Mechanics of Quark Exchange in High-Energy Hadron Reactions at Forward Angles

M.V. Bondarenco
NSC Kharkov Institute of Physics & Technology, 1 Academicheskaya St., Kharkov 61108, Ukraine

Abstract

The 2-quark back-angle scattering mechanism is shown to reproduce main features of high-energy $hh$ flavor-exchange reactions. Prospects for reduction of the reaction matrix element to a form of convolution of hadron wave functions with the hard scattering kernel are discussed. Wave function models suitable for convolution with the kernel, which is singular at small $x$, and the emerging form-factor types, are discussed.

Reactions of one unit of flavor exchange between two hadrons scattering through small angles at high energy can occur, to LO in Mandelstam $s$, due to exchange of a single pair of quarks, which enter into a hard collision and completely interchange their large longitudinal momenta (scatter to 180° in c.m.s.). With no rapidity gap between the hadron remnants and their new comoving quarks created, they should with high probability recombine into a new pair of hadrons. In the binary reaction channel the differential cross-section, indeed, exhibits a forward peak of typical $p_\perp \sim 300$ MeV width. The peak decreases with the energy rather slowly, as $s^{-1\frac{1}{2}} \sim s^{-2}$ – approximately as does the Born-level 2-quark back angle scattering differential cross-section ($\propto s^{-2}$); the rest of the energy dependence must be attributed to reggeization effects.

The Born diagram of the head-on relativistic quark collision is closely similar to that of real photon scattering on a quark, owing to the similarity between hard gluon and hard quark propagators on the light cone. Yet, high energy hadron reactions are much easier to measure than hard forward real Compton scattering is, given the coupling constant factor $\alpha_s^2$ in the cross-section instead of $\alpha_{EM}^2$, and no radiative or diffractive background involved. On the theory side, it brings complications, since eikonal (gauge link, Wilson line) phases in hadron remnant interactions emerge, which spoil factorization in terms of GPDs. However, that may actually happen in photon-hadron interactions as well, and the $A^+ = 0$ gauge condition does not help. As those links are to be dealt with anyway, it would be better to constrain them using cross-checks from different processes.

Forward Compton scattering is, also, not the best playground for studying hadron spin effects, since quark SSA for it cancel in the sum of two LO diagrams. As for 2-quark scattering in flavor exchange, there is only one LO diagram, so transverse polarization may be open, provided soft contributions supply necessary phase shifts. Generally, soft effects can stem from eikonal links, and also from initial and final state wave functions sandwiching the complex hard scattering kernel. Both eikonal and initial/final state contributions are proportional to the interaction strength, but still, it is not excluded that some of them may happen to be more significant. To make a sensible decision, it is instructive first to develop qualitative understanding of the dynamics.
1 Finite Longitudinal Shift + LS-Coupling

Consider the act of the head-on quark collision in the rest frame of one of the hadrons. Instead of backscattering, then, we have braking of a fast quark and knockout of a resting one. The mediating hard gluon, being shed from a fast particle, must propagate along the same direction. Its range (Ioffe time [2]) appears to be collision energy independent and thereby, despite high virtuality of the gluon, finite: $l_z \sim (M_{\text{targ}} x_{\text{targ}})^{-1} \sim R_H$. So, the stop point of the incident quark on average is shifted with respect to the centre of the well by a distance commensurable with the hadron radius, and with the simultaneous finite transverse momentum transfer in the head-on collision, the captured quark acquires an orbital angular momentum in the normal direction.

Of course, if by definition the final state, as well as the initial one, must have zero orbital angular momentum ($s$-state), the quark would be forbidden to orbit. But relativistically, the single-quark $s$-state is the one having upper Dirac bispinor components multiplied by spatial wave functions with $L = 0$, but the lower ones, by parity reasons, must have $L = 1$, composing it with the spin $S = \frac{1}{2}$ to yield the same total momentum $J = \frac{1}{2}$. Now, to flip the spin of the $s$-state bound quark in a head-on collision, it obviously suffices to change projection of $L$ for its lower Dirac bispinor components.

From the emerging spatial picture it seems clear that the effect is, basically, bulk, not due some fine local interference. So, the implementation of real eikonal phases is unlikely to modify the picture drastically. Opacity effects can be of consequence, but their 3d distribution is rather uncertain. Therefore, a reasonable strategy would be to begin with the neglect of eikonal factors, and ultimately look for deriving constraints on them when a nonremovable discrepancy with the data arises. Thus we arrive at the representation of $hh$-collision amplitudes in terms of wave function overlaps.

A popular model of light-front WF with LS-interaction is that of [3]. However, it is of perturbative origin, thence inheriting support $x \in [0, 1]$ and vanishing at the endpoints. Therefore, no imaginary part can result in convolution of such wave functions with the hard kernel $\frac{1}{x_1 x_2 + i \alpha}$. An alternative class of models which tend to give wave functions non-vanishing and continuous at $x = 0$ (with support in $x$ ranging, in principle, from $-\infty$ to $+\infty$, and $x$ normalization being to the single-constituent rest energy, not to that of the whole composite system) assume non-interacting constituents moving in a self-consistent field, static in their c.m.s.\(^3\) The static well is unable to accommodate for recoil effects which must be crucial at $-t > M^2$, but there is no experimental data in that far region anyway. With wave functions continuous at $x = 0$, the imaginary part of the matrix element, in fact, logarithmically diverges in the hard approximation, so it is not purely hard, but that introduces only a weak dependence on the cutoff parameter.

The results obtained with a gaussian WF model were partially presented at the conference, but prior to embarking at model assumptions, it is desirable to exhaust all model-independent means. To this end, one can admit that in general, $x$-dependence of $H$ and $E$

\(^1\)Essentially, this is the same attitude as in DGLAP, where, by the way, the role of eikonal factors, to date, still has not become tangible, at typical $x$.

\(^2\)Note that for elastic scattering that is impossible in principle, because it is caused by soft exchanges. The minimum number of gluons exchanged in the $t$-channel is two, and generally they need not be all attached to the same parton line (though, conditions for the opposite are sought in ‘heavy Pomeron’ models).

\(^3\)At that, GPDs near $x = 0$ are not to be interpreted as quark densities at $x > 0$ and minus antiquark densities at $x < 0$, at any rate not as those extracted from DIS data using the parton model.
GPDs strongly differ\textsuperscript{4}, and analyze consequences of factorization in their terms.

## 2 Factorization With Imaginary Contributions

The factorization of the matrix element proceeds through decomposition of the hard kernel

\[
\frac{1}{x_1 x_2 + i0} = P \frac{1}{x_1} \frac{1}{x_2} - i \pi \frac{1}{|x_1|} \delta(x_2) - i \pi \delta(x_1) \frac{1}{|x_2|},
\]

and leads, for a representative reaction \( np \to pn \), to the matrix element

\[
M_{fi} \propto w_{n}^{t^+} \left( H_{-}(t) + \frac{i \sqrt{-t}}{2M} E_{-}(t) \sigma^N \right) w_{p}^{t^+} \left( H_{-}(t) + \frac{i \sqrt{-t}}{2M} E_{-}(t) \sigma^N \right) w_{n},
\]

\[
- i w_{n}^{t^+} \left( H_{+}(t) + \frac{i \sqrt{-t}}{2M} E_{+}(t) \sigma^N \right) w_{p}^{t^+} \left( H_{0}(t) + \frac{i \sqrt{-t}}{2M} E_{0}(t) \sigma^N \right) w_{n},
\]

\[
- i w_{n}^{t^+} \left( H_{0}(t) + \frac{i \sqrt{-t}}{2M} E_{0}(t) \sigma^N \right) w_{p}^{t^+} \left( H_{+}(t) + \frac{i \sqrt{-t}}{2M} E_{+}(t) \sigma^N \right) w_{n},
\]

\[
+ \left( \tilde{H}_{2}^{2}(t) - 2t \tilde{H}_{+}(t) \tilde{H}_{0}(t) \right) w_{n}^{t^+} \sigma^L w_{p}^{t^+} \sigma^L w_{n},
\]

with

\[
H_{-}(t) = P \int \frac{dx}{x} H_{p}^{u}(x, 0, t), H_{+}(t, \lambda_x) = \int_{|x| > \lambda_x} \frac{dx}{|x|} H_{p}^{u}(x, 0, t) \approx H_{+}(t), H_{0}(t) = \pi H_{p}^{u}(0, 0, t),
\]

and similarly for \( E \) and \( \tilde{H} \). Formally, this structure resembles the sum of \( t \)-channel pole exchanges, but 2 of them having \textit{imaginary} coupling constants. Polarization emerges due to interference between the “-” exchange and “0+” and “+0” exchanges.

Note that \( t \)-dependences of “-” form-factors dominated by typical \( x \) and of “0” and “+” form-factors fed by \( x \approx 0 \) may be of quite different width. Presence of several \( t \)-slopes is typically observed in flavor exchange reactions, the pre-QCD interpretation being the difference in masses of the exchanged mesons and onset of central absorption \textsuperscript{5}. The quark backscattering theory, however, does not contain correlated \( q \bar{q} \) transverse propagation.

Phenomenological consequences of \( u \)-channel gluon reggeization, and of possible contributions involving extra \( t \)-channel color exchanges will be discussed elsewhere.

\textbf{Acknowledgement.} The author is grateful to organizers of the conference for local hospitality during the meeting.

## References

1. S. J. Brodsky \textit{et al}, PRD\textbf{65}, 114025 (2002).

2. A. V. Belitsky, and A. V. Radyushkin, Phys. Rep. \textbf{418}, 1 (2005).

3. S. J. Brodsky, and S. D. Drell, PRD\textbf{22}, 2236 (1980).

4. M. Burkardt, Int. J. Mod. Phys. A\textbf{18} 173 (2003).

5. G. L. Kane, and A. Seidl, Rev. Mod. Phys.\textbf{48}, 309 (1976).

\textsuperscript{4}In contrast to the case of \( H \), \( x \)-dependence of \( E \) is not constrained by DIS data. Note that in some models \textsuperscript{4} those distributions are assumed to be just proportional, but without any motivation beyond the similarity of \( t \)-dependence of their \( x \)-integrals which are associated with EM form-factors.