Reactive Powder Concrete Beam's Behavior in Flexural: Review

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1. Introduction

Reactive Powder Concrete (RPC) can be defined as a fine-grained concrete made of powder and fine aggregate with the absence of coarse aggregate. RPC made of Portland cement, silica fume, or other pozzolanic materials, quartz powder, silica sand, superplasticizer, water, and steel fibers. The range of compressive strength for RPC is about 120-200 MPa with a modulus of rupture of about 30-60 MPa. To reach a compressive strength of 200 MPa and more, specific laboratory conditions should be applied such as high-temperature treatment and applied external pressures before and through setting. The particle size of silica sand is limited to 600µm, but no less than 150µm [1-3]. Portland cement, silica fume, and quartz powder react chemically in presence of water. content of the water/cement ratio used in the mixes varied from 0.15 to 0.22. But water/cement ratio of less than 0.12 will result in poor workability of the concrete. Superplasticizer is needed at this point to offset the flocculation. The superplasticizer ratio must be high because of the minimal amount of water in the mixture [4,5]. Without the addition of the steel, strength would be drawn from ionic forces making the material very brittle. The steel is incorporated either in the form of fibers added to the mixture or by encasing the UHPC in the steel section forming a composite section. The result composite material has the rigidity of concrete with the ductility of steel. The metal

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fibers can delay the localization of active cracks by holding the concrete together as it tries to pull apart. [2], [4]. Heating RPC for two days at 90°C, once the setting has occurred, accelerates these pozzolanic reactions and modifies the hydrate structure [6]. Many researchers presented a state-of-the-art review on reactive powder's production, mechanical properties, durability, development, and applications [7-11]. But the review about stress-strain, structural behavior is hard to find. So, this study tries to collect and view the previously studied and experimental work related to the structural behavior reinforced reactive powder concrete. Before starting a review of strengths, stress-strain relation and ductility are presented because of their importance and effect on the structural behavior of beams under bending.

2. Strengths, Stress-Strain Relations, and Modulus of elasticity of RPC

This section focused on the important properties that are required for structural analysis and design of RPC including Compressive Strength, modulus of rupture, stress-strain relations, and modulus of elasticity of RPC. Also reviewed relations and equations proposed by several researchers based on experimental work. As mentioned before the compressive strengths of RPC range from 120-200 MPa. There are many reasons for this high strength compared to conventional concrete. One of many reasons is the elimination of coarse aggregate and replaced of well-graded, high-quality sub-micron sizes of fine silicon aggregates. With the small size of the particles, the voids between the larger particles are filled resulting in an increased density and reduced porosity and result in compressive strength increase [12,13]. Also, of importance is the large residual strength after the peak, thanks to the presence of the fibers [14]. Many researchers tried to apply different methods of treatment such as steam curing and high-pressure steam curing. The high strength of RPC can be liberated by applying high-pressure steam curing or a high level of thermal curing. Collepardi et al [15] examined three methods of curing; room temperature (always at 20o C), steam curing at 90o C, and high-pressure steam curing at 160o C. results showed that the steam curing at 90o C and especially high-pressure steam curing at 160o C gives a better performance for RPC in terms of higher compressive strength, low drying shrinkage strain and creep strain than the one which curing at room temperature. They concluded that fibers increase the compressive strength in particular with the decreasing distance between the fibers. They ascribed such development to the stronger bond between fibers and cement matrix. Jungwirth [16] showed that compressive strength rises from 120 to 180 MPa for raising curing temperature from 20 to 90o C. and found fibers improve the tensile behavior, increase the ductility of the material and lead to an even distribution of the microcracks. This eliminates significantly the need for passive reinforcement of the structures. A modulus of rupture up to 35 MPa was obtained. Jungwirth [16] stated that the addition of steel fibers as 2% by volume of the RPC, has a significant effect on the reinforcement system.

Sadrekarimi [17] found that both high-temperature heat curing and pressing the samples will increase the compressive strength of concrete. He found that the compressive strength for the specimens treated at 240o C is about (1.5 – 2) times the compressive strength of specimens cured at 90oC. Another method for increasing compressive strength for RPC can be by applying pressure. Roux et al.’s [18] results showed a considerable increase in compressive strength by 35% for pressurized concrete compared to table-vibrated concrete. Adding steel fiber allows RPC material to yield flexural strength up to eight times that of high-performance concrete [19].

Voo et al. [20] studied the effect of the shape of fibers using 2% straight steel fibers and/or hook end steel fibers. Flexural strength up to 30 MPa was obtained. Researchers found that the flexural strength of RPC has ranging from 35–48MPa [21], [22].

The ACI 318M-19 [23] code suggests the following relation for the prediction of modulus of rupture for ordinary concrete from compressive strength:

\[ f_r = 0.62 \sqrt{f_c^2} \]  

where:
fr = modulus of rupture of concrete, MPa
f'c=compressive stress of concrete of standard cylinder, MPa

Al-Ne’aime [24] suggested a linear relationship between compressive strength and flexural strength of prisms of many RPC mixes as shown below:

\[ f_r = af_{cu} + b \]  

Where:

- \( f_{cu} \): average compressive strength of standard cube
- \( a, b \): constants (depending on the mix type)

Relationships similar to the one given by Eq. (2) were proposed by Kasser [25] for plain and steel fiber reinforced SC-RPC which are respectively:

\[ f_r = 0.0462 f_{cu} + 4.2501 \]  
\[ f_r = 0.0901 f_{cu} + 9.4383 \]  

Also, Mahdi [26] established two equations for SC-RPC, one for plain and the other for steel fiber reinforced RPC which are respectively:

\[ f_r = 44.7537/(1 - 0.4979 f'c) \]  
\[ f_r = 44.7537 - 0.4979 f'c + 3.1808 \times 10^{-2} (f'c)^{1.5} \]  

Another reliable empirical relationship has been established by Hannawayya [27]:

\[ f_r = 332.848 f'c + 7.532 V_f \]  

which can be used to estimate modulus of rupture for RPC have f'c between 79 MPa to 119 MPa and content steel fiber ratio 0% to 2%.

Many researchers like Hannawayya [27], Prabha et al. [28], Wang et al [30] Hognestad [31], etc. investigated that factor can affect the stress-strain behavior of RPC.

The effect of length and content of steel fibers stress-strain relationship was tested by Prabha et al. [28]. Results showed that 2% steel fibers (as volume fraction) with 13 mm length gave the highest cylinder compressive strength of 171.3 MPa and static modulus of elasticity about 44.8 GPa. He found that stress-strain curves of RPC have a nearly linear ascending portion and the strain at peak stress increases with increases in strength.

Prabha et al. found from their results the length of fibers and their content affected the behavior of the post-peak curve, which may be more gradually sloping for the high fibers content compared to lower fibers content.

For conventional normal weight concrete, the stress-strain curve before maximum stress can be approximated as a second–degree parabola [29]. Wang et al [30] proposed an equation to predict the stress-strain curve for conventional normal weight and conventional lightweight concrete as follow:

\[ Y = \frac{Ax + Bx^2}{1 + Cx + Dx^2} \]  

where: \( Y = f_c/f'c \) , \( X = \varepsilon_c/\varepsilon_o \)

- \( A, B, C, D \): constants
- \( f_c, \varepsilon_c \): stress and strain at any point, MPa
- \( f'c, \varepsilon_o \): peak strength and corresponding strain, MPa

Hognestad [31] well known fc-\( \varepsilon_c \) relationship, as shown in Figure (2), is an ascending parabola followed by a descending straight line such that:

\[ f_c = f'c \left( 2\varepsilon_c - \frac{\varepsilon_c^2}{\varepsilon_o^2} \right) \text{ for } 0 \leq \varepsilon_c \leq \varepsilon_o \]  
\[ f_c = f'c \left( 1 - \frac{0.15}{\varepsilon_{cu}} \left( \varepsilon_c - \varepsilon_o \right) \right) \text{ for } \varepsilon_o \leq \varepsilon_c \leq \varepsilon_{cu} \]  

where: \( f_c \): compressive stress of concrete, MPa
\( f'c \): compressive strength of cylindrical concrete specimen, MPa
\( \varepsilon_c \): concrete compressive strain corresponding to \( f_c \)
\( \varepsilon_{cu} \): ultimate compressive strain of concrete
\( \varepsilon_o = 2 f'c/Ec \)
\( Ec \): the modulus of elasticity of concrete
Hannawayya [27] modified the equations above to be suitable for RPC according to experimental data from his research as follow:

\[
f_c = f''_c \left( A \left( \frac{\varepsilon_c}{\varepsilon_o} \right)^3 + B \left( \frac{\varepsilon_c}{\varepsilon_o} \right)^2 + C \left( \frac{\varepsilon_c}{\varepsilon_o} \right) \right) \quad \text{for} \quad 0 \leq \varepsilon_c \leq \varepsilon_o \\
\]

\[
f_c = f''_c \left( 1 - \frac{0.15}{\varepsilon_{cu} - \varepsilon_o} (\varepsilon_c - \varepsilon_o) \right) \quad \text{for} \quad \varepsilon_o \leq \varepsilon_c \leq \varepsilon_{cu} \\
\]

Where \(A, B, C\): polynomial depending on mix’s proportion. And he stated that Hognestad’s equation (9) can be adopted for the descending branch. He made a comparison between his experimental results and the proposed model and with that proposed by Wang et al. [30] and Hognestad [31] for different percentages of silica fume (Sf) and steel fibers (Sf) as shown in Figures 3. can be noticed that a steeper ascending part is obtained for RPC having a higher steel fiber ratio except for the case Sf=2% which can be attributed to balling (nesting) effect. It can be seen that adding steel fibers with a ratio of 0.5% to an RPC mix, increases the measured strains by small amounts. But a great jump in these strains can take place when adding steel fibers with a 1.5% ratio. And with an increase in fibers content, increases in strain \(\varepsilon_o\) are found to be greater than increases in ultimate strain \(\varepsilon_{cu}\). This fact is only for low steel fiber ratio (less than 2%) in contrast adding steel fibers by higher volume ratio improves performance significantly for both ascending and descending part [32], and this is attributed to the type of failure which is shear on a plane at about 45±15° to the loading axis [27]. For a constant steel ratio, the silica fume content has little effect on the stress-strain model of RPC mixes.

While the increase in silica fume content led to an increase in both ultimate compressive strength and corresponding strain \(\varepsilon_o\) but at different rates. It is of interest to note that the difference between the two strains \(\varepsilon_o\) and \(\varepsilon_{cu}\) is not critically sensitive to the variation in the silica fume content. But the percentage of such strains difference concerning \(\varepsilon_o\) seems to be larger for smaller silica fume content \(\varepsilon_o\) seems to be larger for smaller silica fume content since an increase in silica fume content will result in a denser composite with less ability to deform when subjected to excessive stresses [27].

Al-Hassani et al [33] also investigated the stress – stress behavior of RPC and study effect of steel fibers (SFs) content (0%, 1%, 2% and 3% as volume fraction of mix (VF)) and silica fume (SF) content (0.0%, 10.0%, 15.0%, 20.0%, 25.0%, and 30.0% by weight as a partial replacement of cement). He used a macro hooked steel fiber with diameter of 0.5mm and length of 30mm. He found that increasing in the

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**Figure 1.** Stress-strain curve proposed by Hognestad [31]
Steel fibers led to increase the cubic compressive strength slightly by 3.72%, 8.36%, and 8.89% for 1.0%, 2.0%, and 3.0% SFs respectively, for cylinder the compressive strength increased by 6.36%, 9.9%, and 11.54% for 1.0%, 2.0%, and 3.0% SFs, respectively.

Improving compressive strength reflects the contribution of steel fibers to the tensile capacity of RPC and crack arrest ability of fibers [34]. While increasing in Sf content caused increasing in cube compressive strength by 13.54%, 18.02%, 24.72%, 29.86%, and 34.17% for 10.0%, 15.0%, 20.0%, 25.0%, and 30.0% Sf, respectively and the cylinder compressive strength increased by 17.56%, 20.30%, 30.92%, 3.79%, and 41.04% for 10.0%, 15.0%, 20.0%, 25.0%, and 30.0% Sf, respectively. The compressive stress-strain curves of RPC specimens with fiber volume fractions of 0%, 1%, 2%, and 3% which are denoted by MFR0, MFR1, MFR2, and MFR3, respectively (see Figure 3). The ascending part of the curves for 0% fiber (MFR0) is steep and almost a straight line and descending part of the curve has almost vanished and the steel fibers changed the mode of failure of the specimen from a brittle failure to a more ductile failure, as shown in Figure 3.

Stress strain curves of RPC specimens with silica fume content (SF) of 0%, 10%, 15%, 20%, 25%, and 30% (by the weight of cement) which are denoted by MSF0, MSF10, MSF15, MSF20, MSF25 and MSF30, respectively, are shown in Figure 4.

The curves show that the shape of the ascending part of the curve in the case of high silica fume content is steeper compared to RPC mix without or with less content of silica fume but its effect is found to be less on strain ε0 or εcu. This is because the mixes with higher Sf content have higher compressive. As mentioned before this is because the increase in silica fume content led to a denser result composite with less ability to deform when subjected to excessive stresses [27]. Al-Hassani et al [33] suggested a proposed equation that describes a complete compressive stress-strain curve based on the results above:

\[ f_c = f' \times \left[ \frac{a}{c} \left( \frac{e_c}{e_o} \right)^b \right] \]

where:

\[ a = 3.805, \quad b = 0.919, \quad c = 2.831, \quad d = 3.970 \]

Graybeal et. al [35] showed that the pre-peak nonlinearity and the post-peak strain capacity both decrease as compressive strength increases. And they found that regardless of the age after applying steam treatment the basic shape of the ascending part of the stress-strain curve remained unchanged.

The results of the compression tests conducted on eighteen different mixes of RPC are presented in the form of stress-strain curves by Ibraheem [36]. The effects of three variable parameters are studied; type of fibers, type of pozzolanic admixtures, and the percentage of fibers.

It can be seen from Figure 5 that for any value of strain, the value of stress is highest for RPC using Silica Fume (Sf), then lesser stress corresponds to RPC using Micro Silica (MS) and the lowest stress is that for RPC using Metakaolin (MK). The results show that using SF gave a higher compressive strength for RPC than MS and much better than MK. From Figure 6 can see that only a little variation during the ascending part can be noticed but the effect is clear with higher discrepancies during the descending part of the \( f_c - e_c \) relationship. However, a 1.25% fiber (whether of the steel or polypropylene type) seems to give a lesser drop in peak stress with increasing strain beyond \( \varepsilon^c \). Based on these results one can see that change in the fibers content has less effect on the ultimate strength, which is higher for low fibers content. The influence of the type of fibers (steel, polypropylene fibers) on the stress-strain relationships is shown in Figure 7. Effect of type is small in the ascending part of the relationship (especially for those using MS or SF) but higher discrepancies are found in the descending part especially for RPC using MK. However, using steel fibers is better than using polypropylene fibers as far as the strength of RPC is concerned.
Figure 2. Comparison among Stress-Strain curves of experimental results and proposed models [27]

Figure 3. Steel fibers effect on the compressive stress-strain behavior of RPC [33]
peak point is reached with coordinates \((\varepsilon_0, f'c)\). Further increments in strain cause the stress to decrease showing a double curvature descending part, with inflection point defined by the coordinates \((\varepsilon_1, f_1)\). The analytical prediction of such descending part requires choosing another point on the curve with coordinates \((\varepsilon_2, f_2)\) and \((\varepsilon_2 - \varepsilon_1)\) equal to \((\varepsilon_1 - \varepsilon_0)\). Two separate sets for the values of the four constants A, B, C, and D have to be obtained to trace the ascending and descending parts of such \((f'c - \varepsilon_c)\) curve.

**Figure 4.** Effect of silica fume stress-strain relationship of RPC

**Figure 5.** Effect of type of Pozzolanic Material on Stress-Strain Curves of RPC [36]
Figure 6. Effect of Vf (Percentage of Fibers) on Stress-Strain Curves of RPC in Compression [36]
Figure 7. Fiber's Types effect on Compression Stress-Strain behavior of RPC

Figure 8. Typical Compression Stress-Strain behavior of RPC [36]
Let $E_c$ and $E_o$ denote the secant modulus of elasticity at stresses $(0.45f'c)$ and $(f'c)$ respectively, such that:

$$E_c = \frac{0.45f'c}{\varepsilon_c}$$  \hspace{1cm} (13)

$$E_o = \frac{f'c}{\varepsilon_c}$$  \hspace{1cm} (14)

Also, let the ratio of these two moduli of elasticity be denoted by $n$ such that:

$$n = \frac{E_c}{E_o}$$  \hspace{1cm} (15)

Therefore, the four constants $A$, $B$, $C$, and $D$ for the ascending part can be evaluated by applying the following four conditions:

1. $\frac{dY}{dX} = n$ at $(Y = 0 , X = 0)$
2. $Y = 0.45$ at $X = \frac{0.45}{n}$
3. $Y = 1$ at $X = 1$
4. $\frac{dY}{dX} = 0$ at $Y = X = 1$

For the descending part the conditions for determining the constants are:

1. $Y = 1$ at $X = 1$
2. $\frac{dY}{dX} = 0$ at $Y = X = 1$
3. $Y = \frac{f_1}{f'c}$ at $X = \frac{\varepsilon_1}{\varepsilon_s}$
4. $Y = \frac{f_2}{f'c}$ at $X = \frac{\varepsilon_2}{\varepsilon_s}$

Applying the first boundary conditions, for any RPC mix, the ratio has to be determined first then the four constants for the ascending part can be evaluated in the respective order $A$, $C$, $D$ than $B$;

$$A = n \quad \text{and} \quad B = D - 1 \quad \text{ (16)}$$

For the descending part, applying the second group of boundary conditions:

$$A = \frac{-B(C + 2)}{(1 - D)} \quad \text{ (19)}$$

$$B = D - 1 \quad \text{ (20)}$$

$$C = \frac{(1 - X_1)^2}{X_1(1 - Y_1)} - \frac{1}{X_1} - DX_1 \quad \text{ (21)}$$

$$D = \frac{1}{X_2} - \left[ \frac{(1 - X_2)^2}{X_2(1 - Y_2)} - \frac{(1 - X_1)^2}{X_1(1 - Y_1)} \right] \quad \text{ (22)}$$

where: $X_1 = \frac{\varepsilon_1}{\varepsilon_o}, \quad X_2 = \frac{\varepsilon_2}{\varepsilon_o}, \quad Y_1 = \frac{f_1}{f'c}, \quad \text{and} \quad Y_2 = \frac{f_2}{f'c}$

By Substituting $Y = \frac{fc}{f'c} = \frac{\varepsilon_c}{\varepsilon_o}$ in eq. 8

This gives the following equation [36]

$$f_c = \frac{ae_c + be_c^2}{1 + ce_c + de_c^2} \quad \text{ (23)}$$

where: $a = \frac{A}{\varepsilon_o f'c}, \quad b = \frac{B}{\varepsilon_o^2 f'c}, \quad c = \frac{C}{\varepsilon_o}$

and $d = \frac{D}{\varepsilon_o^2}$

The new constants $a$, $b$, $c$, and $d$ in this equation were also calculated based on the values of constants $A$, $B$, $C$, and $D$ and therefore explicit dimensional ($fc - \varepsilon_c$) equation (for both ascending and descending parts) of all RPC mixes.

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**Figure 9.** Relation between Steel fibers and modulus of elasticity for RPC [27]
Modulus of elasticity is another factor that affects the flexural behavior of beams and it is affected by concrete constituent and their proportions. It is a function of the modulus of elasticity of mixed individual components and their contents in the composite. Al-Hassani [33] found that increases in steel fibers content led to only a slight increase in the static modulus of elasticity of RPC. This is explained by the high modulus of elasticity of the steel fibers and because of the transfer of stress from the matrix to the fibers thus stress will be shared by both the fibers and surrounding matrix, and a higher load could be applied before the composite cracks [26]. With an increase in Sf content modulus of elasticity increases because of pozzolanic activity of Sf which led to increased density and improve microstructure of RPC composite [37]. Biolzi et al. [38] found that modulus of elasticity practically did not affect by the volume fraction of steel fibers and there is no difference between recorded results under tension and compression. Hannawayya [27] found that 1.5% steel fiber with 15% silica fume gave the highest modulus of elasticity as shown in Figure 9. &10 shows an increase in the tangent modulus of elasticity of RPC as the silica fume content is changed from 5% to 10%, respectively.

![Figure 10. Influence of silica fume's content on the elastic modulus of RPC [27]](image)

The most widely used and simple empirical equation between the compressive strength and the elastic modulus for conventional normal concrete is ACI Committee 318 equation [23];

\[ E_c = 4700\sqrt{f'_c} \]  \hspace{1cm} (24)

For high strength concrete up to 83 MPa ACI Committee, 363[39] adopted the Eq. (25),

\[ E_c = 3320\sqrt{f'_c} + 6900 \]  \hspace{1cm} (25)

while for UHPFRC (Ultra high-performance fiber reinforced concrete) Ma et al. [40] suggested equation 26 as follow:

\[ E_c = 19000\sqrt[10]{f'_c} \]  \hspace{1cm} (26)

Graybeal [35] also suggested an equation for UHPFRC for compressive strength higher than 25 MPa;

\[ E_c = 3840\sqrt{f'_c} \]  \hspace{1cm} (27)

An empirical equation was suggested by Hannawayya [27] by modifying ACI 318 Code equation to estimate the modulus of elasticity of RPC having cylindrical compressive strength between 79 MPa to 119 MPa.

\[ E_c = 4572\sqrt{f'_c} \]  \hspace{1cm} (28)

3. Structural behavior of RCP Beams under bending

Most of the previous review papers focused on production, mechanical properties, and some of them on durability. In this study, a review of the different structural behavior of reactive powder reinforced concrete beams under bending is presented. Because of the unique and ultra-strength of reactive powder concrete, the ACI 318 [23] equation for the evaluation of the nominal bending moment capacity (Mn) is not
valid anymore for RPC. Hassani and Ibraheem derived based on their experimental results an equation for calculating nominal bending moment (Mn) of rectangular section singly reinforced RPC [41].

The ascending part of this curve, covering values of strain ranging between zero and ε or implicitly values of stress fs ranging between zero and fc’ very close to the shape of a straight line. The double curvature descending part represents concrete softening and its end with concrete failure at ultimate strain εcu. To make utilize of this fc −εc relationship in the flexural analysis and design RPC beam, it is converted into an equivalent bi-linear relationship which is denoted as dashed line as shown in Figure 11.

![Figure 11. Stress-Strain Curve of RPC Concrete in Compression [41]](image)

The RPC section details that subjected to a positive bending moment (M), strain and actual stress distributions, and the conversion of actual compressive stress distribution to an equivalent bi-linear stress distribution block as proposed by Al Hassani & Ibraheem [41] are illustrated in Figure 12.

![Figure 12. Proposed Strain and Stress Distributions on RPC Section at Ultimate Stage [41]](image)
Based on this assumption and proposed stress-strain distribution the following equation is proposed for the calculation of Mn of RPC:

\[
M_n = \frac{1}{4} \left( \frac{5}{6} \alpha + 1 \right) \left( \frac{c}{d} \right)^2 f', b d^2 + A_s f_d \left( 1 - \frac{c}{d} \right) + \frac{1}{2} \left( \frac{h}{d} \right)^2 f' - \left( \frac{h}{d} \right) \left( \frac{c}{d} \right) + \frac{1}{2} \left( 1 - \frac{\varepsilon_t^2}{4\varepsilon_o^2} \right) \left( \frac{c}{d} \right)^2 f_i b d^2
\]

where \( c/d \) is as given by Equation

\[
\frac{c}{d} = \frac{\left[ \rho f_y \frac{f'}{f_c} + f_c \left( \frac{h}{d} \right) \right]}{f_c \left( 1 + \frac{\varepsilon_t^2}{2\varepsilon_o^2} \right) + \frac{1}{2} \left( 1 + \frac{\alpha}{2} \right)}
\]

where

- \( A_s \): area of longitudinal steel bars, mm²
- \( b \): width of beam (or prism) cross-section, mm
- \( c \): depth of neutral axis measured from top compression fiber, mm
- \( d \): depth of longitudinal steel bar measured from top compression fiber, mm
- \( f'_c \): compressive strength of cylindrical concrete specimen, MPa
- \( f_y \): yield stress of steel bar, MPa
- \( \alpha \): material constant
- \( \varepsilon_c \): concrete compressive strain corresponding to \( f_c \)
- \( \varepsilon_o \): concrete compressive strain corresponding to \( f'_c \)
- \( \varepsilon_t \): tensile strain
- \( \rho \): longitudinal steel bar ratio = \( A_s / bd \)

For any particular RPC mix, the value of \( \alpha \) can be calculated by substituting \( \alpha f' \) for \( f_c \) and \( 2\varepsilon_o \) for \( \varepsilon_c \) in the appropriate equation of the descending part [36].

The nominal ultimate bending moment capacity \( M_n \) of a singly reinforced RPC section is produced by the combined action of three essential elements in the section which are in their respective order appearing in the three terms of the equation, the concrete in the compression zone represented by \( f_c \), the steel bars represented by \( A_s f_y \) and the steel or polypropylene fibers in the tension zone represented by \( f_t \). The contribution of the latter term to the value of \( M_n \) depends largely on the area of steel bars used in the section. In heavily reinforced RPC sections, the neutral axis shifts towards the tension face of the section reducing the area of the effective tension zone and this results in a lesser contribution of the steel or polypropylene fibers to the value of \( M_n \) [36].

Experimental investigations were carried out by Al-wash and Al-Sultan [42] to investigate the flexural behavior of reinforced RPC beams and normal R.C beams incorporating steel fibers once and polypropylene fibers and compared their efficiency. Regardless of the type of fibers, the utilization of fibers in RPC beams is more effective than the normal strength beams. With increasing steel fibers content the strain and deflection were found to be reduced but the increase with increasing in PPT fibers content compared to beams with no fibers [42].

Hannawayya [27] study the influence of steel fibers ratio (\( V_f \)), the content of silica fume (\( S_i \)), and the longitudinal reinforced ratio (\( \rho \)) on the flexural behavior of singly reinforced RPC simply supported beam. The dimensions of beams were 140*125*1400 mm, which were tested under a two-point load. Experimental and analytical results showed that failure load decreases as each of \( V_f, S_i \) and \( \rho \) increases. For most tested beams, the experimental failure load was lower than the analytical ones, except for beams having \( \rho<0.0819 \). All results showed that within the range of materials used \( V_f, S_i \) and \( \rho \) have a negligible effect on mode of failure while \( \rho \) still plays a major role, where beams having \( \rho<0.0819 \) failed by steel yield while beams with higher \( \rho \) failed by compression crushing. Depending on a new rectangular stress block in compression and tension zones, new expressions are derived for the nominal moment capacity \( M_n \) and balanced steel ratio \( \rho_b \) of singly reinforced rectangular reinforced RPC sections (see Figure13). The internal compressive force of the composite above the neutral axis is to be calculated as the volume of the compression block bounded by the stress-strain curve by direct integration, while the internal tensile force of the composite below the neutral axis is to be calculated as the total volumes of tension zone. As the strains are assumed to be increased gradually through-loading process, for each level of strains a new position of neutral axis is
assumed and corresponding stresses (tension and compression) and internal forces are calculated. The position of the neutral axis is to be adjusted to achieve internal forces equilibrium, then internal moment-resisting is to be calculated. The process of calculations is repeated for each single stage level of loading including first cracking, yielding of steel bars, maximum moment strength, and until complete failure occurs by either full crushing of composite at top compression fiber or rupture of longitudinal steel whichever takes place first.

![Figure 13. Reinforced RPC beam section under bending strains and stresses](image-url)

The equilibrium of the internal forces of the cross-section is obtained from the fact that for the case of pure bending, the compressive force is equal to the total tensile forces:

\[ C = \sum T \]  \hspace{1cm} (31)

\[ C = T_c + T_s \]  \hspace{1cm} (32)

The resisting internal moment of the RPC beam section is the sum of the moments caused by the compressive and tensile forces about the neutral axis;

\[ M_{n} = M_c + M_{tc} + M_s \]  \hspace{1cm} (33)

where: 
- \( C \): compressive force in concrete, N
- \( T_s \): steel bars tensile force, N
- \( T_{tc} \): concrete tensile force, N
- \( M_c \): internal moment made by concrete compressive force \((C)\), N.mm
- \( M_{tc} \): internal moment made by steel bars tensile force \((T_s)\), N.mm
- \( M_s \): internal moment made by concrete tensile force \((T_{tc})\), N.mm

1. The internal moment caused by the compressive force about the neutral axis \((M_c)\) can be calculated as follows;

   - If \( 0 \leq \varepsilon_{ef} \leq \varepsilon_0 \), then the whole compression zone is predicted by the proposed third order polynomial equation;

   \[ M_c = b \int f_c x dx \]  \hspace{1cm} (34)

   \[ f_c = \alpha x^3 + \beta x^2 + \gamma x \]

   where:
   - \( \alpha = A \left( \frac{\varepsilon_{ef}}{c} \right)^3 \)
   - \( \beta = B \left( \frac{\varepsilon_{ef}}{c} \right)^2 \)
   - \( \gamma = C \left( \frac{\varepsilon_{ef}}{c} \right) \)

   \[ M_c = b \int (ax^4 + bx^3 + \gamma x^2) \]  \hspace{1cm} (35)

   A, B, C: factors of third order polynomial suggested stress-strain relationship.

   \( \varepsilon_{ef} \) is the strain in extreme compression fiber

   - If \( \varepsilon_0 \leq \varepsilon_{ef} \leq \varepsilon_{cu} \) then a combination of the third-order polynomial equation and linear equations 11 describe the total compression zone, therefore in this case;
\[ M_c = b \left[ \int_0^{x_1} (\alpha x^4 + \beta x^3 + \gamma x^2) \, dx + \int_{x_1}^c f'_{c'} \left( x \frac{0.15}{\varepsilon_{cu} - \varepsilon_o} \left( \frac{x^2}{c} \varepsilon_{ef} - \varepsilon_o \right) \right) \, dx \right] \]

\[ M_{tc} = b \left[ \frac{f_r x_{cr}^2}{3} + \lambda f_t (h - c - x_{cr}) \left( \frac{h - c + x_{cr}}{2} \right) \right] \]

where: \( x_{cr} = \varepsilon_{cr} C \)

\[ M_{tc} = \frac{b f_r c^2}{6} \left( \frac{\varepsilon_{cr}}{\varepsilon_{ef}} \right)^2 \left[ 2 + 3\lambda \left( \frac{h}{c} - 1 \right)^2 \left( \frac{\varepsilon_{ef}}{\varepsilon_{cr}} \right)^2 - 1 \right] \]  

where: \( \lambda = \) post cracking tensile strength factor

3. Internal Moment Caused by the Tensile Force \( T_s \); two cases are possible depending on whether the steel bars have yielded or not:

- If \( 0 \leq \varepsilon_s \leq \varepsilon_{y,s} \) i.e. reinforcing steel is still elastic, then
  \[ M_s = E_s \varepsilon_s A_s (d - c) \]  

ii. If \( \varepsilon_y \leq \varepsilon_s \leq \varepsilon_{su} \) then the steel bars have yielded and therefore;

\[ M_s = (f_y + H(\varepsilon_s - \varepsilon_y)) A_s (d - c) \]  

where:

- \( f_{su}, \varepsilon_{su} = \) ultimate tensile stress and strain in steel reinforcement bars respectively
- \( f_y, \varepsilon_y = \) yield tensile stress, yield tensile strain in steel bars respectively
- \( E_s = \) modulus of elasticity of steel reinforcement bars

where: \( f_{cr} = \) modulus of rupture and calculated according to Eq. 7, MPa.

- When \( \varepsilon_{tc} > \varepsilon_{cr} \) then the concrete in the tension zone is cracked up to a level \( x_{cr} \) below the neutral axis and the internal moment caused by \( T_c \) is the sum of the moments of the triangular and rectangular blocks about the neutral axis.
\[ H = \text{hardening parameter} = \frac{f_{su} - f_y}{\varepsilon_{su} - \varepsilon_y} \]  

(see Figure 14)

Figure 14. Typical tensile stress-strain diagram of steel bars [27]

4. Conclusions

Referring to the literature review given in this study the following remarks can be concluded:

1. Increasing each of the steel fiber content as volumetric ratio and silica fume content in an RPC mix will be led to an increase in compressive strength.
2. For RPC the modulus of rupture has a significant positive effect on increasing steel fibers content. This positive effect reached 280% by using a steel fiber volumetric ratio of 2%, while the effect of increasing silica fume content from 5% to 15% has a limited effect on the modulus of rupture which increases it by only 7.8% [27]. Several proposed equations based on experimental data have been suggested by different researchers to predict the modulus of rupture of RPC.
3. The stress-strain relationships that the steel fibers affect the ultimate strength (peak stress) value. Also, strain \( \varepsilon_0 \) corresponding to ultimate strength was found to be increased this load to an increase in the ductility of RPC.
4. Increasing in silica fume content was found that lead to a higher modulus of elasticity but no clear influence related to strain \( \varepsilon_0 \).
5. Nonlinear equations were proposed based on experimental results to model complete stress-strain relationship curves of RPC and based on these results empirical proposed equations have been suggested for predicting modulus of elasticity for RPC.
6. Flexural tests of RPC beams showed that each cracking load, ultimate load capacity, and the ratio of first cracking load to ultimate load increases with an increase in steel fibers content. Also increasing in longitudinal steel reinforcement amount led to ultimate load increases. However, the longitudinal steel ratio showed the greater effect while silica fume content may not have a clear effect.
7. Researchers derived equations for calculating the nominal ultimate bending moment capacity \( M_n \) of simply reinforced rectangular RPC sections which is based on an idealized stress block in compression proposed based on experimental data.
8. Based on test results for different studies one can conclude that quantity and type of fibers do not significantly affect cracking load but affected the rate of crack propagation, the failure loads, and stiffness. Also, it affects the failure mode from brittle failure to more ductile behavior.

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