Quantum Logic Gates using q-deformed Oscillators

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Abstract

We show that the quantum logic gates, viz. the single qubit Hadamard and Phase Shift gates, can also be realised using q-deformed angular momentum states constructed via the Jordan-Schwinger mechanism with two q-deformed oscillators.

Keywords: quantum logic gates ; q-deformed oscillators ; quantum computation

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1. Introduction

Quantum logic gates are basically unitary operators (Refs 1-4 and references therein). There are two gates, the Hadamard and Phase Shift gates, which are sufficient to construct any unitary operation on a single qubit\(^5-7\). These gates are constructed using the "spin up" and "spin down" states of \(SU(2)\) angular momentum i.e., the two possible states of a qubit are usually represented by "spin up" and "spin down" states. In this work we show that the Hadamard and Phase Shift gates can also be realised with \(q\)-deformed angular momentum states constructed via Jordan-Schwinger mechanism with two \(q\)-deformed oscillators. We employ the technique of harmonic oscillator realisation of \(q\)-oscillators\(^12-17\).

The motivation of our work comes from the fact that there exists a non-trivial generalisation\(^12-13\) of the harmonic oscillator realisation of \(q\)-oscillators. This generalised scheme allows us to set up an alternate quantum computation formalism at the level of choosing the two basis states. Consequently, this formalism is more general and contains the currently used formalism in quantum computation as a special case, i.e. for \(q = 1\). Let us clarify this further. \(a_q^{\dagger}\) and \(a_q\) are the creation and annihilation operators for \(q\)-oscillators while those for the usual oscillators are \(a^{\dagger}\) and \(a\). These satisfy (with \(q = e^s\), \(0 \leq s \leq 1\)):

\[
\begin{align}
    a_q a_q^{\dagger} - q a_q^{\dagger} a_q &= q^{-N} \ ; \ N^{\dagger} = N \\
    [N, a_q] &= -a_q \ ; \ [N, a_q^{\dagger}] = a_q^{\dagger} \ ; \ a_q^{\dagger} a_q = [N] \ ; \ a_q a_q^{\dagger} = [N+1] \\
    a_q f(N) &= f(N+1) a_q \ ; \ a_q^{\dagger} f(N) = f(N-1) a_q^{\dagger}
\end{align}
\]

where \([x] = (q^x - q^{-x})/(q - q^{-1})\) and \(N\) is the number operator (eigenvalue \(n\)) for the \(q\)-deformed oscillators and \(f(N)\) is any function of \(N\). The above
equations are true for both real and complex $q$. However, we shall confine ourselves to real $q^{10,11}$. The harmonic oscillator realisation of quantum oscillators$^{12,13}$ gives the relationships between $a_q, a_q^\dagger$ and $a, a^\dagger$ as

$$a_q = a\sqrt{\frac{q^N\psi_1 - q^{-N}\psi_2}{\hat{N}(q - q^{-1})}}; \quad a_q^\dagger = \sqrt{\frac{q^N\psi_1 - q^{-N}\psi_2}{\hat{N}(q - q^{-1})}}a^\dagger$$

(2a)

$$N = \hat{N} - (1/s)\ln \psi_2$$

(2b)

$\hat{N}$ is the number operator for usual oscillators with eigenvalue $\hat{n}$; and $\psi_1, \psi_2$ are arbitrary functions of $q$ only with $\psi_{1,2}(q) = 1$ for $q = 1$. The presence of these arbitrary functions allows an alternative formalism:

**Case I** : If all these arbitrary functions are unity, then $N = \hat{N}$. This means that if states are labelled by their occupation numbers, deformed states cannot be distinguished from the non-deformed (i.e. usual) oscillator states. This is the realm of quantum computation with the the usual ”spin-up” and ”spin-down” states and there is no theoretical gain by choosing deformed oscillator states as basis for quantum computation.

**Case II** : However, the harmonic oscillator realisation (2) is general if the arbitrary functions of $\psi_i(q), i = 1, 2$ are not all equal to unity. Let us take $\psi_1 = \psi_2 = \psi(q)$. Now $N = \hat{N} - (1/s) \ln \psi(q)$ (equation (2b)). Hence at the occupation number level states are different as the eigenvalues of the number operator of usual oscillator states (i.e. usual quantum computation) and the eigenvalues of the number operator of deformed oscillator states are now related by $n = \hat{n} - (1/s) \ln \psi(q)$. This shows up in the Jordan-Schwinger construction of angular momentum states and the states in the two cases will be distinguishable through the function $\psi(q)$. So there is this extra functional parameter $\psi(q)$ which is potentially ideal for experimental realisations.
2. Jordan-Schwinger construction for qubits

We now discuss how qubits look in the Jordan-Schwinger construction where two independent oscillators are used to construct the generators of angular momentum.

(a) States are defined by the total angular momentum $j$ and $z$-component of angular momentum $j_z$,

$$|jm> = \frac{(a_1^\dagger)^{j+m}(a_2^\dagger)^{j-m}}{[(j+m)!(j-m)!]^{1/2}}|\phi>$$  \hspace{1cm} (3)

$|\phi> \equiv |0>_{>1} |0>_{>2}$ is the ground state ($j = 0, m = 0$). $|0>_{i}, i = 1, 2$ are the oscillator ground states. $j = (n_1 + n_2)/2$ ; $m = (n_1 - n_2)/2$ and $n_1, n_2$ are the eigenvalues of the number operators of the two oscillators.

(b) For qubits, the only possible states correspond to $(n_1 + n_2)/2 = 1/2$ i.e. $n_1 = 1-n_2$. States characterised by these are therefore $(n_1 + n_2)/2 \equiv |n_1 > |n_2 > \delta_{n_1+n_2,1}$. Since $j = 1/2$ for both qubit states, we suppress $j$ and write the states as

$$|m> = \frac{(a_1^\dagger)^{1/2+m}(a_2^\dagger)^{1/2-m}}{[(1/2 + m)!(1/2 - m)!]^{1/2}}|\phi>$$  \hspace{1cm} (4a)

$$|-m> = \frac{(a_1^\dagger)^{1/2-m}(a_2^\dagger)^{1/2+m}}{[(1/2 + m)!(1/2 - m)!]^{1/2}}|\phi>$$  \hspace{1cm} (4b)

Equivalently, in terms of $n_1, n_2$ these are

$$|n_1 - 1/2 > = \frac{(a_1^\dagger)^{n_1}(a_2^\dagger)^{1-n_1}}{[(n_1)!(1-n_1)!]^{1/2}}|0>$$  \hspace{1cm} (4c)

$$|- (n_1 - 1/2) > = \frac{(a_1^\dagger)^{1-n_1}(a_2^\dagger)^{n_1}}{[(n_1)!(1-n_1)!]^{1/2}}|0>$$  \hspace{1cm} (4d)
(c) In this formalism the two basis states of a single qubit state are \(|1>\equiv |\text{up}>\) state and \(|0>\equiv |\text{down}>\) state

\[
|1> = |1/2, 1/2> = a_1^{\dagger}|\tilde{0}>_1 |\tilde{0}>_2
\]

\[
|0> = |1/2, -1/2> = a_2^{\dagger}|\tilde{0}>_1 |\tilde{0}>_2
\]

(d) The physical meaning of the notation is as follows. The \(|1>\) angular momentum (spin ”up”) state can be constructed out of two oscillator states where the first oscillator state has occupation number 1 while the other has occupation number 0. The \(|0>\) (spin ”down”) state corresponds to the first oscillator having occupation number 0 and the second oscillator having occupation number 1. We thus can write any qubit state in terms of harmonic oscillator states. The column vectors denoting these two basis states may be taken as

\[
|1> = \begin{pmatrix} 1 \\ 0 \end{pmatrix} ; \quad |0> = \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

So we write

\[
|x> = (a_1^{\dagger})^x (a_2^{\dagger})^{1-x} |\tilde{0}>
\]

(Note \(|0>\) represents one of the two possible qubit states while \(|\tilde{0}>)\) represents oscillator ground state i.e. occupation number 0; \(|\tilde{1}>)\) represents an oscillator state with occupation number 1; \(|\tilde{2}>)\) represents oscillator state with occupation number 2 etc. This notation is to avoid confusion).

3. The Hadamard transformation for q-deformed qubits

First consider the case of an ordinary qubit. The Hadamard transformation on a single qubit state \((x = 0, 1)\) is\(^5\)\(^7\) (modulo a normalisation factor of \(1/\sqrt{2}\))

\[
|x> \rightarrow \ (-1)^x |x> + |1-x>
\]
Using (4c), (4d), (5) in (6) gives:

$$|n_1 - 1/2 > ightarrow (-1)^{n_1}|n_1 - 1/2 > + |1/2 - n_1 >$$  (7)

So $n_1 = 0 \Rightarrow |-1/2 > ightarrow | -1/2 > + |1/2 >$ and $n_1 = 1 \Rightarrow |1/2 > ightarrow | -1/2 > - |1/2 >$.

Now consider q-deformed qubits. For states, we have kets $| >$ (or bras $< |$) for the usual oscillator states, while kets $| >_q$ (or bras $<_q |$) denote the corresponding q-deformed states. The general angular momentum q-deformed state in terms two q-deformed oscillators is$^{8,9}$

$$|j m >_q = (a_{1q}^\dagger)^{n_1}(a_{2q}^\dagger)^{n_2}(|\phi >_q)/(n_1)!(n_2)!^{1/2}$$  (8a)

$$|j - m >_q = (a_{1q}^\dagger)^{n_2}(a_{2q}^\dagger)^{n_1}(|\phi >_q)/(n_1)!(n_2)!^{1/2}$$  (8b)

where $|\phi >_q \equiv |\tilde{0} >_q = |\tilde{0} >_{1q} |\tilde{0} >_{2q}$ is the ground state corresponding to two non-interacting q-deformed oscillators. Ground states of q-oscillators in the coordinate representation were studied in Refs. 8 and 9. In our notation a qubit state has either (a) $n_1 = 0, n_2 = 1$ or (b) $n_1 = 1, n_2 = 0$. Thus from (8a), (8b) the q-deformed qubit would look like

$$|n_1 - 1/2 >_q = (a_{1q}^\dagger)^{n_1}(a_{2q}^\dagger)^{1-n_1}(|\tilde{0} >_q)/(n_1)!(1-n_1)!^{1/2}$$  (9a)

$$|-(n_1 - 1/2) >_q = (a_{1q}^\dagger)^{1-n_1}(a_{2q}^\dagger)^{n_1}(|\tilde{0} >_q)/(n_1)!(1-n_1)!^{1/2}$$  (9b)

So the Hadamard transformation for q-deformed state is

$$|n_1 - 1/2 >_q \rightarrow (-1)^{n_1}|n_1 - 1/2 >_q + |1/2 - n_1 >_q$$  (10)
The usual Hadamard transformation for the Jordan-Schwinger construction with usual oscillators is

\[
\frac{(a_1^\dagger)^{n_1}(a_2^\dagger)^{n_2}}{(n_1!n_2!)^{1/2}} \phi \rightarrow (-1)^{(n_1-n_2+1)/2} \frac{(a_1^\dagger)^{n_1}(a_2^\dagger)^{n_2}}{(n_1!n_2!)^{1/2}} \phi \\
+ \frac{(a_1^\dagger)^{n_2}(a_2^\dagger)^{n_1}}{(n_1!n_2!)^{1/2}} \phi 
\]

So the Hadamard transformation in terms of the q-deformed oscillators is:

\[
\frac{(a_1^\dagger_{1q})^{n_1}(a_2^\dagger_{2q})^{n_2}}{([n_1]![n_2]!)^{1/2}} \phi >_q \rightarrow (-1)^{(n_1-n_2+1)/2} \frac{(a_1^\dagger_{1q})^{n_1}(a_2^\dagger_{2q})^{n_2}}{([n_1]![n_2]!)^{1/2}} \phi >_q \\
+ \frac{(a_1^\dagger_{1q})^{n_2}(a_2^\dagger_{2q})^{n_1}}{([n_1]![n_2]!)^{1/2}} \phi >_q
\]

(11)

Note that \(n_1, n_2\) is always 0 or 1 so as to correspond to the qubit. Hence the q-numbers \([n_1], [n_2]\) are always the usual numbers \(n_1, n_2\) in our case. So (12a) becomes

\[
\frac{(a_1^\dagger_{1q})^{n_1}(a_2^\dagger_{2q})^{n_2}}{(n_1!n_2!)^{1/2}} \phi >_q \rightarrow (-1)^{(n_1-n_2+1)/2} \frac{(a_1^\dagger_{1q})^{n_1}(a_2^\dagger_{2q})^{n_2}}{(n_1!n_2!)^{1/2}} \phi >_q \\
+ \frac{(a_1^\dagger_{1q})^{n_2}(a_2^\dagger_{2q})^{n_1}}{(n_1!n_2!)^{1/2}} \phi >_q
\]

(12b)

Using (1),(7), and \(n_1 + n_2 = 1\) in (12b) gives:

\[
[F_1(\hat{N}_1, q)a_1^\dagger]^{n_1}[F_2(\hat{N}_2, q)a_2^\dagger]^{1-n_1}\phi >_q \rightarrow \]

\[
(-1)^{n_1}[F_1(\hat{N}_1, q)a_1^\dagger]^{n_1}[F_2(\hat{N}_2, q)a_2^\dagger]^{1-n_1}\phi >_q \\
+ [F_1(\hat{N}_1, q)a_1^\dagger]^{1-n_1}[F_2(\hat{N}_2, q)a_2^\dagger]^{n_1}\phi >_q
\]

(13a)

where

\[
F_1(\hat{N}_1, q) = \sqrt{\frac{q^{\hat{N}_1}\psi_1 - q^{-\hat{N}_1}\psi_2}{\hat{N}_1(q - q^{-1})}}, \quad F_2(\hat{N}_2, q) = \sqrt{\frac{q^{\hat{N}_2}\psi_3 - q^{-\hat{N}_2}\psi_4}{\hat{N}_2(q - q^{-1})}}
\]

(13b)
For reasons already stated, the eigenvalues of the number operators are constrained to satisfy $n_1 + n_2 = 1$ and the only possibilities are $n_1 = 0, n_2 = 1$ or $n_1 = 1, n_2 = 0$. The same restrictions also apply to usual (i.e. undeformed) oscillators. Hence we restrict the hatted number operators, $\hat{N}_1$ and $\hat{N}_2$, by $\hat{N}_1 + \hat{N}_2 = I$ where $I$ is the identity operator.

In (13b), $\psi_i(q), i = 1, 2, 3, 4$ are arbitrary functions of $q$ only and $\psi_i(1) = 1$. We take $\psi_1 = \psi_3$ and $\psi_2 = \psi_4$. Also $\hat{N}_1 + \hat{N}_2 = I$. Under these circumstances we drop the suffixes from $F_1$ and $F_2$ and take the functional forms to be the same. This means that if one oscillator has the number operator as $\hat{N}$, the other oscillator should be restricted to that described by the number operator $I - \hat{N}$ ($I$, the identity operator). The eigenvalues are $\hat{n}$ and $1 - \hat{n}$ respectively ($\hat{n} = 0, 1$). The harmonic oscillator realisations of the $q$-oscillators are described by the functions $F(\hat{N}, q)$ and $F(1 - \hat{N}, q)$. Then (13a), with $\hat{n}$ replacing $\hat{n}_1$ and using (1c), becomes

$$A|\eta > \rightarrow (-1)^n A|\eta > + B| - \eta >$$

where

$$|\eta >= \frac{(a_1^\dagger)^n(a_2^\dagger)^{1-n}}{(n!(1-n)!)^{1/2}}|\phi > q; \quad - \eta >= \frac{(a_1^\dagger)^{1-n}(a_2^\dagger)^n}{(n!(1-n)!)^{1/2}}|\phi > q$$

and $A = F(\hat{N}, q)^n F(1 + n - \hat{N}, q)^{1-n}$ and $B = F(1 - \hat{N}, q)^{1-n} F(2 - n - \hat{N}, q)^n$. For $n = 0$ this means

$$F(\hat{N}, q)a_{2q}^\dagger|0 >_{1q} |0 >_{2q}$$

$$\rightarrow F(1 - \hat{N}, q)a_{2q}^\dagger|0 >_{1q} |0 >_{2q} + F(\hat{N}, q)a_{1q}^\dagger|0 >_{1q} |0 >_{2q}$$

(16a)
For $n = 1$,

$$F(\hat{N}, q)a_{1q}^\dagger|0 >_{1q}|0 >_{2q}$$

$$\rightarrow -F(\hat{N}, q)a_{1q}^\dagger|0 >_{1q}|0 >_{2q} + F(1 - \hat{N}, q)a_{2q}^\dagger|0 >_{1q}|0 >_{2q} \quad (16b)$$

Obviously $(16a, b)$ would be indistinguishable from the usual Hadamard transformation for $n = 0, 1$ if and only if $F^{-1}(\hat{N}, q)F(1 - \hat{N}, q) = 1$. This operator equation written in terms of the eigenvalues $\hat{n}$ and $1 - \hat{n}$ means

$$\frac{\psi_1(q)}{\psi_2(q)} = \frac{(q^{-\hat{n}} - \hat{n}q^{-\hat{n}} - \hat{n}q^{1-\hat{n}})}{(q^{\hat{n}} - \hat{n}q^{\hat{n}} - \hat{n}q^{1-\hat{n}})} \quad (17)$$

It is simple to check that (17) is always true for $\hat{n} = 0$ and $\hat{n} = 1$ if $\psi_1(q) = \psi_2(q) = \psi(q)$ (say). Therefore the Hadamard transformation can be realised with deformed qubits.

**Case I**

There is only one arbitrary function $\psi(q)$ left and we now discuss its importance. First note that for $\psi_1 = \psi_2 = \psi_3 = \psi_4 = 1$, $(2a, b)$ do not have any arbitrary parameter and just relates the operators $a, a^\dagger$ with $a_q, a_q^\dagger$. Also from $(2b)$ we then have $N = \hat{N}$. This means that at the occupation number level the deformed states cannot be distinguished from the usual states. So this is the realm of quantum computation with the usual "spin-up" and "spin-down" states.

**Case II**

But $(2)$ is general if the arbitrary functions $\psi_i(q), i = 1, 2, 3, 4$ are not all equal to unity. Then $N = \hat{N} - (1/s) \ln \psi(q) \quad [(2b)]$. Hence states labelled by the occupation number are different as the eigenvalues of the number operator of usual oscillator states (i.e. usual quantum computation) and
the eigenvalues of the number operator of deformed oscillator states are now related by \( n = \hat{n} - (1/s) \ln \psi(q) \). This would show up in the Jordan-Schwinger construction.

4. Relation between the states in Case I with those in Case II

Let us denote the angular momentum states in Case I by \(| I \rangle\), and those in Case II by \(| II \rangle\). Remembering that we have suppressed \( j = (n_1 + n_2)/2 \) in the notation (since it is always 1/2) and \( m = (n_1 - n_2)/2 = n_1 - (1/2) \) and relabeling \( n_1 \) as \( \hat{n} \) etc. we have for Case I

\[
| n - 1/2 >_I = | n >_1 | 1 - n >_2 = | \hat{n} >_1 | 1 - \hat{n} >_2
\]

or as \( n = 0, 1 \) and \( n = \hat{n} \), the two states are

\[
| - 1/2 >_I = | \tilde{0} >_1 | \tilde{1} >_2 , \quad | 1/2 >_I = | \tilde{1} >_1 | \tilde{0} >_2
\]

In Case II, the two states are

\[
| n' - 1/2 >_{II} = | n' >_1 | 1 - n' >_2 = | \hat{n} - (1/s) \ln \psi >_1 | 1 - \hat{n} + (1/s) \ln \psi >_2
\]

\[
| - (n' - 1/2) >_{II} = | 1 - n' >_1 | n' >_2 = | 1 - \hat{n} + (1/s) \ln \psi >_1 | \hat{n} - (1/s) \ln \psi >_2
\]

However, here \( n' = \hat{n} - (1/s) \ln \psi(q) \), and the two states are

\[
| - 1/2 >_{II} = | \tilde{0} >_1 | \tilde{1} >_2 = | \hat{n} - (1/s) \ln \psi >_1 | 1 - \hat{n} + (1/s) \ln \psi >_2
\]

\[
| 1/2 >_{II} = | \tilde{1} >_1 | \tilde{0} >_2 = | 1 - \hat{n} + (1/s) \ln \psi >_1 | \hat{n} - (1/s) \ln \psi >_2
\]

**Consistency** now demands that

\[
\hat{n} = (1/s) \ln \psi(q)
\]
5. An alternate formalism for quantum computation

First consider Case I, i.e. (18). It is immediately evident that so far as quantum computation is concerned nothing much is gained by choosing these states because the eigenvalue of the number operators for usual and deformed oscillators are identical. So it will be impossible to distinguish the states in Case I from those of usual oscillators at the level of experimental realisations or consequences.

Now consider Case II, i.e. Eq. (19).

(a) We have (for \( n' = \hat{n} - (1/s)ln\psi(q) \))

\[
|n'>_{II} = \psi(q)^{(n_1+n_2)/2}(a_{1q}^\dagger a_{2q}^\dagger)^{n_2}0 >_{II} = \psi(q)^{(n_1+n_2)/2}|n>_{I} = \psi(q)^{1/2}|n>_{I}
\]

Therefore,

\[
\frac{n'|n'>_{II}}{n|n>_{I}} = \psi(q)
\]

This means that the states in Case II can be distinguished from those in Case I or from the usual oscillator states at the level of experimental realisations or consequences.

(b) \( \hat{n} = (1/s)ln\psi(q) \) means \( \psi(q) = e^{s\hat{n}} = q^{\hat{n}} \), \( \hat{n} \) is the eigenvalue of the number operator and hence \( \hat{n} \geq 0 \) while \( 0 < s < 1 \). Here \( \hat{n} \) cannot be zero because then we will have \( \psi(q) = 1 \) i.e. Case I. So here \( \hat{n} > 0 \). This means that the deformed states in Case II can be related to any usual oscillator
states with occupation number greater than zero. This is a very rich theoretical structure and opens up enormous possibilities for experimental realisations and consequences by suitably choosing the two parameters \( \hat{n} \) and \( s \).

6. The Phase Shift transformation

Let us now consider the Phase Shift transformation of qubit states defined as usual: \( |x\rangle \rightarrow e^{ix\theta}|x\rangle \) which in our notation is \( |n-\frac{1}{2}\rangle \rightarrow e^{i\theta}|n-\frac{1}{2}\rangle \) where \( \theta \) is the phase shift. So denoting initial and final states by \( i, f \)

\[
|n-\frac{1}{2}\rangle_{I} = e^{i\theta}|n-1/2\rangle_{Ii}
\]
\[
|n-\frac{1}{2}\rangle_{II} = e^{i\theta}|n-1/2\rangle_{IIi} = q^{\hat{n}/2}|n-1/2\rangle_{IIi} = q^{\hat{n}/2}|n-1/2\rangle_{IIf}
\]  

(23a)  

Then for \( n = 0 \), \( |n-\frac{1}{2}\rangle_{I,II} \rightarrow |n-\frac{1}{2}\rangle_{I,II} \) and for \( n = 1 \), \( |n-\frac{1}{2}\rangle_{I,II} \rightarrow e^{i\theta}|n-\frac{1}{2}\rangle_{I,II} \). Hence the phase shift transformation can also be implemented for a single deformed qubit. Moreover, note that the two cases I and II can be distinguished from the fact that

\[
\frac{IIf < n - 1/2 |n - 1/2 >_{II} f}{If < n - 1/2 |n - 1/2 >_{IF}} = \psi(q) = q^{\hat{n}} = e^{s\hat{n}}
\]

(24)

So here also the presence of the function \( \psi(q) = q^{\hat{n}} = e^{s\hat{n}} \) gives two parameters (a) a positive integer \( \hat{n} > 0 \) and (b) a positive fraction \( s \) where \( 0 < s < 1 \) that can be exploited for both experimental realisations and consequences.

7. Conclusion

Thus, we have shown that so far as realisation of the single qubit Hadamard and Phase Shift gates are concerned, \textit{q-deformed qubit states can also be used}. A principal advantage over the usual formalism is the occurrence of an arbitrary function of the deformation parameter \( q = e^{s} \). This function
is \( \psi(q) = q^{\hat{n}} = e^{s\hat{n}} \). So we have two free parameters (i) \( s \), \( 0 < s < 1 \) and (ii) \( \hat{n} > 0 \). These can be used to determine whether observed experimental realisations of theoretical predictions obtained from the usual formalism are fully satisfactory or not. If not, then these parameters can be exploited to see whether corrections to the results can be calculated. These aspects require further investigations, but the very possibility that quantum computation may also be done using \emph{q-deformed qubits} is indeed appealing. Whether the difference between quantum computation using usual spin states and quantum computation using q-deformed qubit states is susceptible to experimental observations in the NMR realisation of quantum logic gates [18] is an interesting problem in its own right.

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