Inverse identification of intensity distributions from multiple flux maps in concentrating solar applications

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Abstract. Radiative flux measurements at the focal plane of solar concentrators are typically performed using digital cameras in conjunction with Lambertian targets. To accurately predict flux distributions on arbitrary receiver geometries directional information about the radiation is required. Currently, the directional characteristics of solar concentrating systems are predicted via ray tracing simulations. No direct experimental technique to determine intensities of concentrating solar systems is available. In the current paper, multiple parallel flux measurements at varying distances from the focal plane together with a linear inverse method and Tikhonov regularization are used to identify the directional and spatial intensity distribution at the solution plane. The directional binning feature of an in-house Monte Carlo ray tracing program is used to provide a reference solution. The method has been successfully applied to two-dimensional concentrators, namely parabolic troughs and elliptical troughs using forward Monte Carlo ray tracing simulations that provide the flux maps as well as consistent, associated intensity distribution for validation. In the two-dimensional case, intensity distributions obtained from the inverse method approach the Monte Carlo forward solution. In contrast, the method has not been successful for three dimensional and circular symmetric concentrator geometries.

Nomenclature

\( I(x,y,z,\theta,\phi) \): Intensity at point \((x,y,z)\) in the direction \((\theta,\phi)\)
\( I_0(x,y,z,\theta,\phi) \): Intensity on the solution plane at point \((x,y)\) in the direction \((\theta,\phi)\)
\( q(x,y,z) \): Radiative flux at the point \((x,y,z)\)
\( q_{ij} \): Measured radiative flux at the point corresponding to \((x_j,y_k,z_l)\)
\( w_x \): Weighting factor in the x-direction (equivalent for y)
\( x_j \): x-position at the \(j\)th discretization (equivalent for \(r, y, z, \theta, \) and \(\phi\))
\( x_{0,\text{disc}} \): x-position on the solution plane corresponding to a point \((x_j, y_k, z_l)\) and direction \((\theta_m,\phi_n)\)

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\[ \Delta x \text{: Discretization step in the x-direction (equivalent for } r, y, z, \theta, \text{ and } \varphi \] 
\[ \Delta \Omega \text{: Discrete solid angle} \]

1. Introduction

Solar concentrating systems and high flux solar simulators produce high radiative flux levels. Hirsch and Steinfeld [1] have reported fluxes in excess of 4,250 kW/m\(^2\) for an elliptical trough based solar simulator. Petrasch et al. [2] have reported fluxes as high as 11,000 kW/m\(^2\) from an array of elliptical reflectors coupled to Xe arc lamps. These flux levels are equivalent to blackbody stagnation temperatures of 2945 and 3730 K, respectively. Linear systems such as the EuroTrough [3] parabolic concentrator achieve actual temperatures of roughly 500 °C with concentration ratios of approximately 82:1. The LS-1 and LS-2 parabolic troughs by Luz International Ltd. [4] have geometric concentration ratios around 23:1, achieving temperatures near 400 °C. Flux mapping systems are essential for any experimental concentrating solar energy system. They provide an accurate measurement of the radiative flux distribution in the focal plane and thus form the basis for energy balances and efficiency calculations [5, 6]. Flux mapping systems consist of a diffusely reflecting flux target in the focal plane of the concentrator, a highly linear charge-coupled device (CCD) camera [5–7, 9] and a flux sensor for calibration [7, 8].

Typically, ray-tracing programs such as CIRCE2 [10], and VEGAS [11] are used to simulate flux and intensity distributions. These systems rely on idealized reflector geometries and they are adequate for the design of concentrating systems. However, they fail to predict non-ideal flux and intensity distributions due to surface imperfections introduced during manufacturing or operation [12].

In this paper, an experimentally based, inverse method to determine the intensity distribution at a desired solution plane is presented. The method uses multiple flux maps at varying distances along the concentrator’s axis as an input to solve for the intensity distribution at the solution plane. There are two main approaches to solving inverse radiation problems, (i) iterative and (ii) direct methods. The iterative approach is based on an assumed parametric model of the intensity function. An optimal solution of then intensity is then found by minimizing the associated residual functionals [13]. While iterative methods are often more stable than direct methods, they are typically nonlinear and suffer from larger computation time [13]. Direct methods are based on discretizing the physical relations between measured (flux) and unknown (intensity) quantities and solving the resulting system of equations. Direct methods often result in ill conditioned linear systems of equations. A range of inverse problems involving radiation in participating medium have been studied in [14–19]. In the current paper a direct approach is chosen.

2. Methodology

Flux maps of solar concentrating systems are usually obtained using Lambertian targets and CCD cameras in conjunction with flux sensors for calibration [7]. A water-cooled Lambertian target is positioned at the focal plane. A highly linear digital camera is then used to acquire an image of the irradiated target. The brightness of each pixel of the resulting digital image is proportional to the radiative flux incident at the pixel location. Ulmer et al. detail a flux mapping system installed at the Plataforma Solar de Almeria in Spain [7]. Similar systems have been implemented at the Paul Scherrer Institute [2] and at ETH-Zurich [1]. A flux sensor is then used to establish the relation between pixel brightness and energy flux. Multiple types of flux sensors exist. Kaluza and Neumann have compiled a review of the various options. All directional information is lost in the diffuse reflection from the Lambertian target. Therefore, no predictions of the flux distribution on arbitrarily shaped receivers can be obtained via the flux maps. Furthermore, direct measurement of the directional distribution of radiation is impractical due to the very high flux levels in the focal region. If information about the flux distribution in multiple
planes is available, it can be used to partially restore the directional information [9]. A schematic representation of a typical flux measurement setup is depicted in Figure 1.

![Diagram of flux mapping setup](image1)

**Figure 1.** Flux mapping setup.

2.1. Derivation for the general case

A schematic depiction of the general, three-dimensional case of a ray originating at a solar concentrator and passing through several parallel planes is shown in Figure 2.

Air is assumed to be a non-participating medium, reducing the RTE to

\[
\frac{dl}{ds} = 0
\]

Thus, the intensity, \( I \), is constant along any straight line. According to Figure 2, for a given direction \((\theta, \phi)\) positions of equal intensity are given by

\[
I(x, y, z, \theta, \phi) = I_0(x_0, y_0, \theta, \phi) = I_0(x + z \cos \phi \tan \theta, y + z \sin \phi \tan \theta, \theta, \phi).
\]

The flux at a given position is obtained by integrating the projected intensity over the hemisphere:

\[
q''(x, y, z) = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} I(x, y, z, \theta, \phi) \cos \theta \sin \theta d\theta d\phi
\]

Substituting the result of equation (2) for the intensity in equation (3) one obtains

\[
q''(x, y, z) = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} I_0(x_0, y_0, \theta, \phi) \cos \theta \sin \theta d\theta d\phi
\]

Equation (4) can then be discretized. The measured values of radiative flux, \( q''(x, y, z) \), are known at discrete locations on a Cartesian grid defined by \( j, k, l \):

\[
q_{jkl} = q''(x_{\text{ref}} + j\Delta x_f, y_{\text{ref}} + k\Delta y_f, z_{\text{ref}} + l\Delta z_f)
\]
The unknown intensity distribution in the solution plane \((z = 0)\) is spatially discretized on a uniform rectangular grid in \(x\) and \(y\) while the directional distribution is uniform in elevation \((\theta)\) and azimuthal \((\varphi)\) angles. Equation (4) then becomes

\[
q_{jkl} = \sum_{m=1}^{N_y} \sum_{n=1}^{N_x} \left[w_x w_y I_{j+l,m+n} + w_x (1-w_y) I_{j+l,m+n} + (1-w_x) w_y (1-w_y) I_{j+l,m+n} \right] \cos \theta_m \sin \theta_m \Delta \theta \Delta \varphi
\]  

(6)

This is a linear system of equations of the form \(q = AI\), where the radiative flux, \(q\), is known and the intensity at the solution plane, \(I_o\), is unknown. The system is further defined by equations \((7-9)\).

\[
x_{o,jkmn} = x_j + z \cos \varphi_n \tan \theta_m, \quad y_{o,jkmn} = y_k + z \sin \varphi_n \tan \theta_m
\]  

(7)

\[
J = \text{ceiling} \left( \frac{x_{o,jkmn} - x_o}{\Delta x} \right), \quad K = \text{ceiling} \left( \frac{y_{o,jkmn} - y_o}{\Delta y} \right)
\]  

(8)

\[
w_x = \frac{x_{o,jkmn} - x_o}{\Delta x}, \quad w_y = \frac{y_{o,jkmn} - y_o}{\Delta y}
\]  

(9)

Equations (7) calculate the location of intersection on the solution plane for the \(x\) and \(y\)-directions, respectively. Equations (8) calculate the index of intersection on the solution plane in the \(x\) and \(y\) directions. Equations (9) calculate the weighting factors in the \(x\) and \(y\)-directions as intersections will always occur between two nodal points. The fully three-dimensional case is presented for completeness but is not yet successfully implemented and therefore no results are available.

2.2. Two dimensional case

The high dimensionality of the intensity distribution on the solution plane in the most general case (two spatial and two directional coordinates) leads to very large systems of equations. Therefore an important subclass of problems, the purely two-dimensional case, is explored in detail. The 2D situation is applicable to concentrators such as parabolic, elliptical, and circular troughs and 2D compound parabolic concentrators (CPC’s). Figure 3 depicts the flux distribution at varying locations on the focal axis of a 2D concentrator.

**Figure 3.** Two-dimensional flux maps.

**Figure 4.** Schematic representation of the circular symmetric case.
The derivation of the 2D case is analogous to that of the 3D case. The intensity at an arbitrary position and direction \((x, y, \theta)\) can be related to the intensity in the solution plane according to
\[
I(x, y, \theta) = I_0(x + z \tan \theta, \theta)
\]
(10)

The radiative flux incident at a given location, \(q''(x, z)\), is calculated from the intensity according to the two-dimensional equivalent of equation (4).
\[
q''(x, z) = \int_{\theta=0}^{\pi/2} I(x, z, \theta) \cos \theta d\theta \quad \text{and} \quad \int_{\theta=0}^{\pi/2} I_0(x + z \tan \theta, \theta) \cos \theta d\theta
\]
(11)

Discretization of equation (11) leads to
\[
q_{jl} = \sum_{m=1}^{N_\theta} \left[(1 - w_x)I_{0,j,m} + w_x I_{0,j+1,m}\right] \cos \theta_m \Delta \theta_m
\]
(12)

In equation (12), \(q_{jl}\) represents the radiative flux at location \(j\) on plane \(l\). Using the 2D equivalents of Equations (7), (8), and (9), the weighting factor \(w_x\) is found. The intensity, \(I_{0,j,m}\), is the intensity in direction \(m\) at location \(J\) on the solution plane.

2.3. Circular Symmetric case
Many optical concentrators are circularly symmetric. In these cases, the intensity in the focal plane depends on three independent variables: the radial position, \(r\), and two directional angles, \(\theta\) and \(\phi\). In contrast, the general case requires four independent variables. A system consisting of a circular concentrator and a plane circular target is considered.

From Figure 4 one obtains the geometrical relationship between the radial position and direction \((r, \theta, \phi)\) in any plane \(z\) to a position and direction on the solution plane \((r_0, \theta, \phi_0)\).
\[
\begin{align*}
\rho_o &= r^2 + (z \tan \theta)^2 - 2rz \tan \theta \cos(\pi - \phi) \\
\phi_o &= \cos^{-1}\left(\frac{r_o^2 - r^2 + (z \tan \theta)^2}{2r_o \rho \tan \theta}\right)
\end{align*}
\]
(13)

(14)

Note that the elevation angle, \(\theta\), remains unchanged. As for the general case, the intensity along a straight line is constant leading to
\[
q''(r, z) = \int_{\phi=0}^{\pi} \int_{\theta=0}^{\pi/2} I_0(r_o, \theta, \phi_o) \cos \theta \sin \theta d\theta d\phi
\]
(15)

The flux distribution \(q''(r, z)\) is known from measurement or ray-tracing simulations using
\[
q_{j,l} = q''(j \Delta r, l \Delta z) \\
\]
(16)

discretization then yields
\[
q_{j,l} = \sum_{m=1}^{N_\theta \cdot N_\phi} \left[(1 - w_\rho)(1 - w_\phi)I_{0,j,m,N} + (1 - w_\rho)w_\phi I_{0,j+1,m,N} + (1 - w_\phi)(1 - w_\rho)I_{0,j+1,m,N+1}\right] \cos \theta_m \sin \theta_n \Delta \theta \Delta \phi
\]
(17)

where
2.4. Regularization

Inverse radiation problems often suffer from ill-conditioned coefficient matrices [21]. For continuous solutions, Tikhonov regularization can be employed to impose smoothness onto the solution [22]. It is implemented by appending a smoothing matrix \( L \) to the \( A \) matrix for each discretization variable.

\[
\begin{align*}
J &= \text{ceiling} \left( \frac{r_a - 2}{dr} \right), N = \text{ceiling} \left( \frac{\varphi - d\varphi}{2} \right) \\
\theta &= \frac{r_a - r_j}{dr}, \varphi &= \frac{\varphi_a - \varphi_N}{d\varphi}
\end{align*}
\]

(18)

\[
(20)
\]

\[
(21)
\]

where the smoothing matrix \( L \) connects element \( i \) to its neighbour \( j \) in the x-direction:

\[
L_{x,ii} = 1, \quad L_{x,ij} = -1 \quad \text{all other elements of } L \text{ are zero.}
\]

This method is controlled via one regularization parameter per dimension of the unknown quantity (only \( \lambda_x \) and \( \lambda_\theta \) in the two-dimensional case).

2.5. Monte Carlo Ray Tracing

An in-house Monte Carlo ray tracing program [11] is used to model the concentrating systems. The program features directional binning, which allows for the output of the intensity distribution at any desired location. The program provides the intensity distribution as well as the consistent, associated flux distribution, allowing for in-depth validation of the inverse method.

3. Results

3.1. Parabolic Trough

Parabolic troughs are the most common 2D concentrators. The EuroTrough 150 (ET-150) [4], which was developed under European Commission Project EuroTrough II (5\(^{th}\) Framework Program contract number ERK6-CT-1999-00018), was modelled using the Monte Carlo ray tracing code [11]. Surface and geometry imperfections are modelled through a Gaussian distributed angular error in the direction of reflected rays around the direction of perfectly specular reflection with a standard deviation of 5 mrad.
Table 1. Parameters of the parabolic trough.

| Parameter            | Value |
|----------------------|-------|
| Focal length (m)     | 1.71  |
| Rim angle (°)        | 60.0  |
| Sun Shape (mrad)     | 4.649 |
| Mirror Error (mrad)  | 5.0   |
| Mirror Reflectivity (-) | 0.95 |

The flux distributions in the focal plane and the associated intensity distributions are shown in Figures 6 and 7. The highest intensity values are found at the center of the target, where the concentration is the highest, while the intensity diminishes as the angle of incidence ($\theta$) increases. The flux diminishes as one moves away from the focal plane ($z^*=0.0$).

The inverse method features a range of parameters that influence the accuracy of the solution. These parameters include the number of discretization steps in both the $\theta$ and $x$-direction ($N_\theta$ and $N_x$) and the length of the solution plane ($L_{sol}$). A range of parameter values was set based on the desired resolution of the solution. The inverse solution was generated for each possible combination of parameter values. Solutions were compared based on the RMS difference between each inverse and Monte Carlo solution with the lowest value corresponding to the optimal solution.

Figure 5. 2D parabolic trough as modeled in VeGaS. The useable output from the model is the intensity distribution on the target.

Figure 6. Parabolic trough scaled flux distribution at multiple distances ($z^*=z/f$) from the focal plane.

Figure 8 shows the inverse solution based on the flux distributions from Figure 6.
Figure 7. Scaled intensity distribution ($I/I_{\text{max}}$) along the focal plane of a 2D parabolic trough from Monte Carlo ray tracing.

Figure 8. Scaled inverse solution results ($I/I_{\text{max}}$) at the focal plane ($z^*=0.0$). $N_\theta = 15$, $N_{x_0} = 20$, $L_{x_0} = 0.12$ m.

The inverse solution results closely match the Monte Carlo solutions. The discontinuity at larger angles of incidence is reproduced by the inverse solution. The largest discrepancies between inverse and Monte Carlo solutions are concentrated around areas with large gradients and discontinuities in the intensity. The optimal parameters for the parabolic trough were found to be $N_\theta=15$, $N_{x_0}=20$ and $L_{x_0}=0.12$ m. The inverse solution has a maximum error of 9% with a RMS relative error of 3.1%. The linear system is of full rank with a characteristic number of 1000. Tikhonov regularization did not improve results this is attributed to the discontinuous nature of the solution.

3.2. Elliptical Trough

An elliptical trough-based Vortek-type linear concentrator set-up [1] is studied. The set-up (Table 2, Figure 9) consists of an Argon long arc lamp, an elliptical trough reflector and a cylindrical mirror on the underside of the arc. Secondary mirrors are attached at the outlet of the elliptical trough to reduce losses.

Table 2. Elliptical trough parameters

| Parameter               | Value |
|-------------------------|-------|
| Semi-major Axis, $a$ (m)| 0.277 |
| Semi-minor Axis, $b$ (m)| 0.190 |
| Sun Shape (mrad)        | 4.649 |
| Mirror Error (mrad)     | 5.0   |
| Mirror Reflectivity (-) | 0.90  |
| Collector Length (m)    | 100   |
The flux and intensity distributions from Monte Carlo ray tracing are shown in Figures 10 and 11. The inverse solution is shown in Figure 12. The optimal parameters for the elliptical trough were found to be $N_\theta=5$, $N_{xo}=41$ and $L_{xo}=0.41$ m. The solution has a maximum error of 18% with an RMS relative error of 5.1%. Tikhonov regularization significantly improves the results in the elliptical trough case. The optimal values for the Tikhonov coefficients are $\lambda_\theta=5$ and $\lambda_x=12$. The condition number of the regularized linear system is 16.4.

4. Conclusions
An inverse solution method to identify intensity distributions at the focal plane of two-dimensional concentrating systems has been developed. The method uses multiple flux maps perpendicular to the concentrator axis to predict the intensity distribution in the focal plane. The
mathematical derivation of the circular symmetric and general, three-dimensional cases has been presented. A parabolic trough setup was explored using an in-house Monte Carlo ray-tracing program. The optimal discretization parameters for this setup were found to be $N_\theta=15$, $N_w=20$, and $L_w/f=0.0702$. The results show that the inverse solution process can be applied to a set of two-dimensional flux measurements to successfully predict the intensity distribution. The maximum relative error was less than 10% with an RMS relative error of 3.1%. Furthermore, an elliptical trough setup was also implemented. Tikhonov regularization was used to overcome ill-conditioning of the problem. The elliptical trough solution was optimal for discretization parameter $N_\theta=5$, $N_w=41$, and $L_w=1.7559$ and Tikhonov parameters $\lambda_g=5$ and $\lambda_s=12$ resulting in a relative error of less than 18% and corresponding RMS relative error of 5.1%. In the parabolic trough case Tikhonov regularization has a negative effect due to the discontinuous nature of the solution.

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