A genetic algorithm approach to fitting interferometric data of post-AGB objects: I. the case of the Ant nebula

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ABSTRACT

We present GADRAD, a Python module that adopts heuristic search techniques in the form of genetic algorithms, to efficiently model post-asymptotic giant branch (post-AGB) disc environments. GADRAD systematically constructs the multi-dimensional parameter probability density functions that arise from the fitting of radiative transfer and geometric models to optical interferometric data products. The result provides unbiased descriptions of the object’s potential morphology, component luminosities and temperatures, dust composition, disc density profiles and mass. Correlation in the estimated parameters as well as potential degeneracies are revealed. Estimated probability distributions of the post-AGB environment parameters provide insight into the shaping processes that may occur in the transition from the post-AGB to the planetary nebula phase. We test parameter recovery on simulated artificial data products of a typical post-AGB environment. We then use GADRAD to model the mid-infrared spectrum and visibilities of the Ant nebula (Mz3), taken with the Very Large Telescope Interferometer’s instrument MIDI. Our result is consistent with a large dusty disc with similar parameter values to those previously found by Chesneau et al., except for a larger dust mass of $3.5^{+7.5}_{-2.2} \times 10^{-5} \, M_\odot$. The parameter confidence intervals determined by GADRAD, can however be relied upon to impose additional constraints on all disc and system parameters. Based on our analysis and other considerations, we tentatively suggest that Mz3 is a pre-PN ejected during a magnetic (polar) common envelope interaction, where the binary may or may not have survived at the core of the nebula.

Key words: stars: AGB and post-AGB - planetary nebulae: individual: Menzel 3 - techniques: interferometric - techniques: high angular resolution.

1 INTRODUCTION

A fully consistent narrative that describes the post-Asymptotic Giant Branch (post-AGB) to the planetary nebula (PN) evolutionary transition, has yet to be established. This evolutionary phase is complex with many physical processes taking place simultaneously. Magnetic fields, accretion discs, jets, outflows and binary interactions have all, for example been suggested to act in isolation or in collaboration (e.g., Balick & Frank 2002; De Marco 2009). However, in almost all instances, circumstellar discs are believed to play an important role in the shaping of asymmetric PNe. Discs come in all masses and sizes and can be observed inside young as well as old PN testifying to their formation time and/or their longevity.

The discs of interest here are large enough to be detected by the high spatial resolution of interferometers (a few milliarcseconds). They are not the small accretion discs that sometimes form around one of the stars in a close binary system. They are instead larger (au-scale), cool and dusty discs. Large, cool dusty discs likely form due to a binary interaction process during the AGB and may play an active role in shaping the subsequent outflows. Alternatively they may not directly play a role in shaping the outflow, but may form as a byproduct of the shaping mechanism: for example, jets may inflate lobes that in turn compress material on the equatorial plane forming large disc-like structures (Akashi & Soker 2008). The disc’s parameters, such as its geometry, orientation, mass and chemical makeup (i.e., Balick & Frank...
may give us enough information to determine their origin and whether the disc played an active role in shaping the nebula or if it was itself an outcome of the nebula formation process (e.g., Bright 2013).

However, the study of these discs remains challenging. Direct observation for instance is not typically achievable. Even the closest objects for example remain near the limit of current instruments’ spatial resolution. The required resolution, however, can be obtained via optical interferometric techniques, but such options present their own challenges. For example, the number of interferometric measurements necessary for full image reconstruction is presently unfeasible, because post-AGB discs tend to be far and faint and large telescopes are typically required, limiting the observing time that can be dedicated to individual objects.

One must instead rely on other forms of modelling from interferometric data products to obtain descriptions of the objects, whether geometric/analytic or numeric. In the more complex objects, numerical modelling such as radiative transfer (RT) has become common practise. The models are however typically dependent on numerous input parameters. The simplest environments, for example, can often require parameters describing the stellar properties such as luminosity and temperature, disc density profiles and mass, as well as dust composition. Modelling the discs thus becomes a problem of optimisation, with potentially complex and non-linear dependencies between the parameters. This problem is all too often solved in an ad-hoc, non-systematic fashion, leaving the possibility that better global solutions exist (e.g., Chesneau et al. 2007; Lykou et al. 2011; Bright et al. 2012; Bright 2013). Human parameter choice for example, often considers only small fractions of parameter space. The level of potential under-sampling is evident when considering the number of models required to sample an $n$-dimensional space. For example, consider only a sparsely sampled brute-force approach with just four models sampling a two-dimensional space (i.e., two values per parameter, with a single model sampling each quadrant). To similarly sample the quadrants of an $n$ dimensional space, $2^n$ models are necessary. In the case of RT models of post-AGB objects, were even the simplest models can require 16 parameters, this (very low) level of sampling would require upwards of 65,000 models! Reducing parameter space by fixing variables to literature values reduces the number of models, but this often further degrades the final solutions as this form of parameter limitation necessitates a posteriori sampling (and with incorrect parameter adoption) introduces bias. Furthermore, the $\chi^2$ fitting approach adopted in these studies only give little indication of whether a given parameter set is better than another, they do not for instance, provide insight into the underlying parameter interaction, nor indicate the range of acceptable parameter values that represent good model solutions.

Determining RT models solutions that result in the interferometric data products observed, is an ill-posed, general inverse type problem. It is an inverse type problem as we have to transform the interferometric data products to a set of model parameters, as opposed to the more commonly found forward problem, were the transformation direction is reversed (i.e., given the model parameters, calculate the result). As with many inverse type problems, our problem is also ill-posed, for example the parameters we wish to determine are poorly constrained by the data and many potential solutions exist. The number of parameters available in constructing a RT model though finite, can be numerous, and there is likely non-uniqueness in the solution space (especially when considering the stochastic noise introduced to each model as a result of Monte-Carlo RT simulation). Model approximations need to be made, where the model class adopted hopefully generalises well enough to represent the main characteristic features of the observed object. In determining ranges of acceptable parameter values from such a model, we explore the objective function (the function that maps the parameter input values to the resulting model’s ‘fitness’). Minimisation of the objective function results in good model representations of the object (known as a quasi-solution), the multi-dimensional non-linear nature of the function however, typically requires the application of robust optimisation techniques.

In this paper, we develop a heuristic method, in the form of a genetic algorithm (GA) to determine the disc and stellar parameters of post-AGB objects. Through such an approach we are able to provide much improved fitting to the interferometric data products. We develop GADRAD (Genetic Algorithm Driven RADiative transfer), a Python module that acts as an interface code between the Fortran RT software RADMC-3D and a Python GA. The goal of GADRAD is to characterise the parameter ranges of good fitting RT model solutions in relation to the interferometric data products.

GADRAD differs to previous attempts at obtaining GA solutions to observational data products via RT methods (e.g., Hetem & Gregorio-Hetem 2007; Schechtman-Rook et al. 2012; De Geyter et al. 2013; Menu et al. 2014) by constructing density profiles from the fittest solutions of a number of independent GAs. This allows us to gain an understanding of parameter interaction, explore potential model degeneracies, and provide disc and post-AGB environment constraints through probability density functions (i.e., provide high confidence level error-bar constraints on disc-environment parameters). This approach is ultimately far superior to an approach in which parameters are selected by hand, even where the final parameters of the two methods do not differ significantly. Using a GA gives reasonable certainty that the error bar areas are meaningful, while the by-hand approach does not provide error bars at all.

Here we apply GADRAD to the disc found at the heart of the Ant nebula (Menzel 3, hereafter Mz3). The mechanism that ejected this highly collimated, massive nebula is still unclear. In fact it is not even clear whether it is a pre-planetary nebula (prePN), a nebula that is in the process of being ionised by the heating core of the giant that ejected the nebular material, or a symbiotic system, where a second, white dwarf source is ionising the gas that is being ejected by a giant companion. The hope is that quantifying the disc parameters my lead to an explanation of the evolution of the object and the outflow’s ejection mechanism.

We begin by presenting GADRAD in Sect. 2, where we provide background on the RT code RADMC-3D, and introduce the GA. GA convergence is then tested on an optimisation test function. In Sect. 3 we apply GADRAD to a post-AGB synthetic test object. In this section we assess the ability of the module to handle the stochastic nature of Monte-Carlo RT and parameter recovery over a range of dimensional spaces.
In Sect. 4 we apply GADRAD to the young nebula Menzel 3. Here we obtain parameter probability density estimates, and again review model degeneracies. We also compare our findings with those of Chesneau et al. (2007). In Sect. 5 and Sect. 6 we summarise.

2 GADRAD

2.1 The Radiative transfer code

To simulate our dusty post-AGB environments we use the general-purpose 3-D Monte-Carlo RT RADMC-3D code (Dullemond 2012), based on RADMC (Dullemond 2011). The code is versatile, being applicable to dust continuum RT, molecular and/or atomic line transfer and gas continuum transfer in 1-D, 2-D and 3-D geometries. RADMC-3D was adopted for its ability to create intensity distributions (images) and spectral energy distributions quickly and efficiently.

When running potentially thousands of RT models required for a heuristic optimisation approach, simulation efficiency is paramount. RADMC-3D offers a range of RT approximation methods that save precious computational seconds. Such methods include modified random walk, weighted photon package mode and simplified random walk. These techniques, in addition to controlling the number of photons and model resolution, allow us to balance simulation speed and accuracy. Additionally, RADMC-3D was chosen for its excellent, and comprehensive documentation.

2.2 The genetic algorithm

The number of physical parameters required for the correct RT simulations of even the most simple post-AGB disc can quickly become quite large. Additionally, non-intuitive and complex interaction can sometimes exists between the parameters. The large parameter space, in addition to the not so insignificant RT computational time, rule out brute force grid searching. Instead we must minimise the objective function by way of systematic optimisation methods. Ideally, we seek an algorithm that limits the number of RT runs required, and is able to handle the stochastic nature of Monte-Carlo RT.

For this task we adopt the GA (Holland 1975), which takes inspiration from genetic and natural selection ideas. In mimicking biological concepts such as inheritance, mutation, selection, and crossover, the GA has been shown to be both robust and versatile (e.g., Charbonneau 1995). The GA is known for example to handle large numbers of variables, multiple local optima, as well as complex non-linear objective functions. As is common to some other optimisation techniques the GA does not require calculation of derivatives. Perhaps most importantly however, the GA has been shown to handle well the noisy objective functions that arise in Monte-Carlo RT simulation (i.e., De Geyter et al. 2013). The strength of the GA in handling noisy fitness functions arises from the iteration on a population of candidate solutions. Classical optimisation techniques that iterate on single solutions can be susceptible to the random noise of the objective function for example, and final solutions can be heavily influenced. In comparing the GA with other algorithms (in particular the Levenberg-Marquardt algorithm and Downhill simplex method), De Geyter et al. (2013) found the GA was better at reaching the global minima and handling the noise introduced in the MC models.

The genetic algorithm, at its core, relies on three genetic operators: selection, crossover and mutation. The algorithm proceeds as follows:

(i) Initialisation. Initialise the population, that is create \( n_{\text{pop}} \) parameter sets, known as chromosomes, by selecting random values from within given parameter domain ranges. The parameters in this instance will describe the post-AGB environment, disc scale height, inner-disc radius, etc.

(ii) Evaluation. Evaluate the quality (fitness) of the parameter sets that make up the population, based on a goodness of fit criteria (such as \( \chi^2 \)).

(iii) Selection. Select individuals (sets of parameters) to breed from the given population.

(iv) Recombination. Combine (breed) individuals to form the next generation.

(v) Mutation. Randomly adjust given parameters of the individuals.

(vi) Repeat steps (ii)-(v) until termination criterion has been met, at which point the solution set is obtained. Each iteration is known as a generation.

The initial step (step i) taken in the GA is to produce a population of parameter sets. These parameter sets are known as candidate solutions, or chromosomes. The parameter values that make up the parameter sets are known as genes. For example, a three-dimensional optimisation problem would require a population \( n_{\text{pop}} \) of chromosomes with just three genes. This could be a fit with only 3 freer parameters, say, stellar temperature, disc’s inner-radius and its scale height, where say, 100 combinations of these 3 parameters are chosen as the initial population, the allowable parameter values (gene values) are known as alleles.

A number of methods exists in the creation of the initial population, such as random sampling, uniform sampling or complementary sampling. In GADRAD we adopt random sampling, in which allele values are randomly selected from a defined search range. We select values based on a uniform sampling distribution i.e.,

\[
P(x) = \begin{cases} \frac{1}{b-a} & \text{where } a \leq x \leq b \\ 0 & \text{otherwise}, \end{cases} \quad (1)
\]

where \( a \) and \( b \) are the domain limits of each parameter (allele limit), based upon literature values.

Following initialisation we evaluate the candidate solution (step ii). Determining the fitness of a candidate solution is generally the most computationally expensive task (for example in this instance it is the RT simulation). How the fitness is evaluated is also very important. A fitness function that does not relate in a meaningful way to the observed data will result in poor model representation. We adopt the following weighted function as a fitness measure:

\[
\Phi = \frac{w_1}{N_1} \sum_{i=1}^{N_1} \left( \frac{x_i - \mu_i}{\delta_i} \right)^2 + \frac{w_2}{N_2} \sum_{j=1}^{N_2} \left( \frac{x_j - \mu_j}{\delta_j} \right)^2 + \ldots
\]

\[
+ \frac{w_3}{N_3} \sum_{k=1}^{N_3} \left( \frac{x_k - \mu_k}{\delta_k} \right)^2, \quad (2)
\]

where \( x_i \) is the model value as obtained by the RT sim-
ulation, $\mu_i$ is the observed data product value and $\delta_i$ the error or uncertainty in the observed value. The function is the sum of the data products (i.e. the visibility, spectrum, etc.), $w_i \geq 0$ is the data product weight. If, for example, the spectrum was known to better describe the object than the visibilities, one would set $w_{\text{spec}} > w_{\text{visibility}}$. In this paper we have no indication that one data product better constrains the sources and set $w_1 = w_2 = \ldots = w_n$.

Once the population has been evaluated, we apply the first of the GA operators (step iii). The selection operator acts to determine the candidate solutions (parameter solutions) to be kept, and those that will be discarded. It is thus the job of the selection operator to mimic natural selection processes in which the fitter individuals in the population and allowed to breed. A good selection operator will, however, not remove unfit solutions from the gene pool entirely. For example fit offspring have been shown to not necessarily result from fit parents. However, a selection operator that does not favour the fit chromosomes over the un-fit solutions to some degree, may provide premature, local minima convergence. Ultimately, selection acts under the notion that fit solutions will create better ones. 

Roulette wheel and tournament selection remain the most common selection operators, though there exist many alternative selection methods (see for example Bick 1994; Hancock 1995; Mitchell 1998). In GADRAD we presently adopt tournament selection, in which two or more individuals are picked at random from the population. The individuals compete for selection, with selection favouring (with some probability) the fittest individual from the tournament group. All individuals from the tournament are returned to the population (allowing individuals to be selected more than once) and selection continues until $n_{\text{pop}}$ individuals have been selected. Tournament selection works well for large populations when sorting the entire population is computationally intensive.

Once the candidate solutions have been selected, the second operator is introduced (step iv). The task of the crossover operator (or recombination operator) is to breed the selected solutions to create offsprings. This is done by exchanging parameter values (alleles) between parents. The crossover operator introduces new genotype material to the population. In this way the genetic building blocks of fit members of a generation can be passed onto the next. Crossover occurs with probability $p_c$, thus $(1 - p_c) n_{\text{pop}}$ members of a given generation will remain unmodified, and pass their genetic material intact on to the next stage of the GA process. $K$-point crossover (Holland 1975) is perhaps the most common form of crossover, where $K = 1$ is known as single-point crossover, and $K = 2$ two-point crossover. In $K$-point crossover parental genotypes are passed to the following generations by exchanging allele values between parents, before and after a randomly selected crossover (loci) point(s). For example, with single point crossover ($K = 1$), two parents with six genes and the following allele values 101010 and 111111, would result in following offsprings 101011 and 111110 (if the loci point was between the fourth and fifth gene). In GADRAD we employ $K$-point crossover.

The final genetic operator to apply is the mutation operator (step v). The task of the mutation operator is to introduce new allele values into the population. The mutation operator prevents a loss of diversity (Holland 1975). Without mutation, for example, an allele value may come to dominate the population, and once such a state is reached, no new allele values can be introduced to following generations. This is particularly detrimental to algorithm convergence if good allele values weren’t introduced into the original population. Without mutation, solutions can potentially converge to, and be trapped in, local minimum. It is with the introduction of new genetic building blocks that the genetic structures can be disturbed, and ultimately prevent local minima convergence. The mutation operator is generally applied by giving each gene a low probability ($p_m$; typically between 0.001 and 0.10) of obtaining a new allele value. The choice of value can be selected from a uniform distribution (i.e., Eq. 1), or it can be determined from a normal distribution about the original allele value. Dynamic mutation rates can also be adopted, for example the variance of the normal distribution can be set as a function of generation, such that early populations are given a high variance mutation distribution, which encourages parameter exploration. While in the latter generations (when hopefully nearer the global minima), the distribution is narrowed, such that closer proximity mutations occur. In GADRAD we adopt dynamic mutation based on the Cauchy distribution. The Cauchy distribution introduces a wider mutation scale than the normal distribution, immunising one somewhat against early local convergence. It is employed based on the parameter’s standard deviation, which in general is expected to decrease with each successive generation. The mutation thus becomes:

$$x' = x + \sigma C,$$

where $x'$ is the mutated value, $x$ is the original value, $\sigma$ is the standard deviation of the parameter values for the given population, and $C$ is the Cauchy distribution.

By iterating the three genetic operators we form GA convergence. The solution set is obtained once the termination criteria has been met (step vi), whether based on a specific fitness criteria or on a number of iterations (generations; $n_{\text{gen}}$). A good GA ultimately relies on a correctly balanced contribution from the selection, crossover and mutation operators. Despite numerous studies, seeking an optimum balance seems to remain problem specific. It should be noted, however, that the GA, similar to most other non-brute force techniques, is non-deterministic, such that there is no guarantee that the global minima has been found. The GA is also sensitive to the initial population, such that solution sets are non-reproducible and, as mentioned, parameter tuning is necessary, with the solution quality dependent on the types and implementation of the genetic operators. We describe the termination criterion further in Section 2.3.

The GA in GADRAD was developed in Python, which allows for seamless interfacing between the Python modules that govern the operation of RADMC-3D. The GA is parallelised to run using the OpenMPI library, with each RT simulation run on a separate computational thread. GADRAD also allows regions of parameter space to be disregarded, if for example a heating source must be enclosed within the inner-disc radius, parameter combinations of source temperature, luminosity and inner-disc radius that do not abide by the condition are rejected. Finally, we developed GADRAD to run $N_{\text{GA}}$ independent GAs from which we can produce parameter probability density distributions, and error estimates. GADRAD allows flexibility in how these GAs can be run.
2.3 GA convergence

To determine whether the algorithm is converging efficiently we apply the GA to the Rastrigin function (Rastrigin 1974). The function is frequently used as a test function for optimisation algorithms due to its many local minima and large search space, its multimodal nature combined with the relatively small contrast between global and local minima make it demanding for any optimisation algorithm (especially more classical, gradient-based optimisation methods). The Rastrigin function also benefits from its ability to be scaled to include as many dimensions as is necessary. Its general form is presented in Eq. 4, where the global minima occurs at $x = 0$, with $f(x) = 0$.

$$f(x) = An + \sum_{i=1}^{n} x_i^2 - A \cos(2\pi x_i).$$  \hspace{1cm} (4)

We consider the two-dimensional case, with $A = 10$. The resulting function is plotted (with $x_i \in [-5,12,5,12]$) in Fig. 1. We demonstrate the GA’s convergence in this case by plotting the spatial distribution of the individual search results for the given generations with respect to the function’s contours in Fig. 2. The GA was run for $n_{\text{gen}} = 100$ generations with a population of $n_{\text{pop}} = 100$ individuals. In this test we adopt tournament selection, with 2-point crossover (of frequency $p_c = 0.65$), and a mutation rate of $p_m = 0.05$. The initial population (Fig. 2a), is randomly sampled across the search domain. By $n_{\text{gen}} = 25$ (Fig. 2d) the population can be seen to centre about the global minima. This convergence continues right through to $n_{\text{gen}} = 100$ (Fig. 2f) at which point we obtain a final result of $(x_1, x_2) = (0.00205, -0.0037)$, with $f(x_1, x_2) = 0.0037$ (where the solution is $f(0, 0) = 0$).

To test the convergence properties more thoroughly we repeat the GA 100 times ($N_{\text{GA}} = 100$). We find that the result above is not atypical. After 100 runs we find the average result to be close to the global minima, and with relatively small standard deviation across the solutions, i.e.

$$x_1 = 0.0006 \pm 0.0036, \quad x_2 = 0.0004 \pm 0.0032. \quad \hspace{1cm} (5)$$

It should be noted that due to the low dimensionality of the function, an increase in the mutation rate was found to improve convergence. We now compare the convergence efficiency of our algorithm with the Pyevolve Python package, after 100 such runs, and following the same GA operator parameters, Pyevolve obtains a result of

$$x_1 = 0.0001 \pm 0.0048, \quad x_2 = -0.0004 \pm 0.0040. \quad \hspace{1cm} (6)$$

The similarity is evident, we conclude that our GA is converging in a similar fashion to that of Pyevolve.

2.4 Parameter inference

As indicated, the purpose of GADRAD is not simply the determination of good fitting post-AGB disc solutions to the interferometric data products, but the construction of the multi-dimensional parameter probability density functions that arise in the fitting of these solutions (RT models) to the interferometric data. It is with knowledge of these parameter density distributions, that we can begin to constrain the object’s physical characteristics in the context of an unbiased parameter space. GADRAD allows us to better explore and understand areas of the objective function of interest (i.e., the
areas that return fit RT solutions. The GADRAD process relies on a number \( N_{\text{GA}} \) of independent GAs to search a prescribed area of parameter space of a given model class. The fittest solution from each GA provides a single independent, but stochastic, probe of the underlying objective function. When these solutions are considered collectively we gain some understanding of the objective function (hopefully) near the global minimum.

In approximating the parameter probability density functions we construct density histograms from the fittest \( N_{\text{GA}} \) individuals. The resulting distributions are estimated by way of bootstrapped kernel density estimations. This distribution is a representation of the parameter probability density, however we use point estimation to calculate a single value that best represents the distribution (in this paper this value is the distribution median). We can also calculate error ranges, we do this via interval estimation in which we calculate the 95\% confidence intervals of the distribution. Three values thus represent each parameters density distribution.

3 ALGORITHM CONVERGENCE TO SYNTHETIC TEST OBJECT

In this section we test convergence efficiency and overall algorithm performance by applying the GA to a synthetic test object. We proceed to test the algorithm in a similar to fashion as De Geyter et al. (2013), by setting a sample of the RT parameters fixed to their respective synthetic input value (see Table. 1), while allowing the remaining parameters to freely converge. This approach allows us to identify convergence irregularities and detect potential parameter degeneracies.

Problems of degeneracy in GA solutions to RT problems have been investigated before (e.g., Hetem & Gregorio-Hetem 2007; Schechtman-Rook et al. 2012; De Geyter et al. 2013). It is known that global optimisers such as a GA, when applied to inverse problems of this type, can result in model degeneracies. This is especially true in our case, where the stochastic noise from Monte-Carlo RT can complicate the objective function (i.e., the objective function becomes dynamic). Interferometrically limited coverage of the \( u \)-plane also introduces model degeneracies. For example astrophysically distinct objects can provide non-distinct data product results. That is, one-to-one mapping of the model to its resulting data product is not necessarily always the case. The analysis of these data outputs however, whether they represent over- or under-simplified approximations of the true astronomical source, is beneficial. Parameter correlations and interactions will after all allow us to probe potential parameter degeneracies.

3.1 The model

In testing our synthetic model we apply a simple, azimuthally symmetric stratified disc density structure (e.g., Shakura & Sunyaev 1973), a disc structure common to many similar post-AGB studies (e.g., Chesneau et al. 2007, 2009; Lykou et al. 2011; Bright 2013). In cylindrical coordinates \((r, z)\), we have:

\[
\rho(r, z) = \rho_0 \left( \frac{R_*}{r} \right)^h \exp \left( -\frac{z^2}{2h(r)^2} \right),
\]

where \( \rho_0 \) is a normalisation constant, \( R_* \) is the stellar radius, \( h_\text{a} \) is the mid-plane’s density factor and \( h(r) \) is the disc scale height, increasing with radius as

\[
h(r) = h_0 \left( \frac{r}{R_*} \right)^{h_d},
\]

where \( h_0 \) is the scale height for a given radial distance and \( h_d \) is the vertical-plane density factor. We also define an inner and outer-disc radius \((r_{\text{in}} \text{ and } r_{\text{out}})\). The dust grain size distribution in the disc is set by (Mathis et al. 1977, MRN), in which the dust grains are considered homogeneous spheres, and are distributed between a minimum and maximum grain size \((a_{\text{min}} \text{ and } a_{\text{max}})\) as \( d_\text{grain} \propto a^{-b}\) (the exponent \( b \) is henceforth known as \( d_{\text{pow}} \)). The disc, of mass \( M_{\text{disc}} \), is thus characterised by a total of nine parameters.

The stellar component is approximated as a blackbody, of given temperature and luminosity \((T \text{ and } L)\) respectively. While three final parameters describe the positional properties of the object: distance \((d)\), positional angle \((p)\) and inclination \((i)\). The model thus requires a total of fourteen free parameters. The values adopted were chosen to reflect an object similar in nature to those considered in previous studies (e.g., Chesneau et al. 2007; Lykou et al. 2011), and the orientation of the object was chosen to provide a contrast in resulting visibilities.

In testing our algorithm we adopt typically large parameter ranges and choose non-informative uniform sampling distributions about the input value (i.e. Eq. 1). We adopt a non-symmetric sampling distribution with respect to the synthetic input values (for example we test convergence to the vertical disc density parameter, with \( h_{\text{min}} = a = 1.0, h_{\text{max}} = b = 2.0 \) where \( h_{\text{pow}} = 1.2 \)). Such a distribution, allows us to analyse the performance of the algorithm in a more realistic setting. In testing convergence we also include low level Gaussian noise equivalent to SNR=200 (i.e., add noise to the data \( x_i \) from a normal distribution where \( \sigma = x_i/200 \)) to the resulting images and spectral energy distributions. By doing so we test convergence in a more realistic and rigorous manner. Input values and sampling search spaces are presented in Table. 1.

3.2 Results

3.2.1 Three-parameter test

We begin by considering the objective function of the two-parameter interaction as revealed by a brute-force grid search over the selected domain. We then analyse the convergence of three free parameters with respect to these contours. Parameters are chosen to represent all model categories, i.e. the stellar parameters, orientation, disc characteristics and dust properties. As mentioned, non-symmetric sampling distributions (with respect to the synthetic input values) were chosen.

We test convergence over wide parameter ranges for the following six parameter groups: (i) distance, inclination and
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Table 1. Synthetic model parameters and GA convergence for the 10 and 14-parameter cases.

| Parameter                          | Input,σ | GA Samp. | 10 param. | 14 param. |
|-----------------------------------|---------|----------|-----------|-----------|
|                                   |         | (Min., Max.) | σ        | σ         |
| Stellar parameters                |         |           |           |           |
| Temperature (T)                   | $10^4$K | 30        | (25,35)   | 0.22      | 30.01-3.8 |
| Luminosity (L)                    | $10^8$L⊙| 10        | (7,12)    | 0.55      | 9.65-1.2 |
| Distance (d)                      | kpc     | 1.8       | (1.4,2.4) | -         | 1.93-0.3 |
| Orientation                       |         |           |           |           |
| Inclination (i)                   | deg     | 60        | (50,70)   | 0.33      | 59.61-1.9 |
| Position angle (p)                | deg     | 20        | (15,25)   | 0.58      | 20.20-0.7 |
| Disc characteristics              |         |           |           |           |
| Inner radius ($r_{in}$)           | au      | 10        | (5,25)    | 0.18      | 10.75-1.0 |
| Outer radius ($r_{out}$)          | au      | 600      | (400,800) | 0.03      | 614-0.9  |
| Mid-plane density factor ($h_{m}$)|          | -        | 1.0       | 0.87      | 1.95-0.3 |
| Vertical density factor ($h_{v}$) |          | 1.2      | (1.01,2.0)| 1.00      | 1.37-0.27 |
| Scale height ($h_0$)              | au      | 16        | (10,30)   | 0.44      | 16.39-0.9 |
| Mass ($m_{gal}$)                  |         | 10        | (1,10)$^*$| 0.11      | 9.5-1.2  |
| Dust properties                   |         |           |           |           |
| Minimum size ($a_{min}$)          | μm      | 0.05      | $(10^{-3},10^{-1})$ | -         | 0.05-0.04 |
| Maximum size ($a_{max}$)          | μm      | 1.0       | $(10^{-1},10^{-3})$ | -         | 1.08-0.02 |
| Size distribution ($a_{post}$)     |         | 3.6       | (3,0,4.2) | -         | 3.83 ± 0.46 |

$^*$test model input parameters,
$^*$log sampling

mid-plane density factor; (ii) luminosity, vertical-plane density factor and outer-disc radius; (iii) temperature, disc scale height and dust size distribution; (iv) inner-disc radius, luminosity and maximum grain size in disc; (v) vertical-plane density factor, minimum grain size and distance, and finally (vi) disc scale height, position angle and inclination.

Algorithm convergence for the parameter groups is presented in Fig. 3, for a GA with $n_{pop} = 100$ and $n_{gen} = 25$, where we adopt a crossover rate of $p_c = 0.65$ and mutation rate of $p_m = 0.035$. The GA solutions are represented as dots, with the darker dots representing later generations. It can be seen that the three-parameter GA converges to the synthetic solution efficiently and without too much difficulty. Final best fit solutions, for example, converge to parameter values within 0.1% of their original input values. The argument can be made however (for this single GA run instance) that the following parameter pairings: position angle - scale height, minimum grain size - vertical-plane density factor, and vertical-plane density factor - luminosity, converge quicker than the other parameter pairings (evident in the fact that the mutation operator is constrained to searching parameter space in the region near the synthetic input value in earlier generations when compared to other parameter pairs). However, this is likely explained by the stochastic nature of a single GA run. Additional runs are needed to clarify the situation, but in general, no large scale systematic problems relating to convergence, such as pre-mature convergence (i.e., local minima convergence), or other biases or anomalies are evident. We conclude that in this low-dimension problem, the algorithm performs as well as one would expect for a typical global optimisation algorithm, we therefore proceed to apply the algorithm to the higher dimensional cases.

3.2.2 Ten-parameter test

To test further algorithm performance, we apply the GA to 10 of the 14 parameters. The 10 parameters searched by the GA are those describing the stellar parameters, orientation and disc characteristics only. We fix the parameters which control the distance, minimum and maximum dust grain size and dust size distribution. We set the crossover and mutation rate to the same values as in the three free parameter example above (i.e. $p_c = 0.65$ and $p_m = 0.035$), but increase the population size and generation number. After a number of trial runs the population size was set to $n_{pop} = 600$, with a resulting generation number $n_{gen} = 400$ (dependent on the computational time of the RT runs that made up the specific GA, but in general within 5% of this value).

In this test, we begin to see the underlying statistical discrepancies present in parameter space. As discussed, to build an understanding of the underlying probability density distribution of individual parameters we run $N_{GA}$ such GAs. In this test we run $N_{GA} = 100$ GAs. A histogram of the best individuals of each GA (i.e. 100 final solutions) is constructed using a kernel density estimation.

The resulting parameter solution is found by taking the median point estimate of the distribution, and an estimate to the parameter error is provided by the 95% confidence intervals. The mean parameter values, confidence intervals, and standard deviations with respect to the original input values are presented in Table. 1. Final solutions are deemed acceptable. For example, the mean of all parameter standard deviations is respectable (~ 0.41σ), and reassuringly, median point estimates are found to lie within ~ 1σ of their synthetic values. All estimates are also found to lie within the 95% confidence interval range.

Parameter analysis reveals that the disc’s vertical-plane density factor is the most difficult to constrain, closely followed by the mid-plane density factor with resulting uncer-
tainty of 1.0σ and 0.87σ respectively. This is however not surprising, upon deeper consideration, correlation between model parameters reveals a very strong positive correlation (r = 0.99) between the mid-plane disc density factor (disc compactness; $h_a$) and the vertical-plane density factor (or disc flaring parameter; $h_B$). Some level of degeneracy is thus proposed to exist for the two parameters (at least in the case of near edge-on discs). Physically, this is not surprising. For example, at this orientation it can be seen that similar discs result from high $h_a$ - high $h_B$ values and their low $h_a$ - low $h_B$ counterparts. We conclude that in modelling an edge on disc, $h_a$ and $h_B$ parameters may potentially be replaced by a single parameter. Interestingly however, efficient convergence of the $h_B$ parameter, as found in the three parameter test, was not replicated in this larger 10-parameter study. This may be explained by, again, the stochastic nature of the GA. As in the three parameter test, the algorithm was only run once and with additional runs, degeneracies of the $h_B$ and $h_a$ parameter may begin to surface.

The resulting visibilities and spectrums are presented in Fig. 4, residuals for both the spectrum and visibilities are within 2% of the original model. Considering that the additional artificial noise had to be overcome, the convergence of the GA is considered acceptable. Final parameter values for the 10-parameter case are presented in Table. 1. The resulting probability distributions are illustrated in Fig. 5.

### 3.2.3 Fourteen-parameter fit

In this section we test algorithm performance as applied to all 14 parameters. The crossover and mutation parameters remain unchanged, however the population size was increased to $n_{\text{pop}} = 750$. Again, approximately $n_{\text{gen}} \sim 400$ generations resulted, and the task of constructing an underlying probability density distribution was procured with $N_{\text{GA}} = 100$ runs. The resulting functions are represented in Fig. 5, in which the 10-parameter solutions are also pre-
Algorithm performance in the 14-parameter case is naturally less efficient than the 10-parameter case. For example the average standard deviation between the reference parameter value and the solution value is 0.55σr, (as opposed to 0.41σ in the 10-parameter case). In the 14-parameter case three parameters however exceed 1.0σr, notably the distance, the inner-disc radius and the disc vertical density. The dust grain size distribution is also very close to the 1.0σr, and the mid-plane density factor displays similar high variance to the 10-parameter example.

In the 14-parameter case there are signs of degeneracies. For example, the high standard deviation of the inner-disc radius and distance parameters reflected in the strong correlation (r = 0.79). This degeneracy however may arise from the simple fact that the inner-disc radius was well constrained in the 10-parameter case (i.e., when distance wasn’t considered). Finally it is clear that the correlation makes sense physically: a change in the distance would result in a perceived geometric change to the inner-disc radius. As observed in the 10-parameter test, a very strong correlation between disc compactness (hD) and disc flaring (hF) exists (with r = 0.96). We conclude again (at least for the case of near edge-on discs) that hD and hF may be replaced by a single parameter; this correlation is also evident with paired interactions with other parameters, the disc position angle for example (with r = 0.83 and r = 0.8 respectively), this is also evident in the 10-parameter case (with r = 0.87 and r = 0.89).

No perceived strong correlation between the dust distribution parameter (αpow) and any other parameter is observed, with an average correlation coefficient magnitude of only 0.18. Though a moderate positive correlation (r = 0.64) between the maximum grain size may exist. This finding can be understood. For example, as we increase the maximum grain size, the ratio of large to small grains increases, and an increase in the dust grain distribution αpow, will see this somewhat compensated for, though with more complex higher order effects, the correlation is not strong enough to indicate degeneracy. Another correlation worth noting is the outer-disc radius and disc mass interaction, which is strong in both the 10 and 14-parameter case (r = 0.89 and r = 0.79 respectively), this correlation can be explained, however, as a more massive disc will be larger for a given density. This effect however is expected to not be as strong in non-edge on discs, in which visibility results are likely be affected, and at this point conclude that their inclusion as separate parameter inputs is necessary.

The resulting probability density functions (Fig. 5) show striking similarities between the 10 and 14-parameter test. Perhaps the only parameter to show a difference of note is the inner-disc radius. For example in the 14 parameter case we determine a radius of 10.75±1.30 au, versus 10.1±1.5 au in the 10-parameter case. Interestingly, however, this result reinforces the strong positive correlation between inner-disc radius and distance (r = 0.79). For example, the 10-parameter test did not include distance; as such, it was set to its original input parameter in all GA runs (d = 1800 pc). As a free parameter in the 14-parameter case, a higher distance parameter resulted (1.93±0.33 kpc), such a result is expected to permeate, and in some sense corrupt the other parameters, in particular the strong positive correlation will result in an overestimate of the inner-disc radius. The resulting visibilities and spectrums for the 14-parameter case is presented in Fig. 6, residuals for both the spectrum and visibility are found to lie within 3% of the original model. A correlation matrix of the resulting GA results for the synthetic test object is presented in Fig. 7. We conclude the GA has performed well in this full model reconstruction, and proceed to apply GADRAD to VLTI data products of a post-AGB object.

4 MENZEL 3, THE ANT NEBULA

4.1 Background

The Ant Nebula, Mz3 (Menzel 1922), is a young nebula with numerous large, and highly collimated bipolar outflows (see Fig. 8). At the object’s core resides a proposed symbiotic binary system (e.g., Calvet & Peimbert 1983; Smith 2003), the exact nature of which is however still to be determined. Evidence for a symbiotic Mira core has been suggested (e.g., Schmeja & Kimeswenger 2001; Zhang & Liu 2006), but on constraining the inner dust regions, Chesneau et al. (2007) speculated on the existence of a less luminous, cooler star with a white dwarf companion. As indicated by Guerrero et al. (2004), a symbiotic core would explain the spectacular multipolar structures observed, with the expansion regions being caused by episodic events due to accretion type outbursting. Ages of the outbursts, as estimated by Santander-García et al. (2004), supported this line of thinking. The innermost lobes are proposed to be the youngest outburst region (670 year kpc⁻¹), while the extended column type structure is estimated to have been ejected earlier in the object’s history (875 year kpc⁻¹). Finally, the larger cone structure was found to be older still (1600 year kpc⁻¹). Guerrero et al. (2004) concluded we could be witnessing ongoing evolution, driven by the complex interactions of a binary system.

Spatio-kinematic modelling of the object (e.g., Santander-García et al. 2004; Guerrero et al. 2004) identified a fourth, previously unnoticed structure. The feature, known as the ‘chakram’ (a large flattened disc), is interesting due to its peculiar orientation. The nested pairs of
bipolar lobes already mentioned are estimated to sit at inclination angles between 68° and 78° to the line of sight. The axis perpendicular to the plane of the chakram, in contrast, is inclined in the opposite direction, sitting at an inclination of 115°. The chakram’s axis was also found to have a ~9° clockwise rotation with respect to the projected symmetry z-axis of the nebula (see Figure 8; the projected z-axis has a position angle of 5°). The origin of the chakram is unknown, but unlike the other outflow structures, is proposed to have been ejected over a long time period, in a ‘non-explosive’ type event.

Looking at the core more closely, infrared measurements indicate the presence of a circumstellar dust and gas disc that obscures the inner stellar region to direct imaging (e.g., Cohen et al. 1978; Meaburn & Walsh 1985). In agreement with the gas-phase detected in the nucleus (e.g., Zhang & Liu 2002), a second, flat silicate disc located close to the stellar surface (i.e., well within the chakram structure) was proposed. Chesneau et al. (2007), henceforth C07, observed the inner dust region with the mid-infrared interferometer MIDI at the VLTI. They witnessed a strong dependence on the visibility magnitude with position angle, indicating a disc structure was likely being seen close to edge on. The MIDI spectrum exhibited amorphous silicate signatures, suggesting the structure to be quite young (older, more processed discs tend to show crystalline features). RT modelling was employed in an attempt to constrain the disc geometry further. The disc modelled was simple in nature, but it proved employed in an attempt to constrain the disc geometry further.

Figure 5. Parameter probability density functions for the 14 parameter GA for the synthetic object. The solid vertical line represents the median, with the grey area depicting the 95% confidence interval. The dark regions represent the 2.5% distribution tail. For comparison the confidence distribution for the 10-parameter GA is plotted as the dot dash line.
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The Mz3 data products were obtained with the MIDI (Leinert et al. 2003) instrument of the VLTI, in May and June 2006 by C07. With MIDI being able to combine light from just two telescopes, and with the high sensitivity required, Mz3 was observed with two of the four 8.2m UTs. The measurements were taken in SC1 PHOT mode, such that fringe measurements were taken concurrently with the photometric information. The lower resolution prism mode (R=25) was used. Data reduction was additionally performed by C07, in which MIA and EWS software packages were used. The resulting flux and visibilities measurements are presented in Fig. 9. The observation log is presented in Table. 3 as well as the source calibrators used. For more information regarding the observations please see section 2 of C07.

4.3 Reimplementing the C07 model and the heuristic search

We begin by attempting to reproduce the Mz3 results as determined in the analysis of C07. The C07 model was obtained with the Monte-Carlo RT code MC3D (e.g., Wolf 2003; Wolf et al. 1999), a different RT code from the one we use. Therefore, before blaming result differences on the fitting technique, we compare the two RT codes. The C07 input parameters can be found in Table. 2. The resulting RADMC-3D spectrum and visibilities were found to agree very closely (with differences smaller than 4%) Results are presented in Fig. 9. With confidence in our RT code implementation, we initiate the heuristic search of parameter space by applying the GA. We begin by first testing for the presence of a possible disc aligned with the Santander-García et al. (2004) chakram.

4.3.1 Potential disc-chakram alignment

Santander-García et al. (2004) indicated that Mz3 contains a large chakram or flattened disc surrounding the outflow regions, with an inclination of 115° and a position angle of 9°. What can be considered a ’mis-alignment’ of the chakram structure may in fact be evidence of some complex physical process that may in the same way affect the orientation of a smaller silicate disc structure. In either case, we feel there is enough evidence to begin searching for a small circumstellar disc with a similar orientation to the larger chakram.

In performing a preliminary search of parameter space we set the population size \( n_{\text{pop}} = 1200 \), and iterate for approximately \( n_{\text{gen}} = 500 \) generations. We adopt the stratified disc structure (Eq. 7), however, following 4 GA searches we find only poor fitting results (see Figs. 9 and 10). We conclude that the chakram orientation is likely not mirrored in a smaller internal disc structure. We instead start searching for a disc alignment closer to that suggested by C07. Preliminary attempts suggest that this area of parameter space is more encouraging. We thus apply GADRAD in a similar manner to the synthetic test case as presented in Section 3, and test for convergence of a sample of parameters.

4.3.2 Ten-parameter test

In this section we explore parameter space for only 10 of the 14 simulation parameters. These include: the stellar effective temperature, stellar luminosity, inclination and position angle, inner and outer-disc radii, mid-plane density factor, vertical-plane density factor, scale height and disc mass. By setting the remaining 4 values to the C07 values, we are able to explore parameter space in a similar fashion to C07.
Figure 7. Visualisation of the correlation matrix depicting the correlation coefficient, r, for the synthetic test object. The correlation coefficient is presented in the bottom right corner of each parameter correlation. Darker, more elliptical ellipses indicate a stronger correlation, and less correlated parameters are lighter in colour and more circular. The 10-parameter correlations are presented in the top right corner, while the full 14 parameter simulation is located in the bottom left.

(though in this instance we additionally explore the stellar luminosity and effective temperature search space). Underlying differences in the resulting parameter density distributions between this sample of parameters and full model exploration (i.e. all 14 parameters in Section 4.3.3), will additionally reveal any potential local minima convergence. Parameter correlations can also be compared between the two searches, which may reveal potential degeneracies.

To avoid premature convergence to local minima, and ensure a broad search of parameter space, we set the population size to $n_{\text{pop}} = 750$. Through trial and error, optimal algorithm performance was found with a mutation rate $p_{m} = 0.035$, in combination with a crossover rate $p_{c} = 0.65$. An acceptable level of convergence was reached after approximately $n_{\text{gen}} = 450$ generations. Running GADRAD in parallel on 48 CPUs, resulted in a runtime of approximately 24 hours for each GA. As discussed in Sect. 2.4, in the attempt to gain statistical inference, we construct confidence distributions by running a number of such GAs, in this instance we employ $N_{\text{GA}} = 100$ GAs.

It is evident that a number of parameter distributions are non-uniform, and non-normal, with, for example, a selection of skewed and bimodal results. The distributions were again calculated using a kernel density estimator. The resulting distribution variance was estimated using the median absolute deviation (MAD) measure ($\hat{\sigma}$).\footnote{MAD is a measure of the deviation of the residuals, from the}
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Figure 8. Composite HST image (a), in reference to a schematic depicting the outflow regions (adapted from spatiokinematical modelling from Santander-García et al. 2004); front view (b) and side view (c) where the x axis is directed towards the observer. Images are not of shared scale. The four regions are: 1. the inner lobe core 2. the column structure 3. the steep angled rays, and 4. the chakram ring structure. The chakram is not easily discerned in image (a), outflows 1,2 and 3 are orientated with an inclination between 68° and 78° (the angle between the line of sight and the symmetry axis of the structure). With reference to (c), the chakram (outflow 4) inclination is orientated in the opposite direction (Northern symmetry axis towards the observer) with i = 115°, and with reference to (b) the chakram symmetry axis is rotated ~9° clockwise with respect to the projected symmetry axis z of the nebula.

Table 2. PN Mz3 literature parameter values alongside our point estimates for the 10 and 14-parameter GA fits.

| Parameter | Lit. Values | Ref. | GA Samp. 10 param. | 14 param. | \( \hat{\sigma} \)10 | \( \hat{\sigma} \)14 |
|-----------|-------------|------|-------------------|----------|----------------|----------------|
| Stellar Parameters | | | | | | |
| Temperature \((T)\) | 10^3K | a,b,c | (20,45) | 36^{+11}_{-16} | 0.104 | 35^{+12}_{-15} | 0.161 |
| Luminosity \((L)\) | 10^3L⊙ | a,c,b | (5,16) | 8.6 ± 2.1 | 0.056 | 12 ± 4 | 0.075 |
| Distance \((d)\) | kpc | 1, 1.3, 1.4, 1.8, 2.6, 2.7, 2.85 | (1.0,2.8) | - | - | 1.8^{+0.2}_{-0.3} | 0.039 |

Orientation

| Parameter | Value |
|-----------|-------|
| Inclination \((i)\) | deg | 74 | b | (55,85) | 72 ± 2 | 0.007 | 71 ± 2 | 0.007 |
| Position angle \((p)\) | deg | 5 | b | (0,10) | 5 ± 1 | 0.067 | 5 ± 1 | 0.038 |

Disc Characteristics

| Parameter | Value |
|-----------|-------|
| Inner radius \((r_{in})\) | au | 9 | b | (4,25) | 11 ± 2 | 0.046 | 15^{+3}_{-1} | 0.071 |
| Outer radius \((r_{out})\) | au | 500 | b | (150,600) | 230^{+40}_{-10} | 0.101 | 290^{+20}_{-10} | 0.093 |
| Mid-plane density factor \((h_{n})\) | - | 2.4 | b | (1,5,6,0) | 4.2^{+0.5}_{-1.3} | 0.090 | 4.1 ± 1.0 | 0.041 |
| Vertical density factor \((h_{V})\) | - | 1.02 | b | (1,0,2,5) | 1.1 ± 0.1 | 0.015 | 1.15 ± 0.20 | 0.023 |
| Scale height \((h_{0})\) | au | 17 | b | (5,35) | 23^{+3}_{-1} | 0.053 | 22 ± 6 | 0.038 |
| Mass \((m_{disc})\) | 10^{-3}M⊙ | 0.1, 0.9 | f,b | (0.01,100) | 2.1^{+0.2}_{-1.2} | 0.026 | 3.5^{+2.5}_{-2.2} | 0.025 |

Grain Parameters

| Parameter | Value |
|-----------|-------|
| Composition | AS^2 | 1 | a | 1 | |
| Minimum size \((a_{min})\) | µm | 0.05 | b | (0.001,1) | - | - | 0.4^{+5.0}_{-3.2} | 0.301 |
| Maximum size \((a_{max})\) | µm | 0.2, 1 | f,b | (1,10^4) | - | - | 4^{+120}_{-100} | 0.328 |
| Size distribution \((a_{pow})\) | | 3.5 | b | (3,6.0) | - | - | 4.5^{+2.0}_{-1.5} | 0.127 |

* Cohen et al. (1978); *Chesneau et al. (2007); *Pottasch & Surendiranath (2005); *Lopez & Meaburn (1983); *Cahn et al. (1992); *Smith & Gehrz (2005); *Kingsburgh & English (1992); *Astronomical Silicates; *MAD value; *log sampling. Note: dust grain size confidence intervals reflect the logarithmic sampling of the parameters, in addition to parameters being relatively ill-constrained.

variance provides an indication to how well the parameters are constrained, with the parameter displaying the broadest relative distribution having the higher \( \hat{\sigma} \) value. The largest variance was observed in the parameter controlling the stellar effective temperature \((T_e = 0.104)\), closely followed by the outer-disc radius \((r = 0.101)\), though this is likely due to the MAD estimator being ill-suited to heavy-tailed data
Table 3. Mz3 observing log.

| Label | Time    | Baseline Length (m) | Baseline PA (°) |
|-------|---------|---------------------|-----------------|
| Mz3-1 | 2006-06-11T23 | UT2 – UT3 46.3 | 1.5 |
| Mz3-2 | 2006-05-15T04 | UT2 – UT3 45.4 | 30.5 |
| Mz3-3 | 2006-05-13T08 | UT2 – UT3 31.4 | 73.8 |
| Mz3-4 | 2006-05-14T08 | UT3 – UT4 60.6 | 149.2 |
| Mz3-5 | 2006-06-11T01 | UT3 – UT4 52.0 | 77.2 |
| Mz3-6 | 2006-05-17T06 | UT3 – UT4 62.5 | 122.1 |

Calibrators: HD 151249 5.42 ± 0.06 mas, HD 160668 2.22 ± 0.1 mas, HD 168723 2.87 ± 0.13 mas, HD 188512 1.98 ± 0.1 mas.

As in the case of the synthetic GA test of Section 3, the 10-parameter case here (Fig. 11) shows that a positive correlation exists between the mid-plane density factor \( h_\alpha \) (the parameter that controls disc compactness) and the vertical-plane density factor \( h_\beta \) (the parameter that controls disc flaring). This result is not surprising, as was discussed previously. For a near-edge on disc at this orientation similar intensity distributions result from a highly flared compact disc (i.e. high \( h_\beta \), high \( h_\alpha \)), and a low flaring larger disc (i.e. low \( h_\beta \), low \( h_\alpha \)). In this instance, however, the correlation is moderate to strong \( (r = 0.58) \), as opposed to a very strong correlation as found in the synthetic test case \( (r = 0.99) \). The difference is likely due to more complex underlying parameter interaction, that may be expected from the data products of a real object. Support for correlation between \( h_\beta \) and \( h_\alpha \) is reinforced by the very strong positive correlation existing between the disc scale height \( h_\alpha \) and \( h_\beta \) \( (r = 0.87) \), which is mirrored in the scale height and vertical-plane density factors \( (r = 0.65) \). However, the argument for degeneracy is weakened by the strong correlations that exist between the luminosity and vertical-plane density factor \( (r = -0.77) \), and the inner-disc radius and vertical-plane density factor \( (r = 0.65) \), a result not seen in the mid-plane density factor \( (r = -0.1) \) and \( r = 0.29 \) respectively. Additionally, the vertical-plane density factor, in contrast to mid-plane density factor, was found to be one of the best constrained parameters with \( \sigma = 0.016 \). Contrary to the 10-parameter synthetic case, we conclude that degeneracy is unlikely to exist between the mid-plane and vertical-plane density factors, and propose they remain independent parameters. Other correlations of note is the moderate to strong positive correlation that exists between the inclination angle and luminosity \( (r = 0.58) \), and the luminosity and inner-disc radius \( (r = -0.57) \). Parameter correlations are presented in Fig. 11.

As mentioned previously in this section, the stellar effective temperature was difficult to constrain. We determine the RT model is not particularly sensitive to this parameter. A bimodal distribution is suggested for example. The importance of this is yet to be determined. However, correlations between the temperature parameter and other parameters are weak, suggesting the bimodal structure may be important, however only provide a single point estimate to represent the distribution, as we feel the distribution’s median does an adequate job of representing the probability density function. The outer-disc radius is another instance of a non-normal distribution, yet only very few samples are contained within the tail (i.e. less than 2%). The result could arise due to outlying fitness runs, but with the median appearing very close to the mode the result seems to be of little consequence.

At this point our findings show reasonable agreement with the C07 result. Only the outer-disc radius, mid-plane density factor and disc mass parameters display discrepancies beyond our error estimates. Our model favours a smaller more compact disc than that determined by C07 with an outer-disc radius of \( r_{\text{out}} = 230^{+140}_{-90} \) au, and mid-plane density factor of \( h_\alpha = 4.2^{+1.0}_{-1.5} \). The disc is also suggested to be more massive than that determined by C07 \( (M_{\text{disc}} = 2.1^{+4.2}_{-1.2} \times 10^{-5} \, M_\odot) \) vs 9 \( \times 10^{-6} \, M_\odot \) in C07.

4.3.3 The fourteen-parameter fit

We now apply GADRAD to all 14 of the model parameters, by introducing the distance, minimum dust grain size, maximum dust grain size and size distribution parameters. With the additional search parameters we increase the GA population size to \( N_{\text{pop}} = 850 \). The mutation and crossover rates of the previous section are maintained and, again, sufficient convergence was found after \( N_{\text{gen}} \sim 450 \) generations. However, with the dust parameters requiring additional computational time for the necessary calculation of the dust opacity tables, and with the larger population size, the computational time increased to approximately 36 hours on 48 CPUs. In constructing the parameter confidence distributions we again rely on \( N_{\text{GA}} = 100 \) runs. The resulting distributions are presented in Fig. 12, with the point estimates presented.
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Alongside the 10-parameter result for comparison, in addition to the MAD values in Table 2.

Differences in the resulting spectrums and visibilities become evident when comparing the final 14-parameter GADRAD solution with the 10-parameter result (Fig. 9 and 10). The most notable differences are seen in the parameter controlling the stellar luminosity and inner-disc radius. This however may be explained by the moderate, positive correlation existing for both parameters with the distance parameter \( r = 0.51, r = 0.71 \) respectively. For example, in the 10-parameter case distance was set to the C07 value of 1.4 kpc. The 14-parameter model however favoured a higher value \( 1.8^{+0.6}_{-0.3} \) kpc. With the object located at a closer distance, the correlation suggests that the luminosity and inner-disc radius would likely decrease, and better match the data than 10-parameter distributions.

With GADRAD exploring much more of parameter space, and at a much higher resolution than considered C07, we favour this 14-parameter result, and conclude that Mz3 is likely located at a distance of \( 1.8^{+0.6}_{-0.3} \) kpc, which falls within literature limits. Additionally, we propose that Mz3 has a more luminous star at its core \( (12\,000^{+3500}_{-4900}\, L_\odot\) vs. \( 10\,000\, L_\odot\)), surrounded by a smaller and more compact disc (with \( r_{\text{out}} = 290^{+220}_{-100}\) au vs. 500 au and \( h_r = 4.1 \pm 1.0 \) vs 2.4, respectively). However, our model favours a larger inner-disc radius, with \( r_{\text{out}} = 15^{+1}_{-1}\) au, versus \( r_{\text{out}} = 9.0\) au.

Despite the fact that our 14-parameter model seems to constrain the parameters better, it is evident that parameters controlling the dust properties are ill-constrained. For example, we determine a MAD value for the minimum and maximum dust grain radius of 0.301 and 0.328, respectively. The grain size distribution parameter fairs a little better with \( \sigma = 0.127\), but of the 14 parameters, it still remains one of the poorest constrained. The sensitivity of the model to the dust parameters may be questioned, and the exact bearing they have on the overall parameter distributions is unknown. It is possible that in future GA attempts, the dust parameters can be fixed, this will for one eliminate the need...
to create opacity tables that require additional computational time. Exploring the parameter correlations further, we find moderate negative correlation \((r = -0.61)\) between the minimum grain size and disc inclination, as well as the particle size distribution and inclination angle \((r = -0.53)\). Moreover, the dust parameters seem to have little influence on the well constrained inclination parameter \((\sigma \approx 0.007)\), further supporting the argument that fixed dust values may suffice. At worst the inclusion of GA-derived dust parameters acts to increase the variance of the remaining parameters. This, however, does not seem to be of concern as the MAD values determined are similar to the 10-parameter MAD variances.

Similar to the 10-parameter case, the temperature and luminosity were difficult to constrain, and as mentioned the inclination was well defined. The bimodal nature of the effective temperature distribution, similar to the 10-parameter case, is again evident in the 14-parameter example. Additionally the effective temperature again displays only weak correlations (with an average correlation coefficient magnitude of 0.105). We conclude that our parameter distribution identifies a statistically significant bimodal nature of this parameter, the cause of which is however not known, and would require further analysis. The heavy-tailed distribution of the outer-disc parameter is similarly reproduced here in the 14-parameter solution, though in this instance with more substantive weight, with the median somewhat differing to the mode. However, as was found by Bright (2013) (see also Lykou et al. 2011; Werner et al. 2014), at these in-

Figure 11. Visualisation of the correlation matrix depicting the correlation coefficient, \(r\). The correlation coefficient is given in the bottom right corner of each parameter correlation. Darker, more elongated ellipses indicate a stronger correlation, and less correlated parameters are lighter in colour and more circular. The 10-parameter correlations are presented in the top right, while the full 14-parameter simulation is located in the bottom left.
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1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0

Figure 12. Parameter probability density functions for the 14-parameter GA. The solid vertical line represents the median, with the grey area depicting the 95% confidence interval. The dark regions represent the 2.5% distribution tail. For comparison the confidence distribution for the 10-parameter GA is plotted as the dot dash line. Notice the differences in the $L_1$ and $d_0$ parameters between the 14-parameter and 10-parameter solutions. This can be explained by the differing distance $d$ between the two models (i.e., in the 10-parameter case it was set to 1400 pc. As a free parameter in the 14-parameter instance however, a distance of ~1800 pc was obtained).

4.4 Foreground extinction

The MIDI $N$-band spectrum has a small wavelength coverage ($\lambda \approx 8\mu$m–13$\mu$m) and is virtually independent of reddening. A self-consistent result however requires a model that agrees with the Mz3 spectrum over a broader wavelength range. Literature flux measurements are plotted alongside the MIDI spectrum in Fig. 13, and the extended GADRAD 14-parameter SED.

The GADRAD result is seen to agree closely with the TIMMI2 flux (Smith & Gehrz 2005). However, it is evident that at shorter wavelengths (i.e. the 2MASS data) the observations are fainter than the model. This can be explained by foreground extinction. We approximate the reddening effect using the Cardelli et al. (1989, CCM) extinction law. An extinction value $E(B-V) = 0.625$ provides convincing SED fits, where we have redened the fit rather than de-reddening the data. Much of the reddening has been shown to occur local to Mz3 (i.e., Smith 2003). Interstellar reddening limits can be calculated from the reddening value estimated of the debris surrounding the outer lobe structures.
our adopted reddening value of $E(B - V) = 0.625$ is below this limit $E(B - V) = 0.85$ (Smith 2003).

Additional energy is thought to be provided to the spectrum due to extended structure that is not captured by MIDI, because MIDI spectra only see the flux that can be resolved by the interferometer, effectively simulating a much smaller aperture. However, modelling the extended structure (as seen by 2MASS for example) would require too many additional parameters. Additionally, the MIDI visibilities are essentially insensitive to the over-resolved structure. A fully self-consistent model would however require a much more thorough multi-wavelength study.

### 4.5 Solution interpretation

As indicated, the Mz3 GADRAD solutions are represented by parameter probability density functions, which approximate the underlying objective function near the global minima. The final solution is represented by the median point estimate of the distribution (which approximates the global minima). However, the result is in fact only a quasi-solution, representing an approximate solution of a model class or family only. The model class in this instance represents only a subset of all potential models, in which the GADRAD solution provides an estimate to the good fitting solutions of the model in question. The chosen model however, is formed from empirical prior knowledge (which introduces biasness), that may in fact represent a model class far removed from that that describes the true object.

Evidence of model differences have been acknowledged, for example the systematic differences observed between the model and the observational data products (differences which are similarly seen in C07). However, with the existence of noise, an exact solution to an inverse type problem is inconsequential, as an infinite number of solutions exist. In this instance a better approximation of the true object is a solution that discourages model complexity and avoids data product overfitting, ideas which stem from regularisation theory and maximum entropy arguments. The models chosen to represent Mz3 though not ideal, evident in the systematic differences observed, as well as the introduction of modest parameter degeneracies (the dust parameters for example), is proposed to be simple enough to avoid overfitting, yet represents a model class that generalises well.

### 5 DISCUSSION

#### 5.1 Comparison with C07

In comparing residuals of the visibility and spectrum fits of the 10-parameter GA, 14-parameter GA and the C07 result, no significant differences are seen, though a slight improvement to the fit is observed in both the 10- and 14-parameter cases. However, it is important to stress the fundamental difference between our approach and that of C07. Our results are estimated using the median point estimate of the $N_{GA} = 100$ density distributions, and this measure, while not necessarily being the best solution in terms of the overall $\chi^2$, ultimately provides a better representation of the areas of parameter space that provide acceptable model solutions. When considering an ill-posed inverse type problem of this type, in which many reasonable solutions exist, it is important to estimate the acceptable parameter ranges via the resulting parameter probability density function and the error bars which they represent.

Of additional importance are the systematic similarities that exist between the C07 result, and the 10 and 14-parameter GADRAD findings, because, it is evident, there exist some similarities between the residuals. With similar disc environments results, we conclude that the structures adopted (model class) are too simple an approximation. After all, the perfect symmetry of the disc and the environment adopted, are unlikely to accurately represent the post-AGB object. Introducing more parameters to overcome these systematic differences, such as removing symmetry or adding more complex structure or dust distributions, however, will most likely provide no further information pertaining to the object, but instead result in overfitting of the data products.

Our modelling shows that an inner silicate disc is unlikely to share the alignment (i.e., inclination of $i \sim 115^\circ$, and position angle of $-4^\circ$) of the much larger chakram structure inferred by Santander-García et al. (2004) and Guerrero et al. (2004), and instead favour a disc closely aligned with the equatorial symmetry of the lobes (i.e., with a position angle of $5^\circ$). Our model shows close agreement with the environment described by C07. The distance, outer-disc radius and mid-plane density factor in particular, however, show some level of disagreement, with the C07 equivalent values falling outside the 95% confidence ranges determined here. Overall however, our findings closely agree with those of C07, and confirm that an amorphous circumstellar or circumbinary silicate disc likely resides at the core of Mz3, although our analysis suggests that it is likely more compact (with a smaller outer-disc radius and larger mid-plane density factor), but with a larger inner-disc radius. We also determine that the disc is likely 4 times more massive than what was determined by C07, and the system is overall located farther than the value adopted by C07.
5.2 The shaping history of Mz3

The question of whether Mz3 is a symbiotic nebula is central, because the symbiotic binary is a wide binary and the shaping opportunities it affords are fundamentally different from those of other binary configurations. We argue here, as did C07 based on a different line of reasoning, that the central star is not a giant (RGB nor AGB) and that the system, if indeed a binary is still present at the core of the nebula today, is not a symbiotic. We argue that our star is a post-AGB star, based on the parameters derived from this study and their confidence intervals.

The star was found here to be a 35 000-K, 12 000-L⊙ (3-R⊙), post-AGB star on its way to the white dwarf cooling track. The mass of the star, comparing its luminosity to the stellar evolutionary tracks of Miller Bertolami (2016) should be just larger than 0.66 M⊙. However, this star would, according to the same tracks, reach a temperature of 35 000 K in less than 100 years, making the nebular kinematic ages all too large by approximately one order of magnitude. To reconcile our results with the Miller Bertolami (2016) tracks, we would have to assume that the central star has a luminosity at the lower end of its error range, namely ~8000 L⊙ and a temperature of 55 000 K, which exceeds our upper error bar (47 000 K). In that case the time to transition between the AGB and the current location on the HR diagram would be of the order of 1000 years, more in line with the measured nebular ages. We therefore conclude that our derived parameters are somewhat inconsistent with AGB to post-AGB transition of a single star, though not outside the domain of possibility.

We now consider the possibility that the ejection may have been due to a close binary interaction, which would have disturbed the regular AGB evolution. In particular, taking as an example a common envelope interaction (Ivanova et al. 2013; Paczynski 1976), we know that the envelope removal is almost instantaneous, as is the orbital reduction (e.g., Iaconi et al. 2017), something that would accelerate the left-ward evolution on the HR diagram. This would give us a younger nebula compared to what is inferred using single-star tracks, the opposite of what is observed.

In order to observe a nebula that looks older than explained by the evolution of the central star, the only possibility we are aware of is that the binary interaction caused some post-interaction material fall-back, leading to accretion onto the post-AGB star. This is hypothesised to be able to slow the evolution of post-AGB stars allowing the nebula to expand while the star does not move towards the white dwarf cooling track as quickly (van Winckel et al. 2009).2

The considerations above leave the original question wide open. What collimated the outflow? C07 and, before them, Smith & Gehrz (2005) argued that the small disc at the core of Mz3 is of too low a mass to have influenced the much more massive outflow. Smith & Gehrz (2005) measured the total mass of the dust in the lobes of Mz3 to be 2.6x10⁻³ M⊙, which should be compared to our measured disc dust mass of 3.5x10⁻⁵ M⊙ (see Table. 2).

On the other hand, the estimated densities and velocity contrast of the outflow, assuming typical AGB mass-loss parameters (i.e., an AGB mass loss rate of 10⁻⁷-10⁻⁸ M⊙yr⁻¹, and a velocity of 10-20 km s⁻¹ Renzini 1981; Bloeker 1995), is estimated to be ~100 times larger than that of the disc’s inner rim (3.3x10⁻¹⁵ g cm⁻³ for the outflow, 2.7x10⁻¹⁵ g cm⁻³ for the disc, where we have used a gas-to-dust ratio of 100).3 If so, then the argument can be made that such a disc, if formed before the outflow event, may play a role as a collimating agent.

As an alternative to the collimation-by-disc scenario both Smith & Gehrz (2005) and C07 suggested that the ejection was already bipolar and launched via jets, similar to the scenario described by Sahai & Trauger (1998), Soker & Rappaport (2000) García-Arredondo & Frank (2004). The problem with this scenario is that a jet launched magneto-centrifugally via an accretion disc (Blandford & Payne 1982), presumably formed around the companion during the AGB, needs an accreted mass that is 2.5 to 10 times larger than the mass ejected by the jet. So if the jet launches 0.6 M⊙ (Smith & Gehrz 2005), 1.5-6.0 M⊙ must have been accreted onto the companion. Assuming the original companion to be a low mass main sequence star (~0.5 M⊙), accretion would have made it into a 2-6.5 M⊙ star (where the lower efficiencies are preferred, making the higher masses more likely). While hiding a six-solar-mass main sequence companion may not be out of the question inside the very optically thick disc, another argument against this scenario presents itself.

The accretion rate needed to eject the massive jets is high and such large values are unlikely to be achieved in a wind accretion or even in a Roche-lobe overflow scenario. We have here used equation 6 of Blackman & Lucchini (2014) to determine the minimum accretion rate required to form the lobes of Mz3:

\[ \dot{M}_s \geq 10^{-4} \left( \frac{Q}{2} \right) \left( \frac{M_{\text{a}}}{M_{\text{c}}} \right)^{-1/2} \left( \frac{R_a}{R_\odot} \right)^{1/2} \times \left( \frac{M_{\text{ob}}}{0.1M_\odot} \right) \left( \frac{v_{\text{ob}}}{100 \text{ km s}^{-1}} \right) \left( \frac{t_{\text{acc}}}{500 \text{ yr}} \right)^{-1}, \]

where Q is an efficiency parameter typically between 1 and 5 (Blandford & Payne 1982), \( \dot{M}_s \) is the accretor’s mass, \( R_a \) is the accretor’s radius, \( M_{\text{ob}} \) is the observed outflow mass, \( v_{\text{ob}} \) is the observed outflow velocity and \( t_{\text{acc}} \) is the timescale of the accretion event. Using an outflow mass of 1.9 M⊙, \( v_{\text{ob}} = 90 \text{ km s}^{-1} \) and accretion time of 1800 yr (e.g., Santander-García et al. 2004), we obtain the minimum accretion rate that can cause the observed jets. This is plotted as a function of Q in Fig. 14, which is equivalent to figure 1 of Blackman & Lucchini (2014).

As we can see from Fig. 14, we obtain a limiting value that is only consistent with a common envelope accretion

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2 The post-AGB binaries for which accretion has been hypothesised to have slowed down their evolution tend not to have a visible nebula, and are always in binaries with periods of the order of 100-2000 days. Exceptions exist, for example the Red Rectangle (Bujarrabal et al. 2016). It may be problematic to suggest that Mz3 belongs to this class because many of its characteristics are quite different from those of these post-AGB objects.

3 The outflow density is the calculated instantaneous density at the disc’s inner rim following isotropic mass-loss over the period \( t = r_{\text{in}}/v_{\text{wind}} \). I.e., not accounting for mass accumulation.
Many characteristics identified in Mz3 are common to the bipolar nebula M2-9. These similarities are seen in both the morphological structures of the large scale outflows and their spectroscopic properties (e.g., Lykou et al. 2011). Modelling of MIDI M2-9 data products as realised by Lykou et al. (2011), additionally reveals an equatorially-oriented amorphous silicate disc structure similar to that detected in Mz3. Similar inner-disc radii are also found (~15 au for both M2-9 and Mz3), though in the case of M2-9 the disc is purportedly more extended (900 au vs. ~290 au). Disk masses are however comparable (1.5 × 10^{-5} M_⊙ vs. 3.5 × 10^{-5} M_⊙ respectively). Optical and infrared SEDs of the discs also show similar characteristics, with peak energy occurring at 19 μm in both. Traces of crystalline silicates may however be present in the case of M2-9 (e.g., Lykou et al., 2011), which is typical of longer lived discs surrounding post-AGB stars (e.g., Deroo et al. 2007), suggesting Mz3’s disc may be more recently formed.

However, M2-9 is more likely to be a symbiotic (e.g., Clyne et al., 2015; a white dwarf and giant companion), because of its rotating jet (Corradi et al., 2011) points to a binary with a period of 92±4 years, suggested to be a giant orbiting a cool white dwarf companion. While the central star(s) of M2-9 has never been detected directly, at the very least the binary in this case is far more likely to be a wide binary, rather than a post-common envelope binary, which brings back doubts as to the correct classification of Mz3. We therefore used data from Smith & Gehrz (2005) and Clyne et al. (2015) to place M2-9 on the diagram of Blackman & Lucchini (2014).

Following Smith & Gehrz (2005) and Clyne et al. (2015), we have used a jet mass of 0.78 M_⊙, a timescale of 2000 yr with velocity of 30 km s^{-1} in Eq. 9. The limit we obtained indicates that the ejecta of M2-9 are not consistent with Bondi-Hoyle accretion nor with wind-Roche lobe overflow, the two mechanisms that would operate in a symbiotic binary. They are consistent, however, with Roche lobe overflow, implying a close binary, which may have after a brief phase of Roche-lobe overflow entered a common envelope. In any case, the outflow of M2-9 appears to have substantially less momentum than that of Mz3.

Finally, we point out that accommodating a symbiotic binary (orbital separations of 10-15 au; Gromadzki et al. 2009) within the relatively small inner rims of the discs of Mz3 and M2-9 (15 au in both instances) is problematic, though not impossible: the disc sizes do not preclude the presence of such a symbiotic, particularly considering the uncertainties, but they make them somewhat unlikely. If we

4 Equivalent GADRAD values for M2-9 (e.g., Macdonald et al., 2017; in preparation) are inner-disc radius 18±5 au, outer-disc radius 540^{+260}_{-230} au, and disc mass of 4.6^{+5.1}_{-1.9} × 10^{-6} M_⊙.
were to exclude a symbiotic binary as the cause of the accretion and outflow in the case of M2-9, we would have to invoke a triple star system, where one companion entered a common envelope and one remained farther out. The one farther out, with a period of ∼90 years and an orbital separation of ∼25 au (for a 1-M⊙ primary with a 0.6-M⊙ companion) would reside just inside the disc.

6 CONCLUSIONS

In this paper we have presented an improved, heuristic approach to the ill-posed general inverse type problem that is, determining the characteristics of a compact circumstellar environment of evolved stars from its mid-infrared interferometric data products. We have tested our algorithm on a synthetic post-AGB environment, in which we recover successfully the RT input parameters, and then we have applied the algorithm to the post-AGB object Mz3. In agreement with C07, we conclude that there is an amorphous silicate disc at the centre of Mz3, though we predict it to be more massive and compact. The systematic approach of GADRAD to the fitting of RT models to the telescopes’ data products, additionally allows for the construction of parameter probability density functions, which gives a good understanding of the uncertainties. This approach also allows us to detect potential model degeneracies and parameter correlations.

The exact role of the disc in collimating the outflow remains uncertain, although its density would likely be sufficient to divert, at least in part, an outflow that followed its formation. The broader questions of what shaped the jets of Mz3 and of similar objects such as M2-9 remains open. The most stringent constraint on the engine remains the powerful jet moments observed. These outflows indicate that the jets formed at the time of Roche lobe overflow during the AGB or, later, in a common envelope. The common (magnetic) envelope ejection remains a more appealing option, because while the required mass-loss rates may be achieved in Roche lobe overflow type accretion, the overall jet masses are very large and many Roche lobe overflow events lead to a common envelope in a short timescale, leaving little time in which to expel enough mass. These two objects are among the most powerful outflow when compared with the sample analysed by Blackman & Lucchini (2014).

A common envelope would leave behind a close binary or a merger. In the case of Mz3 the nature of the central system cannot be ascertained so either possibility could work. In the case of M2-9 it is likely that a wide binary resides in the nebula, in which case this system would have been originally a triple. A common envelope would likely leave behind a fall-back disc, material that failed to be fully ejected (e.g., Kuruwita et al. 2016). It is possible that the observed disc could be generated by the fall-back material. In this scenario the observed disc would not partake of the collimating action, which would be instead at the hand of a strong magnetic ejection in the context of the common envelope.

In conclusion, while a magneto centrifugal launch in a strong binary interaction seems to be implied by the outflow power of many collimated objects classified as prePN, the broad variety of many of the characteristics of these objects leave many questions to be answered. It is likely that the VLTI dusty discs still have something to tell us, particularly when their kinematics, particularly their rotation properties and angular momenta are surmised by observations such as those achievable with ALMA.

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