Testing the QCD fragmentation mechanism on heavy quarkonium production at LHC

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We calculate the fragmentation function for charm quark into J/ψ at the QCD next-to-leading-order (NLO) and find that the produced J/ψ is of larger momentum fraction than it is at the leading-order. Based on the fragmentation function and partonic processes calculated at the NLO, the transverse momentum distribution on J/ψ hadroproduction associated with a charm c (or \(\bar{c}\)) jet are predicted. We find that the distribution is enhanced by a factor of 2.0~3.3 at the NLO as \(p_t\) increased from 10 GeV to 100 GeV and it is measurable at the LHC with charm tagger. The measurement at the LHC will supply a first chance to directly test the QCD fragmentation mechanism on heavy quarkonium production where the fragmentation function is calculable in perturbative QCD. It is also applied to J/ψ (T) production in the decay of Z\(^0\) (top quark).

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Quantum Chromodynamics (QCD) is a successful theory to describe strong interaction, but its fundamental ingredients, the quarks and gluons, are not observed freely and must hadronize eventually. The fact makes it impossible to calculate any processes involving detected hadrons in the final or initial states directly. According to the QCD factorization theorem (see Ref. \cite{1} and references therein), in some kinematical regions, the dominant contribution to the cross section can be decomposed into three parts: the partonic part, the part of parton fragmentation into the produced hadron and the part of the parton distributions in the initial hadrons. The partonic part can be calculated perturbatively because of the asymptotic freedom of QCD, while all the long distance physics of the hadrons is put into the parton fragmentation functions and the parton distributions. Therefore, fragmentation functions is one of the most important ingredients to understand QCD or to make predictions for experimental measurements. It is hard to study fragmentation functions directly from QCD because of their non-perturbative nature.

For light hadrons, the fragmentation functions are extracted from global data fits. Recently the transverse-momentum \(p_T\) distribution of inclusive light-charged-particle production measured by the CDF shows significantly exceed on the theoretical prediction based on these fragmentation functions when \(p_T > 80\) GeV \cite{2}. It potentially challenges our understanding of QCD factorization theorem. However, for heavy quarkonium, the non-relativistic QCD (NRQCD) factorization formalism \cite{2} can be used to factorize the fragmentation functions for quarkonium into NRQCD matrix elements and short-distance factors, which are calculable in perturbation theory and have been studied in many works \cite{3 4 5} at QCD leading-order (LO). There are also QCD next-to-leading order (NLO) studies on the color-octet \(^3S_1\) gluon fragmentation function for heavy quarkonium \cite{7}, and the study on relativistic corrections for the fragmentation functions \cite{8}. The QCD factorization on heavy quarkonium production is investigated by many authors \cite{9}. Therefore, we have more prediction power to test QCD fragmentation mechanism on heavy quarkonium production. But until now, there is no available experimental measurement to test it.

In recent years, there is a huge data collection at colliders. Based on that, many J/ψ production processes were observed \cite{10} in the past. Therefore it supplies a very important chance to perform systematical study on J/ψ production both theoretically and experimentally. The large discrepancies for exclusive J/ψ productions at the B factories have been studied and resolved by introducing higher order corrections \cite{11}. The discrepancies for inclusive J/ψ production at the B factories have also been studied \cite{12 13} and the results including NLO QCD corrections can nearly explain the experimental data. Higher order corrections for J/ψ production at hadron colliders were also investigated \cite{14}. Although large improvement have been achieved in theoretical predictions, the experimental data are still unable to fully understand, especially for the polarization of J/ψ.

Higher order contributions have shown their importance and the dominant production mechanism for heavy quarkonium is fragmentation at large transverse momentum region. To achieve reasonable theoretical predictions for J/ψ production at large \(p_t\), it is important to study higher order contribution to fragmentation functions for heavy quarkonium states. Moreover, the most important question here is: is there any chance to directly test the QCD fragmentation mechanism on heavy quarkonium production at the LHC? In this letter, we calculate the fragmentation functions for charm quark into J/ψ at QCD NLO. Based on the fragmentation function and partonic processes calculated at the NLO, the transverse momentum distribution on J/ψ production associated with a charm c (or \(\bar{c}\)) jet are predicted. We find that the distribution is enhanced by a factor of 2.0~3.3 at the NLO as \(p_t\) increased from 10 GeV to 100 GeV and it is measurable at the LHC with charm tagger \cite{15}.
Here we study $J/\psi$ production in $e^+e^-\rightarrow J/\psi+X$ with a very high c.m. energy $\sqrt{s}$, which should be dominated by fragmentation mechanisms. The differential cross section for $J/\psi$ with momentum $p$ is

$$d\sigma[e^+e^-\rightarrow J/\psi(p)+X] = \sum_i \int dz d\sigma[e^+e^-\rightarrow i(p/z)+X,\mu_F] D_i\rightarrow J/\psi(z,\mu_F).$$

Where the factorization scale $\mu_F$ is introduced to maintain this factorized form and the dependence on the arbitrary scale $\mu_F$ cancels between the two factors. And $i$ could represent all quarks and gluon, but we only consider $i=c$ (i.e. $e^+e^-\rightarrow J/\psi+c\bar{c}$ channel) in the theoretical calculation for both side of Eq.(1) to extract charm quark $c$ to $J/\psi$ fragmentation function. Here we choose $\mu_F = 3m_c$ to avoid large logarithms of $\mu_F/m_c$ in the fragmentation function $D_{c(e)}\rightarrow J/\psi(z,\mu_F)$. Hereafter $D(z)$ is used to represent $D(z,3m_c)$. Thus we have

$$\frac{d\sigma_{e^+e^-\rightarrow J/\psi+X}}{dE_{J/\psi}} = 2 \int dE_{c} \frac{d\sigma_{e^+e^-\rightarrow c+X}}{dE_{c}} D_{c\rightarrow J/\psi}(z)$$

up to NLO in $\alpha_s$ with $z = E_{J/\psi}/E_c$. As mentioned in Refs. [1, 2], the initial fragmentation function can be calculated perturbatively as a series in $\alpha_s(2m_c)$ and extracted from Eq. (2) order by order. At leading order (LO) in $\alpha_s$, there are $E_c = E_c = \sqrt{s}/2$ and $z = 2E_{J/\psi}/\sqrt{s}$, and Eq. (2) is simplified into

$$\frac{d\sigma_{e^+e^-\rightarrow J/\psi+X}}{dE_{J/\psi}} = \frac{4}{\sqrt{s}} \sigma_{e^+e^-\rightarrow c+X} D_{c\rightarrow J/\psi}(z).$$

Thus the fragmentation function is extracted as:

$$D_{c\rightarrow J/\psi}(z) = \frac{1}{\alpha_s} \frac{d\sigma_{e^+e^-\rightarrow J/\psi+X}}{dE_{J/\psi}}, \sigma^* = \frac{1}{\sqrt{s}} \frac{d\sigma_{e^+e^-\rightarrow c+X}}{dE_{c}} (4)$$

Under the limitation $m_c/\sqrt{s} \rightarrow 0$, $\sigma^* = (64\pi\alpha^2)/(9s^{3/2})$ is obtained, and by using Eq.(4) in Ref. [13] we obtain

$$D_{c\rightarrow J/\psi}(z) = \frac{8\alpha_s(2m_c^2)|R_s(0)|^2}{27\pi m_c^3} \times \frac{z(1-z)^2(16-32z+72z^2-32z^3+5z^4)}{(2-z)^6} (5)$$

which is exactly the same as the one obtain from $Z_0$ decay in Ref. [13].

In the fix order calculation at NLO in $\alpha_s$, Eq. (2) becomes

$$\frac{d\sigma_{e^+e^-\rightarrow c+X}}{dE_{J/\psi}} = 2 \int dE_{c} \frac{d\sigma_{e^+e^-\rightarrow c+X}}{dE_{c}} D^{NLO}_{c\rightarrow J/\psi}(z)$$

$$+ 2 \int dE_{c} \frac{d\sigma_{e^+e^-\rightarrow c+X}}{dE_{c}} \frac{d\sigma_{e^+e^-\rightarrow c+X}}{dE_{c}} D^{LO}_{c\rightarrow J/\psi}(z). (6)$$

Then the NLO fragmentation function is expressed as

$$D^{NLO}_{c\rightarrow J/\psi}(z) = f_1(z) - f_2(z)$$

$$f_1(z) = \frac{1}{\sigma^*} \int \frac{d\sigma_{e^+e^-\rightarrow J/\psi+X}}{dE_{J/\psi}} \frac{d\sigma_{e^+e^-\rightarrow c+X}}{dE_{c}} D^{LO}_{c\rightarrow J/\psi}(z)$$

$$f_2(z) = \frac{2}{\sigma^*} \int \frac{d\sigma_{e^+e^-\rightarrow J/\psi+X}}{dE_{J/\psi}} \frac{d\sigma_{e^+e^-\rightarrow c+X}}{dE_{c}} D^{LO}_{c\rightarrow J/\psi}(z)$$

$f_1$ is the energy distribution of $J/\psi c\bar{c}$ production at NLO, which have been achieved in our previous work. And $f_2$ needs the LO fragmentation function in Eq. (5). It is hard to obtain an analytic result here and we have to do it numerically. We choose $m_c = 1.5$ GeV, $\alpha_s(2m_c) = 0.26$ and $|R_s(0)|^2 = 0.944$ GeV, and the renormalization scale $\mu_R = 2m_c$. The behaviors of $f_1(z)$ and $f_2(z)$ shown in Fig. 1 strongly depend on $\sqrt{s}$. In Fig. 2, the fragmentation functions extracted numerically at the LO and NLO show very good limitation as $\sqrt{s}$ increasing from 30 GeV to 1000 GeV and even the difference between the $\sqrt{s} = 30$ GeV and $\sqrt{s} = 1000$ GeV is quite small. It means that the fragmentation mechanism can describe the theoretical result even when the c.m. energy is as low as 30 GeV. For the NLO results, there exists unphysical range of negative possibility arising from fix-order perturbative calculation. The result with $\sqrt{s} = 1000$ GeV is a bit unstable even in our quadruple precision FORTRAN calculation because of large numerical cancellation. Therefore we choose the result at $\sqrt{s} = 1000$ GeV to approximate the final $D^{NLO}_{c\rightarrow J/\psi}(z)$. It is clearly shown in Fig. 2 that $J/\psi$ from the fragmentation is of larger momentum fraction at the NLO than that at the LO. On the other hand, $D^{NLO}_{c\rightarrow J/\psi}(z)$ can be expressed as

$$D^{NLO}_{c\rightarrow J/\psi}(z) = D^{LO}_{c\rightarrow J/\psi}(z)$$

$$\times \left\{ 1 + \frac{\alpha_s(\mu_R)}{\pi} \left[ a(z) + \beta_0 \ln \frac{\mu_R}{2m_c} \right] \right\}$$

where $\beta_0$ is the one-loop coefficient of QCD beta function. A 9th-polynomial fitting gives $a(z) = \sum_{i=0}^{9} c_i z^i$ with $c_0 = -4.14253418603 \times 10^1, c_1 = +1.08551074161 \times 10^2, c_2 = +1.6295354180 \times 10^3, c_3 = -1.7703935042 \times 10^4, c_4 = +7.6997417791 \times 10^4, c_5 = -1.8237353780 \times 10^5, c_6 = +3.648505719 \times 10^5, c_7 = -3.962871708 \times 10^5, c_8 = +2.206683868 \times 10^5, c_9 = -7.010907503 \times 10^5$. 

![Fig. 1: Behavior of $f_1(z)$ and $f_2(z)$ with $\mu_F = 3m_c$.](image)
For the top quark decay, the other one is to include the higher order term as a figure. This is the NLO fragmentation function of charm into $J/\psi$ at the initial factorization scale $\mu_F = 3m_c$. The fragmentation function at other factorization scale can be obtained with this initial one by solving the Altarelli-Parisi evolution equation \[16\]. In Fig. 2, the evolutions of the LO and NLO fragmentation functions are presented.

It is easy to do applications with this new fragmentation function. Firstly we study the decay of $Z^0$ into $J/\psi$ in $Z^0 \rightarrow J/\psi + c\bar{c}$. With the additional parameter $\alpha = 1/128$, the decay width at LO is

$$
\Gamma^{LO}_{J/\psi + X} = 2\Gamma^{LO}_{c+X} \int dz D^{LO}_{c\rightarrow J/\psi}(z) = 129 \text{ KeV}.
$$

At NLO, there are two ways to calculate the decay width. One is described by Eq. (6) where the higher order term is neglected

$$
\Gamma^{NLO}_{J/\psi + X} = 2\Gamma^{LO}_{c+X} \int dz D^{NLO}_{c\rightarrow J/\psi}(z) + 2 \int dE_c dz \frac{d\Gamma^{NLO}_{c+X}}{dE_c} D^{NLO}_{c\rightarrow J/\psi}(z) = 136 \text{ KeV}.
$$

The other one is to include the higher order term as

$$
\Gamma^{NLO+}_{J/\psi + X} = 2 \int dE_c dz \frac{d\Gamma^{NLO}_{c+X}}{dE_c} D^{NLO}_{c\rightarrow J/\psi}(z) = 141 \text{ KeV}.
$$

Both the LO and NLO results are consistent with the fully NLO QCD calculation \[17\], which gives 120 KeV at LO and 136 KeV at NLO with same parameters. The differences come from the fact that the limitation is not so well as the mass of $Z^0$ is not large enough to be treated as infinity. Secondly we apply the fragmentation function to $b$ quark case by substituting $m_c \leftrightarrow m_b, n_f = 4 \leftrightarrow n_f = 5, R_{s_{J/\psi}}(0) \leftrightarrow R_{s_{T}}(0)$.

For the top quark decay, $t \rightarrow \Upsilon + W^+ + b$, we have

$$
\Gamma^{LO}_{t \rightarrow \Upsilon + X} = \Gamma^{LO}_{t \rightarrow b + X} \int dz D^{LO}_{b\rightarrow \Upsilon}(z) = 30.9 \text{ KeV}. \quad (9)
$$

And the two corresponding NLO results are

$$
\begin{align*}
\Gamma^{NLO}_{t \rightarrow \Upsilon + X} &= \Gamma^{LO}_{t \rightarrow b + X} \int dz D^{NLO}_{b\rightarrow \Upsilon}(z) \\
&+ \int dE_b dz \frac{d\Gamma^{NLO}_{t \rightarrow b + X}}{dE_b} D^{NLO}_{b\rightarrow \Upsilon}(z) = 40.0 \text{ KeV}, \\
\Gamma^{NLO+}_{t \rightarrow \Upsilon + X} &= \int dE_b dz \frac{d\Gamma^{NLO+}_{t \rightarrow b + X}}{dE_b} D^{NLO+}_{b\rightarrow \Upsilon}(z) = 39.7 \text{ KeV}.
\end{align*}
$$

Here we choose the same parameters as those used in Ref. \[18\]. One should notice that the LO wave function at the origin is used in Eq. (9), while in other two results the NLO one is used. The corresponding LO and NLO results given by Ref. \[18\] are 26.8 and 52.3 KeV.

The most important application is to study the production of $J/\psi + c\bar{c} + X$ at the LHC. Because the fragmentation function $D_{g\rightarrow J/\psi}$ is suppressed comparing with $D_{(c\bar{c})\rightarrow J/\psi}$, we only consider the contribution from the charm quark fragmentation. The cross section is

$$
\begin{align*}
\sigma[pp \rightarrow J/\psi c\bar{c} + X] &= \sum_{i,j=g,q,\bar{q}} \int dx_1 dx_2 dz F_{i/p}(x_1, \mu_f) F_{j/p}(x_2, \mu_f) (10) \\
&\times d\tilde{\sigma}[ij \rightarrow c\bar{c} + X, \mu_f, \mu_r, \mu_F] D_{c\bar{c}\rightarrow J/\psi}(z, \mu_F),
\end{align*}
$$

where $f(x, \mu_f)$ is the parton distribution function, $\mu_f, \mu_r$ and $\mu_F$ are the factorization, renormalization and fragmentation scales respectively. $\tilde{\sigma}$ represents the cross sections of partonic process. The LO and NLO fragmentation functions are used to calculate the final LO and NLO results respectively. In the calculation, there are two subprocesses $gg(q\bar{q}) \rightarrow c\bar{c}$ at the LO, and three real corrections $gg \rightarrow c\bar{c} + g, qg(q) \rightarrow c\bar{c} + q\bar{q}j, q\bar{q} \rightarrow c\bar{c} + g$ and the virtual corrections to $gg(q\bar{q}) \rightarrow c\bar{c}$ at the NLO. The default choice of charm quark mass is $m_c = 1.5\text{GeV}$ and the three scales are set as $\mu_f = \mu_r = \mu_F = \mu$ with the default choice $\mu = \mu_0 = \sqrt{p_T^{(c\bar{c})} + m_c^2}$. We choose $m_c = 1.4 \text{GeV}$, 1.6 GeV and $\mu = \mu_0/2$, 2$\mu_0$ for uncertainty estimation. The Cteq6L1 and Cteq6M \[19\] are used in the LO and NLO calculations respectively, with the corresponding $\alpha_s$ running formula being used. The fragmentation function are evolved from $3m_c$ to $\mu_F$ by solving the Altarelli-Parisi equation numerically. The wave function at the origin of $J/\psi$ is extracted from its leptonic decay as in reference \[17\] at the NLO level and the rapidity cut for $J/\psi$ is $|y_{J/\psi}| > 2.4$.

The predictions for transverse momentum distributions of $J/\psi$ are shown in Fig. 3 with $\sqrt{s} = 7 \text{TeV}$ and $\sqrt{s} = 14 \text{TeV}$ at the LHC. The complete calculation of process $pp \rightarrow J/\psi + c\bar{c} + X$ at the LO are presented for comparison, which was studied in reference \[20\]. It will be dominated over as $p_t \geq 40 \text{GeV}$ by the LO fragmentation result and as $p_t \geq 16 \text{GeV}$ by the NLO fragmentation one, and is about 5 times smaller than the NLO fragmentation one in large $p_t$ region. It is believed.
that the fragmentation mechanism should give a better description on the $J/\psi$ hadroproduction associated with a charm c ($\bar{c}$) jet at large $p_t$ region than the complete calculation result.

We extracted the fragmentation function for charm into $J/\psi$ at the NLO in $\alpha_s$, $D_{c\to J/\psi}(z,3m_c)$, from $J/\psi$ production at $e^+e^-$ annihilation. The fragmentation function for bottom into $\Upsilon$, $D_{b\to \Upsilon}(z,3m_b)$, is also obtained. The Altarelli-Parisi evolution of the fragmentation function is performed to obtain its value at other fragmentation scale. We have applied them to $J/\psi$ production in $Z^0$ decay, and $\Upsilon$ production in top quark decay. The most important application is to predict the transverse momentum distribution of $J/\psi$ production associated with a charm c (or $\bar{c}$) jet at the LHC. We find that the distribution is enhanced by a factor of 2.0-3.3 at the NLO as $p_t$ increased from 10 GeV to 100 GeV and it is measurable at the LHC with charm tagger. In the measurement, it need to identify a $J/\psi$ and a charm jet without the $J/\psi$ in the jet and there will be 20 events to be found if we optimistically assume a 60% charm tagging efficiency, both $e$ and $\mu$ decay channel of $J/\psi$ being detected and $10 fb^{-1}$ of the integrated luminosity at a 14 TeV machine. Of course, this measurement is a very big challenge to the experimental technique on charm tagger. However, the measurement at the LHC will supply a first chance to directly test the QCD fragmentation mechanism on heavy quarkonium production where the fragmentation function is calculable in perturbative QCD.

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