Research on Error Modeling and Identification Technology of CNC Camshaft Grinder

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Abstract. Taking certain type of CNC camshaft grinders as the research object, through the analysis of its structure and movement to determine a total of 21 error parameters, a model describing the motion relations between adjacent body coordinate systems containing the error parameters was established. The machine movements were classified into two kinematic chains: the "bed-workpiece" and "bed-tool ". The topological structure of the machine tool was built and then the multi-body system theory was applied to model the errors of the machine tool. Body coordinate systems and motion reference coordinate systems were set up on each moving body and the corresponding transformation matrix was derived by joining the motion relation models between adjacent body coordinate systems. In order to achieve precision machining in the case of the existence of influential errors, a constraint equation Pw = Pt was put forward and solved. A mathematical model of the X-C axis linkage during camshaft grinding was proposed to work out the grinding point coordinates in the workpiece coordinate system and tool coordinate system. Three kinds of measurement methods were provided to identify 15 error parameters affecting the camshaft machining by using a cue instrument, which provides necessary conditions to research the machine tool error compensation.

1. Introduction
Camshafts are often used to drive the intake and exhaust systems of automotive engines. The machining accuracy of camshafts directly affects the performance of engines. With the rapid development of automobile manufacturing industry, more and more camshafts are needed, and the precision requirements are higher and higher. CNC camshaft grinder is used as the last equipment of camshaft processing [1]. Machining accuracy is one of the most important criteria in the assessment of machine tools’ performance, which directly affects the quality of machined parts, and it is affected by geometric errors, cutting tool deflection errors, thermal-induced errors, servo errors, and so forth. Among them, geometric errors account for about 60% of the total machining errors and are a basic factor influencing the machining accuracy [2]. In order to improve the machining accuracy of NC camshaft grinder, the error compensation is used. The error model of machine tool is established to identify the errors of machine tool, and the influence of geometric error of machine tool on the machining accuracy of workpiece is reduced by modifying NC instructions. The modified NC instructions are used to force
machine tool out of the precise machining trajectory. This paper mainly focuses on the theory of error compensation technology.

The key of error compensation is error modeling and identification. There have been numerous intensive research works on error modeling for machine tools over the past decades. Generally, an error model is established to describe an error mapping relationship between geometric error sources and volumetric error for machining tools. Wang et al. [3] adopted a polynomial fitting and linear fitting method to establish a synthetically model of geometric and thermal errors of NC machine tools and to compensate the errors online and in real time. Kiridena et al. [4] deduced the spatial geometric error model of five-coordinate machine tools by mechanism. Chatterjee et al. [5] established the spatial error model of multi-axis machine tools by using the expression of vector chain. Lin et al. [6] proposed the geometric errors model of 5-axis NC machine tools by using a matrix summation method. Zhu et al. [7] proposed a general error identification method of rotating axle based on multi-body system theory, and six motion error parameters related to rotating axle are identified.

Considering that the application of follow-up camshaft grinder is more and more widespread, and which with both translational and rotational axes and higher precision requirements, the research on error identification of translational axes obtained some positive results at present, but the research on error identification of rotational axes is still rarely discussed. The theory of multi-body system can not only be used for error modeling of machine tools to reduce the difficulty of modeling, but also for translational axes and rotational axes with measuring instruments. Error identification of rotating shaft and their linkage is carried out, but the application of the theory of multi-body system to follow-up camshaft grinder is rarely concerned. In this paper, the error model of follow-up camshaft grinder is established based on the theory of multi-body system. A new method is proposed to identify the translational axis, rotational axis and their linkage by using the theory of multi-body system combined with the variation of the length of the double-ball-bar instrument.

2. The volumetric error modeling based on HTM and MBS theory

An investigated CNC camshaft grinder is shown in Fig. 1. Based on MBS theory, the machine tool can be considered as the combination of various rigid bodies, including machine bed (body 1), Z axis slide carriage (body 2), C axis (body 3 ), workpiece (body 4), X axis slide carriage (body 5), grinding wheel (body 6). Geometric errors are usually composed of position-independent geometric errors (PIGEs), and position-dependent geometric errors (PDGEs). Based on the theory of the rigid body motion, each moving part of a machine tool has six DOFs in the Cartesian coordinate system. Hence, each moving axis of the CNC camshaft grinder has six PDGEs. Therefore, all the geometric errors of the camshaft grinder are listed, as shown in the Table 1.

![Fig. 1 Structure map of follow-up type camshaft grinder](image-url)
**Table 1.** Geometric error parameters of follow-up type camshaft grinder

|                  | Linear error | Angular error |
|------------------|--------------|---------------|
|                  | Along X      | Along Y       | Along Z       | Around X    | Around Y    | Around Z    |
| PDGEs X axis     | $\delta_x$   | $\delta_x$   | $\delta_x$   | $\epsilon_x$| $\epsilon_x$| $\epsilon_x$|
| PDGEs Z axis     | $\delta_z$   | $\delta_z$   | $\delta_z$   | $\epsilon_z$| $\epsilon_z$| $\epsilon_z$|
| PDGEs C axis     | $\delta_c$   | $\delta_c$   | $\delta_c$   | $\epsilon_c$| $\epsilon_c$| $\epsilon_c$|
| PIGEs            |              |               |               | $\epsilon_{xc}$, $\epsilon_{yc}$, $\epsilon_{zc}$ |

The kinematic chain of the CNC camshaft grinder can be divided into two branches by considering the body of machine bed as a reference. One is workpiece branch from machine bed to workpiece, and the other is tool branch from machine bed to cutting tool. In MBS theory, the transformation relationship between the two adjacent bodies can be represented using a $4 \times 4$ HTM.

Supposing that the coordinate of the cutting tool tip position and orientation in the tool coordinate system (TCS) can be expressed as $\mathbf{r}_t = (0 \ 0 \ -l \ 1)^T$, the workpiece coordinate system (WCS) origin in the coordinate system of spindle is $\mathbf{q}_s = (q_{sx} \ q_{sy} \ q_{sz} \ 1)^T$, the coordinate of the cutting tool tip position in WCS can be expressed as $\mathbf{r}_w = (r_{wx} \ r_{wy} \ r_{wz} \ 1)^T$, where $l$ indicates the length of the cutting tool. The centre point in the base coordinate system (BCS) is $\mathbf{q}_b = (q_{bx} \ q_{by} \ q_{bz} \ 1)^T$, $\theta$ donates the rotary angle of spindle. Therefore, the transformation matrices for all of the adjacent bodies are shown in Tables 2–3.

**Table 2.** Position and position error transformation matrix of the CNC camshaft grinder (part).

| Adjacent body | Position transformation matrix | Position error transformation matrix |
|---------------|--------------------------------|-------------------------------------|
| 1-2           | $[S12]_p = E_{4\times4}$       | $[S12]_{pe} = E_{4\times4}$          |
| 2-3           | $[S23]_p = E_{4\times4}$       | $[S23]_{pe} = 
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & -\epsilon_{xc} \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}$ |
| 3-4           | $[S34]_p = E_{4\times4}$       | $[S34]_{pe} = E_{4\times4}$          |
Table 3. Motion and motion error transformation matrix of the CNC camshaft grinder (part).

| Adjacent | Motion transformation matrix | Motion error transformation matrix |
|----------|-----------------------------|---------------------------------|
| body     | \( [S12] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \) | \( [S12]_{se} = \begin{bmatrix} 1 & -\varepsilon_z(z) & \varepsilon_x(z) & \delta_x(z) \\ \varepsilon_z(z) & 1 & -\varepsilon_y(z) & \delta_y(z) \\ -\varepsilon_z(z) & \varepsilon_y(z) & 1 & \delta_y(z) \\ 0 & 0 & 0 & 1 \end{bmatrix} \) |
| 1-2      | \( [S23] = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \) | \( [S23]_{se} = \begin{bmatrix} 1 & -\varepsilon_x(c) & \varepsilon_y(c) & \delta_x(c) \\ \varepsilon_x(c) & 1 & -\varepsilon_y(c) & \delta_y(c) \\ -\varepsilon_x(c) & \varepsilon_y(c) & 1 & \delta_y(c) \\ 0 & 0 & 0 & 1 \end{bmatrix} \) |
| 2-3      | \( [S34]_{se} = E_{4x4} \) | \( [S34]_{se} = E_{4x4} \) |

In workpiece branch, the required tool position of the workpiece cutting point can be described to BCS, as shown in Eq. (1).

\[
P_w = [S12]_{lp}[S12]_{ln}[S12]_{lw}\mathbf{r}_w
\]  

(1)

In addition, in tool branch, the cutting tool tip position in the BCS can be explicitly expressed in Eq. (2).

\[
P = \sum_{j=1}^{r} \left[ \left[ SL(j)E(j) \right]_{lp}\left[ SL(j)E(j) \right]_{ln}\left[ SL(j)E(j) \right]_{lw} \right] \mathbf{r}_j
\]  

(2)

During the practical machining process, the required tool position of the workpiece cutting point in workpiece branch should be coincident with the position of the cutting tool tip in tool branch at any moment in BCS, i.e.,

\[
P_w = P_t
\]  

(3)

Substituting Eqs. (1) and (2) into Eq. (3), then the equation can be represented as

\[
\delta_x(z) + \left( q_{x_i} + x \right) \left[ \left( \varepsilon_x(z) + \varepsilon_x(c) \right) \sin \theta - \cos \theta \right] = \left( q_{x_i} + y \right) \left[ \varepsilon_y(z) + \varepsilon_y(c) \right] \cos \theta
\]  

(4)

As shown in Fig. 2, point \( P \) in BCS can be expressed as

\[
\begin{align}
x_p &= \rho \cos \varphi \\
y_p &= -\rho \sin \varphi
\end{align}
\]  

(5)

Where \( \rho \) indicates camshaft profile curve, \( R \) indicates grinding wheel radius, \( r \) indicates radius of grinding wheel base circle.
\[ K' = \frac{dy}{dx}, \quad K = -\frac{dx}{dy} \]

So the coordinates of point O can be obtained as follow,

\[
\begin{align*}
\{x_o, y_o\} & = \{\rho \cos \varphi - R \cos(\arctan K), -\rho \sin \varphi - R \sin(\arctan K)\}
\end{align*}
\]

3. Error identification of CNC camshaft grinder

A point M \((x_{m0}, y_{m0}, z_{m0})\) in the X-direction slide coordinate system. When X-direction slide moves along the X-axis, the actual and ideal position of point M will be erroneous due to the existence of motion errors, \(\Delta M = (\Delta x(x), \Delta y(x), \Delta z(x), \theta)\)^T

\[
\Delta M = \begin{bmatrix}
\Delta x(x) \\
\Delta y(x) \\
\Delta z(x)
\end{bmatrix}
\begin{bmatrix}
x_n \\
y_n \\
z_n
\end{bmatrix}
\begin{bmatrix}
1 \\
-\epsilon_x(x) \\
-\epsilon_y(x) \\
-\epsilon_z(x)
\end{bmatrix}
\begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
1 \\
-\epsilon_x(x) \\
-\epsilon_y(x) \\
-\epsilon_z(x)
\end{bmatrix}
\begin{bmatrix}
x_m0 \\
y_m0 \\
z_m0
\end{bmatrix}
\]

The error related to X-axis is identified by using the double-ball-bar instrument. The measuring schematic diagram of the Ball-Bar instrument is shown in Fig. 3. The two ends of the spherical rod instrument are fixed on the slide board and the bed respectively. The coordinates of B ball in the coordinate system of the bed are \((x_B, y_B, z_B)\). The initial position of a ball is perpendicular to the connection line and X axis of B ball and moves along the slide board with X. The coordinates of A ball center in the coordinate system of X-direction slide board are \((x_A, y_A, z_A)\). When A ball moves along the X axis, the coordinates of A ball center are \((x(x), y(x), z(x))\). Then the distance X moves to the slide plate X and the length L of the rear rod is
\[ L^2 = \left( x(x) - x_0 \right)^2 + \left( y(y) - y_0 \right)^2 + \left( z(z) - z_0 \right)^2 \]  (9)

Derivative on both sides,
\[ L \Delta L(x) = \left( x(x) - x_0 \right) \Delta x(x) + \left( y(y) - y_0 \right) \Delta y(x) + \left( z(z) - z_0 \right) \Delta z(x) \]  (10)

It can be seen that the length \( L \) of the rod under the ideal situation.
\[ L_0^2 = x^2 + \left[ \sqrt{\left( y_i - y_0 \right)^2 + \left( z_i - z_0 \right)^2} \right]^2 \]  (11)

Six different measuring points on the X-axis are taken and the variation of rod length along the X-direction is measured. Six error parameters related to the X-axis can be obtained through six equations, and the average values can be obtained as the final error parameters through multiple measurements.

The mapping relationship between tool path and NC instruction and between NC instruction and actual tool path is established respectively. Error parameters are substituted into two mapping relationships to correct NC instruction, so as to realize error compensation of machine tools. Considering the space problem, the establishment of two mapping relationships is not described here. The maximum lift error of an automobile engine intake and exhaust camshaft is reduced from 22 to 9 microns by using the machine tool after error compensation. Therefore, the machining accuracy of the machine tool after error compensation has been significantly improved.

4. Conclusion
(1) The structure and movement of NC camshaft grinder are analyzed, and it is determined that the machine tool is a special multi-body system. The machine tool is divided into two open-loop multi-body systems, i.e. workpiece-bed and tool-bed. The error of the machine tool can be modeled by the theory of multi-body system. Finally, the two motion chains are closed-loop by processing points.

(2) The geometric error model of NC camshaft grinder is established by using the theory of multi-body system. The constraint equation of camshaft precision machining condition is deduced by using the model. The equation is solved, which reduces the difficulty of direct error modeling of the whole machine.

(3) A new method of error parameter identification based on multi-body system theory and double-ball-bar instrument is proposed. This method reduces the difficulty of machine tool error identification and provides theoretical guarantee for error compensation. The experiment proves that the machining accuracy of the machine tool after error compensation has been significantly improved.

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