Neutrinoless Double Beta Decay
and Physics Beyond The Standard Model

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Abstract

Neutrinoless double beta decay is a sensitive probe of new physics beyond the standard model. In this review, we begin by describing the various mechanisms for this process and the kind of new physics scenarios where these mechanisms can arise. The present experimental lower bound on the lifetime for $\beta\beta_{0\nu}$ is then used to set limits on the parameters of these new physics scenarios. We then consider the positive indications for neutrino masses present in various experiments such as those involving the solar and atmospheric neutrinos as well as the LSND result. Coupled with other astrophysical and cosmological constraints on neutrino masses and mixings, they restrict the allowed profiles for the Majorana neutrino mass matrices considerably. We then show how ongoing searches for $\beta\beta_{0\nu}$ decay can confirm or rule out the various scenarios for neutrino masses. In the last section, we present the outlook for observable $\beta\beta_{0\nu}$ amplitude in some specific grand unified theories.
Introduction

In the standard electroweak model of Glashow, Weinberg and Salam, the absence of the right-handed neutrinos and the existence of an exact accidental global $B - L$ symmetry guarantees that the neutrinos are massless to all orders in perturbation theory. Any experimental evidence for a non-zero neutrino mass therefore constitutes evidence for new physics beyond the standard model and will be major step towards a deeper understanding of nature[1]. There are at this moment many experiments under way searching directly or indirectly (e.g. via neutrino oscillations) for neutrino masses, one of the most important ones being the search for neutrinoless double beta decay if the neutrino happens to be its own antiparticle (Majorana neutrino) as is implied by many extensions of the standard model. However, Majorana mass of the neutrino is not the only way to get an observable amplitude for neutrinoless double beta decay ($\beta\beta_{0\nu}$), as will be made clear in this article. Since $\beta\beta_{0\nu}$ decay changes lepton number ($L_e$) by two units any time there is violation of electron lepton number $L_e$ in a theory, one can in principle expect this process to turn on. This therefore reflects the tremendous versatility of $\beta\beta_{0\nu}$ as a probe of all kinds of new physics beyond the standard model. Indeed we will see that already very stringent constraints on new physics scenarios such as the left-right symmetric models with the see-saw mechanism[2] and supersymmetric models with R-parity violation[3], scales of possible compositeness of leptons etc are implied by the existing experimental limits[1] on this process.

This talk is organized as follows: In part I, I discuss the basic mechanisms for neutrinoless double beta decay; in part II, I go on to discuss the kind of new physics scenarios that can be probed by $\beta\beta_{0\nu}$ decay and the kind of constraints on the parameters of the new physics scenarios implied by the already existing experimental data; In part III, I address the question of the theoretical and phenomenological outlook for $\beta\beta_{0\nu}$ decay being observable given our present information about neutrinos; in part IV, the question of observability of $\beta\beta_{0\nu}$ in some popular grand unified scenarios for neutrino masses is addressed.

Part I

Mechanisms for $\beta\beta_{0\nu}$ decay

Before starting the discussion of the various mechanisms for $\beta\beta_{0\nu}$, let us write down the basic Four-Fermi V-A interaction resposible for known weak phenomena involving only the first generation:

$$H_{wk} = \frac{G_F}{\sqrt{2}} [\bar{\nu}_e \gamma^\mu (1 - \gamma_5) d \gamma_\mu (1 - \gamma_5) \nu_e + h.c.]$$  (1)

Note that in the second order in $G_F$, the above Hamiltonian leads to the two neutrino double beta decay process which has now been observed in many nuclei [4, 5]. Eq.(1)
immediately implies that if the neutrino is its own antiparticle, then the two neutrinos from the two weak Hamiltonians in [1] can annihilate into vacuum leading to neutrinoless double beta decay. Since in the standard model, neutrino is not its own antiparticle, $\beta\beta_0\nu$ decay probes physics beyond the standard model. It could of course be that there are effective four-Fermi interactions which involve heavier fermions in scenarios beyond the standard models. If such particles are their own antiparticles, again similar arguments as above could also lead to $\beta\beta_0\nu$ decay. Examples of such particles abound in literature: right-handed neutrino, photino, gluino to mention a few popular ones. One could therefore give an arbitrary classification of the mechanisms for $\beta\beta_0\nu$ decay into two kinds: (A) one class that involves the exchange of light neutrinos; and (B) the second class that involves heavy fermions or bosons.

I.A: Light neutrino exchange:

As already mentioned, if the neutrino is considered as a Majorana particle, the process $\beta\beta_0\nu$ can arise. The four-Fermi interaction involving the neutrino however need not be purely $V - A$ type as in Eq.[1] once we entertain physics beyond the standard model. One can therefore contemplate several kinds of mechanisms involving the light neutrino exchange. Before presenting them, it is important to remark that these kind of light neutrino exchange diagrams always lead to a long range neutrino potential inside the nucleons and therefore, crudely speaking the two nucleons ”far” from each other can lead to double beta decay. This has important implications for the evaluation of the nuclear matrix element[13]

I.A.1: Helicity flip neutrino mass mechanism:

If both the four-Fermi interactions involve $V - A$ currents, the $\beta\beta_0\nu$ will be non-vanishing only if there is a flip of neutrino helicity; this can happen only if the neutrino has a mass $m_\nu[7]$. The diagram of Fig.1 can then lead to $\beta\beta_0\nu$ decay. Neglecting nuclear physics complications, one can write down the amplitude $A_{\beta\beta}$ for neutrinoless double beta decay for this case to be:

$$A_{\beta\beta}^{(m)} \simeq \frac{G_F^2}{2} \left( \frac{m_\nu}{k^2} \right)_{\text{Nucl.}} (2)$$

and

The nuclear average in general consists of a three-momentum integral which roughly converts this amplitude to: $A_{\beta\beta}^{(m)} \simeq G_F^2 m_\nu p_F$. The width for the decay can then be written as (again ignoring detailed nuclear factors):

$$\Gamma_{\beta\beta} \simeq \frac{Q^5 |A|^2}{60\pi^3} (3)$$

Here, $Q$ is the available energy for the two electrons. The factors of $\pi$ can also be easily seen from Fermi golden rule combined with the appropriate phase space factors (e.g. a factor $2\pi$ from the golden rule; $(2\pi)^{-6}$ from the phase space for two electrons and
To get a feeling for the kind of restrictions they imply on $m_\nu$ and the $\eta$ parameter, let us use the present bound on the $\Gamma_{\beta\beta}$ indicated by the present Heidelberg-Moscow $^{76}\text{Ge}$ experiment at Gran Sasso i.e. $\Gamma_{\beta\beta} \leq 3.477 \times 10^{-57}$ GeV; using for a rough estimate $Q \simeq 2$ MeV and $p_F \simeq 50$ MeV, we see very easily that one gets an upper limit of .7 eV for the neutrino mass. Of course these limits are very crude; but they do indicate the severity of the constraints on the parameters beyond the standard model from the neutrinoless double beta decay process.

A more careful treatment of the particle physics part of the calculation implies that the $m_\nu$ in Eq.2 should be replaced by $\sum_i U_{ei}^2 \zeta_i m_\nu$ where the $U_{ei}$ denotes the mixing angle of the electron neutrino with other neutrinos and $\zeta_i$ denotes the CP-phase of the $i$-th neutrino. So in principle, if different neutrinos had different CP-phases, then the effective mass that appears in $\beta\beta_{0\nu}$ amplitude could be small while keeping the individual masses bigger.

I.A.2 Helicity nonflip vector-vector mechanism:

If in addition to the usual $V-A$ type four-Fermi interaction, there exist neutrino interactions involving admixture of $V+A$ type leptonic currents, then a helicity conserving mechanism for $\beta\beta_{0\nu}$ (and hence without the need for a neutrino mass) emerges (Fig.2). This for instance can happen, when one replaces $\bar{e}\gamma_\mu(1-\gamma_5)\nu_e$ in Eq.3 by $\bar{e}\gamma_\mu[(1-\gamma_5) + \eta(1 + \gamma_5)]\nu_e$. The amplitude for $\beta\beta_{0\nu}$ (ignoring nuclear physics factors) can then be written as:

$$A^\eta_{\beta\beta} \simeq \frac{G_F^2}{2} \langle \frac{\eta}{\gamma \cdot k} \rangle_{\text{Nucl.}}$$  \hspace{1cm} (4)

Such contributions depend on the value of $\eta$ and are nonzero as long as the neutrino is a Majorana particle regardless of how big its mass is. It is sometimes called the left-right mixing contribution and leads to $0^+ \rightarrow 2^+$ type of transition. Order of magnitude arguments of the type just given for the mass mechanism leads to an upper bound for $\eta$ of about $10^{-8}$ or so. More careful nuclear physics arguments also lead to similar bounds.

I.A.3 Helicity nonflip vector-scalar mechanism:

A completely new class of contributions to $\beta\beta_{0\nu}$ involving the exchange of light neutrinos has been pointed out recently. The new contributions arise from the combination of two effective four-Fermi interactions of the following type:

$$H_{e\text{ff}} = \frac{G_F}{\sqrt{2}} (\bar{e}\gamma_\mu(1-\gamma_5)\nu_e u^\mu(1-\gamma_5) + \epsilon_1 \bar{d}(1-\gamma_5)\nu_e u^T C^{-1}(1-\gamma_5)e + \epsilon_2 \bar{d}(1-\gamma_5)\nu_e u^T C^{-1}(1-\gamma_5)e) + \text{h.c.}$$  \hspace{1cm} (5)

In the above, the first term is the usual $(V-A)$ interaction, the other two are effective lepton number violating terms. The $V-A$ term in the above equation in collaboration with either of the last two terms can lead to $\beta\beta_{0\nu}$ decay via a Feynman diagram which
is similar to Fig. 2. In order to evaluate the matrix elements between nuclear states, we need to do Fierz reordering of the $\epsilon_{ee}^2$ term, which casts it in the form:

$$\frac{G_F}{2\sqrt{2}} \epsilon_{ee}^2 \left( \overline{d}(1-\gamma_5)u \overline{e}(1-\gamma_5)\nu_e + \frac{1}{2} \overline{d} \sigma^{\mu\alpha}(1-\gamma_5)u \overline{e}(1-\gamma_5)\sigma_{\mu\alpha}\nu_e \right).$$

The resulting effective double beta amplitude can be written in momentum space as:

$$A_{\beta\beta}^{\nu_s} \approx G_F^2 (\epsilon_{ee}^i) \langle \frac{1}{\gamma \cdot \vec{k}} \rangle. \quad (6)$$

As a crude estimate, we assume an average value of $k$ as before to be equal to the Fermi momentum $p_F$ of the nucleons in the nucleus ($\approx 50$ MeV). The present upper limits on $m_\nu$ of about 1 eV then translates to an upper limit on the new interaction parameter $\epsilon$ as follows:

$$\epsilon_{1,2} \leq 1 \times 10^{-8}. \quad (7)$$

I.B: Heavy particle exchange:

The second class of mechanisms consists of exchange of heavy particles (such as majorana fermions) which often arise in physics scenarios beyond the standard model. In the low energy limit, the effective Hamiltonian that leads to $\beta\beta_0\nu$ decay in these cases requires point interaction between nucleons; as a result, in general the nuclear matrix elements in these cases are expected to be smaller; nevertheless, a lot of extremely useful information have been extracted about new physics where these mechanisms operate. Symbolically, such contributions can arise from effective Hamiltonians of the following type (we have suppressed all gamma matrices as well as color indices):

$$H^{(1)} = G_{eff} \overline{\pi} \Gamma \overline{d} \Gamma F + h.c. \quad (8)$$

or

$$H^{(2)} = \lambda_\Delta \left( \frac{1}{M^3} \overline{\pi} \Gamma \overline{d} \Gamma d + e^- e^- \right) \Delta^{++} + h.c. \quad (9)$$

Here $F$ represents a neutral majorana fermion such as the right-handed neutrino $(N)$ or gluino $\tilde{G}$ or photino $\tilde{\gamma}$ and $\Delta^{++}$ represents a doubly charged scalar or vector particle. In the above equations, the coupling $G_{eff}$ has dimension of $M^{-2}$ and $\lambda_\Delta$ is dimensionless. The possibility of the doubly charged scalar contribution to $\beta\beta_0\nu$ was first noted in [12] and have been discussed subsequently in [13]. The contributions to neutrinoless double beta decay due to the above interactions arise from diagrams in Fig. 3 and 4 and lead to $\beta\beta_0\nu$ amplitudes as follows:

$$A^{(F)}_{\beta\beta} \approx G_{eff}^2 \frac{1}{M_F} (p^{eff})^3. \quad (10)$$
and

\[ A_{\beta\beta} \simeq \left( \frac{\lambda_{\Delta}^2}{M^3 M_{\Delta}^2} \right) (p_{\text{eff}})^3 \]  

(11)

Here again we have crudely replaced all nuclear effects by the effective momentum parameter \( p_{\text{eff}} \). If we choose \( p_{\text{eff}} \simeq 50 \text{ MeV} \), then the present lower limit on the lifetime for \(^{76}\text{Ge}\) decay leads to a crude upper limit on the effective couplings as follows:

\[ G_{\text{eff}} \leq 10^{-7} \left( \frac{M_{\nu}}{100 \text{ GeV}} \right)^{\frac{5}{2}} \]  

(12)

and

\[ \lambda_{\Delta} \leq 10^{-3} \left( \frac{M}{100 \text{ GeV}} \right)^{\frac{5}{2}} \]  

(13)

In the second equation above, we have set \( M = M_{\Delta} \). Note that these limits are rather stringent and therefore have the potential to provide useful constraints on the new physics scenarios that lead to such pictures.

**Part II**

**Implications for physics beyond the standard model:**

Let us now discuss what kind of new physics scenarios beyond the standard model where the above mechanisms can be realized. Let us first consider the neutrino mass mechanism. Any theory which gives the electron neutrino a significant (\( \simeq \text{eV} \)) Majorana mass or any other species (e.g. \( \nu_\mu \) or \( \nu_\tau \)) a large enough mass and mixing angle with the \( \nu_\ell \) so that \( U_{e\ell}^2 m_{\nu_\ell} \) is of order of an electron volt will make itself open to testability by the \( \beta\beta_0\nu \) decay experiment. There are many theories with such expectations for neutrinos. Below I described two examples: (i) the singlet majoron model and (ii) the left-right symmetric model. This is intimately connected with ways to understand the small neutrino mass in gauge theories.

**II.A: The singlet majoron model:**

This model\(^{[14]}\) is the simplest extension of the standard model that provides a naturally small mass for the neutrinos by employing the the see-saw mechanism\(^{[15]}\). It extends the standard model by the addition of three right-handed neutrinos and the addition of a single complex Higgs field \( \Delta \) which is an \( SU(2)_L \times U(1)_Y \) singlet but with a lepton number +2. There is now a Dirac mass for the neutrinos and a Majorana mass for the right handed neutrinos proportional to the vacuum expectation value (vev) \( \langle \Delta \rangle \equiv v_R \). This leads to a mass matrix for the neutrinos with the usual see-saw form:

\[ M = \begin{pmatrix} 0 & m_D^T \\ m_D & f v_R \end{pmatrix} \]  

(14)
This leads to both the light and heavy (right-handed) neutrinos being Majorana particles with the mutual mass relation being given by the see-saw formula:

\[ m_{\nu_i} \simeq m_{iD} (M_{iR}^{-1}) m_{T} \tag{15} \]

where we have ignored all mixings and \( M_{iR} \simeq f_{ii} v_R \) denote the masses of the heavy right-handed neutrinos. It is clear that the electron neutrino mass can be in the electron-volt range if the values of \( m_{1D} \) are chosen to be of similar order of magnitude to the electron mass. In fact, for \( m_{1D} = m_e \), and \( m_{1R} = 250 \text{ GeV} \), one gets \( m_{\nu_e} = 1 \text{ eV} \) which is the range of masses being probed by the ongoing and proposed \( \beta\beta_{0\nu} \) experiments. This model would predict a hierarchical pattern for neutrino masses with an eV-KeV-MeV masses for the three neutrinos. Cosmological consistency for such a spectrum has been studied in several papers\[16\] and we do not go into details. We simply mention two recent arguments which have brought attention to such scenarios (especially the tau neutrino mass being in the MeV range).

The first point has to do with a recent analysis of the constraints on the number of neutrino flavors imposed by the big bang nucleosynthesis\[17\] (BBN). According to this analysis, the present data on \(^3\text{He}, \ D\) and \(^4\text{He}\) abundances in the universe combined with theoretical models for chemical evolution of \(^3\text{He}\) and \(D\), implies that the total number of neutrino species \( N_\nu \) at the epoch of BBN must be \( \simeq 2 \). Since LEP data has confirmed the existence of \( \nu_\tau \) along with \( \nu_\mu \) and \( \nu_e \), the \( \nu_\tau \) somehow must not contribute to BBN. The only way it can happen is if it has decayed by the BBN time. In the singlet majoron model, such short decay lifetimes have been shown possible within the other existing constraints on the model\[16\].

A second argument arises from considerations of structure formation. Apparently all known data for structure in the universe can be accommodated by assuming the existence of cold dark matter in conjunction with an MeV \( \nu_\tau \) decaying with a lifetime of 10-100 sec\[18\], a lifetime value in the same range as required by the BBN argument.

II.B: Left-right symmetric models:

Let us now consider the minimal left-right symmetric model with a see-saw mechanism for neutrino masses as described in \[4\]. Below, we provide a brief description of the structure of the model. The three generations of quark and lepton fields are denoted by \( Q^a_T \equiv (u_a, d_a) \) and \( \Psi^a_T \equiv (\nu_a, \ e_a) \) respectively, where \( a = 1, 2, 3 \) is the generation index. Under the gauge group \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \), they are assumed to transform as \( \Psi_a \ L \equiv (1/2, 0, -1) \) and \( \Psi_a \ R \equiv (0, 1/2, -1) \) and similarly for the quarks denoted by \( Q^a \ L \equiv (u, d) \). In this model, there is a right-handed counterpart to the \( W^\pm_L \) to be denoted by \( W^\pm_R \). Their gauge interactions then lead to the following expanded structure for the charged weak currents in the model for one generation prior to symmetry breaking (for our discussion, the quark mixings and the higher generations are not very important; so
we will ignore them in what follows.)

\[
L_{wk} = \frac{g}{2\sqrt{2}} [W_{\mu L}^- \left( \bar{d} \gamma^\mu (1 - \gamma_5) u + \bar{e} \gamma^\mu (1 - \gamma_5) \nu_e \right) + L \to R, -\gamma_5 \to +\gamma_5, \nu_e \to N_e ]
\]

(16)

The Higgs sector of the model consists of the bi-doublet field \( \phi \equiv (1/2, 1/2, 0) \) and triplet Higgs fields: \( \Delta_L(1, 0, +2) \oplus \Delta_R(0, 1, +2) \).

The Yukawa couplings which are invariant under gauge and parity symmetry can be written as:

\[
L_Y = \Psi_L h^\ell \phi \Psi_R + \Psi_L \tilde{h}^\ell \tilde{\phi} \Psi_R + Q_L \phi h q Q_R + Q_L \tilde{h} q \tilde{\phi} Q_R + \Psi_L^T f \tau_2 \cdot \Delta_L C^{-1} \Psi_L + L \to R + h.c.
\]

(17)

where \( h, \tilde{h} \) are hermitian matrices while \( f \) is a symmetric matrix in the generation space. \( \Psi \) and \( Q \) here denote the leptonic and quark doublets respectively.

The gauge symmetry is spontaneously broken by the vacuum expectation values:

\(< \Delta^0_R >= V_R \); \(< \Delta^0_L >= 0 \); and \(< \phi >= \left( \begin{array}{cc} \kappa & 0 \\ 0 & \kappa' \end{array} \right) \). As usual, \(< \phi >\) gives masses to the charged fermions and Dirac masses to the neutrinos whereas \(< \Delta^0_R >\) leads to the see-saw mechanism for the neutrinos in the standard way[2]. For one generation the see-saw matrix is in the form given in Eq.14 and leads as before to a light and a heavy state as discussed in the previous section. For our discussion here it is important to know the structure of the light and the heavy neutrino eigenstates:

\[
\nu \equiv \nu_e + \xi N_e \\
N \equiv N_e - \xi \nu_e
\]

(18)

where \( \xi \simeq \sqrt{m_{\nu_e}/m_N} \) and is therefore a small number. Substituting these eigenstates into the charged current Lagrangian in Eq.16, we see that the right-handed \( W_R \) interaction involves also the light neutrino with a small strength proportional to \( \xi \). To second order in the gauge coupling \( g \), the effective weak interaction Hamiltonian involving both the light and the heavy neutrino becomes:

\[
H_{wk} = \frac{G_F}{\sqrt{2}} \left( \bar{\nu} \gamma^\mu (1 - \gamma_5) d [\bar{\nu} \gamma_\mu [(1 - \gamma_5) + \xi \left( m^2_{W_L} / m^2_{W_R} \right)(1 + \gamma_5)] + \xi \bar{e} \gamma_\mu (1 - \gamma_5) N + \xi e \gamma_\mu (1 + \gamma_5) \right) + h.c.
\]

(19)

From Eq. (19), we see that there are several contributions to the \( \beta\beta_{0v} \). Aside from the usual neutrino mass diagram ( Fig.1), there is a contribution due to the wrong helicity admixture with \( \eta \simeq \xi \left( m^2_{W_L} / m^2_{W_R} \right) \) and there are contributions arising from the exchange of heavy right-handed neutrinos (Fig.3). This last contribution is given by:

\[
A^{(R)}_{\beta\beta} \simeq \frac{G_F^2}{2} \left( \frac{m^4_{W_L}}{m^4_{W_R}} + \xi^2 \right) \frac{1}{m_N}
\]

(20)
The present limits on neutrinoless double beta decay lifetime then imposes a correlated constraint on the parameters $m_{W_R}$ and $m_N$. This is shown in Fig. 5 and a correlated constraint on $\xi$ and $m_{W_R}$ shown in Fig. 6 due to Hirsch. It is clear from figure that if we combine the theoretical constraints of vacuum stability then, the present $^{76}Ge$ data provides a lower limit on the masses of the right handed neutrino ($N_e$) and the $W_R$ of 1 TeV, which is a rather stringent constraint. The limits on $\xi$ on the other hand are not more stringent than what would be expected from the structure of the theory. We have of course assumed that the leptonic mixing angles are small so that there is no cancellation between the parameters.

Finally, the Higgs sector of the theory generates two types of contributions to $\beta\beta_{0\nu}$ decay. One arises from the coupling of the doubly charged Higgs boson to electrons (see Fig. 4). The amplitude for the decay is same as in Eq.11 except we have $\lambda_{\Delta} = f_{11}$ and

$$\frac{\lambda_{\Delta}}{M^3} = 2^{7/4} G_F^{3/2} \left( \frac{m_{W_L}}{m_{W_R}} \right)^3$$

(21)

Using this expression, we find that the present $^{76}Ge$ data implies that (assuming $m_{W_R} \geq 1$ TeV)

$$M_{\Delta^{++}} \geq \sqrt{f_{11}} \ 80 GeV$$

(22)

A second type Higgs induced contribution arises from the mixing among the charged Higgs fields in $\phi$ and $\Delta_L$ which arise from the couplings in the Higgs potential, such as $\text{Tr}(\Delta_L\phi\Delta_R^\dagger\phi^\dagger)$ after the full gauge symmetry is broken down to $U(1)_{em}$. Let us denote this mixing term by an angle $\theta$. This will contribute to the four-Fermi interaction of the form given by the $\epsilon_{ee}^{\epsilon_1}$ term through the diagram shown in Fig. 7 with

$$\epsilon_{ee}^{\epsilon_1} \approx \frac{h_u f_{11} \sin 2\theta}{4\sqrt{2} G_F M_{H^+}^2},$$

(23)

where we have assumed that $H^+$ is the lighter of the two Higgs fields. We get $h_u f_{11} \sin 2\theta \leq 6 \times 10^{-9} (M_{H^+}/100 \text{GeV})^2$, which is quite a stringent constraint on the parameters of the theory. To appreciate this somewhat more, we point out that one expects $h_u \approx m_u/m_W \approx 5 \times 10^{-5}$ in which case, we get an upper limit for the coupling of the Higgs triplets to leptons $f_{11} \sin 2\theta \leq 10^{-4}$ (for $m_{H^+} = 100 \text{GeV}$). Taking a reasonable choice of $\theta \sim M_{W_L}/M_{W_R} \sim 10^{-1}$ would correspond to a limit $f_{11} \leq 10^{-3}$. Limits on this parameters from analysis of Bhabha scattering is only of order .2 or so for the same value of the Higgs mass.

II.C: MSSM with R-parity violation:

The next class of theories we will consider is the supersymmetric standard model. As is well-known, the minimal supersymmetric standard model can have explicit violation of the R-symmetry (defined by $(-1)^{3B+L+2S}$), leading to lepton number violating interactions in the low energy Lagrangian. The three possible types of couplings in the superpotential are:
\[ W' = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c. \] (24)

Here, \( L, Q \) stand for the lepton and quark doublet superfields, \( E^c \) for the lepton singlet superfield and \( U^c, D^c \) for the quark singlet superfields. \( i, j, k \) are the generation indices and we have \( \lambda_{ijk} = -\lambda_{jik} \), \( \lambda''_{ijk} = -\lambda''_{ikj} \). The \( SU(2) \) and color indices in Eq. (24) are contracted as follows: \( L_i Q_j D_k^c = (\nu_i^d \alpha_j - e_i^u \alpha_j) D_k^c \alpha \), etc. The simultaneous presence of all three terms in Eq. (24) will imply rapid proton decay, which can be avoided by setting the \( \lambda'' = 0 \). In this case, baryon number remains an unbroken symmetry while lepton number is violated.

There are two types of to \( \beta\beta \) decay in this model. One class dominantly mediated by heavy gluino exchange[20] falls into the class of type II contributions discussed in the previous section. The dominant diagram of this class is shown in Fig.8. Detailed evaluation of the nuclear matrix element for this class of models has recently been carried out by Hirsch et. al.[22] and they have found that a very stringent bound on the following \( R \)-violating parameter can be given:

\[ \lambda'_{111} \leq 3.9 \times 10^{-4} \left( \frac{m_{\tilde{g}}}{100 \text{GeV}} \right)^2 \left( \frac{m_{\tilde{g}}}{100 \text{GeV}} \right)^{1/2} \] (25)

The second class of contributions fall into the light neutrino exchange vector-scalar type[10] and the dominant diagram of this type is shown in Fig.9.(where the exchanged scalar particles are the \( \tilde{b} - \tilde{b}^c \) pair). This leads to a contribution to \( \epsilon^e_e \) given by

\[ \epsilon^e_e \simeq \left( \frac{\lambda'_{113} \lambda'_{131}}{2 \sqrt{2} G_F M_{\tilde{g}}^2} \right) \left( \frac{m_b}{M_{\tilde{b}^c}} \right) (\mu \tan \beta + A_b m_0) \] (26)

Here \( A_b, m_0 \) are supersymmetry breaking parameters, while \( \mu \) is the supersymmetric mass of the Higgs bosons. \( \tan \beta \) is the ratio of the two Higgs vacuum expectation values and lies in the range \( 1 \leq \tan \beta \leq m_t / m_b \approx 60 \). For the choice of all squark masses as well as \( \mu \) and the SUSY breaking mass parameters being of order of 100 GeV, \( A_b = 1, \tan \beta = 1 \), the following bound on \( R \)-violating couplings is obtained:

\[ \lambda'_{113} \lambda'_{131} \leq 3 \times 10^{-8} \] (27)

This bound is a more stringent limit on this parameter than the existing ones [23]. The present limits on these parameters are \( \lambda'_{113} \leq 0.03, \lambda'_{131} \leq 0.26 \), which shows that the bound derived here from \( \beta\beta \) is about five orders of magnitude more stringent on the product \( \lambda'_{113} \lambda'_{131} \). If the exchanged scalar particles in Fig.9 are the \( \tilde{s} - \tilde{s}^c \) pair, one obtains a limit

\[ \lambda'_{121} \lambda'_{112} \leq 1 \times 10^{-6} \] (28)

which also is more stringent by about four orders of magnitude than the existing limits (\( \lambda'_{121} \leq 0.26, \lambda'_{112} \leq 0.03 \)).
II.D: Models with heavy sterile neutrinos

The models we have discussed so far are very strongly motivated by independent physics considerations (other than understanding small neutrino masses). There is however a class of models which one can construct simply to use the see-saw mechanism (or variations of it) to understand the small neutrino mass. A simple example such models can be constructed by taking the singlet Majoron model and eliminating the lepton number carrying scalar boson $\Delta$ and instead adding an explicit heavy majorana mass for the three right handed neutrinos. The see-saw mechanism still operates so that small neutrino masses come out naturally. Let us ask if these models have any interesting implication for $\beta\beta$ decay other than the usual neutrino mass contribution. The only possible new contribution would arise from the exchange of the heavy majorana right-handed neutrino- but as will be clear soon, this contribution is suppressed due to the see-saw mechanism. The point is clear if we look at Eq. and realize that the mixing parameter $\xi$ between the light $\nu_e$ and the heavy $N_e$ is given by $\sqrt{m_\nu/m_N}$ and generic see-saw formula for neutrinos require $m_N$ in the range of tens of GeV. This makes $\xi \leq 10^{-5}$ or so. Therefore, the double beta amplitude contributed by the $N$ exchange is at most of order $G_F^2 \times 10^{-10}/m_N$ which is much too small to be observable.

Since the heavy sterile sector is largely unknown, a possibility to consider is to have two heavy sterile leptons which participate in a $3 \times 3$ see-saw with the light neutrino to make $m_\nu$ small. The analog of the mixing parameter $\xi$ is then not constrained to be small by the see-saw considerations and also a larger range of masses for the heavy sterile particles are then admissible. Such models are however subject to a variety of cosmological and astrophysical constraints. These constraints have been analyzed in detail in [24] and it is found that there is a large range of the parameter space for the sterile particles which can be probed by the ongoing neutrinoless double beta experiments (see Fig.10 from [24]).

II.E: Limits on the scale of lepton compositeness

If the quarks and leptons are composite particles, it is natural to expect excited leptons which will interact with the electron via some effective interaction involving the $W_L$ boson. If the excited neutrino is a majorana particle, then there will be contributions to $\beta\beta$ decay mediated by the excited neutrinos $(\nu^*)$. The effective interaction responsible for this is obtained from the primordial interaction:

$$H_{\nu^*}^{\text{eff}} = g \frac{\lambda_W(\nu^*)}{2m_{\nu^*}} \bar{e} \gamma^\mu (\eta_L(1 - \gamma_5) + \eta_R(1 + \gamma_5))\nu^* W_{\mu\nu} + \text{h.c.}$$  \hspace{1cm} (29)

Here L and R denote the left and right chirality states. This contribution falls into our type II heavy particle exchange category and has been studied in detail in two recent papers[25] and have led to the conclusion that it leads to a lower bound

$$m_{\nu^*} \geq 5.9 \times 10^4 \text{TeV}$$  \hspace{1cm} (30)

for $\lambda_W(\nu^*) \geq 1$. This is a rather stringent bound on the compositeness scale.
Part III

Outlook for $\beta\beta_{0\nu}$ decay given present data on neutrinos:

The present situation in the neutrino physics is rather intriguing. On the one hand, the direct measurements show no evidence for any of the neutrinos to be massive, providing only the upper bounds $m_{\nu_e} < 4.5$ eV ($< 0.7$ eV [$\beta\beta_{0\nu}$]) $m_{\nu_\mu} < 160$ keV and $m_{\nu_\tau} < 20$ MeV. The neutrinos could therefore be massless as far as these experiments are concerned. On the other hand there are several other experiments which provide strong indications in favor of neutrino masses and mixings. Let us describe them now.

III.A: Solar Neutrino Deficit

For massive neutrinos which can oscillate from one species to another, the solar electron neutrino observations\cite{26} can be understood if the neutrino mass differences and mixing angles fall into one of the following ranges\cite{28}, where the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism is included\cite{27}: a) Small-angle MSW, $\Delta m_{\nu_{ei}}^2 \simeq 6 \times 10^{-6}$ eV$^2$, $\sin^2 2\theta_{ei} \simeq 7 \times 10^{-3}$; b) Large-angle MSW, $\Delta m_{\nu_{ei}}^2 \simeq 9 \times 10^{-6}$ eV$^2$, $\sin^2 2\theta_{ei} \simeq 0.6$; c) Vacuum oscillation, $\Delta m_{\nu_{ei}}^2 \simeq 10^{-10}$ eV$^2$, $\sin^2 2\theta_{ei} \simeq 0.9$.

III.B: Atmospheric Neutrino Deficit

The second set of experiments indicating non-zero neutrino masses and mixings has to do with atmospheric $\nu_{\mu}$'s and $\nu_e$'s arising from the decays of $\pi$'s and $K$'s and the subsequent decays of secondary muons produced in the final states of the $\pi$ and $K$ decays. In the underground experiments the $\nu_\mu$ and $\bar{\nu}_\mu$ produce muons and the $\nu_e$ and $\bar{\nu}_e$ lead to $e^\pm$. Observations of $\mu^+$ and $e^\pm$ indicate a far lower value for $\nu_\mu$ and $\bar{\nu}_\mu$ than suggested by naive counting arguments which imply that $N(\nu_\mu + \bar{\nu}_\mu) = 2N(\nu_e + \bar{\nu}_e)$. More precisely, the ratio of $\mu$ events to $e$-events can be normalized to the ratio of calculated fluxes to reduce flux uncertainties, giving $R(\mu/e) = 0.60 \pm 0.07 \pm 0.05$ (Kamiokande); $= 0.54 \pm 0.05 \pm 0.12$ (IMB) and $= 0.69 \pm 0.19 \pm 0.09$ (SoudanII).

Combining these results with observations of upward going muons by Kamiokande\cite{29}, IMB\cite{30}, and Baksan\cite{34} and the negative Fréjus\cite{32} and NUSEX\cite{33} results leads to the conclusion\cite{29} that neutrino oscillations can give an explanation of these results, provided $\Delta m_{\mu\mu}^2 \approx 0.005$ to $0.5$ eV$^2$, $\sin^2 2\theta_{\mu\mu} \approx 0.5$.

III.C: Hot Dark Matter

There is increasing evidence that more than 90% of the mass in the universe must be detectable so far only by its gravitational effects. This dark matter is likely to be a mix of $\sim 20$ to 30% of particles which were relativistic at the time of freeze-out from equilibrium in the early universe (hot dark matter) and $\sim 70$% of particles which were non-relativistic (cold dark matter). Such a mixture\cite{33} gives the best fit of any available model to the
structure and density of the universe on all distance scales, such as the anisotropy of the microwave background, galaxy-galaxy angular correlations, velocity fields on large and small scales, correlations of galaxy clusters, etc. A very plausible candidate for hot dark matter is one or more species of neutrinos with total mass of $m_{\nu H} = 93h^2 F_H \Omega = 5$ eV, if $h = 0.5$ (the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$), $F_H = 0.2$ (the fraction of dark matter which is hot), and $\Omega = 1$ (the ratio of density of the universe to closure density). We shall use the frequently quoted 5 eV below, but different determinations give $h = 0.45 \pm 0.09$ or $h = 0.80 \pm 0.11$ (a value giving difficulties with $\Omega = 1$), making $m_{\nu H} = 2$ or 21 eV.

It is usually assumed that the $\nu_{\tau}$ would supply the hot dark matter. This is justified on the basis of an appropriately chosen see-saw model and a $\nu_e \rightarrow \nu_\mu$ MSW explanation of the solar $\nu$ deficit. However, if the atmospheric $\nu_\mu$ deficit is due to $\nu_\mu \rightarrow \nu_\tau$, the $\nu_\tau$ alone cannot be the hot dark matter, since the $\nu_\mu$ and $\nu_\tau$ need to have close to the same mass. It is interesting that instead of a single $\simeq 5$ eV neutrino, sharing that $\simeq 5$ eV between two or among three neutrino species provides a better fit to the universe structure and particularly a better understanding of the variation of matter density with distance scale.

III.D: Indications of $\nu_{\mu} \rightarrow \nu_{e}$ oscillation from LSND

There appear to be some indications in favor of a possible oscillation of $\nu_{\mu} \rightarrow \nu_{e}$ from the recent LSND experiment. While these results are not completely conclusive, taken at face value a $\Delta m^2 \approx 1 - 6$ eV$^2$ and $\sin^2 \theta \approx 10^{-2}$ appears to be the preferred range of parameters needed to explain observations.

III.E: Nucleosynthesis Limits on Neutrino Species

While the $Z^0$ width limits the number of weakly interacting neutrino species to three, the nucleosynthesis limit on the number of light neutrinos (denoted by $N_{\nu}$) is more useful here, since it is independent of the neutrino interactions with the $Z$-boson. Until a few months ago, the limit on $N_{\nu}$ was 3.3. However, a recent analysis by Hata et al. concludes that after one includes the evolutionary effects on the $^3He$ and Deuterium abundances, the present primordial $^4He$ abundance rules out $N_{\nu} = 3$ at 99.7% confidence level and favors a value close to $N_{\nu} = 2$. Thus one would have trouble understanding present helium abundance using three light neutrinos in the framework of the standard model. One possibility is to have a unstable tau neutrino with mass in the MeV range with a life time of the order of a few seconds decaying to $\nu_e +$ majoron.

Within this set of constraints for example, the atmospheric $\nu_\mu$ problem cannot be explained by $\nu_\mu \rightarrow \nu_s$ because it requires a large mixing angle and in that case, for the $\Delta m^2_{\mu s}$ involved, $\nu_s$ would have contributed as one extra neutrino species. On the other hand, the solar $\nu_e$ problem can be explained by $\nu_e \rightarrow \nu_s$ for either the small-angle MSW or the vacuum oscillation solutions, but not for the less favored large-angle MSW solution.
III.F: Supernova r-Process Constraint

Another set of constraints on neutrino mixings have been derived from the assumption that heavy elements in the universe are produced in the neutron rich environment around the supernovae by rapid neutron capture known as r-process. It has been shown that unless $\nu_e-\nu_\mu$ and $\nu_e-\nu_\tau$ mixing angles are severely restricted ($\sin^2 2\theta \leq 4 \times 10^{-4}$) for $\Delta m^2 \geq 4$ eV$^2$ (with a rapid decrease in $\sin^2 2\theta$ for larger $\Delta m^2$)\[42\], the energetic $\nu_\mu$ and $\nu_\tau$ ($\langle E \rangle \approx 25$ MeV) can convert to $\nu_e$'s which have much higher energy than the thermal $\nu_e$'s ($\langle E \rangle \approx 11$ MeV). The higher energy $\nu_e$'s, having a larger cross section, will reduce the neutron density via $\nu_e + n \rightarrow e^- + p$, diminishing heavy element formation.

III.G: Possible patterns of Neutrino masses consistent with the above constraints

A. Patterns Required by Solar and Atmospheric Neutrino Deficits and Hot Dark Matter

With the above input information, if we stay within the minimal three neutrino picture, then the solar neutrino puzzle can be resolved by $\nu_e \rightarrow \nu_\mu$ oscillations and the atmospheric neutrino deficit by $\nu_\mu \rightarrow \nu_\tau$ oscillations. Note that these observables are controlled only by the mass square difference; on the other hand, the required hot dark matter implies that at least one or more of the neutrinos must have mass in the few eV range. It was pointed out\[43\] in 1993 that, in the minimal picture, this leads to the following scenario, labelled (A):

All three neutrinos are nearly degenerate, with $m_{\nu_e} \approx m_{\nu_\mu} \approx m_{\nu_\tau} \approx 2$ eV, since $\nu_e \rightarrow \nu_\mu$ and $\nu_\mu \rightarrow \nu_\tau$ both require small mass differences, but the required dark matter mass can be shared. The mass matrix for this case in the $\nu_e, \nu_\mu, \nu_\tau$ basis is given by:

$$M = \begin{pmatrix} m + \delta_1 s_1^2 & -\delta_1 c_1 c_2 s_1 & -\delta_1 c_1 s_1 s_2 \\ -\delta_1 c_1 c_2 s_1 & m + \delta_1 c_1^2 c_2^2 + \delta_2 s_2^2 & \delta_1 c_1^2 c_2 s_2 - \delta_2 c_2 s_2 \\ -\delta_1 c_1 s_1 s_2 & \delta_1 c_1^2 s_2 c_2 - \delta_2 c_2 s_2 & m + \delta_1 c_1^2 s_2^2 + \delta_2 c_2^2 \end{pmatrix}$$

(31)

where $c_i = \cos \theta_i$ and $s_i = \sin \theta_i$, $m = 2$ eV; $\delta_1 \approx 1.5 \times 10^{-6}$ eV; $\delta_2 \approx .2$ to .002 eV; $s_1 \approx 0.05$; and $s_2 \approx 0.4$ for the small-angle MSW solution.

In this case, the LSND results cannot be accomodated. However, there will be an observable amplitude for neutrinoless double beta decay mediated by the neutrino mass mechanism. In fact, if the limit on $\langle m_\nu \rangle$ goes below 1 eV ( without any nuclear matrix element uncertainty ), then this model will fail to provide a viable hot dark matter candidate.

B. Mass matrix Accomodating the solar, atmospheric and LSND data and HDM:

In this case, an additional light sterile neutrino ( to be called $\nu_s$ ) is essential\[43\]. The $\nu_e$ and $\nu_\mu$\[44\] are assumed to be quite light to take care of the solar neutrino problem while the $\nu_\mu$ and $\nu_\tau$ share the dark matter role, being $\sim 2.4$ eV each, and explain the
atmospheric $\nu_\mu$ deficit. Recently, several interesting gauge models realizing this texture have been constructed [43].

In this case as in case A, one will have to use the small-angle MSW solution, since the $\nu_s$ has to be very weakly mixed to satisfy the nucleosynthesis bound. Nucleosynthesis also eliminates the large-angle MSW solution and in the vacuum oscillation case forces the $\nu_s$ to be mixed strongly only with the $\nu_e$. The form of the Majorana mass matrix for this case is given by [43]: (in the basis $(\nu_s, \nu_e, \nu_\mu, \nu_\tau)$,

$$M = \begin{pmatrix} 
\mu_1 & \mu_3 & 0 & 0 \\
\mu_3 & \mu_2 & \epsilon_{21} & 0 \\
0 & \epsilon_{21} & m & \delta/2 \\
0 & 0 & \delta/2 & m + \delta 
\end{pmatrix}$$

(32)

As is clear, model can also accomodate the LSND $\nu_e \rightarrow \nu_\mu$ oscillation. Needless to say that the neutrinoless double beta decay will be unobservable in this case.

C. Inverted Mass Hierarchy for Solar Neutrino Puzzle, HDM and LSND

Since the atmospheric neutrino anomaly is perhaps on a somewhat weaker footing due to some experiments (e.g. Frejus and NUSEX) not showing this anomaly, it may be interesting to see what kind of mass pattern is allowed by eliminating it as a constraint. This has been studied in several papers recently [46] and it has been noted that in this case, there is again no need for a sterile neutrino and one can write the following $3 \times 3$ mass matrix for the three Majorana neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$.

$$M = \begin{pmatrix} 
-m\beta - \delta & -\mu_1 & m + \delta \\
-\mu_1 & \mu & -\mu_1 \\
m + \delta & -\mu_1 & m\beta - \delta 
\end{pmatrix}$$

(33)

We assume that $\mu \ll m \simeq 2.4$ eV; $\mu_1/m$ denotes the $\nu_e - \nu_\mu$ mixing angle responsible for the LSND results and $\delta \simeq 10^{-5}$ eV to fit the solar neutrino data. The key new implication here is that the solar neutrino puzzle must be resolved by the large angle MSW solution, a result which can be tested at SuperKamiokande by looking for the day-night variation in the neutrino flux. Note the presence of an exact $L_e - L_\tau$ symmetry of the mass matrix in the limit of vanishing $\beta, \delta, \mu_1$. In this case also the neutrinoless double beta decay is unobservable. It must however be said that if the dominant neutrino masses in this case came by putting the entry $m$ above along the diagonal rather along the antidiagonal positions, then there would be an observable neutrinoless double beta decay amplitude in the ongoing experiments.
Part IV

Theoretical scenarios

Let us note some important qualitative points about the mass matrices described here. Generically, they indicate two kinds of scales: one corresponding to the mass differences which are typically of order \( \approx 10^{-3} \) eV or so and another corresponding to a mass of order of a few eV. In the canonical see-saw models one has \( m_{\nu_i} \approx \frac{m^2_{\nu_{e}}}{f v_R} \) which leads to a hierarchical neutrino mass pattern i.e. \( m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau} \) with \( m_{\nu_\tau} \approx 2 - 4 \) eV. While this simple picture can very easily accomodate a solution to the solar neutrino puzzle and an HDM, it has room neither for the LSND result nor for the atmospheric neutrino data. One has to go beyond this picture to understand all existing observations. It turns out that the canonical see-saw picture is not realized\(^2\) in many popular unified models and the correct see-saw matrix (to be called type II see-saw matrix here) that arises for instance in the SO(10) models helps in generating the matrix texture (A).

IV.A Type II See-Saw and Degenerate Neutrinos in SO(10) GUT

In the early days of the discussion of the see-saw formula for neutrino masses, it was pointed out\(^2\) that implementing it in the simplest left-right or SO(10) models resulted in a \( \nu_L-\nu_R \) mass matrix of the modified see-saw form:

\[
\begin{pmatrix}
fv_L & m_{\nu_D} \\
m^T_{\nu_D} & fv_R
\end{pmatrix}
\]  

(34)

where \( f \) and \( m_{\nu_D} \) are 3 \times 3 matrices and \( v_L \approx \lambda M^2_{W_L}/v_R \). The light neutrino mass matrix that follows from diagonalizing the above mass matrix is (type II see-saw formula)

\[
m_\nu \approx f \lambda M^2_{W_L}/v_R - m_{\nu_D} f^{-1} m^T_{\nu_D}/v_R + \ldots
\]  

(35)

While both terms vanish as \( v_R \to \infty \), the first term always dominates over the second one for neutrino masses. This negates the usual quadratic formula (i.e., the second term) for neutrino masses.

Within the type II see-saw formula, it is clear that if a symmetry dictates that \( f_{ab} = f \delta_{ab} \), then the neutrino masses are degenerate to leading order. For \( v_R \approx 10^{13.5} \) GeV, \( f \lambda \approx 1 \), we get \( m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} \approx 1.5 \) eV, \( m^2_{\nu_\mu} - m^2_{\nu_\tau} \approx 3m^2_\tau/(10fv_R) \approx 10^{-4}/f \) eV\(^2\), and \( m^2_{\nu_e} - m^2_{\nu_\mu} \approx 3m^2_t/(10fv_R) \approx (2/f)(m_t/150 \) GeV\(^2\) eV\(^2\). These mass differences are of the right order of magnitude to explain the solar neutrino (via the MSW mechanism) and the atmospheric neutrino puzzles, while the sum of all the neutrino masses roughly give the needed amount of hot dark matter.

It is also interesting to note that the B-L breaking scale of \( v_R \approx 10^{13} \) GeV emerges naturally from constraints of \( \sin^2 2\theta_W \) and \( \alpha_s \) in non-supersymmetric SO(10) grandunified theories, enhancing the reason for an SO(10) scenario. To guarantee the neutrino degeneracy (i.e., \( f_{ab} = f \delta_{ab} \)), an extra family symmetry is imposed on the model. This family
symmetry will be broken softly by terms in the Lagrangian of dimension two, so that departures from the degeneracy in the neutrino sector are naturally small. An explicit example with an $S_4$ horizontal symmetry was worked out in [47], where it was possible to predict the complete neutrino mixing matrix:

$$V^l = \begin{pmatrix}
-0.9982 & 0.05733 & 0.01476 \\
0.05884 & 0.9334 & 0.3541 \\
-0.00652 & -0.3544 & 0.9351
\end{pmatrix}$$

(36)

This mixing matrix can be tested by the proposed long baseline experiments such as the Fermilab, Brookhaven and KEK experiments. If future experiments bear out a degenerate light neutrino spectrum, this detailed SO(10) model may or may not be the appropriate description of the physics. However, its essential ingredient, the type II see-saw formula, will almost surely be required to fit those data.

**IV.B: Inverted Mass Hierarchy from an $L_e - L_\tau$ symmetric Left-right Model**

We saw in the last section that to get three degenerate neutrinos requires a very elaborate horizontal symmetry structure. On the other hand, it turns out that if only two Majorana neutrinos are to be degenerate with opposite CP properties, it suffices to have a simpler $U(1)$ symmetry involving the two leptons. In the present case, the relevant symmetry is $L_e - L_\tau$ as has been noted in the second paper of [46]. Then one can use the type II see-saw formula in the context of a left-right symmetric model to generate the above mass matrix.

In conclusion, neutrinoless double beta decay provides a very versatile way to probe scenarios of physics beyond the standard model. In this review, we have focussed only on the $0\nu$ mode; there can also be single and multi majoron modes which test for the possibility of lepton number being a spontaneously broken global symmetry. The theory and phenomenology of this type of modes have been discussed in this volume by C. Burgess[48]. The $0\nu$ mode acquires special interest in view of certain SO(10) models predicting such spectra without contradicting the solar and atmospheric neutrino data.

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**References**

[1] R.N. Mohapatra and P.B. Pal, “Massive Neutrinos in Physics and Astrophysics”, World Scientific, Singapore, 1991.

[2] R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980); Phys. Rev. **D23**, 165 (1981).
[3] C. S. Aulakh and R. N. Mohapatra, Phys. Lett. 119B, 136 (1983); F. Zwirner, Phys. Lett. 132B, 103 (1983); L. Hall and M. Suzuki, Nucl. Phys. B231, 419 (1984); G. G. Ross and J. W. F. Valle, Phys. Lett. B151, 375 (1985).

[4] H. Klapdor-Kleingrothaus, Prog. in Part. and Nucl. Phys., 32, 261 (1994); A. Balysh et. al., Phys. Lett. (to appear).

[5] M. Moe and P. Vogel, Ann. Rev. Nucl. Sc. 44, 247 (1994).

[6] see the talks by P. Vogel, K. Muto, S. Stoica and others in this volume.

[7] M. Doi, T. Kotani, E. Takasugi, Prog. Theor. Phys. Suppl. 83, 1 (1985).

[8] H. Primakoff and S. P. Rosen, Rep. Prog. Phys. 22, 121 (1959)

[9] M. Doi et al., W. C. Haxton and G. Stephenson, Prog. in Part. and Nucl. Phys. 12, 409 (1984); H. Grotz and H. Klapdor, The Weak Interactions in Nuclear, Particle and Astrophysics, Adam Hilger, Bristol, (1990); D. Caldwell, Nucl. Phys. Proc. Suppl. B 13, 547 (1990).

[10] K. S. Babu and R. N. Mohapatra, hep-ph/9506354.

[11] A. Halprin, P. Minkowski, S. P. Rosen and H. Primakoff, Phys. Rev. D13, 2567 (1976).

[12] R. N. Mohapatra and J. Vergados, Phys. Rev. Lett. 47, 1713 (1981).

[13] J. Schecter and J.W.F. Valle, Phys. Rev. D25, 2951 (1982); W.C. Haxton, S.P. Rosen and G.J. Stephenson, ibid., D26, 1805 (1982); L. Wolfenstein, ibid., D26, 2507 (1982).

[14] Y. Chikashige, R. N. Mohapatra and R. D. Peccei, Phys. Lett. 98B, 265 (1981).

[15] M. Gell-Mann, P. Ramond and R. Slansky, in ”Supergravity”, Ed. D.Freedman et al (North-Holland, Amsterdam, 1979); T. Yanagida, Prog. Th. Phys. B135 (1978) 66; R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. 44 (1980) 912.

[16] R. N. Mohapatra and S. Nussinov, Phys. Rev. D51, 3843 (1995).

[17] N. Hata et al., Ohio State Univ. preprint, hep-ph/9505319.

[18] S. Dodelson, G. Gyuk and M. Turner, Phys. Rev. Lett. 72, 3578 (1995).

[19] M. Schwarz, Phys. Rev. D40, 1521 (1989).

[20] R.N. Mohapatra, Phys. Rev. D34, 909 (1986).

[21] M. Hirsch, H. Klapdor-Kleingrothaus and S. Kovalenko, Heidelberg preprint (1995).
[22] M. Hirsch, H. V. Klapdor-Kleingrothaus and S. G. Kovalenko, Heidelberg Preprint (1995).

[23] V. Barger, G. Giudice and T. Han, Phys. Rev. D40, 2987 (1989).

[24] P. Bamert, C. Burgess and R. N. Mohapatra, Nucl. Phys. B438, 3 (1995).

[25] O. Panella and Y. N. Srivastava, College de France Preprint, LPC 94-39; E. Takasugi, hep-ph/9506379.

[26] GALLEX Collaboration, Phys. Lett. B327, 377 (1994); SAGE Collaboration, Phys. Lett. B328, 234 (1994); Kamiokande Collaboration, Nucl. Phys. B38 (Proc. Suppl.), 55 (1995); Homestake Collaboration, ibid. 47.

[27] S.P. Mikheyev and A.Yu. Smirnov, Yad. Fiz. 42, 1441 (1985); Nuovo Cim. 9C, 17 (1986); L. Wolfenstein, Phys. Rev. D17, 2369 (1978).

[28] For the recent status see P.I. Krastev and A.Yu. Smirnov, Phys. Lett. B338, 882 (1994); V.S. Berezinsky, G. Fiorentini and M. Lissia, Phys. Lett. B341, 38 (1994); N. Hata and P. Langacker, Phys. Rev. D50, 632 (1994); V. Castellani, S. Degl’Innocenti and G. Fiorentini, Astron. Astrophys. 271 (1993) 601; Phys. Lett. B303, 68 (1993); J.N. Bahcall and H.A. Bethe, Phys. Rev. Lett. 65,2233 (1993); S.A. Bludman, N. Hata and P. Langacker, Phys. Rev. D 49, 3622 (1994); V.S. Berezinsky, Comments Nucl. Part. Phys. 21, 249 (1994); A.Yu. Smirnov, preprint DOE/ER/40561-136-INT94-13-01 (1994).

[29] Kamiokande Collaboration, Y. Fukuda et al., Phys. Lett. B335, 237 (1994).

[30] R. Becker-Szendy et al., Phys. Rev. D 46, 3720 (1992).

[31] P. J. Lichtfield et al., in International Europhysics Conference on High Energy Physics, Marseille, France (1993).

[32] K. Daum et al., Zeit. für Phys. C 66, 417 (1995).

[33] M. Aglieta et al., Europhys. Lett. 8, 611 (1989).

[34] M. M. Boliev et al., in Proc. of Third International Workshop on Neutrino telescopes, Venice ed by M. Baldo-ceolin (1991), p.235.

[35] R. Shaefer and Q. Shafi, Nature 359, 199 (1992); M. Davis. F.J. Summers and D. Schagel, ibid. 396; A.N. Taylor and M. Rowan-Robinson, ibid. 396; E.L. Wright et al., Ap. J. 396, L13 (1992); J.A. Holtzman and J.R. Primack, Ap. J. 405, 428 (1993); A. Klypin et al., Ap. J. 416,1 (1993).

[36] A. Sandage et. al., Astrophys. J. 401, L7 (1992).

[37] M.J. Pierce et al., Nature 371 (1994) 385;
[38] J.R. Primack, J. Holtzman, A. Klypin and D.O. Caldwell, Phys. Rev. Lett. 74, 2160 (1995).

[39] C. Athanassopoulos et al., preprint LA-UR-95-1238 (nucl-ex/9504002).

[40] T. Walker et al., Ap. J. 376 (1991) 51; P. Kernan and L. Krauss, Phys. Rev. Lett. 72 (1994) 3309; K. Olive and G. Steigman, preprint OSU-TA-2/95.

[41] Recently N. Hata et al., preprint OSU-TA-6/95 (May 1995), reconsidered the nucleosynthesis bound and found the upper limit $N_\nu < 2.5$ at For a recent review, see K. Olive and S. T. Scully, UMN-TH-1341/95.

[42] Y-Z Qian and G. Fuller, Phys. Rev. D (in press).

[43] D. Caldwell and R. N. Mohapatra, Phys. Rev. D48, 3259 (1993).

[44] See J. Peltoniemi, Sissa Preprint (1995).

[45] D. Caldwell and R. N. Mohapatra, Ref.[43]; J. Peltoniemi and J. W. F. Valle, Nucl. Phys. B406, 409 (1993); E. Ma and P. Roy, UCRHEP-T145; E. J. Chun, A. Joshipura and A. Smirnov, hep-ph/9505273; Z. Berezhiani and R. N. Mohapatra, hep-ph/9505385.

[46] G. Raffelt and J. Silk, Berkeley preprint, 1995; D. Caldwell and R. N. Mohapatra, Phys. Lett. B (to appear).

[47] D. G. Lee and R. N. Mohapatra, Phys. Lett. B329, 463 (1994).

[48] C. Burgess, this volume.

Figure Caption

Figure 1. Feynman diagram involving neutrino majorana mass that contributes to $\beta\beta_{0\nu}$ decay.

Figure 2. Left-right mixing graph for $\beta\beta_{0\nu}$ decay.

Figure 3. Heavy right handed neutrino contribution to $\beta\beta_{0\nu}$ in the left-right symmetric model.

Figure 4. Contribution of the doubly charged Higgs boson in the left-right symmetric model.

Figure 5. Bounds on the masses of $m_{W_R}$ and $m_N$ from $\beta\beta_{0\nu}$ lifetime and theoretical arguments of vacuum stability [20].

Figure 6. Bounds on the light and heavy neutrino mixing parameter in the left-right model from $^{76}Ge$ data.

Figure 7. Vector-scalar contribution to $\beta\beta_{0\nu}$ decay in the left-right symmetric model.

Figure 8. Gluino mediated contribution in MSSM with R-parity violation.

Figure 9. Vector-scalar contribution in MSSM with R-parity violation.
Figure 10. The shaded area in the figure represents the range of mixing and mass parameters of a specific heavy sterile neutrino which are allowed by low energy and cosmological bounds and where the $\beta\beta_{0\nu}$ amplitude is in the observable range.