Randall-Sundrum scenario from $D=5, \mathcal{N}=2$ gauged Yang-Mills/Einstein/tensor supergravity

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Abstract

In this paper, a new locally supersymmetric two brane Randall-Sundrum model is constructed. The construction starts from a $D=5, \mathcal{N}=2$ gauged Yang-Mills/Einstein/tensor supergravity theory with scalar manifold $\mathcal{M} = SO(1,1) \times SO(2,1)/SO(2)$ and gauge group $U(1)_R \times SO(2)$. Here, $U(1)_R$ is a subgroup of the $R$-symmetry group $SU(2)_R$ and $SO(2)$ is a subgroup of the isometry group of $\mathcal{M}$. Next, the $U(1)_R$ gauge coupling $g_R$ is replaced by $g_R \text{sgn}(x^5)$ and the fifth dimension is compactified on $S^1/\mathbb{Z}_2$. The conditions of local supersymmetry for the bulk plus brane system admit a Randall-Sundrum vacuum solution with constant scalars. This vacuum preserves $\mathcal{N}=2$ supersymmetry in the $AdS_5$ bulk and $\mathcal{N}=1$ supersymmetry on the Minkowski 3-branes.

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1 Introduction

The two brane Randall-Sundrum scenario provides a mechanism for solving the hierarchy problem. To see this, consider the original Randall-Sundrum model \[1\] in which five dimensional pure anti-de Sitter gravity is compactified on an orbifold \(S^1/\mathbb{Z}_2\).\(^1\) There are two 3-branes, one at each orbifold fixed point, with nonzero tension. The 3-brane at \(x^5 = 0\) has tension \(T(0)\) and the 3-brane at \(x^5 = \pi \rho\) has tension \(T(\pi \rho)\). These 3-branes may support \((3 + 1)\) dimensional field theories. The minimal action is

\[
S = S_{\text{bulk}} + S_{\text{brane}}
\]

\[
S_{\text{bulk}} = \int d^5x \ e^{- \frac{1}{2} M^3 R - \Lambda}
\]

\[
S_{\text{brane}} = - \int d^5x e^{(4)} \left[ T(0) \delta(x^5) + T(\pi \rho) \delta(x^5 - \pi \rho) \right]
\]

where \(M, R,\) and \(\Lambda\) are respectively the five dimensional Planck mass scale, five dimensional Ricci scalar, and bulk cosmological constant, \(e \equiv \det (e_{\tilde{\mu}}),\) and \(e = e^{(4)} e^5\) on a brane. A solution to the five dimensional vacuum Einstein equations

\[
R_{\tilde{\mu} \tilde{\nu}} - \frac{1}{2} g_{\tilde{\mu} \tilde{\nu}} R = -M^{-3} \left\{ g_{\tilde{\mu} \tilde{\nu}} \Lambda + g_{\tilde{\mu} \tilde{\nu}} \delta_{\tilde{\rho} \tilde{\sigma}} e^{(4)} e^{\tilde{\mu}} e^{\tilde{\nu}} \left[ T(0) \delta(x^5) + T(\pi \rho) \delta(x^5 - \pi \rho) \right] \right\}
\]

which preserves four dimensional Poincaré invariance is given by

\[
(ds)^2 = a^2(x^5) \eta_{\mu \nu} dx^\mu dx^\nu + (dx^5)^2
\]

where

\[
a(x^5) = e^{-k|x^5|}
\]

is the warp factor and \(\frac{1}{k}\) is the \(AdS_5\) curvature radius. This solution is valid provided \(T(0), T(\pi \rho),\) and \(\Lambda\) are related by

\[
T(0) = -T(\pi \rho) = -\frac{\Lambda}{k} = 6M^3 k.
\]

\(^1\)The \(D = 5\) spacetime manifold has coordinates \(x^\mu = (x^\nu, x^5),\) fünfbein \(e_{\tilde{\mu}},\) and metric \(g_{\tilde{\mu} \tilde{\nu}} = e_{\tilde{\mu}} e_{\tilde{\nu}} \eta_{\tilde{\mu} \tilde{\nu}}.\) Here \(\eta_{\tilde{\mu} \tilde{\nu}} = \text{diag}( -1, 1, 1, 1, 1), \tilde{m}, \tilde{n}, \tilde{\rho}, \ldots = 0, 1, \tilde{1}, \tilde{2}, \tilde{3}, \tilde{5},\) and \(\tilde{\mu}, \tilde{\nu}, \tilde{\rho}, \ldots = 0, 1, 2, 3, 5.\) \(S^1/\mathbb{Z}_2\) is tangent to \(x^5\) with \(\mathbb{Z}_2\) acting as \(x^5 \to -x^5.\) In the upstairs picture the orbifold is a circle \(S^1\) of radius \(\rho\) with \(-\pi \rho \leq x^5 \leq \pi \rho.\) Thus, there exist two hyperplanes (3-branes), one at \(x^5 = 0\) and the other at \(x^5 = \pi \rho,\) which are fixed under the \(\mathbb{Z}_2\) action. In the downstairs picture the orbifold is an interval \(0 \leq x^5 \leq \pi \rho\) with the 3-branes forming boundaries to the spacetime manifold.
The exponential warp factor can generate a large hierarchy of scales. The four dimensional reduced Planck scale $M_P$ can be expressed in terms of $M$ as follows

$$M_P^2 = M^3 \int_{-\pi \rho}^{\pi \rho} dx^5 a^2(x^5) = \frac{M^3}{k} \left( 1 - e^{-2\pi k \rho} \right). \quad (1.8)$$

The effective mass scales on the 3-branes at $x^5 = 0$ and $x^5 = \pi \rho$ are respectively $M_P$ and $M_P e^{-\pi k \rho}$. If the Standard Model fields live on the 3-brane at $x^5 = \pi \rho$, then $M_P e^{-\pi k \rho}$ may be associated with the electroweak scale.

Locally supersymmetric two brane Randall-Sundrum models can be constructed from $D = 5$, $\mathcal{N} = 2$ gauged supergravity theories. The first such models were constructed from $D = 5$, $\mathcal{N} = 2$ gauged pure supergravity theories. Subsequent examples include a model constructed from a $D = 5$, $\mathcal{N} = 2$ gauged Maxwell/Einstein supergravity theory and models constructed from various $D = 5$, $\mathcal{N} = 2$ gauged supergravity theories with hypermultiplets.

In this paper, a new locally supersymmetric two brane Randall-Sundrum model is constructed. The construction starts from a $D = 5$, $\mathcal{N} = 2$ gauged Yang-Mills/Einstein/tensor supergravity theory with gauge group $U(1)^R \times SO(2)$ and the fifth dimension is compactified on $S^1/\mathbb{Z}_2$. The modification of $g_R$ allows a $\mathbb{Z}_2$ invariant bulk theory to be constructed. However, due to the presence of the signum function, the supersymmetry variation of the bulk action vanishes everywhere except at the $\mathbb{Z}_2$ fixed points. The variation of the brane action cancels that of the bulk when the relations are satisfied. In these circumstances, the vacuum solution is given by with warp factor. This vacuum solution preserves $\mathcal{N} = 2$ supersymmetry in the bulk and $\mathcal{N} = 1$ supersymmetry on the branes.

This paper is organized as follows: The ‘starting point’ $D = 5$, $\mathcal{N} = 2$ gauged Yang-Mills/Einstein/tensor supergravity theory with gauge group $U(1)^R \times SO(2)$ is summarized in Section 2. In Section 3 a locally supersymmetric two brane Randall-Sundrum model is constructed from the starting point theory. The results are discussed in Section 4.

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2 The signum function $\text{sgn}(x^5)$ is $+1$ for $0 < x^5 < \pi \rho$ and $-1$ for $-\pi \rho < x^5 < 0$. It obeys $\partial_5 \text{sgn}(x^5) = 2[\delta(x^5) - \delta(x^5 - \pi \rho)].$
2 Starting point theory

This section summarizes the $D=5, \mathcal{N}=2$ gauged Yang-Mills/Einstein/tensor supergravity \cite{11, 12, 13} theory with $U(1)_R \times SO(2)$ gauge group \cite{12} which is taken as the starting point of the construction in Section 3. The theory describes $D=5, \mathcal{N}=2$ pure supergravity \cite{14} coupled to one vector multiplet and one self-dual tensor multiplet, with gauging of:

1. A $U(1)_R$ subgroup of the $R$-symmetry group $SU(2)_R$.
2. An $SO(2)$ subgroup of the isometry group $G = SO(2, 1) \times SO(1, 1)$ of the scalar field target manifold $\mathcal{M} = SO(1, 1) \times SO(2, 1)/SO(2)$.

The fields of the pure supergravity multiplet are the fünfbein $e_{\tilde{m}}^\mu$, an $SU(2)_R$ doublet of symplectic Majorana gravitini $\Psi^\mu_i$, and the graviphoton vector field $A^{0\mu}$.

The vector multiplet contains a vector field $A^{I}_{\mu}$, an $SU(2)_R$ doublet of symplectic Majorana spin-1/2 fermions $\lambda^i_\tilde{a}$, and a real scalar field $\phi$. Finally, the self-dual tensor multiplet contains a self-dual antisymmetric tensor field with real and imaginary parts $B_{\mu\nu}^2$ and $B_{\mu\nu}^3$, respectively, two $SU(2)_R$ doublets of symplectic Majorana spin-1/2 fermions $\lambda^{i\tilde{a}}$ and $\phi^\tilde{x}$, and two real scalar fields $\phi^2$ and $\phi^3$. Thus, the total field content is

$$\{e_{\tilde{m}}^\mu, \Psi^\mu_i, A^{I}_{\mu}, B_{\mu\nu}^M, \lambda^{i\tilde{a}}, \phi^\tilde{x}\} \quad (2.1)$$

where

$I, J, K, \ldots = 0, 1, \quad M, N, P, \ldots = 2, 3, \quad \tilde{a}, \tilde{b}, \tilde{c}, \ldots = 1, 2, 3, \quad \tilde{x}, \tilde{y}, \tilde{z}, \ldots = 1, 2, 3$.

It is convenient for notational purposes to define $\bar{I} \equiv (I, M)$.

The dreibein, metric, and $SO(3)$ spin connection on the manifold $\mathcal{M}$ are denoted by $f_\tilde{a}^\mu$, $g_{\tilde{x}\tilde{y}}$, and $\Omega_{\tilde{a}\tilde{b}}^\tilde{c}$, respectively. This manifold is embedded in a four dimensional ambient space with coordinates $\xi^{\bar{I}}(\phi^\tilde{x}, N)$, cubic norm

\footnote{The \textit{generic Jordan family} of $D=5, \mathcal{N}=2$ gauged Yang-Mills/Einstein/tensor supergravity theories is characterized by scalar manifolds of the form $\mathcal{M} = SO(1, 1) \times SO(n + 2m - 1, 1)/SO(n + 2m - 1)$, where $n$ and $m$ are respectively the number of vector and self-dual tensor multiplets.}

\footnote{The $SU(2)_R$ indices $i, j, k, \ldots = 1, 2$ are raised and lowered according to $X^i = \epsilon^{ij} X_j$, $X_i = X^j \epsilon_{ji}$ with $\epsilon^{ij}$ and $\epsilon_{ij}$ antisymmetric and $\epsilon^{12} = \epsilon_{12} = 1$.}
$N(\xi) = \left(\frac{2}{3}\right) \tilde{\xi} C_{ijK} \tilde{\xi}^i \tilde{\xi}^j \tilde{\xi}^K$, and metric $a_{ij} = -\frac{1}{2} \partial_i \ln N(\xi)$. $\mathcal{M}$ corresponds to the $N(\xi) = 1$ hypersurface

$$C_{ijK} h^i h^j h^K = 1 \quad \text{with} \quad h^i \equiv \sqrt{\frac{2}{3} \xi^i}_{|N=1}. \quad (2.2)$$

A basis $[12]$ can be chosen in which the nonvanishing $C_{ijK}$ are

$$C_{0i\bar{i}} = C_{i0\bar{i}} = C_{i\bar{i}0} \quad \text{with} \quad C_{011} = -C_{022} = -C_{033} = \frac{\sqrt{3}}{2}. \quad (2.3)$$

In this basis, the constraint $N(\xi) = 1$ is solved by

$$h^0 = \frac{1}{\sqrt{3} \|\phi\|^2}, \quad h^1 = \sqrt{\frac{2}{3}} \phi^1, \quad h^2 = \sqrt{\frac{2}{3}} \phi^2, \quad h^3 = \sqrt{\frac{2}{3}} \phi^3$$

with $\|\phi\|^2 \equiv (\phi^1)^2 - (\phi^2)^2 - (\phi^3)^2. \quad (2.4)$

Lowering the index of $h^i$ with $\tilde{a}_{ij} \equiv a_{ij}|_{N=1}$ yields $h_f \equiv \frac{1}{\sqrt{6}} \partial_i N(\xi)|_{N=1}$.

The fermions $\Psi^i_\mu$ and $\lambda^\dagger_{\mu}$ are $U(1)_R$ charged, whereas the fields $\phi^\dagger$, $\lambda^\dagger$, and $B^M_{\mu\nu}$ carry charge under $SO(2)$. Denoting the $U(1)_R$ and $SO(2)$ couplings by $g_R$ and $g$, respectively, the $U(1)_R \times SO(2)$ gauge covariant derivatives of these fields are

$$\mathfrak{D}_\mu \Psi^i_\mu \equiv \nabla_\mu \Psi^i_\mu + g_R A^I_\mu \mathcal{P}^i_\mu \Psi^i_\mu \quad (2.5)$$

$$\mathfrak{D}_\mu \lambda^\dagger_{\mu} \equiv \nabla_\mu \lambda^\dagger_{\mu} + g_R A^I_\mu \mathcal{P}^i_\mu \lambda^\dagger_{\mu} + g A^I_\mu L^\dagger_{\mu\nu} \lambda^\dagger_{\nu} \quad (2.6)$$

$$\mathfrak{D}_\mu \phi^\dagger \equiv \partial_\mu \phi^\dagger + g A^I_\mu K^\dagger_{\mu} \quad (2.7)$$

$$\mathfrak{D}_\mu B^M_{\nu\rho} \equiv \nabla_\mu B^M_{\nu\rho} + g A^I_\mu \Lambda^M_{\mu\nu\rho} B^N_{\nu\rho} \quad (2.8).$$

Here, $\nabla_\mu$ is the spacetime covariant derivative. $K^\dagger$ are the Killing vectors on $\mathcal{M}$ that generate $SO(2)$. $L^\dagger_{\mu\nu} \equiv \partial_\mu K^\dagger_{\nu}$ and $\Lambda^M_{\mu\nu\rho} = \frac{2}{\sqrt{6}} \Omega^{MP} C_{\mu PN}$ are the $SO(2)$ transformation matrices of $\lambda^\dagger$ and $B^M_{\mu\nu}$, respectively.$^5$ The $SU(2)_R$ valued prepotentials $\mathcal{P}^i_\mu$ are chosen along the $\sigma_3$ direction, $^6$ i.e.

$$\mathcal{P}^i_\mu = V_I Q^i_J \quad \text{with} \quad Q^i_j \equiv i(\sigma_3)^j. \quad (2.9)$$

$^5$ $\Omega_{MN} \Omega^{NP} = \delta^P_M$, $\Omega_{MN} = -\Omega_{NM}$, and $\Omega^{23} = -\Omega^{32} = -1$.

$^6$ Note that in $[12]$, the $\sigma_2$ direction is chosen. Gaugings of different $U(1)_R$ subgroups of $SU(2)_R$ can be rotated into each other and hence are physically equivalent.
The $V_I$ are real constants that define the linear combination of the vector fields $A^I_\mu$ that is used as the $U(1)_R$ gauge field

$$A_\mu [U(1)_R] = V_I A^I_\mu. \quad (2.10)$$

The Abelian field strengths $F^I_{\mu\nu} = \partial_\mu A^I_\nu - \partial_\nu A^I_\mu$ and the $B^M_{\mu\nu}$ are combined to form $H^I_{\mu\nu} \equiv \langle F^I_{\mu\nu}, B^M_{\mu\nu} \rangle$.

The Lagrangian is (up to 4-fermion terms)

$$e^{-1} \mathcal{L} = -\frac{1}{2} \mathcal{R} - \frac{1}{2} \bar{\Psi}^I \gamma_\mu \tilde{D}^\mu \Psi^I - \frac{1}{4} \bar{\psi}_I \gamma_{ij} H^I_{\mu\nu} H^j_{\mu\nu}
- \frac{1}{2} \bar{\chi}^I \left( \Gamma_\mu \bar{\psi}_I \gamma_\mu - \Omega_{\mu}^{ij} \gamma_\mu \bar{\phi}_I^j \right) \lambda^I_{\alpha} - \frac{1}{2} \bar{\phi}_I \left( \gamma_\mu \phi^I \right) \tilde{D}^\mu \phi^I
-\frac{i}{2} \bar{\chi}^I \gamma_\mu \bar{\psi}_I \gamma^\mu \phi^I + \frac{1}{4} \bar{\phi}_I \gamma_I \gamma^\mu \bar{\phi}_I H^I_{\mu\nu}
+ \frac{i}{2\sqrt{6}} \left( \frac{1}{4} \delta_{\alpha\beta} h_I \phi^I \right) \bar{\lambda}_{\alpha}^I \gamma_\mu \bar{\psi}_I^\nu H^I_{\mu\nu}
- \frac{3i}{8\sqrt{6}} h_I \left( \bar{\Psi}^I \gamma_\mu \bar{\psi}_I \gamma_\mu H^I_{\mu\nu} + 2\bar{\psi}_I \gamma_\mu \bar{\psi}_I H^I_{\mu\nu} \right)
+ \frac{e^{-1}}{6} C_{IJK} e^{\mu\nu\rho\lambda} \bar{\psi}_I \gamma_\mu F^J_{\rho\lambda} \gamma_\nu \bar{\psi}_K + \frac{e^{-1}}{4g} e^{\mu\nu\rho\lambda} \lambda^I_{\alpha} \Omega_{MN} B^M_{\mu\nu} \bar{\psi}_I^\nu \bar{\psi}_K^\rho \gamma_\lambda
+ g \bar{\lambda}_{\alpha}^I \gamma_\mu \bar{\psi}_I W^\alpha + g \bar{\chi}^I \gamma_\mu \bar{\lambda}^\nu W^\nu
+ \frac{i\sqrt{6}}{4} g_r \bar{\psi}_I \gamma_\mu \gamma^\mu \bar{\psi}_I \gamma^\nu \bar{\psi}_J + g_R \bar{\lambda}^I_{\alpha} \gamma_\mu \bar{\psi}_I \gamma^\nu \bar{\psi}_J - \frac{i}{2\sqrt{6}} g_r \bar{\chi}^I \gamma^\mu \bar{\psi}_I \gamma^\nu \bar{\psi}_J P_{\alpha \beta \gamma \delta}
- g^2 P - g_R^2 P^R \quad (2.11)$$

and the transformation laws are (to leading order in fermion fields)

$$\delta e^I_\mu = \frac{1}{2} \bar{e}^I \tilde{D}^\mu \Psi^I, \quad (2.12)$$

$$\delta \psi^I = \bar{\psi}^{I'} D_\mu \xi^I + \frac{i}{4\sqrt{6}} h_I \left( \Gamma_\mu \bar{\psi}^{I'} - 4 \delta_{\alpha}^I \gamma_\mu \bar{\phi}^I \right) H^I_{\mu\nu} \xi^\nu + \frac{i}{\sqrt{6}} g_r \bar{\psi}_I \gamma^\nu \bar{\psi}_J \xi^j \quad (2.13)$$

$$\delta A^I_\mu = \phi^I_\mu \quad (2.14)$$

$$\delta B^M_{\mu\nu} = 2 \bar{\psi}^I_{\alpha} \gamma^M_\mu \gamma^\nu \xi^I + \frac{\sqrt{6}g}{4} \Omega_{MN} \bar{\psi}^I \gamma_\mu \xi^I + \frac{i\sqrt{6}}{4} \Omega_{MN} h_N \bar{\lambda}^I \gamma_\mu \xi^I \quad (2.15)$$

$$\delta \lambda^I_{\alpha} = -\frac{i}{2} \bar{\psi}^I_{\alpha} \left( \bar{\psi} \gamma^\mu \right) \xi^I + \frac{1}{4} h_I \bar{\lambda}^\nu \gamma_\mu \xi^I H^I_{\mu\nu} + g W^\alpha \xi^I + g_r \bar{\psi}^I \gamma^\nu \xi^j \quad (2.16)$$

$$\delta \phi^I = \frac{i}{2} \bar{\psi}^I \tilde{D}^\mu \lambda^I \quad (2.17)$$
with
\[ \vartheta^i_a = -\frac{1}{2} h^i_a \xi^a \Gamma_{\mu} \lambda^i + \frac{i\sqrt{6}}{4} h^i \bar{\psi}_i \xi^i, \] (2.18)

The scalar field dependent quantities \( h^i, h^i_a, h^i \), \( T_{a\bar{b}c} \), and \( \delta_{i,i} \) are subject to the following constraints imposed by supersymmetry [15]:
\begin{align*}
C_{i\bar{j}k} &= \frac{5}{2} h^i h_j h^k - \frac{3}{2} \delta_{ij} h^k h^l + T_{\bar{z}\bar{y} z} h_j^\bar{z} h^\bar{y} h^z, \\
h^i h^i &= 1, \quad h^i h^i = h^i h^i h^i = 0, \quad h^i h^j \delta_{ij} = g_{\bar{z}y}, \\
\delta_{ij} &= h^i h^j + h^2 h^2 g_{\bar{z}y}, \\
h^i, h^i &= \sqrt{\frac{2}{3}} h^i, \quad h^i, h^i = -\sqrt{\frac{2}{3}} h^i, \\
h^i, h^i &= \sqrt{\frac{2}{3}} (g_{\bar{z}y} h^i + T_{\bar{z}\bar{y} z} h^\bar{z}), \quad h^i, h^i = -\sqrt{\frac{2}{3}} (g_{\bar{z}y} h^i + T_{\bar{z}\bar{y} z} h^\bar{z}).
\end{align*}

The presence of the tensor fields introduces the terms proportional to
\[ W^a = -\frac{\sqrt{6}}{8} h^a M N h_N \] (2.19)
\[ W^{\bar{a}b} = i h^j [\bar{a} \bar{b}] K_{j} + \frac{i\sqrt{6}}{4} h^j K_{j} [\bar{a} \bar{b}] \] (2.20)
and leads to the scalar potential contribution
\[ g^2 P = 2W^a W^a. \] (2.21)

The gauging of \( U(1)_R \) introduces the terms proportional to
\[ P_{ij} \equiv h^j Q_{ij}, \] (2.22)
\[ P_{a\bar{a}} \equiv h^a Q_{ij}, \] (2.23)
\[ P_{\bar{a}a} \equiv 4T_{a\bar{b}c} P^{(R)}_{ij} \] (2.24)
and leads to the scalar potential contribution
\[ g^2 P^{(R)} = g^2 \left( -2P_{ij} P^{ij} + P_{a\bar{a}} P^{(R)}_{ij} \right) \] (2.25)
\[ = g^2 \left( -6W^2 + \frac{9}{2} W_{a \bar{a}} W \bar{a} \right), \] where \( W \equiv \sqrt{\frac{3}{2}} h^j Q_{ij} \) (2.26)

The total scalar potential is
\[ V = g^2 P + g^2 P^{(R)}. \] (2.27)
3 Construction of model

In this section, a locally supersymmetric two brane Randall-Sundrum model is constructed. The starting point is the $D = 5, \mathcal{N} = 2$ gauged Yang-Mills/Einstein supergravity theory with gauge group $U(1)_R \times SO(2)$ which is summarized in Section 2. Next, the $U(1)_R$ gauge coupling $g_R$ is replaced by $g_R \text{sgn}(x^5)$ and the fifth dimension is compactified on $S^1/\mathbb{Z}_2$. The modification of $g_R$ allows a $\mathbb{Z}_2$ invariant bulk theory to be constructed. In this framework, the $\mathbb{Z}_2$ symmetry does not commute with the $SU(2)_R$ R-symmetry and gauging different $U(1)_R$ results in physically different theories.

The $\mathbb{Z}_2$ action on each field is defined by boundary conditions at the $\mathbb{Z}_2$ fixed points. These boundary conditions must be chosen such that the Lagrangian is $\mathbb{Z}_2$ invariant. A consistent set of boundary conditions is obtained as follows. Under reflections $x^5 \rightarrow -x^5$ about the $\mathbb{Z}_2$ fixed point $x^5 = 0$, choose the bosonic fields

\[ \Phi = e^m_{\mu}, e^5_{\mu}, A^I_{\mu}, B^M_{\mu}, \theta^5 \]
\[ \Theta = e^5_{\mu}, e^m_{\mu}, A^I_{\mu}, B^M_{\mu} \]

to satisfy

\[ \Phi(x^\mu, x^5) = +\Phi(x^\mu, -x^5) \] (3.1)
\[ \Theta(x^\mu, x^5) = -\Theta(x^\mu, -x^5) \] (3.2)

and choose the fermionic fields to satisfy

\[ \Gamma^5 \Psi^j_{\mu}(x^\mu, x^5) = +iQ^j_{\mu} \Psi^j_{\mu}(x^\mu, -x^5) \] (3.3)
\[ \Gamma^5 \Psi^j_{5}(x^\mu, x^5) = -iQ^j_{\mu} \Psi^j_{5}(x^\mu, -x^5) \] (3.4)
\[ \Gamma^5 \lambda^i(x^\mu, x^5) = -iQ^i_{\mu} \lambda^i(x^\mu, -x^5) \] (3.5)
\[ \Gamma^5 \varepsilon^i(x^\mu, x^5) = +iQ^i_{\mu} \varepsilon^i(x^\mu, -x^5) \] (3.6)

\[ A \text{ chiral basis is chosen for the Dirac matrices} \]
\[ \Gamma^m = (\Gamma^m, \Gamma^5) = \left( \begin{array}{cc} 0 & -i\sigma^m \\ -i\bar{\sigma}^m & 0 \end{array} \right), \left( \begin{array}{rr} -1 & 0 \\ 0 & 1 \end{array} \right) \]

where $\sigma^m = (1, \bar{\sigma})$ and $\bar{\sigma}^m = (1, -\bar{\sigma})$.

\[ \text{Note that the scalar potential} V = g^2 P + g_R^2 P^{(R)} \text{ is unchanged by} g_R \rightarrow g_R \text{sgn}(x^5) \text{ because sgn}^2(x^5) = 1. \]
where $Q_{ij}$ is given by (2.9). Under reflections $x^5 \to -x^5$ about the $Z_2$ fixed point $x^5 = \pi\rho$, choose the bosonic fields to satisfy
\[ \Phi(x^\mu, \pi\rho + x^5) = +\Phi(x^\mu, \pi\rho - x^5) \quad (3.7) \]
\[ \Theta(x^\mu, \pi\rho + x^5) = -\Theta(x^\mu, \pi\rho - x^5) \quad (3.8) \]
and choose the fermionic fields to satisfy
\[ \Gamma^5 \Psi_i(x^\mu, \pi\rho + x^5) = +iQ_{ij}\Psi_j(x^\mu, \pi\rho - x^5) \quad (3.9) \]
\[ \Gamma^5 \lambda^i(x^\mu, \pi\rho + x^5) = -iQ_{ij}\lambda^j(x^\mu, \pi\rho - x^5) \quad (3.10) \]
\[ \Gamma^5 \varepsilon^i(x^\mu, \pi\rho + x^5) = +iQ_{ij}\varepsilon^j(x^\mu, \pi\rho - x^5). \quad (3.11) \]

The bulk theory is now $Z_2$ invariant. However, due to the presence of the sign function, the supersymmetry variation of the bulk Lagrangian vanishes everywhere except at the $Z_2$ fixed points:
\[ \delta L_{\text{bulk}} = \delta(\Psi)L_{\text{bulk}} + \delta(\lambda)L_{\text{bulk}} \quad (3.13) \]
\[ \delta^{(\Phi)}L_{\text{bulk}} = -6g_RW e^{(4)}\frac{1}{2}\bar{\Psi}_i\Gamma^\mu \Gamma^5(iQ_{ij})\varepsilon^j [\delta(x^5) - \delta(x^5 - \pi\rho)] \quad (3.14) \]
\[ \delta^{(\lambda)}L_{\text{bulk}} = -6i g_RW \bar{\varepsilon} e^{(4)}\frac{1}{2}\bar{\lambda}^i\Gamma^5(iQ_{ij})\varepsilon^j [\delta(x^5) - \delta(x^5 - \pi\rho)]. \quad (3.15) \]

It follows from the boundary conditions (3.16) and (3.17) that
\[ \Gamma^5(iQ_{ij})\varepsilon^j(x^\mu, 0) = \varepsilon_i(x^\mu, 0) \quad (3.16) \]
\[ \Gamma^5(iQ_{ij})\varepsilon^j(x^\mu, \pi\rho) = \varepsilon_i(x^\mu, \pi\rho). \quad (3.17) \]

Thus,
\[ \delta^{(\Phi)}L_{\text{bulk}} = -6g_RW e^{(4)}\frac{1}{2}\bar{\Psi}_i\Gamma^\mu \varepsilon_i [\delta(x^5) - \delta(x^5 - \pi\rho)] \quad (3.18) \]
\[ \delta^{(\lambda)}L_{\text{bulk}} = -6i g_RW \bar{\varepsilon} e^{(4)}\frac{1}{2}\bar{\lambda}^i\varepsilon_i [\delta(x^5) - \delta(x^5 - \pi\rho)]. \quad (3.19) \]

The bulk variation (3.13) can be cancelled by the variation of
\[ L_{\text{brane}} = -e^{(4)}[T^{(0)}\delta(x^5) + T^{(\pi\rho)}\delta(x^5 - \pi\rho)] \]
with $T^{(0)} = -T^{(\pi\rho)} = 6g_RW$. \quad (3.20)
For supersymmetry to hold, the Killing spinor equations must be satisfied. As demonstrated below, these equations admit the vacuum solution

$$(ds)^2 = a^2(x^5)\eta_{\mu\nu}dx^\mu dx^\nu + (dx^5)^2 \quad \text{with} \quad a(x^5) = e^{-k|x^5|} \quad (3.21)$$

at a critical point $\phi_{\text{crit}}$ of the scalar potential in which

$$g_R\mathcal{W}_{\text{crit}} = \sqrt{-\Lambda/6} \equiv k. \quad (3.22)$$

The scalar potential has such a critical point given by

$$\left(\phi^1_{\text{crit}}\right)^3 = \sqrt{2/V_0/V_1}, \quad \phi^2_{\text{crit}} = \phi^3_{\text{crit}} = 0 \quad (3.23)$$

whenever $V_0V_1 > 0$. The associated cosmological constant is

$$\Lambda \equiv \langle V \rangle = -6g_R^2(\phi^1_{\text{crit}})^2(V_1)^2. \quad (3.24)$$

Taking (3.21) as an Ansatz, the Killing spinor equations are

$$0 = \langle \delta \Psi_{\mu i} \rangle = \partial_\mu \varepsilon_i + \frac{1}{2} \left( \frac{a'}{a} \right) \Gamma_\mu \Gamma_{5} \varepsilon_i - \frac{1}{2} \left[ -g_R \text{sgn}(x^5)\mathcal{W}_{\text{crit}} \right] \Gamma_\mu (iQ_{ij}) \varepsilon^j$$

$$0 = \langle \delta \Psi_{5i} \rangle = \varepsilon_i' - \frac{1}{2} \left[ -g_R \text{sgn}(x^5)\mathcal{W}_{\text{crit}} \right] \Gamma_5 (iQ_{ij}) \varepsilon^j$$

$$0 = \langle \delta \lambda_{\tilde{i}} \rangle = -i \frac{\Gamma^5 \phi_{\tilde{i}}' \varepsilon_i + gW_{\tilde{i}} \varepsilon_i + i \frac{3}{2} g_R \text{sgn}(x^5)\mathcal{W}_{\text{crit}} (iQ_{ij}) \varepsilon^j. \quad (3.27)$$

To solve these equations, split $\varepsilon_i$ into $\mathbb{Z}_2$ even ($\varepsilon_i^+$) and $\mathbb{Z}_2$ odd ($\varepsilon_i^-$) parts:

$$\varepsilon_i = \varepsilon_i^+ + \varepsilon_i^- \quad (3.25)$$

$$\varepsilon_i^+ = \frac{1}{2} \left[ \varepsilon_i \pm \Gamma_5 (iQ_{ij}) \varepsilon^j \right] = \pm \varepsilon_i \varepsilon_i^+ \quad (3.26)$$

Writing $\langle \delta \Psi_{5i} \rangle = 0$ in terms of $\varepsilon_i^\pm$ and using $a'/a = -g_R \text{sgn}(x^5)\mathcal{W}_{\text{crit}}$ yields

$$\varepsilon_i^{++} + \varepsilon_i^{--} - \frac{1}{2} \left( \frac{a'}{a} \right) (\varepsilon_i^+ - \varepsilon_i^-) = 0. \quad (3.27)$$

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The only nonvanishing components of the spin connection $\omega_{\mu}^{\tilde{m}\tilde{n}}$ are $\omega_{\mu}^{\tilde{m}5} = -\omega_{\mu}^{5\tilde{m}} = a' \delta_{\mu}^5$, where ' denotes partial differentiation with respect to $x^5$. Thus, $\nabla_{\mu} \varepsilon_i = \left( \partial_\mu + \frac{1}{2} \omega_{\mu}^{\tilde{m}\tilde{n}} \Gamma_{\tilde{m}\tilde{n}} \right) \varepsilon_i = \left[ \partial_\mu + \frac{1}{2} \left( \frac{a'}{a} \right) \Gamma_\mu \Gamma_5 \right] \varepsilon_i$ and $\nabla_{\tilde{5}} \varepsilon_i = \varepsilon_i'$. 

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[12] The only nonvanishing components of the spin connection $\omega_{\mu}^{\tilde{m}\tilde{n}}$ are $\omega_{\mu}^{\tilde{m}5} = -\omega_{\mu}^{5\tilde{m}} = a' \delta_{\mu}^5$, where ' denotes partial differentiation with respect to $x^5$. Thus, $\nabla_{\mu} \varepsilon_i = \left( \partial_\mu + \frac{1}{2} \omega_{\mu}^{\tilde{m}\tilde{n}} \Gamma_{\tilde{m}\tilde{n}} \right) \varepsilon_i = \left[ \partial_\mu + \frac{1}{2} \left( \frac{a'}{a} \right) \Gamma_\mu \Gamma_5 \right] \varepsilon_i$ and $\nabla_{\tilde{5}} \varepsilon_i = \varepsilon_i'$. 

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This determines the $x^5$ dependence of $\varepsilon_i^\pm$ to be

$$\varepsilon_i^\pm = a^{\mp \frac{1}{2}} \varepsilon_i^\pm(x^\mu).$$  (3.28)

Writing $\langle \delta \Psi_{\mu i} \rangle = 0$ in terms of $\varepsilon_i^\pm$ and using $a'/a = -g_R \text{sgn}(x^5)W_{\text{crit}}$ yields

$$\partial_\mu \varepsilon_i^+ + \partial_\mu \varepsilon_i^- + \left(\frac{a'}{a}\right) \Gamma_\mu \Gamma_5 \varepsilon_i^- = 0.$$

(3.29)

This determines that

$$\varepsilon_i^+(x^\mu) = \varepsilon_i^+(0), \quad \varepsilon_i^-(x^\mu) = \left(1 - \frac{a'}{a} x^\mu \Gamma_\mu \Gamma_5 \right) \varepsilon_i^-(0)$$

(3.30)

where $\varepsilon_i^{\pm(0)}$ are constant (projected) spinors. The Killing spinors are thus

$$\varepsilon_i = a^\frac{1}{2} \varepsilon_i^+(0) + a^{-\frac{1}{2}} \left(1 - \frac{a'}{a} x^\mu \Gamma_\mu \Gamma_5 \right) \varepsilon_i^-(0).$$

(3.31)

This solution corresponds to $\mathcal{N} = 2$ supersymmetry in the bulk. On the 3-branes (where the $\mathbb{Z}_2$ odd fields vanish) there is $\mathcal{N} = 1$ supersymmetry. No constraints on the Killing spinors arise from $\langle \delta \lambda_i \rangle = 0$.

4 Discussion

The model of Section 3 is one example of the locally supersymmetric two brane Randall-Sundrum scenarios which can be constructed from $D = 5$, $\mathcal{N} = 2$ gauged Yang-Mills/Einstein supergravity. Here, the construction starts from a $D = 5$, $\mathcal{N} = 2$ gauged Yang-Mills/Einstein supergravity theory with scalar manifold $\mathcal{M} = SO(1,1) \times SO(2,1)/SO(2)$ and gauge group $U(1)_R \times SO(2)$. Next, the $U(1)_R$ gauge coupling $g_R$ is replaced by $g_R \text{sgn}(x^5)$ and the fifth dimension is compactified on $S^1/\mathbb{Z}_2$. The conditions of local supersymmetry for the bulk plus brane system admit the vacuum solution \(^{[5,24]}\) and yield the relations $\mathcal{T}^{(0)} = -\mathcal{T}^{(\pi \rho)} = -\Lambda/k = 6k$. This vacuum preserves $\mathcal{N} = 2$ supersymmetry in the $AdS_5$ bulk and $\mathcal{N} = 1$ supersymmetry on the Minkowski 3-branes. It would be interesting to generalize this construction to the case in which the scalar manifold has the form described in Footnote 3. This is left to future work.
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