Regular and irregular dynamics of spin polarized wavepackets in a mesoscopic quantum dot at the edge of topological insulator

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The dynamics of spin-polarized wavepackets driven by periodic electric field is considered for the electrons in a mesoscopic quantum dot formed at the edge of two-dimensional HgTe/CdTe topological insulator representing a new class of materials with Weyl massless energy spectra, where the motion of carriers is less sensitive to disorder and impurity potentials. It is found that the interplay of strongly coupled spin and charge degrees of freedom creates the regimes of both regular and irregular dynamics with certain universal properties manifested for both free and driven evolution, in the clean limit and in the presence of the moderate disorder. The weak disorder influence is predicted to be overcome by periodic driving while the moderate disorder induces the in-plane spin relaxation, leading to possibility of establishing novel types of driven evolution in nanostructures formed in the topological insulators. The dynamical properties of regular and chaotic behavior of charge and spin in these structures may be of interest for future progress in both quantum nonlinear dynamics on the nanoscale and in the applied nanoscience such as spintronics and nanoelectronics.

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I. INTRODUCTION

One of the most fascinating examples of the coexistence of regular and chaotic dynamics is the manifestation of irregular or chaotic regimes of evolution in quantum systems, where the concepts of quantum stochasticity have been introduced and developed for many years.1–5 The concepts of quantum chaos have been also successfully applied on nanoscale for various condensed matter structures including quantum dots.6 The connections between classical and quantum chaotic systems have been established in such properties like the structure of the energy spectra where the powerful tools of random matrix theory have been applied.4, 5 Also, the concepts of quantum diffusion in the Hilbert space have been introduced where the analogies between the diffusion along the resonance eigenstates and along the separatrices in corresponding classical system have been found.3, 4 Including the analogue of Arnol’d diffusion in quantum systems subject to periodic driving.6 Much less is known for chaos development in quantum systems without a directly observable classical counterpart, such as systems where the spin degrees of freedom play a significant role and which are described in essentially quantum mechanical way like the systems with spin-orbit coupling (SOC). The importance of such systems in both fundamental and applied fields has been recognized during the last decade, and a significant progress can be observed in corresponding field of physics called spintronics.3–10

The knowledge of mutual effect of charge and spin degrees of freedom on the transport and dynamics is crucial for progress in both fundamental aspects of quantum physics and the applications in dynamics of complex systems and in the development of new generations of nanoelectronic devices. While the static properties of energy spectra, eigenfunctions, optical and transport parameters of such systems are known in detail for many cases including the systems with signs of quantum chaos in their spectra5, 6 the dynamics or evolution of spin-resolved states such as spin-polarized electrons or their representation by wavepackets, is known with much less degree of information. It is known that, for example, the combined effects of SOC and the resonance in a multi-level system subject to strong driving may lead to unusual nonlinear behavior in well-known regimes such as classical dynamics of the electron in a double quantum dot11, or the Rabi frequency dependence on the driving strength in the electric dipole spin resonance in a double quantum dot.12 It was shown also that in double quantum dot with SOC other interesting regimes can develop such as phase synchronization or chaotic spin-dependent dynamics.12 Another example can be found in a 2D mesoscopic semiconductor quantum dot with SOC where the non-Poissonian level statistics indicates the presence of quantum chaos.14 In our recent paper12 we have found the development of strongly irregular dynamics in this system under the periodic driving by the electric field, which manifested itself in both charge and spin channels.

The major problem of establishing the quantum-classical correspondence in such spin-dependent systems is the mentioned absence of a direct classical counterpart which create obstacles on describing such systems in terms of the classical chaotic dynamics. The primary tools for overcoming such difficulties are the Floquet analysis for periodic driving3, 4, 7 and the analysis of transport properties analysis reflecting the regular or chaotic structure of energy spectrum and eigenstates.3–17 In the Floquet analysis one may look on the degree of delocalization of the Floquet eigen-

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states in the Hilbert space of the basic functions or the quasienergy level statistics clearly indicating the possibility of diffusion and chaos development,\textsuperscript{2, 16} and on the the direct Fourier analysis of the observables, or quantum mean values.\textsuperscript{2, 16} It is known that periodically driven systems can provide various examples of non-trivial dynamics including the direct transport in certain cases of space-time symmetry found in quantum ratchets,\textsuperscript{13} including the spin ratchets.\textsuperscript{19} Other tools include the analysis of Poincare sections built in various pairs of coordinates for both coordinate and spin degrees of freedom, not necessarily the canonically conjugated ones\textsuperscript{15, 16} or the tracking of the evolution for the variance for the energy levels involved in the dynamics where the growth of such variance indicates the development of chaotic regime, and the saturation points on the transition to the quasi-regular mode with finite number of levels participating in the evolution.\textsuperscript{2, 17}

In the present paper we address the complex driven dynamics of wavepackets in another type of nanostructures which attract considerable attention during the last years, namely, in the 1D quantum dot formed at the edge of 2D topological insulator (TI). Topological insulators are sometimes called a novel state of matter, representing the condensed matter systems where the edge states protected from backscattering by symmetry exist at the edges of the sample and have energies within the bulk gap, producing an efficient transport through edge channels, which is robust against non-magnetic impurities preserving the time-reversal symmetry.\textsuperscript{20} In particular, it is known that the formation of edge states in TI where the spin is coupled to the orbital movement is essentially dependent on the presence of strong SOC. Thus, an example of spin-dependent dynamical phenomena in TI can highlight new important manifestations of SOC on the dynamics of both charge and spin degrees of freedom.

One of the first examples of such structures were the HgTe/CdTe 2D quantum wells where the tuning of the well width may create the phase where topologically protected edge states exist.\textsuperscript{20, 21} Later numerous types of structures with the properties of TI have been proposed and studied both theoretically and experimentally.\textsuperscript{20} The applications of TI in nanoelectronic devices require the fabrication of localized small-to-medium size object like quantum dots (QD). Several models of QD formation at the edge of TI where the symmetry protected state exist have been proposed during the last years.\textsuperscript{22, 23} Most of them relevant to 1D QD on the edge of 2D TI deal with simplified assumptions of non-transparent magnetic barriers which are required to confine the electrons with massless Dirac (or Weyl) spectrum.\textsuperscript{20} Under such assumptions the spectrum of discrete energy levels inside QD forms a set of equidistant levels located in two ladders above and below the Dirac point of TI where two linear dispersion branches cross.\textsuperscript{22} For each level the corresponding eigenstate is a two-component spinor with certain polarization, which makes this system a promising candidate for studying there a driven dynamics excited by external electric field tuned to match the interlevel resonance splitting.

The investigation of wavepacket evolution representing the initially localized or injected electrons in the structures with non-trivial dispersion relations and coupled spin and charge degrees of freedom has been a subject of numerous papers throughout the last years. One can mention the investigation of the evolution of spin polarized wavepackets and their wriggling motion (“zitterbewegung”) in the electron gas with spin-orbit coupling and in the graphene.\textsuperscript{21} It was found that the form and the strength of SOC plays a significant role in the formation of spin polarization in various types of nanostructures, including the superlattices,\textsuperscript{22} TI based in silicon substrate,\textsuperscript{26, 27} and in nanowires with giant Rashba SOC.\textsuperscript{28}

The influence of SOC is also important in determining the properties of the packet free evolution, collapse and revival. Various models of energy spectra have been considered, including the hole Luttinger system in semiconductors\textsuperscript{24} and the 3D Dirac equation\textsuperscript{30, 31}, as well as the evolution of Dirac packets in the magnetic field.\textsuperscript{22, 23} Recently a free motion of wavepackets in thin films of 3D TI insulator formed in Bi\textsubscript{2}Se\textsubscript{3} type has been studied.\textsuperscript{24} It was found that the packet tends to spread along the TI area depending on its initial width and spin polarization, with its velocity demonstrating oscillating behavior in time. Also, the dynamics of wavepackets in such thin films was studied in the presence of terahertz pumping,\textsuperscript{35} and the phenomenon of zitterbewegung was also analyzed.

The effects of the disorder are also of a big interest for the investigation of spin-dependent dynamics since in realistic setups the presence of some form of the disorder is inevitable. If the electron motion is non-localized, i.e. large velocities are present, then the Dyakonov-Perel’ spin relaxation mechanism is effective.\textsuperscript{8, 10} However, if the electron motion is disturbed, for example, by random disorder potential, then the reduction in the electron effective velocities may lead to the slowdown of the spin relaxation, sometimes called a "spin freezing".\textsuperscript{30} A similar effect can be found in systems with spatially random SOC, where after the multiple scatterings in the random potential the spin relaxation becomes much slower, resulting in an algebraic rather than an exponential decay.\textsuperscript{37} On the other hand, in localized systems such as quantum dots the Dyakonov-Perel’ mechanism can be largely suppressed since the mean velocity of the confined electron is equal to zero. The presence of the disorder can still break down the free ballistic propagation of the electron, but the overall outcome of this effect on the externally driven charge and spin evolution, especially for the electrons with Weyl massless dispersion, is presently unclear.

It can be concluded from these studies that the wavepacket dynamics in TI and the TI-based structures is of interest for further research, including the controlled,
or driven evolution in confined systems such as quantum well or quantum dots formed in the TI, as well as the effects of the disorder potential present in realistic experimental conditions.

Here we derive a model of a 1D quantum dot formed at the edge of 2D TI based on the HgTe/CdTe quantum well bounded by magnetic barriers on both ends of the QD which are described by a realistic model of finite barrier height. We obtain the energy spectrum with a nearly equidistant structure and the corresponding two-component spinor eigenstates. The evolution of spin-polarized wavepackets is studied in the basis of these states both for free case and in the presence of periodic driving by the electric field, in the clean limit and in the presence of the random disorder potential. We find that certain properties of driven evolution are sustained for wavepackets of different shape and are not smeared by a moderate disorder potential. Our findings can stimulate new significant proposals in utilizing the dynamic effects in design of new spintronic and nanoelectronic devices.

This paper is organized as following. In Sec.II we introduce a model of 1D QD at the edge of 2D TI formed in HgTe/CdTe quantum well by considering a generalization of previously introduced model\(^\[22\]\) for the case of magnetic barriers with finite transparency where the electron states have nonvanishing exponentially localized tails inside the barriers. We consider a case of macroscopic QD with a length \(L = 3\) nm in order to obtain a large number of levels \((N_{\text{max}} \approx 100)\) in the TI bulk gap which is desirable to capture the quasiclassical traits of chaos development. Such assumption of a long mesoscopic QD is feasible since the experiments report rather high values of mean free paths in such structures, reaching several microns.\(^\[20\]\). In Sec.III we describe the Floquet eigenstates for the periodically driven single-particle evolution which define the presence of the diffusion in the Hilbert space indicating the possibility of chaotic dynamics. In Sec.IV we consider the evolution in the clean limit (no static disorder or noise) for the electron inside the QD, where the electron is represented via the spin polarized wavepacket of various width. We consider both free and driven evolution, and describe it in terms of phase space plots generalized also for pairs of non-conjugate spin variables), Fourier spectra, diffusion in Hilbert and coordinate space, and Lyapunov exponents. In Sec.V we add the random disorder potential representing the non-ideal character of real nanostructure as well as possible noise in the system and study the driven evolution here. Finally, in Sec.VI we present our conclusions.

II. MODEL

We firstly describe an unperturbed Hamiltonian \(H_{\text{QD}}\) for the 1D electron in a quantum dot (QD) confining the edge states in 2D HgTe/CdTe topological insulator (TI). The Hamiltonian is a generalization of the previously derived model for QD with non-transparent barriers\(^\[22\]\) for the more realistic case of the barriers with finite transparency reflected in their finite height \(M_{0,L}:\)

\[
H_{\text{QD}} = A k_y \sigma_z - M_0 \theta(-y) (\sigma_x \cos \theta_0 + \sigma_y \sin \theta_0) - M_L \theta(y-L) (\sigma_x \cos \theta_L + \sigma_y \sin \theta_L).\tag{1}
\]

Here the first term \(H_0 = A k_y \sigma_z\) is the effective Weyl Hamiltonian for massless edge states propagating on the boundaries of the HgTe/CdTe TI with \(A = 0.36\) eV nm,\(^\[21\]\) and the second and third term describe the magnetic barriers located along the TI edge at \(y = 0\) and \(y = L\) forming a confining QD potential, as it is shown schematically in Fig.I(a). The magnitudes \(M_0\) and \(M_L\) of the barriers can be viewed as exchange energies for the interaction between the electron spin and the magnetization inside the barriers. By generalizing previous model\(^\[22\]\) we consider the barriers with finite transparency by choosing finite amplitudes \(M_0\) and \(M_L\) which are taken as to cover the whole band gap of the HgTe/CdTe quantum well which is approximately 40 meV,\(^\[20\]\) and the angles \(\theta_0\) and \(\theta_L\) describe the magnetic domain orientation in the left and right barrier, respectively.

The stationary 1D Schrödinger equation \(H_{\text{QD}} \Psi = E \Psi\) for the two-component electron spinor \(\Psi = (\psi_1(y), \psi_2(y))\) is accompanied by the boundary conditions at \(y = 0\) and \(y = L\) which can be derived from its integration over infinitesimal small region near the boundary, yielding the requirements

\[
\Psi_1(0) = \Psi_r(0),\tag{3}
\]
\[
\Psi_1(L) = \Psi_r(L)\tag{4}
\]

meaning that the wavefunction has to be continuous at the boundaries between the QD and the barriers. The spatial dependence of the solution for a confined state with energy \(E < (M_0, M_L)\) inside the barriers at \(y < 0\) and \(y > 0\) has the form of decaying under-the-barrier exponents,

\[
\psi_{y<0} = B \left[ -i \frac{\sqrt{M_0^2 - E^2}}{M_0} e^{i\theta_0} \right] \exp \left( \frac{\sqrt{M_0^2 - E^2}}{A} y \right), \tag{5}
\]
\[
\psi_{y>L} = D \left[ i \frac{\sqrt{M_L^2 - E^2}}{M_L} e^{i\theta_L} \right] \exp \left( -\frac{\sqrt{M_L^2 - E^2}}{A} y \right), \tag{6}
\]

and the eigenstate inside the QD is the spinor with real wavenumber in its exponents,

\[
\psi_{\text{QD}} = \begin{bmatrix} C_1 e^{iEy/A} \\ C_2 e^{-iEy/A} \end{bmatrix}, \tag{7}
\]

where the coefficients \(B, C_1, C_2, D\) are determined from the boundary conditions \(^\[4\]\), and the energy \(E\) is found...
from the corresponding secular equation with Hamiltonian (2). This equation can be solved analytically for the case of non-transparent barriers where the wavefunction does not enter the under-the-barrier region, where a sequence of up- and down- strictly equidistant energy levels $E_n^{(0)} = \Delta E^{(0)} (n_0 + 1/2), n_0 = \pm 1, \pm 2, \ldots$ is formed with spacing

$$\Delta E^{(0)} = \frac{\pi A}{L}. \quad (8)$$

In the present paper we will consider the case of parallel orientation $\theta_0 = \theta_L = 0$ since different angles of magnetization inside the barriers define mostly the internal structure of corresponding eigenstates and their spin polarization inside the QD, but have only quantitative effects on the structure of matrix elements of the external perturbation and only minor effects on the dynamical properties which are in the focus of our studies. Besides, we choose equal amplitudes of magnetic barriers $M_0 = M_1$, which creates a QD with symmetric potential profile, although various combinations of $M_0, M_1, \theta_0$, and $\theta_L$ can be equally considered if other materials and/or experimental realizations are chosen.

In our model the spectrum cannot be found analytically, and has to be obtained from a transcendental equation, which leads in general to a non-equidistant spectrum with non-uniform level spacing $\Delta E$. However, for the mesoscopic QD with $L = 3$ mkm from where the $\Delta E^{(0)} \approx 0.38$ meV and the condition $\Delta E^{(0)} < M_{0,L}$ is satisfied meaning that there are many levels below the barriers are present ($N_{\text{max}} \approx 100$), we have the level spacing $\Delta E$ being very close to the equidistant value $\Delta E^{(0)}$ from (8).

The schematic representation of the discrete energy levels inside the QD is shown in Fig. (a) with a large interlevel distance which is shown as a guide to the eye and not to scale. Together with the discrete levels we show the linear dispersion branches of the Weyl Hamiltonian $H_0 = A k_0 \sigma_z$ describing the edge states,[20] before the confining barriers are applied, together with corresponding $z$-aligned spin mean values $S_z$ and the boundaries of the bulk energy gap $E_g = 40$ meV. This gap allows to limit the barrier width by $M_0 = M_1 = E_g/2$ since only the edge states within the bulk gap are relevant for the edge QD where they are not masked by the bulk states.

Our final task considering the model of quantum states inside the QD is the choice of the localized initial condition for the dynamical problem representing an electron which has been injected through one of the magnetic barriers into the dot. We model such condition by a Gaussian spin-polarized wavepacket with two different widths and center locations with their probability density distribution shown in Fig. (c) by the solid blue line (1) and dashed red line (2). In terms of the spatial size the packets widths are 1 and 0.1 microns, respectively, which are reasonable values for the semiconductor structures being considered where the mean free path for the electron is about 3 microns. The spin polarization for the corresponding spinor representing the initial packet is chosen to coincide with the magnetization of the magnetic barrier (or electrode) from which the packet has been injected, that is, the $S_z = 1$ polarization of the left barrier, since the majority of the electrons traveling through the magnetic materials without special tuning usually gain the polarization from the host material. The initial condition $\Psi_0(y) = \sum C_n \Psi_n(y)$ inside the QD for further treatment of its evolution, that is, the coefficients $C_n$ in the decomposition $\Psi_0 = \sum_n C_n \Psi_n(y)$ have to be found by standard methods. The structure of their absolute value distribution $|C_n|^2$ in the space of basis states is shown in Fig. (d) and (e) for two initial packets from Fig. (c), respectively. As expected, a wider packet (1) in real space is described
III. FLOQUET STATES

In this and in the following Sections we consider the perturbation for the initial Hamiltonian \(\mathcal{H}_0\) as a scalar potential \(V(y,t)\), in the form of spatially uniform electric field \(E_0\) created by electrostatic gates. The field is harmonic with frequency \(\omega_0 = (E_{n_0+1} - E_{n_0})/\hbar\) matching the level splitting in the region of mostly populated levels by the initial wavepacket, which is the medium part of the spectrum \(n_0 \approx 54\) (see Fig.1d,e), so

\[
V(y,t) = eE_0 y \cos \omega_0 t, \tag{9}
\]

where \(e\) is the elementary charge. The periodic driving allows us to apply some of the tools from the Floquet analysis\([3, 4, 5, 12, 13]\) for understanding the system evolution. The part of it which is the most relevant for our system is the structure of the Floquet states where the \(s\)-th eigenstate is written as a vector \(\bar{A}_s\) in the Hilbert space of the basis states \(\Psi_n(y)\). These vectors are the eigenvectors of the one-period propagator matrix \(U(T_0)\) where \(T_0 = 2\pi/\omega_0\), which can be constructed from the evolution of the state \(\Psi(y,t)\) in the basis of states \(\Psi_n(y)\),

\[
\Psi(y,t) = \sum_n C_n(t) \Psi_n(y), \tag{10}
\]

obeying the non-stationary Schrödinger equation

\[
\frac{i\hbar}{\partial t} \frac{\partial \Psi}{\partial y} = (H_{QD} + V(y,t)) \Psi, \tag{11}
\]

with the initial condition \(C_n(0) = \delta_{n0}\) considered sequentially for all levels \(n_0\)\([3, 4]\). The equation (11) is transformed into a system of ordinary linear differential equations for coefficients \(C_n(t)\) by projecting it on the basis of the states \(\Psi_n(y)\), and this system is solved by standard numerical packages.

The eigenvalues of \(U(T_0)\) labeled by index \((s)\) have the form \(\exp(-iE_{Q}^{(s)} T_0/\hbar)\) where \(E_{Q}^{(s)}\) are the corresponding quasienergies. It is known that the information contained in the quasienergy level spacing distribution can describe the regimes of the driven evolution as regular or chaotic, depending on whether or not such distribution demonstrates the Poissonian or non-Poissonian behavior\([3, 4]\). In Fig.2(a) we show the level spacing distribution \(\rho(\Delta E_Q)\) for three different driving strengths \(E_0 = 0.2\) V/cm (green dash-dotted curve), \(E_0 = 1\) V/cm (red solid curve), and \(E_0 = 2\) V/cm (blue dotted curve). For the weak driving the level statistics looks like the Poissonian one with the most of quasienergy levels grouped with small spacing \(\Delta E_Q\) of the order of 0.005 meV. As the driving increases, the level statistics progressively transforms to a non-Poissonian type with maximum shifted to higher values with increased driving, indicating the possible onset of chaotic regime. (b) Distribution of the Floquet quasienergy eigenstates in the \((\bar{n}, \sigma_n)\) coordinates where \(\bar{n}\) is the mean level number measuring the center of the Floquet state in the basis, and \(\sigma_n\) is the variance (width) in the Hilbert space, for driving strengths \(E_0 = 0.2\) V/cm (green stars), \(E_0 = 1\) V/cm (red filled circles), and \(E_0 = 2\) V/cm (blue open circles). The level variance \(\sigma_n\) in general increases with the driving strength, and the extended states with \(\sigma_n \approx 32\) exist at moderate and strong driving, meaning the presence of the diffusion in the Hilbert space into a substantial part of the spectrum and reflecting the possibility of irregular, or chaotic dynamics.
the order of 0.005 meV. As the driving increases, the level statistics progressively transforms to a non-Poisson type with the maximum located near 0.03 meV for $E_0 = 1$ V/cm and near 0.1 meV for $E_0 = 2$ V/cm. According to the basic concepts of quantum chaos, this result can be viewed as an indication of the transition to chaos in our system with increased amplitude of periodic driving.

Besides the quasienergy spectra, the structure of the Floquet eigenvectors $A_n^{(e)}$ can give a lot of information regarding the possibilities of chaotic regimes for the evolution under periodic driving. In particular, the presence of the states which are extended in the Hilbert space formed by basis functions, that is, described by high values of variance $\sigma_n$,

$$\sigma_n^2 = \sum_n (n - \bar{n})^2 |A_n|^2,$$

where $\bar{n} = \sum_n n |A_n|^2$, corresponds to the regimes of diffusion in the Hilbert space of the initial state along such extended Floquet states, which can be viewed as the quantum counterpart of the classical chaos development. Hence, it is of interest to look at the distribution for all of the quasienergy eigenstates in the $(\bar{n}, \sigma_n)$ coordinates where $\bar{n}$ is the mean level measuring the center of the Floquet state in the basis, and $\sigma_n$ is the variance, or width in the Hilbert space. In Fig.2(b) we plot the $(\bar{n}, \sigma_n)$ distributions for the Floquet eigenstates for three different driving strengths, $E_0 = 0.2$ V/cm (green stars), $E_0 = 1$ V/cm (red filled circles), and $E_0 = 2$ V/cm (blue open circles). It is clear that the level variance $\sigma_n$ in general increases with the driving strength which is an expected effect (although a certain saturation with growing $E_0$ is present), and the extended states with $\sigma_n \approx 32$ exist at moderate and strong driving, meaning the presence of the diffusion in the Hilbert space into a substantial part of the spectrum totaling 108 levels for the present set of model parameters. It is interesting to note that the width of the Floquet eigenstates does not grow constantly with increasing driving amplitude $E_0$, as we can see from Fig.2. The reason can be in the matrix elements of the perturbation which become too small for the states which are far away from each other in the energy spectrum, so their coupling by the electric field considered here is negligibly small. In terms of the applications it means that only moderate driving amplitudes are required for the excitation of the most interesting regimes in our system, which is desirable for the possible device applications.

We thus can conclude that the analysis of the Floquet eigenstates demonstrates the possibility of excitation of the diffusion regimes of driven dynamics if the initial states are located in the region of maximum variance $\sigma_n$ near the center of the spectrum. This situation takes place for both wide and narrow types of initial wavepackets considered in Fig.1(d,e), indicating these states as suitable candidates for modeling the possible irregular dynamics. In the next Section we will confirm this assumption by direct integration of the nonstationary Schrödinger equation with Hamiltonian $H_{qd} + V(y, t)$.

Since the spectrum in our model has a nearly equidistant character, a resonance between many pairs of levels can quickly develop which means the presence of high harmonics of the resonance frequency $\omega_0$. A substantial part of the evolution takes place between the stroboscopic moments of time $T_n = nT_0$ which are in the focus of the Floquet stroboscopic approach. To obtain more detailed picture, we proceed with direct numerical integration for continuous time with suitable time grid catching all of the essential non-vanishing higher harmonics of $\omega_0$, and having also a perfect match between the continuous and Floquet approaches at stroboscopic time points.

### IV. Evolution in the Clean Limit

#### A. Free evolution in the clean limit

We begin with the analysis of the free evolution of the wide wavepacket (see Fig.1(c),(d)) which is described by the Schrödinger equation (11) with zero driving term, $V(y, t) \equiv 0$. The initial state $C_n(0)$ occupies a narrow part of the Hilbert space near the center of the spectrum, as it can be seen in Fig.1(d). We solve the equations of motion for $C_n(t)$ from several hundreds to several thousands of periods $T_0 = \frac{2\pi}{\omega_0}$ which is the unit of time in our model, where $h\omega_0$ is the spacing between a selected pair of levels near the center of the spectrum.

The evolution parameters in which we are interested include the quantum mechanical mean values (observables) both for coordinate and spin degrees of freedom, and their Fourier power spectra

$$I_\xi(\omega) = \left| \int_{-\infty}^{+\infty} \xi(t)e^{-i\omega t} dt \right|^2,$$

where $\xi$ is the variable of interest. Since we have obviously considered large, but finite intervals of time, the Fourier power spectra were calculated by the Fast Fourier Transform with limits of time actually used in our simulations of dynamics. As to the analogue of the classical phase space dynamics, the spin-dependent system with Hamiltonian provides us with an interesting example of the "conjugate" pair of observables, the coordinate one is the usual space coordinate, $y$, while the "velocity"

$$v_y = \frac{1}{\hbar} \frac{\partial H_{qd}}{\partial k_y} = v_F \sigma_z,$$

where $v_F = A/\hbar = 5.3 \times 10^7$ cm/s is the Fermi velocity. This result means that the momentum is effectively represented on the "phase space plot" by the $z$ component of spin, so the first pair of the variables considered by us is $(y, S_z)$. The second pair of variables to be plotted
together is the in-plane spin projection represented by the \((S_x, S_y)\) spin components. This pair of variables has been considered in several studies on the spin-resolved systems and is convenient in representing, for example, the in-plane spin precession. The initial state of the wavepacket injected from the left barrier into the QD is characterized by the spin polarization in units of \(\hbar/2\) as \(S_z = 1, S_y = S_x = 0\). We have found that for all the basis states, the mean spin is always in the plane of the 2D TI, that is, \(S_z = 0\). However, if a time-dependent mixture of such states is considered formed by the initial packet or by the non-stationary driving \(V(y, t)\), the resulting spinor wavefunction may correspond to the state where the out-of-plane \(S_z\) spin component is present. We will discuss it in detail below.

The evolution of the variance of the level number \(\sigma_y(t)\) (see Eq. 12 where \(A_n \rightarrow C_n(t)\)) describes the spreading of the initial state \(C_n(0)\) in the Hilbert space of the basis states. Also, we are interested in the dynamics of the variance \(\sigma_y\) for packet width in the coordinate space,

\[
\sigma^2_y(t) = \langle (y - \langle y(t) \rangle)^2 \rangle, \tag{15}
\]

where \(\langle \ldots \rangle\) stands for standard quantum mechanical averaging with the wavefunction \(\Psi(y, t)\). It is also of interest to look at the spatial distributions along the QD for the charge density \(\rho(y, t_0)\) and some of the spin density components \(S_i(y, t_0), i = x, y, z\), at certain moments of time. The spatial profiles of charge and spin density help in understanding on which spatial scale one can measure the charge and spin spots in actual experimental setups.

All of the evolution parameters described above are calculated and plotted for various initial conditions and driving fields in the same fashion throughout this and the following Sections. The free evolution of the wide initial wavepacket (see Fig.1(c),(d)) is shown in Fig.3 In Fig.3(a) we show the evolution in the "phase space" of the \((y, S_z)\) variables representing classical coordinate and momentum, respectively, and in Fig.(b) the evolution of the in-plane spin components \((S_x, S_y)\) is plotted.

For the free dynamics we look at the evolution for total time of \(T_{\text{tot}} = 2000T\) with 30 grid points per each time domain of the length \(T\). These parameters allow to catch all of the relevant features of the evolution, including the substantial non-vanishing Fourier harmonics of \(\omega_0\). The initial point at \(t = 0\) is shown as black circle "A". One may see that both the coordinate and spin degrees of freedom show rich oscillating behavior, including the out-of-plane component \(S_z\). It can be concluded that the \(S_z\) component indeed plays the role of the particle momentum since its magnitude reaches maximum when the mean value of the coordinate \(y\) passes near the QD center \(y = L/2\), and it goes to zero when \(y\) is near the turning point. The phase trajectory itself looks complex, but without the areas of "chaotic sea" typical for the development of chaos.

(c)-(e) Fourier power spectra for the coordinate \(y\) and two spin components \(S_x\) and \(S_y\). The \(\omega_0\) harmonics are present which is the fundamental frequency of the motion. The number of essential harmonics is limited, and we cannot see the frequency bands completely filled by the Fourier components which is typical for chaotic behavior. (f) The evolution in the Hilbert space plotted as a number of levels effectively pertaining in the dynamics remains the same as at the initial moment of time \(t = 0\) where it was determined by the decomposition of the initial wavepacket. (g) The coordinate variance describing the spreading of the wavepacket in the real space inside the QD. The initial half-width (see Fig.1(c), packet 1)) does not grow with time, and at certain moments of time the packet is narrowed. In the inset a fragment of the \(\sigma_y(t)\) dependence is shown on small time scale indicating the presence of fast oscillations with frequency comparable with the driving frequency \(2\pi/T_0\). (h) Charge density \(\rho(y)\) (green dashed curve) and the \(S_z(y)\) component of spin density (blue solid curve) inside the QD taken at specific moment of time \(t = 1995T_0\). The packet is spread along the whole QD, but the major part of the charge and spin density are concentrated on the width comparable with the width of the initial packet.
spectra \([13]\) for the coordinate \(y\) and two spin components \(S_x\) and \(S_y\) which are shown in Fig.3(c)-(e), respectively. One can see the presence of \(\omega_0\) harmonics which is the fundamental frequency of the motion in our problem as it is determined by the dominating nearly equidistant structure of energy levels. The number of essential harmonics is, however, quite limited, and we cannot see the frequency bands completely filled by the Fourier components which is typical for chaotic behavior of corresponding observables.\([2, 11, 15]\) The evolution in the Hilbert space at the absence of a perturbation is trivial, as it can be seen in Fig.3(f). The number of levels participating in the dynamics remains the same as at the initial moment of time \(t = 0\) where it was determined by the decomposition of the initial wavepacket, see Fig.4(d). The spreading of the wavepacket in the real space inside the QD is described by the coordinate variance \([15]\) which is shown in Fig.3(g). As the other observables, it has a form of a function combined from fast and slow oscillations which is reflected in the Fourier spectra described above. In the inset in Fig.3(g) we show a fragment of the \(\sigma_y(t)\) dependence on small time scale indicating the presence of fast oscillations, with the frequency comparable with the driving frequency \(2\pi/\tau_0\). Such structure of the \(\sigma_y(t)\) function will also take place in all other figures presented below, and thus will not be shown in detail. We see that the changes in the packet half-width can be described as the absence of width growth, i.e. the initial width (see Fig.4(c), packet (1)) does not grow with time, and at certain moments of time the packet is in fact narrowed. This observation is further confirmed by the plots of the charge density \(\rho(y)\) (green dashed curve) and the \(S_z(y)\) component of spin density (blue solid curve) in Fig.3(h) taken at specific moment of time \(t = 1995\tau_0\) when the evolution is well-established. The packet is spread along the whole QD, as it can be seen by its tails, but the major part of the charge and spin density are concentrated on the width comparable with the width of the initial packet, as one can see by comparing Fig.3(h) and Fig.4(c), packet (1). This interesting effect can be attributed, in our opinion, to the nearly equidistant character of the energy levels of the QD Hamiltonian like the ordinary linear oscillator, which may lead to the manifestation of certain effects for the coherent states\([38]\) including the absence of the packet width growth. Such stability of the wavepacket width can play a significant role in the experimental and device applications where the electrons with well-localized charge and spin density inside the nano- and microstructures are desirable to be present.

B. Driven evolution in the clean limit

Here the evolution of both wide and narrow initial spin-polarized wavepackets (see Fig.4(c)) is considered under the presence of driving \([39]\) with moderate driving strength \(\varepsilon_0 = 1\ \text{V/cm}\). Such amplitude of the electric field means that the voltage drop through the mesoscopic QD with length \(L = 3\ \text{nm}\) is about 0.3 mV which provides suitable conditions for experimental and device applications with large currents and/or Joule heating. In the present subsection we consider the clean limit for the QD without the disorder potential which will be included in the next subsection. The assumption of a clean structure is reasonable since the mean free path in high-quality HgTe/CdTe quantum well reaches several microns\([20]\) which covers the whole size of our QD. We consider the time interval of 400 periods of driving field with 200 points per period for the graphical representation. These parameters cover both the time span needful for the stationary regime of the dynamics to be established, and the time grid which catches the significant non-vanishing Fourier components of the evolution of observables.

In Fig.4 we show the results for the driven evolution for the initial state represented by the wide wavepacket from Fig.4(c) presented in the same order as the results on the free evolution in Fig.3. The initial point at \(t = 0\) is marked as the black circle "A". (a) the "phase space" plot of evolution in the \((y, S_z)\) coordinates and (b) the in-plane spin precession in the \((S_x)\) demonstrate the more complex and irregular character of the driven evolution compared to the free evolution shown in Fig.3. In particular, the regular trajectories in Fig.4(a) are accompanied by the surrounding areas of the "chaotic sea", although the general oscillating character of the wavepacket evolution is still visible. The onset of irregular motion is further pronounced in the in-plane spin dynamics shown in Fig.4(b). The spin evolution becomes largely irregular here compared to the free evolution in Fig.3(b). A large clustering area near the origin of spin axes indicates a tendency of reduction of the in-plane spin maximum values during the driven evolution which can be viewed as a manifestation of the spin relaxation phenomena.\([8–10]\)

As to the absence of such relaxation for the out-of-plane \(S_z\) component, it can be attributed to the special form of the initial Hamiltonian \([2]\) where \(S_z\) plays a role of the electron velocity and thus is non-vanishing in the absence of the momentum or energy relaxation mechanisms.

The manifestation of chaotic or at least strongly irregular regimes for the driven evolution is supported by the Fourier power spectra for the coordinate and spin observables plotted in Fig.4(c)-(e). One may see that the driving induces a large number of harmonics of driving frequency \(\omega_0\) both for coordinate and spin, especially the in-plane component \(S_x\) (and similarly for \(S_y\) which is not shown here). Although the dynamics is dominated by the nearly equidistant character of energy levels leading to the nearly equidistant sequence of harmonics of \(\omega_0\) in the power spectra, and thus the Fourier harmonics do not fill frequency bands continuously, the presence of large number of harmonics is a strong indication of irregular dynamics,\([2, 11, 15]\) especially compared to the free evolution shown in Fig.3(c)-(e).

The concept of irregular dynamics or chaos develop-
FIG. 4: (Color online) The same as Fig. 3 but for the driven case with \( E_0 = 1 \) V/cm. (a) Evolution in the “phase space” of the \((y, S_z)\) variables representing classical coordinate and momentum. (b) Evolution of the in-plane spin components \((S_x, S_y)\). The initial point at \( t = 0 \) is shown as black circle marked by “A”. The evolution of both coordinate and spin shows the coexistence of both regular and chaotic regimes seen in the presence of both regular trajectories and the large areas of “stochastic sea”, especially for the spin variables. (c)-(e) Fourier power spectra for the coordinate \( y \) and two spin components \( S_x \) and \( S_y \). Large number of harmonics of the driving frequency indicates the onset of strongly irregular motion. (f) The evolution in the Hilbert space plotted as a number of levels effectively participating in the dynamics shows the linear growth at the beginning of the evolution which corresponds to the chaotic regime. Later the level number saturates near a stable value corresponding to the width of the Floquet states in Fig. 3. (g) The packet half-width variance describing the spreading of the wavepacket in the real space inside the QD. The initial half-width (see Fig. 3(c), packet (1)) does not grow with time, and, as for the free evolution, at certain moments of time the packet is narrowed. (h) Charge density \( \rho(y) \) (green dashed curve) and the \( S_z(y) \) component of spin density (blue solid curve) inside the QD taken at specific moment of time \( t = 395T_0 \). The packet is spread along the QD, but the major part of the charge and spin density are concentrated on the width comparable with the width of the initial packet. (i) Dynamics of the Lyapunov exponent for two initially close wavepackets. At the beginning of the evolution this exponent takes also positive values indicating the presence of chaotic regime, and later it decreases to zero when the quasi-regular quantum dynamics is established.

ment can be supported by analysis of the driven evolution in the Hilbert space of basis states where the onset of chaos usually corresponds to the growth in time of the number of energy levels involved into evolution, which is sometimes called as manifestation of quantum Arnol’d diffusion.\[7\] Our Floquet analysis of the quasienergy eigenfunctions described in Sec.III indicates that the periodic driving with amplitudes of 1…2 V/cm may induce the formation of the Floquet states which are deeply extended into a substantial part of the energy spectrum, see Fig. 2. Thus, we may expect the number of level variance \( \sigma_n \) measuring the number of levels involved into evolution to be as high as the maximum number reached by the Floquet states. This assumption is confirmed by the plot of \( \sigma_n(t) \) in Fig. 4(i) where almost linear growth of level number is present at the initial stage of evolution where the quantum-classical correspondence is mostly pronounced.\[2, 4\] Such growth is usually attributed to the onset of chaotic dynamics, or diffusion in Hilbert space. After some time, however, the discrete character of the quantum mechanical spectrum of a finite motion inside QD leads to the saturation of the level number involved in the evolution, and the diffusion in the Hilbert space effectively stops.\[2\] This can be seen in Fig. 4(f) where the linear growth of \( \sigma_n \) transforms into oscillations with stable mean value. We can say that the chaotic behavior in our quantum system has a transient nature.

As to the dynamics in the real space inside the QD, the evolution of the packet half-width is presented in Fig. 4(g). We may see that the driven evolution here has much more in common with the free evolution shown in Fig. 3(g). Namely, the packet width essentially does not grow with time both for the free and driven case, and the packet at each moment of time occupies effectively only a limited area inside the QD. This finding is illustrated by an example of the spin and charge density distributions inside the QD shown for \( t = 395T_0 \) in Fig. 4(h) as blue solid and green dashed lines, respectively. One may see that the packet occupies a substantial part of the QD, however, its effective width has the value close to the width of the initial packet, see Fig. 3(c), wavepacket (1). As we have mentioned earlier, such stable behavior of the packet width during the driven evolution can be attributed to the nearly equidistant character of the energy levels of the system which can trigger certain properties of the coherent states in the driven evolution.

Another possible manifestation of chaos is the presence of positive Lyapunov exponents which measure the rate of divergence of two initially close trajectories in the phase space,

\[
\lambda = \lim_{t \to \infty} \frac{1}{t} \log \frac{d(t)}{d(0)}, \tag{16}
\]

where \( d(t) \) and \( d(0) \) are the present and initial distances, respectively. The infinite limit in (16) can be tracked also by a continuously monitorong with growing
time where $\lambda = \lambda(t)$ which measures the local transitions between the chaotic and regular regimes. In Fig.5(i) we plot the dependence of $\lambda(t)$ for two initially close wavepackets shifted slightly in their positions along $y$ at $t = 0$. One can see that at the beginning of the evolution the region with positive $\lambda(t)$ indeed exists which corresponds to the linear growth of the level number $\sigma_n$ involved into the dynamics, see Fig.5(f). Both these plots support the presence of chaotic dynamics at the initial stage of the evolution when the discrete character of the quantum spectrum has not yet manifested itself so much. After the initial transient period the evolution tends to transform to a quasi-regular regime with the stable number of levels involved into dynamics, and the Lyapunov exponent reduces to zero, as in can be seen in Fig.5(i). It should be noted that such behavior is known in quantum systems with irregular dynamics[2–4, 7]. However, the results obtained there were mainly for the spinless systems with quadratic spectrum having a certain classical analogue. It is of big importance both for fundamental quantum physics and its applications in spintronics and nanoelectronics to investigate such properties in essentially quantum spin-dependent systems, including the massless Hamiltonians for particles inside topological insulators.

In order to understand the degree of dependence of the evolution patterns on the specific form of the initial conditions we look on the dynamics of another initial state represented by the spin-polarized $(S_x = 1)$ narrow wavepacket shown as the curve (2) in Fig.5(c) which occupies initially a substantial part in the Hilbert space, as it can be seen in Fig.5(c). The evolution under the same conditions as for the wide wavepacket is shown in Fig.5 with all of the parts in the figure having the same meaning as in Fig.4 except the absence of the Lyapunov exponent plot which does not provide essentially new information compared to Fig.4(i). The behavior of the coordinate and spin observables in Fig.4(a), (b) again demonstrates the co-existence of regular and stochastic regions which are more dense for the in-plane spin components $(S_x, S_y)$. The Fourier power spectra in Fig.4(c)–(e) also show a significant number of harmonics for both coordinate and spin, and the evolution in the Hilbert space shown by $\sigma_n$ in Fig.4(f) shows a large number of levels involved into dynamics. Here the stage of linear growth for $\sigma_n$ is less pronounced since the initial value $\sigma_n(0)$ for a narrow packet is already high (see Fig.4(e)) and is close to the maximum available width of the Floquet eigenstates, see Fig.2. It is interesting to note that the effect of packet width stability is also expressed here: the initial packet half-width again does not grow with time, as it can be seen in Fig.5(g). An example of charge and spin density shown in Fig.5(h) for a chosen moment of time $t = 395T_0$ supports this finding, showing the well-localized packet. These predictions of the spatial localization of charge and spin density during the driven evolution may be useful for design of future generations of spintronic devices[8–10] based on quantum dots inside topological insulators.

![Image](image-url)

**FIG. 5:** (Color online) The same as Fig.4 but for the narrow wavepacket shown as the curve (2) in Fig.5(c). (a), (b) Coordinate and spin observables demonstrate the co-existence of regular and stochastic regions which are more dense for the in-plane spin components $(S_x, S_y)$. (c)–(e) Fourier power spectra demonstrate a significant number of harmonics for both coordinate and spin. (f) Evolution in the Hilbert space of $\sigma_n$ shows a large number of levels involved into dynamics from the start of the driving. Here the stage of linear growth for $\sigma_n$ is less pronounced since the initial value $\sigma_n(0)$ for a narrow packet is already high (see Fig.4(e)) and is close to the maximum available width of the Floquet eigenstates, see Fig.2. (g) The packet half-width dynamics shows the stability of maximum width: the initial packet width does not grow with time. (h) An example of charge and spin density shown for a chosen moment of time $t = 395T_0$ shows the well-localized packet.

V. THE EFFECTS OF THE DISORDER ON THE EVOLUTION

In this Sec we insert an additional stationary disorder potential

$$U_d(y) = U_0 f(y)$$

(17)

into the r.h.s. of the non-stationary Schrödinger equation (11) where the potential amplitude $U_0$ is multiplied by a random function $f(y)$ described by a uniform
random distribution from 0 to 1 along the QD where \( 0 \leq y \leq L \). The matrix elements of \( U_d(y) \) contribute to the dynamics of the coefficients \( C_n(t) \) for the wavefunction (11) together with the ones from the driving term \( V(y, t) \). We consider two examples of the amplitude \( U_0 = 0.1 \) meV and \( U_0 = 0.5 \) meV which is comparable with typical interlevel distance \( \delta = 0.38 \) meV, i.e. we consider a moderate disorder. This can be justified by a typical high quality and high mobility of samples usually fabricated and studied in the experiments with TI,[20] which have the mean free path of the order of the QD length \( L \), and low temperatures of around or below 1 K which produces the level broadening of the order of 0.05 meV.

A. Free evolution with disorder

As for the clean case considered in the previous section, we begin the analysis of the evolution at the absence of the driving term by taking \( V(y, t) \equiv 0 \). This situation corresponds to a free evolution of a wavepacket with \( S_x(0) = 1 \) injected from the left barrier into the QD. In Fig.6 we show the evolution of a wide packet (see the packet (1) in Fig.1(c)) viewed with the help of the same tools as the driven evolution considered above. The total observation time is 2000\( T_0 \) where again the unit of time \( T_0 = 2\pi/\omega_0 \) is determined by the level splitting \( \hbar \omega_0 = (E_{n_0+1} - E_{n_0}) \) taken for the pair of levels near the center of the spectrum where most of the initial coefficients \( C_n(0) \) are located, see Fig1(d).

It is clear that the presence of the disorder potential modifies the free evolution significantly. First of all, the disorder induces the relaxation mechanisms which lead to the gradual saturation of the spin components, especially for \( S_y \) and \( S_z \), near the zero values, as it can be see by the most populated areas near \( S_z = 0 \) and \( S_y = 0 \) in Fig6(a),(b). The mean value of \( S_x(t) \) covers with time the areas both near and far the point \( S_x = 0 \) since the electron spinor wavefunction contains parts inside the spin polarized barriers with \( S_z = 1 \). As to the dynamics in the coordinate space inside the QD, one can see in Fig6(a) that it tends to localize at the center of the QD at \( y = L/2 \) which means that the charge distribution approaches a symmetric form with the center located near the geometrical center of the QD.

As for the clean limit, the absence of periodic driving leads to a rather narrow Fourier power spectra shown in Fig6(c)-(e). One can see that the spectrum \( I_y(\omega) \) for the coordinate mean value \( y(t) \) in Fig6(c) demonstrates a more rich structure than the spectra for spin components shown below it. This can be attributed to the dominating relaxation in the coordinate channel where the disorder potential \( U(y) \) acts directly, and the relaxation of spin components accompanies this as a secondary process via the spin-orbit coupling-like form of the initial Weyl Hamiltonian [2].

The evolution in the Hilbert space shown by the level

![FIG. 6: (Color online) Free evolution of the wide wavepacket in the presence of the disorder potential (17). (a),(b) Coordinate and spin observables show the tendency for the mean coordinate relaxation to the QD center and relaxation for the spin components \( S_y, S_z \) to zero. (c)-(e) Fourier power spectra demonstrate a limited number of harmonics for spin components and a greater number of harmonics for coordinate reflecting the action of the disorder potential. (f) Evolution in the Hilbert space of \( \sigma_n \) shows a slow growth of the number of levels involved into dynamics which corresponds to a slow diffusion rather than to a chaos development. (g) The packet half-width dynamics approaches the stable mean value corresponding to a widely spread charge distribution (h) Charge (green dashed curve) and spin (blue solid curve) densities shown for a chosen moment of time \( t = 1995T_0 \) demonstrate a spreaded packet reflecting the presence of the disorder potential. (i) View of an example for the disorder potential \( U_d(y) \) inside the QD.](image-url)
number $\sigma_n$ participating in the dynamics is plotted in Fig. 7(f). We may see a diffusive-like growing function, although without the prolonged stage of a linear growth accompanying the chaos development. As to the diffusion in the coordinate space, we see in Fig. 7(g) that the mean packet half-width quickly saturates near the stationary value close to a maximum achievable width during the driving. The spreading of the initial packet along the QD with the charge and spin probability more or less equally distributed along the QD is shown in Fig. 6(h) for a specific moment of time $t = 1995T_0$. We see that the random potential which typical realization shown in Fig. 6(i) leads to a spreading of the packet on the scale of the QD with equal peak distribution with random heights. It is known that the 1D random potential lead to the localization in space [2–4] however, the length of such localization in our case exceeds the QD size $L$ which leads to a rather uniform distribution of the charge and spin densities inside the QD. Their shapes are in contrast with the distributions for the clean limit shown in Fig. 3(h) where the initial packet-like form is maintained during the evolution in the basis of oscillator-like nearly equidistant levels.

**B. Driven evolution with disorder**

Finally we consider the driven evolution in the presence of the disorder. We combine the disorder potential (17) with the driving $V(y,t)$ with the amplitude $E_0 = 1$ V/cm, and take for this time a narrow wavepacket (see the packet profile shown by curve (2) in Fig. 7(c)) as the initial condition which evolves on 400 periods of driving field with 200 points per period considered as a suitable time grid catching all of the non-vanishing Fourier components.

The results of the evolution modeling are shown in Fig. 7 for the weak disorder strength $U_0 = 0.1$ meV. We see that the driving with the potential amplitude $V \approx \epsilon E_0 L$ reaching 0.3 meV significantly overcomes by its effect the disorder potential with the amplitude of 0.1 meV. In particular, the overall structure of the evolution "phase" plots for the coordinate and spin observables in Fig. 7(a),(b) strongly reminds the corresponding plots in Fig. 5 for the absence of disorder. The same can be said about the Fourier power spectra in Fig. 7(c)-(e) which demonstrate a lot of higher Fourier components of the driving frequency $\omega_0$. The evolution of the variance of the level number in Fig. 7(f) already starts from a high value reflecting the narrow form of the initial condition, see Fig. 5(e), and later evolves near this value which is close to the maximum reachable one. As to the packet half-width evolution shown in Fig. 7(g), it shows the absence of the packet spread which was observed when the disorder potential acted alone. Indeed, the snapshot of the charge and spin density profiles inside the QD shown in Fig. 7(h) shows the well-localized packet which means that the dynamical effect where the mean packet width is maintained by periodic driving can overcome the diffusive tendency to spread the packet by the disorder potential.

As the amplitude of the disorder increases, one can expect certain modifications of the evolution for both coordinate and spin degrees of freedom, when the disorder amplitude $U_0$ begins to exceed the energy of the driving field. In Fig. 8 we show the evolution of the narrow wavepacket under the driving with $\omega_0 = 1$ V/cm, as Fig. 7 but with a stronger disorder amplitude $U_0 = 0.5$ meV which exceed the typical energy of the driving field $\epsilon E_0 L = 0.3$ meV. One can see that the stronger disorder leads to more irregular phase portraits for the evolution in the $(y,S_z)$ and $(S_x,S_y)$ coordinates shown in Fig. 8(a),(b). In particular, the in-plane spin components $(S_x,S_y)$ demonstrate a tendency to cluster near the coordinate origin $(0,0)$ meaning that the effective in-
plane spin relaxation is enhanced by the increased disorder. As to the off-plane spin component $S_z$, it still demonstrates a full-scale oscillating behavior representing the electron velocity in our model, but within a well-established chaotic sea visible in the $(y, S_z)$ plot. This dynamics is supported by the Fourier power spectra shown in Fig. 8(c)-(e) where the spin components demonstrate a more quickly vanishing spectra due to the spin relaxation compared to the coordinate one. The stronger disorder leads to an interesting effect on the number of levels $\sigma_n$ effectively involved in the evolution which is shown in Fig. 8(f). Starting from the initially high number of basis states present in the decomposition of a narrow wavepacket, this number begins to decrease progressively, with the average level number (not shown) moving down from the Dirac point. Such a form of localization in the Hilbert space can be viewed as an example of dynamical localization observed in quantum systems with chaotic behavior.[3, 4] Finally, the variance of the packet half-width shown in Fig. 8(g) shows the oscillating behavior with saturating amplitude which again demonstrates the effect of the wavepacket maximum width stability induced by the periodic driving, which is maintained even in the presence of a strong disorder. An example of the charge $\rho(y)$ and spin density $S_z(y)$ distributions at the end of the observation frame $t = 395T_0$ shown in Fig. 8(h) supports this finding, demonstrating a well-localized packet near the edge of the QD.

We believe that the properties of the wavepacket evolution observed in our model which include the mutual effects of periodic driving and disorder on charge and spin evolution as well on the regular and irregular character of the dynamics can be of interest for further applications of our finding in the development of both fundamental aspects of quantum evolution and for future generations of the spintronic and nanoelectronic devices.

VI. CONCLUSIONS

We have studied the dynamics of spin-polarized wavepackets driven by periodic electric field for the electrons in a mesoscopic quantum dot formed at the edge of two-dimensional HgTe/CdTe topological insulator representing a new class of materials with Weyl massless energy spectra, where the motion of carriers is less sensitive to disorder and impurity potentials. It was found that the interplay of strongly coupled spin and charge degrees of freedom creates the regimes of both regular and transiently irregular dynamics with certain universal properties manifested for both free and driven evolution, in the clean limit and in the presence of the disorder. The effects of random disorder potential are found to be overcome by periodic driving for weak disorder and to produce the in-plane spin relaxation for stronger disorder, leading to possibility of establishing novel types of controlled evolution in nanostructures formed in the topological insulators. The dynamical properties of regular and chaotic behavior of charge and spin in these structures may be of interest for future progress in both quantum nonlinear dynamics on the nanoscale and in the applied nanoscience such as spintronics and nanoelectronics.

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