Can we trust semiclassical description of particle creation?

Hrvoje Nikolić

Theoretical Physics Division, Rudjer Bošković Institute, P.O.B. 180, HR-10002 Zagreb, Croatia

(Dated: February 1, 2008)

The predictions of the semiclassical description of particle creation based on QFT in classical backgrounds may be significantly modified when the source of the classical background is also quantized and backreaction is taken into account. In the cases of a stable charged particle, expanding empty (Milne) universe, and de Sitter universe with a true cosmological constant, the semiclassical particle creation is completely blocked up.

PACS numbers: 03.65.Sq, 04.62.+v

Quantum field theory (QFT) is an appropriate theoretical framework for describing particle creation and destruction. In such processes energy must be conserved, implying that the creation of a new particle is always accompanied by the destruction of an old particle or a transition of the old particle to a lower state. Such processes are most successfully described in perturbative QFT in terms of Feynman diagrams. Nevertheless, in some cases the perturbative methods are not very efficient, forcing us to use different types of approximations.

One such approximation widely used for description of particle creation is the semiclassical approximation. In such an approximation, only the new created particles are described by QFT, while the role of the “old” particles is approximately described by a classical source field. The best known examples of such a semiclassical description of particle creation are the Schwinger effect [1, 2, 3] in which a classical static electric field causes production of electron-positron pairs, the Hawking effect [3, 4, 5] in which the classical gravitational field of a black hole causes production of particles with a thermal distribution of energies, and particle creation caused by a time-dependent classical gravitational field induced by the universe expansion [3, 6]. The best known specific example of the latter is particle creation from a horizon of de Sitter universe [7], which contains only a positive cosmological constant, but not ordinary matter. The Schwinger and Hawking effect are actually closely related [3]. An even closer relation exists between Hawking effect and particle creation by de Sitter universe, as in both cases it is the horizon that is responsible for particle creation with a thermal distribution [3]. Another related effect is the Unruh effect [3, 8], according to which a particle detector accelerated in vacuum detects particles.

All these effects are usually treated in a fixed classical background, thus violating conservation of energy. To fix this problem, one has to study the backreaction of created particles on the source. If the source (i.e., the classical background) is still treated classically, one usually finds that the backreaction does not significantly influence the original particle creation with a fixed background. Nevertheless, the real source is always a quantum object, so, in general, we cannot be sure that the semiclassical approximation is satisfying.

As a simple example, consider the Schwinger effect in which the source of the electric field is a stable particle, say a proton. According to the semiclassical analysis, the static electric field should create electron-positron pairs. (In the case of a proton the electric field is too weak to provide a significant rate of production, but for the sake of argument one may consider a hypothetic particle having the same mass as proton, but a much larger charge producing a much stronger electric field.) Energy conservation implies that, after the pair creation, the energy of the proton and its electric field must be smaller than that in the initial state, which is impossible since the proton is a stable particle. Consequently, in such a case there can be no pair creation, so the semiclassical analysis fails completely.

The purpose of this paper is to study in more detail how, in general, the quantum treatment of the source modifies the semiclassical treatment of particle creation. Our analysis represents a further development and generalization of a recent analysis [10] introduced to study Hawking radiation.

The most convenient method for a semiclassical description of particle creation by a classical source is the Bogoliubov transformation. In general, one finds that the initial vacuum $|0\rangle$ transforms to

$$|0\rangle \rightarrow \sum_e \sum_i c_{e,i} |e, i\rangle,$$

(1)

where $|e, i\rangle$ are energy eigenstates with the energies $e \geq 0$, the label $i$ labels different states having the same energy, and $c_{e,i}$ are the semiclassical probability amplitudes for particle creation, satisfying the normalization condition

$$\sum_e \sum_i |c_{e,i}|^2 = 1.$$  

(2)

The state $|0, i\rangle \equiv |0\rangle$ is the unique vacuum, while states $|e, i\rangle$ with $e > 0$ contain one or more particles.

Clearly, energy is not conserved in (1). To fix that problem, we introduce the energy eigenstates $|E, I\rangle$ of the source. Now in this fully quantized description of particle creation, the initial state is not the vacuum, but
a linear combination of these source energy eigenstates. For simplicity, we assume that the initial state is some energy eigenstate $|E_{0}, I_{0}>$. Then the particle creation can be described as a transition

$$|E_{0}, I_{0} \otimes |0> \rightarrow \sum_{E} \sum_{I} \sum_{e} \sum_{i} D^{(E_{0}, I_{0})}_{E, I; e, i}|E, I > \otimes |e, i>, \quad (3)$$

where $I = 1, \ldots, N_{E}$, and $N_{E}$ is the number of states with energy $E$. The transition of a more general initial state can be easily obtained from (3) by linearity of quantum mechanics. Energy conservation implies that the amplitude $D^{(E_{0}, I_{0})}_{E, I; e, i}$ must have the form

$$D^{(E_{0}, I_{0})}_{E, I; e, i} = \delta_{E_{0}, E} e^{i d_{e, I, i}}, \quad (4)$$

where, for the sake on notational simplicity, the dependence of $d_{e, I, i}$ on $E_{0}, I_{0}$ is suppressed. Therefore, (3) becomes

$$|E_{0}, I_{0} \otimes |0> \rightarrow \sum_{I} \sum_{e} \sum_{i} d_{e, I, i}|E_{0} - e, I > \otimes |e, i>, \quad (5)$$

Strictly speaking, we cannot conclude anything more about the amplitude $d_{e, I, i}$ without knowing the details of the quantum theory of the source. Nevertheless, the knowledge of the semiclassical approximation suggests that (up to a phase) $d_{e, I, i}$ should be approximately proportional to the semiclassical amplitude $c_{e, I}$. Namely, it is reasonable to use the approximation

$$d_{e, I, i} = \frac{e^{i \varphi_{e, I}}}{\sqrt{N}} c_{e, i}, \quad (6)$$

where $\varphi_{e, I}$ are some phases. The factor $1/\sqrt{N}$ is determined by the normalization condition $\sum_{e} \sum_{I} \sum_{i} |d_{e, I, i}|^2 = 1$. This leads to

$$N = \sum_{e} \sum_{I} \sum_{i} |c_{e, i}|^2 = \sum_{e} N_{E_{0} - e} \sum_{i} |c_{e, i}|^2 = \sum_{e} N_{E_{0} - e} P_{e} = \langle N \rangle, \quad (7)$$

where

$$P_{e} = \sum_{i} |c_{e, i}|^2 \quad (8)$$

is the semiclassical probability that the energy of created particles is equal to $e$. The quantity $\langle N \rangle$ is the average number of source states having the same energy, with the average being defined with respect to the semiclassical probability $P_{e}$. Defining

$$|E_{0} - e> \equiv \sum_{I=1}^{N_{E_{0} - e}} \frac{e^{i \varphi_{e, I}}}{\sqrt{N_{E_{0} - e}}} |E_{0} - e, I >, \quad (9)$$

(5) can be written as

$$|E_{0}, I_{0} \otimes |0> \rightarrow \sum_{e} |E_{0} - e > \otimes \sum_{i} \sqrt{\frac{N_{E_{0} - e}}{N}} c_{e, i}|e, i>. \quad (10)$$

Now, to see the relation with the semiclassical result (11), consider the case in which $N_{E}$ does not depend on $E$, i.e., $N_{E} = \langle N \rangle$. Now (10) reduces to

$$|E_{0}, I_{0} \otimes |0> \rightarrow \sum_{e} |E_{0} - e > \otimes \sum_{i} c_{e, i}|e, i>. \quad (11)$$

Thus, the probability that the created particles will be found in the state $|e, i>$ is equal to $|c_{e, i}|^2$, which is the same result as the one obtained from the semiclassical description (1). Furthermore, if energy $e$ is measured, then (11) collapses to

$$|E_{0} - e > \otimes \frac{1}{\sqrt{P_{e}}} \sum_{i} c_{e, i}|e, i>, \quad (12)$$

where $1/\sqrt{P_{e}}$ is the appropriate normalization factor. Similarly, the semiclassical theory based on (11) implies the collapse to

$$\frac{1}{\sqrt{P_{e}}} \sum_{i} c_{e, i}|e, i>, \quad (13)$$

which is nothing but the particle-creation part of the fully-quantized wave function (12). This shows that the case $N_{E} = N$ restores the semiclassical result. From this we conclude that the semiclassical approximation can be trusted when the number of the relevant source states with energy $E$ does not significantly depend on $E$. This condition is expected to be satisfied when the source is in a highly excited state, with the energy $E_{0}$ much larger than typical energies $e$ of created particles. Indeed, such a highly excited state justifies the treatment of the source as a classical object.

As an extreme deviation from the condition above, consider the case $N_{E_{0} - e} = 0$ for $e > 0$. Then (10) implies that the transition amplitude vanishes. In this case the source with the initial energy $E_{0}$ cannot jump to a lower state simply because such a state does not exist. Consequently, a particle with energy $e$ cannot be created, despite the fact that the semiclassical amplitude $c_{e, i}$ does not vanish. Indeed, this is exactly why the electric field of a stable charged particle discussed in the introduction cannot create electron-positron pairs, despite the fact that it should create them according to the semiclassical Schwinger effect.

Now we see that the semiclassical approximation (11) cannot always be trusted. However, the essential effects of quantization of the source can be caught without knowing all the details of the fully quantized theory. Namely, (10) shows that (11) can be improved by replacing the semiclassical transition probability $|c_{e, i}|^2$
by the improved semiclassical transition probability $(N_{E_0\gamma}/N)^2$. Hence, we can replace by the improved semiclassical approximation

$$0 \rightarrow \sum_c \sum_i \sqrt{\frac{N_{E_0\gamma}}{N}} c_{e,i} |e,i\rangle. \quad (14)$$

We see that it is a very useful approximation, because it requires a minimal knowledge on the quantum structure of the source. All we have to know is the number $N_{E_0}$ of states with energy $E$. We have written all the expressions for the discrete spectrum of energies, but it is obvious how to modify them for the (perhaps more interesting) case of continuous energy spectrum.

Now let us discuss various examples. We start with the Schwinger effect. This effect has been confirmed experimentally by experiments involving electric fields produced by heavy ions [11]. A nucleus of a heavy ion, of course, can decay into a large number of lower energy states, which explains why the semiclassical approximation is justified in this case. If, however, the source of the same electric field was a stable elementary particle (the Standard Model of elementary particles does not contain such a particle, but this is not relevant for our theoretical argument), our analysis shows that in this case the Schwinger effect would be completely blocked up.

Perhaps the most interesting example is Hawking radiation from a black hole (BH). This case is studied in more detail in [10]. It is shown how the fact that the number of quantum BH states decreases as the BH mass $M$ decreases (due to the evaporation) resolves the famous BH information puzzle, making complete evaporation consistent with unitarity. We also note that, although the semiclassical Hawking temperature $T = 1/8\pi M$ is infinite for $M = 0$, Hawking radiation stops when the BH mass drops to $M = 0$, simply because lower-energy BH states do not exist.

Next we consider the Unruh effect. The energy source is provided by the external force that accelerates the detector [12], which effectively means that the source can be viewed as always being in the same state. Consequently, our analysis does not modify the semiclassical result. Nevertheless, we stress that physical details specifying a realistic quantum detector may significantly modify the semiclassical result [13].

Now let us consider particle creation by the universe expansion. At the semiclassical level, the metric must satisfy the semiclassical Einstein equation

$$G_{\mu\nu} = 8\pi [T_{\mu\nu}^{\text{bulk}} + \langle \psi | \hat{T}_{\mu\nu}^{\text{creat}} | \psi \rangle], \quad (15)$$

where $G_{\mu\nu}$ is the classical Einstein tensor, $T_{\mu\nu}^{\text{bulk}}$ is the energy-momentum of the bulk matter described classically, $\langle \psi | \hat{T}_{\mu\nu}^{\text{creat}} | \psi \rangle$ is the average energy-momentum of created particles described quantum mechanically, and the Newton gravitational constant is set to 1. If particle creation takes place, then the last term is not conserved, i.e.,

$$\nabla^\mu \langle \psi | \hat{T}_{\mu\nu}^{\text{creat}} | \psi \rangle \neq 0. \quad (16)$$

Then the Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$ and the Einstein equation (15) imply

$$\nabla^\mu T_{\mu\nu}^{\text{bulk}} \neq 0. \quad (17)$$

In other words, the only possible source of energy for particle creation is the bulk matter. Indeed, this is analogous to the energy source responsible for Hawking radiation. However, if the initial universe does not contain bulk matter, i.e., if its initial state is the vacuum state, then a bulk-matter state with an even lower energy does not exist. Consequently, particles cannot be created in an empty universe without initial matter. On the other hand, the semiclassical analysis leads to the result that a time-dependent empty (Milne) universe does lead to particle creation [13]. Therefore, the Milne universe is another example in which the semiclassical particle creation is completely blocked up in a fully quantum treatment.

Finally, let us consider particle creation by de Sitter universe. At the semiclassical level this effect is very similar to Hawking radiation because in both cases it is the existence of the horizon that is responsible for it. Nevertheless, the energy source is very different from that in the case of Hawking radiation. In the de Sitter case, the source is described by a bulk energy-momentum that has a cosmological-constant form

$$T_{\mu\nu}^{\text{bulk}} = \lambda g_{\mu\nu}. \quad (18)$$

Consequently, (17) implies

$$\partial_{\lambda} \lambda \neq 0. \quad (19)$$

In other words, particle creation by de Sitter universe is only possible if the cosmological “constant” is not a true constant, but can decay. Thus, particle creation by de Sitter universe with a true cosmological constant is our final example of semiclassical particle creation completely blocked up in a fully quantum treatment. Nevertheless, it does not mean that the universe with a realistic cosmological “constant” does not produce particles, because there are many dynamical models of dark energy in which $\lambda$ is a dynamical quantity. In fact, a recent result based on linearized quantum gravity [14] indicates that a non-dynamical bare cosmological constant does not contribute to the observed cosmological “constant” at all.

To conclude, we have seen that the semiclassical description of particle creation can be trusted only if the number of source states with given energy does not significantly depend on energy. Otherwise, the semiclassical approximation should be replaced by an improved semiclassical approximation [14]. When the source does not contain a state with an energy lower than the initial
one, then the semiclassical particle creation is completely blocked up. The examples are a stable charged particle that blocks up the Schwinger effect, the expanding empty (Milne) universe that blocks up particle creation caused by the time dependence of the metric, and de Sitter universe with a true cosmological constant that blocks up particle creation caused by the existence of a horizon.

This work was supported by the Ministry of Science of the Republic of Croatia under Contract No. 098-0982930-2864.

* Electronic address: hrvoje@thphys.irb.hr

[1] J. Schwinger, Phys. Rev. 82, 664 (1951).
[2] C. A. Manogue, Ann. Phys. 181, 261 (1988).
[3] R. Brout, S. Massar, R. Parentani, and Ph. Spindel, Phys. Rep. 260, 329 (1995).
[4] S. W. Hawking, Commun. Math. Phys. 43, 199 (1975).
[5] N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space (Cambridge Press, NY, 1982).
[6] L. Parker, Phys. Rev. 183, 1057 (1969).
[7] T. Padmanabhan, Phys. Rep. 380, 235 (2003).
[8] T. Padmanabhan, Phys. Rep. 406, 49 (2005).
[9] W. G. Unruh, Phys. Rev. D 14, 870 (1976).
[10] H. Nikolić, [arXiv:0708.0729].
[11] L. E. Wright, K. K. Sud, and D. W. Kosik, Phys. Rev. C 36, 562 (1987).
[12] R. Mochizuki and T. Suga, gr-qc/9905024.
[13] K.-P. Marzlin and J. Audretsch, Phys. Rev. D 57, 1045 (1998).
[14] H. Nikolić, [arXiv:0708.3563].