Developing the geometrical classification of non-cylindrical folds

N N Haify
Institute of Technology, Middle Technical University, Baghdad, Iraq

Abstract. This classification discusses most of the single and independent folds arisen in nature which are geometrically called non-cylindrical folds. This study worked on developing one of the most accurate classifications to describe the non-cylindrical folds which are presented by previous research. A new variable value was generated; it is called the factor (New) and its symbol (N). This value represents a non-planarity degree of axial surface of folds, factor (N) mathematically depends on the value of angle obtained at the side curvature of the axial surface along the extent of hinge line of folds; this angle is called factor (Wavy) and its symbol (W). Factor (N) and angle (W) were added to factors of geometrical classification of non-cylindrical folds according to previous research. The new, sophisticated, integrated and comprehensive classification was given by adding the new factor (N), where it processed the gaps and defects existed in geometrical classifications dealt with the non-cylindrical folds which have plane axial surface or non-plane axial surface, especially those folds that the classification of previous researches could not give them an accurate geometrical description as the one given by the present classification.

1. Introduction
After the geometrical classifications of folds has been studied, it was found that the classification presented by [1] has mentioned all common folds in nature, simple and complicated depending on the descriptions of hinge line of fold and planarity or non-planarity of axial surface of folds, but no angles or specified mathematical values were used to provide an accurate classification of folds. Researchers [2, 3] have presented much processing for the folds considered in their researches, through the use of specified angles and mathematical values that define the types of folds, but they did not specify a classification of folds which have non-plane axial surface. While [4] have presented a unique classification through using specified angles and mathematical values provide the accuracy and includes folds with non-plane axial surfaces, yet, they couldn't define the geometrical description of some special cases of folds, which have non-plane axial surfaces. They based on many studies and researches of geometric classification of non-cylindrical folds as: [5, 6, 7, 8, 9, 10]. The above mentioned researchers broached to non-cylindrical folds which have a plane and non-plane axial surfaces, but they couldn't specify the correct geometrical description of some folds which exist in nature as non-cylindrical and non-plane that will be discussed later in this research. While this study is based on all the above-mentioned studies and researches, in addition to [11, 12, 13, 14, 15]. First, the classification of [4] will be broached in details including their bases, and then the mathematical values and relations that provided them with an accurate classification of non-cylindrical folds with plane or non-plane axial surfaces will be illustrated. Moreover, we will discuss the gaps and shortfalls of their classification which made it unable to name and describe some non-cylindrical folds with non-plane axial surface. Then, a discussion will be done for this new classification and the evolution that was achieved through inventing and developing new important variable values added to the previous classification presented by [4]. This development is processing all the gaps and shortfalls in the above previous classification. A new original geometrical classification was obtained, which is
comprehensive, integrated and capable of naming and describing all non-cylindrical folds which have non-planar axial surfaces and exist in nature on a large scale.

2. Research Methods
2.1 Geometrical description of non-cylindrical folds

The classification of Williams & Chapman, 1979 [4] used the measurement of interlimb angle alpha (α) within the profile plane of fold according to Fleuty, 1964 [2], and measurement of hinge angle Beta (β) within the hinge line surface (Figure 1). According to Ramsay, 1967 [6], the angle hinge Beta (β) is obtained by a curved hinge line at the points of inflection within the hinge line surface. Through the use of angles (Alpha α) and (Beta β), the unlimited number of cylindrical and non-cylindrical folds with axial plane surfaces can be described. Instead of measuring the interlimb angle (α) when the real side section of a plane axial surface must always be perpendicular to the hinge at any point, which includes unlimited interlimb angles. The surface of a single side section is considered to be perpendicular to the hinge at the inflection point by the hinge angle Beta (β), although the surface of the side section may divide the angle Beta (β). A third factor used for the classification of non-cylindrical non-plane folds is named Gamma (γ). It is the measure of the angle within a plane which is perpendicular to the axial surface at the point of coup (Figure 2), thus, the classification of [4] did not provide a complete treatment for the non-cylindrical non-plane folds, which have non-coup axial surface, but the axial surface has a side wavy curvature along the hinge line of fold, due to the presence of the side wavy curvature in direction of hinge line of folds (Figure 3).

The addition and development provided by the present classification depended basically on the condition of measuring both variable values together. These variables are angle Gamma (γ) and the new angle Wavy (W), which determines the degree of non-planarity of non-cylindrical folds, where (W) represents the angle of a side wavy curvature of axial surface along the hinge line of folds (Figure 4).

Figure 1. Illustrates the measures of angles (α) and (β) due to classification [4]

Figure 2. Illustrates the measure of angle (γ) due to classification [4]

Figure 3. Shows a non-plane axial surface fold with presence of a deviation in the hinge line direction [1]

Figure 4. Shows the measure of angle (W) of non-plane axial surface fold for the deviation in the hinge line direction
2.2 The basis of classifying of non-cylindrical folds on the triangular diagram PQR

The following text is quoted from the classification of [4] in order to explain the scientific basis depended by the present classification, and then the development and innovation presented by the present classification are added: The present classification depends on PQR triangular diagram which is presented by [4], it is a geometric triangular diagram, P represents the degree of planarity of folds, Q represents the degree of domicity of folds, R represents the degree of non-cylindrism of folds. Fold shapes vary infinitely between the three end members: Planes (P); Cylindrical isoclines (Q); and Isoclinal domes (R). Practically, we can locate the shapes of all folds on the triangular diagram (PQR) through the mathematical relation in equation (1), if any two of the three values above are known. Look at the shapes plotted on the triangle PQR in Figure 5. Also, in Figure 6, the geometrical description terms of the folds are illustrated. The varying degree of cylindrical fold tightness is represented along the PQ line in the triangular diagram, while different forms of non-cylindrical folds are represented in the remaining area. Furthermore, the varying degrees of non-cylindrism of isoclinal folds are represented along the QR line in the triangular diagram. The variation of the degrees of the apical angles of domes or basins is represented along the line PR in the triangular diagram. This geometric triangular diagram depends on the interlimb angle value (α), and the hinge angle (β) (Fig. 1), where values (Q, R and P) are calculated by the equations (2, 3 and 4) respectively, and the relations among them should be always integer as in equation (1).

A number of conditions must be satisfied before the fold shape can be plotted on the PQR diagram; the fold must have a planar hinge line surface. Angles (α) and (β) should be represented by only positive values, hence, rubbery and flexible folds are excluded. Also, angle Beta (β) should be greater than angle Alpha (α), so the shapes illustrated in PQR triangular diagram can be presented, (Figure 5).

\[
P + R + Q = 1 \quad \text{(1)}
\]

\[
P = \frac{\alpha}{180} \quad \text{(2)}
\]

\[
R = \frac{(180 - \beta)}{180} \quad \text{(3)}
\]

\[
Q = \frac{180 - [\alpha + (180 - \beta)]}{180} \quad \text{(4)}
\]

Where, the value of P ranged from (0) for narrow folds and (1) for plane folds.

Where, the value of R ranged from (0) for cylindrical fold and (1) for totally non-cylindrical fold.

Rearranging equation (1) by compensating values P and R in equations (1 and 2), we get the value of Q from the following equation:

Where, Q ranges between (0) and (1) which represents the degree of domicity of folds.
Figure 5. Diagrammatic representation of fold shapes on the PQR diagram [4]

Figure 6. Descriptive terminology of fold shapes on the PQR diagram [4]

The classification of [4] was based on the analysis of the geometrical description of each non-cylindrical fold individually and separately, and discusses all its aspects to show integrated results. Although, a complete description for the folded surfaces around the three hinges is difficult, because the value of deviation angle of the axial surface is between ($0^\circ$) and ($180^\circ$) (Figure 7).

Figure 7. Illustrates the positions of different forms of folds on the PQR triangular diagram with representing the factor (S) for these folds due to [4]

The value of the factor (S) can be obtained from the equation (5), through identifying the value of angle ($\gamma$) which is obtained on the axial surface at its coup point. The factor (S) is represented on PQR diagram as a circle surrounds the point which represented the fold, the diameter of a circle is proportional to the value of (S) by either a selected measure or by the degree of angle ($\gamma$) [4] (Figure 8).
The present classification based on the fact that the new factor (N) should be calculated along with the value of the factor (S) through calculating angles wavy (W) and Gamma (γ) respectively. The angle (W) is obtained when the axial surface a side curvature along the hinge line of fold (Fig. 4), it is the measure of the angle within the hinge line surface which is horizontally perpendicular to the axial surface at the point of side wavy curvature (Fig. 4), in order to identify the non-plane axial surfaces in non-cylindrical folds. Angles (β) and (W) should not be measured at the same point. Angle (W) is described in term of the new factor (N) which is calculated from the following equation (6).

\[ N = \frac{180 - W}{180} \]  

The value of the new (N) varies between the (0) for planar non-cylindrical folds and (1) for non-plane non-cylindrical folds, that have a wavy curvature of the axial surface along the hinge line surface. The value of (N) is represented on PQR diagram as a square around the point which represented the fold, and the side length of the square is proportional to the value of (N), by either a selected measure or by the degree of angle (W) (Figure 9), thus, we would have a representation of the non-cylindrical fold shapes on the triangular diagram based on the values of angle (α) and (β), with a visual evaluation to determine the degree of non-planar axial surface.

\[ S = \frac{180 - \gamma}{180} \]  

3. Results and Discussion

3.1 Applications of the PQR diagram and discussion

The triangular diagram is used to classify three-dimensional forms for cylindrical and non-cylindrical folds, exist in nature independently and individually. No information is displayed by the triangular diagram about the directions of structural elements of folds, while it is able to distinguish and describe non-cylindrical folds using self-values methods to analyze the database of directions according to [11]. To indicate the accuracy of this new classification in its overall processing and giving the real geometrical description and names for all non-cylindrical folds, the new classification was applied the anticline Mak-hul mountain fold as an example in nature, then performed a comparison with the application of classification [4] on the same fold, and then the results of both classifications are discussed.

Through studying the geometry of Mak-hul fold, the following mathematical information was obtained by [16] as the following:

- The value of the interlimb angle between the fold sides (α) = 103°
- The value of hinge angle (β) = 170°
- The value of the angle (γ) = 180°
- The value of the angle (W) = 132°

The following equations (7, 8, 9, 10 and 11) were used to obtain the values (P, R, Q, S and N) respectively.
According to the above, the obtained values for the Mak-hul mountain fold, which used in this new classification, it gave accurate and comprehensive results about the geometrical description and represented as Mak-hul fold is considered as a non-cylindrical with a non-plane axial surface (Figure 10). But when the same mathematical values above are used in the classification [4] and applied to the Mak-hul mountain fold, without using the value of angle (W) and new factor (N) mentioned in the equation (11), the following results are obtained: Mak-hul fold is considered as a non-cylindrical with plane axial surface (Figure 11).

The result shown by [4] does not give the proper description and geometrical classification of Mak-hul fold, which is one of the many examples that exist in nature independently and individually, while this new classification gives correct and accurate results.

Considering the different results between the two classifications in the geometric description for the Mak-hul fold, to indicate which classification is more accurate and comprehensive.
Figure 12. A Structural and Topographical map of Mak-hul’s fold showing six cross sections

The other methods were used to the geometrical analysis of this fold, one of these methods was to use the Pi diagram to drop the poles on the axial surface of fold, as well as to use Pi diagram to drop the directions of hinge line in six cross sections [1 (A-B); 2 (C-D); 3 (E-F); 4 (G-H); 5 (I-J); 6 (K-L)] distributed along the axis of fold, and for a distance of (37) km as shown in Figure 12, where the plunge of the fold starts from Fatha region near the town of Baiji heading northern Iraq, and the fold ends near the town of Al-Shirqat [16]. As the results have shown that the axial surface of Mak-hul fold is non-plane. According to the results of the Pi diagram of dropping the poles set on the axial fold surface, also the Pi diagram of dropping the directions of the hinge line in the same six cross sections (Figure 13) and (Figure 14), in addition, the direction readings of the rocky layers on both sides of the fold and along it showed the clear deviation in the hinge line of fold and its axial surface between the area of sections; (E-F), (G-H), (I-J) & (K-L) (Figure 12).
4. Conclusions

It can be concluded that classification of [4] is one of the most important geometric classifications of non-cylindrical folds in nature on a large scale, independently and individually, which is based on values and mathematical factors that give accurate geometric description and classification. However, because of the gaps and deficiencies in applying it on some non-cylindrical folds which made it unintegrated, and needs further developments to address inaccuracies resulting from applying it. The use of a new factor and angle for the first time in the present classification resulted in the emergence of a new geometric classification that is capable of describing and naming all types of non-cylindrical folds which have plane axial surfaces or non-plane axial surfaces which are present in nature in large scales independently and individually. This new innovation has been achieved after a deep and continuous research and investigation about the best and the most common geometric classifications for non-cylindrical folds. After an important development made on classification of [4], which represented through introducing mathematical values as the new factor New (N) and the new angle Wavy (W), to measure and determine the non-plane axial surface for the non-cylindrical folds, when the axial surface as a side wavy curvature along the hinge line of fold.

The achieved results through applying the present classification indicate that it is able to give the correct geometric naming and description to the degree of planarity or non-planarity of axial surface for the non-cylindrical folds. Therefore, it can give these folds a geometrical classification of a high accuracy that were not found in previous geometrical classifications.

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