Abstract

The stochastic gradient descent (SGD) method is the most widely used for deep neural network (DNN) training. However, this method requires the use of appropriate regularization techniques to prevent overfitting and improve the generalization capability. Weight perturbations such as DropConnect and noise injection are widely used techniques. We propose an easy-to-use regularization method that adds symmetrical noises to the DNN weights. The proposed Symmetrical-SGD (S-SGD) algorithm evaluates the loss surface at two separate points during training to avoid convergence to sharp minima. The training stability is considerably improved by injecting fixed-magnitude symmetrical noise. The S-SGD method was applied to image classification using convolutional neural network models, end-to-end speech recognition with Gated ConvNet, and language modeling using recurrent neural network models. In all experiments, S-SGD outperformed conventional weight noise injection and dropout-based regularization techniques.

1. Introduction

Recently, there have been many studies on the shape of the loss function, and it has become clear that local minima that generalize well lie on wide valleys of the loss landscape, rather than in sharp, isolated minima (Hochreiter & Schmidhuber, 1997a; Keskar et al., 2017). In addition, the spectrum of Hessian matrix exhibits that well-generalized networks contain low eigenvalues of Hessian (Ghorbani et al., 2019).

The stochastic gradient descent (SGD) algorithm is widely used for deep neural network (DNN) training, but its convergent properties in non-convex optimizations are explained only in a probabilistic manner. Many types of research have been conducted to help SGD passing through saddle points and sharp minima by providing momentum or changing the learning rate in the weight updates (Jastrzkebski et al., 2018; Kleinberg et al., 2018; Chaudhari et al., 2017; Loshchilov & Hutter, 2017; Keskar & Socher, 2017; Johnson & Zhang, 2013; Smith, 2017; Jastrzebski et al., 2018; Kingma & Ba, 2015). In particular, SGD-based training demands the use of regularization techniques to prevent overfitting and improve the generalization capabilities. Popular regularization techniques include weight decay, Dropout, DropConnect, and noise injection to weights or gradients. (Zur et al., 2009; Ho et al., 2008; Murray & Edwards, 1993; An, 1996; Wen et al., 2018; Jin et al., 2017; Wan et al., 2013; Srivastava et al., 2014).

This study was strongly motivated by recent research on loss surface visualization and sharpness measurements (Li et al., 2018; Keskar et al., 2017). Loss surface flatness can be visualized by inferring a DNN model after injecting random noise to the weights. A flat surface indicates that the loss does not appreciably change when the weights are perturbed by the injected noise (Chaudhari et al., 2017). This suggests that we can improve the training of a DNN model while observing the loss surface or using the error term that depends on the surface flatness.

In this study, we propose a symmetrical SGD (S-SGD) algorithm that computes the loss and flatness to reach flat minima in DNN training. The loss surface flatness is measured using two sets of weights formed by adding and subtracting the same noises to the current model in adaptation. When arriving at minima, we can derive that the gradient for weight updates also depends on the Hessian, or the flatness. To reduce the fluctuation of loss via perturbation, we add symmetrical noises of a fixed amount to the weights. We compare this approach with the conventional SGD and weight noise injection-based methods using convolutional neural network (CNN) and recurrent neural network (RNN) models.

This paper is organized as follows. In Section 2, we review related works on regularization and loss surface measurement methods. The proposed training algorithm is described in Section 3. Section 4 includes the hyperparameter optimization. The experimental results are shown in Section 5. Section 6 discusses this algorithm in comparison to other...
regularization techniques. Section 7 concludes the paper.

2. Related Works

Many regularization techniques have been developed to ensure that DNNs do not overfit the training set. An early study showed that flat minima in the loss surface are closely related to good generalization (Hochreiter & Schmidhuber, 1997a). Recent studies also confirmed that reaching flat minima or wide valleys of loss landscape in DNN training provide robustness against weight and data disturbances, suggesting good generalization capabilities. Many training techniques have been developed to aid the SGD method to escape from sharp minima (Johnson & Zhang, 2013; Keskar & Socher, 2017; Jastrzkebski et al., 2018). One is a gradient update method, such as momentum (Qian, 1999), Adam (Kingma & Ba, 2015), RMSProp (Tieleman & Hinton, 2012), and Adagrad (Duchi et al., 2011). Learning rate scheduling is known to be an effective technique for improving the generalization capability. These techniques include linear learning rate scaling (Smith et al., 2017) and warm-up training (Loshchilov & Hutter, 2017).

Weight perturbation using noise modulation or injection has been studied for regularization for a long time. A well-known method is the Dropout that randomly drops some units during training (Srivastava et al., 2014). DropConnect is based on a similar idea, but it drops weights, not units, using random masks (Wan et al., 2013). Three types of noise injection methods have been studied; input data, weight, and gradient (Zur et al., 2009). Although the concept dates back more than 20 years, research remains quite active (Murray & Edwards, 1993; An, 1996). Several recent studies can be found in (Wen et al., 2018; Chaudhari et al., 2017).

Measuring flatness of local minima can help to understand DNN training and optimization. Eigenvalues of Hessian characterize the loss surface sharpness. However, Hessian computation is not feasible because a DNN typically has millions of parameters. Instead, (Keskar et al., 2017) proposed ϵ-sharpness, which measures the maximum value within a distance ϵ from the local minimum. (Li et al., 2018) visualized loss surfaces by projecting ‘filter normalized’ parameters to the space defined with random directions. This corresponds to adding weight noises of normalized magnitude many times and observing the loss surface. Recently, Entropy-SGD (Chaudhari et al., 2017) was developed to construct a local-entropy-based objective function that favors well-generalized solutions lying in large flat regions of the loss landscape.

3. Algorithm and Operation

In this section, we describe the conventional SGD, weight noise injection, and the proposed S-SGD algorithms. Then, the convergence property of the S-SGD is analyzed.

3.1. Conventional SGD, Weight Noise Injection, and S-SGD Methods

The conventional SGD method that operates with a batch size of B updates the weights using Eq. 1, as follows:

\[
\mathbf{w}_{t+1} = \mathbf{w}_t - \eta \sum_{i=1}^{B} \nabla L_i(\mathbf{w}_t),
\]

where \( L_i \) is the loss computed with the \( i \)-th data in the batch and \( \mathbf{w}_t \) are the trainable parameters. \( \eta \) is proportionally scaled to the batch size \( B \) (Zhu et al., 2018; Jastrzkebski et al., 2018). When the batch size is very large, approximately 1/10 of the total training data size, it is termed large batch training. The SGD method is based on the gradient descent, which was developed to solve convex optimization problems. However, the finite batch size introduces data-dependent variability or noise in inferring the loss function; as a result, the small-batch SGD method is known to be more effective in escaping from sharp minima than the large-batch SGD (Keskar et al., 2017; Goyal et al., 2017).

The weight and gradient noise injection algorithms were developed to provide perturbation to the weights, which can be described as follows

\[
\mathbf{w}_{t+1} = \mathbf{w}_t - \eta \sum_{i=1}^{B} \nabla L_i(\tilde{\mathbf{w}}_t),
\]

where \( \tilde{\mathbf{w}}_t = \mathbf{w}_t + \mathbf{n}_t \), and \( \mathbf{n}_t \) is the weight noise injected. Typically, the noise has uniform or Gaussian distributions (Wen et al., 2018). The weight update is conducted using the derivative of the loss obtained with noise-injected weights. As the weight noise perturbs the network, the loss surface measured is blunt; thus, the training algorithm cannot properly recognize sharp minima, increasing the chance of skipping them.

In this study, we propose a symmetric weight noise injection method, termed S-SGD method. The proposed algorithm injects two symmetrical noises, \( \mathbf{n}_t \) and \( -\mathbf{n}_t \), to the weight, \( \mathbf{w}_t \), to form \( \tilde{\mathbf{w}}_{t+} \) and \( \tilde{\mathbf{w}}_{t-} \). Forward and backward propagation operations are conducted using these two weight sets and the gradients obtained are averaged and used for the weight updates. The magnitude or L2 norm of the injected noise, \( \mathbf{n}_t \), is constant or slowly adapted and is proportional to the L2 norm of the weights for each layer. This scheme demands twice the amount of computation when compared to the conventional weight noise injection method. However, it shows very good ability in finding flat minima as presented in Section 4.
The update equation for S-SGD is as follows:

$$w_{t+1} = w_t - \eta \sum_{i=1}^{N} \nabla(L_i(\tilde{w}_{t+}) + L_i(\tilde{w}_{t-})),$$  \hspace{1cm} (3)

The structure of the S-SGD is shown in Figure 1. Two independent paths, one with $\tilde{w}_{t+}$ and the other with $\tilde{w}_{t-}$, are used for loss measurements and weight updates. Two symmetrical noises are added to the weights, and the magnitude of the noise is fixed or very slowly adjusted. The amount of injected noise is related to loss surface sharpness to escape. The weight update is conducted on the master weight only, $w_t$. In the next sub-section, we explain the S-SGD operation in more detail.

### 3.2. S-SGD Operation

The S-SGD method updates weights using the gradients measured at two symmetrically perturbed weights, $\tilde{w}_{t+}$ and $\tilde{w}_{t-}$. The forward and backward procedure using the proposed method is identical to that of the conventional SGD method; a difference, however, is that both $\tilde{w}_{t+}$ and $\tilde{w}_{t-}$ are used for forward and backward propagation.

For the simplicity of explanation, we assume to have only one variable $w$ instead of $w$, which means that $w$ is a one-dimensional vector and the added noise $n$ is a positive constant.

The loss with the perturbed weight, $\tilde{w} = w + n$, can be approximated as follows:

$$L(w + n) = L(w) + (L(w + n) - L(w))$$

$$= L(w) + \frac{(L(w + n) - L(w))}{n} \cdot n \hspace{1cm} (4)$$

According to the mean-value theorem, we have the following equation.

$$L(w + n) = L(w) + \nabla L(w + n_1) \cdot n,$$  \hspace{1cm} (6)

where $n_1$ is between 0 and $n$.

At the same way,

$$L(w - n) = L(w) - \nabla L(w - n_2) \cdot n,$$  \hspace{1cm} (7)

where $n_2$ is between 0 and $n$.

$$\nabla L_{S-SGD}(w) = \frac{\nabla L(w + n) + \nabla L(w - n)}{2}$$

$$= \nabla L(w) + \frac{\nabla^2 L(w + n_1) - \nabla^2 L(w - n_2)}{2} \cdot n \hspace{1cm} (8)$$

Note that the first term, $\nabla L(w)$, is the same as that of the derivative of SGD loss, while the second term is related to the second order derivatives.

In DNNs, $w_t$ is a multi-dimensional variable and the noise $n_t$ is also a multi-dimensional random number. Note that $n_t$ has a constant magnitude, but is randomly generated for each weight update.

In Eq. 6, we assume a derivative at $w + n_1$, which can easily be determined in the one-dimensional case. However, in the multi-dimensional case, this is a derivative only at the direction of noise $n$. As the dimension is as large as the number of parameters, it is not practical to evaluate the exact derivatives. Thus, we only measure the derivative in the direction of randomly generated noise or use random sampling. If the flatness of the loss function around the current weight in training is fairly smooth, we can assume to have a fairly good gradient estimate or the estimate of $\nabla L_t(w_t + n_t)$ and $\nabla L_t(w_t - n_t)$. Then, the $\nabla^2$ in Eq. 9 can be considered the approximation of the Hessian function obtained via random sampling.

Eq. 9 suggests that if $w$ is at the center of a flat minimum in which Hessian terms at $w_t + n_t$ and $w_t - n_t$ are similar,
it becomes an optimum point to reach; in contrast, if the Hessian terms at $w_t + n_{1,t}$ and $w_t - n_{2,t}$ are considerably different or have the opposite signs, the weight updates continue even if $\nabla L(w_t)$ is zero.

Figure 2 clearly shows the difference of solutions obtained by SGD and S-SGD. This is a one-dimensional misclassification ratio (MCR) function when the weights are perturbed along the direction of $w_{S\text{-}SGD}$ and $w_{S\text{-}SGD}$. This plot is obtained from the ResNet20 training using CIFAR-100 dataset. We can observe that the SGD solution corresponds to the point whose training loss or training MCR is the lowest. S-SGD finds the point whose loss is slightly higher than that of SGD, but its solution is moved toward the center of the flat minimum. The injected noise makes S-SGD maintain some distance from the sharp wall around the solution of SGD. The test MCR curve is also shown in this figure. Surprisingly, the test MCR obtained by S-SGD, which is 30.52%, is much lower than that of SGD, 31.74%. This is because the statistics of test data are not exactly the same as those of the training data. This figure clearly shows the regularization of S-SGD and its effects on generalization. The two-dimensional version of this plot is shown in Section 6.

The amount of computation demanded for S-SGD is about twice that of the conventional SGD. The convergence speed and reduction of computation are discussed in Section 4.

4. Optimization of S-SGD hyperparameters

Hyperparameters need to be carefully selected for DNN training. In this section, we consider the effects of injected noise levels and those of injection. We also perform an ablation study and propose a few variations of the S-SGD method that do not demand increased amount of training time. The large batch training results are also presented.

We conducted experiments with ResNet20, a popular model that employs batch normalization (Ioffe & Szegedy, 2015). CIFAR-10 and CIFAR-100 (Krizhevsky et al., 2010) datasets were used for parameter optimization. We applied channel-wise normalization to the training data and augmented them by random cropping with a size of 4 and horizontal flipping following (Lee et al., 2015). A momentum optimizer with a batch size of 128 was used for training, unless otherwise specified. The initial learning rate was 0.1, which decays by a factor of 10 at epochs of 75 and 125. We trained the models for 175 epochs. All the experiments were implemented in TensorFlow (Abadi et al., 2016).

4.1. Optimum Noise Level

The performance of S-SGD depends on the amount of injected noises. We also observed the effects of selective noise injection to either convolution or dense layers. Note that the Dropout and DropConnect were developed for the regularization of parameter rich dense layers.

We observed the effect of noise level for S-SGD and SmoothOut with ResNet20 on CIFAR-100 dataset. The SmoothOut is one of the weight noise injection algorithms (Wen et al., 2018). The noises were injected into weights of convolution and dense layers. Figure 3 shows that both S-SGD and SmoothOut yield the best results when the noise level is 0.4 $\sigma$ of the weights, where $\sigma$ is the L2 norm of weights for each layer. Although the optimum level is similar, S-SGD injects stronger noise than SmoothOut because it injects two noises and the noise strength of the former is always constant, while that of the latter is uniformly distributed between 0 and the specified noise level. Note that S-SGD shows consistent improvement at broad noise levels. Figure 3 also contains the results when the symmetrical noises are injected only to convolution or dense layers of ResNet20. We can observe that the symmetrical noise injection to convolution layers also yields good results, indicating that S-SGD is a flexible regularization method when compared to Dropout or DropConnect.

4.2. Convergence Curve and Training Time Reduction

The S-SGD method employs two forward and backward passes; thus, the training time for each epoch increases when compared to that of the conventional SGD method. Although this work was intended to find the best model, we examined the convergence speed.

Figure 4 shows the convergence curves of SGD and S-SGD for ResNet20 on CIFAR-10 dataset. The training was conducted for 175 epochs. Figure 4 (a) shows the beginning and intermediate stages of training, and Figure 4 (b) depicts the final stage. Here, we can notice that both SGD and S-SGD
convergences are similar at the beginning stage, but S-SGD outperforms SGD when applied to intermediate and final stages.

Table 1. Test accuracy (%) for ResNet20 on CIFAR-10 and CIFAR-100 under the equal computation budget with SGD. S denotes that the symmetrical noises are injected.

| Train Epochs   | CIFAR-10 | CIFAR-100 |
|----------------|----------|-----------|
| 75 - 50 - 50   | 92.17    | 68.26     |
| 37S - 25S - 25S| 91.70    | 67.89     |
| 75 - 50 - 25S  | 92.33    | 69.32     |
| 75 - 25S - 50  | 92.06    | 68.99     |
| 75 - 25S - 25S | 92.55    | 69.57     |

Figure 4. Convergence curve of the conventional SGD and S-SGD for ResNet20 on CIFAR-10. (a) Test accuracy plot from 0th to 90th epochs, (b) Test accuracy plot from 100th to 175th epochs.

Figure 5. Large-batch training of ResNet20 on CIFAR-10.

4.3. Large Batch Training

Training a DNN using multi-GPUs is an attractive method for reducing the training time, and it typically requires a batch-size increase because a data-parallel approach is widely employed for reducing the communication overhead. The increased batch size decreases the data-dependent noise of the SGD method and hinders the escape from sharp minima. Recent studies suggest that increasing the learning rate proportionally to the batch size relieves this problem (Smith et al., 2017). However, the learning rate cannot be flexibly scaled owing to the stability problem (You et al., 2017).

We also evaluated the performance of the S-SGD method when combined with a learning rate scheduling method that is also intended to reach flat minima. Stochastic gradient descent with warm restarts (SGDR) was used (Loshchilov & Hutter, 2017). The SGDR periodically resets the learning rate to the initial value. Specifically, we used an initial learning rate of 0.1 and decayed with the cosine function for the initial 10 epochs. The learning rate decay period was doubled at every new restart. The experimental results with ResNet20 on the CIFAR-10 dataset are shown in Figure 5. The S-SGDR method is the combination of the symmetrical weight noise injection and the SGDR learning rate scheduling. We used 300 epochs for the SGDR and S-SGDR methods.

Here, we conducted experiments to show that the proposed S-SGD or S-SGDR methods can improve the performance of large-batch training. We used a four-GPU training employing synchronous SGD using Horovod (Sergeev & Del Balso, 2018). The experiments were performed on NVIDIA DGX-1 (NVIDIA, 2018). The batch size for each GPU was one quarter of that shown in Figure 5. Batch normalization statistics are computed for each device in training (Hoffer et al., 2017). The other hyperparameters were identical to those of the single-GPU training.

Figure 5 shows the performance of ResNet20 on the CIFAR-10 dataset, in which batch sizes of 1, 2, 4, and 16 K were
employ. For each batch size, four training methods, SGD, SGDR, S-SGD, and S-SGDR, were employed for performance measurement. For a batch size of 16,384, the proposed S-SGDR method showed the best test accuracy, approximately 2.9% higher than that of the conventional SGD-based training. We found almost no performance decrease compared to that of the small-batch training even at a batch size of 4,096. The performance degradation with increasing batch size is very slow when S-SGDR or S-SGD is used.

The training data size of CIFAR-10 dataset was only 50,000. Thus, the batch size of 4K was almost 1/10 of the total training dataset.

5. Experimental Results

We conducted various experiments to assess the performance of S-SGD algorithm with CNNs for image classification, Simple Gated ConvNet for speech recognition and RNNs for language modeling. All the experiments were implemented in TensorFlow (Abadi et al., 2016).

5.1. Image Classification with CNN Models on CIFAR-10, CIFAR-100, and ImageNet

We measured the performance of S-SGD on various CNN models using CIFAR-10, CIFAR-100, and ImageNet datasets (Russakovsky et al., 2015). The S-SGD method employs noise strengths of 0.4σ and 0.5σ.

The experimental results are presented in Table 2, for which the test accuracies of widely used CNN models were measured. As shown in Table 2, the S-SGD method exhibited approximately 0.45% (92.62% - 92.17%) of accuracy increase for ResNet20 on CIFAR-10, which means that the error rate decreased from 7.83% to 7.38%. An accuracy increase of 0.44% (93.50% - 93.06%) was also observed for ResNet56 on CIFAR-10. A higher performance gain, about 1.5% to 2.0%, can be seen for the CIFAR-100 dataset.

The ResNet18 and ResNet50 training results on ImageNet dataset are exhibited in Table 3. We used an 8-core Cloud TPU for training. A batch size of 1024, assigning 128 for each core, was used. Each model was trained for 90 epoch with learning rate scheduling from (Ying et al., 2018).

5.2. Speech Recognition with Simple Gated ConvNet

The Simple Gated ConvNet (SGCN) is a convolution-based sequence modeling network used for end-to-end speech recognition (Lee et al., 2019). The connectionist temporal classification (CTC) loss is used for training (Graves et al., 2006). Usually, end-to-end speech recognition using CTC loss employs long short-term memory (LSTM) (Hochreiter & Schmidhuber, 1997b) based models, but SGCN is more suitable for embedded applications because it is free from the sequential dependency problem inherent to LSTM RNN models, and allows the processing of multiple output samples at a time. The network configuration and model parameters were imported from (Lee et al., 2019). The SGCN model consists of 12 layers and each layer contains 190 units. Consequently, the number of parameters is about 1 M. The SGCN employs 1-dimensional time-depth convolution to increase the sequential classification capability without considerably increasing the number of parameters. Wall Street Journal si-284 (Paul & Baker, 1992) is used for the training, which contains 81 hours of training data. The original work was trained using Adam. We trained the network using S-SGD Adam, and compared the results.

Figure 6 shows the training loss and validation character error rate (CER) curves when the model is trained with Adam and S-SGD Adam. The training results are summarized in Table 3. ResNet (He et al., 2016), WideResNet (Zagoruyko & Komodakis, 2016) and VGG (Simonyan & Zisserman, 2014) trained with CIFAR-10 and CIFAR-100 using conventional SGD and S-SGD. Average test accuracy values (%) over 5 runs are reported. The value inside of the parentheses in training method indicates the noise level.

Table 2. ResNet (He et al., 2016), WideResNet (Zagoruyko & Komodakis, 2016) and VGG (Simonyan & Zisserman, 2014) trained with CIFAR-10 and CIFAR-100 using conventional SGD and S-SGD. Average test accuracy values (%) over 5 runs are reported. The value inside of the parentheses in training method indicates the noise level.

| Model   | Method      | CIFAR-10 | CIFAR-100 |
|---------|-------------|----------|-----------|
| ResNet20| SGD         | 92.17    | 68.00     |
|         | S-SGD (0.4)| 92.34 (+0.17) | 69.44 (+1.44) |
|         | S-SGD (0.5)| 92.62 (+0.45) | 69.48 (+1.48) |
| ResNet56| SGD         | 93.06    | 70.12     |
|         | S-SGD (0.4)| 93.30 (+0.44) | 71.61 (+1.49) |
|         | S-SGD (0.5)| 93.50 (+0.44) | 72.19 (+2.07) |
| WRNet28-2| SGD         | 95.07    | 77.12     |
|         | S-SGD (0.5)| 95.20 (+0.13) | 77.66 (+0.54) |
|         | S-SGD (0.7)| 95.68 (+0.48) | 77.55 (+0.74) |
| WRNet28-4| SGD         | 95.20 (+0.13) | 77.57 (+0.45) |
|         | S-SGD (0.5)| 95.30 (+0.23) | 77.31 (+0.80) |
|         | S-SGD (0.9)| 95.43 (+1.43) | 75.81 (+1.00) |
| VGG6    | SGD         | 92.94    | 74.81     |
|         | S-SGD (0.7)| 93.30 (+1.36) | 75.61 (+0.80) |
|         | S-SGD (0.9)| 93.43 (+1.43) | 75.11 (+1.00) |
| VGG16   | SGD         | 92.18    | 72.68     |
|         | S-SGD (0.7)| 93.27 (+1.54) | 73.31 (+0.63) |
|         | S-SGD (0.9)| 93.57 (+1.39) | 73.11 (+0.43) |

Table 3. Training of ResNet18 and ResNet50 on ImageNet. The value inside of the parentheses in training method indicates the noise level. The results are the average of three runs.
Table 4. WER (%) of Simple Gated ConvNet trained with S-SGD. The models are trained on WSJ si-284.

| Model | Params. | WER (%) |
|-------|---------|---------|
| 12x190 SGCN | 1.09M  | 21.66   |
| 12x190 SGCN + S-SGD | 1.09M  | 19.90(-1.76) |
| 12x300 SGCN | 2.24M  | 18.30   |
| 12x300 SGCN + S-SGD | 2.24M  | 16.87(-1.43) |

Figure 6. Train loss and valid CER curve of the SGCN. Solid and dashed lines denote the valid CER and train loss, respectively.

Table 4. For the 1M parameter SGCN model, the word error rate (WER) of Adam training with greedy decoding was 21.66%, while the S-SGD result was 19.90%. A similar performance improvement was observed for the 2M parameter model. This clearly indicates that S-SGD is very effective for sequence recognition problems.

5.3. Language Modeling with LSTM RNN

We trained an LSTM network on the Penn Tree Bank (PTB) dataset (Marcus et al., 1994) for word-level language modeling. This dataset contains about one million words divided into training, validation, and test sets of about 930K, 74K and 82K words, respectively, with a vocabulary size of 10K. The network consists of two layers, each with 1,500 hidden units, resulting in about 6.7 million weights. We unrolled the network in 35 time steps for training. Dropout was applied between the layers. We reproduced the training pipeline of (Zaremba et al., 2014) for this network (SGD without momentum) and obtained a word perplexity of around 81.43 and 78.68 on the validation and test sets, respectively, with this setup; these numbers closely match the results of the original work. We ran S-SGD for 55 epochs, and obtained a word perplexity of 76.86 and 73.83 on the validation and test sets, respectively. The performance according to the injected noise level is shown in Table 5. We observed that S-SGD works very well with RNN. Table 5 also shows the results of the small RNN model, which consists of a single LSTM layer containing 300 units (Hubara et al., 2017). In both models, we could obtain very improved results when compared to original RNN-based language models. It is well-known that the Dropout technique does not yield good results when applied to recurrent paths of RNN, and in the original work of (Zaremba et al., 2014) the dropout was only applied to the forward paths. In our implementation, we injected noises to all weights, except for the biases. Thus, in the forward-path, both Dropout and symmetrical weight noise injection were applied, while in the recurrent path, only symmetrical weight noise injection was used.

Table 5. Test perplexity of RNN based language modeling on PTB dataset. The value inside of the parentheses in training method indicates the noise level.

| Model  | Method   | Test Perplexity |
|-------|----------|-----------------|
| Large RNN | SGD     | 78.68           |
|        | S-SGD (0.8) | 74.47 (-4.21)  |
|        | S-SGD (0.9) | 73.95 (-4.73)  |
|        | S-SGD (1.0) | 73.83 (-4.85)  |
| Small RNN | SGD     | 88.85           |
|        | S-SGD (0.6) | 85.56 (-3.29)  |
|        | S-SGD (0.7) | 84.86 (-3.99)  |
|        | S-SGD (0.8) | 86.04 (-2.81)  |

6. Discussion

6.1. Comparison with other regularization methods

Most DNNs currently used employ a large number of parameters, but the amount of training data is insufficient for a good training. Thus, the importance of regularization grows as the network size increases. We compared the proposed S-SGD with the well-known regularization methods, such as Dropout, DropConnect, and SmoothOut.

Dropout is an approach to regularization in neural networks that helps to reduce interdependent learning among the neurons (Srivastava et al., 2014). Individual nodes are either dropped out of the net or kept, with probabilities $1 - p$ and $p$, respectively. Incoming and outgoing edges to a dropped-out node are also removed. Dropout is especially useful when applied to fully connected layers, but it has limitations when applied to batch normalization layers in CNNs or recurrent paths in RNNs (Ioffe & Szegedy, 2015; Zaremba et al., 2014). In Section 4.1, we showed the performance improvement of S-SGD training for ResNet20 using CIFAR-100 dataset. However, for the same setup, Dropout applied to the fully connected layer did not yield any performance improvements regardless of the dropout rate.

DropConnect sets a randomly selected subset of weights
S-SGD: Symmetrical Stochastic Gradient Descent for Reaching Flat Minima in Deep Neural Network Training

Figure 7. Illustrations of loss surface for train and test error on ResNet20 CIFAR-100. The three points are ResNet20 that trained with 125 epochs of SGD (SGD$_m$), SGD$_m$ + 50 epochs of SGD (SGD$_f$), and SGD$_m$ + 50 epochs of S-SGD (S-SGD).

within the network to zero (Wan et al., 2013). Thus, it can be considered a fine-grained version of the Dropout. DropConnect was originally developed for the regularization of fully connected layers. RNN regularization with DropConnect was studied in (Merity et al., 2018). We applied DropConnect to the ResNet20 model using CIFAR-10 data. When DropConnect was applied to convolution layers, the training was not successful, resulting in very low accuracy. Application of DropConnect to the fully connected layers resulted in 92.18% of accuracy, which was, however, just comparable to SGD (92.17%) and lower than S-SGD (92.62%).

The SmoothOut is a weight noise injection method in which the noise injected weights are used for forward- and backward- passes (Wen et al., 2018). As the noise is randomly distributed, the operation may be considered an analog version of the DropConnect. The comparison of S-SGD with SmoothOut was presented in Section 4. There are no application results of SmoothOut on RNNs.

6.2. Visualization of Loss Surface

We attempted to observe the training and test loss surfaces of SGD and S-SGD trained models, using the visualization method developed by (Garipov et al., 2018). This method can show the locations of three different models. We trained ResNet20 on CIFAR-100 dataset using SGD for the first 75 and the next 50 epochs with learning rates (LR) of 0.1 and 0.01, respectively. This half-trained model was stored as SGD$_m$. The SGD$_m$ was trained using SGD with LR of 0.001 for 50 epochs to obtain the fully trained ResNet20. This model was stored as SGD$_f$. In addition, SGD$_m$ was trained using S-SGD with LR of 0.001 for 50 epochs, and it was named S-SGD. The training accuracy values of SGD$_m$, SGD$_f$, and S-SGD were 81.44%, 91.14%, and 86.62%, respectively. The test accuracy values of SGD$_m$, SGD$_f$, and S-SGD were 65.55%, 68.51%, and 69.63%.

Figure 7 (a) shows the training loss surface, where SGD$_m$, SGD$_f$, and S-SGD are located. We observed that SGD$_f$ is near the lowest point in the loss surface. This seems natural considering the operation of SGD seeking minimum. However, we found that SGD$_f$ is located very close to the very steep loss wall, suggesting the possibility of sharp minimum. The loss would sharply increase with variation of input data characteristics, which means overfitting or poor generalization. For training set, loss of S-SGD is a little higher than that of SGD$_f$ but it is located far from the steep loss wall, which shows the possibility of improved robustness to input data perturbation. Figure 7 (b) exhibits the test loss surface, where SGD$_m$, SGD$_f$ and S-SGD are included. The location of S-SGD becomes the minimum point located near the center of the basin. We noticed that S-SGD avoids overfitting by adding weight noise in the training phase. The amount of added noise would determine the distance to the sharp loss wall. Figure 2 is a 1-D sectional view of Figure 7, where the line connecting SGD$_f$ and S-SGD becomes the axis of 1-D plot.

We have included all the training parameters and training results of the experiments in the Appendix. The eigenvalues of Hessian for SGD and S-SGD trained networks are also presented in the Appendix.

7. Concluding Remarks

We proposed the Symmetrical Stochastic Gradient Descent (S-SGD) algorithm, which injects symmetrical weight noises for measuring loss surface flatness in DNN training. This method aids in reaching flat minima during SGD training because the loss values are measured at two points at each weight update. The S-SGD method is very stable because two symmetrical noises are added to the weights. The weight update analysis for this algorithm shows that the gradient for the weight update depends on the flatness of loss surface, or Hessian. We conducted many experiments using several CNN models, datasets, batch sizes, and learning rate scheduling for a performance evaluation of the method. The S-SGD also showed substantial performance improvements when applied to end-to-end speech recognition with Simple Gated ConvNet and language modeling with LSTM RNN. When compared to previously developed regularization methods, such as Dropout and DropConnect,
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S-SGD is considerably more flexible and can be applied to fully connected, convolution and recurrent layers.

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References

Abadi, M., Barham, P., Chen, J., Chen, Z., Davis, A., Dean, J., Devin, M., Ghemawat, S., Irving, G., Isard, M., et al. TensorFlow: A System for Large-scale Machine Learning. In 12th USENIX Symposium on Operating Systems Design and Implementation (OSDI 16), pp. 265–283, 2016.

An, G. The effects of adding noise during backpropagation training on a generalization performance. Neural computation, 8(3):643–674, 1996.

Chaudhari, P., Choromanska, A., Soatto, S., LeCun, Y., Baldassari, C., Borgs, C., Chayes, J., Sagun, L., and Zecchina, R. Entropy-sgd: Biasing gradient descent into wide valleys. International Conference on Learning Representations (ICLR), 2017.

Duchi, J., Hazan, E., and Singer, Y. Adaptive subgradient methods for online learning and stochastic optimization. Journal of Machine Learning Research (JMLR), 12(Jul): 2121–2159, 2011.

Garipov, T., Izmailov, P., Podoprikhin, D., Vetrov, D. P., and Wilson, A. G. Loss surfaces, mode connectivity, and fast ensembling of dnns. In Advances in Neural Information Processing Systems, pp. 8789–8798, 2018.

Ghorbani, B., Krishnan, S., and Xiao, Y. An investigation into neural net optimization via hessian eigenvalue density. In International Conference on Machine Learning, pp. 2232–2241, 2019.

Goyal, P., Dollár, P., Girshick, R., Noordhuis, P., Wesolowski, L., Kyrola, A., Tulloch, A., Jia, Y., and He, K. Accurate, large minibatch sgd: Training imagenet in 1 hour. arXiv preprint arXiv:1706.02677, 2017.

Graves, A., Fernández, S., Gomez, F., and Schmidhuber, J. Connectionist temporal classification: Labelling unsegmented sequence data with recurrent neural networks. In International Conference on Machine Learning (ICML), pp. 369–376. ACM, 2006.

He, K., Zhang, X., Ren, S., and Sun, J. Deep residual learning for image recognition. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR), pp. 770–778, 2016.

Ho, K., Leung, C.-s., and Sum, J. On weight-noise-injection training. In International Conference on Neural Information Processing, pp. 919–926. Springer, 2008.

Hochreiter, S. and Schmidhuber, J. Flat minima. Neural Computation, 9(1):1–42, 1997a.

Hochreiter, S. and Schmidhuber, J. Long short-term memory. Neural computation, 9(8):1735–1780, 1997b.

Hoffer, E., Hubara, I., and Soudry, D. Train longer, generalize better: closing the generalization gap in large batch training of neural networks. In Advances in Neural Information Processing Systems, pp. 1731–1741, 2017.

Hubara, I., Courbariaux, M., Soudry, D., El-Yaniv, R., and Bengio, Y. Quantized neural networks: Training neural networks with low precision weights and activations. The Journal of Machine Learning Research, 18(1):6869–6898, 2017.

Ioffe, S. and Szegedy, C. Batch normalization: Accelerating deep network training by reducing internal covariate shift. In International Conference on Machine Learning (ICML), 2015.

Jastrzkebski, S., Kenton, Z., Ballas, N., Fischer, A., Bengio, Y., and Storkey, A. DNN’s Sharpest Directions Along the SGD Trajectory. arXiv preprint arXiv:1807.05031, 2018.

Jastrzkebski, S., Kenton, Z., Arpit, D., Ballas, N., Fischer, A., Bengio, Y., and Storkey, A. Finding flatter minima with SGD. In Workshop track on ICLR, 2018.

Jin, C., Ge, R., Netrapalli, P., Kakade, S. M., and Jordan, M. I. How to Escape Saddle Points Efficiently. In Proceedings of the 34th International Conference on Machine Learning-Volume 70, pp. 1724–1732. JMLR. org, 2017.

Johnson, R. and Zhang, T. Accelerating Stochastic Gradient Descent using Predictive Variance Reduction. In Advances in Neural Information Processing Systems, pp. 315–323, 2013.

Keskar, N. S. and Socher, R. Improving generalization performance by switching from Adam to SGD. arXiv preprint arXiv:1712.07628, 2017.

Keskar, N. S., Mudigere, D., Nocedal, J., Smelyanskiy, M., and Tang, P. T. P. On large-batch training for deep learning: Generalization gap and sharp minima. International Conference on Learning Representations (ICLR), 2017.
Kingma, D. P. and Ba, J. Adam: A method for stochastic optimization. *International Conference on Learning Representations (ICLR)*, 2015.

Kleinberg, R., Li, Y., and Yuan, Y. An Alternative View: When Does SGD Escape Local Minima? In *International Conference on Machine Learning (ICML)*, pp. 2703–2712, 2018.

Krizhevsky, A., Nair, V., and Hinton, G. Cifar-10 (canadian institute for advanced research). *URL* http://www.cs.toronto.edu/kriz/cifar.html, 2010.

Lee, C.-Y., Xie, S., Gallagher, P., Zhang, Z., and Tu, Z. Deeply-supervised nets. In *Artificial Intelligence and Statistics*, pp. 562–570, 2015.

Lee, L., Park, J., and Sung, W. Simple Gated ConvNet for Small Footprint Acoustic Modeling. In *2019 IEEE Automatic Speech Recognition and Understanding Workshop (ASRU)*. IEEE, 2019.

Li, H., Xu, Z., Taylor, G., Studer, C., and Goldstein, T. Visualizing the loss landscape of neural nets. In *Advances in Neural Information Processing Systems*, pp. 6389–6399, 2018.

Loshchilov, I. and Hutter, F. SGDR: Stochastic Gradient Descent with Warm Restarts. *International Conference on Learning Representations (ICLR)*, 2017.

Marcus, M., Kim, G., Marcinkiewicz, M. A., MacIntyre, R., Bies, A., Ferguson, M., Katz, K., and Schasberger, B. The Penn Treebank: annotating predicate argument structure. In *Proceedings of the workshop on Human Language Technology*, pp. 114–119. Association for Computational Linguistics, 1994.

Merity, S., Keskar, N. S., and Socher, R. Regularizing and optimizing Istm language models. *International Conference on Learning Representations (ICLR)*, 2018.

Murray, A. F. and Edwards, P. J. Synaptic weight noise during multilayer perceptron training: fault tolerance and training improvements. *IEEE Transactions on Neural Networks*, 4(4):722–725, 1993.

NVIDIA. Nvidia dgx-1. *URL*: https://www.nvidia.com/en-us/data-center/dgx-1/, 2018.

Paul, D. B. and Baker, J. M. The design for the Wall Street Journal-based CSR corpus. In *Proceedings of the workshop on Speech and Natural Language*, pp. 357–362. Association for Computational Linguistics, 1992.

Qian, N. On the momentum term in gradient descent learning algorithms. *Neural networks: the official journal of the International Neural Network Society*, 12(1):145–151, 1999.

Russakovsky, O., Deng, J., Su, H., Krause, J., Satheesh, S., Ma, S., Huang, Z., Karpathy, A., Khosla, A., Bernstein, M., et al. ImageNet Large Scale Visual Recognition Challenge. *International journal of computer vision (IJCV)*, 115(3):211–252, 2015.

Sergeev, A. and Del Balso, M. Horovod: fast and easy distributed deep learning in TensorFlow. *arXiv preprint arXiv:1802.05799*, 2018.

Simonyan, K. and Zisserman, A. Very deep convolutional networks for large-scale image recognition. *arXiv preprint arXiv:1409.1556*, 2014.

Smith, L. N. Cyclical learning rates for training neural networks. In *2017 IEEE Winter Conference on Applications of Computer Vision (WACV)*, pp. 464–472. IEEE, 2017.

Smith, S. L., Kindermans, P.-J., Ying, C., and Le, Q. V. Don’t decay the learning rate, increase the batch size. *arXiv preprint arXiv:1711.00489*, 2017.

Srivastava, N., Hinton, G., Krizhevsky, A., Sutskever, I., and Salakhutdinov, R. Dropout: a simple way to prevent neural networks from overfitting. *The Journal of Machine Learning Research (JMLR)*, 15(1):1929–1958, 2014.

Tieleman, T. and Hinton, G. Lecture 6.5—RmsProp: Divide the gradient by a running average of its recent magnitude. COURSERA: Neural Networks for Machine Learning, 2012.

Wan, L., Zeiler, M., Zhang, S., Le Cun, Y., and Fergus, R. Regularization of neural networks using DropConnect. In *International Conference on Machine Learning*, pp. 1058–1066, 2013.

Wen, W., Wang, Y., Yan, F., Xu, C., Wu, C., Chen, Y., and Li, H. SmoothOut: Smoothing Out Sharp Minima to Improve Generalization in Deep Learning. *arXiv preprint arXiv:1805.07898*, 2018.

Ying, C., Kumar, S., Chen, D., Wang, T., and Cheng, Y. Image classification at supercomputer scale. *arXiv preprint arXiv:1811.06992*, 2018.

You, Y., Gitman, I., and Ginsburg, B. Scaling SGD batch size to 32k for ImageNet training. *arXiv preprint arXiv:1708.03888*, 6, 2017.

Zagoruyko, S. and Komodakis, N. Wide residual networks. *arXiv preprint arXiv:1605.07146*, 2016.

Zaremba, W., Sutskever, I., and Vinyals, O. Recurrent neural network regularization. *arXiv preprint arXiv:1409.2329*, 2014.
Zhu, Z., Wu, J., Yu, B., Wu, L., and Ma, J. The anisotropic noise in stochastic gradient descent: Its behavior of escaping from minima and regularization effects. *arXiv preprint arXiv:1803.00195*, 2018.

Zur, R. M., Jiang, Y., Pesce, L. L., and Drukker, K. Noise injection for training artificial neural networks: A comparison with weight decay and early stopping. *Medical physics*, 36(10):4810–4818, 2009.