Collective Strong Coupling with Homogeneous Rabi Frequencies using a 3D Lumped Element Microwave Resonator

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We design and implement 3D lumped element microwave cavities for the coherent and uniform coupling to electron spins hosted by nitrogen vacancy centers in diamond. Our design spatially focuses the magnetic field to a small mode volume. We achieve large homogeneous single spin coupling rates, with an enhancement of the single spin Rabi frequency of more than one order of magnitude compared to standard 3D cavities with a fundamental resonance at 3 GHz. Finite element simulations confirm that the magnetic field component is homogeneous throughout the entire sample volume, with a RMS deviation of 1.54%. With a sample containing $10^{17}$ nitrogen vacancy electron spins we achieve a collective coupling strength of $\Omega = 12$ MHz, a cooperativity factor $C = 27$ and clearly enter the strong coupling regime. This allows to interface a macroscopic spin ensemble with microwave circuits, and the homogeneous Rabi frequency paves the way to manipulate the full ensemble population in a coherent way.

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I. INTRODUCTION

Many different approaches have been proposed for the realization of hybrid quantum systems, ranging from atoms to solid state spins coupled to microwave cavities. Most of these systems are realized by using distributed transmission-line or zero-dimensional lumped element resonators, which allow small confined cavity mode volumes in the microwave domain. This gives rise to strong interactions of the electric or magnetic dipole moment of a two level system with the electromagnetic field of the cavity. In order to allow the coherent exchange of energy on the single photon level, the coupling strength of the single-mode cavity with the two level system has to exceed all dissipation rates in the system.

The interaction strength of the cavity field with a magnetic dipole is small compared to that of an electric dipole, therefore, the interaction has to be collectively enhanced. This can be realized by an ensemble of \( N \) spins in the cavity mode whose collective coupling strength is increased by a factor \( \sqrt{N} \). Strong coupling of such large spin ensembles to one-dimensional cavities has been successfully demonstrated. However, a big disadvantage of these cavities arises when coupling to large ensembles of emitters, because in planar circuits the cavity magnetic field component has a large field gradient. A macroscopic sample placed on top of such a structure is emerged in a magnetic field which changes significantly over the samples spatial extend. This leads to a strongly inhomogeneous distribution of coupling strengths between individual spins and the cavity. The resulting inhomogeneous single spin Rabi frequencies make it impossible to coherently manipulate and control the whole spin ensemble.

A different approach is using 3D microwave cavities. They offer the possibility of high Q values due to their closed structure and only negligible contributions to loss from surface two-level systems, but suffer from their huge mode volume. This results in very weak single spin coupling strengths and requires large ensemble sizes or spin system with larger magnetic moments. Some of theses issues have been addressed already and have shown promising results.

We report the design of a 3D lumped element microwave cavity in order to ensure large and homogeneous single spin Rabi frequencies. In combination with inhomogeneous broadening of the spin ensemble, for which we can account for using spin-echo refocusing techniques, an inhomogeneous field distribution throughout the sample prohibits the uniform and
coherent manipulation of a spin ensemble. Our design addresses this problem by making use of metallic structures to focus the AC magnetic field, such that the resulting field distribution is homogeneous throughout the mode volume while yielding large coupling rates at the same time. This allows us to couple to comparatively small spin ensembles in a uniform and coherent way.

In a numerical study, using COMSOL Multiphysics RF-module, we analyze the electromagnetic field distribution inside the cavity and optimize the coupling strength for an individual spin. The cavity design is fabricated and loaded with a high pressure high temperature (HPHT) diamond sample containing 40 ppm of NV spins. By using our new design we reach the strong coupling limit of cavity QED and open the way for new cavity QED protocols.

II. BOW-TIE CAVITIES

In cavity QED the interaction of light and matter is enhanced by creating boundary conditions for the electromagnetic field inside a spatially confined mode volume. In the optical domain this is usually achieved by using Fabry–Pérot like cavities which create standing waves in free space or inside a waveguide between two mirrors. This principle can be brought to the microwave regime in different ways. One of the most straightforward implementations is a rectangular wave guide cavity where the eigenfrequency is determined by the box dimensions with respect to the wavelength of the cavity photons. This leads to large resonators with mode volumes of at least larger than $\frac{\lambda^3}{8}$ (with $\lambda$ the wavelength of the cavity photons).

However, microwave electronics also uses on chip transmission line waveguides and the realization of one dimensional microwave cavities has proven as to be a powerful implementation from high efficiency single photon detection to quantum information processing. These devices and their spatial extend is however always connected to the wavelength of the fundamental resonance, regardless if they are one or three dimensional distributed.

In contrast an electrical $LC$ oscillator is created by discrete elements for capacitance $C$ and inductance $L$, for which the eigenfrequency is given by $\omega = \frac{1}{\sqrt{LC}}$. In radio and microwave technology the wavelength can become much larger then the actual size of the circuit which is known as lumped element circuit. This means that in a lumped element resonator (LER)
the actual extend is not related to the wavelength which allows to modify the size and shape of the cavity mode volume. This principle can also be applied to three dimensional lumped structures.\textsuperscript{28}

Here we demonstrate how this can be brought one step further by using our so-called "bow-tie" structure within a 3D cavity. These structures not only focus the magnetic field to a small mode volume, but also make it possible to couple strongly and homogeneous to a small spin ensemble (Fig. 1a). The two bow-tie shaped elements are placed in a closed conducting box where the top surfaces form two large capacitors with the lid. The total capacitance of this cavity is then given by two capacitors in a series configuration reading

$$C_{\text{tot}}^{-1} = C_1^{-1} + C_2^{-1} = 2C^{-1} = \frac{2}{\epsilon_0} \frac{d}{A},$$

(1)

where $A$ is the top area of one bow-tie, while $d$ is the separation between the bow-tie and the cover lid.

The inductance $L$ is governed by the path length $l$ the current has to flow between the two capacitors in order to close the $LC$ circuit (Fig. 1b). Numerical simulations show that it can be approximated by the inductance of a flat wire $L_{\text{tot}} \propto l \left( \ln\frac{l}{w} + w/l \right)$ of width $w$ corresponding to the width of the bow-tie structures. The resulting eigenfrequency is then given by

$$\omega_c = \frac{1}{\sqrt{L_{\text{tot}}C_{\text{tot}}}}.$$  

(2)

Thus, the resonance frequency of our 3D LER is only governed by the geometric values of $l$, $w$, $A$ and $d$ and not the dimensions of the enclosing box. This allows to build resonators significantly smaller than the wavelength of our cavity photons. Employing this principle we design the cavity in a way that the distance between the bow-tie structures can be adjusted after manufacturing. With control over the inductance and capacitance we reach a wide range of tune-ability for the resonance frequency and flexibility concerning different sample sizes in the active region.

The electromagnetic field inside the cavity is capacitively coupled via two coaxial ports, which are located below the bow-tie structures. In- and out-coupling capacitances can easily be adjusted by changing the length of these probes, controlling if the cavity is under or over coupled. The current is counter-propagating in the two bow-tie structures, therefore, the magnetic field between them is amplified and focused to a relatively small mode volume.
This results in an enhancement of the magnetic dipole interaction of the cavity mode with our spin system. Moreover, from numeric simulations, we observe that the magnetic field is homogeneous within 1.57\% in the region between the bow-tie structures (see Fig. 2) when calculating the RMS deviation integrated over the sample volume. The improved homogeneity field distribution also ensures homogeneous single spin coupling rates \(g_0\) over the entire sample volume.

From finite element simulations we can extract the electromagnetic field strength and distribution inside the cavity for a given input power. The resulting single spin cavity coupling rate is deduced from the total number of photons in the cavity mode for a specific input power since \(n \approx E_{em}/\hbar\omega\). Using the simulated magnetic field strength \(\vec{B}_{rf}\) normalized by the cavity photon number \(n\) as \(\vec{B}_{rf}^0 = \vec{B}_{rf}/\sqrt{n}\), we can infer the coupling strength of a single spin magnetic moment with the vacuum cavity mode by

\[
|g_0| = \frac{\mu_B g}{2\hbar} \left| \vec{B}_{rf}^0 \right| \left| \vec{S} \right| = \left| \frac{2}{3} \frac{\mu_B g}{2\hbar} \left| \vec{B}_{rf}^0 \right| \left| \vec{S} \right| \right|^3. \tag{3}
\]

Here \(\vec{B}_{rf}^0\) stands for the perpendicular component of the vacuum magnetic field with respect to the spin principle axis, \(\vec{B}_{rf}^0\) denotes the total vacuum magnetic field, \(\mu_B\) the Bohr magneton, \(g\) the gyromagnetic ratio that corresponds to the spin system used, \(|g_0|\) the absolute value for the single spin coupling strength and the factor \(\sqrt{2/3}\) accounts for the angle between cavity mode vector and spin principle axis. This yields values for the single spin coupling strength of \(|g_0| \approx 70\) mHz. These coupling rates are considerably higher compared to the coupling strengths of \(\approx 5\) mHz achievable in a standard 3D rectangular waveguide cavity in the same frequency region.

III. MEASUREMENT

For our measurements we adjust the length of the coupling ports such that the cavity is effectively under-coupled and the total quality factor \(Q\) is dominated by the internal cavity photon damping rate. These Cavity intrinsic losses are dominated by ohmic losses in the metallic structures, which are proportional to surface roughness and skin-depth of the AC magnetic field. To allow external magnetic fields to penetrate the cavity, which is necessary for Zeeman tuning electron spins into resonance with the cavity, we fabricate it using oxygen free copper (99.997\% purity). In order to further reduce ohmic losses from the
metallic structures we perform a mechanical surface treatment and polish all surfaces to a roughness $< 0.25 \mu m$. This yields, for a fundamental frequency of the unloaded cavity of 2.775 GHz a quality factor of $Q = 1920$ at 1 K (see Fig. 3a).

Next we load the cavity with a diamond sample containing a large ensemble of negatively charged nitrogen-vacancy (NV) defect centers. NV centers are formed by a substitutional nitrogen atom and an adjacent vacancy in the diamond lattice. In the following we use a synthetic type-Ib high pressure high temperature (HPHT) diamond with an initial concentration of 100 ppm nitrogen. The sample is irradiated with 2 MeV electrons at 800 °C and annealed at 1000 °C multiple times with a total electron dose of $1.1 \times 10^{19} \text{cm}^{-2}$. After electron irradiation and annealing this results in a NV concentration of approximately 40 ppm.

Four different sub-ensembles depending on the crystallographic location in a diamond cell are pointing in the $\langle 1,1,1 \rangle$ direction. Its electronic spin 1 ground state triplet can be described by a Hamiltonian of the following form

$$H_{NV} = \hbar D S_z^2 + g \mu_B \vec{B}_0 \cdot \vec{S},$$

with a zero-field splitting of $D/\hbar = 2.87 \text{GHz}$ for the axial component along the NV-axes. The second term accounts for the interaction with an static external magnetic field ($\vec{B}_0$), which we use to lift the degeneracy of the $m_s = \pm 1$ manifold and tune the spin transitions into resonance with the cavity mode.

The diamond sample is glued into the cavity using vacuum grease. The (1,0,0) face is placed parallel to the magnetic field mode direction ensuring the projection of the cavity mode onto the four possible NV axes to be equal. The diamond loaded cavity is mounted inside a dilution refrigerator operating at 25 mK in order to achieve thermal polarization rates of our spin ensemble in the ground state well above 99%. A 3D Helmholtz coil configuration provides arbitrary d.c. magnetic fields for Zeeman tuning our spin ensemble into resonance with the cavity later on.

We use a vector network analyzer to probe the transmission of the coupled cavity spin system in the stationary state. The diamond loaded cavity has a fundamental resonance frequency of $\nu = 3.121 \text{GHz}$ with a $Q$ value of 1637 and cavity linewidth of $\kappa = 1.91 \text{MHz}$ (HWHM), measured with all spins far detuned from the cavity resonance frequency. By applying a homogeneous external d.c. magnetic field on the whole ensemble in the $[0,1,0]$
direction, we tune all four possible NV sub-ensembles into resonance with the cavity mode. The measured scattering parameter $|S_{21}|^2$ is compared to an analytical expression for the scattering parameter obtained from a Jaynes-Cummings Hamiltonian reading

$$|S_{21}|^2 = \left| \kappa \frac{(\omega - \omega_s - i\gamma^*)}{(\omega - \omega_c - i\kappa)(\omega - \omega_s - i\gamma^*) - \Omega^2} \right|^2,$$

with $\gamma^*$ the inhomogeneously broadened linewidth of our spin ensemble, $\omega$ the probe frequency and $\omega_c$ the cavity resonance frequency (see Fig. 3). At the point where the central spin transition is in resonance with the cavity mode, we observe an avoided crossing which corresponds to a collective coupling strength of $\Omega = 12.46$ MHz. This is in good agreement with our simulations, assuming a homogeneous single spin coupling strength of 70 mHz and a number of spins coupling to our cavity as $\approx 10^{17}$. The number of spins can be deduced from the sample volume of $4.2 \text{ mm} \times 3.4 \text{ mm} \times 0.92 \text{ mm}$ and an approximate NV density of 40 ppm.

The collective coupling strength is large enough to satisfy the strong coupling condition $\Omega > \kappa, \gamma^*$ (HWHM, see Fig. 3a) with an estimated inhomogeneously broadened spin linewidth of $\gamma^* \approx 3$ MHz (HWHM). Note that we are neglecting effects like the cavity protection effect and treat $\gamma^*$ to be a constant to the first order. This gives a cooperativity parameter of approximately $C = \Omega^2/(\kappa \gamma^*) \approx 27$.

Our results show that we successfully enter the strong coupling regime of cavity QED using our 3D lumped element cavity design. The homogeneous single spin Rabi frequency allows coherent manipulation of the whole ensemble, taking into account the inhomogeneously broadened linewidth. Spin control protocols no longer suffer from excitations diffusing out of the active region via spin-spin interaction, since all spins are emerged in a cavity mode with same field strength.

For experiments without a static external magnetic field, we carried out first measurements with superconducting cavities fabricated out of aluminum yielding quality factors up to $10^5$. They offer much higher $Q$-factors, since dissipation mechanisms are only governed by AC losses in the superconducting material. As a consequence these cavities offer higher sensitivity to emitters in the mode volume, while maintaining the advantages of large and homogeneous coupling rates.
IV. CONCLUSION

We have presented a novel design for a 3D lumped element resonator for cavity QED experiments. They offer high single spin coupling rates combined with a homogeneous field distribution throughout their mode volume. We fabricated the cavity out of copper, which allows Zeeman tuning of the coupled spin system with an external magnetic field. These cavities already yield reasonable high $Q$-values. Even higher $Q$-values, albeit no external magnetic field tunability, are achievable when using superconducting cavities. With the large coupling rates of this cavity we were able to enter the strong coupling regime of cavity QED using an ensemble of NV centers in diamond. In addition, the achieved homogeneity of the cavity field is a requirement to perform coherent spin manipulation on the whole spin ensemble and implement cavity QED protocols where this becomes important.

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FIG. 1. a) Picture of our manufactured cavity with diamond sample glued to the cavity using vacuum grease. Note that for illustrative purposes the top lid that closes the structure and the sidewalls are not shown in this picture (Lighter brown sections in Fig. 1b). b) Schematic cross section of the cavity perpendicular to the mode direction. Capacitors and inductance of the cavity are labeled as $C_1$, $C_2$ and $L$, with the electric and magnetic field generated by this design. The current that generates the focused magnetic field is drawn as red line. The diamond sample is drawn as shaded red rectangle.
FIG. 2. Illustration of the simulated magnetic field distribution. The plot shows a cross section of the mode volume parallel to the direction of the magnetic field mode (y-direction in Fig.1a). For the diamond sample used in this work (shown as dashed area) the deviation for the absolute value of the coupling strength is at most 7%, with a RMS deviation of 1.54%. The green contour lines divide the area in sections wherein the magnetic field strength differs by a certain percentage.

FIG. 3. a) Transmission measurements for the cavity with the spin system far detuned (green curve) and on resonance with an observed normal mode splitting observed for four NV sub-ensembles (blue curve). Fitting the normal mode splitting (red curve) with Eq. 5 yields a collective coupling strength of $\Omega = 12.46$ MHz. b) Cavity transmission spectroscopy as a function of $\nu_P$ versus $\Delta_s$. Here $\nu_P$ stands for the detuning of the probe frequency relative to the spin frequency and $\Delta_s$ for the detuning of the cavity with respect to the spins. The dashed line denotes the detuning for which the normal mode splitting is measured.
REFERENCES

1Xiang, Z.-L., Ashhab, S., You, J. Q. & Nori, F. Hybrid quantum circuits: Superconducting circuits interacting with other quantum systems. *Reviews of Modern Physics* **85**, 623–653 (2013).

2Wallquist, M., Hammerer, K., Rabl, P., Lukin, M. & Zoller, P. Hybrid quantum devices and quantum engineering. *Physica Scripta* **2009**, 014001 (2009).

3Verdú, J. *et al.* Strong Magnetic Coupling of an Ultracold Gas to a Superconducting Waveguide Cavity. *Physical Review Letters* **103**, 043603 (2009).

4Imamoglu, A. Cavity QED Based on Collective Magnetic Dipole Coupling: Spin Ensembles as Hybrid Two-Level Systems. *Physical Review Letters* **102**, 083602 (2009).

5Kubo, Y. *et al.* Hybrid Quantum Circuit with a Superconducting Qubit Coupled to a Spin Ensemble. *Phys. Rev. Lett.* **107**, 220501 (2011).

6Göppl, M. *et al.* Coplanar waveguide resonators for circuit quantum electrodynamics. *Journal of Applied Physics* **104**, 113904 (2008).

7Hatridge, M. *et al.* Quantum Back-Action of an Individual Variable-Strength Measurement. *Science* **339**, 178–181 (2013).

8Wallraff, A. *et al.* Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics. *Nature* **431**, 162–167 (2004).

9Abe, E., Wu, H., Ardavan, A. & Morton, J. J. L. Electron spin ensemble strongly coupled to a three-dimensional microwave cavity. *Applied Physics Letters* **98**, 12–15 (2011).

10Boero, G. *et al.* Room temperature strong coupling between a microwave oscillator and an ensemble of electron spins. *Journal of Magnetic Resonance* **231**, 133–140 (2013).

11Thompson, R. J., Rempe, G. & Kimble, H. J. Observation of normal-mode splitting for an atom in an optical cavity. *Physical Review Letters* **68**, 1132–1135 (1992).

12Brennecke, F. *et al.* Cavity QED with a Bose–Einstein condensate. *Nature* **450**, 268–271 (2007).

13Colombe, Y. *et al.* Strong atom–field coupling for Bose–Einstein condensates in an optical cavity on a chip. *Nature* **450**, 272–276 (2007).

14Amsüss, R. *et al.* Cavity QED with Magnetically Coupled Collective Spin States. *Phys. Rev. Lett.* **107**, 060502 (2011).
15Kubo, Y. et al. Strong Coupling of a Spin Ensemble to a Superconducting Resonator. *Physical Review Letters* **105**, 140502 (2010).

16Schuster, D. I. et al. High-Cooperativity Coupling of Electron-Spin Ensembles to Superconducting Cavities. *Physical Review Letters* **105**, 140501 (2010).

17Zollitsch, C. W. et al. High cooperativity coupling between a phosphorus donor spin ensemble and a superconducting microwave resonator. *Applied Physics Letters* **107**, 142105 (2015).

18Zhang, X., Zou, C.-L., Jiang, L. & Tang, H. X. Strongly Coupled Magnons and Cavity Microwave Photons. *Phys. Rev. Lett.* **113**, 156401 (2014).

19Huebl, H. et al. High Cooperativity in Coupled Microwave Resonator Ferrimagnetic Insulator Hybrids. *Phys. Rev. Lett.* **111**, 127003 (2013).

20Paik, H. et al. Observation of High Coherence in Josephson Junction Qubits Measured in a Three-Dimensional Circuit QED Architecture. *Physical Review Letters* **107**, 240501 (2011).

21Probst, S. et al. Three-dimensional cavity quantum electrodynamics with a rare-earth spin ensemble. *Phys. Rev. B - Condens. Matter Mater. Phys.* **90**, 1–5 (2014).

22Axline, C. et al. A coaxial line architecture for integrating and scaling 3d cQED systems. *arXiv:1604.06514 [quant-ph]* (2016). ArXiv: 1604.06514.

23Reshitnyk, Y., Jerger, M. & Fedorov, A. 3d microwave cavity with magnetic flux control and enhanced quality factor. *arXiv:1603.07423 [quant-ph]* (2016). ArXiv: 1603.07423.

24Walther, H., Varcoe, B. T. H., Englert, B.-G. & Becker, T. Cavity quantum electrodynamics. *Rep. Prog. Phys.* **69**, 1325 (2006).

25Creedon, D. L., Goryachev, M., Kostylev, N., Sercombe, T. & Tobar, M. E. A 3d Printed Superconducting Aluminium Microwave Cavity. *arXiv:1604.04301 [physics]* (2016). ArXiv: 1604.04301.

26Reagor, M. et al. Reaching 10 ms single photon lifetimes for superconducting aluminum cavities. *Applied Physics Letters* **102**, 192604 (2013).

27Floch, J.-M. L. et al. Towards achieving strong coupling in three-dimensional-cavity with solid state spin resonance. *Journal of Applied Physics* **119**, 153901 (2016).

28Creedon, D. L. et al. Strong coupling between P1 diamond impurity centers and a three-dimensional lumped photonic microwave cavity. *Phys. Rev. B* **91**, 140408 (2015).
29Minev, Z. et al. Planar Multilayer Circuit Quantum Electrodynamics. *Physical Review Applied* **5**, 044021 (2016).

30Krimer, D. O., Putz, S., Majer, J. & Rotter, S. Non-Markovian dynamics of a single-mode cavity strongly coupled to an inhomogeneously broadened spin ensemble. *Phys. Rev. A* **90**, 043852 (2014).

31Pozar, M. D. *Microwave Engineering* (John Wiley & Sons, 2011), 4th edn.

32Lita, A. E., Miller, A. J. & Nam, S. W. Counting near-infrared single-photons with 95% efficiency. *Optics Express* **16**, 3032 (2008).

33Kubo, Y. et al. Electron spin resonance detected by a superconducting qubit. *Phys. Rev. B* **86**, 064514 (2012).

34Johnson, B. R. et al. Quantum non-demolition detection of single microwave photons in a circuit. *Nat Phys* **6**, 663–667 (2010).

35Barends, R. et al. Superconducting quantum circuits at the surface code threshold for fault tolerance. *Nature* **508**, 500–503 (2014).

36Proekt, L. & Cangellaris, A. C. Investigation of the impact of conductor surface roughness on interconnect frequency-dependent ohmic loss (2003).

37Putz, S. et al. Protecting a spin ensemble against decoherence in the strong-coupling regime of cavity QED. *Nature Physics* **10**, 720–724 (2014).