Redshift-space distortions from the cross-correlation of photometric populations

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ABSTRACT

Several papers have recently highlighted the possibility of measuring redshift space distortions from angular auto-correlations of galaxies in photometric redshift bins. In this work we extend this idea to include as observables the cross-correlations between redshift bins, as an additional way of measuring radial information. We show that this extra information allows to reduce the recovered error in the growth rate index γ by a factor of ~ 2. Although the final error in γ depends on the bias and the mean photometric accuracy of the galaxy sample, the improvement from adding cross-correlations is robust in different settings. Another factor of 2 – 3 improvement in the determination of γ can be achieved by considering two galaxy populations over the same photometric sky area but with different biases. This additional gain is shown to be much larger than the one from the same populations when observed over different areas of the sky (with twice the combined area). The total improvement of ~ 5 implies that a photometric survey such as DES should be able to recover γ at the 5 – 10% from the angular clustering in linear scales of two different tracers. It can also constrain the evolution of f(z) × σ8(z) in few bins beyond z ~ 0.8 – 0.9 at the 10 – 15% level per-bin, compatible with recent constrains from lower-z spectroscopic surveys. We also show how further improvement can be achieved by reducing the photometric redshift error.

Key words: galaxy clustering; redshift-space distortions; angular correlations; photometric redshift surveys

1 INTRODUCTION

Our understanding of the local Universe and the way it evolved from small perturbations has been reshaped over the past decades with the successful completion of vast observational campaigns for CMB fluctuations, large scale structure and SNIa distances. Yet several still open issues arose from these studies, the most important of which is probably the late-time accelerated expansion of the Universe.

Hence many other cosmic surveys are ongoing or planned for the near future to address these questions with a set of precision measurements never achieved before. Several photometric surveys stand out among these, such as the Dark Energy Survey (DES), the Panoramic Survey Telescope and Rapid Response System (PanStarrs), the Physics of the Accelerating Universe survey (PAU), the future Large Synoptic Survey Telescope or the imaging component of the ESA/Euclid satellite.

Redshift space distortions (RSD) (Kaiser 1987; Hamilton 1998) can be used to understand the (linear) growth of structures, which provides a direct path to study the origin of cosmic acceleration. On large scales, RSD arises from the coherent velocities of galaxies and reveals how perturbations grow in time. Typically this method requires measuring of galaxy clustering in 3 dimensions (3D) in order to sample di-
rections parallel and transverse to the line-of-sight where the effect is maximized or cancels out completely (see e.g. Okumura 2008; Guzzo et al. 2009; Cabré & Gaztaña 2009; Blake et al. 2011; Reid et al. 2012; Kazin et al. 2013 and references therein).

Over the past few years it has been however shown that the effect of RSD is also present, albeit with a smaller contribution, in the angular (2D) clustering of photometric galaxy samples if they are selected in photometric redshift bins, see for instance Nock, Percival, & Ross 2010; Crocce, Cabré, & Gaztaña 2011; Ross et al. 2011. This concrete idea has been already applied to data using a sample of photometric Luminous Red Galaxy (LRG, see Padmanabhan et al. 2007; Thomas, Abdalla & Lahav 2010; Crocce et al. 2011).

Yet all the previous studies focused on the angular clustering from a set of measurements of auto-correlation in one or more redshift bins. The goal of this paper is, on the one hand, to extend these analysis to include also the cross-correlations between redshift bins in order to account for some radial information. This is motivated by the recent findings of Asorey et al. 2012 who show how a tomographic (2D) study involving auto and cross correlations can yield the same constraints on cosmological parameters as a full spatial (3D) study.

On the other hand we will also investigate the improvements brought by considering two different populations (and their cross correlations) in the likelihood analysis for the growth rate. This is motivated by the fact that for the spectroscopic analysis, the combination of different samples tracing the same underlying matter fluctuations can be used to decrease sampling variance and improve considerably the constrains in growth of structure (McDonald & Seljak 2009; White, Song & Percival 2009; Gil-Marín et al. 2010).

This paper is organized as follows. In Sec. 2 we lay out the methodology, including the analytical tools, the definition of the different samples and surveys and the likelihood analysis. In Sec. 3 we present our result, and in Sec. 4 our conclusions.

## 2 METHODOLOGY

Our goal is to study the effect of RSD in angular clustering, especially it usefulness to derive constrains on the growth of structure at large scales. We study angular clustering using auto- and cross- correlations between redshift bins. The inclusion of cross correlations between different radial shells allow us to include the radial modes that account for scales comparable to the bin separation. On the other hand, the angular spectra of each redshift shell includes information mainly from transverse modes.

With the idea of a potential sample variance mitigation in the analysis, we also consider the correlation between the angular clustering of different tracers of matter, considering them either independent (i.e. each tracer in a different patch of the sky) or correlated (same sky).

Throughout this paper we use CAMB sources\(^6\) (Lewis et al. 2000; Lewis et al. 2007; Challinor & Lewis 2011), including cross correlations between radial bins and the correlations between different populations. Let us note that we use the exact \(C_l\) computation in CAMB sources, because in angular clustering the imprint of redshift distortions affect mainly the largest scales, which are not included when using the Limber approximation (Limber 1954; LoVerde & Afshordi 2008; Crocce, Cabré, & Gaztaña 2011). Moreover the Limber approximations does not account for clustering in adjacent redshift bins.

### 2.1 Fiducial survey and galaxy samples

We start by describing the fiducial photometric survey that we assume in our analysis (characterized by a redshift range and a survey area) and the different galaxy samples considered within that volume (characterized by the bias \(b\), the accuracy of photometric redshift estimates \(\sigma_z\) and their redshift distribution).

Our fiducial survey is similar to the full DES, with an area coverage of one octant of the sky (i.e., \(f_{sky} = 1/8\)) and a redshift range \(0.4 < z < 1.4\). We characterize the redshift distribution of galaxies within this survey by

\[
\frac{dN}{dzd\Omega} = N_{gal}^{z} \left( \frac{z}{0.5} \right)^{2} e^{-\left( \frac{z}{0.5} \right)^{1.5}}
\]

where \(N_{gal}^{z}\) is a normalization related to the total number of galaxies of each population sample, denoted by \(\alpha\). We typically consider two types of sample populations, one with bias \(b = 1\) and \(\sigma_z = 0.05(1 + z)\) (Pop1) and another with \(b = 2\) and \(\sigma_z = 0.03(1 + z)\) (Pop2). For simplicity we consider the same redshift distribution for all samples with a fiducial comoving number density, unless otherwise stated, of \(n(z = 0.9) = 0.023 h^3 \text{Mpc}^{-3}\). This value corresponds to a total of \(\sim 3 \times 10^8\) galaxies within the survey redshift range\(^7\).

In Table 1 we show the different redshift binning schemes in which we divide our survey prior to study the clustering either with the auto-correlations or with the 2D tomography that also includes the cross-correlations between bins. Note that we consider consecutive bins with an evolving bin width with redshift, i.e. \(\Delta z \propto (1 + z)\), to match the photometric uncertainty which also assumes a linear evolution with redshift.

| Number of bins | \(\Delta z/(1 + z)\) |
|----------------|---------------------|
| 4              | 0.15                |
| 6              | 0.1                 |
| 8              | 0.08                |
| 12             | 0.05                |
| 19             | 0.03                |

\(^{6}\) camb.info/sources

\(^{7}\) Note that this matches the nominal \(3 \times 10^8\) galaxies expected to be targeted above the magnitude limit of DES \((i < 24)\).
2.2 Angular power spectrum

In our analysis we study angular clustering using the angular power spectrum of the projected overdensities in the space of spherical harmonics. The auto-correlation power spectrum at redshift bin $i$, for a single population, is given by:

$$C^{ii}_\ell = \frac{2}{\pi} \int dk \, k^2 P_\delta(k) \left( \Psi_i^r(k) + \Psi_i^{\ast \ast}(k) \right)^2$$

(2)

where

$$\Psi_i^r(k) = \int dz \, \phi_i(z) b(z) D(z) j_\ell(kr(z))$$

(3)

is the kernel function in real space

$$\Psi_i^{\ast \ast}(k) = \int dz \, \phi_i(z) f(z) D(z) \left[ \frac{2^\ell + 2 - 1}{(2\ell + 3)(2\ell + 1)} j_\ell(kr) - \frac{\ell(\ell - 1)}{(2\ell - 1)(2\ell + 1)} j_{\ell - 1}(kr) - \frac{\ell(\ell + 1)}{(2\ell + 1)(2\ell + 3)} j_{\ell + 1}(kr) \right].$$

(4)

should be added to $\Psi_i^r$ if we also include the linear Kaiser effect (Fisher, Scharf & Lahav 1994; Padmanabhan et al. 2007). In Eqs. (3) and (4), $b(z)$ is the bias (assumed linear and deterministic), $D(z)$ is the linear growth factor and $f(z) = \partial \ln D/\partial \ln a$ is the growth rate. Photo-z effects are included through the radial selection function $\phi(z)$, see below.

For the case of 1 population, there are $N_z$ auto-correlation spectra, one per radial bin. Then, we add to our observables the $N_z(N_z - 1)/2$ cross-correlations between different redshift bins. These are given by

$$C^{ij}_\ell = \frac{2}{\pi} \int dk \, k^2 P_\delta(k) \left( \Psi_i^r(k) + \Psi_j^{\ast \ast}(k) \right) \left( \Psi_j^r(k) + \Psi_i^{\ast \ast}(k) \right)$$

(5)

Therefore, we are considering $N_z(N_z - 1)/2$ observable angular power spectra when reconstructing clustering information from tomography using $N_z$ bins, for a single tracer.

If we combine the analysis of two tracers, $\alpha$ and $\beta$, the angular power spectrum is given by

$$C^{\alpha\beta}_\ell = \frac{2}{\pi} \int dk \, k^2 P_\delta(k) \left( \Psi_\alpha^r(k) + \Psi_\beta^{\ast \ast}(k) \right) \left( \Psi_\beta^r(k) + \Psi_\alpha^{\ast \ast}(k) \right),$$

(6)

where $\Psi_\alpha^r$ and $\Psi_\beta^{\ast \ast}$ characterize each galaxy sample through the radial selection function $\phi_i(z)$ and the bias $b(z)$ in expressions (3) and (4). We use the general notation where $C^{\alpha\beta}_\ell$ is the correlation between redshift bin $i$ of population $\alpha$ with redshift bin $j$ of population $\beta$. By definition,

$$C^{\alpha\gamma}_\ell = C^{\alpha\gamma}_\ell$$

(7)

$$C^{\alpha\beta}_\ell \neq C^{\beta\alpha}_\ell \text{ for } \alpha \neq \beta; i \neq j$$

(8)

Then the total number of observables is $2N_z(2N_z + 1)/2$ if we consider the same redshift bins configuration for both populations, in the case in which both are correlated.

2.2.2 Covariance matrix of angular power spectra

The radial selection functions $\phi_i$ in Eqs. (2) encode the probability to include a galaxy in the given redshift bin. Therefore, they are the product of the corresponding galaxy redshift distribution and a window function that depends on selection characteristics (e.g binning strategy),

$$\phi_i^\alpha(z) = \frac{dN_i}{dz} W_i(z)$$

(9)

where $dN_i/dz$ is given by Eq. (4). We include the fact that we are working with photo-z by using the following window function:

$$W_i(z) = \int d\nu P(\nu-z) W_{ph}(\nu),$$

(10)

where $z_p$ is the photometric redshift and $P(\nu-z)$ is the probability of the true redshift to be $\nu$ if the photometric estimate is $z_p$. For our work we assume a top-hat selection $W_{ph}(\nu)$ in photometric redshift and that $P(\nu-z)$ is Gaussian with standard deviation $\sigma_z$. This leads to,

$$\phi_i^\alpha(z) \propto \frac{dN_i}{dz} \left( \text{erf} \left( \frac{z_p,\text{max} - z}{\sqrt{2}\sigma_z} \right) - \text{erf} \left( \frac{z_p,\text{min} - z}{\sqrt{2}\sigma_z} \right) \right)$$

(11)

where $z_p,\text{min}$ and $z_p,\text{max}$ are the (photometric) limits of each redshift bin considered and $\phi_i^\alpha$ is the photometric redshift error of the given population $\alpha$ at the corresponding redshift.

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2.3 Cosmological model and growth history

Throughout the analyses, we assume the underlying cosmological model to be a flat ΛCDM with cosmological parameters $w = -1$, $h = 0.7$, $n_s = 0.95$, $\Omega_m = 0.25$, $\Omega_b = 0.045$ and $\sigma_8 = 0.8$. These parameters specify the cosmic history as well as the linear spectrum of fluctuations $P_0$. In turn, the growth rate can be well approximated by,

$$f(z) \equiv \Omega_m(z)^\gamma$$  \hspace{1cm} (15)

and $\gamma = 0.545$ for ΛCDM. Consistently with this we obtain the growth history as

$$D(z) \equiv \exp \left[ - \int_0^z \frac{f(z)}{1+z} dz \right]$$  \hspace{1cm} (16)

(where $D$ is normalized to unity today). The parameter $\gamma$ is usually employed as an effective way of characterizing modified gravity models that share the same cosmic history as GR but different growth history (Linder 2005). Our fiducial model assumes the GR value $\gamma = 0.545$. In order to forecast the constrains on $\gamma$ we consider it as a free parameter independent of redshift.

With these ingredients, we do a mock likelihood sampling in which we assume that the theoretical values for the correlations at the fiducial value of the parameters corresponds to the best fit position. The likelihood is based on the $\chi^2$ given in (14). In our case, we keep fixed all the parameters and only allow $\gamma$ to vary, and then we estimate 68% confidence limits of it. In the case in which we show constrains on $f\sigma_8$, we vary this quantity (that now depends on redshift, thus the number of fitting parameters is a function of the bin configuration), fixing the rest of parameters. The maximum $\ell$ considered in the analysis is $\ell_{\text{max}} = r(\bar{z}_{\text{Survey}})k_{\text{max}} \sim 220$ for $k_{\text{max}} = 0.1 \ h \text{Mpc}^{-1}$, while for the largest scales we set $\ell_{\text{min}} = 2$. We had to adapt CAMB sources in order to constrain $\gamma$ or $f\sigma_8$ using the same technique described in the Appendix A of Asorey et al. 2012.

3 RESULTS

In this section we discuss the constrains on the growth index, $\gamma$ defined in Eq. (15) as obtained for the different redshift bin configurations of Table 1. First of all, we study how well we can determine $\gamma$ using different single galaxy populations but including as observables also the cross correlation between bins (for a given single population). We also study how the constrains depend on the bias and in the photometric redshift accuracy of the different galaxy samples. Then, we study the precision achievable when one combines different tracers in the analysis and how this depends on bias, photo-z and in particular, the shot-noise level of the sample.

Lastly we discuss the constrains that we obtain when looking into the more standard $f(z)\sigma_8(z)$ as a function of redshift, and consider auto and cross-correlations of one or two galaxy samples.

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Redshift-space distortions from photometric populations

3.1 Redshift-space distortions with a single photometric population

Let us first consider the constraints on the growth index using single photometric populations. Figure 2 shows the 1-σ errors expected on \( \gamma \) from a combined analysis of all the consecutive photometric redshift bins in the redshift range 0.4 < \( z < 1.4 \) as a function of the bin width (i.e. each of the configurations detailed in Table 1).

In red we show the constraints on \( \gamma \) corresponding to an LRG-type sample, with bias \( b = 2 \) and a photometric redshift \( \sigma_z/(1 + z) = 0.03 \) (Pop2). Blue lines correspond to an unbiased population with \( \sigma_z/(1 + z) = 0.05 \) (Pop1).

Dashed lines correspond to the case in which we only use the auto-correlations in each redshift bin while solid lines correspond to the full 2D analysis that includes all the cross-correlations in our vector of observables. Recall than in the first case the cross-correlations are included in the covariance matrix of the auto-correlations (but not as observables). We see that constrains from a full 2D analysis, including auto and cross-correlations, are a factor \( \sim 2 \) or more better than those from using only auto-correlations.

From Fig 1 it is clear that in all cases the bin configuration can be optimized, with the best results obtained when \( \Delta z \sim \sigma_z \). In addition, there is a competing effect between \( \sigma_z \) and bias. For broad bins (\( \Delta z \gg \sigma_z \)) the photo-z of the populations is masked in the projection and the bias dominates the \( \gamma \) constrains. Smaller bias gives more relevance to RSD and better \( \gamma \) constrains. As one decreases the bin width the population with better photo-z (typically the brighter, with higher bias), denoted Pop2, allows a more detailed account of radial modes improving the derived errors on \( \gamma \) more rapidly than Pop1 until they become slightly better. This optimization is possible until one eventually reaches bin sizes comparable to the corresponding photo-z (what sets an “effective” width) and the constrains flatten out.

In Fig. 2 we study in more detail the dependence of constrains with respect to galaxy bias \( b \) and photo-z accuracy. In the top panel of Fig. 2 we show standard deviation of the growth index, \( \Delta \gamma \), fixing the sample bias to \( b = 1 \) and allowing two values for photo-z accuracy. Red line represents a sample in which \( \sigma_z/(1 + z) = 0.05 \) while blue line has an error of \( \sigma_z/(1 + z) = 0.03 \). In both cases the constraint flattens once \( \Delta z \sim \sigma_z \) and the optimal error improves roughly linearly with \( \sigma_z \). The dependence on the linear galaxy bias, \( b \), is shown in the bottom panel of Fig. 2 (for fixed \( \sigma_z \)). We see that the constrains degrade almost linearly with increasing bias (see also Ross et al. 2011). As discussed before, this is because the lower the bias the larger the relative impact of RSD, which results in better constraints on \( \gamma \).

In summary we have shown that using the whole 2D tomography (auto+cross correlations) allows considerable more precise measurements of \( \gamma \), a factor of 2 or better once the bin width is optimal for the given sample. Hence in what follows we concentrate in full tomographic analysis.

3.2 Redshift-space distortions with 2 photometric populations

We now turn to an analysis combining two galaxy populations as two different tracers of matter. In Figure 3 we compare the constrains from single tracers with respect to the combination of both. As before the populations used in the comparison correspond to \( b = 1 \) and \( \sigma_z/(1 + z) = 0.05 \) (Pop1) and a population with \( b = 2 \) and \( \sigma_z/(1 + z) = 0.03 \) (Pop2). Their respective constrains in \( \gamma \) are the dashed red and blue lines (same as solid lines in Fig. 1).

If we combine both tracers and their cross-correlation in the same analysis we obtain the constrains given by black solid line, notably a factor of 2 – 3 better compared to the optimal single population configuration.

In order to understand how much of this gain is due to “sample variance cancellation”, in analogy to the idea put forward in McDonald & Seljak 2009, we also considered combining the two samples assuming they are located in different parts of the sky (and hence un-correlated). We call this case Pop1+Pop2 in Fig. 3 (solid green line). In such scenario the total volume sampled is the sum of the volumes.
Figure 3. The gain from combining galaxy populations: Comparison of the 68% standard deviations in the growth index from single population analysis (dashed lines) with respect to the combined analysis of these two populations over the same field (black solid), using all the angular auto and cross-correlations. Remarkably the combination yields errors at least 2 times better than any of the single population cases. The solid green line corresponds to the combination of the two samples assuming they are independent (i.e. from different parts of the sky). As shown, the combination of correlated populations (same sky) yield stronger constrains than any other case.

sampled by each population (in our case, two times the full volume of DES). This explains the gain with respect to the single population analysis. Nonetheless, the “same sky” case Pop1 × Pop2 (where cosmic variance is sampled twice) still yields better constrains, a factor of ~1.5 – 2, even though the area has not increased w.r.t. Pop1 or Pop2 alone.

In all, the total gain of a full 2D study with two populations (including all auto and cross correlations in the range 0.4 < z < 1.4) w.r.t. the more standard analysis with a single population and only the auto-correlations in redshift bins (dashed lines of Fig. 1) can reach a factor of ~ 5.

As a next step we show how the combined analysis of two tracers depends on the relative difference on the bias and photo-z errors of the populations. In Fig. 4 we keep Pop1 fix (with b = 1 and $\sigma_z/(1 + z) = 0.05$) and we vary the bias of Pop2 from $b = 2$ (LRG type bias) to $b = 3$ (galaxy clustering like). We keep $\sigma_z/(1 + z) = 0.03$ fixed for Pop2. As expected, increasing the bias difference between the samples improves the constrains on $\gamma$ in a roughly linear way.

If we now have an unbiased tracer and a highly biased one with $b = 3$, while both tracers have the same $\sigma_z/(1 + z) = 0.03$ we obtain constrains given by the black line in Fig. 5. Those constrains are better than the case in which the unbiased galaxies photo-z is worse, $\sigma_z/(1 + z) = 0.05$ (given by the dashed blue line). Therefore, if we determine photometric redshifts of the unbiased galaxies with higher accuracy we will be able to measure the growth rate with higher precision.

One caveat so far is that we have always assumed that biases are perfectly known (bias fixed). Hence, in the top panel of Fig. 6 we show how the constrains on $\gamma$ change if we instead consider them as free parameters and marginalize over. We see that the difference is very small, in particular once the bin configuration is optimal. The reason for this is
clear from the bottom panel that shows the relative error obtained for the bias of each sample in the bias free case. Because the bias is so well determined (sub-percent) the marginalization over them does not impact the error on $\gamma$.

One further concern in our results is that we have assumed a perfect knowledge of the galaxy redshift distributions for both samples. In a more realistic scenario the distribution of photometric errors will be known with some uncertainty. In order to study the impact of this potential unknown we repeated the Pop1 × Pop2 case (for a bin configuration $\Delta (1+z) = 0.1$) this time marginalizing over the value of $\sigma_{1z}$ and $\sigma_{2z}$ when determining the constrain in $\gamma$ (instead of fixing their values to $0.05(1+z)$ and $0.03(1+z)$). We find that the resulting $\Delta \gamma$ only increases by $\sim 10\%$ or less with respect to the error shown by the black solid line in Fig. 5. Such a small change is because the values of $\sigma_{1z}$ and $\sigma_{2z}$ are very well constrained after the marginalization, similar to what happens with the bias free case above.

An even more thoughtfull analysis of this issue would allow for independent errors in the redshift distributions at each redshift bin (with some priors) rather than a global change of the mean photometric error. And possibly also a marginalization over cosmological parameter space. However our results above indicate that this should not have a major impact in our conclusions. Hence we leave this study for future work.

### 3.2.1 The impact of shot-noise

One strong limitation when it comes to implementing the “multiple tracers” technique in real spectroscopic data is the need to have all the galaxy samples well above the shot-noise limit (at the same time as having the largest possible bias difference), see for instance Gil-Marín et al. 2010. This is cumbersome because spectroscopic data is typically sampled at a rate only slightly above the shot-noise (to maximize the area) and for pre-determined galaxy samples (e.g LRGs, CMASS). In a photometric survey these aspects change radically because there is no pre-selection (beyond some flux limits) and the number of sampled galaxies is typically very large (at the expense of course of poor redshift resolution). Therefore is interesting to investigate if the overall density of the samples have any impact in our results.

Figure 7 shows the constrain in $\gamma$ for the combination of two samples, one unbiased population with $\sigma_{1z}/(1+z) = 0.05$ and a population with $b = 2$ and $\sigma_{2z}/(1+z) = 0.03$. We keep the number density for the unbiased population as $n(z = 0.9) = 1.8 \times 10^{-2} h^3$Mpc$^{-3}$ while we vary the number density of the second (typically brighter) sample. The solid black line corresponds to the case in which both populations are correlated (same sky) and the dashed blue line to different areas. In both scenarios we see that decreasing the number

9 Note that we assume the same shape for $N(z)$ as given in Eq. 1 but we vary the overall normalization, which we characterize by the comoving number density at $z = 0.9$. 

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density of the second population does not impact the error on $\gamma$ unless one degrades it by an order of magnitude or more compared to the one of Pop1 ($n_{2} = 0.023 h^{3} \text{Mpc}^{-3}$).

3.3 Constraining the redshift evolution of the Growth Rate of Structure

So far we have used the combined analysis of all the redshift bins to constrain one global parameter, namely the growth rate index $\gamma$ in Eq. (15). We now turn into constraining $f(z)\sigma_{8}(z)$ itself, as a function of redshift. We use a redshift bin configuration given by $\Delta z/(1+z) = 0.1$, in the photometric range $0.4 < z < 1.4$. This configuration consist of 6 bins, and hence we fit $f(z)\sigma_{8}(z)$ evaluated at the mean of these bins. These $f\sigma_{8}$ values are of course correlated, and we include the proper covariance among the measurements (i.e. we do a global fit to the 6 values simultaneously).

In the left panel of Fig. 7 we focus on the gain from adding cross-correlations among the bins, and show the constrain on $f\sigma_{8}$ for a single unbiased population with photometric redshift of $\sigma_{8} = 0.05$ (Pop 1, in blue) and also for a single tracer with bias $b = 2$ and $\sigma_{8} = 0.03$ (Pop 2, in red). Dashed lines corresponds to using only auto-correlations and solid to including also all the redshift bins cross-correlations to the observables. The trend for the errors when we only use auto-correlations are similar to the ones observed in Fig. 8 of Ross et al. 2011. Although in detail we are using different widths for our redshift bins, and we use $C_{l}$ while they used angular correlation functions, $w(\theta)$.

As in Sec. 3.1 there is a gain from the addition of cross-correlations, which is now split across the bins (i.e. 20–30%) for Pop1 in each of the 6 bins, and a bit less for Pop2.

In turn, the right panel of Fig. 7 focuses on the gain from combining the two tracers (and using both auto and cross correlations among redshift bins, as in Sec. 3.2). Here the solid lines correspond to the single population cases discussed above, while the black short-dashed line to the combined analysis assuming these populations are correlated (same sky). For completeness the dashed green line is the result when these two samples are assumed independent. Again, there is a factor of ~2.5 to be gained by combining galaxy samples as opposed to only the unbiased sample.

If we compare our predictions to measurements from spectroscopic surveys like VIPERS (de la Torre et al. 2013) with constrains $f\sigma_{8}(z = 0.8) = 0.47 \pm 0.08$ or WiggleZ (Blake et al. 2011) where $f\sigma_{8}(z = 0.76) = 0.38 \pm 0.04$ we find that DES can achieve the same level of errors ($\sim 15\%$) in determining the growth of structure but extending the constrains beyond redshift of unity. This is quite unique and interesting as there is, to our knowledge, no other spectroscopic survey expected to provide such measurements in the medium term future (before ESA/Euclid or BigBOSS).

As we did for $\gamma$, we checked that when we marginalize over photometric errors we find differences smaller than 1% in the recovered constrains in $f(z)\sigma_{8}(z)$ with respect to the case in which we assume perfect knowledge of the redshift distributions. For concreteness we did this cross-check for the case Pop1 x Pop2 in last two redshift bins shown in Fig. 8.

3.4 The case of high-photometric accuracy

In the previous sections we have focused in galaxy surveys with broad-band photometry for which the typical photometric error achieved is of the order $0.1$ depending on galaxy sample and redshift (we assumed $0.03–0.05 (1+z)$). We now turn to narrow-band photometric surveys such as the ongoing PAU or J-PAS Surveys (Benitez et al. 2009; Gaztaña et al. 2012; Taylor et al. 2013) for which the typical radial accuracy is a factor of 10 times better: $\approx 0.003(1+z)$ (or $10 h^{-1}$ Mpc). This scenario then resembles quite closely a purely spectroscopic survey (Asorey et al. 2012).

We again study two populations, one corresponding to

| Population | b | $\sigma_{8}/(1+z)$ | Auto | Auto + Cross |
|------------|---|-------------------|------|-------------|
| Broad Band (BB) | Pop1 | 1 | 0.05 | 0.869 | 0.564 |
| | Pop2 | 2 | 0.03 | 0.826 | 0.447 |
| | Pop1 x Pop2 | - | - | - | 0.35 |
| | Pop1 + Pop2 | - | - | - | 0.36 |
| Narrow Band (NB) | Pop1 | 1 | 0.003 | 0.047 | 0.027 |
| | Pop2 | 2 | 0.003 | 0.088 | 0.040 |
| | Pop1 x Pop2 | - | - | - | 0.016 |
| | Pop1 + Pop2 | - | - | - | 0.023 |

Table 2. Error in the growth rate $\gamma$ from a combination of 21 narrow bins in the range $0.94 < z < 1.06$. The 4 top entries correspond to a Survey with Broad-Band (BB) filters: Pop1-BB assumes $b = 1$ and $\sigma_{8}/(1+z) = 0.05$ (“main sample”) while Pop2-BB has $b = 2$ and $\sigma_{8}/(1+z) = 0.03$ (“LRG sample”). The 4 bottom entries correspond to a Survey with Narrow-Band (NB) filters. Here Pop1 and Pop2 have the same bias as the BB case but much precise photo-z, both with $\sigma_{8}/(1+z) = 0.003$.
the main sample with bias $b = 1$ and another to the LRG sample with $b = 2$, both with a very good photometric accuracy of $\sigma_z/(1 + z) = 0.003$. We consider a set of 21 narrow redshift bins of width $\Delta z = 0.003(1 + z)$ concentrated in $0.94 < z < 1.06$ (hence we are only looking at a portion of the survey redshift range).

The error on $\gamma$ are given in Table 2 for both the new narrow-band and the broad-band samples discussed previously. For a single population, this table shows that a factor of $\sim 10$ better $\sigma_z$ yields a factor of $\sim 10$ gain in constraining power. The improve in $\gamma$ seems to increase linear with the improvement in $\sigma_z$.

After combining the two populations we see that the errors in $\gamma$ for the broad-band case is similar if samples cover the same region of sky (Pop1 x Pop2) or different regions (Pop1 + Pop2). This is because the redshift range considered ($0.94 < z < 1.06$) is too narrow compared to $\sigma_z$ and the cosmic variance cancelation can not take place. Instead, for the narrow band surveys we find a 43\% improvement for the case Pop1 x Pop2 with respect to Pop1 + Pop2. For the same sky case, the final error is $\Delta \gamma \approx 0.0163 \times (5000 \text{deg}^2/\text{Area})^{1/2}$, in such a way that even a moderate survey of 250 deg$^2$ could achieve $\Delta \gamma \approx 0.7$. In that same narrow redshift range, DES yields an error 5 times worse with 20 times better area (but note that in the case of small areas we could be limited by the $\ell_{\text{min}}$, the largest scales available).

4 CONCLUSIONS

We have studied how measurement of redshift-space distortions (RSD) in wide field photometric surveys produce constrains on the growth of structure, in the linear regime. We focused in survey specifications similar to those of the ongoing DES or PanSTARRS, that is, covering about 1/8 of sky up to $z \sim 1.4$, and targeting galaxy samples with photometric redshift accuracies of 0.03 - 0.05(1 + z) (and hundred of million galaxies prior to sample selection). We also show results for ongoing photometric surveys, such as PAU and J-PAS, that have a much better photometric accuracy.

First, we have found that for a single population we can reduce the errors in half by including all the cross-correlations between radial shells in the analysis. This is because one includes large scale radial information that was missed when only considering the auto-correlations of each bin. The final constraining power depends on the details of the population under consideration, in particular the bias and the photometric accuracy. Less bias gives more relative importance to RSD in the clustering amplitudes. In turn, better photo-z allows for narrower binning in the analysis and more radial information. We find that the $\gamma$ constrains depend roughly linearly in both bias or $\sigma_z$. This means that for 10 times better photo-z errors, such as in PAU, we can improve by 10 the cosmological constrains.

Typically less bias implies a fainter sample, with worse photo-z, therefore these quantities compete in determining the optimal sample. Furthermore we find that opti-
nal constrains are achieved for bin configurations such that $\Delta z \sim \sigma_z$. Although the optimal errors depend on the details of the galaxy sample and binning strategy, the gains from adding cross-correlations are very robust in front of these variations.

In order to avoid sample variance, we have also considered what happens if we combine the measurement of RSD using two different tracers. This is motivated by the idea put forward in McDonald & Seljak 2009 for the case of spectroscopic (hence 3D) redshift surveys, where the over-sampling of (radial + transverse) modes allows a much better precision in growth rate constrains, as long as samples are in the low shot-noise limit. Combining auto and cross angular correlations in redshift bins, we find that if we assume that both tracers are independent, which corresponds to samples from different regions on the sky, the constrains on the growth of structure parameters improve a 30-50% (due to the fact that one has doubled the area). Remarkably if we consider that the populations are not independent, i.e., they trace the same field region, we find an overall improvement of $\sim 2 - 3$ with respect to single populations when constraining $\gamma$. This means that there is a large potential gain when sampling the same modes more than once.

Translating into actual constrains this implies that a DES-like photometric survey should be able to measure the growth rate of structure $\gamma$ to an accuracy of $5 - 10\%$ from the combination of two populations and all the auto-i-cross correlations in the range $0.4 < z < 1.4$ (see Fig. 1). Even though these values correspond to a survey of 5000deg$^2$ ($f_{sky} = 0.125$) they should scale as $f_{sky}^{-1/2}$ for a different area, given our assumptions for the covariance in Eq. (12).

In Fig. 7 we have shown that constrains weaken once one of the populations enter a shot-noise dominated regime, as is typical of spectroscopic samples. However one needs to dilute over 10 times the number densities for a photometric survey, such as DES, for this to happen. Thus, as shown in Section 3.4, by improving on photo-z accuracy without much lost of completeness, a photometric sample can in fact outperform a diluted spectroscopic version with similar depth and area (see also Gaztañaga et al. 2012). In this paper we focused on large angular scales where the approximation of linear and deterministic bias and linear RSD should hold (see for instance Crocce, Cabré, & Gaztañaga 2011). Although we set $\ell_{max} \sim 200$, much of the constraining power in our results, given the typical size of our redshift bins, comes from larger scales, $\ell \lesssim 40$. Yet, a more realistic assessment of these aspects will need to resort to numerical simulations. We leave this for future work.

Lastly, we also investigated what constrains can be placed with this method in the evolution of the growth rate of structure, $f(z) \times \sigma_8(z)$. We found that binning two DES populations into 6 bins in the range $0.4 < z < 1.4$ yields constrains on $f(z) \times \sigma_8(z)$ of $\sim 15\%$ for each bin above $z \sim 0.6$. That case corresponded to bin widths larger than the photometric errors of the samples, which may not be optimal but yield constrains almost uncorrelated between bins ($\rho_{ij} \sim -0.05$). Figure 9 shows instead the results from a narrower binning, $\Delta z/(1 + z) = 0.05$. This leads to better constrains, $\Delta(f\sigma_8) \sim 10\%$, at the expense of more correlation between bins, $0.2 < \rho_{ij} < 0.65$.

In addition to the DES forecast (shadowed region) we over-plot in Fig. 9 current constrain from spectroscopic surveys, 2dFGRS (Percival et al. 2004), LRG’s from SDSS (Tegmark 2006 and Cabré & Gaztañaga 2009), WiggleZ either from power spectrum (Blake et al. 2011) or correlation function (Contreras et al. 2013) and the recent BOSS results (Reid et al. 2012). Note that these values are not expected to improve radically in the near future. This implies that DES will be able to add quite competitive constrains in a redshift regime unexplored otherwise with spectroscopic surveys (i.e. $z \gtrsim 1$), yielding a valuable redshift leverage for understanding the nature of dark energy and cosmic acceleration through the growth of structure.

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$$\rho_{ij} = \frac{\text{Cov}_{ij}}{\sqrt{\text{Cov}_{ii}\text{Cov}_{jj}}}$$

with $\text{Cov}_{ij} = \langle (x - \langle x \rangle)_i(x - \langle x \rangle)_j \rangle$ and where $x$ stands for $f \times \sigma_8$. 

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