Abstract The final state interaction (FSI) contribution to charged $D$ decay into $K\pi\pi$ is computed within a light-front framework, considering $S$-wave $K\pi$ interactions in $1/2$ and $3/2$ isospin states. The convergence of the rescattering series is checked computing terms up to the third perturbative order. The role of the resonances above $K^*_0(1430)$, and the contribution of the $K\pi$ $3/2$ isospin channel to charged three-body $D$ decays, are studied against the available phase-shift analysis.

Keywords Charged 3-body $D$ decays · Final State Interaction · Relativistic Faddeev Equations

1 Introduction

The phases of the $K\pi$ elastic amplitude in partial waves from $\ell = 0$, 1 and 2 were known from experiments using the reactions $K^\mp p \rightarrow K^\pm \pi^+ n$ and $K^\mp p \rightarrow K^\pm \pi^- \Delta^{++}$ at 13 GeV/c [1] and the reaction $K^- p \rightarrow K^- \pi^+ n$ at 11 GeV/c [2]. However, these reactions do not access the low mass region close to the threshold and information on the $K\pi$ phase-shifts become available after the analysis of the charged $D \rightarrow K\pi\pi$ decays from from E791 [3; 4] and FOCUS [5] collaborations. These phase-shift analyses can provide information on the $K\pi$ scattering amplitude starting at the $K\pi$ threshold covering the allowed phase-space for the charged $D$ decay, once three-body rescattering contributions are under control. In this contribution, we further explore theoretically the $S$-wave interaction in the different $K\pi$ isospin states in the calculation of the final state interaction in $D^\pm \rightarrow K^\mp \pi^{\pm} \pi^{\pm}$ decay channel within a light-front relativistic three-body model.

Our work follows the relativistic three-body model for the final state interaction in the charged $D \rightarrow K\pi\pi$ decay based on the three-meson Bethe-Salpeter equation [6; 7; 8]. In the model developed here, the decay amplitude is separated into a smooth term and a three-body fully interacting contribution, which is factorized in the standard two-meson resonant amplitude times a reduced complex amplitude for the bachelor meson, that carries the effect of the three-body rescattering mechanism. The off-shell bachelor reduced amplitude is a solution of an inhomogeneous Faddeev type integral equation, that has as input the $S$-wave isospin 1/2 and 3/2 $K^\mp \pi^{\pm}$ transition matrix. In the previous work [7], the three-body rescattering calculation took into account the isospin 1/2 $S$-wave $K\pi$ interaction within a chiral model fitted to the LASS phase-shift analysis [2] up to the $K^*_0(1430)$ resonance, while the...
available phase-space for the $D$ decay covers energies up to 1.89 GeV. Our work extends the previous three-body rescattering model for charged $D$ decays in order to include the $S$-wave two-body $K\pi$ amplitude in both isospin states, 1/2 and 3/2, for $K\pi$ masses up to 1.9 GeV. We study the role of the resonances above $K_0^*(1430)$ in addition to the contribution of the $K\pi$ isospin 3/2 interaction against the available phase-shift analysis of the charged $D \rightarrow K\pi$ decay from E791 [3, 4] and FOCUS [5] collaborations.

2 Decay Amplitude for $D^\pm \rightarrow K^\mp\pi^\pm\pi^\pm$

The standard technique to study the resonant structure of the three-body decay is the Dalitz plot analysis. In such a plot, the final state of a particle $P$ decaying into three particles $(d_1, d_2, d_3)$ can be described through a bi-dimensional diagram, where the axis are the two-body invariant masses squared. The density of events in this plot is given by

$$d\Gamma(P \rightarrow d_1 d_2 d_3) \propto \frac{1}{M_P^3} |A|^2 ds_1 ds_3$$

and, in such a diagram, we have access to the matrix element $A$. The phase-space density of this three-body decay, $M_P^{-3}$, is constant and if we write $M_P$ in terms of the Mandelstam invariants $s_{12}$ and $s_{13}$, the kinematically allowed event region is delimited. The structure observed in the $D^+ \rightarrow K^-\pi^+\pi^+$ Dalitz plot [4] is an outcome of the decay dynamics and resonances in the decay channels.

The decay amplitude is expressed as a sum over the partial-wave contributions

$$A = \sum_L \left( a_L(s_{K\pi}) e^{i\phi_L(s_{K\pi})} P_L + a_L(s_{K\pi}) e^{i\phi_L(s_{K\pi})} P_L \right)$$

with amplitude $a_L(s_{K\pi})$, phase $\phi_L(s_{K\pi})$. In the above sum $P_L$ is a short-hand notation for the angular functions. We will be interested in the $S$-wave amplitude $A_0(s_A, s_B)$.

3 The $S$-wave $K\pi$ Amplitude

In our model the input for the calculation of the $3 \rightarrow 3$ T-matrix, which brings the final state interaction between the three mesons to the $D^\pm \rightarrow K^\mp\pi^\pm\pi^\pm$ decay is the resonant $S$-wave $I_{K\pi} = 1/2$ and the nonresonant one for $I_{K\pi} = 3/2$. The interaction of the identical pions is neglected. In the resonant $K\pi$ channel, besides the $K_0^*(1430)$ used in parametrization of the LASS data [2] given in Ref. [4], we introduce the resonances $K_0^*(1630)$ (in PDG there is no assignment of spin to $K(1630)$) and $K_0^*(1950)$ in the $S$-wave $K\pi$ scattering amplitude.

The suggestion to include the higher radial excitations of $K_0^*$ comes in analogy of a recent proposal to interpret the scalar meson family ($f_0$) as radial excitations of the $\sigma$ meson [9] and [10]. Then, we make a fit of the LASS $S$-wave phase-shift data in the isospin channel 1/2 with the resonances $K_0^*(1430)$, $K_0^*(1630)$ and $K_0^*(1950)$, which are suggested to be radial excitations of $K_0^*(800)$. In our anzats the $K\pi$ isospin 1/2 $S$-wave S-matrix is given by:

$$S_{1/2}^{k_{K\pi}}(M_{K\pi}^2) = \frac{k \cot \delta + i k}{k \cot \delta - i k} \prod_{\tau=1}^3 \frac{M_{\tau}^2 - M_{K\pi}^2 + iz_\tau i F_\tau}{M_{\tau}^2 - M_{K\pi}^2 - iz_\tau i F_\tau},$$

where $z_\tau = k M_{\tau}^2/(k_\tau M_{K\pi})$, $k \cot \delta = \frac{1}{\pi} + \frac{1}{2} r_0 k^2$, and effective range parameters $a = 1.6$ GeV$^{-1}$, $r_0 = 3.32$ GeV$^{-1}$. The relative momentum of the $K\pi$ pair is $k$ and $k_\tau$ is the momentum at the resonance.

In Fig. [1] we show the results from the three-resonance model, Eq. (3), with parameters given in Table I. The $K\pi$ $S$-wave phase-shift is compared to that of LASS. Although the parameters of the model are not yet optimized, we are able to reproduce the LASS data reasonably well.
Table 1  S-wave resonance parameters for the $K\pi$ isospin 1/2 S-matrix. Particle Data Group (PDG) [11].

| $^1S^0 (0^+)$ | $M_r$ (GeV) | PDG (GeV) | $\Gamma_r$ (GeV) | $\bar{\Gamma}_r$ (GeV) | PDG (GeV) |
|---------------|------------|-----------|-----------------|-----------------|---------|
| $K_0^*(1430)$ | 1.48       | 1.425 ± .050 | 0.25            | 0.25            | 0.27 ± .08 |
| $K_0^*(1630)$ | 1.67       | 1.629 ± .007 | 0.1             | 0.1             | < .025   |
| $K_0^*(1950)$ | 1.9        | 1.945 ± .022 | 0.2             | 0.14            | 0.201 ± 0.086 |

Fig. 1  S-wave $K\pi$ phase as a function of the $K\pi$ mass. Solid line: phase-shift from the $K\pi$ S-matrix, Eq. (3). Circles: LASS data [2].

The non-resonant isospin 3/2 S-wave $K\pi$ S-matrix, $S^{3/2}_{K\pi} = k \cot \delta + k^2/r_0 k^2$, is well parametrized by the effective-range expansion $k \cot \delta = \frac{1}{a} + \frac{r_0}{r_{01}^2} k^2$, with $a = -1.00$ GeV$^{-1}$ and $r_0 = -1.76$ GeV$^{-1}$ according to Ref. [1]. The S-wave $K\pi$ amplitude in the isospin channel $I_{K\pi}$ is written as

$$\tau_{I_{K\pi}}(M_{K\pi}^2) = 4\pi \frac{M_{K\pi}}{k} (S^{I_{K\pi}}_{K\pi} - 1),$$

which is the input of the three-body rescattering model for $I_{K\pi} = 1/2$ and 3/2.

4 Three-Body Rescattering Model

The partonic amplitude for the decay of the $D$ meson into the $K\pi\pi$ channel with off-shell momenta $q^a_i$, and masses $m_i$ ($i = \pi, K, \pi'$) is expressed by the function $D(q_\pi, q_{\pi'})$. It corresponds to a smooth background given by the direct partonic decay amplitude and is represented by the gray blob with three legs at leftmost corner of Fig. 2. This amplitude should be convoluted with the $3 \rightarrow 3$ transition matrix, which take into account the three-meson interacting final state, as shown in Fig. 2 in a form of connected ladder series, where the $2 \rightarrow 2$ scattering process is summed up in the $K\pi$ transition matrix. In addition, the interaction between the two positively charged pions is disregarded.

Fig. 2  Diagrammatic representation of the heavy meson decay process into $K\pi\pi$, starting from the partonic amplitude (gray) and adding the hadronic multiple scattering in the ladder approximation. The input $K\pi$ scattering amplitude (black) is required fully off-mass-shell.

The full decay amplitude represented diagrammatically in Fig. 2 is given by

$$A_0(k_\pi, k_{\pi'}) = a_0(s_A)e^{i\theta_A(s_A)} + a_0(s_B)e^{i\theta_B(s_B)} = D(k_\pi, k_{\pi'}) + a(m_{12}^2) + a(m_{23}^2)$$

$$= D(k_\pi, k_{\pi'}) + \int \frac{d^4q_\pi d^4q_{\pi'}}{(2\pi)^8} T_{3,3}(k_\pi, k_{\pi'}; q_\pi, q_{\pi'}) S_\pi(q_\pi)S_\pi(q_{\pi'})S_K(K - q_{\pi'} - q_\pi)D(q_\pi, q_{\pi'}),$$

(5)
where the momentum of the pions from the decay of the $D$ are $k_\pi$ and $k_\pi'$. The matrix element of the $3 \to 3$ transition matrix is $T(k_\pi, k_\pi'; q_\pi, q_\pi')$. The mesonic Feynman propagators are $S_i(q_i) = i(q_i^2 - m_i^2 + i\epsilon)^{-1}$, in the approximation where self-energies are disregarded. The T-matrix operator acts on the isospin space of the $K\pi\pi$ system, while $D(k_\pi, k_\pi')$ and $D(k_\pi, k_\pi')$ are states in the corresponding isospin space. Our model introduces a two-body transition matrix with matrix elements dependent only on the Mandelstam $s$-variable, which gives a considerable simplification in the Faddeev decomposition of the $3 \to 3$ off-shell transition amplitude. Once this assumption is done, one easily recognizes the factorization

$$a(M^2_{K\pi}) = \tau \left( (K - q_\pi')^2 \right) \xi(q_\pi'), \quad M^2_{K\pi} = (K - q)^2,$$

by inspecting the scattering series that ends with a two-body transition matrix (see Fig. 2), which only carries the dependence on $s_{K\pi} = M^2_{K\pi}$, and remains only the dependence on the on-mass-shell momentum of the bachelor pion ($\pi'$). We have depicted only the $K\pi$ pair, as we additionally assume that the identical pions interact weakly and the corresponding transition matrix disregarded. The total momentum $K$ of the decaying $D$-meson is shared by the $K\pi\pi$ system.

The light-front projection of the covariant equations for the Faddeev components of the decay amplitudes associated with the spectator functions decomposed in isospin states for angular momentum zero are written as:

$$\xi^{I}_{I_T, I_{K\pi}}(y, k_\perp) = \langle I_T, I_{K\pi}, I_{K\pi}|D \rangle \xi_0(y, k_\perp) + \frac{i}{2} \sum_{K'\pi'} R_{I_T, I_{K\pi}, I_{K\pi'}}^{I_{K'\pi}} \int_0^{1-y} \frac{dx}{x(1-y-x)} \int_0^\infty \frac{dq_\perp}{(2\pi)^3} K_{I_{K\pi'}}(y, k_\perp; x, q_\perp) \xi^{I}_{I_T, I_{K\pi'}}(x, q_\perp),$$

where the kinematics was chosen such that the decay plane is transverse to $z$-direction, and rotational invariance is preserved after the light-front projection. The spectator function depends on the total isospin and on the $K\pi$ subsystem isospin quantum numbers, $I_T$ and $I_{K\pi}$ ($I_{K'\pi}$), respectively. The free squared mass of the $K\pi\pi$ system is

$$M^2_{0, K\pi\pi(x, q_\perp, y, k_\perp)} = \frac{k^2_\perp + m_\pi^2}{y} + \frac{q^2_\perp + m_\pi^2}{x} + \frac{q^2_\perp + k^2_\perp + 2q_\perp k_\perp \cos \theta + m^2_\pi}{1-x-y},$$

and the squared-mass of the virtual $K\pi$ system is

$$M^2_{K\pi}(z, p_\perp) = (1-z) \left( M^2_D - \frac{k^2_\perp + m_\pi^2}{z} \right) - p^2_\perp.$$

The isospin recoupling coefficient is

$$R_{I_T, I_{K\pi}, I_{K\pi'}}^{I_{K'\pi}} = \langle I_T, I_{K\pi}, I_{K'\pi} |I_T, I_{K\pi}, I_{K\pi'} \rangle,$$

and the kernel is given by

$$K_{I_{K\pi'}}(y, k_\perp; x, q_\perp) = \int_0^{2\pi} d\theta \frac{q_\perp K_{I_{K\pi'}}(y, k_\perp)}{M^2_D - M^2_{0, K\pi\pi(x, q_\perp, y, k_\perp)}} + i\epsilon.$$  

The driving term is

$$\xi_0(y, k_\perp) = \lambda(\mu^2) + \frac{i}{2} \int_0^1 \frac{dx}{x(1-x)} \int_0^{2\pi} d\theta \int_0^\infty \frac{dq_\perp}{(2\pi)^3} \left( M^2_{K\pi}(y, k_\perp) - M^2_{0, K\pi\pi(x, q_\perp, y, k_\perp)} \right) \mu^2 - M^2_{0, K\pi\pi(x, q_\perp)}.$$  

where the loop-divergence is regulated by a subtraction and the renormalized value of the amplitude at the subtraction scale is $\lambda(0) = 0.12 + 0.06i$ for $\mu^2 = 0$, which matches the driving term obtained with the $\chi$-model of the $D \to K\pi\pi$ decay computed in Ref. [7]. In our actual computations we will allow the subtraction scale to move, by keeping $\lambda(\mu^2)$ fixed to $\lambda(0)$, while moving $\mu^2$. The detailed derivation of the light-front projected Faddeev equations, using the quasi-potential method (see e.g. [12]) will be presented elsewhere.

5 Results for $D^\pm \to K^{\mp} \pi^\pm \pi^\pm$ Model Against Experimental Analysis

The previous calculation of the $\chi$–model for the $D$ decay with only isospin $1/2$ $K\pi$ interaction calculated the spectator function in perturbation theory. The driving term was responsible for about 70–80% of the decay amplitude while the next iteration gave the rest. That calculation stopped in the next to leading order term. We have checked that indeed the third order term contribution to the spectator
function are of the order of few percents of the total and it explicit from is obtained by iterating twice Eqs. (7). To obtain the bachelor amplitude a small and finite imaginary term ($\epsilon = 0.2$ GeV) was introduced in the three-meson propagator, it also represents absorption to other decay channels, which is beyond the model. An arbitrary subtraction point was chosen for the driving term, with values ranging from $-1$ GeV$^2 \lesssim \mu^2 \lesssim (m_\pi + m_K)^2$. Given that, we solve perturbatively up to the second order the spectator function and computed the decay amplitude by fully considering the $K\pi$ interaction in both isospin channels, and written explicitly as:

$$A_0(M_{K\pi}^2) = a_0(M_{K\pi}^2)e^{i\Phi_0(M_{K\pi}^2)} = \sum_{I_T, I_{K\pi}} C_{I_T, I_{K\pi}} \left[ \frac{A_{I_T, I_{K\pi}}}{2} + \tau_{I_{K\pi}}(M_{K\pi}^2)\xi_{I_{K\pi}}(k_{\pi}) \right], \quad (10)$$

where $C_{I_T, I_{K\pi}} = \langle K^-\pi^+\pi^- | I_T, I_{K\pi}, I_T^2 = 3/2\rangle$, and $A_{I_T, I_{K\pi}} = \frac{1}{2} \langle I_T, I_{K\pi}, I_T^2 = 3/2 \rangle D$. Due to the symmetry of the amplitude against the exchange of the identical pions one are left with $A_{3/2, 3/2} = \sqrt{3}$, $A_{3/2, 1/2} = \sqrt{\pi}(W_1 - W_2)$, and $A_{5/2, 3/2} = \frac{2}{\sqrt{5}}$. In the particular case of $|D| = |K^-\pi^+\pi^-|$ one has that $W_1 = W_2 = W_3 = 1$.

The fit found in Ref. [7] below $K_0^*(1430)$ with $K\pi$ interaction in 1/2 isospin state only, suggested that the partonic amplitude has little overlap with the $K^+\pi^-\pi^+$ final state channel, i.e., the first term in left-hand-side of Eq. (10) should vanishes. Here, we also present results computed only by considering $A_0(M_{K\pi}^2) \approx \tau_{I_T}(M_{K\pi}^2)\xi_{I_{K\pi}}(k_{\pi})$. The modulus and phase of this amplitude are shown in the left panel of Fig. 3 and compared to the experimental analysis from E791 [3] and FOCUS collaboration [3]. As in the previous work [7], a reasonable fit to the experimental data below $K_0^*(1430)$ is found. However, note that a structure in the phase is seen in the model which incorporates $K_0^*(1630)$ and $K_0^*(1500)$, as also verified in the LASS data. A better fit of the LASS data above $K_0^*(1430)$ seems necessary to find a better description of the valley in the modulus and the structure of the phase. The conclusion is somewhat independent on the subtraction point, at least for range of values in the figure.

![Fig. 3 Left panel: Single-channel calculation up to 2-loops for $I_T = 3/2$ considering the resonances $K_0^*(1430), K_0^*(1630), K_0^*(1500)$. (a) Modulus and (b) phase of $\tau_{1/2}^{3/2, 1/2}$. Values for $\mu^2$ in GeV$^2$: 0.1 (dotted line), 0.4 (dashed line), and 1 (solid line). Right panel: Coupled-channel calculation up to 2-loops considering $I_T = 3/2$ and $5/2$. $K\pi$ interaction isospin $3/2$ and $1/2$ states with resonances $K_0^*(1430), K_0^*(1630), K_0^*(1500)$. The fitted parameters are $W_1 = 1$, $W_2 = 2$ and $W_3 = 0.2$. (a) Modulus and (b) phase of the $D \rightarrow K\pi\pi$ amplitude for two cases: i) without $K_0^*(1500)$ and ii) without $K_0^*(1500)$ and $K_0^*(1630)$. The experimental data come from the phase-shift analysis of E791 [3] (empty box) and FOCUS collaboration [3] (inverted full triangle). (For reference, the dotted-dashed line gives the previous covariant calculation up to two-loops from Magalhães, et. al. of Ref. [8]).

In the right panel of Fig. 3 we present results for the coupled channel model with $K\pi$ interaction in isospin $1/2$ and $3/2$ states using Eqs. (7) up to two-loops. We considered some variation of $\mu^2 = 0.4, -0.1$ and 1 GeV$^2$ in the driven term and fixed $\epsilon = 0.2$ GeV$^2$. A reasonable fit of the experimental phase and modulus is given by $\mu^2 = -1$ GeV$^2$ and $\mu^2 = -0.1$ GeV$^2$ with $W_1 = 1$, $W_2 = 2$ and $W_3 = 0.2$ in the decay amplitude (10) and spectator functions coupled equations (7). At low $M_{K\pi}$ below 1 GeV, the model tends to underestimate the modulus, where the different analyses of E791 and
FOCUS present a large dispersion. The characteristics valley and the follow-up height is somewhat described by the model, with exception of the region close to the boundary of the decay phase-space, where the data seems to indicates an increase of the amplitude and the model presents a noticeable decrease. Notice also that the effect of the resonances in the fit of the $K\pi$ isospin 1/2 amplitude to the LASS data, in the last model results, is similar to the single channel case we have already discussed. The region close to the valley appearing in the modulus is sensitive mainly to our fit of the LASS data in the neighborhood of $K^0_S(1630)$, while $K^0_S(1950)$ presents a smaller effect in part due to the competition with the interaction in the $I_{K\pi} = 3/2$ state.

6 Summary and Outlook

The final state interaction (FSI) contribution to charged $D$ decay into $K\pi\pi$ is formulated within a three-body model with Faddeev-like equations for the components of the decay amplitude. The dynamical equations for the bachelor amplitudes were computed within a light-front framework assuming the dominance of the valence state composed by a $K\pi\pi$ system. The importance of the $S$-wave $I_{K\pi} = 3/2$ and 1/2 interactions ($I_{K\pi} = 1/2$ is dominant) in $D^\pm \rightarrow K^{\mp}\pi^\pm\pi^\mp$ decay is studied against the experimental phase-shift analysis \cite{4, 3}. The convergence of the rescattering series was checked computing terms up to the third perturbative order, which can be neglected in the region of parameters used in the fits. The role of the resonances above $K^0_S(1430)$ and the contribution of the $K\pi$ isospin 3/2 channel to charged three-body $D$ decays were studied and we found that the not established resonance $K^0_S(1630)$ with quantum numbers $\frac{1}{2}(0^+)$ and width $\Gamma \lesssim 100$ MeV is necessary to improve the fit of the phase and magnitude of the charged $D \rightarrow K\pi\pi$ unsymmetrized decay amplitude above $K^0_S(1430)$.

In addition, we confirmed the importance of the isospin 3/2 contribution to this decay \cite{8}. One interesting application of the formalism is the study of the CP violation in three-body charged $B$ decays, where recent experiments \cite{13} showed a sizable effect, interpreted in Ref. \cite{14} as resulting from the interference between the weak phase and the strong phase from the final state interaction.

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