Modeling of conjugated heat and mass transfer during film desorption

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Abstract. The heat and mass transfer in the process of desorption on a free falling liquid film of the LiBr-water solution is analyzed analytically. The dependences of the Nusselt number on the surface on the longitudinal coordinate downstream of the film were found, and the thickness of the film, which changed due to desorption, was determined. The problem was investigated for different values of Froude and Peclet numbers, for different values of initial film thickness.

1. Introduction
Desorption is the release of one substance from another through the surface. Desorption can occur when an equilibrium situation is violated. We investigate two-phase system: water solution of lithium bromide - water vapor, — under the assumptions corresponding to the conditions of working desorber of a heat pump. The amount of vapour entering and leaving the LiBr-water solution will be the same, — and the vapour concentration in the solution will be constant at the equilibrium state. If the solution temperature increases, the equilibrium and solubility are changed, and the vapour will desorb from the lithium bromide water solution, — lowering the vapour content.

The desorption process enhancement has been the subject of numerous studies. So, the absorption heat pump is a unique device consisting of the desorber, condenser, evaporator, and absorber. In such devices, vapor absorption and desorption in liquid goes along with heat release, so the heat and mass transfer processes are conjugated. In [1] the desorption of CO₂ from supersaturated water solution has been extensively studied using a string-of-spheres apparatus. In [2] the formal methodology of modeling the simultaneous quiescent and bubble desorption of gases from liquids was presented. The method and exemplary results of estimating the model parameters from experimental data were shown, which were restricted to the size independent growth of bubbles. In [3] heat and mass transfer in the processes of absorption, desorption, evaporation and condensation in the initial part of semi-infinite axisymmetric film falling under the pressure was studied. The investigation of thin liquid film flowing down a vertical wall in the roll-wave regime in the presence of heat and mass transfer through the free surface was presented in [4, 5]. In [6] it has been shown that a significant change in the salt concentration leads to different absorption regimes. In [7, 8] it was shown that free convection in the gas affects significantly the desorption (adsorption) of the aqueous solution of the salt.

In this work we obtain the dependence of the Nusselt number on the surface of the free liquid film on the longitudinal coordinate downstream of the film flow and the film thickness dependence on the longitudinal coordinate, which changed due to desorption.
2. Problem statement

2.1. Governing equations

Let us consider that the liquid film falls under the action of gravity with the uniform vertical velocity $u_0$, m/s. We will introduce the Cartesian coordinate system with axis $Ox$, directed downstream, and axis $Oy$, directed normally to the axis of symmetry of the free film from the interface to the liquid depth. The velocity is defined as $u = \sqrt{u_0^2 + 2ghx}$. The initial thickness of the film is $h_0$, m. The Froude number is $Fr = gh_0/u_0^2$, where $g$ is a gravity, m/s$^2$. The process of heat and mass transfer at film absorption is described by the equations of heat conductivity and diffusion:

$$\frac{u}{\partial x} = a \frac{\partial^2 T}{\partial y^2},$$

$$\frac{u}{\partial x} = D \frac{\partial^2 C}{\partial y^2},$$

where $a$ is the thermal diffusivity, $m^2/s$; $D$ is the diffusion coefficient, $m^2/s$. The boundary conditions at the inlet at $x = 0$ are

$$T = T_0, C = C_0, h = h_0.$$ (3)

Similarly to [3], the equilibrium of solution-vapor system occurs on the interface at $y = 0$:

$$C_i = f(T_i), \chi \frac{\partial T}{\partial y} = \frac{\partial C}{\partial y}.$$ (4)

where $\chi = r_a \rho D/(1-C_0)$, $r_a$ is the latent heat of phase transformation, $J/kg$; $\rho$ is solution density, $kg/m^3$, $\lambda$ is heat conductivity, $W/(mK)$, the dynamical viscosity of the liquid; $\sigma$ is the surface tension coefficient. This equilibrium condition joins equilibrium temperature $T_i$ with concentration $C_i$. Also, let us write an asymptotic boundary condition at $y \to \infty$

$$T = T_0, C = C_0.$$ (3)

The vapour desorption occurs, when the concentration of the dissolved gas in the bulk of the liquid is greater than that at the surface. The conditions of desorption are $T_0 > T_s$ and $C_0 > C_s$, where $T_s$ is equilibrium temperature corresponding to initial concentration $C_0$, $C_0 = f(T_s)$ and $C_s = f(T_0)$.

2.2. Non-dimensionalization

To solve the formulated problem, we introduce the following dimensionless variables: $\xi = a(x + 0.5u_0^2/g)/(u_0h_0^2)$, $\bar{y} = y/h_0$; $\Theta = (T - T_0)/(T_s - T_0)$, $\gamma = (C - C_0)/(C_s - C_0)$. Also, we suppose the linear Henry’s law in (4). As a result, we obtain the modified problem with a prolonged initial region described by the system of dimensionless governing equations relatively self-similar variable $\xi = \bar{y}/\bar{\pi}^{1/4}$:

$$\frac{\partial^2 \Theta}{\partial \xi^2} + 2^{-3/2} Fr^{1/2} Pe^{1/2} \xi \frac{\partial \Theta}{\partial \xi} = 0,$$ (5)

$$\frac{\partial^2 \gamma}{\partial \xi^2} + 2^{-3/2} Le^{-1} Fr^{1/2} Pe^{1/2} \xi \frac{\partial \gamma}{\partial \xi} = 0,$$ (6)

with boundary conditions:

$$\xi = 0, \quad \Theta_i + \gamma_i = 1, \quad \frac{\partial \Theta}{\partial \xi} = \frac{KuLe \Delta C}{1-C_0} \frac{\partial \gamma}{\partial \xi},$$ (7)
\[ \xi \to \infty \quad \Theta = 0, \quad \gamma = 0, \]  

(8)

where \( Le = D/a \) is a Lewis number, \( Ku = r_a/(C_p \Delta T) \) is a Kutateladze number, \( Pe = (u_0 h_0)/a \) is a Peclet number.

3. Solutions of the problem

The exact solutions of these ordinary differential equations are obtained in [3]. In this paper we obtain temperature gradient

\[ \frac{\partial T}{\partial y} = \Delta T \Theta_i \frac{Fr^{1/2}Pe^{1/2}}{h_i \sqrt{\pi} (2x Fr h_i^{-1} + 1)^{1/4}}, \]  

(9)

and Nusselt number

\[ Nu(x) = \frac{\Theta_i Fr^{1/2}Pe^{1/2}}{\sqrt{\pi} (2x Fr h_i^{-1} + 1)^{1/4}}, \]  

(10)

where \( x \) and \( y \) are dimensionless. The parameters were chosen close to the characteristic values of the two-phase system: water solution of lithium bromide - water vapor, under the assumptions corresponding to the conditions of working desorber of a heat pump: \( T_s = 114^\circ C, \ T_0 = 140^\circ C, \ C_s = 0.44, \ C_0 = 0.6. \) The pressure in the generator (desorber) is equal 50-100 kPa. In our calculations we have used \( \Delta T = -26K, \ \Delta C = -0.16, \ h_0 = 0.5 \ mm, \ Le = 0.01, \ Ku = -50. \)

Figure 1 shows the dependence of the Nusselt number on the film surface on the longitudinal coordinate for different values the Peclet and Froude number. Fig.2 shows the dependence of the film thickness on longitudinal coordinate for absorption and desorption for different values of the Froude and Peclet numbers.

4. Conclusions

This paper describes the heat and mass transfer in desorption process on the free falling liquid film of LiBr-water solution. The problem was investigated for different values of Froude and Peclet numbers, for different values of initial film thickness. It was shown that the heat transfer is greater for largest initial thicknesses of the film (for initial thickness from 0.25 to 0.5 mm). These calculations of desorption on the free liquid film of the lithium bromide water solution confirm the analytical dependences (9) and (10) of the heat flux at the entrance region on the initial film thickness as a power function \( h_0^{3/4} \) if all dimensionless complexes are substituted. Also, it was shown, that for the large values of Peclet numbers heat transfer from the surface is
greater than for the smaller values. And it is seen that absorption and desorption do not change significantly the film thickness on the longitudinal coordinate.

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