Rectifying acoustic phonons

S Shirota, R Krishnan, Y Tanaka and N Nishiguchi
Department of Applied Physics, Hokkaido University, Sapporo 060-8628, Japan
E-mail: nn@eng.hokudai.ac.jp

Abstract. We propose an acoustic phonon rectifier, utilizing an array of triangular holes drilled in an elastically isotropic material. It is numerically confirmed that rectification of acoustic phonons occurs even in the frequency region with corresponding wavelength comparable to the dimensions of the scatterers. There is a threshold frequency, above which the rectification occurs. The threshold frequency depends on structural periodicity, which enables to realize a tunable rectifier.

1. Introduction
Rectification that converts an alternate flow to a direct flow is an essential function of any devices. Recent studies of geometric effects on the transport in the nano structures revealed that asymmetric scatterers such as triangular potential barriers give rise to rectification of electrons[1, 2, 3, 4], superconducting vortices[5] and thermal solitons[6]. Although the geometric effects are confirmed only for the ballistic-particle flows, we can also expect the geometric effects on rectification of acoustic-wave propagation in the very short wavelength limit. In the extreme case, the wave propagation can be illustrated in terms of ray-acoustics, and specular reflections of the rays from the surface of scatterers govern the transmission through the scatterers. Considering case (I) that rays incoming from the left region toward a matrix with equilateral-triangular (α = \(\pi/3\)) holes of periodically aligned in the \(y\)-direction as shown in Fig. 1, one half of them are scattered backward, and the rest passes between the scatterers. The resultant transmission rate becomes 0.5, where the neighboring holes are uniformly spaced by the same distance as the base of a triangular void. On the other hand, the transmission rate becomes 1 for case (II) where the rays impinge on the summits of the triangular voids from the right region, since the rays reflected from the surface are transmitted to the left region through the passes between the holes. Thus the acoustic phonons are rectified in the very short wavelength limit.

The prediction by the ray-acoustics cannot be immediately applied to the wave propagation at finite wavelength because of a decay in the geometric effects and of interference effects. The waves peculiar to the geometry of scatterers decay near the scatterers and only the azimuthally symmetric waves propagate in the asymptotic field as a cylindrical wave in a two-dimensional (2D) system or a spherical wave in a 3D one. Thus the rectification seems impossible for finite wavelength.

In the present work, we show that the rectification effect vividly survives even in the frequency region with corresponding wavelength comparable to the dimensions of the scatterers, and that there is a threshold frequency, below which rectification does not take place at all. Very interestingly, the threshold frequency stems from quantization of wavevector due to the structural
periodicity of scatterers, indicating that the rectification effects are tunable by adjusting the arrangement of the scatterers.

2. Model and Formalism
The system to be studied is an elastically isotropic material containing a one-dimensional array of isosceles-triangular holes with summit angle $\alpha$ in the $y$ direction, whose axis is in the $z$ direction as shown in Fig. 1. The distance between the neighboring triangles is the same as the base length of the triangles. The holes are left empty in order to get strong reflection of acoustic waves, irrespective of frequency.

We investigate acoustic wave propagation in the $x - y$ plane. We assume transverse acoustic (TA) waves with z-polarization whose displacement is parallel to the hole axis. The TA waves do not give rise to mode conversion when they are scattered, so that we may ignore complicated transmission characteristics caused by the mode coupling between the longitudinal and transverse waves. The wave equation of $u_z$ is given by

$$\frac{\partial^2 u_z}{\partial t^2} = c^2 \nabla^2 u_z,$$  \hspace{1cm} (1)

where $c$ is the sound velocity of TA waves. The displacement also is subject to the boundary condition at the surface of scatterers. Assuming vacuum in the holes, we employ stress free boundary condition on $u_z$, i.e. $\sigma_{za} n_a = 0$. $n_a$ is a component of the unit vector normal to the surface of a scatterer and $\sigma_{za}$ is the stress tensor defined by $\sigma_{za} = \mu \partial_a u_z$. Here $\mu$ is one of the Lamé constants.

The acoustic Poynting vector $\mathbf{J}$ in the present system is defined from the continuity of energy flow as $\mathbf{J} = -\hat{u}_z(\sigma_{xz}, \sigma_{zy}, 0)$. The total energy flow is described in terms of the Fourier components of the displacement $\hat{u}_z(\omega)$ and the stress tensor $\hat{\sigma}_{za}(\omega)$ as

$$\int_{-\infty}^{\infty} \mathbf{J} dt = -4\pi \Im \left[ \int_0^{\infty} \omega \hat{u}_z(\omega)(\hat{\sigma}_{zx}^*(\omega), \hat{\sigma}_{zy}^*(\omega), 0) d\omega \right]. \hspace{1cm} (2)$$

The total energy at $\omega$ is given by,

$$\mathbf{J}(\omega) = -4\pi \Im \left[ \omega \hat{u}_z(\omega)(\hat{\sigma}_{zx}^*(\omega), \hat{\sigma}_{zy}^*(\omega), 0) \right]. \hspace{1cm} (3)$$
Here \( \Im[A] \) means the imaginary component of \( A \). We define the transmission rate \( T(\omega) \) by the ratio of the \( x \)-component of \( \hat{J}(\omega) \) integrated over the \( y-z \) plane to that in the absence of scatterers.

\[
T(\omega) = \frac{\int \hat{J}_x(\omega) dy dz}{\int \hat{J}_x^0(\omega) dy dz} \bigg|_{x=x_D},
\]

where \( x_D \) is the detecting position which is in the right side of the scatterers for case (I) and in the left side of the scatterers for case (II).

By means of a Finite-Difference Time-Domain (FDTD) method[7], we numerically simulate the wave propagation across the scatterers.

3. Transmission Rates and Rectification

Generating a wavepacket having a Gaussian spectral distribution of central frequency \( \omega_C = \frac{5\pi c}{2a} \) with \( \Delta \omega = \frac{3\pi c}{27a} \), we evaluate the transmission rate for the wavepacket. Figure 2(a) shows the transmission rates versus frequency in case of \( \alpha = \frac{\pi}{3} \) for two different incident directions (I) and (II). The two transmission rates coincide for \( \omega a / c < \pi \), and they show distinct dependences on frequency for \( \omega a / c > \pi \); the transmission rate for (I) shows periodic changes with period \( \Delta \omega a / c = \pi \) around \( T = 0.5 \) with increasing frequency. The transmission rate for (II) also shows periodic change with the same period as (I), however, whose average magnitude and amplitude are much larger than those for (I). The obvious difference in the transmission rates above \( \omega a / c = \pi \) between (I) and (II) indicates that the rectification takes place at wavelength comparable to the dimension of the scatterers, i.e. \( \frac{\lambda}{a} > \frac{1}{2} \), using the linear dispersion relation \( \omega = kc = \frac{2\pi c}{\lambda} \). The periodic behavior of the transmission is characterized by the dips, which appear when \( \frac{\omega a}{c} = n\pi \). Hereafter \( n \) indicates an integer. The properties of the transmission rates shown in Fig. 2(a) are not unique to the equilateral-triangular holes(\( \alpha = \frac{\pi}{3} \)), but the characteristic properties of rectification are also found in case of isosceles-triangular holes. Figure 2(b) shows the transmission rates versus frequency for \( \alpha = \frac{2\pi}{9} \), where the distance between the holes are the same as the base of the isosceles triangle. We also confirmed in this case the rectification, the periodic changes in the transmission rates of period \( \pi \) and the threshold frequency. In particular, the transmission rates for (II) of \( \alpha = \frac{2\pi}{9} \) are larger than those for \( \alpha = \frac{\pi}{3} \), which shows the rectification is enhanced with decreasing \( \alpha \).

The threshold frequency for the rectification and the periodic change in the transmission
rate originate from the interference effects. Because of the periodic structure in the \( y \) direction, the scattering is subject to the Bragg diffraction. The reciprocal lattice vector \( \mathbf{G} \) given by \( \mathbf{G} = (\xi, \frac{n\pi}{a}, 0) \) forms a set of lines in the reciprocal lattice space, where \( \xi \) is an arbitrary real number. Because the scattering is elastic, the wave vector of incident wave \( \mathbf{k} \) and that of scattered wave \( \mathbf{k}' \) satisfy \( |\mathbf{k}| = |\mathbf{k}'| \) and the scattering takes place when \( \mathbf{k} = \frac{\omega}{c}(1, 0, 0) \) for (I) and \( \mathbf{k} = -\frac{\omega}{c}(1, 0, 0) \) for (II) satisfies the following diffraction condition \( |\mathbf{k}| = |\mathbf{k} + \mathbf{G}| \). In terms of the “Ewald sphere” [8](actually the Ewald circle in the \( x - y \) reciprocal space plane in the present system), the wave vectors of scattered waves are indicated by the intersections of the Ewald circle and the lines in the reciprocal lattice space. The scattered waves have the quantized \( y \)-component of wave vector \( k_y = \frac{n\pi}{a} \). It is obvious that there is no intersection except for the line with \( n = 0 \) for \( ka < \pi \). On the other hand, the scattered wave for \( ka > \pi \) may have a finite \( k_y \). In the former case, the incident waves in the \( x \) direction are scattered only forward or backward, even if the waves are scattered from the legs of triangles, resulting in the transmission rates independent of the incident-wave directions. Redirection of the incident waves for scattering takes place only in the latter case because of the finite \( k_y \). The geometry of the scatterers enhances or suppresses the redirection depending on the incident directions of the wave. Hence the rectification takes place only for \( ka > \pi \) or \( \omega > \frac{\pi}{a} \). Finally, the dips in the transmission rates are attributed to the divergence of the differential cross section, where the Ewald circle touches the reciprocal lattice vector \( \mathbf{G} \).

4. Conclusion

We proposed in this work an acoustic-phonon rectifier and numerically confirmed that the system rectifies acoustic phonons. The rectification is essentially due to the geometric effects of the asymmetric scatterers like the ballistic-particle flows, therefore, symmetric scatterers including cylinders do not give rise to such a rectification effect. Thus the rectification is owing to the geometric effects, however, which is remarkably enhanced by the Bragg diffraction owing to the periodic arrangement of scatterers. The threshold frequency for rectification originates from the periodic structure. Hence, it is possible to tune the rectifier by adjusting the arrangement of the scatterers. Although we confirmed the rectification effects on the \( z \)-polarized TA phonons, preliminary calculations exhibit the rectification in case of the LA and TA waves polarized in the \( x - y \) plane.

The findings of the present work can be applied not only to sound waves in solids or liquids but also in optical waves, leading to new devices in wave engineering.

Acknowledgments

This work is supported in part by a grant-in-aid for scientific research from the Ministry of Education, Culture, Science and Technology of Japan (Grant No. 1965106507).

References

[1] Song A M, Lorke A, Kriele A, Kotthaus J P, Wegscheider W and Bichler M 1998 Phys. Rev. Lett. 80 3831
[2] Fleischmann R and Geisel T 2002 Phys. Rev. Lett. 89 016804
[3] Linke H, Sheng W, Loefgren A, Xu H-G, Omling P and Lindelof P E 1998 Europhys. Lett. 44 343
[4] Shalom D E and Pastoriza H 2005 Phys. Rev. Lett. 94 177001
[5] Villegas J E, Savalev S, Nori F, Gonzalez E M, Anguita J V, Garcia R and Vicent J L 2003 Science 302 1188
[6] Chang C W, Okawa D, Majumdar A and Zettl A 2006 Science 314 1121
[7] Taflove A and Hagness S C 2000 Computational Electrodynamics: The Finite-Difference Time-Domain Method (Artech House, Boston MA)
[8] Cullity B D 1957 Elements of X-Ray Diffraction (Addison-Wesley)