Cosmography of interacting generalized QCD ghost dark energy

Mohammad Malekjani ∗1,2

1Department of Physics, Faculty of Science, 
Bu-Ali Sina University, Hamedan 65178, Iran

2Research Institute for Astronomy and Astrophysics of 
Maragha (RIAAM), Maragha, P. O. Box:55134-441, Iran

Exploring the accelerated expansion of the universe, we investigate the generalized ghost dark energy (GGDE) model from the statefinder diagnosis analysis in a flat FRW universe. First we calculate the cosmological evolution and statefinder trajectories for non-interacting case and then extend this work by considering the interaction between dark matter and dark energy components. We show that in the non-interacting case the phantom line can not be crossed and also the evolutionary trajectories of model in \( s - r \) plane can not be discriminated. It has been shown that the present location of model in \( s - r \) plane would be close to observational value for negative values of model parameter. In the presence of interaction between dark matter and dark energy, the phantom regime is achieved, the accelerated phase of expansion occurs sooner compare with non-interacting case. The GGDE model is also discussed from the viewpoint of perturbation theory by calculating the adiabatic sound speed of the model. Finally, unlike the non-interacting case, the evolutionary trajectories in \( s - r \) plane can be discriminated in the interacting model. Like non-interacting model, in the interacting case the present location of GGDE model is closer to observational value for negative values of model parameter.

PACS numbers: 98.80.–k, 95.36.+x

Keywords: Cosmology, Dark energy

∗ Email: malekjani@basu.ac.ir
I. INTRODUCTION

The recent astronomical data from SNe Ia, WMAP, SDSS, and X-ray experiments show that our universe experiences an accelerated expansion. The above observational data strongly suggest that the universe is spatially flat and dominated by an exotic component with negative pressure, the so-called dark energy. Dark energy scenario has got a lot of attention in modern cosmology both from theoretical and observational point of view. Observationally, The result of WMAP experiment shows that dark energy occupies about 73% of the energy of our universe, dark matter about 23% and usual baryons occupies only about 4% of the total energy of the universe. Although the nature of dark energy is still un-known, but the ultimate fate of the current universe is determined by this mysterious component. Theoretically, the first and simplest model for dark energy is Einstein’s cosmological constant with constant EoS parameter $w_\Lambda = -1$. Cosmological constant faces with the fine-tuning and cosmic coincidence problems. In recent years a plenty theoretical models have been proposed to interpret the properties of dark energy. Almost all of theoretical dark energy models need to introduce new degree(s) of freedom or modifying general relativity. However it would be better to consider a model of dark energy without a need of new degree(s) or new parameter in its theory. Recently the so-called QCD ghost dark energy has been proposed to interpret the dark energy without any new parameter or new degree of freedom. The Veneziano ghost has been suggested to solve the U(1) problem in low energy effective theory of QCD. The ghost field has no contribution to the vacuum energy density in the flat Minkowski spacetime. However, in the case of curved spacetime it has a small energy density proportional to $\Lambda_{QCD}^3 H$, where $\Lambda_{QCD}$ is QCD mass scale and $H$ is the Hubble parameter. This model does not encounter with some unwanted problems such as the violation of gauge invariance, unitarity and causality. Since the Veneziano ghost field is totally embedded in standard model and general relativity, one needs not to introduce any new degree(s) of freedom or to modify the Einstein’s general relativity. The present value of energy density of dark energy in this model is roughly of order $\Lambda_{QCD}^3 H_0$, with $\Lambda_{QCD} \sim 100 MeV$ and $H_0 \sim 10^{-33} eV$ which is in agreement with observed value $(3 \times 10^{-3} eV)^4$ for energy density of dark energy. This numerical coincidence is adsorbent and gets rid the model from fine tuning problem.
Observationally, the ghost dark energy model has been fitted by astronomical data including SnIa, BAO, CMB, BBN and Hubble parameter data [36]. The cosmological evolution of dark energy in the QCD ghost model has been calculated in [19, 20] and has been resulted that the universe begins to accelerate at redshift around $z \sim 0.6$. Also the squared sound speed of the dark energy for this model is negative, indicating an instability of the model against perturbational theory [19–22]. In [37], the ghost dark energy (GDE, hereafter) has been extended in the presence of interaction between dark matter and dark energy in non-flat universe. The reconstructed potential and the dynamics of scalar fields according the exultation of GDE have been investigated in [38–42]. The reconstructed modified gravity for GDE which describes the late time accelerated expansion has been studied in [43]. The statefinder diagnostic of GDE has been presented in [44]. From the statefinder viewpoint, the evolution of GDE model is similar to holographic dark energy model and present value of statefinder parameters in this model is in good agreement with observation [44].

In all above studies, the energy density of GDE is considered proportional to Hubble parameter as $\rho_d = \alpha H$. However, the energy density of Veneziano ghost field in QCD is generally in the form of $H + O(H^2)$ [45]. In this case the $U(1)_A$ problem in QCD can be solved. Although, up to now, only the leading term $H$ has been assumed for energy density of GDE, but the sub-leading term $H^2$ can also be important in the early evolution of the universe [46]. Including the second term in the energy density of GDE results better agreement with observation in comparison with usual GDE model [47]. Like [48], we call this model as generalized ghost dark energy (GGDE). The energy density of GGDE model is written as $\rho_d = \alpha H + \beta H^2$, where $\alpha$ and $\beta$ are the constants of the model. It has been shown that the GGDE model can result a de-Sitter phase of expansion and also in the presence of interaction between dark matter and dark energy this model results the phantom regime of expansion ($w_d < -1$) [48]. The other features of GGDE model have been presented in [49, 50].

It is well known that in addition of dark energy component which describe the accelerated expansion of the universe, there exist another mysterious component in the universe so-called dark matter. The dark matter component can interpret the flat rotation curve of spiral galaxies and also the scenario of structure formation of universe [51, 52]. Since the nature of these component are un known and they they have different gravitational
treatment, therefore their evolution usually considered independent of each other. However recent observation from galaxy cluster Abell A596 indicates the interaction between these components [56]. Also the observational data from SNIa and CMB experiments is compatible with interacting forms of dark energy models [57]. However the strength of this interaction is not clearly identified [58]. From theoretical viewpoint it is also acceptable to consider the interaction between dark matter and dark energy. In the unified models of field theory dark matter and dark energy can be interpreted by a single scalar field in a minimally interaction. Also considering interaction between dark matter and dark energy can solve the coincidence problem [59–66].

In this work our main task is to investigate the interacting GGDE model in statefinder diagnostic analysis. Different dynamical dark energy models obtain accelerated expansion at the present time \( q < 0 \), where \( q \) is deceleration parameter. Hence we need a diagnostic tool for discriminating these dark energy models. For this aim, Sahni et al. [67] and Alam et al. [68], by using the third time derivative of scale factor, introduced the statefinder pair \( \{s, r\} \). These parameters in flat universe are given by

\[
 r = \frac{\ddot{a}}{aH^3}, \quad s = \frac{r - 1}{3(q - 1/2)}
\]  

The parameters \( s \) and \( r \) are geometrical, because they only depend on the scale factor. In statefinder analysis we plot the evolutionary trajectories of dark energy model is \( s - r \) plane. In recent years the various dark energy models such as quintessence, holographic, new holographic, phantom, tachyon, chaplygin gas, agegraphic, new agegraphic, polytropic gas and ghost dark energy models have been studied in the statefinder analysis [67, 68, 70–83]. These models have different evolutionary trajectories in \( \{s, r\} \) plane, therefore the statefinder tool can discriminate these models. The standard \( \Lambda \)CDM has no evolution in this plane and corresponds to the fixed point \( \{s=0, r=1\} \) [67]. The present observational value for statefinder parameters are \( \{s_0 = -0.006, r = 1.02\} \) [69]. The distance of the current value of statefinder pair \( \{s_0, r_0\} \) of a given dark energy model from the observational value \( \{s_0 = -0.006, r = 1.02\} \) is a valuable criterion to examine of model. Here we see that the location of standard \( \Lambda \)CDM model in \( s - r \) plane is near to observational value. In [83], the evolution of original GDE has been calculated by statefinder diagnostic in \( s - r \) plane and shown that the GDE model mimics the \( \Lambda \)CDM at the late time. Also the behavior of
GDE is similar to holographic dark energy in this plane \[83\].

In this work we first calculate the cosmological evolution of GGDE model and then investigate this model from statefinder diagnostic analysis. The paper is organized as follows: In sect.II, The GGDE model is presented in non-interacting universe. The interacting case of GGDE model is given in sect.III. In sect. IV we obtain the adiabatic sound speed for GGDE model. We calculate the numerical results in sect.V and conclude in sect.VI.

II. NON-INTERACTING GGDE MODEL

A flat Friedmann-Robertson-Walker (FRW) universe dominated by dark matter and dark energy is given by

\[ H^2 = \frac{1}{3m_p^2}(\rho_m + \rho_d) \] (2)

where \( \rho_m \) and \( \rho_d \) are, respectively, the energy density of pressureless dark matter and dark energy and \( m_p \) is the reduced Planck mass. The energy density of GGDE is given by \[48\]

\[ \rho_\Lambda = \alpha H + \beta H^2 \] (3)

where \( \alpha \) and \( \beta \) are constants of model. The Friedmann equation (2) in terms of dimensionless parameters is written as

\[ \Omega_m + \Omega_\Lambda = 1 \] (4)

where

\[ \Omega_m = \frac{\rho_m}{\rho_c} = \frac{\rho_m}{3M_p^2H^2} \quad \Omega_d = \frac{\rho_d}{\rho_c} = \frac{\rho_d}{3M_p^2H^2} \] (5)

The conservation equations for pressureless dark matter and dark energy without interaction read the following equations

\[ \dot{\rho}_m + 3H\rho_m = 0, \] (6)

\[ \dot{\rho}_d + 3H(1 + w_d)\rho_d = 0. \] (7)

Taking the time derivative of Friedmann equation (2) and using (4, 6, 7) obtains

\[ \frac{\dot{H}}{H^2} = -\frac{3}{2}[1 + w_\Lambda \Omega_d] \] (8)

Differentiating Eq.(3) with respect to time yields

\[ \dot{\rho} = \dot{H}(\alpha + 2\beta H) \] (9)
Inserting (9) and (3) in conservation equation for dark energy (7) and using (8), the EoS parameter of GGDE model can be obtained as

\[ w_d = \frac{\xi - \Omega_d}{\Omega_d(2 - \Omega_d - \xi)} \]  

where \( \xi = 8\pi G\beta / 3 \). In the limiting case \( \xi = 0 \), this relation reduces to its original form in [37]. The deceleration parameter \( q \) by using (8) and (10), in GGDE universe is obtained as

\[ q = \frac{1}{2} - \frac{3}{2}\frac{\xi - \Omega_d}{\xi + \Omega_d - 2} \]

The decelerated phase of expansion at the early time is indicated by \( q < 0 \) and accelerated phase is related to \( q > 0 \). Taking the time derivative of dimensionless dark energy density in (5) and using (3), (9), we obtain the equation of motion for the evolution of energy density of GGDE model as

\[ \Omega_d' = -3\frac{(1 - \Omega_d)(\xi - \Omega_d)}{(2 - \Omega_d - \xi)} \]

where prime is derivative with respect to \( \ln a \). Taking a derivative of (10) with respect to \( \ln a \), the equation of motion for EoS parameter can be calculated as

\[ w_d' = \frac{3(1 - \Omega_d)(\xi - \Omega_d)}{\Omega_d(2 - \Omega_d - \xi)^2} \left[ 1 + \frac{(\xi - \Omega_d)(2 - 2\Omega_d - \xi)}{\Omega_d(2 - \Omega_d - \xi)} \right] \]

Using the above relation, in this stage, we calculate the statefinder parameters \( s \) and \( r \) for GGDE model in non-interacting universe. In general form, relation (11) for a given dark energy model in flat universe can be written as

\[ r = 1 + \frac{9}{2}w_d\Omega_d(1 + w_d\Omega_d) - \frac{3}{2}(w_d'\Omega_d + w_d\Omega_d') \]

and

\[ s = 1 + w_d\Omega_d - \frac{1}{3}\frac{w_d' + \Omega_d'}{w_d} \]

Inserting relations (12) and (13) in equations (14) and (15), we obtain the statefinder parameters for GGDE model in spatially flat universe

\[ r = 1 + 9\frac{(\xi - \Omega_d)(1 - \Omega_d)^2}{(2 - \Omega_d - \xi)^3} \]

\[ s = \frac{2(1 - \Omega_d)^2}{(2 - \Omega_d - \xi)^2} \]
In the limiting case of dark energy dominated universe ($\Omega_d \to 0$) the parameters \{s,r\} tends to \{0,1\}, respectively. Hence the GGDE model mimics the $\Lambda$CDM model at the late time when $\Omega_d \to 0$. In sect.V, we calculate numerically the evolution of GGDE model in non-interacting universe from the statefinder viewpoint.

III. INTERACTING GGDE MODEL

In this section we consider the interaction between dark matter and dark energy components. In this case the conservation equations for these components are:

$$\dot{\rho}_m + 3H \rho_m = Q,$$  \hfill (18)

$$\dot{\rho}_d + 3H(1 + w_d)\rho_d = -Q.$$  \hfill (19)

where $Q$ in right hand side indicate the interaction term. The positive value of $Q$ means the transition of energy from dark energy to dark matter component. It should be noted that the left side of (6) and (7) are inversely proportional to time. Therefore the parameter $Q$ can be considered as a function of Hubble parameter $H$ such as following forms:

(i) $Q \propto H \rho_d$

(ii) $Q \propto H \rho_m$

(iii) $Q \propto H(\rho_m + \rho_d)$.

One can assume the above three forms as $Q = \Gamma \rho_d$, where for case (i) $\Gamma = 3b^2H$, for case (ii) $\Gamma = 3b^2H\frac{\Omega_m}{\Omega_d}$ and for case (iii) $\Gamma = 3b^2H\frac{1}{\Omega_d}$. The parameter $b$ is a coupling constant indicating the strength of interaction between dark matter and dark energy [84,86]. In this work we assume the third form of interaction (i.e., $Q = 3Hb^2\frac{\rho_d}{\Omega_d}$).

Substituting $Q$ in (19) and using (3), (8) and (9), the EoS parameter of interacting GGDE model is obtained as

$$w_d = \frac{\xi - \Omega_d - 2b^2}{\Omega_d(2 - \Omega_d - \xi)}.$$  \hfill (20)

Inserting $b = 0$ recovers the EoS parameter of non-interacting case in previous section. Substituting (20) in (8) results the deceleration parameter $q$ for interacting case as follows

$$q = \frac{1}{2} - \frac{3}{2} \frac{(\xi - \Omega_d - 2b^2)}{(\xi + \Omega_d - 2)}.$$  \hfill (21)

It has been shown that for selected parameters ($\xi = 0.03$, $b = 0.15$, $\Omega_{d0} = 0.72$) the deceleration parameter at the present time is $q_0 = -0.38$ which is consistent with observation [48].
Taking the time derivative of dimensionless dark energy density in (5) and using (3), (9) and (20), the equation of motion for the evolution of energy density of interacting GGDE model can be obtained as

\[ \Omega_d' = -3 \left( \frac{(1 - \Omega_d)(\xi - \Omega_d - 2b^2)}{(2 - \Omega_d - \xi)} + b^2 \right) \]  

(22)

In the limiting non-interacting case \((b = 0)\) the respective relation in previous section is retained. Derivative of (20) with respect to \(\ln a\) results

\[ w_d' = \frac{3(1 - \Omega_d)(\xi - \Omega_d - 2b^2) + b^2(2 - \Omega_d - \xi)}{\Omega_d(2 - \Omega_d - \xi)^2} \left[ 1 + \frac{(\xi - \Omega_d - 2b^2)(2 - 2\Omega_d - \xi)}{\Omega_d(2 - \Omega_d - \xi)} \right] \]  

(23)

Once again, the respective relation in previous section can be obtained by setting \(b = 0\).

Finally by substituting relations (22) and (23) in general relations (14) and (15), we obtain the statefinder parameters for interacting GGDE model as follows

\[ r = 1 + 9 \frac{(\xi - \Omega_d - 2b^2)(1 - \Omega_d - b^2)}{(2 - \Omega_d - \xi)^2} - 9 \frac{(1 - \Omega_d)(\xi - \Omega_d - 2b^2) + b^2(2 - \Omega_d - \xi)}{(2 - \Omega_d - \xi)^3} (1 - \xi + b^2) \]  

(24)

\[ s = 2 \left[ \frac{(1 - \Omega_d - b^2)}{(2 - \Omega_d - \xi)} - \frac{(1 - \Omega_d) + b^2 \frac{2 - \Omega_d - \xi}{\xi - \Omega_d - 2b^2}}{(2 - \Omega_d - \xi)^2} (1 - \xi + b^2) \right] \]  

(25)

Setting \(b = 0\) retains the relations for \(s\) and \(r\) in previous section. In section V we investigate the evolution of interacting GGDE model in \(s - r\) plane can calculate the effect of interaction parameter \(b^2\) on the evolution of the model.

**IV. ADIABATIC SOUND SPEED**

In linear perturbation theory, squared sound speed, \(c_s^2\) is a crucial quantity. Stability or instability of a given perturbed mode can be calculated by determining the sign of \(c_s^2\). The positive sign (real value of sound speed) represents the periodic propagating mode for a density perturbation and in this case we have the stability. The negative sign (imaginary value of sound speed) indicates an exponentially growing mode for a density perturbation, meaning the instability \([90, 91]\). Here we obtain the squared sound speed for GGDE model both in non-interacting and interacting cases. The squared sound speed \(c_s^2\) is introduced as

\[ c_s^2 = \frac{dp}{d\rho_d} = \frac{\dot{p}}{\dot{\rho}_d} \]  

(26)
We now differentiate the equation of state, \( p_d = w_d \rho_d \) with respect to time and find

\[
\dot{p}_d = \dot{w}_d \rho_d + w_d \dot{\rho}_d
\]  

(27)

Inserting (27) in (26) and using Eq.(7), we obtain \( c_s^2 \) for non-interacting GGDE model as follows

\[
c_s^2 = w_d - \frac{w'_d}{3(1 + w_d)}
\]  

(28)

where prime is the derivative with respect to \( \ln a \) and \( w'_d = \dot{w}_d / H \). In the case of interacting GGDE model by using Eq.(19), the parameter \( c_s^2 \) can be obtained as

\[
c_s^2 = w_d - \frac{w'_d}{3(1 + w_d) + \frac{3b^2}{\Omega_d}}
\]  

(29)

Here same as previous section we used third form of interaction parameter \( Q = 3Hb^2 \rho_d / \Omega_d \).

In next section, we obtain the evolution of \( c_s^2 \) as a function of cosmic redshift and discuss the stability or instability of GGDE model for both non-interacting and interacting universe.

V. NUMERICAL RESULTS

Here we present numerical description for cosmological evolution and statefinder analysis of GGDE model in the flat FRW cosmology. In numerical procedure we fix the cosmological parameters at the present time as \( \Omega^0_m = 0.3 \) and \( \Omega^0_d = 0.7 \). We first consider non-interacting case and then interacting case of GGDE model.

A. non-interacting case

The EoS parameter of non-interacting GGDE model as a function of density parameter \( \Omega_d \) is given by (10). By solving coupled equations (12) and (10), the evolution of EoS parameter in terms of cosmic redshift \( z = 1/a - 1 \) and for different illustrative values of \( \xi \) is shown in Fig.(1). The cosmic redshift \( z = 0 \) represents the present time, \( z > 0 \) indicates the past times and \( z < 0 \) expresses the future. We see that for any value of \( \xi \) the non-interacting GGDE model can not enter the phantom regime \( (w_d < -1) \) at all. We also see that the EoS of GGDE model with \( \xi > 0 \) is larger than EoS of standard GDE model \( (\xi = 0.0) \). At the future epoch, the EoS tends to \(-1\) which implies that the GGDE model
mimics the cosmological constant at that time.

The deceleration parameter $q$ which indicates the decelerated or accelerated phase of expansion for non-interacting case is given by \[(11)\]. Solving coupled equations \[(11)\] and \[(12)\], the evolution of parameter $q$ as a function of redshift parameter $z$ has been shown in Fig.(2) for different values of model parameter $\xi$. The standard GDE model is indicated by solid line. one can see that for $\xi < 0$ the GGDE model enters the accelerated phase sooner and for $\xi > 0$ later compare with standard GDE model.

The statefinder pair $\{s,r\}$ for non-interacting GGDE model is given by relations\[(16)\] and \[(17)\]. Solving these coupled equations together with \[(12)\], we obtain the evolution of parameters $s$ and $r$ in terms of redshift $z$. In Fig.(3), we plot the evolutionary trajectories of non-interacting GGDE model for different values of model parameter $\xi$ in $s - r$ plane. We see that the parameter $r$ first decreases and then increases and also the parameter $s$ decreases during the history of the universe from past to future. The important note is that in non-interacting case the GGDE model has been shown by single evolutionary trajectory for any value of $\xi$. Hence the evolutionary trajectories can not discriminated by model parameter $\xi$. The present value of statefinder pair $\{s_0, r_0\}$ is indicated by colored circle on the figure. Also the location of $\Lambda$CDM model in $s - r$ plane, i.e., $(s = 0, r = 1)$, has been shown by star symbol. The other feature is that the present value $\{s_0, r_0\}$ is discriminated by model parameter $\xi$. In the case of $\xi < 0$, the distance of $\{s_0, r_0\}$ from observational point $\{s_0 = -0.006, r = 1.02\}$ (red star point on the figure) is shorter compare with standard GDE model (i.e., $\xi = 0.0$). While for $\xi > 0$ the distance is larger than GDE model. Finally we discuss numerically the stability or instability of non-interacting GGDE model form the viewpoint of perturbation theory based on Eq.\[(28)\]. In Fig.(4), the evolution of adiabatic sound speed $c_s^2$ is plotted as a function of redshift $z$ for different values of GGDE model $\xi$. In the case of $\xi \geq 0$, one can see $c_s^2 < 0$ which indicates the instability of GGDE model against perturbation. For $\xi < 0$, we obtain $c_s^2 > 0$ which represents the stability of model against perturbation.
B. interacting case

Here we calculate the numerical description of cosmological evolution and statefinder diagnosis for interacting GGDE model in spatially flat universe. First the evolution of EoS parameter in terms of cosmic redshift is plotted in Fig.(4). For this aim we solved the coupled equations (20) and (22). In left panel, by fixing interaction parameter as $b = 0.2$, the EoS parameter $w_d$ is plotted for different illustrative values of model parameter $\xi$ as described in legend. The solid line represents the original GHDE model. For all cases of $\xi$, the interacting case of GGDE model can cross the phantom line ($w_d = -1$) form upper limit ($w_d < -1$) to lower limit ($w_d < -1$). This behavior of interacting GGDE model in which the phantom line is crossed from up to below is in agreement with recent observations [? ]. In right panel the model parameter is fixed as $\xi = 0.1$ and the interaction parameter $b$ is varied. We see that the non-interacting GGDE model (= $\bar{0}$) cannot enter the phantom regime (solid line), while in other cases ($\xi \neq 0$) the phantom regime has been achieved.

In Fig.(5), by numerical solving of relations (22) and (21), the evolution of deceleration parameter in terms of redshift has been shown in the context of interacting GGDE model. In left panel, by fixing $b = 0.2$, the parameter $\xi$ is varied as indicated in legend. Same as non-interacting case, the parameter $q$ starts from positive value at the earlier (representing the decelerated phase at the past time) and ends to negative value later (indicating the accelerated phase at the present time). Like non-interacting case, transition from decelerated phase to accelerated phase for $\xi < 0$ takes place earlier and for $\xi > 0$ later, compare with original GDE model (see left panel of Fig.(5)). In right panel, for an illustrative value $\xi = 0.1$, the evolution of $q$ has been shown for different values of interaction parameter $b$. The interaction parameter $b$ can influence on the transition epoch from $q > 0$ to $q < 0$. We see that in the framework of interacting GGDE model the accelerated phase of expansion ($q < 0$) can be achieved sooner for larger values of $b$. For all cases, $q \to -1$ at the late time indicating that the GGDE model mimics the standard $\Lambda$ CDM model at that time.

Now, by solving relations (25) and (24), the evolution of interacting GGDE model in $s-r$ plane is plotted in Fig.(6). In left panel, by fixing $\xi = 0.1$, the interaction parameter $b$ is varied as indicated in legend. The evolutionary trajectories starts from right to left and
ended at the ΛCDM fixed point. The distance of present value \(\{s_0, r_0\}\) form the observational point \(\{s_0 = -0.006, r = 1.02\}\) (red star point on the figure) becomes larger by increasing \(b\). In right panel, by fixing \(b = 0.2\), the trajectories have been plotted for different illustrative values of \(\xi\). The important note is that, contrary with non-interacting case, in the presence of interaction between dark matter and dark energy \((b \neq 0)\) the evolutionary trajectories in \(s-r\) plane are discriminated by parameter \(\xi\). Also the distance of \(\{s_0, r_0\}\) from observational point \(\{s_0 = -0.006, r = 1.02\}\) is shorter for \(\xi < 0\) and larger for \(\xi > 0\) compare with original GDE model \((i.e., \xi = 0.0)\). Finally, same as non-interacting case, we investigate the interacting GGDE model from the viewpoint of perturbation theory. The adiabatic sound speed \(c_s^2\) for interacting case is given by Eq.(29). In Fig.(8), we calculate the evolution of \(c_s^2\) as a function of cosmic redshift for different values of model parameter \(\xi\) as well as interaction parameter \(b\). In left panel, by fixing model parameter \(\xi = 0.5\), we obtain the evolution of \(c_s^2\) for different values of interaction parameter \(b\). Here one can interpret that same as non-interacting case, the interacting GGDE model for \(\xi > 0\) is instable \((c_s^2 < 0)\) against perturbation. In right panel, by fixing interaction parameter \(b = 0.1\), the evolution of \(c_s^2\) has been shown for different values of model parameter \(\xi\). Like non-interacting case, we conclude that the interacting GGDE model is stable for \(\xi < 0\) and instable for \(\xi \geq 0\).

VI. CONCLUSION

In summary, we considered the generalized version of QCD ghost dark energy (GGDE) model in both non-interacting and interacting universe. The cosmological evolution and also the statefinder diagnosis of the model have been calculated. We showed that:

(i). In the non-interacting GGDE model the phantom regime can not be achieved and the EoS parameter reaches to asymptotic value \(w_d = -1\) at the late time. We also showed that for negative values of model parameter \(\xi\) the transition from decelerated to accelerated phase takes place sooner compare with original ghost dark energy (GDE) model. The statefinder analysis was also performed for non-interacting GGDE model. In this case, in the absence of interaction between dark matter and dark energy, the evolutionary trajectories of model in \(s-r\) plane can not discriminated. However, the present value of statefinder parameter \(\{s_0, r_0\}\) of the model is diagnosed by parameter \(\xi\). We concluded that \(\{s_0, r_0\}\) is closer to observational value \(\{s_0 = -0.006, r = 1.02\}\) for negative values of \(\xi\) (see
Fig.(3)). We also obtained the stability or instability of the model against perturbation by calculating the adiabatic sound speed and showed that the non-interacting case of GGDE model is instable for $\xi \geq 0$ and stable for $\xi < 0$

(ii). In the presence of interaction between dark matter and dark energy, the GGDE model can cross the phantom line from up to down in agreements with observations [87–89]. Also, in the context of interacting GGDE model the entrance to accelerated phase of expansion occurs earlier compare with non-interacting case. The statefinder diagnosis analysis was also performed for interacting case and we showed that in the presence of interaction the evolutionary trajectories of GGDE model $s - r$ plane are diagnosed. Same as non-interacting model, the present value $\{s_0, r_0\}$ is closer to observational point for negative values of $\xi$ (see Fig.(6)). We also showed that in the presence of interaction, the GGDE model has stability for $\xi < 0$ and instability for $\xi \geq 0$.

Acknowledgements

This work has been supported financially by Research Institute for Astronomy & Astrophysics of Maragha (RIAAM) under research project 1/2782-51.
FIG. 1: The evolution of EoS parameter of non-interacting GGDE model in terms of redshift parameter $z$ for different illustrative values of model parameters $\xi$. Here we take $\Omega_{d}^{0} = 0.70$ and $\Omega_{m}^{0} = 0.30$.

FIG. 2: The evolution of deceleration parameter $q$ in the context of non-interacting GGDE model as a function of redshift $z$ for different illustrative values of model parameters $\xi$. We take $\Omega_{d}^{0} = 0.70$ and $\Omega_{m}^{0} = 0.30$. 
FIG. 3: The statefinder plot of non-interacting GGDE model in flat FRW universe with $\Omega_0^d = 0.70$ and $\Omega_0^m = 0.30$. The present value $\{s_0, r_0\}$ is indicated by colored circle on the curves. The location of $\Lambda$CDM model and observational point are indicated by black and red stars, respectively.

FIG. 4: The adiabatic sound speed $c^2_s$ as a function of cosmic redshift $z$ for different model parameter $\xi$ as described in legend. The horizontal dashed line separates the stability and instability regions.

[1] S. Perlmutter et al., Astrophys. J. 517, 565 (1999).
FIG. 5: The evolution of EoS parameter of interacting GGDE model versus redshift parameter $z$ as described in legend. In left panel, the interaction parameter $b$ is fixed and in the right panel the parameter $\xi$.

FIG. 6: The evolution of parameter $q$ for interacting case of GGDE model as a function of redshift $z$ as indicated in legend.

[2] C. L. Bennett et al., Astrophys. J. Suppl. 148, 1 (2003).
[3] M. Tegmark et al., Phys. Rev. D 69, 103501 (2004).
[4] S. W. Allen, et al., Mon. Not. Roy. Astron. Soc. 353, 457 (2004).
[5] S. Weinberg, Rev. Mod. Phys. 61 (1989) 1.
[6] S. M. Carroll, Living Rev. Rel. 4 (2001) 1.
[7] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75 (2003) 559.
[8] T. Padmanabhan, Phys. Rept. 380 (2003) 235.
FIG. 7: The statefinder plot of interacting GGDE model. In left panel the parameter $\xi$ is fixed and in right panel the parameter $b$ is fixed. The present value $\{s_0, r_0\}$ is indicated by colored circle on the curves. The location of $\Lambda$CDM model and observational point are indicated by black and red stars, respectively.

FIG. 8: The adiabatic sound speed $c_s^2$ as a function of cosmic redshift $z$ for different interaction parameter $b$ (left panel) and various model parameter $\xi$ (right panel) as indicated in legend. The horizontal dashed line in right panel separates the stability and instability regions.

[9] V. Sahni and A.A. Starobinsky, Int. J. Mod. Phys. D 9, 373 (2000).
[10] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006).
[11] M. R. Setare, Phys. Lett. B 644, 99, 2007.
[12] M. R. Setare, Phys. Lett. B 654, 1, 2007.
[13] M. R. Setare, Phys. Lett. B 642, 1, 2006.
[14] M. R. Setare, Eur. Phys. J. C 50, 991, 2007.
[15] M. R. Setare, Phys. Lett. B 648, 329, 2007.
[16] M. R. Setare, Phys. Lett. B 653, 116, 2007.
[17] M. Li, X. D. Li, S. Wang, Y. Wang, Commun. Theor. Phys. 56, 525, 2011.
[18] K. Bamba, S. Capozziello, S. Nojiri and S. D. Odintsov, Astrophys Space Sci, (2012) 342, 155-228.
[19] F. R. Urban and A. R. Zhitnitsky, Phys. Lett. B 688 (2010) 9.
[20] F. R. Urban and A. R. Zhitnitsky, Phys. Rev. D 80 (2009) 063001.
[21] F. R. Urban and A. R. Zhitnitsky, JCAP 0909 (2009) 018.
[22] F. R. Urban and A. R. Zhitnitsky, Nucl. Phys. B 835 (2010) 135.
[23] A. R. Zhitnitsky, Phys. Rev. D 84, 124008 (2011).
[24] N. Ohta, Phys. Lett. B 695, 41 (2011).
[25] R. G. Cai, Z. L. Tuo, H. B. Zhang and Q. Su, Phys. Rev. D 84, 123501 (2011).
[26] E. Witten, Nucl. Phys. B 156, 269 (1979).
[27] G. Veneziano, Nucl. Phys. B 159, 213 (1979).
[28] C. Rosenzweig, j. Schechter and C. G. Trahern, Phys. Rev. D 21, 3388 (1980).
[29] P. Nath and R. L. Arnowitt, Phys. Rev. D 23, 473 (1981).
[30] K. Kawarabayashi and N. Ohta, Nucl. Phys. B 175, 477 (1980).
[31] N. Ohta, Prog. Theor. Phys. 66, 1408 (1981).
[32] A. R. Zhitnitsky, Phys. Rev. D82, 103520 (2010).
[33] B. Holdom, Phys. Lett. B697, 351-356 (2011).
[34] E. Thomas, A. R. Zhitnitsky, Phys.Rev. D85 (2012) 044039.
[35] R. G. Cai, Z. L. Tuo, H. B. Zhang and Q. Su, Phys. Rev. D 84, 123501 (2011).
[36] R. G. Cai, Z. L. Tuo, H. B. Zhang and Q. Su, Phys. Rev. D 84, 123501 (2011).
[37] A. Sheykhi, M. S. Movahed, Gen. Relativ. Gravit. 44 (2012) 449.
[38] E. Ebrahimi and A. Sheykhi, Int. J. Mod. Phys. D20 (2011) 2369.
[39] A. Sheykhi, M. Sadegh Movahed, E. Ebrahimi, Astrophys Space Sci 339 (2012)93.
[40] A. Sheykhi, A. Bagheri, Euro. Phys. Lett., 95 (2011) 39001.
[41] E. Ebrahimi and A. Sheykhi, Phys. Lett. B 706 (2011) 19.

[42] A. R. Fernandez, Phys.Lett.B 709 (2012) 313-321;

[43] A. Khodam-Mohammadi, M. Malekjani, M. Monshizadeh, Mod. Phys. Lett. A, 27 (2012) 1250100.

[44] M. Malekjani, A. Khodam-Mohammadi, Astrophys Space Sci. (2013) 343, 451-461.

[45] A. R. Zhitnitsky, Phys. Rev. D 84, 124008 (2011).

[46] M. Maggiore, L. Hollenstein, M. Jaccard and E. Mitsou, Phys. Lett. B 704, 102 (2011).

[47] R. G. Cai, Z. L. Tuo, Y. B. Wu, Y. Y. Zhao, Phys.Rev. D86 (2012) 023511.

[48] E. Ebrahimi, A. Sheykhi, [arXiv:1209.3147] (2012).

[49] A. Sheykhi, E. Ebrahimi, Y. Yosefi, [arXiv:1210.0781] (2012).

[50] Esmaiel E. Ebrahimi, A. Sheykhi, [arXiv:1211.2686] (2012).

[51] A. Bosma, Astron. J. 86 (1981) 1825.

[52] P. J. E. Peebles, The Large-Scale Structure of the Universe (Princeton University Press, NJ, 1980).

[53] Padmanabhan, T., 1993. Structure Formation in the Universe. Cambridge University Press.

[54] M. Malekjani , S. Rahvar , D.M.Z. Jassur, New Astronomy 14 (2009) 398405.

[55] M. Malekjani, H. Haghi, D. Mohammad-Zadeh Jassur, New Astronomy 17 (2012) 149153.

[56] O. Bertolami, F. Gil Pedro and M. Le Delliou, Phys. Lett. B 654 (2007) 165.

[57] G. Olivares, F. Atrio-Barandela and M. Le Delliou, Phys. Rev. D 71 (2005) 063523.

[58] Feng, C., Wang, B., Gong, Y., Su, R.K.: J. Cosmol. Astropart. Phys. 0709, 005 (2007).

[59] L. Amendola, Phys. Rev. D 62, 043511 (2000).

[60] L. Amendola and C. Quercellini, Phys. Rev. D 68 (2003) 023514.

[61] L. Amendola, S. Tsujikawa and M. Sami, Phys. Lett. B 632 (2006) 155.

[62] D. Pavon, W. Zimdahl, Phys. Lett. B 628 (2005) 206.

[63] S. Campo, R. Herrera, D. Pavon, Phys. Rev. D 78 (2008) 021302.

[64] C. G. Boehmer, G. Caldera-Cabral, R. Lazkoz, R. Maartens, Phys. Rev. D 78 (2008) 023505.

[65] G. Olivares, F. Atrio-Barandela and D. Pavon, Phys. Rev. D 74 (2006) 043521.

[66] S. B. Chen, B. Wang, J. L. Jing, Phys.Rev. D 78 (2008) 123503.

[67] Sahni, V., Saini, T.D., Starobinsky, A.A., Alam, U.: JETP Lett. 77, 201 (2003).

[68] Alam, U., Sahni, V., Saini, T.D., Starobinsky, A.A.: Mon. Not. R. Astron. Soc. 344, 1057 (2003).
[69] Capozziello, S., Cardone, V.F., Farajollahi, H., Ravanpak, A. arXiv: 1108.2789 (2011)
[70] Zimdahl, W., Pavon, D., Gen. Relativ. Gravit. 36, 1483 (2004).
[71] Zhang, X.: Phys. Lett. B 611, 1 (2005).
[72] Zhang, X.: Int. J. Mod. Phys. D 14, 1597 (2005).
[73] Setare, M.R., Zhang, J., Zhang, X.: J. Cosmol. Astropart. Phys. 0703, 007 (2007).
[74] Chang, B.R., Liu, H.Y., Xu, L.X., Zhang, C.W., Ping, Y.L.: J. Cosmol. Astropart. Phys. 0701, 016 (2007).
[75] Malekjani, M., Khodam-Mohammadi, A. and N. Nazari-Pooya, Astrophys Space Sci, 334:193201, 2011.
[76] Zhang, L., Cui, J., Zhang, J., Zhang, X.: Int. J. Mod. Phys. D 19,21 (2010).
[77] Khodam-Mohammadi, A., Malekjani, M.: Astrophys. Space Sci. 331, 265 (2010).
[78] Wei, H., Cai, R.G.: Phys. Lett. B 655, 1 (2007).
[79] Malekjani, M., Khodam-Mohammadi, A.: Int. J. Mod. Phys. D 19,1 (2010).
[80] Malekjani, M., Khodam-Mohammadi, A., Nazari-Pooya, N., Astrophys Space Sci (2011) 332, 515.
[81] Malekjani, M., Khodam-Mohammadi, A., Int. J. Theor. Phys. 51, 3141-3151 (2012).
[82] M. Malekjani R. Zarei M. Honari-Jafarpour, Astrophys Space Sci (2013) 346, 545-552.
[83] M. Malekjani, A. Khodam-Mohammadi, Astrophys Space Sci (2013) 343, 451-461.
[84] A. Sheykhi, Phys. Lett. B 680, 113 (2009).
[85] H. Wei & R. G. Cai, Phys. Lett. B 660, 113 (2008).
[86] L. Zhang, J. Cui, J. Zhang & X. Zhang, Int. J. Mod. Phys. D 19, 21 (2010).
[87] Alam, U., Sahni, V., Starobinsky, A.A.: J. Cosmol. Astropart. Phys. 06, 008 (2004).
[88] Huterer, D., Cooray, A.: Phys. Rev. D 71, 023506 (2005).
[89] Wang, Y., Tegmark, M.: Phys. Rev. D 71, 103513 (2005).
[90] Y. S. Myung, Phys. Lett. B 652 (2007) 223.
[91] K. Y. Kim, H. W. Lee and Y. S. Myung, Phys. Lett. B 660 (2008) 118. 115.