Supersymmetric instantonic D1-branes in AdS$_5 \times$ S$^5$ background

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Abstract: According to the covariant open superstring description of 1/2-BPS D-branes in the AdS$_5 \times$ S$^5$ background, there are two kinds of purely instantonic D-branes. One is the well known D(-1)-brane or D-instanton, and another is the D1-brane spanning a two dimensional subspace inside S$^5$. We identify the actual 1/2-BPS instantonic D1-brane configurations and their supersymmetry structures. From the results, it is concluded that any spherical D1-instanton with the radius equal to that of S$^5$ is 1/2-BPS. After evaluating the 1/2-BPS D1-instanton action, we discuss the relation of D1-instanton with the (p,q) string instanton. We also discuss the possible 1/2-BPS and 1/4-BPS multiple D1-instanton configurations.

Keywords: D-Branes, AdS-CFT Correspondence, Extended Supersymmetry

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1 Introduction

The covariant open superstring description of D-branes [1, 2] has been a useful way in classifying supersymmetric D-branes, especially 1/2-BPS D-branes, in a given supersymmetric background. It has been successfully applied to some important backgrounds in superstring theories such as the flat spacetime [1], IIB plane wave [2], IIA plane wave [3], AdS$_5 \times$ S$^5$ [4–7], and AdS$_4 \times$ CP$^3$ [8] backgrounds.

The data obtained after the classification of supersymmetric D-branes are however ‘primitive’ in a sense that they do not tell us about which configuration of a given D-brane is really supersymmetric and which part of the background supersymmetry is preserved on the D-brane worldvolume. If the background spacetime geometry is for example given by a product of some subspaces, the only information contained in the data is the possible numbers of Neumann directions of open superstring end point in each of the subspaces. In spite of this fact, the data provide us an efficient guideline for further exploration of supersymmetric D-branes, which allows us to avoid a brute force approach. Indeed, in our previous work [9], this has been illustrated in the investigation of 1/2-BPS D-brane configurations in the AdS$_5 \times$ S$^5$ background.

The work of [9] has been based on the result of covariant open superstring description [4–7] given in table 1 and focused on the Lorentzian D-branes. By the way, as one can see from the table, there are two purely Euclidean or instantonic D-branes. One is the well known D(-1)-brane or D-instanton [10–12], which has played an important role in the study of nonperturbative aspects of IIB superstring theory in the AdS$_5 \times$ S$^5$ background. Another is the D1-brane labeled by (0,2) which spans a two dimensional subspace inside S$^5$.

As of now, compared to the D(-1)-brane, the 1/2-BPS instantonic D1-brane looks somewhat exceptional and its role or nature is not obvious. Since instantons are always important in understanding the nonperturbative structure of a given theory, it is certain that such object seems to deserve to be investigated for further development of superstring
Table 1. 1/2-BPS D-branes in the AdS$_5$×S$^5$ background. $n$ ($n'$) represents the number of Neumann directions in AdS$_5$ ($S^5$).

| $(n,n')$ | D(-1) | D1 | D3 | D5 | D7 | D9 |
|----------|-------|----|----|----|----|----|
| (0,0)    | (2,0) | (3,1)| (4,2)| (5,3)|   |    |
| (0,2)    | (1,3) | (2,4)| (3,5)|   |    |    |
| (3,1)    | (4,2) | (5,3)| (6,4)|   |    |    |

theory in the AdS$_5$×S$^5$ background. In this paper, having the information (0,2) of table 1, we will try to identify the 1/2-BPS instantonic D1-brane configurations and their supersymmetry structures. Partly, the present work may be regarded as the continuation of the study carried out in [13] where 1/2-BPS instantonic membrane configurations in the AdS$_4$×S$^7$/Z$_k$ background have been successfully classified.

In the next section, we give a brief description of the AdS$_5$×S$^5$ background and its Killing spinor. The Euclidean action for D1-brane with its basic structure is discussed in section 3. The identification of 1/2-BPS instantonic D1-brane configurations with their supersymmetric structures is worked out in section 4. In section 5, we evaluate the action values of the 1/2-BPS configurations and try to show the relation between D1-instanton and $(p,q)$ string instanton action based on [14, 15]. Finally, we discuss the possibility of supersymmetric multiple D1-instantons in section 6.

2 AdS$_5$×S$^5$ background and Killing spinor

In this section, we briefly describe the AdS$_5$×S$^5$ background and its Killing spinor. In the Poincaré patch coordinate system, the metric of the AdS$_5$×S$^5$ geometry is given by

$$ds^2 = g_{\mu\nu}dX^\mu dX^\nu = \frac{R^2}{x^2} \left[ -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 + (dx^4)^2 + dz^2 \right] + ds^2_{S^5},$$  \hspace{1cm} (2.1)

where $ds^2_{S^5}$ is the metric of five sphere $S^5$ of radius $R$. As usual, the common radius $R$ is written as

$$R^4 = 4\pi g_s N \ell_s^4,$$  \hspace{1cm} (2.2)

where $g_s$ is the string coupling constant, $\ell_s$ the string length scale, and $N$ the number of D3-branes leading to the above geometry in the near-horizon limit. As for the five sphere part, one may parametrize it by using the usual four polar angles and one azimuthal angle. However, because instantonic D1-brane configurations spanning two dimensional subspace inside $S^5$ are of our interest, it is more convenient to take another parametrization in which two dimensional substructures are manifest as follows:

$$ds^2_{S^5} = R^2 \left[ d\alpha^2 + \cos^2 \alpha (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \sin^2 \alpha (d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2) \right],$$  \hspace{1cm} (2.3)

where the ranges of angles are $0 \leq \alpha \leq \pi/2$, $0 \leq \theta_{1,2} \leq \pi$, and $0 \leq \phi_{1,2} \leq 2\pi$. 

- 2 -
From the metric (2.1) with (2.3), we choose the zehnbein to be

\[
\begin{align*}
e^0 &= \frac{1}{z} dx^0, \\
e^1 &= R d\alpha, \\
e^2 &= R \cos \alpha d\theta^1, \\
e^3 &= R \cos \alpha \sin \theta^1 d\phi^1, \\
e^4 &= R \sin \alpha d\theta^2, \\
e^5 &= R \sin \alpha \sin \theta^2 d\phi^2.
\end{align*}
\]

(2.4)

Here, we have adopted hatted numbers and \( z \) as the tangent space index values for the AdS\(_5\) part to distinguish them from those of the S\(_5\) part. With this convention, the ten dimensional tangent space index (denoted by \( A, B, \ldots \)) is denoted as

\[
A = (m, z, a), \quad m = \hat{0}, \hat{1}, \hat{2}, \hat{3}, \quad a = 1, 2, 3, 4, 5,
\]

(2.5)

and the Ramond-Ramond (RR) five-form field strength, another constituent of the AdS\(_5\)xS\(_5\) background in addition to the metric (2.1), is written as

\[
F_5 = 4 e^0 \wedge e^1 \wedge e^2 \wedge e^3 \wedge e^z + 4 e^1 \wedge e^2 \wedge e^3 \wedge e^4 \wedge e^5.
\]

(2.6)

The AdS\(_5\)xS\(_5\) background composed of (2.1) and (2.6) with (2.3) is maximally supersymmetric. Its supersymmetry structure is encoded in the solution of the spacetime Killing spinor equation. The Killing spinor equation itself for the AdS\(_5\)xS\(_5\) background is given by

\[
\left( \nabla \delta^{IJ} + \frac{1}{2R} e^{A} \Gamma_{A} \Gamma_{B} \right) \eta^J = 0,
\]

(2.7)

where \( \nabla = d + \frac{1}{4} \omega^{AB} \Gamma_{AB} \),

\[
\tau_1 = \sigma_1, \quad \tau_2 = i \sigma_2,
\]

(2.8)

(\( \sigma_{1,2} \) are the usual Pauli matrices.) and

\[
\hat{\gamma} \equiv \Gamma^{0123}.
\]

(2.9)

The spacetime Killing spinors \( \eta^I \) (\( I = 1, 2 \)) as the solution of (2.7) are two Majorana-Weyl spinors and taken to have ten dimensional positive chirality in this work, \( \Gamma^{11} \eta^I = \eta^I \).

As demonstrated in [16, 17], the Killing spinor equation (2.7) is solved rather easily if we split \( \eta^I \) as

\[
\eta^I = \eta^I_+ + \eta^I_-,
\]

(2.10)

where \( \eta^I_\pm \) are defined by

\[
\eta^I_\pm = P^I_\pm \eta^J
\]

(2.11)

with the projection operator

\[
P^I_\pm = \frac{1}{2} (\delta^I_J \pm \hat{\gamma} \tau^I_2 J).
\]

(2.12)

In this splitting, we note that \( \eta^I_\pm \) and \( \eta^I_\pm \) are not independent from each other because

\[
\eta^I_2 = \mp \hat{\gamma} \eta^I_1.
\]

(2.13)
Thus, to avoid this redundancy, it is convenient to define
\[ \eta_\pm \equiv \eta_1^\pm, \quad (2.14) \]
to which \( \eta_1^+ \) and \( \eta_2^- \) are related by
\[ \eta_1^+ = \eta_+ + \eta_-, \quad \eta_2^- = -\hat{\gamma}(\eta_+ - \eta_-). \quad (2.15) \]
If we now use \( \eta_\pm \), the solution of the Killing spinor equation (2.7) is obtained as
\[ \eta_+^+ = z^{-1/2}U(\epsilon_+ + \Gamma_m x^m \epsilon_-), \]
\[ \eta_-^+ = z^{1/2}\Gamma_z U\epsilon_-, \quad (2.16) \]
where \( \epsilon_\pm \) are constant spinors and \( U \) is a spinorial function of five angles of \( S^5 \).

The function \( U \) satisfies
\[ \left( d + \frac{1}{4}\omega^{ab}\Gamma_{ab} + \frac{1}{2R}e^a\Gamma_{za}\right)U = 0, \quad (2.17) \]
which can be read off from the Killing spinor equation (2.7). If we took the standard parametrization of \( S^5 \), we could simply adopt the solution of this equation obtained in [18]. However, since we take a different parametrization (2.3), we should solve the equation. Now the necessary ingredients for solving eq. (2.17) are the \( S^5 \) part of zehnbein (2.4) and the corresponding spin connections computed as
\[ \omega^{12} = \sin \alpha d\theta_1, \]
\[ \omega^{13} = \sin \alpha \sin \theta_1 d\phi_1, \]
\[ \omega^{14} = -\cos \alpha d\theta_2, \quad \omega^{15} = -\cos \alpha \sin \theta_2 d\phi_2, \]
\[ \omega^{23} = -\cos \theta_1 d\phi_1, \quad \omega^{45} = -\cos \theta_2 d\phi_2. \quad (2.18) \]

Since the eq. (2.17) is a first order differential equation, it is not so difficult to solve it, and the resulting solution \( U \) is obtained as
\[ U = e^{-\frac{1}{2}\theta_1\Gamma_1}e^{-\frac{1}{2}\theta_2\Gamma_2}e^{\frac{1}{2}\phi_1\Gamma_{24}}e^{\frac{1}{2}\phi_2\Gamma_{15}}. \quad (2.19) \]

### 3 Euclidean D1-brane action

Before moving on to the investigation of 1/2-BPS instantonic D1-branes in the \( \text{AdS}_5 \times S^5 \) background, let us consider the D1-brane action and its basic structure. Since the instantonic D1-brane we are concerned about is an object in the Euclidean spacetime, its action is the Euclidean one given by\(^1\)
\[ S = T \int d^2\zeta \sqrt{|G + F|} - ig_5T \int (C_{(2)} + FC_{(0)}). \quad (3.1) \]

Here, \( T \) is the D1-brane tension given in terms of the string coupling constant \( g_s \) and the string length scale \( \ell_s^2 = \alpha' \),
\[ T = \frac{1}{2\pi g_s \ell_s^2}, \quad (3.2) \]
\(^1\)We note that the bosonic action is sufficient for our purpose.
and $|G + F|$ is the determinant of the sum of two objects which are the induced worldvolume metric
\[ G_{ij} = \partial_i X^\mu \partial_j X^\nu g_{\mu\nu}, \tag{3.3} \]
and the combination of worldvolume gauge field strength $F_{ij}$ and the induced NS-NS two-form gauge field $B_{ij}^{NS}$,
\[ F_{ij} = 2\pi \alpha' F_{ij} - B_{ij}^{NS}. \tag{3.4} \]
The background fields $C_{(0)}$ and $C_{(2)}$ are the induced R-R zero and two-form gauge fields respectively.

As described in the last section, the AdS$_5 \times$S$^5$ background does not have nontrivial profiles for $C_{(0)}$, $C_{(2)}$ and $B^{NS}$. Thus we can eliminate these fields in the action (3.1). From the resulting action, the equation of motion for the worldvolume gauge field is obtained as
\[ \partial_i \left( \frac{F_{ij}}{\sqrt{|G + F|}} \right) = 0. \tag{3.5} \]
It is easy to solve this equation and we see that the solution can be parametrized by a constant $\beta$ as
\[ B \sqrt{|G + F|} = \sin \beta, \tag{3.6} \]
where we have defined
\[ B \equiv F_{12} = 2\pi \alpha' F_{12}, \tag{3.7} \]
and $\beta$ has the range of $-\pi/2 \leq \beta \leq \pi/2$. The solution (3.6) can be rewritten as
\[ B = \sqrt{|G|} \tan \beta, \tag{3.8} \]
and enables us to express the D1-brane action as
\[ S = \frac{T}{\cos \beta} \int d^2 \zeta \sqrt{|G|}. \tag{3.9} \]

4 1/2-BPS D1-instantons

In this section, we identify the 1/2-BPS instantonic D1-brane configurations based on the data from the covariant open string description of D-branes in the AdS$_5 \times$S$^5$ background. For the investigation of instantonic object, the spacetime is taken to be Euclidean. However, the Killing spinor $\eta_\pm$ of (2.16) with (2.19) have been obtained in the Lorentzian signature. Thus, for the expressions in the previous section, we should take the Wick rotation $x^0 \rightarrow -ix^0$, under which
\[ \Gamma^0 \rightarrow -i\Gamma^0, \quad \hat{\gamma} \rightarrow -i\hat{\gamma}. \tag{4.1} \]
In ten dimensional Euclidean spacetime, $\eta_\pm$ and $\epsilon_\pm$ are no longer Majorana-Weyl and become Weyl spinors. This change of nature, however, does not make any annoying issue

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2 The worldvolume indices $i, j$ take values of 1 and 2.
3 We use the same definition for $\hat{\gamma}$ given in (2.9). Thus $\hat{\gamma}^2 = -1$ in the Lorentzian signature, while $\hat{\gamma}^2 = +1$ in the Euclidean one.
in the present work, because our concern is the consistent projection operators acting on 
$\eta_{\pm}$ (strictly speaking $\epsilon_{\pm}$) which identify the 1/2-BPS configurations.

Having the Killing spinor, the 1/2-BPS instantonic D1-brane configurations can be 
investigated by using the usual equation \[19\]
\[
\eta^I = \Gamma^{IJ} \eta^J , \tag{4.2}
\]
which is obtained by combining the spacetime supersymmetry and the worldvolume $\kappa$-
symmetry transformation. The symbol $\Gamma$ represents the spinorial matrix appearing in the $\kappa$ symmetry projection operator and satisfies $\Gamma^2 = 1$ and $\text{Tr} \Gamma = 0$. Its explicit form for the D1-brane in the Euclidean spacetime is
\[
\Gamma^{IJ} = \frac{i}{2\sqrt{|G + F|}} \epsilon^{ij} (\gamma_{ij} \gamma_i^I + F_{ij} \gamma_i^I J) , \tag{4.3}
\]
where $\gamma_{ij} = \partial_i X^\mu \partial_j X^\nu \epsilon^{\mu\nu} \Gamma_{AB}$. Now, by acting the projection operator $P^{IJ}_{\pm}$ of (2.12) on the above equation (4.2), we can express (4.2) in terms of $\eta_{\pm}$ of (2.14) as\footnote{Due to eq. (4.1), the projector $P^{IJ}_{\pm}$ of (2.12) changes to $P^{IJ}_{\pm} = \frac{1}{2} (\delta^{IJ} \mp \i \gamma^I J)$. Because of this, the relation between $\eta^2_\pm$ and $\eta^2_\pm$ of (2.13) becomes $\eta^2_\pm = \mp \i \eta^2_\pm$.}
\[
\begin{pmatrix} \eta_+ \\ \eta_- \end{pmatrix} = \frac{\hat{\gamma}}{2 \sqrt{|G + F|}} \epsilon^{ij} \begin{pmatrix} -F_{ij} \gamma_{ij} \\ \gamma_{ij} - F_{ij} \end{pmatrix} \begin{pmatrix} \eta_+ \\ \eta_- \end{pmatrix} . \tag{4.4}
\]
We have two equations. However, they are actually equivalent, because the equation from the first row is obtained by acting
\[
1 + \frac{\hat{\gamma}}{2 \sqrt{|G + F|}} \epsilon^{ij} F_{ij}
\]
on the equation from the second row. Thus it is enough to consider only one of two equations. Here, we will take the equation from the first row,
\[
\eta_+ = \frac{\hat{\gamma}}{2 \sqrt{|G + F|}} \epsilon^{ij} (\gamma_{ij} \eta_- - F_{ij} \eta_+) . \tag{4.6}
\]
Then, by plugging the Killing spinor (2.16) into this equation, we get
\[
\frac{1}{\sqrt{2}} (1 + \hat{\gamma} \sin \beta) (\epsilon_+ + \Gamma_m x^m \epsilon_-) = \frac{\sqrt{2/2} \cos \beta}{2\sqrt{|G|}} \hat{\gamma} U^{-1} \epsilon^{ij} \gamma_{ij} \Gamma_+ U \epsilon_- , \tag{4.7}
\]
where $U$ is given in (2.19) and we have used (3.6) and (3.8).

The eq. (4.7) is the key for identifying the 1/2-BPS instantonic D1-branes. Since the covariant open string description tells us that the instantonic D1-branes embedded only in the $S^5$ of AdS$_5 \times S^5$ geometry may have the possibility of being 1/2-BPS, we will consider the D1-brane configurations each of which spans a certain two dimensional subspace of the $S^5$ and is a point in the AdS$_5$ space. Thus the coordinates $(z, x^m)$ in (4.7), the position of D1-brane in the AdS$_5$ space, are taken to be constants and the $U$ has a specific expression corresponding to a given configuration. If we evaluate the right hand side of (4.7) for a given
D1-brane configuration and obtain the result which does not depend on any worldvolume coordinate, then the configuration is confirmed to be 1/2-BPS.\(^5\)

Specifying a D1-brane configuration or embedding in \(S^5\) is to choose a static gauge for the worldvolume reparametrization. From the five sphere metric (2.3), we can figure out largely two types of static gauge fixing conditions according to whether or not the coordinate \(\alpha\) is transverse to the D1-brane worldvolume. We first investigate the cases where \(\alpha\) is a transverse direction.

Perhaps, the most immediate one would be the D1-brane that wraps a two sphere parametrized by \((\theta_1, \phi_1)\) or \((\theta_2, \phi_2)\). If we choose the static gauge as
\[
\zeta^1 = \theta_1, \quad \zeta^2 = \phi_1,
\]
with constant \(\alpha\) and \(\theta_2 = \phi_2 = 0\), then \(\sqrt{|G|} = R^2 \cos^2 \alpha \sin \theta_1\) and \(U\) of (2.19) reduces to
\[
U = e^{-\frac{1}{2} \alpha \Gamma_1 z e^{-\frac{1}{2} \theta_1 \Gamma_2} e^{\frac{1}{2} \phi_1 \Gamma_{23}}}.
\]
(4.9)

The right hand side of (4.7) is evaluated by using the following computation:
\[
\frac{1}{2\sqrt{|G|}} U^{-1} \epsilon^{ij} \gamma_{ij} \Gamma_2 U = U^{-1} \Gamma_{23} \Gamma_2 U
= \left( \cos \alpha + \sin \alpha e^{-\frac{1}{2} \phi_1 \Gamma_{23} e^{\theta_1 \Gamma_2} e^{\frac{1}{2} \phi_1 \Gamma_{23}}} \right) \Gamma_{23} \Gamma_2.
\]
(4.10)

We see that this expression depends explicitly on the worldvolume coordinates \(\theta_1\) and \(\phi_1\). Though this seems to make the present configuration non-supersymmetric, we can get rid of the dependence by taking the transverse position of D1-brane in the \(\alpha\) direction to be \(\alpha = 0\), at which the size of the \(S^2\) parametrized by \((\theta_1, \phi_1)\) is maximized as one can see from (2.3). Then, the eq. (4.7) becomes
\[
(1 + \hat{\gamma} \sin \beta)(\epsilon_+ + \Gamma_m x^m \epsilon_-) = z \cos \beta \hat{\gamma} \Gamma_{23} \Gamma_2 \epsilon_-.
\]
(4.11)

If we now introduce a projection operator
\[
P^\pm_x = \frac{1}{2} (1 \pm \hat{\gamma}),
\]
(4.12)
and act on (4.11), we finally get
\[
\epsilon_{++} = \frac{\cos \beta}{1 + \sin \beta} z \Gamma_{23} \Gamma_2 \epsilon_{--} - \Gamma_m x^m \epsilon_{--},
\]
\[
\epsilon_{+-} = -\frac{\cos \beta}{1 - \sin \beta} z \Gamma_{23} \Gamma_2 \epsilon_{--} - \Gamma_m x^m \epsilon_{--},
\]
(4.13)

where we have defined
\[
\epsilon_{++} \equiv P^+_x \epsilon_+, \quad \epsilon_{-} \equiv P^x_- \epsilon_-.
\]

\(^5\)One may argue that any D1-brane embedded in the \(S^5\) is always 1/2-BPS if it is placed at \(z = 0\) or \(\infty\). However, it seems that the object at such position should be handled with care because such supersymmetry structure may not be obtained by taking the limit \(z \to 0\) or \(\infty\) after solving (4.7). In our study, it is presumed that the position of D1-brane in the AdS\(_5\) space is generic and eq. (4.7) is solved preferentially before taking any limit.
The eq. (4.13) clearly shows that $\epsilon_{+\pm}$ is given in terms of $\epsilon_{-\pm}$, while $\epsilon_{-\pm}$ are still free parameters. Thus it is concluded that the D1-brane wrapping the $S^2$ parametrized by $(\theta_1, \phi_1)$ placed at $\alpha = 0$ is 1/2-BPS. We note that this result continues to hold for the D1-brane wrapping another $S^2$ parametrized by $(\theta_2, \phi_2)$. The differences are that $\Gamma_{23}$ in (4.13) is to be replaced by $\Gamma_{45}$ and the position in the $\alpha$ direction should be $\alpha = \pi/2$.

The second case is the configuration corresponding to the following static gauge,

$$\zeta^1 = \phi_1, \quad \zeta^2 = \phi_2, \quad (4.15)$$

with constant $\alpha, \theta_1$ and $\theta_2$. In this gauge choice, $\sqrt{|G|} = R^2 \cos^2 \alpha \sin \theta_1 \sin \theta_2$ and $U$ is just given by (2.19). Then, the important part of the right hand side of (4.7) is computed as

$$\frac{1}{2 \sqrt{|G|}} U^{-1} g^{ij} \gamma_{ij} \Gamma_z U = U^{-1} \Gamma_{35} \Gamma_z U$$

$$= \left( \cos \alpha e^{-\phi_2 \Gamma_{45}} e^{-\frac{1}{2} \phi_1 \Gamma_{23}} e^{\theta_1 \Gamma_{x2}} e^{-\frac{1}{2} \phi_1 \Gamma_{23}}
+ \sin \alpha \Gamma_{x1} e^{-\frac{1}{2} \phi_2 \Gamma_{45}} e^{\theta_2 \Gamma_{14}} e^{-\frac{1}{2} \phi_2 \Gamma_{45}} e^{-\phi_1 \Gamma_{23}} \right) \Gamma_{35} \Gamma_z. \quad (4.16)$$

We see that there are explicit dependencies on the worldvolume coordinates $\phi_1$ and $\phi_2$. Unlike the previous case, one cannot eliminate such dependencies for any choice of $\alpha$, $\theta_1$ and $\theta_2$. Therefore, the only solution of (4.7) is $\epsilon_{\pm} = 0$ and thus the D1-brane wrapping $\phi_1$ and $\phi_2$ is not supersymmetric.

The third case, the last one in which $\alpha$ is a transverse direction, is the D1-brane wrapping the ‘diagonal’ $S^2$ composed of two $S^2$’s parametrized by $\theta_{1,2}$ and $\phi_{1,2}$. If we let

$$\vartheta = \theta_1 = \theta_2, \quad \varphi_{\pm} = \phi_1 = \pm \phi_2, \quad (4.17)$$

the corresponding static gauge is

$$\zeta^1 = \vartheta, \quad \zeta^2 = \varphi_+ \text{ (or } \varphi_-), \quad (4.18)$$

with constant $\alpha$. Let us first consider the configuration where $\zeta^2 = \varphi_+$. It leads us to have $\sqrt{|G|} = R^2 \sin \vartheta$ and

$$U = e^{-\frac{1}{2} \alpha \Gamma_{z1}} e^{-\frac{1}{2} \theta_1 (\Gamma_{z2} - \Gamma_{14})} e^{\frac{1}{2} \Gamma_+ (\Gamma_{23} + \Gamma_{45})}, \quad (4.19)$$

from (2.19), which in turn are used to get

$$\frac{1}{2 \sqrt{|G|}} U^{-1} g^{ij} \gamma_{ij} \Gamma_z U = U^{-1} \Gamma_{23} e^{\alpha (\Gamma_{24} + \Gamma_{35})} \Gamma_z U$$

$$= \Gamma_z \Gamma_{23} e^{\alpha (\Gamma_{24} + \Gamma_{35} - \Gamma_{z1})}. \quad (4.20)$$

Obviously, the last expression in (4.20) is independent from any of the worldvolume coordinates and hence implies that the present configuration is 1/2-BPS. Then, after a bit of

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6This configuration is inspired by that of instantonic D2-brane [20] in the AdS$_4 \times \mathbb{C}P^3$ background.
manipulation with (4.20), eq. (4.7) determining the supersymmetry structure becomes

\[
\epsilon_{++} = \frac{\cos \beta}{1 + \sin \beta} z \Gamma_{123} e^{\alpha (\Gamma_{24} + \Gamma_{35} - \Gamma_{z1})} \epsilon_{--} - \Gamma_m x^m \epsilon_{--},
\]
\[
\epsilon_{+-} = -\frac{\cos \beta}{1 - \sin \beta} z \Gamma_{123} e^{\alpha (\Gamma_{24} + \Gamma_{35} - \Gamma_{z1})} \epsilon_{--} - \Gamma_m x^m \epsilon_{--},
\]

(4.21)

where \(\epsilon_{\pm \pm}\) and \(\epsilon_{\pm -}\) are defined in (4.14) in terms of the projection operator (4.12). These equations clearly show that the D1-brane wrapping the ‘diagonal’ \(S^2\) is 1/2-BPS. Compared to the previous 1/2-BPS configuration (4.8), one distinguishing feature of this configuration is that it is 1/2-BPS for any transverse position in \(\alpha\) and its size is fixed. However, we should notice from (4.21) that the dependence of \(\epsilon_{++}\) on \(\epsilon_{--}\) changes continuously with \(\alpha\).

On the other hand, the configuration corresponding to the static gauge \(\zeta^2 = \varphi_+\) in (4.18) leads to the same result with that of (4.21) but with the replacement \(\Gamma_{35} \rightarrow -\Gamma_{35}\).

Therefore, it is also 1/2-BPS.

We now turn to the configurations where \(\alpha\) is a worldvolume coordinate not a transverse one. Then another worldvolume coordinate is along a circle embedded in the space composed of two \(S^2\)’s parametrized by \(\theta_1, \theta_2\) and \(\phi_1, \phi_2\). The possible candidates for such circle are the circle given by \(\phi_1\) or \(\phi_2\) and the ‘diagonal’ one by \(\varphi_+\) or \(\varphi_-\) defined in (4.17). For the first case, the static gauge is chosen to be

\[
\zeta^1 = \alpha, \quad \zeta^2 = \phi_1,
\]

(4.22)

with constant \(\theta_1\), for which we have \(\sqrt{|G|} = R^2 \cos \alpha \sin \theta_1\) and

\[
U = e^{-\frac{1}{2} \alpha \Gamma_{z1}} e^{-\frac{1}{4} \theta_1 \Gamma_{z2}} e^{\frac{1}{2} \phi_1 \Gamma_{23}}
\]

(4.23)

from (2.19). The next step is to compute

\[
\frac{1}{2 \sqrt{|G|}} U^{-1} e^{ij \gamma_{ij} \Gamma_z} U = U^{-1} \Gamma_{13} \Gamma_z U
\]

\[
= \left( \cos \theta_1 e^{\phi_1 \Gamma_{23}} + \sin \theta_1 \Gamma_{z2} \right) \Gamma_{13} \Gamma_z.
\]

(4.24)

Although the last expression explicitly depends on the worldvolume coordinate \(\phi_1\), the dependence can be removed by taking \(\theta_1 = \pi/2\), corresponding to the great circle in \(S^2\) parametrized by \((\theta_1, \phi_1)\). At such position in \(\theta_1\), eq. (4.7) gives the supersymmetry structure of the configuration as

\[
\epsilon_{++} = \frac{\cos \beta}{1 + \sin \beta} z \Gamma_{123} \epsilon_{--} - \Gamma_m x^m \epsilon_{--},
\]
\[
\epsilon_{+-} = -\frac{\cos \beta}{1 - \sin \beta} z \Gamma_{123} \epsilon_{--} - \Gamma_m x^m \epsilon_{--},
\]

(4.25)

where \(\epsilon_{++}\) and \(\epsilon_{--}\) are defined in (4.14) in terms of the projection operator (4.12). This result shows us that the configuration (4.22) at \(\theta_1 = \pi/2\) is 1/2-BPS. Similarly, we can confirm that another configuration \((\zeta^1 = \alpha, \zeta^2 = \phi_2)\) at \(\theta_2 = \pi/2\) is also 1/2-BPS. Its supersymmetry structure is also given by (4.25) but with the replacement \(\Gamma_{123} \rightarrow \Gamma_{45}\).
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
 & \((\zeta^1, \zeta^2)\) & position \\
\hline
(i) & \((\theta_1, \phi_1)\) & \(\alpha = 0\) \\
(ii) & \((\theta_2, \phi_2)\) & \(\alpha = \pi/2\) \\
(iii) & \((\vartheta, \varphi_+)\) & any \(\alpha\) \\
(iv) & \((\vartheta, \varphi_-)\) & any \(\alpha\) \\
(v) & \((\alpha, \phi_1)\) & \(\theta_1 = \pi/2\) \\
(vi) & \((\alpha, \phi_2)\) & \(\theta_2 = \pi/2\) \\
\hline
\end{tabular}
\caption{1/2-BPS D1-instanton configurations. \((\zeta^1, \zeta^2)\): static gauge for a given D1-brane configuration. position: transverse position. The definitions for \(\vartheta\) and \(\varphi_\pm\) are given in (4.17). All the configurations have the spherical shape with the radius equal to that of \(S^5\), and are related by SO(6) rotations, the symmetry group of \(S^5\). Thus any other configurations not listed here are 1/2-BPS if they can be obtained by SO(6) rotations from, for example, the configuration (i).}
\end{table}

Finally, as for the second configuration where \(\varphi_+\) or \(\varphi_-\) is a worldvolume coordinate, the corresponding static gauge is \((\zeta^1 = \alpha, \zeta^2 = \varphi_+ \text{ or } \varphi_-)\) at constant \(\theta_1\) and \(\theta_2\). However, this configuration turns out to be non-supersymmetric since there is no way to eliminate the dependence on the worldvolume coordinate in the calculation of \(U^{-1} \epsilon_{ij} \gamma_{ij} \Gamma_z U/\sqrt{|G|}\) even by choosing certain values of \(\theta_1\) and \(\theta_2\).

5 Evaluation of action and relation to \((p, q)\) string instanton

So far, we have identified the 1/2-BPS instantonic D1-brane configurations in the AdS\(_5\times S^5\) background, which are summarized in table 2. Having the configurations, if we evaluate the Euclidean action for all the configurations listed in table 2 by using the D1-brane action (3.9) with \(\sqrt{|G|}\)'s computed for the configurations in the last section, we see that the configurations from (i) through (iv) have the same action value as\(^7\)

\[
S_{D1} = \frac{T}{\cos \beta} \int d^2 \zeta \sqrt{|G|} = \frac{T}{\cos \beta} \cdot 4\pi R^2 = \frac{4}{\cos \beta} \sqrt{\frac{\pi N}{g_s}}.
\]  

(5.1)

As for the remaining configurations, (v) and (vi), the action values seem to be half of \(S_{D1}\) at first glance basically because the range of \(\alpha\) is \(0 \leq \alpha \leq \pi/2\) and thus the configurations look like hemispheres. However, this is not the case and the action values are still given by \(S_{D1}\). Let us explain the reason for the configuration (v) by following the prescription of [21] given in a similar parametrization of \(S^5\). If we pick a pair of antipodal points on \(S^2\) parametrized by \((\theta_2, \phi_2)\), which are \(\theta_2 = 0, \pi\) in the present case, then the pair together with the coordinates \(\alpha\) and \(\phi_1\) form an \(S^2\).\(^8\) This \(S^2\) is the actual space that gives the shape of configuration (v). Thus the configuration (v) has the action value of (5.1), not half of it, and importantly its supersymmetry structure (4.25) is not spoiled for the spherical shape.

\(^7\)For the evaluation, the explicit expressions for the AdS radius (2.2) and the D1-brane tension (3.2) have been utilized.

\(^8\)We note that the coordinate \(\theta_1\) is fixed at \(\pi/2\) for the configuration (v).
The same process applies to the configuration (vi). As a result, the whole configurations in table 2 have the spherical shape and have the action value of (5.1). Additionally, one notable fact is that they have the maximum radius equal to that of $S^5$. By using six Cartesian coordinates, $S^5$ of unit radius is described as $\sum_{i=1}^{6} X_i^2 = 1$. Associated with the $S^5$ metric (2.3), we may let $X_1 = \cos \alpha \cos \theta_1$, $X_2 = \cos \alpha \sin \theta_1 \cos \phi_1$, $X_3 = \cos \alpha \sin \theta_1 \sin \phi_1$, $X_4 = \sin \alpha \cos \theta_2$, $X_5 = \sin \alpha \sin \theta_2 \cos \phi_2$, $X_6 = \sin \alpha \sin \theta_2 \sin \phi_2$. (5.2)

Then each configuration appearing in table 2 satisfies an algebraic equation describing a sphere of unit radius: $X_1^2 + X_2^2 + X_3^2 = 1$ for the configuration (i), $X_4^2 + X_5^2 + X_6^2 = 1$ for (ii), $\sum_{i=1}^{6} X_i^2 = 1$ for (iii) and (iv), $X_2^2 + X_3^2 + X_4^2 = 1$ for (v), and $X_1^2 + X_5^2 + X_6^2 = 1$ for (vi). This means that all the configurations appearing in table 2 are related by SO(6) rotations, the symmetry group of $S^5$.

After obtaining the results, one can see that it is enough to find just one 1/2-BPS configuration, for example the configuration (i), because other 1/2-BPS configurations are obtained by SO(6) rotations. However, the investigation of supersymmetry structure would require additional task related to the analysis of associated rotated spinors. Starting from table 1, we got the information that 1/2-BPS D1-instanton should wrap two cycle in $S^5$. But we do not know the geometry of the two cycle, a priori. The geometry is automatically determined when we analyze the 1/2-BPS condition. Before we produce table 2, it is not clear whether the other geometry than the sphere is excluded. But we came to know that the only 1/2-BPS configuration is the sphere with the maximal radius. On the other hand, our analysis leads to the torus like configuration (4.15), but it turns out not to be supersymmetric. If it were 1/2-BPS, we would have different class of configurations with toroidal shape. Even a spherical shape configuration is 1/2-BPS only if it has the certain radius, the radius of $S^5$. Thus, based on our results, it is concluded that any spherical D1-instanton with the radius equal to that of $S^5$ is 1/2-BPS.10

Since the 1/2-BPS D1-instantons have the same shape and size and are related to each other by SO(6) transformations, one may think that the configurations listed in table 2 are redundant and only one of them is important. This is definitely the case if we are only interested in single D1-instanton. However, if we want to study the supersymmetric multiple D1-instantons, the configurations in table 2 with their supersymmetry structures are necessary and important. We will discuss this issue in the next section.

The action value of (5.1) allows us to estimate the saddle-point contribution of D1-instanton to any physical process. For such estimation, it is instructive to rewrite the D1-instanton action in terms of the well known D(-1)-brane or D-instanton action in the AdS$_5 \times S^5$ background [10–12]. Namely, by using the D(-1)-brane action given by

$$S_{D(-1)} = \frac{2\pi}{g_s} = \frac{8\pi^2 N}{\lambda},$$ (5.3)
where \( \lambda = 4\pi g_s N \) is the usual \('t\)Hooft parameter, let us rewrite (5.1) as

\[
S_{D1} = \frac{\sqrt{\lambda}}{\pi} S_{D(-1)},
\]

where the worldvolume gauge field strength has been turned off for simplicity (\( \beta = 0 \)). Because the D(-1)-brane action is proportional to \( N \) and thus its contribution is suppressed by \( e^{-N} \), it is now clear that the contribution of D1-instanton also leads to the same suppression factor. If we turn on the worldvolume gauge field strength and increase it, the suppression becomes much stronger as one can see from (5.1). In this sense, the D1-instanton with vanishing or weak worldvolume gauge field is preferable.

Until now, we have evaluated the D1-instanton action. By the way, the D1-brane action in (5.1) (or (3.9)) has the form of fundamental string action. Since it is well known that the D1-brane is related to the fundamental string or more generally \((p,q)\) string through the SL(2, \( \mathbb{Z} \)) transformation [14, 15], one may be curious as to how the action of (3.9) is written in an SL(2, \( \mathbb{Z} \)) covariant way. For this, we consider a D1-brane and turn on constant non-zero RR 0-form or axion field \( C(0) \) which is allowed in the AdS_5 \( \times \) S^5 background. From (3.1), the corresponding action is given by

\[
S = \int d^2 \zeta \mathcal{L} = T \int d^2 \zeta \sqrt{|G + F|} - i g_s T \int \chi F,
\]

where the constant axion has been denoted as \( \chi \).

Although the relation between the action (5.5) and that of \((p,q)\) string instanton could be obtained simply by Euclideanizing the result of [22, 23], we will show it directly in Euclidean space by following the process described in [20] and let \( q = 1 \) since single D1-brane action is considered. The flux density \( p \) of worldvolume gauge field strength interpreted as the number of fundamental strings dissolved into D1-brane is the momentum dual to the gauge field strength \( F_{12} \),

\[
p = -i \frac{\partial \mathcal{L}}{\partial F_{12}} = -i \frac{F_{12}}{g_s \sqrt{|G + F|}} - \chi.
\]

This flux density is surely a constant because of the equation of motion (3.5)\(^{11}\) and the constant \( \chi \). On the other hand, \( F_{12} \) depends on the worldvolume coordinates as can be seen from eqs. (3.7) and (3.8). Thus it would be appropriate to express the action in terms of \( p \) rather than \( F_{12} \). If we denote such action as \( S_{(p,1)} \), it is related to \( S \) of (5.5) through the Legendre transformation as follows:

\[
S_{(p,1)} = S - \int d^2 \zeta ip F_{12} = \frac{1}{2\pi \alpha'} \sqrt{(p + \chi)^2 + \frac{1}{g_s^2}} \int d^2 \zeta \sqrt{|G|}.
\]

\(^{11}\)The presence of constant axion \( \chi \) does not modify the equation of motion (3.5) because the second term on the right hand side of (5.5) becomes a topological one for constant \( \chi \).
We see that this result is nothing but the Nambu-Goto string instanton action with the \( \text{SL}(2, \mathbb{Z}) \) covariant tension

\[
T_{p,1} = \frac{1}{2\pi\alpha'} \sqrt{(p + \chi)^2 + \frac{1}{g_s^2}},
\]

that is, the \((p, 1)\) string instanton action. From this relation, we see that the D1-instanton action (5.1) is related to \( S_{(p,1)} \) with

\[
\cos \beta = \frac{1}{\sqrt{g_s^2(p + \chi)^2 + 1}},
\]

where the constant axion \( \chi \) has been turned on.

6 Discussion

The single 1/2-BPS D1-instanton has been of our interest. As a next step, we may ask if multiple D1-instanton configuration is supersymmetric or not. The simplest case would be that of two D1-instantons of the same type, one of six types of configurations in table 2, at different positions in the AdS\( _5 \) space. However, it turns out that such D1-instanton configuration breaks all the supersymmetry. For example, let us take two D1-instantons of the configuration (i) in table 2 as a representative whose positions in the AdS\( _5 \) space are \((z^{(1)}, x^{m}_{(1)})\) and \((z^{(2)}, x^{m}_{(2)})\) respectively. Then we have two sets of equations from (4.13):

\[
\begin{align*}
\epsilon_{++} &= z^{(1)} \Gamma_{23} \epsilon_{+-} - \Gamma_m x^m_{(1)} \epsilon_{--}, \\
\epsilon_{++} &= z^{(2)} \Gamma_{23} \epsilon_{+-} - \Gamma_m x^m_{(2)} \epsilon_{--}, \\
\epsilon_{+-} &= -z^{(1)} \Gamma_{23} \epsilon_{--} - \Gamma_m x^m_{(1)} \epsilon_{++}, \\
\epsilon_{+-} &= -z^{(2)} \Gamma_{23} \epsilon_{--} - \Gamma_m x^m_{(2)} \epsilon_{++},
\end{align*}
\]

where the worldvolume gauge field strength has been turned off \((\beta = 0)\) because it does not play any crucial role for the consideration of supersymmetry. We note that the two \( \epsilon_{++}'s \) \((\epsilon_{+-}'s) \) on the left hand side should be the same if some fraction of supersymmetry is preserved. This allows us to get two equations by subtracting the first (last) two equations:

\[
\begin{align*}
0 &= (z^{(1)} - z^{(2)}) \Gamma_{23} \epsilon_{--} - \Gamma_m \left( x^m_{(1)} - x^m_{(2)} \right) \epsilon_{++}, \\
0 &= - \Gamma_{23} \epsilon_{--} - \Gamma_m \left( x^m_{(1)} - x^m_{(2)} \right) \epsilon_{++}.
\end{align*}
\]

By using these, one can check supersymmetry for the three cases, \((z^{(1)} \neq z^{(2)}, \ x^m_{(1)} = x^m_{(2)}), (z^{(1)} = z^{(2)}, \ x^m_{(1)} \neq x^m_{(2)}), \) and \((z^{(1)} \neq z^{(2)}, \ x^m_{(1)} \neq x^m_{(2)}).\) However, as one can see easily, none of them are supersymmetric. Thus, two or more D1-instantons of the same type are supersymmetric (1/2-BPS) only if they are coincident in the AdS\( _5 \) space. This is in contrast to the D\((-1)\)-brane case where multiple D\((-1)\)-branes do not spoil the supersymmetry structure even if they are apart.

Some more generalized case is that of two different types of D1-instantons. From the table 2, we can think of fifteen combinations. However, we will not investigate all of
them but instead intend to show just the possibility for the existence of supersymmetric configuration by taking one example. It is the combination composed of D1-instantons (i) and (ii). If the D1-instantons are taken to be coincident in the AdS$_5$ space$^{12}$ and the worldvolume gauge field strengths on both of them are turned off for simplicity, eq. (4.13) for (i) becomes

$$
\epsilon_{++} = z \Gamma_{23} \epsilon_{-+}, \quad \epsilon_{+-} = -z \Gamma_{23} \epsilon_{-+},
$$

and the corresponding equation for (ii) is

$$
\epsilon_{++} = z \Gamma_{45} \epsilon_{-+}, \quad \epsilon_{+-} = -z \Gamma_{45} \epsilon_{-+},
$$

which follows from (4.13) with the replacement $\Gamma_{23} \rightarrow \Gamma_{45}$ as mentioned below (4.13).

Because $\Gamma_{23}$ and $\Gamma_{45}$ commute with each other, 13 it is convenient to split $\epsilon_{ab}$ ($a, b = \pm$) in terms of their eigenvalues as $\epsilon_{abs}$:

$$
i \Gamma_{23} \epsilon_{abs} = \pm \epsilon_{ab} \epsilon_{s}, \quad i \Gamma_{45} \epsilon_{ab} \epsilon_{s} = \pm \epsilon_{ab} \epsilon_{s}.
$$

Although, generically, the constant spinors $\epsilon_{++}$ ($\epsilon_{+-}$) appearing in (6.3) and (6.4) have different dependences on $\epsilon_{-+}$ ($\epsilon_{+-}$), they should be the same if the present D1-instanton combination is supersymmetric. If we now subtract the first (second) equation of (6.4) from that of (6.3) and project the resulting equation according to the eigenvalues of $\Gamma_{23}$ and $\Gamma_{45}$ by using (6.5), then we get four equations as

$$
0 = 2i z \Gamma_{z} \epsilon_{-++}, \quad 0 = -2i z \Gamma_{z} \epsilon_{-++}, \\
0 = -2i z \Gamma_{z} \epsilon_{--+}, \quad 0 = 2i z \Gamma_{z} \epsilon_{--+}.
$$

This shows that the four constant spinors on the right hand sides, one half of $\epsilon_{-\pm}$, should vanish for generic $z$ position. In the end, the following ones, another half of $\epsilon_{-\pm}$, remain free parameters,

$$
\epsilon_{-++}, \quad \epsilon_{--+}, \quad \epsilon_{-++}, \quad \epsilon_{--+},
$$

and thus the combination of D1-instantons (i) and (ii) coincident in the AdS$_5$ space turns out to be 1/4-BPS.

Having an explicit example of supersymmetric multiple D1-instantons, we may expect other possibilities from the remaining two D1-instanton configurations. However we will not pursue them further since our primary concern is single 1/2-BPS D1-instanton. We hope to have an opportunity to deal with them in a near future.

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$^{12}$Actually, like in the previous case, they do not form a supersymmetric object if they are separated in the AdS$_5$ space.

$^{13}$They also commute with $\hat{\gamma}$ and $\Gamma^{11}$ measuring the ten dimensional chirality.
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