Form factors for semileptonic D decays

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We study the form factors for semileptonic decays of $D$-mesons. That is, we consider the matrix element of the weak left-handed quark current for the transitions $D \to P$ and $D \to V$, where $P$ and $V$ are light pseudoscalar or vector mesons, respectively. Our motivation to perform the present study of these form factors are future calculations of non-leptonic decay amplitudes.

We consider the form factors within a class of chiral quark models. Especially, we study how the Large Energy Effective Theory (LEET) limit works for $D$-meson decays. Compared to previous work we also introduce light vector mesons $V = \rho, K^*, ...$ within chiral quark models. In order to determine some of the parameters in our model, we use existing data and results based on some other methods like lattice calculations, light-cone sum rules, and heavy-light chiral perturbation theory. We also obtain some predictions within our framework.

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I. INTRODUCTION

In the present paper we study the form factors for semileptonic decays of $D$-mesons. Knowledge of the semi-leptonic form factors is of course necessary to calculate factorizable contributions to the non-leptonic decays of mesons. Further, knowledge about these form factors might determine or at least restrict some parameters of our models and thereby indirectly be of importance for our (model dependent) calculations for non-leptonic decays. We are of course aware of the technical challenges when calculating non-leptonic decays of $D$-mesons [1], and we will come back to this in a future publication.

The $D \rightarrow P$ and $D \rightarrow V$ form factors have been calculated by various methods. The various frameworks have their strength in different regions of the momentum transfer $q^2$ squared, from $q^2$ near zero for Light-cone sum rules (LCSR) [2–7] to $q^2 = q_{max}^2$ for Heavy-Light Chiral Perturbation Theory (HL\(\chi\)PT) [8–10]. In the region $q^2 \rightarrow 0$ where the momentum of the outgoing meson is high, one might study form factors within Large Energy Effective Theory (LEET) [11]. This effective theory was later further developed into Soft Collinear Effective Theory (SCET) [12].

In the region of large momentum transfer ($q^2 \rightarrow q_{max}^2$), lattice QCD can be used [13–16]. Form factors have been calculated within HL\(\chi\)PT, which is based on Heavy Quark Effective Theory (HQEFT). Calculations within HL\(\chi\)PT have also been supplemented by calculations within the Heavy-Light Chiral Quark Model (HL\(\chi\)QM) [8, 10, 20–22]. Within the heavy quark symmetry there are corrections of the order $O(1/m_c)$ which will be larger in the $D$ sector than in the $B$ sector. In any case the form factors are influenced by nearby meson poles.

Our intention is to find how well chiral quark models describe the form factors. Namely, in the next step we want to calculate nonfactorizable contributions to non-leptonic decays of $D$-mesons. Then we ought to know how well the chiral quark models work in various energy regions, and specifically we need to know the various form factors within LEET. Some form factors are relatively well known. But some of them might need additional model dependent study beyond leading order. Therefore these models will be shortly presented. Compared to previous work we will in this paper also include light vectors $V = (\rho, \omega, K^*)$. 
II. DECOMPOSITION OF SEMILEPTONIC FORM FACTORS

The $H \to P$ current $J_V^\mu(H \to P)$ is a vector current that depends on the involved momenta $p_H$ and $p$. This current can be decomposed into two form factors. We will consider two commonly used decompositions

\[ J_V^\mu(H \to P) = F_+(q^2) (p_H + p)^\mu + F_-(q^2) (p_H - p)^\mu , \]  

or

\[ J_V^\mu(H \to P) = F_1(q^2) \left[ (p_H + p)^\mu - \frac{(M_H^2 - m_P^2)}{q^2} q^\mu \right] + \frac{M_H^2 - m_P^2}{q^2} F_0(q^2) q^\mu , \]  

where $q = p_H - p$ is the momentum transfer. For decay to leptons $l$, $q^\mu L_\mu$ (where $L_\mu$ is the lepton current) is $\sim m_l$ and the amplitude is dominated by $F_1(q^2)$. The relations between the form factors in (1) and (2) are

\[ F_1 = F_+ \quad ; \quad F_0 = F_+ + \frac{q^2}{M_H^2 - m_P^2} F_- . \]  

The semileptonic decays of type $H \to V$, where $V = (\rho, K^*, \omega, \phi)$, with mass $m_V$, can proceed through both vector and axial currents. These can be decomposed into (in total) four form factors. The vector current depends on only one form factor $V(q^2)$, and is given by

\[ J_V^\mu(H \to V) = \langle V(p, \epsilon)|\bar{q}\gamma^\mu Q|H(p_H)\rangle = \frac{2V(q^2)}{M_H + m_V} \epsilon^{\nu\rho\sigma} \epsilon^*_\nu p_\rho (p_H)_\sigma , \]  

while the axial current includes 3 form factors $A_0, A_1, A_2$:

\[ J_A^\mu(H \to V) = \langle V(p, \epsilon)|\bar{q}\gamma^\mu\gamma_5 Q|H\rangle = (M_H + m_V) \left( \epsilon^*\nu - \frac{(\epsilon^* \cdot q)}{q^2} q^\nu \right) A_1(q^2) - \left( (p + p_H)^\mu - \frac{M_H^2 - m_P^2}{q^2} q^\mu \right) \frac{(\epsilon^* \cdot q)}{M_H + m_V} A_2(q^2) + \frac{2m_V(\epsilon^* \cdot q)}{q^2} q^\mu A_0(q^2) . \]
For the light leptons \((l = \mu, e)\) the amplitudes for \(D \to V l \nu\) are dominated by the form factors \(V(q^2), A_1(q^2),\) and \(A_2(q^2)\). The vector form factor \(V(q^2)\) is dominated by vector resonances, while the \(A_1(q^2)\) and \(A_2(q^2)\) are dominated by axial resonances, and the \(A_0(q^2)\) form factor is dominated by the pseudoscalar resonances.

Bećirević and Kaidalov \[23\] proposed a double pole form for the \(F_+(q^2)\) function. This includes the pole at \(H^*\) for the first pole and a term that includes contributions for higher mass resonances in an effective pole. The form factors, \(F = F_+, V, A_0\) etc. can be written in the generic form:

\[
F(q^2) = \frac{F(0)}{[1 - \frac{q^2}{m_{pole}^2}][1 - \frac{\alpha q^2}{m_{pole}^2}]}, \tag{6}
\]

where the parameter \(\alpha\) parametrizes the contribution of the higher mass resonances into an effective pole.

### III. ASYMPTOTIC BEHAVIOR OF FORM FACTORS

HQET and LEET give constraints on the structure of the form factors. From HQET one can estimate the behavior of the form factors in the limit of zero recoil (see \[20\] and references therein):

\[
F_+ \sim \sqrt{M_H} \quad ; \quad F_- \sim \frac{1}{\sqrt{M_H}}. \tag{7}
\]

The form factors in the LEET limit, with \(p_H^\mu = M_H v^\mu\) and \(p = E n^\mu\), can be parametrized as \[11\]:

\[
\langle P | \bar{q} \gamma^\mu Q_v | H \rangle = 2E (\zeta n^\mu + \zeta_1 v^\mu). \tag{8}
\]

The four vectors \(v, n\) are given by \(v = (1; \vec{0})\) and \(n = (1; 0, 0, 1)\) in the rest frame of the decaying heavy meson. Here the \(\zeta\) should scale with energy \(E\) as:

\[
\zeta \equiv \zeta(M_H, E) = C \frac{\sqrt{M_H}}{E^2}, \quad C \sim (\Lambda_{QCD})^{3/2}, \quad \frac{\zeta_1}{\zeta} \sim \frac{1}{E}. \tag{9}
\]

In the limit \(M_H \to \infty\) and \(E \to \infty\), the ratio \(\zeta_1/\zeta \to 0\). LEET can be used to estimate form factors at large recoil, where the momentum carried by the electroweak bosons (\(W, Z, \gamma\)) is at a minimum, \(q^2 \to 0\). Using \[9\] for small \(q^2\) i.e. for \(E \simeq M_H/2\), one obtains the behavior

\[
F_+ \sim F_0 \sim \frac{1}{M^{3/2}_H}. \tag{10}
\]
For transitions $H(0^-) \to V(1^-)$ one obtains in the LEET limit ($M_H \to \infty$ and $E \to \infty$) for the vector current:

$$\langle V | \bar{q}\gamma^\mu Q_v | H \rangle = 2iE \zeta_\perp \epsilon^{\mu\nu\rho\sigma} v_\nu n_\rho \epsilon_\sigma^* .$$  \hspace{1cm} (11)$$

Here the form factor $\zeta_\perp$ scales in the same way as $\zeta$ in (9), but with a different factor $C$:

$$\zeta_\perp = C_\perp \sqrt{M_H \over E^2} .$$  \hspace{1cm} (12)$$

For the axial current, the corresponding matrix element should have the form

$$\langle V | \bar{q}\gamma^\mu \gamma^5 Q_v | H \rangle = 2E \zeta_\perp^{(a)} [\epsilon^{*\mu} - (\epsilon^* \cdot v) n^\mu] + 2m_V \zeta_{||} (\epsilon^* \cdot v) n^\mu .$$  \hspace{1cm} (13)$$

Here the form factor $\zeta_\perp^{(a)}$ is equal to $\zeta_\perp$ to leading order, and $\zeta_\perp^{(a)}$ and $\zeta_{||}$ scale in the same manner as $\zeta_\perp$ and $\zeta$.

We will need the following relations between the various form factors and the quantities $\zeta_i$ in the LEET case

$$F_1 = F_+ = \zeta + E \over M_H \zeta_1 ; \quad F_- = -\zeta + E \over M_H \zeta_1 ,$$  \hspace{1cm} (14)

It should be noted that in [11] $\zeta_1$ is neglected because it is suppressed by $1/E$.

We will also need the following relations between the various form factors $V, A_0, A_1, A_2$ and the quantities $\zeta_i$ in the LEET case [11]:

$$V = \left( 1 + {m_V \over M_H} \right) \zeta_\perp ; \quad A_0 = {m_V \over M_H} \zeta_\perp^{(a)} + \left( 1 - {m_V^2 \over M_H E} \right) \zeta_{||} ; \quad A_1 = \left( 2E \over M_H + m_V \right) \zeta_\perp^{(a)} ; \quad A_2 = \left( 1 + {m_V \over M_H} \right) \left[ \zeta_\perp^{(a)} - m_V \over E \zeta_{||} \right] ,$$  \hspace{1cm} (15)$$

which should be valid in the $q^2 \to 0$ limit. These form factors are plotted in section V.

**IV. HEAVY-LIGHT CHIRAL PERTURBATION THEORY (HL\(\chi\)PT)**

HL\(\chi\)PT is based on heavy Quark Effective Field Theory(HQEFT), where - to lowest (zeroth) order in $m_Q$ the $0^-$ and the $1^-$ are degenerate and described by the field and $H_v$ is the corresponding heavy $(0^-, 1^-)$ meson field:

$$H_v = P_+ (v \cdot P^* - i\gamma_5 P_v) ,$$  \hspace{1cm} (16)$$
where $P_+(v) = (1 + \gamma \cdot v)/2$ is a projection operator and $v$ is the velocity of the heavy quark. Further, $P_\mu$ is the $1^-$ field and $P_5$ the $0^-$ field. These mesonic fields enter the Lagrangian of HL$\chi$PT:

$$L_{HL\chi PT} = -Tr(\overline{H}_v i\mu \partial_\mu H_v) + Tr(\overline{H}_v^a H_v^b v_\mu \nu_{ba}) - g_A Tr(\overline{H}_v^a H_v^b \gamma_\mu \gamma_5 A_\mu^{ba}^\mu) ,$$

(17)

where $a, b$ are SU(3) flavor indices, and $g_A = 0.59$ is the axial coupling. Further, $\nu_\mu$ and $A_\mu$ are vector and axial vector fields, given by

$$\nu_\mu \equiv \frac{i}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) , \quad A_\mu \equiv -\frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) ,$$

(18)

where

$$\xi = \exp\{i\Pi/f\}, \quad \Pi = \begin{pmatrix}
\pi^0 + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+
\pi^- & -\pi^0 + \frac{\eta}{\sqrt{6}} & K^0
K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}}
\end{pmatrix} ,$$

(19)

where $\eta \equiv \eta_8$.

Based on the symmetry of HQEFT, the bosonized current for decay of a system with one heavy quark and one light quark ($Q_v \bar{q}$) forming $H_v$ is $[9, 24]$:

$$\overline{q}_L \gamma^\mu Q_v \rightarrow \frac{\alpha_H}{2} Tr \left[ \xi^\dagger \gamma^\mu L H_v \right] ,$$

(20)

where $Q_v$ is a heavy quark field, $v$ is its velocity, and $H_v$ is the corresponding heavy meson field. This bosonization has to be compared with the matrix elements defining the meson decay constants (where $H = B, D$) are the same when QCD corrections below $m_Q$ are neglected (see $[22, 25]$):

$$\alpha_H = f_H \sqrt{M_H} .$$

(21)

Using the double pole parametrization, form factors were calculated in $[18]$:

$$F_+(q^2_{\text{max}}) = \frac{\alpha_H}{2\sqrt{M_H f}} g_A \frac{M_H}{m_P + \Delta_H} + \frac{\tilde{\alpha}}{2\sqrt{M_H f}} \tilde{g} \frac{M_H}{m_P + \Delta H^*} .$$

(22)

The term with $\tilde{\alpha}$ and $\tilde{g}$ is the contribution from the higher resonances. (In $[20]$ the higher resonance term was not included. Instead some non-pole terms were included). One can also include light vectors with an effective coupling to heavy mesons, given by $[19]$:

$$L_{HHV} = i \frac{g_V}{\sqrt{2}} \lambda Tr \left( \overline{H}_v H_v \sigma_{\mu\nu} F_\mu^\nu \right) ,$$

(23)
where the coupling $g_V \simeq 5.9$ and

$$F_{V}^{\mu \nu} = \partial^{\mu} V^{\nu} - \partial^{\nu} V^{\mu} + [V^{\mu}, V^{\nu}] \ . \tag{24}$$

This term will give a dominating pole term in the $D \rightarrow V$ form factor similar to the one for $D \rightarrow P$ above. From (23) one obtains [19]:

$$V(q^2_{max}) = - \frac{\alpha_H}{2 \sqrt{M_H f}} \frac{g_V \lambda}{\sqrt{2}} \frac{M_H}{m_V + \Delta H^*} + \frac{\tilde{\alpha}}{2 \sqrt{M_H f}} \tilde{\lambda} \frac{M_H}{m_V + \Delta H^*} \ . \tag{25}$$

where the second term is coming from higher resonances. It might also be calculated in HL$\chi$QM following closely the calculation for $D^* \rightarrow D \gamma$ [26]. The coupling $\tilde{\lambda}$ is a corresponding term for higher resonances.

V. THE VARIOUS CHIRAL QUARK MODELS

Calculating the matrix elements of quark currents, we have used chiral quark models. Within such models one splits the various quark fields into different categories; the ordinary
soft quark fields $q$, the soft flavor rotated fields $\chi$ (representing soft constituent light quarks), the heavy quark field $Q_v$ and the light energetic quark field $q_n$. Moreover, these models might contain ordinary chiral meson fields of $\chi$PT, as well as energetic light meson fields. They contain light soft pseudoscalar mesons via $A_\mu$ and $\xi$, hard light pseudoscalar mesons in an octet filed $M_n$, the heavy meson fields of $H_v$ of HL$\chi$PT, and they might contain vector meson fields $V_\mu$ with low energy - or vector meson fields $V'_\mu$ with high energy. Here the subscript $n$ in $M_n$ and $V'_\mu$ refers to the hard momentum $p^\mu = E n^\mu$ in $[S]$. Below we will give a short descriptions of the various chiral quark models employed.

A. The $\chi$QM for low energy light quarks

For the pure light sector, the chiral quark model gives the interactions between light quarks and light pseudoscalar mesons. The $\chi$QM Lagrangian can be written as $[22, 27–30]$

\[ \mathcal{L}_{\chiQM} = \bar{q}(i\gamma^\mu D_\mu - \mathcal{M}_q)q - m(\bar{q}_R\Sigma^\dagger q_L + \bar{q}_L\Sigma q_R) , \]  

(26)

where $q$ is the light quark flavor triplet, $\mathcal{M}_q$ is the current mass matrix, and $\Sigma = \xi \cdot \xi$ contains the light pseudoscalar mesons. (The current mass term $\mathcal{M}_q$ will often be neglected). The covariant derivative $D_\mu$ contains soft gluons which might form gluon condensates within the model. The quantity $m$ is interpreted as the constituent light quark mass appearing after the spontaneous symmetry breaking $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$. The Lagrangian (26) can be transformed into a useful version in terms of the flavor rotated fields $\chi_{L,R}$:

\[ \chi_L = \xi^\dagger q_L \quad , \quad \chi_R = \xi q_R . \]  

(27)

The Lagrangian in (26) is then rewritten in the form:

\[ \mathcal{L}_{\chiQM} = \bar{\chi} [\gamma \cdot (iD + V) + \gamma \cdot A \gamma_5 - m] \chi - \bar{\chi} \tilde{M}_q \chi , \]  

(28)

where the fields $V$ and $A$ are given in equation (18), and where the term including the current mass matrix $\mathcal{M}_q$ is given by

\[ \tilde{M}_q = \tilde{M}_q^V + \tilde{M}_q^A \gamma_5 , \]  

(29)

where

\[ \tilde{M}_q^V = \frac{1}{2}(\xi \mathcal{M}_q \xi + \xi^\dagger \mathcal{M}_q^\dagger \xi^\dagger) \quad \text{and} \quad \tilde{M}_q^A = \frac{1}{2}(\xi \mathcal{M}_q \xi - \xi^\dagger \mathcal{M}_q^\dagger \xi^\dagger) . \]  

(30)

This term has to be taken into account when calculating $SU(3)$-breaking effects.
B. $\chi$QM including light vector mesons ($V\chi$QM)

The $V\chi$QM adds light vector mesons to the $\chi$QM. The vector meson fields $V_\mu$ are given as $\Pi$ in (19) with pseudoscalars $P = (\pi, K, \eta)$ replaced by vectors $V = (\rho, \omega, K^*, \phi)$:

$$V_\mu = \begin{pmatrix} \frac{\rho^0 + \omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\rho^0 + \omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\phi \end{pmatrix}. \quad (31)$$

These fields are coupled to the light quark fields by the interaction Lagrangian,

$$L_{IV} = h_V \bar{\chi} \gamma^\mu V_\mu \chi. \quad (32)$$

The coupling constant $h_V$ can be determined from the left-handed current for $vac \rightarrow V$ we find the $SU(3)$ octet current

$$J^a_\mu(vac \rightarrow V) = \frac{1}{2} m_V f_V Tr[\Lambda^a V_\mu], \quad (33)$$

where the quantity $\Lambda^a$ is given by $\Lambda^a = \xi \lambda^a \xi^\dagger$, and $\lambda^a$ is the relevant $SU(3)$ flavor matrix. For the currents we obtain

$$m_V f_V = \frac{1}{2} h_V \left( \frac{\langle \bar{q}q \rangle}{m} + f_\pi^2 - \frac{1}{8 m^2} \frac{\alpha_s}{\pi} G^2 \right), \quad (34)$$

which can be used to determine $h_V$. We find, by using $f_\rho \simeq 216$ MeV, that $h_V \simeq 7$ for standard values of $m$, $\langle \bar{q}q \rangle$ and $\langle \alpha_s \pi G^2 \rangle$. [20, 22, 26].

C. The Heavy-Light Chiral Quark Model HL$\chi$QM

The HL$\chi$QM adds heavy meson and heavy quark fields to the $\chi$QM. The heavy quark field $Q_v$ is related to the full field $Q(x)$ in the following way:

$$Q_v^{(\pm)}(x) = P_\pm e^{\mp im_v x} Q(x), \quad (35)$$

where $P_\pm$ are projection operators $P_\pm = (1 \pm \gamma \cdot v)/2$. The heavy quark propagator is $S_v(p) = P_+/ (v \cdot p)$. The Lagrangian for the heavy quarks is:

$$\mathcal{L}_{HQEFT} = \pm Q_v^{(\pm)} i v \cdot D Q_v^{(\pm)} + \mathcal{O}(m_Q^{-1}), \quad (36)$$

where $D_\mu$ is the covariant derivative containing the gluon fields.
To couple the heavy quarks to light pseudoscalar mesons there are additional meson-quark couplings within HLχQM [20]:

\[
L_{\text{int}} = -G_H \left[ \bar{\chi}_a \bar{H}_v^a Q_v + \bar{Q}_v H_v^a \chi_a \right],
\]

where \( a \) is a SU(3) flavor index, \( Q_v \) is the reduced heavy quark field in (35). The quark-meson coupling \( G_H \) is determined within the HLχQM to be [20]

\[
G_H^2 = \frac{2m_f^2}{f^2_\pi} \rho,
\]

where \( \rho \) is a hadronic quantity of order one [20].

The VχQM can be combined with HLχQM, to give a reasonable description of the weak current for \( D \)-meson decays \( D \to V \) [19]. The coupling of \( V^\mu \) to heavy mesons is given by eq. (17) with \( V^\mu \to h_V V^\mu \). In [19] the factor \( \lambda \) is found to be \( \lambda = -0.53 \text{ GeV}^{-1} \). It might also be calculated in HLχQM following closely the calculation for \( D^* \to D \gamma \) [26]. Using the results of [26], we obtain

\[
\lambda = -\frac{\sqrt{2} h_V \beta}{4g_V}
\]

which gives \( \lambda \approx -0.4 \text{ GeV}^{-1} \), in agreement with the value \( \lambda \approx -0.41 \text{ GeV}^{-1} \) in [9].

For the direct term \( J^\mu (H \to V) \) obtained from a quark loop diagram within like in Fig. 4 has the form

\[
J^\mu_{\text{tot}} (H_v \to V) = \text{Tr} \left\{ \xi^\dagger \gamma^\mu LH_v [A \gamma \cdot V + B \nu \cdot V] \right\},
\]

where \( A \) and \( B \) are hadronic parameters containing the couplings \( G_H \) and \( h_V \), gluon condensates and the constituent quark mass. This expression is analogous to eq. (28) in [20] for the case \( H \to P \). However, the \( D \to V \) form factor will be dominated by the pole term in Fig. 3, right, and we will not go further into the detailed structure of \( A \) and \( B \).

### D. The Large Energy Chiral Quark Model (LEχQM)

The LEχQM adds high energy light mesons and quarks to the χQM. Unfortunately, the combination of the standard version of LEET [31] with χQM will lead to infrared divergent loop integrals for \( n^2 = 0 \). Therefore, the following formalism is modified and instead of \( n^2 = 0 \), we use \( n^2 = \delta^2 \), with \( \delta = \nu/E \) where \( \nu \sim \Lambda_{QCD} \), such that \( \delta \ll 1 \). In the following we derive a modified LEET [11] where we keep \( \delta \neq 0 \) with \( \delta \ll 1 \). We call this construction...
LEET$\delta$ and define the *almost* light-like vectors

\[ n = (1, 0, 0, +\eta) \quad ; \quad \tilde{n} = (1, 0, 0, -\eta), \quad (41) \]

where $\eta = \sqrt{1 - \delta^2}$. This gives

\[ n^\mu + \tilde{n}^\mu = 2v^\mu, \quad n^2 = \tilde{n}^2 = \delta^2, \quad v \cdot n = v \cdot \tilde{n} = 1, \quad n \cdot \tilde{n} = 2 - \delta^2. \quad (42) \]

The LEET$\delta$ Lagrangian is \[^{32}L_{\text{LEET}\delta} = \bar{q}_n \left( \frac{\gamma \cdot \tilde{n} + \delta}{N} \right) (in \cdot D) q_n + O(E^{-1}), \quad (43)\]

where $N^2 = 2n \cdot \tilde{n}$. For $\delta \to 0$ this is the first part of the SCET Lagrangian. The quark propagator is

\[ S_n(k) = \frac{\gamma \cdot n}{N(n \cdot k)}, \quad (44) \]

which reduces to the LEET propagator in the limit $\delta \to 0$ (which also means $N \to 2$). For further details we refer to \[^{32}\].

The term $O(E^{-1})$ in (43) contains a term coming from the current mass $m_q$ for the light energetic quark(s). We have found that further development beyond \[^{32}\] gives the SU(3) breaking mass term:

\[ \Delta L_{\text{LEET}\delta}(m_q) = \frac{m_q}{E} \bar{q}_n \left( i\tilde{n} \cdot D - \frac{m_q}{2} \gamma \cdot \tilde{n} \right) q_n. \quad (45) \]

For hard light quarks and chiral quarks coupling to a hard light meson multiplet field $M$, we extend the ideas of $\chi$QM and HL$\chi$QM, and assume that the energetic light mesons couple to light quarks with a derivative coupling to an axial current:

\[ L_{\text{int}q} \sim \bar{q} \gamma_\mu \gamma_5 (i \partial^\mu M) q. \quad (46) \]

The outgoing light energetic mesons are described by an octet $3 \times 3$ matrix field $M = \exp (+iE n \cdot x) M_n$, where $M_n$ has the same form as $\Pi$ in (19):

\[ M_n = \begin{pmatrix} \pi^0_n \sqrt{2} + \frac{m_n}{\sqrt{6}} & \pi^+_n & K^+_n \\ \pi^-_n - \frac{m_n}{\sqrt{2}} & \frac{\pi^0_n}{\sqrt{6}} + \frac{m_n}{\sqrt{6}} & K^0_n \\ K^-_n & K^0_n & -2\frac{m_n}{\sqrt{6}} \end{pmatrix}. \quad (47) \]

Here $\pi^0_n, \pi^+_n, K^+_n$ etc. are the (reduced) meson fields with momentum $\sim E n^\mu$. Furthermore, $q_n$ is related to $M_n$ in the same manner as $Q_v$ is related to $H_v$. 
FIG. 4: Current matrix element in LE\(\chi\)QM. The double dashed line is the (external) heavy meson \(H_v\), and the dashed line with two arrows is the (external energetic light meson. The internal lines (double for heavy quark, single with two arrows is the energetic light quark \(q_n\) and with one arrow is the soft quark \(\chi\).

Combining (46) with the use of the rotated soft quark fields in (27) and using \(\partial^\mu \rightarrow iE n^\mu\) we arrive at the LE\(\chi\)QM interaction Lagrangian \([32]\):

\[
\mathcal{L}_{LE\chi QM} = G_A E\bar{\chi}(\gamma \cdot n) Z_nq_n + h.c.,
\]

(48)

where \(q_n\) is the reduced field corresponding to an energetic light quark having momentum fraction close to one (see (43)), and \(\chi\) represents a soft quark (see Eq. (27)). Further, \(G_A\) is an unknown coupling to be determined later by physical requirements. Further,

\[
Z_n = \xi M_R R - \xi^\dagger M_L L.
\]

(49)

Here \(M_L\) and \(M_R\) are both equal to \(M_n\), but they have formally different transformation properties.

Calculating the matrix elements of quark currents within LE\(\chi\)QM, we obtain \([32]\):

\[
\zeta = \frac{1}{4} m^2 G_H G_A F \sqrt{\frac{M_H}{E}},
\]

(50)

where the quantity \(F\) comes from loop integration in Fig. 4 (with soft gluons forming gluon condensates added) is \([32]\) :

\[
F = \frac{N_c}{16\pi} + \frac{3 f_\pi^2}{8m^2 \rho} (1 - g_A) - \frac{(24 - 7\pi)}{768 m^4} \left(\frac{\alpha_s}{\pi} G^2\right),
\]

(51)

which is numerically \(F \simeq 0.08\). In Fig. 5, the quantity \(F\) is plotted as function of the quark condensate for typical values of the constituent quark mass. We obtain the following
expression for the coupling constant

\[ G_A = \frac{4\zeta}{m^2 G_H F} \sqrt{\frac{E}{M_H}}, \]  

(52)

where \( \zeta \) is numerically known \[2, 7, 33\] to be \( \approx 0.3 \) for the transition \( B \rightarrow \pi \), but is larger, say \( \approx 0.6 \) for \( D \rightarrow \pi \) \[6\].

Within our model, the constituent light quark mass \( m \) is the analogue of \( \Lambda_{QCD} \). To see the behavior of \( G_A \) in terms of the energy \( E \) we therefore write \( C \) in (9) as \( C \equiv \hat{c} m^\frac{3}{2} \), and obtain

\[ G_A = \left( \frac{4\hat{c} f_\pi}{m F \sqrt{2\rho}} \right) \frac{1}{E^\frac{3}{2}}, \]  

(53)

which explicitly displays the behavior \( G_A \sim E^{-3/2} \). In terms of the number \( N_c \) of colors, \( f_\pi \sim \sqrt{N_c} \) and \( F \sim N_c \) which gives the behavior \( G_A \sim 1/\sqrt{N_c} \), i.e. the same behavior as for the coupling \( G_H \) in (37). We consider \( G_A \) to be an auxiliary quantity. Physical results are expressed in terms of form factors, and the bosonized current can now be written as

\[ J^\mu_{tot}(H_v \rightarrow M_n) = -2i\hat{\zeta} \sqrt{\frac{E}{M_H}} \text{Tr} \left\{ \gamma^\mu L H_v [\gamma \cdot n] \xi^\dagger M_L \right\}, \]  

(54)

if \( \zeta_1/\zeta \rightarrow 0 \) is used.

The LE\( \chi \)QM can be extended to include energetic vector mesons \( V^\mu_n \), in analogy with \( M_n \) in (47). In (46) derivative coupling was used for coupling of mesons to quarks through an axial vector field. This is in analogy with light mesons coupling to quarks in (28). As in eq. (46) we will use derivative coupling, here through the field \( F^\mu_\nu \) in (24):

\[ \mathcal{L}_{LE\chi V} \sim \bar{\chi} \sigma \cdot F_V \chi. \]  

(55)

It was found in \[32\] that derivative coupling gave the best description of the \( H \rightarrow P \) current. Using \( V \rightarrow \exp (iEn \cdot x) V_n \), we obtain an interaction (remember that \( \partial^\mu V_\mu = 0 \) implies \( n \cdot V_n = 0 \)):

\[ \mathcal{L}_{LE\chi V} = E G_V \bar{\chi} (\gamma \cdot n \gamma \cdot Z_n) q_n + h.c., \]  

(56)

where

\[ Z^\mu_n = V^\mu_n (\xi R + \xi^\dagger L), \]  

(57)
Here $\rho_n^0$, $\rho_n^+$, $K_n^{*+}$ etc. are the (reduced) vector meson fields corresponding to energetic light vector mesons with momentum $\sim E_n^\mu$. The coupling $G_V$ is determined by the experimental value for the form factors for $B \rightarrow \rho$ (for $B$- decays) or the $D \rightarrow \rho$ (for $D$- decays) at $q^2 = 0$ obtained either from experiment, lattice calculations or eventually LCSR calculations.

In our case where no extra soft pions are going out, we put $\xi \rightarrow 1$, and for the momentum space $V_n^\mu \rightarrow k_M \sqrt{E(\epsilon_V)}^\mu$, with the isospin factor $k_M = 1/\sqrt{2}$ for $\rho^0$ (while $k_M = 1$ for charged $\rho$’s). For the $D$-meson with spin-parity $0^-$ we have $H_v^{(+)} \rightarrow P_+(v)(-i\gamma_5)\sqrt{M_H}$. Using this, the involved traces are easily calculated, and we obtain $J_{tot}^\mu(H_v \rightarrow V_n)$ for the $H_v \rightarrow V_n$ transition.

Using the equations (52), (62), and (59), one obtains [32] the relations between $G_V$ and $\zeta$. The loop factor will also be $F$ in this case. The formulae relating $\zeta_\perp$ and $G_V$ will be exactly those relating $\zeta$ and $G_A$, that is

$$\zeta_\perp = \frac{1}{4} m^2 G_H G_V F \sqrt{\frac{M_H}{E}} ;$$

and so on, obtained by the replacements $\zeta \rightarrow \zeta_\perp$ and $G_A \rightarrow G_V$. in [52] and [53]. Here $\zeta_\perp$ is numerically known for $B \rightarrow \rho$ to be $\simeq 0.3 \ [3,7]$ and for $D \rightarrow \rho$ to be $\simeq 0.59$ from CLEO data [34].

The bosonized current for the vector case can, for $m/E << 1$ (implying also $\zeta_\perp \rightarrow \zeta_\perp$) be written as

$$J_{tot}^\mu(H_v \rightarrow V_n) = -2i \sqrt{\frac{E}{M_H}} \text{Tr} \left\{ \gamma^\mu L H_v \left( \zeta_\perp \gamma \cdot n - \frac{m_V}{m} \zeta_\parallel \right) \sigma \cdot F_n \xi^\dagger \left[ \gamma \cdot n \right] \right\} ,$$

where the tensor $F_n$ is given by [24] with $V_n$ given as in [58]. We find the following predictions within our model:

$$\zeta_\perp^{(a)} = \zeta_\perp + \frac{m}{E} \zeta_\parallel ; \zeta_\parallel = \frac{m F_\parallel}{m_V} \zeta_\perp ; \zeta_1 = \frac{m F_\parallel}{EF} \zeta ;$$

where

$$F_\parallel = \frac{N_c}{16\pi} + \frac{3 f_\pi^2}{8 m^2 \rho} (1 - g_A) + \frac{f_\pi^2}{2 m_\ell} + \frac{1}{48 m^4} (\frac{\alpha_s}{\pi} G^2) \left( \frac{7\pi}{16} - 2 \right) ,$$

and

$$V_n^\mu = \begin{pmatrix} \frac{\rho_n^0}{\sqrt{2}} + \frac{\omega_n}{\sqrt{2}} & \rho_n^+ & K_n^{*+} \\ -\frac{\rho_n^0}{\sqrt{2}} + \frac{\omega_n}{\sqrt{2}} & K_n^* & -\Phi_n \end{pmatrix} \mu .$$

(58)
is a loop function analogous to $F$ in (51). Here the appearance of $\ell = ln(2/\delta)$ is due to the infrared behavior of some of the loop integrals. Numerically one finds $F_{||} \simeq 0.24 \simeq 3F$.

The values for $F$ and $F_{||}$ are obtained with the simplified LEET propagator in (44). For the $B \to D$ case an extra $\Delta$ of order 20 MeV was used in the heavy quark propagator [35]. A similar assumption (which is closer to the SCET propagator) might be used here, leading to modified values of $F$ and $F_{||}$. We observe that although $\zeta_1 / \zeta \sim \frac{m}{E}$ as it should, but the numerical suppression is not strong because $F_{||} \simeq 3F$ and $\frac{m}{E}$ is not very small for $D$-meson decays.

So far we have considered the $SU(3)$-limit $m_q \to 0$. One may also calculate $SU(3)$ corrections from the mass correction Lagrangian in (45), for hard outgoing $s$-quarks. We find that the first order term does not contribute within LE$\chi$QM. The second order term in (45) contributes and gives terms suppressed by $m^2_s/(mE)$ compared to terms already calculated. These will therefore be discarded in this work. For decaying $B_s$ and $D_s$ there will be first order $m_s$ corrections from the ordinary light sector $\chi$QM, through mass terms in (30). However, these corrections must be considered together with meson loops. Some of these loops might be calculated as in chiral perturbation theory, while others are formally suppressed, and anyway problematic to handle within our formalism. Therefore we do not go further into these details.
VI. PLOTTING THE FORM FACTORS

In this section we plot form factors for $D \to P$ and $D \to V$ and as function of $q^2$. We have used input from experimental data \cite{34,36}, lattice gauge calculations \cite{13,14}, Light Cone Sum Rules (LCSR) \cite{2–7} and Light Front Quark Model \cite{37}. The plots do not include error bars because that would make them unreadable. For LEET, $q^2 = 0$ is the reference point, and the shape is determined by a single pole.

For HL$\chi$PT the no-recoil point ($q^2 = (q^2)_{max}$) is the reference point for plots, and the form factors might be calculated within the framework of HL$\chi$QM. The plots for $D \to P$ with $P = \pi, K, \eta$ will be different because of the different masses. But we have not explicitly calculated SU(3)-breaking effects, and (63) below should be valid in the SU(3)-limit $m_s \to 0$. This means the plots for $D \to \pi$ and $D \to \rho$ are most relevant for us. Still other plots are included for comparison. According to our model (see eq. (45)), SU(3) corrections due to hard $s$-quarks (as in $D \to K$ and $D \to K^*$ transitions) should be small, while SU(3) corrections due to soft $s$-quarks (as in decays of $D_s$) should be bigger, as pointed out at the end of section V.

From the plots we extract approximate values for the form factors $F_+(q^2), V(q^2), A_1(q^2)$ and $A_2(q^2)$ at $q^2 = 0$. These are collected in table I and II.

| Decay   | $F_+(0)$ | $F_+(0)_\chi$ | $\zeta$ | $\zeta_1$ |
|---------|----------|----------------|---------|-----------|
| $D \to \pi$ | 0.67     | 0.96           | 0.65    | 0.43      |
| $D \to K$    | 0.74     | 1.06           | 0.74    | 0.49      |
| $D_s \to K$  | 0.74     | 1.12           | 0.74    | 0.49      |
| $D \to \eta$ | 0.55     | 0.66           | 0.55    | 0.37      |
| $D \to \eta'$| 0.45     | 0.55           | 0.45    | 0.30      |

TABLE I: Form factors for $D \to P$ at $q^2 = 0$. The values for $F_+(0)$ are taken from data when available and from sum rules for $D \to \eta, \eta'$. The values for $F_+(0)_\chi$ are obtained from \cite{18}. A similar value (based on similar curve) was obtained in \cite{20}. HL$\chi$QM.

From the plots we extract approximate values for the form factors $F_+(q^2), V(q^2), A_1(q^2)$ and $A_2(q^2)$ at $q^2 = 0$. These are collected in table I and II. But some of the data are uncertain, and we must expect, say, of order 20% uncertainty. Using the relations (15) and
FIG. 6: $D \rightarrow P$ form factors comparing frameworks used: $\text{HL} \chi \text{PT}$ is from [18], LCSR 2000 from [4], LCSR 2009 from [38], LEET from [11], LFQM from [37], and Data is from [36].

| Decay   | $V(0)$ | $V(0)_\chi$ | $A_0(0)$ | $A_0(0)_\chi$ | $\zeta_\perp$ | $\zeta_\perp^{(a)}$ | $\zeta_\parallel$ | $A_1(0)$ | $A_2(0)$ |
|---------|--------|--------------|----------|---------------|----------------|-----------------|-----------------|----------|----------|
| $D \rightarrow \rho$ | 0.84   | 0.72         | 0.65     | 0.78          | 0.59           | 0.68            | 0.44            | 0.56     | 0.47     |
| $D \rightarrow K^*$ | 0.91   | 0.76         | 0.76     | 0.62          | 0.61           | 0.88            | 0.47            | 0.62     | 0.37     |
| $D_s \rightarrow K^*$ | 0.77   | 0.76         | 0.76     | 0.62          | 0.53           | 0.78            | 0.53            | 0.59     | 0.32     |

TABLE II: Form factors for $D \rightarrow V$ at $q^2 = 0$. The values for $V(0)$ and $A_0(0)$ are taken from sum rules for $D_s \rightarrow K^*$ and lattice calculations for $D \rightarrow K^*, \rho$. The values for $V(0)_\chi$ and $A_0(0)_\chi$ are obtained from $\text{HL} \chi \text{QM}$, including vectors.

(61), we will obtain a reasonable overall fit for the $\zeta$’s:

$$
\zeta \simeq 0.5 \ , \ \zeta_1 \simeq 0.3 \ , \ \zeta_\perp \simeq 0.6 \ , \ \zeta_\parallel \simeq 0.5 \ , \ \zeta_\perp^{(a)} \simeq 0.7 \ .
$$

(63)
FIG. 7: $D \to V$ form factors comparing frameworks used: Data CLEO is from [34], LCSR 2006 from [39], LFQM from [37], Lattice 1998 from [13], Lattice 2002 from [14], HLχPT from [19], and QM is from [40].
FIG. 8: $D \rightarrow V$ form factors comparing frameworks used: Data CLEO is from [3, 4], LCSR 2006 from [39], LFQM from [37], Lattice 1998 from [13], Lattice 2002 from [14], HLRPT from [19], and QM is from [40].
VII. CONCLUSIONS

We have collected present information on various form factors for the transitions $D \to P$ and $D \to V$ ($P$ = pseudoscalar, $V$ = vector) obtained from various methods and sources like data, lattice gauge theory, LCSR, etc. From the curves we have as far as possible determined the values of relevant form factors at $q^2 = 0$, and then extracted values for the LEET form factors $\zeta_i$. Our LE$\chi$QM gives relations between the $\zeta_i$’s. We have previously found \cite{32} $\zeta_1/\zeta \sim m/E$. Here we have in addition shown that $\zeta^{(a)}_\perp \to \zeta_\perp$ for $m/E \to 0$ as it should.

The values of $V(0)$ from the plots show a large variation among the various methods used, which makes $\zeta_\perp$ uncertain. However $\zeta_\perp$ is also related to $A_0(0)$ such that we obtain a reasonable fit in eq. (63). We observe what we expected, namely that LEET works best for $q^2$ close to zero, while HL$\chi$QM (eventually supplemented by HL$\chi$QM) works best close to the no-recoil point.

The LEET form factors $\zeta$ and $\zeta_\perp$ will determine the coupling constants $G_A$ and $G_V$, which may be used in calculation of nonfactorizable (color suppressed) non-leptonic $D$-meson decays, in the same manner as have previously been done for $K \to \pi\pi$ \cite{30,41}, $D \to K^0\bar{K}^0$ \cite{42}, $B \to D\bar{D}$ \cite{43,44}, $B \to D\pi$ \cite{32}, and $B \to \pi^0\pi^0$ \cite{45}. Then non-leptonic decay amplitudes might be written in terms of the LEET form factors $\zeta_i$, both for the factorized and the color suppressed cases.

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