Thermal structure of gas in relaxed clusters of galaxies

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Abstract. Gas clouds under the influence of gravitation in thermodynamic equilibrium cannot be isothermal due to the Dufour effect, the energy flux induced by density gradients. In galaxy clusters this effect may be responsible for most of the "cooling flows" instead of radiative cooling of the gas. Observations from XMM-Newton and Chandra satellites agree well with model calculations, assuming equilibrium between energy flux induced by temperature and density gradients, but without radiation loss.

Key words. galaxy clusters – intracluster gas – heat conduction – thermal equilibrium

1. Introduction

It has been known for long years that in most clusters of galaxies the intracluster plasma is far from being isothermal, exhibiting strong decrease of the plasma temperature in the vicinity of individual large galaxies. This effect is commonly attributed to "cooling flows", that means, increasing radiative energy loss with increasing density of the gas, which is gravitationally attracted by the galaxies. But recent high resolution data from the XMM-Newton and Chandra satellites have shown that this cooling flow model does not fit to the observations. This has newly kindled a vivid discussion on the the origin of the discrepancy.

Already in former years the fact was addressed that thermal conduction would wash out the temperature gradients, if the thermal conductivity of the plasma were of the magnitude given in the classical book by Spitzer (1962). As a possible explanation it has been proposed that magnetic fields may reduce the conduction. Several authors claim very different reduction factors of the thermal conductivity (see e.g. Voigt et al. 2002, Markevitch et al. 2003) to fit the data. The existence of magnetic fields in the order of $0.1 - 1 \mu G$ has been confirmed by various observations (see e.g. Dolag et al. 2001). But to produce the required reduction, ordered magnetic fields of a still higher magnitude would be necessary, as the heat flux reduction works only perpendicular to the magnetic field lines and there is not yet a generally agreed model to decide, to which extent magnetic fields can influence thermal conductivity. Narayan and Medvedev 2001 have shown that fields fluctuating on a large range of length scales can produce only minor changes in the thermal conductivity, while others (see Chandran and Maron 2004 and references therein) give reduction factors as high as 10.

One problem with all these reduction factors is, that they would affect the temperature profiles preferably at high temperatures in the outer boundary of the "cooling flow" region, while radiative cooling is most effective near the centre. But observations exhibit just the opposite effect (Kaastra et al. 2004).

In all these discussions it is implicitly assumed that in thermodynamic equilibrium conduction leads to a uniform temperature, though it is well known that density gradients may also cause a flux of energy and thus induce temperature gradients, an effect known from the textbooks as "Dufour effect" (see e.g. Hirschfelder et al. 1964). Under terrestrial conditions this effect is regarded mostly as a second order correction to "normal" conduction, that means, to energy flux due to temperature gradients. We are more familiar with the "reciprocal" effect, the diffusion of particles caused by temperature gradients, known as thermal diffusion. But under the condition of cosmic plasmas, where stationary density gradients are maintained by gravitational fields, the Dufour effect can considerably alter the equilibrium conditions. Even in relaxed systems there will be stationary temperature gradients to balance the energy flux caused by density gradients.

Normally in a gas we associate "equilibrium" with constant pressure, density and temperature. But in a system, which is influenced by volume forces such as gravitation, the equilibrium state may well exhibit gradients of the state variables. Mass flow equilibrium in these systems is obtained by the balance between the gravitational force and the counteracting pressure gradient, which is related to the density gradient by some equation of state.

But if there is a density gradient, energy equilibrium can be obtained only, if there is also a temperature gradient, balancing the Dufour effect, to obtain zero net energy flux. Below we will derive the equilibrium conditions for a
system, which is fully relaxed with respect to flux of mass, momentum and energy, within the scope of a simplified kinetic model.

In the next section we will discuss the equilibrium conditions in cluster plasmas. In the third section a simplified kinetic model is developed to determine the essential consequences of the Dufour effect. In section 4 the results are compared with observational data from XMM-Newton and Chandra, and in section 5 conclusions are drawn with respect to the "cooling flow" region as well as for the equilibrium conditions of complete galaxy clusters.

2. Equilibrium relations

Speaking of equilibrium means that the net fluxes of all properties determining the state of a system are locally balanced. Complete local equilibrium will of course never be established in a system with radiation losses or with irreversible chemical reactions. But as long as these are not the dominant exchange processes, quasi-equilibrium conditions can be formulated, considering only the dominant transport processes.

To identify the dominant transport processes, it is convenient to compare the time constants for the local change of temperature, density or composition due to some loss and production process or flux. Common to all discussions on the energy balance of the "cooling flow" region of intracluster gas is, that radiation is regarded as the only energy loss process. This loss has to be balanced by energy gain from inflowing hot matter, by uptake of gravitational energy from inflowing gas, by heat conduction from outer hot regions and energy production by supernovae in galaxies or active galactic nuclei.

The Dufour effect adds a further transport process to the balance, which acts like an additional cooling, opposite to the heating processes mentioned above. It can be regarded as a thermal sedimentation of low energy particles into the vicinity of a central gravitating mass. Microscopically it is closely connected to the "normal" thermal conduction, as both result from the same process: the transport of kinetic energy of the plasma particles in an inhomogeneous medium. If there is thermal conduction, there is always one fraction caused by temperature gradients and one caused by density gradients. The relation between the two parts will be discussed in more detail in the next section.

To find out, which of the heating or cooling processes preferentially contributes to the balance we compare the time constants of the corresponding processes. In the "classical" cooling flow model only two processes are considered: radiation losses and inward flow of hot matter. The time constant for radiative cooling is \( \tau_r = \frac{(3nkT)}{\Lambda(T) n^2} \), where \( \Lambda(T) \) is the cooling function (see e.g. Voigt et al. 1992). In the temperature and density range observed in the centre of galaxy clusters typical values are \( kT = 3\text{keV} \) and \( n = 10^{-2}\text{cm}^{-3} \), resulting in a cooling time \( \tau_r = 1.2 \times 10^{17}\text{sec} \). As this is much longer than the collapse time \( \tau_c = 1/\sqrt{\rho G} = 3 \times 10^{10}\text{sec} \), equilibrium can always be reached between gas inflow and radiation loss.

There are, however, two major problems with this model. It does not fit to the observations, as the increasing cooling rate should increase with increasing density, so that the temperature profile should exhibit a sharp dip in the centre, which is not observed. Secondly it neglects thermal conduction, which has time constants much shorter than radiation, so that it could wash out all gradients established by radiative cooling. If thermal conduction were at the theoretical value given by Spitzer (1972) \( \lambda = 4.6 \times 10^{-7} T^{5/2}\text{erg cm}^{-2}\text{K}^{-1}\text{sec}^{-1} \), for a cooling flow region of \( r_c = 50\text{kpc} \) the conduction time scale is \( \tau_\lambda = nkT^2/\lambda = 10^{16}\text{sec} \), much less than the cooling time scale. Even if thermal conduction is reduced by the presence of magnetic fields, as proposed by several authors (see Narayan and Medvedev 2001, Chandran and Maron 2004), it remains a dominant transport process.

But as has been mentioned above, this does not imply that the temperature should be constant. Due to the opposite effects of temperature and density gradients also under the condition of local thermal equilibrium the temperature will vary in radial direction. In the next section we will derive this equilibrium condition under the assumption that conduction is dominant and radiation losses can be neglected.

3. Kinetic model

To determine the equilibrium conditions in a plasma under the influence of an outer force field such as gravitation, we determine the net flux of mass and energy through a control area \( \Delta F \) under the condition that the particles in the gas have a Maxwellian distribution everywhere, but with number density and mean energy varying perpendicular to the control area. The net flux is calculated then under the assumption, that any particle crossing the control area retains its velocity and direction, which it has obtained in the last collision, one mean free path from the surface, and in addition is subjected to some acceleration from the force field.

In the energy balance the gain and loss of kinetic energy due to the changing gravitational potential will be omitted, as it cancels out from the balance exactly, when the net particle flux through the control area is zero. As we are interested only in the equilibrium condition, the absolute magnitude of the energy or particle flux is not important, as long as it dominates over other transport, production or loss processes, which appears to be justified for most of the "cooling flow" region in clusters.

Denoting the direction perpendicular to the surface as \( z \) and the angle between this and the direction of particle motion as \( \theta \), the flux of some property \( G \) through the surface is

\[
J = \int \Delta F \cos \theta (j^+ - j^-) d\Omega,
\]

where \( j^+ \) and \( j^- \) are the normal components of the flux of \( G \) carried by particles moving towards the surface
in positive and negative z direction under the angle \( \theta \).
The integration has to be taken over half the solid angle \( \pi/2 \geq \theta \geq 0 \). Denoting the amount of property \( G \) carried by particles moving with velocity \( v \) by \( g(v) \), the flux components of \( g(v) \) are

\[
j^+ = (g(v) - \frac{dg(v)}{dz} \lambda \cos \theta)(v \cos \theta + b \frac{\lambda}{v})
\]

\[
j^- = (g(v) + \frac{dg(v)}{dz} \lambda \cos \theta)(v \cos \theta - b \frac{\lambda}{v})
\]

\[C\]

\( \lambda \) is the mean free path, which may depend also on the velocity. The last term denotes the acceleration of the particles during their flight from the last collision to the control surface. Integrating eq. (1) over the solid angle yields the equation

\[
j = -\pi F \left( \lambda v \frac{dg(v)}{dz} - \frac{2b\lambda}{v} g(v) \right)
\]

\[D\]

To determine mass and energy transport by particles moving with velocity \( v \), \( g \) has to be set to

\[
g_M = n(z) m f(v, z) dv,
g_E = n(z) m v^2 \frac{f(v, z) dv}{2}
\]

\[E\]

In case of a neutral gas \( n(z) \) and \( m \) are the numerical density and mass of atoms, \( f(v, z) \) is the distribution function

\[
f(v) dv = \frac{4}{\sqrt{\pi}} \frac{v^2}{\beta^3} e^{-\frac{v^2}{\beta^2}} dv
\]

\[F\]

\( \beta(z) \) being the abbreviation \( \beta(z) = \sqrt{2kT(z)/m} \). To calculate the quantity \( dg/dz \) we also need the derivative

\[
\frac{df}{dz} = \frac{d\beta}{dz} \frac{4}{\sqrt{\pi}} \left( \frac{2v^4}{\beta^6} - \frac{3v^2}{\beta^4} \right) e^{-\frac{v^2}{\beta^2}} = \frac{1}{\beta^2} \frac{d\beta}{dz} \left( \frac{2v^2}{\beta^2} - 3 \right) f(v)
\]

\[G\]

While in a neutral gas the parameter \( b \) in eq. (1) is the normal gravitational acceleration, in a plasma the action of the force is somewhat indirect. Energy transport is mediated preferably by electrons. Thus in eqs. (5) \( n(z) \) and \( m \) are the electron density and mass. But in this case acceleration by the external force is acting also on the ions, which transfer the force to the electrons by Coulomb interaction. Thus the parameter \( b \) in eqs. (2) and (3) is not the gravitational acceleration of free moving electrons. But for the equilibrium condition of zero transport the absolute magnitude of \( b \) is not relevant, when we only want to know, under which conditions the net flux of electrons and the flux of energy carried by them are in balance.

An additional difference between neutral gas and plasma results from the fact that the collision cross section between neutral atoms is nearly independent of the collision energy, so that the mean free path \( \lambda \) does not depend on \( v \). In plasmas, where Coulomb interaction is dominant, the mean free path increases with \( v \). Thus in the further calculations we set \( \lambda = \lambda_0 (v/v_0)\alpha \) with \( \alpha = 0 \) for neutral gas and \( \alpha = 2 \) for a fully ionised plasma.

Introducing eqs. (5) to (7) into eq. (4), we get for the mass and energy flux

\[
j_M = 2\beta^2 \left( x^2 \frac{dn}{dz} + x(2x^2 - 3)n \frac{d\beta}{dz} - \frac{2bn}{\beta^2} \right) h(x) dx
\]

\[H\]

\[
j_E = \beta^2 \left( x^2 \frac{dn}{dz} + x(2x^2 - 3)n \frac{d\beta}{dz} - \frac{2bn}{\beta^2} \right) h(x) dx
\]

\[I\]

with \( x = v/\beta \) and \( h(x) = 2\sqrt{\pi} m \Delta F(\lambda_0/v_0^2) x^{2+\alpha} e^{-x^2} \). Integrating these equations over all \( x \), we obtain the conditions for zero mass and energy flux:

\[
\beta \frac{dn}{dz} F_{3+\alpha} + n \frac{d\beta}{dz} \left( 2F_2 - 3F_3 \right) - \frac{2bn}{\beta^2} F_{1+\alpha} = 0
\]

\[J\]

\[
\beta \frac{dn}{dz} F_{5+\alpha} + n \frac{d\beta}{dz} \left( 2F_7 - 3F_5 \right) - \frac{2bn}{\beta^2} F_{3+\alpha} = 0
\]

\[K\]

The abbreviation \( F_k \) stands for the integral \( F_k = \int_0^\infty x^k e^{-x^2} dx \). The different arguments of \( F_k \) in eqs. (10) and (11) can be eliminated by the recurrence formula

\[
F_{k+2} = (k+1)/2 \times F_k
\]

\[L\]

so that we finally obtain the equilibrium conditions for mass and energy transport:

\[
\frac{2 + \alpha}{2} \beta \frac{dn}{dz} + \frac{1}{2} (2 + \alpha) (1 + \alpha) \frac{n}{\beta} \frac{d\beta}{dz} - \frac{2bn}{\beta^2} = 0
\]

\[M\]

\[
\frac{4 + \alpha}{2} \beta \frac{dn}{dz} + \frac{1}{2} (4 + \alpha) (3 + \alpha) \frac{n}{\beta} \frac{d\beta}{dz} - \frac{2bn}{\beta^2} = 0
\]

\[N\]

It is immediately evident that this is a set of linear equations for the gradients \( d\beta/dz \) and \( dn/dz \). If there are no external forces \( (b = 0) \), there exists only the trivial solution \( d\beta/dz = 0 \) and \( dn/dz = 0 \). In the presence of volume forces such as gravitation eqs. (12) and (13) are inhomogeneous and allow a non-trivial solution, which constitutes a fixed relation between temperature gradient and particle density gradient or, because of quasi-neutrality, also between temperature and mass density gradient. Eliminating \( b \) leads to the relation

\[
\beta \frac{dn}{dz} + (2\alpha + 5) n \frac{d\beta}{dz} = 0
\]

\[O\]

with the solution \( n\beta(2\alpha+5) = \text{const} \). Replacing the number density by the mass density \( \rho \) and \( \beta \) by \( \sqrt{2kT/m} \) we finally find the condition \( \rho T^{5/2} = \text{const} \) for neutral gas and \( \rho T^{9/2} = \text{const} \) for a fully ionised plasma.

This result is independent from the actual magnitude of the coefficient of thermal conduction, if it is influenced by magnetic fields or not. The only conditions, which have to be satisfied, are the existence of local thermal equilibrium and the condition, that transport of kinetic energy of particles is the dominant transfer mechanism.

4. Observations

During the last years several spatially resolved x-ray emission measurements of galaxy clusters have become available, especially from the Chandra and XMM-Newton...
satellites. Though temperature determinations are still rather poor due to projection effects and necessary background corrections, intensity profiles and from these density profiles have been obtained with high accuracy. But in spite of the remaining error margin the classical cooling flow model can definitely be ruled out as an explanation of the observed data (see Kaastra et al. 2004). Neither the radial temperature profiles fit to those expected from this model nor do the spectra show the expected intensity of metal lines of lower ionisation levels.

Fig. 1. $kT \times \rho^{2/9}$ for consecutive annular channels starting from cluster center for the 7 most luminous clusters given by Kaastra et al. [2004]

Good agreement with observations can, however be obtained with a model based on thermal conduction with inclusion of the Dufour effect. We have applied the model to the data given by Kaastra et al. [2004]. In fig. the quantity $T \times \rho^{2/9}$ is plotted for the seven most luminous clusters, for which data are given by Kaasta et al. Only the clusters with the highest X-ray flux ($\geq 5 \times 10^{-11}$ erg cm$^{-2}$ sec$^{-1}$, 0.1-2.5 keV) have been used, as for these clusters the temperature profiles appear sufficiently accurate. The data points in fig. refer to temperatures in consecutive annular channels starting from centre to the boundary of the cooling region. (For details see Kaastra et al. [2004]) For all seven clusters the constancy of $T \times \rho^{2/9}$ over the cooling region is evident. This can be regarded as a confirmation that thermal conduction with inclusion of the Dufour effect is the dominant energy transfer mechanism and that radiative cooling is less important.

Data from Perseus cluster have also been obtained by Chandra (see Sanders et al. [2004]). Fig. shows a comparison of the temperature profiles obtained from direct measurements by Chandra and XMM-Newton compared with data derived from the corresponding density profiles, applying $\rho T^{3/2} =$ const. The curves are fitted at the cluster centre. As can be seen, the curves derived from density data agree very well with each other and with the direct measurements from XMM-Newton, while the temperatures from Chandra deviate considerably in the outer region. It should be mentioned, however, that the Chandra data are obtained by averaging angular resolved profiles which exhibit a large spread ($> 1.5$ keV) in the outer region.

Whether radiative loss can completely neglected or has at least to be added as a correction to the temperature profile resulting from pure heat conduction, cannot be decided from the present data. For the model this would require a better knowledge of the absolute magnitude of the thermal conductivity. This requires an accurate theory of the reduction effects by magnetic fields and more knowledge on the distribution of these fields over the cluster volume.

5. Conclusions

We have shown that thermal conduction appears to be the dominant energy transport mechanism and that a pure conduction model can describe the observed temperature profiles, when energy flux induced by density gradients is added to the balance. The relative temperature gradients are small compared to the associated density gradients. But temperature changes by a factor of two or three, as they have been found in the central regions of the clusters mentioned above and which are reported also for other "cooling flow" clusters by several authors (see e.g. Voigt et al. 2002), can well be understood from the Dufour effect, as the gas density may change by a few orders of magnitude in the vicinity of large galaxies. It appears likely that most of the observed temperature drop in the "cooling flows" around these galaxies is not caused by radiative cooling but by the "conductive cooling" associated with the density gradients.
It should be noted that the Dufour effect is not restricted to the cores of galaxies, but it should be important also in the hot corona of individual stars, where it can help to explain the observed temperature gradient from the surface into the interstellar medium.

Applying the model to complete galaxy clusters leads to the result that the plasma temperature should continuously rise with increasing distance from the centre. Observational data are contradictory in the outer region. Some authors claim a temperature drop at the outskirts of the observable region, others find constant or increasing temperature (see Mushotzky 2004 and references given there). Most of the recent papers find increasing temperatures at least out to the point, where the increase of the error bars exceeds the expected temperature gradients.

Also the conduction model should be taken with care at the low densities in the outer regions, as the conditions of local thermal equilibrium are no longer satisfied due to the extremely large mean free path of electrons. But it can be stated that, if thermal conductivity were close to the Spitzer value, clusters would have cooled off in the time since formation, unless the clusters are embedded in a very hot but thin intergalactic plasma. The existence of a diffuse high energy x-ray background would be a strong hint to the existence of such an intergalactic plasma.

Measurements of the background in the energy range above that of the intracluster gas are rare. Observations from the HEAO-1 satellite (Boldt 1987) have found a continuous background, which can be interpreted as bremsstrahlung in the temperature range of 40 to 50 keV, but the spatial resolution of these early data is so poor that it cannot be decided, if it is really radiation from a continuously distributed plasma or from the superposition of individual point sources. As a plausible explanation for the formation of a hot continuous background plasma one could think of the relativistic matter jets expelled by quasars. During the time period of several Gyr this matter should have become homogeneously distributed in space.

It has been shown that the conduction model presented in this paper can explain some observations in the central regions of the intracluster plasma, but it deserves more confirmation by detailed measurements, until the astrophysics community will be convinced. Also the extension to the outer regions of galaxy clusters would bring additional confirmation, but this requires the extension of spatially resolved measurements into the high energy range, as they are planned with the launch of future satellites like Astro-E2, which are scheduled for the next years.

References
Boldt, E., 1987, Phys. Rept., 146, 216
Chandran, B.D.G. and Maron, J.L., 2004, ApJ, to be published [astro-ph/0303214]
Dolag, K. et al., 2001, A&A, 378, 777
Hirschfelder, J.O., Curtiss, C.F., Bird, R.B., 1964, Molecular theory of gases and liquids, New York: Wiley
Kaastra, J.S. et al., 2004, A&A, 413, 415
Markevitch, M., Mazotta, P. et al., 2003, ApJ, 586, L19
Mushotzky, R.F., 2004, Carnegie Observatories Astrophysics Series, Vol. 3: Clusters of Galaxies: Probes of Cosmological Structure and Galaxy Evolution, Cambridge: Cambridge Univ. Press
Narayan, R., Medvedev, M.V., 2001, ApJ, 562, L129
Sanders, J.S. et al., 2004, MNRAS, to be published [astro-ph/0311502]
Spitzer L., 1962, Physics of Fully Ionized Gases, New York: Wiley-Interscience
Voigt L.M., Schmidt, R.W. et al., 2002, MNRAS, 335, L7