Laser-induced ultrafast insulator–metal transition in BaBiO$_3$

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We investigate ultra-fast coherent quantum dynamics of undoped BaBiO$_3$ driven by a strong laser pulse. Our calculations demonstrate that in a wide range of radiation frequencies and intensities the system undergoes a transient change from the insulating to the metallic state, where the charge density wave and the corresponding energy spectrum gap vanish. The transition takes place on the ultra-fast time scale of tens femtoseconds, comparable to the period of the corresponding lattice vibrations. The dynamics is determined by a complex interplay of the particle-hole excitation over the gap and of the tunneling through it, giving rise to the highly non-trivial time evolution which comprises high harmonics and reveal periodic reappearance of the gap. The time evolution is obtained by solving the dynamical mean field theory equations with the realistic parameters for the system and radiation. Results are summarized in the phase diagram, helpful for a possible experimental setup to achieve a dynamical control over the conduction state of this and other materials with the similarly strong electron-phonon interaction.

I. INTRODUCTION

Perovskite compounds based on BaBiO$_3$ (BBO) are a special class of the high-temperature superconductors (HTSC) that do not include transition metal ions. As a rule, such structures have a complex phase diagram with many structural phases that depend on the doping level as well as on the temperature. These include dielectric, metallic and superconductive phases, with the critical temperature of the superconductive transition varying between $T_c = 13K$ for the led doped materials$^5$ and $T_c = 34K$ for the potassium doping$^2$.

Unique properties of those materials are related to their peculiar crystal structure and its dynamical distortions (phonons), and to the Fermi surface configurations. For example, when doped with potassium with the concentration $x \geq 0.37$, the local electronic structure is characterised by a spatially separated mixture of free electrons and localized electron pairs$^{3,5}$. The electron pairs determine the charge transfer and formation of the superconductive state, while free electron states play a role in the insulator-metal phase transition and the appearance of the Fermi-liquid state above the percolation threshold$^6$.

Bismuthates are characterized by strong deformations of the crystal lattice associated with the anomalously high amplitude of their phonon oscillations$^7$. In this context the most interesting is the "breathing" phonon mode with alternating sizes of the BiO$_6$ octahedra and Bi–O bonding along three crystallographic directions$^{8,9}$ (see Fig. 1). The size alternation involves the charge transfer by electron pairs tunnelling between the two neighbouring octahedra. The octahedron with the extra electron pair is larger, whereas the one without it is smaller. This migration of electron pairs can be interpreted as the "valence disproportionation" where Bi$^{3+}$ and Bi$^{5+}$ ions are formed instead of two neighbouring Bi$^{4+}$ ions$^{10,11}$. Different electronic filling of the Bi$^6s$–O$^2p$$_x$ orbitals of the neighboring BiO$_6$ octahedra creates a dynamical double-well potential for the vibration of oxygen ions, experimentally observed in EXAFS spectra$^{6,12}$.

The inherently strong coupling between electrons in the conduction band and lattice distortions profoundly affects the carriers state in those materials. Specifically, in the parent compound BaBiO$_3$ with the Fermi level in the middle of the conduction band this coupling gives rise to the charge density wave (CDW) accompanied by the optical gap in the single-particle electronic spectra of the system, estimated in various studies as being $\simeq 2eV$$^{7,13–16}$.

Details of the electronic spectra as well as of the carrier-phonon coupling in those materials can be probed by studying their properties when the system is externally driven from its equilibrium, creating the system dynamical response. For example, the gap structure of the spectrum is investigated by the pump-and-probe measurements of the optical conductivity (see$^{17,18}$), where the charge carriers are excited above the gap leading to a transient metallic state, gradually relaxing towards the original insulating state by the electron-hole recombination. The system can be driven so strongly that its original phase is transiently changed.

An adequate interpretation of such experiments requires modelling the dynamics far from the equilibrium. This has to be done beyond the single-particle picture, since the system state is altered considerably. In addition, many relevant non-equilibrium phenomena take place on the ultra-fast time scale, where quantum coherences play a large role and cannot be neglected.

Sufficiently accurate and consistent analysis of the ultra-fast dynamic in strongly correlated systems, performed on equal footing with the analysis of their equi-
librium state, became possible relatively lately, owing to rapid advances in the computing power. The dynamics of strongly-correlated systems far from equilibrium have been addressed in a number of recent works, that studied, in particular, electron-mediated relaxation after ultra-fast pumping, dynamical redistribution of spectra, the role of the phonon-carrier coupling in the time-resolved optical spectra, dynamics of the photodoped charge transfer insulators, the gap dynamics in excitonic insulators and the dynamics of a periodically driven strongly-correlated Fermi-Hubbard model.

The BBO is a representative example of the systems where the insulating CDW state appears as the result of the strong phonon-carrier coupling. Whether the CDW in this material can be externally manipulated and the metallic state achieved transiently by applying a strong driving is a very interesting problem, which until now has been investigated neither experimentally nor theoretically. Generally, little is known about the dynamics of this structure when it is driven far from equilibrium.

In this work we investigate the ultra-fast dynamics of the BaBiO$_3$ excited by a strong continuous laser pulse. In our analysis we consider a time evolution of the system retaining all pertinent quantum coherences within the dynamical mean field approach adapted to study strongly correlated systems. The chosen level of the theory is sufficient to describe both the metal-insulator transition of the system and its ultra-fast time evolution on the equal footing, ensuring consistency of the results.

Our analysis demonstrates that a sufficiently strong excitation can break the insulating state and the system evolves into the metallic phase on the time scale of tens femtoseconds. It is shown that the dynamics of this regime is highly non-linear, comprises many high harmonics and even develops the continuous high frequency spectra. Although our study focuses on BaBiO$_3$, the results are general and can be used as a guidance for experimental setups to control the state of system with the strong electron-phonon interaction.

The paper is organized as follows. In Section II we introduce the model, describe the used assumptions, present necessary details of the formalism and discuss the choice of the system parameters. Results of the calculations are found in Section III in which we give examples of the various types of the dynamical behaviour of the system as the reaction to the applied excitation field, and give a summary of the obtained results in the form of the phase diagram. Finally, Section IV briefly summarizes results of the work and outlines future perspectives.

II. MODEL AND METHODS

A. Holstein model for BaBiO$_3$

To describe the metal-insulator transition in BaBiO$_3$ we adopt the tight-biding (TB) model for electrons that occupy BiO$_6$ octahedra and can tunnel between them. The octahedra are regarded as "sites" and the hopping elements describe the tunnelling. The coupling to the lattice deformation modes is taken into account within the Holstein model where an electron of an octahedron is coupled to the distortion oscillation (phonon) mode of this octahedron. This model neglects the interaction between phonon modes of different octahedra. The Hamiltonian of the Holstein model is

$$H = -\sum_{ij\sigma} t_{ij\sigma} c_{i\sigma}^\dagger c_{i\sigma} + \text{h.c.} - \mu \sum_i n_i + g \sum_i (b_i^\dagger b_i + b_i^\dagger b_i^\dagger) (n_i - 1),$$

(1)

where $c_{i\sigma}$ denotes the electron operators with spin $\sigma$ located at site $i$, $n_i = n_{i\uparrow} + n_{i\downarrow}$ is the electron number operator with $n_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma}$, $\mu$ is the chemical potential, $t_{ij}$ are the hopping integrals, $b_i$ is a phonon operator corresponding to the deformation mode on site $i$, $\omega_{ph}$ is the mode frequency and $g$ is the phonon-electron coupling constant. Values of $t_{ij}$ are extracted from the ab initio calculations by comparing obtained single-particle
density of states (DOS) with the results for the tight-binding model (1). The comparison yields that the tight-binding model needs to account for the nearest as well as the second- and the fourth-nearest neighbours. The corresponding hopping matrix elements are extracted as \( t_1 = -0.45 \text{eV}, t_2 = -0.09 \text{eV}, t_4 = 0.10 \text{eV}, \) respectively, while the chemical potential is \( \mu = -0.13 \text{eV}. \)

### B. Bethe lattice model

The Holstein model (1) appears simple, however, is still quite difficult to solve, even within the dynamical mean field theory (DMFT). However, one can simplify the calculations considerably by taking into account details of the metal-insulator transition. The transition is associated with the CDW state where the size of the neighbouring octahedra BiO\(_6\) alternate. The alternation of the neighbouring sites can be conveniently described by separating the original system into two sublattices of the double-size elementary cell, each containing octahedra of the same state. Within this model the metal-insulator transition takes place when electronic states of the sublattices become different.

We are not interested in the details of the dynamics within each of the sublattices and, therefore, can use a simplified model that neglects them. A minimal model for each sublattice is a single unit cell with two electron states and a single photon mode. It is, however, not acceptable for the reason it contains only two electronic degrees of freedom, while we need a continuum of such states in order to describe quantum dynamics and relaxation adequately.

In the calculations we adopt the Bethe lattice model for each sublattice, schematically illustrated in Fig. 2 for the coordinate number 3. This model has a continuum of electronic states thereby satisfying the main requirement to describe the quantum relaxation. It is effectively infinite dimensional model and its main advantage in the context of this work is that the DMFT formalism in this case yields exact results. One can thus interpret this approach as choosing the model that is sufficiently close to the original system to describe its physics but for which the mean-field calculations can be trusted. It is commonly assumed that Bethe lattice models offer a good approximation for 3D systems. They are frequently employed to investigate dynamical properties of various many-body problems (see, e.g., 26-31, and also 12 and references therein).

For the BaBiO\(_3\) structure of interest our final model consists of two equivalent Bethe lattices (see Fig. 2) where electron can hope between the nearest neighbours of each sublattice (intra-lattice hopping) as well as between the equivalent sites of different sublattices (inter-lattice hopping). In the following we refer to this model as the double Bethe lattice model (DBLM). The Hamiltonian of the model reads as

\[
H = -t_\parallel \sum_{\langle i,j \rangle} c_{i \alpha \sigma}^\dagger c_{j \alpha \sigma} - t_\perp \sum_{\alpha \sigma} \left( c_{i \alpha \sigma}^\dagger c_{i+1 \alpha \sigma} + \text{h.c.} \right) + \mu \sum_{\alpha \sigma} n_{i \alpha \sigma} + g \sum_{\alpha \sigma} \left( b_{i \alpha}^\dagger b_{i \alpha} \right) \left( n_{i \alpha \sigma} - 1 \right) + \omega_{\text{ph}} \sum_{\alpha \sigma} b_{i \alpha}^\dagger b_{i \alpha},
\]

(2)

where \( \alpha = 0, 1 \) is the sublattice index, the sum with \( \langle i,j \rangle \) goes over the nearest neighbour sites, \( t_\parallel \) and \( t_\perp \) denote the intra- and inter-lattice hopping matrix elements, respectively. Their values are chosen such that the model yields the best possible approximation for the DOS. Figure 3 shows the band structure (Panel a) and the DOS (Panel b) of the TB model as well as the DOS of the DBLM. The best fit of the DOS is achieved for \( t_\parallel = 1 \text{eV} \) and \( t_\perp = 0.5 \text{eV} \). The Bethe lattice gives a qualitatively similar DOS but it does not capture the van Hove singularity in the middle of the band. The van Hove singularity facilitates the instability towards the formation of the CDW state when the Fermi level is close to it, \( \mu \approx 0 \). However, the DBLM demonstrates this instability without the singularity in the DOS.

Finally, the coupling between the laser field and the system carriers is taken into account via the Peierls substitution, where the hopping integrals are assigned with the phase as \( t_{ij} \rightarrow t_{ij} \exp(i \varphi_{ij}) \), calculated as the integral of the vector potential \( \varphi_{ij} = (\mathbf{e}/c) \oint_{ij} (\mathbf{A} \cdot d\mathbf{r}) \) between the sites \( i \) and \( j \). However, as we are interested in the dynamics associated with the movement of electrons between the sublattices, the driving of electrons within the sublattice will be neglected and thus the field will affect only the hopping between the sublattices as

\[
t_\perp(t) = t_\perp e^{i \varphi(t)}, \quad \varphi(t) = f(t) \sin(\omega_0 t),
\]

(3)

where \( \omega_0 \) is the frequency of the driving oscillations and \( f(t) \) is the envelope function of the pulse. In this work we...
assume the constant pulse that arrives at time $t = 0$ and has a constant amplitude, $f(t) = \Theta(t)A_0$, where $A_0$ is the dimensionless driving strength and $\Theta(t) = 1$ at $t > 0$ and zero otherwise.

### C. DMFT for the Bethe lattice model

To calculate the system dynamics we employ the DMFT approach\textsuperscript{25}, which maps the original many-body problem to a model with an effective impurity. Its application to the model with a single Bethe lattice has a number of crucial simplifications with respect to the original TB model\textsuperscript{29}. Here we extend this approach to the DBLM in Eq. (2), where the impurity becomes a dimer, defined on both sublattices. The main DMFT equations in this case retain their general appearance, however, acquire a form of the $2 \times 2$ matrix equations for the corresponding matrix Green functions. For the convenience of the reader we briefly outline the main equations for the dynamics together with the used approximations.

All pertinent physical quantities of the system are found from the two-time electron $G(t, t')$ and phonon $D(t, t')$ matrix Green functions for the dimer impurity, which are obtained by solving the Dyson equations

$$D(t, t') = D_0(t, t') + \int_C d\tau d\tau' D_0(t, \tau) \Pi(\tau, \tau') D(\tau', t'),$$

$$[i\partial_t - \epsilon(t)] G(t, t') - \int_C d\tau \Sigma(\tau, t) G(t, \tau) = \delta_C(t, t'),$$

where the integration is performed along the Kadanoff–Baym (KB) contour $C$ in Fig. 4 and $\delta_C(t, t')$ is the $2 \times 2$ matrix delta function defined on this time contour. The phonon bare Green function is defined by

$$D_0^{-1}(t, t') = -\frac{1}{2\omega_{ph}} \left( \delta^2 + \omega^2_{ph} \right) \delta_C(t, t'),$$

and $\epsilon(t)$ is the one-particle Hamiltonian,

$$\epsilon(t) = \left( \mu + \sum_{\alpha}^{MF} \frac{t_{\alpha}(t)}{t'} \frac{t^*_{\alpha}(t)}{t} \mu + \sum_{\alpha}^{ME}(t) \right).$$

with the mean-field contribution to the self-energy

$$\Sigma_{\alpha}^{MF}(t) = gX_{\alpha}(t),$$

where the sublattice displacement is calculated by averaging the phonon operators as given below. The impurity self-energy for electrons in Eq. (4) comprises two components,

$$\Sigma(t, t') = \Sigma_{\alpha}^{ME}(t, t') + \Delta_G(t, t'),$$

where the hybridization function $\Delta_G(t, t')$ in the case of the Bethe lattice has the form\textsuperscript{25}

$$\Delta_G(t, t') = t^2 I(t, t'),$$

whereas for the contribution $\Sigma_{\alpha}^{ME}$ to the self-energy beyond the mean-field we use the self-consistent Migdal approximation\textsuperscript{33,34}

$$\Sigma_{\alpha}^{ME}(t, t') = ig^2 D(t, t') G(t, t').$$

For the the phonon impurity self-energy this approximation yields

$$\Pi(t, t') \rightarrow \Pi_{\alpha}^{ME}(t, t') = -2ig^2 G(t, t') G(t, t').$$

Dyson equations (4) are solved by introducing four components of the Green functions each corresponding to different time intervals of the KB contour. Namely, we use the lesser $G_{\alpha}(t \in C_1, t' \in C_2)$, the retarded $R_{\alpha}(t \in C_1, t' \in C_2)$ ($t > t'$), the left-mixing $L_{\alpha}(t \in C_1, t' \in C_2)$, and Matsubara $G_{\alpha}^{MF}(t \in C_3)$ components. The same components are introduced for the phonon Green function. Solving the system (4) on the Matsubara time domain $t, t' \in C_3$ gives the equilibrium state of the system for times $t < 0$ before the pulse arrives.

Solutions to the Dyson equations are used to obtain the average lattice displacement needed for the self-consistent calculation of the Dyson equations

$$X_{\alpha}(t) = -\frac{2g}{\omega_{ph}} n_\alpha(0) +$$

$$+g \int_0^t d\tau D_{0,\alpha\alpha}(t, \tau) \left[ n_{\alpha}(\tau) - n_{\alpha}(0) \right],$$

where the electron occupation numbers are obtained from

$$n_\alpha(t) = \text{Im} \left[ G_{\alpha\alpha}^{<}(t, t) \right].$$

Here $n_{\alpha} = 1$ corresponds to the double occupation (electron pairs) on sites of $\alpha$ sublattice.

The occupation numbers as well as the phonon displacement are used to trace the dynamics of the metal-insulator transition. The insulating phase with the CDW is characterized by a non-symmetric occupation $n_0 \neq n_1$ as well as by the stationary lattice distortion manifested in the non-zero phonon displacement averages, $X_0 = -X_1 \neq 0$. The metallic phase does not have the lattice distortion and has no difference between the charge distribution, $X_0 = X_1 = 0$ and $n_0 = n_1 = 0.5$.  

We also analyse the dynamics of the electron spectral function

\[ N(\omega, t) = -\frac{1}{\pi} \text{Im} \left[ \int_0^{\infty} d\tau e^{i\omega \tau} \sum \beta G^{\beta}_{\alpha\beta}(t + \tau, t) \right], \]

which gives us the single-particle DOS and the gap (the choice of \( t_\infty \) is dictated by the numerical efficiency).

Equations (4) are solved numerically using solvers implemented in the Non-Equilibrium Systems Simulation Library (NESSI)\(^{35}\) which were successfully used to study the dynamics of various correlated lattice systems, in particular, excited Mott insulators\(^{36,37}\) and periodically driven cold atom systems\(^{24}\). In this work we extended resources of this library to the case of the two site impurities and the matrix Dyson equations.

D. Material parameters

As noted above the hopping matrix elements of the TB model are extracted from the \textit{ab initio} calculations. The effective matrix elements of the Bethe lattice are obtained using two conditions: 1) the width of the band is equal to that of the TB model \( W \approx 6\text{eV} \), 2) the shape of the DOS for the Bethe lattice model should be as close as possible to the TB model. This yields \( t_\parallel = 1.0\text{eV} \) and \( t_\perp = 0.5\text{eV} \) for the intra- and inter-lattice hopping matrix elements.

Further, the phonon mode frequency is set to \( \omega_{\text{ph}} = 70\text{me} \), which corresponds to the Raman breathing mode of BaBiO\(_3\)\(^{38,39}\). The strength of the electron-phonon coupling is estimated from the value of the gap in the single-particle DOS \( \Delta \approx 2.5\text{eV} \). This yields the coupling constant \( g = 0.35\text{eV} \) which is close to \( g = 0.43\text{eV} \), reported earlier\(^{27}\). In the calculations we consider the case of zero doping with a half-filled band. The calculations are done at room temperature \( T = 290\text{K} \).

In what follows we investigate the dependence of the dynamics of the system on the frequency \( \omega_0 \) and intensity of the excitation laser pulse. We consider a broad interval of optical frequencies \( 1\text{eV} \lesssim \hbar \omega_0 \lesssim 4\text{eV} \). For the excitation intensity we use the dimensionless parameter \( A_0 = ea|E|/(\omega_0c\hbar) \), related to the field amplitude \( |E| \), where \( a = 4.34\text{Å} \) is the lattice constant of the BBO structure, \( e \) is the elementary charge, \( c \) is the speed of light. We consider relatively high intensities with the field amplitude \( |E| \sim \Delta/ea \). This corresponds to the laser intensity \( I_0 = |E|^2/Z_0 \approx 10^{13}\text{W/cm}^2 \), where \( Z_0 = 376.73\text{Ω} \) is the vacuum impedance.

III. DYNAMICS OF A DRIVEN SYSTEM

The dynamics of the system is investigated for the continuous driving, switched on at \( t = 0 \). The time evolution is obtained by solving the DMFT equations (4) briefly described above. Before the pulse arrival, at \( t < 0 \), the system is in the equilibrium state, calculated self-consistently using the same formalism.

The calculations are done by choosing the frequency \( \omega_0 \) and the amplitude \( A_0 \) of the driving pulse in the ranges shown in Fig. 5. From the complete time evolution we extract quantities that characterize the transition from the insulating to the metallic state of the system: electronic occupation numbers \( n_\alpha \) of the sublattice \( \alpha \), which yield the electronic (pair) occupation at different octahedra BaBiO\(_3\), the gap \( \Delta(t) \) in the single-particle DOS, and the phonon displacement \( X_\alpha(t) \) which measures the structural distortion of the octahedra.

Before the arrival of the pulse the system is in the CDW state where the electronic pairs are shifted to one of the sublattice so that \( n_0 = 1 \) and \( n_1 = 0 \), the gap in the DOS has a value \( \Delta(0) = 2.5\text{eV} \) and the octahedra are displaced with the amplitudes \( X_0 = 0.5\text{Å} \) and \( X_1 = -0.5\text{Å} \).

A. General remarks

Before presenting numerical results it is instructive to discuss general factors that determine the dynamical response of the system.

When the excitation pulse is weak the dynamics is defined mostly by the linear response. The system has a gap in the single-particle DOS, equal to the bipolaron binding energy, and thus the response strongly depends...
FIG. 6. Time evolution of the driven system calculated for parameters at points A-E in the diagram in Fig. 5 is shown, correspondingly, in rows of Panels A-E. Panels A1-E1 plot the time evolution of the electronic occupation in sublattices $\alpha = 0, 1$. The second Panel column A2-E2 shows the spectra $I(\omega)$ of the electron dynamics as follows from the analysis of the Fourier component of the inter-lattice current flow. Panels A3-E3 plot the time evolution of the dielectric gap. The square root function $\Delta(t) = \Delta(0)(1 - t/t_0)^{1/2}$ with $\Delta(0) = 2.5 \text{eV}$ is used to fit the time dependence of the gap close to its vanishing point.

on the relation between the excitation frequency and the gap. Here single-particle excitations take place when $\hbar \omega_0 > \Delta$. Analogously to semiconductors the pulse creates electron-hole pairs, which in principle destroys the insulating state making the system conductive. However, the rate of such processes is small so that excited electron-hole pairs recombine back to the ground state and the system remains in the original CDW state with the gap.

The situation becomes more complicated when the in-
tensity of the laser pulse increases and multi-photon assisted tunneling processes start to dominate the time evolution and the dynamics becomes non-linear. When the driving is very strong one can reach a situation where the electronic structure of the system changes so strongly that the lattice distortion disappears together with the gap resulting in the transient metal-insulator transition. One can roughly estimate the onset of such processes as when the field reaches such strength that it overweights the gap-related potential barrier between the different octahedra. Then electrons start moving freely between the sublattices thereby eliminating their asymmetric distribution, lattice static distortion and the spectral gap. This threshold field amplitude is estimated as \( eEa \simeq \Delta \), which yields the field strength interval chosen in this work.

Both the line of the resonance \( \Delta = \hbar \omega_0 \) and the threshold of the driving amplitude is given by \( A_{0} \omega_0 \simeq \Delta \) are shown by the dashed line in Fig. 5 defining the domains of the dynamics types. In this work we are mostly interested in the regime of stronger driving which can change the system state.

### B. Dynamics of the electronic subsystem

Results of the calculations reveal several distinct types of dynamics patterns. Depending on the final state of the system these are divided into two large classes, which occur in the colored domains in Fig. 5. In the regime we call "weak driving" (red domain) the evolution is slow and the system keeps its original insulating state even at very large times. In the regime of "strong driving" (blue domain) the system state rapidly changes to the gapless metallic state. These types interchange abruptly when the solid black crossover line in Fig. 5 is crossed. One notes that the transition line found from the numerical calculations is reasonably close to its simple estimate, discussed above, shown by the dashed line.

Figure 6 shows the time evolution calculated for several representative points A-E, shown in the diagram in Fig. 5. These points are selected to illustrate typical dynamical patterns in both the weak and the strong driving regimes. Panels (A1-E1) in Fig. 6 plot the time evolution of the occupation \( n_\alpha \) for sublattices \( \alpha = 0, 1 \). Panels (A2-E2) show the spectrum of the dynamics, calculated as the frequency dependent absolute value of the Fourier component of the inter-lattice current flow \( I(\omega) \sim |j(\omega)|^2 \), where \( j(\omega) \) is the Fourier transform of the current \( j(t) = \dot{n}_1(t) - \dot{n}_0(t) \). Finally, Panels (A3-E3) plot the time evolution of the gap in the single particle spectrum.

At point A [Fig. 5] the system is in the weak driving regime manifested by the weak time dependence of the electron occupation in Panel (A1). The corresponding changes in gap, shown in Panel (A3), are also small, \( \Delta(t) \) saturates when the time increases and shows no sign of abating. At the same time the spectra of the time evolution in Panel (A2) reveals that the dynamics comprises a large non-linear component with the noticeable contribution of the high harmonics even to the order \( n = 10 \). Appearance of high harmonics in this system is similar to that reported earlier for a strongly driven Mott insulator\(^{40} \). We note in passing that the name "weak driving" for this regime means only that the insulating state does not change in the evolution.

The dynamical pattern changes abruptly when the system crosses the crossover line and goes into the strongly driving regime (blue colour domain in Fig. 5). At
point B, located very close to the crossover line, the electronic occupation decay on the timescale of 50 fs [Panel (B1)]. Furthermore, the occupation rapidly approaches a new quasi-equilibrium state that has equal occupations, \( n_0 = n_1 = 0.5 \) indicating that the CDW is destroyed. This is also seen in Panel (B3) which demonstrates that the gap disappears completely after \( t_0 \simeq 40 \text{fs} \). It is worth noting that the gap disappearance is abrupt. Its time evolution follows not a gradual decay but approaches zero as a square root dependence \((1 - t/t_0)^{1/2}\), reminiscent of a time dependent phase transition.

The spectrum of the occupation dynamics in Panel (B2) demonstrates a large contribution of high harmonics. However, interestingly enough the relative weight of the higher harmonics decreases in comparison with point A [Panel (A2)]. Deeper in the strong driving regime (point C in Fig. 5) the dynamics remains qualitatively similar to that at point B. Still, the electronic occupation relaxes to its new quasi-equilibrium state much faster, the characteristic time for the gap disappearance decreases to \( t_0 \simeq 30 \text{fs} \). One notices a much longer oscillations in the time evolution of the occupation in Panel (C1). These are related to phonons and will be discussed below. It is important to note that point B is still substantially below the resonance line \( \Delta = \hbar \omega_0 \) and, therefore, the dynamics, at least initially, until the gap disappears, is mainly defined by the tunnelling through the potential barrier associated with the gap.

The point D is deeper in the strongly driving regime, approaching the resonance line \( \hbar \omega_0 = \Delta \). Strictly speaking, the resonance condition corresponds to the single-photon picture in the linear response, whereas in our case the dynamics is strongly non-linear and involve many-photon transitions. Nevertheless, the proximity to this line gives rise to noticeable changes in the dynamics because of the much increased contributions of the single-particle excitation processes in the initial stage of the time evolution.

Passing the resonance is manifested in a sharp decrease of the decay time in the electron occupation dynamics [Panel (D1)], accompanied by a similarly fast disappearance of the gap, \( t_0 \simeq 10 \text{fs} \) [Panel (D3)]. Remarkably, the gap reappears after \( t_0 \simeq 30 \text{fs} \). Qualitative changes in the dynamics are manifested in that the spectrum, shown in Panel (D2), becomes essentially continuous (although the peaks at the integer harmonics remain). The spectrum shows that the dynamics also has a significant contribution of a slow component with \( \omega \sim 0 \), related to the dynamics of phonons and the gap reappearance.

With a further increase in the frequency direct excitations over the gap become more and more significant. At point E, the frequency is so high that the laser field moves both electrons at the same time leading to the "overshoot" in the densities, \( n_0(t) < n_1(t) \) [Panel (E1)]. The effect of the gap reappearance becomes more pronounced [Panel (E3)], the gap reappears several times before vanishing completely. The spectrum in Panel (E2) becomes "more continuous"; the integer harmonics peaks are less pronounced. The higher frequency contribution to the spectrum is suppressed in comparison with that in points A-D.

C. Spectral gap and deformation dynamics

A further insight into the system dynamics is obtained by comparing the time evolution of the electronic degrees of freedom with that of the lattice deformation. The colour density plot in Fig. 7 shows the time dependence of the electronic DOS superimposed with the deformation amplitude \( X_\alpha(t) \) mediated by the phonon oscillations. As expected for the CDW state, the dielectric gap in the DOS closely follows the lattice distortion, disappearing precisely at points where it vanishes, \( X_\alpha = 0 \).

The gap reappearance decays on the time scale of \( \sim 100 \text{fs} \), whereas the phonon oscillations do not decay because the driving constantly pumps energy into the system. The phonon decay sets in when the driving is switched off. It has to be noted the Holstein model does not account for mechanisms for the phonon relaxation, and thus phonons decay only via their coupling to electrons. It should also be noted that in this work we are in the limit of very strong phonon-electron coupling. In this case we are in the limit of the reduced phonon relaxations in comparison to that for electrons, as were reported earlier for the Holstein model.

This is shown in Fig. 8, where the laser destroys the CDW state and then is turned off at \( t = 33 \text{fs} \). The phonon dynamics then induces oscillations in the electronic occupations, which decay gradually with the characteristic time of \( \tau_{ph} = 350 \text{fs} \). However, the new equilib-
rium state is not the original insulating CDW but is the metallic state with $n_0 = n_1 = 0.5$ without steady lattice distortions.

IV. CONCLUSIONS

This work studied ultra-fast dynamics of undoped BaBiO$_3$, excited by a strong continuous laser pulse, with a goal to investigate the laser-induced transition between insulating and metallic phases. The material is described by the Holstein model. The dynamics is obtained using the approach that combines the DMFT and the self-consistent Migdal approximation for the self-energy. The method takes into account quantum coherences and allows to calculate time evolution on the ultra-fast time scales. To simplify the calculations the tight-binding model for the electronic states is mapped on the double Bethe lattice model, for which the DMFT equations are greatly simplified.

The dynamics was calculated in a wide range of optical frequencies and excitation intensities. The calculations demonstrated two qualitatively different regimes: the weak driving regime, where the insulating phase remains intact, and the strong driving one, in which the system undergoes a transient change into the metallic state without CDW and lattice deformations. The transition between the two regimes is reliably estimated from the condition the driving field overcomes the gap in the single particle DOS.

The insulating state disappears abruptly: the gap vanishes with time by following the square root law, typical for phase transitions. The transition is of purely quantum nature, it is caused not by the temperature, but by the coherent driven dynamics of electron and phonon degrees of freedom. It is not connected to the nonlinearity: in both cases the dynamics can be highly nonlinear producing a large number of higher harmonics.

Typical characteristic times of the relaxations in the electronic subsystem and that of the insulator-metal transition are found to be in the range of tens of fs, considerably smaller than the characteristic phonon times. Thus the metallic state is achieved on the background of strongly oscillating lattice. However, those oscillations were found to affect the electronic states in a surprising way, causing a periodical reappearance of the gap. This takes place when the driving frequency exceeds the gap. The reappearance time is locked to the period of the lattice oscillations, although the amplitude of the reappearance decays much faster. We also noted that when the laser frequency exceeds the gap the spectrum of the dynamics becomes continuous, although the peaks related to the higher harmonics remain. In this case the spectrum of the electronic dynamics acquires a significant low frequency component, related to the gap reappearance.

Although the adopted model is simplified it is frequently used in the studies of similar structures and is expected to capture the essential physics of the metal-insulator transition, especially on the ultra-fast time scales where other relaxation processes are not important. The model parameters were chosen to match those of the real material. In the calculations we have exploited experimentally available laser frequencies and intensities. We also found that the appearance of different regimes can be understood using simple intuitive estimations, which are, nevertheless, quantitatively not very far from those obtained by the calculations, speaking in favor that the results can describe real dynamics and the transient metal-insulator transition in strongly driven BaBiO$_3$ compounds.

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