Autoregressive Modeling and Forecasts of Degema Monthly Allocation: Buy’s-Ballot and Bartlett’s Transformation

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Author’s contribution

This work was carried out in collaboration among all authors. Author HA designed the study, wrote the protocols, and the first draft of the paper. Authors BOE and ED performed the statistical analysis, supervised the findings and author WDSA verified the analytical methods of the study. All authors read and approved the final manuscript.

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Abstract

The paper focused on Autoregressive modeling and forecasts of Degema Local Government Council Monthly Allocation (DLGCMA) in River State, Nigeria. The Buy’s-Ballot table and Bartlett’s Transformation method were adopted to identify the trend pattern and to determine the best transformation for the series. The logarithmic transformation was adjudged to be the best and was applied to stabilize the variance. Identification of the trend and stationary for the data set was done and the DLGCMA series showed a linear trend that was non-stationary. The stationarity of the DLGCMA series was obtained after the first difference. The ARIMA models were fitted to the series base on the behaviour of autocorrelation function (ACF) and partial autocorrelation function (PACF). Finally, the model selection criteria called Akaike information criterion was used to determine the best model among the predicted models. The AR(3,1,0) model

\[ X_t = 0.56X_{t-1} + 0.17X_{t-2} + 0.64X_{t-3} - 0.37X_{t-4} + e_t \]

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was considered to be the best model because it has the least value of the Akaike information criterion (AIC). Hence, the forecasts for the next allocation of twenty-four (24) months ahead were determined.

Keywords: Autoregressive modeling; forecasts; buys-ballot table; bartlett’s transformation method; Akaike Information Criterion (AIC).

1 Introduction

The first step in making any forecast about the future is that of collecting data on past observation. The ability to forecast optimally is to understand the dynamic relationship between variables. The forecast is essential in both short term and long term business plans as it helps farmers, pilots, and the like to plan their various activities in such a way to reduce unplanned risk. However, making such a forecast implies having a statistical model showing how the economy operates. The need to monitor, evaluate, and make an adequate plan for future compel managers, scientists, and researchers to collect data regularly on processes that vary as time passes. Mathematically, a time series may be defined as a set of observations \( y_1, y_2, \ldots, y_N \) of a variable \( Y \) at time \( t_1, t_2, \ldots, t_N \). Thus, \( y \) is a function of time \( t \). A time series is, however, the body of principles and techniques which deal with the analysis of the observed data \( Y_n = y_{1,n}, y_{2,n}, \ldots, y_{N,n} \). Usually, data are analyzed to find a model that approximates the true function underlying the generating random process. Time series data when analyzed may enhance the understanding of past and current patterns of change. They may also provide clues about future patterns to aid in forecasting.

This paper aims to use the autoregressive model to fit the DLGC data and forecast future value. The specific objectives of the paper are to: 1) Construct the Buys-Ballot table for DLGC monthly allocation and apply Bartlett’s Transformation method to obtain the best transformation for the data set, 2) Identification of trend and stationarity, 3) Build an ARIMA model for the transformed data, 4) Fit several autoregressive integrated moving average models base on the behaviour of autocorrelation function (ACF) and partial autocorrelation function (PACF), and 5) Use the appropriate ARIMA model to generate a series of forecasts values of DLGC monthly allocation.

This paper considered the monthly allocation of Degema Local Government Council (DLGC) for the period between 2010 to 2019. In Nigeria, the bulk of the revenue that accrues to Federal, State, and Local government is gotten from the Federation Account Allocation Committee (FAAC).

The significance of the paper is that it will help develop a statistical model by applying autoregressive modeling that can accurately and reliably estimate or forecast the monthly allocation of DLGC for budgeting. A total of 120 observations of DLGC were obtained via monthly allocation. The paper will be limited to the fitting of the autoregressive model.

This study was divided into different sections. Section 1 contains the introduction of time series, autoregressive model, and forecasts. The aim, objectives, and significance of the paper were also discussed. Section II deals with related works on autoregressive models, Section III contains the methodology as well as the choice of an appropriate transformation of DLGC monthly allocation. Section IV discusses results and discussion, and Section V deals with the conclusion and future direction.

2 Related Works

Forecasting is often the goal of statistical analysis in time series. The use of autoregressive representation of a stationary time series (or the innovation approach) in the analysis of time series has recently been attracting attention to many research workers as it is expected that this time-domain approach will give answers to many problems. Stephen et al., [1] compared the inflation of forecasting power of 19 potential indicators with that of historical inflation data autoregressive. They concluded that “No single indicator in statistical
framework clearly and consistently improved autoregressive projections. The paper provided another indication of the goodness and robustness of a simple autoregressive model for forecasting (inflation). Wright [2] in his title “Forecasting U.S. inflation by Bayesian model averaging” in contrast concluded that inflation forecasts derived from either (1) the equal-weighted output of a large number of different inflation forecasting model or (2) a Bayesian (empirically weight) average of these outputs substantially outperform a simple autoregression. Etuk [3] worked on autoregressive model identification. In his research, he found that when the underlying autoregressive process is of a full order, the partial autocorrelation function invariably process is superior. However, when a subset order autoregressive model generates the data, the inverse autocorrelation function is generally more informative. Gerald, [4] and Brooks, [5] discussed the properties of stationary and nonstationary time series, the calculation, and the use of autocorrelation function for forecasting. The Box-Jenkins methodology does not assume any particular pattern in the historical data of the series to be forecasted. Rather, it uses a three-step iterative approach of model identification, parameter estimation, and diagnostic checking to determine the best parsimonious model from a general class of ARIMA models. This three-step process is repeated several times until a satisfactory model is finally selected. The selected model can be used for forecasting future values of the time series. One criterion in Box-Jenkins methodology for optimal model selection is that the sample ACF and PACF, calculated from the data should match with the corresponding theoretical or actual values. Other widely used measures for model identification are Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC);(Henry) [6]: which are defined below;

\[
\text{AIC (p)} = N \ln \left( \frac{\sigma^2}{n} \right) + 2p \\
\text{BIC (p)} = N \ln \left( \frac{\sigma^2}{n} \right) + p + p \ln(n)
\]

Here \( n \) is the number of effective observations, used to fit the model, \( p \) is the number of parameters in the model, and \( \sigma^2 \) is the sum of sample squared residuals. The optimal model order is chosen by the number of model parameters, which minimizes either AIC or BIC. Other similar criteria have also been proposed in the literature for optimal model identification.

3 Methodology

A model is anything that is used to represent a real-life thing. A model is a simplified description of a complex system. Transformation is the application of a deterministic mathematical function to each point in a data set. It is usually applied to data that appear to meet the assumptions of statistical inference procedure and improve interpretability. Transformation is done to scale down measurements, estimate percentage effects, reducing outliers’ effect, overcome non – linearity, normalizing, variance stability, e.t.c. Bartlett’s [7] transformation technique is summarized in the Table 1.

| Table 1. Bartlett’s transformation technique for the values of \( \beta \) |
|---------|--------|--------|--------|--------|--------|--------|--------|
| S/No.   | 1      | 2      | 3      | 4      | 5      | 6      | 7      |
| \( \beta \) | 0      | 1/2    | 1      | 3/2    | 2      | 3      | -1     |
| Transformation | No Transformation | \( \sqrt{X_i} \) | \( \log(X_i) \) | \( \frac{1}{\sqrt{X_i}} \) | \( \frac{1}{X_i} \) | \( \frac{1}{X_i^2} \) | \( X_i^2 \) |
| Source: Iwueze et al., [8] |

The use of Bartlett’s transformation for time series data is to regress the natural logarithms for the group standard deviation (\( \tilde{\sigma}_i, i = 1,2,\ldots,m \)) against the group means (\( \overline{X}_i, i = 1,2,\ldots,m \)) which determines the slope of the equation

\[
\log_{e} \tilde{\sigma}_i = \alpha + \beta \log_{e} \overline{X} + \text{error}
\]
Table 2. Buys ballot table

| Period (i) | Season (j) | 1   | 2   | 3   | ⋯   | j   | ⋯   | s   | \(T_i\) | \(X_i\) | \(\hat{\sigma}_i\) |
|-----------|------------|-----|-----|-----|-----|-----|-----|-----|---------|---------|--------------|
| 1         | \(X_1\)   | \(X_2\) | \(X_3\) | ⋯   | \(X_j\) | ⋯   | \(X_s\) | \(T_1\) | \(\bar{X}_1\) | \(\hat{\sigma}_1\) |
| 2         | \(X_{s+1}\) | \(X_{s+2}\) | \(X_{s+3}\) | ⋯   | \(X_{s+j}\) | ⋯   | \(X_{2s}\) | \(T_2\) | \(\bar{X}_2\) | \(\hat{\sigma}_2\) |
| 3         | \(X_{2s+1}\) | \(X_{2s+2}\) | \(X_{2s+3}\) | ⋯   | \(X_{2s+j}\) | ⋯   | \(X_{3s}\) | \(T_3\) | \(\bar{X}_3\) | \(\hat{\sigma}_3\) |
| ⋯         | ⋯         | ⋯   | ⋯   | ⋯   | ⋯   | ⋯   | ⋯   | ⋯   | ⋯         | ⋯         | ⋯           |
| \(i\)     | \(X_{(i-1)s+1}\) | \(X_{(i-1)s+2}\) | \(X_{(i-1)s+3}\) | ⋯   | \(X_{(i-1)s+j}\) | ⋯   | \(X_{(i-1)s+s}\) | \(T_i\) | \(\bar{X}_i\) | \(\hat{\sigma}_i\) |
| ⋯         | ⋯         | ⋯   | ⋯   | ⋯   | ⋯   | ⋯   | ⋯   | ⋯   | ⋯         | ⋯         | ⋯           |
| \(m\)     | \(X_{(m-1)s+1}\) | \(X_{(m-1)s+2}\) | \(X_{(m-1)s+3}\) | ⋯   | \(X_{(m-1)s+j}\) | ⋯   | \(X_{ms}\) | \(T_m\) | \(\bar{X}_m\) | \(\hat{\sigma}_m\) |
| \(T_j\)   | \(T_1\)   | \(T_2\) | \(T_3\) | ⋯   | \(T_j\) | ⋯   | \(T_s\) | \(T_\cdot\) | ⋯         | ⋯         | ⋯           |
| \(\bar{X}_j\) | \(\bar{X}_1\) | \(\bar{X}_2\) | \(\bar{X}_3\) | ⋯   | \(\bar{X}_j\) | ⋯   | \(\bar{X}_s\) | \(\bar{X}_\cdot\) | ⋯         | ⋯         | ⋯           |
| \(\hat{\sigma}_j\) | \(\hat{\sigma}_1\) | \(\hat{\sigma}_2\) | \(\hat{\sigma}_3\) | ⋯   | \(\hat{\sigma}_j\) | ⋯   | \(\hat{\sigma}_s\) | \(\hat{\sigma}_\cdot\) | ⋯         | ⋯         | ⋯           |

Source: Iwuzee et al., [8]
Buys Ballot table recaps seasonal variations by summarizing the data thereby each row (i) contains a year and each column (j) contains a quarter or month of that year. A cell (i,j) in the table contains the mean value for all observations made during a particular period i at season j. The periods include the season totals, averages \( \bar{X}_i \) and \( \bar{X}_j \), and standard deviations \( \hat{\sigma}_i \) and \( \hat{\sigma}_j \). The grand mean and pooled standard deviation are also measured in Table 2.

Models for time series data can have many forms and represent different stochastic processes. When modeling variations in the level of a process, the three broad classes of practical importance are the autoregressive (AR) models, integrated (I), and moving average (MA) models. These three ideas produce autoregressive moving averages (ARMA) and autoregressive integrated moving averages (ARIMA) models.

An autoregressive (AR) model is a type of random process which is often used as a model to predict various types of natural and social phenomena. The model is given as:

\[
X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots + \phi_p X_{t-p}
\]

Equation (4) is an autoregressive process of the order p. In practice, the autoregressive process of the first order and second order, that is p=1, and p=2 respectively gives,

\[
X_t = \phi_1 X_{t-1} + \epsilon_t
\]

And

\[
X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \epsilon_t
\]

AR(p), can also take an equivalent form

\[
\phi(\beta)X_t = \epsilon_t
\]

which gives

\[
X_t = 1/\phi(\beta)\epsilon_t = \phi^{-1}(\beta)\epsilon_t
\]

A moving average (MA) model is a common type of univariate time series model. The notation MA(q) refers to the moving model of order q.

\[
X_t = \mu + \epsilon_t + \Theta_1 \epsilon_{t-1} + \Theta_q \epsilon_{t-q}
\]

where \( \mu \) is the mean of the series, the \( \Theta_1 \ldots \Theta_q \) are the parameters of the model and the \( \epsilon_t, \epsilon_{t-1}, \ldots \) are the white noise error terms. The value of q is called the order of the MA model. Sometimes the autocorrelation function (ACF) and partial autocorrelation function (PACF) will suggest that a MA model would be a better model choice and sometime both AR and MA terms should be used in the model Jenkins and Reinsel, [9].

Giving the time series of data \( X_t \) where t is an integer index and \( X_t \) are real numbers, then an ARMA (p, q) model is given by:

\[
(1-\sum_{i=1}^{P} \phi_i L)X_t = (1+\sum_{i=1}^{q} \Theta_i L)\epsilon_t
\]

Where \( L \) is the lag operator, the \( \phi_i \) are the parameters of the autoregressive parts of the model, \( \Theta_i \) are the parameters of the moving average parts and the \( \epsilon_t \) are the error terms. The error terms are generally assumed to be independent and identically distributed. An ARIMA (p, d, q) model is given by:

\[
1-(\sum_{i=1}^{P} \phi_i L)(1-L)^d X_t = 1+(\sum_{i=1}^{q} \phi_i L)\epsilon_t
\]
Thus, this can be thought in a particular case of ARMA \((p, d, q)\) process having the autoregressive polynomial with some roots. For this reason, every ARIMA model with \(d>0\) is not stationary. It is sometimes necessary to include autoregressive and moving average process of order \((p, q)\) in practice to obtain parameters. This sometimes abbreviated to ARMA \((p, q)\) which can be written as

\[
\phi(\beta)X_t = \Theta(\beta)\epsilon_t
\]

\[
X_t = \phi^{-1}(\beta)\Theta(\beta)\epsilon_t
\]

The mixed autoregressive moving average process can be thought of as an input \(X_t\) from a linear filter that transfers function in a ratio of polynomial \(\Theta(\beta)\) and \(\phi(\beta)\) when the input is white noise \(\epsilon_t\). The general representation of an autoregressive model, well-known as AR\((p)\) is

\[
Y_t = \alpha_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + \phi_p Y_{t-p} + \epsilon_t
\]

where the term \(\epsilon_t\) is the source of randomness and is called the white noise. It is assumed to have the following characteristics:

1. \(E[\epsilon_t]=0\)  
2. \(E[\epsilon^2_t]=\sigma^2\)  
3. \(E[\epsilon_t\epsilon_s]=0\) for \(s \neq t\)

The autocorrelation function plot is merely a bar-chart of the coefficient of correlation between a time series and lags of itself. The partial correlation function is the amount of correlation of a stationary time series with its own lagged values that is not explained by correlation at a lower order lag. If the PACF displays a sharp cut off and the ACF has significant spikes at higher lags, then the stationary series displays an “AR signature” meaning that the autocorrelation pattern can be explained more easily by adding AR terms.

A stationary time series \(\{X_t\}\) is said to follow an autoregressive model of order \(p\) if it satisfies the following difference equation.

\[
X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + \epsilon_t
\]

Where \(\{X_t\}\) is a white noise process with the variance \(\sigma^2\), for stationarity is required that;

\[
1- \phi_1 z + \phi_2 z^2 + \ldots + \phi_p z^p \neq 0, \quad |z| \leq 1
\]

### 4 Results and Discussions

The data set used in the paper is presented in the appendix; the data is a monthly allocation to DLGC (2010–2019) in River State, Nigeria. The data comprises of 12 months each year, a total of 10 years, and total observations (or points) of 120. The Buys-Ballot table as shown in Table 3 was used to compute the monthly and yearly means of DLGC series as represented in Figs. 2 and 3 respectively.

Figs. 1, 2, and 3 maintained an upward movement pattern with evidence of nonstationary. Table 3 shows the computed natural logarithm of the yearly means and standard deviation of DLGC series from Table 4. Table 4 is the computed monthly and yearly mean rows and columns with their corresponding standard deviations.

The computation of the natural logarithm of the yearly means and standard deviation of DLGC series from Table 4 is shown in Table 5.
Fig. 1. DMA series plots against time

Fig. 2. Monthly means plot of DMA series

Table 3. Natural logarithm of the yearly means and standard deviation

| Yearly means   | standard deviations | ln(Yearly means) | ln(standard deviations) |
|----------------|---------------------|------------------|-------------------------|
| 69,733,625.08  | 6851279.26          | 18.06019319      | 15.73994595             |
| 82,560,823.69  | 6918646.173         | 18.22904584      | 15.74973067             |
| 78,101,478.63  | 11389953.34         | 18.17351955      | 16.24824224             |
| 108,868,799.95 | 7939538.657         | 18.50565405      | 15.88737341             |
| 123,270,487.26 | 7041796.634         | 18.74288184      | 16.18313402             |
| 138,016,229.90 | 6205750.158         | 18.87248541      | 15.57737055             |
| 157,114,502.71 | 5795388.994         | 18.62996158      | 15.76737390             |
| 160,812,734.43 | 10671999.73         | 18.9074381       | 16.18313402             |
| 175,183,069.13 | 15813436.46         | 18.98134209      | 16.57637055             |

Similarly, the computation of the natural logarithm of the yearly means and standard deviation of the Natural Logarithm transformation Buys-Ballot Table in Table 5 is shown in Table 6.
Table 4. Buys-ballot table with means and standard deviations of DLGC monthly allocation

| MONTHS   | 2009  | 2010  | 2011  | 2012  | 2013  | 2014  | 2015  | 2016  | 2017  | 2018  | Monthly mean | standard deviation |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------------|-------------------|
| JANUARY  | 64,821| 76,098| 69,020| 98,634| 115,876,987| 137,458| 149,160| 158,927| 153,879| 181,376| 120,525 | 41669357.8         |
| FEBRUARY | 75,008| 81,564| 70,668| 100,564| 134,765,876| 129,490| 151,160| 156,282| 177,833| 148,947| 122,628 | 38037939.54       |
| MARCH    | 71,617| 69,030| 87,114| 105,333| 127,345,226| 127,048| 154,432| 162,150| 144,980| 159,700| 120,875 | 35736341.12       |
| APRIL    | 73,810| 86,777| 95,765| 102,356| 115,379,345| 135,021| 157,381| 155,160| 169,664| 171,097| 126,239 | 36063732.35       |
| MAY      | 59,219| 89,693| 74,990| 104,855| 129,843,089| 134,180| 148,445| 163,810| 149,334| 166,041| 122,041 | 37766150.66       |
| JUNE     | 61,469| 92,133| 59,178| 107,577| 113,546,878| 138,021| 154,388| 167,904| 153,908| 198,043| 125,365 | 44762064.05       |
| JULY     | 13,194| 85,733| 64,006| 110,655| 116,907,458| 140,458| 157,381| 152,799| 162,553| 185,036| 124,032 | 42646723.5       |
| AUGUST   | 65,400| 90,413| 79,337| 113,998| 119,392,087| 139,520| 161,451| 149,388| 166,767| 176,990| 126,265 | 38800385.11       |
| SEPTMBR  | 68,506| 83,917| 81,977| 108,988| 122,221,877| 143,834| 159,984| 159,566| 159,298| 153,206| 124,250 | 36101163.51       |
| OCTOBER  | 74,989| 78,431| 86,445| 110,781| 126,448,560| 148,800| 164,455| 173,255| 173,099| 177,543| 131,416 | 41454456.5       |
| NOVMBR   | 65,244| 75,613| 75,267| 114,206| 124,611,005| 137,420| 166,451| 161,733| 164,765| 193,778| 127,909 | 44781757.3       |
| DECEMBER | 85,041| 81,322| 93,444| 128,493| 131,907,453| 144,940| 160,680| 168,773| 176,443| 189,833| 136,087 | 39111289.96       |
| R        | 193.62| 436.61| 521.13| 026.00 | 56.00 | 267.18| 133.57| 087.93| 243.82| 422.23| 878.57    |                   |
| Yearly   | 69,733| 82,560| 78,101| 108,868| 123,270,487| 138,016| 157,114| 160,812| 162,703| 175,183|             |                   |
| mean     | 625.08| 823.69| 478.63| 799.95 | 26.00 | 229.90| 502.71| 177.12| 609.13 |             |                   |
| standard | 6851279.26| 5913646.26| 11389953| 7939599| 7041796.63| 6205750| 5793388.83| 6957043| 6171999.9| 15813436|             |                   |
| deviation| 173   | 34    | 657   | 4     | .158 | 994   | 399   | 73     | .46     |             |                   |
Table 5. Buys-ballot table with means and standard deviations of the natural logarithm transformation

| MONTHS   | 2010    | 2011    | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | Monthly mean | Standard deviation |
|----------|---------|---------|------|------|------|------|------|------|------|-----|-------------|-------------------|
| JANUARY  | 8.99    | 9.07    | 9.02 | 9.20 | 9.28 | 9.37 | 9.41 | 9.44 | 9.43 | 9.51| 9.27 | 0.188428573     |
| FEBRUARY | 9.07    | 9.11    | 9.04 | 9.21 | 9.36 | 9.34 | 9.42 | 9.43 | 9.50 | 9.41| 9.29 | 0.168174132     |
| MARCH    | 9.04    | 9.03    | 9.14 | 9.24 | 9.33 | 9.33 | 9.43 | 9.45 | 9.40 | 9.44| 9.28 | 0.16297254      |
| APRIL    | 9.06    | 9.14    | 9.19 | 9.22 | 9.28 | 9.36 | 9.44 | 9.43 | 9.47 | 9.48| 9.31 | 0.150649969     |
| MAY      | 8.95    | 9.16    | 9.07 | 9.23 | 9.34 | 9.36 | 9.41 | 9.46 | 9.41 | 9.46| 9.28 | 0.176773509     |
| JUNE     | 9.02    | 9.17    | 8.95 | 9.25 | 9.27 | 9.37 | 9.43 | 9.47 | 9.43 | 9.55| 9.29 | 0.196918045     |
| JULY     | 8.99    | 9.13    | 8.99 | 9.26 | 9.29 | 9.38 | 9.44 | 9.42 | 9.45 | 9.52| 9.29 | 0.192758258     |
| AUGUST   | 9.00    | 9.16    | 9.09 | 9.28 | 9.30 | 9.38 | 9.45 | 9.41 | 9.47 | 9.50| 9.30 | 0.17000162      |
| SEPTEMBER| 9.02    | 9.12    | 9.11 | 9.25 | 9.31 | 9.39 | 9.45 | 9.44 | 9.44 | 9.42| 9.30 | 0.161004927     |
| OCTOBER  | 9.07    | 9.09    | 9.14 | 9.26 | 9.33 | 9.41 | 9.46 | 9.49 | 9.48 | 9.50| 9.32 | 0.172404032     |
| NOVEMBER | 9.00    | 9.07    | 9.07 | 9.28 | 9.32 | 9.37 | 9.47 | 9.45 | 9.46 | 9.54| 9.30 | 0.193701566     |
| DECEMBER | 9.13    | 9.11    | 9.18 | 9.34 | 9.35 | 9.40 | 9.45 | 9.47 | 9.49 | 9.53| 9.34 | 0.15553248      |
| Yearly mean | 9.03 | 9.11    | 9.09 | 9.25 | 9.31 | 9.37 | 9.44 | 9.45 | 9.45 | 9.49| 9.49 |                  |
| standard deviation | 0.04800820 | 0.04280222 | 0.07420311 | 0.03532118 | 0.02852968 | 0.02556694 | 0.0184679 | 0.02159733 | 0.0303703 | 0.04588221 | 0.03303703 | 0.04588221 |
Table 6. Natural logarithm of the yearly means and standard deviation

| Yearly means | Standard deviations | ln(Yearly means) | ln(standard deviations) |
|-------------|---------------------|------------------|-------------------------|
| 9.03        | 0.048048209         | 2.200552         | -3.03555                |
| 9.11        | 0.042802226         | 2.209373         | -3.15117                |
| 9.08        | 0.074203119         | 2.206074         | -2.60095                |
| 9.25        | 0.035321182         | 2.224624         | -3.34327                |
| 9.31        | 0.028529684         | 2.231089         | -3.55681                |
| 9.37        | 0.022566945         | 2.237513         | -3.79127                |
| 9.44        | 0.01846797          | 2.244056         | -3.83519                |
| 9.45        | 0.021597333         | 2.246015         | -3.41013                |
| 9.49        | 0.045882213         | 2.250239         | -3.08168                |

The natural logarithms for the group standard deviations ( \( \hat{\sigma}_i, i = 1, 2, ..., m \) ) were regressed against the group means ( \( \bar{X}_i, i = 1, 2, ..., m \) ) to determine the slope of the equation:

\[
\log(e) \hat{\sigma}_i = \alpha + \beta \log(e) \bar{X} + \text{error}
\]

(19)
The slope of the equation in Fig. 4 denoted by \( \beta \) in equation (19) is 1.071 (\( \beta = 1.071 \)). This slope is not different from 1.00, implying that the data set required logarithm transformation from Table 1. The monthly and yearly means, standard deviations of the rows and column were computed from the logarithm transformed data shown in Table 5.

The natural logarithms transformation for the group standard deviations (\( \hat{\sigma}_i, i = 1,2,\ldots,m \)) were regressed against the group means (\( \bar{X}_i, i = 1,2,\ldots,m \)) to determines the new slope of the equation (19) as shown in Fig. 5.

![Fitted Line Plot](image)

**Fig. 5. Fitted line for the natural logarithm transformation series ln(standard deviation) versus ln(Yearly mean)**

The slope of the new transformed series 0.02877, implying that the new data set required no transformation.

The natural logarithm transformation in Fig. 6 below shows an upward trend and Fig. 7 show the differenced series indicating that the series is stationary.

![Trend Analysis Plot](image)

**Fig. 6. Natural log transformation series linear trend plot (lnDMA)**
Fig. 7. N Log transformation series linear trend plot (lnDMA)

The ACF and PACF plot of the differenced series used to determine the order of AR(p) and MA(q) indicates that there was a lag cut off at 1 and 3 respectively.

Based on the ACF and PACF plot, several ARIMA(p,d,q) models were built from the differenced series to determine the suitable (or best) model for prediction of DLGC monthly allocation. The model selection criteria were also computed in Table 7. In Table 7, the identified model is ARIMA(3,1,0), which can be written as

\[(1 - B)(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3)X_t = e_t\]  \hspace{1cm} (20)

Or

\[\left[1 - (1 + \phi_1)B + (\phi_1 - \phi_2)B^2 + (\phi_2 - \phi_3)B^3 + \phi_3 B^4\right]X_t = e_t\]

\[X_t = (1 + \phi_1)X_{t-1} - (\phi_1 - \phi_2)X_{t-2} + (\phi_2 - \phi_3)X_{t-3} + \phi_3 X_{t-4} + e_t\]  \hspace{1cm} (21)

Hence, the model can be written as:

\[(1 - B)\left[1 + 0.44B + 0.27B^2 - 0.37 B^3\right]X_t = e_t\]  \hspace{1cm} (22)

Or

\[X_t = 0.56 X_{t-1} + 0.17 X_{t-2} + 0.64 X_{t-3} - 0.37 X_{t-4} + e_t\]  \hspace{1cm} (23)

Equation 23 was used forecasts of DLGC monthly allocation for the year 2020 and 2021 as shown in Table 8.
### Table 7. ARIMA models, parameter estimates and model selection criteria (AIC and BIC)

| Models         | AR   | MA   | k   | Q(k) [df] | RSS | AIC | \( \sigma^2 \) |
|----------------|------|------|-----|-----------|-----|-----|---------------|
| ARIMA(1,1,0)   | -0.3353 | (0.000) | 1  | 26.4      | 45.7 | 57.5 | 59.7 | 0.9588 | 577.547742 |
| ARIMA(2,1,0)   | -0.3952 | -0.1213 | (0.000) | (0.190) | 2   | 26.6 | 47.6 | 58.7 | 60.9 | 0.9418 | 577.69449 |
| ARIMA(3,1,0)   | -0.4403 | -0.2676 | 0.3676 | (0.000) | (0.005) | (0.000) | 3 | 6.7 | 18.8 | 27.6 | 31.1 | 0.8221 | 592.006198 |
| ARIMA(0,1,1)   | 0.5151 | (0.000) | 1  | 18.2      | 37.0 | 47.2 | 51.1 | 0.8984 | 585.355794 |
| ARIMA(0,1,2)   | 0.4725 | 0.0678 | (0.000) | (0.446) | 2   | 17.8 | 37.3 | 46.9 | 48.9 | 0.8930 | 584.079253 |
| ARIMA(0,1,3)   | 0.4192 | 0.0446 | 0.0917 | (0.000) | (0.659) | (0.327) | 3 | 15.6 | 33.5 | 43.1 | 44.8 | 0.8888 | 582.644974 |
| ARIMA(1,1,1)   | 0.1391 | 0.6062 | 0.5151 | (0.441) | (0.000) | (0.000) | 1 | 18.2 | 37.0 | 47.2 | 51.1 | 0.8984 | 585.355794 |
| ARIMA(2,1,1)   | -1.518 | -0.1555 | -0.9536 | (0.000) | (0.140) | (0.000) | 3 | 22.4 | 34.2 | 44.7 | 50.2 | 0.8978 | Correlated |
| ARIMA(3,1,1)   | -0.4044 | -0.2534 | -0.3595 | 0.0407 | (0.010) | (0.056) | (0.000) | 4 | 6.4 | 18.5 | 27.4 | 30.7 | 0.8221 | Residual |
| ARIMA(1,1,2)   | 0.1070 | 0.5737 | 0.0180 | (0.932) | (0.647) | (0.978) | 3 | 17.7 | 37.1 | 46.7 | 48.6 | 0.8927 | -582.119573 |
| ARIMA(1,1,3)   | -0.9992 | -0.6305 | 0.4950 | 0.1955 | (0.000) | (0.000) | (0.038) | 4 | 9.6 | 20.0 | 28.1 | 33.2 | 0.8118 | Correlated |
| ARIMA(2,1,3)   | -0.5880 | -0.7407 | -0.1614 | 0.4950 | 0.1955 | (0.000) | (0.000) | (0.038) | 5 | 10.2 | 25.6 | 34.2 | 36.1 | 0.8309 | -586.728509 |
| ARIMA(2,1,2)   | 1.2077 | -0.4454 | 1.6791 | -0.8491 | (0.000) | (0.000) | (0.000) | (0.038) | 4 | 18.9 | 36.0 | 44.3 | 46.1 | 0.8491 | Residual |
| ARIMA(3,1,3)   | -0.3529 | 0.1521 | -0.4898 | 0.0346 | 0.4779 | 0.4002 | (0.000) | (0.001) | (0.012) | (0.004) | (0.128) | 6 | 6.3 | 15.2 | 23.3 | 29.6 | 0.7807 | Correlated |
Table 8. Forecasts for DLGC monthly allocations

| YEAR | Month     | Forecasts       | 95% Limits Lower | 95% Limits Upper |
|------|-----------|-----------------|------------------|------------------|
| 2020 | JANUARY   | ₦ 179,968,108.58 | ₦ 161,090,560.86 | ₦ 198,845,656.30 |
|      | FEBRUARY  | ₦ 181,325,941.72 | ₦ 160,324,985.71 | ₦ 202,326,897.73 |
|      | MARCH     | ₦ 184,628,833.59 | ₦ 161,876,178.94 | ₦ 207,381,488.24 |
|      | APRIL     | ₦ 185,481,043.30 | ₦ 161,914,269.32 | ₦ 209,047,817.27 |
|      | MAY       | ₦ 183,694,485.99 | ₦ 157,900,073.92 | ₦ 209,488,898.06 |
|      | JUNE      | ₦ 183,838,810.12 | ₦ 156,017,712.65 | ₦ 210,749,907.59 |
|      | JULY      | ₦ 183,801,264.90 | ₦ 154,877,725.92 | ₦ 212,724,803.87 |
|      | AUGUST    | ₦ 184,207,604.13 | ₦ 154,105,073.78 | ₦ 214,310,134.49 |
|      | SEPTEMBER | ₦ 183,972,176.10 | ₦ 152,494,620.50 | ₦ 215,449,731.69 |
|      | OCTOBER   | ₦ 183,852,315.25 | ₦ 151,116,844.48 | ₦ 210,587,806.02 |
|      | NOVEMBER  | ₦ 183,860,329.44 | ₦ 149,868,848.93 | ₦ 220,216,456.11 |
|      | DECEMBER  | ₦ 183,942,542.57 | ₦ 147,668,629.03 | ₦ 221,290,312.44 |
| 2021 | JANUARY   | ₦ 183,925,753.00 | ₦ 144,441,624.65 | ₦ 223,409,881.36 |
|      | FEBRUARY  | ₦ 183,920,689.22 | ₦ 146,551,066.01 | ₦ 222,290,312.44 |
|      | MARCH     | ₦ 183,907,241.38 | ₦ 145,458,148.59 | ₦ 224,426,106.46 |
|      | APRIL     | ₦ 183,926,615.75 | ₦ 143,427,125.03 | ₦ 225,411,436.60 |
|      | MAY       | ₦ 183,924,871.23 | ₦ 142,438,305.86 | ₦ 226,374,711.92 |
|      | JUNE      | ₦ 183,925,753.00 | ₦ 144,441,624.65 | ₦ 227,321,041.84 |
|      | JULY      | ₦ 183,920,012.90 | ₦ 141,465,313.89 | ₦ 228,246,403.02 |
|      | AUGUST    | ₦ 183,922,746.03 | ₦ 140,524,450.23 | ₦ 229,151,767.20 |
|      | SEPTEMBER | ₦ 183,923,255.44 | ₦ 139,600,107.86 | ₦ 230,038,759.45 |
|      | OCTOBER   | ₦ 183,923,656.45 | ₦ 138,695,545.71 | ₦ 230,909,878.82 |
|      | NOVEMBER  | ₦ 183,922,492.19 | ₦ 137,800,224.93 | ₦ 230,038,759.45 |
|      | DECEMBER  | ₦ 183,922,822.57 | ₦ 136,935,766.32 | ₦ 230,909,878.82 |

5 Conclusion

The paper examined autoregressive modeling and forecast of monthly allocation with special emphases on Degema Local Government Council in River State, Nigeria. The autoregressive model is effective and reliable in the forecasting of monthly allocation. This is because it uses historical data to predict the nearest future. Buys-Ballot table and apply Bartlett’s Transformation was used to stabilize the variance of the series. The transformed series was difference once to obtain stationarity. Several ARIMA models were developed and the suitable model that fit DLGC’s monthly allocation was ARIMA $(3,1,0)$ $(X_t = 0.56X_{t-1} + 0.17X_{t-2} + 0.64X_{t-3} - 0.37X_{t-4} + e_t)$. Forecasts for 24 months were derived from the chosen ARIMA model. The model will help the Degema Local Government Council in the following ways;

(i) To prepare its annual budget based on the forecast result
(ii) It will help the council chairman to be prudent in financial management.
(iii) It will be an eye-opener and make him his resources very well within the period.

Competing Interests

Authors have declared that no competing interests exist.

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