Modelling and practical application of autocorrelation functions of vibroacoustic oscillations in a machine dynamic system

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Abstract. The paper considers applying the residue theory in obtaining mathematical models for autocorrelation functions of vibroacoustic oscillations in a machine dynamic system during the cutting process. It shows that these models are analogous to experimental data and reflects their practical application for assessing the dynamic quality of machine tools and setting a processing mode. Calculating the dynamic system stability margin is carried out automatically according to the oscillation index obtained from a real amplitude-frequency characteristic of the dynamic system. This characteristic, in its turn, is determined from the identified transfer function. The article considers the construction of a theoretical model for the autocorrelation function of vibroacoustic oscillations of a grinding machine dynamic system, that would be equivalent to the autocorrelation function obtained from experimental data. Such a model would be feasible to use to calculate the dynamic system transfer function of the machine with a subsequent evaluation of its stability margin. It substantiates applying the dynamic system stability margin of the machine as an informative characteristic based on measuring vibroacoustic oscillations during the cutting process to evaluate the technological system quality and stating an appropriate processing mode to achieve the required part surface quality. It is shown that the identification of transfer function under the established conditions by the autocorrelation function of vibroacoustic oscillations during cutting enables us, based on the maximum stability margin of the dynamic system, to determine the machine with the highest dynamic quality and set the cutting mode, which ensures high processing quality and reduces tool wear.

1. Introduction

When cutting, the part quality is largely determined by the machine dynamic quality [1-6]. This is important when new machine models and new tool materials are used for machining. An important place in machine dynamics is given to the measuring vibroacoustic (VA) oscillations and their analysis based on the determination of various characteristics: spectra, oscillation levels, correlation functions of VA oscillations in the machine dynamic system (DS) and others [1, 7, 8]. It is also necessary to substantiate an informative characteristic based on measuring the VA oscillations during cutting, which enables evaluating the technical condition of machines and setting an appropriate processing mode to achieve the required part processing quality. A number of studies on grinding and lathe machines show that, in terms of applicability in production conditions, it is promising to calculate the stability margin of the machine DS, based on which its dynamic quality is evaluated. The stability margin is determined by the
DS transfer function [8, 9], which, in turn, is calculated from the autocorrelation function (ACF) of VA oscillations in stationary cutting mode [2-4].

This work is relevant because obtaining the theoretical ACF of VA oscillations at the output of the machine DS when a white noise signal is applied to its input, provided that the transfer function of the machine DS is known, is of scientific and practical interest.

2. Mathematical model of autocorrelation function

It is known from the classical work [10] that the formula for calculating the ACF at the output of the DS has the following form

$$K_{yy} (\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{yy}(\omega) e^{j\omega \tau} d\omega,$$  \hspace{1cm} (1)

where $S_{yy}(\omega)$ is power spectral density (PSD) of a signal at DS output.

The following formula is also known

$$S_{yy}(\omega) = |W(j\omega)|^2 S_{xx}(\omega),$$  \hspace{1cm} (2)

where $|W(j\omega)|^2$ is the squared absolute value of the DS frequency function; $S_{xx}(\omega)$ is signal's PSD at the system input.

In our case, the input signal's PSD is of the white noise type $S_{xx}(\omega) = S_o = 1$, and the machine DS can be represented by an oscillatory link [2, 8] with a transfer function

$$W(p) = \frac{K}{T^2 p^2 + 2gp + 1},$$  \hspace{1cm} (3)

with $0 < g < 1$.

The squared absolute value of the frequency function $|W(j\omega)|^2$ is expressed by the following formula

$$|W(j\omega)|^2 = \frac{K^2}{(1 - \omega^2 T^2)^2 + 4g^2 T^2 \omega^2},$$  \hspace{1cm} (4)

It follows from formulas (1), (2), and (4) that

$$K_{yy} (\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{K^2 e^{j\omega \tau} d\omega}{(1 - \omega^2 T^2)^2 + 4g^2 T^2 \omega^2}.$$  \hspace{1cm} (5)

Jordan’s lemma [11] and residue theorem is to be used for calculating the integral (5), then

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{K^2 e^{j\omega \tau} d\omega}{(1 - \omega^2 T^2)^2 + 4g^2 T^2 \omega^2} = \sum_{k=1}^{2} \text{Res} f(\omega_k),$$  \hspace{1cm} (6)

where $f(\omega)$ is an integrand; $\omega_k$ are poles of function $f(\omega)$, where $\text{Res} f(\omega_k)$ is a residue of function $f(\omega)$ in point $\omega_k$, and the integration is performed over the upper half-plane points, i.e. two roots of the denominator of equation (5) are considered.

To calculate the residue in point $z = z_k$, where $z_k$ is the simple pole and $f(z) = p(z)q(z)$, the formula [11] is used:
\[ \text{Res } f(z_k) = \frac{p(z_k)}{q'(z_k)}, \quad (7) \]

where \( q'(z) \) is a derivative of \( q(z) \).

Then it follows from formulas (5), (7) and (8) that

\[ \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{K^2 e^{\jmath \omega t} d\omega}{(1-\alpha^2 T^2) + 4g^2 T^2 \omega^2} = \sum_{k=1}^{2} p(\omega_K), \quad (8) \]

where \( p(\omega) = e^{\jmath \omega t}, \quad q(\omega) = (1-\alpha^2 T^2) + 4g^2 T^2 \omega^2; \quad \omega_K \) are roots of equation \( q(\omega) = 0 \).

In order to calculate the integral (8), it is necessary to obtain expressions for the complex roots \( \omega_1, \ldots, \omega_4 \) and reveal their location on the complex plane. Denominator’s algebraic transformations enable to obtain four roots of the following form:

\[
\begin{align*}
\omega_1 &= \alpha + j\beta, \\
\omega_2 &= -\alpha + j\beta, \\
\omega_3 &= \alpha - j\beta, \\
\omega_4 &= -\alpha - j\beta.
\end{align*}
\]

(9)

where \( \alpha = \frac{1}{T} \sqrt{1 - g^2} \) ; \( \beta = g/T \).

It follows from (9) that, when calculating integral (8), it is necessary to consider only two residues determined by roots \( \omega_1 = \alpha + j\beta \) and \( \omega_4 = -\alpha + j\beta \) which are located in upper half-plane. In this case, the expression for the ACF (5) is reduced to the following form

\[ K_{yy}(\tau) = K_0 e^{-\beta \tau} (B \cos \alpha \tau + A \sin \alpha \tau), \quad (10) \]

and in time it is a damped cosine wave. Coefficients \( K_0, \alpha, \beta, A, B \) are constant and determined by the values of parameters \( K, T, g \) of the initial transfer function (3). Stated above is true in the stationary cutting mode with a small feed of the wheel during grinding and, therefore, with low cutting forces.

During preliminary processing, the values of cutting mode parameters are large enough, and then the machine DS is represented in the form of two parallel-connected links [12]

\[ W(p) = W_1(p) + W_2(p), \quad (11) \]

where \( W_1(p) \) and \( W_2(p) \) are transfer functions for the spindle unit of the part and the tool, expressed by the following formulas:

\[ W_1(p) = \frac{K_1}{T_1^2 p^2 + 2g_1 T_1 p + 1}, \quad (12) \]

\[ W_2(p) = \frac{K_2}{T_2^2 p^2 + 2g_2 T_2 p + 1}. \quad (13) \]

Based on relations (1) and (2), the expression for the ACF has the following form

\[ K_{yy}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |W_1(j\omega) + W_2(j\omega)|^2 S_{xx}(\omega) e^{\jmath \omega t} d\omega, \quad (14) \]

moreover, PSD \( S_{xx}(\omega) = 1 \), since the input signal is considered as white noise.
After performing mathematical transformations described in detail in [12] integral (14) is reduced to the following form

\[ K_{yy}(\tau) = K_{01} e^{-\beta_1 \tau} \left( B_{01} \cos \alpha_1 \tau + A_{01} \sin \alpha_1 \tau \right) + K_{02} e^{-\beta_2 \tau} \left( B_{02} \cos \alpha_2 \tau + A_{02} \sin \alpha_2 \tau \right) \] (15)

where all the coefficients and frequencies are calculated based on the corresponding coefficients of the initial transfer functions (12) and (13).

It follows from (15) that, in the considered case, the ACF contains two frequency components determined by the values \( \alpha_1, \alpha_2 \). In the time domain, the ACF is a damped cosine wave with amplitude modulation (figure 1), which corresponds to the identified ACF from the experimental data [7-9] (figure 2).

![Figure 1. Calculated ACF.](image1.png)  
![Figure 2. Experimental ACF.](image2.png)

3. Experimental studies

As mentioned above, the DS stability margin is used to set the cutting mode as the main informative characteristic. It is determined from the DS transfer function, obtained from the ACF of VA oscillations at various values of the cutting mode parameters, and then the mode with the greatest stability margin is selected. Such mode corresponds to a vibration minimum in the tool-part system, which ensures the highest processing quality. To determine the DS transfer function, the relation obtained by V.A. Sklyarevich [13] is used to connect the DS transfer function \( W(p) \) with the output VA signal ACF \( K(\tau) \) with an input signal of the white noise type

\[ W(p)W(-p) = K(p) + K(-p), \] (16)

where \( K(p) \) is a Laplace transform \( K(\tau) \).

It is not possible to directly obtain the DS transfer function; therefore, identification is used. In order to do this, during the cutting process, VA oscillations are recorded in the machine DS, the ACF is calculated, it is approximated by the formulas obtained above, and then the transfer function is calculated. The results of a number of well-known works show that, in a stationary mode, a random component of the cutting force affects the DS, which can be considered as a stochastic signal of the white noise type [2-4], which is the condition for identifying the transfer function according to formula (16). Experimental studies of VA oscillations of machines’ DS were carried out under operating conditions on PAB-350 lathe machines, and SIW-4 and SIW-5 grinding machines, processing bearing rings [7-9].

A change in the dynamic quality of machines, determined by their technical condition (figure 3) or technological mode, changes the values of parameters included in the DS transfer function, that causes a corresponding change in the DS stability margin, according to the maximum of which the best machine tool or a suitable process mode is selected.
Figure 3. Relationship between the quality of the machined rolling surface of rings and the dynamic quality of SIW-4 internal grinding machines.

The calculation of the DS stability margin is carried out automatically by the oscillation index [10, 14], obtained from the DS real amplitude-frequency characteristic, which is determined from the identified transfer function. The figure shows that the quality of the bearing rings rolling surface machined on machine No. 247 is significantly higher than on machine No. 333. The quality of the bearing race surface was evaluated by the eddy current method in points (the higher the point, the more homogeneous the surface structure is). Stated above correlates with the machine dynamic quality: the oscillation index for machine No. 247 is almost two times lower, that is, the DS stability margin is higher.

Thus, the identification of transfer function under the conditions established according to the ACF of vibroacoustic oscillations during cutting enables us, based on the maximum of the DS stability margin, to determine the machine with the highest dynamic quality or to set the cutting mode, which ensures high processing quality and reduces tool wear.

4. Conclusions
The theoretical model constructed for the ACF of VA oscillations of a grinding machine DS is equivalent to the ACF obtained from experimental data. Thus, the model is feasible to use to calculate the DS transfer function of the machine with a subsequent evaluation of its stability margin. Then, as shown in [7-9], the DS stability margin is used to evaluate the dynamic quality of machines and to set an effective processing mode for bearing rings.

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