Mathematical analysis of interconnected photovoltaic arrays under different shading conditions

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Abstract: A comprehensive mathematical analysis of interconnected photovoltaic arrays under different shading conditions (opacity) and patterns (column, row, diagonal and corner) has been carried out in this work. The equivalent circuit models for the different shading conditions and patterns, and pseudocode algorithm were developed upon which the performance characteristics of the interconnected arrays were analyzed. Five different interconnections were inclusively considered in this work: series-parallel, total-cross-tied (TCT), bridge-linked, ladder and honey-comb interconnection. The emerging analytical results revealed that TCT is most dominant interconnection and shading patterns across the strings (row and diagonal) have detrimental effect on output power, especially when the opacity is one (signifying perfect shading condition) but shading patterns along the string (column or corner shadings) are less severe to power generation. The formation of double peaks sequel to the presence of shadings are inimical to power generated from the interconnected arrays. Moreover,

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PUBLIC INTEREST STATEMENT
Power generation is crucial to socioeconomic development of our society. Thus, optimization of power generation is geared toward eliminating factors which hamper sustainable power generation from photovoltaic generators; especially, the shading and shadowing effects. Mathematical analysis of this phenomenon is imperative in order to reveal the most effective way to combat this problem. Analytical results show that total-cross-tied technique is superior to other interconnections employed in enhancing power recovery during the occurrence of shades and shadows. Furthermore, the analysis revealed that corner and diagonal shadings of photovoltaic generators are more severe than other shading patterns. Application of interconnection in combating power dissipation in photovoltaic generators has proven to be effective and is capable of boosting power supply to the grid for the purpose of satisfying domestic and industrial power requirements. Thus, artificial shading and shadowing should be avoided in order to increase output power from photovoltaic generators.
increasing the interconnections enhances the output power and further serves as a means of bypassing current in the event of threats to the modules. Thus, the results obtained provide vital information for smooth operation and maximization of output power in interconnected arrays by avoiding shades on the strings.

Subjects: Risk, Science & Technology; Engineering Technology; Educational Technology

Keywords: series-parallel array (SP); interconnected arrays (TCT; BL; LD and HC); shading patterns; maximum output power; output power probability and shading factor (or opacity)

1. Introduction

The traditional series (S), parallel (P) and series-parallel SP (S < P, S = P, S > P) arrays are susceptible to reduction in power generation or tremendous power loss under different shading conditions. This problem raises serious concern and challenges to power-generation engineers who are assiduously proffering economic and viable technical solutions to this imminent problem. Shadows naturally emanate from the position of the sun relative to the earth; especially during the sunrise and sunset, shadows are probably to be casted by tall trees, towers, settlements and neighboring construction (Guerrero, Muñoz, Ibáñez, & Ospino, 2017; Ramaprabha & Mathur, 2012). Shadow produces total darkness which has overall devastating effect on power generation whereas moving clouds cause partial shading of the modules and pose less severity to power generation.

Generally, the occurrence of shades engenders thermal concentration or hot spot in the shaded cells of a module. It is electrically attributed to the development of reverse biased voltage by the unshaded cells in the module; thus, the shaded cells act as a load, which induces power dissipation in lieu of power generation (Liu, Lai, & Yeh, 2013; Moretón, Lorenzo, Lelaux, & Carrillo, 2014). A hot spot is mostly reckoned to absorption of solar radiation and partly due to power dissipation in the shaded cells of a module during short circuit condition (He, Liu, Ji, Zhang, & Chen, 2015). The consequence of this incidence leads to irreparable damage of the module cells in the absence of current bypass mechanisms (diodes or tied-wires). Evidently, reduction in power generation from the arrays becomes prevalent due to ample power dissipation caused by the shaded cells in the modules or arrays (Agarwal & Agarwal, 2014; Chaudhary, Gupta, Pande, Mahfooz, & Varshne, 2015).

Pertinently, previous research works (Gao, 2009; Meyer & Dyk, 2005) have shown that parallel configuration is unsusceptible to shading effect compared to series configuration due to numerous strings, which produce uniform output voltage and accumulated or multiple output currents from the various strings. However, the power generated lacks sufficient voltage necessary to drive the current to an external load which abounds in the series connection at the expense of reduced current. Thus, having equal SP array which is normally characterized by appreciable current and voltage synergistically boot power generation.

Also, pioneer workers (Tam, Van Duong, Tien, & Ngu, 2014; Ziar, Member, Nouri, Asaei, & Farhangi, 2014) in this area have suggested different ways of combating the problem of shades by installing modules either with in-built diode per cell or external bypass diodes per string; introduction of interconnection like total-cross-tied (TCT), bridge-linked (BL), ladder (LD), honeycomb (HC) (Wang & Hsu, 2011), logical patterns, Su Du Ku (Humada, Sulaiman, Hojabri, Hamada, & Ahmed, 2016; Mohammadnejad & Ahmadi, 2017; Vijayalekshmy, Bindu, & Iyer, 2015). According to Ramaprabha & Mathur (2012), TCT and HC interconnections have been proven to be effective under symmetric (S = P) and asymmetry (S ≠ P) configuration, respectively, but generally, TCT is assumed to be most effective interconnection to enhance output power from interconnected arrays (Humada et al., 2016; Ishaque, Salam, & Taheri, 2011; Jazayeri, Uysal, & Jazayeri, 2014; Karatepe, Boztepe, & Metin, 2007). Although, a singular
publication (Kaushika & Gautam, 2003) vehemently disagree with the popular finding, by making a contrary submission that SP configuration is more efficient in maximizing output power from an array. Another nonaligned literature (Thiyagarajan & Somasundaram, 2016) assert that equal performance is achieved among SP, TCT and BL configurations under partial shading condition. Hence, on successful completion of this crucial research work, this paper is bound to unify the divergent opinions about the most efficient interconnection. The tied or linked method of power enhancement similarly function as an internal or external bypass diode but uses electrical wires to create alternate routes for bypassing current from the shaded modules. The former technique is not proficient relative to the latter because of high cost of the all-weather modules which the earlier manufacturers have gone into extinction (Pannebakker, de Waal, & van Sark, 2017). The latter technique trade-off cost and technicality in the bid to solve shading problems. For this reason, evolutions in research are prevalent in this area than in the use of bypass diode and other means of increasing output power from an array.

The advanced techniques of boosting or enhancing power generation in photovoltaic arrays employ fuzzy logic or artificial intelligence and sophisticated algorithm like particle swarm optimization (Burhanudin et al., 2017; Humada et al., 2014a; Karatepe et al., 2007), perturb and observe (Bianconi, Calvente, & Giral, 2013; Rezk & Eltamaly, 2015) and slide mode control (Vázquez, Azaf, Cervantes, Vázquez, & Hernández, 2015) to distinguish local maximum from a global maximum during maximum point power tracking. The benefits accrued from these techniques have been widely published but the magnitude of power generated relative to the tied or linked method is not much compared to the resource input into these methods. To substantiate this point, the number of literature devoted to tied or linked technique far outweigh those on fuzzy logic (Humada et al., 2014b) and algorithms (Burhanudin et al., 2017; Rezk & Eltamaly, 2015; Vázquez et al., 2015).

However, most literature works (Aljafari, 2016; Vijayalekshmy et al., 2015) centered on comparative analysis of interconnected arrays lack comprehensive mathematical model which takes into account the internal resistance (the shunt and series resistance) and holistic shading factor, which delineate a real system. Other published works (Humada et al., 2016; Vijayalekshmy, Bindu, & Iyer, 2015) developed their mathematical models based on an ideal equivalent circuit, in order to escape the rigorous mathematical challenges imposed by the nonlinear model emanating from modelling of real systems. Besides, another literature (Vijayalekshmy et al., 2015) only represents mathematical model for the TCT while unveiling the models for the other interconnection under partial shading conditions.

Thus, the present work is aimed at: developing real generic mathematical models for different interconnection and array size; to investigate the effect of shading factor (opacity, $\alpha; 0 \leq \alpha \leq 1$) on output power; to establish the superiority of interconnection amidst shadings; and the severity of different shading patterns on power generation.

2. Materials and method

2.1. The conceptual model

The interconnected arrays to be developed are equal SP, TCT, BL, LD and HC interconnection. The interconnection is assembled in a manner that parallel modules are interconnected to enhance the certainty of outputting current under shaded conditions. However, all the interconnections have equal certainty to output current in the absence of shading. Figure 1 shows the diagrammatic representation of different interconnection. Figures 2–6, 8 depict the shading patterns to be adopted in the detailed analysis of the five selected interconnection in Figure 1.
**Figure 1.** Interconnected arrays.

1(a) SP  1(b) TCT  1(c) BL  1(d) LD  1(e) HC

**Figure 2.** Shading patterns for series-parallel (SP) interconnection.

2(a) Column  2(b) Row  2(c) Diagonal  2(d) Corner

**Figure 3.** Shading patterns for total-cross-tied (TCT) interconnection.

3(a) Column  3(b) Row  3(c) Diagonal  3(d) Corner

**Figure 4.** Shading patterns for bridge-linked (BL) interconnection.

4(a) Column  4(b) Row  4(c) Diagonal  4(d) Corner
2.2. Problem formulation

Considering Figures 1–6, the unshaded and shaded modules are represented by numbers 1, 2, ⋯, 16 and 1', 2', ⋯, 16', respectively.

2.2.1. Derivation of equivalent circuit model for unshaded interconnected array

Figure 7 shows an equivalent circuit model for unshaded interconnection, the certainty or probability ($P_r$) of outputting currents under the unshaded condition is unity (1) irrespective of the interconnection.

Observing that currents in parallel connection add up

$$I_{1,2,3,4} = (1)(I_1 + I_2 + I_3 + I_4) = 4I_1; \quad I_{5,6,7,8} = (1)(I_5 + I_6 + I_7 + I_8) = 4I_5;$$

$$I_{9,10,11,12} = (1)(I_9 + I_{10} + I_{11} + I_{12}) = 4I_9; \quad I_{13,14,15,16} = (1)(I_{13} + I_{14} + I_{15} + I_{16}) = 4I_{13}$$

The total current, $I_r$ in Figure 1(b) is
\( I_t = \min\{I_{1,2,3,4}, I_{5,6,7,8}, I_{9,10,11,12}, I_{13,14,15,16}\} = 4I_1 \) \hspace{1cm} (2)

and their corresponding voltages, \( V_{i1,i1+i2,i3} : i = 1, 5, 9, 13 \) are:

\[
V_{1,2,3,4} = (V_1 + V_2 + V_3 + V_4)/4 = V_1; \quad V_{5,6,7,8} = (V_5 + V_6 + V_7 + V_8)/4 = V_5;
V_{9,10,11,12} = (V_9 + V_{10} + V_{11} + V_{12}) = V_9; \quad V_{13,14,15,16} = (V_{13} + V_{14} + V_{15} + V_{16}) = V_{13}
\] \hspace{1cm} (3)

The total voltage \( V_i \) is obtained by summing the individual voltages in a string as follows:

\[
V_i = V_{1,2,3,4} + V_{5,6,7,8} + V_{9,10,11,12} + V_{13,14,15,16} = V_1 + V_5 + V_9 + V_{13} = 4V_1; \quad \Rightarrow V_1 = V_i/4
\] \hspace{1cm} (4)

The equivalent circuit model for Equation (2) is giving as:

\[
I_t = 4I_1 = 4I_{ph} - 4I_0 \left( \frac{V_1 + I_1R_s}{AV_T} - 1 \right) - \frac{4V_1 + I_1R_s}{R_p}
\] \hspace{1cm} (5)

Substituting Equation (4) into Equation (5) gives

\[
I_t = 4 \left( I_{ph} - I_0 \left( \frac{V_1 + I_1R_s}{4AV_T} - 1 \right) - \frac{V_1 + I_1R_s}{4R_p} \right)
\] \hspace{1cm} (6)

Equation (6) can be written for different interconnections as

\[
I_{ij} = Pr_j \left[ nI_{ph} - nI_0 \left( \frac{V_{ij} + I_{ij}R_s}{nAV_T} - 1 \right) - \frac{V_{ij} + I_{ij}R_s}{R_p} \right].
\] \hspace{1cm} (7)

where

\( n \) is the number of modules in a string (\(-\)), \( Pr_j \) is the probability of an interconnection to output current (\(-\)), \( I_{ph} \) is the photon current (A), \( I_0 \) is the reverse saturation current (A), \( A \) (\(-\)) is the ideality factor within the range \( 1 \leq A \leq 2 \) (Cubas, Pindado, & Sorribes-Palmer, 2017), \( V_T \) is the thermal voltage (V) and \( V_T = kT/e \) (Bonkoungou, Koalaga, & Njomo, 2013), \( k \) is the Boltzmann constant, \( T \) is the nominal operating temperature (K), \( e \) is the electron charge (C), \( R_s \) is the series resistance (\( \Omega \)) and \( R_p \) is the parallel resistance (\( \Omega \)).

2.2.2. Derivation of equivalent circuit model for shaded and interconnected array

In developing an equivalent circuit model for a shaded interconnected array (Figures 2–6, 8), the probability to output current under partial shading is defined as

\[
Pr_j = \frac{\text{number of connection}}{\text{number of maximum possible connection}}: \quad j = SP, TCT, BL, LD, HC;
\]

\[
Pr_j = \{Pr_{SP} = 12/21, Pr_{TCT} = 1, Pr_{BL} = 17/21, Pr_{LD} = 18/21, Pr_{HC} = 19/21\}
\] \hspace{1cm} (8)
Besides the reduction in total output current caused by the diode and resistance currents, a further reduction on total output current is imposed by the shade current, \( I_{\text{shade}} \), which is defined as (Tam et al., 2014; Vijayalekshmy et al., 2015)

\[
I_{\text{shade}} = \frac{G}{G_0} I_{\text{ph}}
\]  

(9)

where \( G \) is the insolation reaching the earth’s surface and \( \alpha \) is shading factor (opacity); \( 0 \leq \alpha \leq 1 \), \( \alpha = 0 \Rightarrow \) no shading or zero opacity, \( \alpha = 1 \Rightarrow \) total shading or maximum opacity. The shaded modules are represented with a prime ('), \( n' = 1', 2', \ldots, 16' \). For the shading patterns; column, row, diagonal and corner in Figures 2–6, 8, the shaded modules are \( \{1', 5', 9', 13'\}, \{1', 2', 3', 4'\}, \{1', 6', 11'\} \) and \( \{1', 4', 13', 16'\} \), respectively.

For column shading in Figures 2(a)–6(a), the output current is the summation of currents from the different strings\( \{1', 5', 9', 13'\}, \{2, 6, 10, 14\}, \{3, 7, 11, 15\}, \{4, 8, 12, 16\} \), which is defined as

\[
I_t = \min \{I_{1',2,3,4}, I_{5',6,7,8}, I_{9',10,11,12}, I_{13',14,15,16}\} = I_V + 3I_2; \Rightarrow I_V = I_t - 3I_2
\]  

(11)

The corresponding voltages \( V_{i,i+1,i+2,i+3}; i = 1, 5, 9, 13 \) are

\[
V_{1',2,3,4} = (V_1 + V_2 + V_3 + V_4)/4 = \frac{V_1 \pm 3}{4} V_2; \quad V_{5',6,7,8} = (V_5 + V_6 + V_7 + V_8)/4 = \frac{V_5 \pm 3}{4} V_6;
\]

\[
V_{9',10,11,12} = (V_9 + V_{10} + V_{11} + V_{12})/4 = \frac{V_9 \pm 3}{4} V_{10}; \quad V_{13',14,15,16} = (V_{13} + V_{14} + V_{15} + V_{16}) \frac{V_{13} \pm 3}{4} V_{14}
\]  

(12)

The total voltage \( V_t \) is obtained by summing the individual voltages in the strings as follows:

\[
V_t = V_{1',2,3,4} + V_{5',6,7,8} + V_{9',10,11,12} + V_{13',14,15,16} = (V_t + 3V_2);
\]

\[
\Rightarrow V_t = \frac{1}{4} (4V_t - 3V_t) = \frac{V_t}{4} = V_2; \quad \forall V_t = V_t
\]  

(13)

The equivalent circuit model for Equation (11) is given as

\[
I_{e,j} = \frac{P_{\text{column}}}{P_{\text{column}}} \left[ I_{e,j} + 3I_{2,j} \right] = \frac{P_{\text{column}}}{P_{\text{column}}} \left[ I_{e,j} + \frac{3}{4} I_{1,j} \right]
\]

\[
= \frac{P_{\text{column}}}{P_{\text{column}}} \left[ I_{\text{ph}} \left( 1 - \frac{\alpha}{G_0} \right) - I_0 \left( \frac{V_{e,j} + I_{e,j} R_s}{AV_t} - 1 \right) - \frac{V_{e,j} + I_{e,j} R_s}{R_p} + \frac{3}{4} I_{1,j} \right];
\]  

\( j = SP, TCT, BL, LD, HC \)  

(14)

Substituting Equations (11) and (13) into Equation (14) gives

\[
I_{e,j} = \frac{P_{\text{column}}}{P_{\text{column}}} \left[ I_{\text{ph}} \left( 1 - \frac{\alpha}{G_0} \right) - I_0 \left( \frac{V_{e,j} + \frac{4\alpha R_s}{AV_t} I_{1,j} R_s}{R_p} - 1 \right) \right] - \frac{V_{e,j} + \frac{4\alpha R_s}{AV_t} I_{1,j} R_s}{R_p} + \frac{3}{4} I_{1,j}; \quad V_{e,j} = V_t
\]  

(15)

\( j = SP, TCT, BL, LD, HC \)
Equation (15) can be generalized as

\[ I_{\text{r}_j} = \frac{nI_0(1 - \frac{G}{G_0}) - nI_0(\exp\left(\frac{V_{\text{r}_j} + I_{\text{r}_j}R_s}{n\alpha V_T} - 1\right)}{\frac{V_{\text{r}_j} + (\frac{nV_{\text{r}_j} - I_{\text{r}_j}R_s)}{R_p}} + \frac{1}{n}I_j}; \quad V_{\text{r}_j} = V_{\text{ij}}; \]

\[ j = \text{SP, TCT, BL, LD, HC} \]  

(16)

For row shading in Figures 2(b)–6(b), the output current is the summation of the individual output currents from the different strings, which is given as

\[ I_{\text{r}_j} = \frac{\prod_{\text{row}_j} I_{\text{n}_\text{ph}}(1 - \frac{G}{G_0}) - nI_0(\exp\left(\frac{V_{\text{r}_j} + I_{\text{r}_j}R_s}{n\alpha V_T} - 1\right)}{\frac{V_{\text{r}_j} + (\frac{nV_{\text{r}_j} - I_{\text{r}_j}R_s)}{R_p}} + \frac{1}{n}I_j}; \quad V_{\text{r}_j} = V_{\text{ij}}; \]

\[ j = \text{SP, TCT, BL, LD, HC} \]

(17)

For diagonal shading in Figures 2(c)–6(c), the output current is the summation of currents from the different strings, which is derived as

\[ I_{\text{r}_j} = \frac{\prod_{\text{diagonal}_j} I_{\text{n}_\text{ph}}(1 - \frac{G}{G_0}) - (n - 1)I_0(\exp\left(\frac{V_{\text{r}_j} + I_{\text{r}_j}R_s}{n\alpha V_T} - 1\right)}{\frac{V_{\text{r}_j} + (\frac{nV_{\text{r}_j} - I_{\text{r}_j}R_s)}{R_p}} + \frac{1}{n}I_j}; \quad V_{\text{r}_j} = V_{\text{ij}}; \]

\[ j = \text{SP, TCT, BL, LD, HC} \]

(19)

For corner shading in Figures 2(d)–6(d), the output current is the summation of currents from the different strings and is given as

\[ I_{\text{r}_j} = \frac{\prod_{\text{corner}_j} 2I_{\text{n}_\text{ph}}(1 - \frac{G}{G_0}) - \frac{3}{2}I_0(\exp\left(\frac{V_{\text{r}_j} + I_{\text{r}_j}R_s}{n\alpha V_T} - 1\right)}{\frac{V_{\text{r}_j} + (\frac{nV_{\text{r}_j} - I_{\text{r}_j}R_s)}{R_p}} + \frac{1}{n}I_j}; \quad V_{\text{r}_j} = V_{\text{ij}}; \]

\[ j = \text{SP, TCT, BL, LD, HC} \]

(21)

The output currents and voltages from shaded interconnection (SP, TCT, BL, LD, HC) for different shading patterns in Figures 1–6 can be computed by considering the three vital standard test conditions; the short circuit current (I_{sc}), maximum power point current and voltage (I_{mpp}, V_{mpp}) and open circuit voltage (V_{oc}) in conjunction with the Newton-Raphson scheme gives the data for current-voltage (I – V) and power-voltage (P – V) curves (Villalva, Gazoli, & Filho, 2009).

The relative power gained PG_{j} is given in Equation (23) as
Similarly, the relative power loss $PL_k$ is defined in Equation (24) as

$$PL_k = \frac{P_{\text{reg, shaded}, k} - P_{\text{reg, unshaded}, k}}{P_{\text{reg}, \text{sp}, k}} \times 100\%$$

$k = \text{SP, TCT, BL, LD, HC}; \text{shaded}, 0 < \alpha < 1$

2.3. Input data

The input constants and variables are expressed in the first paragraph of the pseudocode algorithm, Algorithm 1. The numerical values are obtained from literature (Solar, 2017) and are summarized in Table 1:

2.4. Computational algorithm

The pseudocode algorithm for the computation of performance curves for the different interconnection and shading patterns is as follows:

**Pseudocode algorithm for computation of output current, voltage and power under different shading conditions and interconnection. Symbols are defined in the main text.**

**start**

function $E_{15}$, $E_{16}$, $E_{17}$, $E_{18}$, $E_{19}$, $E_{20}$, $E_{21}$, $E_{22}$

equivalence $(E_{15}, E_{16}), (E_{17}, E_{18}), (E_{19}, E_{20}), (E_{21}, E_{22})$

get $\{n_{\text{iteration}}, n_{\text{pattern}}, n_{\text{voltage}}, n_{\text{connection}}, n_{\text{pattern}}\}$, $\{n_{\text{connection}}\}_{j=1}^{n_{\text{pattern}}}$, $\{V_j\}_{i=1}^{n_{\text{pattern}}}$; $n_{\text{iteration}} := 5$; $n_{\text{pattern}} := 4$;

$n_{\text{pattern}} := 302$; $n_{\text{connection}, \text{max}} := 21$; $n_{\text{connection}, \text{sp}} := 12$; $n_{\text{connection}, \text{TCT}} := 21$; $n_{\text{connection, BL}} := 17$

$n_{\text{iteration}, \text{LD}} := 18$; $n_{\text{connection, HC}} := 19$; $k := 1$; $m := 1$;

while $i < n_{\text{voltage}}$

do

$k := l_{m+1}$; $P_{l} := l_{V_{l}}$

while $j < n_{\text{pattern}}$

do

$l_{k} := l_{m+1}$; $P_{k} := l_{V_{k}}$

while $l < n_{\text{pattern}}$

do

$k := l_{m+1}$; $l_{m+1} := l_{m} - g_{m}/g_{m}'$

repeat

$l_{m+1} := l_{m} - g_{m}/g_{m}'$

until

$l_{m+1} := l_{m}$

end_repeat

end_do

end_if

end_do

put $(P_{l}, l_{V_{l}})$

stop

3. Results and discussion

3.1. Result presentation

Table 2 shows the estimated values $(I_{ph}, I_{cb}, A, R_{p}, R_{v})$ for the five parameter models, which are capable of simulating the output power approximately to the measured output power at standard test condition with an insignificant dissimilarity between the computed and measured output power.
Figure 9 designates the performance curves for the different interconnections under unshaded condition (\(\alpha = 0\)); it serves as a basis for the comparison of power loss in the shaded interconnection under different shading conditions (0 < \(\alpha\) < 1) and patterns (column, row, diagonal, corner) in Equation (24). The coincidence of the performance curves in Figure 9 supports that the interconnections have equal probability to output current in the absence of shadings.

Figures 10 and 11 show the characteristic curves for column shading pattern under different shading factors (0 < \(\alpha\) < 1) and interconnection (SP, TCT, BL, LD and HC), respectively. Figure 11 is characterized by dual function (linear and exponential) and single function (exponential) in the equivalent circuit model (Equation (15) or (16)), respectively. Also, Figure 10 shows for small perturbation in shading factor (opacity) (the characteristic curves seem to cluster in the shaded string but coincided in the unshaded strings) which implies that column shading is not susceptible to degree of shading (opacity). Figure 11 shows clear separation of the characteristic curves for different interconnection, which portrays that the interconnections have unequal potential to output power.

Similarly, Figures 12 and 13 represent the characteristic curves for row shading pattern under different shading factors (0 < \(\alpha\) < 1) and interconnection (SP, TCT, BL, LD and HC), respectively. Figure 13 is characterized by only global peak because all the strings were shaded; thus, the equivalent circuit model (Equation (17) or (18)) is only made up of shaded function (combination of exponential and linear...
The characteristic curves in Figures 12 and 13 were clearly separated from each other, implying that the row shading pattern is quite sensitive to shading factor and interconnectivity, respectively. The magnitude of power generated in Figures 10 and 11 were higher than those of Figures 12 and 13 because all the strings in Figures 12 and 13 were shaded.
Figures 14 and 15 represent dual-peak characteristic curves for diagonal shading. Generally, the magnitude of maximum global power reduces as the size of local peak increases (or number of shaded string increases). Figure 14 shows first high peak that is sensitive to the shading factor and second small peak that is insensitive to shading factor, corresponding to three shaded strings and one unshaded string (Figures 2(c)–6(c)), respectively. The observed two peaks in Figures 14 and 15 show that the overall equivalent circuit model (Equation (19) or (20)) was synergistically built by two different equivalent circuit model functions (shaded—linear and exponential function, and unshaded—exponential function). The output power is dependent on the magnitude of the shading factor for shaded strings and on the type of interconnection because of unequal probability to output power.

Figures 16 and 17 represent the characteristic curves for the corner shading, which are similar to those of column shading (Figures 10 and 11) with the exception of a steeper or more pronounced local peak, which reduces the magnitude of the global peak compared to column shading patterns. However, the first peaks are sensitive to shading factor more than those of column shading because more strings were shaded in corner than column shading pattern. Moreover, the second peaks were clustered indicating that the remaining strings were unshaded or peaks were formed by single exponential function.

Figure 18 illustrates performance curves for TCT with different shading patterns and low shading factor, whereas Figure 19 depicts performance curves for TCT with different shading patterns and...
Figure 14. I–V and P–V curves for diagonal shading and TCT-interconnection at different shading factors, α.

Figure 15. I–V and P–V curves for diagonal shading and different interconnection at α = 0.25.

Figure 16. I–V and P–V curves for corner shading and TCT-interconnection at different shading factors, α.
high shading factors. These figures are characterized with presence of double peaks for different shading patterns except for row shading as also observed in Figures 12 and 13. At low shading factor ($\alpha \leq 0.1$), row shading (Figure 18) proved to be dominant over other shading patterns due to single peak formation, whereas at high shading factor ($\alpha \leq 0.9$), column shading (Figure 19) proved...
to be dominant to other shading patterns because the output power is slightly influenced by high shading factor but at the same condition, row shading almost vanished. Thus, shading of interconnected array across the strings (row shading) is dangerous to power generation in photovoltaic generators but shading along the strings does not pose detrimental effect to output power in photovoltaic systems especially in the case of one string being shaded.

Data in Table 3 were extracted from Figures 11, 13, 15 and 17 for different interconnections at the maximum power point or global peak power for each interconnection. Using Equation (23), the relative power gained for the different interconnections were obtained by comparing their values with that of SP interconnection.

Similarly, the data in Table 4 were extracted from Figures 9, 11, 13, 14 and 17 for different interconnections at the maximum power point or global peak power for each interconnection. Applying Equation (24), the relative power loss for different interconnections was obtained by comparing their values with those of unshaded interconnection.

Table 3 ranks the order of power gained as follows: TCT > HC > LD > BL > SP. Also, Table 4 represents the order of power loss as follows: TCT < HC < LD < BL < SP. These results simply portray the proportionality between the output power and number of interconnections within an array. Moreover, these results support that TCT-interconnection is superior to other interconnections (with least percentage power loss, 33% or highest percentage power gained, 75%), which is consistent with the submission made by Ramaprabha & Mathur (2012) and Vijayalekshmy et al., 2015.

The order of severity of the shading patterns from Table 5 is as follows for low shading factor: corner > column > diagonal > row; and for high shading factor: row > diagonal column > corner > column. Also, it can be inferred that maximum output power in Table 5 for column and corner shading patterns are constant irrespective of the shading factor, thus, buttressing the fact that it is shading across the strings (row shading) that is prone to power loss in photovoltaic modules. The occurrence of double competing peaks (local maximum) led to colossal power loss in an array.

### Table 3. Relative power gained (PG) at global peak and shading factor of 0.25

| Shading pattern | Global peak power (W) | Relative power gained (%) |
|-----------------|-----------------------|---------------------------|
|                 | SP        | TCT       | BL        | LD        | HC        | SP | TCT | BL | LD | HC |
| Column          | 2,315.0   | 4,051.2   | 3,279.6   | 3,472.8   | 3,665.4   | 0.00 | 75.00 | 41.67 | 50.01 | 58.33 |
| Row             | 2,276.7   | 3,987.1   | 3,227.7   | 3,417.6   | 3,607.4   | 0.00 | 75.13 | 41.77 | 50.11 | 58.45 |
| Diagonal        | 1,899.2   | 3,323.6   | 2,690.5   | 2,848.8   | 3,007.1   | 0.00 | 75.00 | 41.66 | 50.00 | 58.34 |
| Corner          | 1,543.3   | 2,701.0   | 2,186.4   | 2,315.2   | 2,443.6   | 0.00 | 75.01 | 41.67 | 50.02 | 58.34 |

### Table 4. Relative power loss (LP) at global peak and shading factor of 0.25

| Shading pattern | Global peak power (W) | Relative power loss (%) |
|-----------------|-----------------------|-------------------------|
|                 | Unshaded | Shaded                |
|                 | SP        | TCT       | BL        | LD        | HC        | SP | TCT | BL | LD | HC |
| Column          | 5,402    | 2,315     | 4,051     | 3,280     | 3,473     | 3,665 | −133 | −33 | −65 | −56 | −47 |
| Row             | 5,402    | 2,277     | 3,987     | 3,228     | 3,418     | 3,607 | −137 | −35 | −67 | −58 | −50 |
| Diagonal        | 5,402    | 1,899     | 3,324     | 2,691     | 2,849     | 3,007 | −184 | −63 | −101 | −90 | −80 |
| Corner          | 5,402    | 1,543     | 2,701     | 2,186     | 2,315     | 2,444 | −250 | −100 | −147 | −133 | −121 |
which is typical of corner shading at low shading factor. Column shading produced minimal power loss in an array compared to other shading patterns at high shading factor because it had more unshaded strings than the other shading patterns.

4. Conclusion
The development of real generic mathematical models for the different interconnections and array sizes has been successfully carried out. Also, the pseudocode algorithm for the mathematical analysis was developed and detailed performance analysis of the interconnected arrays under different shading conditions and patterns was equally carried out.

It is obvious that the TCT is superior to other interconnections due to its highest number of interconnections, which gave rise to maximum power gained (or minimum power loss) relative to other interconnections.

The present work is able to pinpoint that shading patterns across the strings (row and diagonal) affect the output power from an array of interconnected photovoltaic system.

Also, the present work elucidates that shading patterns along the string (column and corner) do not significantly affect the maximum output power from an array of interconnected photovoltaic system.

Furthermore, the present work affirms that high shading factor (α→1) drastically affects the output power for shading patterns across the strings and output power vanishes under perfect or total shading condition.

Moreover, shading leads to formation of double peaks, which reduces the magnitude of the global maximum power. Thus, equal number or more numbers of shaded strings against unshaded strings tremendously diminish the output power from an array of interconnected photovoltaic system.

However, all the interconnections have equal strength to generate power from the interconnected arrays in the absence of shades (α = 0). Thus, these information are vital for smooth operation and maximum generation of power from interconnected arrays under different shading conditions.

Besides, this paper recommends that arrays should be sited mostly in tropical (and open spaces) in order to minimize the effect of shadings.

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Table 5. Relative power loss (LP) at global peak for shading patterns and TCT interconnection

| Shading factor (opacity) | Global peak power (W) | Relative power loss (%) |
|-------------------------|-----------------------|-------------------------|
|                         | Unshaded (α = 0)      | Shaded (0 < α < 1)      |                         |
|                         | Column | Row | Diagonal | Corner | Col. | Row | Diag. | Corner |
| 0.10                    | 5,402  | 4,051.6 | 4,837.3 | 3,804.3 | 2,701.0 | -25.0 | -10.5 | -29.6 | -50.0 |
| 0.50                    | 5,402  | 4,051.6 | 2,578.4 | 2,519.5 | 2,701.0 | -25.0 | -52.3 | -53.4 | -50.0 |
| 0.90                    | 5,402  | 4,051.6 | 385.5   | 1,350.5 | 2,701.0 | -25.0 | -92.9 | -75.0 | -50.0 |
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