Abstract

The physical meaning of the Levi-Civita spacetime, for some "critical" values of the parameter $\sigma$, is discussed in the light of gedanken experiments performed with gyroscopes circumventing the axis of symmetry. The fact that $\sigma = 1/2$ corresponds to flat space described from the point of view of an accelerated frame of reference, led us to incorporate the C-metric into discussion. The interpretation of $\phi$ as an angle coordinate for any value of $\sigma$, appears to be at the origin of difficulties.
1 Introduction

To provide physical meaning to solutions of Einstein equations, is an endeavour whose relevance deserves to be emphasized [1].

This is particularly true in the case of the Levi-Civita (LC) spacetime [2] which after many years and a long list of works dedicated to its discussion still presents serious challenges to its interpretation ([1], [3]-[14], and references therein).

This metric has two essential constants, usually denoted by $a$ and $\sigma$. One of them, $a$, has to do with the topology of spacetime and, more specifically, refers to the deficit angle. It may accordingly be related to the gravitational analog of Aharonov-Bohm effect [15], [16].

It is however $\sigma$, the parameter which presents the most serious obstacles to its interpretation.

Indeed, for small $\sigma$ ($\sigma \leq 1/4$), LC describes the spacetime generated by an infinite line mass, with mass $\sigma$ per unit coordinate length. When $\sigma = 0$ the spacetime is flat [1].

However, circular timelike geodesics exits only for

$$\frac{1}{4} > \sigma > 0,$$

becoming null when $\sigma = 1/4$ and being spacelike for $\sigma > 1/4$.

Furthermore, as the value of $\sigma$ increases from $1/4$ to $1/2$ the corresponding Kretschmann scalar diminishes monotonically, vanishing at $\sigma = 1/2$, and implying thereby that the space is flat also when $\sigma = 1/2$.

Still worse, if $\sigma = -1/2$ the spacetime admits an extra Killing vector which corresponds to plane symmetry [7] (also present of course in the $\sigma = 1/2$ case).

Thus, the obvious question is: What does LC represents for values of $\sigma$ outside the range $(0,1/4)$?

The absence of circular test particle orbits for $\sigma > 1/4$, and the fact that most of the known material sources for LC, [4], [5], [6], [11] require $\sigma \leq 1/4$, led to think that LC describes the field of a cylinder only if $\sigma$ ranges within the $(0,1/4)$ interval.

However, interior solutions matching to LC exist, [9], [12], [13], [17] with $\sigma > 1/4$.

Furthermore, the absence of circular test particle orbits for $\sigma > 1/4$ may simply be interpreted, as due to the fact that the centrifugal force required
to balance the gravitational attraction implies velocities of the test particle larger than 1 (speed of light) [4].

This last argument in turn, was objected in the past on the basis that Kretschmann scalar decreases as $\sigma$ increases from $1/4$ to $1/2$, suggesting thereby that the gravitational field becomes weaker [8], [11]. However, as it has been recently emphasized [12], [18], Kretschmann scalar may not be a good measure of the strength of the gravitational field. Instead, those authors suggest that the acceleration of the test particle represents more suitably the intensity of the field. Parenthetically, this acceleration increases with $\sigma$ in the interval $(1/4,1/2)$.

On the basis of the arguments above and from the study of a specific interior solution matched to LC [17], Bonnor [18] proposes to interpret LC as the spacetime generated by a cylinder whose radius increases with $\sigma$, and tends to infinity as $\sigma$ approaches $1/2$. This last fact suggests that when $\sigma = 1/2$, the cylinder becomes a plane. This interpretation of the $\sigma = 1/2$ case was already put forward by Gautreau and Hoffman in [7] (observe that theirs $\sigma$ is twice ours), though based on different considerations.

However, in our opinion, the question is not yet solved.

Indeed, the interior solution analyzed in [18] is not valid when $\sigma = 1/2$.

Therefore the vanishing of the normal curvatures of the coordinate lines on the bounding surface when $\sigma \to 1/2$, suggests but does not prove that the exterior solution with $\sigma = 1/2$ has a plane source.

The LC spacetime has no horizons. According to our present knowledge of the formation of black holes, this seems to indicate that there is an upper limit to the mass per unit length of the line sources, and this limit has to be below the critical linear mass, above which horizons are expected to be formed [8].

The anisotropic fluid [14] with $\sigma \leq 1$ matched to LC, produces an effective mass per unit length that has maximum at $\sigma = 1/2$, which might explain the inexistence of horizons. Furthermore, this fact might support the previous acceleration representation of the field intensity. It agrees with the result that the tangential speed $W$ of a test particle [13] in a circular geodesics increases with $\sigma$, attaining $W \to \infty$ for $\sigma \to 1/2$. The source studied in [14] remains cylindrical for $\sigma = 1/2$, producing a cosmic string with finite radius. However, the effective mass density by increasing up to $\sigma = 1/2$, and then decreasing for bigger values of $\sigma$, raises a disturbing situation of a cylindrical distribution mass not curving spacetime exactly at its maximum value.
On the other hand, there exists a puzzling asymmetry between the negative and the positive mass case, for the plane source.

The point is that, as mentioned before, the $\sigma = -1/2$ case possesses plane symmetry and furthermore test particles are repelled by the singularity. Therefore LC with $\sigma = -1/2$, has been interpreted as the gravitational field produced by an infinite sheet of negative mass density [7] (though there are discrepancies on this point [1]). However in this case ($\sigma = -1/2$) the space is not flat, unlike the $\sigma = 1/2$ case.

In other words, if we accept both interpretations, i.e. $\sigma = 1/2$ ($-1/2$) represents the field produced by an infinite plane with positive (negative) mass density, then we have to cope with the strange fact that the negative mass plane curves the spacetime, whereas the positive mass plane does not. This asymmetry is, intuitively, difficult to understand.

In favor of the plane interpretation for the $\sigma = 1/2$ case, point the arguments presented in [18], although as already mentioned, they are not conclusive.

Furthermore, even if we admit the arguments based on the principle of equivalence, leading to the plane interpretation of the $\sigma = 1/2$ case, there is a problem with the localization of the source itself (the plane).

Indeed, it seems reasonable to assume, according to the equivalence principle, that the physical components of curvature tensor of an homogeneous static field, vanish everywhere, except on the source (the plane), where they should be singular. However, when $\sigma = 1/2$ the space is flat everywhere (everywhere meaning the region covered by the patch of coordinates under consideration), and therefore a pertinent question is: Where is the source?

In the $\sigma = -1/2$ case, the plane interpretation is supported by the plane symmetry of the spacetime, although objections to this interpretation have been raised, on the basis that the proper distance between neighbouring paths of test particles changes with time [1]. However, see a comment on this point, below eq.(10).

Also, in this case, the physical components of the curvature tensor, and the Cartan scalars, are singular at $r = 0$, revealing the existence of a source, however they do not vanish (except in the limit $r \to \infty$ ) and therefore the pertinent question here is: Why does the arguments based on the equivalence principle, mentioned above, do not apply, if $\sigma = -1/2$ corresponds to a plane?

So, unless additional arguments are presented, we are inclined to think that either of the interpretations (or both) are wrong.
In order to delve deeper into these questions, and with the purpose of bringing forward new arguments, we propose here to analyze some gedanken experiments with a gyroscope circumventing the axis of symmetry.

The obtained expression for the total precession per revolution, $\Delta \phi$, will depend on $\sigma$. Then, analyzing the behaviour of $\Delta \phi$ as function of different physical variables we shall be able to provide additional elements for the interpretation of LC.

In relation to this, we shall consider also the C-metric (see[19]-[21] and references therein), which, as it is well known, describes, in the limit of vanishing mass parameter, the flat space as seen by an accelerated observer (as the $\sigma = 1/2$ case).

As it will be seen below, the discussion presented here does not lead to conclusive answers to the raised issues, but provides hints reinforcing some already given interpretations and, in some cases, creating doubts about formerly accepted points of view. In particular it appears that the interpretation of the coordinate $\phi$ as an angle coordinate seems to be untenable in some cases. A fact already brought out in [7].

At any rate, it is our hope that the results and arguments here presented, will stimulate further discussions on this interesting problem.

The paper is organized as follows. In the next section we describe the LC spacetime and the C-metric. In section 3 we give the expression for the total precession per revolution of a gyroscope circumventing the axis of symmetry and display figures indicating its dependence upon different variables. Finally, results are discussed in the last section.

2 Notation, conventions and the space time.

We shall first describe the LC line element, together with the notation and conventions used here. Next we shall briefly describe the C-metric.

2.1 The Levi-Civita metric.

The LC metric can be written as [2], [22]

$$ds^2 = -ar^{4\sigma} dt^2 + r^{8\sigma^2-4\sigma}(dr^2 + dz^2) + \frac{r^{2(1-2\sigma)}}{a} d\phi^2,$$ (2)
where $a$ and $\sigma$ are constants.

The coordinates are numbered

$$ x^0 = t, \quad x^1 = r, \quad x^2 = z, \quad x^3 = \phi, $$

and their range are

$$ -\infty < t < \infty, \quad 0 \leq r < \infty, \quad -\infty < z < \infty, \quad 0 \leq \phi \leq 2\pi, $$

with the hypersurface $\phi = 0$ and $\phi = 2\pi$ being identified.

As stressed in [1] neither $a$ nor $\sigma$ can be removed by coordinate transformations, and therefore they have to be considered as essential parameters of the LC metric.

As mentioned before, $a$ has to do with the topology of spacetime, giving rise to an angular deficit $\delta$ equal to

$$ \delta = 2\pi \left(1 - \frac{1}{\sqrt{a}}\right). $$

(5)

Also, as commented in the introduction, the spacetime becomes flat if $\sigma$ is 0 or 1/2.

In the first case, $\sigma = 0$, the line element (2), adopts the usual form of the Minkowski interval in cylindrical coordinates (except for the presence of $a$).

In the second case, $\sigma = 1/2$, the line element becomes

$$ ds^2 = -ar^2 dt^2 + dr^2 + dz^2 + \frac{d\phi^2}{a}, $$

(6)

this last expression corresponding to the flat spacetime described by an uniformly accelerated observer with a topological defect associated to $a$.

Indeed, putting $a = 1$ for simplificity, the transformation

$$ \bar{t} = r \sinh t, \quad \bar{r} = r \cosh t, \quad \bar{\phi} = \phi, \quad \bar{z} = z, $$

(7)

casts (6) into

$$ ds^2 = -d\bar{t}^2 + d\bar{r}^2 + d\bar{\phi}^2 + d\bar{z}^2 $$

(8)

Then, the components of the four-acceleration of a particle at rest in the frame of (6) ($r = r_0 = \text{constant}, z = \text{constant}, \phi = \text{constant}$) as measured by an observer at rest in the Minkowski frame of (8) are
\[ a^\mu = \frac{1}{r_0} (\sinh t, \cosh t, 0, 0), \]  

and therefore

\[ \alpha = \sqrt{a^\mu a_\mu} = \frac{1}{r_0} \]

Thereby indicating that such a particle is accelerated, with proper acceleration \( 1/r_0 \). It is perhaps worth noticing that due to (4) and (7), the range of the Minkowski coordinate \( \gamma \) is rather unusual. Also observe that bodies located at different points, undergo different accelerations. This implies in turn that two bodies undergoing the same proper acceleration do not maintain the same proper distance (see p.176 in [23] for details).

### 2.2 The C-metric.

This metric was discovered by Levi-Civita [24], and rediscovered since then by many authors (see a detailed account in [19]).

It may be written in the form

\[ ds^2 = A^{-2}(x + y)^{-2} \left( F^{-1} dy^2 + G^{-1} dx^2 + Gdz^2 - F d\tau^2 \right), \]

with

\[ F = -1 + y^2 - 2mA^3, \quad G = 1 - x^2 - 2mA^3, \]

where \( m \) and \( A \) are the two constant parameters of the solution.

Introducing retarded coordinates \( u \) and \( R \), defined by

\[ Au = \tau + \int^{u} F^{-1} dy, \]

\[ AR = (x + y)^{-1}, \]

the metric takes the form

\[ ds^2 = -Hdu^2 - 2dudR - 2AR^2 dudx + R^2 \left( G^{-1} dx^2 + Gdz^2 \right), \]

with

\[ H = -A^2 R^2 G (x - A^{-1} R^{-1}). \]
If $A = 0$, and $m \neq 0$, the C-metric becomes Schwarzschild. But, if $m = 0$ and $A \neq 0$, then (15) may be written, with $z = \phi$, $x = \cos \theta$ as
\[
\begin{align*}
  ds^2 &= -\left(1 - 2AR \cos \theta - A^2R^2 \sin^2 \theta \right) du^2 - 2dudR + \\
  &\quad + 2AR^2d\theta \sin \theta + R^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right), \\
\end{align*}
\]
which can be casted into the Minkowski line element by
\[
\begin{align*}
  \bar{t} &= (A^{-1} - R \cos \theta) \sinh Au + R \cosh Au, \\
  \bar{z} &= (A^{-1} - R \cos \theta) \cosh Au + R \sinh Au, \\
  \bar{x} &= R \sin \theta \cos \phi, \\
  \bar{y} &= R \sin \theta \sin \phi.
\end{align*}
\]

Now, for a particle at rest in the $(u, R, \theta, \phi)$ frame ($R = R_0 = \text{constant}$, $\theta = \theta_0 = \text{constant}$, $\phi = \text{constant}$) the components of the four-acceleration as measured by an observer at rest in the $(\bar{t}, \bar{x}, \bar{y}, \bar{z})$ frame, are
\[
\begin{align*}
  a^\mu &= \frac{A}{(1 - AR_0 \cos \theta_0)^2 - A^2R_0^2} \{(1 - AR_0 \cos \theta_0) \sinh Au + AR_0 \cosh Au, \\
  &\quad , 0, 0, (1 - AR_0 \cos \theta_0) \cosh Au + AR_0 \sinh Au \}.
\end{align*}
\]

Then, the absolute value of the four acceleration vector for such particle is
\[
\alpha = \sqrt{a^\mu a_\mu} = \frac{A}{\sqrt{1 - 2AR_0 \cos \theta_0 - A^2R_0^2 \sin^2 \theta_0}}
\]
indicating that the locus $R_0 = 0$ is accelerated with constant proper acceleration $A$. Observe that in this case the $(u, R, \theta, \phi)$ coordinates are only restricted by
\[
\bar{t} + \bar{z} > 0
\]
3 Gyroscope Precession.

3.1 Precession in the Levi-Civita metric.

Let us consider a gyroscope circumventing the symmetry axis along a circular path (not a geodesic), with angular velocity $\omega$. Then it can be shown that the total precession per revolution is given by (see [25] for details)

$$\Delta \phi = 2\pi \left(1 - \frac{n\sqrt{a} \, r^{- (1-n)^2/4}}{(a^2 - \omega^2 r^2)^{1/4}}\right),$$

with $n = 1 - 4\sigma$.

The tangential velocity of particles along circular trajectories (not necessarily geodesics) on the plane orthogonal to the symmetry axis, is given by the modulus of the four-vector (see [26], [27], [28])

$$W^\mu = \left[\left(-g_{00}\right)^{1/2}\left(dx^0 + \frac{g_{0i}}{g_{00}}dx^i\right)\right]^{-1} V^\mu,$$

with

$$V^\mu = (0, 0, 0, d\phi).$$

Then, for a particle in LC spacetime

$$W = (W^\mu W_\mu)^{1/2} = \frac{r^n}{a} \, \omega.$$ 

In terms of $W$, the expression for $\Delta \phi$ becomes

$$\Delta \phi = 2\pi \left[1 - \frac{n \, r^{- (1-n)^2/4}}{\sqrt{a} \, (1 - W^2)^{1/2}}\right].$$

3.2 Precession in the C-metric.

Next, due to the similarity of interpretation, mentioned before, between the $\sigma = 1/2$ case and the C-metric with $m = 0$, we shall also calculate the total precession per revolution of a gyroscope circumventing the axis of symmetry, in the space-time of the C-metric.
Using the Rindler-Perlick method \cite{29}, and writing the C-metric in the form \cite{20}

\[ ds^2 = -H dt^2 + \frac{dR^2}{H} - \frac{2 \sin \theta \cos \theta}{H} AR^2 dR d\theta + \]
\[ + \frac{R^2 \cos^2 \theta}{p^2(1 + 3Amp)^2} \left( 1 + \frac{A^2 R^2 \sin^2 \theta}{H} \right) d\theta^2 + R^2 \sin^2 \theta d\phi^2, \]  
\( (29) \)

with

\[ H = 1 - 2ARp - A^2 R^2(1 - p^2) - \frac{2m}{R}(1 - Arp)^3 = \]
\[ = (1 - Arp)^2 - A^2 R^2 - \frac{2m}{R}(1 - Arp)^3, \]  
\( (30) \)

and

\[ \sin^2 \theta = 1 - p^2 - 2Amp^3, \]  
\( (31) \)

one obtains,

\[ \Delta \phi = 2\pi \left\{ 1 - \left( \sin^2 \theta(1 - Arp)^2 \beta^2 R^2 + Hp^2(1 + 3Amp)^2 \right)^{1/2} \right. \]
\[ \left. \cdot \left( H - \omega^2 R^2 \sin^2 \theta \right)^{-1/2} \right\} \]  
\( (32) \)

with

\[ \beta = R - 3m(1 - Arp). \]

If \( m = A = 0 \) we recover the usual Thomas precession in a Minkowski spacetime.

If \( m = 0 \) and \( A \neq 0 \), on the \( \theta = \frac{\pi}{2} \) plane,

\[ \Delta \phi = 2\pi \left\{ 1 - \left[ 1 - R^2(A^2 + \omega^2) \right]^{-1/2} \right\} \]  
\( (33) \)

which is the Thomas precession modified by the acceleration factor \( A \); while if \( m \neq 0, A = 0 \), we recover the usual Fokker-de Sitter expression for precession of a gyroscope in the Schwarzschild metric \cite{28},

\[ \Delta \phi = 2\pi \left\{ 1 - \left( 1 - \frac{3m}{R} \right) \left( 1 - \frac{2m}{R} - \omega^2 R^2 \right)^{-1/2} \right\} \]  
\( (34) \)

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In the general case $m \neq 0$, $A \neq 0$ (on the $\theta = \frac{\pi}{2}$ plane), we have from (31) that, either $p = 0$ or $p = -1/2$. In the first case ($p = 0$) we obtain

$$\Delta \phi = 2\pi \left\{ 1 - \left[ 1 - \frac{3m}{R} \right] \left[ 1 - \frac{2m}{R} - \left( A^2 + \omega^2 \right) R^2 \right]^{-1/2} \right\}$$  \hspace{1cm} (35)

whereas in the case $p = -1/2$ , the result is

$$\Delta \phi = 2\pi \left\{ 1 + \left( \frac{3m}{R} + 2 \right)^2 + \left( \frac{R}{m} + \frac{3}{2} \right) - \frac{(R + 2m)^2}{32A^2m^3R} \right\}^{1/2} \cdot \left[ -\frac{(R + 2m)^2}{2mR} - \left( A^2 + \omega^2 \right) R^2 \right]^{-1/2}$$  \hspace{1cm} (36)

However, this last case implies $m < 0$, for otherwise $H < 0$ , what would change the signature of the metric.

Finally, the tangential velocity of the gyroscope on the circular orbit calculated from (23) for the C-metric yields

$$W = (W^\mu W_\mu)^{1/2} = H^{-1/2} \omega R \sin \theta$$  \hspace{1cm} (37)

Then replacing $\omega$ by $W$ with (37), into (35), we obtain ($\theta = \frac{\pi}{2}$)

$$\Delta \phi = 2\pi \left\{ 1 - \left[ 1 - \frac{3m}{R} \right] \cdot (1 - W^2)^{-1/2} H^{-1/2} \right\}$$  \hspace{1cm} (38)

where

$$H = 1 - A^2 R^2 - \frac{2m}{R} \quad \text{if} \quad p = 0$$

and

$$\Delta \phi = 2\pi \left\{ 1 + \left( \frac{3m}{R} + 2 \right)^2 + \left( \frac{R}{m} + \frac{3}{2} \right) - \frac{(R + 2m)^2}{32A^2m^3R} \right\}^{1/2} (1 - W^2)^{-1/2} H^{-1/2}$$  \hspace{1cm} (39)

where

$$H = -\frac{(R + 2m)^2}{2mR} - A^2 R^2 \quad \text{if} \quad p = -\frac{1}{2Am}$$

in the last case however, remember that $m$ must be negative.
If \( m = 0 \), (32) may be written (with (37)) as

\[
\Delta \phi = 2\pi \left\{ 1 - \frac{\sqrt{1 + \alpha^2 R^2 \sin^2 \theta}}{\sqrt{1 - W^2}} \right\}
\]

(40)

with

\[
\alpha = (a^\mu a_\mu)^{1/2}
\]

(41)

indicating that the precession is retrograde for any \( \alpha \) and \( \theta \).

In the next section we shall discuss about the meaning of LC in the light of the information provided by (28) and (38).

4 Discussion

Let us now analyze some figures obtained from (28) and (38).

Figure (1) exhibits the dependence of \( \Delta \phi / 2\pi \) on \( n \) for different values of \( W \) (for simplicity all figures are plotted with \( a = 1 \)).

![Figure 1: \( \Delta \phi / 2\pi \) as function of \( n \), for different values of \( W \), for LC.](image)

For \( n < 0 \) (\( \sigma > 1/4 \)) the precession is always forward (\( \Delta \phi > 0 \)) as it obvious from (28). However for \( n > 0 \) (\( \sigma < 1/4 \)) it may be retrograde (\( \Delta \phi < 0 \)) depending on \( r \) and \( W \), as indicated in figure(2), figure(3).
Figure 2: $\Delta \phi / 2\pi$ as function of $r$, for $W = 0.05$ and two different values of $n (-1, 3)$, for LC.

Figure 3: $\Delta \phi / 2\pi$ as function of $W$, for different values of $n$, and $r = 10$, for LC.
Thus the cases \( n = -1 \) (\( \sigma = 1/2 \)) and \( n = 3 \) (\( \sigma = -1/2 \)) induce very different behaviours on gyroscopes. This fact, together with the assymmetry mentioned in the Introduction, reinforces our doubts about the simultaneous interpretation of both cases (\( \sigma = -1/2, 1/2 \)) as due to infinite sheet of either positive or negative mass density.

Next, let us consider the C-metric in the \( m = 0 \) case. Figure (4) shows the behaviour of the gyroscope as function of the acceleration. Observe that the precession is retrograde, in contrast with the \( n = -1 \) case, for which \( \Delta \phi \) is always positive. This behaviour is the opposite for LC and \( n \) equal to \(-1\) (see fig. (5)), and reinforces further the difficulty of interpreting \( \phi \) (in LC with \( n = -1 \)) as the usual azimuthal angle. Still worse, in this later case, \( \Delta \phi \) always exceed \( 2\pi \) indicating that the precession is forward even in the rotating frame.

Figure 4: \( \Delta \phi/2\pi \) as function of the acceleration, for the C-metric, with \( m = 0 \) and \( \theta = \pi/2 \).

Now, since both cases (C-metric with \( m = 0 \) and LC with \( n = -1 \)) represent the same physical situation (i.e. flat space described by an uniformly accelerated observer) then we have to conclude that the meaning of \( \phi \) in LC with \( n = -1 \), is different from its usual interpretation (as an angle). This also becomes apparent from the definition of \( W \) given by (27) (the tangential velocity decreases as \( 1/r \)). Also observe that in the case of the C-metric with
$m = 0$, we recover the Thomas precession in the limit $\alpha = 0$. This however is impossible in the LC case with $n = -1$. In the same order of ideas it is worth noticing that in the case $n = 3$, the meaning of $\phi$ seems to correspond (qualitatively) to that of an azimuthal angle.

On the other hand, it is clear that in the case of a plane source we should not expect $\phi$ to behave like an angle coordinate (see also [12] on this point). Therefore, on the basis of all comments above, we are inclined to think (as in [13]) that the $\sigma = 1/2$ case corresponds to an infinite plane. The absence of singularities in the physical components of the curvature tensor, remaining unexplained, although (probably) related to the restrictions on the covering, of the coordinate system. By the same arguments it should be clear that the interpretation of the $n = 3$ case as due to a plane, seems to be questionable.

![Graph](image)

Figure 5: $\Delta \phi/2\pi$ as function of the acceleration, for $n = -1$ in LC
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