Systematization of tensor mesons and the determination of the 2++ glueball

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Abstract

It is shown that new data on the \((J^{PC} = 2^{++})\)-resonances in the mass range \(M \sim 1700 – 2400\) MeV support the linearity of the \((n, M^2)\)-trajectories, where \(n\) is the radial quantum number of quark–antiquark state. In this way all vacancies for the isoscalar tensor \(q\bar{q}\)-mesons in the range up to 2450 MeV are filled in. This allows one to fix the broad \(f_2\)-state with \(M = 2000 \pm 30\) MeV and \(\Gamma = 530 \pm 40\) MeV as the lowest tensor glueball.

PACS numbers: 14.40.-n, 12.38.-t, 12.39.-Mk

Recent analysis of the process \(\gamma\gamma \to K_S K_S\) [1] and re-analysis of \(\phi\phi\)-spectra [2] observed in the reaction \(\pi^- p \to \phi\phi n\) [3] have clarified the situation with \(f_2\)-mesons in the mass region 1700 – 2400 MeV. Hence, now one may definitely speak about the location of \(q\bar{q}\)-states on the \((n, M^2)\)-trajectories [4], see also [5, 6]. This fact enables us to determine which one of \(f_2\)-mesons is an extra state for the \((n, M^2)\)-trajectories. Such an extra state is the broad resonance \(f_2(2000 \pm 30)\). According to [2, 7, 8], its parameters are as follows:

\[
M = 2050 \pm 30\ \text{MeV}, \quad \Gamma = 570 \pm 70\ \text{MeV} \ [2],
M = 1980 \pm 20\ \text{MeV}, \quad \Gamma = 520 \pm 50\ \text{MeV} \ [7],
M = 2010 \pm 25\ \text{MeV}, \quad \Gamma = 495 \pm 35\ \text{MeV} \ [8].
\]

In [4], we have put quark–antiquark meson states with different radial excitations \((n = 1, 2, 3, 4, \ldots)\) on the \((n, M^2)\)-trajectories. With a good accuracy, the trajectories occurred to be linear:

\[
M^2 = M_0^2 + (n - 1)\mu^2,
\]

with a universal slope \(\mu^2 = 1.2 \pm 0.1\ \text{GeV}^2\); \(M_0\) is the mass of the lowest (basic) state. For the \((I = 0, J^{PC} = 2^{++})\)-mesons, the present status of trajectories (i.e. with the results given by [1, 2]) are shown in Fig. 1.

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The quark states with \((I = 0, J^{PC} = 2^{++})\) are defined by two flavour components, \(n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}\) and \(s\bar{s}\), with \(^{2S+1}L_J = 3P_2, 3F_2\). Generally, all mesons are the mixture of flavour component in the \(P\)- and \(F\)-waves. But, as concern the \(f_2\)-mesons with \(M \lesssim 2\) GeV, they are dominated by the flavour component \(n\bar{n}\) or \(s\bar{s}\) in a definite \(P\) or \(F\) wave. The \(f_2\)-mesons shown in Fig. 1, which belong to four trajectories, are dominated by the following states:

\[
\begin{align*}
&\left[ f_2(1275), f_2(1580), f_2(1920), f_2(2240) \right] \rightarrow 3P_2n\bar{n}, \\
&\left[ f_2(1525), f_2(1755), f_2(2120), f_2(2410) \right] \rightarrow 3P_2s\bar{s}, \\
&\left[ f_2(2020), f_2(2300) \right] \rightarrow 3F_2n\bar{n}, \\
&f_2(2340) \rightarrow 3F_2s\bar{s}.
\end{align*}
\]

To avoid the confusion, in (3) the experimentally observed masses of mesons are shown — these are the magnitudes drawn in Fig. 1 but not those from the compilation [9].

Let us discuss the states which lie on the trajectories of Fig. 1.

**The trajectory \([f_2(1275), f_2(1580), f_2(1920), f_2(2240)]\)**

1) \(f_2(1275)\): This resonance is almost pure \(1^3P_2n\bar{n}\)-state: this is favoured by the comparison of branching ratios \(f_2(1275) \rightarrow \pi\pi, \eta\eta, K\bar{K}\) with quark model calculations. The dominance of \(1^3P_2n\bar{n}\) component is also supported by the value of partial width of the decay \(f_2(1275) \rightarrow \gamma\gamma\) [10, 11].

2) \(f_2(1580)\) (in the compilation [9] it is denoted as \(f_2(1565)\)): About ten years ago, there existed a number of indications to the presence of the \(2^{++}\)-mesons in the vicinity of 1500 MeV [12, 13, 14, 15]. After the discovery of a strong signal in the \(0^{++}\)-wave related to the \(f_0(1500)\) [16, 17] as well as correct account for the interference of \(0^{++}\) and \(2^{++}\) waves, the resonance signal in the \(2^{++}\) wave moved towards higher masses, \(\sim 1570\) MeV. According to the latest combined analysis of meson spectra [6, 18], this resonance has the following characteristics (see Table 1 in [6]):

\[
M = 1580 \pm 6\text{ MeV} , \quad \Gamma = 160 \pm 20\text{ MeV} .
\]

Hadronic decays together with partial width in the channel \(\gamma\gamma\) [10] support the \(f_2(1580)\) as a system with dominant \(n\bar{n}\)-component.

In [9], the \(f_2(1640)\)-state is marked as a separate resonance: this identification is based on resonance signals at \(M = 1620 \pm 16\text{ MeV}\) [19] (Mark 3 data for \(J/\Psi \rightarrow \gamma\pi^+\pi^-\pi^+\pi^-\)), \(M = 1647 \pm 7\text{ MeV}\) [13] (reaction \(\bar{n}p \rightarrow 3\pi^+2\pi^-\)), \(M = 1590 \pm 30\text{ MeV}\) [20], \(1635 \pm 7\text{ MeV}\) [21] (reaction \(\pi^-p \rightarrow \omega\omega n\)). Without doubt, these signals are the reflections of \(f_2(1580 \pm 20)\), and the data [19, 20] do not contradict this fact. In [9], the mass of this state is determined as \(1638 \pm 6\text{ MeV}\) that reflects small errors in the mass definition in [13, 21].

3) \(f_2(1920)\) (in the compilation [9], it is denoted as \(f_2(1910)\)): This resonance was observed in the signals \(\omega\omega\) [20, 21, 22] and \(\eta\eta'\) [23, 24]. In [8], the \(f_2(1920)\) is seen as a shoulder in the
$p\bar{p}(I = 0, C = +1) \to \pi^0\pi^0, \eta\eta, \eta\eta'$ spectra, in the wave $^3P_2p\bar{p}$. According to [6, 18],

$$M = 1920 \pm 40 \text{ MeV}, \quad \Gamma = 260 \pm 40 \text{ MeV}.$$ (5)

A strong signal in the channels with nonstrange mesons surmises a large $n\bar{n}$ component in the $f_2(1920)$.

4) $f_2(2240)$: It is seen in the spectra $p\bar{p}(I = 0, C = +1) \to \pi^0\pi^0, \eta\eta, \eta\eta'$, in the wave $^3P_2p\bar{p}$ [8]. According to [6, 18]:

$$M = 2240 \pm 30 \text{ MeV}, \quad \Gamma = 245 \pm 45 \text{ MeV}.$$ (6)

The decay of $f_2(2240)$ into channels with nonstrange mesons makes it verisimilar the assumption about a considerable $n\bar{n}$ component.

5) The next radial excitation on the $^3P_2n\bar{n}$ trajectory ($n = 5$) is predicted at 2490 MeV.

**The trajectory [$f_2(1525), f_2(1755), f_2(2120), f_2(2410)$]**

This is the meson trajectory with dominant $s\bar{s}$-component. The states lying on this trajectory are the nonet partners of mesons from the first trajectory [$f_2(1275), f_2(1580), f_2(1920), f_2(2240)$]. This suggests a dominance of the $P$-wave in these $q\bar{q}$-systems: $^3P_2q\bar{q}$.

1) $f_2(1525)$: This is the basic state, ($n = 1$), the nonet partner of $f_2(1275)$. The mixing angle of $n\bar{n}$ and $s\bar{s}$ components, which can be determined neglecting the gluonium admixture,

$$f_2(1275) = n\bar{n}\cos \varphi_{n=1} + s\bar{s}\sin \varphi_{n=1},$$

$$f_2(1525) = -n\bar{n}\sin \varphi_{n=1} + s\bar{s}\cos \varphi_{n=1},$$ (7)

may be evaluated from the value of the partial widths $\gamma\gamma$ and ratios of the decay channels $\pi\pi$, $K\bar{K}$, $\eta\eta$ within the frame of quark combinatorics (see [5], Chapter 5 and references therein). Evaluations given in [1, 10] provide us the mixing angle as follows:

$$\varphi_{n=1} = -1^\circ \pm 3^\circ.$$ (8)

2) $f_2(1755)$: This state belongs to the nonet of the first radial excitation, $n = 2$, it is dominantly the $P$-wave $s\bar{s}$ state. The mixing angle $\varphi_{n=2}$ can be evaluated using the data on $\gamma\gamma \to K_S\bar{K}_S$. Neglecting a possible admixture of the glueball component, it was found [1]:

$$f_2(1580) = n\bar{n}\cos \varphi_{n=2} + s\bar{s}\sin \varphi_{n=2},$$

$$f_2(1755) = -n\bar{n}\sin \varphi_{n=2} + s\bar{s}\cos \varphi_{n=2},$$

$$\varphi_{n=2} = -10^\circ \pm 5^\circ -10^\circ.$$ (9)

3) $f_2(2120)$: This resonance was observed in the $\phi\phi$ spectrum in the reaction $\pi^-p \to n\phi\phi$ [3]. At small momenta transferred to the nucleon the pion exchange dominates, so we have the
transition $\pi\pi \rightarrow \phi\phi$. The $f_2(2120)$ resonance is seen in the $\phi\phi$ system in the $S$-wave with the spin 2 (the state $S_2$). According to [2], its parameters are as follows:

$$M = 2120 \pm 30 \text{ MeV} , \quad \Gamma = 290 \pm 60 \text{ MeV} , \quad W(S_2) \simeq 90\% \ ,$$

where $W(S_2)$ is the probability of the $S_2$-wave. The previous analysis [3], that did not account for the existence of the broad $f_2$-state around 2000 MeV, provided one the value $M \simeq 2010$ MeV, $\Gamma \simeq 200$ MeV [3], accordingly, this resonance was denoted as $f_2(2010)$ in [9]. At the same time, there is a resonance denoted in [9] as $f_2(2150)$, which was observed in the spectra $\eta\eta$, $\eta\eta'$, $K\bar{K}$, that assumes a large $s\bar{s}$-component:

| Channel         | $M$ (MeV) | $\Gamma$ (MeV) | $W(S_2)$ (%) | $W(D_0)$ (%) | $W(D_2)$ (%) |
|-----------------|-----------|-----------------|--------------|--------------|--------------|
| $\eta\eta$ [25]| $2151 \pm 16$ | $280 \pm 70$   | 90           | 20           | 30           |
| $\eta\eta'$ [26]| $2105 \pm 10$ | $200 \pm 25$   | 90           | 20           | 30           |
| $\eta\eta'$ [27]| $2104 \pm 20$ | $203 \pm 10$   | 90           | 20           | 30           |
| $K\bar{K}$ [28] | $2130 \pm 35$ | $270 \pm 50$   | 90           | 20           | 30           |

The re-analysis [2] points definitely to the fact that the resonances denoted in [9] as $f_2(2010)$ and $f_2(2150)$ are actually the same state.

4) $f_2(2410)$: It is seen in the reaction $\pi^-p \rightarrow n\phi\phi$ [3]. According to the re-analysis [2], its parameters are as follows:

$$M = 2410 \pm 30 \text{ MeV} , \quad \Gamma = 360 \pm 70 \text{ MeV} ,$$

$$W(S_2) \simeq 50\% , \quad W(D_0) \simeq 20\% , \quad W(D_2) \simeq 30\% .$$

If the contribution of the broad $f_2$-state in the region 2000 MeV is neglected, the resonance parameters move to smaller values: $M \simeq 2340$ MeV, $\Gamma \simeq 320$ MeV [3]; correspondingly, in [9] it was denoted as $f_2(2340)$.

5) The linearity of the $(n, M^2)$ trajectory predicts the next $^3F_2s\bar{s}$ state at 2630 MeV ($n = 5$).

The states with dominant $^3F_2n\bar{n}$ component

At the time being we may speak about the observation of the two states with the dominant $^3F_2n\bar{n}$-component.

1) $f_2(2020)$: It is seen in the partial wave analysis of the reactions $p\bar{p} \rightarrow \pi^0\pi^0, \eta\eta, \eta\eta'$, in the wave $^3F_2p\bar{p}$ [8]. According to [6, 18], its parameters are as follows:

$$M = 2020 \pm 30 \text{ MeV} , \quad \Gamma = 275 \pm 35 \text{ MeV} .$$

In [9], this meson was placed to the Section ”Other light mesons”, it is denoted as $f_2(2000)$. This is the basic $^3F_2$-meson ($n = 1$) with the dominant $n\bar{n}$-component.

2) $f_2(2300)$: It is seen in the partial wave analysis of the reaction $p\bar{p} \rightarrow \pi^0\pi^0, \eta\eta, \eta\eta'$, in the wave $^3F_2p\bar{p}$ [8]. According to [6, 18], its parameters are as follows:

$$M = 2300 \pm 35 \text{ MeV} , \quad \Gamma = 290 \pm 50 \text{ MeV} .$$
This is the first radial excitation of the $^3F_2$-state ($n = 2$), with dominant $n\bar{n}$-component. There is a resonance denoted in [9] as $f_2(2300)$, but this is the state observed in the $\phi\phi$-spectrum [3], the mass and width of which, in accordance with the re-analysis [2], are $2340 \pm 15$ MeV and $150 \pm 50$ MeV — of course, they are different states, see the discussion below.

3) The second radial excitation state ($n = 3$) on the trajectory $^3F_2n\bar{n}$ is predicted to be at $M \simeq 2550$ MeV.

The state with dominant $^3F_2s\bar{s}$ component

This trajectory is marked by one observed state only.

1) $f_2(2340)$: It is seen in the $\phi\phi$-spectrum [3] and $\gamma\gamma \rightarrow K^+K^-$ [29], with the mass $\sim 2330$ MeV and width $275 \pm 60$ MeV. According to [2],

$$M = 2340 \pm 15 \text{ MeV} , \quad \Gamma = 150 \pm 50 \text{ MeV} ,$$

$$W(S_2) \simeq 10\% , \quad W(D_0) \simeq 10\% , \quad W(D_2) \simeq 80\% .$$

(15)

In the previous analysis of the $\phi\phi$-spectrum [3], this resonance had the mass 2300 MeV, in [9] it is denoted as $f_2(2300)$.

2) The next state on the $^3F_2s\bar{s}$ trajectory ($n = 2$) should be located near $M \simeq 2575$ MeV.

The broad $2^{++}$-state near 2000 MeV — the tensor glueball

The averaging over parameters of the broad resonance using the data [2, 7, 8], see (1), gives us the following values:

$$M = 2000 \pm 30 \text{ MeV} , \quad \Gamma = 530 \pm 40 \text{ MeV} .$$

(16)

This broad state is superfluous with respect to $q\bar{q}$-trajectories on the $(n, M^2)$-plane, i.e. it is the exotics. It is reasonable to believe that this is the lowest tensor glueball. This statement is favoured by the estimates of parameters of the pomeron trajectory (e.g. see [5], Chapter 5.4, and references therein), according to which $M_{2^{++}\text{glueball}} \simeq 1.7 - 2.5$ GeV. Lattice calculations result in a close value, namely, $2.2 - 2.4$ GeV [30].

Another characteristic signature of the glueball is its large width, that was specially underlined in [31]. The matter is that exotic state accumulates the widths of its neighbours—resonances due to the transitions meson(1) → real mesons → meson(2).

Just this phenomenon took place with the lightest scalar glueball near 1500 MeV: the decay processes led to a strong mixing of the glueball with neighbouring resonances, so the glueball turned into the broad resonance $f_0(1200 - 1600)$ [32, 33, 34, 35], see also the discussion in [6]. Of course, the width of this scalar isoscalar state is rather large, though its precise value is poorly determined: $\Gamma \simeq 500 - 1500$ MeV. Although the accuracy in the determination of absolute value is low, the ratios of partial widths of this state to channels $\pi\pi, K\bar{K}, \eta\eta, \eta'$
are well defined [36]. So the ratios $\Gamma(\pi\pi) : \Gamma(K\bar{K}) : \Gamma(\eta\eta) : \Gamma(\eta\eta')$ tell us definitely that $f_0(1200-1600)$ is a mixture of the gluonium $(gg)$ and quarkonium $(q\bar{q})$ components being close to the flavour singlet $(q\bar{q})_{\text{glueball}}$. Namely,

$$gg \cos \gamma + (q\bar{q})_{\text{glueball}} \sin \gamma,$$

$$(q\bar{q})_{\text{glueball}} = n\bar{n} \cos \varphi_{\text{glueball}} + s\bar{s} \sin \varphi_{\text{glueball}}$$

with $\varphi_{\text{glueball}} = \arctan \sqrt{\lambda/2} \simeq 26^\circ - 33^\circ$. The mixing angle $\varphi_{\text{glueball}}$ is determined by the fact that the gluon field creates the light quark pairs with probabilities $u\bar{u} : d\bar{d} : s\bar{s} = 1 : 1 : \lambda$, and the probability to produce strange quarks $(q\bar{q})_{\text{glueball}}$ is suppressed $\lambda \simeq 0.5 - 0.85$ (see [37] and the discussion in Chapter 5 of [5]). The mixing angle $\gamma$ for gluonium and quarkonium components cannot be defined by the ratios $\Gamma(\pi\pi) : \Gamma(K\bar{K}) : \Gamma(\eta\eta) : \Gamma(\eta\eta')$ — it should be fixed by radiative transitions, for example, $\gamma \gamma \to f_0(1200-1600)$; such an experimental information is still missing. One may find a detailed discussion of the situation in [5, 6].

If the broad resonance $f_2(2000)$ is the tensor glueball, it must be also the mixture of components $gg$ and $(q\bar{q})_{\text{glueball}}$. Then the decay vertices of $f_2(2000) \to \pi\pi, K\bar{K}, \eta\eta, \eta\eta', \eta'\eta'$, $f_2(2000) \to \omega\omega, \rho\rho, K^*\bar{K}^*$, $\phi\phi, \phi\omega$ should obey the constraints shown in Table.

The decays $\text{glueball} \to \text{two q\bar{q}-mesons}$ may be realized through both planar quark–gluon diagrams and non-planar ones, the contribution from non-planar diagrams being suppressed in terms of the $1/N$-expansion [38]. One may expect that in the next-to-leading order the vertices are suppressed as $G_{NL}^P/G_{LP}^P \sim 1/10$, $G_{NL}^V/G_{LV}^V \sim 1/10$ — in any case such a level of suppression is observed in the decay of scalar glueball $f_0(1200-1600)$ [39]. Therefore, the next-to-leading terms are important for the channel $\text{glueball} \to \omega\phi$ only, for other channels they may be omitted.

In the Particle Data compilation [9] there is a narrow state $f_J(2220)$, with $J^{PC} = 2^{++}$ or $4^{++}$ and $\Gamma \simeq 23$ MeV, which is sometimes discussed as a candidate for tensor glueball, under the assumption $J = 2$ (see [40] and references therein). If this state does exist with $J = 2$, we see that there is no room for it on the $q\bar{q}$-trajectories shown in Fig. 1: in this case it should be also considered as an exotic state.

In the mean time there exist two statements about the value of glueball width: according to [41], it should be less than hadronic widths of $q\bar{q}$-mesons, while, following [6, 31], it must be considerably greater. The arguments given in [41] are based on the evaluation of the decay couplings in lattice calculations. However, such calculations does not take into account the large-distance processes, such as $\text{meson}(1) \to \text{real mesons} \to \text{meson}(2)$ in case of resonance overlapping. And just these transitions are responsible for the large width of a state which is exotic by its origin [31]. The phenomenon of width accumulation for meson resonances has been studied in [32, 33, 34, 35], but much earlier this phenomenon was observed in nuclear physics [42, 43, 44]. Therefore, I think that at present time just the large width of $f_2(2000)$ is an argument in favour of the glueball origin of this resonance. But to prove the glueball nature of $f_2(1200)$ the measurement of decay constants and their comparison to the ratios given in Table is needed.

I am grateful to L.G. Dakhno, S.S. Gershtein, V.A. Nikonov and A.V. Sarantsev for stimulating discussions, comments and help. The paper was supported by the Russian Foundation
Table 1: The constants of the tensor glueball decay into two mesons in the leading (planar diagrams) and next-to-leading (non-planar diagrams) terms of $1/N$-expansion. Mixing angles for $\eta - \eta'$ and $\omega - \phi$ mesons are defined as follows: $\eta = n\bar{n}\cos \theta - s\bar{s}\sin \theta$, $\eta' = n\bar{n}\sin \theta + s\bar{s}\cos \theta$ and $\omega = n\bar{n}\cos \varphi_V - s\bar{s}\sin \varphi_V$, $\phi = n\bar{n}\sin \varphi_V + s\bar{s}\cos \varphi_V$. Because of the small value of $\varphi_V$, we keep in the Table the terms of the order of $\varphi_V$ only.

| Channel | Constants for glueball decays in the leading order of $1/N$ expansion | Constants for glueball decays in next-to-leading order of $1/N$ expansion | Identity factor for decay products |
|---------|---------------------------------------------------------------------|-------------------------------------------------------------------------|----------------------------------|
| $\pi^0\pi^0$ | $G_P^L$ | 0 | $1/2$ |
| $\pi^+\pi^-$ | $G_P^L$ | 0 | 1 |
| $K^+K^-$ | $\sqrt{\lambda}G_P^L$ | 0 | 1 |
| $K^0\bar{K}^0$ | $\sqrt{\lambda}G_P^L$ | 0 | 1 |
| $\eta \eta$ | $G_P^L(\cos^2 \theta + \lambda \sin^2 \theta)$ | $2G_P^{NL}(\cos^2 \theta - \sqrt{\lambda} \sin^2 \theta)^2$ | $1/2$ |
| $\eta \eta'$ | $G_P^L(1 - \lambda) \sin \theta \cos \theta$ | $2G_P^{NL}(\cos \theta - \sqrt{\lambda} \sin \theta) \times (\sin \theta + \sqrt{\lambda} \cos \theta)$ | 1 |
| $\eta' \eta'$ | $G_P^L(\sin^2 \theta + \lambda \cos^2 \theta)$ | $2G_P^{NL}(\sin \theta + \sqrt{\lambda} \cos \theta)^2$ | $1/2$ |
| $\rho^0\rho^0$ | $G_V^L$ | 0 | $1/2$ |
| $\rho^+\rho^-$ | $G_V^L$ | 0 | 1 |
| $K^{*+}K^{*-}$ | $\sqrt{\lambda}G_V^L$ | 0 | 1 |
| $K^{*0}\bar{K}^{*-0}$ | $\sqrt{\lambda}G_V^L$ | 0 | 1 |
| $\omega \omega$ | $G_V^L$ | $2G_V^{NL}$ | $1/2$ |
| $\omega \phi$ | $G_V^L(1 - \lambda) \varphi_V$ | $2G_V^{NL}(\sqrt{\lambda} + \varphi_V (1 - \lambda) / 2)$ | 1 |
| $\phi \phi$ | $\lambda G_V^L$ | $2G_V^{NL}(\lambda / 2 + \sqrt{2} \lambda \varphi_V)$ | $1/2$ |
for Basic Research, project no. 04-02-17091.

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Figure 1: The $f_2$ trajectories of on the $(n, M^2)$ plane; $n$ is radial quantum number of $q\bar{q}$ state. The numbers stand for the experimentally observed $f_2$-meson masses $M$. 