Vortex Core States in Superconducting Graphene.

I. M. Khaymovich(1), N. B. Kopnin (2,3), A. S. Mel’nikov (1), I. A. Shereshevskii (1),

(1) Institute for Physics of Microstructures, Russian Academy of Sciences, 603950 Nizhny Novgorod, GSP-105, Russia
(2) Low Temperature Laboratory, Helsinki University of Technology, P.O. Box 2200, FIN-02015 HUT, Finland
(3) L. D. Landau Institute for Theoretical Physics, 117940 Moscow, Russia

(Dated: January 22, 2009)

The distinctive features of the electronic structure of vortex states in superconducting graphene are studied within the Bogolubov–de Gennes theory applied to excitations near the Dirac point. We suggest a scenario describing the subgap spectrum transformation which occurs with a change in the doping level. For an arbitrary vorticity and doping level we investigate the problem of existence of zero energy modes. The crossover to a Caroli–de Gennes–Matricon type of spectrum is studied.

PACS numbers: 73.63.-b,74.78.Na,74.25.Jb

Recent exciting developments in transport experiments on graphene [1] have stimulated theoretical studies of possible superconductivity phenomena in this material [2,3,4,5]. A number of unusual features of superconducting state have been predicted, which are closely related to the Dirac-like spectrum of normal state excitations. In particular, the unconventional nonnormal electronic dispersion has been shown to result in a nontrivial modification of Andreev reflection [6] and Andreev bound states in Josephson junctions [7]. In this Letter we consider another generic problem illustrating the new physics of Andreev scattering processes in graphene, namely, the electronic spectrum in the core of a vortex that can presumably appear in the superconducting graphene in presence of magnetic field. The electronic vortex structure for Dirac fermions has been previously studied in particle physics for a situation equivalent to the zero doping limit in graphene, when the Dirac point lies exactly at the Fermi level. A number of important results have been obtained, namely, the exact solutions for the zero energy modes [8,9] and the subgap spectrum for some model gap profiles within the vortex core [10]. The problem of zero energy modes has been further addressed in [11,12] for vortices in various condensate phases described by the Dirac theory on a honeycomb lattice. Our goal is to develop a theoretical description of the electronic structure of multiply quantized vortices beyond the zero doping limit and study the transformation of the spectrum under the shift of the Fermi level from the Dirac point.

The energy spectrum is determined by the vortex winding number (vorticity) $\nu$ and labeled by the angular momentum $\nu$ which is a conserved quantity for an axisymmetric vortex. For zero doping the sub-gap spectrum consists of $n$ zero energy states $|0\rangle$. The states with higher energies lie close to the gap edge $\pm |\Delta_0|$. In the present Letter we demonstrate that, with an increase in doping level $\mu$, the distance between the energy levels decreases, so that more and more states fill the sub-gap region. The set of low-energy levels gradually transforms into a set of $n$ “spectrum branches” $E^{(i)}(\nu)$ where $i = 1, \ldots, n$. If $\nu$ is considered as a continuous parameter, these $n$ energy branches cross the Fermi level $\nu = 0$ as functions of $\nu$. The detailed behavior of the energy spectrum as a function of $\nu$ crucially depends on the parity of the winding number. For odd $\nu$ there exists one branch which intersects zero energy at $\nu = 0$. Its crossing point belongs to the spectrum, thus resulting in an exact zero energy mode. The crossing points of other $n-1$ branches do not generally belong to the spectrum for finite $\nu$, thus these zero modes do not exist. However, some of these $n-1$ zero energy modes can appear again for certain doping levels. For high doping $|\nu| \gg |\Delta_0|$, they appear almost periodically with an increase in the Fermi momentum $h k_F = |\nu|/v_F$ by a characteristic value $\delta k_F \sim 1/\xi$. Here $\xi$ is a superconducting coherence length $\xi = h v_F/\Delta_0$, and $\Delta_0$ is a homogeneous gap value far from the vortex center. In a singly quantized vortex the energy spectrum for high doping is given by the Caroli–de Gennes–Matricon (CdGM) expression similar to that for usual s-wave superconductors [13]. For an even winding number the crossing points of all $n$ branches do not generally belong to the spectrum if $\nu \neq 0$. In this case the exact zero modes are absent. Nevertheless, some of these zero energy modes can appear again for certain $\nu$ in the way similar to the odd vorticity case. For high doping, they appear periodically with increasing $\nu$. Indeed, the energy branches cross the Fermi level at certain momenta $\nu(\nu) \sim k_F \xi$. The zero modes appear when these $\nu(\nu)$ become integer (odd $n$) or half-integer (even $n$) for discrete equally spaced (due to equidistant energy levels) values of the Fermi momentum. Because of the symmetry with respect to the sign of energy (see below) the zero energy modes appear and disappear in pairs, for $\nu$ and $-\nu$.

Model. The Bogolubov–de Gennes (BdG) equations in superconducting graphene can be written as two decoupled sets of four equations each [8]. Assuming valley degeneracy we can consider only one of these sets known as “Dirac–BdG equations” [8,13]

$$v_F \sigma \cdot \left( \hat{p} - \frac{\epsilon}{c} \mathbf{A} \right) \hat{u} + \Delta \hat{v} = (E + \mu) \hat{u},$$

(1)

$$-v_F \sigma \cdot \left( \hat{p} + \frac{\epsilon}{c} \mathbf{A} \right) \hat{v} + \Delta^* \hat{u} = (E - \mu) \hat{v}.$$  

(2)
Here $\mathbf{p} = -i\hbar \nabla$, and $\hat{u} = (u_1, u_2)$ and $\hat{v} = (v_1, v_2)$ are two-component wave functions of electrons and holes in different valleys in the Brillouin zone \cite{16}. The indices 1 and 2 denote two sublattices of the honeycomb structure. They form spinors in the pseudo-spin space, in which the Pauli matrices $\sigma_x, \sigma_y, \sigma_z$ are defined; we also introduce a vector $\sigma = (\sigma_x, \sigma_y)$. The energy is measured from the Fermi level. For zero doping, the Fermi level lies exactly at the Dirac point, $\mu = 0$. For nonzero doping, the Fermi level is shifted by $\mu > 0$ upwards (electron doping) or downwards $\mu < 0$ (hole doping) from the Dirac point. For a homogeneous gap $\Delta = \Delta_0$ and zero magnetic field the wave functions take the form of plane waves $\hat{u}, \hat{v} \propto e^{i\mathbf{p}\mathbf{r}}$ and we obtain two noninteracting energy branches $E^2 = \Delta^2_0 + (\mu + ev_F)^2$, which correspond to the pseudospin orientation chosen parallel and antiparallel to the momentum direction, respectively.

**Vortex states.** Consider an $n$-quantum vortex, $\Delta = |\Delta| e^{i\phi}$, where $\rho, \phi$ are cylindrical coordinates. For an axisymmetric magnetic field the eigenstates are labelled by a discrete angular momentum quantum number $\nu$:

$$
\left( \begin{array}{c} \hat{u} \\ \hat{v} \end{array} \right) = e^{i\phi - i\sigma_y \phi/2 + i\sigma_z \pi/4} \left( \begin{array}{c} e^{i\phi/2}U(\rho) \\ e^{-i\phi/2}V(\rho) \end{array} \right). \tag{3}
$$

The extra factor $e^{-i\sigma_y \phi/2}$ as compared to the usual BdG functions $u$ and $v$ comes from the angular dependence of the momentum operator in cylindrical coordinates $p_x \pm ip_y = e^{i\phi} \hbar[ -i\partial/\partial\rho \pm i\rho^{-1}\partial/\partial\phi]$. The transformation properties of the wave functions under rotation around the vortex axis correspond to those for an $s$-wave superconductor with the replacement $n \rightarrow -n - 1$. As a result, $\nu$ is half-integer for even vorticity $n$ and integer for odd $n$ in contrast to standard $s$-wave superconductors.

**General properties of zero energy states.** Equations (1) and (2) are invariant under the transformation

$$
E \rightarrow -E, \ i\sigma_y \hat{u}^* \rightarrow \hat{v}, \ i\sigma_y \hat{v}^* \rightarrow -\hat{u}. \tag{4}
$$

Thus, for arbitrary vorticity and doping level, a set of zero energy modes $\hat{u}, \hat{v}$, labelled by an index $i$, satisfies $\hat{u} = i\sigma_y \hat{u}_i^*$, $\hat{v} = -i\sigma_y \hat{v}_i^*$. This transformation couples the states with opposite angular momenta. One can separate two types of zero energy solutions: (i) The eigenfunction components transform into each other, $i = j$, which can be realized only for $\nu = 0$. (ii) There exists a pair of eigenfunctions $i \neq j$ with opposite $\nu \neq 0$ coupled by the above transformation.

Consider solutions of the first type which can exist only for an odd-quantum vortex. We put $\hat{v} = i\sigma_y \hat{u}^*$ and find:

$$
\left[ e_F \sigma \cdot \left( \frac{\mathbf{p}}{c} - \frac{\mathbf{A}}{c} \right) - \mu \right] \hat{u} + i\Delta \sigma_y \hat{u}^* = 0. \tag{5}
$$

We look for the solution in the form $\hat{u} = \zeta(\rho) \hat{U}^{(0)}$, where $\hat{U}^{(0)}$ is a normal state solution of Eq. (1) with $\Delta = 0$ and $\zeta$ is a real function satisfying

$$
\sigma_z e^{i\sigma_z \phi} \hat{U}^{(0)}(d\zeta/d\rho) = \sigma_y \hat{U}^{(0)*} \Delta \zeta.
$$

In a homogeneous magnetic field $H$ the functions $\hat{U}^{(0)}$ diverge at large distances $\rho$ of the order of the magnetic length $L_H = \sqrt{\hbar c/eH}$ except for a discrete set of $\mu$ which correspond to the Landau energy levels. However, similarly to the case of usual superconductors, this large-distance divergence is cut off either at the magnetic field screening length (isolated vortex) or at the intervortex distance $\sim L_H$ (flux lattice). Thus, we can disregard the above mentioned divergence for the analysis of the states localized near the vortex core at distances of the order of the coherence length $\xi$. Considering weak magnetic fields near the vortex core $H \ll H_{c2}$ such that $L_H \gg \xi = \hbar v_F/\Delta_0$ we can even neglect the vector potential at all. In this way we obtain for $\mu > 0$:

$$
\hat{U}^{(0)} = C \left( \begin{array}{c} e^{-i\pi/4} e^{i(n-1)\phi/2} J_{(n-1)/2}(k_F\rho) \\ e^{i\pi/4} e^{i(n+1)\phi/2} J_{(n+1)/2}(k_F\rho) \end{array} \right),
\zeta(\rho) = e^{-K(\rho)}; \ K(\rho) = \frac{1}{\hbar v_F} \int_0^\rho |\Delta(\rho')| \ d\rho'. \tag{6}
$$

To address the problem of the zero energy solutions of the second type for $\nu \neq 0$ we note that the symmetry transformation Eq. (4) implies that under the transformation $E \rightarrow -E$ and $\nu \rightarrow -\nu$ the functions $\hat{U}$ and $V$ in Eq. (3) change according to $V_2 \rightarrow U_1$ and $V_1 \rightarrow -U_2$. Therefore, for $E = 0$, the functions with opposite momenta coupled by Eq. (1) obey $V_{2,\nu} = U_{1,-\nu}$, $V_{1,\nu} = -U_{2,-\nu}$. Equations for $U_1$ and $U_2$ take the form:

$$
\left( \frac{d}{d\rho} + \frac{N_+}{\rho} - \frac{eA_\phi}{\hbar c} \right) U_{1,\nu} + \frac{|\Delta|}{\hbar v_F} U_{1,-\nu} = \frac{\mu}{\hbar v_F} U_{2,\nu},
\left( \frac{d}{d\rho} + \frac{N_-}{\rho} + \frac{eA_\phi}{\hbar c} \right) U_{2,\nu} + \frac{|\Delta|}{\hbar v_F} U_{2,-\nu} = -\frac{\mu}{\hbar v_F} U_{1,\nu}. \tag{7}
$$

Here $N_\pm = \nu + (n \pm 1)/2$ and $A_\phi(\rho)$ is the $\phi$-component of the vector potential. We multiply the first equation with $U_{2,\nu}$ and the second one with $U_{1,\nu}$. Next we do the same for the opposite sign of $\nu$ and then add all the resulting equations together. We thus find:

$$
\mu \int_0^\infty \left( U_{2,\nu}^2 - U_{2,-\nu}^2 - U_{1,\nu}^2 + U_{1,-\nu}^2 \right) \rho \ d\rho = \hbar v_F \int_0^\infty \frac{d}{d\rho} \left[ \rho (U_{1,\nu} U_{2,\nu} - U_{1,-\nu} U_{2,-\nu}) \right] \ d\rho. \tag{9}
$$

For a solution regular at the origin and decaying at $\rho \rightarrow \infty$ the r.h.s. of Eq. (9) is zero and, thus, the l.h.s. of this equation also should vanish. One can see that for $\nu \neq 0$ and $\mu \neq 0$, the l.h.s. is nonzero in general. Therefore, zero-energy levels do not generally exist.

To illustrate this we analyze the case of small $\mu$ using the perturbation theory. For $\mu = 0$ the equation for $U_1$ decouples from that for $U_2$. As follows from \cite{8} for positive circulation $n \geq 1$, the functions $U_{1,\nu}$ are regular while $U_{2,\nu} = 0$ if $-\frac{1}{2}(n-1) \leq \nu \leq \frac{1}{2}(n-1)$. For negative
circulation, the functions $U_{2,\nu}$ are regular while $U_{1,\nu} = 0$ if $\frac{1}{2}(n + 1) \leq \nu \leq \frac{1}{2}(n + 1)$. In total there exist $|n|$ zero energy levels. For small $\mu$ we can calculate the integral in the l.h.s. of Eq. (9) using the wave functions satisfying Eqs. (7) - (9) with $\mu = 0$. Using the results of Ref. [3] for $n > 0$, one can check that $U_{2,\nu}^2 - U_{1,\nu}^2$ is non-zero unless $\nu = 0$. Therefore, there is no decaying solution and, thus, there is no zero energy level for small $\mu$ and $\nu \neq 0$. However, the zero-energy levels may appear occasionally for certain finite values of $\mu$, as it is shown below.

Large doping levels. Quasiclassical equations. The limit of large $\mu$ is very instructive and helps to get the complete picture of the spectrum transformation. For the analysis we follow a standard quasiclassical scheme (see [13] for details) and introduce a momentum representation: $\psi(p) = (2\pi \hbar)^{-\frac{3}{2}} \int d^2p \exp(\nu p_0/p) \psi(p)$, where $p = p(\cos \theta_p, \sin \theta_p) = p \theta_p$. Let us transform the wave functions so to choose the spin quantization axis along the direction of the momentum $p$: $\hat{u} = \tilde{S} \hat{g}_u$, $\hat{v} = \tilde{S} \hat{g}_v$, where $\tilde{S} = e^{-i \theta_p \sigma_z / 2} (\hat{\sigma}_x + \hat{\sigma}_y) / \sqrt{2}$ and $\tilde{S}^* = (\hat{\sigma}_x + \hat{\sigma}_y) e^{i \theta_p \sigma_z / 2} / \sqrt{2}$. Equations (11), (2) take the form:

\[ [v_F \hat{\sigma}_z p - \mu] \hat{g}_u + \hat{h}_A \hat{g}_u + \hat{h}_\Delta \hat{g}_v = E \hat{g}_u, \quad (10) \]
\[ [\mu - v_F \hat{\sigma}_z p] \hat{g}_v + \hat{h}_A \hat{g}_v + \hat{h}_\Delta \hat{g}_u = E \hat{g}_v, \quad (11) \]

where we define the operators $\hat{h}_A = -(v_F c / \hbar) \hat{S}^* \hat{\sigma} \hat{A} \hat{S}$, as well as $\hat{h}_\Delta = \hat{S}^* \Delta \hat{S}$ and $\hat{h}_\Delta^* = \hat{S}^* \Delta^* \hat{S}$. Here $\Lambda = \Lambda(\rho)$ and $\Delta = \Delta(\rho)$ are functions of the coordinate operator $\hat{p} = i \hbar \delta / \delta \rho$. Generally, the operators $\hat{h}_A$ and $\hat{h}_\Delta$ mix the energy bands with opposite pseudospin orientations with respect to the momentum direction. However, taking the limit of large positive chemical potential $\mu \gg \Delta_0$ and considering only the subgap spectrum one can adopt the single band approximation with a fixed pseudospin orientation: $\hat{\sigma}_z g_u = g_u = g_u(1,0)$, $\hat{\sigma}_z g_v = g_v = g_v(1,0)$. The accuracy of such approximation can be determined using the second order perturbation theory; the corresponding corrections to the energy caused by the off-diagonal pseudospin terms in $\hat{h}_A$ and $\hat{h}_\Delta$ are $\delta E_A \sim (\xi^2 / L_B^2)(\Delta_0 / k_F \xi)$, $\delta E_\Delta \sim \Delta_0 / (k_F \xi)^3$. These corrections are much smaller that the proper energy scale $\Delta_0 / k_F \xi$ for the sub-gap spectrum. One concludes therefore that the single-band approximation is sufficient for the case of large doping $\mu \gg \Delta$. The assumption of large $\mu > 0$ allows us to consider only the momenta close to $k_F$. Thus, we put $p = \hbar k + q (|q| \ll \hbar k_F)$ and perform a Fourier transformation into the $(s, \theta_p)$ representation:

\[ \tilde{g}(\rho) = \left( g_u \right) g_v = k_F^{-1} \int_{-\infty}^{+\infty} ds e^{-isq / \hbar} \tilde{g}(s, \theta_p). \quad (12) \]

We introduce vector $\tilde{g}$ and Pauli matrices $\bar{\tau}_x, \bar{\tau}_y, \bar{\tau}_z$ in electron-hole space. The angular dependence can be separated: $\tilde{g}(s, \theta_p) = \exp(i \nu \theta_p + i \bar{\tau}_z n \theta_p / 2) \bar{F}(s)$.

We start with an odd-quantum vortex and consider the low-energy levels of the first kind which belong to the anomalous energy branch crossing the Fermi level at $\nu = 0$. For angular momenta $\nu / k_F \xi \ll 1$, the solution can be found using a linear expansion of $\hat{h}_A$ and $\hat{h}_\Delta$ in terms of the angular momentum operator $\bar{\nu} = -i \bar{\partial} / \partial \theta_p$. Assuming a small homogeneous magnetic field, we obtain

\[ -i \hbar v_F \bar{\tau}_x \frac{\partial \bar{F}}{\partial s} + |\Delta(s)| \left[ s \left| \bar{\tau}_x + \frac{\nu n}{|s| k_F} \bar{\tau}_y \right| \right] = E' \bar{F}, \quad (13) \]

where $E' = E + \nu \hbar \omega_c / 2$ and $\omega_c = eH v_F / \hbar k_F$ is the cyclotron frequency; $\nu$ is an integer. Considering the last term in the above Hamiltonian as a perturbation we find

\[ E = -\nu \left[ \frac{n}{1k_F} \int_0^\infty \frac{\Delta(s)}{s} e^{-2K(s)} ds + \frac{\hbar \omega_0}{2} \right], \quad (14) \]

where $I = \int_0^\infty \zeta^2(s) ds$ while $K(s)$ and $\zeta(s)$ are defined from Eq. (9). Eq. (14) corresponds to the CdGM result [1] with an account of a magnetic field in the spirit of Ref. [17]. Since $\nu$ can now take zero value, the spectrum has a zero energy mode.

The levels of both first and second kind for any vorticity $n$ can be easily described in the limit of large $\nu$. Here we can replace the angular momentum operator by a classical continuous variable. Instead of Eq. (13) we get from Eqs. (10), (11) the Andreev equations along the rectilinear quasiclassical trajectories:

\[ -i \hbar v_F \bar{\tau}_x \frac{\partial \bar{F}}{\partial s} - \frac{\hbar \omega_0 k_F b}{2} + \bar{D}(\rho) - E \bar{F}(s, \theta_p) = 0, \quad (15) \]

where $\bar{D} = \bar{\tau}_x \text{Re} \Delta(\rho) - \bar{\tau}_y \text{Im} \Delta(\rho)$, $b = -\nu / k_F$ is the trajectory impact parameter, the position vector on the trajectory is $\rho = (s \cos \theta_p - b \sin \theta_p, s \sin \theta_p + b \cos \theta_p)$. Eq. (15) describes the quasiparticle states in an arbitrary gap profile. For the particular case of a multi-quantum vortex the gap operator in Eq. (15) takes the form: $\Delta(\rho) = |\Delta(\sqrt{s^2 + b^2})|[s + \text{ib}] / \sqrt{s^2 + b^2} \equiv e^{i \alpha(\rho)}$. For $|E| < \Delta_0$ the solution of corresponding Andreev equations is known to give $|n|$ anomalous energy branches $E^{(n)}(b)$ crossing zero energy as functions of a continuous parameter $b$ (see [11, 13] and references therein).

The total wave function should be single-valued. The appropriate Bohr–Sommerfeld quantization rule for the angular momentum reads: $k_F \int_0^{2\pi} b(\theta_p) d\theta_p = 2\pi m + \pi (n - 1)$, where $m$ is an integer. The last term accounts for all the phase factors that appear in $\bar{S}$ and $\bar{g}$. As a result, the angular momentum $\nu$ should be integer or half-integer for odd and even vorticity, respectively. This agrees with the conditions imposed by Eq. (3). The expression for the anomalous branches at low energies take the form $E^{(n)}(b) \sim \Delta_0 (b - b_i) / \xi$, where $-\xi \lesssim b_i \lesssim \xi$. For the branches with $b_i \neq 0$ the change in the Fermi momentum is accompanied by the flow of the eigenvalues through the Fermi level: the energy levels cross it by pairs at discrete values $k_F = |\nu b_i|$.
FIG. 1: The subgap spectrum vs the angular momentum $\nu$ for a singly (a) and doubly (b) quantized vortex in superconducting graphene with different doping levels: $\mu/\Delta_0 = 1$ (circles) and $\mu/\Delta_0 = 10$ (squares). The quasiclassical CdGM solutions of Eq. (15) are shown by solid lines (dashed parts of lines are guides for eye). We choose here $R/\xi = 100$.

Discussion. We can summarize that in an s-wave superconducting graphene the subgap spectrum in a vortex core has a set of energy branches which cross Fermi level as functions of $\nu$ treated as a continuous parameter. The number of the branches is determined by the vortex winding number $n$. Whether the crossing points of these branches with zero belong to the energy states or not depends on the parity of the vortex and on the doping level $\mu$. For a degenerate case $\mu = 0$ there are $|n|$ zero energy levels, i.e., all crossing points belong to the spectrum. If the doping level $\mu$ is increased, the spectrum transformation depends strongly on the parity of the winding number. (i) For an odd winding number one of the zero energy modes (type I mode), namely that corresponding to $\nu = 0$, survives with an increase in the doping level. The other $|n| - 1$ (i.e., even number of) levels split off zero with increasing $\mu$ (type II modes). (ii) For even-quantum vortex, all $|n|$ levels belong to the second type and split off zero. In general, the zero energy modes of the second type can appear or disappear with the change in the Fermi momentum and may exist only in pairs.

We are thankful to G.E. Volovik for many stimulating discussions. This work was supported, in part, by Russian Foundation for Basic Research, by the “Dynasty” Foundation and by the Academy of Finland (ASM).

[1] K. S. Novoselov et al., Nature 438, 197 (2005).
[2] B. Uchoa and A.H. Castro Neto, Phys. Rev. Lett., 98, 146801 (2007).
[3] A. M. Black-Schaffer and S. Doniach, Phys. Rev. B, 75, 134512 (2007).
[4] C. Honerkamp, Phys. Rev. Lett. 100, 146404 (2008).
[5] N. B. Kopnin and E. B. Sonin, Phys. Rev. Lett. 100, 246808 (2008).
[6] C. W. J. Beenakker, Phys. Rev. Lett. 97, 067007 (2006); Rev. Mod. Phys. 80, 1337, (2008).
[7] M. Titov, A. Ossipov, and C. W. J. Beenakker, Phys. Rev. B 75, 045417 (2007).
[8] R. Jackiw and P. Rossi, Nucl. Phys. B 190, 681 (1981).
[9] P. Ghaemi and F. Wilczek, cond-mat/0709.2626 (2007).
[10] B. Seradjeh, Nucl. Phys. B 805, 182 (2008).
[11] B. Seradjeh, H. Weber, and M. Franz, cond-mat/0806.0649 (2008).
[12] D. L. Bergman and K. L. Hur, cond-mat/0806.0379 (2008)
[13] G. E. Volovik, Pis’ma Zh. Eksp. Teor. Fiz. 57, 233 (1993) [JETP Lett. 57, 244 (1993)].
[14] C. Caroli, P. G. de Gennes, J. Matricon, Phys. Lett. 9, 307 (1964).
[15] B. Uchoa, G.G. Cabrera, and A.H. Castro Neto, Phys. Rev. B, 71, 184509 (2005).
[16] A.H.Castro Neto, F. Guinea, N.M. Peres, K.S. Novoselov, and A.K. Geim, cond-mat/0709.1163v2 (2007).
[17] Brun E. Hansen, Phys. Lett. A 27, 576 (1968).
[18] A. S. Mel’nikov, D. A. Ryzhov, and M. A. Silaev, Phys. Rev. B 78, 064513 (2008).