Exergy-based model predictive control for design and control of a seasonal thermal energy storage system

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Abstract. In this paper, we investigate the problem of controlling a seasonal thermal energy storage (STES). The STES considered here is a large scale tank of heated water installed in a building and connected to a solar panel. The stored energy in the STES can be used for providing the building with the space heating (SP) and the domestic hot water (DHW). In order to utilize the STES efficiently, we design a suitable model predictive control (MPC) scheme. In this regard, we develop an appropriate model for the system with an emphasis on the computational tractability of problem. Toward this end, we introduce a bilinear model with analytical linearization. Subsequently, we solve the optimization problem using a sequential quadratic programming (SQP) framework in a reasonable computational time. For controlling the system, in addition to solving the corresponding optimization problem, the main challenge is incorporating seasonal features in the MPC. This issue is resolved by augmenting the cost function with an additional term which is defined based on the energy of system. Moreover, we address the challenging question of deriving minimal achievable size of the STES tank while satisfying user demand of DHW and SP. Finally, the efficiency of the proposed method is verified numerically.

1. Introduction

The building sector consumes nearly 40\% of the global energy in the U.S. and Europe with the major part dedicated to heating demand [1]. These newly developed resources are not available uniformly through the time. This highlights the importance of energy storage technologies for satisfying the demand at any time. As an example one can consider the solar panels which are collecting the solar radiation to be used in the building for space heating (SH) [7]. The time of demand for space heating and the time of solar radiation energy harvesting are not completely the same, one is mainly in the winter and the other one is mainly in the summer. The potential solution for this issue is utilizing a seasonal thermal energy storage (STES) which acts as a thermal buffer between the two seasons [5]. Here, we consider a type of STES which is a large scale tank (100 m\textsuperscript{3}) storing energy in the form of heated water. The STES considered here differs from various aspects to the similar types of storage in the literature. For example, despite [8] where the tank is placed in the ground, the STES here is inside the building in order to recover
the heat losses. Also, differently from [11], the system is not endowed with a heat pump and only receives energy from a large solar collector.

In order to utilize the STES efficiently, and consequently minimize the storage volume, it is required to employ a suitable control strategy. In this regard, one may employ a model predictive control (MPC) scheme for the system. Since the STES is charged and discharged according to the conductive and convective heat transfer, MPC helps to plan in advance and therefore, maintain a high level of temperature in the tank. Additionally, the weather forecast and possible constraints can be taken into account. The dynamics of the STES makes the control problem challenging. Therefore, it is necessary to utilize suitable relaxations and approximations of the dynamics [2]. From the control strategy point of view, various works exist investigate a small storage with only one outlet and only one inlet, at the top and at the bottom of the tank, respectively [9]. Moreover, according to the intrinsic difficulty of the corresponding optimal control problem, daily operation of the system is considered merely in these works.

In this paper, we derive a model for the system similar to [2], and subsequently, we design a MPC scheme toward implementing an optimal control strategy for the seasonal operation. In this regard, the model is derived with an emphasis on being suitable for the optimization and MPC implementation. In the modeling, we address the challenging issue of incorporating the natural convection of the water inside the tank. Following this, the optimization problem is formulated where the binary variables are avoided. The details of solving methodology are discussed. Finally, the design considerations for the potentials of storage volume reduction are addressed.

2. System modeling
The general scheme of STES system is shown in Figure 1. The system is composed of a large size tank filled with water, a solar collector connected to the heat exchangers (yellow), a flow circuit for the domestic hot water (blue) connected to the boilers (blue) and the space heating (inlet in red, outlet in orange), a residential building and an external circuitry for exchanging heat with the neighboring building. The heat energy provided by the solar collector and the neighboring buildings is stored in the tank to be used for demand of space heating and the domestic hot

Figure 1: The scheme of seasonal energy storage system with a simple house.
water. The flows and subsequently the system are controlled by the valves and pumps. Here, the main goal is controlling the temperature profile in the tank and meanwhile satisfying the demand.

The system is modeled based on its thermodynamics nature which is described in terms of the energy flows, the temperatures and the mass flows. Similar to [2], we discretize the tank and the corresponding units inside the tank into \( N_s \) slices and subsequently, model their interactions and the heat transfers accordingly. We set the indices of slices with \( i = 1, \ldots, N_s \), from the bottom to the top in an increasing order. Let the vector of states, the vector of inputs, and the vector of disturbances are respectively denoted by \( x \in \mathbb{R}^{n_x} \), \( u \in \mathbb{R}^{n_u} \) and \( v \in \mathbb{R}^{n_v} \), and defined as

\[
x = [T_{sc}, T_{es}, T_1, \ldots, T_{N_s}, T_{inflow}, T_{outflow}; T_{h}, T_{r,in}, T_{ext}; T_{hex,1}, \ldots, T_{hex,N_s}; T_{boil,1}, \ldots, T_{boil,N_s}; T_{p,boil,1, \ldots, T_{p,boil,N_s}}]^T,
\]

\[
u = [f_{out,1}, \ldots, f_{out,N_s}, f_{in,1}, \ldots, f_{in,N_s}, f_{hex,1}, \ldots, f_{hex,N_s}, f_{buy,1}, \ldots, f_{buy,N_s}, f_{MV}, h_{sell}]^T,
\]

\[
v = [I; T_a, T_{fw}, f_{DHW}, h_{fw}]^T,
\]

where \( T_h \) is the average temperature of the house, \( T_{inflow} \) is the average temperature of the house inlet pipe, \( T_{outflow} \) is the average temperature of the house outlet pipe before the mixing valve, \( T_{r,in} \) is the average temperature of outlet pipe of of the house after the mixing valve and after the outlet of the external heat source, \( T_{es} \) is the temperature of the emitter system, \( T_{sc} \) is the temperature of the solar collector, \( J \) is the incoming solar radiation, \( T_{bw} \) is the temperature of the fresh domestic water, \( T_a \) is the ambient temperature, \( f_{sc} \) is the solar collector flow, \( f_{flow} \) is the house inlet pipe inflow, \( f_{MV} \) is the recirculation flow from the outlet to the inlet pipe of the house, \( T_{ext} \) is the temperature of the external exchange source and \( f_{DHW} \) is user demand mass flow of hot water which is equal to the incoming flow of fresh water \( f_{fw} \) due to mass conservation. Moreover, at \( i \)th slice of the tank, we have that \( T_i, T_{hex,i}, T_{boil,i}, T_{p,boil,i} \) is the corresponding temperature of the tank, temperature of the heat exchanger, temperature of the boiler, and temperature of the interconnecting pipe between the two boilers, respectively. Similarly, \( f_{out,i} \) is the outflow, \( f_{in,i} \) is the inflow and \( f_{buy,i} \) is the mass flow of the valve that goes from the tank to the external exchange system. The system has a bilinear dynamics due the multiplication of the mass flows and the temperatures which are the control signals and the state variables, respectively. The resulting dynamics of system is in the following form:

\[
\dot{x} = f(x, u, v) = A(v) \ x + B(v) \ u + N_{ux} \ (u \otimes x) + Dv,
\]

where \( \otimes \) denotes the Kronecker product. Given the dynamics, we can define the problem in the continuous time case as the following optimal control problem

\[
\min_{x, u} \ J(s, x, u) = \int_0^T \left[ \gamma_{sh}(\bar{u}_h + \bar{s}_h) + \gamma_{SDHW}s_{DHW} + f_c^T u + J_{oc}(x, u) \right] dt,
\]

s.t.

\[
\dot{x} = A(v) \ x + B(v) \ u + N_{ux} \ (u \otimes x) + Dv, \quad x(0) = x_0,
\]

\[
Gx + Gu \leq g,
\]

where \( \leq \) is a component-wise inequality, \( T \) is the final time horizon, \( \bar{s}_h, \bar{u}_h, s_{DHW} \) represent the amount of comfort violation, \( \gamma_{sh}, \gamma_{SDHW} \) are the corresponding weights of induced costs, \( f_c \) is the net of the electricity cost induced by the hydraulic pumps and profits earned by selling the heat to the grid, \( J_{oc}(x, u) \) is an additional cost term which is defined in the sequel, \( G, G, g \) linear constraints for a safe system operation and \( A(v), B(v), N_{ux}, D \) matrices of the dynamics.

The continuous time optimal control problem (3) is essentially a difficult problem to be solved, specifically due to the nonlinear dynamics of system and also the uncertainties induced by the
climate variables. In this regard, we use an implicit Euler scheme with zero order hold for deriving a discrete time version of this problem. Based on this discretization scheme, we can maintain the bilinear form of the dynamics and also consider a large discretizing time step (e.g., 1 hour) with guaranteed numerical stability. An important issue to be considered is the self-mixing of water inside the tank which is according to the natural convection. However, using naively the introduced discretization scheme, this fundamental phenomena is not modeled. In order to fix this issue, we slightly modify the continuous time dynamics before the discretization phase as

$$\dot{x} = f(P_{\text{mix}} \cdot x, u, v),$$

where $P_{\text{mix}}$ is the matrix for averaging operation between the slices of water in the tank which are subject to the self-mixing phenomena. Moreover, we need to consider the forced convection phenomena which is modeled in [2] using binary variables. Since mixed-integer optimization problems are not desirable from computational perspective, we introduced the relaxed forced convection, i.e., for slice $i$, we define $h^{\text{relaxed}}_{\text{conv},i}$ as

$$h^{\text{relaxed}}_{\text{conv},i} := \begin{cases} -f_{\text{top},i} c_p \frac{1}{2}(T_{i+1} + T_i) & i = 1 \\ f_{\text{bot},i} c_p \frac{1}{2}(T_{i-1} + T_i) & i = N_a \\ f_{\text{bot},i} c_p \frac{1}{2}(T_{i-1} + T_i) - f_{\text{top},i} c_p \frac{1}{2}(T_{i+1} + T_i) & \text{else} \end{cases}$$

where $f_{\text{top},i}$ and $f_{\text{bot},i}$ represent the flow at the top and the bottom of the slice $i$, respectively, and $c_p$ is the heat capacity of water.

The resulting dynamics is suitable for linearization, and therefore, one can use a Sequential Quadratic Programming (SQP) approach for solving the obtained optimization problem which is the discretized version of the optimal control problem (3). In fact, the model is particularly favorable in the linearization step of SQP, since the linearized bilinear dynamics has an analytical form and $P_{\text{mix}}$ can be efficiently computed. This is also an starting point for formulating the model predictive control scheme.

3. Model Predictive Control

For the MPC setup, we consider the standard scheme. More specifically, we use the model with the relaxed forced convection, the current state of the system $x_0$, the planning horizon of $T = 1$ day and the action horizon of 1 hour. At each iteration, once the optimization problem is solved, the first part of optimal input is applied to the system for the next hour. Though, the model used in the MPC implementation is the relaxed version, the plant is simulated using the mixed-integer forced convection in order to have a realistic simulation.

Being the planning horizon of MPC short, it is well known that the naive implementation of the algorithm will not lead to consideration of the long term objectives and therefore, the MPC will not produce desirable results. Accordingly, the main challenge is the incorporation of the knowledge of long horizon into the current short horizon optimization problem. The main idea is to maintain a reasonable level of stratification of the tank in order to keep the highest temperature the longest possible during the winter. In this regard, similar to [3], we include an extra term in the cost function, denoted by $J_{\text{ex}}$. More precisely, we use the exergy content of the STES tank which is required to be maximized for maintaining a satisfactory stratification, as proposed in [10]. In other words, we define

$$J_{\text{ex}}(x, u) = \int_0^T \sum_{i=1}^{N_c} m_i c_p T_i - m_i c_p T_{\text{ex,ref}} \log \left( \frac{T_i}{T_{\text{ex,ref}}} \right) \, dt$$

where $T_{\text{ex,ref}}$ is the desired reference temperature of hot water delivery (60°C). This is a nonlinear function to be linearized for SQP step. Including $J_{\text{ex}}$ in the cost function does not guarantee
retrieving a full STES tank, defined as \( T_i \geq 95^\circ C \) \( \forall i = 1, \ldots, N_t \). In this regard, we take an approach adapted from \cite{4} where a data-driven scheme is build in order to approximate the dynamic over 1 year. The exact procedure in \cite{4} involves solving the optimization problem for 1 year which is not tractable here. In order to solve this issue, we down-sample for time period after the planning horizon. In fact, starting from the end of planning horizon, \( T \), we can simulated the system until the end of year with a time step of 1 month and using historical weather data of different years. This is referred as the approximated horizon (see Figure 2a) which is used to introduce new constraints. More precisely, for \( j = 1, \ldots, N_{ss} \), let \( \psi^j \) be the realization of weather for year \( j \), \( x_{ss,1year}^j(x(T), \psi^j, u) = \text{vec} \{ T_i \mid i = 1, 2, \ldots, N_t \} \) be the (simulated) states of STES at the end of year \( j \) given the control input \( u \), the weather realization \( \psi^j \), and being the system initialized at end of planning horizon by \( x(T) \). Accordingly, we define the following constraints

\[
T_p - x_{ss,1year}^j(x(T), \psi^j, u) \leq 0^\circ C, \quad \forall j = 1, 2, \ldots, N_{ss},
\]

where \( T_p = 95^\circ C \) is the minimal charged state of the tank using the past data \( \psi^j \). For the case of simplicity, we only consider the case \( N_{ss} = 1 \). In other words, we use the historical weather data recorded at Zurich for year 2010. Also, for the user demand of DHW, we use the hourly averaged data following \cite{6}.

4. Numerical Results

Given the introduced optimal control strategy and subsequent derived MPC scheme, one can address various interesting questions. Here, we focused on the design problem of determining the minimal volume of the tank for which it is possible to supply the house with domestic hot water and space heating during one year of operation. The most restrictive constraint here is the feasibility of delivering the DHW at 60°C. In fact, while a STES tank at temperature below 60°C, e.g. 40°C, can still provide the building with the space heating, it is not feasible to fulfill the requirements for domestic hot water. Given the specifications of a nominal building and also historical weather data, the exergy-based MPC is simulated for different sizes of the tank in order to find the minimum operational volume. This value is estimated as \( 85 m^3 \) and the corresponding operational results are shown in Figure 2b. In order to assess the performance of the introduced MPC and also make a comparison, the optimal control problem is solved for the time horizon of one year (sampled in 2 days) and consequently, the minimum operational volume of tank is
estimated to be 80m$^3$ (see Figure 2a). One can see that the exergy-based MPC performs closely to the optimal operation and in this sense, it is a suitable control strategy. In fact, the tank is almost empty at some point of the year (Figure 2b) since the top slice of the STES reaches 60°C while the temperatures of other slices are at the building minimum allowed temperature (around 20°C). Note that the approximated dynamics is shown in Figure 2b and a suitable planning of the periodicity of the STES is performed accordingly.

5. Conclusion
In this paper, the problem of controlling a seasonal thermal energy storage is investigated. In this regard, a MPC strategy is implemented. First, a model of the system has been developed with an emphasis on having a reasonable computation time. The form of the introduced dynamics is bilinear. The linearization of model has an analytical form. The resulting optimization problem is solved computationally efficient using a SQP framework. In order to incorporate long-term shifts and seasonal features in the MPC, an operational cost is introduced by considering the exergy of system. We have addressed the question of estimating the minimal achievable size of the STES tank while satisfying the demand for DHW and SP is guaranteed. This problem is of significant interest for reducing the investment cost. The proposed control method performed well in the simulated case study.

In the future work, we will investigated more sophisticated cases where low-level controllers will be implemented in addition to the MPC. Moreover, one can include more extended models of the building in the introduced framework. Finally, sharing the STES tank among multiple buildings is an interesting problem to be studied.

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