Topological Mass vs. Dynamical Mass:

Novelty in (2+1)-dimensional Space-Time

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Abstract

The restriction of space-time dimensions to "2+1" leads us to a novel quantum field theory which has the Chern-Simons term in its action. This term changes the nature of gauge interaction by giving a so-called topological mass to a gauge field without breaking the gauge symmetry. We investigate how a dynamical mass of fermion is affected by the topological mass in the non-perturbative Schwinger-Dyson method.

1 Introduction

Generally speaking, dimensions of space-time may become a critical restriction on an allowed law of interaction in that space-time. For example, the inverse-square law of gravitational and electromagnetic forces can be considered to be resulted from the fact that the dimensions of space-time is "3+1". This also is related to the fact that the Huygens’ principle holds only in the (3+1)-dimensional space-time. The number "3+1", the dimensions of our space-time, is really a magic number.

This kind of restriction may be expected in other space-time dimensions. The modern technology of engineering makes possible to produce low-dimensional electron systems in realistic electronic devices. Especially, in (2+1)-dimensional systems, novel phenomena as the Quantum Hall Effect 3 and the High-$T_C$ Superconductivity 4 were discovered. It is plausible that these phenomena may have their origins in the dimensions of the space-time.

In a sense, the dimensions "2+1" is more mysterious because the mathematics tells us that there exists a specific term called a Chern-Simons term 5. This term is allowed just in (2+1) dimensions. As is well known, the Lagrangian density for the electromagnetic field is given by the Maxwell term, which is (i) gauge invariant, (ii) Lorentz invariant, and (iii) bilinear for the gauge field. The Chern-Simons term also satisfies all of (i) ~ (iii). Therefore the Maxwell theory has a natural extension which is
defined by adding the Chern-Simons term to the Maxwell Lagrangian. This extended version is called the Maxwell-Chern-Simons theory. \[5, 6\] Thus the restriction of the space-time dimensions to “2+1” opens a pass to a new type of theory.

In fact, there appeared many approaches to understand these macroscopic quantum effects by using (2+1)-dimensional quantum field theories with and without the Chern-Simons term. For example, the (2+1)-dimensional quantum electrodynamics \(QED_3\) has been used to explain the quantum Hall effect, and \(QED_3\) with the Chern-Simons term \(QED_3\) has provided the anyon model which is expected to give an essential mechanism for the high-\(T_C\) superconductivity. \[7\] These investigations have produced important results and are now still in progress.

What is the physical meaning of the Chern-Simons term? The Chern-Simons term gives the gauge field a mass without breaking the gauge symmetry. \[5, 6\] This mass is called a topological mass because the Chern-Simons term has a topological meaning as the secondary characteristic class. \[4\] Whether the gauge field is massless or massive affects the nature of interactions, e.g., the range of interactions. In this case, the most important effect of the massive gauge field is to rescue the (2+1)-dimensional Maxwell theory from the infrared catastrophe which appears in a self-energy of fermion when the Maxwell field interacts with matters. \[8\]

On the other hand, it is known that nonperturbative radiative corrections can produce a mass of fermion called a dynamical mass. The dynamical mass generation of four-component fermions in \(QED_3\) without the Chern-Simons term has been studied in Ref.\[9\]. One four-component fermion is equivalent to two two-component fermions. The mass term of the two-component fermion breaks the parity \((P)\) symmetry while the one of the four-component fermion breaks \(P \times Z_2\) (flavour) symmetry. In Ref.\[10\], one of the present authors and others have investigated the dynamical mass generation of a single two-component fermion in \(QED_3\) without the Chern-Simons term. A parity-breaking solution which generates the dynamical mass has been found.

Both analyses have been extended to the cases with the Chern-Simons term. The study of the four-component dynamical mass in the Maxwell-Chern-Simons \(QED_3\) has been done in Ref.\[11\]. The dynamical mass generation of a single two-component fermion in the Maxwell-Chern-Simons \(QED_3\) has been studied in Ref.\[12\]. Especially, Ref.\[12\] is motivated to clarify a role of the topological mass in the nonperturbative dynamics.

However, as was pointed out in Ref.\[12\], the estimation of the dynamical mass for a very small value of the topological mass is very difficult technically and also is highly nontrivial. In this paper, we extend further the analysis of Ref.\[12\] to the case in which the topological mass has much more smaller value.

This paper is organized as follows. In Sec. 2, we explain the Maxwell-Chern-Simons \(QED_3\). A perturbative analysis is presented in Sec. 3. The Schwinger-Dyson equation is derived in Sec. 4. An approximated analytical analysis is done in Sec. 5. Results obtained by a numerical method is shown in Sec. 6. Finally we give conclusions with a discussion in Sec. 7.
2 Maxwell-Chern-Simons QED$_3$

We consider the Maxwell-Chern-Simons QED$_3$ with the single two-component Dirac fermion. The Lagrangian density of the theory is given by

$$\mathcal{L} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{\mu}{2} \epsilon^{\mu \nu \rho} A_\mu \partial_\nu A_\rho - \frac{1}{2\alpha} (\partial_\mu A^\mu)^2 + \bar{\psi} (i \not\partial - e \not A) \psi ,$$

where $e$ is the gauge coupling constant and $\alpha$ is the gauge-fixing parameter. The second term in the right-hand side of Eq.(1) is the so-called Chern-Simons term. It is well-known that the term gives the gauge field the mass $\mu$ without breaking the gauge symmetry. In fact, a free propagator of the gauge field $iD_{\mu \nu} (p - k)$ derived from Eq.(1) is written as

$$iD_{\mu \nu} (p) = -i \frac{1}{p^2 - \mu^2} \left( g_{\mu \nu} - \frac{p_\mu p_\nu}{p^2} \right) + \mu \frac{1}{p^2 - \mu^2} \frac{1}{p^2} \epsilon_{\mu \nu \rho} p^\rho - i \alpha \frac{p_\mu p_\nu}{p^2} .$$

We find a massive pole at $p^2 = \mu^2$. $\mu$ is called the topological mass.

$\psi$ is the two-component fermion field which belongs to the irreducible spinor representation in (2+1)-dimensions. The Dirac matrices are defined by $\gamma^0 = \sigma_3, \gamma^1 = i\sigma_1, \gamma^2 = i\sigma_2$ with $\text{diag}(g^{\mu \nu}) = (1, -1, -1)$ where $\sigma_i$'s $(i=1, 2, 3)$ are the Pauli matrices. The $\gamma^\mu$'s satisfy relations as $\{ \gamma^\mu, \gamma^\nu \} = 2g^{\mu \nu}, \gamma^\mu \gamma^\nu = -i\epsilon^{\mu \nu \rho} \gamma_\rho + g^{\mu \nu}$ and $\text{tr}[\gamma^\mu \gamma^\nu] = 2g^{\mu \nu}$. In this representation, there does not exist a matrix which anti-commutes with all of $\gamma^\mu$'s so that we cannot define the chiral transformation. This is a specific aspect of the odd-dimensional space-time. In even-dimensions, the chiral symmetry requires that a fermion is massless. In odd-dimensions, the chiral symmetry itself does not exist. Instead, the mass term of the fermion is forbidden by parity symmetry.

Under the parity transformation $x = (t, x, y) \rightarrow x' = (t, -x, y), \psi(x) \rightarrow \gamma^1 \psi(x'), A^0(x) \rightarrow A^0(x'), A^1(x) \rightarrow -A^1(x'), A^2(x) \rightarrow A^2(x')$.

3 Perturbation

Before proceeding to a nonperturbative analysis, it would be useful to see the lowest order of perturbation. The fermion self-energy in the one-loop approximation, $\Sigma^{(1)}(p)$, is expressed as

$$\Sigma^{(1)}(p) = \int \frac{d^3 k}{(2\pi)^3} \left( -ie \gamma^\mu \right) i S_F(k) \left( -ie \gamma^\nu \right) i D_{\mu \nu} (p - k) ,$$

$\frac{2}{(2+1)}$-dimensions, the parity transformation is defined as $x = (t, x, y) \rightarrow x' = (t, -x, y), \psi(x) \rightarrow \gamma^1 \psi(x'), A^0(x) \rightarrow A^0(x'), A^1(x) \rightarrow -A^1(x'), A^2(x) \rightarrow A^2(x')$. 
where \( iS_F(p) \) is a free fermion propagator written as
\[
  iS_F(p) = \frac{i}{p^2} ,
\]
and \( iD_{\mu
u}(p-k) \) is a free propagator of the gauge field given in Eq. (4).

The allowed form of the fermion propagator in the relativistic theory is written as
\[
iS_F^{(1)}(p) = \frac{i}{A^{(1)}(p) - B^{(1)}(p)} = \frac{i}{\not{p} - i\Sigma^{(1)}(p)} ,
\]
where \( A^{(1)}(p) \) and \( B^{(1)}(p) \) are functions of \( \sqrt{p \cdot p} \), while \( \Sigma^{(1)}(p) \) depends on \( p_\mu \)'s. \( A^{(1)}(p)^{-1} \) is the wave function renormalization and \( B^{(1)}(p)/A^{(1)}(p) \) is a mass induced by dynamical effects at the momentum scale \( p \). The so-called dynamical mass \( m_{phys} \) is defined by \( m_{phys} = B^{(1)}(0)/A^{(1)}(0) \) as usual. It is useful to notice the relations as
\[
  \text{tr} \left[ \Sigma^{(1)}(p) \right] = -2iB^{(1)}(p), \quad \text{tr} \left[ \not{p}\Sigma^{(1)}(p) \right] = 2i\{A^{(1)}(p) - 1\}p^2 .
\]

We substitute Eqs. (4) and (2) into Eq. (3) and use Eq. (6). Then we change the metric to the Euclidean one by the Wick rotation as \( (k^0, k^i) \to (ik^0, k^i) \) and \( (p^0, p^i) \to (ip^0, p^i) \). Then \( k^2 \) and \( p^2 \) are replaced by \( -k^2 = -(k^0)^2 - (k^i)^2 \) and \( -p^2 = -(p^0)^2 - (p^i)^2 \). After that, we transform the integral variables \( k^{\mu} \)'s to the polar coordinates \( (k, \theta, \phi) \). The angular integrations on \( \theta \) and \( \phi \) can be done explicitly. Finally we obtain
\[
  B^{(1)}(p) = \frac{e^2}{8\pi^2 p} \int_0^\infty dk \frac{1}{k} \left[ -\frac{1}{\mu}(p^2 - k^2) \ln \frac{(p + k)^2}{(p - k)^2} \right.
  \left. + \frac{1}{\mu}(p^2 - k^2 + \mu^2) \ln \frac{(p + k)^2 + \mu^2}{(p - k)^2 + \mu^2} \right] ,
\]
\[
  A^{(1)}(p) = 1 + \frac{e^2}{8\pi^2 p^2} \int_0^\infty dk \frac{1}{k} \left[ -2\mu k (\alpha + 1) \right.
  \left. + \left\{ \frac{1}{2\mu^2}(p^2 - k^2)^2 + \frac{1}{2}\alpha(p^2 + k^2) \right\} \ln \frac{(p + k)^2}{(p - k)^2} \right.
  \left. + \left\{ \frac{1}{2\mu^2} - \frac{1}{2\mu^2}(p^2 - k^2)^2 \right\} \ln \frac{(p + k)^2 + \mu^2}{(p - k)^2 + \mu^2} \right] .
\]

The dynamical mass of fermion is defined in the infrared limit so that we are interested in the behaviour of \( A^{(1)}(p) \) and \( B^{(1)}(p) \) in this limit. In the region of \( p \ll 1 \), Eqs. (7) and (8) are written as
\[
  B^{(1)}(p) = \frac{e^2}{\pi^2} \int_0^\infty dk \left[ \frac{\mu}{k^2 + \mu^2} + O(p^2) \right] ,
\]
\[
  A^{(1)}(p) = 1 + \frac{e^2}{\pi^2} \int_0^\infty dk \left[ \frac{1}{3} \left\{ \frac{1}{k^2} - 2\alpha - \frac{\mu^2}{(k^2 + \mu^2)^2} \right\} + O(p^2) \right] .
\]
This infrared approximation makes the integration on k possible and we have

\[ B^{(1)}(0) = \frac{e^2 |\mu|}{2\pi \mu}, \quad A^{(1)}(0) = 1 - \frac{e^2 |\mu|}{6\pi \mu^2} + \frac{e^2 \alpha}{3\pi^2 \epsilon}, \]  

(11)

where \( \epsilon \) is the infrared cutoff in the integration on \( k \).

It should be noticed that \( B^{(1)}(0) \) depends on the sign of \( \mu \). This also may be a specific aspect in (2+1)-dimensions. The dependence of \( A^{(1)}(0) \) on \( \mu \) shows that only the Landau gauge is free from the infrared divergence. On the other hand, \( A^{(1)}(0) \) is singular at \( \mu = 0 \) so that the theory with the Chern-Simons term may not be smoothly connected to the theory without the Chern-Simons term in the perturbation. This situation found in the perturbation motivates us to study the \( \mu \to 0 \) limit of the dynamical fermion mass by a nonperturbative method. This issue is extensively studied in the successive sections.

4 Schwinger-Dyson equation

In this section, we proceed to a nonperturbative analysis, where we use the Schwinger-Dyson technique to evaluate the dynamical mass of the fermion. The Schwinger-Dyson equation for the fermion self-energy \( \Sigma(p) \) is written as

\[ \Sigma(p) = (-ie)^2 \int \frac{d^3k}{(2\pi)^3} \gamma^\mu iS'_F(k) \Gamma^\nu(k,p-k) iD'_\mu\nu(p-k) . \]  

(12)

\( \Gamma^\nu(k,p-k) \) is a full vertex function and \( D'_\mu\nu(p-k) \) is a full propagator of the gauge field. \( S'_F \) is the full propagator of the fermion field which is written as

\[ iS'_F(p) = \frac{i}{A(p) p - B(p)} = \frac{i}{p - i\Sigma(p)} , \]  

(13)

which includes the full correction beyond the perturbative fermion propagator given in Eq. (5).

To analyze Eq. (12) further, we need to introduce any suitable approximation. In this paper, we limit ourselves to use the lowest ladder approximation where the full propagator of the gauge field and the full vertex are replaced by the free propagator and the bare vertex respectively as

\[ iD'_\mu\nu(p-k) \approx iD_{\mu\nu}(p-k) , \quad \Gamma^\nu(k,p-k) \approx \gamma^\nu , \]  

(14)

where \( iD_{\mu\nu} \) has been given in Eq. (2). Thus the Schwinger-Dyson equation in the lowest ladder approximation becomes

\[ \Sigma(p) = (-ie)^2 \int \frac{d^3k}{(2\pi)^3} \gamma^\mu iS'_F(k)\gamma^\nu iD_{\mu\nu}(p-k) . \]  

(15)
We substitute Eqs. (2) and (13) into Eq. (15). Following the same steps as getting Eqs. (7) and (8), we finally obtain the coupled integral equations as

\[
B(p) = \frac{e^2}{8\pi^2 p} \int_0^\infty dk \frac{k}{A(k)^2 k^2 + B(k)^2} \left[ \left\{ \frac{\alpha B(k)}{\mu} - \frac{1}{\mu} (p^2 - k^2) A(k) \right\} \ln \frac{(p + k)^2}{(p - k)^2} \right.
+ \left\{ \frac{1}{\mu} (p^2 - k^2) A(k) + \mu A(k) + 2B(k) \right\} \ln \frac{(p + k)^2 + \mu^2}{(p - k)^2 + \mu^2} , \right.
\]

(16)

\[
A(p) = 1 + \frac{e^2}{8\pi^2 p^3} \int_0^\infty dk \frac{k}{A(k)^2 k^2 + B(k)^2} \left[ -2pk(\alpha + 1)A(k) + \right.
+ \left\{ \frac{1}{2\mu^2} (p^2 - k^2)^2 A(k) + \frac{1}{\mu} (p^2 - k^2) B(k) + \frac{1}{2} \alpha (p^2 + k^2) A(k) \right\} \ln \frac{(p + k)^2}{(p - k)^2} \right.
+ \left\{ \frac{1}{2} \mu^2 A(k) - \frac{1}{2\mu^2} (p^2 - k^2)^2 A(k) + \mu B(k) - \frac{1}{\mu} (p^2 - k^2) B(k) \right\}
\times \ln \frac{(p + k)^2 + \mu^2}{(p - k)^2 + \mu^2} , \right.
\]

(17)

which contain only the integration on the radial variable \( k \). We solve these equations by an approximated analytical method and also numerically by using an iteration method in the following sections.

5 Approximated analytical method

5.1 \( \mu \to 0 \) limit

We can check easily that Eqs. (16) and (17) reduce to the Schwinger-Dyson equations in \( QED_3 \) without the Chern-Simons term if we put the topological mass \( \mu \) equal to zero. In fact, taking the limit as \( \mu \to 0 \) in Eqs. (16) and (17), we obtain

\[
B(p) = (\alpha + 2) \frac{e^2}{8\pi^2 p} \int_0^\infty dk \frac{k B(k)}{A(k)^2 k^2 + B(k)^2} \ln \frac{(p + k)^2}{(p - k)^2} ,
\]

(18)

\[
A(p) = 1 - \alpha \frac{e^2}{4\pi^2 p^3} \int_0^\infty dk \frac{k A(k)}{A(k)^2 k^2 + B(k)^2} \left[ pk - \frac{p^2 + k^2}{4} \ln \frac{(p + k)^2}{(p - k)^2} \right] ,
\]

(19)

which are the Schwinger-Dyson equations in the lowest ladder approximation derived in \( QED_3 \) without Chern-Simons term. We can see that there exists the specific gauge where the wave function renormalization is absent. Thus in the Landau gauge (\( \alpha = 0 \)), Eq. (19) gives us the simple solution as \( A(p) = 1 \) and the problem reduces to solve Eq. (18) with \( A(p) = 1 \).

In the case with the Chern-Simons term, as we seen in Eqs. (16) and (17), there does not exist such a specific gauge where the wave function is not renormalized. So far we cannot find a self-evident reason that the Landau is still specific in \( QED_3 \) with
Chern-Simons term, it must be fair to study Eqs. (16) and (17) for various values of the gauge-fixing parameter $\alpha$.

### 5.2 Constant approximation

Before proceeding to a numerical analysis, it is very useful if we can estimate $A(0)$ and $B(0)$ analytically even under a fairly crude approximation. The kernels of these integral equations are dumped rapidly as the integral variable $k$ increases so that the contribution from $k \approx 0$ is the most dominant one in the integrals. We approximate $A(k)$ and $B(k)$ by $A(0)$ and $B(0)$ in the integrals. We call this approximation "the constant approximation". Of course this approximation might be too crude for our purpose and we only use the result as reference in the numerical analysis. Under this approximation, we can perform the remaining radial integration and obtain

$$B(0) = \frac{e^2}{2\pi} + \frac{e^2}{12\pi} \alpha, \quad A(0) = 1 + \frac{2\alpha}{\alpha + 6},$$

where we have considered the case of $\mu > 0$.

From Eq. (20), we can see that the dependence of $B(0)$ and $A(0)$ on the gauge-fixing parameter, the coupling constant and the topological mass has the following peculiar features:

1) Dependence on the gauge-fixing parameter

$B(0)$ depends linearly on $\alpha$. It is suggestive that $A(0)$ is singular at $\alpha = -6$ where $B(0)$ vanishes. In the Landau gauge ($\alpha = 0$), $A(0) = 1$ and $B(0) = e^2/2\pi$. $A(0) = 1$ is favourable for us because $A(p) = 1$ means that the Ward-Takahashi identity is satisfied.

2) Dependence on the coupling constant

$A(0)$ does not depend on $e^2$. It means that the deviation of $A(0)$ from 1 is independent of the coupling constant. This is crucially different from the perturbative result given by Eq. (11) where the deviation is proportional to $e^2$. On the other hand, $B(0)$ is proportional to $e^2$.

3) Dependence on the topological mass

We recognize that there is no dependence on the topological mass $\mu$ in Eq. (20). In fact, if we apply the constant approximation to the case without the Chern-Simons term, we obtain the same results as Eq. (20). It means that the amount of the explicit parity breaking in the gauge sector by the topological mass does not affect the dynamical mass in the fermion sector in the constant approximation.

Now we proceed to a more precise numerical evaluation in the next section.
6 Numerical method

6.1 Nontrivial solutions and gauge dependence

We solve the two coupled integral equations (16) and (17) numerically by using a method of iteration. First we substitute trial functions into $A(k)$ and $B(k)$ in the right-hand sides of Eqs. (16) and (17) and then calculate the integrals numerically. The outputs so obtained, $A(p)$ and $B(p)$, are substituted back to the right-hand sides until the outputs coincide with the inputs. Finally we obtain convergent functions $A(p)$ and $B(p)$, which satisfy the integral equations, if there exist any solutions of Eqs. (16) and (17).

We have obtained the nontrivial solutions for the various values of the gauge parameter $\alpha$. It has been found that $A(p)$ is fairly close to 1 in the Landau gauge ($\alpha = 0$). In the case of QED$_3$ without the Chern-Simons term, $A(p)$ is exactly equal to 1 in the Landau gauge under the lowest ladder approximation. However, in the case of QED$_3$ with the Chern-Simons term, there may be no apparent reason that $A(p) = 1$ in the Landau gauge. It is surprising that the numerical calculation of so complicated integral equations results $A(p) \approx 1$ in the Landau gauge. There might be a simple reason for explaining a peculiarity of the Landau gauge.

As a way of indicating to what extent gauge symmetry is broken by the bare vertex approximation, it is helpful to study the gauge invariant condensate $\langle \bar{\psi} \psi \rangle$ as a function of $\alpha$. The condensate is defined by $\langle \bar{\psi} \psi \rangle = i \lim_{x \to 0} \text{tr} S'_F(x)$ where $S'_F(x)$ is a propagator in real space-time coordinates. We have found that the $\alpha$ dependence may be considered to be fairly weak. Hereafter we present the results obtained in the Landau gauge.

6.2 Dependence on the topological mass

What we are most interested in is the dependence of the dynamical fermion mass on the topological mass of the gauge field. In the constant approximation, it has been shown that both $A(0)$ and $B(0)$ do not depend on the topological mass. Is this true in the more precise numerical evaluation?

We have studied the dependence of $A(0)$ on the dimensionless parameter $\hat{\mu}$ which is defined by $\hat{\mu} = \mu/e^2$. We have found that the deviation of $A(0)$ from 1 is less than 1 %. We may say that $A(0)$ is almost equal to 1 in all region of $\hat{\mu}$. It means that the $\hat{\mu}$-dependence of $A(0)$ is extremely weak.

The $\hat{\mu}$-dependence of $B(0)$ is nontrivial. One of our motivations is to see how the Maxwell-Chern-Simons QED$_3$ is smoothly connected to QED$_3$ without the Chern-3

\[ < \bar{\psi} \psi > = \frac{1}{\pi^2} \int_0^\infty dk \frac{k^2 B(k)}{A(k)^2 k^2 + B(k)^2}. \]
Simons term in the $\hat{\mu} \rightarrow 0$ limit. Our numerical calculation shows that very small meshes are needed to obtain reliable values of $B(0)$ in the region $\hat{\mu} \ll 1$. Because of the limitation of our machine ability, we take another strategy in which $B(0)$ evaluated on meshes of zero width is estimated by an extrapolation from some $B(0)$'s on meshes of different finite widths. An example of the extrapolation is shown in Fig.1 where $B(0)$ on meshes of zero width is estimated by the curve fitting of three $B(0)$'s on meshes of finite widths.

![Figure 1: The zero mesh-width limit of $B(0)$ by extrapolation.](image)

In Fig.2, we show the $\hat{\mu}$-dependence of $B(0)$ in the region $10^{-5} \leq \hat{\mu} \leq 10^{-2}$. $B(0)$'s on meshes of finite width show an abnormal behaviour in the region $\hat{\mu} \ll 1$ that $B(0)$'s depart from $B(0)$ of $QED_3$ without the Chern-Simons term as $\hat{\mu}$ decreases. This behaviour is improved by the extrapolation. $B(0)$'s obtained by the extrapolation
smoothly tend to the value of $B(0)$ in $QED_3$ without the Chern-Simons term.

The whole shape of $B(0)$ in the region $10^{-5} \leq \hat{\mu} \leq 10^4$ is given in Fig.3. $B(0)$ is almost constant in the region of $\hat{\mu} \gg 1$ and decreases rapidly in the region $\hat{\mu} = 1.0 \sim 0.01$. In the region of $\hat{\mu} \ll 1$, $B(0)$ becomes almost constant again. The upper dotted line in Fig.3 is the result obtained in the constant approximation (Sec.5.2) and also in the lowest order perturbation (Sec.3). The lower dotted line shows the value obtained by a nonperturbative calculation in the case without Chern-Simons term. [10]

Figure 3: $\hat{\mu}$-dependence of $B(0)$ in the region of $10^{-5} \leq \hat{\mu} \leq 10^4$.

In other words, $B(0)$ reproduces the result of perturbation in the region of $e^2 \ll \mu$, and in the region of $e^2 \gg \mu$, $B(0)$ is close to the nonperturbative value obtained in the case without Chern-Simons term.

7 Conclusion and discussion

We have studied the dependence of the dynamical fermion mass on the topological mass in the Maxwell-Chern-Simons $QED_3$ nonperturbatively by using the Schwinger-Dyson method. When the topological mass is larger than the square of the coupling constant, the value of the topological mass remains to be the one obtained by the perturbation. As the topological mass decreases, the value is changed to a nonperturbative value rapidly. The transition from the perturbative value to the nonperturbative one is sharp but not critical. Though it seems not to be a phase transition, the inclusion of the Chern-Simons term changes the nature of the theory drastically.

The most plausible application of our results can be found in condensed matter physics. It will be considered extensively in forthcoming papers. What we would like to stress in this opportunity is an application to a model in the early Universe. In a study of space-time evolution of the Universe, the finite temperature effect may become important. In an imaginary time formalism, an inverse of the time dimension
with a periodic (anti-periodic) boundary condition for a boson (fermion) is interpreted as a temperature. Then in a high-temperature limit, the time dimension vanishes and a (3+1)-dimensional space-time collapses into 3-dimensional space in which the Euclidean version of (2+1)-dimensional models may become effective. It also would be interesting to consider a mechanism which can produce the Chern-Simons term in the high temperature limit.

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References

[1] Private communications of T. M. with Professor M. Kenmoku and Professor Kamal K. Nandi.

[2] K. von Klitzing, G. Dorda, and M. Pepper. Phys. Rev. Lett. 45, 494(1980); D. C. Tsui, H. L. Störmer, and A. C. Gossard, Phys. Rev. Lett. 48, 1559(1982).

[3] G. Bednorz and K. A. Miiller, Z. Phys. 64, 188(1986).

[4] S.S. Chern and J. Simons, Ann. Math. 99, 48(1974).

[5] W. Siegel, Nucl. Phys. B156, 135(1979); J. Schonfeld, ibid. B185, 157(1981).

[6] S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. (N.Y.) 140, 372(1982).

[7] F. Wilczek, Fractional Statistics and Anyon Superconductivity, (World Scientific, 1990).

[8] R. Jackiw and S. Templeton, Phys. Rev. D 23, 2291(1981); T. Appelquist and R. D. Pisarski, ibid. 23, 2305(1981); S. Templeton, ibid. 24, 3134(1981); M. de Roo and K. Stam, Nucl. Phys. B246, 335(1984).

[9] R. D. Pisarski, Phys. Rev. D 29, 2423(1984); T. Appelquist, M. J. Bowick, E. Cohler, and L. C. R. Wijewardhana, Phys. Rev. Lett. 55, 1715(1985); T. Appelquist, M. J. Bowick, D. Karabali and L. C. R. Wijewardhana, Phys. Rev. D 33, 3704(1986).

[10] Y. Hoshino and T. Matsuyama, Phys. Lett. B222, 493(1989); see also Y. Hoshino, T. Matsuyama and C. Yoshida-Habe, in Proceedings of the 1989 Workshop on Dynamical Symmetry Breaking, Nagoya, Japan.

[11] D.K. Hong and S.H. Park, Phys. Rev. D47, 3651(1993); W.-H. Kye and J. K. Kim, ibid. D50, 5398(1994); K.-I. Kondo and P. Maris, Phys. Rev. Lett. 74, 18(1995); ibid. Phys. Rev. D52, 1212(1995).

[12] T. Matsuyama, H. Nagahiro and S. Uchida, Phys. Rev. D60, 105020(1999).