The Role of the Future in Quantum Theory

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ABSTRACT

Interpretation problems are eliminated from quantum theory by picturing a quantum history as having been sampled from a probability distribution over the set of histories which are permitted by all relevant boundary conditions. In laboratory physics, the final measurement plays a crucial role in defining the set of allowed histories by constraining the final state to be an eigenstate of the measurement operator. For the universe itself, a final boundary condition can play a similar role. Together with the special big bang initial state, the final constraint may ensure that the universe admits a classical description in the asymptotic future. Acknowledging the role of a future constraint dispels the mysteries of quantum theory without any amendment to the theory itself.

1 Introduction

In a typical experiment, the initial state of a quantum system is fixed by its preparation and the final state is measured. The eigenstates of the measurement operator constitute the set of allowed final states. The formalism of quantum theory yields a probability distribution on the set of allowed final states, and experiments consistently verify the validity of the formalism. The relationship between experiments of that type and the formalism is clear. Problems of interpretation do not arise if every quantum system is subject to the type of future constraint
which is imposed by a final measurement. This paper considers how a similar final constraint might operate on a closed system like the 4-dimensional universe. There is no external apparatus to perform a final measurement, so the final constraint must be considered as a boundary condition.

The hypothesis of a final constraint offers an approach to quantum theory which is free of interpretation problems and is consistent with any conceivable test. Any condition which is satisfied by the actual final state of the universe could be an operative boundary condition. No experiment in this universe can determine whether or not the satisfied final condition is an imposed boundary condition. The possibility of a future constraint is here taken seriously because it cannot be excluded and because it renders quantum theory understandable. Objective reality is maintained without a “many worlds” interpretation \[1\]. State reductions occur without amending quantum theory with an ad hoc R-process \[2\]. The limitation on causality is no different than for a laboratory quantum system whose final state is constrained by an external measurement apparatus.

Speculation about the actual nature of the final constraint is offered in sections 3-5. The simplest hypothesis is that the final state is required to be sampled from a special set of basis states, exactly as if an external measurement were made. However, due to the highly special initial big bang state, it may be that the final constraint need not be so specific as to select a unique final basis. Many different physically reasonable final bases may define the same probability distribution over a common set of physically equivalent quantum histories. In the context of quantum gravity, the final constraint may be tantamount to requiring that a geometric description should pertain to the universe, at least for asymptotically late times.

The need for a future constraint is suggested by the Einstein-Podolsky-Rosen (EPR) phenomenon \[3\]. As presented by Bohm \[4\], the EPR effect concerns the decay of a spin-0 system into two spinning particles. Measurement of the spins of both particles with respect to a common spin axis always yields equal-but-opposite spin values (as required by angular momentum conservation). The perfect anti-correlation holds regardless of the choice of spin axis for the measurement, and even if the spin axis is selected by a random process after the two spinning particles are well separated \[5\]. Further experimental results verify \[5\], as expected from the non-commutation of spin operators associated with different spin axes, that the two particles cannot carry prepared answers to all possible spin tests \[6, 7\]. That being the case, how can they be certain to give opposite answers to every possible test? No satisfactory answer is available in the context of future-blind evolution. There is nothing paradoxical about the EPR phenomenon, however, if one regards an entire quantum history as having been selected as a single entity. Each allowed history necessarily satisfies all conservation laws. There is no allowed history in which both particles are measured with positive spin components along the z-axis, for example, and so that situation never occurs. The particles do not evolve without reference to the measurements in their future. The entire quantum history is a single entity. Even though
the orientation of the future spin measurement might not be knowable at the time of the spin-0
decay, each of the allowed histories nevertheless includes a specific measurement orientation,
and both particles might be regarded as having been emitted in eigenstates of the corresponding
spin operator. There is no paradox in the EPR phenomenon if one accepts that the emitted
particles are conditioned by their future in this sense.

This approach requires a clear understanding for the term “history” in quantum theory. It is
not simply the Schroedinger evolution of the initial state, since that evolved state is generally a
grotesque superposition of physically interpretable states at late times. The quantum history
of the universe includes a rich sequence of reductions to physically interpretable states. The
occurrence of those state reductions is here attributed to a final constraint condition together
with the special initial state. Before considering specific ideas about the final constraint and
what is meant by a “quantum history,” however, it will first be argued that there is nothing
unacceptable about a future constraint.

2 Boundary conditions and causality

Quantum theory does not favor initial boundary conditions over final boundary conditions or
mixed boundary conditions. The theory itself is time-symmetric. The rules of quantum theory
assign a transition amplitude to any pair of initial and final states. A transition amplitude is
time-symmetric in the following sense: If \( S \) is the transition amplitude for state \( \psi \) to become
state \( \phi \), then \( S \) is also the amplitude for time-reversed \( \phi \) to become time-reversed \( \psi \) (with all
interaction laws operating in their time-reversed form).

Consider the specific example of a photon which passes through a double-slit diffraction and
is absorbed on a subsequent screen. There is a particular amplitude \( S \) for its absorption at a
particular point \( P \) on the screen. Now imagine the time reverse of absorption at \( P \), so a photon
of the same frequency is emitted from \( P \) toward the diffraction. Such a photon can go off in
many directions from the diffraction. Most likely it will be absorbed somewhere on a laboratory
wall. However, the amplitude for its absorption in the lamp is equal to the amplitude \( S \) for the
photon emitted from the lamp to be absorbed at \( P \).

Quantum theory is time-symmetric in this narrow sense of time symmetry for transition
amplitudes. It is not the time symmetry which pertains to deterministic classical physics where
each initial state identifies a unique final state and vice versa.

Time symmetry of quantum theory must also be clearly distinguished from time symmetry
(or not) of particular interaction laws. If all interaction laws were time-symmetric, then the
amplitude for going from state \( \psi \) to state \( \phi \) would be the same as the amplitude for time-
reversed \( \phi \) going to time-reversed \( \psi \), period. If the interaction laws are not time symmetric,
however, then the amplitude for \( \psi \) going to \( \phi \) may have two different values, depending on the
orientation of time. The amplitude for \( \psi \) going to \( \phi \) with one orientation of time is equal to the
amplitude for time-reversed \( \phi \) going to time-reversed \( \psi \) with the opposite time orientation. The
narrow meaning of time symmetry in quantum theory therefore applies also in the presence of interaction laws which are not time-symmetric.

A universe with a final boundary condition can be imagined as easily as a universe with an initial boundary condition. It is customary to assume that the universe has evolved from the low entropy state in the past. In the context of deterministic classical physics, however, it is not possible to reject the interpretation that the universe is evolving backward in time from the very special final state which could be obtained by the classical evolution of the low entropy initial state. In that perverse (but allowed) interpretation, causes are ascribed to future events instead of past events, and one may perversely maintain that no system is influenced by events outside its causal future. The customary time orientation of causality derives from asymmetric boundary conditions. Indeed, were the low-entropy state an imposed final state rather than an imposed initial state, the customary and perverse notions of causality would be interchanged. The theory itself is time symmetric and does not favor any particular boundary conditions or time orientation for causality.

In quantum physics, the initial state of a system does not determine its future. Schroedinger evolution of the initial state yields only an amplitude for transition to any future state. In laboratory experiments, a final measurement defines an orthonormal set of allowed final states for the system. The squared norms of the transition amplitudes to those measurement eigenstates constitute a properly normalized probability distribution. One may identify the set of allowed quantum histories with the set of final measurement eigenstates, so there is a probability distribution over the set of allowed quantum histories. One can picture the final state (and hence the quantum history) as being sampled from that probability distribution. For this same picture to apply to the quantum universe, some final constraint must similarly restrict the allowed histories to a set over which there is a normalized probability distribution. A random sampling from that probability distribution can then give rise to the quantum history of the universe.

The hypothesis that the universe is subject to a future constraint raises questions about causality. The standard point of view is that boundary conditions apply only in the past, and the future is completely unconstrained. Stemming from this, the strong principle of causality asserts that a system cannot be influenced by events or conditions which lie outside its causal past. The existence of a future constraint certainly conflicts with these strong ideas about causality. On the other hand, the final constraint might be analogous to the effect of a final measurement by an external apparatus. How much damage does that do to causality? The EPR experiment can be used to study the question. Whenever the spins of two particles produced by the decay of a spin-0 state are tested with respect to a common spin axis, they behave as if they were produced in eigenstates of that spin operator. For example, the two particles seem to be emitted in x-spin eigenstates if x-spin measurements are subsequently performed on both; they seem to be emitted in y-spin eigenstates if y-spin measurements are subsequently performed on both; etc. It does not matter how or when the measurement axis is selected. The choice of
measurement axis, regardless of how or when it is made, limits the set of possible histories, and the entire quantum history is sampled as a single entity. If one insists on picturing systems as evolving in time, however, it appears that the particles are influenced at their event of production by the measurements performed at some later time. This violates the strong notion of causality cited above. Nevertheless, there is no way to extract the information about the orientation of the future spin measurement axis. The particles’ spin states cannot be tested without disrupting the influence of the future measurements. The influence from the future therefore does not constitute a signal propagating backward in time. Note also that quantum theory respects the light cone structure of relativity in the sense that a single particle cannot be localized at two spacelike-separated events, so particles do not transport signals faster than the speed of light. If backward-in-time signaling does not occur, then the practical notion of causality can be maintained: “The maximum signal speed is the speed of light.” As noted previously, causality is not intrinsic to the theory itself, but derives from boundary conditions. With an imposed low-entropy final state, one would interpret all signals as propagating backward in time. With fully time-symmetric boundary conditions, causality would not be a useful notion at all. Due to asymmetric boundary conditions, causality is a practical notion in the actual universe, but the strongest formulation of the principle may not be tenable.

3 Quantum history as beginning-to-end transition

There is no interpretation problem associated with the EPR phenomenon if a quantum history is sampled from a probability distribution over an allowed set of histories each of which satisfies all conservation laws. This approach requires a specific meaning for the term “quantum history.” In laboratory physics, the set of allowed quantum histories is governed by the measurement chosen to determine the final state as well as the system’s preparation which determines the initial state. Given a fixed initial state $\psi_0$, the allowed quantum histories may be identified with transitions to the possible eigenstates $\psi_k$ of the final measurement operator. By virtue of that identification, the quantum formalism provides a probability for each allowed history. The probability for the history linking $\psi_0$ to $\psi_k$ is $|<\psi_k|H|\psi_0>|^2$, where $H$ maps the initial state to the final time via Schroedinger evolution (or the complex conjugate operator maps the final state to the initial time) and the bracket denotes the Hilbert space inner product. Experiments consistently verify the validity of those probabilities. This paradigm pertains to all laboratory quantum physics. The operative constraints and/or boundary conditions define a set of allowed quantum histories. Only one actual history occurs. The quantum formalism concerns the probability distribution for any set of allowed histories.

For a closed system like the 4-dimensional universe, there is no external measuring apparatus to impose a special set of basis states from which the final state must be drawn. However, by supposing that a final boundary condition similarly requires the final state to be drawn from some special set of final basis states, the same notion of a quantum history as a beginning-to-end
transition can pertain as well to the 4-dimensional universe. A probability amplitude is then associated with each allowed history, and the quantum history of the universe may be regarded as having been sampled from that probability distribution. Only one of the allowed histories actually occurs.

A quantum history is therefore characterized by a transition from the initial state to an allowed final state. In a laboratory scattering experiment, the final measured state may permit a reconstruction of particle worldlines, so the quantum history determines a classical history. That is not always the case, however, as illustrated by a photon which passes through a double-slit diffraction and is detected on a subsequent screen. Simply measuring the final position of the photon does not determine which of the possible classical worldlines it followed. The quantum history does not admit a unique classical description. The situation is different if a measurement of the photon is made at one of the slits. In that case the quantum history is defined by a final state which contains a record of which slit the photon went through, and the photon’s unique worldline is implicit in the quantum history. Now suppose the measurement at the slit is reversible in the sense of Wigner [9]. If the measurement is actually reversed, then there is no record of its outcome in the final state, and the two possible worldlines interfere coherently. Conversely, if the measurement interaction is not reversed, then the final state does contain a record of the photon’s position in one or the other slit, and a unique worldline is determined. The measurement interaction at a slit does not by itself localize the particle, since the measurement could subsequently be reversed. Whether or not the particle is localized at a slit is a property of the whole quantum history, because it is determined by the final state.

The characterization of a quantum history by its initial and final states clarifies the concepts of “successful measurement” and “state reduction.” Let \( \psi_0 \) and \( \psi_f \) denote the initial and final states of a quantum history. Suppose \( \phi \) is a state at some intermediate time with the property that

\[
< \phi | \phi' > = 0 \quad \Rightarrow \quad < \psi_f | H_2 | \phi' > < \phi' | H_1 | \psi_0 > = 0,
\]

where \( H_1 \) is the unitary Schroedinger mapping of states from the initial time to the intermediate time and \( H_2 \) maps states from the intermediate time to the final time. If \( \phi \) is an eigenstate of some measurement interaction between subsystems, for example, then it represents a successful measurement. Condition (1) means that none of the other (orthogonal) eigenstates \( \phi' \) is both accessible from the initial state and compatible with the final state. It also means that the amplitude for transition from \( \psi_0 \) to \( \psi_f \) is not reduced by projection onto \( \phi \) at the intermediate time:

\[
< \psi_f | H_2 H_1 | \psi_0 > = < \psi_f | H_2 | \phi > < \phi | H_1 | \psi_0 >.
\]

In the history defined by \( \psi_0 \) and \( \psi_f \), therefore, the outcome of the measurement is definitely \( \phi \), and one says that the system is reduced to the state \( \phi \) at the intermediate time.

The Schroedinger cat gedanken experiment [10] provides a dramatization of these ideas. A cat is enclosed in a box along with a quantum subsystem which may or may not decay in a
prescribed time interval. Decay of the subsystem would cause a vial of poison to be broken so the cat would die. The box and its contents constitute a quantum system starting in a pure state. When the box is opened after the prescribed time interval, the Schroedinger-evolved pure state entails a superposition of live cat and dead cat. Does any state reduction to either the live-cat state or the dead-cat state occur? The answer lies in the final state, because that is what defines the quantum history. Part of the problem in the Schrödinger cat experiment is deciding what is meant by the final state, since there is a sequence of correlated outcomes. If the atom decays, then the vial is broken, then the cat dies, then an intelligent observer recognizes that the cat is dead, etc. Is there any point beyond which superposition with the alternative outcome is impossible? In principle, the answer is No. The resolution must be sought in the final state $|\Psi_f\rangle$ of the universe itself. If no permanent record exists of whether the cat was alive or dead when the box was opened, then the final state can be reached through either path, and neither classical description by itself pertains to the quantum history. However, the interference of different macroscopic states does not normally occur. It is therefore natural to assume that the final state must contain some record of whether the cat was dead or alive when the box was opened. Symbolically, either $\langle \Psi_f|H_2|\Psi_{live}\rangle = 0$ or $\langle \Psi_f|H_2|\Psi_{dead}\rangle = 0$ must be true, where $H_2$ is the unitary mapping from the time of box opening to the time of the final state. Then, since the initial state is mapped by $H_1$ to a superposition of just $|\Psi_{live}\rangle$ and $|\Psi_{dead}\rangle$, one or the other of those states satisfies the conditions (on $\phi$ in equation 1) for a state reduction at the time of the box opening. The state reduction means the quantum history does not entail a superposition of live cat with dead cat.

It remains to explain why the final state of the universe should be incompatible either with the live cat state or else with the dead cat state. After all, if $|\Psi_1\rangle$ is a final state which is compatible only with the live cat state and $|\Psi_2\rangle$ is another final state which is compatible only with the dead cat, then any superposition of them is a valid quantum state. The quantum history defined by such a superposition final state would not include a state reduction to a live cat or dead cat at the intermediate time. What is it that prohibits a superposition final state of that type? There is nothing in quantum theory itself to exclude superpositions of $|\Psi_1\rangle$ and $|\Psi_2\rangle$. If quantum theory is not to be amended with additional structure, then the exclusion must be attributed to a boundary condition. The states $|\Psi_1\rangle$ and $|\Psi_2\rangle$ may separately satisfy an imposed boundary condition which no superposition of them could satisfy. The following two sections address this issue of what constitutes a physically plausible boundary condition, i.e. why an allowed final state is extremely unlikely to be accessible from both the dead cat state and the live cat state.

4 Toy cosmology

In a simplified cosmology without gravitation or other long-range interactions, one can imagine that at $t = 0$ there is only an extremely massive particle at the origin of Euclidean space. There
are no other particles or radiation. The massive particle is allowed to decay, and a stochastic cascade of inelastic collisions, decays, and radiative processes distributes the energy among more and more particles. In the asymptotic future, there are a vast number of low-energy free particles. The spacetime picture of a cascade is a network of intersecting particle worldlines. The network is similar in some respects to a giant tree stemming from a single root, with a multitude of smaller and smaller branches. Its leaves correspond to classical free particle worldlines in the asymptotic future. Because of the stochastic nature of the decays and interactions, the original massive particle does not determine the cascade. (Many different trees may have an identical root.)

Two or more particle networks which are identical in the asymptotic future are here regarded as a single cascade. The different intermediate developments interfere coherently if the asymptotic cascade contains no record of one or the other development having occurred at the intermediate times. A cascade may be defined as a class of semi-classical (stationary-phase) Feynman paths which are represented in the asymptotic future by the same classical phase space trajectory.

Each cascade can be characterized by some information from its asymptotic classical phase space trajectory. The set of asymptotic trajectories which can result from the special initial state is extremely restricted, so a relatively small amount of information about the final trajectory suffices to identify an individual cascade. In particular, it is plausible that the amount of information which specifies a final quantum state, together with the special initial conditions, is adequate to identify an individual cascade. For example, suppose all particle positions are specified arbitrarily at some late time. Can a momentum be assigned to each final particle so that the final state can be evolved backward semi-classically to the special concentration in a single mass at the origin of Euclidean space at \( t = 0 \)? In general, the answer is No. Only special final particle configurations at the late time can be attained from the various possible cascades. Moreover, a final configuration which can be attained from a cascade comes from (in general) at most one such cascade. It is highly exceptional for two or more distinct cascades to yield identical particle positions at some late time. A cascade can be determined, therefore, by the initial condition together with final particle positions.

The reason why each cascade can be identified by relatively little information about its asymptotic trajectory is that the cascade develops by virtue of particle decays and also inelastic collisions with more than two outgoing particles. The time reverse of such processes requires extremely special kinematic conditions. The time reverse of particle decay is fusion without radiation, a phenomenon which is too special to be observed in nature. Similarly, inelastic collisions which reduce the number of particles can only occur if kinematic conditions are finely tuned. The semi-classical backward evolutions of a generic asymptotic phase space trajectory would afford few opportunities for such processes, so it could not coalesce into the single massive particle at \( t = 0 \). Consequently, a cascade which develops from the single massive particle can
be characterized at some late time by particle information which could be provided by a final quantum state.

The above assertions are supported quantitatively by comparing the degrees of freedom in such a cascade with the amount of information provided by a final quantum state. Suppose the cascade leads to a total of $n$ particles in the final state. A final quantum state can be specified by $3n$ parameters, which might be the final coordinates of all the particles, or the components of their final momenta, etc. The semi-classical cascade can be represented by a network of intersecting particle worldlines in 4 dimensions. There are a countable number of topologically distinguishable networks which start from a given worldline and end with $n$ particles. For each topology, the various possible networks can be parametrized by fewer than $3n$ numbers. Consider, for example, what happens when a particle decays into two. There is one parameter's freedom in specifying when the decay occurs, and two parameters to specify the direction of one of the outgoing particles. With fixed masses for the outgoing particles, the 4-momenta of the two outgoing particles are determined by energy-momentum conservation once the first direction is specified. So there are 3 parameters to specify this decay which increases the number of particles by one. Similarly, if the number of particles increases by $m$ as the result of an inelastic collision of two particles, there are $3m$ additional parameters as a result. (To arrange for the collision of two particles which would otherwise have unconstrained momenta, one of them loses two degrees of freedom. Conserving energy-momentum with particles of fixed mass leaves $3(m + 2) - 4$ degrees of freedom in the 4-momenta of the $m + 2$ outgoing particles. This compensates for the two degrees lost to arrange the collision and also adds $3m$ additional parameters associated with this production of $m$ additional particles.) The final state occurs after an increase of $n - 1$ particles from the initial single mass. The freedom in the cascade development is parametrized by $3(n - 1)$ parameters. In the $3n$-parameter space of allowed final quantum states, therefore, only a special subset admit a semi-classical particle cascade interpretation.

In any of these toy universes, entropy increases from 0 (if there is only one state for the initial massive particle) to a very large value as the energy is distributed among more and more particles. Due to the stochastic nature of the interactions, a specification of the cascade at some intermediate time $t$ is insufficient for reliably predicting the future. On the other hand, if a quantum state at time $t$ is compatible with some cascade from the initial state, then the cascade up to that time is (in general) uniquely determined. As a result, past history can be confidently inferred (using knowledge of the initial state).

It is natural to think of a cascade as a quantum history. This is an apparent departure from the earlier definition of a quantum history as a beginning-to-end transition. However, within the context provided by the highly special initial state of the toy cosmology, each cascade corresponds to a special class of beginning-to-end transitions, and this correspondence endows the set of cascades with a probability distribution. It is thus possible to regard a toy universe as a cascade sampled from that probability distribution.
Each cascade has a unique asymptotic classical phase space trajectory. As a result, it identifies a unique final position eigenstate, or a unique final momentum eigenstate, etc. Moreover, due to the special nature of the initial state, a final eigenstate of position or momentum (etc.) is almost always compatible with at most one cascade. Consider the final momentum eigenstates. There is a subset for which each eigenstate corresponds to at least one cascade. Those are the eigenstates for which there is a non-negligible transition amplitude. (The others have no stationary-phase Feynman path to contribute to their amplitudes in the sum-over-paths method.) Among those final momentum eigenstates which correspond to cascades, there could be ones which are compatible with more than one cascade. (That requires another accessible asymptotic phase space trajectory with identical particle momenta.) Those exceptional eigenstates constitute a set of measure zero, however. The transition probabilities therefore give a normalized probability distribution over the set of those momentum eigenstates which correspond to unique cascades. This probability distribution can then be considered as pertaining to the set of cascades themselves. The same probability distribution results from the correspondence of cascades with final position eigenstates (or some other physically reasonable basis). The probability pertains to the cascade, not just a particular final state in correspondence with it.

The special initial state is responsible for the existence of a probability distribution over the set of cascades. Were the initial state a typical \( n \)-particle position eigenstate, for example, no final \( n \)-particle position eigenstate and no final momentum eigenstate would correspond to a unique cascade. (The amplitude for transition to each such accessible eigenstate would include contributions from multiple stationary-phase Feynman paths with disparate asymptotic classical phase space trajectories.) The almost 1-to-1 correspondence between cascades and the accessible basis states for physically reasonable final bases is a remarkable consequence of the special initial state in the toy cosmology. (The correspondence may be the property which characterizes a “physically reasonable” final basis, a property which no basis would have without the special initial state.)

A cascade identifies many different beginning-to-end transitions since it identifies a final momentum eigenstate, final position eigenstate, etc. All beginning-to-end transitions which correspond uniquely to the same cascade should be regarded as physically equivalent. A transition to a final position eigenstate might imply a sequence of position eigenstate reductions at intermediate times (as defined by condition 1), whereas an equivalent transition (via the same cascade) to a final momentum eigenstate might imply a sequence of reductions to momentum eigenstates at intermediate times. Both sets of state reductions pertain to the same cascade. In a cascade history, the system can be simultaneously reduced to a position eigenstate and a momentum eigenstate. (This is not to say that the state of a system, as defined by its preparation, is ever simultaneously an eigenstate of position and an eigenstate of momentum. The characterization of a state reduction necessarily depends on the future of the system as well as its past.)
Simple laboratory experiments provide examples of physically equivalent beginning-to-end transitions. Consider a single free particle of mass $m$ which is initially measured to be at event $(x_1, t_1)$. Suppose a later measurement at time $t_2$ finds the particle at position $x_2$. This transition admits a classical description given by the straight worldline joining the events $(x_1, t_1)$ and $(x_2, t_2)$. Some other transition would occur if the particle were subject to a final measurement of its momentum instead of its position. However, the same classical description would apply if the particle’s momentum were found to have the value $m \frac{x_2 - x_1}{t_2 - t_1}$, and in that case both transitions have the same probability. Similarly, in a scattering experiment, one may choose to measure either outgoing particle positions or outgoing particle momenta. The set of allowed transitions depends on that choice, but the set of stationary-phase paths may not. Each transition of non-negligible amplitude in one set can determine classical particle worldlines which identify it with a physically equivalent transition in the other set. As in the toy cosmology, each accessible final classical phase space trajectory identifies multiple beginning-to-end transitions which are physically equivalent.

5 Quantum history cascades and Schroedinger’s cat

It is here conjectured that realistic cosmology is in some ways similar to the toy cosmology. In the presence of long-range interactions, Feynman paths are not simply particle networks but also include bound states and classical fields. A cascade may still be defined as a class of stationary-phase Feynman paths which share a common asymptotic trajectory in the classical phase space. The conjecture is that the big bang initial state has special properties in common with the single massive particle initial state in the toy cosmology. In particular, there may be a class of “physically reasonable” final bases, each of which has a subset which is in almost 1-to-1 correspondence with the set of cascades. The set of cascades then inherits a probability distribution from its correspondence with any of those bases. One may picture the universe as a cascade sampled from that probability distribution.

This picture is not intrinsic to the formalism of quantum theory, however. In either the toy cosmology or realistic cosmology, there must be an operative constraint which is responsible for the universe having a semi-classical description. The formalism of quantum theory allows any Hilbert space state to be the final state, so it does not exclude grotesque beginning-to-end transitions which defy any semi-classical description. The transition to a generic final state does not correspond to a cascade. Indeed, if $\Psi_1$ and $\Psi_2$ are physically reasonable final states which separately do correspond to different cascades, then any non-trivial superposition of them would define a beginning-to-end transition whose amplitude would be calculated using paths with both asymptotic classical trajectories. The beginning-to-end transition to a non-trivial superposition of $\Psi_1$ and $\Psi_2$ would not correspond to a single asymptotic trajectory, and so it would not be a cascade. In this sense, a non-trivial superposition of cascades cannot itself be a cascade. A generic beginning-to-end transition does not admit a unique semi-classical description even
asymptotically, but is a grotesque superposition of physically interpretable histories.

A simple hypothesis for the operative constraint is that the universe is sampled from a probability distribution over the set of cascades. This might seem to be inappropriate as a quantum condition. After all, it specifies that the end of the universe corresponds to a classical state. Moreover, it cannot be formulated simply as a final boundary condition because each cascade depends also on the initial state. However, in conjunction with a particular special initial condition (like the single massive particle in the toy cosmology or the big bang initial state in realistic cosmology), this hypothesis can be equivalent to a class of reasonable final quantum constraints. In the toy cosmology, for example, the final quantum constraint could be that the final state must be a momentum eigenstate. Alternatively, the constraint might require the final state to be a position eigenstate, or a basis state of some other physically reasonable basis. Those different constraints have equivalent effects. In particular, transitions to the allowed and accessible final states correspond to cascades.

It is therefore plausible to maintain that a final constraint requires the final state to be sampled from a special set of basis states, exactly like the effect of a final measurement in laboratory physics. An allowed quantum history is simply a beginning-to-end transition from the fixed initial state to one of those final basis states. Due to the remarkable nature of the initial state, however, sampling an allowed transition may correspond to sampling a cascade with its unique classical description at asymptotically late times. Moreover, many different final bases may lead to the same probability distribution over cascades. Any of those final constraints, in conjunction with the highly special initial state, results in the universe being sampled from a probability distribution over the various cascades.

Consider once again cosmology in which a Schroedinger cat experiment occurs. An allowed history is now assumed to correspond to a cascade. It would be remarkable in the extreme for a stationary-phase (semi-classical) path which describes a dead cat at time $t$ to be asymptotically identical to another stationary-phase path which describes a live cat at time $t$. Unless circumstances are cleverly contrived, macroscopically disparate semi-classical paths do not become identical in the asymptotic future. (In the toy cosmology, an asymptotic classical phase space trajectory is generally consistent with at most one particle network starting from the special initial state. It would be exceedingly rare to find two macroscopically distinguishable networks – i.e. distinguished by a set of more than $10^{23}$ interacting particle worldlines that differ in the two networks – which start from the special initial state and are identical in the asymptotic future.) Each cascade has an asymptotic classical description. The superposition of two such cascades would not itself be a cascade. A live-cat path and a dead-cat path do not belong to any single cascade unless they miraculously become identical in the asymptotic future.
6 Quantum theory and spacetime

In the context of quantum gravity, the final constraint should ensure that the universe has an asymptotic spacetime geometry rather than ending in a grotesque superposition of disparate spacetimes. Without a final constraint, there is no natural explanation for the apparent spacetime structure which is the foundation of physics. Spacetime structure at intermediate times may be regarded as due to state reductions which are implicit in a quantum history which starts from the big bang and has a classical geometry in the asymptotic future.

A quantum history need not imply a unique classical spacetime, however. A particular history may be compatible with multiple intermediate spacetime geometries. For example, one can consider the diffraction of a Planck-mass particle by a double-slit diffractor and the particle's detection on a subsequent surface. A Planck-mass particle has a wavelength comparable to its Schwarzschild radius, so wavelike behavior can be expected along with effects of spacetime curvature. If no measurement is made of which path the particle follows, an interference of the possibilities must occur. The initial and final states are compatible with either classical trajectory through the diffractor. The interfering spacetime geometries are disparate, with the spacetime curvature (and corresponding stress-energy) concentrated along one trajectory in one spacetime and along a distinguishable trajectory in the other spacetime. There is not a unique implicit spacetime geometry if there is no record in the final state as to which intermediate geometry occurred. It is inappropriate to use an evolving state vector’s expectation value of stress-energy in an effort to salvage a unique intermediate spacetime. There are disparate intermediate spacetimes contributing to the single quantum history.

7 The Aharonov-Bohm effect

The present approach regards an entire quantum history as a single entity. The initial conditions together with a final constraint determine the set of allowed quantum histories. Only one history occurs. It is not necessary that a quantum history be interpretable as a future-blind evolution from its initial state. This has an important bearing on the Aharonov-Bohm effect as well as the EPR phenomenon.

The Aharonov-Bohm effect concerns an electron which has two possible classical worldlines from event E to event F. The two possible paths interfere, and the interference phase difference depends quantitatively on an imposed electromagnetic field. The two classical worldlines constitute a closed curve in spacetime. The presence of an electromagnetic field contributes a phase difference proportional to the electromagnetic field (2-form) integrated over any 2-surface having the closed worldline curve as its boundary. Since an electromagnetic field is a closed 2-form, that integral is independent of the choice of spanning surface.

What can seem surprising is that the electromagnetic field makes this contribution to the phase difference even if it is identically zero everywhere near the two possible worldlines. Ar-
guing that the electron interference should not be affected by any physical fields which vanish everywhere near the possible paths, Aharonov and Bohm ascribed physical reality to the electromagnetic potential (or at least to the equivalence class generated by adding exact differentials). Using Stokes’ theorem, the surface integral can be re-expressed as a line integral of a potential 1-form over the boundary curve. If the surface integral is non-zero (so the electromagnetic contribution to the phase difference is non-zero), then the potential 1-form cannot vanish everywhere on the electron paths.

In the present approach, the motivation for ascribing physical reality to the potential 1-form is absent. The entire quantum history exists as a single entity. The probability of a given history may depend on the phase difference at event F, and that is determined by the electromagnetic field in the causal future of E and in the causal past of F. It is not necessary that the phase difference be determined exclusively by fields in the neighborhood of the electron paths. There is no reason to prefer the line integral calculation over the surface integral. The phase difference (and hence probabilities for various allowed quantum histories) can be determined by the electromagnetic field without reference to potentials.

8 Discussion

EPR experiments do not pose any problem of interpretation if one imagines that an entire quantum history is selected from a set of allowed histories each of which satisfies all conservation laws. Unfortunately, an initial state, by itself, does not define a suitable set of quantum histories. In laboratory experiments, the set of allowed quantum histories is determined by the initial state together with a final constraint which requires the final state to be sampled from the eigenstates of the final measurement operator. The proposal here is to regard the quantum universe to be subject to an analogous final constraint, even though there is no external apparatus to perform a final measurement. The nature of the final constraint has been the subject of speculation in sections 3-5.

This approach to quantum theory avoids interpretation problems without altering the theory itself. In particular, it is not necessary to introduce a mechanism like von Neumann’s R-process 2 to account for state reductions. There is no modification of the formalism, so there can be no conflict with any experimental result.

The point of view advocated here seems to be shared, in part, by other authors. Griffiths 8, Gell-Mann and Hartle 13, and Omnes 14 have emphasized the use of quantum histories (although they ascribe a different meaning to the term “quantum history”). Schulman 15, Costa de Beauregard 16, Cramer 17, and others have recognized the relevance of future conditions for quantum systems. Future conditions are incompatible with the strong principle of causality. As argued in section 3, however, the type of future constraint suggested here does not lead to faster-than-light signal transmission. Moreover, many authors have concluded that the strong causality principle is incompatible with quantum theory. Even Bohr 18 emphasized that
quantum theory “entails the necessity of a final renunciation of the classical ideal of causality.”

Why there should be a final constraint on the universe may be a metaphysical question. Without such a boundary condition, however, quantum theory poses formidable interpretation problems. The universe is a quantum history with a semi-classical description. Should that be regarded as a fluke, or due to some additional structure which must be incorporated in quantum theory itself, or simply the consequence of an operative boundary condition? Occam’s razor favors the boundary condition. A final constraint can be invoked to explain why quantum states get reduced to physically interpretable states of Hilbert space. It puts the quantum into quantum theory.

The picture presented here of the final state being sampled from a probability distribution over a set of allowed final states can only be a conjecture. It could be, for example, that boundary conditions on the universe include both its definite initial state $\Psi_0$ and also its definite final state $\Psi_f$. Even if one could survey the universe from start to finish, it should not be possible to deduce whether the actual final state is sampled from a probability distribution or is dictated uniquely as a boundary condition. A dictated final state might seem like boring physics since the quantum history would then be fully specified by the imposed boundary conditions. That would be a return to the classical situation where (initial) boundary conditions determine the entire history uniquely. The approach proposed here allows the usual quantum indeterminacy in deriving the actual quantum history from initial conditions. Unless it becomes apparent that the universe is developing toward some highly special final state, there is no compelling reason to suppose that a boundary condition dictates a unique final state. There would be no elegance in some undistinguished final state being imposed as the only allowed final state. Highly specific initial conditions apparently do pertain to the universe. There is no other natural way to account for the low entropy at early cosmological times. The inference of a final constraint is similar. No other natural explanation has emerged to account for the universe having a reasonable semi-classical description rather than a grotesque evolution.

Although the entire quantum history is here presumed to have been selected as a single entity with a unique future, the future is unpredictable because there are an infinite number of allowed histories which include the universe’s sequence of state reductions up to the present time. It is not possible to identify which of the allowed histories is the unique actual complete history. A twin pair of EPR particles may now be in spin eigenstates corresponding to a reference axis to be chosen tomorrow. Their present spin eigenstates are influenced by the future measurements. Those spin states cannot be used to predict the future measurement axis, however, since a spin measurement on either of them today would negate the influence of tomorrow’s measurements.

Essential to this approach is the inference, based on consideration of the EPR phenomenon, that the entire quantum history (through the end of time) has been selected as a single entity. Though the future is not predictable, there is a unique future as well as a unique past. God does not continue to shoot dice. The dice were rolled once to select the entire quantum universe.
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