Collective Rabi Oscillations and Cold Collisions.

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We examine cold atomic collisions within a resonant optical cavity. The quantized cavity mode can be used to manipulate the collisions between the cold atoms, such that periodic exchange of excitations between the atoms and the electromagnetic field strongly alters the collision dynamics. A colliding pair of atoms can thereby oscillate between its ground and excited states during the collision time. Using a semiclassical model, it can be predicted that such Rabi-like oscillations are revealed in the atomic trap-loss probabilities, which show maxima and minima as a function of the detuning between the frequencies of the mode and the atomic transition.

Exchange of excitations between the energy levels of atoms or molecules and the quantum electromagnetic field is one basic interaction process between matter and light. In this context, the Jaynes-Cummings model, describing a single two-level system and a monochromatic lossless field, reveals several characteristics of this interaction. In the quantum Rabi oscillation, in particular, a single energy quantum is periodically exchanged between system and field. On the other hand, when several atoms or molecules interact with the same field, quantum coherence can build among them leading to well known collective behaviors, such as superradiance, for which multiparticle entanglement plays a major role. It should be noticed, however, that the systems with which the field interacts need not be solely composed of stable atoms or molecules. The above characteristics of the matter-field interaction can be present as well when two cold atoms collide in presence of a quantum field. An additional effect is that the very dynamics of the cold atomic collision can be strongly modified by this field. Indeed, pairs of colliding atoms so far apart from each other that their direct mutual interaction is negligible, can be entangled by the field and, thereby, influence one another in a nonlocal way. In the following we study cold atomic collisions within a gas of cold atoms trapped in the center of a high-\(Q\) cavity. It is possible to show that an analogous collective Rabi oscillation can show up in the trap-loss probabilities as a function of the detuning between the cavity mode frequency and the atomic resonance.

The internuclear potential energy of two colliding atoms depends on the electronic states. In particular, the inverse-cube law dipole-dipole potential \(U = -\frac{C_3}{R^3}\) between alkali neutral atoms, separated by a distance \(R\), predominates when the asymptotic atomic states involved are \(nS_{1/2}\) and \(nP_{1/2}\) (Fig. 1). When the collision is slow, the atoms can undergo changes in their electronic states during the collision, either as spontaneous decay or induced transitions. In a previous work, it was examined how cold collisions can be manipulated as spontaneous decay is driven by the colored vacuum of a cavity. The highly increased emission rate of multiply entangled pairs being able to emit coherently to the same cavity mode was then predicted to practically interrupt the collision process. In the present case, we neglect the cavity loss (high-\(Q\)) and allow reabsorptions of the field energy. We study how these induced transitions affect cold collisions.

Cavity Quantum Electrodynamics (CQED) effects on atomic motion in high-\(Q\) cavities have been investigated recently. Modifications of mechanical forces of light acting on atoms, or appearance of new quasi-bound molecular states of two colliding atoms illustrate the interplay between CQED and cold atoms.

A pair of colliding atoms under the attractive potential \(U = -\frac{C_3}{R^3}\) and the ground state van der Waals potential \(1/R^6\), as functions of the internuclear distance \(R\). The energy difference of the latter to the asymptote of \(U\) is the atomic separation \(\hbar\omega_A\) between \(nS_{1/2}\) and \(nP_{1/2}\).

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig1}
\caption{Excited state long range dipole-dipole attractive potential \(U = -\frac{C_3}{R^3}\) and the ground state van der Waals potential \(1/R^6\), as functions of the internuclear distance \(R\). The energy difference of the latter to the asymptote of \(U\) is the atomic separation \(\hbar\omega_A\) between \(nS_{1/2}\) and \(nP_{1/2}\).}
\end{figure}

A pair of colliding atoms under the attractive potential \(U(R) = -\frac{C_3}{R^3}\), with \(C_3\) a constant, is described as in by a two-level system. Its energy splitting \(\hbar\omega_R = \hbar\omega_A + U(R)\) depends on the internuclear distance \(R\) and approaches the atomic energy difference \(\hbar\omega_A\) between \(nS_{1/2}\) and \(nP_{1/2}\) as \(R \to \infty\). In this asymptotic limit, the excited state of the pair denoted by \(|e\rangle\) correlates to \(nS_{1/2} + nP_{1/2}\), whereas the ground state denoted by \(|g\rangle\) correlates to \(nS_{1/2} + nS_{1/2}\) and the van der Waals potential in this latter case is neglected. Since the atoms are weakly bound by \(U(R)\) in \(|e\rangle\) we call them quasi-molecules, extrapolating this denomination to \(|g\rangle\) as well. The formation of this quasi-molecule occurs at the
Condon point $R_C$, the distance at which $\omega_R$ becomes resonant with an excitation laser of frequency $\omega_L < \omega_A$. The nuclei may then attract each other starting from $R_C$. An eventual spontaneous decay from $|e\rangle$ to $|g\rangle$ releases a kinetic energy $\Delta K = \hbar \omega_L - \hbar \omega_R$ (see Fig 4). If $\Delta K$ is more than twice the trap depth $V_0$, i.e., if the decay occurs below $R_e$, the pair is ejected from the trap in the so-called radiative escape collision [14]. This two-level approximation neglects hyperfine splittings of the atomic fine structure and possible (anti) crossings of potential curves, whereas the nuclear motion is treated semiclassically. Nevertheless, this turns out to be reliable for a satisfactory description of cold collisions in the case of $^{85}$Rb, a range of hundreds of MHz for the detuning $\delta = \omega_L - \omega_A$, and a weak excitation laser intensity [15,16]. Trapping the atoms in their ground state, e.g., with a far off-resonance trap (FORT) [18], is most suited since optical pumping effects by the trapping light can be avoided; ejected atoms can then be controlled more efficiently by a separate excitation light field.

For a high-$Q$ cavity, the main difference with our previous work [12] is the exchange of excitations between the cavity mode and the quasimolecules. A set of $N$ identical quasimolecules is described by

$$H_m = \frac{\hbar \omega_R}{2} \sum_{i=1}^{N} (\sigma_{zi} + 1),$$

where each $\sigma_{zi}$ is a Pauli spin matrix. Due to the nuclear motion, $\omega_R$ becomes formally time dependent since $R$ may change with time. The actual value of $N$ depends on the detuning $\delta$, the total number $N_A$ of atoms and their density $n_A$. The Hamiltonian of the quantized field of the cavity mode is in turn given by

$$H_c = \hbar \omega_c a^\dagger a,$$

where $\omega_c$ is the cavity resonance frequency and $a^\dagger$ ($a$), the bosonic creation (annihilation) operator of the field. The interaction hamiltonian in the rotating-wave approximation is

$$H_{int} = \sum_{i=1}^{N} \left( \frac{\hbar \Omega_i}{2} a^\dagger \sigma_i + \frac{\hbar \Omega_i^*}{2} a \sigma_i^\dagger \right).$$

Here, the Pauli matrices $\sigma_i^\dagger$ and $\sigma_i$ are raising and lowering operators, respectively, acting on $|e\rangle$ and $|g\rangle$ of the i-th quasimolecule. The individual Rabi frequencies

$$\Omega_i = 2E(\omega_c) f_e(r_i) \epsilon \cdot d_i / \hbar$$

depend on the field strength per photon $E(\omega) = (2\pi \hbar \omega / V)^{1/2}$ ($V$ being the mode volume), polarization $\epsilon$, mode profile $f_e(r)$, and the molecular dipole moment $d_i$ of the transition $|g\rangle \rightarrow |e\rangle$ (whose absolute value $|d_i| = \sqrt{2} |d_A|$, $d_A$ being the atomic dipole moment of $nS_{1/2} \rightarrow nP_{1/2}$).

If the excitation laser beam is normal to the cavity axis, the quasimolecules are excited independently from each other since the wavelength $2\pi c / \omega_R$ is in the optical domain and is much shorter than the average separation between quasimolecules [13]. This corresponds, for each quasimolecule, to a no-cavity condition since, in the optical domain and for alkalis, typically $\Delta \ll \Gamma$, where $\Gamma$ is the spontaneous decay rate of state $|e\rangle$ [4]. In contrast, using for the excitation the quantized mode of the cavity by injecting the laser through the cavity axis (with $\omega_L = \omega_c$), the quasimolecules become indistinguishable to the mode $a^\dagger a$ and all of them have a quantum probability amplitude to be excited. They therefore end up in a multiparticle entangled excited state. Considering only one excitation, it follows from $\hat{H}_{int}$ that this state is $|E,0\rangle = \sum_i \Omega_i |i,0\rangle / \Omega$, where $|1,0\rangle \equiv |eg \cdots g\rangle |0\rangle$, $|2,0\rangle \equiv |ge \cdots g\rangle |0\rangle$ and so forth, with energy $E_c(R) = \hbar \omega_c$. It couples to the ground state $|G,1\rangle \equiv |g \cdots g\rangle |1\rangle$, with energy $E_0 = \hbar \omega_c$ of one field excitation in the mode and all quasimolecules in their ground state $|g\rangle$, by a coupling constant

$$\hbar \tilde{\Omega} = \left( \sum_i |\hbar \Omega_i|^2 \right)^{1/2}.$$

These two states form a closed subspace for the total hamiltonian

$$H = H_m + H_c + H_{int}.$$

It may be asked whether the evolution of the states $|E,0\rangle$ and $|G,1\rangle$ under this hamiltonian starts immediately after one excitation is injected into the cavity mode. Consider the case of one single pair of free atoms approaching each other in the state $|g\rangle$ (asymptotic to $S_{1/2} + S_{1/2}$). In the presence of a light field, the pair may adiabatically pass from $|g\rangle$ to $|e\rangle$. If a Landau-Zener approximation [20,21] holds, the probability for this to occur is $1 - e^{2\pi \Delta}$, where $\Delta = \hbar \Omega^2 / v_{\infty} |U'(R_C)|$, $\Omega$ being the coupling (Rabi frequency) between $|g\rangle$ and $|e\rangle$, and $v_{\infty}$, the asymptotic relative velocity. This approximation is satisfactory for detunings $|\delta| \gtrsim 10 \Gamma_A$ and small intensity $\Omega^2 [13]$. Analogously, the probability $P_E$ to excite the collective state $|E,0\rangle$ from $|G,1\rangle$ may be estimated by

$$P_E = 1 - e^{-2\pi \Delta}, \quad \Delta = \frac{\hbar \tilde{\Omega}^2}{v_{\infty} |U'(R_C)|},$$

where the collective coupling $\tilde{\Omega}$ replaces $\Omega$. Since $\tilde{\Omega} > \Omega$, an immediate consequence of the cavity mode is to increase the excitation probability. Once $|E,0\rangle$ is formed, the quasimolecules’ evolution is then governed by Eq. (3). Actually, we use a large value for $\tilde{\Omega}$ and the Landau-Zener approximation may fail [13]. Nevertheless, with equal fractions of excited quasimolecules in both situations with and without cavity, the excitation probability
The most notable feature arises as time goes on. One expects a Rabi oscillation between \(|E, 0\rangle\) and \(|G, 1\rangle\) as the excitation is exchanged between the quasimolecules and the cavity mode. This implies that the quasimolecules switch between \(|e\rangle\) and \(|g\rangle\) at the collective rate \(\tilde{\Omega}\), altering completely the collisional encounter compared to how it would proceed without the quantum field; and the larger the number of quasimolecules \(N\), the faster such oscillation will be, since \(\tilde{\Omega}\) scales with \(\sqrt{N}\). This oscillation loses strength as \(\omega_R\) detunes gradually from \(\omega_e\). When \(|\omega_R - \omega_e| \gtrsim \tilde{\Omega}\), we may neglect \(H_{\text{int}}\) and let the quasimolecules evolve freely, subject at most to spontaneous decay. It is interesting to mention here that the resonance condition is determined by \(\tilde{\Omega}\) instead of the resonance condition is determined by \(\Omega\) instead of the cavity mode. This implies that the quasimolecules excitation is exchanged between the quasimolecules and free space is nearly \(\Gamma\). For the quasimolecules and the cavity mode, the Liouville-von Neumann equation with dissipation is

\[
\dot{\rho} = \frac{1}{i\hbar}[H, \rho] + \sum_{i=1}^{N} \frac{\Gamma}{2}(2\sigma_i\rho\sigma_i^\dagger - \sigma_i^\dagger\sigma_i\rho - \rho\sigma_i\sigma_i^\dagger),
\]

where the cavity dissipation is neglected. The dissipator in this equation assumes an incoherent decay of each quasimolecule independently from each other, since the emitted wavelength is shorter than the average separation of quasimolecules \([13]\). When the emission is towards the cavity mode, in contrast, a collective effect in the quasimolecules’ emission is fully contained in the interaction with the field.

For the initial condition \(\rho(0) = |E, 0\rangle\langle E, 0|\), we calculate the probability

\[
p_E(t) = \text{tr}[|E, 0\rangle\langle E, 0|\rho(t)]
\]

that the system remains in the state \(|E, 0\rangle\) after a time interval \(t\). The solution can be found by enlarging the subspace spanned by \(|E, 0\rangle\) and \(|G, 1\rangle\) to include the vacuum state \(|V\rangle = |G, 0\rangle \equiv |gg \cdots g\rangle\) to which the system evolves due to dissipation. In the interaction representation, one has

\[
\rho_I(t) = e^{iH_0t/\hbar}\rho(0)e^{-iH_0t/\hbar} = p_E|E, 0\rangle\langle E, 0| + p_G|G, 1\rangle\langle G, 1| + p_V|G, 0\rangle\langle G, 0| + \text{c.c.}
\]

The time-dependent probabilities \(p_E\), \(p_G\), and \(p_V\), and the coherences \(c_{EG}\), \(c_{EV}\), and \(c_{GV}\) are found by substitution of \(\rho_I(t)\) into the interaction representation of the Liouville-von Neumann equation and projecting into each component of \(\rho_I(t)\). The damping terms generate no new state and we get for the \(p\)’s

\[
\dot{p}_E = -\Gamma p_E + i\tilde{\Omega}(c_{EG} - c_{EG}^\dagger),
\]

\[
\dot{p}_G = -i\tilde{\Omega}(c_{EG} - c_{EG}^\dagger),
\]

\[
\dot{p}_V = \Gamma p_E,
\]

and for the \(c\)’s

\[
\dot{c}_{EG} = -\frac{\Gamma}{2}c_{EG} + i\tilde{\Omega}(p_E - p_G),
\]

\[
\dot{c}_{EV} = -\frac{\Gamma}{2}c_{EV} - i\tilde{\Omega}c_{GV},
\]

\[
\dot{c}_{GV} = -i\tilde{\Omega}c_{EV}.
\]

From the last two equations, it follows that \(c_{EV}(t) = c_{GV}(t) = 0\) if this holds initially. From Eq. (13), Eq. (14), and Eq. (19), \(p_E(t)\) can be solved straightforwardly. Since \(\tilde{\Omega} > \Gamma/4\) (condition for Rabi oscillations) is supposed to be fulfilled with the parameters we adopt, the solution is the underdamped one

\[
p_E(t) = \text{tr}[|E, 0\rangle\langle E, 0|\rho(t)] = e^{-\Gamma t/2}\left(\cos \beta t - \frac{\Gamma}{4\beta} \sin \beta \right)^2,
\]

where

\[
\beta = \sqrt{\tilde{\Omega}^2 - (\Gamma/4)^2}, \quad \tilde{\Omega} > \Gamma/4.
\]
As already mentioned, this solution is to be applied so long as the resonance condition \( |\omega_R - \omega_c| \lesssim \bar{\Omega} \) holds, that is, as the quasi-molecules-field interaction is significant. For \( |\omega_R - \omega_c| \gtrsim \bar{\Omega} \), on the other hand, we neglect this interaction. The solution then is formally equivalent to taking the limit \( \bar{\Omega} \to 0 \) in the overdamped regime, which follows from Eq. (18) by substituting \(-i|\beta|\) for \(\beta\) (i.e., solving Eq. (6) without \(H_{\text{int}}\)).

\[
p_E(t) \approx e^{-t\Gamma}, \quad |\omega_R - \omega_c| \gtrsim \bar{\Omega}. \tag{20}
\]

As expected, this equation implies the value \(\Gamma\) for the decay rate of the collective state \(|E, 0\rangle\), the same as the molecular value for \(|e\rangle\). This can be derived directly from a perturbation calculation à la Fermi’s golden rule by coupling the quasi-molecules to all the electromagnetic vacuum modes except the cavity mode \(\omega_c\), whose small solid-angle is neglected. It reflects the fact that the average separation between quasi-molecules is larger than the wavelength of the emitted radiation, and the indistinguishability of which quasi-molecule emits into free space can no longer hold [19].

We need \(p_\Omega\) to obtain the probability that one pair of atoms is ejected from the trap after excitation by the cavity mode. The transition from \(|E, 0\rangle\) to \(|G, 1\rangle\) should then occur for \(R < R_c\). In the first passage in this region, the probability \(l_1\) that a pair of atoms from any one of the \(N\) quasi-molecules is ejected is constructed as

\[
l_1 = \sum_{i=1}^{N} \left| \frac{\Omega_i}{\bar{\Omega}} \right|^2 p_{\Omega}(t_c) e^{-t\Gamma} (1 - e^{-2t\Gamma})
= p_{\Omega}(t_c) e^{-t\Gamma} (1 - e^{-2t\Gamma}), \tag{21}
\]

where \(t_c\) (\(t'\), or \(t_e\)) is the time interval spent between \(R = 0\) and \(R_c\) (\(R_e\) and \(R'\), or \(R'\) and \(R_c\)). This \(l_1\) is thus a sum of products of conditional probabilities [19], which in our case involve Eq. (13) and Eq. (24). Now, by letting \(R\) evolve further, the quasi-molecules can vibrate between \(R_c\) and \(R = 0\) before emission takes place, that is, they may pass several times across \(R_c\) while still in \(|E, 0\rangle\). Summing over these multiple passages \(10\) and composing them with conditional probabilities, the probability \(L_c\) for a pair of atoms to be ejected at any time is then

\[
L_c = l_1 + l_2 + l_3 + \cdots \\
= \sum_{i=1}^{N} \left| \frac{\Omega_i}{\bar{\Omega}} \right|^2 \left\{ p_{\Omega}(t_c) e^{-t\Gamma} (1 - e^{-2t\Gamma})
+ p_{\Omega}(t_c) e^{-2(t'+t_e)\Gamma} p_{\Omega}(2t_c) e^{-t\Gamma} (1 - e^{-2t\Gamma})
+ p_{\Omega}(t_c) e^{-2(t'+t_e)\Gamma} p_{\Omega}(2t_c) e^{-t\Gamma} (1 - e^{-2t\Gamma})
+ \cdots \right\}
= p_{\Omega}(t_c) \frac{\sinh (\Gamma t_e)}{\frac{1}{2} [e^{(t'+t_e)\Gamma} - p_{\Omega}(2t_e) e^{-(t'+t_e)\Gamma}]}. \tag{22}
\]

As in Eq. (21), the summation over the \(N\) quasi-molecules is factored since the multiple passages probabilities \(l_n\) are the probability that a quasi-molecule exits the region \(R' < R < R_c\) in the excited state \(|e\rangle\) after having been excited at \(R_c\), whereas \(p_{\Omega}(2t_e)\) gives the probability of crossing \(R'\) from \(|e\rangle\) to the right after having crossed it from the left in \(|e\rangle\).

The dependence of \(L_c\) on the collective Rabi frequency \(\bar{\Omega}\) can be observed formally by changing the interaction time \(t_e\) and thus making \(p_{\Omega}\) oscillate. This time interval in turn is a function of the detuning \(\delta = \omega_c - \omega_A\) and can be obtained by integration of the energy conservation condition [19]

\[
\frac{\mu R^2}{2} + U(R) = \text{const.} \tag{23}
\]

Indeed, choosing \(\bar{\Omega}\) such that \(t' \approx 0\) (i.e., \(R' \approx R_c\), see Eq. (8)), we have \(\hbar \delta = 2V_0\) (emission on resonance with the cavity will not lead to trap-loss) and neglecting the initial velocity \(\dot{R}\) at \(R = R_c\), it follows from Eq. (23)

\[
t_e = t_0(\delta) f(\delta), \quad (t' \approx 0, R' \approx R_c)
= t_0(\delta) [1 - f(\delta)] , \tag{24}
\]

where \(t_0 = t_e + t_e\) is the total time interval between \(R_c\) and \(R = 0\), and \(f(\delta)\) is the fraction of \(t_0\) spent between \(R_c\) and \(R_e\).

\[
t_0(\delta) = g_0 \left( \frac{\mu}{2C_3} \right)^{1/2} \left( \frac{C_3}{\hbar |\delta|} \right)^{5/6}, \tag{25}
\]

\[
f(\delta) = \frac{1}{g_0} \int_{r}^{1} \frac{du}{\sqrt{u^3 - 1}}, \quad r \equiv \frac{R_e}{R_c} = \left( 1 + \frac{\bar{\Omega}}{|\delta|} \right)^{-1/3},
\]

\[g_0 = 0.746\] normalizing \(f(0)\) to unity. A slight complication is the dependence of \(\bar{\Omega}\) on \(\delta\) as well via the total number \(N = N(\delta)\) of quasi-molecules, so that \(\bar{\Omega}\) changes as \(t_e(\delta)\) is changed. In order to obtain \(\bar{\Omega} = \bar{\Omega}(\delta)\), we approximate for large \(N\) the summation in Eq. (18) by an average (\(e_i\) being the orientation of \(d_i\), and \(d\) its modulus)

\[
\bar{\Omega}^2 = \sum_{i} |\Omega_i|^2
= N \frac{\varepsilon^2 d^2}{\hbar^2} \left( \frac{1}{N} \sum_{i} |f_e(r_i)|^2 |e \cdot e_i|^2 \right)
\approx N \frac{\varepsilon^2 d^2}{\hbar^2} \langle |f_e(r_i)|^2 \rangle |e \cdot e_i|^2
\sim N \frac{\varepsilon^2 d^2}{6\hbar^2} \equiv N \bar{\Omega}^2, \tag{26}
\]

with \(|f_e(r_i)| \sim \cos(z_i k_c)\) (\(z\) along the cavity axis and \(k_c = \omega_c/c\) and \(e_i\) being randomly oriented, and \(\Omega\) denoting an (averaged) single quasi-molecule Rabi frequency. The total number \(N\) can be estimated by counting all pairs of atoms whose separation \(R\) is such that \(\omega_R = \omega_c\),
with a spread $\Delta R$ about the Condon Point $R_C$ determined by the linewidth $\Gamma$ of the state $|e\rangle$, namely,

$$
N \approx \frac{1}{2} N_A n_A 4\pi R_C^2 \Delta R \\
\sim \frac{1}{2} N_A n_A 4\pi R_C^2 \frac{\Gamma}{|d\omega_R/dR|} \\
\sim N_A n_A \frac{2\pi C_3}{3\hbar} \left( \frac{\Gamma}{\delta} \right)^2,
$$

(27)

having used that $C_3/R_C^3 = \hbar \delta$.

For the sake of comparison, we calculate the trap loss probability $L_o$ that, in the absence of the cavity, $|e\rangle$ decays in the region $R < R_c$ after multiple passages across $R_c$. Both $L_c$ and $L_o$ are similar, except for $p_{11}(t)$ which is substituted for the pure decay $p_e(t) \approx e^{-\Gamma t}$

$$
L_o = \frac{\sinh (t_e \delta \Gamma)}{\sinh (t_e + t_c) \delta \Gamma},
$$

(28)

describing the trap loss probability of a statistical mixture of pairs of atoms colliding independently.

This choice for $\tilde{\Omega}$ allows for the quasimolecules to enter directly the trap escape region $R < R_e$ as they exit the resonant region $R_e < R < R_C$ in which their interaction with the cavity mode is strongest. With such parameters, and $\delta$ going down to $\sim -1.0$ GHz (when $\tilde{\Omega}/2\pi \approx 70$ MHz), we can approximate $\beta \approx \tilde{\Omega}$ and neglect the sine in Eq. (19),

$$
p_{11}(t_e) \approx e^{-t_e \Gamma/2} \cos^2 (\tilde{\Omega} t_e).
$$

(30)

For the detuning $\delta = -350$ MHz, one obtains $t_e = t_0 f = 0.55 t_0$ and $t_c = (1-f) t_0 = 0.45 t_0$. The largest phase becomes then $\tilde{\Omega} t_e \approx 2\pi \times 200$ MHz $\times 0.55 \approx 70$ MHz, and $t_e t_c$ will decrease as $\delta$ decreases. In Fig. (3), we plot $L_o$ and $L_c$, assuming the quasimolecules are equally excited in both cases so that the excitation probabilities can be factored out. Outside the range of detunings shown, the approximations in our model are no longer valid: for smaller $|\delta|$, a full quantum mechanical treatment of the nuclear dynamics is required, whereas the discrete quantum vibrational levels of the dipole-dipole potential $U(R)$ become resolved for $|\delta| \gtrsim 1.0$ GHz.

Within this detuning window, nevertheless, it can be clearly seen the drastic change brought about by the cavity mode. The collective Rabi oscillations begin mildly at a large $|\delta|$ and become more pronounced at smaller values of absolute detuning, where the interaction time $t_e$ is longer. At the minima of $L_c$, the quasimolecules leave the resonant region in the state $|G,1\rangle$ (having undergone a $\pi$-pulse) and the kinetic energy is insufficient to eject a pair of atoms from the trap. The maxima of $L_c$, where the quasimolecules leave the resonant region in the excited state, are above $L_o$ since, without the cavity effect, the probability of a quasimolecule to reach the escape region in the excited state $|e\rangle$ is smaller. These maxima increase with decreasing $\delta$ because the potential becomes steeper and, therefore, the time $t_c$ decreases. The effect of damping during the interaction with the cavity mode becomes less and less important and $p_{11}$ becomes closer to one (see Eqs. (18) and (22)).

It should be noted that an actual realization of such an experiment is quite difficult. For a centimeter sized optical resonator with mirrors separated by $l \approx 1$ cm, the mode volume $V = \pi w_e^2 l \approx 4.0 \times 10^{-5}$ cm$^3$, with a waist $w_e = \sqrt{\delta/\omega_c} \approx 36$ $\mu$m, so that using $d = \sqrt{2} d_A$ and $\Gamma_A = 4d^2 \omega_A^3/3\hbar c^3$ ($\omega_A \approx \omega_c$), the averaged single quasimolecule Rabi frequency in Eq. (26) is $\tilde{\Omega}/2\pi \approx 0.42$ MHz. From Eq. (29) and Eq. (30), one needs at least $N \sim (\tilde{\Omega}/\Omega)^2 \approx 2.3 \times 10^5$ quasimolecules (but less for larger $|\delta|$). Using Eq. (27), this could only be achieved with a gas of $N_A \sim 2.0 \times 10^9$ atoms occupying all the mode volume $V$, i.e., at a high density $n_A \sim 4.0 \times 10^{13}$ cm$^{-3}$. A possible remedy that could do with less atoms at lower densities would imply, however, a smaller $\tilde{\Omega}$ and thus a substantially longer interaction time $t_c$ in order for the phase $\tilde{\Omega} t_c$ to reach $\sim 2\pi$ and show a “collisional Rabi oscillation”; for these longer times, the detuning would fall

**FIG. 2.** Trap loss probabilities $L_c$ and $L_o$ as functions of the detuning $\delta = \omega_c - \omega_A$ (see text for details). Note the “oscillation” in $L_c$ brought about by the interaction with the cavity quantized mode. The curve levels off and approaches $L_o$ for $\delta/2\pi < -800$ MHz, where the interaction time $t_c$ goes to zero.

For numerical estimates, we consider $^{85}$Rb atoms and the atomic transition $5S_{1/2} \rightarrow 5P_{1/2}$. The wavelength of the transition is $\lambda_A = 2\pi c/\omega_A = 795$ nm, with atomic decay rate $\Gamma_A/2\pi = 6$ MHz. The coefficient of the dipole-dipole potential is taken as $C_3 \approx 11 \times 10^{-11}$ erg A$^3$ [23-24] and the detuning $\delta = \omega_c - \omega_A < 0$ is made to decrease, starting from $\delta/2\pi = -350$ MHz. For this $\delta$, the Condon point is $R_C \approx 366$ Â and the total time $t_0 = 1.07 \times 10^{-8}$ s. The trap depth $V_0$ is chosen of order 5 mK ($\sim 100$ MHz) and would have to decrease as $\delta$ is decreased in order to match the condition (see Eq. (24) et seq.)

$$
2V_0 = \hbar \tilde{\Omega}, \quad \text{or} \quad \tilde{\Omega}/2\pi \lesssim 200 \text{ MHz}.
$$

(29)
below the limit $\delta > 10 \Gamma_A$ and our semiclassical approximation for the nuclear motion would break down [17].

In conclusion, it is a remarkable effect that the outcome of a collision encounter between two cold atoms can be so deeply affected when it takes place in the presence of a quantized cavity mode. The slowness of the collision makes possible the exchange of excitation between the colliding pair of atoms and the cavity field. Depending on how long the interaction is effective, the potential energy of the collision can simply be “turned off” as the quasimolecule leaves the excitation within the cavity; in contrast, the quasimolecule may be taken closer to the radiative escape condition if this same excitation is returned back to it. In this process, several quasimolecules get quantum mechanically entangled to each other. The trap-loss probability as a function of the detuning can thus show a Rabi-like collective oscillation.

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[1] H.-I. Yoo, J.H. Eberly, Phys. Rep. 118 (1985) 239.
[2] E.T. Jaynes, F.W. Cummings, Proc. IEEE 51 (1963) 89.
[3] M. Brune, F. Schmidt-Kaler, A. Maali, J. Dreyer, E. Hagley, J.M. Raimond, S. Haroche, Phys. Rev. Lett. 76 (1996) 1800.
[4] D.M. Meekhof, C. Monroe, B.E. King, W.M. Itano, D.J. Wineland, Phys. Rev. Lett. 76 (1996) 1796.
[5] R.M. Dicke, Phys. Rev. 93 (1954) 99.
[6] R. Bonifacio, P. Schwendimann, F. Haake, Phys. Rev. A4 (1971) 302; ibid. A (1971) 854.
[7] M.F.H. Schuurmanns, Q.H.F. Vreken, D. Polder, H.M. Gibbs, Adv. At. Mol. Phys. 17 (1981) 167.
[8] M. Gross, S. Haroche, Phys. Rep. 93 (1982) 301.
[9] S. Haroche, in: G. Gryenberg, R. Stora (Eds.), Les Houches, Session XXXVIII, Vol. I, North Holland, Amsterdam, 1984.
[10] A. Gallagher, D.E. Pritchard, Phys. Rev. Lett. 63 (1989) 957.
[11] J. Weiner, V.S. Bagnato, S. Zilio, P.S. Julienne, Rev. Mod. Phys. 71 (1999) 1.
[12] J.I. Kim, R.B.B. Santos, P. Nussenzveig, Phys. Rev. Lett. 86 (2001) 1474.
[13] T.W. Mossberg, M. Lewenstein, D.J. Gauthier, Phys. Rev. Lett. 67 (1991) 1723.
[14] M. Gangl, H. Ritsch, Phys. Rev. A61 (2000) 043405.
[15] B. Deb, G. Kurizki, Phys. Rev. Lett. 83 (1999) 714.
[16] M.G. Peters, D. Hoffmann, J.D. Tobiason, T. Walker, Phys. Rev. A50 (1994) R906.
[17] K.-A. Suominen, Y.B. Band, I. Tuvi, K. Burnett, P.S. Julienne, Phys. Rev. A57 (1998) 3724.
[18] S. Chu, J. Bjorkholm, A. Ashkin, A. Cable, Phys. Rev. Lett. 57 (1986) 314; J.D. Miller, R.A. Cline, D. J. Heinzen, Phys. Rev. A 47 (1993) R4567.