Pairing fluctuations in a strongly interacting two-dimensional Fermi gas

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Abstract. We theoretically investigate strong-coupling properties of a two-dimensional Fermi gas in the normal phase. Including pairing fluctuations within the framework of a self-consistent $T$-matrix approximation, we calculate the single-particle density of states (DOS) near the observed Berezinskii-Kosterlitz-Thouless phase transition temperature $T_{\text{BKT}}$. We show that the pseudogap phenomenon associated with strong pairing fluctuations appears as an almost fully gapped single-particle density of states in the strong-coupling regime, indicating that amplitude fluctuations of the order parameter are almost absent and the system is dominated by phase fluctuations of the order parameter, as expected in a two-dimensional system near $T_{\text{BKT}}$. On the other hand, although pairing fluctuations are enhanced by the low-dimensionality of the system, they are found to be still not strong enough to open such a full gap in the intermediate coupling (crossover) regime. In this regime, a dip structure is only seen in DOS around $\omega = 0$, indicating that amplitude of the superfluid order parameter is also fluctuating, in addition to phase fluctuations. In theoretically determining $T_{\text{BKT}}$ in a two-dimensional Fermi gas, it is frequently assumed that, while phase fluctuations exist, the amplitude of the superfluid order parameter is fixed. However, our results indicate the necessity of including both amplitude and phase fluctuations of the order parameter in evaluating $T_{\text{BKT}}$ in the crossover region. Since the BKT phase transition has recently been reported in a two-dimensional $^6$Li Fermi gas, our results would be useful for the theoretical study of this observed novel Fermi superfluid and effects of strong low-dimensional pairing fluctuations.

1. Introduction

Experimental techniques in an ultracold Fermi gas enable us to tune various physical parameters \cite{1, 2}. In particular, a tunable pairing interaction associated with a Feshbach resonance has been used for the study of strong-coupling effects from the weak-coupling BCS (Bardeen-Cooper-Schrieffer) regime to the strong-coupling BEC (Bose-Einstein condensation) regime \cite{2, 3, 4}. In the BCS-BEC crossover region, the so-called pseudogap phenomenon has been observed near the superfluid phase transition temperature \cite{5, 6}, which indicates the presence of strong pairing fluctuations.

The high tunability of an ultracold Fermi gas has also realized a two-dimensional system, by using an optical lattice technique. Since pairing fluctuations are enhanced by the low dimensionality of the system, this system is used for the study of, not only low-dimensional physics, strong-coupling physics. As a typical interesting many-body phenomenon, the possibility of Berezinskii-Kosterlitz-Thouless (BKT) phase \cite{7, 8} has recently been discussed in this artificial low-dimensional Fermi system \cite{9, 10, 11, 12, 13, 14, 15, 16}.

In considering the BKT transition in a two-dimensional Fermi gas, a simple approach is that one takes into account phase fluctuations of the superfluid order parameter $\Delta$ with a fixed value of $|\Delta|$. 

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This is analogous to the two-dimensional XY-model, where the magnitude and the direction of spins correspond to $|\Delta|$ and the phase of the order parameter, respectively. Using this approach, it has recently been evaluated $T_{\text{BKT}}$ from the weak- to strong-coupling regime of a two-dimensional Fermi gas [17], where phase fluctuations are included within the framework of the strong-coupling theory developed by Nozières and Schmitt-Rink[18]. In this paper, we assess the importance of amplitude fluctuations of the order parameter near $T_{\text{BKT}}$, from the viewpoint of pseudogap phenomenon associated with pairing fluctuations. Since the magnitude of the order parameter $|\Delta|$ is deeply related to the binding energy of a Cooper pair, while the appearance of a full gap structure in the single particle density of states $\rho(\omega)$ around $\omega = 0$ indicates the establishment of a fixed value of $|\Delta|$, the so-called pseudogap structure (which is characterized by a dip structure with a non-zero DOS around $\omega = 0$) in $\rho(\omega)$ implying the fluctuating $|\Delta|$.

We note that effects of amplitude fluctuations have recently been discussed within the Gaussian fluctuations theory (NSR theory) [19, 20]. However, in addition to the fact that NSR theory cannot explain the recent experiments on thermodynamic quantities, it is also known to unphysically give negative DOS around the Fermi level in the BCS-BEC crossover region. This problem is solved by extending the NSR theory to the so-called non-selfconsistent $T$-matrix approximation (TMA), which has extensively been used to examine various pseudogap phenomena in a three-dimensional ultracold Fermi gas [21, 22]. However, in the present two-dimensional case, it has been shown[23] that TMA unphysically gives a clear gap structure in DOS with the gap size being comparable to the Fermi energy at low temperatures, even in the weak-coupling normal phase. Thus, to overcome this, in this paper, we employ the self-consistent $T$-matrix approximation (SCTMA)[24, 25], to include two-dimensional pairing fluctuations in a consistent manner. Although SCTMA cannot deal with the BKT phase transition, we expect that it can correctly describe strong-coupling effects associated with pairing fluctuations in the low temperature region, where the BKT transition has recently been reported in a $^6$Li Fermi gas[14, 15]. We briefly note SCTMA has been applied to evaluate DOS in the weak-coupling regime of a two-dimensional Fermi gas[25, 26]. In this paper, we investigate the pseudogap phenomenon over the entire interaction regime, to clarify how the pseudogap develops in DOS, with increasing the interaction strength.

Throughout this paper, we take $\hbar = k_{B} = 1$ and the two-dimensional system area is taken to be unity, for simplicity.

2. Formulation
We consider a uniform two-dimensional Fermi gas, described by the model Hamiltonian

$$H = \sum_{p,\sigma} \xi_{p} c_{p,\sigma}^{\dagger} c_{p,\sigma} - U \sum_{p, q} c_{p+q/2,\uparrow}^{\dagger} c_{p+q/2,\downarrow}^{\dagger} c_{-p+q/2,\downarrow} c_{-p+q/2,\uparrow}.$$

(1)

Here, $p = (p_{x}, p_{y})$ is the two-dimensional momentum, and $\xi_{p} = p^{2}/(2m) - \mu$ is the kinetic energy measured from the Fermi chemical potential $\mu$, where $m$ is an atomic mass. $c_{p,\sigma}^{\dagger}$ is the creation operator of a Fermi atom, where pseudospin $\sigma = \uparrow, \downarrow$ describes two atomic hyperfine states. $-U(<0)$ is a pairing interaction, which is related to the two-dimensional s-wave scattering length $a_{2D}$ as [27],

$$\frac{1}{U} = \frac{m}{2\pi} \ln(k_{F}a_{2D}) + \sum_{p \geq k_{F}} \frac{m}{p^2}.$$

(2)

Here, $k_{F} = \sqrt{2\pi N}$ is the Fermi momentum, and $N$ is the total number of Fermi atoms. The strong-coupling regime corresponds to $\ln(k_{F}a_{2D}) \lesssim -1$, and the weak-coupling regime corresponds to $\ln(k_{F}a_{2D}) \gtrsim 1$. The region between the two, $-1 \lesssim \ln(k_{F}a_{2D}) \lesssim 1$, is sometimes called the crossover region in the literature.
In considering single-particle properties of the system, the single-particle thermal Green’s function,

\[ G(p, i\omega_n) = \frac{1}{i\omega_n - \xi_p - \Sigma(p, i\omega_n)} , \]

is useful. Here, \( \omega_n \) is the fermion Matsubara frequency. Within the framework of the self-consistent \( T \)-matrix approximation (SCTMA), the self-energy \( \Sigma(p, i\omega_n) \) is diagrammatically described as Fig. 1, which gives [24, 25, 26],

\[ \Sigma(p, i\omega_n) = T \sum_{q, i\nu_n} \Gamma(q, i\nu_n) G(q - p, i\nu_n - i\omega_n) G(q, i\nu_n) \]

(4)

\[ \Gamma(q, i\nu_n) = -\frac{U}{1 - U\Pi(q, i\nu_n)} , \]

(5)

Here, \( \Gamma(q, i\nu_n) \) is the particle-particle scattering matrix, where \( \nu_n \) is the boson Matsubara frequency, and

\[ \Pi(q, i\nu_n) = T \sum_{p, i\omega_n} G(p + \frac{q}{2}, i\nu_n + i\omega_n) G(-p + \frac{q}{2}, -i\omega_n) \]

(6)

is a pair-correlation function, describing fluctuations in the Cooper channel.

As usual, the Fermi chemical potential \( \mu \) is determined from the equation of the total number \( N \) of Fermi atoms,

\[ N = 2T \sum_{p, i\omega_n} G(p, i\omega_n) , \]

(7)

Using the calculated \( \mu \) from the number equation (7), we evaluate the single-particle density of states \( \rho(\omega) \),

\[ \rho(\omega) = -\frac{1}{\pi} \sum_p \text{Im} G(p, i\omega_n \rightarrow \omega_+ = \omega + i\delta) , \]

(8)
Figure 2. (a) Calculated single-particle density-of-states $\rho(\omega)$ in SCTMA when $T/T_F = 0.3$. $\rho_0 = m/(2\pi)$ is the density of states in a two-dimensional free Fermi gas. $T_F(\varepsilon_F)$ is the Fermi temperature (energy). The inset shows DOS around $\omega = 0$ when $\ln(k_Fa_{2D}) = 0$, to clearly show the pseudogap structure in this case. (b) Density of states $\rho(\omega)$ at various temperatures. We take $\ln(k_Fa_{2D}) = 0$ (crossover region). The lowest temperature $(T/T_F = 0.14)$ is close to the BKT phase transition temperature $(T_{BKT}/T_F = 0.12)$, predicted by a theory that only include phase fluctuations of the order parameter [17]. (c) Calculated single-particle density-of-states $\rho(\omega)$ by using the approximate Green’s function in Eq. (10), when $T/T_F = 0.3$. (d) Chemical potential $\mu$, as well as the effective chemical potential $\mu'$ determined from Eq. (11) as a function of the interaction strength $\ln(k_Fa_{2D})$, when $T/T_F = 0.3$. The interaction strengths considered in panels (a) and (c) are shown as the filled circles. (Color figure online)

to examine the pseudogap phenomenon caused by strong two-dimensional pairing fluctuations.

We briefly note that SCTMA cannot describe the BKT phase transition temperature $T_{BKT}$. Thus, this paper only deals with the normal phase above $T_{BKT}$. $T_{BKT}$ and strong coupling effects have recently been discussed, from the weak-coupling to strong-coupling regime, by only including phase fluctuations of the superfluid order parameter (with a fixed value of the magnitude of the order parameter). In this regard, although the present SCTMA approach cannot treat $T_{BKT}$, both phase and amplitude fluctuations in the Cooper channel are taken into account on an equal footing.
3. Results

Figure 2(a) shows the SCTMA single-particle density of states (DOS) \( \rho(\omega) \) in a two-dimensional Fermi gas \( (T/T_F = 0.3) \). In the strong-coupling regime \( \ln(k_F a_{2D}) = -1 \), one sees a large full gap structure in \( \rho(\omega) \) around \( \omega = 0 \), indicating the existence of large binding energy of a bound pair. Indeed, within a two-particle problem, the binding energy \( E_{\text{bind}} = 1/(ma_{2D}^2) \) is very large as \( E_{\text{bind}}/T_F \gg 1 \). In this case, since the molecular dissociation is difficult to occur when the temperature is much lower than the gap energy seen in Fig. 2. In the language of the superfluid order parameter \( \Delta \), the amplitude is established and the phase is only fluctuating, to destroy the superfluid (BKT) order.

In contrast, such a full-gap is not obtained in the intermediate coupling case \( \ln(k_F a_{2D}) = 0 \), as well as in the weak-coupling regime \( \ln(k_F a_{2D}) = +2 \), as shown in Fig. 2 (a). We only see the so-called pseudogap (dip) in \( \rho(\omega) \) around \( \omega = 0 \) in the former (although \( \rho(\omega \approx 0) \) is also found to be modified by pairing fluctuations when \( \ln(k_F a_{2D}) = +2 \). Even when one decreases the temperature down to be close to the predicted \( T_{BKT} = 0.12T_F \) when \( \ln(k_F a_{2D}) = 0 \). Fig. 2 (b) shows that the non-zero value of \( \rho(\omega = 0) \) continues to remain. These results imply that the assumption of a static magnitude of the superfluid order parameter in evaluating \( T_{BKT} \) should be improved in considering the intermediate and weak-coupling regime \( \ln(k_F a_{2D}) > 0 \).

To relate the presented results in Fig. 2 (a) and (b) to the picture of “static amplitude \( |\Delta|'' \) in a simple manner, it is convenient to employ the static approximation to pairing fluctuations [21, 28, 29], given by

\[
\Sigma(p, \omega_+) = -\Delta_{\text{PG}}^2 G(-p, -\omega_+) + F(p, \omega_+). \tag{9}
\]

When \( \Delta_{\text{PG}} = \sqrt{-T \sum_{q,i \nu} \Gamma(q,i \nu)} \) is frequently referred to as the pseudogap parameter in the literature, which corresponds to \( \Delta \) in the above discussions. We briefly note that \( \Delta_{\text{PG}} \) is also related to the Tan’s contact \( C = \Delta_{\text{PG}}^2 [30] \). For the remaining part \( F(p, \omega_+) \) in Eq. (9), when we treat it with the mean-field level by approximately taking \( F(p, \omega_+) \) to be a constant \( (\equiv \alpha) \), the analytic-continual SCTMA Green’s function in Eq. (3) is approximated to

\[
G(p, \omega_+) = \frac{\omega_+ + \xi_p'}{\omega_+^2 - \xi_p'^2 - 2\Delta_{\text{PG}}^2 \left[ 1 + \sqrt{1 - \frac{4\Delta_{\text{PG}}^2}{\omega_+^2 - \xi_p'^2}} \right]}^{-1}, \tag{10}
\]

Here, \( \xi_p' = \xi_p + \alpha \). In using Eqs. (9) and (10), “ the mean-field shift \( \alpha'' \) is determined so that “ the effective chemical potential \( \mu'' \” shown in Fig. 2 (d) which is determined from the equation,

\[
-\mu'' = -\mu + \text{Re} \left[ \Sigma(p = 0, \omega_+ = -\mu'' + i\delta) \right] \tag{11}
\]

can be reproduced when the approximate self-energy in Eq. (9) is used for SCTMA \( \Sigma(p, \omega_+) \) in Eq. (11). We briefly note that when we ignore \( \Delta_{\text{PG}} \) in \( \cdots \left[ \cdots \right]^{-1} \) in the denominator of Eq. (10), the Green’s function in Eq. (10) has the same form as the diagonal component of the BCS Green’s function when the pseudogap parameter \( \Delta_{\text{PG}} \) is replaced by the BCS superfluid order parameter \( \Delta \).

The Green’s function in the static approximation in Eq. (10) can explain the large gap structure in DOS in the strong coupling region \( \ln(k_F a_{2D}) = -1 \), as shown in Fig. 2 (c). We also find from this figure that the static approximation overestimates the pseudogap phenomenon when \( \ln(k_F a_{2D}) = 0 \) and 2. (Note that \( \rho(\omega = 0) = 0 \) is always obtained in the present static approximation). Thus, as discussed previously, we need to treat both amplitude and phase fluctuating in this regime in considering the BKT transition. In this regard, we briefly note that, when we employ the Gaussian fluctuation theory for this purpose, \( \rho(\omega) \) exhibits a full-gap structure near the predicted \( T_{BKT} \) even in the weak- and intermediate- coupling regime[31]. Thus, to examine amplitude fluctuations, self-consistent treatment of pairing fluctuations as we have done in this paper is important.
4. Summary
To summarize, we have discussed strong-coupling corrections to single-particle density of states $\rho(\omega)$ in a two-dimensional Fermi gas, from the strong-coupling regime to the weak-coupling regime. Including pairing fluctuations within the framework of a self-consistent $T$-matrix approximation (SCTMA), we showed that, while BCS-type gap develops in the strong-coupling region, in the crossover and weak-coupling region, the density of states $\rho(\omega)$ at $\omega = 0$ is still non-zero, exhibiting the pseudogap structure, even in the low temperature region where $T_{\text{BKT}}$ is predicted. So far, it has frequently been assumed a fixed magnitude of the superfluid order parameter in theoretically determining $T_{\text{BKT}}$. However, our results indicate the importance of amplitude fluctuations of the order parameter which lead to, not a fully gapped, but a pseudogapped DOS, in addition to phase fluctuations, in considering the BKT transition in the intermediate coupling regime. For this purpose, extension of the SCTMA to the theory which can deal with BKT transition would be a critical future problem. Since BKT transition is of great interest in cold Fermi gas physics, our results would contribute to the future developments of this field.

5. Acknowledgments
We thank H. Tajima and P. van Wyk for discussions. This work was supported by the KiPAS project in Keio university. M. M. was supported by KLL PhD Program Research Grant. D. I. was supported by Grand-in-Aid for Scientific Research from MEXT and Grant-in-Aid for Young Scientists (B) (No. JP16K17773) from JSPS. R. H. was supported by Grant-in-Aid for Scientific Research from MEXT and JSPS in Japan (No. JP15K00178, No. JP15H00840, No. JP16K05503).

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