Energy and entropy radiated by a black hole embedded in the de-Sitter braneworld

Shao-Feng Wu\textsuperscript{1,2*}, Shaoyu Yin\textsuperscript{3}, Guo-Hong Yang\textsuperscript{1,2}, and Peng-Ming Zhang\textsuperscript{4,5}

\textsuperscript{1}Department of Physics, College of Science, Shanghai University, Shanghai, 200444, P. R. China
\textsuperscript{2}The Shanghai Key Lab of Astrophysics, Shanghai, P. R. China
\textsuperscript{3}Department of Physics, Fudan University, Shanghai 200433, P. R. China
\textsuperscript{4}Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, P. R. China and
\textsuperscript{5}Institute of Modern Physics, Lanzhou, 730000, P. R. China

We study the Hawking radiation of (4+n)-dimensional Schwarzschild black hole imbedded in the space-time with positive cosmological constant. The greybody and energy emission rates of scalars, fermions, bosons, and gravitons are calculated in the full range of energy. The valuable information on the dimensions and curvature of space-time is revealed. Furthermore, we investigate the entropy radiated and lost by black hole. We find their ratio near unit in favor of the Bekenstein’s conjecture.

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I. INTRODUCTION

The concept of extra dimensions is becoming increasingly popular in both particle theory, such as the string theory and the grand unification, and cosmology. It also suggests us a new solution to the hierarchy problem, since the traditional Planck scale can be derived as an effective energy scale from the fundamental higher-dimensional scale \[1, 2\]. This idea opens possibilities for observing strong gravitational phenomena if the fundamental Planck scale is lowered to the order of magnitude around TeV. Based on the well-known braneworld scenario with large or compact extra dimensions, a particularly exciting proposal is the possibility of mini black holes with horizon radii smaller than the size of the extra dimensions centered on our braneworld and extending along the extra dimensions. Even the future possibility has been discussed to produce such mini black holes in super collider, with the center-of-mass energy greater than the fundamental scale \[3\]. It has also been expected to observe the mini black holes via the ultrahigh-energy neutrinos interacting with the atmosphere of the earth \[4\]. Moreover, the mini black holes might have been created in the early universe due to density perturbations or phase transitions. The observational signatures of such mini black holes can be precisely determined by their well-known quantum gravity effect, the Hawking radiation, which encodes the vital information about the structure of the space-time geometrical background. Much effort on the Hawking radiation has been expanded to study the properties of the extra dimensions of space-time \[5\], the string-induced Gauss-Bonnet correction \[6, 7\], and the cosmological constant \[8\]. The attention on the de Sitter space-time is motivated at least by the following three aspects. First, it is important to study non-asymptotically flat space-time since there is a duality between quantum gravity on the (A)dS space and the Euclidean conformal field theory (CFT) \[9\]. Second, recent astronomical observations on supernova indicate that the present universe be dominated by energy with negative pressure \[10\], and a positive cosmological constant is a ready candidate. Third, it is generally accepted that the evolution of the universe is well represented by the FRW cosmology punctuated by a de Sitter-like exponentially inflationary phase \[11\]. Therefore, it is reasonable to ask what effect the inflation might have on the behavior of the primordial black hole.

In this paper, we will study the Hawking radiation of the black hole imbedded in higher-dimensional space-time with the asymptotical dS boundary. Compared with the vast literatures on Hawking radiation, the work accounting for Schwarzschild-de-Sitter (SSdS) black hole is much sparser. For the four-dimensional case, after the pioneer work of Gibbons and Hawking \[12\], the existing literatures mainly considered the two-dimensional Vaidya-de Sitter space-time \[13\], or focused on the evaporating dilatonic final state of a SSdS black hole \[14\]. For the higher-dimensional case, most activities have been focused on the study of the pair creation of black holes \[15\], thermodynamical radiation via tunneling \[16\], the mass and entropy bounds \[17\], and the quasinormal frequencies associated with the perturbations of the higher-dimensional space-time \[18, 19, 20\]. Some possible experimental consequences of the higher-dimensional dS or AdS evaporating black holes have been preliminary studied in Ref. \[21\]. The exact form of the higher-dimensional Hawking radiation spectrum was first studied in Ref. \[3\], where only scalar fields are considered. But it is still unknown for fermions, gauge bosons, and gravitons including tensor, vector, and scalar modes. The gravitons recently have been analytically considered but only at low or at large imaginary frequency \[22\]. One aim of our work is to give the exact energy spectrum radiated by higher-dimensional SSdS black hole and examine the effect of the dimensions of

* Corresponding author. Email: sfwu@shu.edu.cn; Phone: +86-021-66136202.
space-time and the cosmological constant.

Since the key ingredient for the energy spectrum is the greybody factor, or equally, the absorption probability, which distinguishes the black hole radiation from the black body, we should know the greybody factor of higher-dimensional SSdS black hole before we get the energy spectrum. With the spectrum of greybody factor at hand, we will investigate the entropy radiation of higher-dimensional SSdS black hole. It is well-known that one of the most enchanting features of black hole thermodynamics is the Bekenstein’s conjecture [23], positing that a black hole possesses an entropy proportional to its surface area, which can be statistically interpreted as a measure of all possible pre-collapse configurations before the black hole was built. Recovering this entropy is an important success of the quantum gravity [24] and closely connects to the proposal of the (A)dS/CFT correspondence. However, a direct proof of the statistical interpretation of the entropy is still missing. Under the consideration that the evaporation of a black hole is a time-reversal process of its collapse, and that the entropy of the final state of black hole can be accurately estimated, Zurek [25] proposed that Bekenstein’s conjecture can be tested by quantifying the ratio of the evaporated entropy to the entropy lost by the black hole. By numerical calculations based on the greybody factor of four-dimensional Schwarzschild black hole, Page [26] showed the ratio is indeed near unit, which is strongly in favor of the conjecture of Bekenstein, since nothing mathematically prevents the ratio being arbitrarily large or small. The argument of Zurek was immediately generalized to the four-dimensional Schwarzschild black hole with charge [27]. However, because of the sophisticated numerical techniques needed in the greybody factor calculation, similar study on other black hole, especially the higher-dimensional black hole, is not present until very recent work of Barrau et al. [28]. We now wish to extend Zurek’s argument to the case of higher-dimensional SSdS black hole. Quite different from the case of the flat space-time, the greybody factor for scalar field has already been found non-vanishing in the low energy limit [8], and we obtain the similar result for fermion fields. Besides, we also study the behavior of vector and tensor fields for completeness.

The organization of the paper is as follows: In section II, we briefly review the general framework of SSdS black hole. In section III, we give the analytical solutions of master equations for all particles near the event horizon and the cosmological horizon. In section IV, we concentrate on the Hawking radiation of a decaying SSdS black hole. We will show the exact greybody, energy spectrum and entropy variation of all particle species, with the strong effect of the extra dimensions and the cosmological constant. The last section is devoted to conclusions and discussions.

II. SSDS BLACK-HOLE PROPERTIES

We consider the class of black holes that are formed in the presence of a positive cosmological constant \( \Lambda \) in the \( d \)-dimensional \((d = 4 + n)\) space-time. The geometrical background of SSdS black holes is given by the Tangherlini line element [29]

\[
\begin{align*}
\text{d}s^2 &= -f(r)\text{d}t^2 + \frac{\text{d}r^2}{f(r)} + r^2\text{d}\Omega_{n+2}^2
\end{align*}
\]

where

\[
\begin{align*}
f(r) &= 1 - \frac{\mu}{r^{n+1}} - 2\kappa_d^2 \Lambda r^2 \left( \frac{n + 2}{2n + 2} \right),
\end{align*}
\]

The \( \text{d}\Omega_{n+2} \) is the solid angle element. The parameter \( \mu \) is related to the ADM mass of the black hole through the relation

\[
M = \frac{(n + 2)A_{n+2}\mu}{2\kappa_d^2},
\]

where \( A_{n+2} \) is the area of a unit \((n + 2)\)-dimensional sphere. The \( \kappa_d^2 = 8\pi G_d \) stands for the \( d \)-dimensional Newton’s constant, which will be set to unit for convenience. There are two positive roots of equation \( f(r) = 0 \), the larger one \((r_C)\) corresponding to the cosmological horizon, and the smaller one \((r_H)\) to the event horizon. From Fig. 1 one can find that the cosmological horizon will be close to the event horizon when the cosmological constant is big and the space-time dimensions are small. When the two horizons are lying close to each other, Nariai black hole will be arisen, which corresponds to the maximum black hole and minimum de Sitter space [30].

Using the relation between the parameter \( \mu \) and two horizons

\[
\mu = r_H^{n+1} \left[ 1 - \frac{2\Lambda}{(n + 2)(n + 3)} r_H^2 \right],
\]

(3)
we can relate the metric with two horizons
\[ f(r) = 1 - \frac{2\Lambda r^2}{(n+2)(n+3)} + \frac{r_{H/C}^{n+1}}{r_{H/C}^{n+1}} \left[ \frac{2\Lambda}{(n+2)(n+3)} r_{H/C}^2 - 1 \right], \]  
and the ADM mass with the event horizon
\[ M = \frac{(n+2)A_{n+2}}{2} r_{H}^{n+1} \left[ 1 - \frac{2\Lambda}{(n+2)(n+3)} r_{H}^2 \right]. \]  
The temperature of SSdS black hole, defined by the surface gravity, is rather subtle. There are two kinds of temperatures based on the standard and Bousso-Hawking normalizations, respectively. The standard normalization provides the Hawking temperature \( T_0 \), which was derived by the analogy with asymptotically flat space-time [21, 22, 31]. In the case of asymptotically flat space-time, the standard method to obtain the surface gravity is to choose the normalized Killing field at infinity, where an observer does not feel any acceleration. However, in the presence of a cosmological constant, there is no asymptotic flat region and we cannot place the preferred observer at infinity. Usually, the Bousso-Hawking normalization is introduced [8, 32]. Consider the zero point \( r_0 \) of the first derivative of metric function \( f'(r) \),
\[ r_0 = \left[ \frac{(n+1)(n+2)(n+3)\mu}{4\Lambda} \right]^{\frac{1}{n+3}}, \]  
at which the metric function achieves the maximum value. The black hole attraction and the cosmological repulsion exactly cancel out at this point, thus one may achieve the zero acceleration inside the cosmological horizon. Now we can obtain the Bousso-Hawking temperature of the black hole
\[ T_H = \frac{1}{\sqrt{f(r_0)}} T_0, \]  
where
\[ T_0 = \frac{1}{4\pi r_{H}} \left[ (n+1) - \frac{2\Lambda}{n+2} r_{H}^2 \right], \]  
is the Hawking temperature obtained by the standard normalization. Substituting Eq. (3) and Eq. (6) into Eq. (2), we can plot the two temperatures \( T_H \) and \( T_0 \) as the function of extra dimensions \( n \) and the cosmological constant \( \Lambda r_{H}^2 \), see Fig. 2. One can find that the standard Hawking temperature \( T_0 \) increases with \( n \) but decreases with \( \Lambda r_{H}^2 \); however, both parameters cause an increase in the Bousso-Hawking temperature \( T_H \). In most part of this paper, we apply the Bousso-Hawking temperature following the work of Ref. [8]. However, we want to emphasize that the greybody factor and the method to obtain the energy and entropy radiation are not dependent on the different choice of temperatures.
The entropy of the black hole is proportional to its surface area

\[ S = \frac{1}{4G} A_{n+2r_H^{n+2}}. \]

Using Eqs. (5) and (7), we can obtain the loss of entropy

\[ dS = \frac{1}{T_0} dM. \]

With a non-vanishing temperature, SSdS black hole will emit Hawking radiation in the form of elementary particles, which is similar as the blackbody radiation, with a density matrix in each mode

\[ \rho_{kk'} = \delta_{kk'} \left[ \frac{|A_s^{(s)}|^{2k}}{e^{\frac{\omega}{T_0}} - (-1)^{2s}} \right] \left[ e^{\frac{\omega}{T_0}} - (-1)^{2s} + (-1)^{2s} |A_s^{(s)}|^{2k} \right]^{-1} \]

where \( s \) denotes the spin of particle, \( j \) is the angular momentum number, and \( |A_s^{(s)}|^2 \) is the greybody factor. The energy emission rate is \( Tr(-\rho\omega) \), which gives

\[ dE_s^{(s)} = \frac{1}{2\pi} \sum_j N_j^{(s)} |A_j^{(s)}|^2 \frac{\omega}{e^{\frac{\omega}{T_0}} - (-1)^{2s}} d\omega, \tag{8} \]

where \( N_j^{(s)} \) are the multiplicities of states that correspond to the angular momentum number \( j \). For fields localized on brane, the multiplicities of states are \( N_j^{(s)} = 2j + 1 \), while for gravitons, the multiplicities of states are different for three types of perturbation

\[ N_j^{(s)} = \begin{cases} \frac{(2j + d - 3)(j + d - 4)}{j!(d - 3)!} & \text{for scalar mode,} \\
\frac{j(j + d - 3)(2j + d - 3)(j + d - 5)!}{(j + 1)!(d - 4)!} & \text{for vector mode,} \\
\frac{(d - 4)(d - 1)(j + d - 2)(j - 1)(2j + d - 3)(j + d - 5)!}{2(j + 1)!(d - 3)!} & \text{for tensor mode.} \end{cases} \]

Integrating this energy spectrum over all modes gives the exact expression of entropy loss of a black hole

\[ dS = \frac{1}{T_0} dM = \sum_{s,p,j} N_j^{(s)} \int_0^\infty d\omega h_j^{(s)}(w), \tag{9} \]
where \( p \) denotes the helicity, and the integrand of the lost entropy \( h_j^{(s)}(w) \) is

\[
h_j^{(s)}(w) = \frac{1}{\mathcal{Z}} \frac{1}{2\pi} \left| A_j^{(s)} \right|^2 \frac{\omega}{e^{\nu r} - (-1)^{2s}}.
\]

(10)

On the other hand, the entropy emission rate \( Tr(-\rho \ln \rho) \) is expressed as

\[
dS_{rad} = \sum_{s,p,j} N_j^{(s)} \int_0^\infty d\omega g_j^{(s)}(w),
\]

(11)

where the integrand of radiated entropy \( g_j^{(s)}(w) \) is

\[
g_j^{(s)}(w) = \frac{1}{2\pi} \left\{ \left| A_j^{(s)} \right|^2 \ln \left[ \frac{e^{\nu r} - (-1)^{2s}}{\left| A_j^{(s)} \right|^2} \right] + (-1)^{2s} \ln \left[ 1 + (-1)^{2s} \frac{\left| A_j^{(s)} \right|^2}{e^{\nu r} - (-1)^{2s}} \right] \right\}.
\]

(12)

For our aim, we will evaluate the energy spectrum (8) and the ratio of the entropy lost by the black hole to the entropy gained by the radiation

\[ R = \frac{dS_{rad}}{dS}. \]

Obviously, it is necessary to calculate the greybody \( \left| A_j^{(s)} \right|^2 \) at first. It is computed within a full quantum mechanical framework in the aforementioned SSdS black hole background.

III. MASTER EQUATIONS AND THEIR SOLUTIONS NEAR HORIZONS

In this section, we will give the master equations for all elementary particles and the solutions near the event horizon and the cosmological horizon.

A. brane particles

In the braneworld scenarios, standard model fields are confined on the brane, except the gravitons which can propagate in the extra dimensions. We first consider the master equation for particles on the brane. Particles confined on the brane propagated in a background whose geometry is induced by the bulk curvature. The induced geometry on the 4-dimensional brane is given by fixing the values of the extra azimuthal angular coordinates and leads to the projection of the \( d \)-dimensional metric on the 4-dimensional slice that describes our world

\[
ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2.
\]

Here metric function remains the same as in the higher-dimensional line element (2), so the profile of the curvature along the three noncompact spatial dimensions contains a fingerprint of the bulk curvature. Since the line element of a \( d \)-dimensional SSdS black hole has a Schwarzschild-like form, the motion equations for brane particles can be described by the well known Newman-Penrose formalism [35]. Factorizing the propagating field as

\[
\Psi_s(t, r, \theta, \phi) = e^{-i\omega t} \Delta^{-s} P_s(r) Y_{s,j}(\theta, \phi),
\]

where \( \Delta = f^2 \), \( Y_{s,j} \) is the spin-weighted spherical harmonics, the radial part of the master equation of motion has the form [36, 37]

\[
\Delta^{-s} \frac{d}{dr} \left( \Delta^{1-s} \frac{dP_s}{dr} \right) + \left( \frac{\omega^2 r^2}{f} + 2i\omega r - \frac{i\omega r^2 f'}{f} - \tilde{\lambda} \right) P_s = 0,
\]

(13)

where \( \tilde{\lambda} = j(j+1) - s(s-1) \) and the prime denotes the derivative with respect to \( r \). The master equation can be rewritten as the familiar Shrödinger-like form

\[
-\frac{d^2 U}{dr^2} + VU = \omega^2 U,
\]
where $r_s$ is the tortoise coordinate defined by $dr_s = dr/f$, $U = r^{1-s}f^{-s/2}p_s$ is the redefined radial function, and the potential is

$$V = \frac{is\omega (-2f + rf')}{r} + \frac{4s(s-1)f^2 + r^2s^2f'^2 + f(4\lambda + 4r(s-1)f' - 2sf''f'')}{4r^2}.$$  \quad (14)

To find the absorption coefficient for propagation of fields on the brane, we need to solve the radial equation (13) over the whole radial regime. However, even in the absence of a bulk cosmological constant, the general solution of this equation is extremely difficult to be found. It may be analytically solved with some approximations (at the low or high-energy regime). However, a complete solution for general gravitational background inevitably requires numerical methods. To solve numerically the master equation and extract the absorption coefficient under the Schwarzschild background, one needs to check the analytical behavior near the event horizon and infinity; while for the SSdS case, the solution near the cosmological horizon should be considered instead. A simple method will be given to obtain the desired asymptotic solutions. The key point of this method is to express the master equation using the two horizons, respectively. Each equation is used to solve the corresponding asymptotic solution. Substituting the metric functions expressed by two horizons into the master equation respectively, we have

$$C_2(r_{H/C})P'' + C_1(r_{H/C})P' + C_0(r_{H/C})P = 0,$$

where $C_i(r_{H/C})$ are the functions of event horizon/cosmological horizon. In order to find the suitable asymptotic equations which can be analytically solved, we expand the functions $C_i(r_{H/C})$ to the lowest non-vanishing order of $r - r_{H/C}$:

$$C_2^0(r_{H/C}) = -\left[3 - r_{H/C}^2K^2 + d(r_{H/C}^2K^2 - 1)\right] (r - r_{H/C})$$

$$C_1^0(r_{H/C}) = (s - 1)\left[3 - r_{H/C}^2K^2 + d(r_{H/C}^2K^2 - 1)\right] (r - r_{H/C})$$

$$C_0^0(r_{H/C}) = -ir_{H/C}\omega\{s \left[3 - r_{H/C}^2K^2 + d(r_{H/C}^2K^2 - 1)\right] - ir_{H/C}\omega\},$$

where $K^2 = 2\Lambda/(n+2)(N+3)$. Then the asymptotic equations are

$$C_2^0(r_{H/C})P'' + C_1^0(r_{H/C})P' + C_0^0(r_{H/C})P = 0,$$  \quad (15)

with solutions

$$P_s(r) = A_1(r - r_H)^{-\frac{1+ir_H}{d-3-(d-1)r_HK^2}}, \text{ for } r \gtrsim r_H \quad (16)$$

$$P_s(r) = B_1(r - r_C)^{-\frac{1+ir_C}{d-3-(d-1)r_CK^2}} + B_2(r - r_C)^{-\frac{1+ir_C}{d-3-(d-1)r_CK^2} + s}, \text{ for } r \lesssim r_C. \quad (17)$$

In Eq. (16) we keep only the incoming modes in order to satisfy the boundary condition at the black hole event horizon, while at the cosmological horizon in Eq. (17), both incoming and outgoing modes can exist. We will see that Eqs. (16) and (17) are suitable to the numerical computation for all fields on the brane.

### B. gravitons

A graviton can propagate in the bulk, so the gravitational background is described by the complete line element expressed in Eq. (11). Decomposing the graviton into a symmetric traceless tensor, a vector and a scalar part, Kodama and Ishibashi [38] have obtained the master equations for the Schwarzschild-like line element

$$-\frac{d^2\Phi}{dr_s^2} + V\Phi = \omega^2\Phi.$$  \quad (18)
The potential $V$ has a different form for each type of perturbation, namely

$$V_T = f(r) \left[ \frac{(d-2)(d-4)f(r)}{4r^2} + \frac{j(j+d-3)}{r^2} + \frac{(d-2)f'(r)}{2r} \right]$$

for tensor perturbation,

$$V_V = f(r) \left[ \frac{(d-2)(d-4)f(r)}{4r^2} + \frac{j(j+d-3)}{r^2} - \frac{(d-2)f''(r)}{2(r-3)} \right]$$

for vector perturbation, and

$$V_S = \frac{f(r)}{r^2} \left( \frac{q\alpha^3 + \rho\alpha^2 + \omega\alpha + z}{4m + 2(d-1)(d-2)\alpha^2} \right)$$

for scalar perturbation where

$$m = j(j+d-3) - (d-2)$$

$$\alpha = \frac{\mu}{\rho^{d-3}}$$

$$q = (d-2)^4(d-1)^2$$

$$p = (d-2)(d-1) \left[ (d-6)(d-4)(d-2)(d-1) + 4(2d^2 - 11d + 18)m \right] - 2(d-2)^2(d-1)d\Lambda^2$$

$$w = -12(d-2)m \left[ (d-4)(d-2)(d-1) + (d-6)m \right] + 24(d-4)(d-2)m\Lambda r^2$$

$$z = 4(d-2)dm^2 + 16m^3 - \frac{8(d-6)(d-4)m^2\Lambda r^2}{(d-2)(d-1)}.$$

One can find that three potentials have the same form as the ones for Schwarzschild black hole [39, 40] except the additional terms with $\Lambda$ in $p$, $w$, and $z$. The method to obtain the asymptotic equations is similar to the aforementioned one. Interestingly, we find that three different potentials have the same asymptotic behaviour. The asymptotic equations are similar as Eq. (15) but with different functions $C_n^0(r_{H/C})$

$$C_2^0(r_{H/C}) = \left[ 3 - r_{H/C}^2 K^2 + d(r_{H/C}^2 K^2 - 1) \right]^2 (r - r_{H/C})^2$$

$$C_1^0(r_{H/C}) = \left[ 3 - r_{H/C}^2 K^2 + d(r_{H/C}^2 K^2 - 1) \right] (r - r_{H/C})$$

$$C_0^0(r_{H/C}) = \omega^2 r_{H/C}.$$  

The corresponding solutions are

$$\Phi(r) = E_1(r - r_H) - \frac{\omega r_H}{d-3-(d-1)r_{H/C}^2}$$

$$\Phi(r) = E_1(r_C - r) - \frac{\omega r_C}{d-3-(d-1)r_{H/C}^2} + F_2(r_C - r) + \frac{\omega r_C}{d-3-(d-1)r_{H/C}^2}, \text{ for } r < r_H$$

$$\Phi(r) = F_1(r - r_H) - \frac{\omega r_H}{d-3-(d-1)r_{H/C}^2}$$

$$\Phi(r) = F_1(r_C - r) - \frac{\omega r_C}{d-3-(d-1)r_{H/C}^2} + F_2(r_C - r) + \frac{\omega r_C}{d-3-(d-1)r_{H/C}^2}, \text{ for } r < r_C.$$  

IV. ENERGY SPECTRUM AND ENTROPY VARIATION

To evaluate the absorption spectrum in a wide energy range accurately, it is necessary to turn to numerical calculations. The numerical integration of master equations (16) and (18) is performed from the black-hole event horizon, where appropriate boundary conditions (19) and (20) are applied, and extends to the cosmological horizon. Then the absorption coefficient can be extracted by fitting the analytical asymptotic solutions (17) and (20) to the numerical results. In the following, we will evaluate the absorption probability, energy spectrum and entropy variation of each particle species.
The absorption probability is defined as the ratio of the ingoing flux at two horizons. For scalar particles, the flux is

$$F = 2 r^2 \Im \left( \frac{P_0}{r^2} \frac{d}{dr} (r^2 F^+) \right).$$

The absorption probability is found as

$$|A_j^{(0)}|^2 = 1 - \left| \frac{B_2}{B_1} \right|^2.$$

Now we introduce the numerical method to obtain $|A_j^{(0)}|^2$ following Ref. [13]. The asymptotic solution near the cosmological horizon [17] can be written as

$$P_0(x) = B_1 F_- + B_2 F_+,$$

where $F_-$ and $F_+$ denote the ingoing and outgoing waves, respectively. Consider the Wronskians of $F^-$ and $F^+$, which have the following property

$$W[F_-, P_0] = F_-' P_0 - P_0 F_-' = B_2 W[F_-, F_+]$$

$$W[F_+, P_0] = F_+ P_0' - P_0 F_+ = -B_1 W[F_-, F_+] .$$

By solving the radial equation numerically and comparing two Wronskians, we obtain the absorption probability

$$|A_j^{(0)}|^2 = 1 - \left| \frac{W[F_-, P_0]}{W[F_+, P_0]} \right|^2 .$$

In Fig. 3 we depict the absorption probability of scalar field with respect to the energy parameter $\omega r_H$, for different angular momentum $j$, extra dimensions $n$, and the parameter of cosmological constant $\Lambda r_H^2$. It is important to note...
FIG. 4: The differential energy emission rate, for scalar emission on the brane, versus the parameter $\omega r_H$. For the left panel: the dimensionality of space-time is fixed at $n = 0$, while $\Lambda r_H^2$ takes the values $\{0, 0.01, 0.05\}$ from bottom to top; for the right panel: the cosmological constant is fixed at $\Lambda r_H^2 = 0$, and $n$ takes the values $\{0, 2, 4\}$ from bottom to top.

FIG. 5: The integrands of entropy variation, for scalar emission on the brane, versus the parameter $\omega r_H$, with $j = 0$ (red) and 1 (green). The top left panel shows the integrand of radiated entropy $g_j^{(0)}(\omega)$, with $\Lambda r_H^2 = 0.01$, and $n = 0, 1, 2$ from bottom to top; while in the top right panel, $n = 1$, and $\Lambda r_H^2 = 0, 0.01, 0.05$ from bottom to top. The corresponding integrand of lost entropy $h_j^{(0)}(\omega)$ are given in the bottom panel. One can find that the lowest mode of integrand $g_0^{(0)}(\omega)$ diverges as $\omega r_H \to 0$. But for higher modes or the lost entropy, the integrand is finite.

that for the lowest mode ($j = 0$) the absorption probability is nonvanishing in the presence of cosmological constant. This is definitely different from the case of Schwarzschild black hole. As pointed out in Ref. [8], the finite absorption coefficient in the infrared limit can be understood in the following way: in the presence of a second horizon, our universe is within the two horizon and therefore has only finite size, in which the particle propagates with infinite wavelength cannot be localized, and thus has a finite probability to propagate through the barrier and be absorbed in the black hole. The energy spectrum is given in Fig. 4 which is the same as that obtained by Ref. [8].

Now we turn to the entropy variation. One will find that it is not suitable to integrate Eq. (11) directly because it can not be exactly computed near zero frequency. For explicitly, we plot the integrands in Fig. 5 where it is shown that the lowest mode ($j = 0$) of integrands of entropy radiation tends to infinity as $\omega r_H \to 0$. It can be seen clearly from Eq. (12), where the term $|A^{(s)}|^2 \exp(\omega/T_H)/(\omega - 1)^s$ diverges at zero frequency because the denominator is zero for $s = 0$ and
the numerator $|A_j^{(s)}|^2 \neq 0$ for $j = 0$. Obviously, we can not simply truncate the integration since we do not know whether the truncated integration is finite and can be safely omitted. To tackle this problem, we propose to evaluate the lowest mode near $\omega \to 0$ analytically. Fortunately, this lowest mode has been found in [8] as

$$|A_0^{(s)}|^2 = \frac{4 (r_C r_H)^{n+2}}{(r_C^{n+2} + r_H^{n+2})^2}.$$  

Substituting this solution into Eq. (12), one can find that the integration is finite. Comparing the analytical integrand with the numerical result in Fig. 6, one can find they accord very well for small $\omega$, so the truncated integration can be safely performed. In Table I, we list the radiated entropy and lost entropy for different extra dimensions and the cosmological constant.

| $\Lambda r_H^2$ | 0          | 0.01       | 0.05       |
|---------------|------------|------------|------------|
| $n$ | 0 | 1 | 2 | 3 | 0 | 1 | 2 | 3 | 0 | 1 | 2 | 3 |
| $dS_{rad}$ | 6.914$n_0$ | 30.19$n_0$ | 79.12$n_0$ | 160.6$n_0$ | 14.09$n_0$ | 36.87$n_0$ | 85.39$n_0$ | 166.0$n_0$ | 26.76$n_0$ | 48.59$n_0$ | 96.41$n_0$ | 177.3$n_0$ |
| $dS$      | 3.74$n_0$  | 16.73$n_0$ | 44.97$n_0$ | 92.30$n_0$ | 7.535$n_0$ | 19.98$n_0$ | 47.78$n_0$ | 94.98$n_0$ | 15.45$n_0$ | 25.86$n_0$ | 53.46$n_0$ | 99.12$n_0$ |
| $R$       | $1.849$    | $1.805$    | $1.759$    | $1.740$    | $1.870$    | $1.845$    | $1.787$    | $1.748$    | $1.732$    | $1.879$    | $1.803$    | $1.788$    |

Table I: The entropy gained by radiation $dS_{rad}$, lost by black hole $dS$, and their ratio, for scalar field on the brane in different cases with $\Lambda r_H^2 = 0$, 0.01, 0.05 and $n = 0, 1, 2, 3$, respectively. The unit of entropy is $10^{-3}r_H^{-1}$ and $n_0$ denotes the scalar degrees of freedom.

B. fermion field ($s = 1/2$)

To construct the complete solution for the emitted field with non-vanishing spin, one should consider, in principle, both the upper and lower components of the field. However, the determination of either is more than adequate to compute the absorption coefficient. Considering the radial component of conserved current for fermions $J^\alpha = \sqrt{2} \sigma^\alpha_{AB} \Psi^A \bar{\Psi}^B$, Cvetić and Larsen [45] have obtained the ingoing flux

$$F = \left| P_+ \right|^2 - \left| P_- \right|^2.$$

We need to evaluate this formula at two horizons. Similar to the case for Schwarzschild black hole, we find that, from the asymptotic solutions near horizons (16) and (17), the upper and lower components of the emitted field mainly carry the ingoing and outgoing waves respectively. Thus, the ratio between the ingoing flux near two horizons may
In Fig. 7, we plot the absorption probability $|A_j|^2$ of fermion field as function of the energy parameters $\omega r_H$ with different $j$, $n$, and $\Lambda r_H^2$. Similar to the scalar case, one can find that the lowest mode of absorption probability is nonvanishing when $\omega \rightarrow 0$ in the presence of cosmological constant. Comparing Fig. 3 and Fig. 7, it can be noted that the nonvanishing absorption probability for fermion field is about ten times smaller than that of the scalar field. The corresponding energy spectrum is given in Fig. 8. It could be found that the energy emission rate is enhanced, when the space-time dimension or the cosmological constant increases. The peak is shifted towards higher energies when space-time dimension increases, but when the cosmological constant increases, the peak is not shifted significantly. These features are analogous to the behavior found in the asymptotically-flat Schwarzschild space-time and the
In solving the equation of motion for a new unknown radial function, depends on the boundary conditions. A solution to overcome this problem has been proposed in Ref. [44] and consists and can consequently contaminate the ingoing one. As a result, the numerical integration is not stable as it strongly wave-function is very small, thus a tiny error in the numerical solution can easily be mixed with the outgoing solution.

These new variables, the wave equation for gauge boson fields becomes

\[ \omega \rightarrow 0. \]

In Table II, we list the radiated entropy and lost entropy for different extra dimensions and cosmological constants.

| \( n \) | 0 | 0.01 | 0.05 |
|---|---|---|---|
| \( dS_{rad} \) | 3.370\( n \) | 24.41\( n \) | 68.07\( n \) | 13.76\( n \) | 6.020\( n \) | 70.92\( n \) | 140.2\( n \) | 9.823\( n \) | 32.05\( n \) | 75.73\( n \) | 145.0\( n \) |
| \( dS \) | 2.057\( n \) | 14.57\( n \) | 40.71\( n \) | 82.72\( n \) | 4.535\( n \) | 17.23\( n \) | 43.23\( n \) | 85.46\( n \) | 8.632\( n \) | 21.34\( n \) | 47.47\( n \) | 89.72\( n \) |
| \( R \) | 1.638 | 1.676 | 1.672 | 1.664 | 1.352 | 1.597 | 1.641 | 1.641 | 1.138 | 1.502 | 1.595 | 1.616 |

**TABLE II:** The two entropy variations and their ratio, for fermions on the brane in different cases. \( n_{1/2} \) denotes the fermionic degrees of freedom.

**C. gauge boson fields (s = 1)**

The calculation for gauge boson fields is more difficult, since the outgoing mode in the upper component of the wave-function is very small, thus a tiny error in the numerical solution can easily be mixed with the outgoing solution and can consequently contaminate the ingoing one. As a result, the numerical integration is not stable as it strongly depends on the boundary conditions. A solution to overcome this problem has been proposed in Ref. [44] and consists in solving the equation of motion for a new unknown radial function \( F_1 = yF(y)e^{-i\omega r} \) with \( y = r/r_H \). In terms of these new variables, the wave equation for gauge boson fields becomes

\[ f y^2 \frac{d^2F}{dy^2} + 2y(F - i\omega r_H y) \frac{dF}{dy} - j(j + 1)F = 0, \]

with the following boundary conditions:

\[ F(1) = 1, \quad \frac{dF}{dy} \bigg|_{y=1} = \frac{ij(j + 1)}{2\omega r_H}. \]

The ingoing flux is derived from the trace of the energy-momentum tensor \( T^{uv} = 2\sigma^A_{\alpha'\beta'} \Psi^{AB} \bar{\Psi}^{A'B'} \) evaluated over a two dimensional sphere. It has been got [13]

\[ F = \frac{1}{2\sqrt{2}\omega} \left( |P_1|^2 - |P_{-1}|^2 \right). \]

As for fermions, the absorption probability can be determined only by upper component of the field. So we get

\[ \left| A_{y}^{(1)} \right|^2 = \frac{|F_1|^2_{r = r_H}}{|F_{y}^2|_{r = r_C}} = \frac{1}{|F|^2_{y = r_C}}. \]

The last equality has used the new radial wave function and boundary condition.

In Fig. 9 and 10, we plot the absorption probability and energy spectrum. Compared with the scalar particles and fermions, the absorption probability of gauge bosons always vanishes when \( \omega \rightarrow 0 \), which ensures that the integrand of radiated entropy \( (12) \) is not divergent when \( \omega \rightarrow 0 \) although the denominator \( \exp(\omega/T_H) - (-1)^{2s} \) for \( s = 1 \) vanishes as \( \omega \rightarrow 0 \). In Table III, we list the radiated entropy and lost entropy.

**D. graviton**

The absorption probability for the graviton modes can be obtained similar to the scalar field case. We write it directly as

\[ |A_j|^2 = 1 - \left| \frac{F_2}{F_1} \right|^2 = 1 - \left| \frac{W(F_-, \Phi)}{W(F_x, \Phi)} \right|^2, \]
FIG. 9: Absorption probability $|A_j^{(1)}|^2$ of gauge boson field with $j = 1$ (red), 2 (green), 3 (blue). For the top panel: $\Lambda r_H^2 = 0.01$, and $n = 0, 1, 2$ from left to right; for the bottom panel: $n = 1$ and $\Lambda r_H^2 = 0.01$, 0.05 from left to right.

FIG. 10: The differential energy emission rate for gauge boson emission on the brane. For the left panel: the dimensionality of space-time is fixed at $n = 0$, while $\Lambda r_H^2$ takes the values $\{0, 0.01, 0.05\}$ from bottom to top; for the right panel: the cosmological constant is fixed at $\Lambda r_H^2 = 0.01$, and $n$ takes the values $\{2, 4, 6\}$ from bottom to top.

where $F_-$ and $F_+$ denote the ingoing and outgoing waves in asymptotic solution obtained in Eq. (20).

In Fig. 11 we plot the absorption probability of the three perturbations. One can find that the probability for the scalar mode graviton is always the biggest one, followed by the vector mode and tensor mode. It is known that, in the 4d limit, the vector mode corresponds to the gravitational axial perturbation [41], and the scalar mode corresponds to the gravitational polar perturbation [42]. There is no counterpart of the tensor mode in four dimension. An interesting feature is that the absorption probability for the vector mode and scalar mode of graviton by a SdS black hole are exactly the same in four dimensions, which is similar in the case of Schwarzschild black hole [46, 47]. The corresponding energy spectrum is plotted in Fig. 12. We find that the energy emission rate is enhanced for all modes when the space-time dimensions and cosmological constant increase. The peak is shifted towards higher frequency clearly for all modes when space-time dimensions increase. Moreover, we find that the energy emission rate of tensor mode is enhanced more obviously than vector and scalar modes when space-time dimensions increase. Similar to the case of gauge bosons, the integrand of radiated entropy is not divergent when $\omega \to 0$. In Table IV, we list the radiated entropy and lost entropy for different perturbations, cosmological constant, and the numbers of extra dimensions. One may have noticed a strange ratio $R = 0.957 < 1$ for four-dimensional gravitons with large cosmological constant $\Lambda r_H^2 = 0.05$. This is unreasonable since it destroys the generalized second law of thermodynamics. In fact, we find...
scalar particles are rather elusive to be detected, the present analysis on the spectrum for other particles is important. The exact spectrums have not been obtained except for scalars and gravitons at low and asymptotic frequency. Since for all types of particles, including scalars, fermions, gauge bosons, and the gravitons with three modes. Until now, we calculate the greybody factor and energy spectrum of Hawking radiation with a positive cosmological constant. We first studied the greybody factor and find that for fermions, the factor for the lowest mode is nonvanishing in the standard temperature and find that the ratio is always bigger than 1. For examples, the ratio for gravitons with $n_1$ denotes the bosonic degrees of freedom.

| $\Delta r_H H$ | 0 | 0.01 | 0.05 |
|---------------|---|------|------|
| $n$ | 0 | 1 | 2 | 3 | 0 | 1 | 2 | 3 | 0 | 1 | 2 | 3 |
| $d\mathcal{S}_{\text{rad}}$ | 1.268$n_1$ | 17.87$n_1$ | 64.19$n_1$ | 147.51$n_1$ | 2.837$n_1$ | 20.82$n_1$ | 67.41$n_1$ | 150.61$n_1$ | 5.533$n_1$ | 25.30$n_1$ | 72.95$n_1$ | 156.51$n_1$ |
| $d\mathcal{S}$ | 0.845$n_1$ | 11.47$n_1$ | 40.72$n_1$ | 93.23$n_1$ | 2.272$n_1$ | 13.98$n_1$ | 43.46$n_1$ | 95.89$n_1$ | 5.222$n_1$ | 17.98$n_1$ | 48.24$n_1$ | 101.01$n_1$ |
| $R$ | 1.500 | 1.557 | 1.576 | 1.582 | 1.249 | 1.489 | 1.551 | 1.570 | 1.060 | 1.407 | 1.512 | 1.549 |

TABLE III: The two entropy variations and their ratio, for bosons on the brane in different cases. $n_1$ denotes the bosonic degrees of freedom.

![Graph](image.png)

FIG. 11: Absorption probability $|A_j^{(2)}|^2$ for tensor (solid lines), vector (short-dashed lines) and scalar (long-dashed lines) gravitational perturbations in the bulk with $j = 2$ (red), 3 (green), 4 (blue). For the top panel: $\Delta r_H H = 0.01$, and $n = 0, 1, 2$ from left to right; for the bottom panel: $n = 1$ and $\Delta r_H H = 0, 0.01, 0.05$ from left to right.

that, if the cosmological constant is big and the space-time dimensions are small enough, i.e. the black hole is close to the Nariai black hole, the ratio $R$ may be smaller than unit for all types of particles. For example, the ratio $R$ is 0.931 for bosons with $n = 0$ and $\Delta r_H H = 0.1$. Hence, there should be something wrong near the Nariai limit. In the recent work [48] on the thermal stability of Nariai black hole, it was pointed out that black hole thermodynamics favors the standard Hawking temperature rather than the Bousso-Hawking temperature, because the later is inappropriate to describe either the Hawking-Page phase transition or the evaporation process. We evaluate the ratio $R$ using the standard temperature and find that the ratio is always bigger than 1. For examples, the ratio for gravitons with $n = 0$ and $\Delta r_H H = 0.05$ now is $R = 1.376$, and the ratio for bosons with $n = 0$ and $\Delta r_H H = 0.1$ now is $R = 1.407$. We show the comparison between these two temperatures with some indicative cases near the Nariai limit in Table V. These results favor the argument given in [48] but obtained from a different aspect of thermodynamics.

V. CONCLUSION AND DISCUSSION

In this paper, we have studied the radiation of $(4+n)$-dimensional braneworld black hole imbedded in the space-time with a positive cosmological constant. We calculate the greybody factor and energy spectrum of Hawking radiation for all types of particles, including scalars, fermions, gauge bosons, and the gravitons with three modes. Until now, the exact spectrums have not been obtained except for scalars and gravitons at low and asymptotic frequency. Since scalar particles are rather elusive to be detected, the present analysis on the spectrum for other particles is important. We first studied the greybody factor and find that for fermions, the factor for the lowest mode is nonvanishing in the low-energy limit, similar to the scalar field [8], while different from the cases of bosons and gravitons. The energy emission is found to increase significantly for all particles with the increasing cosmological constant and extra
dimensions. And the peak of the spectrum shifts towards high energy when space-time dimensions increase, however, it is not shifted significantly with the increasing cosmological constant. These results show that it is possible to detect two parameters from the energy spectrum of braneworld SSdS black hole. However, compared with the nonvanishing energy emission rate of scalar fields in the low-energy limit, the energy emission rates for other particles vanish as \( w \to 0 \). Therefore, it is difficult to detect the emission of ultra-soft quanta by SSdS black hole which is expected in the Nariai limit, rather than the Bousso-Hawking temperature which is disfavored by the generalized second law of thermodynamics. We find that the ratios of two entropies are near unit for all fields. It should be emphasized that this result can not be foreseen easily, especially for the scalar field, where the integrand of radiated entropy is divergent in the low-energy limit while the integrand of the lost entropy is finite. Our results strongly favor Bekenstein’s conjecture,

\[ \text{FIG. 12: The differential energy emission rate for tensor (solid lines), vector (short-dashed lines) and scalar (long-dashed lines) gravitational perturbations in the bulk. For the top panel: } n = 1, \text{ while } \Lambda r^2 = 0.01, \text{ and } n \text{ takes the values } \{0, 0.01, 0.05\} \text{ from left to right; for the bottom panel: } \Lambda r^2 = 0.01, \text{ and } n \text{ takes the values } \{3, 5, 7\} \text{ from left to right.} \]

| \( n \) | scalar | vector | scalar | vector | tensor | scalar | vector | tensor | scalar | vector | tensor |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| \( \Delta r^2_H \) | 0.130n_{s_1} | 0.130n_{s_2} | 0.130n_{s_3} | 0.130n_{s_4} | 0.130n_{s_5} | 0.130n_{s_6} | 0.130n_{s_7} | 0.130n_{s_8} | 0.130n_{s_9} | 0.130n_{s_{10}} | 0.130n_{s_{11}} |
| \( dS_{rad} \) | 0.097n_{v_1} | 0.097n_{v_2} | 0.097n_{v_3} | 0.097n_{v_4} | 0.097n_{v_5} | 0.097n_{v_6} | 0.097n_{v_7} | 0.097n_{v_8} | 0.097n_{v_9} | 0.097n_{v_{10}} | 0.097n_{v_{11}} |
| \( R \) | 1.347 | 1.347 | 1.347 | 1.347 | 1.347 | 1.347 | 1.347 | 1.347 | 1.347 | 1.347 | 1.347 |
| \( \Delta r^2_H \) | 0.047n_{t_1} | 0.047n_{t_2} | 0.047n_{t_3} | 0.047n_{t_4} | 0.047n_{t_5} | 0.047n_{t_6} | 0.047n_{t_7} | 0.047n_{t_8} | 0.047n_{t_9} | 0.047n_{t_{10}} | 0.047n_{t_{11}} |
| \( dS_{rad} \) | 0.397n_{t_1} | 0.397n_{t_2} | 0.397n_{t_3} | 0.397n_{t_4} | 0.397n_{t_5} | 0.397n_{t_6} | 0.397n_{t_7} | 0.397n_{t_8} | 0.397n_{t_9} | 0.397n_{t_{10}} | 0.397n_{t_{11}} |
| \( R \) | 1.125 | 1.125 | 1.125 | 1.125 | 1.125 | 1.125 | 1.125 | 1.125 | 1.125 | 1.125 | 1.125 |

\[ \text{TABLE IV: The two entropy variations and their ratio, for gravitons in the bulk in different cases. } n_{s_2}, n_{v_2}, \text{ and } n_{t_2} \text{ denote the gravitational degrees of freedom for scalar } (n_{s_2}), \text{ vector } (n_{v_2}), \text{ and tensor } (n_{t_2}) \text{ perturbations, respectively.} \]
TABLE V: The two entropy variations and their ratio for the Boussou-Hawking temperature $T_H$ and the standard Hawking temperature $T_0$. $\lambda_n$ denotes the cosmological constant $\Lambda r_H^2$ when the Nariai limit $r_H = r_C$ arises for different space-time dimensions $n$.

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| $n$ and $s$ | $n=0$ s=2 (vector mode) ($\lambda_0 = 1$) | $n=1$ s=1 ($\lambda_1 = 3$) | $n=2$ s=1 ($\lambda_2 = 6$) | $n=3$ s=0 ($\lambda_3 = 10$) |
|-------------|------------------------------------------|-----------------------------|----------------------------|----------------------------|
| $\Delta r_H^2$ | 0.1$\lambda_0$ | 0.5$\lambda_0$ | 0.9$\lambda_0$ | 0.1$\lambda_1$ | 0.5$\lambda_1$ | 0.9$\lambda_1$ | 0.1$\lambda_2$ | 0.5$\lambda_2$ | 0.9$\lambda_2$ | 0.1$\lambda_3$ | 0.5$\lambda_3$ | 0.9$\lambda_3$ |
| $\Delta S_{rad}$ | 2.456$n_{2v}$ | 49.51$n_{2v}$ | 2215$n_{2v}$ | 43.34$n_{1v}$ | 187.2$n_{1v}$ | 3547$n_{1v}$ | 92.72$n_{1v}$ | 294.7$n_{1v}$ | 4111$n_{1v}$ | 293.7$n_{0v}$ | 710.0$n_{0v}$ | 6208$n_{0v}$ |
| $T_H$ | 2.908$n_{2v}$ | 130.9$n_{2v}$ | 29077$n_{2v}$ | 36.80$n_{1v}$ | 335.8$n_{1v}$ | 36453$n_{1v}$ | 63.79$n_{1v}$ | 454.9$n_{1v}$ | 35043$n_{1v}$ | 181.4$n_{0v}$ | 852.0$n_{0v}$ | 45156$n_{0v}$ |
| $R$ | 0.844 | 0.378 | 0.0762 | 1.178 | 0.558 | 0.0973 | 1.453 | 0.648 | 0.117 | 1.619 | 0.838 | 0.138 |
| $T_0$ | 0.114$n_{2v}$ | 0.048$n_{2v}$ | 0.00589$n_{2v}$ | 0.167$n_{1v}$ | 7.972$n_{1v}$ | 1.182$n_{1v}$ | 64.29$n_{1v}$ | 31.69$n_{1v}$ | 5.460$n_{1v}$ | 194.9$n_{0v}$ | 137.8$n_{0v}$ | 26.61$n_{0v}$ |
| $s$ | 0.0809$n_{2v}$ | 0.0295$n_{2v}$ | 0.00304$n_{2v}$ | 0.916$n_{1v}$ | 4.378$n_{1v}$ | 0.604$n_{1v}$ | 37.84$n_{1v}$ | 15.68$n_{1v}$ | 2.345$n_{1v}$ | 95.9$n_{0v}$ | 68.03$n_{0v}$ | 13.35$n_{0v}$ |
| $R$ | 1.405 | 1.645 | 1.935 | 1.630 | 1.817 | 1.959 | 1.699 | 2.021 | 2.328 | 2.032 | 2.025 | 1.993 |
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even if the extra dimensions and cosmological constant exist. It further assures us the deep relationship between the gravitational entropy and the statistical entropy and is useful to fully understand the entropy of braneworld SSdS black hole.

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