How Should IRSs Scale to Harden Multi-Antenna Channels?

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Abstract—This work extends the concept of channel hardening to multi-antenna systems that are aided by intelligent reflecting surfaces (IRSs). For fading links between a multi-antenna transmitter and a single-antenna receiver, we derive an accurate approximation for the distribution of the input-output mutual information when the number of reflecting elements grows large. The asymptotic results demonstrate that by increasing the number of elements on the IRS, the end-to-end channel hardens as long as the physical dimensions of the IRS grow as well. The growth rate however need not to be of a specific order and can be significantly sub-linear. The validity of the analytical result is confirmed by numerical experiments.

Index Terms—Intelligent reflecting surfaces, channel hardening, large-system analysis.

I. INTRODUCTION

Channel hardening is a fundamental large-scale property of multiple-input multiple-output (MIMO) systems. This property indicates that key performance metrics1 of a MIMO channel become deterministic, as the dimensions of the MIMO channel grow large on at least one side [1, 2]. From the information-theoretic point of view, this is the key property that leads to significant performance gains of massive MIMO systems [3].

This study aims to extend analytically the notion of channel hardening to multi-antenna systems that are enhanced by intelligent reflecting surfaces (IRSs); see [4]–[11] and the references therein for discussions on IRS-aided MIMO systems and their applications. Here, we address this question: How do IRS-aided MIMO systems with finite transmit and receive array sizes behave as the number of IRS elements grows large?

A. Related Work and Main Contributions

Several studies address the above question considering some simplified models. In [12], channel hardening is discussed in the context of an IRS-aided setting with a single-antenna transmitter and receiver. Extensions to scenarios with multiple IRSs and other fading models are given in [13, 14]. The studies in [15] and [16] further investigate channel hardening for IRS-aided non-orthogonal multiple access (NOMA) systems and a fully stand-alone IRS-based transmitter, respectively.

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1For instance, the channel capacity. Such metrics are in general random, due to the randomness of the fading process.
Let \( x_m \) denote the symbol sent by the \( m \)-th element of the transmit array satisfying the transmit power constraint

\[
\sum_{m=1}^{M} E \left[ |x_m|^2 \right] \leq \rho
\]

for transmit power \( \rho \). The received signal is then given by

\[
y = \sum_{m=1}^{M} h_{d,m} x_m + \sum_{m=1}^{M} \sum_{n=1}^{N} e^{-j2\pi n} h_{r,n} t_{nm} x_m + z,
\]

where

- \( z \) is zero-mean and unit-variance additive white Gaussian noise (AWGN),
- \( h_{d,m} \) is the direct channel coefficient between element \( m \) at the transmit array and the receiver,
- \( t_{nm} \) is the channel coefficient between the \( m \)-th element at the transmit array and the \( n \)-th reflecting element,
- \( h_{r,n} \) is the channel coefficient between the \( n \)-th reflecting element and the receiver, and
- \( \beta_n \) is the phase-shift applied by reflecting element \( n \).

The channel state information is assumed to be known at both sides of the channel.

### A. Channel Model

The transmit array and IRS are assumed to be rectangular uniform planar arrays with \( M_x \) and \( N_x \) horizontal and \( M_y \) and \( N_y \) vertical isotropic elements, respectively, i.e., \( M = M_x M_y \) and \( N = N_x N_y \). Each pair of neighboring transmit antennas are distanced with \( \ell_x \) and \( \ell_y \) on the horizontal and vertical axes, respectively. The horizontal and vertical distances between two neighboring reflecting elements are further denoted by \( d_x \) and \( d_y \), respectively. We assume that \( \ell_x, \ell_y, d_x \) and \( d_y \) are smaller than a half wave-length.

We consider a classical scenario in which the line of sight (LoS) link in the direct path is blocked. The direct path between antenna \( m \) at the transmitter and the receiver is hence modeled by a standard Rayleigh fading process, i.e., \( h_{d,m} = \sqrt{\alpha_d A_M} \), where \( \alpha_d \) models the effective path-loss, \( A_M \) denotes the area of a single element on the transmit array, i.e., \( A_M = \ell_x \ell_y \), and \( h_{d,m} \) is zero-mean and unit-variance complex Gaussian, i.e., \( h_{d,m} \sim CN(0,1) \). In the sequel, we compactly denote the direct channel vector as \( h_d = [h_{d,1}, \cdots, h_{d,M}]^T \).

The IRS is often deployed flexibly in the network. We hence assume that the IRS is located at a moderate distance in the transmitter sight, such that the communication link between the transmitter and the IRS is dominated by a LoS component. As a result, we represent the channel from the transmitter to the IRS by \( T \in \mathbb{C}^{N \times M} \), where \( t_{nm} = [T]_{nm} = \sqrt{\alpha_s A_N} \). Here, \( \alpha_s \) is the path-loss, \( A_N \) is the area of a reflecting element, i.e., \( A_N = d_x d_y \), and \( t_{nm} \) denotes the LoS component.

**Remark 1.** In general, the effective area of a single element depends on the wave-length. The considered simple model for \( A_N \) and \( A_M \) however follows from the fact that we assume

\[ \text{the neighboring elements on the transmit array and the IRS to be distanced less than a half wave-length.} \]

In practice, the receiver is in a relatively large distance from both transmitter and IRS; however, it can yet be in the sight of the IRS. We hence assume that the link between the IRS and the receiver has both LoS and non-line of sight (NLoS) components. This means that the coefficient of the channel between the \( n \)-th reflecting element and the receiver is modeled as

\[
h_{r,n} = \sqrt{\alpha_r A_N} \left( \sqrt{\frac{\kappa_r}{\kappa_r + 1}} \hat{h}_{r,n} + \sqrt{\frac{1}{\kappa_r + 1}} \tilde{h}_{r,n} \right),
\]

where \( \alpha_r \) and \( \kappa_r \) are the path-loss and the Rician factor, respectively. The coefficient \( \hat{h}_{r,n} \) further denotes the LoS, and \( \tilde{h}_{r,n} \) models the small-scale fading process in the NLoS link.

In IRS-aided systems, the IRS is typically considered to be filled by a large number of reflecting elements. Consequently, the distance between neighboring elements is rather small, and hence the spatial correlation among the IRS elements cannot be ignored. To capture the spatial correlation, we assume that \( \hat{h}_n = [\hat{h}_{r,1}, \cdots, \hat{h}_{r,N}]^T \) is a zero-mean complex Gaussian process with the covariance matrix \( \mathbf{R} \in \mathbb{C}^{N \times N} \). Note that due to power normalization, we have \( \mathbf{R}_{nn} = 1 \) for \( n \in [N] \).

The LoS components in the channel model can be written in terms of the array responses. Let \( \lambda \) be the wavelength and

\[
i_M(m) = (m-1) \mod M, \quad j_M(m) = \left\lfloor \frac{m-1}{M} \right\rfloor \]

\[
i_N(n) = (n-1) \mod N, \quad j_N(n) = \left\lfloor \frac{n-1}{N} \right\rfloor \]

where \( x \mod L \) determines \( x \) modulo \( L \). We further let the exponent functions at azimuth angle \( \varphi \) and elevation angle \( \theta \) for transmit element \( m \) and reflecting element \( n \) be

\[
\Phi_m(\varphi, \theta) = i_M(m) \ell_x \cos \theta \sin \varphi + j_M(m) \ell_y \sin \theta,
\]

\[
\Pi_n(\varphi, \theta) = i_N(n) d_x \cos \theta \sin \varphi + j_N(n) d_y \sin \theta,
\]

respectively. Denoting the wavelength by \( \lambda \), the transmit and IRS array responses are given at \( (\varphi, \theta) \) respectively by

\[
a_M(\varphi, \theta) = \left[ e^{\frac{2\pi j}{\lambda} \Phi_m(\varphi, \theta)}, \cdots, e^{\frac{2\pi j}{\lambda} \Phi_M(\varphi, \theta)} \right]^T,
\]

\[
a_N(\varphi, \theta) = \left[ e^{\frac{2\pi j}{\lambda} \Pi_1(\varphi, \theta)}, \cdots, e^{\frac{2\pi j}{\lambda} \Pi_N(\varphi, \theta)} \right]^T.
\]

Given the array responses, the LoS components are given by

\[
T = \sqrt{\alpha_s A_N} a_N(\varphi_{t1}, \theta_{t1}) a_M(\varphi_{t2}, \theta_{t2})^H,
\]

and \( \tilde{h}_s = [\hat{h}_{r,1}, \cdots, \hat{h}_{r,N}]^T = a_N(\varphi_{t1}, \theta_{t1}) \). Here, \( (\varphi_{t1}, \theta_{t1}) \) is the angle-of-arrival (AoA) at the IRS, \( (\varphi_{t2}, \theta_{t2}) \) denotes the angle-of-departure (AoD) from the IRS, and \( (\varphi_{t2}, \theta_{t2}) \) is the AoD from the transmitter.
B. Channel Capacity

Consider the end-to-end channel $h = [h_1, \ldots, h_M]^T$, where
\begin{equation}
    h_m = h_{d,m} + \sum_{n=1}^{N} e^{-\beta_n} h_{r,n} t_{nm}.
\end{equation}
For a given realization of $h$, the channel capacity is achieved by maximum ratio transmission and is given by [21, 22]
\begin{equation}
    C = \log_2 \left(1 + \rho \|h\|^2\right).
\end{equation}

Due to fading, the channel capacity expression is a random process whose statistics determine various performance metrics, e.g., ergodic capacity and outage probability. The main goal of this study is to characterize the statistics of $C$, when $N$ is asymptotically large.

III. LARGE-SYSTEM ANALYSIS

The basic limiting scenario is to consider an extreme case in which the number of reflecting elements grows large while the distances between neighboring elements are kept fixed. This is however an unrealistic assumption, as the IRS area in this case scales linearly with the number of reflecting elements. In practice, the IRS area is restricted, and hence by growing the number of IRS elements, the reflecting elements are distanced closer on the surface. This leads to a smaller effective area for IRS elements and therewith to higher spatial correlation [23].

To take the above scaling fact into account, we consider a basic scaling model for the IRS area. Namely, we assume that the area of each IRS element scales as $A_N = A_0 N^{-q}$ for some constant $A_0$ and $0 \leq q \leq 1$. This means that the total area of the IRS scales as $A_{IRS} = A_0 N^{1-q}$. This scaling addresses the limiting scenarios between the two extreme cases:

- $q = 0$ is the idealistic case in which the distances between neighboring elements are kept fixed, i.e., the area of each IRS element is fixed.
- $q = 1$ addresses the case in which the total area of the IRS is fixed. In this case, the area of each reflecting element shrinks reverse-linearly in $N$.

Remark 2. Note that for $q \neq 1$, the above scaling implies that by sending $N \rightarrow \infty$, the area of IRS also grows asymptotically large. One may thus conclude that the far-field model for the IRS array response is no longer valid in the large-system limit. To avoid such inconsistency, we assume that the distances among the terminals, i.e., transmitter, receiver and IRS, are bounded uniformly from below by $D_0 N^{\gamma/2}$ for some $D_0$ and $\gamma > 1 - q$. This assumption guarantees the validity of the far-field model through the asymptotic analyses. More details in this respect can be followed in [23, 24].

Proposition 1 gives the large-system statistics of the capacity term for an arbitrary sub-linear scaling of the IRS surface.

Proposition 1. Let the area of the IRS scale sub-linearly with $N$, i.e., $A_{IRS} = A_0 N^{1-q}$ for some fixed $A_0$ and $0 \leq q < 1$. Let the phase-shifts be set to
\begin{equation}
    \beta_n = \frac{2\pi}{\lambda} (\Pi_n (\varphi_1, \theta_1) + \Pi_n (\varphi_1, \theta_1)).
\end{equation}
Assume the maximum eigenvalue of the IRS covariance matrix $R$, denoted by $\lambda_{max}$, grows with $N$ sub-linearly, i.e.,
\begin{equation}
    \lim_{N \rightarrow \infty} \lambda_{max} N^{-1} = 0,
\end{equation}
and satisfies
\begin{equation}
    \lim_{N \rightarrow \infty} (\lambda_{max} A_{IRS})^{-1} = 0.
\end{equation}
Then, for large $N$, $C$ is well-approximated by a real Gaussian random variable whose mean and standard deviation are
\begin{align}
    \mu_C &= \log_2 (1 + \rho M \mu), \\
    \sigma_C &= \frac{\rho M \log_2 e}{1 + \rho M \mu} \sqrt{\omega \eta + \eta + \frac{M - 1}{M} \alpha_d A_M},
\end{align}
respectively, for
\begin{align}
    \mu &= \alpha_d A_M + \kappa_t \bar{\alpha}_N N^2 + \bar{\alpha}_N \bar{h}_t^M \bar{R}_t, \\
    \eta &= \frac{\alpha_d A_M}{M} + \bar{\alpha}_N \bar{h}_t^M \bar{R}_t, \\
    \omega &= 2\kappa_t \bar{\alpha}_N N^2 + \bar{\alpha}_N \bar{h}_t^M \bar{R}_t,
\end{align}
and $\bar{\alpha}_N$ being defined as $\bar{\alpha}_N = \alpha_t \alpha_a A_N^2 / (1 + \kappa_t)$.

Proof. The proof is given in three steps: First, the distribution of the end-to-end signal-to-noise ratio (SNR) is derived. The mean and variance are then bounded using bounds on the Rayleigh quotient of $R$. By sending $N \rightarrow \infty$, the large-system approximation is derived. Due to page limit, we skip the details and refer the reader to the extended version [25].

Proposition 1 illustrates how IRS-aided multi-antenna channels harden. It also specifies the required scaling order of the IRS to guarantee channel hardening. It is further worth noting few remarks regarding the constraints on $\lambda_{max}$:

1) Proposition 1 assumes a sub-linearly growing $\lambda_{max}$. This can be interpreted as follows: $\lambda_{max}$ is uniformly bounded from above as $\lambda_{max} \leq \alpha N^u$ for some real $\alpha$ and $0 \leq u < 1$. This is not a strong constraint, as $\lambda_{max} \leq \tr \{R\} = N$.

2) In general, the growth order of $\lambda_{max}$ is mutually coupled with $q$; see Section III-C. This is seen by considering the two extreme cases of $q$. For $q = 0$, $R = I_N$ is feasible, and hence $\lambda_{max} = 1$, i.e., the uniform upper-bound is valid for $\alpha = 1$ and $u = 0$. As $q \rightarrow 1$, $R$ tends to a rank-one matrix, and hence $\lambda_{max} = N$, i.e., $\alpha = u = 1$ in the upper-bound of $\lambda_{max}$. For less rank-deficient covariance matrices, the bound is given for some $u \in [0, 1]$.

3) Considering the scaling of the IRS area and $\lambda_{max}$, i.e., $q$ and $u$, the constraint in (12) restricts $u$ in the upper-bound of $\lambda_{max}$ to satisfy $u \geq q$. In the extended version of the work [25, Section III-C], it is further shown that this is the case for an IRS covariance matrix which is derived for the standard Rayleigh fading model in [12].
IV. ASYMPTOTIC CHANNEL HARDENING

The classical channel hardening result, i.e., the initial study in [1], indicates that $C$ converges to a deterministic variable, as the transmit array size $M$ grows unboundedly large. We are however interested in a different asymptotic regime; namely, a scenario with finite transmit antennas but an unboundedly large number of reflecting elements, i.e., fixed $M$ and $N \to \infty$.

Corollary 1. Let the IRS area scale as $A_{\text{IRS}} = A_0 N^{1-q}$ for some fixed $A_0$. Assume that the phase-shifts are set to (10), and let $\lambda_{\text{max}}$ be uniformly bounded from above as $\lambda_{\text{max}} \leq a N^w$, for some $0 \leq q < u < 1$. As $N \to \infty$, the mean of $C$ grows large and its variance converges to zero.

Proof. The proof follows from [25, Proposition 2], where it is shown that under the given constraints, there exist real scalars $b$ and $c$, such that $\mu_C \geq b + (1-q) \log_2 N$ and $\sigma_C^2 \leq c N^{u-1}$. Considering $0 \leq q < u < 1$, we can conclude that $\mu_C$ grows large, and $\sigma_C^2$ converges to zero as $N \to \infty$.

In the extended version [25, Section III-C], it is shown for the Rayleigh fading model that the constraint $0 \leq q < u \leq 1$ is valid with $u = 1$ if $q = 1$. Noting that $q = 1$ represents the case with fixed IRS area, Corollary 1 indicates that the IRS-aided channel hardens, if the physical dimensions of the IRS grows large with $N$. The growth is however sufficient to be sub-linear. From an implementational viewpoint, it is a valid constraint. In fact, due to the restricted physical dimensions of each reflecting element, the distance between two neighboring elements on the surface cannot be set below a certain limit, and hence the overall area of IRS always increases in $N$.

V. NUMERICAL EXPERIMENTS

We now confirm the accuracy of the derivations for practical system dimensions through numerical experiments. We consider a basic scenario in which the transmitter is equipped with a $2 \times 2$ planar array and the IRS contains $N = 256$ reflecting elements. The elements on the IRS are assumed to be aligned on a rectangle with $N_x = 8$ horizontal elements and $N_y = 32$ vertical elements. The elements at both transmitter and receiver are distanced with $\ell_x = d_x = \ell_y = d_y = \lambda/2$, where $\lambda$ denotes the wave-length. We further set $\alpha_{\text{d}} A_M = \alpha_{\text{r}} A_N = \alpha_{\text{d}} A_N = 1$ and $\log \kappa_T = 0$ dB. To generate the covariance matrix, we invoke [12, Proposition 1] and set entry $(n, n')$ of $\mathbf{R}$ to be

$$ [\mathbf{R}]_{nn'} = \text{sinc} \left( \frac{2}{\lambda} \sqrt{\Delta_x^2 + \Delta_y^2} \right) \quad (15) $$

with $\Delta_x = iN_y (n) - iN_y (n')$ and $\Delta_y = jN_x (n) - jN_x (n')$. The AoA and AoDs are further set to $(\varphi_{11}, \theta_{11}) = (\pi/6, \pi/3)$, $(\varphi_{12}, \theta_{12}) = (\pi/8, 2\pi/3)$ and $(\varphi_{21}, \theta_{21}) = (\pi/7, \pi/5)$. The power constraint is set to $\rho = 1$.

We collect $10^6$ realizations of the channel and determine $C$ for each realization. The empirical density is then determined from the collected data and compared with Proposition [1] in Fig. 1. As the figure shows, the analytical result of Proposition [1] almost perfectly matches the empirical density.

As the next experiment, we replace the rectangular IRS in the above setting with a square array, i.e., $N_x = N_y$ and let $N$ grow gradually from $N = 64$ to $N = 1296$ assuming that the distance between each two neighboring elements remains $\lambda/2$. The variance of $C$ is then plotted against $N$ using both numerical data and asymptotic expression in Proposition [1]. As Fig. 2 demonstrates, the analytical results closely approximate numerical simulations, even for rather small choices of $N$. The figure further depicts the drop of $\sigma_C^2$ against $N$ indicating the hardening of the end-to-end channel.

Fig. 2 further compares $\sigma_C^2$ to the analytical upper-bound on the variance of $C$ derived in the extended version of the work [25, Proposition 2]. Interestingly, the suggested upper bound gives a pessimistic approximation of the hardening speed. In fact, the true variance drops much faster than the upper bound. This observation suggests that the end-to-end channel hardens rather fast, even with strongly correlated reflecting elements. The consistency of this conjecture is demonstrated via several numerical investigations in the extended version [25]. Further discussions on this respect are skipped due to page limitation. The interested reader is referred to the extended version [25].

VI. CONCLUSIONS

By increasing the number of reflecting elements, IRS-aided channels harden as long as the physical dimensions of the IRS grow as well. However, the growth order can be significantly sub-linear. This is realistic from the implementational viewpoint, as neighboring elements on an IRS cannot be set closer than a threshold distance, due to their physical dimensions.

The above result indicates that in IRS-aided systems, even by packing the reflecting elements compactly on the IRS, the end-to-end channel hardens as the number of elements grows.
large. This finding shows that by enhancing a multi-antenna system of feasible dimensions via large IRSs, the large-scale properties of a multi-antenna system can be obtained.

The results of this study can be used to investigate IRS-aided multi-antenna systems in various respects. Some examples are addressed in the extended version of this work [25].

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