Single-nucleon removal cross sections on nucleon and nuclear targets

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The eikonal direct-reaction model, as used in spectroscopic studies of intermediate-energy nucleon-removal reactions on light target nuclei, is considered in the case of a proton target and applied to neutron removal from $^{29}$Ne at 240 MeV/nucleon. The computed cross sections and their sensitivities are compared using an earlier detailed analysis of carbon target data. The nuclear structure input, for the $^{29}$Ne ground-state and $^{29}$Ne final states, is that deduced from the carbon target analysis. The comparisons quantify the sensitivity of the two reactions to the angular momenta and binding energies of the active valence orbitals - showing the carbon target to be relatively more efficient for removals from weakly-bound, low-$\ell$, halo-like orbitals. Probing this sensitivity experimentally would provide useful tests of these predictions and of the model’s description of the reaction mechanism.

I. INTRODUCTION

The evolving shell structure and resulting spectroscopy seen in very neutron-rich nuclei in the region of the Island of Inversion [1] near $Z \approx 10$ is complex. Extensive studies in this demanding region of the chart of nuclides, see e.g. [2], have provided challenging tests of large-scale shell-model calculations and have stimulated and guided the development of improved shell-model effective interactions, e.g. Refs. [3–6]. Data sets and analyses of numerous intermediate-energy (80–250 MeV/nucleon) nucleon-removal reactions on both lead and light nuclear targets, such as carbon and beryllium, have proved highly effective in: (a) identifying the active valence single-particle orbitals near the neutron and proton Fermi surfaces, and (b) mapping their migration with $A$ and $Z$ in such nuclei far from stability. Developments in reaction targets and experimental capabilities, e.g. [7], have generated an increased interest in such fast, direct nucleon-removal (or knockout) reactions induced on a proton target. In this work, our focus is on direct-reaction model calculations for fast nucleon removal reactions from the projectile $^{28}$Ne on a proton target at a reaction energy of 240 MeV/nucleon. The $^{28}$Ne system is chosen: (a) as detailed comparisons are possible with an earlier analysis of removal reaction data on a carbon (and lead) target [8] performed at the same beam energy, and (b) $^{28}$Ne is rather typical if weakly-bound, neutron-rich systems in this mass/charge region, with occupancies of both sd-shell and pf-shell (intruder) valence orbitals.

As in the model treatment of reactions on light nuclear targets, usually carbon or beryllium, the basis of calculations is the use of: (i) the eikonal (forward scattering) and sudden (fast collision) approximations to the collision dynamics, combined with (ii) spectroscopic factors from shell-model wave function overlaps, which determine the strengths of the active configurations of the removed nucleons [8–10]. Importantly, the use of a proton target fundamentally alters the dominant direct reaction mechanism and thus the reaction inputs, parameter choices, and also, potentially, the resulting structural and parameter sensitivities.

II. DIRECT REACTION MODEL

In the eikonal-model description of inverse-kinematics single-nucleon removal reactions from a mass $A$ projectile incident on a nuclear target, removal events result from both elastic and inelastic interactions of the struck (removed) nucleon with the target. As the target final-states from these two removal mechanisms are distinct, if only the mass $A–1$ residual nucleus is detected, these two (incoherent) contributions to the single-particle removal cross section must be summed, i.e. $\sigma_{\text{el}} = \sigma_{\text{inel}} + \sigma_{\text{elas}}$. In this sudden, eikonal-model approach, details of which have been presented elsewhere [8], the S-matrices that enter the approximate expressions for $\sigma_{\text{inel}}$ and $\sigma_{\text{elas}}$ describe the effects of the complex optical-model interactions between the removed valence nucleon and the mass $A–1$ reaction residue with the target. These S-matrices account for the loss of flux of the fast, forward-traveling nucleon and residual nucleus due to scattering and absorption by these optical potentials. For reactions on a target of light composite nuclei, the calculated inelastic (also called stripping) mechanism is the more important, and $\sigma_{\text{inel}}$ dominates the removal cross section. These fractions of inelastic and elastic removal events, sensitive to the nucleon’s separation energy, have also been measured in several cases and shown to be very well described by the model calculations for reactions involving nucleon removal from both well-bound ($\approx 17$ MeV) and weakly-bound ($\lesssim 1$ MeV) valence orbitals [11, 12].

By contrast, in the case of a proton target: (i) the removed nucleon-target S-matrix describes the nucleon-nucleon (NN) system, and (ii) at the energies of interest here the proton is inert. The relative importance of the two reaction mechanisms is thus completely altered and the removal cross section is now determined by $\sigma_{\text{elas}}$. Thus, the use of a proton target also permits analyses
based on a number of alternative direct reaction model approaches that are suited to breakup studies - including the continuum-discretized coupled-channels (CDCC) methodology [15] and more recently developed coupled-channels approaches [14] to the (p, 2p) and (p, pn) processes. The CDCC approach was used successfully by Kondo et al. [15] in the case of neutron removal reactions from $^{18,19}$C on a proton target. Quasi-free, distorted-wave approximation (DWIA) nucleon knockout models have also been explored and applied with some success for a limited range of projectiles—primarily the oxygen and carbon isotopes—see for example Refs. [16–18]. Here, the effects of the change in the dominant reaction mechanism are studied using the more computationally efficient eikonal-model approach – that derives the angular momentum of the residue and removed nucleon. The residual nucleus momentum distributions from this model, on a proton target, were used successfully to identify the angular momenta of $^{28}$F final states in Ref. [19], but no comparisons of cross sections were available there. A similar model to that used here was applied to the $^{17}$C($n, n$) reaction at the low beam energy of 70 MeV/nucleon [20], but these calculations were not benchmarked against nuclear target data, the primary interest here. We compare the sensitivity of the modelled cross sections on a proton and nuclear target and quantify their differences, both as potential probes of the reaction mechanisms and for use in spectroscopy.

### A. Nucleon-nucleon description

The required NN S-matrix, describing the removed nucleon-target proton collisions, will be denoted $S_{jp}(b)$ where $j$ denotes the species of the removed nucleon, i.e. $j = n, p$. This NN scattering operator, expressed as a function of the impact parameter, $b$, of the NN relative motion, is conventionally written [21]

$$S_{jp}(b) = 1 - \Gamma_{jp}(b)$$

where $\Gamma_{jp}$, the NN profile function, is determined by the two-dimensional (2D) Fourier transform of the NN scattering amplitude $f_{jp}(q)$. That is

$$\Gamma_{jp}(b) = \frac{1}{2\pi k} \int d^2 q e^{-iq \cdot b} f_{jp}(q),$$

where $q$, the momentum transfer, is in the plane perpendicular to the beam direction. As is often used, the profile function is parameterized as

$$\Gamma_{jp}(b) = \frac{\sigma_{jp}}{2i} (i + \alpha_{jp}) g_2(\beta_{jp}, b)$$

where $g_2(\beta, b)$ is the normalized two-dimensional (2D) Gaussian form factor

$$g_2(\beta, b) = \frac{1}{2\pi \beta} \exp(-b^2/2\beta)$$

representing the finite-range of the NN interaction. Thus, from the inverse transform of Eq. (2)

$$f_{jp}(q) = \frac{k}{4\pi} \sigma_{jp} (i + \alpha_{jp}) \exp(-\beta_{jp} q^2/2)$$

and, from the optical theorem identity, namely

$$\text{Im.} f(0^+) = \text{Im.} f(q = 0) = \frac{k}{4\pi} \sigma_{tot},$$

one notes that the $\sigma_{jp}$ are the np and pp total cross sections. These are computed here from the Charaghi and Gupta parameterization [22] of the experimental NN data, giving $\sigma_{pp} = 22.02$ mb and $\sigma_{np} = 37.14$ mb at the relevant laboratory energy of 240 MeV/nucleon. Also clear from Eq. (5), the parameters $\alpha_{jp}$ are the ratios of the real to the imaginary parts of the NN forward-scattering amplitudes, $f_{jp}(q = 0)$, while $\beta_{jp}$ determines the range of the assumed Gaussian form factor (of Gaussian range $\gamma_{jp} = \sqrt{2/\beta_{jp}}$). Here, the $\alpha_{jp}$ are interpolated from the tabulation (on the interval 100–1000 MeV) of Ray [23], giving values $\alpha_{np} = 0.533$ and $\alpha_{pp} = 0.991$. For the range parameters $\beta_{jp}$, we follow Ref. [24] and dictate, since the NN interaction energy is below the pion production threshold and the NN scattering is entirely elastic, that the total and elastic cross sections derived from the NN S-matrices are equal. This requirement fixes the range parameter $\beta_{jp}$ such that

$$\beta_{jp} = \frac{\sigma_{jp} (1 + \alpha_{jp}^2)}{16\pi}$$

In section III B we assess the sensitivity of our calculated removal reaction cross-sections to this choice of $\alpha_{jp}$ (and hence $\beta_{jp}$). It will be shown there that this NN parameter sensitivity is actually very weak.

### B. Target proton-residue interactions

The remaining dynamical input to the removal cross section is the eikonal S-matrix that describes the interaction of the mass $A - 1$ reaction residue with the proton target. This is computed in the optical limit, or $t_p$ folding approximation to the proton-residue optical potential. This potential, and its S-matrix, includes the effects of the size and neutron-proton asymmetry of the residual nucleus ($r$) through its point-neutron and proton one-body densities $\rho_{np}^{(1)}$. The potential used is thus

$$U_{np}(R) = \sum_{j=n,p} \int dr \rho_{np}^{(1)}(r) t_{jp}(|R + r|),$$

where the NN effective interaction, $t_{jp}$, consistent with our treatment of the NN profile function and S-matrix given above, is

$$t_{jp}(r) = - \frac{\hbar v}{2} \sigma_{jp} (i + \alpha_{jp}) g_2(\beta_{jp}, r).$$

Here, $v$ is the residue-proton relative velocity, $g_2(\beta, r)$ is a normalized 3D Gaussian function with range parameter
\(\beta\), and the parameters \(\sigma_{jp}, \alpha_{jp}\) and \(\beta_{jp}\) are the same as were stated above.

The reaction residue one-body densities, \(\rho_i^{(j)}\), are computed using spherical Skyrme Hartree-Fock (HF) calculations using the Skyrme SkX interaction \([22]\). Such HF calculations have been shown to provide a very good global description of the root mean squared (rms) sizes \([20]\) and radial forms of the matter and charge distributions of both stable and neutron-proton asymmetric nuclei \([23]\).

### C. Removed-nucleon radial overlaps

In common with reactions on a composite target, the geometries of the neutron bound-states potentials, that generate the normalized single-particle overlaps of the removed nucleons in the projectile ground state, were constrained by Skyrme (SkX interaction) HF calculations. As discussed in some detail in Ref. \([28]\), for consistency with the range of the residue-target optical potential, determined by the \(\rho_i^{(j)}\) of the residue, the bound-states potential geometry is adjusted to reproduce the separation energy and the rms radius of each single-particle orbital as obtained using the HF. We generate Woods-Saxon binding potential geometries with reduced radius and diffuseness parameters \((r_0, a_0) = (r_0, 0.7)\) fm. A Thomas form spin-orbit potential with a depth of 6 MeV and the same geometry parameters is also included. For neutron removal from \(^{29}\)Ne, the so-constrained reduced radii \(r_0\) (for further details see Section III of Ref. \([28]\)) are 1.250 fm (\(\nu s_{1/2}, \nu p_{3/2}, \nu f_{7/2}\)) and 1.243 fm (\(\nu d_{5/2}, \pi d_{5/2}\)), as used in Ref. \([8]\), allowing detailed comparisons with the results of that analysis. Having determined these HF-constrained geometries, the bound-state form factors are computed using the empirical ground-state to ground-state separation energy and the excited shell-model final-state energies.

### D. Shell-model calculations

There are a number of large-basis shell-model calculations, using effective interactions developed for \(sdp\)-shell calculations in the mass/charge region of interest, e.g. (i) SDPF-M \([8]\), (ii) SDPF-U-MIX \([4,5]\) and (iii) SDPF-EKK \([6]\). In our methodology, the projectile and residual nucleus shell-model wave functions provide: (a) the spectra of the bound final states of the residues, and (b) from their overlaps, the spectroscopic factors \(C^2 S(\alpha, n\ell_j)\) between the projectile ground state and all bound residue final states \(\alpha = [E^\alpha, J^\alpha_\pi]\) that can be reached by direct removal of a single-neutron with quantum numbers \(n\ell_j\). Here, we will use only the results of the SDPF-M interaction \([8]\), that were used in the carbon and lead target analysis of Ref. \([8]\), and with which we can thus make detailed comparisons. These shell-model calculations included the full \(sd\)-shell plus \(1f_{7/2}, 2p_{3/2}\) states for neutrons while the protons are confined to the \(sd\)-shell. Further comparisons, using the predictions of the alternative shell-model interactions, will be profitable as experimental data becomes available.

### E. Cross-section calculations

Given the model framework and the inputs discussed above, single-particle removal cross sections, \(\sigma_{sp}\) – the cross sections computed assuming a normalized nucleon overlap function – can be computed to each bound-shell-model final state. As clarified earlier, with a proton target this is the elastic breakup (or diffraction dissociation) component \(\sigma_{elas}\), as given by Eq. (6) of Ref. \([4]\), there denoted \(\sigma_{sp}(\text{diff})\).

The contribution to the partial cross section for a given final state \(\alpha\), of excitation energy \(E^\alpha_n\) and spin-parity \(J^\alpha_n\), due to removal of a nucleon with single-particle quantum numbers \(n\ell_j\), is \([10]\)

\[
\sigma_{\ell^{-1}n}(\ell_j, S^{\alpha}_{\ell}) = \left(\frac{A}{A-1}\right)^N C^2 S(\alpha, n\ell_j) \sigma_{sp}(n\ell_j, S^{\alpha}_n)
\]

where \(\sigma_{sp}(n\ell_j, S^{\alpha}_n)\) is the single-particle cross section. Here, \(S^{\alpha}_n(\equiv S_n + E^\alpha_n)\) is the effective neutron separation energy to the final state \(\alpha\) with \(S_n\) the empirical ground-state to ground-state separation energy. \(N\), in the \(A\)-dependent center-of-mass correction to the shell model spectroscopic factor \(C^2 S(\alpha, n\ell_j)\), is the number of oscillator quanta associated with the major shell from which the neutron is removed \([29]\). Since the odd-\(A\), \(^{29}\)Ne initial state has non-zero spin, the total theoretical partial cross section to final state \(\alpha\), \(\sigma_{\ell^{-1}n}(\alpha)\), is the sum

\[
\sigma_{\ell^{-1}n}(\alpha) = \sum \sigma_{\ell^{-1}n}(\ell_j, S^{\alpha}_{\ell})
\]

of the \(\sigma_{\ell^{-1}n}(\ell_j, S^{\alpha}_{\ell})\), of Eq. (10), from all orbitals \(n\ell_j\) with a non-vanishing shell-model spectroscopic factor to that final state.

### III. RESULTS

The earlier carbon target removal reaction analysis of Ref. \([8]\) showed conclusively that the \(^{29}\)Ne ground-state is a deformed, weakly-bound \(p\)-wave neutron-halo system – also exhibiting a large Coulomb dissociation cross section on a lead target. The spin-parity of the \(^{29}\)Ne ground state was unambiguously determined to be \(3/2_-\) using the neutron removal cross sections on both targets and the momentum distribution of \(^{29}\)Ne for the carbon-target induced removal reaction. In particular, the narrow momentum distribution is typical of a \(p\)-wave neutron and excluded the earlier \((3/2^-)\) evaluated assignment \([24]\). The \(3/2^-\) assignment is also in line with the enhanced reaction cross section of \(^{29}\)Ne \([51]\) that favors a low-\(\ell\) valence orbital.
The bound sdpf-M $^{28}$Ne($J^e_z$) shell-model final states, their $C^2 S(\alpha,n\ell_j)$, and the calculated partial cross sections to these states from the $^{29}$Ne(3/2$^-$) projectile are presented in Table I. The results from the analysis of the same transitions on a carbon target can be found in Table VI of Ref. [8], where these shell-model calculations provided an excellent description of the ground-state and excited-states inclusive data. Given these earlier data and its detailed analysis, the present $p(^{28}$Ne,$^{29}$Ne)pm reaction calculations offer a test case to compare the sensitivities of the carbon target calculations with those on a proton. To be fully consistent in these comparisons we take the $^{29}$Ne neutron separation energy to be 963 keV [32] as used previously.

| Reaction | $E_\alpha^*$ (MeV) | $J_\alpha^*$ | $n\ell_j$ | $\sigma_{sp}(n\ell_j,S_j^{\alpha})$ (mb) | $C^2S(\alpha,n\ell_j)$ | $\sigma_{1n}^{th}(n\ell_j,S_j^{\alpha})$ (mb) | $\sigma_{1n}^{th}(\alpha)$ (mb) |
|----------|-------------------|-------------|-----------|--------------------------------|-------------------|-------------------|-------------------|
| $C[^{29}$Ne(3/2$^-$),$^{28}$Ne(\alpha)] | 0.00 | $^2_1$ | $2p_{3/2}$ | 21.19 | 0.438 | 10.31 | 10.31 |
| $S_n(^{29}$Ne) = 0.963 MeV | 1.36 | $^2_1$ | $2p_{3/2}$ | 18.51 | 0.072 | 1.48 | 4.02 |
| | 2.21 | $^2_1$ | $2p_{3/2}$ | 13.69 | 0.167 | 2.54 |
| | 2.76 | $^4_1$ | $1f_{7/2}$ | 17.43 | 0.005 | 0.10 | 0.10 |
| | 2.99 | $^2_2$ | $2p_{3/2}$ | 12.97 | 0.017 | 6.01 | 6.01 |
| | 3.57 | $^2_1$ | $2s_{1/2}$ | 16.63 | 0.066 | 1.22 | 1.43 |
| | 3.69 | $^3_1$ | $1d_{3/2}$ | 15.04 | 0.036 | 0.58 | 1.01 |
| | 3.98 | $^2_3$ | $1d_{5/2}$ | 11.30 | 0.035 | 0.43 | 0.43 |
| | 3.99 | $^4_2$ | $1f_{7/2}$ | 12.46 | 0.005 | 0.07 | 0.07 |

Excited states sum | 22.0

Inclusive | 32.3

We see that the calculated inclusive cross section is 32.3 mb and its ground-state component is 10.3 mb, to be compared with those for the carbon target, of 69.0 mb, and 31.6 mb. In considering these absolute cross-sections and their likely uncertainties with regard the model inputs, we note the most recent evaluated $^{29}$Ne ground-state separation energy is $S_n$ = 971(196) keV [34]. Repeating the present calculations for this value of $S_n$, and its quoted uncertainty, the calculated ground-state and excited-states-inclusive cross sections are 10.3$^{+0.29}_{-0.25}$ mb and 22.0$^{+0.19}_{-0.18}$ mb, respectively, and remain in agreement with the values shown in Table I. So, the $\approx$ 20% uncertainty on the ground-state separation energy results in cross-section changes smaller than the errors on typical measurements by about a factor of three.

A. Neutron orbital sensitivity

It is widely expected, see e.g. Ref. [16] and references therein, that the use of a proton target will increase the sensitivity of the reaction to the overlap functions (removed-nucleon wave functions) in the projectile interior. The extent of this sensitivity will depend on the beam energy and reflects a modest increase in the transparency and penetration of the proton at the projectile surface. The reaction on the proton target is thus less surface localised than when very highly-absorptive optical interactions with a nuclear target are present. We quantify this effect in the case of weakly-bound, neutron-rich, $^{29}$Ne.

The partial cross section results of Table I and those of Table VI of Ref. [5], that involve several $\ell$ values and a range of separation energies, are compared. One clear result from this comparison is that the proton target re-
action is significantly less effective in overlapping with and removing neutrons from the more spatially extended configurations, i.e. the more weakly-bound and low-ℓ orbitals, such as the ground-state to ground-state removal. To elucidate these different sensitivities Table 1 summarizes the two cases and separates the calculated contributions to the bound-states-inclusive cross sections (on the proton and carbon targets) according to their ℓ-value. The table also presents the percentage contributions to the inclusive cross section for removal from the lower ℓ, s + p-state, and higher ℓ, d + f-state, single-particle orbitals; showing the relatively higher percentage of cross section, 49%, due to the d- and f-wave orbitals in the proton target case – as compared to 35% for the carbon target. The ratios of the cross sections from each ℓ on the proton and carbon targets are also shown. The smallest p:C ratio (0.36), for p-wave contributions, is dominated by the weakly-bound ground-state to ground-state 2p3/2 neutron component. This ℓ = 1 ground-state transition alone has ratio 10.3 : 31.6 = 0.33, showing the greater efficacy of the carbon target in removing such halo-like neutrons. The ratios for ℓ = 2 and ℓ = 3 orbitals is 0.65, a factor of two difference. Confirmation of such sensitivities is possible given current experimental capabilities.

TABLE II. Calculated neutron-removal cross sections from 28Ne. The σ1n,ℓ(s) for s,p,d and f-wave orbitals, present the contributions to the inclusive cross-section arising from each ℓ-value of the removed nucleon’s orbital, on the proton (from Table I) and carbon targets (from Table VI of Ref. 3). The ratios of these cross sections on the two targets and the percentage contributions to the inclusive cross section to bound 28Ne shell model final states from low-ℓ (2s1/2 and 2p3/2) and higher-ℓ (1d5/2, 1d3/2 and 1f7/2) neutron orbitals are also shown.

| State | E∗ (MeV) | nℓ | σsp (mb) | σnp (mb) | Ratio(p:C) |
|-------|---------|----|----------|----------|------------|
|       |         |    |          |          |            |
| 0+    | 0.00    | 2p3/2 | 21.19 | 20.67 |            |
| 2+    | 1.36    | 2p3/2 | 18.51 | 18.28 |            |
| +     |         | 1f7/2 | 13.69 | 13.76 |            |
| 4+    | 2.76    | 1f7/2 | 12.97 | 13.07 |            |
| 3−    | 3.69    | 1d5/2 | 11.30 | 11.38 |            |
|       |         | 1d3/2 | 12.37 | 12.44 |            |

Referring to Table I the calculated percentage of the inclusive cross section from the ground-state-excited-states transitions is 32 : 68% on the proton target – whereas on carbon these contributions are comparable, with 46 : 54%. Again, these differences are significant and accessible to experimentation. The ratio from the carbon target data was 49(9) : 51%, consistent with the model calculations. Again, the larger proton target cross-section contribution from excited states confirms the expected increased sensitivity to the more spatially localized, higher-ℓ and more well-bound orbital components in the 28Ne ground-state.

B. Parameter sensitivity: NN S-matrix

As part of this benchmark study, it is also useful to clarify the sensitivity of the calculated removal reaction single-particle cross sections, σsp, to the parameters used in the description of the NN profile function and S-matrix; namely the αnp and the associated βnp. This assessment is presented in Table III which shows the σsp calculated for a selection of the important sdff-M 28Ne shell-model final states from Table I. The calculations use: (i) the αnp values from the Ray tabulation 23; as used above, and (ii) the extreme choice that αnp = 0. In both cases the βnp were determined using Eq. 4. For the Ray value, αnp = 0.533, βnp = 0.0949 fm², which corresponds to a Gaussian range, γnp = 5βnp = 0.436 fm. Taking αnp = 0, then βnp = 0.0739 fm², or γnp = 0.384 fm.

TABLE III. Calculated single-particle cross sections for selected sdff-M shell-model 28Ne final states at 240 MeV/nucleon, when using different parameters αnp, and their associated βnp, computed via Eq. 4, in the NN S-matrix. The αnp = 0.533 cross sections are as in Table I. All calculations use σnp = 37.14 mb.

| State | E∗ (MeV) | nℓ | σsp (mb) | σnp (mb) | Ratio(p:C) |
|-------|---------|----|----------|----------|------------|
|       |         |    |          |          |            |
| 0+    | 0.00    | 2p3/2 | 21.19 | 20.67 |            |
| 2+    | 1.36    | 2p3/2 | 18.51 | 18.28 |            |
| +     |         | 1f7/2 | 13.69 | 13.76 |            |
| 4+    | 2.76    | 1f7/2 | 12.97 | 13.07 |            |
| 3−    | 3.69    | 1d5/2 | 11.30 | 11.38 |            |
|       |         | 1d3/2 | 12.37 | 12.44 |            |

Inspection of these σnp shows that the proton target calculations are extremely insensitive to details of the αnp (and βnp) parameters in the NN S-matrix parameterization, i.e. to details of the NN system beyond the experimentally-constrained NN total cross section σnp. This provides confidence in the stability of the calculated values with respect to realistic variations of these model inputs.

IV. SUMMARY COMMENTS

Intermediate-energy nucleon removal reactions are used as a valuable and efficient tool to study the single-particle degrees of freedom and their evolution with mass and charge in nuclei far from stability. We have presented model calculations for such reactions on a proton target, for which a basic input is the NN collision S-matrix. We show that, in the model used, calculations are highly insensitive to details of the parameters used to describe the NN profile function, beyond the empirically
well-determined NN total cross sections. We compare the computed cross sections to the sdpf-m shell-model $^{20}$Ne final states in the $(^{20}$Ne,$^{20}$Ne)$p$ reaction at 240 MeV/nucleon with the earlier analysis of the reaction on a carbon target at the same incident energy per nucleon.

As anticipated, the calculated final-state cross sections on the proton target show a significantly different sensitivity to the $\ell$-value and separation energy of the orbital of the removed nucleon – with relatively enhanced cross sections for larger-$\ell$ and more bound orbitals. The proton target efficacy is less for removals from low-$\ell$ and more halo-like valence orbitals – compared to reactions on a carbon target. These differences are quantified and measured and calculated partial or inclusive cross sections can be compared, can test these quantitative model predictions. Such data should include both neutron and proton removals from projectile nuclei with an extended range of $(N, Z)$ values. These will include cases of strongly-bound nucleons and so can probe the reaction model predictions over a significant range of nucleon separation energies.

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