Analytic heating rate of neutron star merger ejecta derived from Fermi’s theory of beta decay

Kenta Hotokezaka\textsuperscript{1,2},* Re’em Sari\textsuperscript{1}, and Tsvi Piran\textsuperscript{1}
\textsuperscript{1}Racah Institute of Physics, The Hebrew University of Jerusalem, Jerusalem, 91904, Israel
\textsuperscript{2}Center for Computational Astrophysics, 162 5th Ave, New York, NY, 10010, USA

ABSTRACT
Macronovae (kilonovae) that arise in binary neutron star mergers are powered by radioactive beta decay of hundreds of $r$-process nuclides. We derive, using Fermi’s theory of beta decay, an analytic estimate of the nuclear heating rate. We show that the heating rate evolves as a power law ranging between $t^{-6/5}$ to $t^{-4/3}$. The overall magnitude of the heating rate is determined by the mean values of nuclear quantities, e.g., the nuclear matrix elements of beta decay. These values are specified by using nuclear experimental data. We discuss the role of higher order beta transitions and the robustness of the power law. The robust and simple form of the heating rate suggests that observations of the late-time bolometric light curve $\propto t^{-4/3}$ would be a direct evidence of a $r$-process driven macronova. Such observations could also enable us to estimate the total amount of $r$-process nuclei produced in the merger.

Key words: stars:neutron–gamma-ray burst:general

1 INTRODUCTION

Li & Paczyński (1998) proposed that neutron star mergers will be accompanied by macronovae (kilonovae), which are optical-infrared transients powered by radioactive decay of the merger’s debris. These macronovae are among the most promising electromagnetic counterparts to gravitational-wave merger events (e.g. Metzger & Berger 2014; Nissanke et al. 2013). Recently, macronova candidates have been discovered in the afterglows of several short gamma-ray bursts (Tanvir et al. 2013; Berger et al. 2013; Yang et al. 2013; Jin et al. 2016; Jin et al. 2016) re-analyzed the afterglow light curves of historical nearby short gamma-ray bursts and suggested that macronovae are ubiquitous in short GRBs’ afterglows.

The radioactive heat generated by $r$-process nuclei play an essential role in powering macronovae. Due to strong adiabatic cooling the ejecta’s initial internal energy is practically negligible at the time that the ejecta become optically thin, i.e., when $\tau \approx c/v$, where $\tau$ is the optical depth and $v$ is the velocity of the ejecta. As we discuss later, with typical parameters the peak emission time is around a few days and hence this is the important timescale to focus on. Detailed computations using nuclear database and numerical simulations have been widely used to obtain the radioactive heating rates (Freiburghaus et al. 1999; Metzger et al. 2010; Goriely et al. 2011; Roberts et al. 2011; Korobkin et al. 2012; Wanajo et al. 2014; Lippuner & Roberts 2015; Hotokezaka et al. 2016; Barnes et al. 2016).

Another approach to calculate the heating rate is to consider the $r$-process material as a statistical assembly of radioactive nuclei. Incorporating Fermi’s theory of beta decay with such an approach provides a clear physical understanding of the nuclear heating rate (see, e.g., Way & Wigner 1948 for a discussion on the energy generation by fission products). We follow this approach and use the Fermi theory to estimate the heating rate in neutron star mergers’ ejecta. Colgate & White (1962) considered this approach to estimate the radioactive luminosity of supernovae. But they used only the relativistic regime of Fermi’s theory, which is not relevant on the macronova peak timescale as we discuss later. Furthermore, they assumed that each element decays to a stable one rather than following a decay chain, which we consider in this paper.

In this paper, we analytically derive the nuclear heating rate of macronovae based on Fermi’s theory. In §2 we begin with a brief summary of the basic concepts and the outcome of this work. In §3 we introduce the key ingredients of the theory needed to calculate the heating rate. We derive the heating rate of the beta decay chains of allowed transitions in §4. We discuss, in §5 the role of forbidden transitions and other effects that we ignore and we estimate their possible effect. We summarize the results and discuss the implication to macronova studies in §6.

* E-mail: khotokezaka@simonsfoundation.org
2 A SHORT SUMMARY OF BETA-DECAY HEATING RATE

Radioactive nuclei that are far from the stability valley are produced in $r$-process nucleosynthesis. These nuclei undergo beta decay without changing their mass number. A series of beta decays in each mass number is considered as a decay chain. Because the mean lives of radioactive nuclides typically become longer when approaching the stability valley, the nuclei in a decay chain at given time $t$ stay at some specific nuclide with a mean life $\tau \sim t$. This means that the number of decaying nuclei in a logarithmic time interval is constant, i.e., the decay rate is $\sim N/t$, where $N$ is the total number of nuclei in the chain. Then the beta decay heating rate per nucleus is given by $\dot{q} \sim E(t)/t$, where $E(t)$ is the disintegration energy of the beta decay as a function of the mean life. As we will see later, two important concepts in beta decay theory enable us to determine $E(t)$. First, there are four physical constants in the problem, the Fermi’s constant $G_F$, the electron mass $m_e$, the speed of light $c$, and the Planck constant $\hbar$. The fundamental timescale of beta decay $t_F \approx 9 \cdot 10^{-13}$ s can be obtained from these physical constants. Second, there is a well known relation between the disintegration energy and mean life as $\tau \propto E^{-5}$. Therefore, the heating rate per nucleus can be roughly estimated as

$$\dot{q}(t) \sim \frac{m_e c^2}{t_F} \left( \frac{t}{t_F} \right)^{-1.2}. \tag{1}$$

This gives a correct order of magnitude and a reasonable estimate of the time dependence of the beta decay heating rate of $r$-process material. In the following we refine these ideas.

3 BASICS OF FERMI’S THEORY OF BETA DECAY

The nature of beta decay was successfully described by Fermi [1934]. Here we briefly describe key ingredients of Fermi’s theory needed for obtaining the macronova heating rate. In a beta decay one of the neutrons in a nucleus disintegrated to a proton, an electron, and an anti-neutrino. Using Fermi’s Golden rule, the beta-disintegration probability of a beta-unstable nucleus per unit time in a unit momentum interval of the electron is written as

$$\frac{dw}{dp_e} \rho_e = \frac{2\pi}{\hbar} |H'_{fi}|^2 \rho(E_e), \tag{2}$$

where $p_e$ and $E_e$ are momentum and kinetic energy of the electron, $H'_{fi}$ is the matrix element of the interaction Hamiltonian responsible for the beta disintegration, and $\rho(E_e)$ is the number density of final states of light particles between $E_e$ and $E_e + dE_e$. The number of final states is assumed to be proportional to the volume of the accessible phase space of the light particles:

$$\rho(E_e) dE_e = \frac{(4\pi)^2 V^2}{(2\pi\hbar)^6} p_e^2 dp_e p_e^2 dp_e \approx \frac{(4\pi)^2 V^2}{(2\pi\hbar)^6} p_e^2 dp_e (E_0 - E_e)^2 dE_e, \tag{3}$$

where $E_0$ is the total disintegration energy, $p_e$ is momentum of the neutrino, and energy conservation $E_0 \approx E_e + E_\nu$ has been used. We use the fact that the neutrino mass is sufficiently small compared to $E_0$ and assume that there is no angular correlation between the electron and neutrino. Here we imagine that the whole system is enclosed in a large box with a volume $V$. Hereafter we change the notation as $p_e \rightarrow p$ and $E_e \rightarrow E$.

In Fermi’s theory, the four particles interact at a single point with a coupling constant $G_F$ so that the matrix element is written as

$$H'_{fi} = G_F \int \psi_i^\dagger \mathcal{O} \psi_e \psi_e^\dagger \mathcal{O} \psi_e \psi_e dV, \tag{4}$$

where $\psi_e$ is the wave function of each particle involved in the beta disintegration, $\mathcal{O}$ and $\mathcal{O}$ are operators acting on the light particle’s spin, nucleon’s spin and isospin (see, e.g., Feynman & Gell-Mann [1958] for a discussions on beta interaction). The wave function of the light particles can be evaluated at $r \sim 0$ because their de Broglie wavelengths are much larger than the nuclear size. When the light particles do not carry off orbital angular momentum with respect to the central nucleus, the wave function of each light particle at $r = 0$ is just a normalization factor of $V^{-1/2}$ with a Coulomb correction for the electron’s wave function. Thus the square of the matrix element can be written as

$$|H'_{fi}|^2 = \frac{G_F^2}{V^{1/2}} F(Z, E) |\mathcal{M}_N|^2, \tag{5}$$

where $F(Z, E)$ is the Coulomb correction factor, $Z$ is the proton number of the daughter nucleus, and $\mathcal{M}_N$ is the nuclear matrix element. The transitions described here are allowed transition. More specifically, allowed transitions are transitions which satisfy both conditions that the light particles don’t carry off orbital angular momentum and the parity of the nucleus does not change via its disintegration.

Otherwise the transition is a forbidden transition. Because the population of allowed transitions is larger and because of their simplicity, we focus on allowed transitions in this and the next sections. We will discuss the role of forbidden transitions in §4.

Integrating Eq. (2) over the accessible phase space, the mean-life of a beta-unstable nuclide with the disintegration energy of $E_0$ is obtained as

$$\frac{1}{\tau} = \frac{|\mathcal{M}_N|^2}{t_F} \int_0^{E_0} dp F(Z, E) p^2 (E - E_0)^2, \tag{6}$$

where the variables in the integral are in units of $m_e$ and $c$ and $t_F$ is the fundamental timescale of beta decay:

$$t_F \equiv \frac{2\pi^3}{G_F^2 m_e^2 c^6} \approx 8610 \text{ s}.$$  

Note that, although this fundamental timescale is a characteristic timescale of allowed beta decay, the lifetime of beta unstable nuclides spreads over many orders of magnitude because of the phase space factor of Eq. (3).

The Coulomb correction factor in the matrix element is obtained by evaluating the electron’s wave function at the

\[ We\ employ\ Konopinski’s\ classification\ of\ beta\ decay. \]  

\[ Konopinski\ [1966]. \]
nuclear radius $r_n$ [Fermi 1934]:

$$F(Z, E) \approx \frac{|\psi_n(r_n)|^2}{|\psi_e(r_n)|^2},$$

$$= \frac{2(1 + s)}{|(2s)^2|} (2s^2 - s^2 - i\eta)((s - 1 + i\eta)|^2,$$

where $\eta = Zq^2/\hbar e$, $v$ is the velocity of the electron, $\rho = r_n/(h/m_c)$, $s = 1 - (Z\alpha)^2)/2$. $q$ is the electron charge, and $\alpha \approx 1/137$ is the fine-structure constant. For $E > 1$, the Coulomb correction factor slowly increases with $E$ as $F(Z, E) \sim E^{2s - 2}$.

A simple form of the Coulomb correction factor is obtained in the non-relativistic limit of Eq. (7), $\eta \gg 1$ and $(Z\alpha)^2 \rightarrow 0$:

$$F_N(Z, E) = \frac{2\pi\eta}{1 - \exp(-2\pi\eta)}.$$

The Coulomb correction factor is unity for $\eta \ll 1$ and approaches to $2\pi\eta$ for $\eta \gg 1$. This enhances the transition probability at lower energies. At these energies the electron is pulled by the nucleus due to the Coulomb force and the amplitude of the electron’s wave function is larger near the nucleus. As a result, the lifetime of beta unstable nuclei becomes shorter than the one estimated without the Coulomb correction and the dependence of the lifetime on $E_0$ is weakened. Note that one can also obtain an identical form to (8) by solving the Schrödinger equation and evaluating the electron’s wave function at $r = 0$.

As the integral in Eq. (8) is easily calculated for given $E_0$ and $Z$, comparative half-lives $f_{1/2}$ are often used for comparison with the experimental data:

$$f_{1/2} = \frac{\ln 2}{|M_N|^2} t_F.$$

Although $M_N$ of each beta transition cannot be calculated within Fermi’s theory, $M_N$ can be determined from the measurements of the lifetime and the electron’s spectrum. It is sufficient for our purpose to know the statistical distribution of this quantity. For allowed transitions, the distribution of this quantity. For allowed transitions, the distribution of $f_{1/2}$ is known to have a peak around $10^5$ s corresponding to $|M_N|^2 \sim 0.05$ (e.g. Blatt & Weisskopf 1958), which we take as a reference value in this paper.\footnote{For neutron and mirror nuclides such as $^3$H, the comparative half-lives are $\sim 10^8$ s corresponding to $|M_N|^2 \sim 1$. Such transitions are called as superallowed transitions. These transitions are, however, absent in $r$-process material.}

One can show that $f$ attains simple forms in the following three regimes:

$$f(Z, E_0) = \begin{cases} \frac{1}{16}E_0^2 & \text{(relativistic : } E_0 \geq 1), \\ \frac{1}{16}\frac{E_0^2}{N} & \text{(non relativistic : } E_c < E_0 < 1), \\ 2\frac{Z\alpha}{E_0} & \text{(non relativistic Coulomb : } E_0 < \min(E_i, 1)), \end{cases}$$

where $E_c = (2\pi Z\alpha)^2/2$. The non-relativistic regime exists only for $Z \lesssim 30$, and thus, there is no such a regime in $r$-process material. In previous work [Colgate & White 1966] applied only the relativistic regime $\tau \propto E^{-3}$. However, as we will see later, the mean lives of the nuclei are rather proportional to $E^{-4}$ or $E^{-3}$ on the relevant timescale of macronovae, i.e., a few days.

In the context of macronovae, we are interested in the relation between the lifetime and the mean electron’s energy since the neutrinos don’t contribute to the heat deposition in the merger ejecta. The fraction of energy of the electrons to the total energy is:

$$\epsilon_e = \frac{\langle E_e \rangle}{E_0},$$

$$= \frac{1}{E_0} \int_0^{E_0} F(Z, p)p^2 E_0(e - E)^2 dp.$$\footnote{Figure 1. A decay chain normalized by the total number of nuclei in the chain. The dashed line depicts $e^{-1}/t$, where $e$ is Euler number.}

In the three regimes discussed earlier, $\epsilon_e$ satisfies:

$$\epsilon_e = \begin{cases} 1/2 & \text{(relativistic : } E_0 > 1), \\ \epsilon_0 < E_0 < 1, \\ \epsilon_0 < \min(E_i, 1)), \end{cases}$$

where we assumed $E_c < 1$.

4 THE HEATING RATE: THE IDEAL-CHAINS APPROXIMATION

Neutron-rich nuclei produced via the $r$-process undergo beta decay towards the beta-stable valley without changing their mass number. A series of beta decays of nuclei in each mass number can be considered as a decay chain. Here we consider ideal-chains of radioactive nuclei with a series of mean lives ($\tau_1 < \tau_2 < \tau_3 < \ldots$), in which each chain conserves the total number of nuclei throughout the decay process and sufficiently many chains exist. Within this approximation, the number of decaying nuclei in a logarithmic time interval is constant and the beta decays at a given time $t$ are dominated by nuclides with mean-lives of $\tau \sim t$ (see Fig. 1). This is, of course, valid for $t > \tau_1$, where $\tau_1$ is the mean life of the first nuclide in a decay chain. The heating rate per unit mass is then

$$\dot{Q}(t) = -\sum_i \frac{E_{e,i}}{(A)m_u} dt \approx \frac{e^{-1} \langle E_e(t) \rangle}{(A)m_u} t.$$

where $\langle A \rangle$ is the mean mass number of the $r$-process material, and $m_u$ is the atomic mass unit. Note that $e$ is the Euler number, which arises from the fact that the decay rate
of each nuclide is proportional to $e^{-t/\tau}$. One can obtain $\dot{Q}$ by using Eq. (9) and (11).

In the relativistic and non-relativistic Coulomb regimes, we can derive simple explicit forms of Eq. (13). As the lifetime of beta-unstable nuclides monotonically increases with decreasing $E_0$, the relativistic regime is valid at early times and the non-relativistic Coulomb regime is valid at late times. More specifically, the relativistic regime is valid until $t_R \approx 10^5 \text{s}$ ($0.05/|M_N|^2$) and the non-relativistic Coulomb regime is valid after $t_{NC} \approx 10^6 \text{s}$ ($0.05/|M_N|^2$). Using Eqs. (10) and (12), we obtain the heating rate in these regimes:

$$
\dot{Q}(t) \approx \begin{cases} 
1.2 \cdot 10^{10} \frac{Z}{A} \frac{t_R}{t_{day}} & (t \lesssim t_R), \\
0.3 \cdot 10^{10} \frac{Z}{A} \frac{t_{day}}{t_{R}} & (t \gtrsim t_{NC}),
\end{cases}
$$

where $t_{day}$ is time in units of a day, $\langle A \rangle_{200}$ is the mean mass number normalized by 200, and $\langle Z \rangle_{70}$ is the mean proton number normalized by 70. Note that the overall magnitude of the heating rate is determined by the mean values of the nuclear quantities, $A$, $Z$, and $M_N$. These values should be constant within an order of magnitude, and thus, the magnitude of the heating rate does not depend significantly on the details of the abundance pattern of the r-process nuclei. Furthermore, we emphasize that the formula of Eq. (13) is independent of the distribution of the nuclear decay energy.

Figure 2 depicts the heating rate obtained from Eq. (13) and the one derived using a nuclear database [Hotokezaka et al. 2016; see also similar heating rates in Metzger et al. 2010; Goriely et al. 2011; Roberts et al. 2013; Korobkin et al. 2014; Wanajo et al. 2014; Lipppuner & Roberts 2013]. We find that the heating rate based on the simple analytic formula reproduces the one based on the database remarkably well. In order to see more details, the right panel of Fig. 2 shows the heating rates normalized to the values obtained for the relativistic regime (Eq. (14)). The normalized analytic heating rate (blue solid line) is flat at early times and it approaches the non-relativistic Coulomb regime (magenta dotted line) at late times.

It is worthy noting that the formula with the non-relativistic Coulomb limit reproduces the full heating rate after $10^9 \text{s}$, even though it should be valid only after $\sim 10^9 \text{s}$. This can be understood as follows. The mean life is approximately proportional to $E^{-1}_0$ between the relativistic and the non-relativistic regimes, and thus, the energy generation rate evolves as $E_0/t \propto t^{-3\frac{1}{2}}$. In addition, in this stage, $\epsilon_\text{e}$ changes from 1/2 to 1/4, which approximately corresponds to $\epsilon_\text{e} \propto t^{-\frac{3}{4}}$. As a result, the electron heating rate is $\propto t^{-1.35}$, which is quite similar to the one in the non-relativistic Coulomb regime.

Note that Colgate & White [1968] and Metzger et al. [2010] assume that a nucleus that undergoes a radioactive decay reaches the valley of stability in a single step. In this case the total number of radioactive nuclei decreases with time. This assumption is valid if the radioactive nuclei are distributed just next to the stable nuclei, i.e., at late times. Under such an assumption, the resulting heating rate declines more steeply as $\propto t^{-1.4}$ in the relativistic regime. As we will discuss in the next section, the actual situation is in between these two assumptions.

## 5 Deviation from Our Assumptions

The analytic formula derived in the previous section reproduces remarkably well the result based on the nuclear database. However, there are two important effects that have not been taken into account. Here we discuss the role of these effects.

### 5.1 The role of forbidden transitions

Higher orbital-angular momentum transitions (unique forbidden): The light particles’ wave function in the matrix element Eq. (4) can be expanded in a series of spherical harmonics, of which the $l$th term is proportional to $(Pr/h)^l$, where $P$ is the total momentum of the light particles. The $l$th transition corresponds to the transition in which the light
particles carry off orbital angular momentum of \( \hbar \). This expansion converges rapidly on the energy scale of beta decay on the length scale of nucleus \( r_n \sim f m \). As a result, the \( \hbar \)th transition probability is suppressed by a factor of \((Pr_{\hbar}/\hbar)^2 \lesssim (0.1)^2\). For first unique forbidden transitions, an additional shape factor \( 2(p_{\hbar}\pm p_{\hbar}') \) should be multiplied in the electron spectrum of Eq. \((4)\). This shape factor results in \( \tau \propto E_0^{-5} \) in the non-relativistic Coulomb regime, which can be seen in Fig. \(3\) (a blue dotted line). Even though the number of beta unstable nuclides that disintegrate mainly via unique forbidden transitions is small, they may play a role by increasing the heating rate after a few hours.

**Relativistic transitions (parity forbidden):** Some interactions mix the large and small components of Dirac spinor of the nucleon in the matrix element Eq. \((1)\). A transition due to such an interaction is a parity forbidden transition as it changes the nucleus’ parity without removing the orbital angular momentum (see magenta crosses in Fig. \(3\)). The corresponding amplitude is suppressed by a factor of \( \mathcal{O}(v_{\hbar}/c) \) or \( \mathcal{O}(Za) \) compared to allowed transitions. Here the velocity of nucleons \( v_{\hbar} \) is typically \( v_{\hbar} \approx 0.1c \). As a result, the probability of these transitions is lower than the allowed ones by a factor of \( \mathcal{O}(v_{\hbar}/c^2) \) or \( \mathcal{O}(Z^2a^2) \). The theoretical curves of the first order parity forbidden transitions are shown as the dashed and the dot-dashed lines in Fig. \(3\). Here we use a suppression factor of \( (v_{\hbar}/c^2) \approx 0.01 \) and \( (Za)^2 \approx 0.25 \), respectively. The first order parity forbidden transitions have an electron spectral shape that is similar to the allowed transitions. As one can see in this figure, the curves of these transitions have the same shapes to the allowed one with a constant shift in the half-life. The existence of these transitions in addition to the allowed ones increases the heating rate. The lifetimes of second order parity forbidden transitions, in which angular momentum of \( \hbar \) is carried off by the light particles, are too long to be relevant for the macronova heating rates (see green points in Fig. \(3\)).

5.2 Deviation from the ideal-chains approximation

Beta decay chains terminate when they reach stable nuclides. Once this happens these terminated chains don’t contribute to the heating rate any more. The overall lifetime of a chain, \( T_{1/2} \), can be estimated from the sum of the half-lives of nuclides in the chain. The cumulative distribution of the chains for \( A = 90 \) to \( 210 \) as a function of the chains’ lifetime is shown in Fig. \(4\). The number of the chains begins to decrease slowly as \( \propto T_{1/2}^{-0.1} \) at \( \sim 100s \). After about 10 days it decreases slightly faster as \( \propto T_{1/2}^{-0.2} \). This steep decline at late times due to the termination of the decay chains is consistent with the assumption made by Colgate & White (1966) and Metzger et al. (2010).

In summary, the contribution of forbidden transitions to the heating rate slightly increases the heat generation at late times. On the contrary, at the same time, the termination of the beta decay chains slightly decreases it. As a result, the combined effects on the heating rate somehow cancel out. Note that these corrections to the heating rate depend on the actual abundance distribution of the chains.

Figure 4. The cumulative distribution of the lifetime of decay chains. \( T_{1/2} \) is the sum of the half-lives of beta unstable nuclides of a decay chain.

6 CONCLUSION AND DISCUSSION

We derive an analytic form of the macronova heating rate by considering statistical assembly of radioactive processes nuclides and Fermi's theory of beta decay. The resulting analytic formula reproduces the heating rate derived from the nuclear database remarkably well. Within the assumption that the ideal decay chains of allowed beta transitions generate radioactive heats, we show that the heating rate evolves as \( \propto t^{-6/5} \) at early times and \( \propto t^{-4/3} \) at late times. The overall magnitude of heating rate is determined by the mean value of the nuclear matrix elements, mass and atomic number of beta unstable nuclides involved in the decay chains.

We discuss the role of forbidden transitions and the deviation from the ideal-chains approximation. The former
slightly increases the heating rate at late times and the latter slightly decreases it. As a result, these corrections somehow cancel out with each other.

The robust and simple form of the heating rate suggests that observations of the late-time bolometric macronova light curve can provide and observational evidence that is driven by a radioactive decay of r-process material. Furthermore, determination of the bolometric luminosity will enable us to estimate the total amount of r-process nuclei produced in a merger. Using the non-relativistic Coulomb regime of Eq. (14), the late-time bolometric light curve is written as

$$L(t) \approx 10^{40} \text{erg/s} \times \frac{M}{M_\odot} \text{erg/s} ,$$

(15)

where $M_{-2} = 0.01M_\odot$ is the ejecta mass. This expression is valid after the peak time given by

$$t_{\text{peak}} \approx 5 \frac{\kappa_{10}^2 M_{-2}^{1/2}}{v_0} \text{days} ,$$

(16)

where $v_0 = 0.2c$ is the ejecta velocity and $\kappa_{10} = 10 \text{cm}^2/\text{g}$ is the bound-bound opacity of r-process elements (Kasen et al. 2012; Tanaka & Hotokezaka 2013).

Confirming this behavior by observation may be difficult because the light curve may have large fluctuation due to the temperature and density dependent opacity (Barnes & Kasen 2013; Tanaka & Hotokezaka 2013). However, as suggested in the context of supernovae (Katz et al. 2012; Nakar et al. 2016), the time-weighted integral of the bolometric luminosity after the peak provides a more robust estimate the radioactive power in the ejecta. In this method, the time-weighted integral of the bolometric luminosity should behave as $\propto M \cdot t^{2/3}$.

The bolometric luminosity that we derived here is the total radioactive power emitted in the electrons. At late times, this power is not necessarily thermalized in the ejecta (see Barnes et al. 2013 for a detailed study). The inefficiency of the electron thermalization may reduce the bolometric luminosity by a factor of 2 on the macronova timescale. At the same time, we have ignored, additional heating due to $\gamma$-rays, $\alpha$-particles and fission fragments. The role of these factors in the macronova heating is still under debate. For instance, it has been suggested that the heat generation by spontaneous fission and $\alpha$-decay can be comparable to or even larger than the beta decay heating (see Hotokezaka et al. 2016; Barnes et al. 2016). We explore the role of these effects in the estimating the total amount of r-process material ejected in a macronova from the integrated bolometric light curve in a separate work.

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