Lepton transverse polarization in the $B \to D l \nu_l$ decay due to the electromagnetic final state interaction.

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Abstract

The effect of lepton transverse polarization in the $B^0 \to D^- l^+ \nu_l$, $B^+ \to \bar{D}^0 l^+ \nu_l$ decays ($l = \tau, \mu$) is analyzed within the framework of Standard Model in the leading order of HQET. It is shown that the non-zero transverse polarization appears due to the electromagnetic final state interaction. The diagrams with intermediate $D$, $D^*$ mesons contributing to the non-vanishing $P_T$ are considered. Regarding only the contribution of these mesons, values of the $\tau$-lepton transverse polarization, averaged over the physical region, in the $B^0 \to D^- \tau^+ \nu_\tau$ and $B^+ \to \bar{D}^0 \tau^+ \nu_\tau$ decays are equal to $2.60 \cdot 10^{-3}$ and $-1.59 \cdot 10^{-3}$, correspondingly. In the case of muon decay modes the values of $\langle P_T \rangle$ are equal to $2.97 \cdot 10^{-4}$ and $-6.79 \cdot 10^{-4}$.

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1 Introduction

In spite of the remarkable phenomenological success of the Standard Model (SM) the problem of the $CP$-violation mechanism still remains unexplained. In SM the $CP$-violation appears due to the complexity of the CKM matrix; however, there is a set of models offering other mechanisms of $CP$-violation. For instance, the Weinberg three Higgs boson doublets model stands for one of the simplest SM extensions, where $CP$-violation appears due to the complex Higgs boson couplings to fermions [1]. The investigation of the $CP$-violation phenomenon will help us to understand its mechanism and hence, to clarify one of the fundamental problems of elementary particle physics.

The experimental observables sensitive to $CP$-violating effects are, for example, transverse lepton polarization in weak decays and $T$-odd correlation. Muon transverse polarization in the $K^+\rightarrow\mu^+\nu\pi^0$, $K^+\rightarrow\mu^+\nu\gamma$ processes is the object of intensive study by many theoretical and experimental groups. In some SM extensions a non-zero transverse muon polarization appears already at the tree level [2,3]. The SM contribution to lepton transverse polarization is equal to zero in the leading order, and this fact explains the smallness of the SM background. Final state interaction gives rise to non-zero $CP$-conserving contribution to $P_T$. In the $K^+\rightarrow\mu^+\nu\gamma$ decay the lepton transverse polarization appears at the one-loop level and varies in the range of $(0.0 - 1.1) \cdot 10^{-3}$ on the Dalitz plot. The $P_T$ value averaged over the physical region with the cut on photon energy $E_{\gamma} \geq 20$ MeV is equal to $4.76 \cdot 10^{-4}$ [2]. In the $K^+\rightarrow\mu^+\nu\pi^0$ decay the muon transverse polarization is of order $\sim 10^{-6}$ [4,5], and therefore this decay is rather effective to search for new physics effects. The measurement of muon transverse polarization in this process is carried out by the KEK-E246 experiment, where the following result is obtained [6]:

$$P_T = -0.0042 + 0.0049(\text{stat.}) + 0.0009(\text{syst.}).$$ (1)

This experimental result does not allow to state that the value of $P_T$ is stipulated by new physics effects. However, an increase of experimental accuracy is planned in the nearest future, which seems very promising from the point of $CP$-violation research.

Another experimental observable, suppressed in SM, is the $T$-odd correlation in the charged kaon decays [7] (the distribution of the decay width over the kinematical variable, which is the mixing product of the final particle momenta, for instance, $\vec{p}_\pi \cdot [\vec{p}_\mu \times \vec{q}]$ in the $K^+\rightarrow\pi^0\mu^+\nu\gamma$ decay). The small SM contribution to this observable can be explained by the same reason, as the SM contribution to the lepton transverse polarization in $K_{l2\gamma}$ decay. Here, new perspectives to search for $T$-odd contributions from new physics are connected with the OKA experiment [8], where it is planned to achieve $\sim 7 \cdot 10^5$ events for the $K_{l3\gamma}$ decay.

Aside from $K$-meson decays, it is possible to study the lepton transverse polarization in similar $B$-meson decays. It should be noted that the value of $P_T$ is especially sensitive to $CP$-violating Higgs boson Yukawa couplings in these decays. Obviously, in the case of $B\rightarrow D^{(*)}\tau\nu_\tau$, the value of transverse polarization, due to the complexity of these couplings is $(m_\ell m_\tau)/(m_s m_\mu) \sim 800$ times greater then $P_T$ in analogues $K\rightarrow\pi\mu\nu_\mu$ decay.

In [9,10,11] the effects of $CP$-violating transverse polarization of leptons in the decays $B\rightarrow D^{(*)}\ell\nu$ in various SM extensions are analyzed. From these studies it follows that the $\tau$-
lepton transverse polarization can have the values $P_T \leq 1$ in models with $CP$-violation in the Higgs sector [9,10], and $P_T \leq 0.26$ in the leptoquark models [11]. Thus, one can expect that the value of transverse polarization in various extensions of SM is rather large. However, to estimate the impact of new physics and perform the dedicated study of the polarization phenomenon it is necessary to carry out the calculation of SM contribution to this observable. In this paper we calculate the $CP$-conserving SM contribution to $P_T$ in the $B^+ \rightarrow \bar{D}^{0} l^+ \nu$ decays ($l = \tau, \mu$). For simplicity, the calculations are carried out in the framework of HQET in the leading approximation of $1/m_Q$ expansion. It is shown below that the value of transverse polarization is not equal to zero if and only if there is a non-zero phase shift at one-loop level and that, in turn, results in non-zero value of $P_T$. In our calculations we take into account only the ($D, D^*$) doublet contribution to the transverse polarization.

In the next section we discuss the matrix elements contributing to the polarization value. In section 3 the procedure of transverse polarization calculation is given. The last section contains results and discussion.

2 Matrix elements

The general form for the $\langle D(D^*)|V^\mu(A^\mu)|B \rangle$ matrix elements is as follows:

\[ \langle D(k)|V^\mu|B(p) \rangle = f_+(p^\mu + k^\mu) + f_-(p^\mu - k^\mu), \]
\[ \langle D(k)|A^\mu|B(p) \rangle = 0, \]
\[ \langle D^*(k, \epsilon)|V^\mu|B(p) \rangle = -i\epsilon^\mu_\alpha\epsilon_\beta^* k_\alpha p_\beta, \]
\[ \langle D^*(k, \epsilon)|A^\mu|B(p) \rangle = a_1(\epsilon^*)^\mu + a_2(\epsilon^* p)^\mu + a_3(\epsilon^* k)^\mu, \] (2)

where $V^\mu = \bar{b}\gamma^\mu c$ and $A^\mu = \bar{b}\gamma^\mu\gamma_5 c$.

The $\langle D(k)|A^\mu|B(p) \rangle$ matrix element is equal to zero, since it is impossible to construct the axial vector composed of two momenta available. In our calculations we use the following definition of the Levi-Civita tensor: $\epsilon^{0123} = 1$.

Estimates of transverse polarization are carried out in the leading order of HQET, i.e. under the assumption of $m_b, m_c \rightarrow \infty$. In this approximation the formfactors of the process are expressed in terms of the Isgur-Wise function $\xi(vv')$ [12,13] and, accordingly, expressions (2) can be rewritten as:

\[ \langle D(k)|V^\mu|B(p) \rangle = \frac{\xi(\omega)}{\sqrt{m_D m_B}}(m_D p^\mu + m_B k^\mu), \]
\[ \langle D(k)|A^\mu|B(p) \rangle = 0, \]
\[ \langle D^*(k, \epsilon)|V^\mu|B(p) \rangle = -i\frac{\xi(\omega)}{\sqrt{m_D m_B}}\epsilon^\mu_\alpha\epsilon_\beta^* k_\alpha p_\beta, \]
\[ \langle D^*(k, \epsilon)|A^\mu|B(p) \rangle = \frac{\xi(\omega)}{\sqrt{m_D m_B}}((m_B m_D + p k)\epsilon^* k^\mu - (\epsilon^* p)k^\mu), \] (3)
where $\omega = (pk)/(m_Dm_B)$. Except for the given matrix elements, it is necessary to take into account the matrix elements of the vector current between $D$ and $D^*$ ($D^*$ and $D$) states. In the framework of HQET the formfactors of these matrix elements are also expressed through Isgur-Wise function:

$$\langle D(p')|\bar{c}\gamma^\mu c|D(k)\rangle = \xi(\omega')(p'^\mu + k^\mu),$$
$$\langle D(p')|\bar{c}\gamma^\mu c|D^*(k, \epsilon)\rangle = -i\frac{\xi(\omega')}{m_D}e^{\mu\nu\alpha\beta}\epsilon_\nu p'_\alpha k_\beta,$$  \hspace{1cm} (4)

where $\omega' = (kp')/(m_Dm_B)$. The $D$ and $D^*$ mass difference is neglected in Eqs. (3) and (4) as it does not contribute to these matrix elements in the leading order of HQET.

The function $\xi$ is conventionally parameterized as:

$$\xi(\omega) = 1 - \rho^2(\omega - 1).$$  \hspace{1cm} (5)

In numerical estimates we use the value $\rho^2 = 0.94$ obtained in [14] within the framework of potential quark model. This result is in good agreement with the experimental data [13]. Different values can not drastically change the numerical results, since the kinematical area of the decay is quite narrow. In the $B \rightarrow D\tau\nu_\tau$ decay, $\omega$ varies from 1 to $(m_B^2 + m_D^2 - m_\rho^2)/2m_Bm_D = 1.43$, and in the $B \rightarrow D\mu\nu_\mu$ decay this value varies in the range of 1-1.59.

The $\langle D|J_{em}^\mu|D^*\rangle$ and $\langle D|J_{em}^\mu|D\rangle$ matrix elements, where $J_{em}^\mu$ is the electromagnetic current, are also required. It is possible to write down this operator as the sum of heavy and light quark components:

$$J_{em}^\mu = J^\mu_h + J^\mu_l.$$  \hspace{1cm} (6)

The matrix elements of heavy component of this electromagnetic current are expressed through Isgur-Wise function (4):

$$\langle D(p')|J^\mu_h|D(k)\rangle = -q_c\xi(\omega')(p'^\mu + k^\mu),$$
$$\langle D(p')|J^\mu_h|D^*(k, \epsilon)\rangle = iq_c\frac{\xi(\omega')}{m_D}e^{\mu\nu}\epsilon_\nu p'_\alpha k_\beta,$$  \hspace{1cm} (7)

where $q_c$ is $c$-quark charge. The matrix elements of $J^\mu_l$ have the form:

$$\langle D(p')|J^\mu_l|D(k)\rangle = q_lf^1_i(q^2)(p'^\mu + k^\mu),$$
$$\langle D(p')|J^\mu_l|D^*(k, \epsilon)\rangle = iq_lf^1_i(q^2)e^{\mu\nu}\epsilon_\nu p'_\alpha k_\beta,$$  \hspace{1cm} (8)

where $q_l$ is the light quark charge and $f^1_i(1,2)(0) = 1$. The constant $\beta$, evaluated in [15] is equal to 1.9 GeV$^{-1}$. In our calculations we use $q^2$-dependence of $f^1_i(q^2)$ formfactors ($i = 1, 2$), obtained under the assumption of dominant contribution of the $\omega$ and $\rho$-resonances to these formfactors [16]. Neglecting the $\omega$ and $\rho$ mesons mass difference the expressions for $f^1_i$ take the form:

$$f^1_i = \frac{1}{1 - \frac{q^2}{m_{\rho}^2}}, \hspace{1cm} i = 1, 2$$  \hspace{1cm} (9)

where $m_{\rho}$ is the $\rho$ meson mass.
# Lepton transverse polarization

The amplitude of \( B \rightarrow D(D^*)l^+\nu_l \) decay can be written as follows:

\[
M = \frac{G_F}{\sqrt{2}} V_{cb}^* \langle D|V^\mu - A^\mu|B \rangle \bar{u}(p_\nu)(1 + \gamma_5)\gamma_\mu v(p_l),
\]

where \( G_F \) is the Fermi constant and \( V_{cb}^* \) is the corresponding CKM matrix element. Matrix elements \( \langle D|V^\mu - A^\mu|B \rangle \) are discussed in the previous section. For the case of the \( B \rightarrow Dl\nu_l \) process it is convenient to introduce the following parameterization of the amplitude:

\[
M = \frac{G_F}{\sqrt{2}} V_{cb}^* \bar{u}(p_\nu)(1 + \gamma_5)(C_1 \hat{\rho} + C_2)v(p_l).
\]

It should be noted that Eq. (11) is the most general form of the decay amplitude. The expressions for \( C_1, C_2 \) in the leading order of HQET can be written as follows:

\[
C_1 = \frac{\xi(\omega)}{\sqrt{m_B m_D}}(m_D + m_B), \quad C_2 = \frac{\xi(\omega)}{\sqrt{m_B m_D}}(m_B m_t).
\]

The partial width of the \( B \rightarrow Dl^+\nu_l \) decay in the \( B \)-meson rest frame can be expressed as:

\[
d\Gamma = \frac{|M|^2}{2m_B} (2\pi)^4 \delta(p - p_D - p_l - p_\nu) \frac{d^3p_D}{(2\pi)^32E_D} \frac{d^3p_l}{(2\pi)^32E_l} \frac{d^3p_\nu}{(2\pi)^32E_\nu},
\]

where summation over lepton and photon spin states is performed.

Introducing the unit vector along the muon spin direction in lepton rest frame, \( \hat{s} \), where \( \hat{e}_i \ (i = L, N, T) \) are the unit vectors along the longitudinal, normal, and transverse components of lepton polarization, one can write down the matrix element squared for the transition into the particular lepton polarization state in the following form:

\[
|M|^2 = \rho_0[1 + (P_L\hat{e}_L + P_N\hat{e}_N + P_T\hat{e}_T) \cdot \hat{s}],
\]

where \( \rho_0 \) is the Dalitz plot probability density averaged over polarization states. The unit vectors \( \hat{e}_i \) can be expressed in terms of the three-momenta of final particles:

\[
\hat{e}_L = \frac{\vec{p}_l}{|\vec{p}_l|}, \quad \hat{e}_N = \frac{\vec{p}_l \times (\vec{p}_D \times \vec{p}_l)}{|\vec{p}_l \times (\vec{p}_D \times \vec{p}_l)|}, \quad \hat{e}_T = \frac{\vec{p}_D \times \vec{p}_l}{|\vec{p}_D \times \vec{p}_l|}.
\]

With such definition of \( \hat{e}_i \) vectors, \( P_T, P_L, \) and \( P_N \) denote transverse, longitudinal, and normal components of the muon polarization, correspondingly.

The Dalitz plot probability density has the following form:

\[
\rho_0 = G_F^2 |V_{cb}|^2 (4|C_1|^2(\nu_\nu)(p\nu p_l) + 2|C_2|^2(\nu_\nu p_l) - 2|C_1|^2(\nu_\nu p_l)m_B^2 - 4m_\nu Re(C_2^*C_1)(p\nu_\nu)) \cdot
\]

\[
5
\]
The expression for transverse polarization can be written as follows:

\[
P_T = \frac{\rho_T}{\rho_0},
\]

where \( \rho_T \) has the form

\[
\rho_T = 4G_F^2|V_{cb}|^2 m_B \text{Im}(C_1 C_2^*) |\vec{p}_D \times \vec{p}_l|.
\]

Obviously, the lepton transverse polarization arises only in the case of nonzero phase shift between \( C_1 \) and \( C_2 \) formfactors. At the tree level of SM these formfactors are real and thus the lepton transverse polarization in this case is equal to zero. The non-vanishing \( P_T \) arises due to the effect of final state interaction. To calculate the imaginary parts of the formfactors one can use the S-matrix unitarity as it has been done in [16] for the case of the \( K^0 \to \pi \mu \nu \) decay.

The diagrams inducing the transverse lepton polarization are shown in Fig. 1. As it has been mentioned earlier, in our calculations we take into account only the diagrams with intermediate \( D, D^* \)-mesons. The contribution of these diagrams to the imaginary part of the decay amplitude can be written as follows:

\[
\text{Im}M = \frac{G_F}{\sqrt{2}} V_{cb}^* \alpha \int \frac{d\rho}{k^2} \bar{u}(p_l) (1 + \gamma_5) \gamma_\lambda (\bar{k}_l - m_l) \gamma_\mu v(p_l) \cdot \sum_{n=D,D^*} \langle D| J^\lambda_{em} |n \rangle \langle n | V^\sigma - A^\sigma | B \rangle,
\]

where \( k_l, k_D \) denotes the four-momenta of the intermediate \( D, D^* \) mesons, correspondingly, \( k_\gamma^2 = (k_D - p_D)^2 \) is the squared transferred momentum, \( d\rho \) is the two particle phase space. Expressions for the \( \langle D| J^\lambda_{em} |n \rangle, \langle n | V^\sigma - A^\sigma | B \rangle \) matrix elements are given in the previous section.

It should be noted that \( D \)-meson contribution to the imaginary part of the decay amplitude comprises an infrared divergence, but the latter doesn’t affect the value of transverse polarization. One may explain this fact by the factorization of soft photon contribution, which, in turn, does not lead to non-zero phase difference of formfactors required for a non-vanishing transverse polarization. So, we do not have to take into account divergent terms as it was done in [16]. As for the \( D^* \)-meson contribution it is free from infrared divergence, since in this case the lower bound of transferred momentum is:

\[
(-k_\gamma^2)_{\text{min}} = (m_{D^*}^2 - m_D^2) \frac{m_l}{m_l + m_D}.
\]

which is equal to 500 and 160 MeV for the \( \tau \)-lepton and muon cases, correspondingly. Taking into account this numerical values one can conclude that it is necessary to regard mass difference of \( D \) and \( D^* \)-mesons when integrating over the phase space.

Evidently, there are contributions to transverse polarization coming from excited \( D \)-meson states. But we suppose that this contribution can not change the result dramatically, since the Isgur-Wise function defined in (5) is greater than that in the case of \( B \)-meson decay into excited \( D \)-meson. The Isgur-Wise function for transition into \( (0^+, 1^+) \) doublet, \( \tau_{1/2}(\omega), \)

6
was calculated in [17]. The value of $\tau_{1/2}(1)$ obtained in this paper is equal to 0.24. One can see that the heavy quark contribution to $P_T$ is proportional to second power of $\tau_{1/2}(1)$ and the light quark contribution is linear in $\tau_{1/2}(1)$. Furthermore, the physical region, where the value of transverse polarization is not equal to zero is bounded by inequality $(p_t + p_D)^2 \geq (m_D + m_q)^2$. In the case of intermediate $D^*$-meson this restriction does not cut physical region significantly. Contrary to the case of intermediate state with $D^*$-meson, the physical region for the $(0^+, 1^+)$ case is strongly confined which results in the reduction of average transverse polarization.

We do not give the analytical expressions for imaginary parts of formfactors and integrals entering the imaginary part of one loop decay amplitude for their complexity.

4 Numerical results and discussion

It is convenient to use the following $x, y$ variables

$$E_D = \frac{m_B}{2} x, \quad E_\gamma = \frac{m_B}{2} y,$$

where $E_D$ and $E_\gamma$ are the $D$-meson and photon energies; $m_B$ is the $B$-meson mass.

The three-dimensional distributions of lepton transverse polarization in the kinematical region $(x, y)$ for the processes $B^0 \to D^- \tau \nu_\mu$, $B^0 \to D^- \mu \nu_\tau$, $B^+ \to D^0 \tau \nu_\mu$, and $B^+ \to D^0 \mu \nu_\tau$ are shown in Figs. 2, 4, 6, 8 correspondingly. The contour lines for $P_T$ in these decays are shown in Figs. 3, 5, 7, 9.

Corresponding values of transverse polarization averaged over the physical region are as follows

| Decay               | $\langle P_T \rangle$ |
|---------------------|------------------------|
| $B^+ \to D^0 \tau \nu_\mu$ | $-1.59 \cdot 10^{-3}$ |
| $B^0 \to D^- \mu \nu_\tau$ | $-6.79 \cdot 10^{-4}$ |

As it was mentioned earlier, the lepton transverse polarization can be represented as the sum of heavy and light quarks contributions.

Formfactors of heavy and light electromagnetic current defined in (7) and (8) have different scales. The heavy formfactors scale is of the order of $\sim m_D$ and the light formfactors scale is smaller and it is of the order of $\sim m_\rho$. Due to this fact and $\bar{D}^0$-meson neutrality one can state that the contribution to $P_T$ from the diagram with intermediate $\bar{D}^0$-meson in the $B^+ \to \bar{D}^0 l \nu$ decay is rather small in $k_\gamma^2 \ll m_\rho^2$ region and appreciably large in $k_\gamma^2 \gg m_\rho^2$ kinematical region.

The contribution to $P_T$ coming from the diagram with intermediate $D^*$-meson turns out to be larger than that one of the diagram with intermediate $D$-meson due to $\bar{D}^0$-meson.
neutrality. This fact explains the sign difference of averaged transverse polarization in the $B^+ \rightarrow D^0 l\nu$ and $B^0 \rightarrow D^- l\nu$ decays.

Finally, we would like to remark that the averaged values of $P_T$ in SM is quite small in comparison to some models predictions [9,10,11]. This allows one to conclude that $B$-meson decays provide appealing possibility to search for new physics effects.

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Figure captions

**Fig. 1.** Feynman diagrams contributing to lepton transverse polarization in $B^0 \rightarrow D^- l \nu_l$ decay at one-loop level of SM.

**Fig. 2.** The 3D plot for $\tau$-lepton transverse polarization for the case of $B^0 \rightarrow D^- \tau \nu_\tau$ decay.

**Fig. 3.** The level lines for $\tau$-lepton transverse polarization for the case of $B^0 \rightarrow D^- \tau \nu_\tau$ decay.

**Fig. 4.** The 3D plot for muon transverse polarization for the case of $B^0 \rightarrow D^- \mu \nu_\mu$ decay.

**Fig. 5.** The level lines for muon transverse polarization for the case of $B^0 \rightarrow D^- \mu \nu_\mu$ decay.

**Fig. 6.** The 3D plot for $\tau$-lepton transverse polarization for the case of $B^+ \rightarrow \bar{D}^0 \tau \nu_\tau$ decay.

**Fig. 3.** The level lines for $\tau$-lepton transverse polarization for the case of $B^+ \rightarrow \bar{D}^0 \tau \nu_\tau$ decay.

**Fig. 4.** The 3D plot for muon transverse polarization for the case of $B^+ \rightarrow \bar{D}^0 \mu \nu_\mu$ decay.

**Fig. 5.** The level lines for muon transverse polarization for the case of $B^+ \rightarrow \bar{D}^0 \mu \nu_\mu$ decay.
Fig. 1

Fig. 2

$P_T \cdot 10^3$

Fig. 3
Fig. 4

Fig. 5
Fig. 6

Fig. 7
Fig. 8

Fig. 9