The occurrence of transverse and longitudinal electric currents in the classical plasma under the action of $N$ transverse electromagnetic waves

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Abstract

Classical plasma with arbitrary degree of degeneration of electronic gas is considered. In plasma $N$ ($N > 2$) collinear electromagnetic waves are propagated. It is required to find the response of plasma to these waves. Distribution function in square-law approximation on quantities of two small parameters from Vlasov equation is received. The formula for electric current calculation is deduced. It is demonstrated that the nonlinearity account leads to occurrence of the longitudinal electric current directed along a wave vector. This longitudinal current is orthogonal to the known transversal current received at the linear analysis. The case of small values of wave number is considered.

Key words: Vlasov equation, classical plasma, transversal and longitudinal and transversal electric currents, nonlinear analysis.

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1 Introduction

In the present work formulas are deduced for electric current calculation in classical collisionless plasma. At the solution of the kinetic Vlasov equation describing behaviour of classical degenerate plasmas, we consider as in decomposition distribution functions, and in decomposition of quantity of the self-conjugate electromagnetic field the quantities proportional to square of intensity of an external electric field. In such nonlinear approach it appears

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that the electric current has two nonzero components. One component of an electric current is directed along vector potentials of electromagnetic fields. These components of an electric current precisely same, as well as in the linear analysis. It is a "transversal" current.

Those, in linear approach we receive known expression of a transversal electric current.

The second nonzero an electric current component has the second order of smallness concerning quantities intensity of electric fields. The second electric current component is directed along a wave vector. This current is orthogonal to the first a component. It is a "longitudinal" current.

Occurrence of a longitudinal current comes to light the spent nonlinear analysis of interaction of electromagnetic fields with plasma.

Nonlinear effects in plasma are studied already long time [1]–[10].

In works [1] and [6] nonlinear effects in plasma are studied. In work [6] nonlinear current was used, in particular, in probability questions decay processes. We will notice, that in work [2] it is underlined existence of nonlinear current along a wave vector (see the formula (2.9) from [2]).

In experimental work [3] the contribution normal field components in a nonlinear superficial current in a signal of the second harmonic is found out. In works [4, 5] generation of a nonlinear superficial current was studied at interaction of a laser impulse with metal.

We will specify in a number of works on plasma, including to the quantum. These are works [11]–[17].

2 The Vlasov equation

Let us demonstrate, that in case of the classical plasma described by kinetic Vlasov equation, the longitudinal current is generated and we will calculate its density. It was specified in existence of this current more half a century ago [1].

Let us consider that the $N$ electromagnetic waves are propagated with
strengths
\[ E_j = E_{0j} e^{i(k_j r - \omega_j t)}, \quad H_j = H_{0j} e^{i(k_j r - \omega_j t)}, \]
where \( j = 1, 2, \cdots, N \).

Let us assume that directions of propagation of waves are collinear, that is \( k_1 \parallel k_2 \parallel \cdots \parallel k_N \).

We will consider a case, when the directions electric (and magnetic) fields of waves are collinear \( E_1 \parallel E_2 \parallel \cdots \parallel E_N \), \( H_1 \parallel H_2 \parallel \cdots \parallel H_N \). Corresponding electric and magnetic fields are connected with vector potentials equalities
\[ E_j = -\frac{1}{c} \frac{\partial A_j}{\partial t} = \frac{i \omega_j}{c} A_j, \quad H_j = \text{rot} A_j, \quad j = 1, 2, \cdots, N. \]

We take the Vlasov equation describing behavior of classical collisionless plasma
\[ \frac{\partial f}{\partial t} + v \frac{\partial f}{\partial r} + c \left( \sum_{j=1}^{N} E_j + \frac{1}{c} \left[ v, \sum_{j=1}^{N} H_j \right] \right) \frac{\partial f}{\partial p} = 0. \quad (1.1) \]

In the equation (1.1) \( f \) is cumulative distribution function of electrons of plasma, \( E_j, H_j (j = 1, 2, \cdots, N) \) are components of an electromagnetic field, \( c \) is the velocity of light, \( p = mv \) is momentum of electrons, \( v \) is the electrons velocity, \( f^{(0)} = f_{\text{eq}}(r, v) \) (eq \( \equiv \) equilibrium) is local equilibrium distribution of Fermi–Dirac
\[ f_{\text{eq}}(r, v) = \frac{1}{1 + \exp \left( \frac{\mathcal{E} - \mu(r)}{k_B T} \right)}, \]
or, in dimensionless form,
\[ f_{\text{eq}}(r, v) = \frac{1}{1 + \exp \left( \mathcal{P}^2 - \alpha(r) \right)} = f_{\text{eq}}(r, P), \]
\( \mathcal{E} = mv^2/2 \) is the electron energy, \( \mu \) is the chemical potential of electronic gas, \( k_B \) is the Boltzmann constant, \( T \) is the plasma temperature, \( P = P / p_T \) is dimensionless momentum of the electrons, \( p_T = mv_T \), \( v_T \) is the heat electron velocity, \( v_T = \sqrt{2k_B T/m}, \alpha = \mu/(k_B T) \) is the chemical potential, \( k_B T = \mathcal{E}_T = mv_T^2/2 \) is the heat kinetic electron energy.

Lower local equilibrium distribution of Fermi–Dirac is required to us,
\[ f_0(v) = \left[ 1 + \exp \left( \frac{\mathcal{E} - \mu}{k_B T} \right) \right]^{-1} = [1 + \exp (\mathcal{P}^2 - \alpha)]^{-1} = f_0(P). \]
It is necessary to specify, that vector potential of an electromagnetic field \( \mathbf{A}_j(\mathbf{r}, t) \) is orthogonal to a wave vector \( \mathbf{k}_j \), i.e.

\[
\mathbf{k}_j \cdot \mathbf{A}_j(\mathbf{r}, t) = 0, \quad j = 1, 2, \cdots, N.
\]

It means that the wave vector \( \mathbf{k}_j \) is orthogonal to electric and magnetic fields

\[
\mathbf{k}_j \cdot \mathbf{E}_j(\mathbf{r}, t) = \mathbf{k}_j \cdot \mathbf{H}_j(\mathbf{r}, t) = 0, \quad j = 1, 2, \cdots, N.
\]

For definiteness we will consider, that wave vectors \( \mathbf{k}_j \) of fields are directed along an axis \( x \) and electromagnetic fields are directed along an axis \( y \), i.e.

\[
\mathbf{k}_j = k_j(1, 0, 0), \quad \mathbf{E}_j = E_j(x, t)(0, 1, 0).
\]

Therefore

\[
\mathbf{E}_j = -\frac{1}{c} \frac{\partial \mathbf{A}_j}{\partial t} = \frac{i\omega_j}{c} \mathbf{A}_j, \quad \mathbf{H}_j = \frac{ck_j}{\omega} E_j \cdot (0, 0, 1), \quad j = 1, 2, \cdots, N.
\]

Let us find a vector product from the equation (1.1)

\[
[v, \mathbf{H}_j] = \frac{ck_j}{\omega} E_j (v_y, -v_x, 0),
\]

then

\[
\begin{bmatrix} v, \sum_{j=1}^{N} \mathbf{H}_j \end{bmatrix} = \sum_{j=1}^{N} \frac{ck_j}{\omega_j} E_j (v_y, -v_x, 0).
\]

We find Lorentz force by means of a vector product

\[
e \left( \mathbf{E}_j + \frac{1}{c} [v, \mathbf{H}_j] \right) \frac{\partial f}{\partial \mathbf{p}} =
\]

\[
= \frac{e}{\omega_j} E_j \left[ k_j v_y \frac{\partial f}{\partial p_x} + (\omega_j - k_j v_x) \frac{\partial f}{\partial p_y} \right], \quad (j = 1, 2, \cdots, N).
\]

Let us notice that

\[
[v, \mathbf{H}_j] \frac{\partial f_0}{\partial \mathbf{p}} = 0,
\]

as

\[
\frac{\partial f_0}{\partial \mathbf{p}} \sim v.
\]

Now the equation (1.1) is somewhat simplified:
\[ \frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + e \sum_{j=1}^{N} \frac{E_j}{\omega_j} \left[ k_j v_y \frac{\partial f}{\partial p_y} + (\omega_j - k_j v_x) \frac{\partial f}{\partial p_y} \right] = 0. \] (1.2)

We will search the solution of equation (1.2) in the form
\[ f = f_0 (P) + f_1 + f_2. \] (1.3)

Here
\[ f_1 = E_1 \varphi_1 + E_2 \varphi_2 + \cdots + E_N \varphi_N = \sum_{j=1}^{N} E_j \varphi_j, \] (1.4)

where
\[ E_j \sim e^{i(k_j x - \omega_j t)}, \]

and
\[ f_2 = \sum_{j=1}^{N} E_j^2 \psi_j + \sum_{b,s=1}^{N} E_b E_s \xi_{b,s}, \] (1.5)

where
\[ E_j^2 \sim e^{2i(k_j x - \omega_j t)}, \]
\[ E_b E_s \sim e^{i[(k_b + k_s)x - (\omega_b + \omega_s)t]}. \]

3 The solution of Vlasov equation in first approximation

In this equation exist $2N$ parameters of dimension of length $\lambda_j = \nu_T / \omega_j$ ($\nu_T$ is the heat electron velocity) and $l_j = 1 / k_j$. We shall believe, that on lengths $\lambda_j$, so and on lengths $l_j$ energy variable of electrons under acting correspond electric field $A_j$ is much less than heat energy of electrons $k_B T$ ($k_B$ is Boltzmann constant, $T$ is temperature of plasma), i.e. we shall consider small parameters
\[ \alpha_j = \frac{|eA_j| \nu_T}{c k_B T} \quad (j = 1, 2, \ldots, N) \]

and
\[ \beta_j = \frac{|eA_j| \omega_j}{k_j k_B T c} \quad (j = 1, 2, \ldots, N). \]
If to use communication of vector potentials electromagnetic fields with strengths of corresponding electric fields, injected small parameters are expressed following equalities

$$\alpha_j = \frac{|eE_j| v_T}{\omega_j k_B T} \quad (j = 1, 2, \ldots, N)$$

and

$$\beta_j = \frac{|eE_j|}{k_j k_B T} \quad (j = 1, 2, \ldots, N).$$

We will work with a method consecutive approximations, considering, that

$$\alpha_j \ll 1 \quad (j = 1, 2, \ldots, N)$$

and

$$\beta_j \ll 1 \quad (j = 1, 2, \ldots, N).$$

The equation (1.2) by means of (1.3) is equivalent to the following equations

$$\frac{\partial f_1}{\partial t} + v_x \frac{\partial f_1}{\partial x} = -e \sum_{j=1}^{N} \frac{E_j}{\omega_j} \left[ k_j v_y \frac{\partial f_0}{\partial p_x} + (\omega_j - k_j v_x) \frac{\partial f_0}{\partial p_y} \right] \quad (2.1)$$

and

$$\frac{\partial f_2}{\partial t} + v_x \frac{\partial f_2}{\partial x} = -e \sum_{j=1}^{N} \frac{E_j}{\omega_j} \left[ k_j v_y \frac{\partial f_1}{\partial p_x} + (\omega_j - k_j v_x) \frac{\partial f_1}{\partial p_y} \right]. \quad (2.2)$$

In the first approximation we search the solution of Vlasov equation in the form

$$f = f^{(1)} = f_0(P) + f_1,$$

where \( f_1 \) is the linear combination of vector potentials.

We have the following from the equation (2.1)

$$[E_1 (i\omega_1 + ik_1 v_x) \varphi_1 + E_2 (i\omega_2 + ik_2 v_x) \varphi_2 + \cdots + E_N (i\omega_N + ik_N v_x) \varphi_N] =$$

$$=-e \sum_{j=1}^{N} \frac{E_j}{\omega_j} \left[ k_j v_y \frac{\partial f_0}{\partial p_x} + (\omega_j - k_j v_x) \frac{\partial f_0}{\partial p_y} \right]. \quad (2.3)$$

Let us enter the dimensionless parameters \( \Omega_j = \frac{\omega_j}{k_T v_T} \), \( q_j = k_j k_T \), where \( q_j \) is the dimensionless wave number, \( k_T = \frac{mv_T}{\hbar} \) is the heat wave number, \( \Omega_j \) is
dimensionless oscillation frequency of vector potential electromagnetic field $E_j$.

In the equation (2.3) we will pass to the dimensionless parameters. We obtain the equation

$$[E_1(q_1P_x - \Omega_1)\varphi_1 + E_2(q_2P_x - \Omega_2)\varphi_2 + \cdots + E_N(q_NP_x - \Omega_N)\varphi_N] =$$

$$= -e\sum_{j=1}^{N} E_j \left[ q_jP_y \frac{\partial f_0}{\partial P_x} + (\Omega_j - q_jP_x) \frac{\partial f_0}{\partial P_y} \right].$$  \hspace{1cm} (2.4)

Let us notice that $\frac{\partial f_0}{\partial P_x} \sim P_x$, $\frac{\partial f_0}{\partial P_y} \sim P_y$.

Then

$$\left[ q_jP_y \frac{\partial f_0}{\partial P_x} + (\Omega_j - q_jP_x) \frac{\partial f_0}{\partial P_y} \right] = \Omega_j \frac{\partial f_0}{\partial P_y}.$$

Now the equation (2.4) is somewhat simplified

$$[E_1(q_1P_x - \Omega_1)\varphi_1 + E_2(q_2P_x - \Omega_2)\varphi_2 + \cdots + E_N(q_NP_x - \Omega_N)\varphi_N] =$$

$$= -\frac{e}{kT_p v_T} \sum_{j=1}^{N} E_j \frac{\partial f_0}{\partial P_y}. \hspace{1cm} (2.5)$$

From the equation (2.5) we find

$$\varphi_j = \frac{ie}{kT_p v_T} \cdot \frac{\partial f_0 / \partial P_y}{q_jP_x - \Omega_j}, \hspace{0.5cm} j = 1, 2, \cdots, N. \hspace{1cm} (2.6)$$

Now from the equation (2.6) we find

$$f_1 = \frac{e}{kT_p v_T} \cdot \sum_{j=1}^{N} \frac{E_j}{q_jP_x - \Omega_j}. \hspace{1cm} (2.7)$$

Thus first approximation is defined by equality (2.7).

4 The solution of Vlasov equation in second approximation

In the second approach we search for the decision of Vlasov equation (1.2) in the form of (1.3) in which $f_2$ is defined by equality (1.5). We substitute
(1.5) in the left-hand member of equation (2.2). We receive the following equation

$$\begin{align*}
\sum_{j=1}^{N} E_j^2 (-2i\omega_j + 2ik_j v_x) \psi_j + \\
+ \sum_{b,s=1 \atop b<s}^{N} \sum_{b<s} E_b E_s \left(-i(\omega_b + \omega_s) + i(k_b + k_s) v_x\right) \xi_{b,s} = \\
= -e \sum_{j=1}^{N} \frac{E_j}{\omega_j} \left[ k_j v_y \frac{\partial f_0}{\partial p_x} + (\omega_j - k_j v_x) \frac{\partial^2 f_0}{\partial p_y^2} \right].
\end{align*}$$

(3.1)

Let us pass in this equation to the dimensionless parameters and we will enter the following designations

$$q_{bs} = \frac{q_b + q_s}{2}, \quad \Omega_{bs} = \frac{\Omega_b + \Omega_s}{2}.$$ 

We receive the equation

$$\begin{align*}
\sum_{j=1}^{N} E_j^2 (q_j P_x - \Omega_j) \psi_j + \sum_{b,s=1 \atop b<s}^{N} E_b E_s (q P_x - \Omega) \xi_{b,s} = \\
= -e \sum_{j=1}^{N} \frac{E_j}{\Omega_j} \left[ q_j P_y \frac{\partial}{\partial P_x} \left( \frac{\partial f_0}{\partial P_y} \right) - \frac{\partial^2 f_0}{\partial P_y^2} \right] + \\
+ \sum_{b,s=1 \atop b<s}^{N} \frac{E_b E_s}{\Omega_b} \left[ q_b P_y \frac{\partial}{\partial P_x} \left( \frac{\partial f_0}{\partial P_y} \right) + \frac{\Omega_b - q_b P_x}{q_s P_x - \Omega_s} \frac{\partial^2 f_0}{\partial P_y^2} \right] + \\
+ \sum_{b,s=1 \atop b<s}^{N} \frac{E_s E_b}{\Omega_s} \left[ q_b P_y \frac{\partial}{\partial P_x} \left( \frac{\partial f_0}{\partial P_y} \right) + \frac{\Omega_b - q_s P_x}{q_b P_x - \Omega_b} \frac{\partial^2 f_0}{\partial P_y^2} \right]
\end{align*}$$

We find from this equation

$$\psi_j = -\frac{e^2}{2k^2_T p^2_T v^2_T} \frac{\Xi_j(P)}{q_j P_x - \Omega_j}, \quad (j = 1, 2, \ldots, N) \quad (3.2)$$

and

$$\xi_{b,s} = -\frac{e^2}{2k^2_T p^2_T v^2_T} \left[ \frac{1}{\Omega_b q P_x - \Omega} + \frac{1}{\Omega_s q P_x - \Omega} \right]. \quad (3.3)$$
where

\[ b < s, \quad j = 1, 2, \ldots, N. \]

and

\[
\Xi_j(P) = q_j P_y \frac{\partial}{\partial P_x} \left( \frac{\partial f_0 / \partial P_y}{q_j P_x - \Omega_j} \right) - \frac{\partial^2 f_0}{\partial P_y^2},
\]

\[
\Xi_{bs}(P) = q_b P_y \frac{\partial}{\partial P_x} \left( \frac{\partial f_0 / \partial P_y}{q_s P_x - \Omega_s} \right) + \frac{\Omega_b - q_b P_x}{q_s P_x - \Omega_s} \frac{\partial^2 f_0}{\partial P_y^2},
\]

\[
\Xi_{sb}(P) = q_s P_y \frac{\partial}{\partial P_x} \left( \frac{\partial f_0 / \partial P_y}{q_b P_x - \Omega_b} \right) + \frac{\Omega_s - q_s P_x}{q_b P_x - \Omega_b} \frac{\partial^2 f_0}{\partial P_y^2},
\]

where

\[ b < s, \quad b, s = 1, 2, \ldots, N. \]

Thus the decision of Wigner equation is constructed and in the second approach. It is defined by equalities (1.5) and (3.2)–(3.3).

5 Density of transversal electric current

The density of electric current according his definition is equal

\[
j = e \int v f \frac{2d^3P}{(2\pi\hbar)^3} = \frac{2p_T^2 v_T}{(2\pi\hbar)^3} \int f P d^3P. \quad (4.1)
\]

The vector of a current density has two nonzero components \( j = (j_x, j_y, 0) \), where \( j_x \) is density of transversal current, \( j_y \) is density of longitudinal current.

Let us calculate density of transversal current. It is defined by the following expression

\[
j_y = e \int v_y f \frac{2d^3P}{(2\pi\hbar)^3} = e \int v_y f_1 \frac{2d^3P}{(2\pi\hbar)^3} = \frac{2p_T^2 v_T}{(2\pi\hbar)^3} \int f_1 P_y d^3P. \quad (4.2)
\]

Transversal current is directed along an electromagnetic field. Its density is defined according to (4.2) only first approximation of a cumulative distribution function. The second approximation of a cumulative distribution function does not make a contribution to a current density. Thus, in an explicit form transversal current is equal

\[
j_y = \frac{2ie^2 p_T^2}{(2\pi\hbar)^3 k_T} \int \sum_{j=1}^{N} \frac{E_j}{q_j P_x - \Omega_j} \frac{\partial f_0}{\partial P_y} P_y d^3P. \quad (4.3)
\]
We simplify a formula (4.3)

\[ j_y = \frac{2ie^2p_T^2}{(2\pi\hbar)^3 k_T} \int_{-\infty}^{\infty} \sum_{j=1}^{N} \frac{E_j}{q_j P_x - \Omega_j} \ln(1 + e^{\alpha - P_x^2})dP_x. \] (4.4)

6 Density of longitudinal electric current

We will investigate longitudinal current. By means of decomposition (1.5) we will present longitudinal current in the following form

\[ j_x = \sum_{a=1}^{N} j_a + \sum_{b<s}^{N} j_{bs} + \sum_{b<s}^{N} j_{sb}, \] (5.1)

where

\[ j_a = \frac{e^3p_T E_a^2}{(2\pi\hbar)^3 k_T^2 v_T \Omega_a} \int \frac{\Xi_a(P)P}{q_a P_x - \Omega_a} d^3P, \quad (a = 1, 2, \cdots, N), \] (5.2)

and

\[ j_{bs} = \frac{e^3p_T}{(2\pi\hbar)^3 k_T^2 v_T} \int \frac{E_bE_s}{\Omega_b} \frac{\Xi_{bs}P}{q_x P_x - \Omega} P_x d^3P, \] (5.3)

\[ j_{sb} = \frac{e^3p_T}{(2\pi\hbar)^3 k_T^2 v_T} \int \frac{E_bE_s}{\Omega_s} \frac{\Xi_{sb}P}{q_x P_x - \Omega} P_x d^3P. \] (5.4)

Here

\[ q_{bs} = \frac{q_b + q_s}{2}, \quad \Omega_{bs} = \frac{\Omega_b + \Omega_s}{2}, \quad b < s, \quad b, s = 1, 2, \cdots, N. \]

In these expressions one-dimensional internal integral on \( P_y \) is equal to zero and internal integral for \( P_x \) are calculated piecemeal. Therefore, the previous equalities becomes simpler for components of longitudinal current. Then, internal integral we will integrate on a variable of \( P_y \). Further we will calculate internal integrals in plane \((P_y, P_z)\) in polar coordinates. Equalities (5.2) – (5.4) come down to one-dimensional integral.

\[ j_a = \frac{\pi e^3p_T E_a^2 q_a}{(2\pi\hbar)^3 k_T^2 v_T} \int_{-\infty}^{\infty} \frac{\ln \left(1 + e^{\alpha - P_x^2}\right)}{(q_a P_x - \Omega_a)^3} dP_x, \quad (a = 1, 2, \cdots, N). \]
and
\[
j_{bs} = \frac{\pi e^3 p_T E_b E_s q_b \Omega}{(2\pi \hbar)^3 k_T^2 v_T \Omega_b} \int_{-\infty}^{\infty} \ln \left(1 + e^{a-P_x^2} \right) dP_x \frac{(q_s P_x - \Omega_s)}{(q_s P_x - \Omega_s)^2},
\]
\[
j_{sb} = \frac{\pi e^3 p_T E_b E_s q_b \Omega}{(2\pi \hbar)^3 k_T^2 v_T \Omega_s} \int_{-\infty}^{\infty} \ln \left(1 + e^{a-P_x^2} \right) dP_x \frac{(q_b P_x - \Omega_b)}{(q_b P_x - \Omega_b)^2},
\]

where

\[
b < s, \quad b, s = 1, 2, \ldots, N.
\]

Let us find numerical density the concentration of particles of plasma answering to distribution of Fermi–Dirac

\[
N_0 = \int f_0(P) \frac{2d^3 p}{(2\pi \hbar)^3} = \frac{8\pi p_T^3}{(2\pi \hbar)^3} \int_0^\infty \frac{e^{a-P^2} P^2 dP}{1 + e^{a-P^2}} = \frac{k^3_T}{2\pi^2} l_0(\alpha),
\]

where

\[
l_0(\alpha) = \int_0^{\infty} \ln \left(1 + e^{a-\tau^2} \right) d\tau.
\]

We will enter plasma (Langmuir) frequency in expression before integrals

\[
\omega_p = \sqrt{4\pi e^2 N_0 m}
\]

and numerical density (concentration) \( N_0 \). We will express numerical density through a thermal wave number. Then

\[
\frac{\pi pre^3 q_j}{(2\pi \hbar)^3 k_T^2 v_T} = \frac{e \Omega_p^2}{p_T k_T} \cdot \frac{k_j}{16 \pi l_0(\alpha)} = \sigma_{l, tr} \frac{k_j}{16 \pi l_0(\alpha)}, \quad (j = 1, 2, \ldots, N).
\]

Here

\[
\Omega_p = \frac{\omega_p}{k_T v_T} = \frac{\hbar \omega_p}{m v_T^2}
\]

is the dimensionless plasma (Langmuir) frequency, \( \sigma_{l, tr} \) is the longitudinal-transversal conductivity,

\[
\sigma_{l, tr} = \frac{e \Omega_p^2}{p_T k_T}.
\]

Now we will write down components of longitudinal current in form

\[
j_a = E_a^2 \sigma_{l, tr} k_a J_a, \quad j_{bs} = E_b E_s \sigma_{l, tr} k_b J_{bs}, \quad j_{sb} = E_b E_s \sigma_{l, tr} k_s J_{sb}, \quad (5.5)
\]
where

\[ J_a = \frac{1}{16\pi l_0(\alpha)} \frac{1}{\int -\infty} \ln \left(1 + e^{\alpha - P_x^2}ight) \frac{dP_x}{(q_a P_x - \Omega_a)^3}, \quad (a = 1, 2, \ldots, N), \]

\[ J_{bs} = \frac{\Omega}{16\pi l_0(\alpha) \Omega_b} \frac{1}{\int -\infty} \ln \left(1 + e^{\alpha - P_x^2}ight) \frac{dP_x}{(q_s P_x - \Omega_s) (q P_x - \Omega)^2}, \quad (b < s, \quad b, s = 1, 2, \ldots, N), \]

\[ J_{sb} = \frac{\Omega}{16\pi l_0(\alpha) \Omega_s} \frac{1}{\int -\infty} \ln \left(1 + e^{\alpha - P_x^2}ight) \frac{dP_x}{(q_b P_x - \Omega_b) (q P_x - \Omega)^2}, \quad (b < s, \quad b, s = 1, 2, \ldots, N). \]

Here

\[ q = \frac{q_b + q_s}{2}, \quad \Omega = \frac{\Omega_b + \Omega_s}{2}. \]

In equalities (5.5) \( J_1, J_2, \ldots, J_N, J_{12}, J_{21}, \ldots, J_{bs}, J_{sb} \) are the dimensionless parts of density of longitudinal current. Thus, a longitudinal part of current is equal

\[ j_x = \sigma_{1, tr} \left[ \sum_{a=1}^{N} E_a^2 k_a J_a + \sum_{b<s}^{N} E_b E_s (k_b J_{bs} + k_s J_{sb}) \right]. \quad (5.6) \]

If to enter transversal fields

\[ E_{j_{tr}} = E_j - \frac{k_j (E_j k_j)}{k_j^2}, \]

then equality (5.6) can be written down in an invarianty form

\[ j_{long} = \sigma_{1, tr} \left[ \sum_{a=1}^{N} \left(E_a^{tr}\right)^2 k_a J_a + \sum_{b<s}^{N} E_b E_s \left(k_b J_{bs} + k_s J_{sb}\right) \right]. \]

Let us consider a case of small values of a wave number. From (5.6) follows that at small values of wave numbers for density of longitudinal current we receive

\[ j_x = -\frac{\sigma_{1, tr}}{8\pi} \left[ \sum_{a=1}^{N} E_a^2 k_a^3 + 2 \sum_{b<s}^{N} E_b E_s \frac{k_b + k_s}{\Omega_b \Omega_s (\Omega_b + \Omega_s)} \right]. \quad (5.7) \]
Remark

At calculation of the integrals entering the dimensionless parts of density of longitudinal current it is necessary to use Landau’s rule. According to this rule, for example, for integrals $J_a$ we will have

$$J_a = \frac{1}{16\pi l_0(\alpha)} \left[ -\frac{i\pi}{2q_a^3} \left[ \ln \left( 1 + e^{\alpha - \tau^2} \right) \right]''_{r=\frac{q_a}{q_0}} + \text{V.p.} \int_{-\infty}^{\infty} \frac{\ln \left( 1 + e^{\alpha - \tau^2} \right) d\tau}{(q_a\tau - \Omega_a)^3} \right],$$

where $a = 1, 2, \cdots, N$.

7 Conclusions

We found out dependence of transversal and longitudinal current, generated in classical plasma by $N$ transverse electromagnetic waves. In this work we consider the effect of the nonlinear character of the interaction of electromagnetic fields with collisionless Maxwell classical plasma. We considered the Vlasov equation and his solution, the method of successive approximations has been found of the distributions functions. We found formulas for electric current in collisionless classical plasma.

References

[1] Ginsburg V.L., Gurevich A.V. The nonlinear phenomena in the plasma which is in the variable electromagnetic field//Uspekhy Fiz. Nauk, 70(2) 1960; p. 201-246 (in Russian).

[2] Kovrizhkhyykh L.M. and Tsytovich V.N. Effects of transverse electromagnetic wave decay in a plasma//Soviet physics JETP. 1965. V. 20. №4, 978-983.

[3] Akhmediev N.N., Mel’nikov I.V., Robur L.J. Second-Harmonic Generation be a Reflecting Metal Surface// Laser Physics. Vol. 4. №6. 1994, pp. 1194-1197.
[4] Bezhanov S.G., Urupin S.A. Generation of nonlinear current and low frequency radiation at interaction laser impulse with metal // Quant. Electronics, 43, №11 (2013).

[5] Grishkov V.E., Urupin S.A. Generation of nonlinear currents along direction propagation short laser radiation // XLI Intern. (Zvenigorodskaya) conference on plasma physics and UTS. 10-14 February 2014 (in Russian).

[6] Zytovich V.N. Nonlinear effects in plasmas // Uspekhy Fiz. Nauk, 90(3) 1966; p. 435-489 (in Russian).

[7] Zytovich V.N. Nonlinear effects in plasmas. Moscow. Publ. Leland. 2014. 287 p. (in Russian).

[8] De Andrés P., Monreal R., and Flores F. Relaxation–time effects in the transverse dielectric function and the electromagnetic properties of metallic surfaces and small particles // Phys. Rev. B. 1986. Vol. 34, №10, 7365–7366.

[9] Fuchs R. and Kliewer K.L. Surface plasmon in a semi–infinite free–electron gas // Phys. Rev. B. 1971. V. 3. №7. P. 2270–2278.

[10] Brodin G., Marklund M., Manfredi G. Quantum Plasma Effects in the Classical Regime // Phys. Rev. Letters. 100, (2008). P. 175001-1 – 175001-4.

[11] Latyshev A. V. and Yushkanov A. A. Longitudinal Dielectric Permeability of a Quantum Degenerate Plasma with a Constant Collision Frequency // High Temperature, 2014, Vol. 52, №1, pp. 128–128.

[12] Latyshev A. V. and Yushkanov A. A. Generation of Longitudinal Current by a Transverse Electromagnetic Field in Classical and Quantum Plasmas // ISSN 1063-780X, Plasma Physics Reports, 2015, Vol. 41, No. 9, pp. 715–724. DOI: 10.1134/S1063780X1509007X.
[13] Latyshev A. V. and Yushkanov A. A. Longitudinal electric current in the collisional plasma generated by a transverse electromagnetic field// Theor. and Mathem. Phys. 2016. V. 187(1). P. 559–569.

[14] Latyshev A. V. and Yushkanov A. A. Nonlinear longitudinal current in the Maxwellian plasma generated under the action of a transverse electromagnetic wave// Journal Fluid Dynamics, 50(6), 820-827, 2015.

[15] Latyshev A. V. and Yushkanov A. A. Generation of a Longitudinal Current by a Transverse Electromagnetic Field in Collisional Degenerate Plasma// Comp. Maths and Math. Phys. 2016. V. 9. Pp. 1641–1650. ISSN 0965-5425.

[16] Latyshev A. V., Yushkanov A. A., Algazin O. D., Kopaev A. V., Popov V.S. Nonlinear longitudinal current, generated by two transversal electromagnetic waves in collisionless plasma// arXiv: 1505.06796v1 [physics.plasm-ph] 26 May 2015, 22 p.

[17] Latyshev A. V., Askerova V. I. Nonlinear longitudinal current generated by N transversal electromagnetic waves in quantum plasma// arXiv: 1612.06087v1 [physics.plasm-ph] 19 Dec 2016, 28 p.