Establishment of free surface related multiple migration method based on PSPI theory

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Abstract. Seismic exploration will be affected in many ways, leading to the phenomenon of missing seismic data. Conventional imaging cannot accurately reflect the structure of underground media. This paper is based on the data to data migration which purpose of this method is used to compensate for the missing seismic data. The illumination of the underground media is reduced by rarefying the shot gathers. The seismic data containing primaries and free-surface related multiples are employed in upward and downward extrapolation simultaneously. And the field data has been transformed into frequency domain. Finally, using cross-correlation imaging conditions, the shallow imaging effect obtained is consistent with conventional imaging, and the deep imaging can provide more abundant imaging information. This method has a high imaging accuracy and does not require multiples prediction processing, which greatly reduces the operation process and avoids the possible errors of multiples prediction. Meanwhile, it uses multiples imaging to enhance the processing capacity of underground structures and obtain more structural information. In the case of missing seismic data, the numerical simulation test shows that the data to data migration imaging method can play a key role in the imaging results.

1. Introduction
The high resolution seismic exploration of the target layer is helpful to obtain the accurate image of the underground structure. Most imaging methods of seismic data are based on the assumption that there is only primary in the original earthquake, which means that multiples are always suppressed and eliminated as noise in the conventional processing process[1]. However, the primary and multiples are both reflected from the underground layer, which also means that multiple contains the information about the reflective layer[2]. Due to the multiple reflections experienced at the underground interface, the longer travel of multiples also carries richer information about the underground medium. Therefore, the small propagation angle and long propagation path enable multiples to have wider coverage. For the same seismic record, multiples can make up for the imaging range and illumination coverage that cannot be reached by the primary[3]. Different from the traditional research on the suppression of multiples, the use of multiples has also become the focus of current high-resolution seismic exploration[4]. This paper starts from this aspect and uses the one-way wave theory of multiples migration imaging methods to supplement the underground illumination. To some extent, it can compensate for the seismic data imaging results under special conditions[5].
2. Methods

2.1. Basic principle of PSPI one-way wave migration imaging

At present, most of the reflected results measured by seismic exploration are longitudinal wave components. Therefore, the acoustic wave equation is used instead of the elastic wave equation to simulate the propagation process of corresponding seismic waves in underground media. This process can be simplified and convenient to simulate and calculate the propagation of seismic wave field. For the acoustic wave equation, the unknown density parameter in the observation results are assumed to be a constant value, so that the acoustic wave equation is simplified into a scalar wave equation. This equation describes the main physical characteristics of seismic wave propagation and can be used to simulate seismic waves with high precision. One-way wave migration mainly relies on depth recursion of the wave field to realize the numerical simulation of seismic wave propagation in complex media. The two-dimensional scalar wave equation has the following form:

\[
\frac{1}{v^2} \frac{\partial^2 p(x,z,t)}{\partial t^2} = \frac{\partial^2 p(x,z,t)}{\partial x^2} + \frac{\partial^2 p(x,z,t)}{\partial z^2}
\]

The \( p(x,z,t) \) in the scalar wave equation is represented by a two-dimensional Fourier series:

\[
p(x,z,t) = \sum_{k_x} \sum_{\omega} P(k_x,\omega,\omega) \exp(ik_x x + i\omega t)
\]

The analytical solution of the above equation is:

\[
P(k_x, z + \Delta z, \omega) = P(k_x, z, \omega) \exp(ik_x x + i\omega t)
\]

Where \( k_x \) and \( k_z \) are the horizontal wave numbers and vertical wave numbers respectively. When using seismic records to calculate the wave field extrapolation, \( k_z \) takes a positive value. Another way to express the analytical solution through the dispersion relationship:

\[
P(k_x, z + \Delta z, \omega) = P(k_x, z, \omega) \exp \left( i\omega \left[ \frac{k_x v}{\omega} - \sqrt{\left( \frac{k_x v}{\omega} \right)^2 + 1} \right] \Delta z \right)
\]

The analysis formula shows that the wavefield extrapolation of the one-way wave is equivalent to the phase shift in the frequency-wavenumber domain. However, the phase shift method can only adapt to the vertical velocity changes, and the description of lateral velocity change is no longer accurate. The image wave field is calculated by using the interpolation of the reference wavefield, which is phase shift-plus interpolation migration.

Two or more velocities are selected to interpolate the modulus and phase angles. The final wave field expression obtained from the interpolation results is

\[
P(x,z + \Delta z, \omega) = A \exp(i\theta)
\]

The summation of \( \omega \) is successively superimposed to obtain image result, which is shown in the formula:

\[
p(x,z + \Delta z, 0) = \left( \frac{1}{2\pi} \right)^2 \int_{-\infty}^{\infty} P(k_x, z + \Delta z, \omega) d\omega
\]

2.2. Multiple imaging based on PSPI method

Assuming that the explosive source is excited on the surface, the underground medium is affected by the wave field and can be expressed in the frequency domain as:

\[
D(z_0) = -X_0 S(z_0)
\]

\( D(z_0) \) represents the original seismic data and contains primary and multiples. \( X_0 \) is the real response matrix of the underground medium. \( S(z_0) \) represents the seismic wavelet triggered by the explosion.

At this time, if \( S(z_0) \) can be approximated by the simulated seismic wavelet \( W(z_0) \), the effective wave seismic data of downgoing wave field \( P(z_0) \) can be obtained by using the migration velocity
model and using the phase shift-plus interpolation method:

\[ P(z_0) = XW(z_0) \]  \hspace{1cm} (8)

Regarding \( D(z_0) \) as a virtual seismic source, which is received by the detector buried on the surface after one or more reflections through the relevant media during underground propagation. At this time, the received data volume is surface multiple:

\[ M(z_0) = -XD(z_0) \]  \hspace{1cm} (9)

Combine equation (8) and (9) to obtain a new source function:

\[ P(z_0) + M(z_0) = XW(z_0) + XD(z_0) \]  \hspace{1cm} (10)

Equation (10) can be approximated as the following parts:

\[ P(z_0) + M(z_0) \approx XW(z_0) \]  \hspace{1cm} (11)

\[ P(z_0) + M(z_0) \approx XD(z_0) \]  \hspace{1cm} (12)

Initially, the basic idea of imaging multiples is to separate the surface multiples from the original seismic data and use them. This means that the surface multiples need to be predicted, so it takes a certain amount of time and a considerable amount of calculation. Surface related multiples can also be used for imaging without prediction, which avoids the calculation errors and costly processes introduced in the prediction processing. For a seismic data containing primary and multiples, through the autocorrelation of the data, that is, in the PSPI imaging process, the seismic data is forwarded as a virtual seismic source, and also transmitted back as the wave field at the detection point. Finally, imaging conditions are applied. The results can be expressed as:

\[ \text{Image}(x, z) = \sum_{x_s} \sum_{z_s} D_{up}(x, z, \omega, x_s)D_{down}(x, z, \omega, x_s) \]

\[ = \sum_{x_s} \sum_{z_s} [P_{up}(x, z, \omega, x_s) + M_{up}(x, z, \omega, x_s)] [P_{down}(x, z, \omega, x_s) + M_{down}(x, z, \omega, x_s)] \]  \hspace{1cm} (13)

\[ \text{Image}(x, z) \] is the imaging point at \((x, z)\), \(s_x\) is the source location, and \(\omega\) is the frequency. The subscript of \text{up} and \text{down} represent the upward wave field and the downward wave field in the one-way wave, respectively. \(P\) and \(M\) respectively represent the primary and free surface related multiples. Since the free surface related multiples may generate by reflections on the surface, the multiples can be expanded as follows:

\[ M(x, z, \omega, x_s) = M^1(x, z, \omega, x_s) + M^2(x, z, \omega, x_s) + M^3(x, z, \omega, x_s) + \cdots \]  \hspace{1cm} (14)

Through the above equation, equation (13) can be further expanded into the following form:

\[ \text{Image}(x, z) = \sum_{x_s} \sum_{z_s} [P_{up}(x, z, \omega, x_s)D_{down}(x, z, \omega, x_s)] \]

\[ + \sum_{x_s} \sum_{z_s} [M^1_{up}(x, z, \omega, x_s)D_{down}(x, z, \omega, x_s)] \]

\[ + \sum_{x_s} \sum_{z_s} [M^2_{up}(x, z, \omega, x_s)D_{down}(x, z, \omega, x_s)] + \cdots \]  \hspace{1cm} (15)

Analyzing equation (15) can get the following results:

\[ \text{Image}(x, z) = \sum_{x_s} \sum_{z_s} [P_{up}(x, z, \omega, x_s)M^1_{down}(x, z, \omega, x_s) + M^1_{up}(x, z, \omega, x_s)M^2_{down}(x, z, \omega, x_s) + \cdots] \]

\[ + \sum_{x_s} \sum_{z_s} [P_{up}(x, z, \omega, x_s)M^2_{down}(x, z, \omega, x_s) + M^2_{up}(x, z, \omega, x_s)M^3_{down}(x, z, \omega, x_s) + \cdots] \]

\[ + \sum_{x_s} \sum_{z_s} [M^1_{up}(x, z, \omega, x_s)M^2_{down}(x, z, \omega, x_s) + M^2_{up}(x, z, \omega, x_s)M^3_{down}(x, z, \omega, x_s) + \cdots] \]  \hspace{1cm} (16)

In Equation 16, the first part is the accurate imaging result, and the second and third parts generate the deviation error. If the primary is regarded as the zero-order multiple, the first part can be summarized as the upward extrapolation of n-order multiples and the downward extrapolation of (n+1)-order multiples imaging. Similarly, the second part can be summarized as the error imaging caused by the cross-correlation of the n-order and (n+2)-order multiples during extrapolation. The
third part is the upward extrapolation of the n-order multiples and the downward extrapolation of the m-order multiples and \( m \leq n \), they cannot be imaged.

3. Results

As shown in figure 1, the Marmousi velocity model is adopted in this paper, and a water layer is added to simulate the common free-surface related multiples environment in the ocean. The model has 737 grid points in the horizontal direction and 950 grid points in the vertical direction. The horizontal and vertical grid spacing is 10m and 4m respectively. The shot point and the detection point are set 10m below the water surface, and the sampling interval is 6ms. The total recording time length was 12s.

The results of primary imaging is shown in figure 2(a) and data autocorrelation multiples migration imaging is shown in figure 2(b). It can be seen that the use of multiples for imaging can also provide relatively accurate imaging results, which means multiples imaging can accurately restore the underground medium.

According to the analysis of the two imaging results, the following images are obtained through partial magnification. As shown in figure 3, the partial magnifications above the Marmousi model are shown as (a) and (b) respectively. From this area, it can be seen that in shallow imaging, multiples can obtain the same imaging effect as the primary.

In the area at the bottom of the model, the imaging results is shown in figure 4. The imaging effects at (a) and (b) are different. At this time, the imaging of multiples at this point can more highlight the underground layered structure. The imaging results containing only primary can no longer perform true restoration imaging of the underground medium, and the information contained in primary cannot be fully displayed.
The original data was changed with the distance of shots from 20m to 40m, 80m and 160m. The result of thinning in shot gathers reduces the number of coverage of the underground medium, and the amount of seismic data obtained from the geophone decreases accordingly. On this basis, the imaging results of the seismic data after thinning is carried out, and corresponding imaging results are obtained. It is shown in the figure 5 to figure 7.
4 times thinning

Figure 6

8 times thinning

Figure 7

It can be seen from the observation that the imaging results with primary is difficult to restore the underground structure accurately, and the imaging effect produces huge errors. The imaging results containing multiples can overcome the influence of special conditions. Under the same conditions, multiples can better depict the real underground structure than primary, which is the imaging effect that primary cannot bring about.

4. Conclusion

1) For complex geological conditions, this method has better shallow imaging effect, improved resolution, and can provide consistent information with conventional imaging.

2) Even if part of the seismic data is missing, based on the use of primary and multiples imaging, the results can be compensated to a certain extent to improve the illumination and show more abundant structural information.

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