Stability of shear-jammed granular materials: yielding and stabilization under small-amplitude cyclic shear

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We report on experiments that probe the stability of a two-dimensional jammed granular system formed by imposing a quasistatic simple shear strain $\gamma_0$ on an initially stress free packing. We subject the shear jammed system to quasistatic cyclic shear with strain amplitude $\delta \gamma$. We observe two distinct outcomes after thousands of shear cycles. For small $\gamma_0$ or large $\delta \gamma$, the system reaches a stress-free, yielding state exhibiting diffusive strobed particle displacements with a diffusion coefficient proportional to $\delta \gamma$. For large $\gamma_0$ and small $\delta \gamma$, the system evolves to a stable state in which both particle positions and contact forces are unchanged after each cycle and the response to small strain reversals is highly elastic. Compared to the original shear jammed state, a stable state reached after many cycles has a smaller stress anisotropy, a much higher shear stiffness, and less tendency to dilate when sheared. Remarkably, we find that stable states show a power-law relation between shear modulus and pressure with an exponent $\beta \approx 0.5$, independent of $\delta \gamma$. Based on our measurements, we construct a phase diagram in the $(\gamma_0, \delta \gamma)$ plane showing where our shear-jammed granular materials either stabilize or yield in the long-time limit.

Keywords: Granular matter, suspension, shear jamming, reversibility, yielding, stiffness, dynamic transition

I. INTRODUCTION

Granular materials are ubiquitous in nature and are important components of many industrial products and processes. An apparently stable granular packing may deform plastically in response to a change of the applied stress, a process termed yielding, which initiates the destruction caused in many natural disasters and failures of engineered structures. In addition to the catastrophic yielding of static systems, there are important examples of granular systems that yield slowly and gradually over time when subjected to repetitive perturbations in various forms such as compression/expansion [1–3], shear [4, 5], vibration [6, 7] or thermal cycling [8]. Understanding when, how, and why granular solids yield under such circumstances is a long-standing challenge.

The stability of a granular packing depends on its preparation history. An important class of stress bearing materials consists of noncohesive grains that jam when subjected to a sufficiently strong, fixed volume shear, forming what are known as shear jammed states [3, 5, 9–21]. These are states that support shear stresses and have nonzero pressure despite having a packing fraction below the random close packing value, $\phi_J$, meaning that the system can be in a stress free, unjammed state before the initial shear is applied [3, 18, 22–25]. These shear jammed granular solids exhibit mechanical fragility: subjecting them to sustained shear in a direction different from the original applied shear may lead to immediate plastic deformation [19, 26], or even a complete collapse of the solid [9, 18], and a recent numerical study [27] employing cyclic shear suggests that different types of steady states may appear depending on the parameters of the shearing protocol. Industrial and natural processes that lead to the jamming of dense, particulate matter are rarely isotropic and thus may be expected to exhibit features of shear jamming. An important example is the discontinuous shear thickening of dense suspensions [28, 29], which appears to be controlled by the proximity to a shear jamming transition [30, 31]. To our knowledge, there has been no experimental study of reversibility and yielding under small applied perturbations in amorphous solids that have been created through the process of shear jamming, and the question of how these stress-induced, low-density solids respond to applied stresses or strains remains open.

In this paper, we report on experiments that reveal important features of the response of shear-jammed granular solids to small amplitude cyclic shear, including a transition between yielding and strengthening behavior, depending on both the initial shear that creates the jammed state and the amplitude of the applied perturbations. A cyclic shearing protocol is widely used in studies of reversibility transitions, memory formation, and yielding in amorphous solids, including frictional granular materials, suspensions, and athermal glasses [27, 32–45]. However, most of these studies consider jammed packings with either fixed boundary stress or packing fraction far above $\phi_J$. It is not obvious how the frag-
ile packings formed by shear jamming below $\phi_3$ should respond to cyclic shear perturbations, as the material may completely lose its rigidity and relax to a stress free, unjammed state.

Our experimental granular system is a monolayer of photoelastic discs set in a special shearing device that is capable of imposing homogeneous internal shear strains [5]. (See Fig. 1(a).) Beginning with an unjammed packing, we apply an initial volume conserving shear $\gamma_1$ to create a shear jammed state, then examine the response of that state to cyclic shearing at small amplitude $\delta \gamma$. (See Fig. 1(b).) The cyclic shear has the same principal directions as the initial shear.

Our study highlights a qualitative distinction between classes of shear jammed states, raising a broad theoretical question about the nature of their rigidity. For a given $\delta \gamma$, we find a discontinuous change in the diffusion coefficient and the elastic constants for the states formed after many cycles when $\gamma_1$ is varied. Changing $\delta \gamma$ while fixing $\gamma_1$ also triggers a qualitatively similar transition. For large $\delta \gamma$ and small $\gamma_1$, the system relaxes to a yielding, plastic state with nearly zero stress and diffusive particle motion, a process we term fatigue yielding. (See Fig. 1(c,d).) For $\gamma_1$ and $\delta \gamma$ near the transition, the relaxation dynamics becomes non-smooth, showing transient plateaux in the quantities of interest, corresponding to emergent states we refer to as metastable. The lifetime of the longest lived metastable state grows rapidly as the transition is approached. For small $\delta \gamma$ and large $\gamma_1$, the system settles to a state with a duration longer than our experimental limit, which we refer to as a stable state. (See Fig. 1(e,f).) A stable state is a point on a limit cycle in which all particle positions and contact forces return to the same state after one shear cycle. Compared to the original shear jammed state, this stable state has a much stiffer response to forward shear and shows highly elastic behavior upon shear reversal. We find that the shear modulus of stable states is determined by their pressure in a power-law form $G \propto p^\beta$ with $\beta \approx 0.5$, independent of $\delta \gamma$. This power-law is reminiscent of critical scalings observed near the jamming transition in idealized model systems, where $\beta$ remains the same for compression and shear induced jamming but may differ for systems with different contact force laws [12, 22, 46]. Finally, we provide a phase diagram sketched based on our experimental measurements that specifies the minimal $\delta \gamma$ for a shear-jammed states created by an initial shear $\gamma_1$ to yield under cyclic shear.

This paper is organized as follows. Section II describes our experimental apparatus and protocol. Section III presents our results in six categories: (A) characterisation of the shear jammed solid before the application of cyclic shearing and definitions of fatigue yielding and stable states; (B) the transient evolution to the steady states induced by cyclic shear and identification of the resulting stable states or yielding states; (C) the nature of the dynamical transition from yielding to stable states; (D) comparison of the stiffness of the stable states to the original shear jammed solids; (E) contact network measurements for both the original states and the stable states; and (F) the effect of changing cyclic shear strain $\delta \gamma$. Technical details supporting the data analysis are provided in Appendices. Section IV contains a discussion of the conceptual significance of our results and potential practical implications.

II. MATERIALS AND EXPERIMENTAL PROTOCOL

Our model granular system consists of a bidisperse collection of 1040 photoelastic discs with diameters $d_b = 15.9$ mm and $d_a = 12.7$ mm and thickness $h = 6.77$ mm. The number ratio of large to small discs is 1 : 3, and we keep the system’s packing fraction fixed at $\phi = 0.816$ throughout all experiments. This value is close to, but less than the isotropic jamming packing fraction $\phi_J \approx 0.835$ [5]. The static friction coefficients are $\mu = 0.87 \pm 0.03$ between the particles, $\mu_{\text{base}} = 0.25 \pm 0.05$ between a particle and the base, and $\mu_{\text{wall}} = 0.70 \pm 0.02$ between a particle and boundary wall. The particles are cut using a water jet from a polyurethane sheet (Precision Urethane & Machine, Inc.). The bulk modulus of the material is 5.88 MPa. Under static diametric loading, the normal contact force law is roughly Hertzian. For a small disc squeezed between to rigid surfaces, we measure

$$f_n \approx \frac{\epsilon}{r_s}(\delta/d_s)^{3/2},$$

where $f_n$ is the normal contact force, $\delta/d_s$ is the diametric strain, $r_s$ is the radius of the small disc, and $\epsilon = 2.73$ N.m. This expression slightly overestimates the weak forces. Details on contact force law calibration are given in Appendix A.

We use the multi-slat simple shear apparatus developed by Ren et al. [5] to impose a uniform shear strain field. Our setup avoids the formation of a shear band and the associated density heterogeneity [5]. A schematic top view of the apparatus is shown in Fig. 1(a). The shear cell contains four aluminum walls (white rectangles) as confining boundaries and a bottom formed by 50 parallel acrylic slats (light purple rectangles, shown as only 11). Each slat, as well as wall 1○ and wall 2○, are constrained to move only along the $y$ direction, while the two other walls are constrained to rotate with pivots at $(0, l_y/2)$ and $(0, -l_y/2)$. To impose a uniform shear, a slat (or boundary wall 1○ and 2○) at position $x$ moves with a velocity

$$v(x) = -\dot{\gamma}x u_y$$

where $\dot{\gamma} = 2.1 \times 10^{-3}$s$^{-1}$ is the shear strain rate. $u_x$ and $u_y$ are unit vectors in the horizontal and vertical directions on the figure. The strain rate $\dot{\gamma}$ is held constant for all experiments. Walls 3○ and 4○ rotate in such a way that no slipping occurs at the junctions between
FIG. 1. (a) A schematic of the multi-slot shear cell from top view. (b) The driving strain as a function of time. An initial large forward shear is followed by multiple periods of small amplitude cyclic shear, and then by large amplitude reverse shear. The shear rate is always in the quasistatic regime. (c-f) Snapshots of the force chain network for four independent runs with different $\gamma_I$. Each column shows images in the original configuration, the shear jammed configuration following the initial forward shear, and the configurations reached after 1000 and 2000 shear cycles. As indicated by the shapes of the original configurations, the initial forward shear used to reach the rectangular configuration increases from left to right. All systems in (c-f) have the same packing fraction $\phi = 0.816$. The images are taken through a polariscope and thus only particles that bear finite stress are visible [47].

them and the two other walls, consistent with the motions of the slats. The maximal static friction between a particle and the base slat is 0.0036 N, which is sufficient to entrain the rattler discs to the affine strain field but is negligible compared to the typical contact forces in jammed states. More technical details on this device can be found in Ref. [48, 49]. In cyclic shearing, slight bending of the boundary walls and tiny slipping at the junctions when the direction of shear is changed lead to slightly asymmetric strain cycles, as discussed in detail in Appendix B. The strain rate employed here is considered quasistatic for the following reasons. The two-dimensional inertial number $I = \dot{\gamma} \sqrt{2 m_b/p}$ is less than $10^{-4}$ for pressure $p$ larger than 1 N/m, which is the case for the states of interest, where $m_b = 1.47 \times 10^{-3}$ kg is the mass of a large disc. Also, for nearly stress-free states, a disc with a non-affine velocity $v_{na} = \gamma d_s$ becomes static within a characteristic time $t = v_{na}/\mu_{base}g \approx 10^{-5}$ s that is much smaller than the macroscopic time scale $1/\dot{\gamma} \approx 0.5 \times 10^3$ s. Consistent with these separations of time scales, we find that when the motor is stopped, we observe no relaxation of the particle positions.

A single run of the experiment begins with the preparation of a homogeneous, stress-free, random configuration in a parallelogram frame chosen such that a forward shear strain $\gamma_I$ will yield a rectangular configuration. We then perform three stages of quasistatic shearing, as depicted in Fig. 1(b): (1) the initial forward shear; (2) $N$ cycles of additional shear between $\pm \delta \gamma$; and (3) a large reverse shear strain $\gamma = -0.21$ starting from the end of the last shear cycle. The number of cycles $N$ is typically 1500, but is larger for systems that take longer to relax, up to a maximum of 4800, and some data is collected for the $N = 0$ case (i.e., no cyclic shear is applied between the initial forward shear and reverse shear). We use various $\gamma_I$ ranging from 0% to 21% and $\delta \gamma$ ranging from 0.28% to 1.54%. Our protocol is similar to the one used in the numerical study of Ref. [27].

A high resolution camera (Canon 5D Mark II) accompanied by an automated imaging system with a polariscope is used to take images of the system. Details of the imaging system and post-processing procedures can be found in Ref. [49]. Figure 1(c-f) shows example images obtained from typical runs with different $\gamma_I$. All of the images are taken through a polariscope, so that only the discs supporting finite stress are visible. The second
row shows the stress state after the initial shear $\gamma_1$, and the third and fourth rows show the states after 1000 and 2000 shear cycles with $\delta\gamma = 0.95\%$. The nearly blank images in columns (c) and (d) indicate that the system has relaxed to a steady state with nearly zero pressure. The close similarity between images in the third and fourth rows of columns (e) and (f) indicate that stable states are reached within 1000 cycles.

In experiments involving cyclic shear, we capture images for the initial stress-free state at $\gamma = 0$, the states at the start of each full shear cycle from $n = 0$ to $n = 100$, and at the start of every tenth shear cycle for $n > 100$. For the last two shear cycles and for forward and backward shear experiments without cyclic shear, we capture images during every strain interval 0.0021. Capturing the images for one configuration takes about 30 seconds, during which the strain rate is set to zero.

We measure particle positions and contact forces between particles. The particle centers are detected with an uncertainty around 0.01$d_u$ using a Hough-transform technique [5, 50]. The vector contact forces between particles are measured using a well-known nonlinear fitting algorithm [47, 51–53]. Technical details of our implementation of the algorithm are provided in Appendix. C. We also use an empirical intensity gradient method [47, 54, 55] to estimate the overall pressure only for data shown in Fig. 4(a) and Fig. 12(a). The calibration of this method is described in Appendix. D.

From the contact forces, we construct the stress tensor defined as [9, 56, 57]

$$\hat{\sigma} = \frac{1}{S} \sum_{i=1}^{N_p} \sum_{j=1}^{N_p} \Theta_{ij} f_{ij} \otimes r_{ij},$$

where $i$ and $j$ are indices of particles, $N_p$ is the number of particles excluding the ones that belong to the boundary layer, $S$ is the sum of the Voronoi cell areas of these internal particles, $r_{ij}$ is the vector connecting the center of particle $i$ and the contact point between particle $i$ and particle $j$, $f_{ij}$ is the contact force exerted on particle $i$ by particle $j$, $\Theta_{ij}$ is a contact indicator with value 1 if particles $i$ and $j$ are in contact and 0 otherwise, and $\otimes$ denotes the vector outer product. We exclude contacts with fitted force magnitude less than 0.005N. From $\hat{\sigma}$ we calculate the pressure $p = -\text{Tr}(\hat{\sigma})$ and the off-diagonal element $\sigma_{xy}$, which we term the shear stress in this work. We note that in most cases for our system the principal axes of the stress tensor lie in the $(1,1)$ and $(1,-1)$ directions, so that $|\sigma_{xy}|$ is equal to the second invariant of $\hat{\sigma}$. We also define and measure the stress anisotropy $\Sigma \equiv \sigma_{xy}/p$.

To characterize the contact network structure, we also determine the fabric tensor [9, 57, 58]

$$\hat{R} = \frac{1}{N_{nr}} \sum_{i=1}^{N_p} \sum_{j=1}^{N_p} \Theta_{ij} \frac{r_{ij}}{|r_{ij}|} \otimes \frac{r_{ij}}{|r_{ij}|},$$

where $N_{nr}$ is the number of non-rattlers, defined as particles that have at least one force-bearing contact. We calculate the non-rattler contact number $Z_{nr} = \text{Tr}(\hat{R})$ and the fabric anisotropy $p = |\frac{R_2-R_1}{R_2+R_1}|$, where $R_1$ and $R_2$ are the eigenvalues of $\hat{R}$. We also calculate the non-rattler fraction $f_{nr} = N_{nr}/N$, which plays a key role in theories of shear jamming [9, 10].

### III. RESULTS

We present results on several features of the steady states reached after cyclic shear is applied to an originally shear jammed state. Section III A presents the basic phenomenology of the dynamical transition of interest. Each subsequent section presents detailed results on a particular feature, including more precise discussions of the criteria used to identify the different types of states mentioned in Sec. III A. We present results from experiments using a fixed $\delta\gamma = 0.95\%$ from Sec. III A to Sec. III E and only discuss the effect of changing $\delta\gamma$ in Sec. III F.

#### A. Basic phenomenology and terminologies

Starting from a nearly stress free state, the initial forward shear strain $\gamma_1$ creates shear jammed states with finite shear stress $\sigma_{xy}$ and pressure $p$. These states will be referred to as the original states. We emphasize that the shear box takes the same rectangular shape for all original states. Figure 2 shows $p$ and $\sigma_{xy}$ as functions of $\gamma_1$ for the original states. The inset shows the noise level at $\gamma_1 = 0$ and reveals that both $p$ and $\sigma_{xy}$ increase by more than an order of magnitude above the noise level for $\gamma_1$ as small as 2.1%. Thus, the strain $\gamma_1$ needed to reach the onset of jamming from a stress-free state is less than 2.1%, consistent with measurements from an earlier experiment [18].

When subjected to prolonged, low amplitude cyclic shear, a given original state evolves to one of two types of steady states: (1) a yielding state in which particle motions are diffusive and the stress vanishes; or (2) a stable state in which the particles and stress field return to the same configuration after each shear cycle. We term the transient process in which an original shear jammed state gradually evolves to a yielding state fatigue yielding. An example video of such a process is provided in the Supplementary Materials [59]. During this process the pressure and shear stress remain far below the maximal values that the packing could support under quasi-static continuous shear. A video showing the system evolves to a stable state is also provided in the Supplementary Materials [59].

Given a fixed $\delta\gamma$, increasing $\gamma_1$ would trigger a transition of the steady states from yielding to stable states. This transition is a dynamical transition, and is accompanied by discontinuous changes of stress states and mechanical properties of the steady states. We show the stress states data here and present other results in the
subsequent sections. With a fixed $\delta \gamma = 0.95\%$, figure 3 shows the pressure and stress anisotropy of the steady states as functions of $\gamma_i$, revealing a discontinuous jump in both quantities. For comparison, data for the original states are also shown. We note in particular that the stable states have finite stress anisotropy, although much smaller than the original states.

**B. Transient relaxation and the steady states**

Cyclic shearing leads to transient decay of both pressures and shear stresses measured after each complete shear cycle. Figure 4(a) shows the pressure measured after $n$ shear cycles for several different original states, with the same cyclic shear strain amplitude $\delta \gamma = 0.95\%$ in all cases. The shear stress relaxes in a qualitatively similar manner. After the transient relaxation the system enters a steady state that could be either a yielding state or a stable state. We use a pressure threshold to identify the onset of a steady state. We define this onset to be the smallest cycle number $n_s$ where the pressure deviates no more than $p_{th}$ from the value measured at the end of an experiment, and then study all of the states generated beyond cycle $n_s$. We use $p_{th} = 0.05N/m$, and report steady state results only for experimental runs in which we see at least 500 cycles beyond $n_s$. The threshold values are chosen empirically; changing them to slightly different values does not alter our conclusions. Examples of yielding and stable states are marked using blue and red dashed boxes in Fig. 4(a).

We now show measurements of the mean squared displacements (MSD) of particle centers within the range of steady states, which are later used to define the yielding and stable states. The MSD, $\langle (\Delta r^2) \rangle$, is a function of shear cycle number interval $\delta n$, defined as

$$\langle (\Delta r^2) \rangle(\delta n) = \frac{1}{N-n_s+\delta n} \sum_{n=n_s}^{N-n_s} \langle (\Delta r_{n,n+\delta n})^2 \rangle,$$

where $N$ and $n_s$ are the largest cycle number and the cycle number at which the steady state starts, and

$$\langle (\Delta r_{n,n+\delta n})^2 \rangle = \frac{1}{N_p} \sum_{i=1}^{N_p} | \mathbf{r}_i(n+\delta n) - \mathbf{r}_i(n) |^2,$$

where $N_p$ is the number of particles, $\mathbf{r}_i(n)$ is the position of the center of $i$th particle after $n$ shear cycles. In both equations, we have used the convention that $\langle \cdot \rangle$ means averaging over particles and $\bar{\cdot}$ means averaging over the cycle time $n$.

Figure 4(b) shows $\langle (\Delta r^2) \rangle$ in units of $d_0^2$ as a function of $\delta n$ for both types of steady states, displaying two distinct qualitative behaviors. To quantify this observation, we perform a linear fit for each curve using the form

$$\langle (\Delta r^2) \rangle = 4 \times D \times 4 \delta \gamma \delta n + c_{\text{noise}},$$

where the diffusion coefficient $D$ is the fit parameter and $c_{\text{noise}} = 0.00012$ is the measured noise level of our particle center measurements. In Eq. (7), $4 \delta \gamma \delta n$ is the cumulative absolute value of strain, which plays the role of an effective time. The prefactor of 4 is twice the dimension of the system, introduced to match a similar expression of the MSD for thermal Brownian systems [60]. We plot the fit results using dashed lines in Fig. 4(c), and the resultant diffusion coefficient $D$ are plotted in (d) as functions of the initial shear strain $\gamma_i$.

We define steady states with $D > 10^{-5}$ as yielding states, as the particles do not come back to their original positions after one shear cycle. We define a stable
FIG. 4. (a) The pressure measured after \( n \) full shear cycles, \( p_n \), as functions of shear cycle number \( n \) for several example experiments. The strain amplitude of the shear cycles are all the same \( \delta \gamma = 0.95\% \). The insert panel plots data from more experiments but in log-log scale. The color of each curve or data points in (a-b) labels the initial shear strain \( \gamma_I \) used to create the original states. The red, blue and gray dashed boxes show the spanning of an example stable state, an example yielding steady state, and an example transient stress plateau. (b) The mean square displacements of particle centers defined in Eq. (5) calculated within the steady states spanning at least 500 shear cycles before the experiment is terminated. The unit is \( d_{s}^{2} \), where \( d_{s} \) is the diameter of our small disc. The dashed lines are linear fits on individual curves with form defined in Eq. (7). (c) The diffusion coefficient \( D \) defined in Eq. (7) for the steady states plotted as functions of the initial shear strain \( \gamma_I \).

state to be a steady state with \( D < 10^{-5} \), which indicates that the particles return quite closely to their original positions after one shear cycle. We note that even for the stable states the MSD may grow slightly, as shown in Fig. 4(b), similar to what is observed in an aging glass [61]. Figure 5(a) plots the probability density functions of particle displacements in the last 1000 cycles for an example yielding state and an example stable state, showing that for stable states the displacements are strongly peaked around the center detection noise level, while the yielding state shows a shifted peak and a distribution that includes much larger displacements.

The spatial distribution of the displacements over 1000 shear cycles in a yielding state is shown in Fig. 5(b), which reveals dynamic heterogeneity in the form of chain-like collective motions similar to that reported in other experiments measuring perturbed granular materials near random close packing \( \phi_J \) [2, 62–64]. We do not see the persistent shear banding observed in diffusive steady states of frictionless packings above \( \phi_J \) [40]. A video of particle displacements in an example yielding state is provided in the Supplementary Materials [59]. Figure 5(c) shows the spatial distribution of particle motions for a stable state over the same number of cycles, and we see no obvious spatial pattern. The displacements are small and the spatial distribution in (c) is likely dominated by random noise. We note that there are significant particle rearrangements before the steady states are reached, but analysis of the transient is beyond the scope of the present paper.

C. Dynamical transition of the steady states

Although our system is driven quasi-statically, Fig. 4(c) suggests a “dynamical” transition as the control parameter \( \gamma_I \) is increased, as evidenced by the difference in particle displacement magnitudes in the yielding and the stable states. The transition appears to be
where the repetitive perturbation is thermal cycling. To multiple transient plateaus is also reported in Ref. [8], the transient relaxation process from smooth relaxation bound on the lifetimes of the stable states. A change of end the experiment, these measurements provide a lower red circles. Since the system is in a stable state when we stable steady states detected by same method using Fig. 6(b) using blue circles. We also plot \( n \) of shear cycles in this longest plateau by \( p \).

We now consider how to what extent we can think discontinuous: \( D \) changes more than two orders of magnitude near the transition, as shown in Fig. 4(c). We call the minimum \( \gamma_1 \) for which we ever observed a stable state the transition initial strain \( \gamma_{1, c} \), which is 14.7% for \( \delta \gamma = 0.95\% \). For larger \( \gamma_1 \) some systems relax to yielding steady states, but all experiments with \( \gamma_1 \geq 21\% \) produce stable states. Figure 6(a) shows that systems near \( \gamma_{1, c} \) take the longest time to reach their steady states, as might be expected near a dynamical phase transition.

The dynamical transition is characterized by the emergence of transient stress plateaus whose lifetime grows rapidly and exceeds our experiment duration near \( \gamma_{1, c} \). For larger \( \gamma_1 \) the system is trapped in a plateau state when we stop the experiment. One example of the transient stress plateau is marked by a gray dashed box in Fig. 4(a). Well defined stress plateaus only occur in experiments near or above \( \gamma_{1, c} \). No obvious stress plateau is seen for systems with small \( \gamma_1 \) (See the insert of Fig. 4(a)). We identify a plateau by searching for consecutive shear cycles in which the pressure change between the first and last cycle is below \( p_{th} = 0.05 \) N/m. In any given run there could be multiple plateaus, but we consider only the longest one. We denote the number of shear cycles in this longest plateau by \( n_p \) and plot it in Fig. 6(b) using blue circles. We also plot \( n_p \) measured for the stable steady states detected by same method using red circles. Since the system is in a stable state when we end the experiment, these measurements provide a lower bound on the lifetimes of the stable states. A change of the transient relaxation process from smooth relaxation to multiple transient plateaus is also reported in Ref. [8], where the repetitive perturbation is thermal cycling.

D. Stiffness of the steady states

We now consider the mechanical response of the yielding states and the stable states to perturbations in the form of additional forward or reverse shear strain. We measure the response to forward shear by measuring the stress-strain curves within the last shear cycle of an experiment, focusing on the slopes of the curves near the state of interest. After the last shear cycle, we reverse the direction of the applied shear strain, as shown in Fig. 1(b), and measure the response of the material. Finally, we compare measurements made on stable states and on original states.

Figures 7 (a-b) and (c-d) show pressure and shear stress as functions of strain within one shear cycle for a stable and a yielding state respectively, showing qualitatively different stress responses. Note that after reaching a steady state the stress-strain curve does not change with time. We note the striking similarity between the stress-strain curves for the yielding states to those measured in dense suspensions subjected to oscillatory shear [63].

According to the convention set by Fig. 1(b), the steady states all have \( \gamma = \gamma_1 \). We measure the slope of the stress-strain curves near the steady states during the forward portion of the cycle to quantify the linear response of the system. For shear stress, we denote the slope as \( G \) and call it a shear stiffness. We measure \( G \) for the stable states by applying a linear fit of the form

\[
\sigma_{xy} = G(\gamma - \gamma_1) + \text{constant}
\]

For the yielding states, we use a cubic fit function

\[
\sigma_{xy} = G(\gamma - \gamma_1) + G_2(\gamma - \gamma_1)^2 + G_3(\gamma - \gamma_1)^3 + \text{constant}.
\]

The dashed black curves in Fig. 7(b) and (d) plot two example fits using Eqs. (8) and (9).

It is also instructive to compare the stiffness of a stable state to the stiffness of the original states produced by forward shear alone. To measure the latter, we perform another set of experiments in which we record data during the initial forward shearing processes and measure the corresponding slopes near \( \gamma_1 \), as shown in Fig. 8(a) and (b). In both cases, we find a linear function fits the data well except when a global drop of stress occurs.

We now consider to what extent we can think \( G \) as an elastic constant. When the direction of shear is reversed, it appears that the stable states are highly elastic during such a process, in marked contrast to the original states. Figures 8(a) and (b) plot the response of an original state formed at \( \gamma_1 = 0.105 \) driven by the initial forward shear, followed by a shear reversal with no intervening cyclic shear. Figures 8(c) and (d) show the response of a stable state under the last quarter of a shear cycle, which is a forward shear, followed by a shear reversal shown by the black curve. We quantified the hysteresis in the stress-strain curves by measuring the area enclosed by
FIG. 7. (a,b) Pressure and shear stress within the last shear cycle in a stable steady state. (c,d) Pressure and the shear stress within the last shear cycle in a yielding steady state. The arrows mark the directions of time evolution. The black dashed curves are fits to data on the forward shear branch only using forms given in Eqs. (10), (8), (10), and (9), respectively. The fits are used to extract linear response moduli for the states at the start of a shear cycle, which are marked by the large, filled data points.

the stress strain curve in the range of strain accessed by the last quarter of shear cycle, denoted as $A$. Note that this range is a bit smaller than $\delta \gamma = 0.95\%$ due to the slightly asymmetric driving whose cause is discussed in Appendix B. We expect an elastic material to have $A = 0$. Figure 9(a) plots $A$ as a function of $\gamma$ for the original and steady states and for both the $p$ and $\sigma_{xy}$ curves. For the original states, $A$ grows monotonically for both $p$ and $\sigma_{xy}$, while for steady states $A$ remains near zero. Thus, we say that the stable states are highly elastic and interpret $G$ measured as the slope of stress strain curve as a shear modulus. We note that, in general, the slope of a stress strain curve is not the shear modulus of a sheared granular material [66], in contrast to our stable states prepared by cyclic shearing.

The pressure response is nonlinear for both stable and yielding steady states, and we find that a quadratic form

$$p = R (\gamma - \gamma_I) + R_2 (\gamma - \gamma_I)^2 + \text{constant}.$$  

fits the data well in both cases. We measure $R$, which we refer to as the linear Reynolds coefficient because the growth of pressure during volume-conserved shear is a phenomenon conjugate to the Reynolds dilatancy [5]. We also note that $R_2$ is termed the Reynolds coefficient in Ref. [5] and was found to diverge near $\phi_J$ for the yielding states. Examples of fit results are plotted as black dashed lines in Fig. 7(a) and (c).

The fit results $R$ and $G$ are plotted in Fig. 9(b) and (c). It appears that cyclic shearing process reduces $R$ but enhances $G$ for experiments that end in stable states. Moreover, both $G$ and $R$ exhibit a discontinuous jump across the dynamic transition near $\gamma_I$, and the yielding steady states have vanishingly small values of both.

E. Contact network characteristics

We report measurements of commonly studied features of contact networks for both the original states and the stable states. For the original states, we are interested in any changes that occur near $\gamma_I = 0.147$ that could be related to the transition from yielding to stable states. We are then interested in comparing the stable states and original states to gain insight into the changes in stiffness and elasticity induced by cyclic shearing. The contact forces in yielding states are too small to perform a reliable contact network analysis.
FIG. 9. (a) The area enclosed by stress-strain curves in the range of strain accessed by the last quarter of shear cycle (light brown area in Fig. 8) for the original states formed by initial shear only and the stable and yielding states formed after cyclic shearing, all plotted as functions of the initial shear strain $\gamma_I$. (b-c) The linear Reynolds coefficient $R$ and the shear modulus $G$ for the original states and the steady states. In (a-c) the scattered transparent data points are measurements from individual experiments and the opaque data points are averaged results. All steady state data in (a-c) are for systems with same $\delta\gamma = 0.95\%$.

Fig. 10 shows the non-rattler contact number $Z_{nr}$, the non-rattler fraction $f_{nr}$, and the fabric anisotropy $\rho$ for the original states and the stable states. The scattered faint data points are measurements from individual experiments, and the opaque data points are averages over them. All stable state data are for systems with save $\delta\gamma = 0.95\%$.

F. Effect of the magnitude of $\delta\gamma$

All of the cyclic shear data reported above was obtained for the same amplitude of the cyclic shear straining: $\delta\gamma = 0.95\%$. We have observed qualitatively similar behavior for different $\delta\gamma$ values. Data from experiments at various $\gamma_I$ and $\delta\gamma$, shown in Fig. 11, suggest a phase boundary sketched for the steady states in the control parameter space. The trend of data indicates that the transition occurs at smaller $\gamma_I$ for smaller $\delta\gamma$, or equivalently, that the original shear jammed states created by larger $\gamma_I$ have larger “yield strain”. We expect $\gamma_{I,c}$, the minimum initial forward shear strain required to form a stable state, to approach the shear jamming strain $\gamma_J$ as $\delta\gamma$ zero. Detailed mapping of the phase boundary is beyond the scope of the present work.

In Fig. 11, the dynamical transition described in the previous sections corresponds to the path marked by the vertical arrow. Experiments using larger $\gamma_I$ and larger $\delta\gamma$ than the rightmost and uppermost points shown in Fig. 11 cannot avoid out-of-plane buckling instabilities: stresses become large enough that particles are ejected from the monolayer of interest.

A dynamical transition of same type as reported in previous sections can be induced by fixing $\gamma_I$ and changing $\delta\gamma$, i.e., by following the path shown by the horizontal arrow in Fig. 11. Figure 12 shows the stress relaxation, the MSD in the steady states, and the diffusion coefficient $D$ as functions of $\delta\gamma$ for a set of experiments with fixed $\gamma_I = 18.9\%$. A dynamical transition with a discontinuous jump of $D$ clearly occurs near $\delta\gamma = 0.95\%$.

Figure 13 shows the diffusion coefficient $D$ averaged over yielding states with the same $\delta\gamma$. Remarkably, except for one outlier at the largest $\delta\gamma$ that we measured, $D$ is proportional to $\delta\gamma$. The blue dashed line shows a fit.
FIG. 11. The phase diagram sketched based on about one hundred independent experiments each contains thousands of shear cycles. The size of the circles represents the number of experiments with same control parameter $(\delta\gamma, \gamma_I)$. The smallest circle has one run and the largest one contains 9 runs. The face color of the circles represents the number fraction of experiments that ends in a stable state. The number fraction is denoted as $f_J$ and the color code is shown in the colorbar to the right. The phase boundaries are sketched to guide the eye. For $\delta\gamma = 0$, the minimal strain needed to create a jammed state is marked as $\gamma_J$.

with form

$$D = D_0 \delta\gamma,$$

(11)

with $D_0 = 0.0271 \pm 0.0005$.

Finally, we present a striking result for the stable states produced with different values of $\delta\gamma$. Figure 14 shows that when $G$ is plotted as a function of pressure, without any rescaling, the data falls on a single curve. The collapse of the data for small $\delta\gamma$ suggests that these experiments probe states that are close to the elastic limit of our elasto-plastic material. The solid curve shows a power-law fit of the form

$$G = G_0 p^\beta,$$

(12)

with the fit parameter $G_0 = (95 \pm 15) \text{ N/m}$ and the exponent $\beta = 0.50 \pm 0.06$. Interestingly, shear modulus measured using sound propagation for isotropic systems exhibits similar dependence on pressure as ours [69]. Numerical simulations with frictionless spherical particles interacting through linear spring force laws also show $\beta = 0.5$ near jamming for both isotropic [22, 46, 70] and sheared systems [12]. However, for frictionless Hertzian contact models simulations give $\beta = 2/3$ over the range of dimensionless pressures studied in our system [22, 71]. Simulations of 2D packings of frictional spheres with Hertz-Mindlin forces and with friction coefficients similar to ours show $\beta$ between $1/2$ and $2/3$ [72]. Recent work shows that $\beta$ is also different for deformable particles whose shape is controlled by surface tension rather than bulk stresses [73].

FIG. 12. The dynamical transition induced by changing $\delta\gamma$ for $\gamma_I = 0.189$. (a) The pressure after $n$ shear cycles for experiments under cyclic shear. (b) The mean-square-displacement for steady states. In both (a) and (b) there are some missing data points due to trigger failure of the CCD camera. The dashed curves are linear fit results using Eq. (7). (c) The diffusion coefficient $D$ obtained from the fit results. Red circles represent stable states with $D < 10^{-5}$. Blue triangles represent yielding states with $D > 10^{-5}$.

FIG. 13. The diffusion coefficient $D$ as defined in Eq. (7) averaged over yielding states with same $\delta\gamma$. The transparent scattered data shows measurements on individual experiments. The averaged data from low to high $\delta\gamma$ are averaged over 1, 7, 17, 2, 3, and 3 individual experiments respectively. Except for one outlier, data fits well to a simple form $D = D_0 \delta\gamma$ with fit result $D_0 = 0.0271 \pm 0.0005$.

IV. CONCLUDING REMARKS

We reported above on experiments testing the stability of shear jammed granular materials formed by an initial shear strain $\gamma_I$ imposed on a stress-free configuration. These shear jammed states, referred to as original states, were subjected to cyclic shearing with strain amplitude
δγ ≪ γ₁. We have identified two types of steady states induced by cyclic shearing depending on relative magnitudes of γ₁ and δγ. For large δγ and small γ₁, cyclic shearing leads to yielding steady states with vanishingly small shear stiffness and show irreversible, roughly diffusive strobled particle motions. We refer to the transient relaxation process as fatigue yielding. For small δγ and large γ₁, the system evolves to long-lived (meta)stable states in which both the particle positions and the contact forces return to the original values after a complete shear cycle. Increasing δγ while fixing γ₁ or reducing γ₁ while fixing δγ triggers a discontinuous yielding transition of the steady states. All the experiments are done at same packing fraction below the random close packing φ₁.

We also reported on the mechanical and dynamical properties of the yielding and stable states. For the yielding states, we find that the diffusion coefficient is proportional to δγ for small δγ. The stable states have a larger shear modulus G than the original states, but a smaller linear Reynolds coefficient R. These stable states are highly elastic in the sense that the energy dissipated near the onset of shear reversal is almost zero. For stable states formed by cyclic shearing with different δγ, there is a single power-law relation between their shear modulus G and pressure p with exponent β ≈ 0.5. The physical origin of this value is a compelling topic for further investigation.

The question of why the original states suddenly become capable of surviving cyclic shear as γ₁ is increased or δγ decreased poses a theoretical challenge. In the case of changing γ₁ with fixed δγ, we find no obvious change in the contact network of the shear jammed states near γ₁c: the contact number, fabric anisotropy, and non-rattler fraction all seem to saturate at γ₁ substantially smaller than γ₁c. However, both pressure and shear stress grow monotonically across γ₁c, suggesting that the stability transition is associated with stresses in the original states. Further analysis of mesoscopic features, such as rigid clusters [68, 74], topological features of the contact network [43, 44], or force tile structures [10, 75] may provide additional insights.

We note that the particles used in this work exhibit internal strains comparable to the amplitude of the cyclic strains applied to the system, which may be important for the elasticity properties of the stable states. For the data reported here, δγ is comparable to the strain experienced by an individual particle subjected to the mean contact normal forces observed in the stable states. Applied normal forces induce strains of 0.4% to 1.2% for the range of pressures occurring in our stable states. Technical details are provided in Appendix. A. Moreover, highly stressed contacts become flatter, which may induce additional constraints on the particle rotations [76, 77]. It would be intriguing to study the stable-to-yielding transition in a systems of hard particles, where the packing cannot be elastic but stable limit cycles may still appear.

We close with remarks on three topics described in the literature that may be related to our experiments: jamming phase diagrams for granular materials; the rheology of dense suspensions; and the loss of rigidity in frictionless system. First, we note that jamming phase diagrams in the space of stress and packing fraction, such as the ones reported by Bi et al. [9] and Zhao et al. [18], only describe the original states in our experiments. Our relaxed stable states have a different form of mechanical stability and can sustain unexpectedly robust elastic responses for packings with density below random close packing. In addition, the stability transition described in the present work is different from the fragile to shear-jammed transition in Ref. [18], which captures a change in the response of the system to large-strain shear reversal. For completeness, we note that the jamming phase diagram for long-time steady states of dry granular material under slow periodic shearing was recently studied numerically by Otsuki and Hayakawa [27]. They observed a discontinuous change of storage modulus at a particular γ₁ while fixing φ and δγ, a result that is consistent with our observations. Our yielding states show similar stress-strain curves within a shear cycle as the fragile states reported in Ref. [27].

Second, our experiments probe transitions involving stress-free unjammed states, which are usually not accessible to experiments on three-dimensional dry granular materials [36, 38]. In addition, the weak drag force arising from friction between the particles and the base slats in our system may play a role similar to hydrodynamic drag. Thus, our results may be particularly useful for elucidating the physics of slowly sheared, non-

![FIG. 14. Measurements of the shear modulus G defined in Eq. (8) for the stable states created by cyclic shear with different strain amplitude δγ. Each data point corresponds to one packing that is independent from others. The brown axes show the corresponding dimensionless values for pressure and shear modulus. The insert panel shows same data but in log-log scale. The black curve show a power-law fit with form G ∝ p^β with the fit result β = 0.50 ± 0.06.](image-url)
Brownian suspension systems where gravitational effects are not important. Indeed, the stress-strain relation we observe for the yielding steady states within a shear cycle is strikingly similar to the curves observed in slow cyclic shearing of dense suspensions [65]. Recent experiments and numerical studies have established that cyclic shearing may lead to significant reduction of the viscosity in suspension flow [78–80]. We show here that even in the quasi-static limit within the shear-jammed regime, cyclic perturbation may allow the recovery of such a flow through the process of fatigue yielding. Similar considerations may apply to packings formed through a flow-arrest transition [81, 82]. We emphasize that such yielding can occur while the shear stress and pressure are both far below the maximal values that the system with same packing fraction can statically support – a feature also observed in creep process of dry grains [83]. Fatigue yielding may also be relevant in the liquefaction of water-saturated sand [45, 84–86]. We also note that the transition between stable and yielding states in our system is markedly different from a previously reported reversibility transition in semi-dilute suspensions [32, 65, 87–94]. In the latter case, the reversible behavior occurs when particles in a dilute regime never form contacts during a shear cycle.

Finally, our observations share a number of remarkable features with numerical results from models of athermal glasses involving frictionless soft spheres at packing fractions above $\phi_1$. Kawasaki et al. [37] observed a discontinuous yielding transition similar to ours by increasing the shear cycle strain amplitude $\delta \gamma$. A key difference is that our yielding steady states are stress free and exist only below $\phi_1$. Das et al. [42] reported that there exist isotropic jammed packings that can become unjammed under cyclic shearing within an appropriate range of strain amplitudes, a process that appears similar to the fatigue yielding process in the shear-jammed systems studied here. Dagois-Bohy et al. [95] reported that simple compression leads to “improperly jammed” packings that are unstable to shear and that truly stable states could be constructed when combined with a shear stabilization process. This behavior mirrors the distinction we find between original shear jammed states that are unstable to cyclic shear and those that become stable under the cyclic shearing.

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Appendix A: Contact force law on a single disc

We measure the contact force law of a disc through diametrical loading test using a TA Instruments RSA III Micro-Strain Analyzer. The instrument measures the distance travelled by the upper arm, $d$, and the normal contact force magnitude, $f_n$. A picture of the loading part of this instrument is shown in Fig. 15(b), taken from Ref. [96] with permission. The relation between $d$, rescaled by the radius of the small disc $r_s$, and $f_n$ are plotted in Fig. 15(a) for a small disc. The majority of the contact forces in the work is below 3 N. We note that a Hertzian contact force law fits data reasonably well. Specifically, we fit data between 0.01 N and 3 N to the following form

$$f_n = \epsilon \left( \frac{d - d_c}{d_s} \right)^2,$$

and get $\epsilon = 2.73 \pm 0.03$ N·m and $d_c/d_s = 0.0347 \pm 0.0002$. $d_c$ is the point when the upper arm of the Micro-Strain Analyzer just touches the particle. The fit result is plotted as a blue dashed curve in Fig. 15(a). We note that for small forces a quadratic form fits data better. The red curves in Fig. 15(a) shows a fit for data between 0.01 N and 0.5 N using

$$f_n = \epsilon_1 \left( \frac{d - d_{c,1}}{d_s} \right)^{\alpha_1},$$

and obtain the fit parameters $\epsilon_1 = 5917 \pm 2300$N, $d_{c,1}/d_s = 0.0328 \pm 0.0004$, and $\alpha_1 = 2.1 \pm 0.1$. The closed-form solution for this problem is given in Ref. [97].

Appendix B: Measurement of the global shear strain using particle displacements and minor deviations from uniform shear near the onset of shear reversal

We find that the boundary walls and the base slats impose a uniform simple shear strain field very well in most of the cases both during both forward shear and reverse shear processes, as plotted in Fig. 16(a) for an example cyclic shear experiment. The shear cell is driven by a stepper motor, which precisely controls the position and motion of the left end of wall 4 [48, 49]. A shear step here means that the left end of wall 4 moves a fixed distance 1 mm. When the direction of driving is switched, the release and rebuild of a small elastic deformation of the two long and thin confining aluminum walls 3 and 4, and small relative motion at the joints between different walls and slats may both cause minor deviations from the expected uniform shear strain field. Such a deviation is evidenced from the particle displacements measured at the two shear steps right after the change of shear direction as plotted in Fig. 16(b). The actual shear strain experienced by the system in these two steps is smaller than what is expected from assuming a uniform strain field given the well-controlled motion of the left end of wall 4.
In this work we measure the actual global shear strain of the material using the particle displacements. Given the bin-averaged displacement field during \( i \) th shear step such as the curves in Fig. 16(a) and (b), the global shear strain caused by this shear step, \( \varepsilon_i \), is defined as

\[
\varepsilon_i = \frac{1}{N_{\text{bin}} - 2} \sum_{k=2}^{N_\text{bin} - 1} \frac{\Delta y_{k+1} - \Delta y_{k-1}}{5d_s},
\]

where \( 5d_s \) is the twice of the bin size, \( N_{\text{bin}} = 19 \) is the number of bins used to calculate the averaged data. Then the accumulative shear strain \( \gamma \) is calculated by doing a summation of \( \varepsilon_i \) over consecutive shear steps. Figure 16(c) and (d) plot the accumulative shear strain and shear strain caused by each shear step for an example system experiencing three shear cycles. In Fig. 16(c) purple dashed lines show times at which the direction of shear is changed. We see that the shear cycle is slightly non-symmetric: the minimal accumulative strain is -0.0085, yet the maximal one is 0.0105. We take the average between the absolute value of the two, 0.0095, as the strain amplitude for such a driving. The uncertainty of this strain amplitude measurement is around 0.001.

We note that this small deviation near strain reversal does not affect any of our conclusions: the stiffness measurements in Fig. 7, Fig. 8, Fig. 9(b-c) and Fig. 14 do not include any data points right after shear reversal. Therefore these measurements indeed reflect properties of a uniformly deformed material. The measurements for \( A \) in Fig. 9(a) includes shear steps with small deviations from the uniform strain field. However, the deviation effect is the same for both the steady states and the original shear jammed states. Thus, such an effect would not affect any claim based on the comparison between them.

### Appendix C: Contact force measurements using photoelasticity

We introduce the implementation of the non-linear fitting algorithm we used to measure the contact forces and then report our estimation of the uncertainties of the measurement. We use a Matlab implementation adapted from PEGS [53] with several modifications that improve the quality of the solution for the large forces, including (1) a neural network trained to give initial guesses, (2) the use of reaction forces and/or forces at an earlier strain step to refit particles with a large error at current step, and (3) manually supplied initial guesses determined us-
ing an interactive graphic interface for rare cases. We enforce force and torque balance constraints on individual particles except for rare cases, and use the deviation between action and reaction forces at the contacts to estimate uncertainties of the measurements. In rare cases when some particles are bearing extremely large forces, we do not enforce force and torque balance. Instead we let the algorithm minimize the intensity differences and the residual net force and torque together. Such a method typically leads to smaller intensity differences between the reconstructed photoelastic images and the raw experimental images, and the residual net force and torque are usually negligible. The stress-optic coefficient of the particles is \( F_\sigma = 157 \), defined as in Ref. [53] and measured using a technique detailed in Chapter 3.3.2 of Ref. [96]. The feasible contact positions are detected for each disc before fitting. If the distance between the centers of two particles is less than 1.03 times the sum of their radii, we register a feasible contact and find the fitted contact force carried by it for both of the two contacting discs. If for both discs the magnitude of the contact force is less than 0.005 N, we drop this contact and fit the discs again using only the remaining contacts. Such a process is repeated until all the remaining contacts are bearing forces whose magnitudes are larger than 0.005 N.

We show that our fitting algorithm finds the global minimum of the error function [47, 52, 53] by showing examples of experimental images and reconstructed images in Fig. 17. We show the number of the photoelastic fringes and their overall shapes are very well reproduced even for packings with large pressure. We then compare quantitatively the forces solved by the fitting algorithm and the force measured from a commercial force sensor for a particle under diametric loading (Fig. 18). In such a test, the absolute error is below 0.05 N, and the relative error is less than 10% for forces larger than \( \sim 0.1 \) N.

We estimate that the relative uncertainties for pressure and shear stress measurement to be about 5% by considering the differences between action and reaction forces obtained from the fitting. These differences should be zero for the actual physical contact forces due to Newton’s third law. For the \( k \) th contact of a disc, we denote the absolute difference between the normal components of the action and reaction forces as \( \delta f_{n,k} \), and we denote the absolute difference between the \( y \) component of the action and reaction forces as \( \delta f_{y,k} \). Then, we calculate the uncertainty estimation for \( p \) using

\[
\delta p = \frac{1}{S} \left( \sum_{i=1}^{N_p} \sum_{k=1}^{z_i} r_i^2 \delta f_{n,k}^2 \right)^{1/2},
\]  

\( (C1) \)

where \( r_i \) is the radius of particle \( i \), \( z_i \) is the number of contacts for particle \( i \), and other terms have same meaning as in Eq. 3. We calculate the uncertainty estimation for \( \sigma_{xy} \) using

\[
\delta \sigma_{xy} = \frac{1}{S} \sqrt{ \sum_{i=1}^{N_p} \sum_{k=1}^{z_i} r_{x,i,k}^2 \delta f_{y,k}^2 },
\]  

\( (C2) \)

where \( r_{x,i,k} \) is the \( x \) component of the vector connecting the center of particle \( i \) and its \( k \) th contact, and other terms have same meaning as in Eq. (C1). We plot the relative uncertainty \( |\delta p/p| \) and \( |\delta \sigma_{xy}/\sigma_{xy}| \) for a number of original states and stable states in Fig. 19. It is clear that in most of the cases both uncertainties are below 5%.
FIG. 19. The estimated relative uncertainties for pressure $p$ and shear stress $\sigma_{xy}$ measurements for a number of example original and stable states. The uncertainties $\delta p$ and $\delta \sigma_{xy}$ are calculated using Eq. (C1) and Eq. (C2).

FIG. 20. The pressure $p$ calculated from contact forces solved using the non-linear fitting algorithm plotted as function of $g^2$. The black circles are original states and the colored data points are states formed during cyclic shearing for two different experiments. The black curve plots Eq. (D2).

Appendix D: Intensity gradient method to estimate pressure for states formed during relaxation

The pressure data shown in Fig. 4(a) and Fig. 12(a) are not calculated from contact forces but estimated using an empirical method called intensity gradient method. Such a method is first introduced in Ref. [54] and more information can be found in recent reviews such as Ref. [53] and Ref. [47]. It is a good indicator for pressure only when the tangential forces are small compared to normal forces [55]. In our experiments we find that contacts bearing tangential forces comparable to normal forces are rare, suggesting the applicability of this method.

The mean intensity gradient of the packing, denoted as $g_2$, here, is defined as

$$
g_2 = 10^4 \frac{1}{N_p} \sum_{i=1}^{N_p} \frac{1}{N_{\text{pixel},i}} \sum_{\text{pixel}(i,j) \text{ in disc } i} \frac{1}{4} \left( \frac{(I_{i+1,j} - I_{i-1,j})^2}{2} + \frac{(I_{i,j+1} - I_{i,j-1})^2}{2} + \frac{(I_{i+1,j-1} - I_{i-1,j+1})^2}{2} \right),
$$  

where $I_{i,j}$ is the rescaled intensity in the $(i, j)$ pixel of the polarized image, which ranges from 0 to 1. $N_{\text{pixel},i}$ is the number of pixels in the $i$th disc. The prefactor $10^4$ is introduced in order to adjust $g_2$ value to be at a similar order of $p$.

We calibrate the relation between the system pressure $p$ and $g_2$ using a set of original states where the values of $p$ are calculated using the contact forces solved by the nonlinear fitting algorithm. The relation between $p$ and $g_2$ for these states are plotted using black circles in Fig. 20. We fit these data using

$$
p = ag_2 + b g_2^4,
$$  

which gives $a = 8.0 \pm 0.6$ N/m and $b = 0.11 \pm 0.02$ N/m.

We have solved the contact forces for all states formed in two example cyclic shearing experiments and plot their $p$ and $g_2$ in Fig. 20 as well, showing that these states follow same $p(g_2)$ relation as the original states. Thus, the pressure $p$ shown in Fig. 4(a) and Fig. 12(a) are calculated using Eq. (D2).

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