Tunneling edges at strong disorder

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Scattering between edge states that bound one-dimensional domains of opposite potential or flux is studied, in the presence of strong potential or flux disorder. A mobility edge is found as a function of disorder and energy, and we have characterized the extended regime. In the presence of flux and/or potential disorder, the localization length scales exponentially with the width of the barrier. We discuss implications for the random-flux problem.

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The remarkable transport properties of a 2d electron gas in a strong perpendicular magnetic field are well-known: jumps in Hall resistance and peaks in longitudinal resistance occur as the Fermi level crosses the single energy, at the center of the Landau band, where extended states are believed to lie.

Experiments and numerical studies show that, as the Fermi level $E_c + \delta E$ approaches the center of the band, $E_c$, the correlation length $\zeta$ diverges: $\zeta \propto |\delta E|^{-\nu_q}$, where $\nu_q \approx 7/3$. Disorder plays a crucial role in all models for this transition. In a strong magnetic field $B$, eigenstates are confined to equal-energy contours of a disordered potential, slowly-varying on scales of order the magnetic length $l_B = (\hbar c/eB)^{1/2}$. For a generic random, symmetric potential $V(\vec{r})$ with characteristic length $\lambda_0$, each of these equipotentials, for energies not too close to $E_c = \langle V \rangle$, will traverse a region of finite size, exceeding the cluster diameter $a(\delta E)$ in cross-section with exponential rarity. When $\delta E \to 0$, long-range spatial correlations emerge in the equipotentials as the cluster diameter diverges at the continuum percolation transition.

A model for this transition was proposed by Trugman, who argued that one might neglect tunneling among distinct equipotentials because of the small overlap of their respective wave-functions. If this omission were valid, then the localization length would diverge as the spatial extent of the percolating eigenfunctions. The correlation exponent $\nu_q$ for the classical percolation transition is known to be $4/3$.

Mil’nikov and Sokolov attempted to calculate the correction to Trugman’s classical picture arising from quantum tunneling between equipotentials. They argued that the dominant contribution of tunneling derived from the saddle-points of the potential along the largest eigenstates. Since these saddle-points occur between longest equipotentials that traverse distances of order $a(\delta E) = |\delta E|^{-4/3}$, and the correction they obtained from tunneling was inversely proportional to $|\delta E|$, MS suggested that $7/3 = 4/3 + 1$.

Their proposal, even were it correct, is not supported by their argument. While saddle-points between longest equipotentials are rare, each equipotential is also a (bond) percolation hull, bounding a complex, fractal network of wave-functions with energy $E_c$. MS offered no justification for their neglect of the scattering between the hull and this internal network, which could lead to localization via back-scattering.

The most widely accepted model for the Hall transition, the Chalker network model, yields numerically a $\nu_q$ close to $7/3$. The Chalker model consists of a regular lattice of tunneling saddle-points, to be contrasted with the ‘topologically disordered’ network of equi-contours that underlies the classical percolation transition. This contrast with classical percolation has lead to the claim that quantum tunneling fundamentally alters the classical transition, and that the relation $\nu_q \approx 1 + \nu_c$ is merely coincident.

This issue assumes special importance for understanding eigenstates in a random-flux background (with vanishing mean flux). If quasi-classical arguments applied, one would expect an extended state in the presence of random-flux that exhibited much the same properties as obtained for potential disorder; however, with appropriate modification to incorporate the vanishing net chirality of the random-flux problem, the Chalker model displays no extended states.

Thus, the Chalker model appears to suggest that a picture of of wave-functions, whose spatial extent is dominated by classical equipotentials, is naive. This state of affairs prompted us to reconsider tunneling among distinct equipotentials. In particular, we would like to understand when it may be possible to think of equipotentials as distinct classical units, and to calculate transport properties of a network of eigenstates by perturbing around their spatial configurations.

Such an understanding might be relevant to a variety of physical systems: not only to the QHE and the random-flux problem, but also to gauge-field theories of flux phases, where boundaries between distinct flux domains create a network of one-dimensional edges.
Edges created by nonhomogeneous magnetic field in the absence of disorder or in the context of the quantum Hall effect with small disorder have been studied elsewhere; here we study tunneling between equipotentials in the presence of strong potential and/or flux disorder.

We construct a pair of one-dimensional wave-functions on a strip of length $L$ and width $M$ lattice constants $a$. Our lattice Hamiltonian is:

$$H = \sum_i V_i |i\rangle\langle i| + \left\{ \sum_{i,j} t_{ij} |i\rangle\langle j| + h.c. \right\}$$  (1)

where $V_i$ denotes the on-site energy, and the hopping matrix elements $t_{ij}$ vanish except for nearest neighbors. Finite magnetic flux is included by allowing complex $t_{ij}$, where the local phase of $t_{ij}$ is determined by the flux enclosed within a square plaquette of the lattice. (Energies are in units of $|t| = 1$, and we also take the lattice constant $a = 1$). We calculate localization lengths for this system numerically by computing the largest eigenvalue of a product of $L$ transfer matrices, each of which adds a single row of $M$ lattice sites at the end of the strip, and finite-size scaling. These methods are now standard, although to our knowledge they have not been applied before to the geometries we discuss here.

A pair of parallel edges, separated by roughly $M/2$ lattice sites (one edge centered between sites $M/2$ and $M/2 + 1$, the other between sites $M$ and 1) was constructed in several ways ($i$ denotes transverse coordinate on the strip):

1(a) Two parallel strips of flux $B_0$ with equal magnitude and opposite sign: $B_i = -B_0 \theta(M/2 - i) + B_0 \theta(i - M/2)$, random potential $V$ is chosen on the interval $[-W/2,W/2]$.

1(b) Two parallel stripes of uniform potential $V_0$ and width $M/2$, with equal magnitude and opposite sign in uniform flux $B_0$. Superposed on the potential stripes is random potential, chosen from $[-W/2,W/2]$.

2(a) Two parallel stripes of flux, with flux through plaquette $i$ chosen randomly from $[0,B_0]$ for $i \leq M/2$, and from $[-B_0,0]$ for $i > M/2$.

2(b) Two parallel stripes of uniform potential $V_0$ and width equal in magnitude and opposite sign. Superposed on the potential stripes was random flux with finite mean $B_0$, chosen from the interval $[B_0 - \delta B, B_0 + \delta B]$; $|\delta B| < |B_0|$.

All random quantities were chosen uniformly on the specified intervals, and independently on each lattice site. Periodic boundary conditions were imposed, except where explicitly noted below. We have concentrated in the regime of strong disorder, with $W$ on order of the inter-Landau subband spacing.

Flux and potential configurations differing locally from those listed above over distances of a few lattice constants did not affect the scalings we discuss below. We also allowed our edges to meander randomly in the $y$ direction, and their transverse separation to fluctuate. Provided that the $y$ coordinate of an edge was allowed to vary by at most a few lattice constants for each lattice site traversed in the longitudinal direction, our scalings similarly remained unchanged from those we found for configurations. We emphasize that our results are insensitive to details. The features of our edges that are important for the following discussion appear to be their low curvature, and their local (mean) flux/potential gradient, which remains constant as we increase the width of the barriers. We studied the dependence of the longitudinal localization length upon $S$, the width of the stripes in units of the lattice constant. Although we have for simplicity described geometries with two stripes only ($S = M/2$), our results are unchanged for multiple pairs of stripes.

Figure 1(a) displays, for geometry (1a), $\lambda_M$ as a function of $E$ and stripe width. The intersections of the curves indicate mobility edges: in the localized regime, $\lambda_M \propto \lambda_\infty$, a constant. For $B_0 = \pi/4$, this mobility edge disappears as indicated by figure 1(b) when the potential disorder $W$ exceeds roughly $W_{cr} \sim 5.5$, yielding localization at all energies. Figure 1(a) suggests that in the extended regime, $\lambda_M \propto M^\alpha$ with $\alpha \approx 2$; however, we found that for larger $M$, $\lambda_M$ increases exponentially, scaling as $\lambda_M \propto M^\alpha \exp(M/\Lambda)$, where $\Lambda$ represents a length scale determined by properties of the bulk. This exponential increase is apparent also for smaller values of disorder ($W \ll W_{cr}$). Scaled curves displaying this behavior are shown in figure 2.

The critical properties of this transition will be discussed elsewhere; our present concern is the scaling with stripe width in the extended regime only.

FIG. 2. Scaled localization length in the extended regime for $W = 2.0$, and different energies: $E = 1.45$ (full squares), 1.475 (diamonds), 1.5 (triangles), 2.8 (open squares), 2.6 (crosses). Curves were collapsed by successive displacement along the horizontal axis; at each step the interpolated (not extrapolated) displacements were minimized by least-squares.

The $M^2$ scaling we find for $M < \Lambda$ differs from that obtained for extended bulk states in potential disorder and uniform flux with periodic boundaries, where $\lambda_M \propto M$ is expected at a band center, and is asymptotically constant elsewhere. (This latter geometry, in contrast to the geometries studied here, is isotropic). A $\lambda_M/M$ increasing with $M$ above the band center has been observed previously for uniform flux, Dirichlet boundary conditions, and potential disorder; however, those studies at-
tempted no quantitative analysis of this phenomenon. We have found numerically that this geometry also yields $M^a \exp(M/L)\text{ scaling}$, and that this behavior is consistent with the earlier published data.

We obtained this exponential scaling for all edge configurations we studied, including (1a, b) and (2a, b). $V_0$ was set to 1.0 or 2.0 in cases (1b), (2b); the values of $B_0$ and $V_0$ were unimportant provided that the Landau subbands were not separated by much less than the characteristic scale of variation of $W$.

When boundary conditions of the form $1a–2b$ or Dirichlet are imposed, chiral states traverse the boundaries. In the absence of edge–edge scattering, extended states propagate along these edges; the only mechanism for localization of the edge wave-functions is scattering with states of opposite chirality at the other edge. We now argue that this inter–edge hopping gives rise to the scaling we have observed.

At the quasi–classical level, an edge state lives on a line of constant potential. For strong disorder, an equipotential can wander into the bulk and meet the opposite edge, yielding backscattering. On an infinite plane, these equipotentials (percolation hulls) circumscribe islands of finite extent, the typical size given by $\delta E$, $\delta E$ the energy of the state with respect to the center of a magnetic subband. At the center of a subband, there exists an infinite equipotential that percolates through the sample. In this picture, the edge–to–edge scattering will depend on the probability of finding islands of size larger than $S$. The exponential rarity of these islands, sufficiently far from the percolation threshold, accounts for the exponential increase in localization length. If the typical island size $\ell$ is $\ell \gg S$, then we expect $\ell$ to display algebraic behavior.

When the disorder is small enough, $W \ll \Delta, \Delta$ the gap between magnetic subbands, edge states of energy $E_0$ sufficiently removed from a magnetic subband center $E_c$ may not be connected to one another through equipotential islands. Thus if $|E_0 - E_c| > W$, our quasi-classical argument that neglects tunneling among distinct equipotentials implies a divergent localization length for finite edge separation $S$. The inclusion of quantum tunneling would restore the finite value of the localization length expected at finite $S$, but the spatial attenuation of quantum scattering between edge states as a function of their separation $S$ is then governed by the magnetic length. The resulting localization lengths are of a completely different order of magnitude than those we find for edge states connected by the disorder–generated quasi–classical equipotentials. Indeed, for sufficiently small $W$, as the Fermi energy $E_0$ crosses into the gap $|E_0 - E_c| > W$ the computed localization length increases abruptly by many orders of magnitude, to values beyond our numerical control.

Lee and Chalker [3] have proposed a variant of the original one-channel network model to study localization in a random–flux background. In order to maintain a vanishing $\sigma_{xy}$, they stipulated that each node describe equivalent tunneling for the two channels of opposite chirality, so that the node is represented by scattering parameters $\theta_1, \theta_2$ satisfying $\sinh(\theta_1) = 1/\sinh(\theta_2)$. They obtain no extended state in this model.

Within the framework of edge–edge scattering described in this paper, their conclusion is readily understood: no configuration of their nodes can be equivalent to a pair of edge states on opposite sides of a long flux barrier. Rather, backscattering can occur at every node with amplitude equal to the forward scattering that occurs at that node. In contrast, our backscattering amplitude is exponentially less than our forward scattering amplitude, for the entire length of the flux barrier. In a random background, this length is set by the quenched flux disorder, i.e. the classical path.

This overall picture supports the general notion, mentioned in our introduction, that the classical paths (equipotentials) make up the dominant contribution to transport, and that tunneling among equipotentials may be viewed as a perturbation of the classical background.

This picture is also consistent with unpublished numerical studies of one-dimensional defects in Chalker lattices [11]. In Chalker networks, local isotropy is maintained by an antiferromagnetic choice of node parameter $\theta$ on alternate sublattices. One can create edges in a network by introducing defects in the antiferromagnetic ordering, forming domain boundaries across which the sublattice alternation is displaced by a half unit–cell. It is found that pairs of linear defects (comparable to the edges we study here) in a one-channel Chalker model display an exponential increase of localization length with increasing separation. Two-channel Chalker models display a richer behavior. Nodes of the form described above $(\sinh(\theta_1) = 1/\sinh(\theta_2))$ never exhibit an extended regime, whereas nodes satisfying $\theta = \theta_1 = \theta_2$ with $\sinh(\theta) = s, 1/s$ on the alternate sublattices show decreasing localization lengths as a function of system width $M$ for $s$ close enough to 1, and exponentially increasing localization lengths as a function of $M$ for $s \geq s_c > 1$. Details will be given elsewhere [11].

In summary, we have described numerical studies of tunneling between one-dimensional edge states created by boundaries between regions of differing potential and/or flux. These systems display distinct regimes as a function of energy and disorder; we have characterized the extended regime, in which localization lengths increase exponentially with the spatial separation of the edges, and we have offered a quasi–classical explanation of this behavior. The fundamental contribution of the classical paths, whether formed by magnetic-flux or potential edges, suggests to us that extended states ought to exist in a random–flux background.
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