Wide spread spectrum watermarking with side information and interference cancellation

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ABSTRACT
Nowadays, a popular method used for additive watermarking is wide spread spectrum. It consists in adding a spread signal into the host document. This signal is obtained by the sum of a set of carrier vectors, which are modulated by the bits to be embedded. To extract these embedded bits, weighted correlations between the watermarked document and the carriers are computed. Unfortunately, even without any attack, the obtained set of bits can be corrupted due to the interference with the host signal (host interference) and also due to the interference with the others carriers (inter-symbols interference (ISI) due to the non-orthogonality of the carriers). Some recent watermarking algorithms deal with host interference using side informed methods, but inter-symbols interference problem is still open. In this paper, we deal with interference cancellation methods, and we propose to consider ISI as side information and to integrate it into the host signal. This leads to a great improvement of extraction performance in term of signal-to-noise ratio and/or watermark robustness.

Keywords: Robust watermarking, spread spectrum, side information, interference cancellation

1. INTRODUCTION
First studies in robust watermarking were mostly empirical. The domain became more academic when the watermarking problem was considered as communication over a noisy channel: the watermark is a signal to be transmitted through a channel corrupted by noise due to the cover signal and attacks. Watermarking was then considered as a kind of channel coding. The latest contributions then focused on theoretical studies, inspired by information theory, but not usable as such.

Due to constraints on the embedding distortion (MSE or weighted MSE), the power of the transmitted signal is limited. The communication channel is noisy due to attacks. It has often been modeled as the addition of white Gaussian noise (AWGN channel). The host signal has then often been considered as a noise that limits the performance of the watermarking scheme. But recently, it has been shown that watermarking can be regarded as a problem of communication with side information: a part of the added noise (i.e. the host signal) is perfectly known during the embedding process. Costa studied this kind of channel and gave a limit of capacity, independent of the host signal. He also exhibited a theoretical algorithm (the Ideal Costa Scheme) to reach this limit, considering i.i.d. Gaussian signals and AWGN transmission. However since this scheme relies on exhaustive search among codevectors, practical implementation of this scheme is not realistic. Some implementations inspired by the ICS were then proposed, using structured codebooks: Eggers’s SCS or syndrome based codes.

Costa’s scheme assumes i.i.d. Gaussian signals. Unfortunately, real multimedia signals are not so simple. Moreover, attacks may be not modeled as simple AWGN channels. Several studies proposed to consider non i.i.d. SAWGN channels. Indeed this class of attacks allows to take into account for filtering (such as Wiener filtering for noise removal), scaling, addition of noise correlated to the host signal, noise from compression... Furthermore, it has been shown that optimal attacks are of the kind SAWGN. In order to use ICS properties, watermarking in a linear subspace using wide spread spectrum (WSS) has been considered. While our previous work assumes non i.i.d. Gaussian signals, thanks to the use of spread transform subspace, projected host signal and attack noise are i.i.d and Gaussian. Furthermore, this scheme leads to a practical...
implementation with performances close to optimal.\textsuperscript{13} However adding a watermark in this subspace introduces symbol interference due to the non-orthogonality of the carriers used for the spread transform. This ISI, like the host signal in non-informed watermarking, limits the performance of the scheme.

This paper deals with a practical and complete informed watermarking scheme, using spread spectrum and structured codebooks. It also provides a solution to symbol interference cancellation. In Sec. 2, we recall the subspace-based approach and introduce a structured codebook based on punctured convolutional codes. In Sec. 3, we first study two ISI cancellation methods, and we then provide an iterative algorithm to consider symbol interference as side information, illustrated by experimental results in Sec. 4. We finally conclude this paper in Sec. 5.

2. SPREAD SPECTRUM FOR SIDE INFORMED WATERMARKING

We have shown in our previous work\textsuperscript{12, 13} a practical scheme that achieves performances close to the optimal bounds.\textsuperscript{10} The watermark is embedded in a linear subspace: i.i.d. Gaussian signals are then obtained and ICS can be applied. We first recall in this section the original Costa’s approach. In order to render realistic ICS, we then introduce a structured codebook (dirty paper codes) based on convolutional codes. We finally describe our WSS-based embedding method, optimized using game theory (min-max optimization).

2.1. Channel with side information: Costa’s approach

As seen in the introduction, the watermarking problem can be seen as a communication process with side information available at the encoder.\textsuperscript{3} This kind of channel have been studied by Costa,\textsuperscript{4} which leaded to an upper bound of capacity for this kind of channel.

Let us consider a $n$-long i.i.d. Gaussian host signal $\mathbf{x}$, whose samples are modeled by $X \sim \mathcal{N}(0, Q)$. This signal is perfectly known during the embedding process. We transmit our data with a watermark signal $\mathbf{w} = \{w_1, w_2, \ldots, w_n\}$ as seen on Fig. 1. The energy of $\mathbf{w}$ is bounded so that

$$\frac{1}{n} \sum_{i=1}^{n} w_i^2 \leq P.$$  \hfill (1)

The transmitted signal is then $\mathbf{y} = \mathbf{x} + \mathbf{w}$. This signal is corrupted during the transmission by an added Gaussian noise $\mathbf{z}$, modeled by $Z \sim \mathcal{N}(0, N)$. Receiver then gets the signal $\mathbf{y}' = \mathbf{y} + \mathbf{z}$. If we consider this channel as a classical Gaussian one, two noises are added to the transmitted signal, so the capacity is given by

$$C = \frac{1}{2} \log_2 \left[ 1 + \frac{P}{Q + N} \right].$$  \hfill (2)

Side information $\mathbf{x}$ impacts on the performance of the system, lowering the capacity. Costa showed that the side information does not influence the optimal capacity of the channel, \textit{i.e.}

$$C = \frac{1}{2} \log_2 \left[ 1 + \frac{P}{N} \right].$$  \hfill (3)
when embedding a watermark associated to codevector \( \hat{u} \) is associated to a sub-codebook \( \mathcal{U}^+ \) so that \( \hat{u} \) is chosen. The watermark signal is transmitted signal to go toward the codevector, as illustrated by Fig. 2. At the extraction process, the closest codewords, based on codebooks used for error correcting codes (ECC), are used. During the embedding process, the closest codeword \( \hat{u} \) to be much more simpler than an exhaustive search.

Ha gave a theoretical method to reach this value. He considered a signal \( U \sim \mathcal{N}(0, P + \alpha^2 Q) \), know both from the embedder and the extractor. The capacity of the channel is then given by

\[
C = \max_{\alpha} \left\{ I(U; Y) - I(U; X) \right\},
\]

where \( Y \sim \mathcal{N}(0, Q + P) \) models the transmitted signal \( y \). Costa showed that the previous equation leads to the optimal value \( \alpha = P/(P + N) \), and then to Eqn. 3. The signal \( U \) is obtained using a structured codebook of \( 2^{nI(U; Y) - \epsilon} \) elements\(^1\), designed to be a surjective function between the set of possible messages to embed \( \mathcal{M} \) and the codebook \( \mathcal{U} \): each possible message \( m \) is associated to a sub-codebook \( \mathcal{U}_m \) composed of \( 2^{nI(U; X)} \) codewords. During the embedding process, the closest codeword \( \hat{u} \in \mathcal{U}_m \) is chosen. The watermark signal is then given by

\[
\hat{w} = \hat{u} - \alpha x.
\]

Whereas classical watermarking techniques would have transmitted \( x + \sqrt{nP} \times u^* / \|u^*\| \), the \( \alpha \) term forces the transmitted signal to go toward the codevector, as illustrated by Fig. 2. At the extraction process, the closest codeword \( \hat{u} \in \mathcal{U} \) is computed. The decoded message is then \( \hat{m} \) so that \( \hat{u} \in \mathcal{U} \).

2.2. Dirty paper codes from punctured ones

The original ICS is based on large random codebooks: the only way to decode \( y' \) is by an exhaustive search in \( \mathcal{U} \). Some practical but suboptimal approaches, inspired by the ICS, have been proposed for i.i.d. Gaussian host signals, based on codebooks used for error correcting codes (ECC),\(^{5, 14-16} \) where decoding process is designed to be much more simpler than an exhaustive search.

Each possible \( k \)-long message is associated to \( 2^{nI(U; X)} \) codewords. A simple way to design such a structured codebook would be to insert \( i = n \times I(U; X) \) index bits in the message and to encode it. For an ECC with rate \( r \), this leads to \( n \)-long codewords with \( n = (k + i)/r \) (see Fig. 3). According to Costa, the value of \( i \) is given by

\[
i = n \times I(U; X) = \frac{n}{2} \log_2 \left[ 1 + \frac{PQ}{(P + N)^2} \right].
\]

\(^1\)With \( \epsilon \) chosen to be very small as \( n \to \infty \).
which then depends on \( Q \), i.e. the host signal. Thus the final codeword length \((k + i)/r\) may vary, while the host signal length \( n \) is generally given and fixed (number of pixels for an image, sample size of a sound . . . ). The length of codewords must not depend on \( i \), i.e. the global rate \( k/n \) must be fixed.

We thus propose to use a simple codebook based on punctured convolutional codes and soft trellis decoding. Let us choose an error correcting code in order to get a rate \( r = k/n \). We then design an interleaved pattern composed of the \( k \) bits from the message \( \mathbf{m} \) to be embedded and of \( i \) additional bits, as illustrated in Fig. 4(a). We then expand the host signal from \( n \) to \((k+i)/r\) using neutral values for soft decoding (i.e. 0). This expanded host signal is decoded with a modified soft Viterbi decoding algorithm, using the previous \( k \) bits pattern as a strong a priori in order to force some transitions in the convolutional trellis (see Fig. 4(b)). The output fixes the \( i \) index bits and gives a \((k+i)/r\)-long codeword, which is punctured according to the previous expansion of the host signal, in order to remove \( i/r \) bits and to finally get a \( n/r \) long codeword. This leads to the closest codeword \( \mathbf{u}^* \in \mathcal{U}_m \) to \( \mathbf{x} \). Using BPSK\(^1\), all the obtained codewords are designed to have the same energy, i.e. \( ||\mathbf{u}|| = \sqrt{n} \).

The watermark is finally chosen in order to get the maximum robustness\(^1\): the codeword \( \mathbf{u}^* \) is associated to a hyper-cone of robustness, where \( \mathbf{y} \) must lie into to be correctly decoded. Further, hyperboloids may be defined to represent set of points of given robustnesses (e.g. \( H_{N_1}, H_{N_2} \) . . . on Fig. 2). The watermark \( \mathbf{w} \) is defined in order to maximize robustness, that is

\[
\mathbf{w} = \arg \max_{\mathbf{w}} \left\{ \frac{(\mathbf{x} + \mathbf{w}) \cdot \mathbf{u}^*}{||\mathbf{u}^*||} \right\}^2 \left(1 + \tan^2 \theta \right) - ||\mathbf{x} + \mathbf{w}||^2, \text{ with } ||\mathbf{w}|| = \sqrt{n}P, \tag{7}\]

where \( \theta \) is the angle of the hyper-cone, given by\(^1\)

\[
\tan^{-2} \theta = 2^{2(k+1)/n} - 1. \tag{8}\]

At the receiver, the signal \( \mathbf{y}' = \mathbf{y} + \mathbf{z} \) is expanded from \( n \) to \((k+i)/r\) elements insertion 0 elements, and decoded using the trellis to get \( \mathbf{m} \). Thanks to soft decoding and to the fact that codewords have all same energy, this coding scheme is scale resistant, i.e. \( \mathbf{y}' \) can be scaled \( \mathbf{y}' = \gamma (\mathbf{y} + \mathbf{z}) \) with \( \gamma > 0 \) without loss of robustness.

### 2.3 Game theory applied to spread spectrum

Multimedia signals are not usually i.i.d. and Gaussian. So we consider a non i.i.d. host signal \( \mathbf{x} \) modeled by a set of random variables \( X^m = \{X_1, X_2, \ldots, X_m\} \) with \( X_i \sim \mathcal{N}(0, \sigma_{X_i}^2) \), i.e. signal is modeled as a mixture of Gaussians. We also consider a more general model for attack: SAWGN. The received signal can then be written as \( y_i' = \gamma_i^w x_i + z_i \) where \( \gamma_i^w \geq 0 \) and \( z_i \) is a Gaussian noise modeled by \( Z_i \sim \mathcal{N}(0, \sigma_{Z_i}^2) \). To embed a \( n \) symbols length message in a \( m \)-long signal, wide spread spectrum uses a pseudo-random matrix \( \mathbf{G} \in \{-1; 1\}^{m \times n} \). This can be associated to a spread transform, i.e. the embedding process is made in a linear subspace. In the embedding process, like for ST-DM\(^1\) or ST-SCS.\(^5\) Our previous work\(^12\) demonstrated the interest of Wiener filtering at embedding\(^5\):

\[
y_i = \gamma_i^w x_i + w_i = \gamma_i^w \left[ x_i + \frac{\sigma_{W_i}}{\sqrt{n}P} \sum_{j=1}^{n} w_{ij}^s \times G_{i,j} \right], \tag{9}\]

\(^1\)Binary Phase Shift Keying.

\(^5\)Since attacker would perform Wiener filtering to decrease \( D_{xy'} \), Wiener filtering at embedding allows to decrease \( D_{xy} \) without loss of performance.
Wiener filtering and the scale attack can be grouped: 

\[ \beta_i = \frac{\sigma_{X_i}^2}{\sigma_{X_i}^2 + \sigma_{W_i}^2}. \]  

(10)

The watermark \( w = \{w_1, w_2, \ldots, w_m\} \) is thus non i.i.d. and is modeled by \( W^m \) with \( W_i \sim N(0, \sigma_{W_i}^2) \). The Wiener filtering and the scale attack can be grouped: \( \gamma_i = \gamma_i^b \times \gamma_i^w \). The inverse spread transform (used for extraction) is defined by a weighted linear correlation\(^{12}\): 

\[ x_j^{st} = \sum_{i=1}^{m} \beta_i \gamma_i \times x_i \times G_{i,j} \]  

(11)

\[ y_j^{st} = \sum_{i=1}^{m} \beta_i \times y_i' \times G_{i,j}, \]  

(12)

where \( \beta_i \) is a weighting factor. As demonstrated previously\(^{13}\) considering a SI scheme, the optimal value for \( \beta_i \) can be expressed as 

\[ \beta_i^* \propto \frac{\gamma_i \times \sigma_{W_i}}{\sigma_{Z_i}^2}. \]  

(13)

In this subspace, the embedding process from Eqn. (9) is written as 

\[ \forall j \in \{1, 2, \ldots, n\}, \ y_j^{st} = x_j^{st} + w_j^{st}, \]  

(14)

where \( x^{st} \) is i.i.d. and Gaussian. We can then use Costa’s approach described in Sec. 2.1, and define from Eqns. (9) and (12) the different amounts of energy used as 

\[ Q = \sum_{i=1}^{m} \beta_i^2 \gamma_i^2 \times \sigma_{X_i}^2, \]  

(15)

\[ N = \sum_{i=1}^{m} \beta_i^2 \times \sigma_{Z_i}^2. \]  

(16)
\[ P = \frac{1}{n} \left[ \sum_{i=1}^{m} \beta_i \gamma_i \times \sigma_{W_i} \right]^2. \] (17)

We remark that while \( \sigma_{X_i}^2 \gg \sigma_{W_i}^2 \) to ensure the invisibility of the watermark, the available watermark energy \( P \) is concentrated in the subspace and can then become more important than the energy \( Q \) of the host signal (when \( m/n \gg 1 \)). It also shows that \( P \) is shared by the symbols to be embedded: more symbols (i.e. larger \( n \)) means less watermark energy per symbol.

Given a maximum amount of embedding distortion, we must optimize the embedding energy, i.e. \( \sigma_{W_i} \).
Define an embedding and an attack distortion functions:

\[ D_{xy} = E \left[ \varphi_i (x_i - y_i)^2 \right] = \frac{1}{m} \sum_{i=1}^{m} \varphi_i^2 \frac{\sigma_{X_i}^2 \sigma_{W_i}^2}{\sigma_{X_i}^2 + \sigma_{W_i}^2} \] (18)

and

\[ D_{xy'} = E \left[ \varphi_i (x_i - y_i')^2 \right] = \frac{1}{m} \sum_{i=1}^{m} \varphi_i^2 \left( \sigma_{X_i}^2 (1 - \gamma_i)^2 + \gamma_i^2 \sigma_{W_i}^2 + \sigma_{Z_i}^2 \right), \] (19)

where \( \varphi_i \) is a perceptual weighting factor. The performance of the inverse spread transform can be quantified by the signal-to-noise ratio \( E_b/N_0 \) defined as

\[ \frac{E_b}{N_0} = P = \frac{1}{n} \sum_{i=1}^{m} \gamma_i^2 \times \sigma_{W_i}^2. \] (20)

It should be noted that this value is not the signal-to-noise ratio obtained at the output of the extractor from Eqns. (12) and (13), given by

\[ \text{snr} = \frac{P (P + Q + N)}{N (P + N)}. \] (21)

We now solve the optimization of \( \sigma_{W_i} \) using a min-max game: given a maximal amount of distortion \( D_{xy}^{\text{max}} \), the attacker wants to minimize \( E_b/N_0 \), while the embedder wants to maximize it, for a maximal amount of embedding distortion \( D_{xy}^{\text{max}} \). This is done by two Lagrangian optimizations.12 First, for the attacker, we get the following functional:

\[ \forall i \in \{1, 2, \ldots, m\}, \quad (\gamma_i^*, \sigma_{Z_i}^*) = \arg \min_{\gamma_i, \sigma_{Z_i}} \left\{ J_{\lambda, i} = \frac{\gamma_i^2 \times \sigma_{W_i}^2}{\sigma_{Z_i}^2} + \lambda \left[ \varphi_i^2 \left( \sigma_{X_i}^2 (1 - \gamma_i)^2 + \gamma_i^2 \sigma_{W_i}^2 + \sigma_{Z_i}^2 \right) \right] \right\}, \] (22)

where \( \lambda \) is a Lagrangian multiplier used to respect the constraint on the attack distortion. This leads to the optimal values for \( \gamma_i \) and \( \sigma_{Z_i}^* \):

\[ \gamma_i^* = \frac{\sigma_{X_i}^2}{\sigma_{X_i}^2 + \sigma_{W_i}^2} \text{ if } \sigma_{W_i} \leq \sqrt{\lambda} \varphi_i \sigma_{X_i}^2, \]

\[ = 0 \text{ otherwise}, \] (23)

and

\[ (\sigma_{Z_i}^*)^2 = \gamma_i^* \left( \gamma_i^* \frac{\sigma_{X_i}^2}{\sigma_{W_i}^2} + \sigma_{X_i}^2 \right). \] (24)

The second part of the game consists in optimizing the embedding parameters considering optimal attack, which is also done by a Lagrangian approach:

\[ \forall i \in \{1, 2, \ldots, m\}, \quad \sigma_{W_i}^* = \arg \max_{\sigma_{W_i}} \left\{ J_{\lambda, i} = J_{\lambda, i} - \chi \left[ \varphi_i^2 \frac{\sigma_{X_i}^2 \sigma_{W_i}^2}{\sigma_{X_i}^2 + \sigma_{W_i}^2} \right] \right\}, \] (25)

where \( \chi \) is a Lagrangian multiplier used to respect the constraint on the embedding distortion. This leads to the final optimal embedding parameters

\[ \sigma_{W_i}^* = \frac{\varphi_i^2 (\lambda - \chi) \sigma_{X_i}^2 - 1 + \sqrt{(\varphi_i^2 (\lambda - \chi) \sigma_{X_i}^2 - 1)^2 + 4\lambda \varphi_i^2 \sigma_{X_i}^2}}{2\sqrt{\lambda} \varphi_i}. \] (26)
and also to a particular expression for the optimal correlation factor for inverse spread transform: $\beta_i^* \propto \varphi_i$, when considering optimal attacks.

### 3. INTERFERENCE CANCELLATION

Without side informed watermarking, the signal-to-noise ratio we get is given by $\frac{E_b}{N_0} = \frac{P}{Q + N}$. In practice, this value is correct only if the carriers $\mathcal{G} = \{\mathcal{G}_0, \mathcal{G}_1, \ldots, \mathcal{G}_n\}$ of the spread transform are truly orthogonal, which is not the case with pseudo-random carriers. Thus the signal-to-noise ratio is given by

$$\frac{E_b}{N_0} = \frac{P}{Q + N + I}$$

where

$$I = \sum_{i=1}^{m} \beta_i^2 \gamma_i^2 \times \frac{\sigma_W^2}{nP} (n - 1).$$

The value $I$ is known as the inter-symbols interference. In non-informed watermarking techniques (where the host signal influences the performance of the scheme), this interference is negligible because $Q \gg I$. But in informed watermarking, for a low level of attack, it represents a great amount of noise that limits the robustness and/or the capacity of the scheme. Fig. 5 illustrates the gap between WSS watermarking with pseudo-random carriers and theoretical WSS watermarking (with truly orthogonal carriers). We will see in the remaining part of this section three methods to cancel this interference.

#### 3.1. Insuring orthogonality of the carriers

To avoid interference, a trick is to embed only one symbol per host element,\(^{15}\) i.e., \(i \in \{1, 2, \ldots, m\}\), there is only one element in \(\{\mathcal{G}_{i,1}, \mathcal{G}_{i,2}, \ldots, \mathcal{G}_{i,n}\}\) which is not set to 0. In this case, $I = 0$. However this technique limits the spreading of the bits, especially for important values of $n$, case where interference cancellation is very interesting (low level of attack noise).

Moreover, in the case of smooth signals (see Fig. 6 for an example), the number of well suited host elements (important value of $\gamma_i \sigma_W / \sigma_X$) is limited. The energy of the watermark is mainly located on high energy coefficients. Since this number of coefficients is small, symbols to be hidden may not be equally spread over the host signal (i.e. a symbol may be spread on non significant coefficients whereas an other one will be spread on significant ones). It results in the linear subspace in non i.i.d. signals. Performances are not guaranteed and parallel channels should rather be considered.

#### 3.2. Cancellation at the decoder

If pseudo-random carriers were used at embedding, the received signal in the spread transform subspace can be written as

$$\mathbf{y}^{st} = \gamma \left[ \mathbf{x}^{st} + \mathbf{w}^{st} + i \mathbf{s}(\mathbf{w}^{st}) \right] + \mathbf{z}^{st},$$

but
where isi($w^st$) is the inter-symbols interference. For $Q \ll P$ (very common case for payloads such as $n < 1000$), we can write $x^st + w^* \simeq w^st \simeq \sqrt{P} \times u^*$ and then

$$y^st \simeq \gamma \left[ u^* + isi(\sqrt{P} \times u^*) \right] + z^st.$$  

Thus we can estimate isi($\sqrt{P} \times u^*$) in order to cancel ISI. Receiver first estimates $u^*$. Corresponding interference is then canceled. The new $y^st$ is obtained and used to compute $\tilde{u}^*$. This process iterates until $u^* = \tilde{u}^*$ (see Alg. 1). To be efficient, receiver must know (or estimate) embedding energy $\sigma W_i$. Moreover, optimal scaling factor $\gamma_i^*$ must also be estimated. This may be done by an additional reference signal, leading to a lower capacity for message bits. We will then search for another solution consisting in canceling ISI at embedding.

### 3.3. Interference as side information

As seen in Sec. 2.1, the use of the side information available during the embedding process leads to great improvements, and if no attack is applied during the transmission, the capacity of the channel is infinite. But the spread transform we use to embed the watermark introduces a noise that limits capacity due to ISI. We propose to consider ISI as a kind of side information.

Even if this interference is introduced by the embedder, it is not perfectly known before the embedding. So, it can not be directly considered as side information. The problem is that the interference depends on the watermark signal, which depends on the interference. We use an iterative algorithm to converge to a watermark signal that takes into account its own interference. This algorithm is described by Alg. 2. We first compute $w^st$, as explained in Sec. 2. The interference it produces is computed, and introduced as side information. In a second step, this new side information $\tilde{x}^st$ is used to compute an updated watermark signal. The previous steps are iterated until $w^st$ converges (we observe convergence is attained to after typically less than 3 iterations). At the end of the loop, the watermark signal takes into account the host signal and the symbol interference, and is added using Eqn. (9).

### 4. EXPERIMENTAL RESULTS

The previous studies have been applied to image watermarking. A 3-levels wavelet transform of a gray-scale image generates the host signal $x$ ($m$ is equal to the number of pixels of the host image). We embed $k = 64$ bits using a structured codebook, as described in Sec. 2.2, with a rate equal to $1/2$. This leads to $n = 132$ with some padding bits. We consider a psycho-visual factor inspired from Watson’s model, defined by

$$\varphi_i = \frac{\rho}{\sqrt{\sigma X_i} + 1},$$

### Figure 6. Artic hare, a difficult image to watermark: the number of interesting elements is limited (copyright photos courtesy of Robert E. Barber, Barber Nature Photography).
Algorithm 1 Considering $w_* \simeq \sqrt{P} \times u^*$, search for the closest codeword $u^*$ from $y^{st}$ with ISI canceled

\begin{algorithm}
\begin{algorithmic}
\FOR {$j = 1$ \TO $n$}
\STATE $y_{j}^{st} \leftarrow \sum_{i=1}^{m} \beta_i \times y'_i \times G_{i,j}$
\ENDFOR
\STATE $\tilde{u}^* \leftarrow$ closest codeword to $y^{st}$
\REPEAT
\STATE $u^* \leftarrow \tilde{u}^*$
\FOR {$j = 1$ \TO $n$}
\FOR {$i = 1$ \TO $m$}
\STATE $I_{i,j} \leftarrow \gamma \frac{\sigma_{W_i}}{\sqrt{nP}} \sum_{k=1, k \neq j}^{n} (\tilde{u}^*_{k} \times G_{i,k})$
\STATE $y_{j}^{st} \leftarrow y_{j}^{st} + \beta_i (y'_i - I_{i,j}) G_{i,j}$
\ENDFOR
\ENDFOR
\STATE $\tilde{u}^* \leftarrow$ closest codeword to $y^{st}$
\UNTIL {$u^* = \tilde{u}^*$}
\end{algorithmic}
\end{algorithm}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Signal-to-noise ratio against attacks for Lena (512 $\times$ 512 gray-scale image, 3-levels DWT, $n = 132$ and $D_{xy} = 7$).}
\end{figure}

where $\rho$ is set to get $E[\varphi_i] = 1$ and $\sigma_X$ is a normalized activity measure (based on the variance of $X_i$). We finally tune $\lambda$ and $\gamma$ to obtain an embedding distortion equal to 7.0 (i.e. $\text{wpsnr}(x,y) = 39.7$ dB). Two attacks are tested: Gaussian noise and JPEG lossy compression.

For each attack level (energy of the added noise for AWGN attack and quality factor for JPEG compression), the resulting distortion $D_{xy'}$ is computed and the watermark is extracted to get the signal-to-noise ratio $E_b/N_0$. Figs. 7 and 8\footnote{Both used images are available from F. Petitcolas' web site: \texttt{http://www.cl.cam.ac.uk/~fapp2/watermarking/image_database}.} confirm the interest of interference cancellation, already shown by the theoretical Fig. 5.

\begin{itemize}
\item[(a)] Performance against AWGN attack.
\item[(b)] Performance against JPEG lossy compression ($D_{xy'} = 7$ for 100\% JPEG quality, and $D_{xy'} \simeq 20$ for 15\% JPEG quality).
\end{itemize}
Algorithm 2 Calculate $\mathbf{w}^{st}$ considering ISI as side information

for $j = 1$ to $n$
\begin{align*}
\mathbf{x}_{j}^{st} & \leftarrow \sum_{i=1}^{m} \beta_{i} \gamma_{i}^{*} \times x_{i} \times G_{i,j} \\
\end{align*}
end for

$\mathbf{u}^{*} \leftarrow \text{closest codeword to } \mathbf{x}^{st}$

$\mathbf{w}^{st} \leftarrow \arg \max_{\mathbf{w}^{st}} \left\{ \left( \frac{\mathbf{x} + \mathbf{w}^{st}}{\|\mathbf{u}^{*}\|} \cdot \mathbf{u}^{*} \right)^{2} \left( 1 + \tan^{2} \theta \right) - \| \mathbf{x} + \mathbf{w}^{st} \|^{2}, \text{ with } \| \mathbf{w}^{st} \| = \sqrt{n P} \right\}$

repeat
\begin{align*}
\mathbf{w}^{st} & \leftarrow \mathbf{w}^{st} \\
\end{align*}
for $j = 1$ to $n$
\begin{align*}
\mathbf{x}_{j}^{st} & \leftarrow 0 \\
\end{align*}
for $i = 1$ to $m$
\begin{align*}
\mathbf{w}_{i} & \leftarrow \frac{\sigma W_{i}}{\sqrt{n P}} \sum_{k=1}^{n} w_{k}^{st} \times G_{i,k} \\
I_{i,j} & \leftarrow \mathbf{w}_{i} - \mathbf{w}_{j}^{st} \times \frac{\sigma W_{i}}{\sqrt{n P}} \times G_{i,j} \\
x_{j}^{st} & \leftarrow x_{j}^{st} + \beta_{i} \gamma_{i}^{*} (x_{i} + I_{i,j}) G_{i,j} \\
\end{align*}
end for
end for

$\mathbf{u}^{*} \leftarrow \text{closest codeword to } \mathbf{x}^{st}$

$\mathbf{w}^{st} \leftarrow \arg \max_{\mathbf{w}^{st}} \left\{ \left( \frac{\mathbf{x} + \mathbf{w}^{st}}{\|\mathbf{u}^{*}\|} \cdot \mathbf{u}^{*} \right)^{2} \left( 1 + \tan^{2} \theta \right) - \| \mathbf{x} + \mathbf{w}^{st} \|^{2}, \text{ with } \| \mathbf{w}^{st} \| = \sqrt{n P} \right\}$

until $|\mathbf{w}^{st} - \mathbf{w}^{st}| \leq \epsilon$

(a) Performance against AWGN attack. (b) Performance against JPEG lossy compression ($D_{xy} = 7$ for 100% JPEG quality, and $D_{xy} \approx 20$ for 15% JPEG quality).

Figure 8. Signal-to-noise ratio against attacks for Paper machine (512 × 512 gray-scale image, 3-levels DWT, $n = 132$ and $D_{xy} = 7$).
5. CONCLUSION
We studied in this paper a practical implementation of a watermarking scheme exploiting side information. We propose a method scheme based on a simple structured codebook using a soft Viterbi decoder. A spread transform gets i.i.d. signals from non i.i.d. ones. Embedding in the linear subspace defined by the spread transform generates inter-symbols interference. An iterative algorithm estimates this interference and includes it into the side information. We finally applied this scheme to image watermarking: this leads to important improvements in term of capacity and/or robustness.

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