The Effect of Dynamic Mathematics Software Geogebra on Student’ Achievement: The Case of Differential Equations

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Abstract

In mathematics, a differential equation uses important mathematical concepts like function, derivative, integral, etc. Geogebra is a dynamic mathematical software uniting geometry, algebra and differential calculus. There are two objectives in this study. The first objective is to examine the impact of the use of Geogebra on the students’ understanding of differential equations. The second objective is to see how the students interact with a Geogebra environment according to their styles of mathematical thinking. The subjects of this research were 35 and 32 students for the experimental and control group, respectively of 12th grade at a government school, west of Morocco, in the academic year 2020-2021. These participants have different mathematical thinking styles (MTS): visual, analytic, and integrated. The results show that teaching differential equations with GeoGebra is more efficient in terms of conceptual knowledge than the conventional method. In procedural knowledge, students in both groups were in similar achievement levels. We can conclude that GeoGebra was more beneficial for students with integrated thinking, especially for conceptual knowledge.

Keywords: Differential equations (DE), conceptual knowledge, Geogebra, procedural knowledge, Mathematical Thinking Style (MTS)

1. Introduction

A differential equation is an equation where the unknown is a function, and which takes the form of a relation between this function and its derivatives (Precup, 2018). Many real life problems can be modeled using differential equations (Burges & Huntley, 1981; Hattaf et al, 2012; Krutikhina et al, 2018). In mathematics, differential equations are important, they use important mathematical concepts like function, derivative, integral, etc (Arslan, 2010a, 2010b).

Interactive and animated applications with Geogebra facilitate the differential equations topic
and improve the conceptualization of the students (Funes & Valero, 2018; 2021). For example, Newtonian mechanics provides many examples for differential equations, which can be treated easily with Computer Algebra or Dynamic Geometry Systems, like Geogebra (Kovacs, 2010).

### 1.1 Differential Equations in Moroccan Program

The contents of different mathematics programs in second cycle classes are presented in Table 1. In the first year of the baccalaureate in mathematical sciences or in experimental sciences, the algebraic solution to the equation \( y'' + \omega^2 y = 0 \) is presented without demonstration (accepted). The existence and uniqueness of the solution to the differential equation with an initial condition are presented with an example. We see that there are no real-life applications of this differential equation.

In the program for the second year of the baccalaureate, a whole lesson is reserved for differential equations. We study two types of equations. The first one is \( y' = ay + b \); the student is led to prove the general solution, the existence and uniqueness of the solution to this differential equation with an initial condition. The second one is \( y'' + ay' + by = 0 \), the algebraic solution to the equation \( y'' + ay' + by = 0 \) is presented without demonstration (accepted). Applications in real life of these equations were presented without a modelling activity.

| Level | Lesson | Educational guidelines | Page |
|-------|--------|------------------------|------|
| 1st year of the baccalaureate in Mathematical Sciences and in Experimental Sciences | Derivation and graphical representation of functions | We accept the general solution of the equation: \( y'' + \omega^2 y = 0 \) | 61 and 44 |
| 2nd year of the baccalaureate in Mathematical Sciences and in Experimental Sciences | Differential equations | -Resolution of the equation: \( y' = ay + b \), and use it in situations in special subjects. -Resolution of the equation: \( y'' + ay' + by = 0 \), and use it in situations in special subjects. -We accept the general solution of the equation: \( y'' + ay' + by = 0 \) | 110 and 97 |

### 1.2 Conceptual and Procedural Knowledge

We can distinguish two types of knowledge, conceptual and procedural (Baroody, 2003; Zuya, 2017). Procedural knowledge means definitions, rules, algorithms, and isolated skills (Hiebert & Lefevre, 1986; Engelbrecht et al., 2005). They allow the student to apply procedures to solve mathematical problems. Conceptual knowledge means the individual understanding of the internal relationships between mathematical concepts (New York State Education Department [NYSED], 2005). Procedural knowledge is the basis of conceptual knowledge (Vygotsky, 1986; Baker & Czarnocha, 2002).

The traditional differential equations course focuses on procedures, which reduces students’ understanding (Star, 2005; Miller & Hudson, 2007). The introduction of the different methods of solving differential equations (Algebraic, numerical, and graphical) in Geogebra environment greatly increases the conceptualization of students.

### 1.3 Dynamic Software GeoGebra

Dynamic Geometry Computer Software (DGCS) is a computer software that allows the users to construct geometry figures or shapes, measure the shapes’ variables as well as how to determine their properties. Computer algebra system (CAS), process literal expressions, and allows, when possible, to
perform exact calculations.

With the introduction of Dynamic Geometry software into the education system, mathematics-teaching methods have changed and mathematics has become a science lab. Geometry software with Dynamic functionality has given the opportunity for the students to work on more abstract structures rather than work on widely used traditional paper-and-pencil studies.

GeoGebra is an open-source coded dynamic math software that unites geometry, algebra, and analysis, which is used from primary school until higher education. GeoGebra is an interactive software in which every change of parameters or variables can be seen (Preiner, 2008; Hohenwarter & Lavicza, 2007).

The main feature that differentiates GeoGebra from other mathematics softwares is that it can be considered as a computer algebra system (CAS) in one hand and as a dynamic geometry software (DGS) on the other hand. In mathematics teaching, the success of correlating between geometry and algebra brings this software an important position in the school education program (curriculum) (Tutkun & Ozturk, 2013).

With the program GeoGebra we can plot the direct field of differential equations (DE) and the set of the isoclines. (The isoclines (“iso-cline”) means lines of the “equal slope”) are defined by the equations \( f(x, y) = C \) (\( C = \) constant). We can also sketch the set of the integral curves of our equation. Visualization of these mathematical concepts has a positive impact on the students (Kontrová & Šusteková, 2020).

1.4 Mathematical Thinking Styles

Mathematical thinking styles are the ways in which individuals learn and understand mathematics (Borromeo Ferri, 2012, 2015; Sternberg, 1997). There are several classifications of mathematical thinking styles. In this research, we adopted the definition of Borromeo (2015). Borromeo (2015) defined three thinking styles: visual, analytic, and integrated.

An analytical or formal individual is one who has a preference for the analytical style of thinking, is characterized by internal forms of imagination and by external formal representations (it is also called symbolic). He also prefers to proceed in sequential steps (step by step). A visual individual is one who shows a preference for the visual style of thinking, is characterized by the imagination of internal drawings and the exteriorization of pictorial representations. Also he prefers to understand the facts and mathematical relationships of holistic representations, in this case, internal images are often the effects (consequences) of strong associations with lived situations. The integrated thinking style can be seen in individuals who combine formal and visual thinking styles and are quite flexible between formal or visual representations, the path they choose to solve problems, and the ways of dealing with information (Figure 1).

We used Mathematical thinking style inventory (MTS) developed by Rita Borromeo (Borromeo Ferri, 2012). We used this scale to see the interaction of mathematical thinking styles with the use of Geogebra in the course of differential equations.

![Figure 1: Different solutions depending on the thinking style](image-url)
1.5 Differential Equations Resolution

In student’s minds, for each kind of differential equation there is an algebraic integration (solution). However, most differential equations do not have analytical solutions. In the Geogebra environment one can solve almost any first order differential equation and according to the three modes: algebraic, graphical, and numerical.

To solve a first order differential equation, it is generally a question of finding a function with an arbitrary constant, which satisfies the differential equation in question. To algebraically solve a differential equation, a prelude generally consists of determining the nature (linear, homogeneous, with separable variables, etc.) of it in order to be able to apply the corresponding resolution technique to it. However, the implementation of this approach is not always possible and is dependent on the nature of the differential equation. Several differential equations do not have explicit solutions and cannot apply algebraic resolution to them.

The numerical solution aims to find approximate values of a special solution of a differential equation passing through a given point, and therefore the calculation of the unknown function for the specific values of the independent variable.

Contrary to the analytical solution decided to be applied according to the type of differential equation, the method to be applied in the numerical solution is generally independent of the type of equation, but it varies according to the requirements in the problem.

The graphical solution is to draw the slope field of solutions of equation $y' = f (x, y)$ (Habre, 2000, 2003) (See figure 2).

Figure 2: The slope field and some solutions of the ode $y' = x + y$

With Geogebra environment (Figure 2), slope fields have become a basic component in the understanding of first-order ordinary differential equations.

1.6 Purpose and Importance of the Study

It is not known if there is an interaction between the mathematical thinking styles and learning in
Geogebra environment. In this context, the purpose of this study is to examine the utility of introducing the course of differential equations in the Geogebra environment and to explore the interaction with Students’ mathematical thinking styles.

This study tries to answer the following questions:

1. Do our students have a well developed conceptual image of differential equations?
2. How can learning differential equations with Geogebra enhance students’ conceptualization of differential equations?
3. Does the level of procedural knowledge differ between the group of students who used Geogebra and those who used the conventional method, depending on their mathematical thinking styles?
4. Does the level of conceptual knowledge differ between the group of students who used Geogebra and those who used the conventional method, depending on their mathematical thinking styles?

2. Materials and Methods

2.1 Research design

The experimental group used GeoGebra software (n = 35) while the control group used the conventional method (n = 32).

2.2 Research Goal

The objectives of this study are:

1. To examine the impact of the use of Geogebra on the students’ understanding (conceptual and procedural knowledge) of differential equations.
2. To identify how students interact with a Geogebra environment depending on their mathematical thinking styles.

2.3 Sample and Data Collection

Our research was conducted during the 2020-2021 school year. Sixty-seven students (age 16-18) were selected from a secondary school in the west of Morocco. Our participants are in mathematical sciences (12th grade). Thirty five participants for the experimental group while 32 ones for the control group. Both groups have similar grade levels. The experimental group benefited from Geogebra software, while the control group followed a classical constructivist education. The authors have developed Geogebra’s activities for this purpose.

2.4 Process

The lessons lasted three weeks for both groups. The activities were chosen according to the official program. The only difference between the two groups is the use of Geogebra for the experimental group.

2.5 Achievement Test

The test assessed the ability of students to compose and solve differential equations in different ways, as well as the procedural and conceptual understanding in relation to the differential equations. The test took place one week after the modules.
2.6 Analyzing of Data

Two statistical tests, two-way ANOVA and t-test, were used in this study to determine any enhancement in students’ conceptual and procedural knowledge according to groups and mathematical thinking styles.

3. Results

3.1 Verification of normality and equality of variances for the scores of the two groups (Control and Experimental) at the pre-test and at the post-test

Both groups were administered a Mathematical thinking styles scale (MTS) and a pre-test in order to assess students’ mathematical knowledge as well as their mathematical thinking styles. Before applying the t-test, we checked the normality of the scores. The Shapiro-Wilks test was used. The p-values for students’ scores in the control group and Geogebra group were \( p = 0.271 \) and \( 0.335 > 0.05 \), respectively. Students’ scores were distributed normally in both groups. We also assessed the homogeneity of variances by the Levene test. We found \( p = 0.953 > 0.05 \), which ensured that variances for students’ scores in both groups control and Geogebra were equal. Next, we compared the students’ scores on the pretest (Table 2).

Students’ scores for the experimental group (Geogebra group) \( (M = 11.00, SD = 4.058) \) did not differ significantly from Students’ scores for the control group \( (M = 10.63, SD = 4.014) \); \( t(65) = -0.380, p = 0.705 > 0.05 \) (Table 2).

| Groups          | N  | Mean  | SD     | t value | Df  | Sig  |
|-----------------|----|-------|--------|---------|-----|------|
| Experimental    | 35 | 11.00 | 4.058  | -0.380  | 65  | 0.705|
| Control         | 32 | 10.63 | 4.014  |         |     |      |

Table 2: Students’ scores comparison (Geogebra group and control group) in pre-test

Normality (Shapiro-Wilk Test) and homogeneity of variances (Levene’s Test) were also verified for students’ scores in the post-test (Table 3).

| Groups                  | Shapiro-Wilk Test | Levene’s Test |
|-------------------------|-------------------|---------------|
|                         | Test value | Df  | Sig  | F value | Sig  |
| Conceptual Knowledge Questions |           |     |     |         |      |
| Experimental            | 0.967      | 35  | 0.361| 3.112   | 0.082|
| Control                 | 0.941      | 32  | 0.119|         |      |
| Procedural Knowledge Questions |           |     |     |         |      |
| Experimental            | 0.950      | 35  | 0.153| 0.114   | 0.736|
| Control                 | 0.945      | 32  | 0.102|         |      |

Table 3: Normality and homogeneity of variances for Geogebra and control group students’ scores in post-test

Table 4 shows that most students have an integrated thinking style, and a few have an analytical thinking style. In addition, Table 4 indicates that there is no statistically significant difference between the distributions of mathematical thinking styles between control and experimental group.

| Groups               | Experimental group | Control group | Chi square | Sig   |
|----------------------|--------------------|---------------|------------|-------|
| Visual thinking style| 12                 | 10            | 0.721      | 0.697 |
| Analytical thinking style| 10                | 7             |            |       |
| Integrated thinking style| 13                | 15            |            |       |
3.2 Comparison of students’ scores in conceptual knowledge based on groups and mathematical thinking styles

To study the interaction between the group and the mathematical thinking style we used two-way ANOVA analysis. Table 5 shows the means of students’ conceptual knowledge in post-test for each mathematical thinking styles and for both groups (experimental and control).

The interaction between the use of Geogebra and mathematical thinking styles showed the following results. GeoGebra group showed higher scores in conceptual knowledge comparing to control group. Students with integrated thinking style benefited more from the use of GeoGebra (Table 5). Table 6 indicates that all these interactions are significant.

Table 5: Students’ conceptual knowledge in post-test

| Groups  | Mathematical Thinking Styles | N  | Mean  | Std. Deviation |
|---------|-----------------------------|----|-------|----------------|
| Experimental | Analytic | 10 | 12.30 | 1.567 |
|           | Visual       | 12 | 14.33 | 1.371 |
|           | Integrated   | 13 | 16.08 | 0.954 |
|           | Total        | 35 | 14.40 | 1.988 |
| Control  | Analytic     | 7  | 12.43 | 1.134 |
|           | Visual       | 10 | 12.60 | 1.350 |
|           | Integrated   | 15 | 12.40 | 1.639 |
|           | Total        | 32 | 12.47 | 1.444 |

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Table 6: Two-way ANOVA for interaction between groups and mathematical thinking styles in conceptual knowledge

| Source                      | Type III Sum of Squares | df | Mean Square | F    | Sig.   |
|-----------------------------|-------------------------|----|-------------|------|--------|
| Corrected model             | 143,312a                | 5  | 28,662      | 15,150 | 0.000  |
| Group                       | 48,960                  | 1  | 48,960      | 25,879 | 0.000  |
| Mathematical thinking style | 36,440                  | 2  | 18,220      | 9,631  | 0.000  |
| Group * Mathematical thinking style | 38,499  | 2  | 19,250      | 10,175 | 0.000  |
| Error                       | 115,404                 | 61 | 1,892       |       |        |
| Total                       | 12429,000               | 67 |             |       |        |

a. R-deux = 0.554 (R-deux ajusté = 0.517)

Students with integrated style in the GeoGebra group have the highest conceptual knowledge (Table 7).

Table 7: Scheffe’s test for multiple comparisons based on students’ Mathematical Thinking Styles

| (I) TS | (J) TS  | Mean Difference (I-J) | Std. Error | Sig. |
|--------|---------|-----------------------|------------|------|
| Integrated | Visual | 0.56                  | 0.392      | 0.364 |
|         | Analytic  | 1.75                  | 0.423      | 0.001 |
| Visual | Integrated  | -0.56                 | 0.392      | 0.364 |
|         | Analytic  | 1.96                  | 0.444      | 0.033 |
| Analytic | Integrated  | -1.75                 | 0.423      | 0.001 |
|         | Visual    | -1.19                 | 0.444      | 0.033 |
3.3 Comparison of students’ scores in procedural knowledge based on groups and mathematical thinking styles

To study the interaction between the group and the mathematical thinking styles we used two-way ANOVA analysis. Table 8 shows the mean of students’ procedural knowledge in post-test.

Table 8: Students’ procedural knowledge in post-test

| Groups    | Mathematical Thinking Styles | N  | Mean   | Std. Deviation |
|-----------|------------------------------|----|--------|----------------|
| Experimental | Analytic                     | 10 | 14.20  | 2.936          |
|            | Visual                       | 12 | 13.50  | 2.355          |
|            | Integrated                   | 13 | 14.31  | 2.359          |
|            | Total                        | 35 | 14.00  | 2.485          |
| Control   | Analytic                     | 7  | 14.00  | 2.646          |
|            | Visual                       | 10 | 14.00  | 2.211          |
|            | Integrated                   | 15 | 13.87  | 2.503          |
|            | Total                        | 32 | 13.94  | 2.368          |

There is no statistically significant difference between the Geogebra group and the control group in procedural knowledge (Table 8 and 9).

Table 9: Two-way ANOVA for interaction between groups and mathematical thinking styles in procedural knowledge

| Source                              | Type III Sum of Squares | df | Mean Square | F    | Sig. |
|-------------------------------------|-------------------------|----|-------------|------|------|
| Corrected Model                     | 4,838*                  | 5  | 0.968       | 0.156| 0.978|
| Mathematical Thinking Styles        | 1,711                   | 2  | 0.856       | 0.138| 0.872|
| Group                               | 0.035                   | 1  | 0.035       | 0.006| 0.941|
| Group * Mathematical Thinking Styles| 2,800                   | 2  | 1,400       | 0.225| 0.799|
| Error                               | 379,103                 | 61 | 6,215       |      |      |
| Total                               | 13,460,000              | 67 |            |      |      |

4. Discussion

This research had as objective to study the effects of using the Geogebra software in differential equations topics and how students interacted with this software. Our research revealed the following results. The Control group and the experimental group, are similar for procedural knowledge. For conceptual knowledge, the Geogebra group obtained on average a higher score than the control group.

Students with integrated thinking in GeoGebra group have higher conceptual knowledge than other students. Geogebra was more beneficial for students with integrated thinking, especially for conceptual knowledge.

Through Geogebra, students could see the effect of changes in algebraic formulas on graphic representations. This gives students the opportunity to make the link between the two algebraic and graphical representations of the solution functions of differential equations (Botana & Valcarce, 2001; Hohenwarter & Fuchs, 2007; Tatar & Zengin, 2016).

Secondly, there is no significant difference between the Control group and the experimental group in procedural knowledge. This is normal, since procedural knowledge consists of memorizing formulas, definitions and algorithms (Aspinwell & Miller, 1997; Mahir, 2009).

Other dynamic environment software had positive impacts on learning mathematics subjects (Furner & Marinas, 2007; Healy & Hoyles, 2002).
It emerges from our study that the Geogebra environment improves students' conceptual understanding in the course of differential equations. Other research shares this result with us. For example, Antohe (2009) found that Geogebra helps students understand mathematical concepts and relate these concepts with other mathematical concepts.

Knowledge is developed by students in a constructivist learning environment (Zulnaidi & Zamri, 2017). In mathematics, learning must be linked to the reality of the learner. Geogebra motivates students to learn mathematics and connects algebra and geometry in a dynamic environment which consolidates students' conceptual learning (Kul, 2018).

The students showed a different increase in their conceptual knowledge on the subject of differential equations. It depends on their mathematical thinking styles. GeoGebra software helps to improve conceptual knowledge especially for students with a visual or integrated style.

Considering that differential equations require integrated thinking, integrated students performed better in this topic. Students with analytical thinking perform better in symbolic operations while students with visual thinking perform better in geometry. Differential equations require analytic and graphic interpretations.

Mathematical thinking styles is a factor to consider when creating classroom activities. Students learn better with activities adapted to their styles of thinking (Zhang & Sternberg, 2002, 2005; Borromeo, 2012, 2015).

5. Conclusion

At the end of our research, learning the differential equations in a Geogebra environment considerably improves the students' conceptual understanding. For procedural knowledge, where the student is required to learn facts or algorithms, it is not useful to introduce Geogebra.

Learning differential equations in a constructivist environment with Geogebra improves students' understanding.

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References

Antohe, V. (2009). Limits of Educational Soft “GeoGebra” in a Critical Constructive. Review. Annals. Computer Science Series, 7(1), 47-54.
Arslan, S. (2010a). Traditional instruction of differential equations and conceptual learning. Teaching Mathematics and its Applications, 29 (2), 94–107. https://doi.org/10.1093/teamat/hrq001
Arslan, S. (2010b). Do students really understand what an ordinary differential equation is? International Journal of Mathematical Education in Science and Technology, 41 (7), 873-888. https://doi.org/10.1080/0020739X.2010.486448
Aspinwell, L., & Miller, D. (1997). Students’ positive reliance on writing as a process to learn first semester calculus. Journal of Instructional Psychology, 24, 253-261.
Baker, W., & Czarnocha, B. (2002). Written meta-cognition and procedural knowledge. In I. Vakalis (Ed.), Proceedings of the 2nd International Conference on the Teaching of Mathematics (at the undergraduate level) (pp. 1-7). University of Crete.
Baroody, A. J. (2003). The development of adaptive expertise and flexibility: The integration of conceptual and procedural knowledge. In A. J. Baroody & A. Dowker (Eds.), The development of arithmetic concepts and skills: Constructing adaptive expertise (pp. 1–33). Lawrence Erlbaum Associates Publishers.
Borromeo Ferri, R. (2012). Mathematical thinking styles and their influence on teaching and learning mathematics. In Cho, S., (Ed.), Proceedings of the 12th International Congress on Mathematical Education (pp. abcd-e-fghij). COEX, Seoul, Korea
Borromeo Ferri, R. (2015). Mathematical thinking styles in school and across cultures. In Cho, S.J., (Ed.), Selected Regular Lectures from the 12th International Congress on Mathematical Education (pp. 153-173). Springer, Cham. https://doi.org/10.1007/978-3-319-17187-6_9
Botana, F., & Valcarce, J. L. (2001). Cooperation between a Dynamic Geometry Environment and a Computer Algebra System for Geometric Discovery. In: Ganzha V.G., Mayr E.W., & Vorozhtsov E.V. (Eds.), Computer Algebra in Scientific Computing CASC 2001 (pp. 63-74). Springer, Berlin, Heidelberg. https://doi.org/10.1007/3-540-46666-0_5

Burghes, D., & Huntley, I. (1981). Teaching mathematical modeling— reflections and advice. International Journal of Mathematical Education in Science and Technology, 13 (6), 735-754. https://doi.org/10.1007/BF01996079

Engelbrecht, J., Harding, A., & Potgieter, M. (2005) Undergraduate students’ performance and confidence in procedural and conceptual mathematics. International Journal of Mathematical Education in Science and Technology, 36(7), 701–712. https://doi.org/10.1080/00207390500271107

Funes, J. O., & Valero, E. (2018). Animations and interactive creations in linear differential equations of first order: the case of GeoGebra. IOP Conf. Series: Journal of Physics: Conf. Series, 1141(1), 012126, 1-4. https://doi.org/10.1088/1742-6596/1141/1/012126

Funes, J. O., & Valero, E. (2021). Exploring the exact differential equations with GeoGebra software. IOP Conf. Series: Journal of Physics: Conf. Series, 1730(1), 012063, 1-4. https://doi.org/10.1088/1742-6596/1730/1/012063

Habre, S. (2000). Exploring students' strategies to solve ordinary differential equations in a reformed setting. Journal of Mathematical Behavior, 18(4), 455–472. https://doi.org/10.1016/S0732-3123(00)00024-9

Habre, S. (2003). Investigating students' approval of a geometrical approach to differential equations and their solutions. International Journal of Mathematical Education in Science and Technology, 34(5), 651–662. https://doi.org/10.1080/0020739031000148912

Hattaf, K., Yousfi, N., & Tridane, A. (2012). Mathematical analysis of a virus dynamics model with general incidence rate and cure rate. Nonlinear Analysis: Real World Applications, 13(4), 1866–1872. https://doi.org/10.1016/j.nonrwa.2011.12.015

Healy, L., & Hoyles, C. (2002). Software Tools for Geometrical Problem Solving: Potentials and Pitfalls. International Journal of Computers for Mathematical Learning, 6, 235-256. https://doi.org/10.1023/A:101305627916

Hiebert, J., & Lefèvre, P. (1986). Conceptual and Procedural Knowledge in Mathematics: An Introductory Analysis. In J. Hiebert (Eds.), Conceptual and Procedural Knowledge: The Case of Mathematics (pp. 1-27). Hillsdale, NJ: Erlbaum.

Hohenwarth, M., & Lavicza, Z. (2007). Mathematics teacher development with ICT: Towards an International GeoGebra Institute. In Küchemann, D. (ed.), Proceedings of the British Society for Research into Learning Mathematics, 27(3), 49-54, University of Northampton, UK

Kontrová, L., & Šusteková, D. (2020). Teaching with geogebra versus traditional method. In L. Gómez Chova, A. López Martínez, & I. Candel Torres (Eds.), Proceedings of EDULEARN20 Conference, 5459-5466, University of Zilina, Slovakia.

Kovacs, Z. (2010). Modelling with Difference Equations Supported by GeoGebra: Exploring the Kepler Problem. International Journal for Technology in Mathematics Education, 17(3), 141-146.

Krutikhina, M. V., Vlasova, V. K., Galushkin, A. A., & Pavlushin, A. A. (2018). Teaching of Mathematical Modeling Elements in the Mathematics Course of the Secondary School. Eurasia Journal of Mathematics, Science and Technology Education, 14(4): 1305-1315. https://doi.org/10.29333/iejmste/83561

Kul, U. (2018). Influences of technology integrated professional development course on mathematics teachers. European Journal of Educational Research, 7(2), 233–243. https://doi.org/10.12973/eu-jer.7.2.233

Lavicza, Z. (2006). Factors influencing the integration of Computer Algebra Systems into university-level mathematics education. International Journal for Technology in Mathematics Education, 14(3), 121–129.

Miller, P. J., & Hudson, S. P. (2007). Using evidence-based practices to build mathematics competence related to conceptual, procedural, and declarative knowledge. Learning Disabilities Research Practice, 22(1), 47–57. https://doi.org/10.1177/105986540700230.00230.x

Educational and instructional programs for teaching mathematics in secondary school. (2007). Ministry of National Education, Vocational Training, Higher Education and Scientific Research (Morocco).

New York State Education Department (NYSED). (2005). Learning standards for mathematics. Retrieved from: http://www.emsc.nysed.gov/ciai/mst/mathstandards

Preiner, J. (2008). Introducing dynamic mathematics software to mathematics teacher: the case of GeoGebra. Dissertation in Mathematics Education, University of Salzburg, 2008.21. https://doi.org/10.13410/RG.2.2.15003.05921

Star, J. R. (2005). Reconceptualizing procedural knowledge. Journal for Research in Mathematics Education, 36(5), 404–411. https://doi.org/10.2307/30034943
Sternberg, R. (1997). *Thinking styles*. New York, NY: Cambridge University.

Tatar, E., & Zengin, Y. (2016). Conceptual understanding of definite integral with GeoGebra. Computers in the Schools, 33(2), 120-132. https://doi.org/10.1080/07380569.2016.1177480

Tutkun, O.F., & Ozturk, B. (2013). The effect of GeoGebra mathematical software to the academic success and the level of Van Hiele geometrical thinking. *International Journal of Academic Research Part B*, 5(4), 22-28. https://doi.org/10.7813/2075-4124.2013/5-4/B.3

Vygotsky, L. (1986). *Thought and Language*, Cambridge, MA: MIT Press.

Zhang, L. F., & Sternberg, R. J. (2002). Thinking styles and teachers’ characteristics. *International Journal of Psychology*, 37(1), 3–12. https://doi.org/10.1080/00207590143000171

Zhang, L. F., & Sternberg, R. J. (2005). A threefold model of intellectual styles. *Educational Psychology Review*, 17(1), 1–53. https://doi.org/10.1007/s10648-005-1635-4

Zulnaidi, H., & Zamri, S. N. A. S. (2017). The effectiveness of the GeoGebra software: The intermediary role of procedural knowledge on students’ conceptual knowledge and their achievement in mathematics. *Eurasia Journal of Mathematics, Science and Technology Education*, 13(6), 2155–2180. https://doi.org/10.12973/eurasia.2017.01219a

Zuya, H. E. (2017). Prospective Teachers’ Conceptual and Procedural Knowledge in Mathematics: The Case of Algebra. *American Journal of Educational Research*, 5(3), 310-315. https://doi.org/10.12691/education-5-3-12