Introduction

Thanks to the rapidly decreasing cost of high-throughput genotyping technologies, genome-wide association studies (GWASs) have identified thousands of single nucleotide polymorphisms (SNPs) robustly associated with hundreds of complex human diseases including cancers. However, the large number of GWAS-identified genetic loci only explains a small proportion of the disease heritability. This “missing heritability” problem has been partly attributed to the yet-to-be-identified gene–gene (G × G) and gene–environment (G × E) interactions. In spite of the important roles of G × G and G × E interactions in understanding disease mechanisms and filling in the missing heritability, straightforward GWAS scanning for such interactions has very limited statistical power, leading to few successes. Here we propose a two-step statistical approach to test G × G/G × E interactions: the first step is to perform principal component analysis (PCA) on the multiple SNPs within a gene region, and the second step is to perform Tukey’s one degree-of-freedom (1-df) test on the leading PCs. We derive a score test that is computationally fast and numerically stable for the proposed Tukey’s 1-df interaction test. Using extensive simulations we show that the proposed approach, which combines the two parsimonious models, namely, the PCA and Tukey’s 1-df form of interaction, outperforms other state-of-the-art methods. We also demonstrate the utility and efficiency gains of the proposed method with applications to testing G × G interactions for Crohn’s disease using the Wellcome Trust Case Control Consortium (WTCCC) GWAS data and testing G × E interaction using data from a case–control study of pancreatic cancer.

Keywords: G × E interaction, G × G interaction, GWAS, PCA, SNP, Tukey’s 1-df test

ABSTRACT: Genome–wide association studies (GWASs) have identified thousands of single nucleotide polymorphisms (SNPs) robustly associated with hundreds of complex human diseases including cancers. However, the large number of GWAS-identified genetic loci only explains a small proportion of the disease heritability. This “missing heritability” problem has been partly attributed to the yet-to-be-identified gene–gene (G × G) and gene–environment (G × E) interactions. In spite of the important roles of G × G and G × E interactions in understanding disease mechanisms and filling in the missing heritability, straightforward GWAS scanning for such interactions has very limited statistical power, leading to few successes. Here we propose a two-step statistical approach to test G × G/G × E interactions: the first step is to perform principal component analysis (PCA) on the multiple SNPs within a gene region, and the second step is to perform Tukey’s one degree-of-freedom (1-df) test on the leading PCs. We derive a score test that is computationally fast and numerically stable for the proposed Tukey’s 1-df interaction test. Using extensive simulations we show that the proposed approach, which combines the two parsimonious models, namely, the PCA and Tukey’s 1-df form of interaction, outperforms other state-of-the-art methods. We also demonstrate the utility and efficiency gains of the proposed method with applications to testing G × G interactions for Crohn’s disease using the Wellcome Trust Case Control Consortium (WTCCC) GWAS data and testing G × E interaction using data from a case–control study of pancreatic cancer.

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times larger than that for detecting a genetic main effect.\textsuperscript{11} The lack of power is further exacerbated in the analysis of $G \times G$ interactions due to the curse of dimensionality: one million tests of SNP main effects would correspond to $5 \times 10^{14}$ tests of pairwise SNP interactions. As a result, while a typical GWAS that genotypes around one million SNPs is designed to ensure enough power for detecting genetic main effects, straightforward genome-wide scanning of $G \times G$ and $G \times E$ interactions can be severely underpowered. In response to this pressing challenge, many new statistical methods have been proposed to improve the power of $G \times G$ and $G \times E$ interaction tests. Many new methods are aimed at reducing the burden of multiple testing. For example, Kooperberg and Leblanc\textsuperscript{13} proposed to perform formal $G \times G$ tests only for those SNPs with some marginal/main effects. Other authors proposed some alternative or hybrid strategies to filter SNPs for formal $G \times E$ testing.\textsuperscript{14–16} Another line of research is to group SNPs into genes or biological pathways to aggregate multiple weak/moderate signals and reduce the total number of interaction tests. He et al.\textsuperscript{17} proposed a gene-based $G \times G$ test by first performing principal component analysis (PCA) on multiple SNPs in linkage disequilibrium (LD) in a gene region, and then testing interactions between each pair of PCs in the two genes. In addition to several new gene-based $G \times E$ tests,\textsuperscript{18–20} Tang et al.\textsuperscript{21,22} proposed biological pathway-based $G \times E$ tests and demonstrated their applications to pancreatic cancer.

In this paper, we propose a parsimonious and powerful gene-based interaction test that can be applicable to both $G \times G$ and $G \times E$ testing. Our proposed method is motivated by Tukey’s one degree-of-freedom (1-df) test for interaction,\textsuperscript{23} which was first introduced to statistical genetics by Chatterjee et al.\textsuperscript{24} in the context of testing genetic main effects while adjusting for possible $G \times E$ effects. The test of Chatterjee et al.\textsuperscript{24} is useful in de novo GWAS scanning, aimed at discovering SNPs with any effect, ie, both genetic main and $G \times E$ effects or either effect alone. However, in the post-GWAS era, it is often of primary interest to test and discover $G \times G$ or $G \times E$ interaction effect itself. Here we propose a two-step approach to test $G \times G/G \times E$ interactions: the first step is to perform PCA on the multiple SNPs within a gene region, along the line of He et al.\textsuperscript{17}, and the second step is to perform Tukey’s 1-df test specifically for the interaction effect. We derive the score test for the latter, which is fast and numerically stable to compute. Using extensive simulations, we show that the proposed approach, which combines the two parsimonious models, namely, the PCA and Tukey’s 1-df form of interaction, outperforms other state-of-the-art methods. We also demonstrate the utility and efficiency gains of the proposed method with applications to testing $G \times G$ interactions for Crohn’s disease using the Wellcome Trust Case Control Consortium (WTCCC) GWAS data and testing $G \times E$ (SNP-set by smoking) interaction using data from a case–control study of pancreatic cancer.

**Methods**

**Existing methods:** SNP-based and gene-based $G \times G$ and $G \times E$ tests. We consider a case–control study with a total sample size $n$ including $n_0$ controls and $n_1$ cases ($n = n_0 + n_1$). Let $Y_i$ denote the binary disease status of individual $i$: 0 for controls and 1 for cases ($i = 1, \ldots, n$). Let $Z_i$ denote the covariate vector, including, eg, sex, age, and leading principal components capturing population substructure. Given two SNPs to be tested for $G \times G$ interaction, let $X_{i1}$ and $X_{i2}$ denote the genotypes of the two SNPs in subject $i$, each equal to 0, 1, and 2 for major allele homozygotes, heterozygotes, and minor allele homozygotes, respectively. In this paper, we exclusively focus on multiplicative $G \times G/G \times E$ interactions in the logistic regression framework; see Ref. 25 for tests of additive interactions. A commonly used SNP-based $G \times G$ test is based on the following logistic regression model:

$$
\logit \left( P \left( Y_i = 1 \right) \right) = \beta_0 + \beta_2 Z_i + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_1 2X_{i1}X_{i2} \quad (1)
$$

where we assume the additive genetic model for both SNPs. Alternative genetic models, such as the genotypic and dominant models, can also be assumed; see Ref. 26 for details. To test the null hypothesis of no $G \times G$ interaction, ie, $H_0$: $\beta_1 = 0$, we can perform a 1-df likelihood ratio test (LRT), or its asymptotically equivalent Wald and score tests. As pointed out before, the total number of all pairwise $G \times G$ interaction tests is much larger than that of SNP main effect tests in the GWAS setting, leading to prohibitively high computational cost and low statistical power. To conduct SNP-based $G \times E$ test between a SNP and an environmental exposure, eg, smoking, we can similarly fit a logistic regression model

$$
\logit \left( P \left( Y_i = 1 \right) \right) = \beta_0 + \beta_2 Z_i + \beta_1 X_i + \beta_k E_i + \beta_{Gk} X_i E_i, \quad (2)
$$

where $E_i$ denotes the exposure of subject $i$. We test the null hypothesis $H_0$: $\beta_{Gk} = 0$ with a 1-df LRT.

Gene-based $G \times G$ and $G \times E$ tests have also been proposed to reduce the burden of multiple testing and aggregate weak/moderate signals in genes and biological pathways. Assume two sets of SNPs mapped to two genes of interest, denoted as $X_{1i} = \left( X_{i1,1}, \ldots, X_{i1,J} \right)$ and $X_{2i} = \left( X_{i2,1}, \ldots, X_{i2,J} \right)$ for subject $i$. A SNP is assigned to a gene if it is located within certain range of the gene’s transcription start and end sites, eg, ±20,000 base pairs (20 kb), to include SNPs in regulatory regions.\textsuperscript{27} Most gene-based multilocus $G \times G$ tests are based on the following saturated interaction model:
\[
\text{logit}\left(P\left(Y_i = 1\right)\right) = \beta_0 + \beta'_Z Z_i + \sum_{l_i=1}^{L_i} \beta_{i,1}^{l_i} X_{i,l_i}^{l_i} + \sum_{l_i=1, l_j=1}^{L_i, L_j} \beta_{12,l_i,l_j}^{l_i,l_j} X_{i,l_i}^{l_i} X_{i,l_j}^{l_j}.
\]

The null hypothesis to be tested is \(H_0: \beta_{12} = \left(\beta_{12,11}, \ldots, \beta_{12,L_1,L_2}\right)^T = 0\). A multivariate score test can be performed based on the test statistic \(T_{\text{score}} = U^T V^T U\), where \(U\) is the efficient score vector of length \(L_1 \times L_2\) for \(\beta_{12}\), and \(V\) is its asymptotic covariance matrix under \(H_0\); see Ref. 18 for details. The score test is preferred here over the LRT or Wald test because it requires only fitting the null model without interactions and is more computationally efficient. However, even with the score test, the genetic main effects need to be estimated under the null model and can cause numerical instability due to the multicollinearity among SNPs in high LD within a gene region. In addition, \(T_{\text{score}}\) approximates a \(\chi^2\)-distribution with a large number of degrees of freedom \((L_1 \times L_2)\) under \(H_0\), suffering from loss of power. Pan et al.\(^{18}\) proposed to test \(H_0\) with the sum-of-squared score statistic (SSU) \(T_{\text{SSU}} = U^T U\), assuming an identity covariance matrix in \(T_{\text{score}}\); \(T_{\text{SSU}}\) has an asymptotic distribution of a mixture of \(\chi^2(1)\)'s, which can be approximated by a scaled and shifted \(\chi^2\) distribution.\(^{18}\) As shown by Pan,\(^{28,29}\) the SSU test is equivalent to the permutation-based version of a variance component score test for a random-effects logistic regression model for high-dimensional hypothesis testing.\(^{30}\) In addition, the SSU test is equivalent to the variance component score test in kernel machine regression under a linear kernel, which has been shown to be powerful in rare variant association tests\(^{11}\) and gene-based G \(\times\) E tests.\(^{19}\) Model (3) can be easily modified to perform gene-based G \(\times\) G tests:

\[
\text{logit}\left(P\left(Y_i = 1\right)\right) = \beta_0 + \beta'_Z Z_i + \sum_{l_i=1}^{L_i} \beta_{i,1}^{l_i} X_{i,l_i}^{l_i} + \beta_E E_i,
\]

\[
+ \sum_{l_i=1}^{L_i} \beta_{1l_i}^{l_i} X_{i,l_i}^{l_i} E_i.
\]

Multivariate score test and SSU test for \(H_0: \beta_{E1} = \ldots = \beta_{E1}^{l_i} = 0\) can be similarly derived. Of note, the SSU test may still suffer from numerical instability, as the genetic main effects need to be estimated under \(H_0\).

To overcome the multicollinearity problem, researchers have proposed to first perform PCA on the multiple SNPs in a gene region and then test for interactions based on the leading PCs.\(^{17,21,22,27}\) Specifically, the PCA is used to summarize SNPs in each gene as uncorrelated (orthogonal) linear combinations of the original SNPs accounting for, eg, 90% of the total genetic variation. The number of the resulting PCs, denoted as \(PC_{11}, PC_{12}\) is usually much smaller than the number of the original genotyped SNPs. He et al.\(^{17}\) proposed the following gene-based G \(\times\) G model based on the leading \(K_1\) and \(K_2\) PCs of the two genes of interest:

\[
\text{logit}\left(P\left(Y_i = 1\right)\right) = \beta_0 + \beta'_Z Z_i + \sum_{k_1=1}^{K_1} \gamma_{1,k_1}^{l_1} PC_{1,k_1}^{l_1} + \sum_{k_1=1}^{K_1} \gamma_{2,k_1}^{l_1} PC_{2,k_1}^{l_1}.
\]

\[
+ \sum_{k_1=1}^{K_1} \gamma_{12,k_1}^{l_1} PC_{1,k_1}^{l_1} PC_{2,k_1}^{l_1}.
\]

To test the null hypothesis \(H_0: \gamma_{12} = \left(\gamma_{12,11}, \ldots, \gamma_{12,K_1,K_2}\right) = 0\), He et al.\(^{17}\) proposed to employ an LRT whose test statistic approximately follows a \(\chi^2\)-distribution with \(K_1 \times K_2\) degrees of freedom under \(H_0\). Tang et al.\(^{21,22}\) adapted Model (5) to conduct gene- and pathway-based G \(\times\) E test:

\[
\text{logit}\left(P\left(Y_i = 1\right)\right) = \beta_0 + \beta'_Z Z_i + \sum_{k_1=1}^{K_1} \gamma_{1,k_1}^{l_1} PC_{1,k_1}^{l_1} + \beta_E E_i,
\]

\[
+ \sum_{k_1=1}^{K_1} \gamma_{1,k_1}^{l_1} PC_{1,k_1}^{l_1} E_i,
\]

where \(PC_{1,k_1}^{l_1}\)'s are the leading \(K_1\) PCs for the SNPs mapped to a gene or biological pathway. The null hypothesis \(H_0: \gamma_{1,k_1}^{l_1} = \ldots = \gamma_{E1,k_1}^{l_1} = 0\) can be tested with an LRT or score test.

New method: Tukey’s 1-df interaction test. Although there are typically fewer interaction terms in Model (5) than in Model (3) \((K_1 \times K_2\) versus \(L_1 \times L_2\)), the former can still be large for interactions involving large genes and may lead to loss of power. Here we propose to employ the parsimonious Tukey’s 1-df form of interaction:

\[
\text{logit}\left(P\left(Y_i = 1\right)\right) = \beta_0 + \beta'_Z Z_i + \sum_{k_1=1}^{K_1} \gamma_{1,k_1}^{l_1} PC_{1,k_1}^{l_1} + \theta \sum_{k_1=1}^{K_1} \sum_{k_2=1}^{K_2} \gamma_{12,k_1,k_2}^{l_1} PC_{1,k_1}^{l_1} PC_{2,k_2}^{l_2}.
\]

Note that the above interaction model assumes that the interaction effect is proportional to the sum of the product of genetic main effects. A single parameter \(\theta\) is used to capture the interactions between the PCs of the two genes, leading to the parsimonious Tukey’s 1-df form of interaction. For numerical stability and computational efficiency, we propose a score test for \(H_0: \theta = 0\). We derive the score test statistic as follows:

1. Let \(\psi = \left(\beta_0, \beta'_Z, \gamma_1', \gamma_2'\right)^T\) and \(X_i = \left(1, Z_i', PC_{1,i}, PC_{2,i} \right)^T\) be the nuisance parameters and their corresponding covariate vector in Model (7), where \(\gamma_1 = \left(\gamma_{1,1}, \ldots, \gamma_{1,K_1}\right)\) and \(\gamma_2 = \left(\gamma_{2,1}, \ldots, \gamma_{2,K_2}\right)\). We denote the disease probability under Model (7) by
2. The log likelihood function for Model (7) is

\[
\log L = \sum_{i=1}^{n_y} \left[ Y_i \log P_{\psi,\theta}(X_i) + (1 - Y_i) \log P_{\psi,\theta}(X_i) \right].
\]

The score function for testing \( H_0; \theta = 0 \) is

\[
S\left( \theta = 0, \psi = \hat{\psi}_0 \right) = \sum_{i=1}^{n_y} \left( \sum_{K_{i1}} \sum_{K_{i2}} \hat{\gamma}_{1,i1} \hat{\gamma}_{2,i2} \hat{P}_{PC_{1,i1}PC_{2,i2}} \right)
\]

where \( \hat{\psi}_0 \) is the maximum likelihood estimate (MLE) of \( \psi \) under \( H_0; \theta = 0 \), which can be easily obtained from the reduced (null) model as a standard logistic regression.

3. Obtain the inverse of the asymptotic variance for \( S(\theta = 0, \psi = \hat{\psi}_0) \) based on the observed information matrix:

\[
I^{\theta}_{\psi,\theta} |_{\psi = \hat{\psi}_0, \theta = 0} = \left( I^{\theta}_{\psi,\theta} - I^{\theta}_{\psi,\psi} I^{\theta}_{\psi,\theta} \right)^{-1} |_{\psi = \hat{\psi}_0, \theta = 0},
\]

where

\[
I^{\theta}_{\psi,\theta} |_{\psi = \hat{\psi}_0, \theta = 0} = \sum_{i=1}^{n_y} \left[ \sum_{K_{i1}} \sum_{K_{i2}} \hat{\gamma}_{1,i1} \hat{\gamma}_{2,i2} \hat{P}_{PC_{1,i1}PC_{2,i2}} \right]^2
\]

\[
P_{\hat{\psi}_0,\theta = 0}(X_i) \left[ 1 - P_{\hat{\psi}_0,\theta = 0}(X_i) \right],
\]

and

\[
I^{\theta}_{\psi,\theta} |_{\psi = \hat{\psi}_0, \theta = 0} = \sum_{i=1}^{n_y} \left[ \sum_{K_{i1}} \sum_{K_{i2}} \hat{\gamma}_{1,i1} \hat{\gamma}_{2,i2} \hat{P}_{PC_{1,i1}PC_{2,i2}} \right]
\]

\[
P_{\hat{\psi}_0,\theta = 0}(X_i) \left[ 1 - P_{\hat{\psi}_0,\theta = 0}(X_i) \right] X'_i,
\]

and

\[
I^{\theta}_{\psi,\theta} |_{\psi = \hat{\psi}_0, \theta = 0} = \sum_{i=1}^{n_y} P_{\hat{\psi}_0,\theta = 0}(X_i) \left[ 1 - P_{\hat{\psi}_0,\theta = 0}(X_i) \right] X_i X'_i.
\]

4. The score test statistic for \( H_0; \theta = 0 \) is thus

\[
T_{Tukey} = S\left( \theta = 0, \psi = \hat{\psi}_0 \right) I^{\theta}_{\psi,\theta} |_{\psi = \hat{\psi}_0, \theta = 0} S\left( \theta = 0, \psi = \psi_0 \right),
\]

which approximately follows a \( \chi^2(1) \) distribution under \( H_0; \theta = 0 \).

Model (7) can be easily extended to gene-based testing of \( G \times E \):

\[
\logit \left( P(Y_i = 1) \right) = \beta_0 + \beta_E Z_i + \sum_{k=1}^{K_{i1}} \gamma_{1,k1} P_{PC_{1,k1}} + \beta_E E_i
\]

\[
+ \theta \sum_{k=1}^{K_{i1}} \gamma_{1,k1} \beta_E P_{PC_{1,k1}} E_i,
\]

where \( H_0; \theta = 0 \) corresponds to no interaction between the gene and environmental exposure \( E \). A score test can be similarly derived, whose test statistic follows a \( \chi^2 \)-distribution with one degree of freedom.

Of note, our proposed Models (7) and (8) are conceptually similar to those proposed by Chatterjee et al.24; however, there are two main distinctions. First, while Chatterjee et al directly modeled the genotyped SNPs, we propose to utilize the PCA technique to reduce the dimension and model the uncorrelated PCs, which enjoys numerical stability and improved statistical power (as will be shown in the Results section). Second, and more important, the null hypothesis to be tested in Chatterjee et al.24 was on the genetic main effects of one gene, ie, \( H_0; \gamma_{k1} = \ldots = \gamma_{K_{i1}} = 0 \) in Model (8) while accounting for possible \( G \times E \) interaction effects in Tukey’s 1-df form of interaction. In contrast, our focus here is specifically to test the interaction effect, ie, \( H_0; \theta = 0 \). Note that the nuisance parameter \( \theta \) is not identifiable under \( H_0; \theta = 0 \) and some special treatments were proposed by Chatterjee et al.24; however, all nuisance parameters are identifiable under \( H_0; \theta = 0 \) in our proposed Models (7) and (8), and standard large-sample likelihood theory can be applied.

Results

A simulation study is often used to evaluate and compare different statistical methods’ Type I error rates and powers based on a large number of simulated datasets from some known models, eg, those with or without \( G \times G \) interactions. We performed extensive simulations to evaluate the proposed gene-based Tukey’s 1-df \( G \times G \) interaction test and compared it with the state-of-the-art methods of He et al.17 and Pan et al.19 We also demonstrated the utility of the proposed method with applications to testing \( G \times G \) interactions for Crohn’s disease (CD) using the WTCCC GWAS data and testing \( G \times E \) (SNP-set by smoking) interaction using data from a case–control study of pancreatic cancer.

Simulation setup. We performed simulations based on the HapMap SNP data of two genes \( IL12B \) (interleukin 12B; 5q31.1–q33.1) and \( IL12RB2 \) (interleukin 12 receptor, beta 2; 1p31.3–p31.2), which were found to interact with each other.
influencing the risk of CD in our analysis of the WTCCC GWAS data (results to be shown later). In addition, proteins IL12B and IL12RB2 have been reported to physically interact with each other.\textsuperscript{32} We obtained the genotype and phased haplotype data for 60 HapMap CEU individuals (release 22) for IL12B and IL12RB2. For each gene, we selected common SNPs to be the genotyped/observed SNPs in the simulated datasets if they had MAFs of at least 5%, and were either genotyped in the WTCCC GWAS or picked up as tagSNPs with pairwise tagging $r^2 \geq 0.8$ by the program Tagger.\textsuperscript{33} We ended up with 16 and 21 genotyped SNPs for IL12B and IL12RB2, respectively. We largely followed the simulation setup in He et al.\textsuperscript{17} to simulate a case–control study of 2,000 cases and 2,000 controls in each of the 1,000 simulation replications. We designated either one or two causal SNPs with main/interaction effects in each of the two genes. For the scenario of one causal SNP that was randomly selected in each gene, the case–control status was simulated based on the following logistic regression model:

$$
\text{logit}(P(Y_G)) = \beta_1 Y_i + \beta_2 X_i + \beta_3 Z_i + \epsilon_i,
$$

where $\beta_1 = \log(0.01/0.99)$ for a baseline disease prevalence of 1%, $\beta_2 = 0$ or 0.5, and $\beta_3 = 0$ or 0.5. We let $\beta_1 = 0$ to evaluate the Type I error rate and gradually increased it to evaluate the power. We have listed the parameter values in Table 1 for the six simulation configurations considered here.

We included the following gene–based G $\times$ G tests in the simulation study: (1) the score test for the proposed PC-based Tukey’s 1-df interaction Model (7), (2) the generic LRT for PC-based pairwise interaction Model (5) of He et al.,\textsuperscript{17} and (3) the SSU test for SNP-based pairwise interaction Model (3) of Pan et al.\textsuperscript{18} as a representative of variance–component score tests for high-dimensional hypothesis testing. For PC-based tests, we first applied the PCA to the genotype data of each gene region to derive the top PCs that explained at least 90% of the total genetic variation.

**Simulation results.** As shown in Table 1, all three methods controlled the Type I error rate at the nominal level $\alpha = 0.05$ satisfactorily across the six simulation configurations. We also performed simulations when there was one or two covariates, and the proposed score test for Tukey’s 1-df interaction controlled the Type I error rate satisfactorily as well (results not shown).

Figures 1–6 show the power comparison of the three tests across different simulation configurations. Overall, the proposed Tukey’s 1-df score test was the most powerful one among the three methods under comparison, and the SNP-based SSU test was the least powerful one, suggesting that the proposed strategy combining the two parsimonious statistical models, namely the PCA and Tukey’s 1-df form of interaction, was effective in improving the power of gene-based interaction test. The power difference was larger when there was only one disease locus/causal SNP in each gene (Figs. 1–3) than when there were two disease loci/causal SNPs in each gene (Figs. 4–6) whether or not there was main effect. In particular, there was only a slight power difference between the generic LRT and the SSU in the latter scenario. This suggested that the SSU and its closely related variance–component score tests, such as the kernel machine-based methods, might have more improved power when the interaction signals were less sparse (four true interaction pairs versus one in the simulations here). It is noticeable that the power curve for the SSU test in

Table 1. Empirical Type I error rate based on 1,000 simulation replications (2,000 cases and 2,000 controls in each replication).

| CONFIGURATION | $\beta_1^{(1)}$ | $\beta_2^{(1)}$ | $\beta_1^{(2)}$ | $\beta_2^{(2)}$ | NO. OF INTERACTIONS* | TUKEY’S 1-DF SCORE TEST | GENERIC LRT | SSU |
|---------------|----------------|----------------|----------------|----------------|---------------------|------------------------|-------------|------|
| 1             | 0.5            | 0              | 0.5            | 0              | 1                   | 0.048                  | 0.051       | 0.048|
| 2             | 0.5            | 0              | 0              | 0              | 1                   | 0.051                  | 0.050       | 0.050|
| 3             | 0              | 0              | 0              | 0              | 1                   | 0.047                  | 0.054       | 0.054|
| 4             | 0.5            | 0.5            | 0.5            | 0.5            | 4                   | 0.054                  | 0.052       | 0.049|
| 5             | 0.5            | 0              | 0              | 0              | 4                   | 0.055                  | 0.049       | 0.049|
| 6             | 0              | 0              | 0              | 0              | 4                   | 0.049                  | 0.056       | 0.056|

Notes: *All interaction terms were 0 to evaluate the Type I error rate. Significance level $\alpha = 0.05$. 

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Figure 3 was not monotonically increasing as the interaction signal increased. This may be explained by numerical problems in estimating the genetic main effects of multiple SNPs in high LD in Model (3). On the other hand, we did not observe this abnormal phenomenon for the PC-based Tukey’s 1-df score test and generic LRT, supporting that using PCs can not only reduce the dimension but also ensure numerical stability. In addition, although the proposed Tukey’s 1-df form of interaction is a function of the genetic main effects, it remained more powerful than the other two competing methods even when there was no genetic main effect (Figs. 3 and 6) or only one gene had genetic main effects (Figs. 2 and 5). The reason for this interesting phenomenon is that, even when there was only interaction effect and no main effect in the simulation/true model, the expectation of the marginal genetic effect, ie, the main effect in the fitted null model, was not zero due to the absorption of the true interaction effect into the marginal effect. Therefore, Tukey’s test could still retain the statistical power to detect the interaction effect in the absence of main effects. Finally, the ratio of runtime for Tukey’s 1-df score test, generic LRT, and SSU test was nearly in the ratio 1:2:3, with the proposed Tukey’s test being the fastest.

Application to testing G × G for Crohn’s disease. CD is a type of inflammatory bowel disease and is also considered...
Table 2 lists all pairs of G × G interactions with interaction P-values <0.1 by either Tukey’s 1-df score test or the generic LRT. As a comparison, we also listed the SSU test P-values, which were very similar to those of the generic LRT. As expected with this moderate sample size, none of the tests identified any G × G interaction with a P-value less than 0.01. On the other hand, previous research showed that G × G interactions tend to be enriched among neighboring genes on protein–protein interaction (PPI) networks. While none of the five nominally significant G × G interactions by the generic LRT (P_{lrt} < 0.1) was found to coincide with PPIs in the STRING database, three of the nine nominally significant interactions by Tukey’s 1-df test (P_{Tukey} < 0.1) appeared to be supported by PPIs: the proteins of three pairs of genes, including IL12B–IL12RB2, IL12B–IL23A, and IL12B–IL12RB1, were found to physically interact with each other. Due to the small counts, this enrichment was not statistically significant. Nevertheless, these biologically plausible G × G interactions would be worth following up in independent and larger samples.

**Application to testing G × E in a case–control study of pancreatic cancer.** Pancreatic cancer is the fourth leading cause of cancer-related deaths for both men and women in the US with a 5-year survival rate of 6%. Known risk factors for pancreatic cancer include cigarette smoking, long-term Type 2 diabetes, obesity, heavy alcohol consumption, and family history, with smoking conferring the highest risk. It is of interest to investigate whether there exist genes that interact with environmental exposures, such as smoking, influencing the risk of pancreatic cancer. Here we tested the pancreatic cancer susceptibility SNP-set by smoking interaction (G × E) using data from a case–control study conducted at The University of Texas MD Anderson Cancer Center during 2004–2009. Cases were patients with pathologically confirmed pancreatic adenocarcinoma, and controls were healthy individuals frequency matched to cases by age, race, and sex. These individuals were genotyped for a total of 19 SNPs in 10 susceptibility genes identified in previous GWAS of pancreatic cancer, including ABO, NR5A2, and CLPTM1L–TERT, as well as candidate genes FTO, ACDC, PPARG, PRKAA2, PRKAB2, PRKACB, C4A, C4B, C4C, C2, C3, and B3 . To explore the G × G interactions among known CD susceptibility genes, we retrieved the 31 genes reviewed in Ref. 34. We followed Ref. 27 to assign SNPs to genes: an SNP is assigned to a gene if it is located within 20 kb of the gene’s transcription start and end sites. A total of 653 SNPs were assigned to the 31 genes. Since these genes were identified by large-scale meta-analysis of GWASs and candidate pathway studies, not all of them may be associated with CD in the WTCCC GWAS. To test for genetic marginal associations, we applied PCA to the SNPs in each of the 31 genes, retrieved the leading PCs that explained at least 90% of the genetic variation, and performed PC-based multilocus association test of the genetic main effects. Twelve nominally significant genes that had P-values <0.1 were tested for pairwise G × G interactions, as in Ref. 12. As a proof of concept, we applied the proposed Tukey’s 1-df test for G × G and G × E interactions.
Table 2. Top G × G interactions for Crohn’s disease (P-values <0.1 by either Tukey’s 1-df score test or generic LRT) ordered by Tukey’s 1-df test P-values.

| GENE 1   | $P_{\text{marginal}}$ | GENE 2    | $P_{\text{marginal}}$ | $P_{\text{Tukey}}$ | $P_{\text{SSU}}$ | PPI in STRING |
|----------|-----------------------|-----------|-----------------------|---------------------|------------------|---------------|
| IL12RB2  | 0.0002                | IL12B     | 0.0005                | 0.012               | 0.012            | Yes           |
| STAT3    | 0.0004                | IL12RB1   | 0.058                 | 0.034               | 0.188            | No            |
| IL12B    | 0.0005                | IL23A     | 0.095                 | 0.050               | 0.149            | Yes           |
| IL23R    | <0.0001               | IL18R1    | 0.017                 | 0.058               | 0.358            | No            |
| STAT3    | 0.0004                | RORC      | 0.0048                | 0.061               | 0.281            | No            |
| STAT3    | 0.0004                | IL18R1    | 0.017                 | 0.072               | 0.071            | No            |
| IL23R    | <0.0001               | IL18RAP   | 0.0111                | 0.074               | 0.209            | No            |
| IL12B    | 0.0005                | IL12RB1   | 0.058                 | 0.076               | 0.223            | Yes           |
| RORC     | 0.005                 | IL10      | 0.093                 | 0.096               | 0.792            | No            |
| STAT3    | 0.0004                | IL18RAP   | 0.011                 | 0.194               | 0.016            | No            |
| IL12RB2  | 0.0002                | TNFα      | 0.011                 | 0.295               | 0.029            | No            |
| IL18R1   | 0.017                 | IL23A     | 0.095                 | 0.370               | 0.090            | No            |
| TNFα     | 0.011                 | IL23A     | 0.095                 | 0.929               | 0.075            | No            |

Note: Interaction P-values less than 0.1 are in bold.

PRKAB1, and LOC730242. Genotyping was performed on genomic DNA from peripheral blood samples using the TaqMan method.42

To test the interaction between smoking and the pancreatic cancer susceptibility SNP-set defined by the 19 SNPs in the 10 genes, we applied the proposed Tukey’s 1-df score test (Model 8), the generic LRT [Model (6)] and the SSU test [Model (4)] to the above-described UT MD Anderson study of pancreatic cancer with 534 cases and 552 controls. For the former two PC-based tests, we included the leading 11 PCs that explained 87% of the total genetic variation in the interaction model with smoking status (never/ever). Tukey’s 1-df test and the generic LRT test gave similarly significant results: the $P$-values were 0.019 and 0.018, respectively. On the other hand, the SSU test result was not significant, with a $P$-value = 0.148. When applied to the 11 PCs, the SSU test had a significant $P$-value of 0.024. Of note, one of the 19 SNPs was the top hit in the GWAS of pancreatic cancer41 (rs505922 in gene ABO), and its interaction with smoking on the risk of pancreatic cancer was highly significant ($P$-value = 0.002 by SNP × smoking test based on Model (2) and $P$-value = 0.038 after the Bonferroni correction). This significant interaction identified by the SNP-set-based G × E tests warrants further replication in independent samples.

Discussion

In this paper we proposed a powerful gene-based test for detecting G × G or G × E interactions by combining two parsimonious statistical models, namely, the PCA and Tukey’s 1-df form of interaction. We derived a score test that is computationally fast and numerically stable for the proposed Tukey’s 1-df interaction model. Using extensive simulations based on the HapMap phased haplotype data, we showed that the proposed score test for Tukey’s 1-df interaction model controlled the Type I error rate at the nominal level satisfactorily and was more powerful than the PC-based generic LRT that tests pairwise interactions and the multiple-SNP-based SSU test as a representative of variance-component score tests. We demonstrated the utility and efficiency gains of the proposed test with applications to detecting G × G interactions for CD using the WTCCC GWAS data and to detecting SNP-set by smoking (G × E) interaction using a case–control study of pancreatic cancer. As demonstrated in the latter application, we recommend first using gene-based interaction tests to identify significant genes, and then performing SNP-based interaction tests within the genes to identify which SNPs significantly interact.

We have focused on gene-based G × G/G × E tests for common SNPs (MAF >5%) based on GWAS data. It would be of interest to extend the Tukey’s 1-df test to testing G × G/G × E interactions for rare variants (MAF <5% or 1%) based on the next-generation sequencing (NGS) data. Although PCA is an effective approach to summarize a large number of common SNPs into a few uncorrelated PCs (FPCA), which has been shown to be a powerful method for reducing the dimension of a large number of rare variants.43,44 Further research is warranted.

The proposed method has some potential limitations. First, the PCA step is solely based on the SNP data and does not take the disease–SNP correlations into account. As a result, it is possible that the leading PCs may not capture the information of the most relevant SNP(s) in association with the disease. Alternative dimension reduction techniques, eg,
the partial least squares (PLS) method, could be employed to find linear combinations of the SNPs that are most correlated with the disease status. However, because of the use of the disease status in the first-stage PLS, the test statistic in the second-stage interaction model will no longer follow a distribution under the null hypothesis of no interactions, and computationally intensive permutation or parametric bootstrap procedure is needed to obtain the null distribution of the test statistic. Second, the parsonsimous Tukey’s 1-df form of interaction model assumes that the interaction effect is approximately proportional to the genetic main effects. Although it appeared to be powerful and robust across different simulation scenarios in our study, even in the absence of genetic main effects, it might lose power under some scenarios as investigated in Ref. 45. As the true interaction model is hardly known a priori in real data analysis and likely varies across genes, the proposed method is a competitive and complementary approach to existing gene-based interaction tests.

The current paper focuses on case–control analyses. It is well known that for SNP-based $G \times E$ analysis, the case–only test is more powerful than the standard case–control interaction test [Model (2)] if the assumption of gene–environment independence holds in the general population. Specifically, the case–only analysis tests the association between the environmental exposure and the SNP of interest in the cases. It would be interesting to extend the SNP-based case–only test to a gene-based test by testing the association between the environmental exposure and the leading PCs for the multiple SNPs in a gene. Another possible direction of future research is to extend the existing SNP-based tests for nonremovable interactions to gene-based tests. Finally, as the Tukey’s 1-df interaction model is in the regression framework, it is not limited to binary disease phenotypes and can be easily extended to $G \times G/G \times E$ tests for quantitative traits. R programs implementing the proposed Tukey’s 1-df score test will be posted on our website at: https://sites.google.com/site/ utpengwei/.

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Author Contributions

Conceived and designed the experiment: PW. Analyzed the data: YW, PW. Wrote the first draft of the manuscript: PW. Contributed to the writing of the manuscript: PW, YW, DL. Agree with manuscript results and conclusions: PW, YW, DL. All authors reviewed and approved of the final manuscript.

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