Ultrahigh spin thermopower and pure spin current in a single-molecule magnet

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Using the non-equilibrium Green's function (NEGF) formalism within the sequential regime, we studied ultrahigh spin thermopower and pure spin current in single-molecule magnet (SMM), which is attached to nonmagnetic metal wires with spin bias and angle (θ) between the easy axis of SMM and the spin orientation in the electrodes. A pure spin current can be generated by tuning the gate voltage and temperature difference with finite spin bias and the arbitrary angle except of \( \theta \approx \frac{3}{2} \pi \). In the linear regime, large thermopower can be obtained by modifying \( V_g \) and the angles (θ). These results are useful in fabricating and advantaging SMM devices based on spin caloritronics.

Studies on nanoscale thermoelectric devices have attracted much attention during the past a few years. It is well accepted that nanoscale materials may provide an opening for the thermoelectricity in meeting the challenge of being a sustainable energy source. Huge deviation from the Wiedemann-Franz law in the nanostructure materials makes new opportunities for investigating novel thermoelectric devices with high efficiency. Specially, spin caloritronics (spin Seebeck effect) was observed by Uchida et al. They found that the spin-polarized currents (I<sub>1</sub> and I<sub>2</sub>) can be induced by a temperature gradient and flow in opposite directions. These wonderful discoveries strongly promote research on new energy of thermoelectricity.

A single-molecule magnet (SMM) is a typical nanoscale material. In experiments, controlling the molecular spin and measuring thermopowers of molecule have been realised by using a scanning tunneling microscope. The spin-dependent transport properties, such as tunneling magnetoresistance (TMR) and spin Seebeck effect, were investigated in the sequential, cotunneling, and Kondo regimes using Wilson’s numerical renormalization group and quantum master equation. Many fantastic phenomena have been found in the experimental and theoretical studies, including negative differential conductance, Berry phase blockade, the magnetization of SMM controlled by spin-bias and thermal spin-transfer torque. A SMM in a single spin state is necessary for generating the pure spin current without the magnetic field or magnetic electrodes. Meanwhile, it implies that the system temperature is limited by the blocking temperature of SMM (T<sub>B</sub>). When the symmetry of spin in the leads is broken, the angle (θ) between the easy axis of SMM and the spin orientation in the electrodes will influence the transport properties in the SMM devices. Specially, spin-bias and this angle (θ) are important and crucial on thermoelectric effect.

In this paper, we theoretically investigate the thermoelectric effects of a sandwich structure of NM/SMM/NM with spin-bias and angles between the easy axis of SMM and the spin orientation in the electrodes. We show that, in this system, pure spin currents are observed even though the system temperature is higher than the blocking temperature due to the spin symmetry broken by spin bias. In the linear regime, both thermopower and figure of merit are dependent on the angle and spin bias. It’s worth noting that the angle plays a critical role on generating spin thermopowers. The figure of merit could tend to infinity by tuning the voltage gate at special angles, which implies that this system has an ultrahigh thermoelectric efficiency.
Results

Effective Hamiltonian. The general Hamiltonian is expressed as

\[ H = H_{\text{leads}} + H_{\text{SMM}} + H_{\text{F}} \]

in which

\[ H_{\text{leads}} = \sum_{m,L,R,s'\sigma=(+,-),k} \varepsilon_{m,s'\sigma} c_{m,s'\sigma}^\dagger c_{m,s'\sigma} \]

\[ H_{\text{SMM}} = (\varepsilon_0 - eV_g) \sum_{s\sigma,\ell=1} n_{s\sigma} + U_{1s} n_{1s} - K_2 S_L^2 - J_s S \]

\[ H_{\text{F}} = \sum_{s,k} \left[ \left( \cos \frac{\theta_s}{2} c_{s+,k}^\dagger - \sin \frac{\theta_s}{2} c_{s-,k}^\dagger \right) d_1^\dagger \right. \]

\[ + \left. \left( \sin \frac{\theta_s}{2} c_{s+,k}^\dagger + \cos \frac{\theta_s}{2} c_{s-,k}^\dagger \right) d_1 \right] + H.C. \]

\( H_{\text{leads}} \) describes the free electrons in two leads, with \( c_{m,s'\sigma}^\dagger (c_{m,s'\sigma}) \) being the creation (annihilation) operator for a continuous state in the \( mL \), \( R \) lead with the energy \( \varepsilon_{m,s'\sigma} \) and spin index \( s' = (+, -) \), which denotes spin-majority (spin-minority) electrons. In this paper, wideband approximation is adopted and the density of states of the leads does not depend on the energy of the two leads. The chemical potential of \( \alpha \) lead is defined as \( \mu_\alpha = \pm (eV_\alpha/2 + \delta_\sigma \varepsilon_{\alpha}/2) \) with \( \varepsilon_{\alpha} = \pm \mu_{\alpha}^{(L,R)}/2 \) and \( \delta_\sigma = +1 \) for \( s' = (+, -) \). \( V_c = V_L - V_R \) is the voltage and \( V_i = V_{L,R} \) is the spin voltage. \( P_{\alpha} \) denotes the polarization of \( \alpha \) lead and is defined as \( P_{\alpha} = (\rho_{++} - \rho_{--})/ (\rho_{++} + \rho_{--}) \). \( P_{\alpha} \) describes the molecular degrees of freedom, in which \( n_{s\sigma} = d_{s\sigma}^\dagger d_{s\sigma} \) is the creation (annihilation) operators for the LUMO, \( \epsilon_0 \) is the single-electron energy of the LUMO level, which is tuned by a gate voltage \( V_g \). \( U \) is the on-site Coulomb repulsion. \( J_s \) describes the Hund’s rule coupling between the giant spin \( S \) of SMM and the electron spin in the LUMO, and parameter \( K_2 \) is the easy-axis anisotropy of SMM. \( H_{\text{SMM}} \) describes the tunneling between the LUMO of SMM and the electrodes, and \( \theta_s \) denotes the angle between the spin orientation of lead- \( \alpha \) and the easy-axis of the SMM (as \( z \)-axis).

In the following, we turn to numerical calculations with parameters: \( S = 2, J = 0.1 \) meV, \( K_2 = 0.04 \) meV, \( U = 1.0 \) meV and \( \epsilon_0 = 0.2 \) meV. The tunneling parameters are set to \( \Gamma = \Gamma_L = \Gamma_R = 0.001 \) meV. The properties of the leads are set to \( P_{\alpha} = P_{\alpha,L} = 0. \) Conventionally, \( I_c \) and \( I_s \) are defined as charge current and spin current respectively, and we set \( e = k_B = h = 1 \) and the thermopower and current are scaled in the unit of \( b_0 = \frac{1}{2} \). We can find all of the thermopowers are symmetric about \( \theta = \pi \) because of the spatial symmetry of the sandwich structure.

Transport properties. First, we consider that the left electrode is nonmagnetic with \( V_g = 0.01 \) meV and \( V = 0.0 \) meV. The system temperature is lower than the anisotropy-induced energy barrier (\( 0.16 \) meV). Figure 1a and Figure 1b show \( I_c \) and \( I_s \) as function of \( \theta \) for different values of \( V_g \) with \( \Delta T = 0.0002 \) meV, respectively. In this case, we can find that \( I_c \) is almost ten times of \( I_s \). The maximum or minimum value of \( I_c \) and \( I_s \) depends on the gate voltage \( V_g \) and \( \theta \), but the positions of these extrema only depend on the \( V_g \). When \( \theta = \pi/2, 3\pi/2, \) \( I_c \) is exactly equal to zero due to the coefficient \( \Delta_{\theta/2} = 1/2 \), which leads \( I_c = I_s \). Moreover, we show \( I_c \) and \( I_s \) as a function of \( V_g \) with two types of \( \Delta T \) at \( \theta = 0 \) in Figure 1e. One can find that \( I_s \) is extremely sensitive to the temperature difference. However, \( I_c \) only has a little change. Conventionally, the Fermi-Dirac distributions of spin-up and spin-down electron in the electrode are different due to finite \( V_g \). The higher the temperature is, the less these differences are generated.

In Figure 1c and 1d, we show \( I_c \) and \( I_s \) as a function of \( T \) with different \( \theta \) at \( V_g = 0.1 \) meV and \( T = 0.02 \) meV, respectively. In this case, interesting phenomena can be observed. When \( \theta = 0, \pi, 2\pi, \) \( I_c \) first increases and then decreases with decreasing of \( \Delta T \), but it decreases monotonically when \( \theta = \pi/2, 3\pi/2 \). However, \( I_s \) changes monotonically with decreasing \( \Delta T \) and equals to zero at \( \theta = \pi/2, 3\pi/2 \).

Thermoelectric coefficients. Next, we focus on the thermopower phenomena in the linear response regime and assume spin-bias only exists at the left electrode. Figure 2a and Figure 2b display charge-Seebeck and spin-Seebeck coefficient as a function of \( \theta \) for different values of \( V_g \) respectively. At the point \( V_g = 0.1 \) meV, \( S_c \) and \( S_s \) are zero due to the same weight and the opposite transmission direction of the currents and energy carried by electrons and holes. At the special points \( \theta = 0, 2\pi, \) the contributions of the spin-up and spin-down electrons are the same weight leading to \( G_c = G_s \) and \( S_c = 0 \), but no matter spin bias exists or not, \( S_c \) and \( S_s \) reach the maximum values for each \( V_g \) when \( \theta = \pi \). In Figure 2c, we plot all transport coefficients as a function of \( \theta \) at \( V_g = 0.1 \) meV and find that zero \( k_c \) exists only under special conditions of \( V_g \) and \( \theta \) in Figure 2d. It is interesting that one can have ultrahigh spin thermopower in a single molecular magnetism through manipulating the angle \( \theta \) between the easy axis of SMM and the spin orientation of the electrodes, and also by tuning \( V_g \).

Thermoelectric coefficients have been investigated through solving the non-equilibrium Green’s function (NEGF) in detail. The general formulas are derived to calculate the currents which depend on the angle \( \theta \) between the easy axis of SMM and the spin orientation in the electrodes and spin bias. The spin bias destroys the SU(2) symmetry of electron-spin in nonmagnetic electrodes, which leads the fact that the angle \( \theta \) influences the redistributions of different spin currents. It is amazing that pure spin currents can be obtained by tuning \( V_g \) and \( \Delta T \) with a finite spin bias and arbitrary \( \theta \) except of \( \theta = \pi, \pi/2 \). In the linear regime, infinite figure of merit can be generated by tuning \( V_g \) at special angles \( \theta \) with spin bias. Specially, when the angle \( \theta \) is equal to zero or \( 2\pi \), spin thermopowers vanish identically even though spin bias exists. These phenomena may provide a new approach for the design of SMM devices based spin caloritronics.

Discussion

The details of the constituents of the currents at \( \theta = 0 \) are shown in Figure 1f. When \( V_g \) is equal to 0.1 meV, the lowest-energy states of the isolated SMM are four-fold degenerate: \( 5/2, 1/2 \) and \( 0, 0 \), and the second-lowest-level state is a double degeneracy: \( 1/2, 1/2 \). The energy level difference between the lowest and next-lowest levels is 0.0839 meV. The currents are mainly contributed by the transitions of \((3/2, 3/2) \rightarrow (0, 0)\) and \((1/2, 1/2) \rightarrow (0, 0)\). According to Eq. (9), we can approximate \( I_{1.5/2, 1.5/2} \rightarrow (0, 0, 0) \propto -f \left( \frac{0.0839 + V_g/2}{4} \right) \) and \( I_{1.3/2, 1.3/2} \rightarrow (0, 0, 0) \propto -f \left( \frac{0.0839}{T_L} \right) \). At the electron-hole symmetry point, \( I_{1.5/2, 1.5/2} \rightarrow (0, 0, 0) \) are only controlled by \( V_g \) and temperature \( T_L \).

However, \( I_{1.3/2, 1.3/2} \rightarrow (0, 0, 0) \) depend on the temperatures of the two leads, \( V_g \) and the energy-difference between the lowest and the second-lowest state. From Figure 1f, one can find that \( I_c \) is mainly contributed by \( I_{1.5/2, 1.5/2} \rightarrow (0, 0, 0) \), but \( I_s \) is decided by all transitions. It is amazing that pure spin currents can be obtained at high temperatures. In Figure 1g, it is clear that \( I_c \) first increases and then decreases with the increasing of the average temperature \( T \) at \( V_g = 0.1 \) meV.
Figure 1 | $I_c$ and $I_s$ as a function of $\theta$ for different $V_g$ and $\Delta T$ with parameters $S = 2$, $\epsilon_0 = 0.2$ meV, $J = 0.1$ meV, $K_2 = 0.04$ meV, $U = 1.0$ meV, $k_B = 1$, $P_L = P_R = 0$. In Fig (a) and (b), a tiny temperature difference is considered: $\Delta T = T_L - T_R = 0.0002$ meV and $T = 0.02$ meV. In the (c) and (d), $V_g$ is set to 0.1 meV and the average temperature is fixed: $T = 0.02$ meV. (e) shows the temperature difference influences on the $I_c$ and $I_s$ with $\theta = 0$. (f) displays the details of the constituents of the $I_c$ and $I_s$ with $\theta = 0$. (g) displays $I_c$ and $I_s$ as a function of $T$ for different $V_g$ with $T_L = 0.2$ meV and $V_g = 0.01$ meV at $\theta = 0$. (h) shows $I_c$ and $I_s$ as a function of $T_R$ for different $V_g$ with $T_L = 0.8$ meV and $V_g = 0.27$ meV at $\theta = 0$. Solid lines denote charge currents and dash dot lines mark spin currents in (g) and (h).
0.27 meV. But $I_g$ only has little change and is not equal to zero. Due to the spin splitting induced by spin bias, pure spin currents can be generated at arbitrary temperatures. In Figure 1h, the system temperature is five times as the anisotropy-induced energy barrier and pure spin currents can be obtained by increasing $V_{ld}$.

Interestingly, the numerical results show $k_v$ can be equal to zero with changing $V_g$ and $\theta$ in Figure 2d. It means that $Z_sT(Z_sT)$ may be infinite when $S_r(S_r)$ and $G_{a\sigma}(\sigma = \uparrow, \downarrow)$ are finite. The exact choices of $V_g$ and $\theta$ are related to the details of the system’s parameters. But it is necessary that $S_s$ must be larger than $S_r$. It is well-known that electrons move from high temperature to low temperature, and it is not related to electronic spin. However, the spin-up and spin-down electrons under the spin bias can move in the opposite direction, and it is not possible that there are non-zero amount for thermopower and electrical conductance when thermal conductivity $k_v$ is zero.

**Methods**

Non-equilibrium Hubbard Green function has been used to solve the thermoelectric transport in the sequential and linear response regime. The system Hamilton can be rewritten by the transition operator, i.e. $\delta_e = X^{a^\sigma} + \delta_\sigma X^a$ with $\delta_e = + 1 (-1)$ for $\sigma = \uparrow (\downarrow)$, $s_+ = X^{1^\sigma}$, $s_- = X^{1^\sigma}$, and $s_z = \frac{1}{2}(X^{1^\uparrow} - X^{1^\downarrow})$. For large spin, the operator can be expressed as:

\[
S^\pm = \sum_{m = -\infty}^{\infty} C^\pm_m Y^m + 1, m_s^\pm = \sum_{m = -\infty}^{\infty} C^\pm_m Y^m - 1, m_s = \sum_{m = -\infty}^{\infty} m Y^m.
\]

where $S$ is the spin quantum number, $Y^m = |S_m| < |S_n|$ and $C^\pm = \sqrt{(S \pm m + 1)(S \mp m)}$. Finally, the retarded Green Function is written as

\[
G^\sigma_{a\sigma} = \left(G_{a\sigma}^{\uparrow, \downarrow}\right)^{-1} - \Sigma_{\text{Keldysh}}^{-1}
\]

\[
G^\sigma_{a\sigma} = G_{a\sigma}^{\uparrow, \downarrow}, \Sigma_{\text{Keldysh}}^{\uparrow, \downarrow}
\]

Here, $\Sigma_{\text{Keldysh}}^{\uparrow, \downarrow}$ are the electron self-energy in the second-order approximation and the formulas for calculation are

\[
\Sigma_{\sigma \sigma'} = \sum_{s, s'} A_{s, s'}^{\sigma \sigma'} f_{s, s'}
\]

\[
\Sigma_{\text{Keldysh}}^{\sigma \sigma'} = i \sum_{s, s'} \Gamma_{s, s'} \left(A_{s, s'}^{\sigma \sigma'} + A_{s, s'}^{\sigma' \sigma}\right)
\]

where

Figure 2 | Here, we consider spin-bias only exists at the left lead. (a) and (b) show the $S_r$ and $S_s$ as a function of $\theta$ for different $V_g$ respectively. (c) displays the thermopowers as a function of $\theta$ with $V_g = -0.05$ meV at $T = 0.02$ meV. (d) shows conventional thermal conductance ($k_v$) as a function of $V_g$ and $\theta$ at $T = 0.02$ meV. The other parameters are chosen as same as that in Figure 1.
\[ f_{s\sigma}(e) = \left| \frac{\cos(e - e_0) / \hbar}{\pi T_\varphi} \right|^{1/2} \]

Here \( \mu \) is the chemical potential, \( e V_{12} / 2 + \delta e V_{34} / 2 \) is a chemical potential with \( e V_{12} = \mu - \mu_1 + \mu_2 - \mu_3 \), and \( e V_{34} = \mu - \mu_1 + \mu_2 - \mu_3 \). \( \delta e = 1 \) for \( \varphi = 1 \) (alignment) and \( \delta e = 0 \) for \( \varphi = 0 \) (nonalignment). The function is defined as \( f_{s\sigma}(e) = \left| \frac{\cos(e - e_0) / \hbar}{\pi T_\varphi} \right|^{1/2} \).

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Author contributions
K.Y. put forwards the idea and supervised the whole work. B.L. performed the numerical calculations and wrote the manuscript. B.L. and J.L. entered the discussions and analyzed the results. J.T.L. and J.H.G. made discussion on the referees’ comments and revised the manuscript. All authors reviewed the manuscript.

Additional information
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