Derivation of Earth Geoid Model for Mosul City in Iraq

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Abstract. The quality and efficiency of geoid models have been developed rapidly to provide elevations instead of traditional surveying. The high cost and the long measurement time of surveying instrumentation cause an urgent need for deriving local Earth Geoid Model (EGM). Hundred control points are selected on the study area topographic map. The study area is located in Mosul city in Iraq. Multiple earth geoid models have been derived using different interpolation methods available in ArcGIS environment. The results of statistical analysis determine the best interpolation method. The percentage root mean square errors (RMSEs) for the adapted interpolation methods are 26.54%, 28.43%, 30.13%, 34.79%, 36.75%, and 37.28% in kriging, Inverse Distance Weighted (IDW), Trend, Natural Neighbor, Spline, and Topo To Raster, respectively. Kriging interpolation gives minimum standard deviation (2.226 m) and RMSE (2.654 m) so it is the preferable method. The results indicate that the geoid is departed from GPS Visualizer data by the mean value of minus (1.405 m) in the study area.

Keywords: Geoid Model, GPS Visualizer, Interpolation, Root Mean Square Error, Standard Deviation.

1. Introduction

Many countries have started the derivation of the best fitting geoid model for their regions. Most of surveying measurements are observed based on the geoid. The geoid is an equipotential surface of the gravity field of the earth [1, 2]. "Because of variations in the earth mass distribution and the rotation of the Earth, the geoid has an irregular shape" [3]. The mathematical relation representing the geoid surface with respect to the reference ellipsoid is called the geoid model [4]. It connects the horizontal positioning with their geoid undulations. Regional geoid model may be developed by using an earth gravitational model. An accurate geoid model is substantial for the establishment of precise orthometric heights [5]. The essential application of a geoid model is to transform Global Positioning System (GPS) heights to gravity related elevations above a local datum [6].

The surface of earth is represented mathematically by Digital Elevation Model (DEM). It is interpolated by collecting ground control points. Various procedures may be used to obtain control points like GPS, photogrammetry and remote sensing imagery, laser scanning, and total station field survey. Nowadays, most of surveying receivers are supplied with geoid models like Earth Gravitational Model (EGM): EGM 1996 and EGM 2008 [7]. The field survey has some limitations such as the cost, the long survey time, and the difficult accessibility. Altitudes can be easily obtained from positioning satellites and internet web sites such as GPS Visualizer. The geographic coordinates in World Geodetic System (WGS 1984) can be converted to local map coordinate system using Geographic Information System (GIS).

Both geometric and gravitational methods can be applied to develop local geoid models. In geometric method the ellipsoidal heights of known points are compared with their orthometric heights to determine the geoidal separations. The obtained geoidal separations at the known points are then used to generate a local geoid model using spatial interpolation in condition that the region has a number of known control points distributed appropriately. Geoid Model will efficiently update vertical datum in the future. Interpolation methods are dependence on the spatial auto-correlation and suppose that the closer points are more similar compared to farther ones [8]. "Errors in digital elevation models are undesirable,
especially because they can be perpetuated through derived topographic surfaces, including aspect, slope, hillshade, and surface curvature [9]. Interpolations are commonly implemented using ArcGIS. ArcGIS provides different interpolation techniques. These are Inverse Distance Weighted (IDW), Kriging, Natural Neighbor, Spline, Topo To Raster, and Trend. The relative execution of each interpolation method is based on several ground constants and spatial distribution of specimen points [10]. Earth geoid models for Mosul city in Iraq have been derived using numerous control points. Several interpolation methods must be tested for producing the geoid models. Kriging interpolation is one of the best method used in this study. The accuracy of the output geoid models is determined using statistical criteria.

2. Objective
The classical field surveying requires the employment of instruments and systems such as level, total station, laser scanning, and GPS. It takes long times and high cost combined with obstacles in measurement. The availability of positioning satellites and internet web sites provides an easier way to get altitudes. The objective of this study is to develop a new approach to derive local earth geoid model using ArcGIS software package version (10.2.2). The validation of results requires ground observations represented by well distributed control points. These control points have been used to generate the surface of a geoid model by means of interpolation. Hundred control points are established covering the study area. All interpolations existing in ArcGIS have been examined. These are IDW, Kriging, Natural Neighbor, Spline, Topo To Raster, and Trend. The decision about the competent method is selected by calculating statistical parameters. These parameters include standard deviation and Root Mean Square Error (RMSE).

3. Study Area
The study area is located within the master plan of Mosul city in the north of Iraq approximately (400 km) from Baghdad. It is extended by (36° 15′ 00″ N - 36° 25′ 30″ N) latitudes and (43° 05′ 30″ E - 43° 17′ 30″ E) longitudes based on WGS 1984 coordinate system. Mosul city is the second largest city in Iraq in terms of population after Baghdad. The topographic map of the study area has been scanned to be processed in ArcGIS software package. Many ground control points have been established in the study area. Ten percent of these points are kept as check points. The topographic map is georeferenced to the locations of control points. Figure 1 clarifies the locations of established control and check points on the topographic map. All points are labeled according to their numbers. Control and check points are well distributed to cover the study area. The source data include the map and control points are obtained from surveying department in Iraq. The original map scale is (1: 50000) and its accuracy is suitable for the subject of the study.
4. Application
ArcGIS application and GPS Visualizer web site have been employed in this study. The Microsoft Office Excel file of geographic coordinates (latitudes and longitudes) in decimal degree is uploaded to GPS Visualizer. Plain text is selected for output file. By clicking the icons of "convert" and "add elevation tab", the resulted altitude values have been displayed. The output file is downloaded from the GPS Visualizer web site. The reference datum and ellipsoid of GPS Visualizer in which locations and altitudes are recorded is the WGS 1984.

5. Results and Analysis
Six earth geoid models in Mosul city have been derived using ArcGIS 3D Analyst Toolbar. All interpolations are executed depending on the ArcGIS default setting. Hundred control and check points are involved for analysis. The interpolations have been implemented dependence on 90% from the total number of points (90 control points). The remnants 10% from total points (10 points) are preserved as check points. According to the GPS Visualizer altitudes and control point elevations, Figure 2 shows that the geoid is departed from WGS 1984 ellipsoid by about the mean value of minus (1.405 m) in the study area. Elevations or orthometric heights are ranged from the minimum value of (211.937 m) to the maximum value of (317.865 m), and the mean elevation equals to (249.833 m). Altitudes or ellipsoidal heights are ranged from the minimum value of (209.1 m) to the maximum value of (314.9 m), and the mean altitude equals to (248.428 m). The positions and height data of check points are listed in Table 1. The surface separations in all interpolation methods are illustrated in Table 2. Table 3 shows the residuals and the interpolation statistical parameters.

The derived EGMs using Kriging, IDW, Trend, Natural Neighbor, Spline, and Topo To Raster interpolation methods are represented in Figure 3, Figure 4, Figure 5, Figure 6, Figure 7, and Figure 8 respectively. In these figures, the indicator next to grid denotes the gradient values corresponding to the geoid undulation difference. Kriging is a geostatistical method that relies on several user defined
elements. It requires a knowledge of spatial statistics. IDW depends on the distances between control points. The minimum the distance between the two points is the greater the impact or weight and vice versa. Trend is based on the least square regression and tries to minimize deformation to the least possible value. The natural neighbor supposes that the proximal points have a significant effect when the unknown point value is calculated. Spline interpolation attempts to reduce the surface curvature that will pass through the known sample points. Finally, Topo to Raster depends on a number of iterations to reach the best surface fitting.

The interpolation accuracy of earth geoid model is determined by calculating standard deviation and RMSE. The resulted accuracy is influenced by the use of interpolation method, the distribution and density of control points, and the nature of the earth surface. Interpolation accuracy is increased in a flat surface. Different terrains need better distributed control points to preserve appropriate quality. The residual values against each check point for all interpolation methods are displayed in Figure 9, Figure 10, Figure 11, Figure 12, Figure 13, and Figure 14. The standard deviation and RMSE resulted from applying interpolation methods are evaluated from the sample of check points as illustrated in Figure 15 and Figure 16, respectively. From the statistical results, it becomes clear that kriging is the preferable interpolation method because it gives minimum standard deviation and RMSE equal to (2.226 m) and (2.654 m) respectively. The sequence best interpolation methods are Kriging, IDW, Trend, Natural Neighbor, Spline, and Topo To Raster, respectively.

![Figure 2. Statistics of total control and check points in meter resulted from ArcGIS application.](image-url)
Table 1. Positions and heights of check points.

| Check Point | Latitude ( North ) | Longitude ( East ) | Elevation ( m ) | GPS Visualizer Altitude ( m ) | Separation ( m ) |
|-------------|--------------------|--------------------|----------------|-------------------------------|-----------------|
|             | deg | min | sec | deg | min | sec |                |                  |                 |
| 1010        | 36  | 15  | 49.548 | 43  | 15  | 25.887 | 257.725 | 254.6 | -3.125          |
| 1042        | 36  | 18  | 33.780 | 43  | 14  | 26.872 | 246.429 | 240.7 | -5.729          |
| 1100        | 36  | 24  | 46.590 | 43  | 15  | 35.338 | 285.083 | 285.3 | 0.217           |
| 2041        | 36  | 21  | 05.493 | 43  | 13  | 33.661 | 270.471 | 268.0 | -2.471          |
| 3072        | 36  | 21  | 01.916 | 43  | 11  | 54.431 | 240.923 | 242.0 | 1.077           |
| 4040        | 36  | 20  | 28.802 | 43  | 09  | 22.902 | 224.703 | 221.0 | -3.703          |
| 4052        | 36  | 19  | 27.708 | 43  | 08  | 07.663 | 222.108 | 220.6 | -1.508          |
| 5010        | 36  | 15  | 29.887 | 43  | 07  | 18.925 | 317.865 | 314.9 | -2.965          |
| 5051        | 36  | 19  | 37.074 | 43  | 08  | 46.057 | 218.108 | 216.0 | -2.108          |
| 5100        | 36  | 24  | 35.813 | 43  | 07  | 17.202 | 254.368 | 246.8 | -7.568          |

Table 2. Surface separations.

| Check Point | Separation N ( m ) | Kriging ( m ) | IDW ( m ) | Trend ( m ) | Natural Neighbor ( m ) | Spline ( m ) | Topo to Raster ( m ) |
|-------------|-------------------|---------------|-----------|-------------|------------------------|--------------|---------------------|
| 1010        | -3.125            | -1.831        | -2.190    | -2.186      | -2.465                 | -5.173       | -3.874              |
| 1042        | -5.729            | -1.849        | -1.778    | -1.731      | -1.895                 | -2.319       | -1.910              |
| 1100        | 0.217             | -0.440        | -0.078    | -1.037      | 1.045                  | 3.660        | 2.865               |
| 2041        | -2.471            | -0.711        | 0.287     | -1.313      | 0.737                  | 0.845        | 0.727               |
| 3072        | 1.077             | -0.192        | 0.483     | -1.155      | 0.847                  | 0.857        | 1.335               |
| 4040        | -3.703            | -1.671        | -2.697    | -0.844      | -3.392                 | -5.258       | -5.094              |
| 4052        | -1.508            | -1.461        | -2.726    | -0.852      | -3.303                 | -3.129       | -4.012              |
| 5010        | -2.965            | -2.340        | -3.039    | -1.417      | 0.000                  | -4.720       | -4.233              |
| 5051        | -2.108            | -2.008        | -1.617    | -1.026      | -1.621                 | 0.464        | -1.143              |
| 5100        | -7.568            | -0.931        | -0.248    | -0.229      | 1.516                  | 1.464        | 2.218               |
Table 3. Residuals and statistical indicators for interpolation methods.

| Check Point | Residual in kriging (m) | Residual in IDW (m) | Residual in Trend (m) | Residual in Natural Neighbor (m) | Residual in Spline (m) | Residual in Topo to Raster (m) |
|-------------|-------------------------|---------------------|----------------------|-------------------------------|----------------------|-----------------------------|
| 1010        | 1.294                   | 0.935               | 0.939                | 0.660                         | -2.048               | -0.749                      |
| 1042        | 3.880                   | 3.951               | 3.998                | 3.834                         | 3.410                | 3.819                       |
| 1100        | -0.657                  | -0.295              | -1.254               | 0.828                         | 3.443                | 2.648                       |
| 2041        | 1.760                   | 2.758               | 1.158                | 3.208                         | 3.316                | 3.198                       |
| 3072        | -1.269                  | -0.594              | -2.232               | -0.230                        | -0.220               | 0.258                       |
| 4040        | 2.032                   | 1.006               | 2.859                | 0.311                         | -1.555               | -1.391                      |
| 4052        | 0.047                   | -1.218              | 0.656                | -1.795                        | -1.621               | -2.504                      |
| 5010        | 0.625                   | -0.074              | 1.548                | 2.965                         | -1.755               | -1.268                      |
| 5051        | 0.100                   | 0.491               | 1.082                | 0.487                         | 2.572                | 0.965                       |
| 5100        | 6.637                   | 7.320               | 7.339                | 9.084                         | 9.032                | 9.786                       |
| Min         | -1.268                  | -1.217              | -2.232               | -1.794                        | -2.048               | -2.503                      |
| Max         | 6.636                   | 7.319               | 7.339                | 9.083                         | 9.032                | 9.786                       |
| Mean        | 1.444                   | 1.427               | 1.609                | 1.935                         | 1.457                | 1.476                       |

Standard Deviation | 2.226 | 2.458 | 2.547 | 2.891 | 3.373 | 3.423 |

RMSE | 2.654 | 2.843 | 3.013 | 3.479 | 3.675 | 3.728 |

Figure 3. EGM using kriging interpolation in Mosul city.
Figure 4. EGM using IDW interpolation in Mosul city.

Figure 5. EGM using trend interpolation in Mosul city.
Figure 6. EGM using natural neighbor interpolation in Mosul city.

Figure 7. EGM using spline interpolation in Mosul city.
Figure 8. EGM using Topo To Raster interpolation in Mosul city.

Figure 9. Residuals in kriging interpolation.
Figure 10. Residuals in IDW interpolation.

Figure 11. Residuals in trend interpolation.
Figure 12. Residuals in natural neighbor interpolation.

Figure 13. Residuals in spline interpolation.
Figure 14. Residuals in Topo to Raster interpolation.

Figure 15. Standard deviation resulted from applied interpolation methods.

Figure 16. RMSE resulted from applied interpolation methods.
6. Conclusions
1. Regrettably, the geoid in Iraq is not precisely specified. There is a paucity of gravity data and difficulty to obtain orthometric height information. Nevertheless, plenty of positioning satellites and internet web data have been improved providing DEM. All these reasons make an earnest need to adopt local geoid model. The advantage of deriving geoid models is to get elevations of points without traditional surveying. The high cost and the elapsed measurement time using surveying instrumentation lead up to the need for geoid model generation. The results illustrate that the geoid is departed from GPS Visualizer WGS 1984 data by the mean value of minus (1.405 m) in the study area.
2. Dense points are important for producing rigorous geoid model. Some sample points must be preserved as check points to be used for accuracy assessment. Also, it appropriates to test each interpolation method with all its variables and deduce the unknown values at the check points. The results must be compared with the known values of these points. Residual analysis at check points permits to determine statistic parameters include the standard deviation and RMSE. These residuals are indicators to evaluate the qualification of interpolation. The larger the residuals causes the model becomes weakness and vice versa. These residuals represent unexplained values. It is recommended to eliminate points that have large residual observations in accuracy assessment studies.
3. The accuracy of the resulted geoid model is based on the accuracy of used control points and it is influenced by the spatial interpolation method that generating DEM. There is no suitable interpolation method appropriate for all areas or applications. The user should be test different interpolations in order to select the best surface fitting taking into account the properties of each method and the nature of that region. The judgment of the best interpolation is based on the resulted statistical indicators like standard deviation and RMSE. Kriging interpolation has minimum standard deviation and RMSE in the study area. It gives the best surface fitting and it is favorite method especially in the variation in terrains. Kriging takes into account the spatial distribution of points instead of real values. The priority succession of the interpolation methods are kriging , IDW , Trend , Natural Neighbor , Spline , and Topo to Raster , respectively.

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