HEAVY QUARK FRAGMENTATION INTO BARYONS
IN A QUARK-DIQUARK MODEL

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Abstract

In the framework of the nonrelativistic QCD and a quark-diquark model of baryons we have obtained the fragmentation functions for heavy quark to split into spin-1/2 and spin-3/2 double heavy baryons. It was predicted the production rates as well as the shape of the energy spectra for the $cc$- and $bc$-baryons in the region of $Z^0$ peak at LEP collider.

1 Introduction

In the last decade the great success was obtained in the study of the heavy quarkonium production and decay (see, for example, Refs [1, 2]). The investigation of the processes with the heavy quarks is based on the factorization hypothesis [3]. The mass of heavy quark $m_Q$ is much larger than the scale of strong interactions $\Lambda_{QCD}$. So the presence of the small parameter $\Lambda_{QCD}/m_Q$ has allowed one to separate the effects of small and long distances. The heavy hadron production amplitude may be presented as the product of the partonic part, which can be calculated using the perturbative QCD, and the nonperturbative factor, which describes the free quarks to final hadron transition. In the framework of the nonrelativistic quark model, this nonperturbative part can be presented through the quarkonium wave function at the origin $\Psi(0)$, which is calculated using the potential method (see [4] and References therein). So, the approach based on PQCD and nonrelativistic quark model is known as a nonrelativistic QCD (NR QCD) [5].

The heavy hadron production via fragmentation prevails in the large transverse momentum region at $e^+e^-$ and hadron colliders [6]. Recently it was shown [7] that the fragmentation functions $D_Q \to M(z, \mu)$ and $D_g \to M(z, \mu)$ for quark and gluon to split into a heavy quarkonium with fraction $z$ can be calculated within the framework of NR QCD. The fragmentation functions are the process independent and can be applied to the $e^+e^-$, photonic and hadronic production of heavy quarkonia. Here we demonstrate that the NR QCD approach can be used for the calculation of the fragmentation function $D_Q \to B(z, \mu)$ for heavy quark to split into double heavy baryon.

The estimation of the production rates for baryons containing two heavy quarks was done recently on the basis of the quark-hadron duality [8] as well as on the basis of the PQCD [9]. In spite of the some differences, it was suggested in Refs [10, 11] that the double heavy baryon production has two step. In the first stage, there is the heavy quark $Q$ fragmentation into double heavy diquark ($QQ$) in the colour octet state. The second step consists in the nonperturbative diquark fragmentation into a ($QQq$) baryon. It was mentioned in Ref. [8], that this mechanism of the fragmentation can be factorized into short and long distance contributions as the same in Ref. [10]. The normalization of the fragmentation function $D_{Q \to B}(z, \mu)$ is determined by the model dependent value of the ($QQ$) diquark wave function at the origin $\Psi_{QQ}(0)$ as well as by the additional suggestion about nonperturbative diquark fragmentation mechanism into baryon. Note, that the calculation of the $\Psi(0)$ for the colour object using potential model isn’t grounded and destroys the factorization of the long and short distance effects. It was also suggested in Ref. [8], that the probability of the diquark ($QQ$) fragmentation into baryon is equal to unity and the heavy diquark carries all of the momentum of the baryon $D_{QQ} \to B(z, \mu) \sim \delta(1-z)$. These circumstances raise too high the predicted values of the double heavy baryon production rates.

The model of the quark-hadron duality, which was used in Ref. [7] for the prediction of $cc$-baryon production cross section at $B$-factory, gives us the opportunity to obtain only upper limit of the production rates and doesn’t predict baryon spectra. It was suggested in [8] that all ($cc$)-pairs in the colour octet state with invariant mass from $2m_c$ to $2M_D + \Delta M$ ($M_D$ is $D$-meson mass and $\Delta M = 0.5 \div 1.0$ GeV) split to the $cc$-baryon. However, it is known [9] that the phenomenological application of the quark-hadron duality method for heavy quarkonium photoproduction needs to introduce additional process dependent $K$-factor which is equal to $1/3 \div 1/6$ for description of the experimental data. Therefore the results of Refs [6, 8] may be considered as a rough estimation of the double heavy baryon production rates, which need in more detail analysis.
2 The model

In the present paper we consider a new mechanism of the double heavy baryon production via heavy quark fragmentation based on the hypothesis of the point-like diquarks, which can be produced directly in the hard interactions of quarks and gluons at the short distance [11]. As it was shown in Ref. [12] the data on $e^+e^-$ annihilation into hadrons don’t contradict to the existence of the very small diquark $Qq$ containing one heavy and one light quark. In such a way, the heavy quark ($Q = b, c$) can fragment directly into double heavy baryon spin-1/2 or -3/2 catching the scalar or the vector heavy diquark ($D = (cq), \ q = u, d, s$), correspondingly. In the process of the production quark-diquark system $Q(Qq)$ in the colour singlet state the relevant QCD scale satisfies to the next condition: $\mu \geq m_Q + 2m_D \gg \Lambda_{QCD}$, where $m_D$ is the heavy diquark mass. This fact leads to the factorization of the double heavy baryon production amplitude. The transition of the $Q(Qq)$ into final baryon can be described using the nonrelativistic approximation of the potential model, because the system $Q(Qq)$ contains two heavy particles.

So, for the calculation of the baryon ($B = Q(Qq)$) wave function at the origin we have used equation [12], which effectively takes into account the relativistic effects of the kinematic nature [3]:

$$\left(-\frac{\nabla^2}{2\mu_R} + U_{QD}(r)\right)\Psi(r) = EP(r),$$

where

$$E = \frac{p^2}{2\mu_R}, \quad \mu_R = \frac{M^2 - (m_Q^2 - m_D^2)^2}{4M^4},$$

$$p^2 = \frac{(M^2 - (m_Q + m_D)^2)[M^2 - (m_Q - m_D)^2]}{4M^2}.$$

The potential of the quark-diquark interaction is taken in the conventional form [4]:

$$U_{QD} = -\frac{b}{r} + ar,$$

where $a = 0.183 \text{ GeV}^2$, $b = 0.52$. The results of our numerical calculations for the radial part of the wave function at the origin $|R(0)|^2 = 4\pi|\Psi(0)|^2$, are presented in Tables 1 and 2.

The fragmentation function $D_{Q\rightarrow B}(z,\mu)$ at the scale $\mu = \mu_o = m_Q + 2m_D$ for the production of the baryon $B$, containing the heavy quark $Q$ and the heavy diquark $(Qq)$, is given by the next expression:

$$D_{Q\rightarrow B}(z,\mu_o) = \frac{1}{16\pi^2} \int_{s_{min}}^\infty ds \lim_{q_o \rightarrow \infty} \frac{|M|^2}{|M_o|^2},$$

where $M$ is the matrix element for the production of a baryon $B$ and antiquark $\bar{D}$ with the total four-momentum $q$ and the invariant mass $s = q^2$, $M_o$ is the matrix element for the production of a quark $Q$ with the same three-momentum $\vec{q}$. The lower limit in the integral (3) is

$$s_{min} = \frac{M^2 + \vec{p}_T^2}{z} - \frac{m_D^2 + \vec{p}_T^2}{1-z}.$$

Here $M = m_Q + m_D$ is the baryon mass, $\vec{p}_T$ is the baryon transverse momentum in the reference frame where

$$q = (q_o, 0, 0, q_3), \quad p = (p_o, \vec{p}_T, p_3) \quad \text{and} \quad z = \frac{p_o + p_3}{q_o + q_3}.$$

In the limit of $q_o \rightarrow \infty$:

$$s_{min} = \frac{M^2}{z} - \frac{m_D^2}{1-z}.$$

In the axial gauge associated with four-vector $n = (1, 0, 0, -1)$:

$$d_{\mu\nu}(k) = -g_{\mu\nu} + \frac{k_{\nu}n_{\mu} + k_{\mu}n_{\nu}}{(kn)},$$

the fragmentation contribution comes only from the Feynman diagram for $Q \rightarrow QD\bar{D}$ shown in Fig.1.

The gluon couplings to scalar and vector diquarks are presented by the next expressions:

$$S^b_{\mu} = -ig_sT^b(q - p_D)_{\mu}F_S(k^2),$$

(5)
\[ V_{\mu} = ig_sT^b \left\{ \varepsilon_D^* \varepsilon_D^* (q' - p_D)_{\mu} F_1(k^2) - [(q' \varepsilon_D^* )\varepsilon_D^*_{\mu} - (p_D \varepsilon_D^*)\varepsilon_D^*_{\mu}]F_2(k^2) \right\}, \]

(6)

where \( T^b \) are Gell-Mann matrices, \( \varepsilon_D^*, \varepsilon_D^* \) are the diquark polarization vectors. \( F_s, F_1, F_2 \) and \( F_3 \) are form factors depending on the momentum transfer squared \( k^2 = (q' + p_D)^2 \).

The light diquark form factors may be parametrized as follows [15, 16]:

\[ F_s(Q^2) = \frac{Q^2}{Q^2 + Q_s^2}, \quad F_1(Q^2) = \left( \frac{Q^2}{Q_e^2 + Q^2} \right)^2, \]

\[ F_2(Q^2) = (1 + \kappa)F_1(Q^2), \quad F_3(Q^2) = 0 \]

where \( \kappa \approx 1.39 \) being the anomalous magnetic moment of the vector diquark, \( Q_s^2 \approx Q_s^2/2 \) and \( Q_s^2 \approx 3 \text{ GeV}^2 \) [13].

At present time the form factors of the heavy diquarks are unknown and one has to assume a certain dependence on \( Q^2 \). The general consideration (see, for example, Ref. [11]), based on asymptotic QCD picture and the phenomenology, predicts \( F_s \sim 1/Q^2, \quad F_1 \sim F_2^2, \quad F_3 \gg 3 \text{ GeV}^2 \), corresponding to a very small heavy diquark. We try to take these points into account by using the simplest possible expressions for the heavy diquark form factors:

\[ F_s(Q^2) = \frac{Q^2}{Q^2}, \quad F_1(Q^2) = \left( \frac{Q^2}{Q^2} \right)^2, \]

\[ F_2(Q^2) = (1 + \kappa)F_1(Q^2), \quad F_3(Q^2) = 0, \]

where \( Q_s^2 = k_{\text{min}}^2 = 4m_D^2, \quad Q_v^2 = Q_s^2/2 \). It has \( k_{\text{min}}^2 \approx 20 \text{ GeV}^2 \) for \( (cq) \) diquarks and our parametrization is the same as in Ref. [11]. Note, that such choice of the diquark form factors gives opportunity to obtain the compact analytical expressions for the fragmentation functions.

In the case of the heavy quark fragmentation into spin-1/2 baryon it has fusion of the heavy quark \( Q \) and scalar diquark \( D \). After the some obvious simplifications we have obtained:

\[ \mathcal{M}_{1/2} = \Psi(0) \sqrt{\frac{M}{2m_Qm_D}} g^2 \delta^{ij} \frac{F_s(k^2)}{3\sqrt{3}(s - m_Q^2)^2} \]

\[ 2\bar{U}(p_Q) \left[ -M(\hat{q} + m_Q) + (s - m_Q^2) \frac{(np)}{(nk)} \right] \Gamma, \]

(9)

where \( g_s = \sqrt{4\pi\alpha_s}, \quad p_Q = \bar{r}p \) and \( p_D = r p \) are the momenta of the quark and the diquark in the baryon, correspondingly, \( r = m_D/M, \quad \bar{r} = 1 - r, \) \( p \) is the baryon momentum, \( 4\delta^{ij}/3\sqrt{3} \) is the colour factor of the amplitude shown in Fig.1, the Dirac spinor \( \Gamma \) is the matrix element for the production of a \( Q \) quark of momentum \( q = p + q' \), \( q' \) is the antidiquark momentum, the spinor \( \bar{U}(p) \) describes spin-1/2 baryon in the final state.

The amplitude for heavy quark fragmentation into spin-3/2 baryon, corresponding to fusion of the heavy quark and the vector diquark, can be written as a sum of two parts, which are proportional to \( F_1 \) and \( F_2 \) form factors:

\[ \mathcal{M}_{3/2} = \mathcal{M}_{3/2}^1 + \mathcal{M}_{3/2}^2, \]

\[ \mathcal{M}_{3/2}^1 = \Psi(0) \sqrt{\frac{M}{2m_Qm_D}} g^2 \delta^{ij} \frac{F_1(k^2)}{3\sqrt{3}(s - m_Q^2)^2} \]

\[ 2\bar{\Psi}_{\mu}(p_Q)\varepsilon_D^* \left[ -M(\hat{q} + m_Q) + (s - m_Q^2) \frac{(np)}{(nk)} \right] \Gamma, \]

\[ \mathcal{M}_{3/2}^2 = \Psi(0) \sqrt{\frac{M}{2m_Qm_D}} g^2 \delta^{ij} \frac{F_2(k^2)}{3\sqrt{3}(s - m_Q^2)^2} \]

(10)

(11)

(12)
Here, the spin-vector \( \bar{\Psi}_\mu(p) \) describes the spin-3/2 baryon in the final state and satisfies to the following conditions \([17]\):

\[
(\hat{p} - M)\bar{\Psi}_\mu(p) = 0, \\
\bar{\Psi}_\mu(p)\Psi_\mu(p) = 2M, \quad \gamma_\mu\bar{\Psi}_\mu = p_\mu\Psi_\mu = 0.
\]

In the case of the unpolarized spin-3/2 baryon the summing on the helicity states is carried out by means of the following formula:

\[
\sum_\lambda \Psi^\lambda_\mu(p)\bar{\Psi}^\lambda_\nu(p) = (\hat{p} + M) \left( g_{\mu\nu} - \frac{1}{3} \gamma_\mu\gamma_\nu - \frac{2p_\mu p_\nu}{3M^2} + \frac{p_\mu\gamma_\nu - p_\nu\gamma_\mu}{3M} \right).
\] (13)

### 3 The results

Substituting the amplitudes (9) and (10) to the basic formula (3) we have obtained the fragmentation functions for a heavy quark split into the spin-1/2 and spin-3/2 baryons. Omitting the details of the calculation we write here final result for the heavy quark fragmentation function for spin-1/2 baryon:

\[
D_{1/2}(z, \mu_o) = \frac{8\alpha_s^2(2m_D)}{405r^3} \frac{[\Psi(0)]^2 Q^4}{M^4} F_{1/2}(z, r),
\] (14)

where

\[
F_{1/2}(z, r) = \frac{z^4(1 - z)^3}{(1 - z + rz)^4} \left[ 15 - 6z(7 + 3r) + z^2(44 + 6r + 15r^2) - 22z^3(1 - r) + 5z^4(1 - r)^2 \right].
\]

The fragmentation probability for the production of the spin-1/2 baryon is

\[
\int_0^1 D_{1/2}(z, \mu_o)dz = \frac{8\alpha_s^2(2m_D)}{405r^3} \frac{[\Psi(0)]^2 Q^4}{M^4} I_{1/2}(r),
\] (15)

where

\[
I_{1/2}(r) = \frac{1}{42r^4(1 - r)^6} \left[ 1 - 12r + 75r^2 - 420r^3 - 1827r^4 - 126r^5 + 2037r^6 + 300r^7 - 30r^8 + 2r^9 - 1260r^2(1 + r)^2 \ln(r) \right].
\]

The fragmentation function for \( Q \) split into spin-3/2 baryon is

\[
D_{3/2}(z, \mu_o) = \frac{\alpha_s(2m_D)}{1215r^7} \frac{[\Psi(0)]^2 Q^8}{M^8} F_{3/2}(z, r),
\] (16)

where

\[
F_{3/2}(z, r) = \frac{4}{7} \Phi_{11} + 4(1 + \kappa)\Phi_{12} + (1 + \kappa)^2\Phi_{22},
\]

\[
\Phi_{11}(z, r) = \frac{z^4(1 - z)^3}{(1 - z + rz)^4} \left[ 7z^8(5r^6 - 18r^5 + 51r^4 - 76r^3 + 51r^2 - 18 + 5) + 2z^7(105r^5 - 455r^4 + 1122r^3 - 1038r^2 + 413r - 147) + z^6(105r^6 - 42r^5 + 1596r^4 - 2648r^3 + 5581r^2 - 2198r + 1134) + 2z^5(-21r^5 - 945r^4 + 100r^3 - 4352r^2 + 1505r - 1295) + z^4(847r^4 + 2028r^3 + 8031r^2 - 2170r + 3780) + 2z^3(-446r^3 - 2018r^2 + 343r - 1785) + 7z^2(121r^2 + 2r + 302) + 42z(-r - 17) + 105 \right],
\]

\[
\Phi_{22}(z, r) = \frac{z^4(1 - z)^3}{(1 - z + rz)^4} \left[ 45 - 18z(7 + 3r) + z^2(164 + 84r + 45r^2) - 2z(121 + 42r + 45r^2) - 7z^3(121r^2 + 2r + 302) + 42z(-r - 17) + 105 \right].
\]
The fragmentation probability for the production of the spin-3/2 baryon can be calculated in the same way as for spin-1/2 baryon. We don’t present this result here because of it’s unwieldy.

The fragmentation function \( D_{Q\rightarrow B}(z,\mu) \) satisfies to the Gribov-Lipatov-Altarelli-Parisi (GLAP) evolution equations

\[
\mu \frac{\partial}{\partial \mu} D_{Q\rightarrow B}(z,\mu) = \int_{z}^{1} \frac{dy}{y} P_{Q\rightarrow Q}\left(\frac{z}{y},\mu\right) D_{Q\rightarrow B}(y,\mu),
\]

where

\[
P_{Q\rightarrow Q}(x,\mu) = \frac{4\alpha_s(\mu)}{3\pi} \left(\frac{1+x^2}{1-x}\right), \quad f(x)_+ = f(x) - \delta(1-x) \int_{0}^{1} f(x')dx'.
\]

The boundary condition on the evolution equation is the initial fragmentation function \( D_{Q\rightarrow B}(z,\mu_0) \) at the scale \( \mu_0 = m_Q + 2m_D \). Note, that at leading order in \( \alpha_s \) one has:

\[
\int_{0}^{1} P_{Q\rightarrow Q}(z,\mu)dz = 0,
\]

and the evolution equation implies that the fragmentation probability \( \int_{0}^{1} D_{Q\rightarrow B}(z,\mu^2) \) does not evolve with the scale \( \mu \). Therefore the fragmentation probability is the universal characteristic of the production rates. The evolution only changes the \( z \)-distribution to smaller values of \( z \).

The results of calculation for the fragmentation probabilities and the average values of the momentum fraction for \( Q \) quark to split into double heavy spin-1/2 and spin-3/2 baryons are shown in Tables 1 and 2. The values of \( < z > \) are presented at \( \mu = \mu_0 \) and \( \mu = M_Z/2 \). We used the following set of the mass parameters: \( m_c = 1.7 \text{ GeV}, m_b = 5.1 \text{ GeV}, m_{c_d} = m_{c_d} = 1.9 \text{ GeV}, m_{c_s} = 2.0 \text{ GeV} \). The masses of the vector diquarks are greater than corresponding scalar diquarks by 0.1 GeV. The fragmentation functions \( D_{c\rightarrow cc}(z,\mu) \) and \( D_{b\rightarrow bc}(z,\mu) \), divided by the fragmentation probabilities, are shown in Fig. 2 at \( \mu = \mu_0 \) and \( \mu = M_Z/2 \). In the range of accuracy of our model the shape of \( z \)-spectra for spin-1/2 and spin-3/2 baryons are the same one to other as well as \( z \)-spectra for baryons with strange quark (\( \Omega_{cc},\Lambda_{cc} \)) and without it (\( \Xi_{cc},\Lambda_{bc} \)).

Let compare our results with the estimation [8], where it was founded that fragmentation probabilities for heavy quark to split into double heavy baryons are \( (2 \div 3) \cdot 10^{-5} \) independently on the baryon spin and the flavour content. In our approach the fragmentation probability depends on baryon spin and flavour.

Moreover, there is the relative growth of the production rates of the spin-3/2 baryons for \( b \)-quark fragmentation in comparison with the fragmentation of \( c \)-quark. We have obtained that \( \Xi_{cc}/\Xi_{bc} \sim 0.7 \) and \( \Lambda_{bc}/\Lambda_{bc} \sim 1.8 \). Our results for the fragmentation probabilities for the spin-3/2 baryons are approximately equal to the results of [8], however the fragmentation probabilities for spin-1/2 baryons are smaller about factor 2. Taking into consideration the uncertainties of our calculation, connected with the diquark form factors (8), we can predict the fragmentation probabilities for the \( cc \)- and \( bc \)- baryons are about of \( 10^{-5} \). There are more accurate predictions for baryon \( z \)-spectra in the our approach, which are practically independent on the diquark form factors and other parameters.

We have obtained that the average fraction of the baryon momentum \( < z > \approx 0.54 \) for the \( cc \)-baryons and \( < z > \approx 0.66 \) for the \( bc \)-baryons at \( \mu = M_Z/2 \), and \( < z > \approx 0.75 \) and \( < z > \approx 0.84 \), correspondingly, at \( \mu = \mu_0 \). This results are independent from the baryon spin. By contrast, in the model, based on \( Q \rightarrow (QQ) \) fragmentation mechanism corresponding values at \( \mu = \mu_0 \) are equal: \( < z > = 0.57 \div 0.62 \) for the \( cc \)-baryons and \( < z > = 0.68 \div 0.73 \) for the \( bc \)-baryons.

In conclusion, we sum the results of our paper. In the framework of NR QCD and a quark-diquark model of baryons we have obtained in the leading order in \( \alpha_s \) the fragmentation functions and the probabilities for heavy quark to split into double heavy spin-1/2 and spin-3/2 baryons. Using the QCD evolution equations we have recalculated the fragmentation functions from the initial scale \( \mu_0 \) to \( \mu = M_Z/2 \). Our results can be used for prediction of the double heavy baryon production rates as well as for the description of the energy spectra at \( e^+e^- \)-collider LEP. Contrary to Refs. [4] [8] we have predicted also few precise effects: the nontrivial spin and flavour dependence of the baryon production rates, the value of the average baryon momentum \( < z > \) and it’s dependence from the scale \( \mu \) as well as from the baryon flavours.
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Table 1. The spin-1/2 baryons.

| Baryon | M, [GeV] | $|R(0)|^2$, [GeV$^3$] | $P_{Q \rightarrow B}$ | $< z >_{\alpha}$ | $< z >_{M_{z}/2}$ |
|--------|---------|-----------------|-----------------|--------------|-----------------|
| $\Xi_{cc}$ | 3.6 | 1.2 | $5.8 \cdot 10^{-6}$ | 0.75 | 0.55 |
| $\Omega_{cc}$ | 3.7 | 1.3 | $5.3 \cdot 10^{-6}$ | 0.74 | 0.54 |
| $\Lambda_{bc}$ | 7.0 | 3.8 | $8.9 \cdot 10^{-6}$ | 0.84 | 0.67 |
| $\Omega_{bc}$ | 7.1 | 4.0 | $7.4 \cdot 10^{-6}$ | 0.83 | 0.66 |
Table 2. The spin-3/2 baryons.

| Baryon | M, [GeV] | |R(0)|^2, [GeV^3] | P_{Q\rightarrow B} | <z>_<o> | <z>_<M_Z/2> |
|--------|----------|-----------------|-----------------|----------------|-----------|-------------|
| Ξ^{∗}_{cc} | 3.7      | 1.25            | 1.2 \times 10^{-5} | 0.75         | 0.54      |
| Ω^{∗}_{cc} | 3.8      | 1.35            | 1.1 \times 10^{-5} | 0.74         | 0.53      |
| Λ^{∗}_{bc} | 7.1      | 4.0             | 3.5 \times 10^{-5} | 0.84         | 0.67      |
| Ω^{∗}_{bc} | 7.2      | 4.1             | 3.3 \times 10^{-5} | 0.83         | 0.66      |

Figure captions.

1. Diagram used for description of the heavy quark to split into double heavy baryon.
2. The fragmentation functions normalized to the unity at µ = µ_o (curves 1 and 3) and µ = M_Z/2 (curves 2 and 4). The curves 1 and 2 - D_{Ξ_{cc}}(z, µ), the curves 3 and 4 - D_{Λ_{bc}}(z, µ).
Figure 1:
