On-off skyhook semi-active control via a magneto-rheological (MR) damper for airfoil-based energy harvesting systems

J N I Pang, H G Harno¹ and V C C Lee²
Curtin University Malaysia, CDT 250, 98009, Miri Sarawak, Malaysia.

E-mail:¹hg.harno@curtin.edu.my ²vincent@curtin.edu.my

Abstract. An on-off skyhook semi-active control strategy via MR damper is proposed in this study to enhance the performance of a two degree-of-freedom (DOF) airfoil-based energy harvester. For simplicity, only the plunge mode of the airfoil model is examined. NACA0012 airfoil is selected as the vibrating body where energy is harvested and converted into electricity via piezoelectric transduction. Behavioural performance of an actual MR damper is represented numerically with MATLAB/SIMULINK blocks of a conventional Bouc-Wen model. Simulation of the airfoil model is also performed on MATLAB to acquire its dynamic responses. A comparison between passive and semi-actively controlled airfoil systems demonstrates that the proposed strategy is superior in three different aspects – versatility, energy harvesting and sustaining structural integrity.

1. Introduction

Vibration suppression systems are commonly grouped into three types of control methods, namely passive, semi-active and active [1]. A passive system dissipates unwanted vibration in the form of heat without exerting energy from external sources. Due to their simplicity, passive systems are unable to flexibly react to multiple frequencies and parameter changes [2]. In contrast, active vibration systems do not face this problem and are outstanding in controlling unwanted vibration, but with the cost of sensors and actuators. Active vibration systems also require external sources of power, which means greater costs and complication [3]. Over the decades, semi-active control has been more preferable as it preserves the simplicity of a passive system, yet adapts to performance which is to that of an active system, but with minimal amount of external energy supplies [4]. Semi-active control involves time-varying parameters, such as damping and stiffness, to improve system performance [5].

Vibration is known as an oscillatory mechanical phenomenon that exists ubiquitously. Although the presence of vibration is typically unwanted, researchers have proved the feasibility of cultivating vibratory motions as an alternative source of renewable energy [6]. In fact, techniques to energy harvesting via ambient vibrations have been studied comprehensively. Some related applications are electrostatic micro-electro-mechanical systems (MEMS) vibration energy harvester [7] and large-scale energy harvesters [8].

This study aims to numerically study the performance enhancement of an airfoil-based energy harvesting system by introducing the semi-active control via MR damper. The conventional Bouc-Wen model [9] is used to approximately replicate the behavioral performance of an actual MR damper. Then,
the on-off skyhook semi-active algorithm is incorporated to improve the system performance by controlling aero-elastic vibration of the NACA0012 airfoil. The amount of energy that can be harvested by the semi-active airfoil system is attained and compared against the conventional passive airfoil system via MATLAB. It is demonstrated that the proposed semi-active model has increased the energy harvested in comparison with the passive model. Other benefits that the proposed semi-active model has brought about include versatility and conservation of structural integrity.

2. Numerical Methods
MATLAB/SIMULINK is used in this study to simulate the system’s dynamics. Numerical models are explicitly spelled out in following subsections.

2.1. Dynamic model of an airfoil-based energy harvester
In order to quantify the amount of energy harvested in terms of voltage, a piezoelectric coupling intended for energy conversion from mechanical vibration into electricity is attached onto the plunge DOF of the airfoil-based energy harvester. A typical 2-DOF airfoil section coupled with a piezoelectric transducer subjected to an airflow excitation is shown in Figure 1.

![Figure 1](image)

**Figure 1.** A 2-DOF airfoil section coupled with piezoelectric transducer on the plunge DOF [6].

The equations governing the 2-DOF airfoil section as shown in Figure 1 are expressed in Equations (1) to (3). For the computation of the plunge displacement and output voltage responses, Equations (1) to (3) are re-written in the form of state space equations to be solved via MATLAB’s `ode23` integration function.

\[(m + m_e)\ddot{h} + mb\alpha\ddot{\alpha} + d_\alpha\dot{\alpha}h + k_\alpha h - \frac{\theta}{l} = -L\]  
\[mb\alpha \ddot{h} + I_\alpha \alpha + d_\alpha \alpha + k_\alpha \alpha = M\]  
\[C_p \dot{v} + \frac{v}{R_1} + \theta \dot{h} = 0\]

2.2. On-off skyhook control algorithm
The on-off skyhook control depends on the absolute mass velocity and the damper velocity. When only plunge motion of the semi-active model is concerned, the conditional functions are governed by the plunge displacement to which the airfoil model is confined to move. Since semi-active dampers do not provide energy explicitly, the resulting semi-active damping force must always satisfy the following inequality:

\[F_{sa}(\dot{h} - \dot{\alpha}) \geq 0\]

where \(F_{sa}\) denotes the semi-active damping force to be exerted onto the plunge DOF of the airfoil system and the algorithm describing the on-off skyhook control is expressed as follows:
\[ d_h = \begin{cases} d_{max}, & h > h_{max} \text{ or } h < h_{min} \\ d_{min}, & h \leq h_{max} \text{ and } h \geq h_{min} \end{cases} \]  

(5)

where \( d_{max} \) and \( d_{min} \) denote the maximum and minimum semi-active damping coefficients in the plunge DOF, respectively; \( h_{max} \) and \( h_{min} \) denote the maximum and minimum plunge displacements the airfoil system is confined to move, respectively.

In order to maximize the energy harvested, optimal values are obtained and set for both high damping and low damping during the “on” and “off” states of the semi-active controller, respectively. This ensures consistent aero-elastic vibration amplitude of the NACA0012 airfoil to be maintained within the working range. The time-varying damping coefficient of the airfoil-based energy harvester in the plunge DOF is obtained using the following expression:

\[ d_h(t) = \frac{F_{oa}(t)}{h(t)} \]  

(6)

Median value of the time-varying damping coefficient \( d_h(t) \), which is a constant, is selected as the resulting damping coefficient \( d_h \).

2.3. Conventional Bouc-Wen model

Differential equations which constitute the conventional Bouc-Wen model are expressed as follows [10]:

\[ F = c_o \dot{x} + k_o (x - x_o) + az \]  

(7)

\[ \dot{z} = -\gamma |\dot{x}|z|^{n-1} - \beta \dot{x}|z|^n + A \dot{x} \]  

(8)

where \( c_o, k_o, \alpha, \beta, \gamma, x_o, n \) and \( A \) represent parameters describing the non-linear hysteresis of an MR damper [1]. The SIMULINK block of the Bouc-Wen model [10] is built and shown in Figure 2. In order to compute the semi-active damping force for Equation (6), parameters \( \alpha, \beta, \gamma \) and \( A \) in Equations (7) and (8) are set to be 0, 50, 50 and 100, respectively while the data for the remaining four parameters are shown in Table 1.

| Current level | \( c_o \) | \( k_o \) | \( n \) | \( x_o \) |
|--------------|----------|----------|------|------|
| 0 A          | 5.0743   | 5.106    | 1.1678 | 0    |
| 0.05 A       | 5.8399   | 8.4644   | 1.0858 | 0    |
| 0.1 A        | 6.4582   | 11.955   | 1.0017 | 0    |
| 0.2 A        | 8.6107   | 23.51    | 1.0029 | 0    |
| 0.3 A        | 11.132   | 42.893   | 1.0038 | 0    |
| 0.4 A        | 12.811   | 60.711   | 1.0013 | 0    |
Figure 2. SIMULINK block diagram of the conventional Bouc-Wen model [10].

2.4. Placement of the MR damper

The NACA0012 airfoil is attached onto a cantilever beam which connects it to a fixed point on the right end, as shown in Figure 3. Apparently, the plunge motion is distributed over the entire span length of the cantilever beam. Revisiting Equation (6), the time-varying damping coefficient is a function of the plunge velocity of the airfoil-based energy harvesting system. Therefore, the resulting damping coefficient $d_h$ is also distributed over the whole span length. In order to localize the damping into a particular location to resist only a point, the damping coefficient $d_h$ is multiplied with a dimensionless length. The governing expression is as follows:

$$d_{h\text{actual}} = d_h s$$  \hspace{1cm} (9)

where $s$ denotes the dimensionless length calculated as follows:
\[ s = \frac{a}{200} \]  

(10)

where \( a \) denotes the distance between the MR damper and the fixed point on the right end.

**Figure 3.** The setup of the complete airfoil-based energy harvester [11].

2.5. **Simulation of the semi-active airfoil system**

Depending on each placement of the MR damper over the whole span length, different control algorithms as depicted in Equation (5) are implemented. Equation (5) is then integrated into *ode23* to simulate the semi-active airfoil system to acquire its displacement and voltage responses. Controlled simulations for different current inputs are also performed to examine the effects that they have on the resulting output voltage.

3. **Results and Discussion**

Parametric values of each variable involved in the simulation of the airfoil-based energy harvester are tabulated in Table 2. When the airfoil system is semi-actively controlled, the value of plunge damping coefficient is changed according to the MR damper placement and the inputted current level.

**Table 2.** Simulation parameters for the airfoil system [11].

| Parameter                                                      | Parametric value |
|----------------------------------------------------------------|------------------|
| Airfoil mass (kg)                                              | 0.0175           |
| Equivalent mass (kg)                                          | 0.0349749        |
| Semi-chord length (m)                                         | 0.03             |
| Dimensionless chord-wise offset of elastic axis from centroid  | 0.2              |
| Beam flexural stiffness (N/m)                                 | 77.625           |
| Torsional spring stiffness (Nm/rad)                           | 0.15             |
| Plunge damping coefficient (Ns/m²)                            | 0                |
| Pitch damping coefficient (Ns/rad)                            | 0                |
| Electromechanical coupling (N/V)                              | \(1.55 \times 10^{-3}\) |
| Mass moment of inertia (kgm²)                                 | \(3.76398 \times 10^{-6}\) |
| Equivalent capacitance of piezoelectric layers (F)            | \(113 \times 10^{-9}\) |
| Load Resistance (Ω)                                           | 570              |
| Initial angle of attack (rad)                                 | -0.8727          |
| Simulation time (s)                                           | 5                |
3.1. Dynamic response of the passive airfoil system
It is apparent that both the plunge displacement and the local voltage responses of the passive airfoil system show beating phenomena as depicted in Figure 4. Beating phenomenon occurs when the ratio between the forcing frequency ($\omega_f$) and the natural frequency ($\omega_n$) is close to one [12]. In this case, $\omega_f/\omega_n \approx 1.19$. This signifies that the airfoil-based energy harvester is vibrating at frequency close to the natural frequency, which also explains why the localized voltage achieved is fairly significant, but at a risk of reaching structural destruction.

Apart from that, the pattern of the local voltage response is also closely synchronized with that of the plunge displacement, except it does not fluctuate as much from $t = 2.5$ s to $t = 5$ s. This is due to the mechanism of energy harvesting via a piezoelectric transducer. As the vibrating body of the airfoil system plunges repeatedly, the continuous upwards and downwards motion allows the cantilever beam, which is attached to the energy harvester from the other fixed end, to deform accordingly. This in turn deforms the piezoelectric transducer in the same manner, and generates bi-directional electric current. Hence, this mechanism converts mechanical energy into electricity, and renders the local voltage response to have both positive and negative values for the full duration. Furthermore, the differences between the plunge displacement and the local voltage responses from $t = 2.5$ s to $t = 5$ s is due to the nature of piezoelectric material where voltages are extracted within the range of deflection and not instantaneous deflection. In addition, it is also evident that there is no phase difference between the plunge displacement and the local voltage responses. This is because the electric voltage is generated as soon as the piezoelectric material is deformed.

![Figure 4](image_url)

**Figure 4.** Plunge and voltage responses of the passive airfoil-based energy harvester.

3.2. Dynamic response of the semi-active airfoil system
Inferring from Figures 5 and 6, the incorporation of the semi-active control strategy into the airfoil system has successfully eliminated beating. This is because the natural frequency has been shifted to the extent that the frequency ratio is no longer close to one. In the case where the damping coefficient during the “on” state is 0.64032 Ns/m, i.e. when the input current is maximum, $\omega_f/\omega_d \approx 1.21$, which is slightly...
further away from the natural frequency as compared to the passive airfoil system. This proves the feasibility of an MR damper for the NACA0012 airfoil in vibration attenuation. The relatively low local voltage from $t = 0$ s to $t = 0.2$ s is due to the need for the airfoil system to start up when it is initially subjected to the wind excitation.

Plunge displacement amplitude undergoes suppression each time it exceeds the configured upper and lower bounds, which in this case are $h = 0.14$ m and $h = -0.19$ m, respectively. This is when the skyhook control is activated for the greater damping to be applied to the airfoil system. This proves the viability of the semi-active control in preventing the airfoil system from failing in the midst of cultivating energy. However, it is also visible that the plunge motion has not actually been reduced within the preset bounds. This shows the limitation of the conventional Bouc-Wen model since the provided parametric values, such as $\alpha$ are insufficient and it is apparent that a much larger input current is needed for the on-off skyhook control to provide greater damping. Nevertheless, the maximum plunge displacement has undergone a considerable reduction from 0.3322 m to 0.3205 m, which amounts to about 3.52%. Apart from that, there are also clear signs of minor discontinuities in the plunge displacement and the local voltage. This is due to the occurrences of chatter and jerk when there are immediate switches between the “on” and “off” states. The output voltage shows very consistent trends except the considerable reduction when the semi-active control is activated. This is unavoidable since the greater the plunge motion, the greater the deformation of the piezoelectric transducer, which is why the local voltage is reduced whenever the vibratory motion of the airfoil in the plunge DOF is suppressed.

Simulations of the semi-active model with different MR damper placements are also performed. It is demonstrated that the closer the MR damper is to the vibrating body, more vibration in the plunge DOF can be suppressed. Concurrently, the output voltage that can be cultivated reduces progressively in correspondence with the plunge displacement. Nevertheless, the responses depict similar trends where the semi-active control becomes impactful especially after the plunge motion of the airfoil model reaches its steady state. For the best illustration of how the input current level can affect the plunge displacement of the airfoil system and the cultivated output voltage, the normalized values of the two parameters are acquired with respect to their initial conditions and plotted against the input current as shown in Figure 7. Normalized values are used because their changes over the available range of current are otherwise, too insignificant to notice.

Inferring from Figure 7, the output voltage response of the semi-active airfoil system is actually decreasing at a higher rate as compared to the plunge displacement response. This shows that keeping the amplitude of the aero-elastic vibration at the maximum and without failing is therefore important in order to cultivate more electricity.
Figure 5. Plunge and voltage responses of the semi-active airfoil-based energy harvester ($a = 10\ mm$).

Figure 6. Response comparison between always “off” and semi-active model ($a = 10\ mm$).
Figure 7. Normalized plunge displacement and voltage versus input current graph.

3.3. Comparison between the passive and semi-active model
The passive airfoil model has reached a maximum plunge displacement of 0.3525 m during its beating peak. At the same instant, the semi-actively controlled airfoil system has managed to achieve a maximum plunge displacement of 0.3205 m. This amounts to a total of 9.08% of displacement reduction, which is significantly favorable for a system which requires vibration suppression. Not only does the elimination of beating prolong the airfoil model’s fatigue limit, it also allows a more consistent aero-elastic vibration of the NACA0012 airfoil.

In the case of local voltage response, a maximum of 4.039 V is achieved by the passive system during its beating peak whilst the semi-active airfoil system is only able to hit a maximum of 2.945 V when it peaks at around the fifth second. This brings about voltage reduction of about 27.09%. When observed closely however, it is perceptible that despite the greater voltage that it may acquire, the passive airfoil system spends more than a quarter of its full duration, hovering between 0 to 2 V due to beating. As opposed to this, the semi-active airfoil system peaks rather consistently at approximately 2.4 V. This is proved by acquiring the mean local voltage of the airfoil system for the full duration (from transient to steady state).

Results show that the passive airfoil system has managed to cultivate an output voltage of 45.8 mV whereas the semi-active airfoil system is able to cultivate up to a total of 49.6 mV, amounting to about 8.3% of output voltage increase. This shows that in a long run, the semi-active system is capable of cultivating more energy while being able to suppress vibration simultaneously. Therefore, the semi-active airfoil system is certainly capable of outperforming the passive airfoil system in terms of versatility, vibration suppression, and most importantly energy harvesting.
3.4. Structural integrity

In order to avoid losing structural integrity, systems shall never operate near resonance. Resonance is an undesirable phenomenon which results in the occurrences of extremely large amplitudes, causing structural failure well before a system’s fatigue limit is even reached [13]. Resonance can be meddled with damping and is said to occur only when the forcing frequency coincides with a system’s natural frequency, i.e. when the frequency ratio equates to unity [13]. Based on the simulation results, the incorporation of the proposed strategy has managed to increase the frequency ratio from 1.19 to 1.21. At first glance, an increment of only 0.02 seems fairly insignificant. However, the elimination of beating demonstrates that even the smallest damping is advantageous at putting a system further away from the realm of resonance. This finding is further validated by examining the variation of a system response with frequency ratio as shown in Figure 8. A damping coefficient of 0.64032 Ns/m which is used for the semi-active airfoil model is equivalent to a damping ratio of approximately 0.16. Inferring from Figure 8, smaller damping ratios result in greater system amplitudes. This implies that an improvement of the frequency ratio from 1.19 to 1.21 is definitely significant for vibration suppression.

Essentially, even without damping, a frequency ratio of 1.19 is already within an acceptable working range. In fact, the plunge displacement response of the passive airfoil system in Figure 4 shows only minor beating. However, for the purpose of maximizing and maintaining consistency of energy harvested, a persistent aero-elastic vibration which occurs not too far away from resonance is desirable. Therefore, the 0.02 increment in frequency ratio is certainly a great achievement. Not only does the proposed strategy eliminate beating, a larger amount of energy has been cultivated. Despite the operation of the airfoil system at above the unity frequency ratio, note that its plunge velocity does not actually pass through resonance since the wind speed that the system is subjected to, is constantly 5.32 m/s and is not required to start up from 0 m/s.

Figure 8. Variation of system response with frequency ratio [12].
4. Conclusion
Numerical results show that a semi-actively controlled airfoil system is more valuable than a passive one in terms of versatility as well as simultaneous energy harvesting and conserving structural integrity. Depending on the scale of a desired application, the MR damper can be placed at different locations along the span length of the cantilever beam and be injected with distinct current levels to provide a range of damping coefficients, \( d_h \). It has been demonstrated that the implementation of the semi-active control strategy does not only eliminate beating, but also suppresses up to 9.08% of the maximum plunge displacement of the airfoil system, and thus, prolongs the period of its fatigue life. Moreover, the vibrating body of the airfoil system – NACA0012 airfoil, is also able to vibrate more consistently, which in turn results in a steady and reliable output voltage. Despite its lower maximum voltage magnitude (2.939 V), it is also established that the semi-active model can cultivate more energy in a long run. Unlike the conventional passive model, it does not hover around negligible voltages. Apart from that, an improvement of the frequency ratio from 1.19 to 1.21 is also a significant achievement from two different aspects – protecting the system from losing structural integrity and maintaining the cultivation of a large output voltage. Lastly, the elimination of beating is also noteworthy in terms of terminating thermal stresses and thus, helps conserving the energy storage component for piezoelectric transduction.

Nomenclature

| Symbol | Description |
|--------|-------------|
| \( C_p \) | Equivalent capacitance of piezoelectric coupling. |
| \( I_\alpha \) | Mass moment of inertia. |
| \( R_1 \) | Load resistance. |
| \( d_h \) | Damping coefficient in the plunge DOF. |
| \( d_\alpha \) | Damping coefficient in the pitch DOF. |
| \( k_h \) | Stiffness in the plunge DOF. |
| \( k_\alpha \) | Stiffness in the pitch DOF. |
| \( m_e \) | Effective fixture mass. |
| \( x_\alpha \) | Dimensionless chord-wise offset of the elastic axis from the centroid. |
| \( h, \dot{h}, \ddot{h} \) | Displacement, velocity and acceleration in the plunge DOF. |
| \( c, \gamma \) | Centroid of the airfoil section. |
| \( L \) | Lifting force. |
| \( M \) | Moment. |
| \( b \) | Semi-chord length of the airfoil section. |
| \( l \) | Length of span. |
| \( m \) | Mass of the airfoil. |
| \( v \) | Voltage across the resistant load. |
| \( \alpha, \dot{\alpha}, \ddot{\alpha} \) | Displacement, velocity and acceleration in the pitch DOF. |
References

[1] Yao G Z, Yap F F, Chen G, Li W and Yeo S H 2002 Mechatronics 12 (7) p 963-973.
[2] Gawthrop P J, Neild S A and Wagg D J 2012 J Intel Mat Syst Str 23 (18) p 2103-2116.
[3] Liu Y, Waters T P, and Brennan M J 2005 J Sound Vib 280 (1) p 21-39.
[4] Di Monaco F, Tehrani M G, Elliott S J, Bonisoli, and Tornincasa S 2013 J Sound Vi 332 (23) p 6033-6043.
[5] Dong X M, Yu M, Liao C R and Chen W M 2010 Nonlinear Dyn 59 (3) p 433-453.
[6] De Marqui Jr C and Erteuk A 2013 Advances in Energy Harvesting Methods (New York: Springer) p 269-294.
[7] Wong Z J, Yan J, Soga K and Seshia A 2009 A multi-degree-of-freedom electrostatic MEMS power harvester.
[8] Tang X and Zuo Z 2009 ASME 2009 International Mechanical Engineering Congress and Exposition (American Society of Mechanical Engineers) p 885-896.
[9] Dyke S J and Spencer B F 1997 Intelligent Information Systems, 1997. IIS’97. Proceedings (IEEE) p 580-584.
[10] Peng G R, Li W H, Du H, Deng H X and Alici G 2014 Appl Math Model 38 (15) p 3763-3773.
[11] Lai K H 2015 On the Design and Analysis of an Airfoil-Based Energy Harvester (Curtin University, Malaysia).
[12] Palm W J 2006 Mechanical Vibration (New Jersey: John Wiley & Sons).
[13] Rao S S 1990 Mechanical Vibration (New York: Addison-Wesley).