The Effect of Neutron Decaying to Dark Matter on Properties of Neutron Stars with Hyperons

A B Wahidin\(^1\) and A Sulaksono\(^1\)

\(^1\)Department of Physics, Faculty of Mathematic and Natural Science, University of Indonesia, Depok, Jawa Barat, 16424, Indonesia

E-mail: anto.sulaksono@sci.ui.ac.id

Abstract. It has been reported that there is a discrepancy of 4\(\sigma\) in the neutron lifetime measurement with two methods, i.e., the trap method (measuring the number of neutrons remaining in the various time intervals) and beam method (measuring the number of protons formed from regulated neutrons currents). Fornal and Grinstein (2019) proposed that the possibility neutrons decay into the dark matter can explain this discrepancy. In this study, the author asses the consequence of the proposal by studying the impact on neutron stars with Hyperon. Based on our numerical calculation we found that dark matter may be present in the core of a neutron star because it only appears at very high densities, and the population is negligible with a maximum population of about 0.1\%

1. Introduction

Dark matter (DM) recently becomes a hot topic in the field of astrophysics and particle physics. From various observations, confirmed quite convincingly, dark matter is the dominant material in the universe. Based on data taken from the Review of Particle Physics 2014, the universe consists of 68.3% dark energy, 26.8% dark matter, and only 4.6% known baryonic matter [1]. However, the properties of dark matter such as mass and its interactions are still unknown. Figure out how to detect dark matter directly or indirectly is still an open challenge today.

Recently, discussions about the lifetime of free neutrons can lead us to an alternative way to detect dark matter. The lifetime of free neutrons can be measured by two methods, i.e., traps and beams. The trap method is carried out by trapping a cold neutron on a magnetic trap, then measuring how many neutrons lost within various time. Whereas, the beam method measures the number of protons produced during \(\beta\) decay emitted from cold neutrons. However, both methods provide different lifetime measurement results, which are \(\tau_{\text{trap}} = (879.6 \pm 0.6)\) and \(\tau_{\text{beam}} = (888.0 \pm 2.0)\). Measurement with both methods differs as far as 4\(\sigma\) (see [4] for further study). Previously, this topic did not become a big concern because of the limited accuracy of the measurement instruments. Recently, the accuracy of measurement instruments is getting more reliable. The discrepancy in the lifetime measurements of free neutrons with two kinds of methods is becoming more definite. So we require an explanation of this problem.

B. Fornal and B. Grinstein [4] proposed an exciting proposal. They state that there may be another decay channel alongside \(\beta\) decay that is not detected by the detector, i.e., the possibility of the neutron decaying into dark matter, see Refs. [4] – [8] for further explanation. This event will henceforth refer to as dark decay. This exciting proposal has triggered several studies on the effect of
neutron decay into DM on neutron stars [9] [10] [11]. In this work, the authors will inform our study of the effects of dark decay on neutron stars properties. In this case, the effects of hyperons on neutron stars also are considered.

2. Equations of State of Neutron Stars

Neutron stars with hyperons are described with Lagrangian density. The matter composed of nucleon fields $\psi$, scalar meson fields $\sigma$, vector meson fields $\omega$, isovector meson fields $\rho$, isoscalar meson fields $\delta$, neutral strange scalar meson fields $\sigma^*$ and neutral strange vector fields $\phi$. The explicit form of the Lagrangian is [14, 15, 16]

$$L_{ns} = L_B + L_M + L_{int} + L_L$$

where $L_B$ is the Lagrangian from the baryon field, $L_M$ is the Lagrangian from the meson field, $L_{int}$ is the Lagrangian from the interaction term, and $L_L$ is the Lagrangian from the lepton. Lagrangian for baryon is as follows

$$L_B = \sum_B \psi_B \left( i \gamma_{\mu} \partial_{\mu} - m_B \right) \psi_B,$$

where we add up all the baryon octets (n, p, $\Lambda$, $\Sigma^{-,0,+}$, $\Xi^{0,-}$). Lagrangian for meson fields is

$$L_M = L_\sigma + L_\omega + L_\rho + L_\delta + L_{\sigma^*} + L_\phi,$$

where

$$L_\sigma = \frac{1}{2} \left( \partial_{\mu} \sigma \partial^{\mu} \sigma - m_\sigma^2 \sigma^2 \right),$$

$$L_\omega = \frac{1}{2} \frac{1}{\omega_{\mu\nu}} \omega^{\mu\nu} - m_\omega^2 \omega_{\mu\nu} \omega^{\mu\nu},$$

$$L_\rho = \frac{1}{2} \frac{1}{\rho_{\mu\nu}} \rho^{\mu\nu} - m_\rho^2 \rho_{\mu\nu} \rho^{\mu\nu},$$

$$L_\delta = \frac{1}{2} \left( \partial_{\mu} \delta \partial^{\mu} \delta - m_\delta^2 \delta^2 \right),$$

$$L_{\sigma^*} = \frac{1}{2} \left( \partial_{\mu} \sigma^* \partial^{\mu} \sigma^* - m_{\sigma^*}^2 \sigma^{*2} \right),$$

$$L_\phi = \frac{1}{2} \frac{1}{\phi_{\mu\nu}} \phi^{\mu\nu} - m_{\phi}^2 \phi_{\mu\nu} \phi^{\mu\nu},$$

while for lepton ($e^-, \mu^-$)

$$L_L = \sum_L \psi_L \left( i \gamma_{\mu} \partial_{\mu} - m_L \right) \psi_L.$$

For the interaction terms,

$$L_{int} = - \sum_B g_\sigma \sigma \tilde{\psi}_B \psi_B - \sum_B g_\omega \omega_{\mu} \tilde{\psi}_B Y_{\mu} \psi_B - \sum_B g_\rho \rho_{\mu} \tilde{\psi}_B Y_{\mu} \tilde{\psi}_B$$

$$- \sum_B g_\delta \delta \tilde{\psi}_B \tilde{\psi}_B - \sum_B g_{\sigma^*} \sigma^* \tilde{\psi}_B \psi_B - \sum_B g_{\phi} \phi_{\mu} \tilde{\psi}_B Y_{\mu} \psi_B - \frac{1}{3} b_2 \sigma^3$$

$$- \frac{1}{4} b_3 \sigma^4 + \frac{1}{4} c_3 (\omega_{\mu} \omega_{\mu})^2 + d_2 \sigma (\omega_{\mu} \omega_{\mu}) + \frac{1}{2} d_3 \sigma^2 (\omega_{\mu} \omega_{\mu})$$

$$+ f_2 \sigma (\tilde{\rho}_{\mu} \cdot \tilde{\rho}_{\mu}) + \frac{1}{2} g_3 \sigma^2 (\tilde{\rho}_{\mu} \cdot \tilde{\rho}_{\mu}) + \frac{1}{4} g_4 (\omega_{\mu} \omega_{\mu}) (\tilde{\rho}_{\mu} \cdot \tilde{\rho}_{\mu}).$$
The meson field derivatives are
\[\omega_{\mu\nu} = \partial_{\mu}\omega_{\nu} - \partial_{\nu}\omega_{\mu},\]
\[\rho_{\mu\nu} = \partial_{\mu}\rho_{\nu} - \partial_{\nu}\rho_{\mu},\]
\[\phi_{\mu\nu} = \partial_{\mu}\phi_{\nu} - \partial_{\nu}\phi_{\mu}.\]  (12)

To model DM in stars, a Lagrangian formulation with DM fields \(\chi\), meson scalar fields \(\zeta\), and DM meson vector fields \(\eta\) is written as follows
\[\mathcal{L}_\chi = \bar{\psi}_\chi (i\gamma_\mu \partial^\mu - \mathbf{m}_\chi) \psi_\chi + \frac{1}{2} \left( \partial_\mu \zeta \partial^\mu \zeta - m_\zeta^2 \zeta^2 \right) - \frac{1}{2} \left( \eta_{\mu\nu} \eta^{\mu\nu} - m_\eta^2 \eta \eta \right),\]  (13)
where \(\eta_{\mu\nu} = \partial_{\mu}\eta_{\nu} - \partial_{\nu}\eta_{\mu}\). The Lagrangian above is similar as the Lagrangian in Dark Star case [17]. Then the total Lagrangian is
\[\mathcal{L} = \mathcal{L}_{\text{ns}} + \mathcal{L}_\chi.\]  (14)

We obtain the equation of state of neutron star matter with hyperons and dark matter using standard relativistic-mean-field (RMF) approximation method, and from the energy-momentum tensor, we can extract energy density and pressure. The equation of state is obtained from the relationship between energy density and pressure.

3. Chemical Potential Equilibrium in Neutron Stars
In the chemical potential equilibrium, \(\beta\) decay no longer occurs, and so does the inverse \(\beta\) decay. Hyperon in the star’s core will be stable and will not decay into baryons and other particles. The system will be stable if it is in the chemical potential equilibrium. In general, the chemical potential equilibrium conditions are written as
\[\mu_i = B_i \mu_n - Q_i \mu_e\]  (15)
where \(B\) stands for the baryon charge, \(Q\) is the electric charge and index \(i\) for each baryon. Then we obtained the potential chemical relationship
\[\mu_\Lambda = \mu_\Sigma^0 = \mu_\Xi^0 = \mu_n,\]
\[\mu_\Sigma^+ = \mu_\mu - \mu_e,\]
\[\mu_\mu = \mu_e,\]
\[\mu_\Sigma^- = \mu_\Xi^- = \mu_n + \mu_e.\]  (16)
where the chemical potential is determined from
\[\mu_i = \frac{\partial \epsilon}{\partial \rho_0}, \quad (i = n, p, \Lambda, \Sigma, e, \mu, \chi).\]  (17)

From these equations, we can get the explicit form of chemical potential which contains the term \(k_F^2\). Since \(k_F^2 = (3\pi^2 \rho_0)^{2/3}\), the explicit form of numbers density \(\rho_0\) can be calculated.

Now for the DM, from several proposals submitted by B. Fornal and B. Grinstein [4], we will take a scenario \(n \rightarrow \chi + \phi\), where \(\phi\) is a dark boson that has a very little mass. Then the mass of the DM particle must be in the range of
\[937.900 \text{ MeV} < m_\chi + m_\phi < 939.565 \text{ MeV}.\]  (18)
Motta, Guichon, and Thomas [18] argue that the existence of a dark boson is irrelevant and will leave neutron star quickly. So the chemical potential equilibrium of the reaction $n \leftrightarrow \chi$ is written by the equation $\mu_n = \mu_\chi$. So we obtain

$$\rho_0^\chi = \frac{1}{3\pi^2} \left\{ \left[ \mu_n - \frac{g_\eta^2}{m_\eta^2} \rho_0^\chi \right]^2 - m_\chi^2 \right\}^{3/2}. \quad (19)$$

Where $\rho_0^\chi$ is the number density of dark matter and the effective DM masses is $m_\chi^* = m_\chi + g_\zeta \zeta$. The equation above is self-consistent and be solved numerically to determine the number density of DM particles in the potential equilibrium. For this case, the neutron star we consider is neutral so we can add the constraint to determine the number density from the component such as

$$\rho_0^p = \rho_0^e + \rho_0^\mu + \rho_0^\tau - \rho_0^{\bar{\mu}} - \rho_0^{\bar{\tau}}. \quad (20)$$

We obtain the fraction of each component from $Y_i = \rho_0^i / \rho_0^{tot}$ where $\rho_0^{tot} = \sum_i \rho_0^i$.

4. DM Cross Section

We model dark matter as asymmetric fermionic DM, which interacts by exchanging scalar boson and vector boson. The cross-section is used to obtain the constraint for the coupling constant of DM particle with scalar and vector bosons. First, we consider the interaction as in the Feynman diagram in the following figure.

![Feynman Diagram](image)

**Figure 1.** (Upper panel) $t$-channel and $u$-channel for scalar boson mediators. (Lower panel) $t$-channel and $u$-channels for vector boson mediators.
Because at the non-relativistic limit, \( s = 4M_X^2 \), \( t = 0 \), and \( u = 0 \). Then the total cross-section becomes

\[
\sigma_{\text{tot}} = \sigma_s + \sigma_v = \frac{M_X^2}{16\pi} \left( \frac{7g_s^4}{m_s^4} + \frac{3g_v^4}{m_v^4} \right).
\]

(21)

5. Results and Discussion

We take extreme values from the constraints to determine DM parameters. The parameters varied are DM mass \( m_\chi \), DM scalar coupling strength \( g_\zeta \), and DM vector coupling strength \( g_\eta \). For the mass of scalar boson’s mediator \( m_\zeta \) and vector boson’s mediator \( m_\eta \), we use fixed mass. From equation (21), we calculate the coupling constant required to get the values \( \sigma_\chi/m_\chi \) obeying the constraint \( 0.1 \leq \sigma_\chi/m_\chi \leq 10 \text{ cm}^2/\text{g} \) [6]. All of these EOS are varied to the two extreme values of DM mass constraints proposed by Fornal and Grinstein [4]. More details can be seen in the Table 1.

**Table 1.** The variation of parameters used in the fermionic dark matter model. The masses of dark scalar and vector mediator are fixed to the value \( m_\eta = m_\zeta = 10 \text{ MeV} \).

| EOS | \( \sigma_\chi/m_\chi \) (cm\(^2\)/g) | \( m_\chi = 939.565 \text{ MeV} \) | \( m_\chi = 937.900 \text{ MeV} \) |
|-----|--------------------------------------|-----------------------------------|-----------------------------------|
| I   | 10                                   | 0.300594997                       | 0.300728316                       |
| II  | 0.1                                  | 0.095056484                       | 0.095098644                       |
| III | 10                                   | 0.243213430                       | 0.243321299                       |
| IV  | 0.1                                  | 0.076910840                       | 0.076944951                       |
| V   | 10                                   | 0.222465189                       | 0.222563855                       |
| VI  | 0.1                                  | 0.070349670                       | 0.070380871                       |

From our numerical calculation results, we get the fraction of the particle population shown in figure 2 and figure 3. In figure 2, we utilize the permitted upper mass limit \( m_\chi = 939.565 \text{ MeV} \), whereas, for figure 3, the authors use the permissible lower mass limit \( m_\chi = 937.900 \text{ MeV} \). It appears that variations from EOS I to EOS VI did not produce a significant contrast. With the variation of dark matter masses in that range, we also cannot see any significant differences in results. In general, we observed that the contribution of DM in the neutron star is quite small. Apart from that, the DM emerge at a very high density which is \( \rho/\rho_0 \approx 6.7 \). With \( \rho_0 = \rho_s = k_F^2/3\pi^2 \) is the nuclear saturation density and Fermi’s momentum at nuclear saturation density is \( k_F = 1.30 \text{ fm}^{-1} \). So that indicates that DM may be trapped inside the neutron stars core at very high density if the dark decay exists in nature.

In figure 4, we show the equation of state of neutron stars with the effect of dark decay. We present the plot of the equation of state separately between the contribution of baryonic matter and dark matter. So we can comprehend how significant the effect of dark decay on the energy density of the equation of state. It appears that the contribution of DM is too small to see the hints of dark matter existence. This result is due to the small number of DM arising from the effect of dark decay. So we can state that the effect of dark decay is not significant in neutron stars and the signature of the existence of dark decay is difficult to detect from NS observations.
Figure 2. Particle species fraction appearing in neutron star matter in $\beta$ and chemical equilibrium for case EOS V (left) and VI (right) for DM masses $m_\chi = 939.565$ MeV.
Figure 3. Particle species fraction appearing in neutron star matter in $\beta$ and chemical equilibrium for case EOS I (left) and II (right) for DM masses $m_\chi = 937.900$ MeV.
Figure 4. The equation of state of neutron stars with hyperon by considering the dark decay effect. The $\varepsilon_B$ stands for the contribution of baryonic matter. We see that the contribution of DM on the EoS is negligible to the total equation of state in the neutron stars matter.

Figure 5. The gravitational mass of neutron star (Left panel) and the ratio of gravitational mass with baryonic mass as a function of energy density at the center of the star (Right panel). The tiny dot at the end of the curve is the maximum mass that represents the stability boundary of the star.

Figure 6. The correlation between energy density and number density. The maximum mass is in the energy density $\varepsilon_c \approx 1335$, which correlated to number density $\rho/\rho_0 \approx 7.75$.

In figure 5, we investigated the stellar stability limits. The underlying motivation is that if a star has a stability limit $\rho/\rho_0 < 6.7$, then there will be no DM in neutron stars. Vice versa, if the stability limit is at $\rho/\rho_0 > 6.7$, then DM might be in neutron stars. We found that the maximum mass of the
DS relates to the energy density at the center of the star of $\varepsilon_c \approx 1335$. By looking at the relationship between the energy density and the number density, as shown in figure 6, we found that the stability limit of this neutron star is at $\rho/\rho_0 \approx 7.57$. As shown in figure 7, the shaded area is an unstable part. Then it is found that the DM fraction is around $Y_\chi \approx 10^{-4}$. However, if DM is present inside a neutron star, then the population is less than approximately 0.1%.

![Figure 7](image)

**Figure 7.** The fraction of the particle population in the neutron stars in logarithmic scale. The shaded area represents the unstable region. It means that the maximum fraction of the DM is approximately $Y_\chi \approx 10^{-4}$.

6. Conclusion

We have investigated the effects of dark decay on neutron stars with hyperons. We apply the variation of parameters using the combination of minimum and maximum value of the allowed interaction strength by the cross-section constraint and the mass of dark matter from the proposal of Fornal and Grinstein. However, based on the results of the particle fraction of the population obtained, we do not get a significant contrast to understand the impact of the DM masses, vector coupling strength, and scalar coupling strength. Therefore, all the results lead to the same conclusion that if dark matter particles exist in neutron stars, they will be trapped in the core of neutron stars. Moreover, their population will not be more than $\approx 0.1\%$ of the total number of particles in the star. Also, the effect of dark decay does not significantly change the neutron stars equation of state and might not provide sufficient impact on static property of NS observations.

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