Calculation elements for establishing the design of a rotating machine that transports fluids

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Abstract. The paper presents the operating principle and the constructive solution of a rotating machine with two profiled rotors. The calculation relations between the rotor radius and the height of the rotating piston are established, fundamental relations for the design of the rotating machine. The calculation results on the basis of a computation program are presented, results which specify by the coordinates \((x_i, y_i)\) the contour of the rotor profile. At the end of the paper, the formula for calculating the flow rate transported by the machine and its driving power is analysed.

1. Introduction
The machines are aggregates used to transform the energies from one form to another with the help of a movable member (piston, profiled rotor or blade) [1-2]. The machines, according to their purpose, are divided into two categories [1-4]:

1. Power machines (motor machines), which convert a certain form of energy into mechanical energy (internal combustion engines, steam or gas turbines, etc.).
2. Working machines, which convert mechanical energy into potential pressure energy (fans, pumps, compressors).

Power and working machines that are traversed by fluids; according to the variation of the flow parameters are classified as follows (table 1):

- a. Hydraulic machines, which drive or are driven by liquids, neglecting thermal phenomena;
- b. Thermal machines, which carry gases or vapours (or are driven by it), which does not neglect the thermal phenomena that occur.

| Working machines | Machines with profiled rotors | a) Fans, blowers, pumps |
|------------------|-------------------------------|-------------------------|
| Pallet machines  |                               | b) Fans, blowers        |

| Power machines   | Machines with profiled rotors | c) Internal combustion engines, steam or gas engines, pneumatic engines |
|------------------|-------------------------------|------------------------------------------------------------------------|
| Pallet machines  |                               | d) Steam turbines, gas turbines                                       |

Table 2 presents the classification of rotating machines with profiled rotors according to their objective and the adopted constructive solution.
Table 2. Classification of rotating machines with profiled rotors.

| Working machines                  | Pumps for driving fluids or with suspensions |
|----------------------------------|---------------------------------------------|
|                                  | Fans for transporting gases or vapors       |
|                                  | Blowers for gas and vapor compression       |
| Power machines                   | Hydraulic motor                             |
|                                  | Pneumatic motor                             |
|                                  | Steam engine or combustion gases            |

The term rotating machine in the title of the paper refers to the fact that this machine can be used as:
- force machine if the suction pressure \((p_1)\) is higher than the discharge pressure \((p_2)\);
- working machine when \(p_1 < p_2\) and it can be used as a pump, fan, compressor.

2. The operating principle and the constructive solution of the machine with profiled rotors

The machine consists of two profiled rotors that are identical (3, 4); they rotate in two cylindrical carcases (2, 5), figure 1. The synchronous rotation of the rotors is ensured by two gear wheels mounted on the shafts (7, 8), wheels forming a cylindrical gear located outside the machine.

The rotors have rotational motion in opposite direction so that the rotor pistons enter the cavities of the adjacent rotor. In figure 1.a the rotating piston (6), after a rotation with 180° reaches the position in figure 1.c.

In this rotation movement, the fluid volume between the pistons, carcase (2) and the lateral surface of the rotor (3), denoted by \(V_u\) (figure 1), after a 180° rotation of the rotor (3) will be evacuated to the discharge chamber, then outside the machine. In the following it is considered that the machine functions as a rotating volumetric pump, the fluid pressure increasing from \(p_1\) to \(p_2\).

Figure 1. Position of the rotors after a 180° rotation.

1 - fluid suction chamber; 2 - lower carcase; 3 - lower rotor; 4 - upper rotor; 5 - upper carcase; 6 - rotating piston of the upper rotor; 7 - driven shaft; 8 - driving shaft; 9 - fluid discharge chamber; 10 - cavity in which the piston of the upper rotor enters.

It is specified that the shape of the rotors results from relatively complicated calculations and their construction is carried out with the help of a C.N.C. machine [1-3].

Figure 2 shows a cross section through the rotating volumetric pump. One can observed that between the upper rotor (2) and the lower rotor (1) there is a single point of contact marked with M; if the piston (3) is built with a larger base, it will lock in the rotor cavity (1).
Figure 2. Rotating volumetric pump with profiled rotors
1 - lower rotor; 2 - upper rotor; 3 - rotating pistons; 4 – driving shaft;
5 - rectangular wedge; 6 - oval carcase; 7 – driven shaft.

Figure 3 shows an axonometric sketch of the rotating pump model made of transparent plexiglass; if
the rotors rotate in the direction indicated by the arrows drawn on the rotors, then the fluid is driven
from suction to discharge.

Figure 3. Axonometric view of a volumetric pump model with two profiled rotors.
1 - suction chamber; 2 - discharge chamber; 3 - lower rotor; 4 - upper rotor;
5 - cylindrical gears.

This type of pump has the advantage that it can transport any viscous fluid or suspensions that enter
the suction chamber (1) to the discharge chamber (2).

3. Establishing the connection relations between the rotor radius and the height of the rotating
piston of the machine
To establish this connection mathematically, is considered the two rotors of the machine, tangent to
point K.
Consider a single piston (5) fixed to the lower rotor (figure 4).
Figure 4. Calculation notations for determining the contour of the profiled rotor profile
1 - lower rotor; 2 - upper rotor; 3 - driving shaft; 4 - driven shaft;
5 - rotating piston of triangular shape.

The rotor radius (1) is extended by a length \( z \) and thus the line \( O_1B \) reaches the rotor (2) at point \( A \). Theoretically, when point \( K \) reaches point \( D \), point \( A \) reaches \( K \), respectively point \( N \) reaches \( K \), because the length of the circle arcs \( AK, KD \) and \( KN \) is the same. When the piston (5) exits the gap created in the rotor (2), points \( A \) and \( N \) reach point \( K \); the sealing between the two rotors being ensured by the direct contact between the lateral surfaces of the rotors.

From the rectangular triangle \( O_1O_2A \) results:

\[
O_1O_2^2 = AO_2^2 + AO_1^2 \quad (1)
\]

\[
(2R_r)^2 = R_i^2 + (R_r + z)^2 \quad (2)
\]

relations that becomes:

\[
z^2 + 2R_rz - 2R_i^2 = 0 \quad (3)
\]

\[
z_{1,2} = \frac{-2R_r \pm \sqrt{4R_r^2 + 8R_i^2}}{2} \quad (4)
\]

The negative solution is not suitable and one can obtain:

\[
z = -R_r + R_i \sqrt{3} = 0.732R_i \quad (5)
\]

The relation (5) specifies the relation between \( z \) and \( R_r \) in the sense that \( z \) cannot be greater than 0.732 \( R_r \).

Regarding the carcase radius, the following are observed:

\[
R_c = R_r + z = R_r + 0.732R_i = 1.732R_i \quad (6)
\]

where the casing radius \( (R_c) \) is the sum between the rotor radius \( (R_r) \) and the piston height \( (z) \).

From relation (6) one can observe that there is a delimitation of the carcase radius according to the rotor radius.

A connection between the height of the piston \( (z) \) and the carcase radius can also be established.

\[
R_c = R_r + z = \frac{z}{0.732} + z = 2.366z \quad (7)
\]
Thus, by the relations (5), (6) and (7) a calculation relation between \( R_r, z \), \( R_c \) regarding the architecture of the rotating machine is established.

Performing the calculations, the coordinates \( x_i, y_i \) for the first quarter of the piston from table 3 are obtained.

**Table 3.** The first quarter of the piston (triangular version) specified by the coordinates \( x_i, y_i \)

| \( x_i \) | \( y_i \) | \( x_i \) | \( y_i \) | \( x_i \) | \( y_i \) |
|---|---|---|---|---|---|
| **Punctul A** | | | | | |
| 0 | -0.02 | 0.02191 | -0.04493 | 0.04414 | -0.02347 |
| 0.00104 | -0.02030 | 0.02289 | -0.04455 | 0.04455 | -0.0269 |
| 0.00209 | -0.02013 | 0.02347 | -0.04414 | 0.04493 | -0.02191 |
| 0.00312 | -0.02030 | 0.02424 | -0.04373 | 0.04531 | -0.02113 |
| 0.00415 | -0.02053 | 0.025 | -0.04330 | 0.04567 | -0.02033 |
| 0.00517 | -0.02083 | 0.0257 | -0.04285 | 0.04602 | -0.01953 |
| 0.00618 | -0.02120 | 0.02649 | -0.04240 | 0.04635 | -0.01873 |
| 0.00716 | -0.02163 | 0.02723 | -0.04193 | 0.04667 | -0.01791 |
| 0.00813 | -0.02212 | 0.02795 | -0.04145 | 0.04698 | -0.01710 |
| 0.00907 | -0.02268 | 0.02867 | -0.04095 | 0.04727 | -0.01627 |
| 0.00999 | -0.02330 | 0.02938 | -0.04045 | 0.04755 | -0.01545 |
| 0.01088 | -0.02398 | 0.03009 | -0.03993 | 0.04767 | -0.01511 |
| 0.01174 | -0.02473 | 0.03078 | -0.03940 | 0.0476 | -0.01507 |
| **Punctul B** | | | | | |
| 0.01257 | -0.02553 | 0.03146 | -0.03885 | 0.0476 | -0.01507 |
| 0.01336 | -0.02639 | 0.03213 | -0.03830 | 0.0494 | -0.01424 |
| 0.01411 | -0.02731 | 0.03280 | -0.03773 | 0.0512 | -0.01394 |
| 0.01482 | -0.02828 | 0.03345 | -0.03715 | 0.053 | -0.01357 |
| 0.01549 | -0.02930 | 0.03409 | -0.03656 | 0.0548 | -0.01313 |
| 0.01612 | -0.03038 | 0.03473 | -0.03596 | 0.0566 | -0.01262 |
| 0.01669 | -0.03151 | 0.03535 | -0.03535 | 0.0584 | -0.01205 |
| 0.01722 | -0.03268 | 0.03596 | -0.03473 | 0.0602 | -0.01141 |
| 0.01769 | -0.03390 | 0.03656 | -0.03409 | 0.062 | -0.01071 |
| 0.01811 | -0.03517 | 0.03715 | -0.03345 | 0.0638 | -0.00994 |
| 0.01847 | -0.03647 | 0.03773 | -0.03280 | 0.0656 | -0.00910 |
| 0.01877 | -0.03782 | 0.03830 | -0.03213 | 0.0674 | -0.00819 |
| 0.01902 | -0.03920 | 0.03885 | -0.03146 | 0.0692 | -0.00722 |
| **Punctul C** | | | | | |
| 0.01920 | -0.04062 | 0.03940 | -0.03078 | 0.071 | -0.00618 |
| 0.01932 | -0.04207 | 0.03993 | -0.03009 | 0.0728 | -0.00507 |
| 0.01937 | -0.04355 | 0.04045 | -0.02938 | 0.0746 | -0.00390 |
| 0.01936 | -0.04506 | 0.04095 | -0.02867 | 0.0764 | -0.00266 |
| 0.01931 | -0.04613 | 0.04145 | -0.02795 | 0.0782 | -0.00136 |
| **Punctul D** | | | | | |
| 0.01929 | -0.04612 | 0.04240 | -0.02649 | 0.08 | 0 |
| 0.01953 | -0.04602 | 0.04285 | -0.02575 |  |  |
| 0.02033 | -0.04567 | 0.04330 | -0.025 |  |  |
The contour of the rotor (figure 5) consists of straight lines, circle arcs and curves for which mathematical calculations relation have been established separately and subsequently a quite complicated computation program has been developed [5]. The contour is specified by the coordinates of points \( x_i; y_i \) located as follows (table 3):
- On piston side, the line CD;
- On the rotor contour, the arc BC;
- On the cavity contour in the rotor, the AB curve.

The computation program has an accuracy of 5 decimal, because the rotors are executed on a numerically controlled center (C.N.C) [6].

When developing the computation program as input data or used:
- Rotor radius: \( R_r = 0.05 \) [m];
- Height of the rotating piston: \( z = 0.04 \) [m];
- Case radius: \( R_c = R_r + z = 0.08 \) [m];

In figure 5 is presented the first quarter of the piston specified by the coordinates \( x_i, y_i \).

![Figure 5. The first quarter of the piston](image)

4. The analysis of the formula for calculating the transported flow rate and the driving power of the machine

In previous papers, the expression of calculation of the volumetric flow rate carried by this machine was established [7-8]:

\[
V = \pi \cdot l \cdot z (2R_r + z) \quad [m^3 / s] 
\]

(8)

From relation (8) one can observe that the only geometrical parameter that can be free is the length of the rotor "l".
Therefore, for a certain required flow rate, a value for 'l' is proposed and then z and \( R_r \) are chosen provided that \( z \leq 0.732 R_r \); this choice depends on the execution technology of the respective company performing the rotating machine.

The driving power of the machine will be [9-10]:

\[
P = \dot{V} \cdot \Delta p \ [W] \tag{9}
\]

where:
* \( \dot{V} \) – volumetric flow rate driven by the machine \([m^3/s]\); 
* \( \Delta p \) - increasing pressure between suction and discharge \([N/m^2]\).

The value of \( \Delta p \) can also be expressed according to the geodetic pumping height \((H_g)\) of the water [11-12].

\[
\Delta p = \rho_{H_g} \cdot g \cdot H_g \left[ N/m^2 \right] \tag{10}
\]

as a result:

\[
P = \dot{V} \cdot \rho_{H_g} \cdot g \cdot H_g \ [W] \tag{11}
\]

From relation (11) one can observe that only \( \dot{V} \) influences the machine architecture, and \( \rho_{H_g} \) (or other fluid) and \( H_g \) influence the theoretical power required by the rotating machine.

5. Conclusions

a) For a certain required volumetric flow rate, a certain length of the rotor is chosen according to the execution technology; then the rotor radius is chosen and the height of the rotating piston results.

b) The change of the volumetric flow rate of the rotating machine is made by changing the speed of the engine driving the machine.

c) The calculation results were obtained starting from an experimental prototype built in the laboratory of the Department of Thermotechnics, Engines, Thermal and Refrigeration Equipment’s of University Politehnica of Bucharest.

d) This rotating machine can transport multiphase fluids, viscous fluids; as a result, it can be used in the fields: energy industry, petrochemical industry, food industry, agriculture.

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