Broad enhancement structure in $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ and higher $\rho$ meson excitations

Xiang Liu$^{1,2,3,4}$, Qin-Song Zhou$^{1,3}$, and Li-Ming Wang$^{5,3}$

$^1$School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China
$^2$Joint Research Center for Physics, Lanzhou University and Qinghai Normal University, Xining 810000, China
$^3$Lanzhou Center for Theoretical Physics, Key Laboratory of Theoretical Physics of Gansu Province, and Frontiers Science Center for Rare Isotopes, Lanzhou University, Lanzhou 730000, China
$^4$Research Center for Hadron and CSR Physics, Lanzhou University & Institute of Modern Physics of CAS, Lanzhou 730000, China
$^5$Key Laboratory for Microstructural Material Physics of Hebei Province, School of Science, Yanshan University, Qinhuangdao 066040, China

(Dated: October 27, 2022)

Recently, the BaBar Collaboration reported a broad resonance structure near 2 GeV when analyzing the $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ process, which provides a good opportunity to study the higher $\rho$ mesonic states. When considering the $\rho(1900)$ and $\rho(2150)$ contributions, the experimental data of the cross section of $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ around 2 GeV can be well depicted, especially the observed broad resonance structure in $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ containing two substructures is suggested by the present work. Additionally, we also indicate a possible signal of $\rho(5S)$ around 2.5 GeV in the $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ process. The obtained result may provide valuable hint to identify higher $\rho$ mesons, which will be a new task for the BESIII and Belle II experiments with the accumulation of higher precision data.

I. INTRODUCTION

Very recently, the BaBar Collaboration announced the measurement of the $e^+e^- \rightarrow K^+K^-3\rho^0$, $e^+e^- \rightarrow K_S^0K^+\pi^-\pi^0$, and $e^+e^- \rightarrow K^0_SK^+\pi^-\pi^0$ processes at center-of-mass energies from threshold to 4.5 GeV, where this analysis was performed with initial state radiation method [1]. By the $e^+e^- \rightarrow K^0_SK^+\pi^-\pi^0$ reaction, BaBar reconstructed the $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ process, by which a broad enhancement structure around 2 GeV was observed with resonance parameter

$$M = 2.09 \pm 0.03 \text{ GeV}, \quad \Gamma = 0.50 \pm 0.06 \text{ GeV}.$$

Actually, in 2007, the BaBar collaboration had already studied the $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ process [2], and a broad resonance structure was observed around 2 GeV by a single Breit-Wigner fitting, which has resonance parameter

$$M = 2.15 \pm 0.04 \pm 0.05 \text{ GeV}, \quad \Gamma = 0.35 \pm 0.04 \pm 0.05 \text{ GeV}.$$

Here, the experimental results for the cross section of $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ are collected in Fig. 1. Due to the constraint of conservations of angular momentum and $P$ parity, it requires that the $f_1(1285)\pi^+\pi^-$ system must have a quantum number $J^P(C) = 1^+(1^-)$. Therefore, the measured $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ process may provide a good chance to construct the $\rho$ meson family.

In the mass region around 2 GeV, there are three $\rho$ mesonic states collected in the Particle Data Group (PDG) [3], which are the $\rho(1900)$, $\rho(2150)$, and $\rho(2000)$. Although experiments have reported some $\rho$ states around 2 GeV in the past few decades [3], categorizing these states into the $\rho$ meson family is far from being established. In Fig. 2, we list the resonance parameters of the $\rho(1900)$ and $\rho(2150)$ measured by different experiment groups [3], by which the messy situation of the measured resonance parameters of these two higher $\rho$ states around 2 GeV is presented. Evidently, the resonance parameters of the $\rho(1900)$ and $\rho(2150)$ from different experiments are different. If considering the experimental uncertainties, the obtained widths of the $\rho(1900)$ and $\rho(2150)$ are in the ranges of $5 \sim 180$ MeV and $32 \sim 630$ MeV, respectively. The $\rho(2000)$ is collected in Particle Data Group (PDG) as the “further state” [3], which was first reported by an amplitude analysis of the data of $p\bar{p} \rightarrow \pi\pi$ [4]. After that, its existence was confirmed by a combined analysis of the $e^+e^- \rightarrow \omega\pi\pi$ and $\omega\pi$ [5]. However, many recent experimental measure-
ments with higher precision for the $\rho$ mesonic states show that there is an enhancement structure with the width around 100 MeV and the mass near 2 GeV rather than a broad structure with the width larger than 300 MeV [6, 7]. On the theoretical side, various models have also been used to study the mass spectrum of $\rho$ mesons, including potential model [8–13], Regge trajectory [14, 15], and other methods [16–18], where the $\rho(1900)$, $\rho(2150)$, and $\rho(2000)$ are usually categorized as $\rho(3^3S_1) \equiv \rho(3S)$, $\rho(4^3S_1) \equiv \rho(4S)$, and $\rho(2^3D_1) \equiv \rho(2D)$, respectively. The quark pair creation (QPC) model is widely applied to estimate Okubo-Zweig-Iizuka (OZI)-allowed two-body strong decays of hadron. By using the QPC model [11–15], the widths of higher $\rho$ mesons were obtained, where the calculated widths of the $\rho(1900)$, $\rho(2105)$, and $\rho(2000)$ are in the ranges of $125 \sim 184$ MeV, $102 \sim 122$ MeV, and $179 \sim 228$ MeV, respectively.

As mentioned above, we briefly review the status of experimental and theoretical research on $\rho$ mesonic states around 2 GeV. When facing the BaBar’s measurement of the cross section of the $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ process, some questions are raised as shown in below. Usually, the theoretical results show that the widths of the $\rho(1900)$ and $\rho(2150)$ are about 100 MeV, and the width of the $\rho(2000)$ is less than 300 MeV. Obviously, the broad structure observed by the BaBar Collaboration in the $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ process is not consistent with these reported $\rho$ states. Can we find a solution to alleviate such inconsistency?

To clarify this issue, we conjecture that the reported broad structure in the $f_1(1285)\pi^+\pi^-$ invariant mass spectrum can be still due to the interference effect from some higher $\rho$ mesonic states. According to the theoretical study of higher $\rho$ mesonic states around and above 2 GeV by the modified Godfrey-Isgur model [11–13], we suggest that the $\rho(3S)$, $\rho(4S)$ and $\rho(5S)$ contribution should be introduced when depicting the data of the $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ process, where the $\rho(3S)$ and $\rho(4S)$ correspond to the $\rho(1900)$ and $\rho(2150)$, respectively. In realistic calculation, we adopt the effective Lagrangian approach to calculate the cross section of $e^+e^- \rightarrow \rho^+ \rightarrow f_1(1285)\pi^+\pi^-$. By fitting the experimental data of the cross section of $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$, we find that the reported broad enhancement structure in $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ may contain two substructures, i.e., the $\rho(1900)$ and $\rho(2150)$. In addition, we also find a possible evidence of $\rho(5S)$ when reproducing the line shape of the cross section of $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ well. With more precise data collected by BESIII and Belle II in the near future, we have a reason to believe that our observations can be tested.

This paper is organized as follows. After the Introduction, we illustrate the study of the $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ process by considering the intermediate higher $\rho$ mesons as shown in Sec. II. In Sec. III, we present numerical results for the analysis of the cross section of the $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ process. Finally, the paper ends with a discussion and conclusion in Sec. IV.

II. HIGHER $\rho$ MESONIC STATES CONTRIBUTIONS TO $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$

Since the scatter plot of $m(\pi^+\pi^-)$ vs $m(K^0\bar{K}^0\pi^+\pi^-)$ given by the BaBar Collaboration [1] shows that $\pi^+\pi^-$ is dominantly from $\rho(770)$, we can assume that the reaction $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$ occurs mainly through the two reaction mechanisms shown in Fig. 3. Here, diagram (a) depicts the virtual photon directly coupled with $f_1(1285)\rho$, which provides the background contribution, while diagram (b) reflects intermediate higher $\rho$ meson contribution to $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$. 

FIG. 2: The resonance parameters of the $\rho(1900)$ and $\rho(2150)$ are measured by different experiments collected in PDG [3].

FIG. 3: The schematic diagrams depict the reaction $e^+e^- \rightarrow f_1(1285)\pi^+\pi^-$. Here, diagram (a) depicts the virtual photon directly coupling to $f_1(1285)\rho$, while diagram (b) is due to the intermediate state $\rho^*$ contribution, where $\rho^*$ denotes higher $\rho$ mesonic states.
In this work, we adopt effective Lagrangian approach to describe these interaction vertices shown in Fig. 3. The effective Lagrangians involved in the concrete calculation include [19–23]

\[
\begin{align*}
L_{\gamma\rho\rho} &= g_{\gamma\rho\rho} e^{\rho_0}(\partial\rho_A \partial\rho_B - \partial\rho_B \partial\rho_A) f_{\gamma\rho}, \\
L_{\gamma\sigma\rho} &= g_{\gamma\rho\sigma} e^{\rho_0}(\partial\rho_A \partial\rho_B - \partial\rho_B \partial\rho_A) f_{\gamma\rho}, \\
L_{\gamma\rho\pi} &= g_{\gamma\rho\pi} \rho_0 \partial\rho_A, \\
L_{\gamma\rho\rho} &= -e^2 \rho_0 \rho_0 A_\mu,
\end{align*}
\]

where \( \rho^*, \rho, f_1, \) and \( \pi \) stand for the mesonic fields of higher \( \rho \) meson, \( \rho(770) \), \( f_1(1285) \), and \( \pi \), respectively. With the above preparation, the amplitude of \( e^+e^- \to f_1(1285)\pi^+\pi^- \) corresponding to diagram (a) in Fig. 3 is written as

\[
M_{\text{Diag}} = \frac{g_{\gamma\rho\rho} G_{\rho\pi\pi} v(k_2) (i e_{\gamma\rho}) u(k_1) - g_{\rho\sigma} G_{\rho\pi\pi} e^{\rho_0} (p_1 q^2 + p_1^2 q_\mu)}{q^2 - m^2_{\rho} + i m_{\pi} \Gamma_{\rho}(s)},
\]

where \( p_2, p_3, \) and \( p_4 \) are the four momenta of final states \( f_1(1285), \pi^+, \) and \( \pi^- \), respectively. And, \( p_1 = k_1 + k_2, \) \( q = p_3 + p_4, \) \( g_{\gamma\rho\rho}(q) = -g_{\pi\pi}^{\rho\pi} + g_{\pi\pi}^{\rho\pi}/q^2, \) and the form factor \( F(s) = e^{i\pi\pi(s - m_{\rho} + m_{\pi} - m_{\rho}^2)} \) are defined. The amplitude of the reaction of \( e^+e^- \to f_1(1285)\pi^+\pi^- \) via the intermediate higher \( \rho \) state, which corresponds to diagram (b) in Fig. 3, can be written as the product of the amplitude of \( e^+e^- \to \rho^*_1 \) and the amplitude of \( \rho^*_1 \to f_1(1285)\pi^+\pi^- \)

\[
M_{\rho^*_1} = \frac{M_{e^+e^- \to \rho^*_1} M_{\rho^*_1 \to f_1(1285)\pi^+\pi^-}}{p_1^2 - m_{\rho}^2 + i m_{\pi} \Gamma_{\rho}(s)},
\]

where the amplitude of \( e^+e^- \to \rho^*_1 \) is written as

\[
M_{e^+e^- \to \rho^*_1} = v(k_2) (i e_{\gamma\rho}) u(k_1) - g_{\rho\sigma}^{\rho\rho} e^{\rho_0} e^{\rho_0} (p_1 q^2 + p_1^2 q_\mu),
\]

and the amplitude of \( \rho^*_1 \to f_1(1285)\pi^+\pi^- \) is expressed as

\[
M_{\rho^*_1 \to f_1(1285)\pi^+\pi^-} = g_{\rho\rho} G_{\rho\pi\pi} e^{\rho_0} (p_1 q^2 + p_1^2 q_\mu) \frac{g_{\gamma\rho\rho}(q)}{q^2 - m^2_{\rho} + i m_{\pi} \Gamma_{\rho}(p_1^2 - p_1^2 q_\mu)^2} e^{\rho_0}(p_2).
\]

And then, the \( B(\rho^*_1 \to f_1(1285)\pi^+\pi^-) = \frac{1}{12\pi f_{\rho}^2} \frac{1}{6m_{\rho}^2} |M_{\rho^*_1 \to f_1(1285)\pi^+\pi^-}|^2 \times|p_2||p_3|d\Omega_2 d\Omega_3 dm_{34} \)

III. NUMERICAL RESULTS

In this section, we adopt two schemes to fit the experimental data of the cross section of \( e^+e^- \to f_1(1285)\pi^+\pi^- \) measured by the BaBar Collaboration [1, 2]. In scheme 1, we consider the contributions of these higher \( \rho \) mesonic states near 2 GeV, which include the \( \rho(1900), \rho(2150), \) and \( \rho(2000), \) to decipher the broad resonance structure existing in the \( e^+e^- \to f_1(1285)\pi^+\pi^- \) process. We also notice that there is an event accumulation around 2.5 GeV. Thus, in scheme 2, \( \rho(552) \) associated with the \( \rho(1900), \rho(2150), \) and \( \rho(2000) \) is considered for depicting the data well. In our investigation, these coupling constants, relative phase angles, the resonance parameters of relevant \( \rho \) mesonic states, and the parameter \( b \) in form factor are treated as the fitting parameters.

Although there are three reported \( \rho \) states near 2 GeV, the \( \rho(2000) \) as a typical D-wave state [11, 12, 14, 18] is not included in our study. The main reason is that the dilepton width of the D-wave vector meson is suppressed compared to the case of the S-wave vector meson [8, 12, 24, 25]. Thus, in scheme 1, we only consider the contributions from the \( \rho(1900) \) and \( \rho(2150) \) to reproduce the cross section data of \( e^+e^- \to f_1(1285)\pi^+\pi^- \) process. The fitted result of the cross sections of \( e^+e^- \to f_1(1285)\pi^+\pi^- \) is shown in Fig. 4, where these obtained fitting parameters are collected into Table 1. For convenience, \( g_{\rho\rho} G_{\rho\pi\pi} f_{\rho} \) is abbreviated to \( g_{\rho\rho} \).
With the central values of the fitted parameters, the value of \( \chi^2/\text{n.d.f} \) is obtained to be 1.63.

As shown in Fig. 4, the fitted curve can well reproduce the experimental data of the cross sections for \( e^+e^- \to f_1(1285)\pi^+\pi^- \). Especially, our study shows that the reported broad enhancement structure in \( e^+e^- \to f_1(1285)\pi^+\pi^- \) should at least contain two substructures like the \( \rho(1900) \) and \( \rho(2150) \). We also list the fitted results of the resonance parameters for both \( \rho(1900) \) and \( \rho(2150) \) in Table I, which are within the range of previous theoretical results [10–12, 14, 15].

By such an effort, we provide a possible explanation to understand why there exists the broad structure with width around 300 MeV in \( e^+e^- \to f_1(1285)\pi^+\pi^- \), which is caused by the interference effect of the \( \rho(1900) \) and \( \rho(2150) \). Thus, the inconsistency of the BaBar observation of very broad enhancement structure [1, 2] and the established \( \rho \) states near 2 GeV in PDG can be naturally alleviated.

TABLE I: The parameters involved in scheme 1, which are obtained by fitting the cross sections of the \( e^+e^- \to f_1(1285)\pi^+\pi^- \) process. Here, the experimental data is from BaBar [1, 2].

| Parameters | Values |
|------------|--------|
| \( m_{\rho(1900)} \) (MeV) | 1913 ± 34 |
| \( m_{\rho(2150)} \) (MeV) | 2151 ± 32 |
| \( \Gamma_{\rho(1900)} \) (MeV) | 106 ± 21 |
| \( \Gamma_{\rho(2150)} \) (MeV) | 98 ± 16 |
| \( g_{\eta f_1(1285)} \) (GeV\(^{-2}\)) | 4.11 ± 0.38 |
| \( g_{\rho(1900)} \) (GeV\(^{-2}\)) | 0.088 ± 0.009 |
| \( g_{\rho(2150)} \) (GeV\(^{-2}\)) | 0.024 ± 0.007 |
| \( b \) (GeV\(^{-1}\)) | 3.74 ± 0.18 |
| \( \phi_{\rho(1900)} \) (rad) | 0.63 ± 0.02 |
| \( \phi_{\rho(2150)} \) (rad) | 0.90 ± 0.04 |
| \( \chi^2/\text{n.d.f} \) | 1.63 |

Besides explaining the broad structure reported by BaBar, we notice an event cluster around 2.5 GeV in the measured cross section data. Thus, we propose to include the predicted \( \rho(5S) \) state [11–13] associated with the \( \rho(1900) \) and \( \rho(2150) \), which is as scheme 2 in the present work. The fitted results are shown in Fig. 5, and the corresponding parameters are listed in Table II. Obviously, the fitting result to the cross section of \( e^+e^- \to f_1(1285)\pi^+\pi^- \) can be improved in scheme 2. We obtain the resonance parameter of the \( \rho(5S) \) state, i.e.,

\[
M = 2512 ± 56 \text{ MeV}, \quad \Gamma = 66 ± 38 \text{ MeV},
\]

which is consistent with theoretical predictions in Refs. [11–13]. We suggest future experiment to identify the \( \rho(5S) \) signal with more precise data, which is crucial to construct \( \rho \) meson family.

TABLE II: The parameters for scheme 2 when fitting the cross sections of the \( e^+e^- \to f_1(1285)\pi^+\pi^- \) process [1, 2].

| Parameters | Values |
|------------|--------|
| \( m_{\rho(1900)} \) (MeV) | 1919 ± 38 |
| \( m_{\rho(2150)} \) (MeV) | 2162 ± 31 |
| \( m_{\rho(5S)} \) (MeV) | 2512 ± 56 |
| \( \Gamma_{\rho(1900)} \) (MeV) | 102 ± 41 |
| \( \Gamma_{\rho(2150)} \) (MeV) | 95 ± 28 |
| \( \Gamma_{\rho(5S)} \) (MeV) | 66 ± 38 |
| \( g_{\rho(1900)} \) (GeV\(^{-2}\)) | 3.19 ± 0.10 |
| \( g_{\rho(2150)} \) (GeV\(^{-2}\)) | 0.133 ± 0.019 |
| \( g_{\rho(5S)} \) (GeV\(^{-2}\)) | 0.028 ± 0.008 |
| \( g_{\rho(5S)} \) (GeV\(^{-2}\)) | 0.007 ± 0.002 |
| \( b \) (GeV\(^{-1}\)) | 3.78 ± 0.49 |
| \( \phi_{\rho(1900)} \) (rad) | 0.52 ± 0.21 |
| \( \phi_{\rho(2150)} \) (rad) | 1.47 ± 0.19 |
| \( \phi_{\rho(5S)} \) (rad) | 3.20 ± 0.36 |
| \( \chi^2/\text{n.d.f} \) | 1.52 |

FIG. 4: The fit to the experimental data of the cross sections of \( e^+e^- \to f_1(1285)\pi^+\pi^- \) measured by the BaBar Collaboration in 2007 [2] (blue dots with error bars) and in 2022 [1] (black dots with error bars). In this scheme, only the contributions from the \( \rho(1900) \) and \( \rho(2150) \) are considered. Here, the calculated value of \( \chi^2/\text{n.d.f} \) is 1.63.

FIG. 5: The fit to the experimental data of the cross sections of \( e^+e^- \to f_1(1285)\pi^+\pi^- \) measured by the BaBar Collaboration in 2007 [2] (blue dots with error bars) and 2022 [1] (black dots with error bars). In this scheme, the contributions from \( \rho(1900) \), \( \rho(2150) \), and \( \rho(5S) \) are considered. Here, the calculated value of \( \chi^2/\text{n.d.f} \) is 1.52.

To facilitate comparison with the experimental results, by substituting the values of the coupling constants listed in Ta-
ble II into Eqs. (11) and (12), we can estimate the values of the combined branching ratio $\Gamma_{\psi\to f_1(1285)\pi^+\pi^-}$, which are presented in Table III. The values of these combined branching ratios can be measured by experiments with higher precision in the future, by which our scenario proposed in this work can be tested.

**TABLE III:** The combined branching ratios $\Gamma_{\psi\to f_1(1285)\pi^+\pi^-}$ are estimated by substituting the values of the coupling constants listed in Table II into Eqs. (11) and (12).

| Parameters          | Values (eV)  |
|---------------------|--------------|
| $\Gamma_{\psi\to f_1(1285)\pi^+\pi^-}$ | 2.08 $\sim$ 3.69 |
| $\Gamma_{\psi\to f_1(1285)\pi^+\pi^-}$ | 5.71 $\sim$ 18.48 |
| $\Gamma_{\psi\to f_1(1285)\pi^+\pi^-}$ | 0.46 $\sim$ 13.04 |

**IV. DISCUSSION AND CONCLUSION**

Very recently, the BaBar Collaboration measured $e^+e^- \to f_1(1285)\pi^+\pi^-$ again [1] and reported a very broad enhancement structure around 2 GeV in the $f_1(1285)\pi^+\pi^-$ invariant mass spectrum. In 2007, BaBar once indicated a similar phenomenon [2]. If making a comparison of the BaBar’s observation with these reported $\rho$ states around 2 GeV collected by PDG [3], the broad enhancement structure from BaBar cannot correspond to these reported $\rho$ states like $\rho(1900)$, $\rho(2000)$, and $\rho(2150)$. This puzzling phenomenon stimulates our interest.

Supported by the $\rho$ meson spectroscopy [11, 12, 14, 18], we propose an interference picture of the $\rho(1900)$ and $\rho(2150)$ to depict the observed broad enhancement structure around 2 GeV in the $e^+e^- \to f_1(1285)\pi^+\pi^-$. By this approach, the data of the cross section of $e^+e^- \to f_1(1285)\pi^+\pi^-$ can be reproduced well. Naturally, we provide a natural solution to understand the puzzling phenomenon mentioned above.

In fact, an event cluster around 2.5 GeV can be found if checking the experimental data of the cross section of $e^+e^- \to f_1(1285)\pi^+\pi^-$. We notice that different theoretical groups predicted the existence of the $\rho(555)$ state. Thus, we include the contribution of the $\rho(555)$ state to fit the experimental data again, where the resonance parameter of the $\rho(555)$ state can be obtained, which is consistent with the theoretical predictions [11–13]. Confirming this evidence is a potential issue for our experimental colleagues.

In summary, $e^+e^- \to f_1(1285)\pi^+\pi^-$ is an ideal process to identify higher $\rho$ mesonic states. At present, the $\rho$ meson family is far from being established. As one part of the whole hadronic zoo, we have enough interest in finding out new higher $\rho$ meson states. With the running of Belle II and BES-III, experimentalists do not stop observing new $\rho$ states. We hope that the present work may inspire some insights to construct light flavor vector meson family.

**Acknowledgments**

This work is supported by the projects funded by Science and Technology Department of Qinghai Province (No. 2020-ZJ-728), the China National Funds for Distinguished Young Scientists under Grant No. 11825503, National Key Research and Development Program of China under Contract No. 2020YFA0406400, the 111 Project under Grant No. B20063, and the National Natural Science Foundation of China under Grant No. 12047501.

1. J. P. Lees et al. [BaBar], Study of the reactions $e^+e^- \to K^-K^+\pi^+\pi^-$, $e^+e^- \to K^0_s\bar{K}^0_s\pi^+\pi^-$, and $e^+e^- \to K^+K^-\pi^+\pi^-$ at center-of-mass energies from threshold to 4.5 GeV using ISR, [arXiv:2207.10340 [hep-ex]].

2. B. Aubert et al. [BaBar], The $e^+e^- \to (2\pi^+\pi^-)^0$, $2(\pi^+\pi^-)^0$, $K^-\pi^+\pi^-\pi^0$, and $K^+\pi^-\pi^+\pi^0$ cross sections measured with initial-state radiation, Phys. Rev. D 76, 092005 (2007), (erratum: Phys. Rev. D 77, 119902 (2008)).

3. P. A. Zyla et al. [Particle Data Group], Review of Particle Physics, PTep 2020, no.8, 083C01 (2020).

4. A. Hasan and D. V. Bugg, Amplitudes for $pp \to \pi\pi$ from 0.36 GeV/c to 2.5 GeV/c, Phys. Lett. B 334, 215–219 (1994).

5. D. V. Bugg, Four sorts of meson, Phys. Rept. 397, 257–358 (2004).

6. M. Ablikim et al. [BESIII], Observation of a resonant structure in $\psi(2S) \to \omega\pi\eta$ and another in $\psi(2S) \to \omega\pi^0$ at center-of-mass energies between 2.00 and 3.08 GeV, Phys. Lett. B 813, 136059 (2021).

7. M. Ablikim et al. [BESIII], Measurement of the Born cross sections for $e^+e^- \to \eta\pi^+\pi^-$ at center-of-mass energies between 2.00 and 3.08 GeV, Phys. Rev. D 103, no.7, 072007 (2021).

8. S. Godfrey and N. Isgur, Mesons in a Relativized Quark Model with Chromodynamics, Phys. Rev. D 32, 189-231 (1985).

9. T. Barnes, F. E. Close, P. R. Page and E. S. Swanson, Higher quarkonia, Phys. Rev. D 55, 4157-4188 (1997).

10. D. Ebert, R. N. Faustov and V. O. Galkin, Mass spectra and Regge trajectories of light mesons in the relativistic quark model, Phys. Rev. D 79, 114029 (2009).

11. Z. Y. Li, D. M. Li, E. Wang, W. C. Yan and Q. T. Song, Assignments of the Y(2040), $\rho(1900)$, and $\rho(2150)$ in the quark model, Phys. Rev. D 104, no.3, 034013 (2021).

12. J. Z. Wang, L. M. Wang, X. Liu and T. Matsuki, Deciphering the light vector meson contribution to the cross sections of $e^+e^-$ annihilations into the open-strange channels through a combined analysis, Phys. Rev. D 104, no.5, 054045 (2021).

13. L. M. Wang, S. Q. Luo and X. Liu, Light unflavored vector meson spectroscopy around the mass range of 2.4 to 3 GeV and possible experimental evidence, Phys. Rev. D 105, no.3, 034011 (2022).

14. L. P. He, X. Wang and X. Liu, Towards two-body strong decay behavior of higher $\rho$ and $\rho_1$ mesons, Phys. Rev. D 98, no. 3, 034008 (2013).

15. J. C. Feng, X. W. Kang, Q. F. Liu and F. S. Zhang, Possible assignment of excited light S31 vector mesons, Phys. Rev. D 104, no.5, 054027 (2021).

16. T. Hilger, M. Gomez-Rocha and A. Krassnigg, Light-
quarkonium spectra and orbital-angular-momentum decomposition in a Bethe–Salpeter-equation approach, Eur. Phys. J. C 77, no.9, 625 (2017).

[17] T. Branz, T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Light and heavy mesons in a soft-wall holographic approach, Phys. Rev. D 82, 074022 (2010).

[18] G. L. Yu, Z. G. Wang, X. W. Wang and H. J. Wang, The ground states and the first radially excited states of D-wave vector $\rho$ and $\phi$ mesons, Int. J. Mod. Phys. A 36, 2150197 (2021).

[19] L. Rosenberg, Electromagnetic interactions of neutrinos, Phys. Rev. 129, 2786-2788 (1963).

[20] N. I. Kochelev, D. P. Min, Y. s. Oh, V. Vento and A. V. Vinnikov, A New anomalous trajectory in Regge theory, Phys. Rev. D 61, 094008 (2000).

[21] O. Kaymakcalan, S. Rajeev and J. Schechter, Nonabelian Anomaly and Vector Meson Decays, Phys. Rev. D 30, 594 (1984).

[22] T. Bauer and D. R. Yennie, Corrections to VDM in the Photoproduction of Vector Mesons. 1. Mass Dependence of Amplitudes, Phys. Lett. B 60, 165-168 (1976).

[23] T. Bauer and D. R. Yennie, Corrections to Diagonal VDM in the Photoproduction of Vector Mesons. 2. Phi-omega Mixing, Phys. Lett. B 60, 169-171 (1976).

[24] L. M. Wang, J. Z. Wang and X. Liu, Toward $e^+ e^- \rightarrow \pi^+ \pi^-$ annihilation inspired by higher $\rho$ mesonic states around 2.2 GeV, Phys. Rev. D 102, no.3, 034037 (2020).

[25] Q. S. Zhou, J. Z. Wang, X. Liu and T. Matsuki, Identifying the contribution of higher $\rho$ mesons around 2 GeV in the $e^+ e^- \rightarrow \omega \pi^0$ and $e^+ e^- \rightarrow \rho \eta'$ processes, Phys. Rev. D 105, no.7, 074035 (2022).