Derivation of reliability formula design for cylinder-elliptical head structure

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Abstract: Based on the JC method (reliability calculation method) and the fourth strength theory proposed by Lacvitz et al.[1] and recommended by the Joint Committee on International Security (JCSS), the reliability design formula for pressure vessel elliptical head and thin-walled circular cylinder is deduced. Monte Carlo reliability calculation method of ANSYS is used to verify the correctness of the formula, and the factors that have the greatest impact on the structural reliability are analyzed, which can provide some reference for the design of pressure vessel reliability.

1. Introduction
With the trend toward large-scale petrochemical industry development, the pressure vessel design requirements are getting higher and higher. Under the premise of ensuring the safety of the pressure vessel, taking into account its economy has become increasingly important [2]. In the design of the pressure vessel, the design pressure, material strength and structure size are all random, while existing standards treat these parameters according to the established amount. Using a larger safety factor to ensure the safety of the structure, obviously can’t take into account the economy of the pressure vessel [3, 4]. In this paper, the reliability formula of the pressure vessel elliptical head and circular cylinder is deduced by JC method (reliability calculation method) and fourth strength theory, and applied to the design of the cylinder-elliptical head structure. Monte Carlo reliability calculation method [5] in ANSYS is used to verify the correctness of the formula. The factors that have the greatest influence on the structural reliability are analyzed, which can provide some reference for the design of pressure vessel reliability.

2. Calculation of reliability theory
2.1. Calculation of cylindrical reliability
Radial stress of thin-walled cylinder is much smaller than the axial stress and circumferential stress. According to the elastic mechanics, the three-way stress state of the cylinder can be obtained as follows:

\[ \sigma_1 = \frac{PD}{2t} \]  \hspace{1cm} (2.1)

\[ \sigma_2 = \frac{PD}{4t} \]  \hspace{1cm} (2.2)

\[ \sigma_3 = 0 \]  \hspace{1cm} (2.3)
In the above formulas, P denotes the design pressure, D denotes cylinder diameter, t denotes the wall thickness of cylinder, and $\sigma_{i(1,2,3)}$ denotes the i-th principal stress.

According to the cylinder stress calculation formula and the fourth strength theory, it can be seen that the equivalent stress $S$ (the stress strength of the cylinder) is:

$$S = \sqrt{\sigma_1^2 - \sigma_2 \sigma_3 + \sigma_3^2} = \frac{\sqrt{3}PD}{4t}$$  \hspace{1cm} (2.4)

$P$, $D$, and $t$ in equation (2.4) are treated as random variables, it can be assumed that these random variables conform to the normal distribution according to statistics analysis and design experience. The Taylor formula expansion of $S$ is carried out at the mean of the variable, and more than one level of expand items are neglected. Then according to the functional function of the cylinder (in equation (2.5)), the reliability formula of the cylinder is calculated by JC method as follows:

$$Z = R - S$$  \hspace{1cm} (2.5)

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \frac{\mu_R - \frac{\sqrt{3}\mu_P\mu_D}{4\mu_t}}{\sqrt{\sigma_R^2 + \left(\frac{\sqrt{3}\mu_D}{4\mu_t}\right)^2 \sigma_R^2 + \left(\frac{\sqrt{3}\mu_P}{4\mu_t}\right)^2 \sigma_P^2 + \left(\frac{\sqrt{3}\mu_p\mu_D}{4\mu_t^2}\right)^2 \sigma_t^2}}$$  \hspace{1cm} (2.6)

The intensity $R$ in equation (2.5) is the yield strength of the material at the design temperature, taking 1.5 times the allowable stress at the design temperature as its value. In equation (2.6), $\mu_{i=R,S,P,D,t}$ is the mean value of the variable, $\sigma^2_{i=R,S,P,D,t}$ is the variable variance, $\beta$ is the variable value of the standard normal distribution function.

2.2. Calculation of elliptical head reliability

The stress composition of the elliptical head is similar to that of the spherical head. In addition to the membrane stress caused by internal pressure, there are also discontinuous stresses of the junction. According to a large amount of engineering practice experience and theoretical research, the stress condition of the elliptical head is related to the ratio of the length of the ellipse as $a/b$. When $a/b = 1.0 \sim 2.6$, the following formula is used to calculate the stress enhancement factor $K$ (also known as the shape factor) of elliptical head [6] in the engineering.

$$K = \frac{1}{6}[2 + \left(\frac{D}{2h}\right)^2]$$  \hspace{1cm} (2.7)

Equation (2.7) indicates that 2K equals head total stress divided by circumferential membrane stress. When $a/b = 1.0 \sim 2.6$, the total stress of the elliptical head is $K$ times the membrane stress of the spherical head at the same radius, while the radius of spherical head equals to the long axis of elliptical head. Therefore, the functional function of the elliptical head is:

$$Z = R - S = R - \frac{PD}{2t}K$$  \hspace{1cm} (2.8)

Reliability index of the elliptical head can be obtained after the corresponding mean and variance obtained by JC method.

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \frac{\mu_R - \frac{K\mu_P\mu_D}{2\mu_t}}{\sqrt{\sigma_R^2 + \left(\frac{K\mu_D}{2\mu_t}\right)^2 \sigma_R^2 + \left(\frac{K\mu_P}{2\mu_t}\right)^2 \sigma_R^2 + \left(\frac{K\mu_p\mu_D}{2\mu_t^2}\right)^2 \sigma_t^2}}$$  \hspace{1cm} (2.9)

2.3. Reliability of cylinder-elliptical head structure
The reliability of the structure is:

\[ P_r = \phi(\beta) \]  

(2.10)

The failure probability is:

\[ P_f = 1 - P_r = \phi(-\beta) \]  

(2.11)

In the equations (2.10) and (2.11), \( \phi(x) \) is the probability cumulative distribution function of the standard normal distribution.

3. Calculation of a case

3.1. Problem Description

There is a case of cylinder-elliptical head structure, the parameters and design conditions of a pressure vessel are shown in Table 1.

| Parameters                              | Mean value \( \mu \) | Standard deviation \( \sigma \) |
|-----------------------------------------|------------------------|-------------------------------|
| Design pressure P/MPa                  | 5                      | 0.2                           |
| Design temperature T/℃                 | 100                    | —                             |
| Inner diameter of cylinder D/mm         | 1500                   | 3                             |
| Yield strength of material (16Mn) S/MPa | 282                    | 19.74                         |
| Long axis of head Rt/mm                 | 1500                   | 3                             |
| Length of cylinder L/mm                | 2000                   | —                             |

a The variable here is not a random variable.

According to the experience of pressure vessel design, we can see the reliability of cylinder is 99.999%. Find that \( \beta = 4.27 \) from standard normal function table, and then calculate the wall thickness of the cylinder and the head shown in Table 2 using the iterative method according to the formula (2.6) and formula (2.9).

| Parts                              | Mean value | Standard deviation |
|------------------------------------|------------|--------------------|
| Cylinder thickness t1/mm           | 20         | 0.134              |
| Elliptical head wall thickness t2/mm| 20         | 0.134              |

3.2. Reliability analysis

Take the above-mentioned cylinder-elliptical head structure calculated by theoretical formula as the object of study. The finite element model shown in Figure.1 is established by ANSYS. The reliability analysis of the cylinder - elliptical head structure is carried out. The inner diameter (D), head long axis (Rt), cylinder wall thickness (t1), head wall thickness (t2), design pressure (P) and yield strength (Yies) of the cylinder are taken as random variables that follow the normal distribution during the analysis process.

\[ Z = \text{Yies} - \text{maxstr} \]  

(3.1)

In the formula (3.1), maxstr represents the maximum von Mises stress in the structure. The failure probability \( P_f \) of the structure can be obtained through Monte Carlo Method in ANSYS, which is the probability of \( Z \leq 0 \).
It can be seen from ANSYS reliability analysis results that:

1) The functional function $Z$ sampling process (see Figure 2) is within 95% confidence level, and indicates that the number of simulations is sufficient.

2) The sensitivity analysis of the output variable $Z$ is shown in Figure 3, and it is known from the sensitivity analysis results that $\text{yies}$, $P$ and $t_1$ have a great effect on functional functions, and the influence of other parameters is small. When $\text{yies}$ increases, the carrying capacity of the structure increases, and the reliability increases.

3) The value of $\text{yies}$ is shown in Figure 4. There is no overshoot and step in the histogram and indicates that the number of sampling is sufficient and the simulation results are credible.

4) In the case of 95% confidence, the probability of $Z<0$ is 0%. That is, the failure probability of the container is 0%, so that the reliability of the structure is 100%.

4. Conclusion

In pressure vessel design process, design pressure, diameter of cylinder, yield strength of material and other factors are not independent of each other. There is a certain correlation between each other, so it is necessary to carry out reliability analysis of the pressure vessel. In this paper, the formula for reliability calculation of cylinder-elliptical head is deduced by JC method. And Monte Carlo reliability calculation method is used in ANSYS. The results show that the reliability formula of this paper can be used to design the reliability of cylinder-elliptical head.
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