Doubleverse entanglement in third quantized non-minimally coupled varying constants cosmologies

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Abstract. In this paper we consider a third quantized cosmological model with varying speed of light $c$ and varying gravitational constant $G$ both represented by non-minimally coupled scalar fields. The third quantization of such a model leads to a scenario of the doubleverse with the two components being quantum mechanically entangled. We calculate the two parameters describing the entanglement, namely: the energy and the entropy of entanglement where the latter appears to be a proper measure of the entanglement. We consider a possibility that the entanglement can manifests itself as an effective perfect fluid characterized by the time dependent barotropic index $w_{eff}$, which for some specific case corresponds to the fluid of cosmic strings. It seems that such an entanglement induced effective perfect fluid may generate significant backreaction effect at early times.
1 Introduction

The idea of multiverse assumes that our universe is a part of a larger whole - a multiverse being a collection of many universes. The four different types of the relation between our universe and the rest of the multiverse were defined [1]. The most obvious type of the relation assumes that the rest of multiverse is the space outside the observationally accessible region (level I multiverse). The one more elaborated defines our universe as one of the causally disconnected post-inflationary bubbles with possibly different values of the physical constants (level II multiverse). The other two types involve the idea of Everett’s many-worlds interpretation of quantum mechanics (level III multiverse) or treating large well defined purely mathematical structures as the existing elements of the multiverse (level IV multiverse). An interesting case (level II and III) defining the paradigm of interacting universes describes the interaction between the universes as occurring in the minisuperspace via quadratic terms [2, 3, 4]. The causal disconnection present in level II multiverse in such models can be maintained. Another approach realizing the level I multiverse investigates the effects of the entanglement between different possibly causally disconnected patches of the universe [5, 6]. An extraordinary approach to the concept of multiverse defined in [7, 8] is based on the so-called third quantization procedure which exploits the formal analogy between the Wheeler-DeWitt and the Klein-Gordon equations. In this approach the Klein-Gordon field is substituted by the wave function which is promoted in the course of the third quantization to be an operator acting on the Hilbert space spanned by the orthonormal set of vectors representing occupation with universes characterized by appropriate quantum numbers. A great advantage of this approach is that it naturally introduces quantum entanglement between universes and provides tools to describe an interuniversal entanglement in terms of the thermodynamical quantities [7, 8, 9, 10]. However, the connection between the ordinary thermodynamics and the thermodynamics of quantum entanglement is still not well understood.

Many different cosmological scenarios have been considered so far in the context of the third quantization. We mention here an embedding of Brans-Dicke gravity in the third quantization scheme which interestingly leads to scenarios in which whole multiverse is created out of vacuum [11], an application of third quantization procedure to the varying constants model [12] with non-minimally coupled dynamical scalar fields representing the speed of light and the gravitational constant [13] which results in similar scenario of the multiverse creation or eventually the third quantization of the varying gravitational constant cyclic scenarios [14] in which the naturally arisen interuniversal entanglement leads to interesting behavior of the thermodynamical quantities [9]. The third quantization procedure was also used to discuss the transition from expanding to contracting cosmological phase (and vice-versa) in [15, 16].

Our paper is organized as follows. In Sec. 2 we define the vacuum state of the third quantized varying constants model considered in [12] as the vacuum of instantaneous hamiltonian diagonalization and this way we fix the scenario which describes the creation of pair of entangled universes. We also calculate the reduced density matrix of a single universe as well as its eigenvalues. In Sec. 3 we calculate the energy and the entropy of entanglement where the latter appears to be a proper measure of entanglement. In Sec. 4 we relate the previously calculated energy of entanglement with the classical energy-momentum content.
of the universe and argue that the entanglement can effectively simulate a perfect fluid with
time dependent barotropic index. In Sec. 5 we give our conclusions.

2 Fixing of the vacuum in the third quantized non-minimally
coupled varying constants model

Our considerations are based on the model defined in [12, 13] which describes variation of
the speed of light and the variation of the gravitational constant where both quantities are
represented by the two non-minimally coupled scalar fields. Such a model was originally
inspired by the covariant and locally Lorentz-invariant varying speed of light theories [17]
and is given by the following action:

\[
S = \int \sqrt{-g} \left( \frac{e^{\phi}}{e^{\psi}} \left[ R + \Lambda + \omega (\partial_\mu \phi \partial^\mu \phi + \partial_\mu \psi \partial^\mu \psi) \right] \right) \ d^4 x,
\]

where \( \phi \) and \( \psi \) are some non-minimally coupled scalar fields, \( R \) is the Ricci scalar, \( \Lambda \) plays
the role of the cosmological constant and \( \omega \) is some parameter of the model. By application
of the field redefinition given by

\[
\phi = \frac{\beta}{\sqrt{2\omega}} + \frac{1}{2} \ln \delta,
\]

\[
\psi = \frac{\beta}{\sqrt{2\omega}} - \frac{1}{2} \ln \delta,
\]

the action \( S \) can be rewritten in the form of the Brans-Dicke action which reads:

\[
S = \int \sqrt{-g} \left[ \delta (R + \Lambda) + \frac{\omega}{2} \frac{\partial_\mu \delta \partial^\mu \delta}{\delta} + \delta \partial_\mu \beta \partial^\mu \beta \right] \ d^4 x.
\]

The fields \( \phi(x^\mu) \) and \( \psi(x^\mu) \) are related with the varying speed of light \( c \) and the varying
gravitational constant \( G \) via:

\[
c^3 = e^\phi,
\]

\[
G = e^\psi.
\]

The model based on the action \( S \) was applied to describe the evolution of the flat fried-
mannian background both in the classical and the quantum near big-bang singularity regime
[12, 13]. The classical evolution is given by the following solutions:

\[
a = \frac{1}{D^2 (e^{Fx^0})^2 \sinh^M \left| \sqrt{(A^2 - 9)\Lambda x^0} \right|},
\]

\[
\delta = \frac{D^6 (e^{Fx^0})^6}{\sinh^W \left| \sqrt{(A^2 - 9)\Lambda x^0} \right|},
\]

where \( A = \frac{1}{\sqrt{\omega - 5}}, M = \frac{3}{2} - A^2, W = \frac{2A^2}{2 - A^2}, D \) is some integration constants, \( F \) is a constant
proportional to the kinetic energy contained in the free degrees of freedom and the variable
$x^0$ is the following function of the rescaled proper time $\tilde{x}^0$ defined with its differential $dx^0 \equiv c(\tilde{x}^0)\,d\tau$ with $\tau$ being here the proper time encountered by the comoving observer:

$$x^0 = \frac{2}{\sqrt{(A^2 - 9)}} \text{arctanh} \left( e^{\sqrt{(A^2 - 9)}\Lambda x^0} \right), \quad \text{for } \tilde{x}^0 < 0,$$

$$x^0 = \frac{2}{\sqrt{(A^2 - 9)}} \text{arctanh} \left( e^{-\sqrt{(A^2 - 9)}\Lambda x^0} \right), \quad \text{for } \tilde{x}^0 > 0,$$

where as in [13] we restrict our considerations to the models with $A^2 > 9$. The solutions (7), (8), (9) and (10) describe both the pre-big-bang collapsing and the post-big-bang expanding phase separated by the curvature singularity which occurs for $\tilde{x}^0 = 0$. It should be stressed that with both the phases (branches) in the low curvature regime there are associated the fixed momenta in the minisuperspace - negative for the expanding branch and positive for the collapsing branch (for the detailed explanation of the facts given above see [13]).

The quantum regime of the considered model is described by the following Wheeler-DeWitt equation [12]:

$$\Phi'' - \Delta \Phi + m_{eff}^2 \Phi = 0,$$

where $(\eta') = \frac{2}{\sqrt{(A^2 - 9)}} \frac{\partial}{\partial x_1} + \frac{2}{\sqrt{(A^2 - 9)}} \frac{\partial}{\partial x_2}$ and $m_{eff}^2 = \Lambda e^{-\frac{2}{\sqrt{1}}\eta}$ with $r \equiv 2\sqrt{1\Lambda^2 - 9}$. Here

$$\eta \equiv r \ln \sinh |(A^2 - 9)\Lambda x^0|$$

and it defines both the high and the low curvature regimes which occurs for $\eta \to \infty$ and $\eta \to -\infty$, respectively. The variables $x_1$ and $x_2$ describe the free degrees of freedom in our model. The action that leads to the Wheeler-DeWitt equation given by (11) is:

$$S = \frac{1}{2} \int \left[ (\Phi')^2 - (\nabla \Phi)^2 - m_{eff}^2 \Phi^2 \right] d^2x \, d\eta,$$

where $\nabla$ is a two-dimensional gradient operator associated with the variables $x_1$ and $x_2$. The third quantized hamiltonian corresponding to the action (13) is:

$$H_{3Q}(\eta) = \frac{1}{2} \int \left[ \pi^2 + (\nabla \Phi)^2 + m_{eff}^2 \Phi^2 \right] d^2x,$$

where the conjugated momentum $\pi = \Phi'$. Now we need to specify the actual physical vacuum. A natural choice for the vector representing the physical vacuum is the lowest state of the hamiltonian. In the considered case the hamiltonian given by (14) explicitly depends on the time variable $\eta$ and thus does not possess time-independent eigenstates that could represent the physical vacuum. In this paper we will represent the physical vacuum by the so called instantaneous lowest-energy state (or vacuum of instantaneous hamiltonian diagonalization) which is defined as a ground state of the hamiltonian for a particular value of the time parameter $\eta_0$ (see [18]). Generally the vacuum and the series of excited states can be determined by a set of particular mode functions $v_k(\eta)$ that are included in the usual expansion formula of the field operator $\hat{\Phi}$ given by:

$$\hat{\Phi}(\vec{x}, \eta) = \frac{1}{\sqrt{2}} \int d^2k \left[ e^{ik\cdot\vec{x}} v_k(\eta) a_+^k + e^{-ik\cdot\vec{x}} v_k(\eta) a_-^k \right].$$

4
where \( \vec{k} \equiv (k_1, k_2), d^2 k \equiv dk_1 dk_2 \) and \( |\vec{k}| \equiv k \equiv \sqrt{k_1^2 + k_2^2} \). The mode functions \( v_k(\eta) \) fulfill the following mode equation (a condition imposed by (11)):

\[
v_k(\eta)'' + \omega_k(\eta)^2 v_k(\eta) = 0, \tag{16}
\]

where \( \omega_k(\eta) = \sqrt{k^2 + m_{eff}^2(\eta)} \) and the normalization condition:

\[
W(v_k(\eta), v_k^*(\eta)) = 2i, \tag{17}
\]

where \( W(\cdot, \cdot) \) denotes wronskian. The creation and annihilation operators \( \hat{a}_{\vec{k}}^{-} \) and \( \hat{a}_{\vec{k}}^{+} \) fulfill the standard commutation relations:

\[
[\hat{a}_{\vec{k}}^{-}, \hat{a}_{\vec{k}'}^{+}] = \delta(\vec{k} - \vec{k}'), \tag{18}
\]
\[
[\hat{a}_{\vec{k}}^{-}, \hat{a}_{\vec{k}'}^{-}] = 0, \tag{19}
\]
\[
[\hat{a}_{\vec{k}}^{+}, \hat{a}_{\vec{k}'}^{+}] = 0. \tag{20}
\]

Analysis of the asymptotic behaviour of the classical hamiltonian associated with action (4) leads to the following constraint (for the explanation of this fact see [12, 13]):

\[
\Lambda = k_1^2 + k_2^2. \tag{21}
\]

It can be shown that the mode functions \( v_k(\eta) \) which define the instantaneous ground state of the hamiltonian (14) must fulfill the following conditions [18]:

\[
v_{\vec{k}}^{-}(\eta_0) = \frac{1}{\sqrt{\omega(\eta_0)}}, \quad v_{\vec{k}}^{-}(\eta_0) = i\omega(\eta_0)v_{\vec{k}}^{-}(\eta_0). \tag{22}
\]

It can be checked that the mode functions \( v_{\vec{k}}^{-} \sim H_{-ikr}^{(2)}(x) \) with \( x \equiv \sqrt{\Lambda e^{-\eta/r}} \) being the low curvature \((\eta \to -\infty)\) solutions of the mode equation (16) fulfill the conditions (22) and the vacuum state associated with them describes the instantaneous ground state of the hamiltonian (14). In terms of the creation and annihilation operators \( \hat{a}_{\vec{k}}^{-} \) and \( \hat{a}_{\vec{k}}^{+} \) associated with the mode functions \( v_{\vec{k}}^{-} \sim H_{-ikr}^{(2)}(x) \) the hamiltonian (14) in the low curvature regime for \( \eta \rightarrow -\infty \) can be expressed in the following form [18]:

\[
H_{3Q}(\eta) = \int d^2 k \omega_k(\eta) \left[ \hat{a}_{\vec{k}}^{+} \hat{a}_{\vec{k}}^{-} + \frac{1}{2} \delta(2)(0) \right]. \tag{23}
\]

Let us now specify the boundary conditions related with the considered problem. The initial state of the field describing the multiverse is given by the vacuum associated with the mode functions \( u_k \sim J_{-ikr}(x) \) being the solutions of the mode equation:

\[
u_k(\eta)'' + \omega_k(\eta)^2 u_k(\eta) = 0, \tag{24} \]

in the high curvature regime \((\eta \to \infty)\). Due to the evolving background the vacuum state evolves over time and for the low curvature regime transforms into the state describing the instantaneous lowest-energy state of the hamiltonian (14) completely specified by the
set of mode functions $v_k^n \sim H^{(2)}_{-ikr}(x)$. For each mode $k$ the initial high-curvature vacuum $|_{\text{in}} \langle 0_k, 0_{-k} \rangle$ can be expressed in terms of the the states $|_{\text{out}} \langle n_k, n_{-k} \rangle$ excited with respect to the low-curvature vacuum $|_{\text{out}} \langle 0_k, 0_{-k} \rangle$ [18]:

$$
|_{\text{in}} \langle 0_k, 0_{-k} \rangle = \frac{1}{|\alpha_k|} \sum_{n=0}^{\infty} \left( -\frac{\beta_k^n}{\alpha_k} \right)^n \langle_{\text{out}} n_k, n_{-k} \rangle,
$$

where

$$
|_{\text{out}} \langle n_k, n_{-k} \rangle \equiv \frac{1}{n!} \left( \hat{a}^+_k \right)^n \left( \hat{a}^+_k \right)^n |_{\text{out}} \langle 0_k, 0_{-k} \rangle,
$$

while $\alpha_k$ and $\beta_k$ are the the Bogolyubov coefficients given by:

$$
\alpha_k = \frac{W(u_k, v_k^*)}{2i}, \quad (27)
$$

$$
\beta_k = \frac{W(v_k, u_k^*)}{2i}. \quad (28)
$$

Following [7, 8, 9] we interpret $|_{\text{out}} \langle n_k, n_{-k} \rangle$ as states that represent the pair of wave packets with opposite momenta $k$ and $-k$ that move in the minisuperspace. These, on the other hand, correspond to the collapsing pre-big-bang and the expanding post-big-bang universes [12, 13], respectively. In other words the expression (25) describes the process of creation of the entangled pair of the universes with the excited internal states referred to by the quantum number $n$. We assume the perspective of an observer associated with the expanding branch for which the contracting branch is inaccessible. From his point of view the the state of the expanding branch being a subset of the composite quantum mechanical system made up of both the expanding and the contracting branches is given by the reduced density matrix which is a result of tracing away the degrees of freedom associated with the contracting branch:

$$
\rho_{-k} = \sum_{m=0}^{\infty} \langle_{\text{out}} \langle m_k | \rho |_{\text{out}} \langle m_{-k} \rangle, \quad (29)
$$

where

$$
\rho = |_{\text{in}} \langle 0_{-k} | \langle 0_{-k} |_{\text{in}} \rangle.
$$

By performing the trace in (29) we obtain the reduced density matrix $\rho_{-k}$ in the following form:

$$
\rho_{-k} = \frac{1}{|\alpha_k|^2} \sum_{m=0}^{\infty} \left| \frac{\beta_k^n}{|\alpha_k|} \right|^2 \langle_{\text{out}} \langle m_k | \langle_{\text{out}} \langle m_{-k} \rangle. \quad (31)
$$

By normalizing the mode functions $u_k$ and $v_k$ with conditions $W(v_k(\eta), v_k^*(\eta)) = 2i$ and $W(u_k(\eta), u_k^*(\eta)) = 2i$ and then by calculating the wronskians in (27) and (28) we obtain the Bogolyubov coefficients $\alpha_k$ and $\beta_k$ in the following form:

$$
\alpha_k = \frac{1}{\sqrt{1 - e^{-2\pi kr}}}, \quad (32)
$$

$$
\beta_k = \frac{1}{\sqrt{e^{2\pi kr} - 1}}. \quad (33)
$$
The eigenvalues of the reduced density matrix $\rho_{-\vec{k}}$ given by (31) are:

$$\lambda_n \equiv \frac{\beta_{\vec{k}}}{\alpha_{\vec{k}}} e^{-2\pi kr_n} = \frac{1}{1 - e^{-2\pi kr}}.$$  

The eigenvalues (34) do not fulfill the normalization condition since

$$\sum_{n=0}^{\infty} \lambda_n = \frac{1}{(1 - e^{-2\pi kr})^2}.$$  

The corrected eigenvalues which fulfill the normalization condition (35) are then:

$$\tilde{\lambda}_n \equiv (1 - e^{-2\pi kr}) e^{-2\pi kr_n}.$$  

3 The energy and the entropy of the entangled pair of universes

We will show that with the process of the creation of the pair of entangled universes there is associated a production of the entropy and the energy of entanglement. In order to see that we will first calculate the energy of entanglement (an analog of the internal energy in thermodynamics) defined as [7, 8, 9]:

$$E_{\text{ent}} \equiv Tr \left( \rho_{-\vec{k}} H_d \right) = \sum_{n=0}^{\infty} \langle (\text{out})^n_{-\vec{k}}|\rho_{-\vec{k}} H_d|(\text{out})^n_{-\vec{k}} \rangle,$$

where

$$H_d \equiv \omega_{\vec{k}}(\eta) \left[ \hat{a}^{+}_{-\vec{k}} \hat{a}^{-}_{-\vec{k}} + \frac{1}{2} \right],$$

is a hamiltonian of a single universe of the doubleverse. The explicit form of the energy of entanglement is:

$$E_{\text{ent}} = \frac{\sqrt{\Lambda}}{2} \left( 1 - x^2 \right) e^{-\frac{\eta}{r}},$$

where $x$ is defined by

$$x \equiv \left( \frac{\beta_{\vec{k}}}{\alpha_{\vec{k}}} \right)^2 = e^{-2\pi kr}.$$  

Here $kr = 2\sqrt{\frac{\Lambda}{4\pi^2}}$ which is a consequence of the constraint (21). The energy of the entanglement $E_{\text{ent}}$ (see Fig. 1) grows monotonically together with the value of the cosmological constant $\Lambda$ and it reaches zero as the cosmological constant $\Lambda$ vanishes.

The entropy of entanglement is given by the von Neumann entropy and is defined as [7, 8, 9]:

$$S(\rho_{-\vec{k}}) = -\sum_{n=0}^{\infty} \tilde{\lambda}_n \ln \tilde{\lambda}_n.$$
Figure 1: The energy of entanglement $E_{\text{ent}}$ against the cosmological constant $\Lambda$. The quantity $E_{\text{ent}}$ is a monotonically growing function of the cosmological constant $\Lambda$ and it vanishes for $\Lambda = 0$.

By substituting the corrected eigenvalues $\tilde{\lambda}_n$ of $\rho_{-\vec{k}}$ given by (36) into (41) we obtain that:

$$S(\rho_{-\vec{k}}) = \ln \left[ \frac{x(\frac{x}{x-1})}{1-x} \right].$$

The entropy of entanglement $S(\rho_{-\vec{k}})$ (see Fig. 2) monotonically decreases as the cosmological constant grows. It becomes infinite for the vanishing cosmological constant while tends to zero as the cosmological constant approaches infinity. In other words the pairs of the universes characterized by small values of the vacuum energy are initially much more entangled than those with larger values of the vacuum energy. In fact if the vacuum energy is very large the entanglement disappears and the state of the pair of the universes becomes separable. On the other hand vanishing of the vacuum energy is accompanied by maximal (infinite) entanglement. It seems strange that the energy of entanglement $E_{\text{ent}}$ (compare Fig. 1 and Fig. 2) on one hand vanishes as the entropy of entanglement $S(\rho_{-\vec{k}})$ approaches infinity (maximal entanglement) while on the other hand it goes to infinity as the entropy of entanglement $S(\rho_{-\vec{k}})$ approaches zero value. This means that the energy of entanglement is not a good measure of the strength of entanglement. Remember, however, that the quantum number $n$ enumerates the internal excitation levels of the single universe of the considered doubleverse and the quantity $E_{\text{ent}}$ defined by (37) gives the average value of the energy associated with the internal excitation levels. Given the above, it seems sensible to think of the energy of entanglement as something which is associated with a single universe and whose presence should at least in principle be detectable. In the next section we will postulate that the energy of entanglement $E_{\text{ent}}$ can be noticed by an observer inside a single
universe of the doubleverse as the energy which supplements the energy associated with the matter content.

4 Entanglement effective perfect fluid

Assuming that the energy of entanglement can be a part of the energy-momentum content of the single universe the effect of quantum entanglement can manifest itself in the from of the effective prefect fluid which may affect the evolution of the classical background. We additionally assume that the effective fluid does not interact with the other perfect fluid filling the space. In order to derive the form of the associated barotropic index we start with the ordinary continuity equation:

\[ d\rho + 3\frac{da}{a}(1 + w_{\text{ent}})\rho = 0, \]  \hspace{1cm} (43)

where \( w_{\text{ent}} \) is the barotropic index of the effective fluid associated with the effect of the entanglement. The energy density of the effective fluid scales in the following way:

\[ \rho \sim \frac{E_{\text{ent}}}{a^3}. \]  \hspace{1cm} (44)

Taking into account the expression (39) we can easily calculate that:

\[ \frac{dE_{\text{ent}}}{da} = -E_{\text{ent}} \frac{dl}{dx^0}, \]  \hspace{1cm} (45)
where $a$ is the scale factor and $I \equiv \frac{x}{r}$. By combining (43), (44) and (45) we obtain the effective barotropic index $w_{\text{ent}}$ in the following form:

$$w_{\text{ent}} = \frac{a}{3} \frac{dI}{dx^0}.$$

(46)

Calculating the derivatives in the equation above allows us to plot the effective barotropic index $w_{\text{eff}}$ against the rescaled proper time of the comoving observer $x^0$ for different values of the model parameters (see Fig. 3). In each case the effective barotropic index $w_{\text{ent}}$ suddenly changes its value from zero to a value between $-0.2$ and $-1/3$. For higher values of the cosmological constant the transition occurs earlier and the slope is steeper. Similarly higher value of the $A$ parameter makes the transition occur earlier and the slope is steeper. It also results in more negative value of the effective barotropic index $w_{\text{ent}}$ after the transition. The analysis of the formula (46) shows that for higher value of the kinetic energy related with the free degrees of freedom (determined by the value of the constant $F$) the transition occurs later. On the other hand for sufficiently small value of the kinetic energy the transition disappears and the effective barotropic index maintains a constant value equal to approximately $-1/3$ all the time. An interesting issue here is the effect of an entanglement backreaction which according to (39) introduces the following correction to the value of the cosmological constant:

$$\Lambda \rightarrow \Lambda (1 - x^2)^2.$$

(47)
By equation (40) and Fig. 2 we see that the strong entanglement (for high value of the entropy of entanglement) can largely suppress the value of cosmological constant. On the other hand if the entanglement is weak the effect of backreaction disappears. Interestingly the backreaction on the vacuum energy does not affect the classical orbits of the system in the minisuperspace (eq. (7) and (8)). However, the backreaction of the entanglement induced effective perfect fluid has to be taken into account since for the case with $w_{\text{ent}} = -1/3$ (which corresponds to the cosmic strings) the density of the effective fluid may dominate the vacuum energy at early times.

5 Conclusions

We have shown that the canonical quantization of the Wheeler-DeWitt wave function for non-minimally coupled varying constants model introduced in [13] results in a theory which comprises a scenario that describes the two quantum mechanically entangled - one expanding and one contracting - branches. This is different form the scenario developed in [12] where the third quantization applied to the same model led to a scenario in which a whole multiverse subjected to Bose-Einstein distribution emerged form nothing. The discrepancy in these two scenarios follows form different interpretations of the representation dependent sets of vectors spanning the Hilbert space resulting form the third quantization procedure assumed in both approaches. In scenario given in [12] the orthonormal basis that generates the Hilbert space of the multiverse is assumed to represent an occupation with universes in a given state while in the scenario considered in the present paper the basis that spans the Hilbert space is assumed to represent an excitation levels of one of the two systems which naturally leads to entanglement in a pair of single universes that form the doubleverse (compare with approaches introduced in [7, 8, 9, 19, 20, 21]). Such an approach also facilitates a description of the entanglement in terms of quantities which are formal analogs of the ordinary thermodynamical quantities such as the entropy, the internal energy, heat and work. Including these analogs in the considerations about the multiverse has for the first time been done in [8], however, their relation with the ordinary thermodynamical quantities has never been clearly articulated. This seems to be important since any such relation could possibly equip our models with traits indicating existence of interuniversal entanglement. The postulated relation presented in this paper involves interpreting the energy of entanglement as a form of non-interacting energy homogeneously filling the space. In the framework of our model such assumption results in appearance of perfect fluid with the time dependent barotropic index which may influence the early-time evolution. It should be stressed that our postulate is of a very speculative nature since it was not derived from fundamental principles. However, making such additional assumptions seems to be unavoidable for the interuniversal entanglement to affect in any way the internal properties of a single universe and to become this way an observationally testable phenomenon (compare with the approaches postulating quadratic terms representing an interaction between the universes in the minisuperspace [2, 3, 4]).

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