Shear viscosity of the Quark-Gluon Plasma from a virial expansion

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Abstract. We calculate the shear viscosity $\eta$ in the quark-gluon plasma (QGP) phase within a virial expansion approach with particular interest in the ratio of $\eta$ to the entropy density $s$, i.e. $\eta/s$. The virial expansion approach allows us to include the interactions between the partons in the deconfined phase and to evaluate the corrections to a single-particle partition function. In the latter approach we start with an effective interaction with parameters fixed to reproduce thermodynamical quantities of QCD such as energy and/or entropy density. We also directly extract the effective coupling $\alpha_V$ for the determination of $\eta$. Our numerical results give a ratio $\eta/s \approx 0.097$ at the critical temperature $T_c$, which is very close to the theoretical bound of $1/(4\pi)$. Furthermore, for temperatures $T \leq 1.8T_c$ the ratio $\eta/s$ is in the range of the present experimental estimates $0.1 - 0.3$ at RHIC. When combining our results for $\eta/s$ in the deconfined phase with those from chiral perturbation theory or the resonance gas model in the confined phase we observe a pronounced minimum of $\eta/s$ close to the critical temperature $T_c$.

\begin{pacs}
12.38.Mh Quark-gluon plasma – 25.75.Nq Phase transition in Quark-gluon plasma – 21.65.Qr Quark matter/nuclear matter – 51.20.+d Viscosity, diffusion, and thermal conductivity
\end{pacs}

1 Introduction

The exploration of the phase structure of QCD is a challenging task for modern theoretical physics. Lattice QCD (lQCD) calculations for vanishing quark chemical potentials $\mu_q$ have shown chiral and deconfinement phase transitions at a critical temperature $T_c$ of about 150 to 200 MeV [12]. New methods to extend lQCD calculations to finite chemical potentials $\mu_q$ have been developed in the past decade, i.e. a multi-parameter reweighting [3,4,5], a Taylor expansion around $\mu_q \approx 0$ [6,7,8] and an imaginary chemical potential method [9,10]; however, their validity is limited to a region $\mu_q \lesssim T$ [11]. At finite net quark densities presently some modeling of QCD is needed in order to explore the QCD phase diagram at least on a qualitative level. Indeed, effective approaches indicate a very rich phase structure of QCD, i.e. 1) a quark-gluon plasma (QGP), 2) a color superconducting phase, 3) a color-flavor locking phase and different further combinations (cf. Refs. [12,13,14,15,16,17]). The explicit phase structure, however, is model dependent (except for very high $\mu_q$) and not accessible by experiment in the full $(T, \mu_q)$-plane. Experimental information can be qualitatively obtained from astronomical observations and more quantitatively from relativistic heavy-ion collisions. In the last decade such experiments have been performed at the Relativistic Heavy-Ion Collider (RHIC) at Brookhaven National Laboratory (BNL) and even long before - at lower energies - at the Super-Proton-Synchrotron (SPS) at CERN.

Results from lQCD calculations have early been interpreted as indicating a weakly interacting system of partons since the entropy density $s$ and the energy density $\varepsilon$ are close to the Stefan Boltzmann (SB) limit for a relativistic noninteracting system, i.e. only $\approx 10 - 15\%$ lower. However, more recent experimental observations at RHIC have drastically changed the notion about the QGP. In this context, one of the most intriguing experimental findings is the large elliptic flow $v_2$ of hadrons at RHIC [18,19], which is significantly larger than at SPS energies [20,21]. In particular the transverse momentum $p_T$ dependences of the elliptic flow $v_2(p_T)$ at RHIC is close to predictions from non-dissipative hydrodynamical simulations [22,23,24,25,26] around midrapidity ($|y| \leq 1$). This result has led to the BNL announcement about the discovery of the nearly perfect fluidity of the strongly-coupled quark-gluon plasma (sQGP) [27,28] produced at RHIC. Especially, due to the asymptotic freedom of QCD, the prevailing idea has been an expectation of large shear viscosities in a weakly interacting QGP (wQGP) at very high densities and/or temperatures. Because of the evident failure of these assumptions at RHIC conditions the novel notion of a strong quark-gluon plasma (sQGP) has been put forward to characterize the observed strong coupling properties of the QGP close to (or slightly above) the critical temperature...
$T_c$ that keep viscous effects low at RHIC. Therefore, the inclusion of interactions between partons is mandatory to consistently describe the QGP in the region close to $T_c$.

In going beyond the standard dynamical quasiparticle picture \cite{11,22,30,31,32} - incorporating interactions in terms of a width in the spectral functions of the partons - we have recently developed a generalization of the classical virial expansion approach to calculate the QCD partition function in the partonic phase with an interaction inspired by lattice calculations \cite{33}. We have obtained an Equation-of-State (EoS) for the partonic QGP that is well in line with recent three-flavor QCD lattice data \cite{11} for the pressure, speed of sound and interaction measure at nonzero temperature and vanishing chemical potential ($\mu_q = 0$). Since in the virial expansion approach all thermodynamic quantities are based on an explicit parton interaction in form of a potential, this approach is also the ideal starting point for a consistent description of dynamical properties of the QGP like the shear viscosity $\eta$, which is the stationary limit of a nontrivial correlator \cite{34,35}.

In this work we focus on the calculation of the ratio $\eta/s$, well known as specific viscosity, within the virial expansion approach. It was shown some years ago for supersymmetric Yang-Mills (SYM) gauge theory using the Anti de-Sitter Space/Conformal Field Theory (AdS/CFT) duality conjecture \cite{36} that

$$\frac{\eta}{s}_{\text{SYM}} = \frac{1}{4\pi},$$

which is denoted as KSS bound. This limit is close to the simple bound $\eta/s \geq 1/15$ from the kinetic theory uncertainty principle \cite{37}. Additionally, it has even been speculated that this bound might hold for all substances \cite{38}. Furthermore, it has been found for atomic and molecular substances that the specific viscosity exhibits a minimum in the vicinity of the liquid-gas critical point; this also suggests the possibility of a minimum of $\eta/s$ for QCD at the critical temperature $T_c$ which has been already noted in Refs. \cite{38,39}.

2 Kinetic theory

We start by recalling results well known from literature. Here kinetic theory is a convenient framework to start the investigation of the viscosity coefficient $\eta$ \cite{35}. In an ultrarelativistic quark-gluon plasma, i.e. where the temperature $T$ is much larger than the constituent masses $m_i$, the estimate for the ratio of the shear viscosity $\eta$ and the entropy density $s$ is \cite{37}

$$\frac{\eta}{s} \simeq \frac{1}{15} \frac{4}{s} \sum_i \langle p(p) \lambda_i \rangle,$$  

where $\rho_i$ is the local density of quanta $i$ transporting on average a momentum $\langle p_i \rangle$ over a momentum-degradation length (mean-free-path) $\lambda_i$. Following the more detailed kinetic theory derivations of Ref. \cite{40}, the familiar factor $\frac{1}{2}$ of elementary non-relativistic kinetic theory has to be replaced in the ultrarelativistic domain by $4/15$.

Evidently, two aspects play a crucial role in the expression (2) for $\eta/s$. The first one is the choice of the equation-of-state for the determination of the entropy density $s$. In general - for such calculations - an ideal gas EoS is used. This assumption might be justified for high temperatures $T \gg T_c$, where the deviation of the QGP equation-of-state from the ideal gas limit might be eventually neglected. On the other hand, close to the critical temperature $T_c$, this approximation should be abandoned due to large deviations from the SB limit.

The second crucial point is given by partonic dynamics itself: The effects of the interaction between the constituents directly determine the mean-free-path $\lambda$ and its calculation is the aim of several investigations within different approaches. Following Ref. \cite{11} the mean-free-path is the inverse of the interaction rate for nearly massless quanta $\Gamma_i$, i.e. $\lambda_i = 1/\Gamma_i$, which can be calculated to lowest order in the coupling constant $\alpha$ from the imaginary part of the quark and gluon selfenergy. Another possibility is to relate the mean-free-path to the transport cross section $\sigma_i$ by $\lambda_i = (\rho_i \sigma_i)^{-1}$. This quantity - in a dense partonic system - may be related to the transport parameter $\hat{q}$ governing multiple scattering \cite{42,43} or can be directly modeled in a Debye-screened form \cite{44}; the latter leads to an analytic expression for $\sigma_i$ \cite{45}. Using this last formulation and additionally adopting the approximation $\langle p \rangle \approx 3T$ the specific viscosity becomes

$$\frac{\eta}{s} \approx \frac{4}{5} T \frac{\lambda}{\rho \sigma_i}.$$  

In particular, assuming that the elastic gluon scattering matrix element in a dense partonic medium can be modeled by a Debye screened interaction \cite{44,45} the relevant transport cross section reads

$$\sigma_i(\hat{s}) \equiv \int d\sigma_{ei} \sin^2 \theta_{cm}$$

$$= \sigma_0 \left[ (2z + 1) \left( \frac{1}{2} \frac{1}{1 + 1/z} - 2 \right) \right]$$

with the total cross section $\sigma_i(\hat{s}) = 9\pi\sigma_i^2(\hat{s})/2\mu_{sct}$. Here $\alpha_V = \alpha_V(T)$ and $\mu_{sct}$ are the effective temperature dependent coupling constant and the screening mass, respectively, and $z \equiv \mu_{sct}^2/\hat{s}$. For simplicity, we will assume $\sigma_0$ to be energy independent and neglect its weak logarithmic dependence on $\hat{s}$ in the relevant energy range and set $\hat{s} \approx 17 T^2$. We recall that the $\sin^2 \theta_{cm}$ weight arises in the transport cross section because large-angle scatterings are most effective in momentum degradation \cite{47}. The cross section $\sigma_i$ is a monotonic function of $\mu_{sct}$, which plays a crucial role for the results of $\sigma_i$. It is important to emphasize that the coupling constant and the Debye mass are not independent parameters in the calculation because $\mu_{sct}$ is determined by the value of the coupling $\alpha_V$ itself. In perturbation theory one explicitly obtains

$$\mu_{sct}^2 = 4\pi\alpha_V T^2$$

in gluon dynamics.

For numerical estimates a specific form for $\alpha_V(T)$ has to be chosen. As a quantitative reference the long-range
part of the strong coupling constant extracted from the free energy of a quark-antiquark pair in lQCD [46] has
been used in the literature [47]. We point out that any extraction of a coupling constant $\alpha_V(T)$ from IQCD is
model dependent and deviations (or agreement) of any parametrization from (with) the lattice data have to be
considered with care. Accordingly, several parametrizations can be found in the literature [29,16,32,46,47].

At this point, we emphasize that in previous works [47] these two aspects - equation of state and transport cross
section - are uncorrelated. Therefore a consistent approach is needed which provides $i$) a realistic equation-of-state -
and thus a proper entropy density - as well as $ii$) a transport cross section within the same framework.

3 The virial expansion approach to QCD

As pointed out in the previous Section, a consistent approach requires two fundamental ingredients: $i$) a calculation
of the equation-of-state including the interactions between the constituents and $ii$) an extraction of the effective
coupling that enters the estimate for the transport cross section. Both requirements can be achieved within the virial expansion formalism developed in a previous work [33], where a detailed derivation of the partition function $Z(T, V)$ of all thermodynamic quantities - such as pressure, entropy density, interaction measure and sound velocity - and of the EoS of the QGP at vanishing finite $\mu_q$ has been presented. We achieve an expansion of $\ln Z$ in powers of the logarithm of the partition function in the Stefan Boltzmann limit $\ln Z(0)$, i.e. (cf. [33])

$$\ln Z \approx \ln Z(0) + \frac{b_2}{2} (\ln Z(0))^2$$

(6)

with the second virial coefficient

$$b_2 = \int_V d^3r \left( e^{-\beta W_{12}(r)} - 1 \right).$$

(7)

The pressure is simply obtained as

$$P = T \ln Z.$$  

(8)

The other quantities can be calculated from the pressure $P$ or the partition function $Z$ using thermodynamic relations. In particular, for the entropy density one obtains

$$s = \frac{\partial P}{\partial T}.$$  

(9)

For an application of this formalism to the QGP a specific choice of the interaction potential $W_{12}$ has to be made. Following Ref. [33] we use an effective quark-quark potential inspired by a phenomenological model which includes non-perturbative effects from dimension two gluon condensates (that reproduce the free energy of quenched QCD very well). The effective potential between the quarks explicitly reads

$$W_{12}(r, T) = \left( \frac{\pi}{12} r + \frac{C_2}{2N_c T} \right) e^{-M(T)r},$$

(10)

where $C_2$ is the non-perturbative dimension two condensate and $M(T)$ a Debye mass estimated as

$$M(T) = \sqrt{N_c/3 + N_f/6} g T = \tilde{g} T,$$

(11)

where we have neglected any scale dependence in the coupling constant. By using the potential (10) we assume the interaction to be the same for quarks and antiquarks and neglect explicit gluon contributions. The latter are encoded in parametric form in the interaction (10). In short, we generalize the Yukawa-liquid model for the QGP investigated in Refs. [48,49] before. For a detailed explanation of this interaction we refer the reader to Ref. [33]. A comparison with three-flavor IQCD calculations with almost physical masses from Ref. [11] shows that a coupling parameter $\tilde{g} = 1.30$ allows for a good description of all thermodynamic quantities in the temperature range from $0.8 T_c$ to $5 T_c$. A detailed discussion of the validity of the virial expansion truncated at the second term is given in Ref. [33].

In order to calculate a transport cross section with this interaction the coupling $\alpha_V$ has to be extracted from $V_1$. Following Ref. [50] we define the coupling in the so-called $qq$-scheme,

$$\alpha_{qq}(r, T) \equiv -\frac{12}{\pi} r^2 \frac{dW_{12}(r, T)}{dr}.$$  

(12)

The coupling $\alpha_{qq}(r, T)$ then exhibits a maximum for fixed temperature at a certain distance denoted by $r_{max}$. By analyzing the size of the maximum at $r_{max}$ we fix the temperature dependent coupling, $\alpha_V(T)$, as

$$\alpha_V(T) \equiv \alpha_{qq}(r_{max}, T).$$

(13)

Before we start with the calculation of $\eta/s$ two important aspects have to be pointed out: first we only consider the contribution of our dynamical degrees of freedom (quarks and antiquarks) for $\eta/s$, since here the gluons are massless and interaction free with respect to each other. The fermion-gluon interaction is only space-like and included in the potential (10). Therefore, the gluons do not contribute to the ratio $\eta/s$, in contrast to QCD perturbation theory, where the contribution of the quarks and of the gluons are roughly of the same order [51,52]. We thus consider the quark specific viscosity, i.e.

$$\frac{\eta}{s_q} = \frac{4}{5} \frac{T}{s_q \sigma_t},$$

(14)

where the quark contribution of the entropy density $s_q$ is

$$s_q = s - s_g,$$

(15)

with the gluon contribution to the entropy density in the SB limit given by

$$s_g = \frac{32\pi^2}{45} T^3.$$  

(16)

The second important aspect is the applicability of Eq. (4) to calculate the transport cross section. We recall that this
expression for \( \sigma \) has been derived for a Debye screened interaction [13], whereas the effective potential \( W_{12} \) contains not only such a term but also a purely screened part \( \sim \mathcal{C}_2 \). A preliminary analysis, which actually has lead to the final formulation of Ref. [33], shows that the main contribution to the virial coefficient is given by the Coulomb screened part of the potential. This confirms the observations of Ref. [33], that the exact value of \( \mathcal{C}_2 \), which modulates the purely screened part of the potential, is not so important for a good reproduction of the free energy. Accordingly, we may use Eq. (11) as a good approximation for the transport cross section.

The advantage of this formalism is that for the Debye screening mass no further approximations must be done, as, for example, the assumption of the validity of the perturbation theory that leads to the specific expression given in Eq. (14) and is used often in the literature [17,18]. In our model, the mass is directly given by Eq. (14) and in this way is calculated independently of the coupling \( \alpha_T \) but, at the same time, consistently -in a thermodynamic sense- with the properties of the QGP because the parameter \( \tilde{g} = 1.3 \) is fixed by thermodynamic quantities. This demonstrates the important difference to other effective approaches used to understand the enhancement of the parton transport cross section, that was concluded from the elliptic flow measurements at RHIC.

We, furthermore, point out an analogy between strong electromagnetic and strong quark-gluon plasma coupling presented first in Ref. [18]: The Debye screening length, i.e. \( r_D = \mu_{\text{scr}}^{-1} \), cannot be used as a cut-off parameter for the calculation of \( \sigma_t \) because the interaction range is larger than \( r_D \). Studies for complex non-relativistic electromagnetic plasmas show the importance of these effects as demonstrated in Ref. [54]. Furthermore, by assuming a non-screened Coulomb interaction a modified Coulomb logarithm \( \Lambda^* \) was derived, which leads to a sizeable enhancement of the transport cross section. The heuristic translation of these results for the sQGP (given in [18]) requires the screening mass in the transport cross section to be replaced by

\[
\mu_{\text{scr}} \rightarrow \frac{\mu_{\text{scr}}}{4.6} \simeq 1.3 T \simeq \tilde{g} T. \tag{17}
\]

Surprisingly this is -quantitatively- the same temperature dependence of the Debye mass as that obtained within our formalism (cf. Eq. (11) and discussion below).

Some additional comments are in place: Whereas in our virial expansion approach the screening mass is calculated from the thermodynamic quantities and the formalism includes crucial ingredients such as relativity and an effective screened potential - retaining thermodynamical consistency - the results of Ref. [18] are obtained within several approximations, i.e. a non-relativistic treatment, an unscreened electromagnetic interaction and (though motivated but) \textit{ad hoc} modifications of the transport cross section. In this sense the latter approach is heuristic: the modifications do not automatically follow from the formalism used but have been inserted \textit{ad hoc} to achieve a better description of the experimentally findings.

Before we discuss the results for \( \eta/s \) from the virial expansion approach, we demonstrate the applicability of our formalism for the entropy density in comparison to the lQCD calculations from Ref. [1]. In Fig 1 the entropy density \( s \) (divided by \( T^3 \)) is shown as a function of the temperature divided by \( T^2 \) from the virial expansion approach using Eq. (15) (solid line) as well as in the SB limit (dashed line). The symbols denote the lQCD calculations from Ref. [1]. For completeness we show also the entropy density in the confined phase below \( T_c \), where we have calculated all thermodynamic quantities (cf. Ref. [1]) within a generalized resonance-gas model and matched the different phases at equal pressure. Near \( T_c \) the deviation of our results from the ideal gas limit are sizeable and huge in the confined phase.

The ratio of the viscosity to the entropy density has been calculated using Eq. (14), where the quark contribution of the entropy density \( s_q \) is given by Eqs. (15) and (16). For the transport cross section the general expression given in Eq. (11) has been implemented, where the effective temperature dependent coupling \( \alpha_T(T) \) is given in Eq. (13) and the Debye screening mass is given by Eq. (14). In Fig. 2 our main results are presented in comparison to other estimates. In the deconfined region, \( T/T_c \geq 1 \), the solid red line show the results for \( \eta/s \) as a function of the temperature (in units of the critical temperature \( T_c \)) using the EoS (15)(16) and the coupling \( \alpha_T (13) \) and the Debye mass (11) derived within the virial expansion approach. Additionally, the experimental point (square) from [57] and the lattice data from Ref. [58] (triangles) and from Ref. [59] (full dots) are shown for comparison. In the confined phase, \( T/T_c < 1 \), the purple region...
close to $T_c$ shows the estimates for $\eta/s$ in the resonance-gas model [58], including all the known particles and resonances with masses $m < 2$ GeV and also an exponentially rising level density of Hagedorn states for $m > 2$ GeV. The dotted (red) line shows the scaling $\eta/s \propto T^{-4}$ combined with the requirement that $\eta/s = 1/4\pi$ at $T_c$. This scaling behavior at low temperature has been found within chiral perturbation theory in Ref. [52]. Additionally, the range (0.8–1.5) for $\eta/s$ from perturbative QCD (pQCD) from Ref. [55] is sketched as a blue region. Furthermore, the KSS lowest bound is also indicated by the orange area.

In the deconfined phase several features become evident:

1. At $T_c$, our result for $\eta/s \approx 0.097$ is very close to the theoretical bound of $1/(4\pi)$. Additionally, the convexity of the specific viscosity near $T_c$ suggests a minimum close to $T_c$ as expected in Refs. [55,60].
2. An almost linear increase of $\eta/s$ with the temperature is found for $1.5T_c \lesssim T \lesssim 3T_c$.
3. Currently the different lattice calculations are unable to provide quantitatively reliable results; the large error bars of the lattice data do not allow for a conclusive comparison. Qualitatively, the increasing behavior of the specific viscosity with the temperature might be confirmed. In contrast, the experimental point (full square close to $T_c$) is reproduced very well by our result within the virial approach.
4. At higher temperatures a saturation of the ratio $\eta/s$ is found which roughly coincides within the band for pQCD.

This last finding is supported by the observation that the entropy density $s$ as well as the viscosity scale as $\propto T^{-3}$ for high temperatures. For the entropy density $s$ this is in line with the SB limit, which is approximately reproduced by lattice [11] and model calculations [29,33] at high temperature. For the shear viscosity $\eta$ also very different approaches show this functional dependence: the strong quark-gluon plasma from AdS/CFT [60], the quasiparticle approximation with differently modeled quark self-energy [41,62] as well as the weak coupling estimate from Ref. [41].

With respect to the confined phase the resonance-gas as well as the $\chi$PT calculation suggest a decreasing behavior of $\eta/s$ with increasing temperature. The 'constrained' scaling behavior ($\propto T^{-4}$) slightly underestimates the resonance-gas calculation from Ref. [55]. Note, however, that the entropy density in the Hagedorn resonance model sensitively depends on the level density employed for the continuum states. Accordingly, we show these results in Fig. 2 by a shaded band. Choosing the scaling function ($\propto T^{-4}$) for the confined phase and our virial calculation for the deconfined phase we obtain a well connected description for $\eta/s$ in the whole temperature range (up to $5T_c$) which clearly shows a minimum near $T_c$ close to the KSS bound.

To investigate in more detail the possibility of a minimum of $\eta/s$ at the critical temperature from the virial expansion calculation we search for an approximation of the specific viscosity in the deconfined phase around $T_c$. We employ a Taylor expansion of $\eta/s$ as a function of $t = 1 + T/T_c$ at $t = 1$, i.e.

$$
\frac{\eta}{s} = \sum_{n=0}^{\infty} a_n(t - 1)^n = \sum_{n=0}^{N} a_n(t - 1)^n + R_N,
$$

(18)
We have performed an investigation of the specific viscosity \( \eta/s \) in the QGP in a dynamical way within kinetic theory using the virial expansion approach introduced in Ref. [13]. In this context the investigation of the interaction between the partons in the deconfined phase plays a crucial role to reproduce the thermodynamic properties of the QGP in comparison to lattice QCD. By using a generalized classical virial expansion we have calculated the corrections to a single-particle partition function starting from an interaction potential whose parameters are fixed by thermodynamical quantities. Furthermore, in the virial expansion approach we can directly extract the coupling \( \alpha_Y \) to be employed for the determination of the transport cross section which enters the ratio \( \eta/s \). We find \( \eta/s \approx 0.097 \) at \( T_c \) which is very close to the theoretical lower bound. Furthermore, for \( T \leq 1.8T_c \) the ratio is in the range of the experimental estimates 0.1 – 0.3 extracted from RHIC experiments.

Additionally, a detailed analysis of the temperature dependence of our results for \( \eta/s \) has been performed. Within a Taylor expansion around the critical temperature we found that a power law with a vanishing or very small linear coefficient suggests a simple parametrization for the specific viscosity. This indicates the existence of a minimum in \( \eta/s \) close to \( T_c \).

Since we focus on the deconfined phase only, we do not investigate whether in the vicinity of \( T_c \) a phase transition or a rapid cross over occurs [63]. However, our approach, which provides a unified description of the QGP thermodynamic as well as of its shear viscosity, is a first important improvement in the description of the quark-gluon plasma beyond mean-field models [69].

For further work, using relativistic molecular dynamical simulations, where the phenomenological quark-quark interaction used here can be implemented as well as the QCD equation of state, we may calculate further correlations in the partonic phase by considering partons in a box with periodic boundary conditions at fixed energy density (or temperature). Furthermore, such molecular dynamical calculations will allow to study partonic systems also out of equilibrium and provide important insight on the dynamics of hadronization.

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Table 1. Results for the fit of \( \eta/s \) up to \( T = 2T_c \).

| N | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( \chi^2/\text{dof} \) |
|---|---|---|---|---|
| 2 | 0.0968 | 0.2359 | - | 0.00102 |
| 2 | 0 | 0.3566 | - | 0.03959 |
| 3 | 0.0616 | 0.3528 | -0.08747 | 5.1 \times 10^{-5} |
| 3 | 0 | 0.5373 | -0.2163 | 0.00255 |

Table 2. Results for the fits of \( \eta/s \) up to \( T = 3T_c \).

| N | \( a_2 \) | \( a_3 \) | \( a_4 \) | \( \chi^2/\text{dof} \) |
|---|---|---|---|---|
| 2 | 0.2477 | - | - | 1.4478 |
| 3 | 0.4469 | -0.1194 | - | 0.0164 |
| 4 | 0.5044 | -0.1998 | 0.0268 | 0.0061 |

Table 3. Results for the fit of \( \eta/s \) up to \( T = 4.5T_c \).

| N | \( a_2 \) | \( a_3 \) | \( a_4 \) | \( \chi^2/\text{dof} \) |
|---|---|---|---|---|
| 2 | 0.1227 | - | - | 4.35251 |
| 3 | 0.3756 | -0.0845 | - | 0.8524 |
| 4 | 0.4955 | -0.1781 | 0.01738 | 0.0228 |

5 Conclusions

We have performed an investigation of the specific viscosity \( \eta/s \) in the QGP in a dynamical way within kinetic theory using the virial expansion approach introduced in

\[ a_n = \frac{1}{n!} \frac{d^n \langle \eta/s \rangle}{dt^n} (t = 1). \]

(19)

In [18] \( N \) indicates the order of the Taylor polynomial while \( R_N \) stands for the rest of the corresponding expansion. Our strategy is now to investigate how different polynomial approximations - labeled by different \( N \) - can describe \( \eta/s \) up to \( T = 2T_c, T = 3T_c \) and \( T = 4.5T_c \). For each \( N \) we calculate the \( \chi^2/\text{dof} \) to evaluate the quality of the approximation. Since the expansion coefficients are connected to derivatives of the specific viscosity at \( T_c \) (see Eq. (18)) by requiring that some coefficients vanish, we automatically impose that the corresponding derivatives are equal to zero. In particular, with the requirement \( a_1 = 0 \), we may investigate the possibility of a minimum at a critical temperature. Some polynomial fits to our results are given in the Tables 1, 2 and 3.

As expected, the power \( N = 2 \) cannot describe the behavior of \( \eta/s \) for \( T \geq 2T_c \). However, the inclusion of higher power allows to reproduce very well our data also for higher temperatures. For \( N = 3 \) and \( N = 4 \) we find a very good approximation for our calculated results for \( \eta/s \) up to \( T = 3T_c \) and \( T = 4.5T_c \), respectively. Therefore, a power law approximation with a vanishing or very small linear coefficient suggests a simple parametrization for the specific viscosity which includes a minimum close to \( T_c \).
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