Theoretical and Numerical Simulation of Small High-speed Projectiles Penetrating Brick Masonry

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Abstract. The process of small high-speed projectiles penetrating brick masonry was studied. The cavity expansion theory used in the study of small high-speed projectile penetration was programmed and the penetration process was calculated. At the same time, LS-DYNA was used to simulate the mechanical process of small high-speed projectiles as they penetrate brick masonry. The results show that the numerical simulation results are more consistent with the theoretical calculation results; the numerical simulation of the finite boundary of the brick masonry and the mortar layer will cause some errors, making the brick masonry less resistant to small high-speed projectiles; When the masonry is compressible, the brickwork has less resistance to small high-speed projectiles; the larger the CRH value describing the geometry of the small high-speed projectile head, the maximum penetration depth and remaining of the small high-speed projectile The speed is greater.

1. Introduction

As an important building and protective material, brittle materials have a wide range of military and civilian values. From civilian facilities such as buildings, bridges and tunnels to military facilities such as command posts, protective walls and air-raid shelters, brittle materials are used in large quantities. For safety reasons, people are most concerned about the anti-penetration ability of brittle materials, so a lot of research has been carried out.

Since Bishop proposed the theory of cavity expansion in the 1940s, the theory has been continuously improved and developed. It has been applied to the penetration of metal targets and simplified the target into an incompressible ideal elastoplastic body. The Sandia National Laboratory of the United States began to study penetration in 10, and in the 1990s proposed the revised Young formula, which can be applied to the ideal elastoplastic medium of compressibility. A large number of research experiments have been carried out in the UK and Germany [1].

Research institutions at domestic universities, such as Beijing Institute of Technology, Nanjing University of Science and Technology, and National Defense University of Science and Technology, have also done a lot of research work. Lin Shengling [2] experimented and simulated the projectile
penetrating concrete target, and studied the relationship between the initial velocity of the projectile, the thickness of the target and the penetration depth and overload. Zheng Hao [3] studied the mass erosion effect of the oval projectile penetrating into the concrete target. Jin Xiaochao [4] conducted a numerical simulation study on the penetration depth of the projectile into the concrete target.

In the above studies on brittle materials, there have been many studies on the theoretical and numerical simulation of concrete penetration, but little research has been done on the penetration process of brick masonry. Brick masonry is widely used in non-load-bearing walls of building exterior walls and internal dividing spaces, and is mainly subjected to pressure. Therefore, bricklaying is abundant in both urban and rural areas, and it is more common as a protective shelter. The research on the penetration of small high-speed projectiles into brick masonry has high military value.

Based on the above reasons, this paper intends to program the cavity expansion theory describing the penetration process, and use LS-DYNA software to numerically simulate the penetration of small high-speed projectile into brick masonry. The purpose is to explore the differences between numerical simulation and theoretical calculations in the study of the process of penetrating brick masonry, and to explore the influence of the compressibility and incompressibility of brick masonry on the penetration process, and to explore the effect of head geometry of the small high-speed projectiles on the penetration process.

2. Theory and Model Overview

The cavity expansion theory is used for programming calculation [5-6]. The cavity expansion theory is divided into two parts - the target is incompressible and the target is compressible. The incompressibility is that the target density does not change when the brickwork is subjected to the penetration volume reduction; the compressibility is the target density changes with the volume when the brickwork is subjected to the penetration volume reduction. Each part is further divided into elastic-plastic response at high speed, and elastic-cracked-plastic response at low speed, as shown in figure1 and figure2. The spherical cavity expands outward from the initial zero radius at a defined velocity $V$, $r$ is the radial coordinate in the Euler spherical coordinate system, $t$ is time, $c$ and $c_1$ are the interface velocity, and $c_d$ is the elastic zone expansion velocity.

![Figure 1. Response regions: the elastic-plastic problem.](image-url)
2.1. Elastic-plastic Response under Incompressible Conditions

Plastic zone mass and momentum conservation equation:

\[
\frac{\partial \sigma_r}{\partial r} + \frac{2\sigma_r}{r} = 0 \tag{1a}
\]

\[
\frac{\partial \sigma_r}{\partial r} + \frac{2\sigma_r}{r} (\sigma_r - \sigma_\theta) = -\rho_0 (\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r}) \tag{1b}
\]

Where \(v\) is the particle velocity straight out; \(\sigma_r, \sigma_\theta\) is the radial and circumferential Cauchy stress component; \(\rho_0\) is the density before deformation.

The Mohr Coulomb yield criterion is brought into (1b) for simplification, \(\lambda\) is the material parameter, and \(a\) is a parameter that is convenient for calculation:

\[
\frac{\partial \sigma_r}{\partial r} + \frac{a\sigma_r}{r} + \frac{\sigma_\theta}{r} = -\rho_0 (\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r}) \tag{2a}
\]

\[
a = \frac{6}{3+2\lambda} \tag{2b}
\]

Define the dimensionless parameters \(S, U, \xi, \xi:\)

\[
S = \frac{\sigma_r}{\tau}, \quad U = \frac{v}{c}, \quad \xi = \frac{v}{c}, \quad \xi = \frac{r}{ct}
\]

The cavity surface boundary conditions, elastoplastic surface boundary conditions and incompressibility conditions are brought into the equations and integrated to obtain the cavity surface radial stress formula:

\[
S(\varepsilon) = \frac{2}{\omega\lambda} e^{-a\lambda} \left( \frac{1}{\lambda} + \frac{\rho_0 v^2}{\tau} \left[ \frac{6}{(1-a\lambda)(4-a\lambda)} \frac{2a\lambda}{(1-a\lambda)} \varepsilon^{1-a\lambda} + \frac{2a^4-a\lambda}{(4-a\lambda)} \right] \right) \tag{3}
\]

2.2. Elastic-cracked-plastic Response under Incompressible Conditions

Increased fracture zone. Using the radial stress at the interface between the plastic zone and the fracture zone, the radial stress at the interface of the fracture zone and the elastic zone, and the particle velocity, plus the mass and momentum conservation equations, the radial stress formula of the cavity surface is obtained:
\[ S(\varepsilon) = \left[ \frac{3+4\lambda}{\beta(3-\lambda)} \right] e^{-\omega \lambda} - \frac{1}{\lambda} + \frac{6}{(\alpha-1)(\alpha-4)} \left( \frac{\sigma_0^{1/2}}{\alpha} \right) + 2 \left( \frac{\sigma_0^{1/2}}{\alpha} \right) \left( \frac{1}{(\alpha-4)} - 1 \right) e^{-\omega \lambda} \]  

(4)

2.3. Elastic-plastic Response under Compressible Conditions

The conservation equation of mass and momentum of plastic zone in Euler coordinate system:

\[ \rho \left( \frac{\partial v}{\partial r} + \frac{2v}{r} \right) = - \left( \frac{\partial p}{\partial t} + v \frac{\partial p}{\partial r} \right) \]  

(5a)

\[ \frac{\partial \sigma_r}{\partial r} + \frac{2(\sigma_r - \sigma_0)}{r} = - \rho \left( \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} \right) \]  

(5b)

The Mohr Coulomb yield criterion is brought into the equations, and \( S = \frac{\sigma_r}{r} \) is set. The radial stress and particle velocity at the interface between the elastic zone and the plastic zone are obtained by using the Hugoniot condition, the mass and momentum conservation conditions:

\[ S_2 = S_1 = \frac{2[(1-2\nu)(1+\nu)(1+\beta)+(1+\nu)(\beta \nu)]}{3[(1-2\nu)(1+\nu)-2\lambda(1+\nu)(\beta \nu)]} \]  

(6)

\[ U_1 = U_2 = \frac{3(1+\nu)(1-2\nu)(1+\nu)}{E[3(1-2\nu)(1+\nu)-2\lambda(1+\nu)(\beta \nu)]} \]  

(7)

\[ \gamma^2 = \left( \frac{2c_d^2}{c_d^2} \right)^2 = \frac{1+\nu}{3(1-\nu)} \]  

(8)

\[ c_d^2 = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)\rho_0} \]  

(9)

Finally, the Longg Kutta method is used to obtain the relationship between the dimensionless radial stress \( S \) and the cavity expansion velocity \( V \).

2.4. Elastic-cracked-plastic Response under Compressible Conditions

To determine the radial stress in the plastic zone, it is necessary to obtain the radial stress and the velocity of the particle at the interface between the plastic zone and the fracture zone. In order to obtain the above parameters, it is necessary to solve the response of the elastic zone and the fracture zone, and then link the two by Hugoniot conditions. Using the inverse iterative method to calculate, first give the \( \beta_1 \), calculate \( \beta \), \( U_2 \) and \( S_2 \) by the formula, and calculate the radial stress and particle velocity of the entire plastic zone. If the velocity of the cavity face point satisfies the boundary condition, the relationship between the radial stress of the cavity surface and the expansion velocity of the cavity is obtained.

2.5. Relationship between Acceleration and Stress

The geometric model of the small high-speed projectile head is shown in figure3. \( d \) is the diameter of a small high-speed projectile, \( a \) is the radius, \( S \) is the radius of curvature, \( l \) is the length of the warhead, \( H \) is the penetration depth, and \( \theta \) is the angle between the normal direction of any point on the surface of the small high-speed projectile and the axial direction of the small high-speed projectile. The CRH (Caliber-Radius-Head) is generally used to describe the geometry of a small high-speed projectile head. Define the CRH value as \( S/2a \).
The radial stress $\sigma_r$ at any point on the surface of the small high-speed projectile is obtained by the cavity expansion theory, and $\sigma_r$ is determined by the velocity $V$ of the small high-speed projectile at that time. Under the premise of only considering the influence of resistance, the resistance of each micro surface source on the surface of a small high-speed projectile is:

$$dF = 2\pi S^2 [\sin \theta - \sin \varphi_0] \sigma_r d\theta$$ (10)

Integrate the stress on the entire small high-speed projectile surface to obtain the total resistance in the direction of penetration:

$$F = 2\pi S^2 \int_{\theta_0}^{\theta} (\sin \theta - \sin \theta_0) \cos \theta \sigma_r d\theta$$ (11a)

$$\theta_0 = \arccos \left[ \frac{(S - a)/S}{S} \right]$$ (11b)

Ignore the process when the warhead does not fully enter the target. If the warhead completely enters the target, then $\theta = \pi/2$

The acceleration of the small high-speed projectile at this time is obtained by Newton's second law.

![Figure 3. Small high speed projectile head geometry model.](image)

2.6. Simulation Model

When building a brick wall model, use integral modeling. The brick masonry model is shown in figure4 [7-8]. The brick wall size is 61.5 cm x 36.5 cm x 17.9 cm. The brick size is 240mm × 115mm × 53mm; the mortar layer is 2cm [9-11]. The small high-speed projectile model is shown in figure5.

![Figure 4. Brickwork model.](image)
2.7. Material Constitutive Model

Since the bricks are fired in clay, the interior contains many cavities, and the prepared mortar contains many bubbles, so the brickwork can be regarded as a combination of hard soil and foam to some extent. Therefore, both the brick and the mortar are simulated using the MAT_SOIL_AND_FOAM material model. The constitutive equation is as follows:

\[ J_2 = \frac{1}{2} \sigma_{ij} \sigma_{ij} \]  
\[ \phi = J_2 - [a_0 + a_1 p + a_2 p^2] \]  

Where \( J_2 \) is the first invariant of the stress tensor, \( \phi \) is the plastic potential, \( p \) is the pressure, and \( a_0, a_1, \) and \( a_2 \) are fitting constants.

On the yield surface \( J_2 = \frac{1}{3} \sigma_y^2 \), where \( \sigma_y \) is the uniaxial yield stress:

\[ \sigma_y = \left[ 3(a_0 + a_1 p + a_2 p^2) \right]^{1/2} \]
\[ a_1 = a_2 = 0 \]  
\[ a_0 = \frac{1}{3} \sigma_y^2 \]

Since the penetration process is characterized by instantaneous high speed and large deformation, the metal material model MAT_JOHNSON_COOK should be used. On this basis, in order to simplify the calculation and ignore the influence of temperature, the small high-speed projectile is simulated using the MAT_SIMPLIFIED_JOHNSON_COOK material model. Its constitutive equation is as follows:

\[ \sigma_y = (A + B \varepsilon^p)(1 + C \ln \varepsilon^s) \]

Where \( A, B, C, \) and \( \varepsilon^p \) and \( \varepsilon^s \) are fitting constants. \( \varepsilon^p \) is effective plastic strain and \( \varepsilon^s \) is effective strain rate.

The main material properties of small high-speed projectiles, bricks and mortar layers are shown in table 1, table 2 and table 3. If there is no unit, the parameter is a dimensionless number; \( e \) is a scientific notation unit, and the value after \( e \) represents the power of 10.
Table 1. Main parameters of small high-speed projectile model.

| Parameter | density / (Kg m⁻³) | Elastic Modulus/ Gpa | Poisson's ratio | A/ MPa | B | n | C |
|-----------|---------------------|----------------------|-----------------|--------|---|---|---|
| Value     | 7850                | 210                  | 0.31            | 2000   | 0.5 | 1 | 0 |

Table 2. Main parameters of brick model.

| Parameter | Density/ (Kg·m⁻³) | Shear modulus/Gpa | Poisson's ratio | a₀/ Pa² | a₁/ Pa | a₂ |
|-----------|-------------------|-------------------|-----------------|---------|--------|-----|
| Value     | 1800              | 0.075             | 0.22            | 3e13    | 0      | 0   |

Table 3. Main parameters of mortar model.

| Parameter | Density/ (Kg·m⁻³) | Shear modulus/Gpa | Poisson's ratio | a₀/ Pa² | a₁/ Pa | a₂ |
|-----------|-------------------|-------------------|-----------------|---------|--------|-----|
| Value     | 1900              | 0.0375            | 0.22            | 8.3e12  | 0      | 0   |

2.8. Penetration Process
During the penetration process, a small high-speed projectile was set at a speed of 800 meters per second perpendicular to the target. We simulated the penetration process with the simulation software ANSYS LS-DYNA and obtained the target at the end of the penetration process. The stress cloud diagram of the state is shown in figure3 [12-13].

![Stress cloud diagram](image)

Figure 6. Equivalent stress cloud at 620 microseconds.

3. Results and Analysis

3.1. The variation of velocity obtained by numerical simulation with time and the variation of theoretically calculated velocity with time
The speed change process of small high-speed projectiles under numerical simulation and the speed change process of small high-speed projectiles calculated by theoretical model are shown in the following figure. The two research modes are further divided into brick masonry under incompressible conditions and brick masonry under compressible conditions.
Figure 7. Speed change process of small high-speed projectiles under incompressible conditions and compressible conditions of brick masonry.

As shown in figure7, the velocity profile of the small high-speed projectile under the numerical simulation fits the velocity profile of the small high-speed projectile calculated by the theoretical model. However, the acceleration of small high-speed projectiles under numerical simulation is smaller than that of small-scale high-speed projectiles calculated by theoretical model. This is because the brickwork under numerical simulation is a finite boundary, and the brickwork under the theoretical model is an infinite boundary. It should also be considered that the numerical simulation of the brickwork contains a mortar layer, which causes less resistance than the brick, while the theoretical model only considers the bricks, thus causing a small amount of error.

3.2. Effects of Compressible and Incompressible Conditions
As shown in figure7, in the process of small high-speed projectiles penetrating the brick masonry, the speed is gradually reduced due to the resistance. The acceleration of small high-speed projectiles under incompressible conditions of brick masonry is greater than that of small high-speed projectiles under compressible conditions of brick masonry. This is because the density of the brickwork does not change under incompressible conditions. From the radial stress formula of the cavity surface, it can be seen that under the same conditions, the radial stress of the cavity surface under the incompressible condition is greater than the radial stress of the cavity surface under the compressible condition. That is, the small high-speed projectiles are subjected to greater resistance under the incompressible conditions of the brickwork, and thus the acceleration is greater.

3.3. The Effect of Small high-speed Projectile Head Geometry
The CRH value is used to describe the head geometry of a small high speed projectile. The CRH values were taken as 0.5, 1, 2, 3, 4, and 5 respectively, and numerical simulation calculations were performed and the results were compared:
Figure 8. Penetration depth variation of different CRH values under compressible and incompressible conditions.

Figure 9. Velocity time change graph of different CRH values under compressible conditions.

Figure 10. Velocity time change graph of different CRH values under incompressible conditions.
As shown in figure8, figure9 and figure10, changing the CRH value of the small high speed projectile has a great influence on the speed change process of the small high speed projectile. The larger the CRH, the smaller the acceleration of the small high-speed projectile, and the greater the residual speed, the greater the maximum penetration depth. This is because the larger the CRH, the sharper the head and the greater the penetration.

4. Conclusions

Through numerical simulation analysis and theoretical programming calculation, the following conclusions are obtained:

(1) In the process of small high-speed projectiles penetrating brick masonry, the acceleration of small high-speed projectiles under compressible conditions of brick masonry is smaller than that of small high-speed projectiles under incompressible conditions of brick masonry. It shows that the small high-speed projectiles under the compressible condition of brick masonry are relatively less resistant, and the compressible conditions have a significant impact on the penetration process.

(2) The acceleration of the small high-speed projectile obtained by numerical simulation is smaller than the acceleration of the small high-speed projectile calculated by the theoretical model. It is shown that the numerical simulation of the finite boundary of the brick masonry and the mortar layer will cause certain errors, and the error needs to be taken into account when estimating using the theoretical model.

(3) The larger the CRH value of the small high-speed projectile, the better the penetration, and the greater the residual speed and the maximum penetration depth. Changing the CRH value has a great influence on the process of penetrating the brick masonry.

Through this experiment, a preliminary quantitative analysis of the dynamic process of small high-speed projectiles penetrating brick masonry has been carried out, which provides a reference for the project of small high-speed projectiles penetrating brick masonry.

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