Generation of convective cells by kinetic Alfvén waves

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Abstract. Nonlinear interaction between kinetic Alfvén waves (KAWs) and the electrostatic and magnetostatic convective cells in plasmas is considered here. It is shown that the KAWs in the kinetic regime can excite only magnetostatic convective cells, but those in the inertial regime can excite only electrostatic convective cells. Moreover, there is a preferred spatial scale for the instability-generated electrostatic cells, but not for the magnetostatic cells. The significance of the present results for space plasmas is discussed.
1. Introduction

Convective cells are two-dimensional quasistationary spatially periodic plasma flow structures perpendicular to the external magnetic field \([1, 2]\). They can be electrostatic or magnetostatic \([3–6]\) and are associated with the zonal and other large-scale flows in systems with rotation and/or more complex magnetic fields \([2, 7–9]\). The electrostatic convective cells (ECCs) can cause macroscopic plasma convection and can play an important role in plasma transport and space weather \([1, 2, 6, 10]\). The magnetostatic cells (MCs), on the other hand, are associated with quasistationary bending of the magnetic fields and can lead to the formation of magnetic islands and the enhancement of electron thermal transport \([1, 5, 11]\). The ECCs and MCs can lead to instabilities in the plasma by nonlinearly interacting with higher-frequency modes \([12–20]\). In the present study, we shall consider the nonlinear coupling of the ECC and MC modes with shear or kinetic Alfvén waves (KAWs).

KAWs are shear Alfvén waves having wavelengths (perpendicular to the ambient magnetic field, say \(B_0 = B_0 \hat{z}\)) comparable to the ion gyroradius or the electron inertial length \([21–26]\). The linear dispersion relation of the KAWs of interest here is \(\omega = k_z V_A \sqrt{1 + k_\perp^2 \rho_i^2} / (1 + k_\perp^2 \lambda_e^2)\) \([1, 26, 27]\), where \(V_A\) is the Alfvén speed, \(k_z\) and \(k_\perp\) are the parallel and perpendicular (to \(B_0\)) wave vectors, respectively, \(\rho = (\rho_i^2 + \rho_s^2)^{1/2}\) is an effective gyroradius, \(\rho_i\) is the ion gyroradius, \(\rho_s\) is the ion-acoustic gyroradius and \(\lambda_e\) is the electron inertial length. Thus, we have \(\omega = k_z V_A \sqrt{1 + k_\perp^2 \rho_i^2}\) in the limit \(k_\perp^2 \lambda_e^2 \ll 1\) and \(\omega = k_z V_A / \sqrt{1 + k_\perp^2 \lambda_e^2}\) in the limit \(k_\perp^2 \rho_i^2 \ll 1\) \([28–30]\). These two KAW regimes normally occur in \(m_e/m_i \ll \beta \ll 1\) and \(\beta \ll m_e/m_i\) plasmas, respectively. Here \(\beta\) is the ratio of thermal to magnetic pressure and \(m_e/m_i\) is the electron to ion mass ratio. KAWs in the two regimes have quite different perpendicular dispersion properties and both have been observed in laboratory experiments as well as space plasmas \([26, 31, 32]\). Sources of KAWs include mode conversion \([21]\), instabilities due to parametric effects \([33]\), particle beams and electric currents \([34]\), etc. These waves are closely involved in particle acceleration and heating of space and tokamak plasmas \([12, 35–37]\), and are believed to play important roles in the determination of solar and space weather \([10]\). Furthermore, the coupling of KAWs to the ECC and MC modes may be responsible for the nonlinear dynamics of the ionospheric Alfvén wave resonator and formation of the turbulent Alfvén boundary layer in the topside ionosphere and the auroral current system \([38, 39]\).

It has been shown \([37]\) that KAWs with very long perpendicular wavelengths \((k_\perp \rightarrow 0)\) cannot excite the ECC modes. When the perpendicular wavelength is finite, Sagdeev et al \([40]\),
and others [41] showed that ECCs can be excited by KAWs through a modulational instability in $m_e/m_i \ll \beta \ll 1$ cold-ion plasmas. However, other studies [42–44] found that ECCs can be excited by KAWs only if the ion temperature $T_i$ satisfies $T_i > 4T_e/3 > 0$. Furthermore, Volokitin and Dubinin [41] found that in $\beta \ll m_e/m_i$ plasmas the corresponding inertial KAWs cannot excite ECCs, although Pokhotelov et al [38] and Onishchenko et al [39] found that such an excitation can take place. On the other hand, Yu et al [45] showed that modulation instability of the KAWs can excite MCs in $m_e/m_i \ll \beta \ll 1$ cold-ion plasmas, and Gruzinov et al [46] showed that MC-related (fast) dynamo action can occur in the presence of KAW turbulence. Thus, there is a controversy concerning the conditions of ECC excitation by KAWs, and the excitation of MCs by KAWs has not been investigated in detail. Accordingly, in this paper we shall reconsider the modulational excitation of the ECCs and MCs by KAWs, in particular the role played by the ion temperature. The results show that KAWs can excite ECCs in the inertial regime and can excite MCs in the kinetic regime.

2. The governing equations

To investigate the coupling of KAWs to ECCs and MCs in low-beta ($\beta \ll 1$) plasmas, it is convenient to introduce the scalar potential $\varphi$ and the parallel (to $B_0$) vector potential $A_z\hat{z}$:

$$
E = -\nabla \varphi - \partial_t (A_z\hat{z}) \quad \text{and} \quad B = \nabla \times (A_z\hat{z}),
$$

where $E$ and $B$ are the electric and magnetic field perturbations, respectively. For low-frequency motion ($\partial_t \ll \omega_c$, where $\omega_c$ is the ion cyclotron frequency), one can obtain from the two-fluid equations after assuming quasineutrality [47–50]

$$
\partial_t \nabla^2 \varphi + V_A^2 \mathcal{R}_i \partial_z \nabla^2 A_z = -B_0 \mathcal{R}_{i\perp} \nabla \cdot \left[ \left( v_{i\perp} \nabla v_{i\perp} - \frac{V_A^2}{B_0^2} B_{\perp} \nabla B_{\perp} \right) \times \hat{z} \right],
$$

where $\mathcal{R}_i = 1 - \rho_i^2 \nabla^2$ and

$$
\mathcal{R}_{i\perp} \partial_t A_z + \mathcal{R} \partial_z \varphi = \mathcal{R}_{i\perp} \left( \frac{m_i}{e} v_{e\perp} \nabla v_{e\perp} + v_{e\perp} \times B_{\perp} \hat{z} \right),
$$

where $L = 1 - \lambda_i^2 \nabla^2$, $\mathcal{R} = 1 - \rho_i^2 \nabla^2$, $v_{i\perp}$ is the perpendicular ion velocity, $v_{e\perp}$ is the perpendicular electron velocity and $v_{e\parallel}$ is the parallel electron velocity. The nonlinear terms in equation (2) are from the perpendicular ion convective flow and the magnetic field bending, and the nonlinear terms in equation (3) are from the parallel electron inertial force and Lorentz force.

We decompose $\varphi$ and $A_z$ into their fast (KAW time scale) and slow (ECC or MC time scale) varying components:

$$
\varphi = \psi + \phi, \quad A_z = A + a,
$$

where $\psi$ and $A$ are for the KAWs and $\phi$ and $a$ are for the ECCs and MCs, respectively. The particle velocities $v_{i\perp}, v_{e\perp}$ and $v_{e\parallel}$, as well as the perturbed magnetic field $B_{\perp}$, are also decomposed into their fast and slow varying components, namely $v_{i\perp} = v_{i\perp K} + v_{i\perp C}, v_{e\perp} = v_{e\perp K} + v_{e\perp C}$ and $B_{\perp} = B_{\perp K} + B_{\perp C}$. The linear relations of the KAW quantities are

$$
\mathcal{R}_i v_{i\perp K} \simeq -\frac{1}{B_0} \nabla \psi \times \hat{z},
$$

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magnetic field are of the MCs which include the nonlinear effects arising from the perpendicular and the parallel (to \( L \) and \( R \)) nonlinear terms can be identified as the Reynolds stress tensor \( \nabla_t \) where \( \mu_0 \) is the free-space permeability, \( n_0 \) is the background particle number, the perpendicular velocities \( v_{\perp C} \) and \( v_{\perp L} \) are of the ECCs, and the electron parallel velocity and perturbed magnetic field are of the MCs [1].

One can obtain from equations (2)–(7) the equations governing the nonlinear coupling of the ECCs and KAWs. For the ECCs, we have

\[
\partial_t \nabla_\perp^2 \phi = -B_0 \mathcal{R}_1 \nabla_\perp \cdot \left( \frac{v_{\perp L} \cdot \nabla v_{\perp L} \times \hat{z}}{B_0} - \frac{V_A^2 B_{\perp K} \cdot \nabla B_{\perp K} \times \hat{z}}{B_0^2} \right),
\]

where the over bar denotes averaging over the fast, or Alfvén wave, time scale. The two nonlinear terms can be identified as the Reynolds stress tensor \( v_{\perp L} \cdot \nabla v_{\perp L} \times \hat{z} \) and the Maxwell stress tensor \( B_{\perp K} \cdot \nabla B_{\perp K} \times \hat{z} \), respectively. For the KAWs in the presence of ECCs, we have

\[
\mathcal{L} \partial_t^2 \nabla_\perp^2 \psi - V_A^2 \mathcal{R}_1 \partial_\perp^2 \nabla_\perp^2 \psi = -B_0 \mathcal{R}_1 \mathcal{L} \partial_t \nabla_\perp \cdot \left[ (v_{\perp C} \cdot \nabla v_{\perp L} + v_{\perp L} \cdot \nabla v_{\perp C}) \times \hat{z} \right] - \frac{m_e}{e} v_{\perp C} \cdot \nabla v_{\perp K} + (v_{\perp C} \times B_{\perp K}) \cdot \hat{z},
\]

which include the nonlinear effects arising from the perpendicular and the parallel (to \( B_0 \)) particle motion.

The coupling between the MCs and the KAWs is given by

\[
\mathcal{L} \partial_t a = \frac{m_e}{e} v_{\perp L} \cdot \nabla v_{\perp K} + (v_{\perp L} \times B_{\perp K}) \cdot \hat{z},
\]

and

\[
\mathcal{L} \partial_t^2 \nabla_\perp^2 \psi - V_A^2 \mathcal{R}_1 \partial_\perp^2 \nabla_\perp^2 \psi = \frac{V_A^2}{B_0} \mathcal{R}_1 \mathcal{L} \partial_t \nabla_\perp \cdot \left[ (B_{\perp C} \cdot \nabla B_{\perp K} + B_{\perp K} \cdot \nabla B_{\perp C}) \times \hat{z} \right] - \frac{m_e}{e} v_{\perp C} \cdot \nabla v_{\perp K} + (v_{\perp C} \times B_{\perp K}) \cdot \hat{z},
\]

and

\[
\mathcal{L} \partial_t^2 \nabla_\perp^2 A - V_A^2 \mathcal{R}_1 \partial_\perp^2 \nabla_\perp^2 A = -\frac{V_A^2}{B_0} \mathcal{R}_1 \partial_t \nabla_\perp \cdot \left[ (B_{\perp C} \cdot \nabla B_{\perp K} + B_{\perp K} \cdot \nabla B_{\perp C}) \times \hat{z} \right] + \partial_\perp^2 (v_{\perp K} \times B_{\perp C}) \cdot \hat{z}.
\]
Equation (11) shows that the nonlinear electron parallel convective inertia $v_{eL} \cdot \nabla v_{eK}$ and the Lorentz force $v_{eL} \times B_{L} \cdot \hat{z}$ drive the MCs. Equations (12) and (13) describe the nonlinear KAWs in the presence of the MCs. The nonlinear terms there are from the magnetic field bending and Lorentz force.

3. The generation of electrostatic convective cells (ECCs)

We now consider the modulation of a (pump) KAW by ECCs. The scalar potential of the ECCs can be written as

$$\phi = \phi_0 e^{-i(\Omega t - \mathbf{q} \cdot \mathbf{r})} + \text{c.c.},$$

where $\phi_0$, $\Omega$ and $\mathbf{q}$ are the amplitude, frequency and wave vector of the ECC potential.

The scalar and vector potentials of the pump and sideband KAWs can be written as

$$\psi = \psi_{k_0} e^{-i(\omega_0 t - \mathbf{k} \cdot \mathbf{r})} + \text{c.c.},$$

$$A = A_{k_0} e^{-i(\omega_0 t - \mathbf{k} \cdot \mathbf{r})} + \text{c.c.},$$

where the subscript 0 denotes the pump wave and $\pm$ denotes the upper (+) and lower (−) sidebands. In the interaction, the frequencies and wave vectors of the two sidebands approximately satisfy the matching conditions $\omega_\pm = \Omega \pm \omega_0$ and $\mathbf{k}_\pm = \mathbf{q}_\pm \pm \mathbf{k}_0$.

Substituting equations (14) and (15) into equation (8), we obtain

$$i \Omega \phi_0 = -\frac{R_{\omega} \tilde{q}_0}{B_0 q_{\perp}} \left\{ \frac{k_{\perp}^2 - k_{0 \perp}^2}{R_{\omega} R_{\omega^*}} \psi_{k_0} \psi_{k_0^*} - \frac{k_{\perp}^2 - k_{0 \perp}^2}{R_{\omega} R_{\omega^*}} \psi_{k_\perp} \psi_{k_0} - V_\perp^2 \left( k_{\perp}^2 - k_{0 \perp}^2 \right) A_{k_0} A_{k_0}^* - (k_{\perp}^2 - k_{0 \perp}^2) A_{k_\perp} A_{k_0} \right\},$$

where for simplicity and later use we have defined $\tilde{q}_0 = (q_{\perp} \times k_{0 \perp}) \cdot \hat{z}$, $R_{\omega,0,\pm} = 1 + k_{\perp,0,\pm}^2 \rho_1^2$ and $R_{0,\pm} = 1 + k_{0,\perp,\pm}^2 \rho_1^2$.

The expressions for $\psi_{\pm}$ and $A_{\pm}$ can be obtained from equations (9) and (10),

$$\psi_{k_\perp} = -i \frac{\tilde{q}_0 R_+}{B_0 D_+} \left( \omega_+ \mathbf{z}_1 + \frac{V_\perp^2 k_+}{L_+} \mathbf{z}_2 \right),$$

$$A_{k_\perp} = -i \frac{\tilde{q}_0 R_+}{B_0 D_+ L_+} \left( k_+ \mathbf{z}_1 - \frac{\omega_+}{R_+} \mathbf{z}_2 \right),$$

$$\psi_{k_\perp} = i \frac{\tilde{q}_0 R_-}{B_0 D_-} \left( \omega_- \mathbf{z}_3 + \frac{V_\perp^2 k_-}{L_-} \mathbf{z}_4 \right),$$

$$A_{k_\perp} = i \frac{\tilde{q}_0 R_-}{B_0 D_- L_-} \left( k_- \mathbf{z}_3 - \frac{\omega_-}{R_-} \mathbf{z}_4 \right),$$

where

$$L_{0,\pm} = 1 + k_{0,\perp,\pm}^2 \lambda_e^2,$$

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and
\[ \tilde{\Omega}_1 = \frac{q_1^2 - k_{0,1}^2}{k_{0,1}^2} \frac{1}{R_{10}} \phi_q \psi_{k_0}, \]
\[ \tilde{\Omega}_2 = L_0 \phi_q A_{k_0}, \]
\[ \tilde{\Omega}_3 = \frac{q_1^2 - k_{0,1}^2}{k_{0,1}^2} \frac{1}{R_{10}} \phi_q \psi_{k_0}^*, \]
\[ \tilde{\Omega}_4 = L_0 \phi_q A_{k_0}^*. \]

Note that \( D_\perp = \omega_\perp^2 - V_A^2 k_\perp^2 K_\perp^2 \) with \( K \sim \sqrt{R/L} \) [26–28].

For weak modulational instability, one assumes that the pump KAW is little affected by the wave interaction, so that the relation between \( \psi_{k_0} \) and \( A_{k_0} \) can be obtained from equation (9) as
\[ A_{k_0} = \frac{\omega_0}{R_{10} V_A^2 k_{0,c}} \psi_{k_0}. \] (19)

Substituting expressions (17)–(19) into (16), we obtain
\[ \tilde{\Omega} = -\frac{V_A}{B_0 q_1^2} \frac{R_{10} \tilde{q}_0}{R_0^2} |\psi_{k_0}|^2 \frac{k_{\perp}}{\Sigma_\pm} \frac{(K_{\pm} K_0 - 1)(k_{\perp,0}^2 - k_{\perp,1}^2)}{D_{\pm}} \left( \frac{q_1^2 - k_{0,1}^2}{k_{0,1}^2} K_{\pm} - \frac{L_0}{L_{\perp}} K_0 \right). \] (20)

The typical scale of the convective cells is usually much larger than that of the KAWs, or \( q_\perp \ll k_\perp \), and the perpendicular wavelength of the KAW is much larger than the ion gyroradius or the electron inertial length, i.e. \( \rho_\perp^2 k_\perp^2 \ll 1 \) or \( \lambda_\perp^2 k_\perp^2 \ll 1 \). Accordingly, we obtain
\[ R_{10} \approx R_{10} \approx 1, \]
\[ k_{\perp,0}^2 - k_{\perp,1}^2 = \pm 2 k_{0,1} \cdot q_\perp + q_\perp^2, \] (21)
\[ K_{\pm} K_0 - 1 \approx (\rho^2 - \lambda_\perp^2) k_{0,1}^2, \]
\[ D_{\pm} = \pm 2 V_A k_{0,c} \Omega - q_\perp \cdot v_g \mp \delta, \]

where \( v_g = V_A k_{0,c} (\rho^2 - \lambda_\perp^2) k_{0,1} \) is the group velocity of the pump wave and \( \delta = \frac{1}{2} V_A k_{0,c} (\rho^2 - \lambda_\perp^2) q_\perp^2 \). Equation (20) then reduces to
\[ \tilde{\Omega} = \frac{(\rho^2 - \lambda_\perp^2) k_{0,1}^2}{q_\perp^2} \frac{\tilde{q}_0^2}{B_0^2} |\psi_{k_0}|^2 \frac{\Sigma_\pm}{\Omega - q_\perp \cdot v_g + \delta} \frac{(\pm 2 k_{0,1} \cdot q_\perp + q_\perp^2)}{2 \Omega} \]
\[ = \left( \rho^2 - \lambda_\perp^2 \right) k_{0,1}^2 \frac{\tilde{q}_0^2}{B_0^2} |\psi_{k_0}|^2 \frac{2 \Omega}{\Omega - q_\perp \cdot v_g}^2 - \delta^2, \]
\[ (22) \]

and the solution of equation (22) is
\[ \tilde{\Omega} = q_\perp \cdot v_g \pm \sqrt{\delta^2 + 2 (\rho^2 - \lambda_\perp^2) k_{0,1}^2 \frac{\tilde{q}_0^2}{B_0^2} |\psi_{k_0}|^2}. \] (23)

In the inertial regime \( \beta \ll m_e / m_i \), the electron inertia length is much larger than the ion gyroradius \( \lambda_e \gg \rho \), and equation (23) becomes
\[ \tilde{\Omega} = q_\perp \cdot v_g \pm \sqrt{\delta^2 - 2 \lambda_\perp^2 k_{0,1}^2 \frac{\tilde{q}_0^2}{B_0^2} |\psi_{k_0}|^2}. \] (24)

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where \( \delta_1 = \frac{1}{2} V_A k_0 \lambda_c^2 q_\perp^2 \). Equation (24) shows that the instability can occur when

\[
2 \lambda_c^2 \kappa_\perp \frac{|q_0|}{B_0} |\psi_{k_0}|^2 > \delta_1^2
\]

(25)

or

\[
\frac{B^2_\perp}{B_0^2} > \frac{1}{8 \sin^2 \theta} \lambda_c^2 \kappa_\perp \frac{\kappa_\perp}{\kappa_\perp},
\]

(26)

where \( \theta \) is the angle between \( \mathbf{q}_\perp \) and \( k_{\perp 0} \), and the relation \( \psi_{k_0} \simeq V_A B_{k_0}/k_{\perp 0} \) has been used. The growth rate (24) is a function of the ECC wavenumber, and it is peaked at \( q_\perp = (4k_{\perp 0}^2 \sin^2 \theta / \lambda_c^2 k_{\perp 0}^2)(B_{\perp 0}^2 / B_0^2) \). The maximum growth rate is

\[
\gamma_{\text{max}} = \omega_0 \frac{B^2_\perp \sin^2 \theta}{B_0^2} |\psi_{k_0}|^2.
\]

For the kinetic regime \( n_e / m_i \ll \beta \ll 1 \), where \( \rho \gg \lambda_e \), equation (23) reduces to

\[
\Omega = q_\perp \cdot v_g \pm \sqrt{\delta_k^2 + 2 \rho^2 k_{\perp 0}^2 |q_0|^2 B_0^2 |\psi_{k_0}|^2},
\]

(28)

where \( \delta_k = \frac{1}{2} V_A k_0 \rho^2 q_\perp^2 \). Equation (28) shows that there is no ECC-modulated KAW instability in the kinetic regime.

4. The generation of magnetostatic convective cells

For MC modulation of KAWs, the vector potential of the MCs is given as \( a = a_q e^{-i(\Omega - \mathbf{q}_\perp \cdot \mathbf{r})} + \text{c.c.} \). From equation (11), we obtain

\[
i\Omega a_q = -\frac{q_0}{B_0 L_q} \left( \frac{R_0}{R_i} L_0 \psi_{k_\perp} A_{k_\perp} - \frac{R_+}{R_-} L_0 \psi_{k_\perp} A^*_0 + \frac{R_0}{R_i} L_0 \psi_{k_\perp} A_{k_\perp} - \frac{R_0}{R_i} L_0 \psi_{k_\perp} A^*_0 \right).
\]

(29)

The expressions for \( \psi_{\pm} \) and \( A_{\pm} \) have the same forms as those in equation (17), except that here \( \tilde{\sigma}_{1,2,3,4} \) are given by

\[
\tilde{\sigma}_1 = -V_A^2 q_\perp^2 \frac{q_\perp^2}{k_{\perp 0}^2} a_q A_{k_0},
\]
\[
\tilde{\sigma}_2 = -\frac{R_0}{R_i} a_q \psi_{k_0},
\]
\[
\tilde{\sigma}_3 = -V_A^2 q_\perp^2 \frac{q_\perp^2}{k_{\perp 0}^2} A^*_0 a_q,
\]
\[
\tilde{\sigma}_4 = -\frac{R_0}{R_i} a_q \psi_{k_0}^*.
\]

(30)

Inserting the expressions for \( \psi_{\pm} \) and \( A_{\pm} \) into equation (29) and using relations (19) and (21), we obtain

\[
\Omega = -\frac{q_\perp^2}{k_{\perp 0}^2} \left( \rho^2 - \lambda_c^2 k_{\perp 0}^2 \right) |q_0|^2 B_0^2 |\psi_{k_0}|^2 \left( \frac{\Omega}{\Omega - \mathbf{q}_\perp \cdot v_g} \right)^2 - \delta^2.
\]

(31)
The corresponding solution is
\[ \Omega = q_⊥ \cdot v_g \pm \sqrt{\delta^2 - (\rho^2 - \lambda^2) q_⊥^2 \frac{|\tilde{q}_0|^2}{B_0^2} |\psi_{k_0}|^2}. \] (32)

Thus, the solution for KAWs in the inertial regime is
\[ \Omega = q_⊥ \cdot v_g \pm \sqrt{\delta^2 + \lambda^2 q_⊥^2 \frac{|\tilde{q}_0|^2}{B_0^2} |\psi_{k_0}|^2}, \] (33)
which shows that the MCs cannot be excited by modulation instability of KAWs in the inertial regime.

In the kinetic regime, equation (32) becomes
\[ \Omega = q_⊥ \cdot v_g \pm \sqrt{\delta_K^2 - \rho^2 q_⊥^2 \frac{|\tilde{q}_0|^2}{B_0^2} |\psi_{k_0}|^2}, \] (34)
so that modulation instability of the KAWs can occur if
\[ \frac{|B_{k_0}|}{B_0} > \frac{\rho k_0 z}{2 \sin \theta}, \] (35)
and the growth rate is
\[ \gamma \simeq \omega_0 \rho q_⊥ \frac{q_⊥}{k_0 z} \frac{|B_{k_0}|^2}{B_0^2} \sin^2 \theta - \frac{\rho^2 k_0^2 z}{4}. \] (36)

5. Discussion

We can see from equation (24) that the growth rate of ECC modulation of the KAWs in the inertial regime is a function of the cell scale \( q_⊥ \) and it peaks at \( q_⊥ = q_{⊥\text{max}} \). There is therefore a characteristic or preferred spatial scale of the excited ECCs. Our result is qualitatively consistent with that of Pokhotelov et al [38]. In another work, Pokhotelov et al [42] considered the case \( T_i = 0 \) and showed that ECCs cannot be excited by the KAWs in the kinetic regime. Here we found that this conclusion also holds when the ion temperature is finite.

We also found that only KAWs in the kinetic regime can excite MCs. In contrast to the existence of a character scale for ECC excitation, the growth rate of the MCs increases with \( q_⊥ \), so that within our assumptions there is no characteristic scale for MC excitation. Our results also agree qualitatively with those of Yu et al [45]. However, in the latter the nonlinear effects originating from the parallel electron motion, which as shown here can be comparable to that of magnetic field bending, have been ignored.

Our results show that energy can be transferred from the KAWs to the much larger scaled and much slower varying cell modes. The latter can play important roles in the study of phenomena observed in space plasmas. For example, plasma flow associated with large-scale ECCs can excite Kelvin–Stuart vortex streets in the ionosphere [51], and the electron flow of the MCs can be responsible for the field-aligned auroral currents [38, 52]. The nonlinear coupling between the KAWs and the ECCs and MCs can also play a role in the ionospheric Alfvénic resonator [53, 54]. Moreover, the unstable KAW-driven cells can develop into large-amplitude self-organized structures [1]. The excitation of large-scale convective flows studied here is also
closely related to the excitation of zonal and geodesic flows by KAWs in tokamak plasmas. However, for application to the latter, geometrical effects, as well as background density and temperature gradients, have to be taken into account [2, 12, 13, 37, 55–59].

It should be mentioned that finite-amplitude KAWs can be involved also in other nonlinear interactions. In particular, they can participate in three-wave parametric decay among themselves ([49] and references therein; [60]). In the long-wavelength regime the decay is mainly into two waves propagating in opposite directions. In the inertial regime, the nonlinear growth rate of the KAWs approaches

$$\gamma_{\text{KAW}} \sim 0.1 V_A k_{0,\perp} \lambda_e k_{0,\perp} |B_{k_0}|/B_0,$$

and in the kinetic regime, it is given by

$$\gamma_{\text{KAW}} \sim 0.1 V_A k_{0,\perp} \rho_i k_{0,\perp} |B_{k_0}|/B_0.$$  

Taking into account the electron Landau damping $\gamma_D = \sqrt{\pi/8} v_T e k_z \lambda_e^2 k_{\perp}^2$, the existence condition for the decay is $\gamma_{\text{KAW}} > \gamma_D$, or

$$|B_{k_0}|/B_0 > 6(v_{Te}/V_A) \lambda_e k_{0,z}$$

in the inertial regime and

$$|B_{k_0}|/B_0 > 6(V_A/v_{Te}) \rho_i k_{0,z}$$

in the kinetic regime. Comparing equation (27) with (37) and equation (26) with (39), we obtain the following condition for the dominance of ECC generation over three-wave decay:

$$\frac{k_{0,z}}{k_{0,\perp}} > \frac{|B_{k_0}|}{B_0} > \frac{0.1 \lambda_e k_{0,\perp}}{k_{0,\perp}},$$

$$6 \frac{v_{Te}}{V_A} \lambda_e k_{0,z} > \frac{|B_{k_0}|}{B_0} > \frac{1}{2 \sqrt{2}} \lambda_e k_{0,z} \frac{q_{\perp}}{k_{0,\perp}}.$$  

where we have noted that the fastest growing ECC corresponds to $\theta = \pi/2$, and the left inequality in the first relation comes from the condition $\omega_0 > \gamma_{\text{max}}$. We also found by comparing equation (36) with (38) and equation (35) with (40) that the three-KAW decay always dominates over the parametric generation of MCs. That is, we do not expect large-amplitude KAWs to generate effectively MCs if the three-KAW decay conditions can be met.

There are many observations of kinetic and inertial AWs in space plasmas as well as the Earth’s magnetosphere and ionosphere. For typical parameter values at the altitude $\sim 1000$ km in the ionosphere [61, 62], the perpendicular wavelength is $\sim 0.1–10$ km, the frequency is $\sim 0.2–20$ Hz, the Alfvén speed is $\sim 10^6$ m s$^{-1}$, the electron inertial length is $\sim 80$ m, the perturbed magnetic field is $\sim 50$ nT and the background magnetic field is $\sim 0.1$ G. The parallel wavenumber can be estimated to be $k_z \sim 10$ Hz ($10^6$ m s$^{-1}$)$^{-1} \sim 10^{-5}$ m$^{-1}$. For $q_{\perp} = 10^{-2} k_{\perp}$, the condition for ECC generation obtained from equation (27) is $B_{k_0}/B_0 > 10^{-6}$. This is much smaller than the observed value $\delta B/B_0 \sim 5 \times 10^{-3}$. The maximum growth rate can be estimated to be $\gamma \sim 0.9$ s$^{-1}$ for the KAW with perpendicular wavelength $\sim 10$ km. The relative magnetic field perturbation $\delta B/B_0$ $\sim 5 \times 10^{-3}$ also satisfies the relation (41). Thus, KAWs can indeed generate ECCs in the ionosphere, and it may be worth searching for them in the existing and future data.

It is also of interest to examine if MCs can also be excited in space plasmas. In the plasma sheet boundary layer [63], one finds that the perpendicular wave scale is $\sim 20–120$ km, the wave frequency is $\sim 1$ Hz, the Alfvén speed is $\sim 10^7$ m s$^{-1}$, the ion gyroradius is $\sim 20$ km, the
disturbed magnetic field is $\sim 10 \text{ nT}$ and the background magnetic field is $\sim 400 \text{ nT}$. The parallel wavenumber is then $k_{\parallel} \sim \omega / V_A \sim 10^{-7} \text{ m}^{-1}$, and the condition for the MC generation would be $B_{k_0}/B_0 > 10^{-3}$. The latter is, however, smaller than the observed value $\delta B/B_0 \sim 2 \times 10^{-2}$, so that we do expect MC excitation by the KAWs. But here $\rho k_{\perp} \sim O(1)$, which would not be consistent with the long-wavelength assumption.

Finally, it should be mentioned that some of the assumptions made here, for example that the plasma is isothermal, collisionless and homogeneous and that the KAWs have relatively long perpendicular wavelengths ($k_{\perp} \rho \ll 1$ or $k_{\perp} \lambda_e \ll 1$), may not be satisfied in very-small-scaled local structures in space plasmas or in situations where shorter-wavelength KAWs are dominant [61]. In this case, further quantitative analysis would be needed.

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