We live in the quantum 4-dimensional Minkowski space-time.

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Abstract

We try to define "our world" by stating that "we live in the quantum 4-dimensional Minkowski space-time with the force-fields gauge group $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ built-in from the outset".

We begin by explaining what "space" and "time" are meaning for us - the 4-dimensional Minkowski space-time, then proceeding to the quantum 4-dimensional Minkowski space-time.

In our world, there are fields, or, point-like particles. Particle physics is described by the so-called Standard Model. Maybe I should explain why, how, and what my Standard Model would be everybody's "Standard Model" some day. Following the thinking underlying the minimal Standard Model and based on the gauge group $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$, the extension, which is rather unique, derives from the family concept that there are three generations of quarks, on (123), and of leptons, on another (123). It yields neutrino oscillations in a natural manner. It also predicts a variety of lepton-flavor-violating rare decays.

At the end of the Standard Model, we will provide some clear answers toward two "origin" questions: What is the origin of mass? Another one: what is the origin of fields (point-like particles)?

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1 The Space and Time

I assume that you know about the length measurement and the standard clock. Here we assume that the operations of sticks and clocks does not affect anything. Further, we can make sure that the local space-time which we are working with is flat. If we label an event or point as $(x, y, z, ct)$ or simply $(x, y, z, t)$, then the interval between two points, $(x_1, y_1, z_1, ct_1)$ and $(x_2, y_2, z_2, ct_2)$, would be an invariant quantity, according to relativity. Thus, we, or you and I, could begin to talk about something and in fact set up some common units for space and time.

So, we have, with $\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$, $\Delta z = z_2 - z_1$, and $\Delta t = t_2 - t_1$,

$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2,$$

(1)

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an invariant quantity among all the observers. Or, in infinitesimal forms,

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = \eta_{\mu\nu} dx^\mu dx^\nu.$$  (2)

Here $ds$ is called "the proper length", or "the proper distance", $g_{\mu\nu}$ is the second-rank metric tensor, and $\eta_{\mu\nu} = \text{diag}(1,1,1,-1)$. Similarly, the $d\tau$ (with $d\tau^2 \equiv -ds^2$) is "the proper time". We shall use "the invariant distance" or "the invariant time" for the sake of being precise.

From the measurements of the space and of the time which are rather independent, there is no reason why the Minkowski constraint, the equation specified above, should be valid. Thus, we live in the 4-dimensional Minkowski space-time.

Let’s come to think about it. The so-called "intuition" (classically) could be completely wrong. Because of "Lorentz contraction" and "time dilation" already built in the above Minkowski’s metric, the world of an elementary particle differs from our limited one, based on classical physics or Newton’s. And, for this book on "Relativistic Quantum Mechanics and Quantum Fields" [1], we suggest that we should imagine that our "intuition" of the world be the same as that for an elementary particle. Thus, we try to downplay the importance of the non-relativistic descriptions.

Moreover, the quantities $\Delta x$, $\Delta y$, $\Delta z$, and $\Delta t$ are "observables" in the quantum mechanics sense. Similarly, we should classify $\Delta s$ as a quantum-mechanical observable.

These quantities should be realized as the quantum mechanical observables, as to be indicated by the statements in the next section. In other words, we start out by viewing these quantities as the operators, not simply by commuting real numbers. This is the case, even though in certain representations part of them are real numbers.

As another important example of the 21st-century post-modern physics, let’s start with the Schwarzschild metric,

$$d^2 s = -(1 - \frac{2GM}{r})dt^2 + (1 - \frac{2GM}{r})^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2).$$  (3)

We should try to establish our intuitive explanation of the time and the space, at all $r$ and all $t$. Sign switch may be identified as the important source of confusion. In this example, we use the natural units. We should eventually establish a decent 21st-century physics, with this "black hole" solution as one of the beginning points.

To this end, we identify $t$ as our time and $(x_1, x_2, x_3)$ as our space. The problem with the Schwarzschild metric is the coefficient of $dr^2$ which goes over to $\infty$ at the horizon, i.e., $\frac{2GM}{r} \rightarrow 1$. The coefficient of $dt^2$ would go to zero simultaneously. So, in our space and our time, maybe we never get to this point - that is, black holes in our space-time never form and all the black holes are pre-formed. Einstein equation tells us so.

The other way of saying it: We try to write the Schwarzschild metric as follows:

$$(ds)^2 = -c^2(dt^*)^2 + (dr^*)^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2).$$  (4)

Here we have

$$\frac{dr^*}{dr} = \pm \sqrt{\frac{r}{r-r_h}}, \quad \frac{dt^*}{dt} = \pm \sqrt{\frac{r-r_h}{r}}; \quad r_h = 2GM.$$  (5)
So, it becomes pure imaginary beyond the horizon \((r = r_h)\) rather than the switching between time and space, which might be wrong at the first place. Our space variables and our time variable are real numbers, i.e., \((-\infty, +\infty)\), after all. This may offer an entirely new road for the knowledge of black holes.

We thus suggest that we may stick to the interpretation of viewing \(t\) as the time and \((x_1, x_2, x_3)\) as the space. Thus, these are quantum mechanical observables. Let these variables run from \(-\infty\) to \(+\infty\) to define the observable space-time for us. We should not talk about the events beyond us - not within the \(\pm\infty\). We will be "conservative" in this regard.

We may choose the cgs unit system, the length in \(cm\), the time in \(sec\), and the mass in \(gm\), so that the light velocity \(c = 3.00 \times 10^{10}\ cm/sec\). Or, the natural units may be chosen such that \(\hbar = c = 1\). Unless specified otherwise, the natural units will be used throughout this textbook.

We have to look at the space and the time in particular not too "ideologically". First of all, the space-time changes according to the matter distribution - if we adopt Einstein’s general relativity. In other words, the "definition" of the space-time varies with the matter distribution, which could be anything. Then, how do we "define" the matter in this context?

For example, in the Schwarzschild metric given above which is the solution to Einstein general-relativity equation of a "point" mass, the rationale of the "black-hole" solution still stirs up so many puzzles, even until today (for about a hundred years later). So, the mix-up of the "simple" space-time notion with the matter notion sort of drives these concepts with a lot of ambiguities.

In fact, we might regard the "space-time" as a whole as the "physical system" of some kind. In the simplest case, the physical system possesses the Lorentz invariance and others. When we talk about a "point-like" particle in the space-time, we could try to "define" what the "point-like" means in the physical system of the space-time. In other words, the space-time is more physical than mathematical.

In the last part of the book, we shall study the Standard Model of particle physics. Electrons, other leptons, quarks, etc., the so-called "building block of matter", all are point-like particles - up to the size of \(10^{-20}\ cm\) (the known resolutions), these particles are still point-like.

So, we are studying the behaviors of these "points" in our space-time, physically rather than mathematically, at the scale far small than the atomic size of \(10^{-8}\ cm\).

2 The Point in the Quantum Sense

Mathematically, a "point" does not have any size or volume, according to the geometry. When we talk about a "point-like" particle in the physical sense; it should be different from the "point" in the mathematical sense. When we describe a particle by Dirac equation or by Klein-Gordon equation, that does not have the size parameter, we call it "point-like", or "point-like-ness" in the quantum sense.

Quantum mechanics, or more precisely quantum principle, comes to rescue in a mysterious way. At the scales around 1 cm, we deal with the system macroscopically and we
developed the classical physics. At the scales of about $10^{-8} \text{cm}$, we know that quantum principle has to be there. The uncertainty principle and others are working there, down to the scale of 1 fm, or $10^{-13} \text{cm}$, or much smaller.

The quantum principle states that the position $(x,y,z)$ does not commute with the momentum $m \frac{d}{dt}(x,y,z)$, with the commutator equaling $i\hbar$. Thus, we have to treat the position $(x,y,z)$ and the momentum $m \frac{d}{dt}(x,y,z)$ as operators when we speak of them simultaneously. Thus, we have

$$[x_j,p_j] = i\hbar \delta_{ij}, \quad [x_i,x_j] = 0, \quad [p_i,p_j] = 0.$$  \hspace{1cm} (6)

$h$ sets the scale when the quantum effects are "visible".

So, what is the concept of ”point” in this quantum regime? It is relevant for the atomic size (i.e. $10^{-8} \text{cm}$) or for the size of a nucleus (i.e. $10^{-13} \text{cm}$). Eventually, it has become rather murky at the distance of $10^{-20} \text{cm}$ or shorter. $10^{-20} \text{cm}$ is somewhat smaller than the resolution set by the Large Hadron Collider (LHC) at Geneva.

We should mention the following:

When it becomes much smaller, such as $10^{-25} \text{cm}$ or smaller, which are so small that we never get there, the coordinates $(x,y,z,ct)$ might not commute among themselves – the so-called ”non-commutative geometry”. This phenomenon, if it exists, which we call it the ”super-quantum regime” – it is rather natural in the line of ”reasoning”.

This noncommutative aspect among the coordinates was raised by Hartland S. Snyder in as early as 1947 [Phys. Rev. 71, 38 (1947)] [2]. This aspect was discussed later on in a variety of contexts. The idea could be relevant when we discuss the alternatives of ”the point-like structure” or point-like particles.

In any event, it is quite clear that the ”point-like” in the physical sense could be so different from the ”point” in the mathematical sense. As a physicist, we’d better keep in mind these subtleties, particularly when we ”polish” our theories to eventually describe more peculiar systems or objects.

### 3 Our World: The quantum 4-dimensional Minkowski space-time

Thus, we propose that the differences of the space-time coordinates are the quantum observables which are subject to the measurements for realizing their values. The measurement of the space-time coordinates is the very beginning of everything. As the quantum law requires, the spatial coordinates do not commute with the time derivatives of the spatial coordinates themselves - implying that they are operators, or mappings or functions.

Based on this proposal, the inside of the black hole requires the coordinates to be pure imaginary, thus beyond the observability. Einstein equation relates Ricci’s tensor to the energy-momentum tensor, so mathematically it could cover the ”un-physical” region(s).

In other words, the laws of gravity, when it hits the boundaries of the infinities or of the pure imaginary, remain to be completely open. This is one of the frontiers that we should pursue after, in this 21st century.
To emphasize, we live in the quantum 4-dimensional Minkowski space-time. All the classical notions are in fact used to illustrate our real space-time, nothing any more. In the following, we shall mention that this space-time is also endorsed with the force-fields gauge fields, according to the Standard Model.

4 The Standard Model of the 20th Century

Quantum mechanics and Einstein’s theory of special relativity constitute the basis for the main body of modern physics developed during the first half of the twentieth century (that is, the two pillars of the 20th-Century modern physics). Maybe, in the 21st-century physics, we should, carefully, introduce the third pillar, the fourth pillar, and so on.

Physical systems that can be described by relativistic wave equations include electrons, quarks, and many other elementary particles in the subatomic world. The presentation on the Klein-Gordon equation could be simple since it has the same symmetry as the background, i.e., the 4-dimensional Minkowski space-time. It describes spinless particles such as the Higgs particles, for which the example for its existence was in 2012, and the pions, the composite systems which were discovered in 1940’s. So after we explain the axiom box for the meaning of ”quantization” on the language, we shall present the Dirac relativistic equation in great details. The Dirac theory for the electron has many successes: It is Lorentz covariant; it contains the electron spin with the gyromagnetic ratio $g = 2$; it predicts the anti-particle positron which is experimentally discovered about ten years later; it gives the correct fine structure of the hydrogenic atom levels; etc. Application of the Dirac equation to other leptons such as muons and neutrinos, and to quarks in the context of bag models, also leads to quantitative successes. It is clear that, unlike atomic or molecular physics where relativistic effects can often be treated as small perturbations, a suitable introduction to elementary particle physics and field theories must commence with a description of relativistic wave equations.

Early on, there were ”difficulties” too with the Dirac equation. One was that of extending the theory for a single electron to a system of many electrons. Another was of an even more basic nature, namely, a theory started out to represent a single electron ends up being inseparably bound with a many-body effect on account of the infinite sea of electrons in negative energy states. In fact, with the discovery of the positron (the anti-particle of the electron), the idea of the ”negative-energy sea” could have died off, but it persisted until the end of the 20th Century.

But the antiparticle of the electron, or the positron, was discovered. It explains why the Dirac theory must have four components - two for the electron and another two for the positron. The electron and the positron are born to be described by the same Dirac equation. There is no such thing which are called as ”negative-energy states”. The entire things could have been clear from there, then. The notion of ”negative-energy states” should not appear, and in fact no need to appear — we could call it ”the Mistake of the 20th Century Physics”.

Historically, the story is as follows: The theory of quantized electromagnetic fields begins with the work of Dirac in 1927, followed immediately by the work of Jordan and Wigner, and
by Fermi in 1930. A breakthrough comes only in the mid 1940’s with the work of Tomonaga in Japan and Schwinger, Feynman, and Dyson in the United States. Although infinities still remain, the theory succeeds in ”subtracting” them away in a definite, covariant way so that finite results can be obtained, which have been found to be in excellent agreement with the observed Lamb shifts and the ”$g - 2$ anomaly”. The decade from the mid 1940’s to the mid 1950’s is a period of fervent studies of quantum electrodynamics, both in further calculations on this ”renormalization” theory and in attempts to rid the theory of the infinities (not just to isolate and bury them, so to speak). Till now, there seems not yet a satisfactory solution of this problem and in the meantime physicists have turned their interest to other areas - elementary particles and the nature of their interactions and their unification.

During the decade from the mid 1950’s to the mid 1960’s, serious attempts were made in searching for an alternative means of describing interactions among elementary particles in terms of the so-called ”S-matrix theory”, in which one tries to determine the scattering amplitudes, or the S-matrix elements, using general principles such as unitarity and microscopic causality [which lead to dispersion relations] and a minimal set of dynamical assumptions. Altogether in the S-matrix approach, the question concerning the underlying dynamics must be answered, or postulated, before any quantitative predictions can be consistently made. Therefore, the notion of using ”gauge field theories” to describe interactions among ”building blocks” of matter has scored an amazingly successful comeback since the late 1960’s while progresses in the S-matrix approach, which have since been relatively limited, have become things of the past, at least for the time being.

The development of particle physics of the last half century [since the late 1960’s till the present - 2015] consists of several major breakthroughs which culminate in the general acceptance of the $SU(3)_{\text{color}} \times SU(2)_{\text{weak}} \times U(1)$ gauge field theory of strong, electromagnetic, and weak interactions as the ”standard model”. The Glashow-Salam-Weinberg [GSW] $SU(2)_{\text{weak}} \times U(1)$ gauge theory provides a unified description of electromagnetic and weak interactions. It predicts the existence of neutral weak interactions, the existence of the charm quark, and the existence of weak bosons $W^\pm$ and $Z^0$ [which mediate weak forces], all of which have been substantiated experimentally.

In the meantime, the quantized $SU(3)_{\text{color}}$ gauge theory, or quantum chromodynamics [QCD], has been established as the candidate theory of strong interactions among quarks and gluons, which are believed to be the building blocks of all observed strongly interacting elementary particles or hadrons. QCD supports the dual picture of considering, e.g., a proton as a collection of almost non-interacting quarks, antiquarks, and gluons at high energies [because of the asymptotically free nature of QCD] and as a system of confined, dressed valence quarks at low energies [because QCD is consistent with color confinement].

Dirac believes that in a completely satisfactory theory, infinities should not appear. Since the late 1940’s, he set out to re-examine and reformulate classical dynamics and electromagnetic theory with a view to find a different basic theory for quantization. He believes even some new mathematics not yet known may be needed. He expressed his views only occasionally in writing, but much more freely in private discussions.

QED is the simplest prototype of gauge field theories, in which there exist a class of so-called ”gauge transformations” which do not affect physical observables (classically). The notion of a gauge transformation will be introduced later and it will become the main theme (idea) in the second and third parts of this book.

In a short time span of ten years starting from 1976, Nobel prizes have been awarded three times to discoveries related to the GSW electroweak theory: for the discovery of the charm quark, for the successes of the GSW theory, and for the experimental observation of $W^\pm$ and $Z^0$. 
What is troubling the theoretical physicists over almost a whole century is the occurrence of ultraviolet divergences - see Chapter 10 for example. If we "believe" our Standard Model, the infinite parts should cancel out if all ultraviolet divergences of the same characteristics all are taken account. The answer to this question might be on the positive side.

In Part B of the present volume, we shall present in some detail quantum electrodynamics, the simplest prototype of all quantized gauge field theories. We also describe the conventional standard model, i.e., QCD and the GSW electroweak theory, and the experimental tests of it. In Part C, we move on to describe the Standard Model of the 21st century. We hope to conclude the book with the real Standard Model, the real final chapter.

5 Building Blocks of Matter

We shall for convenient reference begin with a qualitative summary concerning the subatomic and atomic world. In the minimal Standard Model, building blocks of matter are known to include (a) three generations of fermions, (b) mediators of fundamental interactions, and (c) scalar particles which are responsible for spontaneous symmetry breaking related to the physical vacuum. Specifically, fermions consist of leptons, quarks, and their antiparticles:

**Leptons:**
\[
(e^-, \nu_e), (\mu^-, \nu_\mu), (\tau^-, \nu_\tau), \text{ (columns).} \tag{7}
\]

**Quarks:**
\[
\begin{pmatrix}
(u_R, d_R), & (c_R, s_R), & (t_R, b_R), \\
(u_Y, d_Y), & (c_Y, s_Y), & (t_Y, b_Y), \\
(u_B, d_B), & (c_B, s_B), & (t_B, b_B).
\end{pmatrix} \text{ (columns)} \tag{8}
\]

Note that leptons include electrons \(e^-\), muons \(\mu^-\), tau-leptons \(\tau^-\), electronlike neutrino \(\nu_e\), and so on. Quarks come in with six possible flavors: up \(u\), down \(d\), charm \(c\), strange \(s\), bottom \(b\), and top \(t\). A quark of any given flavor is assumed to carry one of three possible colors: red \(R\), yellow \(Y\), blue \(B\), or, \(x, y, \text{ and } z\) with
\[
x = (1, 0, 0), \quad y = (0, 1, 0), \quad z = (0, 0, 1), \text{ (columns).} \tag{9}
\]

Quarks are not observed in isolation presumably because color is strictly confined, a property consistent with the conjectured two-phase picture of QCD.

Mediators of fundamental interactions include (1) the photon \(\gamma\), which mediates the well-known electromagnetic interaction, (2) three weak bosons \(W^\pm, Z^0\), which mediate charged and neutral weak interactions, and (3) eight gluons, which mediate strong interactions among quarks and antiquarks. Gluons carry one of eight possible octet colors and cannot exist in isolation because of color confinement.

**Hadrons**, or strongly interacting elementary particles, are by assumption color-singlet, or colorless, composites of quarks, antiquarks, and gluons. The hadrons include mesons, baryons, glueballs, and so on. Pions \(\pi^\pm, \pi^0\), kaons \(K^\pm, K^0, \bar{K}^0\), etas \(\eta, \eta'\), rho-mesons
(ρ⁺, ρ⁰), ψ/J, and upsilons (Υ, Υ', ....) all are mesons which, at low energies, are believed to be quark-antiquark pairs confined to within the region defined by the meson size. Nucleons (p, n), lambda (Λ), sigmas (Σ⁺, Σ⁰), xi’s (Ξ−, Ξ⁰), deltas (Δ−, Δ⁰, Δ⁺, Δ++), charmed lambda (Λc), and bottomed lambda (Λb) all are baryons which, at low energies, look like systems of three quarks confined to within the region defined by the baryon size. Glueballs are colorless objects consisting of gluons only.

Nucleons, i.e., protons (p) and neutrons (n), are believed to be primary building blocks of the nuclei of the various atoms, ranging from the proton itself [as the simplest nucleus], to the deuteron, α, ¹²C, ⁵⁶Fe, ²⁰⁸Pb, and even to neutron stars [A = ∞ nuclei]. Replacement of a nucleon in an ordinary nucleus by a lambda (Λ) or by a sigma-baryon (Σ±, Σ⁰) results in a hypernucleus. An atom is a system of electrons around an ordinary nucleus, the whole system being often electrically neutral. Molecules are built from atoms. All matter observed terrestrially or celestially are believed to be composed of building blocks conjectured in the standard model.

The GSW SU(2)W × U(1) electroweak theory invokes the so-called "Higgs mechanism", in which the physical vacuum, or the true ground state, differs from the trivial vacuum [where expectation values of all fields vanish identically]. This theory predicts, among other things, the existence of a Higgs particle, which has finally been observed in 2012. Historically, many other important predictions of the GSW electroweak theory have been substantiated experimentally, including the existence of: (1) neutral weak interactions, (2) the charm quark, and (3) weak bosons W± and Z⁰. It is very likely that any new theory which is beyond the standard model must reproduce or explain the successes of the standard model and so will contain the standard model as a limiting case.

Up to the moment of writing, building blocks of matter, as conjectured in the standard model, appear to be structureless [or, less precisely, pointlike] at the highest energy scale (or the smallest distance scale) which we are capable of probing. Qualitatively speaking, quarks and leptons can be described by Dirac equations of some sort while mediators of fundamental interactions (spin-1 particles) and the Higgs particle (spin-0 particle) are described, respectively, by gauge field theories and a generalized Klein-Gordon equation. It is clear that the presentation of relativistic quantum mechanics in Part A is most relevant in the subatomic world. Indeed, it is not clear at all whether a Dirac equation will ever be relevant in the description of a composite spin-¹⁄² system such as a proton or a neutron.

6 The Standard Model of the 21st Century

In the year of 2012, the Standard Higgs particle is finally discovered at CERN, Geneva, Switzerland. Subsequently in 2013, the Nobel prize was given to Englert and Higgs for their realization of the Higgs mechanism (or, the BEH mechanism).⁵

There are two important, and basic, implications involved here. After searching for the Higgs or point-like scalar particles for forty years (i.e. almost half a century), the only thing found is the Standard-Model Higgs particle, which represents some constrained existence

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⁵ The Universe, Vol. 1, No. 4, the Nobel Issue.
of the scalar particles (as described by the Klein-Gordon equation). The constraint for the real world, which is difficult to spell out, amounts to the "minimum Higgs hypothesis".

The important implication is that the story should end at the proper extended Standard Model - particularly, why there are three generations? Basically, the changes among the three generations already happen in neutrino oscillations. This pushed one of us to propose the real Standard Model. The Lamb's joke regarding why there is the second electron (the muon) is indeed there. This indicates that the family idea could be viewed as another family gauge theory.

Therefore, we would like to add the 3rd pillar to the 20th-century modern physics so that we have the 21st-century physics - all building blocks of matter are point-like particles and all the spin-1/2 particles are point-like Dirac particles; referred to as "Dirac similarity principle" to reckon Dirac's invention of the Dirac electron, no size description of the Dirac electron. It seems that all spin-1/2 building blocks of matter, including charged or neutral leptons and quarks of various kinds, follow the route of the Dirac electron.

Maybe we could add another one pillar - the fourth pillar of the 21st Century. Our world seems to know the handedness of these point-like Dirac particles - e.g., it treats the left-handed electron very differently from the right-handed electron. So, the basic units of matter, which treat the right-handed Dirac particles differently from the left-handed Dirac particles, are more appropriate than the so-called "building blocks of matter".

7 The Origin of Mass

Suppose that, before the spontaneous symmetry breaking (SSB), the Standard Model does not contain any parameter that is pertaining to "mass", but, after SSB, all particles in the Standard Model acquire the mass terms as it should — we call it "the origin of mass".

At high enough temperature such at the early Universe, all the mass terms are negligible and so set the stage of the mass generation - as before the mechanism for the origin of mass turns on.

Originally, all the particles, fermions and bosons, are massless. According to the origin of mass, something "ignites" the SSB in the pure family Higgs sector; the SSB in the electroweak sector as well as in the mixed family sector all came out as induced as a result. The Standard-Model Higgs mass would be related to its (electroweak) vacuum-expectation-value (VEV) by a simple factor of two. (Either refers to the articles in "The Universe", or consult with Chapter 14 or later.)

We should remind ourselves of the fact that the generalized Higgs mechanism is "ignited" in the purely family sector $\Phi(3,1)$, but not in the electroweak sector $\Phi(1,2)$ (like in the minimal Standard Model). We believe that there is only one "ignition" point, though in principle there could be more than one point.

Owing to the elegance of the origin of mass, we should settle on the thinking and thus move on by treating the mass as indicated.

This is one basic question of physics, which we should try to answer whenever we can (since we learnt "general physics"). We think that the developments of the Standard Model...
in particle physics are at the point close to actually answering this basic question. Keep in
tune on this question when we are in progresses on this textbook.

8 The Origin of Fields (Point-like Particles)

In our world, i.e., the quantum 4-dimensional Minkowski space-time with the force-fields
gauge group \( SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3) \) built-in from the outset, we realize that
only the Higgs fields \( \Phi(1, 2) \) (the Standard Higgs), \( \Phi(3, 2) \) (the mixed family Higgs), and
\( \Phi(3, 1) \) exist and only they could exist. Here the first label is for \( SU_f(3) \) while the second
for \( SU_L(2) \).

The quark world, having the (123) symmetry, is acceptable by our world while the lepton
world, having another (123) symmetry, is also acceptable by our world. This (123) sym-
metry, or nontrivial under \( SU_c(3) \times SU_L(2) \times U(1) \), makes the quark world asymptotically
free and free of Landau’s ghost.

The magic comes from that, for complex scalar fields \( \phi(x) \), the interaction terms \( \lambda(\phi^\dagger \phi)^2 \)
are repulsive and renormalizable but that, for two “related” complex scalar fields, the
”attractive” interaction \(-2\lambda(\phi_a^\dagger \phi_b) \cdot (\phi_b^\dagger \phi_a) \) could overcome the repulsiveness, to rewrite
the story. The value of the universal \( \lambda \) is determined by the 4-dimensional nature on the
Minkowski space-time, not by the individual field itself. For this magic, we write the story
in ”The Universe”. Altogether, we call it the origin of fields (point-like particles) [?].

So, in view of the repulsive nature of the self-interaction \( \lambda(\phi^\dagger \phi)^2 \), the complex scalar
field \( \phi \) cannot exist by itself. The three related complex scalar fields \( \Phi(1, 2) \), \( \Phi(3, 2) \), and
\( \Phi(3, 1) \) can co-exist, to couple with the force-fields gauge fields (and to make them massive,
if necessary), thus making the whole story.

We believe that the description of point-like (Dirac or Higgs) particles in terms of fields
may be very fundamental, indeed. This point is something against what some of us venture
out for alternate options (such as superstrings).

9 Prelude to Relativistic Quantum Mechanics and Quantum
Fields

When we treat a hydrogen atom, as the example, we have to separate the center-of-mass
(CM) motion from the internal (relative) motion, exhibiting the nontrivial characteristics
associated with the internal motion. The atomic size of the anstron \((10^{-8} \text{ cm})\) scale is
thus showing up. The Einstein’s relativity principle in the Minkowski space-time is mostly
associated with the CM motion.

QCD makes the systems to much smaller in size, about the Fermi \((10^{-13} \text{ cm})\) scale. The
other size making, rather than the atomic size making, can also be understood in terms
of the CM versus internal separation. What is in between is the quantum 4-dimensional
Minkowski space-time, which allows us to chop the space-time at different scales.

Quantum fields are to be used in, e.g., electrodynamics (in Chapters 9 and 10) or the
Standard Model (after Chapter 12 and more). Quantum fields manifest in our real world.
Hopefully, the Standard Model which we are talking about is not only consistent but also complete. Then, we have something in common jointly in mathematics and physics. The commonness is shared by jointly in mathematics in our thinkings (and in the symbolic logic) and in (physical) observations in our real world.

Appendix: Natural Units

The brief summary in the preceding section concerning the building blocks of matter should have made it clear that a knowledge of relativistic quantum mechanics and quantum fields is most relevant in the area of elementary particle and nuclear physics. Since the kinetic energy of a particle under investigation is often more important than its rest mass in a typical particle or nuclear physics problem, it is convenient to measure a given velocity in units of the light velocity $c$.

$$c = 2.9979 \times 10^{10} \text{ cm/sec.} \quad (10)$$

Similarly, it is convenient to express the action in units of $\hbar c$:

$$\hbar c = 197.33 \text{ MeV} - \text{fm}, \quad (11)$$

where $1 \text{ fm} \equiv 10^{-13} \text{ cm}$. The remaining units can be chosen as powers of MeV (or GeV) or as powers of fm or as powers of seconds. For instance, a delta width of 110 MeV corresponds to

$$\frac{110 \text{ MeV}}{\hbar c} \cdot 10^{13} \text{ fm/cm} \cdot c$$

$$= 110 \text{ MeV} \cdot (197.33 \text{ MeV} - \text{fm})^{-1} \cdot 10^{13} \text{ fm/cm} \cdot 2.9979 \times 10^{10} \text{ cm/sec}$$

$$= \{5.98 \times 10^{-24} \text{ sec}\}^{-1}, \quad (12)$$

or to a very short lifetime of $5.98 \times 10^{-24} \text{ sec}$. The system in which quantities are measured in units of $c$ and $\hbar c$ is referred to as "natural units". Customarily, one writes

$$\hbar = c = 1. \quad (13)$$

For problems of present-day particle and nuclear physics, adoption of natural units leads to equations which look simpler than those obtained in ordinary units, although the physical content remains the same. Natural units are used in the present volume. The situation may be considered as different from ordinary quantum mechanics where the role played by $\hbar$ should always be emphasized.

Finally, we note that the metric used by us (Wu and Hwang) is the same as that by W. Pauli, T. D. Lee, H. Primakoff, and others. R. P. Feynman used both notations and vice versa. This is the so-called "old-fashioned" notations. This choice involves the hermitian gamma matrices on the Dirac equation. In fact, one of us (Hwang) began on the notations

\footnote{Note that we do not quote in this book all the significant figures of many basic constants which have been measured with great precision.}
by Bogoliubov but, later, switched to (the mentor) Henry Primakoff - and fixed from there on. The choice of notations does not affect the physics which we are learning. When there is a need to differentiate the upper and lower indices (such as equations in general relativity), we will make it clear when necessary - mainly in a couple of appendices throughout this textbook.

\[ g_{\mu\nu} = \delta_{\mu\nu}; \quad \mu, \nu \in (1, 2, 3, 4). \]  

(14)

For instance, the four-momentum of a particle is specified by

\[ p_\mu = (p, iE), \quad \text{with} \quad p_4 = i p_0 = iE. \]  

(15)

The inner product of two four-vectors \( A_\mu \) and \( B_\mu \) is given by

\[ A \cdot B = A_\mu B_\mu = A \cdot B - A_0 B_0. \]  

(16)

The only complication arises when one tries to take the complex conjugate of a complex four-vector. For instance, we have

\[ \xi^*_\mu = (\xi^*, i\xi^*_0), \]  

(17)

for a complex polarization four-vector \( \xi_\mu = (\xi, i\xi_0) \), so that \( \xi^* \cdot p = \xi^* \cdot p - \xi^*_0 p_0 \). We shall write out the expressions explicitly in the case that some confusion may arise.

As the other final remark, we note that equations and results in the previous volume entitled "Quantum Mechanics" will be used occasionally in this book. As such referencing occurs, we shall use notations such as "(VIII-58), Vol. I" or "p. 12, Vol. I" where "(VIII-58)" is the equation number and "p.12" indicates the page number.

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*As a convention, we use a bold-faced Roman letter or a Greek letter to denote a three-vector in configuration space. The arrow is reserved only for a vector in isospin space or in other internal space.

9To benefit readers in the area of atomic physics, certain equations (up to Chapter 8) are written in ordinary (m.k.s.a.) units as footnotes with primed equation numbers.

10Henceforth referred to as "Vol. I", authored by one of us, T.-Y. Wu, and published by World Scientific, 1986.
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