A denoising algorithm for linear frequency modulation signal based on window function and empirical mode decomposition

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Abstract. The linear frequency modulation signal is widely used in radar, sonar detection and communications as a typical non-stationary signal. In order to better analyse the linear frequency modulation signal received by the receiver, it is necessary to perform denoising processing on the received signal. As a new type of time-frequency analysis method, empirical mode decomposition algorithm is very suitable for processing non-stationary signals. However, due to the large bandwidth of the linear frequency modulation signal, more noise will be retained when the traditional empirical mode decomposition algorithm is used to denoise the linear frequency modulation signal. Under above background, this paper proposes a denoising method based on the window function and the empirical mode decomposition to achieve further denoising of the linear frequency modulation signal. According to the simulation results, when the signal-to-noise ratio is about 0dB, the method proposed in this paper improves the signal-to-noise ratio by about 1.5dB compared with the traditional empirical modal decomposition algorithm.

1. Introduction
The linear frequency modulation (LFM) signal is a very typical non-stationary signal, which is characterized by a large product of time width and bandwidth. Through pulse compression technology, LFM signals can improve the detection capability and anti-jamming capability of the radar system [1], so it has been widely used in the field of radar and sonar. In a complex electronic environment, the LFM signal received by the receiver is usually a noisy signal. In order to reduce the influence of noise on subsequent signal processing such as extracting the radio frequency fingerprint of the LFM signal, it is necessary to denoise the noisy signal first. Generally, time-frequency analysis methods such as short-time Fourier transform [2], wavelet decomposition [3] and empirical mode decomposition (EMD) algorithms [4] are often used to analyse non-stationary signals like LFM signals. Among them, due to the limitation of Heisenberg's uncertainty principle, the short-time Fourier transform method cannot give a good balance between time resolution and frequency resolution in non-stationary signal processing. Wavelet decomposition can solve the problems of short-time Fourier transform very well, but it faces the problem of choosing a suitable wavelet basis function. The choice of wavelet basis function has a great influence on the denoising result. Compared with the above two algorithms, the EMD algorithm has the characteristics of multi-resolution decomposition and adaptive decomposition [5]. This allows it to satisfy both time resolution and frequency resolution, and does not need to choose basis functions like wavelet decomposition, so it is widely used in the analysis and processing of non-stationary signals. However, the traditional EMD denoising algorithm removes noise by separating and discarding the intrinsic mode function whose spectrum is outside the signal spectrum [6-7]. This method can effectively remove the noise whose spectrum is outside the signal spectrum, but it cannot remove the noise whose
spectrum is inside the signal spectrum. Therefore, when denoising the LFM signal by the EMD algorithm, due to the wide spectrum of the LFM signal, more noise is retained. If we decompose a wideband LFM signal into a series of narrowband signals through time domain windowing, since the frequency band of the obtained signal is narrow, applying the EMD algorithm to the narrowband signal can remove most of the in-band noise of the original signal. Therefore, according to the characteristics of the LFM signal, this paper proposes an LFM signal denoising algorithm based on EMD and time-domain windowing.

The remainder of this paper is organized as follows. Section II introduces the related algorithms involved in the method proposed in this paper. Section III shows the denoising result through the simulation process of a noisy LFM signal. Finally, Section IV concludes this paper.

2. Materials and Methods

2.1. Time domain windowing

Linear frequency modulation signal is a non-stationary signal whose frequency changes linearly with time, and the expression is shown in formula 1.

\[
s(t) = \text{rect}\left(\frac{t}{T}\right) \exp(j2\pi(f + 0.5\mu t + \varphi)) \quad 0 \leq t \leq T
\]

where \(\text{rect}(t)\) is a rectangular function, \(T\) is the pulse duration, \(f\) is the initial frequency of the signal, \(\mu\) is the chirp rate, and \(\varphi\) is the initial phase of the signal.

In signal processing, window function is a real function that only has values in a given interval. Therefore, when an LFM signal \(s(t)\) is multiplied by a window function, a short-term signal of \(s(t)\) can be obtained. As the window function move on the time axis, we can get a series of short-term signals \(s_1(t), s_2(t), ..., s_n(t)\). Common window functions include rectangular windows, triangular windows, Hamming windows and so on [8].

2.2. Empirical mode decomposition algorithm

The Empirical Mode Decomposition (EMD) algorithm was proposed by Dr. Huang in 1998. It is a time-frequency analysis method, which makes it very suitable for processing non-stationary signals. The EMD process is similar to applying a high-pass filter to a signal, and then a series of high-to-low frequency components are obtained. These components are called the intrinsic mode function (IMF). Every IMF meets the following two conditions:

- The number of local extreme points and zero-crossing points of the IMF differs by at most one.
- At any time, the upper envelope formed by IMF local maximum interpolation and the lower envelope formed by IMF local minimum interpolation have a mean value of zero.

Based on the above conditions, the process of extracting the intrinsic mode function of the signal \(s(t)\) is shown in Figure 1. In Figure 1, the end condition of the empirical mode decomposition is generally that the residual signal \(r(t)\) is monotonous or less than a small threshold \(SD\), and Dr. Huang gave the \(SD\) expression as formula 2, where \(h_k(t)\) and \(h_{k-1}(t)\) are the \(k\)th and \((k - 1)\)th intrinsic mode function.

\[
SD = \frac{1}{T} \int_0^T \frac{|h_k(t) - h_{k-1}(t)|^2}{|h_{k-1}(t)|^2} d(t)
\]

Through the EMD process in Figure 1, we can get a series of intrinsic mode functions \(IMF_1, IMF_2, ..., IMF_n\) of the signal. These intrinsic mode functions represent different frequency components of the signal. Choosing different intrinsic mode functions to reconstruct the signal can be regarded as applying different filters to the signal as in formula 3.
where \( h(t), b(t), l(t) \) respectively represent the signal obtained by using a high-pass filter, a band-pass filter and a low-pass filter on the original signal.

\[
\begin{align*}
    h(t) &= \sum_{i=1}^{k} IMF_i(t) \\
    b(t) &= \sum_{i=j+1}^{k} IMF_i(t) \\
    l(t) &= \sum_{i=k+1}^{n} IMF_i(t) + r(t), \quad 1 < j < k < n
\end{align*}
\] (3)

Figure 1. EMD algorithm flow chart.

3. Results & Discussion

3.1. Simulation analysis
In this section, we design an LFM signal and superimpose additive white noise with a SNR of 5dB on the signal to verify the effectiveness of the method proposed in this paper. The original signal waveform and the noisy signal waveform are shown in Figure 2.
Figure 2. Real signal and noisy signal.

We can first analyse the denoising effect of the traditional EMD algorithm on the LFM signal. Apply the EMD algorithm to the noisy signal in Figure 2, and the result is shown in Figure 3. In Figure 3, the first signal is the noisy signal to be decomposed, and the last five signals are the obtained intrinsic mode functions $IMF_1 - IMF_5$. It can be seen from Figure 3 that the signal duration in $IMF_6$ and $IMF_7$ is relatively long and the frequency is relatively high. According to the characteristics of the LFM signal, these two intrinsic mode functions must have a relatively large bandwidth, which causes more noise whose spectrum is within the signal bandwidth to be retained. The signal glitches in $IMF_2$ and $IMF_3$ also indicate that a lot of noise components are retained.

Figure 3. Noisy signal and its first five intrinsic mode functions.

Figure 4. Windowed noisy signal and its first five intrinsic mode functions.

Considering that in a short period of time, the spectrum of the LFM signal is very narrow, while the noise spectrum is distributed throughout the frequency band. Therefore, we can first window the LFM signal to obtain a series of short-term signals. The spectrum of these short-term signals is composed of a narrow-band real signal spectrum with a large amplitude and a wide-band noise spectrum with a small amplitude. By applying the EMD algorithm to these short-term signals, we can achieve the target of preserving the narrow-band real signal while removing out-of-band noise as much as possible. The results obtained by applying the EMD algorithm to the windowed noisy signal are shown in Figure 4. In Figure 4, the first signal is the windowed noisy signal to be decomposed, and the last five signals are the obtained intrinsic mode functions $IMF_1 - IMF_5$. It can be seen from Figure 4 that $IMF_6$ basically shows the trend of the original signal and is almost unaffected by noise. Signal denoising can be achieved by reconstructing the signal using the intrinsic mode function of $IMF_6$ and after $IMF_5$.

According to the above description, the basic steps of the algorithm proposed in this paper are as follows:

- Windowing the LFM signal to generate a series of short-term signals.
Apply the EMD algorithm to the short-time windowed signal, and select the appropriate intrinsic mode function to reconstruct the signal.

Combine the denoising results of different short-time windowed signals into a complete signal.

3.2. Denoise effect evaluation

![Image of waveform comparison](image)

Figure 5. Waveform of noisy signal and denoised signal.

We can evaluate the denoising effects of different algorithms from the signal waveform or the signal-to-noise ratio. Figure 5 shows the comparison of the denoising waveform results based on the traditional EMD algorithm and the EMD joint window function algorithm proposed in this paper. It can be seen from Figure 5 that the signal after denoising using the traditional EMD algorithm has more noise in the first 500 signal points. These noises are mainly caused by $IMF_2$ and $IMF_3$ in Figure 3. When using the method proposed in this paper, the waveform of the first 500 signal points of the denoised LFM signal is smoother. This illustrates the superiority of the method proposed in this paper.

Table 1. Denoising results of two denoising methods under different SNRs

| Noisy Signal SNR(dB) | The SNR of the signal after denoising |  |
|----------------------|--------------------------------------|---|
|                      | EMD                                  | EMD and window function |
| -0.35                | 4.78                                 | 6.29 |
| 5.05                 | 9.22                                 | 10.49 |
| 9.87                 | 13.46                                | 14.86 |
| 15.04                | 17.98                                | 18.23 |

The signal-to-noise ratio is the ratio of signal power to noise power, which can characterize the amount of noise. Table 1 shows the denoising effects of the above two methods on LFM signals under different signal-to-noise ratios. It can be seen from Table 1 that compared with the traditional EMD
algorithm, the method proposed in this paper can further improve the signal SNR under different signal-to-noise ratios. When the signal-to-noise ratio is around 0dB, the EMD joint window function algorithm improves the SNR by 1.5dB.

4. Conclusion
In order to solve the shortcomings of traditional EMD algorithm in LFM signal denoising, we propose a new method of LFM signal denoising based on EMD algorithm and window function in this paper. By windowing the LFM signal, this method effectively solves the problem that more noise is retained due to the relatively large bandwidth of the intrinsic mode function obtained by traditional EMD algorithm. The simulation results show that when the SNR of the signal is approximately in the range of 0dB to 10dB, the method proposed in this paper can achieve a SNR improvement of more than 1dB compared with the traditional EMD algorithm. Therefore, when processing some LFM signals in low signal-to-noise ratio environments, such as denoising the receiving radar LFM signals in a complex environment, the method proposed in this paper can achieve better denoising effects. However, during the simulation process, we found that the length of the window function has a great influence on the denoising results, and the length of the window function is manually set through multiple simulation analyses. Therefore, we can further study how to obtain the adaptive window length according to the intrinsic mode function waveform of the original signal, so as to improve the efficiency of the simulation work.

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