Basics of quantum field theory of electromagnetic interaction processes in single-layer graphene

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Abstract

The content of this work is the study of electromagnetic interaction in single-layer graphene by means of the perturbation theory. The interaction of electromagnetic field with Dirac fermions in single-layer graphene has a peculiarity: Dirac fermions in graphene interact not only with the electromagnetic wave propagating within the graphene sheet, but also with electromagnetic field propagating from a location outside the graphene sheet and illuminating this sheet. The interaction Hamiltonian of the system comprising electromagnetic field and Dirac fermions fields contains the limits at graphene plane of electromagnetic field vector and scalar potentials which can be shortly called boundary electromagnetic field. The study of S-matrix requires knowing the limits at graphene plane of 2-point Green functions of electromagnetic field which also can be shortly called boundary 2-point Green functions of electromagnetic field. As the first example of the application of perturbation theory, the second order terms in the perturbative expansions of boundary 2-point Green functions of electromagnetic field as well as of 2-point Green functions of Dirac fermion fields are explicitly derived. Further extension of the application of perturbation theory is also discussed.

Keywords: electromagnetic, graphene, Dirac fermion, perturbation theory, Green function

Classification numbers: 3.00, 5.15

1. Introduction

Soon after the discovery of graphene by Geim and Novoselov [1–4], the research on graphene rapidly developed and became a wide interdisciplinary area of science and technology. It was shown [5] that even in the case when the electron spin plays no role, its quantum states are still described by two-component wave functions satisfying differential wave equations similar to relativistic Dirac equation for a massless particles in (2 + 1)-dimensional Minkowski space–time. Therefore the charge carriers in graphene are called Dirac fermions.

Denote K and K' two nearest corners of the first Brillouin zone in the reciprocal lattice of the hexagonal crystalline structure of a graphene monolayer. They are called Dirac points. In the framework of the quantum field theory the spinless fermions in graphene are described by two-component quantum fields \( \psi^K(r, t) \) and \( \psi^{K'}(r, t) \), \( r = \{ r_1, r_2 \} = \{ x, y \} \). Each of them can be considered as a spinor field of a new SU(2) symmetry group similar to the isospinors in theory of elementary particles [6–9]. Thus the two-component fields \( \psi^K(r, t) \) and \( \psi^{K'}(r, t) \) can be called, for example, quasi-spinors or pseudo-spinors. Three Pauli matrices acting on these spinors of a new type will be denoted...
It was shown [5] that for the system of free Dirac fermions with wave functions having wave vectors in the neighborhood of Dirac points we can use approximations of the Hamiltonian

$$H_G = v_F \int \mathbf{d}r \left[ \psi^K(r, t)^+ \tau^+ (-i \nabla) \psi^K(r, t) + \psi^K(r, t)^+ \tau^* (-i \nabla) \psi^K(r, t) \right],$$

where $v_F$ is the speed of Dirac fermions.

In the study of the interaction between Dirac fermions and electromagnetic field we need to consider electromagnetic field in the physical three-dimensional space. Let us choose to use the Cartesian coordinate system as follows: the plane of graphene monolayer is the $xOy$ coordinate plane and, therefore, the $Oz$ axis is perpendicular to this plane. Then the coordinate of a point in the physical three-dimensional space is denoted $(r, z) = (x, y, z)$. The electromagnetic field is described by the vector potential $\mathbf{A}(r, z, t)$ and scalar potential $\varphi(r, z, t)$. From formula (1) it follows that the interaction Hamiltonian of the system of Dirac fermion fields and electromagnetic field has the expression

$$H_{int}(t) = e v_F \int \mathbf{d}r \left[ \psi^K(r, t)^+ \tau \psi^K(r, t) + \psi^K(r, t)^+ \tau^* \psi^K(r, t) \right] \mathbf{A}(r, o, t) + e \int \mathbf{d}r \left[ \psi^K(r, t)^+ \psi^K(r, t) \right] \varphi(r, o, t).$$

The function $\mathbf{A}(r, o, t)$ and $\varphi(r, o, t)$ of variable $r$ and $t$ are vector field $\mathbf{A}(r, t)$ and scalar field $\varphi(r, t)$ on the graphene plane:

$$\mathbf{A}(r, t) \text{ def. } A(r, o, t),$$

$$\varphi(r, t) \text{ def. } \varphi(r, o, t).$$

Since they are the limits of the vector potential $\mathbf{A}(r, z, t)$ and the scalar potential $\varphi(r, z, t)$ of the electromagnetic field when the point $(r, z)$ tends to the limit $(r, o)$ in the $xOy$ coordinate plane, which is the boundary of the upper or lower half-space above or under the single-layer graphene plane, we shortly call them vector potential and scalar potential of the boundary electromagnetic field on the graphene plane. In terms of $\mathbf{A}(r, t)$ and $\varphi(r, t)$ the interaction Hamiltonian (4) has the expression

$$H_{int}(t) = e v_F \int \mathbf{d}r \left[ \psi^K(r, t)^+ \tau \psi^K(r, t) + \psi^K(r, t)^+ \tau^* \psi^K(r, t) \right] \mathbf{A}(r, t) + e \int \mathbf{d}r \left[ \psi^K(r, t)^+ \psi^K(r, t) \right] \varphi(r, t).$$

The charge current density $\mathbf{J}(r, t)$ and charge density $\rho(r, t)$ are expressed in terms of $H_{int}(t)$ by the definition

$$\mathbf{J}(r, t) = -\frac{\delta H_{int}(t)}{\delta \mathbf{A}(r, t)},$$

and

$$\rho(r, t) = -\frac{\delta H_{int}(t)}{\delta \varphi(r, t)}.$$
studied in [18] and we shall use the results of this work. In the present section we study vector and scalar potentials $A(r, t)$ and $\phi(r, t)$, and boundary 2-point Green function of free electromagnetic field. The study of boundary 2n-point Green functions of electromagnetic field with $n > 1$ by means of the perturbation theory will be carried out in section 3.

Vector field $A(r, t)$ and complex scalar field $i\phi(r, t)$ are the components of the limit at $z = 0$ of a 4-vector field $A_\mu(x)$ in the $(3 + 1)$-dimensional Minkowski space–time: $\mu = 1, 2, 3, 4; x = (r, z, \mathbf{u})$; $A_\mu(x) = (A(x), i\phi(x))$.

For simplifying equations and calculations in classical electrodynamics [19] one frequently imposes on $A_\mu(x)$ the Lorentz condition

$$\frac{\partial A_\mu(x)}{\partial x_\mu} = 0. \quad (10)$$

However, in quantum electrodynamics (QED) this condition cannot hold for the quantum vector field $A_\mu(x)$. Instead of condition (10) it was reasonably proposed to assume another similar but weaker condition imposed on the state vectors $|\Phi_1\rangle$ and $|\Phi_2\rangle$ of all physical states of the system:

$$\langle \Phi_1 | \frac{\partial A_\mu(x)}{\partial x_\mu} | \Phi_2 \rangle = 0. \quad (11)$$

In the fundamental research works on QED [20, 21] it was demonstrated that due to the weak Lorentz condition (11) the electromagnetic waves in the states with longitudinal and scalar polarizations play no role in all physical processes. Therefore in the Hilbert space of state vectors of all physical states of the system the vector potential field $A(r, t)$ has following effective Fourier expansion formula

$$A(r, z, t) = \frac{1}{(2\pi)^{3/2}} \int \frac{dk}{(2\Omega(k, l))^{1/2}} \sum_{\sigma = \pm 1} \frac{1}{\sqrt{2\Omega(k, l)}} \left( \xi_{\pm k} e^{ikr+iz+\Omega(k, l)t} c_{\pm k}^l + \xi_{\pm k}^* e^{-ikr+iz+\Omega(k, l)t} c_{\pm k}^l \right).$$

$$(12)$$

where $\xi_{\pm k}$ with $\sigma = \pm 1$ are two 3-component complex unit vectors characterizing two transversely polarized states of the electromagnetic plane waves with the wave vector $|k, l\rangle$. The vector field $A(r, t)$ of boundary electromagnetic field is

$$A(r, t) = \frac{1}{(2\pi)^{3/2}} \int \frac{dk}{(2\Omega(k, l))^{1/2}} \sum_{\sigma = \pm 1} \frac{1}{\sqrt{2\Omega(k, l)}} \left( \xi_{\pm k} e^{ikr+iz+\Omega(k, l)t} c_{\pm k}^l + \xi_{\pm k}^* e^{-ikr+iz+\Omega(k, l)t} c_{\pm k}^l \right).$$

$$(13)$$

It looks like a linear combination of an un-numerable set of vector quantum fields $A(r, t)_l$ each of them being labeled by a value of the continuous index $l$:

$$A(r, t)_l = \frac{1}{2\pi} \int \frac{dk}{\sigma = \pm 1} \frac{1}{\sqrt{2\Omega(k, l)}} \left( \xi_{\pm k} e^{ikr+iz+\Omega(k, l)t} c_{\pm k}^l + \xi_{\pm k}^* e^{-ikr+iz+\Omega(k, l)t} c_{\pm k}^l \right).$$

$$(14)$$

The vector field $A(r, t)_l$ with an index $l = 0$ looks like a massive free vector field with the mass $|l| l$ in the $(2 + 1)$-dimensional Minkowski space–time.

Note that matrix elements of scalar field $\phi(r, t)$ of boundary electromagnetic field between two state vectors $|\Phi_1\rangle$ and $|\Phi_2\rangle$ of any pairs of two physical states of the system always vanish. Therefore there is no necessity to write the explicit expression of $\phi(r, t)$.

The 2-point Green function of free electromagnetic field at $T = 0$ is defined as follows:

$$D_{\mu\nu}(r, z, t) = -i\langle T \{ A_\mu(r, z, t) A_\nu(0,0,0) \} \rangle, \quad (15)$$

where the symbol $\langle \cdots \rangle$ denote the average of inserted expression (containing field operators) in the ground state $|0\rangle$ of the Dirac fermion gas

$$\langle \cdots \rangle = \langle 01 \ldots 10 \rangle. \quad (16)$$

This ground state can be considered as the vacuum of electromagnetic field.

In QED it was shown that due to the gauge invariance of the theory one always can chose to work in such a gauge that the boundary limit

$$D_{\mu\nu}(r, z, t) = \lim_{z \to 0} D_{\mu\nu}(r, z, t) \quad (17)$$

of the 2-point Green function (16) of free electromagnetic field has following simple formula [20, 21]

$$D_{\mu\nu}(r, t) = \frac{1}{(2\pi)^2} \int dk \int dt \int dk_0 e^{i(kr-k_0t)} D_{\mu\nu}(k, l, k_0) \quad (18)$$

with

$$D_{\mu\nu}(k, l, k_0) = \delta_{\mu\nu} \frac{1}{i(k^2 + l^2 - k_0^2 - io)}. \quad (19)$$

Formula (18) shows that $D_{\mu\nu}(r, t)$ is a linear combination

$$D_{\mu\nu}(r, t)_l = \frac{1}{(2\pi)^2} \int dk \int dt D_{\mu\nu}(k, l, k_0) \quad (20)$$

of an un-numerable set of functions $D_{\mu\nu}(r, t)_l$ labeled by the continuous index $l$.

$$D_{\mu\nu}(r, t)_l = \delta_{\mu\nu} \frac{1}{(2\pi)^2} \int dk \int dt e^{i(kr-k_0t)}$$

$$\times \frac{1}{i(k^2 + l^2 - k_0^2 - io)}. \quad (21)$$

Thus formulae (14) and (15) together with formulae (20) and (21) clearly demonstrated that the boundary vector field $A(r, t)$ and the boundary 2-point Green function $D_{\mu\nu}(r, t)$ are effectively represented as the linear combinations (13) and (20) of the quantum vector field $A(r, t)_l$ and the 2-point Green functions $D_{\mu\nu}(r, t)_l$ labeled by a continuous index $l$ having real values in the whole infinite interval from $-\infty$ to $+\infty$. It is interesting to note that quantum boundary vector fields $A(r, t)_l$ with $l = 0$ look like quantum massive vector fields with the
continuous mass $|l|$, and boundary 2-point Green functions $D_{ij}(\mathbf{r}, t)$ also look like those of those quantum massive vector fields.

3. Perturbation theory

In the present section we develop perturbation theory for studying electromagnetic interaction processes in single-layer graphene. The $S$-matrix is expressed in terms of interaction Hamiltonian (4) as follows

$$S = T \left\{ \exp \left[ -i \int dt H_{\text{int}}(t) \right] \right\},$$

where the integration with respect to the time variable $t$ is performed over the whole real axis from $-\infty$ to $+\infty$. By expanding the exponential function in rhs of formula (22) into power series, we express $S$-matrix in the form of a series

$$S = 1 + \sum_{n=1}^{\infty} S^{(n)},$$

where the $n$th order term $S^{(n)}$ is

$$S^{(n)} = \frac{(-i)^n}{n!} \int dt_1 \int dt_2 \cdots \int dt_n \times T \{ H_{\text{int}}(t_1)H_{\text{int}}(t_2) \cdots H_{\text{int}}(t_n) \}.$$

As the first example of the application of perturbation theory let us study boundary 2-point Green functions of the interacting system comprising electromagnetic field in the whole three-dimensional physical space and the Dirac fermions moving only in the graphene plane. They are expressed in terms of the boundary limits at $z = 0$ of the vector potential field $A(\mathbf{r}, z, t)$ and scalar potential field $\varphi(\mathbf{r}, z, t)$

$$A(\mathbf{r}, t) = \lim_{z \to -0} A(\mathbf{r}, z, t),$$

$$\varphi(\mathbf{r}, t) = \lim_{z \to 0} \varphi(\mathbf{r}, z, t).$$

Dirac fermion fields $\psi^{K',K}(\mathbf{r}, t)$ and $S$-matrix as follows:

$$D_{ij}(\mathbf{r} - \mathbf{r}', t - t') = -i \left\{ \frac{T \{ S A_i(\mathbf{r}, t) A_j(\mathbf{r}', t') \}}{\langle S \rangle} \right\}$$

with $i, j = 1, 2, 3$,

$$D_{00}(\mathbf{r} - \mathbf{r}', t - t') = -i \left\{ \frac{T \{ S \varphi(\mathbf{r}, t) \varphi(\mathbf{r}', t') \}}{\langle S \rangle} \right\}$$

and

$$\Delta_{\alpha\beta}^{K',K}(\mathbf{r} - \mathbf{r}', t - t') = -i \left\{ \frac{T \{ S \psi^{K',K}_\alpha(\mathbf{r}, t) \psi^{K',K}_\beta(\mathbf{r}', t') \}}{\langle S \rangle} \right\}.$$
\[ E(k) = v_F k, \quad k = |k| = \sqrt{k_x^2 + k_y^2}, \]  
\[
\begin{align*}
\eta^K(k) &= \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\theta(k)/2} \\ e^{-i\theta(k)/2} \end{pmatrix}, \\
\eta^F(k) &= \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\theta(k)/2} \\ -e^{-i\theta(k)/2} \end{pmatrix}.
\end{align*}
\]  
\[ \theta(k) = \arctan \frac{k_y}{k_x}. \]  

\eta and \eta' being two arbitrary phase factors \(|\eta| = |\eta'| = 1.\)

In order to calculate \(D_j(r - r', t - t')\) let us consider matrix element

\[
\langle T \{S^{(2)} A_i(r, t) A_j(r', t') \} \rangle = \frac{(-i)^2}{2!} \int \! dt_1 \int \! dt_2 \times \langle T \{H_{\text{int}}(t_1)H_{\text{int}}(t_2)\} \rangle. \]  

Using formula \((4)\) of the interaction Hamiltonian \(H_{\text{int}}(t)\) we rewrite matrix element in the form explicitly containing all quantum field operators of electromagnetic field and Dirac fermion fields:

\[
\langle T \{S^{(2)} A_i(r, t) A_j(r', t') \} \rangle = \frac{(-i)^2}{2!} \int \! dt_1 \int \! dt_2 \int \! dt_2 \int \! dt_2 \sum_{n=1}^2 \langle T \{H_{\text{int}}(t_1)H_{\text{int}}(t_2)\} \rangle \times \langle T \{A_i(r_1, t_1)A_j(r_2, t_2)\} \rangle \]

\[
\langle T \{S^{(2)} A_i(r, t) A_j(r', t') \} \rangle = \frac{(-i)^2}{2!} \int \! dt_1 \int \! dt_2 \int \! dt_2 \int \! dt_2 \sum_{n=1}^2 \langle T \{H_{\text{int}}(t_1)H_{\text{int}}(t_2)\} \rangle \times \langle T \{A_i(r_1, t_1)A_j(r_2, t_2)\} \rangle \]

In order to demonstrate the calculation method let us consider in detail the first term in rhs of equation \((43)\). According to the well-known Wick theorem for the average of any product of quantum free fields in the ground state of the system we have

\[
\langle T \{A_i(r, t)A_j(r', t')A_m(r_1, t_1)A_m(r_2, t_2)\} \rangle \times \langle T \{A_i(r_1, t_1)A_j(r_2, t_2)\} \rangle \]

\[ = \langle T \{A_i(r, t)A_j(r', t')A_m(r_1, t_1)A_m(r_2, t_2)\} \rangle \times \langle T \{A_i(r_1, t_1)A_j(r_2, t_2)\} \rangle \]

\[ = \langle T \{A_i(r_1, t_1)A_j(r_2, t_2)\} \rangle \times \langle T \{A_i(r_1, t_1)A_j(r_2, t_2)\} \rangle. \]

According to formulae \((26)\), in order to find \(D_j(r - r', t - t')^{(2)}\) it is necessary to calculate also the average of 2nd order terms \(S^{(2)}\) of the S-matrix. We have

\[
\langle S^{(2)} \rangle = \frac{(-i)^2}{2!} \int \! dt \int \! dt_2 \langle T \{H_{\text{int}}(t_1)H_{\text{int}}(t_2)\} \rangle. \]  

(44)

(45)

\[ \langle T \{S^{(2)} A_i(r, t) A_j(r', t') \} \rangle = \frac{(-i)^2}{2!} \int \! dt_1 \int \! dt_2 \int \! dt_2 \int \! dt_2 \sum_{n=1}^2 \langle T \{H_{\text{int}}(t_1)H_{\text{int}}(t_2)\} \rangle \times \langle T \{A_i(r_1, t_1)A_j(r_2, t_2)\} \rangle \]

(46)

(47)
It is obvious that the second term in rhs of equation (43) vanishes. The third term comprises following expression
\[
\langle T \{ A_n(r_1, t_1)A_m(r_2, t_2)\psi^K(r_1, t_1)\psi^K(r_2, t_2) \}
\times \psi^K(r_1, t_1)\psi^K(r_2, t_2) \rangle
\]
\[
= - D_{ij} \langle r - r', t - t' \rangle^{(0)} \langle r_i - r_j, t_i - t_j \rangle^{(0)}
\times \text{Tr} \{ [\Delta^K(r_1 - r_2, t_1 - t_2)]^{(0)} \Delta^K(r_2 - r_1, t_2 - t_1) \}
\]
(48)
and a similar one with the replacement \( K \rightarrow K' \).

For calculating \( D_{ij}(r - r', t - t' )^{(2)} \) it is necessary to calculate also matrix element \( \langle S^{(2)}(r', t') \rangle \). The first term in rhs of formula (45) contains following expression
\[
\langle T \{ A_n(r_1, t_1)A_m(r_2, t_2)\psi^K(r_1, t_1)\psi^K(r_2, t_2) \}
\times (r_1, t_1)\psi^K(r_2, t_2) \rangle
\]
\[
= iD_{nm}(r_1 - r_2, t_1 - t_2)^{(0)} \text{Tr} \{ [\Delta^K(r_1 - r_2, t_1 - t_2)]^{(0)}
\times \Delta^K(r_2 - r_1, t_2 - t_1) \}
\]
(49)
and a similar one with the replacement \( K \rightarrow K' \), \( \tau_n \rightarrow \tau_n^* \). The second term vanishes. The third term contains following expressions
\[
\langle T \{ \phi(r_1, t_1)\phi(r_2, t_2)\psi^K(r_1, t_1)\psi^K(r_2, t_2) \}
\times (r_2, t_2)^+\psi^K(r_2, t_2) \rangle
\]
\[
= D_{00}(r - r', t - t' )^{(2)}
\times \text{Tr} \{ [\Delta^K(r_1 - r_2, t_1 - t_2)]^{(0)} \Delta^K(r_2 - r_1, t_2 - t_1) \}
\]
(50)

According to the definition (26) we have
\[
D_{ij}(r - r', t - t' )^{(2)} = - i\langle T \{ S^{(2)}A(r, t)A_j(r', t') \}
\times \phi^K(r_1, t_1)\psi^K(r_2, t_2) \rangle
\]
\[
= - i\langle T \{ A_n(r_1, t_1)A_m(r_2, t_2)\psi^K(r_1, t_1)\psi^K(r_2, t_2) \}
\times (r_2, t_2)^+\psi^K(r_2, t_2) \rangle
= 0.
\]
(51)

On the basis of relations (43)–(51) it can be demonstrated that function (52) has following expression
\[
D_{ij}(r - r', t - t' )^{(2)} = - i e^2 \frac{2}{\hbar^2} \int dt_1 \int dr_1 \int dt_2 \int dr_2 \int dr_3 \int dt_3
\times \sum_{n=1}^{2} \sum_{l=1}^{2} D_{nl}(r - r_1, t - t_1)^{(0)}
\times D_{nm}(r_2 - r', t_2 - t')^{(0)}
\times \text{Tr} [\tau_n \Delta^K(r_1 - r_2, t_1 - t_2)]^{(0)}\tau_m
\times \Delta^K(r_2 - r_1, t_2 - t_1)]^{(0)}
+ \text{Tr} [\tau_n \Delta^K(r_1 - r_2, t_1 - t_2)]^{(0)}\tau_m
\times \Delta^K(r_2 - r_1, t_2 - t_1)]^{(0)}
\times \tau_m \Delta^K(r_2 - r_1, t_2 - t_1)]^{(0)}
\]
(53)

According to the definition (27) we have
\[
D_{00}(r - r', t - t' )^{(2)} = - i\langle T \{ S^{(2)}\phi(r, t)\phi(r', t') \}
\times \phi^K(r_2, t_2)^+\psi^K(r_2, t_2) \rangle
\]
\[
= - i\langle T \{ A_n(r_1, t_1)A_m(r_2, t_2)\psi^K(r_1, t_1)\psi^K(r_2, t_2) \}
\times (r_2, t_2)^+\psi^K(r_2, t_2) \rangle
\]
(54)

According to the definition (26) we have
\[
D_{ij}(r - r', t - t' )^{(2)} = - i\langle T \{ S^{(2)}A(r, t)A_j(r', t') \}
\times \phi^K(r_1, t_1)\psi^K(r_2, t_2) \rangle
\]
\[
= - i\langle T \{ A_n(r_1, t_1)A_m(r_2, t_2)\psi^K(r_1, t_1)\psi^K(r_2, t_2) \}
\times (r_2, t_2)^+\psi^K(r_2, t_2) \rangle
\]
(55)

Let us calculate the first matrix element in rhs of relation (55)
\[
\langle T \{ S^{(2)}\phi(r, t)\phi(r', t') \}\rangle = \frac{(-i)^2}{2\hbar^2} \int dt_1 \int dr_1 \int dt_2 \int dr_2 \int dr_3 \int dt_3
\times \sum_{n=1}^{2} \sum_{l=1}^{2} D_{nl}(r - r_1, t - t_1)^{(0)}
\times D_{nm}(r_2 - r', t_2 - t')^{(0)}
\times \text{Tr} [\tau_n \Delta^K(r_1 - r_2, t_1 - t_2)]^{(0)}\tau_m
\times \Delta^K(r_2 - r_1, t_2 - t_1)]^{(0)}
+ \text{Tr} [\tau_n \Delta^K(r_1 - r_2, t_1 - t_2)]^{(0)}\tau_m
\times \Delta^K(r_2 - r_1, t_2 - t_1)]^{(0)}
\times \tau_m \Delta^K(r_2 - r_1, t_2 - t_1)]^{(0)}
\]
(56)
Consider first term in rhs of equation (56). It contains expression

\[
\langle T [\phi(r, t)\phi(r', t')A_0(r_1, t_1)A_m(r_2, t_2) \\
\times [\psi^K(r_1, t_1) + \tau_\mu^K(r_1, t_1)] \\
+ [\psi^K(r_2, t_2) + \tau^K(r_2, t_2)] \rangle \\
+ \psi^K(r_1, t_1) + \tau^K(r_1, t_1)] \\
\times [\psi^K(r_2, t_2) + \tau^K(r_2, t_2)] \rangle \\
+ \psi^K(r_2, t_2) + \tau^K(r_2, t_2)] \rangle \\
= -D_{00}(r - r', t - t')D_{00}(r_1 - r_2, t_1 - t_2) \\
\times [\langle \tau^K(r_1, t_1)A_0(r_1, t_1)A_m(r_2, t_2) \rangle \\
+ \tau^K(r_1, t_1)] \\
\times [\langle \tau^K(r_2, t_2)A_0(r_1, t_1)A_m(r_2, t_2) \rangle \\
+ \tau^K(r_2, t_2)] \rangle \\
+ \langle \tau^K(r_1, t_1)A_0(r_1, t_1)A_m(r_2, t_2) \rangle \\
\times [\langle \tau^K(r_2, t_2)A_0(r_1, t_1)A_m(r_2, t_2) \rangle \\
+ \tau^K(r_2, t_2)] \rangle \\
\times [\langle \tau^K(r_1, t_1)A_0(r_1, t_1)A_m(r_2, t_2) \rangle \\
\times [\langle \tau^K(r_2, t_2)A_0(r_1, t_1)A_m(r_2, t_2) \rangle \\
+ \tau^K(r_2, t_2)] \rangle \\
\times [\langle \tau^K(r_1, t_1)A_0(r_1, t_1)A_m(r_2, t_2) \rangle \\
\times [\langle \tau^K(r_2, t_2)A_0(r_1, t_1)A_m(r_2, t_2) \rangle \\
+ \tau^K(r_2, t_2)] \rangle \rangle \} \\
= -D_{00}(r - r', t - t')D_{00}(r_1 - r_2, t_1 - t_2) \\
\times \langle \tau^K(r_1, t_1)A_0(r_1, t_1)A_m(r_2, t_2) \rangle \\
\times [\langle \tau^K(r_2, t_2)A_0(r_1, t_1)A_m(r_2, t_2) \rangle \\
+ \tau^K(r_2, t_2)] \rangle \\
+ \langle \tau^K(r_1, t_1)A_0(r_1, t_1)A_m(r_2, t_2) \rangle \\
\times \langle \tau^K(r_2, t_2)A_0(r_1, t_1)A_m(r_2, t_2) \rangle \\
\times [\langle \tau^K(r_1, t_1)A_0(r_1, t_1)A_m(r_2, t_2) \rangle \\
\times [\langle \tau^K(r_2, t_2)A_0(r_1, t_1)A_m(r_2, t_2) \rangle \\
+ \tau^K(r_2, t_2)] \rangle \rangle \} (57) \\
\end{eqnarray}

\[
\begin{align}
\langle T [S^{(2)}(r, t)|\psi^K(r', t')^+ \rangle \rangle \\
= \langle \langle S^{(2)}(r, t)|\psi^K(r', t')^+ \rangle \rangle \\
\times \langle \langle S^{(2)}(r, t)|\psi^K(r', t')^+ \rangle \rangle \\
\times \langle \langle S^{(2)}(r, t)|\psi^K(r', t')^+ \rangle \rangle \} \\
\end{align}
\]

For the fields \(\psi^K(r, t)\) and \(\psi^K(r', t')\) the first matrix element in rhs formula (59) has the form

\[
\langle T [S^{(2)}(r, t)|\psi^K(r', t')^+ \rangle \rangle \\
= \langle \langle S^{(2)}(r, t)|\psi^K(r', t')^+ \rangle \rangle \\
\times \langle \langle S^{(2)}(r, t)|\psi^K(r', t')^+ \rangle \rangle \\
\times \langle \langle S^{(2)}(r, t)|\psi^K(r', t')^+ \rangle \rangle \} \\
\end{eqnarray}

\[
\begin{align}
\langle T [S^{(2)}(r, t)|\psi^K(r', t')^+ \rangle \rangle \\
= \langle \langle S^{(2)}(r, t)|\psi^K(r', t')^+ \rangle \rangle \\
\times \langle \langle S^{(2)}(r, t)|\psi^K(r', t')^+ \rangle \rangle \\
\times \langle \langle S^{(2)}(r, t)|\psi^K(r', t')^+ \rangle \rangle \} \\
\end{align}
\]

The first matrix element in rhs formula (61)

\[
\langle T [A_0(r_1, t_1)A_m(r_2, t_2)|\psi^K(r', t')^+ \rangle \rangle \\
= \langle \langle S^{(2)}(r, t)|\psi^K(r', t')^+ \rangle \rangle \\
\times \langle \langle S^{(2)}(r, t)|\psi^K(r', t')^+ \rangle \rangle \\
\times \langle \langle S^{(2)}(r, t)|\psi^K(r', t')^+ \rangle \rangle \} \\
\end{eqnarray}

Consider now Green functions (28) of Dirac fermions. These Green functions are expanded into the series of the form (31). The second order term in each series is determined by formula

\[
\Delta^K_{\alpha\beta}(r - r', t - t') = -i \langle \langle S^{(2)}(r, t)|\psi^K_{\alpha}(r', t')^+ \rangle \rangle \\
\times \langle \langle S^{(2)}(r, t)|\psi^K_{\beta}(r', t')^+ \rangle \rangle \\
\times \langle \langle S^{(2)}(r, t)|\psi^K_{\gamma}(r', t')^+ \rangle \rangle \\
\times \langle \langle S^{(2)}(r, t)|\psi^K_{\delta}(r', t')^+ \rangle \rangle \} \\
\end{eqnarray}

\[
\begin{align}
\Delta^K_{\alpha\beta}(r - r', t - t') = -i \langle \langle S^{(2)}(r, t)|\psi^K_{\alpha}(r', t')^+ \rangle \rangle \\
\times \langle \langle S^{(2)}(r, t)|\psi^K_{\beta}(r', t')^+ \rangle \rangle \\
\times \langle \langle S^{(2)}(r, t)|\psi^K_{\gamma}(r', t')^+ \rangle \rangle \\
\times \langle \langle S^{(2)}(r, t)|\psi^K_{\delta}(r', t')^+ \rangle \rangle \} \\
\end{align}
\]
The second matrix element in rhs equation (61) vanishes, and the third one is

\[ \langle \{ \varphi(t, r_1) \varphi(t, r_2) \varphi(t, r_3) \}^\dagger \varphi(t, r_4) \rangle \]

and (59). They can be represented by the Feynman diagram in figure 1(a). The second order terms \( \Delta_{\text{A}}(r - r', t - t')\) and \( \Delta_{\text{B}}(r - r', t - t')\) of the Dirac fermion fields are determined by formulae (64) and (65). They can be represented by the Feynman diagram in figure 1(b).

For shortening expressions (53) and (59) of second order terms of boundary 2-point Green functions of electromagnetic field we introduce the self-energy parts of boundary electromagnetic field

\[ \Pi_{\text{nm}}(r_1 - r_2, h - t_2) \]

and

\[ \Pi_{\text{nm}}(r_1 - r_2, h - t_2) \]

Then formulae (53) and (59) become

\[ D_\text{g}(r - r', t - t') = \int \mathcal{D}h \int \mathcal{D}r_1 \int \mathcal{D}r_2 \int \mathcal{D}t_1 \int \mathcal{D}t_2 \sum_{n = 0}^{2} \sum_{m = 0}^{2} \Delta_{\text{A}}^{\text{nm}}(r - r_1, h - t_1) \]

and

\[ D_\text{d}(r - r', t - t') = \int \mathcal{D}h \int \mathcal{D}r_1 \int \mathcal{D}r_2 \int \mathcal{D}t_1 \int \mathcal{D}t_2 \sum_{n = 0}^{2} \sum_{m = 0}^{2} \Delta_{\text{B}}^{\text{nm}}(r - r_1, h - t_1) \]

Similarly, for shortening expressions (64) and (65) of second order terms of 2-point Green functions of Dirac fermion fields we introduce the self-energy parts of Dirac fermion fields

\[ \Sigma_{\text{nm}}^{\text{K, A}}(r_1 - r_2, h - t_2) = \int \mathcal{D}h \int \mathcal{D}r_1 \int \mathcal{D}r_2 \int \mathcal{D}t_1 \int \mathcal{D}t_2 \sum_{n = 0}^{2} \sum_{m = 0}^{2} \Delta_{\text{A}}^{\text{nm}}(r - r_1, h - t_1) \]

and

\[ \Sigma_{\text{nm}}^{\text{K, B}}(r_1 - r_2, h - t_2) = \int \mathcal{D}h \int \mathcal{D}r_1 \int \mathcal{D}r_2 \int \mathcal{D}t_1 \int \mathcal{D}t_2 \sum_{n = 0}^{2} \sum_{m = 0}^{2} \Delta_{\text{B}}^{\text{nm}}(r - r_1, h - t_1) \]
Then formulae (64) and (65) can be rewritten in the new forms

\[
\Delta^K_{\alpha\beta}(\mathbf{r} - \mathbf{r}', t - t')^{(2)} = \int \! d\mathbf{r}_1 \int \! d\mathbf{r}_2 \int \! d\mathbf{r}_3 \int \! d\mathbf{r}_4 \Delta^K_{\alpha\beta}(\mathbf{r} - \mathbf{r}_1, t - t_1^{(0)}) \times \Delta^K_{\alpha\beta}(\mathbf{r}_2 - \mathbf{r}_1, t_2 - t_1^{(0)}) \times \sum_{n=1}^{m=2} (\tau^K_{\alpha\gamma})_{\alpha\gamma} \times (\tau^K_{\beta\gamma})_{\gamma\beta} \times \Delta^K_{\alpha\beta}(\mathbf{r}_3 - \mathbf{r}_2, t_3 - t_2^{(0)}) \times \Delta^K_{\alpha\beta}(\mathbf{r}_4 - \mathbf{r}_3, t_4 - t_3^{(0)}) + \int \! d\mathbf{r}_1 \int \! d\mathbf{r}_2 \int \! d\mathbf{r}_3 \int \! d\mathbf{r}_4 \Delta^K_{\alpha\beta}(\mathbf{r} - \mathbf{r}_1, t - t_1^{(0)}) \times \Delta^K_{\alpha\beta}(\mathbf{r}_2 - \mathbf{r}_1, t_2 - t_1^{(0)}) \times (\tau^K_{\alpha\gamma})_{\alpha\gamma} \times (\tau^K_{\beta\gamma})_{\gamma\beta} \times \Delta^K_{\alpha\beta}(\mathbf{r}_3 - \mathbf{r}_2, t_3 - t_2^{(0)}) \times \Delta^K_{\alpha\beta}(\mathbf{r}_4 - \mathbf{r}_3, t_4 - t_3^{(0)})
\]

(72)

and of self-energy parts

\[
\Pi_{00}(\mathbf{r}, t) = \frac{1}{(2\pi)^2} \int \! d\mathbf{k} \int \! d\omega \, e^{i(kr - \omega t)} \tilde{\Pi}_{00}(\mathbf{k}, \omega), \\
\Pi_{00}(\mathbf{r}, t) = \frac{1}{(2\pi)^2} \int \! d\mathbf{k} \int \! d\omega \, e^{i(kr - \omega t)} \tilde{\Pi}_{00}(\mathbf{k}, \omega), \\
\Sigma^K_{\gamma\gamma}(\mathbf{r}, t) = \frac{1}{(2\pi)^2} \int \! d\mathbf{k} \int \! d\omega \, e^{i(kr - \omega t)} \tilde{\Sigma}^{K'}_{\gamma\gamma}(\mathbf{k}, \omega), \\
\Sigma^K_{\gamma\gamma}(\mathbf{r}, t) = \frac{1}{(2\pi)^2} \int \! d\mathbf{k} \int \! d\omega \, e^{i(kr - \omega t)} \tilde{\Sigma}^{K'}_{\gamma\gamma}(\mathbf{k}, \omega),
\]

(75)

we rewrite relations (68), (69) and (72), (73) in the compact forms of algebraic equations

\[
\tilde{D}_j(\mathbf{k}, \omega)^{(2)} = \sum_{n=1}^{m=2} \tilde{D}_n(\mathbf{k}, \omega)^{(0)} \tilde{\Pi}_{nm}(\mathbf{k}, \omega),
\]

(76)

\[
\tilde{D}_{00}(\mathbf{k}, \omega)^{(2)} = \tilde{D}_{00}(\mathbf{k}, \omega)^{(0)} \tilde{\Pi}_{00}(\mathbf{k}, \omega) \tilde{D}_{00}(\mathbf{k}, \omega),
\]

(77)

and

\[
\tilde{\Delta}^K_{\alpha\beta}(\mathbf{k}, \omega)^{(2)} = \sum_{n=1}^{m=2} \tilde{\Delta}^K_{\alpha\beta}(\mathbf{k}, \omega)^{(0)} \tilde{\Sigma}^{K'}_{\alpha\beta}(\mathbf{k}, \omega) \tilde{\Delta}^K_{\alpha\beta}(\mathbf{k}, \omega)^{(0)} + \tilde{\Delta}^K_{\alpha\beta}(\mathbf{k}, \omega)^{(0)} \tilde{\Sigma}^{K'}_{\alpha\beta}(\mathbf{k}, \omega) \tilde{\Delta}^K_{\alpha\beta}(\mathbf{k}, \omega)^{(0)}.
\]

(78)
\[ \tilde{\Delta}_{\alpha\beta}^{K'}(k, \omega) = \Delta_{\alpha\beta}^{K'}(k, \omega) \]

4. Conclusion and discussion

Using basic formulae for the Hamiltonian of the interacting system comprising the electromagnetic field in the three-dimensional physical space and the Dirac fermion fields in a single-layer graphene sheet we have an efficient method to the theoretical study of electromagnetic interaction processes in single-layer graphene, working in the interaction picture. For this purpose we have determined the boundary free electromagnetic field on graphene as well as boundary limits on the graphene plane of 2-point Green functions of the free electromagnetic field, briefly called boundary Green functions of the free electromagnetic field. Then we have explicitly constructed the S-matrix and applied the perturbation theory to establish expressions of the boundary 2-point Green functions of electromagnetic field and 2-point Green functions of Dirac fermion fields in the second order of the perturbation theory. We have shown that the Fourier transforms \( \tilde{D}_{ij}(k, \omega)^{(2)} \) and \( \tilde{D}_{00}(k, \omega)^{(2)} \) of second order terms of boundary Green functions of electromagnetic field are determined by equations (76) and (77), and the Fourier transforms \( \tilde{\Delta}_{\alpha\beta}^{K'}(k, \omega)^{(2)} \) of second order terms of Dirac fermion Green functions are determined by equations (78) and (79).

The total boundary 2-point Green functions (26) and (27) of electromagnetic field and total 2-point Dirac fermion Green functions (28) will be calculated in a subsequent work. We expect that in the ladder approximation the Fourier transforms \( \tilde{D}_{ij}(k, \omega) \) and \( \tilde{D}_{00}(k, \omega) \) of boundary total 2-point Green functions of electromagnetic field satisfy Dyson equations:

\[ \tilde{D}_{ij}(k, \omega) = \tilde{D}_{ij}(k, \omega)^{(0)} + \sum_{n} \sum_{l} \tilde{D}_{il}(k, \omega)^{(0)} \times \Sigma_{ll}(k, \omega) \tilde{D}_{lj}(k, \omega), \]

\[ \tilde{D}_{00}(k, \omega) = \tilde{D}_{00}(k, \omega)^{(0)} + \tilde{D}_{00}(k, \omega)^{(0)} \times \Pi_{00}(k, \omega) \tilde{D}_{00}(k, \omega), \]

and the Fourier transforms \( \tilde{\Delta}_{\alpha\beta}^{K'}(k, \omega) \) of total 2-point Dirac fermion Green functions satisfy Dyson equations:

\[ \tilde{\Delta}_{\alpha\beta}^{K'}(k, \omega) = \Delta_{\alpha\beta}^{K'}(k, \omega)^{(0)} + \sum_{n} \sum_{l} \Delta_{\alpha\beta}^{K'}(k, \omega)^{(0)} \times \Sigma_{ll}(k, \omega) \Delta_{\alpha\beta}^{K'}(k, \omega), \]