On Behavioral Interpolation in Local LPV System Identification

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Abstract: State-space identification of Linear Parameter-Varying (LPV) models using local data still represents a significant challenge as interpolation of local state-space models suffers from the well-known state-basis coherence problem. Recently, various behavioral-type of interpolation methods, in which a global LPV model is constructed based on matching its input-output behavior with the local models, have been introduced to overcome this issue. However, these methods suffer from high computational complexity and stability restrictions of the local models. In this paper, a novel method is introduced that is based on direct local-matrix norm matching, which, contrary to previous works, does not suffer from basis incoherence of the local models and neither requires their stability. Although these properties are highly desirable, a simulation study on an e-Nose sensor system indicates that the method in its current form is not robust to noise and therefore future research is needed to apply the presented concept in a realistic system identification setting.

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1. INTRODUCTION

Identification and control of Linear Time-Invariant (LTI) systems has been studied in great detail. Among many available approaches (see Ljung (1987) for an overview), the various subspace identification algorithms (Van Overschee and De Moor (1996), Pintelon and Schoukens (2012)) provide reliable methods to estimate a state-space model from Input-Output (IO) data, in both frequency and time domains. These methods have been successfully used in applications, see e.g. Pintelon and Schoukens (2012). When faced with a non-linear plant, a popular control approach has been gain scheduling, in which a family of linearized models is obtained at various operating points and then interpolated, see e.g. Rugh and Shamma (2000). Due to the linearization step, any method designed for the identification of LTI systems can directly be used to obtain such local linear models of the plant. Interpolation of such models over the operating range of the system results in Linear Parameter-Varying (LPV) models, see e.g. Tóth (2010), that aim to capture the nonlinear behavior of the original plant. However, such an interpolation is a delicate process with many pitfalls that may prevent to capture a useful plant model for a desired operating range, see Bachnas et al. (2014); Zhang and Ljung (2017).

There have been many methods proposed for the identification of LPV systems. Roughly, these methods can be divided into local and global approaches. The former is based on local experiments, in which the scheduling variable is fixed at a set of operating points. The resulting LTI models are then combined to form a single global model, see Bachnas et al. (2014) for an overview. The global approach, on the other hand, tries to model the system from one or more experiments where the scheduling variable dynamically changes, see Tóth (2010); Goos (2016) for an overview of available methods. While global methods aim at embedding the overall dynamics of the plant, local methods aim at approximating the local linear dynamics (differential dynamics) of the system. It has been shown that local approaches based control solutions are competitive design methods w.r.t. global models based synthesis. Despite the fact that local LPV identification is an important tool for control synthesis, the current methods still suffer from several challenging interpolation and complexity problems, especially for MIMO (Multiple-Input-Multiple-Output) systems. Roughly, two main approaches that are based on state-space representations exist. The behavioral-type of methods work with system norms, such as the $H_\infty$-norm in Vizer et al. (2013b) or the $H_2$-norm in Petersson (2013) and aim at matching the local frozen frequency response of the LPV model with the available local LTI snapshots of the plant. Because these approaches are based on system norms, they are invariant to the state basis of locally identified models. However, they require the solution of nonlinear optimization problems that have no guarantees of finding the global optimum and their computational time is high when a large number of local models is involved. Furthermore, it is not known how to systematically apply these system norms based methods in case the local models are unstable. The other set of methods directly aim at interpolation of the state-space matrices, see e.g. Lovera and Mercè (2007), Caïgny et al. (2012) and Wassink et al. (2014). Two substantial

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challenges occur when working directly with state-space matrices in LPV identification. First, as shown in Tóth (2010); Kulcsár and Tóth (2011), applying state transformations to bring the true LPV state-space representation of a system to favorable forms for interpolation, e.g., canonical forms, introduces dynamic dependency which is invisible from the local LTI snapshots. Hence, interpolation of local canonical forms can lead to arbitrarily large errors in the LPV model when the scheduling varies over time. The second problem, especially prominent in case a black-box approach is followed as indicated in Tóth (2010); Vizer et al. (2013a); Zhang and Ljung (2017), is that due to the independent identification of the local models, their state-basis are inevitably incoherent, leading to poor performance of the interpolated LPV model in between the operating points that were not included in the data set.

Based on previous considerations, the contributions of this work are as follows:

- A novel matrix interpolation method is introduced that synthesizes scheduling-dependent transformations to both the given local models and the LPV model such that the resulting interpolation does not suffer from the incoherence of the local models.
- For a special class of LPV models, it is shown that the resulting cost function with the proposed matrix-interpolation approach is polynomial for which the complexity of the resulting optimization problem is linear in the number of local models.
- The behavioral method and the proposed matrix approach are compared under several cost functions in a simulation study on an e-Nose system, and recommendations are given on how the behavioral and matrix frameworks can be used together for efficient LPV system identification in practice.

The paper is organized as follows. Section 2 gives an overview of the notation while Section 3 introduces LPV state-space models and certain sub-classes thereof. The proposed methodology is explained in Section 4. The developed algorithms are investigated further through simulations in Section 5. Finally, the work is concluded in Section 6.

2. NOTATION

The $n$-by-$n$ identity matrix is $I_n$. The subscript is omitted when the dimensions are clear from the context. The transpose of $A$ is $A^\intercal$. The pseudo-inverse of $A$ is $A^\dagger$. The maximum singular value is given by $\sigma_{\text{max}}(A)$. $\|A\|_2$ is the 2-norm when $A$ is a matrix and the $H_2$-norm when $A$ is an LTI system. The Frobenius norm of a matrix $A$ is $\|A\|_F$. The $H_\infty$-norm of an LTI system $A$ is denoted by $\|A\|_\infty$. $x[i]$ denotes the $i$th element of a vector $x$. An LTI system $\mathcal{H}$ with state-space realization $(A, B, C, D)$ is denoted by $\mathcal{H} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$.

3. LPV MODELS

An LPV model is a dynamic mapping from an input signal $u(t) \in \mathbb{R}^{n_u}$ to an output signal $y(t) \in \mathbb{R}^{n_y}$. The mapping is governed by a scheduling variable $p(t) \in \mathcal{P} \subset \mathbb{R}^n$. For a fixed scheduling trajectory, the input-output mapping is linear. The LPV systems considered in this work admit continuous-time state-space representations that describe via a first order difference equation how $u$ and $p$ influence the variation of $y$ by the help of a state variable $x(t) \in \mathbb{R}^n$:

$$\Sigma : \begin{cases} \dot{x}(t) = A(p(t))x(t) + B(p(t))u(t), \\
y(t) = C(p(t))x(t) + D(p(t))u(t), \end{cases}$$

with $A, B, C, D : \mathcal{P} \to \mathbb{R}^{n \times n}$ appropriately sized bounded analytic matrix-valued functions of $p$. Note that for any constant scheduling trajectory, i.e. $p(t) \equiv p$, the input-output dynamics of (2) are LTI. In the sequel, the assumption is made that during the identification process the scheduling is held constant. For convenience, the dependence on $t$ will be dropped from subsequent notation. The LPV model (2) is very general and is capable of accurately describing a wide range of physical systems. In the literature, the state-space matrices are often restricted to depend affinely on $p$:

$$\begin{bmatrix} A(p) & B(p) \\ C(p) & D(p) \end{bmatrix} = U_0 + \sum_{i=1}^{n_p} p_i U_i,$$

with $U_i, i = 0, \ldots, n_p$ appropriately sized matrices, resulting in the class of LPV State-Space models with Affine (SSA) dependence. A second model class used in this paper restricts the state-space matrices as

$$\begin{bmatrix} A(p) & B(p) \\ C(p) & D(p) \end{bmatrix} = \left( U_0 + \sum_{i=1}^{n_p} p_i U_i \right) \left( V_0 + \sum_{i=1}^{n_p} p_i V_i \right)^{-1},$$

with $\{V_i\}_{i=0}^{n_p}$ square matrices such that $V_0 + \sum_{i=1}^{n_p} p_i V_i$ is non-singular for all $p \in \mathcal{P}$. This model structure is more general than the LPV-SSA structure (which can be seen and matrix-valued functions of appropriate sizes.

Based on the data-generating LPV system is denoted by $\Sigma$ with state-space representation (2). Given is a set $\{p_i\}_{i=1}^{N}$. The corresponding frozen LTI dynamics of $\Sigma$ is denoted by $G_i := \Sigma(p_i)$. At each operating point, an LTI identification procedure is carried out to obtain a local model $G_i$. A family of LPV model candidates is postulated, parametrized over $\theta \in \mathbb{R}^{n_\theta}$:

$$M(\theta) : \begin{cases} \dot{x}(t) = A_M(p(t), \theta)x(t) + B_M(p(t), \theta)u(t), \\
y(t) = C_M(p(t), \theta)x(t) + D_M(p(t), \theta)u(t), \end{cases}$$

with $A_M, B_M, C_M, D_M : \mathcal{P} \times \mathbb{R}^{n_\theta} \to \mathbb{R}^{n_y}$ matrix-valued functions of appropriate sizes.

### 4. LOCAL MODEL INTERPOLATION

#### 4.1 Problem formulation

The data-generating LPV system is denoted by $\Sigma$ with state-space representation (2). Given is a set $\{p_i\}_{i=1}^{N}$ of operating points at which the corresponding frozen LTI dynamics of $\Sigma$ is denoted by $G_i := \Sigma(p_i)$. At each operating point, an LTI identification procedure is carried out to obtain a local model $G_i$. A family of LPV model candidates is postulated, parametrized over $\theta \in \mathbb{R}^{n_\theta}$:
Assumption 3. Each local state-space model representing \( \hat{G}_i \) is observable and has the same state dimension \( n_x \).

Throughout this work, Assumption 3 is considered to hold. Two approaches to find \( \theta^* \) are discussed. First, the behavioral framework, which is based on local system norm errors, and second, the matrix framework based on local matrix norm errors.

4.2 Behavioral framework

In the behavioral framework, a global LPV model is constructed by matching the input-output behavior of the given local models and the global model. Different formulations exist with varying interpretations of how to characterize the approximation error of the global model. In Vizer et al. (2013a), the authors define a cost function based on the \( H_\infty \)-error:

\[
V(\theta) := \max_{i=1,...,N} \| \hat{G}_i - M(p_i, \theta) \|_\infty.
\]

The global model is then found through a non-linear optimization:

\[
\theta^* = \arg \min_{\theta} V(\theta).
\]

Furthermore, it is shown in Vizer et al. (2013b) that (7) can be cast as a structured \( H_\infty \)-synthesis problem.

4.3 Matrix framework

In the matrix framework, the LPV model is constructed based on the interpolation of the matrices of the SS representations of \( \{\hat{G}_i\}_{i=1}^N \). A state-space representation of each \( \hat{G}_i \) is readily available using the results of LTI system identification (see Ljung (1987) for an overview of available methods). For the sake of simplicity, let us neglect the estimation error and consider \( \{\hat{G}_i = G_i\}_{i=1}^N \). (Assumption 1). However, due to the identification step, the local state-space representations of \( \hat{G}_i \) cannot be assumed to be on the same state basis, i.e.:

\[
\hat{G}_i = \begin{bmatrix} T_i A(p_i) T_i^{-1} & T_i B(p_i) \\ C(p_i) T_i^{-1} & D(p_i) \end{bmatrix},
\]

where \( T_i \) are unknown non-singular matrices. This implies that these representations cannot directly be compared to the local state-space representation of the LPV model \( M(p_i, \theta) \) evaluated at that operating point. The observability matrix \( \mathcal{O}(p_i) \) of \( G_i \) is given by:

\[
\mathcal{O}(p_i) = \begin{bmatrix} C(p_i) \\ C(p_i) A(p_i) \\ \vdots \\ C(p_i) A(p_i)^{n_x-1} \end{bmatrix},
\]

Using (8), the observability matrix of the local models \( \hat{G}_i \) can be written as \( \hat{\mathcal{O}}(p_i) = \mathcal{O}(p_i) T_i^{-1} \). Applying a non-singular state transformation \( \hat{T}_i \) to the state-space representation of \( \hat{G}_i \) gives an equivalent realization

\[
\hat{G}_i = \begin{bmatrix} \hat{T}_i & T_i A(p_i) T_i^{-1} & \hat{T}_i B(p_i) \\ C(p_i) T_i^{-1} & D(p_i) \end{bmatrix},
\]

In many approaches available in the literature, researchers tried to construct \( \hat{T}_i \) such that \( \hat{T}_i^{-1} = T_i \), i.e., making to the local state-basis coherent to recover the global state-basis of a representation of \( \Sigma \). Then, by interpolating the transformed matrices over \( \mathbb{P} \), they argued to recover (2). In Zhang and Ljung (2017), it was shown that restoring coherence of the local state-basis in this way is mathematically not possible, questioning the rationale behind matrix interpolation based methods.

In one of the methods, SMILE, presented in Caigny et al. (2012), the following state transformation is applied:

\[
\hat{T}_i = \hat{O}^T(p_i) \hat{O}(p_i),
\]

which is in fact

\[
\hat{T}_i = (\mathcal{O}(p_i) T_i^{-1})^\dagger \mathcal{O}(p_i) T_i^{-1}.
\]

Without the loss of generality, one can assume that \( T_i = I \) and transform all local models \( \hat{G}_i \) as

\[
\hat{G}_i = \begin{bmatrix} \hat{A}(p_i) & \hat{B}(p_i) \\ \hat{C}(p_i) & D(p_i) \end{bmatrix},
\]

with:

\[
\hat{A}(p) = \mathcal{O}^T(p) A(p) \mathcal{O}(p), \quad \hat{B}(p) = \mathcal{O}^T(p) B(p), \quad \hat{C}(p) = \mathcal{C}(p) \mathcal{O}(p), \quad \hat{D}(p) = D(p),
\]

where \( \mathcal{O} = \mathcal{O}(p_1) \). The SMILE procedure from Caigny et al. (2012) then continues to interpolate the transformed local models \( \hat{G}_i \) directly. However, note that the applied transformation has not solved the coherency issue, as transformation with \( \mathcal{O}(p) \) does not provide an observability canonical form of (2) in the global sense, see Tóth (2010); Zhang and Ljung (2017). However, we can observe that \( \hat{G}_i \) is a transformed form of the local canonical representation of \( G_i \), which is independent of \( T_i \) and uniquely represents the corresponding IO map.

To overcome the incoherence problem, the core idea is that instead of interpolation, a similar transformation is applied to the frozen form \( M(p_i, \theta) \) of the parametrized LPV model \( M(\theta) \) at each \( p_i \) to also arrive locally at a \( (\mathcal{O}_1\text{-transformed}) \) observability canonical form:

\[
T_{M,i}(\theta) := \mathcal{O}_1^T M(p_i, \theta).
\]

Performing the local matching using the observability canonical forms avoids the issue of coherent bases just like in the behavioral case. Approximation error of the LPV model w.r.t. the \( i \)-th local model, can be characterized by

\[
c_i(\theta) := \| \hat{S}_i - T_i^{-1}(\theta) S_{M,i}(\theta) (T_i^R(\theta))^{-1} \|_F,
\]

with:

\[
\hat{S}_i = \begin{bmatrix} \hat{A}_i & \hat{B}_i \\ \hat{C}_i & \hat{D}_i \end{bmatrix}, \quad S_{M,i}(\theta) = \begin{bmatrix} A_{M}(p_i, \theta) & B_{M}(p_i, \theta) \\ C_{M}(p_i, \theta) & D_{M}(p_i, \theta) \end{bmatrix},
\]

\[
T_{iM}(\theta) = \begin{bmatrix} \mathcal{O}_1^T M(p_i, \theta) & 0 \\ 0 & I_{n_y} \end{bmatrix}, \quad T_{iR}(\theta) = \begin{bmatrix} \mathcal{O}_1^T M(p_i, \theta) & 0 \\ 0 & I_{n_u} \end{bmatrix}
\]

The cost (15) associated with each local model can be combined to form a cost function over all local models. For example, the squared sum can be used:

\[
V_{\text{mat}}(\theta) := \sum_{i=1}^{N} c_i^2(\theta).
\]

Calculation of matrix inverses is required when working with (15). To avoid the corresponding complicated optimization problem, the local cost is modified by right-multiplying the argument of the Frobenius norm in (15) with \( T_i^R \):

\[
c_i(\theta) := \| \hat{S}_i T_i^R(\theta) - T_i^{-1}(\theta) S_i(\theta) \|_F
\]
resulting in the overall approximation error function:

\[ V_{\text{mat}}(\theta) := \sum_{i=1}^{N} c_i(\theta)^2. \]  

(18)

In Appendix B, it is shown that \( c_i \) is bounded by \( \bar{c}_i \):

\[ \bar{c}_i \| T_i^R(\theta) \|^{-1}_F \leq c_i \| T_i^R(\theta)^{-1} \|_F \]  

(19)

In the special case of LPV-SSA models, \( V_{\text{mat}}(\theta) \) is a polynomial function due to the use of the Frobenius norm. The degree of the polynomial is at most \( 2(n_x + 1) \). Since both the dimension of \( \theta \) and the maximum degree of the polynomial are fixed, the functional complexity of \( V_{\text{mat}}(\theta) \) is independent of the number of local models, since only the number of polynomial constraints in terms of \( N \) grows linearly by including more local models. Furthermore, with a slight modification, \( V_{\text{mat}}(\theta) \) is a polynomial function for LPV models of the form (4) as well. In this case, the inverse term appearing in the model description can be removed by also right-multiplying the argument of the norm in a similar manner as was done to remove the inverse of \( T_i^R(\theta) \).

4.4 Comparison

Three different cost functions have been introduced that can be used to achieve local model interpolation: the behavioral cost (6) and the matrix-based costs (16) and (18). The behavioral framework at first sight appears to be the most attractive since it minimizes the input-output mismatch between local and global models. The cost function associated with it has a clear physical interpretation, whereas the element-wise error between state-space matrices may not give a representative indication of the input-output error seen in practice. However, the structured \( H_\infty \)-synthesis problem is generally non-convex and although sophisticated solvers exist, e.g., hinfstruct in MATLAB, convergence problems can quickly arise when the number and dimension of local models increases.

A clear advantage of the matrix framework that it requires lower computational effort in general, but had been affected by the local state incoherence problem. With the proposed formulation in (18), the coherence problem is avoided and the model interpolation is cast as a sum-of-squares optimization problem. These types of optimization problems are studied well in literature and may be solved with arbitrary precision using successive convex relaxations, see e.g. Papp and Yildiz (2017). Due to time constraints, this property is not exploited here and instead a gradient-based descent algorithm (fminunc in MATLAB) is used. One additional advantage of the matrix framework is that it is agnostic to the open-loop stability of the local model using (6) must be done in a closed-loop setting. The matrix-based cost functions require no modifications.

In practice, the local frozen behavior of the LPV model is not guaranteed to have constant minimal state dimension for every \( p \in \mathbb{P} \) as postulated in Assumption 3. If observability of a local model drops at a particular operating point, then it is unclear how such models can be used in the proposed basis projection concept. Whether this restriction prevents successful application of the matrix framework is not considered in this work and is left as part of future research.

5. SIMULATIONS

In this section, the presented approaches to local model interpolation are compared on a simulation study using an LPV model of an e-Nose sensor system. The model, from Vizer et al. (2013a), which we treat as the data-generating system to be identified, is a third-order LPV-SSA model with \( n_u = n_y = 1 \). The dimension of \( p \) is \( n_p = 1 \) and \( \mathbb{P} = [-1, 1] \). The exact parameter-varying matrices defining the e-Nose sensor system are given in Appendix A, while the magnitude plots of the frozen frequency responses of the system are displayed in Figure 1.

The estimated model \( M \) is parametrized in terms of \( \theta \) corresponding to the matrices \( A_0, A_1, B_0, B_1, C_0, C_1 \) under affine dependency over \( p \), i.e., besides of the affine structure of the dependency, no prior knowledge is used.

For the identification of the local models \( \{\hat{G}_i\}_{i=1}^{2} \) at operating points -1, 0 and 1, a random-phase multi-sine input is used (see Pintelon and Schoukens (2012)), consisting of 100 frequency components on a logarithmically spaced frequency grid ranging from \( 10^{-3} \) rad/s to \( 10^3 \) rad/s. The output was perturbed by additive white Gaussian noise. The identification was performed using the MATLAB function n4sid. Figure 2 compares the magnitude of the frozen frequency response of the true system and the locally identified model at \( p = 0 \) for a Signal-to-Noise Ratio (SNR) of 20 dB.

The \( H_\infty \)-based cost (6) was optimized using hinfstruct. The matrix-based costs (16) and (18) were optimized using fminunc. \( \theta \) was initialized in each run as follows. The elements of \( \theta \) that correspond to the nominal values \( (A_0, \ldots, D_0) \) were set to the state-space matrices of the estimated \( \hat{G}_1 \) at \( p = 0 \) (since it is assumed that \( T_1 = \) Fig. 1. Frozen frequency response magnitude of the e-Nose sensor model as function of the constant scheduling \( p \).
Fig. 2. Frozen frequency response magnitude of the e-
Nose system and the identified model at p = 0 under
SNR = 20 dB.

To evaluate the quality of the obtained LPV models
w.r.t. the e-Nose sensor system Σ in the local sense, the
maximum relative H∞-error over a dense, linearly spaced
grid of P with Nval = 100 is used, i.e.:
\[
\gamma(\theta) := \max_{i \in \{1, \ldots, N_{\text{val}}\}} \| \Sigma(p_i) - M(p_i, \theta) \|_\infty \cdot \| \Sigma(p_i) \|_\infty^{-1}
\]
(21)

First the noiseless case is treated. When no noise is present,
it is expected that the global minima of all cost functions
converge to the same value of 0, as indicated in (20).
However, the local minimizers and minima may differ.
Using three operating points at -1, 1, and 0, a Monte
Carlo simulation was performed of 100 trials. In each run,
the achieved model quality γ(θ*) was recorded. Box
plots for each of the optimizations are shown in Figure
3. Optimization of V leads to an average error of -45 dB
with small variance. The matrix-based optimization leads
to significantly lower mean cost of -110 dB and -88 dB
for Vmat and Ṽmat, respectively. However, the variance is
larger, with some outliers of Ṽmat leading to the largest γ
of the three optimization procedures.

The experiment was repeated for an SNR of 40 dB. Box
plots for this case are shown in Figure 4. A mean error of
-27 dB is achieved for V, while no stable model is returned
after optimization of Vmat, in which case γ = ∞. For
Vmat, most of the models were unstable as well, while the
stable models were inaccurate (γ > -1 dB). Clearly, V
offers better robustness w.r.t. noise for this problem. One
explanation for this effect is that Vmat and Ṽmat do not
explicitly enforce stability at the operating points. In this
instance, the noise has apparently affected the local models
in such a way that the optimal matching of state-space
matrices leads to unstable LPV models, which indicates
that still further research is needed to restrict the degrees
of freedom in the interpolation to avoid such robustness
problems.

Finally, the computational complexity of the approaches
is compared by measuring the time it takes to perform
optimization of the three cost functions for an increasing
number of operating points. It is meaningless to compare
the duration in absolute terms, because the supporting
back-ends may have been compiled and optimized in com-
pletely different ways. However, the relative increase as a
function of the problem size can give a good indication of
the computational complexity. The results are shown in
Figure 5. Time required to optimize Vmat and Ṽmat pro-
gresses linearly with the number of local models, whereas
for V this relation appears to be quadratic. Vmat shows
the lowest relative increase: after increasing the number of
operating points from 3 to 17, the required time only
increases by a factor 4, whereas V takes 15 times longer
to optimize.

6. CONCLUSION

In this paper, a novel approach for local identification
of LPV state-space models is proposed that (i) exploits
the computational benefits of interpolating the state-space
matrices of locally identified LTI models of the system, but
(ii) is not affected by the basis incoherence problem. The
core idea behind the proposed approach is to formulate a
global parameterization of the matrix functions composing
the state-space model and at each considered operating
point transform the locally identified LTI models and the
frozen state-space representation of the LPV model to the
same basis by the help of observability canonical basis. As
the resulting local state transformations depend on the to-
be-optimized parameters, a reformulated version of the re-
resulting rational cost function of the matrix approximation error is proposed, leading to a polynomial optimization problem that can be efficiently solved.

In a simulation example of LPV identification of an e-Nose sensor, the proposed methodology is compared to a state-of-the-art $H_\infty$ sensor, the developed approach. When noise is not included, the quality of the optimized model is also comparable or even better than the model found by the $H_\infty$ method. However, even when a low amount of noise (SNR = 40 dB) was added, difficulties arise as the proposed method fails to find a stable and accurate model. Thus, future work is required in order to cope with noise while still being able to exploit the reduction in the computational load.

REFERENCES

Bachnas, A. et al. (2014). A review on data-driven linear parameter-varying modeling approaches: A high-purity distillation column case study. Journal of Process Control, 24, 272–28.

Caigny, J., Pintelon, R., Camino, J.F., and Swevers, J. (2012). Interpolated Modeling of LPV Systems Based on Observability and Controllability. In Proc. of the 16th IFAC Symp. on System Identification, 1773–1778. Brussels, Belgium.

Goos, J. (2016). Modeling and Identification of Linear Parameter-Varying Systems. Ph.D. thesis, Vrije Universiteit Brussel.

Kulcsár, B. and Tóth, R. (2011). On the similarity state transformation for linear parameter-varying systems. In Proc. of the 18th IFAC World Congress, 4155–4160.

Ljung, L. (1987). System identification: theory for the user. Prentice-Hall.

Lovera, M. and Mercère, G. (2007). Identification for gain-scheduling: a balanced subspace approach. In Proc. of the American Control Conf., 858–863. New York, USA.

Papp, D. and Yildiz, S. (2017). Sum-of-squares optimization without semidefinite programming. SIAM Journal on Optimization, 29, 822–851.

Petersson, D. (2013). A Nonlinear Optimization Approach to $H_2$-Optimal Modeling and Control. Ph.D. thesis, Linkoping University.

Pintelon, R. and Schoukens, J. (2012). System Identification: A Frequency Domain Approach. Wiley.

Rugh, W. and Shamma, J.S. (2000). Research on gain scheduling. Automatica, 36(10), 1401–1425.

Tóth, R. (2010). Modeling and Identification of Linear Parameter-Varying Systems. Lecture Notes in Control and Information Sciences, Vol. 403. Springer.

Van Overschee, P. and De Moor, B. (1996). Continuous-time frequency domain subspace system identification. Signal Processing, 52, 179–194.

Vizer, D., Mercère, G., Prot, O., and Ramos, J. (2013a). A local approach framework for black-box and gray-box LPV system identification. In Proc. of the European Control Conf., 1916–1921. Zurich, Switzerland.

Vizer, D., Mercère, G., Prot, O., Laroche, E., and Lovera, M. (2013b). Linear fractional LPV model identification from local experiments: An $H_\infty$-based optimization technique. In Proc. of the 52nd IEEE Conference on Decision and Control, 4559–4564.

Wassink, M.G., van de Wal, M., Scherer, C., and Bosgra, O. (2014). LPV control for a wafer stage: beyond the theoretical solution. Control Engineering Practice, 13, 231–245.

Zhang, Q. and Ljung, L. (2017). From structurally independent local LTI models to LPV model. Automatica, 84, 232–235.

Appendix A. SIMULATION SYSTEMS

The e-Nose sensor from Vizer et al. (2013a) can be described by an LPV-SSA model with a single scheduling variable $p \in [-1, 1]$. The state-space matrices are

$$A(p) = \begin{pmatrix} -0.41 & 0.21 & 0 \\ 0.31 & -0.61 & 0.26 \end{pmatrix} + p \begin{pmatrix} -0.09 & 0.09 & 0 \\ 0.09 & -0.09 & 0.09 \end{pmatrix}$$

$$B(p) = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}^\top$$

$$C(p) = \begin{pmatrix} 0.16 & 0.16 & 0.16 \end{pmatrix} + p \begin{pmatrix} 0.09 & 0.09 & 0.09 \end{pmatrix}$$

Note that the values are not equal to the ones from the reference since $p$ has been normalized and centered.

Appendix B. NORM BOUNDS

In this section, $\| \cdot \|$ is any matrix norm that is sub-multiplicative for non-square matrices, such as the Frobenius norm. Based on the sub-multiplicative property:

$$\|AX - B\| \leq \|A\| \|X\| \leq \|A - B\| \|X\|,$$  \hspace{1cm} (B.1)

for any $A, B \in \mathbb{R}^{m \times n}$ and non-singular $X \in \mathbb{R}^{n \times n}$. For the right-hand part, the proof is as follows:

$$\|A - B\| \leq \|A - BX^{-1}\| \leq \|A - B\| \|X^{-1}\|.$$  \hspace{1cm} (B.2)

For the left-hand part, the proof starts from

$$\|AX - B\| \leq \|A - B\| \|X\| \leq \|A - B\| \|X\|.$$  \hspace{1cm} (B.3)

Combining both inequalities gives (B.1).