of participating nucleons. For a sulphur-sulphur collision we have \( dN_B/dy \approx 6 \) [10] and 64 participating nucleons. For the central collision of lead nuclei we can estimate the number of participating nucleons at \( 2A = 416 \), so we have \( dN_B/dy \approx 6 \). Having taken the initial volume in the CRR equal to \( \pi R^3 \), 1 fm, we arrive at \( n_B^0 \approx 0.25 \text{ fm}^{-3} \). This is some underestimation because the S-S collisions were at a beam energy of 200 GeV/nucleon, but Pb-Pb at 158 GeV/nucleon. From the Au-Au data extrapolation one can estimate \( n_B^0 \approx 0.65 \text{ fm}^{-3} \) [1]. These values are for central collisions. So, we estimate \((1)\) for \( n_B = 0.25, 0.65 \text{ fm}^{-3} \) and additionally, to investigate the dependence on \( n_B \) much carefully, for \( n_B = 0.05 \text{ fm}^{-3} \). The results of numerical evaluations of \((1)\) are presented in Fig. [1]. For comparison, we drew also two additional curves: for \( n_B = 0 \) and for a pure massive pion gas. These curves are taken from [3].
FIG. 1. Dependence of the sound velocity squared on temperature for various values of $n_B$: $n_B = 0.65$ fm$^{-3}$ (short-dashed), $n_B = 0.25$ fm$^{-3}$ (dashed), $n_B = 0.05$ fm$^{-3}$ (solid) and $n_B = 0$ (long-dashed). The case of the pure pion gas (long-long-dashed) is also presented.

The “physical region” lies between more or less 100 and 200 MeV on the temperature axis. This is because the critical temperature for the possible QGP-hadronic matter transition is of the order of 200 MeV \cite{13} and the freeze-out temperature should not be lower than 100 MeV \cite{11}. In low temperatures we can see completely different behaviours of cases with $n_B = 0$ and $n_B \neq 0$. We think that this is caused by the fact that for $n_B \neq 0$ the gas density can not reach zero when $T \to 0$, whereas for $n_B = 0$ it can. For the higher temperatures all curves excluding the pion case behave in the same way qualitatively. From $T \approx 70$ MeV they decrease to their minima (for $n_B = 0.65$ fm$^{-3}$ at $T \approx 221.3$ MeV, for $n_B = 0.25$ fm$^{-3}$ at $T \approx 201.5$ MeV, for $n_B = 0.05$ fm$^{-3}$ at $T \approx 176.6$ MeV and for $n_B = 0$ at $T \approx 160$ MeV) and then they increase to cover each other above $T \approx 250$ MeV. To investigate this problem we check the content of the hadron gas when the temperature changes.

FIG. 2. Fraction of mesons in the multi-component hadron gas as a function of temperature for $n_B = 0.65$ fm$^{-3}$ (short-dashed), $n_B = 0.25$ fm$^{-3}$ (dashed), $n_B = 0.05$ fm$^{-3}$ (solid) and $n_B = 0$ (long-dashed).

FIG. 3. Fraction of pions in the multi-component hadron gas as a function of temperature for $n_B = 0.65$ fm$^{-3}$ (short-dashed), $n_B = 0.25$ fm$^{-3}$ (dashed), $n_B = 0.05$ fm$^{-3}$ (solid) and $n_B = 0$ (long-dashed).
We define the fraction of some kind of particles in the hadron gas as

\[ f_p = \frac{\rho_p}{\rho}, \]  

where \( \rho_p \) is the density of these particles and \( \rho \) the density of the hadron gas. We calculate the above-mentioned fractions of: a) mesons; b) pions; c) \( \eta \), \( \rho \) and \( \omega \) mesons and d) strange particles. The results are presented in Figs. 2-5.

Coming back to the behaviour of the sound velocity of the hadron gas above \( T \approx 70 \text{ MeV} \) (see Fig. 1), we can state that the appearance of heavier particles, especially \( \eta \), \( \rho \) and \( \omega \) mesons, is responsible for it. First of all, the curve for the pure pion gas in Fig. 1 is increasing everywhere, so we have to take into account also other species to obtain the sound velocity decreasing in some range of temperature above \( T \approx 70 \text{ MeV} \). If we assume that baryons cause the mentioned behaviour of the sound velocity, it would not agree with the case of \( n_B = 0 \), because the baryon fraction is increasing with the temperature everywhere for this case, see Fig. 2 up-side-down. The same would happen for strange particles (with the only difference that the strange particle fraction is increasing with the temperature for all cases of \( n_B \), see Fig. 5). Only the fraction of \( \eta \), \( \rho \) and \( \omega \) mesons changes its monotonicity above \( T \approx 70 \text{ MeV} \) and at the temperatures similar to those where the sound velocity curves have their local minima (cf. Fig. 1 and Fig. 4), for all cases of \( n_B \).

IV. THE PATTERN OF COOLING AND ITS CONNECTION WITH THE SOUND VELOCITY

In Sect. II we have explained how to obtain the time dependence of the temperature of the longitudinally expanding hadron gas. This dependence proved to be very well approximated by the expression

\[ T(\tau) \approx T_0 \cdot \tau^{-a}. \]  

The above approximation is valid in the temperature ranges \( [T_{f.o., T_0}, T_0 \leq T_{0,\text{max}} \approx 225 \text{ MeV} \). We started from \( T_0 \) equal to 221.8 MeV (for \( n_B^0 = 0.65 \text{ fm}^{-3} \)), 226 MeV (for \( n_B^0 = 0.25 \text{ fm}^{-3} \)) and 226.7 MeV (for \( n_B^0 = 0.05 \text{ fm}^{-3} \)). These values correspond to \( \epsilon_0 = 3.7 \text{ GeV}/\text{fm}^3 \), which is the initial energy density in the CRR slightly above the value \( \epsilon_0 = 3.5 \text{ GeV}/\text{fm}^3 \) estimated by NA50. Then we took several decreasing values of \( T_0 < T_{0,\text{max}} \). For every \( T_0 \) chosen we repeat the procedure of obtaining the approximation (5), i.e. the power \( a \). The values of \( a \) as a function of \( T_0 \) are depicted in Fig. 6 for the case of \( n_B = 0.25 \text{ fm}^{-3} \), together with the corresponding sound velocity curve.
We can see that in the above-mentioned interval of temperature the power $a$ has the meaning of the sound velocity within quite well accuracy. We would like to add that we arrived at the same conclusion also for cases with $n_B = 0.65 \, fm^{-3}$ and $n_B = 0.05 \, fm^{-3}$. Note that the higher temperature, the worse accuracy. This results from the fact that for a higher temperature the wider interval is used to obtain the formula (6). Therefore the approximation is worse.

We can formulate the following conclusion: in the "physical region" of temperature and for realistic baryon number densities, the longitudinal expansion given by (3) results in the cooling of the hadron gas described by (6) with $a = c_2^p(T_0)$. Note that $T(\tau) = T_0 \cdot \left( \frac{\tau}{\tau_0} \right)^{c_2^p(T_0)}$ is the exact expression for a baryonless gas with the sound velocity constant (for details see [2,3]).

V. CONCLUSIONS

In this paper we have calculated the sound velocity in the multi-component hadron gas with non-zero baryon number density. Such a gas, instead of the QGP, could have appeared in the CRR of heavy-ion collisions at AGS and SPS energies. We have compared the results with the sound velocities in a gas with $n_B = 0$ and in a pion gas. We have shown that in the "physical region" (temperatures between roughly 100 and 200 MeV) the sound velocities for various cases of the multi-component hadron gas behave qualitatively the same: firstly they decrease and then they start to increase with the temperature. Comparison with the case of the pion gas and analysis of fractions of particles in the multi-component hadron gas suggest that the appearance of $\eta$, $\rho$, and $\omega$ mesons is responsible for that.

The second result shown here is that for the multi-component hadron gas the cooling imposed by the longitudinal expansion can be described by the approximation

$$T(\tau) \approx T_0 \cdot \left( \frac{\tau}{\tau_0} \right)^{c_2^p(T_0)}$$

where $T_0$ belongs to "physical region". The approximation is valid for the gas with the non-zero baryon number density. The same conclusion was drawn for the case with $n_B = 0$ in [4].

The values of the sound velocity obtained in the "physical region" and the validity of approximation (6) therein, cause that the cooling of the hadron gas is much slower than the cooling of an ideal massless gas with $n_B = 0$ (where $c_2^p = \frac{1}{3}$). This fact has the straightforward consequence for $J/\Psi$ suppression: the longer hadrons last as a gas, the deeper suppression takes place (for details see [4]).

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