Asymmetric double quantum dots as a subminiature mesoscopic cell

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A subminiature mesoscopic cell, consisting of an asymmetric double quantum dot capacitively coupled to a nearby mesoscopic circuit, is proposed, which can transform disordered noise energy to ordered electric energy. Two schemes, the noise originating from the nearby mesoscopic circuit and from the electromagnetic wave disturbance in external environment, are investigated. We found that the proposed cell can manifest as a good constant current source and the output current may not reach its largest value even if the circuit is shorted.

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Mesoscopic physics, a new branch of condensed matter physics, has been developed and become an active field in the last two decades, not only because of the fundamental physics, but also because of the great potential as regards the new generation of electronic and photonic devices. Very recently, by using inelastic transitions in a tunable two-level system, Aguado and Kouwenhoven proposed a detector of high-frequency quantum noise. This detector consists of a double quantum dot which is capacitively coupled to a nearby mesoscopic conductor. Then from the inelastic current through the double quantum dots, one can measure the noise spectrum in the nearby conductor.

Noise always exists in the electronic systems, whether Johnson-Nyquist noise in equilibrium states or shot noise in non-equilibrium states, and has been investigated extensively. In addition, a system usually exists in an electromagnetic environment from that the noise can also be induced in the system from the electromagnetic wave (EMW) disturbance of the environment.

In this letter, we propose a subminiature mesoscopic cell, consisting of an asymmetric double quantum dot (DQD) and a noise offering circuit, capacitively coupled to a nearby mesoscopic circuit via capacitors $C_c$ (Fig.1(b)), by which one can obtain the useful ordered energy from the disordered noise energy. In the proposed structure, the asymmetric DQD is described as a system of three tunnel barriers (See Fig.1(a)) with capacitances $C$. $Z_a$ is the impedance of the appliance load. In this model, the current noise $S_i$ of the noise offering circuit will give rise to the fluctuation of the electric potential of the two quantum dot following the intro-dot energy levels $E_L$ and $E_R$ fluctuate too. This leads that the electron in the level $E_R$ of the right dot maybe absorb a quanta from the noise offering circuit and jump to the level $E_L$ of the left dot (see Fig.1(a)). Therefore, a inelastic current goes through the DQD system from the right side to the left side. We emphasize that no battery exists in this device, but the system can produce ordered current, in other words, the system play a role of electric cell. Specifically, we consider that: (1) Each dot of the DQD has only one electronic level, $E_L$ for the left dot and $E_R$ for the right dot, respectively. (2) The DQD is designed asymmetric to make the two levels $E_L$ and $E_R$ not in line. Notice that no gate voltage applied in this DQD for controlling the intra-dot levels, different from the cases in Ref. (3) $E_L < \mu_L, \mu_R < E_R$, where $\mu_a$ is the chemical potential, so that the level $E_L$ is almost empty and the level $E_R$ is almost occupied by the electron. (4) $\Delta E = E_L - E_R \gg T_c$ with $T_c$ the coupling between two dots. Then the coherent tunneling between the two dots can be neglected while the incoherent tunneling dominates, in which the photon emission or absorption to or from the environment may happen. (5) Let $\Gamma_L$ and $\Gamma_R$ denote the tunneling rates through the left and right barriers, and $\Gamma_i(\epsilon)$ the inelastic transition rate through the middle barrier. Here we consider the situation with $\Gamma_i \ll \Gamma_L, \Gamma_R$, i.e., the middle barrier is much larger than left and right barriers. Then the inelastic current through the DQD system (from the right side to the left side) is mainly decided by $\Gamma_i(\epsilon)$. From perturbation theory, one can obtain this inelastic current as

\begin{equation}
I(\Delta E) = \frac{e}{\hbar} \Gamma_i(\Delta E) f_L(E_L) [1 - f_R(E_R)]
- \frac{e}{\hbar} \Gamma_i(-\Delta E) f_R(E_R) [1 - f_L(E_L)],
\end{equation}

where $f_{L/R}(\epsilon) = \{e^{-(\epsilon-\mu_{L/R})/k_BT_D} + 1\}^{-1}$ is the Fermi distribution function in the left/right lead, $T_D$ is the temperature of DQD’s system. Here $\Gamma_i(\epsilon)$ is related to the coupling strength $T_c$ by $\Gamma_i(\epsilon) = T_c^2 P(\epsilon)$ and the probability $P(\epsilon)$ for the exchange of energy quanta with the environment is given (at $\epsilon /\Omega = 0$) by $P(\epsilon) = \frac{\hbar \Omega}{\epsilon h}[Z(\omega)]^2 S_f(\omega)/\epsilon^2$, $S_f(\omega)$ is the current noise spectrum at frequency $\omega$ ($\omega = \epsilon h$) in the mesoscopic device, $R_K = h/e^2 \approx 25.8k\Omega$ is the quantum resistance, $Z(\omega)$ is the trans-impedance connecting the DQD system and the noise offering circuit. For the circuit of Fig.1(b), $Z(\omega)$ is:

\begin{equation}
Z(\omega) = \frac{C_x/C}{i C_x \omega + \tilde{Z}_a^{-1} + \tilde{Z}_s^{-1} \left(1 + \frac{i C_x \omega + \tilde{Z}_a^{-1}}{\Omega^2 \omega^2}\right)},
\end{equation}
where \( \tilde{Z}_a = Z_a + 1/iC\omega \), \( \tilde{Z}_s = Z_s + 1/iC\omega \), and \( C_c = C(C + C_e)/2(C + C_e) \).

In terms of noise spectrum, the inelastic current through the asymmetric QDQ is:

\[
I = \frac{2e\pi T^2}{hR_K} \frac{|Z(\Delta E/h)|^2}{\Delta E^2} (g_{LR} - g_{RL})
\]  
(3)

where \( g_{\alpha\beta} = S_1((E_n - E_{\beta})/\hbar)f_{\alpha}(E_n)(1 - f_{\beta}(E_{\beta})) \). Notice that the output current of the subminiature mesoscopic cell (i.e. the current through the appliance load \( Z_a \)) is equal to the current through the DQD. In the following, we investigate in detail the output current for the two schemes: (i) the noise originates from the equilibrium noise offering circuit itself. (ii) the noise from the EMW disturbance in a nearby external electromagnetic environment.

(i) the noise from the noise offering circuit. In this case we assume that the noise offering circuit is not affected by EMW disturbance in the external environment; and the mesoscopic device is a quantum point contact (QPC) in equilibrium. Then the noise spectrum \( S_1(\omega) \) can be expressed as:

\[
S_1(\omega) = 4Gh\omega/(1 - e^{-\omega/k_BT_s})
\]

where \( G = 2R_L/\hbar \), and \( Z_s = 0.5R_K \); which are typical experimental values.

(2) Let \( T_c = \Delta E/100 \), so as to satisfy the condition \( \Delta E \gg T_c \). (3) Set \( E_L = -E_R \) and \( \mu_L = -\mu_R \). (4) Let the QPC only has one open channel, leading to \( G = 2/R_K \). First, we investigate the case in which the circuit is shorted, i.e. \( Z_a = 0 \), meanwhile \( \mu_L = \mu_R = 0 \). The output current \( I \) vs. the temperature difference \( \Delta T \) at different \( T_D \) is presented in Fig.2(a), showing an almost linear relation. Fig.2(b) shows the output current \( I \) vs the energy difference \( \Delta E \), exhibiting a non-monotonic variation. In both limits of \( \Delta E \to 0 \) and \( \Delta E \to \infty \), \( I \) becomes to zero. The position \( \Delta E \) at which \( I \) has its maximum, is determined by \( \Delta T \) and \( T_D \). It should be emphasized that \( I \) is exactly zero if \( \Delta T = 0 \), regardless of the values of \( T_D \) and \( \Delta E \). This result is obviously consistent with the second law of thermodynamics. In fact, one can consider the cell as a heat engine, which works between two heat reservoirs with the temperature \( T_a \) and \( T_D \).

(ii) the noise originates from the EMW disturbance in the nearby environment. In this case, we can assume that the temperature difference \( \Delta T \) is zero, and the noise of the EMW disturbance is the only source of the output current \( I \). For simplicity we also assume that the noise spectrum \( S_1(\omega) \) from the EMW disturbance is independent with \( \omega \), i.e. a white noise, \( S_1(\omega) = S_0 \). The inset in Fig.2(b) shows the output current \( I \) vs. \( \Delta E \) at the short circuit case. One can clearly see that the output current \( I \) is nonzero with the following features: \( I \) is zero at \( \Delta E = 0 \). With the increase of \( \Delta E \), the difference of the occupation number becomes larger, leading to a larger current \( I \). Then \( I \) reaches its maximum when \( \Delta E \) at a certain value (several times of \( T_n \)). With the further increase of \( \Delta E \), \( I \) decreases slowly, because that the transition from \( E_R \) to \( E_L \) becomes to very difficult at large \( \Delta E \). It is merit mentioning that: in order to obtain a greater output current \( I \), one can take different ways as: (1) To lower the temperature of the system, \( T_a \) and \( T_D \). (2) To change the noise offering circuit into a LC circuit with the condition of \( 1/\sqrt{LC} = \Delta E/h \).

Up to now, we have studied the output current \( I \) at the short circuit case (i.e. \( Z_a = 0 \)), in which the subminiature mesoscopic cell can produce a current \( I \), but its output power is zero. In the following, we study the case of \( Z_a \neq 0 \), in which the cell has a nonzero output power. Since in this case, \( \mu_L \) and \( \mu_R \) are nonzero, the output current \( I \) has to be solved from Eq.(3) and the equation \( IZ_a = (\mu_L - \mu_R)/e \). Fig.3(a) shows \( I \) vs \( Z_a \) for case (i). One finds that: (1) When \( Z_a \) varies from 0 to a critical value \( Z_{ac} \), \( I \) keeps almost independent with \( Z_a \). This means that this cell is a good constant current source.

The critical appliance impedance \( Z_{ac} \) is also dependent with \( \Delta E \), and is approximately determined by \( Z_{ac} = \Delta E/E \). Neglecting external losses, one has \( \eta = IV\Delta t/(I\Delta ED\Delta t/e) = V/\Delta E \), where \( V = (\mu_L - \mu_R)/e \) is the bias voltage across the appliance impedance \( Z_a \). \( \eta \) vs \( T_a \) at different \( Z_a \) is shown in the inset of Fig.2(a).

For comparison, the limit efficiency of heat engine, \( (T_n - T_D)/T_n \), is also shown by the dotted curve. The efficiency \( \eta \) monotonously increases with the temperature \( T_a \) and the appliance impedance \( Z_a \). Notice that \( \eta \) can never exceed \( (T_n - T_D)/T_n \), the efficiency of the ideal heat engine by the second law of thermodynamics. But the efficiency \( \eta \) may still be quite high, e.g. at \( Z_a = 10^k \Omega \) and \( T_n = 3K \), \( \eta \approx 57\% \); which is close to the limit efficiency of heat engine, 66.7%.

Finally, let us study an interesting result. It is well known that for the ordinary electric cell, the output current \( I \) has its largest value if the circuit is shorted. However, for the proposal cell, the output current \( I \) may be larger if the circuit is loaded (\( Z_a \neq 0 \)) than the circuit is shorted (\( Z_a = 0 \)). Fig.3(b) shows \( I \) vs. \( Z_a \) at very low temperature (\( T_n = 0.1K \) and \( T_D = 0.6K \)). It is obvious that \( I \) is not the largest at \( Z_a = 0 \). For example, for \( \Delta E = 0.2m\text{eV} \), the largest current \( I \) is at \( Z_a \approx 43k\Omega \), not at \( Z_a = 0K \). Because that the current \( I(\Delta E) \) is directly proportional to the modulus square of the trans- impedance, \( |Z(\Delta E/h)|^2 \), which also depends on \( Z_a \) (See Eqs.(2) and (3)). While \( Z_a \) increases, \( |Z(\Delta E/h)|^2 \) increases quickly in a certain range of \( Z_a \), may lead to a larger current \( I \).

In summary, by using an asymmetric double quantum
dots, we propose a subminiature mesoscopic cell, which can change the disordered noise energy in a nearby mesoscopic device to the useful ordered electrical energy. Two different cases of the origin of the noise are studied. The cell manifests a good constant current source and high efficiency. Moreover, an interesting phenomenon, that the output current may have its largest value for the loaded circuit is discussed, which is important from the point of view of the application.

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FIGURE CAPTIONS

Fig. 1 (a) Energy diagram of the asymmetric DQD. (b) A schematic structure of the subminiature mesoscopic cell. The symbols marked with "B" in the asymmetric DQD are the three tunnel barriers. $Z_a$ is the appliance impedance.

Fig. 2 The output current $I$ of the short circuit for the case of the noise from the EMW disturbance in nearby environment. The dotted, dashed, and solid curves correspond to $T_n = T_D = 1K$, $2K$, and $3K$, respectively.

Fig. 3 The output current $I$ vs $Z_a$. The parameters are: (a) $T_D = 0.1K$ and $T_n = 1.5K$; (b) $T_D = 0.1K$ and $T_n = 0.6K$.

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Fig. 1
Fig. 2

(a) $\eta$ vs $T_n (K)$

(b) $I (\mu A)$ vs $\Delta E (meV)$
Fig. 3

(a) $Z_{ac}$ (KΩ) vs. $I$ (µA)

- $\Delta E = 0.2$ meV
- $\Delta E = 0.3$ meV

(b) $Z_{ac}$ (KΩ) vs. $I$ (nA)

- $\Delta E = 0.18$ meV
- $\Delta E = 0.2$ meV