A New Model-independent Method of Determining $|V_{ub}/V_{cb}|$

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Abstract

In order to determine the ratio of CKM matrix elements $|V_{ub}/V_{cb}|$ (and $|V_{ub}|$), we propose a new model-independent method based on the heavy quark effective theory, which is theoretically described by the phase space factor and the well-known perturbative QCD correction only. In the forthcoming asymmetric $B$-experiments with microvertex detectors, BABAR and BELLE, the total separation of $b \rightarrow u$ semileptonic decays from the dominant $b \rightarrow c$ semileptonic decays would be experimentally viable. We explore the possible experimental option: the measurement of inclusive hadronic invariant mass distributions. We also clarify the relevant experimental backgrounds.

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1. Introduction  A precise determination of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements \( |V_{ub}| \) is the most important goal of the forthcoming asymmetric \( B \)-factories \( [2] \), KEKB and SLACB. Their precise values are urgently needed for analyzing CP-violation and for testing the Standard Model (SM) through the unitarity relations among them. Furthermore, the accurate knowledge of these matrix elements can be useful in relating them to the fermion masses and also in the searches for hints of new physics beyond the SM.

The CKM matrix element \( V_{ub} \) is important to the SM description of CP-violation. If it were zero, there would be no CP-violation from the CKM matrix \( (i.e. \) in the SM), and we have to seek for other sources of CP violation in \( K_L \rightarrow \pi \pi \). Observations of semileptonic \( b \rightarrow u \) transitions by the CLEO \( [3] \) and ARGUS \( [4] \) imply that \( V_{ub} \) is indeed nonzero, and it is important to extract the modulus \( |V_{ub}| \) from semileptonic decays of \( B \) mesons as accurately as possible.

Historically, the charged lepton energy spectrum \( (d\Gamma/dE_l) \) has been measured, and the \( b \rightarrow u \) events are selected from the high end of the charged lepton energy spectrum. However, this cut on \( E_l \) is not very effective, since only less than 10% of \( b \rightarrow u \) events\(^1\) survive this cut at the \( B \) meson rest frame. (In the future asymmetric \( B \)-factories with boosted \( B \) mesons, much less than 10% of \( b \rightarrow u \) events would survive the \( E_l \) cut over the \( b \rightarrow c \) threshold.) We also note that the dependences of the lepton energy spectrum on perturbative and non-perturbative QCD corrections \( [5,6] \) as well as on the unavoidable specific model parameters \( (e.g. \) the parameter \( p_F \) of the ACCMM model \( [7] \) ) are strongest at the end-point region, which makes the model-independent determination of \( |V_{ub}|/V_{cb} | \) almost impossible from the inclusive distribution of \( d\Gamma/dE_l \). For exclusive \( B \rightarrow X_u l\nu \) decays, the application of heavy quark effective theory (HQET) is very much limited, since \( u \)-quark is not heavy compared to \( \Lambda_{QCD} \). And the theoretical predictions for the required hadronic matrix

\(^1\)If it were not for the theoretical uncertainty, this \( E_l \) cut would be very effective – it completely suppresses the \( b \rightarrow c \) background.
elements are quite different depending on which model we use \cite{8}. However, in the long run theoretical uncertainties on $V_{ub}$ from the exclusive form factors are possibly reduced to the $10 \sim 15\%$ level through the measurements on $q^2$ dependence of the form factors, and the exclusive semileptonic decays will also be providing valuable information. By using the neutrino reconstruction technique and the beam constrained invariant mass, CLEO \cite{9} has recently succeeded measuring the branching ratio, $\mathcal{B}(B^0 \rightarrow \rho^- l^+ \nu) = (2.5 \pm 0.4^{+0.5}_{-0.7} \pm 0.5) \times 10^{-4}$, where the errors are statistical, systematic and the estimated model-dependence based on the spread of models and individual model errors. And they estimated $|V_{ub}| = (3.3 \pm 0.2^{+0.3}_{-0.4} \pm 0.7) \times 10^{-3}$, which agrees reasonably with the value of $|V_{ub}|$ obtained from the inclusive end point spectrum \cite{3,4}.

Alternatively, the possibility of measuring $|V_{ub}|$ via non-leptonic decays of $B$ mesons to exclusive two meson final states \cite{10} has been theoretically explored. Recently it has also been suggested that the measurements of hadronic invariant mass spectrum \cite{11} as well as hadronic energy spectrum \cite{12} in the inclusive $B \rightarrow X_{c(u)} l\ell \nu$ decays can be useful in extracting $|V_{ub}|$ with better theoretical understandings. In a future asymmetric $B$-factory with microvertex detector, the hadronic invariant mass spectrum will offer alternative ways to select $b \rightarrow u$ transitions that are much more efficient than selecting the upper end region of the lepton energy spectrum, with much less theoretical uncertainties. The measurement of ratio $|V_{ub}|/|V_{ts}|$ from the differential decay widths of the processes $B \rightarrow \rho l\nu$ and $B \rightarrow K^* l\ell$ by using $SU(3)$-flavor symmetry and the heavy quark symmetry has been also proposed \cite{13}. There has also been a recent theoretical progress on the exclusive $b \rightarrow u$ semileptonic decay form factors using the HQET-based scaling laws to extrapolate the form factors from the semileptonic $D$ meson decays \cite{14}. And their prediction is similar to most of quark model predictions \cite{8}. It is urgently important that all the available methods have to be thoroughly explored to measure the most important CKM matrix element $V_{ub}$ as accurately as possible in the forthcoming $B$-factories.
2. Theoretical Proposal

Over the past few years, a great progress has been achieved in our understanding of *inclusive* semileptonic decays of heavy mesons [6], especially in the lepton energy spectrum. However, it turns out that the end-point region of the lepton energy spectrum cannot be described by $1/m_Q$ expansion. Rather, a partial resummation of $1/m_Q$ expansion is required [15], closely analogous to the leading twist contribution in deep inelastic scattering, which could bring about significant uncertainties and presumable model dependences.

Even with a theoretical breakdown near around the end-point region of lepton energy spectrum, accurate prediction of the *total* integrated semileptonic decay rate can be obtained [6] within the HQET including the first non-trivial, non-perturbative corrections as well as radiative perturbative QCD correction [5]. The related uncertainties in calculation of the integrated decay rate have been also analyzed [16–19].

The total inclusive semileptonic decay rate for $B \to X_q l \nu$ is given [17] as

$$
\Gamma(B \to X_q l \nu) = \frac{G_F^2 m_b^5}{192 \pi^3} |V_{qb}|^2 \left\{ z_0(x_q) - \frac{2\alpha_s(m_b^2)}{3\pi} g(x_q) \left( 1 - \frac{\mu_G^2 - \mu_G^2}{2m_b^2} \right) \right.
\left. - z_1(x_q) \frac{\mu_G^2}{m_b^2} + \mathcal{O}(\alpha_s^2, \alpha_s/m_b^2, 1/m_b^3) \right\},
$$

where

- $x_q \equiv m_q/m_b$,
- $z_0(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \log x$,
- $z_1(x) = (1 - x^2)^4$,
- and $g(x) = (\pi^2 - 31/4)(1 - x)^2 + 3/2$ is the corresponding single gluon exchange perturbative QCD correction [3,20]. The expectation value of energy due to the chromomagnetic hyperfine interaction, $\mu_G$, can be related to the $B^* - B$ mass difference

$$
\mu_G^2 = \frac{3}{4} (M_{B^*}^2 - M_B^2) \approx (0.350 \pm 0.005) \text{ GeV}^2,
$$

and the expectation value of kinetic energy of $b$-quark inside the $B$ meson, $\mu_{\pi}^2$, is given from various arguments [21–23],

$$
0.10 \text{ GeV}^2 \leq \mu_{\pi}^2 \leq 0.65 \text{ GeV}^2.
$$
which shows much larger uncertainties compared to $\mu_2^G$. The value of $|V_{cb}|$ has been estimated from the total decay rate $\Gamma(B \to X_c l\nu)$ of Eq. (1) by using the pole mass of $m_b$ and a mass difference $(m_b - m_c)$ based on the HQET. As can be easily seen from Eq. (1), the $m_5^b$ factor, which appears in the semileptonic decay rate, but not in the branching fraction, could be the largest source of the uncertainty, resulting in about $5 \sim 20\%$ error in the prediction of $|V_{cb}|$ via the semileptonic branching fraction and $B$ meson lifetime. However, we note the recent arguments that a consistent treatment of the running masses and the perturbative QCD correction appears to cancel the large uncertainties from (i) the mass term and (ii) the perturbative expansion, which seems to be borne out by the calculations of Ball et al. [18].

We can do a similar exercise to predict the value of $|V_{ub}|$ from the integrated total decay rate of $\Gamma(B \to X_u l\nu)$, to find out

$$|V_{ub}|^2 = \frac{192\pi^3}{G_F^2 m_b^5} \left\{ \left[ 1 - \frac{2\alpha_s(m_b^2)}{3\pi} \right] \left( 1 - \frac{\mu_2^2 - \mu_G^2}{2m_b^2} \right) - \frac{\mu_G^2}{m_b^2} \right\}^{-1}.$$  

And by using the pole mass of $b$-quark $m_b = (4.8 \pm 0.2) \text{ GeV}$ from a QCD sum-rule analysis of the $\Upsilon$-system [24], $x_u \equiv m_u/m_b \simeq 0$, and taking $\alpha_s(m_b^2) = (0.24 \pm 0.02)$, we get numerically as a conservative estimate

$$\gamma_u \equiv \frac{\Gamma_{theory}(B \to X_u l\nu)}{|V_{ub}|^2} \simeq (7.1 \pm 1.5) \times 10^{13}/\text{sec},$$

and

$$|V_{ub}| \simeq (3.6 \pm 0.4) \times 10^{-3} \cdot \left[ \frac{\mathcal{B}(B \to X_u l\nu)}{1.4 \times 10^{-3}} \right]^{1/2} \left[ \frac{1.52 \text{ psec}}{\tau_B} \right]^{1/2}. \tag{4}$$

Note that there exists a similar estimate but with smaller error ($\sim 5\%$) by using the theoretically defined running mass of $m_b$ normalized at the scale about 1 GeV.

We remark that the semileptonic branching fraction of $b \to u$ decay, $\mathcal{B}(B \to X_u l\nu)$, has to be precisely measured to experimentally determine the value of $|V_{ub}|$ from Eq. (4). We will discuss on the experimental possibilities in details in the next Section. 

\footnote{To be conservative, we use a larger error bar (larger by a factor 8) than that of the original analysis [24]. We estimate the largest possible error of $m_b$ as $\mathcal{O}(\Lambda_{QCD})$.}

\footnote{Extrapolating the known 5 \% error of $\alpha_s(m_b^2)$, we estimate about 10 \% error for $\alpha_s(m_b^2)$.}
Once the inclusive branching fraction $\mathcal{B}(B \to X_u l\nu)$ is precisely measured, we can extract the value of $|V_{ub}|$ within the theoretical error similar to those of $|V_{cb}|$.

The ratio of CKM matrix elements $|V_{ub}/V_{cb}|$ can be determined in a model-independent way by taking the ratio of semileptonic decay widths $\Gamma(B \to X_u l\nu)/\Gamma(B \to X_c l\nu)$. As can be seen from Eq. (1), this ratio is theoretically described by the phase space factor and the well-known perturbative QCD correction only,

$$\frac{\Gamma(B \to X_u l\nu)}{\Gamma(B \to X_c l\nu)} \approx \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{1}{3\pi} \left[ 1 - \frac{2\alpha_s}{\pi^2} \left( \frac{25}{4} \right) \right] \left[ z_0(x_c) - \frac{2\alpha_s}{3\pi} g(x_c) \right]^{-1},$$

where we ignored the term $\mu^2_m G_F/m_b^2$, which gives about 1% correction to the ratio. We strongly emphasize here that the sources of the main possible theoretical uncertainties, the factor $m_b^5$ and the still-problematic non-perturbative contributions, are all canceled out in this ratio. By taking $\alpha_s(m_b^2) = (0.24 \pm 0.02)$, and by using the mass difference relation from the HQET [25], which gives $x_c \equiv m_c/m_b \approx 0.25 - 0.30$, the ratio of the semileptonic decay widths is conservatively estimated as

$$\frac{\Gamma(B \to X_u l\nu)}{\Gamma(B \to X_c l\nu)} \equiv \left( \frac{\gamma_u}{\gamma_c} \right) \left| \frac{V_{ub}}{V_{cb}} \right|^2 \simeq (1.83 \pm 0.28) \times \frac{1}{1.2},$$

and the ratio of CKM elements is

$$\left| \frac{V_{ub}}{V_{cb}} \right| \equiv \left( \frac{\gamma_c}{\gamma_u} \right)^{1/2} \simeq (0.74 \pm 0.06) \times \frac{\mathcal{B}(B \to X_u l\nu)}{\mathcal{B}(B \to X_c l\nu)}^{1/2}.$$  \hspace{1cm} (6)

We note here that within a simple spectator model of $b$-quark decay Rosner [26] predicted the ratio of the decay widths as

$$\frac{\Gamma(b \to ul\nu)}{\Gamma(b \to cl\nu)} = (1.85 \sim 2.44) \times \left| \frac{V_{ub}}{V_{cb}} \right|^2.$$  \hspace{1cm} (7)

We find that this free-quark-decay estimate without including any QCD corrections gives a rather similar result to our prediction, Eq. (6), based on the HQET. Once the

**This ratio $x_c$ is calculable from the mass difference $(m_b - m_c)$, which also includes the uncertain parameter $\mu^2_m$ of Eq. (3) as a small correction factor.**
ratio of semileptonic decay widths (or equivalently the ratio of branching fractions $B(B \to X_u l \nu)/B(B \to X_c l \nu)$) is measured in the forthcoming asymmetric $B$-factories, this should give a powerful model-independent determination of $|V_{ub}/V_{cb}|$.

3. Experimental Possibility

As explained in the previous Section, in order to measure $|V_{ub}/V_{cb}|$ (and $|V_{ub}|$) model-independently by using the relations, Eqs. (4,6,7), it is experimentally required to separate the $b \to u$ semileptonic decays from the dominant $b \to c$ semileptonic decays, and to precisely measure the branching fraction $B(B \to X_u l \nu)$ or the ratio $B(B \to X_u l \nu)/B(B \to X_c l \nu)$.

At presently existing symmetric $B$-experiments, ARGUS and CLEO, where $B$ and $\bar{B}$ are produced almost at rest, this required separation is possible only in the very end-point region of the lepton energy spectrum, because both $B$ and $\bar{B}$ decay into the whole $4\pi$ solid angle from the almost same decay point, and it is not possible to identify the parent $B$ meson of each produced particle. Hence all the hadronic information is of no use. However, recently CLEO [9] succeeded measuring the hadronic invariant masses for the fully reconstructed $B \to \rho l \nu$ and $B \to \omega l \nu$ decay events by using the neutrino reconstruction technique and the beam constrained invariant mass. In the forthcoming asymmetric $B$-experiments with microvertex detectors, BABAR and BELLE [2], where the two beams have different energies and the produced $\Upsilon(4S)$ is not at rest in the laboratory frame, the bottom decay vertices will be better identifiable. The efficiency for the full reconstruction of each event could be relatively high (maybe as large as several percentages of efficiency) limited only by the $\pi^0$-reconstruction efficiency of about 60% [2], and this $b \to u$ separation would be experimentally viable.

As of the most straightforward separation method, the measurements of inclusive hadronic invariant mass ($m_X$) distributions in $B \to X_{c,u} l \nu$ can be very useful for the fully reconstructed semileptonic decay events. For $b \to c$ decays, one necessarily has $m_X \geq m_D = 1.86$ GeV. Therefore, if we impose a condition $m_X < m_D$, the resulting events come only from $b \to u$ decays, and about 90% of the $b \to u$ events would survive this cut. This is already in sharp contrast with the usual cut on charged
lepton energy $E_l$. In fact, one may relax the condition\footnote{There are possibly non-negligible contributions at $m_x < m_{D^{**}}$ from the broad $D^{**}$ states and/or from $D^*\pi$ nonresonant decays. Since we know little about the correct hadronic mass shape in this region, we cannot subtract the $b \rightarrow c$ component with absolute certainty. The experimental smearing will further exacerbate this problem. Therefore, it is extremely unlikely that the $m_x < m_D$ condition can be relaxed.} $m_x < m_D$, and extract almost the total $b \rightarrow u$ semileptonic decay rate \footnote{There are possibly non-negligible contributions at $m_x < m_{D^{**}}$ from the broad $D^{**}$ states and/or from $D^*\pi$ nonresonant decays. Since we know little about the correct hadronic mass shape in this region, we cannot subtract the $b \rightarrow c$ component with absolute certainty. The experimental smearing will further exacerbate this problem. Therefore, it is extremely unlikely that the $m_x < m_D$ condition can be relaxed.}, because the $m_x$ distribution in $b \rightarrow c$ decays is completely dominated by contributions of three resonances $D, D^*$ and $D^{**}$, which are essentially like $\delta$-functions,

$$\frac{d\Gamma}{dm_x} = \Gamma(B \rightarrow RL\nu) \delta(m_x - m_R),$$

(8)

where the resonance $R = D, D^*$ or $D^{**}$. In other words, one is allowed to use the $b \rightarrow u$ events in the region even above $m_x \geq m_D$, first by excluding small regions in $m_x$ around $m_x = m_D, m_{D^*}, m_{D^{**}}$, and then by including the regions again numerically in the $m_x$ distribution of $b \rightarrow u$ decay from its values just around the resonances. There still is a non-resonant decay background at large invariant-mass region $m_x \geq m_D + m_\pi$ from $B \rightarrow (D + \pi)\ell\nu$ in using this inclusive $m_x$ distribution separation. To avoid this non-resonant background, we have to impose a condition $m_x < m_D + m_\pi$, and we would still get about 95% of the total $b \rightarrow u$ semileptonic decay events. For more details on this inclusive hadronic invariant mass distribution $d\Gamma/dm_x$, please see Ref. \footnote{There are possibly non-negligible contributions at $m_x < m_{D^{**}}$ from the broad $D^{**}$ states and/or from $D^*\pi$ nonresonant decays. Since we know little about the correct hadronic mass shape in this region, we cannot subtract the $b \rightarrow c$ component with absolute certainty. The experimental smearing will further exacerbate this problem. Therefore, it is extremely unlikely that the $m_x < m_D$ condition can be relaxed.}.

We would like to note the difficulties on this inclusive separation of the $b \rightarrow u$ from the dominant $b \rightarrow c$ decays, when the neutral particles, such as $K_L, n, \pi^0$, are produced as final decay products. A small rate of mis-handling of these particles could lead to very long tails on the invariant mass distribution. Therefore, the hadronic invariant mass has to be precisely measured, even for the masses well below $m_D$, in order to separate out the true $b \rightarrow u$ decays from the dominant $b \rightarrow c$. Being able to reconstruct correctly several percentages of the events would be only the first step.
one must be able to suppress the mis-reconstructed $b \rightarrow c$ events, which is a much harder challenge.

We also note that there is possibly a question of bias. Some classes of final states (e.g. those with low multiplicity, few neutrals) may be more susceptible to a full and unambiguous reconstruction, as previously explained. Hence an analysis that requires this reconstruction may be biased. However, the use of topological information from microvertex detectors should tend to reduce the bias, since vertex resolvability depends largely on the proper time of the decay and its orientation relative to the initial momentum (that are independent of the decay mode). Also such a bias can be allowed for in the analyses, via a suitable Monte Carlo modeling. There also possibly is a source of background from the cascade decay of $b \rightarrow c \rightarrow sl\nu$. Recently ARGUS and CLEO [28] have separated this cascade decay background from the signal events to extract the model-independent spectrum of $\frac{d\Gamma}{dE_{l}}(B \rightarrow X_{c}l\nu)$ for the whole region of electron energy, by taking care of lepton charge and $B - \bar{B}$ mixing systematically. In the future asymmetric $B$-factories with much higher statistics, this cascade decay may not be any serious background at all except for the case with very low energy electron production. We should also note that the decay channel $b \rightarrow cl\nu$ with $c \rightarrow sl\nu$ is another background source in a sense that is similar to the $K_{L}, n, \pi^{0}$ backgrounds mentioned earlier. Identifying a track as a muon or an electron will be problematic in the momentum range below 0.5 to 1 GeV. This means that a large portion of the $c \rightarrow sl\nu$ decays will not have the lepton identified. These events can thus appear experimentally as single-lepton events with a low hadronic mass. This kind of semileptonic cascade background where the secondary lepton is not identified would also be very serious problem unless the experimentalists find the solution to avoid systematically.

4. Summary The precise value of $V_{ub}$ is urgently needed for understanding the origin of CP-violation, for testing the SM through the unitarity relations among them, and also in the searches for hints of new physics beyond the SM. We propose that the ratio of CKM matrix elements $|V_{ub}/V_{cb}|$ can be determined in a model-independent
way by taking the ratio of semileptonic decay widths $\Gamma(B \to X_u l\nu)/\Gamma(B \to X_c l\nu)$, which is theoretically described by the phase space factor and the well-known perturbative QCD correction only, and which is conservatively estimated as

$$\frac{\Gamma(B \to X_u l\nu)}{\Gamma(B \to X_c l\nu)} = \left(\frac{\gamma_u}{\gamma_c}\right) \times \left|\frac{V_{ub}}{V_{cb}}\right|^2 \simeq (1.83 \pm 0.28) \times \left|\frac{V_{ub}}{V_{cb}}\right|^2,$$

and

$$\frac{|V_{ub}|}{|V_{cb}|} = \left(\frac{\gamma_c}{\gamma_u}\right)^{1/2} \times \left[\frac{B(B \to X_u l\nu)}{B(B \to X_c l\nu)}\right]^{1/2} \simeq (0.74 \pm 0.06) \times \left[\frac{B(B \to X_u l\nu)}{B(B \to X_c l\nu)}\right]^{1/2},$$

based on the heavy quark effective theory. Once the ratio of semileptonic decay widths (or equivalently the ratio of branching fractions $B(B \to X_u l\nu)/B(B \to X_c l\nu)$) is measured, this ratio will give a powerful model-independent determination of $|V_{ub}/V_{cb}|$.

In the forthcoming asymmetric $B$-factories with microvertex detectors, the total separation of $b \to u$ semileptonic decays from the dominant $b \to c$ semileptonic decays to determine the ratio would be experimentally viable. We explore the possible experimental option: the measurement of inclusive hadronic invariant mass distributions. We also clarify the relevant experimental backgrounds. In view of the potential importance of $B(B \to X_u l\nu)/B(B \to X_c l\nu)$ as a new theoretically model-independent probe for measuring $|V_{ub}/V_{cb}|$, we would like to urge our experimental colleagues to make sure that this $b \to u$ separation can indeed be successfully achieved.

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