Predictions of the most minimal see-saw model

M. Raidal\textsuperscript{1,2} and A. Strumia\textsuperscript{1,3}

\textsuperscript{1}Theoretical Physics Division, CERN, CH-1211 Genève 23, Switzerland
\textsuperscript{2}National Institute of Chemical Physics and Biophysics, Tallinn 10143, Estonia
\textsuperscript{3}Dipartimento di Fisica dell’Università di Pisa and INFN, Italy

We derive the most minimal see-saw texture from an extra-dimensional dynamics. It predicts $\theta_{13} = 0.078 \pm 0.015$ and $m_{ee} = 2.6 \pm 0.4$ meV. Assuming thermal leptogenesis, the sign of the CP-phase measurable in neutrino oscillations, together with the sign of baryon asymmetry, determines the order of heavy neutrino masses. Unless heavy neutrinos are almost degenerate, successful leptogenesis fixes the lightest mass. Depending on the sign of the neutrino CP-phase, the supersymmetric version of the model with universal soft terms at high scale predicts BR($\mu \rightarrow e\gamma$) or BR($\tau \rightarrow \mu\gamma$), and gives a lower bound on the other process.

Introduction

The minimal see-saw texture that allows to explain the solar and atmospheric neutrino anomalies in terms of oscillations contains two heavy singlet neutrinos $N_{\text{atm}}$ and $N_{\text{sun}}$ coupled as

\begin{equation}
\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{M_{\text{atm}}}{2} N_{\text{atm}}^2 + \frac{M_{\text{sun}}}{2} N_{\text{sun}}^2 + \lambda_{\text{sun}} H N_{\text{sun}} (s L_e + c e^{-i\phi/2} L_\mu + 0 L_\tau) + \lambda_{\text{atm}} H N_{\text{atm}} (0 L_e + s_{\text{atm}} L_\mu + c_{\text{atm}} L_\tau),
\end{equation}

where $M_i$, $\lambda_i$, $\phi$, $s$ and $s_{\text{atm}}$ are free parameters. We abbreviate $s_i = \sin \theta_i$, $c_i = \cos \theta_i$, $t_i = \tan \theta_i$. Nothing can be removed from this 'most minimal texture' without generating conflicts with present data. The phase $\phi$ is the unique source of CP-violation in the lepton sector. Possible connection between the sign of the observed baryon asymmetry of the universe and the CP-violation in neutrino oscillations via $\phi$ was the original motivation of the model. This texture is predictive also if the zeros are replaced by sufficiently small numbers.

In this letter we motivate the model and show in detail how it can be tested using low and high-energy observables. After deriving predictions for neutrino experiments, we clarify that the sign of the baryon asymmetry, together with the sign of the neutrino CP-violation in oscillations, determines a discrete ambiguity of the model: the order of $M_{\text{sun}}$ and $M_{\text{atm}}$. Successful leptogenesis fixes the mass of the lightest heavy neutrino. In the supersymmetric version of the model, either $\text{BR}(\mu \rightarrow e\gamma)$ or $\text{BR}(\tau \rightarrow \mu\gamma)$ is predicted, depending on the sign of the neutrino CP-phase. The other process is a function of the heaviest singlet neutrino mass only, and has a lower bound.

At first sight the texture looks quite artificial: e.g. we do not know how a U(1) flavour symmetry could justify it. However, the can be easily obtained from extra-dimensional models. Following we consider a 5-dimensional fermion $\Psi(x)$ in presence of a domain wall $\varphi(x_5)$. The system is described by the action

$$S = \int d^5x \bar{\Psi} [i\gamma^\mu \partial_\mu + \lambda \varphi(x_5) - m] \Psi.$$  

The Kaluza-Klein spectrum of $\Psi$ contains a massless chiral mode localized around $x_5 = x_5^*$, where $\lambda \varphi(x_5^*) = m$. When $\varphi(x_5)$ can be approximated with a linear function, the chiral zero mode has a Gaussian profile in the extra dimension with $\lambda$-dependent width. Assuming that the Higgs $H$ is not localized, small Yukawa couplings between $N$ and $L$ are naturally given by a small overlap between their wave functions as depicted in the figure below.

\begin{center}
\includegraphics[width=0.5\textwidth]{diagram.png}
\end{center}

This setup naturally generates the desired matrix of
Yukawa couplings

\[
N_{\text{sun}} \begin{pmatrix} \mathcal{O}(\epsilon^2) & \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon^{11}) \\ \mathcal{O}(\epsilon^{11}) & \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon) \end{pmatrix} N_{\text{atm}} \begin{pmatrix} \mathcal{O}(\epsilon^2) & \mathcal{O}(\epsilon^2) & \mathcal{O}(\epsilon^{11}) \\ \mathcal{O}(\epsilon^{11}) & \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon) \end{pmatrix}
\]

(\text{where } \epsilon \text{ is a free parameter) and } \text{suppresses } N_{\text{sun}}N_{\text{atm}} \text{ mixing mass terms. While we do not gain any new insight proceeding along this route, we are motivated to study the implications of the model.}

**Neutrinos**

The model \(^1\) predicts the following Majorana mass matrix for the light neutrinos:

\[
m_{\nu} = m_{\text{atm}} \begin{pmatrix} \epsilon s^2 & \epsilon s c e^{-i\phi/2} & 0 \\ \epsilon s c e^{-i\phi/2} & s_{\text{atm}}^2 + \epsilon^2 c^2 e^{-i\phi} & s_{\text{atm}} c_{\text{atm}} e^{i\phi} \\ 0 & s_{\text{atm}} c_{\text{atm}} e^{-i\phi} & c_{\text{atm}}^2 \end{pmatrix},
\]

where

\[
m_{\text{atm}} = \frac{\lambda_{\text{atm}}^2}{\lambda_{\text{sun}}^2} m_{\text{sun}}, \quad \epsilon = \frac{\lambda_{\text{sun}}^2}{\lambda_{\text{atm}}^2} m_{\text{sun}} / M_{\text{atm}}.
\]

\(N_{\text{atm}}\) plays the rôle of ‘dominant right-handed neutrino’ \(^2\). Neutrinos have a hierarchical mass spectrum and the lightest neutrino is massless \(^3\). At leading order

\(^1\)An alternative minimal texture, where \(N_{\text{sun}}\) couples to \(L_{\tau}\) rather than to \(L_{\mu}\), is equally acceptable. Its predictions concerning neutrinos can be obtained exchanging in the equations below \(\theta_{23} \leftrightarrow \pi/2 - \theta_{23}\). At the moment atmospheric data do not distinguish between them.

\(^2\)Neutrino masses are \(m_{\nu_{\text{atm}}} = m_{\nu_{\text{sun}}} / \lambda_{\text{sun}}^2 / \lambda_{\text{atm}}^2 m_{\text{sun}} / M_{\text{atm}}\), with

\(^3\)If the singlet neutrinos have a pseudo-Dirac mass term \(M_{N_{\text{sun}}N_{\text{atm}}}\), rather than the masses of eq. \(^1\), one gets light neutrinos with inverted mass hierarchy. Without fine-tuning its parameters, the resulting texture predicts \(\theta_{12} \approx \pi/4\) which is strongly disfavoured by the present data.

in \(\epsilon\), the oscillation parameters are

\[
\Delta m_{\text{atm}}^2 = m_{\text{atm}}^2 > 0, \quad \Delta m_{\text{sun}}^2 = R \Delta m_{\text{atm}}^2,
\]

with \(R = \epsilon^2(s^2 + c^2 e^{-i\phi})^2\). We define \(m_{\text{sun}} = \sqrt{\Delta m_{\text{sun}}^2}\).

The mixing angles in the standard notation are

\[
\theta_{23} = \theta_{\text{atm}}, \quad \theta_{13} = \epsilon s c_{\text{atm}}, \quad \tan \theta_{12} = \frac{s}{c_{\text{atm}}^{\text{atm}}}.
\]

The neutrino mixing matrix \(V\) relating the mass eigenstates \(\nu_i\) to the flavour eigenstates, \(\nu_i = V_{ij} \nu_j\), is

\[
V = \text{diag}(1, e^{-i\phi/2}, e^{-i\phi/2}) \cdot R_{23}(\theta_{23}) \cdot \text{diag}(1, 1, e^{i\phi}),
\]

where \(R_{ij}(\theta_{ij})\) represents a rotation by \(\theta_{ij}\) in the \(ij\) plane. The first phase matrix in \(V\) is unphysical and can be absorbed into the phases of \((L_e, L_\mu, L_\tau)\). The last phase matrix contains a practically unmeasurable Majorana phase. The phase matrix in the middle determines that the CP-violating phase in oscillations (observable in the planned experiments) is exactly the phase \(\phi\) in \((\text{I})\). A ‘positive’ phase, \(0 < \phi < \pi\) induces \(P(\nu_e \rightarrow \nu_\mu) > P(\nu_\mu \rightarrow \nu_e) = P(\nu_e \rightarrow \nu_e)\) in vacuum oscillations with baseline \(L < 2\pi E_\nu / \Delta m_{\text{sun}}^2\).

Therefore this model predicts (see also \(\text{I})\)

\[
\theta_{13} \approx \frac{\sqrt{R}}{2} \sin 2\theta_{12} \tan \theta_{23}, \quad m_{ee} = m_{\text{sun}} \sin^2 \theta_{12}, \quad (2)
\]

where \(m_{ee}\) is the \(ee\) element of the neutrino mixing matrix to be measured in neutrino-less double-beta \((0\nu\beta\beta)\) decay experiments. Present atmospheric neutrino data indicate \(\Delta m_{\text{atm}}^2 \approx 2.7 \times 10^{-3} \text{eV}^2\) and \(\theta_{13} \approx 1 \text{I})\). Solar and reactor data \(\text{I})\) indicate \(\Delta m_{\text{sun}}^2 \approx 7.0 \times 10^{-5} \text{eV}^2\) and \(\tan^2 \theta_{12} \approx 0.45\) (another solution with a slightly higher value of \(\Delta m^2\) is somewhat disfavoured by data, see Fig. \(\text{I})\). Combining present astrophysical and solar data \(\text{II})\) we obtain the predictions

\[
\theta_{13} = 0.078 \pm 0.015, \quad m_{ee} = 2.6 \pm 0.4 \text{meV}. \quad (3)
\]

The predicted value of \(m_{ee}\) is below the sensitivity of the planned next-generation \(0\nu\beta\beta\) experiments \(\text{III})\). Therefore we focus on studying \(\theta_{13}\).

Fig. \(\text{II})\) shows the \(\Delta \chi^2\) distribution for the predicted \(\theta_{13}\), compared with the present bound from CHOOZ \(\text{IV})\).
and SK [13] ($\theta_{13} < 10^\circ$ at 90% CL). Its structure reflects the presence of local minima at different values of $\Delta m_{\text{sun}}^2$ in our global fit of solar and KamLAND data, see Fig. 3a. With more statistics, KamLAND will be able to measure the solar oscillation parameters with few % error [12]. Long baseline experiments will measure the atmospheric parameters with few % error [15], allowing to predict $\theta_{13}$ within $\sim 10\%$ error. First-generation long-baseline experiments will be sensitive to $\theta_{13} > 0.08$ [15]. The whole predicted range for $\theta_{13}$ can be covered at second-generation experiments, such as JHF [15].

So far we have discussed the predictions for the light-neutrino mass matrix $m_\nu$. Within this model, oscillation experiments have already fixed it, except for the CP-violating phase. Future experiments will test the model. To get information on the heavy neutrino masses in (1), and to test the high-energy part of the model, we need an additional input from leptogenesis.

**Leptogenesis**

The decays of the lightest right-handed neutrino,

$$N_1 = N_{\text{sun}} \text{ or } N_{\text{atm}},$$

generate a lepton asymmetry only in $L_\mu$ (see (2)). The generated lepton asymmetry is then converted into a baryon asymmetry by sphalerons [14]. The baryon-to-entropy ratio in the non-supersymmetric model is given by

$$\frac{n_B}{s} = (0.85 \pm 0.15) \times 10^{-10} = \frac{3\epsilon \eta}{2183}, \quad (4)$$

where $\epsilon$ is the CP-asymmetry in $N_1$ decays and $\eta < 1$ is an efficiency factor, determined by solving the relevant set of Boltzmann equations [17].

In Fig. 3b, we show the iso-curves of the predicted $n_B/s$ in the $(M_{\text{sun}}, M_{\text{atm}})$ plane assuming the best-fit values of oscillation parameters. Unless the heavy neutrinos are extremely degenerate, which we regard as a fine tuning, Fig. 3 implies that the $N_1$ Yukawa couplings are sufficiently large that $N_1$ quickly reaches the thermal abundance and washes out the lepton asymmetry eventually generated by the heavier singlet neutrino.

The main features of leptogenesis in this model can be understood by simple analytic approximations as follows.

- If $M_{\text{sun}} \ll M_{\text{atm}},$

$$\epsilon = \frac{3}{16\pi} \frac{m_{\text{atm}} M_{\text{sun}}}{v^2} \frac{s_{23}^2}{1 + c_{23}^2 t_{12}} \sin \phi.$$

For $M_1 < 10^{14}$ GeV only $\Delta L = 1$ washout scatterings contribute to the efficiency factor, and $\eta$ is approximately given by [17]

$$\eta \approx 1.5 \times 10^{-4} \text{eV}/\tilde{m},$$

where the effective mass $\tilde{m}$ is given only in terms of the $L_\mu$ interactions in (1). The texture predicts at the best-fit point

$$\tilde{m} = m_{\text{sun}} \frac{c_{12}^2}{c_{23}^2} \approx 0.01 \text{ eV}.$$

Thus $\eta \sim 0.01$. The observed baryon asymmetry is obtained for $\phi < 0$ and $M_{\text{sun}} \approx 10^{11}$ GeV$/|\sin \phi|$ independently of $M_{\text{atm}}$.

- If $M_{\text{atm}} \ll M_{\text{sun}},$

$$\epsilon = -\frac{3}{16\pi} \frac{m_{\text{atm}}}{v^2} \frac{t_{23}^2}{1 + t_{12}^2} \sin \phi,$$

$$\tilde{m} = s_{23}^2 m_{\text{atm}} \approx 0.03 \text{ eV},$$

where the effective mass $\tilde{m}$ is obtained from an accurate numerical computation, performed along the lines of [15]. We include important thermal corrections to the Higgs mass $m_H$ and use the values of top Yukawa coupling $\lambda_t$, $\tilde{m}$ and $m_{\text{atm}}$ renormalized at the $N_1$ mass ($\Delta L = 1$ washout scatterings depend on $m_H$ and $\lambda_t$).
thus $\eta \sim 0.003$. The observed baryon asymmetry is obtained for $\phi > 0$ and $M_{\text{atm}} \approx 10^{12}$ GeV$/|\sin \phi|$ independently of $M_{\text{sun}}$.

- If $M_{\text{atm}} \approx M_{\text{sun}}$ the CP-asymmetry is enhanced [18] by $1/|M_{\text{atm}} - M_{\text{sun}}|$ and reaches a maximum $\epsilon \sim 1$ when the mass difference is comparable to the decay widths. The observed baryon asymmetry can be obtained for a large range of relatively low heavy neutrino masses. Its sign still depends on which singlet neutrino is heavier, and it does not fix the sign of CP-violation in oscillations.

To summarize, Fig. 3 implies that we need to know both the sign of $\phi$ and the sign of the baryon asymmetry to determine the discrete ambiguity of the model: the mass ordering of the heavy neutrinos. For hierarchical heavy neutrinos leptogenesis determines the mass of the lightest one, but does not test the model.

**Supersymmetry and lepton flavour violation**

If nature is supersymmetric, it could be possible to fix and test the high-energy part of the model. For leptogenesis and neutrino masses the presence of supersymmetry changes only few $O(1)$ coefficients: (i) the vacuum expectation value $v$ is replaced by $v \sin \beta$; (ii) the CP-asymmetry $\epsilon$ becomes 2 times larger when $M_{\text{sun}}$ and $M_{\text{atm}}$ are hierarchical [19] (iii) numerically eq. (4) remains practically unchanged since the number of model degrees of freedom is about doubled; (iv) washout becomes more efficient [20]:

$$\eta \approx 0.3 \ 10^{-4} \text{eV}/\tilde{m}.$$ 

The final result is shown in Fig. 3b which differs from Fig. 3a by a small factor. Both for $\phi > 0$ and $\phi < 0$ we observe a potential conflict between obtaining a successful thermal leptogenesis and avoiding overproduction of gravitinos [21] in this model. If gravitinos do exist, they either must be heavier than $m_{\tilde{G}} > 10$ TeV in order to allow the mass-scales of Fig. 3b, or, for $m_{\tilde{G}} \sim 1$ TeV, one must have $M_1 < 10^8$ GeV. The last condition is satisfied only when $M_{\text{sun}}$ and $M_{\text{atm}}$ are almost degenerate.

In supersymmetric extensions of the see-saw model, the renormalization effects due to the neutrino Yukawa couplings imprint lepton flavour violation in the slepton masses [22]. Assuming that soft terms are universal at the unification scale (a hypothesis that collider experiments can partly test), in a generic see-saw model

$$\mathcal{W} = \mathcal{W}_{\text{MSSM}} + \frac{M_{ij}}{2} N_i N_j + \lambda_N^i N_i^T L^j H_u,$$

the correction to the $3 \times 3$ mass matrix of left-handed sleptons is given by

$$m_{\tilde{L}}^2 = m_0^2 - \frac{1}{(16\pi)^2} (3m_0^2 + A_0^2) \lambda_N^i \ln \left( \frac{M_{\text{GUT}}^2}{M M} \right) \lambda_N^i + \cdots.$$
In general see-saw models the presence of too many uncontrollable neutrino parameters does not allow to make real predictions on lepton flavour violation (LFV). The present model allows us to compute the $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ rates \cite{23} (and related LFV processes \cite{23}) in terms of the two high-energy parameters $M_{\text{sun}}$ and $M_{\text{atm}}$. Assuming that thermal leptogenesis generates the observed baryon asymmetry, we get predictions more sharp than what suggested by a naive counting of the number of free parameters. Barring the case of almost degenerate singlet neutrinos $M_{\text{sun}} \approx M_{\text{atm}}$ (where only the ratio

$$
\frac{\text{BR} (\mu \rightarrow e\gamma)}{\text{BR} (\tau \rightarrow \mu\gamma)} = \frac{m_\mu^5 m_\mu \Delta m_{\text{sol}}^2}{m_\tau^2 m_\tau \Delta m_{\text{atm}}^2 \sin^2 \theta_{12} \sin^2 \theta_{23} \cos \theta_{23}} \approx 0.2 \tag{5}
$$

can be predicted), leptogenesis fixes the mass of the lightest singlet neutrino allowing to compute its Yukawa couplings, and consequently, the LFV rates that it induces. The predictions depend on the sign of the CP-violating phase $\phi$ measurable in oscillations.

- If $\phi < 0$, $N_1$ is $N_{\text{sun}}$, $\text{BR} (\mu \rightarrow e\gamma)$ can be predicted while $\text{BR} (\tau \rightarrow \mu\gamma)$ remains a function of a single unknown parameter, $M_{\text{atm}}$. Since $M_{\text{atm}} > M_{\text{sun}}$ the model also predicts a lower bound on $\text{BR} (\tau \rightarrow \mu\gamma)$.

- If instead $\phi > 0$, $N_1$ is $N_{\text{atm}}$, $\text{BR} (\tau \rightarrow \mu\gamma)$ can be predicted, together with a lower bound on $\text{BR} (\mu \rightarrow e\gamma)$. The latter is a function of the unknown $M_{\text{sun}} > M_{\text{atm}}$.

As usual, the predicted LFV rates depend on sparticle masses which can be measured at colliders. Taking into account naturalness considerations and experimental bounds, we give our numerical examples for $m_0 = 100 \text{ GeV}$, $M_{1/2} = 150 \text{ GeV}$, $A_0 = 0$ and $\tan \beta = 10$. In Fig. 3 we show the iso-curves of the LFV processes for this input, assuming the best-fit oscillation parameters. The branching ratios are calculated by solving numerically the renormalization group equations and using exact formulae in \cite{23}. Both $\text{BR} (\mu \rightarrow e\gamma)$ and $\text{BR} (\tau \rightarrow \mu\gamma)$ can be in the reach of future experiments \cite{23}. Their behavior is approximately given by

$$
\begin{align*}
\text{BR} (\mu \rightarrow e\gamma) & \approx 2.7 r 10^{-12} \left( \frac{M_{\text{sun}}}{10^{12} \text{ GeV}} \right)^2, \\
\text{BR} (\tau \rightarrow \mu\gamma) & \approx 1.5 r 10^{-11} \left( \frac{M_{\text{atm}}}{10^{12} \text{ GeV}} \right)^2,
\end{align*}
$$

where the logarithmic dependence on the heavy masses is neglected and we have introduced an approximate scaling factor

$$
 r \approx \left( \frac{\tan \beta}{10} \right)^2 \left( \frac{150 \text{ GeV}}{m_{\text{SUSY}}} \right)^4,
$$

($r = 1$ at our reference point) in order to show the dominant dependence on supersymmetric model parameters. In particular, the branching ratios decouple as $1/m_{\text{SUSY}}^4$ if sparticles are heavy. When sparticle masses will be measured, it will be possible to present more precise predictions.

For hierarchical heavy neutrinos, for $|\sin \phi| = 1$, and for the LMA best-fit oscillation parameters, the predictions are

$$
\begin{align*}
\text{BR} (\mu \rightarrow e\gamma) & \approx 2 r 10^{-13} \quad \text{for} \quad \phi < 0, \\
\text{BR} (\tau \rightarrow \mu\gamma) & \gtrapprox 3 r 10^{-12} \quad \text{for} \quad \phi < 0.
\end{align*}
$$

These results imply that, if also $\tau \rightarrow \mu\gamma$ is observed the ratio $\text{BR} (\mu \rightarrow e\gamma)/\text{BR} (\tau \rightarrow \mu\gamma)$ according to (5). Observation of the LFV processes allows in principle to fix all the model parameters in (5), and to test its high-energy part.

\textbf{Conclusions}

Unlike the general see-saw model \cite{27}, the most minimal see-saw model \cite{8} allows to determine the low energy neutrino mass matrix entirely from neutrino oscillation experiments. The model predicts eq.s (2), (3). The sign of the oscillation CP-phase, together with the sign of the baryon asymmetry, fixes the order of the two heavy neutrino masses. Unless they are almost degenerate, successful thermal leptogenesis determines the lightest of them. The supersymmetric version of the model predicts either $\text{BR} (\mu \rightarrow e\gamma)$ or $\text{BR} (\tau \rightarrow \mu\gamma)$, depending on the sign of $\phi$. The other process remains a function of the heavier neutrino mass only, and has a lower bound on its branching ratio. If the heavy neutrinos are almost degenerate, the model predicts only the ratio $\text{BR} (\mu \rightarrow e\gamma)/\text{BR} (\tau \rightarrow \mu\gamma)$ according to (5). Observation of the LFV processes allows in principle to fix all the model parameters in (5), and to test its high-energy part.
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