Research and Application of Path Planning Algorithm in Complex Terrain

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Abstract. Aiming at the failure of path planning in complex three-dimensional terrain, a path planning algorithm based on Hopfield neural network is proposed. According to the three-dimensional terrain, the undulating terrain and obstacles are transformed into a calculable regular figure, and the terrain function model is established according to the flow field control equation; the terrain function model is effectively integrated with Hopfield neural network algorithm. The algorithm is applied to complex three-dimensional terrain environment, can avoid undulating terrain and obstacles, and find an optimal path, which lays an important foundation for the navigation vehicle path planning.

1. Introduction

The path planning problem is to find an optimal collision-free path from the initial state to the target state according to one or more optimization rules, such as the minimum working cost, the shortest walking distance, and the shortest walking time [1]. Path planning under complex terrain is a hot issue in recent years. At present, the algorithms for path planning mainly include D* algorithm [2-3], genetic algorithms, etc. These algorithms are relatively simple and only for simple terrain, which cannot be applied in complex three-dimensional terrain. Neural network algorithms have been rapidly developed in recent years and are suitable for solving complex terrain input problems [4]. Yu Jianli et al. [5] solved the shortest path planning problem by applying neural networks to path planning. However, these algorithms still lack the consideration of undulating terrain and obstacles in the three-dimensional special terrain with large undulating terrain.

In summary, although many people have studied path planning issues, they have not considered the impact of undulating topography and obstacles on path planning under complex terrain [6-7]. In other words, many path planning algorithms are disconnected from the actual and complex terrain [8]. This article uses the current popular neural network algorithms, and considers the terrain and obstacles in complex 3D terrain environment, the neural network and terrain function model are combined to lay the foundation for autonomous navigation research.

2. Terrain Function Model

2.1 Terrain Preprocessing

During the process of the complicated three-dimensional terrain, the navigation vehicle must avoid obstacles and the terrain with large fluctuations in order to reach the optimal path [9-10]. Due to the complexity of the terrain, it cannot be calculated directly, so the preprocessing of the terrain is the first step in modeling. For this reason, in this study, the undulating terrain is transformed into regular...
cylinders, and obstacles are transformed into spheres, which facilitates the navigation vehicle to avoid these obstacles and find an optimal path during the travel. The functional form of regular geometry is as follows:

\[
f(x, y, z) = \left(\frac{x-x_0}{a}\right)^{2p} + \left(\frac{y-y_0}{b}\right)^{2q} + \left(\frac{z-z_0}{c}\right)^{2r} = 1
\]  

(1)

Where, \(x_0, y_0\) and \(z_0\) represent the fluctuation of the terrain and the coordinates of the center point of the obstacle; \(a, b, c\) is a constant used to control the size of the obstacle; equation (1) can describe different geometries according to different \(p, q, r\) values. When \(p = q = r = 1\), represents a sphere, that is, the path that needs to be avoided at this time is an obstacle. When \(p = q = 1\) and \(r > 1\), it is a cylinder, that is, the path that needs to be avoided at this time is the top and bottom of the terrain.

2.2 Terrain Function

The terrain function model is an easy-to-handle model built to avoid the undulating terrain and obstacles during the navigation of the car. Different terrains must have different functional models. This article mainly uses the basic idea of the flow function, that is, the flow from the obstacle around learning from phenomena, topography is considered as a boundary condition and is established by using fluid mechanics to calculate the flow field distribution in the planning area. The flow control equation is as follows:

\[
\nabla^2 \Phi = 0
\]  

(2)

One of the boundary conditions of the flow control equation is the surface of the obstacle \(n\) \(\frac{\partial \Phi}{\partial n} = 0\) (3), the second boundary condition is at infinity \(\nabla \Phi = \mu_\infty\) (4). \(\nabla\) differential symbol; equation (2) represents the Laplace equation, in this paper, this function represents the functional form of the regular geometry, that is, the processed terrain \(f\) function; \(n\) represents unit normal vector of obstacle surface outward; \(\mu_\infty\) represents the speed. By solving equations (2), (3) and (4), a terrain function model suitable for avoiding obstacles and undulating terrain will be obtained.

3. Continuous Hopfield Neural Network

Hopfield neural network is a recursive neural network that combines storage systems and binary systems. It is implemented by Hopfield and others using analog electronic circuits. It successfully solves some of the most representative problems in the optimal combination problem and also opens up information. New methods of applying artificial intelligence technology in processing. The activation function of neurons in this network is a continuous function, so it is also called a continuous Hopfield neural network [11-12]. In a continuous Hopfield neural network, each neuron generally works in parallel, and the input and output are analog.

In biological systems, due to the cell membrane input capacitance \(C_i\) of neuron \(i\), transmembrane resistance \(R_i\) and determined impedance \(R_{ij} = W_{ij}^{-1}\) (5) (\(W\) is the weight between \(i\) and \(j\)), state \(U_i\) will lag behind the instantaneous output of other neurons \(V_j\), so the output of the neuron will be a continuous value between 0 and 1, rather than the binary value of the discrete model. In the continuous Hopfield neural network, the basic components of the circuit are
Figure 1. The basic components of the circuit

a. Operational amplifier with co-directional and inverting outputs, S-type input-output relationship with saturation nonlinearity

\[ V_i = f(U_i) = \frac{1}{2} \left[ 1 + \tanh \left( \frac{U_i}{U_0} \right) \right] \quad (6) \]

where \( U_0 \) is equivalent to the amplification factor of the input signal, and also controls the slope of the activation function. When \( U_0 \) approaches 0, \( f \) becomes a binary threshold function.

b. The product of the input capacitance \( C_i \) of the amplifier and the input resistance \( R_i \) is the time constant of the neuron, describe the dynamic characteristics of neurons.

\[ T_i = \frac{1}{R_i} \quad (7) \]

representing the weights of network neuron connections, \( R_{ij} \) is connection resistance.

c. The applied bias current \( I_i \) corresponds to the threshold of the neuron \( \theta_i \). The dynamic equation of the entire circuit system is

\[ C_i \frac{dU_i}{dt} = \sum_{j=1}^{n} T_{ij} V_j - \frac{U_i}{R_i} + I_i \quad (8) \]

if this equation has a solution, it means that the system state change will eventually stabilize. In the case of symmetric connections and no self-feedback, define the energy function of the system as

\[ E = -\frac{1}{2} \sum_{j=1}^{n} \sum_{i=1}^{n} T_{ij} V_i V_j - \sum_{i=1}^{n} V_i I_i + \sum_{i=1}^{n} \int_{0}^{U_i} f^{-1}(V) dV \quad (9) \]

when steady state, can ignore the last integral item, that is

\[ E = -\frac{1}{2} \sum_{j=1}^{n} \sum_{i=1}^{n} T_{ij} V_i V_j - \sum_{i=1}^{n} I_i V_i \quad (10) \]

when the nonlinear action function \( f^{-1} \) is a continuous monotonically increasing function, it can be proved that the energy function \( E \) is monotonically decreasing and bounded. That is

\[ \frac{dE}{dt} \leq 0 \quad (11) \]

At this point, with the state equation and energy function of the Hopfield neural network model, we can try to solve the problem of path planning in 3D terrain.

4.3D Terrain Modeling

In order to effectively establish a three-dimensional terrain model, this paper introduces a height map as
a method of input data in the study. The height map is the standard method for creating three-dimensional terrain. It is a two-dimensional array of values [13]. Each value in the array represents the height of the terrain at the location of the value [14]. For example, Figure 2 shows the cells of the height map and the value of each cell, while Figure 3 shows the terrain wireframe view generated by this map.

![Figure 2. Cells and values for height map](image)

![Figure 3. Wireframe view of generated terrain](image)

The establishment of three-dimensional terrain requires height-based maps and iterative use of different algorithms. This article chooses the hill algorithm to build a three-dimensional terrain model. The basic algorithm steps are as follows:

1. Initialize all height values to zero;
2. Select random points on or near the terrain and a random radius between a predetermined minimum and maximum. Selecting this minimum and maximum will make the terrain rough or smooth;
3. Rise the hill at the center of a point with a given radius;
4. Return to step 2 and repeat iterations as many times as necessary. The selected number of iterations will affect the appearance of the terrain;
5. Standardize the terrain.

At this point, the model of the three-dimensional terrain is completed. This 3D terrain will be used to study the path planning algorithm.

5. Path Planning Algorithm In 3D Terrain

The continuous Hopfield neural network works synchronously when processing data. If the objective function of a path planning problem is converted into the energy function of a continuous Hopfield neural network, and the variable of the problem corresponds to the state of the neuron in the network, then the continuous Hopfield neural network can be well applied to the path planning problem. In addition, considering the terrain function model into the energy function, Hopfield neural network can better deal with the obstacles and undulating topography of 3D terrain. When the neuron state of the network approaches the equilibrium point, the energy function of the network also tends to the minimum, and the process of the network converging from the initial state to the steady state is the optimal solution process of the path.

The energy function of the Hopfield neural network designed in this algorithm corresponds to the objective function. Therefore, the energy function of the network contains the target term and the constraint term. Here, the energy function of the network is defined as

\[
E = \frac{A}{2} \sum_{x} \sum_{i} \sum_{j \neq i} V_{ix} V_{ij} + \frac{B}{2} \sum_{x} \sum_{i} \sum_{j \neq i} V_{ix} V_{ij} + \frac{C}{2} (\sum_{x} \sum_{i} V_{ix} - N)
\]

(12), in addition, in order to better deal with obstacles and undulating terrain in 3D terrain, you must add terrain function model information:
\[
\frac{D}{2} \sum_i \sum_{j \neq i} \sum_{l} \Phi(V_{y,i+l} + V_{y,j-l})
\]
(13) can be expressed. From this, the network energy function of the path planning problem can be obtained
\[
E = \frac{A}{2} \sum_{x} \sum_{i \neq j} \sum_{l} V_{x,i} V_{y,j} + \frac{B}{2} \sum_{x} \sum_{i \neq j} \sum_{l \neq m} V_{x,i} V_{x,j} + \frac{C}{2} \left( \sum_{x} \sum_{i} V_{x,i} - N \right) + \frac{D}{2} \sum_{x} \sum_{i \neq j} \sum_{l} \Phi(V_{y,i+l} + V_{y,j-l})
\]
(14) and
\[
V_{x,i} = g(u_{x,i})
\]
(15).

The specific algorithm steps are as follows:
1. Step 1: Import 3D terrain coordinates;
2. Step 2: Initialize the network;
3. Step 3: Using network dynamic equations to calculate \( \frac{du_{x,i}}{dt} \), and using the first-order Euler method to calculate \( U_{x,i}(t+1) = U_{x,i}(t) + \frac{du_{x,i}}{dt} \Delta T \);
4. Step 4: Using \( V_{x,i}(t) = g(U_{x,i}(t)) = \frac{1}{2} \left[ 1 + \tanh \left( \frac{U_{x,i}(t)}{U_0} \right) \right] \)
to calculate \( V_{x,i}(t) \);
5. Step 5: Computing network energy function \( E \);
6. Step 6: If the number of iterations \( P > 10000 \), the program is ended. Then \( P = P + 1 \) to return to step 3.

6. Experimental
The experiments were carried out in Unity 2018. First, a three-dimensional topographic map is created using the hill algorithm, as shown in Figure 4. Among them, the regular geometry represents obstacles in the path of travel, and the rest is undulating terrain.

Hopfield nerves are used before undulating terrain and obstacles are processed the path of the network is shown in Figure 5; then, the terrain function model is used to process the undulating terrain and obstacles, and the path that is incorporated into the Hopfield neural network algorithm for path planning is shown in Figure 6. The first three experiments were performed. Tables 1 and 2 respectively show the comparison of path length and time before and after terrain processing.

Figure 4.3D topographic map
Table 1. Comparison of path length before and after terrain processing

| Comparison item (/km) | Before terrain processing | After terrain processing |
|-----------------------|---------------------------|--------------------------|
| experiment one        | 32.6                      | 28.3                     |
| experiment two        | 58.9                      | 47.32                    |
| experiment three      | 63.1                      | 60.7                     |

Table 2. Comparison of path times before and after terrain processing

| Comparison item (/s) | Before terrain processing | After terrain processing |
|----------------------|---------------------------|--------------------------|
| experiment one       | 12.3                      | 9.6                      |
| experiment two       | 18.7                      | 12.1                     |
| experiment three     | 27.9                      | 19.9                     |

7. Conclusions
This paper uses Hopfield neural network as the background to study an effective path planning problem in 3D terrain. The hill algorithm is used to establish a three-dimensional terrain model. In this three-dimensional terrain, the undulating terrain and obstacles are first processed to obtain the terrain model function, and then the terrain function model is integrated in the Hopfield neural network algorithm. Experiments show that the algorithm can avoid undulating terrain and obstacles in complex three-dimensional terrain, find an effective path, and save time. The research in this paper also lays the foundation for autonomous navigation without driver.

References
[1] Zhang Jian. Mobile robot path planning method based on genetic algorithm[J]. Journal of Anqing Teachers College.2016,22(4):48-52.
[2] Wu Jian, Zhang Donghao. Dynamic target route planning based on Kalman filter and D* algorithm[J]. Electro-optic and control. 2014,21(8):50-53.

[3] Liu Maohua, Wang Yan, Zhou Hanzhuang. Application Research of D* Algorithm Shortest Path in Digital Campus[J]. Surveying and mapping bulletin. 2012:624-625.

[4] Huang X Y. Oscillatory behavior of n-th-order neutral dynamic equations with mixed nonlinearities on time scales[J]. Electronic Journal of Differential Equations, 2016, 2016(16):1-18.

[5] Yu J L, Kroumov V, Sun Z Y. A fast neural network path planning algorithm[J]. Robot. 2001, 20(33): 201-205.

[6] Wang Zhongmin, Yao Liqin, Zhang Hansong. Research on Path Planning of Mobile Robot Based on Neural Network[J]. Journal of Tianjin Vocational and Technical Teachers College. 2003, 13(1):10-12.

[7] Zhang Guoliang. A Survey of Research on Path Planning of Mobile Robots in Dynamic Environment[J]. Machine tool and hydraulic. 2013, 41(1):157-162.

[8] Yu Jianli, V. Kroumov, Sun Zengyi. A Fast Neural Network Path Planning Algorithm[J]. Robot. 2001, 20(33): 201-205.

[9] Zhang Hui, Rong Xuewen, Li Bin. Quadruped robot terrain recognition and path planning algorithm[J]. Robot. 2015, 9(5): 546-556.

[10] Zhang Huaye. Design and Implementation of Path Planning Parallel Algorithm Based on Hopfield Neural Network[D]. Master of Science, South China University of Technology, 2016.

[11] Zhang Ying, Lu Shouyin. Autonomous navigation method for substation inspection robot based on Hopfield neural network[J]. Manufacturing automation. 2015, 37(11): 36-41.

[12] Kang T, Ding W, Zhang L, et al. A biological network-based regularized artificial neural network model for robust phenotype prediction from gene expression data [JJ. BMC Bioinformatics, 2017, 18(1): 565.

[13] Hasircinglu I, Topcunglu H R, Ermis M. 3-D path planning for the navigation of unmanned aerial vehicles by using evolutionary algorithms[C]/Proceedings of the 10th annual conference on Genetic and evolutionary computation. New York: ACM, 2008: 1499-1506.

[14] Khansari-Zadeh S M, Billard A. A dynamical system approach to real-time obstacle avoidance[J]. Autonomous Robots, 2012, 32(4): 433-454.