Loss-tolerant state engineering for quantum-enhanced metrology via the reverse Hong–Ou–Mandel effect

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Highly entangled quantum states, shared by remote parties, are vital for quantum communications and metrology. Particularly promising are the N00N states—entangled N-photon wavepackets delocalized between two different locations—which outperform coherent states in measurement sensitivity. However, these states are notoriously vulnerable to losses, making them difficult to both share them between remote locations and recombine in order to exploit interference effects. Here we address this challenge by utilizing the reverse Hong–Ou–Mandel effect to prepare a high-fidelity two-photon N00N state shared between two parties connected by a lossy optical medium. We measure the prepared state by two-mode homodyne tomography, thereby demonstrating that the enhanced phase sensitivity can be exploited without recombining the two parts of the N00N state. Finally, we demonstrate the application of our method to remotely prepare superpositions of coherent states, known as Schrödinger’s cat states.
In the current race towards the practical implementation of quantum techniques for information processing and communications, a strong trend is to design loss-tolerant quantum protocols, such as the preparation of non-local superpositions of quasi-classical light states, discord-assisted remote state preparation, quantum illumination, undoing the effect of losses on continuous-variable entanglement and the preparation of single-qubit entangled states over a long distance.

In this article we are interested in N00N states $|N::0\rangle = \frac{(|N,0\rangle + |0,N\rangle)}{\sqrt{2}}$, which are useful in linear-optical quantum computation, quantum-optical state engineering and the preparation of photon-number path entanglement. But the most important potential application of these states is as a resource for quantum enhanced metrology. Interference measurements with N00N states exhibit super-resolving properties: the number of fringes per wavelength equals $N$, in contrast to a single fringe in the case of coherent states. This property can be exploited for precise measurement of diverse physical quantities. Widespread application of N00N states for metrology is however precluded by their extreme sensitivity to losses. When exposed to even moderate losses, the degree of entanglement and hence the super-resolution potential of the N00N states dramatically degrade to an extent that eliminates any advantage.

In the present work, we address this challenge by developing a technique to losslessly produce N00N states between parties that are separated by a lossy quantum channel. In addition, using N00N states usually requires bringing back together the two entangled parts, introducing more propagation losses. But we will show that this second step is not required, and that super-resolution for the optical phase can be obtained remotely, by using homodyne detection.

**Results**

**Concept.** For N00N state production, we exploit some peculiar properties of the Hong–Ou–Mandel (HOM) effect, a well-known quantum interference phenomenon in which two indistinguishable photons that are overlapped on a symmetric beam splitter (BS) always emerge in the same output mode, preparing the N00N state

$$|11\rangle \rightarrow |2::0\rangle = \frac{|2,0\rangle + |0,2\rangle}{\sqrt{2}} \quad (1)$$

in the beam splitter output. Our experiment relies upon the reverse version of the HOM effect, in which the measurements in the two output modes of the BS project them onto single-photon states. Because of the time-reversible nature of quantum mechanics, such projection is equivalent to projecting the state of the input onto the two-photon N00N state (1). If each of the beam splitter inputs is, in turn, entangled with other modes, these modes become entangled with each other, thanks to entanglement swapping. Whereas the original HOM setting creates a two-photon N00N state, extension to any even $N$ is straightforward (see Supplementary Section ‘Preparation of high-order N00N states’).

Specifically, consider a set-up in which two pairs of modes (A,C) and (B,D) are prepared in two-mode squeezed states:

$$|\psi_{AC/BD}\rangle \propto |0,0\rangle + \gamma|1,1\rangle + \gamma^2|2,2\rangle + \cdots, \quad (2)$$

by means of non-degenerate parametric down-conversion. Modes C and D are then mixed on a symmetric BS, the outputs of which are subjected to measurement in the photon-number basis via single-photon counting modules (SPCMs), as shown in Fig. 1.

In the weak-squeezing limit $|\gamma|^2 \ll 1$, every SPCM click is likely to be caused by no more than one photon. Then, a coincidence click in both SPCMs correspond to projection on state $|1,1\rangle_{CD}$. Due to the unitary nature of the BS operation, this event assures that modes C and D were initially in the N00N state (1). This corresponds to two pairs of photons having been produced in either of two crystals; therefore, now the remaining modes A and B also share a two-photon N00N state. The application of the reverse HOM effect to N00N state preparation has been proposed by Kok et al., albeit not in a remote fashion as we do here.

A remarkable feature of this scheme is its robustness to losses in channels C and D. Such losses only reduce the probability of the two SPCMs to click, but, if the clicks do occur and the down-conversion amplitude $\gamma$ is sufficiently small, the leading term in the state of channels A and B is still the two-photon N00N state.

**N00N state tomography.** Conditioned on coincidence clicks of the two SPCMs, we characterize the state of modes A and B by means of homodyne tomography (Methods section). The measured states in both cases are very close to the ideal state (1) that has suffered a $1-\eta = 45\%$ loss due to imperfect detection efficiency ($\eta$) (Fig. 2a).

To illustrate the reverse HOM effect and the critical significance of the matching between modes C and D for proper N00N state preparation, we measured the behaviour of the state in modes A and B as a function of increasing temporal mismatch between modes C and D. As evidenced by Fig. 3, the fraction of the biphoton component $|1,1\rangle$ in that state exhibits a dip that is characteristic as a signature of the HOM effect. The tomographic state reconstruction for the case of complete mismatch is shown in Fig. 2b. In addition to a macroscopic biphoton component, no off-diagonal terms are present in this case because no coherence between modes A and B can emerge in the absence of interference.

![Figure 1 | Conceptual scheme of the experiment.](image-url)
The statistical error of the reconstruction. The emerged $|1, 1\rangle$ component is shown in grey. Green hats show the phase quadratures on the difference $\Delta \theta = \theta_A - \theta_B$ and two-photon N00N states one has, respectively,

$$\langle 1 : 0 | X_A X_B | 1 : 0 \rangle = -\frac{\eta}{2} \sin \Delta \theta,$$

$$\langle 1 : 0 | X_A^2 X_B^2 | 1 : 0 \rangle = \frac{1 + 2\eta}{4}$$

and

$$\langle 2 : 0 | X_A X_B | 2 : 0 \rangle = 0,$$

$$\langle 2 : 0 | X_A^2 X_B^2 | 2 : 0 \rangle = \frac{1}{4} + \eta + \frac{\eta^2}{2} \cos(2\Delta \theta)$$

where $\Delta \theta = \theta_A - \theta_B$ and we assumed the phases in modes C and D to be constant.

We see that in order to compare between the one- and two-photon N00N states using quadrature measurements, we need to use different observables. This notwithstanding, the phase dependence of the appropriate observable in each state is as expected, that is, with period $2\pi/N$ for each $N$-photon N00N state.
An experimental check of this behaviour is demonstrated in Fig. 4. In agreement with the results of state reconstruction, the second-order interference of the $|2 \leftrightarrow 0\rangle$ state, generated in the presence of the loss, exhibits the same visibility as without loss.

**Loss-tolerant preparation of Schrödinger’s cat states.** Being path-entangled, the N00N state can also be used for single-mode quantum-state engineering. Associating modes A and B with fictitious observers Alice and Bob, we consider a setting in which Alice, by performing quadrature measurements on her mode, remotely prepares a state in Bob’s mode. Neglecting inefficiencies, Alice’s quadrature outcome $X$ measured at phase angle $\theta$ brings the Bob’s mode to the state

$$\lambda \langle X_\theta | 2 \leftrightarrow 0 \rangle_{AB} = \lambda \langle X_\theta | 2 \rangle_A | 0 \rangle_B + \lambda \langle X_\theta | 0 \rangle_A | 2 \rangle_B$$

(5)

where

$$\langle X_\theta | m \rangle_\lambda = e^{i m \theta} e^{-X^2} \frac{H_m(X)}{\sqrt{2^m m!}}$$

(6)

are Fock state wavefunctions, with $H_m(X)$ being Hermite polynomials. In this way, by postselecting specific values of Alice’s observed quadrature, one can generate arbitrary superpositions of the 0- and 2-photon states, which approximate the even coherent-state superpositions (CSS) $|z\rangle + |\bar{z}\rangle$, sometimes viewed as a quantum-optical implementation of the Schrödinger’s cat paradox\cite{11,12,28,29}.

The states of Bob’s mode for different outcomes of Alice’s homodyne measurement are displayed in Fig. 5. Projection on $X = 0$ (Fig. 5, first column) results in superposition $0.052|0\rangle - 0.85\langle 2|$ partially mixed due to losses in Alice’s channel. After correcting for Bob’s homodyne detection inefficiency, this state has a fidelity of 0.88 with the even CSS state of amplitude $z = 1.84$, squeezed with parameter $\varepsilon = 0.48$. This value compares favourably with state-of-the-art results\cite{12,29}, with the added advantage that our protocol is tolerant to the losses in the optical channel between Alice and Bob.

Effective CSS amplitudes and approximation fidelities for other values of $X$ are shown in Fig. 5b. With increasing $X$, the two-photon fraction initially increases relative to the vacuum because the two-photon state wavefunction $X|2\rangle$ decreases faster than the vacuum wavefunction $X|0\rangle$. Wavefunction $X|2\rangle$ changes sign near the value $X = 0.7$, where ideally a pure two-photon state in Bob’s mode should be observed. In practice, due to the losses and finite width of Alice’s post-selection window, a phase-insensitive mixture of $0.4|0\rangle + 0.6|2\rangle$ is produced (second column). In this region, the remotely prepared state approximates the CSS poorly because of the high two-photon component. For higher values of $X$, this two-photon component in Bob’s state reduces again, resulting in increasingly
faithful approximation of even CSSs with decreasing amplitudes (third column). For very low amplitudes, this state approximates a weakly squeezed vacuum state. This is the case for \( X = 2 \) (fourth column): Bob’s quadrature spectrum exhibits squeezing by 0.65 \( \pm 0.24 \) dB (without efficiency correction). The corresponding quadrature histogram is shown in the upper right panel of Fig. 5.

**Discussion**

The protocol developed here addresses the primary challenge in the way of employing nonclassical states of light, particularly the N00N state, for quantum metrology: optical losses. With conventional optical fibres, it allows establishing nearly ideal N00N entanglement over large distances, with a considerably increased tolerance to the loss in the channel connecting the parties. Due to its extreme sensitivity to the overall phase between Alice and Bob (see Supplementary Section ‘Phase behavior’ for details), our scheme could be used for ultra-precise measurements of the distance between Alice’s and Bob’s homodyne detectors. Its further advantage is that there is no need to recombine the two parts of the N00N states, thanks to the homodyne detections.

In many cases, however, it is likely that a simple laser interferometric measurement would do better than our scheme due to the much larger number of available photons. Our scheme becomes advantageous in settings where the amount of light transmitted between the two parties is limited. Suppose, for instance, that a lossy biological medium, very sensitive to the light intensity, is inserted in channels C and/or D. Then it will be possible to get high-precision interferometric measurements through this medium, with almost negligible light intensity, as long as channels A, B and the associated homodyne detections have a high quality.

At this stage our experiment is a proof-of-principle only, and its application in practical settings would involve a number of challenges—in particular, the synchronization of the photons in channels C and D within the inverse down-conversion bandwidth. Furthermore, the production of higher-order N00N states would be required to achieve a significant practical gain over interferometry with coherent states, which are not limited in photon number. Our scheme can be generalized to enable producing N00N states with an arbitrary even \( N \). To this end, we can use a sequence of \( N/2 \) reverse HOM measurements to subject modes C and D to operator \( 2 \hat{N}^\dagger \hat{N}^\dagger \) using the recipe of Kok et al.\(^{13} \) An event in which the application of this operator has been successful (all \( N \) SPCMs have clicked) implies that either C or D initially contained at least \( N \) photons, but is unable to distinguish which. This means, in turn, that modes A and B are in the \( |N = 0 \rangle \) state. Similarly to the two-photon case, the down-conversion amplitude \( \gamma \) must be sufficiently low in order to reduce the contribution of higher-order terms in the output state.

We present a detailed theoretical and numerical analysis of this scheme in the Supplementary Section ‘Preparation of high-order N00N states’. In particular, we find that the fidelity of the prepared state for a given \( \gamma \) does not strongly depend on the losses in modes C and D. This enables preparation of high N00N states at reasonable rates, even with quite lossy channels.

In addition to interferometry, the generalization of our scheme to higher-order N00N states may have other applications, based on ultraremotely preparation of quantum states of light, as illustrated by the ‘cat state’ generation presented above. In that case, fibre transmission over long distances may be used, with possible quantum cryptography applications. The present scheme is therefore a valuable addition to the quantum-state engineering toolbox, for both discrete- and continuous-variable degrees of freedom of optical modes\(^ {11,30,31} \).

**Methods**

**Experimental details.** We employ a pulsed Ti:Sapphire laser (Coherent Mira 9005) with a wavelength of 780 nm, mean power 1.3 W, repetition rate of 76.6 MHz and a pulse width of \( \sim 1.6 \) ps. Most of the laser output is directed into a lithium triborate crystal for frequency doubling. We obtain up to 300 mW second harmonic; after subsequent spectral cleaning, about 100 mW remain. Then we implement parametric down-conversion in two periodically poled potassium titanyl phosphate crystals in a type II spectrally and spatially degenerate, but polarization non-degenerate configuration. The single-photon detection is implemented using SPCMs by Excelitas. The modes entering the SPCMs are spectrally filtered and delivered to the detectors by means of single-mode fibres, which ensures proper preparation of the heralded photon mode\(^ {26,27} \). Including the fibre and filters, the quantum efficiency of these detectors is estimated as \( \eta_{\text{SPCM}} \sim 0.15 \).

Without losses, the experimental double-coincidence event rate is \( \sim 100 \) Hz, which corresponds to a probability of \( \sim 10^{-8} \) per pulse and the down-conversion amplitude \( \sqrt{\gamma} \sim 0.007 \). With a total of 10 dB loss equally distributed between modes C and D, the coincidence rate decreases by a factor of 10 due to the reduction of the equivalent SPCM efficiency down to \( \sim 0.1 \).

A total of 500,000 quadrature samples have been acquired in each setting for the tomographic reconstruction of the N00N state. The density matrix was obtained from these data using the iterative maximum likelihood algorithm\(^ {32–34} \), which in case of N00N states requires the knowledge of the phase difference between two local oscillators at each moment in time. This information was collected from the quadrature correlations exhibited by \( \{ 1 | 0 \} \) state (Fig. 4, top), which was produced upon a single click of one of the SPCMs. The rate of these events was about 100 kHz, which allowed us to evaluate the quadrature correlation and hence the phase at any moment in time with a sufficient precision.

**Data availability.** The data that support the findings of this study are available from the corresponding author upon request.

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**Author contributions**

All authors participated in the conception and planning of the project, theoretical analysis and writing of the paper. The experiment was performed by A.E.U., I.A.F. and D.S. The data were analysed by A.E.U., I.A.F., D.S. and A.I.L.

**Additional information**

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