A Natural Mechanism for Supersymmetry Breaking with Zero Cosmological Constant

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ABSTRACT

Supersymmetric theories in four dimensions with chiral superfields have very rich BRS cohomology, which gives rise to potential anomalies in theories that contain composite antichiral spinor superfields. Assuming the coefficients are non-zero, absence of the anomalies would generate new constraints on theories. In addition, the anomalies give rise to a new kind of supersymmetry breaking which is quite different from the known kinds, and also naturally yields a zero cosmological constant after supersymmetry breaking.
1 Introduction

There are some major problems in superstring theory that call for some new ideas. What is special about four dimensions of spacetime? What picks out the standard model at low energy? Why is supersymmetry broken? Why is the cosmological constant zero after supersymmetry breaking?

Some recent results in the BRS cohomology of N=1, D=4 supersymmetry suggest that the answers to some of these questions may be hidden in the subject of supersymmetry anomalies. The purpose of this article is to discuss the possibilities, and to try to point out what is known, what seems possible, and what is unknown but worth finding out.

One of the remarkable features of N=1, D=4 global supersymmetry is that when it is unbroken it naturally gives a zero energy density to the vacuum. Another feature that looks promising is that this is not spoiled by the spontaneous breaking of gauge and internal symmetries, which happens naturally and in many interesting ways in globally supersymmetric theories.

However, when global supersymmetry is spontaneously broken by the Fayet-Iliopoulos [1] or O’Raifeartaigh [2] mechanisms, the vacuum must acquire an energy density.

The unfortunate consequence of this is that while unbroken supersymmetry is not observed, so that supersymmetry must somehow be broken, the vacuum energy density generated when supersymmetry is spontaneously broken generates a cosmological constant that is far too large to be consistent
with the experimental size of the universe. And when global supersymmetry is extended to supergravity and superstring theories, this problem still tends to be present in the sense that a zero cosmological term requires an ‘unnatural’ fine tuning of the parameters of the theory.

A second problem of spontaneous supersymmetry breaking is that it is rather hard to achieve. Models tend to have a contrived look, with invariant chiral superfields or Abelian gauge groups needed. For phenomenological purposes, non-Abelian theories with simple gauge groups are more appealing in many ways. Furthermore, in global supersymmetry, there are sum rules regarding masses of broken supermultiplets that give trouble with phenomenology. Is there a way to break supersymmetry that is not ‘spontaneous’ or explicit?

As is well known, freedom from gauge and gravitational anomalies in the heterotic string puts stringent requirements on the possible Yang-Mills gauge group in ten space-time dimensions. Are there any more anomalies than these?

The existence of anomalies is governed by the BRS cohomology of the relevant symmetry. This is entirely unknown for any supersymmetric theory except rigid N=1, D=4 supersymmetry, where partial results have been obtained after a decade during which incorrect results were generally accepted. The incorrect results stated that the cohomology was trivial—no new anomalies were possible. The correct results indicate that the cohomol-
ogy is very complex in D=4. It is entirely possible that for D=10 supersymmetry, there are anomalies that render the theory inconsistent. This kind of result could account for the observed dimension of space-time. The calculation of the BRS cohomology for this case is a very non-trivial task however. It is also possible that the D=4 superstring may be subject to higher order constraints relating to the presently known cohomology of supersymmetry. To determine this one needs to extend the known results to local supersymmetry. This requires a careful demonstration, probably in component form, that antichiral spinor superfields couple to Poincare supergravity. It is shown in [6] that they do not couple to anti-de-Sitter supergravity. When this is done, it is probably straightforward to show that the rigid cohomology carries over to the local case. Let us assume that both of these work out.

In this situation, one can construct an explanation of supersymmetry breaking which accounts for the zero cosmological constant in a natural way. Let us assume that we have a four dimensional locally supersymmetric effective action (derived from the superstring) which is free of anomalies, including supersymmetry anomalies—this may be a very stringent requirement of course.

Then we introduce composite gauge-invariant antichiral spinor superfield operators to the theory by coupling them to external superfield sources. The BRS cohomology states that these new terms may be subject to anomalies. Recall that the observable states in our theories are in fact usually composite
and gauge-invariant. For example the proton is constructed from quarks, and the electron and photon are also composite in the sense that one must write them as composite using a Higgs field. The VEV of the Higgs allows a ‘single particle’ to appear from the composite field. These operators do not normally appear in the action.

It appears from the BRS cohomology of supersymmetry in D=4 that all these composite operators are subject to supersymmetry anomalies. So we would not expect to observe supersymmetry in the masses and interactions of the corresponding ‘bound states’. On the other hand, because the fundamental theory is anomaly free, the theory is consistent and unitary. So this is a new mechanism of supersymmetry breaking, and we could envisage using it alone without any spontaneous supersymmetry breaking at all.

The appealing feature of this is that this mechanism of supersymmetry breaking leaves the vacuum energy untouched. Now the vacuum energy is normally zero in supertheories that come out of the superstring. It is still zero after spontaneous breaking of gauge symmetries, which naturally occurs in supersymmetric theories as noted above, Hence, using the superanomaly mechanism above, the cosmological constant is naturally zero after gauge and supersymmetry breaking.

It is clear from the BRS cohomology that a very complicated supersymmetry breaking would indeed be induced in this way. Moreover supersymmetry breaks itself—the breaking occurs in all supersymmetric theories and it is not
a consequence of special tinkering with the theory to break it so that it can match experiment.

So where is the ‘catch’? The catch is that it is not yet known how to compute the coefficients of these anomalies—admittedly, if these are all zero, that is a very big catch indeed. It is probable that non-zero coefficients do not appear in one-loop calculations. However, despite the rough arguments in [3] to the effect that all the coefficients should be zero, I now believe that they do appear in higher orders of perturbation theory.

This is essentially because there does not appear to exist a gauge-invariant and supersymmetric regularization procedure for supersymmetric theories. It would be hard to define one without simultaneously eliminating the known axial anomalies, which would contradict the accepted view that these are present. Evidently this is a delicate problem.

Obviously it is somewhat unsatisfactory to offer physical interpretations before finding the coefficients. The excuse offered is that one needs a motivation to do the tricky calculations of the anomalies, and also of the BRS cohomology of rigid and local supersymmetry in various dimensions, and this paper is designed to supply that motivation.

Several objections might be made not counting the above catch.

1. The first objection is that it would be strange to have observable states that are not supersymmetric when the underlying theory is supersymmetric. I agree but do not yet see that it is also impossible— in fact it
is rather similar in some ways to spontaneous symmetry breaking.

2. It is totally unclear whether the splitting obtained in this way would match experiment, and that seems to involve an interesting and complex mixing problem. But presumably the predictions are at least testable.

3. Is it reasonable to state that the observable states of a theory are not the fundamental fields of the theory, but are actually composite operators? I think this is natural in the context of the standard model and our accepted beliefs about QCD and spontaneous breaking of gauge symmetry.

4. The supersymmetry anomalies probably occur only beyond one loop and they probably have higher order corrections. They also seem to involve mass parameters in general. This is all quite different from what happens for the known chiral anomalies. The chiral anomalies do not get renormalized because they are connected to the index theorem. Hence we might not expect an index theorem interpretation for the present anomalies.

5. This might also raise worries that the effects are too small to account for phenomenology. However, since the anomalies probably involve mass parameters as well as coupling constants, there is hope that large mass splittings can be generated.
Now let us get to facts and leave speculation.

2 Calculation of Coefficients of Anomalies

As is well known, the anomalies that can occur in a symmetry in field theory correspond to the local integrated polynomials with ghost charge one that lie in the BRS cohomology space of the BRS operator corresponding to the symmetry. These polynomials cannot be removed by adding renormalization counterterms. So once one knows the BRS cohomology of a theory, one knows the possible anomalies that can arise in that theory.

As mentioned above, the BRS cohomology of rigid N=1, D=4 supersymmetry is very rich and complex—it contains many potential anomalies ('holes') in the BRS cohomology [3] [4].

The only set of holes that have been studied so far are those which are coupled to a spin $\frac{1}{2}$ antichiral superfield. A completion of the BRS analysis would probably reveal that there are actually potential anomalies special to supersymmetry for all half integer spins $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$, and that these anomalies mix these operators with operators of all integer spins 0, 1, 2, ...

In order to truly qualify these holes as anomalies however, it is necessary that the relevant coefficients be calculated and that at least some of them are not zero. In this section we illustrate the general situation by trying to calculate a simple example.

One thing which is clear from the outset is that no supersymmetry anoma-
lies can occur unless there are chiral terms in the action of the general form:

\[
S = \int d^4x\, d^2\theta \left[ \frac{m}{4} e_{ij} S_i S_j + \frac{1}{6} g_{ijk} S_i S_j S_k \right]
+ \int d^4x\, d^2\theta \left[ \frac{m}{4} \bar{e}_{ij} \bar{S}_i \bar{S}_j + \frac{1}{6} \bar{g}_{ijk} \bar{S}_i \bar{S}_j \bar{S}_k \right]
\]  

(1)

where either \(e\) or \(g\) is not zero. Here \(m\) is a parameter with the dimension of mass, and \(\bar{\tau}\) is the complex conjugate of \(e\) etc. \(S\) are chiral superfields \((\bar{D}_\alpha S = 0)\) labelled by an index \(i\), which might be an isospin Yang-Mills index. These terms are needed because in all examples of possible supersymmetry anomalies it is necessary to start with chiral fields and end with antichiral fields and the only way to convert one to the other is with these chiral terms in the action.

So let us start with the above and the kinetic terms:

\[
S = \int d^4x\, d^4\theta \frac{1}{4} [\bar{S}_i S_i]
\]  

(2)

The simplest example of a potential anomaly in this theory arises when we add the following term to the action:

\[
\int d^4x\, d^4\theta \frac{1}{8m} e^{ijk} \Phi_\alpha^i S_j D_\alpha S_k = \int d^4x\, d^2\theta \frac{1}{8m} e^{ijk} \Phi_\alpha^i D^2(S_j D_\alpha S_k)
\]  

(3)

Here \(\Phi_\alpha\) is the massive antichiral spinor superfield discussed at length in [3]. For present purposes it can be regarded as an external superfield source subject to the (antichirality) constraint: \(D_\alpha \Phi_{\beta i} = 0\). Here and below we assume that \(\Phi_\alpha\) has its canonical dimension of \(\frac{1}{2}\) and we add the appropriate power of \(m\) to make the other coefficients like \(e^{ij}\) dimensionless. The fact
that a negative power of $m$ is needed in (3) shows that this coupling is non-renormalizable. The canonical dimension $(\frac{1}{2})$ of $\Phi_{\alpha i}$ is determined from its kinetic action, which is discussed in [3].

According to the BRS cohomology, this coupling could give rise to anomalies of the form:

$$\left[ c^\alpha Q_\alpha + \overline{\sigma} \overline{Q}_\alpha \right] \Gamma \equiv \delta \Gamma = \sum H_i$$

where ($c_\alpha$ is the constant supersymmetry ‘ghost’)

$$H_2 = \int d^4x \ d^2\overline{\theta} \ a_{ij}^2 \ m^2 \Phi_i^\alpha c_\alpha \overline{\sigma}_j$$

$$H_1 = \int d^4x \ d^2\overline{\theta} \ a_{i} \ m \Phi_i^\alpha c_\alpha \overline{\sigma}_j \overline{\sigma}_k + \cdots$$

$$H_0 = \int d^4x \ d^2\overline{\theta} \ a_{ijkl} \ m \Phi_i^\alpha c_\alpha \overline{\sigma}_j \overline{\sigma}_k \overline{\sigma}_l$$

The ‘hole’ $H_2$, for example, can come from the variation of the following non-local $\Gamma$:

$$\Gamma_{2-\text{anom}} = \frac{1}{4} a_{ij}^2 m^2 \int d^4x \ d^4\theta \ \Phi_i^\alpha \frac{D^2}{\Box} \sigma_{\alpha \beta} \partial_{\mu} \overline{Q}^\beta \overline{S}_j$$

$$= -\frac{1}{4} \ a_{ij}^2 m^2 \int d^4x \ d^4\theta \ \Phi_i^\alpha \frac{\theta^2}{\Box} \sigma_{\alpha \beta} \partial_{\mu} \overline{Q}^\beta \overline{S}_j$$

$$= a_{ij}^2 m^2 \int d^4x \ d^2\overline{\theta} \left\{ \Phi_i^\alpha \frac{1}{\Box} \sigma_{\alpha \beta} \partial_{\mu} \overline{Q}^\beta \overline{S}_j \right\}$$

Clearly this term violates supersymmetry and introduces a non-supersymmetric mass splitting between the two fields. If the theory gives rise to such a term, supersymmetry is violated.
In terms of components, this has the form:

$$\Gamma_{2-anom} = 2a^i \frac{1}{2} m^2 \int d^4 x \left[ W^\mu_i \frac{\partial}{\partial x^\mu} F_j + \chi_i^\alpha \sigma^\mu_{\alpha\beta} \frac{\partial}{\partial x^\mu} \tilde{\psi}^\beta \right]$$  \hspace{1cm} (10)$$

where we define $W^\mu = W^{\alpha\beta} \sigma^\mu_{\alpha\beta}$ and

$$\Phi_\alpha (x) = \phi_\alpha (y) + W_{\alpha\beta} (y) \bar{\theta}^\beta + \frac{1}{2} \chi_\alpha (y) \bar{\theta}^\beta \bar{\theta}^\beta \hspace{1cm} (11)$$

$$S(x) = A(y) + \theta \cdot \psi(y) + \frac{1}{2} \theta^2 F(y)$$  \hspace{1cm} (12)$$

and the variable $\bar{y}^\mu = x^\mu - \frac{1}{2} \theta^\alpha \sigma^\mu_{\alpha\beta} \bar{\theta}^\beta$ is the solution to

$$D_\alpha \bar{y}^\mu = \overline{D}_\alpha y^\mu = 0. \hspace{1cm} (13)$$

Note that an interesting and novel feature of the present situation is that it is possible that anomalies might arise that have this mass parameter in them as indicated above. This does not happen with the known chiral anomalies, which have parameters like $e_i$ only. It does happen in ten-dimensional gravity in a sense however. The difference is that in gravity (and also non-Abelian Yang-Mills) there are only a few holes corresponding to the chiral anomalies for a given dimension of spacetime, whereas here we have an infinite number of possibilities.

The relevant part of the action in components is:

$$S = \int d^4 x \left\{ A_i \bar{A}_i + \psi_i^\alpha \sigma^\mu_{\alpha\beta} \frac{\partial}{\partial x^\mu} \psi_\beta^{\dot{\beta}} + F_i \bar{F}_i ight\}$$

$$+ \frac{m}{2} (2A_i F_i + \psi_i \cdot \psi_i) + g^{ijk} [A_i A_j F_k + A_i \psi_j^{\alpha} \psi_{\alpha k}]$$

10
\[ + \frac{1}{m} e^{[i|j]} [\chi_j^\alpha F_{j\alpha} - W_i^{\alpha\dot{a}} (\sigma_{\dot{a}\dot{b}} \partial_\mu \psi_{\dot{b}}^\beta \psi_{\dot{a}}^\beta - \sigma_{\alpha\dot{a}} F_j \partial_\mu A_k) \\
+ \phi_i^\alpha \partial_\mu (\partial_\mu A_j \psi_{\dot{a}}^\alpha)] + \text{c.c.} \] (14)

The next stage is to see what the BRS identity predicts for the various two point functions that could give rise to the anomaly. A convenient form of the BRS identity is:

\[
\int d^4 x \left\{ e^\alpha \psi_{\dot{a}}^\alpha \delta \Gamma \Theta_i \delta A_i + \left[ -F_i e^\alpha + \partial_\mu A_i \sigma_{\alpha\beta} \sigma_{\dot{a}\dot{b}} \frac{\delta \Gamma}{\delta \psi_{\dot{a}}^\alpha} \right] \delta \Gamma \\
- \partial_\mu \psi_{\dot{a}}^\alpha \sigma_{\dot{a}\dot{b}} e^\beta \frac{\delta \Gamma}{\delta F_i} \\
+ c^\beta W_{\alpha\dot{b}i} \frac{\delta \Gamma}{\delta \phi_{\alpha i}} + \left[ -\chi_i^\alpha e^\beta + \partial_\mu \phi_{\alpha i} (\sigma^\mu)^{\beta\dot{a}} c_{\dot{a}} \right] \frac{\delta \Gamma}{\delta W_{\alpha\dot{b}i}} \\
- \partial_\mu W^{\alpha\dot{a}i} e^\beta \frac{\delta \Gamma}{\delta \chi_i^\alpha} + \text{c.c.} \right\} = H_2 (15)
\]

where the component form of \( H_2 \) is:

\[
H_2 = 2m^2 a_{ij} \int d^4 x \{ \chi_i^\alpha \overline{A}_j - W_i^{\alpha\dot{a}} \psi_{\dot{a}}^\beta + \phi_i^\alpha \overline{F}_j \} c_{\alpha} (16)
\]

and from this we can deduce, using functional differentiation and Fourier transform, identities in momentum space relating various n-point functions to the anomalies (if they are present).

So, for example, taking functional derivatives with respect to \( \overline{F}, \phi_{\alpha} \) and \( c_{\alpha} \), we get:

\[
k^{\mu} \Gamma_{ij\mu}(\overline{F}, W) = 4a_{ij} m^2 \] (17)

where \( a \) is the coefficient of the anomaly,

\[
\Gamma_{ij\mu}(\overline{F}, W) = \frac{\delta^2 \Gamma}{\delta F_i \delta W_j^\mu} \] (18)
So, for this example, we only have to calculate one 1PI diagram:

\[
\begin{array}{c}
A & \bar{A} \\
\hline
W_\mu & \bar{F} \\
F & \bar{A}
\end{array}
\]

This diagram is clearly zero because it would require a propagator which converts \( F_i \) into \( \bar{A}_j \) and this does not exist in the theory as it stands. The relevant bosonic kinetic term is:

\[
V^\dagger K V = \left( \bar{A}_i, A_i, \bar{F}_i, F_i \right) \begin{pmatrix}
-k^2 & 0 & 0 & m \\
0 & -k^2 & m & 0 \\
0 & m & 1 & 0 \\
m & 0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
A_i \\
\bar{A}_i \\
\bar{F}_i \\
F_i
\end{pmatrix}
\]

whose inverse is:

\[
K^{-1} = \begin{pmatrix}
-k^2 + m^2 & 0 & 0 & m \\
0 & -k^2 + m^2 & m & 0 \\
0 & m & k^2 + m^2 & 0 \\
m & k^2 + m^2 & 0 & k^2 + m^2
\end{pmatrix}
\]

which clearly has no term that can give rise to the indicated two point functions. Diagrams with different component fields behave in a similar way.

Since there are no diagrams that can contribute to the relevant two point functions, it would seem unnecessary to regulate these amplitudes, and hence no anomaly should develop. This kind of result was behind the arguments made in \[3\] that no anomalies should arise for this cohomology—it seems to
be quite general in fact. But is this computation really correct? Note that if one calculates the usual axial anomaly naively in an unregulated theory, one incorrectly obtains zero. So at the very least we should regulate the theory. But how? The most reasonable suggestion might be to try ‘supersymmetric Pauli-Villars regularization’ in superspace—that ought to show that there are no supersymmetry anomalies, at least when it can be used. But even supersymmetric Pauli-Villars for chiral superfields is in doubt for three reasons: (1) massless fields cause a problem, (2) when implemented with a regularized action, Pauli-Villars requires the wrong connection between spin and statistics for the regulator fields, which in turn plays havoc with the supersymmetry algebra—the Hamiltonian is no longer positive definite, which means that $\sum_i Q_i^2 = H$ must be modified somehow. (3) Pauli-Villars breaks gauge invariance when supersymmetric Yang-Mills is present.

My guess is that the result is correct, in spite of these worries. However it is much less clear what happens at higher orders in a theory with Yang-Mills and chiral matter and spontaneous gauge breaking. To keep the Yang-Mills symmetry manifest, one would like to use dimensional regularization. But it is clear that dimensional regularization is very hard to reconcile with supersymmetry, and that the problems start to show up at multiloop orders. It is also easy to convince oneself that some anomalies could not possibly arise at one loop simply because it requires more than one loop to even generate
the appropriate outgoing fields. For example, one could take:

\[ \Psi_\alpha = D^2 \{ U^i_L D^j_L [e^{-V} D_\alpha e^V U_L]_k \epsilon_{ijk} \} \] (21)

as the type of operator that could create a proton. It requires two loops to convert all the chiral fields to antichiral ones, so could not possibly have a supersymmetry anomaly before two loops.

Similarly one could make an operator to create an electron from the Higgs chiral superfield and the electron matter chiral superfield. The VEV of the Higgs would allow the electron flavour to ‘emerge’. Again a supersymmetry anomaly could split its mass from the mass of its superpartners.

### 3 Conclusion

The non-trival BRS cohomology of N=1, D=4 supersymmetry naturally gives rise to a number of speculations concerning the origin of the dimension of spacetime, the uniqueness of our own universe and the reason that supersymmetry breaking occurs with a zero cosmological constant. It would be interesting to determine whether the known supersymmetry cohomology carries over to local supersymmetry and higher dimensions, and it is desirable that the coefficients of some of the anomalies be calculated. It would also be interesting to work out the supersymmetry breaking consequences of the anomalies, assuming that they are indeed present at some order of perturbation theory.
How can one compute the coefficients of the holes in supersymmetric theories? In order for the theory to exhibit supersymmetry anomalies, one must be forced to use a regularization which explicitly violates supersymmetry. Indeed, this appears to be happen if one insists on regularizing in a way that preserves gauge symmetry, which is exactly the case of interest for phenomenology.

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