CASVI: Computer Algebra System Aimed at Visually Impaired People. Experiments

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ABSTRACT The limitations in access to mathematical resources faced by visually impaired people (VIPs) are undoubtedly one of the most significant obstacles for them to obtain, e.g., a degree in science. In this context, computational tools such as Computer Algebra Systems (CAS) are not blind-friendly, which causes solving elementary mathematical problems to become a challenging task for this group of people. This paper presents the Casvi system, a CAS for people with visual disabilities that allows carried out basics and advanced numerical calculations using the Maxima mathematical engine. Twenty-five VIPs tested the Casvi system to assess its functionality and usability. As a result, VIPs achieved a 92% accuracy when executing mathematical operations through Casvi. Finally, our system outperforms the LAMBDA system regarding the time needed by VIPs to perform mathematical operations correctly.

INDEX TERMS Assistive technologies, mathematics, people with visual disabilities.

I. INTRODUCTION

At the least 2,200 million people worldwide have a visual impairment or blindness [1]. In the US, the dropout rate of high school students who have some form of disability is almost 40% [2]. Only 13.7% of students with visual disabilities pursuing higher education obtained a degree [3].

On the other hand, the population in Ecuador (the country in which part of this work is carried out) exceeds 17 million people [4], of which 481,392 are registered persons who have some disability, having an annual prevalence of 2.74%. Of this population group, 11.60% (55,843 people) have visual disabilities. Two thousand nine hundred six students with visual disabilities are studying in either primary, middle, or high school education, and 1,188 are enrolled in Universities or Polytechnic Schools. Also, there is a registry of 147 people with visual disabilities enrolled in Technical and Technological Institutes.

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Bachelor of Science majors (e.g., engineering) are more difficult for them since most of the resources (e.g., specialized software) commonly used in science (e.g., math textbooks) lack accessibility for the visually impaired, limiting their academic and career options.

Among these resources, Algebraic Computational Systems (CAS) such as MATLAB, Wolfram Mathematica, and Maxima have become indispensable tools in engineering and related areas. However, they are not accessible to the visually impaired, which means that executing the most basic mathematical operation in this type of software becomes a challenging task, even with the help of screen readers. It is also known that the difficulty for visually impaired people increases, and the degree of access decreases as the level of technical information in the document increases.

Therefore, the most significant barrier for people with visual disabilities to understand mathematical semantics is not blindness but access to mathematical content. This barrier makes it imperative to build a bridge between existing CAS tools and visually impaired people to allow writing, editing,
evaluating and, solving mathematical expressions. Furthermore, as visually impaired students increasingly attend regular schools, these tools must also be accessible to teachers who are not particularly familiar with braille [5].

In the search for a solution to the previously posed problem, this work presents the Casvi computational algebraic system as an alternative support tool for people who have some degree of visual impairment in their academic training process in engineering and exact sciences careers.

This work is organized as follows. In Section III, Related Work, we describe the software tools that create and edit mathematical expressions and solve mathematical operations. In Section IV, System Description, we present the Casvi system, a Computer Algebra System for people with visual disabilities. The experiments and results obtained with this CAS are presented in Section V and Section VI, respectively. In Section VII, a discussion of our work takes place. Finally, in Section VIII, we conclude this work and discuss various possible solutions to improve accessibility for VIPs and therefore encourage future research.

II. INTERVENTION AREA

In Quito, Ecuador, the experiments were carried out. There are more than 8000 people with visual disabilities registered according to the National Council for Disability Equality. This institution directs public policies on disabilities in Ecuador [6].

In search of participants who can carry out the experiments with our Computational Algebraic System, was conducted an initial investigation on the Web of educational institutions, associations, and foundations that work with people with visual disabilities. Likewise, were visited the offices of the Ministry of Education of Ecuador to obtain information on inclusive educational institutions in the country. After defining a preliminary list with more than 90 institutions (colleges, associations, and foundations), contact was established with the institutions of the city of Quito, achieving a collaboration agreement with 5 of them. Of a total of 91 people with visual disabilities (distributed in these five institutions), 67 voluntarily agreed to carry out a survey that would allow us to know some aspects of this population group; and 25 of them decided to carry out the experiments with the Casvi system.

The survey consisted of 14 questions which allowed us to know the particular characteristics of this population group. Hence, some of the questions asked about their age group, their percentage of disability, if they study or are interested in exploring a university degree, and if they have a computer program for learning mathematical analysis. Next, are presented the results obtained from this survey.

- It was identified, five age groups. Sixty-six percent of those surveyed are male; the remaining 34% are female. Under 18 years (11%), 18-39 (37%), 40-50 (21%), 51-60 (19%) and 61-79 (12%).
- Most people surveyed (30 people) have 85% to 100% visual impairment. Furthermore, 85% of the participants acquired their visual impairment. Only 15% are visually impaired from birth. People with acquired visual impairment do not like to learn the Braille language, so we should seek an alternative way to access information with mathematical content.
- Nine percent of people surveyed have a college education, 49% high school education, and 42% primary education (see Fig. 1). The vast majority of this group of people (61 people) do not have a university education. Moreover, of these 61 people, only 47 have the desire to pursue university studies.
- When asked which engineering career they would like to pursue, 34 people leaned towards STEM careers; Science, Technology, Engineering, and Math careers. The use of Computer Algebra Systems is every day in these types of university careers.
- When asked if they can perform mathematical operations, 100% of the respondents answered affirmatively. Ninety-one percent do not require the help of another person, while the remaining 9% do. Furthermore, 87% can only perform basic mathematical operations (addition, subtraction, multiplication, and division) and 13% intermediate-level mathematical operations (trigonometric operations, among others). Likewise, the instruments used to perform this type of mathematical operations are the following:: 42% Mind and fingers, 31% Calculator, 16% Abacus, 4% Cellphone, 3% Computer, and Tokens 3%.
- The approximate time it takes to perform an essential mathematical operation (addition, subtraction, multiplication, and division) is presented in the Fig. 2). More than 50% of respondents take less than 3 minutes to perform one of these operations. The division is the operation that takes the longest to complete of the four basic mathematical operations.

In the context of education for VIPs in engineering fields, the goal is to read mathematical expressions and be able to evaluate them. Data from the survey shows us that 84% do not have a computer tool that allows them to perform
mathematical operations. Some people make use of voice assistants and/or screen readers such as JAWS to access well-known computer programs such as Microsoft Excel.

### III. RELATED WORK

This section describes several related works that focus on accessing and creating mathematical content and executing mathematical operations.

Mackowski in [7], presents a Platform for Math Learning with Audio-Tactile Graphics for Visually Impaired Students. This research aims to enable VIPs to learn mathematics independently through mathematical formulas rendered using audio.

Moreover MathType [8] is a popular plugin for inserting math equations into Microsoft Word documents and recently Google Docs. Most of the MathML [9] and LaTeX standards are available in this application. The mathematical expressions entered can be exported to image formats. MathType integrates with the math Duxbury Braille Translator and the open-source NVDA [10] screen reader.

The work done by Asebriy et al. [11] presents a Retrieval Mathematical Equations System for VIPs; this ongoing project allows searching mathematical equations from the web, showing encouraging preliminary results.

The Benetech Math Editor [12], currently known as Mathshare [13], is an open-source math editor that allows solving basic math problems step by step. Unlike well-known editors such as DUDAMATH [14] and LiveMath [15], Mathshare is accessible to the visually impaired due to its support for screen readers such as NVDA and JAWS [16].

InftyReader and InftyEditor are two popular tools first presented by Suzuki et al. [18]. InftyReader [19], [20] accepts as input non-accessible digital documents, processes them using OCR techniques and finally generates an accessible document in different popular formats such as Microsoft Word and ISfêX. On the other hand, InftyEditor allows a sighted person to manually edit InftyReader documents to correct errors from the automatic conversion stage (such as equations incorrectly detected by OCR) or to augment new information [17].

There are other tools similar to InftyReader and InftyEditor such as i-Math [21] and the SZSLatex Editor [22], however they focus only on Microsoft Word documents and LaTeX code, respectively. Specifically, SZSLatex offers a simplified version for entering LaTeX code. The main disadvantage is that users must have some prior knowledge of LaTeX syntax.

The web application Pearson’s Accessible Equation Editor (AEE) [23] allows creating mathematical expressions from a Braille display. It relies on external screen readers and supports a wide range of mathematical Braille notations.

From the creators of InftyReader and InftyEditor, ChattyInfty [20], [24], [25] is a tool that allows VIPs to access, write, or edit scientific documents, including math expressions. ChattyInfty does not rely on external screen readers and offers several popular output formats such as XHTML, EPUB3, LaTeX, and DAYSIL.

The L-MATH System [26] allows editing and inspecting mathematical formulas. The writing and reading of mathematical expressions are achieved through the BlindMath [27] and TalkingMath, respectively. With BlindMath, the visually impaired student can enter mathematical formulas using a computer keyboard. TalkingMath uses an original adaptive algorithm to read formulas. For 2D graphics exploration, L-Math incorporates the AudioTac and BlindGraph modules.

LAMBDA (Linear Access to Mathematics for Braille Device and Audio-Synthesis) [28], [29] allows access to mathematical expressions through Braille code and synthetic speech. This system introduces its own Braille code with 256 unique characters (LAMBDA Code) based on the representation in 8-point Braille, which includes new symbols that allow the representation of mathematics linearly. The system offers the possibility of inserting normal text and equations within a single document as well as solving basic math operations.

DOSVOX [30] is an autonomous system designed by the Federal University of Rio de Janeiro. This system encompasses more than 80 open source tools accessible through spoken menus that allow VIPs to perform various activities such as sending emails. Two tools within the DOSVOX system allow the execution of mathematical operations: MATVOX and FINANVOX. MATVOX [31]–[33] is a computer algorithm interpreter that helps to write and compile pseudocode from a text editor called EDIVOX [34], FINANVOX [35]–[37] allows to perform financial and statistical calculations by emulating the HP-12C calculator.

In summary, Table 1 indicates the essential features of most of the previously reviewed solutions. For further information about the challenges and tools used by VIPs for accessing and creating mathematical content, refer to the surveys in [38], which focuses in general on smartphone-based assistive solutions, and [17], which specifically focuses on mathematical tools for VIPs.

As far as we know, only LAMBDA, FINANVOX, and MathShare allow VIPs to solve math operations. In this
TABLE 1. Comparison of the main characteristics of mathematical tools partially taken from [17].

| Name             | Synthetic Speech feedback | Non-speech feedback | Haptic feedback | External Screen Reader Reliance | Mathematical Operations | Input Formats | Output Formats | Usability tests |
|------------------|---------------------------|---------------------|-----------------|--------------------------------|------------------------|---------------|----------------|-----------------|
| IFeReader        | Simple TTS                | –                   | –               | ✓                              | –                      | PDF, Image formats | –              | –               |
| InfyEditor       | Simple TTS                | –                   | –               | ✓                              | –                      | IML, LaTeX, Image formats | –              | –               |
| i-Math           | Simple TTS                | –                   | –               | –                              | –                      | XML           | Audio formats | 84 users        |
| SSIlatex Editor  | Simple TTS                | –                   | –               | ✓                              | –                      | LaTeX         | PDF            | 3 users          |
| Pearson’s Accessible Math Editor | Simple TTS | –         | –               | ✓                              | –                      | Text, Nemeth braille | –              | –               |
| ChattyInfty      | Simple TTS                | –                   | –               | –                              | –                      | IXML, IML, XML, KML, ITML, LaTeX | –              | –               |
| L-MATH           | Simple TTS                | –                   | ✓               | ✓                              | –                      | LaTeX         | Audio formats, LaTeX | 4 users          |
| LAMBDA           | Simple TTS                | –                   | –               | ✓                              | Basic                  | LAMBDA code, MathML, XHTML | LAMBDA code, Braille, MathML, XHTML | 151 users        |
| MATVOX           | Simple TTS, Lexical cues  | –                   | –               | ✓                              | Basic                  | Own plain text format | Audio formats, Plain text | 6 users          |
| FINANVOX         | Simple TTS, Lexical cues  | –                   | –               | –                              | Basic                  | Own plain text format | Audio formats, Plain text | –               |
| Casvi            | Simple TTS, Lexical and Prosodic cues, Browsing | Barcons | –         | –                              | Advanced               | Own plain text format | Audio formats, Braille, MathML, Plain text | 25 users          |

IV. SYSTEM DESCRIPTION

The implementation of accessible computer systems for VIPs has become one of the biggest challenges today, although it really should be said a necessity. This section details Casvi; a tool that allows people with visual disabilities to develop basic and advanced mathematical operations (algebra, linear algebra, differential calculus, integral calculus, among others). In this paper, our main goal is to extensively describe the Casvi system, whose preliminary version was introduced in [40], as well as its validation through several experiments, including usability tests. Finally, we compare Casvi to LAMBDA, which is the most feature-rich tool from prior works to solve math operations.

C# has been used as the main programming language. This allowed to include all the advantages that an Object-Oriented Paradigm provides (Abstraction, Inheritance, Polymorphism, and Encapsulation); in addition to a simple, secure, and distributed language. The graphical interface has been studied and designed with great care, taking into account two premises:
- It is essential to mention that blind people cannot use a two-dimensional screen.
- Visually impaired users should receive as much information as possible on each interface element as they learn to use the application.

The Casvi interface maintains controls such as text boxes and buttons, all through agile keyboard access. However,
the access to the spoken information is sequential, so the user has to wait for the voice to reach the fragment of the destination information. Without a doubt, this process has been one of the most significant challenges in developing this system. Long training tests were required; however, was took each test to refine functional user requirements and improve non-functional user requirements step by step, in this case, user-friendliness and usability. The final prototype shown in this article was accepted in the testing stage by the target users.

This interface, implemented on the IDE (Integrated Development Environment) proprietary of Visual Studio .NET, maintains agile access through the combination of keys to access its functions. For example, the user can access the Algebra menu by pressing “Alt + J” or the Calculus menu by pressing “Alt + F”. The Alt modifier key is used to access most of the functions of the Casvi system.

Access to all the functions and menus of the Casvi system as well as the entry of mathematical operations is done through the computer keyboard. In the case of special commands, each one has a different function. The Table 3 shows the role of each one.

**TABLE 3. Casvi system special commands.**

| Key | Function | Description |
|-----|----------|-------------|
| F1  | Text reading | Allow the user to know what they wrote |
| F2  | Arithmetic operators | Presents the arithmetic operators available in the Casvi system |
| F3  | Mathematical operations | Presents the mathematical operations available in the Casvi system |
| F4  | Specials constants | Presents the specials constants available in the Casvi system |
| F5  | Stored expressions | Presents the expressions stored in the Casvi system |
| F6  | Stored variables | Presents the variables stored in the Casvi system |
| F7  | Results obtained | Presents the result of the last operations carried out |
| F8  | Transformations | Allows transforming from degrees to radians and vice versa |
| F9  | Fraction | Allow the entry of a fraction |

In each procedure performed by the user, auditory feedback is generated, which will allow the user to better navigate through the system. This system is composed of 8 modules detailed below:

1) File: it has the options to open, save, and print, among others. The Casvi system has the option of printing in Braille format and transforming the results obtained into MathML code.

2) Casvi menu: presents essential functions used directly for system configuration. The user has 6 configuration options: audio (volume level), language, speech synthesizer speed, significant numbers, decimal numbers, and fractional numbers. It also has an option for users to know and memorize the location of each of the keys.

3) Equations: it allows finding the roots of a polynomial and calculates the root of an expression in a given interval. It also allows solving a linear system of up to 10 equations.

4) Algebra: this module aims to facilitate the user to solve mathematical problems related to linear algebra. The user will be able to generate and introduce a matrix of dimensions $n \times m$. The user can also invert a matrix, find its determinant, find the adjoint matrix or find its transposed form.

5) Calculus: allows the user to perform advanced mathematical operations such as determining the LaPlace transform (direct or inverse), developing a Taylor series, or performing differential and integral calculus operations on a variable.

6) Simplify: this module helps the user reduce mathematical expressions that initially contain several terms or complex expressions into shorter or simplified expressions. It also allows factoring algebraic expressions.

7) Graphics: this module focuses on the sound presentation of some mathematical functions. Play non-speech files that vary over time. The graph of the function is interpreted in a 2D plane.

8) Help: offers the user information on the operation of each module and the System’s functions or sub-modules. The user will find general descriptions and examples of the Casvi system.

**TABLE 4. Keys combinations for entering interactive menus.**

| Menu | Keys |
|------|------|
| File | alt + a |
| Casvi | alt + s |
| Simplify | alt + d |
| Calculus | alt + f |
| Graphic | alt + g |
| Help | alt + h |
| Algebra | alt + j |
| Equations | alt + k |

Table 4 shows the key combinations that the user must press to enter each of the interactive menus of the Casvi system.

Two figures are under a Unified Modeling Language (UML) (Fig. 3 and Fig. 5). The first (see Fig. 3) refers to a Component Diagram detailing its main entities and their communication.

Casvi includes the Mathematical Expressions Editor as the primary entity. In the Component Diagram’s left part, there is a connection to Maxima’s complex expression manipulation system. Maxima is a computer algebra system that produces highly accurate results. In addition, it has the great advantage that its source code can be compiled in the operating systems with support in the Windows and Linux community, which has allowed it to be included as a mathematical engine within the Casvi system. Next, there is the Logical component Math Expression, which represents the Casvi command input console. This console has been designed following the same structure as other mathematical programs such as MATLAB. On the left side, there is the text editor where
all the mathematical operations are performed. On the right side, there are three windows: the first and second shows the history of the operations carried out (the second in Braille) and the third, a sequence of the results obtained. The above can be visualized in Fig. 4.

Next to the Mathematical Expressions Editor are the Casvi system outputs for the Human-Machine interaction. First, the “Synthetic Voice and Non-Speech-sound” module has been made under mature technology (TTS) in .NET. The System.Speech library that was used has methods to personalize audio parameters such as synchronous or asynchronous output, speed, volume, and language. Second, the export of files in two formats is suitable for the target audience: Plain text (which can be modified under any text editor) and Braille. The latter was programmed through a letter-to-letter translation, defined as constants within an interface in class design, giving the possibility that sighted and blind people can review the code made. Lastly, Casvi has the ability to generate the MathML code for each of the operations performed (one after another).

On the other hand, an Activity Diagram is presented in which the actors and their main actions are detailed. An Activities Diagram that allows visualizing the control flow (sequences) followed by the System’s actors. Two actors have been defined: the first and intuitively, people with visual disabilities (VIPs), and a second actor called System, which is the internal entity of the system. The System actor was designed in order to accurately program the internal processes of the Casvi system, such as validations, error control, connection, and interfaces. The main actor presents actions within its 8 modules, and as a representative example, a diagram corresponding to the Calculation module, Fig. 5. This Activity Diagram shows that if a user wants to calculate the derivative of $\sin(x)$, the user can do it in 2 ways:

- First, through its text editor that allows the entry of numbers, letters, symbols, mathematical expressions, and commands supported by the system. Several of the mathematical operations performed by this computer tool have the structure shown in Fig. 6. In this way, the user should type `diff(sin(x),x)` (see Fig. 7), where `diff` implements the differential operator.
- The second, with interactive voice-assisted menus (available in Portuguese, English, and Spanish) that guide the user during the development of a mathematical expression.
calculation. In this way, the user must press “Alt + F” to activate the interactive menu “Calculus”, navigate through the menu (with the keyboard arrows) and choose the “Differentiate” function. The user must enter the variable and its expression (see Fig. 8).

It is emphasized that each mathematical operation entered by the user is validated before being executed by the Casvi system. Also, an error message will be generated if the required parameters have been entered incorrectly (e.g. incorrect syntax); otherwise Casvi will generate the corresponding code so that Maxima can interpret it, evaluate it, and deliver a result once the operation is executed.

Finally, the system proceeds to call the Maximum engine for mathematical evaluation. The system sends its response to both the Text and Audio Editor and waits for a new call again. It is worth emphasizing that as the Activity Diagram shows, each user action will call the System.Speech library to generate audio that guides the user in each operation. The generation of synthetic voice corresponds to 3 well-defined stages: Text Analysis, Prosody Analysis, and Voice Synthesis. In addition to assigning syntactic categories to words, the first stage identifies and expands abbreviations, recognizes and analyzes mathematical expressions. The second stage aims to avoid ambiguity by assigning phonological characteristics, for example, those that have a variable relationship with words: frequency (that is, the perception of pitch), amplitude (that is, loudness), and time (that is, signal duration). A concatenated speech synthesis is used to generate the final speech signal. In addition to speech, non-speech sounds are used (that is, lexical signals) that have been shown to be effective in transmitting information to people with visual disabilities regarding the structure of mathematical expressions.

Next, we show an example of how Casvi deals with a complex algebraic expression depicted by Equation 1, which is composed of three sub-expressions labeled as (a), (b) and (c):  

\[
\frac{2x}{x-1} + \frac{x-1}{x+2} = 2x
\]

Equation 1 is presented in a linear way in Equation 2:

\[
2x/((x-1)/(x+2)) + ((x-1)/(x+2))/\sqrt{x+1} = 2x
\]

The procedure to enter this equation is described in the Tables 5, 6, and 7. First, is entered the sub-expression (a).

| Step | User action | Auditory feedback |
|------|-------------|-------------------|
| 1    | Press F9    | "Fraction, enter the numerator" |
| 2    | Press F9    | "Fraction, enter the numerator" |
| 3    | Type (x−1)  | "Open parenthesis, x, minus, one, close parentheses" |
| 4    | Press TAB   | "Enter the nominator" |
| 5    | Press TAB   | "Enter the nominator" |
| 6    | Press ENTER| "Fraction entered" |
| 7    | Press ENTER| "Fraction entered" |

Are detailed the steps to enter this sub-expression in Table 5. Column 2 (User Action) of Table 5 specifies each of the keys that the user must press to enter sub-expression (a). Each time the user presses the indicated key (or combination of keys) in column 2, they will receive the Auditory Feedback indicated in column 3.

Similarly, the steps to enter the sub-expression (b) are detailed in Table 6.

Once the sub-expressions (a) and (b) have been entered and stored (see Fig. 9), Table 7 details the additional steps that the user must perform to enter equation 1 completely.

It is important to emphasize that this example presents the entry of a complex algebraic expression. Therefore, the user must carry out a process of prior learning of the operation of the Casvi system. Nevertheless, the result of entering Equation 1 linearly through the Casvi system is showed in Fig. 10.

V. EXPERIMENTS

This section describes the experiments performed to test and verify the operation of the Casvi system. The test group is described in Section V-A. The research protocol is presented
in Section V-B. The test protocol is described in Section V-C. In addition, in Section V-D, the user test developed is detailed.

A. TEST GROUP
A total of 25 VIPs participated in the experiments carried out with the *Casvi* system; 7 female persons (28%) and 18 male persons (72%) (see Fig. 11).

Additionally, in Fig. 12 it can be seen that 12 people in the test group have primary studies (48%), 10 with secondary studies (40%) and only 3 people have third-level studies (12%).

B. RESEARCH PROTOCOL
The research protocol implemented for the execution of the experiments with the *Casvi* system was submitted and approved by the Ophthalmology Unit, and the Research Ethics Committee in Human Beings of the Carlos Andrade Marín Specialty Public Hospital.

C. TEST PROTOCOL - EXECUTION OF MATHEMATICAL OPERATIONS
1) OBJECTIVE
To test if and how the *Casvi* computational algebraic system developed for learning mathematical analysis can facilitate the execution of mathematical operations for engineering students with different degrees of visual impairment.

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1 Title: Contribution in the academic training stage of students with visual impairment through the use of a computer program for learning mathematical analysis. Approved 26/03/2020.
2) HYPOTHESIS
- $H_0$: The user cannot perform mathematical operations through the computational algebra system.
- $H_1$: The user can perform mathematical operations through the computational algebra system.

3) VARIABLES
- Independent: mathematical operation complexity.
- Dependent: time to perform the execution of the mathematical operation.

4) ELIMINATION OF BIASES
While the participant takes the tests of execution of mathematical operations, the other participants will receive an induction regarding the tests to be carried out. After carrying out the tests, the participant will evaluate the computational algebraic system.

5) MATERIALS
- Registration sheet
- Informed consent form
- User manual
- List of mathematical operations. A list with more than 100 mathematical operations was implemented. These are divided into 3 degrees of difficulty:
  - Level I (low difficulty): basic operations (addition, subtraction, multiplication, division, square root, factorial, among others), operations with fractions, operations with trigonometric functions (sine, cosine, tangent) and, compound operations (conformed by basic operations, fractions, and trigonometric functions).
  - Level II (medium difficulty): operations with polynomials, factoring, matrices, derivatives, and integrals.
  - Level III (high difficulty): simplify radicals; expand logarithms, LaPlace transform, partial fractions.
- Computer with Casvi system installed
- Headphones
- Video camera
- Photographic camera
- Chronometer
- Results sheet
- Evaluation system to perform the Self-Assessment Manequin-based test

6) STEP-BY-STEP
1) Verify the number of participants. Where $P$ is the total number of participants.
2) Make an explanation to all participants of the tests to be carried out with the software.
3) Select a participant ($P_n$) to perform the software tests.
4) ($P_n$) will be instructed so that the participant can perform and start (without any problem) the respective software tests.
5) It will indicate the mathematical operations\(^2\) that the participant must execute.
6) The chronometer will be activated the moment that ($P_n$) is ready to start the execution of the mathematical operations using the computational algebraic system.
7) When ($P_n$) finishes the execution of the mathematical expressions, the chronometer will be stopped.
8) Which mathematical expressions were executed correctly and incorrectly will be determined.
9) Repeat steps 3 to 9 for all $n$ participants who have not yet test the Casvi system.

7) SCHEDULE
- Presentation and signing of the informed consent form (20 minutes).
- Explanation of how the program works (30 minutes)
- Test (30 minutes).
- Final discussion (15 minutes).

8) METRICS
- For each $P_n$ check the time it took to execute all the mathematical expressions, (it does not matter if it was done correctly or not).
- How many and which of the mathematical expressions were executed correctly and incorrectly.

D. USER EXPERIENCE EXPERIMENT (USABILITY TEST)
In search of new forms of inclusive assessment, in 1994 an emotional measurement method called SAM (Self-Assessment Manikin) was developed [41]. This method consists of a nonverbal pictorial rating technique that directly measures the pleasure, arousal, and dominance that a person experiences over a wide spectrum of stimuli that requires 18 different ratings [42]. The system has come to be considered as a valid qualification tool for products or services; it can also present symbolic variants depending on its application.

An evaluation system based on the SAM system was implemented to assess the user experience with the Casvi system. This evaluation system was designed so that VIPs can use it with or without knowledge of the Braille language. Furthermore, this evaluation tool allows the evaluator to enter or edit the questions to be asked; these can be entered in three languages Spanish, English, or Portuguese. Fig. 13 shows the evaluation system implemented. The device has two modes for use:
- Braille text: raised typeface to be recognized by a user with knowledge of Braille writing, and
- Buttons in high relief: emoticons to relate the level of acceptance in each question.

\(^2\)In this work, experiments were carried out only with mathematical operations of the level of difficulty I.
FIGURE 13. Evaluation system based on the SAM system.

1) EVALUATION PROCESS

Levels of pleasure, interest, and dominance, on the part of the users, were evaluated. A scale from 1 to 5 was used; 1 being the lowest weight and 5 the highest. Nine people participated in this test. The questions asked were the following:

1) Do you think that the Casvi system arouses interest?
2) Do you consider Casvi to be innovative?
3) To what extent do you consider the Casvi system to be intuitive?
4) Do you consider that the Casvi system is a useful tool in the mathematical learning process?
5) How did you feel about learning the Casvi system?
6) Do you consider that the learning time was optimal?
7) How would you rate the method of entering variables to the program?
8) How would you rate the inclusion of this teaching method for schools and colleges?

VI. RESULTS

A. EXECUTION OF MATHEMATICAL OPERATIONS

In the first session, 24 people tested the Casvi system; each one performed the input and execution of 10 mathematical operations. The distribution of the operations carried out is presented in Table 8.

TABLE 8. Distribution of operations carried out.

| Operations    | Percentage (%) |
|---------------|----------------|
| Basics        | 30             |
| Fractions     | 18             |
| Trigonometric | 7              |
| Compound      | 45             |

Two hundred forty operations were performed. Of these, users performed 109 compound operations. In Table 9, it can be seen that most of the basic operations performed by users were done correctly (of 73 operations executed, only 3 were done incorrectly). In addition, although only 16 operations of evaluation of trigonometric functions were carried out, 25% of these were carried out incorrectly. In general, 92% of the operations carried out were done correctly (220 out of 240 operations in total).

Fig. 14 shows that the median of the execution time of basic mathematical operations is the lowest for the execution time of the other three operations performed by users. No outliers were presented in the execution time of basic operations and trigonometric functions. Likewise, both in the execution time of operations with fractions and the execution time of compound operations, atypical values were presented; 2 and 5 respectively. Also observed was that users found it challenging to execute trigonometric operations, which is reflected in the time it took them to carry out this type of operation.

The shortest execution time of a mathematical operation was 5s, corresponding to the execution of a basic operation and a maximum time of 251s, which corresponds to the execution of a compound operation; an average of 67s. Of the 220 operations carried out correctly, only 8 operations present atypical execution times (see Fig. 15).

FIGURE 14. Median execution time of all mathematical operations.

In each box, the middle mark indicates the median, and the bottom and top edges of the box indicate the 25th and 75th percentiles, respectively. Whiskers are extended to the most extreme data points that are not considered outliers, and outliers are plotted individually using the symbol ‘+’.

FIGURE 15. Median execution time of operations performed correctly and incorrectly.
Likewise, Fig. 16 shows that of the 24 users, 50% correctly carried out 100% of the operations assigned to each one of them.

More than 50% of users have a median greater than 60s. User 15 presents the lowest value, 23.5s. The latter performed 100% of the mathematical operations correctly; performing nine of them between 8 and 29s, only presenting an outlier outside the 67s range (see Fig. 17).

A person with visual impairment (see Fig. 18) performed mathematical operations with the Casvi system and with the LAMBDA system in a second session. Table 10 presents the results obtained.

In Fig. 19, it can be seen that the reduction in the execution time of mathematical operations with the Casvi system is statistically significant (with a confidence level of 95% and p value = 0.0159) concerning the LAMBDA system. The execution time was reduced on average from 84.4s to 34s, which is a reduction of 59.7%. It is worth mentioning that the user who carried out the tests with the two systems has solid knowledge regarding the subject of mathematics (the user currently teaches mathematics to people with visual disabilities). Also, the user has experience in the use of computer tools for people with visual disabilities (including the LAMBDA software), so he had no problems in entering the dictated operations. Furthermore, it can be observed that the shortest time (53s) that the user required to perform a mathematical operation with the LAMBDA system is greater than the longest time (43s) that the user required to perform the same operations with the Casvi system.

Once this test session was finished, the user made some suggestions about the Casvi system. The user mentioned that it would be of great help if the Casvi system allows breaking down (step-by-step) the procedure for solving the mathematical operations performed. Furthermore, he encountered a certain degree of difficulty when accessing the system menus.
TABLE 11. Usability test results.

| Questions                                                                 | Metrics   | Results (scale over 5) |
|---------------------------------------------------------------------------|-----------|------------------------|
| 1. Do you think that the Casvi system arouses interest?                   | Arousal   | 4.11 (σ = 0.6)         |
| 2. Do you consider Casvi to be innovative?                                | Dominance | 2.67 (σ = 0.5)         |
| 3. To what extent do you consider the Casvi system to be intuitive?      | Dominance | 2.89 (σ = 0.6)         |
| 4. Do you consider that the Casvi system is a useful tool in the mathematical learning process? | Dominance | 3.33 (σ = 0.7)         |
| 5. How did you feel about learning the Casvi system?                      | Pleasure  | 4.11 (σ = 0.3)         |
| 6. Do you consider that the learning time was optimal?                    | Pleasure  | 1.89 (σ = 0.3)         |
| 7. How would you rate the method of entering variables to the program?   | Pleasure  | 2.78 (σ = 0.4)         |
| 8. How would you rate the inclusion of this teaching method for schools and colleges? | Dominance | 4.11 (σ = 0.6)         |

FIGURE 19. Median execution time of mathematical operations with the Casvi system and the LAMBDA system.

via the “Alt” key; since, on certain occasions, he had to place one hand on the other when requesting access to Casvi’s menus. As a suggestion, he mentioned that the “Alt Gr” key should be configured to fulfill the same function as the “Alt” key within the Casvi system whenever possible.

B. USER EXPERIENCE

Table 11 presents the results obtained. It can be seen that the value of Arousal is high, which confirms that the Casvi system arouses interest in the target public. Likewise, the Dominance mean is low (3.25), which is related to the learning curve (it describes the degree of success obtained during learning over time), making it clear that the longer the learning time, the better results achieved. Finally, the lowest mean was pleasure. Because the Casvi system is relatively new and a tool that allows the resolution of advanced mathematical operations, the user must have previous knowledge or a learning process related to math.

VII. DISCUSSION

The system has been designed to take advantage of previous works cited in the state of the art. In the first instance, Casvi makes use of earcons (non-speech sounds) to represent specific events (i.e., start and end of line.). Prior works [28], [31], [37] focus on the execution of basic mathematical operations, while Casvi allows the execution of basic and advanced mathematical operations (i.e., algebra, linear algebra, differential, and integral calculus).

Second, one of the advantages of Casvi is that it can be installed and used simply and quickly, without requiring additional software resources such as NVDA or JAWS screen readers (such as [22], [23]), which, in many cases, it would become a limitation. However, for test users who maintain an experience using screen readers, they argue that the Casvi system could improve even more if it were 100% compatible with screen readers.

Finally, as has been observed in the results section, Casvi has managed to surpass one of the most representative works in the area, LAMBDA. Casvi improves on average execution times of mathematical operations by almost 60% to the values obtained by LAMBDA. In terms of usability, Casvi has been fairly well evaluated obtaining a 3.2/5 average. This might be explained by the fact that Casvi requires a learning curve. However, we think that the learning effort is worth it, considering the benefits of solving complex mathematical operations. Note that this complex math were previously unavailable for VIPS and even with the advantage of seeing, it is not straightforward to operate over complex math expressions.

VIII. CONCLUSION

In Ecuador, 37.9% of the population with disabilities have not received formal education and only 1.8% have had access to higher education. In this way, it is a challenge for people with visual disabilities to study engineering and complete and get the degree. It is worth mentioning that there are 9,304 people with visual disabilities active in the workplace in Ecuador. There are 298 working people with a degree of visual impairment from 85% to 100%, which indicates that only 3.20% of this group of people have a job.

This work presents the Casvi Computational Algebraic System as a computer aid tool for VIPs in their higher education stage. Researchers in Brazil and Ecuador have designed Casvi. The system has mainly been developed to take care of the user’s stimuli, with simple shortcuts and outputs in audio and Braille format. Casvi, implemented on Visual Studio in an Object-Oriented language (C#), maintains a permanent connection with the Maxima mathematical engine which allows evaluating advanced mathematical expressions.

Our system presented encouraging results in terms of functionality and usability, as evidenced by the experimental
results. Furthermore, the execution time of a mathematical operation in our system is statistically lower than that of the LAMBDA system, which shows the potential of the Casvi system to execute basic and advanced mathematical operations efficiently such as Linear Algebra, Differential, and Integral Calculus, among others.

This tool is the beginning of an investigation favoring of social inclusion, whose primary purpose is to reduce the percentage of student dropouts of blind people in engineering careers due to the lack of accessible computational tools. The inclusion of computational tools such as the Casvi system within the VIPs’ academic training stage will allow the student dropout rate to decrease over the years. In addition, higher education institutions will benefit from this type of tool which will collaborate in the incorporation of this group of people into the professional field.

Finally, the implementation of this type of assistive technology (AT) will allow this group of people to get involved in the development of the same activities carried out by people without any type of disability. Casvi seeks to improve the self-esteem of students with visual disabilities, which can be affected when the blind student cannot carry out any activity at the same speed or with the same efficiency as their sighted peers.

A. FUTURE WORK
In light of this work, we have identified the following possible research lines to improve how VIPs deals with solving math operations.

1) PERFORM NEW EXPERIMENTS
A list with more than 100 mathematical operations that can be executed with the Casvi system was implemented; divided into 3 levels of difficulty (low, medium and high). For the execution of the experiments in this work, only low-level difficulty level mathematical operations were considered. In the future, it is proposed to carry out experiments with mathematical operations of medium-level difficulty operations such as polynomials, factorization, matrices, derivatives, or integrals and high-level difficulty (e.g. simplify radicals, expand logarithms, Laplace transform, or partial fractions).

2) WEB APPLICATIONS
Nowadays computer systems such as MATLAB and Mathematica present online web applications that can be accessed through a standard web browser. Making the Casvi system an online tool is a great challenge allowing, e.g., to keep the system up to date automatically.

3) SPEECH CONTROL FUNCTION
The Casvi system has been designed and implemented so that 100% of its functions are activated through the computer keyboard. Every time the user presses a key or a combination of keys, they receive Synthetic Speech feedback. In a new version of our system, the aim is to implement the speech control function, where the user can activate the functions of the Casvi system through voice input.

4) JUPYTER-LIKE NOTEBOOKS
Finally, as a major challenge, it is intended that this system can act as a Jupyter Notebook where text, images, code, and mathematical operations can be combined in a single document.

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