Analysis of the $\frac{1}{2}^+$ doubly heavy baryon states with QCD sum rules

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Abstract

In this article, we study the $\frac{1}{2}^+$ doubly heavy baryon states $\Omega_{QQ}$ and $\Xi_{QQ}$ by subtracting the contributions from the corresponding $\frac{1}{2}^-$ doubly heavy baryon states with the QCD sum rules, and make reasonable predictions for their masses. Those doubly heavy baryon states maybe observed at the Tevatron, the LHCb and the PANDA.

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1 Introduction

In 2002, the SELEX collaboration reported the first observation of a signal for the doubly charmed baryon state $\Xi_{cc}^+$ in the charged decay mode $\Xi_{cc}^+ \to \Lambda_c^+ K^− \pi^+$ [1], and confirmed later by the same collaboration in the decay mode $\Xi_{cc}^+ \to pD^+ K^−$ with the measured mass $M_{\Xi} = (3518.9 \pm 0.9)$ MeV [2]. However, the Babar and Belle collaborations have not observed any evidence for the doubly charmed baryons in $e^+e^−$ annihilations [3, 4].

The charmed and bottom baryons which contain one (two) heavy quark(s) are particularly interesting for studying dynamics of the light quarks in the presence of the heavy quark(s), and serve as an excellent ground for testing predictions of the quark models and heavy quark symmetry. There have been several approaches to deal with the doubly heavy baryon masses, such as the relativistic quark model [5, 6], the non-relativistic quark model [7, 8, 9, 10], the three-body Faddeev method [11], the potential approach combined with the QCD sum rules [12], the quark model with AdS/QCD inspired potential [13], the MIT bag model [14], the full QCD sum rules [15, 16], and the Feynman-Hellmann theorem and semiempirical mass formulas [17], etc.

The QCD sum rules is a powerful theoretical tool in studying the ground state heavy baryons [18, 19, 20]. In the QCD sum rules, the operator product expansion is used to expand the time-ordered currents into a series of quark and gluon condensates which parameterize the long distance properties of the QCD vacuum. Based on the quark-hadron duality, we can obtain copious information about the hadronic parameters at the phenomenological side [18, 19]. There have been several works on the masses of the heavy baryon states with the full QCD sum rules and the QCD sum rules in the heavy quark effective theory (one can consult Ref. [21] for more literatures).

In Ref. [22], Jido et al introduce a novel approach based on the QCD sum rules to separate the contributions of the negative parity light flavor baryon states from the positive parity light flavor baryon states, as the interpolating currents may have non-vanishing couplings to both the negative parity and positive parity baryon states [23]. Before the work of Jido et al, Bagan et al take the infinite mass limit for the heavy quarks to separate the contributions of the positive parity and negative parity heavy baryon states unambiguously [24].

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In Refs. [21, 22, 23, 28], we study the heavy baryon states \( \Omega_Q, \Xi_Q^*, \Sigma_Q, \Lambda_Q \) and \( \Xi_Q \) with the full QCD sum rules, and observe that the pole residues of the \( \frac{1}{2}^+ \) heavy baryon states from the sum rules with different tensor structures are consistent with each other, while the pole residues of the \( \frac{1}{2}^- \) heavy baryon states differ from each other greatly. In Refs. [21, 23], we follow Ref. [21, 23] and study the masses and pole residues of the \( \frac{1}{2}^+ \) heavy baryon states \( \Omega_Q, \Xi_Q^*, \Sigma_Q, \Lambda_Q \) and \( \Xi_Q \) by subtracting the contributions of the negative parity heavy baryon states to overcome the embarrassment. Those pole residues are important parameters in studying the radiative decays \( \Omega_Q^* \to \Omega_Q \gamma, \Xi_Q^* \to \Xi_Q^* \gamma \) and \( \Sigma_Q^* \to \Sigma_Q \gamma \) [23, 24, etc.]

In this article, we extend our previous works to study the \( \frac{1}{2}^+ \) doubly heavy baryon states \( \Xi_{QQ} \) and \( \Omega_{QQ} \) with the QCD sum rules.

The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the doubly heavy baryon states \( \Xi_{QQ} \) and \( \Omega_{QQ} \) in Sec.2; in Sec.3, we present the numerical results and discussions; and Sec.4 is reserved for our conclusions.

## 2 QCD sum rules for the \( \Xi_{QQ} \) and \( \Omega_{QQ} \)

The \( \frac{1}{2}^+ \) doubly heavy baryon states \( \Xi_{QQ} \) and \( \Omega_{QQ} \) can be interpolated by the following currents \( J_\Xi(x) \) and \( J_\Omega(x) \) respectively,

\[
J_\Xi(x) = \epsilon^{i j k} Q_i^T(x) C \gamma_\mu Q_j(x) \gamma_5 \gamma^\mu q_k(x), \\
J_\Omega(x) = \epsilon^{i j k} Q_i^T(x) C \gamma_\mu Q_j(x) \gamma^\mu s_k(x),
\]

where the \( Q \) represents the heavy quarks \( c \) and \( b \), the \( i, j \) and \( k \) are color indexes, and the \( C \) is the charge conjunction matrix. In the heavy quark limit, the doubly heavy baryon states can be described by the (heavy)diquark-(light)quark model [12].

The corresponding \( \frac{1}{2}^- \) doubly heavy baryon states can be interpolated by the currents \( J_- = i \gamma_5 J_+ \) because multiplying \( i \gamma_5 \) to \( J_+ \) changes the parity of \( J_+ \) [22], where the \( J_+ \) denotes the currents \( J_1(x) \) and \( J_\Xi(x) \). The correlation functions are defined by

\[
\Pi_\pm(p) = i \int d^4 x e^{ipx} \langle 0 | T \{ J_\pm(x) J_\pm(0) \} | 0 \rangle,
\]

and can be decomposed as

\[
\Pi_\pm(p) = \eta \Pi_1(p^2) \pm \Pi_0(p^2),
\]

due to Lorentz covariance. The currents \( J_+ \) couple to both the positive parity and negative parity baryon states [23], i.e. \( \langle 0 | J_+ | B^- \rangle \langle B^- | J_+ | 0 \rangle = -\gamma_5 \langle 0 | J_- | B^- \rangle \langle B^- | J_- | 0 \rangle \gamma_5 \), where the \( B^- \) denote the negative parity baryon states.

We insert a complete set of intermediate baryon states with the same quantum numbers as the current operators \( J_+(x) \) and \( J_-(x) \) into the correlation functions \( \Pi_\pm(p) \) to obtain the hadronic representation [18, 19]. After isolating the pole terms of the lowest states of the doubly heavy baryons, we obtain the following result [22]:

\[
\Pi_+(p) = \lambda_+^2 \frac{M_+}{M_+^2 - p^2} + \lambda_+^2 \frac{p - M_-}{M_-^2 - p^2} + \cdots,
\]
where the $M_\pm$ are the masses of the lowest states with parity $\pm$ respectively, and the $\lambda_\pm$ are the corresponding pole residues (or couplings). If we take $\vec{p} = 0$, then

$$\lim_{\epsilon \to 0} \frac{\text{Im} \Pi_+(p_0 + i\epsilon)}{\pi} = \lambda_+^2 \frac{\gamma_0 + 1}{2} \delta(p_0 - M_+) + \lambda_-^2 \frac{\gamma_0 - 1}{2} \delta(p_0 - M_-) + \cdots$$

$$= \gamma_0 A(p_0) + B(p_0) + \cdots,$$

where

$$A(p_0) = \frac{1}{2} \left[ \lambda_+^2 \delta(p_0 - M_+) + \lambda_-^2 \delta(p_0 - M_-) \right],$$

$$B(p_0) = \frac{1}{2} \left[ \lambda_+^2 \delta(p_0 - M_+) - \lambda_-^2 \delta(p_0 - M_-) \right],$$

the $A(p_0) + B(p_0)$ and $A(p_0) - B(p_0)$ contain the contributions from the positive-parity and negative-parity baryon states respectively.

We calculate the light quark parts of the correlation functions $\Pi_+(p)$ in the coordinate space and use the momentum space expression for the heavy quark propagators, i.e. we take

$$S_{ij}(x) = \frac{i \delta_{ij} x - \delta_{ij} m_s}{2 \pi^2 x^4} - \frac{\delta_{ij} (\bar{s}s)}{4 \pi^2 x^4} + \frac{i \delta_{ij} m_s (\bar{s}s) x}{48}$$

$$- \frac{i}{32 \pi^2 x^4} G^{ij}_{\mu\nu}(x) [f \sigma^{\mu\nu} + \sigma^{\mu\nu} f] + \cdots,$$

$$S_Q^{ij}(x) = \frac{i}{(2\pi)^4} \int d^4k e^{-ikx} \left\{ \frac{\delta_{ij}}{k - m_Q} - \frac{g_s G^{\alpha\beta}_{ij}}{4} \frac{\sigma_{\alpha\beta}(k + m_Q) + (k + m_Q) \sigma_{\alpha\beta}}{(k^2 - m_Q^2)^2} \right\} + \pi^2 \left( \frac{\alpha_s GG}{\pi} \right) \delta_{ij} m_Q \frac{k^2 + m_Q k}{(k^2 - m_Q^2)^2} + \cdots,$$

where $\langle \frac{\alpha_s GG}{\pi} \rangle = \langle \frac{\alpha_s G_{\mu\nu}G^{\mu\nu}}{\pi} \rangle$, then resort to the Fourier integral to transform the light quark parts into the momentum space in $D$ dimensions, take $\vec{p} = 0$, and use the dispersion relation to obtain the spectral densities $\rho^A(p_0)$ and $\rho^B(p_0)$ (which correspond to the tensor structures $\gamma_0$ and 1 respectively) at the level of quark-gluon degrees of freedom. Finally we introduce the weight functions $\exp \left[ -\frac{p_0^2}{T^2} \right]$, $p_0^2 \exp \left[ -\frac{p_0^2}{T^2} \right]$, and obtain the following sum rules,

$$\lambda_+^2 \exp \left[ -\frac{M_+^2}{T^2} \right] = \int_\Delta \sqrt{\gamma_0} dp_0 \left[ \rho^A(p_0) + \rho^B(p_0) \right] \exp \left[ -\frac{p_0^2}{T^2} \right],$$

$$\lambda_+^2 M_+^2 \exp \left[ -\frac{M_+^2}{T^2} \right] = \int_\Delta \sqrt{\gamma_0} dp_0 \left[ \rho^A(p_0) + \rho^B(p_0) \right] p_0^2 \exp \left[ -\frac{p_0^2}{T^2} \right],$$

(8) and (9) respectively.
where

\[
\rho^A_\Omega(p_0) = \frac{3p_0}{8\pi} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (1 - \alpha - \beta)(p_0^2 - \tilde{m}_Q^2)(5p_0^2 - 3\tilde{m}_Q^2) + \frac{3m^2_p}{8\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (1 - \alpha - \beta)(p_0^2 - \tilde{m}_Q^2) - \frac{m_Q^2}{24\pi^2} \frac{\alpha G G}{\pi} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (1 - \alpha - \beta) \left[ \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right] \left[ 1 + \frac{p_0}{4T} \right] \delta(p_0 - \tilde{m}_Q)
\]

\[
\frac{-m_Q^2}{192\pi^2 p_0 T} \frac{\alpha G G}{\pi} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (1 - \alpha - \beta) \left[ \frac{1}{\alpha^3} + \frac{1}{\beta^3} \right] \delta(p_0 - \tilde{m}_Q) + \frac{m^2_s}{32\pi^2} \frac{\alpha G G}{\pi} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (1 - \alpha - \beta) \left[ \frac{1}{\alpha^2} + \frac{1}{\beta^2} \right] \delta(p_0 - \tilde{m}_Q) + \frac{m_s^2 m_Q^2}{8\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \alpha (1 - \alpha) \left[ 6p_0 + p_0^2 \delta(p_0 - \tilde{m}_Q) \right] + \frac{m_s^2 m_Q^2}{64\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \frac{d\alpha}{\alpha \beta} \delta(p_0 - \tilde{m}_Q),
\]

(10)

\[
\rho^B_\Omega(p_0) = \frac{3m_s}{8\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (p_0^2 - \tilde{m}_Q^2)(2p_0^2 - \tilde{m}_Q^2) + \frac{3m_s m_Q^2}{4\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (p_0^2 - \tilde{m}_Q^2) - \frac{m_s m_Q^2}{96\pi^2} \frac{\alpha G G}{\pi} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left[ \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right] \left[ \frac{1}{m_q^2} + \frac{1}{2T} \right] \delta(p_0 - \tilde{m}_Q)
\]

\[
\frac{-m_s m_Q^2}{96\pi^2 p_0 T} \frac{\alpha G G}{\pi} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left[ \frac{1}{\alpha^3} + \frac{1}{\beta^3} \right] \delta(p_0 - \tilde{m}_Q) + \frac{m_s m_Q^2}{16\pi^2} \frac{\alpha G G}{\pi} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left[ \frac{1}{\alpha^2} + \frac{1}{\beta^2} \right] \delta(p_0 - \tilde{m}_Q) - \frac{\langle \bar{s}s \rangle}{2\pi^2} 2 \int_{\alpha_i}^{\alpha_f} d\alpha \alpha (1 - \alpha) \left[ 3p_0^2 - 2\tilde{m}_Q^2 \right] - \frac{m^2_Q \langle \bar{s}s \rangle}{2\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha 
\]

\[-\frac{m_s}{16\pi^2} \frac{\alpha G G}{\pi} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left[ 1 + \frac{m_Q^2}{4} \delta(p_0 - \tilde{m}_Q) \right],
\]

(11)

the \( s_0 \) are the threshold parameters, \( T^2 \) are the Borel parameters, \( \alpha_f = \frac{1 + \sqrt{1 - 4m_Q^2/p_0^2}}{2} \), \( \alpha_i = \frac{1 - \sqrt{1 - 4m_Q^2/p_0^2}}{2} \), \( \beta_i = \frac{\alpha m_Q^2}{\alpha p_0 - m_Q} \), \( \tilde{m}_Q^2 = \frac{(\alpha + \beta) m_Q^2}{\alpha \beta} \), \( \bar{m}_Q = \frac{m_Q^2}{\alpha(1 - \alpha)} \), and \( \Delta = 2m_Q + m_s \).
With the simple replacements, \( \langle \bar{s}s \rangle \to \langle \bar{q}q \rangle \) and \( m_s \to 0 \), we can obtain the corresponding spectral densities of the \( \Xi_{QQ} \) at the level of quark-gluon degrees of freedom.

3 Numerical results and discussions

The input parameters are taken as \( \langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3 \), \( \langle \bar{s}s \rangle = (0.8 \pm 0.2) \langle \bar{q}q \rangle \), \( \langle \bar{q}g_s \sigma Gq \rangle = m_0^2 \langle \bar{q}q \rangle \), \( \langle \bar{g}g_s \sigma Gs \rangle = m_0^2 \langle \bar{s}s \rangle \), \( m_0 = (0.8 \pm 0.2) \text{ GeV}^2 \), \( \langle \sigma_{GG} \rangle = (0.012 \pm 0.004) \text{ GeV}^4 \), \( m_s = (0.14 \pm 0.01) \text{ GeV} \), \( m_c = (1.35 \pm 0.10) \text{ GeV} \) and \( m_b = (4.7 \pm 0.1) \text{ GeV} \) at the energy scale \( \mu = 1 \text{ GeV} \).

The value of the gluon condensate \( \langle \sigma_{GG} \rangle \) has been updated from time to time, and changes greatly \([20]\). At the present case, the gluon condensate makes tiny contribution (see Table 1); the updated value \( \langle \sigma_{GG} \rangle = (0.023 \pm 0.003) \text{ GeV}^4 \) \([20]\) and the standard value \( \langle \sigma_{GG} \rangle = (0.012 \pm 0.004) \text{ GeV}^4 \) \([31]\) lead to a slight difference and can be neglected safely.

The \( Q \)-quark masses appearing in the perturbative terms are usually taken to be the pole masses in the QCD sum rules, while the choice of the \( m_Q \) in the leading-order coefficients of the higher-dimensional terms is arbitrary \([20, 33]\). The \( \overline{MS} \) mass \( m_c(m_c^2) \) relates with the pole mass \( m_c \) through the relation \( m_c = m_c \left[ 1 + \frac{C_F \alpha_s(m_c^2)}{\pi} + \cdots \right]^{-1} \).

In this article, we take the approximation \( m_c(m_c^2) \approx \hat{m}_c \) without the \( \alpha_s \) corrections for consistency. The value listed in the Particle Data Group is \( m_c(m_c^2) = 1.27_{-0.07}^{+0.07} \text{ GeV} \) \([32]\), it is reasonable to take \( \hat{m}_c = m_c(1 \text{ GeV}^2) = (1.35 \pm 0.10) \text{ GeV} \). For the \( b \) quark, the \( \overline{MS} \) mass \( m_b(m_b^2) = 4.20_{-0.07}^{+0.17} \text{ GeV} \) \([32]\), the gap between the energy scale \( \mu = 4.2 \text{ GeV} \) and 1 GeV is rather large, the approximation \( \hat{m}_b \approx m_b(m_b^2) \approx m_b(1 \text{ GeV}^2) \) seems rather crude. It would be better to understand the quark masses \( m_c \) and \( m_b \) at the energy scale \( \mu^2 = 1 \text{ GeV}^2 \) as the effective quark masses (or just the mass parameters).

In calculation, we neglect the contributions from the perturbative \( \mathcal{O}(\alpha_s^2) \) corrections. Those perturbative corrections can be taken into account in the leading logarithmic approximations through the anomalous dimension factors. After the Borel transform, the effects of those corrections are to multiply each term on the operator product expansion side by the factor, \( \left[ \frac{\alpha_s(T)}{\alpha_s(x)} \right]^{2 \Gamma_J - \Gamma_{\mathcal{O}_n}} \), where the \( \Gamma_J \) is the anomalous dimension of the interpolating current \( J(x) \), and the \( \Gamma_{\mathcal{O}_n} \) is the anomalous dimension of the local operator \( \mathcal{O}_n(0) \), which governs the evolution of the vacuum condensate \( \langle \mathcal{O}_n(0) \rangle_{\mu} \) with the energy scale through the renormalization group equation.

If the perturbative \( \mathcal{O}(\alpha_s) \) corrections and the anomalous dimension factors are taken into account consistently, the spectral densities in the QCD side should be replaced with

\[
\mathcal{O}_0(0) \to \left[ \frac{\alpha_s(T)}{\alpha_s(\mu^2)} \right]^{2 \Gamma_J} \left[ 1 + A(p_0^2, m_Q^2) \frac{\alpha_s(T)}{\pi} \right] \mathcal{O}_0(0),
\]

\[
\langle \mathcal{O}_n(0) \rangle_{\mu} \to \left[ \frac{\alpha_s(T)}{\alpha_s(\mu^2)} \right]^{2 \Gamma_J - \Gamma_{\mathcal{O}_n}} \left[ 1 + B(p_0^2, m_Q^2) \frac{\alpha_s(T)}{\pi} \right] \langle \mathcal{O}_n(0) \rangle_{\mu},
\]

where the \( A(p_0^2, m_Q^2) \) and \( B(p_0^2, m_Q^2) \) are some notations for the coefficients of the perturbative corrections, the average virtuality of the quarks in the correlation functions is characterized by the Borel parameter \( T^2 \). We cannot estimate the corrections and the uncertainties originate from the corrections with confidence without explicit calculations. In
Figure 1: The contributions of the pole terms with variations of the Borel parameters $T^2$, the $A$, $B$, $C$ and $D$ correspond to the channels $\Xi_{cc}$, $\Omega_{cc}$, $\Xi_{bb}$ and $\Omega_{bb}$ respectively, the $\beta$ corresponds to the central values of the threshold parameters, the energy gap among $\alpha$, $\beta$ and $\gamma$ is 0.1 GeV.

In this article, we carry out the operator product expansion at the special energy scale $\mu^2 = 1$ GeV$^2$, and set the factor $\left[\frac{\alpha_s(T^2)}{\alpha_s(\mu^2)}\right]^{2P_J - \Gamma_{\Omega_n}} \approx 1$ for consistency, as the $\alpha_s$ corrections have not been calculated yet. Such an approximation maybe result in some scale dependence and weaken the prediction ability. In this article, we study the $J^P = \frac{1}{2}^+$ doubly heavy baryon states systemically, the predictions are reasonable as we take the analogous criteria in those sum rules.

In the conventional QCD sum rules\cite{18, 19}, there are two criteria (pole dominance and convergence of the operator product expansion) for choosing the Borel parameter $T^2$ and threshold parameter $s_0$. We impose the two criteria on the doubly heavy baryon states to choose the Borel parameter $T^2$ and threshold parameter $s_0$.

In Fig.1, we plot the contributions from the pole terms with variations of the Borel parameters $T^2$ and the threshold parameters $s_0$. The pole contributions are larger than (or about) 50% at the values which are denoted by the vertical lines for central values ($\beta$) of the threshold parameters $s_0$. From the figure, we can set the upper bound of
the Borel parameters $T_{\text{max}}^2$, $T_{\text{max}}^2 = 3.8\text{GeV}^2$, $4.0\text{GeV}^2$, $9.1\text{GeV}^2$ and $9.3\text{GeV}^2$ in the channels $\Xi_{cc}$, $\Omega_{cc}$, $\Xi_{bb}$ and $\Omega_{bb}$, respectively. The lower bound of the Borel parameters $T_{\text{min}}^2$ can be determined in the regions where the contributions from the perturbative terms are larger than (or equal) the ones from the quark condensates, $T_{\text{min}}^2 = 2.8\text{GeV}^2$, $1.6\text{GeV}^2$, $7.7\text{GeV}^2$ and $<6.5\text{GeV}^2$ in the channels $\Xi_{cc}$, $\Omega_{cc}$, $\Xi_{bb}$ and $\Omega_{bb}$, respectively. The convergent behaviors in the channels $\Omega_{QQ}$ are better than the corresponding ones in the channels $\Xi_{QQ}$, this is mainly due to the fact that the values of the quark condensates, $|\langle \bar{q}q\rangle| > |\langle \bar{s}s\rangle|$. The Borel windows $T_{\text{min}}^2 - T_{\text{max}}^2$ in the channels $\Xi_{QQ}$ and $\Omega_{QQ}$ overlap with each other. In this article, we can take the uniform intervals for the Borel windows (IBW), i.e. IBW = $1.0\text{GeV}^2$ and $1.4\text{GeV}^2$ in the doubly charmed and doubly bottom channels respectively, which warrant the pole contributions are analogous. The values of the threshold parameters $s_0$ and the Borel parameters $T^2$ are shown in Table 1, from the table, we can see that the two criteria of the QCD sum rules are fully satisfied \cite{18, 19}.

In Ref.\cite{21}, we study the $\frac{1}{2}^+$ sextet heavy baryon states $\Omega_b$, $\Omega_c$, $\Xi'_b$, $\Xi'_c$, $\Sigma_b$ and $\Sigma_c$ by subtracting the contributions from the corresponding negative parity heavy baryon states with the QCD sum rules, the predicted masses are in good agreement with the experimental data for the well established mesons, $\Omega_b$, $\Sigma_b$, $\Omega_c$, $\Xi'_c$ and $\Sigma_c$. In those sum rules, the contributions from the pole terms are about $(45 - 65)\%$ and $(45 - 80)\%$ for the bottom and charmed baryon states respectively. In this article, we take analogous pole contributions, see Table 1, the predictions are reasonable although the contributions from the perturbative continuum are somewhat large.

In Fig.2, we plot the predicted masses with variations of the threshold parameters $s_0$. From the figure, we can see that the predicted masses are not sensitive to the threshold parameters, although they increase with the threshold parameters. In calculation, we take uniform uncertainties for the threshold parameters, $\delta s_0 = \pm 0.1\text{GeV}$.

From Table 1, we can see that the contributions from the quark condensates are large, even comparable with the perturbative terms, this is an indication of the non-perturbative origin of the masses of the baryon states. In the chiral limit, the spectral densities on the QCD side come from the quark condensates only. An astonishingly simple expression can be obtained in case of the proton \cite{35, 36},

$$M_p = \sqrt[3]{-8\pi^2 \langle \bar{q}q\rangle_{\mu=1\text{GeV}}} \approx 1\text{GeV}. \quad (12)$$

Taking into account all uncertainties of the relevant parameters, we can obtain the values of the masses and pole residues of the doubly heavy baryon states $\Xi_{QQ}$ and $\Omega_{QQ}$, which are shown in Figs.3-4 and Tables 2-3. In Table 2, we also present the predictions of other theoretical approaches and the values of the experimental data, the present predictions are consistent with them.

The fractions

$$R = \frac{\int_{\Delta s_0} dp_0 \left[ \rho^A(p_0) - \rho^B(p_0) \right] \exp \left[ -\frac{p_0^2}{T^2} \right]}{\int_{\Delta s_0} dp_0 \left[ \rho^A(p_0) + \rho^B(p_0) \right] \exp \left[ -\frac{p_0^2}{T^2} \right]} \quad (13)$$

are shown explicitly in Fig.5. At the value $T^2 = (2.5 - 4.0)\text{GeV}^2$, $R = (-7 \sim 17)\%$ in the doubly charmed baryon channels; and at the value $T^2 = (7.5 - 9.5)\text{GeV}^2$, $R = (-4 \sim 10)\%$ in the doubly bottom baryon channels. The contributions from the negative parity doubly
Table 1: The Borel parameters $T^2$ and threshold parameters $s_0$ for the doubly heavy baryon states, the "pole" stands for the contribution from the pole term, and the "perturbative" stands for the contribution from the perturbative term in the operator product expansion, etc. In calculating the contributions from the pole terms, we take into account the uniform uncertainties of the threshold parameters, $\delta \sqrt{s_0} = \pm 0.1$ GeV.

|       | $T^2$(GeV$^2$) | $\sqrt{s_0}$(GeV) | pole      | perturbative | $(\bar{q}q)$ | $(\bar{\alpha}_sGG\pi)$ |
|-------|----------------|-------------------|-----------|--------------|--------------|--------------------------|
| $\Xi_{cc}$ | 2.8 – 3.8     | 4.2               | (46 – 78)%| (48 – 55)%   | (43 – 50)%   | $\approx 2\%$             |
| $\Omega_{cc}$ | 3.0 – 4.0     | 4.3               | (45 – 75)%| (67 – 72)%   | (26 – 30)%   | $\approx 2\%$             |
| $\Xi_{bb}$ | 7.7 – 9.1     | 10.8              | (46 – 69)%| (48 – 53)%   | (46 – 51)%   | $< 1\%$                  |
| $\Omega_{bb}$ | 7.9 – 9.3     | 10.9              | (46 – 68)%| (69 – 72)%   | (28 – 31)%   | $< 1\%$                  |

Table 2: The masses $M$(GeV) of the doubly heavy baryon states, where the star * denotes the masses of the negative parity doubly heavy baryon states predicted by the non-relativistic quark model.

| References | $\Xi_{cc}$ | $\Omega_{cc}$ | $\Xi_{bb}$ | $\Omega_{bb}$ |
|------------|------------|--------------|------------|---------------|
| [5]        | 3.620      | 3.778        | 10.202     | 10.359        |
| [7]        | 3.676      | 3.815        | 10.340     | 10.454        |
| [8]        | 3.612      | 3.702        | 10.197     | 10.260        |
| [11]       | 3.579      | 3.697        | 10.189     | 10.293        |
| [12]       | 3.48       | 3.59         | 10.09      | 10.18         |
| [13]       | 3.547      | 3.648        | 10.185     | 10.271        |
| [14]       | 3.520      | 3.619        | 10.272     | 10.369        |
| [15]       | 3.48       |              | 9.94       |               |
| [16]       | 4.26       | 4.25         | 9.78       | 9.85          |
| [32]       | 3.5189     |              | ?          | ?             |
| This work  | 3.57 ± 0.14| 3.71 ± 0.14  | 10.17 ± 0.14| 10.32 ± 0.14  |
| [7]*       | 3.910      | 4.046        | 10.493     | 10.616        |

Table 3: The pole residues $\lambda_+$ of the doubly heavy baryon states.
Figure 2: The masses $M$ of the doubly heavy baryon states with variations of the threshold parameters $s_0$, the $A$, $B$, $C$ and $D$ correspond to the channels $\Xi_{cc}$, $\Omega_{cc}$, $\Xi_{bb}$ and $\Omega_{bb}$ respectively, the Borel parameters $T^2$ are taken to be the central values.

Figure 3: The masses $M$ of the double heavy baryon states, the $A$, $B$, $C$ and $D$ correspond to the channels $\Xi_{cc}$, $\Omega_{cc}$, $\Xi_{bb}$ and $\Omega_{bb}$ respectively.
Figure 4: The pole residues $\lambda_+$ of the doubly heavy baryon states, the $A$, $B$, $C$ and $D$ correspond to the channels $\Xi_{cc}$, $\Omega_{cc}$, $\Xi_{bb}$ and $\Omega_{bb}$ respectively.

Figure 5: The ratios between the contributions from the negative parity and positive parity doubly heavy baryon states with variations of the Borel parameters $T^2$, the $A$, $B$, $C$ and $D$ correspond to the channels $\Xi_{cc}$, $\Omega_{cc}$, $\Xi_{bb}$ and $\Omega_{bb}$ respectively.
heavy baryon states are not very large, although they are considerable in some sense. In this article, the central values of the threshold parameters are \( s_0 = 4.2 \text{ GeV}, 4.3 \text{ GeV}, 10.8 \text{ GeV} \) and \( 10.9 \text{ GeV} \) in the channels \( \Xi_{cc}, \Omega_{cc}, \Xi_{bb} \) and \( \Omega_{bb} \) respectively, which are larger than the masses of the corresponding negative parity doubly heavy baryon states, see Table 2. If we take the tensor structures \( \tilde{p} \) and 1, the contributions from the negative parity baryon states are included in. In the case of the doubly bottom channels, the threshold parameters are slightly larger than the thresholds of the corresponding negative parity baryon states, the contaminations are small, see Fig.5. In fact, without separating the contributions of the positive parity baryon states from the negative parity baryon states explicitly, the two criteria (pole dominance and convergence of the operator product expansion) for choosing the Borel parameter \( T^2 \) and threshold parameter \( s_0 \) in the conventional QCD sum rules do not work efficiently; we maybe (or maybe not) choose the Borel windows where the contaminations from the negative parity doubly heavy baryon states are large. With suitable Borel parameters, we can choose the tensor structures \( \tilde{p} \) or 1 to study the masses and pole residues freely. If we choose the tensor structure \( \gamma_0 + 1 \), the contaminations from the negative parity doubly heavy baryon states are excluded explicitly.

Those doubly heavy baryon states \( \Xi_{cc}, \Omega_{cc}, \Xi_{bb} \) and \( \Omega_{bb} \) may be observed at the Tevatron, the LHCb and the PANDA, especially at the LHCb. For example, the \( \Xi_{cc} \) and \( \Omega_{cc} \) can be produced at the high energy \( pp \) or \( p\bar{p} \) collisions through the gluon-gluon fusion mechanism and the intrinsic charm mechanisms, \( g + g \rightarrow (cc)^3S_1\bar{3} + \bar{c} + \bar{c}, \ g + g \rightarrow (cc)^3S_1\bar{3} + \bar{c} + \bar{c}, \ g + g \rightarrow (cc)^3S_1\bar{3} + \bar{c} + \bar{c}, \ g + g \rightarrow (cc)^3S_1\bar{3} \) and \( g + c \rightarrow (cc)^1S_0\bar{6} + \bar{c} + c, \ c + c \rightarrow (cc)^3S_1\bar{3} + g, \ c + c \rightarrow (cc)^3S_1\bar{3} + g \) where the \( (cc)^3S_1\bar{3} \) (in color anti-triplet 3) and \( (cc)^1S_0\bar{6} \) (in color sextet 6) are two possible \( S \)-wave configurations of the doubly charmed diquark pair \( (cc) \) inside the baryon states \( \Xi_{cc} \) and \( \Omega_{cc} \). The LHCb is a dedicated \( b \) and \( c \)-physics precision experiment at the LHC (large hadron collider). The LHC will be the world’s most copious source of the \( b \) hadrons, and a complete spectrum of the \( b \) hadrons will be available through gluon fusion. Furthermore, once reasonable values of the pole residues \( \lambda_{\Omega} \) and \( \lambda_{\Xi} \) are obtained, we can take them as basic input parameters and study the relevant hadronic processes with the QCD sum rules.

4 Conclusion

In this article, we study the \( \frac{1}{2}^+ \) doubly heavy baryon states \( \Omega_{QQ} \) and \( \Xi_{QQ} \) by subtracting the contributions from the corresponding \( \frac{1}{2}^- \) doubly heavy baryon states with the QCD sum rules, and make reasonable predictions for their masses. Those doubly heavy baryon states maybe observed at the Tevatron, the LHCb and the PANDA, especially at the LHCb. Once reasonable values of the pole residues \( \lambda_{\Omega} \) and \( \lambda_{\Xi} \) are obtained, we can take them as basic input parameters and study the relevant hadronic processes with the QCD sum rules.

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