Exploring students’ adaptive reasoning skills and van Hiele levels of geometric thinking: a case study in geometry

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Abstract. This study aims to explore junior high school students’ adaptive reasoning and the Van Hiele level of geometric thinking. The present study was a quasi-experiment with the non-equivalent control group design. The participants of the study were 34 seventh graders and 35 eighth graders in the experiment classes and 34 seventh graders and 34 eighth graders in the control classes. The students in the experiment classes learned geometry under the circumstances of a Knisley mathematical learning. The data were analyzed quantitatively by using inferential statistics. The results of data analysis show an improvement of adaptive reasoning skills both in the grade seven and grade eight. An improvement was also found for the Van Hiele level of geometric thinking. These results indicate the positive impact of Knisley learning model on students’ adaptive reasoning skills and Van Hiele level of geometric thinking.

1. Introduction

To cope with the complex demands of the 21st century, individuals need to have a number of competences. In this respect, the Partnership for the 21st Century Skills (P21) [1] proposed a so-called 21st century skills comprising critical thinking, creativity, communication, and collaboration. Among these four skills, the critical thinking skill is the focus of the present study. Critical thinking skills include interpretation and analysis, reasoning, constructing argument, and problem solving. Reasoning takes a crucial role in the process of thinking critically. The importance role of mathematical reasoning has been considered in various curricula across the world that mandate the integration of reasoning in mathematics learning [2-3].

In 2011, the National Research Council introduced the term adaptive reasoning skills (ARS) which connect all types of reasoning and guide students’ learning process. ARS is seen as a general skill of reasoning because it includes not only informal explanation and justification, but also intuitive reasoning and inductive reasoning on the basis of pattern, analogy, and metaphor [4]. According to Kilpatrick et al [4], ARS is the ability to reflect, to explain, to justify, and to logically think about the relation between concept and situation. The logical thinking skills refer to the ability to generate conjectures, whereas the ability to reflect refers to the ability to check the truth of a procedure or mathematical argument. Students with good ARS could use logic to explain and justify solutions for problems and also to generalize and extend the solution to new situation [4]. A combination of ARS and metacognitive skills could help students to describe complex problem situation, do self-reflection, and to explain argument to themselves or to others [5]. Furthermore, students with good ARS also have the ability to evaluate whether a solution is correct or incorrect and to give logical explanation and justification [4,6].

Another important mathematical thinking that needs to be considered in the learning of mathematics is Van Hiele level of geometric thinking (VHLGT). Van Hiele level of geometric thinking plays an important role in the learning of geometry [8-9]. Van Hiele level consists of five levels, i.e. visualization, analysis, order, deduction, and rigor. The visualization level refers to
students’ ability to recognize shapes and their name. The analysis level deals with students’ ability to identify the characteristics of geometrical shapes. The order level concerns students’ ability to categorize and connect geometrical shapes in accordance with their characteristics. The deduction level refers to students’ ability to make deduction and to understand theorem and their proof. Lastly, students who achieve the rigor level could give correct geometrical proof and abstract deduction.

Although ARS and VHLGT were important for students, some studies indicated a low level of both students' abilities. According to Kilpatrick's research, the percentage of 13-year-olds with ARS was comparatively proportional to the percentage of students who did not have ARS [4]. The results of Gunhan's research also showed that the ability of grade VIII students in geometry was still weak because students could not balanced conceptual and procedural information [10]. Students also unable to provided rational mathematical arguments and unable to visualized the incorporation of geometric objects well [10]. The results of Isiksal, Koc, and Osmanoglu's research was known that the grade VIII students were still problematic in problem solving about conceptual understanding of reasoning and measuring of cylinder surface area and volume [11]. In addition, students also have difficulty making generalizations concerning the relationship between lateral surface area and volume of other geometrical structures [11].

In order to increased ARS and VHLGT students’, it is necessary to mathematics learning that facilitates both capabilities. Developing students’ mathematical thinking skills requires more than a procedural and mechanistic approach of teaching mathematics and one of the lessons that could increased ARS and VHLGT students’ is Knisley Learning Mathematics (KLM). The principles of KLM are in line with the philosophy of learning as understanding that could reduce students’ workload, develop students’ transfer ability, and increase students’ beliefs [12-13]. Furthermore, KLM facilitates the learning process of students from concrete level to abstract level. KLM consists of four levels of learning, i.e. allegorization, integrator, analysis, and synthesis [14]. The first two levels deal with the concrete level of mathematical thinking whereas the last two levels deal with abstract level of mathematical thinking. In the allegorization level, a new concept is described figuratively in a familiar context in terms of known concepts. In the integration level students compare, measure, and explore the topics in order to distinguish the new concept from known concepts or their prior knowledge. In the analysis level, the new concept becomes part of students’ existing knowledge base. In the synthesis level, students acquire the new knowledge as a unique identity and can use it as a tool for strategy development and further allegorization. Teacher and students play different role during the four levels of KLM. In the first two levels teacher takes a greater role in the learning process, whereas in the last two levels the students become more active. So, KLM is not only important for ARS, but it is also potential to develop the students’ VHLGT.

Considering the aforementioned rationale, the present study is aimed to explore the adaptive reasoning skills and Van Hiele level of geometric thinking of students who learn geometry through Knisley learning model.

2. Methods

The present study was a quasi-experiment with the non-equivalent control group design. The participants of the study comprised two groups of students. The first group of students were seventh graders of SMP Negeri 7 Mataram, i.e. 34 students in the experiment class and 34 students in the control class. The second groups of students were eighth graders of SMP Negeri 4 Cilacap, i.e. 35 students in the experiment class and 34 students in the control class. Students in experiment classes learned geometry through KLM, whereas students in the control classes learned geometry through an expository approach. The topic taught in grade seven was triangle and quadrilateral, whereas the topic taught in grade eight students was circle.

Students’ ARS were measured before and after the intervention by using a test. Students’ responses to the test were analyzed quantitatively for which Two Way Anova test, Tukey test, and independent sample one tile t-test were used to analyze the data. The gain between pretest score and posttest score was analyzed on the basis of students’ initial ability in mathematics (kemampuan awal matematika or KAM). The categorization of KAM was done by using norm references evaluation (Penilaian Acuan Normatif or PAN).

Students’ VHLGT was measured by using a test. The test results were scored and analyzed qualitatively to describe students’ VHLGT in grade VII. Students’ VHLGT in grade VIII as an
additional finding in the study of geometry capabilities with the questions used for the analysis of students’ VHLGT eighth grade students are shown in Figure 1.

![Figure 1. Problem VHLGT Test Eighth Grade](image)

3. Result and Discussion

Summary of Two way Anova test against ARS N-Gain based learning factor in terms of KAM students are presented in Table 1.

| Factor                | VII Class |     | VIII Class |     |
|-----------------------|-----------|-----|------------|-----|
|                       | F         | Sig| H₀         | F   | Sig| H₀ |
| Learning (1)          | 102.2     | 0.000 | rejected   | 4.830 | 0.032 | rejected |
| KAM (2)               | 8.56      | 0.001 | rejected   | 0.507 | 0.605 | received |
| Learning and KAM (3)  | 2.75      | 0.072 | received   | 0.633 | 0.534 | received |

Based on Table 1, it can be concluded that the increase of ARS of the seventh and eighth graders got KLM significantly higher than that of the students who received expository approach. After testing independent sample one tile t-test, it is concluded that KLM has a significantly positive effect on the improvement of student’ ARS, i.e. for the seventh graders by sig. 1-tailed at 0.0005 and for the eighth graders sig. 1-tailed at 0.0015.

Table 1 also informs that the difference among students’ initial ability in mathematics (KAM) significantly influence the increase in seventh graders’ ARS, but there is no significant influence for the eighth graders. Considering this finding, an advanced Tukey Test was performed to find out the difference in seventh grade (Table 2).

| KAM (I) | KAM (J) | Diff Mean | Sig |
|---------|---------|-----------|-----|
| High    | Medium  | 0.1519    | 0.092 |
|         | Low     | 0.2670 *  | 0.010 |
| Medium  | Low     | 0.1151    | 0.266 |

Table 2 shows that there is no significant difference in the improvement of ARS between students with high KAM and medium KAM and between students with medium KAM and low KAM. However, a significant difference in improvement was found between students with high KAM and low KAM. Based on independent sample one tile t-test, sig. 1-tailed at 0.0425 was obtained that means the increase of ARS of students with high KAM is significantly higher than that of the students with low KAM.
Table 1 also shows that there is no interaction between learning and KAM factors to the increase of students’ ARS seventh grade and eighth grade. Furthermore, it can be concluded that KLM is the most suitable for students with high KAM because the increase of their ARS is the greatest among all students.

In addition to effective improvement of ARS, with regard to students’ VHLGT, KLM is found to give a positive influence on the increase of seventh graders’ VHLGT (see Table 3). Table 3 shows that there are differences on the improvement of students’ VHLGT at analysis level. In the group of students who received expository approach, the number of students who reached the analysis level increased from 19% to 33%. However, there was a decrease for the number of students who reached the order level, i.e. from 3% to 0%. In the group of students who got KLM, the number of students who reached the analysis level increased from 41% to 76%. In other words, KLM has a positive effect on improving students’ achievement in the analysis levels (Table 3).

### Table 3. Summary of Percentage Students’ VHLGT for Seventh Grade

| Class   | Pretest | Visualization | Analysis | Order | Deduction | Rigor |
|---------|---------|---------------|----------|-------|-----------|-------|
| Control | Pretest | 19%           | 47%      | 33%   | 0%        | 0%    |
| Postest |         | 46%           | 35%      | 16%   | 3%        | 0%    |
| Class   | Pretest | 15%           | 44%      | 41%   | 0%        | 0%    |
| Experiment | Postest | 3%            | 21%      | 68%   | 9%        | 0%    |

The results of qualitative analysis of students' answers to questions in Figure 1 show KLM’s positive effect on VHLGT. Before students got KLM, the dominant students think at the visualization level because students answer the same-sized circle only through the form (Figure 2). None of the students used the concept of radius of a circle to answer. Students who have linked the size of the circle area to square size, all have errors in determining the size of the area of a small circle on the square C with an answer of 1/8 the size of the circle A. The researchers guessed the error is that students only count the number of small rectangles so that no students do the level of analysis correctly.

![Figure 2. Answer Visualization Level before Students Obtained KLM](image)

After students got KLM, the students' dominance at the visualization level moves to the analysis (Figure 3) and order levels (Figure 4). However, some of them are still less perfect at the analysis level by answering the size of the area of a small circle in the square C of 1/8 the size of the circle A. But there are already some students who write the relationship between the circles on the three squares and answer questions b by listing the correct reasons for the answers given. In addition, there are also students who try to answer by using the concept of radius of a circle.

![Figure 3. Answer Analysis Level](image)

![Figure 4. Answer Order Level](image)
Based on the above explanation, there is a movement of students' geometric abilities after obtaining KLM. In seventh grade, the percentage of students who reached the level of analysis increased. In eighth grade, the students’ ability to answer also experienced a positive change.

The results of this study indicate that KLM is significantly more effective in increasing ARS and has a positive impact on VHLGT students compared with expository learning. The effectiveness of KLM is due to students' enthusiasm built up well at the stage of allegorization [14] through the activity Let's Observe using a familiar context and related to the early knowledge of the students to improve the learning readiness [4] because the learning experience contributes to the development of academic skills [15]. The "Ayo Mengamati" activity also facilitates VHLGT because learning focuses on identifying images and drawing activities [16, 17] so that students are facilitated to reason geometry on the basic concept [18].

Through KLM, students' thinking skills continue to be improved in the next learning phase. The integrator stage through the "Ayo Praktik" activity can facilitate students performing Visualization, experimentation, and exploration [14] to acquire mathematical concepts through real experience [19]. The activities can also build mathematical understanding through questions or observations [20] and is relevant to the learning of mathematics for middle-aged students who combine direct activity with thinking activity [21], consequently the level of analysis on VHLGT can be facilitated because the students focus on the activity reaching that level i.e observing, measuring, experimenting, drawing and building a flat build [17].

The analytical phase undertaken through "Ayo Mendalami" is designed as a structured learning that facilitates students in analyzing problems and sharing solutions to the problem [22]. At this stage, students' curiosity is built and facilitated [14] so that students can think deeply and apply the mathematical concepts studied in different cases [23]. A well-facilitated curiosity is able to encourage students to make new connections between ideas, perceptions, concepts, and representations of learned mathematical concepts [24]. As a result, order levels in VHLGT can be facilitated because students are trained to link mathematical concepts [16]. In addition, the level of deduction and rigor is also exercised, although the intensity of the training is not as much as the level of analysis and order.

In the last stage of the KLM, students are given the opportunity to apply mathematical concepts and evaluate the understanding gained through concluding activities and practice questions. Students combine facts, concepts, and learning processes into new knowledge [21] that is communicated singly to explain the mathematical pattern or structure of conclusion [25]. Students who have mastered the new concept can use the concept to solve problems on exercise questions as a form of student knowledge evaluation activities to assess students' understanding and ability and provide opportunities for teachers to evaluate student progress.

The implementation of KLM resulted in stronger student retention as it reduced the material to be memorized as the Learning as Understanding paradigm [13]. In addition KLM is also implemented without any differences in treatment that distinguishes KAM students so that the increase in ARS grade VIII students in various categories KAM and grade VII students between high and medium KAM has no significant difference. The difference in ARS increase in grade VII students between high and low KAM was thought to be due to the high cognitive ability gap in both groups and the habit of mind of low KAM students who used to imitate the teacher's behavior so that they are not accustomed to communicate their mathematical ideas independently. One consequence is that KAM students have low difficulty on giving indicators of reason so that students only assert the argument is right or wrong without providing supporting reasons. However, low KAM students in grade VIII can actually experience an increase in ARS that is not significantly different from high and medium KAM so that does not cover the possibility of KLM conducted more intensively then the increase in students’ ARS can be better. Based on the above explanation it can be concluded that KLM can be used to facilitate the increase of students’ ARS on various levels of mathematical ability.

4. Conclusions
Based on the results and discussion, it can be concluded that KLM is significantly more effective in increasing student ARS and has a positive impact on VHLGT students compared with the expository approach. In addition, based on KAM it can be concluded that KLM can facilitate grade VIII students in different categories of KAM and grade VII students in high and medium categories of KAM.
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