Numerical analysis of THz radiation wave using upper hybrid wave wiggler

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Abstract. A theory for upper hybrid wave induced by relativistic electron beam in magnetized plasma emits tuneable and coherent terahertz radiation. The nonlinear interaction with REB is used to generate terahertz radiation. The enhancement in the amplitude of THz wave is also observed when pre-bunched REB is used. The ponderomotive force applied on beam electrons due to radiation wave and upper wave wiggler modifies the dispersion relation. By solving the dispersion relation, we have derived the growth rate of the radiation wave. Numerical studies indicate that by increasing the beam energy the growth rate of the radiation wave decreases, while it increases with wiggler frequency. Besides this, the growth rate of the radiation wave increases with beam density and decreases with radiation frequency and static magnetic field.

1. Introduction
For past few decades the production of terahertz (0.3-10 THz) radiation has been witnessed by the researchers in the broad range of spectroscopy, military security, sensing and communications [1-3]. Since several approaches have been demonstrated to generate the terahertz radiation such as laser based sources, electron beam based sources. In electron beam based sources: surface plasmons [4] and free electron lasers (consist of an electron beam and electromagnetic wiggler i.e., provides magnetic field) [5-6], which are the perfect sources for the generation of tuneable high frequency radiation. On account of high cost and larger size the implication of such kind of wigglers are replaced by the plasma wigglers [7]. In this scheme, the upper hybrid wave is employed as an electrostatic wiggler to generate the THz radiation.

2. Enhancement of THz radiation
We consider an upper hybrid wave of potential $\psi = \phi \exp(-i(\omega t - \vec{k}.\vec{r}))$ propagates in x-z plane is being dipped in magnetized plasma ($B_z$). The interaction of relativistic electron beam ($v_b, \xi$) with upper hybrid wave ($\vec{E}_u$) produces the nonlinear current density and responsible to obtain the nonlinear dispersion relation. The ponderomotive force and nonlinear REB induced upper hybrid wave wiggler is the main mechanism for the enhanced THz radiation emission. We have derived the results using fluid equation model and the equations as mentioned as below.
The interaction of REB with upper hybrid wave produces the electron oscillatory velocity \( \vec{v}_0 \) and taking into account the equation of motion as

1. \[ \frac{\partial}{\partial t} (\gamma v) + (\vec{v} \cdot \vec{\nabla}) (\gamma v) = -\frac{e}{m} \vec{E} - \vec{v}_0 \times (\omega_0 \vec{z}) \] (1)

2. Equation of continuity

\[ \frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v}) = 0 \] (2)

3. The magnetic field of the radiation wave

\[ B = \frac{c}{\omega_0} (\vec{k}_e \times \vec{E}) \] (3)

Here the terms \( n, \vec{v} \) and \( m \) are defined as density of electrons, velocity and mass of the electron respectively, relativistic gamma factor \( \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \) and beam velocity \( \vec{v} = v_{bo} \hat{z} + \vec{v}_0 \). However, the cyclotron frequency is represented as \( \omega_0 = (eB_0 / mc) \).

The electromagnetic wave with electric field \( \vec{E} = \hat{x} \vec{E} \exp(-i(\omega_0 t - \vec{k}_b \cdot \vec{z})) \) is assumed the perturbation field for the equilibrium. In addition, the potential offered by negatively charged beam wave is

The frequency of THz radiation wave is calculated as \( \omega_b = 2\gamma_{bo}^2 (k_b v_{bo} + \omega) - 2\gamma_{bo}^{1/2} \omega_b \), where \( \omega_b = (4\pi n e^2 / m) \) is the beam plasma frequency. We obtain the relations as \( \omega_1 = k_c \) and \( \omega_b = k_b v_{bo} - \omega_b / \gamma_{bo}^{1/2} \).

However, the beam wave is characterized as \( n = n_{b1} + n_{b2} \exp(-i \omega_0 (t - z / v_{bo})) \) and \( n_{b2} = \Delta n_{b1} \), where \( n_{b1} \) is the beam velocity at equilibrium. However, \( n_{b2} \) is the perturbed density due radiation wave and \( \Delta \) is the modulation index.

Therefore, the ponderomotive force \([8]\) produced by radiation wave \((\omega_1, \vec{k}_1)\) and wave wiggler \((\omega_b, \vec{k}_b)\) on beam electrons at \((\omega_b, \vec{k}_b)\) is obtained as

\[ F_p = -\frac{m}{2} [\vec{v} \cdot \vec{\nabla}(\gamma v)] - \frac{e}{2c} (\vec{v}_0 \times B) \] (4)

and \[ F_p = -e \nabla \phi_p \]

After using Eqs. (1), (2) and Eq.(4) and solving them, the nonlinear current density due to ponderomotive force and non-linearity, becomes

\[ J^{nl}_{(\omega_1, \vec{k}_1)} = -\frac{1}{2} n_{b1} e v_0 \] (5)

Now, solving Maxwell’s wave equation and simplifying the terms after putting the value of current density and approximate value of static magnetic field, we get instability of the radiation wave and derived the dispersion relation. Thus, the amplification of the radiation wave is perceived as long as the beam velocity is larger than the phase velocity of the radiation wave. In this case, we get the small frequency mismatch i.e., the imaginary part of the radiation frequency \( \omega_1 = k_c \omega_b + \delta \). This leads to the enhancement in the radiation wave.

The growth rate of the radiation wave i.e., the imaginary part of \( \delta \)

\[ \alpha = \text{Im} \delta = \frac{\sqrt{3}}{2} \left[ \frac{\omega_b^2 \omega_{pb} (1 + \Delta k_{bo}) + \Delta k_{bo}}{32 \omega_1 \omega_b \gamma_{bo}^{1/2}} \right]^{1/3}. \] (6)
where $v_{os} = e\phi k_x / me\gamma_{bo}$ is the measurement of beam oscillatory velocity due to wiggler wave along x-direction. The Equation (6), represents the relation for the growth rate $\alpha$ (in sec$^{-1}$) of the unstable mode of THz radiation wave.

3. Results and Discussions

The typical parameters are used to analyse the behaviour of terahertz radiation wave in the magnetized plasma: electronic mass $m = 9.1x10^{-28}$ g, electronic charge $e = 4.8 \times 10^{-10}$ Statcoulomb, electron plasma density $n_p = 4.1x10^{18}$ cm$^{-3}$, wiggler wave frequency $\omega = 1.6x10^{14}$ rad/sec, electron cyclotron frequency $\omega_c = 0.2\omega$, $0.15\omega$, and $0.09\omega$ etc., static magnetic field $B_1 = 100kG$, $105kG$ and $111kG$ etc. 

The modulation index $\Delta$ ranges from 0 to 1 in the steps of 0.1.

Using Equation (6), we have plotted Figure 1, which illustrates the variation of growth rate $\alpha$ (in sec$^{-1}$) of the radiation wave with beam energy $\gamma_{bo}$ and radiation frequency $\omega_1$ (in rad/sec). This figure reveals that the growth rate decreases with beam energy and radiation frequency. The physical mechanism may be: As long as the beam velocity is larger than the phase velocity of the radiation wave, the amplification in the growth rate of unstable radiation mode keeps on rising.

![Figure 1: Variation of growth rate $\alpha$ (in sec$^{-1}$) of radiation wave with radiation frequency $\omega_1$ (in rad/sec) and beam energy $\gamma_{bo}$ at modulation index $\Delta = 0.8$.](image)

However, Figure 2 depicts the variation of growth rate $\alpha$ (in sec$^{-1}$) of the unstable radiation mode with static magnetic field $\omega_2$ (in rad/sec) and beam density $n_{bo}$ (in cm$^{-3}$) of electrons. In this sketch, we see that the growth rate of radiation unstable mode increases with beam density but decreases as static magnetic field increases [9]. The reason may be understood as: for the increased values of magnetic field, the amplitude of the radiation wave is suppressed.
Figure 2: Variation of growth rate $\alpha$ (in sec$^{-1}$) of radiation wave with cyclotron frequency $\omega_c$ (in rad/sec) or magnetic field $B_1$ and beam density $n_{b1}$ (in cm$^{-3}$) at modulation index $\Delta = 0.6$.

4. Conclusion
The present work shows the possibility of upper hybrid wave as a wiggler for producing the coherent terahertz radiation. The interaction of relativistic electron beam with such wiggler is analogous to free electron laser, which shows the coherence and tuneability of the radiation in THz frequency range, if phase matching conditions are satisfied. The nonlinear current is involved to obtain the nonlinear dispersion relation and that leads to the instability of growth rate of the radiation wave.

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