Confinement and chiral symmetry breaking in heavy-light quark systems

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Abstract

Assuming a Gaussian approximation for the QCD gluodynamics, all the nonperturbative physics can be encoded into two parameters: the gluon correlation length $T_g$ and the gluon condensate $G_2$. These parameters are sufficient in order to describe the heavy-heavy quark nonperturbative interaction. In this work we adopt the same framework in order to study heavy-light bound states in the non-recoil limit. Spontaneous chiral symmetry breaking and a confining chiral non-invariant interaction emerge quite naturally. The gap equation is solved and discussed. In particular a relation between the light quark condensate and $T_g$ is derived. The energy spectrum for the bound state equation is evaluated and commented.
1 INTRODUCTION

The study of heavy-light quark bound states has absorbed in the last years a huge amount of effort either on the experimental side, where a lot of new data have been produced (compare, for instance, with [1]), either on the theoretical side, due to the use of heavy quark symmetries [2]. Around the limit where the symmetries are exact (infinitely heavy quark) a systematic expansion in the small parameters $\Lambda_{\text{QCD}}/M_Q$ and $\alpha_s(M_Q)$ hold, $M_Q$ being the mass of the heavy quark. This expansion contains the long-distance physics of several observables encoded into few hadronic parameters, which in general can be defined in terms of operator matrix elements in heavy quark effective theory (HQET) [3]. In particular in HQET the heavy-light meson mass is given by

$$M_{q\bar{Q}} = M_Q + \bar{\Lambda} + O\left(\frac{1}{M_Q}\right)$$

corrections.

The parameter $\bar{\Lambda}$ is one of the nonperturbative parameters of the HQET. It is the sum of all the contributions coming from the terms independent on the heavy-quark mass $M_Q$. $\bar{\Lambda}$ can be fixed on the data. In order to calculate it from QCD one needs some dynamical input. This dynamics, being the dynamics of the light quark and the glue, is inherently nonperturbative. Some approaches resort to calculation via phenomenological potential models [4,5], sum rules [6] or relativistic phenomenological equations [7]. To have a well founded calculation of $\bar{\Lambda}$ is of great importance since its value affects the determination of many phenomenological quantities (cf e.g. [8]).

An attempt in this direction has been done in [9]. In that work an unified description of the heavy-heavy and the heavy-light dynamics in the non-recoil limit was suggested on the basis of the so-called Stochastic Vacuum model of QCD [10]. This consists in a Gaussian approximation of the QCD gluodynamics which is assumed to be governed by the two-point (non-local) gluon condensate. This assumption is confirmed by lattice simulations data [11] in the case of quasi-static quarks. In this way a new dynamical scale explicitly appears: the gluon correlation length $T_g$. In [9] the heavy mass limit ($m > T_g^{-1}$) was explicitly explored. Under this condition two different situations occur. If $T_g^{-1}$ is bigger than all the other scales of the problem, then the one-body limit of the heavy quark potential is recovered with the famous Eichten–Feinberg–Gromes spin-orbit scalar-like interaction [12]. The other case, $T_g^{-1}$ smaller than all the other scales of the problem, is the typical heavy quark sum-rule situation. As a check the well-known expression of the heavy quark condensate as a function of the gluon condensate was obtained. Realistic heavy-light systems are characterized by $m < T_g^{-1}$. In this case the quark propagator cannot be considered as a free one and its dynamics is inherently nonperturbative. For $m = 0$ our effective Lagrangian is chiral symmetric. Then the most appro-
appropriate approach is to solve the non-linear bound state equation as suggested in \[9,13\] or, in a different language, to determine the physical chiral broken vacuum of the problem. This will be done in the present work by means of the Bogoliubov–Valatin variational method. Since the Gaussian approximation of the QCD gluodynamics works very well in the heavy quark limit \[14\], one expects reasonable results for, at least, \(B_s\) and \(D_s\) mesons.

More from the point of view of principles this work would like to shed some light on the QCD nonperturbative dynamics. It is clear that when we deal with light quarks, chiral symmetry breaking and confinement are the two relevant facts. But it is also true that it is difficult to get hold of both effects. On one side, Nambu–Jona–Lasinio models, as well as instantons, explain the main physics of chiral symmetry breaking but do not have the color confinement property of QCD. On the other side, the confining models that try to explain the light quark phenomenology are mainly of two types: 1) models with a scalar confining interaction \[7\] that explicitly breaks chiral symmetry but insures the spin-orbit interaction known in the heavy quark limit \[12\]; 2) models with a vector confining interaction \[15,16\] which are chiral symmetric and allow spontaneous breaking of chiral symmetry but then are in trouble with the spin-orbit interaction. The heavy-light interaction which we propose is obtained cleanly from QCD in a gauge invariant approach and under the assumption of the dominance of the bilocal correlator. It has the feature of being able to reproduce the expected confining linear potential and the expected nonperturbative spin-orbit coupling in the heavy quark limit as well as of being chiral symmetric in the chiral limit \[9\]. More in detail, our interaction kernel turns out to be more complicated than a simple convolution kernel. It depends on two dimensional parameters, the gluon condensate and the correlation length that control the nonperturbative physics. The string tension \(\sigma\) arises as a function of these two parameters in the potential limit. We stress that the presence of these two parameters is relevant in order to reproduce the flux tube structure which is the main physical fact of the heavy quark nonperturbative dynamics. For heavy-light systems chiral symmetry and confinement appear to be strongly related as also suggested by lattice simulations at non-zero temperature where confinement and chiral symmetry breaking show up at the same temperature \[17\]. This is because in the proposed approach chiral symmetry breaking effects emerge in the physical framework of a quark-antiquark bound state. As a consequence chiral symmetry breaking modifies the binding interaction, which turns out to be non–chiral invariant on the physical vacuum. This is very different with respect to the traditional Dyson–Schwinger methods where chiral symmetry breaking is associated with the (gauge dependent) non-linear dynamics of a single quark \[18\]. But it differs also from the traditional phenomenological motivated Dirac equations used in the literature for describing heavy-light systems in the non-recoil limit \[7\]. In those works a scalar confining interaction which explicitly breaks chiral symmetry is introduced by hand and phenomenologically justified. Here a chi-
ral non-invariant binding interaction emerges naturally on the chiral broken
physical vacuum. The main pieces of the puzzle seem to go at the right place.

In the present work a simplified expression for the non-local gluon condensate
with respect to the parameterization derived from lattice simulations [19] will
be used [20]. This is only a technical assumption which in this exploratory work
helps to increase the transparency of the calculation, leaving the main physics
untouched. The main point is that we are able to give an unified description of
the heavy-heavy and of the non-recoil heavy-light bound states in terms of few
parameters (the gluon condensate, the gluon correlation length) determined
by the nonperturbative dynamics of QCD.

The plan of the paper is the following. In section 2 we set up the formalism
and discuss with some extension the Gaussian approximation of the QCD
 gluodynamics. In section 3 we derive the gap equation and evaluate the quark
 condensate for different values of the mass. In section 4 we write the heavy-
light bound state equation in the non-recoil limit and calculate the spectrum.
Conclusions and outlook are given in section 5.

2 THE HAMILTONIAN

A system made up by a quark $q$ of mass $m$ and an antiquark $\bar{Q}$ of mass $M_Q$
is described by the 4-point Green function:

$$G_{\text{inv}}(x, u, y, v) = \langle 0 | \bar{q}(y) U(y, v) Q(v) \bar{Q}(u) U(u, x) q(x) | 0 \rangle. \quad (1)$$

The Schwinger strings $U(y, x) \equiv \text{P} \exp \left\{ i g \int_{0}^{1} ds (y - x)^{\mu} A_{\mu}(x + s(y - x)) \right\}$
have been added in order to have gauge invariant initial and final bound
states. P stands for the path ordering of the color matrices.

Let us consider the limit in which the antiquark is infinitely heavy ($M_Q \to \infty$).
The heavy quark behaves then like a static source propagating from $u = (-T/2, 0)$
to $v = (T/2, 0)$. Let us take $x = (-T/2, x)$ and $y = (T/2, y)$. For
infinitely heavy $\bar{Q}$ we have then

$$G_{\text{inv}}(x, u, y, v) = e^{-i M_Q T} \int \mathcal{D}q \mathcal{D}\bar{q} \left\{ \exp \left\{ i \int d^4 x \mathcal{L}(x) \right\} \bar{q}(y) U(y, v) U(v, u) \right.$$ 
$$\times U(u, x) \left( \frac{1 - \gamma^0}{2} \right)^{t(2)} q(x) \right\}, \quad (2)$$

$$\mathcal{L}(x) \equiv \bar{q}(x) (i \not{D} - m) q(x),$$
\( \langle O \rangle \equiv \frac{1}{N} \int DA e^{iS_{\text{YM}}} O(A), \quad S_{\text{YM}} = \text{Yang–Mills } SU(3) \text{ action,} \)

where the superscript \( (2) \) refers to the second heavy fermion line, \( D_\mu = \partial_\mu - igA_\mu \) and \( N \) is a normalization factor. A “natural” gauge choice is that one which eliminates the contributions coming from the straight-line Schwinger strings \( U \) in Eq. (2) [21]:

\[ A_\mu(x_0, 0) = 0, \quad x^j A_j(x_0, \mathbf{x}) = 0. \]  

(3)

We then have

\[ G_{\text{inv}}(x, u, y, v) = e^{-iMqT \int Dq \, D\bar{q} \left\langle \exp \left\{ i \int \! d^4x \, \mathcal{L}(x) \right\} \right\rangle \bar{q}(y) \left( \frac{1 - \gamma^0}{2} \right)^{t(2)} q(x). \]  

(4)

In the Wilson loop language, the Wilson loop \( W(\Gamma) \) made up by the heavy quark trajectory, a generic light quark path \( z \) connecting \( x \) with \( y \) and the initial and final point Schwinger strings (see Fig. 1) simply reduces in the gauge (3) to the light quark contribution only:

\[ W(\Gamma) \equiv \text{Tr } P \exp \left\{ \left. ig \int_\Gamma dz \, A_\mu(z) \right\} \right\} = \text{Tr } P \exp \left\{ ig \int_x^y dz^\mu A_\mu(z) \right\}. \]

Moreover with this gauge choice the gauge field \( A_\mu \) can be expressed in terms of the field strength tensor \( F_{\mu\nu} \):

\[ A_0(x) = \int_0^1 d\alpha \, x^k F_{k0}(x_0, \alpha \mathbf{x}), \quad A_j(x) = \int_0^1 d\alpha \, x^k F_{kj}(x_0, \alpha \mathbf{x}). \]

We notice that, though this gauge appears to be natural in this problem, its choice is completely arbitrary and motivated only by convenience. Being the starting Green function (1) gauge invariant, by proper handling we would obtain exactly the same results within any gauge.

In order to go on we need an assumption on the gauge field average in Eq. (4). Up to now this is unavoidable since the light quark sector does not provide us with an obvious expansion parameter.\(^1\) We assume that

\[ \left\langle \exp \left\{ ig \int d^4x \, \bar{q}(x) \gamma^\mu A_\mu(x)q(x) \right\} \right\rangle = \exp \left\{ ig \int d^4x \, \bar{q}(x) \gamma^\mu T^a q(x) \langle gA^a_\mu(x) \rangle \right\} \]

\(^1\) For instance, in the case of heavy quarks the system is essentially non-relativistic and we can expand in the quark velocity.
\[ -\frac{1}{2} \int d^4x \int d^4y \overline{q}(x) \gamma^\mu T^a q(x) \overline{q}(y) \gamma^\nu T^b q(y) \langle g A^a_{\mu}(x) g A^b_{\nu}(y) \rangle + \text{higher order clusters} \]
\[ \simeq \exp \left\{ -\frac{1}{2} \int d^4x \int d^4y \overline{q}(x) \gamma^\mu T^a q(x) \overline{q}(y) \gamma^\nu T^b q(y) \langle g A^a_{\mu}(x) g A^b_{\nu}(y) \rangle \right\} \] (5)

The first equality is exact. It follows from interpreting the gauge field average as a statistical average and from performing a cluster expansion [10]. The second is an approximation. It corresponds to stop the expansion with the first non-vanishing cluster. This approximation was used in this context in [9,13]. Although it has been very successful in the last years either in applications to the heavy quark potential where it was first proposed [10], as well as in the study of soft high-energy scattering problems (for some recent reviews see [14]). Equation (5) is our key assumption. It has been proven to work for quasi-static quarks as the phenomenological success of the quoted papers seems to show and as it is confirmed by lattice data (see discussion in [11] and [22]). For systems with a light quark there is no a priori reason nor to accept it nor to discard it. At this stage of our understanding we will assume that approximation (5) works and see how far we can go with it. In particular in this work we will see how chiral symmetry breaking emerges in a heavy-light bound state and how it leads to a non-chiral invariant confining interaction. Since we are confident that our assumption works at least for heavy quark bound states we will assume a conservative attitude and realistically expect that our picture might be able to describe \( B_s \) and \( D_s \) mesons. Nevertheless in the following we will not put a lower bound on the light quark mass.

The effective Lagrangian density we will use is therefore

\[ \mathcal{L}(x) = \overline{q}(x)(i \slashed{\partial} - m)q(x) \]
\[ + \frac{i}{2} \int d^4y \overline{q}(x) \gamma^\mu T^a q(x) \overline{q}(y) \gamma^\nu T^b q(y) \langle g A^a_{\mu}(x) g A^b_{\nu}(y) \rangle , \]
with

$$\langle gA_\mu^a(x)gA_\nu^b(y) \rangle = x^k y^l \int_0^1 d\alpha \alpha^n \int_0^1 d\beta \beta^n \langle gF_{k\mu}^a(x^0, \alpha x)gF_{l\nu}^b(y^0, \beta y) \rangle, \quad (6)$$

where \(n(0) = 0\) and \(n(i) = 1\). The quantities in Eq. (6) are gauge invariant since, due to the gauge choice, all the field strength tensors can be thought to be connected by straight-line Schwinger strings. The corresponding Hamiltonian is given by:

$$H = \int d^3x \bar{q}(x)(-i\beta \cdot \nabla + m)q(x) - \frac{i}{2} \int d^3x \int d^4y \bar{q}(x)\gamma^\mu T^a q(x)\gamma^\nu T^b q(y)\langle gA_\mu^a(x)gA_\nu^b(y) \rangle, \quad (7)$$

where we have introduced the traditional notation \(\beta = \gamma^0\) and \(\alpha^i = \gamma^0\gamma^i\).

We notice that the interaction with the heavy quark source only apparently disappeared from \(H\). Actually, it manifests itself via the breaking of translational invariance in the 4-fermion term due to the gauge choice (3). Therefore the effective Hamiltonian given in Eq. (7) is simply another way to write the bound state equation already given in [9,13]. It can, and in principle it should, be used as it is. The whole nonperturbative physics is contained into the nonlocal gluon condensate \(\langle g^2 F_{\mu\nu}^a(x)F_{\rho\lambda}^b(0) \rangle\) and a parameterization of it is known from lattice measurement [19]. Being interested only in the nonperturbative contributions, a good parameterization is (in Euclidean space)

$$\langle g^2 F_{\mu\nu}^a(x)F_{\rho\lambda}^b(0) \rangle = \frac{1}{96} \delta^{ab}(\delta_{\mu\rho}\delta_{\nu\lambda} - \delta_{\mu\lambda}\delta_{\nu\rho})\langle g^2 F^2(0) \rangle e^{-|x|/T_g}. \quad (8)$$

In particular it manifests a long range exponential fall off in \(|x|\) with a gluonic correlation length \(T_g \sim 0.3 \div 0.4\) fm. Once the form of the non-local gluon condensate is plugged in Eq. (6), the Hamiltonian describing the nonperturbative dynamics of heavy-light mesons in QCD (under the crucial assumption (5)) is in principle completely defined.

At this point we are left with the usual technical problems connected with the actual solving of the bound state equation. In the following we will bypass all these by not keeping the “realistic” lattice parameterization of Eq. (8), but by making a rough approximation on it. This will allow us to perform almost all the calculations analytically and to take advantage from some existing literature. The approximation will be very rough and we do not expect that it will allow us to make quantitative predictions. Nevertheless the qualitative features of the Hamiltonian (7) will be preserved. In particular we will see how
the mechanism of chiral symmetry breaking is expected to work in the bound
state framework and how it leads to a non chiral invariant effective bound
state interaction. Let us, first, neglect all the perturbative contributions to
\[ \langle g^2 F_{\mu\nu}(x) F_{\rho\lambda}^a(0) \rangle. \]
They are irrelevant since they do not lead to confinement
and to chiral symmetry breaking. We assume that the leading contribution
to it is given by the electric fields only and that they are allowed to vary
only in time. Moreover we approximate the exponential fall off in time with
correlation length \( T_g \) with an instantaneous delta-type interaction. This

\[ \langle g A_\mu^a(x) g A_\nu^b(y) \rangle \simeq -i \frac{\delta^{ab} \delta_\mu^\alpha \delta_\nu^\beta}{24} \mathbf{x} \cdot \mathbf{y} \delta(x_0 - y_0) T_g \langle g^2 E^2(0) \rangle. \]  

(9)

A similar assumption can be found in [20]. The effective Hamiltonian (7)
becomes now:

\[
H = \int d^3 x q^\dagger(x)(-i\alpha \cdot \nabla + m\beta)q(x) \\
- \frac{1}{2} V_0^3 \int d^3 x \int d^3 y r^2 q^\dagger(x)T^a q(x)q^\dagger(y)T^a q(y) \\
+ 2 V_0^3 \int d^3 x \int d^3 y R^2 q^\dagger(x)T^a q(x)q^\dagger(y)T^a q(y),
\]

(11)

\[ V_0^3 \equiv -\frac{T_g}{96} \langle g^2 E^2(0) \rangle = \frac{T_g}{96} \pi^2 G_2, \quad G_2 \equiv \left\langle \frac{\alpha_s}{\pi} F^2(0) \right\rangle, \]

where \( R = x/2 + y/2 \) and \( r = x - y \). Since the interaction is instantaneous
from now on we will consider the fermion fields as a function of the spa-
tial coordinates only. From Eq. (11) it is very clear the role played by the
approximation (9). It has allowed us to disentangle trivially in our effec-
tive Hamiltonian the self interacting part (function of \( r \)) from the external source
interacting term (function of \( R \)). In the starting Hamiltonian (7) with a “real-
istic” lattice parameterization of the non-local gluon condensate, like Eq. (8),
these terms might be mixed up in a very complicate way.

3 THE GAP EQUATION

In this section we concentrate on the chiral properties of the Hamiltonian (7).
Following [15,16] we will use a Bogoliubov–Valatin variational method in order
to select the chiral broken vacuum. Let us expand the quark fields on a trial
basis of spinors:

\[ q(x) = \sum_s \int \frac{d^3k}{(2\pi)^3} e^{ikx} \left[ u_s(k)b_s(k) + v_s(k)d_s^\dagger(-k) \right], \tag{12} \]

where the trial spinors \( u_s \) and \( v_s \) satisfy the usual orthonormality relations

\[ u_{s'}^\dagger(k)u_s(k) = v_{s'}^\dagger(k)v_s(k) = \delta_{ss'}, \quad u_{s'}^\dagger(k)v_s(k) = v_{s'}^\dagger(k)u_s(k) = 0. \]

A possible choice is

\[
\begin{align*}
    u_s(k) &= \frac{1}{\sqrt{2}} \left[ \sqrt{1 + \sin \phi(k)} \left( 1 +\sin \phi(k) \alpha \cdot \mathbf{k} \right) \right] u_s^0, \tag{13} \\
    v_s(k) &= \frac{1}{\sqrt{2}} \left[ \sqrt{1 + \sin \phi(k)} \left( -\sin \phi(k) \alpha \cdot \mathbf{k} \right) \right] v_s^0, \tag{14}
\end{align*}
\]

where \( u_s^0 \) and \( v_s^0 \) are the usual rest-frame spinors for the free particle on the chiral invariant vacuum. In a moving frame the spinors on the chiral invariant vacuum are given by \( \sin \phi(k) = m/\sqrt{m^2 + k^2} \) and \( \cos \phi(k) = k/\sqrt{m^2 + k^2} \). In particular, in the limiting case \( \phi = 0 \) the trial spinors reduce to the free massless one, while for \( \phi = \pi/2 \) they reduce to free static sources. Let us also define the projectors

\[
\Lambda_+(k) \equiv \sum_s u_s(k)u_s^\dagger(k) = \frac{1}{2} \left[ 1 + \beta \sin \phi(k) + \alpha \cdot \mathbf{k} \cos \phi(k) \right],
\]
\[
\Lambda_-(k) \equiv \sum_s v_s(k)v_s^\dagger(k) = \frac{1}{2} \left[ 1 + \beta \sin \phi(k) - \alpha \cdot \mathbf{k} \cos \phi(k) \right] = 1 - \Lambda_+(k).
\]

The operators \( b, d \) and \( b^\dagger, d^\dagger \) are annihilation and creation operators respectively. They define the trial vacuum. Eventually after minimizing the vacuum energy they will define the chiral broken vacuum. Expanding the Hamiltonian (11) on (12) we obtain

\[
H = \mathcal{E} + H_2^r + H_2^R + H_4
\]
\[
\mathcal{E} = \mathcal{V} \left\{ 3 \int \frac{d^3k}{(2\pi)^3} \text{Tr} \left[ (\alpha \cdot k + m\beta) \Lambda_- (k) \right] -2 V_0^3 \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} \text{Tr} \left[ \Lambda_- (k) \Lambda_+ (k') \right] \int d^3r e^{-i(k-k')\cdot r} \right\}, \tag{16}
\]
\[
H_2^r = \int d^3x : q^\dagger (x) (-i\alpha \cdot \nabla + m\beta) q (x) : \\
- \frac{2}{3} V_0^3 \int d^3R \int d^3r R^2 \int \frac{d^3k}{(2\pi)^3} e^{ik \cdot r} \cdot q^\dagger (x) \left[ \Lambda_+ (k) - \Lambda_- (k) \right] q (y) :, \tag{17}
\]

8
\[ H^R_2 = \frac{8}{3} V^3_0 \int d^3 R \int d^3 r \int \frac{d^3 k}{(2\pi)^3} e^{ik \cdot r} \cdot q^\dagger (x) [\Lambda_+(k) - \Lambda_-(k)] q(y) : \quad (18) \]

\[ H_4 = -\frac{1}{2} V^3_0 \int d^3 R \int d^3 r \cdot (r^2 - 4R^2) : q^\dagger (x) T^a q(x) q^\dagger (y) T^a q(y) : \quad (19) \]

where : : is the normal ordering operator and \( V \) is the volume of the space.

\( E \) is the vacuum energy. On our trial basis it is a function of the chiral angle \( \phi \). The Bogoliubov–Valatin variational method consists in choosing \( \phi \) which minimizes \( E \):

\[ \delta E(\phi) = 0. \quad (20) \]

This is the gap equation. From Eq. (16) more explicitly we get

\[ m \cos \phi(k) - k \sin \phi(k) + \frac{2}{3} V^3_0 \left[ \phi(k)^{''} + \frac{2}{k} \phi(k)^{'} + \frac{2}{k^2} \cos \phi(k) \sin \phi(k) \right] = 0. \quad (21) \]

This is just the equation solved in [16]. As noticed there, it is identical to the Dyson–Schwinger equation we would obtain for the self-mass of the light quark interacting via a harmonic oscillator potential in the instantaneous ladder approximation. The solution \( \phi \), which is a function of \( k \), defines the new chiral broken vacuum. A plot of \( \phi \), solution of Eq. (21), for different values of the mass \( m \) is shown in Fig. 2. As the mass increases as much the solution approaches the infinitely massive limit \( \phi = \pi/2 \). Finally, the explicit calculation shows that \( \phi \), solution of Eq. (21), not only is a stationary point of the vacuum energy, but also that his vacuum is energetically favored with respect to the chiral symmetric one (in the massless case \( E(\phi) - E(0) < 0 \)).

The light quark condensate can be calculated explicitly from Eq. (12):

\[ \langle 0 | \bar{q} q | 0 \rangle = -\frac{3}{\pi^2} \int \limits_0^\infty dk \ k^2 \sin \phi(k). \]

On the solution of Eq. (21) we get for \( m = 0 \) (see Ref. [16])

\[ \langle 0 | \bar{q} q | 0 \rangle = -\frac{1}{24} T_g G_2 \times 0.3722 \quad (22) \]

The result (22) looks appealing. It establishes a connection between the gluon condensate and the light quark condensate. The connection is possible since the non-local gluon condensate has introduced into the game a finite correlation length \( T_g \). Substituting to \( T_g \) the lattice value 0.35 fm [19] and to the
Fig. 2. Chiral angles (solution of the gap equation) for different quark masses. The momentum $k$ and the mass $m$ are in units of $4V_0^3/3$.

gluon condensate $G_2$ the value of $0.048 \text{ GeV}^4$ \cite{23} we get $\langle 0|\bar{q}q|0 \rangle \simeq -(110 \text{ MeV})^3$ which is a rather low value. This is not surprising and is entirely due to our crude assumption (9). If used for static sources Eq. (9) would lead to an harmonic oscillator confining potential systematically below the expected linear rising potential with slope $\sigma = 0.2 \text{ GeV}^2$ in the range of interest from 0.1 to 1 fm. This suggests that, if we would use the covariant parameterization of the non-local gluon condensate given in Eq. (8), that we know from Ref. \cite{10} leads to a phenomenological correct linear confinement between static sources, we would presumably enhance the light quark condensate to the usually accepted value of $\langle 0|\bar{q}q|0 \rangle \simeq -(250 \text{ MeV})^3$.

Let us briefly discuss the $H_2'$ term (17). Expanding on (12) we get:

$$H_2' = \sum_{ss'} \int \frac{d^3k}{(2\pi)^3} \left[ A(k) \sin \phi(k) + B(k) \cos \phi(k) \right]$$

$$\times \left[ b_s^\dagger(k)b_{s'}(k) + d_s^\dagger(k)d_{s'}(k) \right] \delta_{ss'}$$

$$- \left[ A(k) \cos \phi(k) - B(k) \sin \phi(k) \right]$$

$$\times \left[ b_s^\dagger(-k)d_{s'}^\dagger(-k) + d_s(-k)b_{s'}^\dagger(k) \right] (\sigma \cdot \hat{k})_{ss'},$$

$$A(k) \equiv m + \frac{2}{3} V_0^3 \Delta \sin \phi(k),$$

$$B(k) \equiv k + \frac{2}{3} V_0^3 \left( \Delta \cos \phi(k) - \frac{2}{k^2} \cos \phi(k) \right).$$

The stationarity condition (20) or (21) cancels the so-called Bogoliubov anomalous term (proportional to $A \cos \phi - B \sin \phi$) which would destabilize the
vacuum. On the physical vacuum we then have
\[ H_2^r = \sum \int \frac{d^3k}{(2\pi)^3} E(k) \left[ b_s^\dagger(k)b_s(k) + d_s^\dagger(k)d_s(k) \right], \]  
(23)
with \( E^2 \equiv A^2 + B^2 \). Eq. (23) simply gives the light quark kinetic energy on the physical vacuum.

4 THE BOUND STATE EQUATION

The binding is given in the effective Hamiltonian (15) by the terms \( H_2^R \) and \( H_4 \). As far as the bare \( q\bar{Q} \) mass is concerned we do not need to evaluate \( H_4 \) matrix elements. For instance, a term like \( \langle q | H_4 | q\bar{q} \rangle \) would be responsible for the coupling of mesons to the bare \( q\bar{Q} \) state [24]. Notice that, terms like \( \langle 0 | H_2^R | q\bar{q} \rangle \) which would lead to coupled channels and terms contributing only for baryons, have been also neglected. Expanding Eq. (18) on the quark field (12) and taking the matrix element between a one-particle state of momentum \( p \) and a one-particle state of momentum \( q \), we have

\[ H_2^R(p, q)_{ss'} \equiv \langle 0 | b_s(p) H_2^R b_{s'}^\dagger(q) | 0 \rangle = \frac{8}{3} V_0^3 u_s^\dagger(p) \left\{ \beta \sin \left[ \phi \left( \frac{p + q}{2} \right) \right] + \frac{\alpha \cdot \hat{p} + \alpha \cdot \hat{q}}{2} \cos \left[ \phi \left( \frac{p + q}{2} \right) \right] \right\} u_{s'}(q) \times \left[ -\Delta (2\pi)^3 \delta^3(p - q) \right]. \]  
(24)
As expected the binding interaction would be chiral invariant (\( \sim \alpha \cdot \hat{p} + \alpha \cdot \hat{q} \)) for a massless particle on the perturbative vacuum (\( \phi = 0 \)). While for a infinitely massive particle (\( \phi = \pi/2 \)) chiral invariance would be maximally broken. In our case the solution of the gap equation (21) gives rise to a binding interaction which contains two pieces. One is chiral invariant and the other, proportional to \( \beta \), breaks explicitly chiral invariance. The existence of such a term is suggested by the spin-orbit structure of the heavy quarkonium potential whose relativistic origin may be traced back to a scalar confining Bethe–Salpeter kernel [25]. In a Hamiltonian language this would just correspond to an interaction proportional to \( \beta \). Indeed such a kind of interaction has been used, also recently, in phenomenological applications to the heavy-light spectrum [7]. As already argued in [9], what one obtains following the approach discussed in this work is an interaction which turns out to be not only proportional to \( \beta \). It manifests, also under the strong simplifying assumption (9), a more complicate structure which interpolates between a chiral invariant vector interaction and a scalar interaction generated both by the mechanism of chiral condensation and by the initial light quark mass.
Substituting into Eq. (24) the explicit expression for the spinors given by Eq. (13) and (14) and integrating over a wave function \( \Phi \) we get:

\[
\int \frac{d^3q}{(2\pi)^3} H^R_2(p, q) \Phi(q) = \frac{8}{3} V^3_0 \left\{ \frac{1}{2p^2} [1 - \sin(\phi(p))]^2 - \frac{i}{p^2} [1 - \sin(\phi(p))] \sigma \cdot (p \times \nabla) - \Delta \right\} \Phi(p).
\]

Summing up the contributions coming from the pieces \( H^R_2 \) and \( H^R_2 \) of the Hamiltonian, the bound state equation on the physical vacuum reads

\[
\left\{ E(p) + \frac{8}{3} V^3_0 \left( \frac{1}{2p^2} [1 - \sin(\phi(p))]^2 + \frac{2}{p^2} [1 - \sin(\phi(p))] \mathbf{S} \cdot \mathbf{L} - \Delta \right) \right\} \Phi(p) = \tilde{\Lambda} \Phi(p),
\]

where we have introduced the spin operator \( \mathbf{S} = \sigma/2 \) and the orbital angular momentum operator \( \mathbf{L} = \mathbf{r} \times \mathbf{p} \). The eigenvalues \( \tilde{\Lambda} \) of the equation are the energy levels of the bound state in the non-recoil limit, i.e. the difference between the mass \( M_{qQ} \) of the considered heavy-light meson and the mass \( M_Q \) of the corresponding heavy quark.

In the heavy quark limit \( (m = M_Q \to \infty, \phi \to \pi/2) \) we get

\[
\left[ \frac{p^2}{2M_Q} - \frac{8}{3} V^3_0 \Delta \right] \Phi(p) = \tilde{\Lambda} \Phi(p).
\]

This is the bound state equation for two static sources. As already announced, assumption (9) leads to a confining potential between infinitely heavy quarks which is a harmonic oscillator \( \sim V^3_0 r^2 \).

The general spectroscopic properties of Eq. (25) are simple to derive. Since \( 2 \mathbf{S} \cdot \mathbf{L} = J^2 - S^2 - L^2 \), where \( \mathbf{J}_l \) is the total angular momentum of the light quark, and \( \Delta = d^2/dp^2 + (2/p)d/dp - L^2/p^2 \), the energy levels are sensitive only to \( J_l \) and \( L \). In particular for each orbital angular momentum quantum number \( \ell \) we have two different levels, one with \( j_l = \ell + 1/2 \) (i.e. \( 2 \mathbf{S} \cdot \mathbf{L} = \ell \)) and the other with \( j_l = \ell - 1/2 \) (i.e. \( 2 \mathbf{S} \cdot \mathbf{L} = -1 - \ell \)). Of course, since in this treatment the heavy quark symmetry is exact, there is no dependence of the energy levels on the heavy quark spin. In Tab. 1 we list up to \( n = 4 \) the energy spectrum obtained by solving numerically Eq. (25). This spectrum exhibits a small deviation from the pseudo-spin symmetry (i.e. the spectrum becomes for \( J \) excited states, almost parity independent) which measures the extent of chiral symmetry breaking. The value of \( \tilde{\Lambda}_{u,d} \approx 0.6 \) GeV which we obtain with the values of the parameters given in section 3, appears quite reasonable.
Table 1
\[ \bar{\Lambda} \text{ in } 4 V_0^3/3 \text{ units as obtained from Eq. (25) for } m = 0. \] J and P are respectively the total angular momentum and the parity of the meson.

5 COMMENTS AND CONCLUSIONS

In this work we have studied the heavy-light non-recoil dynamics of QCD adopting a Gaussian approximation for the gluodynamics. All the heavy quark limits \((m > T_g)\) were already studied in [9] and give the expected results. Here we do not have given any constraint on the light quark mass. Chiral symmetry breaking and a chiral non-invariant binding interaction emerge quite naturally in our approach and a link is established between chiral symmetry breaking properties and confining interaction. In particular with Eq. (22) we establish a relation between the order parameter of chiral symmetry (the quark condensate \(\langle 0|\bar{q}q|0\rangle\)) and that one which in our framework describes confinement (the gluon correlation length \(T_g\)). A similar relation can be found in [13].

The actual calculations were performed under the rough approximation (9). This is unrealistic since it gives in the heavy quark limit a confining potential which is not linear. Moreover all magnetic contributions were not considered. This is expected to affect the levels listed in Tab. 1. Nevertheless, as discussed in section 2, we expect that the main features presented in this work (chiral
symmetry breaking, chiral non-invariant binding interaction, relation between
the order parameters) will still hold also by using a more realistic parameter-
ization of the bilocal gluon condensate, for instance that one suggested by
lattice simulations. Indeed, the encouraging results obtained in such a simple
framework, not only support but also make urgent the extension in this sense
of the present analysis. Work is in progress in this direction.

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References

[1] Particle Data Group, C. Caso et al. European Physical Journal C 3 (1998) 1;
see also http://pdg.lbl.gov/.
[2] N. Isgur and M. Wise, Phys. Lett. B 232 (1989) 113.
[3] M. Neubert, Lectures given at the International School of Subnuclear
Physics: 34th Course: Effective Theories and Fundamental Interactions, Erice,
Italy, (1996) CERN-TH-96-281; Phys. Rep. 245 (1994) 259; T. Mannel in
“Schladming 1996, Perturbative and nonperturbative aspects of quantum field
theory”, (Springer, Berlin, 1996); B. Grinstein, Lectures given at 6th Mexican
School of Particles and Fields, Villahermosa, Mexico, (1994).
[4] E. J. Eichten, C. Hill and C. Quigg, Phys. Rev. Lett. D 71 (1993) 4116.
[5] W. Kwong and J. Rosner, Phys. Rev. D 44 (1991) 212 and refs. therein.
[6] M. Neubert, Phys. Rev. D 46 (1992) 1076; E. Bagan, P. Ball, V. M. Braun
and H. G. Dosch, Phys. Lett. B 278 (1992) 457.
[7] V. D. Mur, V. S. Popov, Yu. A. Simonov and V. P. Yurov, J. Expt. Theor.
Phys. 78 (1994) 1; M. G. Olsson, S. Veseli and K. Williams, Phys. Rev. D 51
(1995) 5079; M. R. Ahmady, R. Mendel and J. D. Talman, Phys. Rev. D 52
(1995) 254.
[8] I. I. Bigi and N. G. Uraltsev, Z. Phys. C 62 (1994) 623.
[9] N. Brambilla and A. Vairo, Phys. Lett. B 407 (1997) 167; Nucl. Phys. Proc.
Suppl. B 64 (1998) 423.
[10] H. G. Dosch, Phys. Lett. B 190 (1987) 177; H. G. Dosch and Yu. A. Simonov, Phys. Lett. B 205 (1988) 339.

[11] G. Bali, N. Brambilla and A. Vairo, Phys. Lett. B 421 (1998) 265.

[12] E. Eichten and F. Feinberg, Phys. Rev. D 23 (1981) 2714; D. Gromes, Z. Phys. C 22 (1984) 265.

[13] Yu. A. Simonov, Phys. Atom. Nucl. 60 (1997) 2069.

[14] H. G. Dosch, Prog. Part. Nucl. Phys. 33 (1994) 121; O. Nachtmann, in “Schladming 1996, Perturbative and nonperturbative aspects of quantum field theory”, (Springer, Berlin, 1996).

[15] P. Bicudo and E. Ribeiro, Phys. Rev. D 42 (1990) 1611; Phys. Rev. D 42 (1990) 1625.

[16] A. le Yaouanc, L. Oliver, O. Pene and J.-C. Raynal, Phys. Lett. B 134 (1984) 249, Phys. Rev. D 29 (1984) 1233; A. le Yaouanc, L. Oliver, S. Ono, O. Pene and J.-C. Raynal, Phys. Rev. D 31 (1985) 137.

[17] F. Karsch, in “QCD, 20 years later” eds. P. M. Zerwas and H. A. Kastrup (World Scientific, Singapore, 1993).

[18] C. D. Roberts and A. G. Williams, Prog. in Part. and Nucl. Phys. 33 (1994) 477.

[19] M. D’Elia, A. Di Giacomo, E. Meggiolaro, Phys. Lett. B 408 (1997) 315.

[20] H. G. Dosch and U. Marquard, Nucl. Phys. A 560 (1993) 333.

[21] I. I. Balitsky, Nucl. Phys. B 254 (1985) 166.

[22] Yu. A. Simonov, Phys. Usp. 39 (1996) 313.

[23] S. Narison, “QCD spectral sum rules”, (World Scientific, Singapore, 1989).

[24] P. Bicudo and E. Ribeiro, Phys. Rev. D 42 (1990) 1635.

[25] W. Lucha, F. F. Schöberl and D. Gromes, Phys. Rep. 200 (1990) 127.