Hard exclusive $J/\psi$ leptoproduction on polarized targets

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Abstract:
We consider the exclusive production of $J/\psi$ mesons in polarized virtual-photon–proton collisions. We derive helicity amplitudes for the dominant subprocess $\gamma^* g \rightarrow J/\psi g$ and show that polarization asymmetries vanish in the case of collinear scattering. Thus, contrary to what has been suggested earlier, this process is not a good probe of the polarized gluon distribution $\Delta g(x, Q^2)$ of the proton.

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1 Introduction

Considerable attention has recently been devoted to study exclusive meson production in QCD. In particular, there has been renewed interest \[1, 2\] in the concept of nonforward (also called off-forward, off-diagonal or asymmetric) parton distributions of nucleons. The nonforward distributions are defined as matrix elements of non-local twist-2 QCD string operators between nucleon states of unequal momenta. Thus, they present a generalization of the concept of ordinary parton distributions on one hand and of nucleon form factors on the other hand.

In physical terms, leading-twist (twist-2) nonforward parton distributions characterize a process where one parton is extracted from a nucleon and another is returned, carrying a different light-cone momentum fraction. Such processes can be probed in exclusive reactions like the (virtual) photoproduction of mesons \[2, 3, 4, 5, 6\] and deeply virtual Compton scattering \[7\]. There is currently significant experimental activity on these topics at both collider \[8\] and fixed-target \[9, 10, 11\] energies.

The nonforward distributions become equal to their forward counterparts only in the exact forward limit i.e., when the difference between initial and final nucleon momenta vanishes. On the other hand it has been argued by many authors \[3, 12, 13\] that nonforward distributions which determine the amplitude for exclusive meson production in the hard diffractive limit have shape similar to corresponding forward distributions.

Because the probability amplitudes of exclusive reactions are proportional
to nonforward distributions, exclusive cross sections depend quadratically on the distributions. These cross sections should therefore be highly sensitive to the form of the parton distributions involved. A well-known example is the proposed relation \cite{3,14} between the unpolarized gluon distribution $g(x,Q^2)$ of the proton and the exclusive $J/\psi$ leptoproduction cross section. As opposed to light meson production, $J/\psi$ production amplitudes depend only negligibly on quark distributions, which makes the $J/\psi$ a very useful probe of the gluon content of nucleons.

Similarly, the polarized gluon distribution $\Delta g(x,Q^2)$ could possibly be investigated by means of measuring photon-proton polarization asymmetries in exclusive $J/\psi$ leptoproduction. This was considered in the small Bjorken $x$ limit in \cite{15,16}. In the present paper, we discuss the general case where the process probes nonforward twist-2 gluon distributions.

In section 2 we write down the amplitude for $\gamma^* + p \rightarrow J/\psi + p$ in terms of nonforward parton distributions. We restrict here to the case of collinear scattering and follow the treatment of \cite{2,4,6,7}. In order to evaluate polarization asymmetries one then needs the helicity amplitudes for the subprocess $\gamma^* + g \rightarrow J/\psi + g$. These are derived in section 3 using the nonrelativistic approximation for the $J/\psi$ wave function.

Contrary to the results of \cite{15,16}, the polarization asymmetry in hard exclusive $J/\Psi$ production turns out to be zero at the twist-2 level. In addition, we have found that the spin-flip transition between hard photon and $J/\psi$ is similarly suppressed. It implies that hard exclusive $J/\psi$ production is

\textit{Theorem:} hard exclusive $J/\psi$ production is suppressed at the twist-2 level.

\textit{Proof:}
not a good probe of either the polarized gluon distribution or the nonforward
gluon helicity flip distribution of the nucleon [19].

2 Nonforward distributions

Below we discuss the amplitude for the process

\[ \gamma^*(q, \lambda) + p(P, S) \rightarrow J/\psi(K, \lambda') + p(P', S') \, . \]  

(1)

Our presentation follows closely that of Refs. [2, 4, 7] as well as the calculation
of hard exclusive production of light mesons in [6]. Our notation is the same
or analogous to that in [7].

In Eq. (1) \( q \) is the four-momentum of the hard photon with virtuality
\( Q^2 = -q^2 \) and polarization \( \lambda \). \( P, S \) and \( P', S' \) denote momenta and spins
of initial and final nucleons, respectively, and \( K, \lambda' \) is the momentum and
polarization of the produced \( J/\psi \). We shall first consider the amplitude for
the process (1) in the limit when \( Q^2 \) is large, but our results are presumably
also valid in the photoproduction limit (\( Q^2 = 0 \)), where a hard scale is still
provided by the charm quark mass.

In a general, covariant gauge, arbitrary number of gluons can be ex-
changed between the charm quark loop and nucleon, all contributing to the
same order of the twist expansion. To avoid this complication, in the fol-
lowing we choose the light-cone gauge for the gluon field. In this case the
dominant contribution comes from the exchange of two gluons, as illustrated
by the Feynman diagram on Fig. 1. The amplitude is first written as

$$A_{\lambda\lambda'SS'} = \int \frac{d^4k_1}{(2\pi)^4} H^{\mu\nu}_{\lambda\lambda'}(q, K, k_1) \int d^4z e^{ik_1 \cdot z} \langle P', S'| A_\mu^a(-z/2) A_\nu^a(z/2) | P, S \rangle.$$  

(2)

Here $H^{\mu\nu}_{\lambda\lambda'}(q, P, k)$ is the perturbative amplitude for the process

$$\gamma^* (q, \lambda) + g(k_1) \rightarrow J/\psi (K, \lambda') + g(k_2),$$  

(3)

$k_1$ and $k_2$ are momenta of the incoming and outgoing gluons, respectively.

Calculating $H^{\mu\nu}_{\lambda\lambda'}$ to the leading twist accuracy one can neglect the momentum transfer $t = (K - q)^2$ and the nucleon mass $M_N$. The scattering amplitude then becomes collinear, i.e., all momenta can be expressed as linear combinations of two light-like vectors $P$ and $n$. Assuming that the space-like component of the virtual photon momentum is directed along the negative direction of the $\hat{z}$ axis, we can take $P = |P|(1, 0, 0, 1)$ and $n = (1, 0, 0, -1)/(2|P|)$. The gluon momenta $k_1$ and $k_2$ become collinear as well, $k_i = (k_i \cdot n) P$. The amplitude can then be simplified to

$$A_{\lambda\lambda'SS'} = \int_{-1}^{1} dx \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} H^{\mu\nu}_{\lambda\lambda'}(q, K, xP) \times \langle P', S'| A_\mu^a(-\lambda n/2) A_\nu^a(\lambda n/2) | P, S \rangle.$$  

(4)

Because of collinearity, the angular momentum conservation reduces to helicity conservation. We can therefore select the case $S = S'$ in (4) by requiring that the photon and $J/\psi$ helicities are equal. In the light-cone gauge $n \cdot A = 0$ the matrix element with equal proton spins is parametrized by the unpolarized and polarized nonforward gluon distributions as

$$\langle P', S = \pm 1/2 | A_\mu^a(-z/2) A_\nu^a(z/2) | P, S = \pm 1/2 \rangle \bigg|_{z^2 = 0}$$  

4
\[ \frac{1}{2} \int_{-1}^{1} dx \frac{e^{-ix(P+P') \cdot z/2}}{(x - \xi + i \epsilon)(x + \xi - i \epsilon)} \times \sum_{\lambda_1, \lambda_2} \delta_{\lambda_1 \lambda_2} \epsilon_\mu(k_1, \lambda_1) \epsilon^*_\nu(k_2, \lambda_2) \left[ G(x, \xi; \mu^2) \pm \lambda_1 \Delta G(x, \xi; \mu^2) \right]. \tag{5} \]

Here
\[ \xi = \frac{(P - P') \cdot n}{(P + P') \cdot n} \tag{6} \]

and
\[ \epsilon_\mu(k, \lambda) = -\frac{1}{\sqrt{2}}(0, \lambda, i, 0) \tag{7} \]

are polarization vectors for transverse gluons moving along the z axis in the positive direction. They coincide with the polarization vectors for gluons in the light-cone gauge because \( n \cdot \epsilon = 0 \) in the collinear kinematics.

The polarized and unpolarized nonforward gluon distributions \( G(x, \xi; \mu^2) \) and \( \Delta G(x, \xi; \mu^2) \) are related to the usual gluon distributions \( g(x, \mu^2) \) and \( \Delta g(x, \mu^2) \) as
\[ G(x, 0; \mu^2) = x g(x, \mu^2), \tag{8} \]
\[ \Delta G(x, 0; \mu^2) = x \Delta g(x, \mu^2). \tag{9} \]

Inserting (5) into (4) we obtain
\[ A_{\lambda_1 \lambda_2 \lambda_1'} = \frac{1}{2} \int_{-1}^{1} dx \frac{1}{(x - \xi + i \epsilon)(x + \xi - i \epsilon)} \times \sum_{\lambda_1} A_{\lambda_1 \lambda_1} \left[ G(x, \xi; \mu^2) \pm \lambda_1 \Delta G(x, \xi; \mu^2) \right], \tag{10} \]

where
\[ A_{\lambda_1 \lambda_1 \lambda_2} = \epsilon_\mu(k_1, \lambda_1) H_{\lambda_1 \lambda_1'}^{\mu
u}(q, K, k_1 = xP) \epsilon^*_\nu(k_2, \lambda_2) \tag{11} \]
is the helicity amplitude for the perturbative subprocess.
Eq. (10) shows that in order to access the polarized distribution $\Delta G(x, \xi; \mu^2)$, the structure $\lambda \lambda_1 \delta_{\lambda_1 \lambda_2} \delta_{\lambda \lambda'}$ must be present in $A_{\lambda \lambda' \lambda_1 \lambda_2}$.

3 Helicity amplitudes for the subprocess

Let us now study the helicity amplitudes $A_{\lambda \lambda' \lambda_1 \lambda_2}$. To make contact with results existing already in the literature [17, 18] we have chosen to consider the general case i.e., retaining the full kinematics. Using the standard non-relativistic approximation of the $J/\psi$ wavefunction [18], we write

$$H_{\lambda \lambda'}^{\mu \nu}(q, K, k_1) = e Q c g_s^2 \frac{R_S(0)}{\sqrt{16\pi M}} \frac{T_R}{\sqrt{N_C}} \epsilon_\alpha(q, \lambda) \epsilon^{*\alpha'}(K, \lambda')$$

$$\times \left\{ \text{Tr} \left[ \gamma^\alpha S_F(K/2 - q) \gamma^\mu S_F(-K/2 - k_2) \gamma^\nu \gamma^{\alpha'}(K' + M) \right] + \ldots \right\}$$

(12)

where $e$ is the electromagnetic and $g_s$ the strong coupling constant, $Q_c = 2/3$, $R_S(0)$ is the radial wavefunction of the $J/\psi$ at the origin, $M = 2m_c$ is the $J/\psi$ mass, $S_F(k) \equiv (k - m_c)^{-1}$, and the dots stand for five other permutations of the gauge bosons on the charm quark line. The term shown explicitly in (12) corresponds to the diagram shown in Fig. 1.

It is convenient to express the polarization vectors in terms of four-momenta of the problem. Clearly, it is not possible if the scattering is collinear, but our expressions have the correct limit as $\theta \to 0$. We work in the Gottfried–Jackson frame, i.e. the $J/\psi$ rest frame where the $J/\psi$ spin quantization direction is determined by the virtual photon three-momentum. The $y$ axis is chosen in the direction of $P \times q$. In this frame the polarization
vectors are
\[\epsilon^\mu(q, \lambda = 0) = \frac{s + u}{T \sqrt{Q^2}} \left( q^\mu + \frac{2Q^2}{s + u} K^\mu \right), \tag{13}\]
\[\epsilon^\mu(K, \lambda' = 0) = \frac{s + u}{MT} \left( -K^\mu + \frac{2M^2}{s + u} q^\mu \right), \tag{14}\]
\[\epsilon^\mu(q, \lambda = \pm 1) = N^{-1} \left[ \lambda R^\mu(\gamma^*) - i I^\mu \right], \tag{15}\]
\[\epsilon^\mu(K, \lambda' = \pm 1) = N^{-1} \left[ \lambda' R^\mu(\gamma^*) + i I^\mu \right], \tag{16}\]
\[\epsilon^\mu(k_1, \lambda_1 = \pm 1) = N^{-1} \left[ \lambda_1 R^\mu(g_1) - i I^\mu \right], \tag{17}\]
\[\epsilon^\mu(k_2, \lambda_2 = \pm 1) = N^{-1} \left[ \lambda_2 R^\mu(g_2) + i I^\mu \right], \tag{18}\]
where
\[T^2 = (s + u)^2 + 4M^2Q^2, \tag{19}\]
\[N = -\sqrt{2t(M^2Q^2 + su)}, \tag{20}\]
\[R^\mu(\gamma^*) = \frac{s + u}{T} \left[ \left( s + u + \frac{4M^2Q^2}{s + u} \right) k_1^\mu - \left( s + Q^2 - \frac{2Q^2(u - M^2)}{s + u} \right) K^\mu \right.\]
\[\left. + \left( u - M^2 + \frac{2M^2(s + Q^2)}{s + u} \right) q^\mu \right], \tag{21}\]
\[R^\mu(g_1) = \left( s + u + \frac{2M^2(s + Q^2)}{u - M^2} \right) k_1^\mu + (s + Q^2) K^\mu + (u - M^2) q^\mu, \tag{22}\]
\[R^\mu(g_2) = \left( s + u + \frac{2M^2(u + Q^2)}{s - M^2} \right) k_2^\mu - (u + Q^2) K^\mu - (s - M^2) q^\mu, \tag{23}\]
\[I^\mu = 2\epsilon^{\mu\nu\rho\sigma} K_\nu k_1^\rho q_\sigma, \tag{24}\]
and \(s, t, u\) are the usual Mandelstam variables for the subprocess. Writing
\[A_{\lambda\lambda'\lambda_1\lambda_2} = \sqrt{\frac{\alpha_{em}}{27M}} \frac{32\pi\alpha_s R_5(0) a_{\lambda\lambda'\lambda_1\lambda_2}}{(s - M^2)(t - M^2 - Q^2)(u - M^2) T^2} \tag{25}\]
we find the following helicity dependence:
\[a_{00\lambda_1\lambda_2} = \delta_{\lambda_1\lambda_2} \cdot 2\sqrt{Q^2} \left( M^2Q^2 + su \right) \left[ (M^2 + Q^2)^2 - t(t + 4M^2) \right]. \]
+ \delta_{\lambda_1, -\lambda_2} \cdot 2 \sqrt{Q^2} t M^2 T^2, \quad (26)
\begin{align*}
a_{0\lambda_1 \lambda_2} & = \delta_{\lambda_1 \lambda_2} \cdot N \left[ \lambda_1 T + \lambda (s - u) \right] \frac{1}{2} \left[ (M^2 + Q^2)^2 + t(Q^2 - M^2) \right], \quad (27) \\
a_{\lambda\lambda' \lambda_1 \lambda_2} & = \delta_{\lambda_1 \lambda_2} \delta_{\lambda\lambda'} \cdot 2 M (M^2 Q^2 + s u) \left[ -(M^2 + Q^2)^2 + t(M^2 - 3Q^2) \right] \\
& \quad + \delta_{\lambda_1 \lambda_2} \delta_{\lambda\lambda'} \cdot t M (t - M^2 - Q^2) \\
& \quad \times \left[ \lambda_1 \lambda(s - u) T + s^2 + u^2 + 2 M^2 Q^2 \right] \\
& \quad + \delta_{\lambda_1, -\lambda_2} \delta_{\lambda\lambda'} \cdot t M T^2 \left[ \lambda_1 \lambda T - (s + u) \right]. \quad (29)
\end{align*}

In Eqs. (26-29) the indices $\lambda, \lambda'$ refer to transverse polarization of the photon and $J/\psi$, while longitudinal polarization is denoted by an index "$0$".

In the limit $s \gg M^2 \sim Q^2 \gg |t|$ our helicity amplitudes agree with Eq. (31) of [17]. (Ref. [17] also presents an exact expression for the helicity amplitudes, but it is given in the parton-level recoil frame rather than in the Gottfried–Jackson frame and therefore cannot be directly compared with our result at $t \neq 0$.) Our amplitudes are also valid at $Q^2 = 0$, in which case the charm quark mass provides a large momentum scale, and reproduce the unpolarized cross section of [18].

4 Discussion

First, from Eqs. (26-29) one recovers immediately the well known results for the leading parts of the amplitudes $a_{\lambda\lambda' \lambda_1 \lambda_2}$ and $a_{00\lambda_1 \lambda_2}$. They lead to helicity-conserving $J/\psi$ production amplitudes proportional to nucleon unpolarized gluon distributions [3, 14]. Moreover, from (29) one finds that the...
structure $\lambda\lambda_1\delta_{\lambda_1\lambda_2}\delta_{\lambda\lambda'}$ is absent. We thus predict photon-proton polarization asymmetries to vanish in the collinear limit relevant for Eq. (4).

Let us briefly discuss the role of the helicity-flip structures $\delta_{\lambda_1\lambda_2}\delta_{\lambda,-\lambda'}$ and $\delta_{\lambda_1,-\lambda_2}\delta_{\lambda\lambda'}$ in Eq. (29), which are potentially significant for $J/\psi$ polarization studies and for the study of gluon helicity-flip nonforward distributions [19], respectively. In the Bjorken limit ($s \sim Q^2 \gg M^2$, $|t|$) the helicity-flip terms are suppressed by a factor of $O(t/Q^2)$ with respect to the helicity-conserving $\delta_{\lambda_1\lambda_2}\delta_{\lambda\lambda'}$ term. In the photoproduction limit, the situation depends on whether the subprocess occurs close to the threshold ($s-M^2 \ll M^2$) or far from the threshold ($s \gg M^2$). Close to the threshold the ratio of the helicity-flip to the helicity-conserving term becomes $t/(s-M^2+t)$. Far from the threshold the $\delta_{\lambda_1\lambda_2}\delta_{\lambda,-\lambda'}$ term is suppressed by a factor $t/M^2$ and the $\delta_{\lambda_1,-\lambda_2}\delta_{\lambda\lambda'}$ term by a factor $t(t-M^2)^2/(M^2s^2)$.

Hence the $J/\psi$ photo- or leptoproduction process is not a good probe of the gluon helicity-flip distribution, except possibly close to the subprocess threshold. A helicity flip between the photon and the $J/\psi$ should become significant in photoproduction at $|t| \gtrsim M^2$.

In order to cross-check our results we have calculated the amplitude (4) directly in the collinear limit using the polarization vectors (7). We have found the contribution from the unpolarized gluon distribution which agrees exactly with Refs. [3, 14]. As far as the polarized gluon distribution is concerned, we have found that the contribution of each individual diagram has exactly the same magnitude as in the unpolarized case, but half of them have
opposite signs due to asymmetry with respect to exchange of \( \mu \leftrightarrow \nu \) in the part of the matrix element (3) which is proportional to \( \Delta g(x, \xi, \mu^2) \). As a consequence, the total contribution is zero. Our result thus corrects earlier work \([15, 16]\) where there was a mistake, as confirmed by the author \([20]\).

In summary, we have discussed the calculation of exclusive \( J/\psi \) production off nucleons. We have found that the contribution of twist-2 polarized gluon distribution vanishes in the hard virtual photon limit. It will thus be difficult to determine polarized gluon distribution from polarization asymmetry in hard exclusive \( J/\psi \) production. We note, however, that the present calculation has been performed neglecting the relative velocity \( v \) of charmed quarks in the \( J/\psi \) bound state. It is possible that relaxing this approximation, like e.g., in \([4]\), will lead to non-vanishing polarization asymmetry as well as a non-zero coupling to gluon helicity-flip distribution at the leading twist level, but at the subleading order in the \( v/c \) expansion.

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Figure 1: One of 6 Feynman diagrams which contribute to the amplitude (2).