From LO to NLO in the parton Reggeization approach.

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Quarks 2018

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Outline.

1. Motivation
2. LO Framework
3. Examples of applications at LO and NLO
4. DIS@NLO: Matching to the NLO CPM result
5. Virtual NLO corrections and rapidity divergences
Parton Reggeization Approach (PRA) is a hybrid scheme of $k_T$-factorization which combines gauge-invariant matrix elements with off-shell (Reggeized) partons in the initial state with the unintegrated PDFs resumming doubly-logarithmic corrections $\sim \log^2(q_T^2/Q^2)$ (Kimber-Maritn-Ryskin unPDFs).

- **The aim** of PRA is to improve the description of multi-scale correlational observables at the energies accessible at the LHC in comparison with the fixed-order NLO/NNLO calculations.

- **The wider task** is to understand the role of transverse momentum in Parton Showers (PS) and put the *Recoiling Scheme* ambiguity of PS under theoretical control.

- To provide predictions with controllable accuracy and understand our formalism better we should go to NLO.

- The LO/NLO calculation in PRA should describe single-scale observables with the same accuracy as LO/NLO calculation in CPM. ⇒ We will use single-scale observables (DIS structure functions) to fix the scheme of calculations at NLO in PRA.
LO framework
Derivation of the LO factorization formula

See [A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, Phys.Rev. D96 096019 (2017)] for details.

Inclusive process ($\mu^2$ – hard scale of $\mathcal{Y}$):

$$p(P_1) + p(P_2) \rightarrow \mathcal{Y}(P_A) + X,$$

auxiliary hard ($2 \rightarrow 3$) subprocess:

$$g(p_1) + g(p_2) \rightarrow g(k_1) + \mathcal{Y}(P_A) + g(k_2),$$

Sudakov (Light-cone) decomposition:

$$n_-^\mu = P_1^\mu / \sqrt{S}, \quad n_+^\mu = P_2^\mu / \sqrt{S}:$$

$$k^\mu = \frac{1}{2} (k^+ n_-^\mu + k^- n_+^\mu) + k_T^\mu,$$

where $k^\pm = (n_\pm k) = k^0 \pm k^3$, $n_\pm k_T = 0$ and

$$k q = \frac{1}{2} (k^+ q^- + k^- q^+) - k_T q_T.$$
From LO to NLO in the parton Reggeization approach.

Derivation of the LO factorization formula

See [A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, Phys.Rev. D96 096019 (2017)] for details.

Auxiliary hard CPM subprocess:

\[ g(p_1) + g(p_2) \rightarrow g(k_1) + \Upsilon(P_A) + g(k_2), \]

where \( p_1^2 = 0, \ p_1^- = 0, \ p_2^2 = 0, \ p_2^+ = 0. \)

Kinematic variables (\( 0 < z_{1,2} < 1 \)):

\[ z_1 = \frac{p_1^+ - k_1^+}{p_1^+}, \quad z_2 = \frac{p_2^- - k_2^-}{p_2^-}, \]

Two limits where \( |\mathcal{M}|^2 \) factorizes:

1. **Collinear limit**: \( k_{T1,2}^2 \ll \mu^2, \ z_{1,2} \) arbitrary,

2. **Multi-Regge limit**: \( z_{1,2} \ll 1, \ k_{T1,2}^2 \) arbitrary.
Derivation of the LO factorization formula

See [A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, Phys.Rev. D96 096019 (2017)] for details.

Auxiliary hard CPM subprocess:

\[ g(p_1) + g(p_2) \rightarrow g(k_1) + \Upsilon(P_A) + g(k_2), \]

1 Collinear limit: \( k_{T1,2}^2 \ll \mu^2, \ z_{1,2} \) - arbitrary:

\[ |\mathcal{M}|^2_{\text{CL}} \simeq \frac{4g_s^4}{k_{T1}^2 k_{T2}^2} P_{gg}(z_1) P_{gg}(z_2) \frac{|\mathcal{A}_{CPM}|^2}{z_1 z_2}, \]

where \( |\mathcal{A}_{CPM}|^2 \) - amplitude \( g + g \rightarrow \Upsilon \) with on-shell initial-state partons, \( P_{gg}(z) \) - DGLAP \( g \rightarrow g \) splitting function.
Derivation of the LO factorization formula

See [A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, Phys.Rev. D96 096019 (2017)] for details.

Auxiliary hard CPM subprocess:

\[ g(p_1) + g(p_2) \rightarrow g(k_1) + \Upsilon(P_A) + g(k_2), \]

2 Multi-Regge limit: \( z_{1,2} \ll 1 \)  
\( \Leftrightarrow \Delta y_{1,2} \gg 1 \), \( k_{T1,2}^2 \) - arbitrary:

\[
|M|^2_{\text{MRK}} \simeq \frac{4g_s^4}{k_{T1}^2 k_{T2}^2} \tilde{P}_{gg}(z_1) \tilde{P}_{gg}(z_2) \frac{|A_{PRA}|^2}{z_1 z_2},
\]

where \( \tilde{P}_{gg}(z) = 2C_A/z \) and \( |A_{PRA}|^2 \) is the gauge-invariant amplitude  
\( R_+(q_1) + R_-(q_2) \rightarrow \Upsilon \) with Reggeized  
(off-shell) partons in the initial state.
Derivation of the LO factorization formula

See [A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, Phys.Rev. D96 096019 (2017)] for details.

Auxiliary hard CPM subprocess:

\[ g(p_1) + g(p_2) \rightarrow g(k_1) + \Upsilon(P_A) + g(k_2), \]

**Modified MRK approximation:** \( z_{1,2} \) and \( k_{T1,2}^2 \) – arbitrary:

\[
|\mathcal{M}|^2 \text{mMRK} \approx \frac{4g_s^4}{q_1^2 q_2^2} P_{gg}(z_1) P_{gg}(z_2) \frac{|A_{PRA}|^2}{z_1 z_2},
\]

where \( q_{1,2}^2 = q_{T1,2}^2 / (1 - z_{1,2}) \), has correct **collinear** and **Multi-Regge** limits!

**Conjecture:** mMRK approximation is reasonable zero-approximation for exact \( |\mathcal{M}|^2 \) away from collinear limit.

At least, it is better than collinear limit itself.
Numerical test

Ratio $\sigma_{\text{mMRK}}/\sigma_{\text{exact}}$ of cross sections of the subprocess

$$g + g \rightarrow g(y_1 < y_2) + g(y_2, p_{TL}) + g(y_3 > y_2),$$

in the mMRK approximation vs. exact result in CPM.

Curves from top to bottom: no constraint on $\Delta y$; $\min(\Delta y_{12}, \Delta y_{23}) > 1.5$; $> 3.5$. The ratio is almost flat vs. $p_{TL}$!
Factorization formula

Substituting the $|\mathcal{M}|^2_{mMRK}$ to the factorization formula of CPM and changing the variables we get:

$$d\sigma = \int_0^1 \frac{dx_1}{x_1} \int \frac{d^2q_{T1}}{\pi} \tilde{\Phi}_g(x_1, t_1, \mu^2) \int_0^1 \frac{dx_2}{x_2} \int \frac{d^2q_{T2}}{\pi} \tilde{\Phi}_g(x_2, t_2, \mu^2) \cdot d\hat{\sigma}_{PRA},$$

where $x_1 = q_1^+/P_1^+$, $x_2 = q_2^-/P_2^-$, $\tilde{\Phi}(x, t, \mu^2)$ – “tree-level” un-integrated PDFs, the partonic cross-section in PRA is:

$$d\hat{\sigma}_{PRA} = \frac{|\mathcal{A}_{PRA}|^2}{2Sx_1x_2} \cdot (2\pi)^4 \delta \left( \frac{1}{2} (q_1^+ n_- + q_2^- n_+) + q_T1 + q_T2 - P_A \right) d\Phi_A.$$

Note the usual flux-factor $Sx_1x_2$ for off-shell initial-state partons.
LO unintegrated PDF

The “tree-level” unPDF:

\[ \tilde{\Phi}_g(x, t, \mu^2) = \frac{1}{t} \frac{\alpha_s}{2\pi} \int_{x}^{1} dz \ P_{gg}(z) \cdot \frac{x}{z} f_g \left( \frac{x}{z}, \mu^2 \right). \]

contains collinear divergence at \( t \to 0 \) and IR divergence at \( z \to 1 \).

In the “dressed” unPDF collinear divergence is regulated by Sudakov formfactor \( T(t, \mu^2) \):

\[ \Phi_i(x, t, \mu^2) = \frac{T_i(t, \mu^2, x)}{t} \times \frac{\alpha_s(t)}{2\pi} \int_{x}^{1} dz \ \theta_{z}^{\text{cut}} P_{ij}(z) \frac{x}{z} f_j \left( \frac{x}{z}, t \right) \]

where: \( \theta_{z}^{\text{cut}} = \theta ((1 - \Delta_{KMR}(t, \mu^2)) - z) \), and the Kimber-Martin-Ryskin(KMR) cut condition [KMR, 2001]:

\[ \Delta_{KMR}(t, \mu^2) = \frac{\sqrt{t}}{\sqrt{\mu^2} + \sqrt{t}}, \]

follows from the rapidity ordering between the last emission and the hard subprocess.
LO unintegrated PDF

\[ \Phi_i(x, t, \mu^2) = \frac{T_i(t, \mu^2, x)}{t} \times \frac{\alpha_s(t)}{2\pi} \int_0^1 dz \theta_{z}^{\text{cut}} P_{ij}(z) \frac{x}{z} f_j \left( \frac{x}{z}, t \right) \]

\[ = \frac{\partial}{\partial t} \left[ T_i(t, \mu^2, x) \cdot x f_i(x, t) \right] \leftrightarrow \text{derivative form of unPDF} \]

⇒ LO normalization condition:

\[ \int_0^{\mu^2} dt \ \Phi_i(x, t, \mu^2) = x f_i(x, \mu^2) \leftrightarrow \text{Holds exactly!} \]

Because \( T(0, \mu^2, x) = 0 \) and \( T(\mu^2, \mu^2, x) = 1 \).
Sudakov formfactor

\[ T_i(t, \mu^2, x) = \exp \left[ - \int_{t}^{\mu^2} \frac{dt'}{t'} \frac{\alpha_s(t')}{2\pi} (\tau_i + \Delta \tau_i) \right] , \]

\[ \tau_i = \sum_j \int_0^1 dz \, \theta_{z}^{\text{cut}} \cdot zP_{ji}(z) , \]

\[ \Delta \tau_i = \sum_j \int_0^1 dz \, (1 - \theta_{z}^{\text{cut}}) \cdot \left[ zP_{ji}(z) - \frac{x}{z} f_j \left( \frac{x}{z}, t' \right) \underbrace{xf_i(x, t')}_{P_{ij}(z) \cdot \theta(z - x)} \right] . \]

similar structure in the non-emission probability in ISR PS
Gauge-invariant off-shell amplitudes

$|A_{PRA}|^2$ is obtained from Lipatov’s gauge-invariant effective theory for MRK processes in QCD [Lipatov 1995; Lipatov, Vyazovsky, 2001].

Some Feynman rules for Reggeized gluons:

\[
\begin{align*}
\frac{a}{q} & \rightarrow b = \frac{-i\delta_{ab}}{2q^2} \\
\frac{a}{q} & \rightarrow \frac{b}{\mu} = (-iq^2) n_\mu \delta_{ab}
\end{align*}
\]

\[
g_s f_{aa1a2} (n_\mu n_\nu) \frac{q^2}{k_1^+}
\]

\[
ig_s^2 (n_{\mu_1} n_{\mu_2} n_{\nu_3}) \frac{q^2}{k_3^+} \left[ \frac{f_{aba1f_{ba2a3}}}{k_1^+} + \frac{f_{aba2f_{ba1a3}}}{k_2^+} \right]
\]
Some Feynman rules for Reggeized quarks:

|   |   |   |   |
|---|---|---|---|
| $\leftarrow k$ | $\rightarrow q$ | $i g_s T^a \left( \gamma_\mu + \hat{q} n_k^+ \right)$ | $q_1 \rightarrow k$ | $q_2 \rightarrow$ | $i g_s T^a \left( \gamma_\mu + \hat{q}_2 n_k^+ + \hat{q}_1 n_k^- \right)$ |
| $q \uparrow$ | $2$ | $k$ | $q_1 \uparrow$ | $2$ | $q_2 \downarrow$ |
| $q_1 \uparrow$ | $1$ | $k$ | $i g_s^2 (n_{\mu_1}^+ n_{\mu_2}^+) \hat{q} \left[ \frac{T_{a_1} T_{a_2}}{k_1^+ (k_1 + k_2)^+} + k_2^+ (k_1 + k_2)^+ \right]$ | $q_1 \uparrow$ | $3$ | $q_2 \downarrow$ |
| $q \uparrow$ | $2$ | $1$ | $i g_s^2 \left[ \hat{q}_2 (n_{\mu_1}^+ n_{\mu_2}^+) \left( \frac{T_{a_1} T_{a_2}}{k_1^+ (k_1 + k_2)^+} + \frac{T_{a_2} T_{a_1}}{k_2^- (k_1 + k_2)^+} \right) \right] - \hat{q}_1 (n_{\mu_1}^- n_{\mu_2}^-) \left( \frac{T_{a_2} T_{a_1}}{k_1^- (k_1 + k_2)^-} + \frac{T_{a_1} T_{a_2}}{k_2^- (k_1 + k_2)^-} \right)$ |
| $q_1 \uparrow$ | $3$ | $2$ | $k$ | $i g_s^3 \hat{q} (n_{\mu_1}^+ n_{\mu_2}^+ n_{\mu_3}^+) \left[ \frac{T_{a_1} T_{a_2} T_{a_3}}{k_1^+ (k_1 + k_2)^+ (k_1 + k_2 + k_3)^+} + (1 \leftrightarrow 2 \leftrightarrow 3) \right]$ |
| $q_2 \downarrow$ | $1$ | $3$ | $q_1 \uparrow$ | $3$ | $q_2 \downarrow$ |
| $q_1 \uparrow$ | $2$ | $2$ | $i g_s^3 \left[ \hat{q}_2 (n_{\mu_1}^+ n_{\mu_2}^+ n_{\mu_3}^+) \left( \frac{T_{a_1} T_{a_2} T_{a_3}}{k_1^+ (k_1 + k_2)^+ (k_1 + k_2 + k_3)^+} + (1 \leftrightarrow 2 \leftrightarrow 3) \right) \right] + \hat{q}_1 (n_{\mu_1}^- n_{\mu_2}^- n_{\mu_3}^-) \left( \frac{T_{a_3} T_{a_2} T_{a_1}}{k_1^- (k_1 + k_2)^- (k_1 + k_2 + k_3)^-} + (1 \leftrightarrow 2 \leftrightarrow 3) \right)$ |
Implementation in **FeynArts**.

The Feynman rules of Lipatov’s EFT, up to the order $O(g_s^3, e g_s^2, e^2 g_s, e^3)$ are implemented in our model-file **ReggeQCD** for the package **FeynArts**.
BCFW recursion and Lipatov’s EFT

The approach to derive gauge-invariant scattering amplitudes with off-shell initial-state partons, using the spinor-helicity techniques and Britto-Cachazo- Feng-Witten-like recursion relations for such amplitudes, was introduced in Refs.:
1) A. van Hameren and M. Serino, BCFW recursion for TMD parton scattering, J. High Energy Phys. 07 (2015) 010; K. Kutak, A. Hameren, and M. Serino, QCD amplitudes with 2 initial spacelike legs via generalised BCFW recursion, J. High Energy Phys. 02 (2017) 009.
2) A. van Hameren, KaTie: For parton-level event generation with kT -dependent initial states, arXiv:1611.00680.

This formalism is equivalent to Lipatov’s EFT at the tree level, but for some observables, e.g., related with heavy quarkonia, or for the generalization of the formalism to NLO, the explicit Feynman rules and the structure of EFT are more convenient.
Azimuthal decorrelations of dijets, diphotons and $B$-mesons
From LO to NLO in the parton Reggeization approach.

Dijet azimuthal decorrelations at the LHC

[M.A.Nefedov, V.A.Saleev, A.V.Shipilova, Phys. Rev. D87, 094030 (2013)].

Dijet production at the LHC ($\sqrt{S} = 7$ TeV). *Open points* – only 2 jets with $p_T > 30$ GeV at $|y| < 0.8$, *Closed points* – inclusive data.

The description in PRA is given by $2 \rightarrow 2$ subprocesses and is dominated by:

$$R_+ + R_- \rightarrow g + g.$$

To reject the jets from the last stage of unPDF evolution, the cut $\max(|q_{T1}|, |q_{T2}|) < p_T^{\text{subleading jet}}$ is necessary.
From LO to NLO in the parton Reggeization approach.

**D-meson pair production @ LHC**

[A. V. Karpishkov, V. A. Saleev, A. V. Shipilova, Phys. Rev. D 94, 114012 (2016)]

Production of **charge-conjugated** states: mostly through

$$R_+ + R_- \rightarrow c + \bar{c}.$$
From LO to NLO in the parton Reggeization approach.

\textit{D-meson pair production @ LHC}

[A. V. Karpishkov, V. A. Saleev, A. V. Shipilova, Phys. Rev. D 94, 114012 (2016)]

Production of \textbf{same-sign} pairs: through gluon fragmentation

\[ R_+ + R_- \rightarrow g + g. \]
Diphoton production at Tevatron and the LHC

[M.A.Nefedov, V.A.Saleev, Phys. Rev. D92, 094033 (2015)].

The PRA calculation is at the NLO* level. 2 → 2 subprocesses:

\[ Q_+ + \bar{Q}_- \rightarrow \gamma + \gamma, \]  
\[ R_+ + R_- \rightarrow \gamma + \gamma, \]  

the subprocess (2) goes through quark box, and \( t_{1,2} \)-dependence of this amplitude has been calculated.

The NLO 2 → 3 subprocesses:

\[ Q_+ + Q_- \rightarrow \gamma + \gamma + g, \]  
\[ Q_\pm + R_\mp \rightarrow \gamma + \gamma + q, \]

+mMRK double-counting subtraction, which practically removes the contribution of the subprocess (3) and greatly reduces the contribution of the subprocess (4).
From LO to NLO in the parton Reggeization approach.

Diphoton production at Tevatron and the LHC

[M.A.Nefedov, V.A.Saleev, Phys. Rev. D92, 094033 (2015)].
Comparison with CDF data:

![Graph showing comparison of CDF-2012 data with theory and CDF-2012 data at different pT values.](image-url)
B\bar{B}-production with a jet @ LHC

[A.V.Karpishkov, M. A. Nefedov, V.A.Saleev, Phys. Rev. D 96, 096019 (2017)].

Complementarity of $2 \rightarrow 2$ and $2 \rightarrow 3$ contributions in the Phase-space:

**Complicated kinematical situation!**
The $B\bar{B}$-pair is searched in the events with a hard jet.
Our solution – “merging” of two contributions:

1. The hard jet = $b$-jet:

   \[ R_+(q_1) + R_-(q_2) \rightarrow b(q_3) + \bar{b}(q_4), \]

   \[ \Downarrow_{\bar{B}} \]

2. The hard jet $\neq b$-jet:

   \[ R_+(q_1) + R_-(q_2) \rightarrow b(q_3) + \bar{b}(q_4) + g(q_5), \]

   \[ \Downarrow_{\bar{B}} \]
From LO to NLO in the parton Reggeization approach.

Comparison with CMS data

Green line – 2 → 2, blue line – 2 → 3, orange line – sum. The dependence of normalization and shape of the $B\bar{B}$ azimuthal decorrelation spectrum on $p_T$ of the leading hard jet is described!
Towards NLO calculations
From LO to NLO in the parton Reggeization approach.

**Differences with NLO of CPM**

- **Real NLO corrections:** No collinear divergences when integrating over $k_T$ of additional emitted parton down to 0, unlike in CPM. Initial-state collinear divergences are regularized by transverse momentum of initial-state partons.

- Instead, the **double-counting** of region $k_T \to 0$ or $\Delta y \to \infty$ should be subtracted from NLO contribution to the hard-scattering coefficient.

- Vertices of Lipatov’s theory are **nonlocal**: contain *eikonal denominators* – $1/k^\pm \Rightarrow$ Rapidity divergences in real and virtual corrections!

- Rapidity divergences are related with BFKL resummation of contributions $\sim \log 1/x$. So some elements of BFKL resummation necessarily enter at NLO.
Motivation for PRA is the multi-scale correlational observables. ⇒ NLO CPM accuracy for single-scale observables is enough.

We DO NOT want to do our own fit of unPDFs ⇒ using (MS) PDFs of CPM as collinear input. But NLO PDFs are scheme-dependent!

For the single-scale observables (e.g. $F_{2/L}(x, Q^2)$) collinear factorization is a theorem (up to corrections $\sim (\Lambda_{QCD}^2/Q^2)^\#$).

⇒ Physical normalization condition at NLO:

$$f_i^{MS}(x, \mu^2)$$

$$F_{2q}^{(NLO \ PRA)}(x, Q^2) \leftarrow \downarrow F_{2q}^{(NLO \ CPM)}(x, Q^2) + O(\alpha_s^2(Q^2)) + \text{Higher twist},$$

Basic idea: Collect all terms in PRA, contributing $O(\alpha_s)$ to the normalization condition and match them to the NLO CPM result.
Simplest example: gluon contribution to $F_2$ @ NLO of PRA

[M. A. Nefedov, V. A. Saleev, hep-ph/1709.06378 ]

The $F_2$ structure function at LO ($x_B = \frac{Q^2}{2qP}$, $q_1^+ = x_1P^+$):

$$F_2(x_B, Q^2) = \sum_j \int_0^1 \frac{dx_1}{x_1} \int dt_1 \Phi_j(x_1, t_1, Q^2) \times C_{2j}^{(LO)} \left( z = \frac{x_B}{x_1}, \frac{t_1}{Q^2} \right)$$

where

$$C_{2q}^{(LO)} \left( z, \frac{t_1}{Q^2} \right) = e_q^2 \cdot z \delta \left( \frac{(Q^2 + t_1)z}{Q^2} - 1 \right)$$
is calculated with the use of gauge-invariant Fadin-Sherman $\gamma Qq$-vertex [Fadin, Sherman, 1977]:

$$\Gamma^{\mu}(q_1, k) = \gamma^{\mu} + \hat{q}_1 \frac{n_\mu}{k^-}.$$  

The $O(\alpha_s)$ contributions comes from:

1. $t_1$-dependence of LO contribution,
2. NLO subprocess

$$R_+(q_1) + \gamma^*(q) \rightarrow q + \bar{q},$$
3. Double-counting subtraction term
From LO to NLO in the parton Reggeization approach.

**Role of the $q_{T1}$-dependence**

[M. A. Nefedov, V. A. Saleev, hep-ph/1709.06378]

\[ F_{2}^{(LO)}(x_{B}, Q^{2}) = (e_{q}^{2} x_{B}) \left[ f_{q}(x_{B}, Q^{2}) + \Delta F_{2}^{(T)}(x_{B}, Q^{2}) + \Delta F_{2}^{(f)}(x_{B}, Q^{2}) + \text{pow. suppr.} \right] + O(\alpha_{s}) , \]

where:

\[ \Delta F_{2}^{(T)}(x_{B}, Q^{2}) = \int_{0}^{1} dx_{1} \ f_{j}(x_{1}, Q^{2}) \int_{0}^{Q^{2}} dt_{1} \left[ 1 - T_{j}(t_{1}, Q^{2}, x_{1}) \right] \frac{\partial}{\partial t_{1}} C_{j}^{(LO)} \left( \frac{x_{B}}{x_{1}}, \frac{t_{1}}{Q^{2}} \right) + O(\alpha_{s}) \]

\[ \Delta F_{2}^{(f)}(x_{B}, Q^{2}) = \int_{0}^{1} dx_{1} \int_{0}^{Q^{2}} dt_{1} \ T_{j}(t_{1}, Q^{2}, x_{1}) \left[ f_{j}(x_{1}, Q^{2}) - f_{j}(x_{1}, t_{1}) \right] \frac{\partial}{\partial t_{1}} C_{j}^{(LO)} \left( \frac{x_{B}}{x_{1}}, \frac{t_{1}}{Q^{2}} \right) + O(\alpha_{s}) \]
**O(α_s)-term from q_T-dependence**

[M. A. Nefedov, V. A. Saleev, hep-ph/1709.06378]

One obtains:

\[ ΔF_2^{(fg)} = \left( \frac{α_s(Q^2)}{2π} \right) \int_0^1 \frac{dx_1}{x_1} f_g(x_1, Q^2) \cdot ΔC_{qg}^{(fg)} \left( \frac{x_B}{x_1} \right) \]

where:

\[ ΔC_{qg}^{(fg)}(z) = T_R \left[ ξz ((4 + ξ)z - 2) + (1 - 2z + 2z^2) \log ξ \right], \]

where \( T_R = 1/2 \) and \( ξ = \min (1, (1 - z)/z) \).

Other corrections:

\[ ΔF_2^{(fq)}, ΔF_2^{(T)} \]

contain only \( f_q(x_1, Q^2) \), and will be important for the \( γ^* + Q \to q + g \) subprocess and one-loop correction.
NLO subprocess $\gamma^* + R \rightarrow q + \bar{q}$.

Kinematics ($\tilde{z} = (q_1^+ - q_3^+)/q_1^+$,
$\psi$ – azimuthal angle of $k_T$):

$q_{T2} = k_T + q_{T1} \frac{\tilde{z} - z}{1 - z}$,

$k_T^2 = \frac{(\tilde{z} - z)(1 - \tilde{z})}{1 - z} \left[ \frac{Q^2}{z} - \frac{t_1}{1 - z} \right]$,

$F_2^{(NLO,g)} = e_q^2 \cdot \frac{\alpha_s}{2\pi} \int \frac{dx_1}{x_1} \int dt_1 \Phi_g(x_1, t_1, \mu^2)$

$\times \int_z^1 \frac{z \cdot d\tilde{z}}{(1 - z)} \int_0^{2\pi} d\psi \frac{C_2^{(NLO,g)}}{2\pi} C_2^{(NLO,g)} \left( z = \frac{x_B}{x_1}, \frac{t_1}{Q^2} \right)$
Double-counting subtraction

[M. A. Nefedov, V. A. Saleev, hep-ph/1709.06378]
LO contains (horizontal blue line denotes mMRK approximation):

\[ p \rightarrow q \uparrow \quad k_1 \rightarrow \quad + \quad + \]

\[ q_1 \uparrow \quad - \quad k_2 \rightarrow \quad - \quad \]

\[ p_1 \uparrow \]

\[ P \rightarrow \quad + \quad + \]

\[ q \downarrow \quad k_1 \rightarrow \quad + \quad + \]

\[ q_t \uparrow \quad k_2 \rightarrow \quad + \quad + \]

\[ q_1 \uparrow \]

\[ \Rightarrow \]

\[ 2 \]

\[ 2 \]

\[ 2 \]
From LO to NLO in the parton Reggeization approach.

**Physical normalization condition**

\[ [\text{M. A. Nefedov, V. A. Saleev, hep-ph/1709.06378} ] \]

Physical normalization, \( O(\alpha_s) \) gluon contribution only:

\[
F_{2q}^{\text{NLO CPM}} (x_B, Q^2) = (e_q^2 x_B) \left[ f_q^{\overline{MS}} (x_B, Q^2) + \left( \frac{\alpha_s(Q^2)}{2\pi} \right) f_g^{\overline{MS}} \otimes C_{2g}^{\overline{MS}} + \text{c.c.} \right]
\]

\[
F_{2q}^{\text{NLO PRA}} (x_B, Q^2) = (e_q^2 x_B) \left[ f_q^{\text{PRA}} (x_B, Q^2) + \left( \frac{\alpha_s(Q^2)}{2\pi} \right) f_g^{\text{PRA}} \otimes \Delta C_{qg}^{(f_g)} \right.
\]

\[
+ \left( \frac{\alpha_s(Q^2)}{2\pi} \right) \Phi_g^{\text{PRA}} \otimes \left( C_2^{(NLO,g)} - \Delta C_2^{(t+u,g)} \right)
\]

PDFs in the **PRA scheme**:

\[
f_g^{\text{PRA}} = f_g^{\overline{MS}} + O(\alpha_s^2), \quad f_q^{\text{PRA}} = f_q^{\overline{MS}} + \left( \frac{\alpha_s}{2\pi} \right) f_g^{\overline{MS}} \otimes (\Delta C_{qg} - \Delta C_{qg}^{(f_g)}) + O(\alpha_s^2),
\]

where **matching coefficient** \( \Delta C_{qg} \) can be computed analytically \( \Rightarrow \)

unPDFs in PRA scheme:

\[
\int_0^{\mu^2} dt \, \Phi_i^{\text{PRA}} (x, t, \mu^2) = x f_i^{\text{PRA}} (x, \mu^2),
\]
Numerical test of Physical Normalization Condition

[M. A. Nefedov, V. A. Saleev, hep-ph/1709.06378]

Plot of the ratio $- F_{2q}^{NLO \, PRA}(x_B, Q^2)/F_{2q}^{NLO \, CPM}(x_B, Q^2)$:
Numerical test of Physical Normalization Condition

[M. A. Nefedov, V. A. Saleev, hep-ph/1709.06378]

Plot of the ratio $- F_{2q}^{NLO \text{ PRA}}(x_B, Q^2)/F_{2q}^{NLO \text{ CPM}}(x_B, Q^2)$:

$Q^2 = 10^4 \text{ GeV}^2$

Magenta lines $= 1 \pm \alpha_s^2(Q^2)$
Virtual corrections in PRA
What such EFTs as SCET or Lipatov’s theory are (needed for)?

- EFT = formalism which **explicitly implements** certain factorization properties of QCD amplitudes:
  - Factorization in the **soft** and **collinear** limits – *Soft-Collinear Effective Theory (SCET).*
  - Factorization in the **Multi-Regge limit** – *Lipatov’s theory.*
- As a result of kinematic approximations, **artificial logarithmic divergences** arise in different factors, but they should cancel in order-by-order in PT.
- Factorization + cancellation of artificial divergences ⇒ **Renormalization group.** The latter allows to resum **large logarithms** of scale ratios.
Rapidity divergences and regularization.

Due to the presence of the $1/q^\pm$-factors in the induced vertices, the loop integrals in EFT contain the light-cone (Rapidity) divergences:

\[
\hat{\Sigma}_1 = \int \frac{d^D q}{(2\pi)^D} \frac{\gamma_{\mu} - \hat{p} \frac{n_{\mu}^+}{q^+}}{q^2(p-q)^2} \left( \gamma_{\mu} - \hat{p} \frac{n_{\mu}^-}{q^-} \right)
\]

The regularization by explicit cutoff in rapidity was proposed by Lipatov [Lipatov, 1995] ($q^\pm = \sqrt{q^2 + q_T^2 \epsilon^{\pm y}}$):

\[
\int \frac{dq^+ dq^-}{q^+ q^-} = \int_{y_1}^{y_2} dy \int \frac{dq^2}{q^2 + q_T^2},
\]

then

\[
\hat{\Sigma}_1 = C_F g_s^2 \hat{p} \int \frac{\mathbf{p}_T^2 d^{D-2} q_T}{q_T^2(q_T^2 - \mathbf{p}_T)^2} (y_2 - y_1) + \text{finite terms}
\]

The dependence on the regulators $y_i$ have to cancel between the contributions of the neighbouring regions order-by-order in $\alpha_s$, building up the $\frac{1}{n!} (\log(s)\omega(t))^n$-terms.
Covariant regularization.

The regularization and **pole prescription** was introduced in a series of papers [Hentschinski, Vera, Chachamis *et. al.*, 2012-2013]. The regularization of the light-cone divergences is achieved by the shifting the $n^\pm$ vectors from the light-cone:

\[
\tilde{n}^\pm = n^\pm + e^{-\rho} n^\mp, \quad \tilde{k}^\pm = k^\pm + e^{-\rho} k^\mp, \quad \rho \to +\infty,
\]

and for the lowest-order ($Rgg, Qqg$) induced vertices the PV prescription is at work:

\[
\frac{1}{[\tilde{k}^\pm]} = \frac{1}{2} \left( \frac{1}{\tilde{k}^\pm + i\varepsilon} + \frac{1}{\tilde{k}^\pm - i\varepsilon} \right).
\]

For the higher-order induced vertices, there is the nontrivial interplay between **color and kinematics** in the pole prescription.

Recently, this prescriptions has been tested at one loop for the case of **Reggeized quarks**: [M. A. Nefedov, V. A. Saleev, Mod. Phys. Lett. A, 32 1750207 (2017)].
The Reggeized quark self-energy.

[M. A. Nefedov, V. A. Saleev, Mod. Phys. Lett. A, 32 1750207 (2017)]

The self-energy graph is computed in the central kinematics \((p^+ = p^- = 0)\):

\[
\hat{\Sigma}_1 = q \downarrow \ = g_s^2 C_F \int \frac{d^D q}{(2\pi)^D} \left( \gamma_\mu - \hat{p} \frac{n_\mu^+}{[\tilde{q}^+]} \right) \frac{\hat{p} - \hat{q}}{q^2(p - q)^2} \left( \gamma_\mu - \hat{p} \frac{n_\mu^-}{[\tilde{q}^-]} \right),
\]

the result is:

\[
\hat{\Sigma}_1 = (i\hat{p}) \frac{C_F \bar{\alpha}_s}{4\pi} \left( \frac{\mu^2}{t_1} \right)^\epsilon \left[ \frac{-i\pi + 2\rho}{\epsilon} + \left( \frac{1 + \epsilon}{1 - 2\epsilon} \right) \frac{1}{\epsilon} \right],
\]

where \(\bar{\alpha}_s = (4\pi)^\epsilon r_\Gamma \alpha_s\).
From LO to NLO in the parton Reggeization approach.

The unsubtracted $\gamma Qq$-scattering vertex.

\[ C_F \bar{\alpha}_s \hat{\Gamma}_1^\mu = p^\uparrow \rightarrow \alpha_s \]

where $k^2 = 0$, $p^2 = -t_1$, $(k + q)^2 = 0$.

The $e^\rho$-divergences cancel in the sum of diagrams. The terms proportional $e^{-\epsilon \rho} \rightarrow 0$ for $\epsilon > 0 \Rightarrow$ only logarithmic singularities $\sim \rho$ are left.

[M. A. Nefedov, V. A. Saleev, Mod. Phys. Lett. A, 32 1750207 (2017)]
The unsubtracted $\gamma Qq$-scattering vertex.

[M. A. Nefedov, V. A. Saleev, Mod. Phys. Lett. A, 32 1750207 (2017)]

The one-loop result can be expressed in terms of two gauge-invariant Lorentz structures:

$$\hat{\Gamma}_0^\mu = e e_q \bar{u}(p + k) \left( \gamma^\mu + \hat{p} \frac{n_+^\mu}{k^+} \right) \hat{n}^+,$$

$$\hat{\Delta}_0^\mu = e e_q \left( p^\mu - \frac{t_1}{2k^+ n_+^\mu} \right) \left( \bar{u}(p + k) \hat{k} \hat{n}^+ \right)$$

The result:

$$\hat{\Gamma}_1^\mu = \frac{2}{t_1} \hat{\Delta}_0^\mu + \hat{\Gamma}_0^\mu \left[ \begin{array}{c} \frac{1}{\epsilon^2} - \frac{L_1}{\epsilon} + (\rho - i\pi) \left( \frac{1}{\epsilon} + L_1 \right) + \frac{2L_2}{\epsilon} - \\ \text{IR part} \\ \text{High-Energy part} \\ \frac{1}{\epsilon} + L_1 + 3 \end{array} \right] + 2L_1L_2 - \frac{L_1^2}{2} + \frac{\pi^2}{2} + O(\epsilon),$$

where $L_1 = \log \left( \frac{\mu^2}{t_1} \right)$, $L_2 = \log \left( \frac{k^+}{\sqrt{t_1}} \right)$. 
Subtracted $\gamma Qq$-scattering vertex.

The subtracted result:

$$\hat{\Gamma}^\mu_{1S} = \hat{\Gamma}^\mu_1 - \delta \hat{\Gamma}^\mu_1 = \frac{2}{t_1} \hat{\Delta}^\mu_0 + \hat{\Gamma}^\mu \left[ -\frac{1}{\epsilon^2} - \frac{L_1}{\epsilon} - \rho \left( \frac{1}{\epsilon} + L_1 \right) + \frac{2L_2}{\epsilon} - 2 \left( \frac{1}{\epsilon} + L_1 + 3 \right) + 2L_1 L_2 - \frac{L_1^2}{2} + \frac{\pi^2}{2} \right] + O(\epsilon).$$

The subtraction term:

$$C_F \bar{\alpha}_s \delta \hat{\Gamma}^\mu_1 = \frac{2}{t_1} \hat{\Delta}^\mu_0 + \hat{\Gamma}^\mu \left[ (2\rho - i\pi) \left( \frac{1}{\epsilon} + L_1 \right) + \left( \frac{1}{\epsilon} + L_1 + 3 \right) \right] + O(\epsilon).$$
The cross-check.

[M. A. Nefedov, V. A. Saleev, Mod. Phys. Lett. A, 32 1750207 (2017)]

The terms $\sim \rho$ cancel in the sum of graphs $(-\rho - \rho + 2\rho)$, as it was for gluons!

The result is the MRK asymptotics ($s \to \infty$, $t$-fixed) for the $\gamma\gamma \to q\bar{q}$-amplitude at $O(\alpha_s)$. To compare the EFT prediction with QCD result, let's compute:

$$
\alpha_s \text{ LO} = \left(8N_c(ee_q)^4\right) \frac{C_F \tilde{\alpha}_s}{4\pi} \left[ \frac{1}{\tau} C^{(-1)}_{HE} + C^{(0)}_{HE} + O(\tau) \right],
$$

where $\tau = -t/s$. The EFT predicts $C^{(-1)}_{HE}$. 

\[ \begin{array}{c}
\text{LO} \\
\alpha_s \\
\end{array} \]
From LO to NLO in the parton Reggeization approach.

\[ \gamma \gamma \rightarrow u_k u_l \]
From LO to NLO in the parton Reggeization approach.

The Regge limit of the $\gamma\gamma \to q\bar{q}$- amplitude.

[M. A. Nefedov, V. A. Saleev, Mod. Phys. Lett. A, 32 1750207 (2017)]

Both QCD(FeynArts+FeynCalc) and EFT agree on the result for $C_{HE}^{(-1)}$:

\[
\text{Re } C_{HE}^{(-1)} = -\frac{2}{\epsilon^2} - \frac{2}{\epsilon} \log \frac{\mu^2}{(-t)} + \left(1 + 2 \log \frac{1}{\tau}\right) \left(\frac{1}{\epsilon} + \log \frac{\mu^2}{(-t)}\right) \\
- \left[3 - \pi^2 + \log^2 \frac{\mu^2}{(-t)} + 4 \log \frac{1}{\tau}\right],
\]

\[
\text{Im } C_{HE}^{(-1)} = -\pi \left(\frac{1}{\epsilon} + \log \frac{\mu^2}{(-t)} - 2\right).
\]
Fermionic Glauber Operators and Quark Reggeization

Ian Moult, Mikhail P. Solon, Iain W. Stewart, and Gherardo Vita. 
**Fermionic Glauber Operators and Quark Reggeization**, JHEP 1802 (2018) 134.

Note added: As this paper was being finalized, Ref. [Nefedov, Saleev, 2017] appeared, which studies $\gamma\gamma \rightarrow q + \bar{q}$ amplitudes at one-loop in the Regge limit by constructing the quark Reggeization terms in the effective action formalism of Lipatov [Lipatov, 1995]. In the SCET language this corresponds to formulating an auxiliary field Lagrangian for the offshell Glauber quarks, while using the full QCD Lagrangian for other fields (without defining EFT fields for the n-collinear, soft and \(n^-\)-collinear sectors). Since having distinct fields for these sectors enables their factorization properties to be easily determined and studied, such as in our BFKL calculation, we believe there are certain advantages to our approach. It would be interesting to make a more explicit comparison between these formalisms.
Conclusions

- Parton Reggeization Approach has been shown to be a reliable phenomenological tool at LO.
- The extension to NLO is possible and is under active development.
Thank you for your attention!