Seismic barriers: mathematical foundations and dimensional analysis

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Abstract. The concept of a vertical barrier embedded in soil to protect from seismic waves of the Rayleigh type is discussed. The principle idea for such a barrier is to reflect and scatter wave energy by the barrier, thus decreasing amplitude of surface vibrations beyond the barrier. Numerical FE simulations of a plane model are presented and discussed.

1. Introduction

Ground vibrations generated by the external sources, such as earthquakes, blasts, railroads, etc. can affect structures and cause their damage. During recent few decades, several approaches were suggested to mitigate effects of the ground vibrations inside the protected regions by introducing barriers of different nature; see (Adam and v.Estroff, 2005; Herle, 2006; Jesmani et al., 2012; Ketchart and Wu, 2001; Kim and Das, 2013; Kusakabe et al. 2008; Kuznetsov, 2009, 2011; Kuznetsov and Nafasov, 2011; Motamed et al., 2008; Takahashi et al., 2001).

Most of these works concern with vertical barriers filled in with the acoustically softer material than the ambient soil. However, as was observed in (Kuznetsov, 2011) horizontal barriers filled in with acoustically more stiff material than the ambient soil can produce even stronger effect of vibration protection. The discussed effect relates to Chadwick’s theorem (Chadwick and Smith, 1977; Chadwick and Borejko, 1994) stating that no Rayleigh waves can propagate over a clamped surface of a halfspace or a halfplane.

Herein, different materials for filling in the vertical barriers are analyzed with respect to their ability to mitigate ground vibrations beyond the barrier. The main attention is paid to Rayleigh waves, as the major factor causing ground surface vibrations at regions sufficiently distant from the buried dynamic sources (Kuznetsov and Terentjeva, 2014).

2. Basic notations

The starting point for the analysis of interaction of the surface acoustic waves (in the considered case of a homogeneous halfplane the surface waves are reduced to Rayleigh waves) with the vertical barrier, is analysis of the equation of motion

\[ c_p \nabla \text{div } \mathbf{u} - c_S \text{ rot rot } \mathbf{u} = \frac{\partial^2 \mathbf{u}}{\partial t^2} \]  

(1)
where \( u \) is the displacement field, \( c_P \) and \( c_S \) are velocities of the longitudinal and transverse bulk waves respectively:

\[
c_P = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad c_S = \sqrt{\frac{\mu}{\rho}}.
\]  

In (2) \( \lambda \) and \( \mu \) are Lamé constants and \( \rho \) is the material density.

Due to Helmholtz decomposition, the displacement field can be represented in terms of scalar (\( \Phi \)) and vector (\( \Psi \)) potentials

\[
u = \nabla \Phi + \text{rot} \Psi.
\]  
The potentials are assumed harmonic in time

\[
\Phi(x,t) = \Phi'(x)e^{i\omega t}, \quad \Psi(x,t) = \Psi'(x)e^{i\omega t}.
\]  

Substituting representation (4) into Eq. (1) yields two independent Helmholtz equations

\[
\left( \Delta + \frac{c_P^2}{c_P^2} \right) \Phi' = 0, \quad \left( \Delta + \frac{c_S^2}{c_S^2} \right) \Psi' = 0.
\]  

To define plane waves and to simplify the analysis, the splitting spatial argument is needed

\[
x = (x \cdot n) n + (x \cdot v) v + (x \cdot w) w.
\]  

The further assumption relates to the periodicity of the potentials in the direction of propagation

\[
\Phi'(x) = \varphi(x^*) e^{x^*}, \quad \Psi'(x) = \psi(x^*) e^{x^*},
\]  

where the dimensionless complex coordinates \( x' \) and \( x'' \) are

\[
x' = ir x \cdot n, \quad x'' = ir x \cdot v.
\]  

In (8) \( i = \sqrt{-1} \) and \( r \) is the wave number related to the wavelength \( l \) by

\[
r = \frac{2\pi}{l}.
\]  

Substituting representations (7) into Eq. (5) results in the decoupled system of two ordinary differential equations

\[
\frac{d^2 \varphi}{dx'^2} + \left( 1 - \frac{c_P^2}{c_P^2} \right) \varphi = 0, \quad \frac{d^2 \psi}{dx''^2} + \left( 1 - \frac{c_S^2}{c_S^2} \right) \psi = 0,
\]  

where the phase speed \( c \) relates to the frequency and the wave number by the following relation

\[
c = \frac{\omega}{r}.
\]  

The boundary surface \( x \cdot v = 0 \) is assumed free from the surface tractions:

\[
t_v = \left( \lambda \text{tr} (\nabla u) I + \mu (\nabla u + \nabla u^T) \right) \cdot v = 0, \quad x \cdot v = 0.
\]  

Substitution representation (3) into boundary conditions (12) yields boundary conditions written in terms of potentials \( \Phi'' \) and \( \Psi'' \)

\[
\left( \lambda \Delta \Phi' I + 2\mu \left( \nabla \nabla \Phi' + \frac{1}{2} \left( \nabla \text{rot} \Psi' + (\nabla \text{rot} \Psi')^T \right) \right) \right) \cdot v = 0, \quad x \cdot v = 0.
\]  

Equation (13) is one, we are looking for; it describes propagation of Rayleigh waves along free surface of a halfspace/halfplane.

Equation (13) should be supplemented with equation of motion for the barrier, analogous to Eq. (1) and boundary conditions at the interface between barrier and soil. The ideal mechanical contact is imposed at the interface:
In the next section the FE approach for solving the considered equations will be developed, allowing us to analyse the interaction of Rayleigh waves with the vertical seismic barrier.

3. FE modeling of a system “soil-vertical barrier”

Herein, some results based on numerical modeling of seismic waves propagation as well as their interaction with vertical seismic barriers are presented. The given outcomes were carried out by the explicit FE code.

3.1. Basic Remarks

Our analyses showed that similarly to the horizontal barriers (Kuznetsov, 2011), the vertical barriers should satisfy several important conditions in order to defend the given area from seismic waves effectively: (i) height of the barrier should be comparable with the lengths of the waves which it protects from; (ii) material of the barrier should have larger Young’s module and density than the ambient soil has (iii).

3.2. 2D Model

In connection with the complexity of this problem, 2D model was used in order to simplify the subsequent studies. These are models consisting of a symmetric plate with sizes which were chosen lest the waves reflected from the boundaries of the model should return to the points of observation during the calculation time. The condition of symmetry (3) is applied on the left edge of the plate while, the lower and the right edges were fixed. The source of waves was simulated as a harmonic load (1) applied on the upper edge in the center of the plate (on the top of the axe of symmetry). Vertical barrier (2) was created at a distance from the axe of symmetry so that the wave picture might stabilize. Figure 1 represents the picture of wave propagation in the model.

Comparing the kinetic energy of a piece of the plate beyond the barrier with the energy of the same area without barrier provides us with the information on the efficiency of this barrier. The same comparison may be carried out with the magnitudes of displacement of the observation points behind the barrier.

![Figure 1](image_url)

**Figure 1.** Finite element model with a vertical round-shaped thin barrier filled in with a granular metamaterial

a) 3D model;  b) Cross section

Similarly, it can be shown by a corresponding finite element simulation of a horizontal barrier, that it also has the ability to prevent Rayleigh waves in entering the protecting zone. The concept of the
horizontal barrier utilizes Chadwick’s theorem on non-propagating Rayleigh waves in a clamped halfspace. In view of this theorem, the modeled horizontal barrier had either larger Young’s modulus than the halfspace, or larger density, or both. The latter case, as numerical computations reveal, appears the best in terms of reduction vibrations behind the barrier: Rayleigh waves are almost completely eliminated in the protecting zone.

Another view of 3D FE model of the round-shaped barrier used for seismic protection against Rayleigh waves, is given in figure. 2.

![3D model of round-shaped barrier](image)

![Cross-section of round-shaped barrier](image)

**Figure 2.** Round-shaped vertical barrier protecting from Rayleigh waves: a) 3D model; b) cross-section

Along with vertical barriers, horizontal barriers can also be considered, however, herein these types of seismic barriers are not studied. For analysis of horizontal barriers see [12], where horizontal barriers made of elastic materials are studied.

Similarly, there are ideas to use pile fields to scatter seismic Rayleigh waves [21 – 24]. The pile

### 3.3. Dimensional analysis

In accordance with the π-theorem (Gibbings, 2011) which states that physical law does not depend on the form of units, the kinetic energy field $E_{kin}^{bar}$ of an area $\Delta$ beyond the barrier can be described by the following group of dimensionless parameters:

$$E_{kin}^{bar} = \frac{E_{bar}}{E_{soil}}, \frac{\rho_{bar}}{\rho_{soil}}, \frac{d \times h}{\Delta}, \frac{d \times \Delta}{\lambda^2}, \frac{d \times \Delta}{\lambda}, \frac{\omega \lambda}{\sqrt{E_{soil} \rho_{soil}}}, v_{bar}, v_{soil},$$

(15)

where index $soil$ marks the ambient material of the half-space, while index $bar$ corresponds to the parameters of the barrier; $\lambda$ is the wavelength of the Rayleigh wave in a half-space (this wavelength can be solved from the Bergmann-Viktorov equation); $E_{bar}, E_{soil}$ are corresponding Young’s modules; $v_{bar}, v_{soil}$ are Poisson’s ratios; $\rho_{bar}, \rho_{soil}$ are the densities; $d$ and $h$ are the thickness and the height of the barrier accordingly; $\omega$ is the circular frequency of the exciting load (here it is equal to the wave circular frequency).

According to the analyses performed in (Kuznetsov, 2011) as well as this research, both Poisson’s ratios almost do not have the influence on the kinetic energy field of the area $\Delta$, therefore, we can eliminate both Poisson’s ratios. Apart from that, the frequencies of considered waves remain constant (because the applied harmonic load has a constant frequency). That is why the expression (15) can be simplified to the following:
\[
E_{\text{kin}}^{\text{bar}} \left( \frac{E_{\text{bar}}}{E_{\text{soil}}} ; \frac{\rho_{\text{bar}}}{\rho_{\text{soil}}} ; \frac{d \times h}{\lambda^2} ; \frac{d}{h} \right).
\]

For performing the dimensional analyses in similar problems, see also works (Kuznetsov, 2018a,b; Djeran-Maigre et al., 2014; Ilyasov et al. 2017).

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References
[1] Adam, M., & Estorff, O. 2005. Reduction of train-induced building vibrations by using open and filled trenches. *Computers and Structures*, vol. 83, pp. 11 – 24.
[2] Chadwick, P. & Borejko, P. 1994. Existence and uniqueness of Stoneley waves, *Geophys. J. Int.* 118. 279-284.
[3] Chadwick, P. & Smith, G.D. 1977. Foundations of the theory of surface waves in anisotropic elastic materials. *Adv. Appl. Mech.* 17. Pp. 303 – 376.
[4] Gibbings, J.C. 2011. Dimensional Analysis. Springer. ISBN 1-84996-316-9.
[5] Herle, V. 2006. Long-term performance of reinforced soil structures. *Proceedings of the 13. Danube-Conference on Geotechnical Engineering, Slovenian Geotechnical Society, Ljubljana,* Slovenia, Vol. 2, pp. 251-256.
[6] Jesmani, M., Fallahil, M.A. & Kashani, H.F. 2012. Effects of geometrical properties of rectangular trenches intended for passive isolation in sandy soils. *Earth Science Research;* Vol. 1, No. 2; pp. 137 – 151.
[7] Ketchart, K. & Wu, J.T.H. 2001. *Performance test for geosynthetic reinforced soil including effects of preloading.* Federal Highway Administration, McLean, VA, USA, Report No. FHWA-R-01-018.
[8] Kim, S.-H. and Das, M.P. 2012. Artificial seismic shadow zone by acoustic metamaterials. *Modern Physics Letters, B* 27(20).
[9] Kusakabe, O. Takemura, J. Takahashi, A.Izawa, J. and Shibayama, S. 2008. Physical modeling of seismic responses of underground structures. In: *Proceedings of the 12th International Conference of International Association for Computer Methods and Advances in Geomechanics* (Goa, India, 2008), pp. 1459–1474.
[10] Kuznetsov, S.V. 2009. A new principle for protection from seismic waves. In: *Performance-Based Design in Earthquake Geotechnical Engineering. International Conference, Tsukuba,* Japan, pp. 1-8. 82/2839.
[11] Kuznetsov, S.V. 2011. Seismic waves and seismic barriers. *Acoustical Physics, Vol. 57,* No. 3, pp. 420–426.
[12] Kuznetsov, S.V. and Nafasov, A.E. 2011. Horizontal acoustic barriers for protection from seismic waves. *Advances in Acoustics and Vibration, Volume 2011,* Article ID 150310, pp. 1-8, doi:10.1155/2011/150310.
[13] Kuznetsov, S.V. and Terentjeva, E.O. 2014. Plane inner Lamb problem: waves in the near epi zone by the vertical point source. *Acoustical Physics, Vol. 60,* No.4.
[14] Motamed, R., Itoh K., Hirose S., Takahashi A., & Kusakabe O. 2008. Evaluation of Wave Barriers on Ground Vibration Reduction through Numerical Modeling in ABAQUS, *Proceedings of SIMULIA Customer Conference, 2009,* London, UK, pp. 402-41.
[15] Takahashi, A., Takemura, J., & Shimodaira T. 2001. Seismic performance of reinforced earth wall with geogrid, *Proceedings of the 15th International Conference on Soil Mechanics and Geotechnical Engineering,* Istanbul, Turkey, pp. 1265-1268.
[16] Kuznetsov, S.V. 2018. Abnormality of the longitudinal Pochhammer–Chree waves in the vicinity of C2 phase speed. *JVC/Journal of Vibration and Control*, 24, 5642-5649.

[17] Kuznetsov, S.V. 2018. Lamb waves in functionally graded plates with transverse inhomogeneity. *Acta Mechanica*, 229, 4131-4139.

[18] Ilyashenko, A.V., and Kuznetsov, S.V. 2018. Pochhammer–Chree waves: polarization of the axially symmetric modes. *Archive of Applied Mechanics*, 88, 1385-1394.

[19] Djeran-Maigre, I., Kuznetsov, S.V. 2014. Velocities, dispersion, and energy of SH-waves in anisotropic laminated plates. *Acoustical Physics*, 60, 200-207.

[20] Ilyasov, K. K., Kravtsov, A. V., Kuznetsov, S. V., & Sekerzh-Zenkovich, S. Y. 2016. Exterior 3D Lamb problem: Harmonic load distributed over a surface. *Mechanics of Solids*, 51, 39-45.

[21] Aviles, J. and Sanchez-Sesma, F.J. 1983. Piles as barriers for elastic waves. *Journal of Geotechnical Engineering*, 109, 1133–1146.

[22] Kuznetsov, S.V. 1995. Elastic wave scattering in porous media. *Mechanics of Solids*, 30, 71-76.

[23] Tsai, P.-H., Feng, Z.-Y., and Jen, T.-L. 2008. Three-dimensional analysis of the screening effectiveness of hollow pile barriers for foundation-induced vertical vibration. *Computers and Geotechnics*, 35, 489–499.

[24] Huang, J. and Shi, Z. 2013. Attenuation zones of periodic pile barriers and its application in vibration reduction for plane waves. *Journal of Sound and Vibration*, 332, 4423–4439.

[25] Kuznetsov, S.V. 2014. Lamb waves in anisotropic plates (Review). *Acoust. Physics*, 60, 95-103.