Study of Geologically Consistent History Matching Peculiarities by Means of Gradient-Free Optimization Methods

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Abstract. This paper presents some results and distinctive features of the algorithm for automated history matching of 3D reservoir flow models using gradient-free methods. Widely used Nelder-Mead method was chosen for optimization. In order to preserve geological consistency of a model within the history matching process, control parameters are comprised of “porosity-to-permeability” relation parameters, anisotropic (semi)variogram parameters, and reservoir properties at pilot points. The control parameters are used at each iteration of the history matching process to generate porosity and permeability distributions based on static well data and perform flow simulations. Test studies were conducted for a specially developed synthetic model representing a inhomogeneous five-spot well pattern element. Oil flow rates at production wells and water flow rate at the injection well, as well as bottomhole pressures of all wells were chosen as measured dynamic data in the objective function. The influence of initial guess of parameter values and relative weights in the objective function on the quality of reconstruction of the ‘true’ solution and convergence rate were evaluated. Problems of gradient-free optimization methods application and specific features of residuals normalization in the objective function, as well as influence of pilot points are discussed.

1. Introduction

Reservoir flow models are based on two main data sources: geological data and well operation data. Quantitative geological information about reservoir parameters is acquired from core studies and / or interpretation of well logging data. All this information forms the basis for reservoir properties distribution within a geological model. At the same time, geological, or static data can only be obtained along a wellbore. Therefore, estimating reservoir property values, such as porosity and permeability, in the inter-well space represents itself a complex and ambiguous problem.

Based on a geological model, a 3D flow model is constructed. By means of the 3D flow model, dynamic data of well operation can be predicted. Usually it includes data on phases flow rates and bottomhole pressures at different times – for example, corresponding to the moments of measurement. Simulated well data predicted by the 3D flow model most often greatly differ from actually measured well operation data. The history matching procedure is aimed at minimizing this difference by estimating unknown or uncertain reservoir properties within reservoir volume based on the dynamic data measured during field development [1, 2].

Most often, history matching, or data assimilation, or identification of model parameters, is carried out manually. This approach introduces a significant share of subjectivity, and quality of results...
strongly depends on experience and knowledge of a reservoir engineer. Recently, automated or assisted history matching methods are becoming more common. Their application greatly speeds up the procedure, but usually does not allow to preserve important geological information about spatial distribution of reservoir properties [3, 4]. This paper presents some results of a study in the framework of geologically consistent history matching, aimed at solving this problem.

It was widely accepted after the paper [5] that PVT properties of reservoir fluids have minimal uncertainty among all input data for reservoir simulation – with the single exception of near-critical fluids where special efforts should be undertaken to describe volumetric properties properly [6]. Later in the paper we consider that reservoir properties (porosity and permeability) are the most uncertain parameters in flow simulation. However, relative permeabilities, reservoir geometry, aquifer properties and other parameters can also be taken into account in the general case [2].

In the next section, the concept of geologically consistent history matching will be introduced, and definition of the (semi)variogram being the main tool of two-point geostatistics will be given. Next, the inverse problem will be formulated and optimization methods for its solution will be presented. After the theoretical part, basic parameters of the synthetic model and a general description of the undertaken studies will be delivered. At the end, the main results and conclusions will be drawn, including some uncertainty analysis.

2. Geological consistency

In the proposed approach, preservation of geological principles employed in a 3D geological model during the data assimilation process should be achieved by identifying parameters of (semi)variograms involved in generation of reservoir property distributions within the 3D model, instead of direct estimation of reservoir property values in the model cells. Semivariogram, or simply variogram, is a quantitative measure of uncertainty in spatially distributed data. It represents variance of the difference in parameter values at two points as a function of distance and direction between them [7, 8]. Semivariogram values increase from its minimum value at zero, which is due to the variance of measurement errors and the nugget effect [7], to the maximum one at a distance where mutual correlation of parameter values at two points vanishes. To describe differences in spatial data distribution along different directions, anisotropic semivariogram is applied. The main (and usually the most uncertain) parameters of the anisotropic semivariogram are two perpendicular radii (ranges) in the horizontal plane of the model (along the major direction $R_1$ and minor direction $R_2$), radius (range) along the vertical axis $R_3$, and rotation angle of the major direction in the horizontal plane relatively to one of the coordinate axes. In this case, the direction of the major range $R_1$ is usually chosen along the main trend of property propagation in the reservoir.

An experimental semivariogram is constructed from static data along wells. It reflects spatial structure of reservoir properties distribution. And it is later fitted by the most appropriate theoretical model. To date, many different theoretical models of semivariograms have been developed, which make it possible to fairly well approximate different data. Throughout this study, the exponential variogram model was routinely applied in all calculations [8].

Semivariogram is permanently involved in all methods of the so-called two-point geostatistics [7]. There are both deterministic algorithms that represent interpolation of static well data and produce the most probable distribution model of reservoir properties, as well as stochastic methods that generate a set of equiprobable models (realizations). In this paper the most well-known among deterministic methods, namely, the point-wise kriging method was implemented. Within the kriging method, unknown value of a reservoir property is calculated as follows:

$$Z(x_0) = \sum_{i=1}^{N} \lambda_i z(x_i).$$  \hspace{1cm} (1)

Here $\lambda_i$ are the kriging weights, $x_0$ is a point where the property value is assessed, and $z(x_1), z(x_2), ..., z(x_N)$ are known values of the property at other points (static data at wells). The weights are chosen in accordance with the requirement that the estimate (1), in statistical sense, must
be unbiased and have minimal variance. This leads to the well-known set of \( N + 1 \) linear equations – the linear ordinary kriging system:

\[
\begin{bmatrix}
\gamma_{11} & \gamma_{12} & \cdots & \gamma_{1N} \\
\gamma_{21} & \gamma_{22} & \cdots & \gamma_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\gamma_{N1} & \gamma_{N2} & \cdots & \gamma_{NN}
\end{bmatrix}
\begin{bmatrix}
\lambda_1 \\
\lambda_2 \\
\vdots \\
\lambda_N
\end{bmatrix}
= \begin{bmatrix}
\gamma_{10} \\
\gamma_{20} \\
\vdots \\
\gamma_{N0} \\
\mu
\end{bmatrix},
\]

and the variance of the resulting estimate (1) can be expressed in terms of the variogram values:

\[
\sigma^2 = -\sum_i \sum_j \lambda_i \lambda_j \gamma_{ij} + 2 \sum_i \lambda_i \gamma_{i0} - \gamma_{00},
\]

where \( \gamma_{ij} \) is the variogram value between static data points \( i \) and \( j \), \( \gamma_{i0} \) – between the static data point \( i \) and the assessed point, \( \gamma_{00} \) is the magnitude of the nugget effect [7], \( \mu \) is the Lagrange multiplier for the condition of unbiased estimate.

3. Inverse problem

The inverse problem is formulated as a problem of minimizing the objective function (quality criterion). This function reflects the measure of consistency between the simulation results from a current 3D model and actually observed dynamic well data. The quality criterion can be represented as follows:

\[
J(\vec{u}) = \sum_{i=1}^{n} \sum_{j=1}^{k} w_i (y(\vec{u})_i^j - Y_i^j)^2 .
\]
4. Test model and problem statement

For the purpose of this study, a synthetic heterogeneous model of a five-spot pattern waterflood element (figure 1) was prepared with four production wells at the corners and a single injector in the center of the model. Model dimensions are $25 \times 25 \times 10 = 6250$ active cells, with the size of each cell equal to $100 \times 100 \times 10$ m. The top of the reservoir was set at the depth of 2400 m, the bottom is at 2500 m. Initial porosity values were set constant along the wellbores for the entire reservoir thickness. Permeability was calculated based on porosity. Table 1 shows specified values of reservoir properties at wells (static data) corresponding to a hypothetical ‘true’ model. This model was considered as the real one, and ‘actual’ well data dynamics were generated for it. Those dynamic data would be later considered as measured (observed), and they would be assimilated in the model through the inverse problem solution.

![Figure 1. The element of a five-spot waterflood pattern.](image)

| Well location | PROD11 | PROD125 | PROD2525 | PROD251 | INJ1313 |
|---------------|--------|---------|----------|---------|---------|
| Well          | Southwest corner | Northwest corner | Northeast corner | Southeast corner | Center  |
| Porosity      | 0.25   | 0.18    | 0.25     | 0.1     | 0.25    |
| Permeability, md | 900    | 255     | 900      | 60      | 900     |

Figure 2 displays porosity and permeability distributions for the ‘true’ model generated in accordance with the static well data. Porosity distribution was generated by kriging with the following variogram parameters: $R_1 = 1800$, $R_2 = 500$, $R_3 = 50$ m, $\varphi = 135^\circ$. Permeability $K$ (in millidarcy, md) was calculated through porosity $\varnothing$ by the relation $K = a \cdot \exp(b \cdot \varnothing)$, where $a = 10$, $b = 18$.

The element is located within the pure oil zone. Initial reservoir pressure was set to 250 bar. Other parameters of the reservoir and fluids were specified in accordance with data of a real reservoir.

Considered period of model operation comprised 15 years with monthly measurements of dynamic well data, which gave 180 measurement points in total. From the beginning of production, waterflood was implemented. At the injector, flow rate was specified at 14,400 m$^3$/day, with bottomhole pressure limited to not exceed 400 bar. At the production wells, liquid flow rates were set at 2700, 5500, 900, and 5500 m$^3$/day respectively, going clockwise from the northwest to the southwest corner.
Bottomhole pressures for all producing wells were limited to being not less than 130 bar. Figure 3 reflects initial and final states of the model in terms of oil saturation.

To solve the inverse problem, a software module has been developed that implements an iterative procedure for minimizing the objective function according to the Nelder-Mead method. At each iteration, distributions of reservoir properties are revised, and the OPMFlow simulator is run to solve the forward problem. The maximum number of optimization algorithm iterations was limited to 100, while the number of iterations without improvement of the objective function value was limited to 10.

The following parameters were selected as the control parameters: semivariogram ranges in the major ($R_1$) and minor ($R_2$) directions within the horizontal plane, the rotation angle $\varphi$ of the major direction, as well as the $a$ and $b$ parameters of the porosity-to-permeability dependency.

Two different initial guesses were considered for the inverse problem. For the first initial guess, all parameters, with exception for $R_2$, were underestimated as compared to their ‘true’ values; for the second initial guess, vice-versa, they were overestimated, and only the $R_2$ was underestimated (table 2).

**Figure 2.** The ‘true’ distributions of porosity (left) and permeability (right, md).

**Figure 3.** Initial (left) and final (right) oil saturation distributions in the ‘true’ model.
Terms within the objective function (4) correspond to the model misfit by oil production / water injection rates at the production / injection wells, as well as misfit by bottomhole pressures for all wells. Relative weights of these terms in the objective function were specified based on the following principle. For the initial guess model, the ratio between contributions to the objective function from total misfit in production / injection flow rates and total misfit in bottomhole pressures was set equal to a specified value from 0 to 100%, with 5 options available. Thus, the inverse problem was solved for 10 different cases with 2 initial guesses and 5 values of the ratio.

Table 2. **Results of the inverse problem solution for different initial guesses and weights distribution between terms in the objective function**

| Summary table of history matching results for the synthetic model with different initial guesses and different weights distribution between objective function terms | Weights distribution between objective function terms | Relative weight for production well flow rates and injection rate | Relative weight for bottomhole pressures at production wells and injector |
|---|---|---|---|
| Initial guess 1 | 1 | 0.75 | 0.5 | 0.25 | 0 | 0 | 0.25 | 0.5 | 0.75 | 1 |
| Model number: | 1-1 | 1-2 | 1-3 | 1-4 | 1-5 | 1-6 | 1-7 | 1-8 | 1-9 | 1-10 |
| $R_1$ | 1600 | 1500 | 1650 | 1785 | 1800 | 1875 | 1900 | 1925 | 1950 | 1975 |
| $R_2$ | 700 | 930 | 800 | 416 | 404 | 364 | 342 | 320 | 298 | 276 |
| $\varphi$ | 110 | 124 | 127 | 128 | 129 | 117 | 113 | 109 | 105 | 101 |
| $a$ | 8 | 10 | 12 | 14 | 16 | 9 | 11 | 13 | 15 | 17 |
| $b$ | 16 | 17.8 | 17.8 | 18 | 18 | 18 | 18 | 18 | 18 | 18 |
| misfit\textsubscript{total} | 2.47 | 1.78 | 0.17 | 0.12 | 1.5 | 2.47 | 1.66 | 0.14 | 0.08 | 0 |
| misfit\textsubscript{rates} | 2.47 | 1.66 | 0.14 | 0.08 | 0 | 0 | 0.12 | 0.03 | 0.04 | 1.5 |
| misfit\textsubscript{pressure} | 0 | 0.12 | 0.03 | 0.04 | 1.5 | 0 | 0.12 | 0.03 | 0.04 | 1.5 |
| iterations | 75 | 59 | 64 | 56 | 27 | 16 | 22 | 27 | 42 | 62 |
| Initial guess 2 | 1 | 2-1 | 2-2 | 2-3 | 2-4 | 2-5 | 2-6 | 2-7 | 2-8 | 2-9 |
| Model number: | 2-1 | 2-2 | 2-3 | 2-4 | 2-5 | 2-6 | 2-7 | 2-8 | 2-9 | 2-10 |
| $R_1$ | 2000 | 1860 | 2015 | 1175 | 2100 | 2070 | 2100 | 2070 | 2100 | 2070 |
| $R_2$ | 300 | 784 | 795 | 1000 | 785 | 782 | 785 | 782 | 785 | 782 |
| $\varphi$ | 160 | 157 | 145 | 211 | 188 | 159 | 188 | 159 | 188 | 159 |
| $A$ | 12 | 14.4 | 11 | 8.9 | 9.7 | 10.5 | 9.7 | 10.5 | 9.7 | 10.5 |
| $B$ | 20 | 16.3 | 17.4 | 18.6 | 18.2 | 17.7 | 18.2 | 17.7 | 18.2 | 17.7 |
| misfit\textsubscript{total} | 5.26 | 2.33 | 3.59 | 1.96 | 0.46 | 5.26 | 2.26 | 2.92 | 1.34 | 0 |
| misfit\textsubscript{rates} | 5.26 | 2.26 | 2.92 | 1.34 | 0 | 0 | 0.07 | 0.67 | 0.62 | 0.46 |
| misfit\textsubscript{pressure} | 0 | 0.07 | 0.67 | 0.62 | 0.46 | 0 | 0.07 | 0.67 | 0.62 | 0.46 |
| iterations | 56 | 16 | 22 | 62 | 35 | 36 | 22 | 62 | 35 | 36 |

$R_1$ – semivariogram range in the major direction, m; $R_2$ – semivariogram range in the minor direction, m; $\varphi$ – rotation angle of the major direction; $a$ and $b$ - parameters of the ‘porosity-to-permeability’ dependence, $K = a \cdot \exp(b \cdot \varphi)$; misfit\textsubscript{total} – final (residual) objective function value; misfit\textsubscript{rates} – total residual misfit for production and injection flow rates; misfit\textsubscript{pressure} – total residual misfit for pressures; iterations – the number of algorithm iterations.
5. Results for the test model

Results of inverse problem solution for the cases considered are illustrated by comparing reservoir properties distributions: the ‘true’ one, the initial guess and the final (history matched) one. Also distributions of kriging variance for reservoir properties were calculated, which reflect uncertainty of estimated property values in each model cell. The farther from points with measured static data (wells), the higher the variance [7]. Variance distributions make it possible to visually assess the influence of each semivariogram parameter on the final distribution of reservoir properties (see figure 4c).

The summary of the inverse problem solution results is presented in table 2 for the cases considered. In figure 4b, permeability distributions, being the most informative, are presented for all the models (cases). Comparing figure 4b with the ‘true’ permeability distribution (figure 2), one can clearly see that the initial guess No. 1 (see figure 4a) makes it possible to solve the inverse problem much more accurately. Here, in general, correct configuration of reservoir properties distribution was restored in all the 5 cases, with expected improvement in the cases where both types of measured well data contribute to the objective function. At the same time, a fairly good approximation to the ‘true’ permeability distribution was provided, even despite the fact that not all the individual control parameters were restored quite accurately (see table 2).

For the second initial guess, the Nelder-Mead algorithm coped much worse with solving the inverse problem both in terms of general configuration of the resulting reservoir properties distribution (see figures 4b, 4c), and final values of individual control parameters (see table 2). Unexpectedly, inclusion of misfits in both types of measured well data into the objective function does not improve the solution, but even worsens the result. The best result was achieved in the case when only bottomhole pressure measurements at wells were taken into account (contribution of misfit in production / injection flow rates was zeroed).

Figure 5 displays dependences of residual misfit values in the objective function – total and for each of the two measured data types – versus weights specified for them. Pictures for the two initial guesses differ significantly. If for the first initial guess best results were expectedly obtained by combining the both types of measurements, for the second initial guess the better was the weight of the bottomhole pressure misfit, the better were the results. On the one hand, this shows greater sensitivity of the problem to bottomhole pressure observations. On the other hand, the final point found by the method was clearly far from the optimum of the objective function. The final residuals were several times higher than those for the initial guess 1, and the obtained estimates of control parameters and general configuration of reservoir properties distribution were far from the ‘true’ ones (see table 2 and figures 4b-4c).

These conclusions are confirmed by figures 6a and 6b. Those figures depict flow rates and pressure dynamics for the cases 1-3 and 2-3 (initial guesses 1 and 2, weights equally allocated between misfits for rates and pressures). Here dynamics of well data for the ‘true’ model, the initial model and the matched model are shown. It can be clearly seen that numerical values of the residual misfits decently reflect the correspondence between the ‘actual’ measurements and simulation results according to the history matched model. This also confirms that the initial misfit in bottomhole pressures for the initial guess No. 2 is much higher than for the initial guess No. 1.

To analyze the effectiveness of solution search by the Nelder-Mead algorithm, figures 7 and 8 are presented. They show misfits and control parameter values versus iteration number for the models 1-3 and 2-3. In figure 8, the parameters are normalized in such a way that 1 corresponds to the parameter value in the ‘true’ model. It is clearly seen that in the case of initial guess No. 1, most of the parameters approached their ‘true’ values by the 20th iteration and were only slightly adjusted further. Quite different situation was observed in the case of initial guess No. 2. The algorithm fell into a local minimum and could not get out of it, although misfits and parameter values were far from optimal. This feature characterizes the problem of finding a minimum by the Nelder-Mead method: despite the fact that it is gradient-free, it displays a possibility of stagnation at a non-optimal point and a strong sensitivity to the initial guess.
**Figure 4a.** Distributions of kriging variance (above) and permeability (below) for the initial guesses No. 1 (left) and No. 2 (right).

**Figure 4b.** Final permeability distributions for initial guesses No. 1 (above) and No. 2 (below) for different allocation of weights in the objective function according to table 2.

**Figure 4c.** Final kriging variance distributions for initial guesses No. 1 (above) and No. 2 (below) for different allocation of weights in the objective function according to table 2.
Figure 5. Dependence of misfits on weights allocation for initial guesses No. 1 (above) and No. 2 (below)

Figure 6a. Oil flow rate dynamics of wells PROD11 and PROD125 for initial guesses No. 1 (above) and No. 2 (below)
Figure 6b. Bottomhole pressure dynamics at wells PROD11 and PROD125 for initial guesses No. 1 (above) and No. 2 (below).

Figure 7. Misfits versus algorithm iteration number. Model with initial guesses No. 1 (above) and No. 2 (below). Weights equal to 0.5 for flow rates / bottomhole pressures.
Figure 8. Control parameter values versus algorithm iteration number. Model with initial guesses No. 1 (left) and No. 2 (right). Weights equal to 0.5 for flow rates / bottomhole pressures

It is also worth noting that final values of $R_2$ for all the cases differ significantly from the ‘true’ one. This fact indicates low sensitivity of well dynamics on $R_2$. This is confirmed by figures 2 and 4: change in this parameter affects practically only the width of the highly permeable ‘diagonal’. And its variation over a fairly wide range of values results in no significant change in reservoir properties in vicinity of production wells and rate of water breakthrough along the primarily SW-NE direction.

6. New version of the pilot-point method

History matching of 3D flow models is a complex, complicated problem. In order to reduce the number of control parameters, speed up the history matching procedure and make it more robust, various parametrization methods have been proposed. Parameterization is commonly used to match reservoir properties distributed across a 3D reservoir model, such as porosity and permeability. Zoning method [11], gradual deformation method [12], probability perturbation method [13], co-simulation perturbation method [14] and others are used for parameterization.

In this paper, another widely adopted approach to parameterization is applied, namely, the method of reference (pilot) points. Pilot points correspond to a set of model cells in the inter-well space. Values of control parameters in those points are included in the set of parameters identified through the inverse problem solution. Values in other model cells at each iteration of the inverse problem solution are computed by kriging on the basis of static well data and values at pilot points.
In the framework of geostatistically consistent history matching discussed in this paper, it is proposed to include the values of reservoir properties at individual pilot points in the vector of control parameters for the objective function (4), in addition to the variogram parameters (ranges, angle, and possibly sill) and parameters of porosity-to-permeability dependency for each facie. This ensures geological consistency of history matched models, but provides additional flexibility to identify reservoir properties in the inter-well space.

Initially, the method of pilot points was used to match groundwater models [15-17] and was later applied to history matching of hydrocarbon systems [18]. The main problem while applying this method consists in pilot point locations. In some papers, it was proposed to locate pilot points relatively evenly over the model. To select only those points that affect the objective function, it was proposed to conduct a sensitivity analysis of the objective function for different pilot points in a semi-automatic mode. This process was later automated: at each step, one point was added, and a sensitivity analysis was conducted to select its location zone [17]. Further modification of this method was proposed by adding the likelihood function to the objective function [19], which made the method more stable and improved the quality of reservoir inhomogeneities localization, which in turn made it possible to increase the number of pilot points applied. In modern studies, it is proposed to combine parametrization methods, for example, the method of gradual deformation and the pilot point method [20] or the pilot point method with the ensemble Kalman filter [21].

Below in this paper, a comparative analysis is performed for model history matching with standard control parameters and with inclusion of pilot points. Several options are considered, both with inclusion of single or several pilot points in different zones of the model.

7. Test study of geostatistically consistent history matching with pilot points

All test simulations described below were undertaken on the same synthetic two-phase (oil-water) model of a five-spot waterflood pattern as in previous sections of the paper (see figure 1), with four production wells at corners and a single injection well in the center. Details of the inverse problem statement and solution procedure were also already presented. The second (problematic) initial guess was chosen, which corresponded to the following parameter values: $R_1 = 2000$ m, $R_2 = 300$ m, $\varphi = 160^\circ$, $a = 12$, $b = 20$. Weights in the objective function (4) were set equal for misfits in production/injection flow rates and bottomhole pressures.

History matching was carried out for the model with different options: namely, without pilot points, with different locations of a single pilot point, with two and three pilot points (figure 9). Results of the inverse problem solution for the cases considered can be comprehensively illustrated by maps of reservoir properties distributions: the ‘true’ one, the initial guess and the history matching result. Also kriging variance maps for reservoir properties were calculated to reflect the uncertainty of estimated reservoir property values in each cell of the model with and without pilot points.

![Figure 9. Schematic of the five-spot waterflood pattern model with locations of pilot points](image-url)
Summary of the inverse problem solution results for the cases considered are presented in table 3. Figure 10 displays permeability and variance maps for models with a single pilot point in comparison with the model without pilot points. It can be clearly seen that the general pattern of the permeability distribution was accurately retrieved only in the case of pilot point 7-7 located on the diagonal along the major direction of properties propagation in the true model. Variation of parameter value at the pilot point has a strong impact on dynamic well data, and it makes possible to properly recover the general form of the permeability field. However, the property value at the pilot point itself was not accurately restored. In all the other cases, pilot points also have strong influence on the general form of the permeability field, but they create zones with overestimated permeability values in the neighborhood of their location and could not help to improve history matching results in terms of total misfit.

For cases with 2 and 3 pilot points (figure 11), similar results were obtained if compared to the single pilot point case. The permeability distribution was correctly retrieved only if pilot points were located at cells 7-7 and 7-19, namely, on diagonals between wells.

To further analyze the effectiveness of solution search with the Nelder-Mead algorithm we present figures 12 and 13. Those figures display misfit and parameter values versus iteration number for all cases with a single pilot point. Again, in figure 13 parameters were normalized in such a way that 1 corresponds to the parameter value in cases with a single pilot point. Again, in figure 13 parameters were normalized in such a way that 1 corresponds to the parameter value in the ‘true’ model. It can be clearly seen that in the case of the pilot point 7-7 the objective function is significantly less than in cases with any other pilot point location or in the case without pilot points at all. But the control parameters in all cases were restored approximately equally good. And in the case of the pilot points 7–13 and 7–19, the algorithm even fell into the same local minimum and cannot get out of it, although parameter values were far from optimal. So, we face again the same problem of the Nelder-Mead method: stagnation at a non-optimal location or in the case without pilot points at all. But the control parameters in all cases were restored approximately equally good. And in the case of the pilot points 7–13 and 7–19, the algorithm even fell into the same local minimum and cannot get out of it, although parameter values were far from optimal. So, we face again the same problem of the Nelder-Mead method: stagnation at a non-optimal point with strong sensitivity to the initial guess.

Table 3. Parameters of the ‘true’ model, initial guess and history matching results for test cases with different pilot points.

| Pilot point locations (cell indices) | True model | Initial guess | Without pilot points | 1 pilot point | 2 pilot points | 3 pilot points |
|--------------------------------------|------------|---------------|---------------------|--------------|--------------|--------------|
| 7-7                                  | 1800       | 2000          | 1175                | 2112         | 1703         | 1703         |
| R_1                                 | 500        | 300           | 1000                | 691          | 670          | 670          |
| R_2                                 | 135        | 160           | 211                | 149          | 187          | 187          |
| var misunderstand                     | 10         | 12            | 8.9                 | 12           | 10.2         | 10.2         |
| A                                    | 18         | 20            | 18.6               | 17.2         | 18.3         | 18.3         |
| B                                    | 1.0        | -             | 0.87               | 1.00         | 1.07         | 0.8/1.0      |
| PORO                                 | -          | -             | -                  | 0.87         | 0.59         | 1/0.11       |
| misfit_total                         | 33.58      | 3.59          | 1.87               | 5.65         | 5.83         | 3.55         |
| misfit_rates                         | 16.74      | 2.92          | 1.42               | 4.05         | 4.05         | 2.81         |
| misfit_pressure                      | 16.84      | 0.67          | 0.45               | 1.6          | 1.77         | 0.74         |
| iterations                           | 22         | 32            | 15                 | 15           | 16           | 16           |

R_1 – semivariogram range in the major direction, m; R_2 – semivariogram range in the minor direction, m; \( \varphi \) – rotation angle of the major direction; 
\( a \) and \( b \) – parameters of the ‘porosity-to-permeability’ dependence, \( K = a \cdot \exp(b \cdot \varphi) \);
PORO – porosity value(s) at reference point(s), normalized to the ‘true’ one(s);
misfit_total – final (residual) objective function value;
misfit_rates – total residual misfit for production and injection flow rates;
misfit_pressure – total residual misfit for pressures;
iterations – the number of algorithm iterations.
Figure 10. Final permeability and kriging variance maps for cases with a single pilot point.

Figure 11. Final permeability and kriging variance maps for cases with two and three pilot points.
Figure 12. Misfits versus iteration number for cases without pilot points or with one pilot point.
Figure 13. Control parameter values versus iteration number for cases without pilot points or with one pilot point.
8. Conclusions

Results of the performed studies demonstrate some distinctive features of inverse problems solution for history matching 3D flow models in geologically consistent formulation with simultaneous evaluation of anisotropic semivariogram parameters and parameters of “porosity-to-permeability” dependency for several facies. In particular, the following conclusions are confirmed and clearly shown.

- Correct restoration of principal direction and, in general, overall map of reservoir properties distribution in case of significant anisotropy is quite possible.
- Choice of relative weights for misfits of various measurement types within the objective function has strong impact on results of the inverse problem solution.
- Accuracy (uncertainty) of individual control parameters evaluation is highly dependent on their influence on well performance dynamics through key configuration features of reservoir properties distribution.

At the same time, regarding application of gradient-free methods for this type of problems, in particular, the popular Nelder-Mead method, we can draw the following conclusions.

- Gradient-free minimization methods are simple to implement and allow an easy use of existing code for 3D flow simulation, but they do not always provide satisfactory results for inverse problem solution.
- Despite the lack of theoretical link to local extremum points, effectiveness of history matching using gradient-free methods can be strongly dependent on initial guess. At the same time, lack of information on the norm of the objective function gradient makes it impossible to assess whether the point of stagnation is really a local minimum.
- On the one hand, this should be taken into account in real cases, and various initial guesses should be considered. It is also fruitful to carry out trial ‘perturbations’ of parameters for exiting from such points and to investigate influence of measurement weights in the objective function.
- On the other hand, this feature deprives gradient-free methods of corresponding advantages over the more efficient, in other aspects, methods of smooth optimization. Their application to inverse problems solution in geologically consistent statement was also described in papers [3, 4, 10].

As well, results of the performed studies demonstrate several distinctive features of inverse problems solution with the use of pilot points.

- Inclusion of pilot points into the control parameter vector, together with variogram parameters, makes it possible to find adequate solution of the data assimilation problem. This solution can be further improved by gathering and matching new dynamic data.
- Pilot points make significant contribution to the quality and speed of model history matching. However, they can both improve and worsen the results. It is necessary to apply the criterion for selection of pilot points.
- The proposed algorithms provide a possibility of geologically consistent history matching with simultaneous preservation of both original geological principles used for construction of the 3D model and its compliance with dynamic production data.
- Simultaneous adjustment of variogram parameters and property values at pilot points makes it possible to extend capabilities of automated history matching methods, subject to introduction of automatic criteria for pilot points selection. The corresponding criteria could be developed, for example, by means of calculating objective function derivatives with respect to porosity and permeability of grid cells.

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