The process of schematic representation in mathematical problem solving

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Abstract. Representation plays an important role in solving mathematical problems, but not a few students who create difficulties in shaping it. Therefore, this study aims to reveal the process of formation of schematic representation by students during the realization of the word problem. The subjects involved in this study were 54 Junior High School students. See the schematic representation process in this research using the think aloud technique. In addition, task-based interviews were also conducted. The results obtained in this study are a schematic representation that can be formed as long as the students understand the problem. By establishing a scheme, students can solve problems so that students can receive the information contained in the problem. Schematic representation process begins with; a) read the problem repeatedly, b) identify the problem by forming a schematic, and c) create a schematic drawing. This schematic representation process is very effective in helping students understand the problem. Students' success in understanding the problem affects the next stages of resolution so that students can solve the words problem well.

1. Introduction

Although the representation has an important role in solving problems [1,2,3], many students have difficulty in shaping it [4,5,6]. Forming representations can strengthen students' understanding to construct concepts and solve mathematical problems, especially the word problem [7,8,9]. Therefore, it is very important to use representation as long as students learn mathematical concepts or solve mathematical problems.

Mathematical representation can manifest itself in various forms, including verbal forms, diagrams, graphs, tables, schema, and symbolic. These forms of representation are important modalities in the expression of mathematical thinking [10]. One of the most difficult types of student representation during problem-solving is the schematic representation. The schematic representation is a representation that describes the relations that explain the situation in the problem [11]. Using the schematic representation, the student can issue the main data of the problem [12].

Several studies have found the types of schematic representations used in solving mathematical problems. The type of schematic representation in the form of diagrams and schematic drawings is used by four-year students in solving arithmetic problems [13]. In another study, we found two types of schematic representations i.e. accurate schematic representation and inaccurate schematic representation.
used by sixth-year students in solving word problems. Accurate schematic representations contain images or diagrams relevant to the problem. While the schematic representation is inaccurate it occurs when there are one or more missing or imperfect parts in the drawing or diagram [14].

The ability of students to form schematic representations is still very limited. Even many students make inaccurate schematic representations during problem solving. Thus, students cannot solve mathematical problems, because, in the formation of an inaccurate schematic representation, information exists in a problem that does not exist [14]. Thus, this study intends to reveal the student process in the formation of a schematic representation during the solving of mathematical problems. Thus, obtaining a schematic representation process can be used to evaluate the student's representation skills, especially the schematic representations.

2. Method
The research is a qualitative study involving 54 Junior High School students. Instruments in research are problem-solving tasks and interview guides. Before auxiliary instruments are used, the instrument is validated by two validators who are experts in mathematics and mathematics learning. The result of the validation performed shows that the problem-solving tasks and interview guidelines task tool is valid and feasible to use. The process of data collection is done about think aloud technique, in which the student, when thinking about solving the problem, in addition to verbalize the results of his thinking. During the process, the researchers observe, record, and record all student activities using camera recording. After the students finish solving the problem, the researcher conducts a task-based interview to find out things that the researcher has not yet understood.

3. Result and discussion
The following is a description of the process of schematic representation by the students during the completion of the word problem. This process is initiated by the student, reading the entire sentence in a given problem using a loud voice. After reading the problem aloud, students re-read the whole problem in a low voice. When reading in a low voice, the students analyze the problem by writing some information contained in the problem. The student returns reading the subject carefully in all sentences. After reading the problem for the third time, while reading the first sentence, students are encouraged to form a schema. The scheme is manifested in a sketch pointing south and given a 40-meter witness, followed by a line pointing east and 30 meters (figure 1). This process shows that understanding the problem by creating a schema is one of the strategies that students can use to solve word problems, because using the student schema can unpack the problem and simplify the complex problem [15,16]. Then the students continue reading the second sentence. After reading the second sentence, students draw a mark on both ends of the square line. The plaque was used to mark Faiz's house and Farhan's house. Then the students read the third sentence and the fourth one contains the problem question. The process of digging information by reading and identifying problems in each sentence facilitates the student to understand the information contained in the problem. The results of this student's work indicate that the scheme can be used as a tool to understand the information students face verbally [8].

![Figure 1. Written answers of the student in the understanding of the problems.](image-url)

After receiving some information, students navigate the schematic drawing using the tip of the pencil. Then the students mutter saying part of the second sentence of the problem, that is, "there is a shortcut passing the garden between their houses". The students paused and the students made a square sketch
between the two lines. The students say grumbling "this garden". After knowing the position of the garden, students draw the line connecting the two ends of the line in the schematic drawing. So That obtained image triangle (figure 2). With this triangular image, students understand the problem issue that the length of the shortcut between Faiz's house and Farhan's house is the oblique side of the triangle. The use of good representation can link the information learned with the collection of information that students already have [7].

Figure 2. Written answers from students in understanding the question problem.

In addition to the above data, the process of understanding the problems assumed by the students is shown in the following interview:

R: What information exists in this problem?
S: The information is for the south 40 meters and then 30 meters.
R: 40 meters is what?
S: Distance traveled Faiz to the south and then to the east 30 meters.
R: The distance Faiz is going to Farhan's house, right?
S: Yes, so that means Faiz's house and this is Farhan's house (pointing to the box mark at the end of the picture). Then there is the garden, there is a shortcut passing the garden between their houses. Meaning of the garden here (pointing to the side of the slopes and rectangular image)
R: Ok ..so the question of this problem what?
S: What is the length of the shortcut (pointing to the oblique lateral line of the triangle), then from the end of this line here.
R: So you think the shortcut is on this side?
S: yes

The students then calculate the length of the shortcut between Faiz's house and Farhan's house, surveying the length of the triangular oblique side. Students plan a plan to solve the problem using the concept of Pythagoras Theorem. Students then calculate the squares at a distance of 40 meters to the south and squared at a distance of 30 meters to the east. After getting the squared values of each distance, students are confused when finding long-distance shortcuts. This process is shown by students in the following thinking process:

S: 1600 less than 900 equals 700
700 divided by two equal to 350, emmm is not
25 multiplied by 25 is equal to 625, emm is not
Phytagoras Emmmm what more or less already? (Mumble)
(in silence for a while) Emm added

Then the students calculate summing 1600 plus 900 to get 2500. To find the root value of 2500, students try to multiply several numbers, among others:

\[
\begin{align*}
10 \times 10 &= 100 \\
30 \times 30 &= 900 \\
40 \times 40 &= 1600 \\
50 \times 50 &= 2500 \\
\end{align*}
\]
This shows that the use of schematic representation can provide important help to students in solving problems [13].

After obtaining a shortcut length, the students again create a schematic drawing of triangular images. Using the student scheme scanned the path Faiz used to go to Farhan's house with a distance of 40 meters to the south and turn east 30 meters. Students repeatedly thicken the beveled side of a triangular image using a pencil accompanied by a soft sound “Faiz wants to take a shortcut by passing through the garden to get as fast as possible. How long is the shortcut?”. Until finally students read the question of the problem "How long a shortcut between the house Faiz and Farhan?", Then with confidence then answered 50 meters. This process shows that using representations embodied in schematic drawings may help the subject to monitor or review the troubleshooting process [17]. It is also important to modify or recreate the schematic drawing if necessary [18], because it communicates the mathematical idea by using extremely stony images in connecting ideas to the problems faced [19].

4. Conclusion
The schematic representation can be formed as long as students understand the problem. By establishing a scheme, students can analyze the problem so that students can receive the information contained in the problem. It can be concluded that the schematic representation process begins with; a) read the problem repeatedly, b) identify the problem by forming a schematic, and c) create a schematic drawing. This schematic representation process is very effective in helping students understand the problem. Students’ success in understanding the problem affects the next stages of resolution so that students can solve the problem of words well.

Acknowledgments
The authors would like to thank the Ministry of Research, Technology and Higher Education Republic of Indonesia who has supported financially this research through Doctoral Dissertation Research.

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