Measurement of the branching ratio for the decay $K^{\pm} \rightarrow \pi^\pm \pi^0 \pi^0$ with the KLOE detector

The KLOE Collaboration

A. Aloisio\textsuperscript{g}, F. Ambrosino\textsuperscript{g}, A. Antonelli\textsuperscript{c}, M. Antonelli\textsuperscript{c}, C. Bacci\textsuperscript{m}, M. Barva\textsuperscript{m}, G. Bencivenni\textsuperscript{c}, S. Bertolucci\textsuperscript{c}, C. Bini\textsuperscript{k}, C. Bloise\textsuperscript{c}, V. Bocci\textsuperscript{k}, F. Bossi\textsuperscript{c}, P. Branchini\textsuperscript{m}, S. A. Bulchjov\textsuperscript{f}, R. Caloi\textsuperscript{k}, P. Campana\textsuperscript{c}, G. Capon\textsuperscript{c}, T. Capussela\textsuperscript{g}, G. Carboni\textsuperscript{f}, F. Ceradini\textsuperscript{m}, F. Cervelli\textsuperscript{i}, F. Cevenini\textsuperscript{g}, G. Chieftari\textsuperscript{g}, P. Ciambrone\textsuperscript{c}, S. Conetti\textsuperscript{o}, E. De Lucia\textsuperscript{c}, A. De Santis\textsuperscript{k}, P. De Simone\textsuperscript{c}, G. De Zorzi\textsuperscript{k}, S. Dell’Agnello\textsuperscript{c}, A. Denig\textsuperscript{d}, A. Di Domenico\textsuperscript{k}, C. Di Donato\textsuperscript{g}, S. Di Falco\textsuperscript{i}, B. Di Micco\textsuperscript{m}, A. Doria\textsuperscript{g}, M. Dreucci\textsuperscript{c}, O. Erriquez\textsuperscript{a}, A. Farilla\textsuperscript{m}, G. Felici\textsuperscript{c}, A. Ferrari\textsuperscript{d}, M. L. Ferrer\textsuperscript{c}, G. Finocchiaro\textsuperscript{c}, C. Forti\textsuperscript{c}, P. Franzini\textsuperscript{k}, C. Gatti\textsuperscript{k}, P. Gauzzi\textsuperscript{k}, S. Giovannella\textsuperscript{c}, E. Gorini\textsuperscript{e}, E. Graziani\textsuperscript{m}, M. Incagli\textsuperscript{i}, W. Kluge\textsuperscript{d}, V. Kulikov\textsuperscript{f}, F. Lacava\textsuperscript{k}, G. Lanfranchi\textsuperscript{c}, J. Lee-Franzini\textsuperscript{c,n}, D. Leone\textsuperscript{d}, F. Lu\textsuperscript{c,b}, M. Martemianov\textsuperscript{c,f}, M. Martini\textsuperscript{c}, M. Matsyuk\textsuperscript{c,f}, W. Mei\textsuperscript{c}, L. Merola\textsuperscript{g}, R. Messi\textsuperscript{f}, S. Miscetti\textsuperscript{c}, M. Moulson\textsuperscript{c}, S. Müller\textsuperscript{d}, F. Murtas\textsuperscript{c}, M. Napolitano\textsuperscript{g}, F. Nguyen\textsuperscript{m}, M. Palutan\textsuperscript{c}, E. Pasqualucci\textsuperscript{k}, L. Passalacqua\textsuperscript{c}, A. Passeri\textsuperscript{m}, V. Patera\textsuperscript{j,c}, F. Perfetto\textsuperscript{g}, E. Petrolo\textsuperscript{k}, L. Pontecorvo\textsuperscript{k}, M. Primavera\textsuperscript{e,2}, P. Santangelo\textsuperscript{c}, E. Santovetti\textsuperscript{f}, G. Saracino\textsuperscript{g}, R. D. Schamberger\textsuperscript{m}, B. Sciascia\textsuperscript{c}, A. Sciuabba\textsuperscript{j,c}, F. Scuri\textsuperscript{i}, I. Sfiligoi\textsuperscript{c}, A. Sibidanov\textsuperscript{c,h}, T. Spadaro\textsuperscript{c}, E. Spiriti\textsuperscript{m}, M. Testa\textsuperscript{k}, L. Tortora\textsuperscript{m}, P. Valente\textsuperscript{k}, B. Valeriani\textsuperscript{d}, G. Venanzoni\textsuperscript{i}, S. Veneziano\textsuperscript{k}, A. Ventura\textsuperscript{e,1}, R. Versaci\textsuperscript{m}, I. Vilella\textsuperscript{g}, G. Xu\textsuperscript{c,b}

\textsuperscript{a}Dipartimento di Fisica dell’Università e Sezione INFN, Bari, Italy.
\textsuperscript{b}Permanent address: Institute of High Energy Physics of Academica Sinica, Beijing, China.
\textsuperscript{c}Laboratori Nazionali di Frascati dell’INFN, Frascati, Italy.
Abstract

We have measured the absolute branching ratio $\text{BR}(K^\pm \rightarrow \pi^\pm \pi^0 \pi^0)$ with the KLOE detector at the DAΦNE $e^+e^-$ collider. We collected $\sim 3.3 \times 10^7$ tagged charged kaons, from the reaction $e^+e^- \rightarrow \phi \rightarrow K^+K^-$. We find $\text{BR}(K^\pm \rightarrow \pi^\pm \pi^0 \pi^0) = (1.763 \pm 0.013_{\text{stat}} \pm 0.022_{\text{syst}}) \times 10^{-2}$.

Recently there has been renewed interest in three pion decays of charged kaons [1]. Because of the small energy available in the reaction, $K \rightarrow 3\pi$ is an ideal process where to apply the notion of the Goldstone-boson nature of the pseudoscalar mesons, by testing the predictions obtained from the chiral lagrangian realization of the $\Delta S = 1$ weak interactions [2].

The branching ratio for $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ ($\tau'$) decays is the least well known of the hadronic $K^\pm$ decay modes. The most accurate measurement to date of $\text{BR}(\tau')$, performed with a sample of $\sim 1300 K^+ \rightarrow \pi^+ \pi^0 \pi^0$ decays in flight [3,4], has an accuracy of 3.3% and dates back to more than thirty years ago. We present a new determination of $\text{BR}(\tau')$, performed with the KLOE detector [5].

1 Corresponding author: Andrea Ventura, e-mail ventura@le.infn.it
2 Corresponding author: Margherita Primavera, e-mail primavera@le.infn.it
3 In the following text, this old notation will be often used to refer to $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ decay.
at the Frascati φ−factory DAΦNE [6]. DAΦNE is an \(e^+e^−\) collider operated at a CM energy \(W = m_\phi \sim 1020\) MeV/\(c^2\). About 50% of the \(\phi\) mesons decay to \(K^+K^-\) pairs. Detection of a \(K^\pm\) meson tags the presence of a \(K^\mp\). Samples of \(N_K\) (positive and negative) tagged kaons can be identified and used to search for the \(\tau'\) decay. The branching ratio is then given by \(\text{BR} = N(\tau')/N_K\). This procedure allows to determine directly the absolute branching ratio of interest. The one necessary condition is that the trigger of the tagging kaon should be only slightly dependent on the decay mode of the tagged kaon. We ensure this to be the case by verifying that the tagging kaon did in fact trigger by itself, using the complete set of information, recorded for each event, concerning all signals produced and processed by the trigger system [7]. The measurement described in the following is based on data collected at the \(\phi\) peak in 2001 and 2002, corresponding to an integrated luminosity of \(441\) pb\(^{-1}\) or, equivalently, to the production of \(\sim 1.5 \times 10^9\) \(\phi\) mesons.

The KLOE detector consists of a large cylindrical drift chamber surrounded by a lead-scintillating fiber sampling calorimeter. A superconducting coil outside the calorimeter provides a 0.52 T magnetic field parallel to the beam axis. The drift chamber [8] is 4 m in diameter and 3.3 m long. It uses aluminium field wires, a 90% helium–10% isobutane gas mixture and is constructed entirely with carbon fiber-epoxy composites. Transparency to photons is thus maximized and multiple Coulomb scattering minimized. Single point resolutions are \(\sim 150\) \(\mu\)m in the transverse plane and \(\sim 2\) mm longitudinally. Typical momentum resolution is \(\sigma(p_T)/p_T \leq 0.4\%\). Vertices are reconstructed with a spatial resolution of \(\sim 3\) mm.

The calorimeter [9], divided into a barrel and two endcaps, covers 98% of the solid angle. The readout granularity is \(\sim 4.4\times4.4\) cm\(^2\) with 2440 “cells” arranged in five layers. Cells with signals close in time and space are grouped into a “calorimeter cluster”. For each cluster, the energy deposit is the sum of the single cell energies. Arrival time and position are calculated as energy-weighted averages over the fired cells. The energy resolution is \(\sigma_E/E = 5.7%/\sqrt{E \text{ (GeV)}}\) and the time resolution \(\sigma_t = 54\text{ ps}/\sqrt{E \text{ (GeV)}} \oplus 50\text{ ps}\). The KLOE trigger [7] requires two local energy deposits in the calorimeter above threshold (50 MeV in the barrel, 150 MeV in the endcaps) or an appropriate hit multiplicity in the chamber. The trigger system also produces a cosmic-ray veto using the signals from the outer calorimeter layers.

Charged kaons are identified by observation of their two-body decays: \(\mu^\pm\nu_\mu\) (\(K_{\mu2}^\pm\)) and \(\pi^\pm\pi^0\) (\(K_{\pi2}^\pm\)), comprising \(\sim 85\%\) of all \(K^\pm\) decays. The monochromatic charged particle momentum in the kaon CM produces a very clean signature and is exploited to identify \(K^\pm\) production and thus tag the presence of another charged kaon, whose decay can be investigated.

As mentioned above, since all information on how the trigger was formed is
available, we require that the tagging kaon indeed satisfied the trigger requirements. Charged kaon decays are found by looking for a charged particle originating from the interaction point and ending in a decay vertex reconstructed inside the drift chamber. We require that: 1) the radial distance of the decay vertex from the beam axis be between 40 and 150 cm; 2) the kaon momentum at the decay point be between 70 and 130 MeV/c and the point of closest approach, $x_c, y_c, z_c$, of its track to the beam axis satisfy $\sqrt{x_c^2 + y_c^2} < 10$ cm and $|z_c| < 20$ cm (the origin of the reference frame is the collider interaction point); 3) the momentum of the decay product $p_d$, assumed to be a pion, satisfy $180 < p_d < 270$ MeV/c in the kaon CM. In Fig. 1 the $p_d$ distribution is shown for data and Monte Carlo: the two well distinct peaks correspond to $K^{\pm}\pi^0$ and $K^{\pm}\mu^0$ decays.

$K^{\pm}_{\pi^0}$ decay identification requires a decay particle momentum in the $K$ meson rest frame compatible with the expected value of 205.14 MeV/c and detection of a $\pi^0$. The latter requires observation of two energy clusters in the calorimeter, photons, with energies $> 15$ MeV and times of flight consistent with the decay vertex, which we refer to as “on-time” clusters in the following. In addition, we require $85 < m_{\gamma\gamma} < 185$ MeV/c$^2$. $K^{\pm}_{\mu^0}$ decay identification requires the decay particle momentum in the $K$ meson rest frame to be compatible with 235.53 MeV/c. Furthermore, the missing mass $m_m$ at the decay vertex must satisfy $|m_m^2| < 5000$ MeV$^2/c^4$, which removes residual background from $\pi^0\pi^0$ decays. The main background in the two samples is mostly due to $K^{\pm}_{\pi^0}$ and $K^{\pm}_{\mu^0}$ decays, especially for $K^{\pm}_{\pi^0}$ decays. From Monte Carlo simulation we find: $f_{\text{bg}}(\pi^0\pi^0) = (0.37 \pm 0.02)\%$, $f_{\text{bg}}(\mu^0\nu) = (0.21 \pm 0.02)\%$, where the quoted errors include statistical and systematic contributions.

$\tau'$ decays are events in which a decay vertex is observed for the tagged kaon, as for the tagging kaon above. In addition, at least four on-time energy deposits with $E > 15$ MeV must be detected. The maximum charged pion momentum
in the $K^\pm$ frame is $132.95$ MeV/c for $\tau'$ decays. We accept only pions with $p_{\pi^\pm} < 135$ MeV/c. The on-time condition ensures that the arrival times are consistent with photons originating at the same point, the decay vertex. Since the cluster time $t_i$ is well known but the decay instant is not, we construct the differences $\Delta_{jk}$ between the quantities $t_i - r_i/c$ for all cluster pairs $j, k$, with $j, k = 1, \ldots, N$, $k > j$ and $r_i$ the distance from the centroid of the cluster $i$ to the decay vertex. Each difference is normalized using the appropriate error so that its variance is unity:

$$\Delta_{jk} = \frac{t_j - t_k - (r_j - r_k)/c}{\sqrt{\sigma^2_{t,j} + \sigma^2_{r,k}}}.$$

We require $|\Delta_{jk}| < 4$ for all pairs. Accidental on-time background clusters are effectively rejected by this cut. Events with more than four on-time clusters in the final sample, apart from the decay $K^\pm \rightarrow \pi^\pm \pi^0\pi^0\gamma$ (BR < $10^{-5}$), are due to residual on-time background and photon showers reconstructed as multiple clusters. We finally require that $\sum E_i < 450$ MeV, for the four most energetic energy deposits. Data from 188 pb$^{-1}$ are used to search for $\tau'$ decays. The remainder of the data is divided into three subsets for the purposes of efficiency evaluation. We find 41896 $K^+$ and 41155 $K^-$. We get 52253 and 30798 $\tau'$ decays which pass all requirements above, tagged by $1.9925 \times 10^7 K^\pm_{\mu2}$ and $1.2753 \times 10^7 K^\pm_{\pi2}$ decays, respectively.

As background in the $K^\pm \rightarrow \pi^\pm \pi^0\pi^0$ sample, we have considered $K^\pm_{\pi2}$, $K^\pm_{e3}$ and radiative decays like $K^\pm \rightarrow \pi^\pm \pi^0\gamma$, in which a “spurious” cluster in the calorimeter has been paired with the cluster of the radiated $\gamma$ (the probability for this to happen has been estimated from a $K^\pm_{\pi2}$ data sample to be $\sim 8\%$). The nuclear interaction of a charged kaon (mainly $K^-$) with the beam pipe and the chamber walls can produce secondaries that simulate the $\tau'$ signal. Finally, we include $K^\pm \rightarrow l^\pm \pi^0\pi^0\nu_l$ ($K'_\ell$) decays. The relative contributions of these backgrounds have been estimated by Monte Carlo simulation, while the total amount of background events has been determined by fitting the observed missing mass $(m_m)$ spectrum at the decay vertex (assumed to be $K \rightarrow \pi + x$) with the sum of the simulated signal and background spectra. The background fraction estimated from the Monte Carlo simulation had to be corrected by a factor $1.13 \pm 0.07$. After applying this correction we find that the average total background fraction is $f_{\text{bkgd}}(\tau') = (0.75 \pm 0.11)\%$, where the quoted error includes the contributions from the finite statistics of the Monte Carlo spectra and from the fit. Contamination from other decays not originating from $K^\pm$ is negligible.

The overall $\tau'$ efficiency $\epsilon(\tau')$ is given by the product $\epsilon = \epsilon_K\epsilon_v\epsilon_{\gamma}$, $\epsilon_K$ is the efficiency to reconstruct the track of the charged kaon that undergoes $\tau'$ decay. $\epsilon_v$ includes the efficiency for decay vertex finding and losses due to the pion momentum cut. $\epsilon_{\gamma}$ is the efficiency for observing at least four photons in the
calorimeter satisfying the energy and timing requirements above. Each factor has been measured separately for positive and negative kaon charge and for $K^\pm_{\mu2}$ and $K^\pm_{\pi2}$ tags by using special control samples [10] extracted from data taken in run periods close in time but different from the one used for signal selection. Control samples selected using calorimeter variables have been used to compute the efficiencies involving the drift chamber and vice-versa.

Where necessary, the efficiencies have been corrected for contamination of the control samples, as estimated by Monte Carlo simulation. For all samples the purities are > 95%. The efficiency $\epsilon_K$ has been found, within errors, to be the same for the two tags and kaon charges and its average value is equal to $0.466 \pm 0.001_{\text{stat}} \pm 0.002_{\text{syst}}$. It is dominated by the geometrical acceptance (more than one third of the charged kaons decay before reaching the drift chamber). The effect of the above mentioned nuclear interactions of $K^\pm$ has been taken into account by properly weighting, to get $N_{K^\pm}$, the number of tagging kaons with a factor containing the probability for charged kaons to interact with the materials in front of the chamber. These weights have been extracted from data by using $K^\pm_{\pi2}$ and $K^\pm_{\mu2}$ samples: $w^+ = 0.9696 \pm 0.0036$ and $w^- = 0.9970 \pm 0.0034$ respectively for positive and negative tags.

The efficiency for reconstructing the $K^-\pi^+$ vertex, $\epsilon_{\text{vtx}}$, has been parameterized as a function of the charged pion momentum, as shown in Fig. 2 (left). After including the effect of the cut on the pion momentum, the average value $\epsilon_{\text{vtx}} = 0.539 \pm 0.003_{\text{stat}} \pm 0.002_{\text{syst}}$ is obtained for the signal. The systematic error includes small differences between the two tags.

The cluster efficiency is evaluated from a control sample and corrected by using a Monte Carlo simulation to consider the effects of accidental clusters. Fig. 2 (right) shows the single cluster efficiency $\epsilon_{\text{clu}}$ vs energy. The only appreciable source of the difference in $\epsilon_{\gamma}$ between the $K^\pm_{\mu2}$ and $K^\pm_{\pi2}$ tags results from the on-time cluster requirement, due to the different cluster multiplicities in each event type. As average value on positive and negative kaon charge we find: $\epsilon_{\gamma} = 0.645 \pm 0.001_{\text{stat}} \pm 0.004_{\text{syst}} (0.625 \pm 0.002_{\text{stat}} \pm 0.005_{\text{syst}})$ in the case of $K^\pm_{\mu2} (K^\pm_{\pi2})$ tag.

Finally, $\epsilon(\tau') = 0.1621 \pm 0.0010_{\text{stat}} \pm 0.0016_{\text{syst}} (0.1573 \pm 0.0011_{\text{stat}} \pm 0.0018_{\text{syst}})$ combining positive and negative kaons for $K^\pm_{\mu2} (K^\pm_{\pi2})$ tags.

The residual dependence of our tagging procedure on the tagged kaon decay mode is expressed by the ratio $R_{\text{tag}}$ of the average (over all channels) tagging efficiencies (including trigger) and the values of the same efficiencies in the case of $\tau'$ signal. $R_{\text{tag}}$ is determined from Monte Carlo and the averages on the two kaon charges are: $R_{\text{tag}} = 1.058 \pm 0.005 (1.099 \pm 0.009)$ for $K^\pm_{\mu2} (K^\pm_{\pi2})$ tags, where the error includes systematic effects due to small differences between data and Monte Carlo, as estimated on a set of variables to which $R_{\text{tag}}$ is found to be sensitive.

The $\tau'$ branching ratio is computed separately for $K^\pm_{\mu2}$ and $K^\pm_{\pi2}$ tags and for
Fig. 2. Vertex efficiency as a function of the pion momentum in the lab frame (left). Single cluster efficiency corrected as described in the text as a function of the photon energy (right).

the two signs of the kaon charge, from:

$$\text{BR}(K^\pm \rightarrow \pi^\pm \pi^0\pi^0) = \frac{N(\tau')}{N_K} \times \frac{1 - f_{\text{bckgd}}(\tau')}{1 - f_{\text{bckgd}}(\tau')} \times \frac{R_{\text{tag}}}{\varepsilon(\tau')} \times \frac{1}{[\text{BR}(\pi^0 \rightarrow \gamma\gamma)]^2}$$

where $$\text{BR}(\pi^0 \rightarrow \gamma\gamma) = 0.98798 \pm 0.00032$$ [3]. We find:

$$\text{BR}(K^+ \rightarrow \pi^+ \pi^0\pi^0) = (1.760 \pm 0.024_{\text{stat}} \pm 0.017_{\text{syst}}) \times 10^{-2} \ (K^-_{\mu_2 \text{ tag}})$$

$$\text{BR}(K^- \rightarrow \pi^- \pi^0\pi^0) = (1.793 \pm 0.024_{\text{stat}} \pm 0.026_{\text{syst}}) \times 10^{-2} \ (K^-_{\mu_2 \text{ tag}})$$

$$\text{BR}(K^+ \rightarrow \pi^+ \pi^0\pi^0) = (1.769 \pm 0.027_{\text{stat}} \pm 0.020_{\text{syst}}) \times 10^{-2} \ (K^+_{\pi_2 \text{ tag}})$$

$$\text{BR}(K^- \rightarrow \pi^- \pi^0\pi^0) = (1.724 \pm 0.027_{\text{stat}} \pm 0.027_{\text{syst}}) \times 10^{-2} \ (K^-_{\pi_2 \text{ tag}})$$

yielding the average value:

$$\text{BR}(K^\pm \rightarrow \pi^\pm \pi^0\pi^0) = (1.763 \pm 0.013_{\text{stat}} \pm 0.022_{\text{syst}}) \times 10^{-2}$$

where the final systematic error takes into account correlations. All contributions to the uncertainty are listed in Table 1. The statistical error is dominated by the uncertainty in determining the efficiencies, due to the limited statistics of the data samples used in their evaluations. The stability of the result has been verified using data sets from different running periods. It should be pointed out that this measurement is fully inclusive of the radiative decays $$K^\pm \rightarrow \pi^\pm \pi^0\pi^0\gamma$$ (there is no cut on the radiated photon energy $$E_\gamma$$), since we require at least four (rather than exactly four) on-time clusters in the signal selection. An upper limit to the contribution of this decay is estimated to be 0.1% of the selected events, by taking into account possible differences in the efficiency for the two channels.
| Source of uncertainty                                      | Fractional error (10^{-3}) |
|-----------------------------------------------------------|-----------------------------|
| $N$($\tau'$)/$N_K$ statistics                            | 3.5                         |
| Charged kaons nuclear interaction probability             | 3.5                         |
| Charged kaon reconstruction/identification efficiency    | 5.2                         |
| Vertex reconstruction efficiency                          | 6.6                         |
| Cluster algorithm efficiency                              | 2.3                         |
| Four-cluster acceptance                                  | 3.6                         |
| On-time requirement for clusters                          | 7.5                         |
| Total energy cut                                          | 1.4                         |
| Background subtraction                                   | 1.1                         |
| Ratio of tag efficiencies $R_{tag}$                      | 6.6                         |
| $BR(\pi^0 \rightarrow \gamma \gamma)^2$                  | 0.7                         |

Table 1
Summary of all contributions to the total error on $BR(K^\pm \rightarrow \pi^\pm \pi^0 \pi^0)$.

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