Angular analysis of charged and neutral $B \rightarrow K \mu^+ \mu^-$ decays

The LHCb collaboration

E-mail: p.owen10@imperial.ac.uk

Abstract: The angular distributions of the rare decays $B^+ \rightarrow K^+ \mu^+ \mu^-$ and $B^0 \rightarrow K^0_S \mu^+ \mu^-$ are studied with data corresponding to 3 fb$^{-1}$ of integrated luminosity, collected in proton-proton collisions at 7 and 8 TeV centre-of-mass energies with the LHCb detector. The angular distribution is described by two parameters, $F_H$ and the forward-backward asymmetry of the dimuon system $A_{FB}$, which are determined in bins of the dimuon mass squared. The parameter $F_H$ is a measure of the contribution from (pseudo)scalar and tensor amplitudes to the decay width. The measurements of $A_{FB}$ and $F_H$ reported here are the most precise to date and are compatible with predictions from the Standard Model.

Keywords: Rare decay, B physics, Flavour Changing Neutral Currents, Flavor physics, Hadron-Hadron Scattering

ArXiv ePrint: 1403.8045

doi:10.1007/JHEP05(2014)082
1 Introduction

The $B^+ \to K^+\mu^+\mu^-$ and $B^0 \to K_S^0\mu^+\mu^-$ decays are rare, flavour-changing neutral-current processes that are mediated by electroweak box and penguin amplitudes in the Standard Model (SM). In well motivated extensions of the SM [1, 2], new particles can introduce additional amplitudes that modify the angular distribution of the final-state particles predicted by the SM.

In this paper, the angular distributions of the final-state particles are probed by determining the differential rate of the $B$ meson decays as a function of the angle between the direction of one of the muons and the direction of the $K^+$ or $K_S^0$ meson in the rest frame of the dimuon system. The analysis is performed in bins of $q^2$, the dimuon invariant mass squared. The angular distribution of $B^+ \to K^+\mu^+\mu^-$ decays has previously been studied by the BaBar [3], Belle [4], CDF [5] and LHCb [6] experiments with less data.

For the decay $B^+ \to K^+\mu^+\mu^-$, the differential decay rate can be written as [2, 7]

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta_l} = \frac{3}{4} (1 - F_H)(1 - \cos^2\theta_l) + \frac{1}{2} F_H + A_{FB} \cos\theta_l,$$

where $\theta_l$ is the angle between the direction of the $\mu^-$ ($\mu^+$) lepton and the $K^+$ ($K_S^0$) meson for the $B^+$ ($B^-$) decay. The differential decay rate depends on two parameters, the forward-backward asymmetry of the dimuon system, $A_{FB}$, and a second parameter $F_H$.

\footnote{The inclusion of charge conjugated processes is implied throughout.}
which corresponds to the fractional contribution of (pseudo)scalar and tensor amplitudes to the decay width in the approximation that muons are massless. The decay width, $A_{FB}$ and $F_H$ all depend on $q^2$.

The structure of eq. (1.1) follows from angular momentum conservation in the decay of a pseudo-scalar $B$ meson into a pseudo-scalar $K$ meson and a pair of muons. In contrast to the decay $B^0 \to K^{*0} \mu^+ \mu^-$, $A_{FB}$ is zero up to tiny corrections in the SM. A sizable value of $A_{FB}$ is possible in models that introduce large (pseudo)scalar- or tensor-like couplings \cite{1, 2}. The parameter $F_H$ is non-zero, but small, in the SM due to the finite muon mass. For eq. (1.1) to remain positive at all lepton angles, $A_{FB}$ and $F_H$ have to satisfy the constraints $0 \leq F_H \leq 3$ and $|A_{FB}| \leq F_H/2$.

Since the $B^0$ and $\bar{B}^0$ meson can decay to the same $K^0_S \mu^+ \mu^-$ final state, it is not possible to determine the flavour of the $B$ meson from the decay products. Without tagging the flavour of the neutral $B$ meson at production, it is therefore not possible to unambiguously chose the correct muon to determine $\theta_l$. For this reason, $\theta_l$ is always defined with respect to the $\mu^+$ for decays to the $K^0_S \mu^+ \mu^-$ final-state. In this situation any visible $A_{FB}$ would indicate that there is either a difference in the number of $B^0$ and $\bar{B}^0$ mesons produced, $CP$ violation in the decay or that the $A_{FB}$ of the $B^0$ and $\bar{B}^0$ decay differ. Any residual asymmetry can be canceled by performing the analysis in terms of $|\cos \theta_l|$, 

$$
\frac{1}{\Gamma} \frac{d\Gamma}{d|\cos \theta_l|} = \frac{3}{2} (1 - F_H)(1 - |\cos \theta_l|^2) + F_H,
$$

(1.2)

where the constraint $0 \leq F_H < 3$ is needed for this expression to remain positive at all values of $|\cos \theta_l|$. This simplification of the angular distribution is used for the $B^0 \to K^0_S \mu^+ \mu^-$ decay in this paper.

2 Data and detector description

The data used for the analysis correspond to 1 fb$^{-1}$ of integrated luminosity collected by the LHCb experiment in $pp$ collisions at $\sqrt{s} = 7$ TeV in 2011 and 2 fb$^{-1}$ of integrated luminosity collected at $\sqrt{s} = 8$ TeV in 2012. The average number of $pp$ interactions, yielding a charged particle in the detector acceptance, per bunch crossing was 1.4 in 2011 and 1.7 in 2012.

The LHCb detector \cite{8} is a single-arm forward spectrometer covering the pseudorapidity range $2 < \eta < 5$, designed for the study of particles containing $b$ or $c$ quarks. The detector includes a high-precision tracking system consisting of a silicon-strip vertex detector surrounding the $pp$ interaction region, a large-area silicon-strip detector located upstream of a dipole magnet with a bending power of about 4 Tm, and three stations of silicon-strip detectors and straw drift tubes \cite{9} placed downstream of the magnet. The combined tracking system provides a momentum measurement with relative uncertainty that varies from 0.4% at 5 GeV/$c$ to 0.6% at 100 GeV/$c$, and impact parameter resolution of 20 $\mu$m for tracks with large transverse momentum. Different types of charged hadrons are distinguished by information from two ring-imaging Cherenkov detectors \cite{10}. Photon, electron and hadron candidates are identified by a calorimeter system consisting of scintillating-pad and preshower detectors, an electromagnetic calorimeter and a hadronic
calorimeter. Muons are identified by a system composed of alternating layers of iron and multiwire proportional chambers [11].

Samples of simulated $B^+ \rightarrow K^+\mu^+\mu^-$ and $B^0 \rightarrow K^0_S\mu^+\mu^-$ decays are used to understand how the detector geometry, the reconstruction and subsequent event selection bias the angular distribution of the decays. In the simulation, $pp$ collisions are generated using PYTHIA [12] with a specific LHCb configuration [13]. Decays of hadronic particles are described by EvtGen [14], in which final state radiation is generated using PHOTOS [15]. The interaction of the generated particles with the detector and its response are implemented using the GEANT4 toolkit [16, 17] as described in ref. [18].

3 Selection of signal candidates

The LHCb trigger system [19] consists of a hardware stage, based on information from the calorimeter and muon systems, followed by a software stage, which applies a full event reconstruction. In the hardware stage of the trigger, candidates are selected with at least one muon candidate with transverse momentum, $p_T > 1.48 (1.76)$ GeV/c in 2011 (2012). In the second stage of the trigger, at least one of the final-state particles from the $B^0$ or $B^+$ meson decay is required to have $p_T > 1.0$ GeV/c and impact parameter larger than 100 $\mu$m with respect to any primary vertex (PV) from the $pp$ interactions in the event. Tracks from two or more of the final-state particles are required to form a secondary vertex that is displaced from all of the PVs.

The $K^0_S$ mesons from the decay $B^0 \rightarrow K^0_S\mu^+\mu^-$ are reconstructed through their decay $K^0_S \rightarrow \pi^+\pi^-$ in two different categories: the first category contains $K^0_S$ mesons that decay early enough that the final-state pions are reconstructed in the vertex detector; and the second contains $K^0_S$ mesons that decay later, such that the first track segment that can be reconstructed is in the large-area silicon-strip detector. These categories are referred to as long and downstream, respectively. Candidates in the long category have better mass, momentum and vertex resolution.

Reconstructed tracks that leave hits in the LHCb muon system are positively identified as muons. Two muons of opposite charge are then combined with either a track ($K^+$) or a reconstructed $K^0_S$ to form a $B^+$ or $B^0$ candidate. The $\pi^+\pi^-$ pair from the reconstructed $K^0_S$ is constrained to the known $K^0_S$ mass when determining the mass of the $B^0$ candidate. Neural networks, using information from the RICH detectors, calorimeters and muon system, are used to reject backgrounds where either a pion is misidentified as the kaon in the $B^+$ decay or a pion or kaon are incorrectly identified as one of the muons.

An initial selection is applied to $B^+$ and $B^0$ candidates to reduce the level of the background. The selection criteria are common to those described in ref. [20]: the $\mu^\pm$ and the $K^+$ candidates are required to have $\chi^2_{IP} > 9$, where $\chi^2_{IP}$ is defined as the minimum change in $\chi^2$ of the vertex fit to any of the PVs in the event when the particle is added to that PV; the dimuon pair vertex fit has $\chi^2 < 9$; the $B$ candidate is required to have a vertex fit $\chi^2 < 8$ per degree of freedom; the $B$ momentum vector is aligned with respect to one of the PVs in the event within 14 mrad, the $B$ candidate has $\chi^2_{IP} < 9$ with respect to that PV and the vertex fit $\chi^2$ of that PV increases by more than 121 when including the $B$ decay products. In addition, the $K^0_S$ candidate is required to have a decay time larger than 2 ps.
The initial selections are followed by tighter multivariate selections, based on boosted decision trees (BDTs) [21] with the AdaBoost algorithm [22]. The working points for the BDTs are chosen to maximise $N_S/\sqrt{N_S + N_B}$, where $N_S$ and $N_B$ are the expected numbers of signal and background candidates within $\pm 50\text{MeV}/c^2$ of the known $B^0$ or $B^+$ meson masses, respectively. For the $B^+ \rightarrow K^+\mu^+\mu^-$ decay, the variables used in the BDT are identical to those of ref. [20]. In contrast to that analysis, however, the multivariate selection is trained using a sample of simulated events to model the signal and candidates from the data with $K^+\mu^+\mu^-$ invariant masses in the range $5700 < m(K^+\mu^+\mu^-) < 6000\text{MeV}/c^2$ for the background. This background sample is not used in the subsequent analysis, where the invariant mass of the candidates is restricted to the range $5170 < m(K^+\mu^+\mu^-) < 5700\text{MeV}/c^2$. The multivariate selection has an efficiency of 89% for signal and removes 94% of the background that remains after the initial selection. For the $B^0 \rightarrow K_S^0\mu^+\mu^-$ decay, two independent BDTs are trained for the long and downstream categories. Samples of simulated events are used in the signal training and candidates from the data with masses $5700 < m(K_S^0\mu^+\mu^-) < 6000\text{MeV}/c^2$ for the background training. The following information is used in the classifiers: the $B^0$ candidate momentum and $p_T$, its vertex quality ($\chi^2$) and decay time, the $K_S^0$ candidate $p_T$, and the angle between the $B^0$ candidate momentum and the direction between the PV and the $B^0$ decay vertex. For the long category, the $K_S^0$ candidate $\chi^2_{PV}$ is also included. The multivariate selection removes 99% of the combinatorial background and is 66% and 48% efficient for the long and downstream signal categories.

Combinatorial backgrounds for the $B^+ \rightarrow K^+\mu^+\mu^-$ decay, where the $K^+$, $\mu^+$ and $\mu^-$ candidates do not all come from the same $b$-hadron decay, are reduced to a small level by the multivariate selection. After applying the multivariate selection, the signal-to-background ratio in a $\pm 50\text{MeV}/c^2$ range around the known $B^+$ mass is better than six-to-one. Remaining backgrounds mainly come from $b$-hadron decays that are fully or partially reconstructed in the detector. The $B^+ \rightarrow J/\psi K^+$ and $B^+ \rightarrow \psi(2S)K^+$ decays\(^2\) are rejected by removing the regions of dimuon mass around the charmonium resonances $(8.0 < q^2 < 11.0\text{GeV}^2/c^4$ and $12.5 < q^2 < 15.0\text{GeV}^2/c^4$). These decays can also form a background to the $B^+ \rightarrow K^+\mu^+\mu^-$ decay if the kaon is incorrectly identified as a muon and the muon with the same charge is incorrectly identified as a kaon. This background is removed by rejecting candidates with a $K^+\mu^-$ pair whose invariant mass (under the $\mu^+\mu^-$ mass hypothesis) is consistent with that of the $J/\psi$ or $\psi(2S)$ meson, if the reconstructed kaon can also be matched to hits in the muon system. A narrow range in $q^2$ from $0.98 < q^2 < 1.10\text{GeV}^2/c^4$ is also removed to reject $B^+ \rightarrow \phi K^+$ decays, followed by the $\phi \rightarrow \mu^+\mu^-$ decay. The region $m(K^+\mu^+\mu^-) < 5170\text{MeV}/c^2$ is contaminated by partially reconstructed $b$-hadron decays such as $B^0 \rightarrow K^{*0}\mu^+\mu^-$ where the pion from the $K^{*0} \rightarrow K^+\pi^-$ decay is not reconstructed. This region is not used in the subsequent analysis and dictates the lower bound of the $5170 < m(K^+\mu^+\mu^-) < 5700\text{MeV}/c^2$ mass range. Backgrounds from fully hadronic $b$-hadron decays, such as the decay $B^+ \rightarrow K^+\pi^+\pi^-$, are reduced to a

\(^2\)Throughout this paper the decays $B^+ \rightarrow J/\psi K^+$ and $B^0 \rightarrow J/\psi K_S^0$ refer to decays of $B^+$ and $B^0$ mesons to $K^+\mu^+\mu^-$ and $K_S^0\mu^+\mu^-$ final-states, respectively, through the decay $J/\psi \rightarrow \mu^+\mu^-$. \(\)
negligible level using stringent muon-identification selection criteria. A further requirement is applied on the $K^+\mu^-$ pair to remove a small contribution from $B^+ \to D^0 \pi^+$ decays with $D^0 \to K^+\pi^-$, where the pions survive the muon-identification requirements. Candidates are rejected if the mass of the $K^+\mu^-$ pair, computed under the $K^+\pi^-$ hypothesis, is in the range $1850 < m(K^+\pi^-) < 1880 \text{ MeV}/c^2$. After the application of all selection criteria, the background from other $b$-hadron decays is reduced to $\mathcal{O}(0.1\%)$ of the level of the signal. The total efficiency for reconstructing and selecting the $B^+ \to K^+\mu^+\mu^-$ decay is around 2%.

Due to the long lifetime of the $K^0_S$ meson, there are very few $b$-hadron decays that can be mistakenly identified as $B^0 \to K^0_S \mu^+\mu^-$ decays. The largest source of fully reconstructed background is the decay $\Lambda^0_b \to \Lambda \mu^+\mu^-$, where the proton from the $\Lambda \to p\pi^-$ decay is incorrectly identified as a $\pi^+$. This background is removed by rejecting $K^0_S$ meson candidates if the mass of the $\pi^+\pi^-$ pair, under the $p\pi^-$ mass hypothesis, is consistent with that of a $\Lambda$ baryon within $\pm10 \text{ MeV}/c^2$ ($\pm15 \text{ MeV}/c^2$) for long (downstream) candidates. This veto is 95% efficient on genuine $K^0_S$ meson decays and removes more than 99% of $\Lambda$ baryons. The total efficiency for reconstructing the $B^0 \to K^0_S \mu^+\mu^-$ decay is about 0.2%, which is a factor of ten lower than for the charged decay. This is due to a combination of three effects: the long flight distance of $K^0_S$ mesons in the detector, the $K^0_S \to \pi^+\pi^-$ branching fraction, and the requirement of having four, rather than three, tracks within the detector acceptance.

After applying the selection procedure, the signal-to-background ratio in a $\pm50 \text{ MeV}/c^2$ range around the known $B^0$ mass is better than three-to-one for the $B^0 \to K^0_S \mu^+\mu^-$ decay.

After applying the full selection criteria, more than 99% of the selected events contain only one $B^+$ or $B^0$ candidate. Events containing more than one candidate have all but one candidate removed at random in the subsequent analysis.

4 Angular acceptance

The geometrical acceptance of the LHCb detector, the trigger and the event selection can all bias the $\cos \theta_l$ distribution of the selected candidates. The angular acceptance is determined using a sample of simulated signal events. The acceptance as a function of $\cos \theta_l$ is parameterised using a fourth-order polynomial function, fixing the odd-order terms to zero so that the acceptance is symmetric around zero. Any small asymmetry in the acceptance for $B$ and $B$ mesons, due to charge asymmetries in the reconstruction, cancels when combining $B$ and $\overline{B}$ meson decays.

At small values of $q^2$, there is a large reduction of the signal efficiency at values of $\cos \theta_l$ close to $\pm1$, as seen in figure 1. This results from the requirement for muons to have $p \gtrsim 3 \text{ GeV}/c$ to reach the muon system. Smaller reductions of the signal efficiency also arise from the $p_T$ requirement of the hardware trigger and the impact parameter requirements on the $\mu^\pm$ in the selection.

For the decay $B^+ \to K^+\mu^+\mu^-$, the $D^0$ veto described in section 3 introduces an additional bias to the angular acceptance: at a fixed value of $q^2$, there is a one-to-one correspondence between $\cos \theta_l$ and the reconstructed $D^0$ mass, and the $D^0$ veto therefore removes a narrow region of $\cos \theta_l$ in each $q^2$ bin. The $D^0$ veto results in the dip in the
Angular acceptance as derived from simulation in the dimuon mass squared ranges $1.1 < q^2 < 6.0 \text{GeV}^2/c^4$ and $15.0 < q^2 < 22.0 \text{GeV}^2/c^4$. The dip in the acceptance for $B^+ \rightarrow K^+ \mu^+ \mu^-$ decays results from the veto used to reject $B^+ \rightarrow D^0 \pi^+$ decays (see text). The acceptance is normalised to unit area to allow a comparison of the shape of the distributions.

5 Angular analysis

The $m(K^+ \mu^+ \mu^-)$ and $m(K^0_s \mu^+ \mu^-)$ invariant mass distributions of candidates that pass the full selection procedure are shown in figure 2, for two $q^2$ intervals. The long and downstream categories are combined for the decay $B^0 \rightarrow K^0_s \mu^+ \mu^-$. The angular distribution of the candidates is shown in figure 3.

For the $B^+ \rightarrow K^+ \mu^+ \mu^-$ decay, $A_{FB}$ and $F_H$ are determined by performing an unbinned maximum likelihood fit to the $m(K^+ \mu^+ \mu^-)$ and $\cos \theta_l$ distributions of the candidates in bins of $q^2$. The signal angular distribution is described by eq. (1.1), multiplied by the acceptance distribution described in section 4. The signal mass distribution is parameterised by the sum of two Gaussian functions with power-law tails, with common most probable values and common tail parameters, but different widths. The parameters of the these signal functions are obtained fitting the $m(K^+ \mu^+ \mu^-)$ distribution of $B^+ \rightarrow J/\psi \ K^+$ candidates in data. The peak position and width parameters are then corrected, using simulated events, to account for kinematic differences between the decays $B^+ \rightarrow K^+ \mu^+ \mu^-$ and $B^+ \rightarrow J/\psi \ K^+$. The $m(K^+ \mu^+ \mu^-)$ distribution of the combinatorial background is parameterised by a falling exponential function. Its angular distribution is parameterised by a third-order polynomial function multiplied by the same angular acceptance function used for the signal.

Decays of $B^0$ and $\bar{B}^0$ mesons to the $K^0_s \mu^+ \mu^-$ final state cannot be separated based on the final-state particles. The angular distribution of $|\cos \theta_l|$ is described by eq. (1.2), which depends only on $F_H$. Simultaneous unbinned maximum likelihood fits are then performed to the $|\cos \theta_l|$ and $m(K^0_s \mu^+ \mu^-)$ distributions of the two categories of $K^0_s$ meson (long and downstream). The only parameter that is common between the two simultaneous fits is $F_H$. The $m(K^0_s \mu^+ \mu^-)$ shape parameters of the two categories are determined in the same way as that of the decay $B^+ \rightarrow K^+ \mu^+ \mu^-$, using $B^0 \rightarrow J/\psi \ K^0_s$ decays. Information on the angular
Figure 2. Top, reconstructed mass of $B^+ \rightarrow K^+ \mu^+ \mu^-$ candidates in the ranges (a) $1.1 < q^2 < 6.0$ GeV$^2$/c$^4$ and (b) $15.0 < q^2 < 22.0$ GeV$^2$/c$^4$. Bottom, reconstructed mass of $B^0 \rightarrow K^0_S \mu^+ \mu^-$ candidates in the ranges (c) $1.1 < q^2 < 6.0$ GeV$^2$/c$^4$ and (d) $15.0 < q^2 < 22.0$ GeV$^2$/c$^4$. The data are overlaid with the result of the fit described in the text. The long and downstream $K^0_S$ categories are combined for presentation purposes. The shaded region indicates the background contribution in the fit.

The likelihood fits for the $B^+ \rightarrow K^+ \mu^+ \mu^-$ decay and the two categories of $K^0_S$ meson in the $B^0 \rightarrow K^0_S \mu^+ \mu^-$ decay are performed in two dimensions, treating $m(K^+ \mu^+ \mu^-)$ and $\cos \theta_l$ as independent variables. In total, there are 4746$\pm$81 reconstructed signal candidates for the $B^+ \rightarrow K^+ \mu^+ \mu^-$ decay and 176$\pm$17 for the $B^0 \rightarrow K^0_S \mu^+ \mu^-$ decay, summing the yields of the individual $q^2$ bins.

6 Results

For the decay $B^+ \rightarrow K^+ \mu^+ \mu^-$, the results are presented as two-dimensional confidence regions for $A_{FB}$ and $F_H$ and as one-dimensional 68% confidence intervals for $A_{FB}$ and $F_H$. The two-dimensional confidence regions demonstrate the correlation between $A_{FB}$ and $F_H$. 

shape of the background in the likelihood fit is obtained from the upper mass sideband, $5350 < m(K^0_S \mu^+ \mu^-) < 5700$ MeV/c$^2$. For candidates in the long $K^0_S$ category, the number of candidates in the sideband is so small that the shape is assumed to be uniform. For the downstream category, the shape is parameterised by a second-order polynomial. The signal and background angular distributions are then both multiplied by the signal angular acceptance distribution. The $m(K^0_S \mu^+ \mu^-)$ distribution of the background candidates is parameterised by a falling exponential function.
Figure 3. Top, angular distribution of $B^+ \rightarrow K^+ \mu^+ \mu^-$ candidates with (a) $1.1 < q^2 < 6.0 \text{GeV}^2/c^4$ and (b) $15.0 < q^2 < 22.0 \text{GeV}^2/c^4$. Bottom, angular distribution of $B^0 \rightarrow K^0_s \mu^+ \mu^-$ candidates with (c) $1.1 < q^2 < 6.0 \text{GeV}^2/c^4$ and (d) $15.0 < q^2 < 22.0 \text{GeV}^2/c^4$. Only candidates with a reconstructed mass within $\pm 50 \text{MeV}/c^2$ of the known $B^+$ or $B^0$ mass are shown. The data are overlaid with the result of the fit described in the text. The long and downstream $K^0_s$ categories are combined for presentation purposes. The shaded region indicates the background contribution in the fit.

arising from eq. (1.1). The one-dimensional intervals are intended for illustration purposes only. Two-dimensional confidence regions, for the $q^2$ ranges $1.1 < q^2 < 6.0 \text{GeV}^2/c^4$ and $15.0 < q^2 < 22.0 \text{GeV}^2/c^4$ are shown in figure 4; the other $q^2$ bins are provided in the appendix, with the numerical values available in the attachment. The one-dimensional confidence intervals for $B^+ \rightarrow K^+ \mu^+ \mu^-$ decays are shown in figure 5 and given in table 1. The result of the fits to $|\cos \theta|_l$ for the decay $B^0 \rightarrow K^0_s \mu^+ \mu^-$ are shown in figure 6 and given in table 2. Results are presented in 17 (5) bins of $q^2$ for the $B^+ \rightarrow K^+ \mu^+ \mu^-$ ($B^0 \rightarrow K^0_s \mu^+ \mu^-$) decay. They are also presented in two wide bins of $q^2$: one at low hadronic recoil above the open charm threshold and one at large recoil, below the $J/\psi$ meson mass.

The confidence intervals on $F_H$ and $A_{FB}$ are estimated using the Feldman-Cousins technique [23]. Nuisance parameters are incorporated using the so-called plug-in method [24]. At each value of $F_H$ and $A_{FB}$ considered, the maximum likelihood estimate of the nuisance parameters in data is used when generating the pseudoexperiments. For the $B^+ \rightarrow K^+ \mu^+ \mu^-$ decay, $A_{FB}$ ($F_H$) is treated as if it were a nuisance parameter when determining the one-dimensional confidence interval on $F_H$ ($A_{FB}$). The physical boundaries, described in

---

\[^3\text{Data files are provided as supplementary material and are available at this article’s web page.}\]
Figure 4. Two-dimensional confidence regions for $A_{FB}$ and $F_H$ for the decay $B^+ \to K^+ \mu^+ \mu^-$ in the $q^2$ ranges (a) $1.1 < q^2 < 6.0 \text{GeV}^2/c^4$ and (b) $15.0 < q^2 < 22.0 \text{GeV}^2/c^4$. The confidence intervals are determined using the Feldman-Cousins technique. The shaded (triangular) region illustrates the range of $A_{FB}$ and $F_H$ over which the signal angular distribution remains positive in all regions of phase-space.

section 1, are accounted for in the generation of pseudoexperiments when building the confidence belts. Due to the requirement that $|A_{FB}| \leq F_H/2$, statistical fluctuations of events in $\cos \theta_l$ have a tendency to drive $F_H$ to small positive values in the pseudoexperiments.

For the $B^0 \to K^0_S \mu^+ \mu^-$ decay, fits are also performed to $\cos \theta_l$ allowing for a non-zero $A_{FB}$ using eq. (1.1). The value of $A_{FB}$ determined by these fits is consistent with zero, as expected, and the best fit value of $F_H$ compatible with that of the baseline fit.

The data for $F_H$ in figures 5 and 6 are superimposed with theoretical predictions from ref. [25]. In the low $q^2$ region, these predictions rely on the QCD factorisation approaches from ref. [2], which lose accuracy when the dimuon mass approaches the $J/\psi$ mass. In the high $q^2$ region, an operator product expansion in the inverse $b$-quark mass, $1/m_b$, and in $1/\sqrt{q^2}$ is used based on ref. [26]. This expansion is only valid above the open charm threshold. A dimensional estimate of the uncertainty associated with this expansion is discussed in ref. [27]. Form-factor calculations are taken from ref. [28].

Two classes of systematic uncertainty are considered for $A_{FB}$ and $F_H$: detector-related uncertainties that might affect the angular acceptance, and uncertainties related to the angular distribution of the background.

The samples of simulated events used to determine the detector acceptance are corrected to match the performance observed in data by degrading the impact parameter resolution on the kaon and muons by 20%, re-weighting candidates to reproduce the kinematic distribution of $B^+$ candidates in the data and re-weighting candidates to account for differences in tracking and particle-identification performance. Varying these corrections within their known uncertainties has a negligible impact on $A_{FB}$ and $F_H$ ($\lesssim 0.01$).

The acceptance as a function of $\cos \theta_l$ is determined from simulated events in each bin of $q^2$. This assumes that the distribution of events in $q^2$, within the $q^2$ bin, is the same in simulation and in data. To assess the systematic uncertainty arising from this assumption,
Figure 5. Dimuon forward-backward asymmetry, $A_{FB}$, and the parameter $F_H$ for the decay $B^+ \rightarrow K^+ \mu^+ \mu^-$ as a function of the dimuon invariant mass squared, $q^2$. The inner horizontal bars indicate the one-dimensional 68% confidence intervals. The outer vertical bars include contributions from systematic uncertainties (described in the text). The confidence intervals for $F_H$ are overlaid with the SM theory prediction (narrow band). Data are not presented for the regions around the $J/\psi$ and $\psi(2S)$ resonances.

Figure 6. Results for the parameter $F_H$ for the decay $B^0 \rightarrow K^0_S \mu^+ \mu^-$ as a function of the dimuon invariant mass squared, $q^2$. The inner horizontal bars indicate the one-dimensional 68% confidence intervals. The outer vertical bars include contributions from systematic uncertainties (described in the text). The confidence intervals are overlaid with the SM theory prediction (narrow band). Data are not presented for the regions around the $J/\psi$ and $\psi(2S)$ resonances.

the acceptance as a function of $\cos \theta_l$ is determined separately for simulated events in the lower and upper half of the $q^2$ bin, and the average acceptance correction for the bin is re-computed varying the relative contributions from the lower and upper half by 20%. This level of variation covers any observed difference between the differential decay rate as a function of $q^2$ in data and in simulation and introduces an uncertainty at the level of 0.01 on $A_{FB}$ and $F_H$.

In order to investigate the background modelling, the multivariate selection requirements are relaxed. With the increased level of background in the upper mass sideband, an alternative background model of a fourth-order polynomial is derived. Pseudoexperiments are then generated that explore the differences between the $A_{FB}$ or $F_H$ values obtained with
Table 1. Forward-backward asymmetry, $A_{FB}$, and $F_H$ for the decay $B^+ \to K^+ \mu^+ \mu^-$ in the $q^2$ bins used in this analysis. These parameters are also given in a wide bin at large ($1.1 < q^2 < 6.0 \text{ GeV}^2/c^4$) and low ($15.0 < q^2 < 22.0 \text{ GeV}^2/c^4$) hadronic recoil. The column labelled $\text{stat}$ is the 68% statistical confidence interval on $F_H$ ($A_{FB}$) when treating $A_{FB}$ ($F_H$) as a nuisance parameter. The column labelled $\text{syst}$ is the systematic uncertainty.

| $q^2$ (GeV$^2$/c$^4$) | $F_H$ (stat) | $F_H$ (syst) | $A_{FB}$ (stat) | $A_{FB}$ (syst) |
|----------------------|--------------|--------------|-----------------|-----------------|
| 0.10 − 0.98          | [+0.01, +0.20] | ±0.03        | [−0.09, −0.01]  | ±0.01           |
| 1.10 − 2.00          | [+0.00, +0.21] | ±0.03        | [−0.00, +0.10]  | ±0.01           |
| 2.00 − 3.00          | [+0.05, +0.30] | ±0.03        | [−0.01, +0.11]  | ±0.01           |
| 3.00 − 4.00          | [0.00, +0.04]  | ±0.02        | [−0.02, +0.01]  | ±0.01           |
| 4.00 − 5.00          | [0.00, +0.09]  | ±0.03        | [−0.01, +0.05]  | ±0.01           |
| 5.00 − 6.00          | [0.00, +0.14]  | ±0.02        | [−0.04, +0.04]  | ±0.01           |
| 6.00 − 7.00          | [0.00, +0.08]  | ±0.02        | [−0.01, +0.04]  | ±0.01           |
| 7.00 − 8.00          | [0.00, +0.03]  | ±0.03        | [−0.02, +0.02]  | ±0.01           |
| 11.00 − 11.75        | [+0.06, +0.23] | ±0.03        | [−0.03, +0.12]  | ±0.01           |
| 11.75 − 12.50        | [+0.00, +0.10] | ±0.02        | [−0.00, +0.05]  | ±0.01           |
| 15.00 − 16.00        | [+0.06, +0.20] | ±0.02        | [−0.10, −0.03]  | ±0.01           |
| 16.00 − 17.00        | [+0.00, +0.12] | ±0.02        | [−0.05, +0.00]  | ±0.01           |
| 17.00 − 18.00        | [+0.01, +0.16] | ±0.02        | [−0.06, +0.00]  | ±0.01           |
| 18.00 − 19.00        | [+0.05, +0.23] | ±0.02        | [−0.03, +0.05]  | ±0.01           |
| 19.00 − 20.00        | [0.00, +0.10]  | ±0.04        | [−0.02, +0.05]  | ±0.02           |
| 20.00 − 21.00        | [0.00, +0.14]  | ±0.04        | [−0.01, +0.07]  | ±0.02           |
| 21.00 − 22.00        | [+0.04, +0.41] | ±0.05        | [+0.03, +0.19]  | ±0.02           |
| 1.10 − 6.00          | [0.00, +0.06]  | ±0.02        | [−0.01, +0.02]  | ±0.01           |
| 15.00 − 22.00        | [0.00, +0.07]  | ±0.02        | [−0.03, +0.00]  | ±0.01           |

Table 2. The 68% confidence interval on the parameter $F_H$ for the decay $B^0 \to K_S^0 \mu^+ \mu^-$ in $q^2$ bins. In addition to the narrow binning used in the analysis, results are also given in wide bins at large ($1.1 < q^2 < 6.0 \text{ GeV}^2/c^4$) and low ($15.0 < q^2 < 22.0 \text{ GeV}^2/c^4$) hadronic recoil. The column labelled $\text{stat}$ is the 68% statistical confidence interval. The column labelled $\text{syst}$ is the systematic uncertainty.
the default and the alternative background model. A systematic uncertainty is assigned based on the sum in quadrature of the root-mean-square of these differences and the mean bias. The method introduces an uncertainty at the level of 0.01 on $A_{FB}$ and 0.02 on $F_H$ for the $B^+ \rightarrow K^+ \mu^+ \mu^-$ decay and 0.04 − 0.20 for the $B^0 \rightarrow K^0_s \mu^+ \mu^-$ decay.

The dependence of the one-dimensional $A_{FB}$ ($F_H$) confidence interval on the assumed true value of the $F_H$ ($A_{FB}$) nuisance parameter is negligible ($< 0.01$).

The fitting procedure for $B^+ \rightarrow K^+ \mu^+ \mu^-$ ($B^0 \rightarrow K^0_s \mu^+ \mu^-$) decays is also tested using samples of $B^+ \rightarrow J/\psi K^+$ ($B^0 \rightarrow J/\psi K^0_s$) decays where $A_{FB} = F_H = 0$, due to the vector nature of the $J/\psi$ meson. These samples are more than one hundred times larger than the signal samples. Tests are also performed splitting these samples into sub-samples of comparable size to the data sets in the individual $q^2$ bins. No indication of any bias is seen in the fitting procedure in either set of tests.

## 7 Conclusion

In summary, the angular distributions of charged and neutral $B \rightarrow K \mu^+ \mu^-$ decays are studied using a data set, corresponding to an integrated luminosity of 3 fb$^{-1}$, collected by the LHCb experiment. The angular distribution of the decays is parameterised in terms of the forward-backward asymmetry of the decay, $A_{FB}$, and a parameter $F_H$, which is a measure of the contribution from (pseudo)scalar and tensor amplitudes to the decay width.

The measurements of $A_{FB}$ and $F_H$ presented for the decays $B^+ \rightarrow K^+ \mu^+ \mu^-$ and $B^0 \rightarrow K^0_s \mu^+ \mu^-$ are the most precise to date. They are consistent with SM predictions ($A_{FB} \approx 0$ and $F_H \approx 0$) in every bin of $q^2$. The results are also compatible between the decays $B^+ \rightarrow K^+ \mu^+ \mu^-$ and $B^0 \rightarrow K^0_s \mu^+ \mu^-$. The largest difference with respect to the SM prediction is seen in the range $11.00 < q^2 < 11.75$ GeV$^2/c^4$ for the decay $B^+ \rightarrow K^+ \mu^+ \mu^-$. Even in this bin, the SM point is included at 95% confidence level when taking into account the systematic uncertainties on the angular observables.

The results place constraints on (pseudo)scalar and tensor amplitudes, which are vanishingly small in the SM but can be enhanced in many extensions of the SM. Pseudoscalar and scalar amplitudes were already highly constrained by measurements of the branching fraction of the decay $B^0_s \rightarrow \mu^+ \mu^-$ [29, 30]. The results presented here, however, also rule out the possibility of large accidental cancellations between the left- and right-handed couplings of the (pseudo)scalar amplitudes to the $B^0_s \rightarrow \mu^+ \mu^-$ branching fraction. Tensor amplitudes were previously poorly constrained.

## Acknowledgments

We express our gratitude to our colleagues in the CERN accelerator departments for the excellent performance of the LHC. We thank the technical and administrative staff at the LHCb institutes. We acknowledge support from CERN and from the national agencies: CAPES, CNPq, FAPERJ and FINEP (Brazil); NSFC (China); CNRS/IN2P3 and Region Auvergne (France); BMBF, DFG, HGF and MPG (Germany); SFI (Ireland); INFN (Italy); FOM and NWO (The Netherlands); SCSR (Poland); MEN/IFA (Romania); MinES, Rosatom, RFBR and NRC “Kurchatov Institute” (Russia); MinECo, XuntaGal
and GENCAT (Spain); SNSF and SER (Switzerland); NASU (Ukraine); STFC and the Royal Society (United Kingdom); NSF (U.S.A.). We also acknowledge the support received from EPLANET, Marie Curie Actions and the ERC under FP7. The Tier1 computing centres are supported by IN2P3 (France), KIT and BMBF (Germany), INFN (Italy), NWO and SURF (The Netherlands), PIC (Spain), GridPP (United Kingdom). We are indebted to the communities behind the multiple open source software packages on which we depend. We are also thankful for the computing resources and the access to software R&D tools provided by Yandex LLC (Russia).

A Two-dimensional confidence intervals

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{plots/figure7.pdf}
\caption{Two-dimensional confidence regions for $A_{FB}$ and $F_H$ for the decay $B^+ \to K^+ \mu^+ \mu^-$ in the $q^2$ ranges (a) $0.10 < q^2 < 0.98 \text{GeV}^2/c^4$, (b) $1.10 < q^2 < 2.00 \text{GeV}^2/c^4$, (c) $2.00 < q^2 < 3.00 \text{GeV}^2/c^4$ and (d) $3.00 < q^2 < 4.00 \text{GeV}^2/c^4$. The confidence intervals are determined using the Feldman-Cousins technique and are purely statistical. The shaded (triangular) region illustrates the range of $A_{FB}$ and $F_H$ over which the signal angular distribution remains positive in all regions of phase-space.}
\end{figure}
Figure 8. Two-dimensional confidence regions for $A_{FB}$ and $F_H$ for the decay $B^+ \rightarrow K^+ \mu^+ \mu^-$ in the $q^2$ ranges (a) $4.00 < q^2 < 5.00 \text{GeV}^2/c^4$, (b) $5.00 < q^2 < 6.00 \text{GeV}^2/c^4$, (c) $6.00 < q^2 < 7.00 \text{GeV}^2/c^4$ and (d) $7.00 < q^2 < 8.00 \text{GeV}^2/c^4$. The confidence intervals are determined using the Feldman-Cousins technique and are purely statistical. The shaded (triangular) region illustrates the range of $A_{FB}$ and $F_H$ over which the signal angular distribution remains positive in all regions of phase-space.
Figure 9. Two-dimensional confidence regions for $A_{FB}$ and $F_H$ for the decay $B^+ \rightarrow K^+ \mu^+ \mu^-$ in the $q^2$ ranges (a) $11.00 < q^2 < 11.75$ GeV$^2$/c$^4$, (b) $11.75 < q^2 < 12.50$ GeV$^2$/c$^4$ and (c) $15.00 < q^2 < 16.00$ GeV$^2$/c$^4$. The confidence intervals are determined using the Feldman-Cousins technique and are purely statistical. The shaded (triangular) region illustrates the range of $A_{FB}$ and $F_H$ over which the signal angular distribution remains positive in all regions of phase-space.
Figure 10. Two-dimensional confidence regions for $A_{FB}$ and $F_H$ for the decay $B^+ \rightarrow K^+ \mu^+ \mu^-$ in the $q^2$ ranges (a) $16.00 < q^2 < 17.00 \text{GeV}^2/c^4$, (b) $17.00 < q^2 < 18.00 \text{GeV}^2/c^4$ and (c) $18.00 < q^2 < 19.00 \text{GeV}^2/c^4$. The confidence intervals are determined using the Feldman-Cousins technique and are purely statistical. The shaded (triangular) region illustrates the range of $A_{FB}$ and $F_H$ over which the signal angular distribution remains positive in all regions of phase-space.
Figure 11. Two-dimensional confidence regions for $A_{FB}$ and $F_H$ for the decay $B^+ \rightarrow K^+ \mu^+ \mu^-$ in the $q^2$ ranges (a) $19.00 < q^2 < 20.00 \text{ GeV}^2/c^4$, (b) $20.00 < q^2 < 21.00 \text{ GeV}^2/c^4$ and (c) $21.00 < q^2 < 22.00 \text{ GeV}^2/c^4$. The confidence intervals are determined using the Feldman-Cousins technique and are purely statistical. The shaded (triangular) region illustrates the range of $A_{FB}$ and $F_H$ over which the signal angular distribution remains positive in all regions of phase-space.
Open Access. This article is distributed under the terms of the Creative Commons Attribution License (CC-BY 4.0), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

[1] A.K. Alok, A. Dighe and S.U. Sankar, Large forward-backward asymmetry in $B \to K \mu^+ \mu^-$ from new physics tensor operators, Phys. Rev. D 78 (2008) 114025 [arXiv:0810.3779] [inSPIRE].

[2] C. Bobeth, G. Hiller and G. Piranishvili, Angular distributions of $B \to K \bar{\ell} \ell$ decays, JHEP 12 (2007) 040 [arXiv:0709.4174] [inSPIRE].

[3] BaBar collaboration, B. Aubert et al., Measurements of branching fractions, rate asymmetries and angular distributions in the rare decays $B \to K \ell^+ \ell^-$ and $B \to K^* \ell^+ \ell^-$, Phys. Rev. D 73 (2006) 092001 [hep-ex/0604007] [inSPIRE].

[4] BELLE collaboration, J.-T. Wei et al., Measurement of the Differential Branching Fraction and Forward-Backward Asymmetry for $B \to K(\ast) \ell^+ \ell^-$, Phys. Rev. Lett. 103 (2009) 171801 [arXiv:0904.0770] [inSPIRE].

[5] LHCb collaboration, The LHCb Detector at the LHC, 2008 JINST 3 S08005 [inSPIRE].

[6] LHCb collaboration, Differential branching fraction and angular analysis of the $B^+ \to K^+ \mu^+ \mu^-$ decay, JHEP 02 (2013) 105 [arXiv:1209.4284] [inSPIRE].

[7] A. Ali, P. Ball, L.T. Handoko and G. Hiller, A Comparative study of the decays $B \to (K, K^*) \ell^+ \ell^-$ in standard model and supersymmetric theories, Phys. Rev. D 61 (2000) 074024 [hep-ph/9910221] [inSPIRE].

[8] I. Belyaev et al., Handling of the generation of primary events in GAUSS, the LHCb simulation framework, IEEE Nucl. Sci. Symp. Conf. Rec. (NSS/MIC) 2010 (2010) 1155.
[17] GEANT4 collaboration, S. Agostinelli et al., GEANT4: a simulation toolkit, *Nucl. Instrum. Meth.* A 506 (2003) 250 [SPIRE].

[18] M. Clemencic et al., The LHCb simulation application, Gauss: design, evolution and experience, *J. Phys. Conf. Ser.* 331 (2011) 032023 [SPIRE].

[19] R. Aaij et al., The LHCb Trigger and its Performance in 2011, 2013 JINST 8 P4022 [arXiv:1211.3055] [SPIRE].

[20] LHCb collaboration, Observation of a resonance in $B^+ \rightarrow K^+ \mu^+ \mu^-$ decays at low recoil, *Phys. Rev. Lett.* 111 (2013) 112003 [arXiv:1307.7595] [SPIRE].

[21] L. Breiman, J.H. Friedman, R.A. Olshen and C.J. Stone, *Classification and regression trees*, Wadsworth international group, Belmont U.S.A. (1984).

[22] R.E. Schapire and Y. Freund, A decision-theoretic generalization of on-line learning and an application to boosting, *J. Comp. Syst. Sci.* 55 (1997) 119.

[23] G.J. Feldman and R.D. Cousins, Unified approach to the classical statistical analysis of small signals, *Phys. Rev. D* 57 (1998) 3873 [physics/9711021] [SPIRE].

[24] B. Sen, M. Walker and M. Woodroofe, On the unified method with nuisance parameters, *Statistica Sinica* 19 (2009) 301.

[25] C. Bobeth, G. Hiller, D. van Dyk and C. Wacker, The decay $B \rightarrow K^\pm \ell^\pm$ at low hadronic recoil and model-independent $\Delta B = 1$ constraints, *JHEP* 01 (2012) 107 [arXiv:1111.2558] [SPIRE].

[26] B. Grinstein and D. Pirjol, Exclusive rare $B \rightarrow K^* \ell^+ \ell^-$ decays at low recoil: controlling the long-distance effects, *Phys. Rev. D* 70 (2004) 114005 [hep-ph/0404250] [SPIRE].

[27] U. Egede, T. Hurth, J. Matias, M. Ramon and W. Reece, New observables in the decay mode $\bar{B}_d \rightarrow \bar{K}^{(*)0} \ell^+ \ell^-$, *JHEP* 11 (2008) 032 [arXiv:0807.2589] [SPIRE].

[28] A. Khodjamirian, T. Mannel, A.A. Pivovarov and Y.-M. Wang, Charm-loop effect in $B \rightarrow K^{(*)} \ell^+ \ell^-$ and $B \rightarrow K^{*}\gamma$, *JHEP* 09 (2010) 089 [arXiv:1006.4945] [SPIRE].

[29] LHCb collaboration, Measurement of the $B_s \rightarrow \mu^+ \mu^-$ branching fraction and search for $B^0 \rightarrow \mu^+ \mu^-$ decays at the LHCb experiment, *Phys. Rev. Lett.* 111 (2013) 101805 [arXiv:1307.5024] [SPIRE].

[30] CMS collaboration, Measurement of the $B_s^0 \rightarrow \mu^+ \mu^-$ branching fraction and search for $B^0 \rightarrow \mu^+ \mu^-$ with the CMS Experiment, *Phys. Rev. Lett.* 111 (2013) 101804 [arXiv:1307.5025] [SPIRE].
1 Università di Genova, Genova, Italy
2 Università di Milano Bicocca, Milano, Italy
3 Università di Roma Tor Vergata, Roma, Italy
4 Università di Roma La Sapienza, Roma, Italy
5 Università della Basilicata, Potenza, Italy
6 LIFAELS, La Salle, Universitat Ramon Llull, Barcelona, Spain
7 Hanoi University of Science, Hanoi, Viet Nam
8 Università di Padova, Padova, Italy
9 Università di Pisa, Pisa, Italy
a Scuola Normale Superiore, Pisa, Italy
b Università degli Studi di Milano, Milano, Italy