Correlation effects in multiple hard scattering

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Introduction

- multiparton interactions are ubiquitous in hadron-hadron collisions
- populate characteristic part of phase space
  there they can be substantial part of rate
- important theory progress for hard double scattering
- but many open questions:
  - size of correlations between partons
  - parton splitting contributions evolution of DPDs
- promising experimental developments:
  - different processes
  - kinematic distributions
- use $\sigma_{\text{eff}}$ as a handy tool, not as a precision instrument

summary of my talk yesterday
Double parton scattering: cross section formula

\[
\frac{d\sigma_{\text{double}}}{dx_1 d\bar{x}_1 dx_2 d\bar{x}_2} = \frac{1}{C} \hat{\sigma}_1 \hat{\sigma}_2 \int d^2 b \ F(x_i, b) F(\bar{x}_i, b)
\]

- \(C\) = combinatorial factor
- \(\hat{\sigma}_i\) = parton-level cross section
- \(b\) = transv. distance between partons
- \(F(x_i, b)\) = double parton distribution (DPD)
Double parton scattering: cross section formula

\[
\frac{d\sigma_{\text{double}}}{dx_1 \, d\bar{x}_1 \, d^2 q_1 \, dx_2 \, d\bar{x}_2 \, d^2 q_2} = \frac{1}{C} \hat{\sigma}_1 \hat{\sigma}_2 \left[ \prod_{i=1}^{2} \int d^2 k_i \, d^2 \bar{k}_i \, \delta(q_i - k_i - \bar{k}_i) \right] \times \int d^2 b \, F(x_i, k_i, b) \, F(\bar{x}_i, \bar{k}_i, b)
\]

\[F(x_i, k_i, b) = k_T \text{ dependent two-parton distribution}\]

- \[F(x_i, b) = \int d^2 k_1 \int d^2 k_2 \, F(x_i, k_i, b) \text{ up to issues of regularization}\]
- analogous to TMD formalism

see talks in WG6, Wed from 11:20
Double parton scattering: pocket formula

- if two-parton density factorizes as
  
  \[ F(x_1, x_2, b) = f(x_1) f(x_2) G(b) \]

  where \( f(x_i) = \text{usual PDF} \)

- if assume same \( G(b) \) for all parton types
  then cross sect. formula turns into

  \[
  d\sigma_{\text{double}} \over dx_1 \over d\bar{x}_1 \over dx_2 \over d\bar{x}_2 = \frac{1}{C} \frac{d\sigma_1}{dx_1 \over d\bar{x}_1} \frac{d\sigma_2}{x_2 \over \bar{x}_2} \frac{1}{\sigma_{\text{eff}}}
  \]

  with \( 1/\sigma_{\text{eff}} = \int d^2b \ G(b)^2 \)

  \(
  \sim \) scatters are completely independent

- analogous derivation for cross sect. dependent on \( q_i \)

- pocket formula fails if any of the above assumptions is invalid
  and if further terms must be added to original expression of cross sect.
Parton correlations

- if neglect correlations between two partons

\[ F(x_1, x_2, b) = \int d^2 b' \ f(x_1, b' + b) \ f(x_2, b') \]

where \( f(x_i, b) = \) impact parameter dependent single-parton density

and if neglect correlations between \( x \) and \( b \) of single parton

\[ f(x_i, b) = f(x_i) F(b) \]

with same \( F(b) \) for all partons

then \( G(b) = \int d^2 b' \ F(b' + b) \ F(b') \)
Parton correlations

- if neglect correlations between two partons
  
  \[ F(x_1, x_2, b) = \int d^2 b' \ f(x_1, b' + b) \ f(x_2, b') \]
  
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with same \( F(b) \) for all partons

then \( G(b) = \int d^2 b' \ F(b' + b) \ F(b') \)

- correlations in \( b \) space need not invalidate pocket formula

- for Gaussian \( F(b) \) with average \( \langle b^2 \rangle \)

\[ \sigma_{\text{eff}} = 4\pi \langle b^2 \rangle = 41 \text{ mb} \times \langle b^2 \rangle/(0.57 \text{ fm})^2 \]

  determinations of \( \langle b^2 \rangle \) range from \( \sim (0.57 \text{ fm} - 0.67 \text{ fm})^2 \)

  is \( \gg \sigma_{\text{eff}} \sim 10 \) to 20 mb from experimental extractions

  if \( F(b) \) is Fourier trf. of dipole then 41 mb → 36 mb

  complete independence between two partons is disfavored

  or something is seriously wrong with \( \sigma_{\text{eff}} \)

  cf. Calucci, Treleani 1999; Frankfurt, Strikman, Weiss 2003, 2004
Correlations involving $x$

- $F(x_1, x_2, b) = f(x_1) f(x_2) G(b)$ cannot hold for all $x_1, x_2$
- Most obvious: energy conservation $\Rightarrow x_1 + x_2 \leq 1$
  Often used: $F(x_1, x_2, b) = f(x_1) f(x_2) (1 - x_1 - x_2)^n G(b)$
  To suppress region of large $x_1 + x_2$
- Significant $x_1 - x_2$ correlations found in constituent quark model
  \text{Rinaldi, Scopetta, Vento: arXiv:1302.6462}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{plot.png}
\caption{Plot showing $\int d^2 b \ F_{uu}(x_1, x_2, b) / f_u(x_2)$ is $x_2$ independent if factorization holds}
\end{figure}

- Unknown: size of correlations when one or both of $x_1, x_2$ small
Correlations involving $x$ and $b$

- single-parton distribution $f(x, b)$ is Fourier trf. of generalized parton distributions at zero skewness
  - information from exclusive processes and theory
- HERA results on $\gamma p \rightarrow J/\Psi p$ give

$$\langle b^2 \rangle \propto \text{const} + 4\alpha' \log(1/x)$$

with $\alpha' \approx 0.15 \text{ GeV}^{-2} = (0.08 \text{ fm})^2$ for gluons at $x \sim 10^{-3}$

- lattice simulations $\rightarrow$ strong decrease of $\langle b^2 \rangle$ with $x$ above $\sim 0.1$
  seen by comparing moments $\int dx x^{n-1} f(x, b)$ for $n = 0, 1, 2$

- precise mapping of single-parton distributions $f(x, b)$ over wide $x$ range in future lepton-proton experiments

  JLab 12, COMPASS, EIC, LHeC

  $\rightarrow$ parallel talks in WG6 and WG7

- expect similar correlations between $x_i$ and $b$ in two-parton dist’s
  even if factorization $F(x_1, x_2, b) = f(x_1, b) f(x_2, b)$ does not hold
Consequence for multiple interactions

- indications for decrease of $\langle b^2 \rangle$ with $x$
- if interaction 1 produces high-mass system
  → have large $x_1, \bar{x}_1$
  → smaller $b$, more central collision
  → secondary interactions enhanced

Frankfurt, Strikman, Weiss 2003
study in Pythia: Corke, Sjöstrand 2011
Spin correlations

- polarizations of two partons can be correlated even in unpolarized target already pointed out by Mekhfi (1985)
  - quarks: longitudinal and transverse pol.
  - gluons: longitudinal and linear pol.

have eight ($k_T$ integrated leading-twist) distributions for each combination $qq$, $qg$, $gg$

- fulfil positivity constraints analogous to Soffer bound for usual PDFs, e.g.
  \[
  F_{qq} - F_{\Delta q\Delta q} \geq 2|F_{\delta q\delta q}|
  \]
  \[
  q = \text{unpol.}, \ \Delta q = \text{long.}, \ \delta q = \text{transv.; schematic notation MD, Kasemets 2013}
  \]

- in general not suppressed in hard scattering consequences for rate and distributions
Spin and angular distributions

- detailed calc’n for gauge boson pair production followed by leptonic decay
  
  T. Kasemets, MD 2012; see also A. Manohar, W. Waalewijn 2011

- longitudinal quark spin correlations
  - overall rate and distribution in lepton rapidities

- transverse quark spin correlations
  - azimuthal correlation between lepton planes
  - two hard scatters are not independent

- expect similar effects for gluon initiated processes (esp. for jets)
  - linear gluon pol. azimuthal correlation between scattering planes

- note: independent scattering planes sometimes assumed as criterion to characterize double parton scattering
Spin correlations

how important are spin correlations?
large effects expected in valence quark region

study in bag model: Chang, Manohar, Waalewijn: arXiv:1211:3132

plots show $F(x_1, x_2 = 0.4, k_\perp)$ for different pol. combinations
$k_\perp =$ Fourier conjugate to $b$

unknown: size of correlations when one or both of $x_1, x_2$ small
Color structure

- Quark lines in amplitude and its conjugate can couple to color singlet or octet:

\[ 1F \to (\bar{q}_2 \mathbb{1} q_2) (\bar{q}_1 \mathbb{1} q_1) \quad \quad 8F \to (\bar{q}_2 t^a q_2) (\bar{q}_1 t^a q_1) \]

- \( 8F \) describes color correlation between quarks 1 and 2 is essentially unknown (no probability interpretation as a guide)

- For two-gluon dist’s more color structures: 1, 8_S, 8_A, 10, \overline{10}, 27

- For \( k_T \) integrated distributions:
  - Color correlations suppressed by Sudakov logarithms
  - But not necessarily negligible for moderately hard scales

Mekhfi 1988; Manohar, Waalewijn 2011
Sudakov factors

- for $k_T$ dependent distributions, i.e. measured $q_i$: Sudakov logarithms for all color channels close relation with physics of parton showers
- for double Drell-Yan process can adapt Collins-Soper-Sterman formalism for single Drell-Yan $\rightsquigarrow$ include and resum Sudakov logs in $k_T$ dependent parton dist’s MD, D Ostermeier, A Schäfer 2011
- at leading double log accuracy: singlet and octet dist’s $^1F$ and $^8F$ have same Sudakov factor as in single scattering
- for $q_T \sim \Lambda$
  - Sudakov factors mix singlet and octet dist’s mixing only suppressed by $1/N_c$
  - generically Sudakov factors of same size for singlet and octet
- for $q_T \gg \Lambda$ and $|b| \sim 1/\Lambda$
  - singlet $^1F$ decouples from octet $^8F$
  - octet contribution has extra suppression by fractional power of $\Lambda/q_T$
Conclusions

- various two-parton correlations can affect rate and kinematic distributions of double parton interactions
- correlations in $b$ dependence, between $x_1, x_2$, and between $x_1, x_2$ and $b$
- correlations in spin and color give rise to new double parton dist's not included in usual double scattering formula
- for $x_i$ in valence region expect strong correlations between $x_1, x_2, b$ and strong spin correlations situation for small $x_i$ not known
- note: at small $b$ double parton distr's dominated by splitting graphs give strong correlations in $x_1, x_2$, spin and color
- physics of color correlations closely connected with Sudakov factors