Rate difference between $b \rightarrow s\mu^+\mu^-$ and $b \rightarrow se^+e^-$ in SUSY with large $\tan\beta$

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Abstract

We study the inclusive semileptonic rare decay $b \rightarrow sl^+l^-$ in minimal supergravity model (mSUGRA). If $\tan\beta$ is large, down-type quark mass matrices and their Yukawa couplings cannot be diagonalized at the same basis. This induces the flavor violating neutral Higgs boson couplings. These couplings contribute significantly to decay $b \rightarrow s\mu^+\mu^-$ and $b \rightarrow s\tau^+\tau^-$, but negligible to $b \rightarrow se^+e^-$ decay because of its negligible $m_e$ mass. The ratio $R \equiv B(b \rightarrow s\mu^+\mu^-)/B(b \rightarrow se^+e^-)$ can be very different from its corresponding value in the Standard Model. We find that part of parameter space can accommodate a large $R$ value, and that maximum $R$ value can be larger than 2. We also present our results in $b \rightarrow s\tau^+\tau^-$ decay channel. Although it can not be detected now, it is potentially a new channel for the future observation of new physics.

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I. INTRODUCTION

Rare semileptonic B decays provide an extremely helpful tool to search new physics beyond the Standard Model (SM). New physics contributions, which enter through one loop radiative corrections, may be observed whenever the SM contributions absent or suppressed. Thus, the measurement of \( B \to X_s l^+ l^- \) has a very good chance to reveal new physics beyond Standard Model.

One way in which new physics may reveal its presence is for there to be a deviation from the Standard Model prediction for \( B \to X_s l^+ l^- \). In the absence of a deviation in the cross section, it is still possible that new physics will be show itself in the details of the \( B \to X_s l^+ l^- \) signal. The detailed study of this signal can also provide important clues as to the nature of the new physics.

In this paper, we suggest that a significant discrepancy between \( B \to X_s e^+ e^- \) and \( B \to X_s \mu^+ \mu^- \) could arise in some instances of the two Higgs doublet model. Indeed such a discrepancy would strongly suggest new physics that couples to the mass of the lepton. In this paper we discuss how observable signals of this sort could arise in the popular framework of minimal supergravity.

The SM predicts the \( b \to se^+ e^- \) and \( b \to \mu^+ \mu^- \) branching ratio for \( M_{l^+ l^-} > 2m_\mu \) to be \([1, 2]\):

\[
\mathcal{B}(b \to se^+ e^-) = 6.5 \times 10^{-6} \tag{1}
\]
\[
\mathcal{B}(b \to s\mu^+ \mu^-) = 6.2 \times 10^{-6} \tag{2}
\]
\[
\mathcal{B}(b \to s\tau^+ \tau^-) = 4.4 \times 10^{-7} \tag{3}
\]

So that the ratio of branching fractions, \( R_{SM} \), is:

\[
R_{SM} \equiv \left. \frac{\mathcal{B}(b \to s\mu^+ \mu^-)}{\mathcal{B}(b \to se^+ e^-)} \right|_{SM} \sim 0.95 \tag{4}
\]

SM predicts \( R_{SM} \sim 1 \) unless it is significantly altered by new physics. Thus the measurement of \( R \) provides a signal for specific classes of new physics.

Supersymmetry (SUSY) is a promising candidate for new physics beyond Standard Model [3]. In SM, the decay \( b \to sl^+ l^- \) occurs through electroweak penguins and box diagrams. SUSY introduces several additional classes of contributions showing in Fig. 1:

- a gluino, down-type squark loop,
- b chargino, up-type squark loop,
- c chargino, up-type squark loop, (Higgs field attaching to charginos.)
- d neutralino down-type squark loop.
There are many papers exploring the photon penguin, Z penguin, gluino penguin and box diagram contributions in SUSY [4–6]. In our paper, we focus on the penguin contributions to $b \to s l^+l^-$ mediated by neutral Higgs bosons. At the one-loop level, couplings of the “up-type” Higgs field $H_u$ to down-type quarks \([7–9]\) are induced. This coupling gives a new contribution to the down type fermion mass matrix, and induces flavor violating couplings of neutral Higgs bosons. This induced flavor violating coupling increases with $\tan\beta$. If $\tan\beta$ is sufficiently large, this flavor violating coupling has significant contribution to the branching fraction of $B_s \to \mu^+\mu^-$ [10–20] and $b \to s l^+l^-$ decays [1,5,6,21–25].

In this paper, we would like to emphasize the contributions mediated by neutral Higgs exchange. As we shall see, the ratio $R$ is particularly sensitive to new physics which contributes through this kind of mechanism, because other contributions which couple to $e$ and $\mu$ equally contribute to both the numerator and denominator of Eq.4, therefore giving little contributions to $R$.

Higgs mediated SUSY contributions to the amplitude increase with lepton mass $m_l$. Therefore such contributions to $b \to se^+e^-$ are negligible while, given a sufficient large $\tan\beta$, the contribution to $b \to s\mu^+\mu^-$ may be comparable to the SM and so observable deviations of $R$ from $R_{SM}$ are possible. We will explore these effects as a function of SUSY parameter space which, in some places, are even larger than the SM contributions. In such cases, the ratio $R \equiv \frac{B(b \to s\mu^+\mu^-)}{B(b \to se^+e^-)}$ alters substantially providing a possible way to detect new physics beyond SM.

The main object of this paper is to examine the prospects for observing a deviation in $R$ within the framework of minimal supergravity (mSUGRA) [26]. In Sec. II, we review the effective Hamiltonian for quark transition $b \to s l^+l^-$. A detailed formula to calculate the branching fraction of $b \to s l^+l^-$ is presented. Sec. III contains a brief description of the mSUGRA model along with our main results. We analyze the ratio $R$ in mSUGRA framework and also discuss the large contribution of such effects to the decay $b \to s\tau^+\tau^-$. We present our conclusions in the last section.

II. EFFECTIVE HAMILTONIAN

The effective Hamiltonian for $b \to s$ is derived by integrating out the heavy degrees of freedom at the electroweak scale or above. This Hamiltonian [27,13] can be written as:

$$H = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left[ \sum_{i=1}^{10} C_i(\mu) O_i(\mu) + C_{Q1} Q_1 + C_{Q2} Q_2 \right]$$

As in Ref. [9,13], the operators we choose are in basis\(^1\):

\[
O_1 = (\bar{s}_a \gamma_\mu P_L c_\beta)(\bar{c}_\beta \gamma^\mu P_L b_\alpha),
\]

\[
O_2 = (\bar{s}_a \gamma_\mu P_L c_\alpha)(\bar{c}_\beta \gamma^\mu P_L b_\beta),
\]

\[
O_3 = (\bar{s}_a \gamma_\mu P_L b_\alpha) \sum_{q=u,d,s,c,b} (\bar{q}_\beta \gamma^\mu P_L q_\beta),
\]

\(^1\)Only $O_{7,9,10}$ and $Q_{1,2}$ are obviously relevant to $b \to s l^+l^-$ directly.
There is a one-loop diagram with internal stop and chargino which gives a large contribution from the photon penguin diagrams; the Z penguin diagram; and the box diagram, contributing to the SM contributions to these coefficients, there are several classes of SUSY contributions.

$$O_1 = (\bar{s}_\alpha \gamma^\mu P_L b_\beta) \sum_{q=u,d,s,c,b} (\bar{q}_\beta \gamma^\mu P_L q_\alpha),$$
$$O_5 = (\bar{s}_\alpha \gamma^\mu P_L b_\alpha) \sum_{q=u,d,s,c,b} (\bar{q}_\beta \gamma^\mu P_R q_\beta),$$
$$O_6 = (\bar{s}_\alpha \gamma^\mu P_L b_\beta) \sum_{q=u,d,s,c,b} (\bar{q}_\beta \gamma^\mu P_R q_\alpha),$$
$$O_7 = \frac{e}{4\pi^2} m_b (\bar{s}_\alpha \sigma_{\mu\nu} P_R b_\alpha) F^{\mu\nu},$$
$$O_8 = \frac{g_s}{4\pi^2} m_b (\bar{s}_\alpha T^a_{\alpha\beta} \sigma_{\mu\nu} P_R b_\beta) G^{a\mu\nu},$$
$$O_9 = \frac{e^2}{4\pi^2} (\bar{s}_\alpha \gamma^\mu P_L b_\alpha) (\bar{l}_\gamma^\mu l),$$
$$O_{10} = \frac{e^2}{4\pi^2} (\bar{s}_\alpha \gamma^\mu P_L b_\alpha) (\bar{l}_\gamma^\mu \gamma_5 l),$$
$$Q_1 = \frac{e^2}{4\pi^2} (\bar{s}_L b_R) (\bar{l} l),$$
$$Q_2 = \frac{e^2}{4\pi^2} (\bar{s}_L b_R) (\bar{l} \gamma_5 l),$$

\(\alpha, \beta\) being color indexes, \(a\) labels the SU(3) generators, and \(P_{L,R} = (1 \mp \gamma_5)/2\). \(C_i\) are Wilson coefficients.

\(O_1\) and \(O_2\) are current-current operators, \(O_3...O_6\) are QCD penguin operators. The contributions of these four quark operators to \(b \to s l^+ l^-\) are proportional to the tree level matrix elements of operators \(O_7, O_8\) and \(O_9\) at one loop level. The chromomagnetic dipole operator, \(O_8\), gives no contributions to \(b \to s l^+ l^-\). As a result, the \(O_1...O_6\) contributions can be absorbed by appropriately modifying the Wilson coefficients of operator \(O_7\) and \(O_9\), which originated in the \(Z^0\) and \(\gamma\) penguin diagrams with external \(l^+ l^-\) pairs. The neutral Higgs couplings SUSY contributions are mainly through \(Q_1\) and \(Q_2\). The effective Hamiltonian for \(b \to s l^+ l^-\) is thus:

$$H = -\frac{G_F \alpha}{\sqrt{2}} V_{ts} V_{tb}^\ast [-2i C_i^{eff} \bar{s} \sigma_{\mu\nu} (m_b P_R + m_s P_L) \frac{q^\nu}{q^2} b \bar{l} \gamma^\mu l + C_9^{eff} \bar{s}_L \gamma_\mu b_L \bar{l} \gamma_\mu^\prime l + C_{10} \bar{s}_L \gamma_\mu b_L \bar{l} \gamma_\mu \gamma_5 l + C_{Q1} \bar{s}_L b_R \bar{l} l + C_{Q2} \bar{s}_L b_R \bar{l} \gamma_5 l]$$

The Wilson coefficient \(C_9^{eff}\) includes leading-order (LO) and next-to-leading order (NLO) logarithms, while \(C_7^{eff}\) and \(C_{10}\) enter only at the NLO level. In addition to the SM contributions to these coefficients, there are several classes of SUSY contributions. The photon penguin diagrams contribute to \(C_7^{eff}\). Three types of diagram: the photon penguin diagram; the Z penguin diagram; and the box diagram, contribute to \(C_9^{eff}\). The \(C_{10}\) is induced by the Z penguin diagram and the box diagram.

In the calculation of \(C_9^{eff}\), we find that \(C_7^{eff}\) can be quite different from its SM value. There is a one-loop diagram with internal stop and chargino which gives a large contribution when \(\tan \beta\) is large. This stop-Higgsino diagram is proportional to the product of the top and bottom Yukawa coupling constant, \(m_t m_b / (\sin \beta \cos \beta)\), which grows with \(\tan \beta\). There are no such terms in the calculation of \(C_9^{eff}\) and \(C_{10}\). The corresponding stop-Higgsino diagram is proportional to the square of the top Yukawa coupling constants, \(m_t^2 / \sin^2 \beta\), which does not grow for large \(\tan \beta\). As pointed out by Ref. [5], SUSY
contributions to $C_9^{\text{eff}}(m_b)$ and $C_{10}(m_b)$ are very small and alter these coefficients by $\leq 5\%$ over the whole parameter space on $C_9^{\text{eff}}(m_b)$ and $C_{10}(m_b)$. Because of this, SUSY contributions on $C_9^{\text{eff}}$ and $C_{10}$ can be ignored.

We must consider the SUSY contribution to $C_7^{\text{eff}}$. We write,

$$C_7^{\text{eff}}(M_w) = C_7^{\text{SM}}(M_w) + C_7^{\text{SUSY}}(M_w)$$

where $C_7^{\text{SM}}(M_w)$ is given in Ref. [27], and $C_7^{\text{SUSY}}(M_w)$ is taken from Ref. [6,28] with mass insertion approximation. For complete calculation, see Ref. [5]. At the $m_b$ scale:

$$C_7^{\text{eff}} = \eta^{16/23}C_7^{\text{eff}}(M_w) + \frac{8}{3}(\eta^{14/23} - \eta^{16/23})C_8(M_w) + \left(\sum_{i=1}^{8} h_i \eta^{a_i}\right)C_2(M_w)$$

where $\eta = \frac{a_s(M_w)}{a_s(m_b)}$, $C_8(M_w)$, $C_2(M_w)$, $h_i$ and $a_i$ can be found in Ref. [27]. The expressions of $C_9^{\text{eff}}$, and $C_{10}$ can be found in Refs [13,27]. Within SM, they are:

$$C_9^{\text{eff}} = C_9 + Y(\hat{s}), \quad C_9 = 4.138, \quad C_{10} = -4.221 \quad (8)$$

where $\hat{s} = q^2/m_b^2$, and $q$ denotes the invariant momentum of the lepton pair. The expression for function $Y(\hat{s})$, coming from the one loop contributions of operators $\mathcal{O}_1 - \mathcal{O}_6$, can be found in Ref [13]. The Wilson coefficients $C_{Q_1}$ and $C_{Q_2}$, can be found in Ref. [9]. In terms of the Wilson coefficients, the differential decay rate is [21,29]:

$$\frac{d\Gamma}{d\hat{s}} = \frac{G_F^2 \alpha^2 m_b^5}{128\pi^5} |V_{ts}|^2 |V_{tb}|^2 \sqrt{\lambda(1, \mu_s, \hat{s}) \lambda(1, \frac{\mu_l}{\hat{s}}, \frac{\mu_l}{\hat{s}})} \left[ \frac{1}{6} (|C_9^{\text{eff}}|^2 + |C_{10}|^2) \right.$$  

$$\left( \hat{s}(1 + \mu_s - \hat{s}) \lambda(1, \frac{\mu_l}{\hat{s}}, \frac{\mu_l}{\hat{s}}) + (1 + \hat{s} - \mu_s)(1 - \hat{s} - \mu_s)(1 + \frac{2\mu_l}{\hat{s}}) \right)$$

$$+ (|C_9|^2 - |C_{10}|^2) \mu_l (1 + \mu_s - \hat{s}) + \frac{2}{3} |C_7^{\text{eff}}|^2 \hat{s}(1 + \frac{2\mu_l}{\hat{s}})$$

$$\left( 2(1 + \mu_s)(1 - \mu_s)^2 - \hat{s}(1 + 14\mu_s + \mu_s^2) - \hat{s}^2(1 + \mu_s) \right)$$

$$+ 2C_7^{\text{eff}} Re(C_9^{\text{eff}})(1 + \frac{2\mu_l}{\hat{s}})(1 - \mu_s)^2 - \hat{s}(1 + \mu_s)$$

$$+ \frac{1}{4} C_Q^2 (1 + \mu_s - \hat{s})(\hat{s} - 4\mu_l) + \frac{1}{4} C_Q^2 \hat{s}(1 + \mu_s - \hat{s})$$

$$+ C_{10} C_Q \sqrt{\mu_l}(1 - \mu_s - \hat{s}) \right] \quad (9)$$

In Eq.9, $\mu_l = m_l^2/m_b^2$, and $\mu_s = m_s^2/m_b^2$. The function $\lambda(x, y, z)$ is:

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$$

Integrating out $\hat{s}$ from $4m_l^2/m_b^2$ to $(1 - m_s/m_b)^2$, the branching ratios of decay $b \rightarrow s l^+ l^-$ are easily obtained by:

$$\mathcal{B}(b \rightarrow s l^+ l^-) = \frac{1}{\Gamma} \int_{4m_l^2/m_b^2}^{(1-m_s/m_b)^2} \left( \frac{d\Gamma}{d\hat{s}} \right) d\hat{s} \quad (10)$$
III. RESULTS

The minimal supergravity model [26] has been a popular model for SUSY phenomenology. It provides a well motivated realization of the Minimal Supersymmetric Standard Model (MSSM). In this model, SUSY is broken in a hidden sector. The fields in the hidden sector interact with usual particles and their superpartners only via gravity. In this way, SUSY breaking is communicated to the observable sector of Standard Model particles and their superpartners.

The mSUGRA framework assumes that at the GUT scale ($M_{GUT} \approx 2 \times 10^{16}$ GeV), all scalar fields have a common SUSY breaking mass $m_0$, all gauginos have a mass $m_1^+$, and also all soft SUSY breaking trilinear scalar couplings have a common value $A_0$. The resulting MSSM will have various soft SUSY breaking terms unified at $M_{GUT}$. The soft SUSY breaking parameters are evolved from $M_{GUT}$ to weak scale using renormalization group equations (RGE). Minimization of the Higgs potential gives a relation between $\mu^2$, $m_Z^2$, B and $\tan\beta$, therefore $\mu^2$ can be determined by $m_Z^2$ [30] and the B parameter is traded in favor of $\tan\beta$, so that the model with radiative electrical symmetry breaking is completely specified by the SM parameters together with:

$$m_0, \quad m_1^+, \quad A_0, \quad \tan\beta, \quad \text{sign}(\mu)$$

To evaluate the mass spectrum of the MSSM resulting from mSUGRA, we use the ISASUGRA program from the ISAJET package [31]. With these sparticle masses and mixing angles, we can get the Wilson coefficients $C_{Q_1}$ and $C_{Q_2}$. Then the branching ratio of $b \to s l^+ l^-$ can be computed using Eq. 9 and 10.

We calculated the branching ratios of $b \to s e^+ e^-$ and $b \to s \mu^+ \mu^-$. We find that if $\tan\beta$ is small, SM contributions dominate and SUSY contributions are very small. As $\tan\beta$ grows, SUSY contributions become large. $O_7$ gives a sizable SUSY contributions with a little dependent on the lepton mass. The $O_7$ and $O_9$ interfere term also gives a comparable contributions. They together alter the branching fraction around 15% to 20%. $Q_1$ and $Q_2$ terms are induced by Higgs-mediated contributions. Their contributions to $b \to s l^+ l^-$ branching fraction increase with $\tan^6\beta$ and in some parameter space, they are even bigger than SM contributions. The corresponding Wilson coefficients $C_{Q_1}$ and $C_{Q_2}$ are proportional to the lepton mass so that the branching ratio is proportional to lepton mass square. Therefore, Higgs-mediated SUSY contributions in decay $b \to s \mu^+ \mu^-$ are about $10^4$ bigger than that in $b \to s e^+ e^-$ decay and alter the ratio $R \equiv B(b \to s \mu^+ \mu^-)/B(b \to s e^+ e^-)$ deviating from its corresponding SM value. We require that the lepton pair mass is larger than $2 m_\mu$ to reduce the dependences of the branching fraction on the lepton pair mass in small s range.

In Fig. 2, we show the dependence of $R \equiv B(b \to s \mu^+ \mu^-)/B(b \to s e^+ e^-)$ on $\tan\beta$ and $m_0$ for $A_0 = 0$ and $A_0 = -300$ cases. We fixed $m_{1/2} = 300$ GeV in all the frames. Frame a) is $R$ vs. $\tan\beta$ with $\mu < 0$ and $m_0 = 300$ GeV. Frame b) is $R$ vs. $m_0$ with $\mu < 0$ and $\tan\beta = 45$. Frames c) and d) are for $\mu > 0$ case. Frame c) is $R$ vs. $\tan\beta$ with $m_0 = 300$ GeV, while frame d) has $\tan\beta = 53$ with $R$ vs $m_0$ plot. The solid (dashed) line is for $A_0 = -300$ (0) GeV, and the dotted line is the corresponding R value in SM. Values of $\tan\beta (m_0)$ larger than the corresponding values denoted by circles on the curves are where $m_h$ falls below its experimental bound 113 GeV, which gives out the most stringent
constraint. In frame b), $m_h > 113$ GeV for all the regions in the graph along $A_0 = 0$ line. In frames c) and frame d), $m_h$ is always larger than 113 GeV. The main reason to show this figure is to understand the behavior of the ratio $R$. Several features need to be noted:

- The $R$ value is significantly larger for negative value of $\mu$. As explained in ref. [9], changing the sign of $\mu$ changes the denominator of $\chi_{FC}$ in Eq.10 of ref. [9], so that a suppression for positive $\mu$ changes to enhancement for negative $\mu$.

- For $\mu < 0$ case, in low $\tan\beta$ regions, SM contributions dominate, so that all curves are close the SM value: $R \sim 0.95$. Same thing happens in $m_0$ larger than 400 GeV range.

- Again in $\mu < 0$ case, as $\tan\beta$ is larger than 44, there is a sharp rise for both $A_0 = 0$ and $A_0 = -300$. This is because Higgs-mediated SUSY contributions are expected to behave as $\tan^6\beta/m_\chi^2$ [9] \(^2\). The solid and dashed curves indeed show this behavior as long as $\tan\beta$ is large. Clearly then, SUSY contributions are dominated by Higgs mediated penguin when $\tan\beta$ is large.

- $R$ deviates significantly from the SM value when $\tan\beta$ larger than 45 in $\mu < 0$ case. Unfortunately, in the mSUGRA scenario, these ranges are excluded by the experiment bound on the Higgs mass, $m_h > 113$ GeV [32].

- As shown in frame c) and frame d), in $\mu > 0$ case, $R$ changes very slowly over the whole parameter space. Higgs-mediated SUSY contributions are very small [9]. The fact that solid line and dashed line are almost overlap and parallel to the dotted line tells us a constant $R$ value exists. We even increase the value of $A_0$ to around 300 GeV. Still we get a $R$ value, which is very close to its SM prediction. This excludes the possibility to find a signal of new physics in $\mu > 0$ case.

Fig. 2 illustrates that typically large contributions to $R$ occur in the $\mu < 0$ scenario with $\tan\beta \geq 40$. For $\mu > 0$, $R$ is almost constant over the whole parameter space and very close to the SM prediction, so that no room has left for new physics. Although $\mu < 0$ is generally thought to be disfavored by the determination of the muon anomalous magnetic moment by the E821 experiment [33], a conservative estimate [34] of the theoretical error suggests that there is a region allowed [35] by this constraint, though perhaps in conflict with $B(b \to s\gamma)$ \(^3\), where $b \to s\ell^+\ell^-$ may provide the first hint of new physics if $\tan\beta \geq 40$. This region would expand as more experimental data are accumulated.

\(^2\)The contribution from $h$ exchange is very small as long as $h$ is a SM-like Higgs boson, and $m_H \sim m_A$.

\(^3\)It is worth reminding the reader that SUSY contribution, special for large values of $\tan\beta$, may have considerable theoretical uncertainty. Unlike constraints from direct searches, constraints from $B(b \to s\gamma)$ are very sensitive to details of the model. A small amount of flavor mixing in the squark sector could lead to large differences in the predictions of $b \to s\gamma$ decays. For this reason, the constraints from $B(b \to s\gamma)$ should be interpreted with some care.
The large values of $R$ in frame a) and b) in Fig. 2 encourage us to do further exploration in $\mu < 0$ case. Therefore, we plot a contours of $R$ values in $A_0 - m_{1/2}$ plane in Fig. 3 for $\mu < 0$ case. We choose $m_0 = 300$ GeV and in frame a), $\tan\beta = 45$, while in frame b), $\tan\beta = 42$. The dark-shaded regions are excluded on theoretical grounds because the overall theory does not lead to electroweak symmetry breaking. The slant-hatched region are excluded because of $\tilde{Z}_1$ is not the lightest supersymmetric particle (LSP). If the neutralino is the LSP, it will be stable and therefore a dark matter candidate. Within the slant-hatched region, charged sparticle is the LSP which disagrees with cosmologies models. Below the dashed line labeled “$m_h = 113$ GeV” are the regions $m_h$ is smaller than its experimental bound 113 GeV. Below the dotted line, branching fractions of $B_s \to \mu^+\mu^-$ are larger than $5 \times 10^{-10}$, but smaller than $10^{-7}$. The later is the maximum limit the experiment, Tevatron or B factories, can explore in detecting $B_s \to \mu^+\mu^-$ decay. The contours of $R \equiv B(b \to s\mu^+\mu^-)/B(b \to se^+e^-)$ are labeled by the values of corresponding ratio $R$. From frame a) we see that in the allowed region, $R$ can be as large as 2. Even in frame b), $R$ can be as large as 2. The outermost curves is for $R$ equals to 1.1. Turning our attention to the sensitivity with respect to $\tan\beta$, we show in frame b), the same contours, but with $\tan\beta = 42$. Although the range for $\tan\beta = 42$ is smaller than that for $\tan\beta = 45$, it has still regions where B factories may be expected to detect it. The ratio can be significantly larger below the $R = 2$ contour.

Recently, Belle experiment has observed a signal for exclusive B decays with branching fraction $B(B \to K^+\pi^-) = (0.75_{-0.21}^{+0.25} \pm 0.09) \times 10^{-6}$ averaged over electron and muon channels and $B(B \to K\mu^+\mu^-) = (0.99_{-0.32}^{+0.40} \pm 0.13) \times 10^{-6}$ in muon channel [36]. This is consistent with our SM value for the inclusive rate $\sim 6.4 \times 10^{-6}$. The inclusive B decay branching ratio is measured by Belle group with $\mathcal{B}(B \to X_{s}\mu^+\mu^-) = (6.1_{-1.4}^{+1.4}(stat)_{-1.1}^{+1.1}(syst)) \times 10^{-6}$ [37,38], which is consistent with the SM prediction. We also estimate that approximately $\sim 10^7$ B mesons are required to reach the $R = 1.2$ limit.

Due to the $m_0^2$ dependence of these branching fractions, one would clearly expect contributions to $b \to s\tau^+\tau^-$ [29,39] two orders of magnitude greater than $b \to s\mu^+\mu^-$. Turning to Fig. 4, we see that this is indeed the case. In Fig. 4, we show the contours of $B(b \to s\tau^+\tau^-)$ in $m_0 - m_{1/2}$ plane with $A_0 = 0$ GeV. In frame a) $\mu < 0$ and $\tan\beta = 45$, while in frame b) $\mu > 0$ and $\tan\beta = 53$. As in Fig. 3, the dark-shaded regions are excluded by theoretical constraints: charge-breaking minimal or lack of electroweak symmetry breaking. $\tilde{Z}_1$ is not the lightest supersymmetric particle (LSP) in the slant-hatched region. The contours are labeled by the corresponding values of $b \to s\tau^+\tau^-$ branching fraction. The outermost contour corresponds to a branching fraction of $10^{-6}$. The SM branching fraction is around $B(b \to s\tau^+\tau^-) \sim 3.66 \times 10^{-7}$. In Fig. 4, at the most parameter space, SUSY contributions is much larger than the SM background, reaching a limit of $\sim 10^{-4}$ within the allowed region. Here we remind the readers that some region perhaps is excluded by $B_S \to \mu^+\mu^-$ constraint. In ref. [9], we give out detailed calculations on $B_s \to \mu^+\mu^-$. Unfortunately $\tau$ identification is very difficult. The branching fraction about $10^{-3}$ for $\tau$ process may be accessed at B factory. Clearly then, at least one order of magnitude improvement in the acceptance for $b \to s\tau^+\tau^-$ is required to have a significant impact on this parameter space. It is interesting to note that any new physics which contribute significantly to $b \to s\tau^+\tau^-$ will also give a signal in $B_s \to \tau\tau$ as discussed in Ref. [9].
Another aspect of new physics contributions to the $b \to s \mu^+ \mu^-$ signal is the forward-backward lepton asymmetry $A_{FB}$. This is considered in ref. [25] and found to be on the order of a few percent hence difficult to observe at current B factory luminosities. We do not explore this feature in this paper, but it is potentially another channel for new physics detection.

IV. SUMMARY AND CONCLUSIONS

We have explored the rare semileptonic decay $b \to se^+e^-$, $b \to s\mu^+\mu^-$ and $b \to s\tau^+\tau^-$ within mSUGRA framework where some Higgs-mediated SUSY contributions to the branching fraction of $b \to l^+l^-$ are proportional to corresponding lepton mass squared. When such contributions in the muon case become comparable to the SM, this opens a new channel for new physics detection. We explored these decays in detail and find that while in $\mu > 0$ case, SUSY contribution is suppressed, for negative $\mu$ with large $\tan \beta$ the flavor violating couplings of neutral Higgs bosons to down type quarks leads to substantial SUSY contributions to the branching fraction of decay $b \to s\mu^+\mu^-$. In particular observably large contributions to the branching fraction of $b \to s\mu^+\mu^-$ are possible when $\tan \beta > 40$. In particular there can be leading a large discrepancy between the $b \to s\mu^+\mu^-$ and $b \to se^+e^-$ branching ratios. The resultant deviations in $R \equiv B(b \to s\mu^+\mu^-)/B(b \to se^+e^-)$ from the SM values in some regions of the mSUGRA parameter space are as shown in Fig. 3.

In this framework the large value of $m_\tau$ would give a huge deviation from the SM prediction for $B(b \to s\tau^+\tau^-)$ in the region of large $\tan \beta$. We explore the branching fraction for $b \to s\tau^+\tau^-$ decay in $m_0 - m_{1/2}$ plane and in the region of parameter space considered the maximum values of $B(b \to s\tau^+\tau^-)$ can be up to $8 \times 10^{-4}$. Unfortunately, because of the difficulties in separating signals of $\tau$ decay from the backgrounds, the $\tau$ signal is undetectable at B factories. Thus, the most promising signal of the new physics considered in this paper is a deviation in the value of $R$ from its SM value. Conversely, if such a deviation in $R$ were observed it would strongly indicate new physics coupling to mass through the Higgs sector as in the case of mSUGRA model considered in this paper.

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FIG. 1. One loop SUSY contributions.
FIG. 2. Ratio $R \equiv B(b \rightarrow s\mu^+\mu^-)/B(b \rightarrow se^+e^-)$ in mSUGRA model with $m_{1/2} = 300$ GeV and a) $R$ vs $\tan\beta$ with $\mu < 0$, $m_0 = 300$ GeV, b) $R$ vs $m_0$ with $\mu < 0$, $\tan\beta = 45$, c) $R$ vs $\tan\beta$ with $\mu > 0$, $m_0 = 300$ GeV, d) $R$ vs $m_0$ with $\mu > 0$, $\tan\beta = 53$. The solid line is for $A_0 = -300$ GeV, and $A_0 = 0$ GeV for dashed line. The dotted line is the corresponding SM line. The circles mark the limits of the experimentally allowed regions discussed in the text.
FIG. 3. Contours of constant ratio $R \equiv B(b \to s\mu^+\mu^-)/B(b \to se^+e^-)$ with $\mu < 0, m_0 = 300$ GeV in $A_0 - m_{1/2}$ plane with a) $\tan\beta = 45$ b) $\tan\beta = 42$ in mSUGRA model. The dark-shaded region is excluded by the theoretical constraints on electroweak symmetry breaking (EWSB). and the slant-hatched region is excluded for $\tilde{Z}_1$ is not LSP. Below the dashed line is for $m_h < 113$ GeV. Below the dotted line, the branching fractions of $B_s \to \mu^+\mu^-$ decay are larger than $5 \times 10^{-10}$.
FIG. 4. Contours of constant branching fraction for decay $b \rightarrow s\tau^+\tau^-$. The results are showed in $m_0 - m_{1/2}$ plane with $A_0 = 0$ GeV; in frame a). $\tan\beta = 45$, $\mu < 0$ and in frame b). $\tan\beta = 53$, $\mu > 0$. The dark-shaded region is excluded by the theoretical constraints on EWSB. and the slant-hatched region is excluded for $\tilde{Z}_1$ is not LSP.