Theory of Dyakonov–Tamm waves at the planar interface of a sculptured nematic thin film and an isotropic dielectric material

Kartiek Agarwal$^{1,2}$, John A. Polo, Jr.$^3$, and Akhlesh Lakhtakia$^{2,4}$

$^1$ Department of Electrical Engineering, Indian Institute of Technology Kanpur, Kanpur 208016, India
$^2$ NanoMM—Nanoengineered Metamaterials Group, Department of Engineering Science and Mechanics, Pennsylvania State University, University Park, PA 16802, USA
$^3$ Department of Physics and Technology, Edinboro University of Pennsylvania, Edinboro, PA 16444, USA
$^4$ Department of Physics, Indian Institute of Technology Kanpur, Kanpur 208016, India

Abstract. In order to ascertain conditions for surface–wave propagation guided by the planar interface of an isotropic dielectric material and a sculptured nematic thin film (SNTF) with periodic nonhomogeneity, we formulated a boundary–value problem, obtained a dispersion equation therefrom, and numerically solved it. The surface waves obtained are Dyakonov–Tamm waves. The angular domain formed by the directions of propagation of the Dyakonov–Tamm waves can be very wide (even as wide as to allow propagation in every direction in the interface plane), because of the periodic nonhomogeneity of the SNTF. A search for Dyakonov–Tamm waves is, at the present time, the most promising route to take for experimental verification of surface–wave propagation guided by the interface of two dielectric materials, at least one of which is anisotropic. That would also assist in realizing the potential of such surface waves for optical sensing of various types of analytes infiltrating one or both of the two dielectric materials.

Keywords: Dyakonov wave, optical sensing, sculptured nematic thin film, surface wave, Tamm state, titanium oxide

‡ Corresponding author; e-mail:akhlesh@psu.edu
1. Introduction

In 1988, Dyakonov [1] theoretically predicted the propagation of electromagnetic waves guided by the planar interface of two homogeneous dielectric materials, one of which is isotropic and the other uniaxial with its optic axis aligned parallel to the interface. Since then, the existence of Dyakonov surface waves has been theoretically proved for many sets of dissimilar dielectric partnering materials, at least one of which is anisotropic [2, 3]. The possibility of the anisotropic partnering material being artificially engineered, either as a photonic crystal with a short period in comparison to the wavelength [4] or as a columnar thin film (CTF) [5], has also emerged. Just like that of many surface phenomena [6], the significance of Dyakonov surface waves for optical sensing applications is obvious: the disturbance of the constitutive properties of one or both of the two partnering materials—due to, say, infiltration by any analyte—could measurably change the characteristics of the chosen Dyakonov surface wave.

However, the directions of propagation of Dyakonov surface waves parallel to any suitable planar interface are confined to a very narrow angular domain, typically of the order of a degree or less [2, 7]. Not surprisingly, Dyakonov surface waves still remain to be experimentally observed. Clearly then, the potential of Dyakonov surface waves cannot be realized if they cannot be demonstrably excited; a wider angular domain is needed.

Recently, Lakhtakia and Polo [8] examined surface-wave propagation guided by the planar interface of an isotropic dielectric material and a chiral sculptured thin film [9] with its direction of periodic nonhomogeneity normal to the planar interface. The authors named the surfaces waves after Dyakonov and Tamm, the latter being the person who indicated the possibility of finding surface states of electrons at exposed planes of crystals and other periodic materials [10, 11]. The angular domain formed by the directions of propagation of the Dyakonov–Tamm waves turned out to be very wide, as much as $98^\circ$ in one case for which realistic constitutive parameters of the two partnering materials were employed [8]. This implies that Dyakonov–Tamm waves could be detected much more easily than Dyakonov surface waves.

Chiral sculptured thin films (STFs) are structurally chiral materials§ the

---

§ An object is said to be chiral if it cannot be made to coincide with its mirror-image by translations and/or rotations. There are two types of chiral materials: (i) microscopically or molecularly chiral materials, and (ii) structurally chiral materials. The first type of chiral materials either have chiral molecules or are composite materials made by embedding, e.g., electrically small helixes in a host material. Materials comprising chiral molecules have been known for about two hundred years, as a perusal of an anthology of milestone papers [12] will show to the interested reader. These materials are generally (optically) isotropic [13]. Composite materials comprising electrically small [14, 15] chiral inclusions were first reported in 1898 [16], and these materials can be either isotropic [17, 18] or nonisotropic [20]. The first type of chiral materials can be considered as either homogeneous or nonhomogeneous continua at sufficiently low frequencies. In contrast, the second type of chiral materials can only be nonhomogeneous and anisotropic continua at the length–scales of interest, their constitutive parameters varying periodically in a chiral manner about a fixed axis. Take away the nonhomogeneity of a structurally chiral material, and its (macroscopic) chirality will also vanish.
Dyakonov–Tamm waves at nematic thin film/isotropic dielectric interface

Figure 1. Schematic of a collimated vapor flux responsible for the growth of tilted straight nanowires growing at an angle $\chi \geq \chi_v$.

permittivity dyadic rotates at a uniform rate along the direction of nonhomogeneity. As a result, chiral STFs are periodically nonhomogeneous. Thus, in a chiral STF, structural chirality and periodic nonhomogeneity are inseparable. Is the huge angular existence domain of Dyakonov–Tamm waves for the case investigated by Lakhtakia and Polo [8] due to the periodicity or due to the structural chirality of the chiral STF? In order to answer this question, we devised a surface–wave–propagation problem wherein the chiral STF is replaced by a periodically nonhomogeneous and anisotropic material that is not structurally chiral. Specifically, we replaced the chiral STF with a sculptured nematic thin film (SNTF) [9].

CTFs, chiral STFs, and SNTFs are all fabricated by physical vapor deposition [9, 22, 23]. In the simplest implementation of this technique, a boat containing a certain material, say titanium oxide, is placed in an evacuated chamber. Under appropriate conditions, the material evaporates towards a substrate such that the vapor flux is highly collimated. The collimated vapor flux coalesces on the substrate as an ensemble of more or less identical and parallel nanowires, due to self–shadowing. If the substrate is held stationary during deposition, a CTF grows wherein the nanowires are straight and aligned at an angle $\chi \geq \chi_v$ with respect to the substrate plane, where $\chi_v$ is the angle between the collimated vapor flux and the substrate plane, as shown in Fig. 1. If the substrate is rotated about an axis passing normally through it, a chiral STF comprising helical nanowires grows. Finally, if the substrate is rocked about an axis tangential to the substrate plane, $\chi_v$ and $\chi$ are not constant and an SNTF grows. Scanning-electron-microscope images of a CTF, an SNTF, and a chiral STF are presented in Fig. 2.

This paper is organized as follows. Section 2 presents the boundary–value problem and the dispersion equation for the propagation of Dyakonov–Tamm waves guided by the interface of a homogenous, isotropic dielectric material and a periodically nonhomogeneous SNTF. Section 3 contains numerical results when the SNTF is chosen to be made of titanium oxide [4, 24]. An $\exp(-i\omega t)$ time–dependence is implicit, with $\omega$ [9, 21].
denoting the angular frequency. The free–space wavenumber, the free–space wavelength, and the intrinsic impedance of free space are denoted by 

\[ k_o = \omega \sqrt{\varepsilon_o\mu_o}, \lambda_o = 2\pi/k_o, \text{ and } \eta_o = \sqrt{\mu_o/\varepsilon_o}, \]

respectively, with \( \mu_o \) and \( \varepsilon_o \) being the permeability and permittivity of free space. Vectors are underlined, dyadics underlined twice; column vectors are underlined and enclosed within square brackets, while matrices are underlined twice and similarly bracketed. Cartesian unit vectors are identified as \( u_x, u_y \) and \( u_z \). The dyadics employed in the following sections can be treated as \( 3 \times 3 \) matrixes \([25]\).

2. Formulation

2.1. Geometry and permittivity

Let the half–space \( z \leq 0 \) be occupied by an isotropic, homogeneous, nondissipative, dielectric material of refractive index \( n_s \). The region \( z \geq 0 \) is occupied by an SNTF with a periodically nonhomogeneous permittivity dyadic given by \([9]\)

\[ \varepsilon(z) = \varepsilon_o S_z(\gamma) \cdot S_y(z) \cdot \varepsilon^{\circ \text{ref}}_z(z) \cdot S_T(z) \cdot S_T(\gamma), \quad z \geq 0. \quad (1) \]

The dyadic function

\[ S_z(\gamma) = (u_x u_x + u_y u_y) \cos \gamma \]
\[ + (u_y u_x - u_x u_y) \sin \gamma + u_z u_z \quad (2) \]

contains \( \gamma \) as an angular offset, and the superscript \( T \) denotes the transpose. The dyadics

\[ S_y(z) = (u_x u_x + u_z u_z) \cos [\chi(z)] + (u_y u_x - u_x u_y) \sin [\chi(z)] + u_y u_y \quad (3) \]

and

\[ \varepsilon^{\circ \text{ef}}(z) = \varepsilon_a(z) u_x u_x + \varepsilon_b(z) u_y u_x + \varepsilon_c(z) u_y u_y \quad (4) \]
Dyakonov–Tamm waves at nematic thin film/isotropic dielectric interface

depend on the vapor incidence angle
\[
\chi_v(z) = \tilde{\chi}_v + \delta_v \sin \left( \frac{\pi z}{\Omega} \right)
\]  
(5)

that varies sinusoidally as a function of \(z\) with period \(2\Omega\). Whereas \(\tilde{\chi}_v \geq 0\), we have to ensure that \(\chi_v(z) \in (0, \pi/2]\).

As no experimental data connecting \(\epsilon_{a,b,c}(z)\) and \(\chi(z)\) to \(\chi_v(z)\) have been reported for SNTFs (\(\delta_v \neq 0\)), we decided to use available data for CTFs (\(\delta_v = 0\)). Optical characterization experiments on CTFs of titanium oxide at \(\lambda_o = 633\) nm [24] lead to the following expressions for our present purpose:

\[
\begin{align*}
\epsilon_a(z) &= \left[ 1.0443 + 2.7394 v(z) - 1.3697 v^2(z) \right]^2 \\
\epsilon_b(z) &= \left[ 1.6765 + 1.5649 v(z) - 0.7825 v^2(z) \right]^2 \\
\epsilon_c(z) &= \left[ 1.3586 + 2.1109 v(z) - 1.0554 v^2(z) \right]^2 \\
\chi(z) &= \tan^{-1}[2.8818 \tan \chi_v(z)] \\
v(z) &= 2\chi_v(z)/\pi
\end{align*}
\]  
(6)

Let us note that the foregoing expressions—with \(v(z)\) independent of \(z\)—are applicable to CTFs produced by one particular experimental apparatus, but may have to be modified for CTFs produced by other researchers on different apparatuses.

Without loss of generality, we take the Dyakonov–Tamm wave to propagate parallel to the \(x\) axis in the plane \(z = 0\). There is no dependence on the \(y\) coordinate, whereas the Dyakonov–Tamm wave must attenuate as \(z \to \pm \infty\).

2.2. Field representations

In the region \(z \leq 0\), the wave vector may be written as
\[
k_s = \kappa u_x - \alpha_s u_z,
\]  
(7)

where
\[
\kappa^2 + \alpha^2 = k_o^2 n_s^2,
\]  
(8)

\(\kappa\) is positive and real–valued for unattenuated propagation along the \(x\) axis, and \(\text{Im}[\alpha_s] > 0\) for attenuation as \(z \to -\infty\). Accordingly, the field phasors in the region \(z \leq 0\) may be written as \[8\]

\[
E(r) = \left[ A_s u_y + A_p \left( \frac{\alpha_s}{k_o} u_x + \frac{\kappa}{k_o} u_z \right) \right] \exp(ik_s \cdot r), \quad z \leq 0,
\]  
(9)

and
\[
H(r) = \eta_o^{-1} \left[ A_s \left( \frac{\alpha_s}{k_o} u_x + \frac{\kappa}{k_o} u_z \right) - A_p n_s^2 u_y \right] \exp(ik_s \cdot r), \quad z \leq 0,
\]  
(10)

where \(A_s\) and \(A_p\) are unknown scalars representing the amplitudes of \(s\)– and \(p\)–polarized components.
The field representation in the region \( z \geq 0 \) is more complicated. It is appropriate to write
\[
\begin{align*}
E(z) &= e(z) \exp(i\kappa x) \\
H(z) &= h(z) \exp(i\kappa x)
\end{align*}
\]
and create the column vector
\[
[f(z)] = [e_x(z) \quad e_y(z) \quad h_x(z) \quad h_y(z)]^T.
\]
This column vector satisfies the matrix differential equation \( \frac{d}{dz} f(z) = i [P(z, \kappa)] \cdot f(z) \), \( z > 0 \),
where the \( 4 \times 4 \) matrix
\[
[P(z, \kappa)] = 
\begin{bmatrix}
0 & 0 & 0 & \mu_o \\
0 & 0 & -\mu_o & 0 \\
\epsilon_o [e_c(z) - e_d(z)] \cos \gamma \sin \gamma & -\epsilon_o [e_c(z) \cos^2 \gamma + e_d(z) \sin^2 \gamma] & 0 & 0 \\
\epsilon_o [e_c(z) \sin^2 \gamma + e_d(z) \cos^2 \gamma] & -\epsilon_o [e_c(z) - e_d(z)] \cos \gamma \sin \gamma & 0 & 0
\end{bmatrix}
\]
\[
+ \kappa \frac{\epsilon_d(z)}{\epsilon_a(z) \epsilon_b(z)} [e_a(z) - e_b(z)] \sin \chi(z) \cos \chi(z)
\]
\[
\begin{bmatrix}
0 & 0 & 0 & -\kappa^2 \omega \epsilon_o \epsilon_b(z) \\
0 & 0 & 0 & 0 \\
0 & \kappa^2 \omega \mu_o & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
and
\[
\epsilon_d(z) = \frac{\epsilon_a(z) \epsilon_b(z)}{\epsilon_a(z) \cos^2 \chi(z) + \epsilon_b(z) \sin^2 \chi(z)}.
\]
For \( \sin \gamma = 0 \), \( (13) \) splits into two autonomous matrix differential equations, each employing a \( 2 \times 2 \) matrix \( \underline{26} \).

Equation \( (13) \) has to be solved numerically. We used the piecewise uniform approximation technique \( \underline{9} \) to determine the matrix \( \underline{[N]} \) which appears in the relation
\[
[f(2\Omega)] = \underline{[N]} \cdot [f(0+)]
\]
to characterize the optical response of one period of the chosen SNTF. By virtue of the Floquet–Lyapunov theorem \( \underline{27} \), we can define a matrix \( \underline{[Q]} \) such that
\[
\underline{[N]} = \exp \left\{ i2\Omega[\underline{Q}] \right\}.
\]
Both $[N]$ and $[Q]$ share the same eigenvectors, and their eigenvalues are also related. Let $[\mathbf{t}]^{(n)}$, $(n = 1, 2, 3, 4)$, be the eigenvector corresponding to the $n$th eigenvalue $\sigma_n$ of $[N]$; then, the corresponding eigenvalue $\alpha_n$ of $[Q]$ is given by

$$\alpha_n = -i \frac{\ln \sigma_n}{2\Omega}.$$  \hfill (18)

2.3. Dispersion equation for Dyakonov–Tamm wave

For the Dyakonov–Tamm wave to propagate along the $x$ axis, we must ensure that $\text{Im}[\alpha_{1,2}] > 0$, and set

$$[f(0+)] = [\mathbf{t}]^{(1)} \cdot \begin{bmatrix} B_1 \\ B_2 \end{bmatrix},$$  \hfill (19)

where $B_1$ and $B_2$ are unknown scalars; the other two eigenvalues of $[Q]$ describe waves that amplify as $z \to \infty$ and cannot therefore contribute to the Dyakonov–Tamm wave. At the same time,

$$[f(0-)] = \begin{bmatrix} 0 & \frac{\alpha}{k_o} \\ 1 & 0 \\ \frac{\alpha}{k_o} \eta_o^{-1} & 0 \\ 0 & -n_s^2 \eta_o^{-1} \end{bmatrix} \cdot \begin{bmatrix} A_s \\ A_p \end{bmatrix},$$  \hfill (20)

by virtue of (9) and (10). Continuity of the tangential components of the electric and magnetic field phasors across the plane $z = 0$ requires that

$$[f(0-)] = [f(0+)],$$  \hfill (21)

which may be rearranged as

$$[M] \cdot \begin{bmatrix} A_s \\ A_p \\ B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$  \hfill (22)

For a nontrivial solution, the $4 \times 4$ matrix $[M]$ must be singular, so that

$$\text{det} [M] = 0$$  \hfill (23)

is the dispersion equation for the Dyakonov–Tamm wave. The value of $\kappa$ satisfying (23) was obtained by employing the Newton–Raphson method [28].

3. Numerical Results and Discussion

We set $\lambda_o = 633 \text{ nm}$ in accordance with (6), since calculations were performed only for CTFs or SNTFs composed of titanium oxide. All calculations for the periodically nonhomogeneous SNTFs were performed for $\bar{\chi}_v = 19.1^\circ$ and $\Omega = 197 \text{ nm}$ with two oscillation amplitudes: $\delta_v = 7.2^\circ$ and $16.2^\circ$. For comparison, calculations were
performed on CTFs for $\chi_v(z) \equiv 7.2^\circ$ and $19.1^\circ \forall z \geq 0$. The former is the approximate lower limit of $\chi_v$ obtainable with current STF technology. For a chosen set of values of $n_s$, $\tilde{\chi}_v$, and $\delta_v$, surface–wave propagation was found to occur in four separate $\gamma$-ranges: $\gamma \in [\pm \gamma_m - \Delta \gamma/2, \pm \gamma_m + \Delta \gamma/2]$ and $\gamma \in [\pm \gamma_m + 180^\circ - \Delta \gamma/2, \pm \gamma_m + 180^\circ + \Delta \gamma/2]$. Here the angle $\gamma_m \in (0^\circ, 90^\circ)$ is used to describe the mid–point of a $\gamma$-range while $\Delta \gamma \leq 90^\circ$ is the extent of that range. Since the surface–wave characteristics are the same in all four $\gamma$-ranges, results are displayed only for $\gamma \in [\gamma_m - \Delta \gamma/2, \gamma_m + \Delta \gamma/2]$.

The magnitude of the phase velocity of the Dyakonov–Tamm wave was compared with that of the phase velocity of the bulk wave in the isotropic partnering material. For this purpose, we defined the relative phase speed

$$v = v_{DT} n_s \sqrt{\epsilon_o \mu_o}, \quad (24)$$

where $v_{DT} = \omega / \kappa$ is the phase speed of the Dyakonov–Tamm wave.

Let us begin the presentation and discussion of results with the characteristics of Dyakonov waves guided by the interface of a CTF ($\delta_v = 0^\circ$) and an isotropic dielectric material [5]. Figure 3 shows plots of $v$ versus $\gamma$ with: $\chi_v = 7.2^\circ$ for $n_s = 1.57, 1.59, 1.61, 1.63, 1.65, 1.67, 1.69, 1.71, \text{and } 1.73$; and $\chi_v = 19.1^\circ$ for $n_s = 1.80, 1.82, \text{and } 1.84$. Three characteristics of Dyakonov waves can be garnered from this figure:

(i) the nearly vertical curves demonstrate the extremely small width $\Delta \gamma$ of the $\gamma$-range, leading to very narrow angular existence domains as we remarked upon in Section [1];

(ii) higher values of $n_s$ result in higher values of $\gamma_m$; and

(iii) lower values of $\chi_v$ also result in higher values of $\gamma_m$.

Figure 4 shows $v$ as a function of $\gamma$ when $\tilde{\chi}_v = 19.1^\circ$ for three groups of parameters:

• $\delta_v = 0^\circ$ for $n_s = 1.80, 1.82, \text{and } 1.84$;

• $\delta_v = 7.2^\circ$ for $n_s = 1.73, 1.77, 1.82, 1.88, 1.92, \text{and } 1.96$; and

• $\delta_v = 16.2^\circ$ for $n_s = 1.80 \text{and } 1.84$.

The first group has a CTF as the anisotropic partnering material, whereas the second and the third groups have a periodically nonhomogeneous SNTF serving that role. Immediately apparent is the dramatic increase in $\Delta \gamma$ brought about by the introduction of sinusoidal oscillation in the vapor incidence angle. With $\Delta \gamma$ on the order of tens of degrees when an SNTF is the anisotropic partnering material, the width of the $\gamma$-range is orders of magnitude greater than that obtained with a CTF as the anisotropic partnering material. As $n_s$ increases, the $\gamma$-range supporting surface–wave propagation widens and the mid-point $\gamma_m$ shifts to higher values. When $\delta_v = 16.2^\circ$ and $n_s$ is increased to 1.84, $\Delta \gamma$ actually increases to $90^\circ$, which means that the four separate $\gamma$-ranges merge into one, thereby allowing surface–wave propagation to occur for any value of $\gamma$.

Figure 4 also indicates that the average value of $v$ over the $\gamma$-range rises as $n_s$ increases, for both $\delta_v = 7.2^\circ$ and $16.2^\circ$. For a given value of $\gamma$, $v$ takes on similar values for CTFs and SNTFs.
Figure 3. Relative phase speed $\overline{\nu}$ as a function of $\gamma$ when $\delta_v = 0^\circ$. The values of $\tilde{\chi}_v$ are as follows: (1)-(9) 7.2°, (10)-(12) 19.1°. The values of $n_s$ are as follows: (1) 1.57, (2) 1.59, (3) 1.61, (4) 1.63, (5) 1.65, (6) 1.67, (7) 1.69, (8) 1.71, (9) 1.73, (10) 1.80, (11) 1.82, (12) 1.84.

Figure 4. Relative phase speed $\overline{\nu}$ as a function of $\gamma$ when $\tilde{\chi}_v = 19.1^\circ$. The values of $\delta_v$ are as follows: (1)-(3) 0°, (4)-(9) 7.2°, (10) and (11) 16.2°. The values of $n_s$ are as follows: (1) 1.80, (2) 1.82, (3) 1.84, (4) 1.73, (5) 1.77, (6) 1.82 (7) 1.88, (8) 1.92, (9) 1.96, (10) 1.80, (11) 1.84.
Figure 5. Normalized decay constants $\text{Im}[\alpha_1]/k_o$ and $\text{Im}[\alpha_2]/k_o$ as functions of $\gamma$ when $\delta_v = 0^\circ$. The values of $\tilde{\chi}_v$ are as follows: (1)-(9) $7.2^\circ$, (10)-(12) $19.1^\circ$. The values of $n_s$ are as follows: (1) 1.57, (2) 1.59, (3) 1.61, (4) 1.63, (5) 1.65, (6) 1.67, (7) 1.69, (8) 1.71, (9) 1.73, (10) 1.80, (11) 1.82, (12) 1.84.

The localization of the surface wave about the bimaterial interface is described by the decay constants $\text{Im}[\alpha_1]$ and $\text{Im}[\alpha_2]$ in the anisotropic partnering material and by $\text{Im}[\alpha_s]$ in the isotropic partnering material. Figure 5 displays the two decay constants in the CTF normalized to the free-space wavenumber, $\text{Im}[\alpha_1]/k_o$ and $\text{Im}[\alpha_2]/k_o$, as functions of $\gamma$ for $\tilde{\chi}_v = 7.2^\circ$ and $19.1^\circ$ for the same values of $n_s$ as in Figure 3. The values of $\text{Im}[\alpha_1]/k_o$ show considerable variation over the $\gamma$-range for a given set of $\chi_v$ and $n_s$ even though the $\gamma$-range is very narrow, as shown by the nearly vertical curves in Figure 5a. Unlike $\text{Im}[\alpha_1]/k_o$, $\text{Im}[\alpha_2]/k_o$ shows little variation over the $\gamma$-range and, for most values of $n_s$, appears as a single point in Figure 5b. Furthermore, $\text{Im}[\alpha_2]/k_o$ is roughly a linear function of $\gamma$ with a positive slope, for a given value of $\chi_v$. The slope increases as $\chi_v$ increases.

Figures 6 and 7 show the decay constants in the anisotropic partnering material as functions of $\gamma$ for $\tilde{\chi}_v = 19.1^\circ$, the remaining parameters being the same as in Figure 3. The values of $\text{Im}[\alpha_1]/k_o$ in Figure 6 are larger in the SNTFs than they are in the CTFs for the same values of $n_s$ and $\gamma$. The values of $\text{Im}[\alpha_1]/k_o$ appear to decrease towards zero at the upper range of $\gamma$ for each value of $n_s$, except for those cases where the range of $\gamma$ extends to $90^\circ$. In the two cases where the $\gamma$-range extends to $90^\circ$, $\text{Im}[\alpha_1]/k_o$ shows a minimum at $90^\circ$. It should be kept in mind that neither $0^\circ$ nor $90^\circ$ truly represent the limits of a $\gamma$-range for surface-wave propagation, since curves which seem to end at these points are joined to curves of one of the other three, in general, separate $\gamma$-ranges, thereby resulting in only two distinct $\gamma$-ranges.

Values of $\text{Im}[\alpha_2]/k_o$ in the SNTF when $\delta_v = 7.2^\circ$ are nearly identical to those in the CTF at a given value of $\gamma$, as shown in Figure 6b, for $\tilde{\chi}_v = 19.1^\circ$. The curves of $\text{Im}[\alpha_2]/k_o$ versus $\gamma$ for the various values of $n_s$ nearly join to form a single smooth continuous curve, except near $\gamma = 0^\circ$ where the curves for $n_s = 1.73, 1.77$ and $1.82$ bifurcate. The value of $\text{Im}[\alpha_2]/k_o$ increases as $n_s$ and $\gamma$ increase.
Dyakonov–Tamm waves at nematic thin film/isotropic dielectric interface

Figure 6. Normalized decay constants $\text{Im}[\alpha_1]/k_0$ and $\text{Im}[\alpha_2]/k_0$ as functions of $\gamma$ when $\tilde{\chi}_v = 19.1^\circ$. The values of $\delta_v$ are as follows: (1)-(3) 0°, (5)-(9) 7.2°, (10) and (11) 16.2°. The values of $n_s$ are as follows: (1) 1.80, (2) 1.82, (3) 1.84, (4) 1.73, (5) 1.77, (6) 1.82, (7) 1.88, (8) 1.92, (9) 1.96, (10) 1.80, (11) 1.84.

Figure 7. Normalized decay constant $\text{Im}(\alpha_2)/k_0$ as a function of $\gamma$ when $\tilde{\chi}_v = 19.1^\circ$ and $\delta_v = 16.2^\circ$. The values of $n_s$ are (1) 1.80, (2) 1.84.

The behaviour of $\text{Im}[\alpha_2]/k_0$ for higher modulation amplitude, $\delta_v = 16.2^\circ$, is presented in Figure 4 for $n_s = 1.80$ and 1.84. At low values of $\gamma$, $\text{Im}[\alpha_2]/k_0$ is larger for $\delta_v = 16.2^\circ$ than for $\delta_v = 7.2^\circ$. As $\gamma$ approaches 90°, however, the value of $\text{Im}[\alpha_2]/k_0$ for $\delta_v = 16.2^\circ$, for both $n_s = 1.80$ and 1.84, approaches a value only slightly less that observed when $\delta_v = 7.2^\circ$ and $n_s = 1.96$.

The polarization state of the surface wave in the isotropic partnering material was also investigated. First, let us present data when the anisotropic partnering material is a CTF ($\delta_v = 0^\circ$). Figure 8 shows the ratio of the $s$-polarization amplitude to the $p$-polarization amplitude for $n_s = 1.57, 1.59, 1.61, 1.63, 1.65, 1.67, 1.69, 1.71,$ and
Figure 8. Ratio $|A_s|/|A_p|$ as a function of $\gamma$ when $\delta_v = 0^\circ$. The values of $\tilde{\chi}_v$ are as follows: (1)-(9) 7.2°, (10)-(12) 19.1°. The values of $n_s$ are as follows: (1) 1.57, (2) 1.59, (3) 1.61, (4) 1.63, (5) 1.65, (6) 1.67, (7) 1.69, (8) 1.71, (9) 1.73, (10) 1.80, (11) 1.82, (12) 1.84.

1.73 when $\tilde{\chi}_v = 7.2^\circ$; and for $n_s = 1.80, 1.82$ and 1.84 when $\tilde{\chi}_v = 19.1^\circ$. When the vapor incidence angle is low ($7.2^\circ \forall z \geq 0$), the surface wave in the isotropic partnering material is predominantly $s$-polarized with $4 < |A_s|/|A_p| < 5$. When the vapor incidence angle is increased to $19.1^\circ \forall z \geq 0$, the surface wave is only mildly polarized with $1 < |A_s|/|A_p| < 2$, and $|A_s|/|A_p|$ shows a positive slope as a function of $\gamma$.

Figure 9 shows the ratio $|A_s|/|A_p|$ versus $\gamma$, when $\tilde{\chi}_v = 19.1^\circ$, for CTFs ($n_s = 1.80$, 1.82, 1.84) and SNTFs with $\delta_v = 7.2^\circ$ ($n_s = 1.73, 1.77, 1.82, 1.88, 1.92, 1.96$). At the same values of $\gamma$, the values of $|A_s|/|A_p|$ are slightly smaller for the SNTF than the CTF. As with the curves describing $\text{Im}[\alpha_2]/k_o$ in Figure 6b, the curves describing $|A_s|/|A_p|$ for the SNTFs nearly join to form a single, continuous curve. At $\gamma = 0^\circ$, the surface wave is entirely $p$-polarized in the isotropic partnering material. The $s$-polarization state intensifies rapidly but then levels off at larger values of $\gamma$. The value of $|A_s|/|A_p|$ is nearly constant at $\sim 1.6$ for $\gamma > 60^\circ$. For $\delta_v = 19.1^\circ$, the behaviour of $|A_s|/|A_p|$ is similar, as shown in Figure 10, but plateaus at a value of about 1.3 at large values of $\gamma$.

4. Concluding Remarks

To conclude, we examined the phenomenon of surface–wave propagation at the planar interface of an isotropic dielectric material and a sculptured nematic thin film with periodic nonhomogeneity. The boundary–value problem was formulated by marrying the usual formalism for the Dyakonov wave at the planar interface of an isotropic dielectric material and a columnar thin film with the methodology for Tamm states in solid–state physics. The solution of the boundary–value problem led us to predict the existence of
Figure 9. Ratio $|A_s|/|A_p|$ as a function of $\gamma$ when $\chi_v = 19.1^\circ$. The values of $\delta_v$ are as follows: (1)-(3) 0°, (4)-(9) 7.2°. The values of $n_s$ are as follows: (1) 1.80, (2) 1.82, (3) 1.84, (4) 1.73, (5) 1.77, (6) 1.82, (7) 1.88, (8) 1.92, (9) 1.96.

Figure 10. Ratio $|A_s|/|A_p|$ as a function of $\gamma$ when $\chi_v = 19.1^\circ$ and $\delta_v = 16.2^\circ$. The values of $n_s$ are (1) 1.80 and (2) 1.84.

Dyakonov–Tamm waves.

Dyakonov surface waves may propagate guided by the bimaterial interface of an isotropic dielectric material and a columnar thin film (or any other biaxial dielectric material). The angular domain of their existence is very narrow, of the order of a degree. Although several techniques have been suggested [2, 7], the widening of that domain has not been impressive. By periodically distorting the CTF in two ways, either as a chiral STF [8] or now as a periodically nonhomogeneous SNTF, the angular existence
domain for surface (Dyakonov–Tamm) waves can be widened dramatically, even to the maximum possible. A search for Dyakonov–Tamm waves is, at the present time, the most promising route to take for experimental verification of surface-wave propagation at the interface of two dielectric materials at least one of which is anisotropic.

In Section 1 we posed the following question: Is the huge angular existence domain of Dyakonov-Tamm waves for the case investigated by Lakhtakia and Polo due to the periodicity or due to the structural chirality of the chiral STF? Although the two attributes are inseparable in a chiral STF, our results in Section 3 indicate that the periodic nonhomogeneity is the responsible attribute.

Acknowledgment. This work was supported in part by the Charles Godfrey Binder Endowment at Penn State. KA thanks the Department of Engineering Science and Mechanics, Penn State, and AL is grateful to the Department of Physics, IIT Kanpur, for hospitality.

References

[1] D’yakonov M I 1988 Sov. Phys. JETP 67 714
[2] Takayama O, Crasovan L-C, Johansen S K, Mihalache D, Artigas D and Torner L I 2008 Electromagnetics 28, 126
[3] Crasovan L C, Artigas D, Mihalache D and Torner L I 2005 Opt. Lett. 30 3075
[4] Artigas D and Torner L I 2005 Phys. Rev. Lett. 94 013901
[5] Polo J A Jr, Nelatury S R and Lakhtakia A 2007 J. Nanophoton. 1 013501
[6] Abdulhalim I, Zourob M and Lakhtakia A 2007 In: Marks R, Cullen D, Karube I, Lowe C R and Weetall H H (eds) Handbook of Biosensors and Biochips (Chichester, United Kingdom: Wiley) pp. 413–446
[7] Nelatury S R, Polo J A Jr and Lakhtakia A 2008 Microw. Opt. Technol. Lett. 50 2360
[8] Lakhtakia A and Polo J A Jr 2007 J. Eur. Opt. Soc.—Rapid Pub. 2 07021
[9] Lakhtakia A and Messier R 2005 Sculptured Thin Films: Nanoengineered Morphology and Optics (Bellingham, WA, USA: SPIE Press).
[10] Tamm I 1932 Phys. Z. Sowjetunion 1 733
[11] Ohno H, Mendez E E, Brum J A, Hong J M, Agulló-Rueda F, Chang L L and Esaki L 1990 Phys. Rev. Lett. 64 2555
[12] Lakhtakia A (ed) 1990 Selected Papers on Natural Optical Activity (Bellingham, WA, USA: SPIE)
[13] Chynoweth E 1985 The Molecular Basis of Optical Activity (Malabar, FL, USA: Krieger)
[14] van de Hulst H C 1981 Light Scattering by Small Particles (New York, NY, USA: Dover) Sec. 6.4.
[15] Mackay T G 2008 J. Nanophoton. 2 029503
[16] Bose J C 1898 Proc. R. Soc. Lond. A 46 146
[17] Ro R 1991 Determination of the electromagnetic properties of chiral composites, using normal incidence measurements, Ph.D. dissertation, Pennsylvania State University, University Park, PA, USA
[18] Lakhtakia A 1994 Beltrami Fields in Chiral Media (Singapore: World Scientific)
[19] Gómez A, Lakhtakia A, Margineda J, Molina-Cuberos G J, Núñez J, Ipiña J S, Vegas A and Solano M A 2008 IEEE Trans. Microw. Theory Tech. 56 2815
[20] Whites K W and Chung C Y 1997 J. Electromagn. Waves Applics. 11 371
[21] Chandrasekhar S 1992 Liquid Crystals (Cambridge: Cambridge University Press)
[22] Hodgkinson I J and Wu Q-h 1997 Birefringent Thin Films and Polarizing Elements (Singapore:
Dyakonov–Tamm waves at nematic thin film/isotropic dielectric interface

[23] Mattox D M 2003 The Foundations of Vacuum Coating Technology (Norwich, NY, USA: Noyes Publications)
[24] Hodgkinson I J, Wu Q H and Hazel J 1998 Appl. Opt. 37 2653
[25] Chen H C 1992 Theory of Electromagnetic Waves (Fairfax, VA, USA: TechBooks).
[26] Motyka M A and Lakhtakia A 2008 J. Nanophoton. 2 021910
[27] Yakubovich V A and Starzhinskii V M 1975 Linear Differential Equations with Periodic Coefficients (New York, NY, USA: Wiley).
[28] Jaluria Y 1996 Computer Methods for Engineering (New York, NY, USA: Brunner–Routledge)